

4D Blood Flow Reconstruction Over the Entire Ventricle From Wall Motion and Blood Velocity Derived From Ultrasound Data

Alberto Gomez*, Adelaide de Vecchi, Martin Jantsch, Wenzhe Shi, Kuberan Pushparajah, John M. Simpson, Nicolas P. Smith, Daniel Rueckert, Tobias Schaeffter, and Graeme P. Penney

Abstract—We demonstrate a new method to recover 4D blood flow over the entire ventricle from partial blood velocity measurements using multiple 3D + t colour Doppler images and ventricular wall motion estimated using 3D + t BMode images. We apply our approach to realistic simulated data to ascertain the ability of the method to deal with incomplete data, as typically happens in clinical practice. Experiments using synthetic data show that the use of wall motion improves velocity reconstruction, shows more accurate flow patterns and improves mean accuracy particularly when coverage of the ventricle is poor. The method was applied to patient data from 6 congenital cases, producing results consistent with the simulations. The use of wall motion produced more plausible flow patterns and reduced the reconstruction error in all patients.

Index Terms—Doppler measurement, echocardiography.

I. INTRODUCTION

ACCURATE assessment of true intracardiac 3D velocities is required to fully understand three-dimensional flow structures. Availability of such velocity fields improves the understanding of haemodynamics and remodelling in response to abnormal cardiac conditions in native and operated structural and functional heart disease, for example before and after interventions in HLHS patients. A thorough review on clinical importance of the ventricular vortex was recently published by Pedrizzetti *et al.* [1]. In particular, there are several functional

parameters that can be calculated from vortical structures, such as kinetic energy and viscous losses [2], vortex position and orientation, flow transport mechanisms [3], delayed ejection [4] momentum thrust and overall efficiency of the ejection mechanism. Unfortunately, estimation of such parameters using ultrasound has only been approached in 2D [5], [3]. This assumes that the vortex is in plane with the acquisition slice, which might not be the case, particularly in the case of structural abnormalities. Availability of 3D blood velocities would also provide valuable information for constraining and validating patient specific cardiac models [6].

Time resolved, 3D cardiac blood velocity can be currently imaged using phase-contrast magnetic resonance imaging (4D flow MRI) and echocardiographic particle image velocimetry (echo-PIV). 4D flow MRI remains the technique of reference and is clinically established [7]. However, 4D flow MRI is relatively expensive, takes long time, requires general anaesthesia in young children and paediatric hearts are difficult to image in detail due to their small size and high heart rate. Echo-PIV is an accurate way of measuring 2D blood velocity but it needs the injection of contrast agent and a main limitation for 3D PIV is that it would need very high frame rates for frame-to-frame tracking of the required contrast agent which is not achievable with current commercial systems.

Latest advances in echocardiographic (echo) systems allow for acquisition of 3D images of anatomy (BMode) and flow (colour Doppler). In addition, echo is widely available and accepted, comfortable for patients, cost effective and compatible with cardiac implants. Doppler techniques allow measure of blood velocity without the need for any contrast agent. However, Doppler measured velocity is only a 1D projection of the true 3D velocity vector along the beam direction and thus constitutes only a partial velocity measurement.

A. Previous Work

Over the past years there has been increasing interest in recovering all components of intraventricular blood velocity using echo. Unfortunately, most of the previously proposed approaches only consider the reconstruction of a 2D projection of the 3D velocities over a 2D slice of the ventricle. Crossed-beam techniques, which use multiple views to reconstruct the 2D velocity information, have the potential to overcome the 1D limitation [8]. More recently, alternative approaches to recover 2D intracardiac velocity maps have been proposed, either by 2D

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blood tracking [9] or by assuming zero flow variation orthogonal to the scan plane and imposing incompressibility of the 2D flow [10], [11]. Arigovindan *et al.* proposed a crossed-beam method [12], which used a B-spline framework to recover 2D blood flow in a carotid from two 2D colour Doppler images. In a previous paper [13] inspired by the former we proposed the first crossed-beam method to recover 3D blood velocity from multiple registered 3D echo Doppler views. An important limitation of this method is that 3D velocity can only be recovered where at least 3 views overlap. Achieving such a coverage in the entire ventricle is difficult in clinical practice, for two reasons: firstly 3D colour Doppler images have reduced field of view (FoV) within a small colourbox compared to standard BMode images; and secondly the echocardiographer has no means of knowing what regions have been already covered by previous images. Partial coverage limited to a small volumetric region of the ventricle or to a single 2D plane prevent the calculation of global quantities such as kinetic energy and cardiac efficiency which require 3D velocities over the entire ventricle. Moreover, correct alignment of a 2D acquisition plane with the vortex plane is challenging with structural abnormalities where the vortex plane can change over time.

Early results presented in [14] suggested that information on ventricular wall motion and its coupling with measurements of intracardiac blood velocity has the potential to enable full ventricular velocity reconstruction even in suboptimal coverage conditions. This paper extends and further validates the method introduced in [14]. The novel contribution introduced in this paper is three-fold: first we extend the 3D formulation from [14] to 4D, which allows the cyclic behaviour of blood flow to be imposed and to incorporate data which is non-uniformly distributed over time; second we provide a quantitative and qualitative analysis on the ability of wall motion to enable full velocity field coverage of the ventricle when there are gaps between colour Doppler views; and third we validate the method on synthetic data from simulations and real data from 6 paediatric congenital patients.

This paper is organised as follows. Section II describes the reconstruction method and details how wall motion may be incorporated into the proposed framework. Section III describes the experiments carried out in synthetic and patient data, including a description of how realistic simulated data is generated from computational models of the heart. Section IV shows both qualitative and quantitative results on simulated and patient data. A critical discussion on the method and its limitations is presented in Sections V and VI.

II. METHODS

An overview of the proposed method is represented in Fig. 1. In summary, a 3D BMode sequence and three or more 3D colour Doppler images of the ventricle are acquired from different views. Wall motion is extracted from the BMode sequence (Section II-A), and a 1D projection of blood velocity is extracted from each Doppler image. All images are registered into the same coordinate space (Section II-B). The registered data is combined using a B-spline framework to yield 4D blood velocity over the entire ventricle (Sections II-C, II-D).

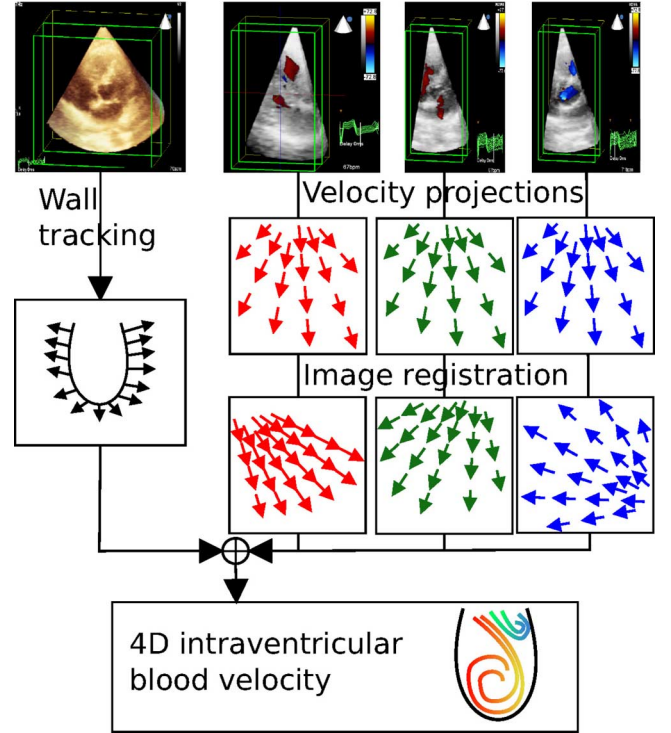


Fig. 1. Overview of the proposed method. Wall motion from a 3D + t BMode image is combined with Doppler velocities from n views to recover full 4D intraventricular flow.

Conveniently, the 4D formulation enables the imposition of cyclic flow (Section II-F) and the incorporation of temporally scattered input data.

