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A 2D Atom Interferometer Accelerometer in the Horizontal Plane

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the requirements for the degree of Doctor of Philosophy

Abstract

The thesis presented here describes the construction and early testing of an atom interferometer able to measure horizontal accelerations along two orthogonal axes. An ultra-high vacuum chamber is built for this cold atom interferometry. Rubidium 87 atoms are prepared in a 2D⁺ MOT and then directed into the main chamber where they are captured in a 3D MOT. The atoms are cooled from the MOT equilibrium temperature of ~ 140 mK down to a few μ K by means of an optical molasses. The atoms are then prepared with a narrow velocity distribution in a magnetically-insensitive ground state using a sequence of optical and microwave pulses. A matter-wave equivalent of the classical Mach-Zehnder interferometer is formed by use of Raman transitions in a $\pi/2 - \pi - \pi/2$ pulse sequence. To achieve the maximum sensitivity to acceleration the fringe contrast of the interferometer should be maximised. Adjustments to the chirp rate and alignment of the laser beams improved the fringe contrast by over a factor of 3. Analysis of the noise sources reveals that the noise floor is currently due to instability in the fluorescence detection but will ultimately be due to phase noise in the light driving the Raman transitions. This work demonstrates that the new 2-axis interferometer has better sensitivity than the mechanical accelerometers with which it is compared and points the way to a further tenfold improvement in sensitivity by reducing the noise and increasing the repetition rate.

Declaration

I declare that this thesis has been composed solely by myself and that it has not been submitted, in whole or in part, in any previous application for a degree. Except where states otherwise by reference or acknowledgement, the work presented is entirely my own.

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For Mum, Dad, and Jason

My constants in a world of variables



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Chapter 1

Introduction

1.1 Motivation

Methods of navigation date as far back as 1000 BC when Polynesian explorers used stars, ocean currents, and wind patterns to make sea voyages spanning thousands of kilometers [?]. For centuries thereafter, many cultures relied on the stars as a means of navigation. It wasn't until the 11th century that a magnetic compass was developed in China to allow for journeys across the sea in overcast conditions [?]. Over the next 900 years more sophisticated navigation techniques were developed driven by the desire to exploit or establish trade routes. With the development of artificial satellites, most methods of navigation over both land and sea became obsolete in favour of the more universally available and reliable Global Navigation Satellite System (GNSS).

1.1.1 Global navigation satellite system

The launch of Russia's first satellite, Sputnik, in 1957 gave birth to the idea of navigating using artificial satellites. The complete trajectory of Sputnik's orbit was calculated from Doppler shift data by Guier and Wieffenback working at the John Hopkins Applied Physics Laboratory [?]. Their director, Dr. F. T. McClure, realised that this technique could be inverted. If one knows the precise orbit of a satellite, then a receiver's unknown location could be determined from the Doppler data. In April 1960 the Transit satellite system was launched into orbit by the US [?]. Transit was the first satellite-based positioning system to be used and provided the US Navy location information for their ballistic missile submarines. Today, a number of GNSS systems are in operation: GPS (USA), GLONASS (Russia), BeiDou (China), and Galileo (EU).

A Blackett review published by the UK government [?] provides a comprehensive overview

of the various sectors that are currently vulnerable due to their over-reliance on GNSS. These include sectors that make up the national infrastructure such as telecommunications, emergency services, energy, finance, and food and transport. A study by London economics estimated that if the UK's GNSS signal were to be disrupted for just 5 days the cost to the economy could exceed £5 billion [?]. A key outcome of the Blakett review was an understanding that “loss or compromise of GNSS-derived position, navigation, and time should be added to the National Risk Assessment in its own right,... ”.

This loss of signal can be caused by ionospheric scintillation [?], signal masking/reflection (trees, tall buildings, etc), severe weather, jamming, and “spoofing”. We have already seen evidence of GNSS interference in 2017 [?] where several ships in the Black Sea reported that their on-board GPS receivers were giving their positions as inland. Later suggestions point to this being the first occurrence of GNSS “spoofing”. Whilst this particular loss of GNSS signal caused no damage, such occurrences in military situations pose serious security threats and could cost lives.

Despite advances in methods of preventing GNSS signal loss, intentional interference of satellite signals remains a significant vulnerability. In addition, GNSS-denied environments, such as deep under the sea's surface where satellite signal is not available, will always pose difficulties if a reliable alternative to GNSS is not put forward. Therefore, a solution free of GNSS reliance has become of high interest.

1.1.2 Inertial navigation system (INS)

Inertial navigation units (INUs), which pre-date GNSS, are the most common choice for supplementing GNSS information for navigation. INUs were developed in the early 1920s and then experienced a surge in research beginning in the 1940s due to the need for unaided guidance for aircraft, ships, and later missiles [?]. Crucially, INUs operate without the need for external signals in order to calculate position and orientation. As such, they can be used when GNSS signal is temporarily lost or unavailable.

INU's typically contain 3 accelerometers and 3 gyroscopes orientated along 3 orthogonal axes. Gyroscopes measure angular velocity, $\vec{\Omega}(t)$, so by integrating this signal one can obtain the change in orientation, $\Delta\vec{\theta}(t)$,

$$\Delta\vec{\theta}(t) = \int \vec{\Omega}(t)dt. \quad (1.1)$$

Accelerometers measure the rate of change of velocity, $\vec{a}(t)$, and so they can be integrated twice to give a change in position, $\Delta\vec{x}$,

$$\Delta\vec{x} = \int \int \vec{a}(t)dt dt. \quad (1.2)$$

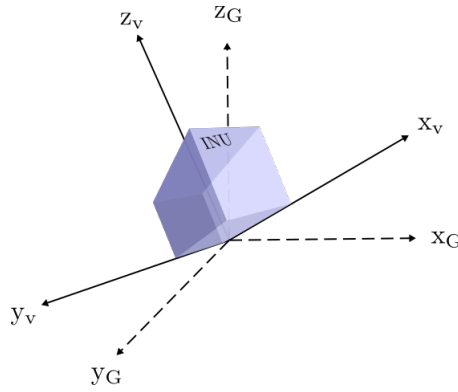


Figure 1.1: Diagram depicting the vehicle and global reference frames for an INU mounted in a strapdown configuration. The set of axes $\{x_G, y_G, z_G\}$ define the global reference frame and $\{x_v, y_v, z_v\}$ define the vehicle's reference frame.

By combining these two measurements, one can calculate the distance moved by a craft and in which direction.

In a stable platform system the INU is mounted to a platform which is isolated from external rotational motion by a series of gimbals. This allows the platform to move freely in all directions. In such a system, measurements from the gyroscopes are fed back to torque motors which rotate the gimbals. This keeps the platform aligned to a (arbitrary) global reference frame. The need for gimbals as well as motors make stable platform systems quite large and mechanically complex.

With a strap-down system, the INU is mounted rigidly to the vehicle. Figure 1.1 shows an INU in the vehicle's reference frame along with a global reference frame and fig. 1.2 shows a flow diagram for calculating position and orientation from a strapdown INU. A computer calculates the orientation of the vehicle's reference frame relative to the global reference frame using the gyroscope signals. This allows one to resolve the three components of acceleration in the vehicle's reference frame, $\vec{a}(t)$, onto the global reference frame. Subtraction of gravity and double integration allows the updated position of the vehicle to be obtained. A strap-down inertial navigation system (INS) is less mechanically complex than a stable platform INS at the cost of higher computational complexity. With the exponentially decreasing cost of computation in the last few decades, strap-down systems are now the dominant type of INS in use today.

INU accelerometers

All INU accelerometers rely on the same basic principle to measure acceleration. An INU accelerometer consists of a test mass suspended by some sort of spring. Through Newton's second law of motion, acceleration causes a force. When the body that an INU accelerometer

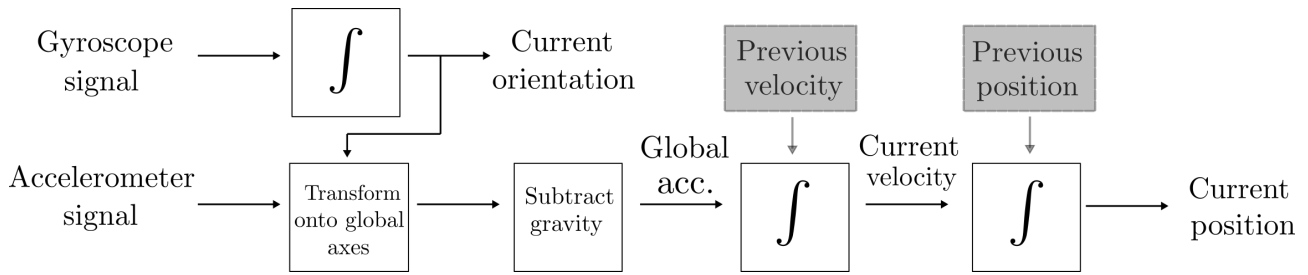


Figure 1.2: Flow diagram for the generic inertial navigation algorithm in a strap-down INS.

is attached to accelerates this force displaces the test mass from it's null position. By sensing the displacement of the test mass, along with knowledge of the characteristics of the spring, one is able to infer the acceleration that was experienced. This simple idea has been developed into more sophisticated configurations but whatever the spring used or displacement sensed, the general working principle is the same.

Micro-electromechanical systems (MEMS) accelerometers refer to a production technique used to produce small and low cost accelerometers. They use the same batch fabrication techniques used to create integrated circuits allowing to miniaturise the springs, displacement sensor, electrical readout, and other components in a conventional accelerometer.

A limiting factor for both conventional and MEMS sensors is that they possess scale factors and biases that drift over time. The scale factor is the conversion from the output of the accelerometer, such as a current, to the acceleration. The bias is a small offset of the accelerometer from the real acceleration. These two characteristics are carefully calibrated by the manufacturer on a per-device basis. Despite this, over time these two quantities drift due to temperature, humidity, mechanical shock, corrosion, thermal stress, wear from adhesion and abrasion to name a few [?]. Typical values for the scale factor and bias one-year repeatability are 300 ppm and $160 \mu\text{g}$. Any error in acceleration due to either of these quantities is double integrated in the calculation of position as shown in fig. 1.2. As a result, the position error grows quadratically with time limiting INUs to short missions. Because of this, INUs are used when GNSS signal is unavailable for a short period of time. Therefore INUs are often operated in conjunction with GNSS rather than instead of it.

Figure 1.3 gives an overview of the current accelerometer technologies and their applications placed by their scale factor and bias stability. For applications requiring high performance mechanical accelerometers are used whereas in the mid to low performance end MEMS and quartz accelerometers are preferred due to their low cost and small size. For long term navigation the better the scale factor and bias stability the more accurate the positioning. As fig. 1.3 shows, for navigation lasting for up to a day an accelerometer with a scale factor and bias stability of 50 ppm and $100 \mu\text{g}$ respectively is required. For navigation lasting up to a week, such as submarine navigation, the equivalent figures are 1 ppm and $10 \mu\text{g}$. However, there is a desire to

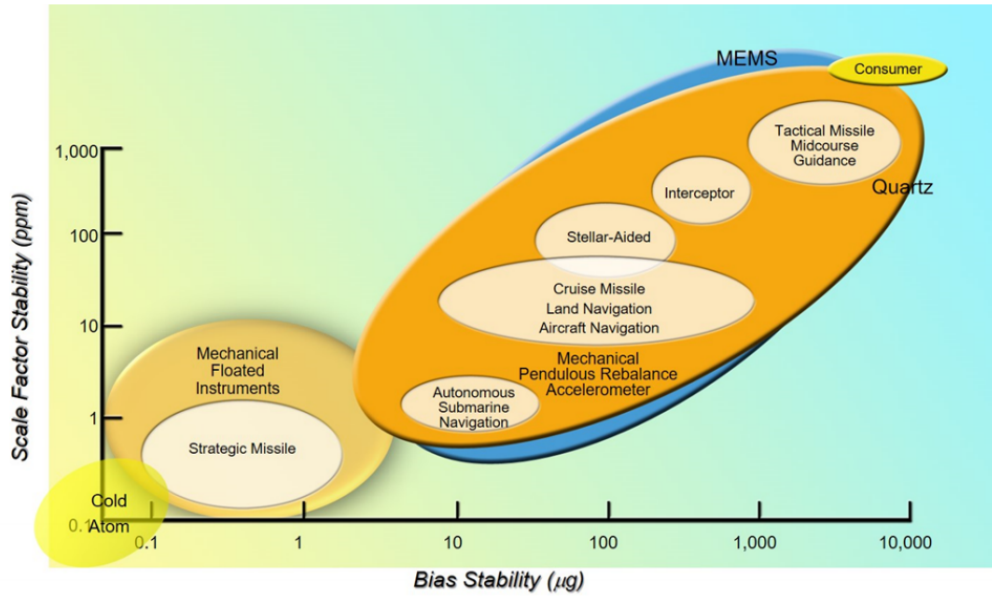


Figure 1.3: Current accelerometer technology capability is shown as coloured regions. The requirements for different applications is shown as cream coloured regions. Both are classified by their bias and scale factor stability capability/requirements. ([?]).

extend the navigation period to the order of months which current accelerometer technology cannot achieve.

1.2 Cold Atom Inertial Sensors

Atom interferometry offers an attractive alternative to conventional inertial sensing. Atoms in free-fall are a near perfect inertial observer. They do not require independent calibration and they do not age. As a result, atoms offer the possibility of completely eliminating bias instability and keeping scale factor instability on the part per billion level as depicted in fig. 1.3. Development of an INU that utilises atom interferometry could offer a 2-5m/h [?] navigation system suitable for long term applications without the reliance on GNSS signal.

Matter-wave interferometers

Atom interferometers are part of the broader class of matter-wave interferometers. They exploit the wave-particle duality of particles first postulated by [?] in 1923. De Broglie hypothesised that all particles can be described by a de Broglie wavelength and as such display wave-like

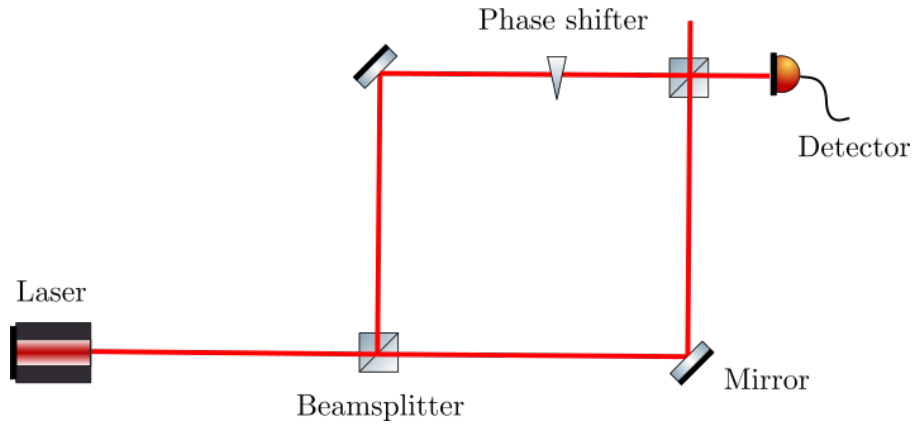


Figure 1.4: Schematic of a classical Mach-Zehnder interferometer. The measured intensity at the output depends on the phase difference between the two paths. An optical element such as a wedge can induce a phase difference by changing the path length.

properties. Hence it is possible to interfere a particle with itself such as in a double slit experiment or interferometer.

A matter-wave interferometer can be analogous to an optical Mach-Zehnder (MZ) interferometer. Figure 1.4 shows a schematic of a MZ interferometer. In a MZ interferometer coherent light from a laser is split into two paths by a beamsplitter. After freely propagating, the two paths are reflected by mirrors, and finally they are recombined on a second beamsplitter. One then measures the intensity of the output. The measured intensity is sensitive to the phase difference between the two arms. A phase difference between the two arms can be introduced by making one of the paths longer due to the presence of an optical element.

In fig. 1.4, matter implements the splitting, reflection, and recombination of light. In contrast, the matter-wave interferometer in this work reverses these roles using atoms as the wave and light to produce mirrors and beamsplitters.

Inertial sensitivity

A matter-wave interferometer was first shown to be sensitive to inertial forces in 1975 in which [?] used a neutron interferometer to measure a phase shift due to the Earth's gravitational field. When one considers the affect of an acceleration, a , on a matter-wave interferometer it can be shown that the phase shift is proportional to [?],

$$\delta\Phi \propto \left(\frac{m^{3/2}}{E^{1/2}} L \right) a. \quad (1.3)$$

Here m is the mass of the matter subject, E is it's external kinetic energy, and L is the length of the interferometer arms. This indicates that the highest sensitivity to inertial acceleration is achieved when using high mass and low velocity particles.

Atom interferometers

[?] demonstrated the first measurement of an inertial force using an atom interferometer. In their experiment, they measured the acceleration due to gravity, g , of a cloud of cesium with a sensitivity of $\Delta g/g = 3 \times 10^{-8}$. In the following years atom interferometry has proven its ability to make measurements of inertial forces countless times [?] with sensitivities up to $\Delta g/g = 6.7 \times 10^{-12}$ recently reported [?]. In addition, atom interferometers have been applied to the measurement of fundamental constants [?], dark energy [?], gravimetry [?], gradiometry [?], underground surveying [?], and inertial navigation [?].

This use of atom interferometers for precision measurements has been enabled by advances in the field of laser cooling. It is now relatively straightforward to cool a thermal stream of atoms to thermal velocities in the cm/s range and lower. Such cooling techniques have been applied to a range of atoms including, but not limited to, Na [?], Cs [?], Mg [?], Ca [?], K [?], Li [?], Rb [?], Sr [?], and Yb [?]. This has been greatly helped by the availability of stable and robust lasers at the appropriate wavelengths for cooling and interferometry.

To implement the optical equivalents of mirrors and beamsplitters two-photon stimulated Raman transitions can be used [?]. Raman transitions couple two ground states of an atom through an intermediate state. Crucially, this intermediate state is never populated which eliminates the possibility of spontaneous emission. This would destroy the coherence of the atom's wavefunction which is required to observe interference in the same way that a coherent light source is required for a MZ interferometer. Therefore, one can construct matter-wave MZ interferometers using such Raman transitions with long coherence times. This increases the sensitivity to acceleration by allowing large arm lengths (see eq. (1.3)) to be constructed. In addition the requirements on laser stability are on the order of the ground state splitting which is in the microwave frequency for certain atomic species. Microwave stability is much easier to achieve than optical stability which would be required in the case of one-photon transitions which couple a ground and excited state.

1.3 Transportable Cold-Atom Inertial Sensors

Numerous high precision laboratory-scale gravimeters [?], gravity gradiometers [?], and gyroscopes [?] have been demonstrated. However, these systems have been built with the intention of operating them within laboratories. As a result, they are large and unsuitable for field-deployment.

To bridge this gap a global effort to push such quantum technology from the laboratory into the real world is currently on-going. For example, in the UK the National Quantum Technologies

Programme has been set up to bring quantum devices towards commercialisation. Within this programme £23 million has been put towards a Quantum Technology Hub for Sensors and Timing highlighting the value of developing such a quantum navigation device which is also transportable.

When selecting an INU for an application one is usually concerned with the cost, size, weight, and power (CSWaP) of the device in addition to its performance metrics (e.g. bias, scale factor, axis misalignment, etc). An INU intended to be embedded into a smart phone will be distinct from one required to navigate a submarine, with the latter allowing for higher CSWaP values. Atom interferometers are unlikely to lower their values to even the higher end of the CSWaP spectrum of classical INUs. However, they should be within reasonable bounds such that operation, movement, and maintenance within the field is of minimal effort for the end user. Coupled with their superior stability, this would encourage their adoption into the commercial space.

Much commercial progress has been made in developing field-deployable gravimeters and gradiometers. AOSense, a spin off from Stanford University, was the first commercial company to offer atomic gravimeters in 2004. Since then several companies have emerged into the field of inertial sensors based on atom interferometers including MuQuans (France), ONERA (France), iXBlue (France), AtomSensors (Italy), Rafael Ltd. (Israel), Teledyne e2v (US), and Atomionics (Singapore). Outside of these commercial offerings academic groups have demonstrated working gravimeters aboard a ship [?] and aircraft [?] showing that these devices can successfully operate even in the presence of significant environmental noise.

1.4 Cold Atom Accelerometer

To achieve the highest sensitivity to acceleration, the matter wave needs to be split for a long time. However, that restricts the bandwidth of the accelerometer. Several groups have accepted lower sensitivity in order to demonstrate higher bandwidth. For example, [?] [?] have realised an atom interferometer with cycling rates[†] up to 330 Hz and a sensitivity of $36.7\mu\text{g}/\sqrt{\text{Hz}}$ by recapturing atoms at the end of the interferometer sequence. This sensitivity is four orders of magnitude above the best MEMS accelerometers [?] which offer high bandwidth and high sensitivity. However, even the best MEMS sensors (low bias and scale factor stability) suffer from significant drifts, which limits their performance for long range navigation applications.

On the other hand atom interferometers offer long term stability with lower bandwidth. A

[†]The cycling rate of the interferometer is $1/T$ where T is the time required to run one complete cycle of the experiment.

hybridisation of these complementary technologies would allow one to make high bandwidth measurements with a MEMS whilst making periodic corrections with the much lower bandwidth atom interferometer. This would lead to improved long term stability than operating the MEMS alone.

Our project at Imperial College London has already demonstrated such a hybridisation for measurements of acceleration along a single horizontal axis. Ultimately we wish to develop a field-deployable quantum-classical INU which offers superior long-term stability compared to classical INUs. Such a device will present a significant step forward in GNSS-free navigation and would pave the way for commercialisation.

1.5 Thesis Outline

The main focus of my project has been to extend the previous one-axis cold atom accelerometer into a two axis system to progress us towards a complete quantum-classical INU. In addition, I have designed a novel Raman beam expander to progress us towards our acceleration uncertainty goal of 100 ng at 1 second of integration. This uncertainty will, for example, extend the period of inertial navigation for a submarine from days to several months. Finally, I have quantified the various sources of noise which currently limit the uncertainty of our system. The next stage of this project will be to remedy these sources of noise and then to extend the system to allow for rotation measurements. This will allow us to realise a full quantum inertial navigation system.

Chapter 2 gives an overview of how we are able to measure acceleration using atom interferometry. I start by describing the two-photon Raman transition, which we use to coherently manipulate the atoms in a manner similar to a Mach-Zehnder interferometer. In §2.2 I explain how we use a combination of such transitions to construct an interferometer for sensing acceleration. I conclude by detailing some experimental considerations for our setup in §2.3.

In chapter 3 I detail the experimental setup. In §3.1 I describe the components of the novel laser system and measure the phase noise which is crucial in achieving long term stability of acceleration measurements. In §3.2 the vacuum chamber geometry is presented as well as the pressure achieved. §3.3 details the light distribution module setup, as well as its frequency and power output. In §3.4 I present the design of the in-vacuum detection optics and a measurement of the technical noise of the detection system. Finally §3.5 details the design of the coils for achieving a 3D magneto-optical trap.

Chapter 4 gives a description of the Raman optics and the retro-reflecting module. In §4.1 I explain the design considerations for the new Raman optics setup followed by a description of the new design in §4.2. In §4.3 I characterise the collimation and spot size of the Raman beam

to show agreement with the design goals. Finally in §4.4 I describe the components of the retro-reflecting module which include a quarter wave-plate, mirror, and MEMS accelerometer, concluding with a characterisation of the MEMS accelerometer performance in §4.4.2.

In Chapter 5 I explain optimisation of the preparation sequence which produced 10^7 atoms ready for interferometry. These are: §5.1 optimisation of the 2D MOT parameters and magnetic field ramp of the 3D MOT coil, §5.2 frequency and power ramps to cool the atoms to $8\,\mu\text{K}$ in optical molasses, §5.3 frequency shifts to achieve both a vertical and horizontal launch whilst cooling the atoms in a moving molasses, and finally §5.4 our 5 step state preparation sequence.

Chapter 6 presents the fringes obtained with both interferometers. In §6.1 I start with the fringes that were first obtained, then describe how we improved them. In §6.2 I study the coherence and contrast of the fringes. Finally, §6.3 describes how I attributed the various sources of noise in our system and in the process identified the factors responsible for uncertainty in a measurement of acceleration using the interferometers.

Chapter 2

Raman Interferometry

2.1 Two-Photon Raman Transitions

Two-photon stimulated Raman transition are used to split, reflect and recombine matter waves in our atom interferometers. These transitions impart momentum to the matter wave packets, without populating the excited state which could spontaneously decay thus destroying the coherence of the atom's wavefunction.

2.1.1 Propagator for atom-light interaction

This section is not intended to be a complete description of atom-light interaction. The interested reader is directed towards textbooks where a more thorough treatment of the subject can be found (e.g. [?]).

Figure 2.1 shows the structure of a two-photon Raman transition between two ground states, $|1\rangle$ and $|2\rangle$, split by an amount ω_0 . Two laser beams are incident on the atom with electric fields

$$\vec{\mathcal{E}}^{(1)}(\vec{z}, t) = \vec{\mathcal{E}}_0^{(1)} e^{i(\vec{k} \cdot \vec{z} - \omega_1^{(L)} t + \phi_1)} \quad (2.1)$$

$$\vec{\mathcal{E}}^{(2)}(\vec{z}, t) = \vec{\mathcal{E}}_0^{(2)} e^{i(\vec{k}_2 \cdot \vec{z} - \omega_2^{(L)} t + \phi_2)}. \quad (2.2)$$

Here $\vec{\mathcal{E}}_0^{(i)}$, $\vec{k}_j$, ω_j , ϕ_j are the field amplitude (real), wavevector, angular frequency, and offset phase of the field originating from laser j . Each beam couples a ground state to an intermediate state displayed as a black dotted line in fig. 2.1. The intermediate state is frequency shifted down by Δ_3 from an excited state denoted $|3\rangle$. Thus each beam is red-detuned from $|3\rangle$ by Δ_3 and laser beam 2 is shifted by an additional amount δ . For $\delta \ll \omega_0$, beam 1 mainly couples states

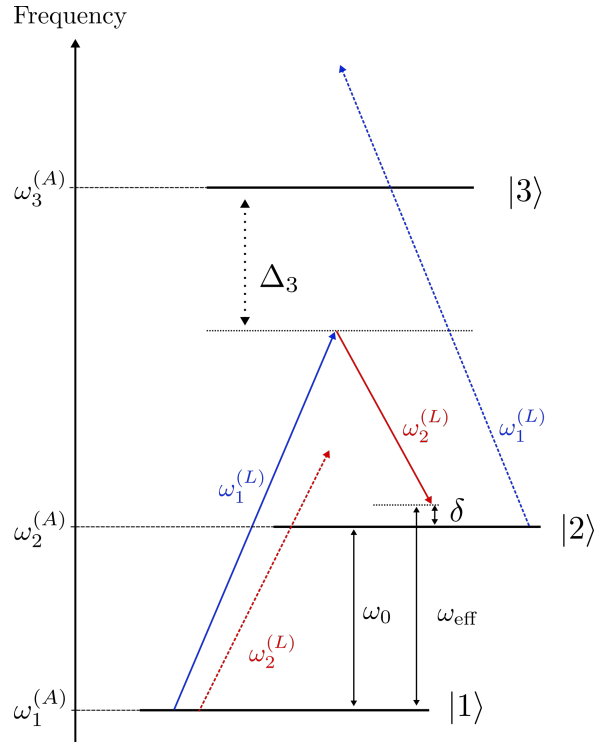


Figure 2.1: Diagram showing a two-photon Raman transition process within an atom with three energy levels. The axis on the left shows the frequency of the various energy levels. The two frequencies emitted by a laser are shown in red and blue with solid lines representing frequencies that resonantly drive the atom between energy levels and dotted lines showing off-resonant excitation.

