

Finite-volume CFD modelling of fluid-solid interaction in EHL contacts

By

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Abstract

Classically in an elastohydrodynamic (EHD) problem, the Reynolds equation is the most widely used PDE to describe the behaviour of lubricants in high-pressure non-conforming contacts, and elastic deformation is usually calculated using the Hertzian theory of elastic contacts. This thesis outlines the development of a new method for modelling of fluid-solid interactions in elastohydrodynamic lubrication (EHL) contact based on Finite Volume (FV) techniques.

A Computational Fluid Dynamics (CFD) approach to solve the Navier-Stokes equations is implemented to model lubrication in roller bearings using the open-source package OpenFOAM. This has first been applied to simulate full film hydrodynamic lubrication (HL), enabling an accurate description of the flow within the entire domain surrounding the contact region. The rheology is assumed to be non-Newtonian and shear-thinning. The phenomenon of cavitation is modelled by implementing a homogenous equilibrium cavitation model, which maintains specified lubricant saturation pressure in cavitating region. The current fluid solver involves the solution of the full momentum and energy equations, and satisfying continuity. The aim is firstly to demonstrate the range of applicability and the limitations of traditional formulations of the Reynolds equation and secondly to highlight areas where Navier-Stokes based approaches are necessary for accurate solution of lubrication problems. Subsequently, a finite volume solid solver is fully coupled with the fluid solver in a forward iterative manner to take into account elastic deflection effects using Navier-Lamé equation. The advantage of using a single numerical tool enables an internal transfer of information at the fluid-solid interface through one common data structure. The stability of the model, in the presence of high contact pressures, is enhanced by incorporation of multigrid method, implicit coupling and improved mesh adaption and motion techniques. The developed model has been applied to a series of lubricated metal on metal smooth line contact with slide to roll ratios ranging from 0 to 2 and is stable for a wide range of industrial operating conditions (pressures up to 4 *GPa*). The model is further improved to account for

time-dependent transient behaviour of an EHL rough contact. The results for a travelling ridge, dent and sinusoidal wave through EHL conjunction are presented.

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Nomenclature

Greek symbols

α	Vapour fraction	
α_p^*	Roelands pressure viscosity index	Pa.s
$\alpha_{T,s}$	Thermal diffusivity of solid	W/mK
β	Thermoviscous constant	1/K
Δt	Time step	s
γ	Shear rate	1/s
η	Piezoviscous shear-thinning and thermal viscosity	Pa.s
η_0	Initial viscosity	Pa.s
η_{Barus}	Barus viscosity	Pa.s
η_{Eyring}	Eyring viscosity	Pa.s
η_{Houpert}	Houpert viscosity	Pa.s
η_{Roelands}	Roelands viscosity	Pa.s
Γ_ϕ	Diffusivity	
μ	Mixture viscosity	Pa.s
μ_l	Liquid dynamic viscosity	Pa.s
μ_v	Vapour dynamic viscosity	Pa.s
ν	Poisson's ratio	
ϕ	Flow quantity	
ψ	Mixture compressibility	s ² /m ²
ψ_l	Compressibility of liquid	s ² /m ²
ψ_v	Compressibility of vapour	s ² /m ²
$\psi_{l,\text{dow}}$	Compressibility of Dowson liquid	s ² /m ²
ρ	Density	kg/m ³
ρ_s	Solid density	kg/m ³
$\rho_{l,0}$	Liquid density at ambient pressure	kg/m ³
$\rho_{l,\text{dow}}$	Dowson pressure density	kg/m ³
$\rho_{l,\text{sat}}$	Liquid density at p_{sat}	kg/m ³
$\rho_{v,\text{sat}}$	Vapour density at p_{sat}	kg/m ³

σ	Stress	Pa
τ	Viscous stress	Pa
τ_0	Eyring stress	Pa
λ	Roughness wavelength	m
Roman symbols		
A	Roughness amplitude	m
A_d	Diagonal matrix coefficients	
A_c	Off diagonal matrix coefficients	
B	Doolittle viscosity constant	
E	Young's modulus	Pa
E'	Reduced young modulus	Pa
F	Surface and body force	N
f_b	Body force	N
d	Length vector between two neighbouring cell centres	m
$H(u)$	Off diagonal matrix coefficients multiplied by their corresponding velocities	
$p_{r,0}$	Roelands reference pressure	Pa
V_{occ}	Occupied volume	m ³
t	Time	s
v	Solid body displacement	m
T_0	Reference temperature	K
T	Temperature	K
Z	Oil viscosity constant	
k	Fluid thermal conductivity	W/(mK)
k_s	Solid thermal conductivity	W/(mK)
x	Spatial coordinate perpendicular to x and film-thickness h	
z	Spatial coordinate	
C_p	Heat capacity	J/(kg K)
$C_{p,l}$	Heat capacity of fluid	J/(kg K)
$C_{p,v}$	Heat capacity of vapour	J/(kg K)
$C_{s,v}$	Heat capacity of solid	J/(kg K)
h_{evap}	Heat of evaporation	
L	Force	N/m

S Surface area vector

Chapter 1

1.1 Introduction

Lubricating regimes can generally be categorised into boundary lubrication, mixed lubrication and full-film lubrication. Boundary lubrication occurs when the load is carried by the solid surfaces which are only protected by a thin boundary layer formed through the action of lubricant additives. In mixed lubrication, the load is carried by both lubricant pressurised film and the solid surfaces. Full fluid-film lubrication occurs when there are two solid surfaces in relative rolling-sliding motion and the lubricant film is sufficiently thick to prevent the opposing solids from coming into contact. Load is fully supported by the pressurised fluid film. Full fluid-film lubrication helps to protect surfaces from damage as well as minimize friction between the contacting components. This in turn leads to improved energy efficiency and reliability of machines. There are two types of fluid-film lubrication: Hydrodynamic Lubrication (HL) and Elasto-hydrodynamic Lubrication (EHL). Hydrodynamic lubrication occurs in conforming contacts, such as those in journal bearings, where pressures are low enough not to cause any significant elastic deformation or increase in lubricant viscosity. On the other hand, Elasto-hydrodynamic lubrication occurs in non-conforming contacts where very high pressures are encountered so that elastic deformation of the solids as well as increase in lubricant viscosity becomes significant. This form of lubrication occurs in many machine elements including rolling bearings, gear teeth contacts and cam-follower contacts. EHL can also occur at relatively lower pressures in contacts of compliant solids when it is often termed soft-EHL.

The key elements that affect EHL are:

- Rheology and compressibility of the lubricant: The viscosity and density of the lubricant strongly depend on pressure, temperature and shear-rate.

- Thermal effects: At high sliding speed and/or load, the heat generation in the lubricant becomes important.
- Cavitation: In the diverging contact exit, pressure drops below vapour pressure and the fluid cavitates and forms oil-streamers and air fingers.

1.2 Research objectives

This study is concerned with predicting the physical behaviour of EHL contact by considering full Navier-Stokes equations, solid deformation (Navier-Lamé equation) and thermal effects. The study uses computational fluid dynamics (CFD) methodology to provide an EHL solution with realistic physics and improved accuracy. The work is a continuation of a previous PhD project at Imperial College on the same general topic.

The first part of this work will investigate the accuracy and limitations of different computational approaches generally used for numerical modelling of EHL. This is split into two parts: (a) Applicability and accuracy of different governing equations for the lubricant and the solid (b) Stability and efficiency of different numerical schemes, computational models and coupling strategies. The Reynolds equation, which is an integrated version of simplified Navier-Stokes equation through film thickness, is the most widely used partial differential equation for prediction of lubricant flow in contact. The elastic deformation is usually calculated using the Hertzian contact theory. These two equations have been coupled successfully using different numerical schemes and the results are in good agreement with experimental measurements over a limited range of contact conditions. However, the simplifying assumptions made in derivation of the Reynolds equation reduce the accuracy level of the Reynolds based approaches with the error becoming significant at certain contact conditions. These simplifying assumptions can be summarized as follows:

- Pressure is constant through the film thickness.
- Gradients of fluid properties and velocities through the film thickness are either assumed to be zero or neglected.

On the other hand, there are alternative approaches and coupling techniques that can be implemented in order to (a) maintain stability and (b) model fluid-solid interaction

more precisely in an EHL contact within a wider range of contact operating conditions. In this respect, the use of full Navier-Stokes equations provides the ability to solve for all gradients through the fluid-film as well as to incorporate more complex behaviour of the lubricating fluid, ultimately resulting in a more accurate and flexible EHL model. In addition, a CFD methodology offers a fast and convenient way of implementing such a solution to Navier-Stokes equations.

The second part of this study is concerned with the development of a new general CFD based EHL model that can predict the behaviour of both fluid and solid domains as well as simultaneously account for the interaction between the two domains. This is mainly focused on the dynamic mesh movement, data exchange and coupling methods between solid and fluid domains to enhance stability and computational efficiency. The idea is to develop a flexible platform, with the capability to incorporate different governing equations relative to the physics of EHL to investigate both fluid and solid behaviours in industrial operating conditions.

Finally, the developed CFD model is expanded to incorporate lubricant rheology, cavitation, compressibility, thermal and surface roughness effects. This complete model is then applied to EHL contacts operating at relatively high pressures in order to study the physical behaviour of the contact at these conditions but also to illustrate the model stability at such high pressures.

1.3 Thesis outline

This thesis is organized in 6 main chapters.

Chapter 2 contains a comprehensive introduction to HL phenomena, fluid flow governing equations, lubricant physical properties, CFD and the Reynolds based approaches and cavitation treatment.

In Chapter 3 different existing approaches for EHL modelling are described and a new fluid-solid interaction CFD model is presented. The results of the model are validated against existing EHL solutions based on the Reynolds equation and Hertzian contact theory.

In Chapter 4, the range of EHL contact conditions over which the presented model is applicable are also investigated.

Chapter 5 presents model results for selected EHL cases with surface roughness and transient effects. Case studies are presented to show steady state and transient surface

roughness behaviour including the effects of a surface dent, ridge and a sinusoidal roughness profile.

Chapter 6 lists the general conclusions of this study and briefly outlines some ideas for potential future developments of the model.

Chapter 2

2.1 Modelling fluid film lubrication

This chapter first provides an overview of existing literature on modelling of lubricated contacts. It starts with an introduction to the Reynolds approach to solving Hydrodynamic films. Existing models for piezoviscosity, viscosity dependence on temperature and shear-thinning are described in a critical manner.

Following this, the developed models based on Reynolds and modified-Reynolds solutions are presented. Then, the CFD based fluid solver developed in this study is described. Finally, results comparing CFD and Reynolds based approaches are presented. It should be noted that the elastic deformation of contacting surfaces is not considered in this Chapter. This will be described in next Chapter.

2.1.1 Literature review

In an HL contact, where two contiguous surfaces form a converging wedge and their relative motion causes lubricant entrainment into the contact, the lubricant becomes pressurised and therefore able to support load. Journal and pad bearings are examples of many mechanical components that operate in hydrodynamic lubrication. Generally, the HL film thicknesses are in the order of micrometres and supported pressures range from tens to hundreds of MPa.

Tower [1] noted the presence of pressurised lubricant for the first time and Reynolds [2] developed the first partial differential equation governing pressure distribution in HL contacts. This equation was derived from the simplified form of the momentum and mass continuity equations by assuming that the lubricant is an isoviscous, incompressible and Newtonian fluid and also that the viscosity and density are constant through the film thickness. Based on the solution of the Reynolds equation for a finite plain bearing, Cameron et al. [3] proposed a diagram where the journal bearing eccentricity ratio could be obtained from the load criterion for any bearing of the diameter to length ratio of 0 to 4. Sassenfeld et al. [4] developed the first Reynolds-based computerised numerical solution for hydrodynamic lubrication in a finite journal bearing. Dowson [5] provides a comprehensive historical review of numerical studies of HL contacts. Since then, several studies show that, for low sliding speeds and/or loads, physically acceptable results can be predicted by Reynolds equation. However, it is notable that linear Newtonian behaviour is only observed at relatively low shear stress [6], while the range of operating speeds and contact pressures in many applications lead to much higher shear rates. Johnson and Tevaarwerk [7] showed that the Newtonian model overestimates the shear-stress in comparison to the experimental data. Najii et al. [8] derived a generalised Reynolds equation for non-Newtonian fluid which can accommodate both implicit and explicit rheological models for an incompressible fluid in a steady-state flow. Conry et al. [9] derived another modified Reynolds equation based on non-Newtonian rheological model referred to as ‘Eyring sinh law’ [10]. Although Conry’s equation is for non-Newtonian lubricants, the Newtonian Roelands [11] viscosity-pressure relation was used, and therefore the underlying effects of shear-thinning were not accounted for. Yang and Wen [12] assigned all the non-Newtonian effects of the lubricant to a set of functions called ‘the equivalent viscosity’ and proposed a generalized Reynolds

equation similar to Dowson [5]. A Newtonian-Eyring rheological model is used to incorporate the effects of shearing stress, shearing rate, velocity, pressure and viscosity in the algorithm simultaneously. Wolff and Kubo [13] implemented an empirical relationship between velocity gradients, shear stress and visco-plasticity to capture non-Newtonian behaviour of the lubricant in an EHL contact. Visco-plastic models of Bair and Winer [14] and the Circular model of Lee and Hamrock [15] were considered. This work showed that the visco-plastic models can only predict the traction value correctly for high viscosity oil under heavy loading conditions. Also, the visco-elastic behaviour of the lubricant is only important at a very low slip condition and it can slightly reduce the traction value. Rajagopal et al. [16] proposed a modified Reynolds equation with an extra pressure gradient term to properly account for the variation of viscosity with pressure. The modified equation results show a small difference in pressure and significantly higher viscosity values than the classical Reynolds equation. Bair et al. [17] claim that the Reynolds equation accurately predicts the mechanics of the piezoviscous lubricant only when the shear-stress is much less than the reciprocal of the pressure-viscosity coefficient. Their model of lubricant properties accounts for pressure and temperature dependency of density through an equation of state. Schafer et al. [18] noted that appropriateness of applying the Reynolds equation to EHL problems is questionable at large pressures and in high shear-stress regions of the contact. These suggested that deficiencies of the Reynolds's approach can be traced back to the assumptions made for derivation of the classical Reynolds and also modified Reynolds equations. Specifically, the gradients of velocity, viscosity and pressure through the film thickness are either significantly simplified or completely neglected.

An alternative approach to Reynolds is to consider continuity and momentum equations, which describe the fundamentals of fluid dynamics, to truly resolve all these gradients. This approach can be pursued through implementing a CFD method. The CFD method has an added benefit as it also describes the entire fluid and solid domains whereas the Reynolds-based models are limited to near-parallel contact region. Moreover, a more sophisticated treatment of cavitation can be implemented in the CFD approach than is possible with highly simplified Reynolds equation approaches. Finally, CFD also offers greater flexibility in terms of fluid rheology relationships that can be implemented. The main disadvantage of CFD method is the high computational cost relative to the problem size and domain resolution.

Tucker and Keogh [19] applied CFD techniques to predict hydrodynamics of a journal bearing while accounting for thermal and viscous effects. The energy equation is solved and thermal conduction into the rotating and orbiting shaft is considered. The cavitation regions are not set and are predicted through pressure calculation. Chen et al. [20] incorporated full Navier-Stokes equation in a CFD solution for a HL contact. The lubricant was assumed to be iso-viscous and incompressible. The results presented are for relatively low pressures and match with Reynolds approach. Brajdic-Mitidieri [21] applied CFD method to model lubricant flow in a linear pad bearing. It was assumed that the lubricant is an iso-thermal, Newtonian fluid, and considered a mixture single-phase cavitation phenomenon. The results presented show a complete agreement with the classical Reynolds approach for pressures up to 50 MPa.

2.1.2 Reynold's equation

Reynolds [2] proposed the first differential equation governing pressure distribution in a lubricant film. The following assumptions are necessary for derivation of the Reynold's equation from the full Navier-Stokes equation:

- (a) Body and surface tension forces are negligible
- (b) Viscosity and pressure are constant through the film thickness
- (c) Lubricant is Newtonian with low Reynolds number (laminar flow)
- (d) Velocity and shear stress gradients are only considered across the film thickness
- (e) No slip at boundary surfaces

The momentum equation for an incompressible unsteady flow can be written as:

$$\rho \frac{Du_a}{Dt} = -\frac{\partial p}{\partial x} + \eta \nabla^2 u_a + f_b \quad \text{Equation 2-1}$$

where p , u_a , t , ρ , f_b and η represent pressure, x-component of velocity, time, fluid density, body forces and dynamic viscosity respectively.

Equation 2-1 can be simplified to give:

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial z} \left(\eta \frac{\partial u_a}{\partial z} \right) \quad \text{Equation 2-2}$$

The Reynolds equation results from integrating Equation 2-2 and subsequently substituting boundary conditions at the surfaces:

$$\begin{aligned} \frac{\partial}{\partial x} \left(\frac{\rho h^3}{12\eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\rho h^3}{12\eta} \frac{\partial p}{\partial y} \right) = \frac{\partial}{\partial x} \left[\frac{\rho(u_a + u_b)h}{2} \right] \\ + \frac{\partial}{\partial y} \left[\frac{\rho(v_a + v_b)h}{2} \right] \end{aligned} \quad \text{Equation 2-3}$$

where u_a , u_b , v_a and v_b are lower (a) and upper (b) plane velocities in x and y directions respectively.

An alternative method is to derive the equation from the first principle, i.e. by considering the equilibrium of forces on a lubricant element.

The analytical solution for the pressure distribution of an infinitely wide journal bearing have been known since 1904 when Sommerfeld [22] made the complete solution of the Reynolds equation. For an isoviscous lubricant, one-dimensional Reynolds equation gives:

$$\frac{dp}{dx} = 6u_a\eta_0 \frac{(h - h_c)}{h^3} \quad \text{Equation 2-4}$$

where h is film thickness, h_c is the film thickness at maximum pressure, U is the surface speed and η_0 is the lubricant viscosity. By integrating Equation 2-4 the Full Sommerfeld formulation [22] can be obtained:

$$p(x) = \frac{2u_a\eta_0 x}{h^2 \left(1 + \left(\frac{x^2}{2Rh_0} \right) \right)^2} \quad \text{Equation 2-5}$$

2.1.3 Fluid properties

The Reynolds equation can be adapted to incorporate non-Newtonian and compressible behaviours of the lubricant. This section provides an overview on lubricant rheology and compressibility models that are necessary for accurate prediction of EHL. The main aim is to determine the effects of lubricant pressure and temperature variation on viscosity and density.

2.1.3.1 Oil viscosity

Viscosity of the fluid may be thought of as the amount of resistance to flow that is arising from intermolecular forces and internal friction due to relative movement of molecules. Newton, in 1687, proposed the formula for internal friction in viscous fluids:

$$\tau = \eta \frac{du}{dz} \quad \text{Equation 2-6}$$

where τ is internal shear stress in the fluid and η is the dynamic viscosity.

2.1.3.2 Viscosity dependence on pressure and temperature

Viscosity is a determining factor in pressure build up and film formation between tribological elements, therefore extensive research continued to better understand the rheological behaviour of the fluid in an EHL contact. Barus [23] proposed an exponential relationship between viscosity and pressure:

$$\ln\left(\frac{\eta}{\eta_0}\right) = \alpha p \quad \text{Equation 2-7}$$

where η_0 is viscosity at atmospheric pressure and α is a constant depending on the oil, called pressure-viscosity coefficient, which is a function of temperature (not pressure). It varies from 10^{-8} to $2 \cdot 10^{-8} \text{ Pa}^{-1}$ for mineral oil. Equation 2-7 predicts viscosity inaccurately for pressures above 0.5 GPa. Following the work by Barus, a number of empirical equations have been suggested; for instance, Walther's equation and Appeldorn's [24] equations have been implemented to study temperature and pressure dependence respectively. Roelands [25], in 1966, proposed a more comprehensive expression which includes effect of both pressure and temperature on the viscosity of lubricants and it was further developed by Houpert [26] in 1985 as:

$$\eta_R = \eta_0 \exp \left\{ (\ln(\eta_0) + 9.67) \left[\left(\left[1 + \frac{p}{p_0} \right]^Z - 1 \right) \left(\frac{T - 138}{T_0 - 138} \right)^{-\left(\frac{\beta(T_0 - 138)}{\ln(\eta_0) + 9.67} \right)} \right] \right\} \quad \text{Equation 2-8}$$

$$\eta_{Roelands-Houpert} = \eta_R \exp(-\beta^*(T - T_0))$$

Equation 2-9

where Z and β^* are given by:

$$Z = \frac{\alpha}{5.1 \times 10^{-9} (\ln(\eta_0) + 9.67)}$$

$$\beta^* = [\ln(\eta_0) + 9.67][1 + 5.1 \times 10^{-9} p]^Z \left[\frac{\beta}{\ln(\eta_0) + 9.67} \right]$$

α is pressure-viscosity coefficient (usually given for well-known oil), β is thermo-viscous constant, η_0 is the atmospheric viscosity, T_0 is a reference or ambient temperature, Z is constant for any oil independent of temperature and pressure.

Despite the fact that Roelands equation is the most widely used piezoviscous model in EHL contact studies, it fails to model the real piezoviscous behaviour at pressures as high as those encountered in typical EHL contact problems. Bair [27, 28] investigated the accuracy of pressure-viscosity and temperature-viscosity behaviours of the Roeland's equation. It was shown that the Roeland's equation fails to recover the greater than exponential pressure-viscosity behaviour at high pressures and the temperature-viscosity behaviour is questionable when compared to experimental data. Free volume model, first introduced by Doolittle [29] in 1951, is based on the fact that the resistance to flow in a liquid depends on the relative volume of molecules existence per unit free volume. The volume variation with pressure was initially described by Tait's equation that involves the occupied volume of the closest packed liquid molecules and the free volume available in the liquid for molecular transitions. Free-volume viscosity model simply suggests that the viscosity is infinite, since no molecular motion is possible and the molecules are packed together at their maximum density i.e. when the specific volume is V_0 (volume at ambient pressure). Increase in temperature create free volume, $V - V_0$, which provides space for molecular motion and hence for viscous flow. This model was originally developed by Eyring and co-workers in 1940 [10], but Doolittle [29] proposed the first free-volume model based on a physically-grounded concept:

$$\eta = \eta_0 \exp \left[B \frac{V_{occ}}{V_0} \left[\frac{1}{\frac{V}{V_0} - \frac{V_{occ}}{V_0}} - \frac{1}{1 - \frac{V_{occ}}{V_0}} \right] \right]$$

Equation 2-10

where the volume variation is described by the Tait's equation of state [30]:

$$\frac{V}{V_0} = 1 - \frac{1}{K_0' + 1} \ln \left[1 + \frac{p}{K_0} (1 + K_0') \right] \quad \text{Equation 2-11}$$

B , K_0 , K_0' , V_{occ} and V_0 are Doolittle parameter, Bulk modulus at $p=0$, pressure rate of change of bulk modulus at $p=0$, occupied volume and volume at ambient pressure respectively. The free-volume is a compressible rheological model and, therefore, enables derivation of the relationship between density and pressure. Cook et al. [31] modified Tait's equation of state to produce models to better capture the fluid physical response. Williams, Landel and Ferry [32] derived an empirical equation (WLF) with the focus on temperature dependence of viscosity. Following their work, Yasutomi et al. [33] developed a modified-WLF equation which accounts for the pressure dependence of the glass transition temperature and for the thermal expansion of the free volume. Bair [34] recently has carried out series of experimental investigations at high pressures and confirmed the validity of the Yasutomi free-volume model for EHL contact studies. Liu et al. [30], in 2006, used Yasutomi [33] free-volume model for EHL contact simulations and it was shown that there is an increasing difference between Roelands and Yasutomi for pressures higher than 300 MPa.

2.1.3.3 Viscosity dependence on shear rate and shear stress

At high load and high strain rates, there is a large discrepancy between experimental data and numerical results obtained from implementing Newtonian viscosity models. Therefore, several research studies attempt to investigate the non-Newtonian behaviour of lubricants. The non-Newtonian fluid models proposed for EHL contacts study can be categorised into: shear-thinning and limiting shear stress behaviours. The former refers to declining trend in the effective lubricant viscosity, $\tau/\dot{\gamma}$, with increasing shear rate, whereas the latter involves a nearly constant value of shear stress at adequately high shear rate. Ree and Eyring, in 1955, used a hyperbolic sine (sinh) law to represent the lubricant shear-thinning behaviour. In 1961, Bell [35] used

the Ree-Eyring sinh-law, for the first time, to demonstrate shear-thinning behaviour of lubricants:

$$\dot{\gamma} = \frac{\tau_0}{\eta} \sinh\left(\frac{\tau}{\tau_0}\right) \quad \text{Equation 2-12}$$

where $\dot{\gamma}$ is the shear rate, τ is the shear stress, η is the viscosity and τ_0 represents the Newtonian limit of the lubricant (Eyring stress). The elastic component is dropped from shear-rate equation since the time dependency in viscous response of the lubricant is shown to be negligible, in the time scale of an EHL contact, by Bair et al. [36]. Also, according to Johnson [37], viscoelastic effects occur when the relaxation time of the fluid ($\lambda = \frac{\eta \text{ (Viscosity Pa.s)}}{G \text{ (Elastic shear modulus } \sim 10^9 \text{ Pa})}$) exceed the time of passage of the fluid through the contact which requires viscosities in excess of 10^6 Pa.s for typical size of an EHL contact.

The shear stress of the fluid, τ can be expressed as:

$$\tau = \eta \dot{\gamma} \quad \text{Equation 2-13}$$

Substituting Equation 2-12 into Equation 2-13, gives the well-known Eyring equation for shear-thinning viscosity of the fluid:

$$\eta_{\text{eyring}} = \frac{\tau_0}{\dot{\gamma}} \sinh^{-1}\left(\frac{\eta_0 \dot{\gamma}}{\tau_0}\right) \quad \text{Equation 2-14}$$

In this study, Equation 2-14 is used in the domain regions where the shear rate is larger than 10^{-8} s^{-1} and is adapted to incorporate effects of pressure and temperature by replacing η_0 with $\eta_{\text{Roelands-Houpert}}$. In other regions where shear thinning is less likely to occur, viscosity is determined through Roelands-Houpert equation only.

A rheological model based on the sinh law was suggested by Wang et al. [38] to highlight that the non-Newtonian behaviour originates from a physical mechanism which cannot be related to the limiting shear stress. They concluded that, in the non-Newtonian regimes, constitutive models that account for shear-rate dependence of viscosity are clearly necessary for accurate modelling of the rheological behaviour. Tanner [39] proposed the first generalised Maxwell model for lubrication using a simple power law form of the viscosity function. Although this model is quite accurate for high shear stresses, it does not capture the linear Newtonian behaviour at low shear stresses. Johnson and Tevaarwerk [40] presented a non-linear Maxwell

rheological model for a lubricant under isothermal conditions, where the total shear strain rate is the sum of an elastic term and a nonlinear viscous term (based on Eyring's theory of viscosity). The works of Bair and Winer [14], Gecim and Winer [41], Iven and Hamrock [42], Lee and Hamrock [15] and Carreau-Yasuda [43] provide further information on non-Newtonian behaviour of lubricants.

2.1.3.4 Comparison of the most common rheology models

The piezoviscous models described here are compared in Figure 2-1 for pressures up to 500 MPa. The following rheology models are shown: Barus (Equation 2-7), Roelands (Equation 2-8), Doolittle (Equation 2-10) and Eyring-Roelands (Equation 2-14). From Figure 2-1 (b), it is evident that all the models show similar behaviour for pressures lower than 70 MPa. Barus, Roelands and Eyring-Roelands predictions are similar for pressures up to 140 MPa. However, Eyring-Roelands relationship differs from Roelands and Barus at pressures over 140 MPa where the lubricant shear-thinning behaviour becomes significant. Only Doolittle and Eyring-Roelands models suggest physically acceptable values for pressures larger than 300 MPa (Figure 2-1 (a)). Furthermore, Doolittle and Eyring-Roelands models show a good agreement with majority of experimental data available in the literature. A comparison between mathematical models and experimental measurement, by ASME [44] is shown in Figure 2-1 (c).

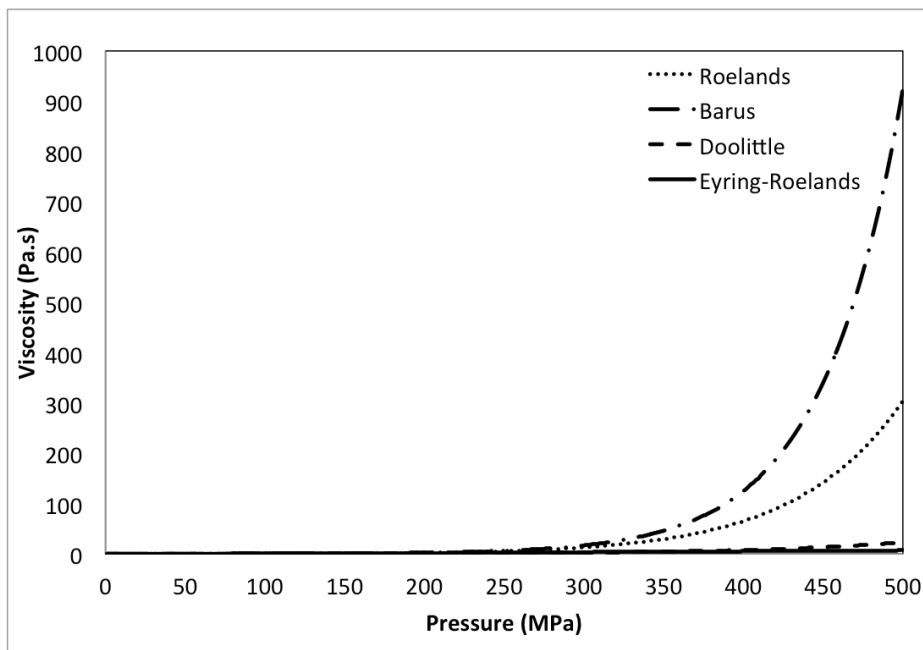


Figure 2-1. (a)

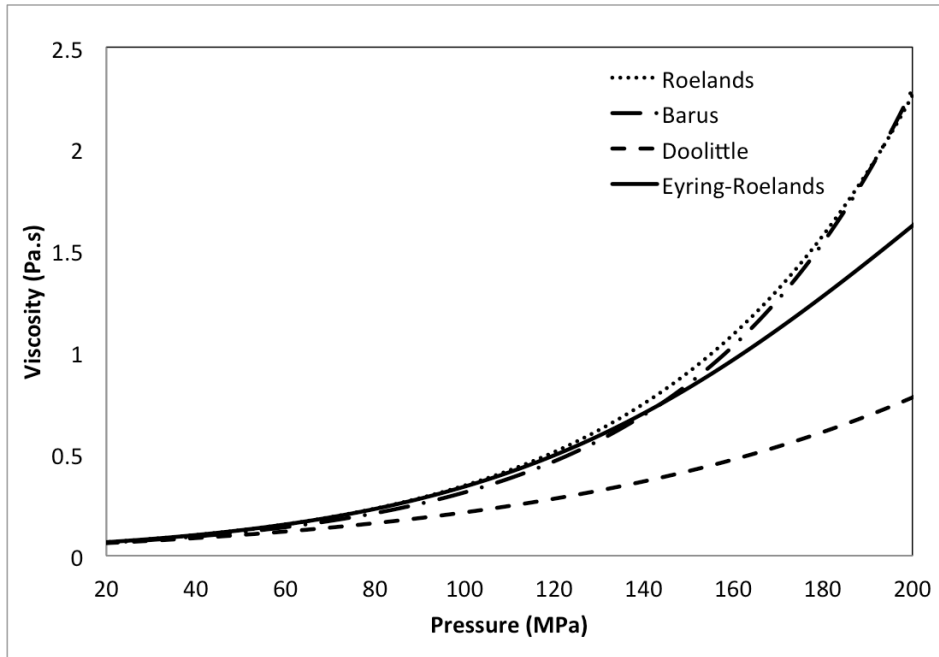


Figure 2-1. (b)

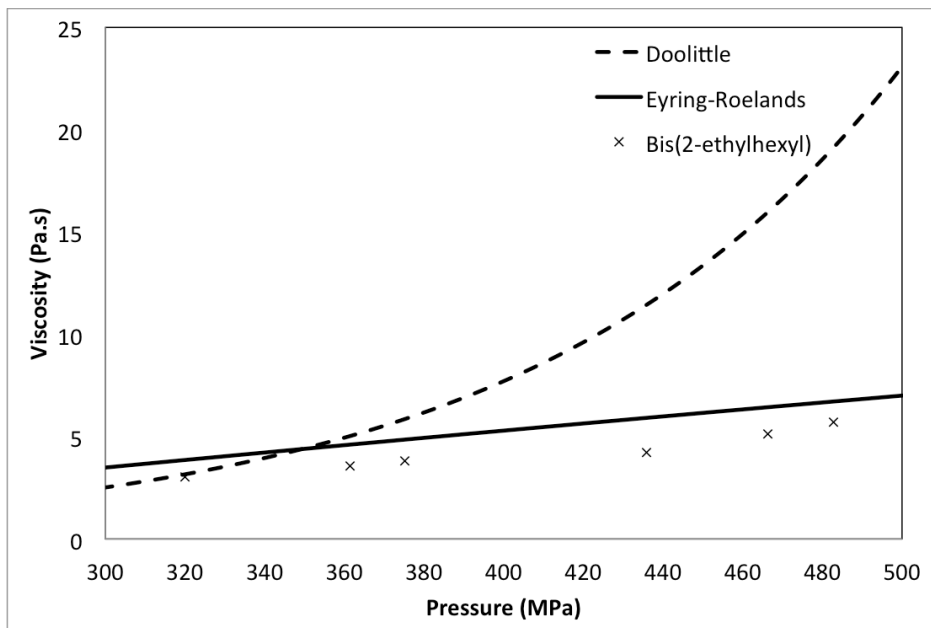


Figure 2-1. (c)

Figure 2-1. Viscosity versus pressure for different rheology models and comparison with experimental data at 25°C [44] (a) 0-500 MPa (b) 20-200 MPa (c) 300-500 MPa

2.1.3.5 Oil compressibility

For an equivalent change in pressure, the change in viscosity is much larger than change in density of the lubricant. Therefore, research has been mainly directed towards obtaining an accurate prediction of the rheological behaviour rather than the compressibility of lubricants in EHL contacts. However, an understanding of how the lubricant density varies with pressure is necessary for the numerical solution of EHL contact problems, specifically in the cavitating and converging areas of the contact. In the early development of EHL theory, the iso-thermal Dowson and Higginson [45] density-pressure relation was widely used for compressibility of the lubricating oils:

$$\rho_{l,dow} = \rho_0 \frac{5.9 \times 10^8 + 1.34p}{5.9 \times 10^8 + p} \quad \text{Equation 2-15}$$

where ρ_0 is the lubricant density at atmospheric pressure. Equation 2-15 was obtained from curve fits of experimental data for mineral oil with pressures up to about 0.4 GPa. It was later shown by Wong et al. [46] and Feng et al. [47] that the Dowson-Higginson equation overestimates the density of lubricant at pressures higher than 400 MPa. Hamrock et al. [48] and Ramesh [49] proposed empirical relationships for compressibility based on experimental data.

At high sliding speed and/or load, the heat generation in the lubricant becomes important. Zhu [50] and Yang [51] modified Dowson-Higginson expression by adding a linear temperature correction to incorporate thermal effects on density. Hutton [52] proposed an empirical model for the pressure-temperature-density relation for a mineral oil (LVI260) based on series of experiments covering a large temperature range. Wong et al. [46] introduced a new pressure-temperature-density formula based on the molecular interactions in liquid lubricants under elevated pressures and temperatures. The model was validated for both mineral and synthetic oils for pressures up to 900 MPa by comparison to experimental data obtained by impact microviscometry technique. The use of a compressible fluid model becomes necessary in case of using the free-volume rheology model. The pressure-density relationship can be derived from Tait equation as [53]:

$$\frac{\rho_0}{\rho} = 1 - \frac{1}{K_0 + 1} \ln \left[1 + \frac{p}{K_0} (1 + K_0) \right] \quad \text{Equation 2-16}$$

Equation 2-15 is used in the present work to describe compressibility of the lubricant.

2.2 Numerical investigations on HL contact problems

In this section, a model for simulation of a roller on a flat surface contact is considered under simplified hydrodynamic conditions i.e. elastic deformation is neglected. The presented model is two-dimensional and is envisaged to represent a contact of a roller element against a bearing ring in oil lubricated cylindrical roller bearing. First, the classical Reynolds and a non-Newtonian modified Reynolds equations are used to predict the pressure and viscosity distributions. Then, the same problem is solved using a CFD method which accounts for oil rheology more accurately. Finally, the differences in results from the two approaches are highlighted and discussed. The work presented here therefore serves to illustrate the benefits offered by a full CFD model when compared to classical Reynold's solutions.

2.2.1 Reynold's equation based model

Finite-Difference (FD) method is applied to obtain the pressure distribution in a 1-Dimensional HL contact problem using the classical Reynold's equation. FD methods determine an approximate solution of the differential equations governing the physics of the problem. There are several differential schemes available for discretisation, which are all derived from the Taylor series expansion. The first-order backward, second-order central, and second-order backward approximations are the most widely used schemes for the discretisation of the Reynold's equation. The truncation error, for above-mentioned differential schemes, is evaluated for types of derivatives contained in the Reynold's equation.

For the first-order backward scheme:

$$\left. \frac{\partial u}{\partial x} \right|_{i,j} = \frac{u_{i,j} - u_{i-1,j}}{\Delta x} + O(\Delta x) \quad \text{Equation 2-17}$$

where $O(\Delta x)$ is the truncation error, which can be expressed, using Taylor expansion, as:

$$O(\Delta x) = \frac{\Delta x}{2} \frac{\partial^2 u}{\partial x^2} \Big|_{i,j} + O[(\Delta x)^2] \quad \text{Equation 2-18}$$

For the second-order central scheme:

$$\frac{\partial u}{\partial x} \Big|_{i,j} = \frac{u_{i+1,j} - u_{i-1,j}}{2\Delta x} + O(\Delta x^2) \quad \text{Equation 2-19}$$

where:

$$O(\Delta x^2) = -\frac{(\Delta x)^2}{6} \frac{\partial^3 u}{\partial x^3} \Big|_{i,j} + O[(\Delta x)^4] \quad \text{Equation 2-20}$$

Finally, for the second-order backward scheme

$$\frac{\partial u}{\partial x} \Big|_{i,j} = \frac{3u_{i,j} - 4u_{i-1,j} + u_{i-2,j}}{2\Delta x} + O(\Delta x^2) \quad \text{Equation 2-21}$$

where:

$$O(\Delta x^2) = \frac{(\Delta x)^2}{3} \frac{\partial^3 u}{\partial x^3} \Big|_{i,j} + O[(\Delta x)^3] \quad \text{Equation 2-22}$$

It can be concluded that, for a given mesh resolution, results obtained from second-order central scheme are the closest approximation to the exact solution.

The one-dimensional Reynold's equation can be derived from Equation 2-3 as:

$$\frac{\partial}{\partial x} \left(\frac{\rho h^3}{12\eta} \frac{\partial p}{\partial x} \right) = \frac{\partial}{\partial x} \left[\frac{\rho(u_a + u_b)h}{2} \right] \quad \text{Equation 2-23}$$

Replacing $\frac{\rho h^3}{12\eta}$ with ε^x and using a second-order central differencing scheme gives:

$$\left[\frac{\partial}{\partial x} \left(\varepsilon^x \frac{\partial p}{\partial x} \right) \right]_{i,j} = \frac{1}{\Delta x^2} \left[\varepsilon_{i+1/2,j}^x p_{i+1,j} - (\varepsilon_{i+1/2,j}^x + \varepsilon_{i-1/2,j}^x) p_{i,j} + \varepsilon_{i-1/2,j}^x p_{i-1,j} \right] + O(\Delta x^2) \quad \text{Equation 2-24}$$

where:

$$\varepsilon_{i+1/2,j}^x = \frac{1}{2}(\varepsilon_{i,j}^x + \varepsilon_{i+1,j}^x) \quad \text{Equation 2-25}$$

and

$$\varepsilon_{i-1/2,j}^x = \frac{1}{2}(\varepsilon_{i-1,j}^x + \varepsilon_{i,j}^x) \quad \text{Equation 2-26}$$

The domain of study is set to consist of a half-cylinder on a flat surface as shown in Figure 2-2. The lubricant film thickness for a rigid case study can be evaluated from:

$$h = h_0 + R - \sqrt{R^2 - x^2} \quad \text{Equation 2-27}$$

where h_0 is central film thickness and R is the cylinder radius.

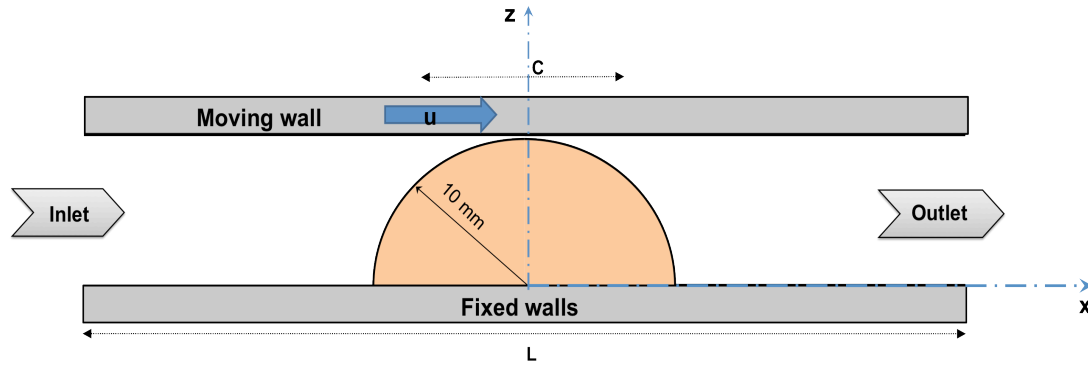


Figure 2-2. Domain of study

For a one-dimensional Reynolds approach, the computation was performed along the discretised top-moving wall and close to the area where the pressure is expected to build up due to hydrodynamic action of the lubricant and translation of the top wall in x-direction. The domain should be long enough to avoid influence of remote boundaries on predicted pressure distribution.

The boundary conditions for the current problem are: velocity of the fluid at the boundary is equal to that of the wall and pressure has zero gradient at the boundary, since the flux through the wall is zero.

The parameters for an iso-viscous case are summarised in Table 2-1.

Parameter	Value
Cylinder radius	R=10 mm
Moving wall velocity	u=1 m/s
Minimum film thickness	$h_0=10^{-7}$ m
Viscosity	$\eta=0.04201$ Pa.s

Table 2-1. Case parameters

For an incompressible and iso-viscous flow, Equation 2-23 can be re-written as:

$$\frac{1}{\Delta x^2} \left[\epsilon_{i+1/2}^x p_{i+1}^{old} - (\epsilon_{i+1/2}^x + \epsilon_{i-1/2}^x) p_i + \epsilon_{i-1/2}^x p_{i-1}^{new} \right] = \frac{\rho u_a}{2} \frac{\partial h}{\partial x} \quad \text{Equation 2-28}$$

The right-hand side of Equation 2-28 can be derived from Equation 2-27. A C++ code is developed to solve the pressure matrix and the newest updated-value for each node is concurrently used in order to reduce the computational time. All the nodes are initialised with pressure value of 10^5 Pa as the first guess. The solution is considered converged when the relative error in pressure at all nodes is less than 10^{-6} so that:

$$Error = \frac{P_{New} - P_{Old}}{P_{New}} < 10^{-6} \quad \text{Equation 2-29}$$

The pressure distributions are symmetric for isoviscous-incompressible cases and the existence of negative pressures implies the presence of cavitation in the outlet region. The first step is to perform mesh and domain length investigations with constant convergence tolerance for the pressure (10^{-6}). Figure 2-3 shows model results for different number of nodes and $C = 1$ mm. It is evident that the pressure distribution and pressure spike values are the same for $C = 1$ mm with 1000 and 2000 nodes (See Figure 2-3).

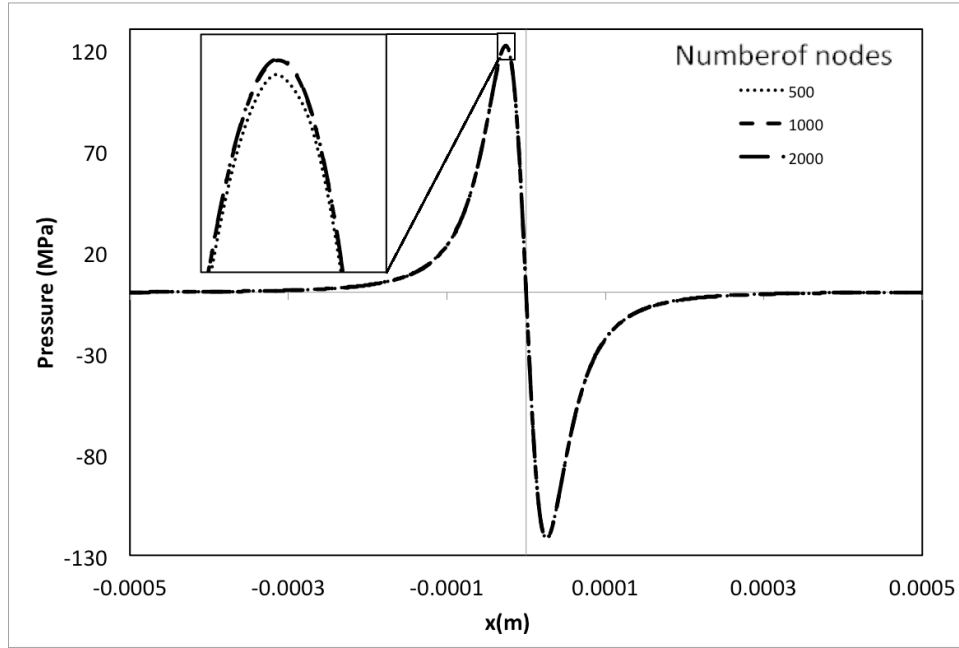


Figure 2-3. Pressure distribution for $C = 1 \text{ mm}$

The behaviour of the model at different domain lengths was also checked. The pressure starts to build up at $x = -0.5 \text{ mm}$ and the converged results are perfectly matched for the cases that begin the calculation at/or before $x = -0.5 \text{ mm}$ (i.e. minimum domain length of 1 mm). It can be concluded that the domain length of 1 mm with 1000 nodes is appropriate to produce accurate results with current configuration.

The Roelands viscosity model shown in Equation 2-8, is used to account for piezoviscous effects. Viscosity is calculated in every iteration loop using the Roelands relationship. Additionally, the viscosity convergence check criterion is added to the model where viscosity calculation error was such that:

$$Error = \frac{\eta_{New} - \eta_{Old}}{\eta_{New}} < 10^{-6} \quad \text{Equation 2-30}$$

In order to investigate the capability of the model, results were obtained at different surface speeds. Pressure and viscosity profiles are presented in Figure 2-4 for surface speed of 0.36 m/s . Since only the piezoviscous effect is included in applied Roelands rheology model, maximum viscosity occurs right at pressure spike as may intuitively be expected. The developed code converges at surface speeds up to 0.39 m/s

(Maximum pressure of 570 MPa and a maximum viscosity of 858 Pa.s). Although the code converged for surface speed of 0.39 m/s, the sudden increase in the viscosity values, from 0.5 Pa.s at 0.36 m/s to 858 Pa.s at 0.39 m/s, illustrates the deficiencies of this approach using Roelands piezoviscous model and the classical Reynold's equation.

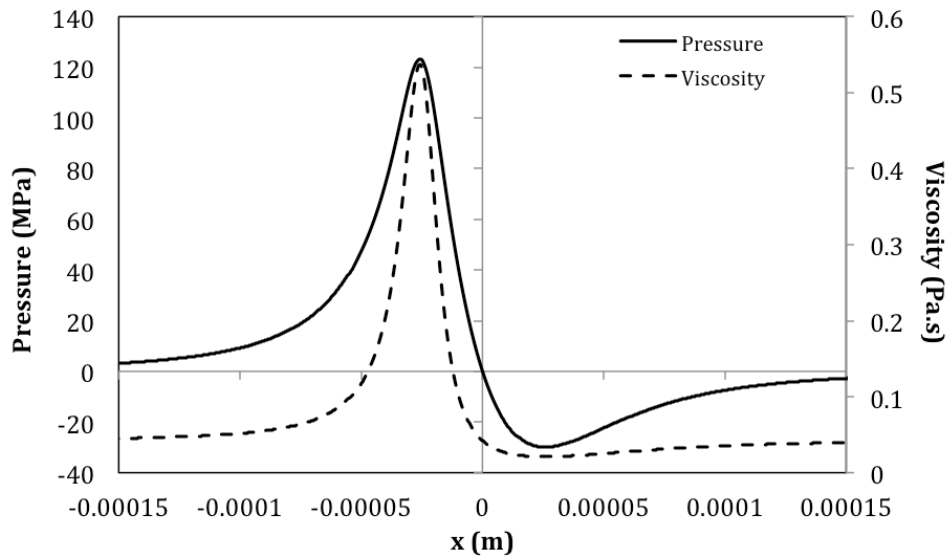


Figure 2-4. Predicted pressure and viscosity distributions– pure sliding at 0.36 m/s (rigid)

To improve on these predictions, the Eyring-Roelands rheology model can be implemented. In order to achieve this, Equation 2-23 is first integrated to calculate shear-rate:

$$\frac{\partial u}{\partial z} = \frac{2z - h}{2\eta} \frac{\partial p}{\partial x} + \frac{u_b - u_a}{h} \quad \text{Equation 2-31}$$

For a pure sliding case this reduces to:

$$\dot{\gamma} = \frac{-h}{2\eta} \frac{\partial p}{\partial x} - \frac{u_a}{h} \quad \text{Equation 2-32}$$

Equation 2-32 can be discretised as:

$$\dot{\gamma} = \frac{-h}{2\eta} \frac{p_{i+1} - p_{i-1}}{x_{i+1} - x_{i-1}} - \frac{u_a}{h} \quad \text{Equation 2-33}$$

Equation 2-33 is added to the Reynolds solver described above in order to calculate lubricant viscosity through the Eyring-Roelands viscosity model, instead of the Roelands equation as in the above example. For surface speeds up to 0.36 m/s, the results are similar to implementing the Roelands formula (Equation 2-8). At higher

speeds, lubricant starts to shear-thin and the equivalent shear-rate corresponding to the start of shear-thinning is 10^7 s^{-1} . In this case, the classical Reynolds solution converges for speeds up to 0.42 m/s , although results are not physically acceptable in the shear-thinning region.

The order investigation of different terms in the shear rate equation is presented in Figure 2-5 for sliding speed of 0.36 m/s . The pressure gradient term is dominant in the region before the pressure build-up and in the vicinity of the pressure spike.

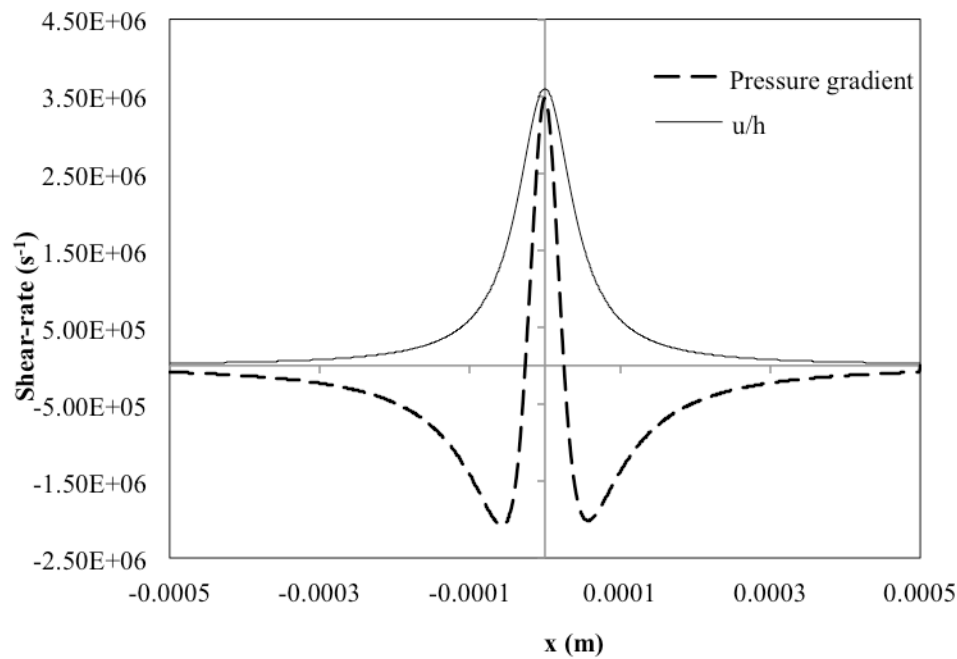


Figure 2-5. Order investigation of different terms of the shear-rate equation

In order to tackle the numerical divergence issues in the implemented model a set of measures were taken including under-relaxation of both pressure and viscosity (separately), implementation of first-order and second-order backward differential schemes and interpolation of the pressure for high error nodes. However, none of these numerical methods eliminates divergence problems.

The non-dimensionalised form of the Reynolds equation is applied to reduce the order of pressure values. The following non-dimensional variables are defined:

$$P^* = \frac{ph_0}{\eta_0 u}, \quad H^* = \frac{h}{h_0}, \quad U^* = \frac{u}{u_0}, \quad \eta^* = \frac{\eta}{\eta_0}, \quad X^* = \frac{x}{h_0}.$$

With these parameters, the non-dimensionalised Reynolds equation can be re-written as:

$$\frac{\partial(\frac{H^{*3}}{\eta^*})}{\partial X^*}(\frac{\partial P^*}{\partial X^*}) = 6U^* \frac{\partial H^*}{\partial X^*} \quad \text{Equation 2-34}$$

Although the insertion of Equation 2-34 in the code results in numerical order relaxation of pressure value, the code still diverges at higher sliding speeds. Here, it was shown that the classical Reynolds equation fails to predict correct HL pressure distribution for non-Newtonian lubricants at high speeds and shear-rates. The failure of the classical Reynolds equation approach leads to the use of a non-Newtonian modified-Reynolds equation proposed by Conry et al. [9]. Figure 2-6 shows two solid surfaces (S1 and S2) that are separated by a thin fluid film.

