

HEAT TRANSFER IN LOW SPEED
SEPARATED FLOWS

by

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Dedicated to

My family

Yuk Ying

With love and affection

SUMMARY

This thesis is divided into two parts. The first part contains heat transfer measurements on the forward face of an isothermal normal flat plate. An energy balance between the heat supplied to the model and the heat convected in the wake region is provided in order to assess the accuracy of the heat transfer measurement technique. The effect of free-stream turbulence on the stagnation point heat transfer has been investigated in detail. The separate effects of turbulence intensity ($\frac{\sqrt{u'^2}}{U_\infty}$), integral length scale (L_x) and Reynolds number have been studied.

The second part of the thesis contains heat transfer measurements in the separated, reattached and redevelopment regions of two-dimensional flow over an isothermal flat plate with a blunt leading edge. Mean and fluctuating quantities of velocity and temperature in the flow field have been measured. Measurements of velocity and temperature spectra have been carried out. An energy balance between the heat input and the convective heat in the shear layer downstream has also been obtained. The effect of free-stream turbulence on separated flow heat transfer has been studied. The present measurements of velocity and temperature are restricted to the reattachment region and the centre of the separation bubble.

The present investigations were confined to low speed flow ($U_\infty \sim 15$ m/s) with a small temperature difference between the model and the free-stream ($T_w - T_\infty \sim 10^\circ\text{C}$). In the turbulent free-stream studies, the intensity ($\frac{\sqrt{u'^2}}{U_\infty}$) of the free-stream did not exceed 10%.

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LIST OF SYMBOLS

Principal Notation

A	Area
b	Breadth of the model
c	Constant of proportionality
C_p	Specific heat of fluid at constant pressure
C_p	Pressure coefficient
C_{p_b}	Base pressure coefficient
$\tilde{C}_{p_R}$	Pressure recovery factor
C_{p_s}	Pressure coefficient at separation
d	Diameter of the model
D	Drag of the model
D	Model thickness of the blunt flat plate
$E(k)$	Velocity spectrum function
f	Function of
h	Half the normal flat plate height
h	Heat transfer coefficient
H	Half the blunt flat plate thickness
k	Wave-number
k	Thermal conductivity
k_f	Thermal conductivity of fluid
k_{air}	Thermal conductivity of air
l	Mixing length
l	Thickness of shear layer
L_x	Integral length scale
n	Reynolds number dependence
n	Wave-number
$\tilde{n}$	Non-dimensionalised wave-number
$\tilde{n}_v$	Viscous wave-number based on the results of Suter et al. ¹⁶
$\tilde{n}_i$	Inviscid wave-number based on the results of Bearman ⁷²
Nu	Nusselt number
Nu_H	Nusselt number based on half the blunt flat plate thickness
Nu_R	Nusselt number based on the reattachment length
Nu_x	Nusselt number based on the x distance

P	Mean pressure
P_b	Base pressure
Pr	Prandtl number
PSD	Power spectral density
Q	Total rate of heat transfer through an area
$\dot{q}$	Heat transfer rate
q^2	Twice the turbulent kinetic energy $\equiv u^2 + v^2 + w^2$
R	Reattachment length
R	Resistance
Re	Reynolds number
Re_H	Reynolds number based on half the blunt flat plate thickness
Re_{u_s}	Reynolds number based on the separation velocity
Re_δ	Reynolds number based on boundary layer thickness
T_w	Temperature of the wall
T_∞	Temperature at infinity
T_f	Free-stream temperature
T	Mean temperature
T_b	Temperature at the base of the model
u_i, u_e	Velocity just outside the edge of the shear layer
u, v, w	Fluctuating velocity in the x, y, z directions
U, V, W	Total velocity in the x, y, z directions
$\sqrt{u^2}/U_\infty$	u-component turbulence intensity based on the free-stream velocity
$\sqrt{u^2}/U$	u-component local turbulence intensity
U	Free-stream velocity
U_s	Separation velocity
U_e	Velocity at the edge of the separated shear layer
U_c	Convection velocity
$-\overline{uv}$	Reynolds stresses
$\overline{v\theta}$	Transfer of enthalpy in the v direction
V_E	Entrainment velocity
x	Distance from the stagnation point
x	Distance from the leading edge of the blunt flat plate

Greek Symbols

α	Temperature coefficient of resistance
β	Strain gauge factor
δ	Boundary layer thickness
δ_T	Thermal layer thickness
ϵ	Dissipation rate
Δ	Denotes difference
ϵ_m	Eddy diffusivity of momentum
ϵ_h	Eddy diffusivity of heat
η	Similarity variable
θ	Angle
θ	Fluctuating temperature
$\sqrt{\theta^2}$	r.m.s. value of fluctuating temperature
κ	Thermometric conductivity
μ	Viscosity of fluid
ν	Kinematic viscosity of fluid
ρ	Density of fluid
σ	Prandtl number
ν_t	Eddy viscosity
τ	Shear stress
τ_w	Shear stress at the wall
ϕ	Velocity potential
ψ	Stream function

1. INTRODUCTION

1.1 General Introduction to Heat Transfer Phenomenon

1.1.1 The Importance of Heat transfer

There are numerous engineering applications which involve the science of heat transfer. For example, in the design of heat exchangers such as condensers, boilers and radiators, heat transfer analysis is essential for sizing such equipment. In aerospace technology, the temperature-distribution and heat-transfer problems are crucial because of weight limitations and safety considerations. In the design of nuclear reactor cores, the heat transfer analysis of the fuel elements is essential for proper sizing of fuel elements to prevent burnout. In the field of electronics, the heating effects of transistors, transformers, resistors etc. are important. In heating and air-conditioning applications for buildings, a heat transfer analysis is necessary to estimate the amount of insulation required to prevent excessive heat losses or gains.

1.1.2 The Basic Modes of Heat Transfer

There are three basic modes of heat transfer, i.e. conduction, convection and radiation. In industrial processes all three modes often participate simultaneously in the transmission of heat. For a study of heat-transfer processes, however, it is necessary to distinguish clearly among the various modes, since they are subject to different laws.

Conduction is the mode of heat transfer in which

energy exchange takes place from the region of high temperature to the region of low temperature by the kinetic motion or by direct impact of molecules as in the case of fluid at rest, and by the drift of electrons, as in the case of heat conduction in metals. The law of heat conduction is normally known as Fourier rate equation.

$$Q_x = -kA \frac{\partial T}{\partial x}$$

where Q_x is the rate of heat flow through area A in the positive x direction. The positive constant of proportionality k (thermal conductivity) is a property of the material. The minus sign is used to ensure that Q_x is positive in the direction of a negative temperature gradient.

Convection is described as the heat transfer mechanism which occurs in a fluid by the mixing of one portion of the fluid with another portion due to the gross motion of the fluid itself. Convection also takes place when fluid flows over a solid surface at different temperature, as a result of the motion of the fluid relative to the surface. If the fluid motion is induced by external mechanical means (e.g. a pump, a fan etc.), the process is called forced convection. If the fluid motion is caused by buoyancy effect resulting from density difference created by temperature differences existing in the fluid, the process is called free convection. The law which governs convective heat transfer is sometimes called Newton's law of cooling.

$$Q_x = hA(T_f - T_w)$$

where T_f and T_w are the temperatures of the fluid and the surface respectively, and h is called the heat transfer coefficient.

Thermal radiation is the mode of heat transfer by which thermal energy is transported in the form of electromagnetic waves. Solid bodies, liquids and gases are capable of absorbing or releasing thermal energy in this manner. The law which governs the process of radiation is called Stefan - Boltzmann equation.

$$Q = \xi \sigma (T_1^4 - T_2^4)$$

where σ is the Stefan - Boltzmann constant, T_1 , T_2 are the temperatures of the body and the surrounding respectively, and ξ is the coefficient of emissivity.

1.1.3 The Governing Equations of Convective Heat Transfer

The analysis of heat transfer is complicated by the fact that fluid motion plays an important part in heat transfer. The temperature field in the fluid is dependent on the velocity field (and vice versa). For simplicity the flow is assumed to be steady, incompressible ($\rho \approx \text{constant}$), constant viscosity (assuming temperature variations to be sufficiently small - Boussinesq assumption). The equations governing the fluid motion and heat transfer are the momentum, continuity, state and energy equations.

The momentum equations (Navier Stokes equations) take the form (ref. Schlichting'),

$$\begin{aligned} u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu (\Delta^2 u + \frac{1}{3} \frac{\partial \text{div } w}{\partial x}), \text{ where } \text{div } w = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \\ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu (\Delta^2 v + \frac{1}{3} \frac{\partial \text{div } w}{\partial y}) \\ u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu (\Delta^2 w + \frac{1}{3} \frac{\partial \text{div } w}{\partial z}) \end{aligned}$$

The continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

The equation of state is

$$p/\rho = RT$$

The energy equation is

$$\begin{aligned} u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = & \frac{k}{\rho C_p} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \\ & + \frac{1}{\rho C_p} \left(u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} + w \frac{\partial p}{\partial z} \right) + \mu \phi \end{aligned}$$

where ϕ is the dissipation function (containing terms like $(\frac{\partial v}{\partial y})^2$).

It is extremely difficult to solve the above equations except in a few simple and idealised cases. By using similarity arguments, i.e. non-dimensionalising the above equations, the flow field is shown (ref. Schlichting') to be dependent only on Reynolds number ($Re = \frac{U_\infty d}{\nu}$), and the temperature distribution depends on Re and Prandtl number ($Pr = \frac{\nu}{\kappa}$). For engineering purposes, the amount of heat transfer occurring between the fluid and the wall is normally expressed as the Nusselt number Nu

$$Q = \frac{Nu \, k_f \, A \, (T_w - T_f)}{d} \quad \text{or} \quad Nu = \frac{h \, d}{k_f}$$

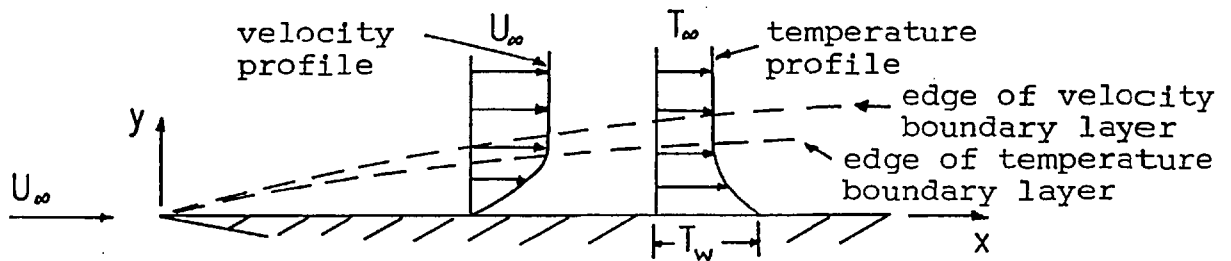
where k_f is the thermal conductivity of the fluid, d is a representative length.

For forced convection (no free-stream turbulence), $Nu = f(Re, Pr)$ is generally true.

1.1.4 Forced Convection in Boundary Layer Flows (Laminar)

The boundary layer concept was originally proposed by Prandtl. In many flow situations, where there is a temperature difference existing between the wall and the fluid a temperature gradient of appreciable magnitude is confined to a thin layer of fluid adjacent to the wall. At greater distances from the wall the temperature gradients may be assumed to be negligible. One may think in terms of

a temperature boundary layer within which thermal effects are concentrated, together with the velocity boundary layer within which frictional effects are concentrated (see diagram below). The heat transfer rate is determined by the intensity of the conductive and convective transport of fluid within the boundary layer. The heat transfer coefficient is essentially determined by the thickness of this layer. The boundary layer thickness is in turn dependent upon the fluid flow which occurs over the surface.



In general the boundary layer concept provides a good description of the velocity and temperature fields, provided that the velocity and temperature gradients in the stream direction are much smaller than those in the direction perpendicular to the wall. The equations of momentum and energy are much simplified.

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{dp}{dx} + \nu \frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{u}{\rho C_p} \frac{dp}{dx}$$

The above equations are applicable to two dimensional,

steady, incompressible flows (with laminar boundary layer) with constant values of C_p, k, μ (Boussinesq assumption).

The velocity fields u, v and the temperature field T may be obtained from the three equations, together with the boundary conditions. The momentum and continuity equations are uncoupled from the energy equation for incompressible flow

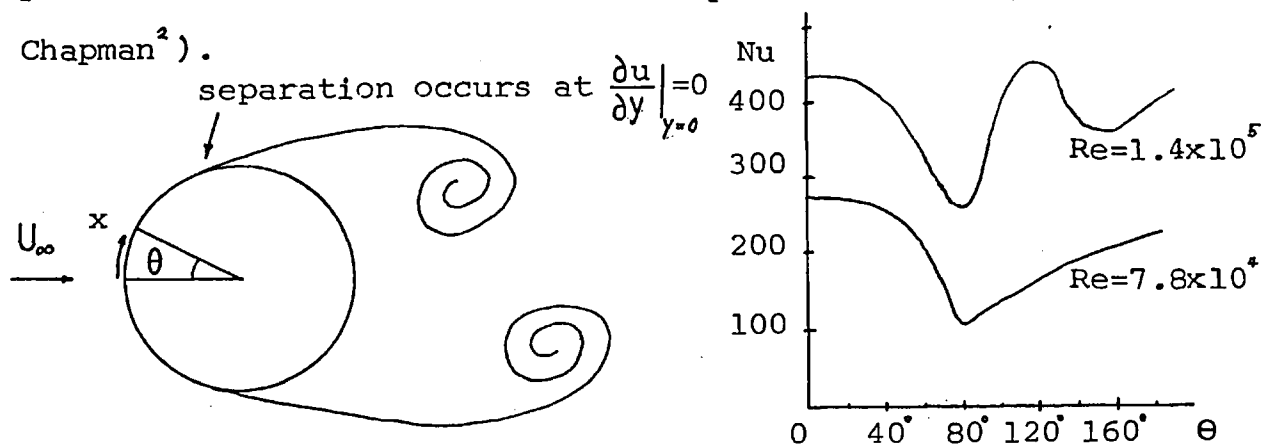
(assume the temperature effects are confined solely to the energy equation), i.e. solve the momentum and continuity equations for the velocity field, then solve the energy equation for the temperature field.

1.1.5 Forced Convection in Separated Flows

There are many flow situations where the boundary layer separates from the surface under certain conditions (e.g. a sharp change in body geometry, adverse pressure gradient, a sharp change of flow direction). Beyond the point of separation, the flow field becomes so complicated that the simplified boundary layer equations are no longer applicable. We consider two types of flow separation here. The first type is the flow separation from a surface (it does not reattach to the surface), e.g. the wake of a circular cylinder. The second type is that after separation the flow reattaches to the surface again, e.g. the flow behind a step.

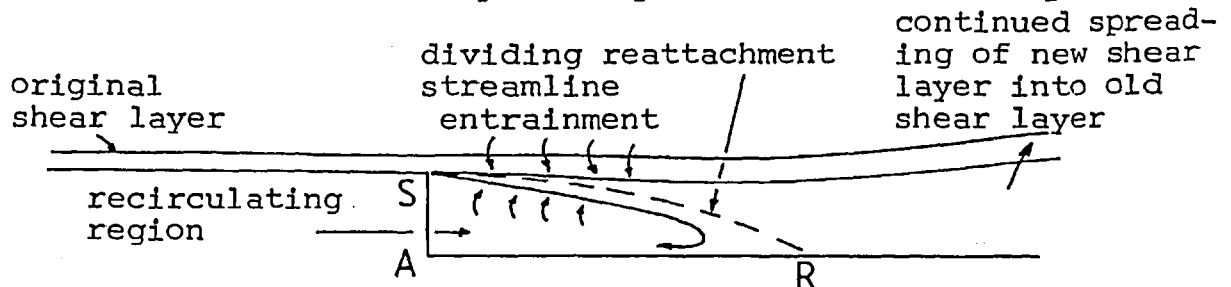
In the case of the flow field around a circular cylinder, a boundary layer is formed on the surface of the cylinder. The pressure decreases from the stagnation point as the flow speeds up around the surface. In the rear half of the cylinder, the pressure again increases. The retarded layer (deficient in momentum) has increasing difficulty in overcoming the rise in pressure. Eventually a point is reached at which $\left. \frac{\partial u}{\partial y} \right|_{y=0} = 0$. Downstream of this point reverse flow sets in, and the boundary layer separates from the surface. The separation phenomenon is associated with the creation and shedding of vortices in the wake region. The local heat transfer for a heated cylinder is strongly affected by the

above flow behaviour. At lower Reynolds numbers the boundary layer is laminar and a minimum value of the local Nusselt number ($Nu = \frac{\dot{q} \theta}{k(T_w - T_f)}$) occurs at the separation point. After separation, a rise in local Nusselt number occurs as transverse mixing is enhanced by the creation of eddies. At higher Reynolds numbers a first minimum in Nusselt number is observed when the laminar to turbulent transition takes place, and a second minimum when separation occurs (ref. Chapman²).



In the case of flow behind a step, due to the sudden change in geometry the shear layer separates from the surface at the corner S (see diagram below). The shear layer grows by entrainment from the adjacent fluid, entrainment from the cavity (between the shear layer and the wall) causes the shear layer to curve and eventually reattach (time-averaged) to the surface at R. At the reattachment region, part of the fluid continues downstream as a decaying wake and the remainder sweeps upstream to be eventually entrained by the shear layer. One important aspect of this type of flow separation is that the flow does not shed vortices after separation (in the sense of strong periodic shedding). The local heat transfer at the wall is strongly affected by this flow behaviour. The local Nusselt number ($Nu = \frac{\dot{q} x}{k(T_w - T_f)}$) increases from the point A and

reaches a maximum at the reattachment point R. After reattachment it decreases gradually(ref. Filetti and Kays³).



The momentum and heat transport processes of separated flows are so complex that a concise analytical description is virtually impossible. Empirical relation of the form $Nu = c Re^n$ is normally used to express the relationship between Nusselt number and Reynolds number, where c and n are functions of Reynolds number(ref. Chang⁴).

The section above presents a brief account of the different types of convective(forced) heat transfer. The next section will discuss the common calculation methods used in laminar and turbulent flows.

1.2 Engineering Calculation Methods for Laminar and Turbulent Flows

The calculation of the temperature field depends on the knowledge of the velocity field. The velocity field must therefore be evaluated first. A lot of effort has been spent especially in the development of calculation methods for turbulent flows in recent years. Due to the advent of high speed computers, it is now possible to solve some turbulent flow problems with high precision(e.g. thin shear layers, one should note that they can only be solved with high precision for cases where turbulence models are known to be accurate). This section is divided into four parts(i) laminar shear layers(ii) turbulent shear layers(iii) recirculating flows(iv) numerical methods.

1.2.1. Analysis of Laminar Shear Layers

For the sake of simplicity, the present discussion is confined to thin shear layers (two-dimensional, steady, constant property laminar flows). The equations of motion are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{dp}{dx} + \nu \frac{\partial^2 u}{\partial y^2}$$

We consider both the internal and external flows here. In both cases a solution can be obtained either by a 'differential' method, solving the partial differential equations (continuity and momentum) numerically, or by an 'integral' method, solving ordinary differential equations already integrated in the y direction.

For an external boundary layer flow subjected to boundary conditions $u = v = 0$ at the wall ($y=0$), the solution of velocity is $\frac{u}{u_e} = g(x, y)$; g = function of (x, y) . There are some cases in which $\frac{u}{u_e} = g(\eta)$, where η (similarity variable) is a specific function of x and y . The number of independent variables is reduced from two (x, y) to one (η) so that the equations of motion become ordinary differential equations for u and v . The similarity variable is often in the form of $\eta = \left(\frac{u_e}{\nu x}\right)^{\frac{1}{2}} y$.

For the case of non-similar flows (i.e. $\frac{u}{u_e} = f(x, \eta)$) the Pohlhausen's method (ref. Cebeci and Bradshaw⁵) is often used to obtain solutions. In this method the velocity profile is assumed to be $\frac{u}{u_e} = a_0 + a_1 \eta + a_2 \eta^2 + a_3 \eta^3 + a_4 \eta^4$ where a_0, a_1 , etc. are constants and $\eta = \frac{y}{\delta}$. The extra boundary conditions generated are $\frac{\partial u}{\partial y}, \frac{\partial^2 u}{\partial y^2}$ etc. $\rightarrow 0$ at $y \rightarrow \infty$. A solution for the velocity field can be obtained by evaluating the momentum equation with the boundary conditions (ref. Cebeci

and Bradshaw⁵).

1.2.2 Analysis of Turbulent Shear Layers

Due to the complexity of the Navier Stokes equations, solutions of these exact equations are virtually impossible. The Navier Stokes equations are therefore replaced by the Reynolds-averaged mean-motion equations and the Reynolds stress transport equations (model equations). These are not exact equations, but they do contain enough information about the turbulence to form a closed, soluble set.

The transport equations are

$$\begin{aligned} \frac{DU_i}{Dt} &= \frac{1}{\rho} \frac{\partial}{\partial x_j} (-\overline{\rho u_i u_j}) + \dots\dots \\ \frac{D}{Dt} (\overline{u_i u_j}) &= - \frac{\partial}{\partial x_i} (\overline{u_i u_j u_i}) + \dots\dots \end{aligned}$$

plus other higher order equations.

Most calculation methods model one or more of the Reynolds stress transport equations, by expressing the unknown turbulence quantities on the right hand side as empirical functions of $\overline{u_i u_j}$, the mean rate of strain, etc.. The dimensions of $\frac{D(\overline{u_i u_j})}{Dt}$ are (velocity)³/length and a length scale must be introduced into at least one of these terms. The length scales normally used are

$$\begin{aligned} \ell &\equiv (-\overline{uv})^{1/2} / (\partial U / \partial y) && \text{mixing length} \\ \nu_t &\equiv -\overline{uv} / (\partial U / \partial y) && \text{eddy viscosity} \\ L_x &\equiv \int_0^\infty \overline{u(x)u(x+r_1)} dr_1 / \overline{u^2}(x) && \text{integral scale} \\ L_\epsilon &\equiv (\overline{u_i^2}/2)^{3/2} / \epsilon && \text{where } \epsilon = \frac{\nu}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2 \end{aligned}$$

There are 26 more integral scales like L_x .

There are several levels of turbulent flow partial differential equation models. These are

(a) Zero-equation models: $\nu_t = c_\mu \ell^2 \left| \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \right|$

l from algebraic formula.

(b) One-equation models: $\nu_t = c_\mu k^{\frac{1}{2}} l$, k from transport equation, l from algebraic formula.

(c) Two-equation models: $\nu_t = c_\mu k^2 / \epsilon = c_\mu k^{\frac{1}{2}} l$, k and ϵ (or l) both from transport equations.

(d) Stress-equation models: $\overline{u_i u_j}$ from transport equation. where $k = \overline{u_i^2} / 2$, $c_\mu = \text{constant}$.

The zero-equation models with k not included in the viscosity formulation imply that the mean motion is unaffected by turbulence. The length scale l is presumed known in that it can be specified by an algebraic equation. In two dimensional boundary layer flows, Prandtl's mixing length may be used $\nu_t = c_\mu l^2 \left| \frac{dU}{dy} \right|$. The advantage of the zero-equation models is that no differential equations other than those representing conservation of momentum and mass need to be solved.

The one-equation models require the solution of an equation for turbulent energy and, as a result, allows for its transport. The length scale is specified algebraically and is flow dependent.

The two-equation models assume that the Reynolds shear stress is directly proportional to the kinetic energy squared and to the mean velocity gradient, and is inversely proportional to the dissipation rate. These assumptions are more realistic than those of the zero-equation and one-equation models. The solution of equations for k and ϵ implies that both are calculated, rather than specified.

The stress-equation models use transport equations for eddy viscosity, i.e. $-(\overline{u_i u_j} - \frac{1}{3} \overline{u_i^2} \delta_{ij}) = \nu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$.

Any eddy viscosity transport model can be converted to a Reynolds stress transport model by using the exact 'rate of strain equation', i.e. $\frac{\partial}{\partial x_j} \frac{DU_i}{Dt} + \frac{\partial}{\partial x_i} \frac{DU_j}{Dt} \rightarrow \frac{D}{Dt} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)$. The advantage of these models is that they always give qualitative plausible (laminar like) solutions (for details see Bradshaw et al⁶).

In order to utilize the above models the following conditions are assumed to hold in the flow field: (i) Production = dissipation, in near equilibrium shear flows, (ii) Production = $-\overline{uv} \partial U / \partial y$ and mixing length $\ell = (-\overline{uv}/k)^{1/2} L_\epsilon$ for a simple shear layer, and (iii) Boussinesq's concept of eddy viscosity $-\overline{u_i u_j} = \nu_t \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} k$.

Another type of closure methods is the mean flow integral methods. These methods are based on ordinary differential equations with local assumptions of turbulent properties. One of the widely used integral methods is Head's method (ref. Bradshaw⁷). He assumed an entrainment velocity $V_E = u_e f(H) = \frac{d}{dx} (u_e (\delta - \delta^*))$ and $\frac{\delta - \delta^*}{\theta} = H_1 = f(H)$. Then by using the momentum integral equation, an equation of the form $\theta \frac{dH}{dx} = f \left(\frac{\theta}{u_e} \frac{du_e}{dx}, H, \frac{u_e \theta}{\nu}, \dots \right)$ is obtained which can be solved empirically.

1.2.3 Analysis of Turbulent Recirculating Flows

For turbulent recirculating flows, the process of diffusion is very important. The momentum equation is often expressed in the form of

$$\frac{\partial \phi}{\partial t} + U \frac{\partial \phi}{\partial x} + V \frac{\partial \phi}{\partial y} + S_\phi - \left(\frac{\partial}{\partial x} \left(\Gamma_\phi \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left(\Gamma_\phi \frac{\partial \phi}{\partial y} \right) \right) = 0$$

where $\phi = U, V, W$ etc., $S =$ source terms and $\Gamma_\phi =$ diffusion

coefficient. It can also be expressed in terms of vorticity ω

and stream function ψ ,

$$a_{\phi} \left(\frac{\partial \phi}{\partial x} \frac{\partial \psi}{\partial y} - \frac{\partial \phi}{\partial y} \frac{\partial \psi}{\partial x} \right) + S_{\phi} - \frac{\partial}{\partial x} \left(b_{\phi} \frac{\partial}{\partial x} (c_{\phi} \phi) \right) - \frac{\partial}{\partial y} \left(b_{\phi} \frac{\partial}{\partial y} (c_{\phi} \phi) \right) = 0$$

where $\phi = \omega, \psi$ etc., a_{ϕ} , b_{ϕ} etc. are constants.

The above equations are for two-dimensional, unsteady recirculating flows with constant properties. Most calculation methods for recirculating flow treat the fluid as though it were a laminar non-Newtonian one. The above equations are therefore the same as for laminar flow, but the shear stresses are non-linearly related to the shear strain. They are elliptic equations and therefore can cause upstream influence, they must be treated numerically (ref. Gosman et al⁸).

1.2.4 Numerical Methods for Turbulent Flows

The finite difference methods have found to be very successful in calculating turbulent flows especially thin shear layers. In this class of methods a grid (usually rectangular) is placed on the flow field and the flow quantities are to be calculated only at the nodes of the grid (the net points). Algebraic equations at each net point are obtained from the partial differential equations by approximating each derivative by an appropriate difference quotient (ref. Bradshaw et al⁶). The boundary conditions must be enclosed so that there are as many equations as unknowns.

For parabolic and hyperbolic problems (e.g. turbulent thin shear layer), the difference schemes can be devised that are solved by 'marching' through the net and determining the unknowns at each net point. For flow problems the marching direction will be in the predominant flow direction. Some marching procedures will be implicit in that all the unknowns on some mesh line (perpendicular to the marching direction)

must be simultaneously computed, e.g. the Crank-Nicholson method and Keller box method (Bradshaw et al⁶). In the explicit methods the equations for each point on the mesh line can be solved independently.

For elliptical problems such as recirculating flows, the difference equations are always implicit. Solution schemes will have to repeat many iterations so that upstream influence can be gradually propagated. Two dimensional recirculating flows can now be solved with some degree of precision for many engineering applications, e.g., the flow behind a step.

In the past few decades, a fair amount of theoretical and experimental work has been done on the field of heat transfer in boundary layers and separated flows which will be reviewed in the next two sections.

1.3 Stagnation Point (Laminar Boundary Layer) Heat Transfer

1.3.1 Theoretical Studies of Stagnation Point Heat Transfer

The stagnation point heat transfer of a cylinder has been studied by Goldstein⁹. He assumed that the tangential velocity u , just outside the edge of the stagnation point boundary layer is proportional to the distance from this point x , i.e., $u = cx$. He then adopted the following similarity variables (used by Howarth¹⁰ to work out the velocity field at the stagnation point of a cylinder),

$$\eta = \left(\frac{c}{\nu}\right)^{\frac{1}{2}} y, \quad u = c x f'(\eta), \quad v = -(\nu c)^{\frac{1}{2}} f(\eta)$$

and assumed the temperature distribution in the boundary layer to have the form $T = T_w - (T_w - T_\infty) \theta(\eta)$. The energy

equation (for two-dimensional, steady, inviscid, incompressible flow) was simplified by neglecting the heating effect due to pressure gradient and the dissipation function. By assuming that the energy equation and the momentum equations were uncoupled, the energy equation was reduced from a partial differential equation to an ordinary differential equation. Together with the boundary conditions, he was able to obtain a solution for the temperature distribution. Finally, the average Nusselt number ($Nu = \frac{\dot{q} d}{k(T_w - T_\infty)}$) was found to be a function of Reynolds number ($Re = \frac{U_\infty d}{\nu}$) and Prandtl number ($Pr = \frac{C_p \mu}{k}$). At the stagnation point region of a cylinder ($u_s = cx$ holds), Nusselt number was found to be proportional to $Re^{\frac{1}{2}}$, i.e., $Nu/Re^{\frac{1}{2}} = \text{constant}$. This relationship is valid for all two-dimensional laminar stagnation points (if $u_s = cx$ at the stagnation point). The above relationship has certain limitations, i.e., it only holds for low speed subsonic flow with constant fluid properties.

Frössling¹¹ proposed a more general method to calculate the temperature distribution in the laminar boundary layer for bodies with arbitrary shape (two-dimensional and axially symmetric cases). He assumed a power series for the potential velocity distribution around the body expanded in terms of the length of arc (Blasius series), i.e., $U = u_s x + u_3 x^3 + \dots$. The velocity distribution in the boundary layer was assumed to be $u(x, y) = u_s x f_1(y) + u_3 x^3 f_3(y) + \dots$, and the temperature distribution was of the form $T(x, y) = T_0 + x T_1(y) + x^3 T_3(y) + \dots$. It was found that the functions $T_1(y)$, $T_3(y)$ etc. satisfied ordinary differential equations which included functions of f_1 , f_3 etc. of the velocity

distribution, and T_1, T_3 etc. also depended on Prandtl number. The method considered here is quite cumbersome by its nature, as for the case of slender bodies, a large number of terms in power expansion are needed.

1.3.2 Experimental Studies of Stagnation Point Heat Transfer

The stagnation point heat transfer of a circular cylinder has been studied experimentally by Schmidt and Wenner¹². The local rate of heat transfer ($Nu/Re^{1/2}$) was found to be dependent on Reynolds number, and it increased with an increase in Reynolds number. The base pressure of the circular cylinder was Reynolds number sensitive, the velocity gradient (du_x/dx) at the stagnation point was therefore affected by the change in base pressure and hence $Nu/Re^{1/2}$ was also Reynolds number sensitive.

Experiments on the heat transfer behaviour of a normal flat plate were carried out by Sogin et al¹³ on a model with an isothermal front surface. Tests were made on both the front and back of the model. They found that the Reynolds number effect had significant influence on the local rate of heat transfer ($Nu/Re^{1/2}$) along the model. In fact an increase in Re tended to increase the heat transfer. The cause of this effect may be due to the high model/wind tunnel blockage. Tests on the effect of a splitter plate placed symmetrically behind the model were also made ($3/16$ " gap between model and splitter plate). It was found that the total heat transfer was reduced by 35% due to the presence of the splitter plate and induced a maximum rate of heat

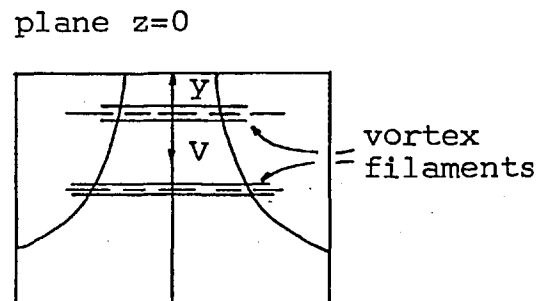
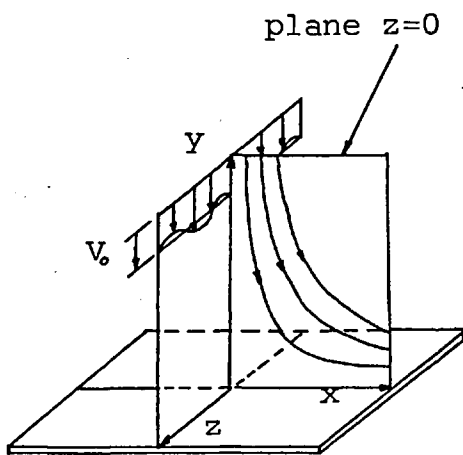
transfer at the centre of the model (rear portion). The researchers suggested that the splitter plate inhibits the process of vortex shedding (it changes C_{p_i} and $du_i/dx|_{x=0}$) and hence the amount of heat transfer (it should be noted that they did not provide the base pressure, so the heat transfer rate cannot be calculated).

1.3.3 Theoretical Studies of the Effects of Free-Stream Turbulence on Stagnation Point Heat Transfer

The interactions between a turbulent flow and a laminar boundary layer are not well understood nor are they easy to calculate from equations. Interaction between turbulence and a body are also difficult to calculate (Hunt¹⁴). Despite these difficulties, an approximated theory for the effect of stream turbulence on stagnation point heat transfer has been developed by Kestin et al¹⁵. They simulated the free-stream turbulence by imposing a simple harmonic oscillation in the free-stream progressing with a velocity equal to the mean flow velocity, i.e., $U = U_\infty \sin \omega t$. The equations of motion and energy were assumed to be two-dimensional, incompressible and time dependent. Effects of the oscillation on laminar boundary layer heat transfer were found to be small for small amplitudes, but the behaviour of such a boundary layer agreed with that observed in heat transfer experiments in the presence of free-stream turbulence. The draw back of this analysis is that such a simple oscillation does not represent the true behaviour of turbulence. Most important of all this approximate theory has not taken full account of the interaction between the turbulent stream and the body,

(i.e., the distortion of turbulence), also quantities such as the body diameter and the size of the turbulent eddies have not been considered.

Sutera and other researchers^{16,17,18} developed a mathematical model for the interaction of vorticity in the oncoming flow with a two-dimensional stagnation point boundary layer. The model was based on two facts, first, large effects due to turbulence occurring outside the stagnation boundary layer and, second, the vorticity carried by the turbulent stream constitutes one of its most important physical characteristics. The flow field is represented by a uniform stream of velocity V_0 approaching a flat, infinite wall placed at right angles to it.



Characteristics of flow problem treated by Sutera et al¹⁶

It is assumed that the stream carries with it a distributed vorticity whose axis is parallel to the x axis. Since fluid elements follow the curved stream lines (see diagram above), the axes of the vortices become stretched, thus causing the associated rotation velocity to increase. Simultaneously,

the action of viscosity exerts a dissipating effect, and only vorticity with a wavelength λ in the z direction (which exceeds a certain magnitude, i.e., the neutral wavelength) amplifies. This vorticity can enter into the boundary layer and induces substantial effects. It was found that the thermal boundary layer is more sensitive to the induced effects than the velocity boundary layer. The reason is possibly because the harmonic components of the periodic temperature variation are larger than the corresponding harmonic components of periodic velocity (Suter et al ¹⁶).

The vortex amplification theory has been demonstrated experimentally by Sadeh et al ¹⁹ (with a normal flat plate) and Sadeh and Brauer ²⁰ (with a circular cylinder). The experimental results yielded close agreement with the theory. The flow visualization (smoke) pictures gave substantial support to the flow development suggested by the theory. One must bear in mind that the experimental turbulence field was highly idealized (turbulence was generated by placing a row of vertical rods upstream of the model), only vorticity in a certain direction was amplified. The advantage of vortex amplification theory is that it provides a good physical explanation of the effect of stream turbulence near a stagnation point. It is true to assume that vortex lines are being stretched as they approach the body. However, in turbulent flows there are large interactions between vortex lines via their induced velocity field (the motion of vortex lines are stretching, tilting and rotating). It is therefore wrong to assume that vortex lines are distributed in a sinusoidal fashion as in the vortex amplification theory.

A different theory was proposed by Galloway²¹ using both the eddy viscosity and the mixing length to formulate solutions for the equations of motion. The effect of turbulence intensity on stagnation point heat transfer of a circular cylinder was evaluated using this theory and was compared with the experimental results, but the agreement is poor (about 50% difference).

Traci and Wilcox²² developed a two-equation model of turbulence based on the formulation of eddy viscosity. In order to complete the analysis, empirical information from the experiments of Smith and Kuethe²³ on circular cylinders was used. Owing to the complexity of the analysis, certain simplifying assumptions and operations were made in order to facilitate the evaluation of the solutions. A similarity type solution was used. They were able to deduce the Nusselt number and friction factor at the stagnation point of a cylinder in a uniform cross flow. The theoretical data were in good agreement with the results of Smith and Kuethe²³.

More recently, Miyazaki and Sparrow²⁴ formulated a model for eddy diffusivity induced by free-stream turbulence using an empirical formula for the local turbulence velocity fluctuation in the boundary layer. The advantages of this method over Traci and Wilcox's method are that the expression for eddy diffusivity ($\sqrt{u^2} = l^* \frac{\partial U}{\partial y} + \sqrt{u_\infty^2} \frac{y}{\delta}$ — the formulation of eddy diffusivity induced by the free-stream turbulence, where l^* is a function of the mixing length) contains a single unknown constant which can be determined from experimental stagnation point heat transfer results and fewer assumptions are made.

As a whole, the eddy viscosity and diffusivity models can give a more quantitative description (i.e., $Nu/Re^{\frac{1}{2}} = f(\text{intensity, length scale, } Re)$) of the effects of free-stream turbulence. However, they do not describe the qualitative effects of turbulence (i.e., the transport of turbulence, the structure of turbulence, the interaction between the stream and the boundary layer) on stagnation point heat transfer, they are merely closure equations. It is difficult to incorporate the eddy viscosity and diffusivity models with the vortex amplification theory, the reason is that it is not possible to relate the eddy viscosity (or mixing length) with the vorticity of the turbulence by a simple relationship.

1.3.4 Experimental Studies of the Effects of Free-Stream Turbulence on Stagnation Point Heat Transfer

Experiments concerning the effects of free-stream turbulence on stagnation point heat transfer of circular cylinders have been carried out by the following researchers: Kestin et al.²⁵, Kestin²⁶, Lowery and Vachon²⁷, Smith and Kuethe²³, Sarma and Sukhatame²⁸. They all concluded that the higher the turbulence intensity the higher the heat transfer rate. Although they were able to present some quantitative results, they were unable, however, to give definite reasons for this effect.

Sogin et al.¹³ obtained some experimental results on the effects of turbulence on stagnation point heat transfer of a normal flat plate. Unfortunately, they studied only one intensity of turbulence. In view of all the above experimental

work, no one has actually studied the effect of length scale on heat transfer. An isotropic turbulent field may be characterised by its intensity and length scale (or in terms of energy spectrum). Previous researchers have only considered the effect of intensity which is not sufficient to describe the full effects of turbulence.

1.4 Separated Flow Heat Transfer

1.4.1 Theoretical Studies of Heat Transfer in Separated Flows

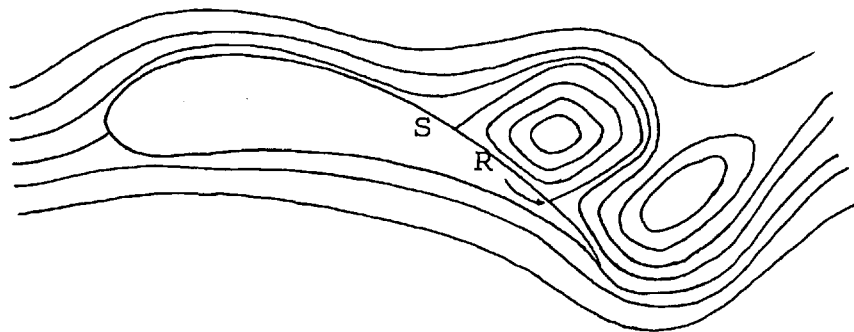
To date, a completely detailed theoretical analysis of subsonic separated flows has not been performed due to the degree of complexity of the problem (the best available numerical analyses of the present time are given in section 1.2). The addition of heat transfer in the separated flow region makes the problem even more difficult to investigate. On the other hand, theoretical analysis of supersonic separated flows with high Reynolds numbers (separated flows over steps, grooves, etc.) have been relatively successful. This success is attributed to the possible assumptions such as (i) non interference of the separated flow region with the external flow and (ii) the flow being steady.

Chapman²⁹ made a considerable effort to solve the heat transfer problem for a laminar separated shear layer. The flow field analysed consisted of a thin, constant pressure, viscous mixing layer separated from a solid surface by an enclosed region (cavity) of low velocity air. The governing momentum, continuity and energy equations were assumed to be two-dimensional, compressible, laminar and without pressure

gradient. These partial differential equations were reduced to ordinary differential equation by similar variables^a (Chang⁴). Together with the boundary conditions, the temperature distribution in the separated flow region was obtained. The disadvantage of this method is that it only applies to laminar separated flow with fairly low Reynolds number. Chung and Viegas³⁰ developed Chapman's analysis further. They were able to obtain the heat transfer rate in the reattachment zone of a two-dimensional separated laminar boundary layer. They found that the average Nusselt number varies with $(Re)^{\frac{3}{2}}$ in the reattachment zone and a slightly lower power of Re outside this zone.

Further attempts at solving the separated laminar flow heat transfer problem were made by Debruge³¹. He formulated an analytical method depicting the flow configuration over the suction side of a Joukowski aerofoil, on which a large separation bubble had developed. Firstly, the potential flow around a circular cylinder with circulation in cross flow (with two equal and opposite vortices at rest behind the cylinder) was transformed into a Joukowski aerofoil with separation bubble (generated by the two vortices).

S=separation point
R=reattachment point



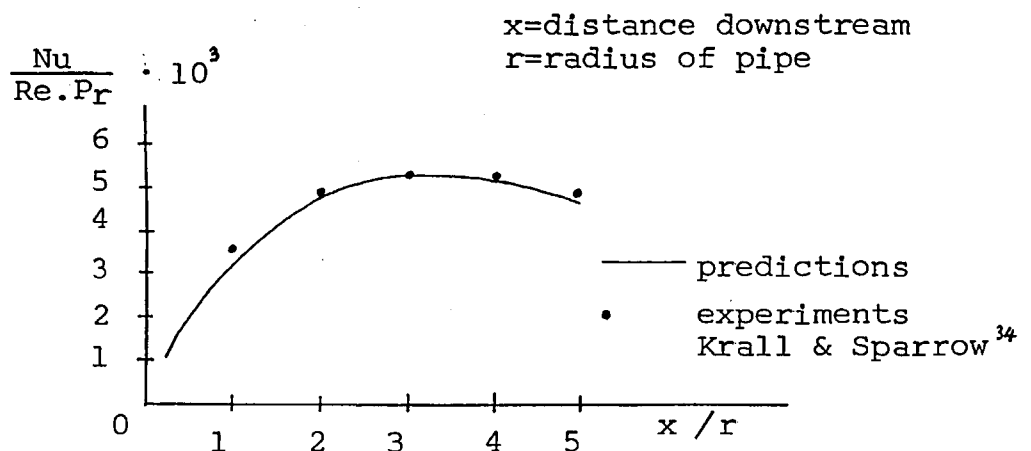
Streamline pattern of the flow over the aerofoil

Secondly, the heat transfer distribution in the laminar boundary layer was then calculated, and the analysis was finally extended to the separated flow region (Debruge³¹). The limitation of this analysis is that a large separation bubble is required to achieve success. In practice, the bubble on an aerefoil is reasonably long compared with its thickness.

Heat transfer in turbulent separated flows has been treated theoretically by Spalding³². He used a one-dimensional model for the transport of turbulence. This model was based on the assumptions that the state of the turbulence of the fluid at a particular point may be governed by the turbulent kinetic energy and the distance from the wall. He related the turbulent energy dissipation, diffusion and generation to these two quantities. The turbulent kinetic energy becomes: $0 = \frac{d}{dy} \left(\frac{\nu_t}{\sigma_q} \frac{d(\frac{1}{2}\overline{q^2})}{dy} \right) - \xi$ (assuming a gradient diffusion law) where σ_q denotes the effective Prandtl number for turbulence energy diffusion, ξ is the dissipation rate and $\nu_t = \text{constant}(\frac{1}{2}\overline{q^2})^2 / \xi$. The turbulence energy and temperature distributions in the fully turbulent region were then calculated (Spalding³²). The Nusselt number was found to depend on $Re^{\frac{4}{3}}$. The results for a heated circular cylinder were in agreement with experiment. The main defect of this model was that it rested on an equation for the transport of turbulence in one dimension; whereas the flow behind a bluff body is at least two dimensional (in the mean).

Runchal and Spalding³³ developed the one-dimensional model of turbulence into two-dimensional model. They selected the separated flow produced by an abrupt enlargement in a pipe of circular cross-section. Predictions were made of the

distributions of velocity, turbulence energy and other properties within the fluid, and of shear stress and heat transfer coefficients along the wall. The computational procedure was carried out as follows: the turbulent flow was treated as being of a laminar non-Newtonian type, the differential equations for the time mean velocity, vorticity, enthalpy and stream function were the same as for laminar flow (except shear stress was non-linearly related to strain). These equations were solved with the general boundary conditions by numerical methods, i.e., finite difference (Gosman et al⁸). In order to develop the analysis further, the eddy length scale was determined empirically to ensure that the recirculation region was of the right length and that the maximum amount of the wall heat flux occurred in the appropriate region. The theoretical prediction was found to be in good agreement with the experimental data of Krall and Sparrow³⁴.



Computed and experimental $Nu / Re.Pr$ downstream of enlargement $Re=51500$, $Pr=3$

1.4.2. Experimental Studies of Separated Flow Heat Transfer

Circular cylinders were amongst the first bluff bodies to be investigated. Fage and Falkner³⁵ studied the wake of a heated cylinder. Schmidt and Wenner¹² examined the effects of Reynolds number change on heat transfer since the Reynolds number governs the state of transition for a circular cylinder. Heat transfer at the rear stagnation point was also correlated with Reynolds number, the correlation was $Nu = cRe^{\frac{2}{3}}$. It was found that blockage has an effect on the constant c . McAdams³⁶ measured the average heat transfer for a circular cylinder.

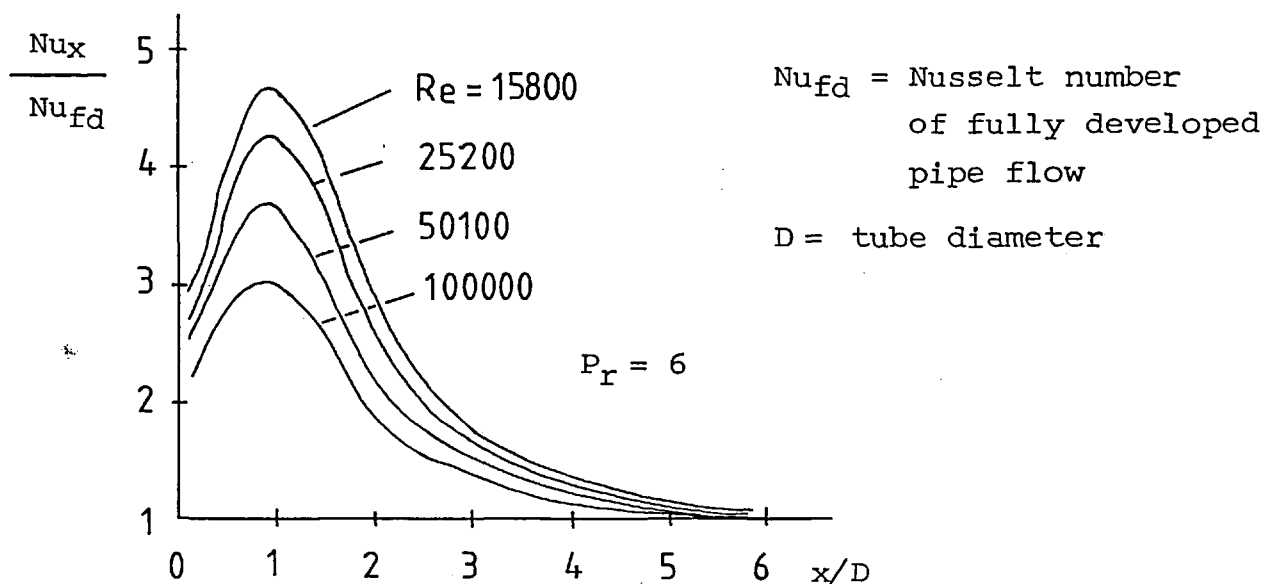
Other bluff body shapes such as a normal flat plate have been studied by Sogin et al¹³ and Sogin³⁷. They found that the $Nu-Re$ correlation at the back of the model was similar to that of circular cylinders, i.e., $Nu = cRe^{\frac{2}{3}}$, except that the constant c for both cases differs. The presence of free-stream turbulence had little effect on the back of the model.

Richardson³⁸ performed experiments on circular cylinders. He studied the effects of the free-stream turbulence level, wind tunnel blockage, cylinder diameter and aspect ratio on the rear stagnation point. He found that the free-stream turbulence intensity and aspect ratio did not have significant effects on the rear-stagnation point heat transfer ($Nu/Re^{\frac{2}{3}}$). The effects of tunnel blockage ratio and cylinder diameter were quite significant ($Nu/Re^{\frac{2}{3}}$ increased almost 100% with a three fold increase in blockage and decreased by about 50% as the cylinder diameter doubled). He suggested that these could be due to errors in the evaluation of conduction losses, the misalignment of the cylinder in the stream and systematic

errors. The tunnel blockage rather than cylinder diameter should have a systematic effect because the effect of the tunnel wall would be to constrain the wake region and cause a different distribution of free-stream velocity from that occurring in the vicinity of the cylinder in an infinite stream.

Seban et al³⁹ and Seban⁴⁰ measured the local heat transfer rate and the recovery factor in the separated and redevelopment regions of a backward/forward facing step. They found that the heat transfer rate reached a maximum at the reattachment point, and the heat transfer rate was dependent on $Re^{0.8}$.

Krall and Sparrow³⁴ performed heat transfer measurements with a circular tube. The flow separation was induced by an orifice situated at the inlet of the heated circular tube. The results showed that the local heat transfer rates in the separated, reattached and redevelopment regions were several times higher than those of a fully developed flow. The increase in heat transfer rate relative to a fully developed flow was accentuated as Reynolds number decreased (see diagram below).

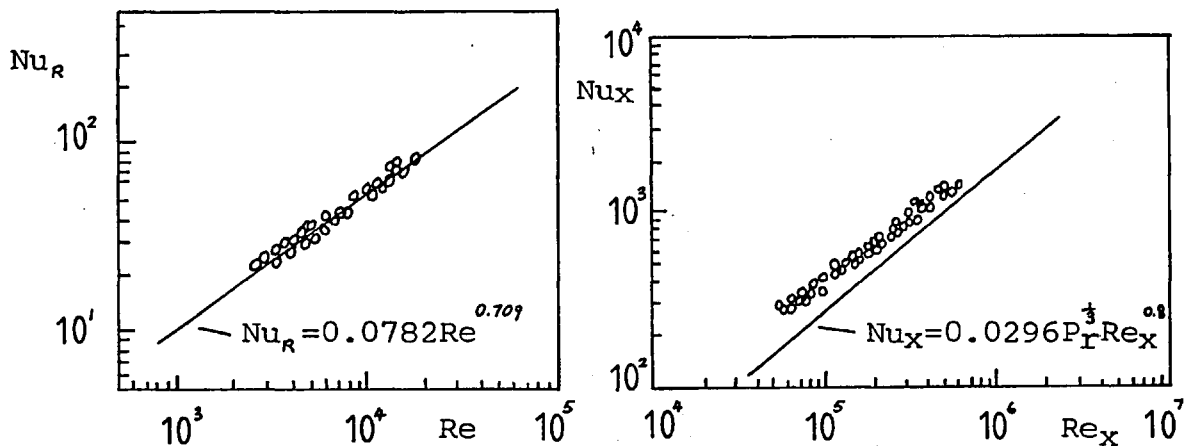


Local heat-transfer results in the separated, reattached, and redevelopment regions

The reattachment point was found to be at maximum in heat transfer. Similar findings were obtained by Filetti and Kays³ in an experiment behind a double step at the entrance to a flat duct.

Seki et al⁴¹ studied effects of both turbulent fluctuations and heat transfer on the flow field behind a downward facing step. They concluded that the heat transfer rate (on the back surface of the step) in the separated flow region was closely related to the behaviour of the transported heat which is represented by the product of normal velocity and temperature fluctuations (i.e., $\overline{v\theta}$) in the shear layer. They also proposed an empirical equation for the local heat transfer coefficient in the separated and reattached regions (i.e., $Nu/Re^{1/2} = 0.45 - 0.24x/L$, where x is the distance from the step and L is the duct height). But the discrepancy with experimental results was about 20%. The experimental results deviated significantly from the separated flow region. A special technique (Shigetomi and Seki⁴²) was developed by these researchers in order to measure the velocity and temperature fluctuations. A single hot wire was placed in the shear layer (hot) and it was allowed to rotate at small angles with respect to the mean flow. The output voltage from the hot wire was the sum of the voltages due to temperature and velocity fluctuations. Since the voltage outputs, angle of inclination and temperature characteristics of the wire were known, the researchers carried out an analysis and obtained fluctuating quantities such as $\overline{uv}$ and $\overline{v\theta}$. The disadvantage of this technique is that the temperature characteristics of the wire have to be determined fairly accurately.

Heat transfer experiment on a blunt leading edge flat plate have been carried out by Ota and Kon⁴³. The experiments were performed with a constant heat flux model. The Nusselt numbers at the reattachment position correlate well with Reynolds numbers to give $Nu_R = 0.0782 Re^{0.709}$. In the redeveloped flow region, a different correlation was required. They tried to correlate the results in the redeveloped flow region using the turbulent boundary layer heat transfer equation, i.e., $Nu_x = 0.0296 Pr^{1/3} Re_x^{0.8}$, but the results did not agree (see diagrams below).



Correlation of Nu_R with Re

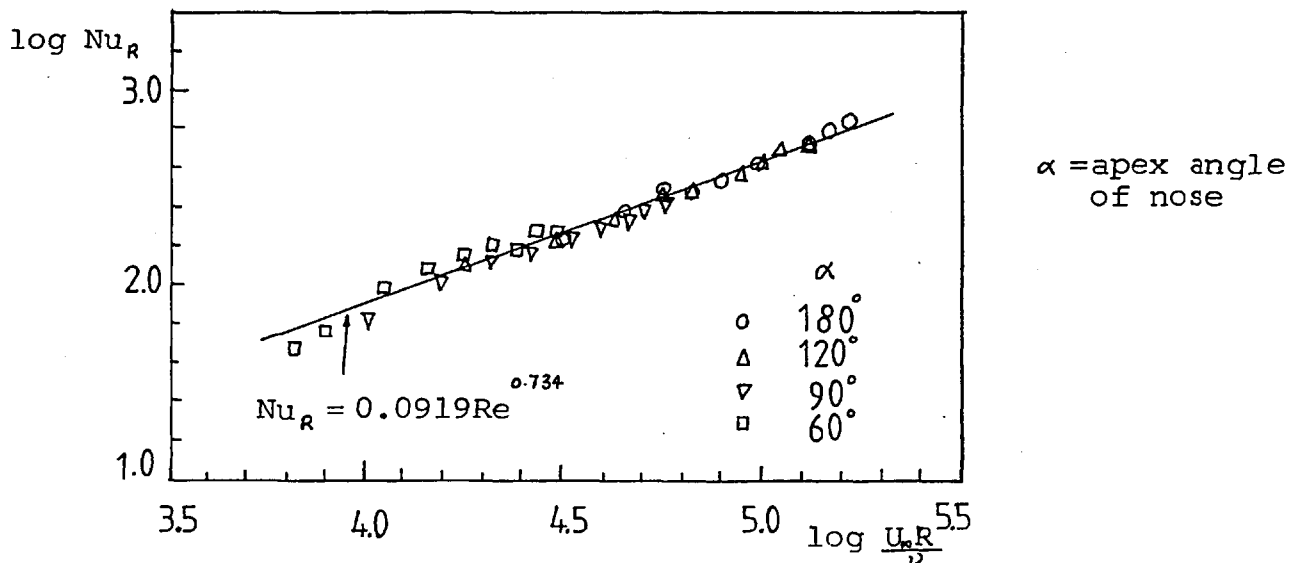
Local Nu_x distributions in downstream region $x/H \geq 20$

In the separated flow region, the local heat transfer distribution showed a peculiar effect, the distribution had a small trough near the centre of the separated flow region. However, no explanation was put forward for this behaviour. It should be noted that the experimental data of Ota and Kon are used for comparison with the present data in separated flow heat transfer because the geometry of their model is identical to the model used in present work.

Experiments performed by Ota and Kon⁴⁴ on a blunt

circular cylinder (with its axis aligned with the flow direction) showed that $Nu-Re$ at reattachment correlates differently from the blunt leading edge flat plate. The correlation obtained for the streamwise circular cylinder was $Nu_R = 0.0875 Re^{0.75}$. They used $Nu/Re^{0.75}$ to evaluate the heat transfer distribution in the separated flow region. But the distribution appeared different from that of the blunt flat plate. There was no trough in the middle of the separated flow region. They observed that the temperature profiles in the separated flow region showed a peculiar deviation from other regions. The temperature $((T-T_\infty)/(T_w-T_\infty))$ decreased sharply from the shear layer region to the wall.

Ota and Kon⁴⁵ also examined the effect of nose shape on blunt leading edge flat plate heat transfer. They suggested that the behaviour of the separated shear layer played a very important part upon the heat transfer in the reattached flow region. The reattachment $Nu-Re$ correlation was found to be independent of nose shape when the reattachment length was used as the reference length. The correlation obtained was $Nu_R = 0.0919 Re^{0.734}$.



Ota et al⁴⁶ made further measurements in the shear layer. They found that the temperature, velocity and turbulence intensity distributions in the redeveloped flow region were strongly dependent upon the nose shape, ^{but} on the contrary, these quantities showed little variations with nose shape in the reattachment region.

Smyth⁴⁷ studied the heat transfer characteristics of an axisymmetric two dimensional turbulent separated flow. The configuration of the test model was a circular disc attached to a circular cylinder. He found that the empirical parameter $Nu/Re^{0.71}$ for the reattachment position correlated fairly well in the separated and redeveloped flow regions. The local $Nu/Re^{0.71}$ distribution was found to be independent of Reynolds number effect. This finding was in agreement with Daigo et al⁴⁸ (same geometry model).

Recently, Sam et al⁴⁹, Test and Lessmann⁵⁰ performed experiments on an inclined rectangular plate. They found that for small angles of incidence (between 0° to 20°) the flow field was similar to that of a blunt flat plate. In the separated flow region, the local heat transfer rate increased with downstream position and reached a maximum in the vicinity of the reattachment point. For large angles of incidence (above 30°) the flow field resembled to that of a normal flat plate. In the laminar flow region the heat transfer rates were about 50% higher than those determined by theory. They suggested that the difference could be due to free-stream disturbances (caused by wake oscillations) which had the same effect as free-stream turbulence.

1.5 The Present Work

1.5.1 The Purpose of the Present Work

From the above review of previous work, it has been shown that the effect of free-stream turbulence on stagnation point heat transfer is still little understood. Although there are a fair amount of experimental results on separated flow heat transfer, the mechanism of heat transfer in separated flows is still not clear. Therefore, the purpose of this research was to carry out experimental studies to develop a better understanding of some of the problems concerning fluid flow with heat transfer.

1.5.2 A General Description of the Present Work

The present research was mainly concentrated on separated flow heat transfer. The logical steps of the present research were: firstly, to study the heat transfer effects of a normal flat plate model (stagnation point heat transfer) and secondly, to study the heat transfer effects of a blunt leading edge flat plate (separated flow heat transfer).

The reasons for studying the normal flat plate first were:

- (i) Possible analogies between stagnation point and reattachment point.
- (ii) Since there are theoretical and experimental studies of stagnation point heat transfer, it seems a good check of the measurement techniques.
- (iii) Very little experimental information on the effect of free-stream turbulence on the stagnation point heat transfer.

The reasons for choosing the blunt leading edge flat plate to study separated flow heat transfer were:

- (i) The blunt flat plate has fixed separation points. The advantages of having fixed separation points are that the origin of the separated flow region is fixed and the effect of Reynolds number change on the bubble structure is small (i.e. C_p is not Re sensitive).
- (ii) There is a lot of experimental data obtained by Cherry⁵¹ on the effects of free-stream turbulence on the mean and fluctuating velocity fields.
- (iii) There are heat transfer data obtained by Ota and Kon⁴³ on a constant heat flux blunt flat plate.

The present research was started off by calculating the stagnation point heat transfer of a normal flat plate which would provide a basic check of the experimental data. This calculation (which will be discussed in Chapter 2) was based on the work of both Goldstein⁹ and Frössling¹¹.

One of the objectives of the present investigation was to design constant wall temperature heat transfer models (isothermal models). The design details are presented in Chapter 3. The idea of studying constant wall temperature heat transfer is important. For momentum consideration, $V=0$ at the wall, the analogy for the temperature field is that $T=\text{constant}$ at the wall. Most researchers have performed experiments with constant heat flux models, the momentum analogy to constant heat flux is that the skin friction is constant at the wall. In reality, this condition ($\tau=\text{constant}$) rarely occurs.

From the review of the previous work (section 1.3),

the present author felt that a fair amount of experimental work needed to be done on the normal flat plate. The basic heat transfer characteristics of the model should be checked first, i.e., the Nu-Re relationship, effects of power input and temperature difference ($T_w - T_\infty$). An energy balance could provide a useful check of the accuracy of the experimental technique, i.e., by measuring the power input and the temperature profiles in the wake. As the previous studies have not provided a better understanding on the effect of turbulence, it was intended that the present investigation should concentrate on the detailed structure of the turbulent flow.

In view of all the experimental work mentioned in section 1.4.2, it appears that these researchers were unable to answer some of the questions concerning separated flow heat transfer. The Nu-Re dependence does not show consistency between different experiments, i.e., Nu varies with Re over the range $Re^{\frac{2}{3}}$ to $Re^{\frac{3}{4}}$ depending on the geometry of the model. Initially, the present author decided to investigate the Nu-Re correlation for the blunt flat plate (in a non-turbulent free-stream). The effect of Reynolds number may change the separated flow structure and hence affect the heat transfer characteristics. Thus, the possibility of Reynolds number effects should be considered. The maximum heat transfer rate found by previous researchers occurred at the reattachment region. In most cases, the reattachment position was determined by means of wool tufts. The author felt that it was necessary to determine this position accurately because the maximum heat transfer position is very important to a designer. The surface oil flow technique

is an obvious choice for measurement as it is accurate and reliable (de Brederode⁵²).

Seki et al⁴¹ and Ota and Kon⁴⁵ pointed out that the shear layer plays an important part in the heat transfer from the surface of the model. Ota et al⁴⁷ provided some mean temperature and velocity profiles in the flow field. The mean quantities can only describe the mean characteristics of the flow field, they can not describe the details of the transport of turbulent energy in the shear layer. Therefore, it is important for the present investigation to study the shear layer structure in detail. The attainment of an energy balance between the power input into the model and the convective heat transport by the shear layer plus the heat conduction along the model will provide a good check on the accuracy of the results.

Experimental work concerning the effect of free-stream turbulence on separated flow heat transfer is limited. However, the effect of turbulence on the shear layer structure has been studied by Bradshaw⁵³, Hillier and Cherry⁵⁴. The entrainment process of the shear layer was found to be significantly affected by the presence of turbulence. It is possible that the presence of free-stream turbulence will increase the amount of heat transfer from the wall. The mean and fluctuating temperature and velocity profiles may be informative.

1.6 Outline of the Thesis

The following chapter of the thesis deals with the theoretical calculation of stagnation point heat transfer of

the normal flat plate. The design of the heat transfer models is described in Chapter 3. The experimental techniques and instrumentation are presented in Chapter 4. Chapter 5 contains the experimental results of the normal flat plate. Chapter 6 describes the experimental results of the blunt leading edge flat plate. Chapter 7 contains the shear layer measurements of the blunt leading edge flat plate. The conclusions and suggestions for further work are given in the last chapter.

2. Theoretical Calculation of the Stagnation Point Heat Transfer of the Normal Flat Plate

The objective of this section is two fold. Firstly to calculate the stagnation point heat transfer, and secondly to evaluate the heat transfer distribution along the front face of a normal flat plate. Initially, the velocity field for the front face of the plate was evaluated using Parkinson and Jandali's⁵⁵ wake source model for bluff body potential flow. The stagnation point heat transfer was obtained by the method given by Goldstein⁹. The heat transfer distribution over the whole front face was evaluated by the series solution given by Frössling¹¹.

2.1 The Velocity Distribution along the Front Face Parkinson and Jandali⁵⁵ - a wake source model for bluff body potential flow.

This theory is applicable for two dimensional incompressible potential flow external to a symmetrical bluff body and its wake. The desired flow-separation points are made the critical points of a conformal transformation to a complex plane in which the surface sources in the wake create stagnation conditions at the critical points. The stagnation streamlines then transform to tangential separation streamlines in the physical plane, with separation at the desired pressure. The position and strength of the sources are determined by the requirement of separation position and pressure coefficient. The flow inside the separation streamline is ignored and base pressure is assumed constant at the separation value. Features of this model include a finite

wake width, a pressure distribution on the separation streamlines decreasing asymptotically towards the free-stream value at infinity and a simple analytical expression for the pressure distribution on the body.

The theory of this wake source model is given in Appendix A. The velocity distribution on the front face is found to be $-\frac{v}{U} = \frac{2y/h}{1/k - (1 - (2y/h)^2)^{1/2}}$, where $k = (1 - C_{pb})^{1/2}$ (7b)

The expression of equation (7b) using the Binomial theorem is rather cumbersome, so a series expansion of $-\frac{v}{U}$ was adopted. For simplicity, write

$$v^* = c_1 y^* + c_3 y^{*3} + c_5 y^{*5} + c_7 y^{*7} + \dots \dots \dots (8)$$

where $v^* = -v/U$ and $y^* = y/h$, c_1, c_3 etc. are functions of C_{pb} (or $\frac{1}{k}$) differentiate equation (8), get

$$\frac{dv^*}{dy^*} = c_1 + 3c_3 y^{*2} + 5c_5 y^{*4} + \dots \dots \dots (9)$$

at $y^* = 0$, $\frac{dv^*}{dy^*} = c_1$, (i.e., at stagnation point)

$$\text{From equation (7b)} \quad \frac{dv^*}{dy^*} = \frac{(\cos\theta(\cos\delta - \cos\theta) - \sin^2\theta) \cdot 2}{(\cos\delta - \cos\theta)^2 \cos\theta} \quad (10)$$

At the stagnation point $\theta = \pi$

$$\frac{dv^*}{dy^*} = \frac{2}{(\cos\delta + 1)} \quad \text{where } \sec\delta = k$$

Therefore, c_1 may be found if the base pressure C_{pb} is known.

If one selects nine terms in equation (9), then

$$\frac{dv^*}{dy^*} - c_1 = 3c_3 y^{*2} + 5c_5 y^{*4} + \dots \dots \dots + 17c_{17} y^{*16}$$

if c_1 is known, and if one selects eight values of θ eg., $\theta = 175^\circ, 170^\circ, 160^\circ$ etc., there will be eight corresponding values of y^* .

It is therefore possible to generate an 8×8 matrix of coefficients c_i ($i=1,3,\dots,17$), i.e., the unknowns.

The above numerical method is rather useful. The value of v^* (velocity distribution on the front face) may be determined very accurately by selecting more terms,

provided that the base pressure is known. Although v^* may be determined by knowing θ and C_{p_b} , it can be seen later that Frössling's method in evaluating the heat transfer distribution of the velocity field, the values of c_i are needed.

The theory of Parkinson and Jandali⁵⁵ works well in comparison with experimental data. The base pressure of bluff bodies (e.g., normal flat plates, circular cylinders) is normally found to be nearly constant, so the assumption of constant base pressure in the theory is correct. But the theory is conceptually incorrect as it ignores the flow process at the back of the body. i.e., the process of vortex shedding. It is intended to be an inviscid method, so the boundary layer on the front face has been ignored. Nonetheless, the velocity distribution can be determined accurately using the theory.

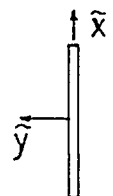
After knowing the velocity distribution on the front face, it is now possible to calculate the stagnation point heat transfer.

2.2 Stagnation Point Heat Transfer using Goldstein's⁹ Method

Close to the forward stagnation point in two dimensional flow past a cylinder, the velocity u , outside the boundary layer increases linearly with distance from that point $\tilde{x}$, i.e., $u_1 = c\tilde{x}$. The equations for steady motion in the boundary layer are

$$\tilde{u} \frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v} \frac{\partial \tilde{v}}{\partial \tilde{y}} = c^2 \tilde{x} + \nu \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2} \quad \dots\dots\dots (1)$$

$$\tilde{u} = \frac{\partial \psi}{\partial \tilde{y}} \quad , \quad \tilde{v} = \frac{\partial \psi}{\partial \tilde{x}} \quad \dots\dots\dots (2)$$



where $\tilde{y}$ is the normal distance from the wall, $\tilde{u}$ is the $\tilde{x}$

component velocity and $\tilde{v}$ is the $\tilde{y}$ component velocity.

If the body is heated to a temperature T_1 , the energy equation

$$\text{is: } \tilde{u} \frac{\partial T}{\partial \tilde{x}} + \tilde{v} \frac{\partial T}{\partial \tilde{y}} = \kappa \frac{\partial^2 T}{\partial \tilde{y}^2} \dots\dots\dots(3), \text{ where } \kappa = \frac{k}{\rho C_p}$$

The term $\tilde{u} \frac{\partial p}{\partial \tilde{x}}$ is ignored (assuming the heat generated by pressure gradient is small) and the dissipation function is neglected (assuming the heat generated by viscosity is small).

The boundary conditions are $T=T_1$ at $\tilde{y}=0$, $T=T_0$ at $\tilde{y}=\infty$.

Equation (3) has a solution which is a function of $\tilde{y}$ only.

Putting $\eta = (\frac{C}{\nu})^{\frac{1}{2}} \tilde{y}$, $\tilde{u} = c \tilde{x} f'(\eta)$, $\tilde{v} = -(\nu c)^{\frac{1}{2}} f(\eta)$, $T=T_1 - (T_1 - T_0) \theta(\eta)$.

Substitute the above terms into equation (3), get

$$\theta'' + \sigma f \theta' = 0 \dots\dots\dots(4) \text{ where } \sigma = \frac{\nu}{\kappa} \text{ Prandtl number.}$$

The new boundary conditions are $\theta(0) = 0$, $\theta(\infty) = 1$.

The solution of equation (4) is

$$\theta(\eta) = \alpha_3(\sigma) \int_0^\eta (\exp(-\sigma \int_0^\eta f d\eta)) d\eta, \text{ where } \frac{1}{\alpha_3(\sigma)} = \int_0^\infty (\exp(-\sigma \int_0^\eta f d\eta)) d\eta.$$

The rate of heat transfer from a section of the cylinder of breadth b and length $\tilde{x}$ measured from the stagnation point is

$$Q = \alpha_3(\sigma) k b \tilde{x} (T_1 - T_0) (c/\nu)^{\frac{1}{2}}$$

and the Nusselt number is

$$Nu = \alpha_3(\sigma) (c d^2 / \nu)^{\frac{1}{2}}, \text{ Where } d \text{ is a representative length.}$$

For the normal flat plate, if the base pressure $C_{p_b} = -1.48$,

$$\frac{dv^*}{dy^*} = c_1 = 1.21 \text{ and } \alpha_3(\sigma) = 0.495 \text{ (for air).}$$

$$\text{It can be deduced that } \frac{d\tilde{u}}{d\tilde{x}} = \frac{dv}{dy} = 1.21 \frac{U}{h} = c$$

$$\text{Therefore, } Nu = 0.495 \left(1.21 \frac{Uh}{\nu} \right)^{\frac{1}{2}} = 0.55 \left(\frac{Uh}{\nu} \right)^{\frac{1}{2}}$$

Goldstein's⁹ analytical solution for stagnation point heat transfer is very accurate if the velocity gradient at the stagnation point region is accurately determined. The limitation of the equation is that the difference in $(T_1 - T_0)$ must not be too large, so that the properties of the fluid

can assume to be constant. The flow is assumed to be incompressible (low subsonic speed), so that the dissipation and the heat generated by change in pressure may be neglected. The advantage of this method is that it can be applied to any two-dimensional bodies.

Another analytical method to calculate the stagnation point heat transfer rate of two-dimensional bodies was given by Frössling⁸, he extended the calculation to positions further away from the stagnation point.

2.3 The Heat Transfer Distribution along the Normal Flat Plate using Frössling's Method

Frössling's⁸ method is in actual fact an extension of Goldstein's⁹ method. It is applicable to bodies of arbitrary shape. In his calculations, the $\tilde{u} \frac{\partial p}{\partial \tilde{x}}$ term and the dissipation are ignored. He assumed a power series for the potential velocity distribution around a body expanded in terms of distance from the forward stagnation point, i.e., $U = c_1 \tilde{x} + c_3 \tilde{x}^3 + c_5 \tilde{x}^5 + \dots$. The velocity distribution in the boundary layer is assumed to have the form: $\tilde{u}(x,y) = c_1 \tilde{x} f_1(\tilde{y}) + c_3 \tilde{x}^3 f_3(\tilde{y}) + \dots$ and the temperature distribution is $T(x,y) = T_0 + \tilde{x} T_1(\tilde{y}) + \tilde{x}^3 T_3(\tilde{y}) + \dots$. The functions $T_1(\tilde{y})$, $T_3(\tilde{y})$, etc. are found to satisfy the ordinary differential equations which include the functions f_1 , f_3 etc. of the velocity distribution. The expression for the heat transfer distribution is found to be

$$\frac{Nu}{Re^{\frac{1}{2}}} = (-F_0' c_1^{\frac{1}{2}} - \frac{4c_3}{c_1^{\frac{3}{2}}} F_2' \tilde{x}^2 - \frac{6c_5}{c_1^{\frac{5}{2}}} (G_4' + \frac{c_3^2}{c_1 c_5} H_4') \tilde{x}^4 + \dots)$$

where F_0 , F_2 etc. are functions generated by T_1 , T_3 etc.. For the normal flat plate $\sigma = 0.7$ (air), $-F_0' = 0.495$ (from Frössling) and if $Cp_b = -1.48$, $c_1 = 1.21$.

At the stagnation point $\tilde{x} = 0$

$$\frac{Nu}{Re^{\frac{1}{2}}} = 0.495 (1.21)^{\frac{1}{2}} = 0.55$$

This value is in exact agreement with Goldstein's value. The reason is that the first term of Frössling's equation is simply Goldstein's solution for stagnation point heat transfer.

For the heat transfer distribution on the front face of the flat plate, the values of c_1 , c_3 etc. may be determined by the 'matrix method' (given in section 2.2) if the base pressure C_{pb} is known. The other functions F_2' , G_4' , H_4' etc. have been obtained numerically by Frössling. So a complete description of the heat transfer distribution may be found quite easily.

The limitation of Frössling's method is that it is quite cumbersome in nature, especially in the case of slender bodies where a large number of terms in the power expansion are required.

3. The Development of Heat Transfer Measurement Techniques and Models

This chapter deals with the development of heat transfer measurement techniques and the design of the test models, namely, the normal flat plate model for stagnation point heat transfer measurements and the separated flow model.

Initially it had been hoped to use thin film gauges to measure heat transfer rate, and one objective of the stagnation point measurements was to provide a reliable test for checking the measurement technique. In the event serious problems were encountered, and section 3.2 will deal with the likely sources of error. Because of the failure of the thin film gauges, heat flux data were eventually obtained using surface heating strips whose power dissipation and temperature were directly monitored. This was checked to be a reliable technique for the laminar/stagnation point flow, and was therefore used for the main experimental programme. A detailed discussion of the technique is given in section 3.3.

At the beginning of this research, a detailed survey of the current heat transfer measurement techniques was made, which is reviewed in the following section.

3.1 Background Study of Heat Transfer Measurement Techniques

3.1.1 Constant Surface Heat Flux Models

For the low speed flow heat transfer experiments, many researchers base their models on the condition of constant surface heat flux because this enables the experiments to be performed more easily. The model normally consists of two

thin sheets of heating strips which are heated electrically. The temperature distribution of the model is determined by a large number of thermocouples buried underneath the heating strips. The amount of current supplied to the heating strips and the resistance of the strips can be measured, so the heat flux/unit area is known ($Q = I^2 R$, $\dot{q} = \frac{I^2 R}{A}$, A = area of strip). Hence the local Nusselt number may be determined if the local temperature of the model is known. Daigo et al⁴⁸, Ota and Kon⁴³, Seki et al⁴¹, Smyth⁴⁷, Test and Lessmann⁵⁰ used the above method in their heat transfer experiments. Simonich and Bradshaw⁵⁶ used a slightly different heat transfer model, the model was heated by a large number of heating strips sandwiched between two sheets of insulator. The heat flux distribution was likely to be non-uniform because there were large gaps between each heating strip.

3.1.2 Isothermal Surface Models

Experiments with a constant surface temperature model (normal flat plate) have been performed by Sogin et al¹³. Their model consisted of ten heating strips, each strip was equipped with a thermocouple and a heat flow meter to measure heat flux. Junkhan and Serovy⁵⁷ used a similar technique, the heat transfer rate was obtained by measuring the power input into each strip. Filetti and Kays³ carried out heat transfer tests on a double step (backward facing). The instrumental plate was made up of a large number of copper cells which were filled with water. The thermocouples and heat flux measurement devices were installed to measure the heat transfer rate which was found to be accurate to $\pm 2\%$.

3.1.3 Thin Film Resistance Thermometry

The thin film gauge is a resistance thermometer deposited on the surface of an insulating substrate. Generally heat transfer measurements employ either steady state or transient techniques. The latter were primarily developed for short duration tunnels (eg. shock tubes) where run-lengths were sufficiently short that gauges could usually be regarded as of infinite extent from the view point of the diffusion of a heat pulse into them. Surface heat transfer rates could then be inferred solely from a knowledge of material properties and of the variation with time, of a point measurement(eg. the surface) of temperature. Such transient techniques are dealt with exhaustively by Schultz and Jones⁵⁸, Richards⁵⁹. In the present experiment the flow is not transient, nor is the present research interested in the measurements of heat transfer fluctuation. The present research is concerned only with time averaged heat transfer, therefore, the transient techniques are not appropriate. However, the thin film gauge has two special characteristics which satisfy the the present requirements, (i) rapid response (ii) minimal surface interference. The general principles of operation of the thin film gauge in a steady flow will be given in the next section.

3.2 The Utilization of Thin Film Resistance Gauges for the Stagnation Point Heat Transfer Measurements of The Normal Flat Plate

3.2.1 The General Method

The resistance-temperature relationship of a thin

film is linear for a small temperature range (0° - 100°C), i.e., $R = R_0(1 + \alpha(T - T_0))$. It is possible to construct a gauge with a film on the top side and on the bottom side of the substrate (Figure 3.1). If heat flows steadily through the substrate, the resistance and temperature of the two films are:

$$R_1 = R_{o_1}(1 + \alpha_1(T_1 - T_0)) \quad \dots\dots\dots \text{top film}$$

$$R_2 = R_{o_2}(1 + \alpha_2(T_2 - T_0)) \quad \dots\dots\dots \text{bottom film}$$

For steady heat conduction, $\dot{q} = k \frac{\Delta T}{\Delta y}$

where k is the thermal conductivity of the substrate, Δy is the thickness of the substrate and $\Delta T = (T_1 - T_2)$.

If the resistance of the two films can be determined, the heat transfer rate across the substrate can then be obtained.

The Calibration of Gauges

The resistance-temperature calibration of thin film gauges was carried out in oil in a 'Grant Instrument Calibration Bath' over a temperature range (20°C - 100°C). Gauges were fully submerged into the bath. The resistance of the thin films (typically 50 - 100 Ω) were measured using a digital multimeter. The oil temperature was registered by a Mercury-in-glass thermometer and a Chromel/Alumel thermocouple. A typical resistance-temperature calibration of thin film gauges is given in Figure 3.2. The resistance/temperature variation is fairly linear for both films (maximum deviation of about 5%), and good repeatability of data is also obtained.

The Normal Flat Plate Model

The dimensions of the normal flat plate used in the present studies were 50cm in length, 5cm in height, 1cm in thickness. The main body of the plate was made of Dural.

The front face of the plate was equipped with four thin film gauges. These gauges were flush-mounted on the surface of the plate. Seven pressure tappings were located at various positions on the model (Figure 3.3). Two ribs on the back of the plate were obtained by machining the plate. The Nickel/Chromium heating tape was glued onto the back of the Dural plate. The back face of the plate was pasted with a uniform layer of thermosetting cement (as insulator). The flat plate was supported by two insulated side plates (as those shown in Figure 4.1).

The Measurement Technique

The Peekel strain-indicator is accurate in strain measurements (it is able to measure up to 1μ strain, i.e., about 2×10^{-6} change of resistance for a gauge factor of 2). It consists of a Wheatstone bridge system. A change in resistance of one arm of the bridge is directly proportional to a change in strain, i.e., $\frac{\Delta l}{l} = \beta \frac{\Delta R}{R}$ (where β is the gauge factor). It is therefore feasible to use this device to measure temperature (since $\Delta T = \frac{1}{\alpha} \frac{\Delta R}{R}$).

The method used in the present investigation to obtain heat flux across a gauge is described as follows: When there is no heat flow across a gauge (the wind tunnel switched off), the temperature of the two films is assumed equal. The two films are connected to the two arms of the Wheatstone bridge. The bridge is then balanced and this reading (of the Peekel strain-indicator) is designated as the null reading μ_0 . When heat is allowed to flow through the gauge (the wind tunnel running), the temperature of the two films will be different ($T_2 > T_1$). The difference in temperature

will offset the bridge balance, and the strain-indicator will register a new reading μ_{new} . The temperature difference across the two films $(T_2 - T_1)$ may be obtained by subtracting the two readings $(\mu_{new} - \mu_o)$, i.e., $\frac{\mu_{new} - \mu_o}{\mu_o} = \beta \left(\frac{R_2}{R_{o2}} - \frac{R_1}{R_{o1}} \right)$, therefore $(T_2 - T_1) = \frac{1}{\alpha \beta} \left(\frac{\mu_{new} - \mu_o}{\mu_o} \right)$, assuming α is constant for both films.

3.2.2 Discussion of Thin Film Gauge Technique

A series of heat transfer experiments using the normal flat plate were carried out in the 3'x2' wind tunnel. The aims were to provide a basic check of the technique and to prove the Nu-Re dependence for the stagnation point heat transfer of the normal flat plate. The effect of varying the power input to the model at constant wind speed and the effect of varying the wind speed at a constant power input were investigated. The results of these experiments are presented in Figures 3.4 and 3.5. It was found that the experimental results were largely different from the present theoretical prediction (section 2.2) of stagnation point heat transfer, i.e., $Nu = 0.55 Re^{\frac{1}{2}}$ (based on an experimental C_p of -1.48).

The experimental stagnation point Nusselt numbers

($Nu = \frac{\dot{q} d}{k_{air}(T_w - T_\infty)}$) were about 4 times the theoretical value (Figures 3.4 and 3.5).

The Possible Sources of Error

(1) The value of α was assumed constant for the two films in the evaluation of results. Later calibrations found that α differed by about 10% for the two films (possibly due to deterioration). A correction was made to account for the difference in α 's (see Appendix B 9). From Appendix B 9,

it can be seen that $\frac{\alpha_1 - \alpha_2}{\alpha_1} \sim 0 \left(\frac{T_1 - T_2}{T_1} \right)$, for a 10% difference in $(\alpha_1 - \alpha_2)$, the value of $(T_1 - T_2)$ can be increased 3 times. In other words the experimental Nu is 3 times the theoretical value (since $Nu \propto \dot{q} \propto (T_1 - T_2)$).

(2) Initially T_1 was assumed equal to T_2 . But the outer film (the one exposed to air) was more sensitive to a change in wind tunnel temperature. A difference in initial temperature of 0.5°C could impose an error of 20% in Nu ($(T_1 - T_2)$ was typically about 3°C during an experiment).

(3) Some disturbance to the heat flux distribution was expected from the insertion of the gauges, which had thermal conductivity very different from the dural plate.

(4) Failure of electrical insulation of the gauges could lead to a change in film resistance.

(5) Other factors might possibly bear on the experiment were radiation losses (heat losses due to radiation were estimated to be 5% for $T_w \sim 60^\circ\text{C}$) and lateral conduction (about 1% of the total heat input).

The Disadvantages of Using Thin Film Gauges

- (1) Thin film gauges can easily be damaged.
- (2) Resistance/temperature calibration changes with time.
- (3) Recalibration is difficult after installation into the model.

3.3 The Development of a Constant Surface Temperature Normal Flat Plate for Stagnation Point Heat Transfer

The practical problems which the author encountered in thin film gauges could be difficult to overcome and

probably time consuming. It was therefore decided to develop an alternative method to measure heat transfer rate. The new method was evolved from the method used by Junkhan and Serovy⁵⁷. A normal flat plate was then designed which enabled the surface temperature to be monitored.

3.3.1 The Development of the Heat Transfer Measurement Technique

The use of thermocouples to measure the surface temperature of a model has been widely used by researchers in heat transfer experiments (section 3.1.1). The heat transfer rate at the surface can be obtained by measuring the power input into a heating strip.

The method of obtaining a constant surface temperature normal flat plate was proposed by the author. If a large heating strip was fixed on the front face (facing the direction of wind) of the normal flat plate model, owing to the presence of the laminar boundary layer on the surface, the amount of heat transfer from the surface would be different at different positions across the surface (and hence the temperature distribution). But if a number of small heating strips were fixed on the surface of the model, then the power input into each strip could be monitored. The surface temperature of the model could therefore be adjusted to be approximately constant. It was important that the heating strips should conduct heat efficiently, so that the temperature distribution remained uniform. The power input of each heating strip was measured and its temperature was determined by a thermocouple. The local

Nusselt number could be evaluated by the following equation,

$$\dot{q} = \frac{VI}{\text{area of strip}} \quad \text{and} \quad Nu = \frac{\dot{q} d}{k_{air}(T_w - T_\infty)}$$

where d is the breadth of the plate.

3.3.2 The Design Factors of the Constant Temperature Normal Flat Plate

The design was divided into two parts, firstly, the general dimensions of the model based on aerodynamic considerations and secondly, the thermal characteristics of the model.

The General Dimensions of the Model (Figures 4.2 and 4.3)

The 3'x2' wind tunnel was selected for the heat transfer experiments. The general dimensions of the model (mounted horizontally across the tunnel) were therefore decided under the restrictions of the wind tunnel. The vertical height of the tunnel was 1.02m. In order to minimize error due to blockage of the wind tunnel, a model/tunnel height ratio of 0.05 was used. The calculated optimum model height was therefore 5cm..

Bearman and Trueman⁶⁰ showed that for a model thickness/model height ratio in the order of 0.2 - 0.4, the base pressure C_{p_b} behind a rectangular cylinder remained almost constant across the span. A thickness to height ratio of 0.3 was selected for the normal flat plate, the thickness of the plate was 1.5cm..

The width of the wind tunnel was 0.61m.. Side plates were used to eliminate the influence of the tunnel wall boundary layers. An active model length of 50cm. was

selected so that the model would be well outside the region of the influence of the walls. An aspect ratio(model length/model height) of 10 would be sufficient to ensure the flow field was two dimensional in the mean.

The required length of the side plates was estimated to be about 3 times the length of the recirculating flow region which would be sufficient for the flow to recover (Donoso⁶¹). The height of the side plates was about 5 times the height of the separation bubble which would ensure no influence from the tunnel walls. The dimensions of the side plates were estimated to be 35cm. x 30cm..

The Thermal Characteristics of the Model

The fundamental structure of the model consisted of a rectangular plate (made of insulator) and heating strips. When the model was heated on the front face, most of the heat was conducted through the heating strips into the front face boundary layer. However, there would still be significant conduction losses back into the substrate (the rectangular plate), and consequential heat transfer from the back and side of the model. If the model was very thin compared with its width, then heat losses from the back became more severe. Fortunately, these losses could be calculated fairly well from one dimensional considerations if base temperature was known(see Appendix B 1). If the model was thicker, however, losses were reduced but now varied in a two-dimensional manner from the stagnation point and hence were more difficult to predict. An estimation was carried out to study the effect of a single heating strip on the stagnation region of the model. The heat loss through the model

was evaluated by solving the two-dimensional steady heat conduction equation(see Appendix B 2). The analysis also indicated that the best design was to have the width of the strip greater than the thickness of the substrate, this would ensure that the heat flux through the back to approach the one-dimensional manner.

