

Errata

Page 75.

Fig. 31 should read $p_{o_1} = 14.2 \text{ p.s.i.a.}$

Subtract .010 from each figure given in ordinate
i.e for $\frac{p}{p_{o_1}} = .120$ read .110 etc.

AN INVESTIGATION OF THE INCREASE OF TOTAL PRESSURE
OF A HOT GAS STREAM BY THE INJECTION OF WATER

by

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ABSTRACT

Studies in compressible fluid flow have shown that it is possible to obtain a total pressure rise in a hot fast moving gas stream by cooling it. The aerothermopressor, a device designed to achieve this by injecting cold water, has been extensively investigated both theoretically and experimentally but until now the maximum pressure rise possible has been undecided. This has made its utility uncertain.

The following work has been carried out to determine maximum performance more precisely.

An examination of liquid atomization principles indicates the types of water injection nozzles suitable for maximum performance. Tests on these in a small scale aerothermopressor reveal which are best. These are shown to differ for subsonic and supersonic gas flow.

Numerical solutions of the governing equations are given for a small scale supersonic aerothermopressor together with a corrected method for extrapolating to large scale solutions.

Subsonic experiments show that agreement between theory and experiment may be improved by slight modification of the theoretical model assumed for analysis.

Supersonic experimental results differ widely from their predicted numerical solutions. Shock wave formation which contracts the water spray is demonstrated to be the cause of the discrepancy. This makes the supersonic aerothermopressor inferior to the subsonic aerothermopressor.

These results together with those of previous workers indicate that maximum total pressure rise is achieved at the highest possible Mach Number obtainable with best possible water/air ratio in subsonic flow. This corresponds to $M = 0.78$ and $\Omega = 0.24$ at $T_0 = 1500^\circ\text{R}$. With these parameter values a maximum total pressure rise of about 20% is possible in a 12" diameter duct.

Experiments also show that the aerothermopressor has fouling problems which complicate its practical application. Its best use is probably not as a means of raising total pressure but rather for Mach Number control and reduction of friction loss in large high speed flow gas ducts.

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CONTENTSPage No

Abstract	2
Acknowledgements	4
Contents	5
List of Figures	6
Nomenclature	7
1. Review of Previous Work.	10
2. Aims of the Research.	19
3. The Atomization of Water.	21
4. The Supersonic Aerothermopressor.	28
5. The Design and Construction of Apparatus.	42
6. Experimental Procedure and Development Problems.	64
7. Experimental Results.	68
8. Discussion.	96
9. Conclusions.	112
10. Appendix A. Derivation of the Fundamental Equation.	114
11. Appendix B. Derivation of Influence Coefficients.	117
12. Appendix C. Specimen Calculation 2nd Solution.Stage.16.	120
13. Appendix D. Conduction Loss in Thermocouple Leads.	124
14. Appendix E. Governing Physical Equations for Droplets.	127
15. Bibliography.	130

LIST OF FIGURESPage No

Fig.1.	Water Characteristics at High Pressures	27
Figs. 2 - 9	Theoretical Curves of Supersonic Aerothermopressor Performance	35 - 37
Plate 1 - 2	Photographs of Apparatus	50 - 51
Figs.10 - 14	Diagrams of Apparatus	52 - 56
Figs.16 - 22	Diagrams of Apparatus	57 - 63
Figs.24 - 49	Graphs of Experimental Results	73 - 83
Plates 3- 14	Water Spray Photographs	84 - 95

NOMENCLATURE

A	=	cross-sectional area of duct
C	=	speed of sound in gas phase
C_p	=	specific heat at constant pressure of gas phase
C_{pl}	=	specific heat of water in droplet
$\overline{C_{p12}}$	=	mean value of c_p between 1 and 2
C_D	=	drag coefficient of droplet
d	=	volume-surface mean droplet diameter
d_o	=	value of d immediately after atomization
d_{oj}	=	value of d_o given by Nukiyama and Tanasawa formula
D	=	duct diameter
$\mathcal{D}$	=	diffusivity of water vapour into air
E	=	$\frac{dp_o}{p_o M^2}$
f	=	skin-friction coefficient of pipe
h_a	=	specific enthalpy of air
h_l	=	specific enthalpy of liquid water in droplet
h_v	=	specific enthalpy of water vapour
h_r	=	film coefficient of heat transfer
h_D	=	film coefficient of mass transfer
h_o	=	stagnation enthalpy of gas phase
h_{oa}	=	stagnation enthalpy of air
h_{ov}	=	stagnation enthalpy of water vapour
k	=	ratio of specific heats for gas phase
k	=	thermal conductivity of gas
L	=	length of evaporation section
$\mathcal{L}$	=	latent heat of water vapour at temperature
M	=	Mach Number of gas phase = $\frac{V}{C}$
Nu_D	=	Nusselt Number for mass transfer = $\frac{h_D \cdot d}{\mathcal{D}}$
Nu_r	=	Nusselt Number for heat transfer =
p	=	static pressure of gas phase
p_a	=	atmospheric pressure
p_o	=	stagnation pressure of gas phase

P_0	=	stagnation pressure of mixture
Pr	=	Prandtl Number = $\frac{C_p \cdot \mu}{k}$
Q	=	heat added per unit mass of gas
R	=	gas constant of gas phase
R_a	=	gas constant of air
R_v	=	gas constant of water vapour
$\bar{R}$	=	universal gas constant
Re	=	relative Reynolds number of droplet = $\frac{\rho \cdot d V - v_e }{\mu}$
Sc	=	Schmidt Number = $\frac{\mu}{\rho \cdot D}$
t	=	time
T	=	absolute temperature of gas phase
T_d	=	temperature of droplet
T_0	=	stagnation temperature of gas phase
V	=	gas velocity
v_e	=	droplet velocity
w	=	mass rate of gas flow
w_a	=	mass rate of air flow
w_e	=	mass rate of water flow
w_{e0}	=	mass rate of water injected
W	=	molecular weight of gas phase
W_a	=	molecular weight of air
W_v	=	molecular weight of water vapour
X	=	fraction evaporated = $\frac{\omega - \omega_0}{\Omega(1 + \omega_0)}$
y	=	$\frac{v_e}{V}$
z	=	longitudinal distance from inlet plane
θ_0	=	mixture stagnation temperature
μ	=	viscosity of gas phase
ρ	=	mass density of gas phase
ρ_e	=	mass density of water in droplet
ρ_{ve}	=	spatial mass density of saturated water at droplet surface
$\rho_{v\infty}$	=	spatial mass density of water vapour far from droplet surface

- τ_w = shear stress at pipe wall
- ω = specific humidity of gas phase lbs of water per lb of air
- Ω = initial water/gas ratio

Subscripts

- 1 = at inlet of evaporation section
- 2 = at exit of evaporation section
- 3 = at exit of diffuser
- a = air
- ℓ = water
- v = water vapour

1. REVIEW OF PREVIOUS WORKSummary

The origin of the idea that it is theoretically possible to raise the total pressure of a hot fast moving gas stream by cooling is first demonstrated. It is then shown that cooling by injection of cold water is the most practicable method for achieving this. The development of a device designed specifically for this purpose and called the Aerothermopressor is reviewed. The governing equations are given, and their principal characteristics explained. Relation between theory and experiment is presented for subsonic flow. Finally practical application of the Aerothermopressor is discussed.

1.1 The Fundamental Equation

In 1947 Shapiro and Hawthorne (ref.1.) developed the following equation showing how simultaneous changes in various factors affect the isentropic stagnation pressure of a gas stream.

$$\frac{dp_o}{p_o} = -\frac{kM^2}{2} \left[\frac{dT_o}{T_o} + \frac{4f dz}{D} + 2(1-y) \frac{dw}{w} \right] \quad (1.)$$

This equation derived in appendix A, is based on a simplified analysis of flow in a duct assuming, one dimensional flow, stepwise injection and evaporation of liquid, and negligible changes in specific heat and molecular weight.

The most significant feature of equation (1) is that if the change in total temperature $\frac{dT_o}{T_o}$ has a sufficiently large negative value to make the whole term in the brackets negative, then the change in total pressure $\frac{dp_o}{p_o}$ will be positive. In physical terms; if the gas stream can be cooled sufficiently quickly, the adverse effects of pipe friction $\frac{4f dz}{D}$ and injected liquid drag $2(1-y) \frac{dw}{w}$ will be overcome, and the stagnation pressure will rise.

This result, though contrary to intuitive ideas based on cooling a gas under static conditions, is more familiar as the reverse of the steady flow combustion process where a total temperature increase always results in a loss of total pressure.

In order to cool the gas stream two methods are available.

- a) By heat exchangers
- b) By evaporation of a volatile liquid

1.2 Cooling with Heat Exchangers

Cooling with Heat Exchangers only, equation (1) reduces to

$$\frac{dp_o}{p_o} = -\frac{kM^2}{2} \left[\frac{dT_o}{T_o} + \frac{4f dz}{D} \right] \quad (1.a)$$

Reynolds analogy between heat transfer and wall friction demonstrates that (1a) can only be positive if the wall temperature of the duct is negative.

Since negative temperatures violate the Second Law of Thermodynamics it is impossible to raise the total pressure of a gas stream by cooling with heat exchangers.

1.3 Cooling by Evaporation of a Volatile Liquid

To determine whether evaporative cooling can raise the total head pressure, assume that the injected liquid has an initially negligible forward velocity and that wall friction is absent. Equation (1) then becomes:

$$\frac{dp_o}{p_o} = -\frac{kM^2}{2} \left[\frac{dT_o}{T_o} + 2 \cdot \frac{dw}{w} \right] \quad (1.b)$$

The steady flow energy equation for the combined liquid and gas streams may be written:

$$w h_g + \frac{w V^2}{2g} + dw h_l + \frac{dw V_l^2}{2g} = (w+dw)(h+dh) + \frac{(w+dw)(V+dV)^2}{2g}$$

which reduces to:

$$-w \cdot d \left(h_g + \frac{V^2}{2g} \right) = dw(h_v - h_l) + \frac{dw(V^2 - V_l^2)}{2g}$$

Neglecting the kinetic energy increase of the injected fluid and assuming the vapour to be a perfect gas, this becomes:

$$-w.c_p.dT_o = (h_v - h_l).dw \quad (2)$$

Substituting (2) in (1b):

$$\frac{dp_o}{p_o} = -\frac{k.M^2}{2} \left[2 - \frac{(h_v - h_l)}{c_p.T_o} \right] \frac{dw}{w}$$

where h_v = Enthalpy of evaporated liquid in equilibrium with gas phase.

h_l = Enthalpy of injected liquid.

The criterion for a total pressure rise is therefore:

$$\frac{h_v - h_l}{c_p.T(1 + \frac{k-1}{2} M^2)} > 2 \quad (3)$$

Injecting a liquid of high latent heat appears the most likely way of raising the total pressure. This suggests the use of water. The following table gives values for the left hand side of equation (3) with water as the injected liquid.

Table 1

<u>T</u> <u>R</u>	<u>M=0</u>	<u>M=1</u>	<u>M=2</u>	<u>M=3</u>	<u>M=4</u>
500	8.0	8.0	7.7	7.4	7.4
1000	5.0	4.9	4.6	4.3	4.2
2000	3.5	3.3	3.0	2.7	2.7

Figures in Table 1 are so much greater than 2, that even allowing for the adverse effects of friction and change in molecular weight, an overall increase in total pressure indicated by the simplified theory still appears possible. Approximate numerical solutions of equation (1) given in ref (1) confirm this and show the rise to be a maximum at values of M between 1 and 2.

Equation (1) was derived to demonstrate the effect of simultaneous change of all possible variables in an infinitesimal length of duct. Such a system involving stepwise injection and evaporation of water was not intended specifically for raising total pressure. For this purpose Shapiro, Wadleigh, Gavril and Fowle (ref.2) 1956, presented a paper discussing the possibilities of a device which they called the Aerothermopressor. This essentially comprised a length of duct with water injection nozzles at the entry, the entire water content being injected at the duct inlet to ensure maximum residence time for all the water. Evaporation was thereby achieved in the shortest possible length keeping wall friction losses to a minimum.

1.4 Equations of the Aerothermopressor

Ref. 2 derived equations predicting Aerothermopressor performance and change in stream properties as indicated in Appendix B. These were obtained for an infinitesimal length of duct by simultaneous solution of the equations demonstrating, the First Law of Thermodynamics, the Conservation of, Mass, Momentum, and Energy, the Equation of State of a perfect gas mixture obeying the Gibbs Dalton law, and the definitions of Mach Number and Stagnation properties.

It was assumed that the flow is one dimensional throughout and that the water is injected as a cloud of spherical droplets of equal diameter uniformly distributed over the duct inlet plane having velocity and temperature with which it leaves the injection nozzles.

The result of this analysis with two algebraic corrections to the original is reproduced in Table 2 as a table of influence coefficients. This constitutes a series of 8 total differential coefficients forming 8 simultaneous 1st order non-linear differential equations with 7 independent derivatives and 8 dependent derivatives. Though these cannot be solved analytically, examination of the influence coefficients together with information gained from numerical solutions indicates the nature of the process.

TABLE 2. INFLUENCE COEFFICIENTS

	$\frac{dA}{A}$	$\frac{dQ - \frac{w_e}{w} \cdot dh_e}{c_p \cdot T_0}$	$\frac{dw}{w}$	$4.5 \frac{dz}{D}$	$\frac{w_e}{w} \cdot \frac{dV_e}{V}$	$\frac{dW}{W}$	$\frac{dk}{k}$
$\frac{dM^2}{M^2}$	$-\frac{2s}{1-M^2}$	$\frac{(1+\eta) \cdot s}{1-M^2}$	$\frac{-s\{[1+\eta]\beta - 2[1+(1-y)\eta]\}}{1-M^2}$	$\frac{\eta \cdot s}{1-M^2}$	$\frac{\eta}{1-M^2} \left[2s - \frac{k-1}{k} (1+\eta)y \right]$	$\frac{1+\eta}{1-M^2}$	-1
$\frac{dV}{V}$	$\frac{-1}{1-M^2}$	$\frac{s}{1-M^2}$	$\frac{-s\beta + 1 + (1-y) \cdot \eta}{1-M^2}$	$\frac{\eta}{2(1-M^2)}$	$\frac{\eta - (k-1)M^2 y}{1-M^2}$	$-\frac{1}{1-M^2}$	0
$\frac{dp}{p}$	$\frac{\eta}{1-M^2}$	$\frac{-\eta \cdot s}{1-M^2}$	$\frac{s \cdot \eta}{1-M^2} \left[\beta - 2 + \frac{1+(k-1)M^2}{s} y \right]$	$\frac{\eta [1+(k-1)M^2]}{-2(1-M^2)}$	$-\frac{(k-1)M^2}{1-M^2} \left[1 + (k-1)M^2(1-y) \right]$	$\frac{\eta}{1-M^2}$	0
$\frac{dT}{T}$	$\frac{(k-1) \cdot M^2}{1-M^2}$	$\frac{(1-\eta) \cdot s}{1-M^2}$	$\frac{(1-\eta) \cdot s\beta + (k-1)M^2 [1+\eta(1-y)]}{-(1-M^2)}$	$\frac{\eta (k-1)M^2}{-2(1-M^2)}$	$-\frac{(k-1)M^2}{1-M^2} [y + \eta(1-y)]$	$\frac{(k-1)M^2}{1-M^2}$	0
$\frac{dT_0}{T_0}$	0	1	$-\beta$	0	$-\frac{(k-1)M^2 y}{s}$	$\frac{(k-1)M^2}{2s}$	$\frac{M^2}{2s}$
$\frac{d\theta_0}{\theta_0}$	0	$\frac{s}{\alpha}$	$\frac{s\beta + \frac{k-1}{2} M^2 (1 + \frac{w_e}{w}) \cdot y^2}{-\alpha}$	0	0	$\frac{(k-1)M^2 (1 + \frac{w_e}{w} y)}{2\alpha}$	$\frac{M^2 (1 + \frac{w_e}{w} y^2)}{2\alpha}$
$\frac{dp_0}{p_0}$	0	$-\frac{\eta}{2}$	$\frac{\eta}{2} [\beta - 2(1-y)]$	$-\frac{\eta}{2}$	$-\frac{\eta}{2} \left[2 - \frac{(k-1)M^2 y}{s} \right]$	$\frac{\eta}{2s}$	$\frac{\eta}{2(k-1)s} - \frac{k}{(k-1)^2} \eta \cdot s$
$\frac{dP_0}{P_0}$	0	$-\frac{\eta}{2}$	$\frac{\eta}{2\alpha} \left[s(1 + \frac{w_e}{w}) y^2 \cdot \beta - (1 + \frac{w_e}{w} y) \right] - k \cdot M^2 (1-y)$	$-\frac{\eta}{2}$	$-\eta(1-y)$	$\frac{\eta (1 + \frac{w_e}{w} y^2)}{2\alpha}$	$\frac{k}{(k-1)^2} \left[\frac{\alpha-1}{\alpha} - \eta \alpha \right]$

DEFINITIONS:

$$\alpha = \frac{\theta_0}{T} \equiv 1 + \left(1 + \frac{w_e}{w} y^2\right) \frac{k-1}{2} M^2$$

$$\beta = \frac{h_v - h_e}{c_p \cdot T_0} + \frac{(k-1)M^2}{2+(k-1)M^2} \cdot (1-y^2)$$

$$s = 1 + \frac{k-1}{2} \cdot M^2$$

$$\eta = k \cdot M^2$$

$$\frac{P_0}{p} = \left(\frac{\theta_0}{T}\right)^{\frac{k}{k-1}}$$

The table is to be interpreted as follows:

$$\frac{dM^2}{M^2} = -\frac{2s}{1-M^2} \cdot \frac{dA}{A} + \frac{(1+\eta) \cdot s}{1-M^2} \cdot \frac{dQ - \frac{w_e}{w} \cdot dh_e}{c_p \cdot T_0} + \dots \text{etc}$$

Influence coefficients with the term $(1-M^2)$ in the denominator indicate that the corresponding parameter has opposite effects in subsonic and supersonic flow. Thus friction $4f \frac{dz}{D}$ and liquid acceleration $\frac{w_e}{w} \cdot \frac{dV_e}{V}$ drive Mach Number towards unity so reducing the static temperature in subsonic flow and increasing it in supersonic flow. Similarly evaporation $\frac{dw}{w}$ drives Mach Number away from unity with corresponding effect on temperature.

Numerical solutions show that though all properties change simultaneously, the net effect can be qualitatively described by dividing the process into 3 regimes in each of which one parameter is dominant. For simplicity consider a duct of constant cross sectional area. The sequence of events may then be described as follows.

Regime 1 Droplet Drag Predominant

Initially the water droplets are accelerated to gas stream speed. This is achieved in a very short distance in which friction and evaporation effects are small. Total pressure is reduced and Mach Number driven towards unity.

Regime 2 Droplet Evaporation Predominant

Acceleration being virtually complete, the hot unsaturated gas stream permits high evaporation rates raising the total pressure and driving the Mach Number away from unity. Wall friction effects, though significant, are still small.

Regime 3 Wall Friction Predominant

As evaporation proceeds, the water droplet surface area decreases and the gas cools. Ultimately wall friction takes control, driving Mach Number back towards Unity and reducing the total pressure. Regime 3. should be avoided and the fluid stream made to enter a diffuser while still in Regime 2.

The whole process is indicated diagrammatically in Fig.A.

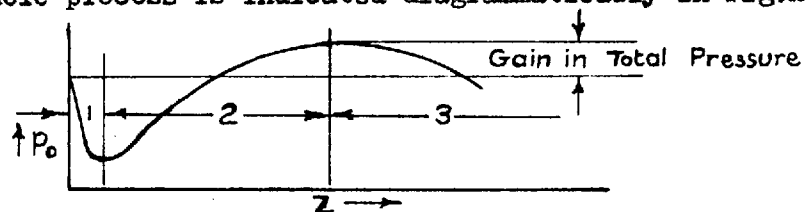


Fig A

From this summary of Aerothermopressor behaviour three important features may be demonstrated.

1. Transition through the speed of sound is possible in a constant area duct.
2. The Aerothermopressor must be a large scale unit.
3. For maximum possible total pressure rise, the cross sectional area of the duct must vary along its length.

1.4.1 Transition through the speed of sound

The opposing effects of droplet drag in Regime 1. which drives M towards unity and evaporation in Regime 2. which drives M away from unity can be so balanced that provided correct initial values of Mach Number, Stagnation Temperature, and water/air ratio are chosen, continuous transition through the speed of sound can be achieved in a duct of constant cross sectional area. Refs.(2 and 4) give experimental confirmation of this in increasing from subsonic to supersonic speeds, but report failure in the opposite direction.

1.4.2 Scale Effect of the Aerothermopressor

Given the initial temperatures, velocities, and relative masses of the gas and water streams, the length of the evaporation section is proportional to the residence time required for the droplets to evaporate. Since this residence time is independent of the absolute mass flow, a large scale Aerothermopressor requires no greater length than a small scale unit. However, the greater the duct diameter the smaller the wall friction losses which are proportional to $4.f \cdot \frac{dx}{D}$. Numerical solutions in ref (2) show that a total pressure rise is impossible if $D \geq 2"$. An effective Aerothermopressure must therefore be a large scale unit.

1.4.3 The Importance of Area Control

The last 2 lines of influence coefficients show that:

$$\begin{aligned} \frac{dp_0}{p_0} &= M^2 \text{ (terms giving difference between Wall Friction, Drag,} \\ &\hspace{15em} \text{Evaporation etc.)} \\ &= M^2.E \end{aligned}$$

In Regime 2. E is governed by evaporation which is dependent on the heat transfer rate. This in turn depends on the temperature difference between the gas and liquid streams. As explained previously in Section 1.4. the effect of evaporation is to drive M away from unity. In subsonic flow, M^2 will be reduced increasing E while in supersonic flow, M^2 will be increased so reducing E. The constant area duct is thus self cancelling.

Area change shown in Table 2 by $\frac{dA}{A}$ has no direct effect on p_0 but it can be used to control M and hence keep a favourable balance between M^2 and E. In general it appears that M should tend towards unity as evaporation proceeds and the duct area should decrease before reaching the diffuser.

There appears to be no direct analytical method for determining best area variation, for by sacrificing the gains at one section of the duct a greater return may possibly be achieved elsewhere, but ref (2) indicates a design method for maintaining $\frac{dp_0}{p_0}$ a maximum at every point along the duct. This method requires extensive numerical computation.

1.4.4 The Length of the Evaporation Section

As stated in Section 1.4.2 the length of duct required is proportional to the water residence time. Ignoring Regime 1. which is of the order of 1" - 2" and assuming that the enthalpy rise of the evaporated water is approximately proportional to the temperature difference between the gas and liquid streams, it can be shown that evaporation time $t \propto d_0^2$ where d_0 is the initial droplet diameter. Atomization data show that $d_0 \propto \frac{1}{V - V_L}$ where V is the air velocity and V_L the liquid velocity.

If V_L is small then $d_0 \propto \frac{1}{V}$

$$\text{Evaporation Length } L \propto V \cdot t \propto \left(\frac{1}{d}\right) \cdot d^2 \propto d \propto \frac{1}{V}$$

This demonstrates that duct length is decreased;

- a) by increasing the initial gas velocity.
- b) by decreasing the initial droplet size.

1.5 Relation Between Theory and Experiment

To assess the worth of the Aerothermopressor, a large number of numerical solutions are required for varying values of M , Ω , T_0 , D , and area variation. Ref (2) gives several numerical solutions in subsonic flow. In addition a simplified integral relation for overall total pressure rise was developed which though not accurate, served as a guide for further computation. The result of these solutions indicated a probable total pressure rise of 10-20% in a subsonic Aerothermopressor.

Experiments on a small scale subsonic unit showed fair quantitative agreement between theory and experiment.

1.6 Application

Ref (2) considered the most likely practical application of the Aerothermopressor to be as a power and efficiency boost to the Gas Turbine. The expansion ratio of the turbine could be increased by reducing the back pressure so raising power output and efficiency. The exhaust gases at sub-atmospheric pressure could be discharged to atmosphere by passing through an Aerothermopressor.

Though less effective than a heat exchanger for raising cycle efficiency it boosts the power of a given size gas turbine and has the advantage of small weight and volume and mechanical simplicity. It was also claimed to be free from fouling and corrosion.

2. AIMS OF THE RESEARCH

2.1 Object

The aim of the research is to make an assessment of the Aerothermopressor by more precise determination of its maximum performance limits. This would be best achieved by:

- a) determining the best method of injecting the water
- b) investigating the possibilities of a supersonic Aerothermopressor.

2.2 The Method of Water Injection

Improvements in atomization technique reduce the constant of proportionality between gas velocity and droplet diameter. As shown in Section 1.4.4. this reduces duct length. Hence total friction loss is reduced and performance thereby improved for any given initial parameter values. It is possible that determination of the best method of injection may limit the range of initial parameter values practicable. This would simplify finding their values for maximum performance. Alternatively given these values as independent of injection technique, the maximum possible total pressure rise can only be determined quantitatively when the best method of water injection is established.

2.3 The Supersonic Aerothermopressor

Determination of the ideal initial parameter values and duct shape is necessary for estimating maximum total pressure rise. Unfortunately, as Section 1.4.3. indicates, the selection of these by direct analysis is impossible.

Ref.(2) gives only one numerical solution of the equations in supersonic flow for a full scale parallel duct and no experimental results in this region.

Section 4. of this thesis will show the probability that the best Aerothermopressor is entirely supersonic. It will also be shown that in the supersonic region theory and experiment may not

agree as closely as in the subsonic case.

An exhaustive quantitative investigation of the supersonic Aerothermopressor is beyond the scope of this research but examination of a particular supersonic Aerothermopressor both theoretically and experimentally, is useful. By this means agreement between supersonic theory and experiment in the whole supersonic range may be verified. In addition, though not determining the best values of M_1 etc., together with results in ref.(2) a particular solution should indicate more precisely in what range these values are likely to be found and, together with results of atomization tests, what order of magnitude the maximum possible total pressure rise will be. By performing the investigation on a small scale Aerothermopressor and extrapolating to full scale, an idea of the effect of area control may be obtained. Since the increased friction effect in a small scale unit drives Mach Number to unity more rapidly than in a large scale unit of similar shape, provided account is taken of the additional total pressure losses incurred, the Mach Number variation obtained by this means is equivalent to that obtained in a full scale duct with favourable cross sectional area variation.

3. THE ATOMIZATION OF WATER

Summary

This section reviews the water injection requirements of the Aerothermopressor and describes the basic theory of liquid atomization. From these considerations, possible injector designs are derived.

3.1 The Requirements of the Water Injection System

Immediate, fine, atomization and diffusion of the injected water is of prime importance. The water spray may be compared to a large heat exchanger. The smaller and more widely dispersed the droplets, the greater the heat transfer surface. Evaporation is thereby completed in a shorter length of duct and performance improved as described in Section 2.2.

From Section 1.4 it may be seen that the net rise in total pressure in the Aerothermopressor is the difference between acceleration losses and evaporative gains. By decreasing the relative velocity between the injected water and gas streams the total pressure loss due to acceleration may be reduced. Either reduction in M_1 or increase of liquid injection velocity achieves this. Since the Aerothermopressor effect is due to the compressibility of the gas, the Mach Number must be high. The best means of reducing momentum transfer to the water stream would therefore be to inject the water at high velocity.

Section 1.4.4. shows that droplet size is inversely proportional to the initial air/water relative velocity. Reducing momentum loss increases droplet size. Evaporation gains are therefore simultaneously decreased.

With these conflicting tendencies, the best injection system must produce rapid break up and uniform distribution to conform with the theoretical assumptions, and in addition either:

- a) Use low velocity injection to yield minimum droplet size; or
- b) Inject the water at high velocity by some means which does not produce too large a droplet size.

Ref.(3) describes 20 different methods of injection tried. Final results for Aerothermopressor performance are given in it and Ref. (2) using the best of these. The method of choice of the original 20 appears to be quite arbitrary and unrelated to atomization theory. There are many methods of injecting and distributing liquids, but after consideration of the basic principles of atomization, it may be seen that only few of these are suitable for our purpose. The following outline of these principles serves as a guide to selection and design of injection nozzles meeting Aerothermopressor requirements.

3.2 The Mechanism of Liquid Atomization

The principal method of atomizing a liquid is to increase the surface of a rod or sheet until it becomes unstable and disintegrates. This is usually achieved by the interaction of the liquid and its surrounding atmosphere.

In the case of a rod or jet of liquid, internal turbulence causes wave motion which is increased by the local accelerations and shearing action of the air. Thin ligaments of liquid are rapidly drawn off. These attenuate and break up into small droplets.

When the liquid is injected in the form of a flat or conical sheet it attenuates as it travels from the orifice. Instability forms waves in the sheet which are intensified by the atmosphere. This produces holes in the sheet. Under the action of surface tension the holes rapidly grow larger and the surrounding liquid contracts into ligaments which quickly break up into droplets.

Although the shear action between the liquid and atmosphere is the most important, other factors affect atomization.

These are:

Liquid Surface Tension: If reduced, atomization is improved.

Liquid Viscosity: Increase coarsens droplets. This effect is slight.

Atmospheric Viscosity: Increase produces finer droplets. Effect is slight.

Liquid Density: Increase improves atomization.

Atmospheric Density: Increase improves atomization.

Liquid Turbulence: Increase yields smaller droplets in a jet.

From this discussion the following conclusions may be demonstrated.

- a) Liquid atomization is not normally achieved by sudden disruption but by a progressive though very rapid process.
- b) The main cause of liquid break up is shear between liquid and atmosphere.
- c) Given initial Aerothermopressor parameters, the only fluid property that could be varied easily to improve atomization is the water surface tension.
- d) Given the properties of the liquid and atmosphere and velocities, the atomization of a jet of liquid can be improved either by increasing the liquid turbulence or by promoting break up by some means additional to the shear action.

3.3 Design of Nozzles

The requirements of Sections 3.1 and 3.2 may now be applied to a rationalized design of Aerothermopressor water injection nozzles.

Conclusions (a) and (b) of Section 3.2 show that all water injection in a moving gas stream should be parallel to the gas flow. Conical sheets will be collapsed by the gas dynamic head resulting in a large droplet size while radial injection will reduce the contact area and shearing effect by producing air eddies on one surface. The inferiority of radial injection has been confirmed experimentally by Hrubecky ref. (14) 1958.

Conclusion (b) suggests that to obtain maximum shear forces the injected liquid should present as much surface area as possible.

3.3.1 Low Velocity Water Injection

To meet requirement (a) of Section 3.1 one of the following nozzle designs is likely to be the best.

- a) Axial jet.
- b) Axial Attenuating Sheet.
- c) Axial Non Attenuating Sheet.

Axial Jet

This was used in refs (2) and (3) and results gave good agreement between Aerothermopressor theory and experiment in subsonic flow using the Nukiyama and Tanasawa droplet size equation ref. (13).

Axial Attenuating Sheet

This is usually formed by a fan spray nozzle. In view of its large contact surface it may be better than the jet. It should be noted that attenuating sheets are primarily designed for a relatively fast moving liquid discharging into a slow moving or static gas where they give superior atomization to jets at velocities up to 400 ft/sec. In the Aerothermopressor where the effect is reversed, the external air forces may reduce the fan angle excessively so contracting the sheet and coarsening the droplets.

Axial Non Attenuating Sheet

The possible contracting effects of the air stream on a fan spray can be prevented by forming a continuous non attenuating surface. Injecting the water as an annular sheet gives large contact surface, good distribution and minimum injection velocity. The nozzle together with the surrounding duct could be area controlled to give identical Mach Number to the air passing through and around it. There are no known atomization data for such a nozzle but it appears to offer the best atomization conditions possible.

Reduction of Surface Tension

The most obvious way of doing this is to add detergent to the water, "Tide", a typical product in 0.1% solution reduces the surface tension of water from 72 dynes/cm. to 28.5 dynes/cm. (Ref 20). The main drawback to this is that it takes approximately 0.01 secs from the formation of a free surface until the surface tension is actually reduced. If the water entry velocity is sufficiently low, this may be effective before break up occurs. If successful, performance of all three low speed nozzles could be improved.

3.4.2 High Velocity Water Injection

Requirement (b) of section 3.1 is difficult to meet for high velocities generally rule out large contact surfaces and consequent shearing forces. Reduction of surface tension using detergents is not likely to succeed. Increasing the turbulence of a jet or promoting additional break up are the only remaining likely methods. These could be achieved by:

- a) Using nozzles with rough surfaces.
- b) Injecting the water at high pressure with Carbon Dioxide or some similar soluble gas dissolved in it.

Nozzles with Rough Surfaces

By using nozzles with rough edges and surfaces the water turbulence will be increased. This method is not to be recommended as results will be difficult to reproduce.

Injection with High Pressure Carbon Dioxide

High injection velocities require high injection pressures. If the pressure is obtained by compressing with a highly soluble gas such as carbon dioxide, a "soda syphon effect" could be obtained at the nozzle exit. Atomization would be improved by increasing the initial turbulence as the gas comes out of solution, by promoting more rapid ligament formation due to the foaming of the jet, and possibly at sufficiently high pressures by disrupting the jet immediately and hence presenting the air stream with a larger contact

surface. This effect would increase with injection pressure.

Fig. 1 (a) shows the solubility of carbon dioxide in water at various pressures while Fig. 1 (b) shows the corresponding water velocity due to these pressures.

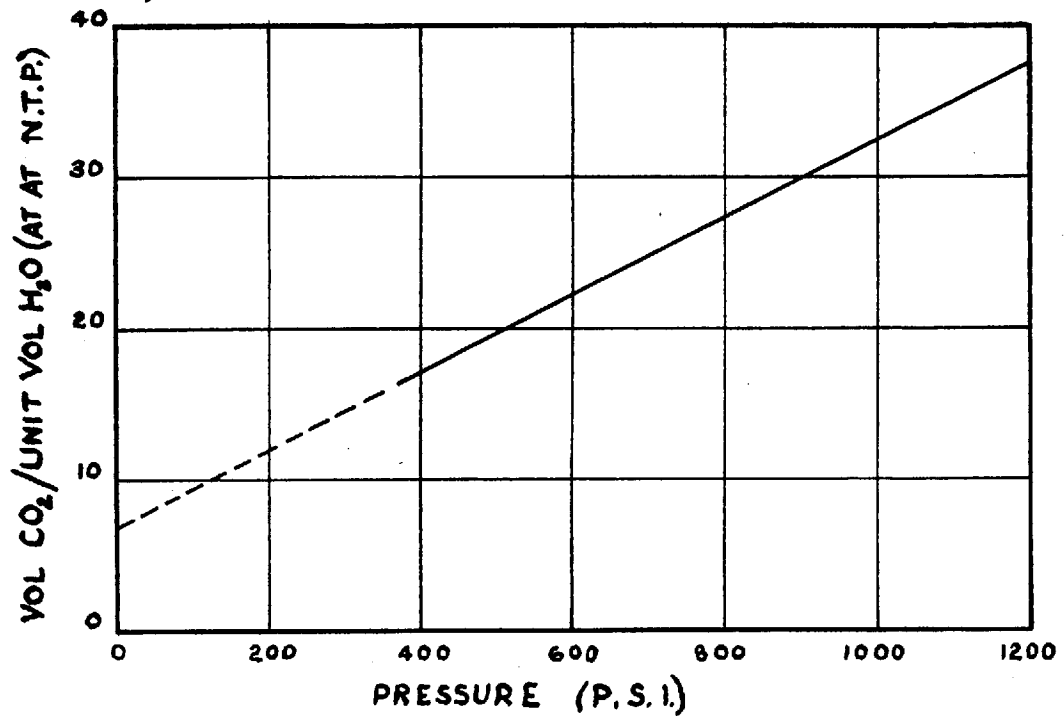
The success of such an injection method depends on the rate at which carbon dioxide comes out of solution and the nature of the disturbances it causes the liquid. The rate at which the carbon dioxide comes out of solution and the degree of disturbance it causes are likely to be proportional to the volume of gas contained in unit volume of water. The nature of the disturbance, i.e. the manner in which the carbon dioxide comes out of solution must be determined by inspection.

If P_1 is the compressed water pressure and P_2 the gas stream pressure, then the injected water velocity $V_e \propto \sqrt{P_1 - P_2}$

The amount of carbon dioxide in solution $\propto P_1$ and its final specific volume $\propto \frac{1}{P_2}$

$\therefore$ Final volume of Carbon Dioxide / Unit Volume of Water $\propto \frac{P_1}{P_2}$.
By reducing P_2 , which in a gas turbine exhaust will be less than atmospheric pressure, the degree of disturbance is greatly increased without much increase in injection velocity. Such a mode of injection applied to an axial jet will therefore be more successful the higher the initial Mach Number.

a) SOLUBILITY OF CARBON DIOXIDE IN WATER



b) WATER VELOCITY VERSUS PRESSURE

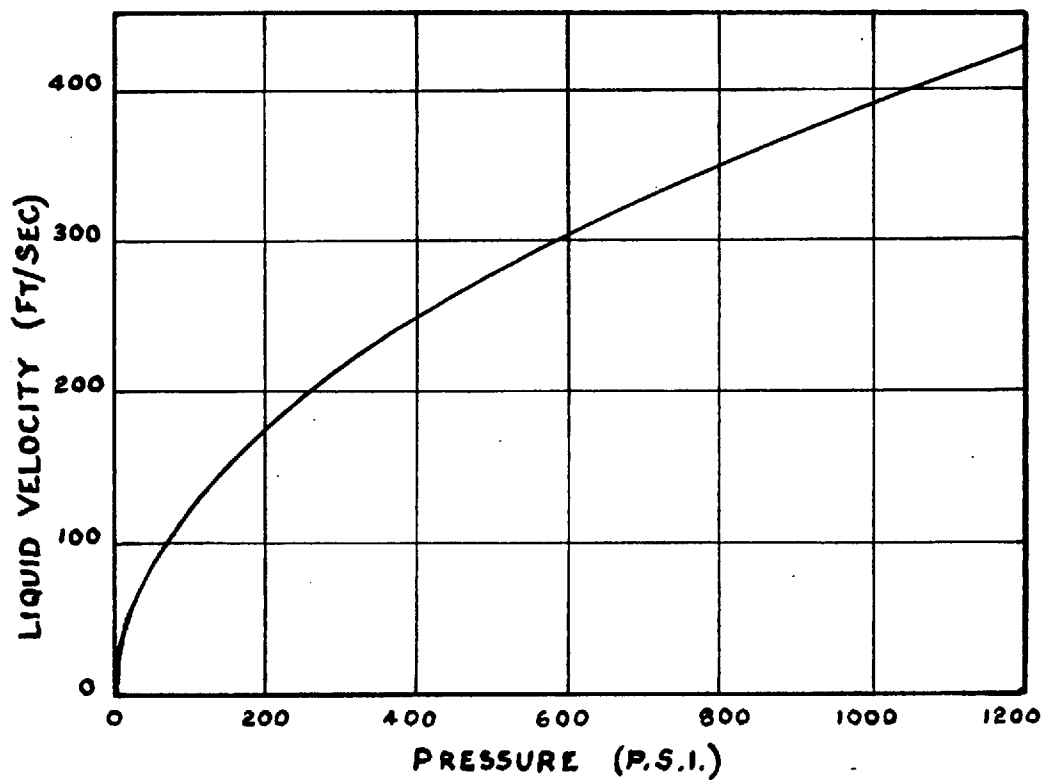


FIG.1. WATER CHARACTERISTICS AT HIGH PRESSURES

4. THE SUPERSONIC AEROTHERMOPRESSOR

Summary

The case for an entirely supersonic Aerothermopressor is first made. It is shown that experimental confirmation of supersonic theory is necessary. Two exact solutions for a typical small scale supersonic Aerothermopressor are given and the extrapolation of results to a large scale unit discussed. Extrapolation by the method of ref (2) is shown to be incorrect and likely to overestimate large scale total pressure gains for most cases in both subsonic and supersonic flow.