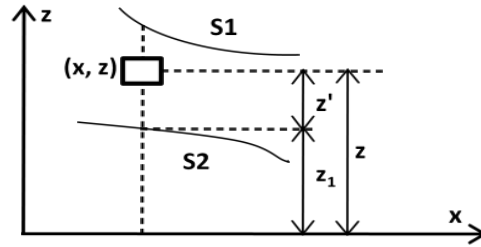


Figure 2-6. Description of the contact region in two dimensions

The equation of equilibrium for a given element of fluid, located at (x, z) , can be written as:

$$\frac{\partial \tau_{xz}}{\partial z} = \frac{\partial p}{\partial x} \quad \text{Equation 2-35}$$

The lower surface height is z_1 and the distance from the lower surface to the fluid volume is z' ($0 \leq z' \leq h(x)$). The integration of Equation 2-35, by neglecting pressure variation in z -direction, results in:

$$\tau_{xz} = \tau_1 + z' \left(\frac{dp}{dx} \right) \quad \text{Equation 2-36}$$

By inserting Equation 2-36 into Equation 2-12 (U_1 and U_2 are upper and lower surfaces speed), the velocity profile can be written as:

$$u(x, z') = U_1 + \frac{\tau_0 h}{2\eta} \frac{1}{\xi} \left[\cosh\left(\frac{\tau_m}{\tau_0} - \xi + \left(\frac{2z'}{h}\right)\xi\right) - \cosh\left(\frac{\tau_m}{\tau_0} - \xi\right) \right] \quad \text{Equation 2-37}$$

where τ_m is the shear stress on the mid-plane and ξ is a dimensionless function:

$$\xi = \frac{h}{2\tau_0} \left(\frac{dp}{dx} \right) \quad \text{Equation 2-38}$$

The mass flux per unit width, $M(x)$, can be calculated from:

$$M(x) = \int_0^{h(x)} \rho u(x, z') dz' \quad \text{Equation 2-39}$$

Substitution of Equation 2-37 into 2-39 results in:

$$M(x) = \rho h (U_1 + U_2) / 2 + \frac{\rho h^3}{12\eta} \frac{dp}{dx} \left[\frac{3(\sinh \xi - \xi \cosh \xi)}{\xi^3} \cosh\left(\frac{\tau_m}{\tau_0}\right) \right] \quad \text{Equation 2-40}$$

The solution of continuity equation together with Equation 2-40 results in the one-dimensional Reynolds-Eyring equation:

$$\frac{d}{dx} \left\{ \frac{\rho h^3}{12\eta} \frac{dp}{dx} \left[\frac{3(\xi \cosh \xi - \sinh \xi)}{\xi^3} \cosh\left(\frac{\tau_m}{\tau_0}\right) \right] \right\} = \frac{(U_1 + U_2)}{2} \frac{d}{dx} (\rho h) \quad \text{Equation 2-41}$$

By defining the following parameter:

$$S(x) = \frac{3(\xi \cosh \xi - \sinh \xi)}{\xi^3} \cosh\left(\frac{\tau_m}{\tau_0}\right) \quad \text{Equation 2-42}$$

and replacing $\frac{\rho h^3}{12\eta}$ with ε^x , Equation 2-41 can be re-written as:

$$\frac{d}{dx} \left\{ \varepsilon^x \frac{dp}{dx} S(x) \right\} = \frac{(U_1 + U_2)}{2} \frac{d}{dx} (\rho h) \quad \text{Equation 2-43}$$

The representative stress, τ_0 , of a lubricant characterises the changeover from Newtonian to non-Newtonian behaviour; an infinitely large τ_0 characterises a Newtonian fluid when using Eyring model. As τ_0 approaches infinity, the values of the

dimensionless function, ξ , approaches zero and by using the L'Hospital's rule it can be shown that $S(x)$ approaches unity, which reduces Equation 2-43 to the classical Reynold's equation. The value of $S(x)$ is always greater than or equal to one for all values of ξ or τ_m . For pure rolling, $\cosh(\tau_m/\tau_0)$ reaches one, and the resulting Equation 2-43 is identical to the Reynolds equation derived by Bell in 1962 [54].

Equation 2-43 can be discretised as:

$$\varepsilon_{i+1/2}^x S(x)_{i+1/2}^x \left(\frac{dp}{dx} \right)_{i+1/2} - \varepsilon_{i-1/2}^x S(x)_{i-1/2}^x \left(\frac{dp}{dx} \right)_{i-1/2} = \rho \frac{(U_1 + U_2)}{2} \frac{dh}{dx} \quad \text{Equation 2-44}$$

The C++ code is modified to start the iteration by calculating ξ , ε and τ_m using the initial values of pressure and viscosity. The viscosity is then calculated using the updated $S(x)$ and ε^x parameters obtained from pressure matrix solution. The solver is converged for the objective sliding speed of 1 m/s (and higher). The results are matched with the classical Reynolds equation solution for sliding speed of up to 0.36 m/s (Newtonian region). The results for sliding speeds of 0.5 m/s and 1 m/s are presented in Figure 2-7 and Figure 2-8.

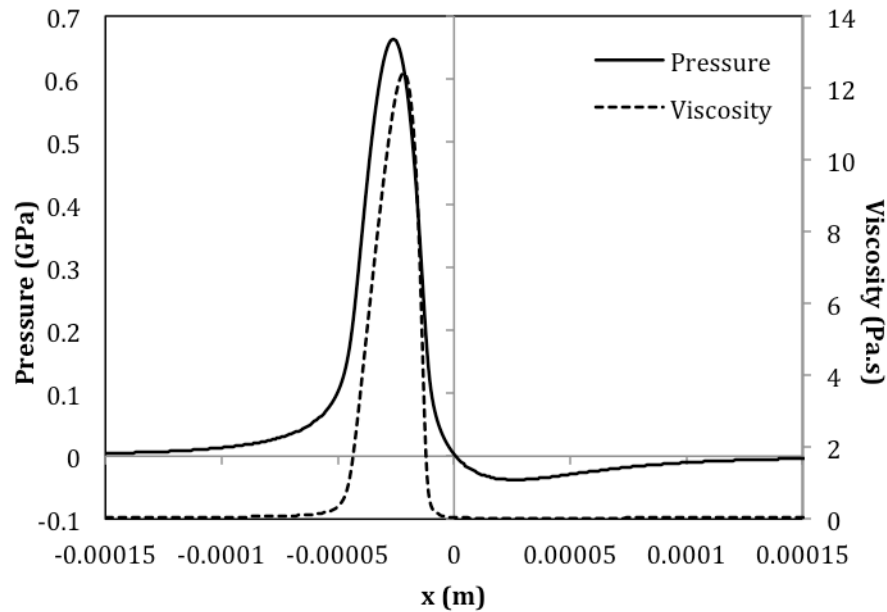


Figure 2-7. Pressure and viscosity distributions - pure sliding at 0.5 m/s by using Eyring-Roelands model

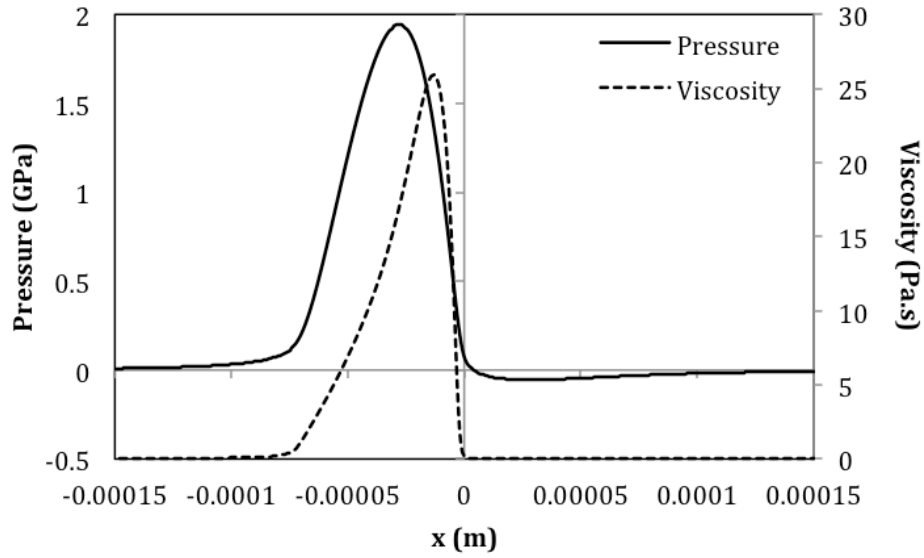


Figure 2-8. Pressure and viscosity distributions - pure sliding at 1 *m/s* by using Eyring-Roelands model

It can be seen from Figure 2-7 and Figure 2-8 that the pressure is not the main parameter affecting the viscosity at high sliding speeds. The pressure dominates the viscosity calculation in regions where the lubricant behaves as a Newtonian fluid (for pressures lower than 140 *MPa* in this configuration) and therefore the use of Roelands model is identical to the Eyring-Roelands model. The shear-rate effects on viscosity become significant at shear rate values past $\dot{\gamma} = 10^7 / s$. The conventional Reynold's equation results for sliding speed between 0.36 *m/s* to 0.4 *m/s* are not equivalent to their corresponding results from the modified-Reynolds equation as the former predicts incorrect pressure and shear-rate values.

In the next section, CFD modelling of HL contacts particularly for a rolling element bearing case study is described. A comparison between the results obtained using the Reynolds and Navier-Stokes based approaches is performed.

2.2.2 CFD based approach

CFD refers to the numerical solution of the flow equations that describe a phenomenon of interest. The fluid flow is described by a set of partial differential equations that cannot be solved analytically. Therefore, discretised version of fluid flow governing equations, a system of algebraic equations, should be used to obtain an approximate solution. This solution produces values at discrete locations in space and time. The accuracy of the numerical approximation depends on: (a) quality of discretisation and (b) convergence criteria.

In this section, the fluid governing equations and finite volume discretisation method (FVM) are introduced.

2.2.2.1 Fluid governing equations

The governing equations of fluid flow are: conservation of momentum, continuity and energy. For a general variable ϕ , the conservative form of fluid flow equation, namely transport equation, can be written as:

$$\underbrace{\frac{\partial \rho \phi}{\partial t}}_{\text{temporal}} + \underbrace{\nabla \cdot (\rho u \phi)}_{\text{convection}} - \underbrace{\nabla \cdot (\rho \Gamma_{\phi} \nabla \phi)}_{\text{diffusion}} = \underbrace{S_{\phi}(\phi)}_{\text{source}} \quad \text{Equation 2-45}$$

where ρ is the density, t is time, u is velocity, Γ_{ϕ} is diffusivity and S_{ϕ} is a source term.

The continuity equation, conservation of mass, can be derived from transport equation by setting $\phi = 1$:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0 \quad \text{Equation 2-46}$$

According to Bird [55], by setting ϕ equal to u , the momentum equation (neglecting gravitational effects) is:

$$\frac{\partial \rho u}{\partial t} + \nabla \cdot (\rho u u) - \nabla \cdot \tau = -\nabla p \quad \text{Equation 2-47}$$

where τ is viscous stress tensor defined as:

$$\tau = -\eta(\nabla u + (\nabla u)^T) + \eta \frac{2}{3} I \nabla \cdot u \quad \text{Equation 2-48}$$

where η is the viscosity of fluid. It was previously mentioned that the lubricant does not behave as a Newtonian fluid and this will be modified in the CFD model as of the Reynolds code.

The energy equation can be written as [55]:

$$\frac{D\rho H}{Dt} = \nabla \cdot (k\nabla T) - \tau : \nabla U + \frac{Dp}{Dt} \quad \text{Equation 2-49}$$

where H is the enthalpy and k is the thermal conductivity of the fluid.

2.2.2.2 FVM

There are different discretisation methods available to be applied in CFD modelling:

1. Finite difference: easy to implement, restricted to simple grid.
2. Finite element: highest accuracy on coarse grids, slow for large problems.
3. Finite volume: not limited to cell shape, conserve transport equations even on coarse grids, false diffusion in the case of using simple numerical methods.

Finite volume is the most widely used method in CFD studies and iterative solvers are well developed for it.

The following elements should be defined for a numerical solution setup:

- Mathematical model

A set of equations and boundary conditions that define a particular problem.

- Coordinate system

There are different coordinate systems available: Cartesian, cylindrical, spherical and etc. that can be used depending on the geometry and form of the governing equations.

- Discretisation methods

Based on the mathematical model, an appropriate discretisation method should be selected. For all CFD calculations in this study, FVM is employed which consists of discretisation of the domain of study and equations discretisation. The steps toward FVM discretisation can be outlined as follow:

- Spatial discretisation

The flow domain is subdivided into a finite number of subdomains called control volumes (CV). In contrast to the finite difference method, control volumes define

boundaries rather than computational nodes that completely fill the domain with any polyhedral shape without any overlap. Two neighbouring cells must only share one face (internal face) and the boundary face belongs to one cell only. A cell is surrounded by a set of arbitrarily unstructured flat faces with no limitation on the number. In this study, block-structured grid is used in order to introduce much finer grids in the areas where high resolution is required. Basically, the domain is divided into large segments or blocks that are then sub-divided into CV. The following information is required to define the mesh:

- Points: defined by positions in three dimensions.
- Faces: defined by a list of points.
- Cells: defined by a list of faces.
- Boundary patches: defined by a list of boundary faces that can be only member of one boundary patch.

- Finite approximations

The governing equations are integrated over all control volumes by approximating the variation of flow properties between mesh points. The finite approximation for surface and volume integrals greatly influences the accuracy of numerical solution. This will be briefly described in the following.

- Convergence criteria

The convergence criteria must be set for iterative method by accounting for efficiency and accuracy concurrently. In this study, there are inner and outer loop iterations where different convergence criterion is considered.

- Solution method

The resulted system of algebraic equations, resulted from using FVM, can be solved using:

SIMPLE (Semi-Implicit Method for Pressure Linked Equations), used for steady state problems.

PISO (Pressure Implicit Splitting of Operators), used for time dependent flows.

- Errors and Residuals [56]

The solution obtained from numerical methods is always an approximation. The errors produced by the numerical solution can be divided into the following three main categories [57]:

- Modelling errors: the difference between the actual flow and the exact solution of the mathematical model.
- Discretisation errors: the difference between the exact solution of the conservation equations and the exact solution of the algebraic system of discretised equations.
- Iteration errors: the difference between the iterative and the exact solutions of the algebraic equations systems.

The residual is a function that shows how well the governing differential equations are approximated over the computational cell. Sparse matrix solvers are iterative, i.e. they are based on reducing the equation residual over a succession of solutions. The residual is allegedly a measure of the error in the solution so that the smaller it is, the more accurate the solution. More specifically, the residual is calculated by substituting the current solution into the equation and taking the magnitude of the difference between the left and right hand sides; the residual is usually normalised to make it independent of the scale of model being analysed. Before solving an equation for a particular field, the initial residual is evaluated based on the current values of the field and it is then re-evaluated after each iteration. The solver stops if either of the following conditions occurs:

- The residual falls below the solver tolerance;
- The ratio of current to initial residuals falls below the solver relative tolerance;

2.2.2.3 Discretisation of the governing equations

The integrated form of the conservation equation (Equation 2-45) is used as the starting point:

$$\begin{aligned} \frac{\partial}{\partial t} \int_{V_p} \rho \phi \, dV + \int_{V_p} \nabla \cdot (\rho u \phi) \, dV - \int_{V_p} \nabla \cdot (\rho \Gamma_\phi \nabla \phi) \, dV \\ = \int_{V_p} S_\phi(\phi) \, dV \end{aligned} \quad \text{Equation 2-50}$$

The convective and diffusive volume integrals are converted into integrals over the cell surface, bounding the volume, using the Gauss divergence theorem:

$$\int_V \nabla \phi = \int_S dS_\phi \quad \text{Equation 2-51}$$

where S is the surface area vector and ϕ can represent any variable.

The net flux through the CV boundary is the sum of integrals over CV faces:

$$\int_S k dS = \sum_f \int_{S_f} f dS \quad \text{Equation 2-52}$$

where k is the component of the convective or diffusive flux vector in the direction normal to CV face.

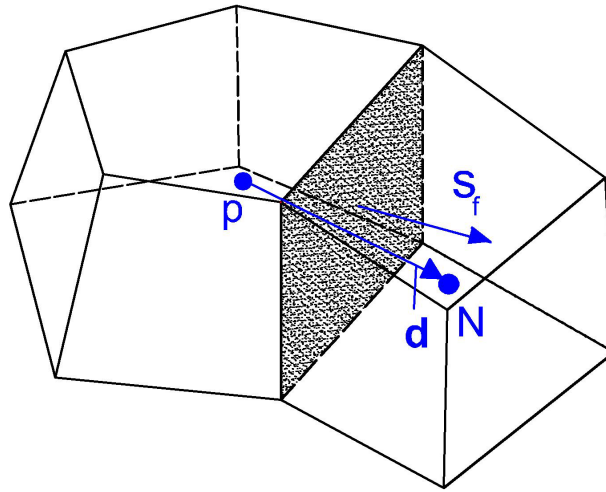


Figure 2-9. Parameters in finite volume discretisation

The diffusion (Laplacian) term is integrated over a control volume and linearised as follows:

$$\int_V \nabla \cdot (\Gamma \nabla \phi) \, dV = \int_S dS \cdot (\Gamma \nabla \phi) = \sum_f \Gamma_f S_f \cdot (\nabla \phi)_f \quad \text{Equation 2-53}$$

In case of an orthogonal length vector d (i.e. parallel to S_f in Figure 2-9), the face gradient discretisation is implicit between the centre of the cell of interest P and the centre of neighbouring cell N :

$$S_f \cdot (\nabla \phi)_f = |S_f| \frac{\phi_N - \phi_P}{|d|} \quad \text{Equation 2-54}$$

Additional explicit term is introduced in the case of non-orthogonal meshes [58].

The convection term is integrated over a CV and linearised as follows:

$$\int_V \nabla \cdot (\rho u \phi) dV = \int_S dS \cdot (\rho u \phi) = \sum_f S_f \cdot (\rho u)_f \phi_f = \sum_f F \phi_f \quad \text{Equation 2-55}$$

The face field ϕ_f can be evaluated using a variety of schemes:

Central differencing (CD) is second order accurate but unbounded:

$$\phi_f = f_x \phi_P + (1 - f_x) \phi_N \quad \text{Equation 2-56}$$

where $f_x = \frac{\overline{fN}}{PN}$.

Upwind differencing (UD) evaluates ϕ_f from the direction of flow and is bounded at the expense of accuracy:

$$\phi_f = \begin{cases} \phi_P & \text{for } F \geq 0 \\ \phi_N & \text{for } F < 0 \end{cases} \quad \text{Equation 2-57}$$

These two schemes can be blended to preserve boundedness with reasonable accuracy:

$$\phi_f = (1 - \gamma)(\phi_f)_{UD} + \gamma(\phi_f)_{CD} \quad \text{Equation 2-58}$$

In OpenFOAM, the Gamma differencing scheme is incorporated through implementation of several blending coefficient methods along with well-known schemes such as SUPERBEE, MINMOD and etc.

In this study, time derivatives are evaluated only by using Euler implicit scheme:

$$\frac{\partial}{\partial t} \int_V \rho \phi dV = \frac{(\rho_P \phi_P V)^n - (\rho_P \phi_P V)^0}{\Delta t} \quad \text{Equation 2-59}$$

where n and 0 denotes new (current time step) and old (previous time step) values respectively.

The gradient term is an explicit term and can be determined in a variety of ways. The Gauss integration scheme is performed as follows:

$$\int_V \nabla \phi \, dV = \int_S dS \phi = \sum_f S_f \phi_f \quad \text{Equation 2-60}$$

The reader is referred to Programmer's Guide [58] for further details.

Since ϕ is a function of time and space and spatial in a transient problem and also the spatial derivatives are averaged over one or more time steps. A transient partial differential equation can be expressed as:

Using the Euler implicit method of Equation 2-59, the first and second terms can be expressed as:

$$\int_t^{t+\Delta t} \left[\frac{\partial}{\partial t} \int_V \rho \phi \, dV \right] dt = \frac{(\rho_P \phi_P V)^n - (\rho_P \phi_P V)^0}{\Delta t} \Delta t \quad \text{Equation 2-61}$$

$$\int_t^{t+\Delta t} \left[\int_V \ddot{A} \phi \, dV \right] dt = \int_t^{t+\Delta t} \ddot{A}^* \phi \, dt \quad \text{Equation 2-62}$$

where $\ddot{A}^*$ represent spatial discretisation of $\ddot{A}$. This integral can be discretised in three ways:

- Euler implicit uses current time values, ϕ^n , and is first order accurate in time and guarantees boundedness and is unconditionally stable.

$$\int_t^{t+\Delta t} \ddot{A}^* \phi \, dt = \ddot{A}^* \phi^n \Delta t \quad \text{Equation 2-63}$$

- Explicit uses old values, ϕ^0 , and is first order accurate in time and is unstable if the courant number (Co) is greater than 1.

$$Co = \frac{U_f \cdot d}{|d|^2 \Delta t} \quad \text{Equation 2-64}$$

where U_f is characteristic velocity of the flow.

- Crank Nicholson uses mean of current and old values. It is second order accurate in time and is unconditionally stable but does not guarantee boundedness.

$$\int_t^{t+\Delta t} \dot{A}^* \phi dt = \dot{A}^* \frac{\phi^n + \phi^0}{2} \Delta t \quad \text{Equation 2-65}$$

Among the time schemes presented above, Euler implicit is found to be the most stable one for the use in this study.

2.2.2.4 Numerical Boundary conditions

In order to obtain the solution of the transport equation in discretised form, the value of ϕ_f (represents the conserved property per unit mass) $S_f \cdot (\nabla \phi)_f$ (diffusion term) should be determined for boundary faces as well as within the main fluid stream. Two types of boundary conditions are conventionally adopted in CFD:

- Dirichlet boundary condition prescribes the value of the dependent variable on the boundary.
- Neumann boundary condition prescribes the gradient of the variable normal to the boundary.

In order to perform discretisation on terms that include the sum over faces $\sum_f$, the following situations are considered for a boundary face:

- Fixed value boundary condition prescribes a fixed value at the boundary ϕ_b either by substitution in cases where the discretisation requires the value on a boundary face or in terms where the gradient is required, e.g. Laplacian, it is calculated using boundary face value and cell centre value.
- Fixed gradient boundary condition is a specification on inner product of the gradient and unit normal to the boundary either through the cell centre value by interpolation in cases when discretisation requires the value on a boundary face or by direct substitution in cases where the discretisation requires the face gradient to be evaluated.

2.2.2.5 Physical boundary conditions

The specification of boundary conditions is usually an engineer's translation of the real behaviour. Real boundary conditions are defined by physical attributes rather than the numerical description.

The following physical boundary conditions are used in this study:

No-slip impermeable walls: The velocity of the fluid on the wall is equal to the velocity of the wall itself. There is no flux through the wall and therefore the pressure and velocity boundary conditions are zero gradient.

Symmetry plane: The components of the gradient normal to the plane are zero and the ones parallel to it are projected to the boundary face from the inside of the domain.

Total pressure: The total pressure is fixed and p adjusted in accordance with U changes.

$$p_0 = p + \frac{1}{2}\rho|U|^2 \quad \text{Equation 2-66}$$

2.2.2.6 CFD mesh convergence and non-Newtonian piezoviscous results

The governing equations and their numerical solutions are implemented using OpenFOAM, a freely available open-source CFD package based on FVM.

The case considered is a sliding rigid wall on a rotatory rigid half cylinder. The developed model is capable of simulating 3D cases, however 2D cases are only considered in this section. The computational domain is shown in Figure 2-10 where multi-block approach is used for meshing in order to enhance computational efficiency. The computation was performed along a multi-block discretised domain and there are 88,000 cells near to the region where large pressure gradient is expected to build up due to hydrodynamic action of the lubricant.

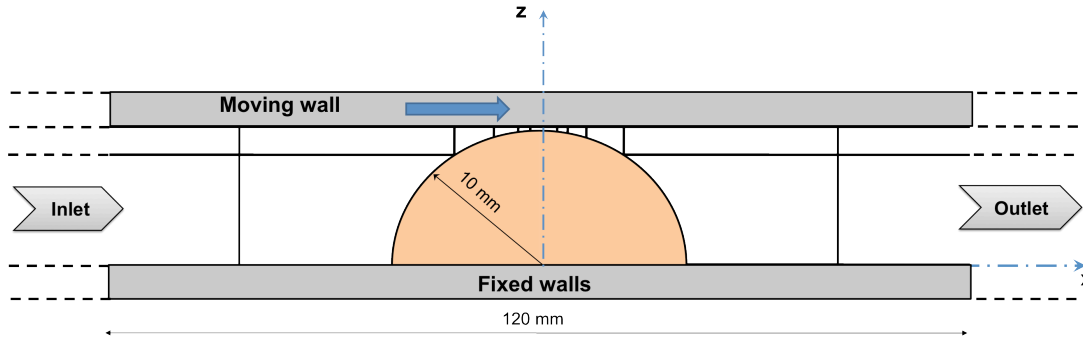


Figure 2-10. CFD domain of grid study

The boundary conditions for pressure are zero gradient at walls and fixed to ambient pressure at inlet and outlet. The boundary conditions for velocity is dependent on slide-to-roll ratio. In a roller bearing, there are series of cylinder-flat surface contacts which makes periodic boundary condition physically more acceptable. Therefore, parallel cases with periodic boundary conditions at the inlet and the outlet have been also tested.

The simpleFoam solver, a steady-state solver for incompressible and isoviscous fluids, is employed for computation at this stage. The governing equations for simpleFoam are continuity and momentum. The parameters for the isoviscous case are summarised in Table 2-2. The viscosity is that of a typical lubricant in a bearing.

Parameter	Value
Cylinder radius	$R=10 \text{ mm}$
Moving wall velocity	$u=1 \text{ m/s}$
Film thickness	$h_0=10^{-7} \text{ m}$
Viscosity	$\eta=0.04201 \text{ Pa.s}$

Table 2-2. CFD isoviscous-incompressible case parameters

The equations were discretised using second-order central differencing. ICCG (incomplete-Cholesky preconditioned conjugate gradient) and BICCG (incomplete-Cholesky preconditioned biconjugate gradient) solvers were used for pressure and velocity matrices respectively. The residual tolerances are set to 10^{-6} for both velocity and pressure. The main loop execution takes 16 seconds on a single Intel Core i7 3.2 GHz processor.

The mesh development is complicated considering the very low value of h_0 in comparison with the overall geometry of the domain. Different expansion ratios are set in different blocks to maintain aspect ratios of less than 5. The finest mesh consists of 139840 cells and elements are equally spaced into 20 parts in y direction in between x $[-7.07 \text{ mm}, 7.07 \text{ mm}]$. For mesh convergence study, coarser meshes are prepared with lower number of cells in both directions (x and z) with total 69920, 34960 and 8740 cells. The last step is to calculate the error of calculation by comparing the simulation results with the values from the Reynolds equation approach (bearing in mind that the mesh convergence check on the Reynolds approach results has been done already). The maximum error in pressure values occurs at $x=\pm 0.026 \text{ mm}$ where the highest pressure gradient or maximum and minimum pressure values build up. The maximum error related to each mesh size scale is presented in Table 2-3.

	xFactor=1	xFactor =2	xFactor =4	xFactor =8
zFactor=1	1.813	2.145	2.463	2.601
zFactor=2	0.983	0.426	0.447	0.521
zFactor=4	1.361	0.296	0.112	0.121
zFactor=8	1.781	0.312	0.106	0.109

Table 2-3. Different mesh size relative errors (%)

The extrapolated relative error is 0.72% and the fine-grid convergence index is 0.89%. These are calculated [56] using the first three mesh sizes in Table 2-3. Beyond the 4/4 scaling case, the error does not change very significantly since the point of grid-independent solution is reached. It can be seen from Table 2-3 that the error is growing by an increase in the resolution in x -direction. This proves the importance of an appropriate aspect ratio for the computational cells.

A comparison between the pressure distributions from CFD (the finest mesh) and the Sommerfeld (Equation 2-5) analytical solution (see Figures 2-11 and 2-12) shows a perfect match for a Newtonian lubricant under pure sliding at 1 m/s . The solution is point symmetric around the origin and the pressure is zero in the centre and maximum pressure value is 120 MPa . The region that undergoes with negative pressure values is

where cavitation occurs. In the next section, the cavitation treatment for the fluid modelling is described.

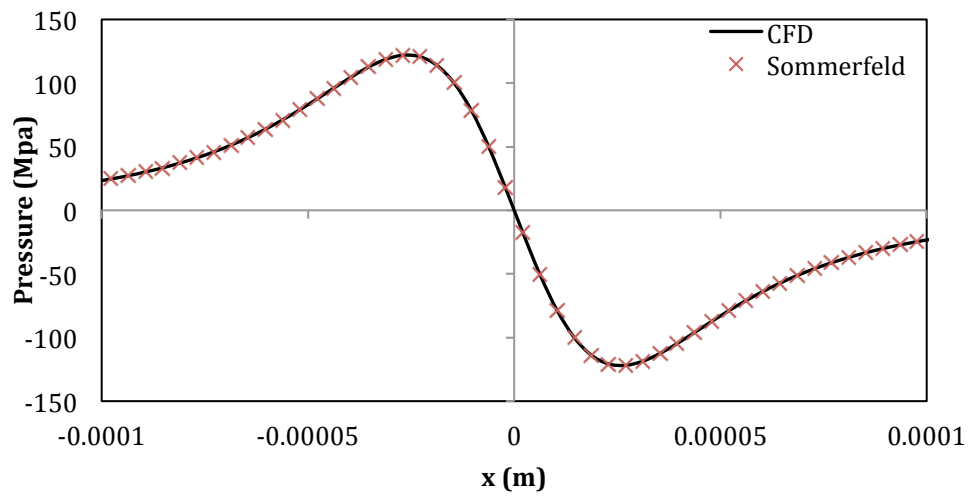


Figure 2-11. Comparison between CFD and Sommerfeld solutions for pure sliding at 1 *m/s*

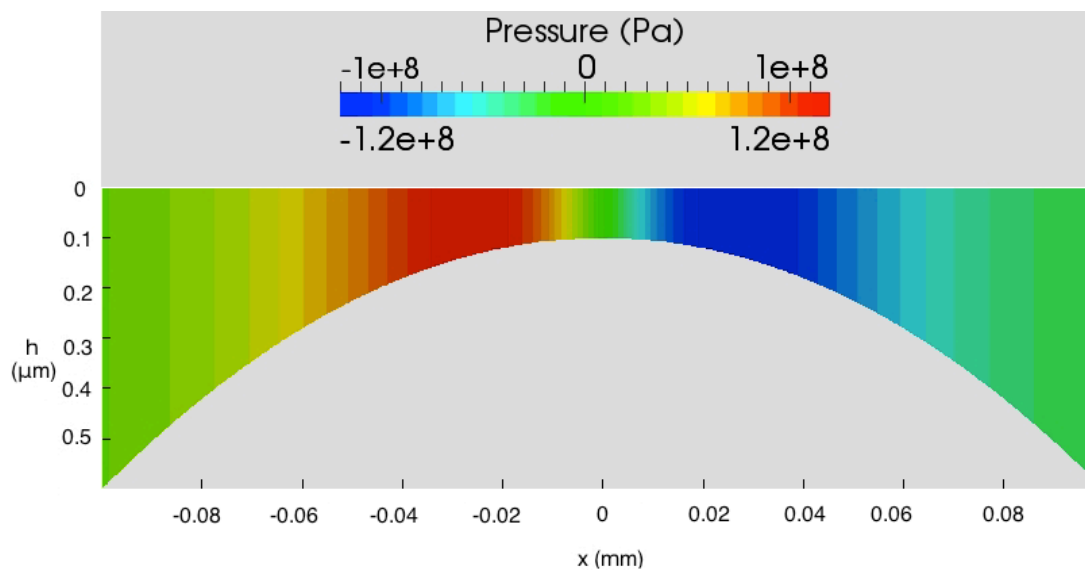


Figure 2-12. Pressure distribution for a Newtonian lubricant - pure sliding at 1 *m/s*

The next step is to incorporate the effects of pressure on viscosity. Firstly, Roelands viscosity model is implemented for different sliding speed starting from 0.1 *m/s* and aiming for 1 *m/s*. The code is modified in order to calculate the viscosity of each cell using Equation 2-8. An under-relaxation factor of 0.4 is used in order to control the pressure and eventually calculation. However, the code diverges at sliding speeds of higher than 0.15 *m/s*. In order to control the viscosity build up, a time variant boundary condition is applied to the sliding wall to achieve stability at higher sliding

speeds. The new boundary condition creates a series of velocity values dependent to the time instance. As a result, pressure and viscosity build up gradually and therefore avoid quick changes in viscosity values at higher pressure. The velocity function is defined as in Equation 2-67:

$$U_B(t) = U_{BS} + (3 - \cos(\alpha t - 1))(U_{BF} - U_{BS}) \quad \text{Equation 2-67}$$

where U_B , U_{BS} , U_{BF} and α are surface speed at time t , initial speed, aim speed and a speed factor.

The sliding speed changes smoothly from the start to the end of simulation (see Figure 2-13) to avoid sharp increments in viscosity.

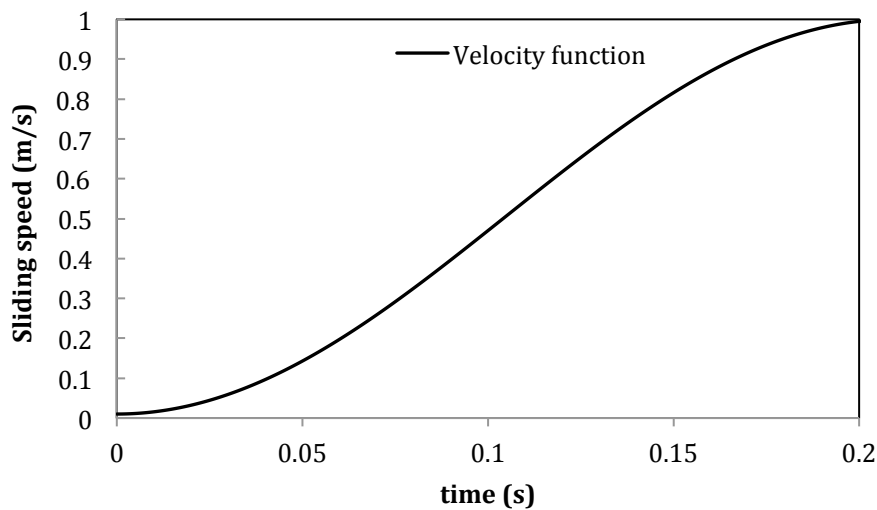


Figure 2-13. Velocity ramp-up profile for stability enhancement

GAMG (Generalised Algebraic Multi-Grid) matrix solver algorithm is used to solve pressure equation aiming for faster convergence and higher stability. The key idea behind using a multi-grid approach is to use sequence of coarser and coarser grids. Therefore, after a number of iterations on the finest mesh, it is more efficient to solve an equation for remaining error on a coarser mesh than to continue on the finest mesh. The error equation solution would be then interpolated back onto the finest grid to correct the solution. The GAMG solver decreases significantly the initial fluctuations occurred using the ICCG solver. It is also notable that the time step should be small enough (0.00001 s) to maintain Courant number around 0.5 for different sliding speeds.

By implementing above-mentioned numerical techniques, the Reynolds-code converges for sliding speeds of up to 0.41 m/s. The divergence at higher speeds can be

traced back into the inappropriateness of Roelands viscosity model. CFD matches with the classical Reynolds equation predictions for sliding speed of up to 0.36 m/s with maximum relative errors of 0.42% for pressure and 0.84% for viscosity.

The next step is to implement Eyring-Roelands viscosity model to avoid unphysical viscosity prediction at high pressures. Equation 2-14 is used for the viscosity calculation where the shear-rate can be evaluated directly from the velocity components. The time variant velocity profile, GAMG solver and under-relaxation factors of 0.4 for pressure and 0.6 for viscosity are applied in order to control the solution near the contact area. The classical Reynolds equation approach was shown to be limited to Newtonian regions and therefore the modified-Reynolds approach results are used for higher sliding speed investigations. A comparison between CFD and the modified-Reynolds results shows significant differences between the two solutions pressure (see Figure 2-14) and viscosity (see Figure 2-15) predictions. The lubricant behaviour changes in shear-thinning regions.

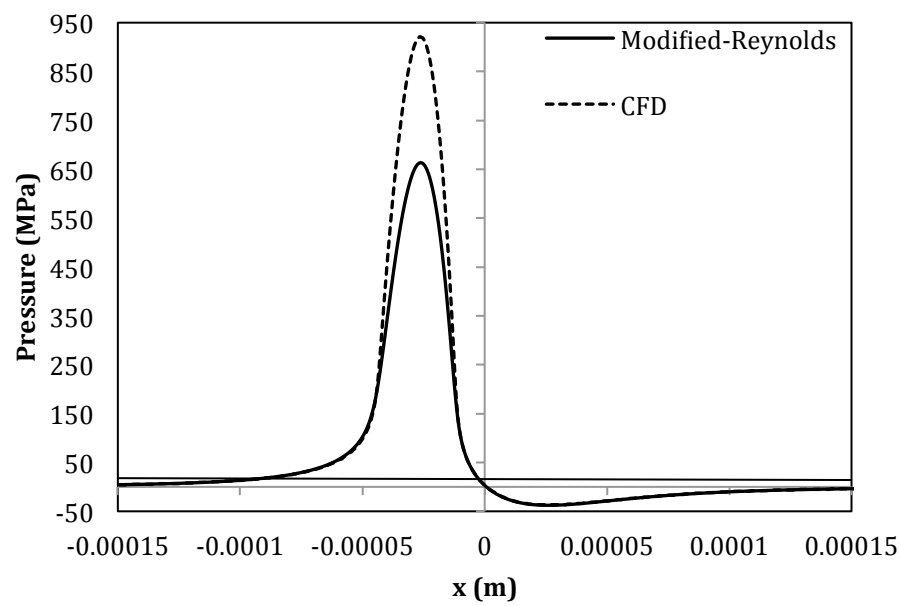


Figure 2-14. Pressure distributions of CFD and modified-Reynold's solution for pure sliding at 0.5 m/s

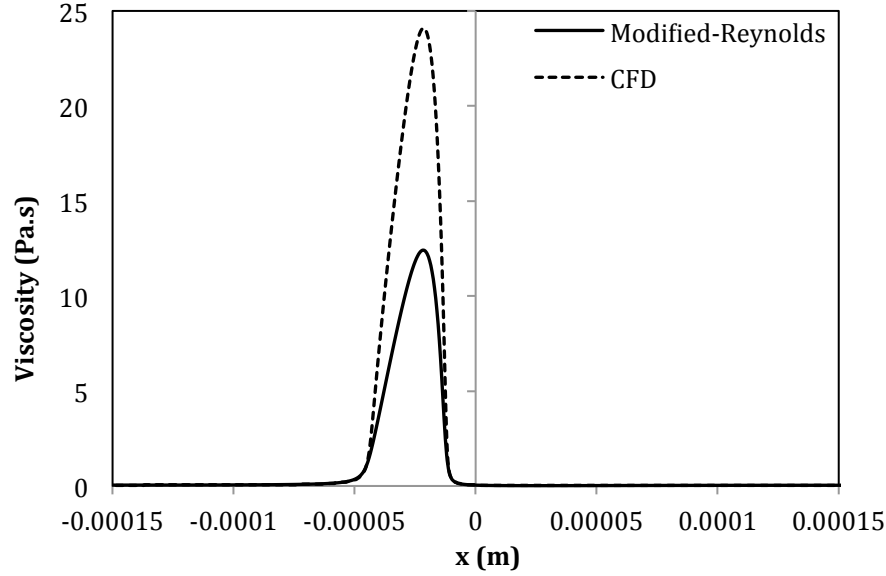


Figure 2-15. Viscosity distributions of CFD and modified-Reynold's solution for pure sliding at 0.5 m/s

The viscosity and pressure distributions, from CFD, are also presented in Figure 2-16 to Figure 2-21 in order to understand the underlying behaviour of lubricant at shear-thinning regions. Since both surfaces assumed to be rigid at this stage, the hydro dynamically pressurised lubricant film build up very large pressure values. The lubricant pressure and viscosity distributions are both constant through the film thickness within the Newtonian regime (see Figure 2-16 and Figure 2-17). However, lubricant viscosity varies through the film in non-Newtonian regime while the pressure is still constant through the film thickness (see Figure 2-19 and Figure 2-21). The localised maximum viscosity is notable near to the stationary surface with lower range of shear rate values.

The results obtained using CFD have been compared to various modified-Reynolds equation finite difference solvers for problems characterised by piezoviscous and non-Newtonian lubricant behaviour in the presence of high pressures (up to 3 GPa), and high-pressure gradients. The difference between the solutions can be traced back to the derivation of the modified-Reynolds equation that relies on finding the correct velocity profile through the film thickness.

The devised CFD model has been shown to provide an improved prediction compared to that obtained using modified-Reynolds equations by correctly capturing

the three dimensional distributions of pressure and viscosity in non-Newtonian regime.

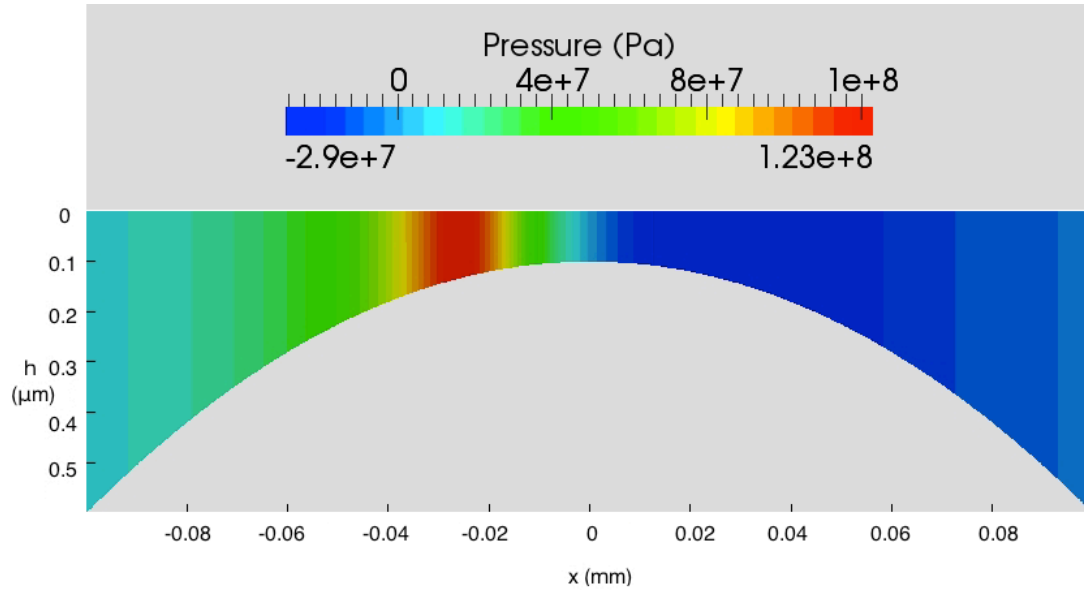


Figure 2-16. Pressure distribution for a non-Newtonian lubricant – pure sliding at 0.36 *m/s*

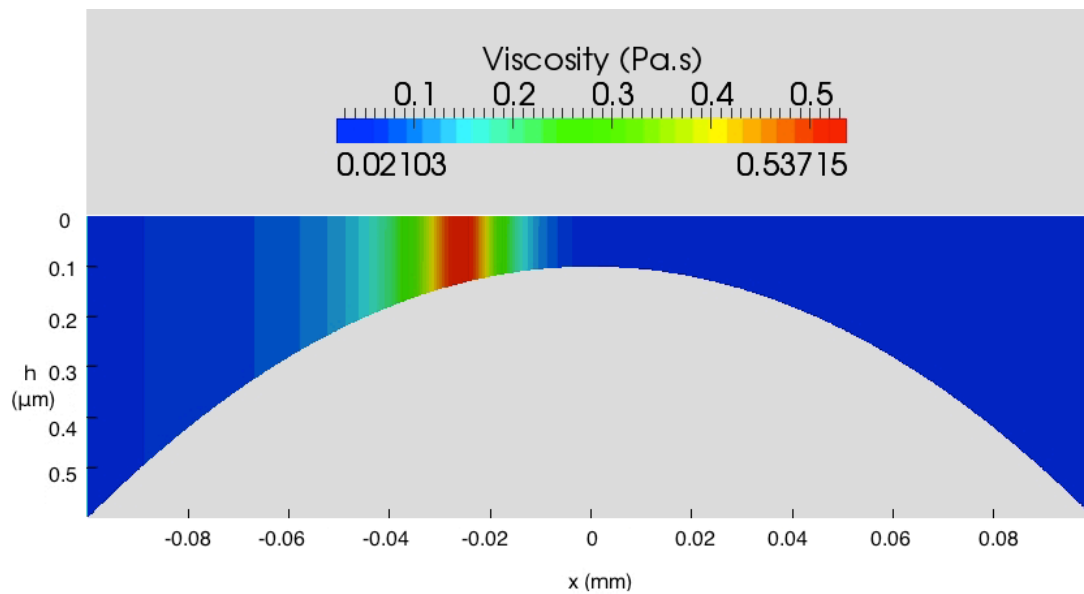


Figure 2-17. Viscosity distribution for a non-Newtonian lubricant – pure sliding at 0.36 *m/s*

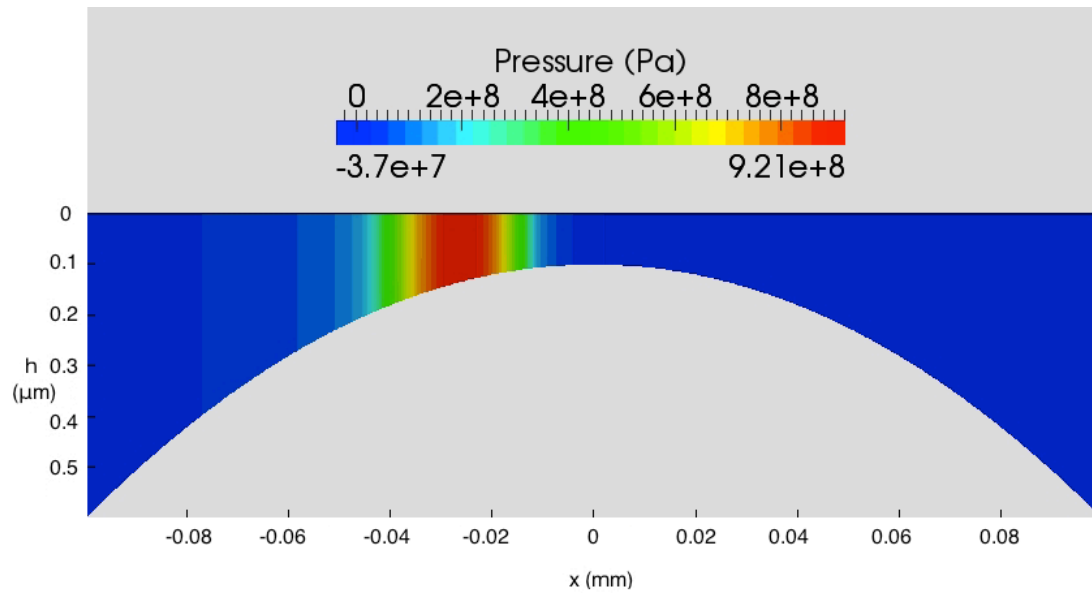


Figure 2-18. Pressure distribution for a non-Newtonian lubricant – pure sliding at 0.5 m/s

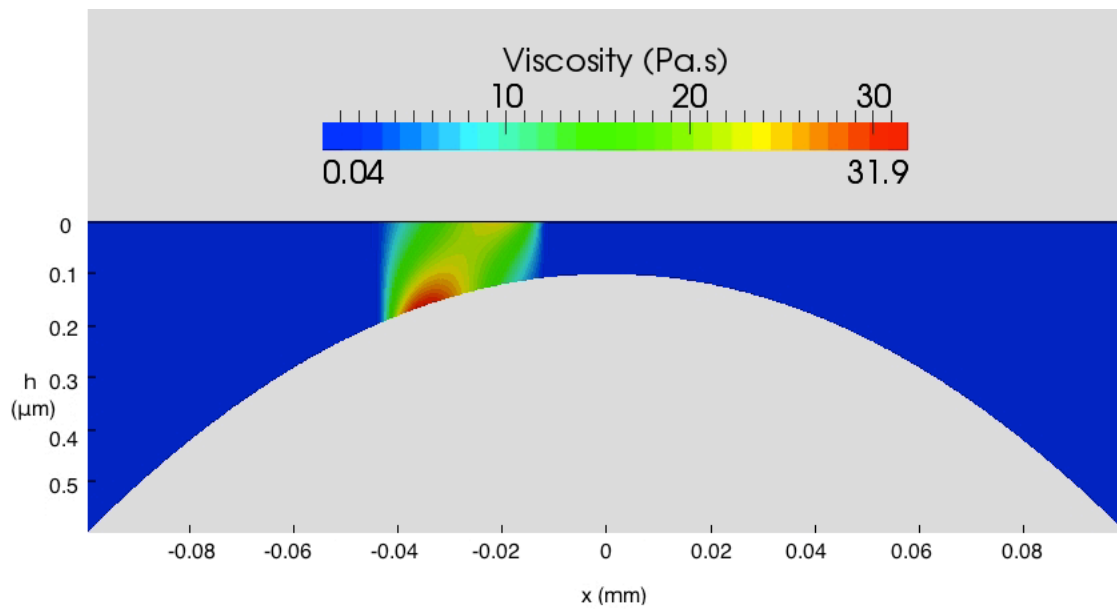


Figure 2-19. Viscosity distribution for a non-Newtonian lubricant – pure sliding at 0.5 m/s

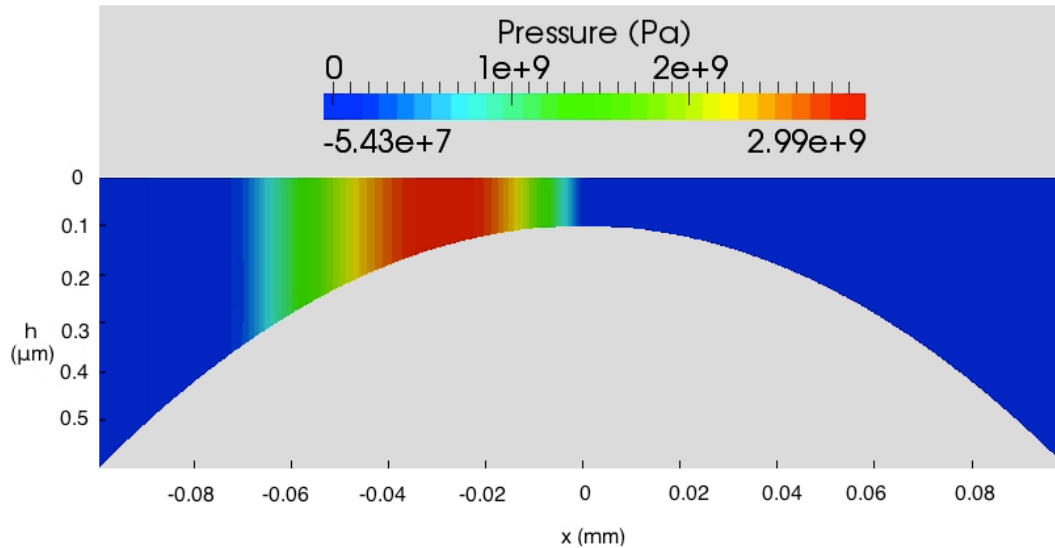


Figure 2-20. Pressure distribution for a non-Newtonian lubricant – pure sliding at 1 m/s

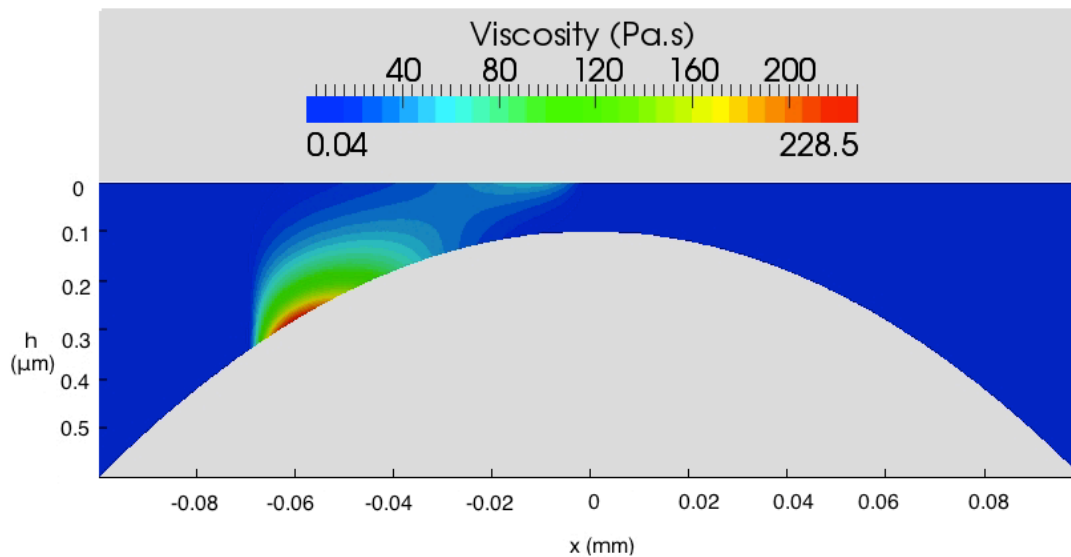


Figure 2-21. Viscosity distribution for a non-Newtonian lubricant – pure sliding at 1 m/s

2.2.2.7 Cavitation

Cavitation is formation and activity of bubbles (or cavities) in a liquid being driven by pressure change without any heating. In other words, it can be explained as rupture of the liquid continuum due to stresses. Cavitation can be divided into four different types: Hydrodynamic, Acoustic, optic and particle cavitation. In bearings, we are only concerned with hydrodynamic cavitation and there are three sub-categories to it:

1. Travelling cavitation: cavities and bubbles travel within the liquid.

2. Fixed cavitation: cavity or pocket attached to the rigid boundary.
3. Vortex cavitation: cavities form in the cores of vortices.

Cavitation treatment is greatly important for accurate solution of EHL contact problems. For ball bearing, it was shown experimentally that fixed cavitation occurs by formation of oil fingers, separated by pockets of gas [59]. There are three possible sources of disruption to the continuous liquid phase flow: (a) air might be drawn from the atmosphere (b) dissolved gas may come out of solution until the pressure is maintained close to the saturation level (c) the liquid pressure falls to the vapour pressure. However, whether that gas is vapour or dissolved gas escaping from the liquid is unclear.