A metal backing plate was added to the model. The purpose of the backing plate was two-fold, firstly, it gave additional strength to support the model and secondly, it provided an isothermal rear surface. An estimation (see Appendix B 4) of the insulator thickness was carried out based on a one-dimensional heat loss calculation through the back(accounting for the metal backing plate). It was estimated to be about 0.8cm..

The width of the heating strips was evaluated (see Appendix B 3) by basing the analysis on the heat transfer characteristics of the stagnation region together with the velocity field on the front face. It was found that five heating strips were enough to give a reasonably isothermal surface.

The maximum operating model temperature was also estimated(see Appendix B 5). It was based on the maximum power available (about 8W) and a wind speed of about 20m/s. A temperature difference ($T_w - T_\infty$) of 17°C was obtained.

Radiation loss from the front face was estimated for the maximum model temperature (for ($T_w - T_\infty$) about 17°C- see Appendix B6). It was found to be insignificant for the present design (about 1.5%).

The side plates were also used as insulators to reduce lateral conduction from the model. Two insulating blocks were added between the model and the side plates. The heat loss by lateral conduction was further reduced.

The present model was later found to give accurate data of stagnation point heat transfer(Chapter 5). The same methods in measuring heat transfer rate (using thermocouples) and in controlling surface temperature (by controlling the power input to the heating strips) were therefore applied to the separated flow model(the blunt leading edge flat plate).

3.4 The Design of a Constant Surface Temperature Blunt Flat Plate for Separated Flow Heat Transfer

The design of the constant temperature blunt flat plate was similar to that of the constant temperature normal flat plate. Both the aerodynamic and thermal effects of the blunt flat plate were considered.

The General Dimensions of the Model(Figure 4.8)

The flow field around a blunt flat plate is very complicated, separation occurs at the leading edges and two separation bubbles are generated on the upper and lower faces of the plate. These bubbles will cause blockage in the tunnel, i.e., wake blockage. So the total blockage is the sum of the blockage due to the model and the wake.

In order to minimize the blockage effect but still maintain a reasonably large model, a model height/wind tunnel height ratio of 0.25 was selected(the model height was about 2.5cm.). Previous research (Cherry⁵¹) showed that the length

of the separation bubble was about five times the model height (in a non-turbulent free-stream). The length (in the direction of the flow) of the present model was about five times the reattachment length, i.e., the flow field downstream of reattachment was fully developed.

Side plates were used to ensure a reasonably two-dimensional flow over the length of the model and to eliminate the effect of tunnel wall boundary layers.

The Thermal Characteristics of the Model

Due to the large surface area of the model, the total power required to heat up the model would be fairly large compared with the normal flat plate. An estimation was made to decide whether it was possible to provide the power in practice (see Appendix B 7). The heat transfer data of Ota and Kon⁴³ were used as a guideline for this estimation because their model geometry was identical to the present one. The heat transfer data were divided into three flow regions, i.e., the separated flow region, the reattachment region and the redevelopment region. The total estimated power of these regions was found to be about 600W for a non-turbulent free-stream. The amount of power required for a turbulent free-stream was expected to be more than for a non-turbulent free-stream. This problem could be overcome by lowering the operating temperature (T_w) of the model, so an estimation of the power requirement in a turbulent free-stream was not necessary.

The width of the heating strips was also 0.9cm. (same as for the heating strips of the normal flat plate). There were at least ten heating strips in the separated

flow region which would give reasonable data resolution. The power requirement of each heating strip was also estimated (about 10W - see Appendix B 8).

The main structure of the model consisted of a metal plate (backbone) sandwiched between two layers of insulator(substrate). The heating strips were fixed onto the surfaces of the insulator (thermocouples buried beneath the heating strips - see Figure 4.8). The amount of heat loss through the insulator was little in this design and hence reduced the errors in working out the surface heat transfer rate.

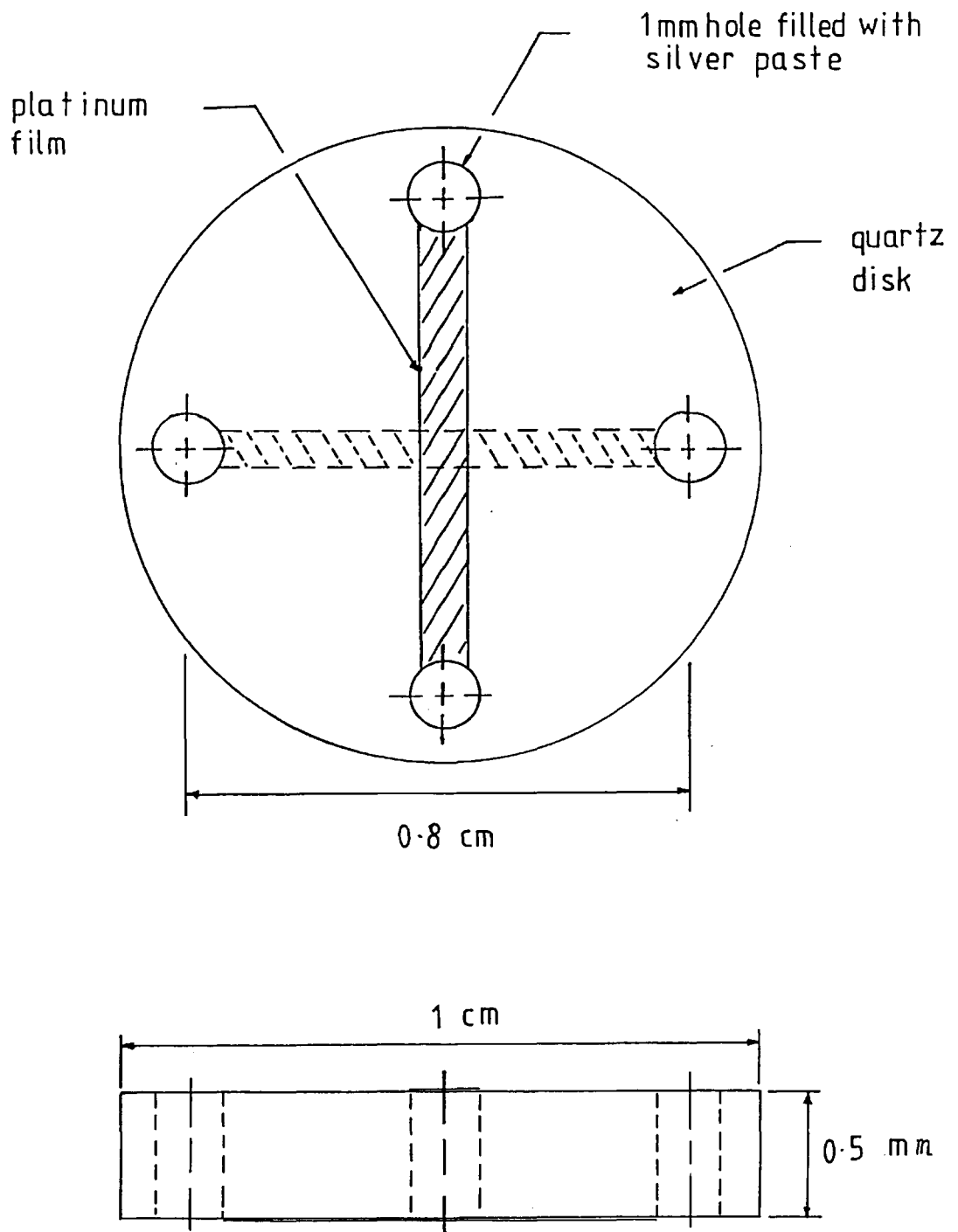


Figure 3.1 The dimensions of the thin film resistance gauge

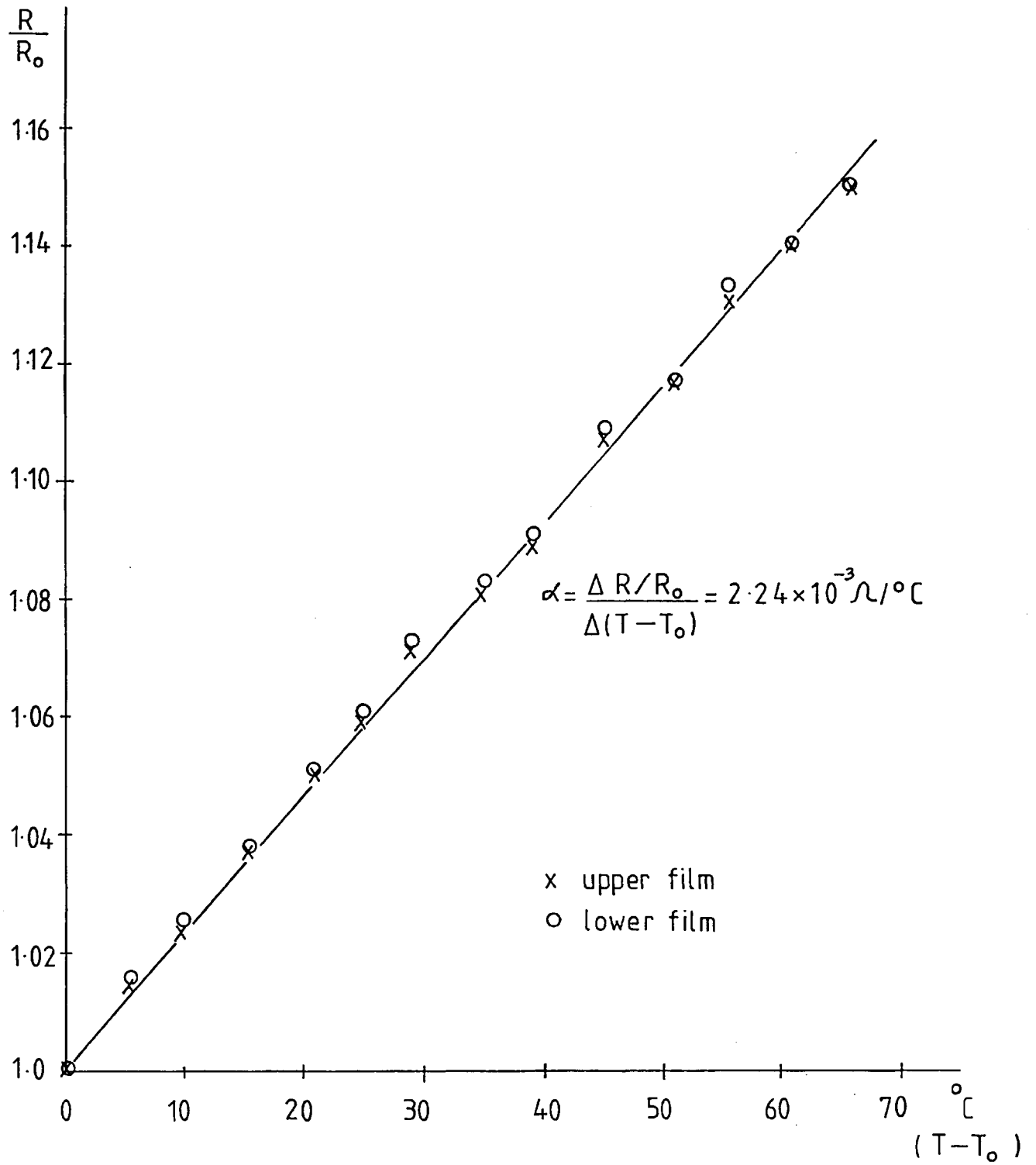
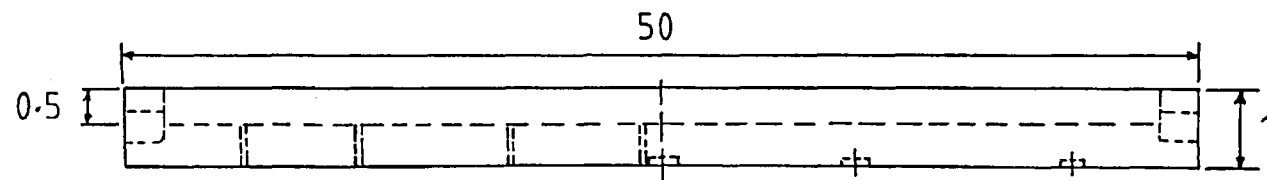
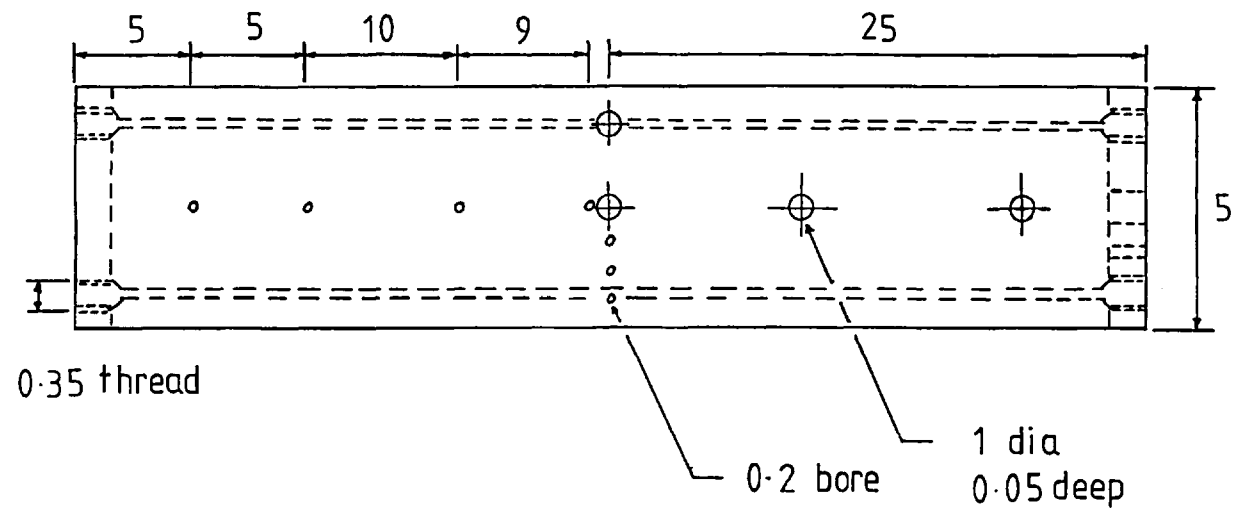
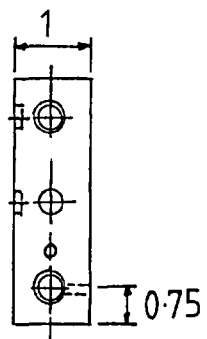


Figure 3.2 The resistance/temperature calibration of a thin film resistance gauge



all dimensions in cms.
not to scale

Figure 3.3 The dimensions of the normal flat plate

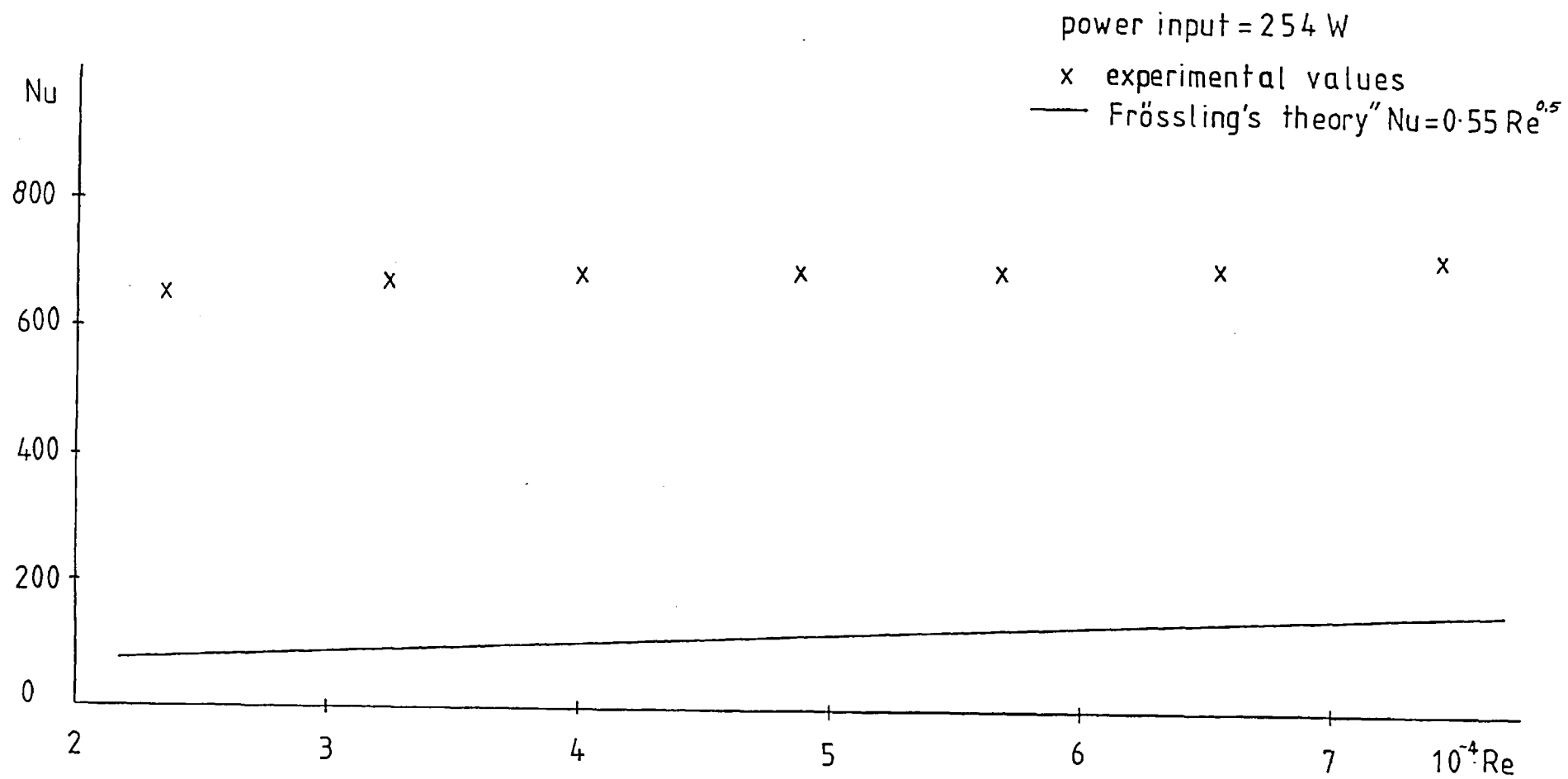


Figure 3.4 The effect of Reynolds number at a constant power input

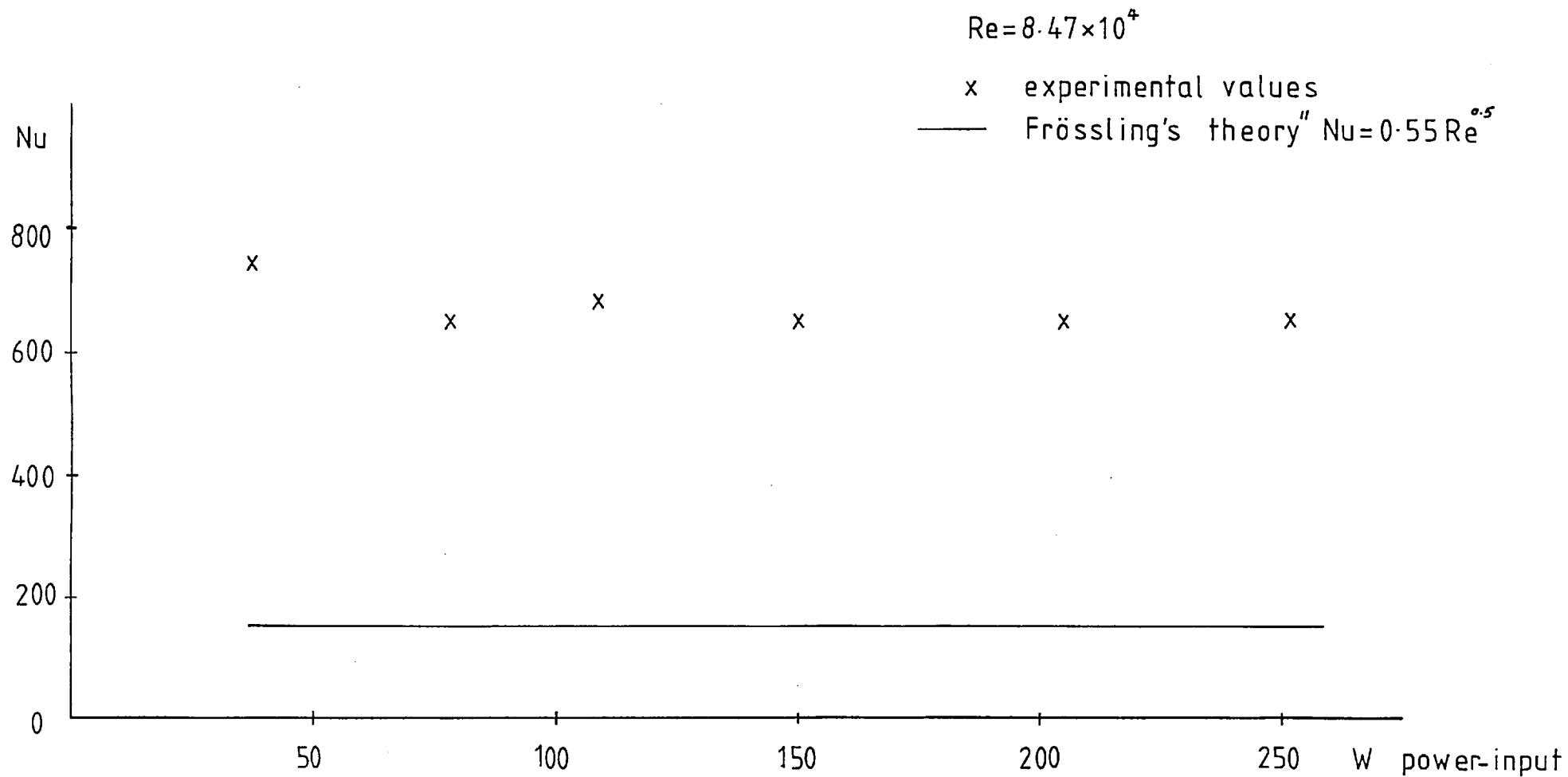


Figure 3.5 The effect of varying the power input at a constant Reynolds number

4. Instrumentation and Experimental Techniques

The present research is divided into three main experimental programmes, (i) Stagnation point heat transfer of the normal flat plate; (ii) Surface heat transfer and static pressure measurements of the blunt flat plate; (iii) Velocity and temperature measurements in the separated, reattached and redeveloped flow regions of the blunt flat plate. The different experimental arrangements and instrumentation used in each programme are described in the following sections. Some of the sections in this chapter are devoted to explain the techniques used in collecting data and the calibration procedures used in connection with the different instruments are described. The general procedures of each programme are also included in this chapter.

4.1 Instrumentation and Experimental Techniques for Stagnation Point Heat Transfer

4.1.1 Description of the Constant Surface Temperature Normal Flat Plate

The dimensions of the normal flat plate were 50cm x 5cm x 1.6cm. It consisted of three main units, the first unit was the heating strips (foils), then two layers of insulator (glued together) and finally the metal backing plate. Thermocouples were installed in the model to measure temperatures.

The model was heated by means of five heating strips glued onto the front surface of the insulator with 'Araldite'. There were small gaps of about 1mm between each strip to insulate them electrically (Figure 4.1). The gaps

were then filled with 'Araldite', so a smooth model surface was retained. The heating strips were produced from a 'Nickel/Chromium' alloy and were 50cm x 9cm x 0.0125cm in dimensions.

The insulation was made of 'Nylatron' which is a good insulator of heat and electricity. Eleven V-shape grooves of 0.005 inch deep were machined onto the face attached to the heating strips (Figure 4.2a) to enable 'Copper/Constantan' thermocouples (0.0015" in diameter) to be mounted onto the insulator. Three more grooves were machined onto the back face of the insulator to allow three thermocouples to be located into the insulator (Figure 4.2b).

The metal backing plate was made of 'Dural' due to its high strength and good conduction capabilities (isothermal rear surface). Three static pressure taps on the back face of the plate were used to record the base pressure on the model (Figure 4.3a). Seven pressure taps drilled through the model (Figure 4.1) were used to record the pressure distribution on the front face of the model.

The two side plates were made from a layer of 'Nylatron' and a layer of 'Dural' (Figure 4.1). The 'Nylatron' was used to prevent heat losses from the model (i.e., the required sideplate condition was zero conduction) while the 'Dural' provided greater strength. Two rectangular blocks of 'PTFE' were located between the side plates and the model which were used to hold the electrical connectors between the heating strips and the power supplies (Figure 4.3b).

4.1.2 Instrumentation

The 3'x2' Wind Tunnel

All heat transfer experiments were carried out in the 3'x2' wind tunnel. The working section is of about 10' in length. The free-stream turbulence intensity in the empty working section is about 0.06%. Turbulence grids can be introduced at the inlet of the working section. The maximum working flow speed of this tunnel is about 35m/s.

The Data Amplifier and Digital Voltmeter

An amplifier with a gain of 50x was connected to the thermocouples. A digital voltmeter was used to record the e.m.f. from the thermocouples.

The Power Supplies

Three 'Coutant' power supply units were used to provide the power to heat up the model. Each power supply was capable of providing 30V at 4A. Two 'Avometers' were used to measure the voltage and current from the power supplies.

The Thermocouple Probe

A 'Copper-Constantan' thermocouple probe was specially made for measurement of the free-stream temperature. The diameter of the thermocouple wires was about 0.0015". The hot junction was the free-stream and the cold junction (0°C) was placed inside a 'Thermos' flask containing an ice/water mixture.

Hot Wire Anemometry^{62,63}

Turbulence velocities were measured using constant temperature hot wires. Longitudinal turbulence intensity $\overline{u}$ was measured using DISA Uprobes (type 55P01) with department made 5μm Wollaston wires. The standard static calibration

procedures were carried out. For most of the measurements, a DISA '55D01' anemometer with a low pass filter set at 20KHz, a high pass filter at 1 Hz and a 'Datron' r.m.s. meter were used. Integral length scale L_x was measured by passing the anemometer output into a Hewlett Packard '3721 A' correlator. The hot wire analogue output was recorded on 1" magnetic tapes using an Ampex 'FR 1300' 14 channel tape recorder. The hot wire signal was always monitored on an oscilloscope.

4.1.3 Experimental Techniques

Constant Surface Temperature Adjustment of the Model

The model consisted of five 'Nickel-Chromium' heating strips numbered from 1 to 5. The centre strip (number 3) was heated independently by a 'Coutant' d.c. power supply unit. The two outer strips (numbered 2 and 4) were connected in series and heated by a power supply unit. Similarly, the outermost strips (numbered 1 and 5) were heated by another unit. The heating arrangement was based on the assumption of flow symmetry along the centre line.

For constant surface temperature adjustment in heat transfer experiments, each power supply unit was adjusted independently. Firstly, the centre strip was allowed to heat up to a known temperature. Secondly, the outer strips were allowed to heat up to that temperature. Finally, the outermost strips were heated. The power input into each strip was carefully adjusted until the temperature of all the strips were matched (a temperature difference between the strips of $\pm 0.05^\circ\text{C}$, for a typical $(T_w - T_\infty)$ of 10°C).

Surface Temperature Measurement of the Model

The hot junctions of the thermocouples were located in the model, while the cold junctions were kept in the ice-water mixture (the cold junctions were not placed in the free-stream due to practical difficulties in mounting them). When the model was heated, the e.m.f. generated by the thermocouples (hot and cold junctions) were amplified and displayed on the digital voltmeter. A uniform cold junction temperature of 0°C was maintained by continuous stirring of the ice-water mixture. Temperatures registered by the thermocouples were therefore the model temperatures.

Calibration of Thermocouples

The calibration of thermocouples was carried out in an oil bath (already mentioned in section 3.2.1), three thermocouples were used. The hot junctions were immersed in the oil bath and the cold junctions were placed in the ice-water mixture. The oil temperature was recorded by a 'Chromel-Alumel' thermocouple and a Mercury-in-glass thermometer. The oil temperature was raised slowly from 20°C to about 70°C (stirred continuously). The oil was then allowed to cool down slowly to room temperature. The output e.m.f. readings of the thermocouples and the temperature of the oil bath were recorded. A typical calibration of the thermocouples is shown in Figure 4.4. It can be seen that the calibration is linear over the temperature range and no hysteresis between heating and cooling.

Wind Tunnel Calibration

A full scale calibration (velocity calibration) of the 3'x2' wind tunnel was performed. For the non-

turbulent free-stream, the tunnel was calibrated at the position of the normal flat plate. For the turbulent free-streams, three turbulence grids of mesh size 6", 3" and 1.5" (denoted as grids A, B, C respectively) were employed in order to vary the length scale and intensity. For all the cases, a wide range of tunnel speeds (300 rpm to 1200 rpm) were used. A standard pitot-static tube was placed at the required positions in the working section. The calibration factor of the tunnel was defined as $\frac{P_s - P_{st}}{P_1 - P_2}$, where P_{st} , P_s were the total and static pressures of the pitot-static tube, P_2 and P_1 were the pitot pressure of the reservoir and the static pressure at the contraction respectively. A typical tunnel calibration is shown in Figure 4.5. A table containing all the calibration factors is given below.

	distance from upstream				
	55"	66"	81"	99"	114"
tunnel calibration factor					
non-tur. free-stream	1.037				
grid A		0.917	0.906	0.920	0.925
grid B	0.968	0.966	0.958	0.975	0.977
grid C	1.056	1.054	1.063	1.061	1.068

Turbulence intensity ($\frac{\sqrt{u^2}}{U_\infty}$) and integral length scale (L_x) calibrations of the empty tunnel were also obtained (Figures 4.6, 4.7). The present calibrations agreed fairly well with the results of Cherry⁵¹. The hot wire was placed at the centre of the cross-section of the tunnel. The instrumentation used to measure intensity and length scale was described in section 4.1.2.

4.1.4 Experimental Programme and General Procedures

Experimental Programme

The experimental programme was divided into three parts. The first part was to study the heat transfer effect of the model in a non-turbulent free-stream by keeping the surface temperature at the front of the model constant. The second part was to measure the mean quantities (velocity and temperature) in the wake. Finally the effect of free-stream turbulence on stagnation point heat transfer was studied.

General Procedures

On each occasion before commencing an experiment the tunnel was left running for about half an hour until the tunnel temperature became more or less constant. The model was then heated, the power input into each heating strip was adjusted until the surface temperature was approximately constant. The model normally took about half an hour to reach equilibrium.

The first part of the experimental programme was mainly concerned with the measurement of the model temperatures. The free-stream temperature was recorded by a separate thermocouple located above the model. The wind speed was recorded using the Betz manometer and the pressure distribution on the model was recorded by a multitube manometer. The power input into the model was measured by means of 'Avometers'. The temperature of the thermocouples were obtained from the data amplifier and digital voltmeter.

The general procedures for the second part of the experimental programme was identical to the first. A

pitot-static probe was mounted at 1" from a thermocouple probe, the pressure and temperature fields in the wake were measured by these probes.

For the last part of the programme, turbulence grids were introduced into the wind tunnel. The experimental procedures were identical to that for the non-turbulent free-stream, except that the model was moved along the wind tunnel to provide different turbulence conditions. For each of these above mentioned experiments, special care was taken to ensure a constant tunnel speed.

4.2 Instrumentation and Experimental Technique for Separated Flow Heat Transfer

4.2.1 Description of the Blunt Leading Edge Flat Plate

The model consisted of three components, the instrumental plate, the trailing edge section and the side plates.

The instrumental plate(50cm. x 48cm. x 2.48cm.) consisted of a 'Dural' plate sandwiched between two layers of 'Nylatron' (see Figure 4.8). Twenty eight heating strips were glued on the upper and lower surfaces of the 'Nylatron'. Two strips were glued onto the front surface of the model (Figure 4.9). The dimensions of the heating strips were 48cm. x 0.9cm. x 0.002cm.. The strips were separated by the 1 mm gaps. Pressure taps were located in the gaps on the 'Nylatron' plates. The upper surface of the model had 16 pressure taps, while the lower and front surfaces had 3 each(Figure 4.10b). 'Copper-Constantan' thermocouple probes were located in the grooves of the 'Nylatron' plates

underneath the heating strips(Figure 4.10a). There were 34 thermocouples altogether in the model. Electrical connection to the heating strips were made via Copper plugs.

The wooden trailing edge section was screwed into the instrumental plate. A small flap was attached to the trailing edge section to align the flow fields on the upper and lower surfaces of the model(Figure 4.8).

The side plates were each about 60cm long and 30 cm wide. The material used was 'fire-proof fibre', two layers of this material were glued together to form a side plate of 2mm thick(Figure 4.9).

4.2.2 Instrumentation

Broadly speaking the apparatus used in the blunt flat plate experiments was the same as for section 4.1.2.

The Power Supply Units

Two multi-channel power supplies(department-made) were constructed to provide part of the heating of the model. There were eight channels on one power supply (controlling 8 heating strips on the front section of the model), and twelve channels on the other(controlling the heating of the upper surface of the model). Three 'Coutant' power supply units were used to heat the lower surface of the model (Figure 4.10a).

The first twelve pairs of heating strips(upper and lower surfaces) were heated in pairs(Figure 4.10a)

because the flow field around the model was assumed to be symmetrical along the centre line. These strips were located in the separated flow region where the surface temperature had to be controlled extremely accurately. After the first twelve pairs, the rest of the heating strips on the upper surface of the model were heated in pairs. For the rest of heating strips on the lower surface, four pairs were heated by two 'Coutant' power supplies(15Vx4A), and another four pairs were heated by means of a 30Vx5A 'Coutant' supply. The heating arrangement of the model is shown in Figure 4.10a.

The multi-channel power supply units were designed to provide the current and voltage requirement of the heating strips. Two types of transformers(12V, 6V) were used to provide the correct voltage. The '12V' transformers were used to heat up the first four pairs of heating strips. Twenty two power supply modules were constructed according to a designed circuit(Figure 4.11). The a.c. output from the transformer into the module was rectified. The transistor and potentiometer sections were used to control the output voltage. When the heating strips(pairwise)were connected to the module, the amount of current flowing through the strips were equivalent to that flowing in the circuit. The amount of current could be determined by measuring the voltage across the three resistors($2 \times 0.22 \Omega$ and 3.3Ω).

4.2.3 Experimental Techniques

Constant Surface Temperature Adjustment of the Model

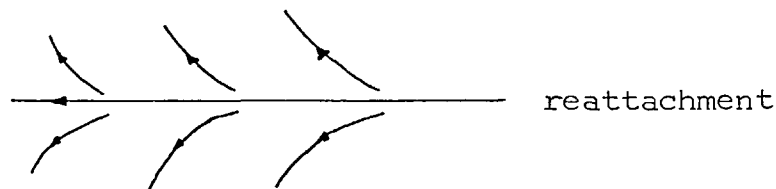
For constant surface temperature heat transfer experiments, a systematic method was used to control the surface temperature of the model. Firstly, the two heating strips at the front section of the model were heated until a required constant temperature was reached. Then the next eleven pair of strips (Figure 4.10a) were heated one after the other. At the same time the temperature of the strip was monitored by selecting the required thermocouple. The power input into the strips was adjusted until the surface temperature of this section of the model was roughly constant. The rest of the heating strips on the upper surface of the model were then allowed to heat up. Finally the rest of the heating strips on the lower surface of the model were heated as well. The power input into all the strips were adjusted until the temperature of the heating strips were found to be constant (temperature difference between the strips of $\pm 0.01^{\circ}\text{C}$). The total time required to reach steady state conditions (i.e. a constant temperature model) was about 45 minutes. The effect of heat input on the tunnel temperature was small because the total heat input of the model never exceeded 350W and the wind tunnel speed was about 20 m/s (max.). In fact the tunnel temperature was found to increase by about 3% over an experiment.

Constant Power Adjustment of the Model

For heat transfer experiments with constant power input, the power supplied to the heating strips were adjusted one by one until the power input was the same. The model was allowed about half an hour to reach equilibrium. Then the temperatures of the strips were recorded.

Determination of Reattachment Length by Visualization of Surface Oil Flow

The surface oil flow technique was used to determine the reattachment length of the separated flow region. The oil mixture as recommended by de Brederode⁵² was also used in the present work. The approximate oil mixture proportion was as follows: 4 parts of 'plus-gas' formula 'A' penetrating oil, 2 parts of 'speedivac' grade 16 vacuum pump oil, 1 part of titanium dioxide and small drops of oleic acid. The mixture was painted onto a thin rectangular polythene film(about 10cmx7cm). The film was then stuck onto the upper surface of the model near the reattachment region. The wind tunnel was switched on at high speed(about 35m/s) for about half an hour. In the region of reattachment the shear stress was minimum($\tau_w \rightarrow 0$). The direction of the 'surface streamlines' (skin friction lines) was away from the reattachment line(de Brederode⁶²). An accumulation of oil was obtained along the reattachment line. The reattachment length was then measured from the leading edge.



4.2.4 Experimental Programme and General Procedures

Experimental Programme

The experimental programme was divided into two parts (i) to carry out basic studies of the flow field and to determine the heat transfer characteristics of the model in the non-turbulent free-stream, and (ii) to study the effect of turbulence on the surface heat transfer of the model.

General Procedures

For the non-turbulent free-stream experiments, the flow field around the model was made symmetrical by adjusting the flap, until the pressure distributions on the upper and lower surfaces of the model were nearly equal (better than 0.02 in C_p). The power supplies were then switched on. When a constant temperature model was required, the power inputs were carefully adjusted until the surface temperature was constant. When a constant power model was required, the power supplies were adjusted to provide a fixed power.

For the turbulent free-stream experiments, the model was placed in various position in the tunnel (at various distance from the grid). The experimental procedures were identical to those for the non-turbulent free-stream.

4.3 Velocity and Temperature Measurements in the Shear Layer Region of the Blunt Flat Plate

This section describes the equipment and the experimental techniques used to measure the mean and fluctuating quantities in the shear layer. For velocity measurements the model was not heated because (i) heating of the model would not significantly affect the velocity field (due to low operating model temperature). (ii) heating the model might affect the hot wire operation. Hot wire equipment (as in section 4.1.2) was used to measure turbulence fluctuations. For temperature measurements the model was heated to the condition of constant temperature.

4.3.1 Instrumentation and Experimental Techniques Fluctuating Temperature Measurement

Temperature fluctuation measurements in the shear layer were made by 1 μ m 'Wollaston' Platinum wire soldered onto a DISA U probe type '55P01'. The temperature probe was constructed in exactly the same way as a hot wire probe (for details of construction see Bradshaw⁶⁴).

The temperature probe was operated as a cold wire in the constant current mode, and it was connected to the constant current anemometer (Appendix C 1). In the constant current mode, the current flowing through the wire was about 1mA, so the current did not heat the wire appreciably. The resistance of the wire was small (about 80 Ω) compared with the resistance of the anemometer circuit (about 6K Ω), so the wire resistance did not affect the current (Appendix C 1). If the variation in wire voltage (e) were due to

the variations of the temperature of the wire alone, i.e. $e = I \alpha R_w \theta$, or $e = \beta \theta$ (the constant of proportionality β could be obtained algebraically or by experimental calibration). The fluctuating output signal (e) was then amplified by a 'Brookdeal 431' preamplifier. The preamplifier operated at a fixed gain of 60dB, it had a lower frequency limit of 1Hz (high pass filter) and an upper frequency limit of 20KHz (low pass filter).

It was recommended by Hancock⁶⁵ that two conditions had to be satisfied in order to achieve good temperature sensitivity and negligible velocity sensitivity: (i) the wire resistance had to be between 40Ω and 120Ω and (ii) the heating current of the wire had to be approximately 1mA. An analysis is presented in Appendix C 2 to show that the wire is sensitive only to temperature fluctuation.

The temperature probe was mounted on a DISA probe holder. The distance of the wire from the surface of the model was controlled by the 3'x2' tunnel traverse gear.

When the surfaces of the model were heated to a constant temperature, the probe was slowly traversed into the shear layer. The output voltage from the Brookdeal preamplifier was fed into a DISA r.m.s. meter and an oscilloscope. The r.m.s. temperature fluctuation ($\sqrt{\theta^2}$) was displayed on the r.m.s. meter (in terms of voltage). The mean temperature of the flow was obtained from the output voltage of the constant current anemometer.

The present set up to measure fluctuating temperature was found to have two advantages. Firstly, it

was easy to operate and, secondly, it had a very low noise level. The main disadvantage of the present circuit was that the upper limit of the frequency response of the Wollaston wire was about 1KHz (Andreopoulos⁶⁶), so that the probe could not be used in the leading edge region. The reason for this being that the high wave-number components of turbulence are important in this region.

The temperature signal was recorded by means of the Ampex tape recorder. The Brookdeal preamplifier low pass filter was set at 10KHz and high pass filter at 1Hz. The output signal was then passed through a data amplifier. The recording procedure was identical to that for hot wire signal.

Mean Temperature Measurement

The mean temperature of the flow was measured using both the constant-current temperature wire and the 'Copper-Constantan' thermocouple. The mean temperature could be determined accurately by the temperature wire (an error of 2 to 3%) provided the calibration of the wire was known. The thermocouple probe (mentioned in section 4.1.2) was mounted on the traverse gear and traversed vertically across the shear layer.

4.3.2 Experimental Programme

The experimental programme was divided into two parts for studying the velocity and temperature fields in the shear layer in (i) a non-turbulent free-stream and (ii) a turbulent free-stream. The 3" grid (grid B) was used to generate the appropriate condition in the tunnel because

it produced turbulence of moderate intensity ($\sqrt{u^2}/U_\infty \sim 5\%$) at an integral length scale of about 3.5cm. ($L_x/2H \sim 1.4$).

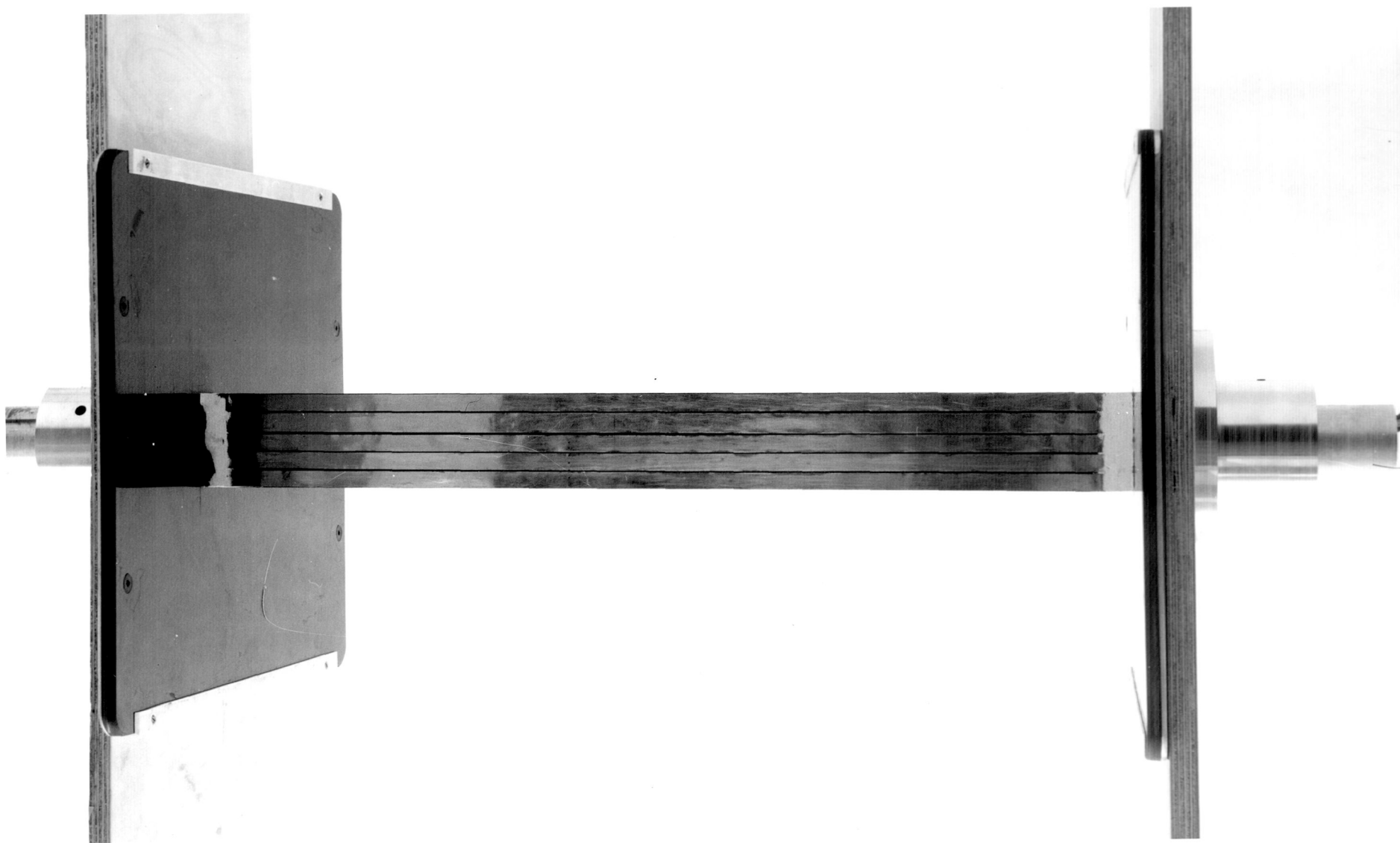


Figure 4.1 The whole assembly of the normal flat plate

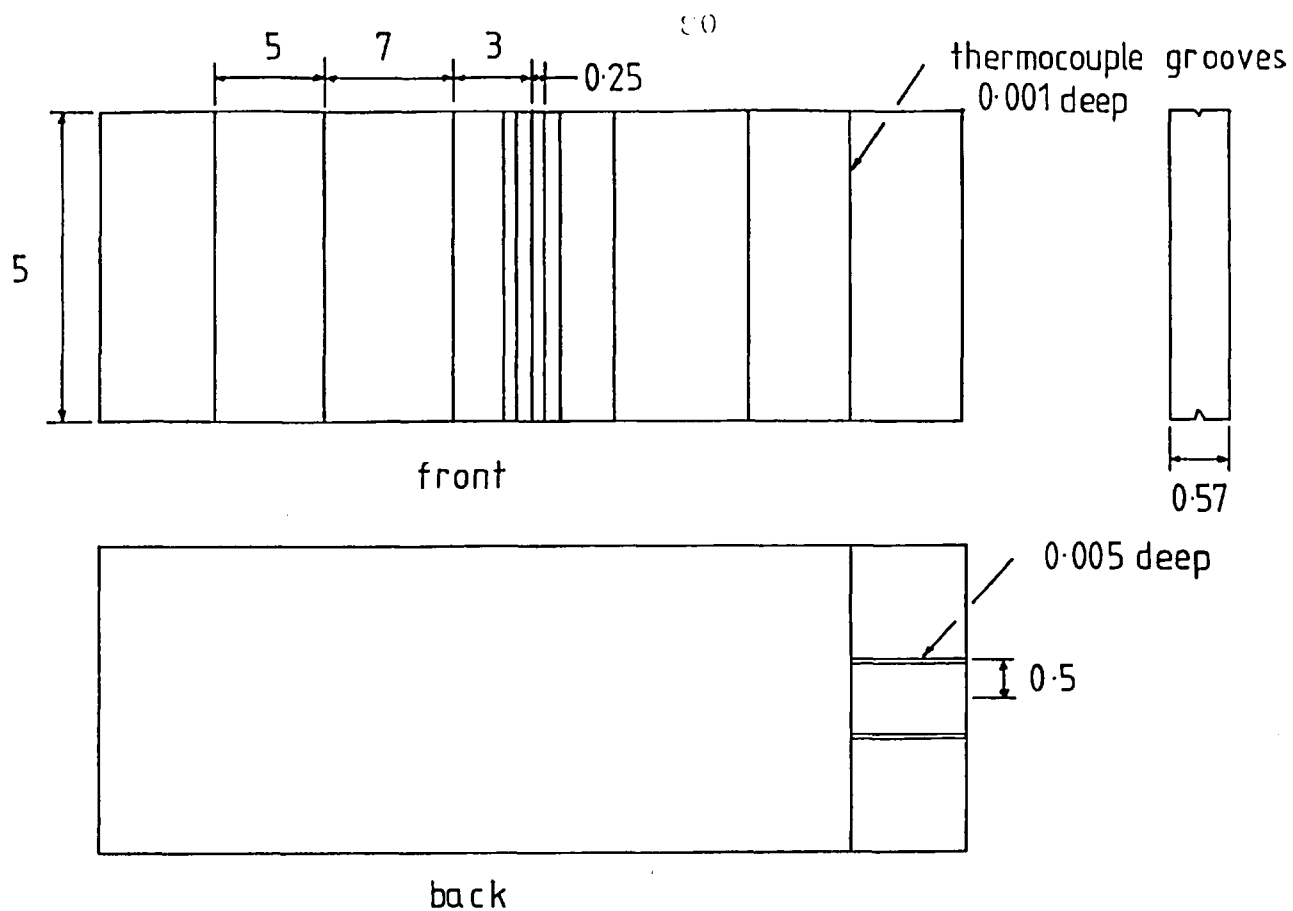


Figure 4.2a The dimensions of the first layer of insulator

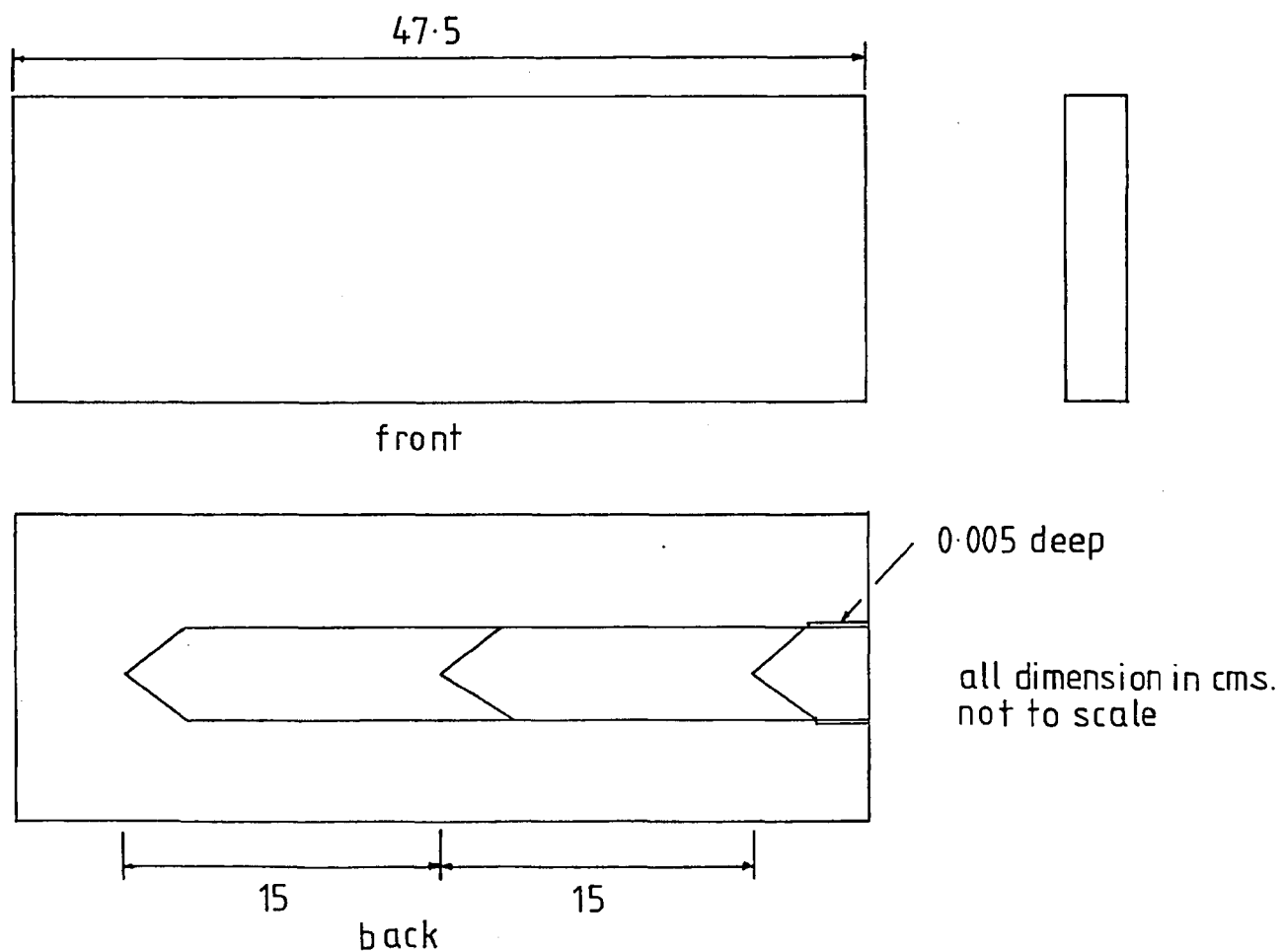


Figure 4.2b The dimensions of the second layer of insulator

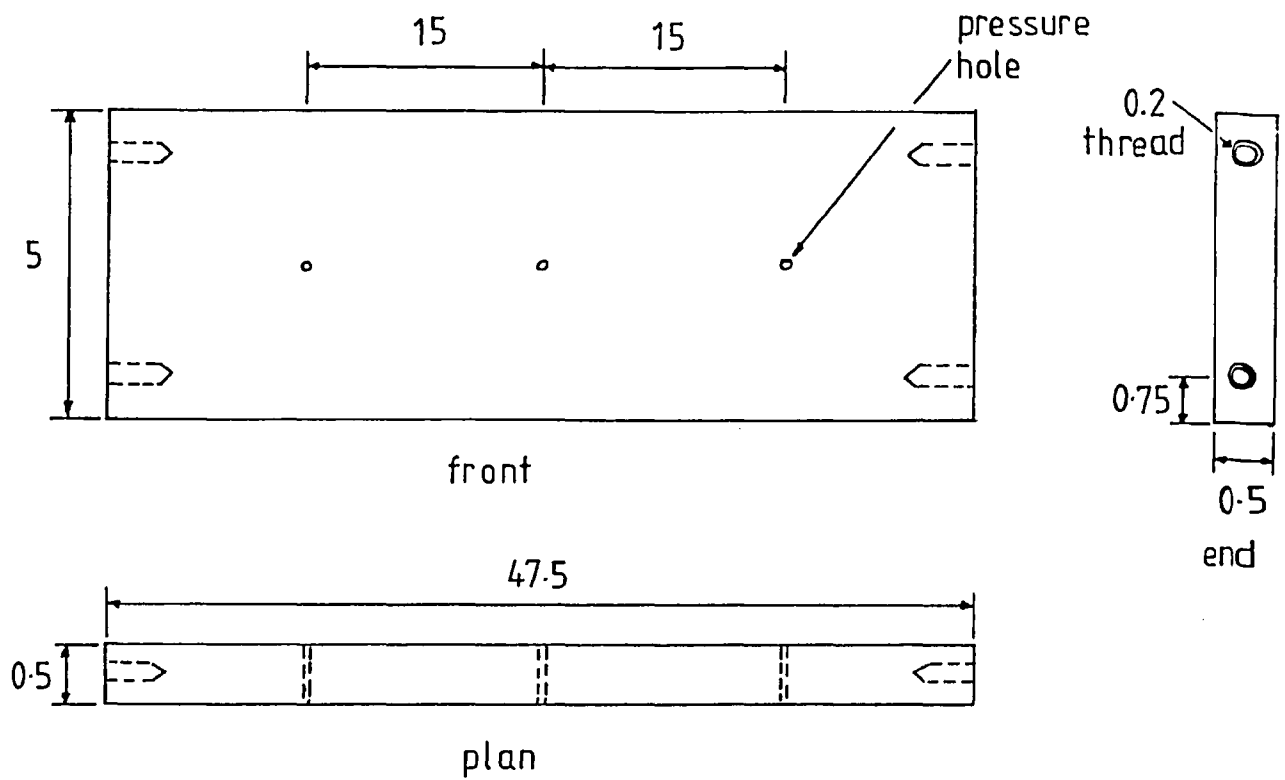


Figure 4.3a The dimensions of the dural back plate

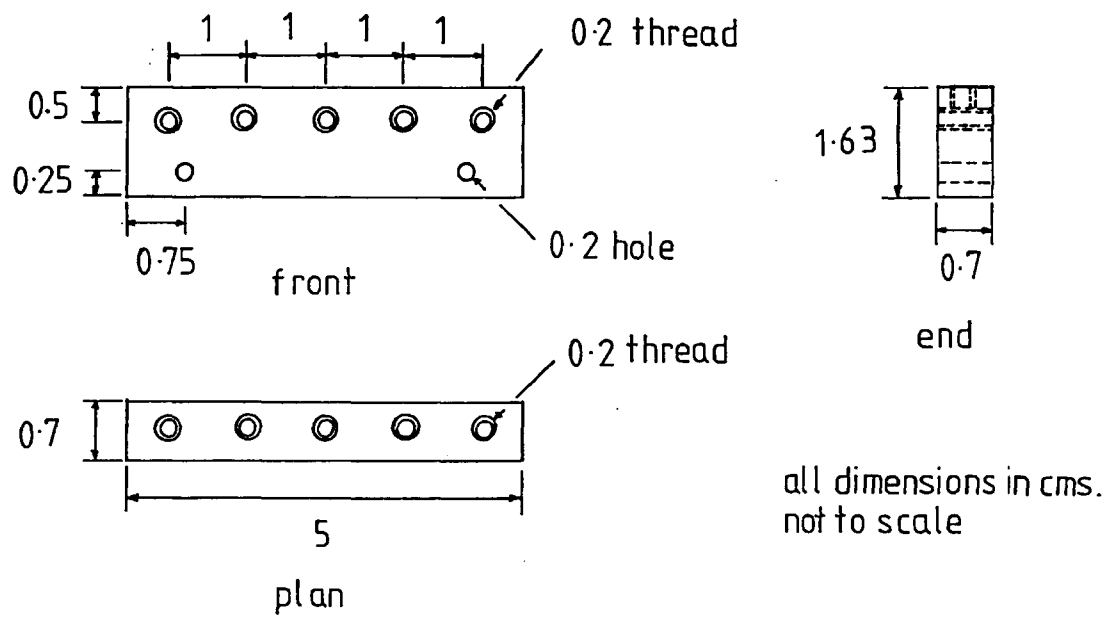


Figure 4.3b The dimensions of the PTFE block

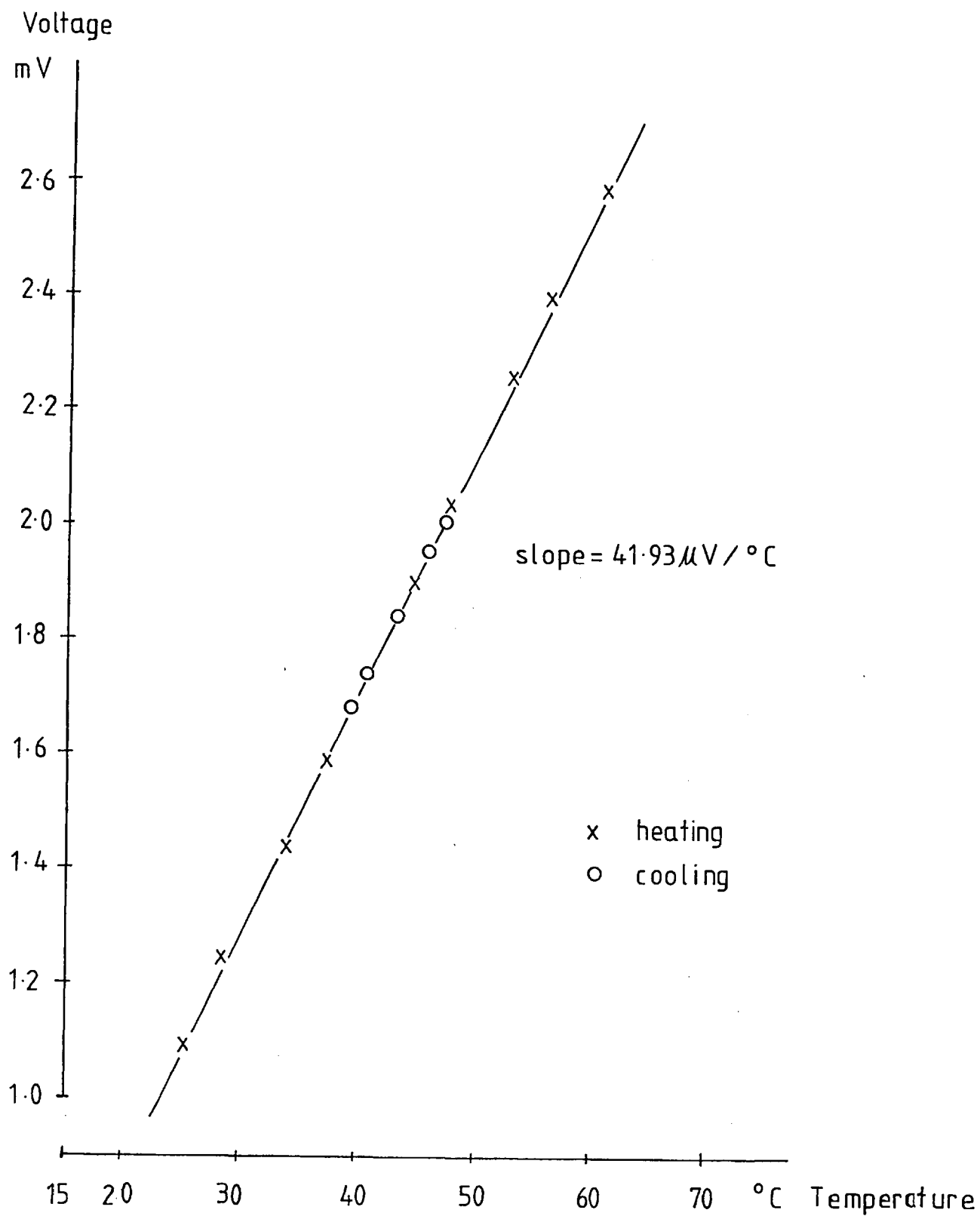


Figure 4.4 The voltage/temperature calibration for a Copper/Constantan thermocouple

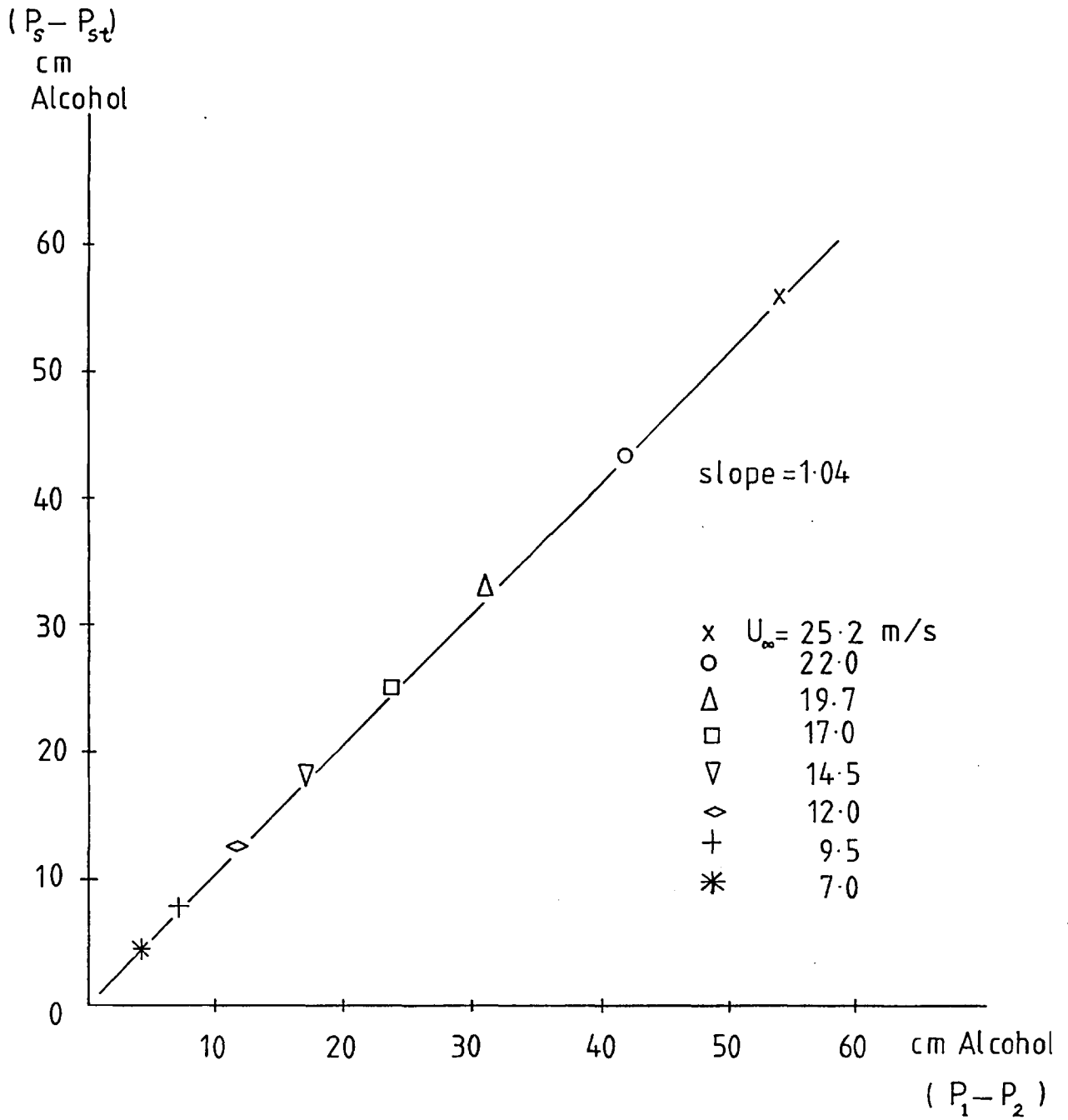


Figure 4.5 The 3'x2' wind tunnel calibration in non-turbulent free-streams

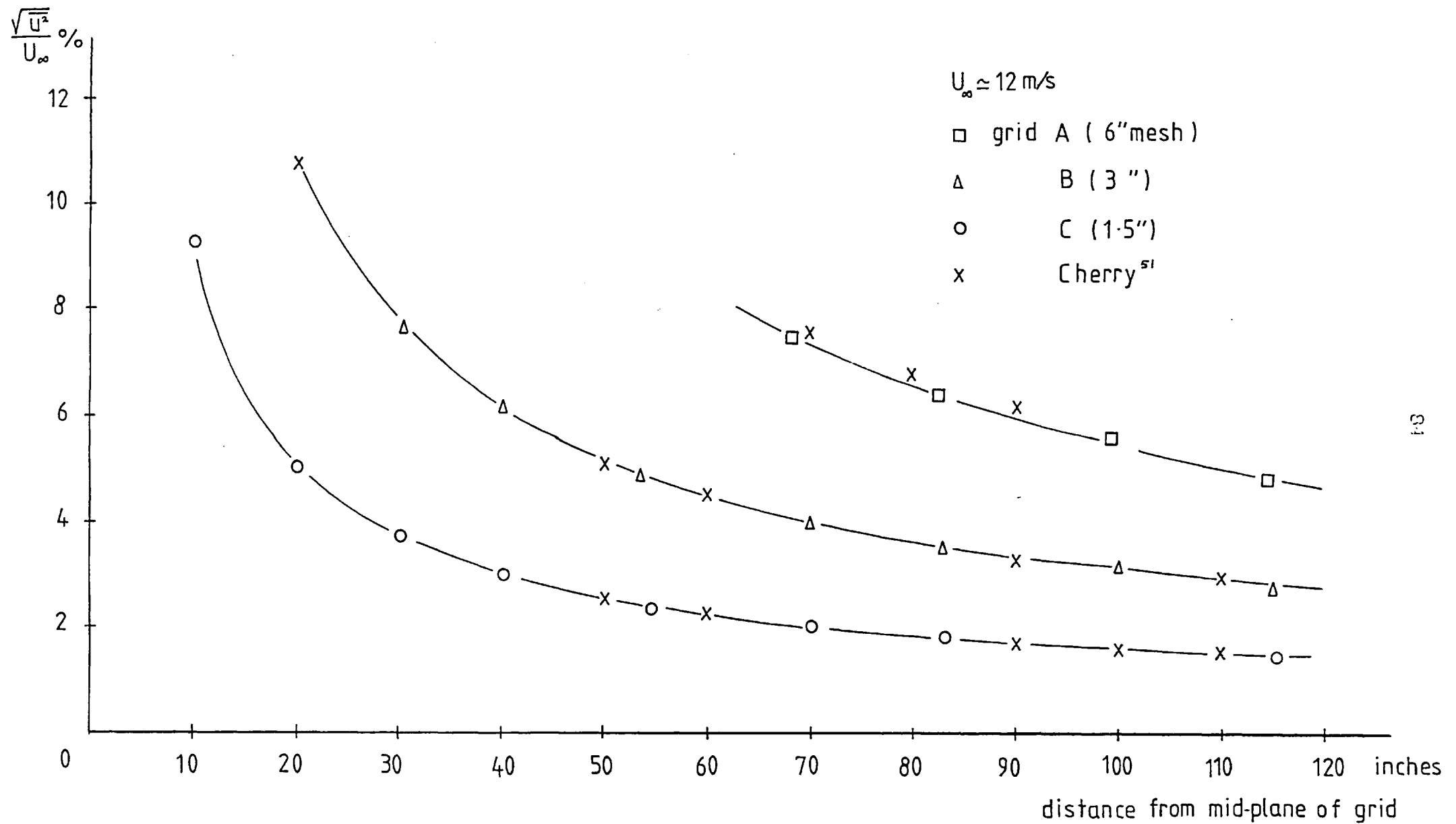


Figure 4.6 The variations of turbulence intensity along the wind tunnel

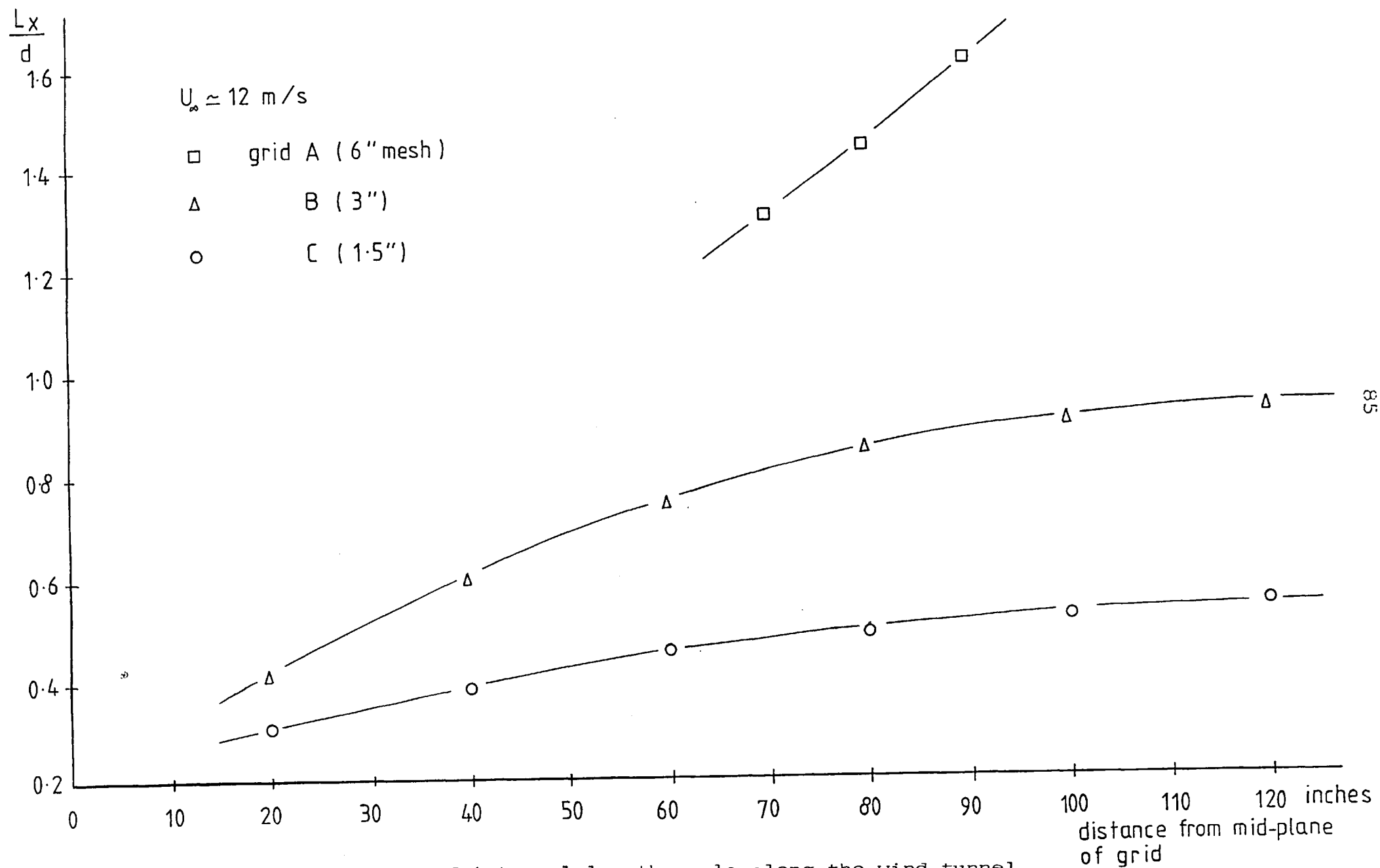


Figure 4.7 The variations of integral length scale along the wind tunnel

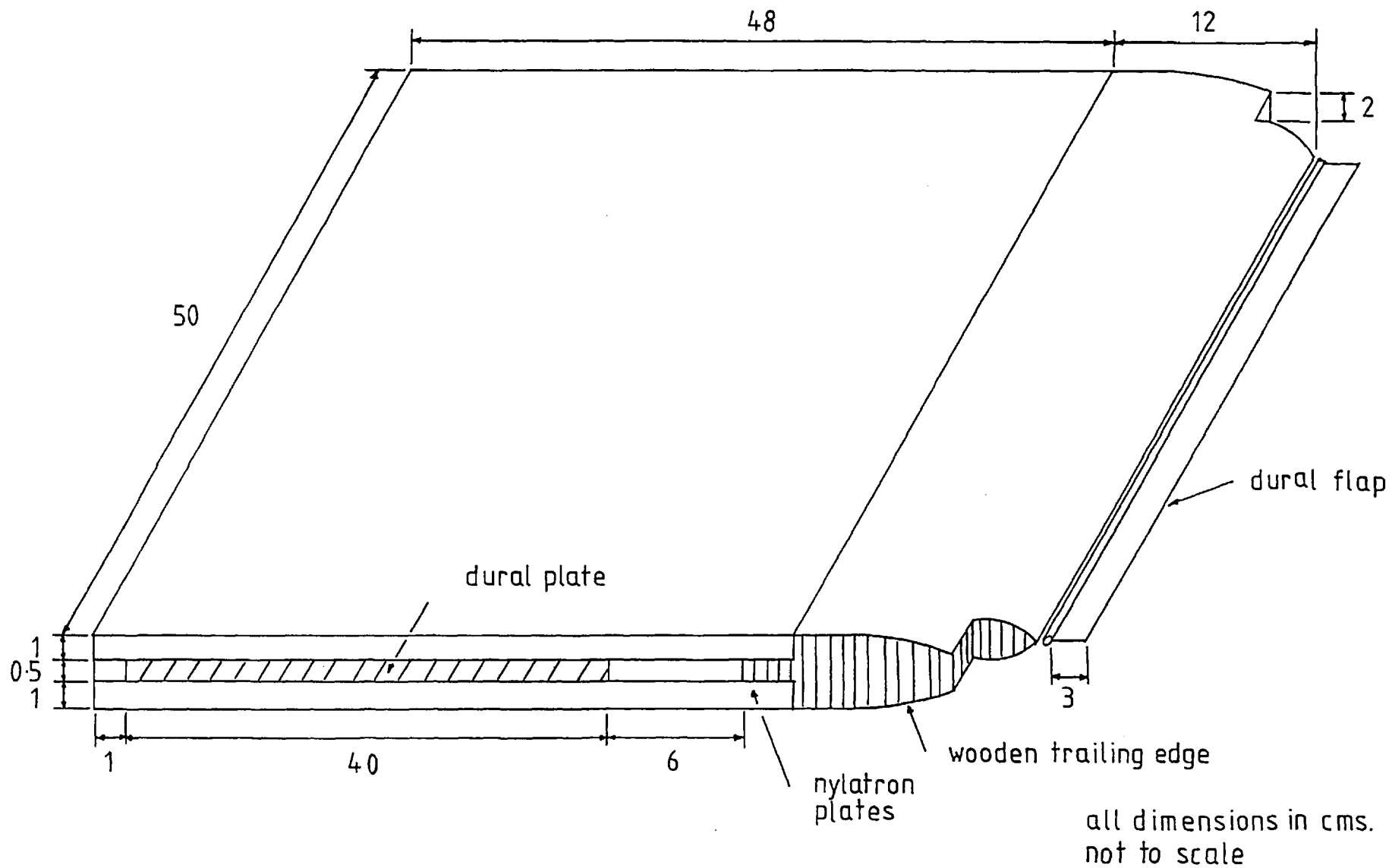


Figure 4.8 The dimensions of the blunt flat plate

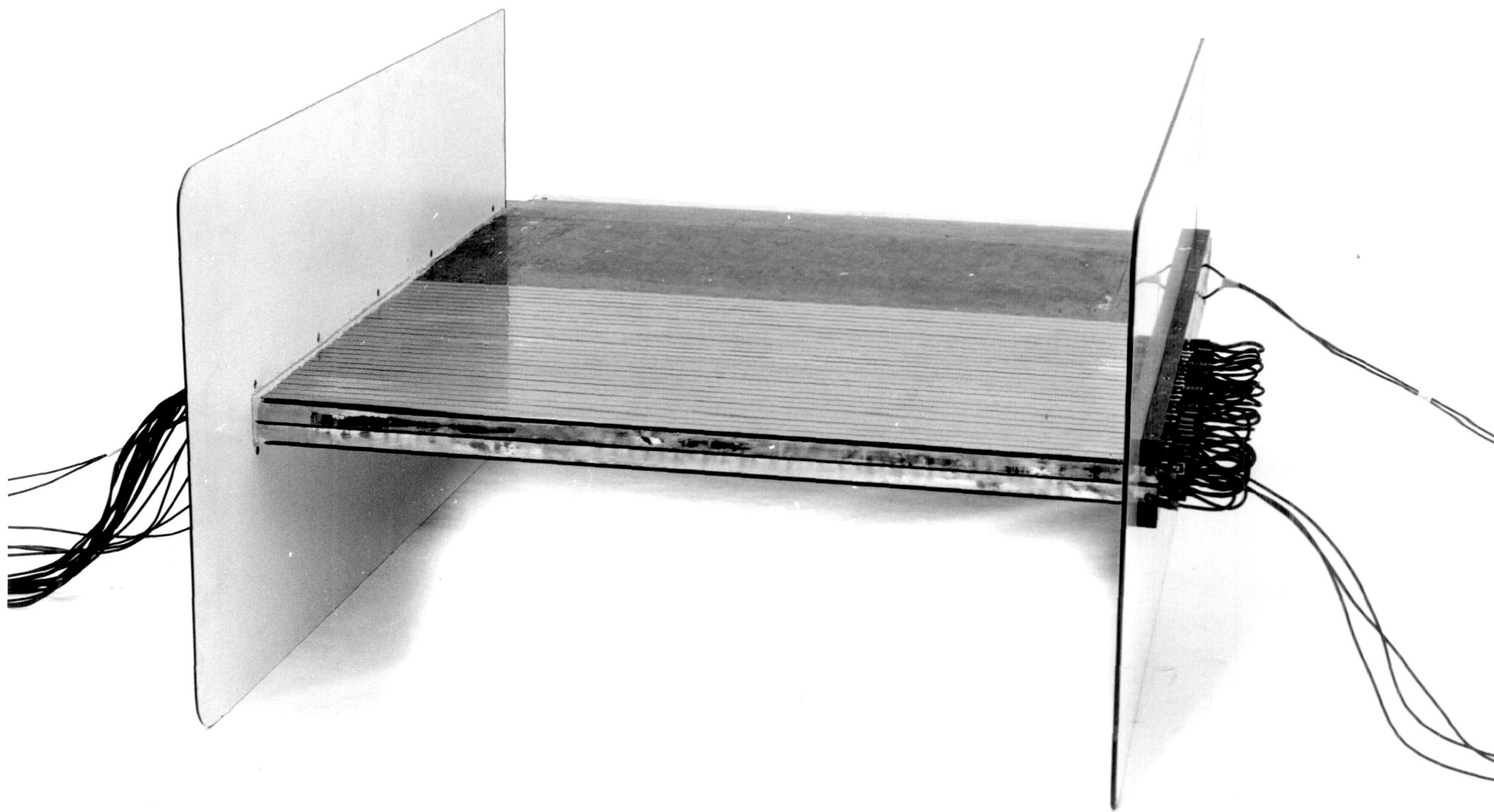


Figure 4.9 The blunt flat plate model

The numbering systems of the blunt flat plate

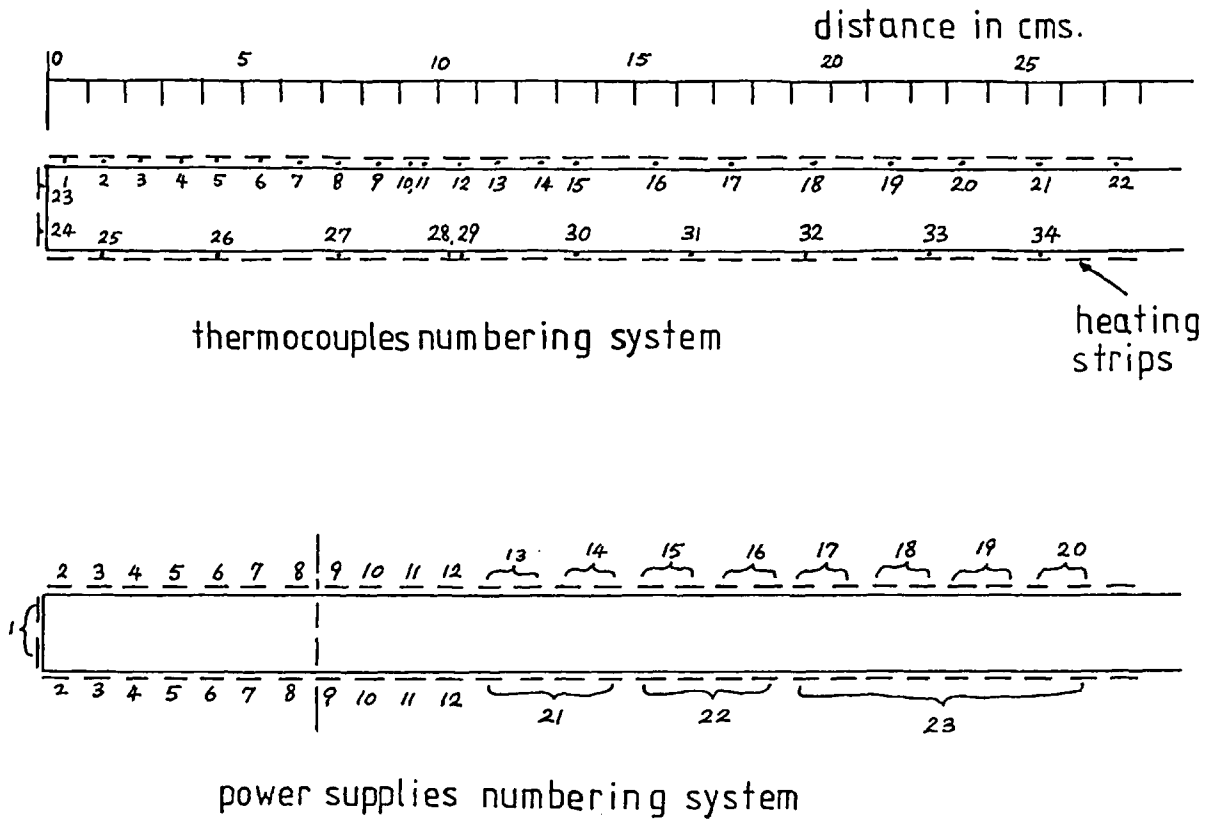


Figure 4.10a The numbering systems for the power supplies and thermocouples

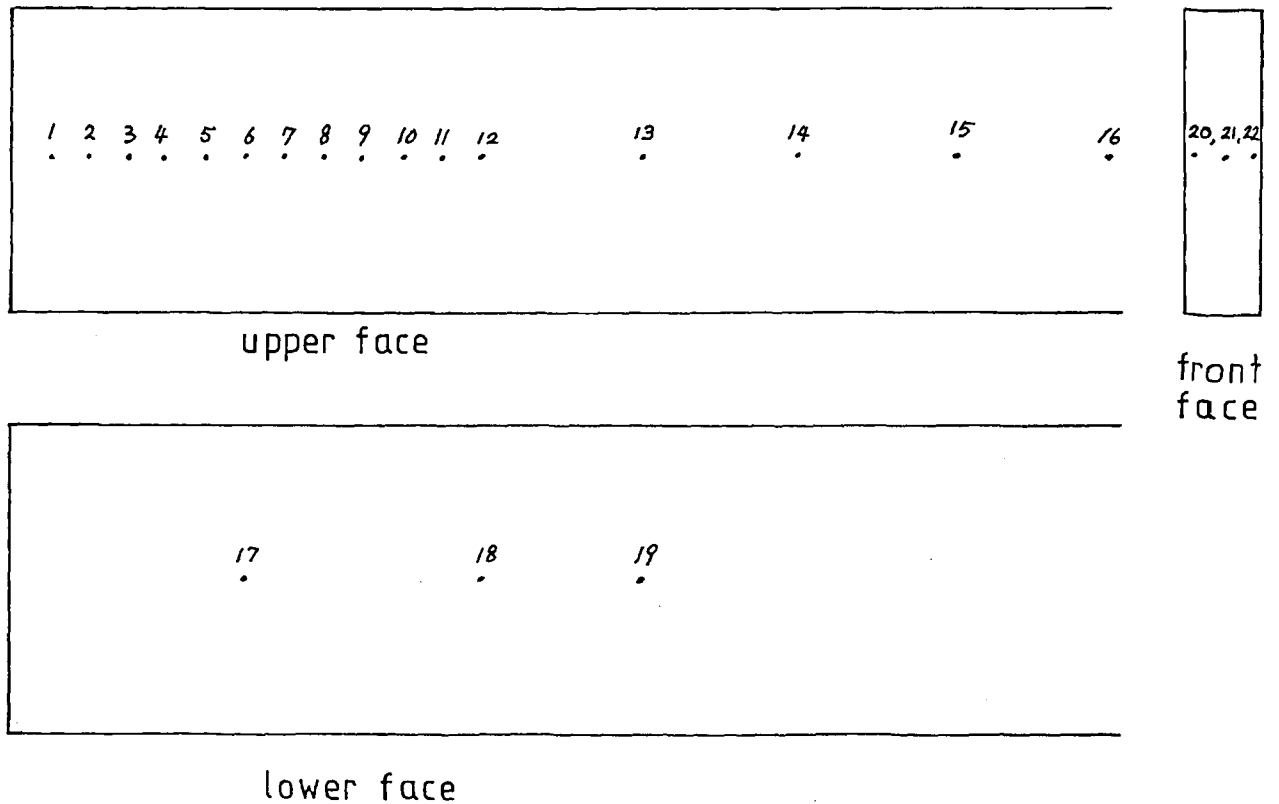
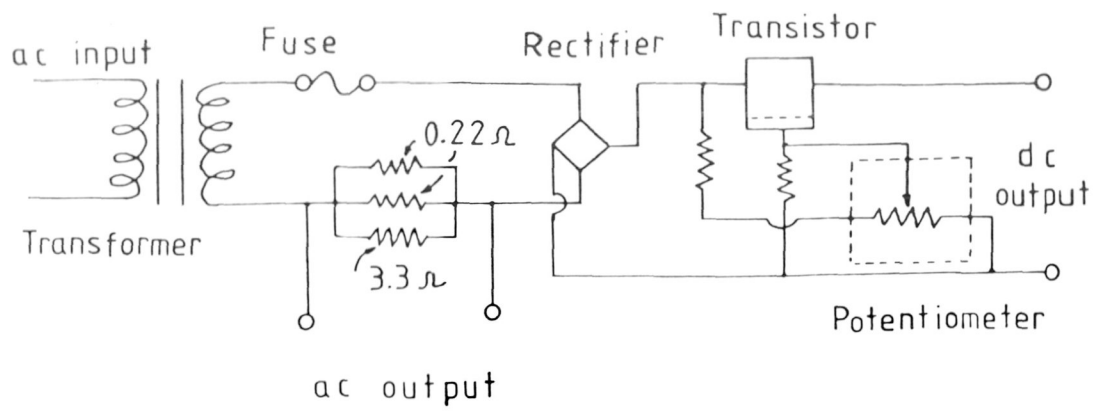


Figure 4.10b The numbering system for the pressure holes



Module circuit

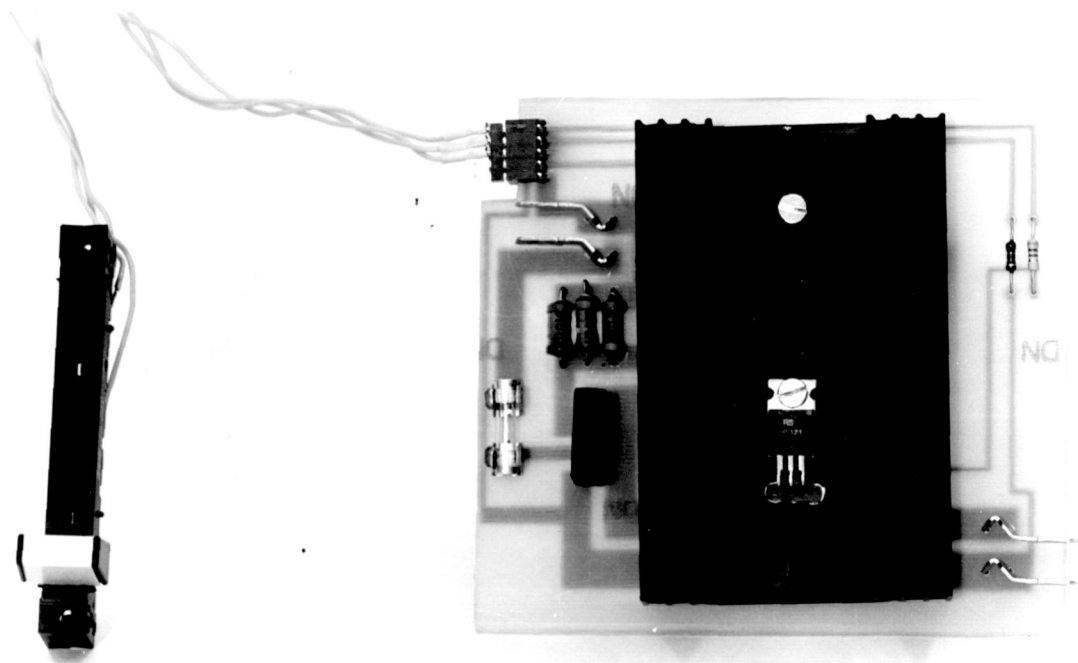


Figure 4.11 The power supply module

5. PRESENTATION OF THE STAGNATION POINT(Laminar
BOUNDARY LAYER) HEAT TRANSFER RESULTS FOR THE
NORMAL FLAT PLATE

5.1 Discussion of Results for the Non-turbulent
Free-stream

5.1.1 The Stagnation Point

Three different experiments were performed to investigate the effects of power input, temperature difference ($T_w - T_\infty$) and Re changes on the Nu/Re relationship. The stagnation point Nusselt numbers(log) were plotted against Reynolds numbers(log) as shown in Figure 5.1. The gradient was found to be +0.50, i.e. $Nu = c Re^{0.5}$. The constant c was approximately 0.50(± 0.01) for the three experiments(Figure 5.2). These experimental results showed conclusively that the stagnation point heat transfer parameter $Nu/Re^{0.5}$ was independent of power input, ($T_w - T_\infty$) and Re (under the present experimental conditions). This finding is in general support of the underlying assumptions in theories such as Goldstein⁹ or Frössling¹⁰, where it is assumed that temperature effects are confined solely to the energy equation, and that this in consequence can be uncoupled from the continuity/momentum equations(section 1.3.1); hence linear dependence of $\dot{q}$ on ($T_w - T_\infty$). Only when temperature changes are sufficiently large to affect either ρ or μ would one expect to see a departure from this conclusion. The maximum temperature difference ($T_w - T_\infty$) was about 12°C in the present investigation. The density of air(near the model surface) was estimated to change by about 3% and the viscosity by about 2% due to heating of the model. Therefore ρ and μ

could be assumed constant.