A. Calculation of Ventricular Wall Motion

Ventricular wall motion was calculated from the 3D BMode sequence using the Temporal-Sparse Free Form Deformation (TS-FFD) method proposed in [15]. This method provides a smooth and diffeomorphic velocity field over the entire 3D + t BMode image. The endocardial wall was segmented at one reference time phase (the first acquired frame) using the MITK software [16] and the resulting binary segmentation was converted into a closed mesh. The segmentations were carried out manually by experts, thus we assume the error in boundary delineation is minimum. The inflow and outflow tracts were manually removed from the closed mesh. The output from the TS-FFD was used to propagate the resulting mesh throughout the cardiac cycle. An example of a 2D slice of the propagated mesh for patient 1 is shown in Fig. 2. Wall velocity (shown as coloured vectors) was derived from the propagated mesh by using cubic interpolation between consecutive positions of corresponding mesh nodes.

We assume that blood flow slips freely on the ventricular wall and that there is no fluid penetration through the wall (Figs. 2(c) and 2(d)). This condition is preferred to no-slip condition because normal components of the wall velocity can be calculated more accurately than tangential components. The free-slip condition can be expressed as:

$$\mathbf{v}_{b \perp} = \mathbf{v}_{w \perp} \quad (1)$$

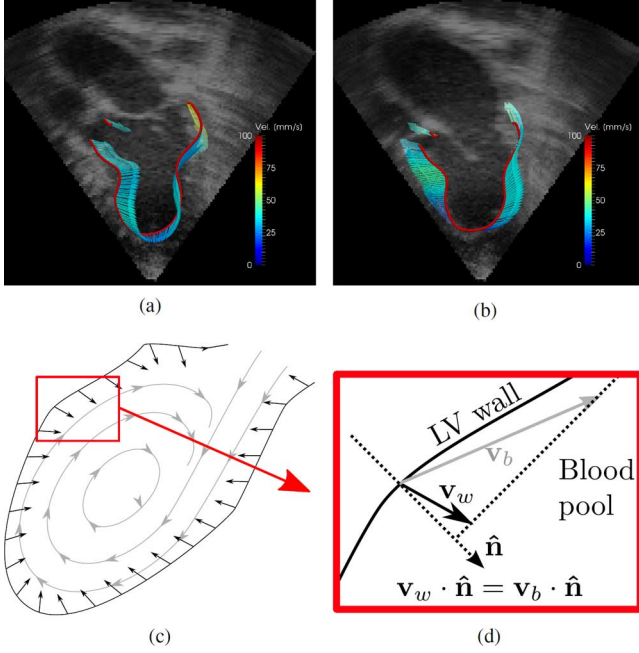


Fig. 2. Ventricular wall motion. (a),(b) 2D slice of the propagated mesh and wall velocity vectors, for a systolic phase (a) and a diastolic phase (b). (c) Wall and blood motions, detailed in (d). (a) $t = 3/15$ (b) $t = 11/15$ (c) (d).

where $\mathbf{v}_{b\perp}$ is the component of blood velocity orthogonal to the wall and $\mathbf{v}_{w\perp}$ is the component of the wall velocity orthogonal to the wall. There is no permeation because the relative velocity of blood with respect to the wall is zero in the orthogonal dimension, this is, the component of blood velocity orthogonal to the wall equates the component of wall velocity orthogonal to the wall. Thus, once the wall velocity is known, we know a projection of the blood velocity along the direction orthogonal to the wall at the position where the wall motion is known (i.e., the mesh nodes). Therefore, we define the boundary velocity data set $\{m_k, \mathbf{p}_k, t_k, \hat{\mathbf{d}}_k\}$, $k = 1 \dots K_w$, where m_k is the orthogonal component of the estimated wall velocity at node k of the wall mesh at coordinates $\mathbf{p}_k$, corresponding to the cardiac phase at time t_k and $\hat{\mathbf{d}}_k$ is the unit vector orthogonal to the wall at $\mathbf{p}_k$. The key remark is that $\mathbf{v}_{w\perp} = \mathbf{v}_{b\perp}$ provides one component of the blood velocity at one location, in the same way as any Doppler measurement does; therefore the $\mathbf{v}_{w\perp}$ component of wall motion can be incorporated into the reconstruction problem as if it was another Doppler measurement.

B. Image Registration

Each colour Doppler image includes a reduced FoV BMode anatomical image in the background and velocity measurements as a colour overlay. The background image was used to rigidly register the three colour Doppler images and the standard BMode. Registration was initialised by picking four corresponding points in each image and using landmark-based registration. The image registration itself used a similarity metric based on phase congruency proposed in [17]. The initialisation landmarks depended on the FoV of each view but were typically picked from among: the valve hinges; the apex; the middle of the inflow valve; the middle of the outflow tract; the middle of the interventricular wall; and the insertion

of the papillary muscles. A more detailed description of the registration process can be found in [13].

Each registered Doppler dataset i contributes to the reconstruction problem with a Doppler velocity dataset $\{m_k, \mathbf{p}_k, t_k, \hat{\mathbf{d}}_k\}_i$ where for each point $k = 1 \dots K_i$, m_k is the (projected) velocity measurement at position $\mathbf{p}_k$ and time t_k along the echo beam direction $\hat{\mathbf{d}}_k$ for that view.

C. Least Squares Formulation of the Reconstruction in a B-Spline Grid

The proposed method consists in solving a B-spline based linear system. Let $\mathbf{v}(\mathbf{p}, t) = [v_x(\mathbf{p}, t) \ v_y(\mathbf{p}, t) \ v_z(\mathbf{p}, t)]^\top$ be the blood velocity field evaluated at position $\mathbf{p} = [p_x \ p_y \ p_z]^\top$ and time t . $\mathbf{v}(\mathbf{p}, t)$ can be expressed in the space of uniform B-splines of degree d :

$$v_\gamma(\mathbf{p}, t) = \sum_{i,j,k,n} c_{i,j,k,n}^\gamma \beta_{s,i}^d(p_x) \beta_{s,j}^d(p_y) \beta_{s,k}^d(p_z) \beta_{\tau,n}^d(t) \quad (2)$$

where $\gamma \in \{x, y, z\}$, the B-spline grid has a size of $N_g^i \times N_g^j \times N_g^k \times N_g^n$, and $\{c^x, c^y, c^z\}_{i,j,k,t}$ are the B-spline coefficients. To improve clarity notation for B-Spline functions has been shortened as $\beta_{s,i}^d(\cdot) = \beta^d(\cdot/s - i)$, where $\beta^d(\cdot)$ is the B-spline of degree d and s is the distance between grid nodes and determines the finest resolution of the resulting vector field (i.e its scale).

Appending input data from all N colour Doppler views and the K_w points from wall motion, the resulting $K_w + \sum K_i$ input data points can be used to calculate the 4D velocity field $\mathbf{v}(\mathbf{p}, t)$ by minimising the following energy term:

$$j_{proj}(\mathbf{v}) = \sum_k^{K_w + \sum K_i} \|\hat{\mathbf{d}}_k \cdot \mathbf{v}(\mathbf{p}_k, t_k) - m_k\|^2 \quad (3)$$

where $\hat{\mathbf{d}}_k = [d_{x,k} \ d_{y,k} \ d_{z,k}]$ are the beam directions and m_k and the Doppler measurements as defined in Section II-B. The problem can be posed in matrix form by defining the beam direction block-diagonal matrix $\mathbf{D}$ (4), the coefficient column matrix $\mathbf{C}$ (5) and the B-Spline sampling matrix $\mathbf{S}$ (6) defined below:

$$\mathbf{D} = [\mathbf{D}_x \ \mathbf{D}_y \ \mathbf{D}_z] \quad \{\mathbf{D}_x\}_{kk} = d_{x,k} \ \{\mathbf{D}_y\}_{kk} = d_{y,k} \ \{\mathbf{D}_z\}_{kk} = d_{z,k} \quad (4)$$

$$\mathbf{C} = [\mathbf{C}_x \ \mathbf{C}_y \ \mathbf{C}_z]^\top \quad \{\mathbf{C}_x\}_{((nN_g^k+k)N_g^j+j)N_g^i+i,1} = c_{i,j,k,n}^x \text{ and so on for } y, z \quad (5)$$

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_s & 0 & 0 \\ 0 & \mathbf{S}_s & 0 \\ 0 & 0 & \mathbf{S}_s \end{bmatrix} \quad \{\mathbf{S}_s\}_{r,((nN_g^k+k)N_g^j+j)N_g^i+i} = \beta_{s,i}^d(p_{r,x}) \beta_{s,j}^d(p_{r,y}) \beta_{s,k}^d(p_{r,z}) \beta_{\tau,n}^d(t_r). \quad (6)$$