4.1 The Case for the Supersonic Aerothermopressor

As explained in Sections 1.4 and 1.4.3. it is not possible to determine the best possible values for M_1 and Ω analytically but the following considerations favour the entirely supersonic Aerothermopressor.

a) Results of subsonic experiments given in Ref (2) show that maximum total pressure rise increased with M_1 up to subsonic choking conditions.

b) The qualitative results of Ref (1) give $\frac{P_{02}}{P_{01}}$ to be a maximum for $1 < M_1 < 2$.

c) If $M_1 > 1.8$ f/D values would be reduced below subsonic results for $M_1 > 0.5$ because of the decreasing flow/unit area. Duct diameter for a given mass flow would thus increase.

d) Experiments of Keenan and Neumann, ref (22), indicate that the value of f for a dry duct is more than 25% lower in supersonic flow than in subsonic flow for equal values of Re .

e) In subsonic experiments, injecting water raised the value of f from 10 - 20% above the dry duct values. The effect of water injection on friction coefficient in supersonic flow will probably be less since the larger thermal gradients in the boundary layer will greatly increase the local evaporation rate and reduce droplet contact with the duct walls.

f) Given a subsonic and a supersonic Aerothermopressor of equal performance based on droplet size calculated from the Nukiyama and Tanasawa formula, any fractional reduction in drop size equally effective in both will raise the performance of the supersonic unit more than that of the subsonic unit. This is because the net total pressure rise in supersonic flow is a smaller percentage of the total evaporative gain than in the subsonic case.

g) Section 3.4.2 shows that high velocity water injection is more likely to be effective the higher the value of M_1 .

Disadvantages of the Supersonic Aerothermopressor are:

a) Increased friction losses

$$(P_{01} - P_{02})_{\text{Friction}} \propto L \cdot M^2$$

for a given size duct. From Section 1.4.4.

$$L \propto \frac{1}{V} \propto \frac{\sqrt{1 + \frac{k-1}{2} M^2}}{M}$$

$$\therefore (P_{01} - P_{02})_{\text{Friction}} \propto M \sqrt{1 + \frac{k-1}{2} M^2}$$

It is not certain where increased friction plus increased acceleration losses become larger than increased evaporation gains.

b) Due to normal shock, diffuser losses may be higher than in the subsonic case. It is likely that this difference may be small since normal shock will also raise the evaporation rate by both increasing temperature difference between gas and water streams and raising the local Re of the droplets. In addition momentum transfer from the unevaporated drops to the gas stream will then raise P_0 .

Uncertainties of the Supersonic Aerothermopressor are:

a) Lack of atomization data in supersonic flow.

b) Possible shock formation between water and air having unforeseen effects.

It is clear from the foregoing that existing theory and experiment need further correlation in supersonic flow.

4.2 Numerical Solutions for a Typical Supersonic Aerothermopressor

4.2.1 Selection of Initial Parameter Values

The solutions were carried out not only to give theoretical values for possible Aerothermopressor performance, but also as a basis for comparison with experiments.

Selection of M_1 was determined as follows. If $M_1 < 1.4$ instability due to choking effects is likely. If $M_1 > 2.2$ the temperature difference between gas and water becomes too small for rapid evaporation. Considerations (c) and (g) favouring the supersonic Aerothermopressor in Section 4.1. require M_1 to be as high as possible. At the time this research was begun a 2" square supersonic nozzle designed for $M_1 = 1.86$ together with some parallel ducting was available. This was considered to be suitable for the experimental work. $M_1 = 1.86$ was therefore taken as the initial value of Mach Number for the solution.

Water/air ratio is virtually independent of M_1 being roughly a function of T_{o1} only. Subsonic results showed $\frac{P_{o1}}{P_{o1, \text{Maximum}}}$ to be fairly insensitive to Ω . A value of $\Omega = 0.2$ was selected as being not only approximately correct but also directly comparable to subsonic results.

Two solutions were attempted. The first was based on assumed duct properties and the second on measured values. The initial values are listed as follows:

	<u>1st Solution</u>	<u>2nd Solution</u>
M_1	1.86	1.86
T_{01}	1500°R	1500°R
Ω	0.2	0.2
P_{01}	14.7 p.s.i.a	14.1 p.s.i.a
$4f/D$	.008/ins	.0095/ins
V_e	8 ft/sec	7 ft/sec
Initial Gas Condition	Dry Air	Air and Town Gas Combustion Products.
Duct Shape	2" Square Parallel	2" Square Parallel

4.2.2 The Method of Solution

The influence coefficients of Table 2. relate 8 dependent derivatives to 7 independent derivatives. Given $M_1, \Omega, T_{01}, P_{01}, f$, and duct diameter and shape, the independent derivatives become functions of a single variable. The choice of independent variable is arbitrary, but for convenience the fraction evaporated $\chi = \frac{\omega - \omega_1}{\Omega(1 + \omega_1)}$ is selected. The method of solution adopted was by stepwise integration, i.e. successive determination of the increments of all the variables for a given increment of χ .

Euler's method was adopted for the solution. This is based on the approximation that if $y = f(x)$ such that $\frac{dy}{dx} = f(x, y)$ then given x_0, y_0 , the increment in y corresponding to an increment Δx is given by: $\Delta y_1 = f(x_0, y_0) \Delta x_1$.

The solution was completed in 31 stages for increments of $\Delta \chi$ varying between 0.005 and 0.03 depending on the rate of change of properties. It is shown graphically in figs 2-7 where it is referred to as the 1st Solution.

Later, comparison with theory and experiment showed wide differences. Apart from other reasons to be discussed in Section 7 the difference was thought to be partly due to the numerical solution being non representative of the true experimental conditions. This was because:

a) Subsonic experimental results showed that for $\Omega = 0.2$, $\frac{f_{Wet}}{f_{Dry}} = 1.13$. Assuming this to be true for supersonic flow, the value assumed for f was 19% low.

b) The gas stream was assumed to be initially dry air, and its properties, i.e. viscosity, specific heat, and thermal conductivity, remained those of dry air throughout the process. The latter assumption was not considered serious but the initial assumption was. Since $\Omega = 0.2$ and $\chi_{Max} \approx 0.6$, maximum fraction of water vapour in the gas stream ≈ 0.12 . The combustion products contain roughly 2½% moisture. Since conditions approach saturation towards the end of the solution, the error in the moisture content of the gas stream of 21% is significant.

c) In the last 8 stages of the solution, Euler's approximation, wherein properties for the stage were considered to be those at the beginning of the stage, led to stage errors of up to 8% in the heat and mass transferred per stage. It is important to differentiate between stage error and total error since the former is to some extent self cancelling. The overall computation error for the solution derived from integral momentum and energy balances assuming mean average values for Mach Number etc., was as follows.

	<u>Error</u>	<u>Percentage Error</u>
Pressure	-0.05 p.s.i.	-0.02%
Temperature	-32°R	-5%

It was somewhat reluctantly decided that a more accurate solution was needed bearing closer relation to the true duct and fluid properties.

This is shown graphically in figs. 2-7 where it is referred to as the 2nd Solution. This assumes that: The average friction coefficient = 1.13x measured dry friction coefficient. The initial properties of the gas stream are air plus combustion products.

The viscosity, specific heat, and thermal conductivity of the gas are conservative properties of the air-water vapour mixture.

A modified version of Runge's method was used for solution. This assumes that if $\frac{dy}{dx} = f(x, y)$, given x_0, y_0 and Δx , then:

$$\Delta y = f \left[x_0 + \frac{1}{2} \Delta x, y_0 + \frac{1}{2} f(x_0, y_0) \Delta x \right] \Delta x$$

In this case Δy was obtained by plotting values of the second differential for the previous terms against x and extrapolating. The final calculated values of Δy were always within 2% of the assumed values.

The solution was completed in 42 stages, $X = 0 - 0.20$
 $\Delta X = 0.01$, and $X = 0.20 - 0.64$ $\Delta X = 0.02$.

Both solutions had a tendency to become unstable. This was due to the method of obtaining change in liquid temperature T_ℓ . Given:

$$\frac{dT_\ell}{dX} = \frac{-1}{C_{p\ell}(1-X)} \left[\mathcal{L} - \frac{h_r(T-T_\ell)}{h_D(\rho_{v\ell}-\rho_{v\infty})} \right]$$

ΔT_ℓ is the small difference between the large quantities $\mathcal{L}$, the latent heat of evaporation, and $\frac{h_r(T-T_\ell)}{h_D(\rho_{v\ell}-\rho_{v\infty})}$, the heat transferred per unit mass evaporated. Calculation of ΔT_ℓ involves small errors. According to ref. (2) $\rho_{v\ell} \propto T_\ell^{15}$ in the range of values considered. A small error in ΔT_ℓ produces a large error of similar sign in $\rho_{v\ell}$ in the following stage. This in turn gives $\frac{h_r(T-T_\ell)}{h_D(\rho_{v\ell}-\rho_{v\infty})}$ a large error causing ΔT_ℓ of the following stage a large error of opposite sign. Oscillations in T_ℓ of increasing magnitude so produced, in turn affects the whole solution.

It was found that once T_ℓ reached the local wet bulb conditions its subsequent variation is small. The solution was modified to assume T_ℓ constant for a given increment.

$$\therefore \mathcal{L} h_D (\rho_{v_2} - \rho_{v_\infty}) = h_T (T - T_\ell)$$

for each stage. T_ℓ was found from this by iteration. Sufficient accuracy was obtained on either the first or second attempt as the solution proceeded. By this means the error feed back was eliminated and the solution remained stable.

4.2.3 Comparison of Solutions

The two solutions are based on different assumptions and are therefore not easily comparable, but certain trends may be observed. In the second solution the increased value of f reduces the average value of Mach Number (fig. 4) but this keeps the static temperature higher (fig. 5). Therefore despite the increased humidity, the evaporation rate is maintained and even overtakes that of the first solution as the computation proceeds (fig. 7). The lower average value of M to some extent counters the higher value of f so that total pressure loss due to friction which is proportional to $f.M^2$ is not much reduced. The final value of p_0 in the second solution is therefore only 3% less than that of the first solution (fig. 2). The higher static temperatures in the 2nd solution keep up the velocities even though the Mach Number is reduced (fig. 6). Most important for the purpose of comparing theory and experiment, the second solution has a final static pressure 9% higher than the first solution.

4.2.4 Extrapolation of Solutions

The utility of small scale numerical solutions and experiments is greatly increased if they may be extended to predict full scale performance. Ref. (2) suggested the following method of extrapolation.

$\frac{p_{03} - p_{01}}{p_{01} M_1^2}$ is defined as the Coefficient of Performance, where
 p_{03} = Total Pressure at Diffuser Exit.

M_1^2 is made a factor in the denominator to reduce the sensitivity of performance to Mach Number effects. The significance of this will become more clear subsequently.

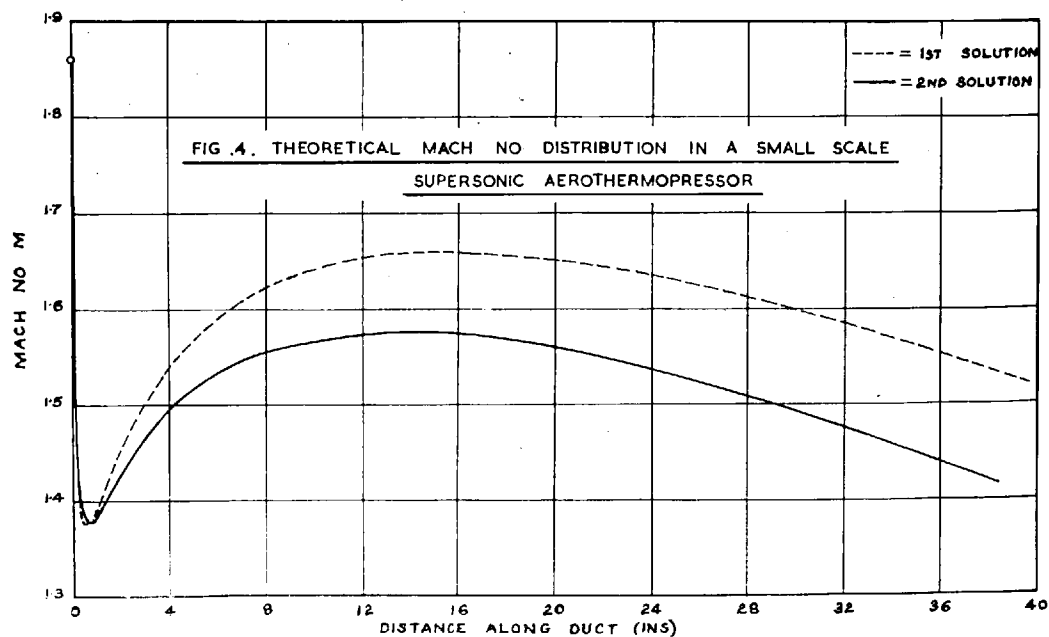
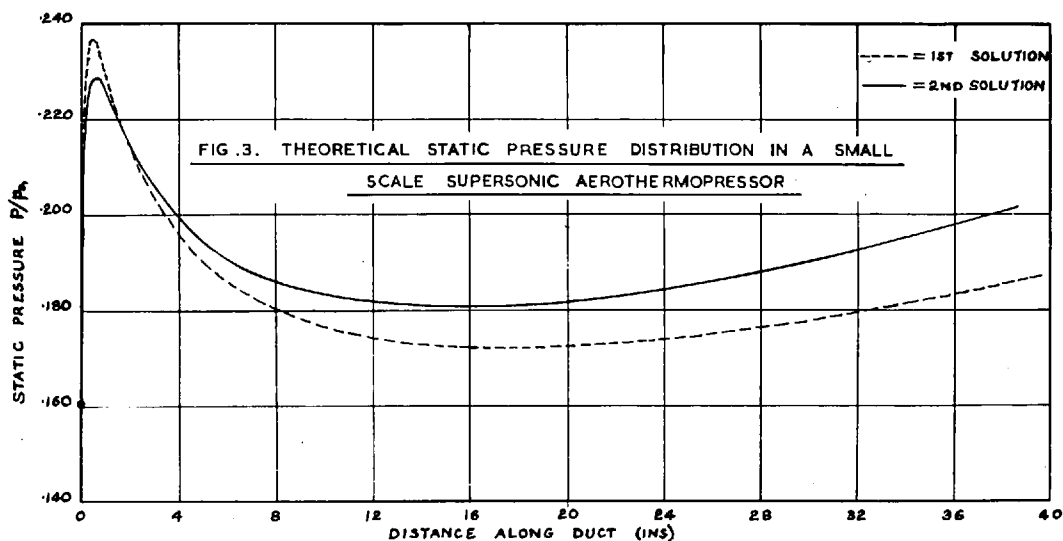
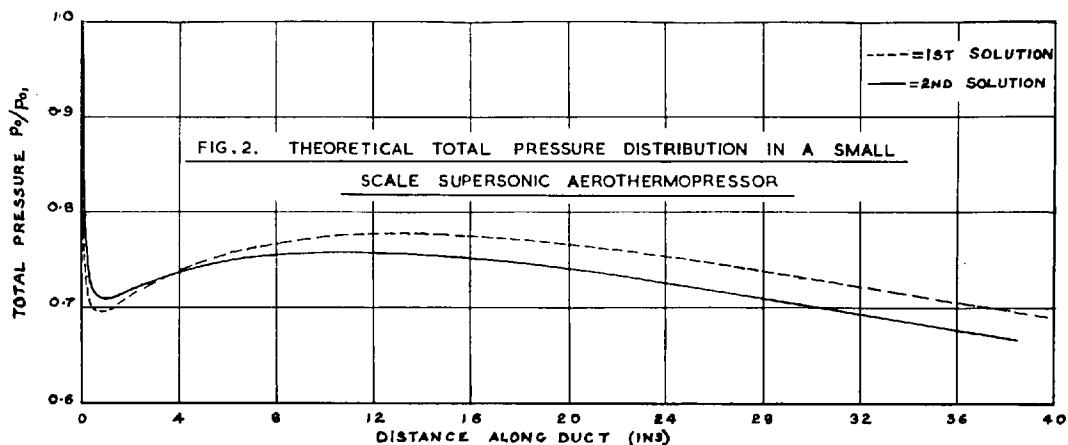


FIG. 5. THEORETICAL TEMPERATURE DISTRIBUTION IN A SMALL SCALE

SUPERSONIC AEROTHERMOPRESSOR

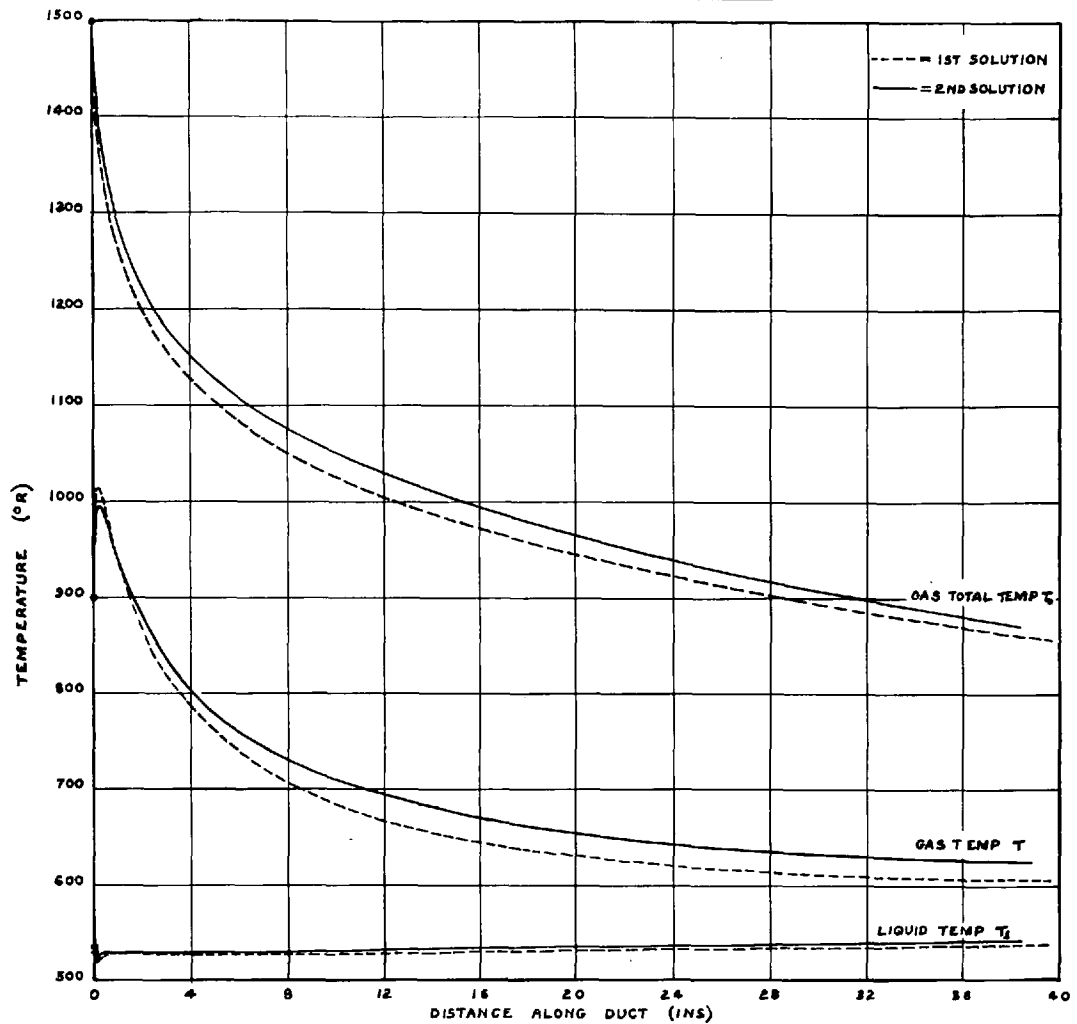
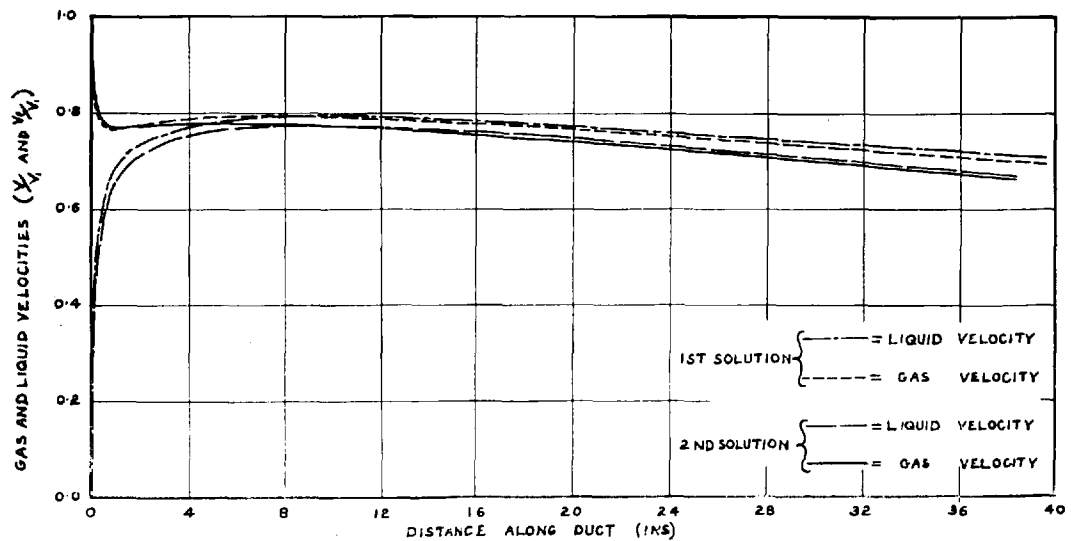
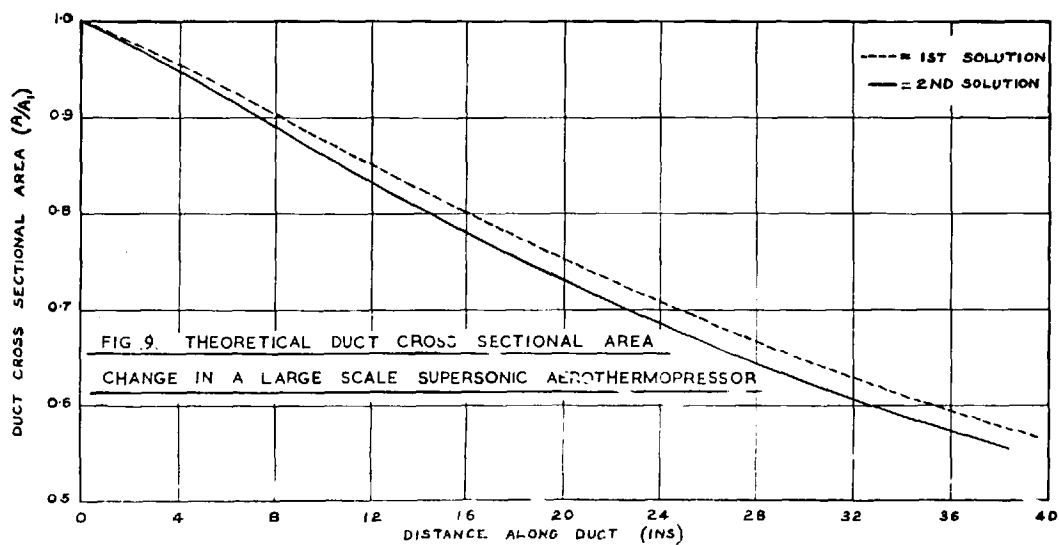
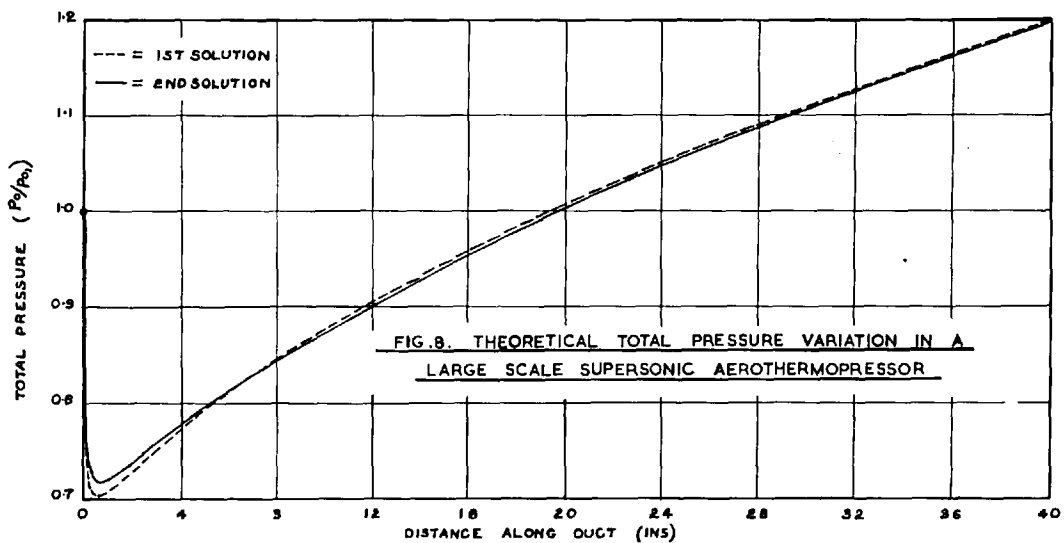
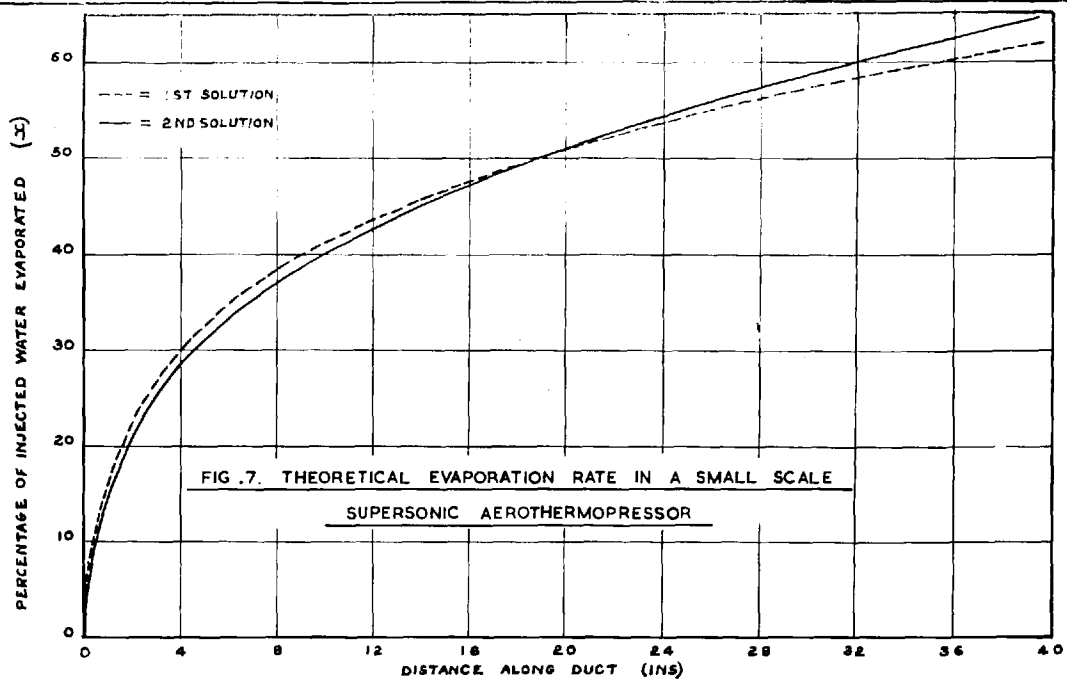


FIG. 6. THEORETICAL VELOCITY DISTRIBUTION IN A SMALL SCALE

SUPERSONIC AEROTHERMOPRESSOR





Coefficient of Performance may be expressed as:

$$\frac{P_{03} - P_{01}}{P_{01} \cdot M_1^2} = Y - \frac{2.f.k.L}{D}$$

By computing or measuring the Coefficient of Performance for a small scale unit, Y may be obtained. Y was claimed to be the effect of all terms other than friction and therefore independent of size. The total pressure rise for any size of Aerothermopressor with the same Mach Number distribution as the small scale unit could then be obtained by substituting different values of $\frac{L}{D}$ in the expression

$$\frac{P_{03} - P_{01}}{P_{01}} = Y \cdot M_1^2 - k \cdot M_1^2 \cdot \frac{2.f.L}{D}$$

By this means it was claimed that an Aerothermopressor with $M_1 = 0.78$, $T_{01} = 1500^\circ R$, and $\Omega = 0.24$ would give a total pressure rise of 13% without any friction losses i.e. $\frac{L}{D} = 0$ and 10% rise with $\frac{L}{D} = 5$ which corresponds roughly to a 12" diameter duct or the exhaust of a 1500 H.P. Gas Turbine. This was thought to be an almost certain minimum since duct area variation though favourable, was unlikely to be the best possible.

Using this method of extrapolation in the two solutions described in this section we get:

		<u>1st Solution</u>	<u>2nd Solution</u>
$\frac{P_{03} - P_{01}}{P_{01} \cdot M_1^2}$		-0.0906	-0.0969
$2.f.k.\frac{L}{D}$		0.2167	0.2515
Y		0.1261	0.1546
$\frac{L}{D} = 0$	$\therefore \frac{P_{03} - P_{01}}{P_{01}}$	0.436	0.535
$\frac{L}{D} = 3$	$\therefore \frac{P_{03} - P_{01}}{P_{01}}$	0.324	0.405

$\frac{L}{D} = 3$ corresponds roughly to the size of Aerothermopressor needed for a 1500 H.P. Gas Turbine.

These gains in total pressure are so much larger than the results of ref. (2) that at first sight the supersonic Aerothermopressor offers revolutionary opportunities for the Gas Turbine. A closer examination of this method of extrapolation shows the error.

The term $k \cdot M_1^2 \cdot 2 \cdot f \cdot \frac{L}{D}$ is given as the frictional loss in a duct with Mach Number variation identical to that of a small scale Aerothermopressor. In fact it is the frictional loss of a duct of constant Mach Number M_1 and mean diameter D .

The true method of extrapolation is as follows:

For any two Aerothermopressors with the same initial parameter values and Mach Number distribution, differing only in size.

$$\begin{aligned} \frac{P_{03} - P_{01}}{P_{01}} &= (\text{Evaporative Gains Term} - \text{Friction Loss Term}) \\ &= (\text{Constant} - \int_0^L 2 \cdot f \cdot \frac{k \cdot M^2}{D} \cdot dz) \\ \frac{P_{03} - P_{01}}{P_{01} \cdot M_1^2} &= \left(\frac{\text{Constant}}{M_1^2} - \frac{1}{M_1^2} \int_0^L 2 \cdot f \cdot \frac{k \cdot M^2}{D} \cdot dz \right) \end{aligned}$$

Given M_1^2 and assuming $k = \text{Constant}$,

$$\therefore \frac{P_{03} - P_{01}}{P_{01} \cdot M_1^2} = \left(Y_{True} - \frac{2 \cdot f \cdot k}{M_1^2} \int_0^L \frac{M^2}{D} \cdot dz \right) \quad (4)$$

Given $\frac{P_{03} - P_{01}}{P_{01}}$ for a parallel duct then Y calculated by ref.(2) will be incorrect since

$$Y - Y_{True} = 2 \cdot f \cdot k \cdot \frac{L}{D} \cdot M_1^2 \left(1 - \frac{M_m^2}{M_1^2} \right)$$

$$\text{where } M_m^2 = \frac{\int_0^L M^2 \cdot dz}{L} \quad \text{i.e. (Mean Mach No)}^2 \text{ /Unit length.}$$

How M_m differs from M_1 can be seen from fig. 4 where it is roughly 15% less than M_1 in Solution 1. In general this result together with the small scale results for an almost similar size duct given in ref. (2) shows that M_1 is greater than M_m for all

values of M_1 except for the unique case where M passes from subsonic flow in Regime 1 to supersonic flow in Regime 2. In all cases of an Aerothermopressor of this scale and shape other than this singular solution Y will be overestimated.

Y_{True} may be found from a small scale numerical solution for a parallel duct by finding M_m graphically but extrapolation to a large scale unit is not easy for the duct will no longer be parallel. Equation (4) cannot be solved because $\int_0^L \frac{M^2}{D} dz$ is now a function of three variables. Average values of $\frac{L}{D}$ may be selected for insertion but what size of Aerothermopressor they correspond to on a mass flow basis, the true size criterion, is not exactly certain. From this discussion it is clear that extrapolation of results from small scale to large scale is only possible if the governing equations can be solved analytically. This is what the method sought to avoid.

The two solutions given in this section were obtained by manual computation. It was possible, having knowledge of the property values at every step, to calculate the true theoretical total pressure rise and duct shape for a large scale unit of similar Mach Number variation where $\frac{L}{D} = 0$ without much effort.

The true total pressure rise was obtained by adding the term $\frac{kM^2}{2} \cdot 4f \cdot \frac{\Delta z}{D}$ to each calculated increment $\frac{\Delta P_0}{P_0}$. The corresponding area change was obtained by ensuring that $\frac{\Delta M^2}{M^2}$ remained unchanged. From Table 2

$$\therefore \frac{\Delta A}{A} = - \frac{kM^2}{2} \cdot \frac{4f \Delta z}{D}$$

The results of these solutions are given in figs 8 and 9. The true full scale total pressure rise is almost identical for the two solutions and is given below together with the small scale results and interpolated practical values.

		<u>1st Solution</u>	<u>2nd Solution</u>
$\frac{L}{D} = 20$	$\therefore \frac{P_{03} - P_{01}}{P_{01}}$	-0.314	-0.3353
$\frac{L}{D} = 0$	$\therefore \frac{P_{03} - P_{01}}{P_{01}}$	0.20	0.20
$\frac{L}{D} = 3$	$\therefore \frac{P_{03} - P_{01}}{P_{01}}$	0.123	0.120

From the last row the penalty of high Mach Number may be seen and hence the significance of the form of coefficient of performance becomes more apparent. Despite early promise, it appears at least in this case that supersonic results are not significantly better theoretically than subsonic though area reduction is more severe being 45% in our case while only 25% in the subsonic case.

We may conclude that unless the variation of each influence coefficient is obtained separately as with manual computation, small scale solutions have limited usefulness and cannot be extrapolated to large scale results simply, without serious error, while small scale experiments are equally restricted. ~~Both should only be used for comparative purposes to verify the validity of the theoretical assumptions.~~

5. THE DESIGN AND CONSTRUCTION OF APPARATUS

5.1 The Technique

Experimental investigation was carried out on a small scale Aerothermopressor. The general layout is shown in fig.10 and Plate 1. It consisted of a gas fired combustion chamber as an air heating source connected through a nozzle to a parallel duct used as a test section. Water was admitted at the entry plane of the test section. The hot air, water, and vapour were drawn through the test section via a cooler by an exhaust pump which discharged the mixture to atmosphere. The whole unit thus operated at sub-atmospheric pressure. The desired Mach Number in subsonic flow could be obtained by varying the air flow through the duct with the aid of a by-pass control valve connected directly to the pump. In supersonic flow no air was by-passed and Mach Number was determined by fitting a convergent - divergent nozzle of suitable dimensions. Water was supplied from a pressure vessel and the flow maintained by compressing with gas from a high pressure cylinder.

Performance was estimated by measuring static pressure variation along the duct and atomization techniques compared by performance results. A short length of duct with quartz walls could be fitted immediately after the nozzle to give further information on spray formation with the aid of high speed flash photographs.

5.2 Gas Flow Component Design.

Gas ducting was adapted from components originally made by undergraduates for a final year project (ref.23), and considered to be suitable for this research.

5.2.1 Combustion Chamber

The combustion chamber used was a single can of a Whittle W.2. 700 jet engine adapted to burn town gas. This is of the reverse flow type shown in fig.10 and Plate 2.

Modification was achieved as follows. On the outer casing the interconnectors were blanked off, and the torch igniter removed and replaced by a detachable plugged socket. The kerosene burner

attachment was sawn off and a boss fitted to take various types of burner. The burner fitted was of the "Hypac" type milled out to permit sufficient gas flow. A transition piece was fitted to the inner flame tube as shown in fig. 10 and 17 and Plate 2. reducing its size from $6\frac{1}{2}$ " diameter to 2" square.