The experimental constant c ($\approx +0.50$) was about 9% lower than the theoretical value ($\approx +0.55$ based on an experimental Cp_b of -1.48 , uncorrected for blockage) calculated using Frössling's theory. The difference between theory and experiment was due to a combination of the following factors (i) heat losses from the model such as radiation (it was estimated to be about 2% of the total heat input), heat conduction to the end plates (about 3%) and lateral conduction of the model (about 0.2%). (ii) error due to temperature measurement (the measurement accuracy of the thermocouples was about $\pm 0.05^\circ\text{C}$, for a typical $(T_w - T_\infty)$ of 10°C ; the error in temperature measurement was about 1%).

5.1.2 The Heat Transfer Distribution across the Plate

The variation of $Nu/Re^{0.5}$ across the front face of the plate is shown in Figure 5.3. The experimental results were plotted against Frössling's theoretical prediction $(Nu/Re^{0.5} = 0.55 + 0.60(y/d)^2 + 0.91(y/d)^4 + \dots)$ based on the experimental $Cp_b = -1.48$ and the experimental results of Sogin et al.¹³ (heat transfer of a normal flat plate) were also included. The present experimental results show that $Nu/Re^{0.5}$ increases steadily away from the stagnation point and is insensitive to Reynolds number change. The agreement between experimental results and Frössling's prediction is fairly good (about 9% difference) around the central region of the plate ($-0.2 < y/d < +0.2$), the theory over-predicting in this region. Further outboard the

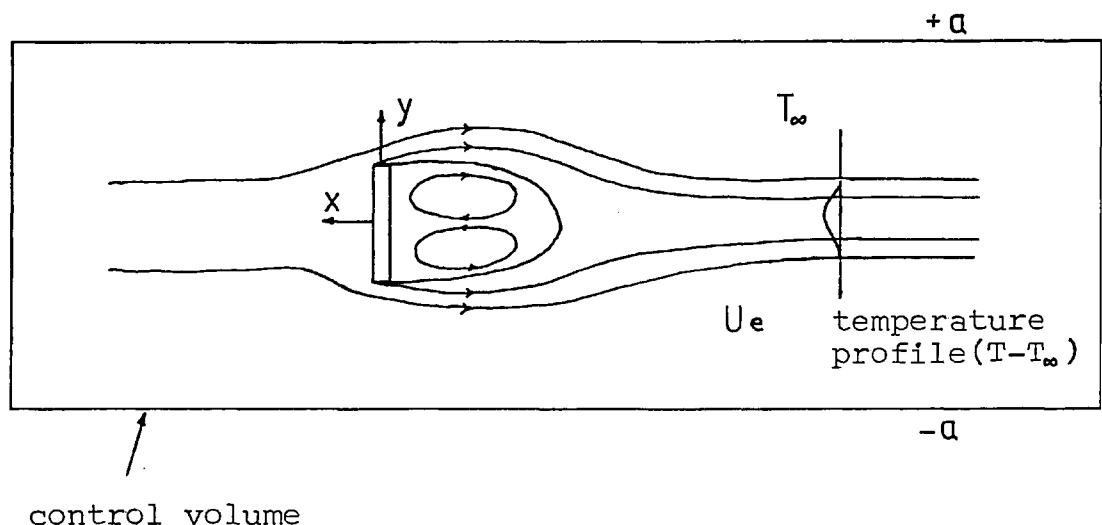
experimental $Nu/Re^{0.5}$ increases faster than theory, and the two curves cross at $y/d = \pm 0.25$. At the positions of $y/d = \pm 0.4$ the experimental values are about 15% higher than those calculated theoretically. The reason for the larger difference between theory and experiment at these positions may be due to the fact that a small amount of heat was lost through the edges of the model, so more power was required to maintain constant temperature at this region. It was not possible to calculate this heat loss because the temperature distribution at the edges of the model was not known (as a rough estimation the heat loss through the edges was about 3% of the total heat input). It should be noted that Frössling's theory is accurate near edges provided sufficient number of terms are taken to describe the velocity and temperature distributions (the error in ignoring higher order terms in $Nu/Re^{0.5}$ is about 1%).

The experimental results of Sogin et al.¹³ were very much higher than the present results and showed a significant Reynolds number effect (Figure 5.3), for which Sogin et al. did not provide any explanation. They provided no base pressure measurements over the Re range, so that it cannot be established whether the $Nu/Re^{0.5}$ variation correlates with a variation in $d(v/U_\infty)/d(y/d)$ (this would probably require a large C_p change any way). Sogin's experiments were conducted at a low aspect ratio (model span/model width ≈ 3.3) and without end plates, so that clearly three-dimensional effects may have been important - bleed of tunnel wall boundary layers into the wake region and separation of tunnel wall boundary layers by the model

(both are Reynolds number dependent).

5.1.3 Temperature and Velocity Measurements in the Wake Region

The purpose of these measurements was to check the balance between the input power and the convected thermal energy in the wake, so that an assessment of the accuracy of the heat transfer measurement technique could be made. The mean temperature and velocity profiles at 9 diameters downstream were measured because this position was downstream of the recirculating region (a standard pitot-static tube was used to measure the mean flow velocity) and the temperature difference ($T - T_\infty$) was still large enough to be measured by the thermocouple probe. In order to determine the thermal energy balance, a set of imaginary control planes at large distances from the plate was assumed (see diagram below).



The amount of thermal energy convection in the wake was obtained by integrating the thermal energy in the section

where measurements were made (assuming mean temperature distribution was two-dimensional),

$$\text{i.e. } Q = \rho C_p b \int_{-a}^a U (T - T_{\infty}) dy \quad (\text{Appendix D1})$$

minus the kinetic energy dissipated into thermal energy (due to viscous effect). In order for all the kinetic

energy to be converted into thermal energy a distance very far downstream is required. Ultimately, this contribution and the thermal energy flow rate must be equal to the drag of the plate i.e. $D \cdot U_e = b \rho U_e \int_{-a}^a U (U_e - U) dy$.

At the position of measurement ($\sim 9d$) it was expected that only part of the kinetic energy was dissipated into thermal energy. A rough estimation of the amount of kinetic energy dissipation was carried out in Appendix D1. The turbulence intensity $\sqrt{u^2}/U_e$ in the wake was assumed to be about 20% (also that $\overline{u^2} \sim \overline{v^2} \sim \overline{w^2}$) - ref. Bearman⁶⁷. The kinetic energy dissipation was estimated to be about 12% of the ultimate value. Under the present experimental conditions ($U_e = 7\text{m/s}$, $(T_w - T_{\infty}) \simeq 10^\circ\text{C}$), the amount of power due to drag was found to be about 2.7W and the integrated thermal energy was about 31.8W. The kinetic energy dissipation was hence about 0.3W (about 1% of the amount of thermal energy convection and was therefore negligible). One must bear in mind that the power due to drag and the integrated energy were functions of U_e and $(T_w - T_{\infty})$ respectively, the ratio of these two quantities was therefore dependent on the magnitude of U_e and $(T_w - T_{\infty})$. Finally, the input power ($\simeq 33\text{W}$) and the thermal energy convection ($\simeq 31.5\text{W}$) were found to differ by about 5%. The measurement technique was thus fairly accurate. The main error could be due to the temperature

measurement in the wake(the error in temperature was about 10% for $(T - T_\infty) \sim 1^\circ\text{C}$).

5.2 Discussion of the Effects of Free-stream Turbulence on Stagnation Point Heat Transfer

5.2.1 The Effect of Reynolds number

With stream turbulence the experimental stagnation point Nusselt numbers(log) were plotted against Reynolds numbers(log) as shown in Figure 5.4. The gradient was found to be approximately 0.58 for all three turbulence grids, i.e. $Nu \propto Re^{0.58}$. The data were evaluated in the form of $Nu/Re^{0.58}$, the following results being obtained(see Table below).

	grid A(6"mesh)	grid B(3")	grid C(1.5")
$\frac{Nu}{Re^{0.58}}$ (stagnation point)	$c \approx 0.30 \pm 0.05$	$c \approx 0.29 \pm 0.05$	$c \approx 0.28 \pm 0.05$
$\frac{\sqrt{u^2}}{U_\infty}$ (averaged value)	7.7%, 6.7%	4.9%, 4.1%	2.5%, 2.1%
Re range	$1.7 \times 10^4 - 4.9 \times 10^4$	$1.7 \times 10^4 - 5.4 \times 10^4$	$1.7 \times 10^4 - 5.4 \times 10^4$

It can be seen that $Nu/Re^{0.58}$ remains roughly constant for each grid, in other words $Nu/Re^{0.58} = c Re^{0.08}$. In free-stream turbulence $\frac{Nu}{Re^{0.58}} = f\left(Re, \frac{L_x}{d}, \frac{\sqrt{u^2}}{U_\infty}\right)$. From the above results, it appears that c is a function of $\frac{L_x}{d}$ and $\frac{\sqrt{u^2}}{U_\infty}$, i.e. $\frac{Nu}{Re^{0.58}} = Re^{0.08} f\left(\frac{L_x}{d}, \frac{\sqrt{u^2}}{U_\infty}\right)$ is a reasonable way to represent the present data. Moreover, the stagnation point heat transfer is also dependent on base pressure(i.e. $\frac{d(v/U_\infty)}{d(y/d)}$), hence c is a function of $\left(\frac{\sqrt{u^2}}{U_\infty}, \frac{L_x}{d}, \frac{d(v/U_\infty)}{d(y/d)}\right)$.

According to Frössling's theory the stagnation point $Nu/Re^{0.5}$ varies as $(1 - Cp_b)^{1/4}$. The base pressure is found to change only slightly (less than 0.05 in Cp_b) between the different turbulent free-streams (Figure 5.7). Therefore $\frac{d(v/U_\infty)}{d(y/d)}$ is expected to change very slightly (because $\frac{d(v/U_\infty)}{d(y/d)} \propto (1 - Cp_b)^{1/4}$). Hence the effect of $\frac{d(v/U_\infty)}{d(y/d)}$ on c is likely to be negligible, i.e. $c = f\left(\frac{L_x}{d}, \frac{\sqrt{u^2}}{U_\infty}\right)$. The stagnation point heat transfer data were then plotted in the form of $Nu/Re^{0.5}$ against Re (Figure 5.5). It can be seen that the relationship $Nu/Re^{0.5} = c Re^{0.08}$ gives a good correlation of data (the amount of scatter is less than 2% in $Nu/Re^{0.5}$). Although the turbulent free-streams were Reynolds number sensitive, one would not expect the Reynolds number dependence to increase suddenly from 0.50 (non-turbulent free-stream) to the value of 0.58 (turbulent free-streams). There may be some intermediate dependence (between 0.50 and 0.58) at lower intensities ($\sqrt{u^2}/U_\infty < 2\%$) which the present investigation did not study. At very low Reynolds numbers ($Re \rightarrow 0$) the relationship $Nu/Re^{0.5} = c Re^{0.08}$ breaks down (as $Nu/Re^{0.5} \rightarrow c$), and at very low intensities ($\sqrt{u^2}/U_\infty \rightarrow 0$) it does not tend towards the free-stream value ($Nu/Re^{0.5} = 0.50$). The above relationship does not explain how the transition from non-turbulent to turbulent free-stream can occur and is not a universal function, it merely provides a good correlation of data. However, it is physically more plausible to express the effect of Reynolds number in the form of $Nu/Re^{0.5} = a_0 \text{ (non-turb.)} + a_1 \text{ (turb.)} + a_2 \text{ (turb.) } Re$ (where $a_0 = 0.50$ for non-turbulent free-stream, a_1 and a_2 being functions of $\frac{\sqrt{u^2}}{U_\infty}$ and $\frac{L_x}{d}$). The above form is able to

account for low intensities ($\frac{Nu}{Re^{0.5}} \rightarrow a_0$ as $\frac{\sqrt{u^2}}{U_\infty} \rightarrow 0$) and low Reynolds numbers ($Nu/Re^{0.5} \rightarrow a_0 + a_1$, as $Re \rightarrow 0$). It should be noted that the present investigation has insufficient data to verify a_1 and a_2 , further measurements are therefore necessary.

The frequency spectrum of a typical turbulent free-stream (grid B $\frac{\sqrt{u^2}}{U_\infty} = 4.9\%$) at different Reynolds number was studied (Figure 5.6). It can be seen that there is a steady progression from high to low Re . High Re shows a more rapid fall off at the highest wave-numbers ($\tilde{n} = 2\pi n \frac{L_x}{U_\infty}$). In consequence, since spectra are normalised and each should have the same area, the high Re case shows higher values at low wave-numbers ($\tilde{n} \sim O(1)$). The change between the two occurs at about $\tilde{n} = 20$. The turbulence intensity and integral scale of the turbulent free-stream at different Reynolds numbers were also measured (see Table).

Re	5.3×10^4	4×10^4	1.7×10^4
$\sqrt{u^2}/U_\infty$	4.93% ($\pm 0.05\%$)	4.86% ($\pm 0.05\%$)	4.81% ($\pm 0.05\%$)
L_x	3.81cm (± 0.1 cm)	3.70cm (± 0.1 cm)	3.84cm (± 0.1 cm)
$Nu/Re^{0.5}$	0.790	0.675	0.643

The intensity was found to increase roughly with increasing Re , whilst the length scale did not show a consistent trend. However, the changes in intensity and length scale with Re were small (within the measurement accuracy). The increase in intensity ($\sim 0.12\%$) with Re is unlikely to be the cause

for the increase ($\sim 10\%$) in stagnation point heat transfer (see Table). The reason is that in order for a 10% increase in $Nu/Re^{0.5}$ (0.643 to 0.709) to occur an increase in intensity of 2%(roughly) is required-see Figure 5.9. The change in length scale is likely to have an even smaller effect than intensity on heat transfer. Therefore, the increase in stagnation point heat transfer $Nu/Re^{0.5}$ with Re may be due to the change in free-stream turbulence structure as the Reynolds number varies.

The stagnation point heat transfer parameter $Nu/Re^{0.5}$ in turbulent free-stream is expected to be a function of $(Re, \sqrt{u^2}/U_\infty, L_X/d)$. The boundary layer on the front face of the model was likely to be laminar because (i) the intensity $(\sqrt{u^2}/U_\infty \sim 7.7\%$ at the most) and length scale(maximum $L_X/d \sim 2.2$) of the turbulent free-stream were fairly moderate (ii) a strong favourable pressure gradient (iii) large L_X/δ ($L_X \sim 50 \delta$, $Re_\delta \sim 260$ to 500 for $Re = 1.7 \times 10^4$ to 5.3×10^4 -Appendix D2). The boundary layer thickness δ and the thermal layer thickness δ_T are functions of Re . Although they are very thin compared with L_X ($L_X \sim 50 \delta$ or δ_T), they may be comparable to the scale of the higher wave-number eddies (assuming $n \sim 5\text{KHz}$ -filtered frequency; $L_X \sim 3.8\text{cm}$; $U_\infty \sim 12\text{m/s}$, then $\tilde{n} \sim 100$ - Figure 5.6). If these eddies are partly responsible for the stagnation point heat transfer, then the parameters L_X/δ and L_X/δ_T could become important. Hence the stagnation point heat transfer $\frac{Nu}{Re^{0.5}}$ is a function of $(Re, \sqrt{u^2}/U_\infty, L_X/d, L_X/\delta, L_X/\delta_T)$, i.e., the effect of Re in heat transfer is important.

5.2.2 The Effect of Pressure Distribution

The pressure distribution along the front face and the back face of the model is shown in Figure 5.7. It was found that the pressure distribution of the front face was similar to that in the case of a non-turbulent free-stream. The presence of turbulence had only a small effect on the front face pressure distribution (one should note that the front edge velocities change like $(1 - C_{p_b})^{\frac{1}{2}}$, but the velocity gradient is very large at the edge region, so the effect on the rest of the C_p distribution is not apparent). The base pressure was found to reduce slightly in turbulent free-streams. This finding was in agreement with Petty⁶⁸ (experiments with a two-dimensional rectangular cylinder) and Donoso⁶⁹ (experiments with a sharp edge normal flat plate). They found that the base pressure was slightly affected by the free-stream intensity $\sqrt{u^2}/U_\infty$, but it showed no dependence on the scale of turbulence (L_x/d). The present experimental C_{p_b} (-1.59) for turbulent free-stream (grid A, $\sqrt{u^2}/U_\infty \sim 7.7\%$) was used to calculate the theoretical heat transfer. It was found that $Nu/Re^{0.5}$ increased only by 0.5% due to the change in base pressure ($C_{p_b} = -1.48$ for non-turbulent free-stream). The reason for this small increase is because $Nu/Re^{0.5}$ varies approximately as $(1 - C_{p_b})^{\frac{1}{4}}$.

5.2.3 The Effect of Intensity and Length Scale

The frequency spectra of all the turbulent free-streams are shown in Figure 5.8. The power spectral density is made non-dimensional by multiplying it with $U_\infty/2\pi L_x$ and

the non-dimensional frequency is $\tilde{n} = \frac{2\pi L_x n}{U_\infty}$. All the experiments were carried out at a fixed Reynolds number ($Re = 4 \times 10^4$), so the Reynolds number effect was eliminated. It can be seen that all the spectra fit quite well onto the von Kármán interpolation formula ($\theta_n = \frac{2}{\pi} (1 + 1.8\tilde{n}^2)^{-\frac{5}{6}}$; Martin¹⁰). The degree of scatter is very small. The turbulent free-stream can therefore be adequately described by their intensities and length scales.

In free-stream turbulence the stagnation point heat transfer parameter $Nu/Re^{0.5}$ depends on Re , $\sqrt{u^2}/U_\infty$, L_x/d . Turbulence tests were conducted at a fixed Reynolds number ($Re = 4 \times 10^4$). Essentially in the present investigation it is desirable to study the variation of intensity (with fixed L_x/d) and the variation of L_x/d (with fixed intensity). This aim is difficult to carry out precisely (since both intensity and scale change with distance downstream of a given grid), but the appropriate interpolations are shown in Figures 5.9 and 5.10. It can be seen that $Nu/Re^{0.5}$ increases with increasing turbulence intensity and with decreasing L_x/d . The sensitivity to turbulence intensity does not appear to be as strong as that found in experiments on circular cylinders where increases of 100% (for 3% intensity) have been noted (Kestin et al²⁵, Smith and Kuethe²³, Lowery and Vachon²⁷). As an empirical correlation of data, the combined parameter $(\frac{\sqrt{u^2}}{U_\infty}) (\frac{L_x}{d})^{-\frac{1}{2}}$ appears to give the best fit of data points (Figure 5.11). It can be seen that $Nu/Re^{0.5}$ increases steadily with $(\frac{\sqrt{u^2}}{U_\infty}) (\frac{L_x}{d})^{-\frac{1}{2}}$ which was similar to Taylor's parameter $(\frac{\sqrt{u^2}}{U_\infty}) (\frac{L_x}{d})^{-\frac{1}{5}}$ (Kestin and Maeder⁷¹). The physical significance of $(\frac{\sqrt{u^2}}{U_\infty}) (\frac{L_x}{d})^{-\frac{1}{2}}$

indicates that intensity has the dominant effect on stagnation point heat transfer (a massive L_x change is required to see its effect).

It should be noted that free-stream turbulence also has a significant effect on the skin friction (τ_w) near the stagnation region. The measurements of Smith and Kuethe²³ (heat transfer of a circular cylinder) showed that the increase in skin friction of the wall was nearly within 70% of that of heat transfer. However, no measurements of skin friction were taken in the present investigation .

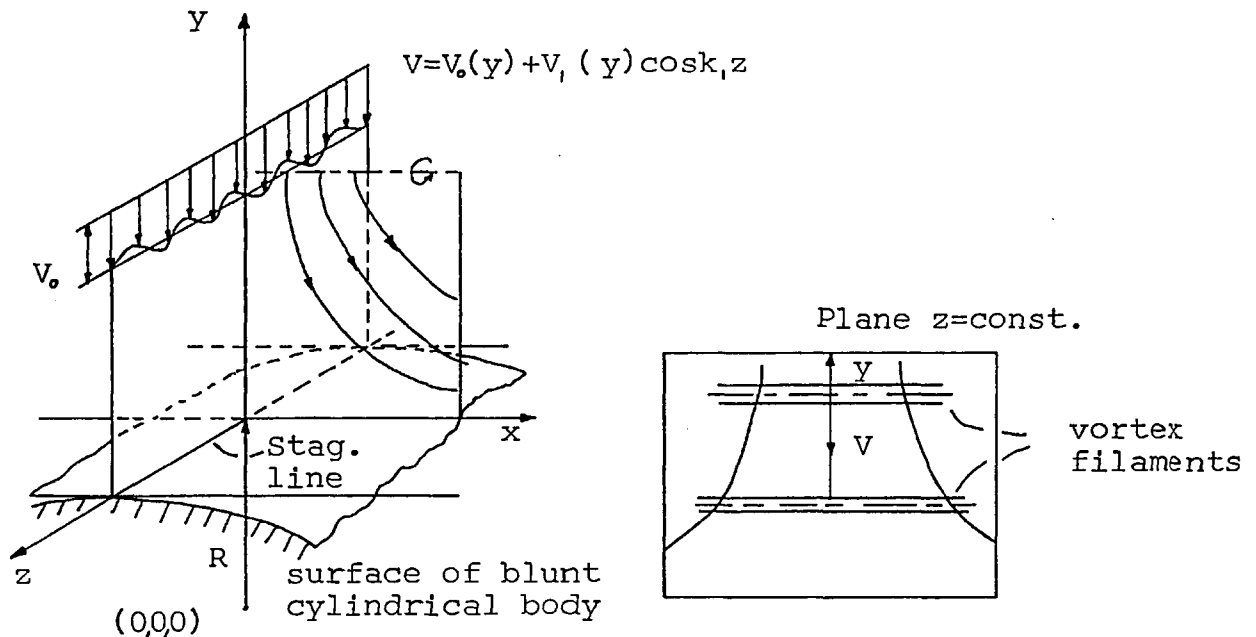
5.3 Discussion on the Physics of Free-stream Turbulence on Stagnation Point (Laminar Boundary Layer) Heat Transfer

When a bluff body is placed in a turbulent free-stream, the mean flow field around the body distorts the approaching free-stream turbulence in a complex way. This interaction influences the relationship between upstream velocity fluctuations and the resulting pressure fluctuations on the body surface (and velocity fluctuations near it). The approaching free-stream turbulence can be divided into three regions (i) the region far upstream of the body where the presence of the body is not felt by the free-stream ($x > 5d$ roughly) (ii) the region ahead of the body where large distortion of the free-stream occurs ($x < 5d$ roughly) (iii) the region near the surface of the body where large interaction between the distorted free-stream turbulence, the body surface and boundary layer occurs ($x \sim 0(\delta)$). There is no exact theory which describes the distortion of

free-stream turbulence round two-dimensional bluff bodies, but there are two approximate theories, namely (i) the vortex amplification theory developed by Sutra and others and (ii) the rapid distortion theory of Hunt⁴. The usefulness and limitation of these two theories are described in turn.

The Vortex Amplification Theory

Sutra et al¹⁶ analysed the effect of a simple spatially varying fluctuation on the mean velocity (i.e., a sinusoidal pattern of vorticity) in the stagnation point boundary layer (Hiemenz) flow (see diagram below).



Physical situation and coordinate system

Stretching of vortex filaments

By considering a Fourier component with dimensionless wave-numbers k (k 's are positive numbers, with each number there is an associated dimensionless wave-length $\lambda = 2\pi/k$) in the direction parallel to the axis of the cylinder which receives the maximum amplification, they then calculated the effect of this disturbance on the mean velocity. They found that only those vorticity components with a wavelength greater

than a neutral wavelength λ_0 ($\lambda_0 \sim 0(\delta)$ boundary layer thickness) amplify and can therefore affect the boundary layer and as a consequence augment the mean heat transfer. It should be noted that the only scales of disturbance considered in the theory were of the order δ . But this theory does include viscous effects and ignore quadratic terms (i.e., a type of rapid distortion theory with the assumptions of $\sqrt{u^2}/U_\infty \ll 1$, $\sqrt{u^2}/U_\infty \ll L_x/d$ and the mean flow is rotational). The analysis of Sutra et al.¹⁶ was extended by Sadeh et al.¹⁹ who considered a similar kind of fluctuation but now analysed its behaviour outside the boundary layer. Taking disturbance with k of order δ^{-1} they showed that some disturbances starting upstream decay owing to viscous dissipation before amplification begins near the stagnation point. Although the theory has demonstrated the importance of vorticity amplification and provided some explanation for the effect of turbulence on heat transfer, it is a crude model of turbulence. It has not accounted for all components of vorticity and has not considered time dependent disturbances.

The neutral wavelength ($\lambda_0 \sim 2.6\delta$) of the vortex amplification theory for the present problem is estimated to be equivalent to a wavenumber $\tilde{n}_v \sim 30$ (let the neutral wavenumber be $\tilde{n}_v$). The analysis is as follows:

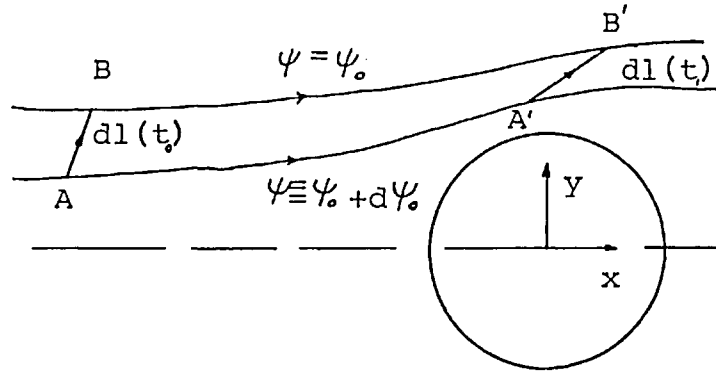
$$n \sim \frac{U_\infty}{2.6\delta} \sim \frac{5 \times 10^3}{2.6}, \quad \tilde{n}_v = 2\pi n \frac{L_x}{U_\infty} \sim 30 \text{ roughly.}$$

According to the theory, eddies of scale greater than $\tilde{n}_v \sim 30$ will amplify and can affect the stagnation point heat transfer. The wavenumber $\tilde{n}_v \sim 30$ is plotted onto the non-dimensional u component spectra (Figure 5.6). It can be seen that eddies with $\tilde{n} > 30$ dissipate very rapidly. Since the turbulence vorticity at

these wavenumbers is viscosity dependent (not susceptible to vortex stretching), it is unlikely that they will be important in heat transfer. It should be noted that the neutral wave-number $\tilde{n}_v \sim 30$ is obtained only for a particular turbulent free-stream (grid B, $\sqrt{u^2}/U_\infty = 4.4\%$, $L_x/d = 0.72$). Since $\tilde{n}_v$ depends on the ratio of L_x/U_∞ , these two quantities (L_x , U_∞) do vary between different turbulent free-streams. The wind speed U_∞ changes by a factor of 3 (5m/s to 15m/s) and the length scale L_x/d is in the range of 0.44 to 2.2 for the present investigation. As a rough approximation the neutral wave-numbers $\tilde{n}_v$ are in the range of 15 to 150. The energy containing eddies of the present experiments lie in the region of $\tilde{n} < 2$ (Figure 5.8). The neutral wavenumbers ($\tilde{n}_v \sim 15$ to 150) are situated well outside the energy containing range (i.e., in the dissipating range). It therefore appears that the neutral wavenumbers are not very appropriate for the scale of the present experiments.

The Rapid Distortion Theory

Hunt⁴ has formulated a theory for analysing the distortion of turbulence in a flow sweeping past a body (based on the rapid distortion theory of Batchelor and Proudman). The principal assumption made in the theory is that, in the time it takes for turbulence to be swept past the body, the changes in the mean flow around the body and the effect of its boundaries distort the turbulence far more than its own internal viscous and non-linear inertial forces. The turbulence will be distorted by the stretching and rotating of vortex line filaments as they are convected past the body (see diagram on the following page). The assumptions



The motion of a fluid element in a flow round a bluff body showing how it is stretched and rotated

made in the theory are (i) the mean flow is irrotational (ii) $\sqrt{u^2}/U_\infty \ll 1$, so that the dominant contribution to the distortion comes from changes in the mean flow (and not the turbulence itself). (iii) $\sqrt{u^2}/U_\infty \ll L_x/d$. It follows from these assumptions that this problem involving a random process is linear and therefore tractable by analytical methods. The success of Hunt's theory lies in the fact that (i) it can consider non-homogeneous disturbances (ii) a boundary condition is imposed on the turbulence that the velocity normal to the surface of the body be zero (iii) the analysis consists of calculating the velocity field for each wavenumber of turbulence (iv) it is applicable to other two-dimensional bluff body shapes. The limitations of the theory are (i) difficulty in calculating the r.m.s. values and the spectra of velocity when $L_x = 0(d)$ (ii) difficulty in accounting for separation (iii) break down of rapid distortion theory assumptions.

The turbulent free-streams of the present investigation satisfy the assumptions of the rapid distortion theory quite well. The present turbulent free-stream intensities are in the range of $\sqrt{u^2}/U_\infty \sim 0.02$ to 0.08 , i.e., $\sqrt{u^2}/U_\infty \ll 1$.

The length scales are in the range of $L_x/d \sim 0.44$ to 2.2 , hence $L_x/d \gg \sqrt{u^2}/U_\infty$. It is therefore possible to apply the rapid distortion theory to the present problem. Although no measurements of the velocity field just ahead of the normal flat plate were made in the present investigation, the measurements obtained by Bearman⁷² can be used as a direct comparison. The reason is that the bluff body used by Bearman simulated a flat plate plus wake. The only difference between the two is that vortex shedding was prohibited in Bearman's model (free streamline after body shape). Bearman's results showed that the local intensity ($\sqrt{u^2}/U$) was amplified by 5 to 9 times the free-stream intensity ($\sqrt{u^2}/U_\infty$) very near the stagnation point ($x/d = 0.1$), depending on the ratio of L_x/d (1.2 to 2.6). The spectral measurements of the u component showed that a shift of energy to higher wave-numbers at the stagnation point region (at $x/d = 0.18$). The shift of energy occurred at $\tilde{n} \sim 1$ (call it $\tilde{n}_I$). This value of $\tilde{n}$ is plotted onto the non-dimensional u component spectra of the present investigation (Figure 5.6). It can be seen that $\tilde{n}_I$ lies well away from the viscous region (because of the scale of the present experiments, $\tilde{n}_I \sim \tilde{n}$ (energy containing eddies)). According to the u component spectra of Bearman⁷² the value of $\tilde{n}_I$ did not vary significantly with L_x/d ($\tilde{n}_I \sim 1$ to 2). Since the range of L_x/d studied by Bearman ($L_x/d \sim 1.2$ to 2.6) was roughly in the same range as the present investigation ($L_x/d \sim 0.44$ to 2.2). Therefore, $\tilde{n}_I$ (~ 1) should hold for all turbulent free-streams in the present case.

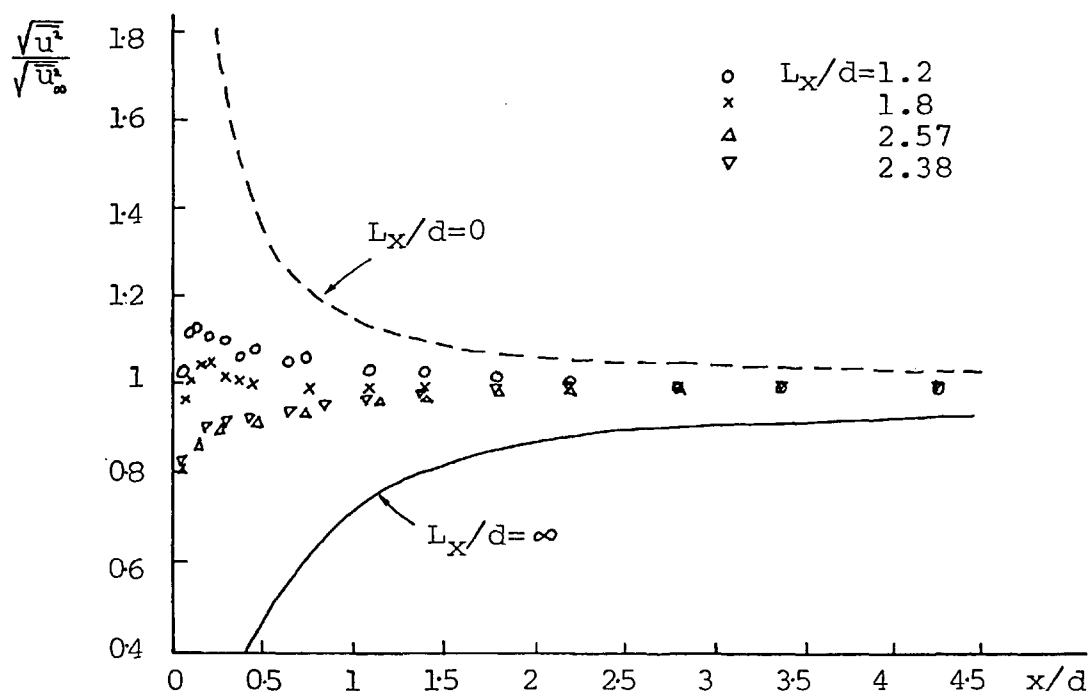
The Effect of Free-stream Turbulence on Stagnation
Point Heat Transfer of the Normal Flat Plate

The present discussion is based upon the usefulness of the vortex amplification theory, the rapid distortion theory and the experimental data of Bearman⁷². One must bear in mind that the process of vortex shedding has been omitted in these three cases. As a result of vortex shedding (or unsteadiness in the wake) the stagnation point of the normal flat plate will oscillate (about a mean position). The present results showed that free-stream turbulence had increased the stagnation point heat transfer $Nu/Re^{0.5}$ by as much as 40% at an intensity of about 8% (compared with the non-turbulent free-stream data). It was estimated that the stagnation point (in non-turbulent free-stream) had to fluctuate to a distance of $0.3d$ in order to achieve the same amount of heat transfer (as in turbulent free-streams). It was unlikely that the stagnation point would fluctuate that much in turbulent free-streams because the present pressure distributions (Figure 5.7) showed that the mean pressure C_p at the stagnation point region remained the same ($C_p = 1$) for both turbulent and non-turbulent free-streams. Therefore the increase in stagnation point heat transfer in turbulent free-streams was not due to the movement of the stagnation point.

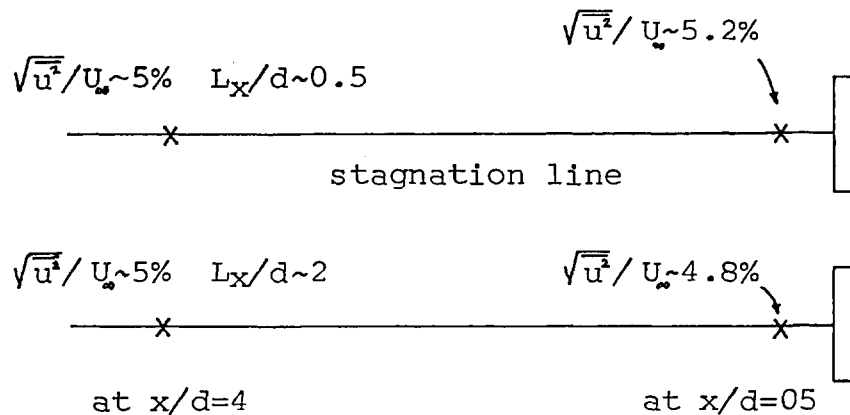
The present results showed that the stagnation point heat transfer parameter $Nu/Re^{0.5}$ increased in free-stream turbulence. The effect of Reynolds number on $Nu/Re^{0.5}$ was quite significant ($Nu/Re^{0.5} \propto Re^{0.08}$), an increase in $Nu/Re^{0.5}$ of 10% over the range of Re investigated ($Re = 1.7 \times 10^4$ to 5.4×10^4).

At a fixed Re , the effect of intensity and length scale were important ($\frac{Nu}{Re}$ varied as $(\frac{\sqrt{u^2}}{U_\infty})(\frac{L_x}{d})^{-\frac{1}{2}}$). As a rough guide, $\frac{Nu}{Re}$ increased by 12% for a 3% increase in $\sqrt{u^2}/U_\infty$ and decreased by 5% for a 3 fold increase in L_x/d . Although the measurements of intensity, length scale and u component spectrum were made in the absence of the model (i.e., an empty wind tunnel), these quantities must be determined first before the study of the distortion of free-stream turbulence (due to the model). Since no measurements of turbulence were made in the presence of the model, the data of Bearman⁷² provided the basis of comparison.

The present investigation has L_x/d in the range of 0.44 to 2.2, i.e., $L_x/d = O(1)$. Bearman⁷² found that the distortion of free-stream turbulence was dependent on L_x/d . His results showed that for $L_x/d = 1.2$ and 1.8 the local r.m.s. component of turbulence ($\sqrt{u^2}/\sqrt{u_\infty^2}$) amplified ($\sqrt{u_\infty^2}$ was taken at $x = 4d$), whereas for $L_x/d = 2.38$ and 2.57 the local r.m.s. component of turbulence attenuated (see diagram below).



The conclusion which one can draw from his results is that 'the turbulence field (along the stagnation line) ahead of the body ($0 < x/d < 2$) is different from that of the free-stream ($x/d > 2$), and it is the distorted turbulence ahead of the body which is responsible for the stagnation point heat transfer'. The wave-number of importance is therefore $\tilde{n}_1$ (the wavenumber at which the shift of energy occurs). The present $\tilde{n}_1$ is about 1 (because L_x/d 's are similar to those obtained by Bearman) and the region of the spectrum which is important for heat transfer is around $\tilde{n} < 10$ (energy containing region). If two turbulent free-streams have the same intensity (5% say) but different length scale ($L_x/d = 0.5$ and 2 say), the local intensity is expected to be higher for the smaller L_x/d (according to the results of Bearman⁷²), see diagram below. The stagnation point heat

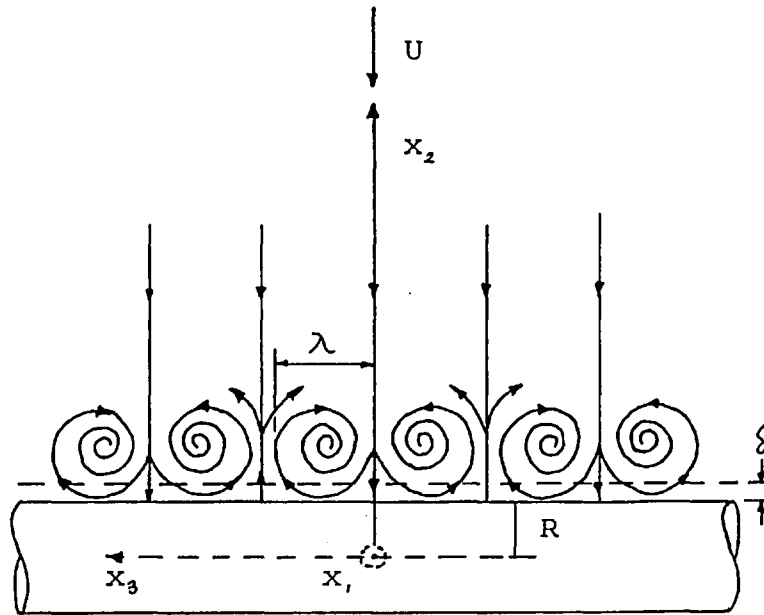


transfer parameter $Nu/Re^{0.5}$ is therefore length scale dependent. For two turbulent free-streams of approximately the same intensities ($\sqrt{u^2}/U_\infty \sim 4.9\%$ and 5.3%) and different length scales ($L_x/d \sim 0.72$ and 2.2), the present results showed that $Nu/Re^{0.5}$ decreased by 5% for a 3 fold increase in length scale.

As one would expect the local intensity (ahead of the model) would be higher for the case of lower L_x/d (0.72) and hence the heat transfer rate was higher. For two turbulent free-streams of approximately the same length scales ($L_x/d \sim 0.54$ and 0.72) and different intensities ($\sqrt{u^2}/U_\infty \sim 1.5\%$ and 4.9%), the present results showed that $Nu/Re^{0.5}$ increased by 20% (for a 3.4% increase in intensity). Since the ratios of L_x/d were about the same, one would expect the distortion of turbulence to be about the same. But since the free-stream intensities were different, the local intensities would also be different (as a rough estimate, $\sqrt{u^2}$ varies directly as $\sqrt{u_\infty^2}$). The case of higher intensity ($\sim 4.9\%$) should have local intensity many times higher than the case of lower intensity (1.5%), the heat transfer rate was therefore higher. As the turbulence approaches the body, the local intensity (ahead of the body) is likely to increase and the length scale decreases. These two factors are inter-related and therefore should be considered together. This is in agreement with the present investigation, the stagnation point heat transfer parameter $Nu/Re^{0.5}$ data were found to correlate in the form of $(\sqrt{u^2}/U_\infty)(L_x/d)^{-\frac{1}{2}}$ (this is not an universal form, but in most laboratory conditions $L_x/d \sim 0(1)$ and $\sqrt{u^2}/U_\infty \sim 0(10\%)$, this form may be valid).

According to the vortex amplification theory the neutral wavenumber $\tilde{n}_v$ is the important wave-number in heat transfer. The neutral wavenumber of the present investigation varied over a wide range of wavenumbers ($\tilde{n}_v \sim 15$ to 150 roughly). For the high $\tilde{n}_v (>30)$ cases it is unlikely that the viscous effect will be important, but one might expect this

effect to be important for $\tilde{n}_v < 30$ (because this region contains most of the energy - see Figure 5.6). The above statement is contradictory to the present experimental results because the present stagnation point heat transfer data tend to show a slight Reynolds number effect for all the turbulence conditions (i.e., $\frac{Nu}{Re^{0.8}} \propto Re^{0.08}$ - approximately true for all cases). It is therefore not possible to use $\tilde{n}_v$ to explain the Reynolds number effect on heat transfer. But a change in Reynolds number will certainly change the boundary layer thickness which in turn will affect the turbulence/boundary layer interaction (the turbulence seen by the boundary layer is an inviscid distortion of the turbulence actually far upstream of the model). The interaction of turbulence/boundary layer has been studied by Kestin and Wood⁷³, Sadeh and Brauer⁷⁴. The researchers demonstrate experimentally (using the smoke-visualization technique) the existence of a coherent substructure near the stagnation zone of a circular cylinder. The circular cylinder is placed at a distance of 18 mesh length from the grid. Free-stream turbulence is produced in a controlled manner by means of a grid consisting of 24 regularly spaced vertical cylindrical bars. The smoke pictures reveal the persistence of standing cross-vortex tubes whose axes are in the x , direction and whose cores are outside the boundary layer (this coherent substructure represents an array of energy-containing eddies-see diagram below). The standing cross-vortex tubes are most probably caused by the grid rather than by the interaction of the model with the side walls because (i) the vertical orientation of the rods is specially chosen for generating mainly vorticity



Schematic picture of the coherent structure of energy-containing eddies near the stagnation zone

susceptible to undergoing amplification by stretching, i.e., vorticity in the x_1 direction, (ii) the model has an aspect ratio of 11 and low blockage ratio(8%), the side wall boundary layers are unlikely to cause the vortical pattern as shown in the above diagram. Most of the turbulence amplification occurs at a most amplified scale λ which is characteristic of this coherent substructure (as λ in the vortex amplification theory). Penetration of the energy containing eddies into the boundary layer affects its nature, such as wall pressure fluctuations, boundary layer thickness, local intensity. The spacing λ is proportional to $Re^{-\frac{1}{2}}$. As the Reynolds number increases, the spacing λ decreases; the vorticity of the eddies increases and the accompanied streamwise turbulence velocity is amplified. An increase in local intensity thus results in an increase in stagnation point heat transfer.

Some measurements of the v and w components of turbulence along the stagnation line have been obtained by Bearman⁷². The local r.m.s. values of $\sqrt{v^2}/\sqrt{v_\infty^2}$ generally show an opposite effect to $\sqrt{u^2}/\sqrt{u_\infty^2}$ (i.e., if $\sqrt{u^2}/\sqrt{u_\infty^2}$ attenuates, $\sqrt{v^2}/\sqrt{v_\infty^2}$ amplifies), and $\sqrt{w^2}/\sqrt{w_\infty^2}$ shows only amplification. The three components (u , v , w) of turbulence are different along the stagnation line, whereas in the undistorted free-stream the turbulence is nearly isotropic since $\overline{u^2} \sim \overline{v^2} \sim \overline{w^2}$. The v and w components should be considered in stagnation point heat transfer. However, in the present investigation, no measurements of the v and w components are made and future work is necessary.

5.4 Conclusions

The main findings of the present investigation may be summarised as follows:-

Non-turbulent Free-stream

- (i) The experimental stagnation point Nusselt number/Reynolds number relationship is $Nu = 0.50Re^{0.5}$. It is in close agreement with the theoretical prediction of Frössling⁷³, i.e., $Nu = 0.55Re^{0.5}$ (based on experimental C_{p_b}).
- (ii) $\frac{Nu}{Re^{0.5}}$ is independent of power input, temperature difference ($T_w - T_\infty$) and Re . This is in agreement with Frössling's theory.
- (iii) $\frac{Nu}{Re^{0.5}}$ increases away from the stagnation point, it is insensitive to Reynolds number. This is in disagreement with the results of Sogin et al⁷³ which are strongly affected by Reynolds number.

Turbulent Free-streams

- (i) The stagnation point $\frac{Nu}{Re^{0.5}}$ increases with increasing Re , the relationship $\frac{Nu}{Re^{0.5}} = cRe^{0.08}$ provides a good correlation of data.
- (ii) The effect of pressure change on $\frac{Nu}{Re^{0.5}}$ is insignificant.
- (iii) $\frac{Nu}{Re^{0.5}}$ increases with intensity and decreases with length scale. The parameter $\frac{\sqrt{u^2}}{U_\infty} (\frac{L_x}{d})^{-\frac{1}{2}}$ give the best fit of data, i.e., intensity being the dominant effect on heat transfer.
- (iv) The distorted turbulence ahead of the model is responsible for the stagnation point heat transfer. The important wave number is $\tilde{n}_I$ and the region of the spectrum which is important for heat transfer is about $\tilde{n} < 10$.
- (v) The neutral wave number $\tilde{n}_v$ of vortex amplification theory is irrelevant for the scale of the present experiments.
- (vi) The v and w components of the distorted turbulence are important.

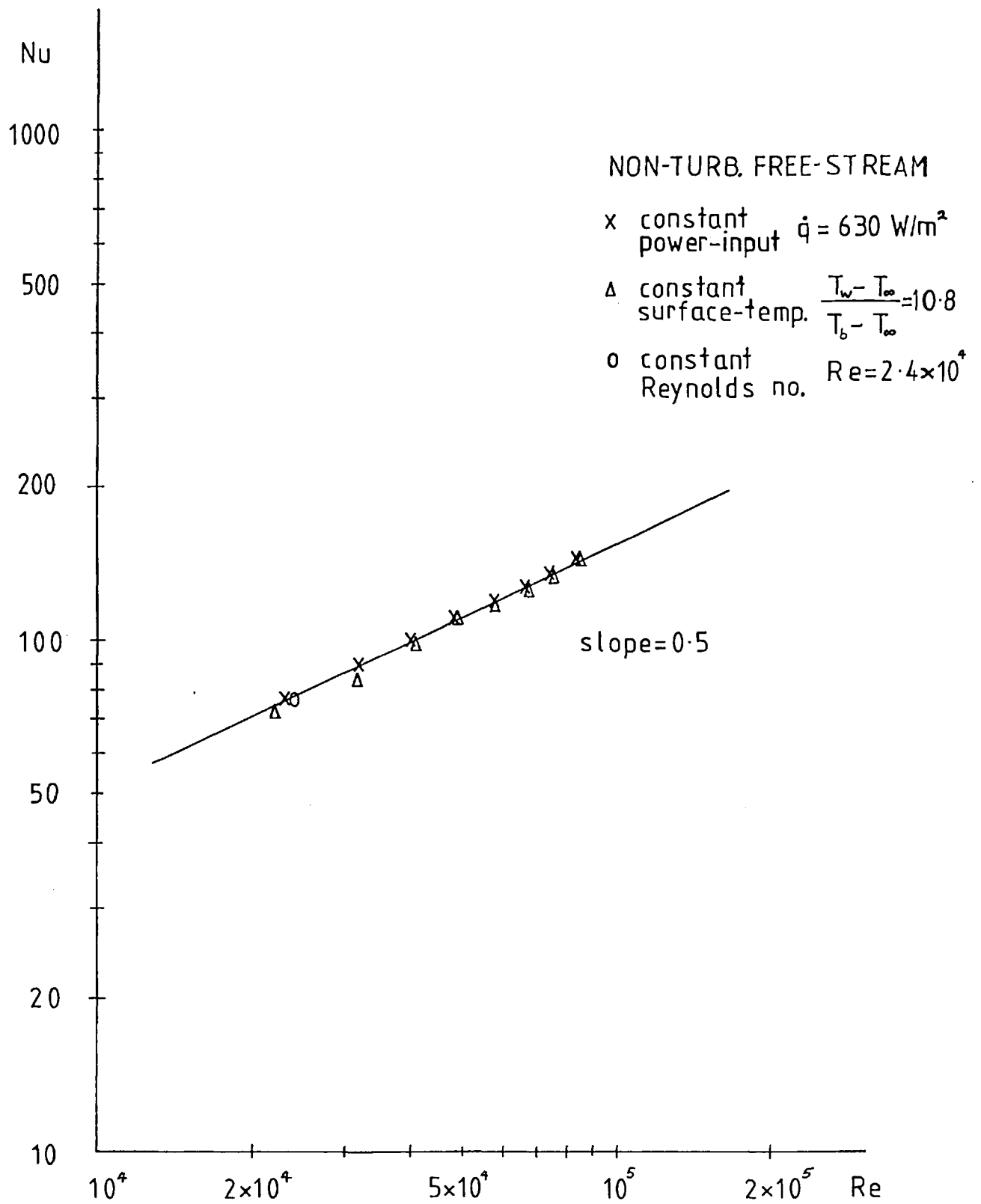


Figure 5.1 log Nu versus log Re at the stagnation point

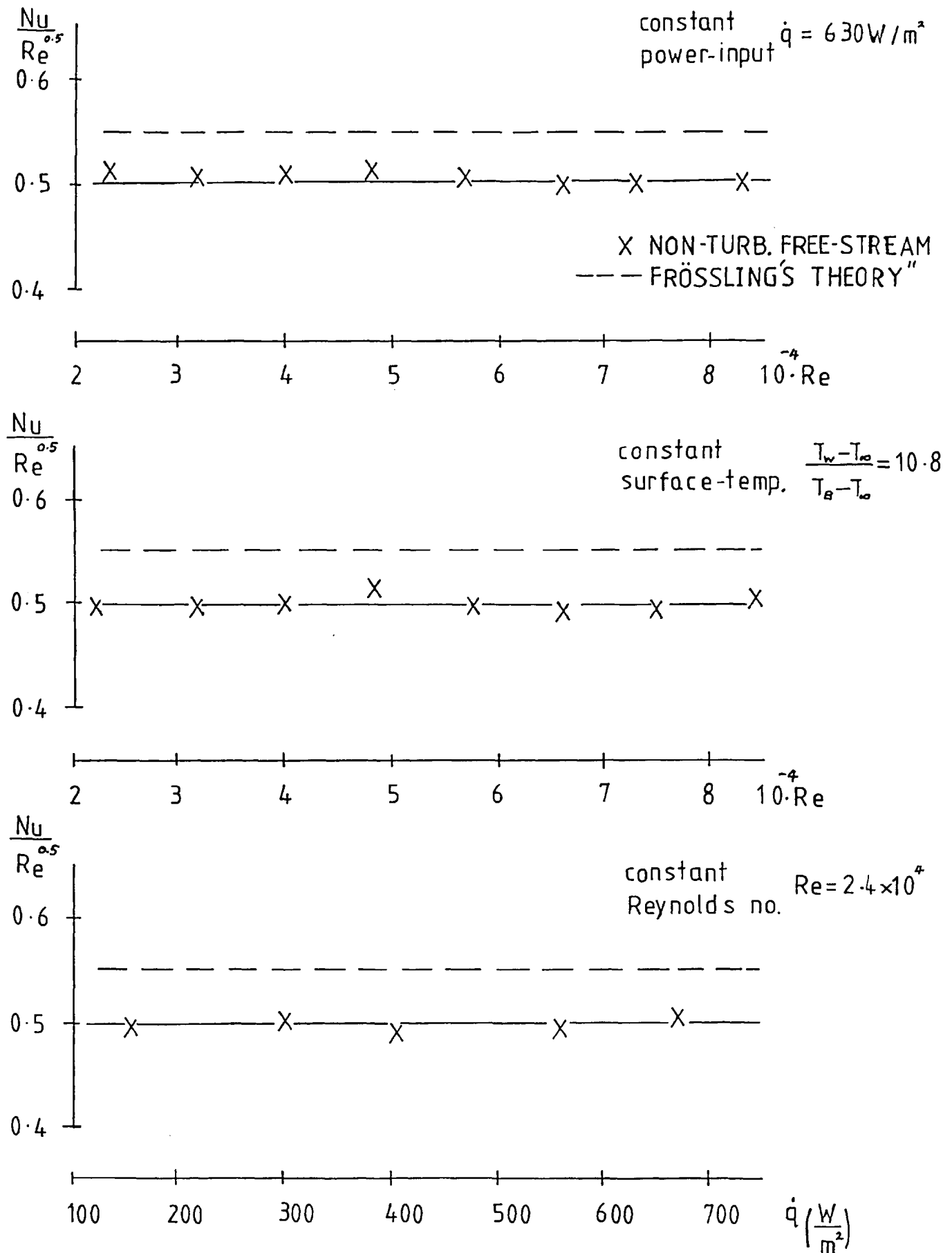


Figure 5.2 The effects of power input, surface temperature and Reynolds number on stagnation point heat transfer

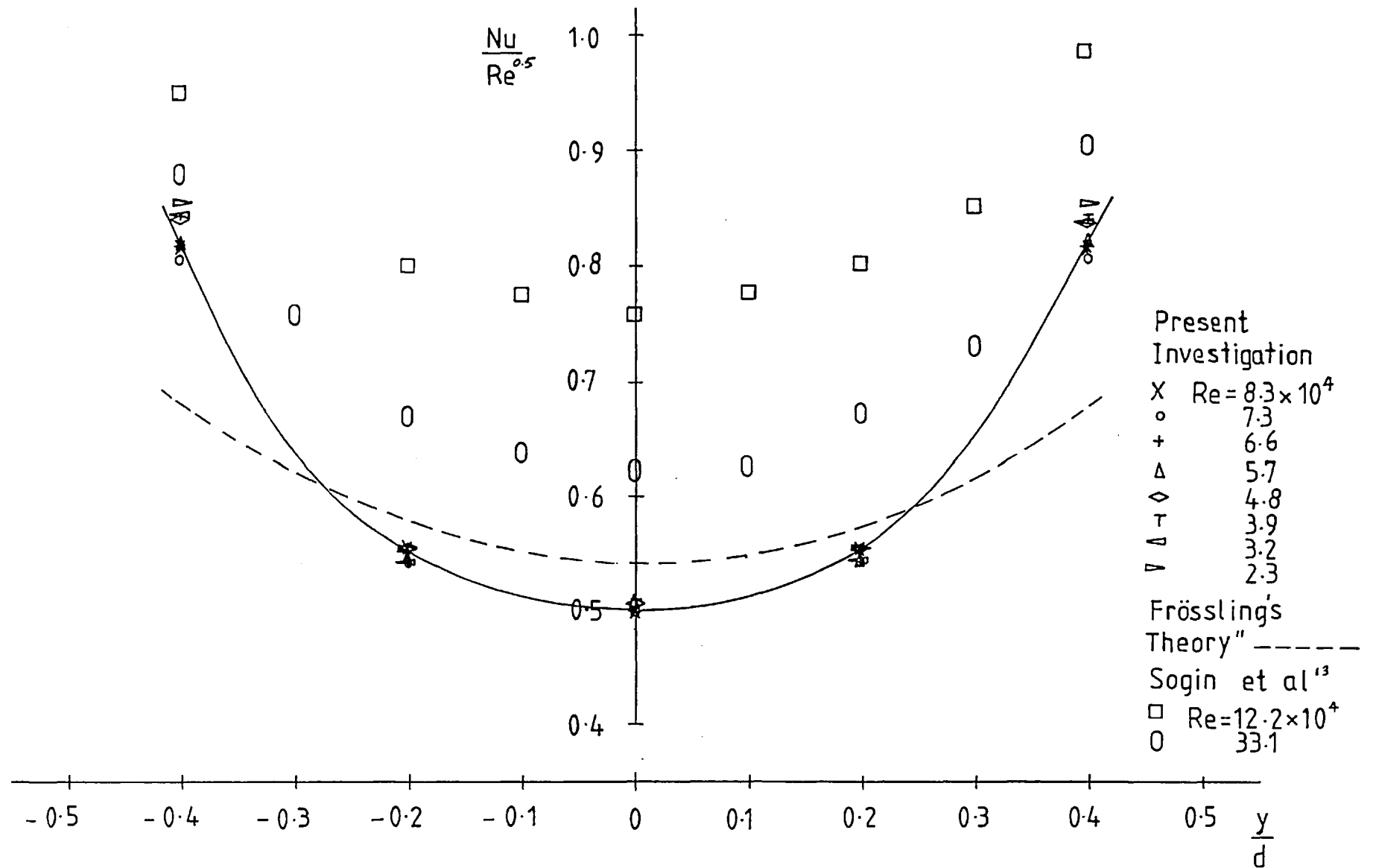


Figure 5.3 The heat transfer distribution along the front face

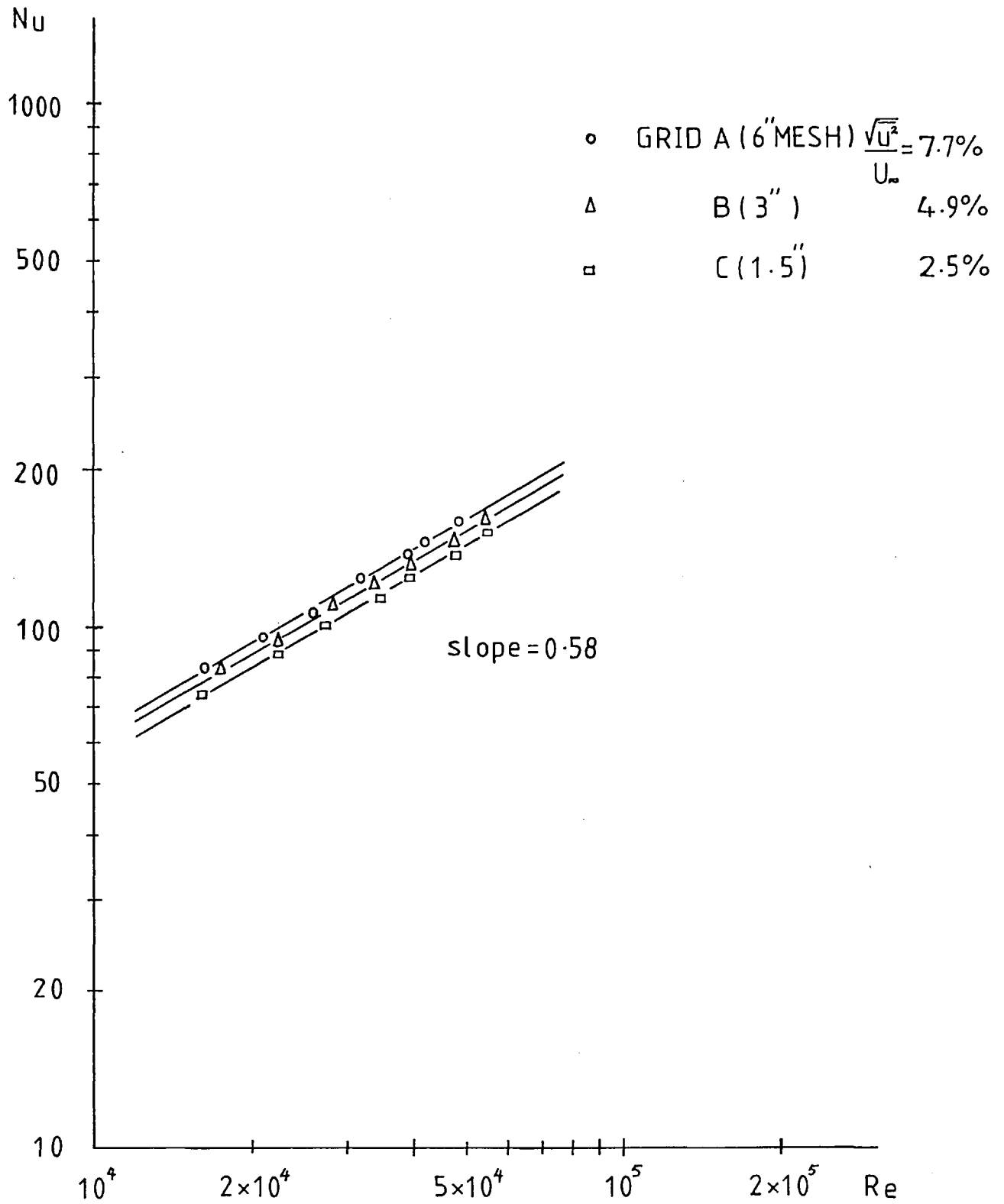


Figure 5.4 log Nu versus log Re at the stagnation point for the turbulent free-streams

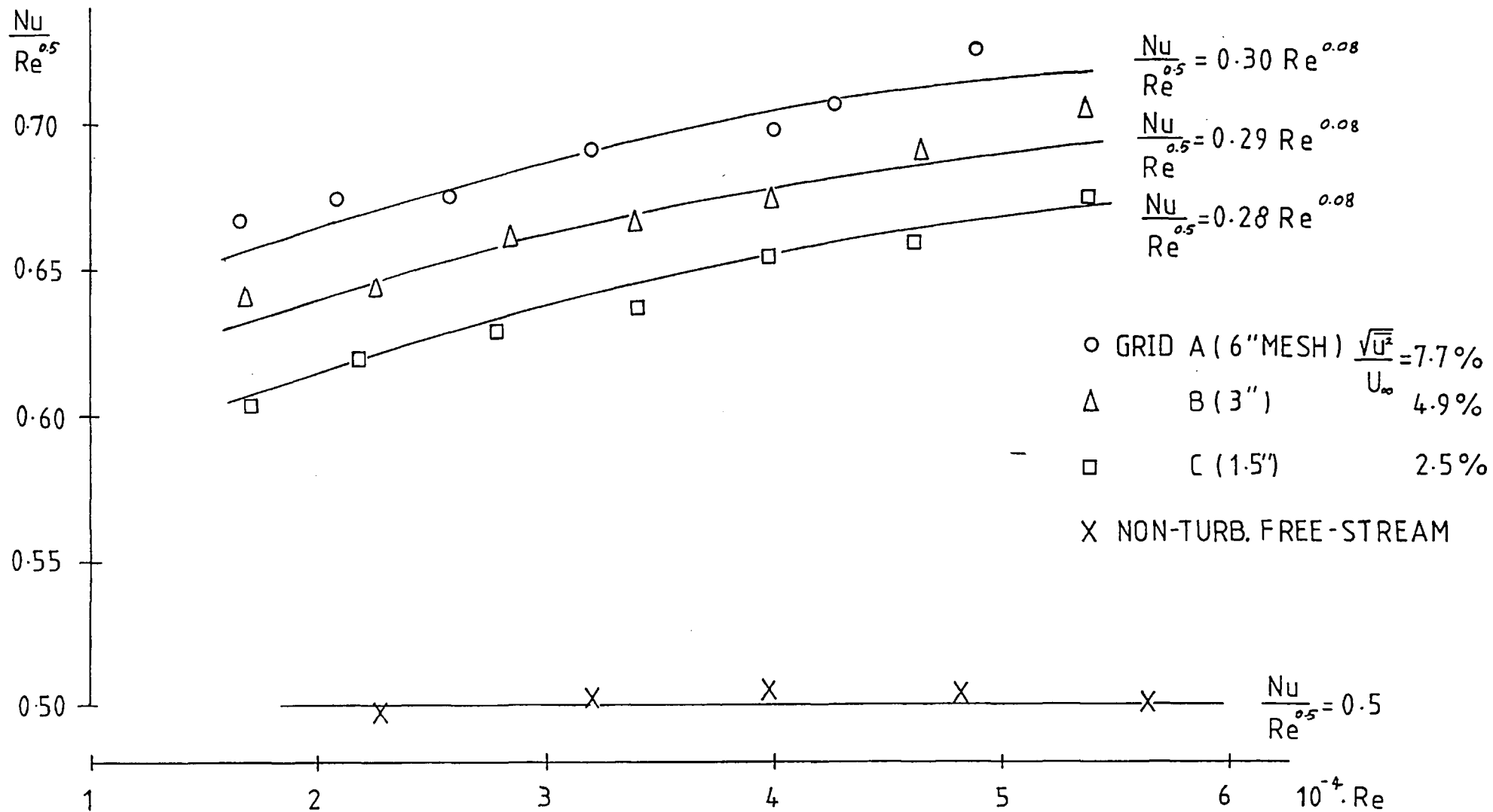


Figure 5.5 The effect of Reynolds number on stagnation point heat transfer

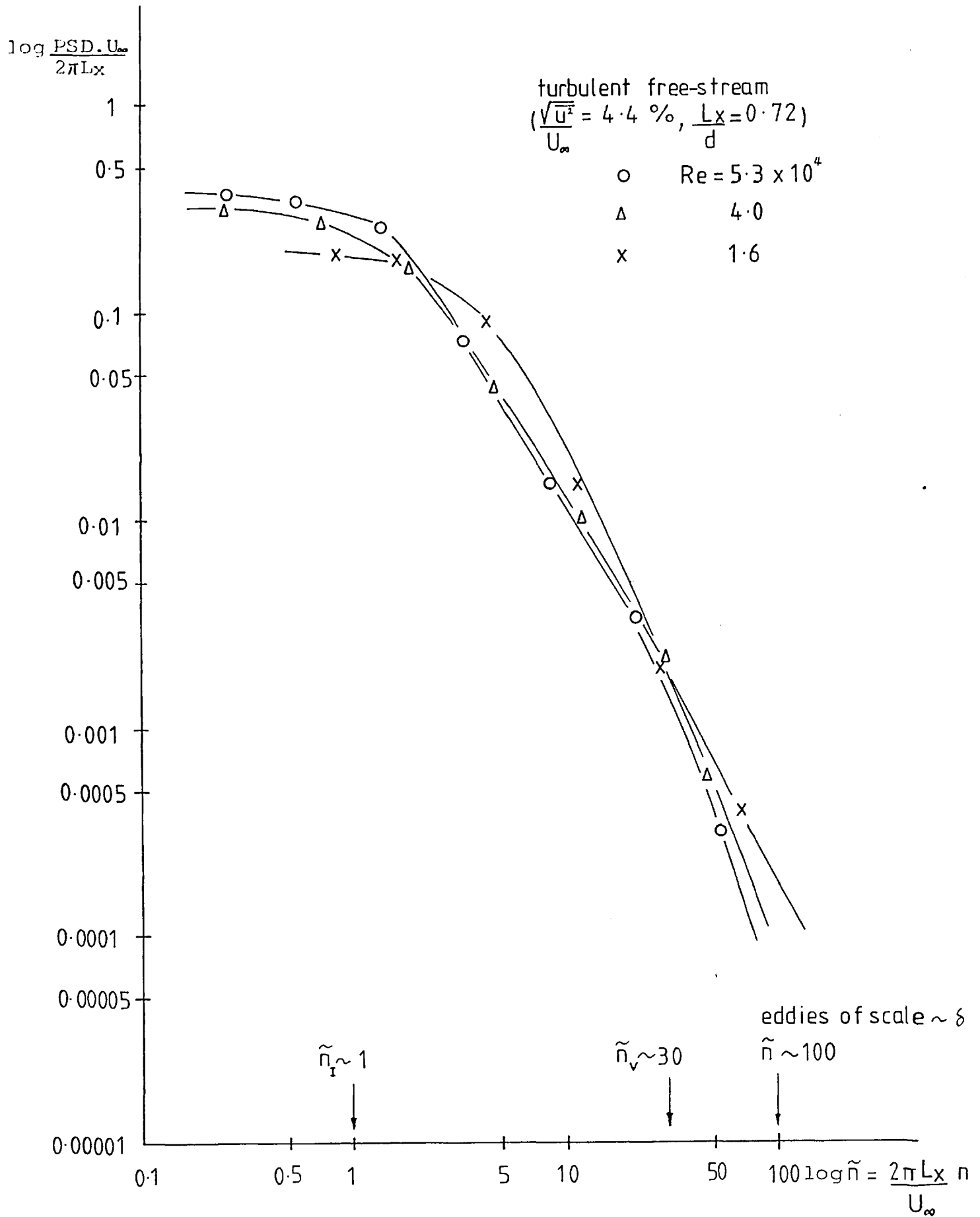


Figure 5.6 The effect of Reynolds number on the power spectrum

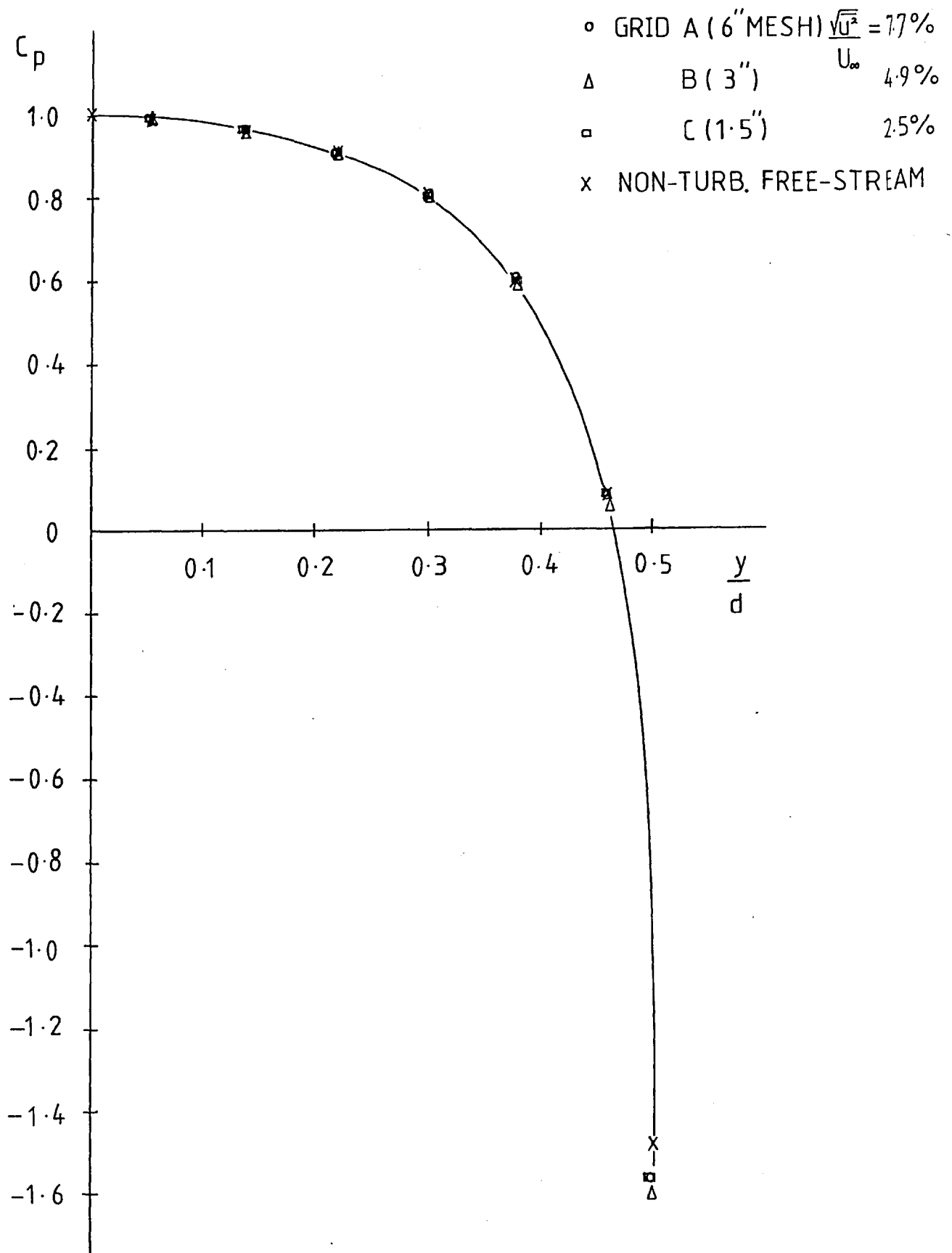


Figure 5.7 The effect of free-stream turbulence on the pressure distribution

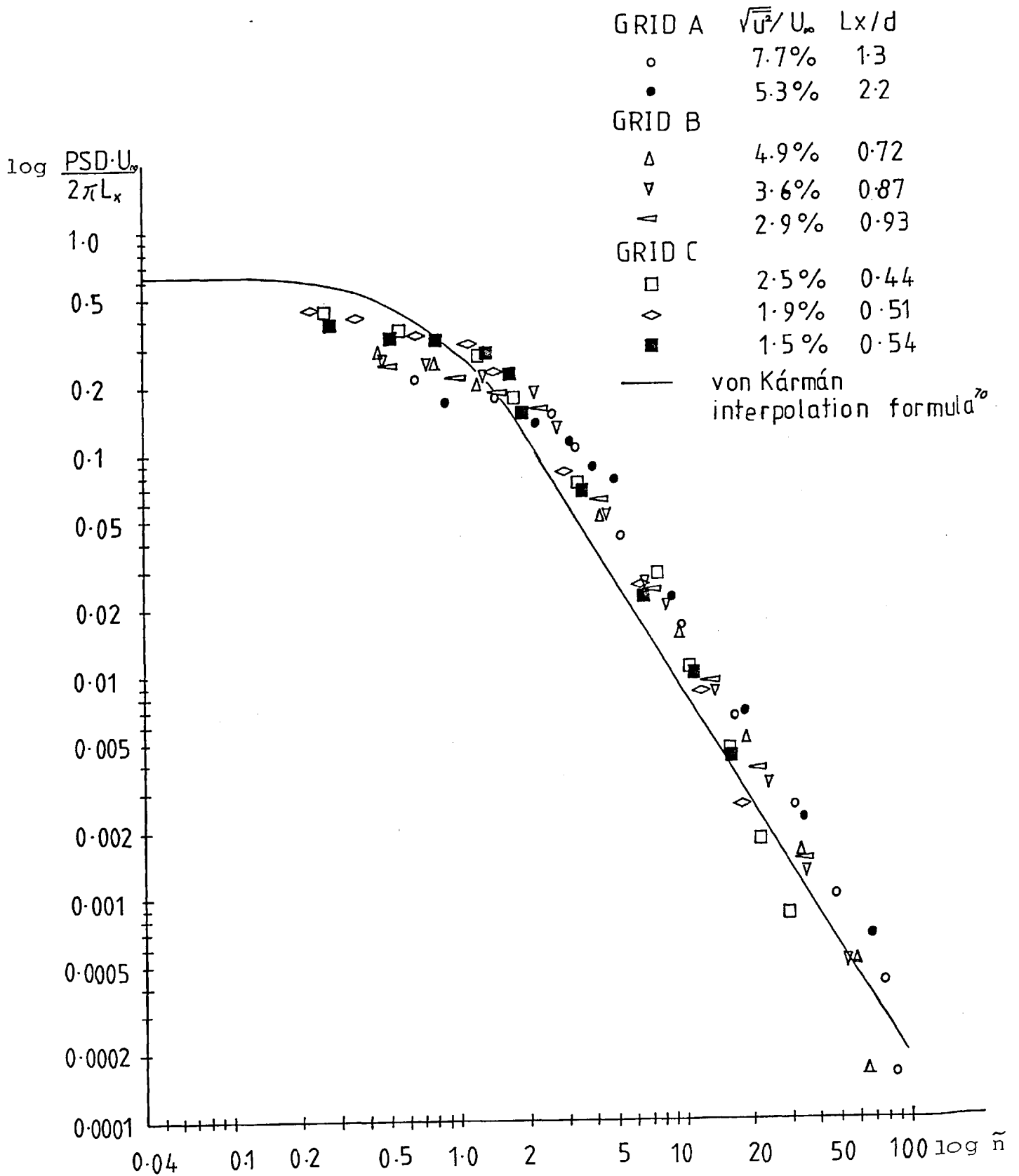


Figure 5.8 The power spectral density distributions for turbulent free-streams

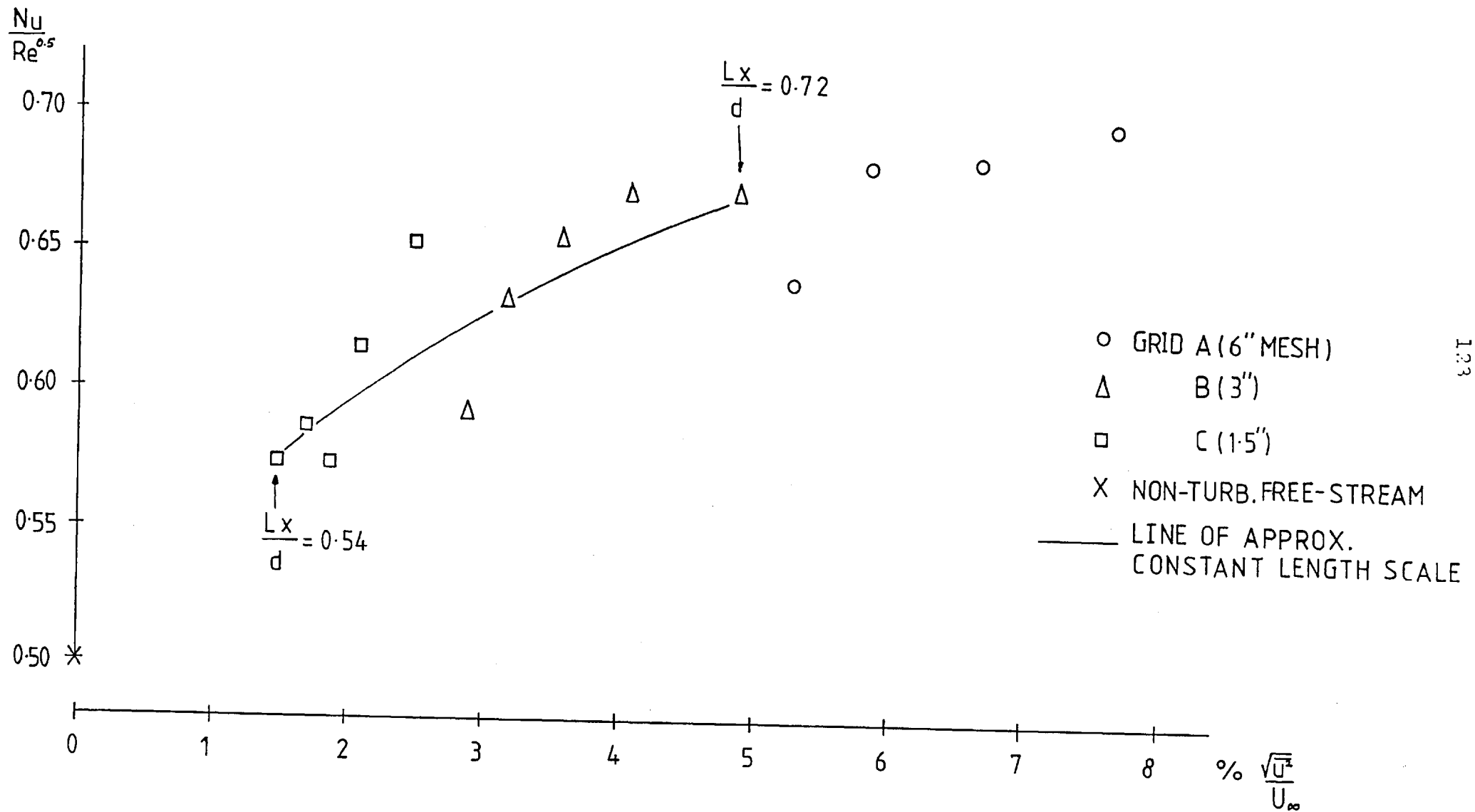


Figure 5.9 The effect of intensity on stagnation point heat transfer

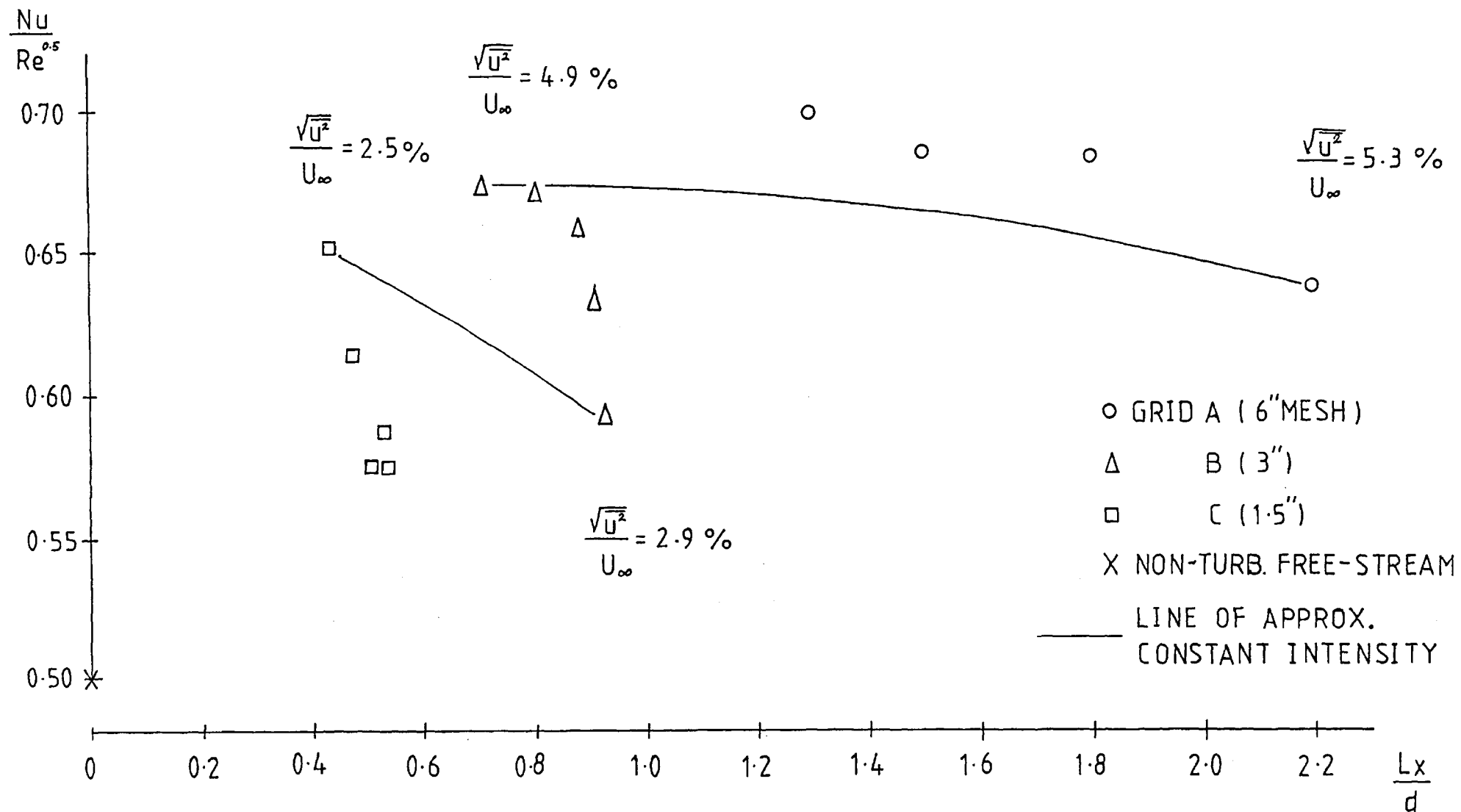


Figure 5.10 The effect of length scale on stagnation point heat transfer

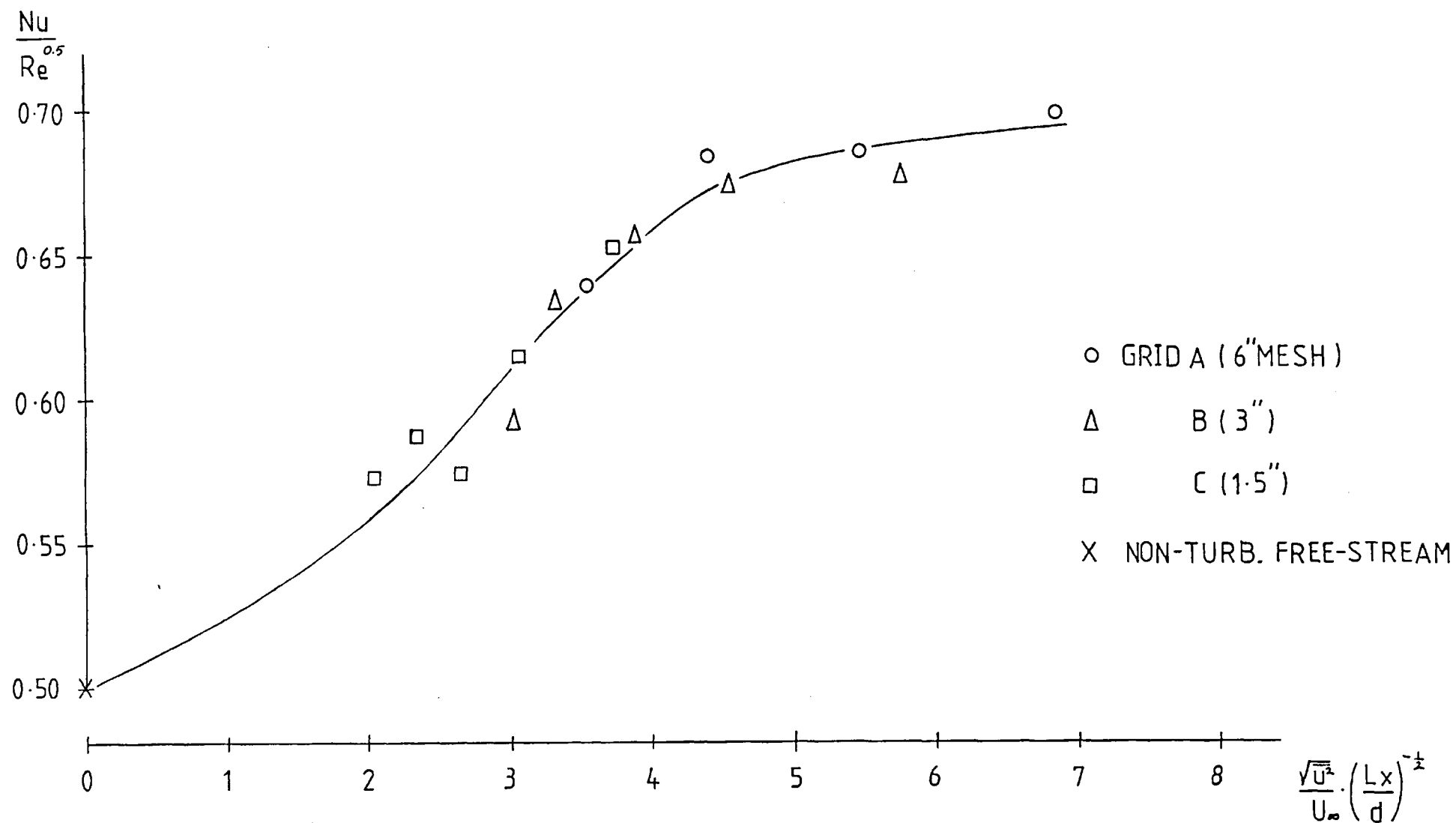


Figure 5.11 The combined effect of intensity and length scale on stagnation point heat transfer

6. Presentation of Heat Transfer and Mean Flow Measurements in the Separated and Reattached Flow over the Blunt Leading-edge Flat Plate

6.1 Discussion of Experimental Results

6.1.1 Results of the Preliminary Tests

Some preliminary tests were carried out with the model in non-turbulent free-streams. The purpose of these tests was to study the effect of heating the model on the flow field. The pressure distributions of both the heated and unheated model are shown in Figure 6.1. The figure shows that the two distributions are nearly identical and one may conclude that heating the model has no effect on the pressure distribution. It was also found that heating the model did not change the reattachment length ($R/2H = 5.0$ for both cases; where R is the reattachment length and H is the half model thickness).