There are four classes to the cavitation modelling:

1. Micro bubbles dynamic: These models focus on the growth and collapse of micro-bubbles in the flow. The notion of cavitation inception is related to the integrated form of an empirical formulation accounting for lift, drag, pressure and inertial forces.
2. Interface tracking method: the flow in liquid phase is studied and is supposed to circumvent the cavity that is assumed to be continuous and attached to the blade.
3. The two-phase models: Two sets of conservation equations are applied along with simplifying assumptions for the calculation of the mass and momentum exchange rate. Non-equilibrium effects of vaporisation and condensation are considered.
4. Homogenous equilibrium models: One set of conservation equations is applied to one phase along with an empirical state law that defines the density and mixture condition between liquid and vapour phase.

A common approach is the use of homogenous equilibrium model. Variety of methods proposed for mixture density calculation. One of the first homogenous models introduced by Delannoy [60] who experienced problems with liquid/vapour density ratios higher than 1:100. Avva et al. [61] employ thermodynamic equilibrium and therefore the volume fraction of vapour is calculated from mixture, saturated liquid and vapour enthalpies. Schmidt [62] assumed an isentropic compression in

energy equation and thermal conduction and viscous effects are considered to be negligible. All above-mentioned models are characterised by high Reynolds and high Weber numbers and negligible viscous effects are assumed. However, none of these assumptions are applicable to a bearing case study where there are low Reynolds and Weber numbers present and viscous heating is significant.

In the Reynolds based approach, cavitation is simply modelled by forcing pressure to be greater or equal to zero by modifying the Gauss-Seidel matrix solver. In a CFD approach however, any unphysical tampering of pressure causes violation in the continuity equation. The current cavitation model, originally developed by Weller [63], is a homogenous equilibrium model where a single set of density and momentum equation for the mixture is solved. The saturation pressure (p_{Sat}) of the liquid is maintained in the cavitating region. The liquid is converted into vapour where the pressure drops below p_{Sat} . If the pressure rises above cavitating pressure, vapour is converted into liquid. In case there is no liquid to convert, pressure will drop below p_{Sat} .

The derivation of Weller isobaric cavitation model equations can be re-written as outlined by Hartinger [63] to include non-linear and unsteady term of Equation 2-47.

The vapour fraction is defined as:

$$\alpha = \frac{\rho - \rho_{l,sat}}{\rho_{v,sat} - \rho_{l,sat}} \quad \text{Equation 2-68}$$

where $\rho_{l,sat}$ and $\rho_{v,sat}$ are the saturated liquid and vapour density at p_{sat} . The density of vapour is given by:

$$\rho_v = \psi_v p \quad \text{Equation 2-69}$$

where ψ_v is the compressibility of vapour. The density of the liquid phase reads:

$$\rho_l = \rho_{l,0} + \psi_l p \quad \text{Equation 2-70}$$

where $\rho_{l,0}$ is the liquid density at zero pressure and ψ_l is the compressibility of liquid.

The density of the vapour-liquid mixture is given by:

$$\rho = \alpha \rho_v + (1 - \alpha) \rho_l \quad \text{Equation 2-71}$$

and the mixture dynamic viscosity is assumed to be:

$$\mu = \alpha\mu_v + (1 - \alpha)\mu_l \quad \text{Equation 2-72}$$

The discretized momentum equation can be expressed in terms of diagonal matrix coefficients (A) and all off-diagonal matrix coefficients (H(u)) multiplied by their corresponding velocities:

$$A \cdot u = H(u) - \nabla p \quad \text{Equation 2-73}$$

Subsequently, the velocity predictor $\tilde{u}$ is defined in terms of the pressure of the previous timestep:

$$\tilde{u} = A^{-1}H - A^{-1}\nabla p^{t-1} \quad \text{Equation 2-74}$$

The mixture density equation is inserted into the continuity equation:

$$\frac{\partial(\psi p)}{\partial t} + \frac{\partial \alpha}{\partial t} \left((\psi_v - \psi_l)p_{sat} - \rho_{l,0} \right) - \frac{\partial \psi}{\partial t} p_{sat} + \nabla \cdot (\rho u) = 0 \quad \text{Equation 2-75}$$

The final form of the pressure equation is derived from replacing the term $\nabla \cdot (\rho u)$ in Equation 2-75 with:

$$\nabla \cdot (\rho u) = \nabla \cdot (\rho \tilde{u}) + \nabla \cdot (\rho A^{-1} \nabla p^{t-1}) - \nabla \cdot (\rho A^{-1} \nabla p) \quad \text{Equation 2-76}$$

The pressure equation needs to be adapted to use the non-linear Dowson density-pressure relationship. The sonic velocity in general can be calculated from:

$$a = \sqrt{\left(\frac{\partial p}{\partial \rho} \right)_s} \quad \text{Equation 2-77}$$

where $\left(\frac{\partial p}{\partial \rho} \right)_s$ represent an isentropic change. Inserting Dowson pressure density equation into Equation 2-77 results in:

$$a_{l,dow} = \sqrt{\left(\frac{(5.9 \cdot 10^8 + p)^2}{2.006 \cdot 10^8} \rho_0 \right)} \quad \text{Equation 2-78}$$

The liquid compressibility is then given by:

$$\psi_{l,dow} = \frac{1}{(a_{l,dow})^2} \quad \text{Equation 2-79}$$

Temporal differentiation of Equation 2-71 results in:

$$\frac{\partial \rho}{\partial t} = \frac{\partial \alpha}{\partial t} (\rho_v - \rho_l) + \alpha \frac{\partial \rho_v}{\partial p} \frac{\partial p}{\partial t} + (1 - \alpha) \frac{\partial \rho_l}{\partial p} \frac{\partial p}{\partial t} \quad \text{Equation 2-80}$$

By inserting $\rho_v = \psi_v p$, $\rho_l = \rho_{l,dow}$ and $\psi_{l,dow}$ into the equation above:

$$\frac{\partial \rho}{\partial t} = \frac{\partial \alpha}{\partial t} (\psi_v p - \rho_{l,dow}) + \alpha \psi_v \frac{\partial p}{\partial t} + (1 - \alpha) \psi_{l,dow} \frac{\partial p}{\partial t} \quad \text{Equation 2-81}$$

The convection term in the continuity equation is expressed as in Equation 2-75 and temporal term as in Equation 2-81 together form the pressure equation:

$$\begin{aligned} \frac{\partial \alpha}{\partial t} (\psi_v p^{t-1} - \rho_{l,dow}) + \alpha \psi_v \frac{\partial p}{\partial t} + (1 - \alpha) \psi_{l,dow} \frac{\partial p}{\partial t} + \nabla \cdot (\rho \tilde{u}) \\ + \nabla \cdot (\rho A^{-1} \nabla p^{t-1}) - \nabla \cdot (\rho A^{-1} \nabla p) = 0 \end{aligned} \quad \text{Equation 2-82}$$

where $\rho_{l,dow}$ and $\psi_{l,dow}$ are evaluated at p^{t-1} .

The Euler implicit is used for discretisation of time derivatives, which is first order accurate in time, therefore discretisation error reduces with smaller time steps. The divergence terms are discretised using the upwind scheme. The gradient terms are calculated based on Gauss linear discretization. All Laplacian terms are evaluated using Gauss linear corrected.

For stability reasons, sonic velocity of vapour and density of vapour should be within a certain range using:

$$a_{v,min} = \sqrt{\left(\frac{\Delta p}{\Delta \rho}\right)_s} \quad \text{Equation 2-83}$$

where in the case of $\rho_l = 870 \text{ kg/m}^3$ maximum $\Delta \rho$ is 870. If maximum pressure occurring in a case is 1 GPa, therefore $a_{v,min} = 1072 \text{ m/s}$.

If the saturated density of vapour is set below a certain range, the singularity in pressure equation occurs which is accompanying with the Mach number ($Ma = u/a$) approaching unity in cavitating region.

According to Wallis [64], the sound velocity for homogenous flows can be derived

from:

$$\frac{1}{a^2} = (\alpha\rho_v + (1 - \alpha)\rho_l)\left(\frac{\alpha}{\rho_v a_v^2} + \frac{1 - \alpha}{\rho_l a_l^2}\right) \quad \text{Equation 2-84}$$

By assuming $\rho_l \gg \rho_v$, this can be simplified to:

$$\frac{1}{a^2} \approx \left(\frac{\alpha(1 - \alpha)\rho_l}{\rho_v a_v^2}\right) \quad \text{Equation 2-85}$$

When Equation 2-85 is maximum at $\alpha = 0.5$, minimum sonic velocity and maximum mach number occur. Replacing a with minimum allowable sonic velocity a_{min} :

$$a_{min} = \frac{u_{max}}{Ma_{max}} \quad \text{Equation 2-86}$$

where $Ma_{max} = 0.5$ is the stable upper limit of Mach number.

Therefore, the minimum saturated density of vapour can be evaluated using:

$$\rho_{v,Sat,min} = \frac{1}{4} \rho_{l,sat} \frac{a_{min}^2}{a_v^2} \quad \text{Equation 2-87}$$

2.2.2.8 Cavitation result

The domain of study described in Figure 2-10 is used for the simulation of an isothermal isoviscous compressible lubricant flow by accounting for the cavitation phenomenon in the diverging part of the contact. The boundary conditions are: zero gradient for density at all boundaries, zero gradient for pressure at the moving and fixed walls, constant 10^5 Pa for pressure at inlet and outlet, set to zero at fixed walls and calculated from fluxes normal to patch at inlet and outlet for velocity. The top wall is sliding at 1 m/s and the case parameters are given in Table 2-4. The test case is run until a steady-state solution is obtained (fully developed cavitated region observed). GAMG algorithm (with 5 coarse levels with the diagonal incomplete-Cholesky preconditioner) is used for the solution of pressure and the continuity equations. The use of GAMG algorithm is effective both on reducing computational time and increasing accuracy. The time-step criterion is of great importance in isobaric cavitation modeling and is set to 10^{-8} - 10^{-9} second depending on the case configuration. All the divergence terms are discretised using upwind (first order

accurate) scheme. The Laplacian terms are discretised using Gauss linear corrected (second order accurate) which preserve conservation and unboundedness. All gradient terms evaluated using a Gauss linear discretisation. The solution algorithm is PISO with 4 outer corrector and 8 inner corrector loops for stability reasons. The convergence residual is set to 10^{-12} for pressure and 10^{-8} for velocity and density.

Parameter	Value
Cylinder radius	$R = 10 \text{ mm}$
Minimum film thickness	$h_0 = 0.1 \text{ }\mu\text{m}$
Roelands reference pressure	$p_{r,0} = 1.98 \cdot 10^8 \text{ Pa}$
Liquid density at p_0	$\rho_{l,sat} = 870 \text{ kg/m}^3$
Roelands pressure index	$z = 0.6$
Eyring stress	$\tau_0 = 7 \cdot 10^6 \text{ Pa}$
Dynamic viscosity vapour	$\mu_v = 8.97 \cdot 10^{-6} \text{ m}^2/\text{s}$
Environment pressure	$p_0 = 10^5 \text{ Pa}$
Vapour pressure	$p_{vapour} = 3000 \text{ Pa}$
Vapour density at p_{vapour}	$\rho_{v,sat} = 0.0288 \text{ kg/m}^3$

Table 2-4. Cavitation case parameters

The vapour fraction and the density distributions along the sliding wall are shown in Figure 2-22 for steady state solution at $t=0.00024 \text{ s}$. The cavitation formation at $x = 0.023 \text{ mm}$ and reformation at $x = 0.0545 \text{ mm}$ can be seen. The density rises, due to hydrodynamic pressure build up, at the inlet and drops in the cavitating region due to vapour phase mixture effects. The density, the pressure and the vapour fraction contour plots are shown in Figure 2-23, Figure 2-24 and Figure 2-25.

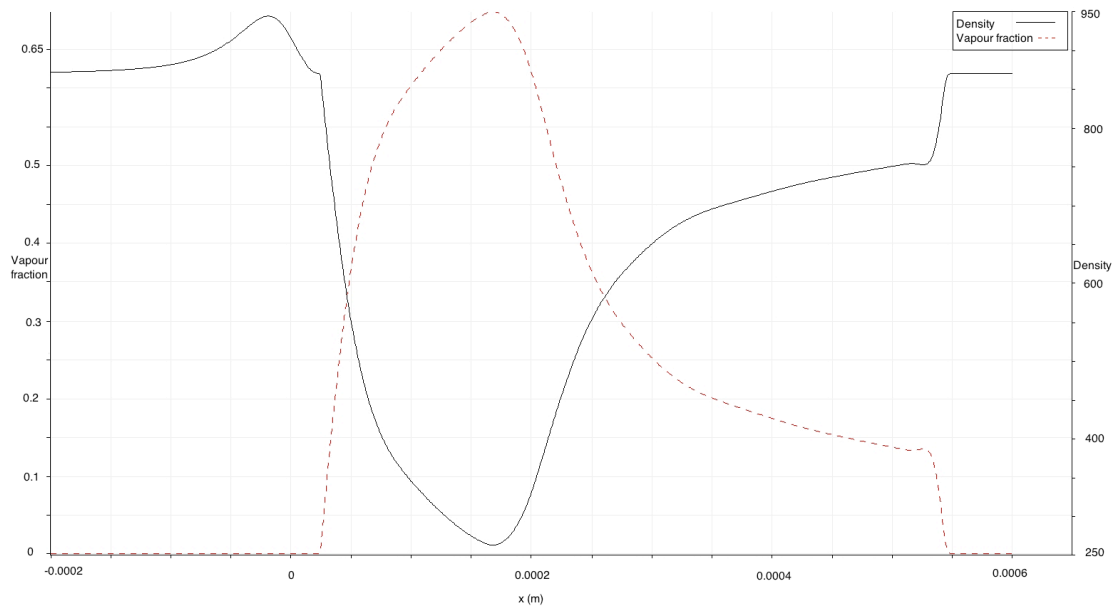


Figure 2-22. 1-D distributions of density and vapour fraction along the sliding wall

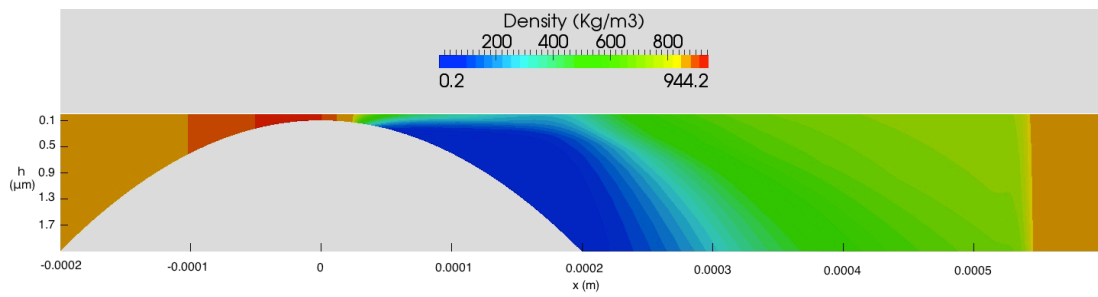


Figure 2-23. Density distribution of an isoviscous pure sliding case at 1 m/s

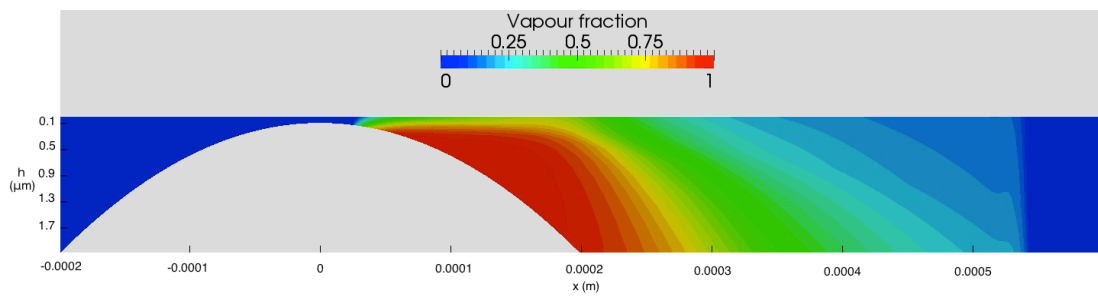


Figure 2-24. Vapour fraction distribution of an isoviscous pure sliding case at 1 m/s

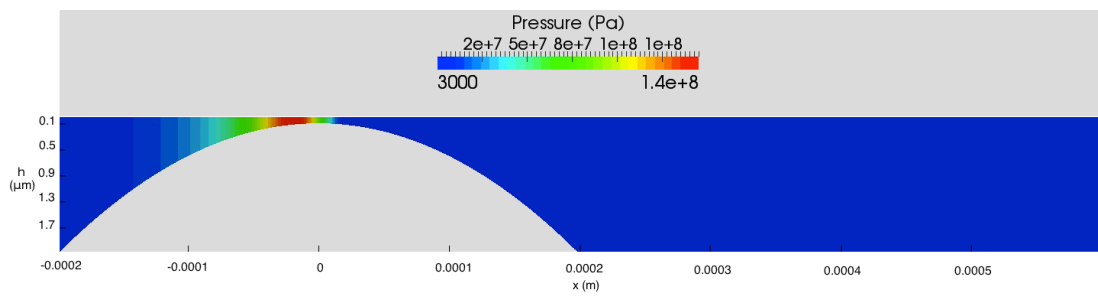


Figure 2-25. Pressure distribution of an isoviscous pure sliding case at 1 m/s

2.3 Closure

In this chapter, numerical approaches for simulation of HL contact problems are described. Fluid flow governing equations, rheology, compressibility and cavitation modelling techniques are presented. Applicability and implementation of FVM and FD methods are described for HL modelling. A comparison between the Reynolds based and a CFD approach is performed in order to investigate the limitation of the traditional methods. The cavitation treatment for a compressible case study shows the capability of CFD techniques for physically acceptable description of fluid flow behaviour. Since negative pressures are previously visualized in experimental studies by Brown [65], the isobaric cavitation treatment can be expanded for the purpose of capturing negative pressures inside the cavitating region.

Chapter 3

3.1 EHL Model

This chapter outlines a brief review of elasto-hydrodynamic lubrication modelling, including Hertz contact theory based methods, and fluid-solid interaction (FSI) solvers based on different algorithms to solve the equations governing the solid and fluid domains. The development of a finite volume solid solver based on the Navier-Lamé equation and a fully coupled finite volume thermal solver for CFD modelling of FSI with particular application to rolling element bearings is then described.

3.2 Literature review

When elastic deformations induced by the fluid pressurisation are important, *e.g.* in non-conforming contacts, elasto-hydrodynamic lubrication (EHL) takes place. The first contribution which enabled EHL modelling is by Hertz [66] who studied the contact between two sphere-shaped bodies, to identify the surface deformation due to high local pressure. The assumptions in Hertz theory are:

- The contact area dimensions are fairly small in comparison to the radii of curvature of undistorted bodies.
- The contact is frictionless and so only normal stresses exist at the interface.
- The solids are semi-infinite and in equilibrium.

His contribution allowed to relate the elastic deformation of the equivalent surface body to the fluid pressure acting on the surface. Martin [67] and Gumbel [68] applied Reynolds' equation to the lubrication of gear teeth, found that the predicted film thickness was far too small in comparison to the surface roughness. The difference to the previous studies was that, in gears, there is a non-conformal (concentrated) contact. Non-conformal contacts are considerably different with respect to conformal contact due to the high pressure that can be generated in between elements. The load is concentrated over a small contact area in a line or point contact and generates high pressures of the order of gigapascals. This was followed by Ertel [69] and Grubin et al. [70] who proposed semi-analytical solution to the Reynolds equation coupled with the Hertz (elastic deformation) and Barus (piezoviscous) formulations. Since then, EHL has entailed three key elements: the Reynolds equation, Hertz elastic deformation, and Barus viscosity dependence on pressure. Dowson et al. [71] proposed a film thickness equation as the basis of modern numerical solution of EHL which is applicable to several operating conditions. The model predicts accurate pressure distribution within the pressure ranges below 0.5 *GPa*. Brandt [72], in 1977, proposed a multi-grid technique in order to speed-up the convergence of the non-linear elliptical equations governing EHL contact problems. Evans and Sindle [73] developed refined numerical techniques to solve the Reynolds equation inversely under point contact conditions. It was possible to obtain point contact solution at very high contact loads. Following to Brandt and Evans works, Lubrecht et al. [74] were the first to develop line and point contact model using multi-grid techniques. In the presence of high loading condition, the model was still limited by numerical

instabilities. Venner's [75] work should be considered as the main contribution to multilevel solution of EHL contact problems. He was the first to suggest the use of distributive relaxation scheme to overcome numerical errors due to the localised high pressure peaks as a result of the integral formulation used for the elastic-deformation governing equations.

The vast majority of the works on HL, discussed in Chapter 2, and EHL modelling implement the Reynolds equation using finite difference method discretisation. The application of CFD methods on EHL contact problems started in the late 70s. During 80s, Blahey [76] solved a set of simplified Navier-Stokes equations using the control volume method to examine thermal effects in elliptical EHL contacts and proposed an explicitly coupled numerical model. During the 90s, Chang [77] and Chen et al. [20] continued CFD-based studies where the former used a similar solution to Blahey [76] by adding non-Newtonian effects. In particular, Chen et al. [20] investigated the capability of CFD method for steady state EHL contact problems. The geometries considered were slider bearings, step pad bearings and journal bearings. Schafer [78] used CFD approach to simulate EHL in smooth surface line contacts assuming the lubricant to be isothermal and Newtonian. The implementation of the Reynolds equation proved not to be accurate in the presence of partial or pure sliding. Almqvist [79], in 2000, developed a thermo-hydrodynamic model for lubricated thrust bearings based on CFD. The model was capable of capturing three-dimensional temperature distribution in the oil film. Viscosity and density assumed to be a function of temperature and pressure. The results are in a good agreement with theoretical and experimental data. Almqvist et al. [80] continued the application of the Navier-Stokes equations in the solution of thermal EHL line contact. Dowson-Higginson formulation was incorporated for pressure-density dependency in the contact inlet. Near to the contact outlet, where the pressure is below the specified cavitation pressure, a second order polynomial was used to interpolate density to zero. The model is stable for thermal EHL contact simulations within pressure ranges lower than 0.7 *GPa*. The incorporation of thermal effects was helpful for the prediction of EHL film behaviour at higher loads compared to the isothermal case studies due to the occurrence of lower viscosity at higher temperature. Van Odyck and Venner [81] applied the Navier-Stokes equations, neglecting inertia term, to solve EHL contact problems. Since the boundaries are non-rectangular, independent variables are transformed to curvilinear coordinates. In the exit of an EHL contact, the pressure drops below vapour pressure

and the lubricant cavitates. The difference between the two-phase cavitation model and the Reynolds cavitation model was discussed: two-phase cavitation model is preferred as it can be used inside the contact, while the latter is only applicable in the inlet and outlet regions. Large differences were found between the results from the Reynolds and the Navier-Stokes equations solution. Almqvist and Larsson [82], in 2004, compared CFD and the Reynolds equation approaches for simulation of transient EHL line contacts using multilevel techniques. Dowson-Higginson expression, see Hamrock [83], was used to model cavitation and the lubricant assumed to be Newtonian. The Boussinesq expression was employed to compute elastic deformations. They found good agreement with the Reynolds theory but they did not discuss thermal effects. The pressure solution obtained from the Reynolds equation solution was used for the film thickness calculation. By comparing both approaches result, the deviation in pressure is approximately 10-20 times higher in comparison with the deviation in film thickness. This is also the case for numerical errors in each approach. Almqvist and Larsson [84], in 2008, modified a commercial code (CFX4) to simulate thermal, transient, rough EHL line contacts. The surface roughness is superimposed with surface irregularities using a cosine wave profile (ridge) which is propagating along the contact surface. Ree-Eyring rheology model is employed and transient study has been carried out later on. The highest-pressure peak was about 1.5 *GPa*. It was shown that Eyring stress has significant effect on the thermal and rheological behaviours. Their results showed that surface roughness could highly affect the film thickness and pressure distribution. Also in 2007, Hartinger [85] modelled EHL contact in roller bearing considering thermal and viscous effects along with deflection of the solid. Viscosity and density were treated as functions of temperature and pressure. The cavitation models tested in this study are isentropic and isobaric, where the former follows the approach in Schmidt [62]. The isobaric model is based on the principle that it tries to maintain the specified cavitation pressure p_{sat} inside the cavitating region (if the pressure rises above p_{sat} vapour is converted into liquid). The deformation of the solid bodies was evaluated using the Hertzian contact theory and it was assumed, in the deflection algorithm, that the surface points are moving only in one direction and that movement causes deformation of the internal mesh. Finally, cavitation and elastic deflection models were coupled for different ratios of rolling/sliding values, considering both thermal and isothermal cases where it was found that the solution is more stable in thermal

cases at high speeds. Temperature impact on friction force shows difference of up to 88.5% compared to isothermal condition.

Bruyere et al. [86] used Finite Element method to solve Navier-Stokes equations coupled with elastic deformation. They employed non-Newtonian rheology model along with compressibility and thermal effects. The results were in a good agreement (up to 0.7 GPa) with the Reynolds solution and Hartinger's model. Film thickness and pressure profiles for different SRR are compared and the dimple formation is shown for SRR greater than 4. The viscosity, temperature and pressure gradients through film thickness are found for $SRR=\infty$.

3.3 Solution for Solid Domain

Most EHL simulations involve a combination of finite volume (FV) or finite difference (FD) solver for the fluid phase coupled with an analytical equation or a finite element (FE) solver for the solid domain. While the modelling of the fluid has been dealt with in the previous chapter, the solid solver modelling is presented in the following section.

The key features of using FEM can be summarised as:

- (i) Use of pre-defined shape functions dependent on the topology of the element.
- (ii) It is easily extendable to higher order discretisation, large block-matrices.
- (iii) Based on direct solvers. The current model presents a new approach where FVM is used for the discretisation of both solid and fluid domains in three-dimensions. FVM is characterised by second order accuracy and is based on the integral form of the governing equations; furthermore, the coupling and non-linearity can be treated by the use of a segregated solution procedures and suitable diagonally dominant matrices can be generated for iterative solvers.

The motivations for the use of FVM for stress analysis are:

- The advantage of using a single numerical tool for multiple domains, which incorporate computationally less expensive solver in performing iterations and enables an internal transfer of information at the fluid-solid interface through a single, common data structure.

- The capability to use complex and full-size geometries, which is necessary for industrial applications modelling.
- The implementation of non-linear and complex mathematical models for accurate prediction of physical phenomena.
- The mesh size can be optimized based on the solution time. Therefore, the results are in a finer mesh for a faster solving FVM with higher accuracy and reduced numerical diffusion.

3.3.1 Solid deformation

The developed model can accommodate complex laws to describe different solid behaviour, including the transition from elastic to plastic response of the solid phase. In this study, the solid solver incorporates the linear elastic equation (moderate stresses and strains) using an iterative segregated approach and a Lagrangian formulation. The numerical implementation in OpenFoam is based on the linear structural equations presented by Jasak [87].

The force balance of a solid element reads:

$$\frac{\partial^2(\rho_s v)}{\partial t^2} - \nabla \cdot \sigma = \rho_s f_b \quad \text{Equation 3-1}$$

where v is the displacement vector, ρ_s is the solid density, f_b is the body force and σ is the stress tensor.

The plane strain assumption is applied since line contact cases are studied in this work. The strain tensor is defined in terms of v :

$$\varepsilon = \frac{1}{2}(\nabla v + \nabla v^T) \quad \text{Equation 3-2}$$

The stress and strain tensor relation closes the system of equations (Hooke's law):

$$\sigma = 2\mu_s \varepsilon + \lambda_s \text{tr}(\nabla v) I \quad \text{Equation 3-3}$$

where I is the unit tensor and μ and λ (for plain strain and 3-D) are Lamé's coefficients, relating to Young's modulus of elasticity, E , and the Poisson's ratio, ν , as:

$$\mu_s = \frac{E}{2(1+\nu)} \quad \text{Equation 3-4}$$

$$\lambda_s = \frac{\nu E}{(1+\nu)(1-2\nu)} \quad \text{Equation 3-5}$$

Equation 3-1 can be re-written as:

$$\frac{\partial^2(\rho_s v)}{\partial t^2} - \nabla \cdot [\mu_s \nabla v + \mu_s (\nabla v)^T + (\lambda_s I \text{tr}(\nabla v))] = \rho_s f_b \quad \text{Equation 3-6}$$

Two neighbouring control volumes (CV) are shown in Figure 3-1 where the computational points (P and N) are in their centroid, the internal face f is shared only between two CVs and S is the face area vector.

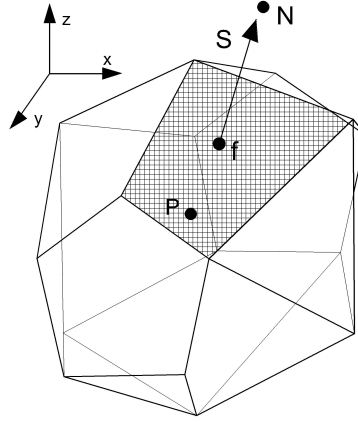


Figure 3-1. Control volume

The equations are discretized in the integral form over the CV and solved in a segregated manner, where each component of the displacement vector is solved separately and the inter-component coupling is treated explicitly. Segregation enables partial convergence and memory use optimization through solving three smaller matrices consecutively rather than a large matrix consisting of all the three components of displacement. The convergence is achieved when the residuals of all equations drop below a prescribed level.

In order to perform FVM discretisation, Gauss's theorem is applied to the integrated form of Equation 3-6 over the CV associated to point P and characterised by volume V_P :

$$\int_{V_P} \frac{\partial^2(\rho_s v)}{\partial t^2} dV = \oint_{\partial V_P} ds \cdot [\mu_s \nabla v + \mu_s (\nabla v)^T + \lambda_s I \text{tr}(\nabla v)] + \int_{V_P} \rho_s f_b dV \quad \text{Equation 3-7}$$

The first term on the left hand side is discretised using first-order scheme (two old-time levels of v). The temporal derivative is calculated as:

$$\frac{\partial^2 v}{\partial t^2} = \frac{v^n - 2v^0 + v^{00}}{\Delta t^2} \quad \text{Equation 3-8}$$

where $v^n = v(t + \Delta t)$, $v^0 = v(t)$, $v^{00} = v(t - \Delta t)$. It preserves boundedness and is

first order accurate in time. The use of second-order time accurate operator causes unphysical stress peaks due to un-boundedness.

The volume integrals are evaluated through the mid-point rule:

$$\int_{V_P} \phi dV = \phi_P V_P \quad \text{Equation 3-9}$$

The surface integrals of Equation 3-7 are evaluated using the sum of integrals over the cell faces. For instance, Divergence-Gradient term is discretised as:

$$\int_{V_P} \nabla \cdot (\mu \nabla v) dV = \oint_{\partial V_P} ds \cdot (\mu \nabla v) = \sum_f \mu_f s_f \cdot (\nabla v)_f \quad \text{Equation 3-10}$$

The implicit and explicit discretisation and the gradient evaluation for FVM are the same as discussed in Chapter 2.

The resulting matrices are sparse diagonally dominant and the system of algebraic equation on a single cell is given by:

$$a_P v_P + \sum_N a_N v_N = b_P \quad \text{Equation 3-11}$$

where a_P is a diagonal coefficient that includes contributions from the transient and diffusive terms, a_N is an off-diagonal coefficient generated by the diffusion term and b_P is the right-hand side coefficient that depends on v values from previous time step and previous iteration.

Since v_P depends on the values of the neighbouring cell, Equation 3-11 can be written as a system of algebraic equations:

$$[A_d][v] = [r] \quad \text{Equation 3-12}$$

where A_d is the sparse matrix (diagonally dominant and symmetric) with coefficients a_P on the diagonal and coefficients a_N off the diagonal, $[v]$ is vector of v s consisting of displacement increments for all the CVs and $[r]$ is the right hand side vector consisting of source terms for all CVs. The boundary conditions can be of the following types:

- Fixed displacement (prescribed displacement value)
- Traction displacement where the boundary condition specifies the surface traction, $t_f = \hat{n} \cdot \sigma$, on the boundary face. This can include the specification of both pressure and tangential tractions (e.g. frictional terms). For numerical purposes, a traction boundary specifies a fixed gradient on displacement vector:

$$\hat{n} \cdot \nabla v = \frac{t_f - \hat{n} \cdot [\mu_s (\nabla v)^T - (\mu_s + \lambda_s) \nabla v] - \hat{n} \lambda_s \nabla \cdot v}{2\mu_s + \lambda_s} \quad \text{Equation 3-13}$$

where $\hat{n}$ is the outward pointing boundary face unit vector.

The system of equations is solved by using a segregated algorithm where three components of displacement vector increment are solved separately from each other. The matrix A_d is symmetric and diagonally dominant and is solved with a conjugate gradient Cholesky preconditioned solver for accuracy and computational time efficiency purposes.

3.3.2 Energy equation

The dimensionless parameter describing conduction vs. convection is the Peclet number:

$$Pe = \frac{Du}{\alpha_T} \quad \text{Equation 3-14}$$

where D is the characteristic length scale, u is the velocity and α_T is the thermal diffusivity of the solid defined as:

$$\alpha_T = \frac{k_s}{\rho_s C_{s,v}} \quad \text{Equation 3-15}$$

For a solid element passing against a heat source, there are two extremes: When $Pe \gg 1$, the temperature field in the solid is dominated by convection. At the other extreme, for $Pe \ll 1$, the conduction term is dominant.

Several studies on EHL contact problems rely on the use of Carslaw and Jaeger equation [88] for solid thermal behaviour, which is limited to certain range of Peclet numbers ($Pe > 5$).

For the complete treatment of the solid domain, thermal effects and heat conduction need to be incorporated into the solver. This is achieved by implementing an algorithm which treats the energy equation. The energy conservation for a solid cell can be written as [88]:

$$\rho_s C_{s,v} \frac{\partial T}{\partial t} = \nabla \cdot (k_s \nabla T) - \rho_s C_{s,v} \dot{v} \cdot \nabla T + \frac{\partial k_s}{\partial T} (\nabla \cdot T)^2 \quad \text{Equation 3-16}$$

where $\dot{v}$ is the velocity of solid cell. For the presented cases in this work, the solid thermal conductivity is assumed to be constant and therefore the last term on the right hand side of Equation 3-16 is neglected. The integrated form of Equation 3-16 can be written as:

$$\int_{V_P} \rho_s C_{s,v} \frac{\partial T}{\partial t} dV = \oint_{\partial V_P} ds \cdot (k_s \nabla T) - \int_{V_P} (\rho_s C_{s,v} \dot{v} \cdot \nabla T) dV \quad \text{Equation 3-17}$$

The Laplacian term (first term on the right hand side) is discretised using Gauss linear

corrected scheme. The gradient terms are calculated based on Gauss linear discretisation. The Euler implicit scheme is used for discretisation of time derivatives (see Chapter 2). The resulting matrix is solved using the PCG (preconditioned conjugate gradient) solver [89] for symmetric matrices, with DIC (diagonal incomplete-Cholesky) pre-conditioner.

3.3.3 Load balance

The traction forces, exerted by the lubricant, load the solid elements at the interface. In order to mimic EHL experimental configurations, where applied load is prescribed rather than a fixed geometry, the pressure distribution force is evaluated numerically at the end of each time step to compare the current load (L), carried by the EHL film, with the target load (L_{aim}). The current load is calculated through the integral form of pressure distribution acting on the deformed surface.

The rigid displacement increment prescribed at the remote boundaries of the solid body to achieve the prescribed load reads from:

$$\Delta h_d = (v_{max} - v_{min}) \frac{L_{aim} - L}{L_{aim}} \Delta t_d r_d \quad \text{Equation 3-18}$$

where v_{max} and v_{min} are the maximum and minimum deflections in the solid domain and r_d is the under-relaxation factor for deflection. A characteristic deformation time is defined to introduce a dependency on time-step and the solid body size:

$$\Delta t_d = \Delta t \frac{R}{a_s} \quad \text{Equation 3-19}$$

where R is the radius (for the case studied here) or another characteristic length of the solid and a_s is the sonic velocity of the body (taken as 5000 m/s for the material under examination in this thesis). For stability reasons, Δh_d is limited to the prescribed maximum velocity of the solid body 2.10^{-3} m/s. The final form of load balance equation can be written as:

$$\Delta h_{Final} = \begin{cases} \min[2.10^{-3} \Delta t, |\Delta h_d|] & \Delta h_d > 0 \\ -\min[2.10^{-3} \Delta t, |\Delta h_d|] & \Delta h_d < 0 \end{cases} \quad \text{Equation 3-20}$$

3.4 Fluid Solid Interaction (FSI)

An FSI model is used to describe a dynamic system influenced by the interaction of a moving fluid and a deforming solid. The balance between stability, generality and

programming effort parameters for different FSI coupling methods is shown in Figure 3-2.

There are generally two main approaches for FSI problems:

- The monolithic approach which involves a simultaneous solution of fluid and solid governing equations in a single solver (see Figure 3-2 (c)). This approach is suitable for very strong fluid-solid interaction problems due to stability and convergence properties. However, a single equation system may lead to ill-conditioned matrices with zero entries on the diagonal in some cases.
- The partitioned approach which solves the governing equations in two distinct solvers. It requires communication between two solvers at the interfaces using one-way or two-way coupling algorithms. Data transfer in one-way coupling algorithms is unidirectional. Two-way coupling can be divided into explicit and implicit methods. Data are exchanged only once per time-step for explicit schemes (see Figure 3-2 (a)), while the implicit method allows for several FSI iterations within each time-step (see Figure 3-2 (b)). Using an additional iteration loop over the partial solvers can extensively improve the coupling stability.

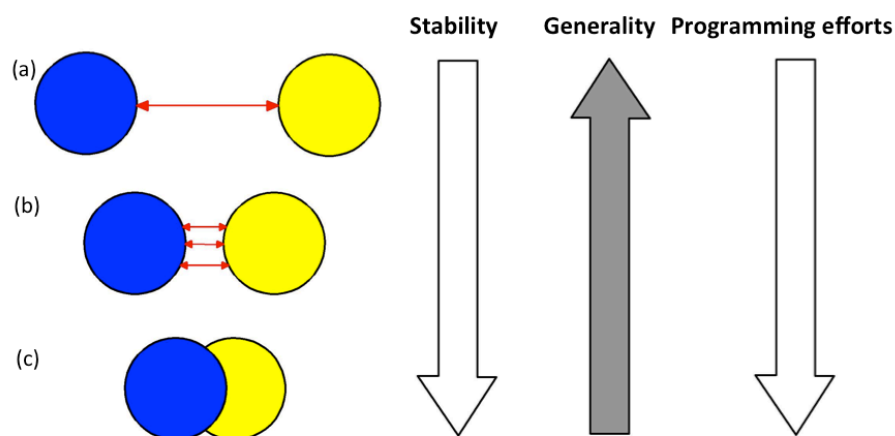


Figure 3-2. Coupling approaches for FSI (a) weak (b) strong (inner iteration) (c) one system of equations

In an EHL contact problem, the most important aspect of FSI analysis is the coupling of the fluid and solid components, which requires the following aspects to be considered:

- Deformed shape of the domain (interface position) and internal mesh movement linked to the elastic deflections;
- Moving mesh governing equations;
- Heat transfer between the fluid and the solid domains;
- General Grid Interface (GGI) interpolation;
- FSI algorithm.

3.4.1 Automatic mesh motion

The mesh motion can be categorised into boundary motion and internal point movement. In this study, boundary motion (updated domain shape) can be evaluated from the solid body elastic deformation. The objective of internal node motion is to conform to boundary motion while preserving the validity and quality of the mesh. The following requirements are necessary for an automatic mesh motion solver:

- The method must be vertex-based to avoid interpolations;
- The resulting matrices from discretisation should be diagonally dominant;
- None of the tetrahedral or triangular elements should be inverted.

The validity and quality of the mesh from FVM point of view is briefly summarised in the following. The first assumption is the existence of a topologically and geometrically valid mesh as a starting point. The required validity checks for the updated mesh can be categorised into topological and geometrical tests. Topological validity of mesh definition tests for a face-addressed FVM mesh are:

- A point can appear in a face only once.
- A face can appear in a cell only once and can belong to maximum two cells and only one patch.
- Two cells cannot share more than one face.

The following tests are necessary to make sure that all cells and boundary hull are topologically closed:

- Decomposing cells into edges: Every edge must appear in two cell faces.
- Decomposing boundary faces into edges: Every edge must appear in exactly two boundary faces.

The geometrical tests mainly check the positivity of face areas and cell volumes plus convexity and orientation requirements.

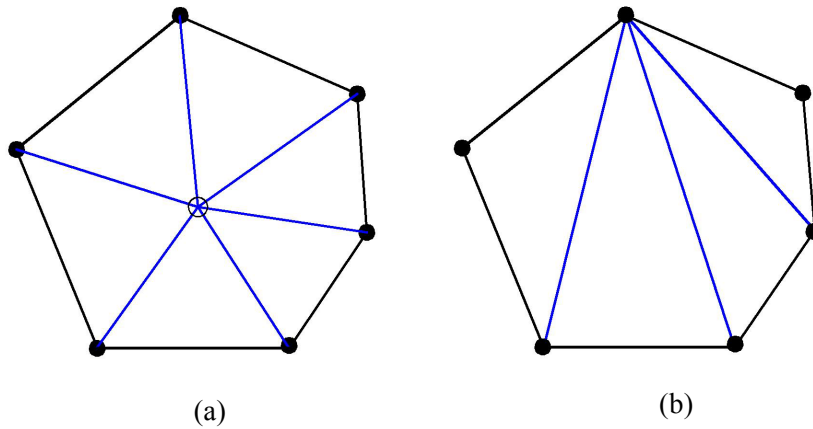


Figure 3-3. Decomposing the face into triangles (a) using face centroid (b) using internal edges

Geometrical measures (face area, normal vector, face and cell centroid, volume change by the dynamic face) are calculated using a decomposed face. A polygon face can be decomposed into triangles using face centroid node or internal edges (see Figure 3-3). A face is convex if all triangles normals point in the same direction. For a cell to be geometrically closed, the sum of outward-pointing face area vectors for the faces of a cell must be zero (relative to prescribed tolerance).

The most widely used automatic mesh motion models adopted in FV models are the cell based motion and the spring analogy. The deficiency of cell-based methods is the necessity to interpolate from cell centre values to the edge points. Moreover, motion of corner points cannot be constructed reliably as they only belong to one cell. The spring analogy consists of all edges in the mesh being replaced by elastic spring loaded with boundary motion. The simplest mode of failure is when two edges of a triangular cell degenerate into a line (see Figure 3-4).

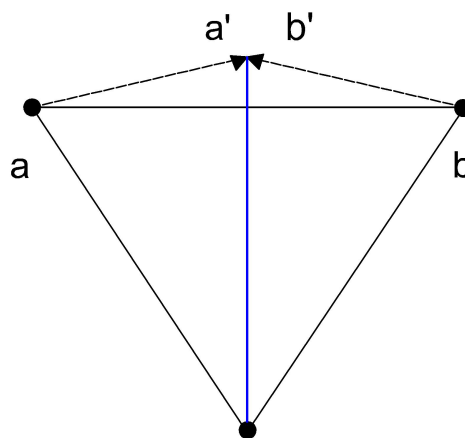


Figure 3-4. Failure mode for coincident points

The above-mentioned methods have been proven to be weak particularly for arbitrarily unstructured meshes that are common in FV simulations [90].

The use of tetrahedral finite volumes for a Laplacian operator results in diagonally dominant matrices and second-order discretisation [91]. This will be allowing the use of iterative solvers with bounded motion variable irrespective of mesh quality. The choice of cell decompositions from polyhedron into tetrahedral are (see Figure 3-5):

- (a) Cell decomposition: An additional point is introduced only in the cell centre;
- (b) Cell-and-face decomposition: Additional points are introduced at faces and cell centres.

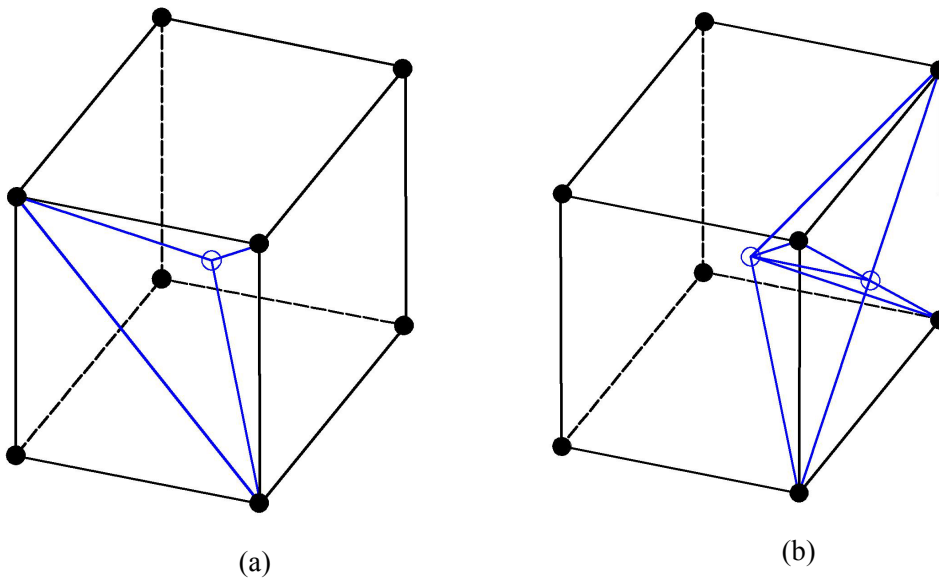


Figure 3-5. Decomposing a polyhedral face into tetrahedral (a) cell decomposition (b) cell and face decomposition

The choice of decomposition method is a balance between quality of the resulting tetrahedral and the computational cost. In this study, cell decomposition is used to increase computational efficiency.

The current method is devised for arbitrary un-structured polyhedral mesh composition. A vertex-based mesh motion solver is adopted in the current model which is similar to the FE dynamic mesh motion solver proposed by Jasak [91]. This is based on the Laplacian operator, which is a perfect mesh motion solver as it is always bounded. The Laplace operator with distance based diffusion field (γ) is chosen to govern the mesh motion:

$$\nabla \cdot (\gamma \nabla u) = 0 \quad \text{Equation 3-21}$$

where u is the point velocity field used to modify point positions:

$$x_{new} = x_{old} + u\Delta t \quad \text{Equation 3-22}$$

where x_{old} and x_{new} are the point positions before and after mesh motion. The mesh updates according to displacement increment.

To avoid the largest mesh motion happens near to the moving boundary a variable diffusivity is used to confine local deterioration. A linear distance based method is implemented where the diffusion field γ is a function of cell centre distance, l , to the nearest prescribed boundary patches.

$$\gamma = \frac{1}{l} \quad \text{Equation 3-23}$$

Equation 3-21 is discretised over the tetrahedral decomposition using a FE second-order accurate method. OpenFOAM architecture allows a separate FEM implementation of tetrahedral discretisation from the rest of the code. The FEM adaptive handling parts are incorporated through tetMatrices, tetPointFields and faceTetPolyhedral libraries in OpenFOAM. The resulted matrix from discretisation of Equation 3-21 is solved using an iterative linear equation solver Incomplete Cholesky preconditioned Conjugate Gradient (ICCG) [89]. The boundary condition is enforced from the known boundary motion through traction forces induced on cells at their mutual boundary.

3.4.2 FVM for moving meshes

A general form of the transport equation is modified to accommodate the effects of moving cell's faces. In the case of moving mesh, the control volume is no longer fixed in space and its motion is captured through the bounding surface velocity.

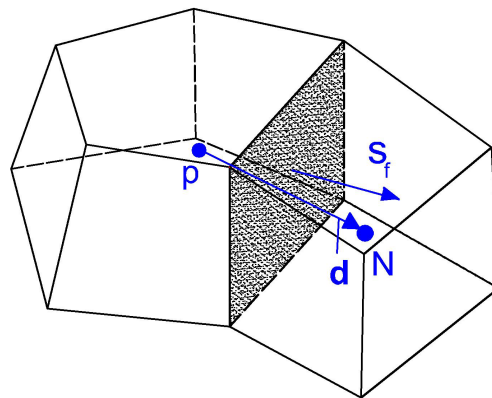


Figure3-6. Two neighbouring control volumes

The transport equation (Equation 2-50) for an arbitrary moving volume V (see Figure 3-6) bounded by a surface S can be re-formulated as:

$$\frac{\partial}{\partial t} \int_{V_p} \rho \phi dV + \oint_S ds. \rho(u - u_b) \phi - \oint_S ds. (\rho \Gamma_\phi \nabla \phi) = \int_{V_p} S_\phi(\phi) dV \quad \text{Equation 3-24}$$

where u_b is the boundary (bounding surface S) velocity. The differences compared to static mesh are:

1. The temporal derivative which introduces the rate of change of cell volume:

$$\frac{\partial}{\partial t} \int_{V_p} dV \quad \text{Equation 3-25}$$

2. The mesh motion flux which accounts for the grid convection:

$$\oint_S n. u_b dS \quad \text{Equation 3-26}$$

where n is the outward pointing unit normal to the surface S. The space conservation law governing the relationship between Equations 3-25 and 3-26 reads:

$$\frac{\partial}{\partial t} \int_{V_p} dV - \oint_S n. u_b dS = 0 \quad \text{Equation 3-27}$$

The discretised form of Equation 3-27 needs to be preserved:

$$\frac{V_p^n - V_p^0}{\Delta t} - \sum_f F_s = 0 \quad \text{Equation 3-28}$$

The mesh motion flux (F_s) is calculated using the volume change associated with the face (f) motion during the current time-step rather than from the grid velocity u_b , therefore making it consistent with the cell volume calculation.

3.4.3 Heat transfer

A multi-region coupling strategy is implemented for the thermal modelling of the fluid-solid interaction algorithm. By the use of a similar decomposition method, both meshes decompose at the same coordinate location. The fluid temperature equation is solved subject to the appropriate Dirichlet boundary condition ($T_f = T_s$) at the coupled interface (see Figure 3-7). Subsequently, the solid temperature equation is solved subject to Neumann boundary condition $q_s'' = q_f''$ at the coupled interface. This enforces flux matching at the mutual boundary using interpolated patch fluxes. This is also known as Dirichlet-Neumann partitioning.

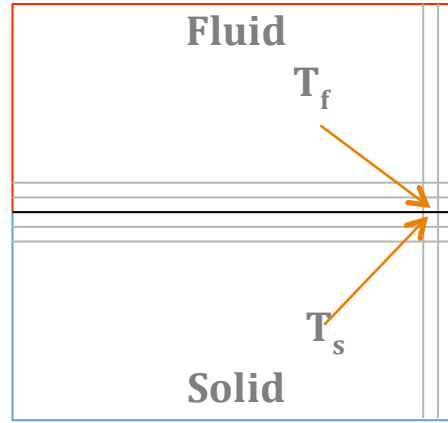


Figure 3-7. Fluid-Solid heat conduction

Now, the energy equations need to be solved in each region multiple times to obtain a fully coupled solution (final thermal convergence). An inverse distance weighted interpolation scheme can be used for temperature and heat fluxes. However, this method is only appropriate in the case of conformal meshes. Since non-conformal meshes are implemented in this study, the following section explains the implemented coupling of multiple regions into a single contiguous domain at matrix level.

3.4.4 General Grid Interface (GGI) interpolation

Implicit coupling interfaces are mainly designed to join conformal mesh (see Figure 3-8 (a)) regions where the patch nodes are matching one by one at the interface. In the case of non-conformal meshes, a conservative scheme such as GGI (General Grid Interface), an existing interpolation scheme available in OpenFOAM extended version, must be used. The advantage of GGI is that you can solve problems which involve boundary motion where different parts of the mesh are sliding relative to each other without having to wade through all the problems of mesh quality, mesh deformations, continuity of mesh regions, sliding interfaces etc. The GGI method helps to properly balance fluxes at the interface.

For a quick overview, each GGI pair consists of a master and a slave patch. The interpolation weights across the interface for each face of the master patch are calculated based on which faces lie on the shadow side of that face, and how much area the master face covers of each of those slave faces. For instance, assuming the yellow interface boundary patches (Figure 3-8 (b)) as master, each master consists of 1 complete and two partial slave patches. The weights are always calculated relative

to the master. When the time comes to interpolate the fields, the value of the internal field on one side is transferred to the other side after multiplying by the weights.

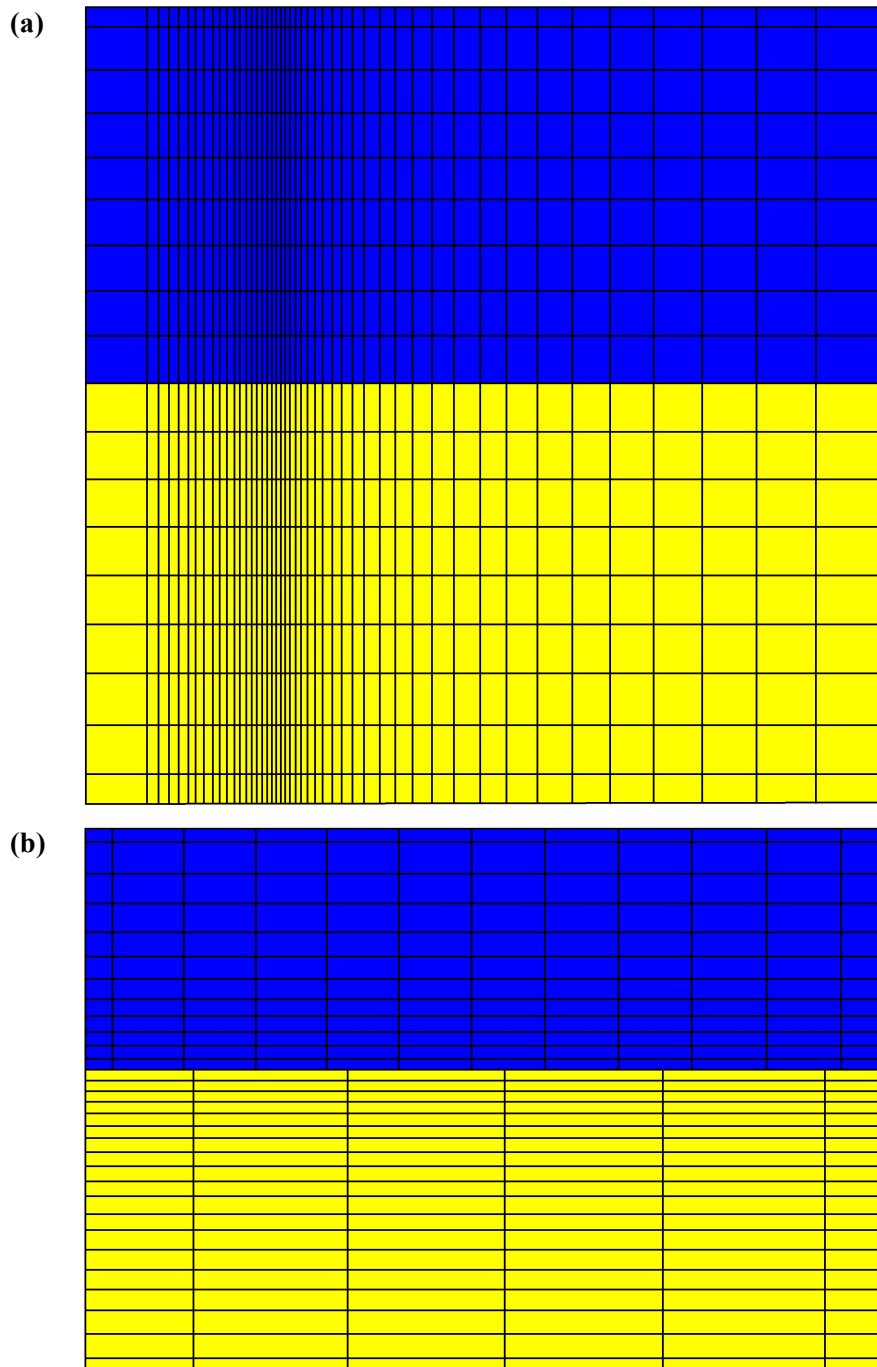


Figure 3-8. (a) Conformal and (b) non-conformal meshes.