5.2.2 Gas Nozzles

The supersonic nozzle available was taken from a design given in ref. (25) and scaled to 2" square at outlet. A 2" square parallel nozzle was also made for subsonic experiments and for supersonic experiments with the annular water injection nozzle. These are shown in fig. 11. Both were made from 20 S.W.G. stainless steel bent to shape and welded. Neither was wholly satisfactory as the weld seams distorted and penetrated the thin metal causing entry effects in subsonic flow described in Section 6.4. and oblique shocks in supersonic flow which are discussed in Section 7.1.2 (b)

5.2.3 Test Section

The original was made in two parts as shown in fig.12. The short section had been intended to be detachable to permit access to the water injectors but accessibility was later found to be unnecessary. The construction was of 20 S.W.G. stainless steel bent and welded as for the nozzles. Consequently the duct suffered from similar defects which together with lack of sufficient pressure tapplings led to its ultimate rejection. It was replaced by a new and more adaptable duct shown in fig.12. The old duct was not entirely discarded for use was made of its two component construction to substitute a spray photographing section shown in figs. 13 and 21 in place of the short section. As this was only used for examining initial spray break up, previously raised objections to its use no longer applied.

5.2.4 Cooler

This followed the test section as shown in fig.10 and was inserted to prevent erosion of the exhaust pump by the hot gas. It was composed of two water cooling units, one consisting of staggered water tubes perpendicular to the flow and the other of the honeycomb film type.

5.2.5 Exhaust Pump

The Holland Exhaust pump was of the rotary vane type comprising 24 vanes sliding in a rotor eccentrically mounted. It was driven by a 90 H.P. squirrel cage motor. Control of the gas throughput was by two upstream valves shown in fig.10. One admitted atmospheric air, and the other admitted gas from the cooler.

5.3 Water Injection System

5.3.1 Initial Flow Control System

This is shown in fig.14. but was never satisfactory because it was designed for low pressure injection. The large pressure drop across the main flow control valve caused entrained air to come out of solution despite insertion of a downstream pressure regulating valve and a bleed-off cock to drive out residual air. The entrained air upset the level of the manometer and flow readings could never be relied on.

5.3.2 Injection with Carbon Dioxide

A small pressure vessel was constructed from a 3" bore drawn steel pipe as shown in fig. (15). This was used to determine

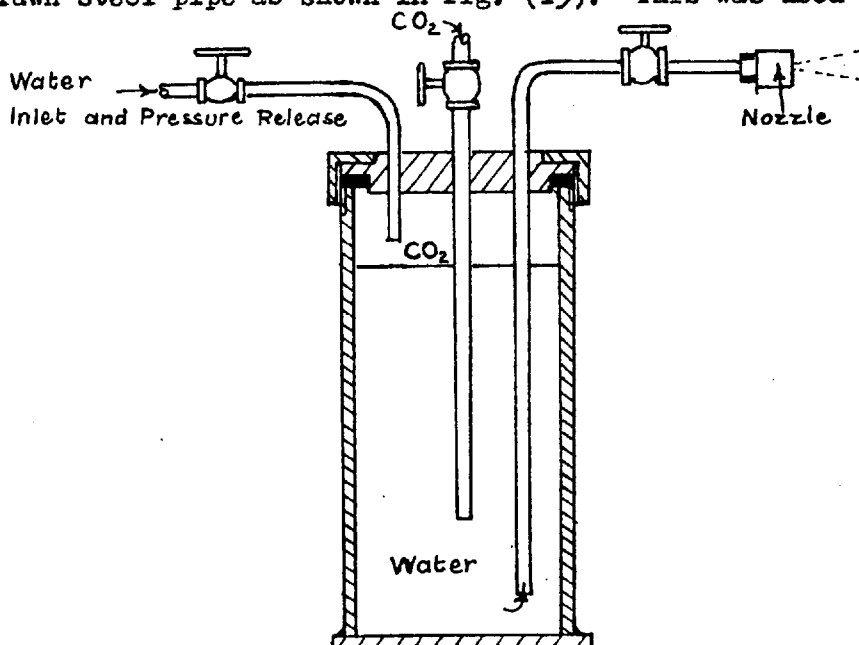


Fig. 15.

whether to proceed with construction of full scale apparatus for high velocity injection tests with carbon dioxide. A stroboscope was used to view the spray formation. Using carbon dioxide at pressures up to 35 p.s.i., in addition to the characteristic foaming action, large drops were seen to grow into bubbles and finally burst into fine droplets. This final bursting took place rather far downstream about 3 ft from the nozzle implying that carbon dioxide comes out of solution rather slowly. Above 35 p.s.i. the water velocity became too rapid to permit observation with a stroboscope but the low pressure tests appeared so encouraging that full scale high pressure injection apparatus was constructed with the hope that the rate of separation of the carbon dioxide would increase with pressure.

5.3.3 Final Flow Control System

This is shown in fig. 16. It was built for pressures up to 1,000 p.s.i. and was used for all injection tests in view of the deficiencies of the original system. A pressure vessel used for other tests many years earlier was found and adapted for this purpose. The water needed considerable agitation to ensure that it was fully saturated with carbon dioxide. Tests showed that a propeller type agitator giving upward thrust approximately 2" below the surface and rotating at high speed would give considerable surface break up. The stirring motor and baffles shown, were fitted to do this. To prevent overfilling and consequent damage to the motor, the tube leading to the pressure release valve extended down to the maximum water surface level. On filling the vessel with water, the valve was opened at intervals and if too full, the excess water would be driven out by the air pressure built up in the vessel. Details of the high pressure fittings are given in fig.17.

5.3.4. Water Tube Fittings

According to ref. (2) the number of injectors required was between 0 and 2 per square inch of duct cross section. Taking 1 per square inch as a mean, 4 were required. These were fitted to

the duct upstream of the nozzle to permit axial injection free from shocks and downstream disturbances. Originally the tubes were secured individually with collets and nuts. This had many fitting disadvantages but was ultimately rejected due to persistent gas leaks round the collets. The final construction and fitting of the tubes is shown in fig.18.

5.3.5 Water Nozzle Construction

a) Open Tubes

To make flow rate and pattern identical for each tube all were lapped internally and on the ends.

b) High Speed Injectors

These were designed by calculating water flow rates and assuming $C_D = 0.6$. The nozzles were drilled undersize and lapped on all surfaces in contact with the final jet. The small pressure vessel described in Section 5.3.2. was adapted to flow check each nozzle and to determine that flow patterns were identical. The nozzle bores were then increased till each gave the correct flow to $\pm 1\%$. Final dimensions are given in fig.19. The nozzles were fitted to the tubes by brazing.

c) Fan Spray Nozzles

These are shown in fig.19 and were manufactured by Messrs H.T. Watson & Co.Ltd., Attachment to the tubes was also by brazing.

d) Annular Nozzle

The annular nozzle was designed to be fitted in the parallel subsonic gas nozzle. Originally it was intended to test it in subsonic flow, but minimum material requirements showed that it was impossible to make without forming a throat in the gas duct. It was therefore designed for supersonic tests at $M_1 = 1.86$. No known design technique was available for the outer gas nozzle formed between the square duct and the outer diameter of the annulus. It was realised that flow both inside and outside the annulus would be three dimensional producing weaker downstream oblique shocks than

two dimensional flow. Bearing the deficiencies of the existing square convergent divergent nozzle in mind, the design was simplified by making the area ratios in the inner and outer nozzles identical to those of the two dimensional nozzle and cutting short where the wall became too thin for machining. The resulting design is shown in fig. 20 and Plate 2 (b). The annulus was originally made $0.001" \pm 0.0005"$ thick but machining distortion and blockage through fine particles in the water stream made it necessary to increase the gap to $0.004"$. The water contacting surfaces were lapped to size. Waterproof sealing of the inner and outer parts and set screws was by "Quigley" high temperature porcelain cement. Attachment of the nozzle was by cutting back the four water tubes and brazing into position.

5.4 Instrumentation and Measurements

Measurement of the fluid properties in a two phase stream presents many difficulties. To avoid these, it was decided to compare theory and experiment initially by recording only static pressure variation along the duct. This restricted comparison between theoretical and experimental total pressure rise to a qualitative basis. Results given in section 7 did not justify further downstream measurements. These would have required construction of a special probe described in ref. (2). The problem was to determine flow conditions at the test section inlet and downstream static pressure variation. For this purpose measurements were made of:

- a) Total Temperature at test section inlet.
- b) Total Pressure at test section inlet.
- c) Static Pressure at test section inlet
- d) Downstream Static Pressure
- e) Water Flow Rate.

5.4.1 Inlet Total Temperature

This was measured by a chromel/alumel thermocouple 2 $\frac{1}{2}$ " upstream of the nozzle in position shown in fig.17 and assuming that flow through the duct was adiabatic. The thermocouple was made to traverse the duct in order to get a true mean temperature. At first a shielded couple with high recovery factor and low radiation loss was made according to ref. (29). Two subsidiary thermocouples were fitted to account for conduction loss near the walls. The results were unsatisfactory as the heat transfer coefficient to the main couple could not be estimated accurately. The design was modified to the form shown in fig.21. The main thermocouple was unshielded but recovery factor was easily obtained from cold static and dynamic tests while heat losses were calculated according to Appendix D.

5.4.2 Inlet Total Pressure

This was measured by a total head pitot shown in fig.21. It was fitted in the test section inlet plane and for correlation purposes on dry runs at various downstream positions shown by the ports and blanking off plugs in fig.12. Like the thermocouple, it was made to traverse the duct to obtain a mean value.

5.4.3 Inlet Static Pressure

This was taken from the mean of 4 wall statics in the test section entry. For comparison, 2 or 4 statics for mean values were measured in any total pressure traversing plane as shown in fig.12.

5.4.4 Downstream Static Pressure

Wall Statics connected to a multiple manometer board were used. Tappings were made from the top of the duct to prevent water accumulation. The spacing was staggered according to fig.12 to give more readings in the region of sudden change near the test section inlet.

5.4.5 Water Flow Rate

An orifice plate designed according to B.S.S. 1042 and scaled down for a $\frac{3}{4}$ " bore pipe was made and calibrated. It was used in conjunction with an inverted U manometer shown in figs 14 and 15.

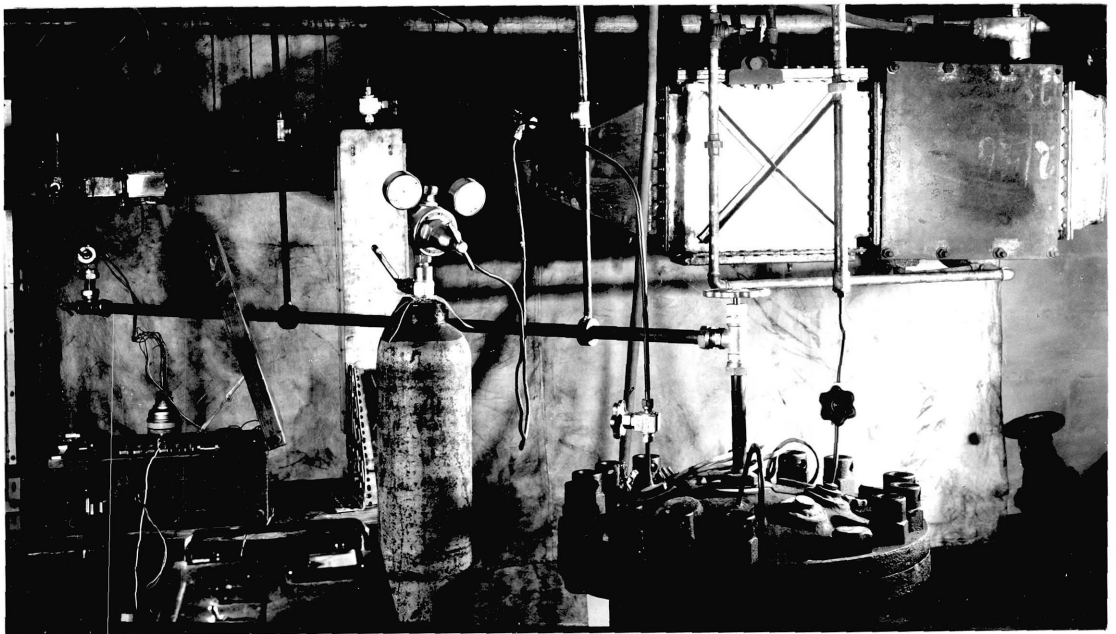
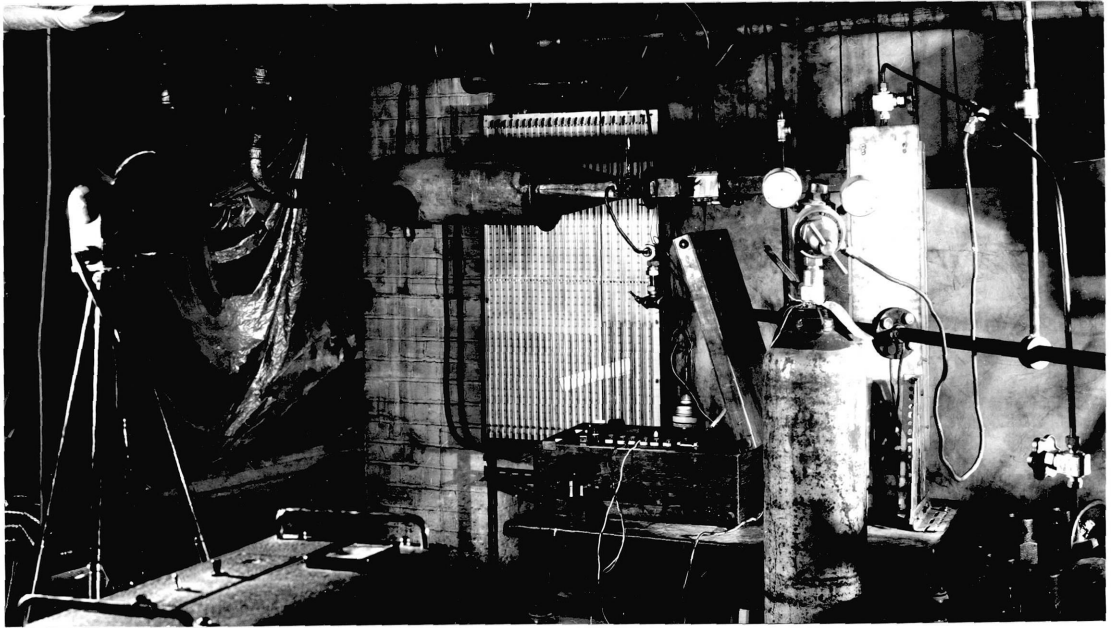
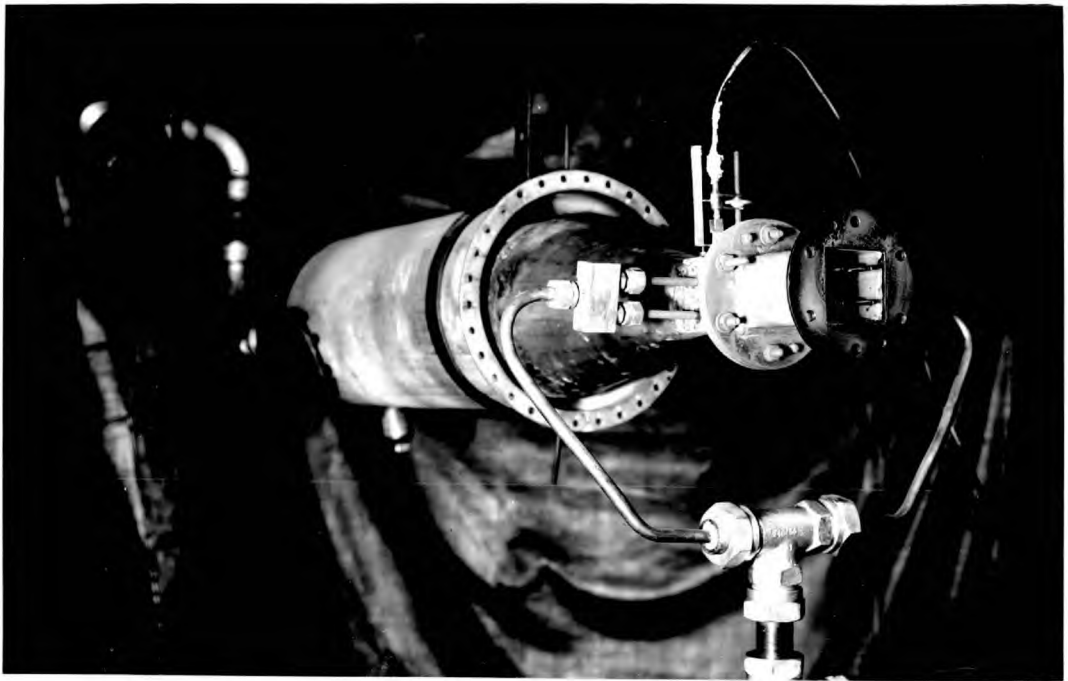
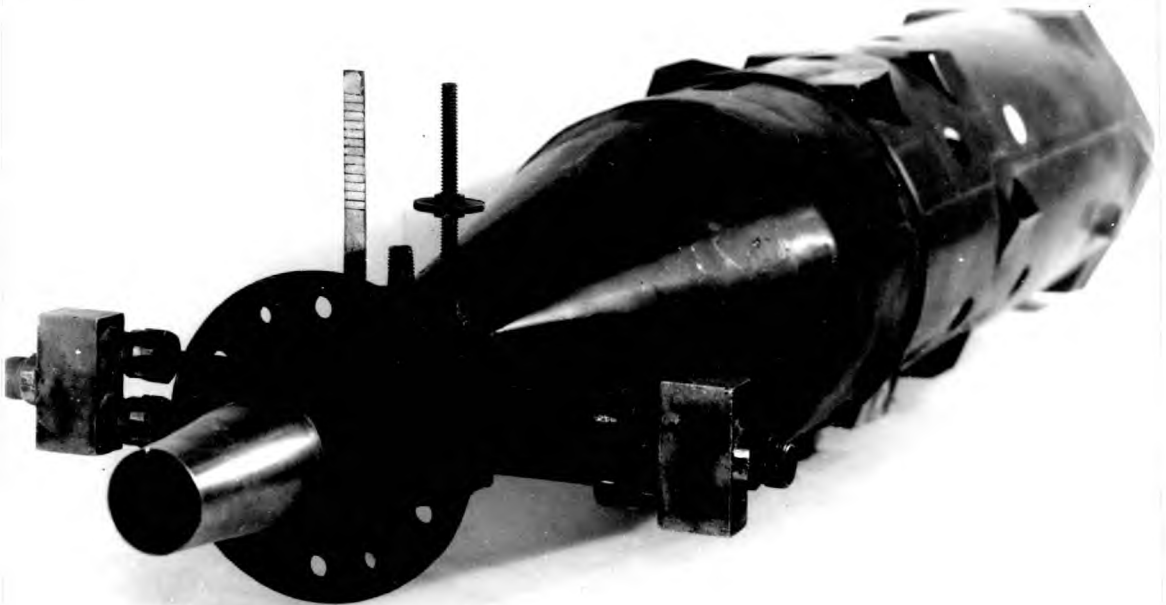


PLATE .I. GENERAL VIEWS OF APPARATUS
SET UP FOR SPRAY PHOTOGRAPHY



a) OPEN TUBES COMBUSTION CHAMBER ASSEMBLED



b) ANNULAR NOZZLE INNER FLAME TUBE ONLY

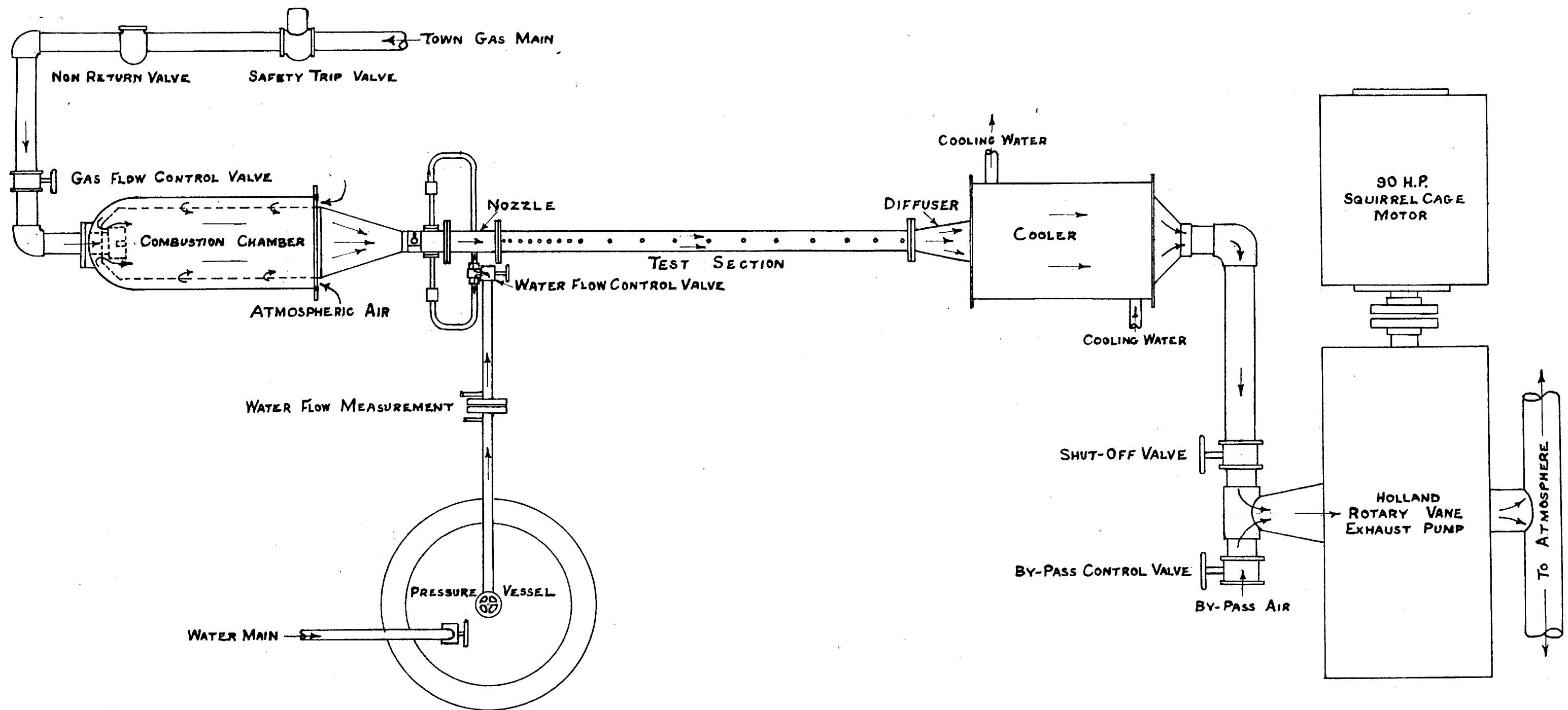
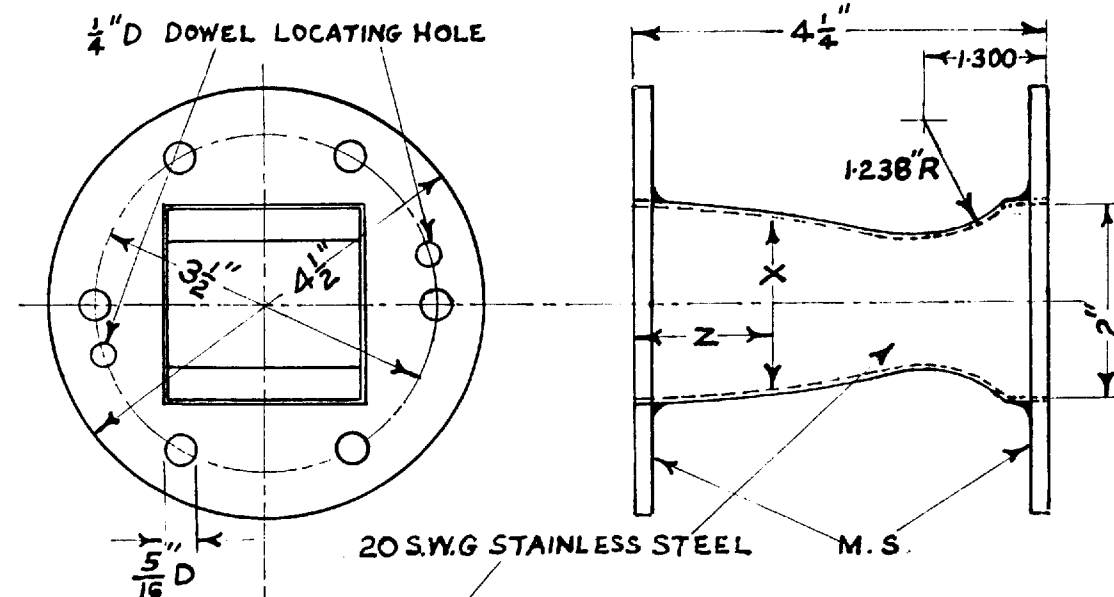
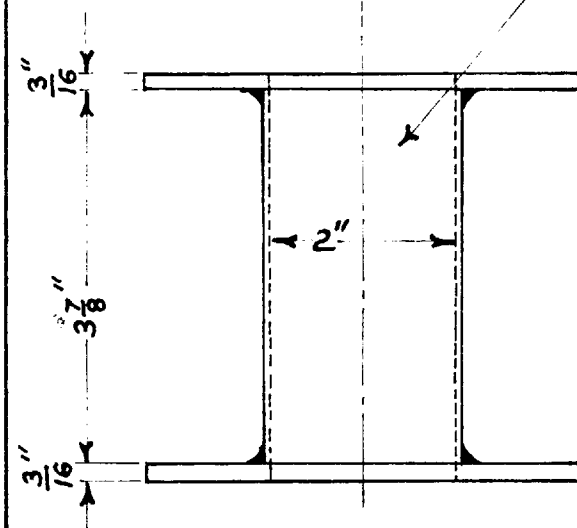


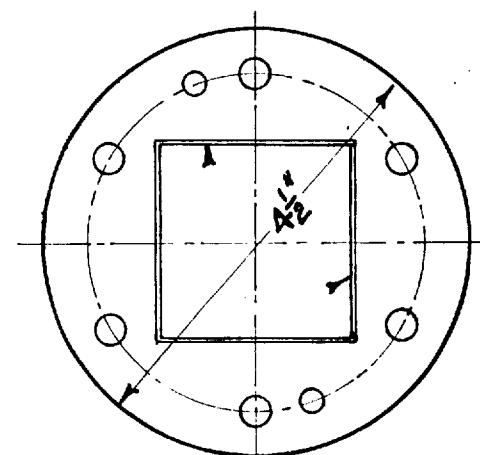
FIG. 10.
 GENERAL LAYOUT OF APPARATUS
 NOT TO SCALE



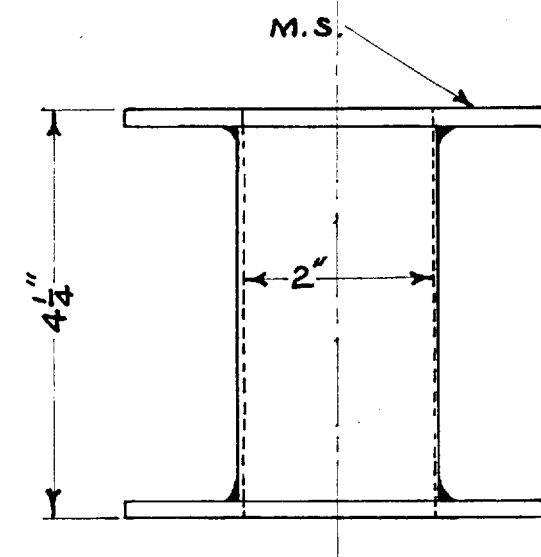
Z(INS)	X(INS)
0.000	2.000
0.298	1.986
0.398	1.974
0.497	1.966
0.597	1.956
0.697	1.944
0.797	1.930
0.964	1.904
1.140	1.864
1.326	1.826
1.512	1.776
1.707	1.718
1.910	1.650
2.100	1.550
2.270	1.514
2.482	1.426
2.690	1.342
2.950	1.290



SUPERSONIC NOZZLE



20 S.W.G. STAINLESS STEEL



SUBSONIC NOZZLE

FIG. 11.
GAS NOZZLES
SCALE: HALF FULL SIZE

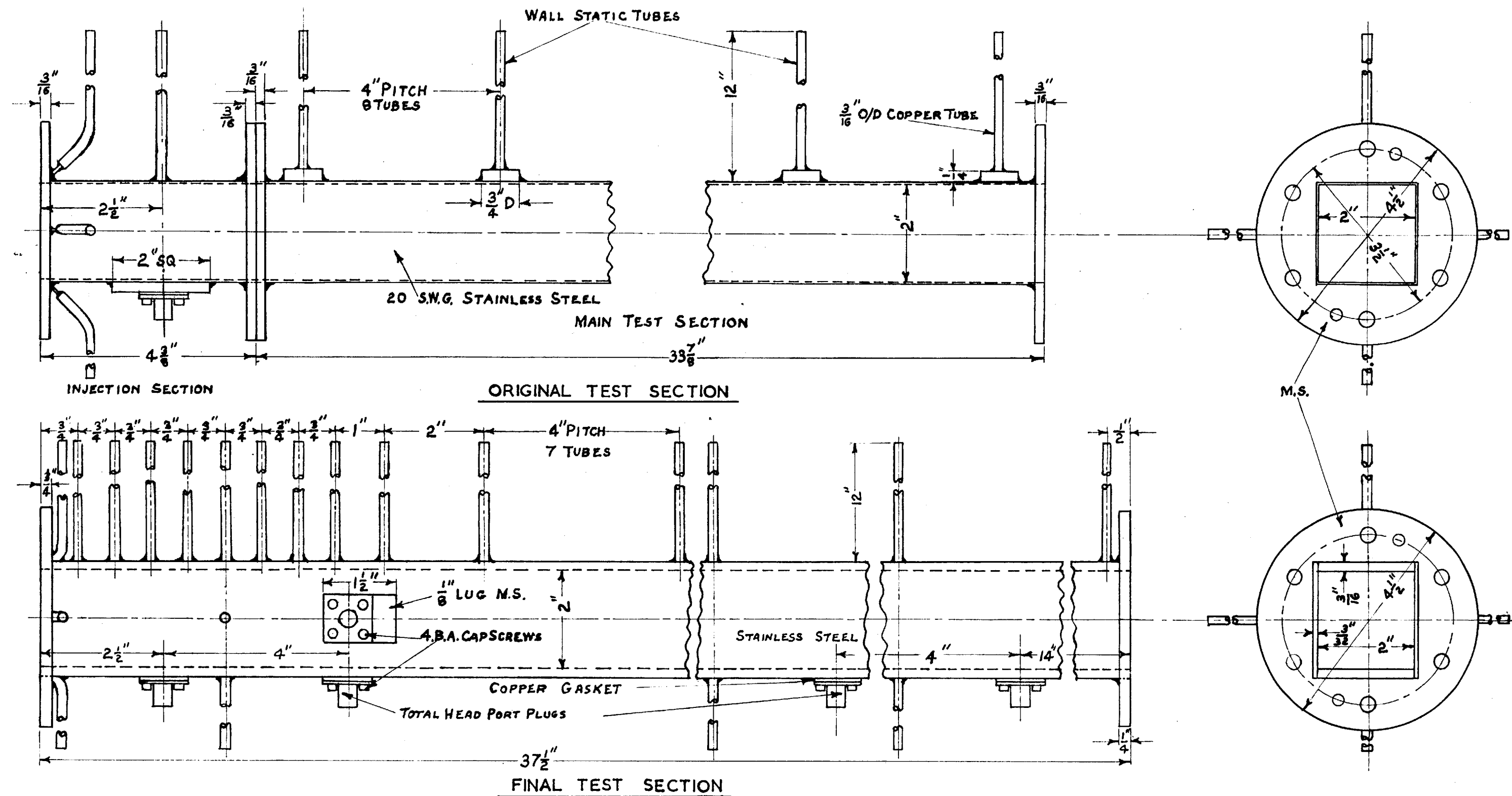


FIG .12.
GAS DUCTS
SCALE: HALF FULL SIZE

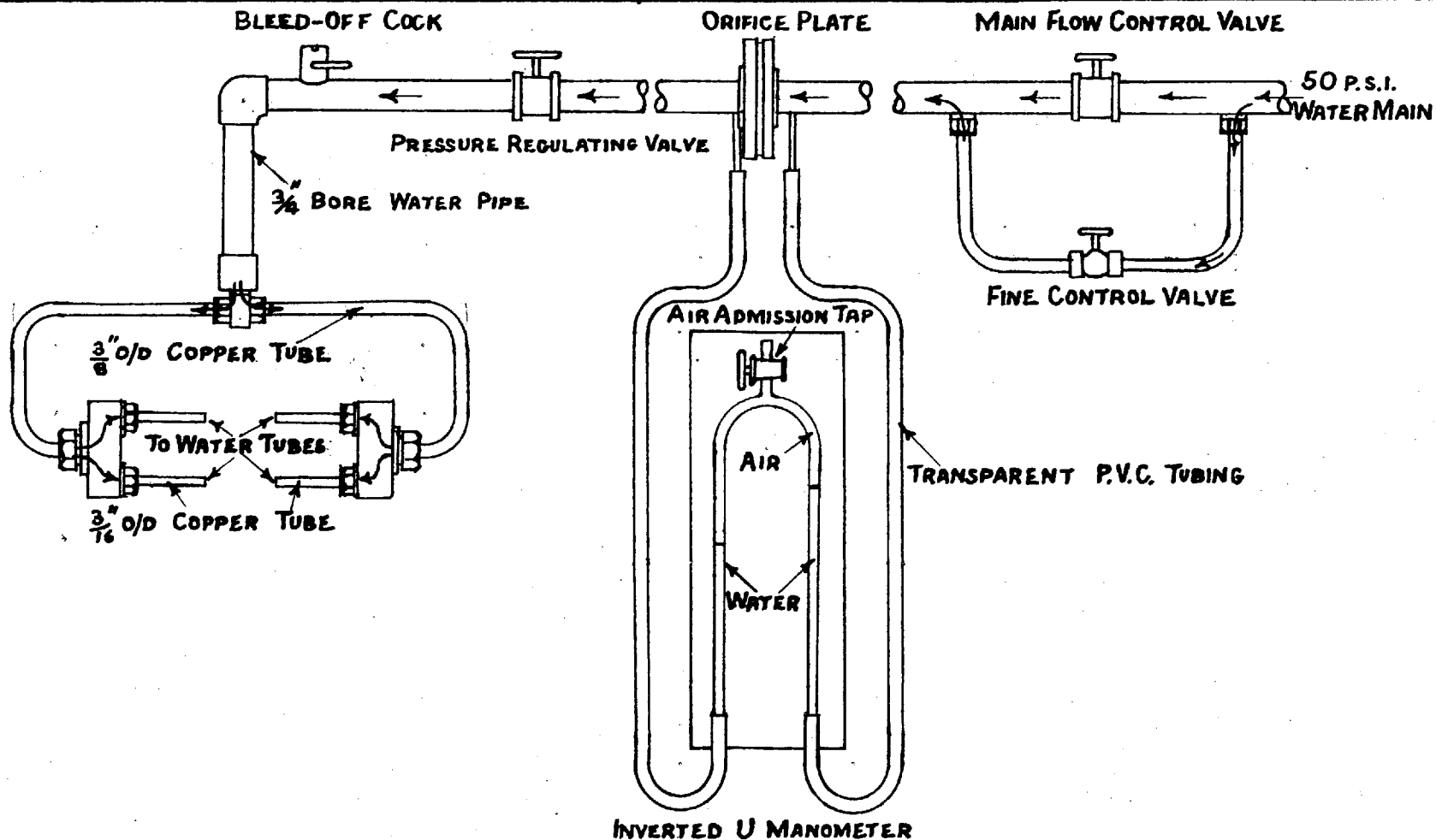


FIG .14.

INITIAL WATER FLOW SYSTEM

NOT TO SCALE

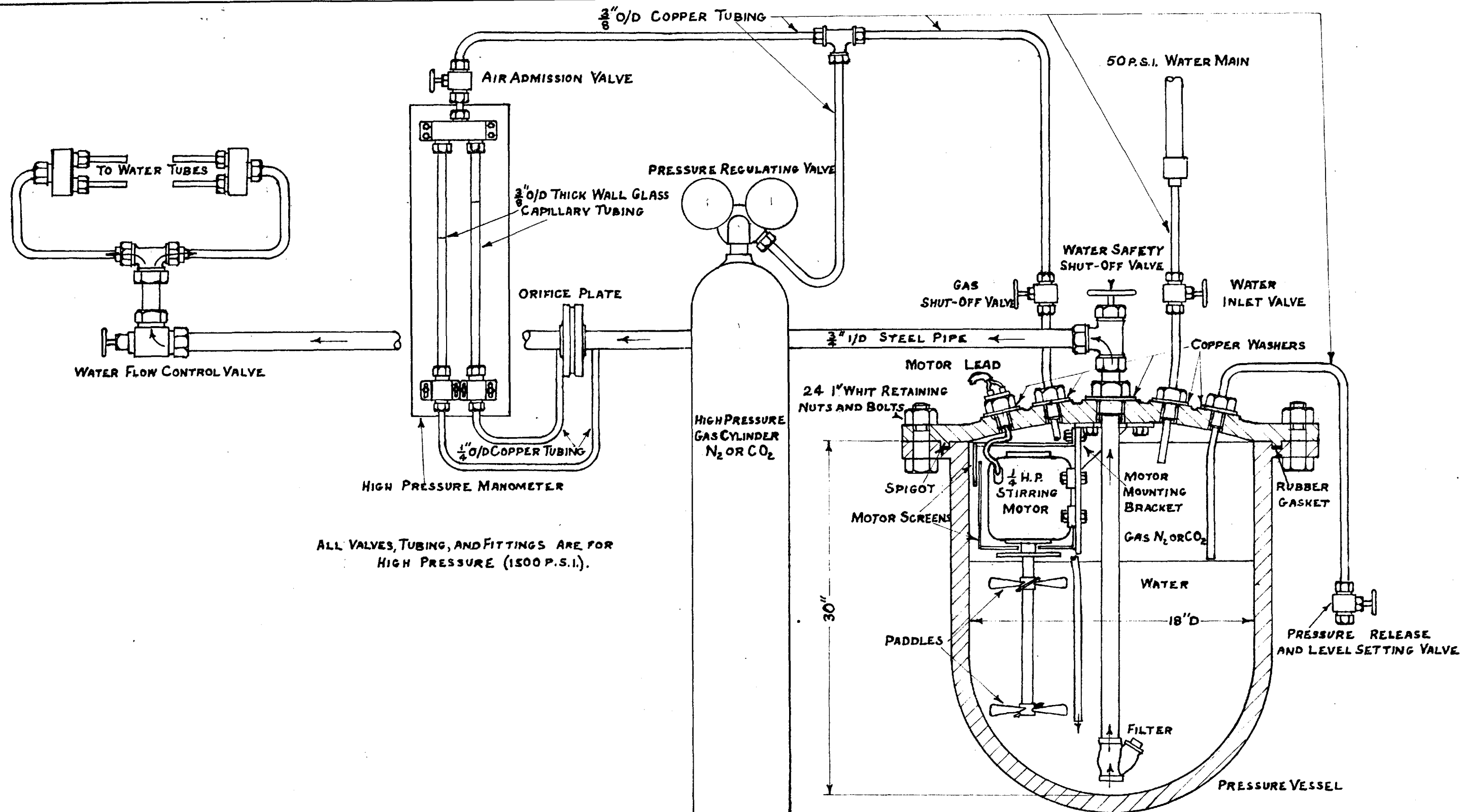


FIG. 16.