1 and 3, while beam 2 mainly couples states 2 and 3. These couplings are characterised by the Rabi frequencies

$$\Omega_{3j} = \frac{\langle j | e\vec{r} \cdot \vec{\mathcal{E}}_0^{(j)} | 3 \rangle}{\hbar}. \quad (2.3)$$

Here $e\vec{r}$ is the electric dipole operator. If $\Delta_3 \gg \Omega_{31}, \Omega_{32}$, and δ , then population of the intermediate state is strongly suppressed [?] effectively eliminating the intermediate state from the equations of motion. The state vector of the remaining two-level system is

$$|\Psi(t)\rangle = c_1 |1, \vec{p}_0\rangle + c_2 e^{-i(\delta - \phi_{\text{eff}})t} |2, \vec{p}_0 + \hbar\vec{k}_{\text{eff}}\rangle \quad (2.4)$$

where $|1, \vec{p}_0\rangle$ represents an atom in internal state $|1\rangle$ with centre-of-mass momentum $\vec{p}_0$. In undergoing a two-photon Raman transition from $|1\rangle$ to $|2\rangle$, the momentum changes by $\hbar\vec{k}_{\text{eff}}$ as a result of absorbing a photon from one beam and emitting a photon into the other beam. This effective two-level system can be thought of as being coupled by a single effective field having frequency and phase given by

$$\omega_{\text{eff}} = \omega_1^{(L)} - \omega_2^{(L)} \quad (2.5)$$

$$\phi_{\text{eff}} = \vec{k}_{\text{eff}} \cdot \vec{z} + \phi_1 - \phi_2 = \vec{k}_{\text{eff}} \cdot \vec{z} - \Phi_{\text{off}}. \quad (2.6)$$

Here $\vec{k}_{\text{eff}} = \vec{k}_1 - \vec{k}_2$ is the effective wavenumber and $\Phi_{\text{off}} = \phi_2 - \phi_1$ is the difference offset phase. The coupling of the two level system is given by the effective Rabi frequency

$$\Omega_{\text{eff}} = \frac{\Omega_{31}^* \Omega_{32}}{2\Delta_3}. \quad (2.7)$$

To obtain a unitary operator, $\mathcal{U}(t_1, t_0)$ that describes the interaction of an atom with this effective field I write the Hamiltonian of the atom-light system in the Schrödinger picture as

$$\mathcal{H}_S = \mathcal{H}_{0,S} + \mathcal{H}_{1,S}(t) \quad (2.8)$$

$$= \hbar\omega_1^{(A)} |1\rangle \langle 1| + \hbar\omega_2^{(A)} |2\rangle \langle 2| - \frac{1}{2}\hbar\Omega_{\text{eff}}(e^{i(\omega_{\text{eff}}t + \phi_{\text{eff}})} + e^{-i(\omega_{\text{eff}}t + \phi_{\text{eff}})})(|1\rangle \langle 2| + |2\rangle \langle 1|). \quad (2.9)$$

$\mathcal{H}_{0,S}$ is the time independent Hamiltonian of the atom, $\mathcal{H}_{1,S}$ is the time dependent interaction Hamiltonian, and $\mathcal{H}_S$ is the total Hamiltonian. In the interaction picture, state vectors, $|\Psi_I(t)\rangle$, and operators, $A_I(t)$, are defined using the unitary operator $\mathcal{U}_0(t) = e^{-i\mathcal{H}_{0,S}t/\hbar}$:

$$|\Psi_I(t)\rangle = \mathcal{U}_0^\dagger(t) |\Psi_S(t)\rangle, \quad A_I(t) = \mathcal{U}_0^\dagger(t) A_S(t) \mathcal{U}_0(t). \quad (2.10)$$

Starting from the Schrödinger equation one can obtain an expression for the Hamiltonian of the atom-light system in the interaction picture,

$$\mathcal{H}_S |\Psi_S(t)\rangle = i\hbar \frac{d}{dt} |\Psi_S(t)\rangle \quad (2.11)$$

$$\mathcal{H}_S \mathcal{U}_0(t) |\Psi_I(t)\rangle = i\hbar \frac{d}{dt} \mathcal{U}_0(t) |\Psi_I(t)\rangle \quad (2.12)$$

$$= i\hbar \left[\left(\frac{d}{dt} \mathcal{U}_0(t) \right) |\Psi_I(t)\rangle + \mathcal{U}_0(t) \frac{d}{dt} |\Psi_I(t)\rangle \right] \quad (2.13)$$

$$\mathcal{U}_0^\dagger(t) \mathcal{H}_S \mathcal{U}_0(t) |\Psi_I(t)\rangle = i\hbar \left[\mathcal{U}_0^\dagger(t) \left(\frac{d}{dt} \mathcal{U}_0(t) \right) |\Psi_I(t)\rangle + \frac{d}{dt} |\Psi_I(t)\rangle \right] \quad (2.14)$$

$$i\hbar \frac{d}{dt} |\Psi_I(t)\rangle = \left[\mathcal{U}_0^\dagger(t) \mathcal{H}_S \mathcal{U}_0(t) - i\hbar \mathcal{U}_0^\dagger(t) \left(\frac{d}{dt} \mathcal{U}_0(t) \right) \right] |\Psi_I(t)\rangle \quad (2.15)$$

$$\therefore \mathcal{H}_I(t) = \mathcal{U}_0^\dagger(t) \mathcal{H}_S \mathcal{U}_0(t) - i\hbar \mathcal{U}_0^\dagger(t) \frac{d}{dt} \mathcal{U}_0(t). \quad (2.16)$$

The interaction Hamiltonian, after making the rotating wave approximation (RWA)[†] is therefore,

$$\mathcal{H}_I(t) = \frac{\hbar}{2} \begin{pmatrix} 0 & \Omega_{\text{eff}} e^{i\phi_{\text{eff}}} e^{i\delta t} \\ \Omega_{\text{eff}} e^{-i\phi_{\text{eff}}} e^{-i\delta t} & 0 \end{pmatrix}, \quad (2.17)$$

[†]In the RWA terms that rapidly oscillate at $\omega_0 + \omega_{\text{eff}}$ are neglected and only terms oscillating at $\omega_0 - \omega_{\text{eff}}$ are considered. The RWA is a good approximation when $\Omega_{\text{eff}} \ll \omega_0$ which is indeed the case for us.

giving a propagator for a pulse of duration τ of

$$\mathcal{U}(\tau) = \begin{pmatrix} \cos\left(\frac{\Omega_G \tau}{2}\right) - \frac{i\delta}{\Omega_G} \sin\left(\frac{\Omega_G \tau}{2}\right) & -\frac{i\Omega_{\text{eff}}}{\Omega_G} \sin\left(\frac{\Omega_G \tau}{2}\right) e^{-i\phi_{\text{eff}}} \\ -\frac{i\Omega_{\text{eff}}}{\Omega_G} \sin\left(\frac{\Omega_G \tau}{2}\right) e^{i\phi_{\text{eff}}} & \cos\left(\frac{\Omega_G \tau}{2}\right) + \frac{i\delta}{\Omega_G} \sin\left(\frac{\Omega_G \tau}{2}\right) \end{pmatrix}. \quad (2.18)$$

I have dropped an irrelevant over-all phase factor here and

$$\Omega_G = \sqrt{\delta^2 + \Omega_{\text{eff}}^2} \quad (2.19)$$

is the generalised Rabi frequency.

Consider an atom initially in state $|1\rangle$. The state amplitudes are $c_1(t=0) = 1$ and $c_2(t=0) = 0$. To find the probability of finding an atom in state $|2\rangle$ after a Raman interaction of length τ we apply the propagator and project onto the $|2\rangle$ state to obtain,

$$P_2(\tau) = |\langle 2 | \mathcal{U}(\tau) | \Psi(t) \rangle|^2 \quad (2.20)$$

$$= \frac{|\Omega_{\text{eff}}|^2}{\Omega_G^2} \sin^2\left(\frac{\Omega_G \tau}{2}\right) = \frac{|\Omega_{\text{eff}}|^2}{\delta^2 + \Omega_{\text{eff}}^2} \sin^2\left(\frac{\Omega_G \tau}{2}\right). \quad (2.21)$$

Equation (2.21) shows that the probability of driving an atom into state $|2\rangle$ depends on the value of δ . The explicit form of the Raman detuning, δ is

$$\delta = \omega_{\text{eff}} - \left(\omega_0 + \frac{\vec{p}_0 \cdot \vec{k}_{\text{eff}}}{m} + \frac{\hbar \vec{k}_{\text{eff}}^2}{2m} + \delta_{\text{DLS}} \right) \quad (2.22)$$

$$= \omega_{\text{eff}} - (\omega_0 + \omega_D + \omega_r + \delta_{\text{DLS}}) \quad (2.23)$$

where m is the mass of the atom and δ_{DLS} ,

$$\delta_{\text{DLS}} = \frac{|\Omega_{32}|^2}{4\Delta_3} - \frac{|\Omega_{31}|^2}{4\Delta_3} \quad (2.24)$$

is the differential light shift (DLS). Equation (2.23) shows that the two laser frequencies must cancel the splitting of the energy levels, ω_0 , the Doppler shift, ω_D , the recoil shift, ω_r , and the DLS to be resonant with the transition. In §2.3.5 it will be shown that for each value of the detuning Δ_3 there exists a “magic” ratio of the laser intensities that cancels the differential light shift. Thus we can set $\delta_{\text{DLS}} = 0$, which is helpful in achieving a large, well-defined P_2 when the light intensity is not constant across the cloud of atoms.

2.1.2 Mirrors and beamsplitters

Raman transitions change the internal state of an atom whilst also imparting a momentum of $\hbar \vec{k}_{\text{eff}}$. One can form a mirror by completely inverting the state of the atom so that the path of

the atom changes direction. Or form a beamsplitter by putting the atom into a superposition of both internal states. On setting $\delta_{\text{DLS}} = \delta = 0$, eq. (2.18) gives the state of the atom after interaction with the field for a time τ as

$$\begin{pmatrix} c_1(\tau) \\ c_2(\tau) \end{pmatrix} = \begin{pmatrix} \cos\left(\frac{\Omega_{\text{eff}}\tau}{2}\right) & -i \sin\left(\frac{\Omega_{\text{eff}}\tau}{2}\right) e^{-i\phi_{\text{eff}}} \\ -i \sin\left(\frac{\Omega_{\text{eff}}\tau}{2}\right) e^{i\phi_{\text{eff}}} & \cos\left(\frac{\Omega_{\text{eff}}\tau}{2}\right) \end{pmatrix} \begin{pmatrix} c_1(0) \\ c_2(0) \end{pmatrix}, \quad (2.25)$$

where the coefficients $c_1(\tau)$ and $c_2(\tau)$ are defined in eq. (2.4). When the pulse area $\Omega_{\text{eff}}\tau = \pi$, the state amplitudes are given by

$$c_1(\tau) = -ie^{-i\phi_{\text{eff}}} c_2(0) \quad (2.26)$$

$$c_2(\tau) = -ie^{i\phi_{\text{eff}}} c_1(0). \quad (2.27)$$

An atom initially in state $|1\rangle$ before the interaction is now in state $|2\rangle$ after the interaction. We have produced an optical equivalent of a mirror and such a pulse is called a π -pulse. Similarly, when $\Omega_{\text{eff}}\tau = \pi/2$ the state amplitudes are

$$c_1(\tau) = \frac{1}{\sqrt{2}} [c_1(0) - ie^{-i\phi_{\text{eff}}} c_2(0)] \quad (2.28)$$

$$c_2(\tau) = \frac{1}{\sqrt{2}} [c_2(0) - ie^{i\phi_{\text{eff}}} c_1(0)]. \quad (2.29)$$

An atom initially in state $|1\rangle$ before the interaction is now in an equal superposition of states $|1\rangle$ and $|2\rangle$ after the interaction. This is the optical equivalent of a 50:50 beamsplitter and such a pulse is called a $\pi/2$ -pulse.

2.2 Three Pulse Raman Interferometry

As in fig. 1.4, we are concerned with measuring the difference in the phases accumulated along the two paths - the two arms of the interferometer. The phase accumulated along a single arm can be decomposed into contributions from two sources: the propagation phase and the phase impressed on the atom by the laser light. The total phase difference between the two arms of the interferometer is

$$\Phi_{\text{int}} = \Delta\Phi_{\text{prop}} + \Delta\Phi_{\text{laser}}. \quad (2.30)$$

Here $\Delta\Phi_{\text{prop}}$ is the difference in propagation phase and $\Delta\Phi_{\text{laser}}$ is the difference in laser induced phase.

2.2.1 Propagation phase

The propagation phase, $\Phi_{\text{prop}}^{(p)}$ along a path p is given by

$$\Phi_{\text{prop}}^{(p)} = \frac{1}{\hbar} \mathcal{S}^{(p)}. \quad (2.31)$$

Here $\mathcal{S}^{(p)}$ is the classical action. The action along a single segment is,

$$\mathcal{S}(z_f, t_f, z_i, t_i) = \int_{t_i}^{t_f} \mathcal{L}(z_f, t_f, z_i, t_i) dt. \quad (2.32)$$

Here z_i and z_f are the start and end positions of the segment and t_i and t_f are the start and end times. The classical action of a particle in the presence of a time-independent acceleration, a_z , is described by [?]]

$$\mathcal{S}(z_f, t_f, z_i, t_i) = \int_{t_i}^{t_f} \left[\frac{1}{2} M \dot{z}(t)^2 - M a_z z(t) \right] dt \quad (2.33)$$

$$= \frac{M}{2} \frac{(z_f - z_i)^2}{(t_f - t_i)} - \frac{M a_z}{2} (z_f + z_i)(t_f - t_i) - \frac{M a_z^2}{24} (t_f - t_i)^3. \quad (2.34)$$

Using eq. (2.34), the difference in action along the two paths in fig. 2.2 is given by

$$\mathcal{S}^{(L)} - \mathcal{S}^{(U)} = \frac{M}{T} (z_D - z_C)(z_C + z_D - z_A - z_B - a_z T^2). \quad (2.35)$$

For the paths shown in fig. 2.2

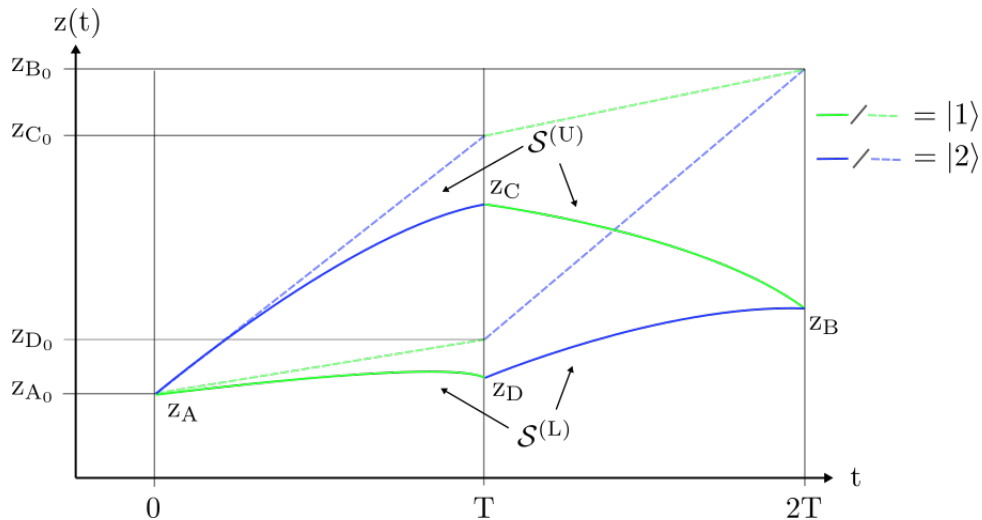


Figure 2.2: Trajectories taken by a atomic wave packet in the presence (solid lines) and absence (dashed lines) of acceleration. The upper path is denoted $\mathcal{S}^{(U)}$ and the lower path $\mathcal{S}^{(L)}$.

$$z_A = z_{A_0} \quad z_B = z_{B_0} - 2a_z T^2 \quad (2.36)$$

$$z_C = z_{C_0} - \frac{1}{2}a_z T^2 \quad z_D = z_{D_0} - \frac{1}{2}a_z T^2. \quad (2.37)$$

Using these equations in the final term of eq. (2.35) leads to

$$z_C + z_D - z_A - z_B - a_z(t)T^2 = z_{C_0} + z_{D_0} - z_{A_0} - z_{B_0} = 0 \quad (2.38)$$

because the paths of the atom in the absence of acceleration trace out a parallelogram for which opposites edges have equal lengths. Therefore, the phase difference arising from the propagation phase along the two paths of the interferometer is zero

$$\Delta\Phi_{\text{prop}} = \Phi_{\text{prop}}^{(L)} - \Phi_{\text{prop}}^{(U)} = \frac{1}{\hbar} [\mathcal{S}^{(L)} - \mathcal{S}^{(U)}] = 0. \quad (2.39)$$

The interferometer phase is completely determined by the difference in laser phase accumulated along the two paths, $\Phi_{\text{int}} = \Delta\Phi_{\text{laser}}$.

2.2.2 Interferometer phase

§2.1.1 derived a general propagator for atom-light interaction and §2.1.2 showed how one can construct mirrors and beamsplitters using such interactions. Using a $\pi/2 - \pi - \pi/2$ pulse sequence we are able to construct a matter-wave MZ interferometer. Figure 2.3a and fig. 2.3b display the space-time diagrams in the absence and presence of acceleration respectively.

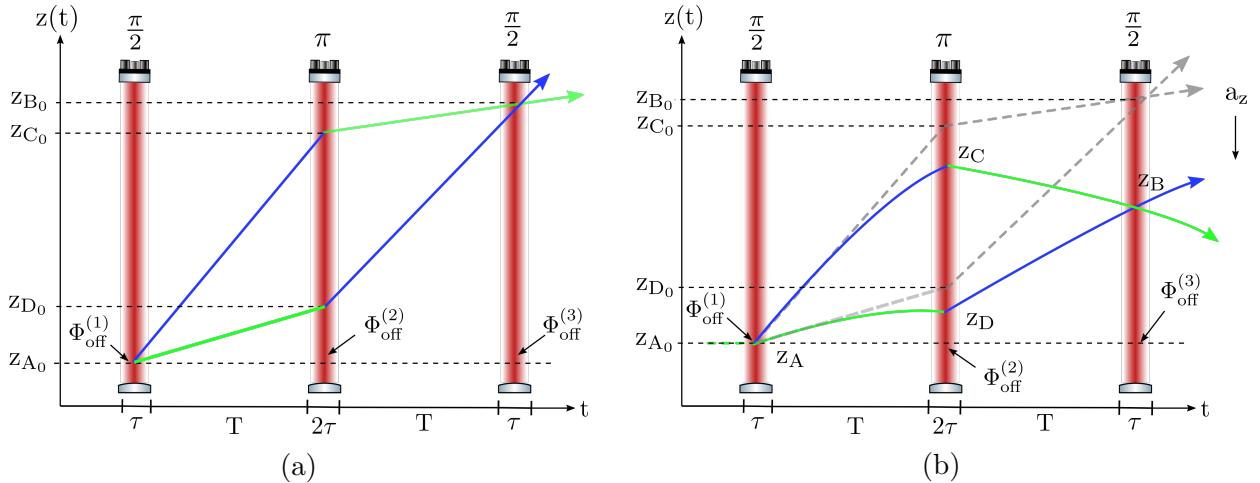


Figure 2.3: (a) Space-time diagram for a $\pi/2 - \pi - \pi/2$ interferometer in the absence of any acceleration. (b) Space-time diagram for a $\pi/2 - \pi - \pi/2$ interferometer in the presence of acceleration, a_z . The light grey dashed lines represent the paths of the atomic wavepacket from (a).

Between pulses the atom freely evolves for a time T . The propagator for this free evolution can

be found by setting $\Omega_{\text{eff}} = 0$ in eq. (2.18) to obtain

$$\mathcal{U}_{\text{free}}(T) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (2.40)$$

The state of the atom after the three pulse sequence is

$$|\Psi\rangle = \mathcal{U}_{\pi/2}(3\tau + 2T, \tau) \cdot \mathcal{U}_{\text{free}}(T) \cdot \mathcal{U}_{\pi}(\tau + T, 2\tau) \cdot \mathcal{U}_{\text{free}}(T) \cdot \mathcal{U}_{\pi/2}(0, \tau) |\Psi_0\rangle. \quad (2.41)$$

If we initially prepare an atom in the $|1\rangle$ state, the probability of an atom being in state $|2\rangle$ after the interferometer is

$$P_2 = |\langle 2|\Psi\rangle|^2 = \frac{1}{2} (1 - \cos \Phi_{\text{int}}). \quad (2.42)$$

Using eqs. (2.27) and (2.29) we can obtain an expression for Φ_{int} . An atomic wavepacket travelling along the upper path has an atomic state that has followed the sequence $|1\rangle \rightarrow |2\rangle \rightarrow |1\rangle \rightarrow |2\rangle$. Thus the total accumulated phase from the laser is

$$\Phi_{\text{laser}}^{(U)} = \phi_{\text{eff}}^{(1)} - \phi_{\text{eff}}^{(2)} + \phi_{\text{eff}}^{(3)} \quad (2.43)$$

$$= \left[k_{\text{eff}} z_A - \Phi_{\text{off}}^{(1)} \right] - \left[k_{\text{eff}} z_C - \Phi_{\text{off}}^{(2)} \right] + \left[k_{\text{eff}} z_B - \Phi_{\text{off}}^{(3)} \right] \quad (2.44)$$

$$= k_{\text{eff}} (z_A + z_B - z_C) - \left(\Phi_{\text{off}}^{(1)} - \Phi_{\text{off}}^{(2)} + \Phi_{\text{off}}^{(3)} \right). \quad (2.45)$$

An atomic wavepacket following the lower path has followed a state sequence of $|1\rangle \rightarrow |1\rangle \rightarrow |2\rangle \rightarrow |2\rangle$ and therefore accumulates a phase from interaction with the light field of

$$\Phi_{\text{laser}}^{(L)} = \phi_{\text{eff}}^{(2)} \quad (2.46)$$

$$= k_{\text{eff}} z_D - \Phi_{\text{off}}^{(2)}. \quad (2.47)$$

Using eq. (2.36) and eq. (2.37) in the above expressions for $\Phi_{\text{laser}}^{(U)}$ and $\Phi_{\text{laser}}^{(L)}$, the interferometer phase can be expressed as

$$\Phi_{\text{int}} = \Phi_{\text{laser}}^{(L)} - \Phi_{\text{laser}}^{(U)} \quad (2.48)$$

$$= \left(\Phi_{\text{off}}^{(1)} - 2\Phi_{\text{off}}^{(2)} + \Phi_{\text{off}}^{(3)} \right). \quad (2.49)$$

Equation (2.49) shows that in the absence of acceleration, the interferometer phase is determined by the difference in offset phases at the three pulses. In the presence of acceleration, eqs. (2.45) and (2.47) become

$$\Phi_{\text{laser}}^{(U)} = k_{\text{eff}} \left(z_A + z_B - z_C - \frac{3}{2} a T^2 \right) - \left(\Phi_{\text{off}}^{(1)} - \Phi_{\text{off}}^{(2)} + \Phi_{\text{off}}^{(3)} \right) \quad (2.50)$$

$$\Phi_{\text{laser}}^{(L)} = k_{\text{eff}} \left(z_D - \frac{1}{2} a T^2 \right) - \Phi_{\text{off}}^{(2)} \quad (2.51)$$

and the interferometer phase becomes

$$\Phi_{\text{int}} = k_{\text{eff}} a_z T^2 + \left(\Phi_{\text{off}}^{(1)} - 2\Phi_{\text{off}}^{(2)} + \Phi_{\text{off}}^{(3)} \right). \quad (2.52)$$

Figure 2.3 show that the interferometer arms form a closed area. However, if there is any rotation around an axis normal to this area, that changes the arm lengths and thus induces a further phase shift known as the Sagnac phase. Rotations will not be covered in this thesis but are left as the work of the next student on this project.

2.2.3 Hybridisation of an atom interferometer with a MEMS accelerometer

Equation (2.42) is the output signal for the ideal case. In practice, when working with an ensemble of atoms several factors can affect the measured signal. For example, eq. (2.21) shows that imperfect π and $\pi/2$ can occur due to imperfect choice of τ , or of δ , or to the inhomogeneity of these resulting from inhomogeneous laser intensity. These lead to a reduction of the interferometer fringe contrast C , giving a transition probability

$$P_2 = \frac{1}{2} (1 - C \cos(\Phi_{\text{int}})). \quad (2.53)$$

Suppose when making a measurement of P_2 we make an error of σ_{P_2} . This leads to an error of the measured interferometer phase of

$$\sigma_{\Phi_{\text{int}}} = \sigma_{P_2} / \frac{dP_2}{d\Phi_{\text{int}}} = \frac{2\sigma_{P_2}}{C \sin(\Phi_{\text{int}})}. \quad (2.54)$$

Therefore, phase errors can be minimised by operating with high contrast at the point of steepest slope on the interferometer fringe - $\Phi_{\text{int}} = \pi/2$.

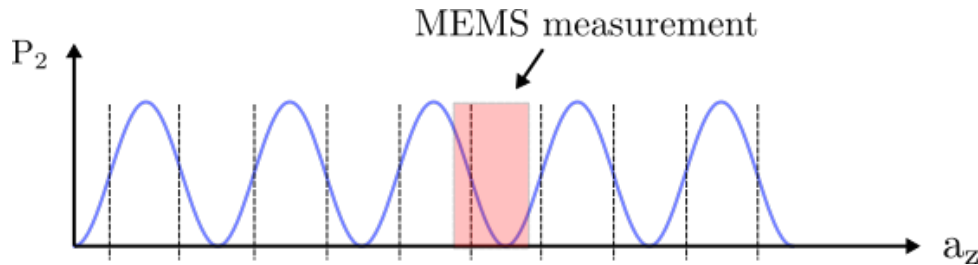


Figure 2.4: Plot of the interferometer fringe obtained as a function of acceleration.

Using eq. (2.53) and eq. (2.52) one can see that P_2 is periodic in the acceleration with a period $\Delta a = 2\pi/(k_{\text{eff}} T^2)$. The periodic nature of P_2 means measurements of Φ_{int} are multi

valued: adding any integer multiple of $\Delta a/2$ will produce the same P_2 . Figure 2.4 illustrates this idea. The black dashed lines represent the acceleration values that could give rise to the same interferometer phase. We identify the fringe order of the interferometer using a MEMS accelerometer with an uncertainty $\ll \Delta a/2$. This is shown as a red region on fig. 2.4. The accurate quantum measurement can then be used to correct the MEMS bias drift. In this way, one can benefit from the high bandwidth of a conventional MEMS accelerometer and the inherent zero bias drift of a cold atom interferometer. It is this configuration which we operate our device in.

2.3 Experimental Considerations

2.3.1 Rubidium 87 - D₂ line

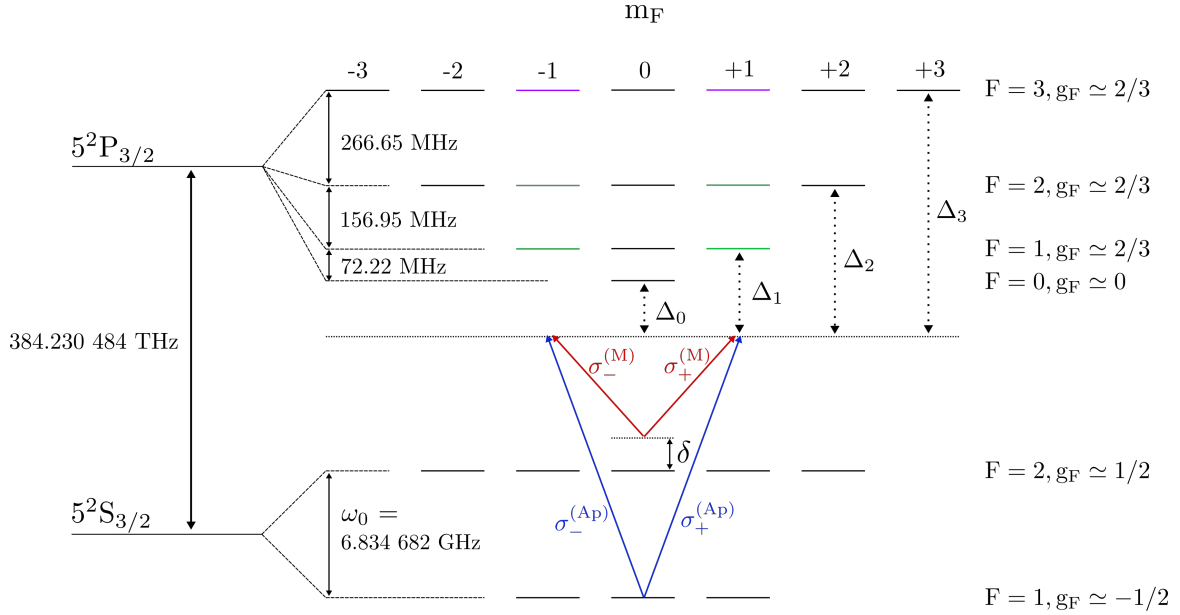


Figure 2.5: Energy level diagram of the D₂ line in ⁸⁷Rb. The polarisation and frequency of the Master laser (superscript (M))) are shown by the red arrow while those of the Apprentice are shown in blue. We choose to have the polarisations both σ₊ or both σ₋. The energy levels highlighted in green show the m_F = ±1 intermediate states that induce the Δm_F = 0 clock transition in the interferometer. The two purple energy levels show m_F = ±1 states that cannot drive the transition because the F=1 ground state does not couple to the F=3 excited state.

Figure 2.5 shows the energy level structure of ⁸⁷Rb. We operate on the D₂ line due to the presence of a “magic” ratio that cancels the DLS. The detuning of each laser from the excited state |j⟩ is labelled Δ_j. The states that allow two photon Raman transitions are shown in green. I refer to the laser that couples ground state |1⟩ to the intermediate state the Apprentice laser and the laser that couples the ground state |2⟩ the Master laser. The two states in purple

show states that can be off-resonantly excited to by the Master laser but do not allow Raman transitions. The green and purple states contribute to the DLS, δ_{DLS} , whereas only a subset of green states contribute to Ω_{eff} .

2.3.2 Polarisation considerations for Raman transitions

To minimise sensitivity to magnetic fields we use the clock states $|F = 1, m_F = 0\rangle$ and $|F = 2, m_F = 0\rangle$ as the two levels for the interferometer. Careful consideration must be taken over polarisation in order to drive the $\Delta m_F = 0$ transition. The Rabi frequency in eq. (2.3) becomes

$$\Omega_{g,j}^q = \frac{1}{\hbar} \langle g | e\vec{r} \cdot \vec{\mathcal{E}}_0^q | j \rangle. \quad (2.55)$$

Here $g = \{F, m_F\}$, $j = \{F', m_{F'}\}$, F indicates a hyperfine state, m_F a Zeeman sub-level, a prime indicates a state in the $5^2P_{3/2}$ manifold, and $q = +1, 0, -1$ indicates the Δm_F transition driven by the field which has amplitude $\vec{\mathcal{E}}_0^q$.

Consider the case where both lasers are parallel circularly polarised such that they drive σ^+ ($q=+1$) transitions. When one carries through eq. (2.55) through the equations of motion and eliminates the intermediate state, the effective Rabi frequency describing the two-photon coupling becomes a summation over excited states,

$$\Omega_{\text{eff}}^{++} = \sum_{F'=1,2} \frac{\left(\Omega_{10,\{F',+1\}}^{+1} \right)^* \Omega_{20,\{F',+1\}}^{+1}}{2\Delta_{\{F',+1\}}} \quad (2.56)$$

$$= \sum_{F'=1,2} \frac{\Omega_{\{F',+1\},10}^{-1} \Omega_{20,\{F',+1\}}^{+1}}{2\Delta_{\{F',+1\}}}. \quad (2.57)$$

Similarly, for the case of σ^-/σ^- polarised beams,

$$\Omega_{\text{eff}}^{--} = \sum_{F'=1,2} \frac{\left(\Omega_{10,\{F',-1\}}^{-1} \right)^* \Omega_{20,\{F',-1\}}^{-1}}{2\Delta_{\{F',-1\}}} \quad (2.58)$$

$$= \sum_{F'=1,2} \frac{\Omega_{\{F',-1\},10}^{+1} \Omega_{20,\{F',-1\}}^{-1}}{2\Delta_{\{F',-1\}}}. \quad (2.59)$$

In eqs. (2.57) and (2.59) the resonant Rabi frequencies induce $\Delta m_F = -1$ and $\Delta m_F = +1$ and thus an overall $\Delta m_F = 0$. Therefore, to drive the desired transitions requires parallel circularly polarised light.