Using weighted interpolation, the flow values between the GGI master patch to the GGI slave patch can be written as:

$$\phi_{S_i} = \sum_n \omega_{M_n \rightarrow S_i} \phi_{M_n} \quad \text{Equation 3-29}$$

For flow values from the slave patch to the master patch:

$$\phi_{M_j} = \sum_m \omega_{S_m \rightarrow M_j} \phi_{S_m} \quad \text{Equation 3-30}$$

where

ϕ_S : Slave patch variable

ϕ_M : Master patch variable

i : i^{th} slave patch face

j : j^{th} master patch face

n : number of master face neighbours for slave patch i

m : number of slave face neighbours for master patch j

$\omega_{M \rightarrow S}$: Master facet to slave facets weighting factor

$\omega_{S \rightarrow M}$: Slave facet to master facets weighting factor

In order for the interface discretisation to remain conservative, the following three conditions should be satisfied. For conservation, the sum of all the weights for a given face adds up to 1.0.

$$\sum_n \omega_{M_n \rightarrow S_i} = 1 \quad \text{Equation 3-31}$$

$$\sum_m \omega_{S_m \rightarrow M_j} = 1 \quad \text{Equation 3-32}$$

Perceived facet area must be the same:

$$\omega_{M \rightarrow S} |S_M| = \omega_{S \rightarrow M} |S_S| = |S_{\cap M-to-S}| \quad \text{Equation 3-33}$$

where

$|S_M|$: surface area of master facet

$|S_S|$: surface area of slave facet

$|S_{\cap M-to-S}|$: intersection surface area between master and slave facets.

However, in general:

$$\omega_{M \rightarrow S} \neq \omega_{S \rightarrow M} \quad \text{Equation 3-34}$$

In order to finalise the GGI interpolation scheme, a robust and precise algorithm needs to be developed to evaluate weighting factors for both master and shadow patch faces and determine the number of neighbours for each facet.

The weighing factors, which are basically the percentage of surface intersection between two overlapping faces, can be deduced from:

$$\omega_{M \rightarrow S_i} = \frac{|S_{\cap M \rightarrow S_i}|}{|S_{M_n}|} \quad \omega_{M \rightarrow S_i} \in]0, 1] \quad \text{Equation 3-35}$$

$$\omega_{S \rightarrow M_j} = \frac{|S_{\cap S \rightarrow M_j}|}{|S_{S_m}|} \quad \omega_{S \rightarrow M_j} \in]0, 1] \quad \text{Equation 3-36}$$

$|S_{\cap M \rightarrow S_i}|$ and $|S_{\cap S \rightarrow M_j}|$: Surface intersection area between a master and a slave patch faces

$|S_{S_m}|$ and $|S_{M_n}|$: Surface area of a master and slave patch face

i : i^{th} slave patch face for a given master patch face

j : j^{th} master patch face for a given slave patch face

The GGI implementation in OpenFOAM uses Sutherland-Hodgman algorithm for computing the master and the slave face intersection surface area. An Axis Aligned Bounding Box (AABB) algorithm is implemented in order to determine neighbouring faces [92].

3.4.5 FSI solver algorithm

Based on data exchange, the methods for solving FSI problems can be divided into weakly and strongly coupled algorithm. For weak FSI problems, solvers for fluid and solid are applied sequentially only once per time step (see Figure 3-9). Firstly, the fluid field is solved at the current time-step and the forces, induced by the fluid, are applied as boundary conditions for the solid solver. The new structural position is considered in the next time-step. Therefore, this staggered solution procedure can be considered as an explicit coupling method. There is a potential time lag between fluid and solid solution for simulation with large time-steps. Therefore, due to the explicit nature of weak coupling method convergence problem may arise.

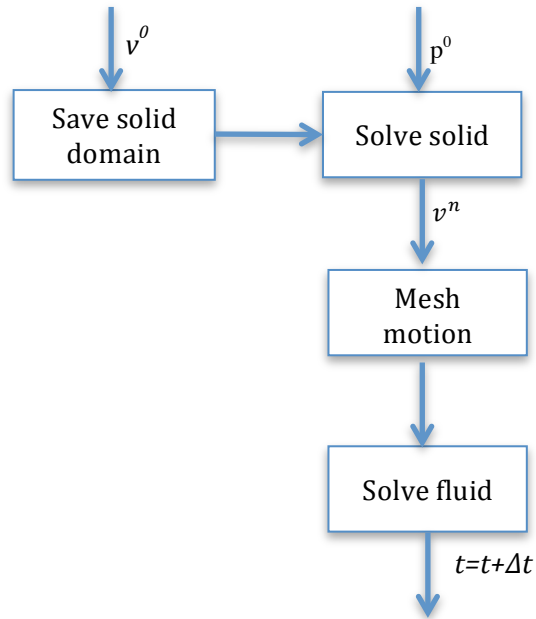


Figure 3-9. Weak FSI algorithm

Instability occurs when weak coupling is used on FSI problems with incompressible fluid and/or light structures. For strong FSI problems, a fully coupled algorithm for solid displacement can be implemented. In each time step, solid and fluid solvers progress together with the mesh motion solver until the convergence is reached for all variables (see Figure 3-10).

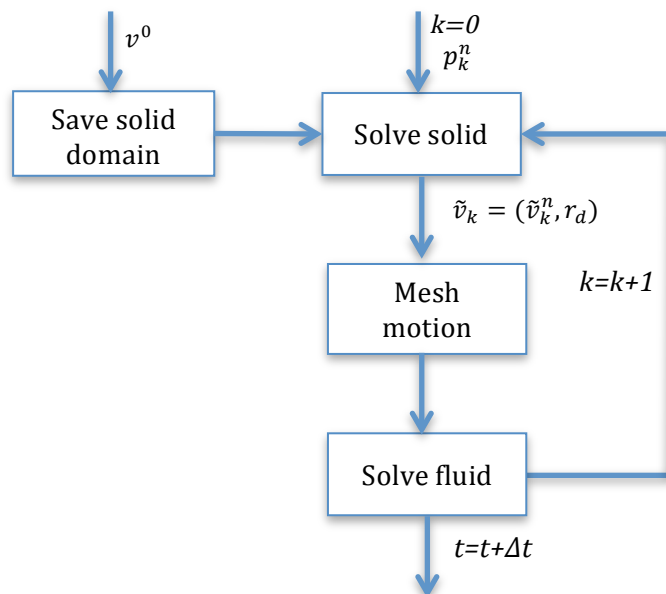


Figure 3-10. Strong FSI algorithm

Furthermore, it has been observed that very strong interactions require solution under-relaxation that reduces significantly the computational efficiency of the solver.

The following criteria are essentially effective on an FSI problem's mode selection:

- Time step size
- Solid-fluid density ratio
- Fluid viscosity and incompressibility
- Solid stiffness

The current model implements a semi-implicit fully coupled method, which involves sub-iteration at each time step to ensure that the fluid-solid interaction is converged. The variables are transferred between the fluid and solid surface using patch-to-patch interpolation. The temperature, pressure and viscous force increments at the fluid side of the interface are transferred to the solid side. Displacement increment (v), temperature (T_s), heat flux and velocity (u_s) at the solid side of the interface are transferred to the fluid side of the interface (see Figure 3-11).

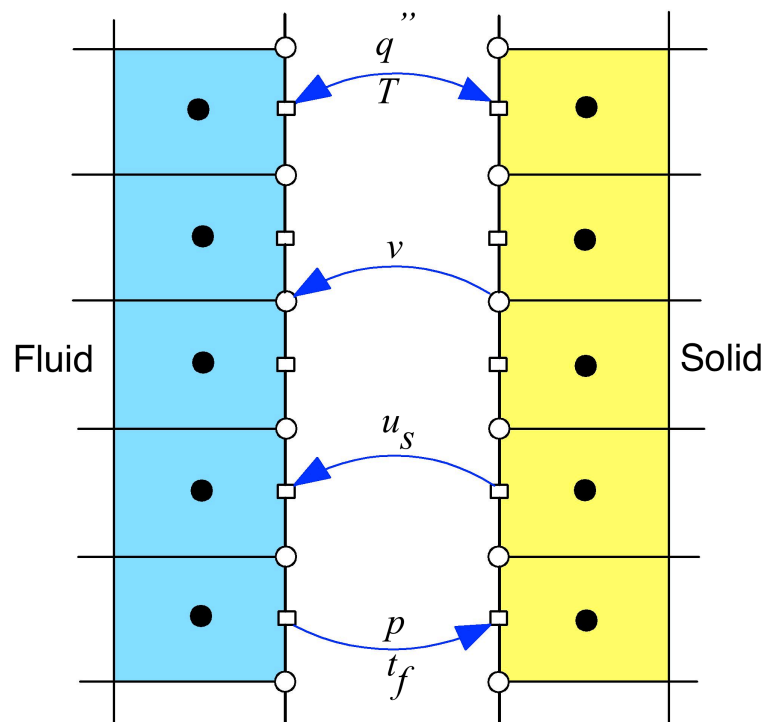


Figure 3-11. Data transfer between fluid and solid

The initialization of the solver in Reynolds-based approaches is normally done with dry Hertzian contact solution. However, the current cavitation model becomes unstable with prescribed deflection initialization. In the presented model, solid and fluid bodies are separated by an initial gap ($0.1 \mu m$ was found to be numerically stable

for most of the simulations presented in the following chapters). The bodies are pressed together, using Equation 3-20, until the target load is reached.

The fluid domain is solved using an algorithm similar to the Pressure-Implicit Split-Operator (PISO) [93]. The overall procedure of the fluid-solid structure solver is illustrated in Figure 3-12. The first and second inner loops are dedicated to fully coupled FSI calculations and implicit thermal solvers. All fluid governing equations, continuity, temperature and pressure equations are solved using GAMG matrix solver. The GAMG solver can often be the optimal choice, particularly for solving the pressure equation. GAMG uses the principle of generating a quick solution on a mesh with a small number of cells; mapping this solution onto a finer mesh and using it as an initial guess to obtain an accurate solution on the fine mesh. The aim is to reduce computational time compared to standard methods by solving first on coarser meshes: this outweighs the additional costs of mesh refinement and mapping of field data. The switching from a coarse mesh to the fine mesh is handled by agglomeration of cells; this is performed either by a geometric agglomeration, where cells are joined together, or by an algebraic agglomeration, where matrix coefficients are joined. Solid governing equations are solved using the Cholesky preconditioned conjugate gradient solver.

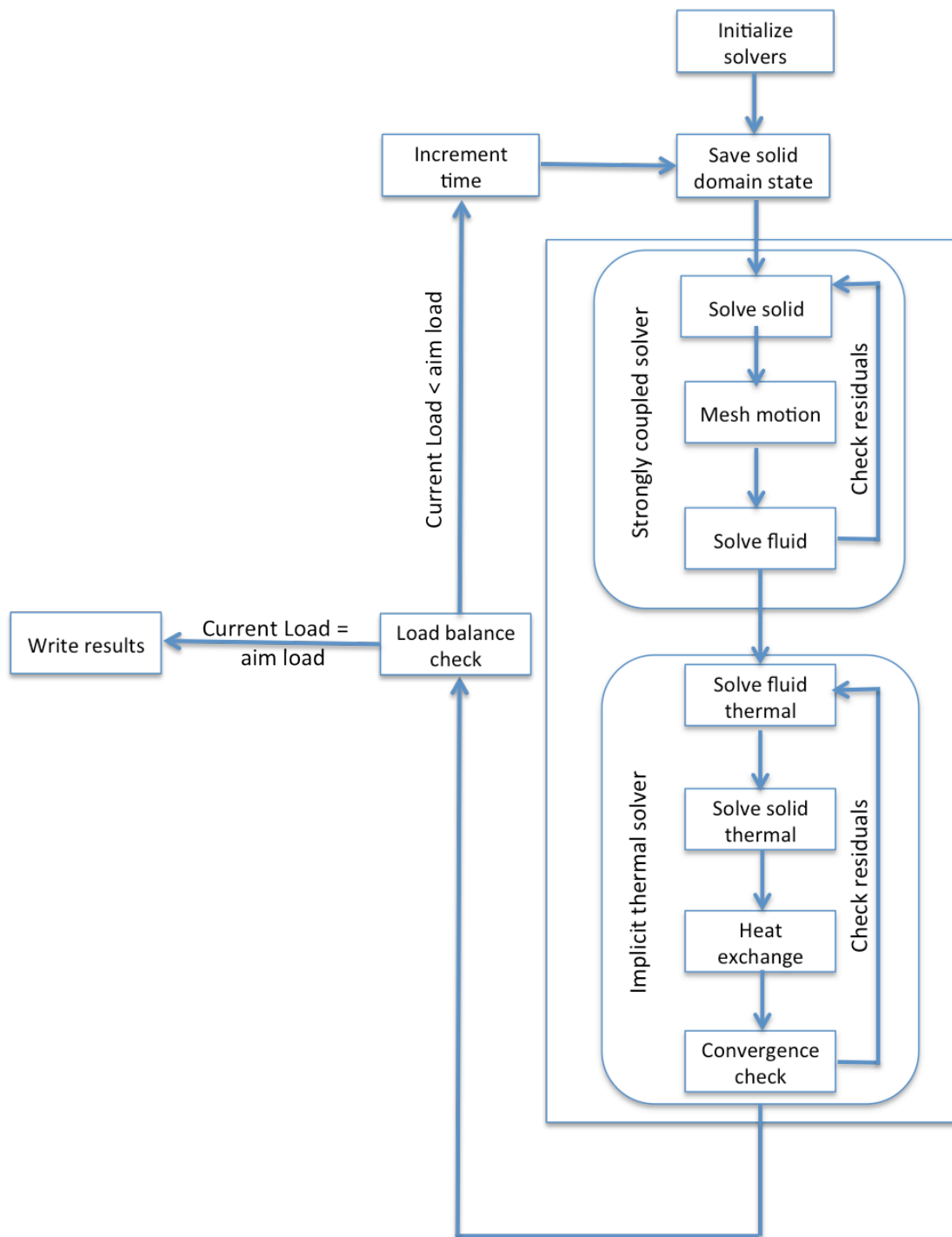


Figure 3-12. FSI model algorithm

3.5 Model Validation

The geometry considered in all case studies in this section is that of an elastic half-cylinder in a sliding/rolling contact with a rigid wall. The half-cylinder is assigned material properties typical of roller bearing steel (see Figure 3-13). The model assumes that all the elastic deformation is accommodated by the half-cylinder while the other surface is assumed to be rigid. The case parameters are listed in Table 3-1. The thermal properties of the lubricant and the solid are listed in Table 3-2 and Table 3-3 respectively.

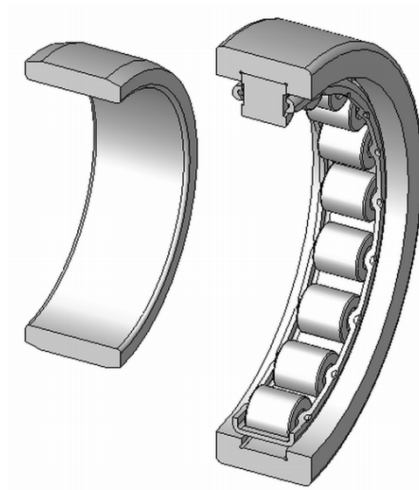


Figure 3-13. Roller bearing

The developed model is capable of simulating 3D cases, however only 2D cases are presented in this study. The computational domain is shown in Figure 3-14. The computation was performed using a multi-block discretised domain. There are 88,000 cells describing the fluid region where large pressure gradients are expected to occur due to the hydrodynamic action of the lubricant. The main loop execution takes 3 minutes on a single core Intel i7 3.2 *GHz* processor with 32 *GB* RAM.

The boundary conditions for density are zero gradient at all boundaries. The pressure gradient is zero at the walls and pressure is fixed to ambient pressure at inlet and outlet. The boundary conditions for velocity are set in accordance with the imposed slide-to-roll ratio. The velocity at the walls is applied as shown in Figure 3-14. The velocity at the inlet and outlet is calculated according to the mass-flux. The boundary

condition for the solid cylinder are fixed displacement of 0 at the flat surface and traction displacement at the deforming wall.

An actual roller bearing encompasses a series of cylinder-raceway contacts and therefore a periodic boundary condition is physically more acceptable. Consequently, additional cases with periodic boundary conditions at the inlet and the outlet have also been tested to ensure the validity of the chosen boundary conditions. These tests have shown that the boundary conditions outlined above are representative for the configuration considered. The convergence criterion for each parameter is set separately and iterations are carried out until all residuals are below their respective convergence criteria (pressure and velocity 10^{-12} , temperature and density 10^{-10} and deformation 10^{-9}). To improve stability, the deflection under-relaxation factor is set to values between 0.01 to 0.05, depending on speed and pressure ranges. The time step value is varied between 10^{-10} and 10^{-8} seconds, depending on the particular conditions of the case studied. It should be noted that it is possible to partition the solution domain into sub-domains, in which case different time steps may be used in different parts of the domain in order to reduce computation time. The number of inner correctors for the solid deformation loop is related to the range of pressure values.

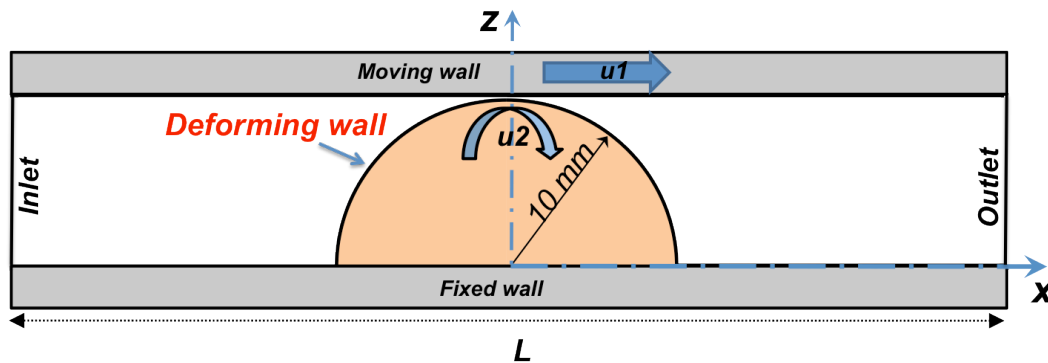


Figure 3-14. Domain of study

Table 3-1 EHL case parameters

Parameter	Value
Cylinder radius	$R = 10 \text{ mm}$
Domain length	$L = 120 \text{ mm}$
Young's reduced modulus	$E_r = 3.4523 \cdot 10^{11} \text{ Pa}$
Un-deformed film thickness	$h_i = 0.1 \text{ }\mu\text{m}$
Thermo viscous constant	$\beta = 0.0476 \text{ 1/K}$
Roelands reference pressure	$p_{r,0} = 1.98 \cdot 10^8 \text{ Pa}$
Liquid density at p_0	$\rho_{l,sat} = 870 \text{ kg/m}^3$
Roelands pressure index	$z = 0.6$
Eyring stress	$\tau_0 = 7 \cdot 10^6 \text{ Pa}$
Dynamic viscosity, vapour	$\mu_l = 8.97 \cdot 10^{-6} \text{ m}^2/\text{s}$
Environment pressure	$p_0 = 10^5 \text{ Pa}$
Vapour pressure	$p_{vapour} = 5000 \text{ Pa}$
Vapour density at p_{vapour}	$\rho_{v,sat} = 0.0288 \text{ kg/m}^3$

Table 3-2 Thermal properties of the lubricant

Parameter	Value
Specific heat capacity liquid	$C_{p,l} = 2300 \text{ J/(kg K)}$
Specific heat capacity vapor	$C_{p,v} = 1800 \text{ J/(kg K)}$
Ambient temperature	$T_0 = 353 \text{ K}$
Thermal conductivity liquid	$k_l = 0.15 \text{ W/(m K)}$
Thermal conductivity vapor	$k_v = 0.025 \text{ W/(m K)}$
Heat of evaporation	$h_{evap} = 287 \text{ kJ/kg}$

Table 3-3 Thermal properties of the solid cylinder (based on properties of AISI 52100 bearing steel)

Parameter	Value
Density solid	$\rho_s = 7850 \text{ kg/m}^3$
Specific heat capacity solid	$C_{v,s} = 450 \text{ J/kg K}$
Thermal conductivity solid	$k_s = 47 \text{ W/m K}$
Thermal diffusivity solid	$\alpha_{T,s} = 1.31 \cdot 10^{-5} \text{ W/m K}$

3.5.1 Isothermal Solution

In order to validate the current model, its predictions are compared with the equivalent results obtained by Hartinger et al. [63] who also used a CFD based finite-volume approach. In addition, the current predictions are also compared to those obtained by Bertocchi et al. [94] based on a mass-conserving formulation of the Reynolds equation proposed by Giacomini et al. [95]. The first case studied is an iso-

thermal pure sliding case ($SRR = 2$) with entrainment velocity ($u_e = 0.5(u_1 + u_2)$) of 2.5 m/s. The lubricant is assumed to be non-Newtonian and shear-thinning with initial dynamic viscosity of $\eta_0 = 0.01$ (Pa.s) with target load of 10^5 N/m.

Figure 3-15 compares isothermal pressure distributions and film thickness predictions from the current model to those of Hartinger et al. [63] and Bertocchi et al. [94]. A complete agreement with Hartinger's predictions is evident in both the film thickness and pressure predictions.

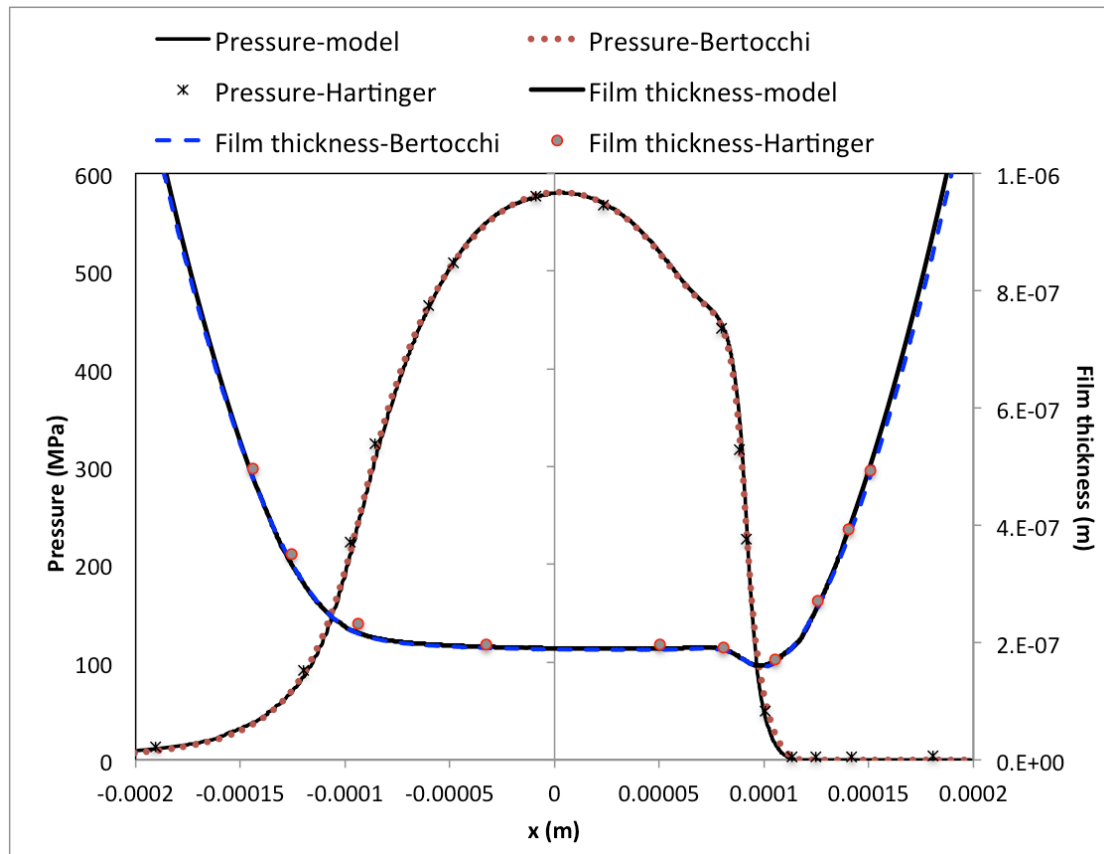


Figure 3-15. Film thickness and pressure distributions for an isothermal case as predicted by the current model compared with equivalent results of Hartinger and Bertocchi models

It is evident from Figure 3-16 (b) that viscosity varies through the film thickness in between $x = 0.05$ mm to 0.1 mm. The reason for slight difference between pressure distributions of the current and Bertocchi et al. models can be traced back into neglecting viscosity gradient term through film thickness in the latter.

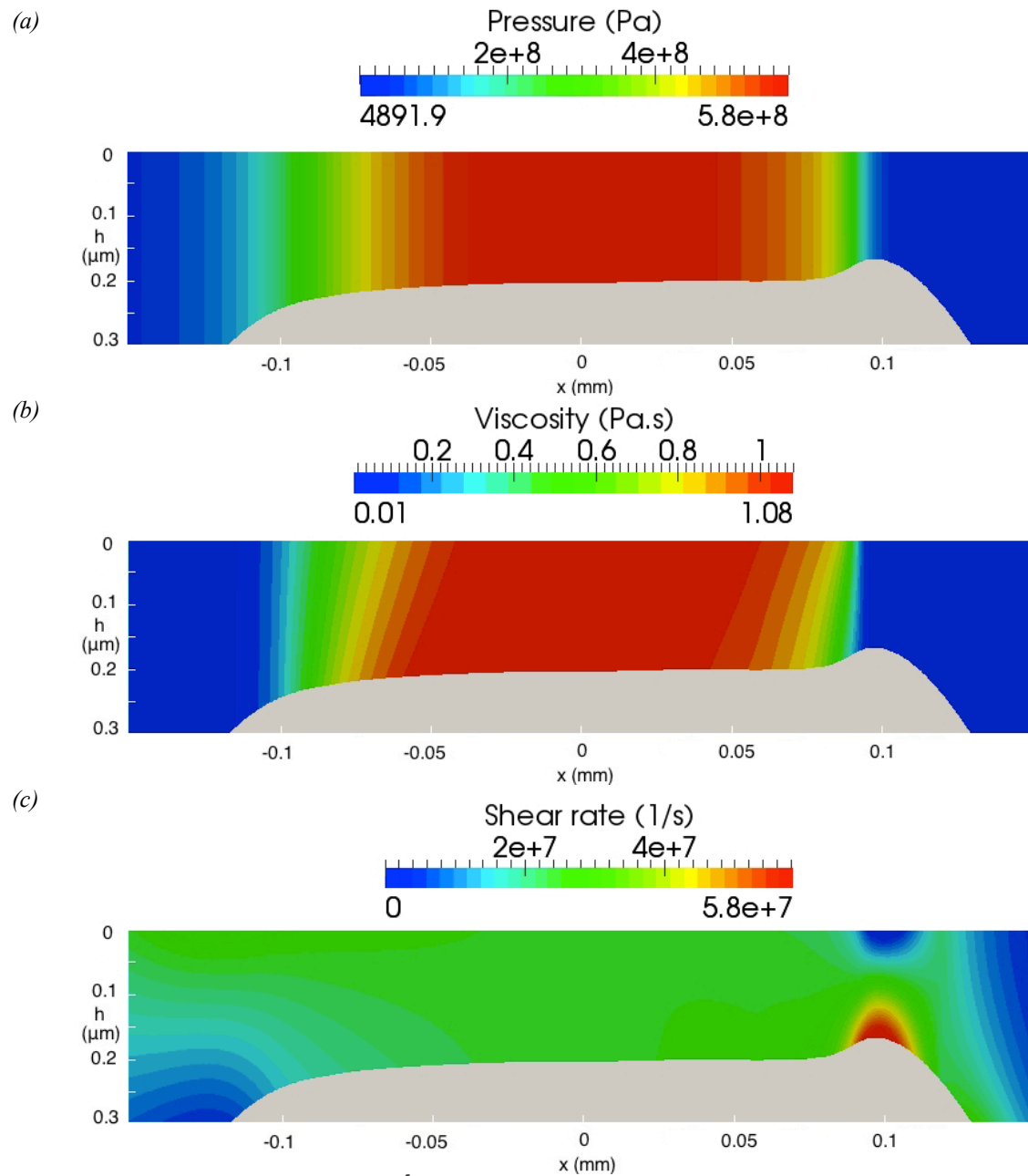


Figure 3-16. Isothermal SRR=2, $L= 10^5$ kN/m, $\eta_0=0.01$ Pa.s (a) Pressure (b) Viscosity (c) Shear rate distributions

Pressure, viscosity and shear rate distributions for this case are shown in Figure 3-16 (a), (b) and (c) respectively. Maximum viscosity within the contact reaches 1.08 Pa.s and some variation of viscosity through the film is evident in the centre of the contact due to shear-thinning. The maximum shear rate is seen to occur at the exit constriction, near the stationary cylinder. The fluid film pressure is constant through the film thickness for this case, characterised by relatively low viscosity. These

observations are fully in line with those predicted by Hartinger [85] for the equivalent case of pure sliding with relatively low viscosity lubricant.

3.5.2 Thermal Solution

In order to validate the full model, including thermal effects, the results from a thermal case study are compared with equivalent thermal results of Hartinger et al. [85]. Figure 3-17 plots the pressure and film thickness results obtained from the current model for a pure sliding case with $\eta_0=0.01 \text{ Pa.s}$ together with equivalent results of Hartinger's model. The difference between two models, in the contact centre, can be traced back into adiabatic assumption on top sliding wall.

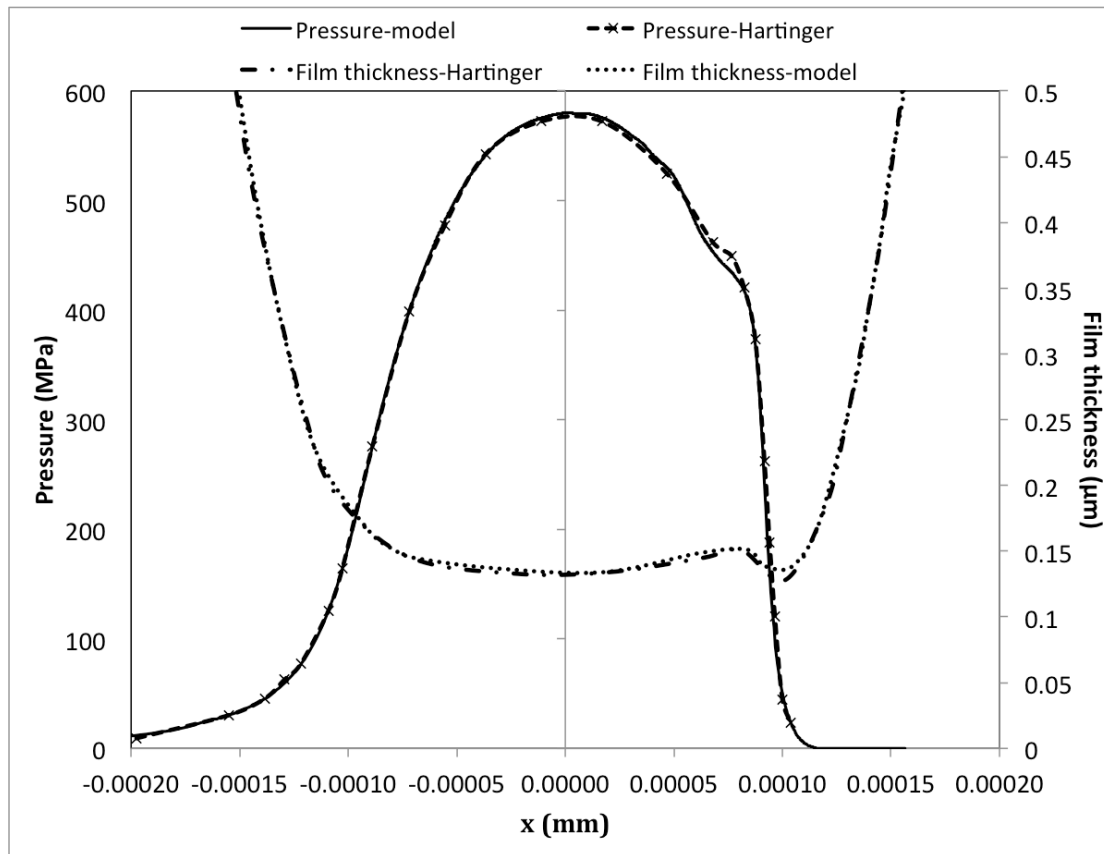


Figure 3-17. Pressure distribution for a thermal case compared with equivalent results of Hartinger et al [85] - $\text{SRR}=2$ and $\eta_0=0.01 \text{ Pa.s}$

Figures 3-18 and 3-19 show the viscosity, temperature, shear rate and pressure distributions for the same case ($\text{SRR}=2$ and $\eta_0=0.01 \text{ Pa.s}$). The maximum viscosity of 4.2 Pa.s , which occurs on the faster moving wall, is higher than the maximum viscosity predicted in the equivalent isothermal solution (1.08 Pa.s , Figure 3-16 (a)). The difference occurs due to the fact that less shear-thinning is occurring when the

thermal effects are accounted for. It should also be noted that lubricant viscosity varies significantly through the film due to non-Newtonian and thermal effects. This is in contrast to standard Reynolds solution which assumes that the viscosity is constant through the film thickness. Lubricant temperature rises up to 28°K in the central region of the contact with the maximum temperature occurring at lower stationary surface. As a consequence of this, the viscosity in this region is lower. Some downstream conduction of heat-flux occurs which results in the lower lubricant viscosity at the inlet and less lubricant being dragged into the contact. As was observed for the isothermal case, the maximum shear rate occurs at the stationary surface in the constriction region while there is no significant variation in pressure through the film. Since the other surface assumed to act adiabatic with constant temperature, the predicted behaviours of viscosity and temperature are slightly different to those shown by Hartinger [85] for the same thermal case.

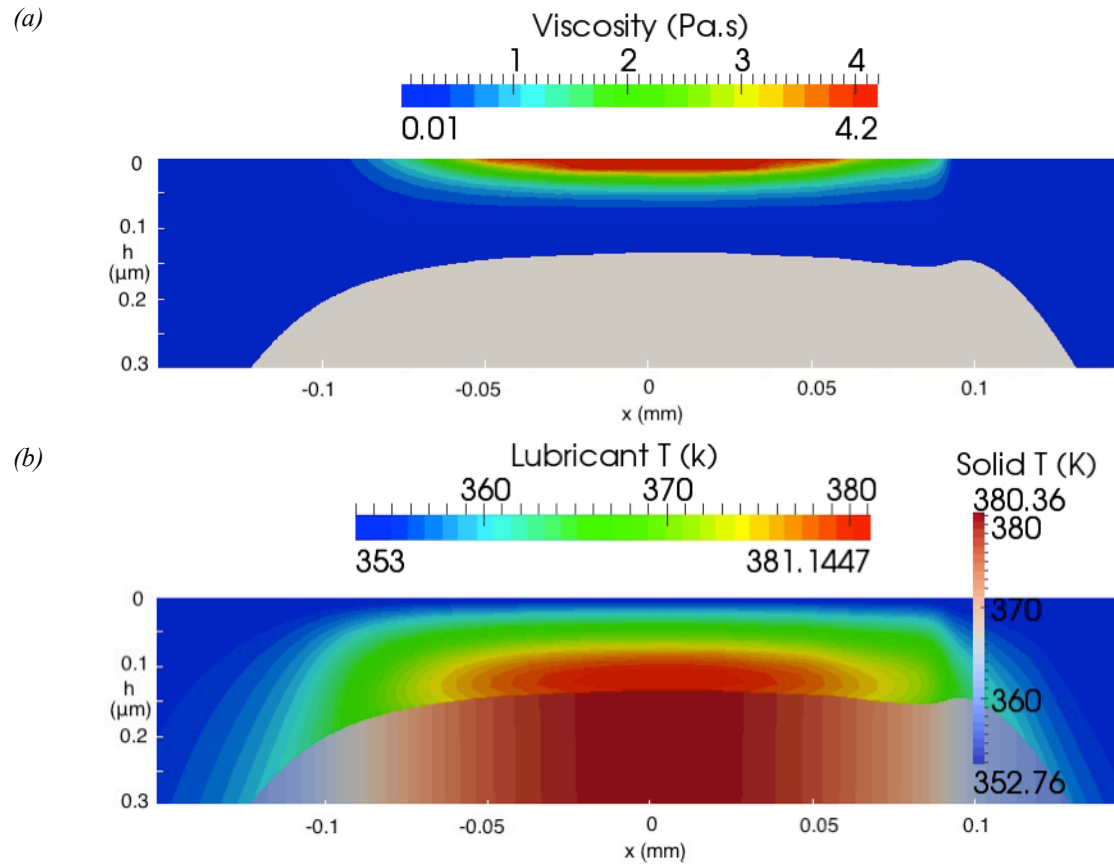


Figure 3-18. $SRR=2$, $L=10^5$ kN/m and $\eta_0=0.01$ Pa.s: (a) Viscosity (b) Temperature distributions

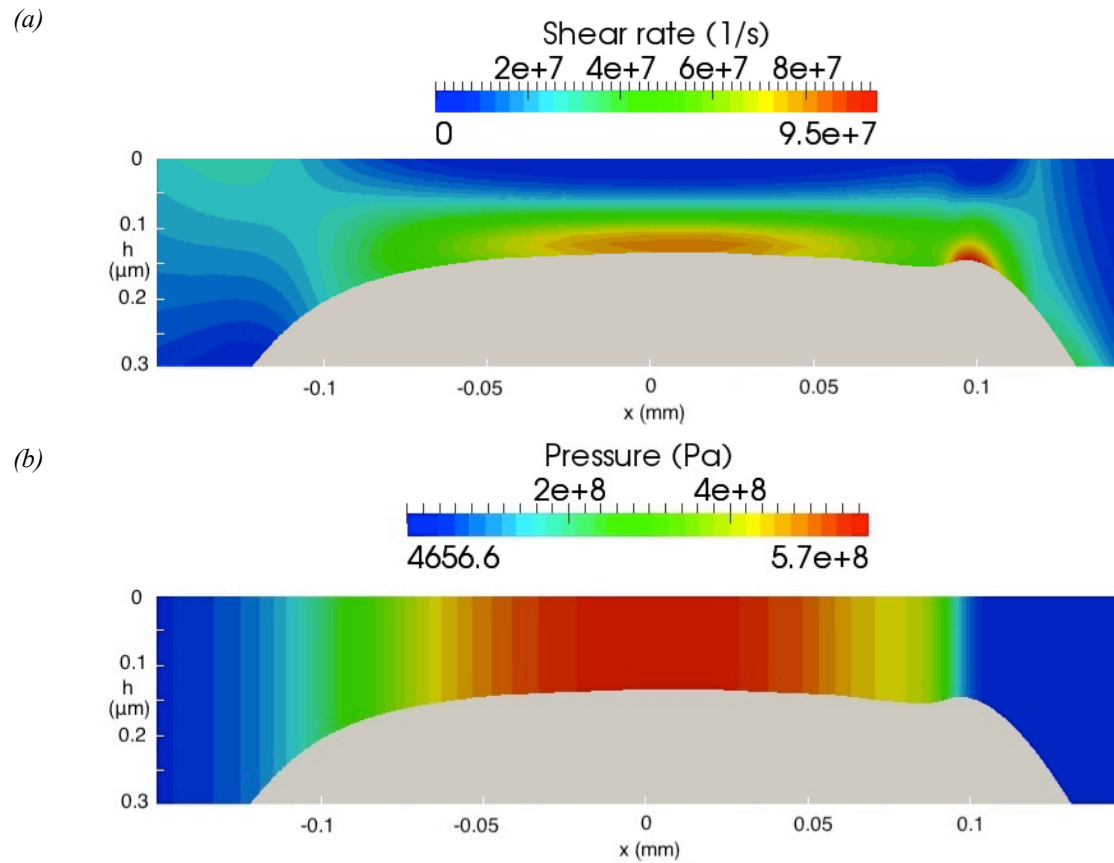


Figure 3-19. $L=10^5$ kN/m $SRR=2$ and $\eta_0=0.01$ Pa.s: (a) Shear rate (b) Pressure distributions

3.6 Closure

In this chapter the FSI model implemented and adopted by the author with particular application to EHL modelling of roller bearings, has been presented. Details are provided for solvers and strategies implemented to include solid deformation and energy equations, automatic mesh motion, data transfer between solid and fluid continua, and non-conformal mesh interface interpolation schemes. The developed fully coupled finite volume thermal FSI algorithm is described and the model is benchmarked against existing Reynolds-based and CFD solutions for both iso-thermal and thermal case studies.

Chapter 4

4.1 Smooth EHL Contacts

This chapter contains CFD modelling results of a roller bearing case study using the FSI model presented in Chapter 3. The cases are set up for different initial viscosity values and are characterised by variable slide-to-roll ratios (ranging between 0 and 2). The aim is to highlight the features of the FSI methodology implemented in OpenFoam and the stability of the solver under a wide range of operating conditions.

4.1.1 Case study setup

The hydrodynamic modelling of a full-cylinder (see Figure 4-1) is performed to compare the lubricant behaviour with that of a half-cylinder case study. The simplification from a full-cylinder domain of study to a half-cylinder case is performed based on the observations that the flow far from the contact does not influence the contact flow much. Therefore, in order to save computational time, the geometry considered in all case studies in this section is that of an elastic half-cylinder in a sliding/rolling contact with an adiabatic-rigid wall (only one case with both solid domains as non-adiabatic surfaces). The half-cylinder is assigned material properties with higher elastic modulus than typical bearing steel for the first three initial case studies. The case parameters, the thermal properties of the lubricant and the solid are listed in Tables 3.1, 3.2 and 3.3 respectively. The boundary conditions are outlined in section 3.5. All divergence terms are discretised using an upwind scheme. All gradient terms are discretised using Gauss linear discretisation. The laplacian terms are evaluated using the Gauss linear corrected scheme. Temperature, pressure and continuity equations are solved using GAMG (generalized geometric-algebraic multigrid) matrix solver.

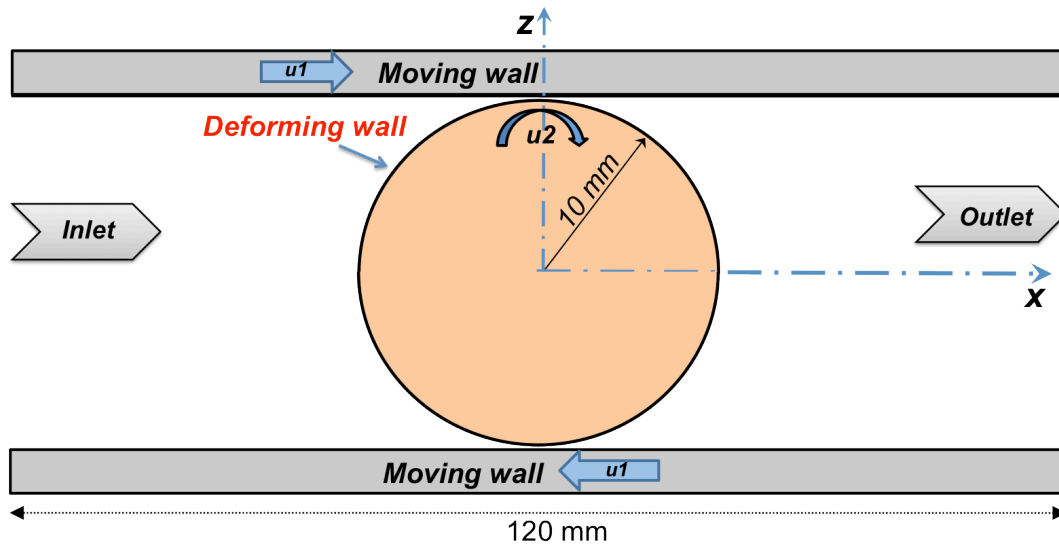


Figure 4-1. Full cylinder domain of study

4.1.2 Stability at high pressures and/or loading conditions

The usual weakness of coupled CFD models, for EHL contact problems, is the stability at high pressures. For example maximum Hertz pressure achievable in

Hartinger et al. [85] model was 0.8 *GPa*. The stability of the current model is examined through running a series of cases with increased initial dynamic viscosity of the lubricant ($\eta_0=0.04$ and 0.5 *Pa.s*), higher elastic modulus of the solid body and operating loading conditions.

Figure 4-2 shows the the thermal solution results for pure rolling case with initial viscosity of $\eta_0=0.04$ *Pa.s* and the cylinder is assigned elastic modulus of $E=200$ *GPa* and therefore equivalent elastic modulus of the bodies in contact is not representative of typical steel (it is higher than the cases presented in Chapter 3). The cylinder and the flat plane are both moving at a speed of 2.5 m/s in the proximity of the contact. The target load is set to 40 *kN/m* (Figure 4-2 (a)) and 65 *kN/m* (Figure 4-2 (b)) to highlight the effect of target load variation on pressure distribution and the film thickness. The higher target load results in lower film thickness and in larger pressure values induced by the cylinder movement. Due to lower shear rates the maximum lubricant temperature rise is only 7.3°K and occurs near the constriction (see Figure 4-2 (c)). The maximum viscosity occurs at the boundary of the moving surface due to relatively lower temperature and shear thinning in this region (Figure 4-2 (d)). Local maximums in shear rate occur on the surfaces of both bodies at the constriction where the flow has to accelerate. This is in contrast to pure sliding cases, where the maximum shear rate is located on the stationary surface. The zoomed-in plot of the viscosity contours near the local maximum in Figure 4-2 (e) shows how simultaneous shear-rate and thermal effects produce localised rise in viscosity up to 3200 *Pa.s*.

Since turbulent and surface tension effects are neglected in this study, the following non-dimensionalised number evaluation is necessary to establish the validity of the approximations introduced by these assumptions to the solution. In fluid mechanics, turbulent and surface tension effects are evaluated using the Reynolds and the Weber dimensionless numbers. The Reynolds number is a measure for the importance of inertial and viscous forces:

$$Re = \frac{\text{inertial force}}{\text{viscous force}} = \frac{\rho u l}{\eta} \quad \text{Equation 4-1}$$

where u is the characteristic velocity, l is the characteristic length and $\rho = 850$ *kg/m³*. The extension of the cavitation bubble can be taken as the largest characteristic length (for this case 1.8 *mm* corresponds to the extension of cavitation bubbles). This leads to the Reynolds number of 191.25 which represents laminar regime for a channel flow ($Re < 1350$). For the outer flow field with $l=120$ *mm*, the Reynolds

number goes up to 12750 which is in the transitional regime to fully turbulent behaviour ($Re > 18000$) [96].

The relationship between surface tensions and inertia (mass (m) $\times$ acceleration (a)) is usually described by Weber number:

$$We = \frac{\text{fluid inertia}}{\text{surface tension}} = \frac{ma}{\sigma_s l} = \frac{\rho u^2 l}{\sigma_s} \quad \text{Equation 4-2}$$

where σ_s is the surface tension. Surface tension is important when the solid boundaries of a liquid surface are in close proximity. For minichannels (diameter between 3 mm to 200 μm), the flow is inertia dominated when $We \geq 11$. The characteristic length ranges from minimum film thickness value (1 μm) to the extent of cavitation bubble (1.8 mm). Using the surface tension of 0.028 N/m, gives the Weber number range [0.75, 1366]. Therefore, surface tensions can be important for the lubricant in the cavitating region.