FINAL WATER FLOW SYSTEM

NOT TO SCALE

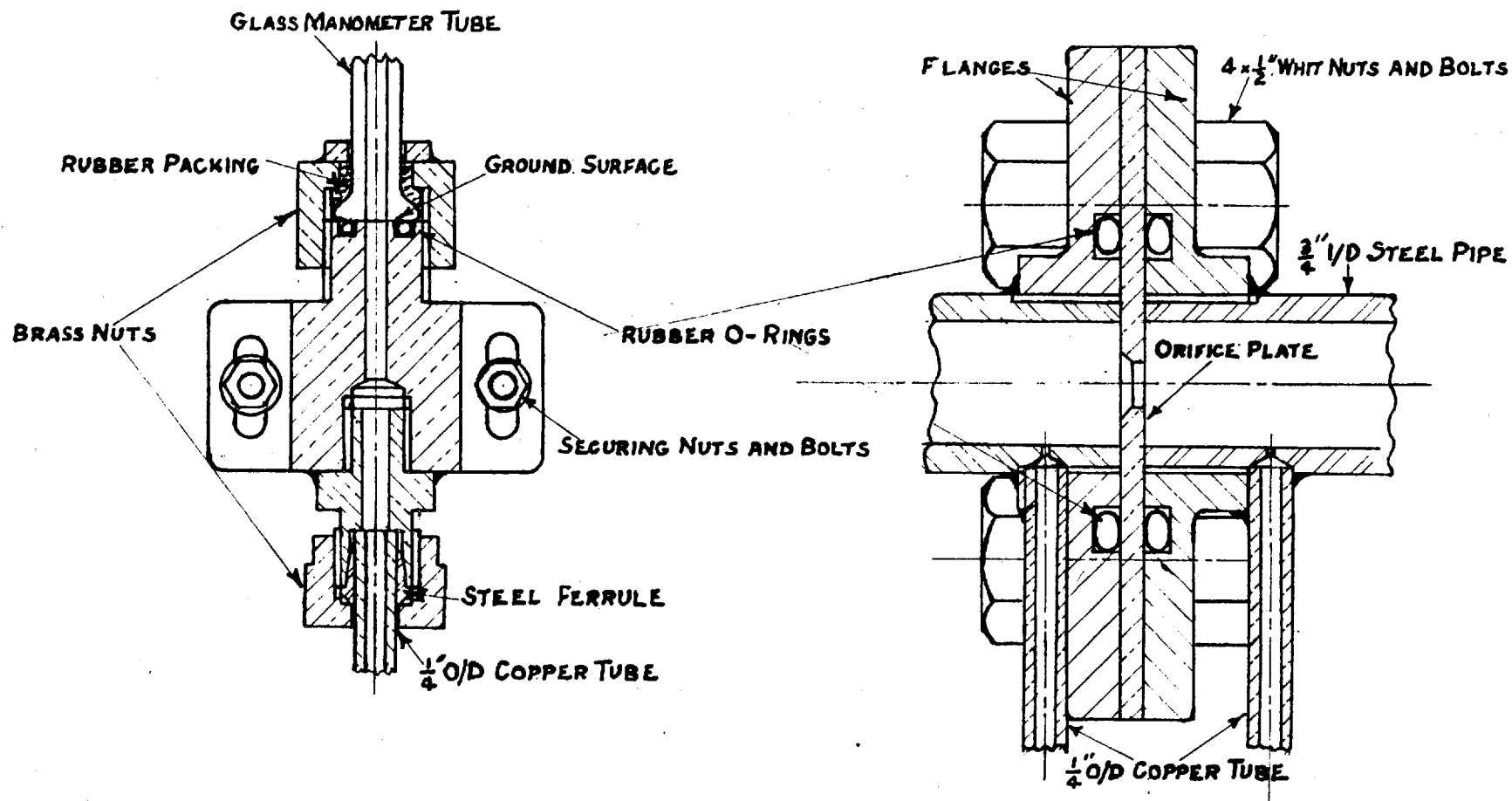


FIG. 17.

PRESSURE FITTING DETAILS

SCALE : FULL SIZE

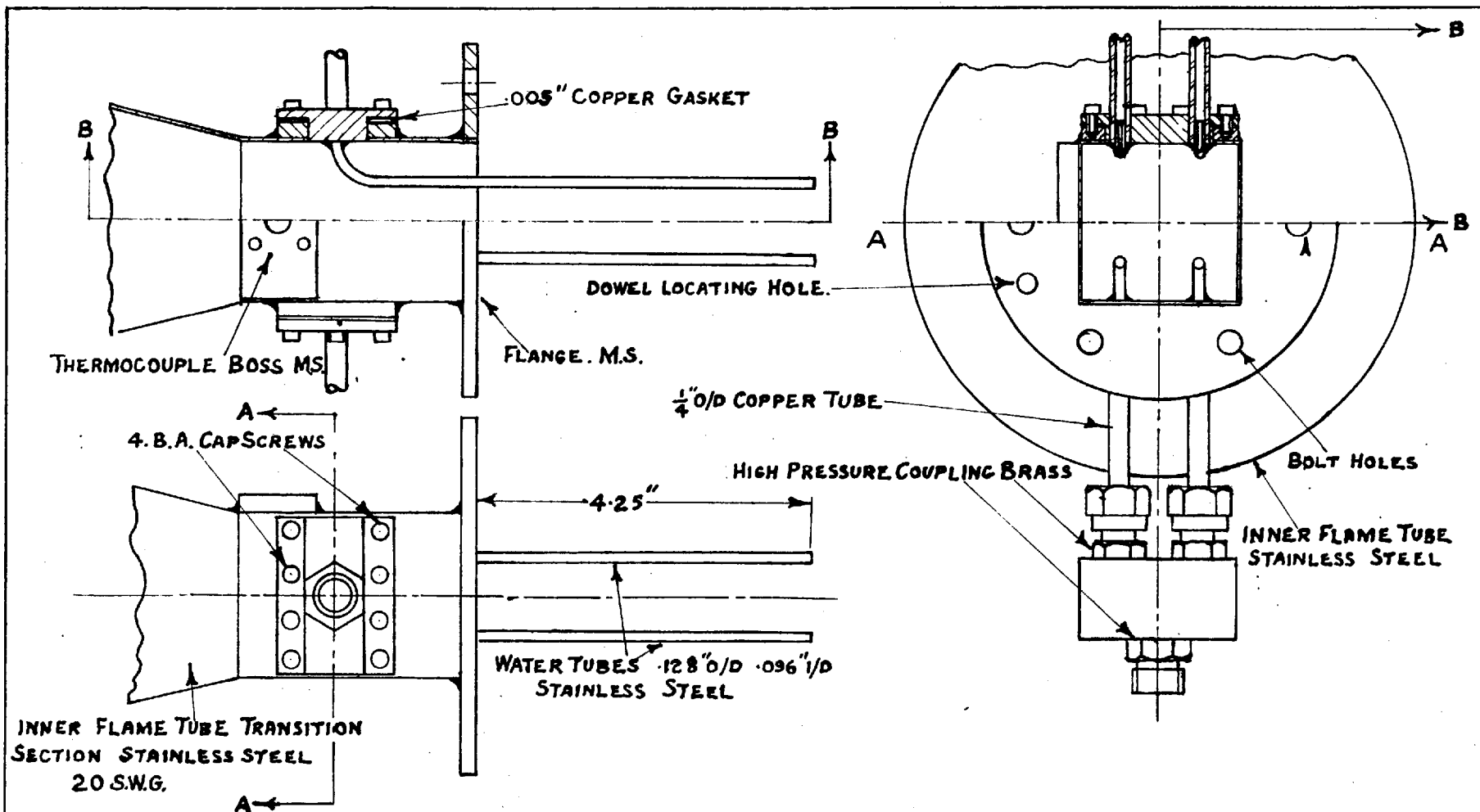
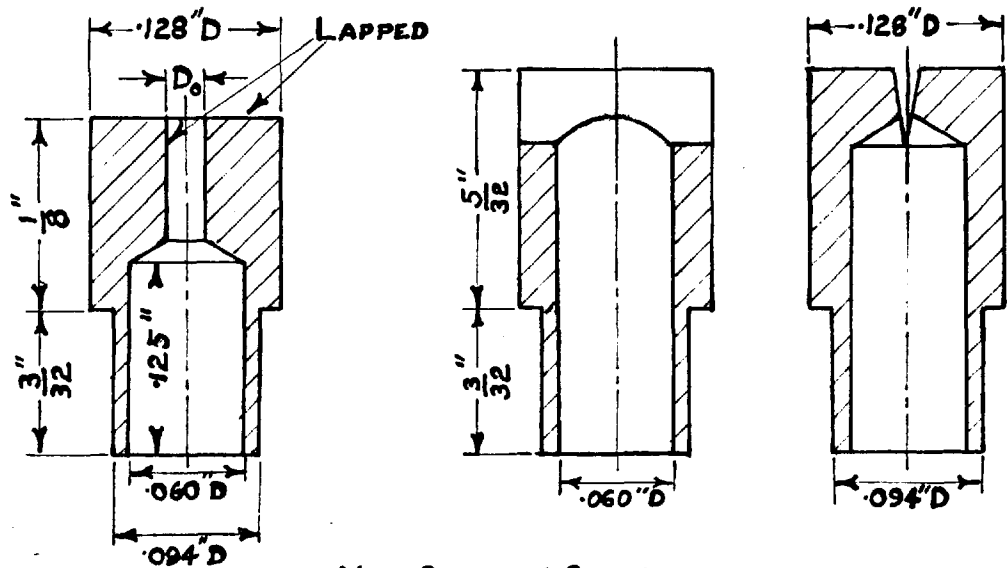


FIG. 18.
WATER TUBE DETAILS
 SCALE: HALF FULL SIZE



NORMAL HIGH PRESSURE NOZZLE

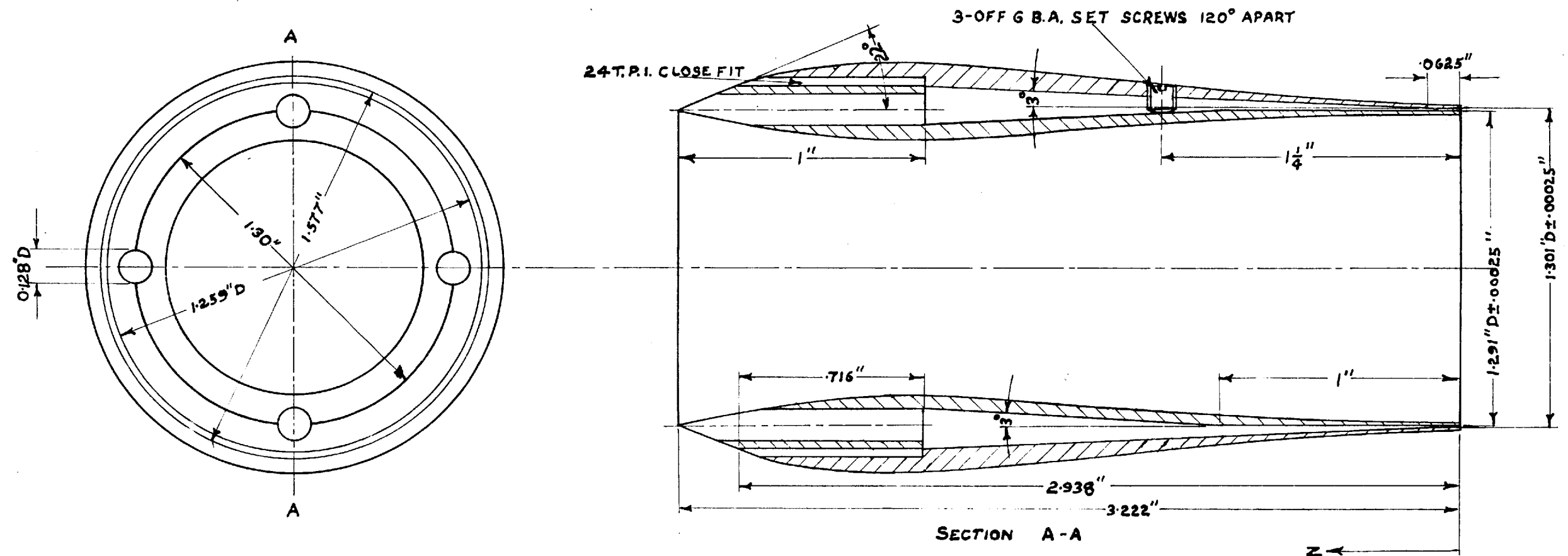
FAN SPRAY NOZZLE

FLOW CONDITIONS	NO-OFF	D _o (INS)
0.116 LBS/SEC 50 P.S.I.	4	0.036
0.116 LBS/SEC 100 P.S.I.	4	0.031
0.116 LBS/SEC 200 P.S.I.	4	0.025
0.082 LBS/SEC 50 P.S.I.	4	0.028
0.082 LBS/SEC 100 P.S.I.	4	0.025
0.082 LBS/SEC 200 P.S.I.	4	0.021
0.116 LBS/SEC 15 P.S.I.	4	0.055

FIG .19.

WATER NOZZLES

SCALE: 8x FULL SIZE



Z (Ins)	0.000	0.367	1.110	1.673	2.093	2.353	2.938	3.222
INNER DIAM (Ins)	1.270	1.250	1.206	1.132	1.065	1.051		1.300
OUTER DIAM (Ins)	1.330	1.365	1.480	1.600	1.680	1.700	1.529	1.300

FIG .20.
ANNULAR NOZZLE
 SCALE : TWICE FULL SIZE

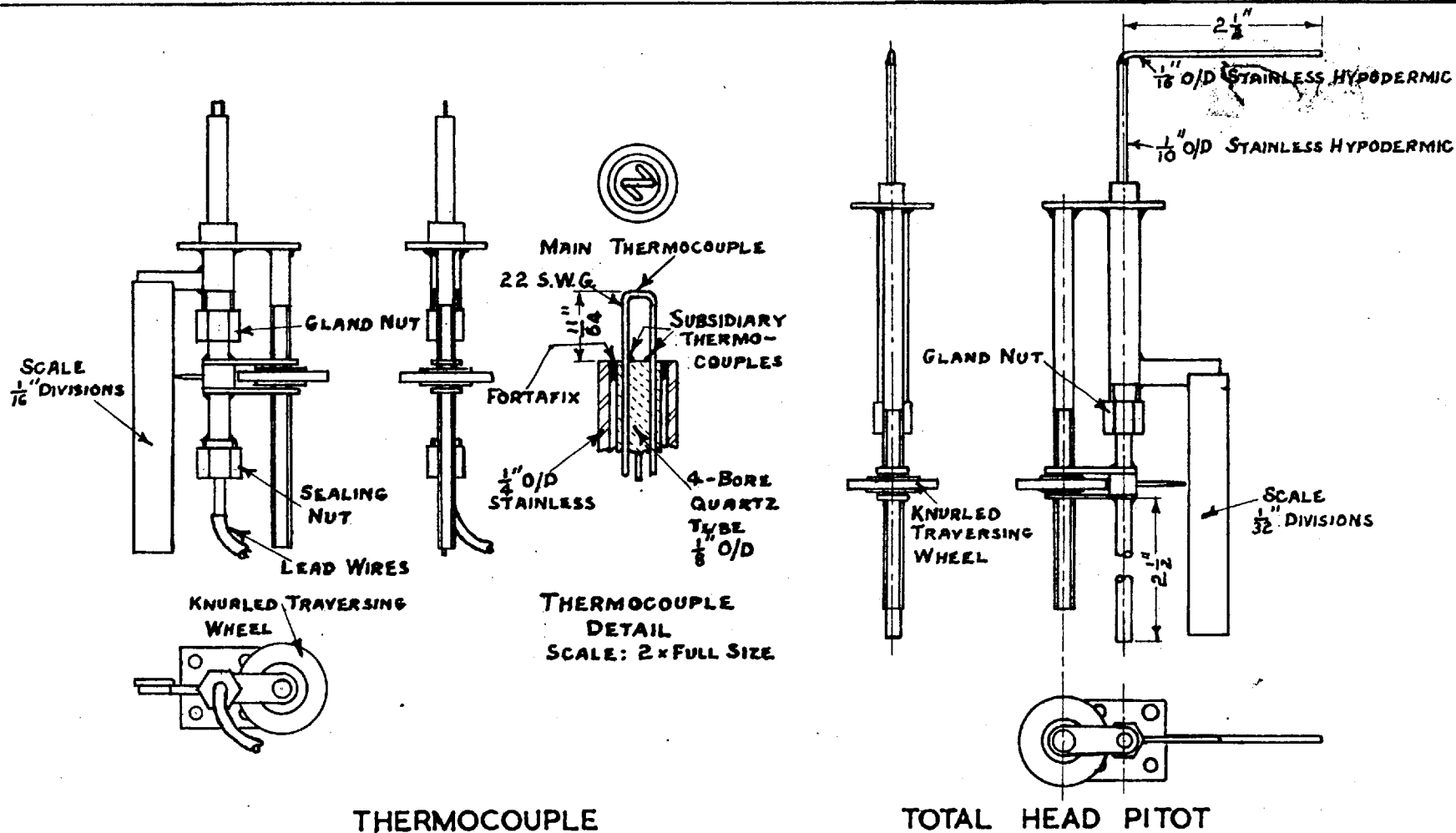


FIG. 21.
INSTRUMENTS
SCALE: $\frac{1}{2}$ FULL SIZE

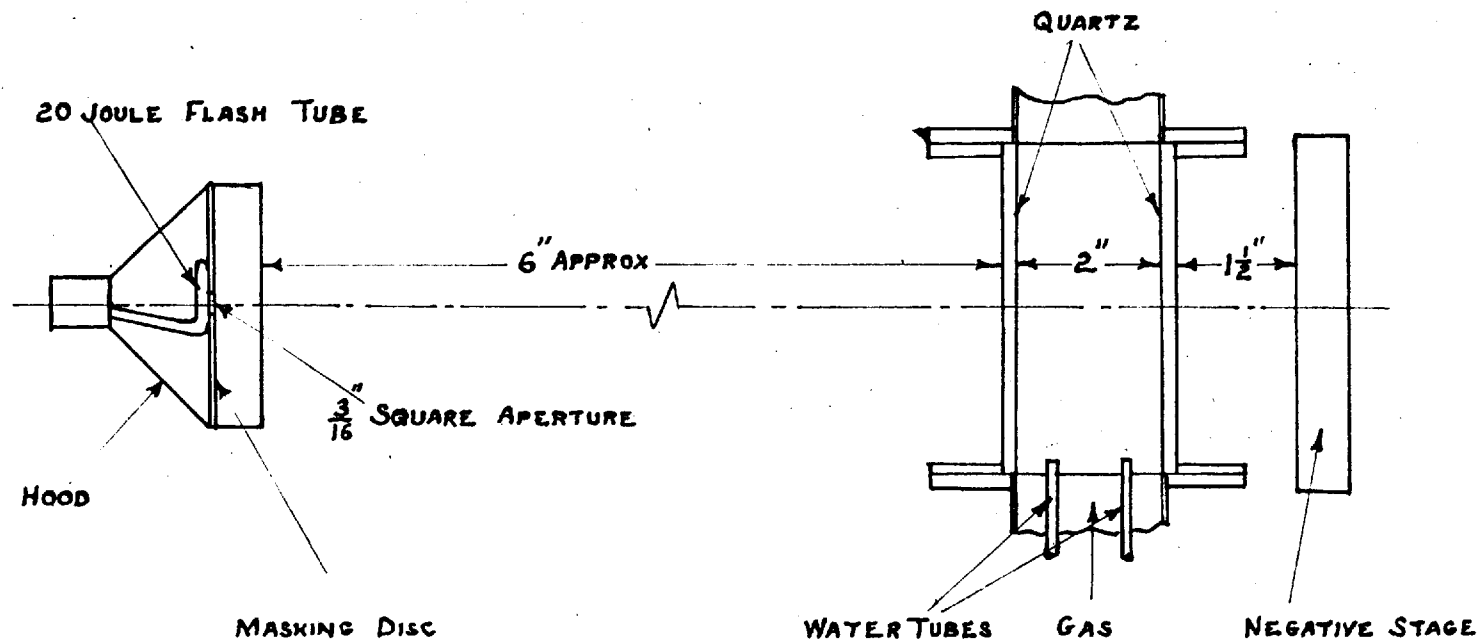


FIG .22.
 SHADOWGRAPH SYSTEM
 NOT TO SCALE

6. EXPERIMENTAL PROCEDURE AND DEVELOPMENT PROBLEMS

Summary

In addition to the developments described in Section 5, the following gives some of the outstanding problems and shows how experimental techniques were finally derived.

6.1 Starting Procedure

Dangerous explosive mixtures of air and gas formed if the gas was lit when air was already flowing through the combustion chamber. The most satisfactory starting procedure was to start the exhaust pump with the shut-off and by-pass control valves wide open to reduce the starting load. When running, the shut-off valve was closed. The gas was then lit by inserting a taper through the torch igniter boss. The shut-off valve was then opened fully. Desired Mach Number and temperature were obtained by regulating the gas flow and by-pass control valves.

6.2 Leaks at the Duct Flanges

High gas temperatures burned up most jointing compounds while insertion of gaskets between components of the ducting disturbed the flow pattern. Leaks between metal to metal surfaces were hard to eliminate. These were eventually cured by lapping all flanges flat to give good contact.

6.3 Flow Disturbances caused by Thermocouple and Pitot

After temperature traverses indicated the position of the thermocouple for mean temperature measurement, pitot traverses showed that the thermocouple was causing downstream disturbances in subsonic flow. Accordingly the thermocouple was withdrawn after setting to the desired temperature and the gas flow and by-pass control valves were adjusted to restore initial conditions. To eliminate disturbances due to the pitot, traverses determined the static pressure at the test section inlet corresponding to a given Mach Number. The pitot was then removed completely and a plug substituted for it. Downstream correlating traverses were performed at identical inlet static pressures.

6.4 Irregularities in Static Pressure Distribution

Static Pressure distribution in early subsonic tests had an upward kink at entry when running dry. This was increased when water was injected. Investigation showed this to be a double fault. Small area change between test section and nozzle produced the kink. Injecting water improved the suction of the Holland raising the Mach Number. Both faults were overcome by putting the subsonic nozzle after the test section. Initial disturbances were then smoothed out before the water injection plane was reached. After turning on the water, the by-pass control valve was adjusted to give the same static pressure distribution upstream of the water injection plane as in the dry run. This had the disadvantage of reducing the instrumented length of the test section.

6.5 Fluctuating Pressures in Subsonic Flow

About 30 seconds after turning on the water, the static pressure started to fluctuate and if the water flow continued for more than 1 minute the Holland tripped out on overload. This was attributed to water collecting within the Holland. In order to get all readings down before fluctuation began, the entire manometer board was photographed. Though this instability did not persist in supersonic flow this technique was continued throughout the tests as it improved the accuracy of the readings.

6.6 Liming up of Duct

After 45 minutes running time with water injected, a calibration check in supersonic flow showed that the normal shock position on a dry run had moved 1 diameter upstream. This was found to be due to roughening of the duct walls due to deposition of dissolved solids in the water. Subsequently the duct was rinsed with boiler descaling fluid for every 10 gallons of water injected. This eliminated water hardness problems but due to acid attack frequent wall static tube replacement was required.

6.7 Spray Photography

As soon as first results were obtained, it became apparent that their interpretation would be aided by observing

the water spray break up. The aim of this was to give information on how quickly the water was atomized, how uniformly it was distributed, and in the case of carbon dioxide injection, how the dissolved gas came out of solution. For these purposes shadowgraph photography was considered to be most suitable. This also had the advantage of simplicity as it required no lens system.

As mentioned in Section 5.2.3. a short section of duct with quartz walls together with the old test section were fitted for photographic runs. In order to get a clear picture of the spray formation only 2 tubes were used for injecting water while the other pair were blanked off. The lighting system is shown in fig. 22. While exposing the plate the apparatus was run in darkness. The 20 joule flash unit had a 5μ sec discharge duration which is slow for these gas speeds. The exposure time was reduced by using Process N.40 plates and developing with a high contrast developer.

As the image was formed by a direct shadow, it was not possible to synchronize the flash with a shutter mechanism. Photographs were first taken by sliding back the plate holder exposing the plate, and then triggering the flash. Hot runs produced fogged negatives as the glow from the combustion chamber was reflected down the duct and illuminated the spray. This was overcome by arranging the sliding portion of the plate holder to press a bomb release switch operating the flash. In this way the overall exposure time was reduced to approximately $\frac{1}{2}$ sec.

It was seen that water evaporated before reaching the quartz walls but the dissolved solids deposited a film on them which interfered with the passage of light. To avoid this the two tubes discharging water were bent as shown in fig.23. This procedure was only adopted during the spray photographing runs.

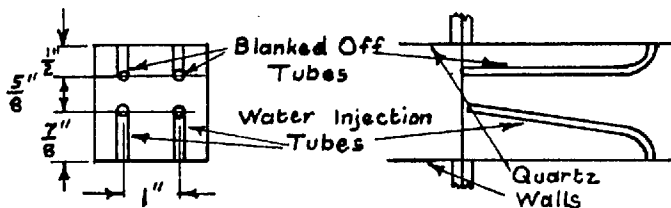


Fig 23.

It was found to be impossible to take cold spray photographs without evaporation in supersonic flow as the temperature drop in the expansion through the gas nozzle caused moisture condensation on the quartz walls. As a compromise, the temperature was set so that the static temperature of the gas was equal to that of the atmosphere.

When spray photographs were completed, shut down of the apparatus had to be performed with care to avoid failure of the quartz in thermal shock. For subsonic tests, where the mechanical stresses due to pressure difference between the inside and outside of the duct were low, slow steady closing of the gas flow control valve followed by switching off the compressor was satisfactory. In supersonic flow, before reducing the gas flow it was necessary to relieve the quartz of mechanical stress by opening the compressor by-pass control valve slightly first. This induced a normal shock in the convergent divergent nozzle. Shut down was then completed as for subsonic flow.

7. EXPERIMENTAL RESULTS

7.1 Selection of Mean Values

Realistic comparison of one dimensional flow theory and experiment is only possible if suitable mean values are taken which allow for boundary layer growth and flow profile irregularities in the duct. Tyler (ref 30) demonstrated that mass flow is the controlling parameter in one dimensional compressible flow. If mean Mach Number and other parameters are calculated from mass flow and static pressure measurements, the one dimensional assumption is accurate for all parameters except total pressure to within 2% even for fully developed flow. Estimation of total pressure by this means is liable to up to 10% error. Fortunately, change of total pressure, which we are concerned with, is also accurately derived on a mass flow basis to within 2%. This assumes that total temperature and static pressure are constant across any cross section.

7.1.1 Mean Values in Subsonic Flow

a) Total Temperature

Results of subsonic total temperature traverses for $M_1 = 0.5$ are given in fig. 24(b) for $T_{o_1} = 1500^\circ R$. As can be seen, temperature profile is fairly independent of the absolute temperature values and has a variation of approximately $\pm 5\%$ from the mean value. This is a large variation for results to be based on Tyler's assumption. The measurements were taken just past the entry of the 2" square section of the ducting where there are still radial components of acceleration while the water injection plane is $3\frac{3}{8}$ diameters downstream. It is assumed that in this extra distance the profile would even out to more acceptable limits.

b) Mass Flow, Mach Number, and Total Pressure

Total pressure traverses giving Mach Number distribution at the water injection plane and 21" downstream are shown in fig. 25 (a) and (b). These are based on the assumption that measured static pressure is constant across the duct cross section and that the relation between p_o and p is isentropic. Mean Mach Number was

obtained by numerical integration of the curves assuming symmetric flow in the duct and the mass flow was calculated from the mean Mach Number. Trial calculations show this method to give results differing negligibly from those obtained by numerical integration of mass flow per unit area and subsequent derivation of Mach Number. Therefore due to ease of calculation this area based mean is used for all traverses. Results of the traverses plotted as functions of atmospheric pressure are as follows:

	P_a	$\underline{M}$	$\underline{W}_A$
Water Inlet Plane	.948 P_A	0.50	0.188 P_A
21" Downstream	.925 P_A	0.514	0.186 P_A

The two independently derived values of mass flow agree within 1.1%.

Final values on which water/air ratios and non dimensional static pressure variation is based are for $T_{o_1} = 1500^{\circ}R$ and $M_1 = 0.5$ at duct inlet:

$$W_a = 0.187 P_A \quad P_{o_1} = 0.943 P_A$$

c) One Dimensional Friction Coefficient

This was obtained for the dry duct from the mean curve of static pressure variation down the duct given in fig.30 and assuming adiabatic flow. From this we get $f_{FANNO} = 0.0043$

$$\text{Assuming from ref. (2) } \frac{f_{WET}}{f_{DRY}} = 1.13 \quad \text{for } \Omega = 0.2$$

$$\therefore \frac{f_{WET}}{D} = 0.00241 / \text{ins}$$

Deviation of points from the mean curve beyond the entry length in fig.30 is less than $\pm 1\%$.

7.1.2 Mean Values in Supersonic Flow Convergent-Divergent Nozzle

a) Total Temperature

Results of total temperature traverses for $M_1 = 1.86$ are given in fig. 24 (a). In shape and variation from the mean values they are similar to the subsonic traverses in 24 (b). It is assumed that the profile evens out as for subsonic flow.

b) Mass Flow Mach Number and Total Pressure

Figs 26 and 27 give results of total pressure traverses at $T_{o_1} = 1500^\circ\text{R}$ assuming relation between p_{o_1} and p is according to the Rayleigh pitot tube equation. These are summarized below.

	p_{mean}	M	w_a
Water Inlet Plane Vertical	.143 p_a	1.68	.0139 p_a
3 " Downstream Vertical	.166 p_a	1.69	.0162 p_a
3 " Downstream Horizontal	.166 p_a	1.61	.0150 p_a

The large variation in mass flow calculation implies either a wrong assumption or incorrect measurement. The error is in assuming that the mean of the 4 wall statics is the mean static pressure across the duct. Examination of the shape of the profiles shows a rather complex flow pattern. Fig. 26 (a) shows a pattern of flow similar to that anticipated and on which water/air ratios were originally erroneously calculated. The two minima correspond to the wake of the water tubes. Fig. 26 (b) shows that downstream in the same plane the area change after the termination of the water tubes has produced a Prandtl Meyer expansion while fig. 27 shows that at the same distance downstream but in the horizontal plane there is evidence of a two dimensional oblique shock pattern. The static pressure is therefore varying across the cross section and the measured value depends on the position of the shock wave. Since this is indeterminate with the given instrumentation the following procedure was adopted for obtaining mass flow. From fig. 29 where a shock wave was introduced in the diverging portion of the annular nozzle and the traverse performed in the subsonic region, assuming choking at the throat we get $p_{o_1} = 0.97 p_a$. This

value is assumed identical for the two dimensional nozzle since the throat area only differs by the small area of the water tubes. Since an oblique shock pattern is regular in form a periodicity may be detected in the static pressure variation in the dry duct given in fig. 31. The mean line in this figure upstream of the shock is obtained from a least squares plot and the value of p at which it cuts the vertical axis is taken as the mean static pressure at the test section entry. These assumptions give the final values at test section entry on which interpretation of wet runs was based as:

T_{01}	P_{01}	M_1	w_a
1500° R	0.97 p_A	1.87	0.0165 p_A

The high value of Mach Number is attributed to the change in area ratio produced by insertion of the water tubes. The accuracy of these assumptions is of the order of $\pm 3\%$ due mainly to the large scatter of points in fig. 31.

c) One Dimensional Friction Coefficient

This is based on the slope of the mean line upstream of the normal shock in fig 31 and assuming adiabatic flow. Then

$$f_{FANNO} = 0.0042$$

Assuming unfavourably that $\frac{f_{WET}}{f_{DRY}}$ is identical with subsonic experiments, for $\Omega = 0.2$ we get $\frac{f_{WET}}{D} = 0.0024/\text{ins.}$

7.1.3. Mean Values in Supersonic Flow Annular Nozzle

a) Total Temperature

This profile is assumed to be identical with that of the Convergent-Divergent two dimensional nozzle.

b) Mass Flow, Mach Number, and Total Pressure

Results of traverses $3\frac{3}{4}$ " downstream given in figs 28 and 29 are :

	M	P	w_a
Supersonic Vertical	1.55	.196 p_A	.0169 p_A
Supersonic Horizontal	1.54	.195 p_A	.0167 p_A
Subsonic Vertical	0.49	.743 p_A	.0170 p_A
Subsonic Horizontal	0.48	.744 p_A	.0168 p_A

Assuming $w_a = .0168 p_A$ and choking at the throat $p_{0,} = 0.97 p_A$ neglecting friction loss from nozzle throat to nozzle exit. P_1 is obtained from a least squares plot of static pressure variation down the dry duct (Fig. 48). This gives final result as:

$T_{0,}$	M_1	$p_{0,}$	w_a
1500°R	1.86	0.97 p_A	.0168 p_A

The accuracy of these assumptions is of the same order as for the two dimensional convergent-divergent nozzle i.e. $\pm 3\%$.

7.2 Results of Subsonic Aerothermopressor Tests

These are given in figs 32 - 40 and Plates 3 - 8.

7.3 Results of Supersonic Aerothermopressor Tests

These are given in figs 41 - 49 and Plates 9 - 14.