These matrices allow (3) to be expressed in matrix form:

$$j_{proj}(\mathbf{C}) = (\mathbf{DSC} - \mathbf{m})^\top (\mathbf{DSC} - \mathbf{m}) \quad (7)$$

where $\mathbf{m} = [m_1 \ m_2 \ \dots]^\top$ are the Doppler and wall motion measurements. B-spline coefficients $\mathbf{C}$ are calculated by minimising

(7) by taking the derivative of the energy function $j_{proj}(\mathbf{C})$ with respect to $\mathbf{C}$, and equating to zero:

$$\frac{\partial}{\partial \mathbf{C}} j_{proj}(\mathbf{C}) = 2\mathbf{C}^\top (\mathbf{DS})^\top (\mathbf{DS}) - 2\mathbf{m}^\top (\mathbf{DS}) = 0 \quad (8)$$

which yields

$$\begin{aligned} \mathbf{C}^\top &= \mathbf{m}^\top (\mathbf{DS}) [(\mathbf{DS})^\top (\mathbf{DS})]^{-1} \\ \mathbf{C} &= \mathbf{A}_{proj}^{-1} \mathbf{b} \end{aligned} \quad (9)$$

where $\mathbf{A}_{proj} = \mathbf{S}^\top \mathbf{D}^\top \mathbf{DS}$ and $\mathbf{b} = \mathbf{S}^\top \mathbf{D}^\top \mathbf{m}$. The reconstructed 4D velocity field at the points and times sampled by $\mathbf{S}$ can be obtained as $\mathbf{V} = \mathbf{SC}$.

D. Incorporation of Physical Constraints

The proposed linear system will in general need some regularization. We have proposed [13] to introduce a penalty term proportional to the divergence of $\mathbf{v}$ in 3D, which enforces incompressibility of blood flow. In this paper, velocity is reconstructed over time, so this penalisation applies for all t . The solution to the regularised problem consists of the minimisation of a new energy function:

$$j(\mathbf{v}) = j_{proj}(\mathbf{v}) + \frac{\lambda}{1-\lambda} j_{div}(\mathbf{v}) \quad (10)$$

where $\lambda \in [0, 1)$ controls the trade-off between compliance with input data and amount of regularization, and

$$j_{div}(\mathbf{v}) = \sum_k \|\nabla \cdot \mathbf{v}(\mathbf{p}_k, t_k)\|^2. \quad (11)$$

The term on the right hand side of (11) can be expressed in matrix form, using the derivative B-spline sampling matrix $\dot{\mathbf{S}}$ extended from [13] to 4D, as detailed in Appendix A. The energy term corresponding to the divergence of the velocity (11) can be written in matrix form as

$$j_{div}(\mathbf{C}) = \mathbf{C}^\top \dot{\mathbf{S}} \mathbf{C} \quad (12)$$

and the full energy term, equivalent to (10) is $j(\mathbf{C}) = j_{proj}(\mathbf{C}) + \lambda/(1-\lambda) j_{div}(\mathbf{C})$. Taking the derivative and equating to zero yields

$$\begin{aligned} \frac{\partial}{\partial \mathbf{C}} j(\mathbf{C}) &= 2\mathbf{C}^\top (\mathbf{DS})^\top (\mathbf{DS}) - 2\mathbf{m}^\top (\mathbf{DS}) + \\ &\quad \frac{\lambda}{1-\lambda} 2\mathbf{C}^\top \dot{\mathbf{S}} = 0 \end{aligned} \quad (13)$$

and therefore

$$\begin{aligned} \mathbf{C}^\top (\mathbf{DS})^\top (\mathbf{DS}) + \frac{\lambda}{1-\lambda} \mathbf{C}^\top \dot{\mathbf{S}} &= \mathbf{m}^\top (\mathbf{DS}) \Rightarrow \\ \mathbf{C} &= \left[(\mathbf{DS})^\top (\mathbf{DS}) + \frac{\lambda}{1-\lambda} \dot{\mathbf{S}} \right]^{-1} (\mathbf{DS})^\top \mathbf{m} \Rightarrow \\ \mathbf{C} &= \left[\mathbf{A}_{proj} + \frac{\lambda}{1-\lambda} \mathbf{A}_{div} \right]^{-1} \mathbf{b}. \end{aligned} \quad (14)$$

E. Multi-Scale Refinement of the B-Spline Reconstruction

The input data will be non-uniformly distributed in space and time. In particular, some regions of the ventricle will lack input

data. These “empty” regions occur typically near the ventricular wall, since the FoV of the Doppler images is small and the sonographer aims at the valves and at the centre of the ventricle to capture the main velocity features.

A coarse-to-fine B-spline refinement reconstruction can handle the sparsely distributed input data in an efficient way. At coarser levels the result approximately fits large features while at a finer scale the result in regions with low density of input data is driven by the regularization term $\nabla \cdot \mathbf{v} = 0$.

F. Imposing Temporally Cyclic Reconstruction

The different colour Doppler views are acquired one after the other, thus the proposed method relies on the assumption that the cardiac cycle does not vary during the echo examination except for slight changes in heart rate that can be compensated by temporally scaling each colour Doppler sequence. As a result, the reconstructed velocity field should also be cyclic over time.

Cyclic motion was achieved in [15] by calculating the grid node influencing each input data point modulo grid size along the time direction. In this paper, the same approach is adopted, which results in redefining the sampling matrix $\mathbf{S}$. Provided that the cycle is defined in the interval $t \in [t_0, t_1)$, the cyclic B-spline sampling matrix $\mathbf{S}_c$, which replaces $\mathbf{S}_s$ from (6), is defined as:

$$\{\mathbf{S}_c\}_{r,((nN_g^k+k)N_g^j+j)N_g^i+i} = \beta_{s,i}^d(p_{r,x})\beta_{s,j}^d(p_{r,y})\beta_{s,k}^d(p_{r,z})\beta_{\tau,n}^d([t_r-t_0] \pmod{t_1}). \quad (15)$$

Since the t axis is now periodic the spacing between grid nodes along time τ is restricted to integer fractions of $t_1 - t_0$.

III. EXPERIMENTS

We carried out experiments on both patient and synthetically generated data. The experiments on synthetic data, where a ground truth was available, were aimed to ascertain the ability of the method to recover whole-ventricle 3D flow under different scenarios of volume coverage. In all cases, B-splines of degree 3 and $\lambda = 0.1$ were used, following the findings in [13]. The whole reconstruction took approximately three hours on a standard workstation. Recent improvements in hardware plus the fact that matrix operations and patch calculations could be run in parallel could yield a much faster execution time.

A. Experiments From Simulated Data

The aim of the experiments on simulated data were to evaluate to what extent the combination of wall motion and colour Doppler information could help filling the gaps in input data, in order to recover 3D velocity over the entire ventricle.

Accuracy was measured through mean error in velocity magnitude and mean error in velocity direction (angle). In addition, a fourth independent 3D colour Doppler view was compared to the reconstructed velocity, as in a leave-one-out (LOO) strategy. This was to help interpret the results obtained from patient data, where no ground truth was available, and so a LOO accuracy measurement was carried out.

The same algorithm parameters were used for synthetic and patient data: $\lambda = 0.1$ and 4 refinement levels, with the grid

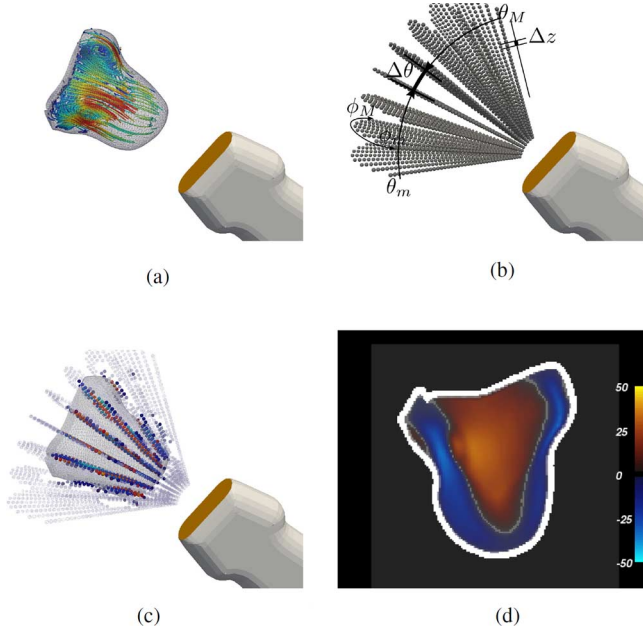


Fig. 3. Simulation of colour Doppler images from a computational model of a normal ventricle. (a) Location of the virtual ultrasound transducer with respect to the computational model of the heart. (b) The acquisition parameters: angular and longitudinal extent and resolution. (c) Velocity projection sampled along the virtual scan lines. (d) A slice of a scan-converted 3D simulated colour Doppler image, acquired during ventricular filling.

spacing ranging from 40 mm at the coarsest level to 4 mm at the finest level.