2.3.3 Raman beam configuration

We produce the parallel circularly polarised beam pair by coupling orthogonal linearly polarised beams into a single polarisation-maintaining (PM) fibre, pass them through a quarter waveplate (QWP) and then retro reflect the beams through a second QWP. Figure 2.6 is a schematic of our retro-reflecting setup. Orthogonal linear polarised light exits the PM fibre and passes through a QWP to produce orthogonal circular polarisation. After passing through a QWP, reflecting off of a mirror, and passing through the QWP again, the returning light field rotates in the opposite direction from the incoming light, so drives the opposite Δm_F transition. Right and left handed helicity are shown in red and blue respectively. This leads to four beams uniquely described by their frequency and polarisation.

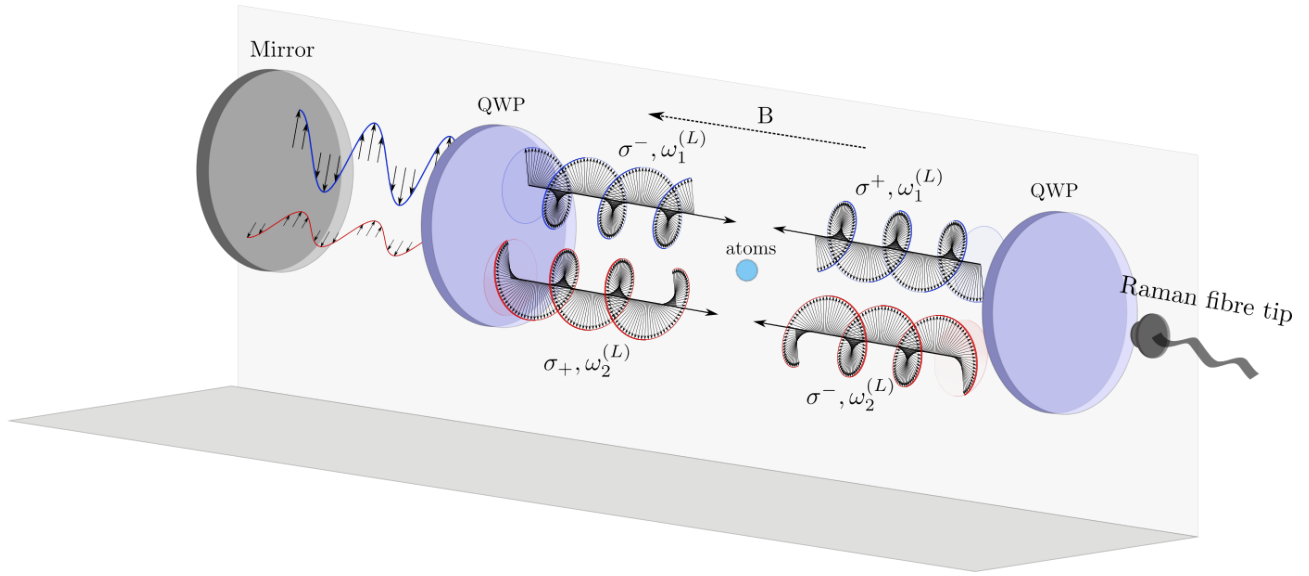


Figure 2.6: Schematic of the retro-reflecting setup for a single Raman beam.

The key advantage of this setup is that the phase noise acquired as the light propagates from the lasers to the vacuum chamber is common to both beams so cancels out in the Raman phase Φ_{off} .

Two issues arise when using a retro-reflecting configuration. The light shifts caused by the presence of additional beams which are not used in driving the desired transitions and the so-called “double interferometer”.

2.3.4 Double interferometer

For the interferometer shown fig. 2.6 there are two pairs of beams that can drive the Raman clock transition: σ^+/σ^+ and σ^-/σ^- . Each beam pair is composed of counter-propagating fields thus their effective wavevectors are in opposite directions. As a result, an atom resonant with

both counter-propagating pairs is driven into a superposition of the states $|1, \vec{p}_0\rangle$, $|2, \vec{p}_0 + \hbar\vec{k}_{\text{eff}}\rangle$ and $|2, \vec{p}_0 - \hbar\vec{k}_{\text{eff}}\rangle$ after the first pulse. Figure 2.7 shows the trajectory for an atom subject to a $\pi/2 - \pi - \pi/2$ pulse sequence in this situation. Two closed loop interferometers are formed with oppositely signed phases. Therefore, the population P_2 is insensitive to acceleration.

In order to avoid this problem, we detune ω_{eff} away from ω_0 . Because the beams are counter-propagating $\vec{k}_{\text{eff}} = \vec{k}_1 - \vec{k}_2 \approx 2\vec{k}_1$ and so the Raman detuning is sensitive to Doppler shifts (see eq. (2.23)). Because of this, an atom with non-zero velocity is shifted closer to resonance with one beam pair and further from resonance from the other. The difference between the resonance frequencies for the two beam pairs is now $2|\vec{k}_{\text{eff}} \cdot \vec{v}|$. By launching the atoms with velocity $\vec{v}$ such that the difference in resonance conditions is larger than the velocity distribution of the atomic cloud, only a single beam pair is able to drive transitions. In this way we avoid the double interferometer.

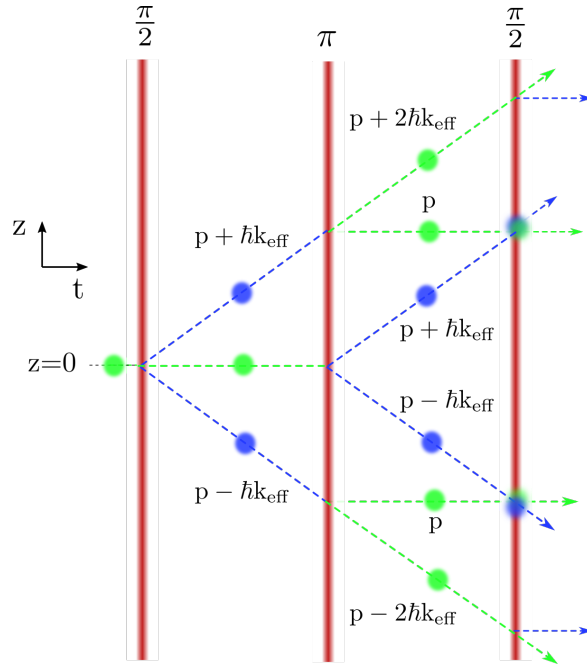


Figure 2.7: Space-time diagram showing the trajectories when an atom is resonant with both pairs of beams. The two pairs of beams impart the same momentum to the atom but in opposite directions. The interferometer phase shifts given by eq. (2.52) have equal magnitude but opposite signs due to the dependence on k_{eff} . Therefore, the double interferometer is insensitive to accelerations.

2.3.5 Cancelling the light shift

If there is a fluctuating DLS during the interferometer then we are not resonant at each pulse, i.e. $\delta \neq 0$. This leads to an additional term in the interferometer phase of eq. (2.52). By selecting the appropriate ratio of intensities for the two Raman beams we can cancel the DLS.

The frequency of the clock transition is shifted by the external magnetic field (Zeeman shift) and by the intensity of the laser field (AC Stark shift or light shift). By choosing states with $m_F = 0$, the shift due to magnetic fields is of second order[†]. By selecting an appropriate intensity ratio of the two lasers, we are able to cancel the light shift.

To account for the presence of two lasers, I rewrite the Rabi frequency as

$$\Omega_{g,j}^{(L)} = \frac{1}{\hbar} \langle g | e\vec{r} \cdot \vec{\mathcal{E}}_0^{(L)} | j \rangle. \quad (2.60)$$

Here L denotes either the Master or Apprentice laser. The light shift for state $|F = 1, m_F = 0\rangle$ and $|F = 2, m_F = 0\rangle$ is given by

$$\delta_{LS}^{F=1} = \frac{1}{\hbar} \sum_{j=\{F', m_{F'}\}} \frac{|\Omega_{10,j}^{(M)}|^2}{2\Delta_j^{(M)}} + \frac{|\Omega_{10,j}^{(Ap)}|^2}{2\Delta_j^{(Ap)}} \quad (2.61)$$

$$\delta_{LS}^{F=2} = \frac{1}{\hbar} \sum_{j=\{F', m_{F'}\}} \frac{|\Omega_{20,j}^{(M)}|^2}{2\Delta_j^{(M)}} + \frac{|\Omega_{20,j}^{(Ap)}|^2}{2\Delta_j^{(Ap)}}. \quad (2.62)$$

Here $\Delta_j^{(L)}$ denotes the detuning of laser L from the excited state $j = |F', m_{F'}\rangle$. The DLS can therefore be expressed as

$$\delta_{DLS} = \delta_{LS}^{F=2} - \delta_{LS}^{F=1} \quad (2.63)$$

$$= \frac{1}{\hbar} \sum_{j=\{F', m_{F'}\}} \left[\frac{|\Omega_{20,j}^{(M)}|^2}{2\Delta_j^{(M)}} - \frac{|\Omega_{10,j}^{(M)}|^2}{2\Delta_j^{(M)}} \right] + \left[\frac{|\Omega_{20,j}^{(Ap)}|^2}{2\Delta_j^{(Ap)}} - \frac{|\Omega_{10,j}^{(Ap)}|^2}{2\Delta_j^{(Ap)}} \right]. \quad (2.64)$$

Using that the Rabi frequency is proportional to the field's intensity leads to,

$$\delta_{DLS} \propto I_0^{(M)} \alpha + I_0^{(Ap)} \beta \quad (2.65)$$

$$\propto \alpha + R\beta. \quad (2.66)$$

Here $I_0^{(L)}$ denotes the intensity of laser L, α and β are constants that depend on the polarisation of the lasers, the internal states of the atom, and the detuning of the lasers, and $R = I_0^{(Ap)} / I_0^{(M)}$. Equation (2.66) shows that if α and β have opposite sign there can be a ratio R that cancels the DLS. This is indeed the case for the D₂ line in ⁸⁷Rb.

[†]For our typical magnetic field values the Zeeman shift is of the order of a couple hundred Hz.

The “magic” intensity ratio

Equation (2.64) shows that the DLS is a function of the detuning $\Delta_3^\dagger$. For a range of detunings there exists a “magic” ratio, R_0 such that $\delta_{\text{DLS}} = 0$. In fig. 2.8 I plot the detuning against the “magic” ratio where fig. 2.8a displays R_0 for a blue detuned Δ_3 and in fig. 2.8b for red detuned Δ_3 .

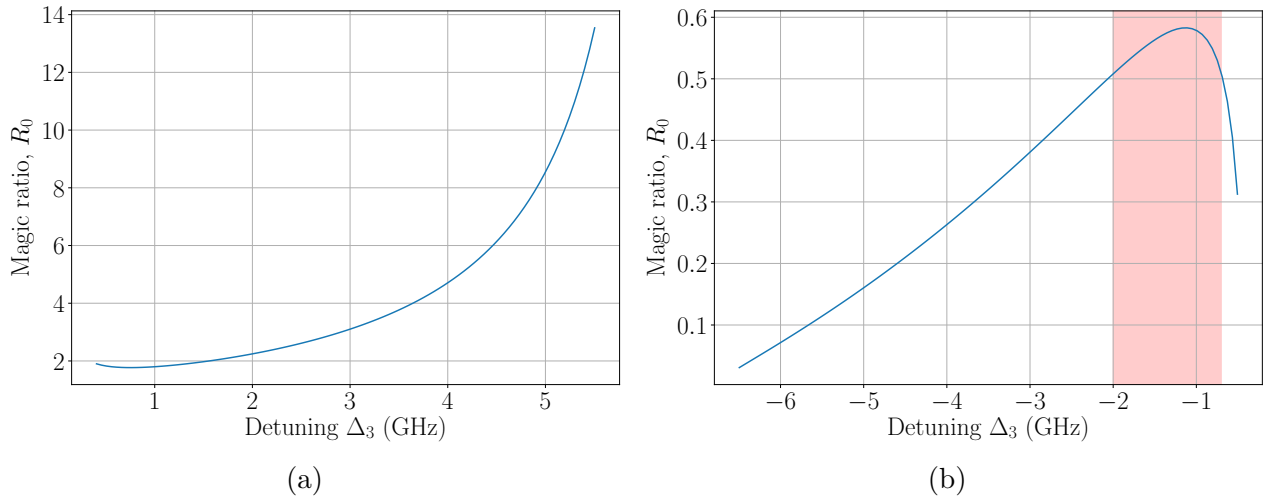


Figure 2.8: Variation of “magic” ratio as a function of (a) blue detuning and (b) red detuning. The red region shows the approximate range of Δ_3 that we would like our system to be set at due to the weak dependence on Δ_3 . The gradient for this region is plotted in fig. 2.9.

In fig. 2.8b the red region shows where the “magic” ratio is least sensitive to changes in the detuning. Figure 2.9 shows how the gradient $dR_0/d\Delta_3$ varies with detuning around the peak in this region. The gradient is zero at a detuning of $\Delta_3 = -1.13$ GHz. At this detuning the “magic” ratio is 0.583. As a result, the power of the Apprentice laser is set to be $P^{(\text{Ap})} = 0.583 P^{(\text{M})}$ where $P^{(\text{Ap})}$ and $P^{(\text{M})}$ are the powers of the Master and Apprentice lasers respectively.

[†]Because $\delta \ll \Delta_3^{(\text{M})}, \Delta_3^{(\text{Ap})}$, I drop the specification of the laser.

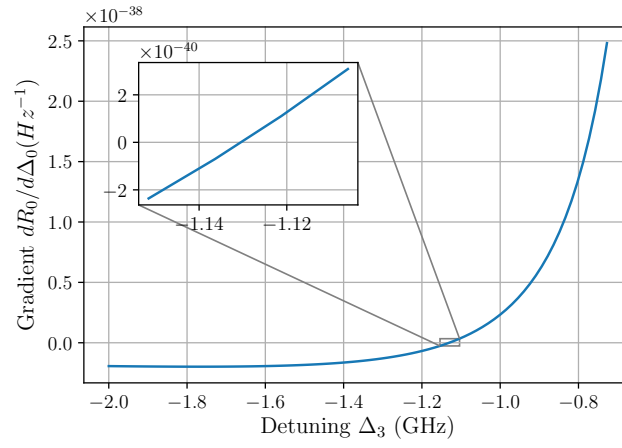


Figure 2.9: Graph showing variation in the gradient $dR_0/d\Delta_3$ with Δ_3 around the turning maximum. Inset: linear dependence of $dR_0/d\Delta_3$ with respect to the detuning Δ_3 . The zero in the gradient occurs at a detuning of -1.13 GHz.

Chapter 3

Experimental Setup

This chapter describes my experimental setup, which is illustrated as an overview in fig. 3.1. In §3.1 I present the laser system, §3.2 the vacuum chamber, §3.3 the apparatus to distribute the light throughout the experiment, §3.4 the detection system, and §3.5 the 3D magneto-optical trap (MOT) setup.

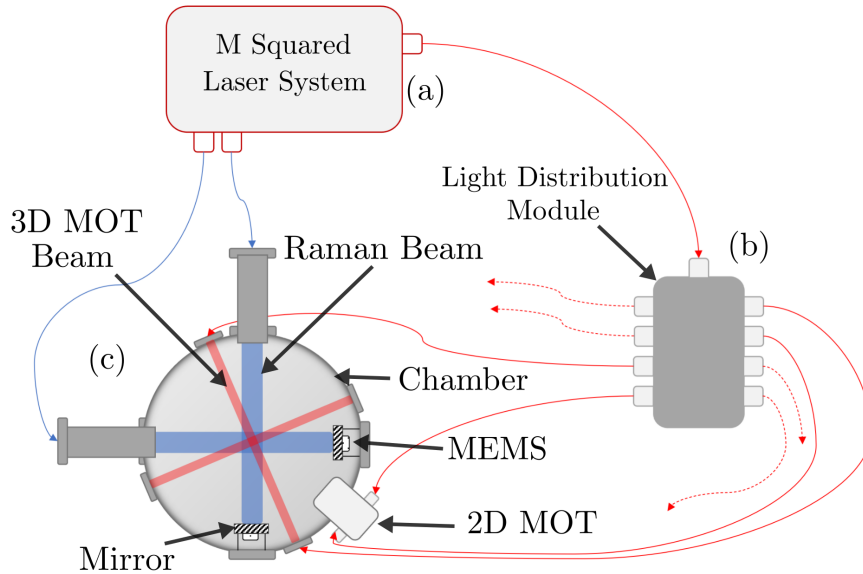


Figure 3.1: Experimental overview. (a) laser system - single laser system to produce light for two interferometers and cooling and trapping. (b) light distribution module - a single input from the laser system is distributed into eight separate outputs used for cooling and trapping. (c) Vacuum chamber.

3.1 Laser System

Inertial sensing using atom interferometry requires large ensembles of ultra-cold atoms and the ability to drive stimulated Raman transitions with a well controlled phase. There was no commercially available laser system that could provide the high optical powers and low phase-noise properties we required. In partnership with M Squared a new high power Raman laser system was developed. Figure 3.2 shows a schematic of the laser system.

Two Lighthouse Photonics DPSS Sprout lasers pump two titanium-sapphire lasers (SolsTiS-R-PSX-2000 and SolsTiS-R-PSX-EOM-1000). These lasers are combined on a polarising beam-splitter to produce the Raman beams. The frequency that addresses state $|1\rangle$ originates from SolsTiS 1 and for state $|2\rangle$ from SolsTiS 2. These are the Apprentice and Master lasers respectively. They are each linearly polarised and coupled into orthogonal axes of two polarisation-maintaining (PM) fibres. They are then delivered to the Raman collimators attached to the vacuum chamber. A description of the Raman collimators is deferred until chapter 4.

To minimise the differential light shift, the Master laser is locked to a frequency $\Delta_3 = -1.13$ GHz (see fig. 2.5) by offsetting it from the $|F = 3\rangle \longleftrightarrow |F' = 3/4\rangle$ crossover in ^{85}Rb using saturated absorption spectroscopy. The blue sideband from an electro-acoustic modulator (EOM) driven by a voltage controlled oscillator (VCO) is used as the reference signal for the offset lock. The ratio of Apprentice to Master laser power is fixed at 0.583 through active feedback to the power of the Sprout lasers. The Master and Apprentice are combined on a photodiode to produce a beat-note. This is down-sampled to 3.417 GHz and mixed with an oven controlled crystal oscillator to provide a phase lock error signal close to DC. This error signal is used as an input to a P-I controller which provides feedback to a piezo-actuated mirror for frequency correction and an intra-cavity EOM inside of the Apprentice for phase control. The Master and its phase-locked Apprentice are offset in frequency by 6.834 682 GHz with the ability to tune this value. In addition, the system is capable of applying a frequency chirp during operation.

3.1.1 Frequency output

The interferometer sequence has three frequency configurations illustrated in fig. 3.3. In addition to driving the Raman transition, shown as configuration I in fig. 3.3, the laser system can also be configured to produce other frequencies, shown as configurations II and III.

In changing between any of the configurations shown in fig. 3.3 the following process is followed: the vapour cell lock must be disengaged, the VCO driving the EOM ramped, and then the vapour cell lock reapplied. This process takes ≈ 4 ms and is limited by the tuning speed of the SolsTiS lasers (500 MHz/ms).

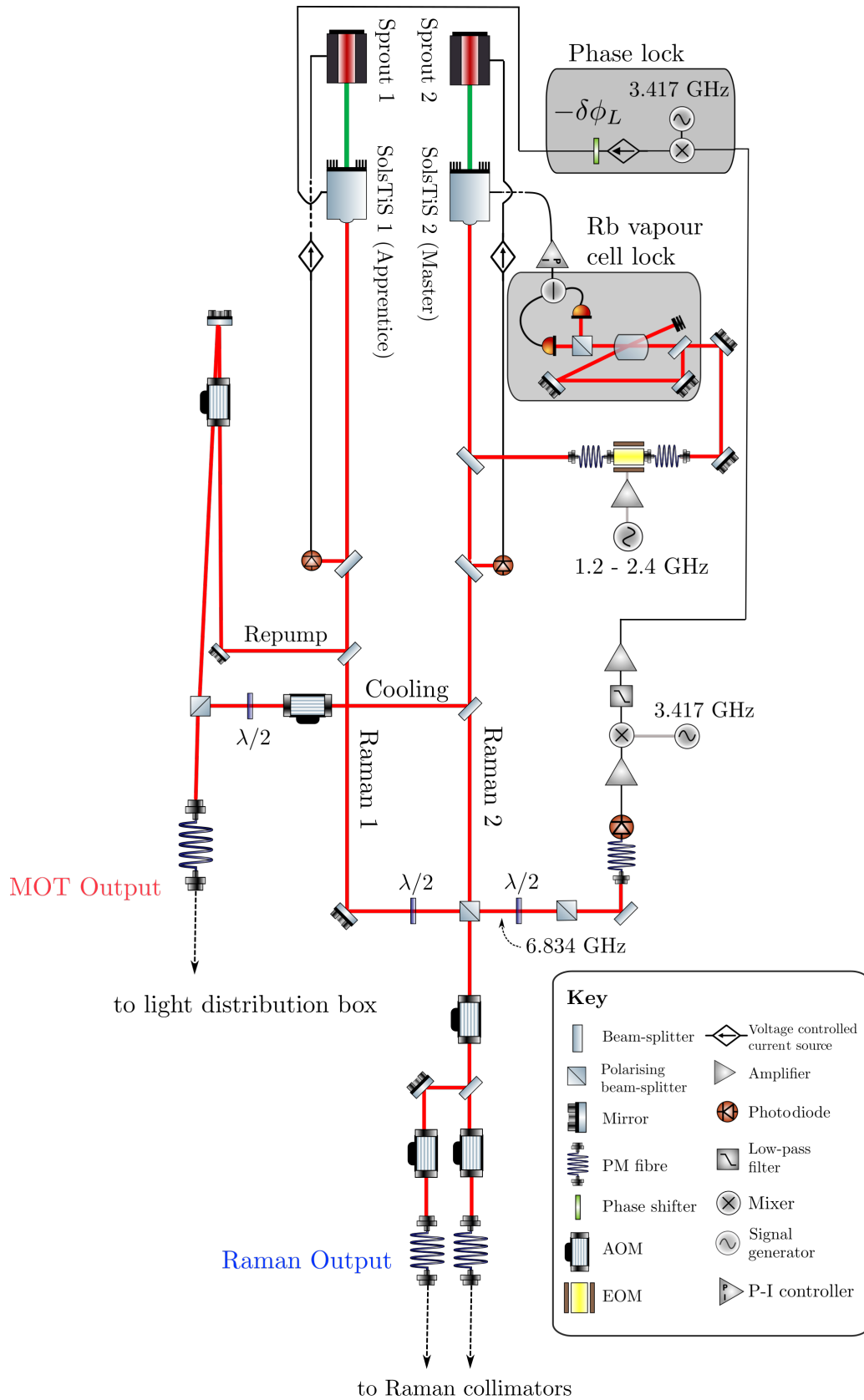


Figure 3.2: Laser system: 2 Lighthouse Photonics DPSS Sprout lasers, 2 titanium-sapphire lasers (SolsTiS), vapour cell lock in series with electro-acoustic modulator (EOM), phase lock, acousto-optic modulators (AOMs), photodiodes, and fibres. Laser system produces 3 outputs: 2 are used for driving Raman transitions and 1 is used for cooling and trapping.

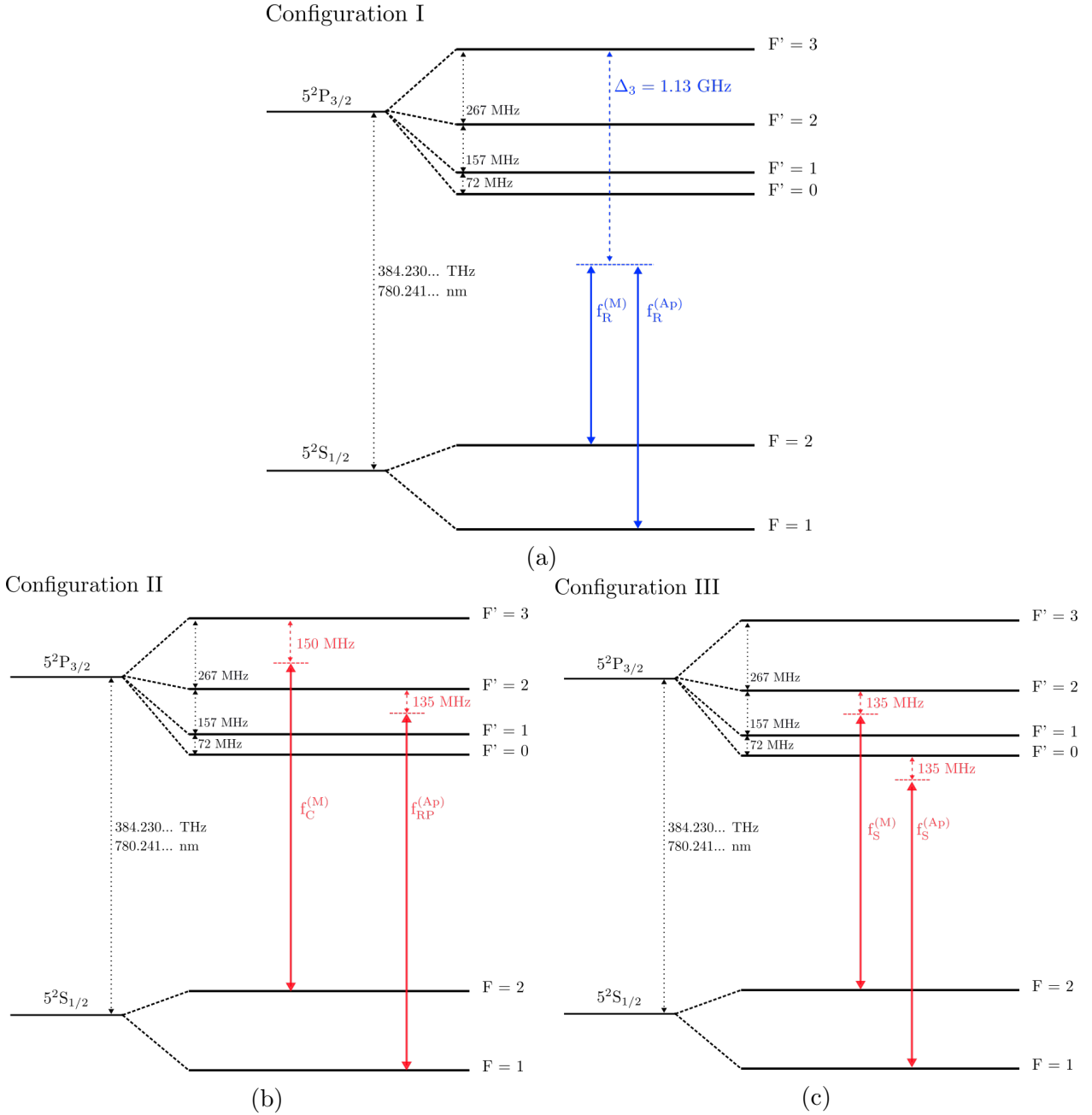


Figure 3.3: Frequency configurations. (a) Raman transitions. The two blue lines indicate that the ‘Raman Output’ is used in this configuration. Both Master and Apprentice lasers are detuned by -1.13 GHz from $|F' = 3\rangle$. (b) Cooling, trapping, launching, blow-away and detection. The two red lines show the two frequencies that are output by the ‘MOT Output’ fibre in fig. 3.2. The light from the Master laser is used for cooling and trapping the atoms and then later for blowaway and state detection. It is red detuned from the $|F = 2\rangle \longleftrightarrow |F' = 3\rangle$ transition by 150 MHz . The output from the Apprentice laser that is used as a re-pump is detuned by 135 MHz from the $|F = 1\rangle \longleftrightarrow |F' = 2\rangle$ transition. These are both shifted up as described below, before being used. (c) State preparation. Both Master and Apprentice lasers are detuned by 135 MHz from the transitions $|F = 2\rangle \longleftrightarrow |F' = 2\rangle$ and $|F = 1\rangle \longleftrightarrow |F' = 0\rangle$ respectively.

3.1.2 Power output

Sprout 1 pumps the Apprentice SolsTiS 1 with 9.7 W producing 2.8 W of light at 780 nm. Sprout 2 pumps the Master SolsTiS 2 with 14.7 W producing 5.5 W. After splitting the output from each SolsTiS laser, the MOT fibre outputs from the Master and Apprentice lasers are 990 mW and 23 mW respectively. In §3.3 I will detail how this power is further sub-divided amongst the 2D MOT, 3D MOT, and push beam. At the output of each Raman fibre the Master and Apprentice lasers produce 450 mW and 262 mW for Raman X and 490 mW and 283 mW for Raman Y respectively. These powers are summarised in table 3.1.

Fibre output [Laser]	Power (mW)
Raman X [Master]	450
Raman X [Apprentice]	262
Raman Y [Master]	490
Raman Y [Apprentice]	283
MOT [Master]	990
MOT [Apprentice]	23.4

Table 3.1: The powers out of the MOT fibre and each of the two Raman fibres with the Master and Apprentice contributions specified in square brackets. Raman X and Y are located in the equator of the chamber and perpendicular to each other, as shown in fig. 3.1.