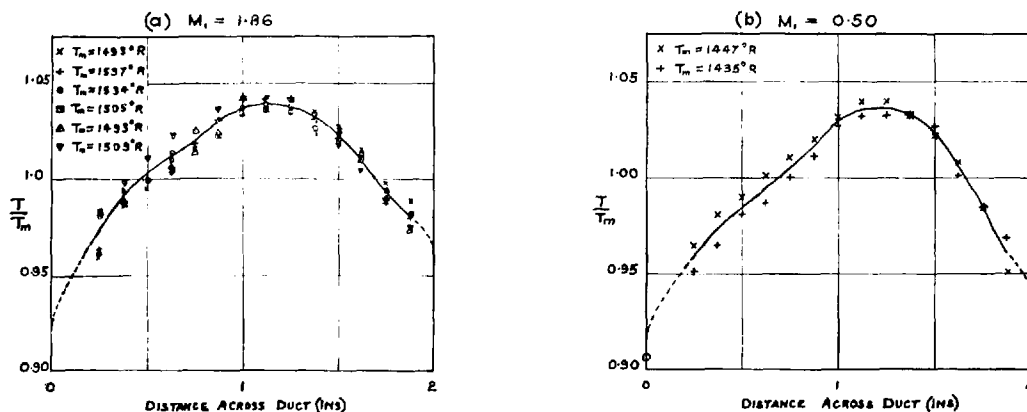
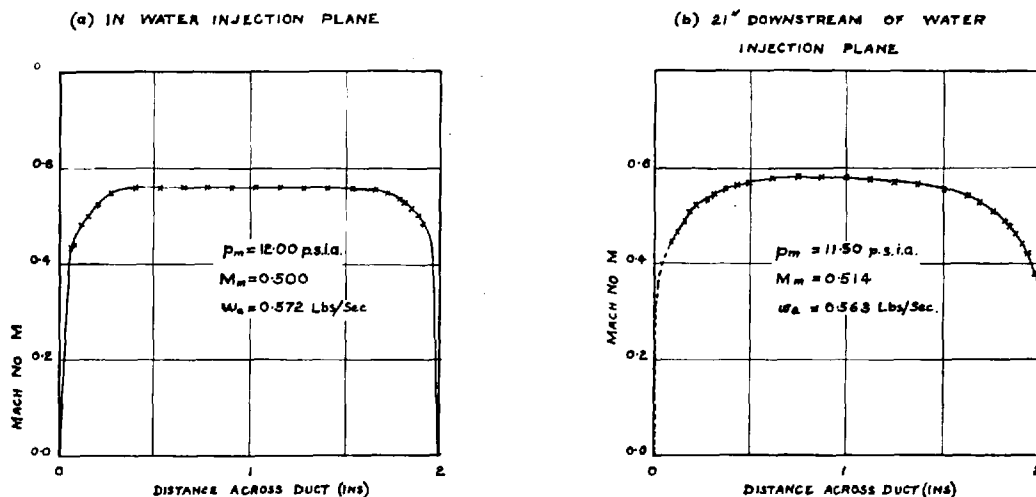
FIG.24. TOTAL TEMPERATURE TRAVERSES OF DUCT $6\frac{3}{4}$ " UPSTREAM OF WATER INJECTION PLANE $T_0 \approx 1500^\circ\text{R}$ FIG.25. SUBSONIC TOTAL PRESSURE TRAVERSES OF DUCT $T_0 = 1500^\circ\text{R}$, $M_1 = 0.5$, $\Omega = 0.00$ 

FIG.26. SUPERSONIC TOTAL PRESSURE TRAVERSES IN VERTICAL PLANE OF DUCT

$$T_0 = 1500^\circ\text{R}, \quad M \approx 1.86, \quad \Omega = 0.00$$

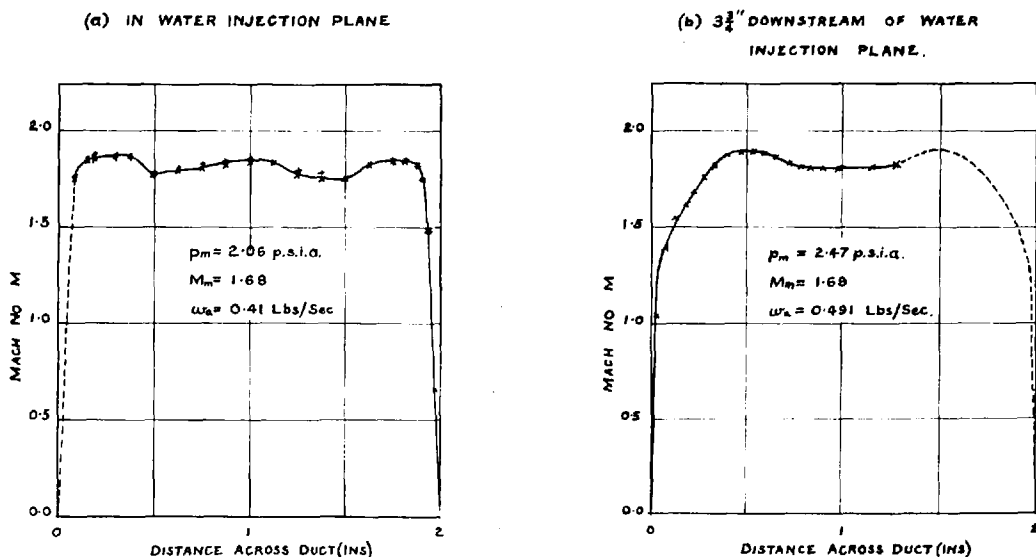


FIG. 27. SUPERSONIC TOTAL PRESSURE TRAVERSES IN HORIZONTAL PLANE OF DUCT

$$T_0 = 1500^\circ\text{R} \quad M_1 = 1.86 \quad \Omega = 0.00$$

3 3/4" DOWNSTREAM OF WATER
INJECTION PLANE

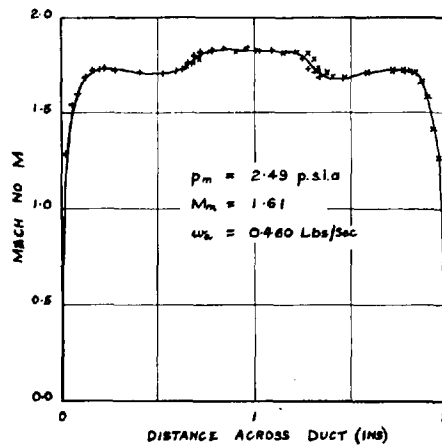
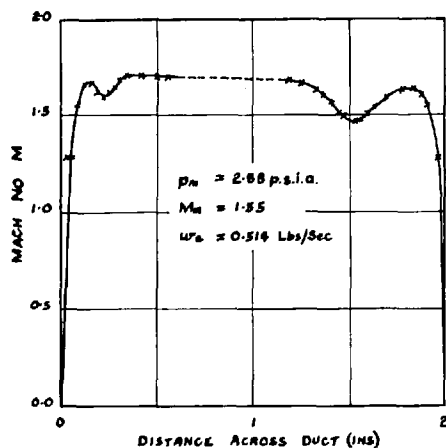


FIG. 28. SUPERSONIC TOTAL PRESSURE TRAVERSES ANNULAR INJECTION NOZZLE FITTED

TRAVERSING POSITION 3 3/4" DOWNSTREAM OF WATER INJECTION PLANE $T_0 = 1500^\circ\text{R}$ $M_1 = 1.86$ $\Omega = 0.00$

(a) HORIZONTAL TRAVERSE



(b) VERTICAL TRAVERSE

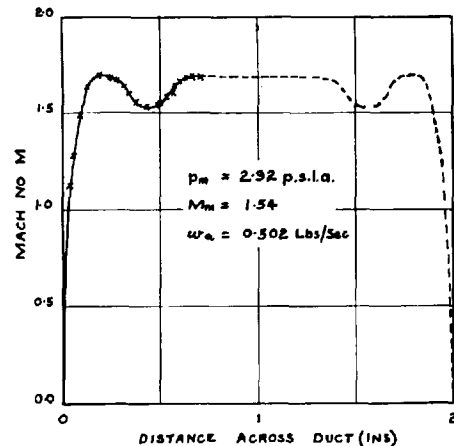
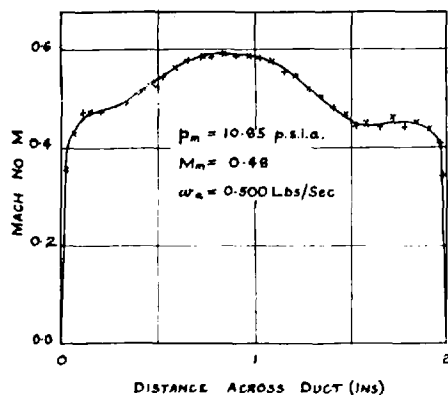


FIG. 29. SUBSONIC TOTAL PRESSURE TRAVERSES ANNULAR INJECTION NOZZLE FITTED

TRAVERSING POSITION 3 3/4" DOWNSTREAM OF WATER INJECTION PLANE $T_0 = 1500^\circ\text{R}$ $M_1 = 1.86$ $\Omega = 0.00$

(a) HORIZONTAL TRAVERSE



(b) VERTICAL TRAVERSE

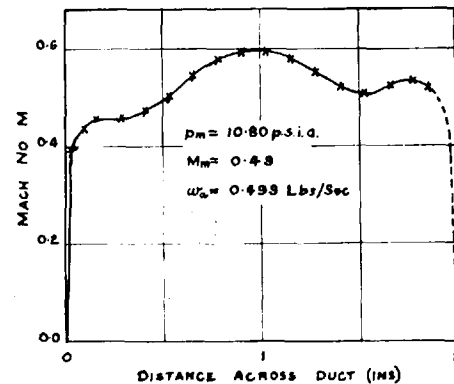


FIG. 30. DETERMINATION OF DUCT FRICTION COEFFICIENT IN

SUBSONIC FLOW

$$T_0 = 1500^\circ\text{R} \quad M_1 = 0.50 \quad p_{01} = 14.15 \text{ p.s.i.a.} \quad \Omega = 0.00$$

$$f_{\text{FANNO}} = 0.0043$$

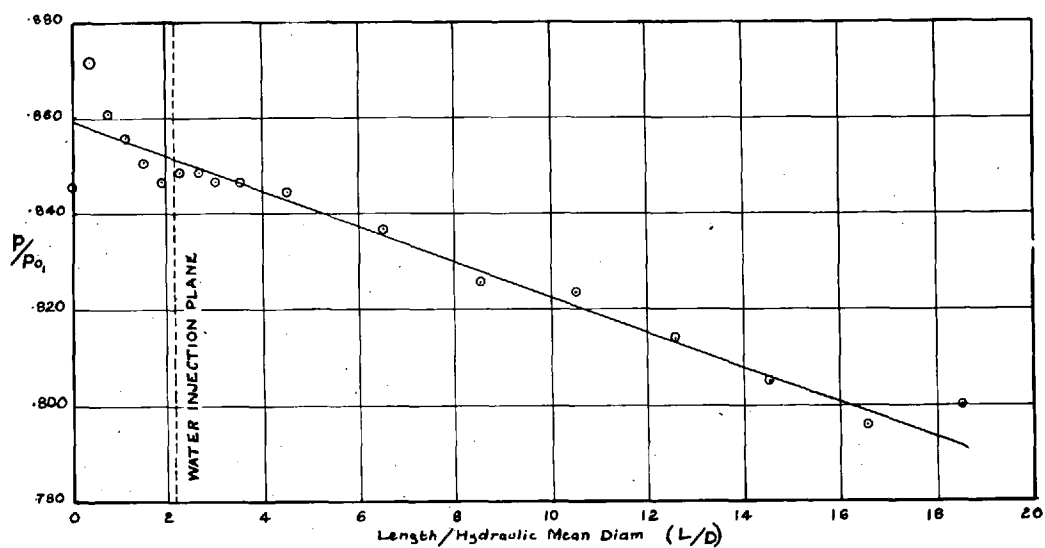


FIG. 31. DETERMINATION OF DUCT FRICTION COEFFICIENT IN

SUPERSONIC FLOW

$$T_0 = 1500^\circ\text{R} \quad M_1 = 1.86 \quad p_{01} = 13.60 \text{ p.s.i.a.} \quad \Omega = 0.00$$

$$f_{\text{FANNO}} = 0.0042$$

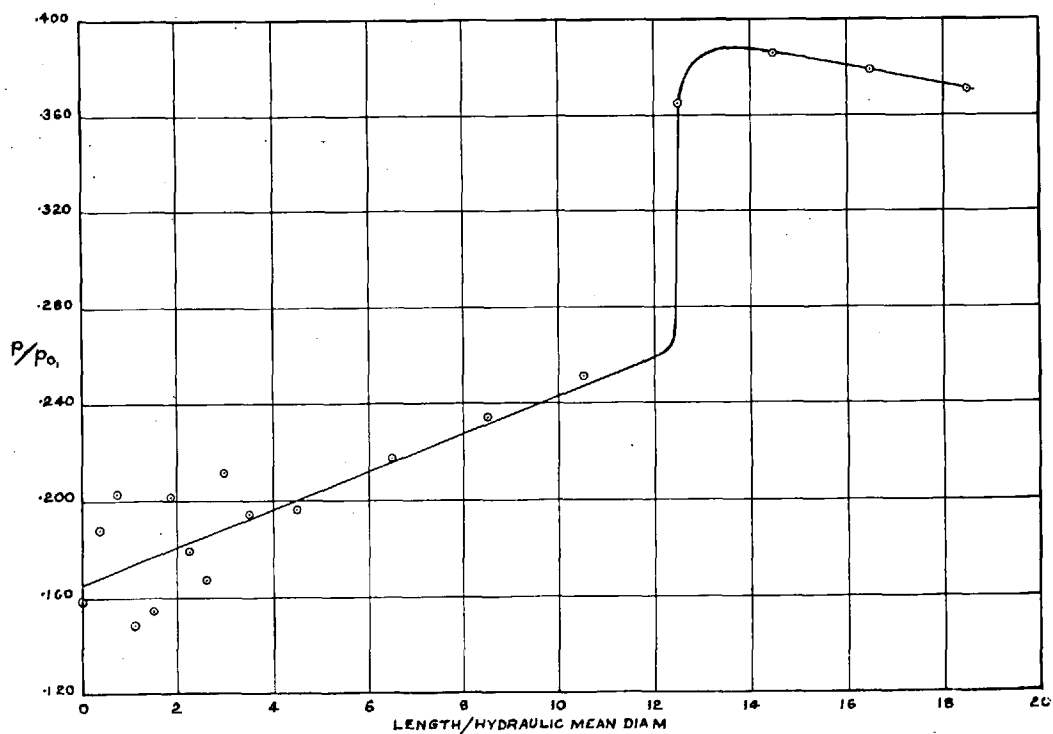


FIG. 32. $T_0 = 1500^\circ\text{R}$ $M_1 = 0.50$ $\Omega = 0.25$ $T_e = 530^\circ\text{R}$ $V_e/V_1 = 0.0$

SMITH: $p_0 = 14.15$ p.s.i.a. $f/D = 0.0241/\text{ins}$

SHAPIRO: $p_0 = 14.70$ p.s.i.a. $f/D = 0.0235/\text{ins}$

MODE OF INJECTION:

SMITH: 4 OPEN TUBES 0.125" O/D 0.096" I/D

SHAPIRO: 7 OPEN TUBES 0.125" O/D 0.110" I/D

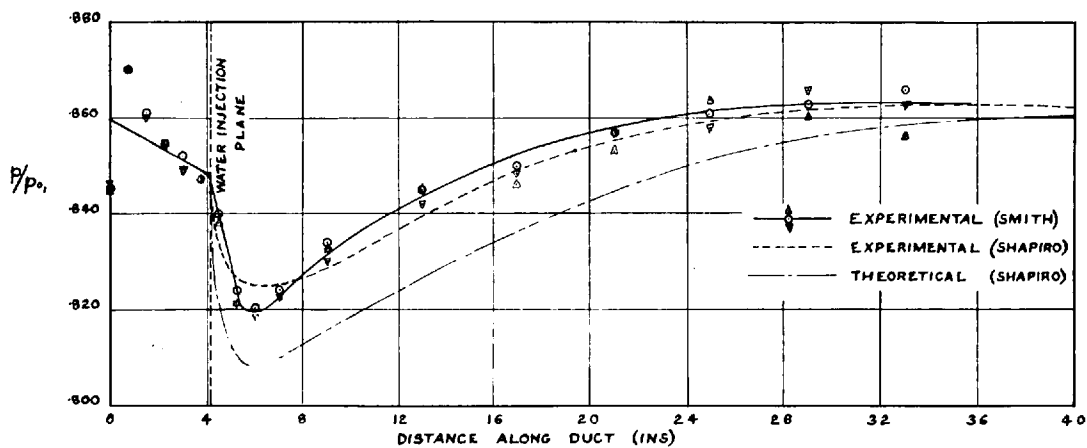


FIG. 33. $T_0 = 1500^\circ\text{R}$ $M_1 = 0.50$ $p_0 = 14.15$ p.s.i.a. $\Omega = 0.20$ $T_e = 530^\circ\text{R}$ $V_e/V_1 = 0.0$ $f/D = 0.00237/\text{ins}$

MODE OF INJECTION: 4 OPEN TUBES 0.125" O/D 0.096" I/D

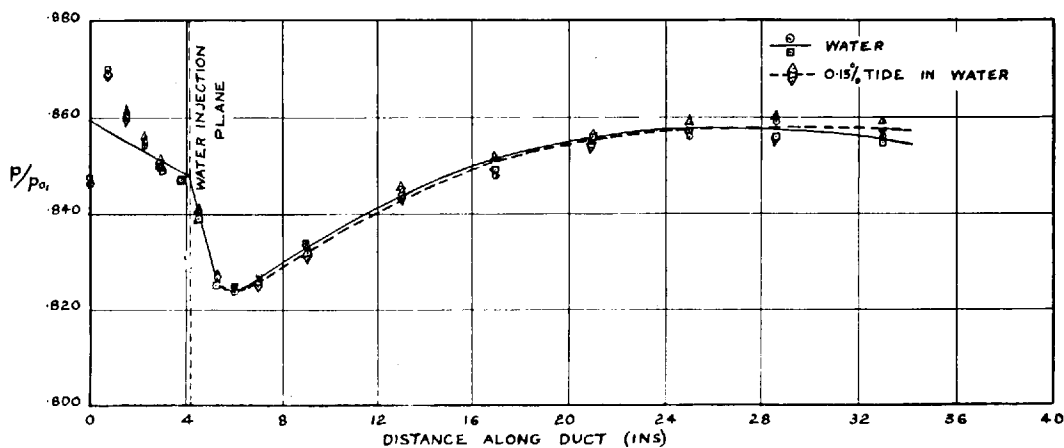


FIG. 34. $T_0=1500^\circ\text{R}$ $M_1=0.50$ $p_0=14.15\text{ ps.i.a.}$ $\Omega=0.20$ $T_c=530^\circ\text{R}$ $V_1/V_2=0.046$ $f/D=0.00237/\text{ins}$

MODE OF INJECTION: 4 NOZZLES $0.125''\text{q/d}$ $0.055''/\text{d}$ AT 12 ps.i.a.

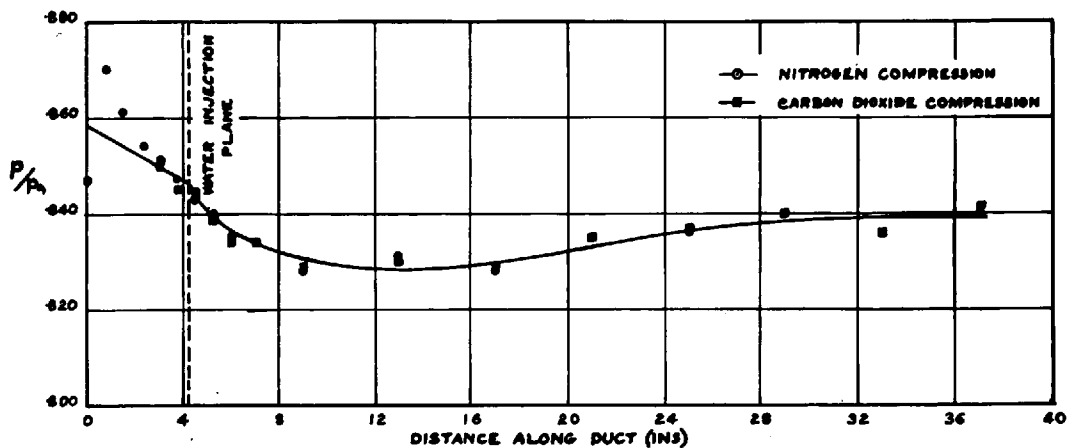


FIG. 35. $T_0=1500^\circ\text{R}$ $M_1=0.50$ $p_0=14.15\text{ ps.i.a.}$ $\Omega=0.25$ $T_c=530^\circ\text{R}$ $V_1/V_2=0.052$ $f/D=0.00241/\text{ins}$

MODE OF INJECTION: 4 NOZZLES $0.125''\text{q/d}$ $0.055''/\text{d}$ AT 15 ps.i.a.

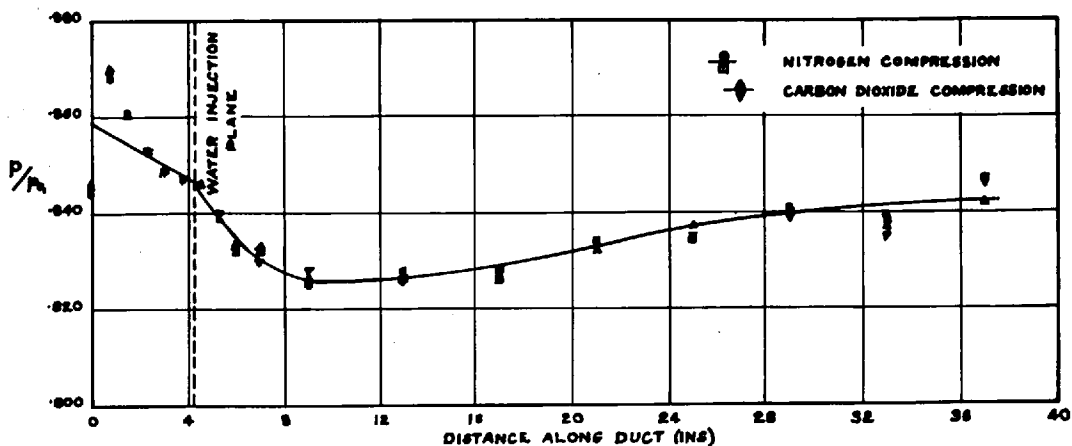


FIG. 36. $T_0=1500^\circ\text{R}$ $M_1=0.50$ $p_0=14.10$ p.s.i.a. $\Omega=0.20$ $T_1=530^\circ\text{R}$ $V_1/V_2=0.094$ $f/D=0.00241/\text{ins}$

MODE OF INJECTION: 4 NOZZLES $0.125''\text{O/D}$ $0.036''/\text{D}$ AT 50 p.s.i.

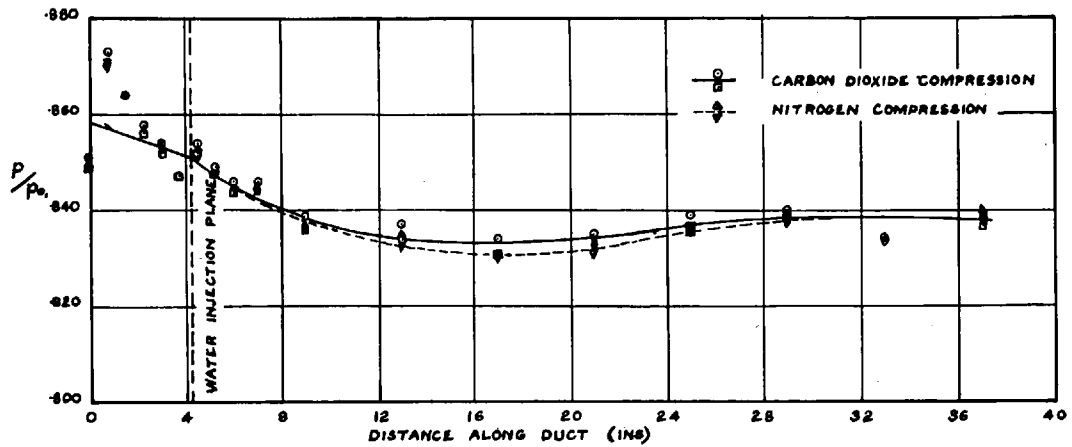


FIG. 37. $T_0=1500^\circ\text{R}$ $M_1=0.50$ $p_0=14.15$ p.s.i.a. $\Omega=0.20$ $T_1=530^\circ\text{R}$ $V_1/V_2=0.134$ $f/D=0.00241/\text{ins}$

MODE OF INJECTION: 4 NOZZLES $0.125''\text{O/D}$ $0.031''/\text{D}$ AT 100 p.s.i.

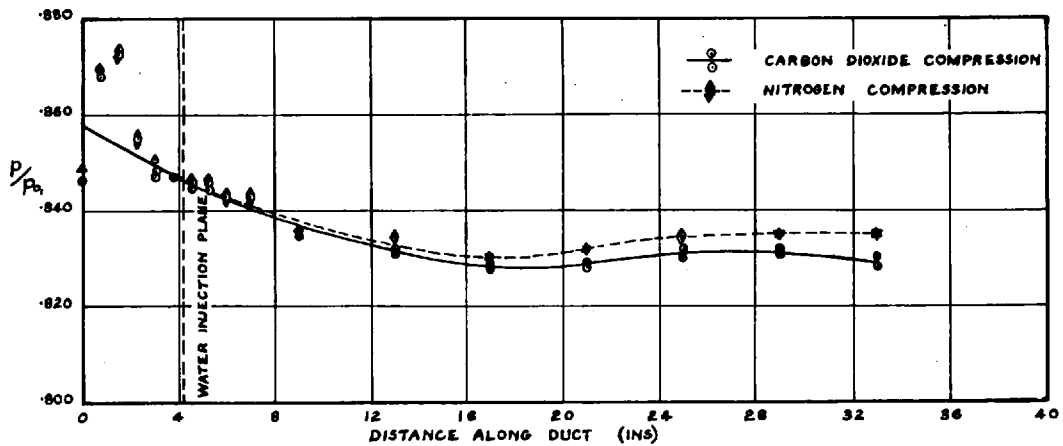


FIG. 38. $T_0=1500^\circ\text{R}$ $M_1=0.50$ $p_0=14.10$ p.s.i.a. $\Omega=0.20$ $T_1=530^\circ\text{R}$ $V_1/V_2=0.189$ $f/D=0.00241/\text{ins}$

MODE OF INJECTION: 4 NOZZLES $0.125''\text{O/D}$ $0.025''/\text{D}$ AT 200 p.s.i.

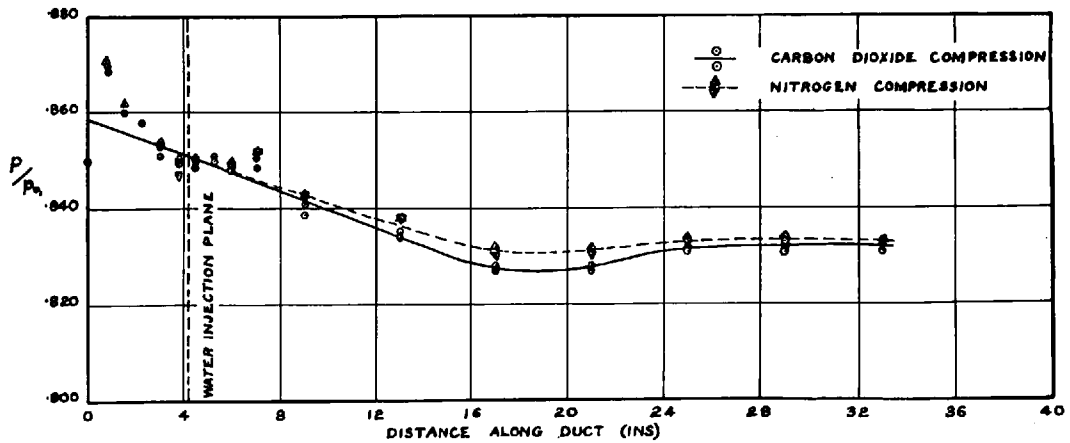


FIG. 39. $T_0=1500^{\circ}\text{R}$ $M_1=0.50$ $p_0=14.2\text{ p.s.i.}$ $\Omega=0.20$ $T_1=530^{\circ}\text{R}$ $V_0/V_1=0.101$ $f/D=0.00237/\text{ins}$

MODE OF INJECTION: 4 FAN SPRAY NOZZLES 0.125" O/D AT 75 p.s.i.

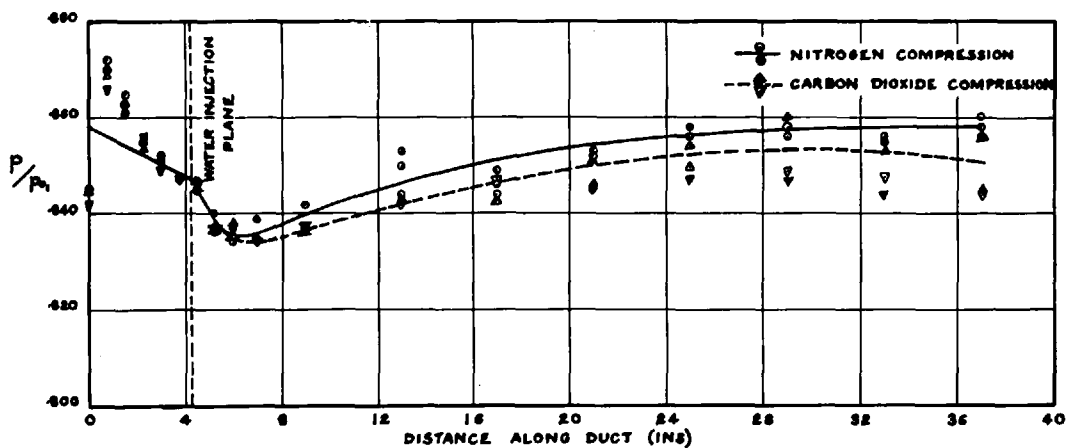


FIG. 40. $T_0=1500^{\circ}\text{R}$ $M_1=0.50$ $p_0=14.20\text{ p.s.i.}$ $\Omega=0.25$ $T_1=530^{\circ}\text{R}$ $V_0/V_1=0.116$ $f/D=0.00241/\text{ins}$

MODE OF INJECTION: 4 FAN SPRAY NOZZLES 0.125" O/D AT 118 p.s.i.

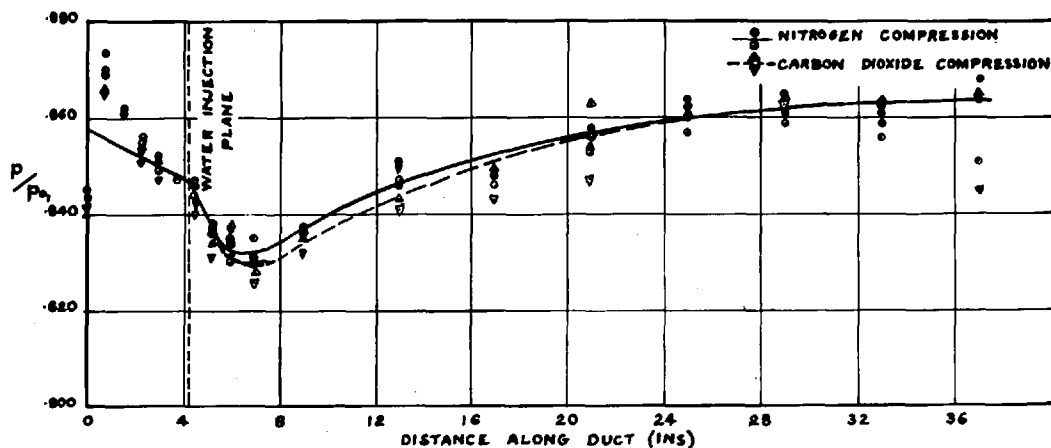


FIG. 41. $T_0 = 1500^\circ\text{R}$ $M_1 = 1.87$ $p_{01} = 14.25 \text{ p.s.i.a.}$ $\Omega = 0.205$ $T_t = 530^\circ\text{R}$ $V_t/V_1 = 0.002$ $f/D = 0.0024/\text{ins}$

MODE OF INJECTION: 4 OPEN TUBES $0.125''\text{O/D}$ $0.096''\text{I/D}$

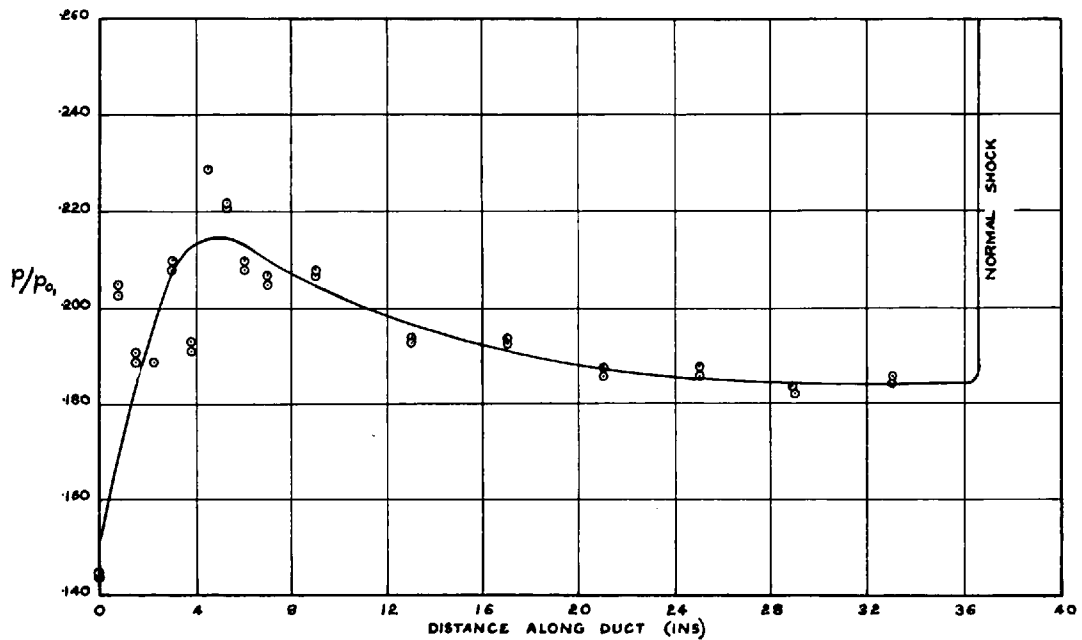


FIG. 42. $T_0 = 1500^\circ\text{R}$ $M_1 = 1.87$ $p_{01} = 14.3 \text{ p.s.i.a.}$ $\Omega = .085, .125, .165, .205, .250$ $T_t = 530^\circ\text{R}$ $V_t/V_1 = .002$ $f/D = .0024/\text{ins}$

MODE OF INJECTION: 4 OPEN TUBES $0.125''\text{O/D}$ $0.096''\text{I/D}$

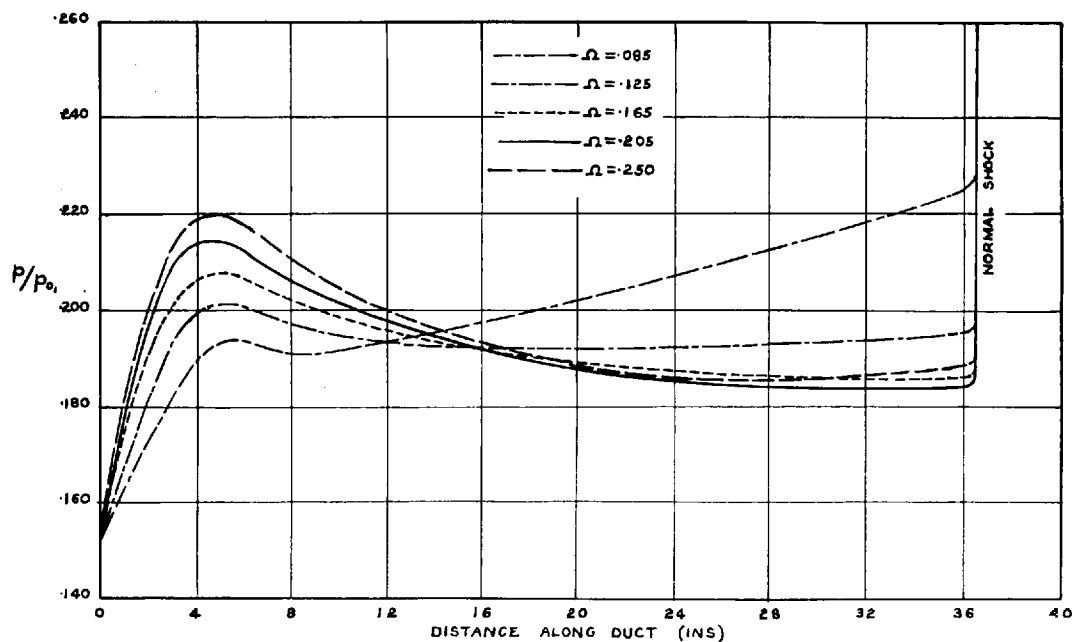


FIG. 43.

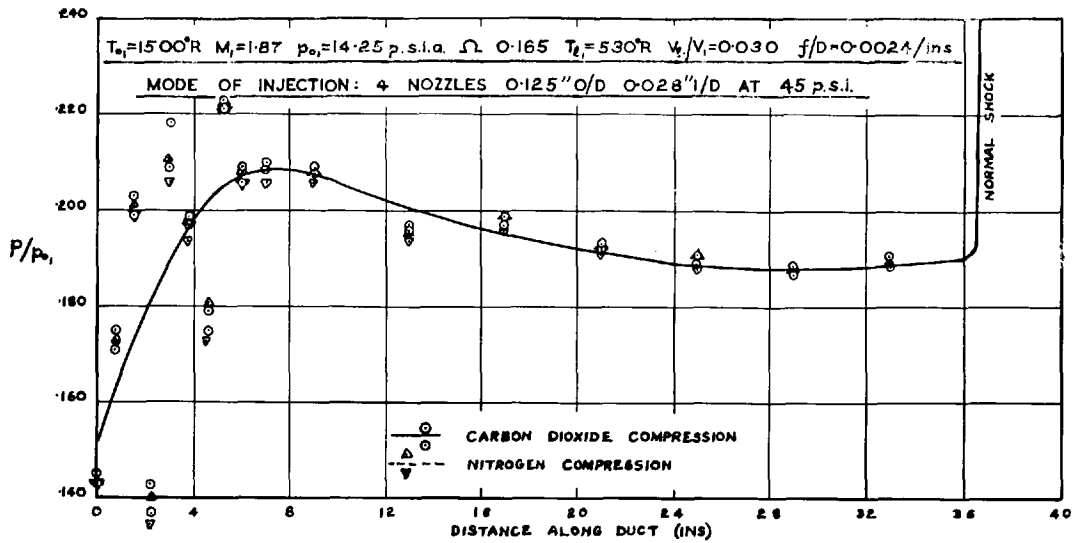


FIG. 44.

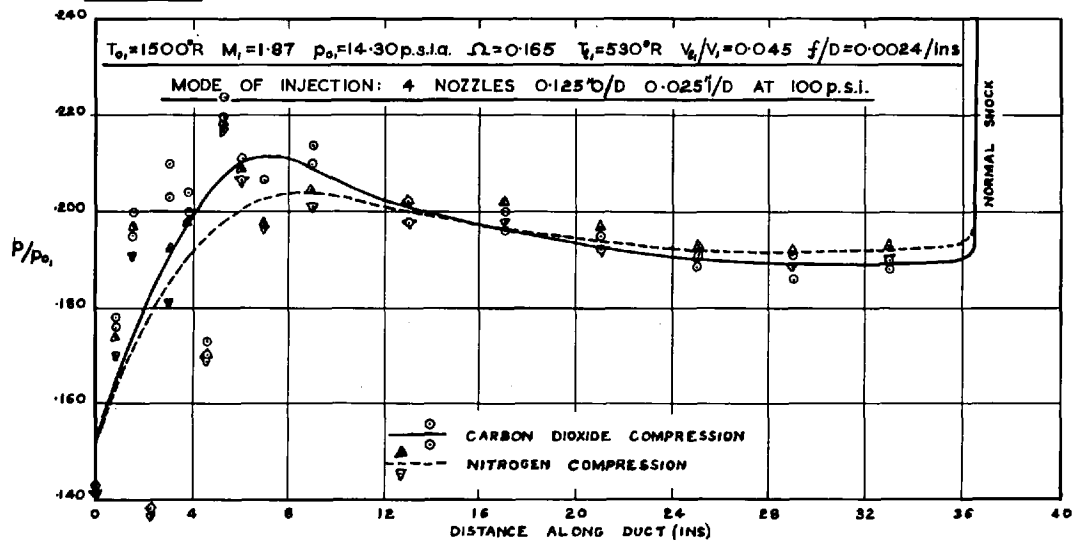


FIG. 45.

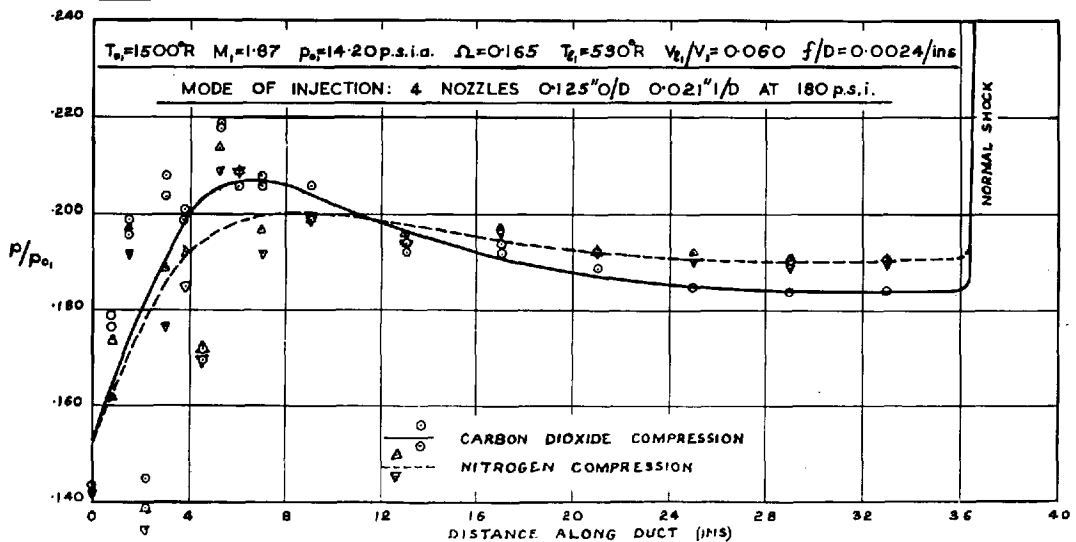


FIG. 46. $T_{01}=1500^{\circ}\text{R}$ $M_1=1.87$ $p_{01}=14.25\text{ p.s.i.}$ $\Omega=0.165$ $T_{02}=530^{\circ}\text{R}$ $V_0/V_1=0.015$ $f/D=0.0024/\text{ins}$

MODE OF INJECTION: 4 FAN SPRAY NOZZLES 0.125" O/D AT 20 p.s.i.