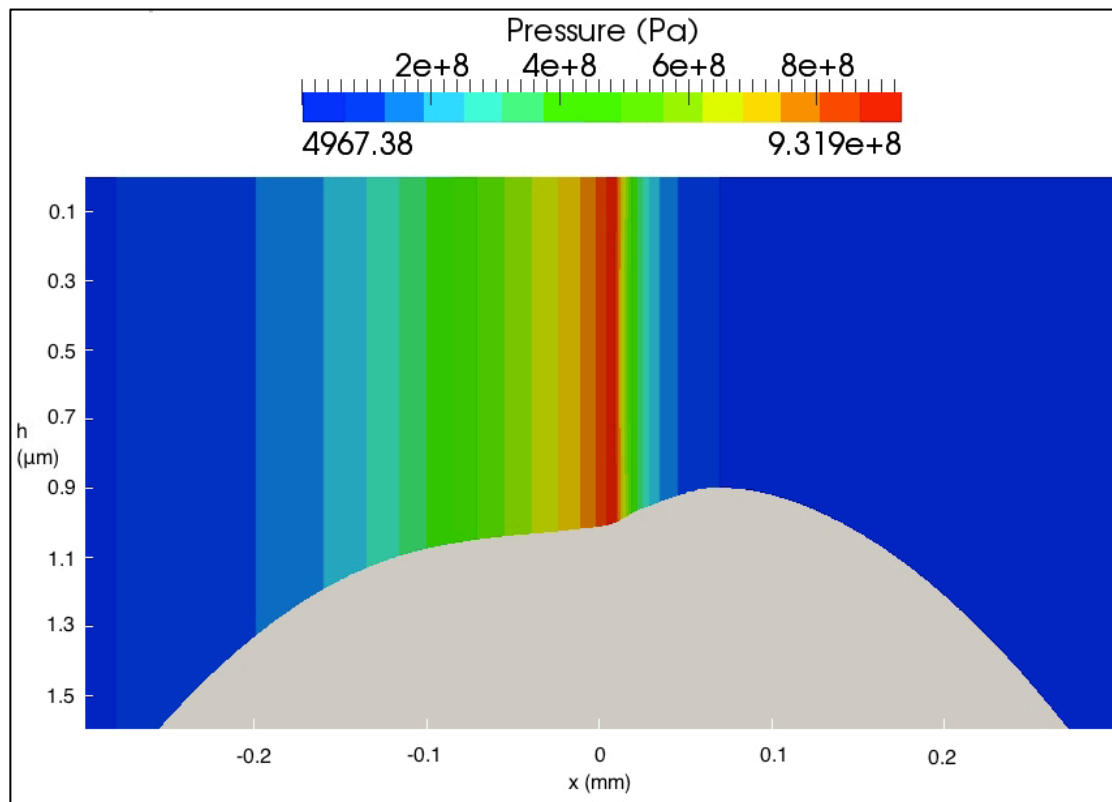


Figure 4-2. (a)

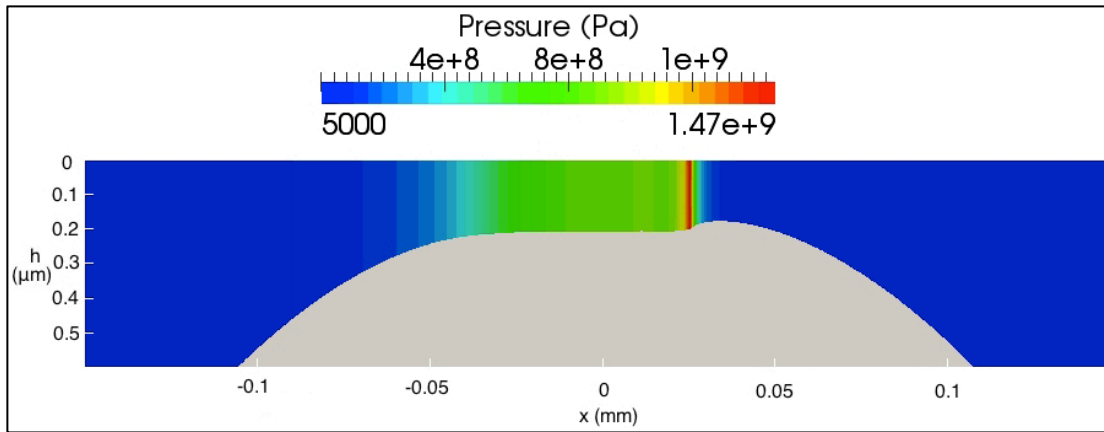


Figure 4-2. (b)

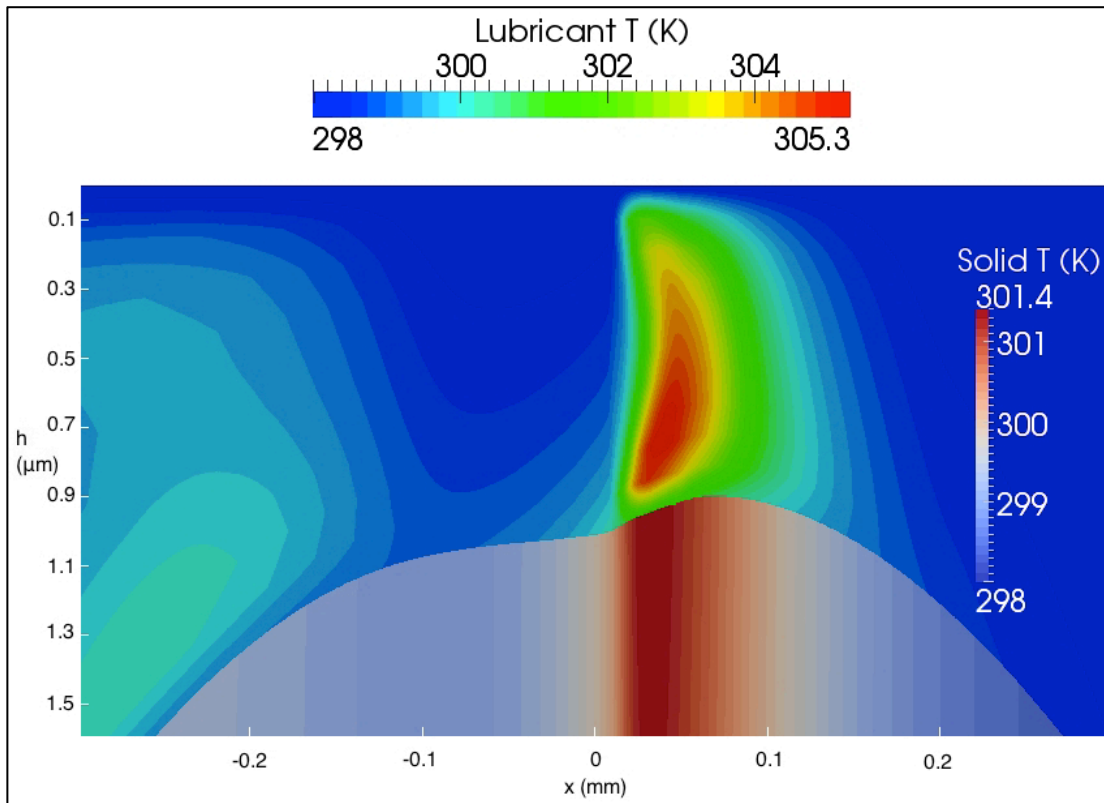


Figure 4-2. (c)

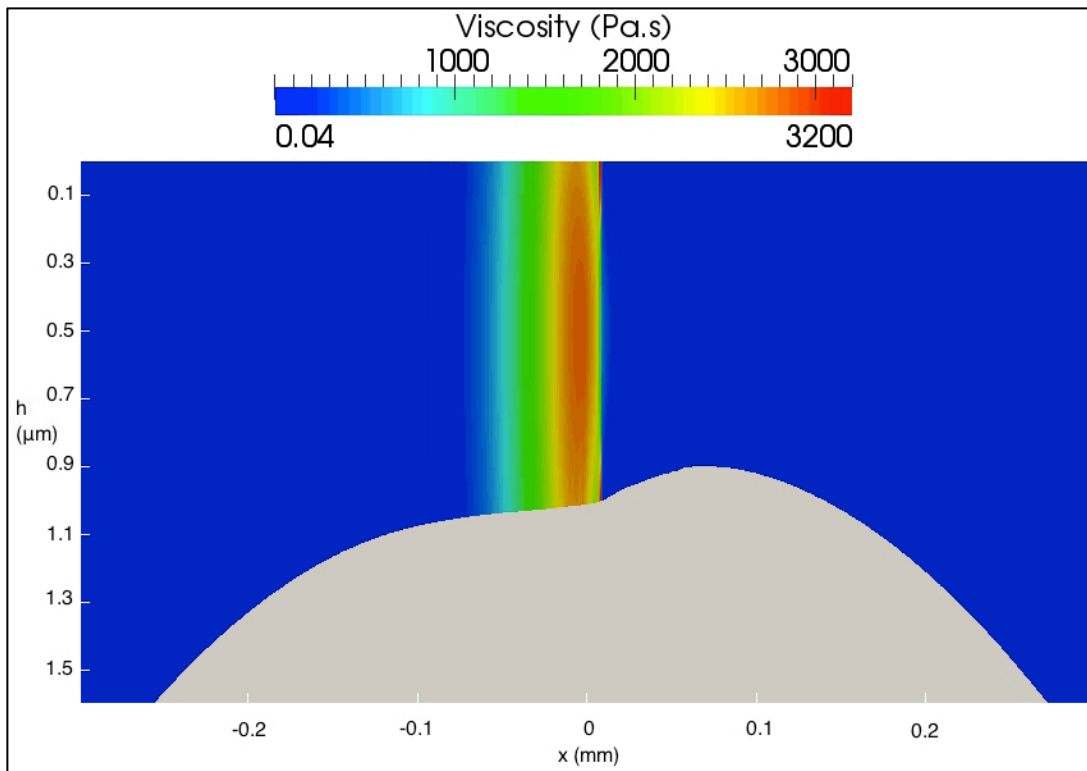


Figure 4-2. (d)

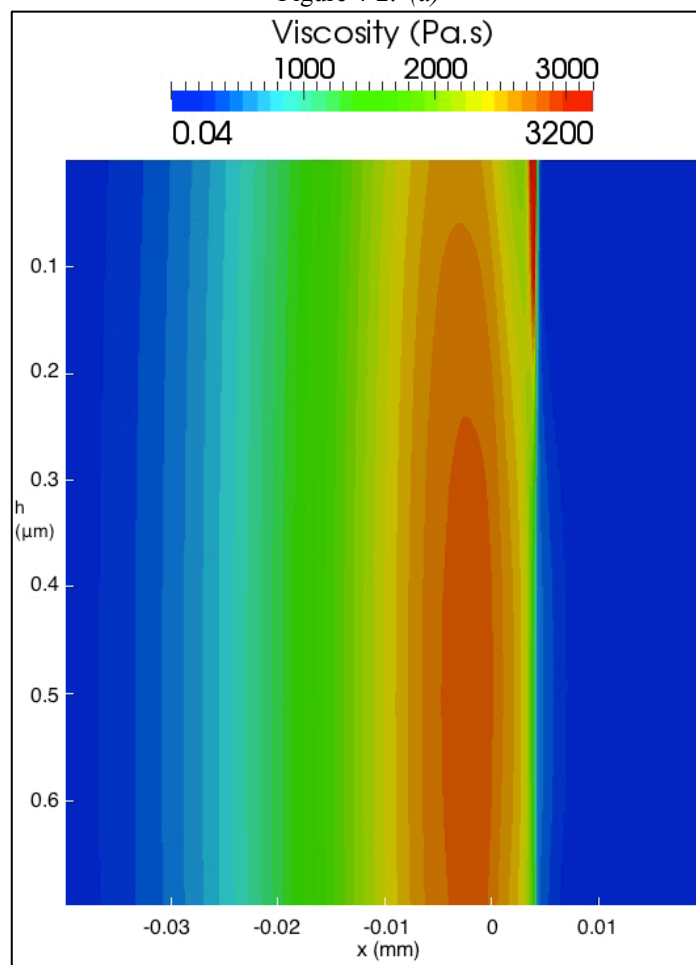


Figure 4-2. (e)

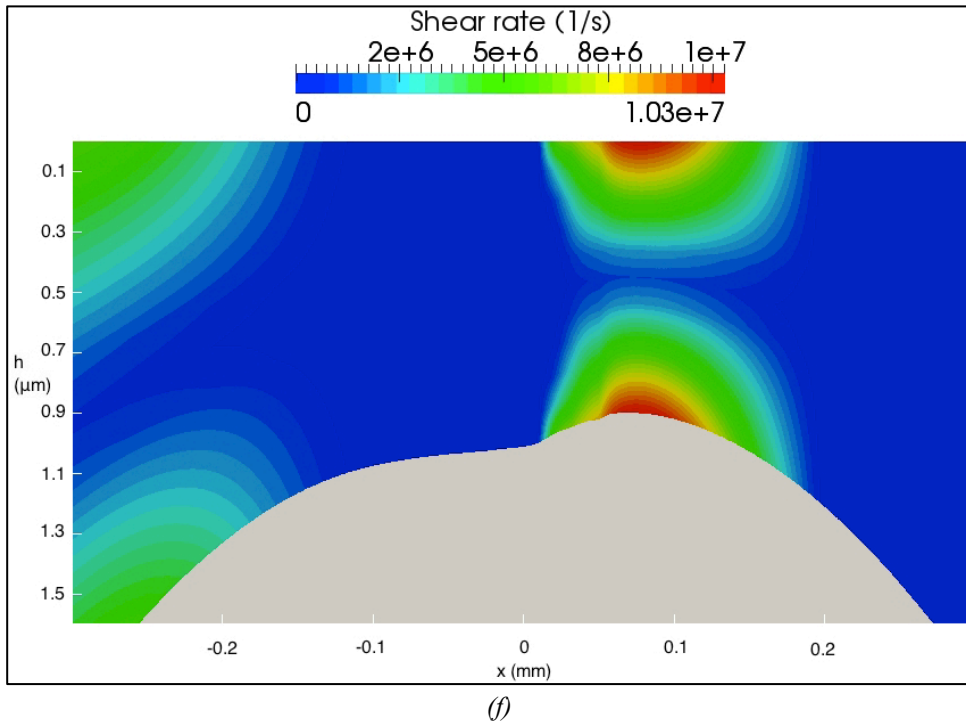
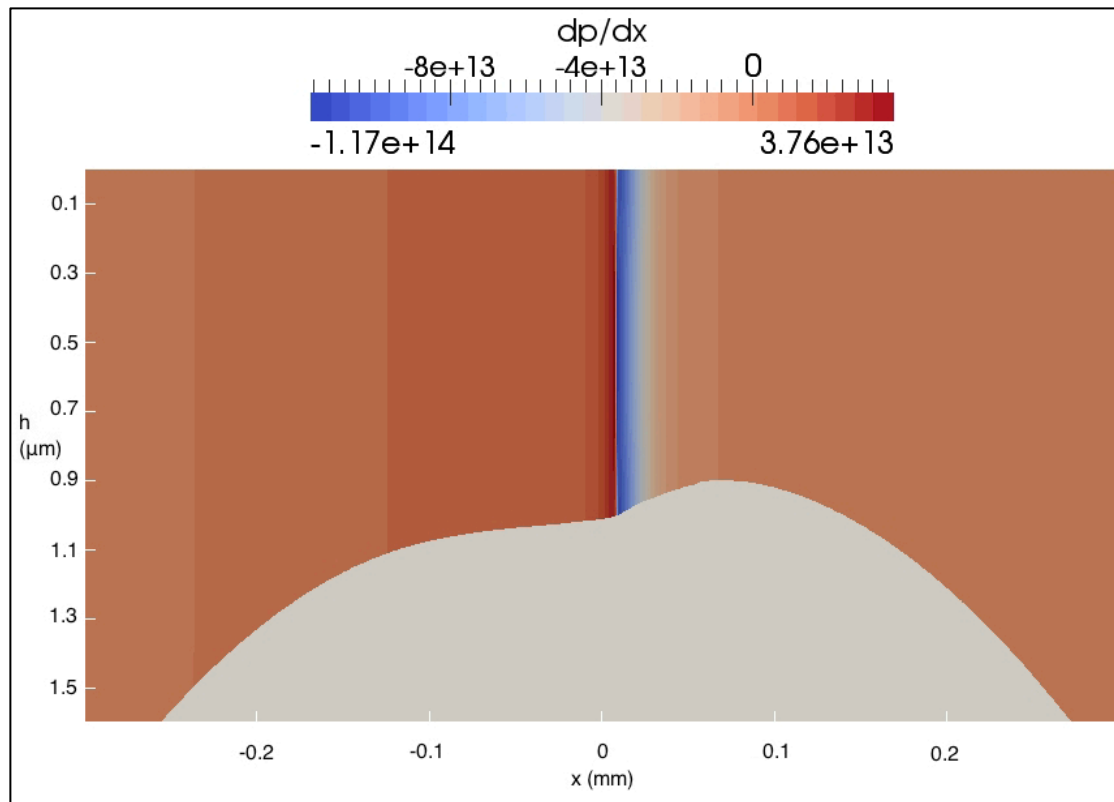
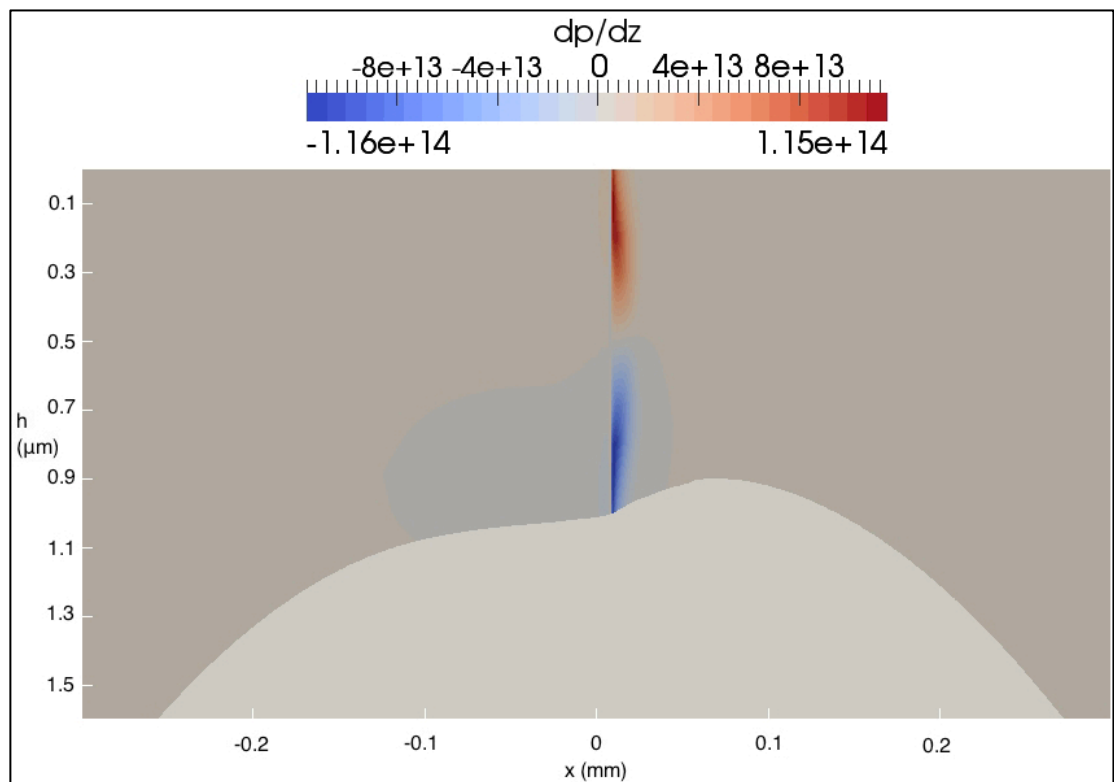


Figure 4-2. Results for the thermal solution of the case with $SRR=0$, $\eta_0=0.04 \text{ Pa.s}$, $E=200 \text{ GPa}$. (a) Pressure (40 kN/m) (b) Pressure (65 kN/m) (c) Temperature (d) Viscosity (e) Zoomed-in viscosity (f) Shear rate distributions

Reynolds solution of EHL contacts assumed that there is no pressure variation through the lubricant film i.e. $\delta p / \delta z = 0$. With this in mind, it is interesting to consider the level of pressure variation through the film as predicted by full CFD solution. Figure 4-3 illustrates pressure gradient along and through the contact, $\delta p / \delta x$ and $\delta p / \delta z$ respectively, for the above-mentioned pure rolling case. The plot clearly illustrates that under these conditions the variation in $\delta p / \delta z$ through the film is significant so the usual Reynold's assumption would produce inaccurate results in this case. The variation in $\delta p / \delta z$ is closely related to the variation of viscosity, temperature and shear rate. The range of $\delta p / \delta z$ values is comparable and occasionally higher than that of $\delta p / \delta x$.



(a)



(b)

Figure 4-3. Pressure gradient across the domain (a) along the contact (b) through the film thickness

Figure 4-4 shows the results of a thermal case with $SRR=0.4$ and $\eta_0=0.5 \text{ Pa.s}$. The pressure goes up to 1.6 GPa (see Figure 4-4 (a)). The integration of pressure profile gives the accumulated load of 420 kN/m . The lubricant temperature (Figure 4-4 (b)) rises, in the centre of the contact, by up to 141K and this effect results in lower viscosity values in that region. The location of the local maxima in the temperature rise coincides with the locations of high shear rate gradients. The increased temperature in the central region between the surfaces results in a lower viscosity and consequently leads to the development of a marked shear band. The maximum viscosity of 36758 Pa.s occurs at the rigid wall. The viscosity peaks near to the highest pressure gradient regions are shown in Figure 4-4 (c). The localised viscosity rise just before the constriction is linked to the high temperature gradient (Figure 4-4 (d)). The maximum shear stress is 142 MPa , which is bounded to the viscosity and shear-rate distribution maxima (Figure 4-4 (e)). The isothermal result from Bertocchi model, Reynolds based approach, is shown in Figure 4-5. The thermal effects, viscosity change through the film thickness, pressure gradient and shear-thinning effects are resulted into completely different prediction of lubricant behaviour using 1-D Reynolds based solution.

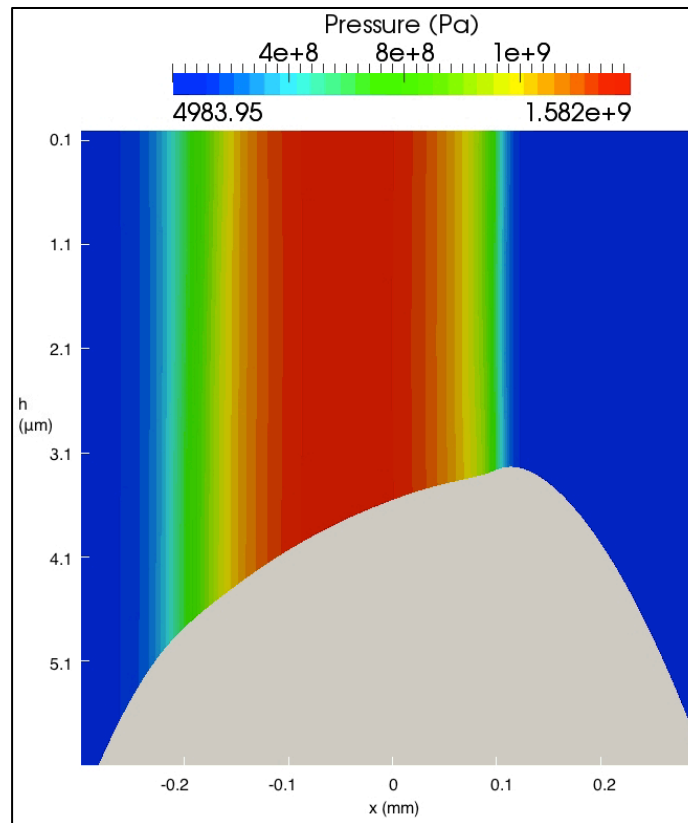


Figure 4-4. (a)

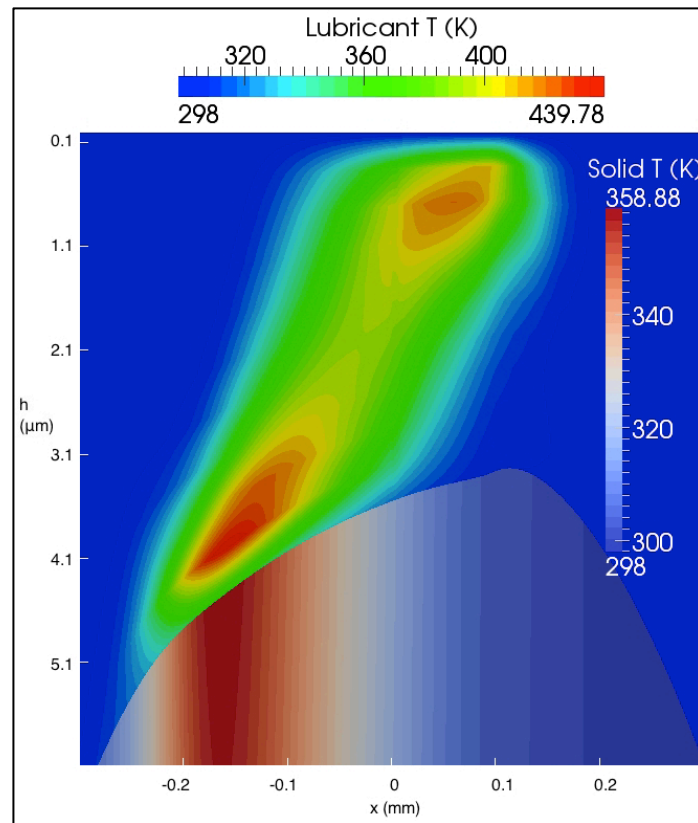


Figure 4-4. (b)

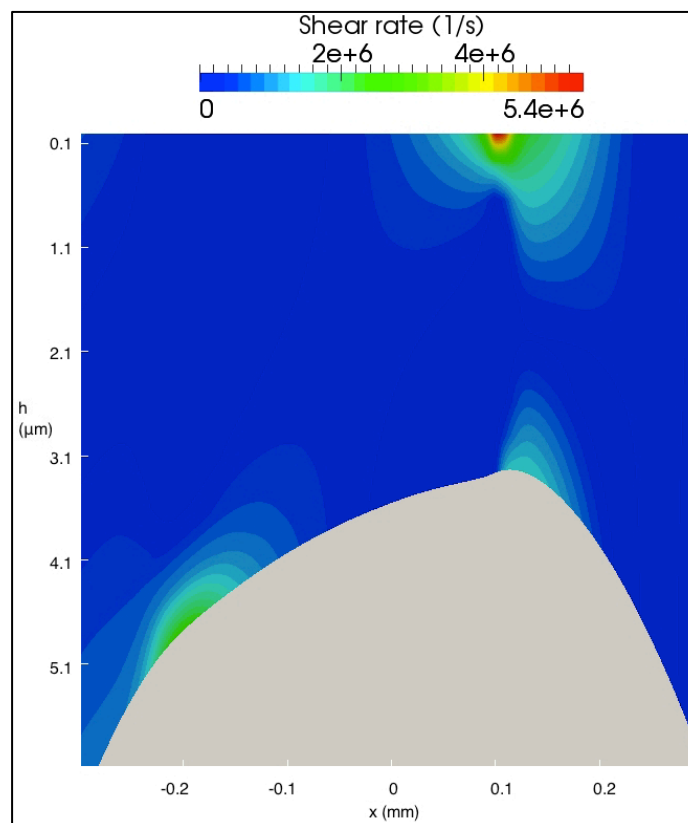


Figure 4-4. (c)

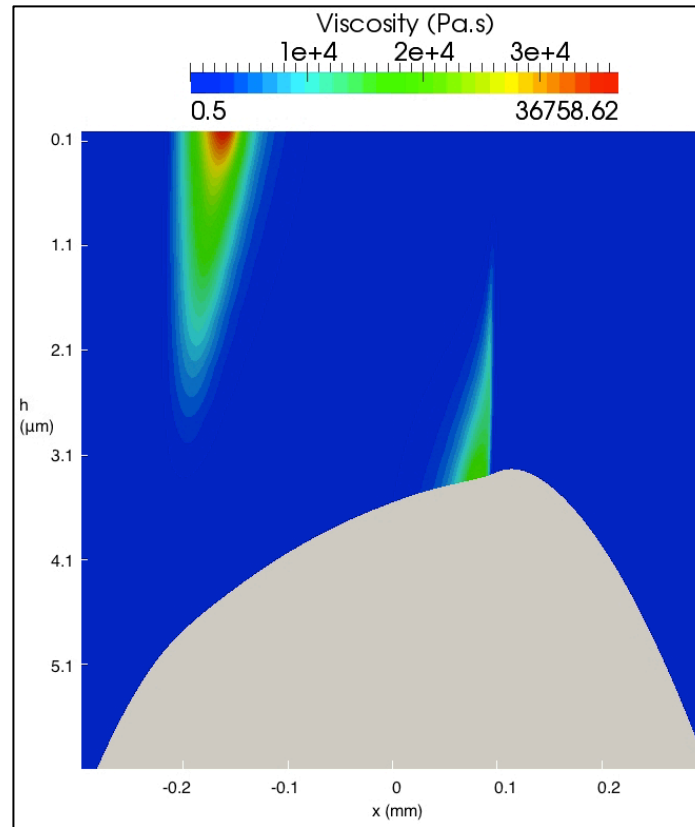


Figure 4-4. (d)

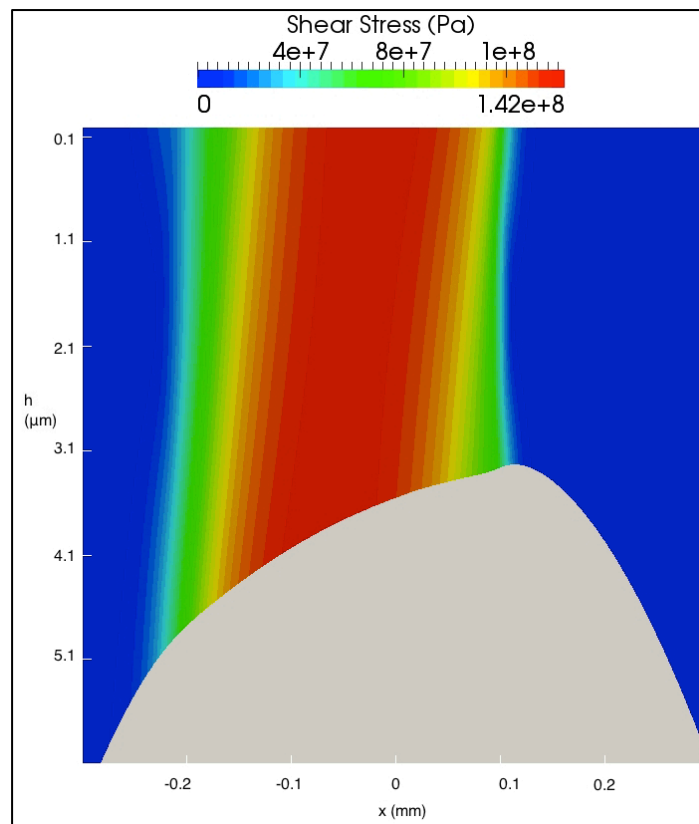


Figure 4-4. (e)

Figure 4-4. Thermal solution results for the case $L = 420 \text{ kN/m}$, $\text{SRR} = 0.4$, $\eta_0 = 0.5 \text{ Pa.s}$, $E_{\text{Cylinder}} = 200 \text{ GPa}$. (a) Pressure (b) Temperature (c) Shear-rate (d) Viscosity (e) Shear stress distributions

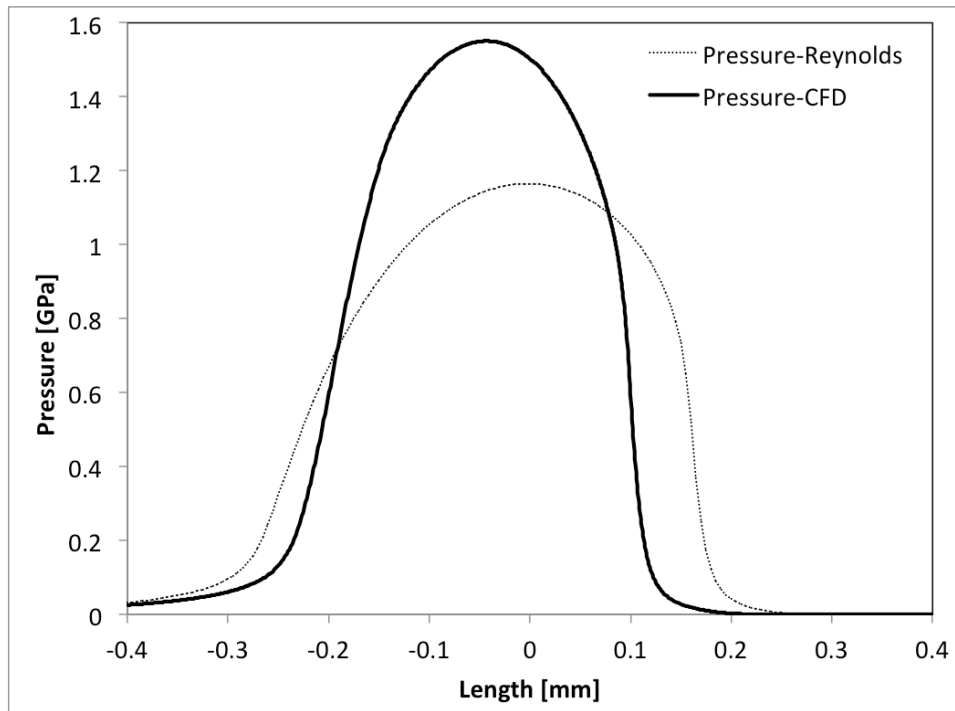


Figure 4-5. Isothermal pressure distribution from Bertocchi model compared to CFD thermal solution

An equivalent solution for a pure sliding case ($SRR = 2$, $\eta_0 = 0.5 \text{ Pa.s}$) is shown in Figure 4-6. Figure 4-6 (a) and (b) show that the maximum pressure is about 1.3 GPa and that the maximum viscosity of 658 Pa.s occurs at the lower stationary surface. This is caused by low shear rates and temperature values. There is considerable amount of shear-thinning in the middle of the contact induced by temperature rise which can be seen from localised shear rate rise in the middle of the contact. The target load of 400 kN/m is reached. The maximum shear rate occurs on the stationary surface near the constriction (Figure 4-6 (d)), as was observed for the equivalent case with lower initial viscosity. However, there is another localised maximum shear-rate on the moving wall near to the contact inlet. The maximum temperature rise is 181 K and there is a significant heating of the middle of the inlet region through heat conduction upstream, which in turn leads to a lower viscosity in the inlet near the surface of the sliding wall.

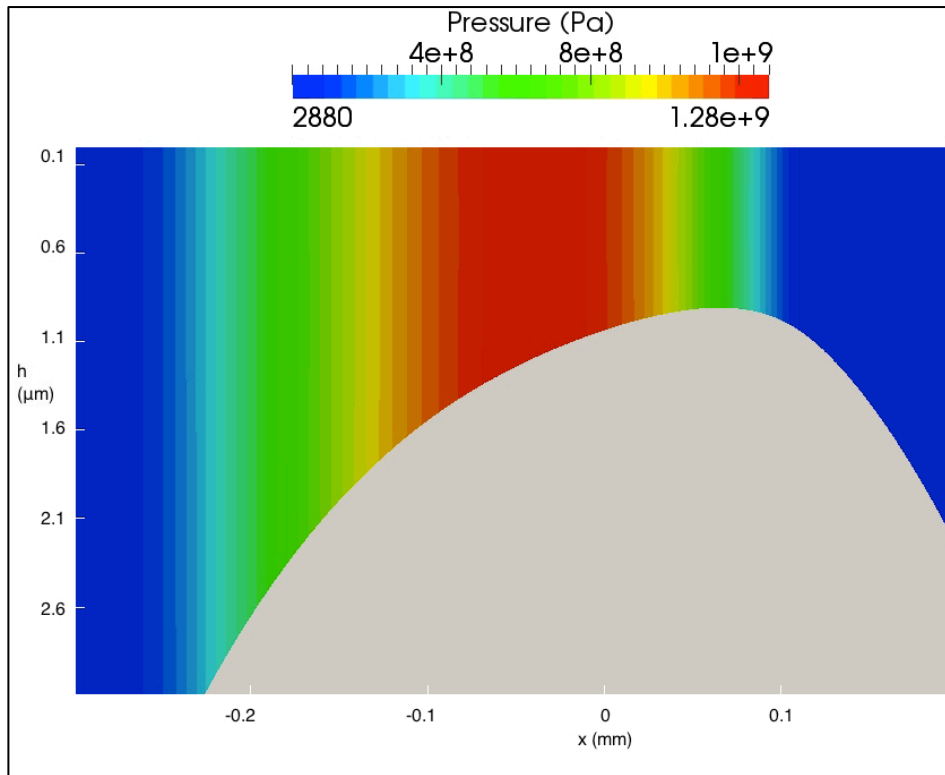


Figure 4-6. (a)

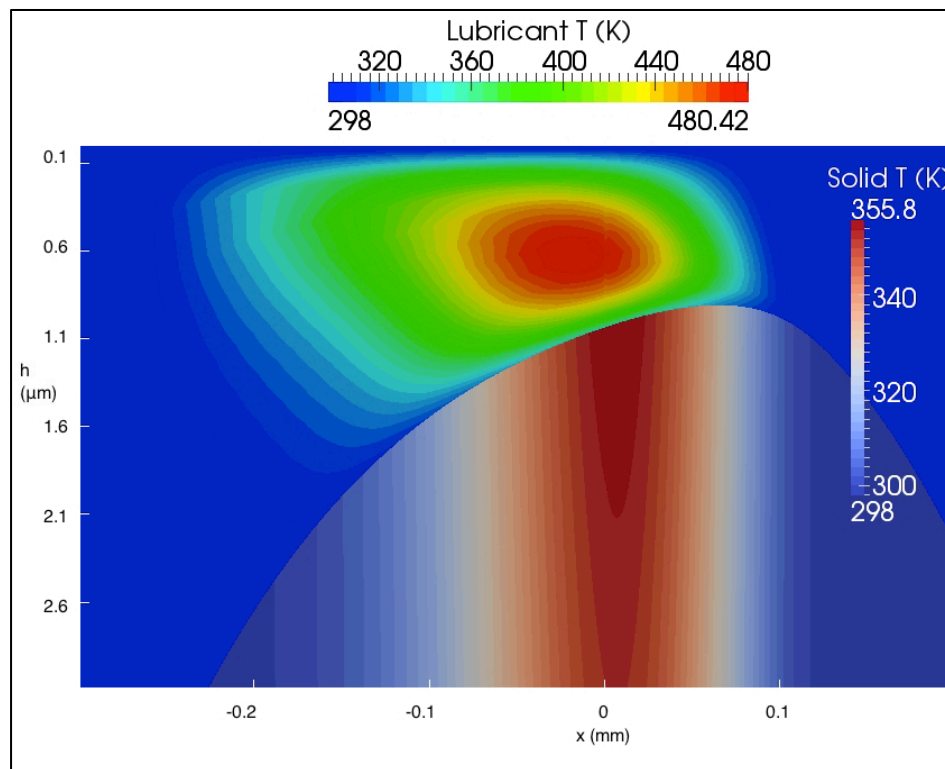


Figure 4-6. (b)

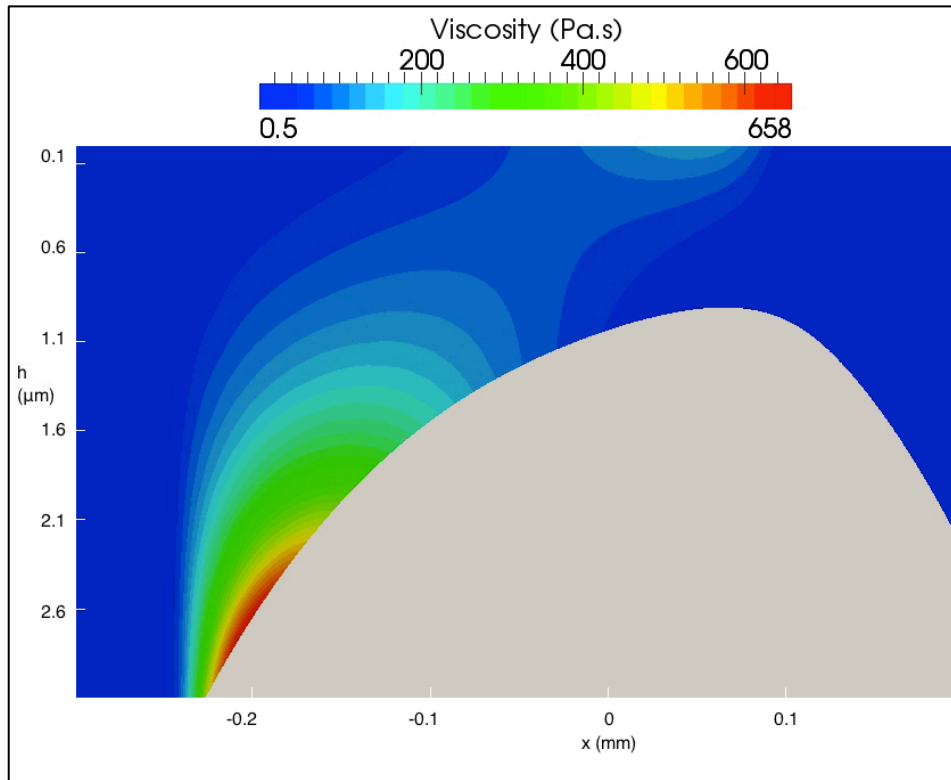


Figure 4-6. (c)

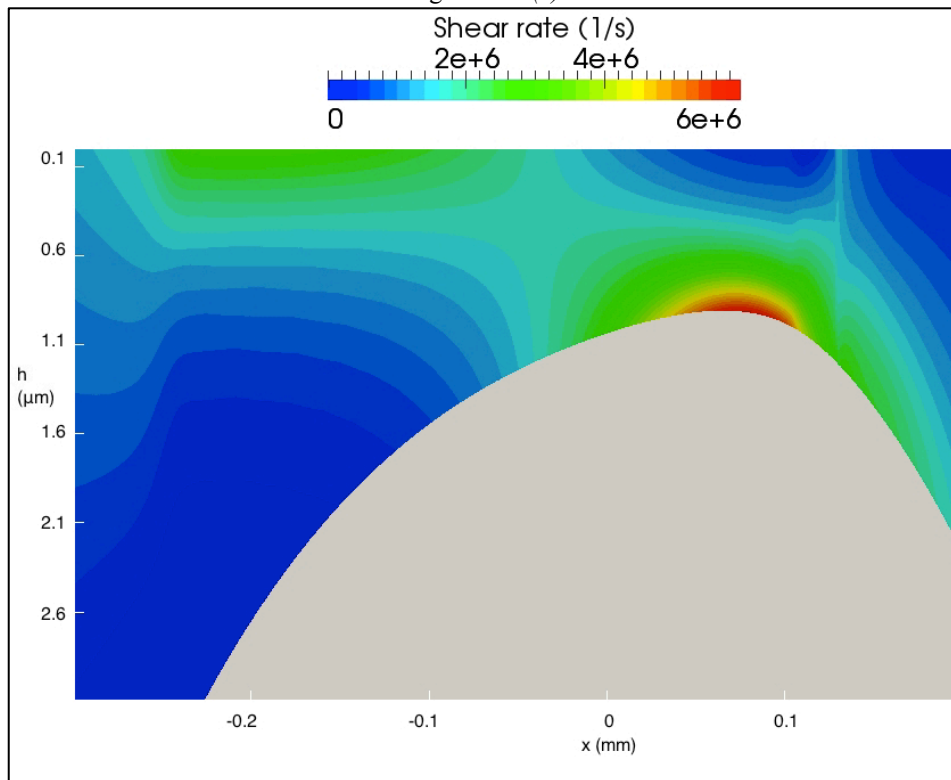


Figure 4-6. (d)

Figure 4-6. Thermal solution results for the case $\text{SRR} = 2$, $L = 400 \text{ kN/m}$, $\eta_0 = 0.5 \text{ Pa.s}$, $E_{\text{Cylinder}} = 200 \text{ GPa}$
 (a) Pressure (b) Temperature (c) Viscosity and (d) Shear rate distributions

The last case examined and discussed is a smooth-surface thermal solution for a pure rolling case with initial viscosity value of 1 Pa.s (Young's modulus is lower compared to the previous case studies in this chapter and the assigned material properties are representative of typical bearing steel in here). Pressure, shear rate and temperature distributions are shown in Figure 4-7. The fluid pressure reaches 4 GPa (Figure 4-7 (a)), which corresponds to the development of high viscosity. The viscosity distribution contour plot is not presented here since the rheology model produced un-physical viscosity values during the convergence process. This is mainly related to large viscosity gradient resulted from a small change in pressures higher than 2 GPa . It can be concluded that Roelands equation, implemented as piezoviscous part of the rheology model, fails to capture the right physical behaviour of the lubricant in the presence of large pressure gradients; this leads to fluctuations in viscosity values. Habchi et al. [97] measure viscosity behaviour at pressures up to 1 GPa using a falling body viscometer. The proposed Tait-Doolittle viscosity model is then validated against experimental data for pressures up to 1 GPa . The extended viscosity-pressure plots at $75 \text{ }^\circ\text{C}$ (similar to the current case) show a sharp increase in viscosity from the order of 10^4 Pa.s at 1 GPa to the order of 10^6 Pa.s at 1.2 GPa . In contrary, a change in the pressure from 0.8 GPa to 1 GPa results in pressure rise from the order of 900 Pa.s to the order of 10^4 Pa.s . This trend suggests that the change in viscosity at higher pressures are expected to be much sharper based on mathematical pieoviscosity models. However, the real response of the lubricant at such pressures is not reported in the literature and accurate data is required for precise modelling. The temperature of the lubricant is shown to relax this trend significantly. As in the pure rolling cases discussed above, the shear rate localised maxima occur in the proximity of the minimum film thickness (Figure 4-7 (b)). Temperature rises in the center of the contact by up to $10 \text{ }^\circ\text{K}$. The rheological model also affects the temperature and, therefore the temperature distribution may not capture the real behaviour of the contact. This needs to be addressed in future studies. The location of the maximum temperature is towards the surface of the cylinder. The viscosity behaviour is dominated by pressure rise; thermal and shear rate effects on viscosity are negligible in this case.

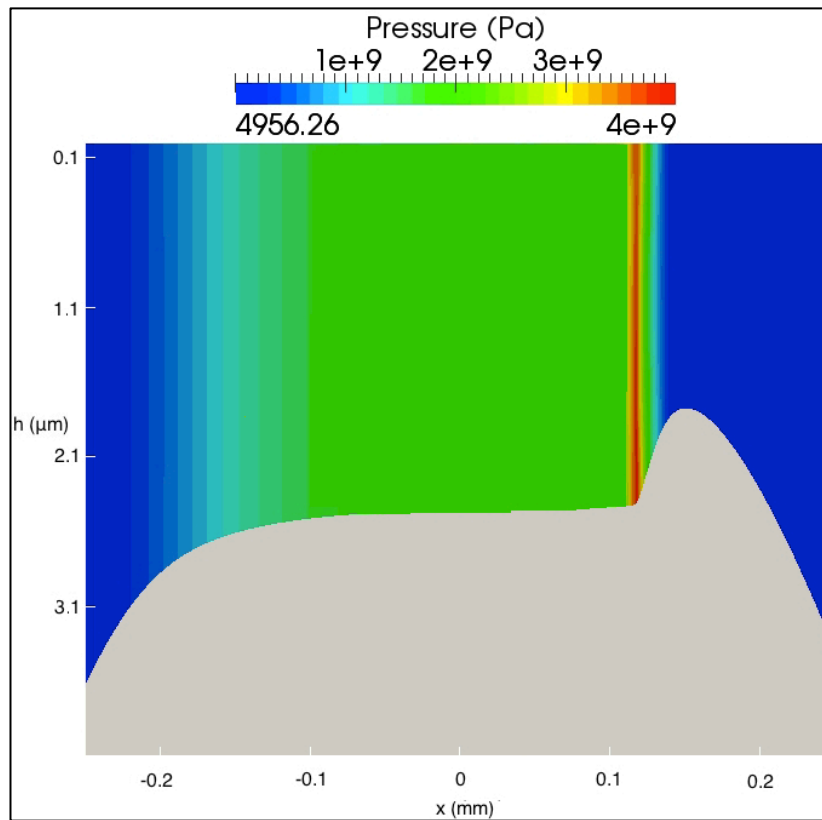


Figure 4-7. (a)

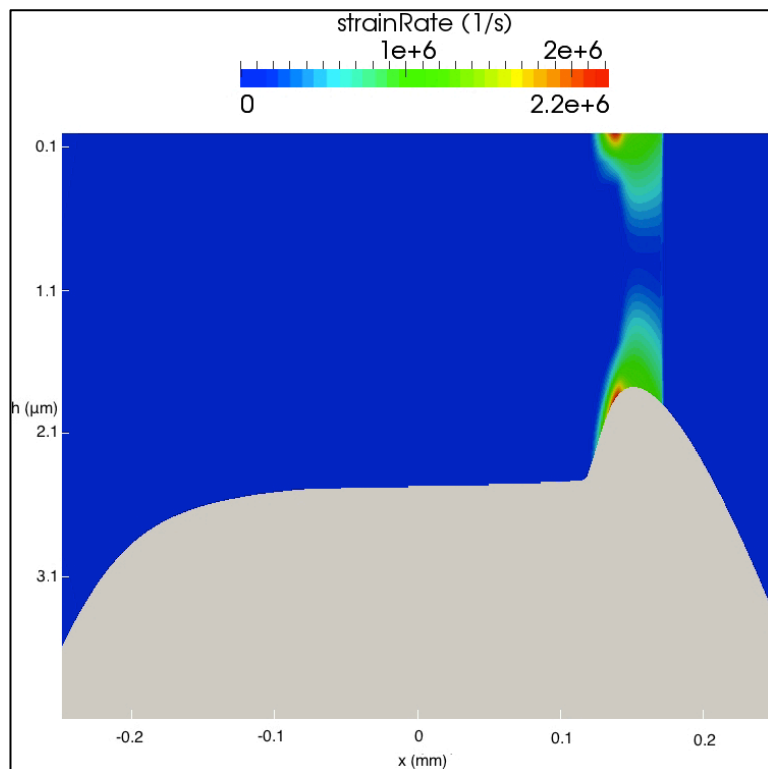


Figure 4-7. (b)

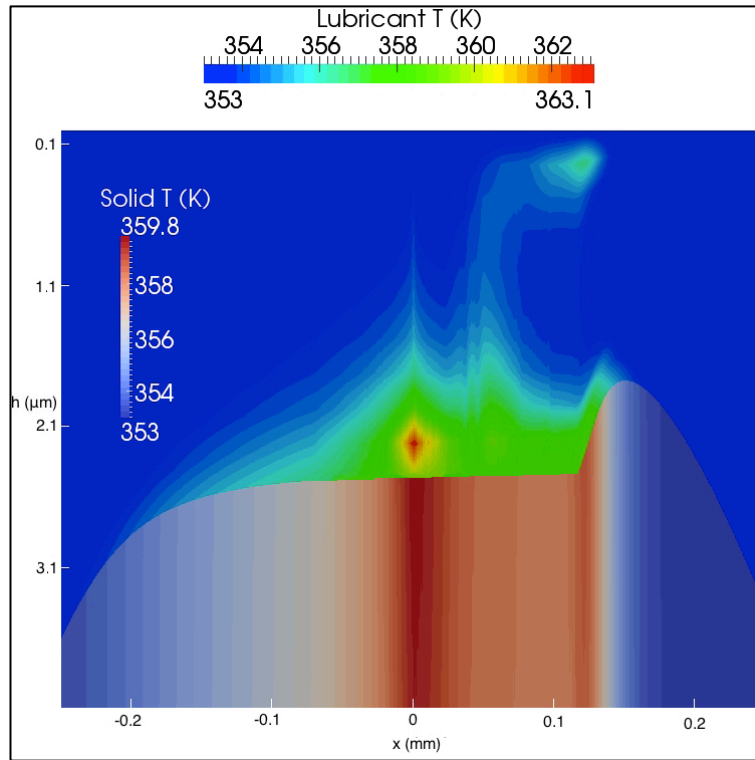


Figure 4-7. (c)

Figure 4-7. Thermal solution results for the case $SRR = 0$, $\eta_0 = 1 \text{ Pa.s}$, $E_{\text{Cylinder}} = 100 \text{ GPa}$, $L = 600 \text{ kN/m}$
 (a) Pressure (b) Shear rate (c) Temperature distributions

4.2 Frictional forces

The current model is capable to predict friction forces in an EHL contact for given loading condition and lubricant properties (the validity of the results is subject to appropriateness of piezo-viscosity and non-Newtonian rheology models). This makes it a powerful tool for improving operational efficiency through the optimisation of contact in terms of lubricant properties for given contact conditions.

The wall shear stresses at the rigid wall of thermal and iso-thermal cases are shown in Figure 4-8. The predicted shear stresses can be linked to the frictional forces experienced within the contact. The thermal pure sliding case (B) has interfacial shear forces (the integral of the shear stresses) 76% lower than the iso-thermal case (A). This is a consequence of the relative increase in lubricant temperature near the top sliding wall, which in turn results in lower viscosity and shear stress values. For case C ($SRR=1$), the predicted wall shear stress is higher than $SRR=2$ due to different shear rate and temperature distributions. The wall shear stresses of cases D and E,

both characterised by larger initial viscosity of 0.5 Pa.s and highr loads, are also presented. According to Björling et al. [112], friction force is not considerably varies with lubricant viscosity grade changes and the difference here is mainly related to the imposition of higher loads (400 kN/m) rather than higher viscosity grades.

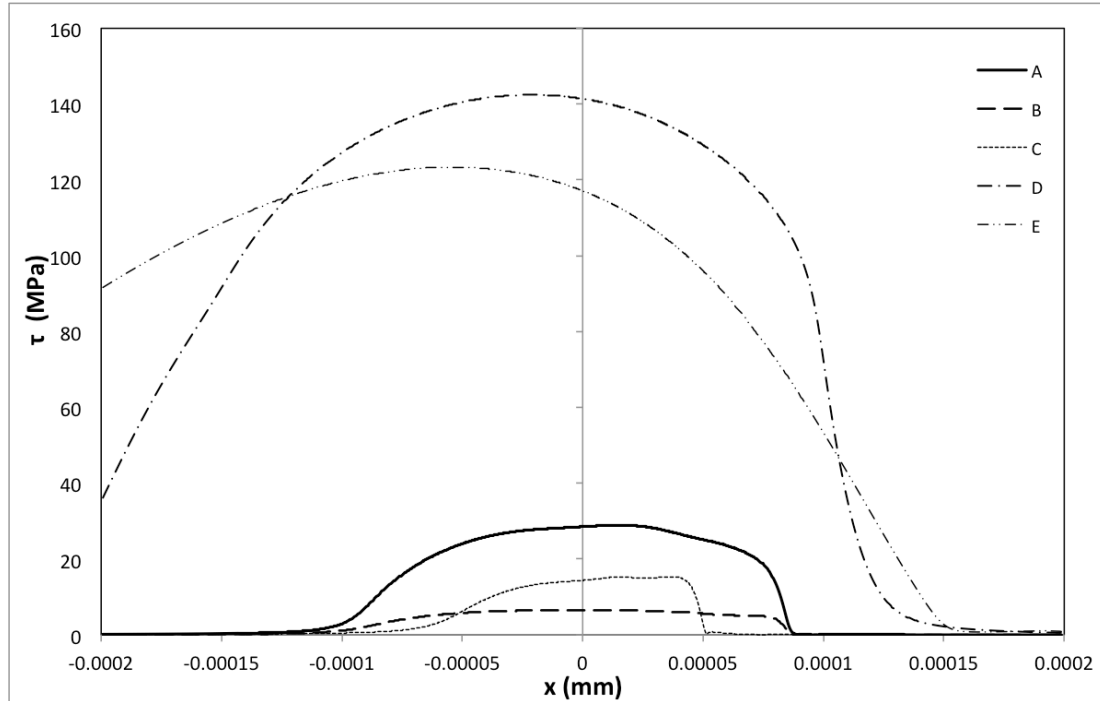


Figure 4-8. Predicted shear stress along the top sliding wall for *A*) isothermal pure sliding ($\eta_0=0.01$ Pa.s) *B*) thermal pure sliding ($\eta_0=0.01$ Pa.s) *C*) thermal SRR=1 ($\eta_0=0.01$ Pa.s) *D*) thermal SRR=0.4 ($\eta_0=0.5$ Pa.s) *E*) thermal pure sliding ($\eta_0=0.5$ Pa.s)

4.3 Cavitation

Density and vapour fraction distributions along the rigid wall are shown in Figure 4-9 for the full sliding case (SRR=2). Cavitation formation and reformation are evident in the plot. The density of the lubricant drops to as low as 40 kg/m^3 in the region very close to the end of the cavitating zone. The cavitating zone starts immediately after the constriction ($x = 0.1 \text{ mm}$) and ends at $x = 0.75 \text{ mm}$.

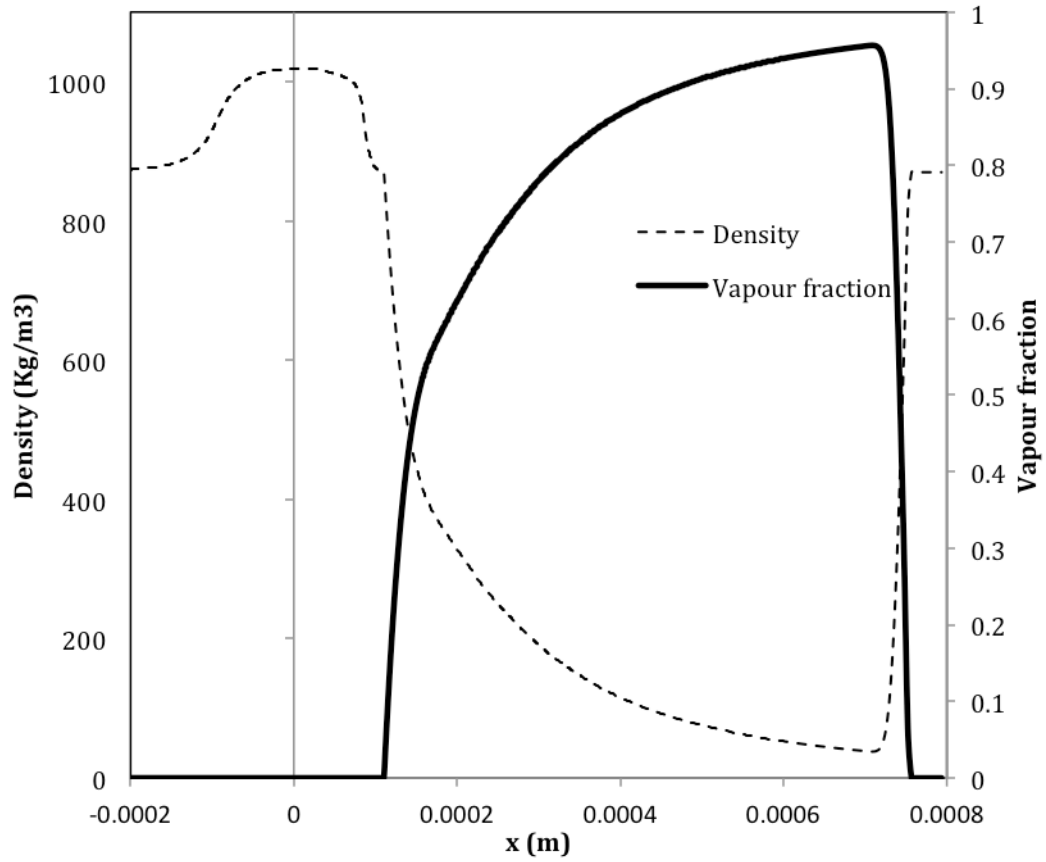


Figure 4-9. Density and vapour fraction along the rigid moving wall for an isothermal solution showing the cavitation and reformation along the contact (SRR=2 and $\eta_0=0.01 \text{ Pa.s}$)

It has been noted that cavitation starts at the stationary or lower speed surface in the contact. In order to further investigate the origin and propagation of the cavitation, vapour fraction distributions for two cases, (1) SRR=2, $\eta_0 = 0.01 \text{ Pa.s}$; and (2) SRR=0.4, and $\eta_0 = 0.5 \text{ Pa.s}$ (with equal entrainment velocity of 5 m/s) are presented in Figure 4-10 for both the cavitation initiation phases and the fully developed steady-state conditions. In the case of the flat wall sliding (SRR = 2), shown in Figure 4-10 (a) and (b), fully cavitated region sticks to the stationary cylinder surface. Cavitation originates on the stationary cylinder surface after $1.2 \mu\text{s}$ and the fully cavitated region is visible at 0.2157 ms . For the case of SRR=0.4 (wall speed = 2 m/s , cylinder speed = 3 m/s), illustrated in Figure 4-10 (c) and (d), it is evident that the cavitation starts on the slower wall surface at $3.2 \mu\text{s}$. The comparison of the two fully cavitated regions for the two cases (Figure 4-10 (b) and (d)) indicates that the length of the cavitation zone in the pure sliding case (SRR=2) is about 25% of that for SRR = 0.4.