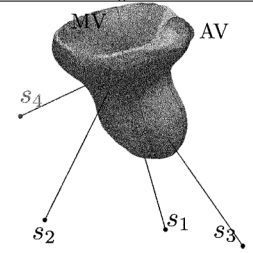
1) *Generation of Synthetic Colour Doppler Images:* Simulated data was extracted using a computational model [2] of a morphologically normal left ventricle. The anatomical model generation required segmentation of the endocardium at end systole from echo data. The volume mesh of the blood pool was subsequently generated using the Cubit software package (<https://cubit.sandia.gov>) and consisted of 101,716 tetrahedra. The wall motion computed from the echo data throughout the cardiac cycle was interpolated to the nodes on the surface of the mesh and applied as a Dirichlet boundary condition. A similar boundary condition was imposed at the valves: the inflow and outflow velocity vectors were aligned to the normal directions at the corresponding valve planes and their magnitude was adjusted to be consistent with the volume change dictated by the wall motion at every frame. Finally, direct numerical simulations were performed on the model, using a solver based on finite element methods whose application to haemodynamics problems has been extensively validated [18].

The simulated colour Doppler data was generated as illustrated in Fig. 3. Four realistic view positions, obtained during a standard echo imaging procedure, were used to generate four synthetic colour Doppler views from the computational model (Fig. 3(a)). A 3D synthetic image was generated every 30ms. The view positions relative to the ventricle are shown in the figure accompanying Table I. Views s_1 , s_2 and s_3 were used for reconstruction and view s_4 was used for the LOO validation.

Simulated images were generated by a uniform distribution (in spherical coordinates) of scan lines, represented as a succession of sampling points in Fig. 3(b), scanning a FoV de-

TABLE I
ACQUISITION PARAMETERS OF THE SYNTHETIC DOPPLER IMAGES. THE ABBREVIATIONS ON THE LEFT COLUMN REPRESENT: $\Delta\theta$: θ RANGE; $\Delta\phi$: ϕ RANGE; Δd : DEPTH RANGE; %V: RELATIVE FRACTION OF THE VENTRICULAR VOLUME COVERED BY EACH VIEW; % V_n : RELATIVE FRACTION OF THE VENTRICULAR VOLUME COVERED BY $\leq n$ VIEWS (% V_0 : FRACTION WITH NO COVERAGE AT ALL). ALL ANGLES ARE IN DEGREES. BOTTOM RIGHT: DIAGRAM WITH THE LOCATION $\{s_i\}_{i=1,\dots,3}$ OF THE VIRTUAL VIEW POINTS, TAKEN FROM A PATIENT SCAN. IMAGES GENERATED FROM VIEW POINT s_4 WERE USED FOR LOO VALIDATION

View	Setup 1			Setup 2			Setup 3		
	s_1	s_2	s_3	s_1	s_2	s_3	s_1	s_2	s_3
$\Delta\theta$	60	65	60	33	34	34	33.5	34	33
$\Delta\phi$	60	55	50	34	34	34	34	33.5	34
Δd	55	45	45	52	52	59	53	53	58
%V	100	100	100	76.5	70.0	82.4	64.3	50.3	53.1
% V_3	100			49.02			17.1		
% V_2	0			30.27			42.8		
% V_1	0			16.15			28.1		
% V_0	0			4.56			12.0		
View	Setup 4								
	s_1	s_2	s_3						
$\Delta\theta$	33.5	34	34						
$\Delta\phi$	33.5	33.5	33.5						
Δd	53.5	53	58						
%V	46.0	48.5.6	24.1						
% V_3	1.39								
% V_2	27.6								
% V_1	58.03								
% V_0	12.9								



fined by the angular widths $\Delta\theta$ and $\Delta\phi$ and by the total depth Δd . At each sampling point, the projection of the velocity from the computational model along the direction of the scan line was computed, resulting in the synthetic Doppler velocity value. Fig. 3(c) shows the synthetic Doppler velocities at their sampled locations. Fig. 3(d) shows a slice of the resulting scan-converted synthetic colour Doppler image.

2) *Experiments Design:* A total of 4 different setups, each providing 3 views (the acquisition parameters are detailed in Table I) were generated covering a number of different scenarios.

The four sets of parameters shown in Table I cover a range of possible acquisitions, represented in Fig. 4. Setup 1 represents the ideal case, in which all the views cover 100% of the heart volume. Setups 2 to 4 have been generated using acquisition parameters taken from real echo images acquired on a paediatric patient. The generated views have been grouped in sets of three views, so that the setups represent realistic cases of decreasing coverage. Setup 2 represents a realistic best-case scenario, where the overlap of the three views covers approximately half of the ventricle. Setup 3 represents a scenario where little coverage is achieved with three views simultaneously but where at least two views cover 60% of the ventricular cavity. Setup 4 represents a case in which only 30% of the ventricle is covered with at least 2 views.

Full 4D flow was recovered in the simulated data using setups 1 to 4 with and without incorporating wall motion.

B. Experiments on Patient Data

Full 4D intraventricular velocity was reconstructed for 6 paediatric patients, with and without the incorporation of wall motion. For each of these two cases, blood velocity was reconstructed using three images each covering the central part of

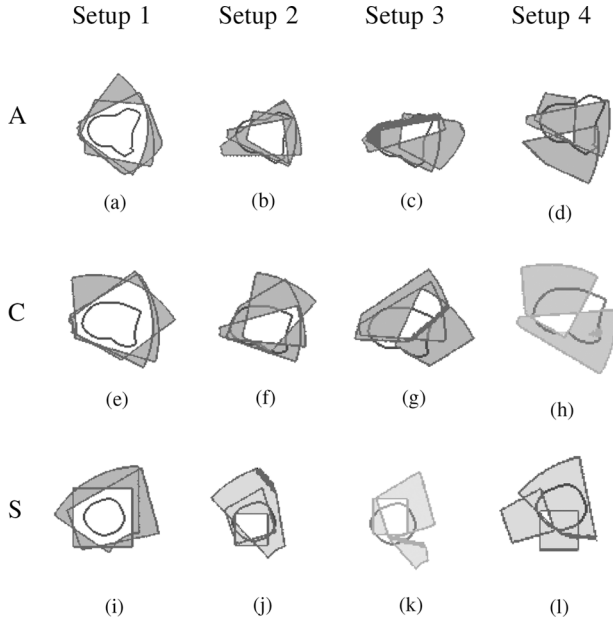


Fig. 4. View overlap for each simulated setup. Each row represents a 2D slice of the schematic overlap on the axial (A), coronal (C) and sagittal (S) planes. The region where all three views overlap is highlighted in white.

the ventricle. Patient 1 had a repaired critical aortic stenosis, and reconstruction was carried out in the anatomically normal left ventricle (LV). Patients 2 to 6 had Hypoplastic Left Heart Syndrome (HLHS), and therefore flow was reconstructed on the right ventricle (RV). The average age of the patients was 3 years old ± 3 years. Patients 2 and 6 had a slit-like LV, so the shape of the RV was close to the shape of a normal LV. Patients 3, 4 and 5 had a globular LV so the shape of the RV was abnormally elongated and curved (Fig. 12).