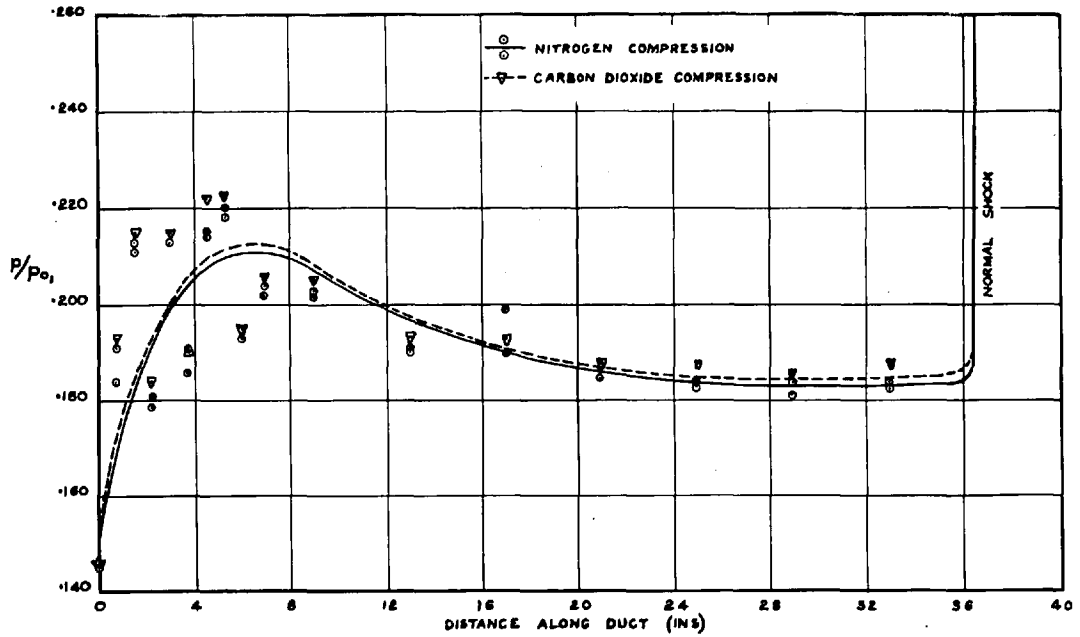


FIG. 47. $T_{01}=1500^{\circ}\text{R}$ $M_1=1.87$ $p_{01}=14.25\text{ p.s.i.}$ $\Omega=0.205$ $T_{02}=530^{\circ}\text{R}$ $V_0/V_1=0.021$ $f/D=0.0024/\text{ins}$

MODE OF INJECTION: 4 FAN SPRAY NOZZLES 0.125" O/D AT 41 p.s.i.

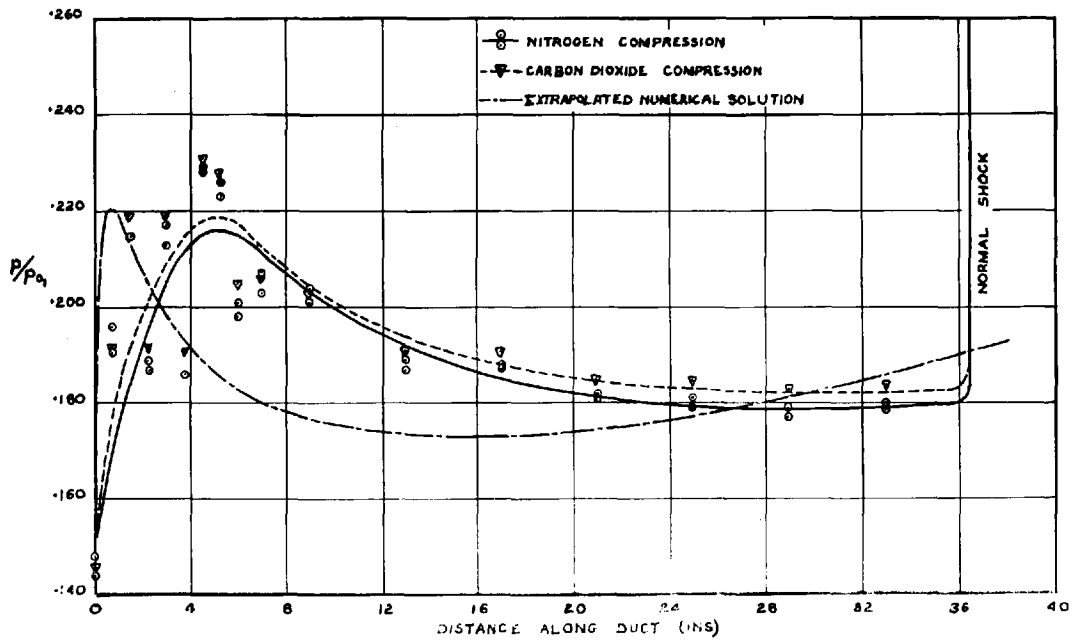


FIG. 48. $T_0=1500^\circ\text{R}$ $M_1=1.86$ $p_{01}=14.20\text{ p.s.i.a.}$ $\Omega=0.00$ $T_t=530^\circ\text{R}$ $V_t/V=0.000$ $f/D=0.0024/\text{ins}$

MODE OF INJECTION: ANNULAR NOZZLE.

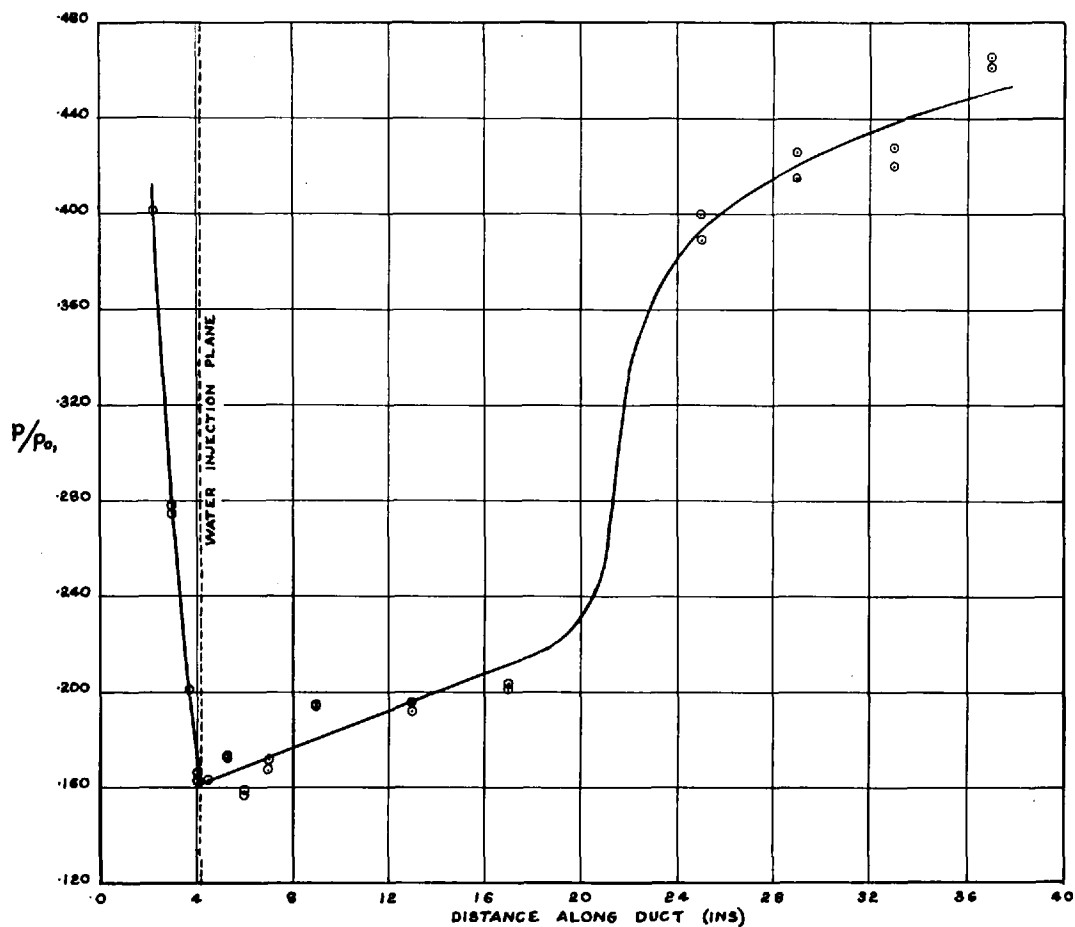
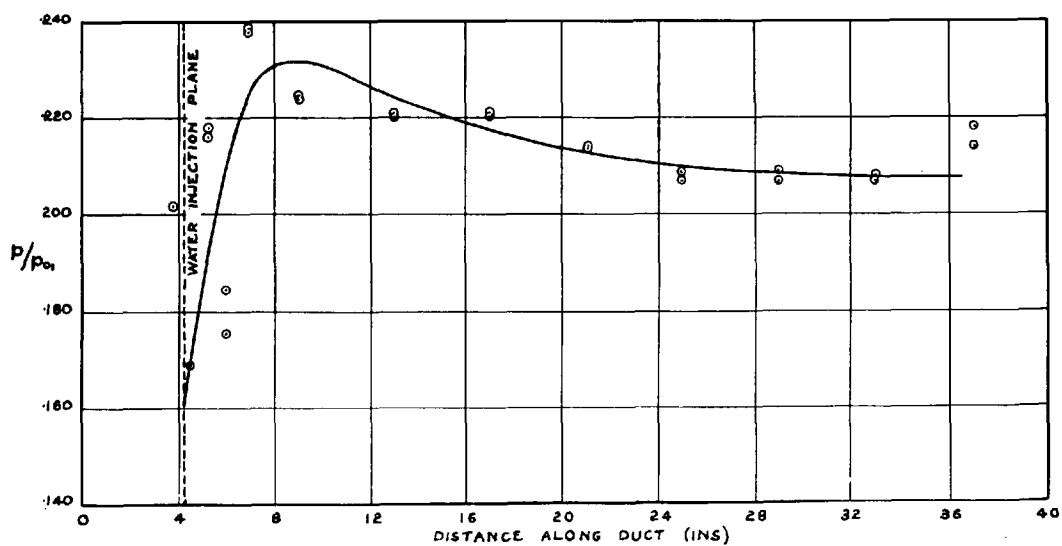


FIG. 49. $T_0=1500^\circ\text{R}$ $M_1=1.86$ $p_{01}=14.25\text{ p.s.i.a.}$ $\Omega=0.205$ $T_t=530^\circ\text{R}$ $V_t/V_1=0.001$ $f/D=0.0024/\text{ins}$

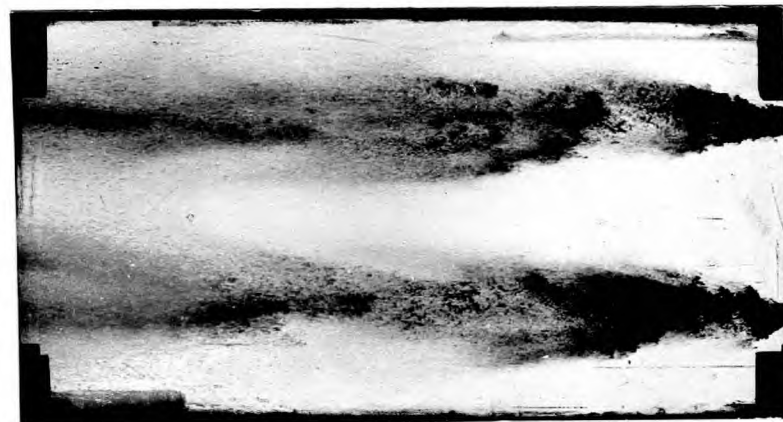
MODE OF INJECTION: ANNULAR NOZZLE.





(a)

$M_1 = 0.0$



(b)

$M_1 = 0.5$

$w_e = 0.146 \text{ Lbs Sec } T_{01} = 530^\circ \text{R}$



(c)

$w_e = 0.116 \text{ Lbs/Sec } (\Omega = 0.2)$

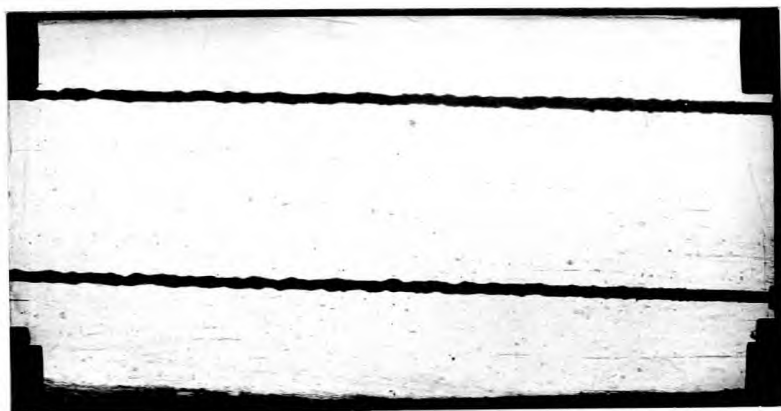


(d)

$w_e = 0.146 \text{ Lbs/Sec } (\Omega = 0.25)$

$M_1 = 0.5 \quad T_{01} = 1500^\circ \text{R}$

PLATE.3. OPEN TUBES SUBSONIC FLOW



(a)

$M_1 = 0.0$



(b)

$M_1 = 0.5$

$15 \text{ p.s.i. } \omega_e = 0.146 \text{ Lbs/Sec } T_{01} = 530^\circ \text{R}$



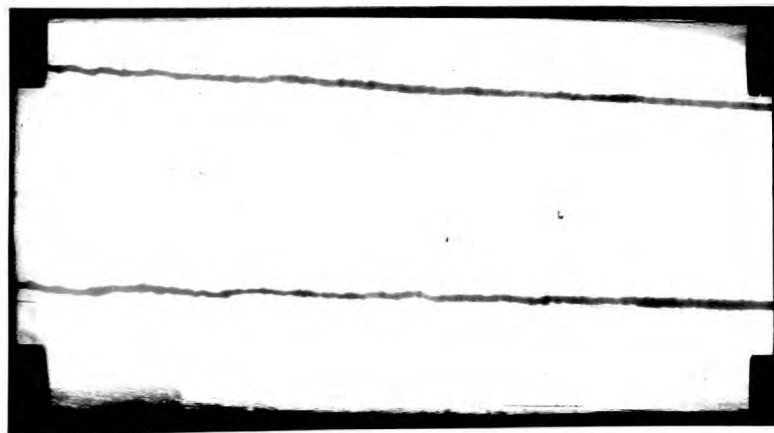
(c) $\omega_e = 0.116 \text{ Lbs/Sec } (\Omega = 0.2) \text{ 12 p.s.i.}$



(d) $\omega_e = 0.146 \text{ Lbs/Sec } (\Omega = 0.25) \text{ 15 p.s.i.}$

$M = 0.5 \quad T_{01} = 1500^\circ \text{R}$

PLATE.4. LOW PRESSURE NOZZLES SUBSONIC FLOW



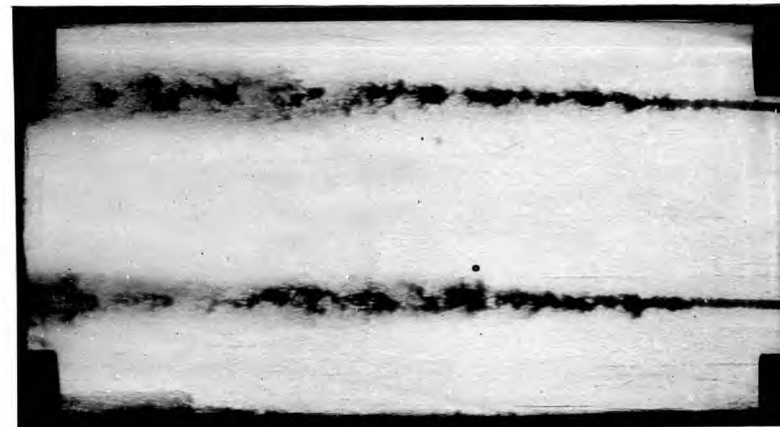
(a) $M_1 = 0.0$ $w_2 = 0.116 \text{ Lbs/Sec}$ $T_{01} = 530^\circ\text{R}$

CO₂ Compression



(b)

N₂ Compression



(c)

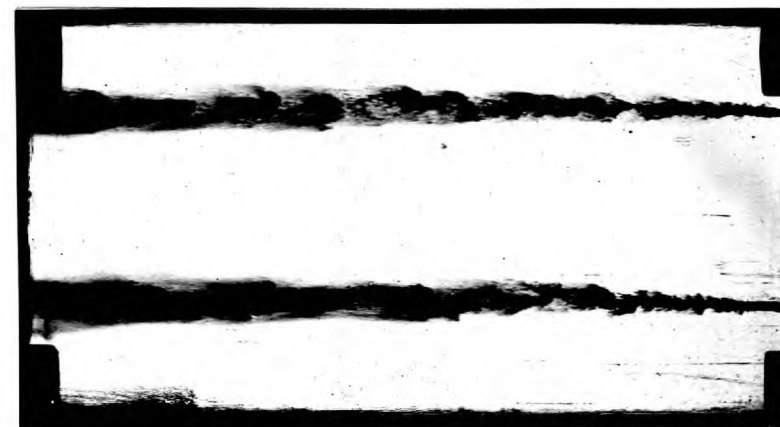
CO₂ Compression

$M_1 = 0.5$ $w_2 = 0.116 \text{ Lbs/Sec}$ $T_{01} = 530^\circ\text{R}$



(d)

N₂ Compression



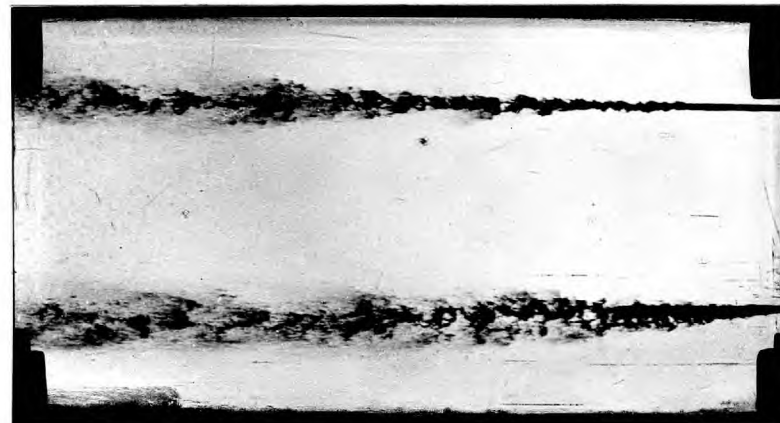
(e)

CO₂ Compression

$M_1 = 0.5$ $w_2 = 0.116 \text{ Lbs/Sec}$ ($\Omega = 0.2$) $T_{01} = 1500^\circ\text{R}$



(a) N₂ Compression

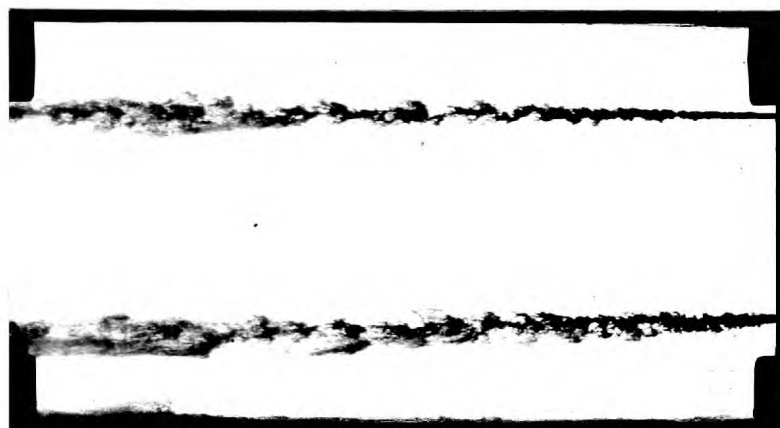


(b) CO₂ Compression

M₁ = 0.5 w_e = 0.116 Lbs/Sec T₀₁ = 530°R



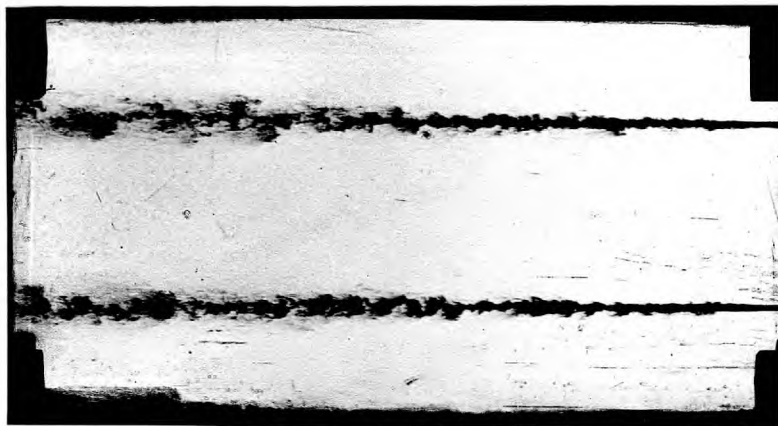
(c) N₂ Compression



(d) CO₂ Compression

M₁ = 0.5 w_e = 0.116 Lbs/Sec (Ω = 0.2) T₀₁ = 1500°R

PLATE .6. 100 P.S.I. NOZZLES SUBSONIC FLOW

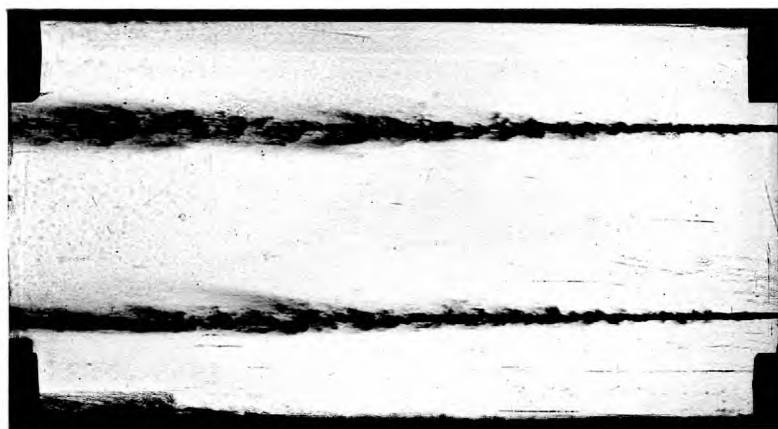


(a) N₂ Compression



(b) CO₂ Compression

M₁ = 0.5 w_g = 0.116 Lbs/Sec T₀₁ = 530°R



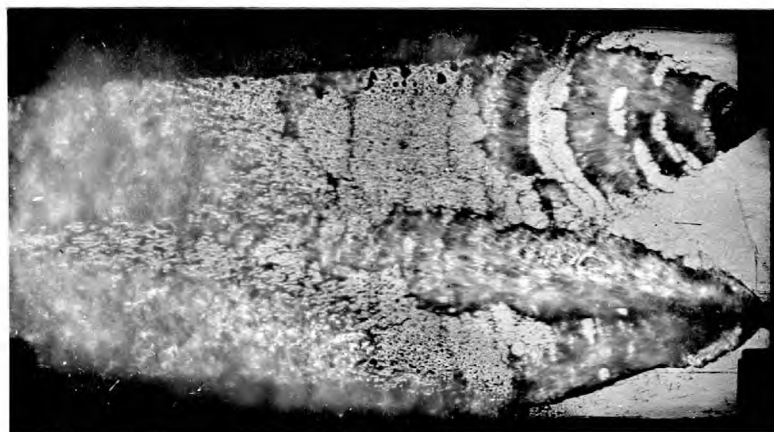
(c) N₂ Compression



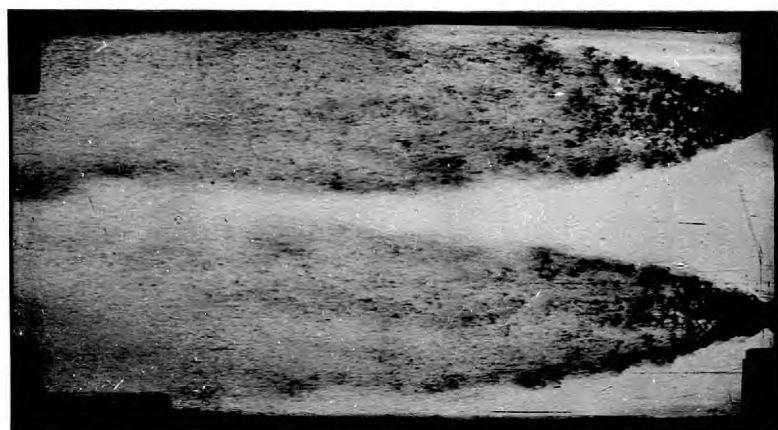
(d) CO₂ Compression

M₁ = 0.5 w_g = 0.116 Lbs/Sec (Ω = 0.2) T₀₁ = 1500°R

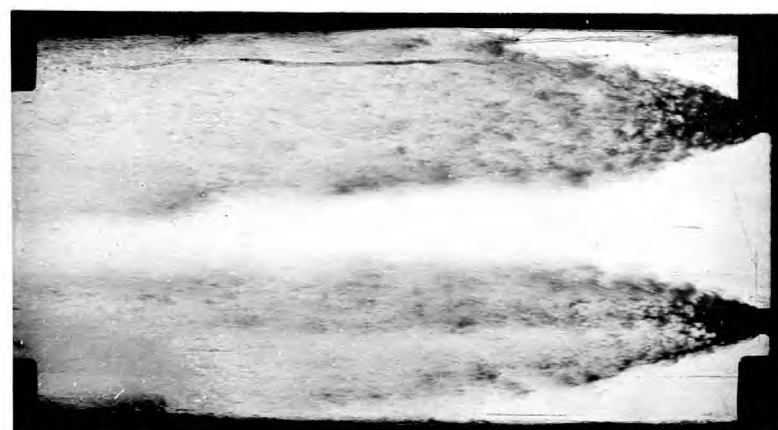
PLATE 7. 200 P.S.I. NOZZLES SUBSONIC FLOW



(a) $M_1 = 0.0$ $\dot{w}_2 = 0.146 \text{ Lbs/Sec}$ $T_{01} = 530^\circ\text{R}$

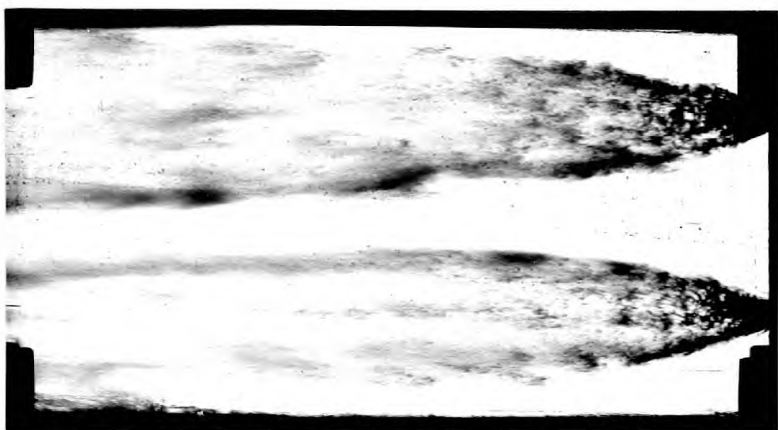


(b) N_2 Compression

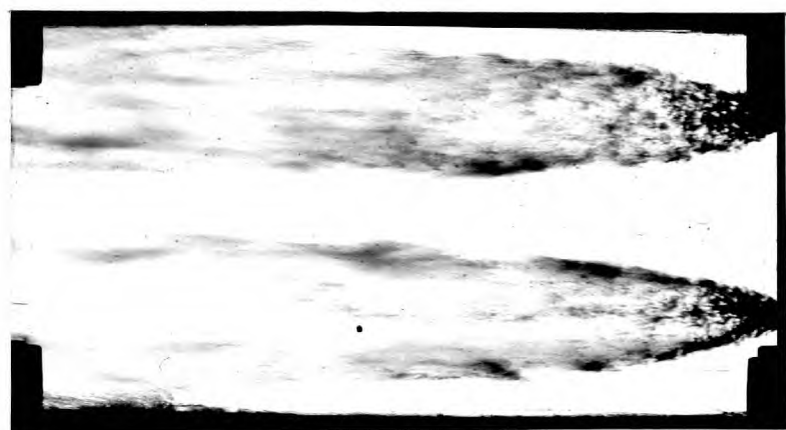


(c) CO_2 Compression

$M_1 = 0.5$ $\dot{w}_2 = 0.146 \text{ Lbs/Sec}$ $T_{01} = 530^\circ\text{R}$



(d) N_2 Compression



(e) CO_2 Compression

$M_1 = 0.5$ $\dot{w}_2 = 0.146 \text{ Lbs/Sec}$ ($\Omega = 0.25$) $T_{01} = 1500^\circ\text{R}$



(a)

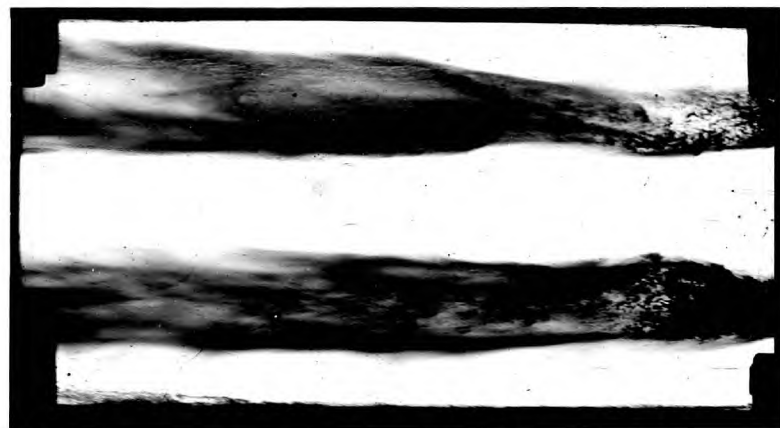
$M_1 = 0.0$



(b)

$M_1 = 1.87$

$w_e = 0.102 \text{ Lbs/Sec}$ $T_1 = 530^\circ\text{R}$



(c)

$w_e = 0.082 \text{ Lbs/Sec}$ ($\Omega = 0.165$)

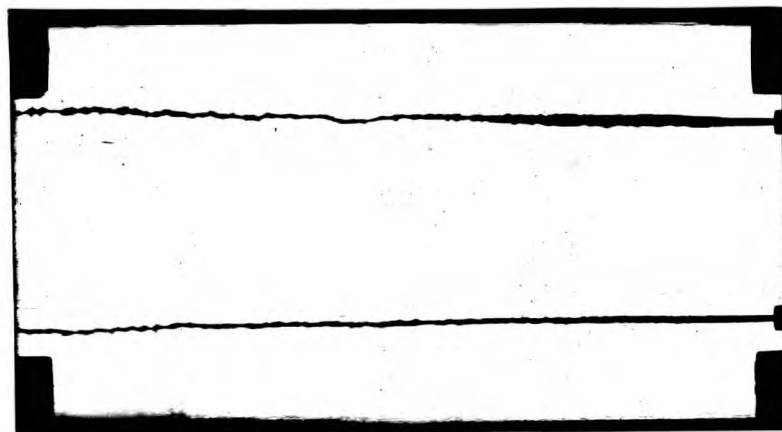


(d)

$w_e = 0.102 \text{ Lbs/Sec}$ ($\Omega = 0.205$)

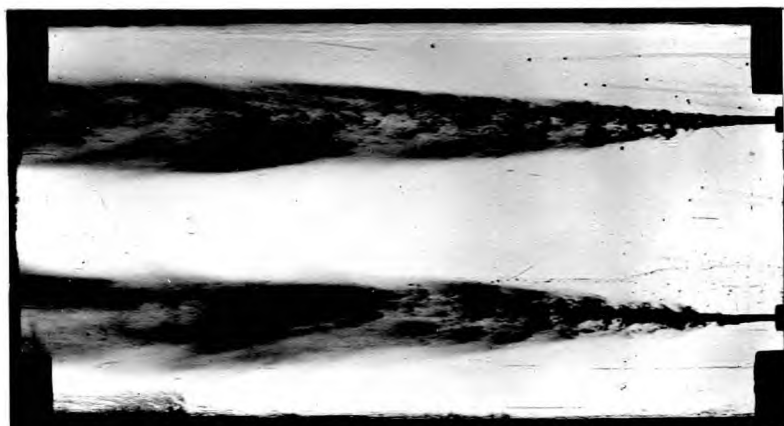
$M_1 = 1.87$ $T_{0,1} = 1500^\circ\text{R}$

PLATE .9. OPEN TUBES SUPERSONIC FLOW



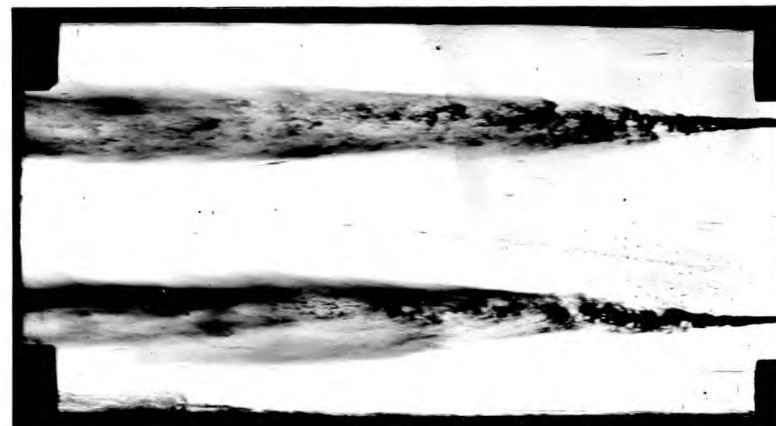
(a) $M_1 = 0.0$ $w_0 = 0.082$ Lbs/Sec $T_{01} = 530^\circ R$

CO₂ Compression



(b)

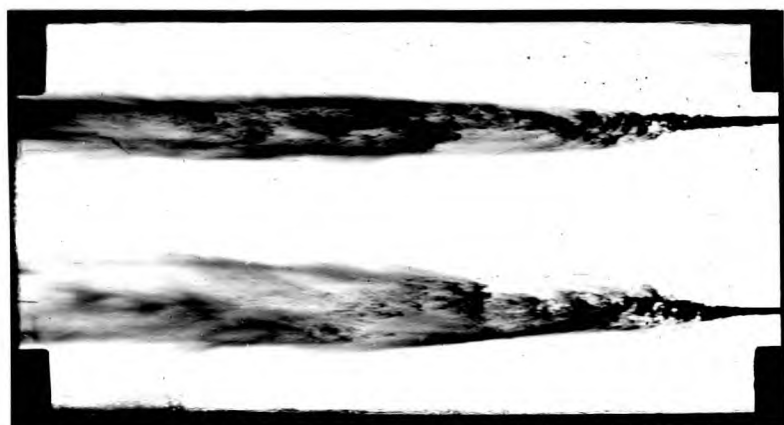
N₂ Compression



(c)

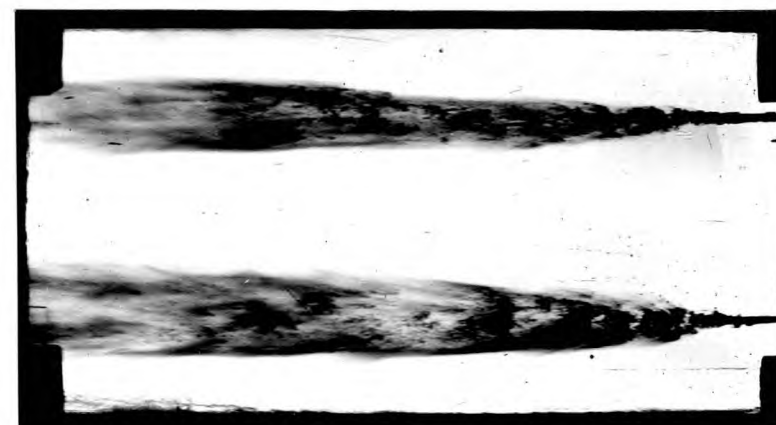
CO₂ Compression

$M_1 = 1.87$ $w_0 = 0.082$ Lbs/Sec $T_1 = 530^\circ R$



(d)

N₂ Compression

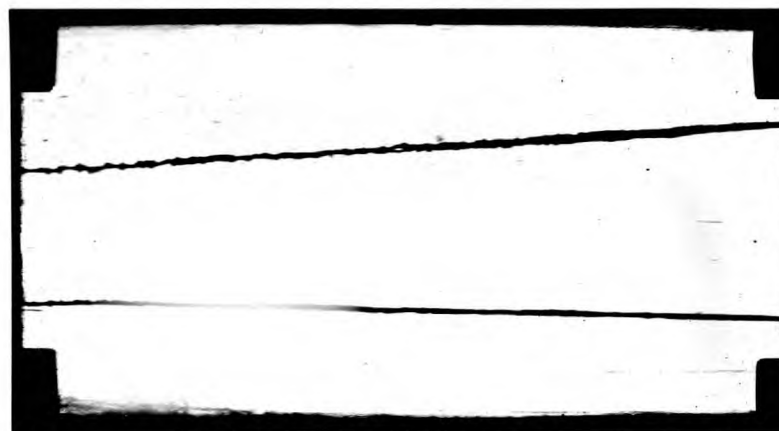


(e)