Two separate tests were performed to study the effect of heating or not heating the front face of the model on the local Nusselt number distribution in the separated flow region - Figure 6.2 ($Nu_H = \frac{\dot{q} H}{k_{air} (T_W - T_\infty)}$ - the evaluation of Nu_H is given in Appendix E4). It was found that the local Nu_H distributions did not differ significantly (Nu_H increased by less than 4% in most positions due to heating of the front face) except near the leading edge. As a rough estimate, one can assume that 50% of the heat flux from half of the front face enters the bubble. There are 27 heating strips on each surface and two on the front face. If one also assumes that the heat flux of each strip is roughly equal (although the front face boundary layer is laminar, as a crude estimate,

the heat transfer rate is of the same order as the separated flow region), then the amount of additional heat flux enters the bubble due to heating the front face is about 4% ($1/27 \times 100\%$) of the total heat flux in the bubble. Since heating the front face of the model had little effect on the local Nu_H distribution along the model, it was therefore decided that later experiments should be performed with the model front face heated. The advantage of heating the front face was that the error in evaluating the heat loss from it would be eliminated.

For small temperature differences, the Boussinesq assumption suggests that density (ρ) and viscosity (μ) of fluids will not be significantly altered due to heating. This was tested by changing the temperature difference ($T_W - T_\infty$) and measuring the local Nu_H distribution (Figure 6.3). Two tests were performed at a fixed Reynolds number ($Re_H = \frac{U_\infty H}{\nu} = 0.99 \times 10^4$) with temperature differences ($T_W - T_\infty$) $\sim 12^\circ\text{C}$ and 5°C . It was found that there was no obvious effect on the local Nu_H distributions (differences of 3% in Nu_H at the most were recorded). The change in free-stream temperature T_∞ over the course of an experiment was also investigated. It was found that T_∞ altered by about 3% over the duration of the experiment (when an equilibrium state was reached, the error in Nu_H as a result of T_∞ change was estimated to be less than 1%).

6.1.2 The Accuracy of the Results

The Nusselt number is defined as $Nu_H = \frac{\dot{q} H}{k_{air} (T_W - T_\infty)}$. The accuracy of the temperature measurements (T_W and T_∞) was

therefore of prime importance. The thermocouple and data amplifier system was estimated to be capable of measuring temperature to an accuracy of 0.05°C (provided that the thermocouple calibration was linear and the gain of the amplifier was accurately determined), for a typical $(T_w - T_{\infty})$ of about 10°C . The error in temperature measurements was of the order of 0.5%. Nusselt number is a function of the heat transfer rate ($\dot{q}$) which in turn depends on the current flowing through the heating strip ($\dot{q} = \frac{I^2 R}{A}$). The current ($I = V/R$) was obtained by measuring the voltage (V) and resistance (R) of the power supply module. The tolerance of the resistors in the module was about 5% ($0.22\Omega \pm 0.01\Omega$). As a rough estimate the error in the evaluation of $\dot{q}$ was of the order of 5%. Therefore, the combined error (due to errors in heat transfer rate and temperature measurement) in the evaluation of Nu_{μ} was of the order of 5%.

Heat loss due to radiation and lateral conduction along the heating strip were estimated (Appendix E5). They were found to be negligible (1% of the total input for radiation loss and less than 1% for lateral conduction). Conduction loss through to the rear of the model and through the two sides of the model was found respectively to be about 6% and 2% of the total power input. The latter loss was considered to be insignificant (Appendix E2). Chordwise conduction across the gap between heating strips was estimated to be less than 1% of the power input into the heating strips and was therefore neglected (Appendix E5).

In order to assess the accuracy of the heat transfer measurement technique, a comparison was made between

the heat flux transferred from the surface and that convected past a station somewhat downstream of reattachment (i.e., $\int_0^{\infty} \rho C_p U (T - T_{\infty}) dy \cdot b$ at $x/R = 1.5$, where b is the breadth of the plate, $x = 18.7\text{cm}$ and $R = 12.5\text{cm}$.), plus the thermal energy loss ($Q = -kA \frac{\Delta T}{\Delta x}$) through the rear of the model (Appendix E2). The accuracy of the integral depended on the accuracy of the velocity and temperature measurements. As the temperature probe was accurate to 0.05°C (typically $T - T_{\infty} \sim 1^{\circ}\text{C}$) and the hot wire error in mean velocity measurement was about 2%, the likely error in the integral was therefore about 5%. The difference between the thermal energy input and output was found to be about 4% (total input power = 71.4W, thermal energy convection = 63.6W and conduction loss = 4.5W). The measurement technique was therefore fairly accurate. The difference was most likely due to conduction loss through the sides of the model (about 2% of the power input) and experimental errors. It should be noted that the difference between the thermal energy input and output was slightly larger (~9%) for the normal flat plate. The main reason was that $(T - T_{\infty})$ was typically about 0.5°C in the wake, so that errors in temperature measurement could be as much as 10%.

6.1.3 The Correlated Results

In order to determine the Reynolds number dependence on Nusselt number for the separated flow region, the basic experimental results were plotted in the form of $\log Nu_{\mu}$ versus $\log Re_{\mu}$ (Figures 6.26 - 6.32). These plots suggested that the Nusselt number varies as a power of Reynolds number (the majority of the log plots appear to be linear for both

turbulent and non-turbulent free-streams), i.e., $Nu_H = cRe_H^n$; where c and n vary with position and the level of free-stream turbulence. For the non-turbulent free-stream case (Figure 6.26), n was found to vary between 0.71 and 0.72 over most positions in the separation bubble (except near the leading edge $n=0.60$), the maximum value of n (0.75) occurred at the reattachment position. This value of n was identical to that of Ota and Kon⁴⁴ (heat transfer of a blunt circular cylinder), but it was slightly greater than that obtained by Smyth⁴⁷ ($n=0.71$ - heat transfer in axisymmetric separated and reattached flows), Ota and Kon⁴³ ($n=0.71$ - blunt flat plate). For the free-stream turbulence cases (Figures 6.27-6.32), n was found to vary between 0.72 and 0.76 over most of the separation bubble (except grid A $\sqrt{u^2}/U_\infty = 8.5\%$, $L_x/2H=2.52$; where n varied between 0.68 and 0.75), lower values of n (~ 0.6) were obtained near the leading edge. At the reattachment position n was found to be about 0.75 for most cases (except grid A $\sqrt{u^2}/U_\infty = 8.5\%$, $n=0.68$).

The reattachment Nusselt numbers ($\log Nu_H$) for both turbulent and non-turbulent free-streams were plotted against Reynolds numbers ($\log Re_H$)-Figure 6.33. The 'constant heat flux' data were also included in the plot. All the data points were found to correlate fairly well onto a straight line (with a small amount of scatter), the relationship $Nu_H = \text{constant } Re_H^{0.75}$ was generally valid.

In order to account for the Reynolds number effect on the surface heat transfer, the parameter Nu_H/Re_H^n was used to correlate data. The reattachment value of n ($n=0.75$) was selected for both turbulent and non-turbulent free-streams

because it was generally true for all the cases. The $\frac{Nu_H}{Re_H^{0.75}}$ distributions for both turbulent and non-turbulent free-streams at a fixed Reynolds number ($Re_H = 0.99 \times 10^4$) were plotted together (Figure 6.42). In the separation bubble region, the distributions were quite distinct from one another. As the reattachment length decreased the slope of the distribution became steeper. The highest $Nu_H/Re_H^{0.75}$ was obtained for the flow with the highest intensity. In the redeveloped flow region the distribution curves were less distinct, $Nu_H/Re_H^{0.15}$ decreased gradually.

In free-stream turbulence the pressure distribution in the separation bubble was found to contract (as shown in Figure 6.7-6.12) and the reattachment length (R) shortened. The reattachment length and the separation velocity ($U_s \approx (1 - C_{p_s})^{\frac{1}{2}} U_\infty$) at the leading edge were different from that of the non-turbulent free-stream. In order to compare the surface heat transfer of the separated flow region between turbulent and non-turbulent free-streams, Nusselt number and Reynolds number based on U_s and R might provide a better correlation of data. The idea that the reattachment length is a suitable characteristic length to correlate the reattachment Nusselt number and Reynolds number has been shown by Ota and Kon⁴⁵ (nose effect on blunt flat plate heat transfer). The present Nusselt number based on the reattachment length R is defined as $Nu_R = \frac{\dot{q} R}{k_{air} (T_W - T_\infty)}$ and the Reynolds number based on U_s and R is $Re_{U_s} = U_s R / \nu$. The relationship $Nu_R = C Re_{U_s}^n$ should give a better data correlation (provided that U_s/U_∞ is constant for all Reynolds number, c may vary but not n).

The log of the reattachment Nu_R for both turbulent and non-turbulent free-streams were plotted against the log of Re_{U_s} . The 'constant heat flux' data were also included (see Figure 6.43). The data were found to correlate well in the form of $Nu_R = 0.09 Re_{U_s}^{0.75}$ (with a small amount of scatter). Hence $\frac{Nu_R}{Re_{U_s}^{0.75}}$ was used to correlate the data. The $\frac{Nu_R}{Re_{U_s}^{0.75}}$ distributions for both turbulent and non-turbulent free-streams at a fixed Reynolds number ($Re_H = 0.99 \times 10^4$) were obtained (Figure 6.44). It was found that the data points correlated quite well in the separation bubble region, but with a larger degree of scatter in the region downstream of reattachment. It appeared from this that the influence of free-stream turbulence had little direct influence upon the heat transfer distribution. The presence of free-stream turbulence in actual fact only changed the structure of the bubble (characterised by U_s and R) and basing Nusselt number and Reynolds number on U_s and R took into account this influence of free-stream turbulence on the bubble structure. Ota and Kon⁴⁵ have studied the influence of nose shape on blunt flat plate heat transfer. Although a change in nose shape altered the reattachment length, they found that the correlation between the reattachment Nusselt number and the Reynolds number was independent of the nose shape when the reattachment length was employed as the reference length. From the stand point of the present results and Ota and Kon's results, it seems probable that the heat transfer data of other flow configurations (such as flows behind steps) may also be correlated if Nusselt number and Reynolds number are based on U_s and R .

6.2 Discussion of Results for the Non-turbulent Free-stream

6.2.1 The Mean Pressure Distributions

The mean (static) pressure distributions are shown in Figure 6.4. It can be seen that the pressure distributions are not significantly affected by changes in Reynolds number (less than 0.02 in C_p). The present distributions in general agree with those of Hillier and Cherry⁵⁴ (the model geometry and wind tunnel used were identical), except in the front section of the separation bubble (their results have higher values, i.e., lower pressure). The difference is probably due to different blockage ratios (2.5% for the present study and 3.7% for that of Hillier and Cherry). The reattachment positions are also in close agreement ($\frac{R}{2H} = 5.0$ - present case, $\frac{R}{2H} = 4.9$ - Hiller and Cherry).

The pressure recovery factor $\tilde{C}_{p_R}$ (defined as $C_{p_R} - C_{p_s}) / (1 - C_{p_s})$) at the reattachment position ($\frac{R}{2H} = 5.0$) was also evaluated. It was found to be 0.32 ($\tilde{C}_{p_R}$) and was in agreement with the values obtained from Ota and Itasaki⁷⁵ and Hillier and Cherry⁵⁴.

6.2.2 The Local Nusselt number Distributions

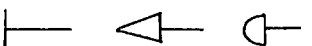
The local Nusselt number (Nu_μ) distributions at different Reynolds numbers are shown in Figure 6.5. The distributions can be divided into 4 regions in general, (i) the leading edge region (ii) the separated flow region (iii) the reattachment region (iv) the redeveloped flow region.

The Leading Edge Region

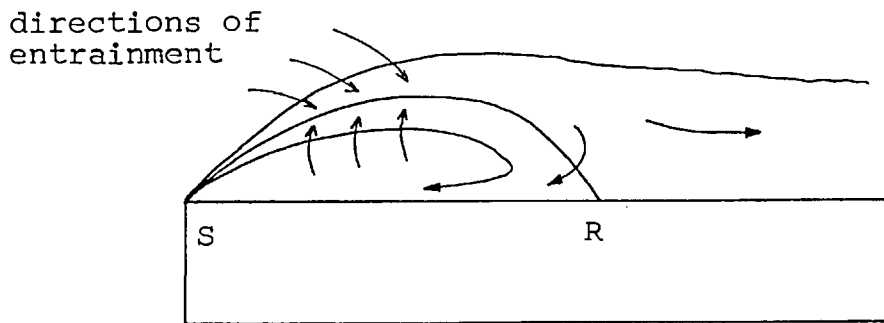
There are small troughs (in nearly all the cases)

in the Nu_μ distributions around the leading edge (Figure 6.5). The values are higher at the position nearer to the separation point. This behaviour has also been reported by Filetti and Kays³ (heat transfer behind a double step). The reason for this may be due to the fact that the shear layer was laminar at separation, it then went through a stage of transition to become turbulent (indeed Roshko and Lau⁹⁶ found that transition occurs within a few shear layer thickness of separation) so the heat transfer rate at the position nearer to the separation position was different. However, it should be noted that the heat transfer rate on the front face (laminar boundary layer) is about three times greater than that for small x (distance from the leading edge). It therefore appears that the surface heat transfer rate of the separated shear layer is very different from that of the attached boundary layer on the front face. The pressure fluctuations and the rate of growth of the separated shear layer can both affect the wall heat transfer. The pressure fluctuations can cause velocity fluctuation and hence affect the $\overline{v\theta}$ transport. As the shear layer grows the shear stress ($-\overline{uv}$) also increases (Ota and Narita⁹¹) which can in turn increase $\overline{v\theta}$ transport (it is shown in Chapter 7 that heat transport is via turbulence transport).

The Separated Flow Region

In the separated flow region the local Nu_μ increases steeply with distance from the leading edge (Figure 6.5). The growth of shear layers has been studied by Hillier and Latour⁷⁶ (backward facing steps ). They found that the shear layer thickness $\tilde{Y}$ (defined as the distance between

5% and 100% deficit of total pressure) was growing linearly with distance up to 60% of the separation bubble(the shear layer grows by the process of entrainment of fluid from the cavity and the external free-stream). After this position the growth rate increased rapidly, so that the shear layer was increasingly distorted by the approaching reattachment.



The growth of a two-dimensional mixing layer has been studied by Wagnanski and Fiedler⁷⁷, Champagne et al⁷⁸ (unheated mixing layers). In their experiments one side of the mixing layer was of high velocity and the other side was of low velocity. They found that the mixing layer grew linearly with distance, maximum turbulent fluctuations ($\overline{u^2}$, $\overline{v^2}$ and $-\overline{uv}$) remained constant and velocity profiles were similar at various sections of the shear layer. The shear layer of the present blunt flat plate is somewhat similar to that of a mixing layer, (i.e., regions of high and low velocities exist). The density difference between the external stream and the recirculating region should not affect the growth rate of the shear layer(Boussinesq assumption); as this is true for a two-dimensional mixing layer(Brown and Roshko⁷⁹). If the shear layer grows like a mixing layer, the heat transfer rate across it will be constant (as for a turbulent mixing

layer, Keffer et al⁸⁰ showed that the heat transfer rate was constant with distance). The above statement clearly violates the present results (Nu_h increases with distance). The rapid increase in Nu_h along the surface of the present blunt flat plate may not simply be caused by the growth of the shear layer, the recirculating fluid may also play an important part in heat transfer. Temperature and velocity measurements in the separated flow region are therefore necessary (see Chapter 7 for temperature and velocity measurements).

The Reattachment Region

The local Nu_h reaches a maximum at the reattachment position (Figure 6.5). This finding is in agreement with Krall and Sparrow³⁴ (backward facing step), Seban et al³⁹ (backward facing steps), Ota and Kon⁴³ (blunt flat plate). Reattachment is a 'stagnation-like' process, the shear layer dividing in the mean so that some fluid sweeps downstream whilst the remainder deflects upstream to satisfy entrainment requirement of the near wake. Ota and Kon⁴¹ reported that large turbulent shear stress and heat flux were generated at the reattachment region. It seems probable that the large change in flow structure is the cause of maximum heat transfer. Detailed measurements of fluctuating temperature and velocity should be concentrated in this region (see discussion given in Chapter 7).

The Redeveloped Flow Region (downstream of reattachment)

The local Nu_h shows a gradual decrease after reattachment (Figure 6.5). Similar results were obtained by Filetti and Kay³ (backward facing step), Ota and Kon⁴³.

Downstream of reattachment the flow field is gradually recovering (as shown by the present static pressure distribution in Figure 6.4). Ota and Narita¹¹ reported that there was a considerable difference in the shear layer downstream of reattachment as compared with a turbulent boundary layer. The values of their velocity measurements ($\frac{\sqrt{u^2}}{U_\infty}$, $\frac{\sqrt{v^2}}{U_\infty}$, and $\frac{-\overline{uv}}{U_\infty^2}$) were at least an order of magnitude higher than those of the turbulent boundary layer. Bradshaw and Wong⁸² (backward facing step) also observed that even at a distance of 52 step heights, the boundary layer was still not a fully developed boundary layer. Ota and Kon⁴⁵ found that the longitudinal distance of the model was not sufficient for the Nusselt number to reach the value as predicted by Colburn's formula ($Nu_x = 0.0296 Pr^{1/3} \left(\frac{U_\infty x}{\nu} \right)^{0.8}$ - heat transfer in turbulent boundary layer). This fact suggests clearly that the separation and reattachment of flow have severe influence upon the heat transfer characteristics for a long distance downstream of reattachment. The flow in the redeveloped region of the present model is therefore unlikely to reach the characteristics of a turbulent boundary layer (the heat transfer rate cannot be predicted by Colburn's formula - Chapman²).

6.2.3 The Effect of Reynolds number on the Heat Transfer Distributions

The effect of Reynolds number on $\frac{Nu_H}{Re_H^{0.75}}$ is shown in Figure 6.34. It can be seen that $\frac{Nu_H}{Re_H^{0.75}}$ differs by 5% at the most between different Re_H values. There is no consistent trend on the distribution ($\frac{Nu_H}{Re_H^{0.75}}$ increases and decreases with

Re_H). The effect of Reynolds number on $\frac{Nu_H}{Re_H^{0.75}}$ is fairly weak. Heat transfer (like skin friction) is associated with a boundary layer. It is therefore necessary to obtain temperature profiles at different Re_H (see discussion given in Chapter 7).

6.2.4 The Reynolds number Dependence

In the separation bubble the Reynolds number dependence n (over most positions along the model) was about 0.71, i.e., $Nu_H = cRe_H^{0.71}$ (see Figure 6.26), with one exception—a lower n (~ 0.60) was obtained at $x/R = 0.04$. A slightly higher n (~ 0.75) was obtained near the reattachment position. The special feature of separated flow is that the locations of maximum shear stress are remote from the wall, indeed the greatest stresses are found in the shear layer (Arie and Rouse⁸³, Mueller and Robertson⁸⁴). A consequence of this feature is that the turbulence which is generated in the shear layer region must be conveyed to the vicinity of the wall by the action of convection and diffusion (unlikely to be significant for a reattaching shear layer). The turbulence intensity in the vicinity of the wall is a main determinant of heat transfer, is governed by the interaction of these two factors with turbulence dissipation. The balance between convection and dissipation of turbulence energy controls the value of n (Spalding³²).

6.2.5 The Results of Ota and Kon

The heat transfer results of Ota and Kon⁴⁵ (constant heat flux model) were plotted against the present constant heat flux results (Figure 6.6). The general trend of the Nu_H

distributions was similar, but the magnitudes were higher in the present case (due partly to higher Re_H). $\frac{Nu_H}{Re_H^{0.75}}$ was therefore used to compare results (Figure 6.45). It was found that their results were in general about 20% lower in the separated flow region and the difference increased to 40% in the redeveloped region. The position of maximum $\frac{Nu_H}{Re_H^{0.75}}$ also differed ($\frac{x}{2H} = 5$, the present case; $\frac{x}{2H} = 4.5$, Ota and Kon). Since the blunt leading edge flat plate used by Ota and Kon had an aspect ratio (span/thickness) of 5 and no end plates, the mean flow field was unlikely to be two-dimensional (because bleeding of tunnel wall boundary layers into the bubble and separation of tunnel wall boundary layers by the model could induce three-dimensional effect). Their mean pressure distribution had higher values in the separated flow region (Ota and Itasaki⁷⁵). The Reynolds number dependence ($n = 0.71$) at reattachment was also different from the present case ($n = 0.75$). The discrepancy in Nu_H between the two cases was perhaps a consequence of the above results.

6.3 Discussion of Results for the Turbulent Free-Stream

6.3.1 The Mean Pressure Distributions

The mean pressure distributions at a fixed Reynolds number ($Re_H = 0.99 \times 10^4$) for both turbulent and non-turbulent free-streams are shown in Figure 6.46. The separation pressure (C_{ps}) is found to reduce in turbulent free-streams. It reduces further with increasing free-stream intensity. An increase in intensity also tends to contract the pressure distribution. This finding is in agreement with Hillier and

Cherry⁵⁴. They also concluded that intensity has the dominant effect on pressure distribution and the effect of length scale is not significant. Their conclusion is confirmed by the present results. The mean pressure distributions of grid A ($\frac{\sqrt{u^2}}{U_\infty} = 5.4\%$, $\frac{Lx}{2H} = 3.66$) and grid B ($\frac{\sqrt{u^2}}{U_\infty} = 5.4\%$, $\frac{Lx}{2H} = 1.37$) are identical.

The reattachment length is found to reduce in free-stream turbulence (Appendix E3). In general it decreases with increasing free-stream intensity. The length of the bubble arises from an equilibrium between the fluid recirculating into the cavity region, by the reattaching shear layer, and entrainment from this cavity into the shear layer closer to separation. From this stand point, stream turbulence tends to provide some mechanism for increasing these entrainment rates. The increase of suction within the bubble can be regarded as a consequence of the increased curvature of the shear layer trajectory and of the external streamlines as the bubble contracts (increased suction is necessary to draw more fluid back into the cavity from reattachment and/or shorten bubble length). The effect of free-stream turbulence on a plane mixing layer has been reported by Mohammadian et al⁸⁵, Pui and Gartshore⁸⁶. Their results show some evidence that turbulence in one or other of the streams can increase spreading rates and turbulent stresses. It should be noted that plane mixing-layer experiments may not adequately represent some features of the separated flow problem such as the effects of flow reattachment and the mean curvature of the shear layer.

6.3.2 The Local Nusselt number Distributions

The local Nu_{η} distributions of the separation bubble contracts in free-stream turbulence (relative to bubble length which contracts). The amount of contraction increases with increasing intensity (Figures ^{6.13-}6.18). It can be seen that the distance from the leading edge to the position of maximum Nu_{η} contracts more rapidly than does the bubble length. This probably arises from the increasing spreading rate of the shear layer since the reattachment heat transfer rate is controlled to some extent by the shear layer thickness at reattachment (heat transfer is associated with boundary layers).

6.3.3 The Reynolds number dependence

The Reynolds number dependence of heat transfer in the separated flow region is in general higher in free-stream turbulence ($n \approx 0.71$ for non-turbulent free-stream, $n \approx 0.75$ for turbulent free-streams). The higher values of n in the case of free-stream turbulence are most probably due to the more thorough mixing of fluid in the shear layer. The reattachment n is approximately constant ($n \approx 0.75$) for both turbulent and non-turbulent free-streams. The physical implication which one can draw from the latter fact is that the reattachment process is identical for both turbulent and non-turbulent free-streams (i.e., free-stream turbulence does not alter the process of shear layer attachment). The above observation is supported by temperature and velocity measurements at the reattachment region (further discussion is given in Chapter 7).

6.3.4 The Effect of Reynolds number on the Heat Transfer Distribution

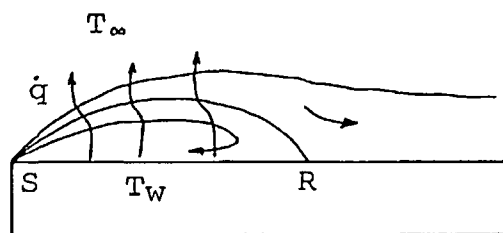
The effect of Reynolds number on the $\frac{Nu_H}{Re_H^{0.75}}$ distributions is in general small (differed by 8% for different Re_H) in free-stream turbulence (Figures 6.35-6.40). As a rough guide $\frac{Nu_H}{Re_H^{0.75}}$ decreases with increasing Re_H . It was shown in Chapter 5 that the free-stream itself is Reynolds number sensitive in grid generated turbulence (as shown by the energy spectrum). As a result of this effect the interaction of the free-stream turbulence and the separation bubble may also be dependent on Reynolds number, which in turn influences the surface heat transfer $\frac{Nu_H}{Re_H^{0.75}}$.

6.3.5 The Effect of Intensity and Length Scale

In free-stream turbulence the surface heat transfer $\frac{Nu_H}{Re_H^{0.75}}$ depends on Re_H , $\frac{\sqrt{u^2}}{U_\infty}$ and $\frac{Lx}{2H}$. In order to see the effect of any one of them alone, the other two must be kept constant. The effect of free-stream turbulence on $\frac{Nu_H}{Re_H^{0.75}}$ is shown in Figure 6.42 (it should be noted that the effect of Reynolds number was eliminated in these investigations, i.e., $Re_H = 0.99 \times 10^4$ for all the cases, so that the effect was solely due to intensity and length scale). It can be seen that the distributions do not overlap (in general) in the separated flow region, and the slope becomes steeper with decreasing reattachment length. It appears that the maximum $\frac{Nu_H}{Re_H^{0.75}}$ position increases with increasing free-stream intensity. In the leading edge region the small troughs in the distributions (mentioned in section 6.2.2 for non-turbulent free-stream) still can be seen at lower intensities ($\frac{\sqrt{u^2}}{U_\infty} \sim 2-5\%$), but the

troughs appear to be getting shallower with increasing intensity and they seem to disappear at higher intensities ($\frac{\sqrt{u^2}}{U_\infty} \sim 5-8\%$, except grid A $\frac{\sqrt{u^2}}{U_\infty} = 5.4\%$). The presence of free-stream turbulence is known to produce earlier transition (Hinze⁸⁷), i.e., the transition region moves closer to the separation point. It seems probable that the movement of the transition region has a significant effect on heat transfer in the leading edge region.

The effect of length scale alone on the $\frac{Nu_H}{Re_H^{0.75}}$ is also shown in Figure 6.42 (by cross plotting two cases of nearly the same intensity). It can be seen that the distributions for grid A ($\frac{\sqrt{u^2}}{U_\infty} \approx 5.4\%$, $\frac{Lx}{2H} \approx 3.66$) and grid B ($\frac{\sqrt{u^2}}{U_\infty} \approx 5.4\%$, $\frac{Lx}{2H} \approx 1.37$) are nearly identical (differed by about 3% in $\frac{Nu_H}{Re_H^{0.75}}$). One may therefore conclude that the effect of length scale on separated flow is weak, i.e., free-stream intensity has the dominant effect. This finding is in agreement with the mean pressure distributions (section 6.3.1). The reason for the insensitivity to length scale of the mean pressure has been suggested by Hillier and Cherry⁵⁴. They proposed that the shear layer is stimulated by the stream turbulence over a wide range of wave-numbers, from the finest eddy scales to the largest eddy scales in the flow field (comparable to or greater than R). It should be noted that heat transfer is a surface effect (associated with boundary layer), whilst the mean pressure is essentially controlled by the inviscid flow field. It is therefore not possible to draw a simple analogy for the effect of length scale between heat transfer and mean pressure. However, the integrated heat flux over the bubble length must all pass through the shear layer,



which in turn is controlled by the inviscid flow field (heat transfer depends on $T_w - T_\infty$). It therefore appears that heat transfer is similar to the mean pressure (to a certain extent) which does not show a significant scale effect.

The effect of intensity on the reattachment $\frac{Nu_H}{Re_H^{0.75}}$ and the maximum $\frac{Nu_H}{Re_H^{0.75}}$ is shown in Figure 6.47. It can be seen that the reattachment $\frac{Nu_H}{Re_H^{0.75}}$ increased by 9% for an 8.5% increase in $\frac{\sqrt{u^2}}{U_\infty}$ and the maximum $\frac{Nu_H}{Re_H^{0.75}}$ increases by about 18%. The effect of intensity is therefore important. It should be noted that the effect of intensity is more pronounced for the stagnation point heat transfer (Chapter 5), $\frac{Nu}{Re^{0.5}}$ increases by as much as 40% for an 8% increase in intensity.

One of the most important findings of the present research is that it is possible to correlate the heat transfer data (in the form of $\frac{Nu_R}{Re_{u_s}^{0.75}}$) to a certain extent in the separated flow region ($\frac{Nu_R}{Re_{u_s}^{0.75}}$ differs by about 10% at the most) - see Figure 6.44. The effect of intensity appears to have little influence on $\frac{Nu_R}{Re_{u_s}^{0.75}}$. According to the heat transfer theory of Spalding³², heat transfer in separated flows may be modelled by a length scale and a velocity (intensity). Therefore, the action of free-stream turbulence on the surface heat transfer is almost entirely through its effect on the velocity field (such as bubble length and U_s) and not through any direct

interaction with the mechanics of heat transfer itself. The heat transfer data of Ota and Kon^{44,45} (blunt flat plate and axisymmetric blunt circular cylinder $\rightarrow \text{O} \text{---} \text{O}$) are also plotted onto Figure 6.44 for comparison. It can be seen that the blunt flat plate data overlap fairly well with the present data, but the axisymmetric blunt circular cylinder data are roughly 20 - 30% lower than the present values. It therefore appears that flows of the same configuration (two-dimensional flows in this case) have common heat transfer characteristics, and heat transfer data can be correlated using R and U_s .

6.3.6 The Artificially Shortened Bubble

The separation bubble on the upper surface of the model (of a non-turbulent free-stream) was shortened by moving the tail-flap upwards. The purpose of this test was to study whether heat transfer increases as a result of the effects of free-stream turbulence (such as changing entrainment rates and bubble structure) or simply due to a change in bubble length. The pressure distributions (C_p) at various Reynolds numbers ($Re_H = 1.2 \times 10^4$ to 2.1×10^4) are shown in Figure 6.24. The effect of Reynolds number is generally small (less than 0.03 in C_p). However, the pressure in the bubble is found to increase (compared with the non-turbulent free-stream) as the suction is reduced. The local Nu_H distribution at different Reynolds numbers and the 'constant heat flux model' Nu_H distributions are shown in Figure 6.25. The variations are in general similar to that of the non-turbulent free-stream. The $\frac{Nu_H}{Re_H^{0.75}}$ distributions are shown in Figure 6.41.

Although the bubble length is shorter than that of the non-turbulent free-stream ($\frac{R}{2H} = 3.8$ and $\frac{R}{2H} = 5.0$ respectively), the values of maximum $\frac{Nu_H}{Re_H^{0.75}}$ are approximately the same in both cases ($\frac{Nu_H}{Re_H^{0.75}} \sim 0.063$ and 0.065 respectively). This may imply that shortening the bubble in a non-turbulent free-stream does not increase the heat transfer. The value of 0.063 (for the artificially shortened bubble) is much lower than the values of free-stream turbulence (maximum $\frac{Nu_H}{Re_H^{0.75}}$ varies between 0.072 to 0.078). In one particular turbulent free-stream (grid C, $\frac{\sqrt{u^2}}{U_\infty} = 2.7\%$, $\frac{Lx}{2H} = 0.86$), the reattachment length ($\frac{R}{2H} = 3.85$) is approximately the same as that of the artificially shortened bubble ($\frac{R}{2H} = 3.8$), but the values of maximum $\frac{Nu_H}{Re_H^{0.75}}$ differ considerably (0.072 and 0.063 respectively). The implication of the above result is that although the bubble lengths are approximately the same, the structure of the bubble and the process of entrainment may be different. This implication is supported by the measurements of separation velocity (U_s), U_s for the artificially shortened bubble ($\frac{U_s}{U_\infty} = 1.25$) is lower than that of the non-turbulent free-stream ($\frac{U_s}{U_\infty} = 1.35$ for grid C $\frac{\sqrt{u^2}}{U_\infty} = 2.7\%$, $\frac{Lx}{2H} = 0.86$).

6.4 Conclusions

Non-turbulent Free-stream

The mean pressure distribution is in good agreement with Hillier and Cherry⁵⁴. The effect of Reynolds number on mean pressure is insignificant and the pressure recovery factor $\tilde{C}_{p_R}$ at reattachment is 0.32 .

The local Nusselt number Nu_H increases rapidly from the leading edge and reaches a maximum at the reattachment position. After reattachment it decreases gradually downstream. In the separation bubble, Nusselt number varies with

Reynolds number in the form of $Nu_H = cRe_H^n$. The Reynolds number dependence n is approximately constant ($n \approx 0.71$) in the separation bubble and is a maximum at the reattachment position ($n = 0.75$). The Reynolds number effect on $\frac{Nu_H}{Re_H^{0.75}}$ is insignificant.

The constant heat flux data (in the form of $\frac{Nu_H}{Re_H^{0.75}}$) of Ota and Kon⁴⁵ are roughly 20% lower than the present data, but the general trend of the Nusselt number distributions is similar.

Turbulent Free-stream

The mean pressure distribution in the bubble contracts in free-stream turbulence. The reattachment length reduces with increasing intensity. There is little effect of length scale on the mean pressure distribution and the effect of Reynolds number is generally weak.

The local Nusselt number increases more rapidly in the bubble region than the non-turbulent free-stream case owing to the contraction of the bubble. The location of maximum heat transfer does not in general correspond to the reattachment position. Inside the separation bubble, the relationship $Nu_H = cRe_H^n$ is still valid. The values of n ($n \approx 0.75$) in the separation bubble are greater than those for the non-turbulent free-stream ($n = 0.71$). However, the reattachment n is identical for both the turbulent and non-turbulent free-streams ($n = 0.75$). The reattachment Nusselt numbers (Nu_R) and Reynolds numbers (Re_{u_s}) are found to correlate in the form of $Nu_R = 0.09Re_{u_s}^{0.75}$.

The effect of intensity is found to increase the maximum heat transfer (for an increase of 8% in

intensity, $\frac{Nu_H}{Re_H^{0.75}}$ increases by 20%). An increase in intensity is also found to increase the heat transfer rate in the bubble region. The effect of length scale on heat transfer is insignificant. The effect of Reynolds number on $\frac{Nu_H}{Re_H^{0.75}}$ is generally weak, $\frac{Nu_H}{Re_H^{0.75}}$ decreases with increasing Re .

Correlation of Nusselt number distributions (in the form of $\frac{Nu_R}{Re_{U_s}^{0.75}}$ against $\frac{X}{R}$) for both turbulent and non-turbulent free-stream is obtained (Figure 6.44). The action of free-stream turbulence on the surface heat transfer is almost entirely through its effect on the velocity field (such as U_s and R). It appears that flows of the same configuration have common heat transfer characteristics, and heat transfer data can be correlated using R and U_s .

The heat transfer characteristic of the artificially shortened bubble does give similar results on the basis of the non-dimensional presentation of Figure 6.44.

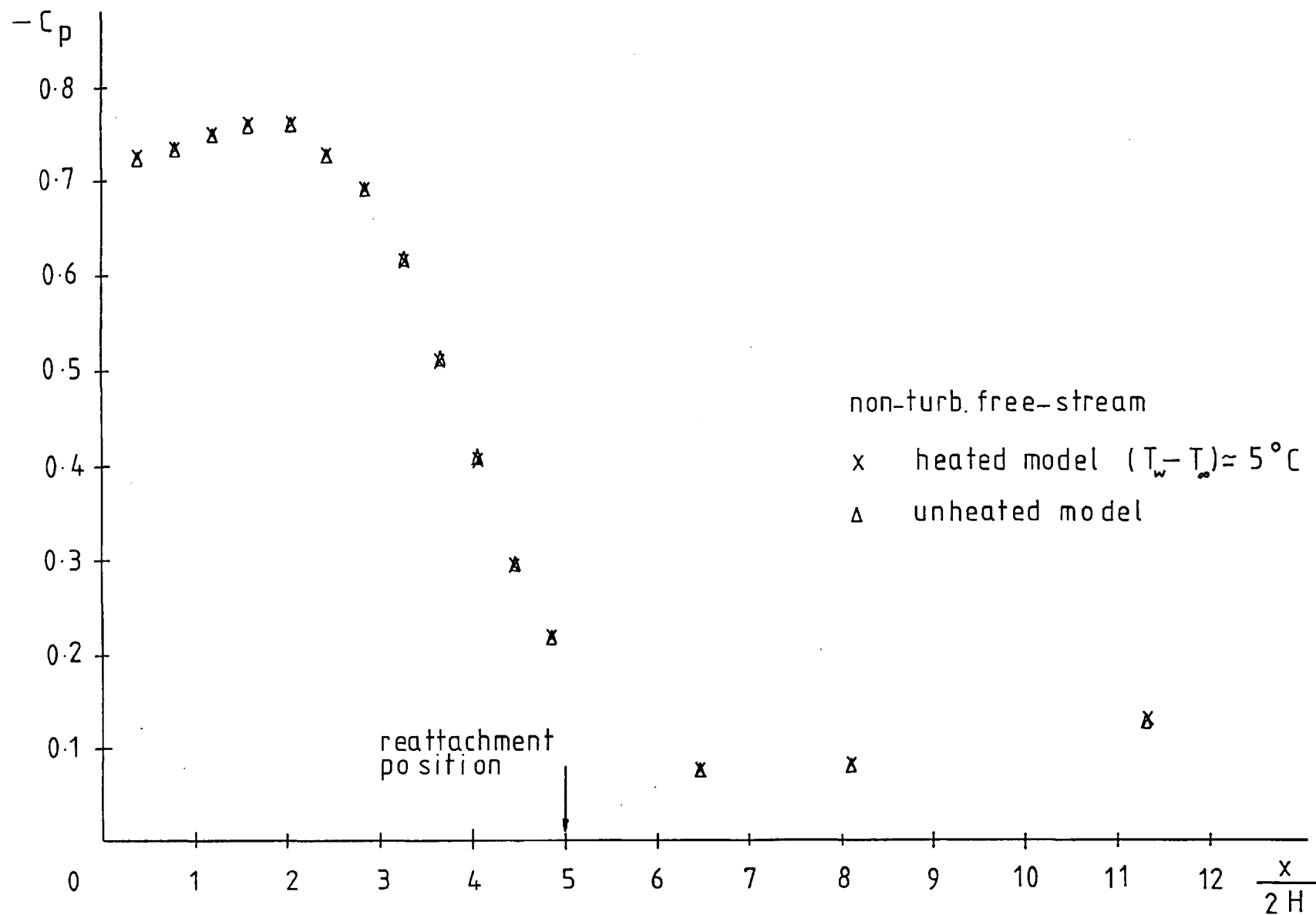


Figure 6.1 The effect of heating on pressure distribution

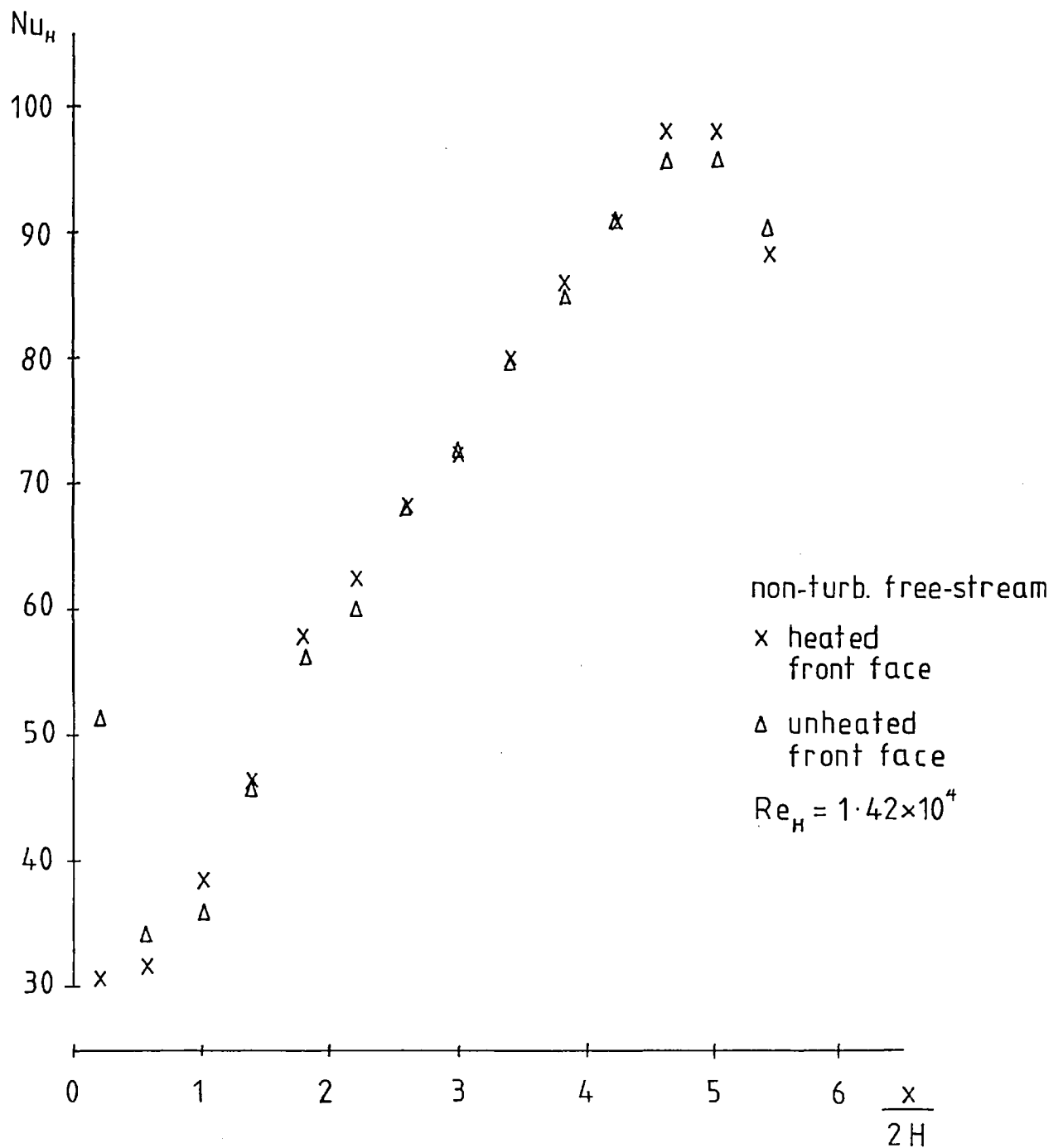


Figure 6.2 The effect of heating the front face on the local Nu_H distribution

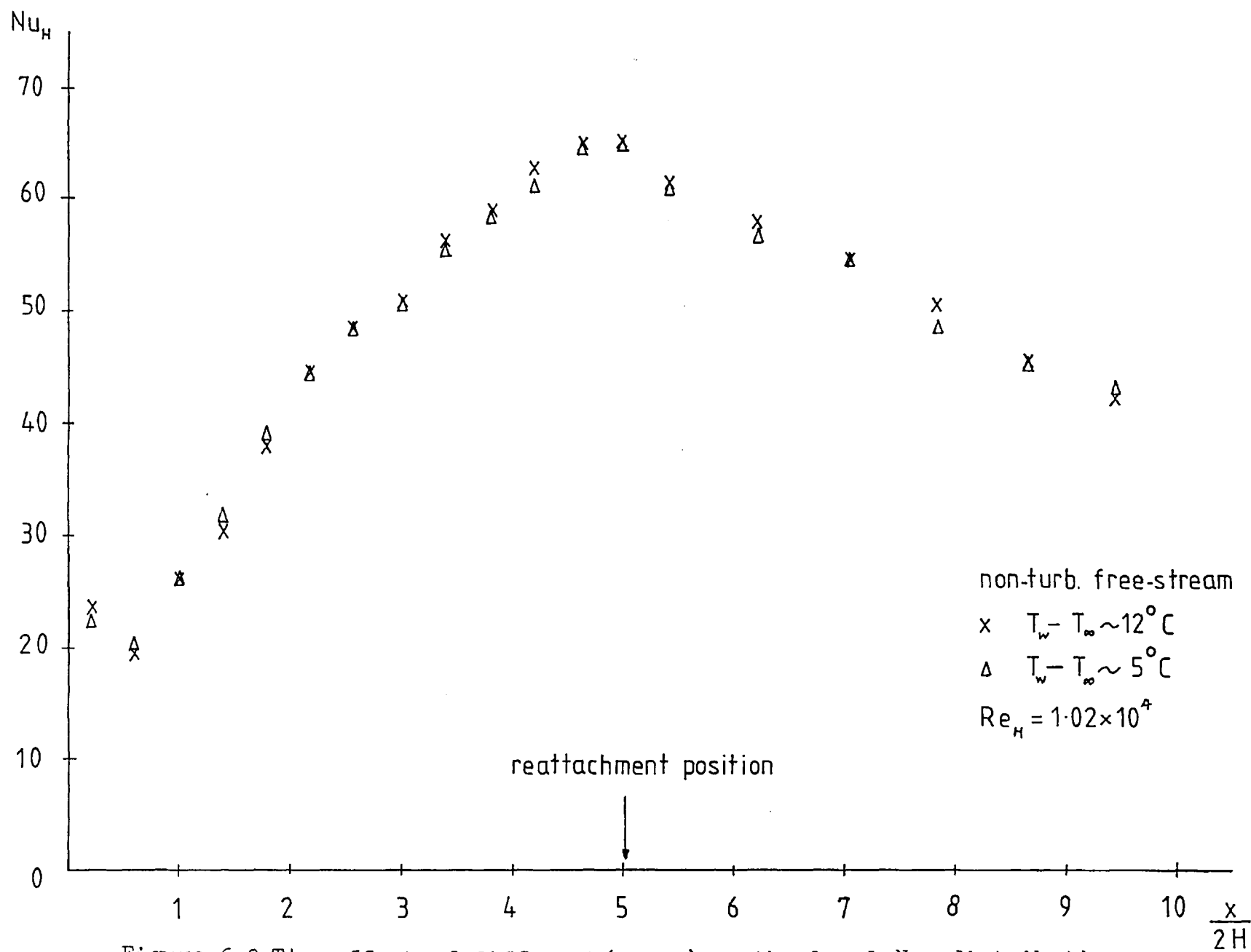


Figure 6 3 The effect of different $(T_w - T_\infty)$ on the local Nu_H distribution

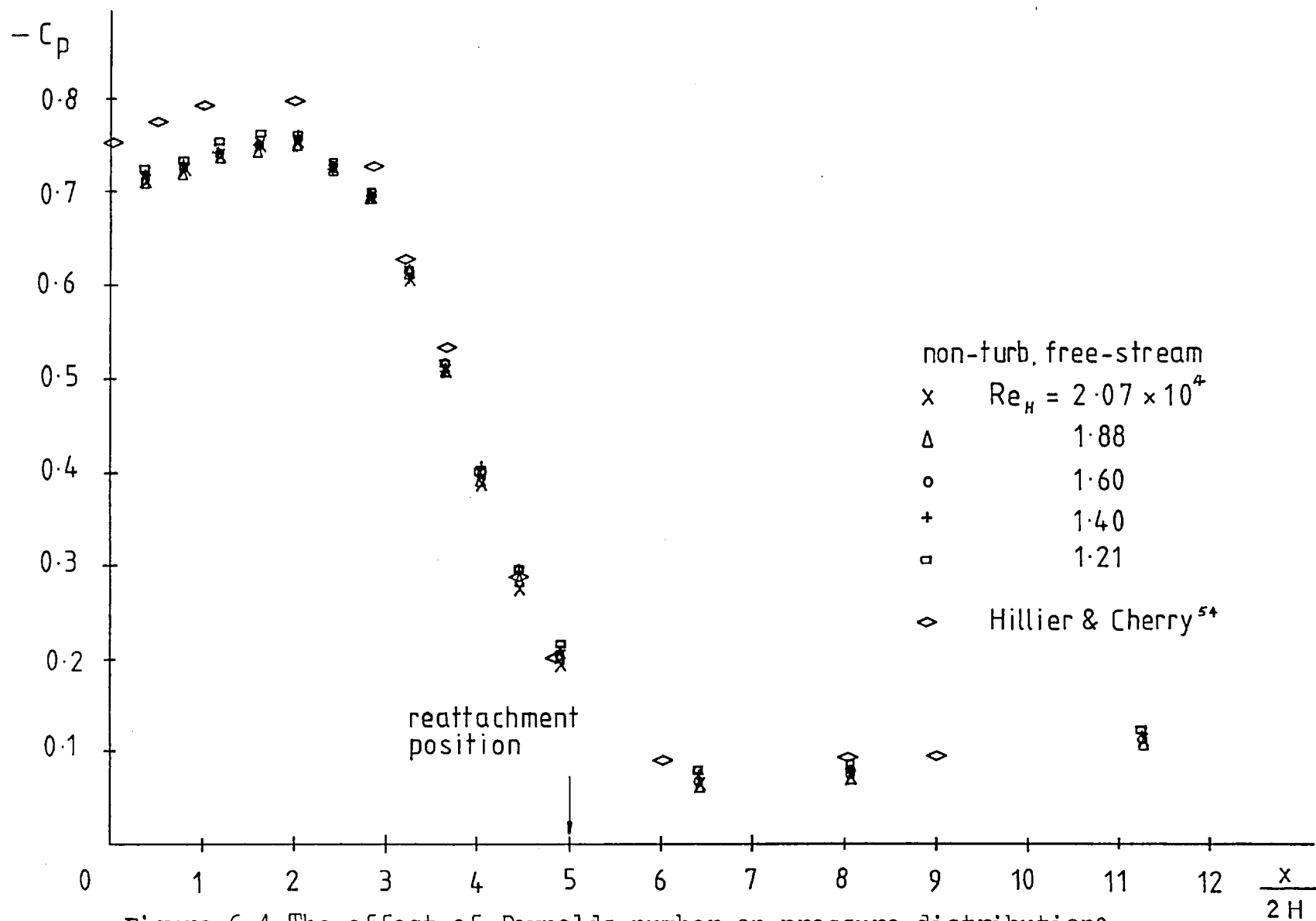


Figure 6.4 The effect of Reynolds number on pressure distributions

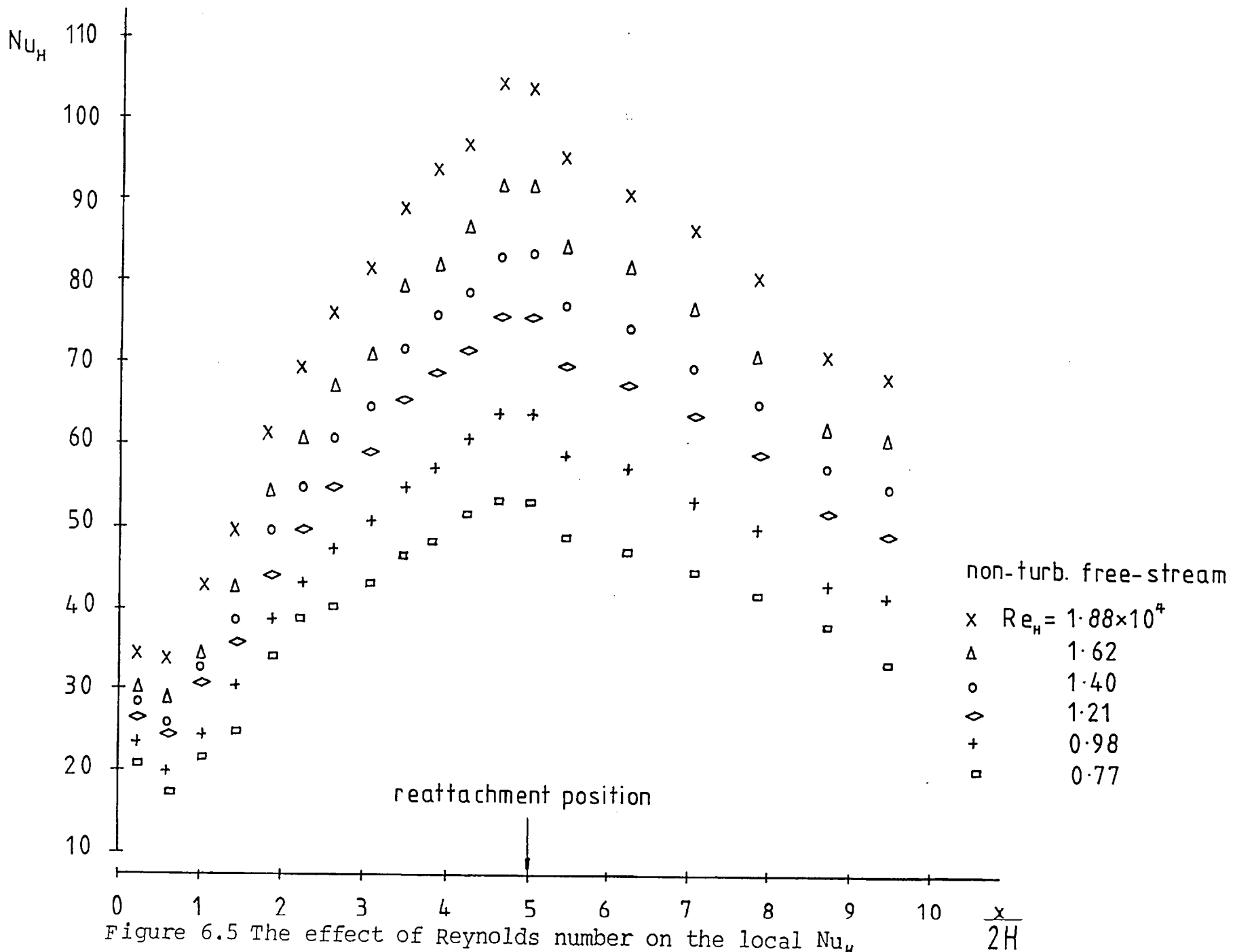


Figure 6.5 The effect of Reynolds number on the local Nu_H distributions

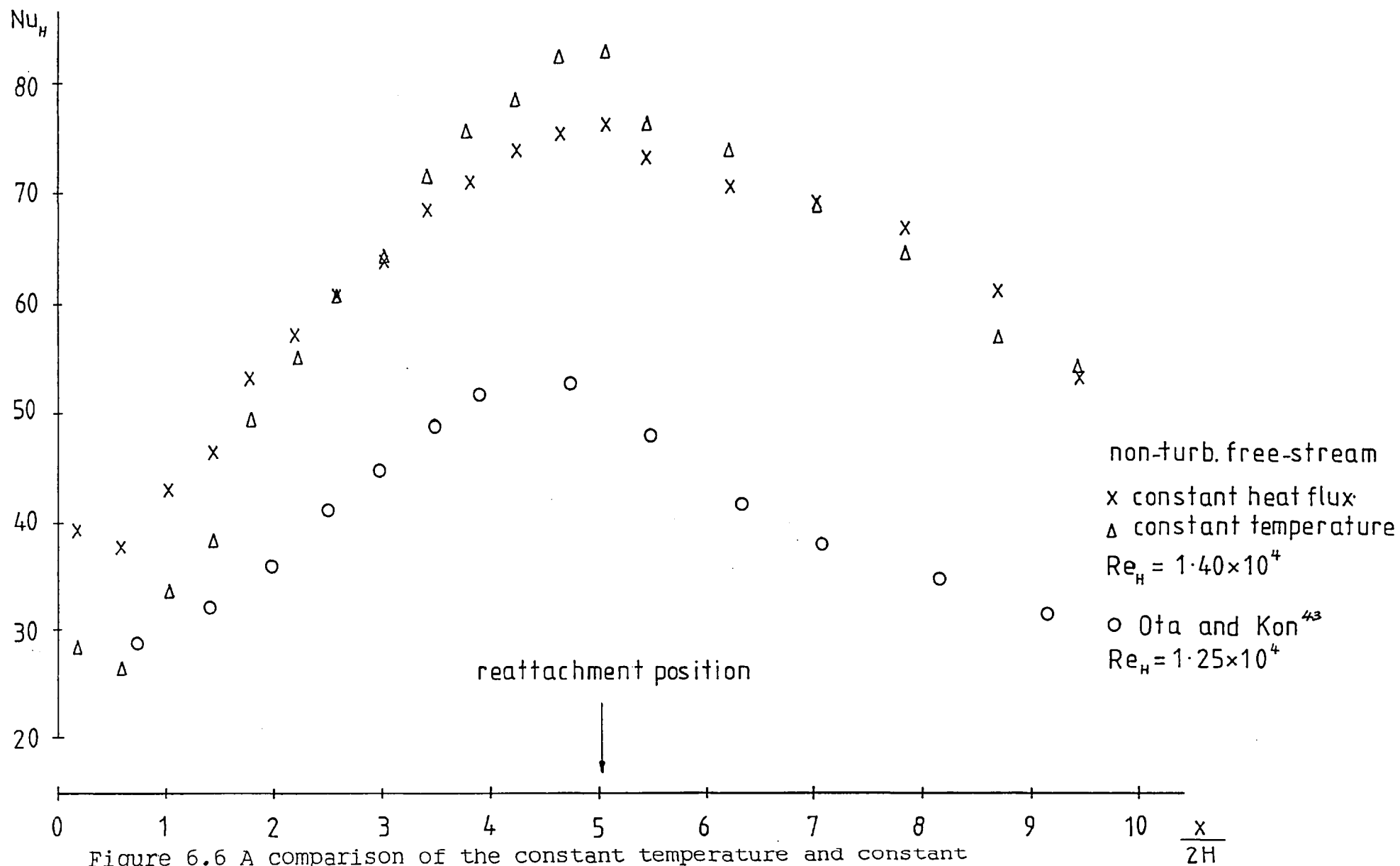
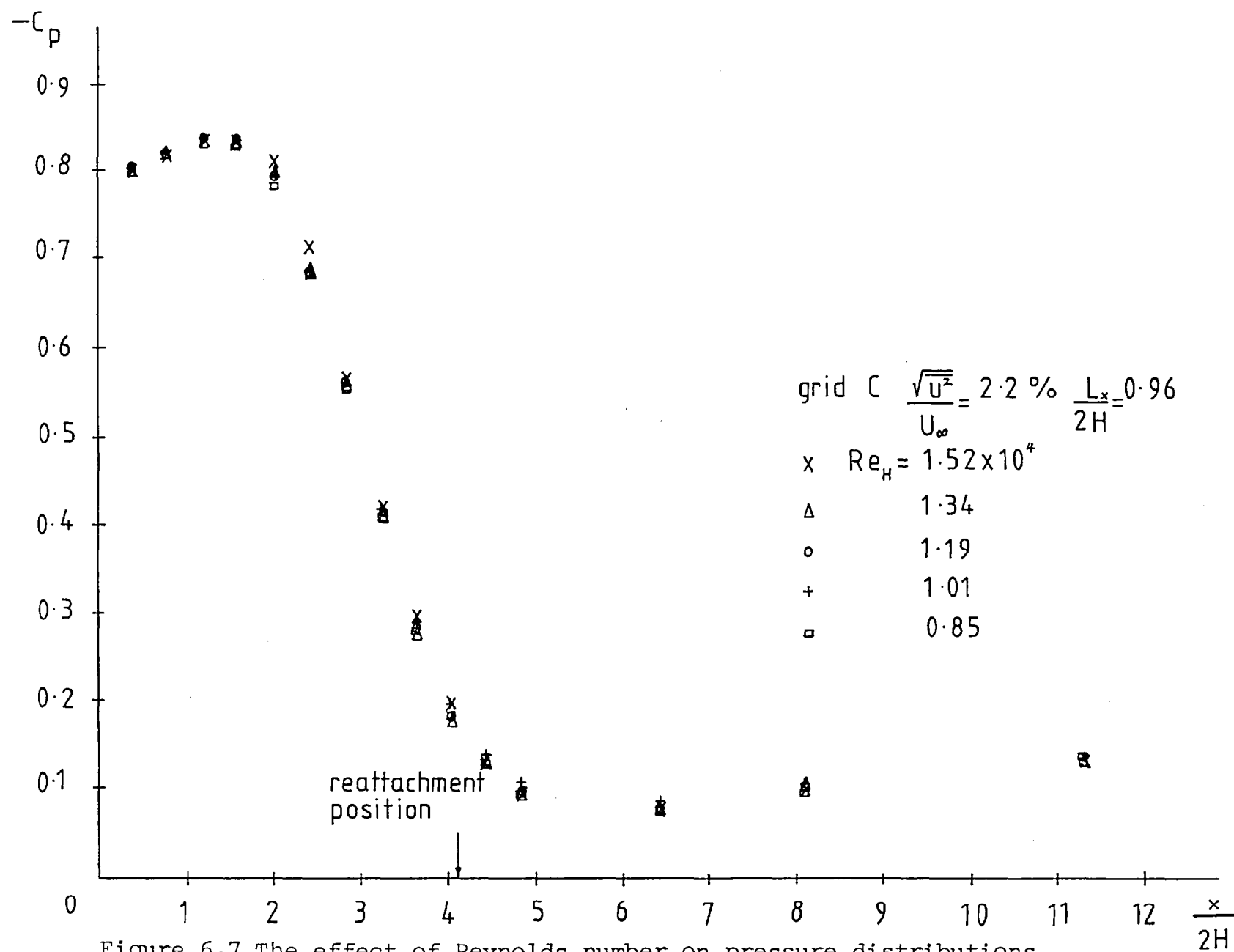


Figure 6.6 A comparison of the constant temperature and constant heat flux Nu_h distributions



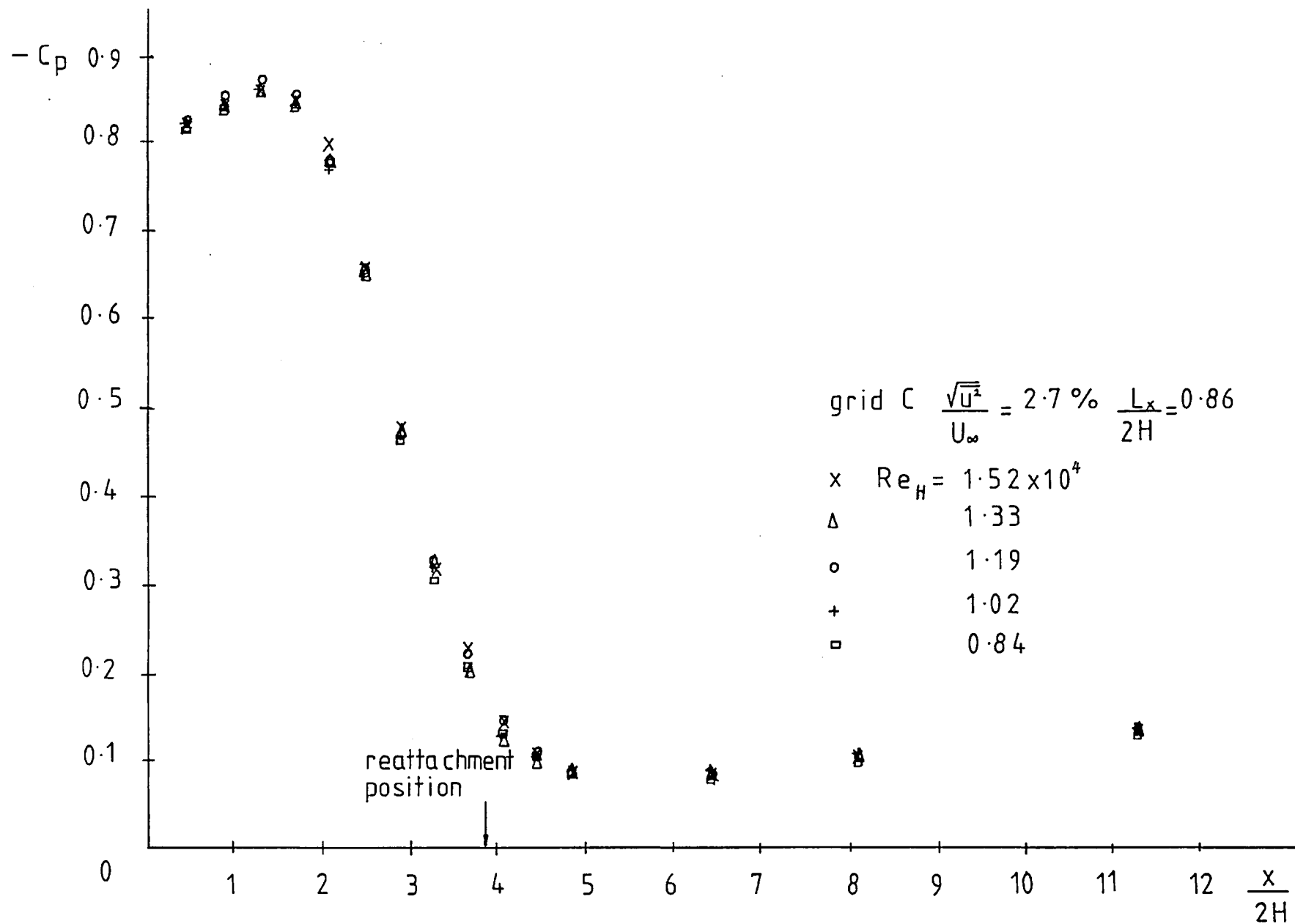
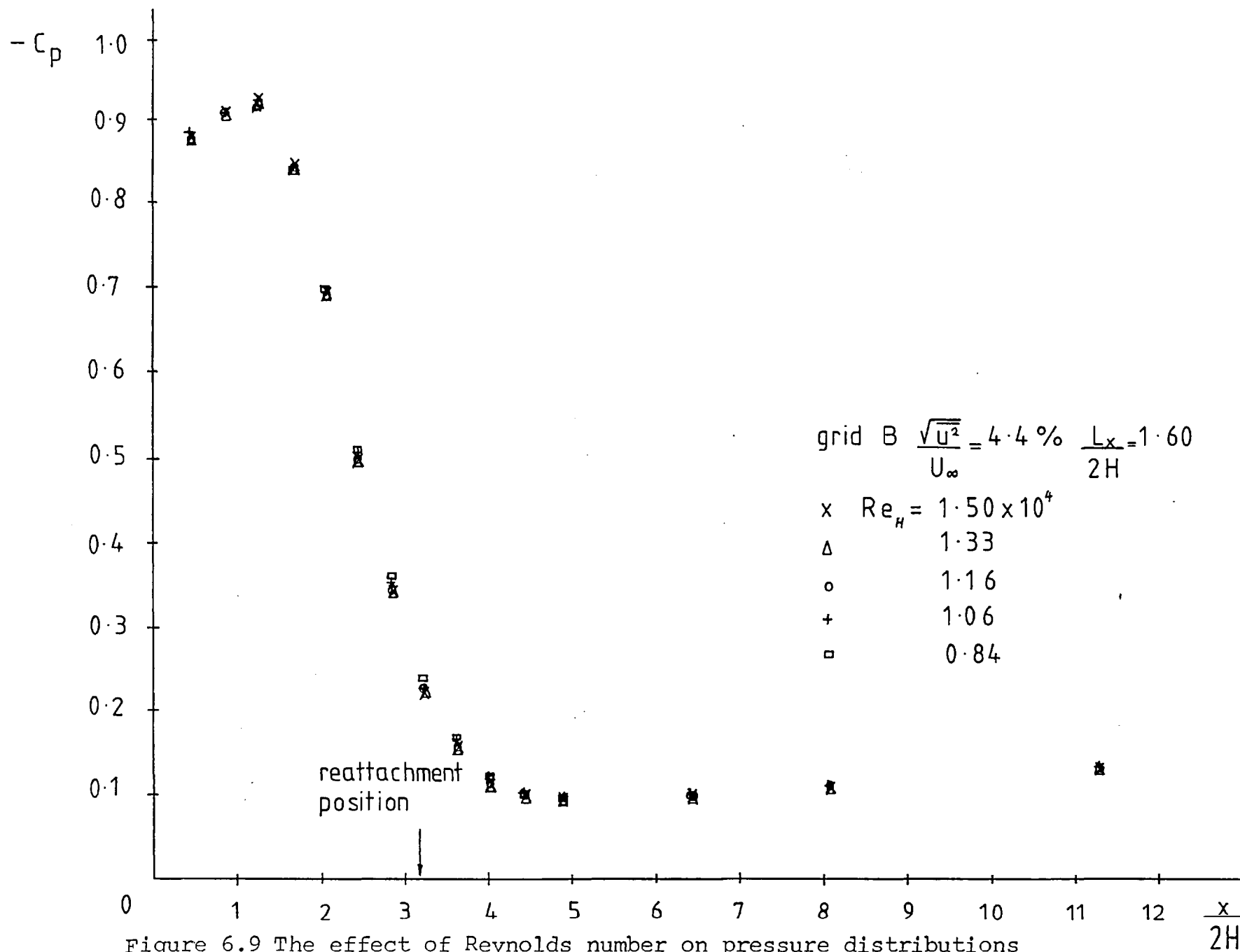