We carried out three quantitative experiments. In the first experiment, we calculated the average reconstruction error using a LOO strategy as for the simulated data. In the second experiment, we separated the regions where the number of overlapping input views was 0, 1, 2 and 3+, and calculated the average error independently. In the third experiment we calculated the velocity magnitude at the centre of the ventricle inflow valve over the cardiac cycle. In addition to the quantitative analysis, we qualitatively analysed the vortex formation for the ellipsoidal ventricles (patients 1, 2, 6) and the curved ventricles (patients 3, 4, 5).

1) Acquisition Protocol: Three colour Doppler views were acquired with a Philips X7-2 (patient 1 and 4) or a Philips X3-1 (patients 2, 3, 5 and 6) probe during breath-holds, targeting the systemic ventricle: 1 apical, 1 parasternal and 1 apical from a medial position. Since the colour Doppler FoV was smaller than the ventricle in all cases, the colour box was set to include as much as possible of the central part of the ventricle and the valves. One additional view was acquired for validation and was not used for reconstruction.

Doppler range was set to values from ± 77 cm/s to ± 134.3 cm/s depending on the view while trying to maximise colour filling and avoiding aliasing. A standard BMode sequence was acquired in each view for image registration. The system was set to 4 beat acquisition for the BMode, 7 beat

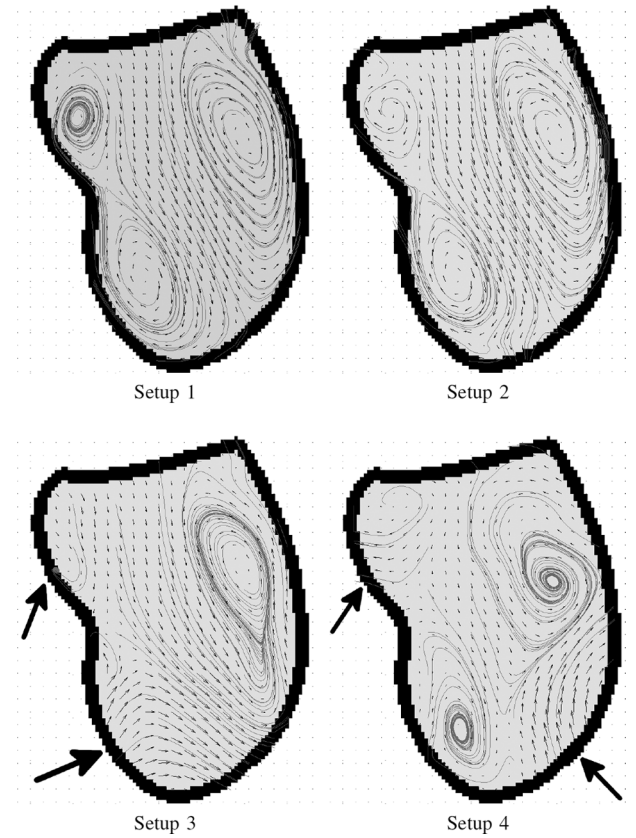


Fig. 5. Reconstructed velocity (1 slice) from simulated data using 4 levels of grid refinement, showing the difference between the different setups with no wall motion.

acquisition for the colour Doppler. Image data in spherical coordinates was extracted from the DICOM files. Patients were under general anaesthesia with active respiratory control. Acquisitions where variation in heart rate was greater than 10% to the average was discarded. Images were acquired with institutional ethical approval after informed parental consent.

2) Data Pre-processing: Colour Doppler images were semi-automatically dealiased. All sequences were temporally normalised to the duration of an arbitrary sequence by scaling the time between frames. As a result, input data was non-uniformly distributed along time as opposed to the data in [13], [14] which was resampled in a uniform time grid. Temporally non-uniform data was usable because in this paper the B-spline reconstruction was extended to 4D.

IV. RESULTS

A. Results on Simulated Data

3D flow was reconstructed from simulated data using the virtual 3D Doppler views defined in the four setups in Table I, with and without incorporating wall motion. These setups correspond to the four different degrees of coverage shown in Fig. 4. Fig. 5 shows a slice of the 3D reconstructed velocity (without using wall motion) for each setup after four refinement levels. The small arrows in the ventricle represent the direction and relative magnitude of blood velocity. Additionally, flow structures are illustrated with streamlines. As the coverage with two and

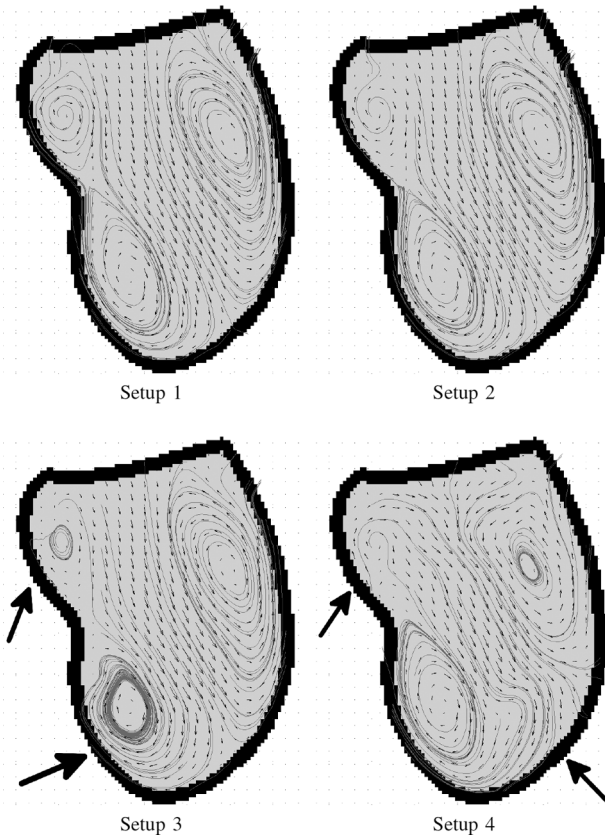


Fig. 6. Reconstructed velocity (1 slice) from simulated data using 4 levels of grid refinement, showing the difference between the different set-ups with wall motion.

three views decreases from setup 1 to setup 4, some regions of the ventricle (indicated with large black arrows) present clearly incorrect flow patterns, for example in setup 3 the apical region shows implausible patterns and setup 4 shows chaotic vortices and incorrect arrows near the walls. As a result, realistic flow is only achieved in setups 1 and 2. Fig. 6 shows the same results as Fig. 5 this time achieved when incorporating wall motion to the reconstruction problem. It can be noticed how flow patterns are now plausible in all setups. In particular, flow reconstruction near the wall has improved and the vortices are correctly positioned.

Quantitative results support the qualitative observations from Figs. 5 and 6. Fig. 7 shows the reconstruction error in magnitude (top left) and direction (top right) with respect to the ground truth velocity. The whiskers represent the standard deviation. The bottom left chart in Fig. 7 shows the relative reconstruction error using a leave-one-out (LOO) strategy. The results with the LOO measurement are consistent with the results with respect to the ground truth, showing an increase in accuracy when using wall motion information. The LOO experiment is also carried out on real data so this experiment allows us to relate real and synthetic data. We carried out a 2-way ANOVA test on the error in magnitude using the setup number and reconstruction type (with/without wall motion) as independent variables. This showed a statistically significant difference between samples ($p < 0.01$) both between the reconstruction type and between the setups. Since the mean error with wall motion is con-

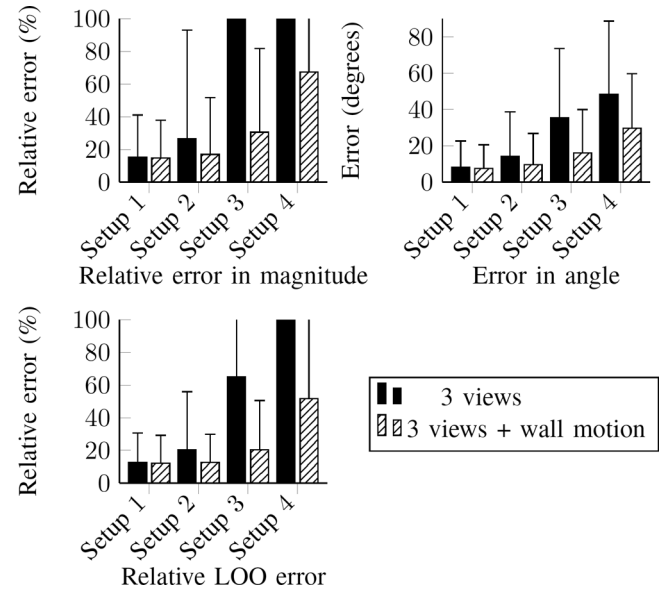


Fig. 7. Error in flow reconstruction from synthetic data. The bars show mean relative error, the whiskers show the standard deviation of the relative error. First row shows the error in velocity magnitude and angle with respect to the ground truth. Bottom left shows the relative error with respect to an independent (unused) view.

sistently less than without wall motion for all cases, this shows that using wall motion statistically significantly improves reconstructed flow accuracy.