CO₂ Compression

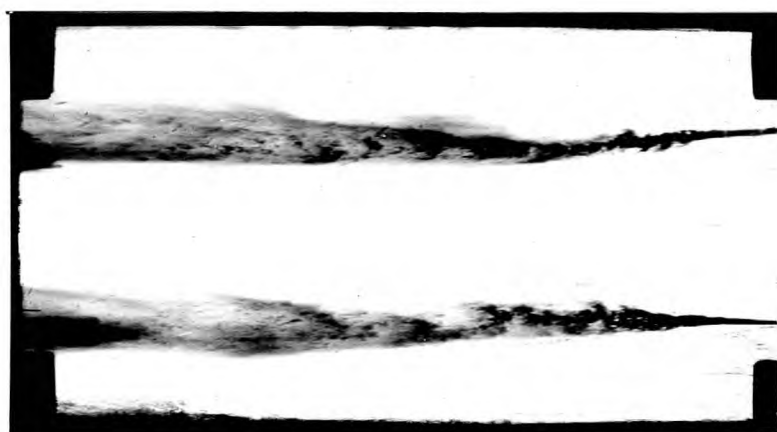
$M_1 = 1.87$ $w_0 = 0.082$ Lbs/Sec ($\Omega = 0.165$) $T_{01} = 1500^\circ R$

PLATE .10. 50 P.S.I. NOZZLES SUPERSONIC FLOW



(a) $M_1 = 0.0$ $w_2 = 0.082 \text{ Lbs/Sec}$ $T_0 = 530^\circ\text{R}$

CO₂ Compression



(b)

N₂ Compression



(c)

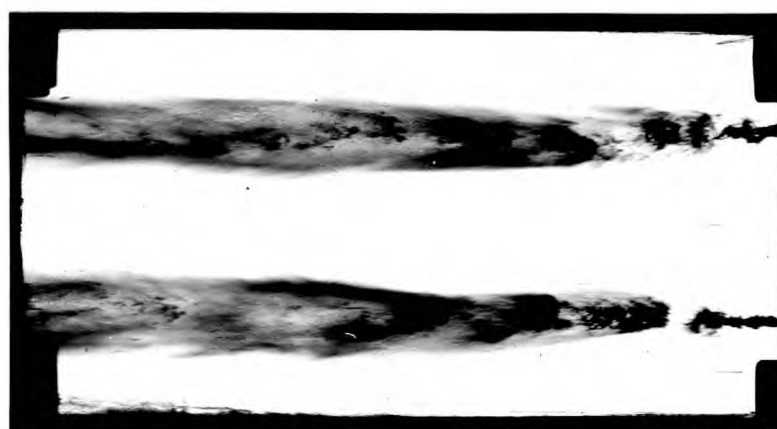
CO₂ Compression

$M_1 = 1.87$ $w_2 = 0.082 \text{ Lbs/Sec}$ $T_1 = 530^\circ\text{R}$



(d)

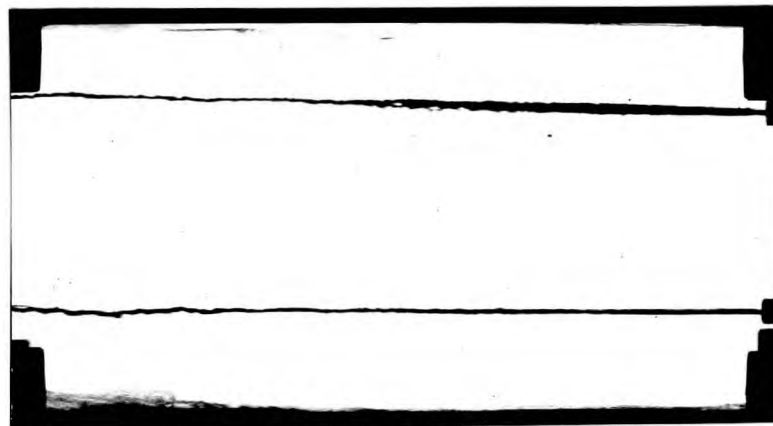
N₂ Compression



(e)

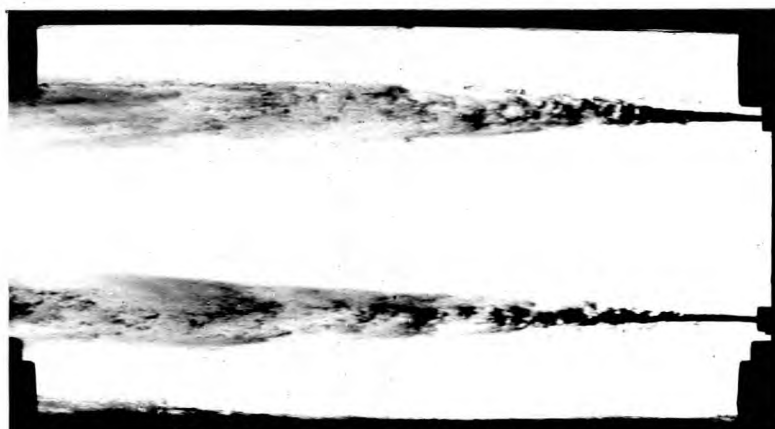
CO₂ Compression

$M_1 = 1.87$ $w_2 = 0.082 \text{ Lbs/Sec}$ ($\Omega = 0.165$) $T_0 = 1500^\circ\text{R}$



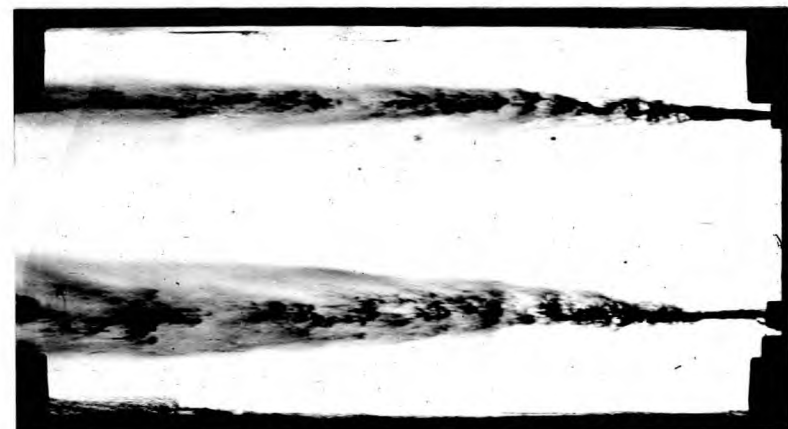
(a) $M_1 = 0.0$ $w_0 = 0.082 \text{ Lbs/Sec}$ $T_{01} = 530^\circ\text{R}$

CO₂ Compression



(b)

N₂ Compression



(c)

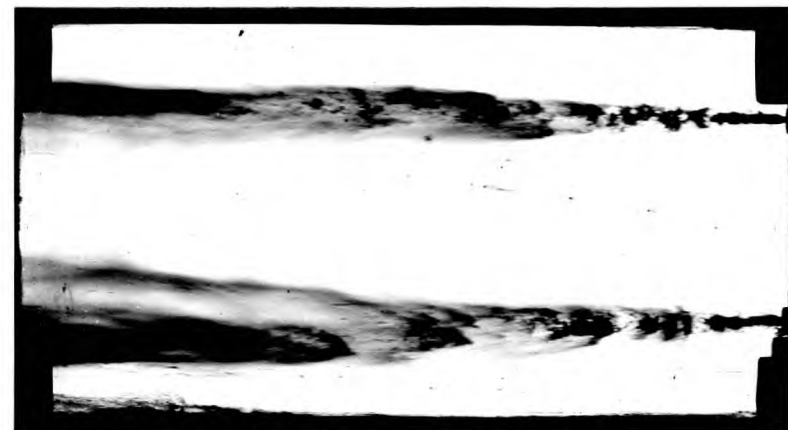
CO₂ Compression

$M_1 = 1.87$ $w_0 = 0.082 \text{ Lbs/Sec}$ $T_1 = 530^\circ\text{R}$



(d)

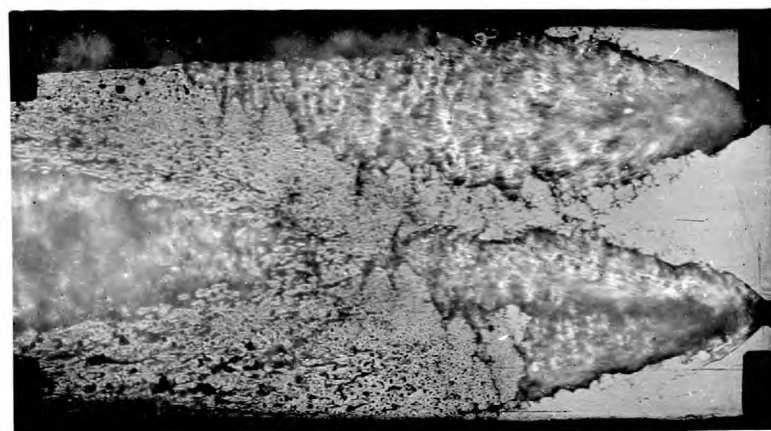
N₂ Compression



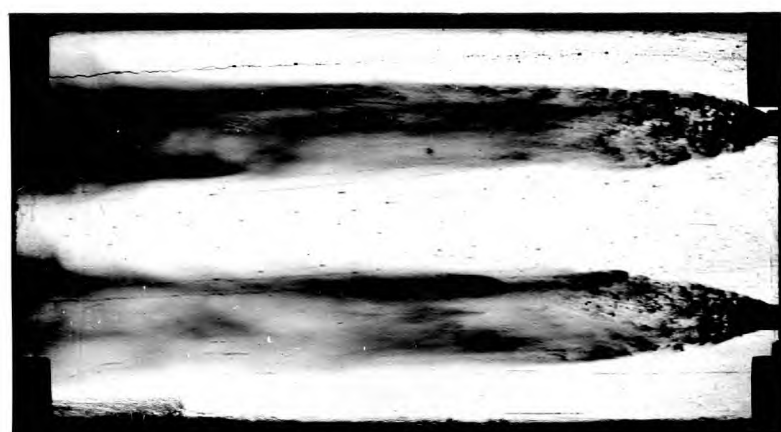
(e)

CO₂ Compression

$M_1 = 1.87$ $w_0 = 0.082 \text{ Lbs/Sec}$ ($\Omega = 0.165$) $T_{01} = 1500^\circ\text{R}$



(a) $M_1 = 0.0$ $w_e = 0.082 \text{ Lbs/Sec}$ $T_{01} = 530^\circ\text{R}$
 CO_2 Compression



(b) $w_e = 0.082 \text{ Lbs/Sec}$

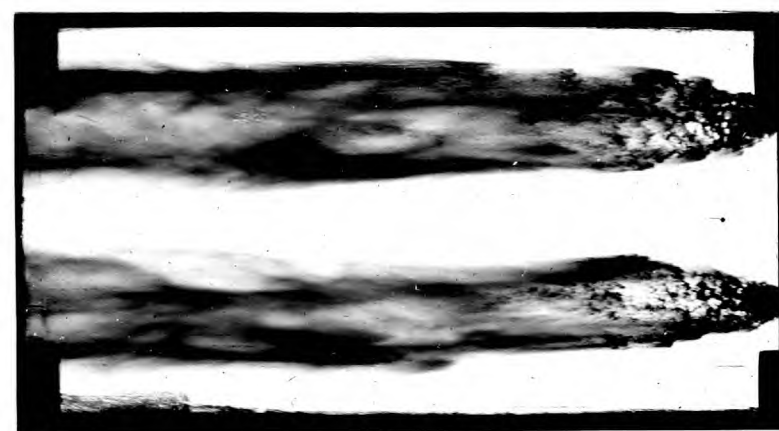


(c) $w_e = 0.102 \text{ Lbs/Sec}$

$M_1 = 1.87$ 530°R N_2 Compression



(d) $w_e = 0.082 \text{ Lbs/Sec}$ ($\Omega = 0.165$)



(e) $w_e = 0.102 \text{ Lbs/Sec}$ ($\Omega = 0.205$)

$M_1 = 1.87$ $T_{01} = 1500^\circ\text{R}$ N_2 Compression



$M_1=1.86 \quad w_t=0.102 \text{ Lbs/Sec } (\Omega=0.205) \quad T_{01}=1500^\circ\text{R}$

PLATE .14. ANNULAR NOZZLE SUPERSONIC FLOW

8. DISCUSSION

Summary

Results of subsonic experiments show that of the injector types tested, low velocity axial jets give the best results. A closer agreement between subsonic theory and experiment using this method of water injection is forecast by using a revised drop-let size equation and a slightly modified model for theoretical analysis. Supersonic experimental results show that in this range fan spray nozzles are best. Agreement between supersonic theory and experiment is poor and this is attributed to shock formation between the water and air streams. From these experiments and results of ref. (2) the maximum performance of the Aerothermopressor is estimated. Finally, application of the Aerothermopressor is reconsidered.

8.1 Subsonic Flow Experiments

8.1.1 Injection with low Velocity Axial Jets

As a basis for comparison, water was injected under conditions as nearly as possible identical with those of ref. (2) i.e. with low velocity axial jets from open tubes. By fortunate coincidence the value for $\frac{f}{D}$ of the author is so close to that of ref. (2) that direct quantitative comparison is permissible. This is given in fig. 32.

Agreement between the two values of p_o , is probably closer than that given in fig. 32 since ref (2) appears to have ignored total pressure loss due to combustion. Differences in friction coefficient do not exceed likely experimental error.

The two results show remarkably close agreement well within the limits of experimental error. It appears that the number of water tubes is not critical within the limits of 1 and 2 per square inch of duot cross section. The author's results are probably more accurate where they deviate most from those of ref. (2) namely near the water injection plane. This is because the number of wall statics in the first 6" length of test section is 6 compared to 3 in ref. (2)

Comparison of theory and experiment shows that acceleration losses are lower in the experiment than in the theory, while examination of $\frac{dp}{dz}$ shows that evaporation gains are at first higher but, as the process continues, ultimately lower than predicted.

Plate 3 gives the corresponding spray photographs. It may be seen that break up is complete in approximately $\frac{3}{4}$ " - 1" while distribution becomes fairly uniform within 3". This is sufficiently soon after break up is complete to make closer tube spacing unnecessary. Hence the close agreement between the two experimental curves in Fig. 32. The anticipated effect of this difference between theory and experiment is a "flattening off" of the total and static pressure curves near the beginning of the process as a result of the merging of Regime 1 into Regime 2. This accounts for the reduced acceleration losses, but not for the increased rate of evaporation at the beginning of Regime 2.

A preferable explanation is that the effects of extended acceleration and gradual distribution are to some extent self cancelling in Regime 1. Apart from the resulting decrease in the total amount evaporated in this region the difference between theory and experiment is rather due to incorrect calculation of droplet size and heat transfer rates.

The experiments of Hrubecky ref (14) indicate that the Nukiyama and Tanasawa droplet equation gives too large a drop size. In the experiments of Nukiyama and Tanasawa the water was injected at the entry of a converging nozzle while the gas velocity on which the equation is based was measured at the nozzle exit. Hrubecky showed that by injecting the water in the nozzle exit plane the droplet size was reduced. Since atomization begins immediately the water is injected, the higher relative velocities at the nozzle exit produced smaller droplets. Injection upstream gives earlier break up at lower velocities which results in some of the droplets merely being accelerated without further splitting up. The experiments of

Lane (ref.15) confirm this. Tanasawa in conversation with the author agreed with the findings of Hrubycky. It is not certain how much the true droplet size is less than that predicted by ref. (13). The biggest discrepancies were found at air/water volume ratios (Q_A/Q_L) of about 20×10^3 and at velocities very close to $M = 1$ where evaporation and conditions of imminent shock were thought to be partly the cause. In these experiments Q_A/Q_L is $10 - 13.5 \times 10^3$ at which values the maximum possible Mach Number in subsonic flow is 0.78. At $V - V_\ell = 228$ metres/sec corresponding to $M \simeq 0.75$ and $Q_A/Q_L = 13.5 \times 10^3$

$d_{O \text{ MEASURED}} = 0.8 d_{Oj}$. This is not directly comparable as results shown in fig.32 are based on $M = 0.5$ and $V - V_\ell = 278$ metres/sec. However it appears likely that the true droplet diameter $d_o \simeq (0.8 - 0.9)d_{Oj}$ for most subsonic Aerothermopressor conditions.

The model for analysis in ref.(2) assumes a mean droplet size such that the total mass and surface area of the uniform spray is equal to that of the actual spray. As evaporation proceeds the number of droplets remains constant but the diameter decreases. In fact the droplets are not of equal size but vary over a spectrum, so that the smaller droplets will evaporate first and the total number will decrease. Shapiro, in notes at the end of ref.(2a), shows that for the evaporation of a stationary droplet cloud, to which the Aerothermopressor approximates for most of the process, a constant mean droplet diameter and a varying number of droplets should be assumed. This will give a different heat transfer rate, for consider the following:

$$w_e = N \cdot \rho_e \cdot \frac{\pi \cdot d^3}{6}$$

where N = Number of Droplets

If N is constant:

$$\therefore \frac{dw_e}{dd} = N \cdot \frac{\rho_e \cdot \pi \cdot d^2}{2}$$

$$\text{Total Heat Transfer Surface} = N \cdot \pi \cdot d^2$$

$$\therefore \frac{d(H.T.S)}{dd} = 2 \cdot \pi \cdot N \cdot d$$

$$\therefore \left[\frac{d(H.T.S)}{dw_e} \right]_{N=Const} = \frac{d(H.T.S)}{dd} \cdot \frac{dd}{dw_e} = \frac{4}{\rho_e \cdot d}$$

If d is constant:

$$\frac{dw_e}{dN} = \frac{\rho_e \cdot \pi \cdot d^3}{6}$$

$$\text{and} \quad \frac{d(H.T.S)}{dN} = \pi \cdot d^2$$

$$\left[\frac{d(H.T.S)}{dw_e} \right]_{d=Const} = \frac{6}{\rho_e \cdot d}$$

For a given increment of evaporation dw_e

$$\frac{d(H.T.S)_{N=Const}}{d(H.T.S)_{d=Const}} = \frac{2}{3}$$

By assuming a droplet cloud of constant diameter instead of constant number, neglecting Reynolds Number effects which will be small, the heat transfer rate will be reduced for the process though initially both rates will be identical.

The effect of the smaller droplet size will be to reduce losses in Regime 1 and raise the evaporation rate at the beginning of Regime 2 while as evaporation proceeds, assuming constant droplet size will result in a more rapidly reducing evaporation rate. The governing physical equations reformulated to assume constant droplet size are given in Appendix E. Time considerations did not permit a further numerical solution to confirm the accuracy of these new assumptions. Examination of the results in ref.(2) shows that the experimental curve follows the numerical solution corresponding to $d_o \approx 0.8 - 0.9 d$ at the beginning but deviates towards the curve corresponding to $d_o = d_{oj}$ reaching its maximum in about $\frac{3}{4}$ the distance calculated by the numerical solution for $d_o = d_{oj}$ and then falling away more rapidly as seen in fig.32. In view of this an improved agreement between theory and experiment based on a new droplet size equation and the revised model for analysis may be almost guaranteed using this form of water injection.

8.1.2 Reduction of Surface Tension

Fig. 33 gives the results of injection of water containing 0.15% by mass of the detergent "Tide". In accordance with Section 3.3.1 this was used in the axial jets at low velocity. Results are virtually indistinguishable from those obtained with water. This indicates that even at low injection speeds the wetting action is not rapid enough to be effective. In addition, a practical objection to the use of detergent is that it deposited on the duct walls as evaporation proceeded with caustic effects tending to roughen the wall surface. Reduction of surface tension does not appear to be a practicable means of improving water atomization.

8.1.3 High Velocity Injection with Carbon Dioxide

Figs. 34 and 35 give results for injection of water at low pressures using nitrogen and carbon dioxide. Compared to fig.33 they show a greatly extended acceleration length and a much inferior evaporation rate. Carbon dioxide dissolved in the water has no effect. Plate 4 gives photographs of the spray formation. It appears that the increased velocity of the jet gives it greater stability. This is in agreement with the findings of Tanasawa and Toyoda ref. (17). Break up is therefore prolonged and the drawing off of ligaments continues far down the duct. This results in large droplets and poor distribution which reduces evaporation rates.

Results of injection of water at 50,100 and 200 psi are shown on figs 36, 37 and 38. These are essentially similar to the results at 12 and 15 psi but with the acceleration zone increasingly prolonged and evaporation rates much reduced. The effect of carbon dioxide at 50 psi is negligible while at 100 and 200 psi it appears to slightly impair the performance. Plates 5, 6 and 7 show the corresponding spray photographs. These confirm the increasing stability of the jet with increasing pressure. Plates 5 and 7 show no difference between nitrogen and carbon dioxide compression. Plates 6 (b) and(d) at 100 psi show slight evidence of additional break up caused by carbon dioxide but the effect is confined to one jet and might be only transient. There is no evidence to explain why carbon dioxide should impair the performance. A possible explanation prompted by the visual examination of the carbon dioxide compressed water described in Section 5.3.2 is that the carbon dioxide comes out of solution far down stream where its effect may be to agglomerate already formed droplets.

The effect of high velocity water injection in subsonic flow is therefore much worse than a mere increase in droplet size, as it greatly increases break up distance and leads to very uneven distribution. Up to 200 psi the effect of dissolved carbon dioxide is insignificant.

8.1.4 Injection with Fan Spray Nozzles

Fan spray nozzles gave results shown in figs. 39 and 40. The two water/air ratios were used in order to get the effect of carbon dioxide at different pressures. Break up distance is almost identical with that of the open tubes and acceleration losses are reduced due to the higher injection velocities. Evaporation rates are not so good as with the open tubes and the overall performance is inferior. The use of carbon dioxide reduces performance at 75 psi while at 118 psi it has no effect. The spray photographs are shown in Plate 8. It should be noted that the fan sheets are not parallel to the quartz walls but are in diagonal planes joining opposite edges of the duct walls. Plate 8 (a) shows the typical break up pattern for this type of jet described in Section 3.2. Plate 8 (b), (c), (d) and (e) show that the break up is rapid and distribution 4" downstream good though not quite equal to that of open tubes.

Recently Hooper ref. (18) 1959, has produced evidence that at high velocities the fan spray gives larger droplets than the jet. Increased relative velocity between water and gas stream decreases the wave length and the sheet breaks up nearer the nozzle. The sheet therefore does not attenuate so much and is thicker at break up. Though initially shorter wave length produces finer droplets, a point is reached where the thickness of the sheet at break up has greater effect than the decreased wave length and from then onwards higher relative velocities produce larger droplets. The inferior performance of the fan spray to the jet is probably due to this.

8.2 Supersonic Experiments

8.2.1 Open Tubes

Fig. 41 shows results of supersonic experiments with axial jets similar to those described in Section 8.1.1. Compared with fig. 31 which gives results of flow without water injection, the initial scatter of points from the mean curve is increased by

injecting water. Shock free conditions are reached in about the same distance as in the dry duct. Compared with the numerical solutions in fig. 3 it is obvious that theory and experiment only agree qualitatively. Regime 1 is approximately 8 times longer in the experiment than in the numerical solution. In Regime 2 the slope $\frac{dp}{dz}$ shows a much inferior rate of evaporation, and the static pressure reaches a minimum in twice the distance predicted. The experiment very roughly appears to follow the theory at half the rate in Regime 2 and $\frac{1}{8}$ the rate in Regime 1 while increased wall friction losses raise the minimum value of p. Reasons for this could be:

- a) Wrong estimate of droplet size.
- b) Errors in Regime 1 due to wrong estimation of droplet drag coefficients in supersonic flow.
- c) Incorrect values of droplet heat and mass transfer coefficients.
- d) Water air interaction unique to supersonic flow.

Droplet Size

The droplet size was calculated assuming the Nukiyama and Tanasawa equation is true for supersonic flow. The experiments of Bitron ref.(16) 1955 confirmed this, but these were carried out on a small scale rig similar to that of ref.(13) i.e. with the water injected in the nozzle inlet plane. This mode of injection is unsuitable for the Aerothermopressor, since in a large scale unit, break up would be complete before reaching the nozzle throat. It is therefore unlikely that such an Aerothermopressor would be supersonic at all. There is no record of droplet size measurement for truly supersonic injection but subsonic findings suggest that the mean droplet size may be smaller than predicted.

Droplet Drag Coefficient

No accurate values of drag coefficients for spheres are available in supersonic flow at Reynolds Numbers less than 20. The value of droplet Re in the injection plane for this experiment

= 19.6. Extrapolating the values of Bartman, Chaney, Jones and Liu ref (32) at $M_1 = 1.87$ and $Re = 19$, $C_D \simeq 2.0$. Assuming the incompressible values for C_D given in ref. (2) and used in the numerical solution, $C_D = 3.52$. It should be noted that the drag coefficient is only in error when gas flow is compressible relative to the droplet. This distance is theoretically extremely short being of the order of 0.2" according to the numerical solution while the total calculated length of Regime 1 is approximately 0.6". It is therefore apparent that errors in calculation of drag coefficient cannot account for the large difference between theory and experiment either in Regimes 1 or 2.

Droplet Heat and Mass Transfer Coefficients

The values assumed in the numerical solutions were those given in ref. (2) which were taken from Ranz and Marshall ref.(33). Since these were apparently satisfactory in the subsonic solution they also would only be in serious error in the first 0.2". In that distance approximately 8% of the water is evaporated. Errors in this amount cannot seriously affect the solution.

Water Air Interaction Unique to Supersonic Flow

Plate 9 gives photographs of the liquid break up. It is clear from (b), (c) and (d) that break up is extremely rapid being in the order of $\frac{1}{2}$ ". Distribution is far inferior to that in subsonic flow shown in Plate 3. The unusual feature is that though the water spray starts to diverge at first at a larger angle than in subsonic flow, it turns in sharply and starts to converge after travelling about $\frac{3}{4}$ ". At $1\frac{1}{2}$ " from the nozzles it reaches a minimum and then the process repeats itself but in a greater distance and much less sharply. 4" from the nozzles the water flow is parallel but by then most of the water has reached gas stream speed and no further divergence takes place.

The reason for this phenomenon is undoubtedly that the water initially diverging causes oblique shock waves to form in the gas stream deflecting the stream lines. The reflected waves then

divert the stream lines inwards so contracting the spray. As the water stream reaches the gas speed the shock pattern fades and the flow continues uniform and parallel. The effect of this on the jet break up is two-fold. Contraction of the jet must almost certainly cause agglomeration particularly of the outer droplets. This reduces heat and mass transfer rates of the outer droplets, while the hot gas cannot penetrate to accelerate the inner droplets. Acceleration and evaporation are both greatly reduced so extending Regime 1. In Regime 2 near the beginning the large droplets and poor distribution which leads to local air saturation reduce the evaporation rate. Further downstream the rate of evaporation is greater than the theoretical value for the same distance along the duct as the gas stream is overall less saturated. Mach Number continues to rise for nearly twice the length of duct predicted, but the increased friction loss reduces the maximum value attained.

Fig. 42 shows the effect of varying the water/air ratio. As in subsonic flow, given in ref.(2), the effect is less sensitive to excess water than lack of it. It is interesting to note that the evaporative effect is such that at $\Omega = 0.085$, when a total pressure rise could not be expected even theoretically, the flow remains supersonic to the end of the duct.

8.2.2 High Velocity Injection with Carbon Dioxide

Results of high velocity injection experiments are given in figs. 43, 44 and 45. Carbon dioxide has no effect at 50 psi. At 100 and 180 psi there is a marked difference between compression with carbon dioxide and nitrogen. The effect of the carbon dioxide is to increase the rate of the whole process by reducing the length of Regime 1, increasing the static pressure rise in this distance and increasing heat transfer rates in Regime 2.

Comparison of fig. 45 with the corresponding curve in fig. 42 shows that though Regime 1 is longer than with open tubes, using high velocity injection with carbon dioxide compression the evaporation rate is slightly better in Regime 2. Plates 10, 11 and 12 give

the corresponding spray photographs. Plate 10 confirms the experimental result at 50 psi by showing no difference in spray formation between nitrogen and carbon dioxide compression. Plates 11 and 12 show the effect of the carbon dioxide on the spray. Injection with nitrogen produces a spray similar to that of the subsonic nozzles except that the break up distance is less than in the subsonic case because of the higher relative velocity between the water and gas streams. The effect of oblique shocks is hardly visible because the divergence angle of the jet is much less and therefore the induced shocks are much weaker. The carbon dioxide does not produce foaming or turbulence in the jet. Instead it comes out of solution in bursts disrupting the jet into short lengths and so accelerating the water disintegration by increasing the water surface area. The narrower divergence angle permits the jet to continue expansion further downstream than with open tubes without much effect from the oblique shock waves, but since the divergence angle of the spray is less, the distribution at the end of acceleration is still poor.

The slight superiority of high velocity injection at 180 psi with carbon dioxide over low velocity injection with open tubes is probably not the direct result of finer atomization or reduced acceleration loss but since the water/air shock interaction is less sharp, agglomeration of droplets is reduced.

8.2.3 Injection with Fan Spray Nozzles

Fan spray nozzle test results are shown in figs 46 and 47. Effects of carbon dioxide compression are indeterminate as both acceleration and evaporation maximum pressures are raised leaving gradients almost identical. This effect is therefore not considered.

The static pressure gradients together with the low minimum pressure reached in evaporation show this method of injection to be the best attempted so far in supersonic flow. In Fig. 47 it is compared directly with the 2nd numerical solution of Fig. 3 extrapolated for $M_1 = 1.87$ instead of $M_1 = 1.86$.

Despite the probable errors due to the large scatter of points in Regime 1 and difficulty in assessing true mean values indicated in Section 7.1.2 the difference between theory and experiment is too large to be reconciled. Due to the poor distribution of water in supersonic flow together with consideration (e) in Section 4.1 it is probable that f has been overestimated and the true numerical solution is probably nearer the 1st Solution in fig. 3. Thus apart from experimental errors the difference between theory and experiment is probably larger than that shown. Plate 13 shows the spray photographs. As in the open tubes, initial break up is very rapid, initial divergence is large and shock effects are clearly visible.

At first the suggested inferior atomization of the fan spray in Section 8.1.4 and the superior results in supersonic flow appear to be contradictory. The fan however injects a sheet and not a "solid cone" of water. The shock effects and probable resulting agglomeration are therefore limited to the fringes of the sheet only while rapid evaporation proceeds from the centre of the sheet which is still in free contact with the unsaturated air stream. This is confirmed in plate 13 by accounting for the high density of the spray at the edges in (b), (c), (d) and (e).

The true total pressure rise using this form of injection may be approximately deduced as follows. Assume that the position of experimental and theoretical static pressure minima give lengths of duct corresponding to the same amount of water evaporated. Assume also that equal amounts of water evaporated give equal gains in total pressure. Difference in total pressure rise will then correspond to difference in friction loss which in turn will be proportional to the duct length. Fig. 47 shows

$$L_{\text{Experimental}} = L_{\text{Theoretical}} \times 1.75$$

Extending the final results of Section 4.2.4.

$$\frac{L}{D} = 3 \text{ corresponds to } \frac{L}{D} = 5.25$$

	$\frac{L}{D} = 0$	$\frac{L}{D} = 3$	$\frac{L}{D} = 5.25$
$\frac{P_{02} - P_{01}}{P_{01}}$	0.20	0.12	0.06

This is inferior to the subsonic results.

8.2.4 Annular Nozzle

Static pressure variation in the duct with annular nozzle fitted is given in figs. 48 and 49. Flow without water is given in fig. 48. Due to three dimensional shocks being weaker than two dimensional shocks, the scatter about the mean line is less than in fig. 31 despite the approximation of the nozzle design method. Normal shock occurs in a shorter length of duct and this is due to an increase in the one dimensional friction coefficient. The friction coefficient increases because boundary layer growth on the walls of the annulus causes flow profile irregularities shown in fig. 28 which result in radial momentum transfer.

Fig. 49 shows the result of flow with water injected. There is a large increase in scatter in Regime 1 due to the water injection. This verifies the suggestion in Section 8.2.1 that the water injection causes shocks. Acceleration losses, even allowing that $M_1 = 1.86$ and not 1.87 as with the two dimensional nozzle are high while evaporation rates in Regime 2 are disappointingly low. A lower evaporation rate throughout the process than with any other form of injection is evident. Plate 14 gives immediate explanation of this. Due to the extremely thin walls and eccentricity of the inner and outer sections, slightly uneven water flow rates around the annulus have caused variation in the cooling rate of the metal. The metal in turn distorted and exaggerated the effect. This results in water flowing from the top and bottom of the annulus only. The two sheets so formed contract through the action of surface tension and thicken. Atomization and distribution of the water are therefore both poor. It is possible that more careful design of the

nozzle with internal webs to stiffen the walls might overcome this but it is doubtful if the enormous temperature gradients near the water injection plane could be prevented from closing the gap of only 0.004" partially. Despite its theoretical desirability such a nozzle cannot be recommended.

8.4 Maximum Performance of the Aerothermopressor

Results of these experiments together with those of ref. (2) permit a more precise assessment of the performance limits. The best Aerothermopressor is clearly subsonic and the best method of water injection in subsonic flow is with open tubes as used in ref (2).

As already mentioned in section 4.1 (a) the maximum attainable coefficient of performance in subsonic flow was attained at $M_1 = 0.78$ and $\Omega = 0.24$ at $T_{01} = 1500^\circ\text{R}$ in a small scale parallel duct. This corresponds to subsonic choking conditions. Since no advantage is to be obtained in the supersonic region, we may conclude that for $T_{01} = 1500^\circ\text{R}$ these initial conditions are the best possible. Estimation of the maximum possible total pressure rise, using these parameter values, is more difficult for not only may the duct have a varying cross section but due to the possibility of continuous transition through the speed of sound a variety of flow patterns with normal shocks at various positions may be obtained. By experiment with a small scale rig and using the incorrect extrapolation method described in Section 4.2.4 ref (2) forecast a 10% gain in a full scale rig. It is not certain how incorrect this extrapolation is since there is no evidence given of the position of the normal shock in the duct and hence it is not known whether M_m is greater or less than M_1 . Results of a numerical solution which takes account of normal shock and area ^{in a 12" diameter duct} ruling given in ref (2) indicate a possible total pressure rise of 18%. Though this may not be quoted as an absolute maximum since there is no guarantee that either the normal shock is in the best position or the duct shape the best possible it is clear that the maximum total pressure rise is of this order of magnitude and almost certainly will not much exceed 20%.

This is far from reversible flow analyses which show that at $T_{01} = 1500^{\circ}\text{R}$ a total pressure rise of 540% is possible. However it should be realised that reversible flow analyses only approximate to practical results when processes may be considered as approximately reversible. The Aerothermopressor process is a synthesis of several simultaneous processes all of which are highly irreversible. This large discrepancy may therefore be expected.

8.5 Application

The main advantages claimed for the Aerothermopressor as a means of boosting power and efficiency of a gas turbine are its mechanical simplicity and small size and weight. In these respects it would be superior to the regenerator. The claim that it is free from fouling is not substantiated by this research. The effect of fouling in a regenerator is to reduce the heat transfer coefficient and hence the efficiency of the device. Carbonate deposits on the duct walls described in Section 6.6. raise the duct friction coefficient and hence reduce the efficiency of the Aerothermopressor. Insofar as the performance of both devices is reduced by deposition of solids both may be said to have fouling problems. In the case of the Aerothermopressor this may be overcome by insertion of water softening equipment. Unfortunately this robs the Aerothermopressor of the advantage of weight and simplicity. As a device for raising the power and efficiency of a gas turbine it is therefore not recommended.

As pointed out by Spalding & Cole (ref.6) from the purely thermodynamic point of view, the Aerothermopressor is a particular case of the mixing of a hot compressible fluid stream with a cold incompressible fluid stream. It is therefore identical thermodynamically with the steam injector, a device invented approximately 100 years ago. Its only differences are that the equation of state for steam is different from that of air and that the mass ratio of water to gas is such that the final phase of the mixture is entirely liquid, whereas in the Aerothermopressor it is entirely vapour.

The thermodynamic efficiency of a steam injector considered as a pump is only of the order of 1% and its use is justified in locomotives because of mechanical simplicity.

As an efficient means of actually raising total pressure the Aerothermopressor has little worth but this does not rule out its use entirely for other purposes.

It may be useful in any application requiring high velocity flow in long ducts without choking or normal shock.

9. CONCLUSIONS

Results of this research lead to the following conclusions.

9.1 The Subsonic Aerothermopressor

- a) The best means of injecting the water is by low velocity axial jets spaced approximately 1 per square inch of duct cross section.
- b) The best initial parameter values are for subsonic choking conditions which at $T_{o_1} = 1500^{\circ}\text{R}$ correspond to $M_1 = 0.78$ and $\Omega = 0.24$.
- c) The maximum possible total pressure rise at $T_{o_1} = 1500^{\circ}\text{R}$ is of the order of 20% in a 12" diameter duct.
- d) Theory and experiment could be made to agree more closely by assuming a droplet size 0.8 - 0.9 of the value predicted by the equation of Nukiyama and Tanasawa and altering the theoretical model for analysis to assume a cloud of uniform droplets of constant diameter and diminishing number.
- e) Correct extrapolation of results from small scale to large scale involves lengthy calculation and is not generally recommended. Large scale numerical solutions should preferably be attempted directly.
- f) Any application of the Aerothermopressor to raise the power and efficiency of a gas turbine will have to take account of its fouling problems.
- g) The subsonic Aerothermopressor may have use as a means of obtaining high speed flow through long ducts without choking.

9.2 The Supersonic Aerothermopressor

- a) The best means of injecting the water is by fan spray nozzles parallel to the gas flow using one to two per square inch of duct cross section.
- b) Maximum performance of the supersonic Aerothermopressor is uncertain but due to oblique shock waves formed by water/air it interaction it is much inferior to the subsonic Aerothermopressor and it is even possible that an overall total pressure rise may be unattainable.

c) Due to oblique shock wave formation resulting from water and gas stream interaction, agreement between theory and experiment is only qualitative. Using the Nukiyama and Tanasawa droplet size equation the calculated overall rate of evaporation is roughly twice that achieved.

d) The Supersonic Aerothermopressor has possible applications for increasing the length of duct in which supersonic flow may be maintained given any initial Mach Number.

10. APPENDIX A

The Derivation of the Fundamental Equation.