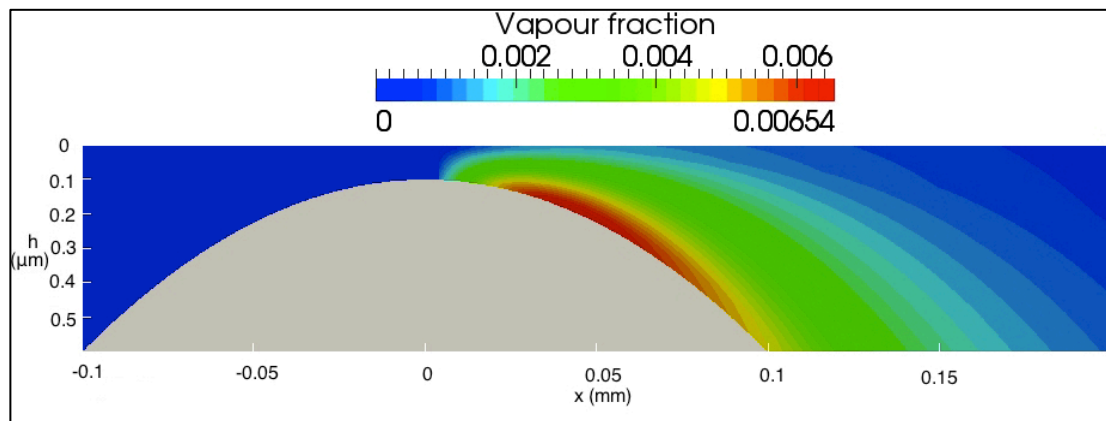


Figure 4-10. (a)

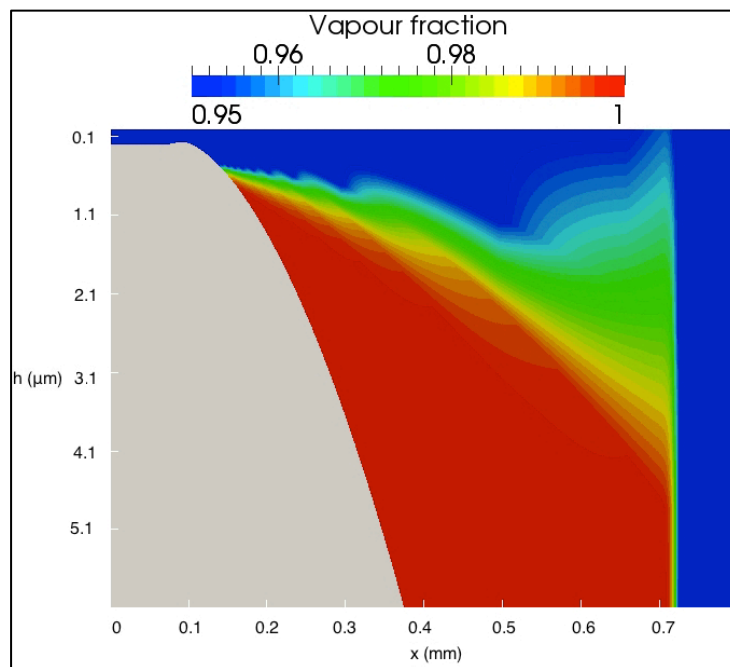


Figure 4-10. (b)

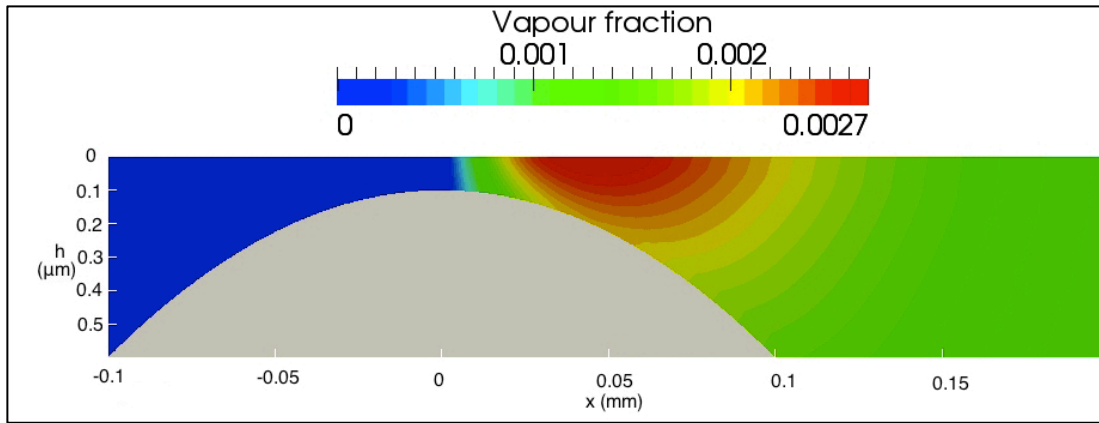


Figure 4-10. (c)

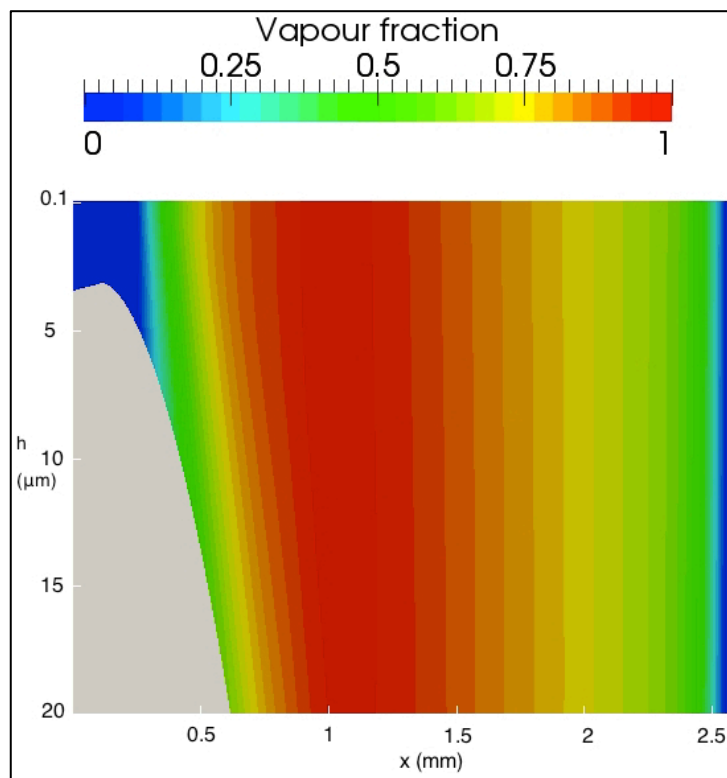


Figure 4-10. (d)

Figure 4-10. Vapour fraction distributions for: (a) $SRR = 2$, $\eta_0 = 0.01 \text{ Pa.s}$, $t = 1.2 \mu\text{s}$, (b) $SRR = 2$, $\eta_0 = 0.01 \text{ Pa.s}$, $t = 0.0002157 \text{ s}$, (c) $SRR = 0.4$ and $\eta_0 = 0.5 \text{ Pa.s}$ $t = 3.2 \mu\text{s}$, (d) $SRR = 0.4$ and $\eta_0 = 0.5 \text{ Pa.s}$, $t = 0.0004094 \text{ s}$

Most of the fluid passing near by the faster moving surface and this is in accordance with the fact that cavitation sticks to the slower moving surface (see Figure 4-11).

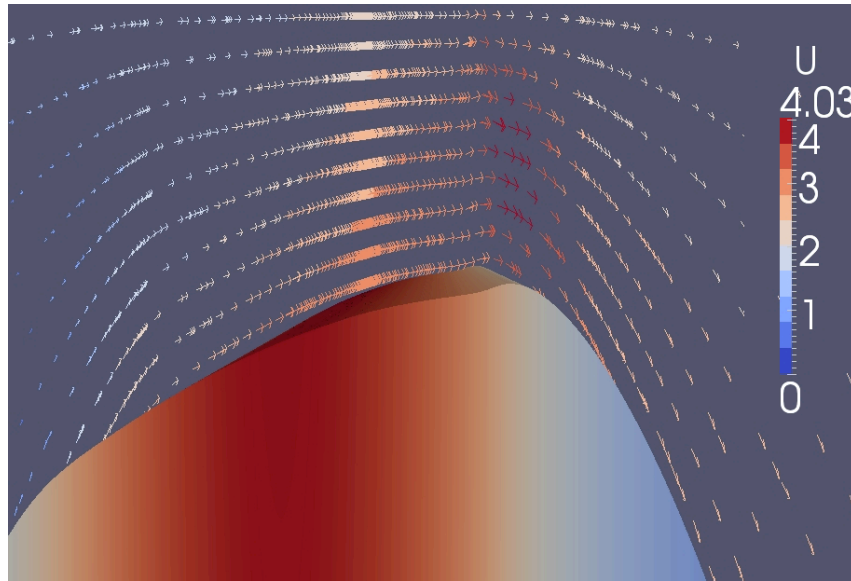


Figure 4-11. Velocity vectors over the cylinder surface

4.4 Non-adiabatic top wall

The thermal effects study is completed by accounting for heat conduction from the lubricant to the top moving wall (see Figure 4-12). This has been done by adding a flat plane of similar material as of the cylinder to the domain of study. The implementation of thermal solution is similar to cylinder heat conduction. The inclusion of non-adiabatic plane surface is tested for the case presented in Figure 4-4. It can be seen in Figure 4-13 that most of the heat are conducted through the faster moving wall (cylinder).

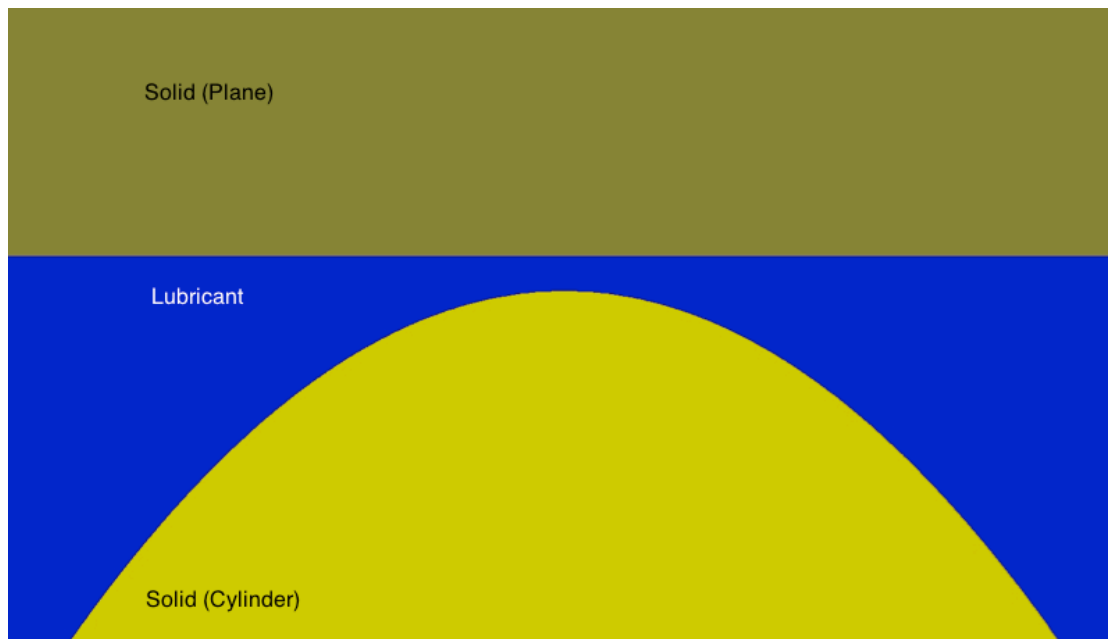


Figure 4-12. Full domain of study

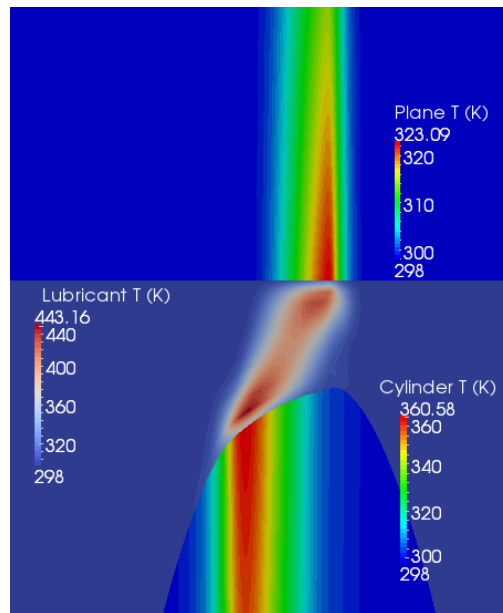


Figure 4-13. Temperature distributions at cylinder, lubricant and plane wall

4.5 Discussion

The viscosity behaviour is significantly different between a sliding and a pure rolling case. In a sliding case, the viscosity changes through the film thickness considerably due to shear-thinning; this is not the case for a pure rolling case. Moreover, the thermal effect is essential to be accounted for accurate prediction of lubricant viscosity, especially for the sliding case that undergoes higher temperature changes. Higher initial viscosity results in more complex flow patterns, characterised by high pressure, shear rates, viscosity and temperature gradients. High pressure and viscosity cases show small pressure variations through the film thickness whereas low viscosity cases show no significant pressure variation through the film thickness. The temperature rise is not significant in rolling conditions. Frictional forces predictions are very different in thermal and iso-thermal cases. The implemented rheology model is valid up to certain pressure range and shows its limitation in cases where the maximum pressures exceed 2 GPa . An improved rheology model must be implemented to overcome this drawback; however, the results presented in this chapter demonstrate the suitability of the solver, which has been demonstrated to be stable for very large values of pressures, to study conditions similar to those experienced in service by rolling element bearings.

4.6 Closure

The results for CFD modelling of a roller bearing (2-D case study) have been presented in this Chapter. The developed FSI model provides the following capabilities:

- Finite Volume solver for both solid and fluid domain in an EHL contact.
- The solver is not limited to the contact area; complete fluid and solid domains can be investigated, including the extent of the cavitated region and the development of the inlet conditions.
- Stability at high pressures and large deformations by using improved numerical tools; these include efficient coupling schemes and the implementation of high efficiency multi grid methods.
- Improved insights into EHL contact problems where Reynolds solution is not capable to predict lubricant film behaviour correctly (gradients through the fluid film and non-Newtonian effects).

Chapter 5

5.1 Surface Roughness Effects on EHL

Surface irregularities (scratches, dents, surface finishes due to various machining processes etc.) are known to be a potential source of damage in rolling/sliding contacts. Surface imperfection reduces the life of tribological components specifically in EHL contacts with relatively low film thicknesses. This chapter provides an introduction to the effects of surface roughness on EHL performance. In particular, the proposed FSI model is modified to investigate steady and transient surface roughness effects.

5.2 Literature review

The main aim of EHL studies is to reduce friction, damage and catastrophic failures of concentrated contacts. Friction is proportional to contact area and therefore on the number, size and shape of asperities. Moreover, since machined surfaces are not perfectly smooth on the scale of the film thickness and recent designs rely on higher loads, lower viscosity and higher temperatures, which result in lower film thicknesses, surface roughness effects are significantly important for accurate EHL modelling. Under highly loaded EHL contact and in the presence of high pressures, both the solid surface and the roughness deform. This can lead to a smoother or rougher surface depending on the contacting solid and fluid characteristics. Therefore, the features of this behaviour are necessary to be accounted for an accurate prediction of the response of bearing surfaces working in the EHL regime.

Tallian [98] proposed a formulation that relates bearing fatigue life to the film thickness/roughness ratio. In his work, the probability of fatigue failure is mainly related to the increased traction and surface stresses due to surface roughness and pressure fluctuations. Since this early work, there has been a large amount of work carried out to study the effects of roughness on EHL contacts. Lubrecht [99] studied EHL film behaviour in the presence of rough surfaces by introducing a travelling indentation and transverse wave. The numerical model is based on the Reynolds equation. The influence of the transient indentation and the effect of the transverse wave are investigated for both line and point contacts. Greenwood and Morales-Espejel [100] obtained the solution of an infinite EHL line contact with transverse sinusoidal roughness profile. The wavelength to semi-contact width ratio was assumed to be short. The results are presented for 1-D waviness in steady state sliding contacts. Greenwood and Morales-Espejel did not consider non-Newtonian effects while Greenwood and Johnson [101] assumed complete flattening of roughness under non-Newtonian conditions. Venner [102] studied the effects of waviness amplitude to film thickness ratio on the pressure and the film thickness predictions under rolling/sliding conditions assuming Newtonian, iso-thermal lubricant. Amplitude reduction was found to depend on amplitude, wavelength and SRR. Almqvist [103] studied the effects of a ridge passage through the contact using both CFD and the Reynolds equation based approaches. Furthermore, two-sided roughness was

investigated by considering different asperities profiles including ridge-ridge, dent-ridge and dent-dent. The predicted pressure, film thickness and over-taking effects are described for different SRR. Hooke [104] implemented a Reynolds based approach to study the effects of a travelling cosine wave and ridge through the contact. The calculated film thickness showed to be in agreement with measured data near to the fast-travelling ridge for different rolling/sliding conditions. However, there are differences in slow-travelling ridge calculated results and experimental data which can be related to the idealised ridge profile adopted in the calculation. It was shown that complete flattening of roughness in non-Newtonian conditions is invalid. Kumar [105] found that the polymeric fluid additives could significantly reduce surface roughness effects and recommended a Newtonian-power law lubricant mixture to minimise surface roughness effects. Holmes et al. [106] developed a finite difference based method to study experimental rough surface profile effects on pressure and film thickness distributions in point and line contacts. It was shown that line contact solution is sufficient to determine pressure and film thickness on the centre-line of the contact. Mourier et al. [107] simulated a transient deep and shallow micro-cavity propagating along micro-machined steel ball (point contact) using a 2D-multigrid Reynolds based solver. Almqvist and Larsson [108] showed that the Reynolds based approach solutions are valid for rough surfaces with film thickness to wavelength ratios down to 0.01. Almqvist and Larsson [109] studied two-sided surface roughness effects for different sliding/rolling conditions. It was noted that film breakdown would probably occur due to low pressure or internal cavitation. However, no actual breakdown was observed in the presented cases. Wang [113] used Hertzian contact theory coupled with the Reynolds equation to study surface waviness effects. It was shown that the roughness effect is mainly dominated by the wave-to-inlet length ratio and SRR. The behaviour is defined by a proposed master curve that relates deformed-amplitude to initial-amplitude ratio as a function of the roughness wavelength.

5.3 Transient governing equations

In order to capture transient effects due to the presence of surface features present at the contact interface, the FSI methodology developed in chapter 3 and used in chapter 4 to study smooth contacts requires to be modified. There are three necessary modifications to the FSI model to add transient effects. The first is the inclusion of cell boundary movement speed (u_b) to the convection term (see Equation 2-55).

$$\int_V \nabla \cdot (\rho(u - u_b)\phi) dV = \sum_f S_f \cdot (\rho(u - u_b)_f) \phi_f \quad \text{Equation 5-1}$$

Secondly, the mass conservation (for moving meshes) is enforced using space conservation law (the change in volume of a cell has to be equal to the volume swept by the faces of a cell).

Thirdly, the temporal derivative of density term at environment pressure ($\rho_{l,0}$), in Equation 2-82 and 2-83, is necessary to be accounted for transient study since the time derivatives are evaluated using the Euler implicit method as:

$$\frac{\partial}{\partial t} \int_V \rho \phi dV = \frac{(\rho_P \phi_P V)^n - (\rho_P \phi_P V)^0}{\Delta t}, \quad \text{Equation 5-2}$$

where the volume V changes in the case of mesh movement. Assuming the mixture density equation as:

$$\rho = \alpha \rho_v + (1 - \alpha)(\rho_{l,0} + \psi_l p), \quad \text{Equation 5-3}$$

the modified temporal density derivative can be written as:

$$\frac{\partial \rho}{\partial t} = \frac{\partial \alpha}{\partial t} (\rho_v - \rho_l) + \alpha \frac{\partial \rho_v}{\partial p} \frac{\partial p}{\partial t} + (1 - \alpha) \psi_l \frac{\partial p}{\partial t} + (1 - \alpha) \frac{\partial \rho_{l,0}}{\partial t} \quad \text{Equation 5-4}$$

and the pressure Equation 2-83 is modified accordingly.

5.3.1 Adaptive time-step

In order to maintain stability for highly transient case studies, it is necessary to adjust the time step dynamically. The time step is calculated based on the Courant–Friedrichs–Lewy (CFL) condition:

$$CFL = \frac{U_f \cdot d}{|d|^2} \Delta t \quad \text{Equation 5-5}$$

where Δt is the time step, d is the length vector between two neighbouring cell centres and U_f is the velocity of the flow. The time-step is set according to the maximum CFL number of 1. There is also the maximum time step value prescribed to capture time-dependent features which is set to 10^{-9} s for all cases presented in this

section. The solver relative tolerance (the ratio of current to initial residuals) is set to 0 to force the solution to converge to the solver tolerance at each time step.

It is worth mentioning that as the deflection is under-relaxed for stability reasons, the elastic deformation solution and the fluid solver are iterated within a time step until equilibrium is reached.

5.4 Elementary surface features

The first part of the rough-transient EHL study presented in this thesis focuses on the elementary surface features, such as ridges and dents, which are passing through the conjunction of an EHL line contact. In a pure rolling case, since both surfaces are not moving relative to each other, the surface roughness can be assumed to be only on one surface of the contact. Therefore, the film thickness changes transiently just as a result of the surface feature motion. In mixed rolling-sliding conditions the roughness profiles of each surface are moving relative to each other and asperity “overtaking” phenomena occur. The height range of surface features used in this section is selected in accordance to experimental studies carried out by Choo et. al. [110].

5.4.1 Ridge effects on EHL film behaviour

The basic ridge geometry is schematically represented in Figure 5-1. The asperity-lubricant interaction is investigated while the surface feature enters the contact and is passing along the EHL conjunction. The ridge travels inside the contact with the same speed as the surface to which it is attached (cylinder rotation speed). For every time-step, an un-deformed ridge is moved to the new position. To better understand the ridge interaction with the surrounding lubricant, two different film thickness sizes are considered in the presence of a ridge which belongs to the elastic surface. To obtain meaningful results and understand the effect that the ridge has on the system response, the ridge amplitude is chosen based on the minimum film thickness predicted for the equivalent smooth cases. The first case study is a sliding-rolling contact with a ridge attached to the slower moving surface. The case conditions are summarised in Table 5-1:



Figure 5-1. Single travelling ridge

Parameter	Value
Cylinder radius	$R = 10 \text{ mm}$
Domain length	$L = 120 \text{ mm}$
Young's modulus	$E_{\text{Cylinder}} = 100 \text{ GPa}$
Minimum film thickness	$h_i = 0.14 \text{ } \mu\text{m}$
Initial viscosity	$\eta_0 = 0.01 \text{ Pa.s}$
Ridge amplitude	$A = 0.05 \text{ } \mu\text{m}$
Ridge length	$\lambda = 2 \cdot 10^{-5} \text{ m}$
Sliding speed	$u_1 = 3.75 \text{ m/s}$
Rotating speed	$u_2 = 1.25 \text{ m/s}$

Table 5-1. Thin film ridge parameters

The effects of the ridge passage through the EHL conjunction on pressure, viscosity and shear rate are given for intermediate time steps (Figure 5-2 to Figure 5-5). The ridge entrance to the contact region causes a pressure peak near to the inlet, since the film thickness is reduced locally. Higher shear rates near the ridge result in shear thinning and lower viscosity in the contact area (see Figure 5-2 (c)). The localised high viscosity near to the ridge (towards the inlet) is induced by the local pressure rise. The ridge motion towards the contact centre results in a secondary pressure spike right before the ridge (moving constriction). The increase in secondary pressure spike value is related to the film thickness profile difference between the ridge upstream and downstream. The sharp pressure rise in the contact makes it necessary to use rough surface (cylinder) speed in CFL calculations for larger amplitude to wavelength ratios (in order to preserve $u_{\text{ridge}} \Delta t \leq \Delta x$). The choice of time step is greatly important to maintain stability and to capture flow behaviour at every time step. The localised minimum shear rate results in viscosity rise (up to 37 Pa.s) opposite to the ridge location on the faster moving surface. The maximum value of the shear rate near to

the ridge increases when the surface feature moves towards the contact centre; this corresponds to a relative reduction of film thickness (see Figure 5-4 (d)). The pressure spike value (right before the constriction) rises initially and starts to drop when the ridge passes $x=0.05 \text{ mm}$ (see Figure 5-4 (a) and (b)). The pressure spike disappears as the ridge reaches the contact centre. The pressure decreases since less fluid passes through the contact and the ridge is moving slower than the sliding wall. As the ridge approaches toward the constriction (see Figure 5-5), the pressure drops significantly near to the centre of the contact due to double constriction occurrence.

In comparison to the results presented by Almqvist et al. [103] (where minimum film thickness is $0.21 \text{ }\mu\text{m}$, $A=0.2 \text{ }\mu\text{m}$ and $\lambda=100 \text{ }\mu\text{m}$), the ridge passage effect on pressure distribution is lower in their work. This can be traced back into the differences in ridge profile size and equation. The implemented ridge profile by Almqvist, increases the instantaneous film thickness at the start of ridge profile which leads to lower increase in pressure values right before the ridge. Also, the film thickness to wavelength ratio is 3.33 times higher in the results presented here which leads to larger deviations.

As the rough surface is moving slower than that of the smooth surface, the film thickness downstream is remarkably affected by the ridge motion. It was also noted by Kaneta et al. [111] through direct observations on different types of patterns of transversely oriented ridges (maximum height of about $0.6 \text{ }\mu\text{m}$) using the optical interferometry technique under sliding rolling conditions.

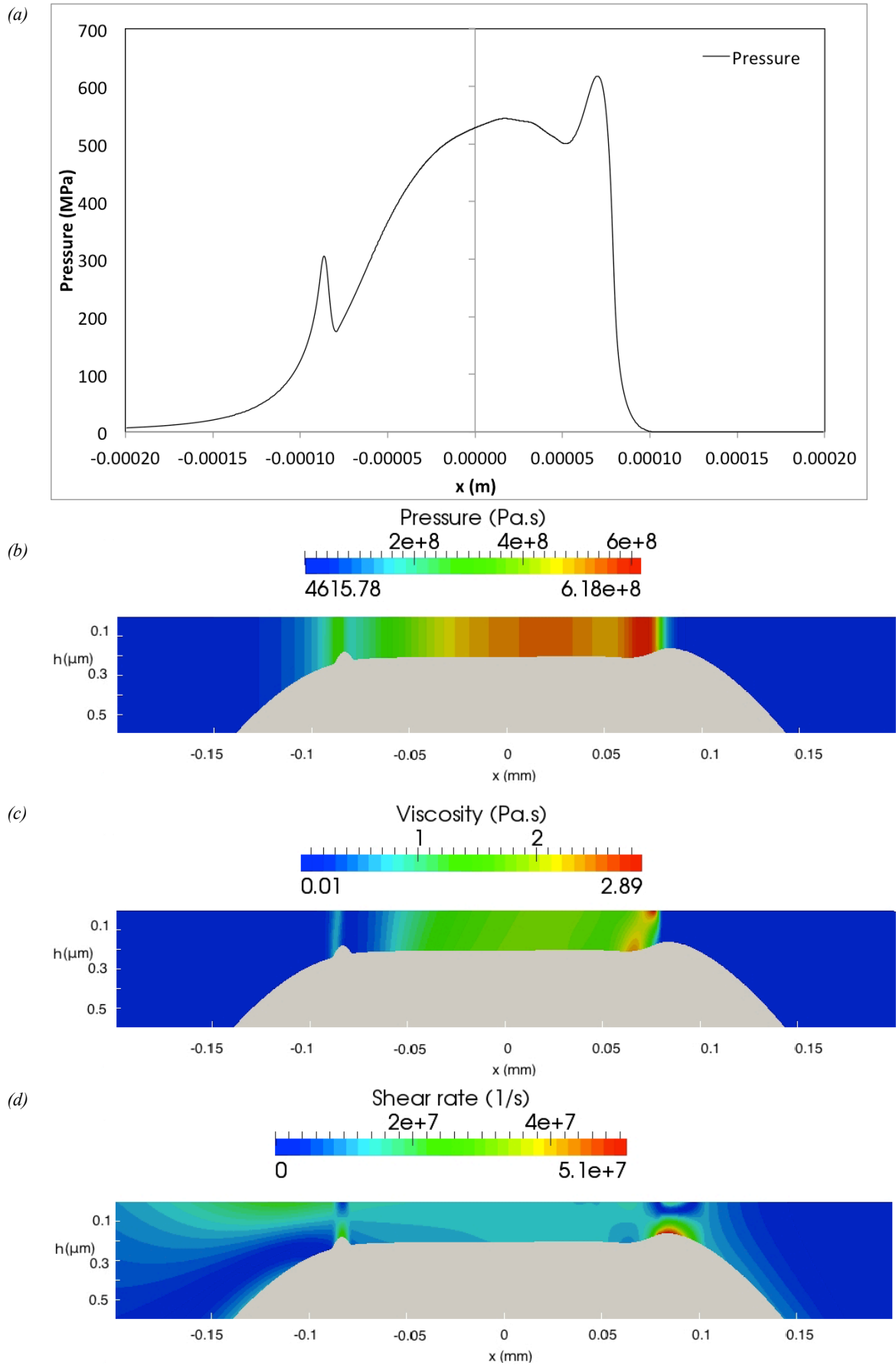


Figure 5-2. (a) Pressure along top wall (b) pressure (c) viscosity (d) shear rate distributions at $t=0.0008884$ s

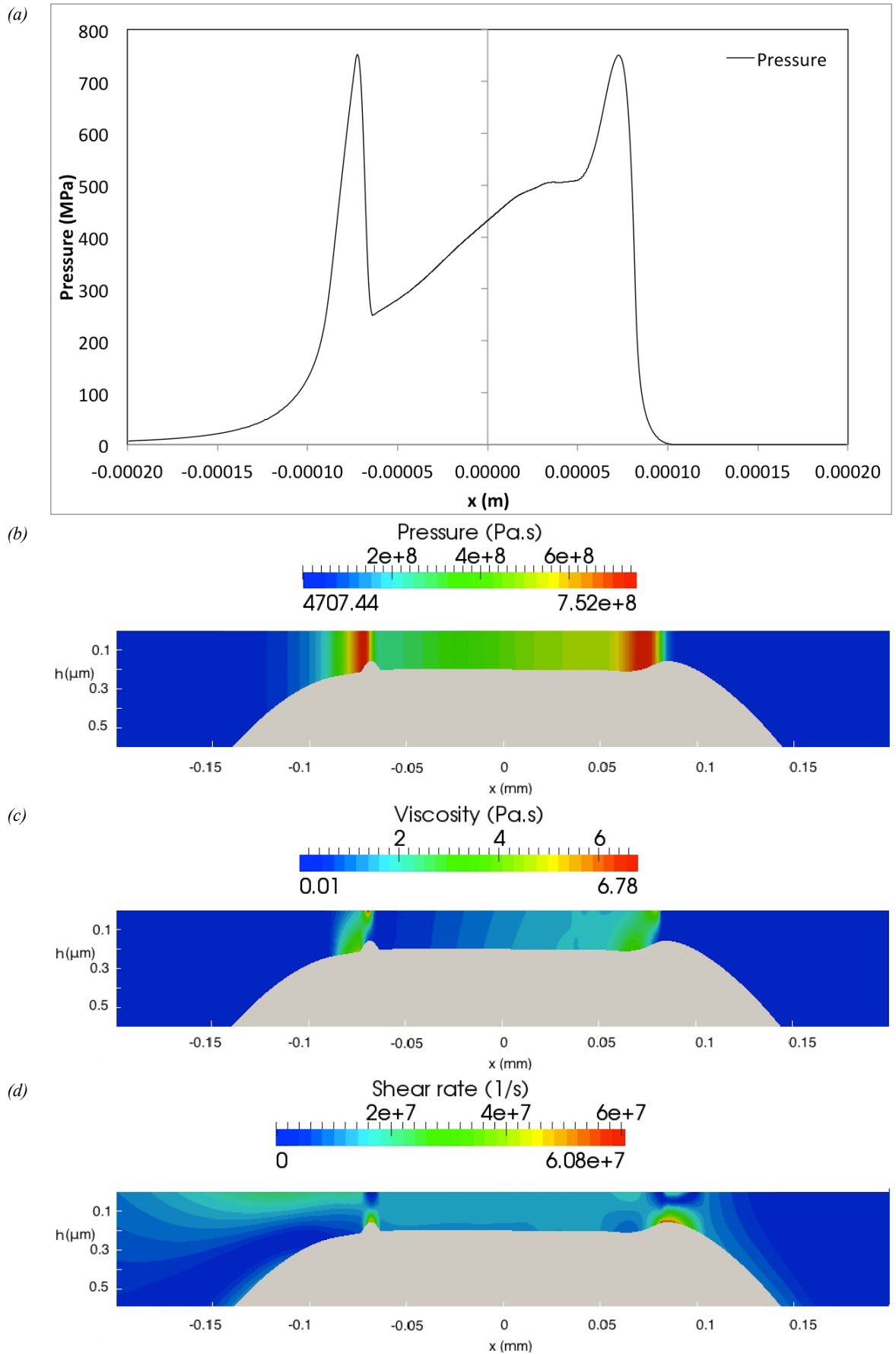


Figure 5-3. (a) Pressure along top wall (b) pressure (c) viscosity (d) shear rate distributions at $t=0.0009004$ s

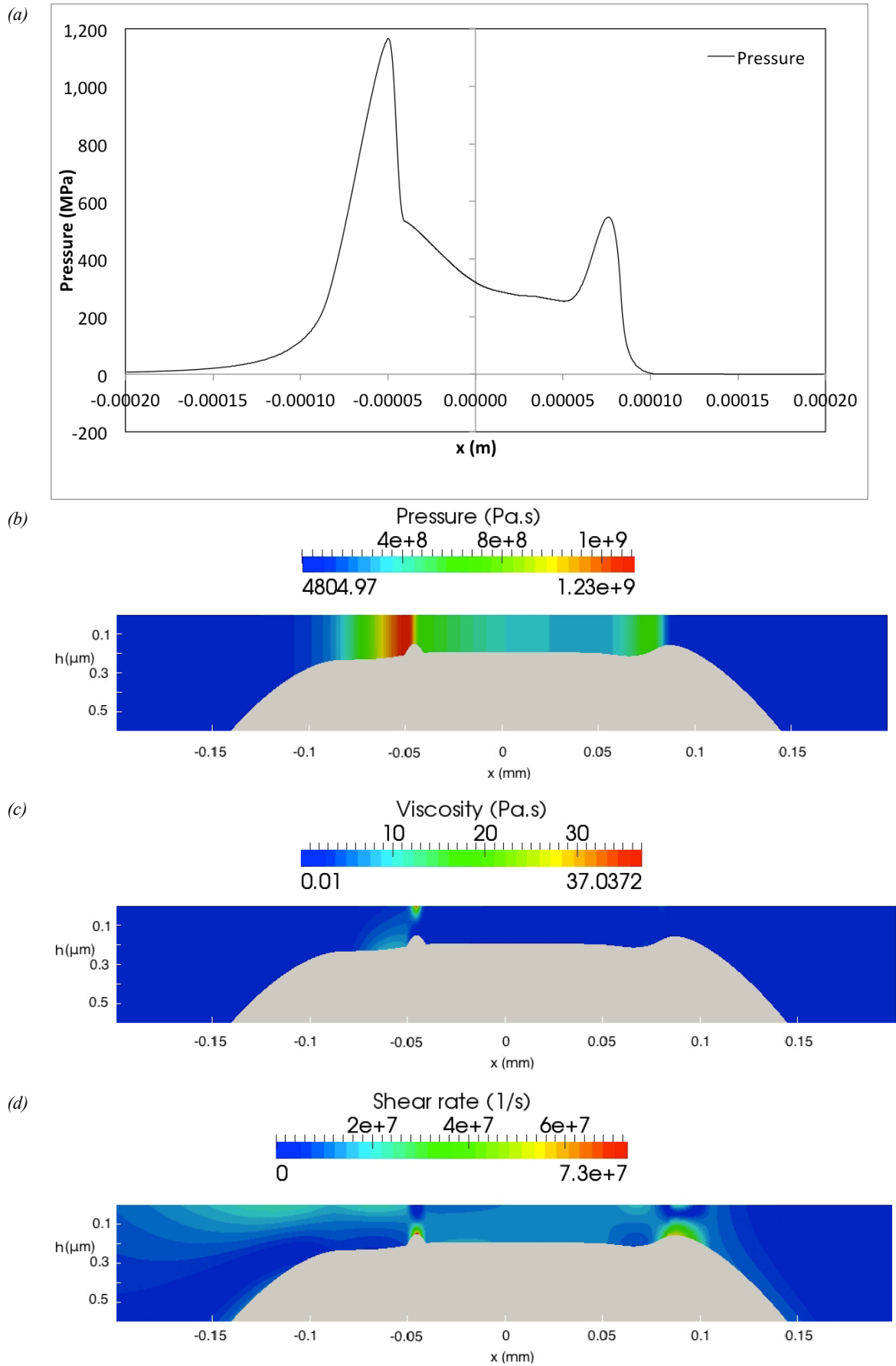


Figure 5-4. (a) Pressure along top wall (b) pressure (c) viscosity (d) shear rate distributions at $t=0.0009188\text{ s}$

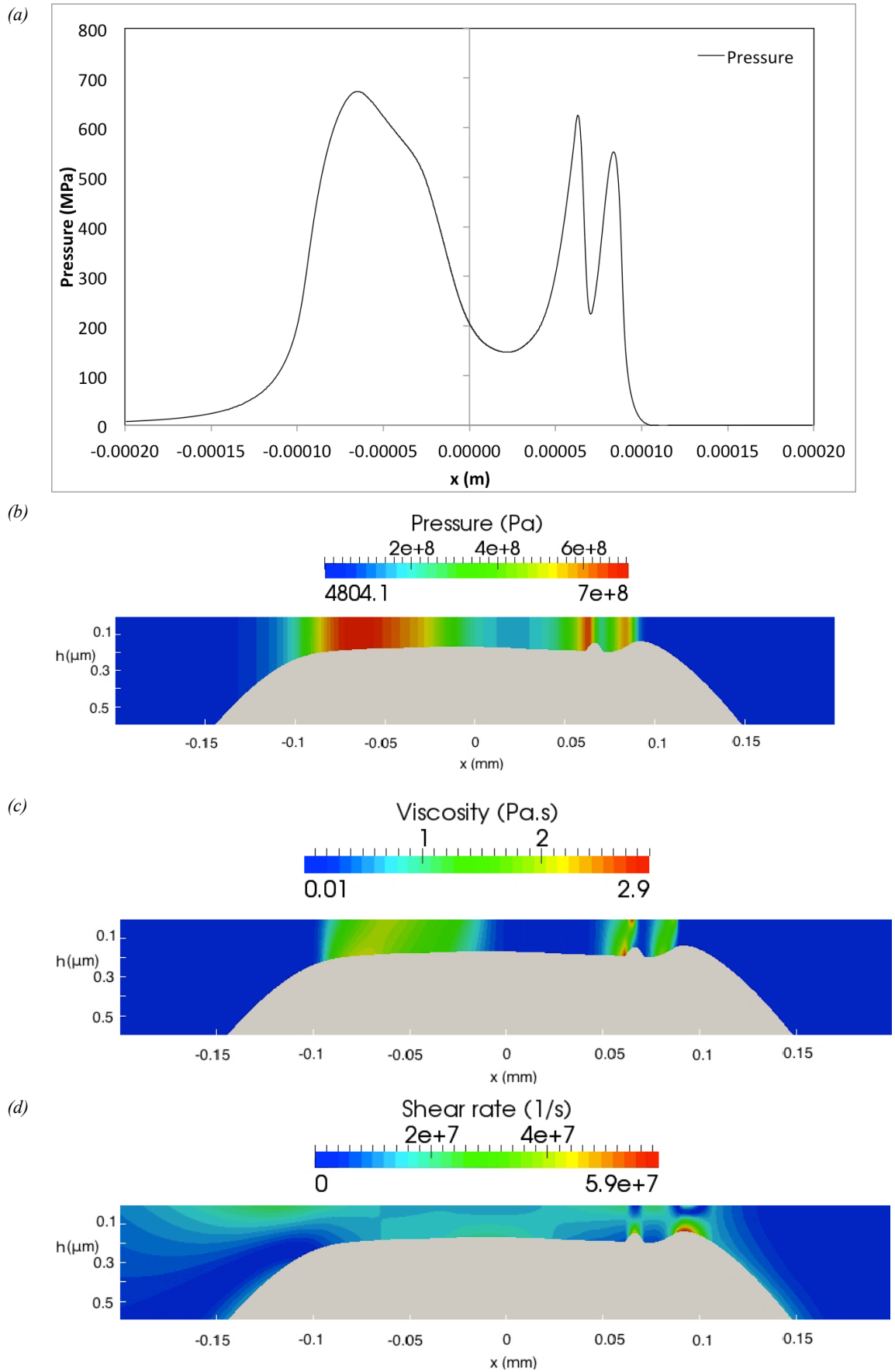


Figure 5-5. (a) Pressure along top wall (b) pressure (c) viscosity (d) shear rate distributions at $t = 0.0010084 \text{ s}$

The effects of the ridge's passage through a relatively large EHL film are analysed for one of the cases studied in Chapter 4. The case parameters are summarized in Table 5-2.

Parameter	Value
Cylinder radius	$R = 10 \text{ mm}$
Domain length	$L = 120 \text{ mm}$
Young's modulus	$E_{\text{Cylinder}} = 200 \text{ GPa}$
Minimum film thickness	$h_i = 3.35 \text{ }\mu\text{m}$
Initial viscosity	$\eta_0 = 0.5 \text{ Pa.s}$
Ridge amplitude	$A = 0.2 \text{ }\mu\text{m}$
Ridge length	$\lambda = 2 \cdot 10^{-5} \text{ m}$
Sliding speed	$u_1 = 2 \text{ m/s}$
Rotating speed	$u_2 = 3 \text{ m/s}$

Table 5-2. Thick film case parameters

The pressure, viscosity and shear rate distributions of different time instants are shown in Figures 5-6 and 5-7. There is pressure variation through the film thickness near to the ridge location (see Figure 5-6 (a)). The localized maximum viscosity distribution is located near the slower moving surface. Large changes in viscosity through the film thickness can be identified which are related to the pressure gradients induced by the presence of the ridge (see Figure 5-7 (a)). As a result, viscosity drops from the order of 20000 *Pa.s* near the moving ridge surface to 100 *Pa.s* near to the middle of the film thickness (see Figure 5-7 (b)). The ridge approach towards the constriction results in significant pressure variation through the film thickness. It can be seen from Figure 5-8 that Reynolds-based calculations, which neglect the variation of pressure and viscosity through thickness, can lead to pressure under-estimation in the presence of surface features. The three traces identified by Pressure-ridge, Pressure-plane and Pressure-middle indicate pressure distributions along a straight line at $z=3.3 \text{ }\mu\text{m}$, at the top wall and along a straight line at $z=1.7 \text{ }\mu\text{m}$ respectively where z is measured from the rigid sliding wall. The changes in pressure through the film are significant and of the order of 500 *MPa*. The comparison to low film thickness case study reveals the effect of amplitude to film thickness ratio on lubricant film behaviour in the presence of ridge. The lower is the ratio; the lower is the deviation in pressure distribution. However, the pressure change through the film was only evident in large film thickness case where the amplitude to film thickness ratio is

lower. Since larger pressure gradient can be essentially effective on pressure distribution in the contact, further studies are necessary for the evaluation of amplitude to film thickness ratio effect on lubricant behaviour. Such pressure gradient will change the contact stress profiles, leading to stress concentration at the surface. In highly loaded surfaces in contact, this can be a critical parameter for failure prediction.

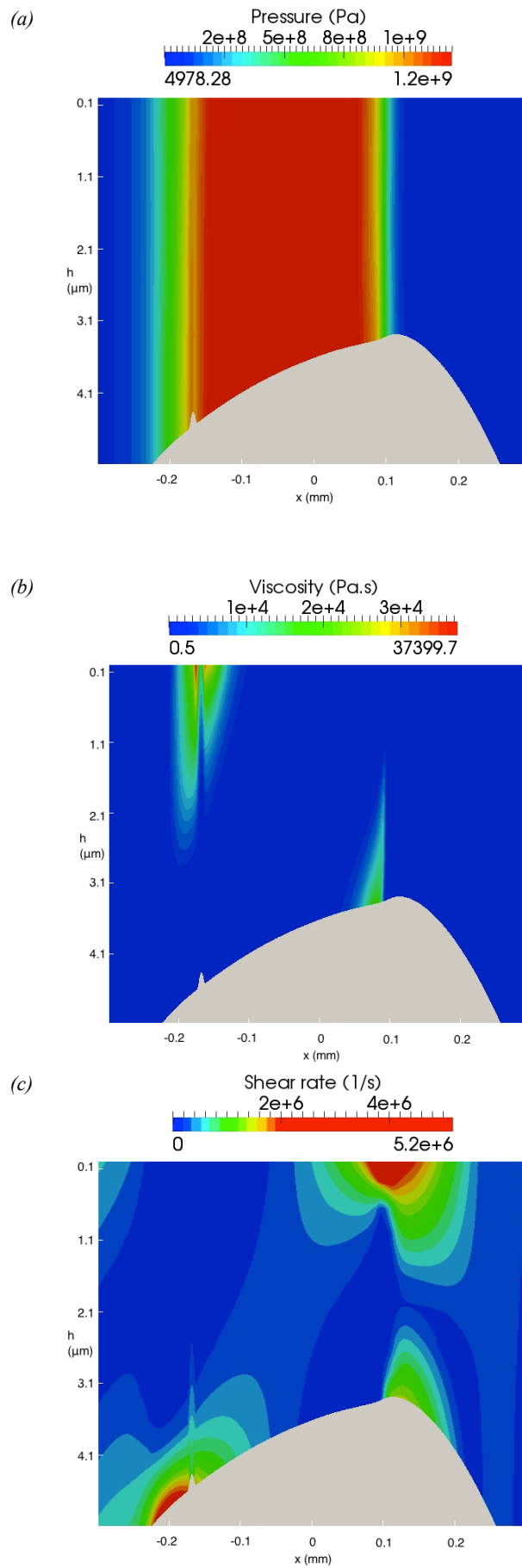


Figure 5-6. (a) Pressure (b) viscosity (c) shear rate distributions at $t=0.0004498\text{ s}$

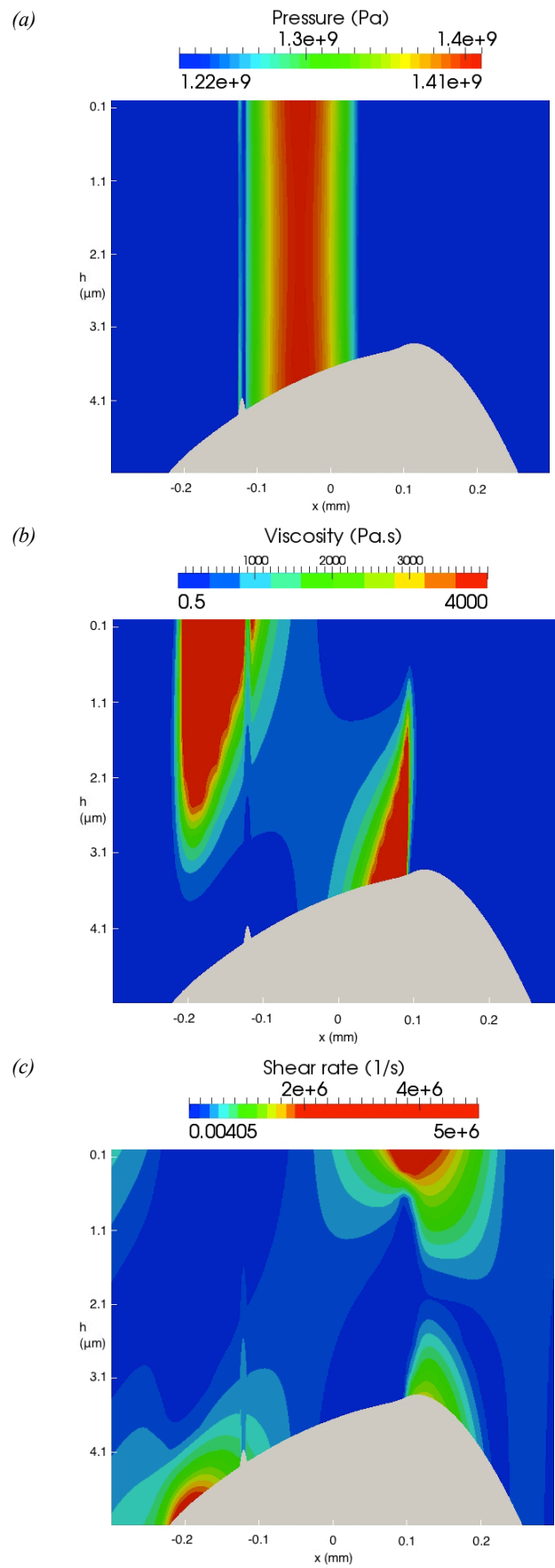


Figure 5-7. (a) Pressure (b) viscosity (c) shear rate distributions at $t=0.0004524$ s

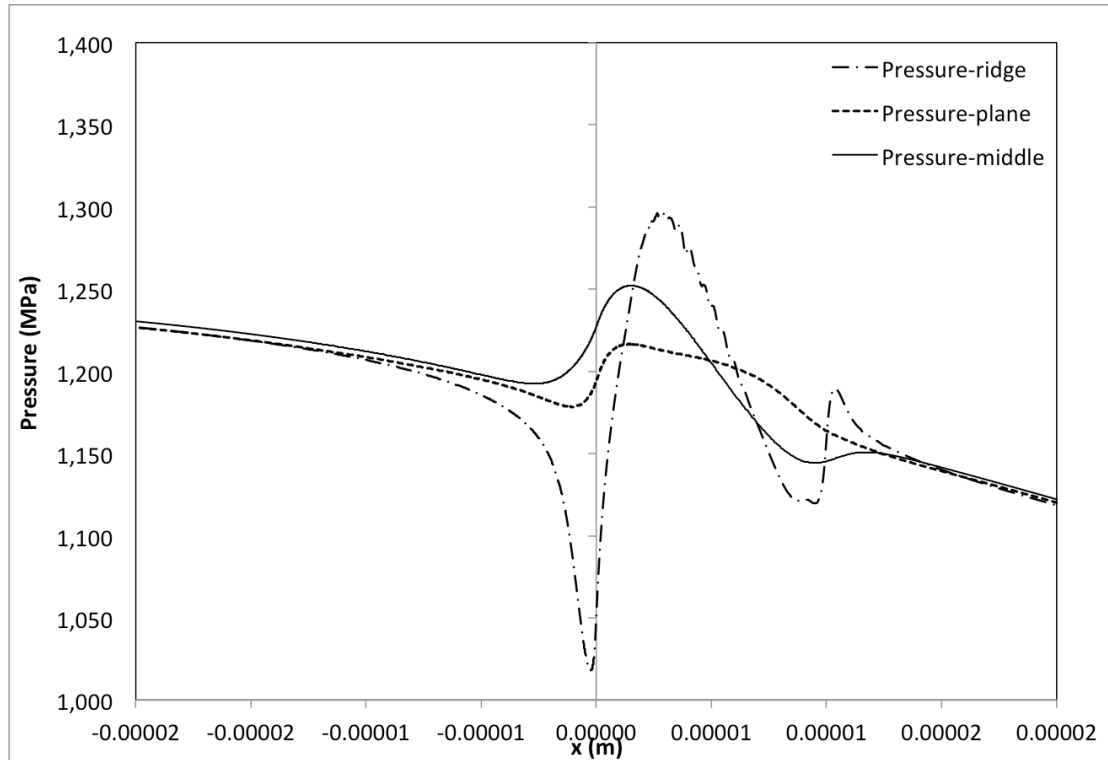


Figure 5-8. Pressure distributions at the top wall middle of the film and near to the ridge - $t=0.000494$ s

5.4.2 Dent effects on EHL film behaviour

The basic dent geometry analysed in this thesis is shown in Figure 5-9. The dented surface belongs to the moving deformable cylinder. The effects that the presence of a dent travelling through the contact have on the behaviour of the EHL film are investigated for two case studies with parameters similar to those adopted in section 5.4.1 for the ridged surface.