The 2-way ANOVA test also showed a significant interaction between the two factors. To look at the interaction of the two main factors separately, we applied a 1-way ANOVA test using 8 groups consisting on all combinations of the two factors above, which resulted in a rejection the null hypothesis ($p < 0.01$). A post-hoc Sheffe test failed to show statistically significant difference in the reconstruction error between setups 1, 2, 3 and 4 when using wall motion, however there was a statistically significant difference between setups 3 and 4 (low Doppler coverage) with no wall motion and all the other setups. This backs up our visual observations where much better reconstructions could be seen in setups 3 and 4 with wall motion (Fig. 6) compared to these setups without wall motion (Fig. 5).

B. Results on Real Data

1) *Quantitative Results:* Reconstruction error for the six patients is presented in Fig. 8. The bars in the chart represent the average LOO error and the whiskers represent the standard deviation of the error with and without incorporating wall motion into the reconstruction. In all cases, the use of wall motion significantly reduced the error.

Fig. 9 shows the average error for each patient over the regions of the ventricle where $n_{overlap} = 0, 1, 2, 3$ views overlapped. A two way ANOVA test using the patient number and the number of overlapping views ($n = 0, 1, 2, 3$) as independent variables yielded $p < 0.01$ for both factors. Using a Sheffe post hoc test along the number of overlapping views, we obtained significant difference between $n = 0$ and $n = 3$ ($p < 0.05$), but the difference between $n = 0$ and $n = 2$ ($p = 0.11$) and $n = 1$ and $n = 3$ ($p = 0.06$) could not be considered significant. The

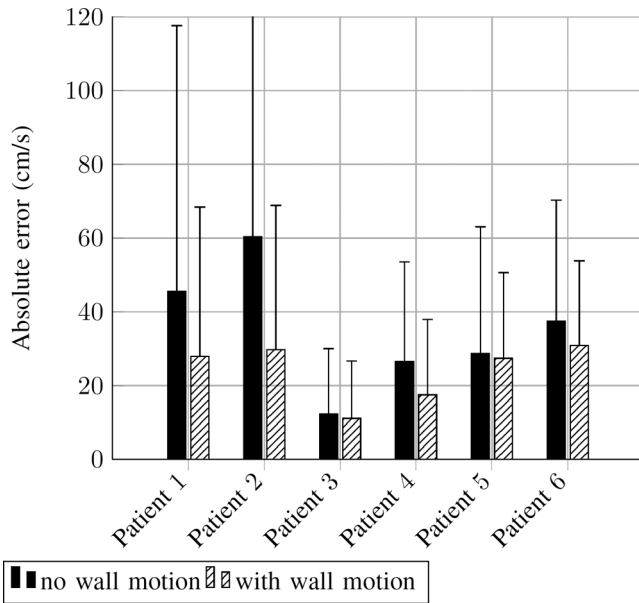


Fig. 8. Error in flow reconstruction from 6 congenital patients with respect to an independent and unused view. The bars indicate the average absolute error and the whiskers represent the standard deviation.

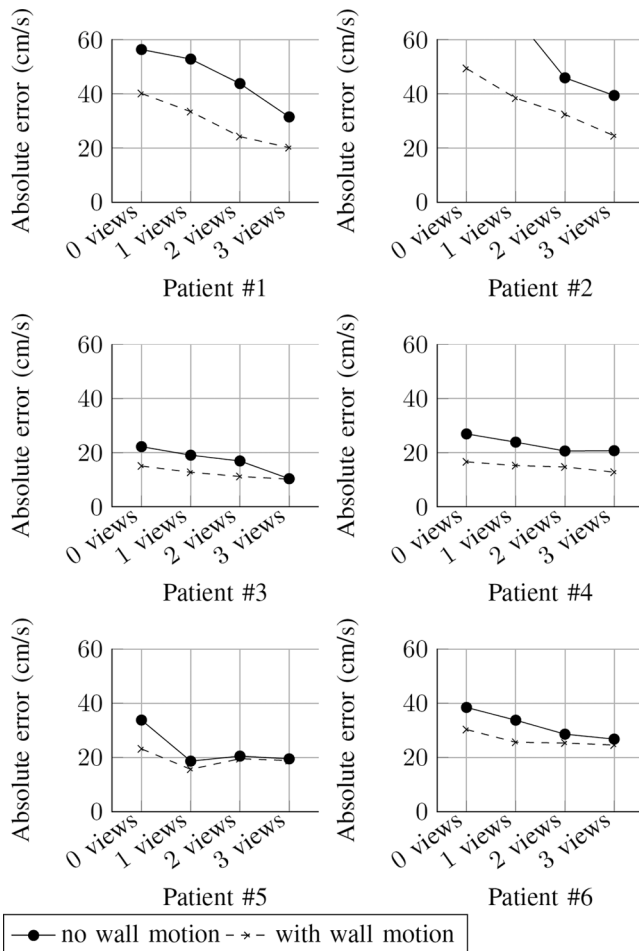


Fig. 9. Error in flow reconstruction from 6 congenital patients with respect to an independent and unused validation view, classified by the number of overlapping views intersecting the validation view.

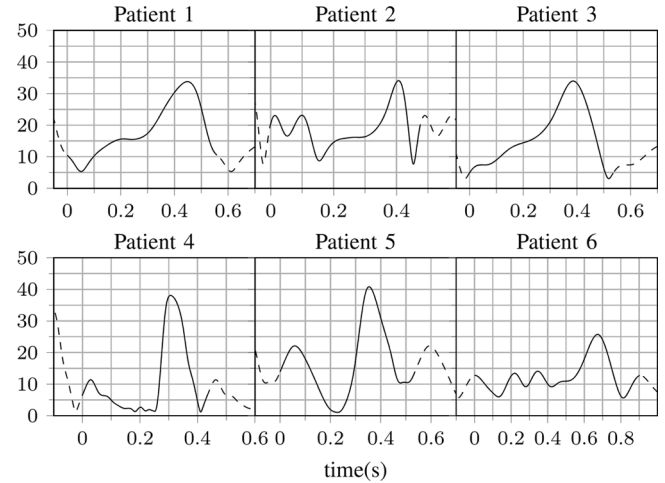


Fig. 10. Inflow velocity magnitude reconstructed on 6 patients. Vertical axis indicates velocity magnitude, in cm/s. The solid line indicates one cycle, which is prolonged with a dashed line to demonstrate periodicity of the reconstruction.

test failed to show statistically significant difference between $n = 0$ and $n = 1$ ($p = 0.9$), $n = 1$ and $n = 2$ ($p = 0.27$), or $n = 2$ and $n = 3$ ($p = 0.84$). These results are consistent with the observations on synthetic data and indicate that cases with worse coverage benefit more significantly from wall motion.

Fig. 10 shows the velocity magnitude at the ventricular inflow from the 6 patients. The solid line shows the velocity over the cardiac cycle (starting at end diastole). The dashed line extends the trace to the previous and the next cycle, demonstrating the cyclic behaviour of the reconstruction.

2) *Qualitative Results:* We show the effect of using wall motion in the reconstructed velocity, particularly near the ventricular wall. Additionally, we show examples of reconstructed vortices for different types of ventricular geometries.

Fig. 11 shows a 2D slice of the reconstructed velocity at different phases of diastole for patient 1, when using Doppler data only (left column) and when incorporating wall motion (right column). Velocity is represented by vectors coloured by velocity magnitude. The black arrows indicate the regions near the wall where reconstructed velocity benefits most from using wall motion. The incorporation of wall motion forces blood flow to remain confined inside the ventricle and forces the blood close to the wall to move parallel to the wall.