3.1.3 Phase noise

The stability of the phase-lock is crucial as the atoms pick up the phase difference between the Master and Apprentice lasers during a Raman transition. To test the stability, a small portion of the 6.8 GHz beat-note between the Master and Apprentice is directed onto a fast photodiode. This is then mixed down to ≈ 3.4 GHz and mixed with a 3.4 GHz signal from an ultra-stable dielectric resonant oscillator (DRO). The resulting voltage signal[†], $V(t)$, is then of the form,

$$V(t) = V_1 \cos(\omega_1 t + \phi + \delta\phi(t)). \quad (3.1)$$

Here, V_1 is the signal amplitude, ω_1 is the frequency of the mixed DRO and laser beat-note signal, ϕ is a constant offset phase, and $\delta\phi(t)$ is a time-dependent phase noise. We are interested in assessing the characteristics of $\delta\phi(t)$. By performing the Fourier Transform on eq. (3.1), setting the negative frequency components to zero, and applying the Inverse Fourier Transform, one can obtain a signal, $s(t)$,

$$s(t) = \frac{1}{2} e^{-i(\omega_1 t + \phi + \delta\phi(t))}. \quad (3.2)$$

[†]Data provided by Stephen Webster at M-Squared Lasers.

I then obtain $\phi + \delta\phi(t)$ using

$$\phi + \delta\phi(t) = -\text{Im}\{\ln[2s(t)]\} - \omega_1 t. \quad (3.3)$$

Figure 3.4a shows the phase deduced using eq. (3.3) and $V(t)$ sampled every $100\mu\text{s}$. One can see a phase that slowly varies in time. This could be due to the stability of the DRO oscillator or a real slow drift of the phase lock. Because the typical interferometer time is of the order of ms, we are not interested in a slowly varying phase, but instead, in the much faster phase fluctuations in fig. 3.4a.

To estimate the effect on the interferometer I construct strings of length $2T$ from the data in fig. 3.4a. Within each string I use the first, middle, and last data point to calculate the interferometer phase $\Phi_{\text{int}} = \Phi_{\text{off}}^{(1)} - 2\Phi_{\text{off}}^{(2)} + \Phi_{\text{off}}^{(3)}$. The rest of the data within each string is ignored. Figure 3.4b shows the interferometer phase calculated in this way for the first 4s of data from fig. 3.4a for three different values of $2T$ - 1 ms (blue), 25 ms (orange), and 50 ms (red). The interferometer phase for all three $2T$ times is centered around 0 rad. The standard deviations for the full 40s of interferometer phase are 47 mrad, 36 mrad, and 50 mrad for the $2T = 1\text{ ms}$, 25 ms, and 50 ms data-sets respectively.

Now that I have a time series of interferometer phase a useful tool for characterising the noise is the (M-sample) Allan variance, $\sigma_x(\tau, f, N)^2$ [?],

$$\sigma_x(\tau, f, N)^2 = \frac{1}{2(N - \tau f - 1)} \sum_{k=1}^{N - \tau f - 1} [\langle x(f)_\tau \rangle_{k+1} - \langle x(f)_\tau \rangle_k]^2. \quad (3.4)$$

Here τ is the averaging time, f is the sampling rate of the signal, N is the total number of samples, and $x(f)_\tau$ is a group of successive measurements sampled at a rate f which span a length of τ . Figure 3.4c shows the Allan deviation of the interferometer phase, $\sigma_{\Phi_{\text{int}}}$, calculated from the data in fig. 3.4b with a sampling rate $f = 1/(2T)$. The Allan deviation is shown as a function of N . For $2T = 25\text{ ms}$ and 50 ms the Allan deviation follows a $N^{-2/3}$ -dependence, which is slightly steeper than the $N^{-1/2}$ of white noise. For $2T = 1\text{ ms}$ there is a short period of N^0 -dependence up to around 10 measurements. After this the Allan deviation follows the same $N^{-2/3}$ trajectory but with a strong periodic modulation. To understand this behaviour I look directly at the square root of the power spectral density of fig. 3.4a plotted in fig. 3.4d. Here one sees a spectrum that follows a f^{-1} -dependence at frequencies less than 100 Hz which is characteristic of white frequency noise. We also observe spikes in the spectrum, with the largest occurring at 34 Hz, 947 Hz, and 3.8 kHz. The spike at 947 Hz is the carrier frequency, ω_1 in eq. (3.1). The oscillation in the blue (1 ms) curve in fig. 3.4c can be understood as the beat between this frequency and the sampling frequency. At this time we are unsure as to the origin of the other spikes at 34 Hz and 3.8 kHz and they are likely the reason that the three curves in fig. 3.4c depart from the $N^{-1/2}$ expected of white noise. In the next stage of the project we plan

to re-take the phase noise measurement to check whether these oscillations in the laser phase are real or whether this was an artefact of the measurement made at M-Squared lasers. In either case, these measurements show that our laser system produces a Raman beat-note with extremely low phase noise, such that the interferometer phase noise averages down ~ 10 mrad after 10 shots. For an interferometer having $2T = 50$ ms, this corresponds to an acceleration uncertainty of 100 ng. Assuming a cycling rate of the experiment of 5 Hz, this represents 2 s of operation.

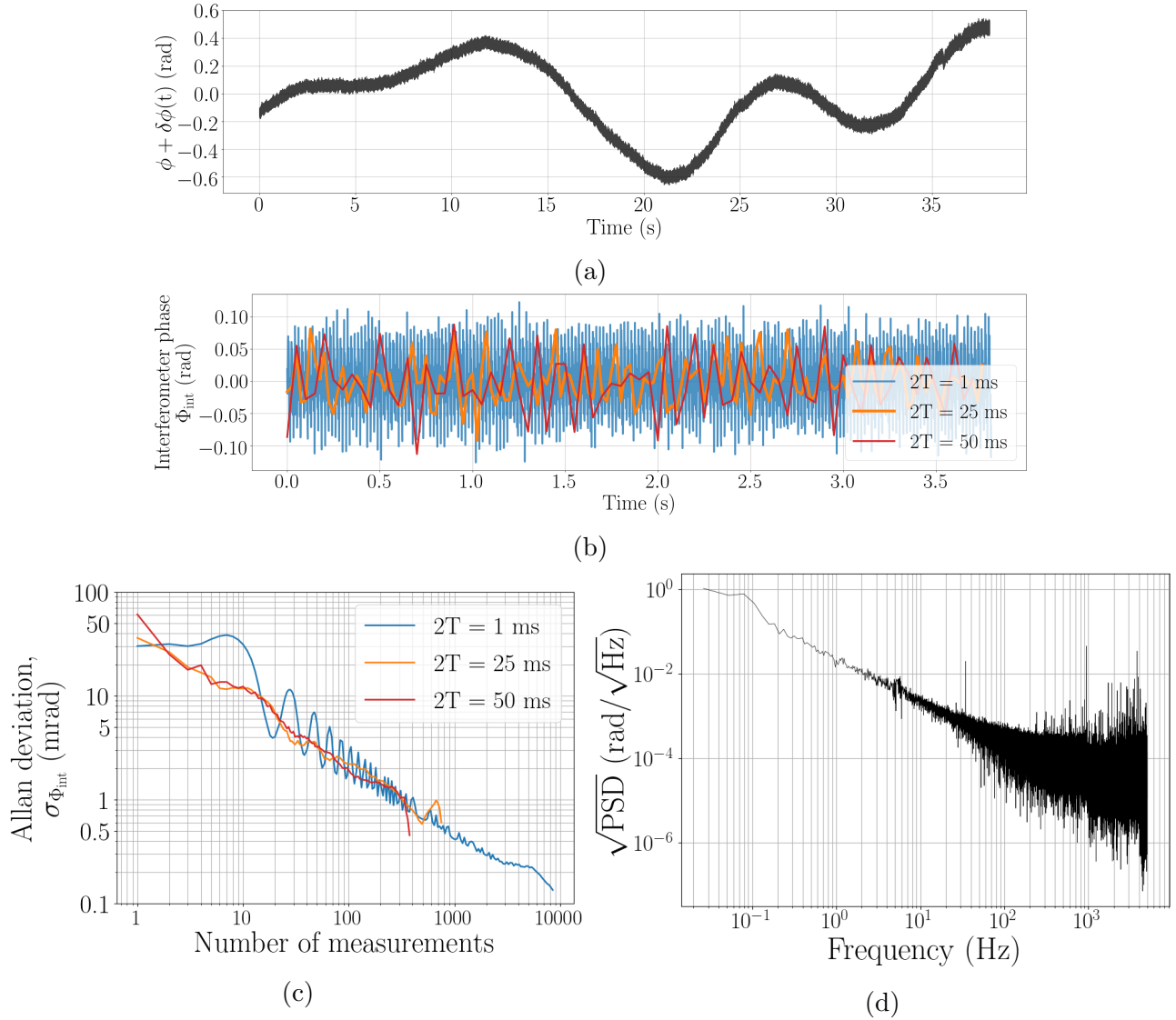


Figure 3.4: (a) Phase of the beat-note between the DRO oscillator and the two lasers beat-note as a function of time. (b) Interferometer phase for the first 4 s of data in (a). (c) Allan deviation of the interferometer phase as a function of the number of measurements. (d) Square root of the power spectral density of $\phi + \delta\phi(t)$, the phase of the Raman beat-note.

3.2 Vacuum Chamber

Figure 3.5 shows CAD images of our custom chamber. The chamber is an oblate spheroid with inner equatorial diameter of 450 mm and height of ± 150 mm from the chamber centre. The body of the chamber contains a total of 28 DN40 flanges: 12 sitting along the equator, and the remaining 16 displaced from the equator at an angle of $\pm 19^\circ$. Also on the equator are four DN60 flanges. Those located at the south and west in fig. 3.5c are for mounting the retro-reflection optics (see fig. 2.6) and the MEMS accelerometer. The other two DN60 flanges on the chamber are each connected to a nipple that contains the Raman optics and feed-through for the Raman fibre. These are located at the north and east in fig. 3.5c. The MOT coils are mounted to the top and bottom DN160 flanges. The equator of the chamber defines our x-y plane.

The atoms are cooled and trapped by a magneto-optical trap (MOT), formed by three pairs of counter-propagating laser beams and a magnetic quadrupole field. The light passes through six vacuum windows as follows: the Z MOT beams use viewports on the top and bottom flanges of the chamber mounted on a zero length DN160-DN40 adapter (vertical red beam in fig. 3.5d). The X and Y MOT beams use viewports on the equator displaced from the DN60 ports by 22.5° in the anti-clockwise direction with respect to fig. 3.5c. In fig. 3.5 these MOT beams are shown in red. One of the magnetic quadrupole coils for the MOT can be seen in fig. 3.5a, near the centre of the chamber. These coils are supported by the top and bottom flanges.

In figs. 3.5a and 3.5b the two black attachments on the DN40 flanges located out of the equator show where the microwave horns used for state preparation are located. The horns can be seen clearly in fig. 3.5c inside of the chamber.

Finally the viewports where the detection optics are mounted can be seen in both figs. 3.5a and 3.5c. In fig. 3.5c they are located displaced by $\pm 45^\circ$ from the Raman beam axis at the top of the figure. The yellow beams in each of these images show the fluorescence light from the atoms that is collected and focused to our photodiode located outside of the chamber.

The 2D MOT and ion pump are not shown in fig. 3.5. However they are both mounted to DN40 flanges - one above the equator and one below.

To achieve ultra-high vacuum (UHV) the empty chamber was baked to release any contaminants accrued during manufacture, while being pumped using a turbo molecular pump attached to one of the out-of-equator DN40 flanges. After baking, the turbo pump was valved off and the chamber was pumped by the SAES NEX Torr D300-5 ion pump. This is composed of a passive ‘getter’ cartridge and a diode ion pump. These two elements work in tandem such that gases not pumped by the diode ion pump are efficiently absorbed into the getter cartridge.

Figure 3.6 shows heating tape wrapped around the chamber. Before baking, the windows were

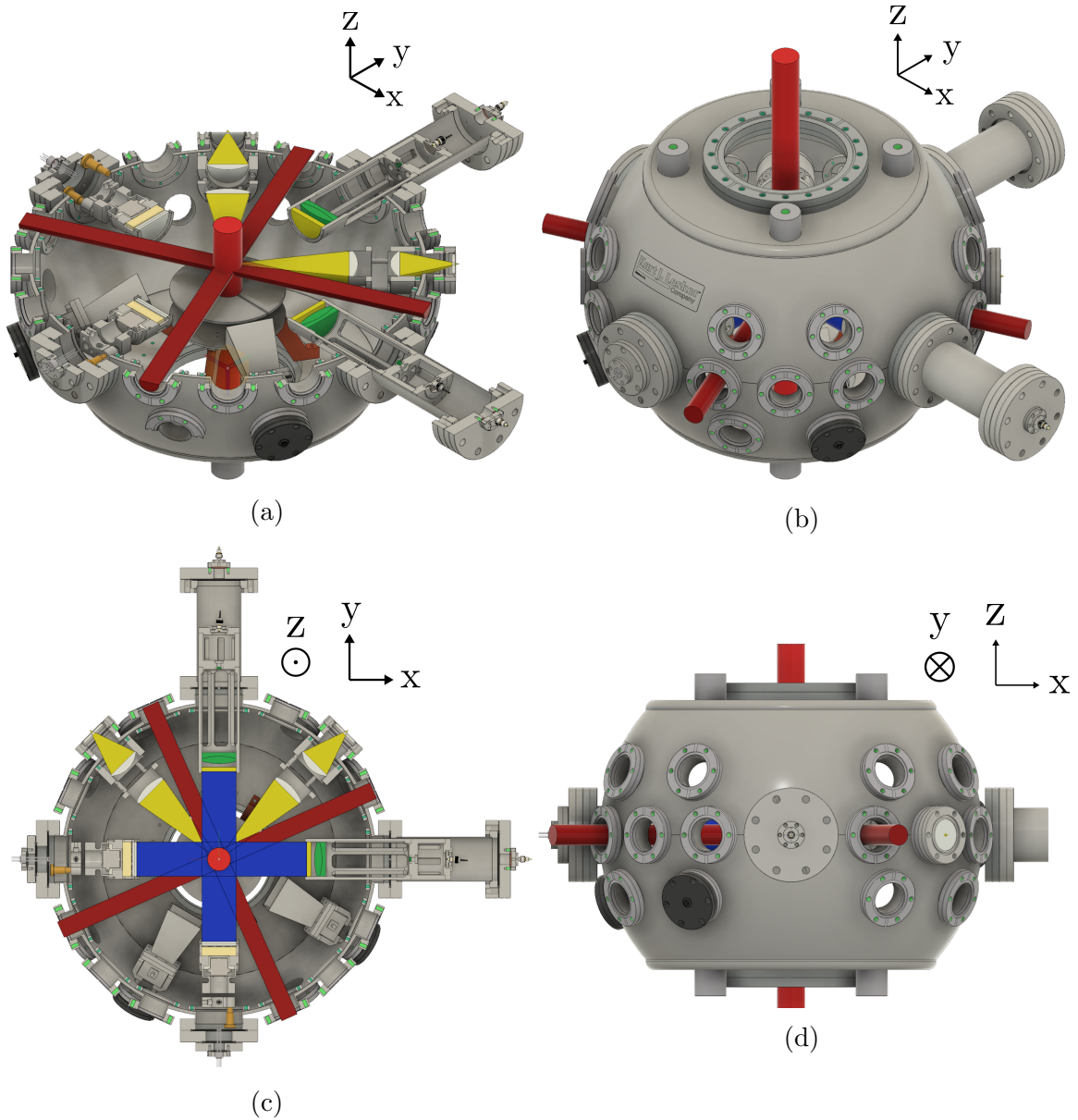


Figure 3.5: Vacuum chamber. (a) Tilted view of chamber with the upper half cut to show the origin and interaction of the various beams and of the components housed within the chamber. The two long arms at the upper right and bottom right of the image are the Raman axes. The semi-circle moons on each Raman axis represent the achromat lens (green) and the QWP (yellow) respectively. See chapter 4 for a full description of the Raman optics. The red beams represent the MOT beams whilst the yellow show the fluorescence light that is collected by our detection optics and focused to a focal point outside of the chamber. The Raman laser beams are not shown for clarity. (b) Same as in (a) but with the top half of the chamber included to show the optical access points. (c) A bird's eye view of the chamber with the top half cut. The Raman laser beams are shown here in blue. (d) is a side view of the chamber looking down a Raman axis. The other Raman axis is located on the right and has been cropped for clarity.

covered in aluminium foil, the shield and magnets of the ion pump were removed and the whole assembly was wrapped in an insulating blanket to ensure a uniform temperature. This enabled us to raise the temperature of the chamber to around 250°C. Thermocouples attached onto the

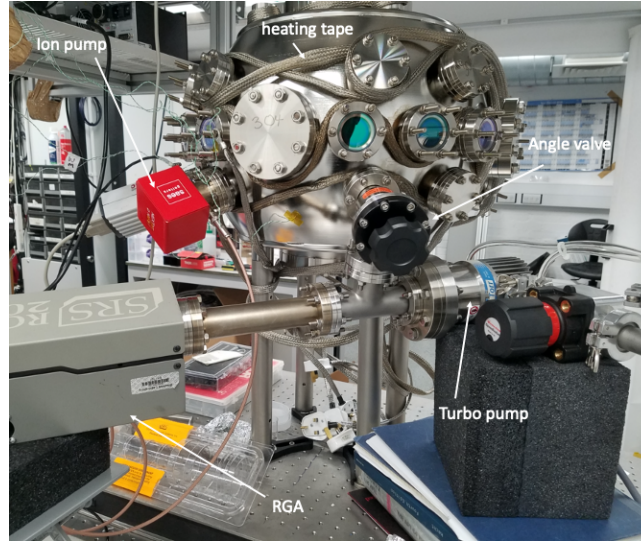


Figure 3.6: Photo of the chamber wrapped in heater tape that was used to heat the chamber to 250°C . A residual gas analyser produced by Stanford Research Systems allowed us to monitor the contaminants being released.

surface of the chamber, and some of the external components such as the ion pump, allowed us to monitor the temperature of specific parts of the chamber to ensure a gradual rise. After a week of baking the empty chamber, the heating was stopped and the ion pump was switched on once the pressure was below 10^{-8} mbar. The main chamber was then isolated by closing an angle valve connecting the chamber to the turbo pump. Within 24 hours the pressure fell below 5×10^{-10} mbar, the minimum pressure that could be inferred from the ion pump current.

After establishing the base pressure we could achieve with this chamber and pumps, we vented, using dry nitrogen, and installed the Raman tubes, retro-reflecting module, detection optics, 3D MOT coils, and microwave horns. We then baked the chamber, including internal components, again, this time at 100°C , which was limited by the maximum temperature that the lenses in the Raman tube and detection system could withstand. The populated chamber reached a base pressure of 5×10^{-9} mbar, suggesting some out-gassing from the internal components. A low pressure is needed to maintain a cold cloud of atoms as any collisions with background atoms heats up the cloud. A more stringent requirement on pressure comes from the need for the mean time between atomic collisions to be longer than the time of the interferometer. For an interferometer with $2T = 50$ ms this necessitates having a pressure of $\approx 10^{-10}$ mbar[†]. As we are above this pressure, we expect to see some loss of contrast of the interferometry fringes as a result of collisional decoherence.

[†]This assumes Rb-N₂ to be the dominant collision process and that the inelastic collision rate, τ^{-1} , is described by [?] $\tau^{-1} = \sigma P \sqrt{\frac{16}{\pi m k_B T}}$, where σ is the collision cross-section for Rb-N₂ interactions, P is the vacuum pressure, m is the mass of the Rb atoms, k_B is the Boltzmann constant and T is the temperature of the atomic cloud.

3.3 Light Distribution

The light distribution module (LDM) splits light from a single fibre output of the laser into 8 separate beams used for cooling and trapping. Each of the 8 beams has a dedicated AOM, which can be used to control the intensity and frequency of the light. There are 6 3D MOT beams, 1 beam for the 2D MOT and 1 to act as a push beam for the 2D MOT. Figure 3.7 shows a schematic of the layout of the LDM. The radio-frequency (RF) driving each of the AOMs is labelled in fig. 3.7 as $f_{\text{AOM}}^{(i)}$ where i represents the specific output of the LDM.

The MOT output fibre from fig. 3.2 is fed into the LDM and is initially split into two arms of equal power by a beamsplitter. Each of these arms then passes through a half-wave plate to align the linear polarisation of the light fields with the horizontal and vertical. Each arm is directed toward a series of beam splitters to pick off light to couple into a fibre AOM. Each fibre AOM's center frequency is 135 MHz.

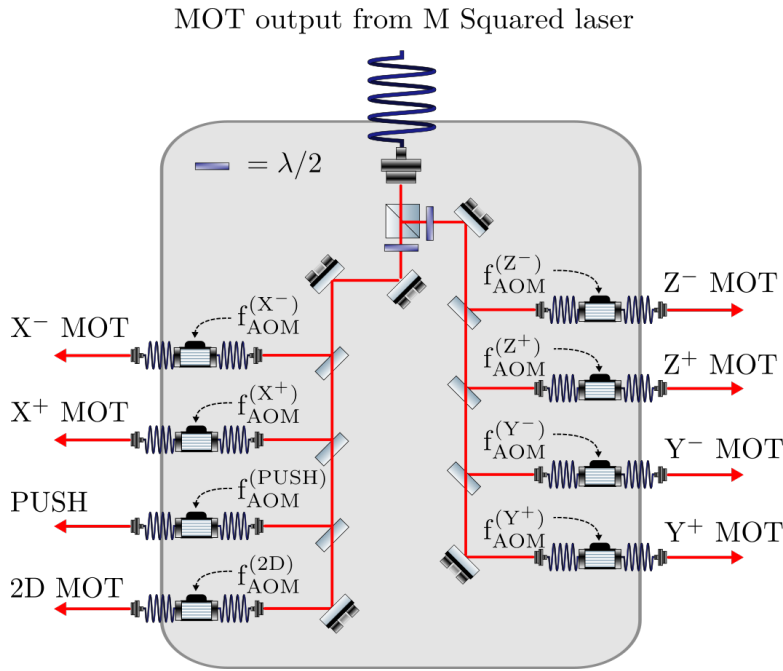


Figure 3.7: The light distribution module. A beamsplitter divides the light originating from the MOT output fibre of the M Squared laser system into two arms of equal power. Each arm is then coupled into 4 fibre AOMs. The module produces 8 separate beams each with individual control of their intensity and frequency through their dedicated AOM. The frequency driving each of the fibre AOMs is denoted $f_{\text{AOM}}^{(i)}$ where i identifies which output of the module it affects.

3.3.1 Frequency output

A 3D MOT with a large number of atoms is achieved with the cooling light red-detuned by ~ 15 MHz from the $|F = 2\rangle \longleftrightarrow |F' = 3\rangle$ transition [? ?]. The output from the laser is red-

detuned by 150 MHz, as shown in fig. 3.3b, but this is shifted to the desired -15 MHz by the AOMs in the LDM. In addition, a ramp of both the intensity and detuning of the MOT beams is required to cool the atoms in our sub-Doppler cooling technique (see §5.2). Finally, we use moving molasses (see §5.3) to launch the atoms, to avoid a double interferometer. Thus, the AOMs in fig. 3.7 are crucial in our setup.

The AOMs in fig. 3.7 modifies the frequency before the LDM from that of fig. 3.3 to fig. 3.8. The RF driving each of the AOMs from fig. 3.7 is shown in purple on fig. 3.8. The frequency labelled by $f_{\text{AOM}}^{(3D)}$ represents any of the 6 AOMs that control the outputs of the 3D MOT beams (i.e. $f_{\text{AOM}}^{(3D)} = \{f_{\text{AOM}}^{(X+)}, f_{\text{AOM}}^{(X-)}, f_{\text{AOM}}^{(Y+)}, f_{\text{AOM}}^{(Y-)}, f_{\text{AOM}}^{(Z+)}, f_{\text{AOM}}^{(Z-)}\}$). The re-pump beam is now approximately resonant with the $|F = 1\rangle \longleftrightarrow |F' = 2\rangle$ transition. Figure 3.8b shows the output of the 6 MOT beams with the addition of the AOMs for configuration III. The 2D MOT and push beam AOM frequencies are not shown because they are not used in this configuration. The frequency originating from the Master laser is now approximately resonant with the $|F = 2\rangle \longleftrightarrow |F' = 2\rangle$. Similarly, the frequency from the Apprentice laser is now resonant with the $|F = 1\rangle \longleftrightarrow |F' = 0\rangle$ transition.

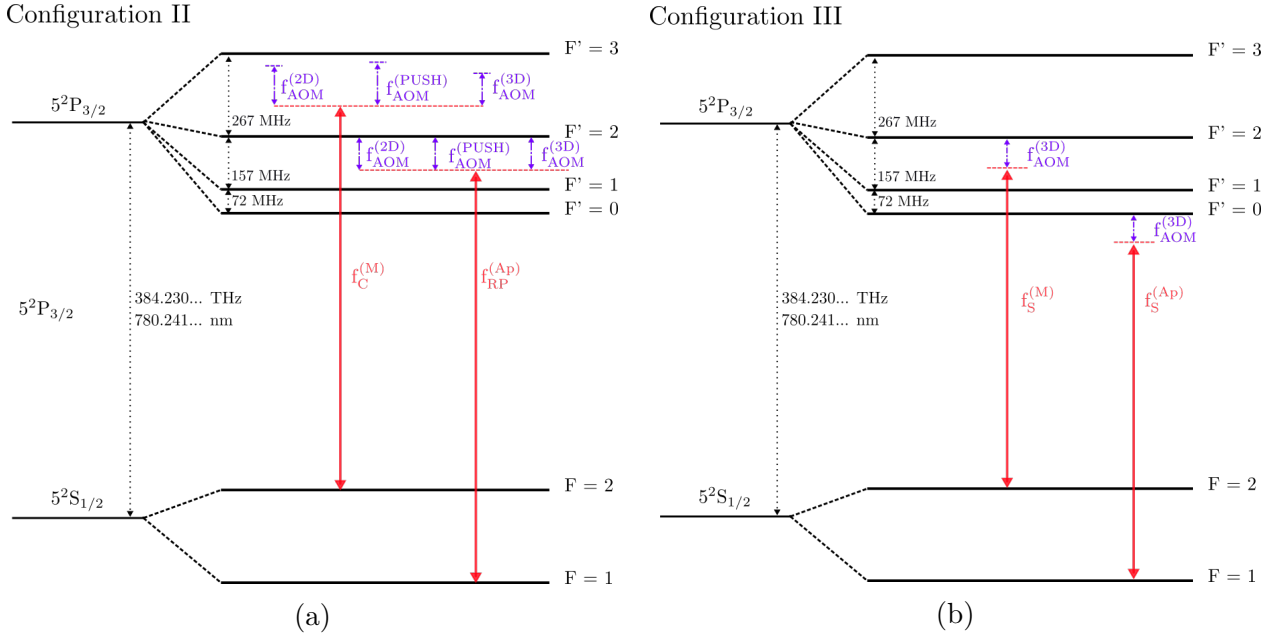


Figure 3.8: Energy level diagram including the frequency shift added by the AOMs in fig. 3.7 shown in purple. Configuration I is not shown as the Raman output is not affected by the addition of the AOMs in the LDM. $f_{\text{AOM}}^{(3D)}$, $f_{\text{AOM}}^{(2D)}$ and $f_{\text{AOM}}^{(\text{PUSH})}$ represents the RF driving each of the 6 3D MOT beams, the 2D MOT output and the push beam output respectively shown in fig. 3.7. (a) The frequency output for configuration II after the LDM. Similarly, (b) shows the output of the system after the LDM for configuration III. Here the frequency driving the 2D MOT and push beam is not shown because they are not used when the laser is in this configuration.

3.3.2 Power output

After passing through the LDM the powers out of each of the 8 outputs are as detailed in table 3.2. This is the combined power of the Master and Apprentice lasers in each output. Most of the power dissipation between the MOT output of the laser system and the output of the individual fibres occurs in the fibre AOMs which have an efficiency of $\sim 50\%$.

Fibre output	Power (mW)
X ⁺ MOT	22.3
X ⁻ MOT	22.3
Y ⁺ MOT	22.3
Y ⁻ MOT	22.3
Z ⁺ MOT	22.3
Z ⁻ MOT	22.3
2D MOT	60
PUSH	5

Table 3.2: Table showing the powers out of each LDM fibre shown in fig. 3.7.

3.4 Detection System

3.4.1 Detection noise - Theory

We read out from our interferometer by, first measuring the fluorescence from atoms in $F=2$, V_2 , followed by the fluorescence from all the atoms, V_{tot} . The photodiode outputs a voltage proportional to the number of atoms so we are then able to construct $P_2 = N_2/N_{\text{tot}} = V_2/V_{\text{tot}}$. The precision of our measurements is affected by the quantum projection noise (QPN), the photon shot-noise (PSN), and noise in our detection system.

On detecting the state of the atom it is projected into either $F=1$ or $F=2$. This is a binomial process which introduces uncertainty into the value of N_2 called the quantum projection noise [?]. The resulting uncertainty in P_2 is given by (see ??)

$$\sigma_{P_2}^{(\text{QPN})} = \sqrt{\frac{P_2}{N_{\text{tot}}}(1 - P_2)}. \quad (3.5)$$

The counting of the fluorescence photons is subject to Poissonian statistics. If we detect on average n fluorescence photons per atom, the number of photons measured has standard deviations $\sqrt{nN_2}$ and $\sqrt{nN_{\text{tot}}}$ in measuring N_2 and N_{tot} respectively. The corresponding uncertainty

in P_2 is (see ??)

$$\sigma_{P_2}^{(\text{PSN})} = \sqrt{\frac{P_2}{nN_{\text{tot}}}(1 + P_2)}. \quad (3.6)$$

In our experiments we typically detect 27 photons per atom. Therefore we neglect the photon counting noise henceforth as it is always small in comparison with the quantum projection noise.

Finally, let us consider the noise in P_2 due to the electronic noise in the detector. Using straightforward error propagation this is

$$\sigma_{P_2}^{(\text{DET})} = \sqrt{\frac{1}{V_{\text{tot}}^2} \sigma_V^2 + \frac{V_2^2}{V_{\text{tot}}^4} \sigma_V^2} \quad (3.7)$$

$$= \frac{\sigma_V}{V_{\text{tot}}} \sqrt{1 + P_2^2}. \quad (3.8)$$

Here, I have assumed that, $\sigma_V = \sigma_{V_2} = \sigma_{V_{\text{tot}}}$. Ideally we would also like this noise to be small in comparison with the quantum projection noise. At the steepest part of an interferometer fringe, where $P_2 = 0.5$, this means we want

$$\sigma_V < \sqrt{\frac{1}{5N_{\text{tot}}}} V_{\text{tot}}. \quad (3.9)$$

With typical numbers of $N_{\text{tot}} = 10^7$ and the corresponding $V_{\text{tot}} \sim 0.2 \text{ V}$, that means we would like to have $\sigma_V \leq 30 \mu\text{V}$.