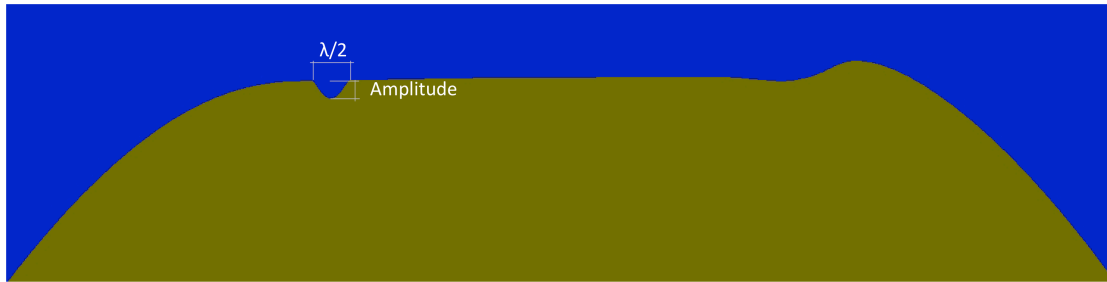


Figure 5-9. Single travelling dent

The first case study parameters are shown in Table 5-1. The dent entrance to the EHL conjunction reduces the pressure throughout the contact and increases the value of the pressure spike (see Figure 5-11 (a) and (b)). A similar behaviour is observed while the dent is moving toward the middle of the contact (see Figure 5-12(a)). The pressure spike value starts to decrease after $x_{\text{dent}}=0$ and significant pressure rise before the dent is observed (see Figure 5-13 (a)). As less lubricant flow towards the dent upstream, the overall pressure drops in the centre of the contact while the dent approaches the middle of the contact (see Figure 5-11 (a) and Figure 5-12 (a)). Viscosity rises inside the dent up to 8.43 Pa.s due to large pressure and low shear rates (see Figure 5-13 (c)). Pressure drop near to the leading edge of the dent results in lower viscosity distribution. The pressure distribution in leading and trailing edge of the dents suggest that the local stresses are higher in trailing edge. The film thickness downstream of the dent is essentially affected (see Figure 5-13).

The second case study parameters are shown in Table 5-2 (section 5.3.1). The results for a moving dent at one time instant are shown in Figure 5-14. The change in the effective heat conduction area results in different thermal distribution (see Figure 5-14 (c)). The shear-thinning (see Figure 5-14 (b)) and thermal behaviour of the lubricant

affect the viscosity distribution through the film thickness near to the dent (see Figure 5-14 (c)). Such rheological behaviour can significantly influence lubricant flow in the inlet.

The results for dent and ridge passage through an EHL conjunction for a relatively large film thickness reveal different effects on pressure and viscosity distributions that needs to be considered for surface finishing accuracy in accordance to the operating conditions. Several ridges and dents can largely influence the pressure and shear rate profiles and consequently viscosity distribution which leads to substantial increase in friction.

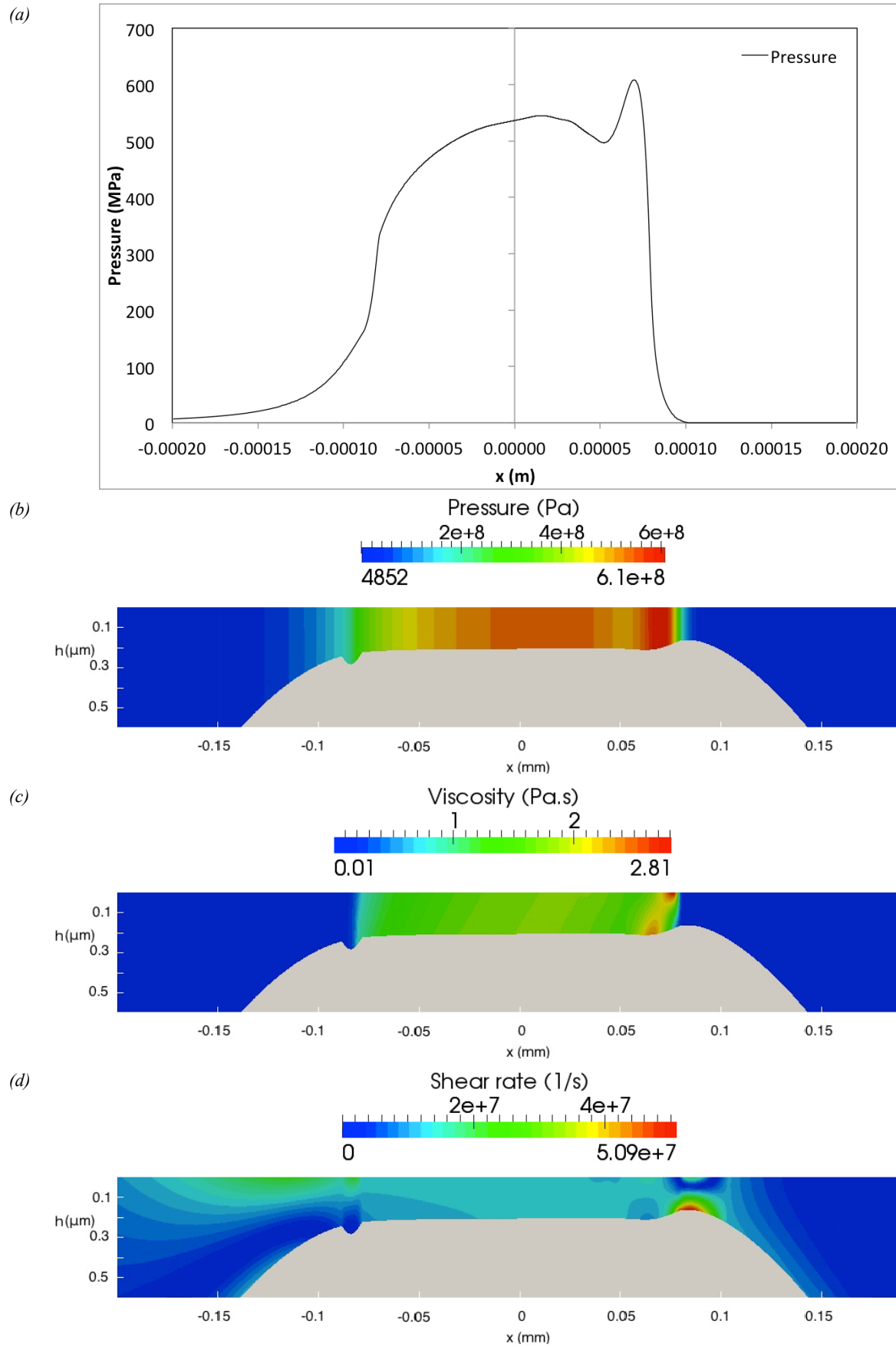


Figure 5-10. (a) Pressure top sliding wall (b) pressure (c) viscosity (d) shear rate distributions at $t=0.0008884$ s

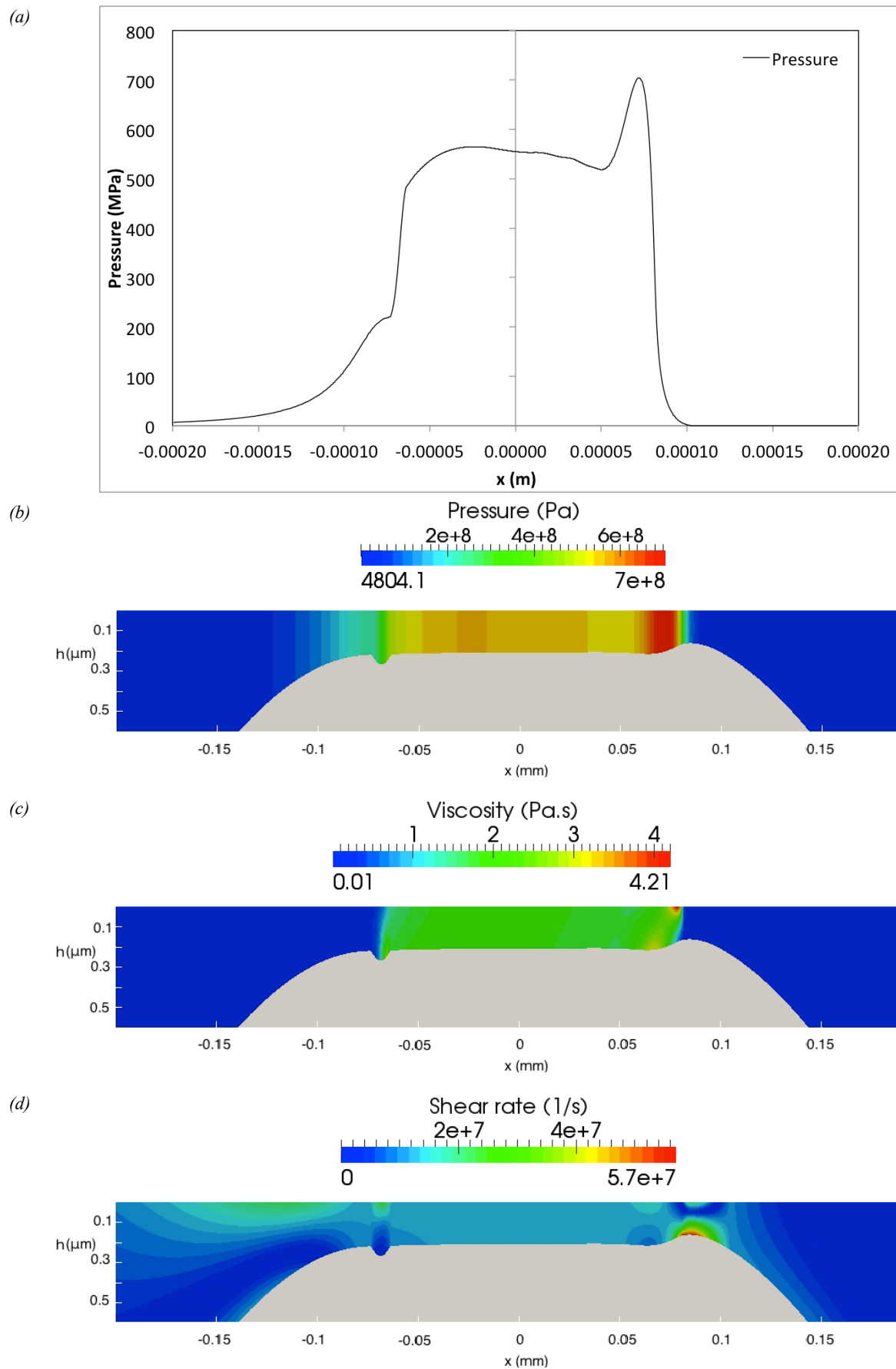


Figure 5-11. (a) Pressure top sliding wall (b) pressure (c) viscosity (d) shear rate distributions at $t = 0.0009004 \text{ s}$

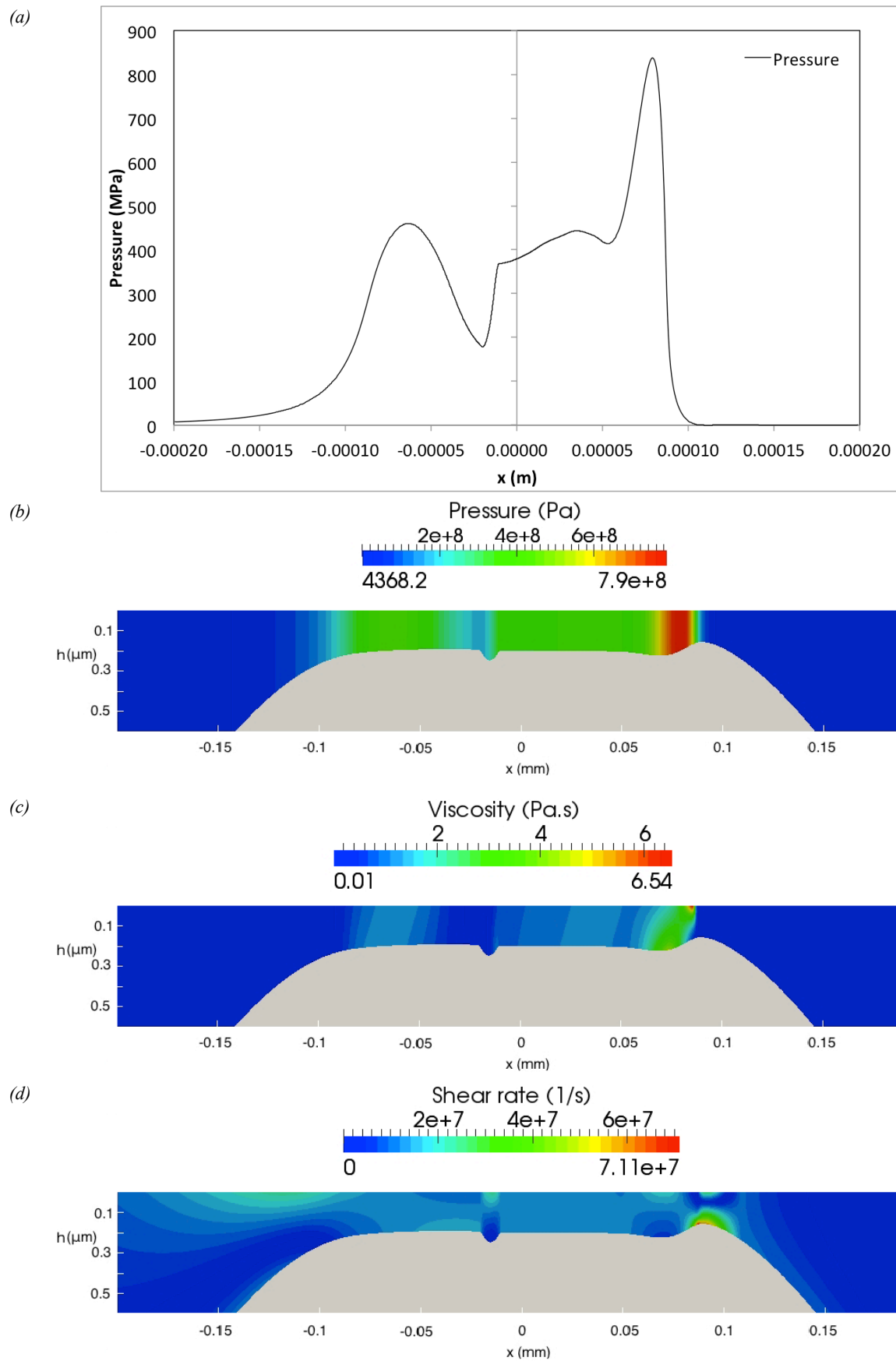


Figure 5-12. (a) Pressure top sliding wall (b) pressure (c) viscosity (d) shear rate distributions $t = 0.0009428$ s

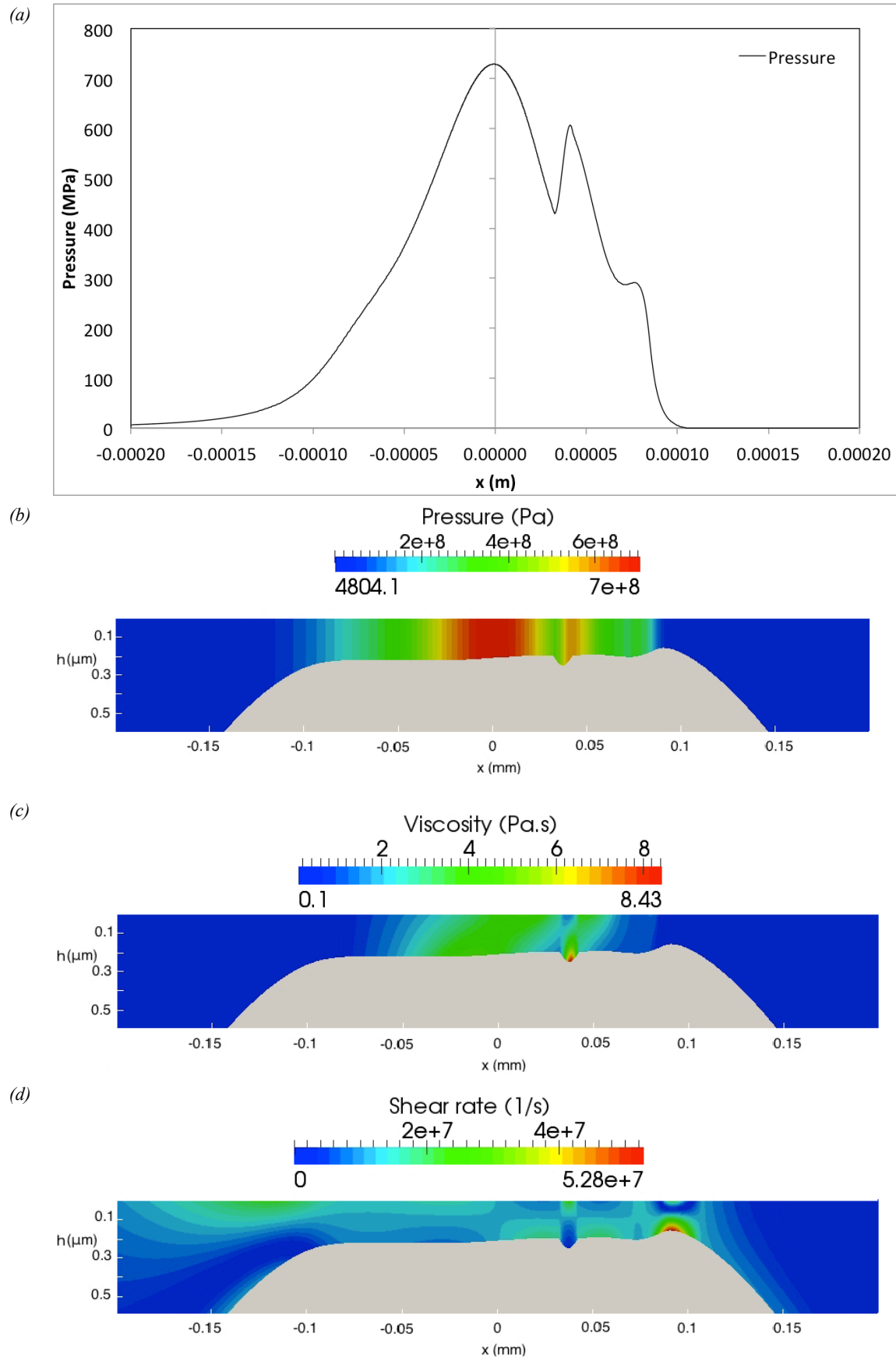


Figure 5-13 (a) Pressure top sliding wall (b) pressure (c) viscosity (d) shear rate distributions
 $t=0.0009258$ s

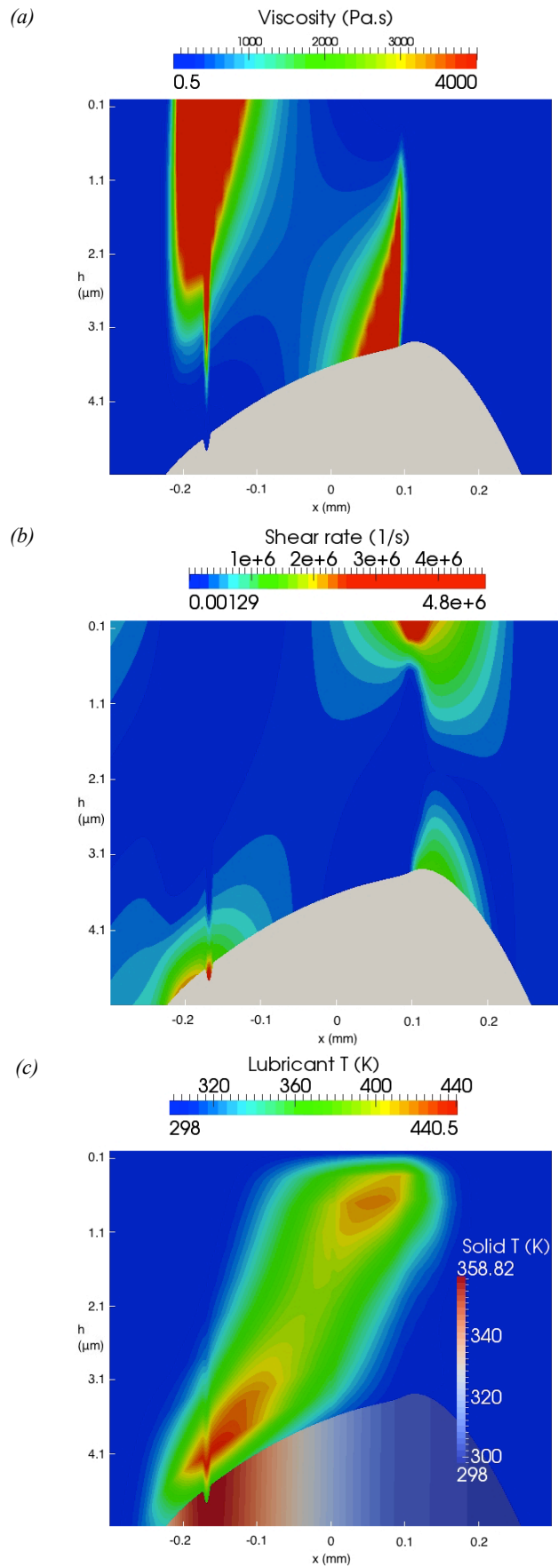


Figure 5-14. (a) Pressure (b) viscosity (c) shear rate distributions $t = 0.0004368 \text{ s}$

5.5 Surface waviness

A single frequency wave takes the form of a sine or a cosine wave. Figure 5-15 shows a one-side rough surface contact where a single frequency wave can be characterised by λ (wavelength) and A (amplitude). A cosine wave function can be expressed as:

$$\text{Surface roughness} = A \cos\left(\frac{2\pi}{\lambda}(x)\right) \quad \text{Equation 5-6}$$

where x is the boundary cell centre position.

In this study only the cylinder surface is assumed to be rough in order to better understand and investigate the wave parameters variation effects.

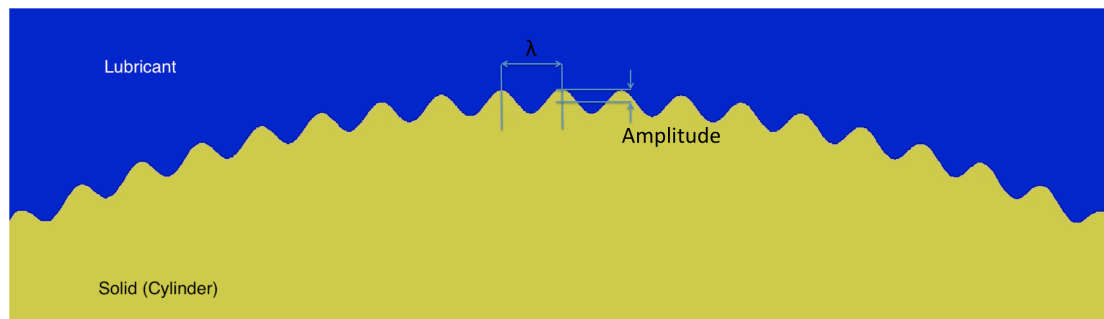


Figure 5-15. One side surface waviness

5.5.1 Stationary waviness results

In this section results for a stationary or steady-state surface roughness case studies, under sliding and rolling running conditions are shown. The sinusoidal roughness profile in Equation 5-6 is superimposed to the smooth surface of the deformable cylinder.

The first stationary roughness results are related to a pure sliding case (top wall is moving at 2 m/s). A roughness profile with amplitude of 0.02 μm and wavelength of 20 μm is imposed. The main difference between this case and the equivalent smooth case studies is the presence of multiple localised maxima of viscosity, temperature and vapour fraction. The results reveal the origination of cavitation at multiple zones; one in every wavelength (see Figure 5-17 (c)). This will be further discussed in the next section. The presence of two localised viscosity peaks is associated with temperature rise and shear rate increase due to roughness induced film thickness fluctuations (see Figure 5-17 (b)). The comparison to similar smooth case studies

shows that the location of the minimum film thickness is approximately the same. According to the flow pattern related to the top wall sliding, the pressure rippling occur once for every wavelength inside the contact. However, it is not obviously visible which is related to low amplitude to film thickness ratio.

Parameter	Value
Cylinder radius	$R = 10 \text{ mm}$
Domain length	$L = 120 \text{ mm}$
Young's modulus	$E_{\text{Cylinder}} = 2 \cdot 10^{11} \text{ Pa}$
Target load	$L_{\text{aim}} = 60 \text{ kN/m}$
Undeformed film thickness	$h_i = 0.08 \text{ }\mu\text{m}$
Thermo viscous constant	$\beta = 0.0476 \text{ 1/K}$
Roelands reference pressure	$p_{r,0} = 1.98 \cdot 10^8 \text{ Pa}$
Liquid density at p_0	$\rho_{l,\text{sat}} = 870 \text{ kg/m}^3$
Roelands pressure index	$z = 0.6$
Eyring stress	$\tau_0 = 7 \cdot 10^6 \text{ Pa}$
Dynamic viscosity, vapour	$\mu_l = 8.97 \cdot 10^{-6} \text{ m}^2/\text{s}$
Environment pressure	$p_0 = 10^5 \text{ Pa}$
Vapour pressure	$p_{\text{vapour}} = 5000 \text{ Pa}$
Vapour density at p_{vapour}	$\rho_{v,\text{sat}} = 0.125 \text{ kg/m}^3$

Table 5-3. Case parameters

Parameter	Value
Specific heat capacity liquid	$C_{p,l} = 2084 \text{ J/(kg K)}$
Specific heat capacity vapour	$C_{p,v} = 1800 \text{ J/(kg K)}$
Ambient temperature	$T_0 = 353 \text{ K}$
Thermal conductivity liquid	$k_l = 0.11735 \text{ W/(m K)}$
Thermal conductivity vapor	$k_v = 0.025 \text{ W/(m K)}$
Heat of evaporation	$h_{\text{evap}} = 287 \text{ kJ/kg}$

Table 5-4. Thermal properties of the lubricant

Parameter	Value
Density solid	$\rho_s = 7000 \text{ kg/m}^3$
Specific heat capacity solid	$C_{v,s} = 450 \text{ J/kg K}$
Thermal conductivity solid	$k_s = 47 \text{ W/m K}$
Thermal diffusivity solid	$\alpha_{T,s} = 1.31 \cdot 10^{-5} \text{ W/m K}$

Table 5-5. Thermal properties of the solid cylinder

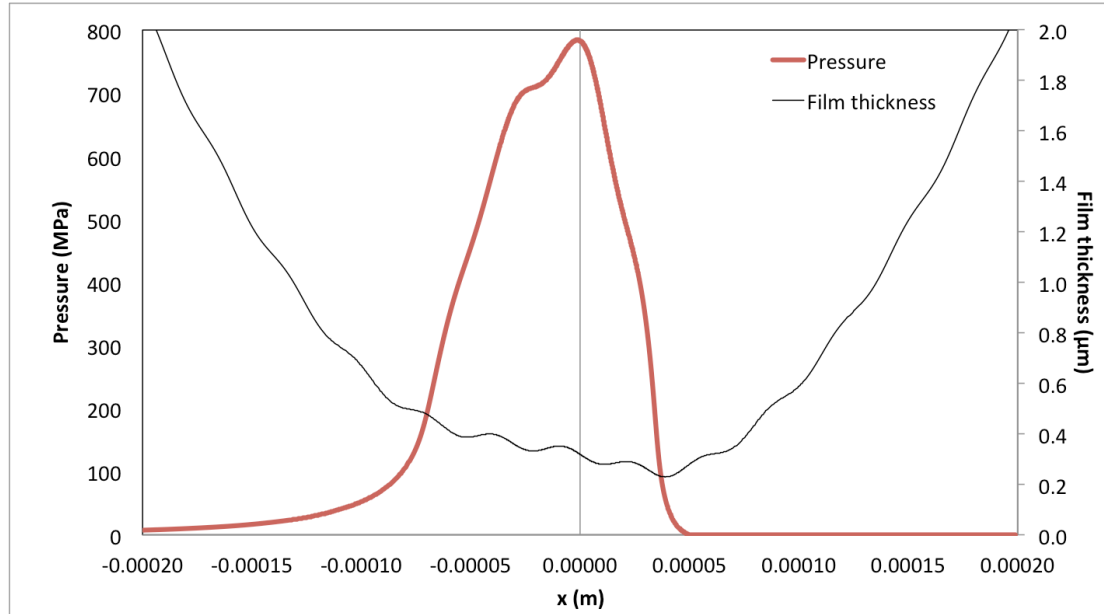


Figure 5-16. Pure sliding at 2 m/s, $\lambda=20 \mu\text{m}$, $A=0.02 \mu\text{m}$ – Pressure distribution and film thickness

Hooke [104] proposed a formulation to predict surface roughness flattening extend for an Eyring fluid:

$$|Q| = \frac{6}{\pi^2} \frac{\tau_0}{E_r} \frac{\lambda^2}{h^2} \quad \text{Equation 5-7}$$

where the sign of Q is as that of Δu . It is also proposed that for pure rolling the original roughness will be un-deformed. For the current case study, Q is 0.073 which suggests that the roughness will pass through the contact unaltered.

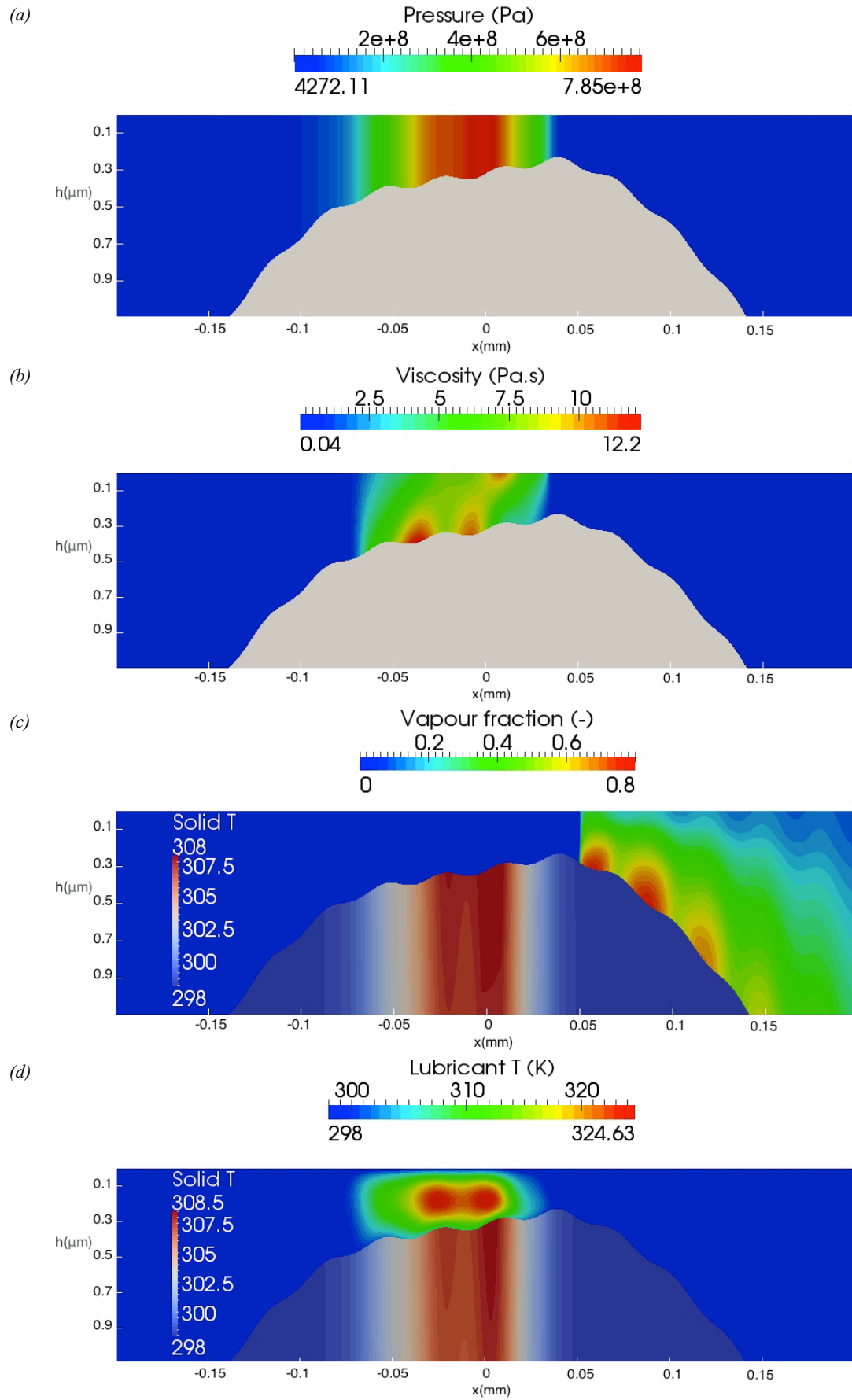


Figure 5-17. Pure sliding at 2 m/s, $\lambda=20 \mu\text{m}$, $A=0.02 \mu\text{m}$ (a) pressure (b) viscosity (c) vapour fraction (d) temperature

The second case study is a pure rolling case (parameters are summarised in Tables 5-3, 5-4 and 5-5). The low temperature rise near to the constriction region (as expected for $SRR=0$) and the shear rate distribution in the centre of the contact reduces multiple viscosity localisations due to the surface roughness (see Figure 5-19 (b) and (c)). In contrary to pure sliding case, the vapour fraction distribution is not localised on the surface roughness (see Figure 5-19 (d)).

The main differences with respect to previous pure sliding case are the frequency and the maximum and minimum values of the pressure rippling (see Figure 5-18). This results in amplitude reduction of the surface waviness which is also observed by Venner et al. [102] for a stationary waviness profile with amplitude of $0.11 \mu m$.

Existing treatments of the surface waviness using Reynolds approaches usually superimpose the roughness contribution to a smooth contact steady state solution; this implies that the hydrodynamic pressure is already present when the roughness is considered and this can lead to unphysical start-up effects and an underestimation of roughness-induced pressure gradient.

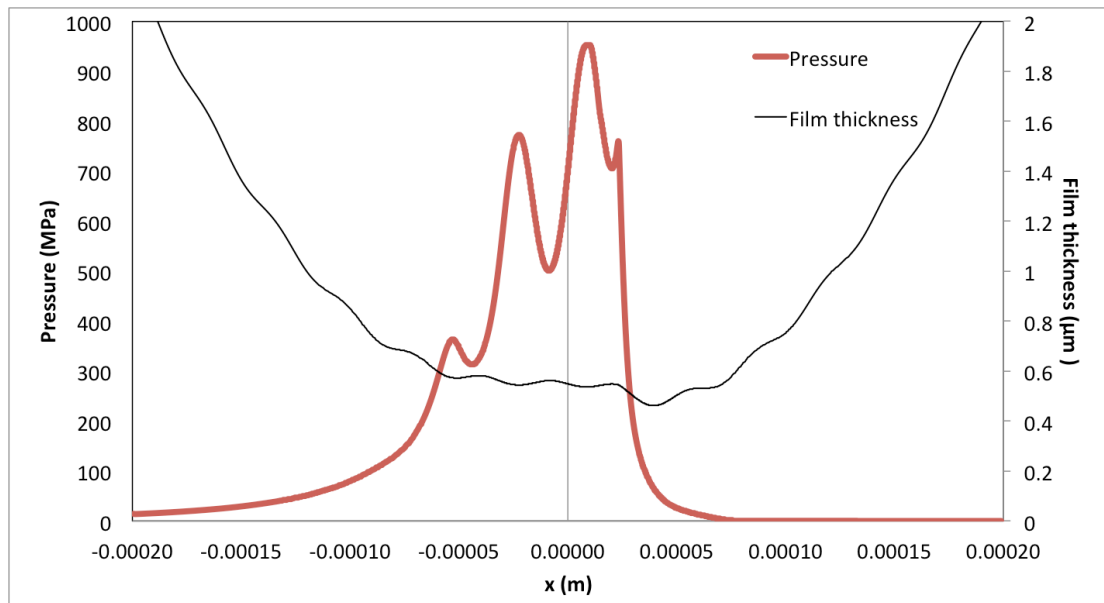


Figure 5-18. Pure rolling $\lambda=20 \mu m$, $A=0.02 \mu m$ – Pressure distribution and film thickness

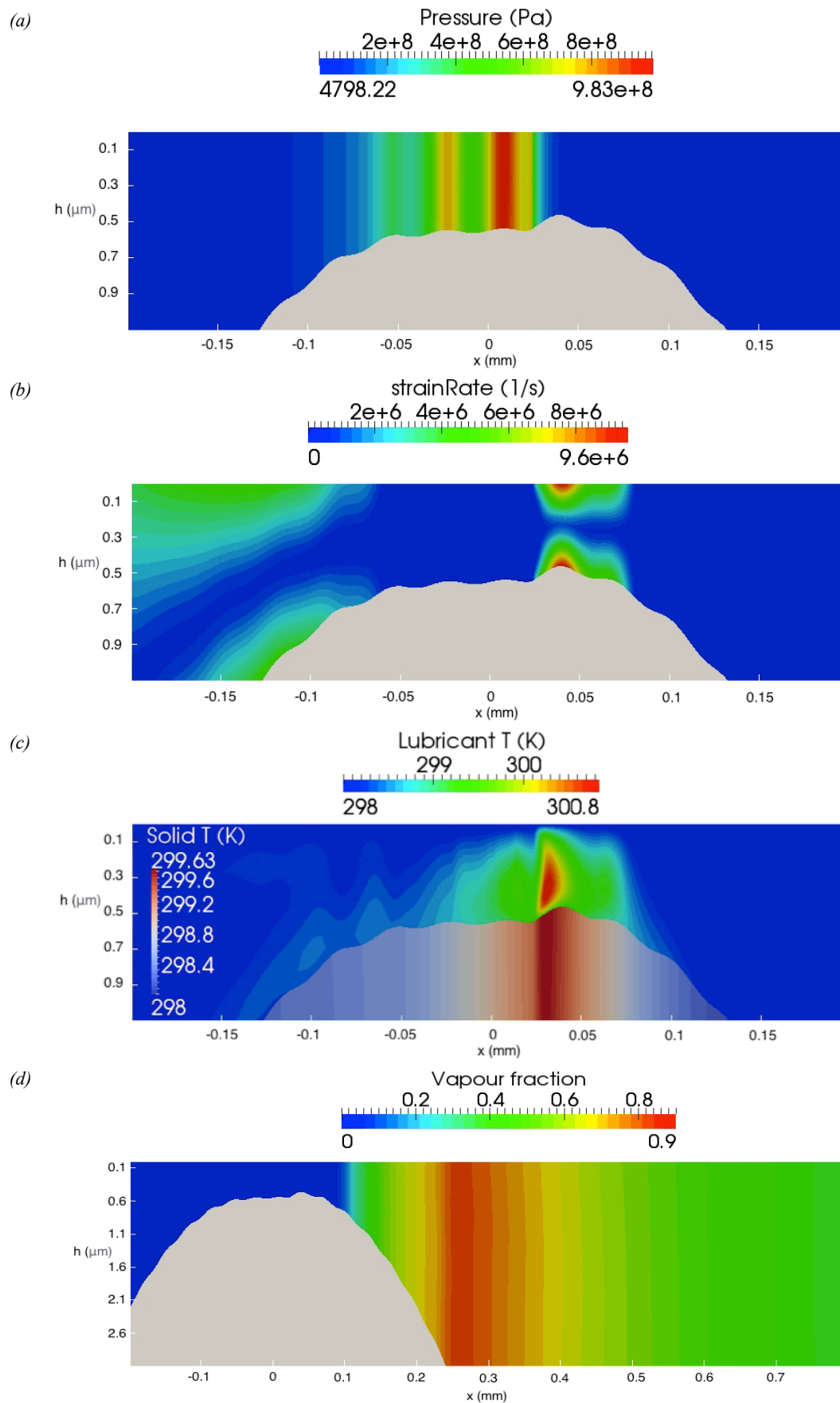


Figure 5-19. Pure rolling $\lambda=20\ \mu\text{m}$, $A=0.02\ \mu\text{m}$ (a) pressure (b) shear rate (c) temperature (d) vapour fraction

5.5.2 Moving sinusoidal wavy surface

The numerical results for a single-sided moving sinusoidal surface roughness are presented in this section. The only required change to the FSI model is to superimpose numerically the surface feature motion to the fluid-solid mutual boundary.

To mimic surface roughness motion, the rotational speed of the cylinder is inserted into the surface roughness function and it can be re-written as:

$$\text{Travelling wavy roughness} = R(x, t) = A \cos\left(\frac{2\pi}{\lambda}((x) - u_2 t)\right) \quad \text{Equation 5-8}$$

where u_2 is the surface travelling speed (cylinder speed) and x is the cell position. Since the waviness function is time dependent, a new vector field (called geometry) is defined to store roughness profiles at each time step. The number of fluid cells is increased to 20 through the film thickness for stability reasons. To mimic the physics of a contact start-up phase, the velocity is ramped up linearly based on an end-time and end-speed for sliding and rotating speeds. Therefore, the roughness rotates in accordance to the instantaneous speed of the cylinder. The speed remains constant when it reaches to the end speed. The load is also ramped up based on current and target loads to mimic hydrodynamic pressure build up physically. The roller is moved up or down based on the current load value.

The simulation is performed for pure rolling ($u_1=u_2=1 \text{ m/s}$) and pure sliding ($u_2=1 \text{ m/s}$) conditions and roughness profiles characterised by amplitudes of $0.02 \text{ }\mu\text{m}$ and $0.01 \text{ }\mu\text{m}$ and wavelengths of $10 \text{ }\mu\text{m}$ and $5 \text{ }\mu\text{m}$. The initial viscosity is set to 0.1 Pa.s .

The results for amplitude of $0.01 \text{ }\mu\text{m}$ are not presented since the EHL film behaviour was similar to cases with amplitude of $0.02 \text{ }\mu\text{m}$. The only difference observed was roughness flattening at wavelength and amplitude of $10 \text{ }\mu\text{m}$ and $0.01 \text{ }\mu\text{m}$ respectively. Top wall sliding is found to be an essential element to build up large pressure in the presence of roughness on the cylinder. Therefore, the results for a pure sliding case (cylinder rotation) is not presented since high-pressure rise and subsequently significant roughness deformation are not observed.

Figures 5-20 to 5-25 show lubricant behaviour at different time instants for $A=0.02 \text{ }\mu\text{m}$ and $\lambda=10 \text{ }\mu\text{m}$. Hydrodynamic pressure build up strongly interacts with the surface roughness initially (see Figures 5-21 and 5-23) and flattening of surface roughness takes approximately $21.4 \text{ }\mu\text{s}$. However, un-deformed surface roughness motion results in rough profile appearance in the contact at $27 \text{ }\mu\text{s}$ later (see Figure 5-25). A similar

behaviour is noted by Choo et al. [110] by using SLIM optical technique with an accuracy of $\pm 1 \text{ nm}$. Choo investigated the effects of ridges on thin film lubrication under pure rolling and rolling sliding conditions. The ridge peak to base height sizes are 270 nm , 165 nm and 60 nm with feature width of $47 \text{ }\mu\text{m}$, $42 \text{ }\mu\text{m}$ and $45 \text{ }\mu\text{m}$ respectively. The ratio of nominal film thickness to the un-deformed feature height is in the range of $0.06 - 7.7$. The ridges orientation is such that there are 5 ridges within the Hertzian footprint of the contact. It was shown that all the ridges retained their approximate uncompressed height by the increase of entrainment speed.

The size of the surface roughness (λ/A ratio) and nominal film thickness to unreformed feature height ratio are in the range close to the results presented by Choo et al. [110] where similar phenomena is captured.

The pressure profile changes continuously due to rough surface motion. The distance between pressure spikes is approximately equal to the roughness wavelength. Pressure fluctuations at the highest pressures do not seem to be associated to numerical errors and disappear at later time steps. The reason for the existence of pressure rippling near the pressure spikes (see Figures 5-20 and 5-22) in early time steps is likely to be due to the instability induced by the use of Roelands equation to describe the pressure-viscosity law for large pressure (this has also been discussed in Chapter 4); however, further investigations will need to be carried out to study the development of such fluctuations in the pressure profiles. Cavitation occurs at multiple regions and continuously changes according to the roughness profile location (see Figures 5-21 (b), 5-23 (c), 5-25 (c), 5-27 (c) and 5-28 (c)). Viscosity distribution is mainly dominated by pressure. Viscosity rises to very high values due to high-pressure gradient and values (see Figure 5-23 (b) and 5-30 (a)). The continuous pressure spikes fluctuations result in a very complex viscosity distribution despite the fact that the pressure distribution stabilises and produces smooth results apart from the fluctuations near the pressure peaks discussed above. The drawbacks of using Roelands piezoviscosity models at pressures higher than 2 GPa are discussed in Chapter 4. The vapour fraction goes up until it reaches 1.0 at steady state. However, the movement of the cavitating regions results in vapour fraction fluctuations in a transient rough surface EHL contact. For instance, vapour fraction maximums are 0.78 and 0.66 at $t=11.4 \text{ }\mu\text{s}$ and $t=48.33 \text{ }\mu\text{s}$ respectively.

Figures 5-27 to 5-30 show lubricant behaviour at different time instants for $A=0.02 \text{ }\mu\text{m}$ and $\lambda=5 \text{ }\mu\text{m}$. The main difference to the longer wavelength is that the roughness

profile is not notably deformed even in the presence of very high pressures (see Figure 5-28). The viscosity behaviour at pressures lower than 2 *GPa* is more stable (see Figure 5-27 (b)). The occurrence of pressure rippling is more significant, as the wavelength is shorter (see Figure 5-26 (a) and (b)), while surface deflection occurs in a faster manner compared to $\lambda=10\ \mu\text{m}$. Wavelength attenuation is not considerable neither for pure rolling nor for pure sliding conditions when $\lambda=5\ \mu\text{m}$.

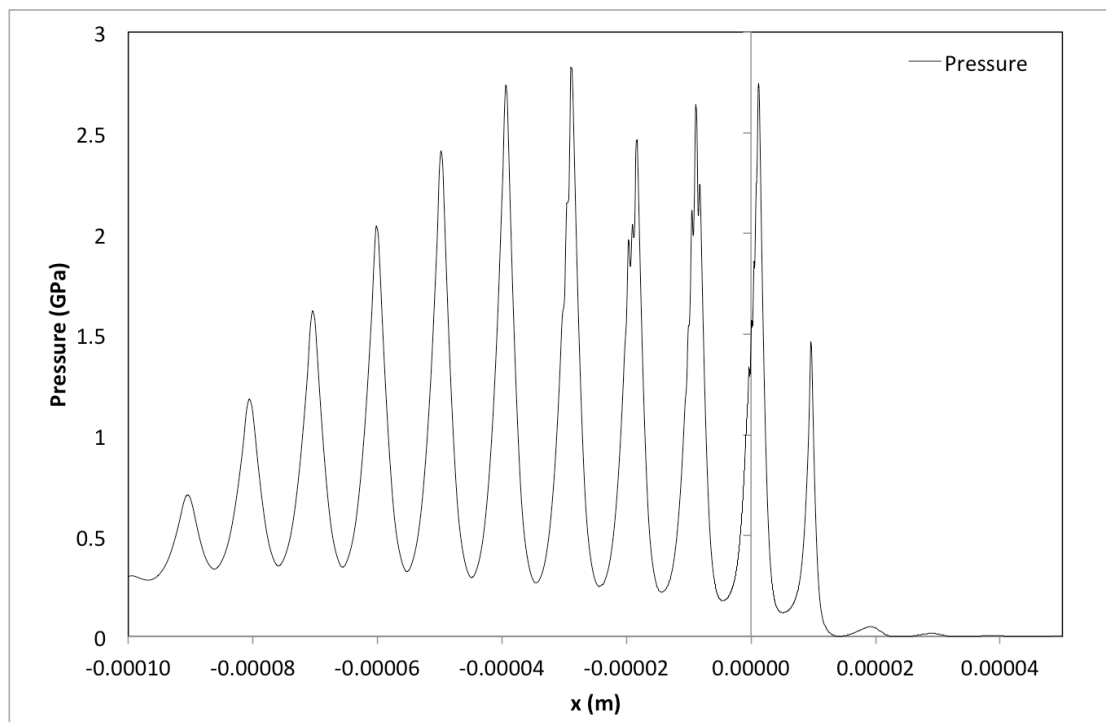


Figure 5-20. Pressure distribution at $t=1.14\text{e-}05\ \text{s}$, $A=0.02\ \mu\text{m}$, $\lambda=10\ \mu\text{m}$

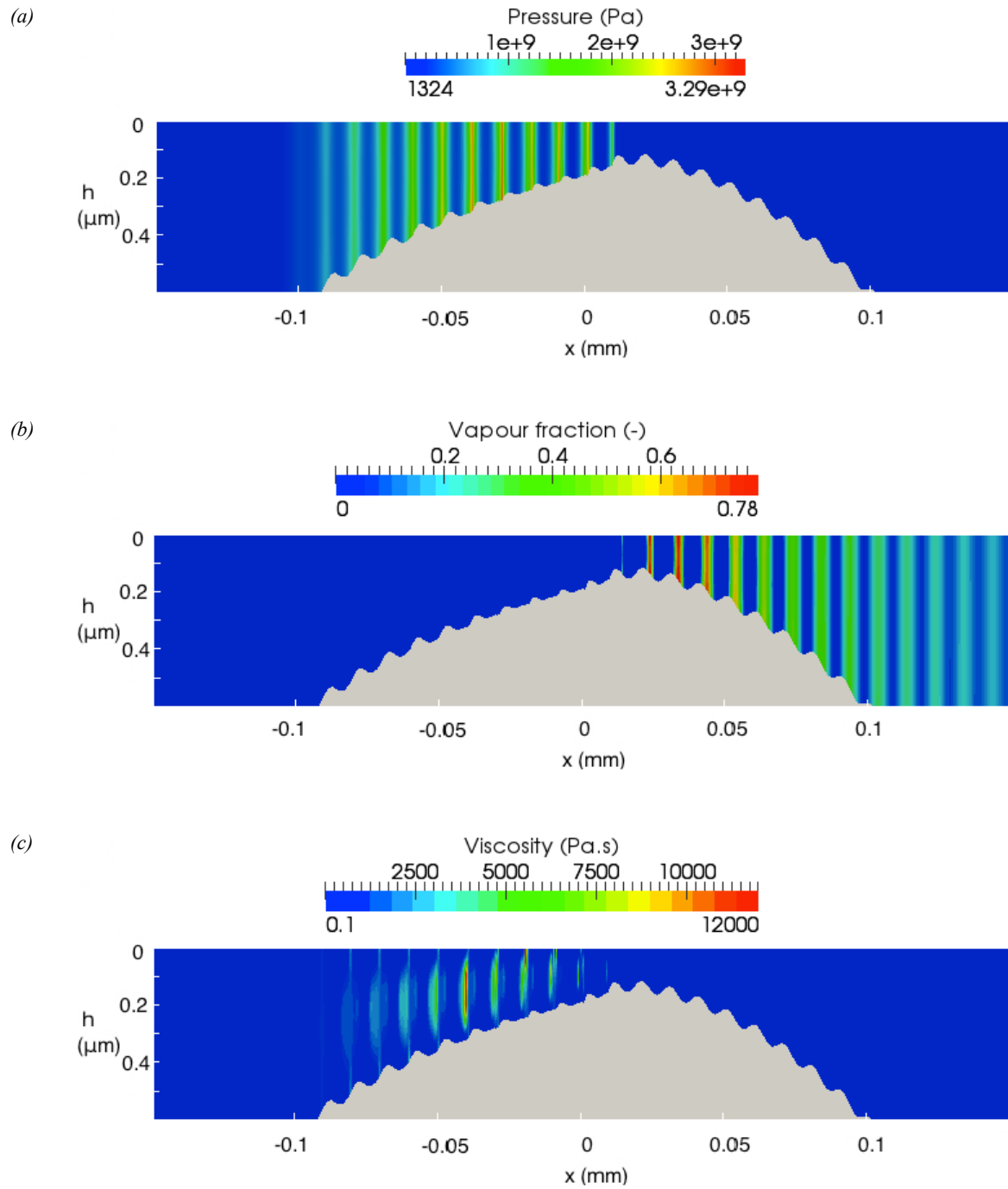


Figure 5-21. $t=1.14\text{e-}05\text{ s}$, $A=0.02\text{ }\mu\text{m}$, $\lambda=10\text{ }\mu\text{m}$ (a) pressure (b) viscosity (c) vapour fraction