Figure 6.8 The effect of Reynolds number on pressure distributions



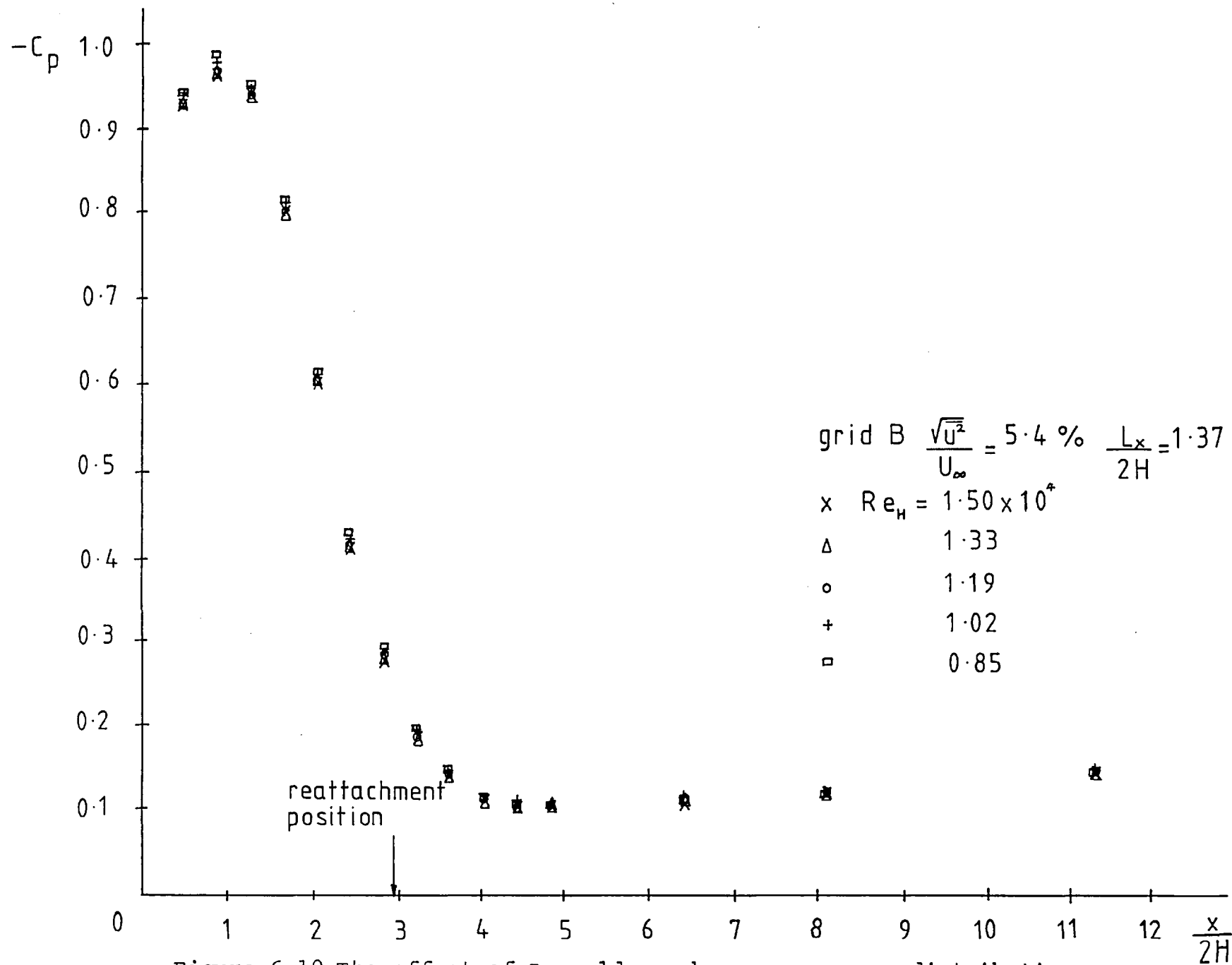


Figure 6.10 The effect of Reynolds number on pressure distribution

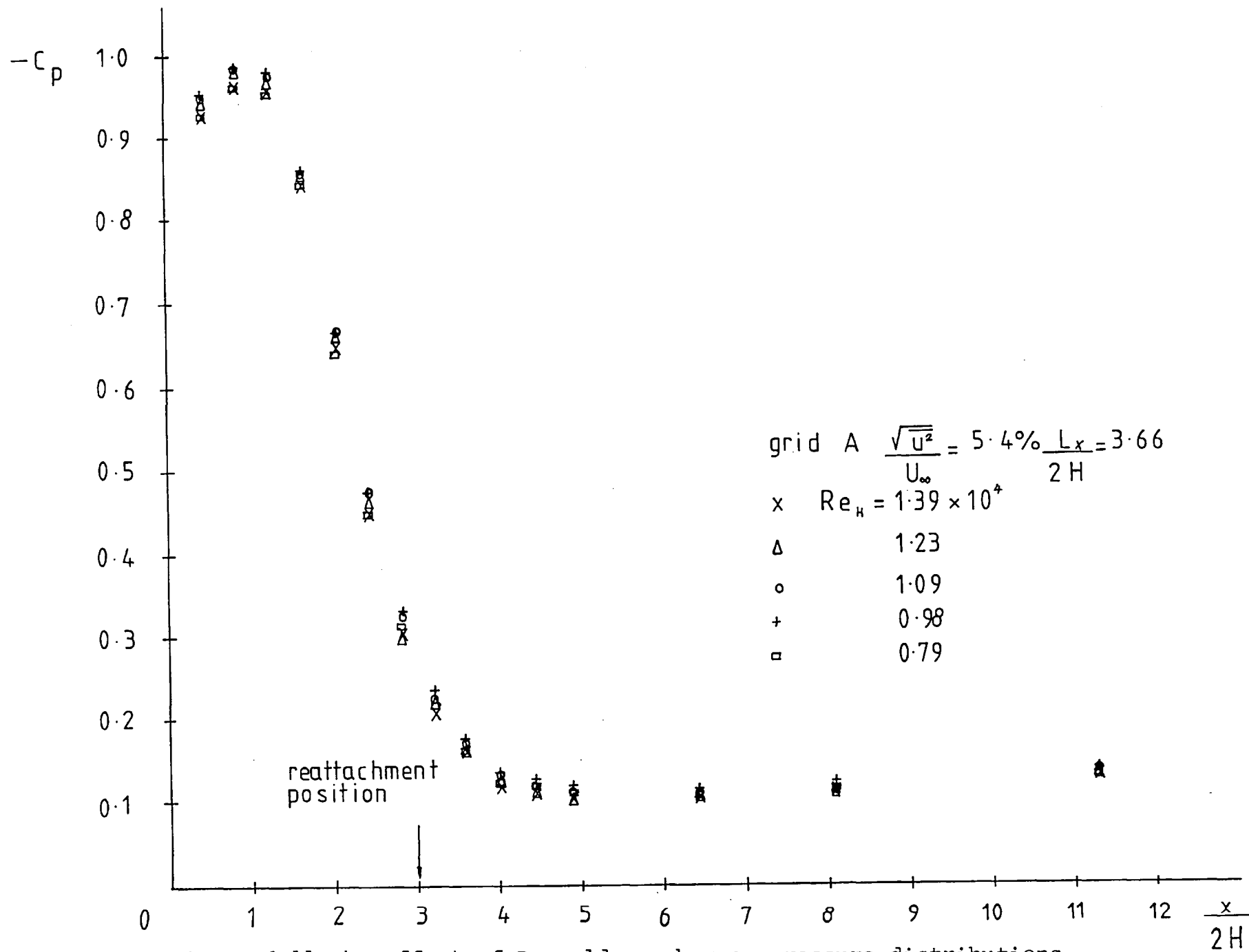


Figure 6.11 The effect of Reynolds number on pressure distributions

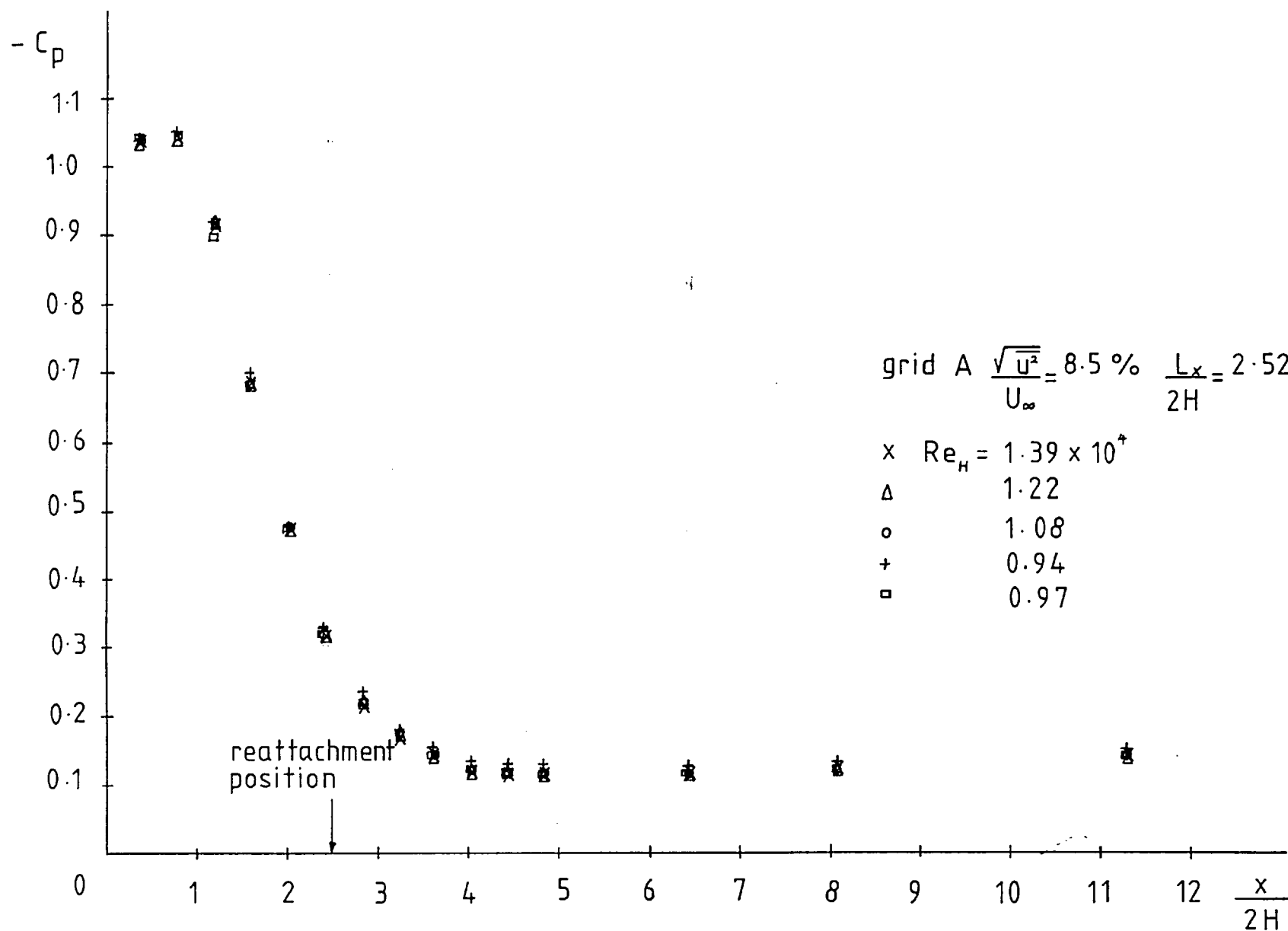


Figure 6.12 The effect of Reynolds number on pressure distributions

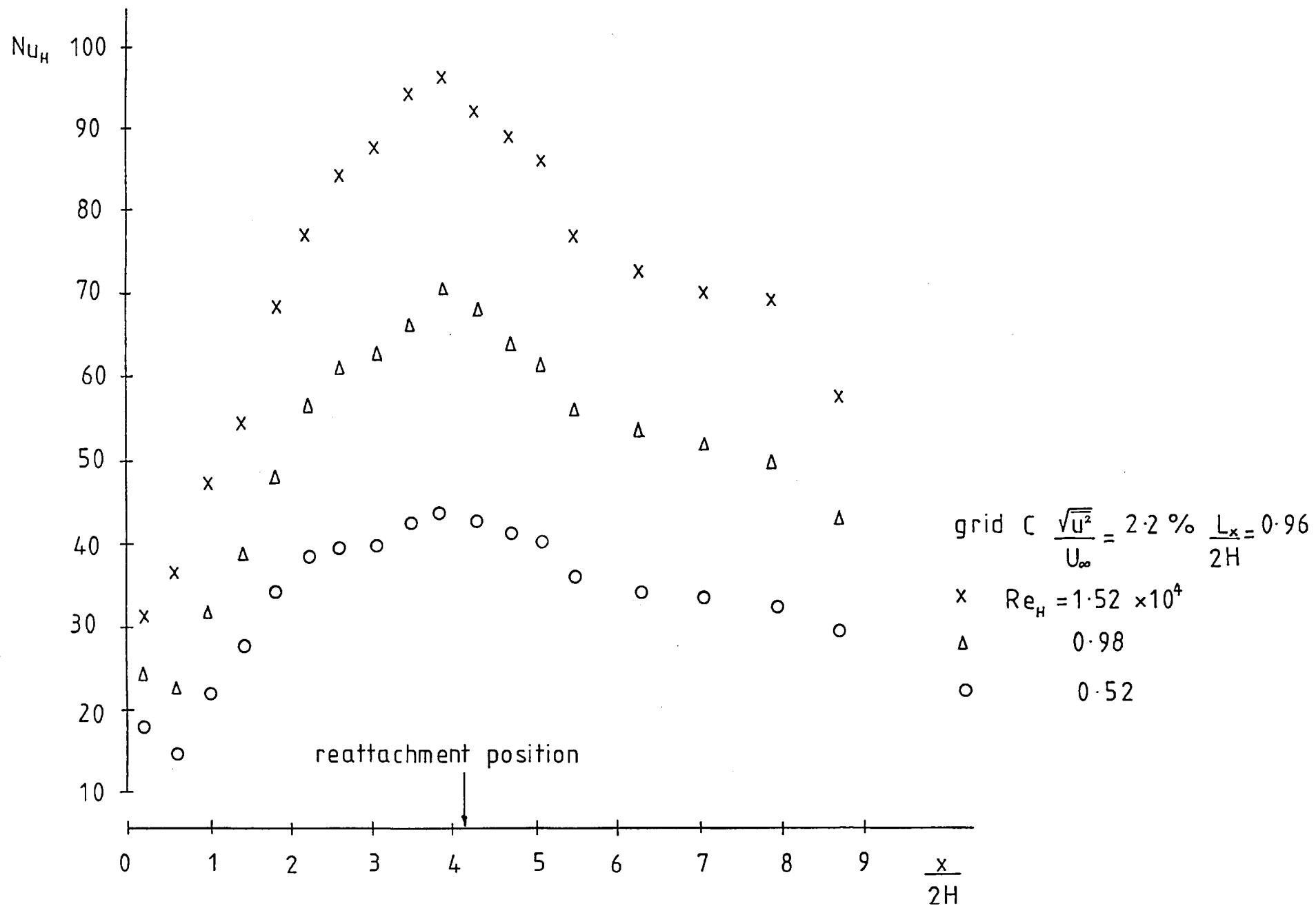


Figure 6.13 The effect of Reynolds number on the local Nu_H distributions

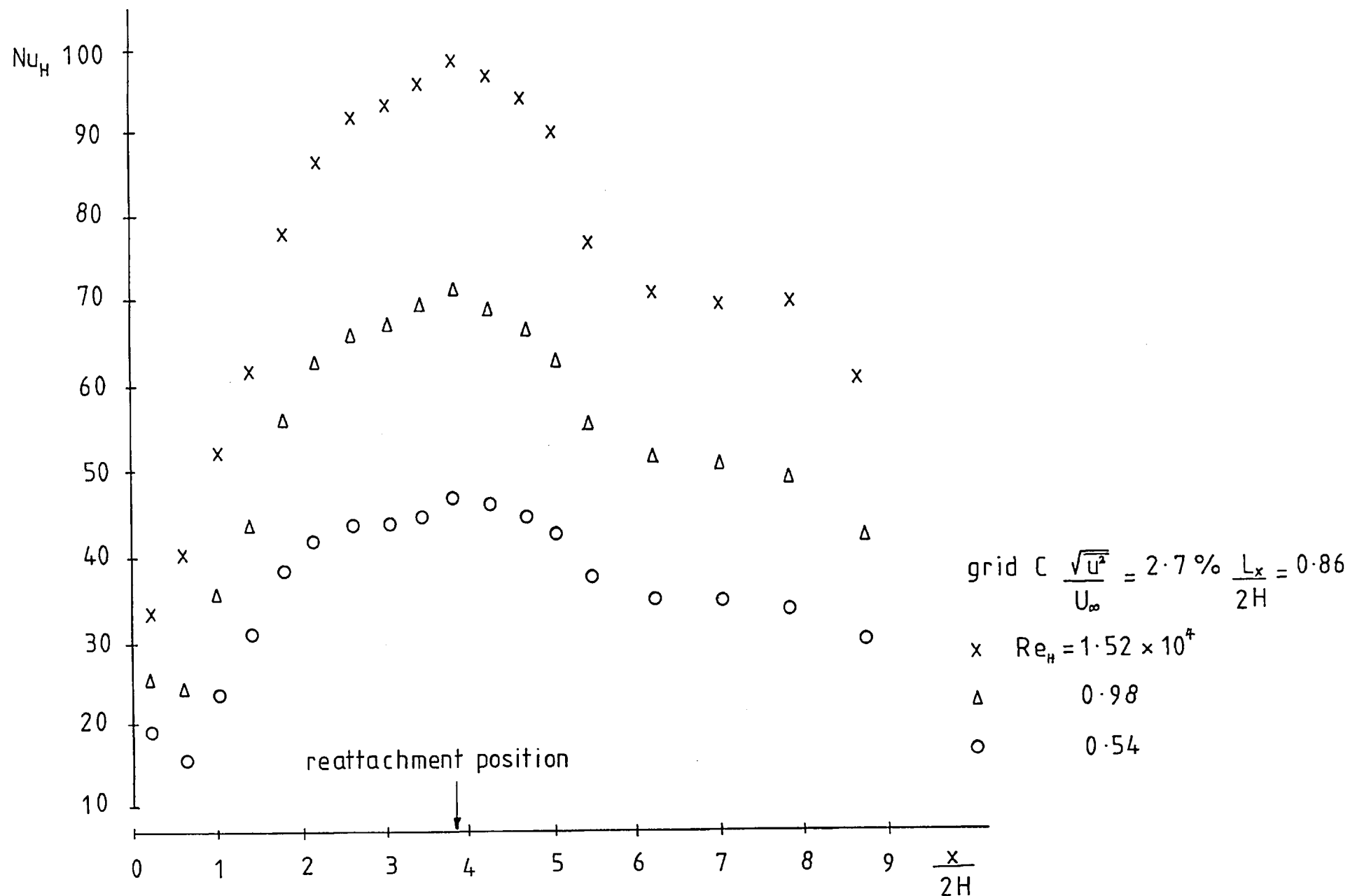


Figure 6.14 The effect of Reynolds number on the local Nu_H distributions

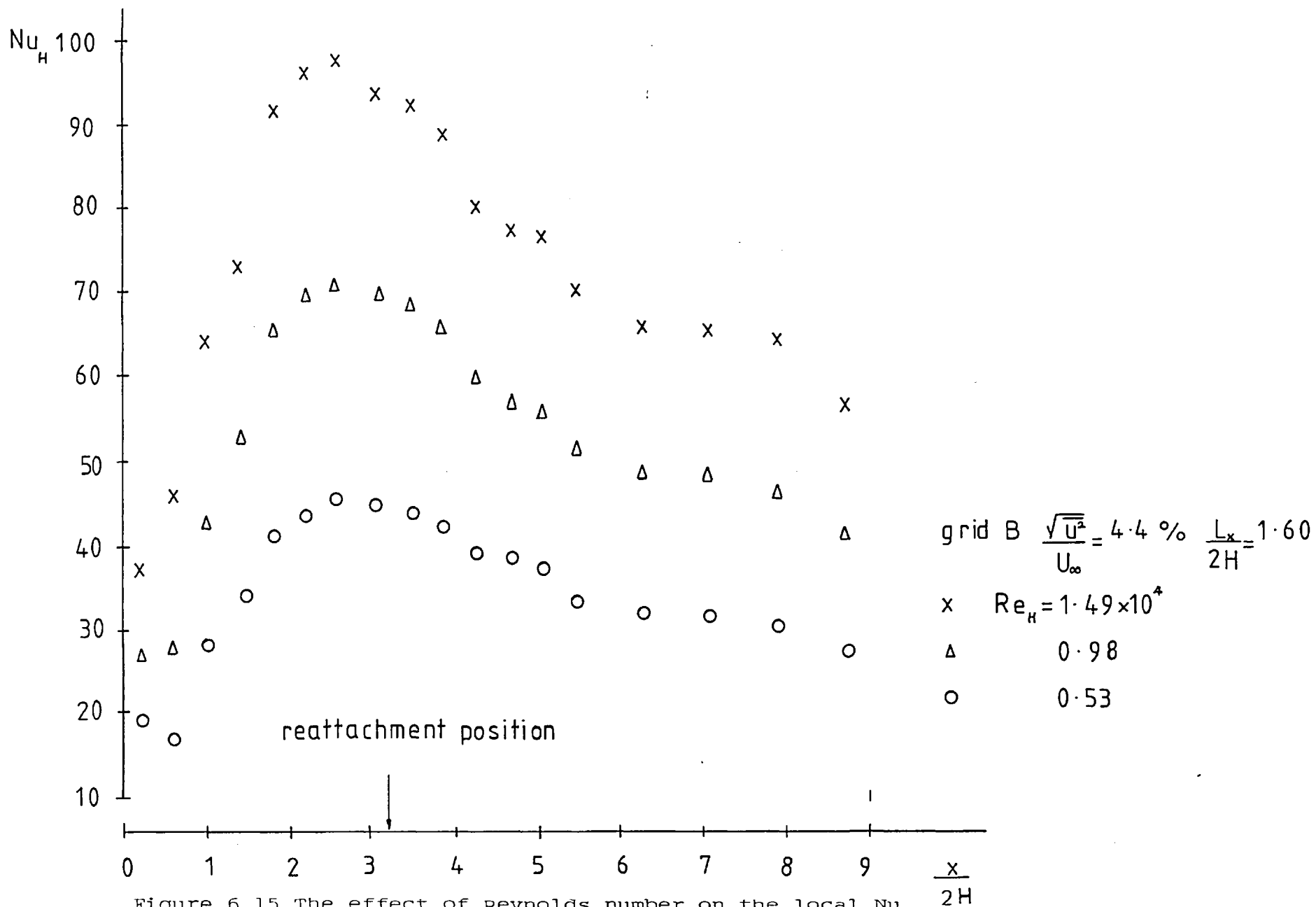


Figure 6.15 The effect of Reynolds number on the local Nu_H distributions

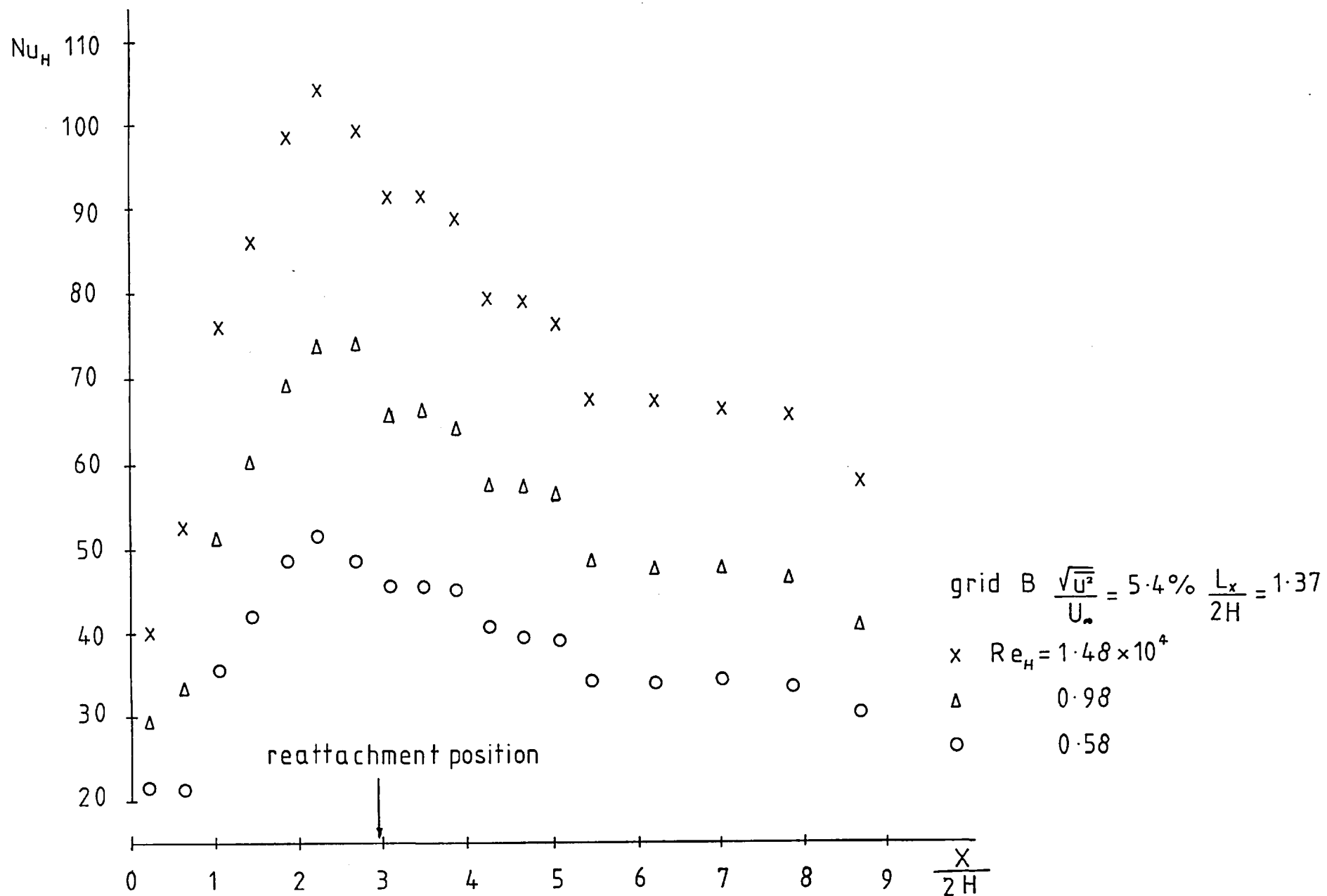


Figure 6.16 The effect of Reynolds number on the local Nu_H distributions

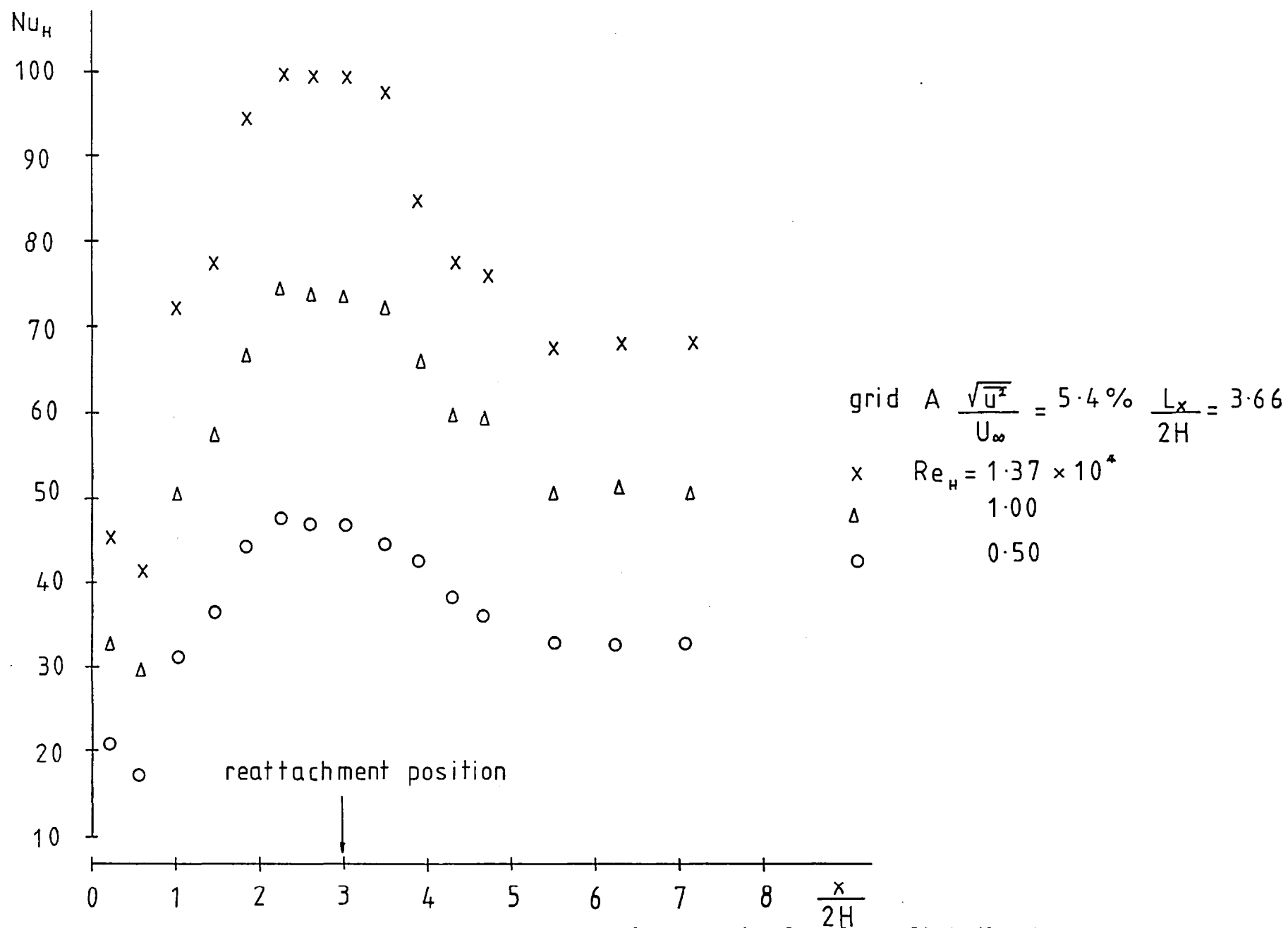


Figure 6.17 The effect of Reynolds number on the local Nu_H distributions

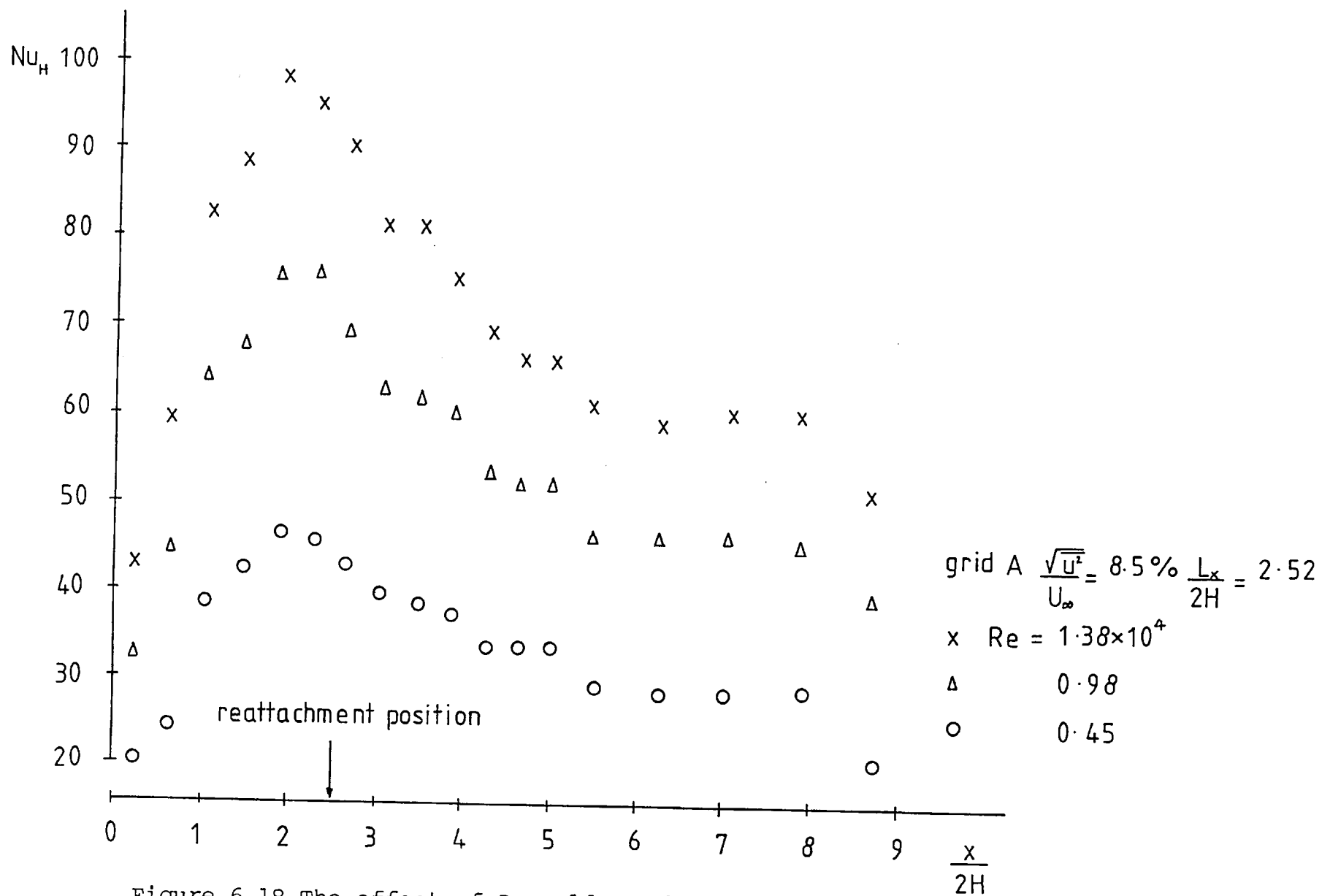


Figure 6.18 The effect of Reynolds number on the local Nu_H distributions

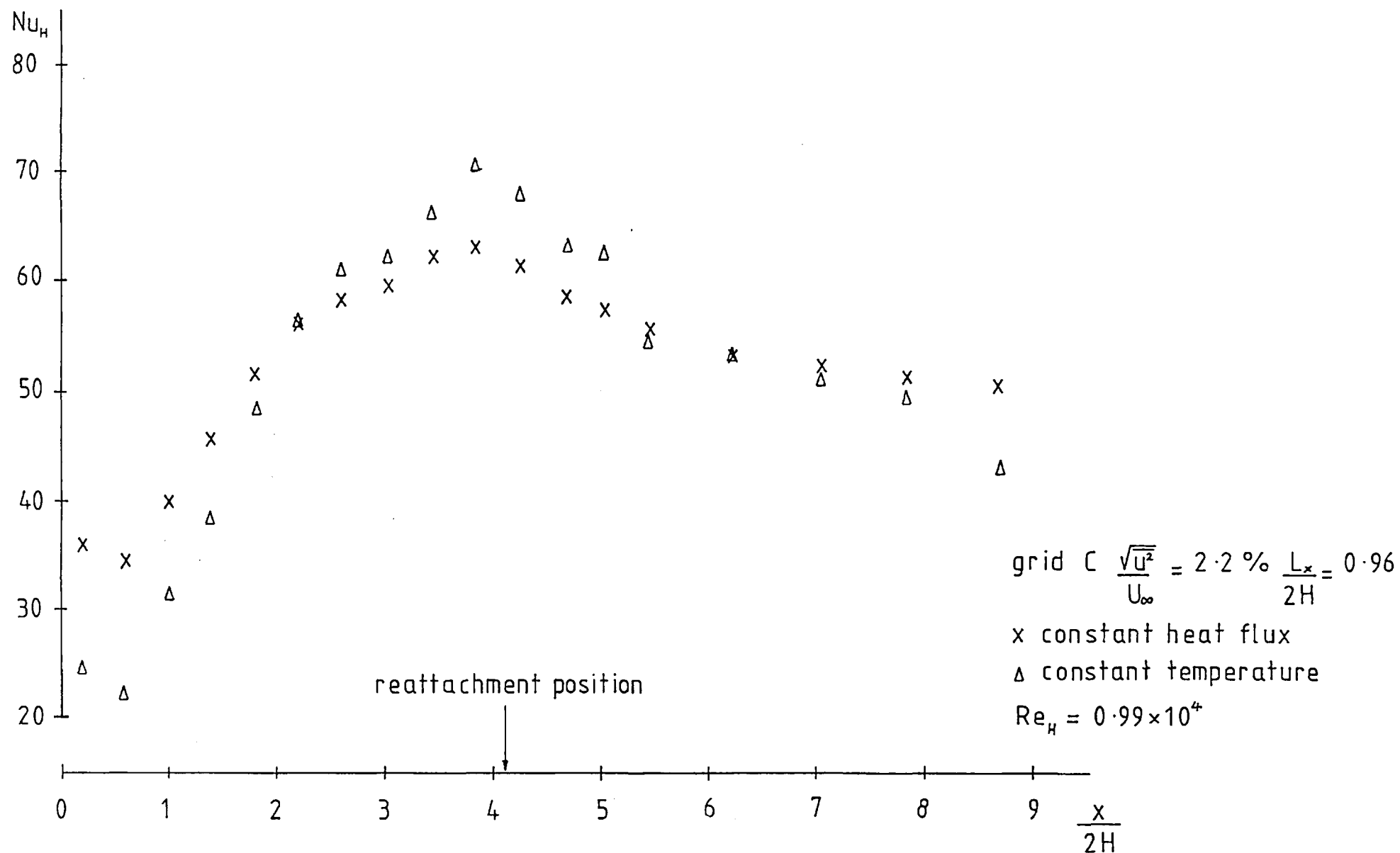


Figure 6.19 A comparison of the constant temperature and constant heat flux Nu_h distributions

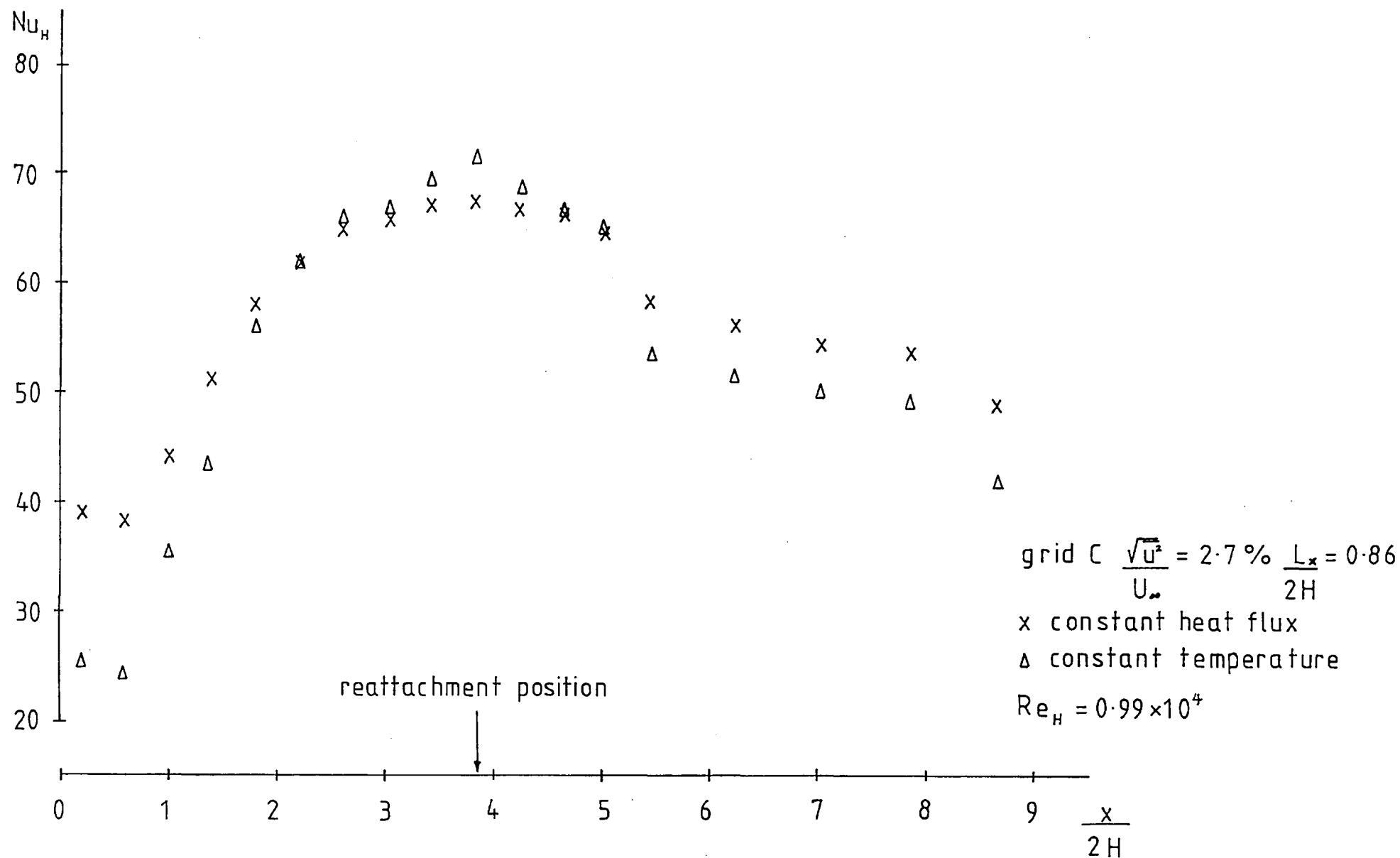


Figure 6.20 A comparison of the constant temperature and constant heat flux Nu_H distributions

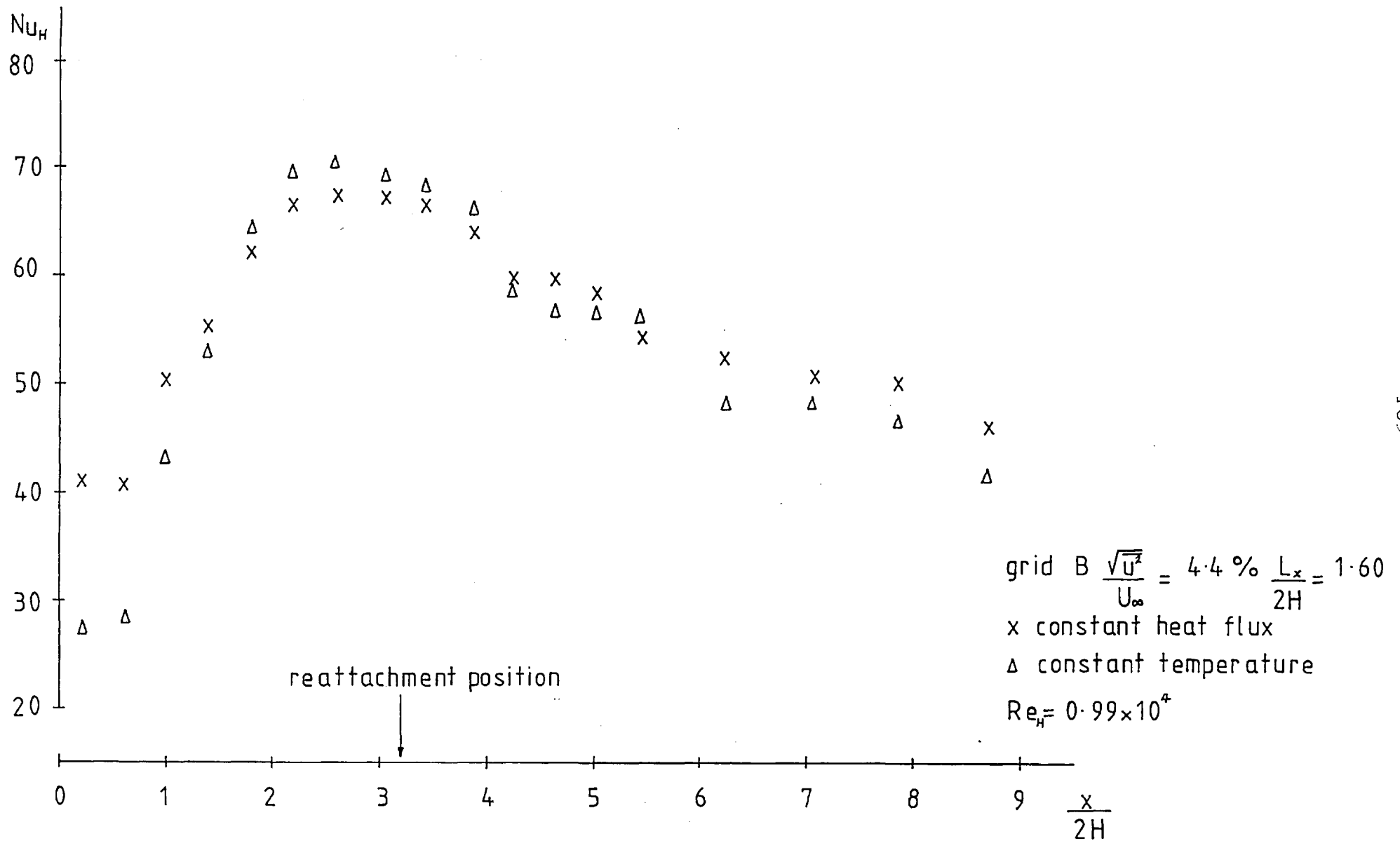


Figure 6.21 A comparison of the constant temperature and constant heat flux Nu_H distributions

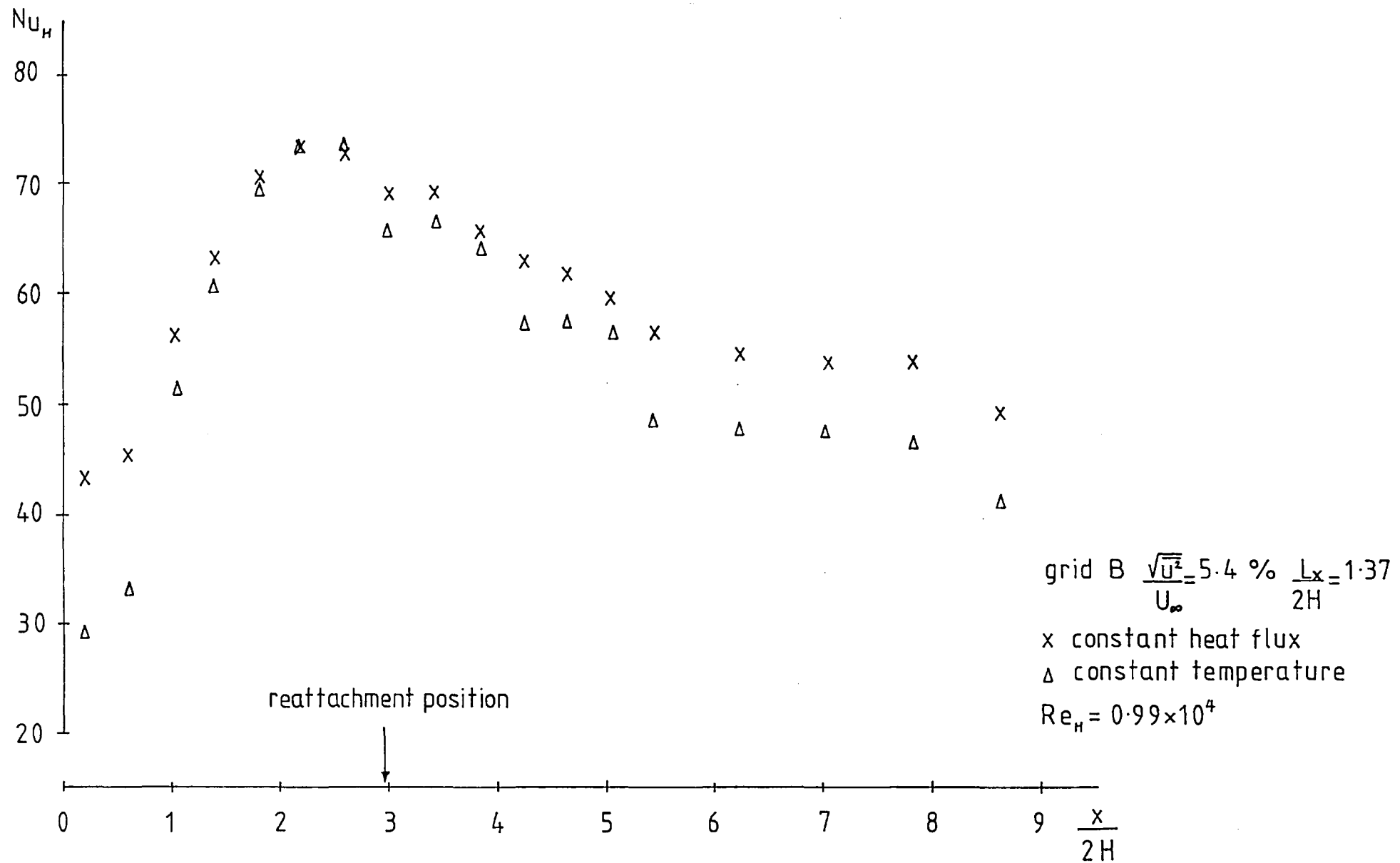


Figure 6.22 A comparison of the constant temperature and constant heat flux Nu_H distributions

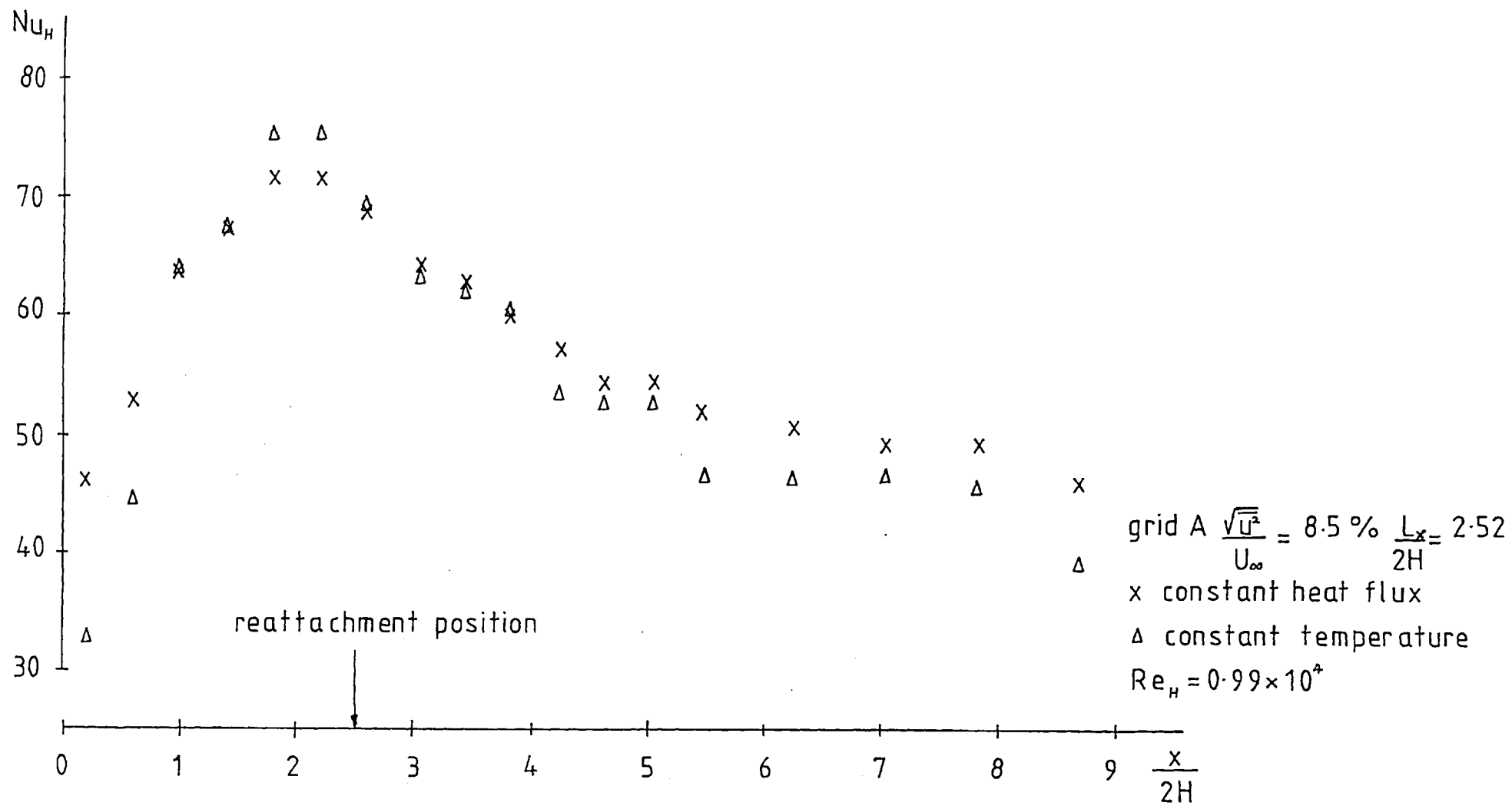
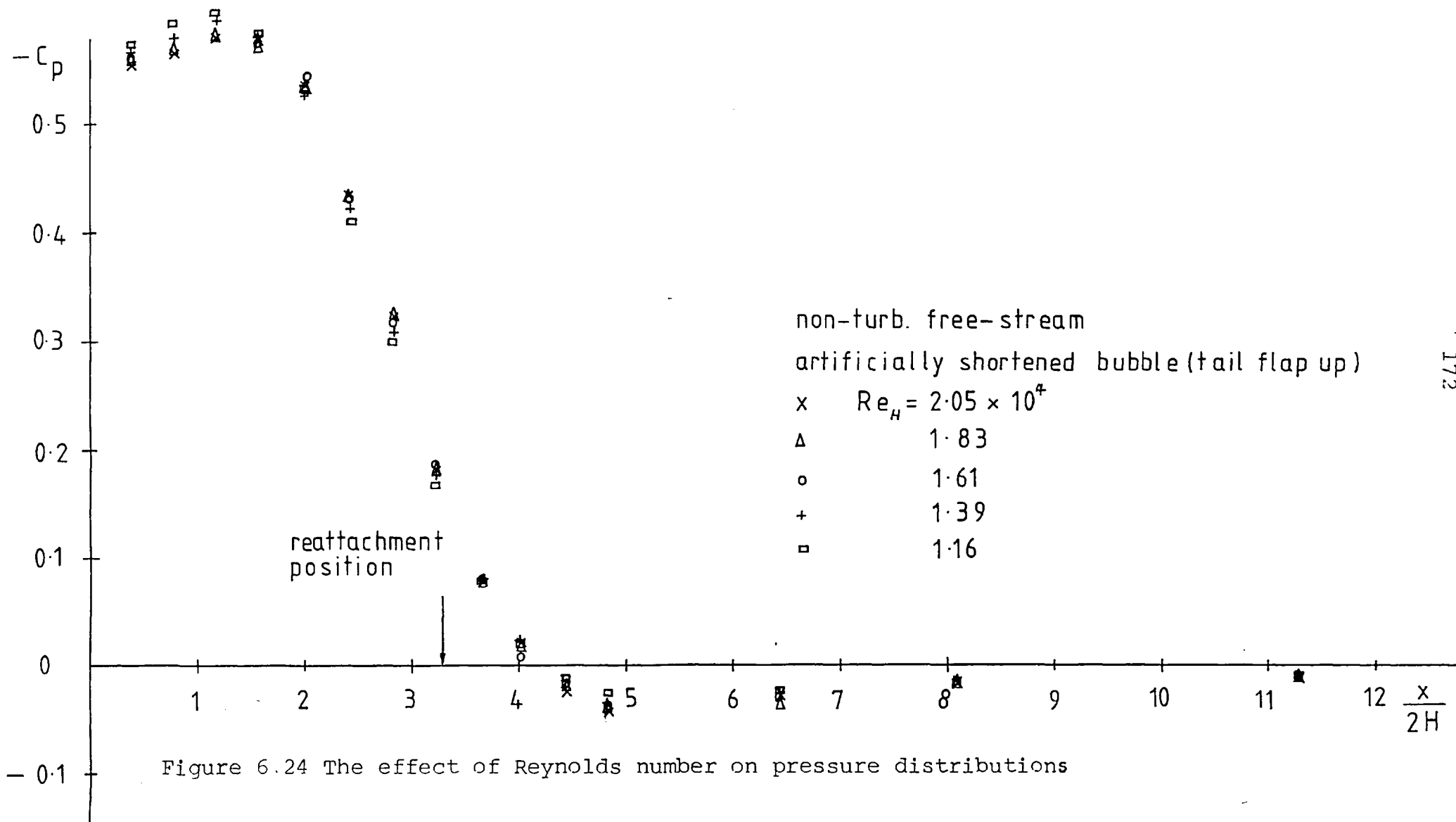


Figure 6.23 A comparison of the constant temperature and constant heat flux Nu_H distributions



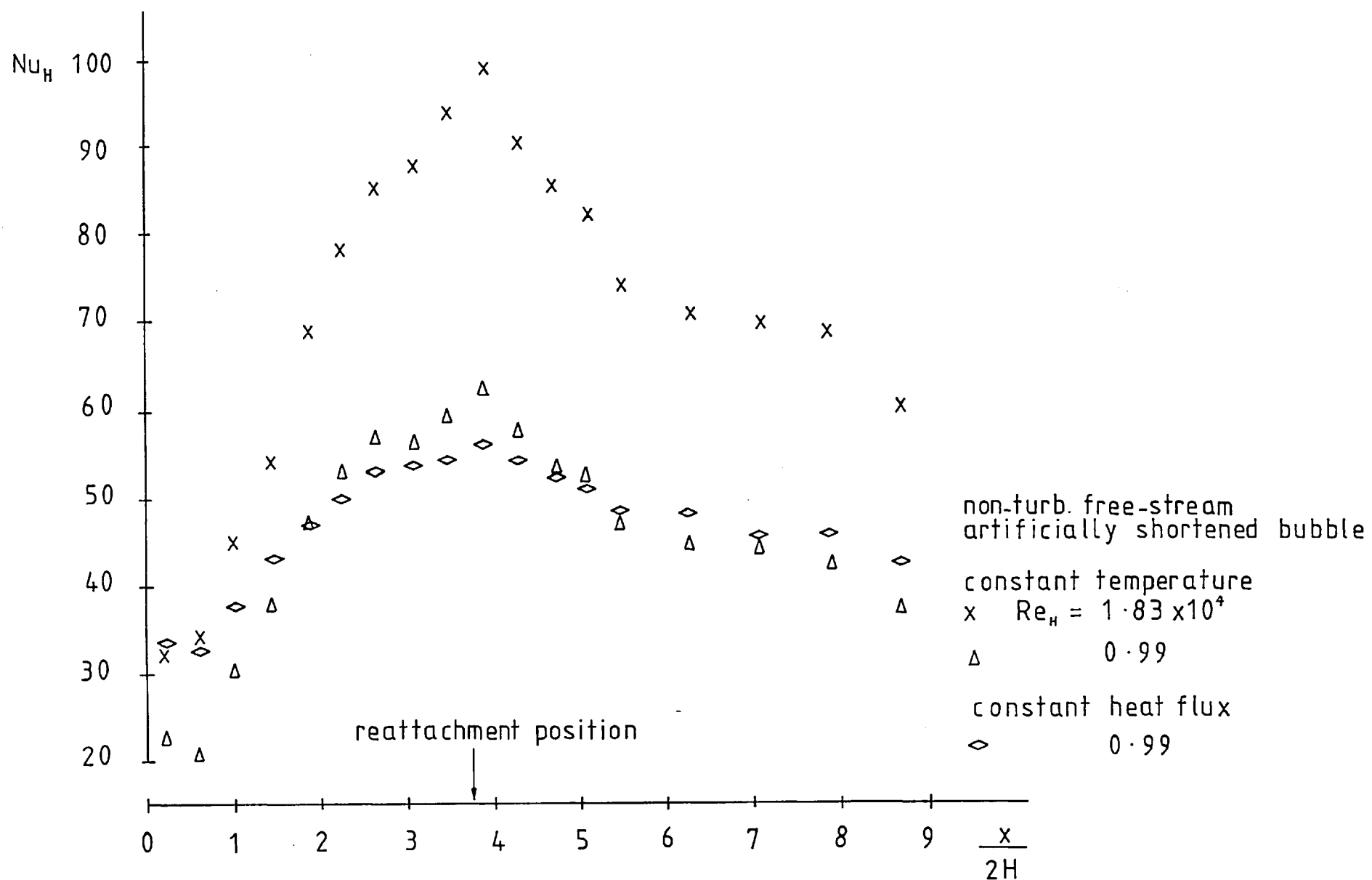
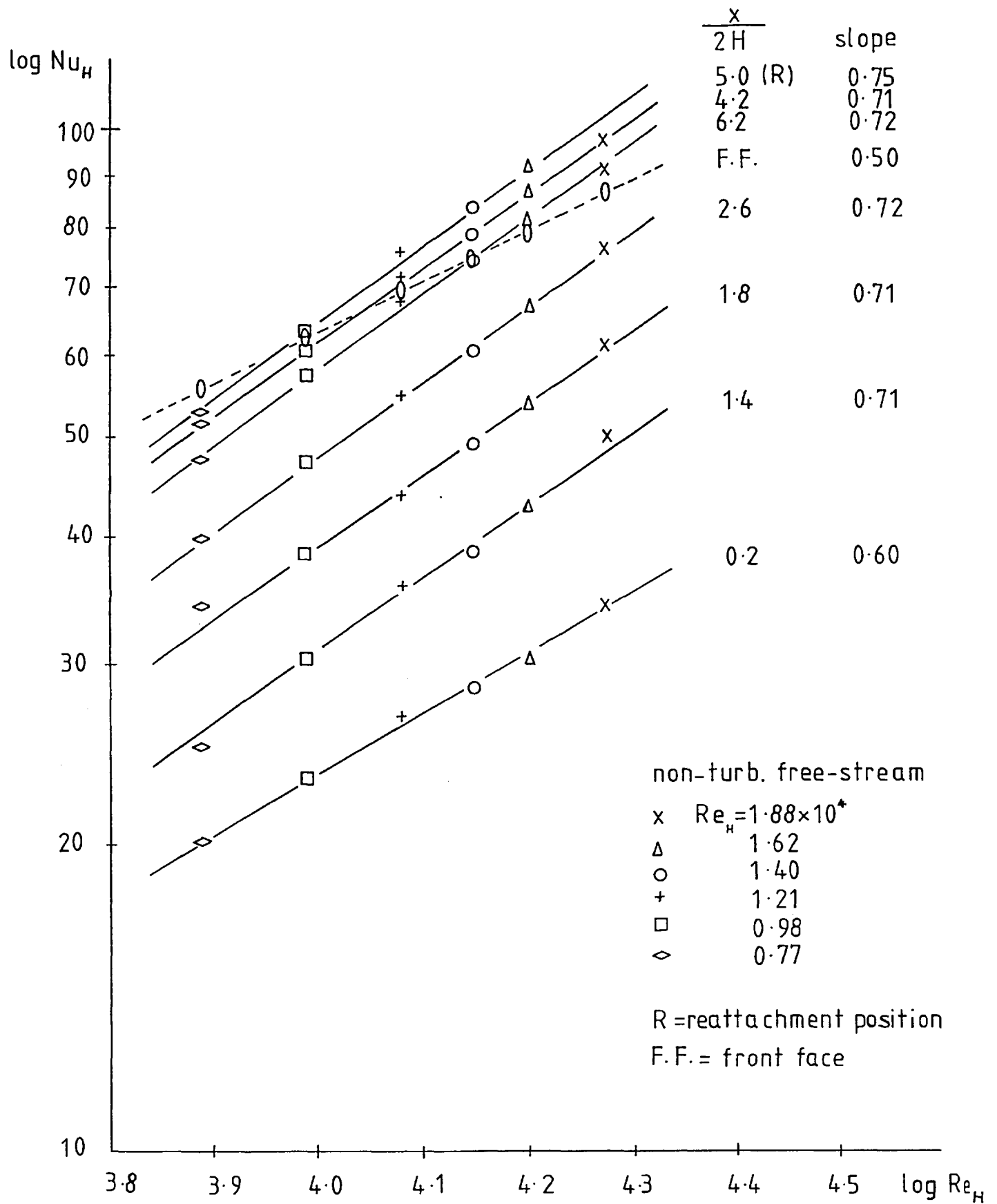
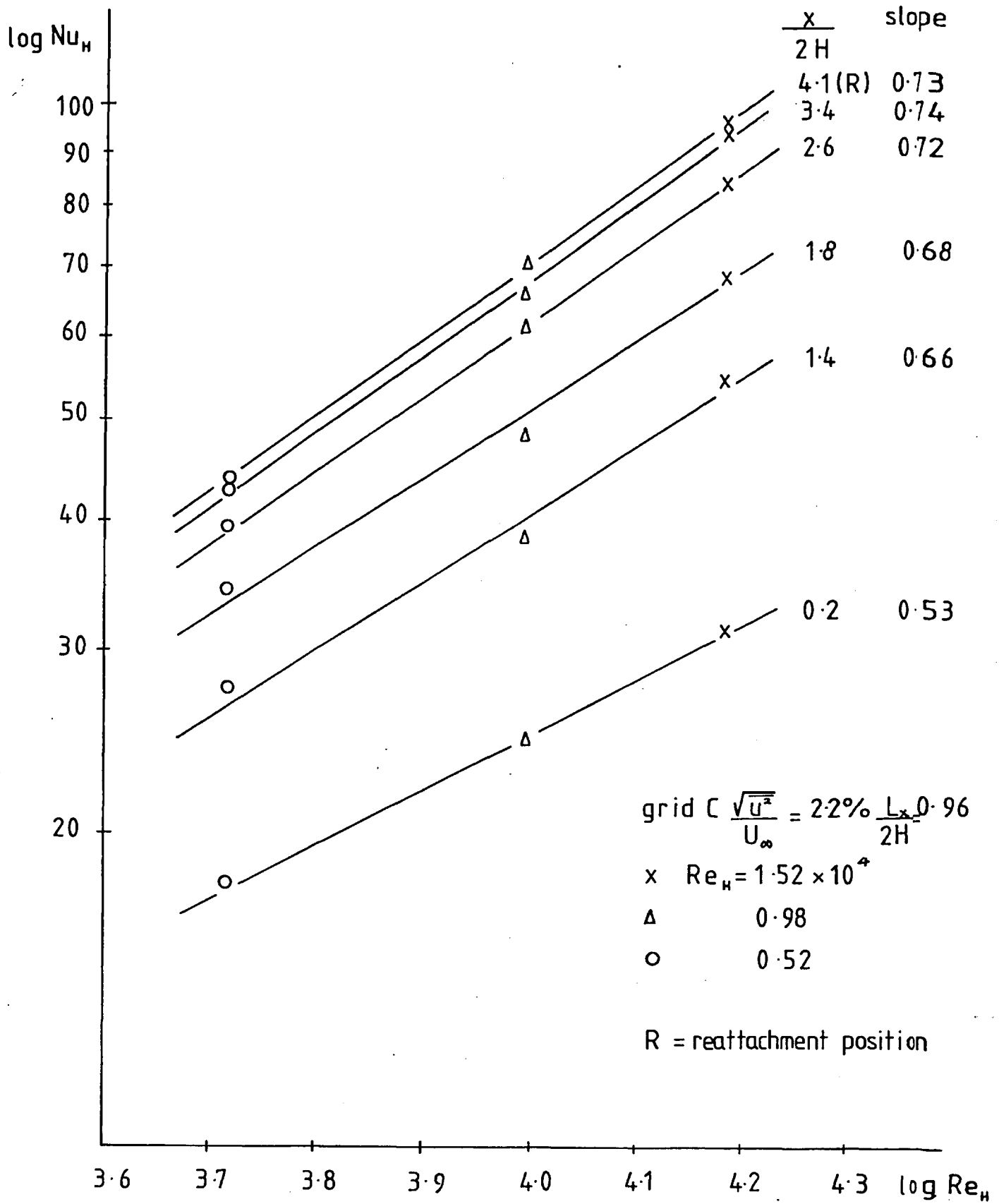
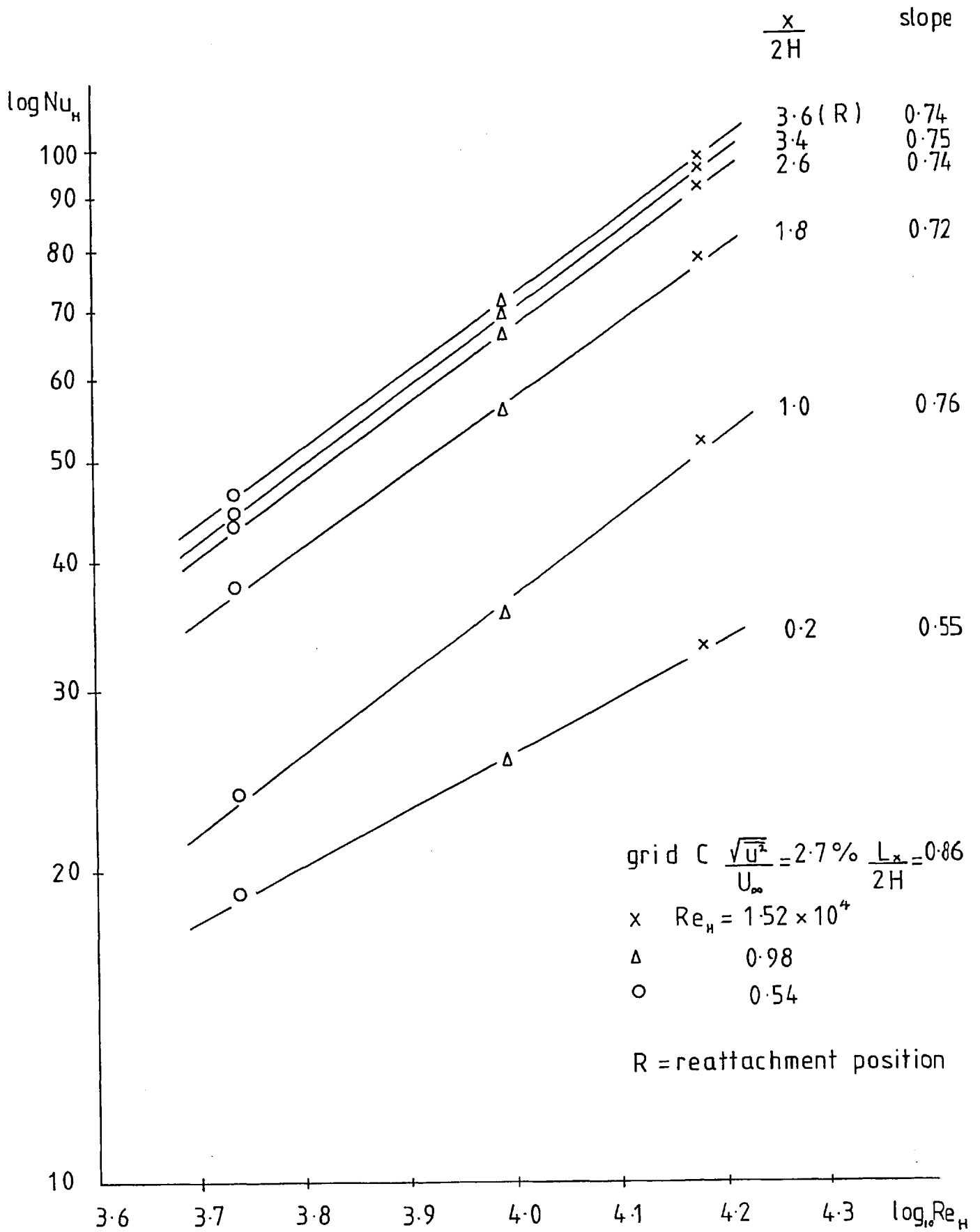
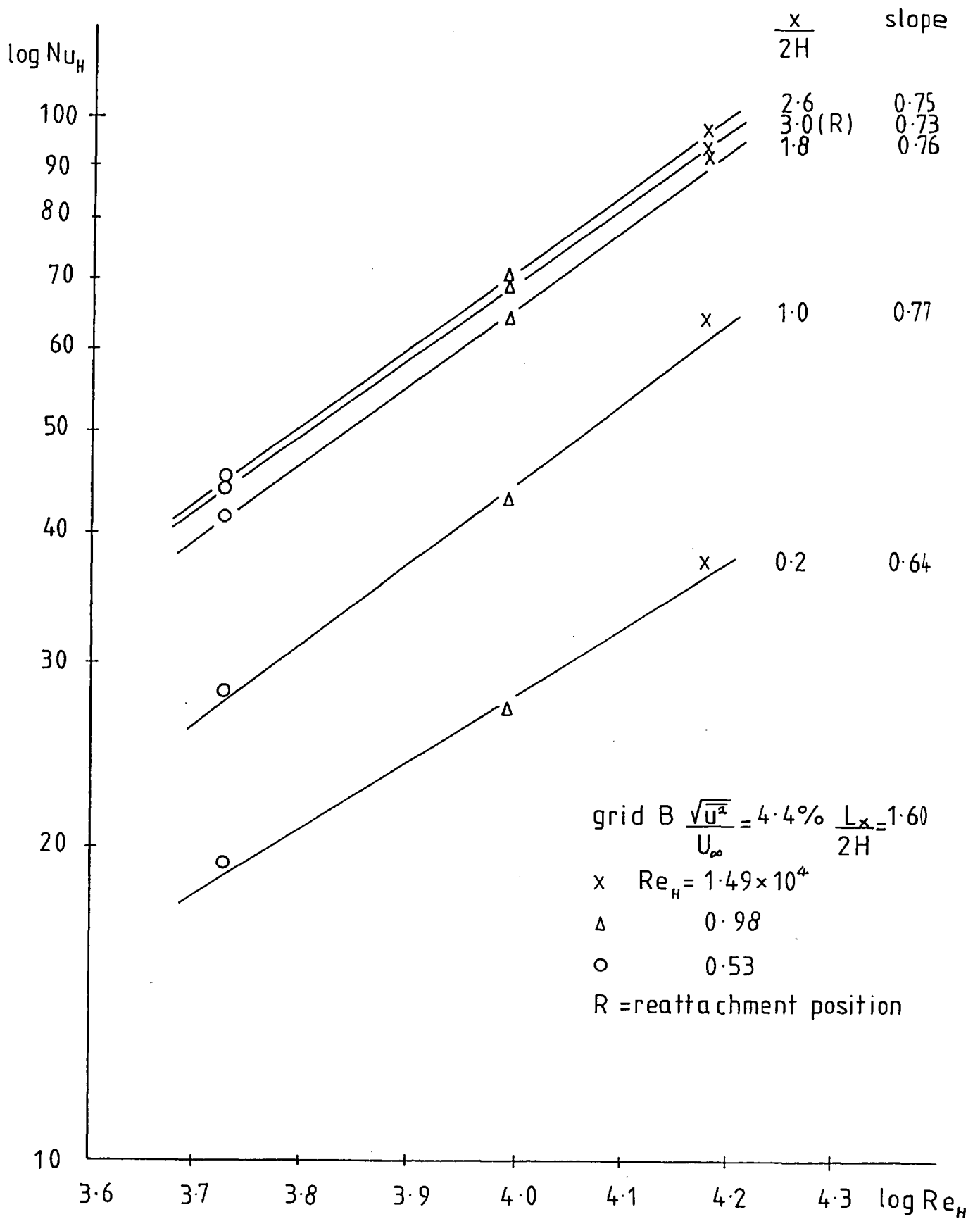


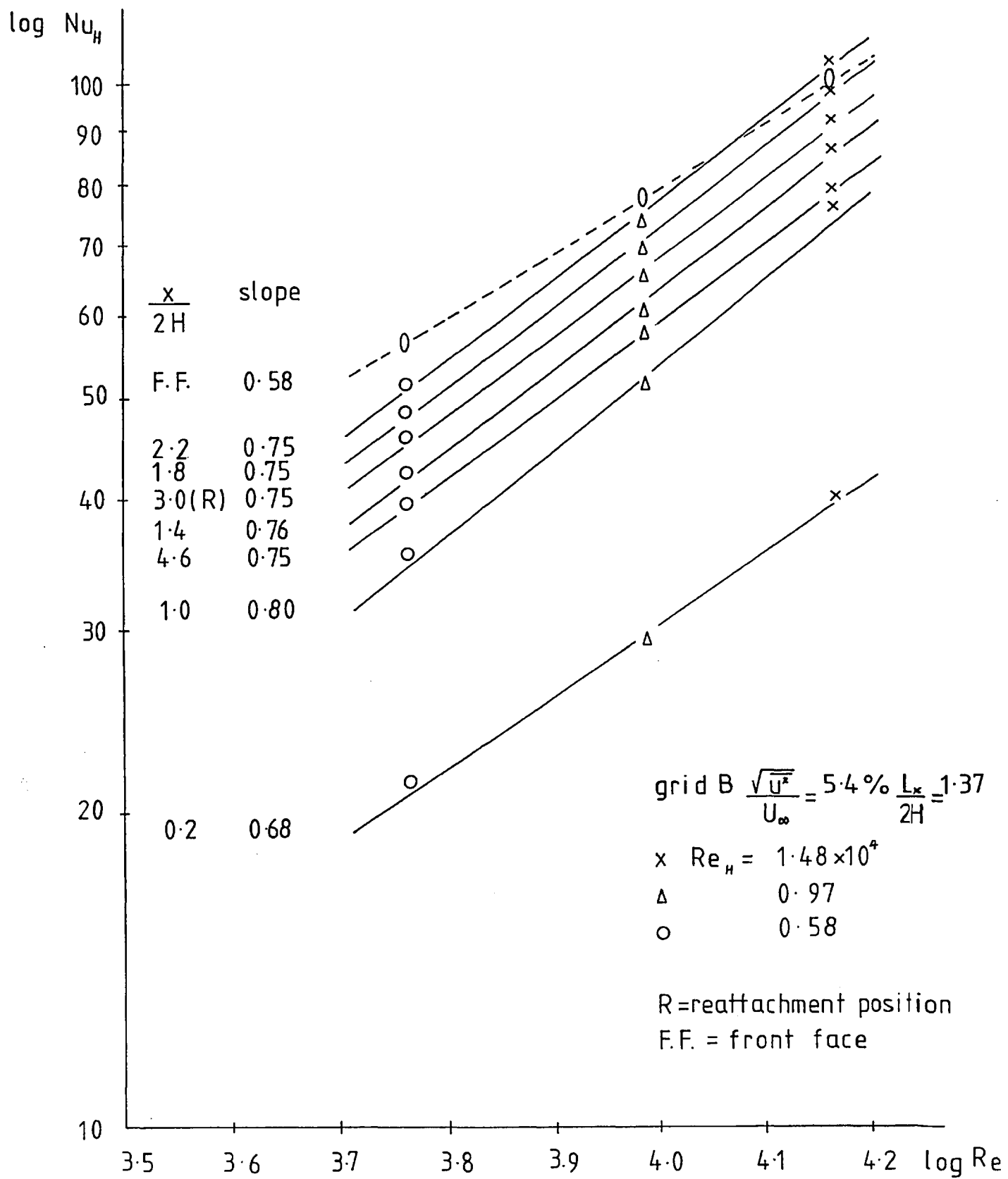
Figure 6.25 The effect of Reynolds number on the local Nu_H distributions

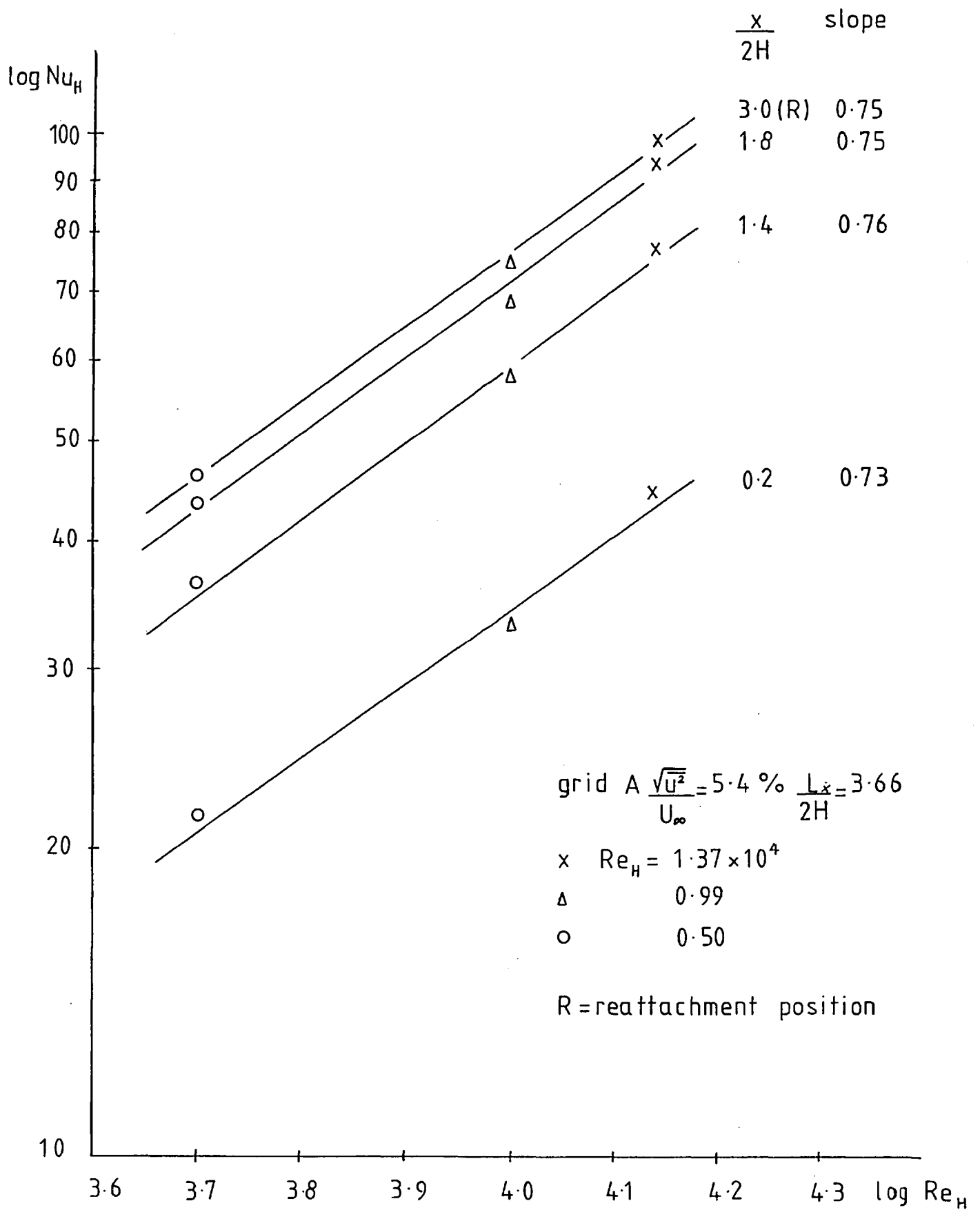
Figure 6.26 Log Nu_H versus log Re_H

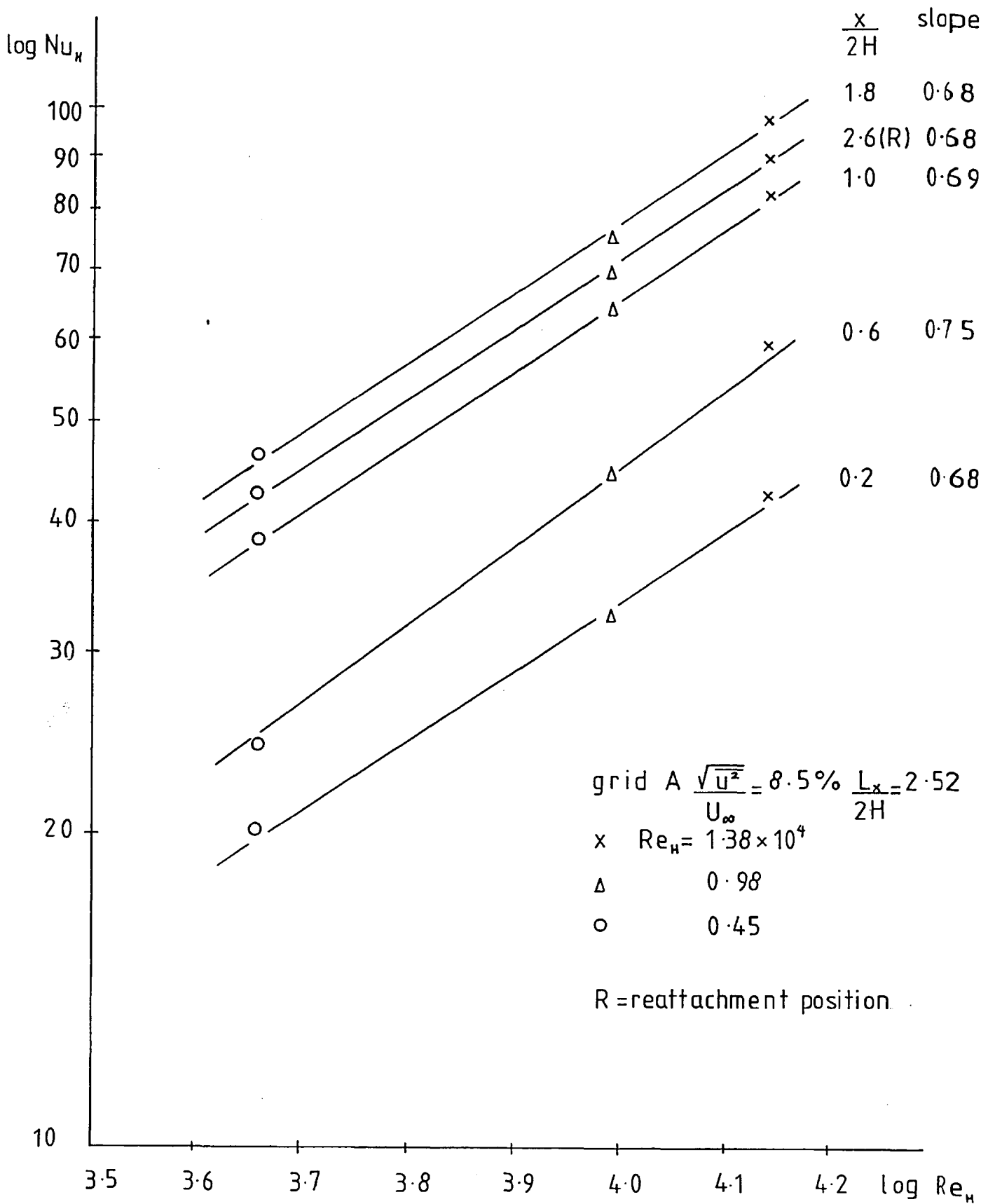
Figure 6.27 Log Nu_H versus log Re_H

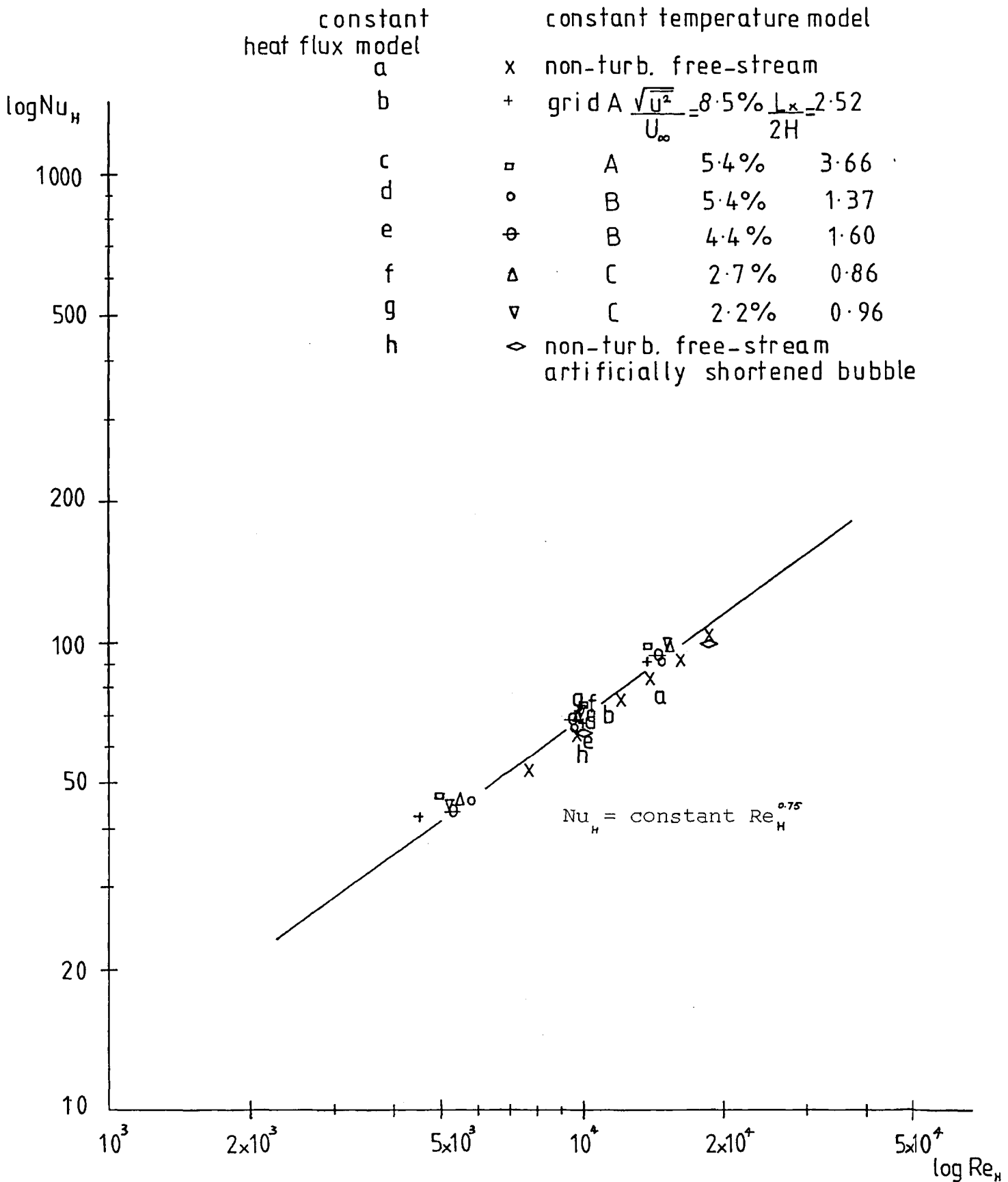
Figure 6.28 Log Nu_H versus log Re_H

Figure 6.29 Log Nu_H versus log Re_H

Figure 6.30 Log Nu_H versus log Re_H

Figure 6.31 Log Nu_H versus log Re_H

Figure 6.32 $\log Nu_h$ versus $\log Re_h$

Figure 6.33 The correlation of Nu_H and Re_H at reattachment

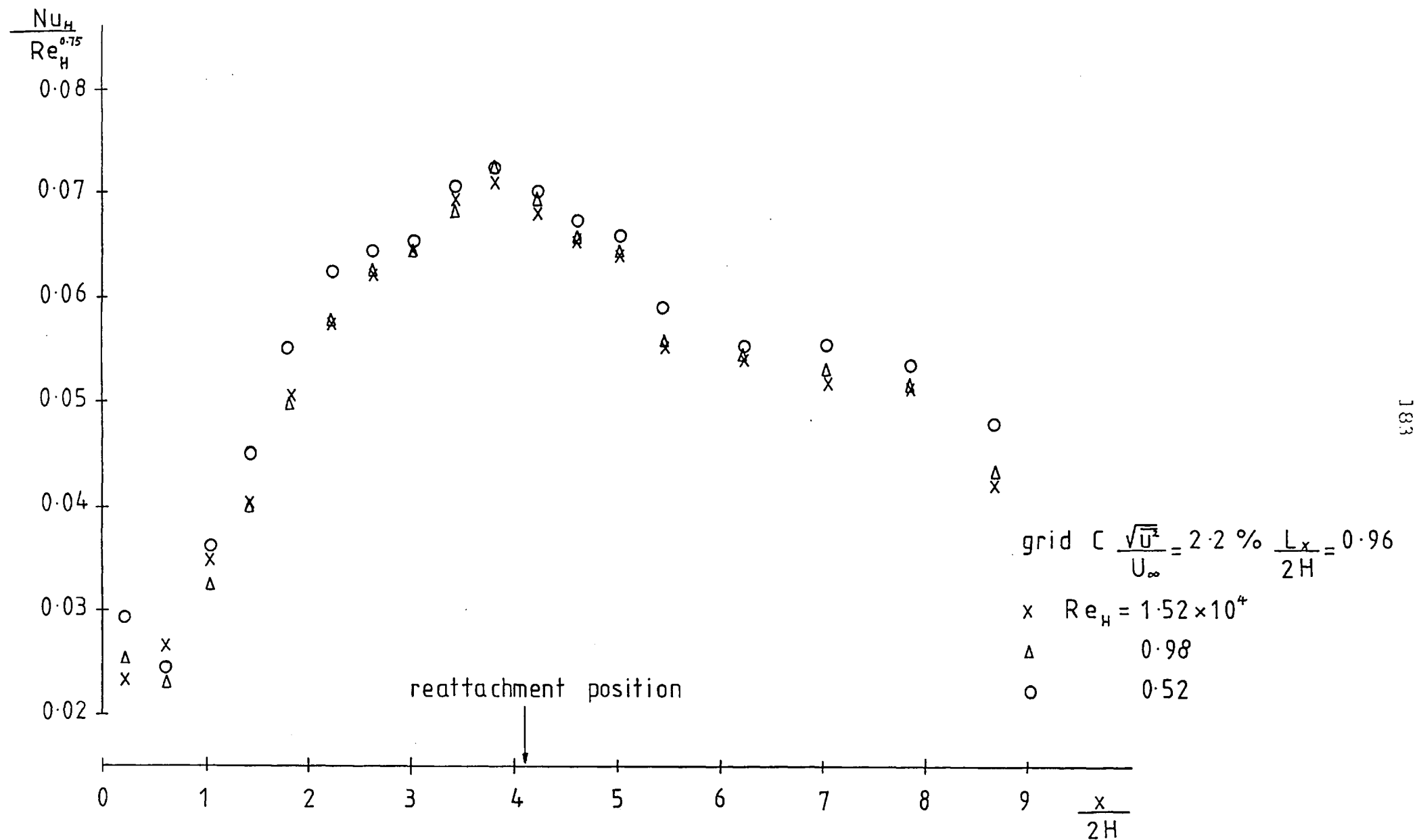


Figure 6.35 The effect of Reynolds number on the local $Nu_H / Re_H^{0.75}$ distributions

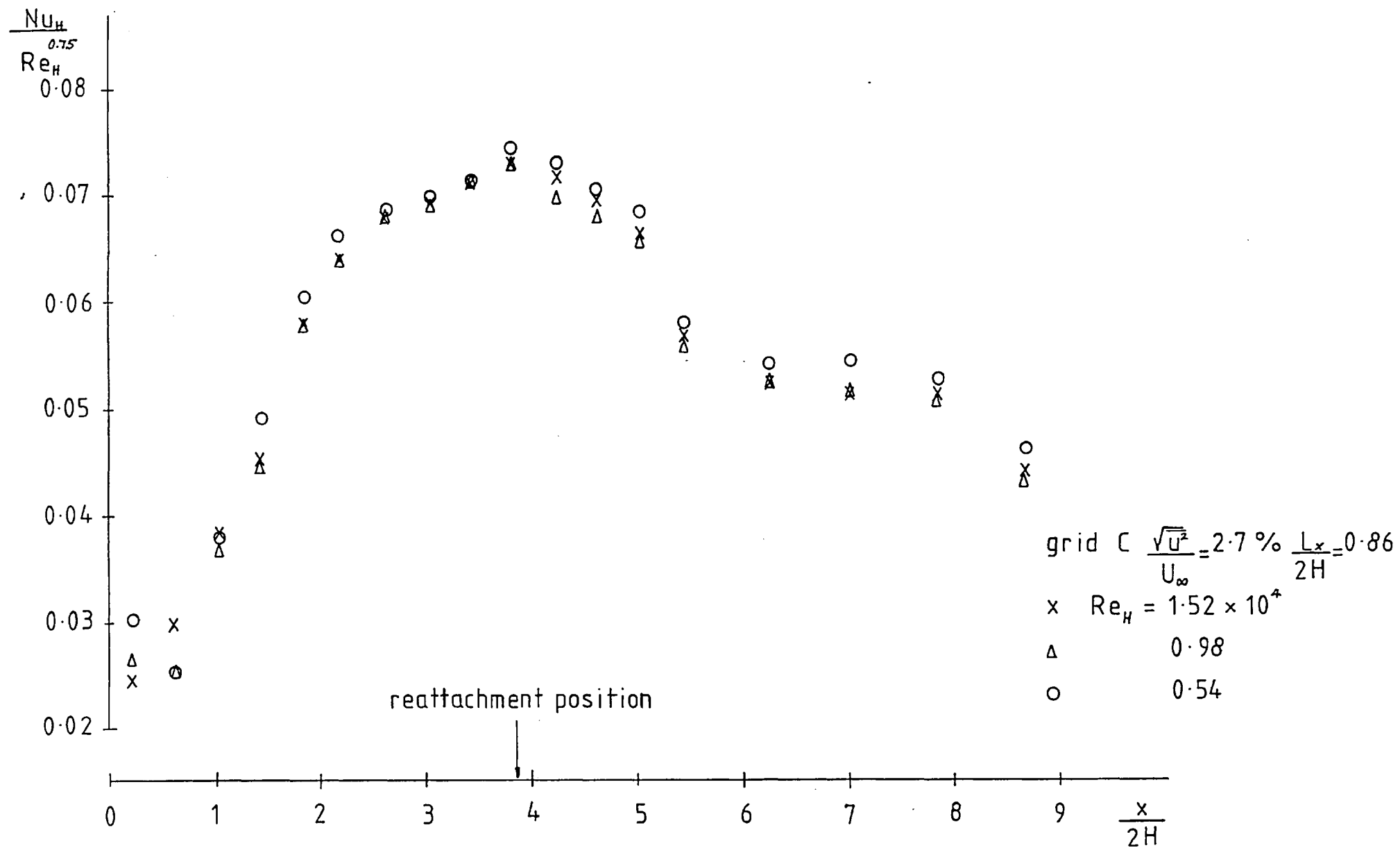


Figure 6.36 The effect of Reynolds number on the local $Nu_H/Re_H^{0.75}$ distributions

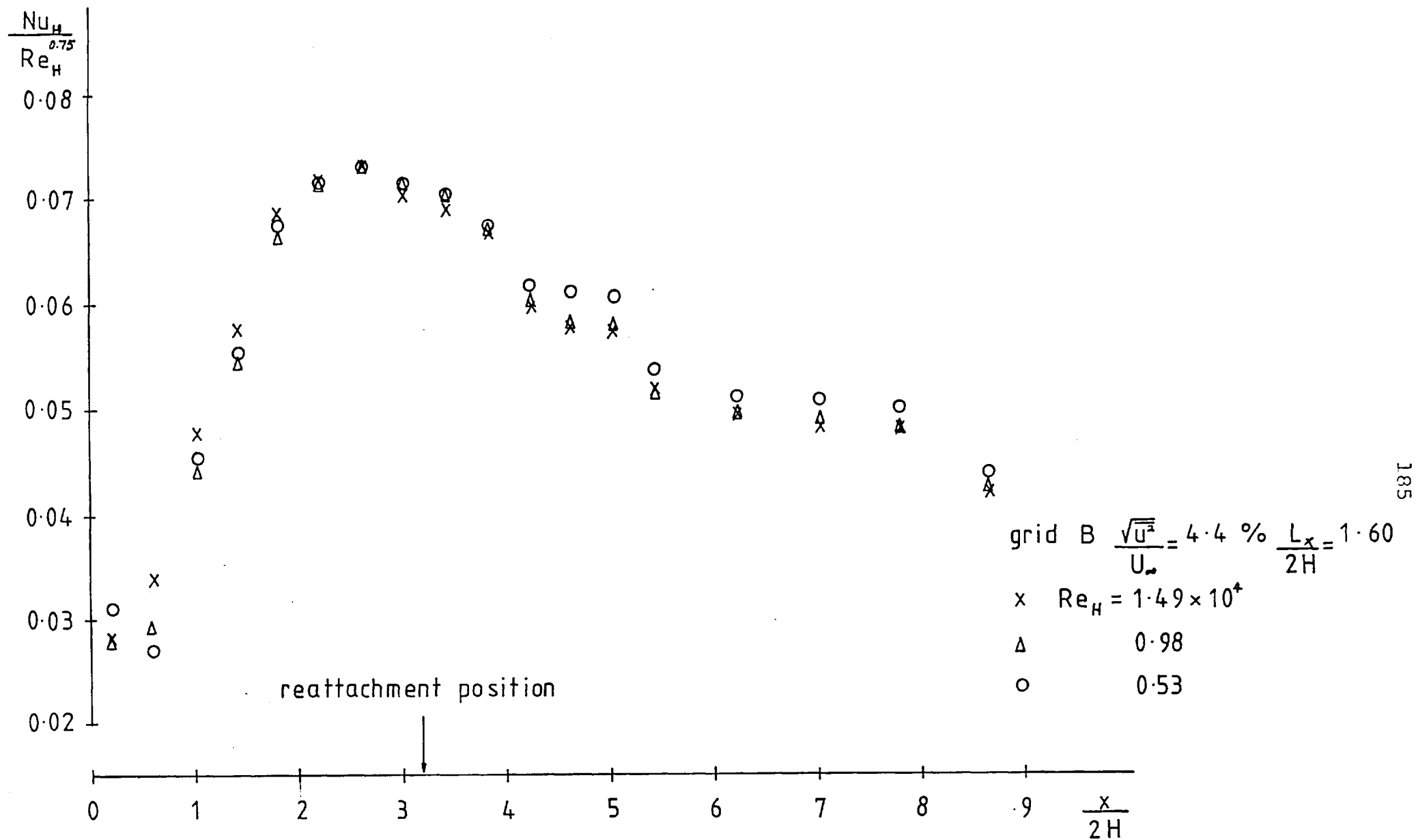


Figure 6.37 The effect of Reynolds number on the local $Nu_H/Re_H^{0.75}$ distributions

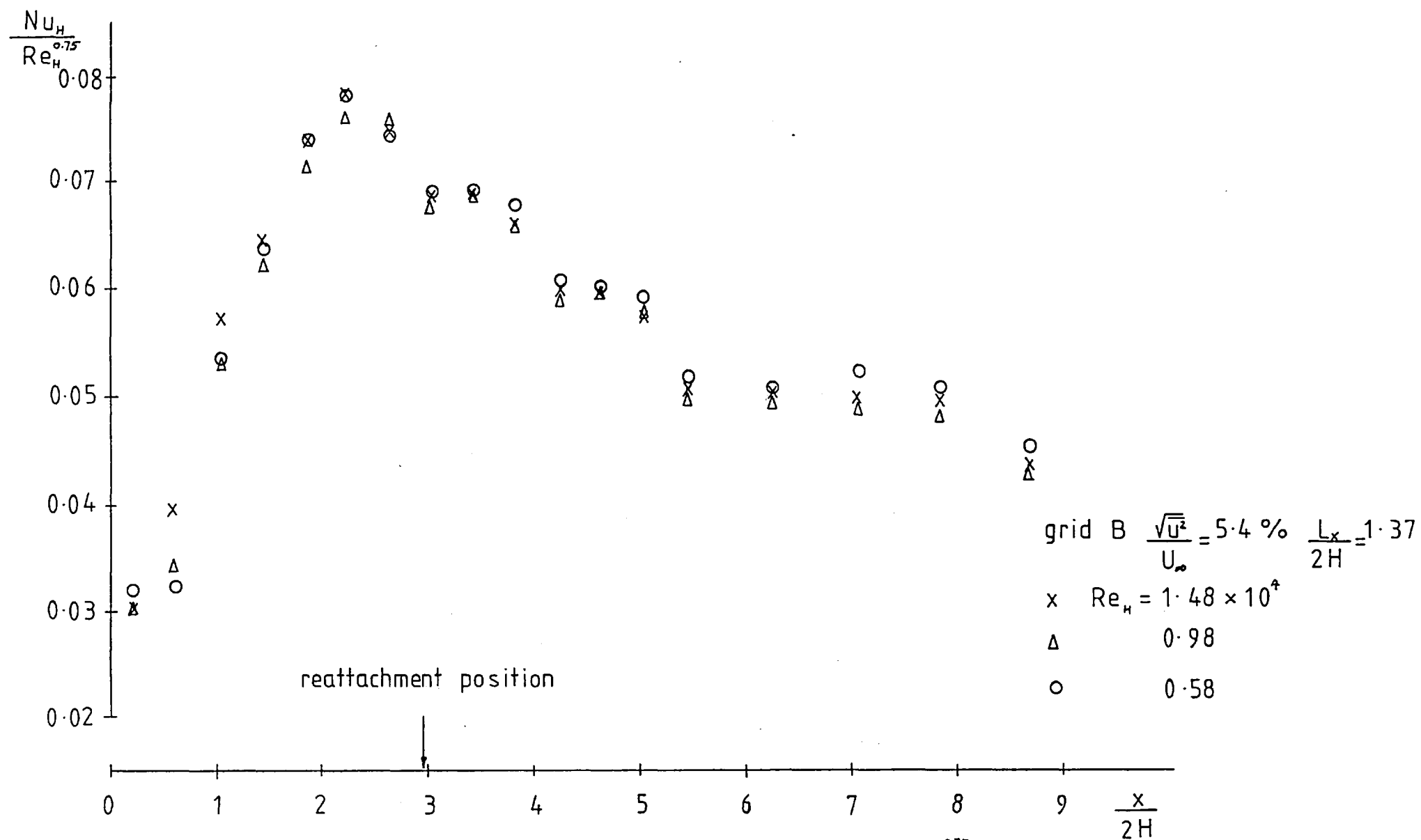


Figure 6.38 The effect of Reynolds number on the local $Nu_H/Re_H^{0.75}$ distributions