3.4.2 Detection optics

From eq. (3.8) we see that the detector contribution to the noise in P_2 reduces with increasing number of photons detected. It is therefore advantageous to subtend the largest possible solid angle with the first lens of the detection optics. Figure 3.9 shows the layout of the detection optics in our chamber. Labelled in both diagrams are L_1 , which is the distance from the centre of the chamber to the front surface of lens 1, and L_3 , which is the distance from the back surface of lens 2 to the inside surface of the viewport's window. We initially chose $L_1 = 118 \text{ mm}$ and had a 125 mm focal length plano-convex lens with a diameter of 50 mm as the first lens to ensure high collection efficiency without clipping the MOT beam.

In the Earth's gravitational field atoms will fall a distance of $\approx 1 \text{ cm}$ for $2T = 50 \text{ ms}$ interferometer. Therefore, we wanted our detection system to be insensitive to movement of the atomic cloud by $\pm 1 \text{ cm}$. We use photodiodes (Hamamatsu S2744-09) that have an active area of $10 \text{ mm} \times 20 \text{ mm}$.

Considering a typical atomic cloud with a width of 5 mm, and that the shortest edge of the

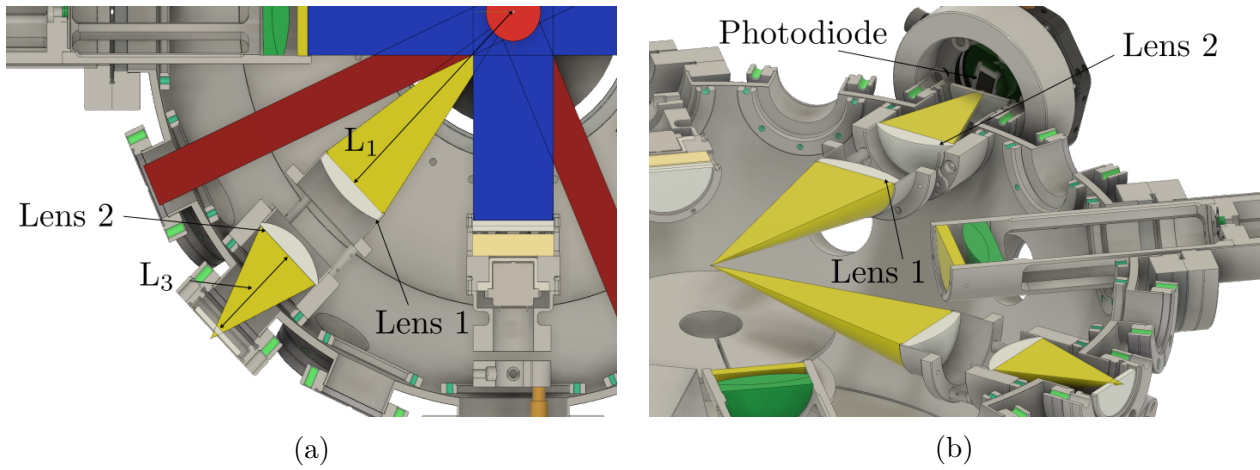


Figure 3.9: (a) Detection optics viewed from above. L_1 and L_3 are the distance from the centre of the chamber to the first surface of lens 1, and the length from the back surface of lens 2 to the front surface of the window of the viewpoint respectively. (b) View along the detection optics axis. The Hammamatsu S2744-09 photodiode that is mounted on the outside of the chamber is shown with the light collected (yellow) from the atoms being focused onto the center of its active area. The second detection setup is shown at the bottom right of the image. The two detection setups are mounted on DN40 flanges along the equator of the chamber at 90° to each other.

photodiode is 10 mm, in order to allow for movement of ± 1 cm in the object plane the system should have a magnification of $m = 0.4$. Any optical system must obey the conservation of étendue [?]. For our system this amounts to the following

$$\Omega_{\text{atoms}} A_{\text{atoms}} = \Omega_{\text{image}} A_{\text{image}}. \quad (3.10)$$

Here, Ω_{image} is the solid angle subtended by the second lens as viewed from the photodiode, Ω_{atoms} is the solid angle subtended by the first lens as viewed from the atoms, A_{image} is the area of the image of the cloud on the photodiode, and A_{atoms} is the area of the cloud. Étendue can only increase in an optical system and never decrease. Rearranging eq. (3.10) gives

$$\Omega_{\text{atoms}} = \frac{A_{\text{image}}}{A_{\text{atoms}}} \Omega_{\text{image}} = m^2 \Omega_{\text{image}}. \quad (3.11)$$

Equation (3.11) shows that there is a fundamental limit on the achievable solid angle one can collect using an optical system with a magnification of m . For a 5 mm cloud moving ± 1 cm, we are trying to image a region of 25 mm diameter. This leads to a maximum possible solid angle, Ω_{atoms} , of 16% of 4π .

Figures 3.10a and 3.10b[†] shows ray tracing diagrams with the second lens chosen to focus the fluorescence just in front of the photodiode. On each figure 3 ray bundles with origins of 0 mm (blue), 2.5 mm (green) and 12.5 mm (orange) from the optic axis are drawn. The bundles

[†]All ray tracing diagrams produced using a Mathematica package written by Prof. M. Boshier.

originating from 0 mm and 2.5 mm represent the centre and edge of a 5 mm atomic cloud which is perfectly centered on the optic axis. The bundle at 12.5 mm represents the fluorescence of the edge of a cloud that has moved by 1 cm from the optic axis. In Figure 3.10a from left to right, the first two red lines represent the surfaces of the first lens and next two represent the surfaces of the second lens. The fourth and fifth lines represent the surfaces of the viewport in the DN40 flange whilst the final line represent the plane of the photodiode located outside of the chamber.

In fig. 3.10a, some rays in the orange bundle (not drawn) miss the bottom of the second lens and so are not focused onto the photodiode. The black solid lines in fig. 3.10b represent the opening in the chamber wall for the DN40 flange (see fig. 3.9). For all three positions of the cloud the collected light is clipped by the chamber wall. To avoid these sorts of clipping, so that the measured detection signal is position independent, we had to move the objective lens out to 140 mm, thereby slightly reducing the collection solid angle, but avoiding the clipping. Figure 3.10c shows the path of rays from 0 mm (blue), 2.5 mm (green), and 10.5 mm (orange) from the optic axis. The atomic cloud can now move up to 8 mm without clipping the chamber wall whilst still being imaged onto the area of the photodiode, as shown in fig. 3.10d.

3.4.3 Detection noise - Measured

The photodiode is a Hamamatsu S2744-09. It has a photo-sensitive area of 10 mm x 20 mm with a spectral response of 0.58 A/W at our operating wavelength of 780 nm. Typical dark current is 3 nA for the photodiode with a maximum of 10 nA. The signal from the photodiode is amplified using a Femto DLPCA-200 low noise, variable gain amplifier with a bandwidth up to 500 kHz.

I recorded the output signal from both readout systems sampled at 200 kHz with a gain setting of 10^6 V/A over the course of 3 seconds with no light incident on either photodiode. Figure 3.11 displays the time series for (a) photodiode 1 and (b) photodiode 2. From these sets of data I calculate the power spectral density (PSD) for each photodiode. Figures 3.12a and 3.12b shows the square root of the PSD for photodiode 1 and 2 respectively. The spectrum for photodiode 1 is flat up until around 20 kHz at which point there is a small region of f^{-1} -dependence. This indicates white phase-modulation noise and is produced by amplification of a signal [?]. For photodiode 2 we observe a similar behaviour except that the onset of the f^{-1} -dependence happens around 4 kHz. This is due to the smaller amount of noise in the low frequency portion of the PSD.

Each step in our detection sequence averages the fluorescence signal over $300 \mu\text{s}$. To estimate the noise in a single detection step I therefore partition the data in fig. 3.11 into bins of length $300 \mu\text{s}$. Within each bin I calculate the mean and then take the standard deviation of those

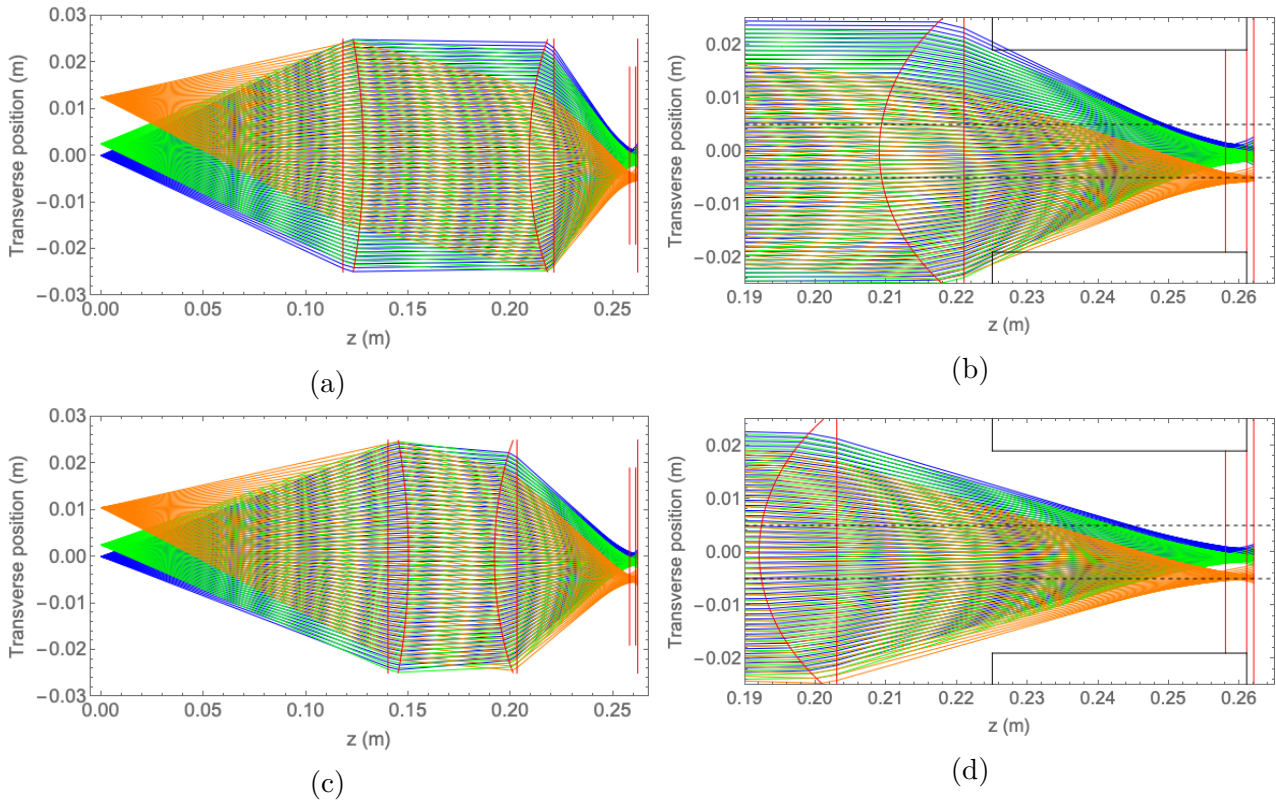


Figure 3.10: (a) Ray tracing diagram for ray bundles originating from 0 mm, 2.5 mm and 12.5 mm from the optic axis for a configuration of $L_1 = 118$ mm and $L_3 = 43$ mm with a first lens of focal length 125 mm and second lens of focal length 50 mm. (b) Zoomed in version of (a) with the chamber wall (black solid lines) and the width of the photodiode shown (black dashed). (c) Ray tracing diagram for ray bundles originating from 0 mm, 2.5 mm and 10.5 mm from the optic axis for a configuration of $L_1 = 140$ mm and $L_3 = 59$ mm with a first lens of focal length 125 mm and second lens of focal length 75 mm. L_3 is set to 59 mm to avoid light collected from clipping the chamber wall. (d) Zoomed in version of (c) with the chamber wall (black solid lines) and the width of the photodiode shown (black dashed).

means. This results in $\sigma_V = 58 \mu V$ and $\sigma_V = 13 \mu V$ for photodiode 1 and 2 respectively. This puts us within a factor of 2 of the quantum projection noise with photodiode 1 being above it and photodiode 2 below. We are unsure as to the discrepancy between the two noise values, σ_V , between the photodiodes at this time. I leave this for further investigation in the next stage of the project.

It is interesting to see how the uncertainties in P_2 from quantum projection noise and detection noise scale with atom number. Knowing the geometry and detection efficiency I expect to detect 9×10^4 photons per atom per second (with a saturation parameter $s = 10$), giving a signal of 15 nV per atom during each 300 μs detection step with a gain of 10^6 V/A. I am then able to plot eq. (3.8) as a function of atom number. Figure 3.13 shows how the uncertainty in P_2 due to noise in photodiodes 1 (blue) and 2 (orange) scales with atom number. Also shown is the quantum projection noise of eq. (3.5) for comparison (green). One can see from the graph that above 4×10^6 atoms, the measured noise of photodiode 2 is below the quantum projection

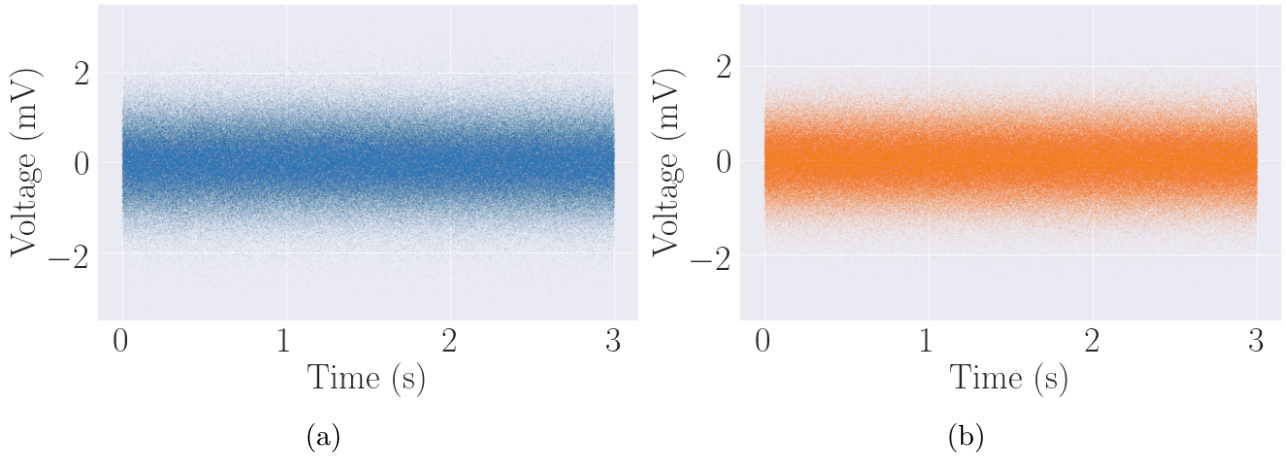


Figure 3.11: Voltage measured from (a) photodiode 1 and (b) photodiode 2 placed in the dark as a function of time.

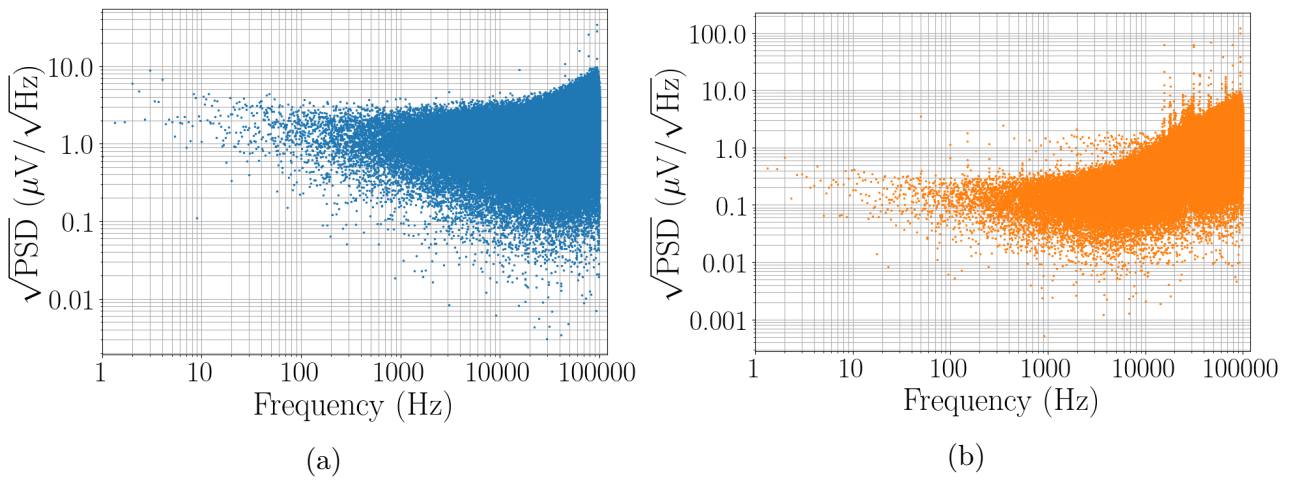


Figure 3.12: Square root of the power spectral density for (a) photodiode 1 and (b) photodiode 2.

noise. Due to the higher noise level observed in fig. 3.12a, the equivalent atom number for photodiode 1 is 10^8 . These numbers are to be compared with the 10^7 atoms that typically make the interferometer fringes.

3.5 3D MOT

Figure 3.14a shows our 3D MOT setup. The red beams represent the MOT light. The coils used to produce the quadrupole magnetic field for the MOT are wrapped around formers and then mounted to the top and bottom DN140 flanges. Only the former, and not the coils, is shown in fig. 3.14a. They produce a quadrupole field such that the minimum of the field overlaps with the intersection of the 6 MOT beams. Figure 3.14b shows one of the 6 3D MOT collimators used to produce the MOT beams. Each collimator consists of a mirror located at the elbow

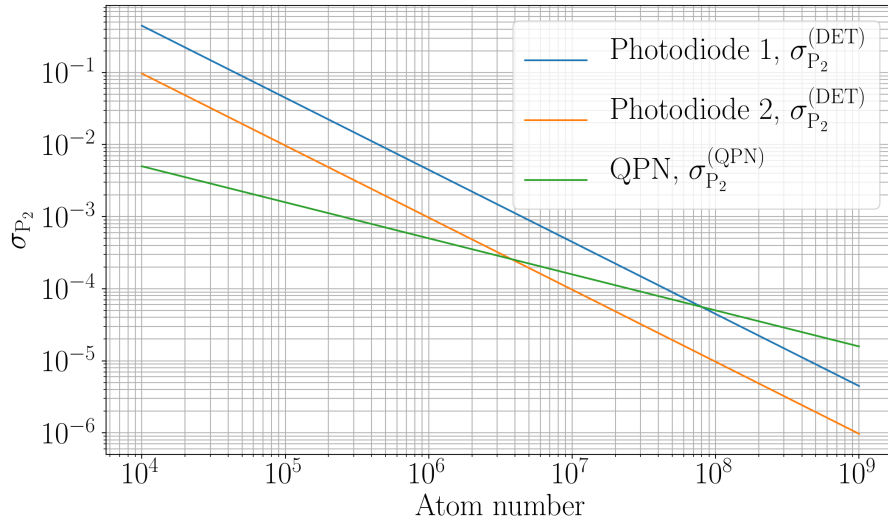
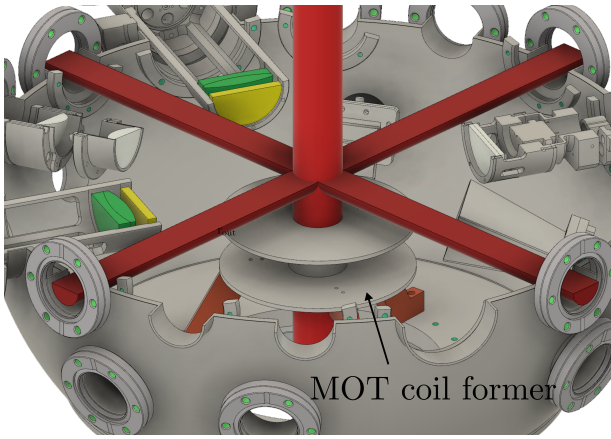
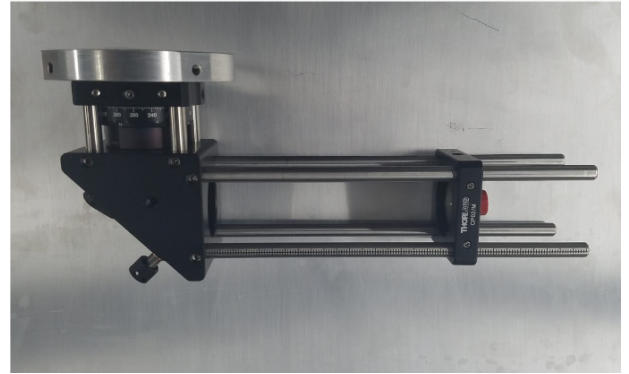


Figure 3.13: σ_{P_2} as a function of atom number for both the detector noise, $\sigma_{P_2}^{(DET)}$ and the quantum projection noise, $\sigma_{P_2}^{(QPN)}$.



(a)



(b)

Figure 3.14: (a) 3D MOT beams inside the chamber are shown in red. The magnetic coil former holds the windings for the MOT coil wire which produces the quadrupole magnetic field needed to form a 3D MOT. A second identical coil former is mounted to the top half of the chamber (not shown). (b) photo of one of the MOT collimators.

of the collimator mount which reflects the diverging light coming from the fibre. This light is then directed through a collimating lens with a focal length of 150 mm and a QWP. The MOT beams have a Gaussian $1/e^2$ intensity radius of 12.7 mm. With this radius and the powers listed in table 3.2, each MOT beam has a peak intensity of $I_0 \approx 5I_s$ where $I_s = 1.669 \text{ mW/cm}^2$ is the saturation intensity for the $|F = 2, m_F = \pm 2\rangle \leftarrow |F' = 3, m'_F = \pm 3\rangle$ transition in ^{87}Rb .

3D MOT Coil

A quadrupole field gradient of $\approx 15 \text{ G/cm}$ allows us to produce a dense and cold 3D MOT [? ?]. To achieve this with only a modest current, the coils are positioned inside the vacuum chamber.

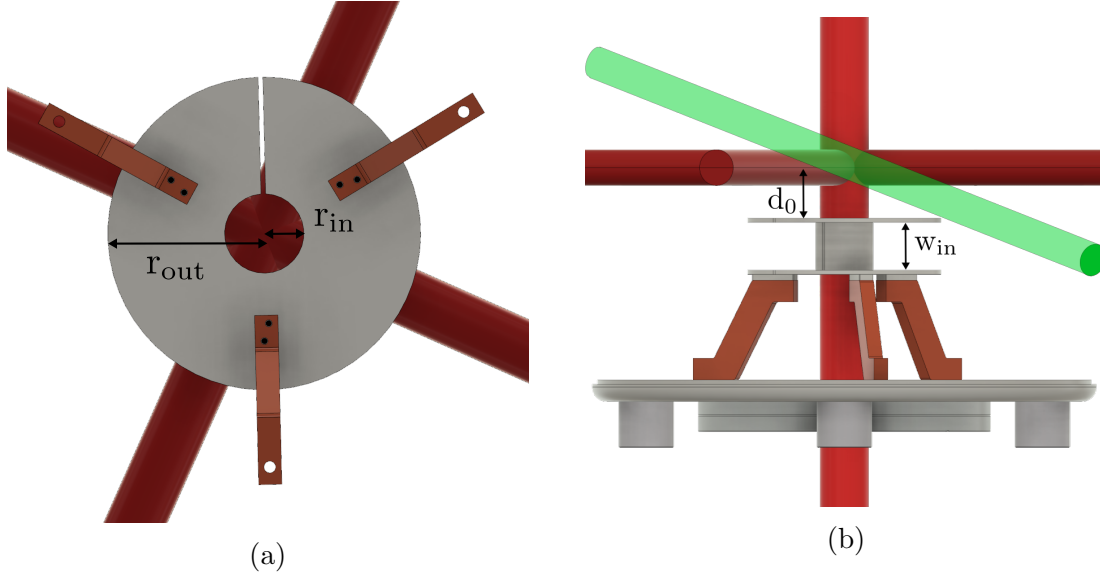


Figure 3.15: (a) Bottom view of the 3D MOT coil former and beams with the inner and outer radius of the former labelled as r_{in} and r_{out} respectively. (b) Side view of the 3D MOT coil former and beams. The inner width of the former is labelled as w_{in} , the distance from the former to the centre of the chamber is labelled d_0 , and the detection beam is shown in green. The legs attaching the former to the chamber are made from copper to draw heat into the body of the chamber.

Figure 3.15a shows a schematic of the coil former as viewed from below the chamber. The inner and outer radii are labelled as r_{in} and r_{out} respectively. We choose the inner radius to be 20 mm so that only 0.7% of the MOT beam power is clipped by the aperture. In addition, to avoid clipping the detection beam transmitted through flanges out of the equator (see fig. 3.15b), the distance d_0 in fig. 3.15b is chosen to be $d_0 = 38 \text{ mm}$.

The numbers of radial and axial turns for the quadrupole coils were carefully chosen in order to minimise power dissipation and therefore heating. For various numbers of radial turns, fig. 3.16a shows a graph of the dissipated power versus the number of axial turns needed to achieve a gradient of 15 G/cm with copper wire having a 1.1 mm square cross section. For each number of radial turns, there is a minimum in this curve. If the turns could all be in the same loop at the top of the former, the power required would simply scale inversely with the number of turns. However, once each radial layer of turns is full, the next layer must sit below and be further away from the atoms. This results in a need for higher current to achieve the same gradient and eventually to diminishing returns. The minima are not all the same because the

Parameter	Value	Parameter	Value
r_{in}	20 mm	d_0	38 mm
r_{out}	64 mm	P	1.5 W
n_r	40	I	0.544 A
n_z	30	R	5.2Ω
w_{in}	33 mm	V	2.8 V
A_{wire}	$1.1 \text{ mm} \times 1.1 \text{ mm}$		

Table 3.3: 3D MOT coil parameters.

coil is most efficient at producing a gradient when the radius is comparable with the distance away from the atoms. We see that the lowest of the minima occurs with about 50 radial turns. Figure 3.16b shows a graph of the minimum height the former must sit vertically to avoid clipping the detection beam as a function of radial turns. We use 40 radial turns to ensure our selected $d_0 = 38 \text{ mm}$ is comfortably above the minimum distance indicated by fig. 3.16b. This number is less than the optimal 50, but at 1 W per coil, the power is still very low. With these parameters set, the coil bundle design can be fully specified and is summarised in table 3.3. All values specified relate to a single coil of which we have two mounted 38 mm from the centre of the chamber. P, I, R, and V represent the power, current, resistance, and voltage of the coil bundles respectively, and A_{wire} is the cross sectional area of the copper wire.

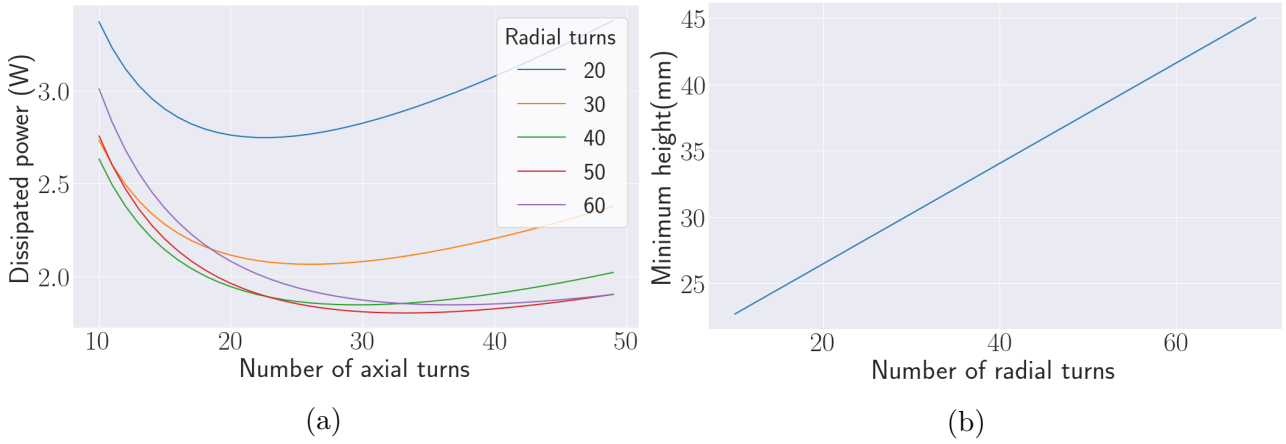


Figure 3.16: (a) Power dissipation against the number of axial turns of 1.1 mm square wire in the MOT coil for varying numbers of radial turns. (b) Minimum distance that the coil former must be located from the centre of the chamber in order to avoid clipping a detection beam.

The final setup for our 3D MOT can be seen in fig. 3.17, where the black dotted lines indicate the direction of the magnetic field and the red arrows represent the direction of each of the 3D MOT beams. The helicity of each beam, either left or right, is denoted by L or R respectively. The subscript denotes which output of the light distribution module in fig. 3.7 the light originates from. For example, L_{X^+} indicates a left-handed beam which originated from the X^+ output port of the light distribution module.