Fig. 12 show a three dimensional representation over time of the reconstructed velocities for patient 3 respectively, when using 3 colour Doppler views and wall motion extracted from BMode images. Ventricular anatomy is represented by a semi-transparent 3D mesh, and blood velocity is represented by three parallel slices where velocity magnitude is colour coded plus streamlines tangent to the velocity field. Each subfigure shows the reconstructed flow at a different phase of the cardiac cycle, starting at end diastole.

Fig. 13 shows the end diastolic state for the six patients. The patients have been separated between those who had an ellipsoid shaped systemic ventricle (left) and those who have a curved ventricle (right). In the former, the vortex is centred and takes over the majority of the ventricular volume. In the latter, the vortex is displaced and smaller with respect to the ventricular volume.

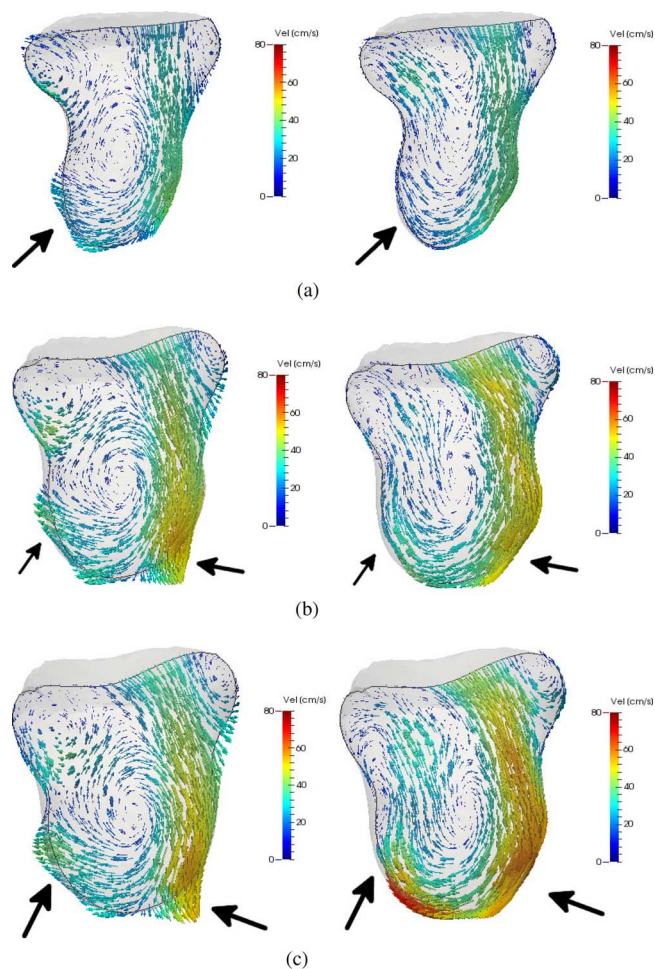


Fig. 11. Reconstructed velocity from real data from patient 1, showing the difference between using solely Doppler data (left column) and incorporating wall motion (right column), for different time phases relative to the length of the cardiac cycle. Velocity reconstructed in 3D, only single slice shown. (a) $t = 8/15$ (b) $t = 11/15$ (c) $t = 14/15$.

V. DISCUSSION

We have presented a method to incorporate wall motion estimated from 3D BMode images into the blood velocity reconstruction problem first described in [13], by assuming that there is no blood flow through the endocardial wall.

We have demonstrated the applicability and accuracy of the method on synthetic data generated from a computational model of the left ventricle and on real data from six paediatric patients. The results on simulated data show that incorporation of wall motion improves the reconstruction accuracy in all cases, and that the improvement is statistically significant when Doppler coverage is poor. This is often the case for data acquired in-vivo where acoustic access is limited by the patient's anatomy and some regions of the ventricle can be impossible to access from three different views. As a result it is advised that the acquired images cover at least the central part of the ventricle and the valves area, so that if areas with no Doppler coverage exist, they will be close to the ventricular wall and will be filled using wall motion information. The new 4D formulation enables the use of temporally scattered data and ensures a cyclic velocity field.

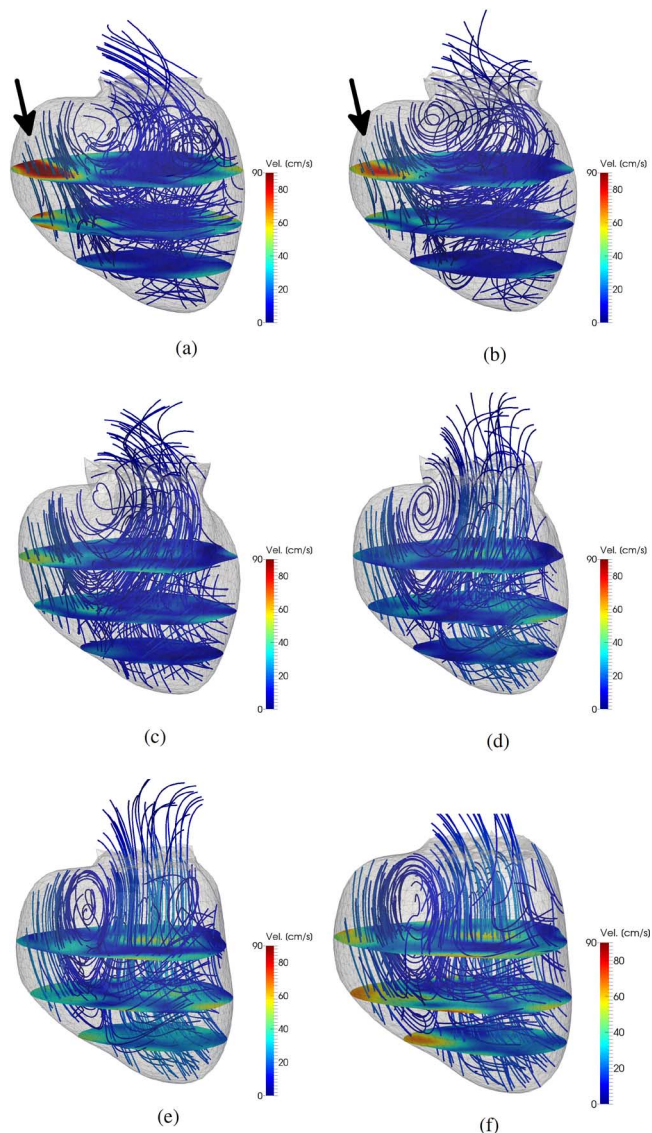


Fig. 12. Reconstructed velocity from patient 3, showing the velocity magnitude in three colour coded parallel slices and the flow patterns represented by streamlines. The black arrows in the systolic frames indicate the high velocity outflow jet. (a) $t = 5/20$ (b) $t = 8/20$ (c) $t = 12/20$ (d) $t = 15/20$ (e) $t = 18/20$ (f) $t = 20/20$.

The results from the six paediatric patients support the findings from synthetic data, following a similar trend: an improvement in reconstruction accuracy when wall motion is used. In particular, the improvement in accuracy is larger in the regions where fewer views overlap, which is desirable and also consistent with our simulations. Moreover, the presented results show how vortex formation depends on the ventricular geometry; For example, patients 1, 2 and 6 show circular vortices that grow to occupy the entire ventricle, which is a fairly spherical shape. Patients 3, 4 and 5, where the ventricle is elongated and asymmetric with respect to inflow direction, presents an eccentric vortex which grows on one side. This is consistent with findings on the same patient group using computational models [2].