Consider an infinitesimal control volume bounded by duct walls as shown in Fig 50. Gas and injected liquid flow through it such that at the exit plane all the liquid has evaporated and attained equilibrium with the gas stream. The resultant mixture may be regarded as a homogeneous gas.

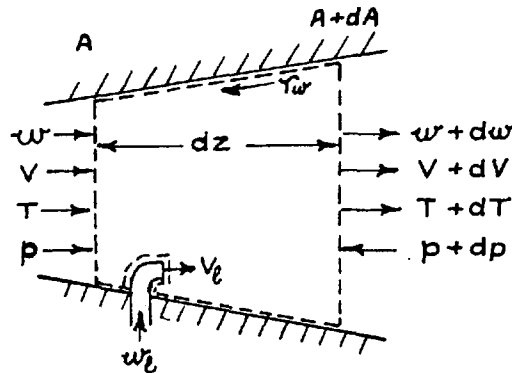


Fig. 50.

The original analysis given in Ref (1) allowed for : change in specific heat and molecular weight due to evaporation and chemical reaction, external heat transfer and work. Equation (1) was then obtained as a particular result when these factors were neglected.

The following analysis illustrates the method by deriving equation (1) assuming initially: constant specific heats and molecular weights, and no external heat transfer or work. Assume that the boundaries of the control volume are the walls of a duct of circular cross section

Equation of State :

$$\frac{P}{\rho} = \frac{\bar{R}}{W} \cdot T \quad (4)$$

Equation of Continuity:

$$w + w_e = w + dw$$

$$\therefore w_e = dw \quad (5)$$

$$\text{also} \quad w = \rho \cdot A \cdot V \quad (6)$$

$$\text{Definition of Mach No:} \quad M^2 \equiv \frac{V^2}{C^2} = \frac{V^2 \cdot w}{k \cdot R \cdot T} \quad (7)$$

$$\therefore \quad \frac{dM^2}{M^2} = \frac{dV^2}{V^2} - \frac{dT}{T} \quad (8)$$

$$\text{Definition of Total Temperature:} \quad T_0 \equiv T + \frac{V^2}{2 \cdot c_p} = T \left(1 + \frac{k-1}{2} M^2 \right)$$

$$\therefore \quad \frac{dT_0}{T_0} = \frac{dT}{T} + \frac{\frac{k-1}{2} \cdot M^2}{1 + \frac{k-1}{2} \cdot M^2} \cdot \frac{dM^2}{M^2} \quad (9)$$

$$\text{Definition of Total Pressure:} \quad \frac{p_0}{p} \equiv \left(\frac{T_0}{T} \right)^{\frac{k}{k-1}} = \left(1 + \frac{k-1}{2} \cdot M^2 \right)^{\frac{k}{k-1}}$$

$$\therefore \quad \frac{dp_0}{p_0} = \frac{dp}{p} + \frac{\frac{k \cdot M^2}{2}}{1 + \frac{k-1}{2} \cdot M^2} \cdot \frac{dM^2}{M^2} \quad (10)$$

Conservation of Momentum:

$$p \cdot A + p \cdot dA - (p+dp)(A+dA) - \tau_w \cdot \pi \cdot D \cdot dz = (w+dw)(V+dv)w \cdot V - w \cdot V \cdot w \quad (11)$$

$$\text{Defining } f \equiv \frac{2 \cdot \tau_w}{\rho \cdot V^2} \text{ and } y = \frac{V \ell}{V} \text{ and substituting (5) in (11)}$$

$$-A \cdot dp = (1-y) \cdot V \cdot dw + w \cdot dV + \rho \cdot V^2 \cdot \pi \cdot D \cdot dz \quad (12)$$

$$\text{Dividing through by } A = \frac{\pi d^2}{4} \text{ and substituting (6) in (12)}$$

$$-dp = \frac{\rho V^2}{2} \left[\frac{dV^2}{V^2} + \frac{4 \cdot f \cdot dz}{D} + 2(1-y) \cdot \frac{dw}{w} \right] \quad (13)$$

Combining (4) and (7) and substituting in (13)

$$\frac{dp}{p} = - \frac{k \cdot M^2}{2} \left[\frac{dV^2}{V^2} + \frac{4 \cdot f \cdot dz}{D} + 2(1-y) \cdot \frac{dw}{w} \right] \quad (14)$$

Combining (8) and (9) and substituting in (14)

$$\frac{dp}{p} = - \frac{k \cdot M^2}{2} \left[\frac{dT_0}{T_0} + \frac{4 \cdot f \cdot dz}{D} + 2(1-y) \frac{dw}{w} \right] - \frac{\frac{k \cdot M^2}{2}}{1 + \frac{k-1}{2} \cdot M^2} \cdot \frac{dM^2}{M^2} \quad (15)$$

Combining (10) and (15):

$$\frac{dp_0}{p_0} = -\frac{k.M^2}{2} \left[\frac{dT_0}{T_0} + \frac{4.f.dz}{D} + 2(1-y) \frac{dw}{w} \right]$$

11. APPENDIX B

DERIVATION OF INFLUENCE COEFFICIENTS

The following analysis indicates the procedure for deriving the equations given in Table 2. This is an abridged form of that given in Ref (2).

Consider an infinitesimal control volume through which water, air and vapour flow as shown in Fig. 51.

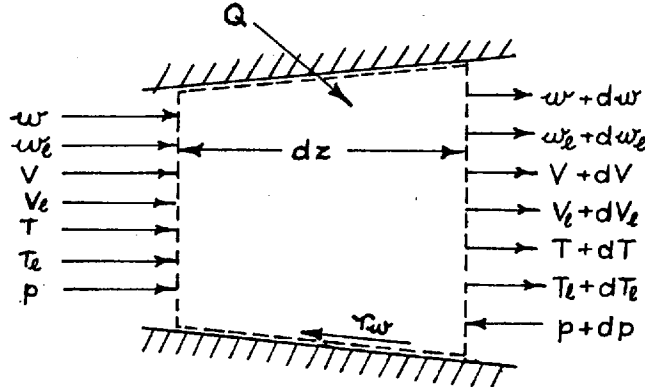


Fig. 51.

The gas mixture is assumed to obey the perfect gas law and the Gibbs-Dalton law but with variable specific heats.

$$\begin{aligned} \text{Equation of State: } \frac{p}{\rho} &= \frac{\bar{R} \cdot T}{W} \\ \frac{dp}{p} &= \frac{d\rho}{\rho} + \frac{dT}{T} - \frac{dW}{W} \end{aligned} \quad (16)$$

$$\text{Equation of Continuity: } w = \rho \cdot A \cdot V$$

$$\frac{-dw_e}{w} = \frac{dw}{w} = \frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} \quad (17)$$

$$\text{Definition of Mach No: } M^2 \equiv \frac{V^2}{C^2} = \frac{W \cdot V^2}{k \cdot \bar{R} \cdot T}$$

$$\frac{dM^2}{M^2} = \frac{dV^2}{V^2} + \frac{dW}{W} - \frac{dk}{k} - \frac{dT}{T} \quad (18)$$

Conservation of Momentum:

$$pA + p.dA - (p+dp)(A+dA) - T_w \pi D dz = (w+dw)(V+dV) + (w_\ell + dw_\ell)(V_\ell + dV_\ell) - wV - w_\ell V_\ell$$

Hence as in Section 10.

$$\frac{dp}{p} + kM^2 \frac{dV}{V} + \frac{kM^2}{2} \left(4f \frac{dz}{D} + 2 \frac{w_\ell}{w} \frac{dV_\ell}{V} \right) + kM^2 (1-y) \frac{dw}{w} = 0 \quad (19)$$

Conservation of Energy:

$$w.dQ + w.h + w_\ell.h_\ell + w \frac{V^2}{2} + w_\ell \frac{V_\ell^2}{2} = (w+dw)(h+dh) + (w_\ell + dw_\ell)(h_\ell + dh_\ell) + (w+dw) \frac{(V+dV)^2}{2} + (w_\ell + dw_\ell) \frac{(V_\ell + dV_\ell)^2}{2}$$

For a mixture of Gases following the Gibbs-Dalton law:

$$w.dh = w_a.dh_a + w_v.dh_v = w.c_p.dT$$

Since from continuity:

$$w = w_a + w_v$$

$$\therefore dw = dw_v = -dw_\ell$$

Combining these, simplifying and rearranging, the energy equation becomes:

$$\frac{dT}{T} + (k-1)M^2 \frac{dV}{V} + \left[\frac{h_v - h_\ell + \frac{V^2}{2}(1-y^2)}{c_p T} \right] \frac{dw}{w} - \frac{dQ \frac{w_\ell}{w} dh_\ell}{T} + \frac{w_\ell}{w} (k-1)M_y^2 \frac{dV_\ell}{V} = 0 \quad (20)$$

Formation of Influence Coefficients: Equations 16, 17, 18, 19 and 20.

constitute five simultaneous linear algebraic equations among twelve differential variables:

For convenience the first five are chosen as dependent and the remaining seven as independent variables. Solving these equations simultaneously the first four lines in Table 2 are obtained.

Definition of Gas Stagnation Temperature:

$$T_0 \equiv T + \frac{V^2}{2.c_p} = T \left(1 + \frac{k-1}{2} M^2 \right)$$

$$\therefore \frac{dT_0}{T_0} = \frac{dT}{T} + \frac{\frac{k-1}{2} M^2}{1 + \frac{k-1}{2} M^2} \cdot \frac{dM^2}{M^2} + \frac{\frac{k M^2}{2}}{1 + \frac{k-1}{2} M^2} \cdot \frac{dk}{k} \quad (21)$$

from which the fifth line of Table (2) is found.

Definition of Gas Stagnation Pressure:

$$\frac{p_o}{p} \equiv \left(1 + \frac{k-1}{2} M^2\right)^{\frac{k}{k-1}}$$

$$\frac{dp_o}{p_o} = \frac{dp}{p} + \frac{\frac{k \cdot M^2}{2}}{1 + \frac{k-1}{2} M^2} \cdot \frac{dM^2}{M^2} + \left[\frac{\frac{k^2 M^2}{2}}{(k-1)(1 + \frac{k-1}{2} M^2)} - \frac{k}{(k-1)^2} \cdot \log\left(1 + \frac{k-1}{2} M^2\right) \right] \frac{dk}{k} \quad (22)$$

from which the seventh line of Table (2) is found.

Definition of Mixture Stagnation Temperature:

$$\theta_o \equiv T \left[1 + \left(1 + \frac{w_e}{w} \cdot \frac{V_e^2}{V^2}\right) \frac{k-1}{2} M^2 \right]^{\frac{k}{k-1}} = \alpha \cdot T \quad (23)$$

$$\begin{aligned} \frac{d\theta_o}{\theta_o} = \frac{1}{\alpha} \left(1 + \frac{k-1}{2} M^2\right) \frac{dT_o}{T_o} + (k-1) \frac{M^2 y}{\alpha} \left(\frac{w_e}{w} \cdot \frac{dV_e}{V} \right) - \frac{k-1}{2} \cdot \frac{M^2 y^2}{\alpha} \left(1 + \frac{w_e}{w}\right) \frac{dw}{w} \\ - \frac{k-1}{2} \cdot \frac{M^2 y^2}{\alpha} \cdot \frac{w_e}{w} \frac{dW}{W} + \frac{M^2 y^2 \cdot w_e}{2\alpha w} \cdot \frac{dk}{k} \end{aligned} \quad (24)$$

from which the sixth line of Table (2) is found.

Definition of Mixture Stagnation Pressure:

$$\frac{p_o}{p} \equiv \left(\frac{\theta_o}{T}\right)^{\frac{k}{k-1}} = \left[1 + \left(1 + \frac{w_e}{w} \cdot \frac{V_e^2}{V^2}\right) \frac{k-1}{2} M^2 \right]^{\frac{k}{k-1}} \quad (25)$$

$$\frac{dp_o}{p_o} \equiv \frac{dp}{p} + \frac{k}{k-1} \left(\frac{d\theta_o}{\theta_o} - \frac{dT}{T} \right) - \frac{k}{(k-1)^2} \left(\log \frac{\theta_o}{T} \right) \frac{dk}{k}$$

from which the last line of Table (2) is found.

12. APPENDIX C

SPECIMEN CALCULATION 2ND SOLUTION. STAGE 16.

M_{16}	T_{016}	P_{016}	T_{16}	p_{16}	V_{16}	w_{16}	C_{p16}	k_{16}	W_{16}	x_{16}	ω_{16}	w_{216}	d_{16}	V_{e16}	T_{e16}	Z_{16}	$\frac{w_e}{w_{16}}$
1.381	1294	9.504	941.4	3.033	2088	.5058	2.596	1.381	28.48	0.15	5.745×10^{-2}	0.0832	2.000×10^{-3}	1761	528.1	9.772×10^{-1}	.1645

STAGE : 16

$dx = 0.01$

True Calculated Values

Extrapolated Values

$$dd = -\frac{3.132 \times 10^{-15}}{d^2} \cdot dx = \frac{-3.132 \times 10^{-17}}{3.983 \times 10^{10}} = -7.863 \times 10^{-8}$$

$$dd \approx -7.87 \times 10^{-8}$$

$$\therefore d_m \approx 1.996 \times 10^{-5}$$

$$d\omega = \Omega \cdot dx (1 + \omega_0) = 0.2054 \cdot dx = 2.054 \times 10^{-3}$$

$$dT \approx -10.0$$

$$\therefore T_m \approx 936.4$$

$$\mu \text{ (From Charts)} = \frac{\mu_a + \omega \mu_v}{1 + \omega} = \frac{(183.8 + 0.0585 \times 1226) \times 10^{-7}}{1.058} = 180.5 \times 10^{-7}$$

$$\omega_m \approx 5.848 \times 10^{-3}$$

$$W_m \approx 28.48$$

$$V = \frac{10.73 \cdot \mu \cdot T}{W \cdot p} = \frac{10.73 \times 180.5 \times 10^{-7} \times 936.4}{28.47 \times 3.025} = 2.106 \times 10^{-3}$$

$$dp \approx -0.018$$

$$\therefore p_m \approx 3.025$$

$$Re = \frac{IV - V_{e1} \cdot d}{V} = \frac{307 \times 1.996 \times 10^{-5}}{2.106 \times 10^{-3}} = 2.910$$

$$dV \approx -3$$

$$\therefore V_m \approx 2087$$

$$\phi = \frac{3.525}{p} \left(\frac{T}{460} \right)^{1.81} \times 10^{-3} = \frac{3.525}{3.025} \times \left(\frac{936.4}{460} \right)^{1.81} \times 10^{-3} = 4.220 \times 10^{-3}$$

$$dV_e \approx 37$$

$$\therefore V_{em} \approx 1780$$

$$Sc = \frac{V}{\phi} = \frac{2.106}{4.220} = 0.4991$$

$$Nu_D = 2 + 0.6(Sc)^{1/3}(Re)^{1/2} = 2 + 0.6(2.910)^{1/2}(0.4991)^{1/3} = 2.812$$

$$h_D = \frac{Nu_D \cdot \phi}{d} = \frac{2.812 \times 4.220 \times 10^{-3}}{1.996 \times 10^{-5}} = 594.5$$

$$Pr \text{ (From Charts)} = \frac{Pr_a + \omega Pr_v}{1 + \omega} = \frac{.6663 + 0.0585 \cdot .9635}{1.058} = 0.6830$$

$$Nu_T = 2 + 0.6(Pr)^{1/3}(Re)^{1/2} = 2 + 0.6(2.910)^{1/2}(.6830)^{1/3} = 2.901$$

$$k \text{ (From Charts)} = \frac{k_a + \omega k_v}{1 + \omega} = \frac{(6.82 + 0.0585 \times 5.96) \times 10^{-6}}{1.058} = 6.776 \times 10^{-6}$$

$$h_T = \frac{Nu_T \cdot k}{d} = \frac{2.901 \times 6.776 \times 10^{-6}}{1.996 \times 10^{-5}} = 0.9848$$

$$q_{v\infty} = 2.702 \times \frac{p}{T} \times \frac{\omega}{1 + \omega} = 2.702 \times \frac{3.025}{936.4} \times \frac{0.05848}{1.094} = 4.666 \times 10^{-4}$$

$$\text{Try } T_{em} = 528.2 \quad \therefore \text{From Charts } q_{v2} = 11.09 \times 10^{-4} \text{ and } \mathcal{L} = 1050.5$$

$$\therefore h_T(T - T_e) = 0.9848 \times 408.2 = 401.9$$

$$\therefore \mathcal{L} \cdot h_D(p_{v2} - p_{v\infty}) = 1050.5 \times 0.3819 = 401.2$$

$$dt = \frac{-31.2 \times dd}{h_D(p_{v2} - p_{v\infty})} = \frac{-31.2 \times -7.863 \times 10^{-8}}{0.3819} = 6.424 \times 10^{-6}$$

$$dV_e = 18 \times (Re)^{1/3} \cdot \frac{\mu}{\rho_e \cdot d^2} \cdot (V - V_e) \cdot dt = 18 \times 1.428 \times 180.5 \times 10^{-7} \times 307 \times 6.424 \times 10^{-6} = 36.8$$

$$dz = V_e \cdot dt = 1779 \times 6.424 \times 10^{-6} = 1.371 \times 10^{-1}$$

$$\frac{-w_e \cdot dh_e}{w \cdot C_p \cdot T_0} =$$

$$\frac{dw_e}{w} \approx -0.0023$$

$$\therefore \frac{w_e}{w_m} \approx 0.1633$$

$$dC_p \approx 0.0002$$

$$\therefore C_{pm} \approx 0.2597$$

$$\frac{dw}{w} = \frac{d\omega}{1 + \omega} = \frac{2.054 \times 10^{-3}}{1.058} = 1.941 \times 10^{-3}$$

$$dT_0 \approx -11.4$$

$$\therefore T_{0m} \approx 1288$$

$$\frac{w_e \cdot dV_e}{w \cdot V} = 0.1633 \times \frac{36.8}{2087} = 2.879 \times 10^{-3}$$

$$\frac{dW}{W} = \frac{-379}{0.621 + \omega} \times \frac{d\omega}{1 + \omega} = \frac{-379}{.6795} \times 1.941 \times 10^{-3} = -1.083 \times 10^{-3}$$

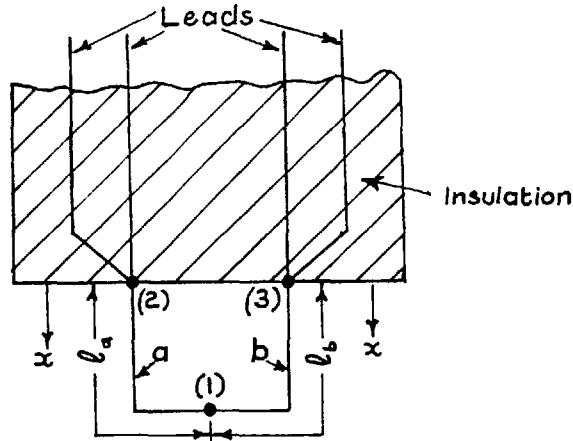
$$dT = -10.2$$

$$\frac{dk}{k} = \frac{2.1 \times 10^{-5} dT + .475 \cdot d\omega}{2.273 + 2.1 \times 10^{-5} T + .475 \cdot \omega} - \frac{2.1 \times 10^{-5} dT + .3646 \cdot d\omega}{.1589 + .0197 + .0213}$$

$$= \frac{(9.757 - 2.142) \times 10^{-4}}{2.273 + .0197 + .0278} - \frac{(7.489 - 2.142) \times 10^{-4}}{.1589 + .0197 + .0213} = 9.6 \times 10^{-5}$$

True Calculated Values				Extrapolated Values	
γ	$= 1 - M^2$	$= -0.914$		$dM^2 \approx +0.012$	
η	$= k \cdot M^2$	$= 1.381 \times 1.914$	$= 2.641$	$\therefore M_m^2 \approx 1.914$	
$\frac{k-1}{2} \cdot M^2$	$= 0.1905 \times 1.914$	$= 0.3646$		$dc_p \approx 0.0002$	
δ	$= 1 + \frac{k-1}{2} \cdot M^2$	$= 1.365$		$\therefore c_{pm} \approx 0.2597$	
h_{vr} (From Steam Tables)	$= 1281.5$	Similarly $h_e = T_e - 32 = 36$		$dT_0 \approx -11.4$	
$c_p \times T_0$	$= .2597 \times 1288$	$= 334.5$		$\therefore T_{0m} \approx 1288$	
β	$= \frac{h_{vr} - h_e}{c_p T_0} + \frac{\frac{k-1}{2} M^2}{1 + \frac{k-1}{2} M^2} (1 - \gamma^2)$	$= \frac{1245}{334.5} + \frac{.3646}{1.365} \times .2726$	$= 3.795$	$y_m \approx \frac{1780}{2087}$	
α	$= 1 + (1 + \frac{\omega^2}{\omega^2} \cdot \gamma^2) \cdot \frac{k-1}{2} M^2$	$= 1 + (1 + .1633 \times .7274)$	$= 1.408$	≈ 0.8529	
$\frac{dM^2}{M^2}$	$= .032030 - .005140 - .015600 - .004318 - .000096$	$= .00688$			
$\frac{dV}{V}$	$= .008051 - .001883 - .006368 - .001185$	$= -.00138$			
$\frac{dp}{p}$	$= -.022030 + .033255 + .009213 + .003132$	$= -.00643$			
$\frac{dT}{T}$	$= -.015920 + .001372 + .002852 + .000864$	$= -.01083$			
$\frac{dT_0}{T_0}$	$= -.007366 - .001311 - .000289 + .000067$	$= -.00890$			
$\frac{d\theta_0}{\theta_0}$	$= -.007566 - .000314 + .000073$	$= -.00781$			
$\frac{dp_0}{p_0}$	$= .008981 - .001720 - .005879 - .001049 - .000068$	$= .00027$			
$\frac{dP_0}{P_0}$	$= .008263 - .001720 - .001119 - .001138 - .000048$	$= .00424$			
M_{17}^2	$= M_{16}^2 (1 + \frac{dM^2}{M^2})$	$= 1.908 (1 + .00688)$	$= 1.921$	True Increments	
V_{17}	$= V_{16} (1 + \frac{dV}{V})$	$= 2088 (1 - .00138)$	$= 2085$	$dM^2 = .013$	
p_{17}	$= p_{16} (1 + \frac{dp}{p})$	$= 3.033 (1 - .00643)$	$= 3.013$	$dV = -3$	
T_{17}	$= T_{16} (1 + \frac{dT}{T})$	$= 941.4 (1 - .01083)$	$= 931.2$	$dp = -.020$	
T_{017}	$= T_{016} (1 + \frac{dT_0}{T_0})$	$= 12.94 (1 - .00690)$	$= 12.62.5$	$dT = -10.2$	
p_{017}	$= p_{016} (1 + \frac{dp_0}{p_0})$	$= 9.504 (1 + .00027)$	$= 9.507$	$dT_0 = -11.5$	
ω_{17}	$= \omega_{16} + d\omega$	$= (5.745 + 2.504) \times 10^{-2}$	$= 5.950 \times 10^{-2}$	$dc_p = .0002$	
c_{p17}	$= c_{p16} + \frac{1}{1+\omega} \left[2.1 \times 10^{-5} dT + \frac{d\omega}{1+\omega} (2.477 - 2.1 \times 10^{-5} T) \right]$	$= .2596 + \frac{1}{1.058} (-2.142 \times 10^{-4} + 1.941 \times 2280)$	$= .2598$		
k_{17}	$= k_{16} (1 + \frac{dk}{k})$	$= 1.381 (1 + 9.6 \times 10^{-5})$	$= 1.381$		
ω_{e17}	$= \omega_{e16} - \omega_{16} \times \frac{d\omega}{\omega}$	$= .0832 - .0010$	$= .0822$		
ω_{r17}	$= \omega_{r16} (1 + \frac{d\omega_r}{\omega_r})$	$= .5058 (1 + 1.941 \times 10^{-3})$	$= .5068$		
d_{17}	$= d_{16} + dd$	$= (2.000 - .0078) \cdot 10^{-5}$	$= 1.992 \times 10^{-5}$		
W_{17}	$= W_{16} (1 + \frac{dW}{W})$	$= 28.48 \times .9989$	$= 28.45$		
x_{17}	$= x_{16} + dx$	$= 0.160$			
V_{e17}	$= V_{e16} + dV_e$	$= 1761 + 36.8$	$= 1797.8$		
T_{e16-17}	$= 528.2$	at $z = 1.041$			
Z_{17}	$= z_{16} + dz$	$= 1.109$			

	$\frac{-w_e \cdot dh_e}{w \cdot c_p \cdot T_0}$		$\frac{dw}{w}$	1.941×10^{-3}	$\frac{4 \cdot f \cdot dz}{D}$	1.302×10^{-3}	$\frac{w_e}{w} \times \frac{dV_e}{V}$	2.879×10^{-3}	$\frac{dW}{W}$	-1.083×10^{-3}	$\frac{dk}{k}$	9.6×10^{-5}
$\frac{dM^2}{M^2}$	$(1+\eta) \times \frac{\delta}{\gamma}$	$\frac{\delta}{\gamma} [(1+\eta)(2-\beta) - 2\eta \cdot y]$	$\eta \times \frac{\delta}{\gamma}$	$\frac{\eta}{\gamma} [2\delta - \frac{k-1}{k}(1+\eta)y]$	$\frac{1+\eta}{-\gamma}$	-1						
		16.50	3.203×10^{-2}	-3.948	-5.140×10^{-3}	-5.417	-1.560×10^{-2}	3.987	-4.318×10^{-3}	-1	-9.6×10^{-5}	
$\frac{dV}{V}$	$\frac{\delta}{\gamma}$	$\frac{1}{\gamma} [1 + (1-y)\eta - \delta \cdot \beta]$	$\frac{\eta}{2\gamma}$	$\frac{\eta - (\eta - M^2)y}{\gamma}$	$-\frac{1}{\gamma}$	0						
		4.148	8.051×10^{-3}	-1.446	-1.863×10^{-3}	-2.212	-6.368×10^{-3}	1.094	-1.185×10^{-3}	0	0	
$\frac{dp}{p}$	$-\eta \times \frac{\delta}{\gamma}$	$\eta \cdot \frac{\delta}{\gamma} [\beta - 2 + \frac{1+\eta-M^2}{\delta} \cdot y]$	$\frac{\eta(1+\eta-M^2)}{-2\gamma}$	$\frac{\eta[1+(\eta-M^2)(1-y)]}{-\gamma}$	$\frac{\eta}{\gamma}$	0						
		-11.35	-2.203×10^{-2}	2.500	3.255×10^{-3}	3.200	9.213×10^{-3}	-2.892	3.132×10^{-3}	0	0	
$\frac{dT}{T}$	$(1-\eta) \times \frac{\delta}{\gamma}$	$\frac{(1-\eta)\delta \cdot \beta + (\eta-M^2)[1+\eta(1-y)]}{-\gamma}$	$\frac{\eta(\eta-M^2)}{-2\gamma}$	$\frac{\eta-M^2}{-\gamma} [y + \eta(1-y)]$	$\frac{\eta-M^2}{\gamma}$	0						
		-8.204	-1.592×10^{-2}	1.054	1.372×10^{-3}	.9906	2.852×10^{-3}	-7.977	8.639×10^{-4}	0	0	
$\frac{dT_0}{T_0}$	1	$-\beta$	0	$\frac{(\eta-M^2)y}{-\delta}$	$\frac{\eta-M^2}{2 \cdot \delta}$	$\frac{M^2}{2 \cdot \delta}$						
		-3.795	-7.366×10^{-3}	0	0	-4.555	-1.311×10^{-3}	.2671	-2.893×10^{-4}	.7011	6.731×10^{-5}	
$\frac{d\theta_0}{\theta_0}$	$\frac{\delta}{\alpha}$	$\frac{\delta \cdot \beta + k-1/2 \cdot M^2(1+w_e/w)y^2}{-\alpha}$	0	0	$\frac{(\eta-M^2)(1+w_e/w)y^2}{2 \cdot \alpha}$	$\frac{M^2(1+w_e/w)y^2}{2 \cdot \alpha}$						
		-3.898	-7.566×10^{-3}	0	0	0	0	.2897	-3.137×10^{-4}	.7607	7.303×10^{-5}	
$\frac{dp_0}{p_0}$	$-\frac{\eta}{2}$	$\frac{\eta}{2} [\beta - 2(1-y)]$	$\frac{-\eta}{2}$	$\frac{\eta}{2} [\frac{(\eta-M^2)y}{\delta} - 2]$	$\frac{\eta}{2 \cdot \delta}$	$\frac{\eta}{2(k-1)\delta} - \frac{k}{(k-1)^2} \ln \alpha$						
		4.627	8.981×10^{-3}	-1.321	-1.720×10^{-3}	-2.042	-5.879×10^{-3}	.9683	-1.049×10^{-3}	-.7130	-6.845×10^{-5}	
$\frac{dP_0}{P_0}$	$-\frac{\eta}{2}$	$\frac{\eta}{2\alpha} (1 + \frac{w_e}{w} y^2)(\delta \cdot \beta - 1) - \eta(1-y)(1 - \frac{1+y}{2\alpha})$	$\frac{-\eta}{2}$	$-\eta(1-y)$	$\frac{\eta(1+w_e/w \cdot y^2)}{2\alpha}$	$\frac{k}{(k-1)^2} [\frac{\alpha-1}{\alpha} - \ln \alpha]$						
		4.257	8.263×10^{-3}	-1.321	-1.720×10^{-3}	-.3888	-1.119×10^{-3}	1.051	-1.138×10^{-3}	-.4976	-4.777×10^{-5}	

13. APPENDIX DCONDUCTION LOSS IN THERMOCOUPLE LEADS

The thermocouple consists of one main couple (1) measuring the gas stream temperature, and two subsidiary couples (2) and (3) to account for conduction losses.

It is assumed that radiation losses are negligible, that the gas stream temperature is constant between (1) - (2) and (1) - (3), that the main loop (2) - (1) - (3) corresponds to a uniform rod in cross flow of materials a and b of lengths l_a and l_b and that the recovery factor of the rod is known.

$$\text{The True Gas Temperature } T_g = T + \frac{rV^2}{2gJc_p}$$

where r = recovery factor of the rod.

The equation for conduction loss in a rod is of the form

$$T - T_g = C_1 e^{mx} + C_2 e^{-mx} \quad (\text{D.1})$$

where x is the length from a point of known temperature to the point of temperature measurement in the gas stream.

Equation D.1 may be written as:

$$\theta = C_1 e^{mx} + C_2 e^{-mx} \quad \text{where } \theta = T - T_g$$

$$\therefore \frac{d\theta}{dx} = m \cdot C_1 e^{mx} - m \cdot C_2 e^{-mx} \quad (D.2)$$

We may solve for C_1 , C_2 , and T_g as follows:-

For wire a at (2) where $x = 0$

$$\begin{aligned} T_2 - T_g &= C_{1a} + C_{2a} \\ \therefore C_{1a} &= (T_2 - T_g) - C_{2a} \end{aligned} \quad (D.3)$$

At (1) where $x = \ell_a$

$$T_1 - T_g = C_{1a} e^{m_a \ell_a} + C_{2a} e^{-m_a \ell_a} \quad (D.4)$$

Substituting Equation D.3 in Equation D.4 we get

$$T_1 - T_g = (T_2 - T_g) e^{m_a \ell_a} - C_{2a} e^{m_a \ell_a} + C_{2a} e^{-m_a \ell_a}$$

which may be arranged to give:

$$C_{2a} = \frac{(T_1 - T_g) - (T_2 - T_g) e^{m_a \ell_a}}{-(e^{m_a \ell_a} - e^{-m_a \ell_a})} \quad (D.5)$$

and

$$C_{1a} = \frac{(T_1 - T_g) - (T_2 - T_g) e^{-m_a \ell_a}}{(e^{m_a \ell_a} - e^{-m_a \ell_a})} \quad (D.6)$$

Similarly for wire b:

$$C_{1b} = \frac{(T_1 - T_g) - (T_2 - T_g) e^{-m_b \ell_b}}{(e^{m_b \ell_b} - e^{-m_b \ell_b})} \quad (D.7)$$

and

$$C_{2b} = \frac{(T_1 - T_g) - (T_2 - T_g) e^{m_b \ell_b}}{-(e^{m_b \ell_b} - e^{-m_b \ell_b})} \quad (D.8)$$

Also at (1) assuming that $\frac{d\theta}{dx}$ is a continuous function

$$m_a \cdot C_{1a} e^{m_a \ell_a} - m_a C_{2a} e^{-m_a \ell_a} = m_b \cdot C_{1b} e^{m_b \ell_b} - m_b \cdot C_{2b} e^{-m_b \ell_b}$$

$$\therefore C_{1a} e^{m_a \ell_a} - C_{2a} e^{-m_a \ell_a} = \frac{m_b}{m_a} \left[C_{1b} e^{m_b \ell_b} - C_{2b} e^{-m_b \ell_b} \right] \quad (D.9)$$

Now

$$m = \sqrt{\frac{S \cdot h}{k_r \cdot A}}$$

where h = heat transfer coefficient of Rod
 s = Rod Circumference
 A = Rod Cross Sectional Area
 k_r = Thermal Conductivity of Rod

For a circular rod this becomes

$$m = \sqrt{\frac{4 \cdot h}{k_r \cdot d}} \quad (D.10)$$

For a cylinder in Cross Flow we are given (Ref.7)

$$Nu = 0.615 (Re)^{0.466} \quad (D.11)$$

where $Re = \frac{V \cdot d}{\nu}$ and $h = \frac{Nu \cdot k}{d}$

k = Thermal Conductivity of Gas stream

Given Gas Stream Velocity and Properties and the properties of the metals and using Equations (D.5, 6, 7, 8, 9, 10 and 11) the heat loss due to conduction may be calculated and the true value of T_g obtained.

14. APPENDIX EGOVERNING PHYSICAL EQUATIONS FOR DROPLETS

Consider a cloud of N droplets of uniform diameter d_o , Velocity V_l , and Temperature T_l . These are travelling in a gas stream of velocity V , Temperature T , pressure p , and specific humidity ω . Mutual interaction between the droplets is ignored and the gaseous medium surrounding each droplet is considered as infinite in extent.

14.1 Droplet Number versus Stream Humidity

Since the droplets are incompressible and of uniform and of unchanging diameter per unit time, the mass rate of liquid flow at any section is proportional to their number N . Local humidity which is a measure of loss in liquid mass flow is given by

$$X = \frac{\omega - \omega_o}{\Omega(1 + \omega_o)} = 1 - \frac{N}{N_o}$$

Change in number of droplets is related to change in humidity by

$$-\frac{dN}{N_o} = dX = \frac{d\omega}{\Omega_o(1 + \omega_o)} \quad (E.1)$$

14.2 Mass Transfer

Mass transfer rate due to evaporation is, according to Ranz and Marshall (Ref.33) expressed in terms of mass transfer coefficient, surface area and difference in spatial mass concentration.

$$-\frac{d}{dt} \left(N \cdot \frac{\pi \cdot d_o^3}{6} \cdot \rho_l \right) = h_D \cdot N \cdot \pi \cdot d_o^2 (\rho_{ve} - \rho_{v\infty})$$

$$\therefore \frac{dN}{dt} = -\frac{6 \cdot N \cdot h_D}{d_o \rho_l} (\rho_{ve} - \rho_{v\infty}) \quad (E.2)$$

$$\text{where } Nu_D = 2 + 0.6 \cdot (Sc)^{\frac{1}{3}} \cdot (Re)^{\frac{1}{2}} \quad (E.3)$$

$$\text{and } h_D \equiv \frac{N_{uD} \cdot \varnothing}{d_o}, \quad Sc \equiv \frac{\mu}{\rho \cdot \varnothing}, \quad \text{and } Re = \frac{\rho \cdot d_o |V - V_e|}{\mu} \quad (\text{E.3b})$$

ρ_{v_e} = Reciprocal of the specific volume of saturated water vapour at temperature T

$$\text{and } \rho_{v\infty} = \frac{p}{R_a \cdot T} \cdot \frac{\omega}{1 + \omega \cdot \frac{W_v}{W_a}} \quad (\text{E.4})$$

14.3 Heat Transfer

From the first law of thermodynamics:

$$h_T (N \pi d_o^3) (T - T_e) = \frac{N \pi d_o^3}{6} \cdot \rho_e \cdot \frac{d}{dt} (c_{pe} \cdot T_e) - \mathcal{L} \cdot \frac{d}{dt} \left(N \cdot \frac{\pi d_o^3}{6} \cdot \rho_e \right)$$

$$\therefore \frac{dT_e}{dt} = \frac{6 \cdot h_T (T - T_e)}{d_o \cdot \rho_e \cdot c_{pe}} + \frac{\mathcal{L}}{N \cdot c_{pe}} \cdot \frac{dN}{dt} \quad (\text{E.5a})$$

From Equations (E.1) and (E.2) this may be written as

$$\frac{dT_e}{dX} = \frac{-1}{c_{pe}(1-X)} \left[\mathcal{L} - \frac{h_T (T - T_e)}{h_D (\rho_{v_e} - \rho_{e\infty})} \right] \quad (\text{E.5b})$$

$$\text{and } Nu_T = 2 + 0.6 (Pr)^{\frac{1}{3}} \cdot (Re)^{\frac{1}{2}} \quad (\text{E.6})$$

$$\text{where } Nu_T \equiv \frac{h_T \cdot d_o}{k}, \quad Pr \equiv \frac{c_p \cdot \mu}{k} \quad (\text{E.7})$$

14.4 Droplet Cloud Acceleration

Using the conventional definition of drag coefficient for spheres we get for the droplet cloud

$$C_D \cdot \frac{N \pi d_o^2}{4} \cdot \frac{\rho}{2} (V - V_e) |V - V_e| = \rho_e \cdot \frac{N \pi d_o^3}{6} \cdot \frac{dV_e}{dt}$$

$$\frac{dV_e}{dt} = \frac{3}{4} (C_D \cdot Re) \frac{\mu}{\rho_e \cdot d^2} (V - V_e) \quad (\text{E.8})$$

14.5 Displacement of Droplet Cloud

The distance travelled by the droplet cloud in time t is given by

$$\frac{dz}{dt} = V_d \quad (E.9)$$

From the above set of equations it may be seen that assuming a constant droplet diameter leaves the governing equations unchanged from those assuming a constant number of droplets. Assuming identical conditions at the start, the numerical solutions solved on the different assumptions of constant droplet number and constant droplet diameter will diverge due to the progressive divergence of Reynolds Number and heat transfer coefficient.

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