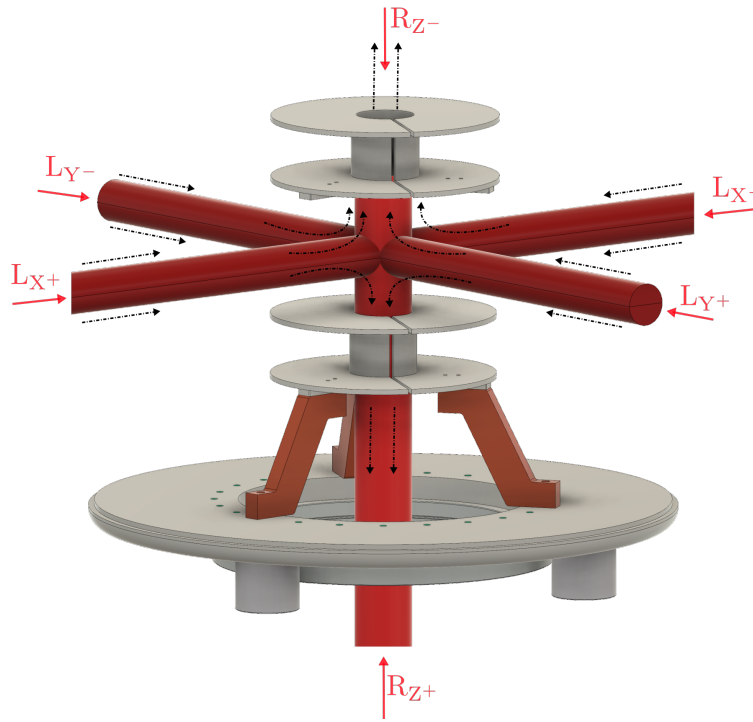


Figure 3.17: 3D MOT setup. The two coils are driven in an anti-Helmholtz configuration, producing a quadrupole field (black dotted lines). The red arrows indicate the propagation. The label on each MOT beam indicate its handedness and the output port of the light distribution module the beam originated from.

The field gradient $\frac{\partial B_z}{\partial z}$ at the centre is 27.6 G/A. Figure 3.18 shows the field and field gradient over the central region $-1 \text{ cm} < z < 1 \text{ cm}$ with a current of 544 mA, which produces a gradient of 15 G/cm at the centre.

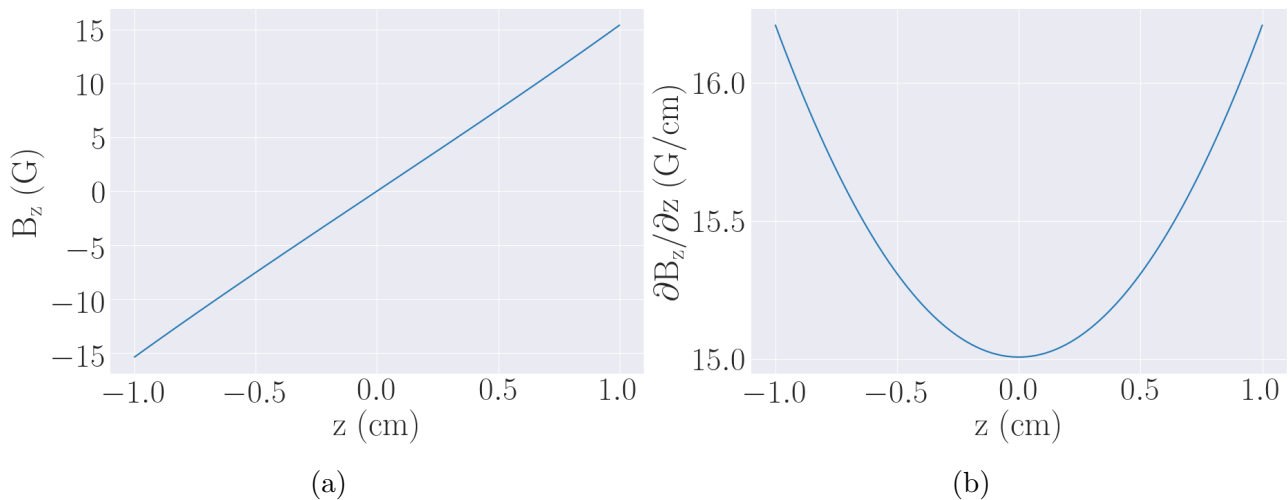


Figure 3.18: (a) Axial magnetic field and (b) field gradient as a function of the axial distance from the centre of the chamber.

Bias coils

To compensate the Earths magnetic field and other ambient fields in the lab I used three orthogonal pairs of Helmholtz coils mounted to the outside of the chamber. Each of these bias coils consists of approximately 10 layers of axial turns and 4 layers of radial turns with an inner radius of 15 cm. Two of the bias coil pairs produce fields in the equatorial plane at $\pm 45^\circ$ to the x-axis, while the third pair makes a field along z. Each pair produces 330 mG/A at the centre of the chamber. The rise and fall times were measured by monitoring the magnetic field after switching the current drivers supplying the bias coils on and off. We find the rise time of each bias field to be ~ 1 ms and the fall time to be ~ 0.4 ms.

Chapter 4

Raman Optics

4.1 Design considerations

4.1.1 Spot size

In order to maximise the fringe contrast achievable we carefully select the spot size of the Raman beam.

Throughout this chapter I use the following definition for the spot size, $w(z)$, of a Gaussian beam

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2}. \quad (4.1)$$

Here, w_0 is the waist ($1/e^2$ intensity radius), z is the distance along the beam from the waist, and z_R is the Rayleigh range. For our setup the Rayleigh range is $\approx$ km, and thus $w \approx w_0$.

Rabi oscillation - single atom

Figure 4.1a shows a single atom falling through the Gaussian beam used to drive Raman transitions. The atom starts at the centre of the beam at $r = 0$ mm. An initially stationary atom falling in the Earth's gravitational field travels a distance $\Delta r = \frac{1}{2}gt^2$ in the time t . For an interferometer with pulses separated by 25 ms, this corresponds to positions -3 mm and -12 mm for the second and third pulses. The intensity decreases at each of these positions, leading to a decrease in the effective Rabi frequency as the atom falls. With an atom initially prepared in $F=1$, the probability of finding the atom in $F=2$ after interaction with a single

resonant ($\delta = 0$) Raman pulse of length τ is given by

$$P_2(r, \tau) = \sin^2 \left(\frac{\Omega_{\text{eff}}(r)\tau}{2} \right). \quad (4.2)$$

Here, $\Omega_{\text{eff}}(r)$ indicates that the effective Rabi frequency is a function of the radial position of the atom within the beam. Figure 4.1b shows a plot of eq. (4.2) as a function of the pulse duration for the three positions of the atom shown in fig. 4.1a. I have considered a Raman beam with a Gaussian intensity profile corresponding to a spot size of $w = 20$ mm. The x-axis is referenced to the first pulse such that a pulse length of π is when there is a complete population inversion at $r = 0$ mm. The graph shows that as the atom falls the time needed to achieve a π -pulse increases due to the decreasing effective Rabi frequency.

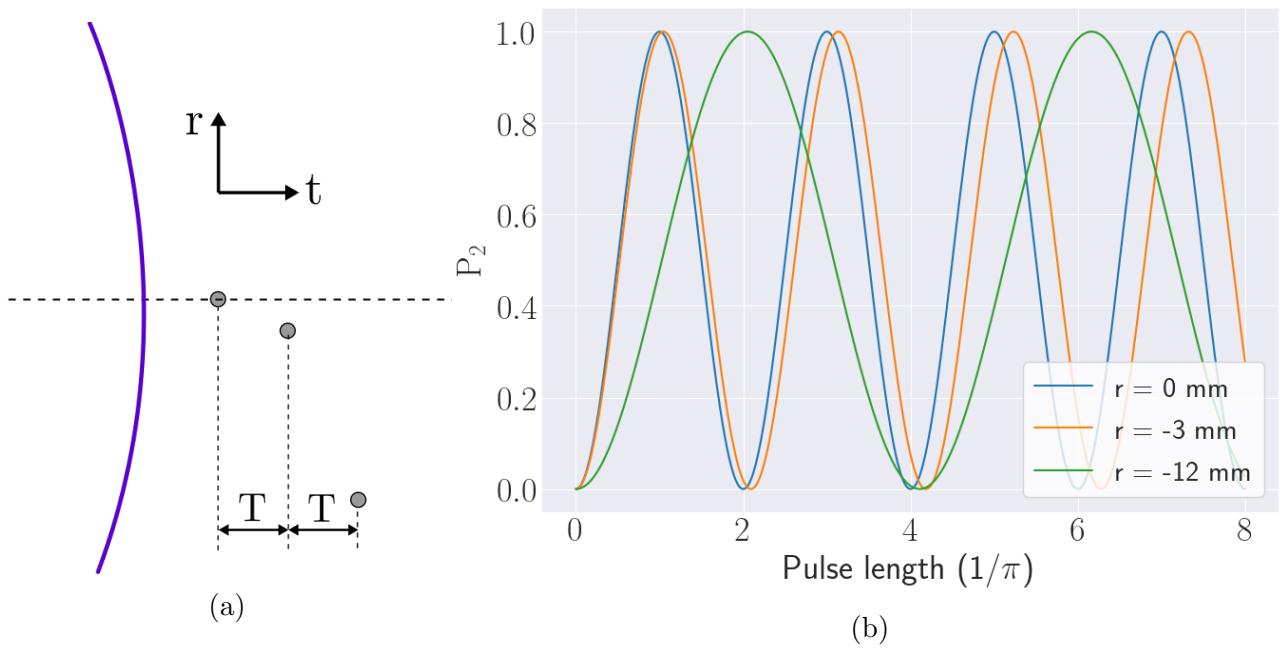


Figure 4.1: (a) Atom's position relative to the centre of a laser beam used to drive Raman transitions in the interferometer. The solid blue line represents the Gaussian intensity variation across a laser beam with a 20 mm spot size. (b) Probability of an atom being in $|F = 2\rangle$ as a function of pulse length for the three different positions for an atom falling through this beam. The x axis is referenced to the first position of the atoms, i.e. pulse length of π is defined as the pulse length for a complete state inversion at $r = 0$ mm.

Rabi oscillation - multiple atoms

In the interferometer we apply pulses to a cloud of atoms, therefore one must consider the range of Rabi frequencies across a spatial extent. The ensemble average probability for a finite sized

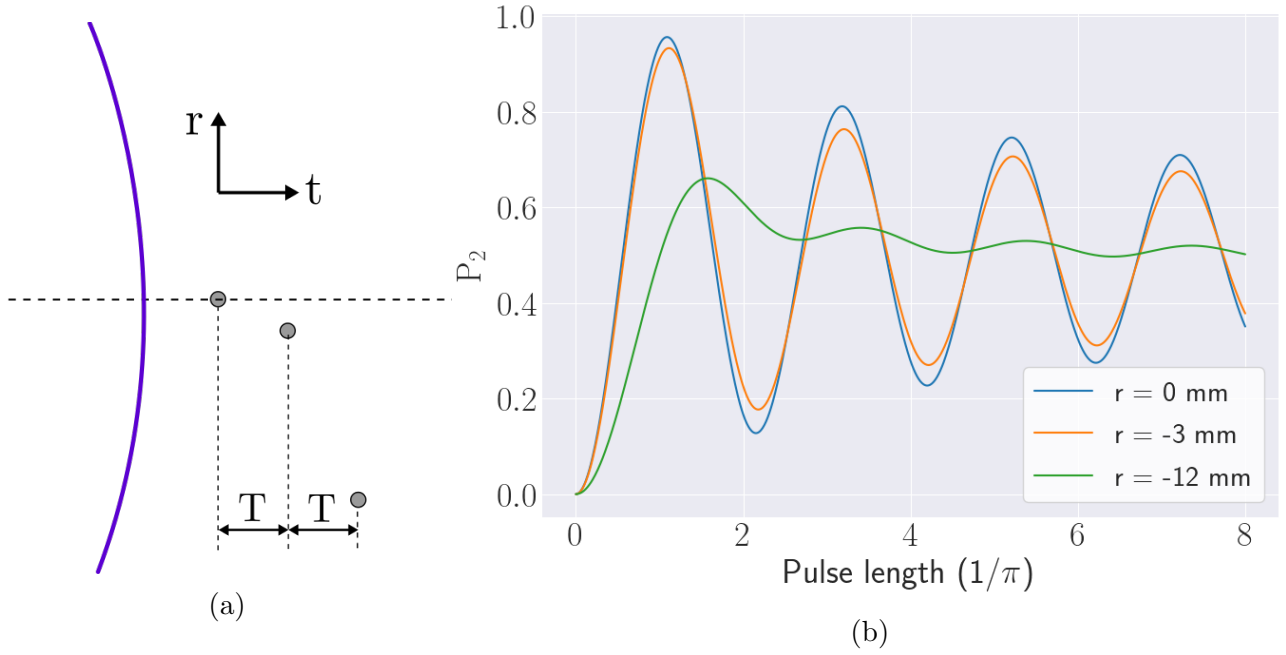


Figure 4.2: (a) Atomic cloud's position relative to the centre of a laser beam used to drive Raman transitions in the interferometer. The solid blue line represents the Gaussian variation across a laser beam with a 20 mm spot size. (b) Probability of an atom being in $|F = 2\rangle$ as a function of pulse length for the three different positions of the atomic cloud shown in (a).

atom cloud with a Gaussian distribution of positions is

$$P_2(r, \tau, \sigma) = \int_{-\infty}^{+\infty} \sin^2\left(\frac{\Omega_{\text{eff}}(r)\tau}{2}\right) p_G(r, \sigma) dr. \quad (4.3)$$

Here, $p_G(r, \sigma)$ is the normalised Gaussian density distribution of the atomic cloud. Figure 4.2b shows a plot of the population transfer for an atomic ensemble with rms radius $\sigma = 2.5$ mm at $r = 0$ mm (blue), -3 mm (orange), and -12 mm (green) as a function of the pulse length. The finite size of the atomic ensemble and beam's spot size result in damped Rabi oscillations. As before, the pulse length required to maximise P_2 increases with increasing $|r|$, but now it is not possible to completely invert the population.

A large spot size allows one to minimise the range of Rabi frequencies across the atomic cloud but at the cost of needing higher laser power. In addition, we launch the cloud before the interferometry sequence in order to keep the atoms close to the flattest part of the intensity distribution, as discussed below. Both of these help us to avoid the damping seen in fig. 4.2b.

Launching

Launching the atoms vertically ensures the atom-light interaction during each pulse occurs when the cloud is close to the centre of the beam, where it is most uniform. Figure 4.3a

illustrates a launch velocity of 24.5 cm/s such that the atoms return back to the centre after 50 ms, the three pulses occurring when the atomic cloud has a mean position of 0 mm, 3 mm, and 0 mm. Figure 4.3b shows that launching the atoms increases the peak value of P_2 for the third pulse to 96% instead of the 66% obtained without launching.

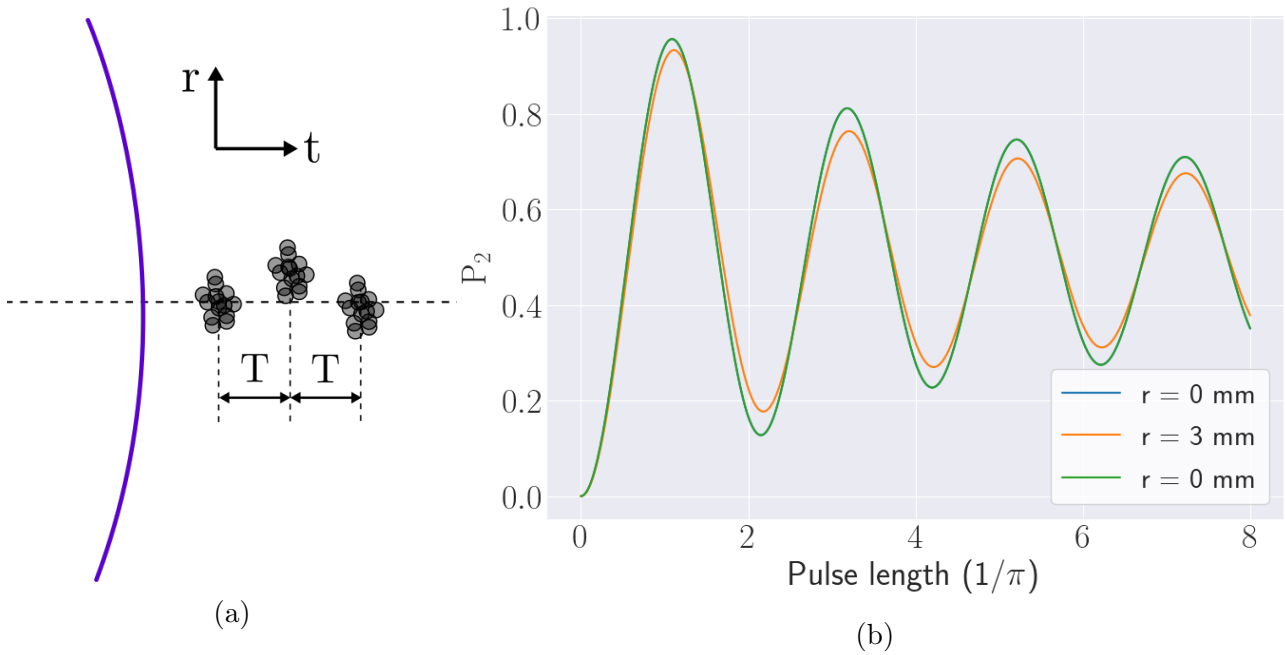


Figure 4.3: (a) Atomic cloud's position relative to the centre of a laser beam used to drive Raman transitions in the interferometer. The solid blue line represents the Gaussian intensity variation across a laser beam with a 20 mm spot size. (b) Proportion of atoms in $|F=2\rangle$ as a function of pulse length for the three different positions of the atomic cloud shown in (a).

Raman beam spot size

Figure 4.4a shows a Raman beam with a 60 mm spot size. This produces Rabi oscillations with significantly reduced damping compared with those for the 20 mm beam shown in fig. 4.3b. There is, however, a difficulty with increasing the spot size. Given a fixed laser power, this reduces the intensity, and hence increases the duration, τ , of the π -pulse. Although the π -pulse gives $P_2 = 1$ for an atom at resonance, P_2 drops to zero for a detuning (in Hz) of $\sqrt{3}/(2\tau)$, as given by the sinc^2 line shape in eq. (2.21). With the laser power that we have available, a 40 mm spot size gives $\tau = 20 \mu\text{s}$ and hence a full width for the first lobe of the sinc^2 of 90 kHz. This is roughly equal to the Doppler width of the transition due to the thermal velocity spread. For that reason, it is necessary to select a small subset of atoms for use in the atom interferometer, with a narrow velocity distribution whose Doppler width is small compared with $1/\tau$. Increasing the spot size therefore reduces the number of atoms that can participate in the interferometer and that makes the quantum projection noise worse. In the experiment, we therefore choose a spot size of 40 mm as a good compromise between the competing requirements.

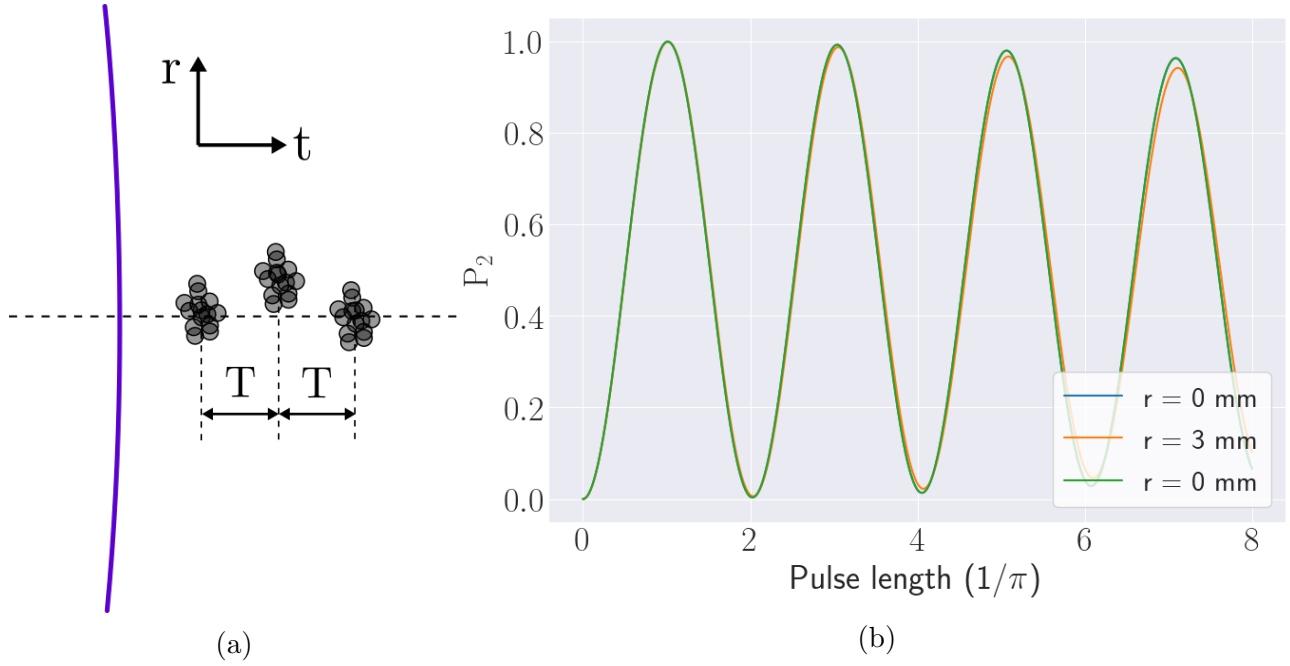


Figure 4.4: (a) Atomic cloud's position relative to the centre of a laser beam used to drive Raman transitions in the interferometer. The solid blue line represents the Gaussian intensity variation across a laser beam with a 60 mm spot size. (b) Proportion of atoms in $|F = 2\rangle$ as a function of pulse length for the three different positions of the atomic cloud shown in (a).

For the moment let us assume the velocity selection is so narrow that we can neglect the Doppler shifts. In fig. 4.5 I show the fringes I calculate for a full three pulse interferometer having a spot size of 40 mm and a launch height of 3 mm. We see that these fringes go fully between 0 and 1. We conclude that with our 40 mm spot size and with vertical launching, the inhomogeneity of the Rabi frequency causes negligible reduction of the fringe visibility. As already noted, that will not be true for the inhomogeneity of the velocity.

4.1.2 Collimation

In chapter 2 we considered plane-wave Raman beams with phase $\vec{k} \cdot \vec{z}$, but in reality, the light is close to a paraxial Gaussian, whose phase at a position (r, z) is

$$\phi(r, z) = k z + \frac{k r^2}{2R(z)} - \Psi(z). \quad (4.4)$$

Here, z is the longitudinal position measured from the waist, r is the radial position, $R(z) = z[1 + (\frac{z_R}{z})^2]$ is the radius of curvature of the wavefront, $\Psi(z) = \arctan\left(\frac{z}{z_R}\right)$ is the Gouy phase, and $z_R = \frac{\pi w_0^2}{\lambda}$ is the Rayleigh length. For a waist size of 40 mm, the Rayleigh length is 6 km, and the effect of the Gouy phase on measuring accelerations can be neglected. For our retro-reflecting setup, properties of the outgoing Raman beam have the subscript “out”, while those

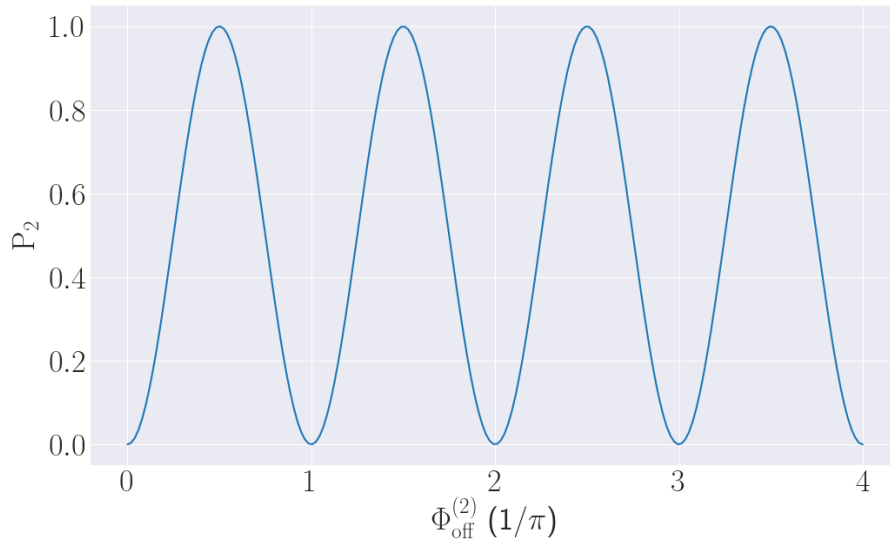


Figure 4.5: Calculated interferometer fringe for a Raman beam spot size of 40 mm and a launch height of 3 mm. The fringe is scanned over by varying the phase of the second Raman pulse.

of the reflected beam are sub-scripted “refl”. Figure 4.6 illustrates a beam with a divergence angle θ reflecting from a mirror a distance $L = L_1 + L_2$ away from the collimating lens. Because

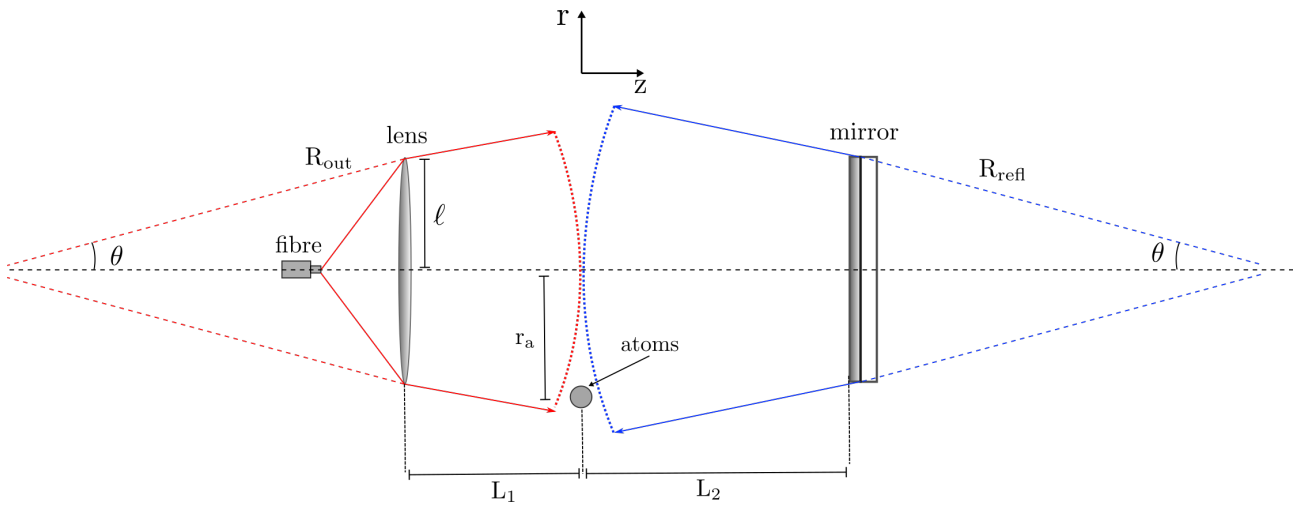


Figure 4.6: Schematic of a diverging beam reflecting from a mirror.

$L_2 \ll z_R$, it is an excellent approximation to consider that the divergence angle of the reflected beam is also θ . On axis, the phase difference between the two beams is exactly as it would be for a plane wave, apart from the (virtually) constant Gouy phase. However, if the atom is displaced to radius r there is an additional phase shift of

$$\delta\phi(r) = \frac{r^2}{2} \left(\frac{k_{\text{out}}}{R_{\text{out}}} - \frac{k_{\text{refl}}}{R_{\text{refl}}} \right). \quad (4.5)$$

In our experiment the fractional difference between k_{out} and k_{refl} is much less than that between R_{out} and R_{refl} , so we can approximate $\delta\phi$ as,

$$\delta\phi(r) = \frac{k r^2}{2} \left(\frac{1}{R_{\text{out}}} - \frac{1}{R_{\text{refl}}} \right) \quad (4.6)$$

where k no longer makes the negligible distinction between k_{out} and k_{refl} . On noting that $R_{\text{refl}} = R_{\text{out}} + 2L_2$, and that $L_2 \ll R_{\text{out}}$ we conclude that

$$\delta\phi(r) = k \frac{r^2}{R_{\text{out}}^2} L_2. \quad (4.7)$$

To a good approximation $R_{\text{out}} = \ell/\theta$, hence,

$$\delta\phi(r) = k L_2 \left(\frac{r}{\ell} \right)^2 \theta^2. \quad (4.8)$$

Equation (4.8) represents the error in phase that is imprinted onto the atoms at a single pulse. As discussed in §4.1.1, we launch the atoms with a vertical velocity of 24.5 cm/s such that the position of the atom at the three pulses are $r = 0$ mm, 3 mm, and 0 mm. Thus all of the error due to the wavefront curvature will be accumulated at the second pulse. Using eq. (2.52), which relates the interferometer phase to the acceleration, we see that the phase error in eq. (4.8) corresponds to an acceleration error of

$$\delta a = \left(\frac{r\theta}{\ell} \right)^2 \frac{L_2}{T^2}. \quad (4.9)$$

Our Raman lens has a diameter of 50 mm and we have set $L_2 = 140$ mm in our chamber. Therefore, with $T = 25$ ms, we need the divergence angle out of the Raman lens to be $\theta \leq 560 \mu\text{rad}$ if the acceleration error is to be kept below 100 ng. In short, the Raman beams needs to be very well collimated.

4.1.3 Intensity inhomogeneity

In a previous experimental setup aberrations caused by the use of a low-quality lens reduced the atomic fringe contrast that could be achieved. Figure 4.7a shows an interferometer fringe taken with the previous Raman optics setup. The fringe has a contrast of only 4%. Figure 4.7b shows the measured Rabi oscillation (black dots) at a height of 1.2 mm above the optical axis of the Raman beam. Overlaid on fig. 4.7b are simulated Rabi oscillations for three different values of an intensity change across the size of the atomic cloud (~ 4 mm). These intensity changes are $\Delta I/I_0 = 0.1$ (red), 0.2 (blue), and 0.9 (green). The data observed are most easily explained by a intensity change of 90% across the atomic cloud. We imaged the beam with a

CCD camera with a sensor of size 6.5 mm x 4.9 mm. After stitching together the individual images the composite image shown in fig. 4.7c was produced. We indeed observe a region with very large intensity gradients just above the centre of the beam.