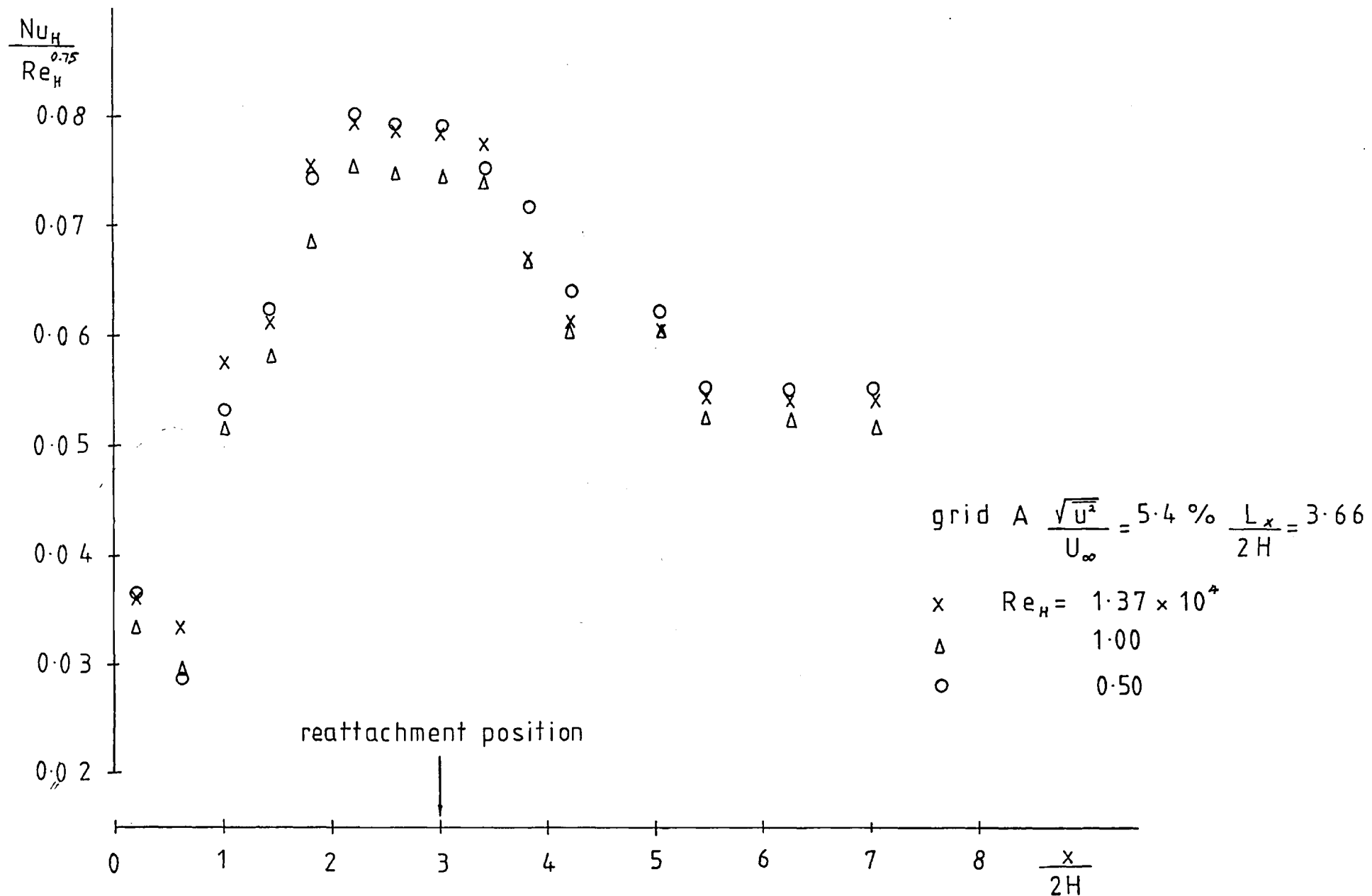


Figure 6.39 The effect of Reynolds number on the local $Nu_H/Re_H^{0.75}$ distributions

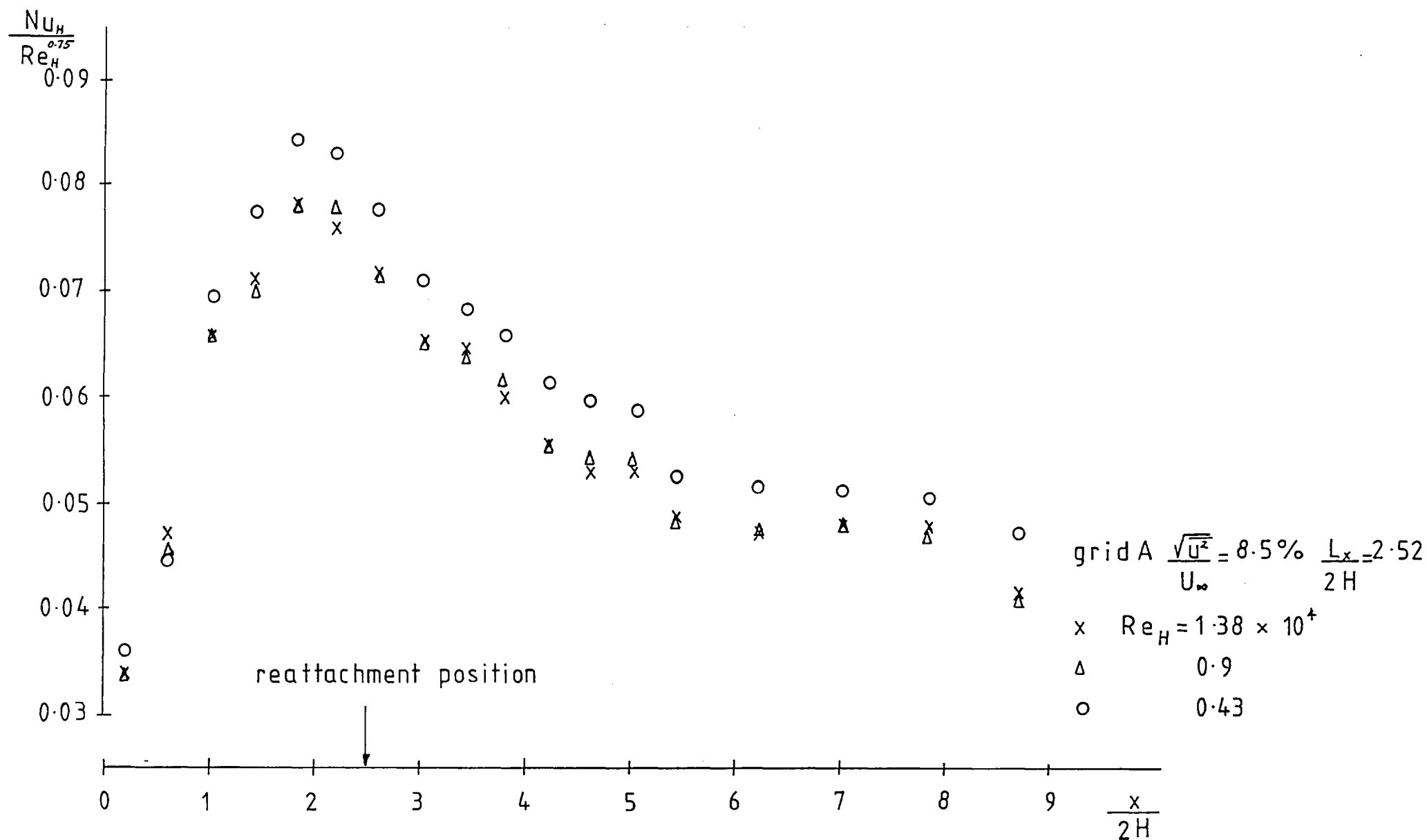


Figure 6.40 The effect of Reynolds number on the local $Nu_H / Re_H^{0.75}$ distributions

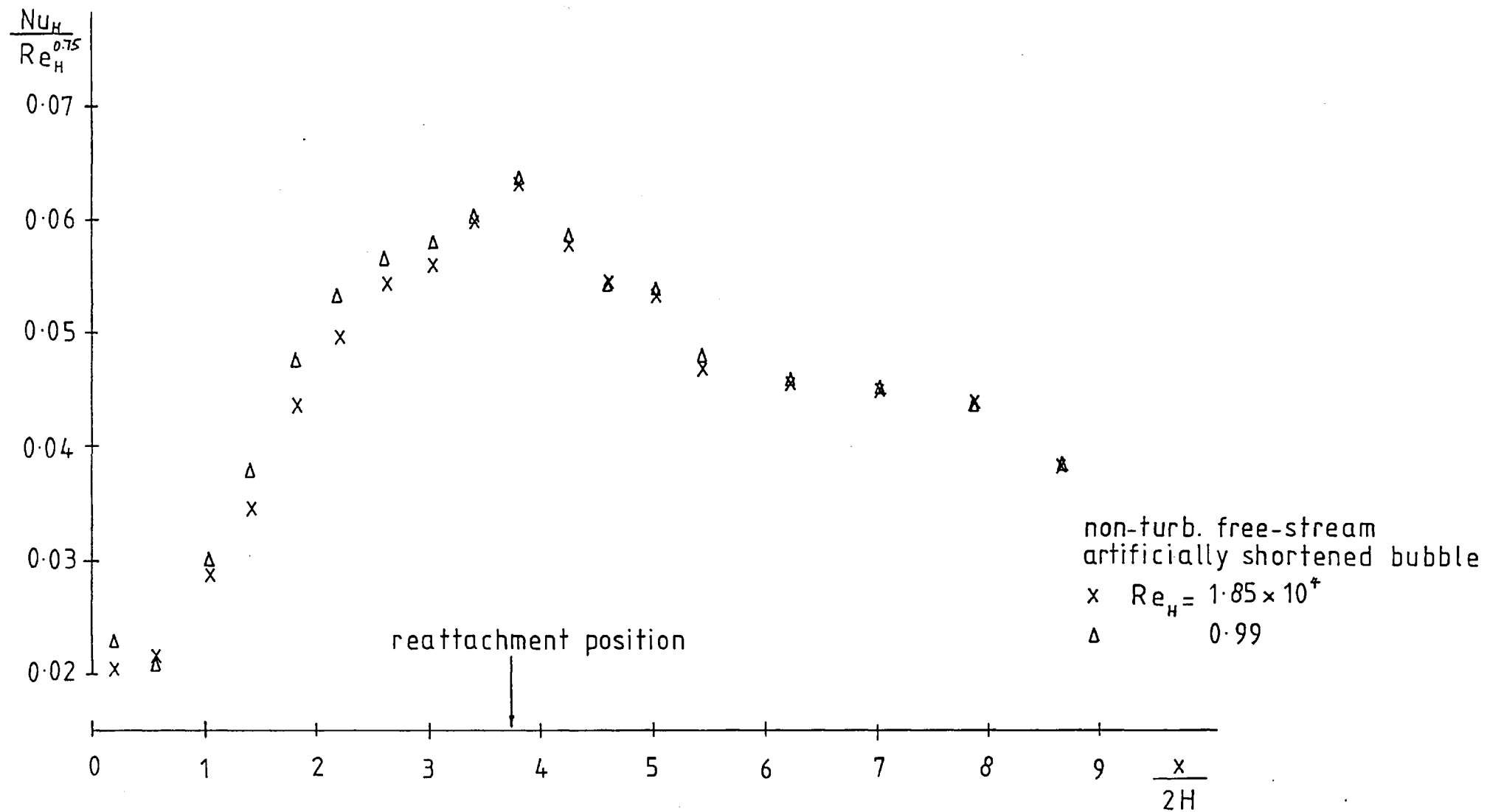


Figure 6.41 The effect of Reynolds number on the local $Nu_H/Re_H^{0.75}$ distributions

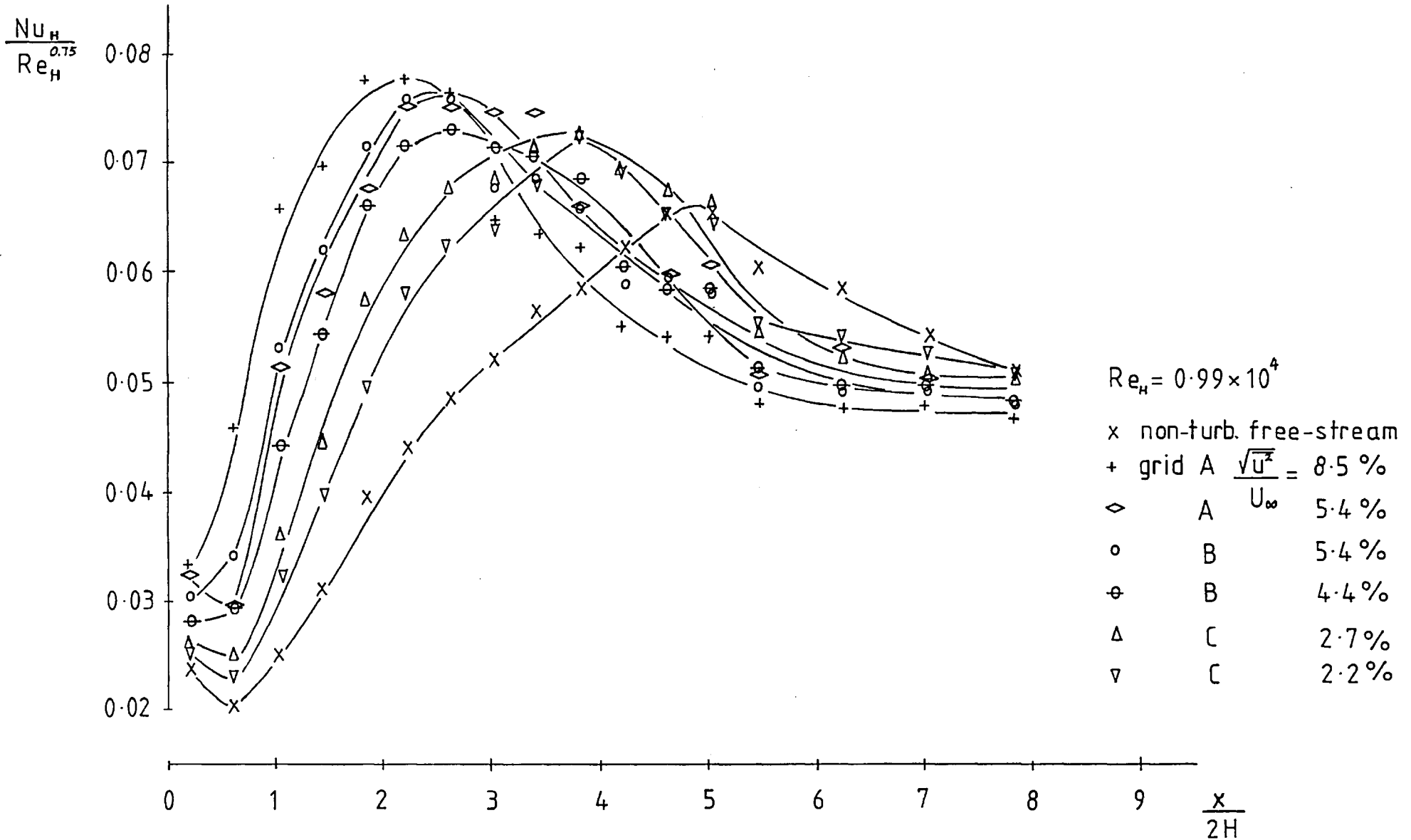
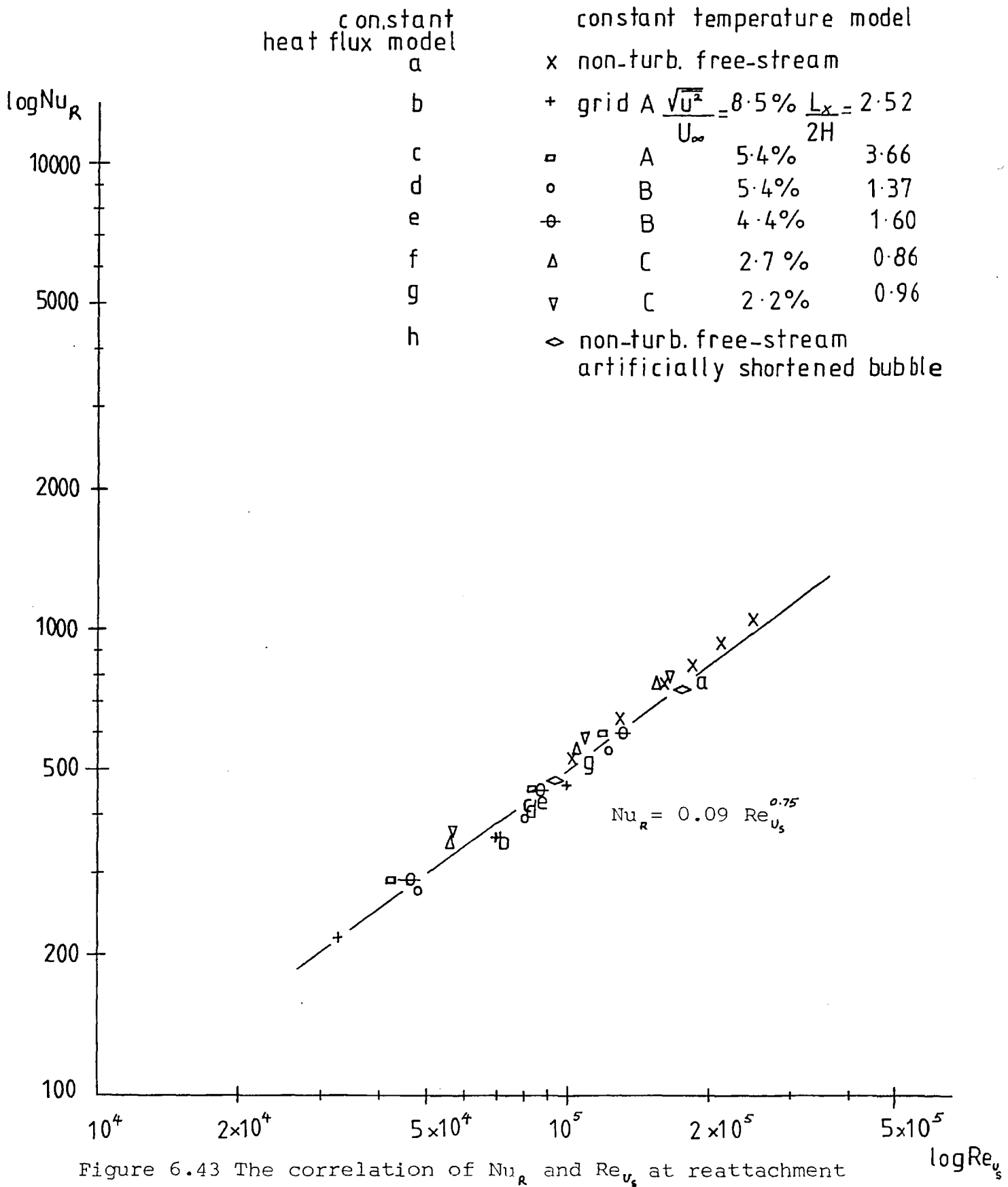


Figure 6.42 The local $Nu_H/Re_H^{0.75}$ distributions for turbulent and non-turbulent free-streams

Figure 6.43 The correlation of Nu_R and Re_{v_s} at reattachment

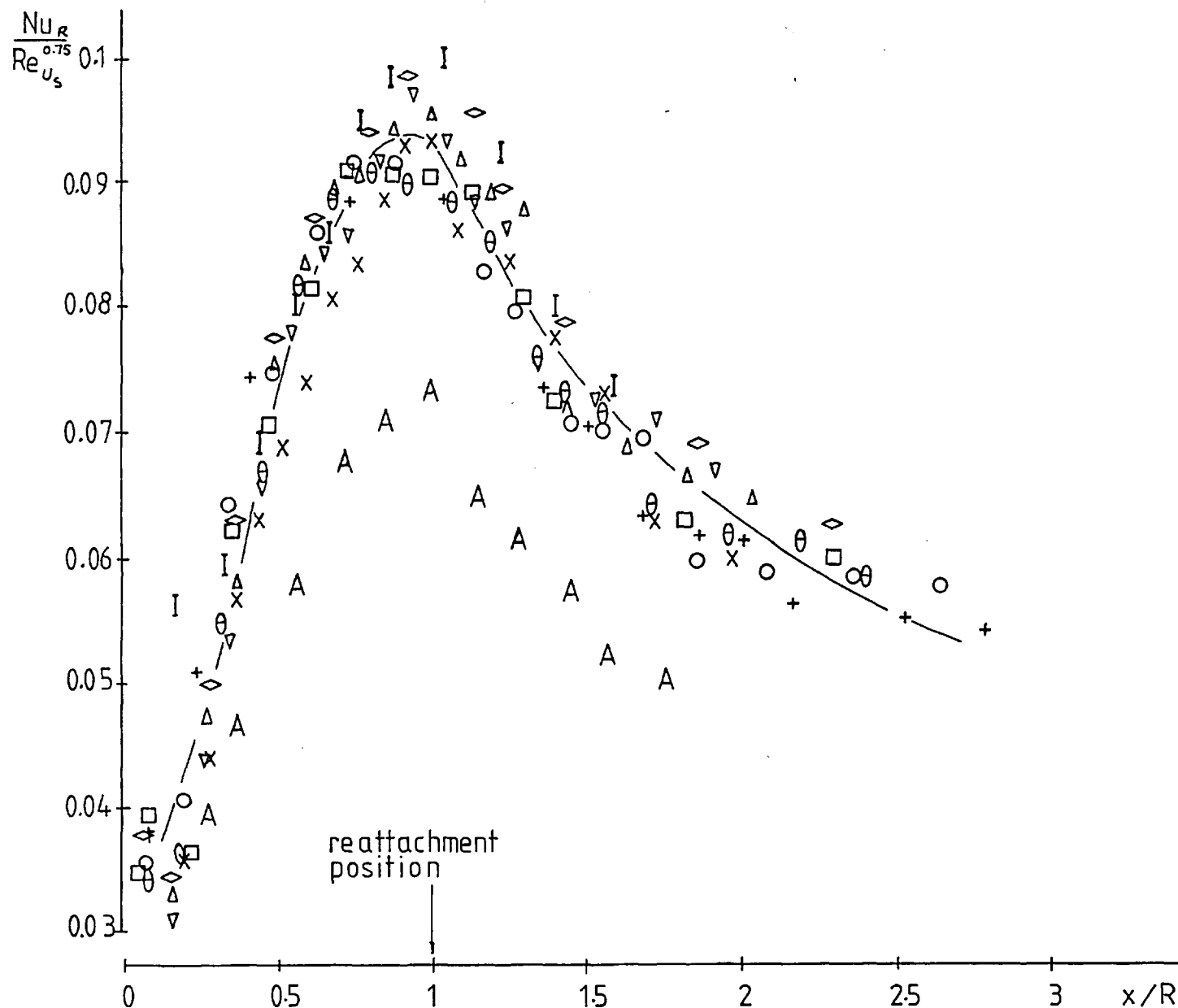


Figure 6.44 The local $Nu_R/Re_{u_s}^{0.75}$ distributions for turbulent and non-turbulent free-streams

$$Re_H = 0.99 \times 10^4$$

x non-turb. free-stream

+ grid A $\frac{\sqrt{u^*}}{U_\infty} = 8.5\%$ $\frac{L_x}{2H} = 2.52$

□ A 5.4% 2.66

○ B 5.4% 1.37

○ B 4.4% 1.60

△ C 2.7% 0.86

▽ C 2.2% 0.96

◇ non-turb. free-stream
arti. shortened bubble

I Ota & Kon⁴⁵ blunt flat plate
 $Nu_R = \text{const.} \cdot Re_{u_s}^{0.71}$ $Re_H = 1.24 \times 10^4$

A Ota & Kon⁴⁴ blunt circular cylinder
 $Nu_R = \text{const.} \cdot Re_{u_s}^{0.75}$ $Re_H = 1.25 \times 10^4$

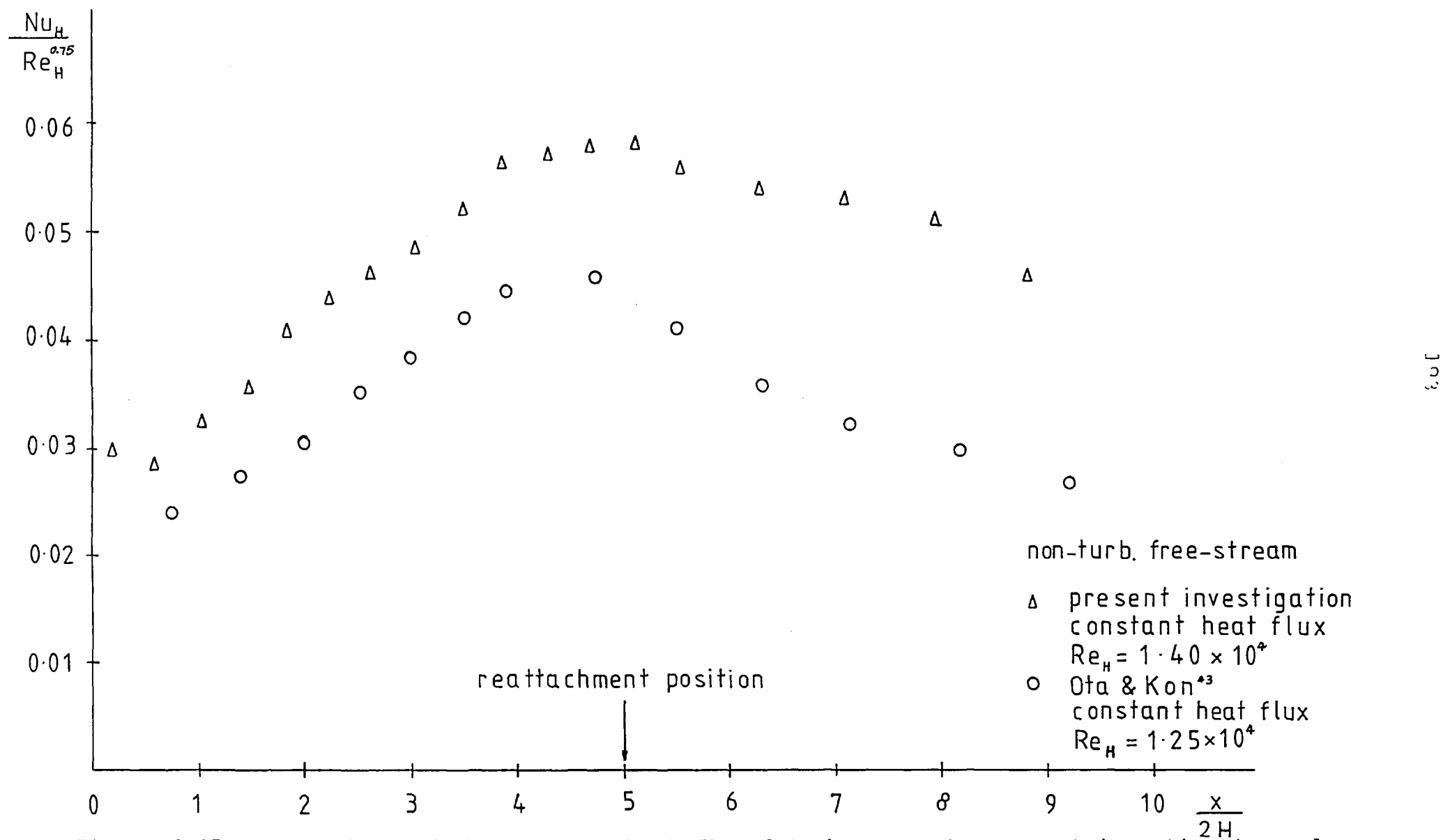


Figure 6.45 A comparison of the constant heat flux data between the present investigation and Ota&Kon

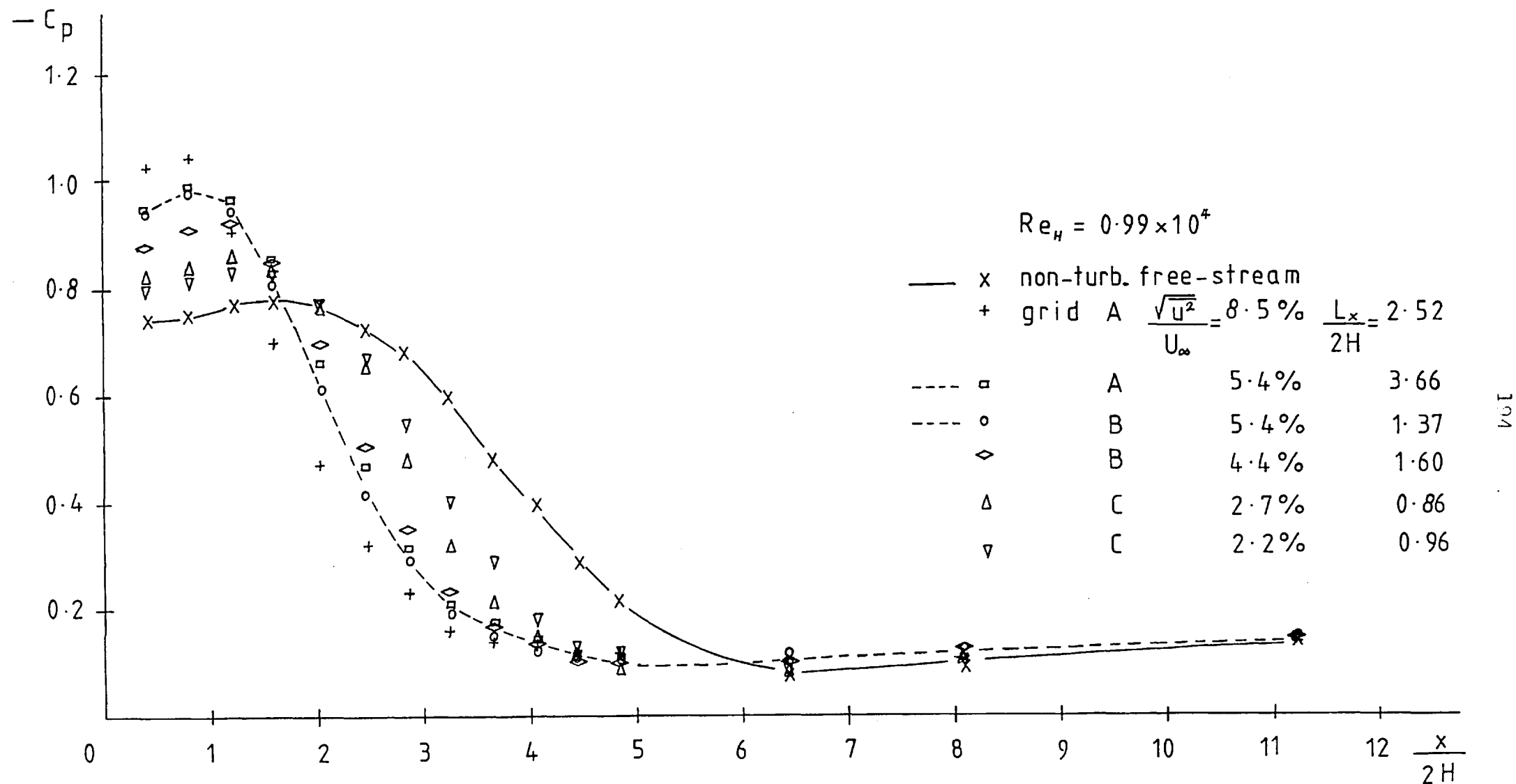


Figure 6.46 The pressure distributions for the turbulent and non-turbulent free-streams

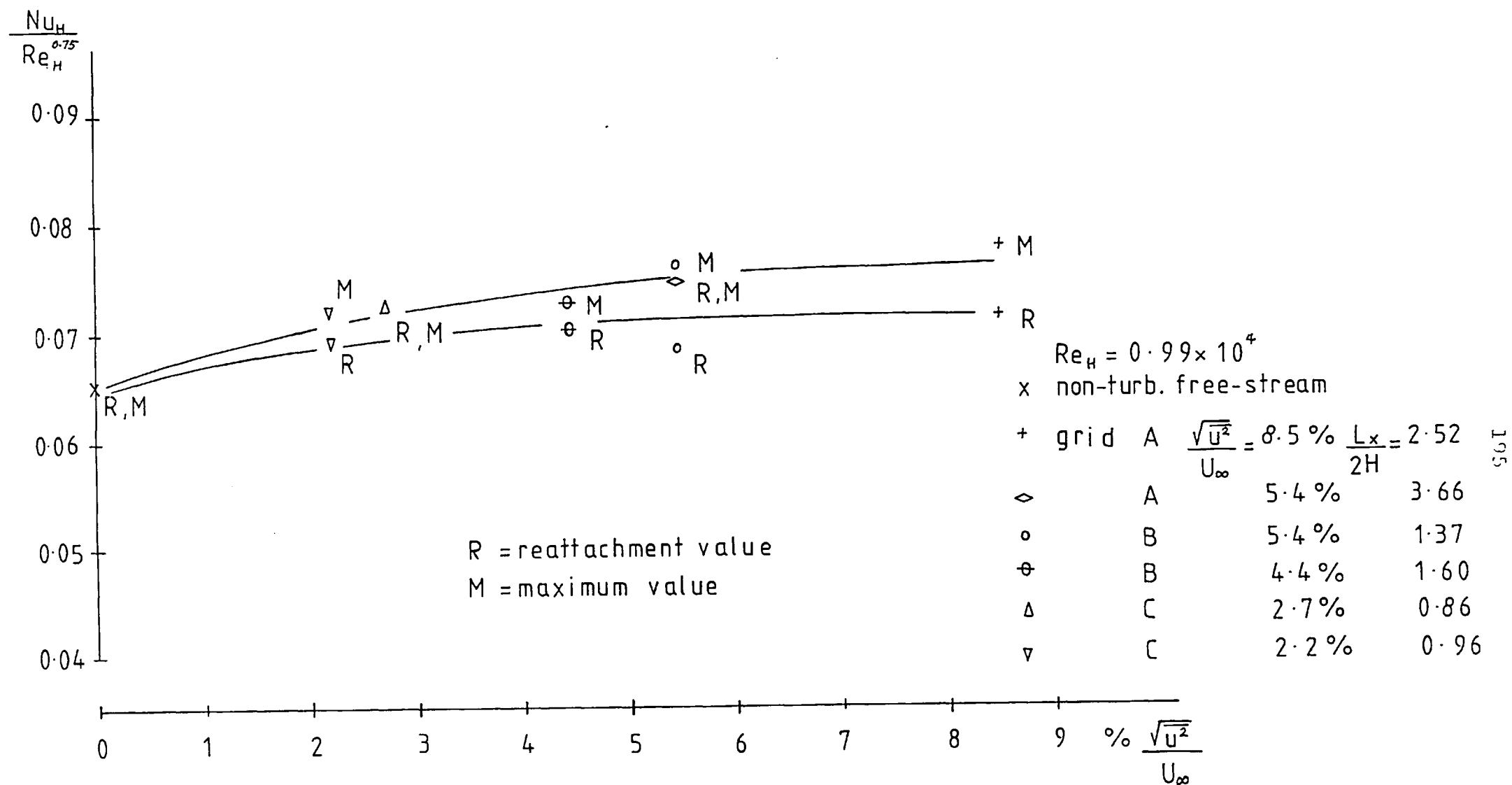


Figure 6.47 The effect of intensity on $Nu_H / Re_H^{0.75}$ at reattachment

7 Presentation of Velocity and Temperature Measurements in the Separated and Reattached Flow over the Blunt Leading-edge Flat Plate

Detailed measurements of mean and fluctuating quantities of both temperature and velocity were carried out at several stations in the separated and reattached flow regions. The effect of free-stream turbulence was also studied (only one condition of turbulence was studied, $(\sqrt{u^2}/U_\infty \simeq 5.3\%; L_x/2H \simeq 1.37)$). Temperature and velocity measurements were concentrated at the reattachment region. These measurements were aimed at providing a basic understanding of heat transfer in separated flows.

7.1 Experimental Results for both Non-turbulent and Turbulent Free-streams

7.1.1 The Longitudinal Turbulent Fluctuating Velocity Non-turbulent Free-stream

The profiles of the longitudinal component of turbulent fluctuating velocity $\sqrt{u^2}/U_\infty$ across the flow field at various distances along the plate are shown in Figure 7.1. In the separated flow region at $x/R = 0.2, 0.35, 0.5, 0.75$ (where x is the distance from leading-edge and R is the reattachment length), the turbulence intensity (fluctuating velocity $\sqrt{u^2}/U_\infty$) generally increases rapidly from the free-stream value ($\sqrt{u^2}/U_\infty \sim 0.4\%$) when the shear layer is reached. A maximum intensity of about 22% is attained around the central region of the shear layer. The intensity then decreases gradually from the maximum and remains roughly constant ($\sqrt{u^2}/U_\infty \sim 10\%$) in the recirculating region (it

should be noted that the hot wire readings are fairly unreliable in this region due to rectification errors). The region of maximum intensity generally broadens with increasing distance downstream (as the shear layer grows downstream). In the reattachment region ($x/R=1.0$) the intensity increases steadily from the free-stream and a maximum intensity of about 24% is reached in the shear layer, it then decreases gradually towards the wall. Downstream of reattachment ($x/R=1.25, 1.5, 1.8$) the profiles flatten progressively away from it. The maximum intensity also decreases downstream ($\sqrt{u^2}/U_\infty \sim 21\%$ at $x/R=1.25$, $\sqrt{u^2}/U_\infty \sim 19\%$ at $x/R=1.5$ and 1.8) as the flow tends towards the characteristics of a turbulent boundary layer.

Turbulent Free-stream

The turbulence intensity profile at reattachment ($x/R=1.0$, $R=7.4\text{cm}$) of the turbulent free-stream ($\sqrt{u^2}/U_\infty = 5.3\%$, $L_x/2H=1.37$) is plotted against that of the non-turbulent free-stream in Figure 7.2. The general shape of the profiles is similar except that the maximum intensity in the shear layer is higher in the turbulent free-stream ($\max \sqrt{u^2}/U_\infty \sim 27\%$) than in the non-turbulent free-stream ($\max \sqrt{u^2}/U_\infty \sim 24\%$).

The Accuracy of Results

For turbulence intensity measurements exceeding 10% the hot wire response may be non-linear. It is necessary to estimate the error if a linear response is assumed. The correction for the mean velocity is (Hinze⁸⁷)

$$\bar{U}_{\text{meas}} = \bar{U} \left[1 + (n-1) \frac{\bar{u}^2}{2\bar{U}^2} + \frac{\bar{v}^2}{2\bar{U}^2} + O\left(\frac{\bar{u}^3}{\bar{U}^3}\right) \right]$$
 where $\bar{U}_{\text{meas}}$ is the measured velocity and $\bar{U}$ is the actual velocity.

and for the turbulence intensity is

$$\frac{\bar{u}^2}{\bar{U}^2} = \left(\frac{\bar{u}^2}{\bar{U}^2} \right)_{\text{meas}} \left(\frac{\bar{U}_{\text{meas}}}{\bar{U}} \right)^{2n} \frac{A_1 + B\bar{U}^n}{A_1 + B\bar{U}_{\text{meas}}^n} [1 - \phi(u, v)]^{-1}$$

where A_1 , B , n are calibration factors of the hot wire, $\phi = f(u, v)$. For the shear layer region well out of the recirculating flow of the present model, typically $\sqrt{\bar{u}^2}/U_\infty \sim 0.2$; assuming $\sqrt{\bar{u}^2} \sim \sqrt{\bar{v}^2}$, hence the error in $\bar{U}_{\text{meas}}$ is about 2%. If one also assumes a Gaussian distribution for u and v , then $\phi(u, v) = \frac{-7}{128} \frac{\bar{u}^2}{\bar{U}^2}$. The error in $(\sqrt{\bar{u}^2}/\bar{U})_{\text{meas}}$ is estimated to be about 2% of $(\sqrt{\bar{u}^2}/\bar{U})$.

For the recirculating region, it is possible that the hot wire anemometer rectifies the signal (as well as non-linear response, the v and w components are comparable to the u component). As the mean velocity $\bar{U} \rightarrow 0$, the King's law relationship $E^2 = A + B\bar{U}^n$ can no longer hold because natural convection of the wire becomes important. It is difficult to assess the degree of accuracy in this region because of the uncertainties associated with hot wire errors (for details on hot wire errors, see Chandrsuda⁹³), thus the results can only be described qualitatively.

It should be noted that the model was not heated when the hot wire velocity measurements were made (because heating the model did not affect the flow field - see section 6.1.1, but would have made the measurements very difficult to interpret). The change in temperature of the fluid was therefore only due to the heating up of the wind tunnel by friction losses. The maximum wind tunnel temperature rise was about 2°C during each run. According to the correction method due to Bearman⁸⁸,

$$\left(\frac{\sqrt{u^2}}{\bar{U}}\right)_{\text{meas}} \approx 2n \frac{E_m(e_{rms})_m}{(E_m^2 - E_o^2)} (1 + \epsilon/\sigma)$$

where E_m , $(e_{rms})_m$ are the measured voltages.

$$\epsilon = (T_1 - T_2)/T_1, \quad \sigma = \theta - 1 \quad (\theta = \text{overheat ratio})$$

The error in $(\sqrt{u^2}/\bar{U})_{\text{meas}}$ due to a temperature rise of 2°C ($T_1 - T_2$) is estimated to be about 1% of $(\sqrt{u^2}/\bar{U})$. Therefore no corrections on the fluctuating velocity measurements were made for temperature change of fluid.

7.1.2 The Mean Velocity

Non-turbulent Free-stream

The mean velocity profiles were obtained using the hot-wire anemometer. The mean velocity profiles along the plate are shown in Figure 7.3. In the separated flow region ($x/R = 0.2, 0.35, 0.5, 0.75$) the mean velocity U/U_∞ increases gradually from the free-stream value ($U/U_\infty \approx 1$) towards the edge of the shear layer (U/U_∞ reaches 1.2). It then decreases fairly rapidly across the shear layer (from $U/U_\infty \sim 1.2$ down to $U/U_\infty \sim 0.2$). The rate of fall in U/U_∞ in the shear layer region generally decreases with increasing distance downstream (as the width of the shear layer increases). In the recirculating region the mean velocity remains fairly constant (in this region the hot wire rectified the signals). In the reattachment region ($x/R = 1.0$) the mean velocity U/U_∞ decreases almost linearly from the edge of the shear layer towards the wall. Downstream of reattachment ($x/R = 1.25, 1.5, 1.8$) the profiles flatten progressively with increasing distance downstream (as the flow tends towards the characteristics of a turbulent boundary layer).

Turbulent Free-stream

The effect of free-stream turbulence on the mean velocity profiles is shown in Figure 7.4. The general shape of the profiles for the turbulent free-stream and the non-turbulent free-stream is similar. The local velocity (U) inside the shear layer is about 10% higher for the turbulent free-stream. Near the wall region the difference in mean velocity between the two streams is less significant.

The Accuracy of Results

In the shear layer region, the correction for the mean velocity is $\bar{U}_{\text{meas}} = \bar{U} \left[1 + \frac{(n-1) \bar{u}^2}{2\bar{U}^2} + \frac{\bar{v}^2}{2\bar{U}^2} + O\left(\frac{\bar{u}^3}{\bar{U}^3}\right) \right]$.

If one assumes that $\sqrt{\bar{u}^2}/\bar{U} \sim 0.2$ and $\sqrt{\bar{u}^2} \sim \sqrt{\bar{v}^2}$. The error in $\bar{U}_{\text{meas}}$ is therefore about 2%. In the recirculating region the hot wire is unable to differentiate between forward flow and reversed flow due to rectification errors. It is not possible to estimate the hot-wire error in this region, so the mean velocities measured in this region are fairly unreliable.

7.1.3 The Turbulent Fluctuating Temperature

Non-turbulent Free-stream

The turbulent fluctuating temperature of the flow field was measured using a constant current hot wire (1 μ m Wollaston wire). The operating temperature difference ($T_w - T_\infty$) for the experiments was about 10°C. The turbulent fluctuating temperature profiles along the plate are shown in Figure 7.5. In the separated flow region ($x/R = 0.2, 0.5, 0.75$) the turbulent fluctuating temperature $\sqrt{\bar{\theta}^2}/(T_w - T_\infty)$ is about zero in the free-stream (there is no real equivalent to the

irrotational velocity fluctuations). It then increases rapidly towards the shear layer until a maximum is reached (the position of maximum $\sqrt{\theta^2}/(T_w - T_\infty)$ corresponds roughly to the position of maximum $\sqrt{u^2}/U_\infty$). After the maximum, the fluctuating temperature starts to fall slightly and then remains nearly constant in the recirculating region. But it increases fairly rapidly again towards the wall and very high magnitudes ($\sqrt{\theta^2}/(T_w - T_\infty) \sim 0.08$) are reached close to the wall. The region of maximum $\sqrt{\theta^2}/(T_w - T_\infty)$ generally broadens with increasing distance downstream. In the re-attachment region ($x/R = 1.0$) and downstream of reattachment ($x/R = 1.8$) the fluctuating temperature increases gradually from the edge of the shear layer towards the wall.

Turbulent Free-stream

The effect of free-stream turbulence on the fluctuating temperature profile is shown in Figure 7.6. The profile with the turbulent free-stream is found to match well onto that of the non-turbulent free-stream. There is little difference in magnitude between the two profiles.

The Accuracy of Results

The change in voltage (e) across a constant current wire due to a change in the resistance (ΔR_w) is simply $e = I \Delta R_w$. Therefore, for small fluctuations in R_w due to small fluctuations in temperature θ , one gets $e = \alpha I R_w \theta = \alpha \bar{E} \theta$ (see Appendix C1). The accuracy of the fluctuating temperature measurement is therefore dependent upon the accuracy of α , $\bar{E}$ and e . The current applied to the wire was checked to be constant. The Brookdeal amplifier was found to have a fixed gain of 1000. The temperature coefficient of resistance α of platinum does

not change significantly with small temperature changes. The output voltage (e) from the amplifier was measured by the DISA rms voltmeter. The error in reading e is probably 2%. The error in fluctuating temperature due to large fluctuating velocity is about 2% (this estimation was carried out in Appendix C2). The total error in fluctuating temperature is therefore roughly 4%.

7.1.4 The Mean Temperature

The mean temperature was obtained using both the thermocouple probe and the constant current hot wire. The mean temperature $\frac{T - T_w}{T_\infty - T_w}$ was found to be in close agreement ($\frac{T - T_w}{T_\infty - T_w}$ differed by 5% at the most) between the two probes (see Figure 7.7). In the separated flow region ($\frac{X}{R} = 0.2, 0.5, 0.75$) the mean temperature $\frac{T - T_w}{T_\infty - T_w}$ decreases steadily from the edge of the shear layer to the vicinity of the wall, it then decrease rapidly very close to the wall ($\frac{T - T_w}{T_\infty - T_w} \rightarrow 0$ at the wall). The rate of fall of $\frac{T - T_w}{T_\infty - T_w}$ in the separated flow region generally decreases with increasing distance downstream. In the reattachment region ($\frac{X}{R} = 1.0$) and downstream of reattachment ($\frac{X}{R} = 1.8$) the mean temperature generally decreases gradually across the shear layer and very rapidly near the vicinity of the wall.

Turbulent Free-stream

The mean temperature profiles for the turbulent free-stream (grid B $\sqrt{u^2}/U_\infty = 5.3\%$, $L_x/2H = 1.37$) and the non-turbulent free-stream are shown in Figure 7.8. The two profiles in general coincide with each other (differing by about 1% in

$\frac{T - T_w}{T_\infty - T_w}$ in a few positions).

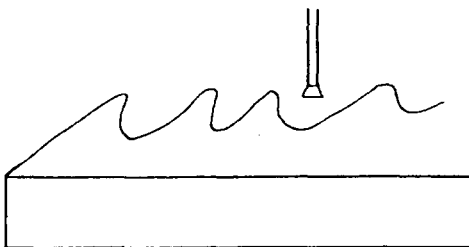
The Accuracy of Results

The thermocouple probe has been estimated to be capable of measuring temperature up to 0.05°C (see section 6.1.2). Typically, $(T_w - T_\infty)$ is about 10°C . The error in mean temperature measurements is about 0.5%.

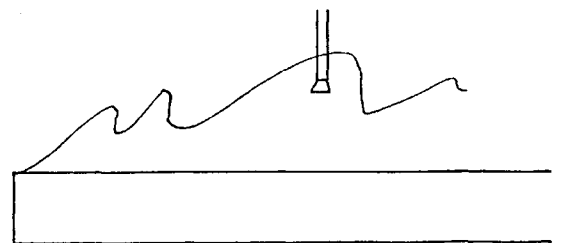
7.1.5 The Fluctuating Velocity and Temperature Oscillograms

The fluctuating velocity and temperature signals were recorded (using the Ampex tape-recorder) across the flow field at the reattachment position ($\frac{X}{R} = 1.0$) and the centre of the separation bubble ($\frac{X}{R} = 0.5$) in a non-turbulent free-stream).

At a position just inside the edge of the shear layer ($\frac{Y}{2H} = 1.07$), the velocity oscillogram contains moderate velocity fluctuations with occasional large velocity fluctuations (spikes), see Figure 7.9b. This region of the flow field is highly intermittent, the moderate velocity fluctuations correspond to irrotational fluctuations induced by the turbulent eddies in the adjacent shear layer. The occasional large spikes correspond to patches of turbulent fluid in the shear layer passing the wire. This effect is illustrated in the figure below.



probe in irrotational flow



probe in turbulent flow

However, the temperature oscillogram has a definite base line with occasional spikes (see Figure 7.9a). The base line corresponds to the mean temperature of the non-turbulent fluid since the temperature does not fluctuate in the irrotational flow, and unlike the velocity field no fluctuations can be induced by the hotter turbulent eddies. The spikes correspond to the hot fluid in the turbulent eddies of the shear layer.

At the centre of the shear layer ($\frac{Y}{2H} = 0.54$), both the temperature and velocity signals fluctuate randomly about the mean (since the shear layer is turbulent in this region) - see Figures 7.10a and b. At the position close to the wall ($\frac{Y}{2H} = 0.10$), the temperature and velocity oscillograms are turbulent with high amplitudes of fluctuations (see Figures 7.11a and b).

At the Centre of the Separation Bubble

At the position just inside the outer edge of the shear layer ($\frac{Y}{2H} = 0.97$), both the temperature and velocity oscillograms show the characteristic of intermittent flow - see Figures 7.12a and b. At the central region of the shear layer ($\frac{Y}{2H} = 0.56$) and in the recirculating flow region ($\frac{Y}{2H} = 0.17$), the temperature and velocity signals are highly irregular (see Figures 7.13a and b and 7.14a and b) which exhibit the characteristic of turbulent flow.

7.1.6 The Temperature and Velocity Spectra

The fluctuating temperature and velocity spectra across the flow field at the reattachment position ($\frac{x}{R} = 1.0$) and at the centre of the separation bubble ($\frac{x}{R} = 0.5$) in a

non-turbulent free-stream were obtained. The power spectral density (PSD) was made dimensionless by multiplying it by $\frac{U_\infty}{2\pi H}$ and the dimensionless frequency $\tilde{n} = \frac{2\pi H n}{U_\infty}$.

At the Reattachment Position

At the position just inside the outer edge of the shear layer ($\frac{Y}{2H} = 1.07$), the shapes of the temperature and velocity spectra are different (see Figures 7.15a and b). The velocity spectrum has a distinct peak around $n = \frac{2\pi n H}{U_\infty} \sim 0.6$

.. The dimensionless power spectral density (PSD $\frac{U_\infty}{2\pi H}$) falls rapidly after the peak. The temperature spectrum has a less obvious peak region around $\tilde{n} \sim 0.7$ and the rate of decay in dimensionless PSD is slower than the velocity spectrum (at $\tilde{n} > 0.7$). The spectra thus clearly indicate that there is a difference between the temperature and velocity fields in this region. The peak in the velocity spectrum perhaps indicates that most of the turbulence energy is generated (via shear action) by eddies of low wave-numbers ($\tilde{n} < 2$) and there is not much energy generated by the high wavenumber eddies (i.e. a rapid fall in PSD). Unlike the turbulence energy (which is generated locally), the thermal energy originates from the wall and is then transported across the shear layer. It is already well mixed when it reaches the edge of the shear layer, so the thermal energy is more evenly distributed over a range of eddy scales (hence the temperature spectrum does not show a peak).

At the central region of the shear layer ($\frac{Y}{2H} = 0.54$), the velocity spectrum has three distinct regions (see Figure 7.16b), i.e., a plateau, a linear region and a region where the

dimensionless PSD decays rapidly. The plateau (energy containing range) extends from $\tilde{n} \sim 0.04$ to $\tilde{n} \sim 0.7$. The linear region ($\tilde{n} \sim 0.7$ to $\tilde{n} \sim 4$) obeys the $-\frac{5}{3}$ law. The region of rapid decay in PSD occurs at $\tilde{n} > 4$. The temperature spectrum has a similar shape to the velocity spectrum (see Figure 7.16a). The temperature and velocity spectra at the position closer to the wall ($\frac{Y}{2H} = 0.10$) are shown in Figures 7.17a and b. These spectra resemble those of the central region of the shear layer (figures 7.16a and b).

At the Centre of the Separation Bubble

At the outer edge of the shear layer ($\frac{Y}{2H} = 0.97$), the velocity and temperature spectra (Figures 7.18a and b) are similar to those obtained at the edge of the shear layer in the reattachment position (Figures 7.15a and b).

In the central region of the shear layer ($\frac{Y}{2H} = 0.56$) and in the recirculating region ($\frac{Y}{2H} = 0.17$), the temperature and velocity spectra are similar in shape (see Figures 7.19a and b and 7.20a and b). Each spectrum consists of a plateau ($n \sim 0.1$ to 0.4) followed by a decay in PSD. The spectra do not exhibit the linear region (i.e., they do not obey the $-\frac{5}{3}$ law).

The Accuracy of Results

Assessment of the accuracy of a spectral estimate is difficult in general since turbulence is non-Gaussian. The accuracy of the spectra is therefore dependent on the linearity of the calibration, the accuracy of the recorded signature and the digitizing system. The temperature and velocity spectra were obtained using the 'Fast Fourier Transform' programme described by Davies⁸⁹.

Davies⁸⁹ showed that the normalised standard error (ϵ_r) of a spectral estimate is given by $\epsilon_r = (\frac{1}{k\ell})^{\frac{1}{2}}$ where $k = 2K + 1$ (K is the frequency averaging parameter) and $\ell = B_e T$ (B_e is the bandwidth which is equal to $\frac{\text{sampling rate}}{N}$, T is the sampling time and N is the number of points). For the present spectral analysis $K = 3$, $T \sim 30\text{sec.}$, $N = 1024$, the bandwidth for the velocity spectra $= \frac{10^4}{1024} \sim 9.8\text{Hz}$ and for the temperature spectra $\sim 19.6\text{Hz}$. The normalised standard error for the velocity spectra is hence about 2% and for the temperature spectra is about 1.5%.

7.2 Discussion of Results for the Non-turbulent Free-stream

7.2.1 The Temperature and Velocity Profiles

In the separated flow region, the present results show that the u component fluctuating velocity (turbulence intensity $\sqrt{u^2}/U_\infty$) is highest in the shear layer (see Figure 7.21). The fluctuating temperature ($\frac{\sqrt{\theta^2}}{T_w - T_\infty}$) is also found to have a first maximum in the shear layer and remains fairly constant in the recirculating region (see Figure 7.22). But the highest temperature fluctuations occur in the vicinity of the wall (not in the shear layer) and the fluctuating temperature increases towards the wall. The present results also show that the mean temperature gradient $\frac{\partial(T - T_w/T_\infty - T_w)}{\partial Y/2H}$ is maximum at the wall (see Figure 7.24) and the local intensity ($\sqrt{u^2}/U$) can be as high as 50% in this region. It seems probable that the largest fluctuations in temperature are a consequence of the maximum mean temperature gradient and the high local velocity fluctuations.

According to Reynolds analogy (Townsend⁹⁰), if heat and momentum are convected in the same way by turbulent motion, the velocity and temperature profiles should be proportional to each other. A comparison of the mean velocity and temperature profiles (see Figures 7.23 & 7.24) indicates that they do not have any noticeable similarity. Even across the mixing layer at the first half of the bubble, there is no similarity in the profiles, whereas for a plane mixing layer the profiles are similar (Andreopoulos⁶⁶). At the reattachment region, the shear stress at the wall is zero ($\tau_w = 0$), but the surface heat transfer rate is maximum ($\dot{q} \neq 0$), Reynolds analogy clearly can not hold in this region.

The effect of a Reynolds number change on the mean temperature field was also obtained. The mean temperature profiles ($(T - T_w) / (T_\infty - T_w)$) across the shear layer at the reattachment region are shown in Figure 7.25. The profiles are approximately identical over most regions of the shear layer, except close to the wall where most changes in mean temperature occur. The highest temperature gradient ($\frac{d(T - T_w / T_\infty - T_w)}{d(y/2H)}$) corresponds to the highest Reynolds number.

The experimental results of Ota and Kon^{43, 45} (blunt flat plate heat transfer) are plotted against the present results. Their mean velocity and temperature profiles at reattachment ($x/R = 1$) are in good agreement with the present measurements (see Figures 7.23 & 7.24). The fluctuating velocity ($\sqrt{u}/U_\infty$) profile at reattachment of Ota and Kon is slightly different from the present profile (Figure 7.21), the fluctuating velocities of Ota and Kon are in general higher (as much as 4% difference in $\sqrt{u}/U_\infty$ at the outer edge of the shear

layer). The reason for the higher values of $\sqrt{u'}/U_\infty$ in Ota and Kon seems likely to be due to the fact that the mean flow field is three-dimensional (low aspect ratio of 5, no end plates). Hence the fluctuating velocity distribution is considerably higher than the present results (the mean flow is two-dimensional).

The recent experimental results of Ota and Kon⁹ showed that the characteristic features of turbulent heat flux $\overline{v\theta}/U_\infty (T_w - T_\infty)$ are nearly equal to those of turbulent shear stress $-\overline{uv}/U_\infty$ in the separated and reattached flow regions. At the centre of the separation bubble, their results tend to show that $-\overline{uv}$ (and $\overline{v\theta}$) is zero in the recirculating region ($-\overline{uv}$ and $\overline{v\theta}$ were measured using a hot wire probe at two inclination angles). This finding is in disagreement with the work of Mueller and Robertson⁸⁴, Etheridge and Kemp⁹² (experiments with backward facing steps). These researchers found that the shear stress is small ($-\overline{uv}/U_\infty^2 \sim 0.001$) in this region, but it is not negligible. The experimental results of Ota and Kon⁹ are questionable because the degree of uncertainty in their $-\overline{uv}$ and $\overline{v\theta}$ measurements is by as much as 30%. Also since the recirculating region has strong re-entrainment of fluid from the shear layer, the shear stress is certainly not zero.

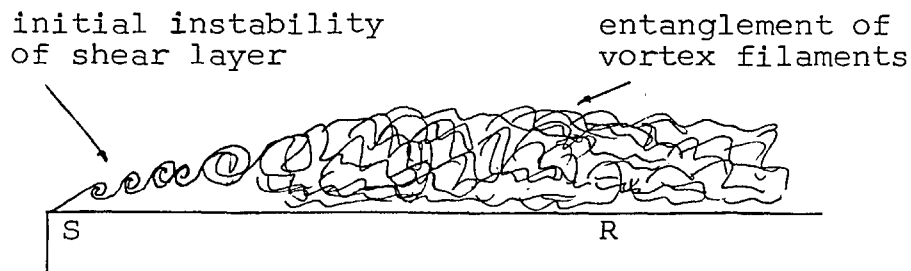
The mean and fluctuating velocity profiles at various distances (from upstream) of a two-dimensional mixing layer (self-preserving) have been obtained by Wygnanski and Fiedler⁷⁷, Champagne et al⁷⁸. The mean velocity profiles were found to be self-similar in the fully developed region. Each component of velocity fluctuations (u , v , w) was also

self-similar. The mean velocity and temperature profiles of the two-dimensional thermal-velocity mixing layer (grid generated turbulence) have been studied by Keffer et al⁸⁰. They found that the dimensionless temperature profiles overlapped together(this is not surprising since the temperature and velocity fields(without pressure gradient) satisfy identical equations and boundary conditions). The mean and fluctuating profiles of velocity and temperature of the present research differ significantly at each section of the flow field. There is no similarity between temperature and velocity profiles as in a self-preserving mixing layer. The conclusion is therefore that the temperature and velocity characteristics of separated-reattached flows are different from a self-preserving mixing layer. Indeed, Chandrsuda⁹³ pointed out that the mean difference between a separated-reattached shear layer and a plane mixing layer is the re-entrainment of fluid from the reattachment zone.

7.2.2 The Fluctuating Temperature and Velocity Oscillograms

The temperature and velocity oscillograms obtained in the present research reveal certain interesting observations. At the outer edge of the shear layer the flow is highly intermittent. The present velocity oscillograms(Figures 7.9b & 7.12b) contain moderate velocity fluctuations, but with occasional large fluctuations many times the local r.m.s. values in magnitude. At the reattachment region, if one assumes that the shear layer has a convection velocity (U_C) of $0.5 U_\infty$, and an average period $\Delta T U_\infty / D$ between successive peaks in the oscillograms of about 6 ($\Delta T \sim 12 \text{msec}$, $U_\infty \sim 12 \text{m/s}$,

$D = 2H \sim 2.5 \text{ cm}$), then an average spacing λ (U_c/n , where $n \sim 1/\Delta T$) of about $3D$ is produced. Similar results (an average spacing of $3.2D$) have also been obtained by Hillier and Cherry⁹⁵ on the model of the same geometry. Data obtained on mixing layers (Brown and Roshko⁷⁹) show that such spacing ($\sim 3D$) is sufficient for a full shear layer development up to reattachment. The present results therefore indicate that fairly large scale disturbances comparable to the bubble length have been developed at reattachment. The shear layer initially separates from the surface of the model at S (see diagram below). Owing to the instability of the shear layer, it rolls up into vortices which entrain fluid from the irrotational flow and the recirculating region. The growth of these vortices is presumably by the amalgamation of vortices forming in the instability process, i.e., vortex pairing (the mechanism of vortex pairing for turbulent mixing layers has been reported by Winant and Browand⁹⁷). Further downstream ($0.2 < x/R$), the disturbances grow and amalgamate with other disturbances. The lateral correlation of pressure and velocity obtained by Hillier and Cherry⁹⁵ falls monotonically ^{in about $4D$} to zero which might imply that the flow structure could be three-dimensional as reattachment is approached.



Schematic diagram of the shear layer instability

Johnson⁹⁴ pointed out that if the velocity layer is thicker than the thermal layer, then at the edge of the thermal layer the signal will become intermittent even if the velocity signal is highly turbulent. The temperature and velocity oscillograms (Figures 7.9a and b) of the present research look similar in the outer edge of the shear layer when the velocity fluctuations due to the irrotational flow are ignored. This fact suggests that the temperature and velocity layers are nearly of the same thickness.

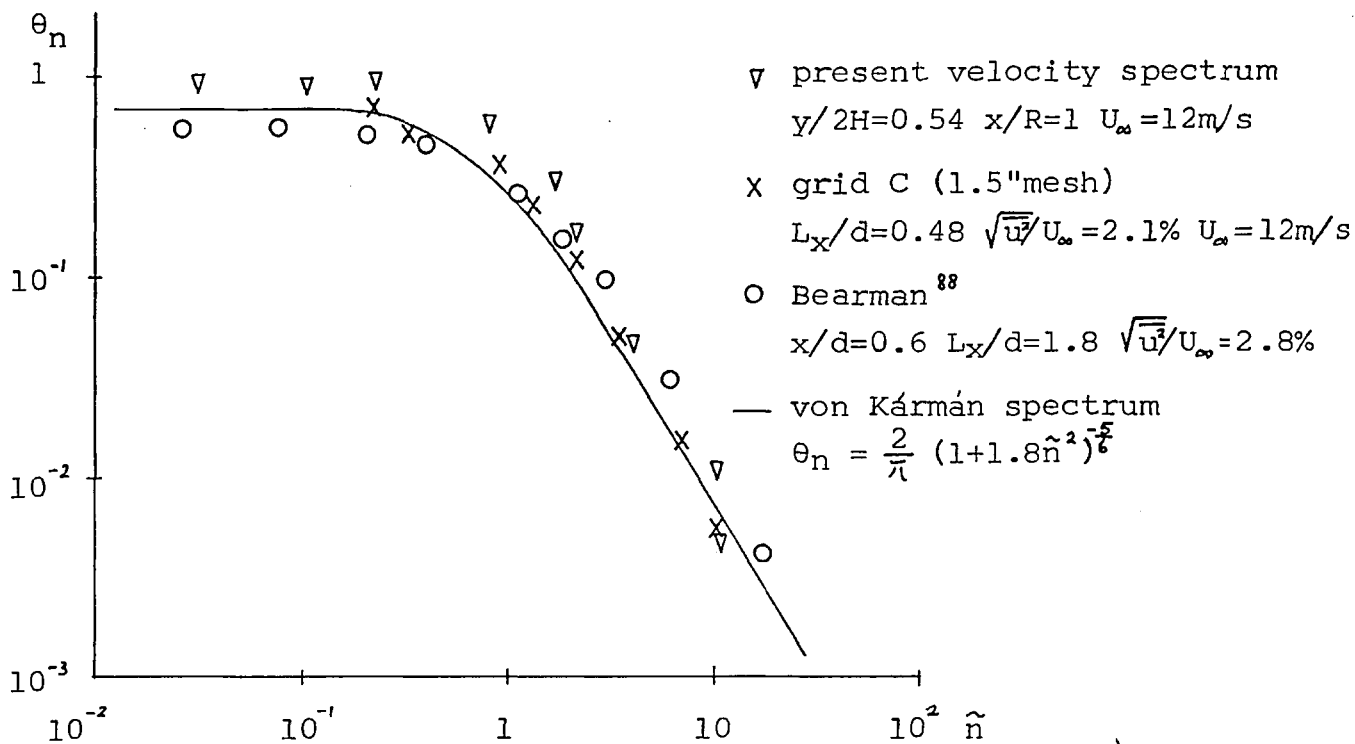
7.2.3 The Temperature and Velocity Spectra

At Reattachment

In the outer edge of the shear layer the velocity spectrum (Figure 7.15b) shows that most of the energy is contained by a number of eddy sizes ($\tilde{n} \sim 0.1-2$), a distinct peak in dimensionless PSD distribution can be seen. This peak corresponds roughly to the peak in the pressure spectra obtained by Hillier and Cherry⁹⁵ (centred upon $\tilde{n} \sim 0.14$). The flow field in this region is intermittent and consists mostly of energy containing eddies. The distribution probably suggests the existence of a large scale vortical structure with weak periodicity in this region of the flow field. The temperature spectrum does not have the peak (Figure 7.15a). It seems probable that the turbulent thermal energy is distributed more evenly over a range of eddy sizes ($\tilde{n} \sim 0.1 - 0.65$). The velocity spectra at positions further into the shear layer are found to be similar (Figures 7.16b and 7.17b). There are three regions in each spectrum, i.e., the energy containing range, the inertial subrange and the energy dissipating range.

The inertial subrange obeys $-\frac{5}{3}$ law. The temperature spectra are similar to each other (Figures 7.16a and 7.17a) and contain the three regions (i.e., the thermal energy containing range, the inertial convective range and the inertial conductive range - see Townsend⁹⁰). It should be noted that it is not possible to overlap the temperature and velocity spectra because the three regions occur at different wave-numbers.

The present velocity spectrum is plotted against the von Kármán spectrum, the grid turbulence spectrum and Bearman's⁸⁸ spectrum for distorted free-stream turbulence -see diagram below. It can be seen that the grid turbulence



spectrum follows the von Kármán spectrum fairly well, but the velocity spectrum and the distorted free-stream turbulence spectrum differ from it slightly. The velocity spectrum shows amplification of turbulence energy all the way except at high wavenumbers ($\tilde{n} > 10$), whereas the distorted free-stream turbulence spectrum shows attenuation at low wavenumbers

($\tilde{n} < 1$) and amplification at high wavenumbers ($\tilde{n} > 1$). The above results therefore suggest that there is some similarity between distorted free-stream turbulence and distortion of turbulence structure at reattachment. The difference in spectra can partly be due to the difference in length scale (at reattachment, assume that the largest eddy scales are of the order of shear layer thickness, typically $L_x/2H \sim 1$, whereas $L_x/d = 1.8$ for the distorted free-stream turbulence). But most important of all large changes in turbulence structure occur when the shear layer bifurcates at reattachment.

At the Centre of the Bubble

At the outer edge of the shear layer the velocity and temperature spectra are completely different in shape (Figures 7.18a and b), the velocity spectrum has a distinct peak but not the temperature spectrum. At the central region of the shear layer and in the recirculating region, the velocity and temperature spectra do not contain the $-\frac{5}{3}$ law region (Figures 7.19 and 7.20), the energy containing range for the temperature spectrum ($\tilde{n} \sim 0.1 - 0.65$) is wider than the velocity ($\tilde{n} \sim 0.04 - 0.26$). It therefore appears that the transfer of heat and momentum in the separated flow region is different from that at reattachment, the scale of eddies in these two regions is slightly different.

7.3 Discussion of Results for the Turbulent Free-stream

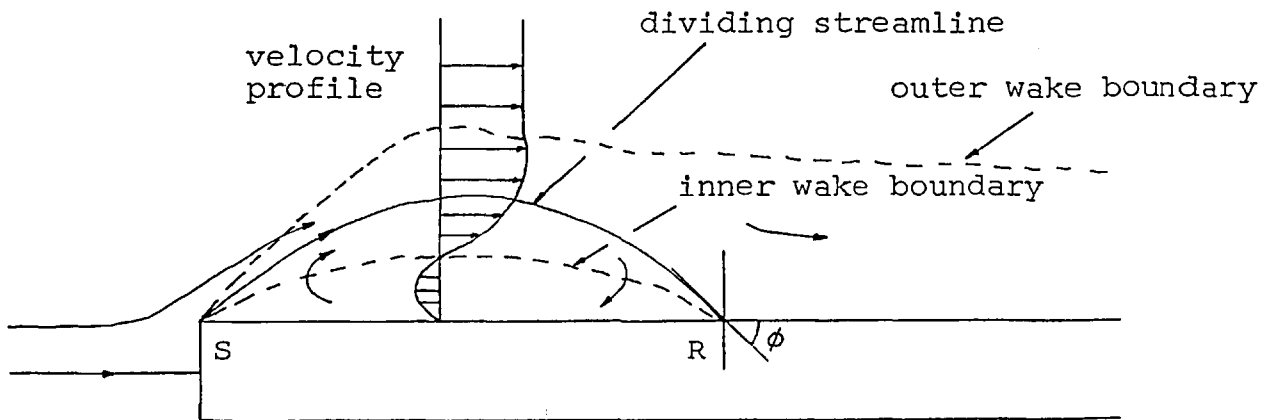
The effect of free-stream turbulence at the reattachment region of the flow field was studied. The mean and fluctuating profiles of temperature and velocity in the turbulent free-stream (grid B $\sqrt{u^2}/U_\infty = 5.3\%$, $L_x/2H = 1.37$) were plotted against those of the non-turbulent free-stream. The

mean velocity (U/U_∞) across the shear layer is found to increase by 2 to 10% due to the presence of free-stream turbulence (in response to the change of $U_s = (1 - C_{P_s})^{\frac{1}{2}} U_\infty$) - see Figure 7.4. It seems probable that the increase in entrainment rate in this region causes the increase in mean velocity. The fluctuating velocity ($\sqrt{u^2}/U_\infty$) across the shear layer is generally increased by 1 to 3% in the turbulent free-stream. Similar results were obtained by Mohammadian et al.⁸⁵ for a two-dimensional mixing layer with two unequal turbulent streams. They found that the effect of turbulence had penetrated through the mixing layer. It is interesting to find that the mean and fluctuating temperature profiles over most regions of the shear layer are little affected by the presence of free-stream turbulence (Figures 7.6 and 7.8). The increase in the amount of fluid entrainment at reattachment does not seem to have much effect on the temperature distribution in the shear layer. As it was found out earlier, most of the temperature change occurred within a thin layer of fluid close to the wall. Since no measurements were taken close to the wall, it was therefore not possible to see the difference.

7.4 Discussion on the Physics of Heat Transfer in Separated Flows

The equilibrium of the shear layer is maintained by a process of entrainment of fluids, entrainment occurs both inside and outside the shear layer. Downstream of separation the shear layer grows by turbulent mixing between the cavity region and the external stream. Entrainment from the cavity

causes a general reduction of pressure in this region, which reflects in the initial trajectory of the shear layer and in its subsequent curvature towards the surface and the reattachment zone. Reattachment is a 'stagnation like' process, the shear layer dividing in the mean so that some fluid sweeps downstream whilst the remainder deflects upstream to satisfy the entrainment requirement of the near wake (see Figure below).

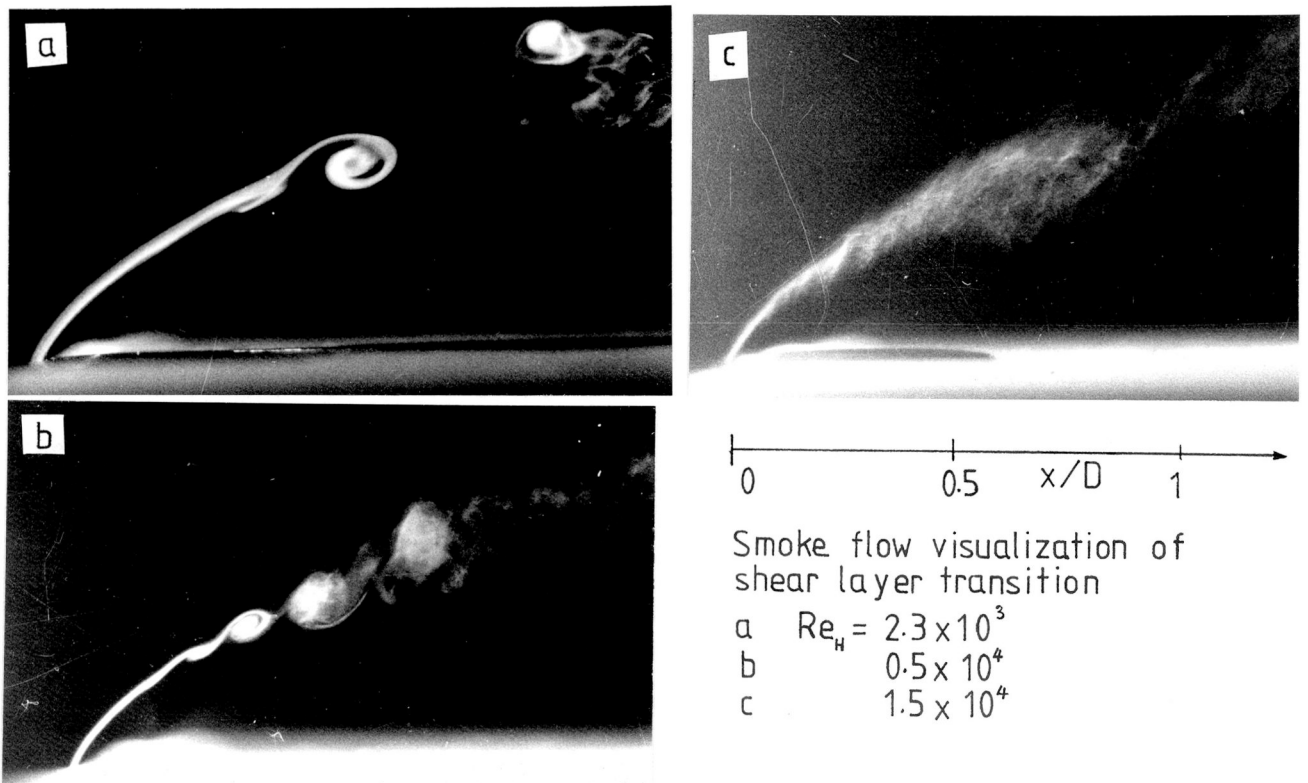


The dividing streamline connects the separation and reattachment points. The mean mass transfer across this line is zero at all points for a truly two-dimensional flow, but there is a vigorous exchange of fluid along its length due to turbulent diffusion.

According to this free shear layer reattachment mechanism, an increase in entrainment rate will cause a general increase in the typical velocities inside the bubble (and a decrease in base pressure). The larger resulting transverse pressure gradient will originate an increase in the shear layer curvature, hence an increase in the reattachment angle ϕ (see Figure above), the larger ϕ being associated with a larger amount of returned flow to the separation bubble in order to balance the increase in entrainment rate.

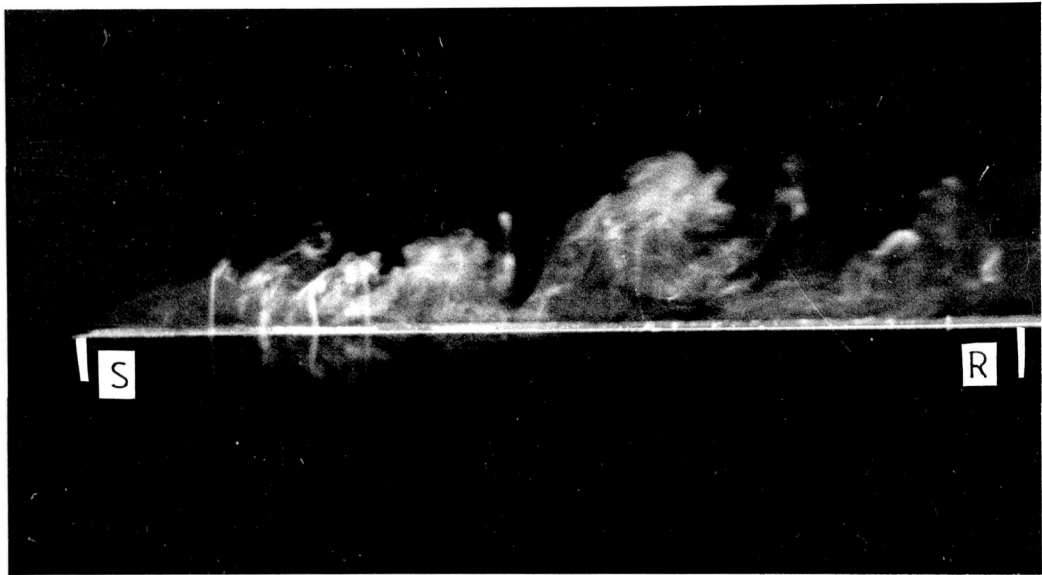
The instantaneous flow fields of the separated/

reattached regions differ overwhelmingly from the mean. The recent measurements of Hillier and Cherry⁵⁴ found that the structure appears to be three dimensional (from the lateral correlation of fluctuating pressure). The degree of three-dimensionality of the unsteady flow field is important because three-dimensionality is a prerequisite for the development of a vortex stretching energy cascade, and distinguishes turbulence from various quasi-two-dimensional random motions (such as large scale motion in the atmosphere). Their smoke visualization photographs demonstrate the importance of transitional effect in the separated shear layer (see Figures below). At the two lowest Re_s , rolling up of vortices from the leading edge can be seen, at the higher Re there is no apparent organisation and a rapid onset and growth of disturbances after separation. Large vortical structures are obtained at the

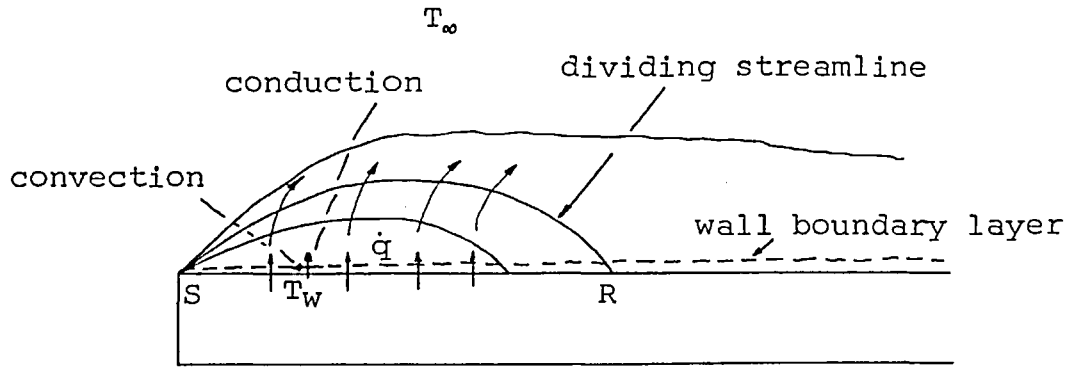


middle of the bubble (see Figure overpage). At the reattachment region there are no clear patterns (possibly due to the

process of mixing and branching of eddies). It should be noted that it is not possible to give a precise description of the instantaneous flow field here because there has been relatively little attempt to investigate the unsteady structure of reattaching flows.



In a non-turbulent free-stream, the present results indicate that the largest fluctuations in temperature and most of the change in mean temperature occur in a thin layer of fluid near the wall region (Figures 7.5 and 7.7). This layer appears to have a dominant influence on heat transfer, in other words heat transfer in separated flows is associated with a wall boundary layer. In practice, it is difficult to measure its thickness. However, it is possible to estimate its thickness using the present results. If one assumes that the bubble has a mean 'dividing' streamline, and at equilibrium all the heat leaving the surface of the model must cross this line (see Figure overpage). The surface heat transfer rate is thus $\dot{q} = k_{\text{air}} \frac{\partial T}{\partial y}$ (assuming heat transfer by convection is small). As a rough approximation the wall boundary layer thickness is

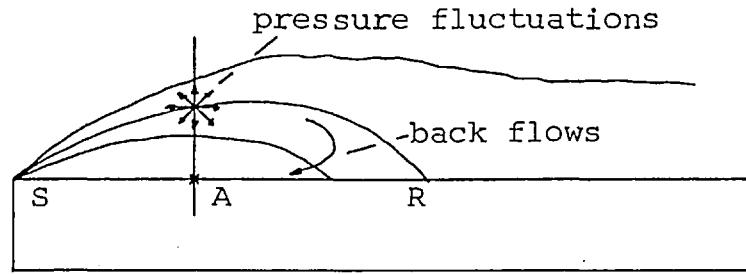


$\Delta y \sim \frac{k_{air} \Delta T}{\dot{q}}$. Typically $\dot{q} \sim 600 \text{ W/m}^2$, $\Delta T \sim 10^\circ \text{C}$, $k_{air} \sim 0.0262 \text{ W/m}^2 \text{K}$, therefore Δy is about 0.4 mm (or $y/2H \sim 0.02$). The wall boundary layer is thus very thin (but its thickness is Re dependent). In this region, one would expect the viscous effect to be dominant and heat transfer is most likely via conduction. As a rough estimate, assume the wall boundary layer is laminar, the amount of heat convection is thus $Nu_x \sim 0.3 Pr^{1/3} Re_x^{1/2}$. Typically $Pr \sim 0.7$, $\Delta T \sim 10^\circ \text{C}$, $U \sim 0.5 \text{ m/s}$, $x \sim 5 \text{ cm}$. Hence $\dot{q} \sim 50 \text{ W/m}^2$, i.e., the convective component is small compared with conduction ($\sim 600 \text{ W/m}^2$).

The present results show that the fluctuating temperature reaches a maximum close to the centre of the shear layer and the fluctuating velocity ($\sqrt{u^2}/U_\infty$) is also a maximum there. The profiles of fluctuating temperature correspond roughly to the profiles of fluctuating velocity except at the wall region (where the highest fluctuating temperature attains). It appears that the shear layer plays an important part in the surface heat transfer. The shear layer can affect the surface heat transfer in two ways (see Figure overpage)

- (i) via pressure fluctuations.
- (ii) via the back flow.

A is a point on the surface



The pressure fluctuations (irrotational fluctuations) do not directly affect the temperature field, but they can cause velocity fluctuations (hence affect the $\overline{v\theta}$ transport). The back flow from the reattachment region directly affects the rate of cooling on the surface, the rate of entrainment of back flow determines the typical velocities (and base pressure) inside the bubble and it may be the major determinant of surface heat transfer.

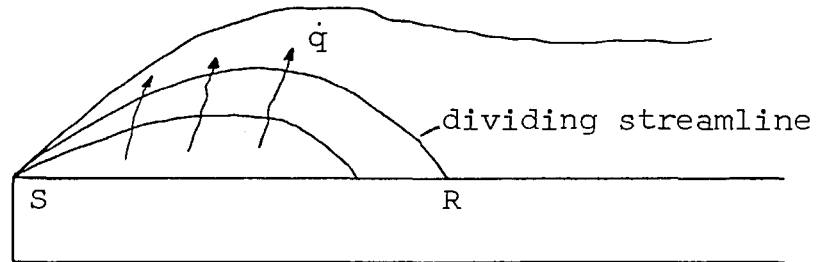
The present measurements of spectra reveal certain interesting observations. At the reattachment region, although the temperature and velocity spectra do not overlap, the profiles are in general similar. But the temperature and velocity spectra contain the energy-containing range, the $-\frac{5}{3}$ law region and the rapid decay region (it should be noted that the filtered frequency for temperature spectra are much lower, so that PSD values around the cut-off frequency should be considered inaccurate). The present flow Reynolds number is fairly large, viscous dissipation is probably negligible over a large wavenumber and viscous forces should have no direct influence on the motion. In the inertial subrange, it is not surprising that the velocity spectrum obeys the following

law $E(k) \sim c \xi^{\frac{1}{3}} k^{-\frac{5}{3}}$, where k is the wavenumber; ξ is the dissipation function; c is a constant. But the temperature spectrum also tends to obey a similar law, this probably suggests that the temperature spectrum function depends on the velocity spectrum function (Townsend⁹⁰). In other words the transport of heat is perhaps via this transport of turbulence.

In Chapter 6, the present research found that Nusselt number is a function of Reynolds number ($Nu_H \propto Re_H^{0.75}$ approximately - this is generally true over the whole bubble). The mean temperature profiles at different Reynolds numbers are shown in Figure 7.25. It can be seen that the mean temperature across the shear layer is approximately the same at different Re_H , except at the wall region where the mean temperature gradient $\frac{dT}{dy}$ changes considerably at different Re_H . The highest $\frac{dT}{dy}$ corresponds to the highest Re_H . Since the surface heat transfer rate ($\dot{q}$) is proportional to $\frac{dT}{dy}$, hence $\dot{q}$ (or Nu_H) is a function of Re_H . The shear layer measurements are thus in agreement (to a certain extent) with the surface heat transfer.

In Chapter 6, in general Nu_H was found to increase with distance from the leading edge. The present mean temperature profiles at various distances (Figure 7.24) also indicate that the mean temperature gradient $\frac{dT}{dy}$ is increasing with distance (i.e., Nu_H increases with distance). It should be noted that it is not possible to evaluate Nu_H using $\frac{dT}{dy}$ in the present research because of the lack of mean temperature measurements very close to the wall, at least it shows that the trend is correct. It is hoped that future measurements can measure T closer to the wall.

It should be noted that no measurements of $\overline{v\theta}$ and $\overline{uv}$ correlations were made in the present research. In actual fact the rate of heat transport by turbulence is $\rho C_p \overline{v\theta}$, and the total heat transport in the bubble is thus $\dot{Q} \approx \int_0^R \rho C_p \overline{v\theta} dx$. Since no measurements of $\overline{v\theta}$ were made, an accurate account of the heat transport by turbulence and the heat input to the model cannot be obtained. However, a very rough estimation can be made as follows. If one assumes that the shear layer grows approximately linearly with distance from the separation point (ignoring the thickening of shear layer at reattachment), i.e., $l \approx kx$. Typically the size of the energy containing eddies is of the order l . If one also adopts the mixing length theory for the transport of turbulence and assumes that $u = l \frac{\partial U}{\partial y}$, $v = l \frac{\partial U}{\partial y}$. If the thermal layer grows like the velocity shear layer, the temperature fluctuation (θ) due to turbulence can be written as $\theta \approx l \frac{\partial T}{\partial y}$. The temperature velocity correlation is thus $\overline{v\theta} \approx l^2 \frac{\partial U}{\partial y} \cdot \frac{\partial T}{\partial y}$. Since all the heat flux must



cross the mean streamline, so the integrated heat flux in the bubble equals the total heat input, i.e., $\dot{Q} \approx \int_0^R \rho C_p \overline{v\theta} dx$, or $\dot{Q} \sim \int_0^R \rho C_p k^2 x^2 \frac{\partial U}{\partial y} \cdot \frac{\partial T}{\partial y} dx$. From the results of the present experiments, $\frac{\partial U / U_\infty}{\partial y / 2H} \sim 2.8$ and $\frac{\partial (T - T_w) / (T_\infty - T_w)}{\partial y / 2H} \sim 0.3$ (these quantities were obtained at the centre of the shear layer at

$x/R = 0.5$, $T_w - T_\infty \sim 10^\circ\text{C}$, $U_\infty \sim 12\text{m/s}$). The value of k is about 0.05 for a mixing layer (Wyganski and Fiedler⁷⁷). The integrated heat flux is thus about 300W/m^2 . The heat input into the model is typically about 600W/m^2 . The mixing length theory has therefore under-estimated the heat flux, but the important fact is that it gives the correct order of magnitude. It should be noted that the mixing length theory assumes the flow is quasi-steady and eddies of only one size. As it is shown by the present spectrum measurements the shear layer is in fact turbulent, the turbulent and thermal energies distribute over a wide range of wavenumbers (or eddy scales). Thus the mixing length theory is clearly inadequate.

The Effect of Free - stream Turbulence

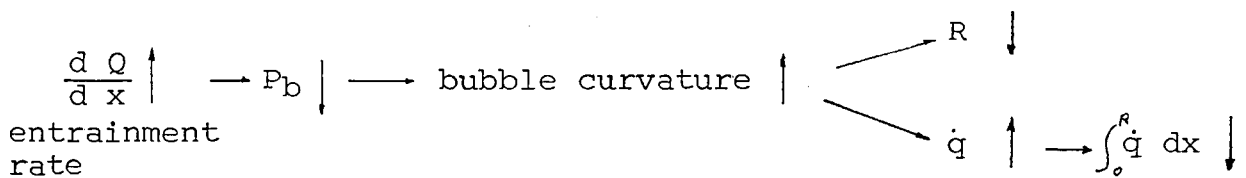
In free-stream turbulence the reattachment length of the bubble reduces as the base pressure decreases, and the surface heat transfer (Nu_μ) increases (from Chapter 6). It can be seen from figures 7.2 and 7.4 that the mean and fluctuating velocities increase in general in free-stream turbulence (as a result of the increase in entrainment rate). However, it is difficult to see from the mean temperature profiles (in Figure 7.8) whether $\left. \frac{dT}{dy} \right|_{y=0}$ is greater in free-stream turbulence (because no measurements were made close to the wall), but one can easily derive that it is greater from the $\dot{q}$ measurements. Due to the complexity of the problem of heat transfer in separated flows and the number of variables (such as $\overline{uv}$, $\overline{v\theta}$ etc.)^{Appendix E1} which need to be measured, it is not possible to construct a mathematical model using the present data. But some tentative suggestions can be made.