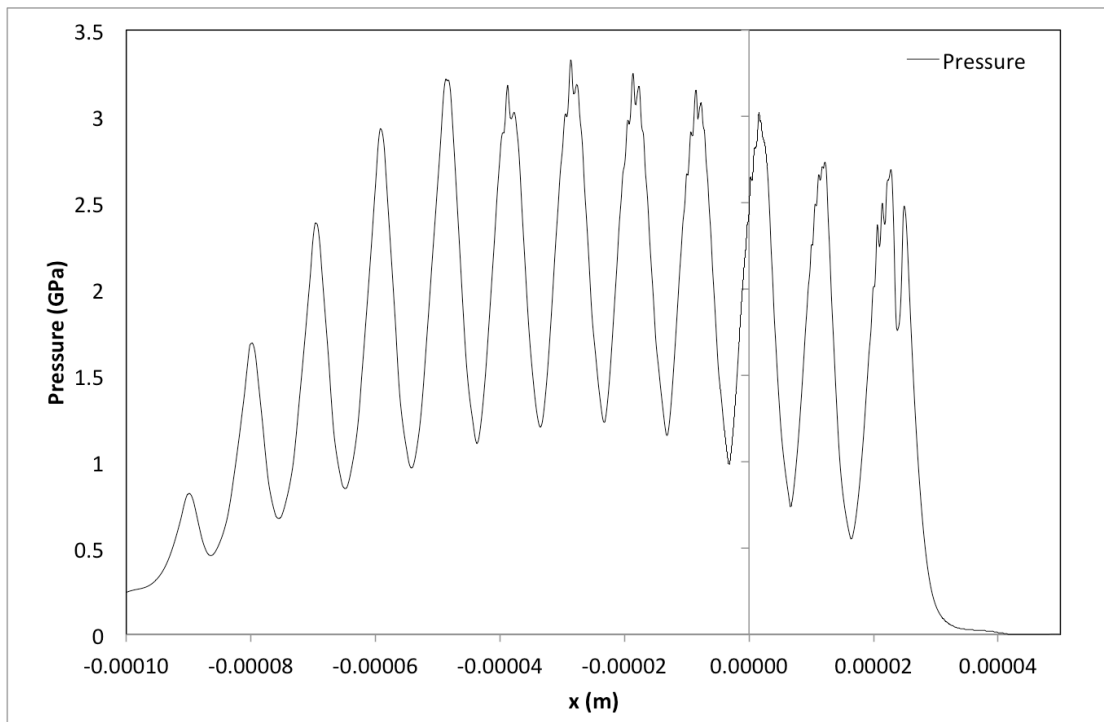
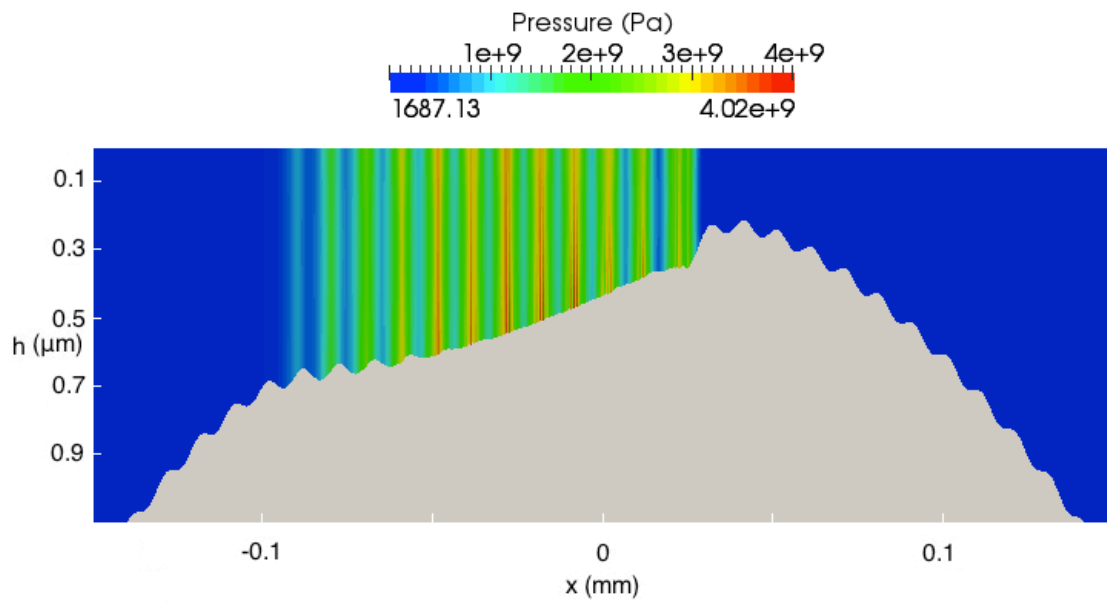
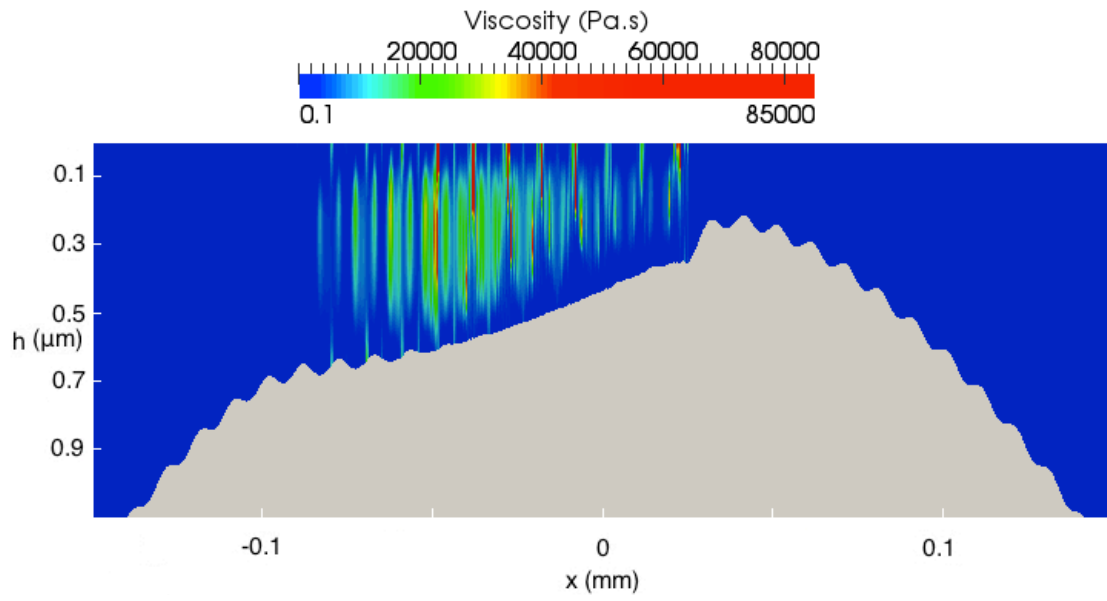


Figure 5-22. Pressure distribution at $t = 21.432 \mu\text{s}$, $A = 0.02 \mu\text{m}$, $\lambda = 10 \mu\text{m}$

(a)



(b)



(c)

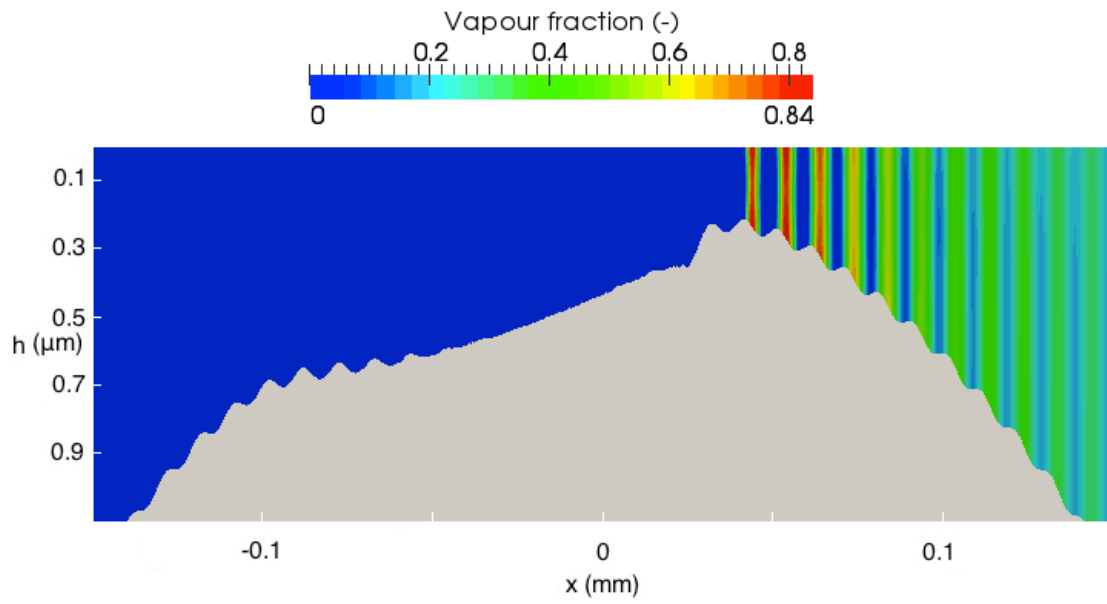


Figure 5-23. $t = 21.432 \mu\text{s}$, $A = 0.02 \mu\text{m}$, $\lambda = 10 \mu\text{m}$ (a) pressure (b) viscosity (c) vapour fraction

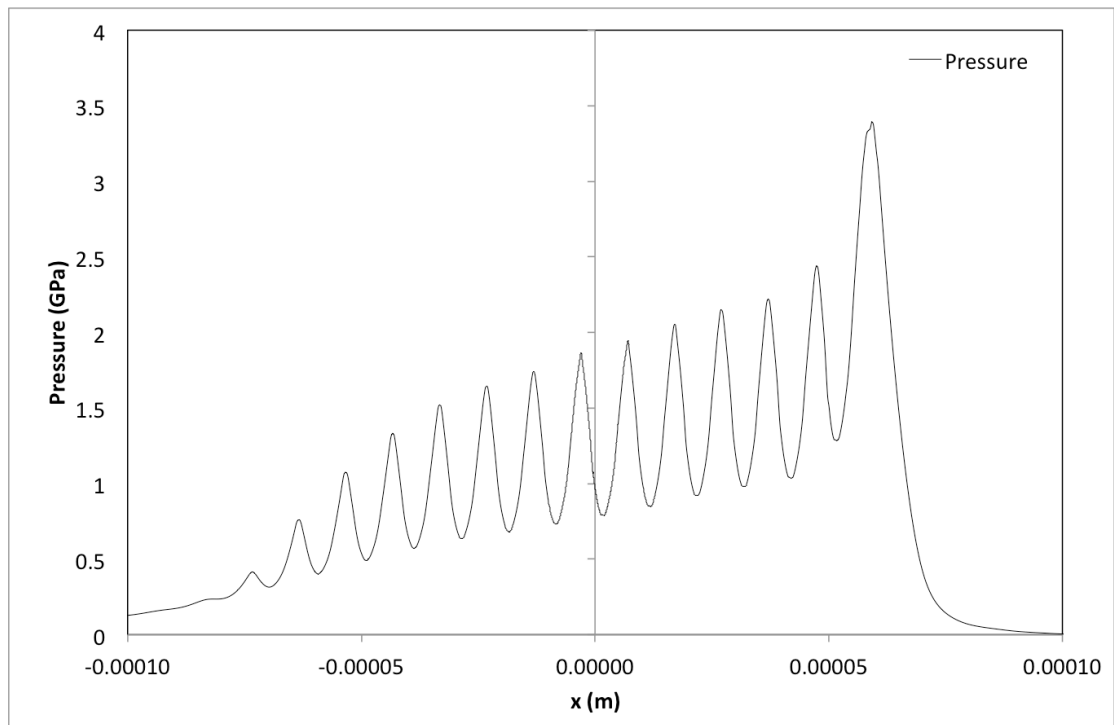
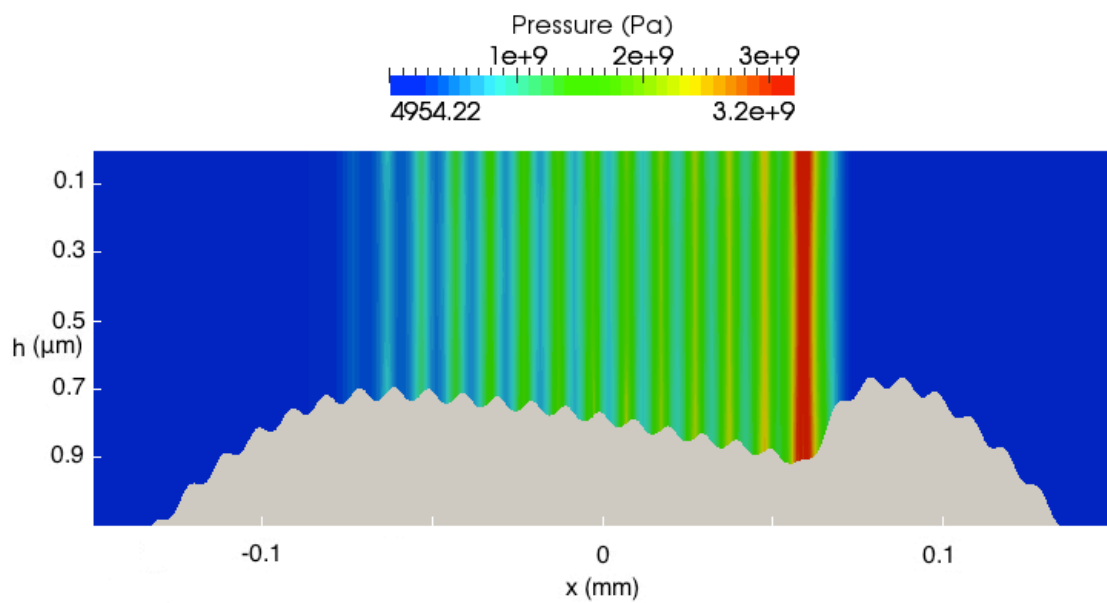


Figure 5-24. Pressure distribution at $t=48.336 \mu\text{s}$, $A=0.02 \mu\text{m}$, $\lambda=10 \mu\text{m}$, $L=200 \text{ kN/m}$

(a)



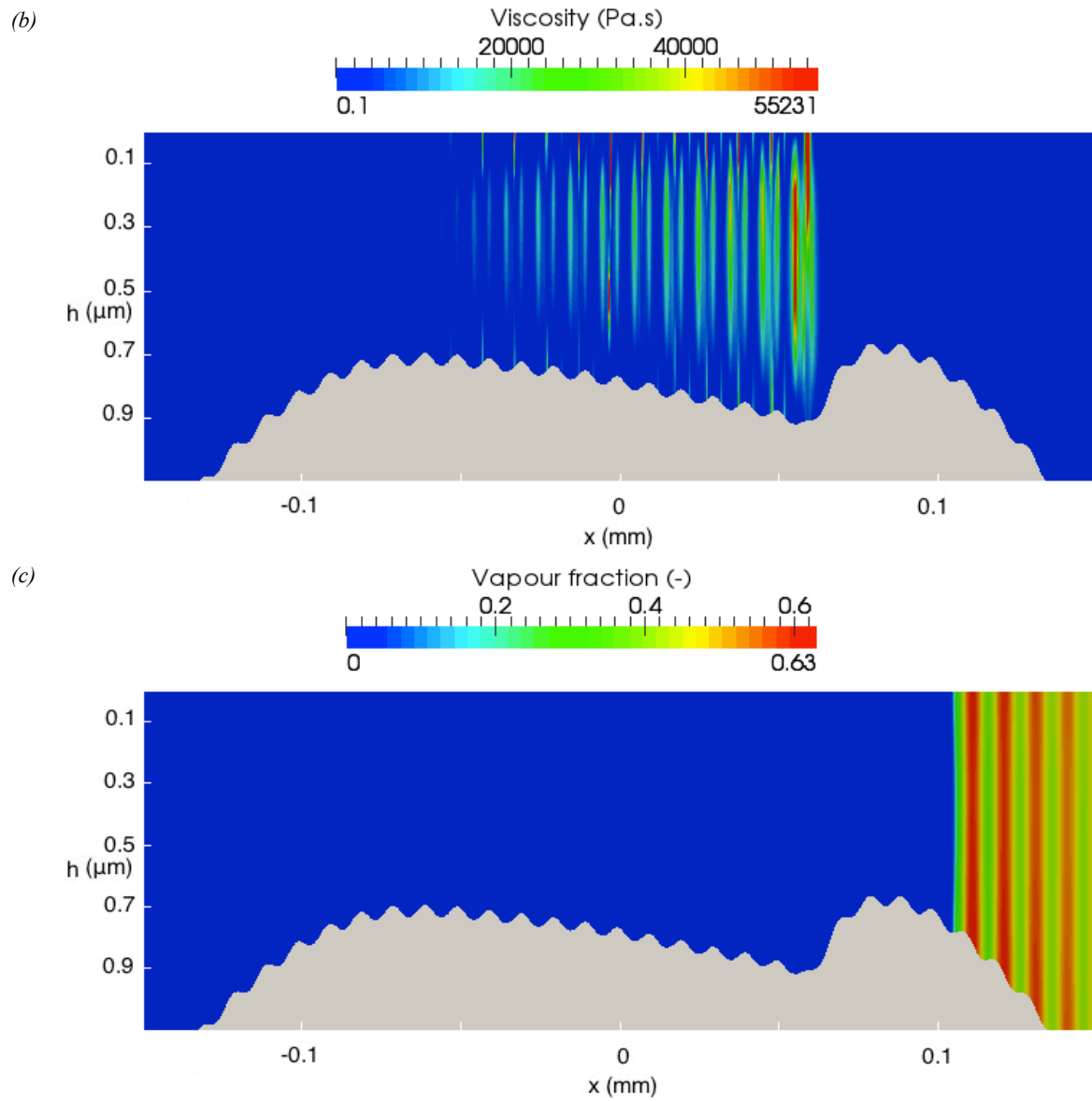


Figure 5-25. $t=48.336 \mu\text{s}$, $A=0.02 \mu\text{m}$, $\lambda=10 \mu\text{m}$, $L=200 \text{ kN/m}$ (a) pressure (b) viscosity (c) vapour fraction

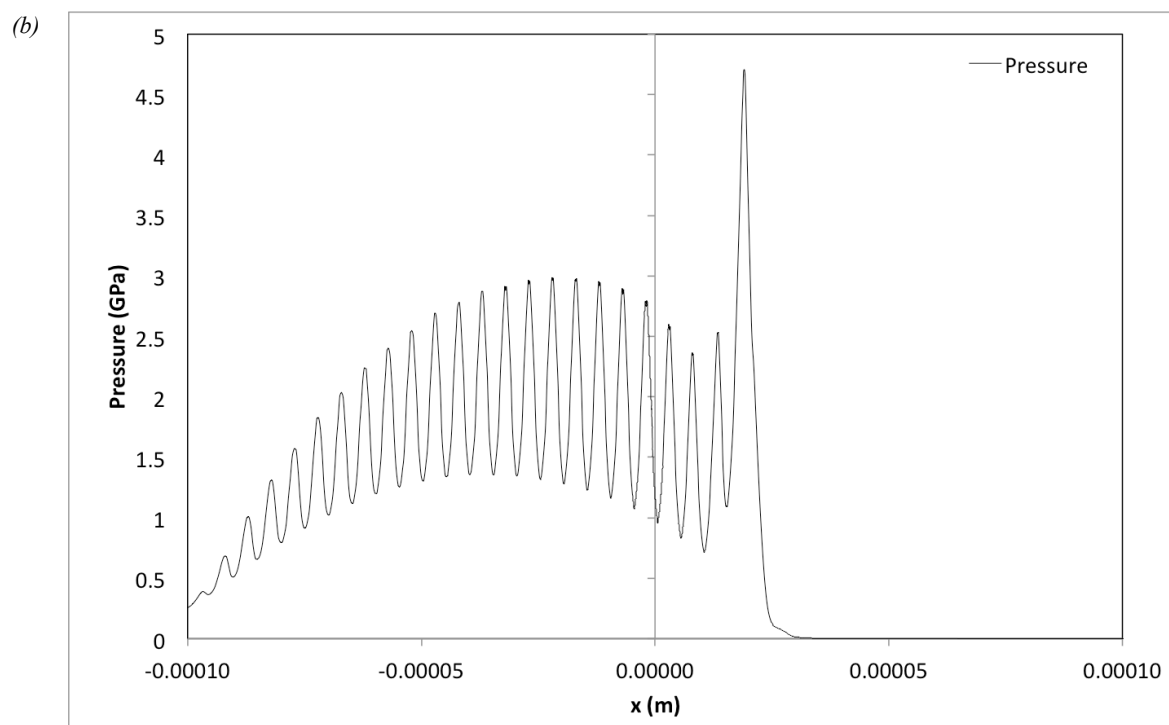
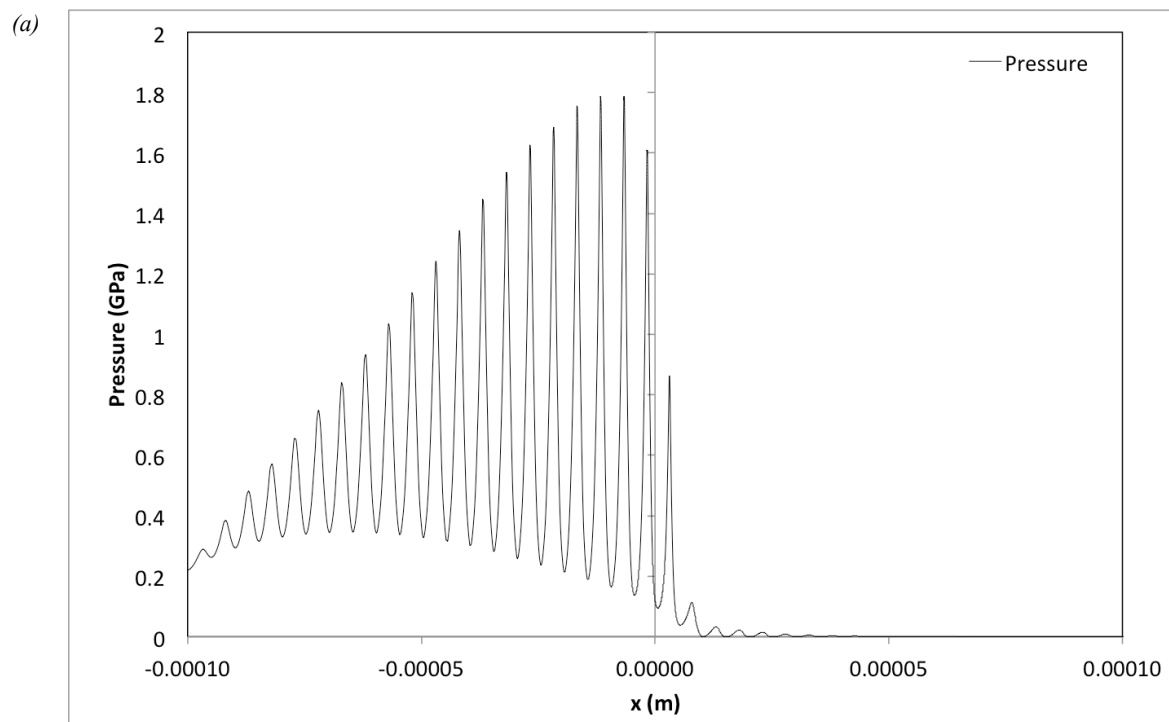


Figure 5-26. Pressure distributions at (a) $t=9.12 \mu s$ (b) $t=18.81 \mu s$

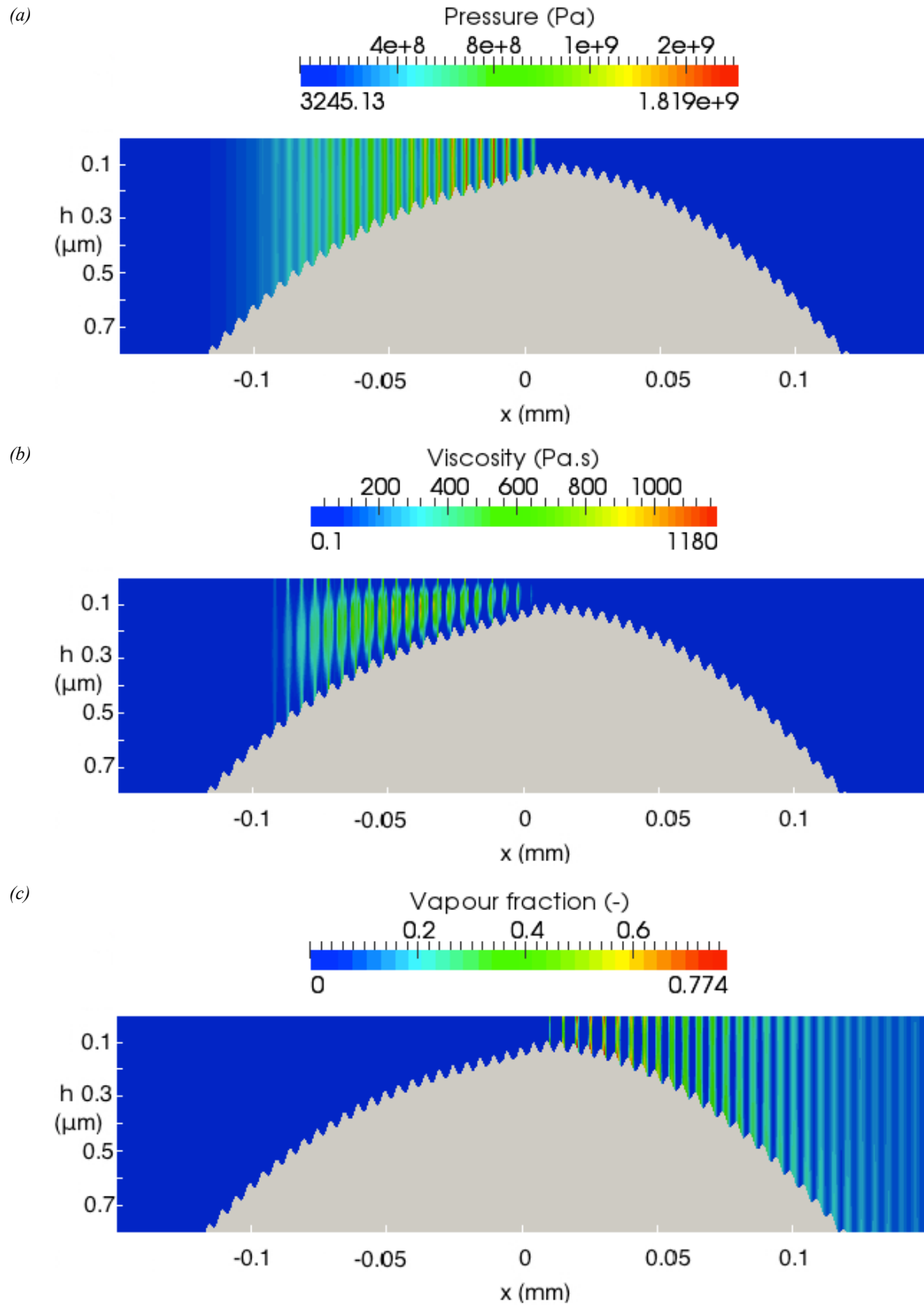


Figure 5-27. $t=9.12 \mu s$, $A=0.02 \mu m$, $\lambda=5 \mu m$ (a) pressure (b) viscosity (c) vapour fraction

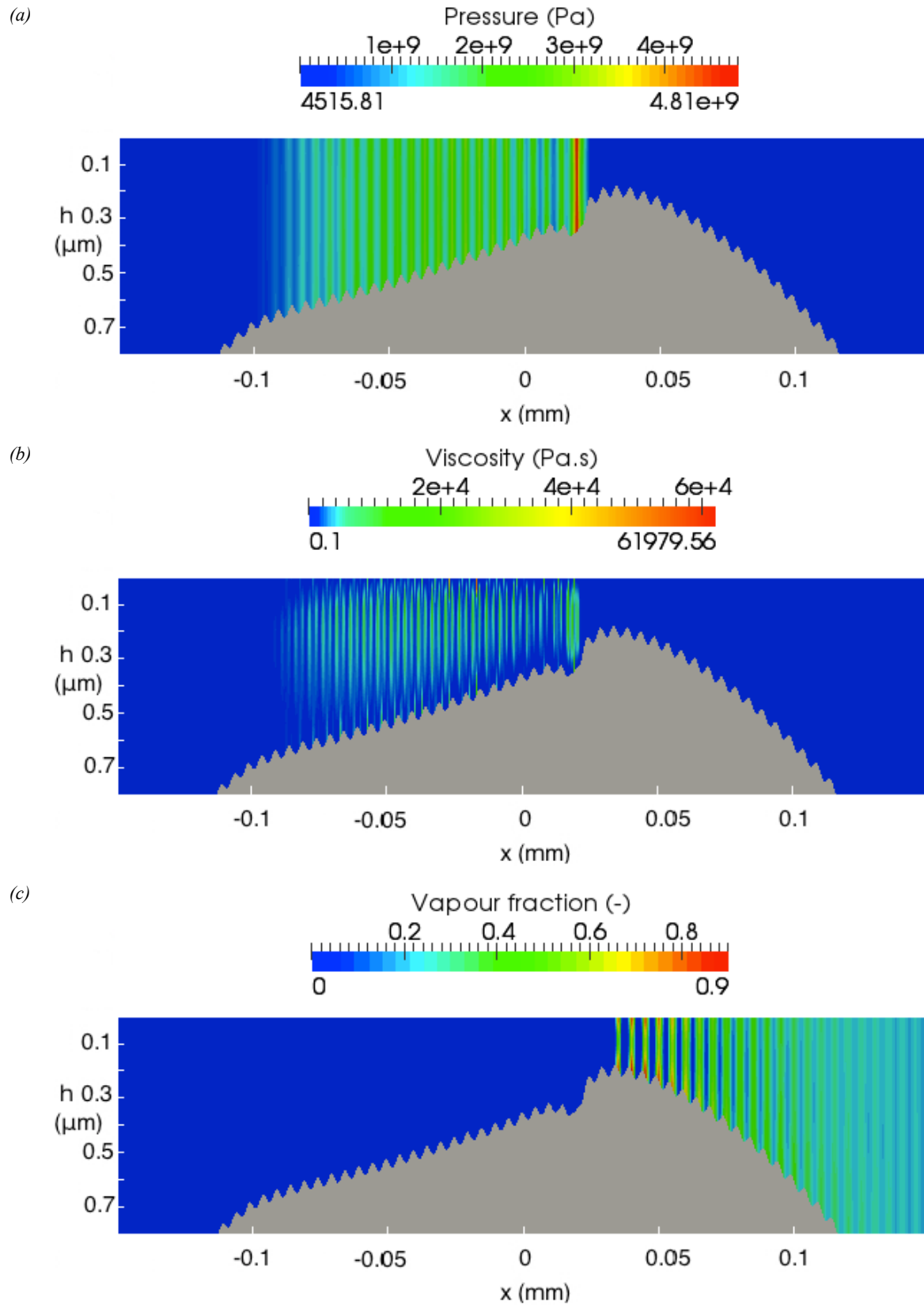


Figure 5-28. $t=18.81 \mu s$, $A=0.02 \mu m$, $\lambda=5 \mu m$ (a) pressure (b) viscosity (c) vapour fraction

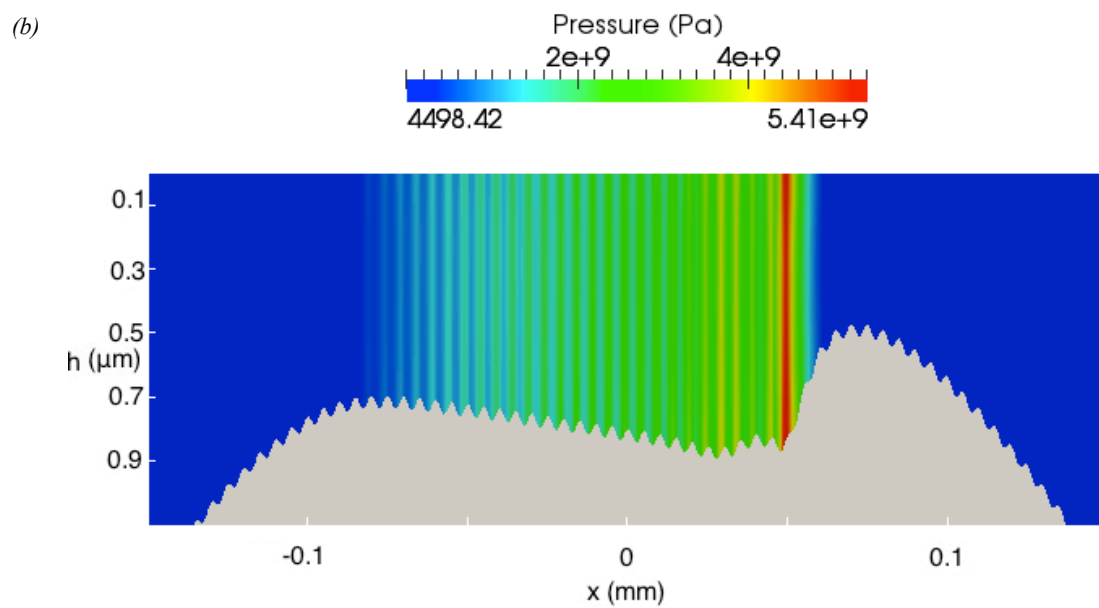
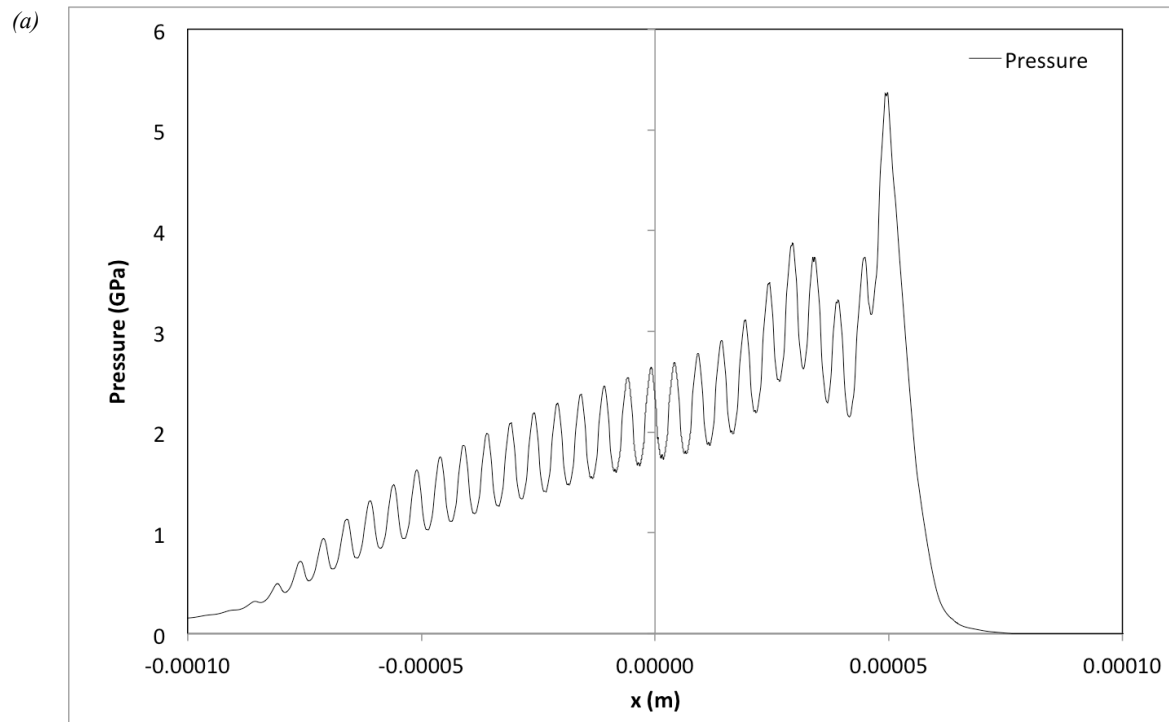


Figure 5-29. $t=34.998 \mu\text{s}$, $A=0.02 \mu\text{m}$, $\lambda=5 \mu\text{m}$, $L=300 \text{ kN/m}$ (a) pressure along top wall (b) pressure

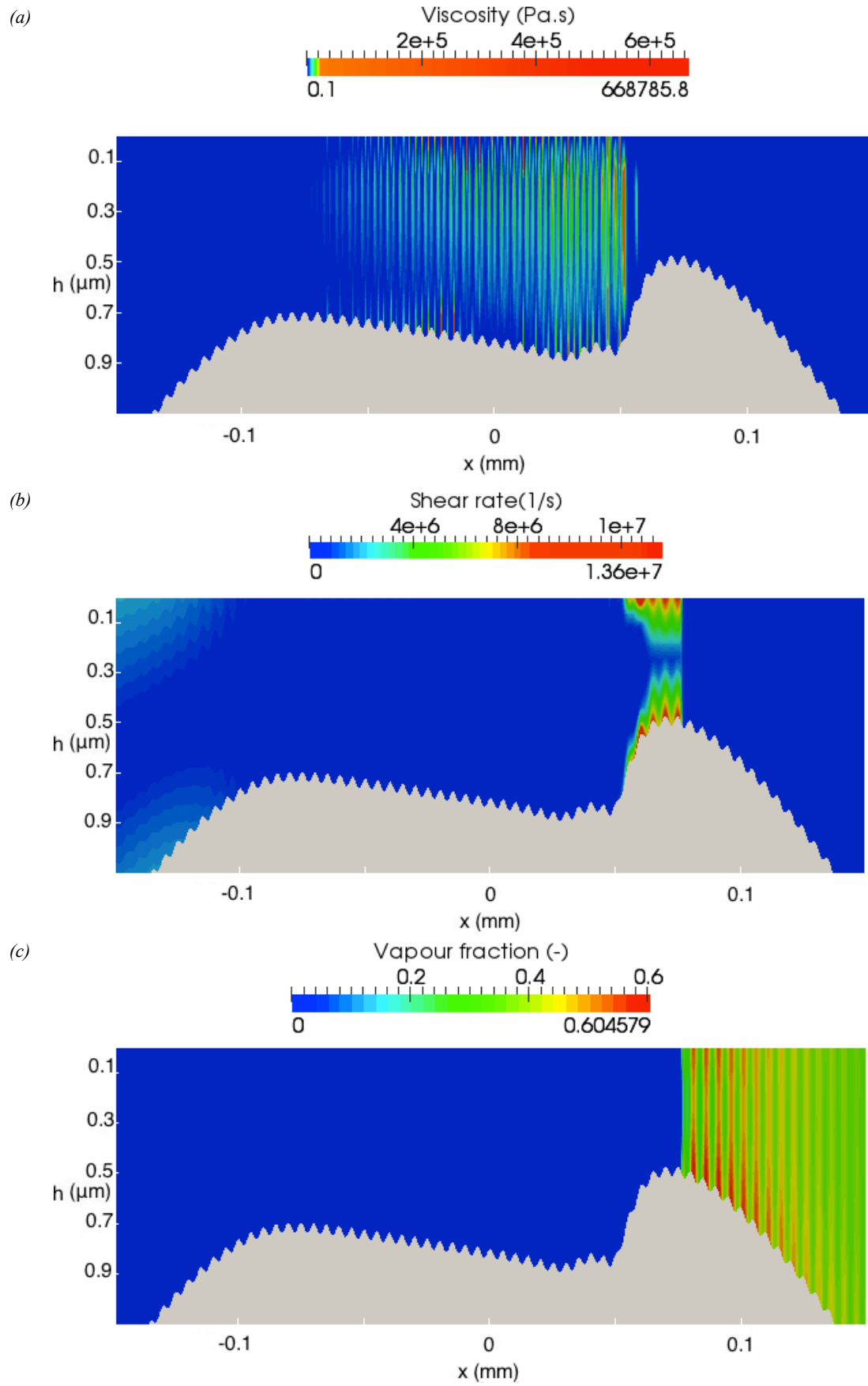


Figure 5-30. $t=34.998 \mu\text{s}$, $A=0.02 \mu\text{m}$, $\lambda=5 \mu\text{m}$ (a) viscosity (b) shear rate (c) vapour fraction

Figure 5-31 shows vapour fraction distributions at $t=27.93 \mu s$ and $t=24.966 \mu s$. Cavitation origination differs relative to the location of the first dent after the EHL constriction. The localised vapour fraction peaks correspond to the position of the roughness peaks. Maximum vapour fraction locations at $t=27.93 \mu s$ correspond to minimum vapour fraction locations at $t=24.966 \mu s$ as it is to be expected due to the transient nature of the calculations.

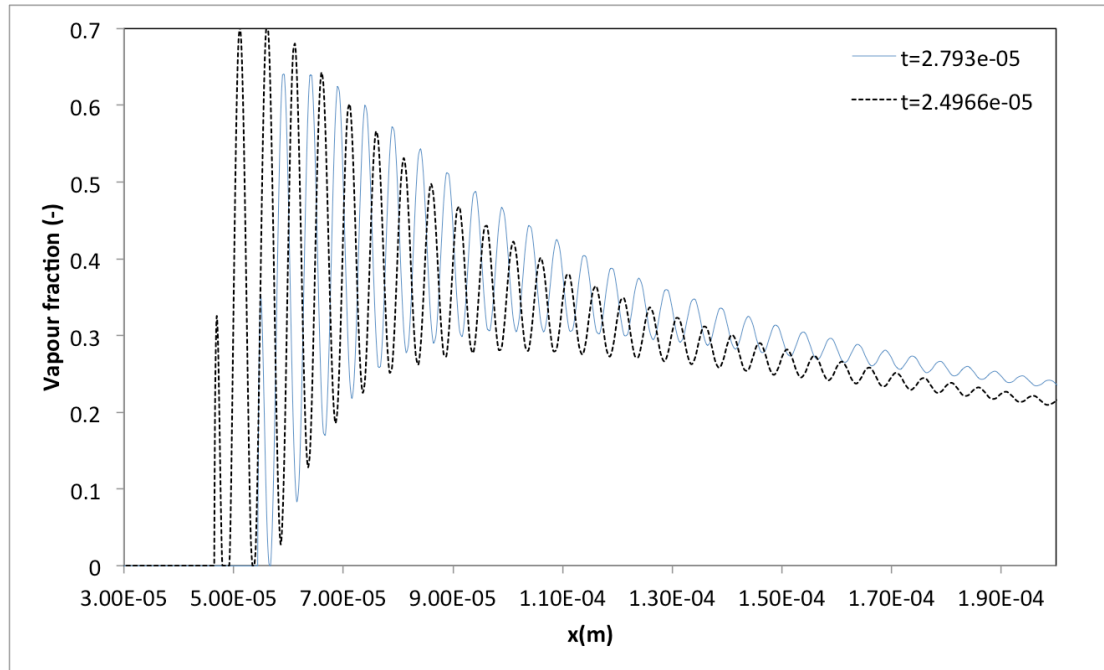


Figure 5-31. Vapour fraction distribution at $t=27.93 \mu s$ and $t=24.966 \mu s$

In order to study elastic fracture mechanics of the solid, the stresses are required to be calculated in the solid. The stress field tensor can be computed from linear combinations of the displacement field's derivatives:

$$\sigma = \rho_s(\mu_s(\nabla v + (\nabla v)^T) + \lambda_s Itr(\nabla v + (\nabla v)^T)) \quad \text{Equation 5-9}$$

For instance, σ_{xx} is shown in Figure 5-32.

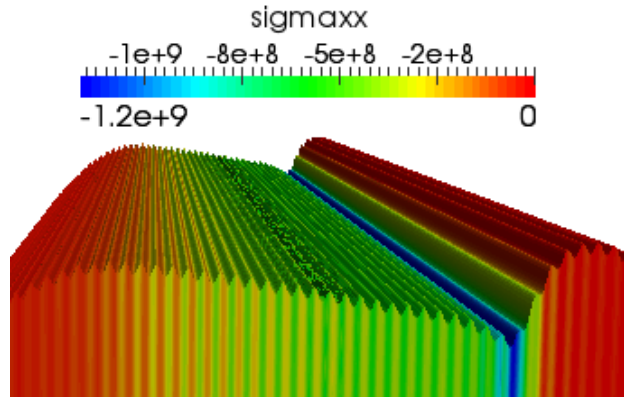


Figure 5-32. σ_{xx} at $t=4.3668 \cdot 10^{-5} \text{ s}$, $A=0.02 \mu\text{m}$, $\lambda=5 \mu\text{m}$

5.6 Closure

Elementary surface features (dent and ridge) passage effects on an EHL film behaviour have been presented. The effects were found to be significant even for low ratios of surface feature size to film thickness. In particular, pressure and viscosity distributions are affected. The stationary roughness case studies show basic effects of surface waviness on lubricant behaviour. The transient surface waviness case studies show the effects of different roughness profile, SRR, wavelength and amplitude sizes on lubricant and the solid behaviours. The capability of the model at very high pressures is examined and the main drawback for further improvement is the rheology model. The transient surface waviness results are in agreement with experimental observations on surface roughness.

Chapter 6

6.1 Conclusions and Future work

Chapter 6 summaries the main achievements of the research carried out by the author. This thesis has reported the development of an efficient and accurate model to simulate fluid-solid interactions in EHL contacts. This methodology has been shown capture the complex behaviour of the lubricant and the solid while shedding light on the mechanisms that govern EHL at high pressures and in the presence of surface features.

6.1.1 Summary of the main results

A summary of the work presented in this thesis is reported below:

- In Chapter 2, the essential fluid properties required for the accurate modelling and prediction of EHL film behaviour were introduced. Viscosity and density dependence on pressure, non-Newtonian and thermal effects and cavitation treatment were discussed. The use of the Reynold's equation for fluid flow and lubrication modelling were presented together with the relevant numerical methods commonly adopted in this area. Non-Newtonian modified Reynold's equation was used to develop a finite difference model.

The finite volume method and the Navier-Stokes equation discretisation were then outlined. Hydrodynamic lubrication modelling, using CFD, the Reynolds and modified Reynolds based approaches, is performed. The CFD fluid solver stability at very high pressure is achieved through implementation of multigrid and high-resolution mesh. It was shown that when the lubricant enters non-Newtonian regime at entrainment velocities higher than a certain threshold, the results obtained using Reynolds-based and CFD approaches show significant difference in pressure and viscosity distributions prediction due to the fact that the contact is characterised by non-Newtonian shear-thinning regimes. Despite the use of the modified Reynolds equation for non-Newtonian fluid, the results differ from the CFD approach. The viscosity variation through the film thickness is found to be the source of deviation since viscosity is taken out of the derivatives through the film thickness as a constant.

- The use of different numerical methods for fluid and solid domains is common in EHL modelling. In contrast, the present study set out to model the lubricant and solid mechanics of an EHL contact problem using a single numerical tool. In Chapter 3, a finite volume based solid solver using Navier-Lamé equation was introduced and coupled with the cavitation solver developed to discretise the fluid domain. The thermal solvers of the fluid and the solid domains were coupled in an implicit manner by considering heat conduction between solid and fluid domains. The final version of the model includes fluid domain and two non-adiabatic elastic solids. Vertex based mesh motion method is implemented. The complete FSI model offers significant improvement on

stability at high pressures through implementing fully coupled scheme and finite volume techniques. The model was validated against Hertzian based solutions in the regimes when alternative solutions were available.

- In Chapter 4, the developed FSI model was used to simulate smooth and rough surfaces of 2D roller bearing for different slide to roll ratios and loading conditions. Results were presented for pure sliding, pure rolling and sliding-rolling cases. The thermal effects are shown to be most significant in pure sliding and sliding-rolling cases. The temperature rise of up to 180 °K is captured in a pure sliding case. Viscosity dependency on temperature is necessary to be accounted in order to capture accurate non-Newtonian behaviours and shear-rate evaluation. The viscosity rises up to 40000 *Pa.s* in SRR=0.4 condition due to thermal, non-Newtonian effects and high-pressure values. The above-mentioned parameters are very important for accurate prediction of friction forces. The cavitation origination and development was found to be on the stationary or slower moving surface.
- In Chapter 5, elementary surface roughness features passage along the EHL conjunction are shown for large and small amplitude to minimum film thickness ratios. For amplitude to minimum film thickness ratio of 0.05, the ridge passage can greatly change the pressure distribution and generates pressure gradients of up to 500 MPa. Travelling and stationary surface waviness results were shown for relatively small amplitudes. Besides the geometric reasons, surface roughness effects were attributed to sharp pressure rise at the asperity tips caused by the reduction of area available for the flow. The pressure distribution and the effects of pressure rippling are accurately predicted by considering surface imperfections from the first time-step. The superposition of surface roughness to already converged steady-state solutions lead to an under-estimation of asperities effect on pressure distribution since larger pressure gradient are built before the roughness deforms significantly due to hydrodynamic effects. In both steady and transient case studies, surface roughness reduces the value of minimum fluid film thickness. The roughness produces pressure and viscosity fluctuations inside the conjunctions. These effects are found to increase with increasing amplitudes and decreasing wavelengths. Long wavelength components are attenuated and short

wavelength roughness profiles are passing unaltered (or with very small deflections) through the contact. The pressure distribution changes continuously as the wave propagates for pure rolling and sliding cases while the position of the roughness changes.

6.1.2 significant achievements

In this work, a CFD based FSI solver was developed to model fluid and solid interaction in elastohydrodynamic lubrication contacts using the open-source software package OpenFoam. The developed model is applied to a series of line contact problems at high contact loads which lead to large surface deformations and correspondingly high contact pressures in the region of 1 to 4 *GPa*.

For thermal and isothermal cases characterised by low and moderate loads and pressures, the Reynolds solution predicts the EHL film behaviour accurately. However, the solutions obtained using Reynolds-based approaches was found to be different from CFD-based calculations in the presence of thermal effects and high loading conditions. The existence of different fluid properties and pressure gradients through the film thickness was found to be the source of such difference.

The rough surface results were presented and pressure spikes of up to 4.5 *GPa* were calculated. The viscosity distribution was questionable at such pressure range and it was argued that the Roelands piezoviscosity model, implemented in this study, is not capable of predicting physical viscosity distribution at pressures higher than 2 *GPa*. Higher stability can be achieved by solving a single set of equations for both solid and fluid domains.

Overall, a fully coupled finite volume FSI solver was developed and applied to a wide range elastohydrodynamic lubrication problems; the stability of the code to reach the pressures encountered in the prototypical application of interest in this thesis was demonstrated. The developed model is capable to provide a better understanding of EHL film formation and development and solid deformations at operating conditions.

6.2 Future works

The present work provides a model to study FSI in the presence of high pressures with particular application to roller element bearings. Future efforts should be devoted to the improvement of the implemented algorithms and constitutive laws for the applicability of the model in wider range of EHL contact problems and also to obtain physically sound and acceptable results.

6.2.1 Fluid

The fluid solver can be improved significantly through incorporating more sophisticated cavitation treatment, rheology and compressibility models.

6.2.1.1 Rheology, density and thermal conductivity models

The implementation of new rheology model is strongly recommended; in particular, it is extremely important to better capture the piezoviscous behaviour of the lubricant at pressures higher than 2 *GPa*. This needs to go hand in hand with the development of experimental techniques which can provide data at such high pressures (this information is currently unavailable in the literature as mentioned in Chapter 4).

On the modelling front, one option would be to implement Doolittle piezoviscosity model together with Tait equation of state pressure-density relationship. This would require to re-write the pressure equation based on the density gradient $\frac{\partial \rho}{\partial p}$ to be introduced to capture this. In order to account for thermal dependency of density, temporal density derivative can be re-written as:

$$\frac{\partial \rho}{\partial t} = \frac{\partial \rho}{\partial p} \frac{\partial p}{\partial t} + \frac{\partial \rho}{\partial T} \frac{\partial T}{\partial t} \quad \text{Equation 6-1}$$

A Vogel-like thermal piezoviscous rheology model can be incorporated together with Carreau-Yasuda shear dependence equation as outlined by Habchi et al. [97]. The heat capacity and thermal conductivity of lubricant were measured and their variation with pressure and temperature were represented.

6.2.1.2 Volume of Fluid method for cavitation modeling

The volume of fluid (VOF) formulation can be used to model cavitation phenomenon. This model originally developed for two-phase flows without phase transition. The method can be modified to model the growth and collapse of bubbles. In VOF technique, phase fraction base function (F) is defined as unity for liquid cell and 0 for vapour cell. The transient governing equation for F can be written as:

$$\frac{\partial F}{\partial t} + u \frac{\partial F}{\partial x} + v \frac{\partial F}{\partial y} = 0 \quad \text{Equation 6-2}$$

VoF is implemented along with local-time stepping algorithm in OpenFoam for two incompressible and immiscible fluid (LTSInterFoam solver).

6.2.2 Fluid-Solid Coupling

The computational efficiency and instability are the main limitations for FSI models. Solving a single set of continuity and momentum equations for both solid and fluid domains using a single mesh can optimize the stability and computational time. The implementation of FV discretization by adopting constitutive equation for a Hookean solid to a fluid-momentum like equation can lead to derivation of such equations. This can be pursued by assigning a material property factor to each of the interacting subdomains. The use of single mesh can substantially reduce mesh update time.

6.2.3 Solid

The following development can greatly improve the capability of the model to study fracture mechanics.

6.2.3.1 Solid plastic deformation and thermal stress

When the stress is sufficient to deform the metal permanently, it is called plastic deformation. The governing equation for solid domain can be modified as follows to include plasticity effects:

$$\begin{aligned} \nabla \cdot [(2\mu + \lambda)\nabla(u)] = & -\nabla \cdot [\mu[\nabla(u)]^T + \lambda \text{tr}[\nabla(u)] - (\mu + \lambda)\nabla(u)] \quad \text{Equation 6-3} \\ & + \underbrace{\nabla \cdot [2\mu(\epsilon_p) + \lambda \text{tr}(\epsilon_p)]}_{\text{plastic term}} \end{aligned}$$

The plastic deformation can significantly affect lubricant behavior and contact characteristics. The thermal stress effect on solid deformation can be added to the model by modifying the stress equation as:

$$\sigma = 2\mu_s\varepsilon + \lambda_s tr(\varepsilon)I - \beta\tau \quad \text{Equation 6-4}$$

where β is thermo-elastic constant.

6.2.3.2 Real surface roughness profile and overtaking effects

The measured surface roughness profile superimposition to the model is necessary to observe physical behaviour of the lubricant in the presence of machined rough surfaces profiles. Further more, for two machined surfaces of a concentrated contact, both surfaces are rough and a double side surface waviness is shown in Figure 6-1. The implementation of transient roughness solver to the double-sided rough surfaces enables investigation on asperity overtaking phenomenon. A. Almqvist and R. Larsson [109] studied two-sided surface roughness effects for different sliding/rolling conditions. It was noted that film breakdown would probably occur due to low pressure or internal cavitation. However, no actual breakdown was observed in the presented cases.

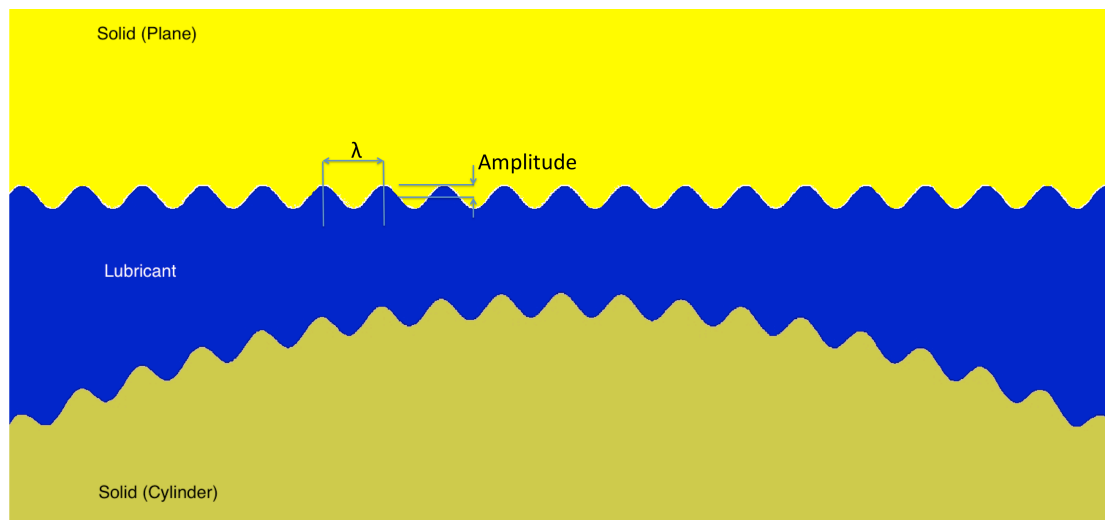


Figure 6-1. Double-side surface waviness

6.2.4 Study 3-D EHL point contact

The developed FSI model is currently based on 3D equations for both solid and fluid. The interaction between two entities is defined for 3D modeling as well. However, due to the focus of this thesis on the development of an accurate and efficient FSI methodology, point contacts were not considered and further developments are needed to test the code in 3D. The first step towards 3D EHL modeling would be to create three-dimensional domains for both solid and fluid phases. This would enable to capture the entrainment of lubricant into the contact, which is significantly different for a point contact when compared to the 2D line contact problem analysed in this thesis (see [85]) The implementation of local time stepping and block mesh orientation can be essentially effective on computational cost.

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