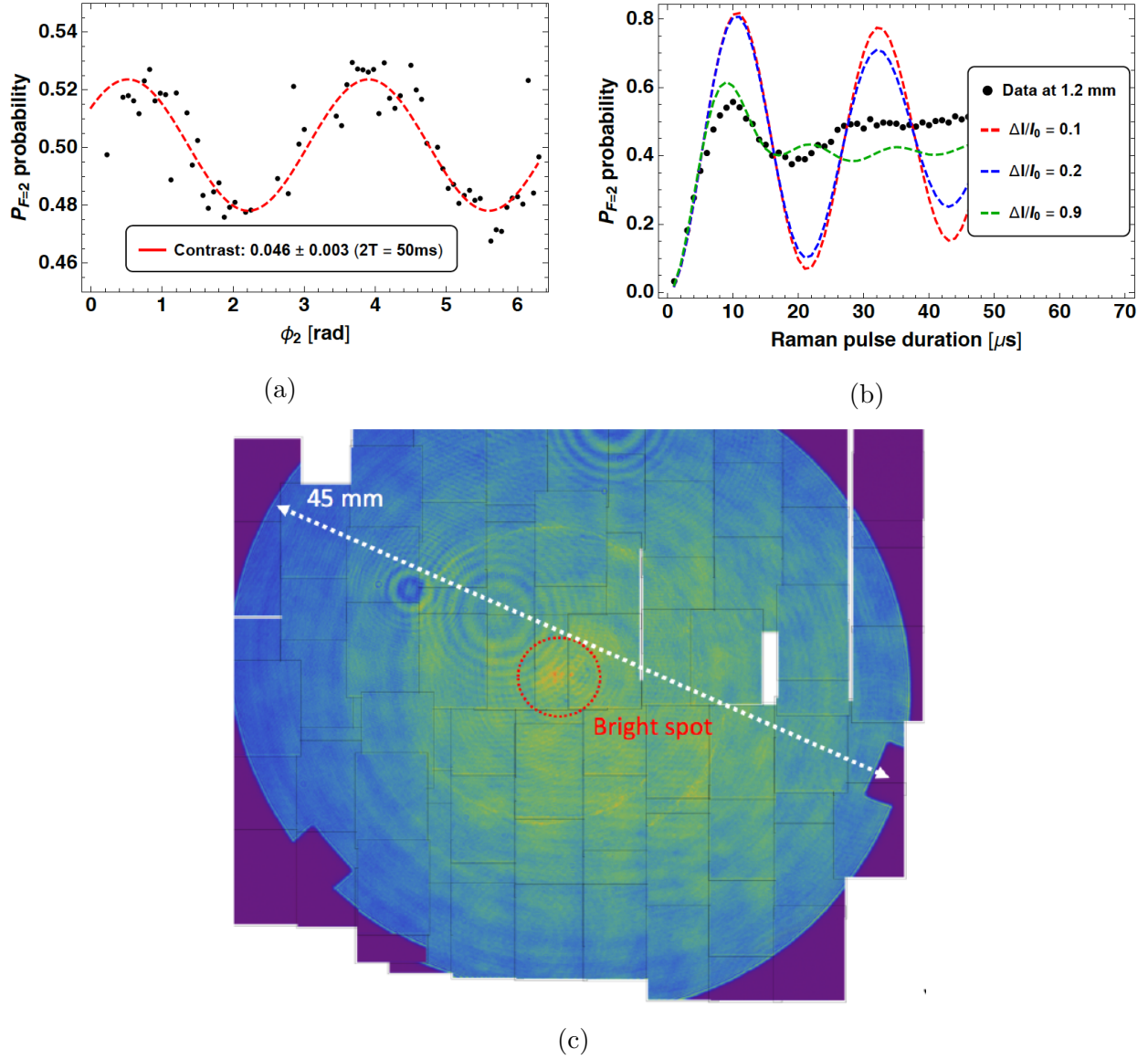


Figure 4.7: (a) Initial interferometer fringe with the previous Raman optics setup. The fringe shows a severely reduced contrast value of 0.04. (b) Rabi oscillation (black dots) taken at a height of 1.2 mm above the optical axis of the Raman beam. Simulated Rabi oscillations for beams with intensity changes across the atomic cloud of $\Delta I/I_0 = 0.1$ (red), 0.2 (blue) and 0.9 (green) are shown. (c) Image of the previous Raman beam using a series of stitched images taken with a CCD camera. Image and both plots taken from [?] and reproduced with permission.

To correct this I have designed a new expander for the Raman beams.

4.2 New Raman optics setup

4.2.1 OpticStudio design

For a Gaussian beam with a spot size of 40 mm propagating through optical elements with a diameter of 50 mm, 46% of the power falls outside this radius and is lost. In the new design I chose to clip the light at the first lens to avoid stray light being propagated to the atoms. Figure 4.8 is a schematic of this new setup.

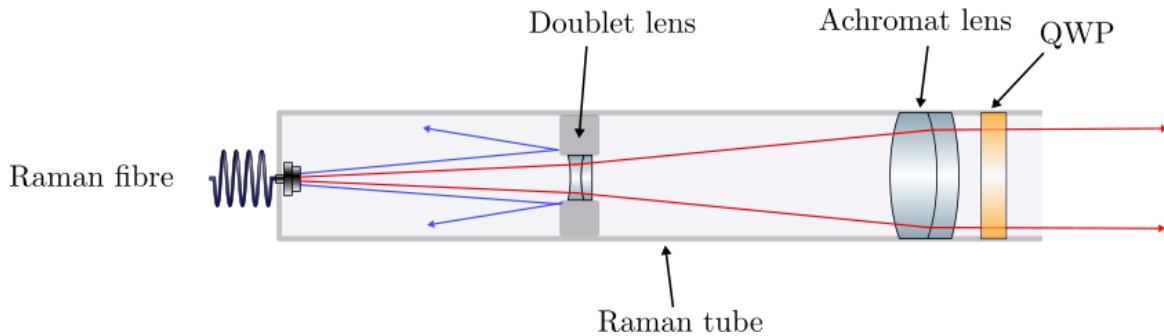


Figure 4.8: New Raman tube setup. The light is clipped at the first lens which leads to the reflected light (blue) propagating away from the atoms.

The optical design software OpticStudio was used to generate simulations for the new Raman optics setup. The new design is a combination of a doublet lens with a focal length of -25 mm and 6.25 mm diameter and an achromat 50 mm diameter lens with a focal length of 150 mm. The distance from the fibre to the first lens is 52.5 mm and from the first lens to the second lens is 122 mm. The system is designed in a Galilean configuration in which there is no intermediate focal point.

OpticStudio is able to produce optical path difference (OPD) plots which show the difference in path length relative to the chief ray. Figure 4.9a shows a contour plot of the OPD for the new Raman optics setup. The x and y axes are in normalised units relative to the entrance pupil[†] which has a diameter of 6.25 mm. The graph displays the OPD relative to the chief ray in units of wavelength. The clipping of the light leads to substantial reduction in wavefront quality near the edge of the simulated beam. Figure 4.9b shows a 1-dimensional plot of the OPD where the x-axis coordinate has been transformed into the plane where the atoms reside. The inset shows that the simulated beam is flat to $< \lambda/100$ over the central region 11.5 mm in diameter. This corresponds to a divergence angle θ of less than $12 \mu\text{rad}$ indicating that the error due to wavefront curvature can be reduced to a very low level.

[†]The entrance pupil of an optical system is the image of the physical aperture stop as viewed from the image plane. In the case of the new design the aperture stop is the first lens. Therefore, pupil coordinates of ± 1 represent the very edge of the first lens.

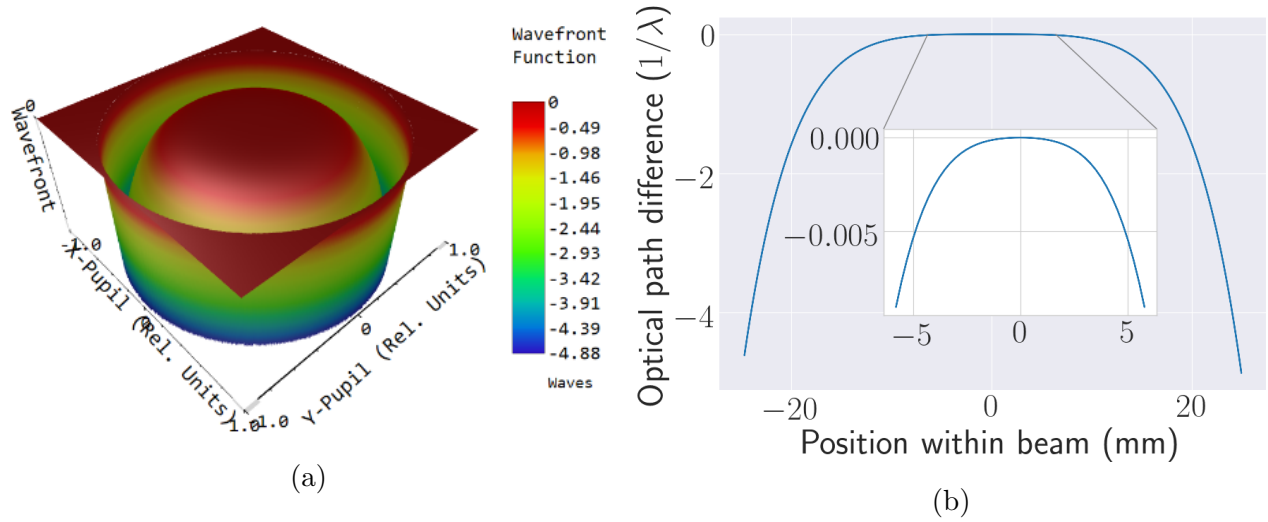


Figure 4.9: Optical path difference across the new Raman beam in units of the 780 nm wavelength λ (a) at the first lens. The x and y axes units are in normalised pupil coordinates where ± 1 corresponds to the edge of the aperture. (b) In the plane of the second lens. The inset shows the region of the simulated beam where the OPD is less than $\lambda/100$.

4.2.2 Identification of bright spot

Upon constructing this new Raman optics setup the same bright spot as in the previous design was observed. Figure 4.10a shows a series of images taken of the Raman beam reflecting from a screen at varying distances from the end of the Raman tube. The upper left image is taken at 10 cm from the end of the tube, and each image thereafter, going from left to right and top to bottom, is taken at increments of 10 cm. The series of images show a very bright spot at approximately the centre of the beam with its focus at around 10 cm from the end of the Raman tube. This is also close to the plane of the atomic cloud at 15 cm from the Raman tube. Because of this observation I concluded that the bright spot must be light reflecting from the edges of the lenses rather than from the Raman tube.

To combat this bright spot, a small circular aperture was placed before the first lens in the Raman tube to clip the edges of the incoming light. Figure 4.10b shows an image taken of the beam before inserting an aperture and fig. 4.10c shows the beam imaged after inserting the aperture. The bright spot is no longer visible. A much fainter intensity focus was observed after inputting the aperture before the first lens. After also using an aperture before the second lens, this too was resolved.

4.2.3 Raman tube construction

The optical elements used to expand and collimate the Raman beams are aligned using a three-part mounting tube. The first section contains the fibre tip mount, the middle section holds the

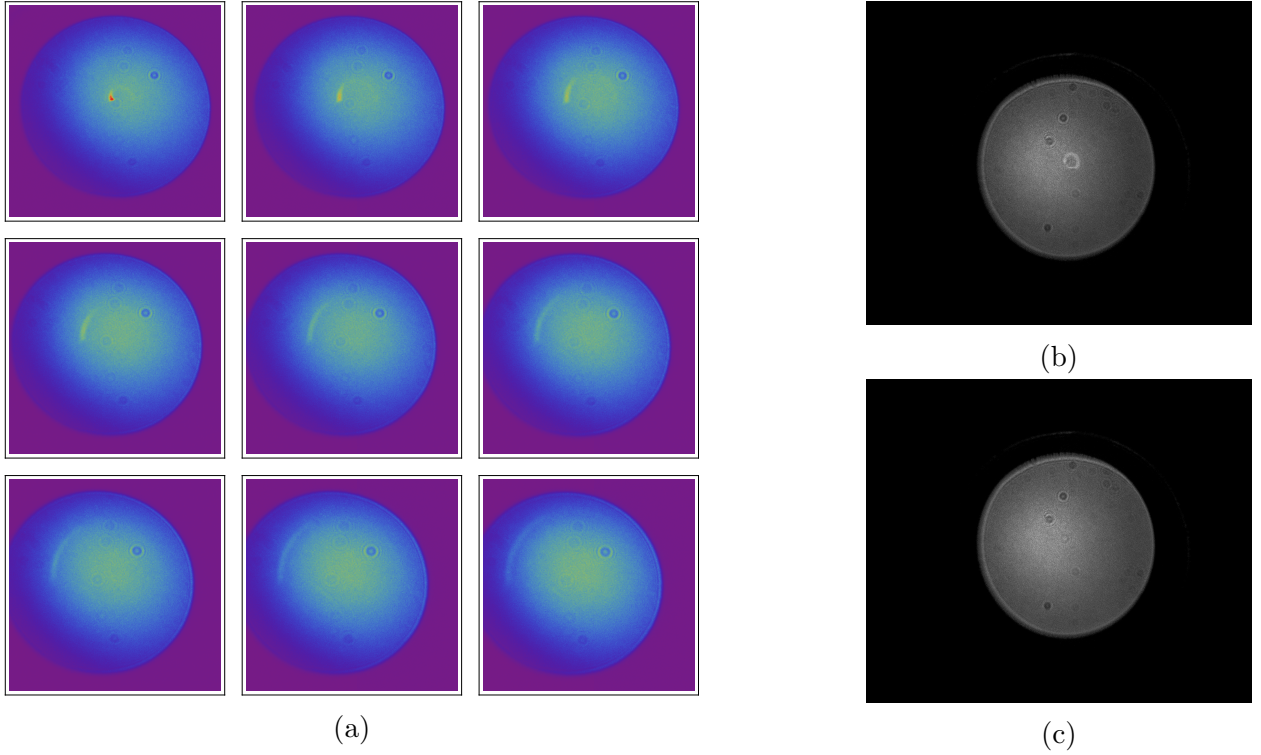


Figure 4.10: (a) Series of images taken of the Raman beam reflecting from a screen at various distance from the end of the Raman tube shown in fig. 4.11a. The upper left image is taken at 10 cm and subsequent photos are taken at increments of 10 cm going from left to right and top to bottom. the atomic cloud is roughly at 15 cm. The bright intensity spot is most in focus near the plane of the atomic cloud. Image of the Raman beam (b) before and (c) after an aperture was inserted before the first lens.

doublet lens, and the final section holds the achromat lens, a large 4 mm spacer, and the QWP. The mounting tube is attached to a DN60 flange, as illustrated in fig. 4.11a. We purchased a custom built continuous fibre from Diamond SA that is attached to the chamber using a DN16 flange to deliver the light coupled from the laser system straight to the Raman optics. Figure 4.11b shows the new fibre feedthrough and the vacuum flange.

4.3 Collimation & Spot Size Characterisation

The peak intensity for a Gaussian beam with power P and spot size $w(z)$ is,

$$I_{\max} = \frac{2P}{\pi w(z)^2}. \quad (4.10)$$

Figure 4.12 shows a schematic of an experiment where I measured the peak intensity at two different distances using an optical power meter. The power-head is mounted to a 2D translation stage which allows us to raster the power-head until the position of the intensity maximum is found. I aimed to collimate the beam by putting thin shims before the fibre tip or be-

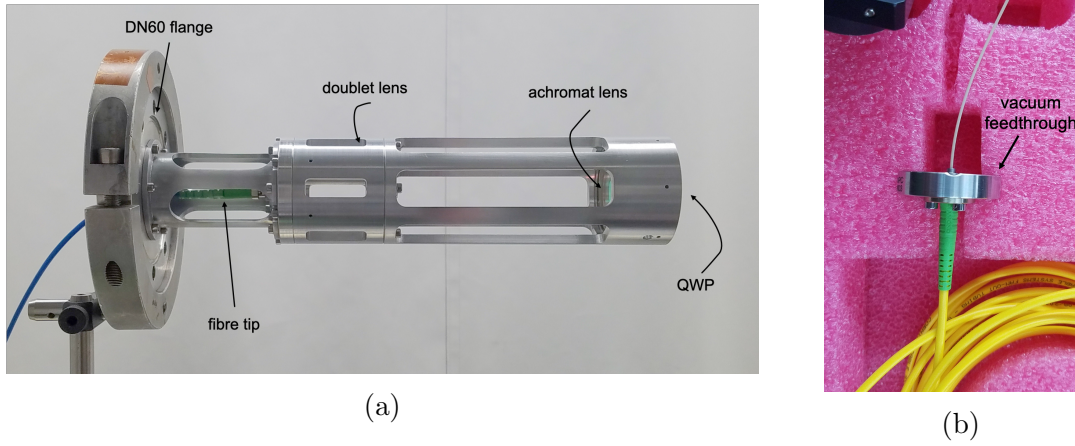


Figure 4.11: (a) Photo of the mounted Raman optics. The tube is made up of three parts. The first section houses the fibre tip, the second holds both the doublet lens and the aperture, whilst the final section contains the achromat lens, a large spacer, and finally the QWP. (b) Photo of the fibre feedthrough which comprises of a single continuous fibre attached to a DN16 flange.

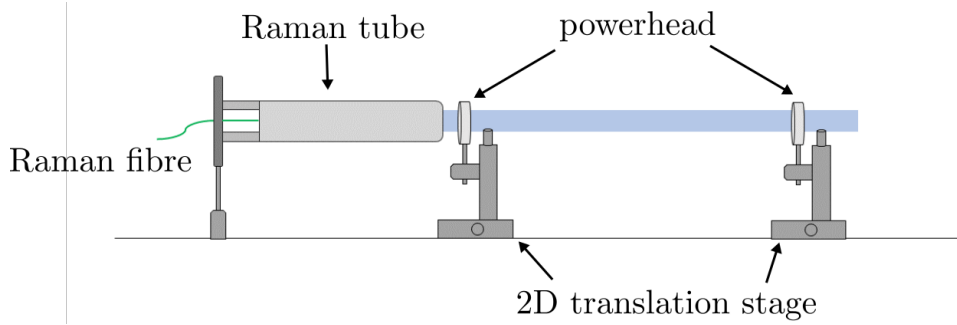


Figure 4.12: Setup used to take power-head measurements of each of the Raman beams. The power-head is mounted on a 2D translation stage allowing us to raster it through the beam until the intensity maximum is found. Measurements taken close and far from the Raman tube end were used to get an estimation of the collimation.

fore the second lens to achieve the same maximum intensity at both distances. The optimal configuration used a 2.6 mm shim before the second lens.

To assess the collimation of the beam more accurately I measured the width at which the intensity (counts) falls below a threshold value. The setup was similar to fig. 4.12 but with the power-head replaced by a Sony $\alpha 7$ mirrorless CCD camera. Images were taken 30 cm and 105 cm from the end of the Raman tube. The ‘centre of mass’ of the beam was calculated and a 1 pixel slice across the ‘centre of mass’ of each image was taken. Figure 4.13 shows the graph resulting from this process.

Using the data in fig. 4.13, I vary the threshold, work out the collimation using the two calculated beam widths, and average the collimation values. This gives a collimation angle θ (defined in fig. 4.6) of $65 \mu\text{rad} \pm 68 \mu\text{rad}$ for tube 1 and $42 \mu\text{rad} \pm 82 \mu\text{rad}$ for tube 2. This gives me

95% confidence that $\theta < 200 \mu\text{rad}$ in both cases, and hence from eq. (4.9) that the systematic error due to wavefront curvature is less than 15 ng on both axes with $T = 25 \text{ ms}$.

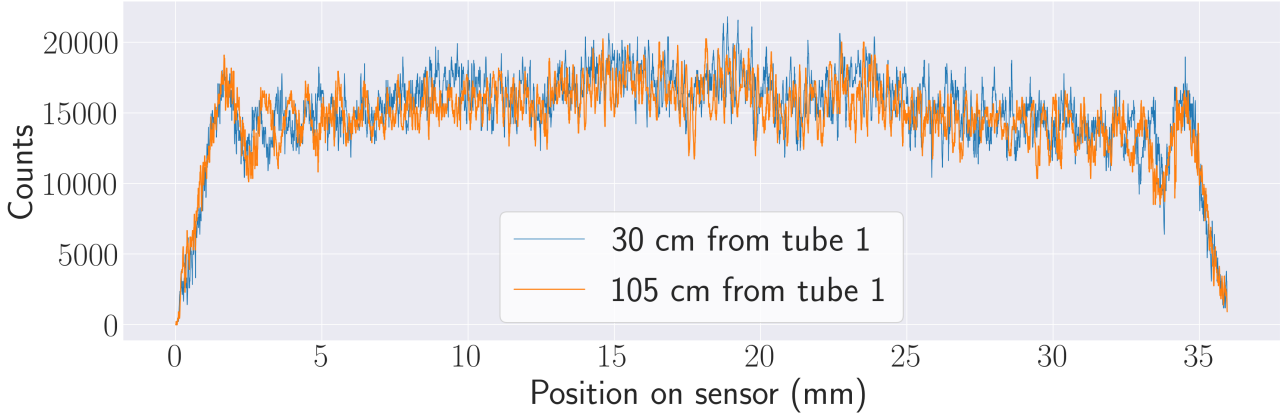


Figure 4.13: Collimation of tube 1. Number of counts versus the position on the CCD sensor. Counts are shown for a slice through an image taken 30 cm (blue) and 105 cm from the end of the Raman tube.

To illustrate the improved intensity uniformity of my Raman beam compared with our previous setup I used a Pike F-505B CCD camera and stitched together multiple images. In order to avoid ruining the image with interference fringes, it was necessary to remove the protective glass plate that covers the sensor. Figure 4.14 shows the new intensity profile. The small airy ring patterns that can be seen are thought to be due to dust particles on either one of the two lenses. Figure 4.14 shows that the bright spot at the centre of the beam from fig. 4.7c is no longer present. Another noticeable feature of the new beam is the much more homogeneous intensity profile across the whole of the beam compared with fig. 4.7c.

Figure 4.15 shows a 1 pixel slice taken from one of the central images of fig. 4.14, which exhibits a 5% standard deviation of the intensity across the CCD. With ~ 1800 photons per pixel in the image, this noise is roughly double the 2.4% expected from photon counting statistics. The excess noise is presumably the speckle in the Raman beam, which can be seen in fig. 4.10. This appears in fig. 4.15 as fluctuations with a typical correlation length of order $100 \mu\text{m}$. The atomic cloud is a few millimetres in size with a Gaussian spatial distribution. As a result, the atomic cloud samples a few millimetre portion of the intensity profile with a Gaussian weighting. The intensity variations over this region are small enough that they do not compromise the Rabi flopping.

In order to check the spot size of the beam, I fit the quadratic $a + bx + cx^2$ to the profile in fig. 4.15. The Gaussian Ae^{2x^2/w^2} expands to $A - \frac{2Ax^2}{w^2}$ near it's centre, so the equivalent spot size is given by $\sqrt{-2a/c}$. This procedure yields the result $w = 41.5 \text{ mm}$ which is satisfactorily close to the 40 mm spot size we had hoped to produce.

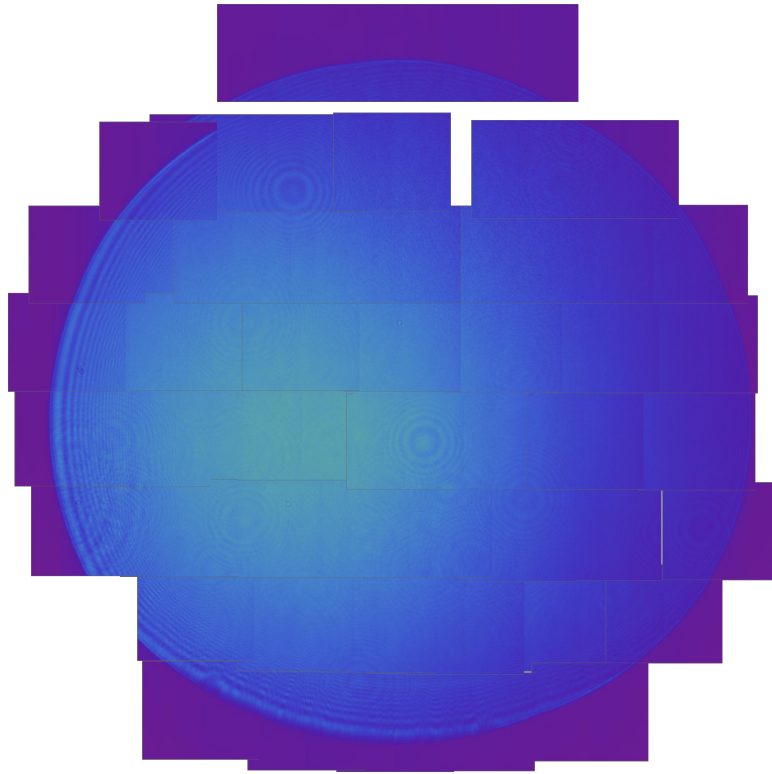


Figure 4.14: Composite photo of Raman beam produced by stitching together multiple $8.5 \text{ mm} \times 7.1 \text{ mm}$ images taken using a Pike F-505B CCD camera.

4.4 Retro-reflecting module

The complete Raman optics setup includes a retro-reflecting module as shown in fig. 2.6 and fig. 3.5c, which contains a QWP, mirror, MEMS accelerometer, and a kinematic mount. I now describe these components and present measurements of the MEMS' noise performance.

4.4.1 Quarter wave-plate and mirror

The $\Delta m_F = 0$ two-photon Raman transition requires circularly polarised light in pairs of either σ^+/σ^+ or σ^-/σ^- . The two different frequencies from the M Squared laser system are coupled into orthogonal axes of a polarisation maintaining (PM) fibre. On exiting the fibre in vacuum, the light is transformed from linear to circular by a QWP at the end of the Raman tube. On retro-reflection at the mirror side, the light passes twice through an identical QWP, which flips the handedness of each polarisation. Figure 4.16 shows CAD images of the retro-reflecting module with fig. 4.16a showing the module installed inside the chamber and fig. 4.16b showing the module with just its mounting DN40 flange. The QWPs at the end of the Raman tube and on the retro-reflecting module have an anti-reflection coating that gives less than 0.5% reflectance at 780 nm. Although we would ideally wish these wave-plates to distort the wave-front as little as possible, we settled in this first trial of the two-axis accelerometer for a less

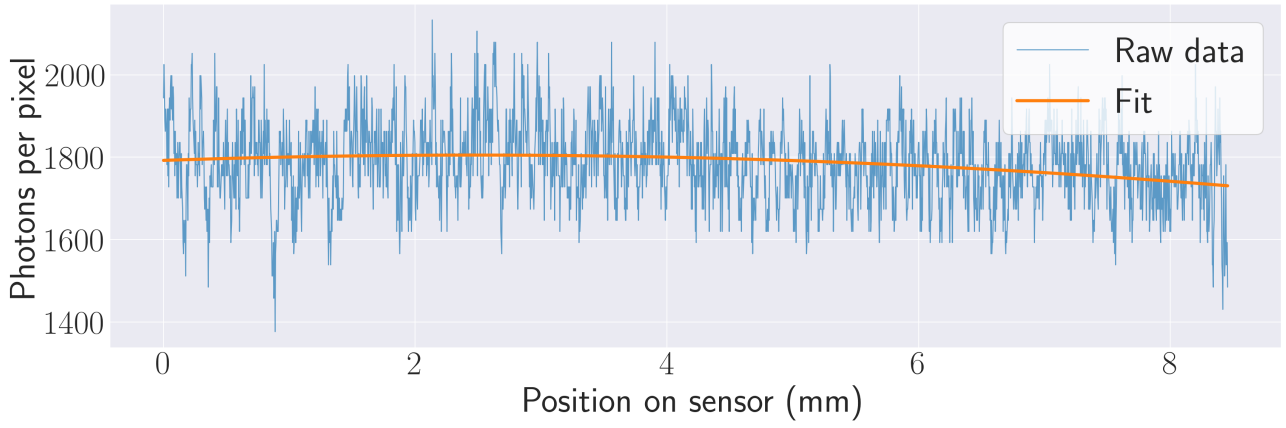


Figure 4.15: Photon counts detected on a 1 pixel slice taken from the central CCD image of fig. 4.14 (blue) plotted as a function of position on the sensor. Also shown is a plot of the fit to a quadratic. This has the same curvature as the centre of a Gaussian with a spot size of 41.5 mm.

stringent specification. The manufacturer's specified better than $\lambda/4$ at 633 nm.

The mirror is made from ZERODUR[®] providing an extremely low coefficient for thermal expansion, has a reflectance of $> 98\%$ at 780 nm, and a surface flatness of $\lambda/20$.