In free-stream turbulence, the integrated areas

under the $\frac{Nu_H}{Re_H^{0.75}}$ distributions (figure 6.42) show an interesting trend. The integrated area (integrated heat flux) decreases slightly with decreasing bubble length (or increasing intensity) -see Table below.

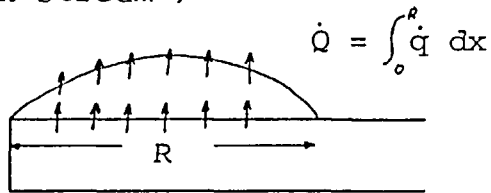
condition	$\sqrt{u^2}/U_\infty$	$R/2H$	$\int_0^R \frac{Nu_H}{Re_H^{0.75}} d \frac{x}{H}$
non-turbulent free-stream	0%	5.0	4.45
grid C	2.2%	4.1	4.10
grid C	2.7%	3.9	4.00
grid B	4.4%	3.2	3.60
grid B	5.4%	3.0	3.50
grid A	8.5%	2.5	3.10

The bubble length has reduced by 50% from the non-turbulent free-stream to grid A, but the integrated area decreases only by 30%, i.e., total loss from bubble decreases, $\dot{q}$ per unit area increases. In free-stream turbulence the surface heat transfer increases as the entrainment rate increases. But the bubble shortens more than the heat transfer rate increases, so the integrated area is in fact less. This reasoning can be summed up diagrammatically as shown below.

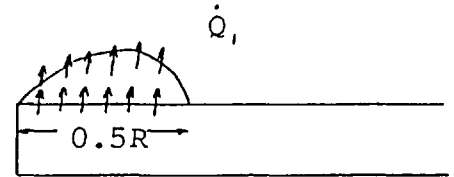


It is possible to pursue the argument one stage further by assuming the integrated heat flux to be $\dot{Q} (= \int_0^R \dot{q} dx)$ and the bubble length R in the non-turbulent free-stream; the bubble length is halved and the integrated heat flux is $\dot{Q}_1$ in the turbulent free-stream (see figure overpage). For the non turbulent free-stream the heat flux per unit length of bubble

(assumed all the heat leaves the surface must cross the mean stream)



non-turbulent free-stream



turbulent free-stream

is thus $\dot{Q}/R$. For the turbulent free-stream (grid A $\sqrt{u}/U_\infty \sim 8.5\%$, the bubble length is halved) $\dot{Q}_t$ is about $0.7\dot{Q}$, so the heat flux per unit length of bubble is $\frac{0.7\dot{Q}}{0.5R}$ ($\sim 1.4\frac{\dot{Q}}{R}$). Hence it shows that the heat transfer rate actually increases as the bubble shortens.

In Chapter 6, the heat transfer data based on R and U_s tend to correlate well ($\frac{Nu_R}{Re_{U_s}^{0.75}}$ vs $\frac{x}{R}$ - see Figure 6.44). It appears that the action of free-stream turbulence on the surface heat transfer is almost entirely through its effect on the velocity field (R , U_s etc.). In free-stream turbulence, the bubble length shortens and the separation velocity increases (the increase in U_s is a result of increasing entrainment in the bubble), the bubble length shortens more than the increase in U_s . As it was shown previously that the heat flux per unit length of bubble increase in free-stream turbulence, hence higher U_s is needed to transfer more heat down the bubble to maintain equilibrium. The growth of the shear layer is also governed by the separation velocity. It is therefore more plausible to use U_s rather than U_∞ as the characteristic velocity.

7.5 Conclusions

Non-turbulent Free-stream

Very large velocity and temperature fluctuations occur at the shear layer region. It appears that the generation of turbulence in the shear layer can provide a mechanism for the surface heat transfer. The shear layer can affect the surface heat transfer in two ways:

- (i) via the back flow
- (ii) via pressure fluctuations.

Heat transfer in separated flow is associated with a wall boundary layer where most of the change in mean temperature and the largest fluctuations in temperature occurs. The thickness of this layer is estimated to be 2% of the model thickness and depends on the free-stream Reynolds number (and intensity). The largest fluctuations in temperature at the wall region appears to be a consequence of the maximum mean temperature gradient and the high local velocity fluctuations.

At the outer edge of the shear layer, the temperature oscillogram has a smooth base line (with distinct spikes), whilst the velocity oscillogram base line consists of fluctuations. This suggests that the irrotational flow does not cause temperature fluctuations. The temperature and velocity oscillograms match fairly well in this region which probably indicate that the temperature and velocity layers are nearly of the same thickness.

At the reattachment region, the temperature and velocity spectra do not overlap, but the profiles in general are similar. It appears that the temperature and velocity spectra (in the inertial subrange) obey similar laws, and the

temperature spectrum function depends on the velocity spectrum function. Thus it seems probable that the transport of heat is via the transport of turbulence.

Turbulent Free-stream

At the reattachment region, the mean and fluctuating velocities increase in general as a result of the increase in entrainment rate. The mean and fluctuating temperature over most regions of the shear layer (except near the wall) are not significantly affected by the presence of free-stream turbulence, the reason is that most of the temperature changes occur in the wall boundary layer.

Finally, the shear layer measurements are generally in agreement with the surface heat transfer data.

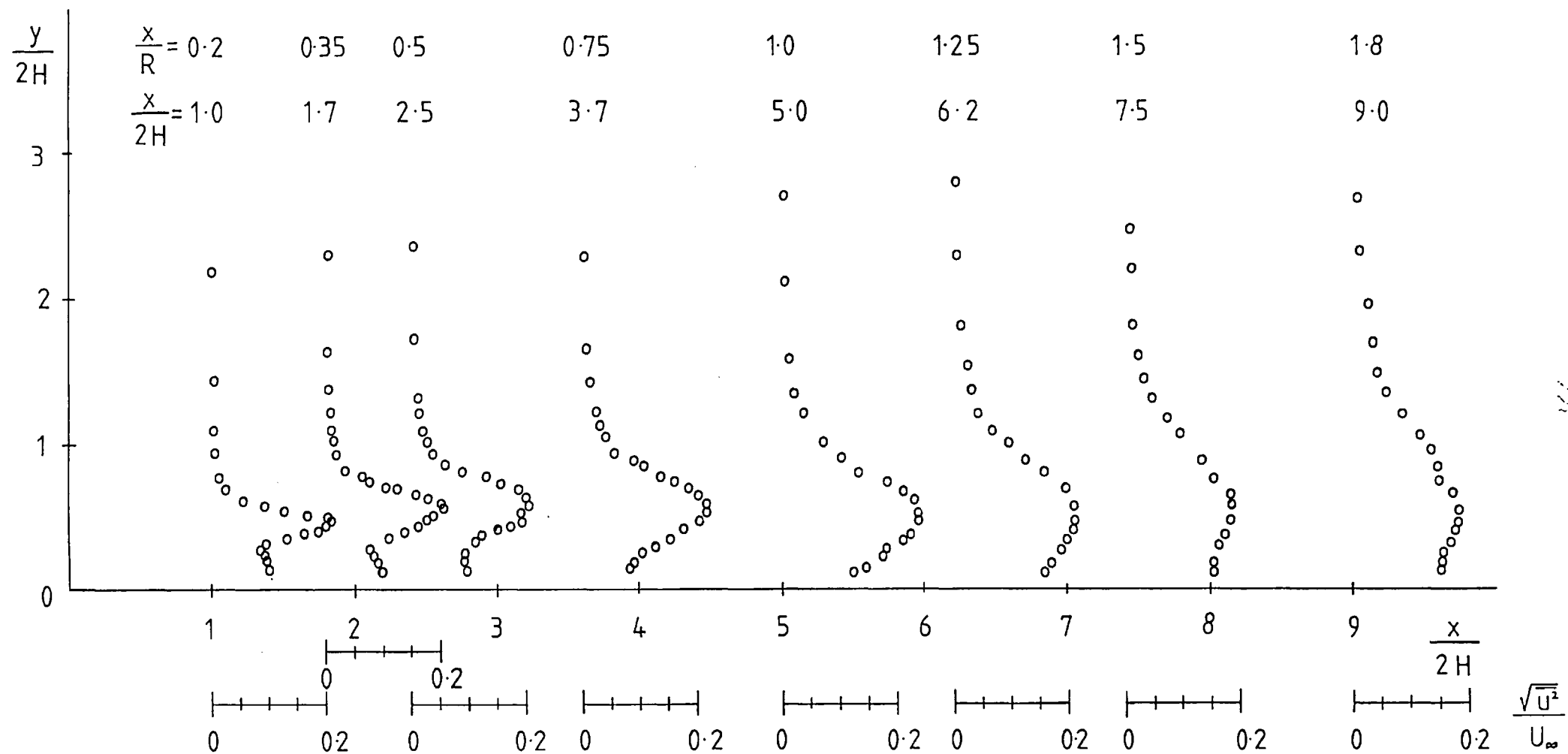


Figure 7.1 The u-component turbulent fluctuating velocity profiles along the blunt flat plate

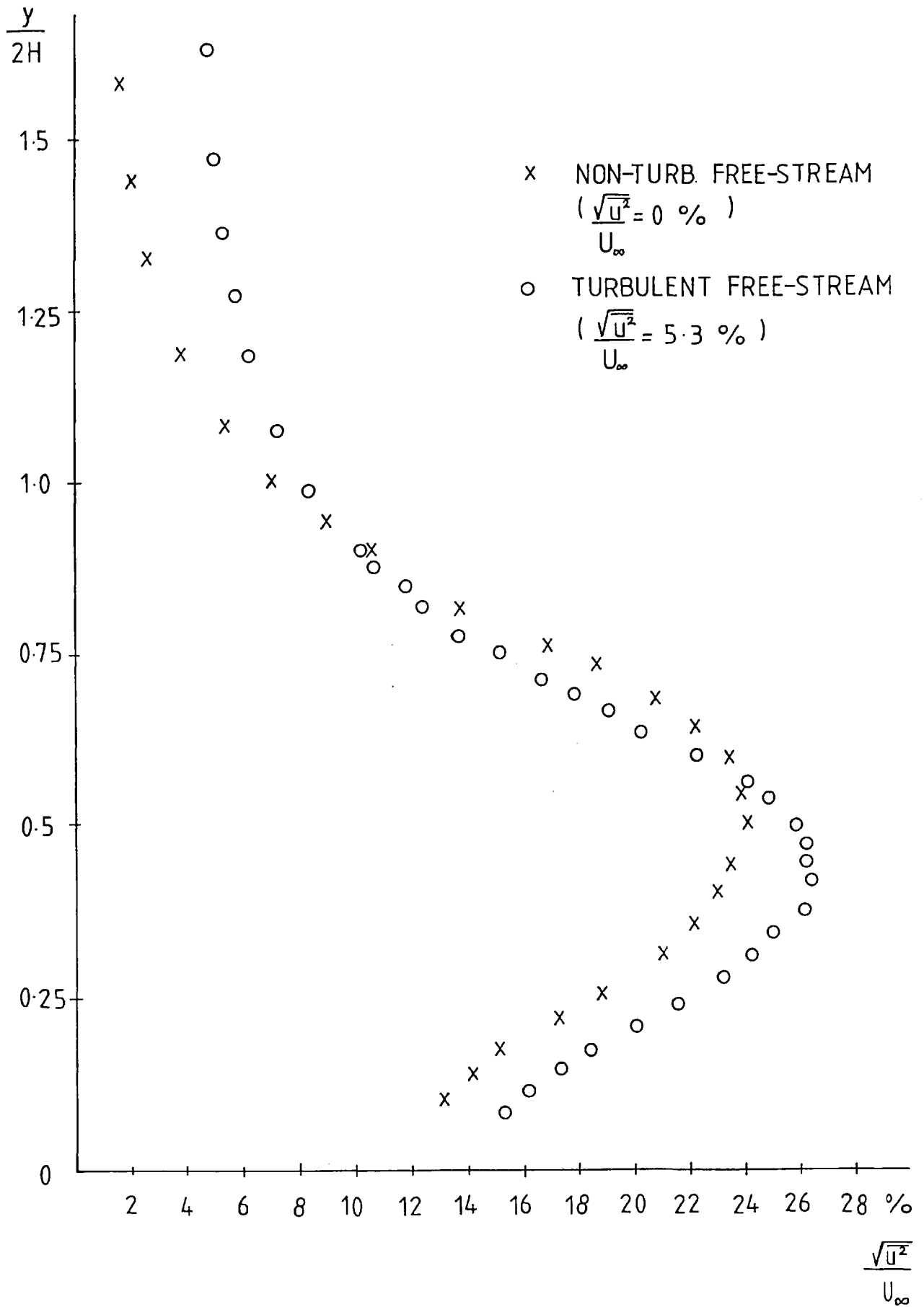


Figure 7.2 The u-component turbulent fluctuating velocity profiles at reattachment

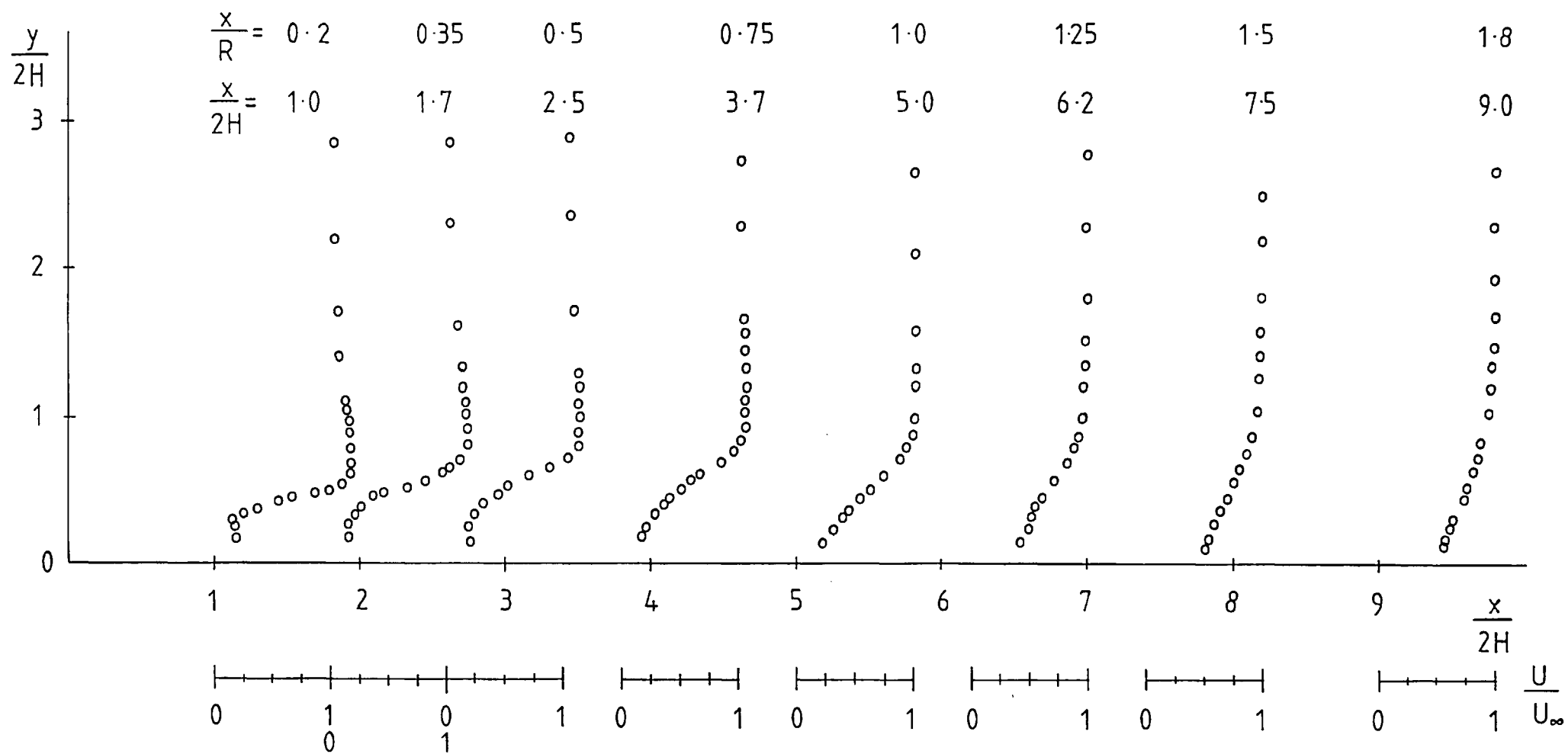


Figure 7.3 The mean velocity profiles along the blunt flat plate

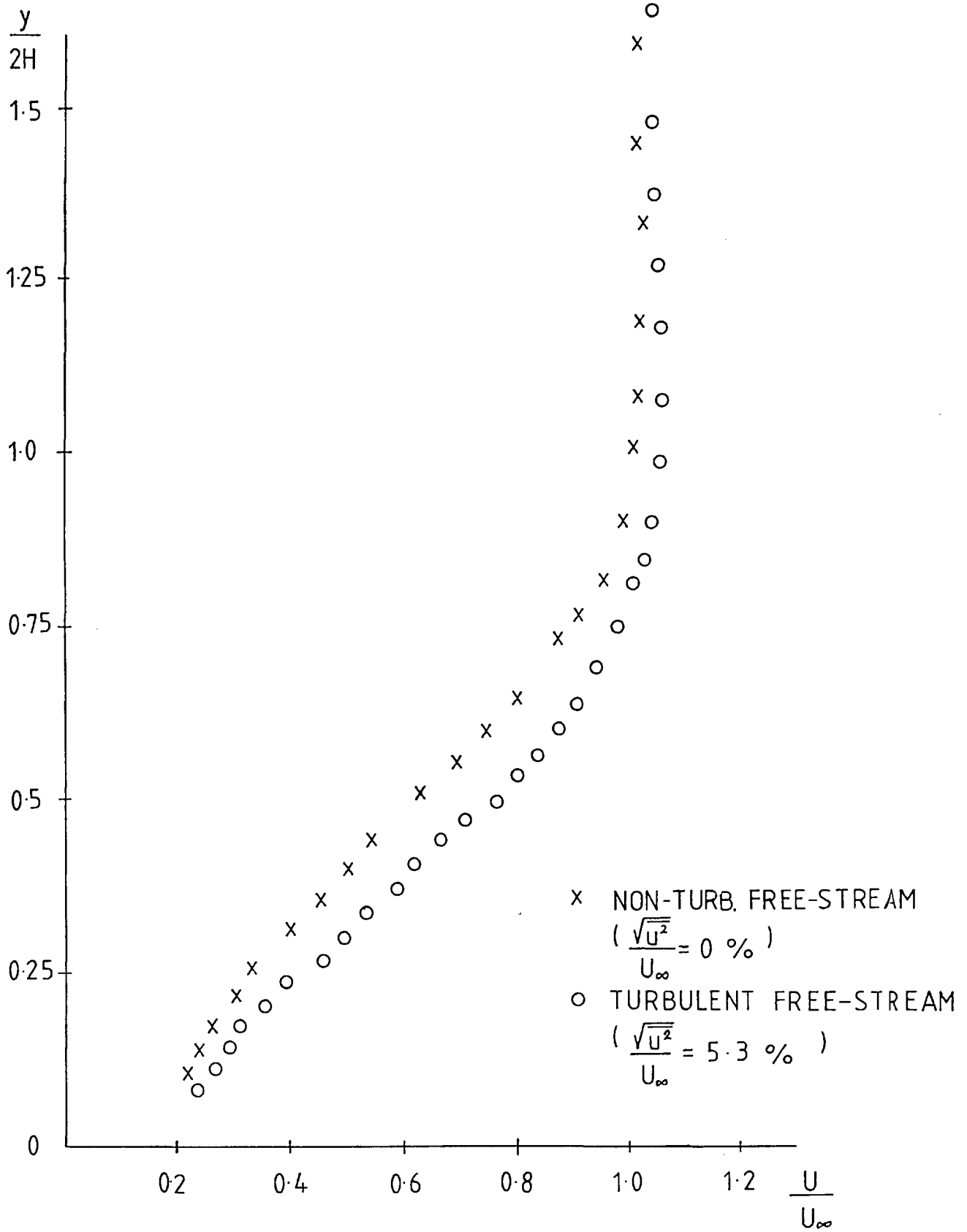


Figure 7.4 The mean velocity profiles at reattachment

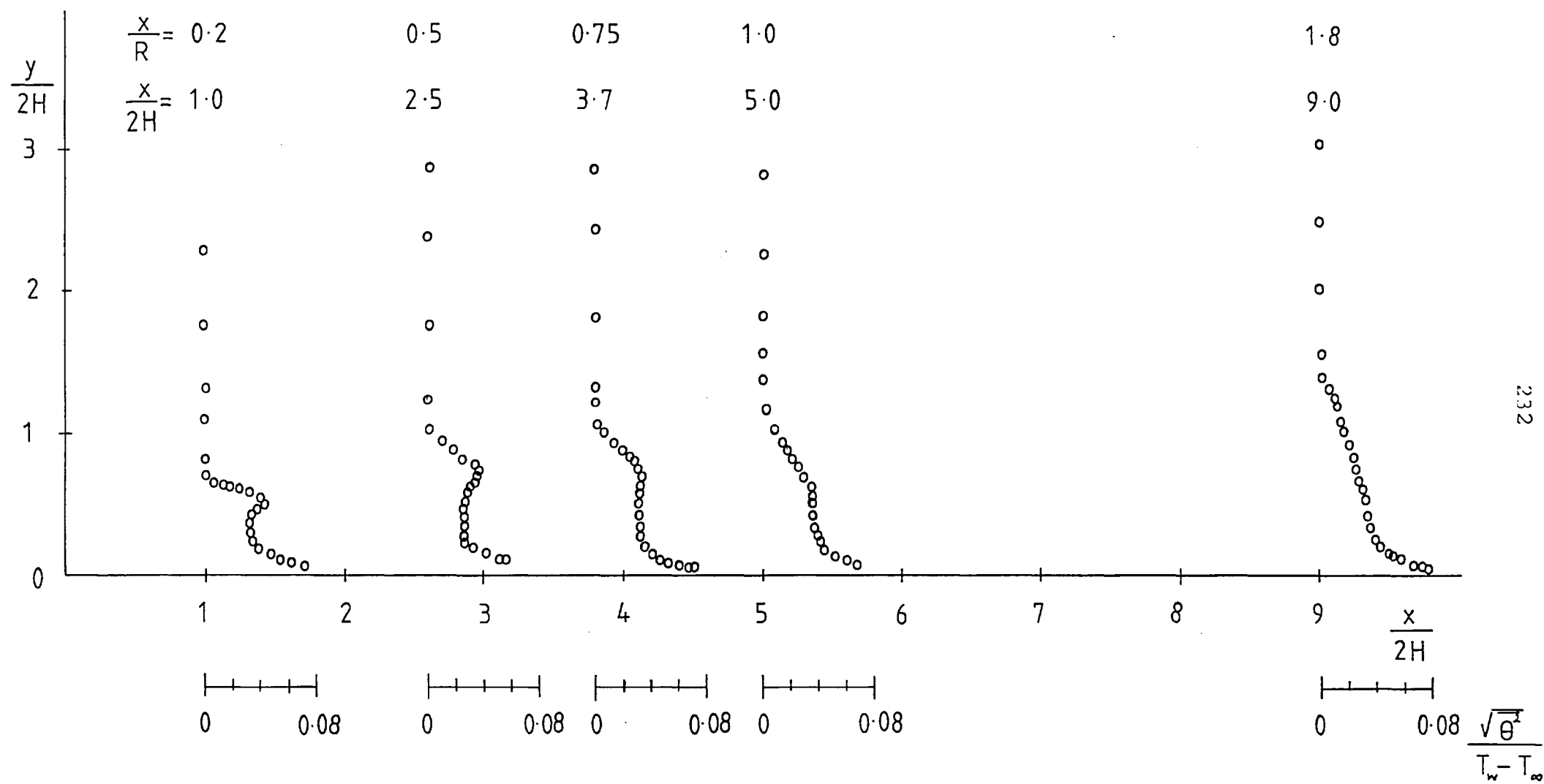


Figure 7.5 The turbulent fluctuating temperature profiles along the blunt flat plate

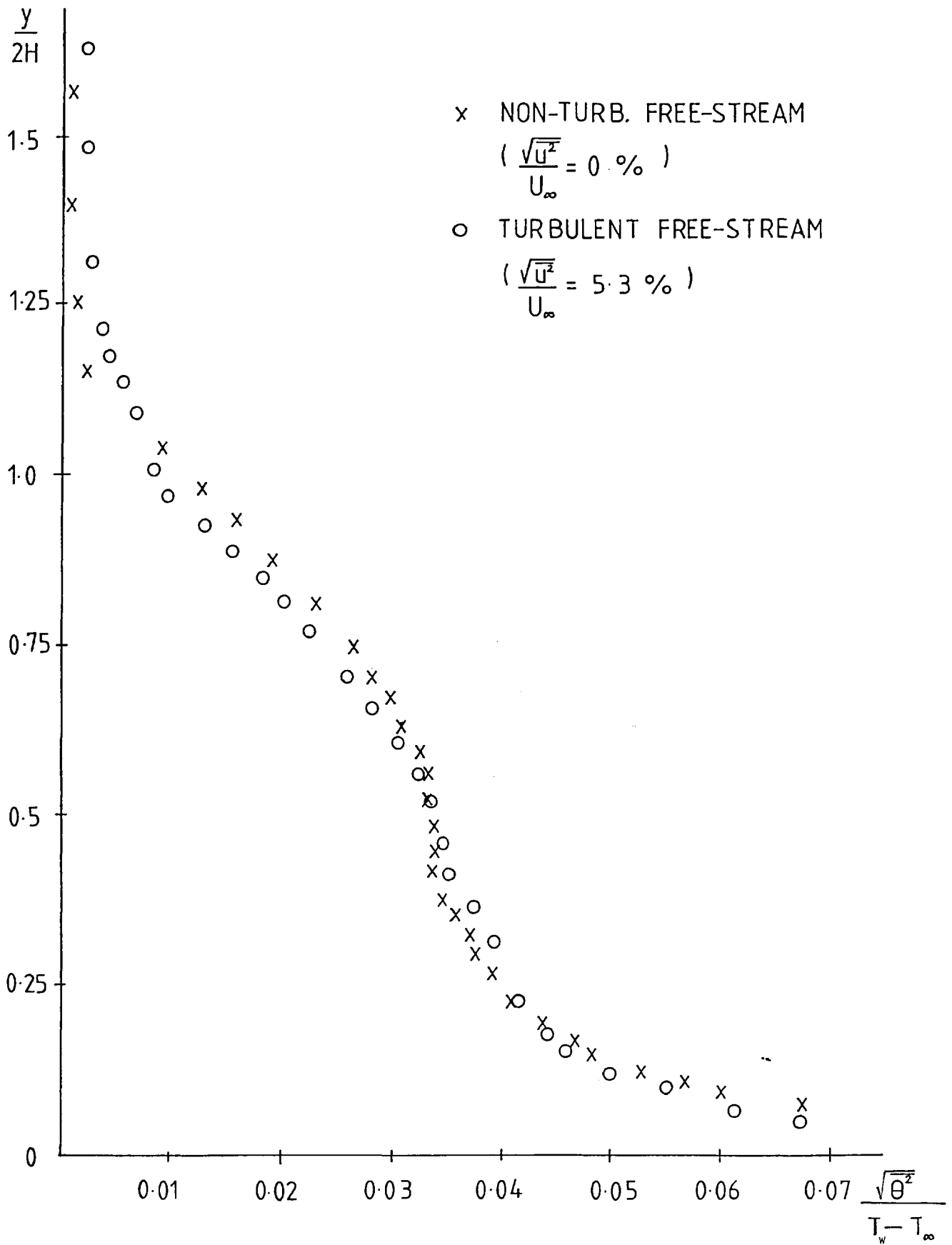


Figure 7.6 The turbulent fluctuating temperature profiles at reattachment

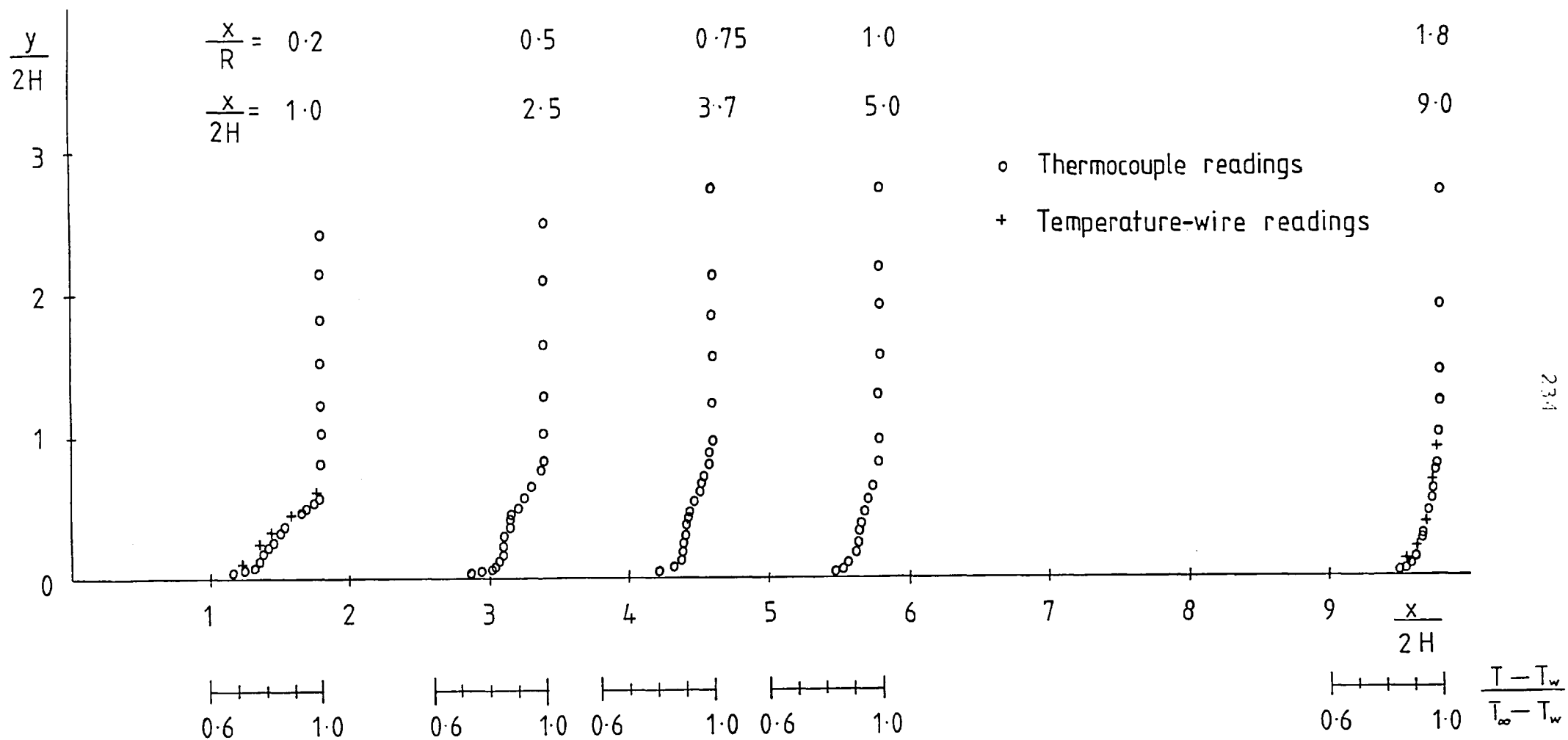


Figure 7.7 The mean temperature profiles along the blunt flat plate

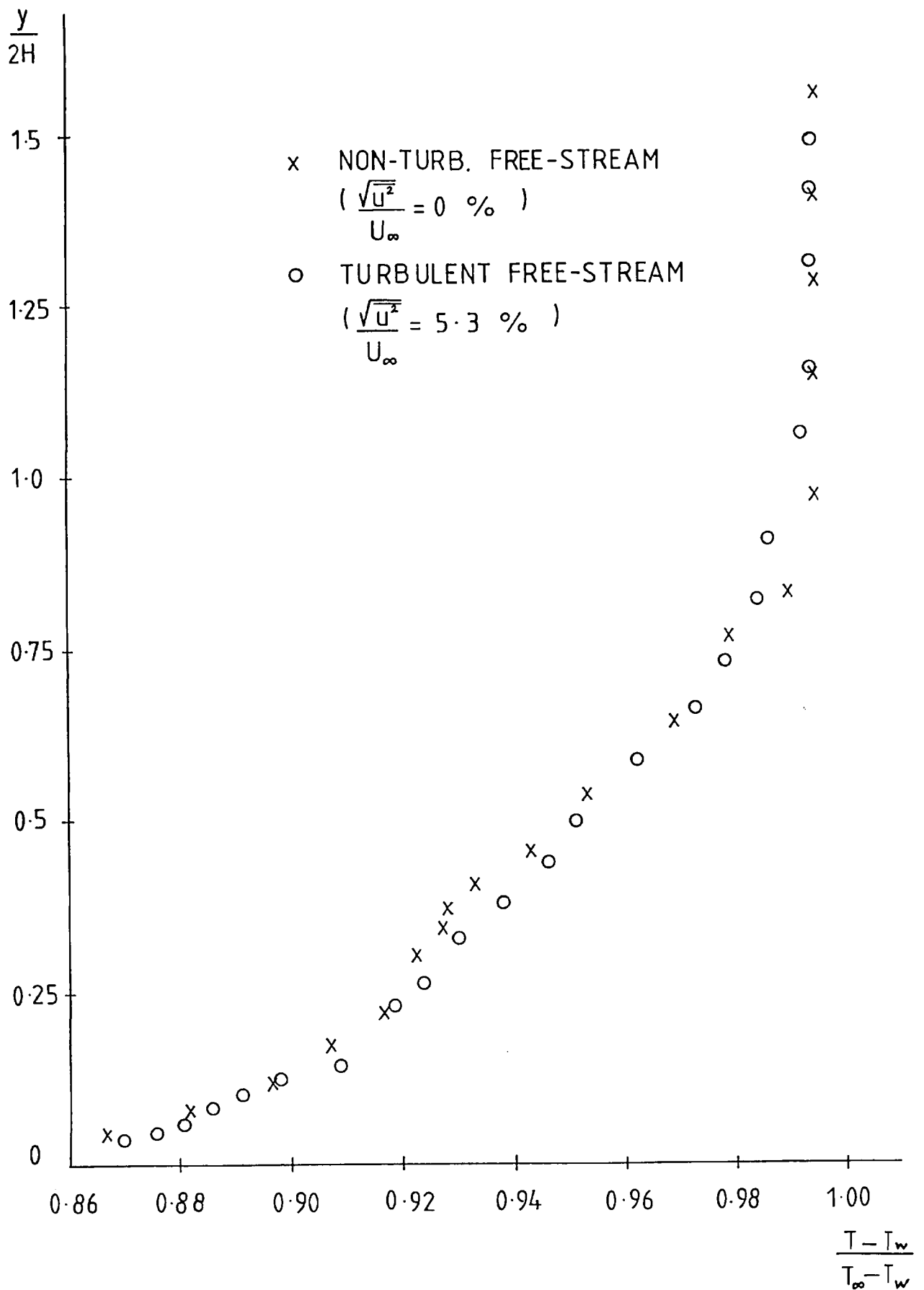
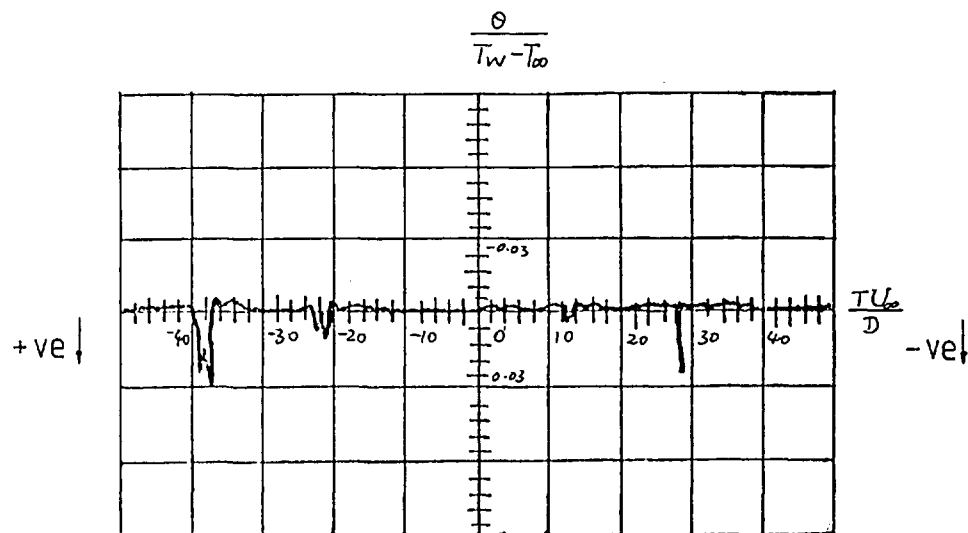
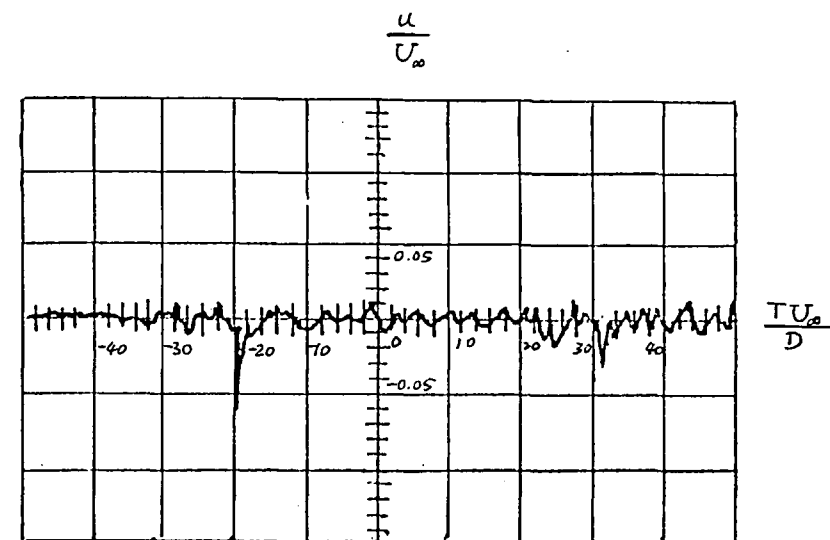


Figure 7.8 The mean temperature profiles at reattachment



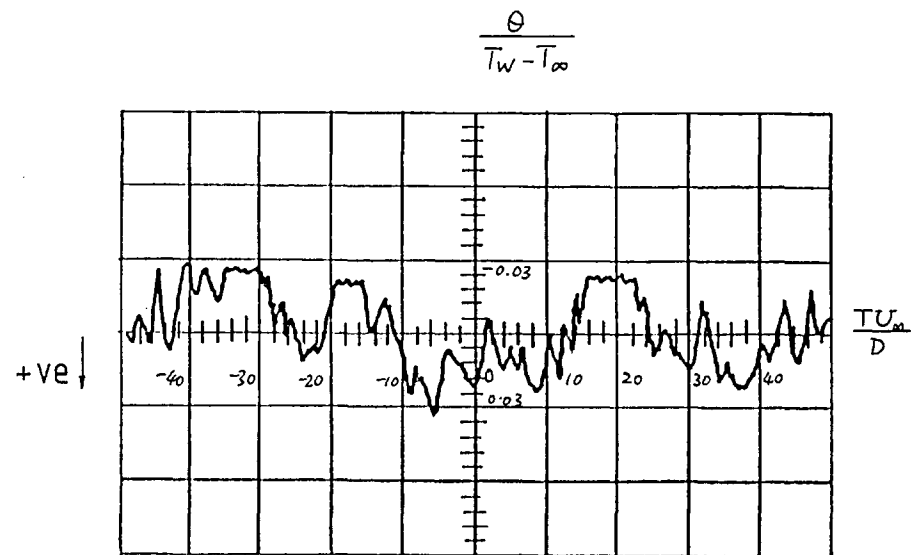
horizontal scale : 20 msec/cm
vertical scale : 0.5 V/cm

Figure 7.9a Turbulent fluctuating temperature
oscillogram at $y/2H=1.07$ $x/R=1$



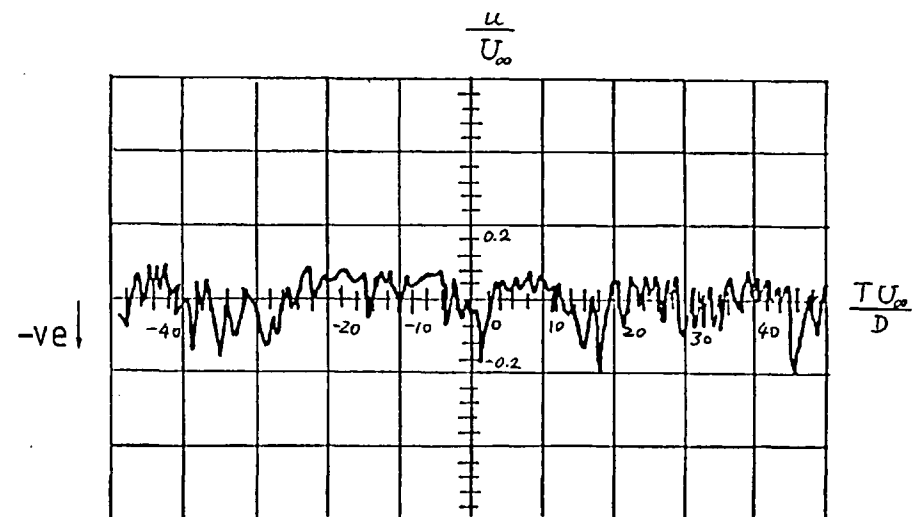
horizontal scale : 20 msec/cm
vertical scale : 0.5 V/cm

Figure 7.9b Turbulent fluctuating velocity
oscillogram at $y/2H=1.07$ $x/R=1$



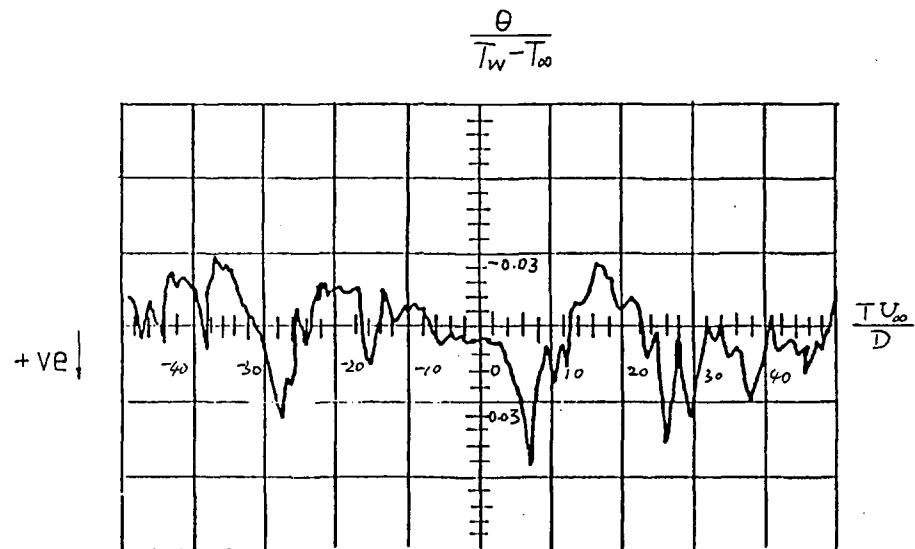
horizontal scale : 5 msec/cm
vertical scale : 0.5 V/cm

Figure 7.10a Turbulent fluctuating temperature
oscillogram at $y/2H=0.54$ $x/R=1$



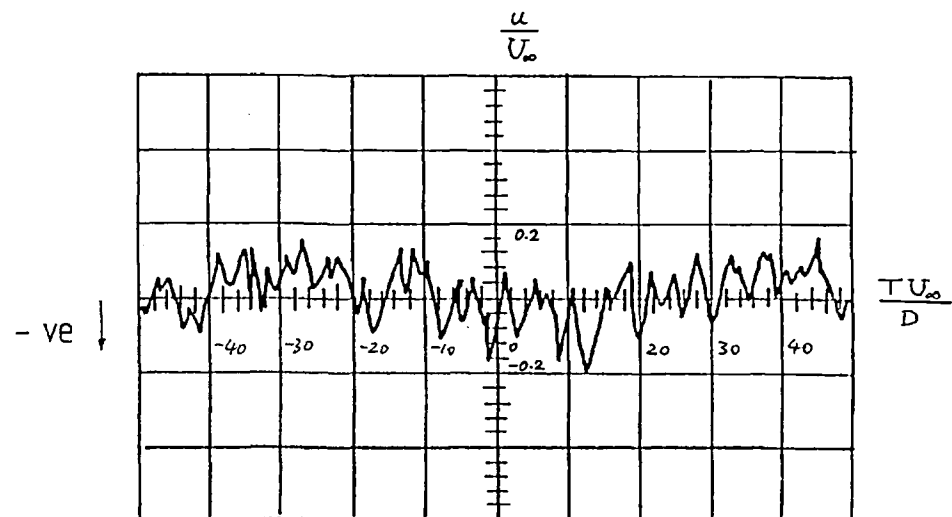
horizontal scale : 5 msec/cm
vertical scale : 0.5 V/cm

Figure 7.10b Turbulent fluctuating velocity
oscillogram at $y/2H=0.54$ $x/R=1$



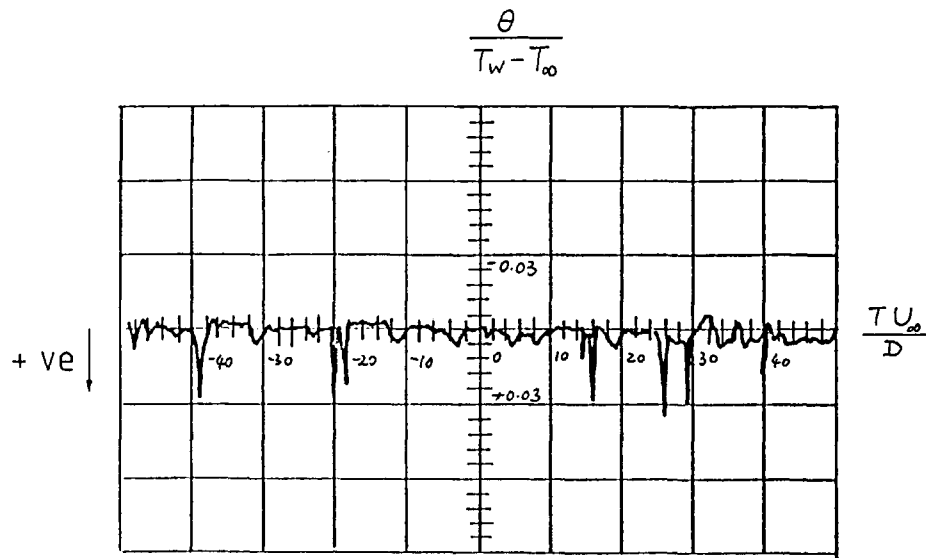
horizontal scale : 5 msec/cm.
vertical scale : 0.5 V/cm

Figure 7.11a Turbulent fluctuating temperature
oscillogram at $y/2H=0.10$ $x/R=1$



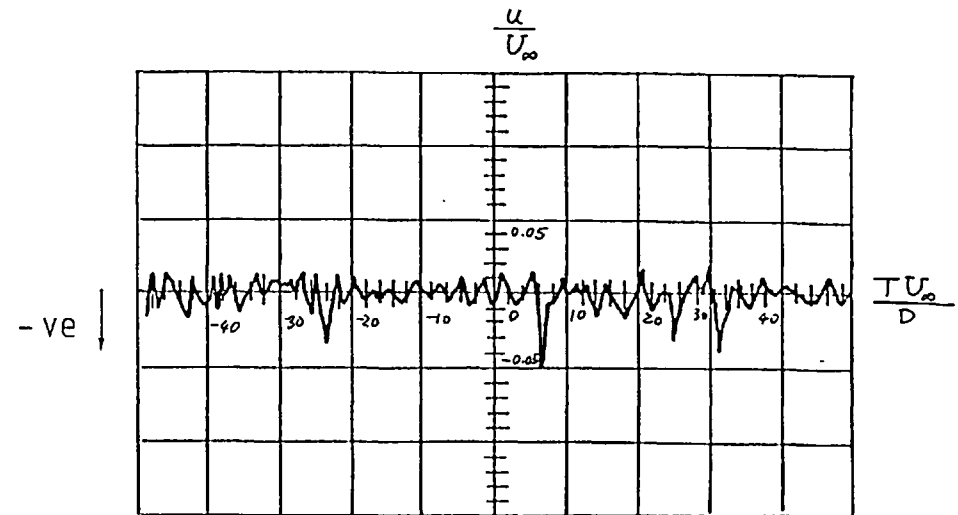
horizontal scale : 5 msec/cm
vertical scale : 0.5 V/cm

Figure 7.11b Turbulent fluctuating velocity
oscillogram at $y/2H=0.10$ $x/R=1$



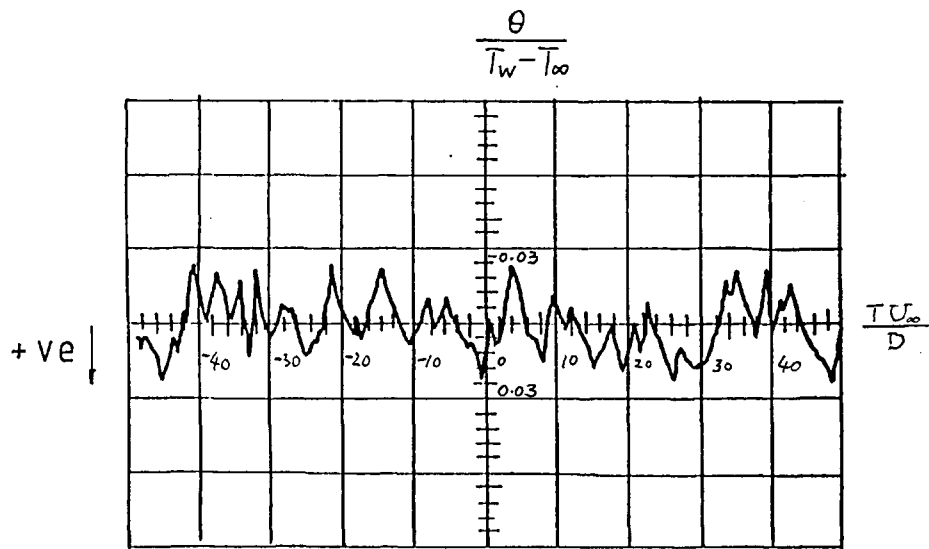
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vertical scale : 0.5 V/cm

Figure 7.12a Turbulent fluctuating temperature
oscillogram at $y/2H=0.97$ $x/R=0.5$



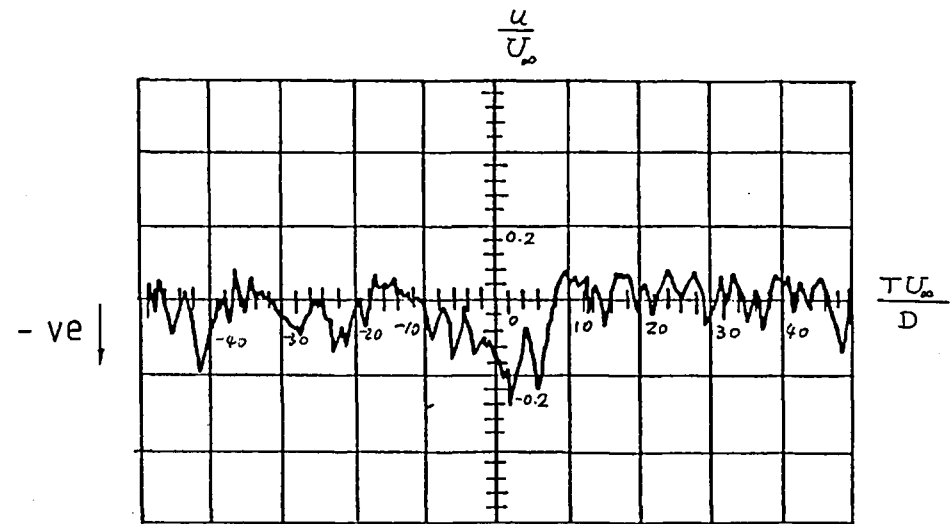
horizontal scale : 20 msec/cm
vertical scale : 0.5 V/cm

Figure 7.12b Turbulent fluctuating velocity
oscillogram at $y/2H=0.97$ $x/R=0.5$



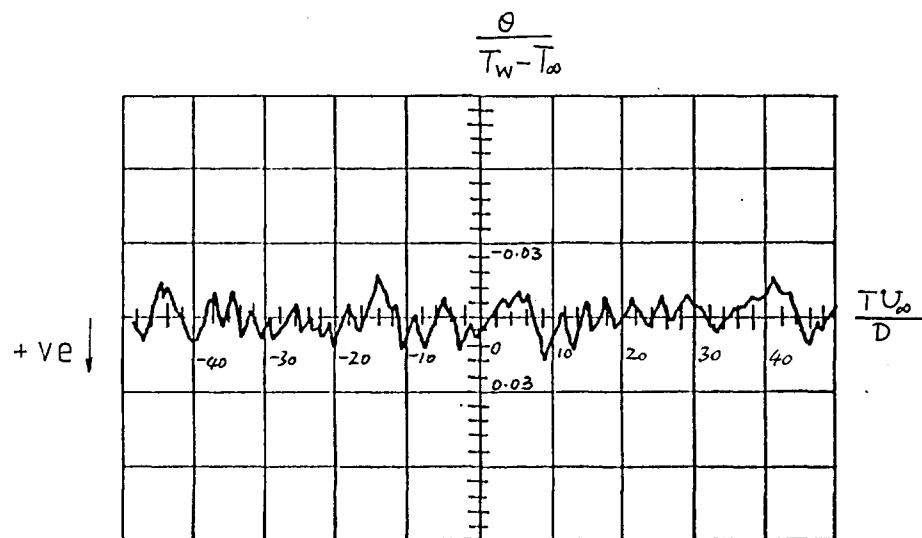
horizontal scale : 5 msec/cm
vertical scale : 0.5 V/cm

Figure 7.13a Turbulent fluctuating temperature
oscillogram at $y/2H=0.56$ $x/R=0.5$



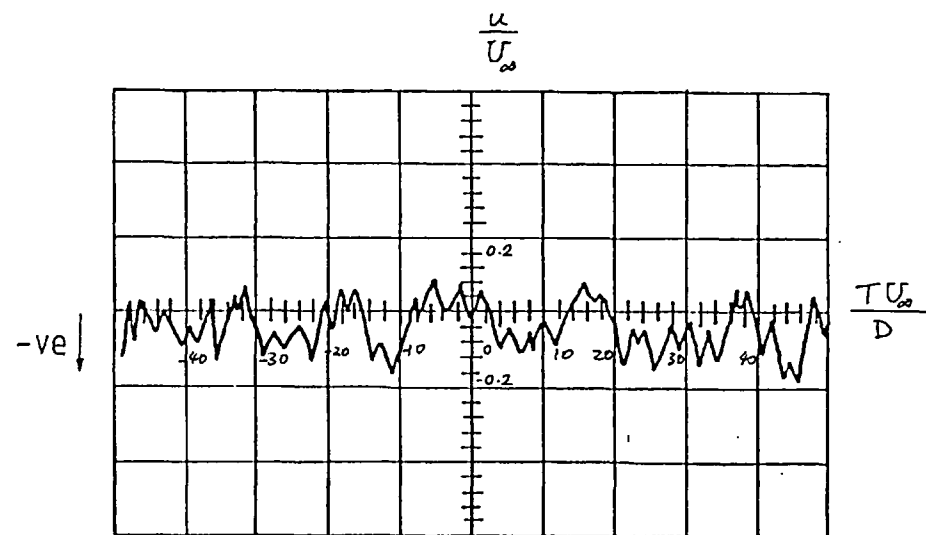
horizontal scale : 5 msec/cm
vertical scale : 0.5 V/cm

Figure 7.13b Turbulent fluctuating velocity
oscillogram at $y/2H=0.56$ $x/R=0.5$



horizontal scale : 5 msec/cm
 vertical scale : 0.5 V/cm

Figure 7.14a Turbulent fluctuating temperature
 oscillogram at $y/2H=0.17$ $x/R=0.5$



horizontal scale : 5 msec/cm
 vertical scale : 0.5 V/cm

Figure 7.14b Turbulent fluctuating velocity
 oscillogram at $y/2H=0.17$ $x/R=0.5$