Wall motion was calculated using the TSFFD method [15]. However, the proposed reconstruction method is agnostic to the

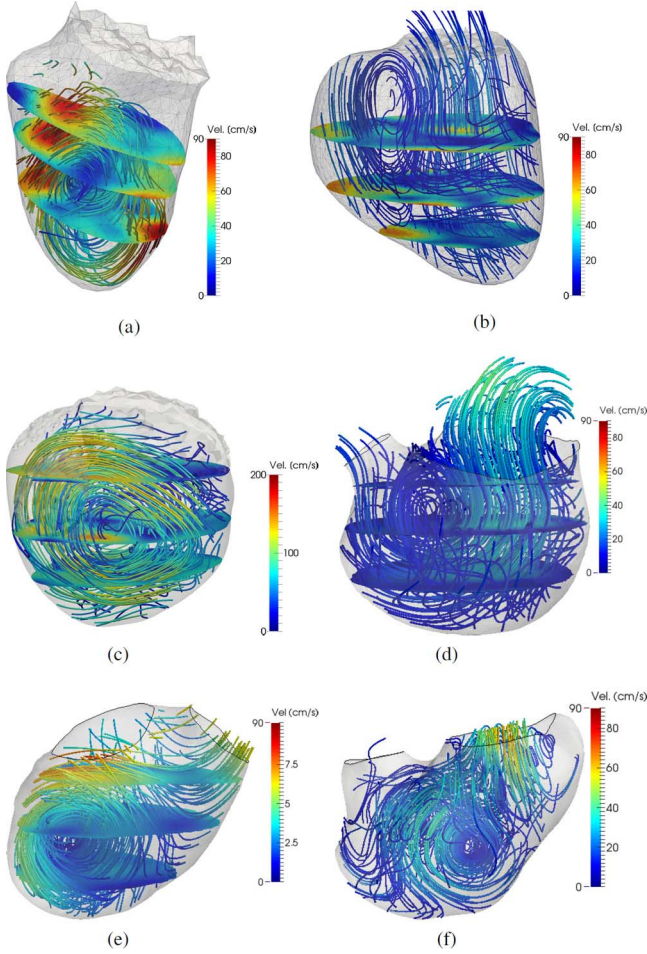


Fig. 13. Reconstructed velocity from six congenital patients, showing the velocity magnitude in three colour coded parallel slices and the flow patterns represented by streamlines. Three colour Doppler views and wall motion estimated from a 3D BMode sequence were used. Patients 1, 2 and 6 (left) have an ellipsoid shaped systemic ventricle, yielding a centred vortex. Patients 3, 4 and 5 (right) had a curved systemic ventricle which yield an eccentric vortex. (a) Patient #1 (b) Patient #3 (c) Patient #2 (d) Patient #4 (e) Patient #6 (f) Patient #5.

wall motion estimation technique, so if accuracy of wall motion estimation is a concern other sources of motion (tissue Doppler, tagged-MRI, etc) or other tracking methods could be used. The boundary was manually traced by experts, which should introduce minimum errors in segmentation.

Future work will investigate the incorporation of more sophisticated spatio-temporal physical constraints to reduce the amount of Doppler data required, especially the number of independent views needed. Fewer required views could extend the applicability of the method to patient groups where acquisition of three independent views is more difficult.

VI. LIMITATIONS

The proposed method relies on the assumption that the cardiac cycle is perfectly periodic and that therefore blood velocity is consistent across consecutive cycles. If there are significant changes in heart rate between acquisitions, the velocity reconstruction might show large errors, which we have not characterised in this paper.

The accuracy of the result depends on several factors, including the temporal alignment of the different acquired sequences. In this paper we use a rather simple method to temporally align the datasets: we assume that there is only a temporal scaling factor between the sequences. Other more efficient approaches [19] could be used.

Another factor that affects accuracy is the quality of the wall motion estimation. In order to minimise the error propagation of wall motion into the reconstructed flow, wall velocity is not imposed but simply included in the same way as the Doppler information, therefore our framework allows deviation from the measured wall motion, when it conflicts with the input Doppler data. Additionally, clinical translation of the proposed method would require a more automated approach to segmentation.

Regarding the clinical applicability of the proposed method, one must consider two required manual inputs: the initial alignment of the datasets and the dealiasing of aliased Doppler data. Landmark picking is too time consuming for regular clinical use, so for clinical translation of the proposed method automatic optical or electromagnetic trackers should be used [20]. Aliasing is a limitation in any Doppler based technique and dealiasing is an active area of research [21].

The use of the proposed method in adult patients remains challenging because the ventricular coverage and the image quality will be worse than in children. However, the proposed method would improve the results obtained with previous methods since can better manage suboptimal coverage. Patients with significant aliasing artefacts should not be considered for this method because input data is unreliable, unless a robust and accurate de-aliasing method is available.

VII. CONCLUSION

We have presented a new method to recover 4D intraventricular blood velocities from colour Doppler images and ventricular wall motion. This method allows recovery of 3D time resolved blood flow patterns and enables an improvement in reconstruction accuracy especially in cases where view coverage is poor.

The proposed method is the first to provide 4D velocity fields over the entire ventricle from echo data and introduces the ability to study 4D flow dynamics from ultrasound data acquired with commercially available systems.

APPENDIX

IMPLEMENTATION OF DIVERGENCE-FREE REGULARIZER

In this appendix we detail the calculation of the divergence regularizer, adapted from [22]. The regularization term is

$$\begin{aligned}
 j_{div} &= \int_{\Omega} \|\nabla \cdot \mathbf{v}\|^2 d\Omega = \int_{\Omega} \|\partial_x v_x + \partial_y v_y + \partial_z v_z\|^2 d\Omega \\
 &= \underbrace{\int_{\Omega} \partial_x v_x \partial_x v_x d\Omega}_{(i)} + \underbrace{\int_{\Omega} \partial_y v_y \partial_y v_y d\Omega}_{(ii)} + \underbrace{\int_{\Omega} \partial_z v_z \partial_z v_z d\Omega}_{(iii)} \\
 &\quad + 2 \underbrace{\int_{\Omega} \partial_x v_x \partial_y v_y d\Omega}_{(iv)} + 2 \underbrace{\int_{\Omega} \partial_x v_x \partial_z v_z d\Omega}_{(v)} + 2 \underbrace{\int_{\Omega} \partial_y v_y \partial_z v_z d\Omega}_{(vi)}
 \end{aligned} \tag{16}$$

where $\Omega \subset \mathbb{R}^4$. Using the B-spline relation in (2)

$$\begin{aligned}
 (i) &= \int \left(\sum_{i,j,k,n} c_{i,j,k,n}^x \dot{\beta}_{s,i}^d(x) \beta_{s,j}^d(y) \beta_{s,k}^d(z) \beta_{\tau,n}^d(t) \right)^2 d\Omega \\
 &= \sum \sum c_{i,j,k,n}^x c_{u,v,w,m}^x \underbrace{\int \dot{\beta}_{s,i}^d(x) \dot{\beta}_{s,u}^d(x) dx}_{(i.a)} + \dots d\Omega \quad (17) \\
 (i.a) &= \int \dot{\beta}^d\left(\frac{x}{s-i}\right) \dot{\beta}^d\left(\frac{x}{s-u}\right) dx =_{[x/s-u=\xi]} \\
 &= \int s \dot{\beta}^d(\xi - (i-u)) \dot{\beta}^d(\xi) d\xi \\
 &= s \dot{\beta}^d(i-u) * \dot{\beta}^d(u-i) \quad (18)
 \end{aligned}$$

where $*$ indicates the convolution product. Proceeding analogously for each term, (17) becomes

$$\begin{aligned}
 (i) &= \sum \sum c_{i,j,k,n}^x c_{u,v,w,m}^x s \left(\dot{\beta}^d(i-u) * \dot{\beta}^d(u-i) + s \beta^d(j-v) \right. \\
 &\quad \left. * \beta^d(v-j) + s \beta^d(k-w) * \beta^d(w-k) + \tau \beta^d(n-m) * \beta^d(m-n) \right) \\
 &= \mathbf{c}^\top \dot{\mathbf{S}}_{xx} \mathbf{c}. \quad (19)
 \end{aligned}$$

An analogous development can be carried out for (ii) to (vi) to calculate matrices $\dot{\mathbf{S}}_{yy}$, $\dot{\mathbf{S}}_{zz}$, $\dot{\mathbf{S}}_{xy}$, $\dot{\mathbf{S}}_{xz}$ and $\dot{\mathbf{S}}_{yz}$. Finally, (16) can be rewritten as

$$\dot{\mathbf{j}}_{div}(\mathbf{c}) = \mathbf{c}^\top \begin{bmatrix} \dot{\mathbf{S}}_{xx} & \dot{\mathbf{S}}_{xy} & \dot{\mathbf{S}}_{xz} \\ \dot{\mathbf{S}}_{xy} & \dot{\mathbf{S}}_{yy} & \dot{\mathbf{S}}_{yz} \\ \dot{\mathbf{S}}_{xz} & \dot{\mathbf{S}}_{yz} & \dot{\mathbf{S}}_{zz} \end{bmatrix} \mathbf{c}. \quad (20)$$

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