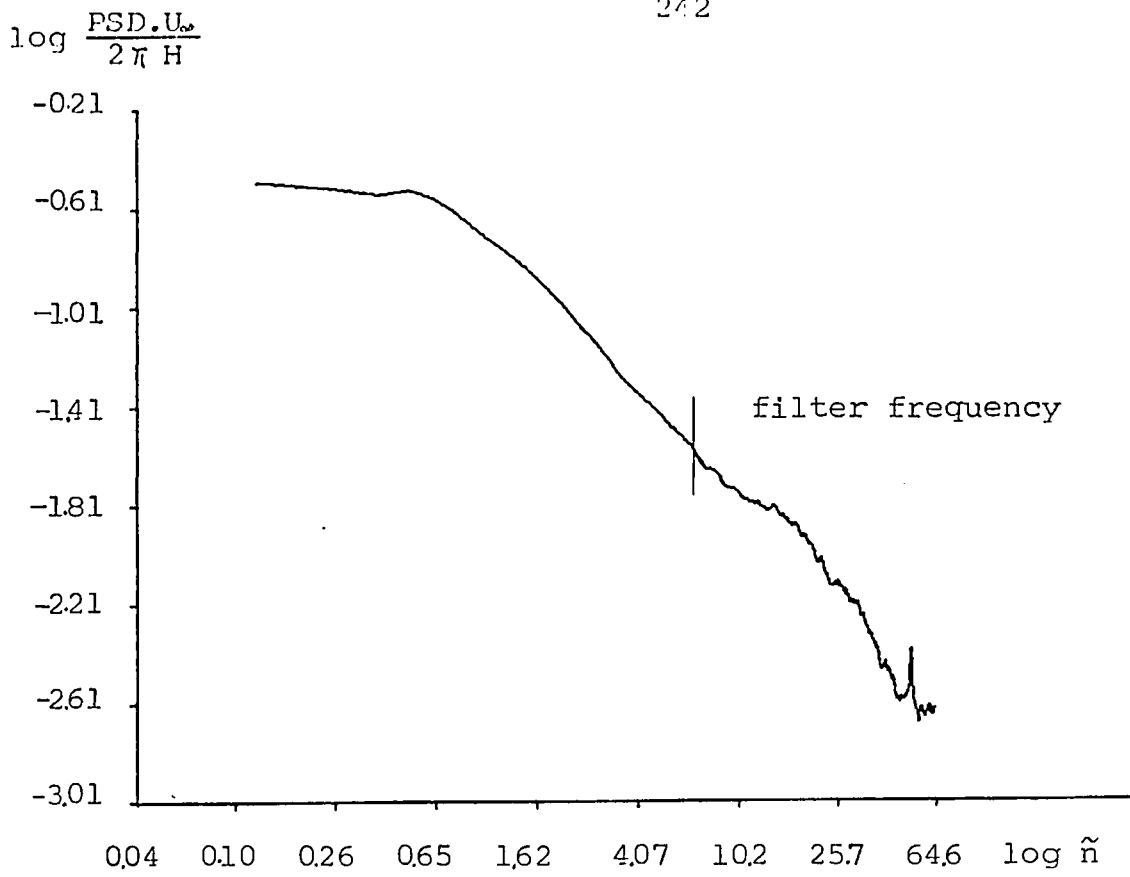


Figure 7.15a Turbulent temperature spectrum at $y/2H=1.07$ $x/R=1$

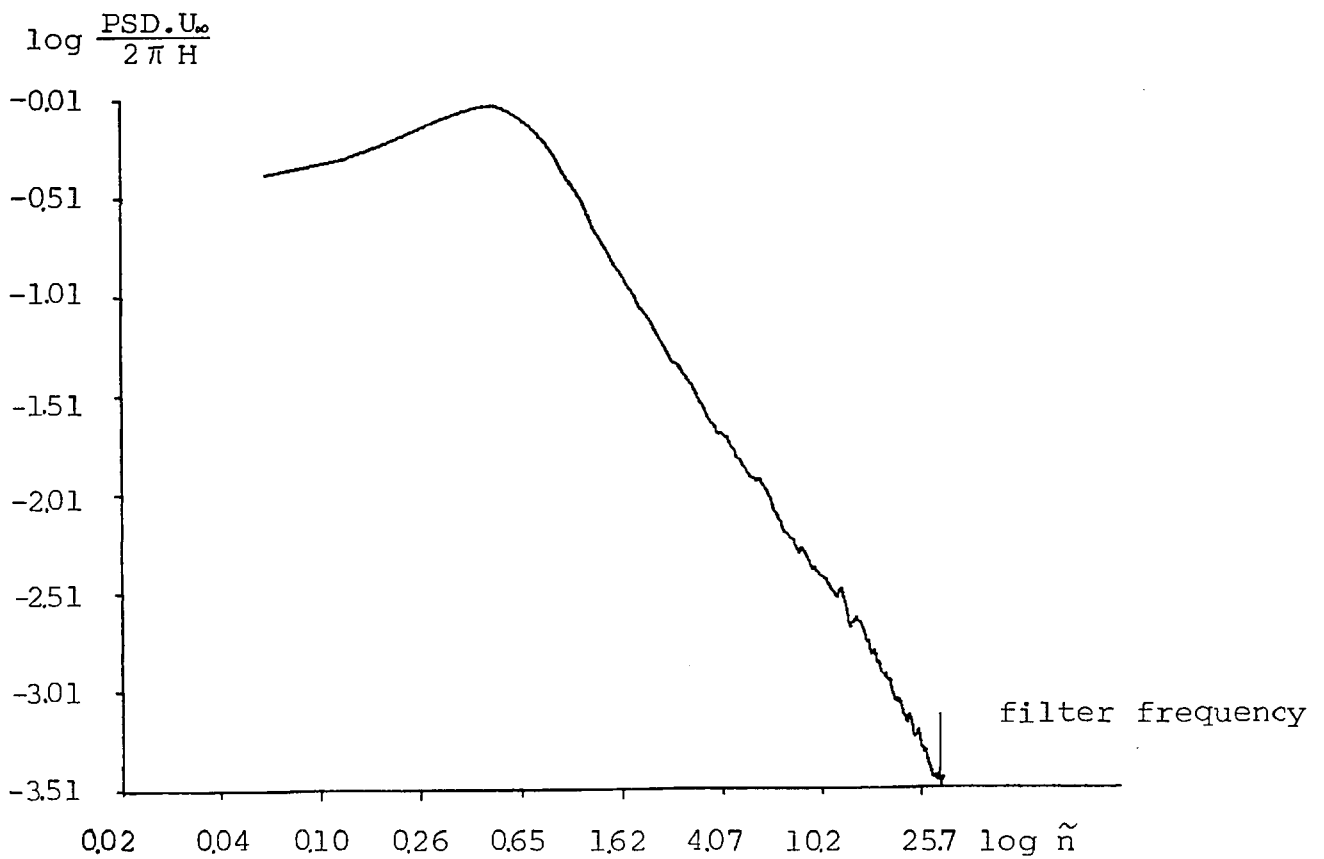


Figure 7.15b Turbulent velocity spectrum at $y/2H=1.07$ $x/R=1$

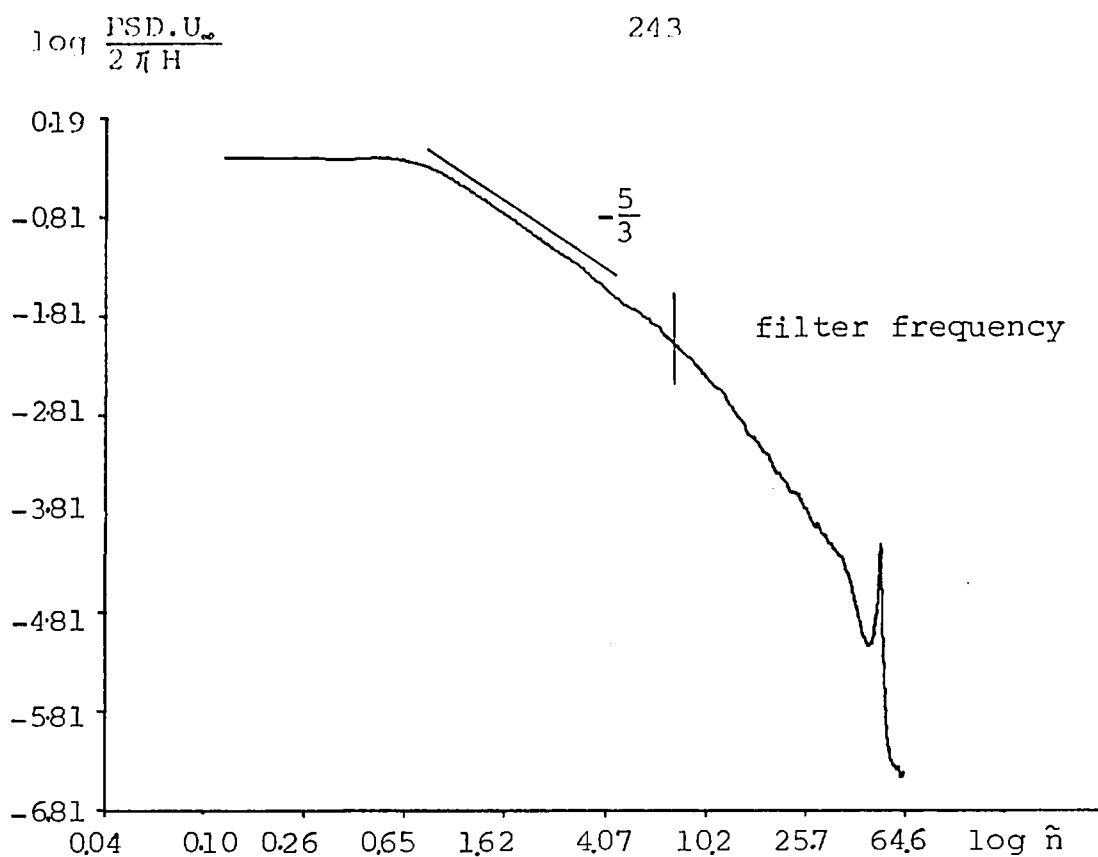


Figure 7.16a Turbulent temperature spectrum at $y/2H=0.54$ $x/R=1$

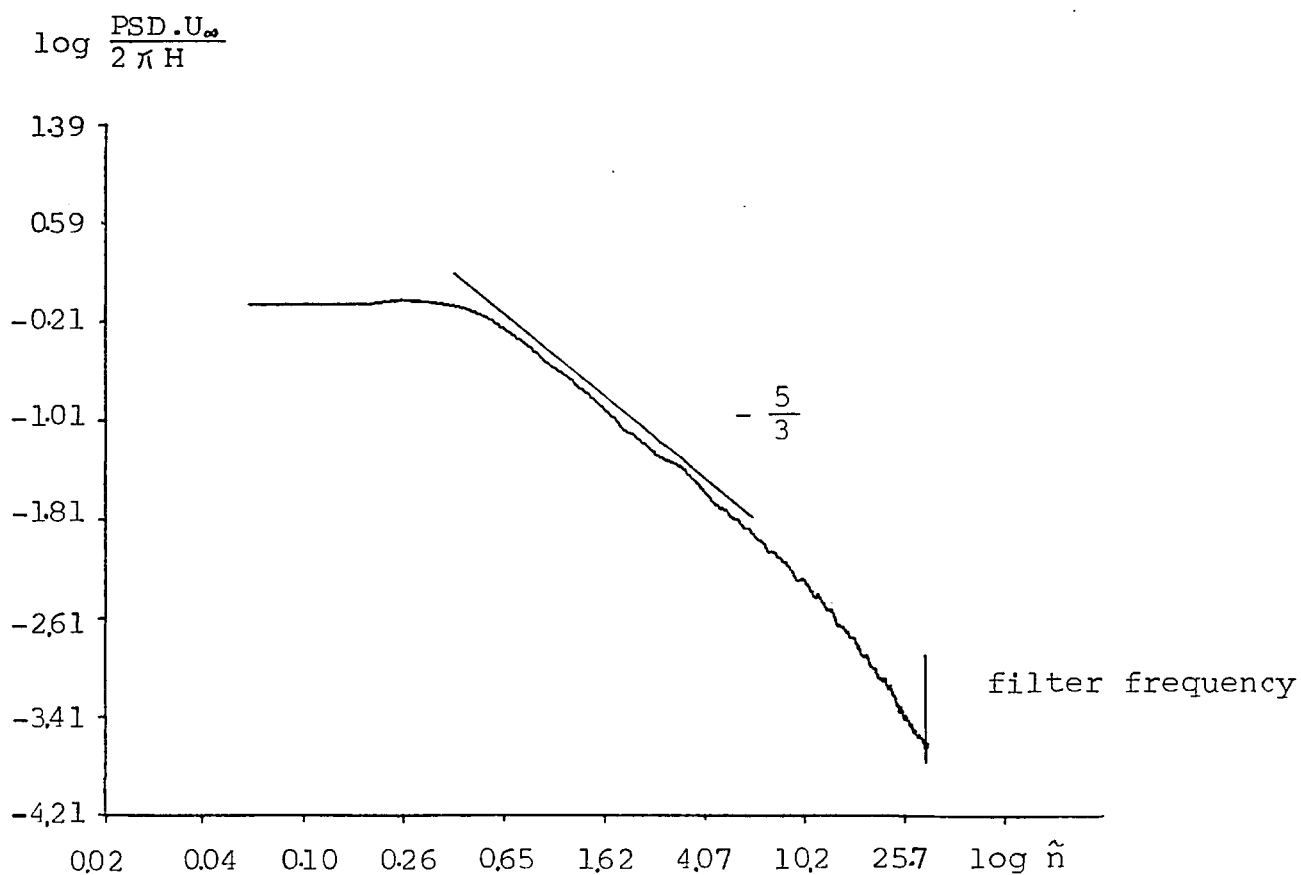


Figure 7.16b Turbulent velocity spectrum at $y/2H=0.54$ $x/R=1$

$$\log \frac{\text{PSD} \cdot U_{\infty}}{2 \pi H}$$

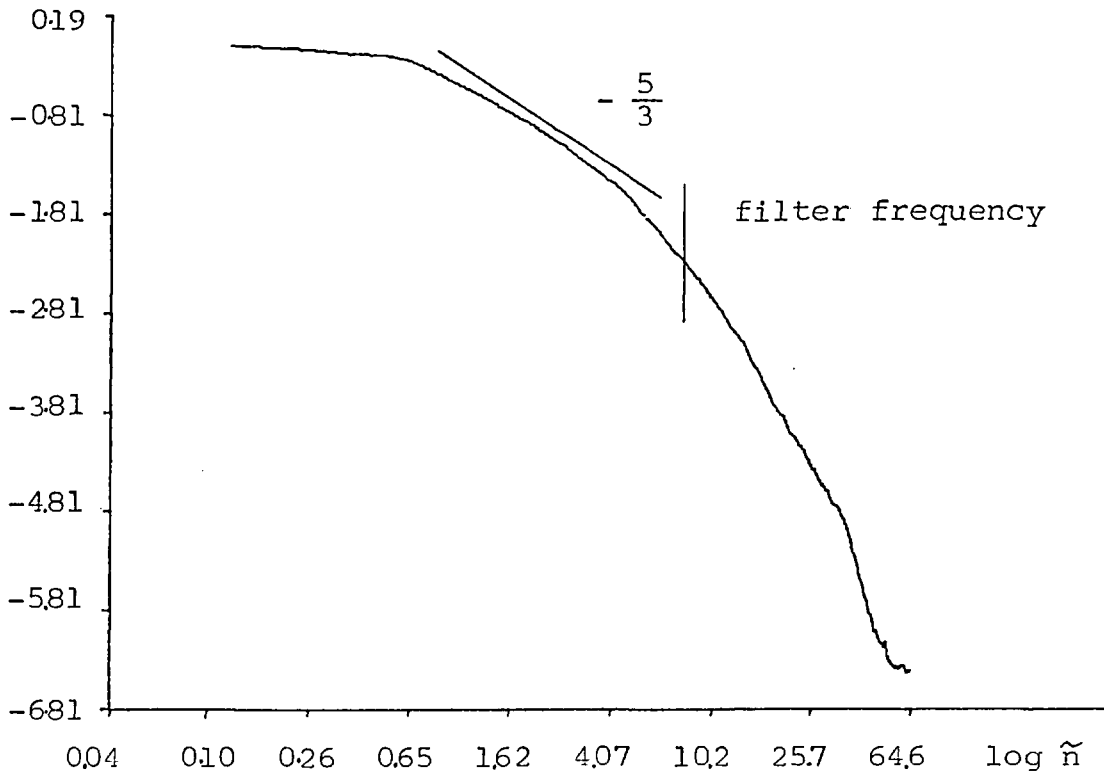


Figure 7.17a Turbulent temperature spectrum at $y/2H=0.10$ $x/R=1$

$$\log \frac{\text{PSD} \cdot U_{\infty}}{2 \pi H}$$

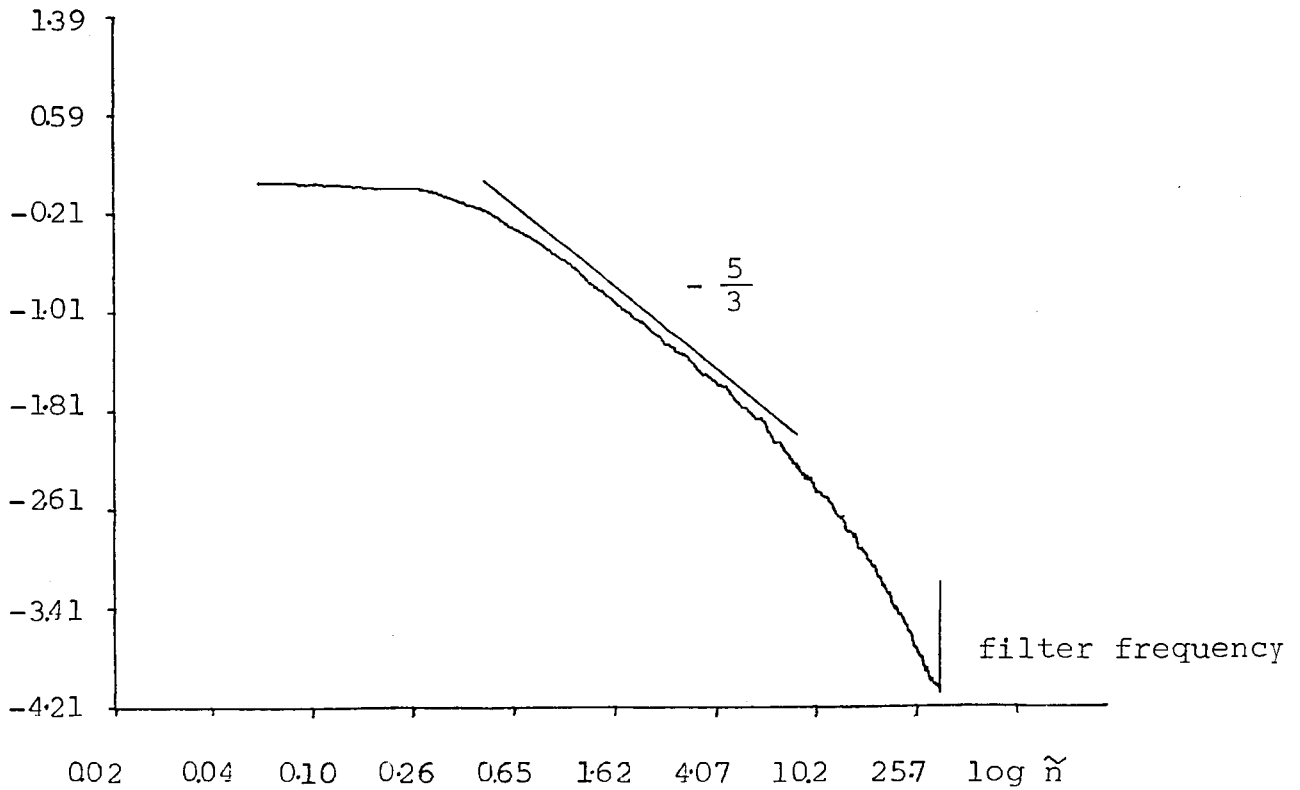


Figure 7.17 b Turbulent velocity spectrum at $y/2H=0.10$ $x/R=1$

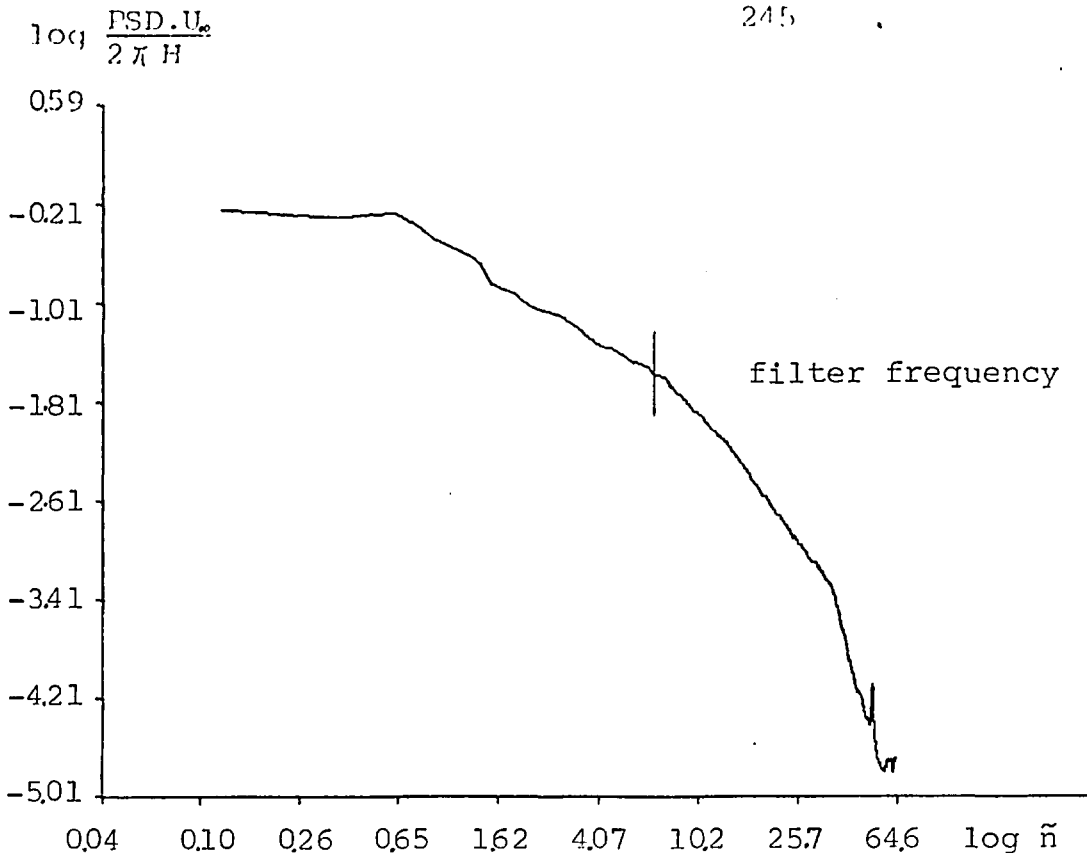


Figure 7.18a Turbulent temperature spectrum at $y/2H=0.97$ $x/R=0.5$

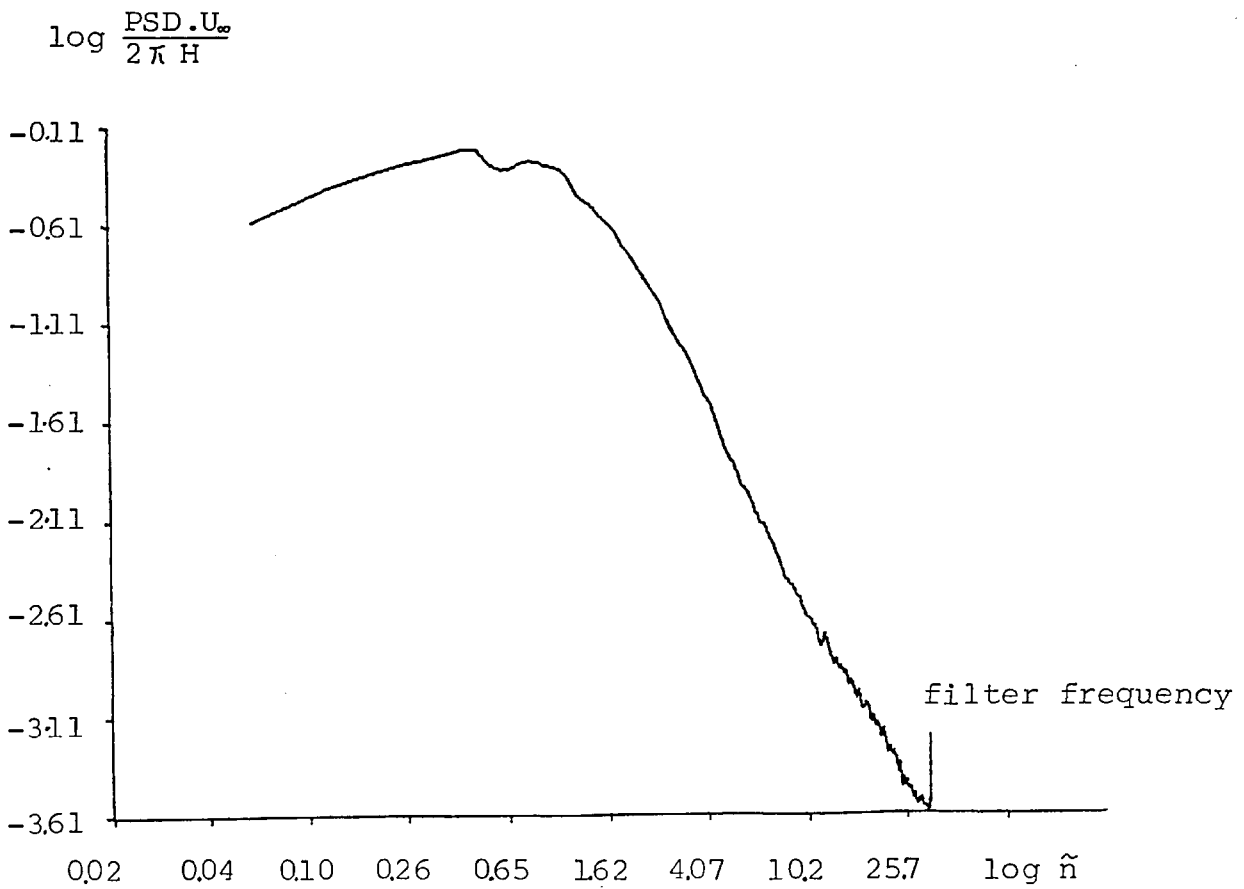


Figure 7.18b Turbulent velocity spectrum at $y/2H=0.97$ $x/R=0.5$

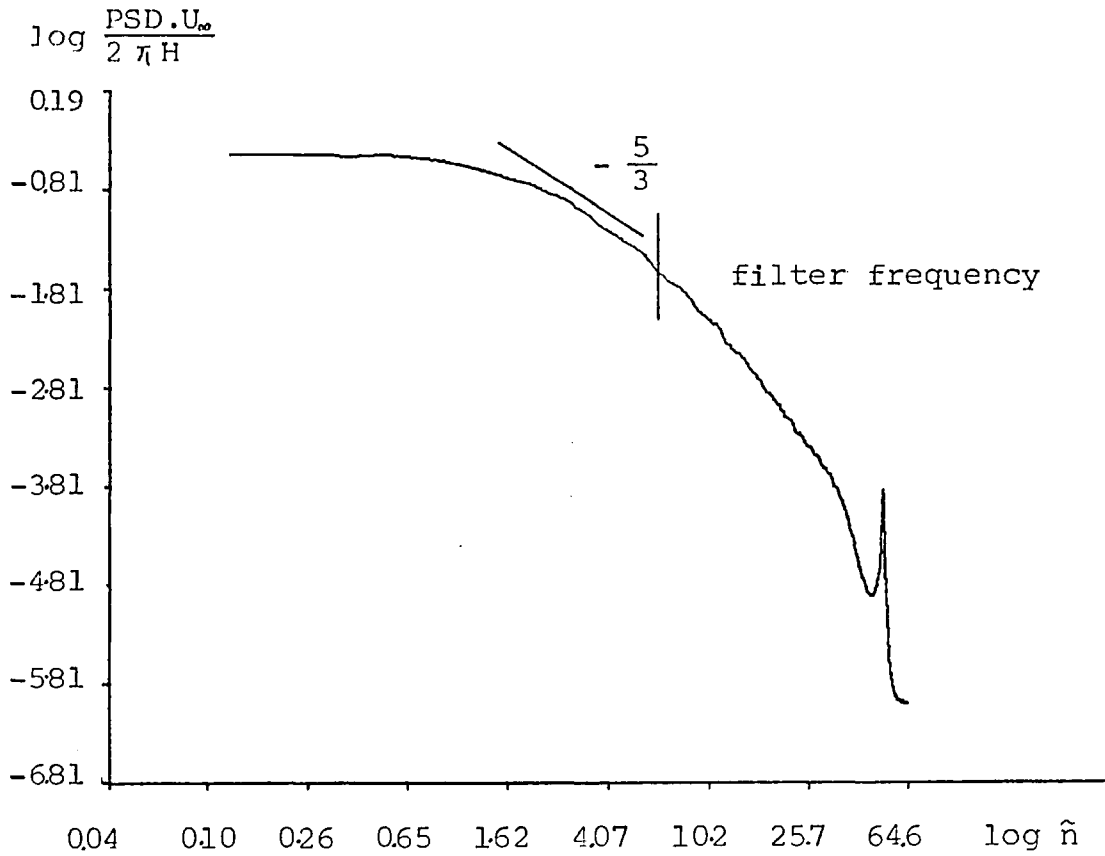


Figure 7.19a Turbulent temperature spectrum at $y/2H=0.56$ $x/R=0.5$

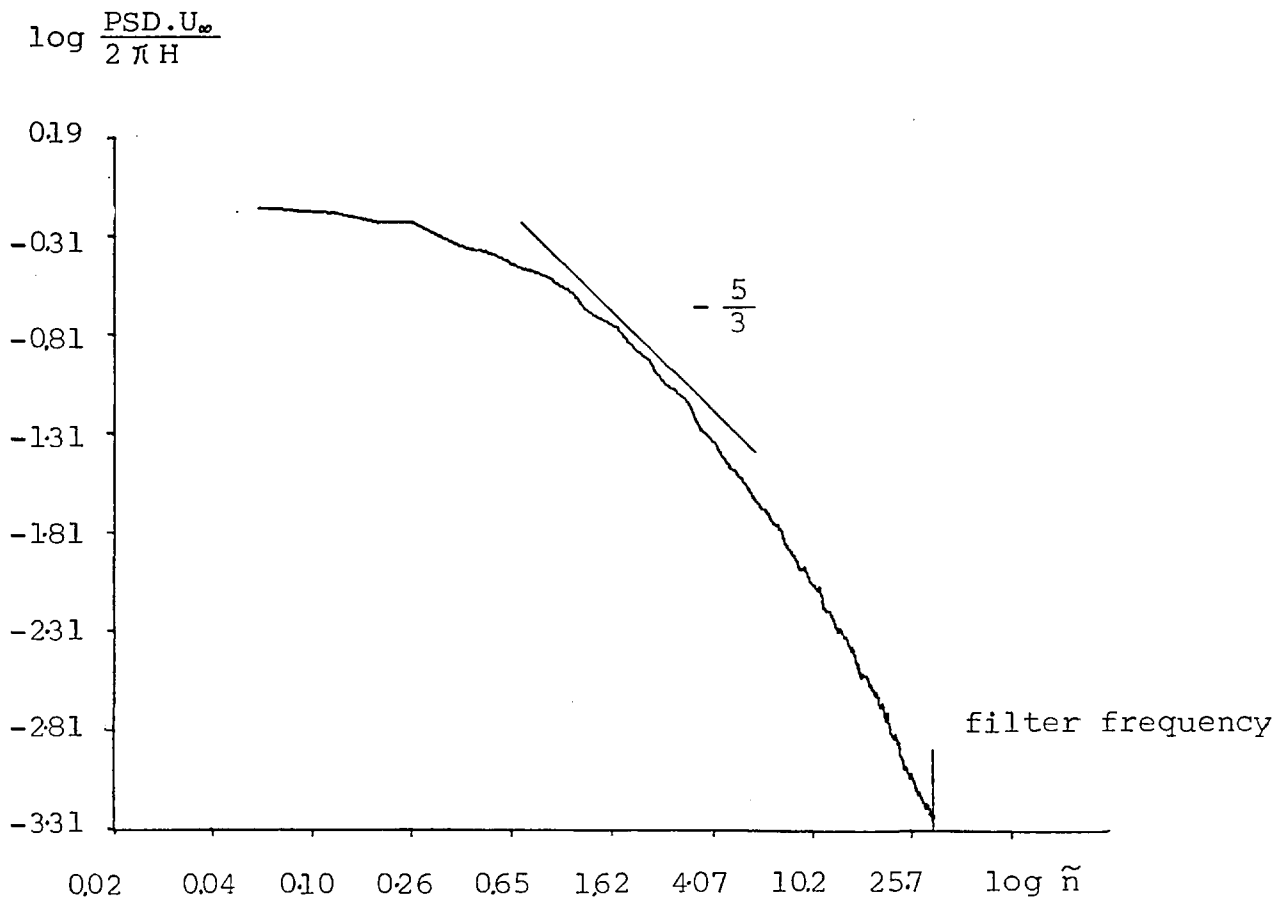


Figure 7.19b Turbulent velocity spectrum at $y/2H=0.54$ $x/R=0.5$

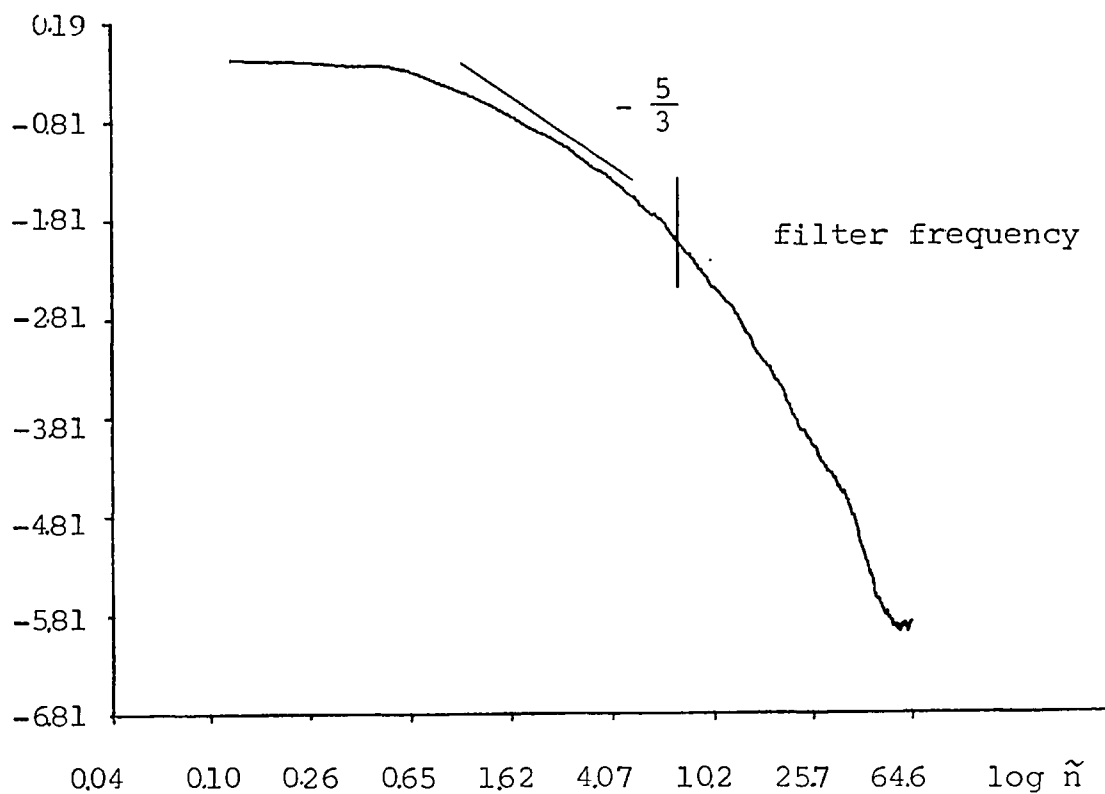


Figure 7.20a Turbulent temperature spectrum at $y/2H=0.17$ $x/R=0.5$

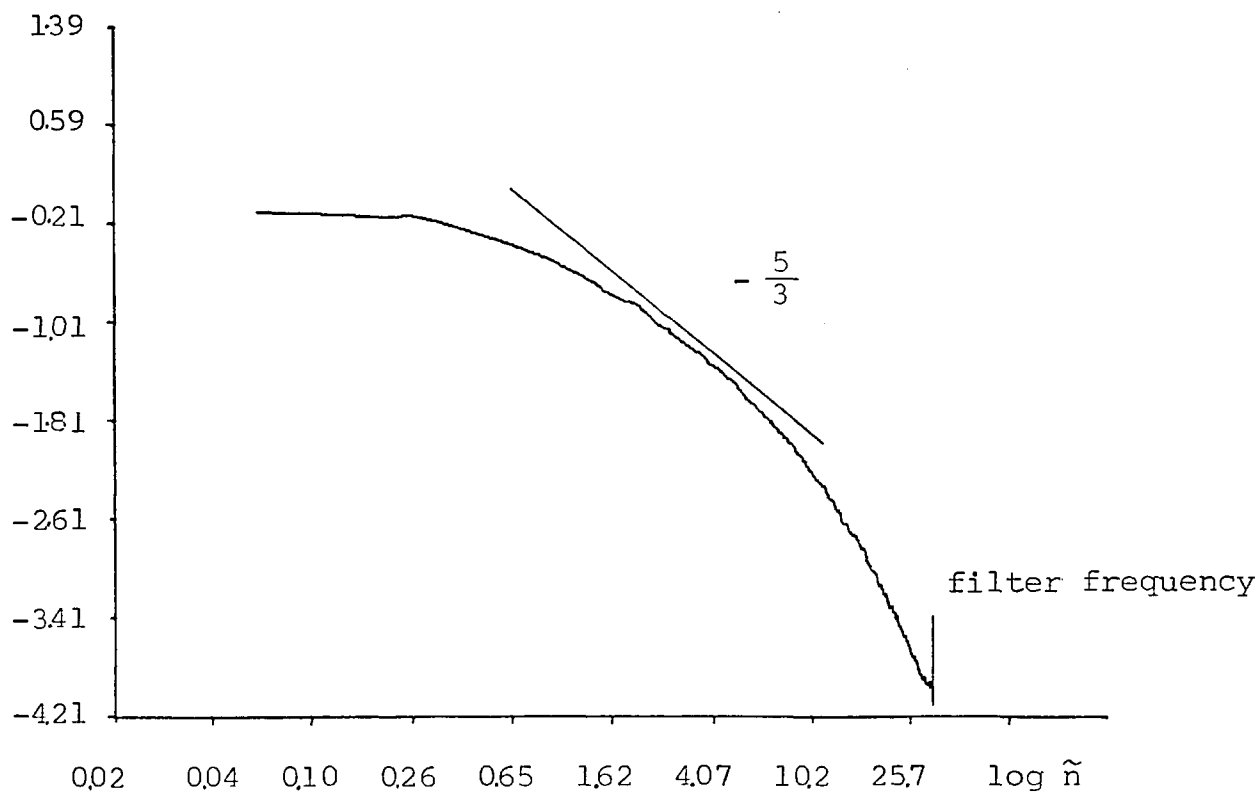


Figure 7.20b Turbulent velocity spectrum at $y/2H=0.17$ $x/R=0.5$

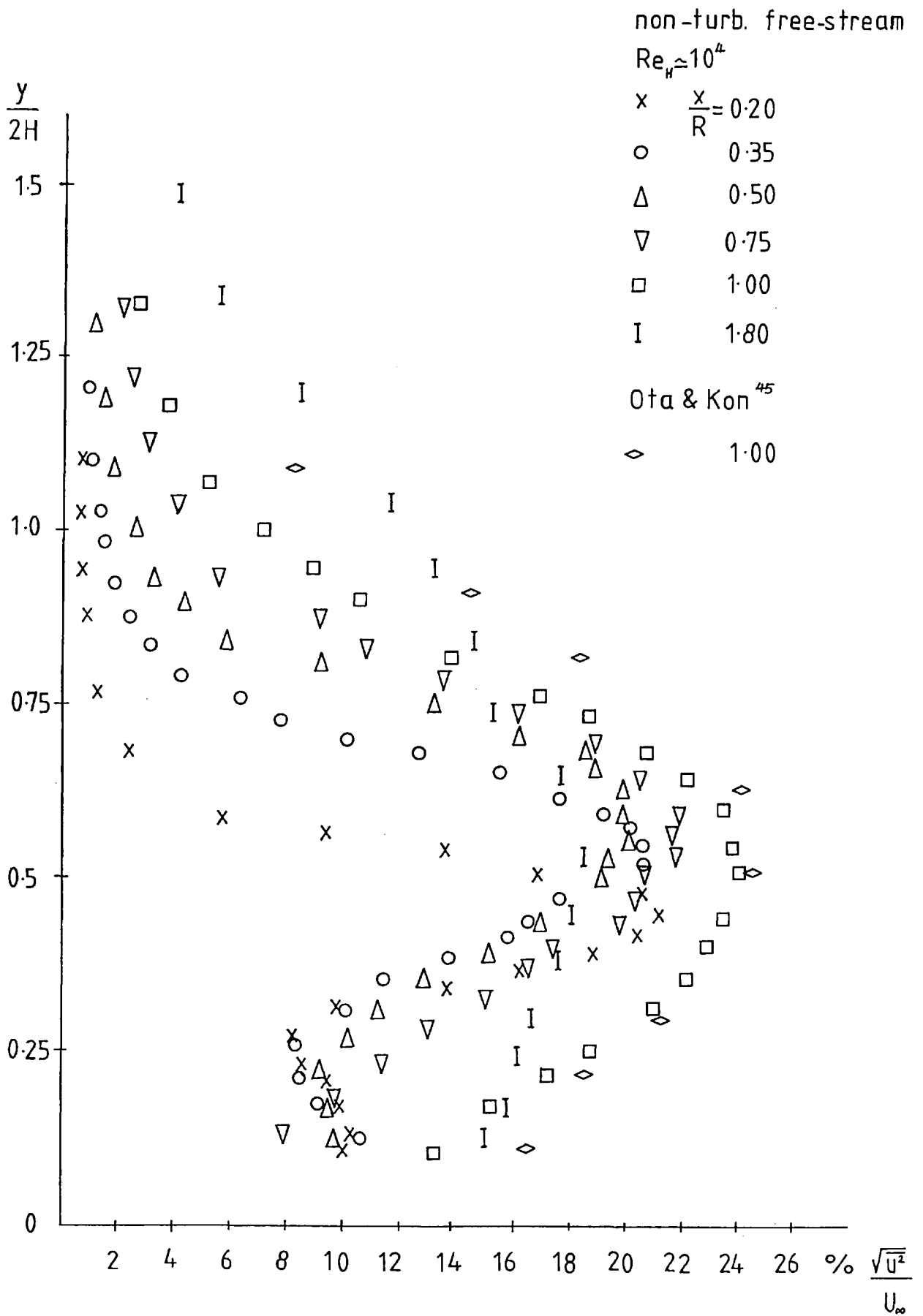


Figure 7.21 The u -component turbulent fluctuating velocity distributions

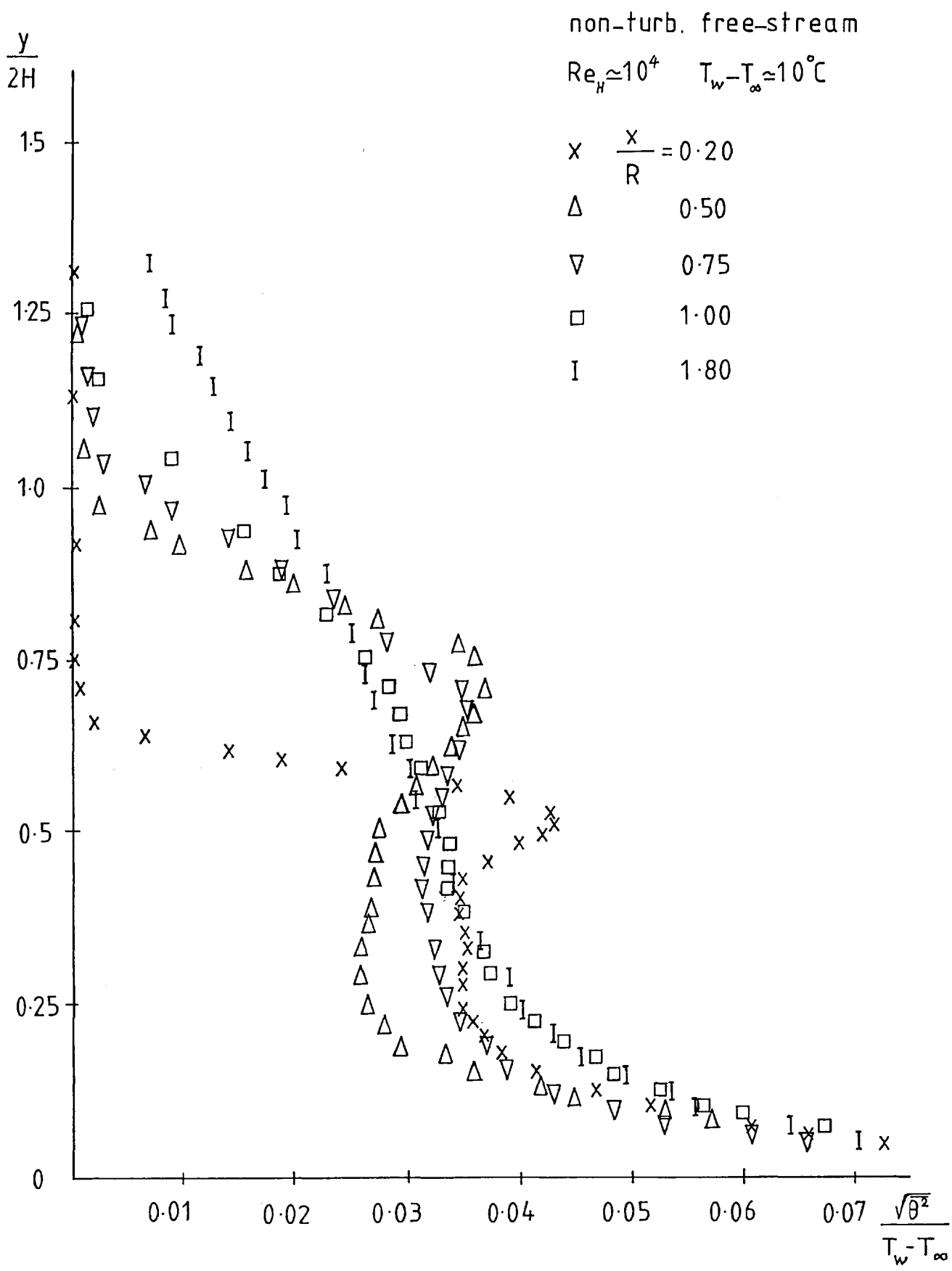


Figure 7.22 The turbulent fluctuating temperature distributions

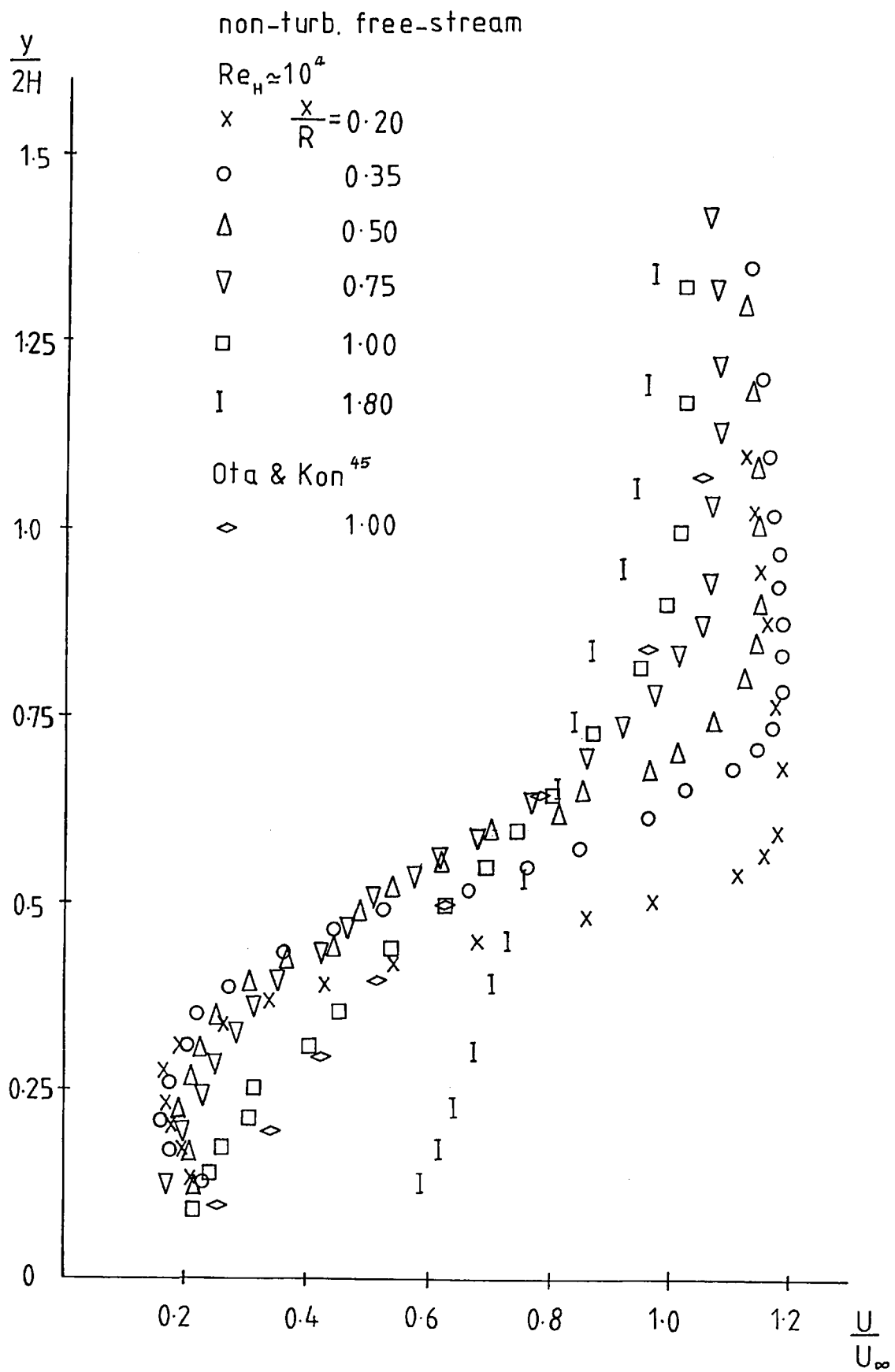
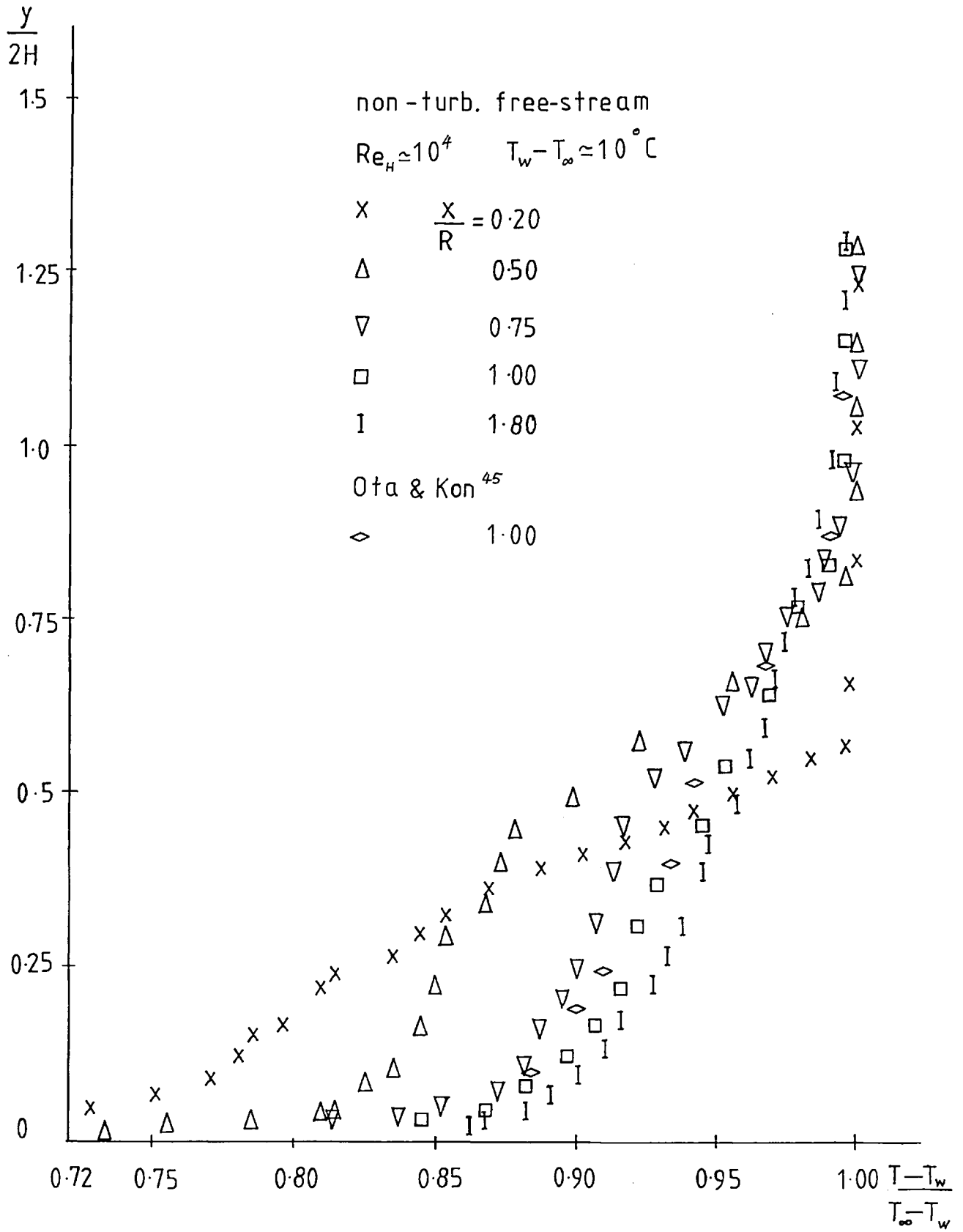


Figure 7.23 The mean velocity distributions



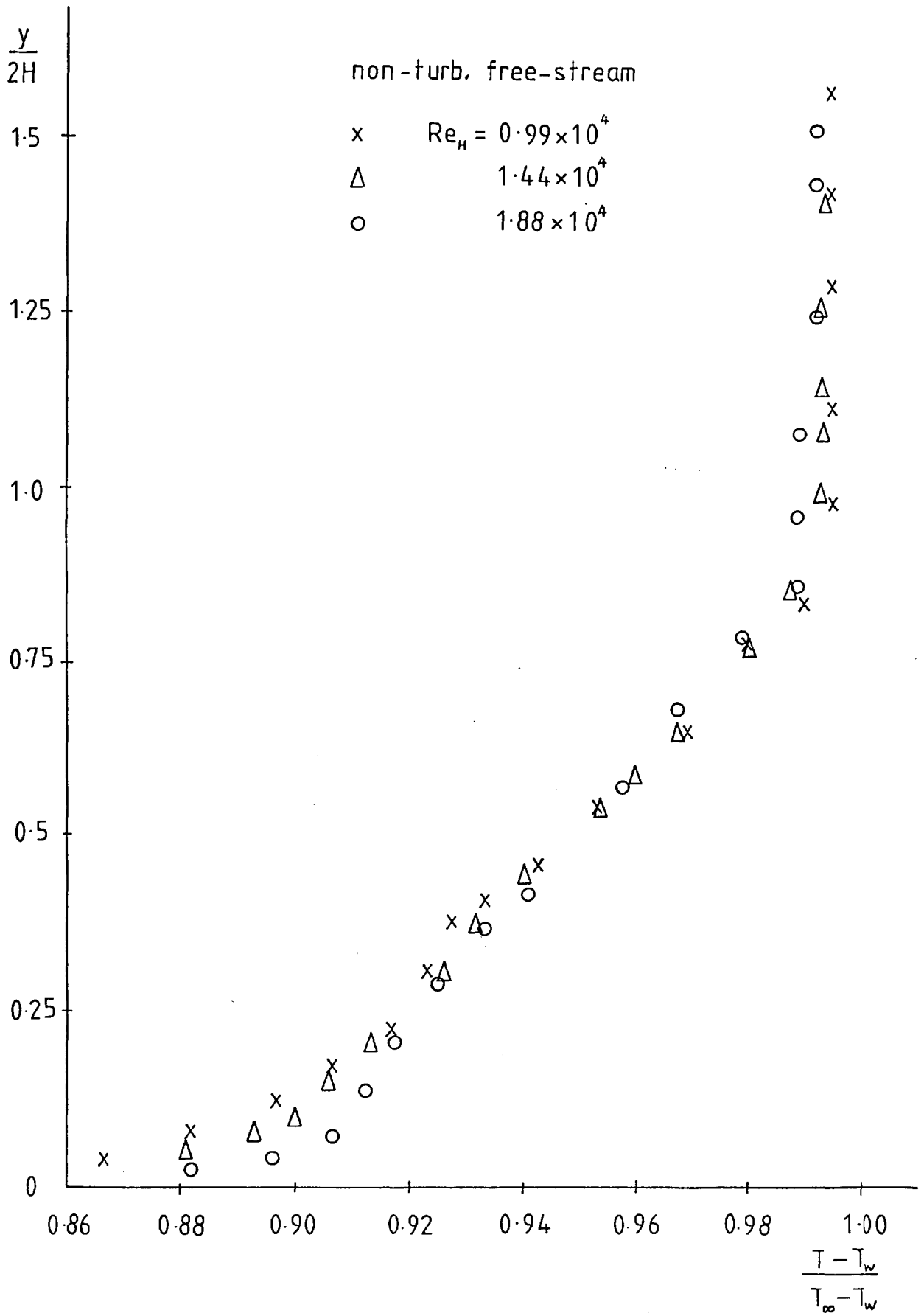


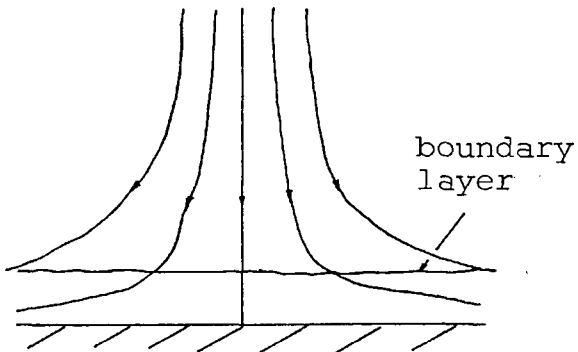
Figure 7.25 The effect of Reynolds number on the mean temperature profiles at reattachment

General Discussion and Conclusions

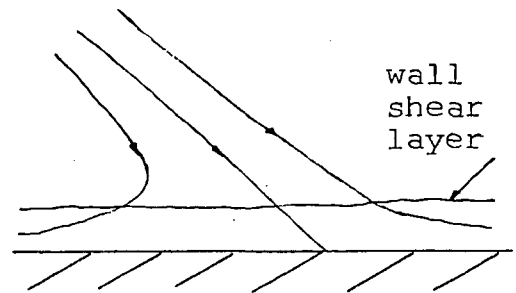
The objective of this chapter is to bring together the similarities and differences between heat transfer at the stagnation point region and at the reattachment region. The main points of the results of the present investigation are recapitulated in this chapter. The present investigation in separated flow heat transfer is by no means complete, some suggestions for further work are given.

8.1 Heat Transfer at the Stagnation Point and the Reattachment Position

Superficially, the flow fields at the stagnation point region of the normal flat plate and at the reattachment region of the blunt flat plate are quite similar (see figures below). However, the present research finds a great deal of differences in heat transfer between the two cases. For the stagnation point, Nu varies as $Re^{0.5}$ and has the minimum heat transfer rate, whereas, Nu_H varies as $Re_H^{0.75}$ in the reattachment position and has the maximum heat transfer rate. It appears that the mechanics of heat transfer is different between the



Streamline pattern at the stagnation point region



Streamline pattern at the reattachment region

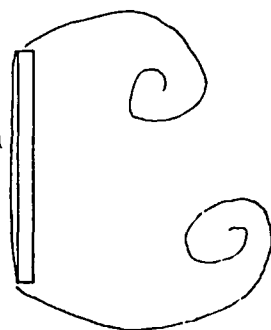
two cases. In order to explain the difference in heat transfer behaviour between the two cases, it is best to analyse the flow fields separately. At the stagnation point region of the normal flat plate, fluid approaching the surface is being divided into two streams. A laminar boundary layer grows along the surface. The fluid velocity at the edge of the boundary layer increases away from the stagnation point, but the velocity gradient is constant at this point. When the plate is heated, a thermal layer of finite thickness is developed on the surface. The rate of heat transfer at the stagnation point is governed by the velocity gradient $(\frac{d v / U_{\infty}}{d y / d})$ which in turn is a function of the base pressure. The Nusselt number in fact varies as $(\frac{d v / U_{\infty}}{d y / d})^{0.5}$ and hence it varies as $Re^{0.5}$. At the stagnation point the velocity gradient is a minimum, therefore the heat transfer rate is minimum (for other geometries such as circular cylinders, similar arguments can be applied). At the reattachment region of the blunt flat plate, the rate of surface heat transfer is governed by the turbulence intensity at the wall region which in turn is governed by the interaction of convection and dissipation of turbulence. The Reynolds number dependence is determined by the combined effect of the above factors. However, it is not possible for the present research to measure the terms in each quantity to prove the Reynolds number dependence. At the reattachment region the shear layer splits into two streams (in the mean), the highest turbulent fluctuations are generated which in turn produce the maximum rate of heat transfer. The present spectral measurements also show that turbulence transport plays an important role in the heat

transport at the reattachment zone.

In free-stream turbulence, the pressure distribution around the normal flat plate is little affected. The reason is that the vortex shedding process is so strong so that the effect of turbulence on the velocity becomes less significant. The changes in $C_{p,b}$ and $\frac{d v/U_\infty}{d y/d}$ cannot cause the large increase in stagnation point heat transfer. The increase in heat transfer appears to be the result of the interaction of free-stream turbulence with the boundary layer. As turbulence approaches the plate, large distortion of the turbulence field occurs due to the presence of the plate. A coherent substructure is formed at the vicinity of the surface as a result of the amplification of free-stream turbulence. This coherent substructure (consisting of energy-containing eddies) can penetrate into the boundary layer and causes the local intensity to amplify. An increase in local intensity can cause an increase in heat transfer (for details see section 5.3).

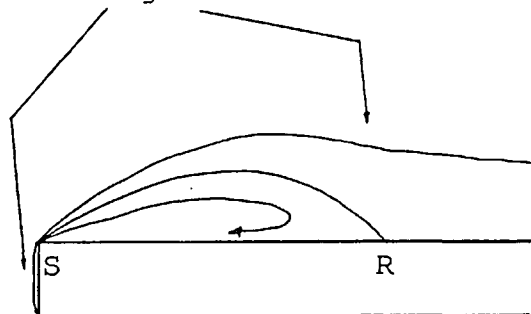
In free-stream turbulence the pressure distribution along the blunt flat plate reduces significantly as the bubble contracts. The presence of turbulence increases the entrainment rate and causes the typical velocity inside the bubble to increase. The free-stream turbulence field can be distorted in two regions (see figure below) (i) the region prior to separation of the shear layer and (ii) the region approaching reattachment. However, the present intensity measurements at the reattachment region (see Figure 7.2) show that the maximum $\sqrt{u^2}/U_\infty$ increases from 25% (non-turbulent free-stream) to 27% (grid B $\sqrt{u^2}/U_\infty = 5.4\%$). This finding

distortion of free-stream turbulence due to the plate



Stagnation point
heat transfer

distortion of free-stream turbulence occur in these regions



Separated flow
heat transfer

perhaps suggests that the distortion of free-stream turbulence at reattachment is not significant. The present blunt plate has low blockage ratio of 2.5%, the free-stream turbulence intensity around the plate will not be amplified as significantly as in the case of stagnation point heat transfer (5% blockage)- see figure above. The presence of free-stream turbulence may enhance the process of entrainment and increases the turbulence generation in the shear layer which in turn increases the turbulent fluctuations at the wall and hence increases the surface heat transfer rate. But the basic mechanism in heat transfer may not be altered in free-stream turbulence (as it is shown by the present results, Re^n stays the same).

8.2 Conclusions

The effect of free-stream turbulence on stagnation point heat transfer has been studied in detail. Measurements of intensity ($\sqrt{u^2}/U_\infty$), length scale (L_x), Reynolds number and power spectral density of the free-stream have been obtained. The pressure and surface heat transfer distributions of the blunt leading edge flat plate in non-turbulent and turbulent

free-streams have been evaluated. Measurements of the mean and fluctuating temperature, velocity in the separated/reattached regions of the flow field have been obtained. The possible mechanics governing heat transfer at the stagnation point and the separated/reattached flow regions are proposed.

The main findings of the present investigations are as follows:-

- (i) In free-stream turbulence, the stagnation point heat transfer increases with increasing Reynolds number, the relationship $Nu/Re^{0.5} = c Re^{0.08}$ provides a good correlation of data, but this relationship breaks down as $Re \rightarrow 0$ and $\sqrt{u^2}/U_\infty \rightarrow 0$. A physically more plausible form is $Nu/Re^{0.5} = a_0(\text{non-turb.}) + a_1(\text{turb.}) + a_2(\text{turb.})Re$. It is not possible to verify a_1 and a_2 using the present data, further measurements are necessary.
- (ii) The stagnation point heat transfer ($Nu/Re^{0.5}$) increases with intensity and decreases with length scale. The parameter $(\sqrt{u^2}/U_\infty)(L_X/d)^{-\frac{1}{2}}$ provides the best fit of data (it is not a universal parameter). The physical significance of this parameter is that intensity has the dominant effect on heat transfer.
- (iii) The distortion of free-stream turbulence appears to be responsible for the increase in stagnation point heat transfer. The amplification of turbulence ahead of the body can interact with the boundary layer and affects its nature. The important wave-number of the distorted turbulence is $\tilde{n}_t$ (in the energy containing range) above which is unlikely to be important in heat transfer (viscous dissipation dominates).
- (iv) At the reattachment region, the local Nusselt number (Nu_H) varies approximately as $Re_H^{0.75}$ for both turbulent and

non-turbulent free-streams. The physical implication is that the flow reattachment process is identical for both cases.

(v) In free-stream turbulence, $Nu_H/Re_H^{0.75}$ increases with increasing intensity, length scale and Reynolds number do not seem to have any major effect. Turbulence intensity reduces both the pressure at separation and the bubble length.

Although heat transfer is a surface effect, the integrated heat flux over the bubble length must all pass through the shear layer which in turn is controlled by the inviscid flow field.

(vi) Correlation of Nusselt number (Nu_R) and Reynolds number (Re_{U_s}) based on U_s and R for both turbulent and non-turbulent free-stream cases gives an approximately universal fit of data for two dimensional flows. It appears that surface heat transfer in separated flow is almost entirely through its effect on the velocity field. The basic mechanism of heat transfer is perhaps unchanged between turbulent and non-turbulent free-streams.

(vii) Heat transfer in separated flows is associated with a wall boundary layer where most of the change in mean temperature and the largest fluctuations in temperature occurs. The thickness of this layer is about 2% of the model thickness and depends on the free-stream Reynolds number and intensity.

(viii) Very large turbulent velocity and temperature fluctuations occur at the shear layer region. It appears that the generation of turbulence at the shear layer can provide mechanism for the surface heat transfer. The shear layer can affect the surface heat transfer via two ways (a) via the back flow (b) via pressure fluctuations.

8.3 Suggestions for Future Work

(a) attempts should be made to measure the turbulent shear stress ($-\rho \overline{uv}$) and the turbulent heat flux ($\rho C_p \overline{\theta v}$). These two quantities determine the eddy diffusivities of momentum ($\epsilon_m = -\overline{uv} / \frac{\partial U}{\partial y}$) and heat ($\epsilon_h = -\overline{v\theta} / \frac{\partial T}{\partial y}$) which are used to estimate the degrees of transfer of momentum and heat by turbulent fluctuation .

(b) the effect of free-stream turbulence on separated flow heat transfer is still not fully understood, more measurements in the temperature and velocity fields of the separation bubble are therefore necessary.

(c) Turbulent energy balance and enthalpy balance in the separation bubble would be very informative, the importance of each term in the turbulent energy equation ($\frac{D}{Dt} \frac{1}{2} \overline{q^2} = \text{R.H.S.}$) and turbulent enthalpy equation ($\frac{D}{Dt} \frac{1}{2} \overline{\theta^2} = \text{R.H.S.}$) can be assessed.

(d) It is hoped that the present data will aid in the development of a numerical method for separated flow heat transfer. As a starting point, it will be useful to compare the present heat transfer data with theoretical values obtained using the Runchal and Spalding method³³.

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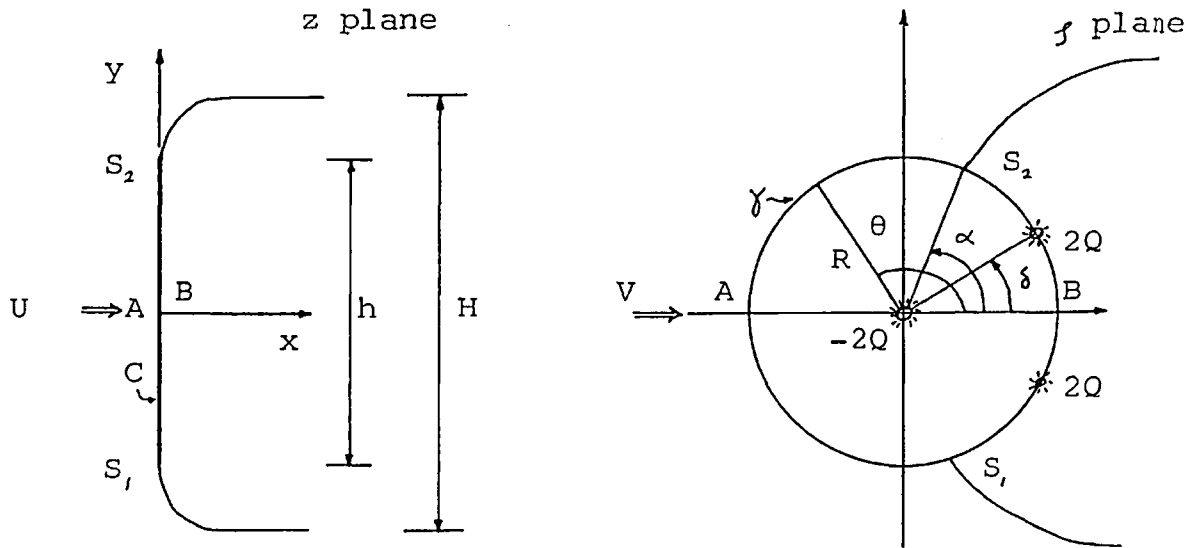
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APPENDIX A

PARKINSON AND JANDALI - A WAKE SOURCE MODEL FOR BLUFF
BODY POTENTIAL FLOW



Physical and basic transform plane

The normal flat plate is mapped from the circle γ in the f plane by the Joukowski transformation.

$$z = f(f) = f - \frac{R^2}{f} \quad \text{where } R = \frac{1}{4}h \dots\dots\dots(1)$$

and since separation occurs at the edges of the plate,

$$\alpha = \frac{\pi}{2}$$

Since $f'(f) = 1$ at infinity, $V = U$

The complex velocity at f plane

$$w(f) = 2i e^{-i\theta} \left[U \sin \theta + \frac{Q}{4\pi R} \cot\left(\frac{\delta + \theta}{2}\right) - \cot\left(\frac{\delta - \theta}{2}\right) \right] \dots\dots(2)$$

Equating (2) to zero for $\theta = \pm \frac{\pi}{2}$ give

$$Q = \frac{1}{2} \pi U h \cos \delta \dots\dots\dots(3)$$

After some rearrangement, the complex velocity $w(z)$ is

$$\frac{w(z)}{U} = \frac{-i \sin \theta}{\cos \delta - \cos \theta} = \frac{u+iv}{U} \dots\dots\dots(4)$$

and $\sec \delta = k$ where $k = \frac{|w(z)|}{U} = (1 - C_{p_b})^{\frac{1}{2}} \dots\dots(5)$

The pressure distribution on the front of the plate is

$$C_p(\theta) = 1 - \frac{\sin^2 \theta}{(\cos \delta - \cos \theta)^2} \dots\dots\dots (6)$$

where $y = \frac{1}{2}h \sin \theta$

The velocity distribution on the front face is obtained by equating (4)

$$- \frac{v}{U} = \frac{\sin \theta}{\cos \delta - \cos \theta}, \quad \frac{u}{U} = 0 \dots\dots (7a)$$

$$\text{or in terms of } \frac{y}{h}, \quad -\frac{v}{U} = \frac{2y/h}{1/k - (1 - (2y/h)^2)^{1/2}} \dots\dots (7b)$$

Therefore the velocity distribution is a function of $\frac{y}{h}$ and the base pressure.

APPENDIX B

THE DESIGN FACTORS OF THE MODELS

Appendix B1

The Evaluation of One-dimensional Heat Loss through the Substrate

The substrate plate is assumed to be infinitely long, i.e. one-dimensional, it has only a heating strip glued onto its surface.

The total amount of heat transfer from the heating strip is

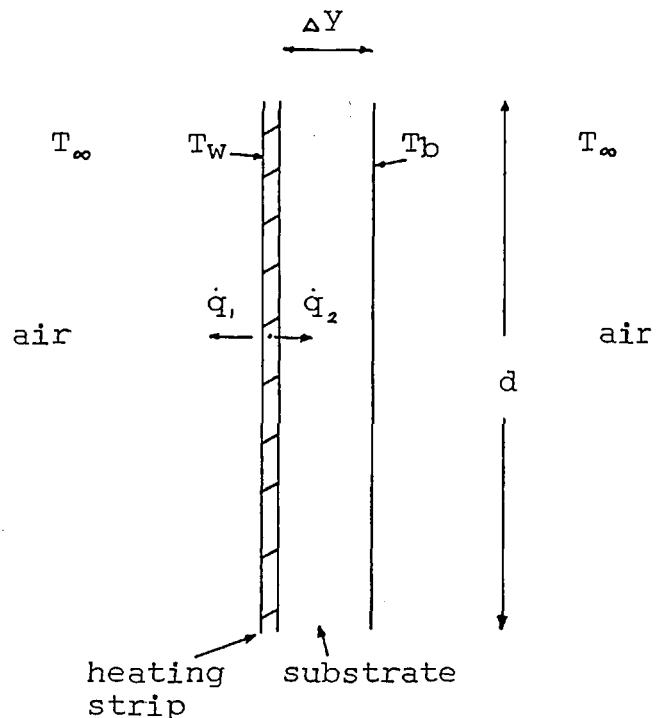
$$\dot{q} = \dot{q}_1 + \dot{q}_2$$

For one-dimensional heat conduction ($d \gg \Delta y$)

$$\dot{q}_2 = k \cdot \frac{T_w - T_b}{\Delta Y}$$

Assuming that T_b is known, as a rough approximation $T_b \sim T_\infty$

$$\text{i.e. } \dot{q}_2 \sim k \cdot \frac{T_w - T_\infty}{\Delta Y}$$



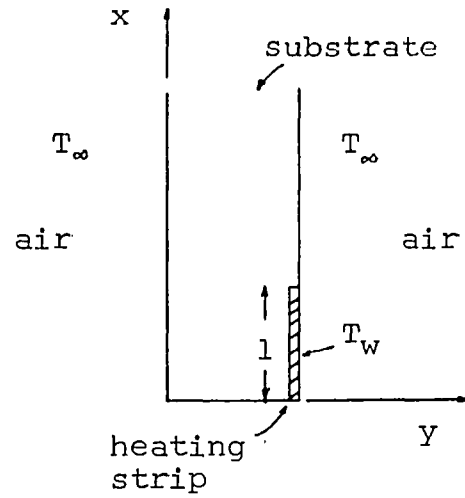
Appendix B2

The Evaluation of Two-dimensional Heat Loss through the Substrate

For simplicity, a two-dimensional semi-infinite substrate with a narrow heating strip is considered.

Assuming that $T - T_{\infty} = \theta$,
where T is the temperature
distribution in the substrate

The boundary conditions for
this problem are



$$\begin{aligned}
 T - T_{\infty} &= 0 \quad \theta = 0 && \text{at } x = \infty \\
 T &= T_w \quad \theta = T_w - T_{\infty} && \text{at } y = \Delta y, \quad 0 \ll x \ll l \\
 T - T_{\infty} &= 0 \quad \theta = 0 && \text{at } y = \Delta y, \quad l \ll x \ll \infty \\
 T - T_{\infty} &= 0 \quad \theta = 0 && \text{at } y = 0, \quad 0 \ll x \ll \infty
 \end{aligned}$$

The equation to be solved is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \quad \text{or} \quad \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0$$

The solution of θ is

$$\theta(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{2\sin kl}{k} \cdot \frac{\sin ky}{\sin k\Delta y} \cdot (T_w - T_{\infty}) e^{-ikx} \cdot dk$$

where k is the Fourier kernal

As $x \rightarrow \infty$

$$\frac{\partial \theta}{\partial y} = \frac{2}{\pi} \cdot \frac{(T_w - T_{\infty})}{\Delta y} \left[\frac{\pi}{2} \cdot \text{Coth} \frac{\pi}{2} \cdot \frac{1}{\Delta y} - \frac{\Delta y}{1} \right]$$

If $l \gg y$

$$\therefore \frac{\partial \theta}{\partial y} \Big|_{x=0} \rightarrow \frac{(T_w - T_{\infty})}{\Delta y} \quad \text{i.e. the one-dimensional value}$$

The criterion for model design was therefore to
have l greater than Δy (because $\frac{\partial \theta}{\partial y} \Big|_{x=0} \rightarrow$ one-dimensional value)

Appendix B3Estimation of the Width of the Heating Strips

Near the stagnation point region, the velocity field is assumed to be $u_1 = c x$

From Chapter 2, $Nu \propto (Re)^{\frac{1}{2}} \propto \frac{(u_1 x)^{\frac{1}{2}}}{(\nu)^{\frac{1}{2}}}$

$$\text{i.e. } Nu \propto (c)^{\frac{1}{2}}$$

Assuming that $Nu \propto \left(\frac{du_1}{dx}\right)^{\frac{1}{2}}$, so a 5% change in Nu

yields a 10% change in $\frac{du_1}{dx}$ (approximately).

At the stagnation point $\frac{du_1}{dx} \sim 1.21$ for $Cp_b \sim -1.48$ (using the results of Chapter 2).

Hence $\frac{du_1}{dx} \sim 1.33$, for a 10% change in $\frac{du_1}{dx}$

The corresponding value of x at $\frac{du_1}{dx} \sim 1.33$ is

$x \sim 0.16d$ (obtained from chapter 2).

Therefore the width of heating strip (maximum width permitted) is about $0.16 \times 5 \times 2 \sim 1.6$ cm

It was decided that the width of the heating strips to be 0.9 cm rather than 1.6 cm. The advantages were two fold, firstly the resolution error was reduced (the change in $\frac{du_1}{dx}$ was reduced) and secondly the model surface was able to accommodate five heating strips (provided sufficient control on the surface temperature).

Appendix B4

Estimation of the Thickness of Insulator

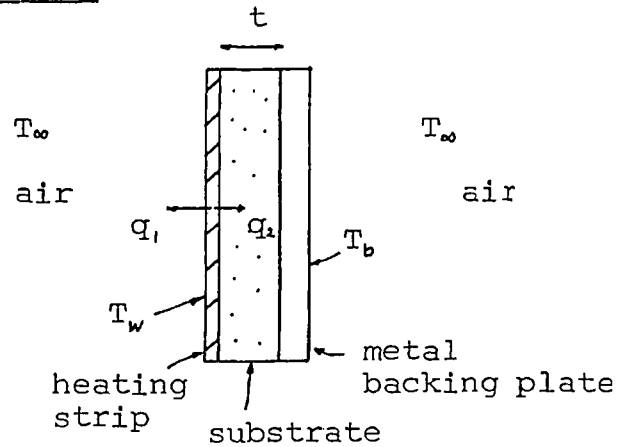
The total amount of heat transfer from the heating strip is

$$\dot{q} = \dot{q}_1 + \dot{q}_2$$

$$\text{Let } \xi \dot{q} = \dot{q}_1 = h (T_w - T_\infty),$$

$$\text{and } (1 - \xi) \dot{q} = \dot{q}_2 = h (T_b - T_\infty)$$

$$= k \frac{(T_w - T_b)}{t}$$



where ξ is a fraction existed between 0 - 1
and h is the heat transfer coefficient

Assuming that h is the same for both the front and back of the model.

$$\therefore \dot{q}_2 = \frac{k}{t} \cdot (T_w - \frac{\dot{q}_2}{h} - T_\infty) = \frac{k}{t} \cdot (\frac{\dot{q}_1}{h} - \frac{\dot{q}_2}{h})$$

$$\therefore (1 - \xi) \dot{q} = \frac{k}{ht} \cdot (\xi \dot{q} - \dot{q} + \xi \dot{q})$$

$$\therefore (1 - \xi) = \frac{k}{ht} \cdot (2\xi - 1)$$

From Frössling's theory in Chapter 2:

$$\begin{aligned} \text{At the stagnation point, } Nu &= 0.55 (Re)^{\frac{1}{2}} \quad (\text{for a } Cp_b \sim -1.48) \\ &= 0.55 \left(\frac{U_\infty d}{\nu} \right)^{\frac{1}{2}} \quad \text{average experimental value} \end{aligned}$$

Assuming that $U_\infty \sim 20 \text{ m/s}$ and $d = 5 \text{ cm}$ (width of the model)

$$Nu = 0.55 \cdot \frac{20 \times 5 \times 10^{-2}}{1.5 \times 10^{-5}} \sim 144$$

$$\text{and } h = \frac{Nu \cdot k_{\text{air}}}{d} \sim \frac{144 \times 0.026}{5 \times 10^{-2}} \sim 75 \frac{\text{W}}{\text{m}^2 \text{K}}$$

Rearrange the above equation, get

$$t = \frac{k}{h} \cdot \frac{(2\xi - 1)}{(1 - \xi)}$$

For $\xi = 0.8$, i.e. a 20% heat loss through the substrate.
and $k \sim 0.2 \text{ W/mK}$

$$t \sim \frac{0.2}{75} \times 3 \sim 0.8 \text{ cm}$$

Appendix B5

Estimation of $(T_w - T_\infty)$

Assuming that the heating strips are made of Nickel/Chromium and have dimensions $0.125 \text{ mm} \times 0.9 \text{ cm} \times 50 \text{ cm}$.

Resistance of each strip, $R = \frac{\rho l}{A}$ (where l and A are the length and Area of the strip respectively)

Let ρ = resistivity of Nickel/Chromium = $112 \times 10^{-8} \text{ m}$ at 20°C ,

$$A = 0.9 \times 0.125 \times 10^{-5} \text{ m}^2, \text{ and } l = 50 \text{ cm}$$

$$\therefore R \sim 0.5 \Omega$$

Assuming that the maximum current the power-supply can provide is 4 A

$$\text{Therefore } Q = I^2 R \sim 4 \times 4 \times 0.5 \sim 8 \text{ W}$$

$$\text{and } \dot{q} = \frac{Q}{A'} \sim \frac{8}{0.9 \times 10^{-2} \times 50 \times 10^{-2}} \sim 1800 \frac{\text{W}}{\text{m}^2}$$

$$\text{For } \xi = 0.8, \therefore \dot{q}_1 \sim 1800 \times 0.8 \sim 1400 \frac{\text{W}}{\text{m}^2}$$

From Appendix B4 , $Nu \sim 144$ (at $U_\infty = 20 \text{ m/s}$)

$$\text{Since } Nu = \frac{\dot{q}_1 \cdot d}{k_{\text{air}} (T_w - T_\infty)}$$

$$T_w - T_\infty = \frac{\dot{q}_1 \cdot d}{k_{\text{air}} \cdot Nu} \sim \frac{1400 \times 5 \times 10^{-2}}{0.0262 \times 144} \sim 17^\circ\text{C}$$

Appendix B6

Estimation of Radiation Losses

$$\dot{q} = \xi \sigma (T_w^4 - T_\infty^4) \quad \text{where } \sigma = \text{Stefan Boltzmann constant}$$

and ξ = coefficient of emissivity

For the case of $(T_w - T_\infty) \sim 17^\circ\text{C}$, $T_\infty \sim 20^\circ\text{C}$, $U_\infty \sim 20 \text{ m/s}$

and $\epsilon \sim 0.1$ (for a polished surface)

$$\dot{q} \sim 0.1 \times 56.7 \times 10^{-12} \times (330^4 - 293^4)$$

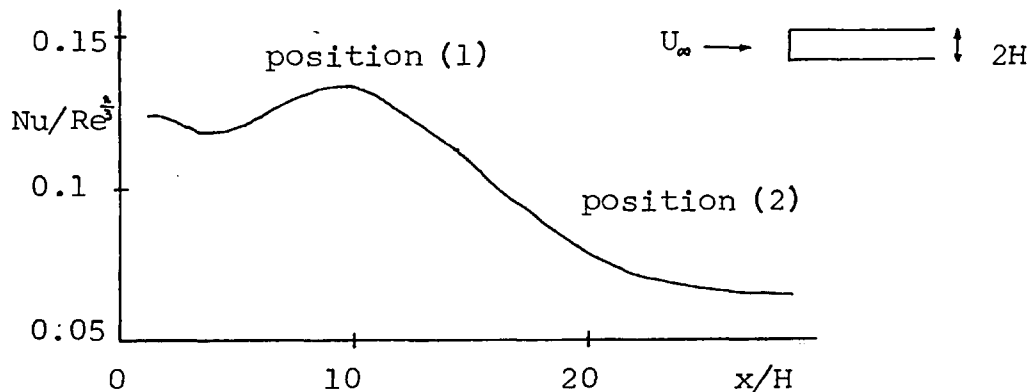
$$\dot{q} \sim 25 \text{ W} \quad \text{for the front surface}$$

From Appendix B5 , $\dot{q}_r \sim 1400 \frac{\text{W}}{\text{m}^2}$. The amount of heat loss due to radiation is about 1.5% $(\frac{25}{1400} \times 100\%)$.

Appendix B7

Estimation of the Total Power Requirement of the Blunt Flat Plate

From the heat transfer data of Ota and Kon³⁸



In the separated flow region

$$\text{Nu} \sim 0.12 (\text{Re})^{\frac{2}{3}}$$

In the reattachment region

$$\text{Nu} \sim 0.12 (\text{Re})^{\frac{2}{3}} \quad \text{.....Position (1)}$$

$$\text{Nu} \sim 0.07 (\text{Re})^{\frac{2}{3}} \quad \text{.....Position (2)}$$

In the redeveloped flow region

$$\text{Nu} \sim 0.06 \text{Re}^{\frac{2}{3}}$$

Assuming that $U_\infty \sim 30 \text{ m/s}$, $2H \sim 2.5 \text{ cm}$, $(T_w - T_\infty) \sim 10^\circ \text{C}$

$$\text{Re} = \frac{U_\infty \cdot 2H}{\nu} \sim 2.5 \times 10^4$$

From the definition of Nu , $\dot{q} = \frac{\text{Nu} \cdot k_{\text{air}} \cdot (T_w - T_\infty)}{H}$

Therefore the power dissipation in the separated flow region is

$$\dot{q} \sim 107 \text{ W} \quad (\text{for one surface})$$

In the reattachment region $\dot{q} \sim 128 \text{ W}$

In the redeveloped flow region $\dot{q} \sim 67 \text{ W}$

The total power dissipation of the model is:

$$2 \times (107 + 128 + 67) \sim 600 \text{ W}$$

Appendix B8

Estimation of the Power Dissipation of a Heating Strip

The heating strips of the blunt flat plate are made of Nickel/Chromium and have dimensions $0.020\text{mm} \times 0.9\text{cm} \times 50\text{cm}$.

Since $R = \frac{\rho l}{A}$, $\rho = 112 \times 10^{-8} / \Omega \text{ m}$ at 20°C

$$R \sim \frac{112 \times 10^{-8} \times 50 \times 10^{-2}}{0.02 \times 0.9 \times 10^{-5}} \sim 3.1 \Omega$$

In the separated flow region, the power dissipation of ten heating strips is about 107 W, therefore the power dissipation of one heating strip is about 10.7 W

The current requirement of each heating strip is,

$$I = \sqrt{\frac{Q}{R}} \sim \sqrt{\frac{10.7}{3.1}} \sim 1.8 \text{ A}$$

and the voltage requirement is

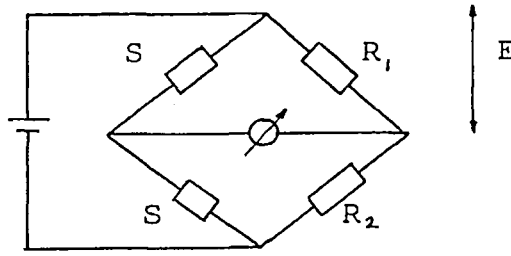
$$V = \frac{Q}{I} \sim \frac{10.7}{1.8} \sim 5.7 \text{ V}$$

Appendix B9

A Correction Method on the Effect of Difference in Temperature Coefficient of Resistance of Thin Film Gauges

In order to speculate the seriousness of taking the average value of α for the two gauges (top and bottom),

the following analysis is performed.



R_1 = top gauge
 R_2 = bottom gauge
 S = standard resistor

$$E = \frac{R_1}{R_1 + R_2} \quad , \quad E + \Delta E = \frac{R_1 + \Delta R_1}{(R_1 + R_2) + \Delta(R_1 + R_2)}$$

$$\text{Let } T_2 = T_1 + \Delta T \quad \Delta T \ll T_1 \text{ or } T_2$$

$$\alpha_2 = \alpha_1 + \Delta \alpha \quad \Delta \alpha \ll \alpha_1 \text{ or } \alpha_2$$

$$\text{Therefore } \frac{\Delta E}{E} = \frac{-R_2 T_1 \alpha_1 (\Delta T/T_1 + \Delta \alpha/\alpha_1)}{(R_1 + R_2) + (1 + \alpha_1 T_1)}$$

$$\text{Hence } \frac{\alpha_2 - \alpha_1}{\alpha_1} \text{ is of the order of } \frac{T_2 - T_1}{T_1}$$

$$\text{Assuming that } \alpha_{av} = \alpha_1 + \frac{\Delta \alpha}{2} \quad , \quad \Delta \alpha \rightarrow 0 \text{ and } \Delta T_m = \text{measured } \Delta T$$

$$\text{Hence } \frac{\Delta E}{E} = \frac{-R_2 T_1 (\alpha_{av} - \Delta \alpha/2) (\Delta T_m / T_1)}{(R_1 + R_2)}$$

$$\text{By substitution } \frac{\Delta T_m}{T_1} = \left(\frac{\Delta T}{T_1} + \frac{\Delta \alpha}{\alpha_1} \right) / \frac{\alpha_{av} (1 + \alpha_1 T_1)}{\alpha_1 (1 + \alpha_{av} T_1)}$$

$$\text{For a typical case, } \alpha_{av} = 2.23 \times 10^3 \Omega/^{\circ}\text{C} \quad , \quad \alpha_1 = 2.1 \times 10^3 \Omega/^{\circ}\text{C} \quad , \quad T_1 = 64^{\circ}\text{C} \quad ,$$

ΔT_m was found to be 10°C for a ΔT of 3°C . In other words a 10% difference in α had caused the measured ΔT to be 3 times the actual value.

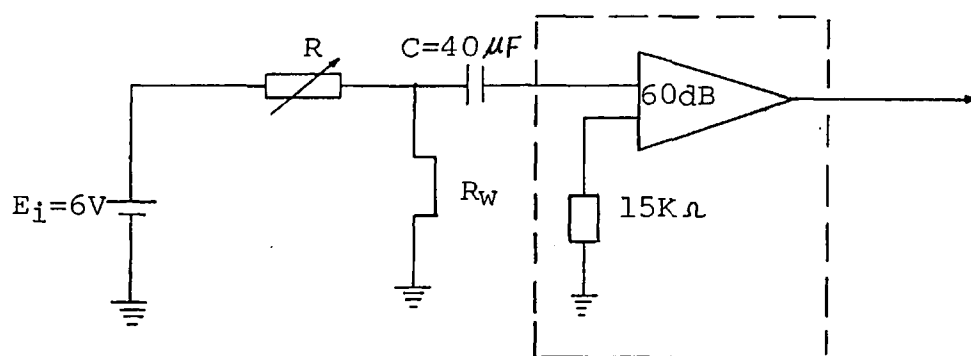
APPENDIX C

THE CONSTANT CURRENT ANEMOMETER

Appendix C1

Temperature Fluctuations

The circuit used to measure fluctuating temperature is shown below. The probe current is $I = E_i / (R + R_w)$, but since $R_w \ll R$, $I \simeq E_i / R$ is independent of R_w .



Brookdeal 431

Constant current anemometer

The voltage across the 'cold' wire operating in constant current mode is:

$$E = I R_w$$

and any variation in R_w causes a small change in E ,

$$\text{i.e. } \Delta E = I \Delta R_w$$

If the variations in E are due to variations of the temperature of the wire alone,

$$\text{then } \Delta R_w = -\alpha R_w \Delta T$$

where α is the temperature sensitivity of the Wollaston wire.

$$\text{therefore } \Delta E = -I \alpha R_w \Delta T$$

$$\text{For small } \Delta T, \quad e = I \alpha R_w \theta \quad \text{or} \quad e = \beta \theta$$

The determination of the constant β can be done either by the calculation $\beta = I\alpha R_w$ or by measuring the voltage decrease (ΔE) due to a temperature increase ΔT . The experimental determination of β was done in the Aeronautics department hexagonal wind tunnel. The experimental value agreed favourably with the theoretical value.

Appendix C2

The Error due to Velocity Fluctuations

Corrsin first derived the relation

$$\Delta E = \frac{\partial E}{\partial T} \Delta T + \frac{\partial E}{\partial U} \Delta U$$

where $\partial E / \partial T$ and $\partial E / \partial U$ are the temperature and velocity sensitivity of the wire.

If $\frac{\partial E}{\partial T} \Delta T \gg \frac{\partial E}{\partial U} \Delta U$, then the velocity fluctuations

do not contribute to ΔE .

For a typical case, $R_w \approx 80 \Omega$, $I = 1.0 \text{ mA}$, $\frac{\partial R}{\partial U} = 0.005 \Omega/\text{m/s}$,

$$\text{so } \frac{\partial E}{\partial U} = I \frac{\partial R}{\partial U} \approx 0.0005 \text{ mV/m/sec}$$

$$\text{and } \frac{\partial E}{\partial T} = \alpha I R_w = 0.304 \text{ mV/sec}$$

$$\text{typically } \Delta U = \sqrt{u_{\max}^2} \approx 4 \text{ m/s}$$

$$\Delta T = \sqrt{\theta_{\max}^2} \approx 0.4^\circ \text{C}$$

$$\frac{\partial E}{\partial T} \Delta T = 0.12 \quad \text{and} \quad \frac{\partial E}{\partial U} \Delta U = 0.002$$

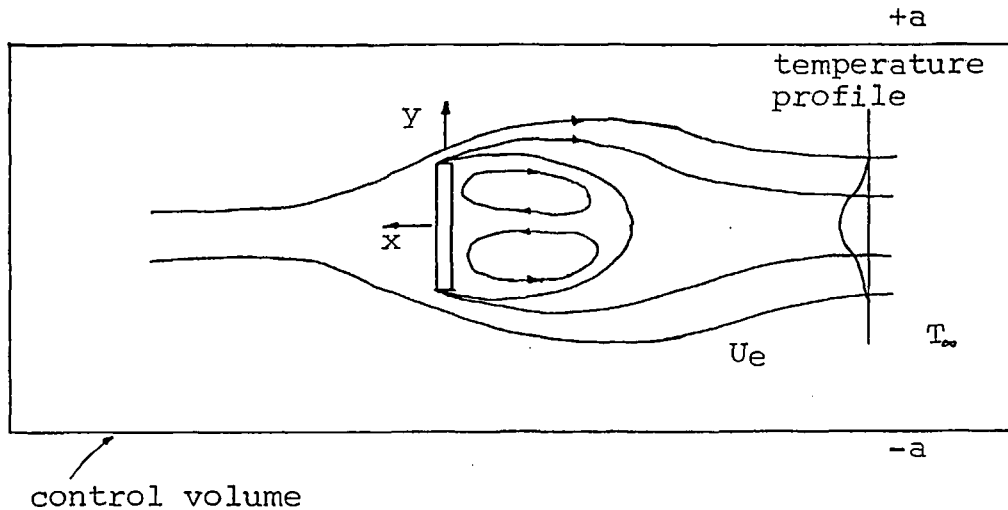
Therefore the error due to velocity fluctuation is approximately 2%.

APPENDIX D

CALCULATIONS FOR STAGNATION POINT HEAT TRANSFER

Appendix D1

Heat Balance between Power Input and Heat Convection in the Wake



By the law of energy conservation:

Total power input into the model = total heat convection in the wake

Consider a control volume, assume ρ = density of air = constant, C_p = constant, neglect the effect of viscosity,

Total heat convection $Q = b \int_{-a}^a \rho C_p U \Delta T dy$, b = breadth of the model

$$\rho = 1.2 \text{ kg/m}^3, C_p = 1.0 \text{ KJ/KgK}, b = 0.5 \text{ m}, \Delta T = (T - T_\infty)$$

$U_e = 6.85 \text{ m/s}$ (velocity at the edge of wake)

$$\therefore Q = 1.2 \times 10^3 \times 0.5 \int_{-a}^a U \Delta T dy$$

$$\text{From Figure on page 284 : } \int_{-a}^a U \Delta T dy = 5.3 \times 10^{-2} \frac{\text{m}^2}{\text{KS}}$$

$$\therefore Q \sim 31.8 \text{ W}$$

$$\text{Drag force due to model } D = b \rho \int_{-a}^a U (U_e - U) dy$$

$$D = 0.5 \times 1.2 \times 0.65 = 0.39 \text{ N}$$

The amount of power due to drag = $0.39 \times 6.85 \sim 2.7W$

At a distance very far downstream, all the turbulent kinetic energy is dissipated into heat, i.e. finite drag of the body $\approx$ heat dissipation.

At a distance of nine plate diameters downstream, only part of the turbulent kinetic energy is dissipated into heat.

Total kinetic energy / Area in the wake (U direction)
 $= \frac{1}{2} \rho \int_{y/d = -1.5}^{y/d = 1.5} \overline{U^2} dy$, (where d is the diameter of the plate),
 assume $U = \overline{U} + u$ and $\frac{\sqrt{\overline{u^2}}}{\overline{U}} \sim 0.2$ in the wake at $\frac{y}{d} \approx 9$

$$= \frac{1}{2} \rho \int_{-y/d}^{y/d} (\overline{U^2} + 2\overline{U}u + \overline{u^2}) dy$$

Also assume $\overline{u^2} \sim \overline{v^2} \sim \overline{w^2}$ in the wake

As a rough approximation the total turbulent kinetic energy

= 3×0.04 of the total kinetic energy

Hence the turbulent kinetic energy dissipated into heat

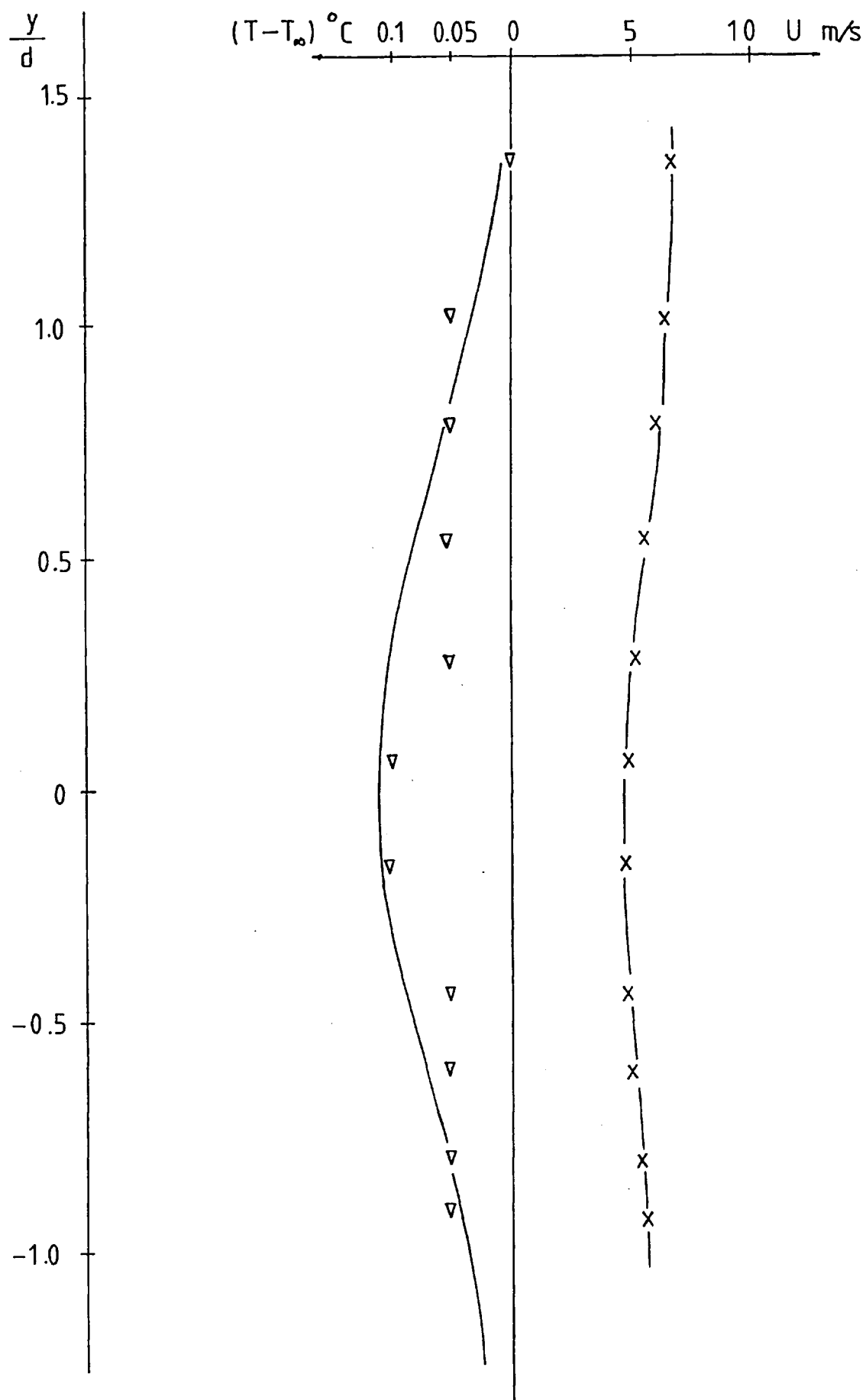
is $\sim 12\%$ of the total power due to drag

$\sim 0.3 W$

Total power input = 33W (from experiment)

Total power output = $31.8 - 0.3 \sim 31.5W$

Hence there is a difference of about 5% between heat input and heat output.



Velocity and temperature profiles at $y/d = 9$ in the wake

Appendix D2Estimations of the Velocity and Thermal Layer Thickness

At the stagnation point region $u_1 = cx$, where

u_1 = velocity just outside the boundary layer

From Goldstein³: $\eta = \left(\frac{c}{\nu}\right)^{\frac{1}{2}} y$, $u = cxf'(\eta)$

Define $y = \delta$ at $u = 0.995u_1$, δ = boundary layer thickness

$$\therefore 0.995cx = cxf'(\eta)$$

$$\therefore f'(\eta) = 0.995 \quad \text{and } \eta = 2.6$$

From Parkinson and Jandali⁵⁰ :

$$c \approx 1.20 \frac{U}{d} \quad \text{for an experimental } C_{p_b} \text{ of } -1.48$$

$$\therefore \eta = \left(\frac{1.20}{\nu} \cdot \frac{\nu}{d}\right)^{\frac{1}{2}} y$$

$$\therefore 2.6 = (1.20 \text{ Re})^{\frac{1}{2}} \frac{\delta}{d} \quad \text{at } y = \delta \quad (d = 5\text{cm})$$

For a low Re flow, $\text{Re} = 2.3 \times 10^4$ (experimental)

$$\delta = 0.8\text{mm}$$

For a high Re flow, $\text{Re} = 8.5 \times 10^4$

$$\delta = 0.4\text{mm}$$

From Goldstein: $T = T_1 - (T_1 - T_o)\theta(\eta)$ $\theta(\eta)$ is the temperature distribution

$$\text{The solution of } \theta(\eta) = \frac{\int_0^\eta \left(\frac{e^{-\sigma \int_0^\eta f d\eta}}{e^{-\sigma \int_0^\eta f d\eta}} \right) d\eta}{\int_0^\infty \left(\frac{e^{-\sigma \int_0^\eta f d\eta}}{e^{-\sigma \int_0^\eta f d\eta}} \right) d\eta}$$

at $T = T_o$, $\theta(\eta) = 1$

Define $\theta(\eta) = 0.995$, $y = \delta_T$ thermal layer thickness

From Frössling⁵ $\theta(\eta) = 0.995$, $\eta = 3.8$

$$\therefore 3.8 = (1.20 \text{ Re})^{\frac{1}{2}} \delta_T/d$$

For $\text{Re} = 2.3 \times 10^4$, $\delta_T = 1.1\text{mm}$

$$\text{Re} = 8.5 \times 10^4 \quad , \quad \delta_T = 0.6\text{mm}$$

For $\text{Re} = 1.7 \times 10^4$, $\delta \sim 0.9\text{mm}$; $\text{Re}_\delta = \left(\frac{5 \times 0.9 \times 10^{-3}}{1.5 \times 10^{-5}} \right) \sim 260$ ($\text{Re}_\delta = \frac{U_\infty \delta}{\nu}$)

$$\text{Re} = 5.4 \times 10^4 \quad , \quad \delta \sim 0.5\text{mm} ; \text{Re}_\delta = \left(\frac{16 \times 0.5 \times 10^{-3}}{1.5 \times 10^{-5}} \right) \sim 500$$

APPENDIX ECALCULATIONS FOR SEPARATED FLOW HEAT TRANSFERAppendix E1The Governing Equations of Heat Transfer in Turbulent Separated FlowsThe equations of motion and energy (Townsend⁹⁰)

Continuity $\frac{\partial U_k}{\partial x_k} = 0$

Mean flow $\frac{\partial U_i}{\partial t} + U_k \frac{\partial U_i}{\partial x_k} = -\frac{\partial P}{\partial x_i} - \frac{\partial \overline{u_i u_k}}{\partial x_k} + g_i \frac{T}{T_a} + \nu \frac{\partial^2 U_i}{\partial x_k^2}$

Mean temperature $\frac{\partial T}{\partial t} + U_k \frac{\partial T}{\partial x_k} = -\frac{\partial \overline{u_k \theta}}{\partial x_k} + k \frac{\partial^2 T}{\partial x_k^2}$

Kinetic energy of velocity fluctuations

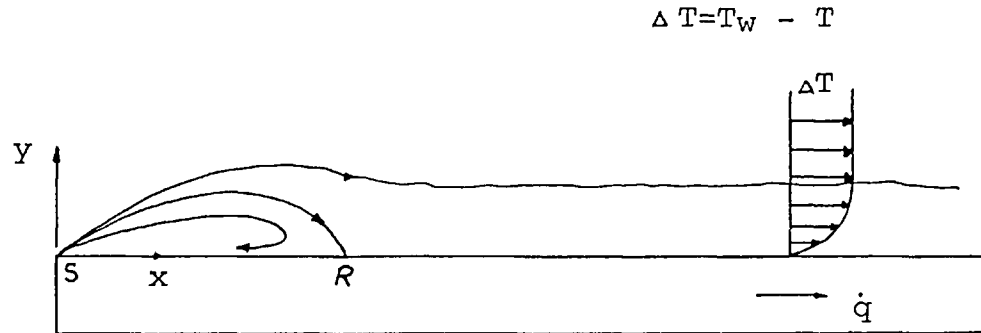
$$\begin{aligned} \frac{\partial}{\partial t} (\frac{1}{2} \overline{q^2}) + U_k \frac{\partial}{\partial x_k} (\frac{1}{2} \overline{q^2}) + \frac{\partial}{\partial x_k} (\overline{p u_k} + \frac{1}{2} \overline{q^2 u_k}) \\ \text{mean advection} \qquad \qquad \qquad \text{diffusion} \\ = -\overline{u_i u_k} \frac{\partial U_i}{\partial x_k} + \frac{g_i}{T_a} \overline{\theta u_i} + \nu \overline{u_i \frac{\partial^2 u_i}{\partial x_k^2}} \\ \text{production} \qquad \qquad \qquad \text{dissipation} \end{aligned}$$

Mean heat flux

$$\begin{aligned} \frac{\partial}{\partial t} (\overline{\theta u_i}) + U_k \frac{\partial}{\partial x_k} (\overline{\theta u_i}) + \frac{\partial}{\partial x_k} (\overline{\theta u_i u_k}) = -\overline{u_i u_k} \frac{\partial T}{\partial x_k} - \overline{\theta u_k} \frac{\partial U_i}{\partial x_k} \\ - \overline{\theta \frac{\partial P}{\partial x_i}} + \frac{g_i}{T_a} \overline{\theta^2} + \frac{\partial}{\partial x_k} (\kappa u_i \frac{\partial \theta}{\partial x_k} + \nu \overline{\theta \frac{\partial u_i}{\partial x_k}}) - (\nu + \kappa) \overline{\frac{\partial u_i}{\partial x_k} \frac{\partial \theta}{\partial x_k}} \end{aligned}$$

Intensity of temperature fluctuations

$$\begin{aligned} \frac{\partial}{\partial t} (\frac{1}{2} \overline{\theta^2}) + U_k \frac{\partial}{\partial x_k} (\frac{1}{2} \overline{\theta^2}) + \frac{\partial}{\partial x_k} (\frac{1}{2} \overline{\theta^2 u_k}) \\ \text{enthalpy} \qquad \text{spatial transport of } \overline{\theta^2} \\ = -\overline{\theta u_k} \frac{\partial T}{\partial x_k} + \kappa \frac{\partial^2}{\partial x_k^2} (\frac{1}{2} \overline{\theta^2}) - \kappa \overline{(\frac{\partial \theta}{\partial x_k})^2} \\ \text{production} \qquad \text{convection, diffusion and} \\ \qquad \qquad \qquad \text{dissipation} \end{aligned}$$

Appendix E2Energy Balance Check

Temperature profile and velocity profile are taken at $x = 18.7\text{cm}$ (see Figure on page 288).

Total power input into heating strips = Total amount of heat convected by the shear layer + conduction loss due to model.

Total power = 71.4W obtained from experiment for upper surface of the model.

Total amount of heat convected by the shear layer

$$Q = \int_0^{\infty} \rho C_p U (T - T_{\infty}) dy \cdot b \quad (b \text{ is the breadth of plate})$$

$$Q = 1.2 \times 1.0 \times 10^3 \times 48 \times 10^{-2} \int_0^{\infty} U (T - T_{\infty}) dy$$

$$Q = 600 \times A'$$

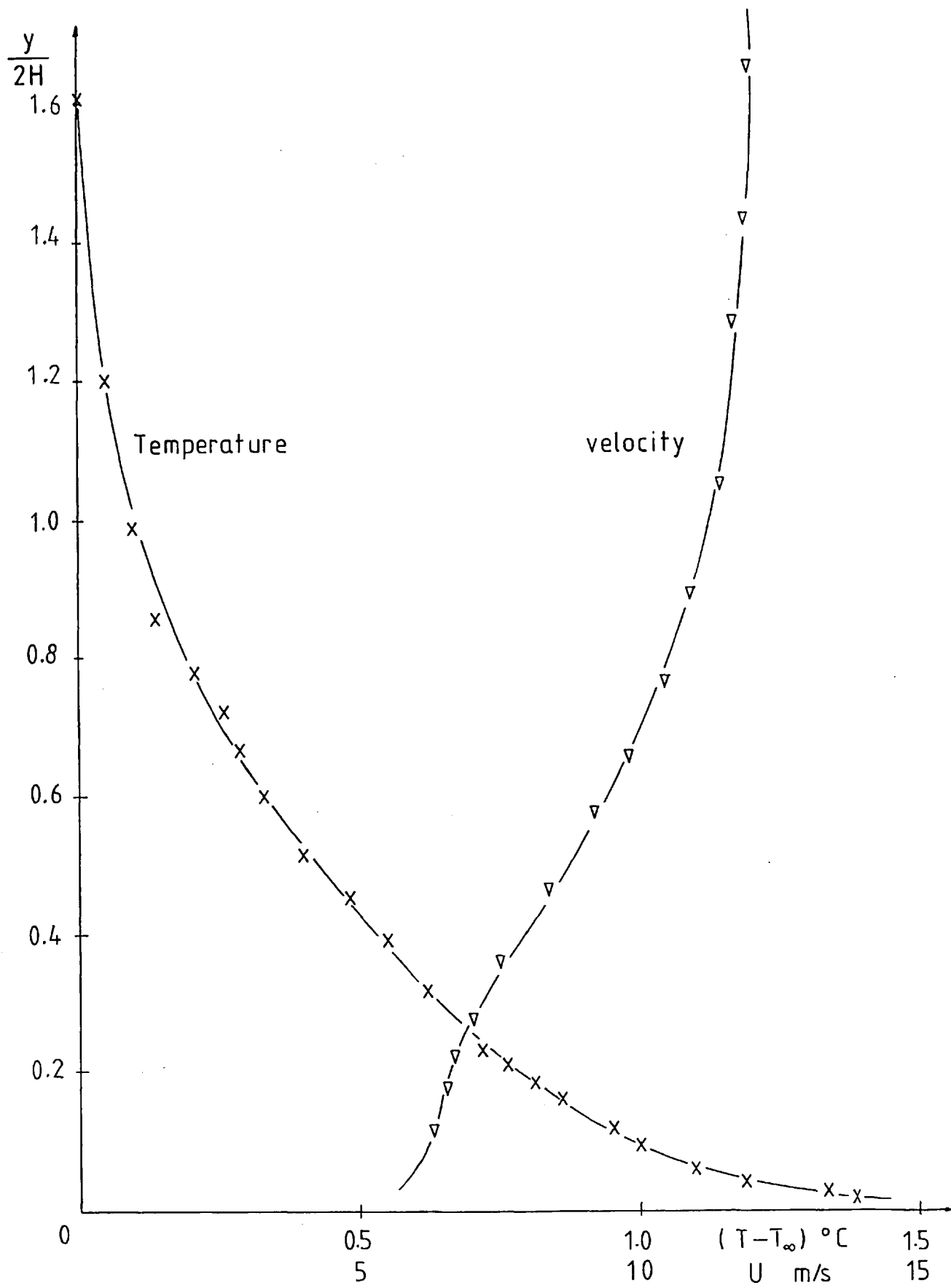
$$\therefore Q = 600 \times 0.106 = 63.6\text{W}$$

Conduction loss along the model $\dot{q} = -k \frac{\partial T}{\partial x}$

$$\therefore Q = -kA \frac{\Delta T}{\Delta x} = \frac{0.2483}{2 \times 2 \times 10^{-2}} (2.48 \times 10^{-2} \times 48 \times 10^{-2} \times 5.06) = 4.5\text{W}$$

Total power of heat output = $63.6 + 4.5 = 68.1\text{W}$

$$\text{difference between } \frac{\text{output power}}{\text{input power}} = \frac{71.4 - 68.1}{71.4} \times 100\% = 4\%$$



velocity and temperature profiles at $x/R = 1.5$

Appendix E3Characteristics of Turbulent Free-stream

grid	distance from grid	$\frac{\sqrt{u^2}}{U_\infty}$	$\frac{Lx}{2H}$	$\frac{R}{2H}$	$\frac{C_{p_r} - C_{p_s}}{1 - C_{p_s}}$
A(M=6")	63"	8.5%	2.52	2.5	0.35
A	100"	5.4%	3.66	3.02	0.36
B(M=3")	47"	5.4%	1.37	2.98	0.35
B	63"	4.4%	1.60	3.22	0.34
C(M=1.5")	46"	2.7%	0.86	3.85	0.36
C	63"	2.2%	0.96	4.1	0.34
non-turb. free-stream				5.0	0.32

Appendix E4The Evaluation of Nusselt number

The current I_1 , flowing through the heating strip was equivalent to the current flowing through the power supply module. The voltage V_1 , and resistance R_1 , of the module circuit were measured. Therefore, the current I_1 , could be evaluated ($I_1 = V_1/R_1$). Hence the power dissipation is

$$Q = I_1^2 R$$

where $R = \rho l/A'$ - resistance of
heating strips

ρ = resistivity

A' = cross-sectional area

l = length

$$\therefore \dot{q} = Q/A$$

A = surface area of strip

Therefore Nusselt Number $Nu_H = \frac{\dot{q} H}{k_{air} (T_W - T_\infty)}$

The numerical values for the parameters used in the evaluation were:

$$R_1 = 0.106 \, \Omega$$

$$R = 3.01 \, \Omega$$

$$A = 0.02 \times 10^{-3} \times 0.9 \times 10^{-2} \, m^2$$

$$k_{air} = 0.0262 \, \frac{W}{m \, K}$$

$$H = 1.24 \, cm$$

Appendix E5

The Estimation of Heat Losses

Radiation loss

$$\dot{q} = \epsilon \sigma (T_W^4 - T_\infty^4) \quad \text{Stefan Boltzmann Law}$$

$$\text{where } \sigma = 56.7 \times 10^{-9} \, W/m^2 K$$

In the present investigation $\epsilon = 0.1$ for a polished surface

$$T_W = 310K$$

$$T_\infty = 298K$$

$$\therefore \dot{q} = 7.7 \, W/m^2$$

$$\text{Total radiation loss} = \frac{\dot{q}}{\text{area of model}} = \frac{7.7}{25 \times 50 \times 10^{-4}} = 0.96W$$

Total power input $\approx 104W$ (obtained from experiment)

$$\frac{\text{radiation loss}}{\text{total power}} = \frac{0.96 \times 100}{104} \sim 1\%$$

Conduction loss

$$\begin{array}{ccc} T_2 & \dot{q} & T_1 \\ \hline & \leftarrow & \end{array}$$

$$Q = kA \frac{T_1 - T_2}{\Delta Y} \quad \text{where } \Delta Y \text{ is the distance between the two ends } \sim 50cm$$

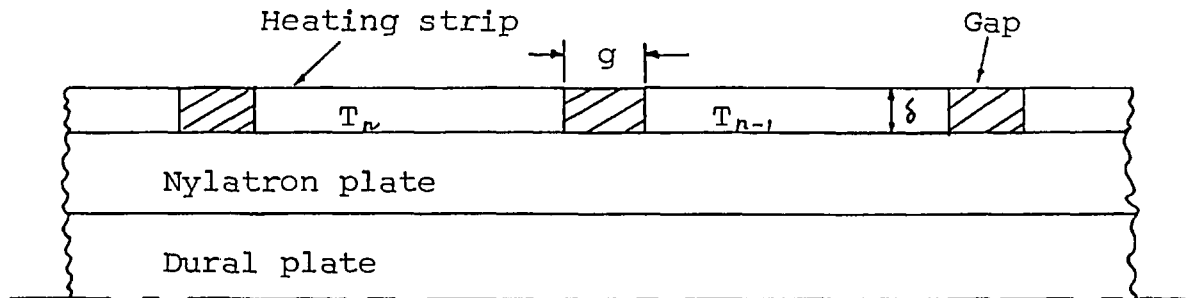
$$\text{Let } (T_2 - T_1) = 0.5 \, C, \quad k \sim 60 \, W/mK, \quad A = 0.02 \times 10^{-3} \times 0.9 \times 10^{-2} \, m^2$$

$$\therefore Q \sim 1.08 \times 10^{-5} \, W$$

Power input into heating strip $\sim 5W$

Therefore the heat loss due to conduction is negligible.

Chordwise Conduction across the gap between heating strips



A cross-section of the model

Assume that each heating strip is individually isothermal and the plate is at uniform temperature, the amount of heat conduction (Q) across the gap due to a temperature difference ($T_n - T_{n-1}$) between adjacent heating strip is thus

$$Q = kA \frac{(T_n - T_{n-1})}{g} \quad \text{where } A \text{ is the length of heating strip } \times \delta$$

Let $k \sim 0.2W/m \text{ K}$, $T_n - T_{n-1} \sim 0.1^\circ C$, $g = 10^{-3}m$, and $A = 0.48 \times 0.02 \times 10^{-3}m^2$

$$\therefore Q \sim \frac{0.2 \times 0.96 \times 10^{-5} \times 0.1}{10^{-3}} \sim 2 \times 10^{-4}W$$

Power input into heating is $\sim 5W$

$\therefore$ Heat conduction across the gap is negligible.