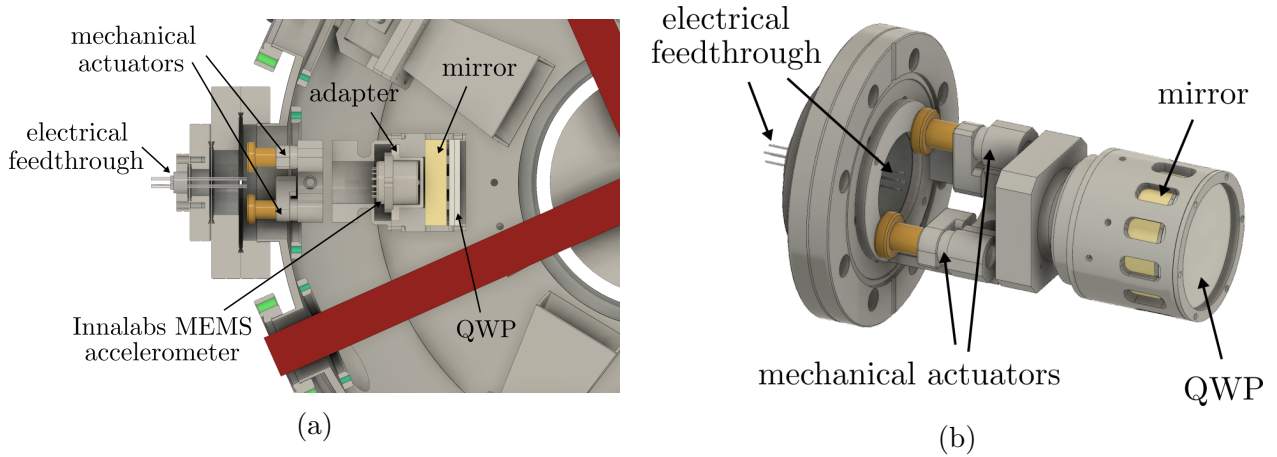


Figure 4.16: (a) CAD drawing of the retro-reflecting module mounted inside the chamber. (b) CAD drawing of the retro-reflecting module along with the DN40 used to mount it to the chamber. The MEMS accelerometer is mounted to the back of the mirror. Fine in-situ adjustment of the orientation of the mirror is made possible by mounting the mirror, QWP, and MEMS accelerometer to a kinematic mirror mount with two mechanical actuators. Electrical feedthroughs on the mounting flange allow readout of the MEMS accelerometer signal as well as control of the actuators.

4.4.2 MEMS accelerometer

The MEMS accelerometers used for hybridisation are Innalabs AI-Q-2010 quartz pendulous accelerometers. Figure 4.17 shows a schematic of the device and illustrates its operating principle. It contains a proof mass which is a high purity fused quartz flapper. This acts as a pendulum and is positioned between two high-stability magnets that are attached to a nickel-iron alloy case. Under an acceleration the flapper moves closer to one side of the casing, causing a change in the capacitance, which is detected and used to derive an error signal. A servo loop circuit uses this error signal to apply a current in two coils which are wrapped around the two magnets. This restores the pendulum to its null position over a frequency range of input accelerations $\sim 2\text{ kHz}$. The current required to maintain the null position is proportional to the acceleration. We record this current by measuring the voltage drop across a high precision resistor network produced by Vishay Foil Resistors. We have measured the resistance of the resistor networks as $6000.40 \pm 0.01 \Omega$ and $5999.71 \pm 0.02 \Omega$ for the accelerometers used on the X and Y axis respectively.

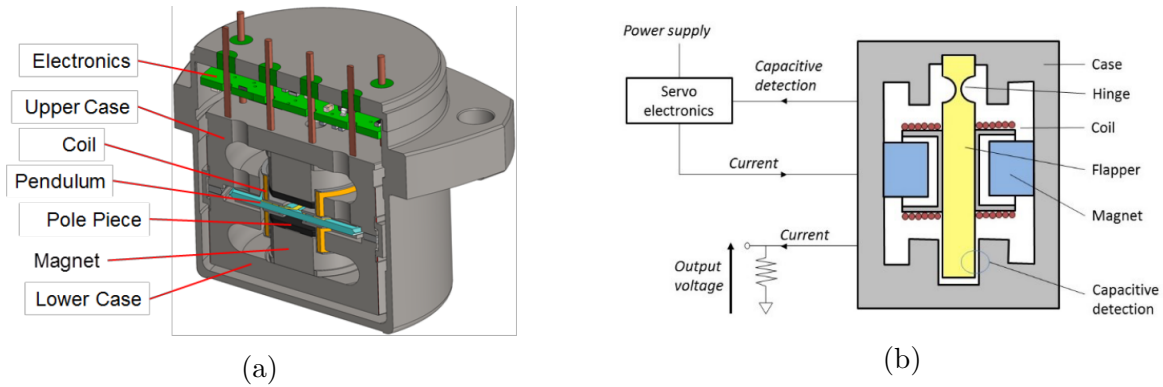


Figure 4.17: (a) Schematic of the Innalabs AI-Q-2010 quartz pendulous accelerometer. (b) Diagram illustrating the working principle. Both figures from [?].

Because the atom interferometer measures the acceleration of the mirror relative to the freely-falling atoms, it is important to mount the MEMS accelerometer in contact with the mirror in order to ensure that both are measuring the same thing. It was not possible to mount the MEMS accelerometer directly on the mirror. Instead, an adapter annulus, which mounts flush to the back of the mirror, was made by the mechanical workshop at Imperial College London. This adapter piece can be seen in fig. 4.16a.

The mirror, QWP, and MEMS accelerometer are all attached to a Radiant Dyes Laser & Accessories GmbH kinematic mount, whose two mechanical actuators allow fine in-situ adjustment of the orientation of the retro-reflecting module. The electrical feedthroughs shown in fig. 4.16 allow control of the mechanical actuators as well as reading out the MEMS accelerometer signal.

With the XY plane of the apparatus approximately horizontal, the X and Y MEMS accelerom-

eters registered mean values of 0.82 m/s^2 and -0.26 m/s^2 respectively. Figure 4.18a shows the deviation from the mean of both accelerometers over a period of 60 s, sampled every 0.01 ms. These have rms noises of $\sigma_X = 3 \text{ mm/s}^2$ and $\sigma_Y = 2 \text{ mm/s}^2$, which are entirely consistent with the manufacturer's specification of $< 15 \text{ mm/s}^2$. I note that the period of the atom interferometer fringes is $\Delta a = \lambda/(2T^2)$, which has the value 1 mm/s^2 when $T = 20 \text{ ms}$. For the MEMS accelerometer to tell with certainty which fringe the interferometer is on, we need its uncertainty to be less than 0.2 mm/s^2 , say, which requires some integration of the signal as we discuss later in this section.

Figure 4.18b displays the deviations measured over 8 hours, sampled every 4 ms, but averaged using a moving window with a width of 1 second. In this long scan, the measurements drift in opposite directions by about $40 \mu\text{g}$, while the temperature (axis on the right) drifts by about 3.5°C . This corresponds to a bias variation of $\sim 10 \mu\text{g}/^\circ\text{C}$, which is consistent with the manufacturer's specification of $< 30 \mu\text{g}/^\circ\text{C}$. When operating as an inertial sensor, this slow drift will be corrected by the atom interferometer.

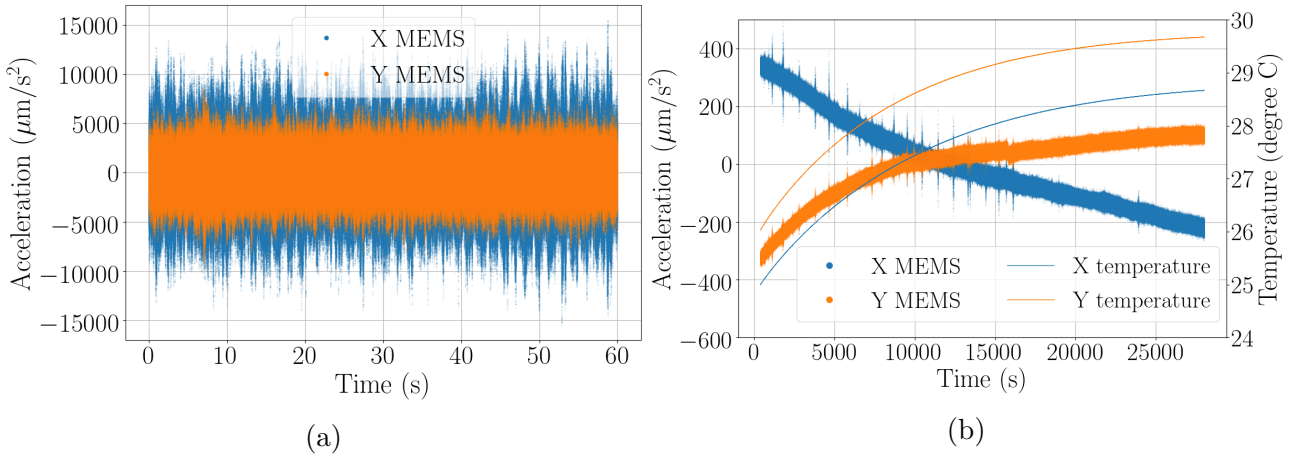


Figure 4.18: Time traces for MEMS accelerometers. X and Y axis are shown in blue and orange respectively. (a) 60 s sampled every 0.01 ms, and (b) approximately 8 hours sampled every 4 ms and averaged with a sliding window with a width of 1 s. Also shown are the readings from the integrated temperature sensors.

Figure 4.19 shows the square root of the power spectral density (PSD) computed from the data shown in fig. 4.18a. To check that this noise was not coming from the National Instruments ADC cards (NI card 9251) used to read the output of the MEMS accelerometers, I also measured the noise of the cards without any input. This gave the black lines in fig. 4.19, which are indeed negligible. From fig. 4.19 we see that the response of these accelerometers rolls off at about 2 kHz, which is consistent with the 2.6 kHz -3 dB stated in the manufacturer's data sheet. We also see some peaks in the spectrum, which are due to mechanical vibrations of the apparatus.

Using eq. (3.4) and the data in fig. 4.18b with $f = 250 \text{ s}^{-1}$ and $N = 3 \times 10^4 \times 250$, I plot

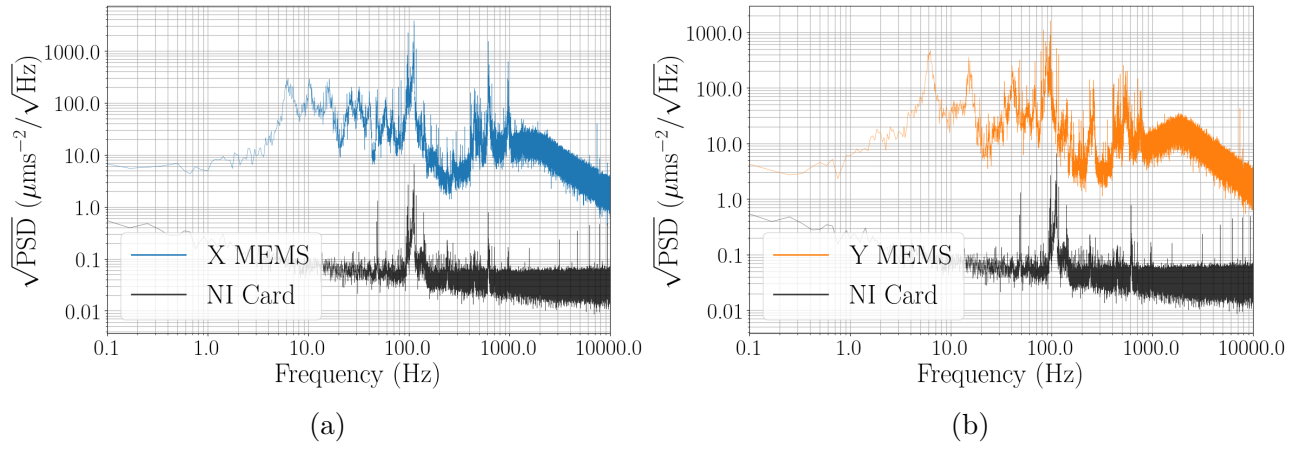


Figure 4.19: Square root of power spectral density for the (a) X MEMS and the (b) Y MEMS. The square root of the power spectral density for the acquisition card is shown in black on both graphs.

the Allan deviation for the acceleration, $\sigma_a(\tau)$ in fig. 4.20. This shows that the acceleration measured by the MEMS accelerometer averages down to 100 ng over 30 s before the temperature-dependent bias drift takes over. That sets the time scale for correcting the bias by means of the atom interferometer. We also see that integration for 100 ms reduces the uncertainty in the acceleration below 0.2 mm/s^2 . This means that on every shot of the atom interferometer we can expect to know with certainty which fringe the interferometer is on, at least for interferometer times up to $2T = 40 \text{ ms}$.

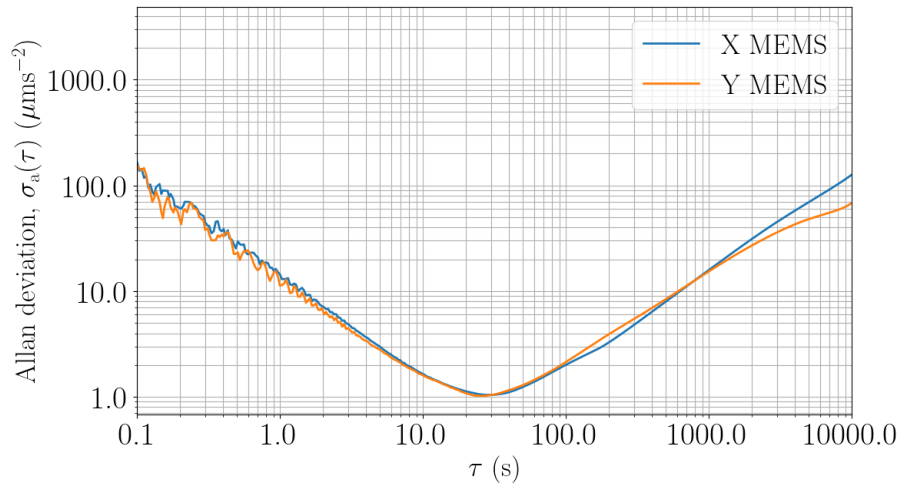


Figure 4.20: Allan deviation as a function of averaging time τ for the X MEMS (blue) and Y MEMS (orange).

Chapter 5

Atomic Cloud Preparation

5.1 2D & 3D MOT

Three orthogonal pairs of counter propagating beams overlapped with the centre of a quadrupole magnetic field are used to create a 3D magneto-optical trap (MOT) (see fig. 3.17). To cool the atoms, each beam in the 3D MOT is red detuned so that atoms scatter the light more rapidly when moving toward the light source than when moving away. This provides a velocity dependent scattering force that acts to slow atoms to within a small velocity range centred around zero thus cooling the atomic cloud. The magnetic field on the other hand provides a position dependent trapping force by lifting the degeneracy of the Zeeman sublevels of the atom. As an atom moves away from the trap centre, the Zeeman level that is able to be driven by the beam that the atom is moving towards is shifted closer to resonance. Conversely, the Zeeman sublevel that could be addressed by the counter-propagating beam is shifted further from resonance. This results in more frequent scattering from the beam the atom is moving towards resulting in a momentum transfer in the direction of the trap centre. Therefore, the combination of both the red detuned beams and the magnetic field provides a strong spatially dependent trapping force on the atoms as well as a cooling effect.

Preparation of the atomic cloud begins with a glass cell connected to the main vacuum chamber by a small (0.7 mm diameter) hole. Rubidium vapour in this cell is captured and cooled by a 2D MOT, whose principle of operation is the same as the 3D MOT except the atoms are cooled and trapped along two orthogonal axes. A stream of thermal atoms exits the 2D MOT through the small hole, flying into the main chamber, where it is recaptured by the 3D MOT. In addition, we apply a push beam along this third axis, which increases the number of atoms leaving the 2D MOT per unit time. This configuration is called a 2D⁺ MOT and our setup has previously been described in [? ?]. The high pressure of Rb in the cell allows fast loading of the 2D MOT. By separating the 2D MOT setup from the main chamber we can keep a high partial

pressure in the glass cell whilst maintaining the low pressure in the chamber needed for sensitive interferometer measurements. Using a $2D^+$ MOT significantly increases the speed at which we can load the 3D MOT, and hence reduces the cycling time of the interferometer. This increases the rate at which we can make measurements and leads to a reduction in the acceleration uncertainty. This comes at the expense of deteriorating the CSWaP figures. However, because optimisation of the CSWaP does not form a part of the current project we chose to continue with this slightly more complex setup due to the aforementioned advantage it affords us.

Figure 5.1 shows the control sequence used to load the 3D MOT. We start off by loading both the 2D and 3D MOT. This includes having the push beam on (not shown). After 100 ms, the 2D MOT coil, 2D MOT beams and push beam are switched off simultaneously. Following this, a period of 10 ms allows the number of atoms in the 3D MOT to stabilise. Finally we switch off the coil of the 3D MOT by applying a reverse current and then a linear ramp back to 0 A in order to reduce the magnetic field to zero as quickly as possible. During this stage the laser is set to configuration II and the AOMs in the light distribution module set such that the cooling beams of the 3D MOT are -15 MHz detuned (see fig. 3.8a). In addition we equalise the powers of the 3D MOT beams to ensure balanced radiation forces from each counter-propagating beam pair. Having first formed a 3D MOT we then used this to adjust the parameters of the 2D MOT so as to load the largest number of atoms in the 3D MOT.

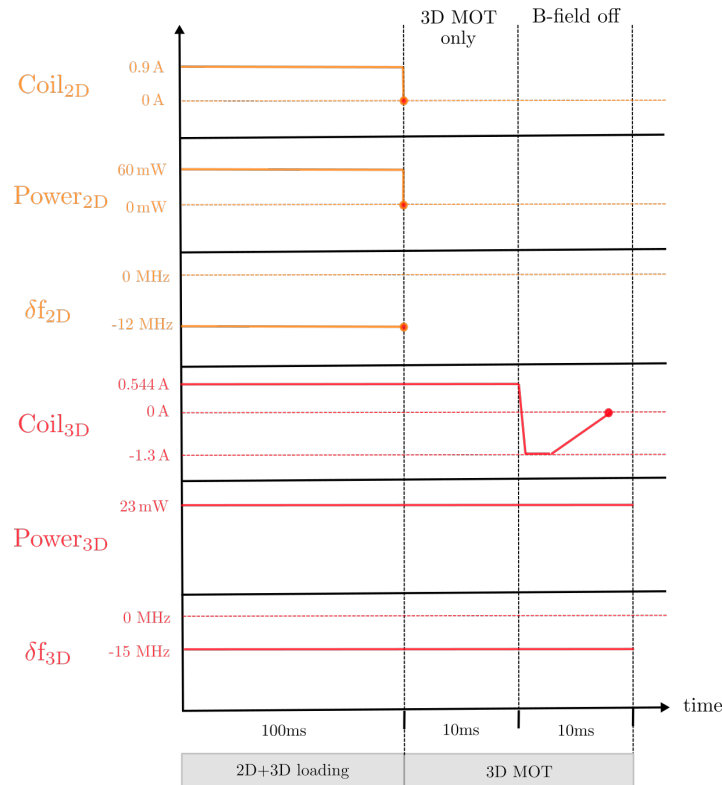


Figure 5.1: Sequence loading the 3D MOT. In this diagram dots represent that the output has been turned off. For instance, the orange dot at the end of the “2D+3D loading” block for δf_{2D} represents that the switch controlling the output of the 2D MOT beams is off.

5.1.1 2D MOT optimisation

2D MOT beams

Figure 5.2 shows the sequence of steps used in optimising the 2D MOT. In the diagram, δf_{2D} and δf_{3D} are the detunings of the 2D and 3D MOT cooling beams from the $|F = 2\rangle \longleftrightarrow |F' = 3\rangle$ transition (see fig. 3.8a). We load both the 2D and 3D MOT for 100 ms. Following this, we switch off the 2D MOT beams and detect the fluorescence from the atoms in the 3D MOT to measure the number of atoms. Figure 5.3a shows the result of this process for δf_{2D} between

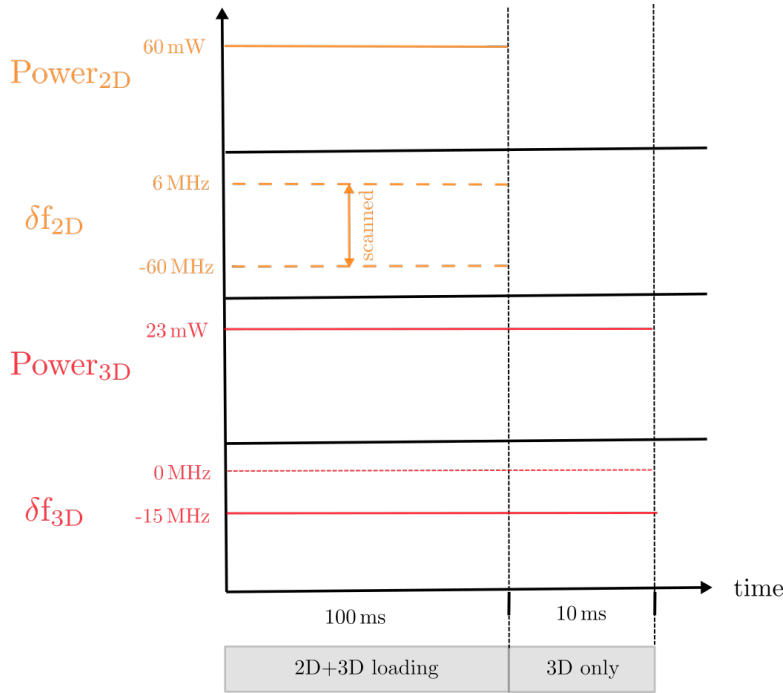


Figure 5.2: Sequence diagram for the optimisation of the 2D MOT beam frequency.

-60 MHz and 6 MHz. From fig. 5.3a we see that the number of atoms in the 3D MOT is maximised for a 2D MOT detuning of -12 MHz.

To optimise for the power of the 2D MOT beams we use the same sequence as in fig. 5.2 but set the detuning of all the 2D MOT beams to be -12 MHz. We then varied the power of the 2D MOT beams and observed optimal loading at the maximum available power for the 2D MOT output of the light distribution module. Figure 5.3b shows that we would probably benefit from increasing the power in the 2D MOT beams.

Push beam

Following a similar procedure to fig. 5.2 we optimised the number of atoms in the 3D MOT by varying the detuning and power of the push beam. Figure 5.4a shows the result of varying just

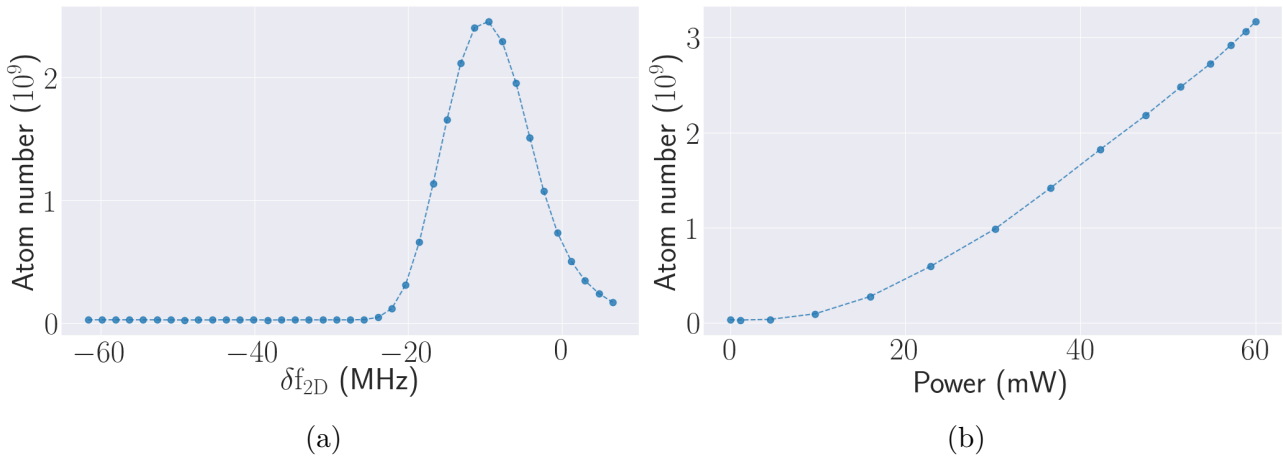


Figure 5.3: Number of atoms in the 3D MOT versus (a) detuning of the 2D MOT beams from the $|F = 2\rangle \longleftrightarrow |F' = 3\rangle$ cycling transition and (b) the power of the 2D MOT fibre of the light distribution module.

the detuning, whilst fig. 5.4b shows the result of varying just the power. These graphs indicate that a detuning of $+8$ MHz and a power of 4.5 mW is optimal. The positive detuning makes sense because we have to compensate for the Doppler shift of the accelerating atomic beam. We are unsure as to why there appears to be an optimal power for the push beam.

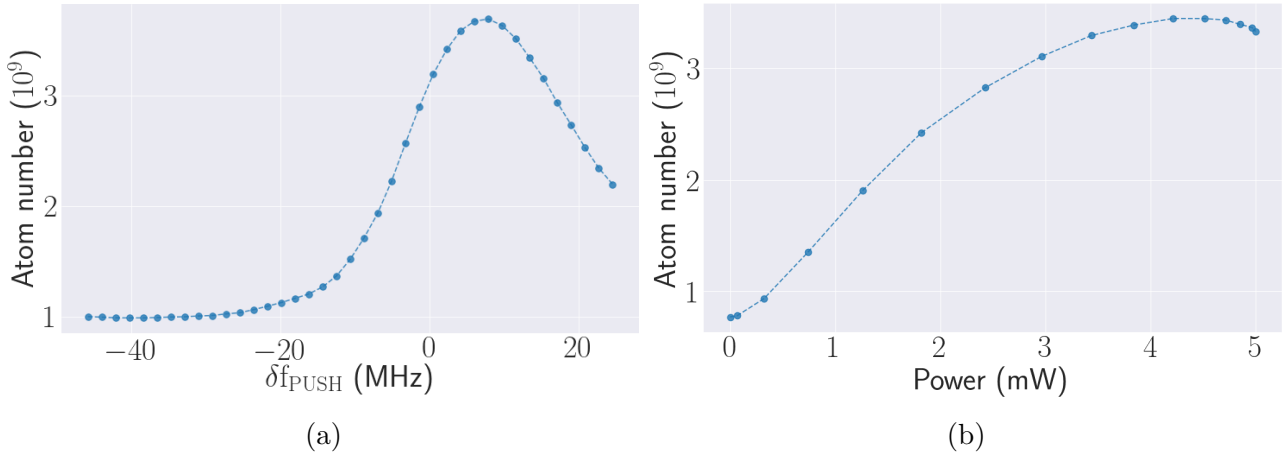


Figure 5.4: Number of atoms in 3D MOT versus (a) detuning of the push beam from the $|F = 2\rangle \longleftrightarrow |F' = 3\rangle$ cycling transition and (b) the power of the push beam.

2D MOT quadrupole field

I adjusted the 2D MOT quadrupole field gradient to maximise the loading rate of the 3D MOT, which is observed by measuring the fluorescence of the 3D MOT during the “2D+3D loading” period in fig. 5.2. We load both MOTs for 1.75 s and measure the fluorescence as a function of time for a range of 2D MOT coil currents. Figure 5.5a shows some of the results of this process (circles) for various 2D MOT coil currents. These curves are described very well (solid lines)

by a simple exponential approach to equilibrium. One can see that the loading rate increases between currents of 0.3 A to 0.9 A and then decreases as the current is increased further to 1.1 A. Figure 5.5b shows the measured loading rate. We find that a coil current of 0.9 A gives both the fastest loading rate of 2.2×10^{10} atoms/s and the largest number of atoms, which is 3.4×10^9 .

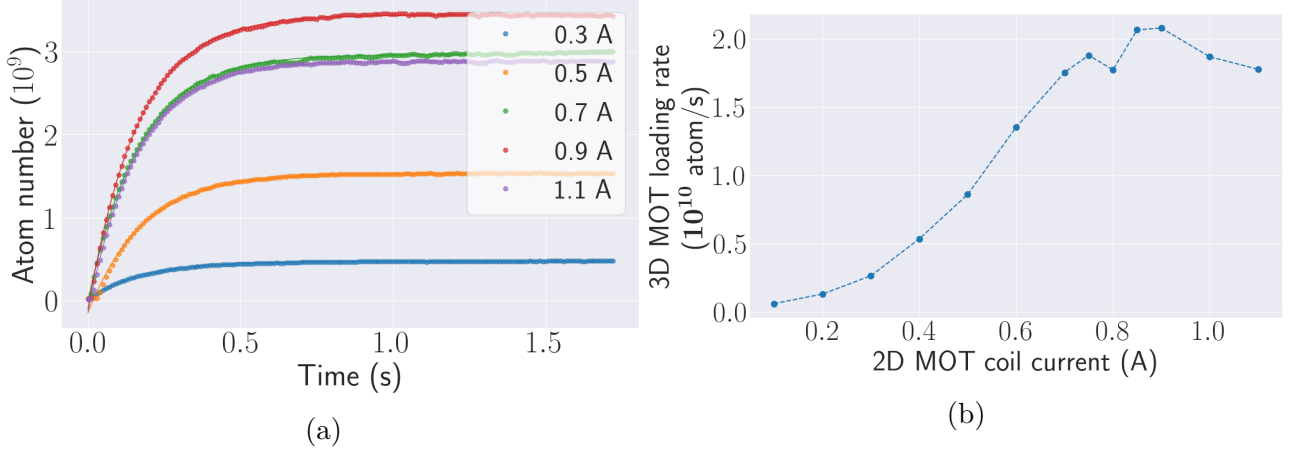


Figure 5.5: (a) Number of atoms in the 3D MOT versus time for a range of 2D MOT coil currents. The measured data (circles) and exponential fits (solid) are both shown. (b) 3D MOT loading rate as a function of 2D MOT coil current.

Dispenser current

Atoms are loaded into the 2D MOT from a background vapour of Rb provided by a dispenser. The amount of rubidium released by the dispenser, and therefore the loading rate of the 3D MOT, is controlled by the current running through it. Due to the compact setup of the 2D MOT there is no access point to allow for optics to measure the number of atoms. Instead we measure the number of atoms in the 3D MOT as a proxy for the loading efficacy of the 2D MOT. Figure 5.6a shows how the number of atoms captured by the 3D MOT varies as a function of time for different dispenser currents. The exponential fits are also shown (solid). In fig. 5.6b I plot the loading rates for the curves in fig. 5.6a. At the optimal dispenser current of 2.9 A, we find a loading rate of 5×10^{10} atoms/s. For higher currents, collisions between atoms in the 2D MOT and the background vapour reduce the 2D MOT flux. Also shown in fig. 5.6b are the loading rates without the use of the push beam in the 2D MOT. At the optimal dispenser current the difference in loading rate between using the push beam and not using it is over a factor of 6, which is a significant advantage.

After the optimisation steps detailed above, the 3D MOT is able to load $> 10^9$ atoms in 100 ms.

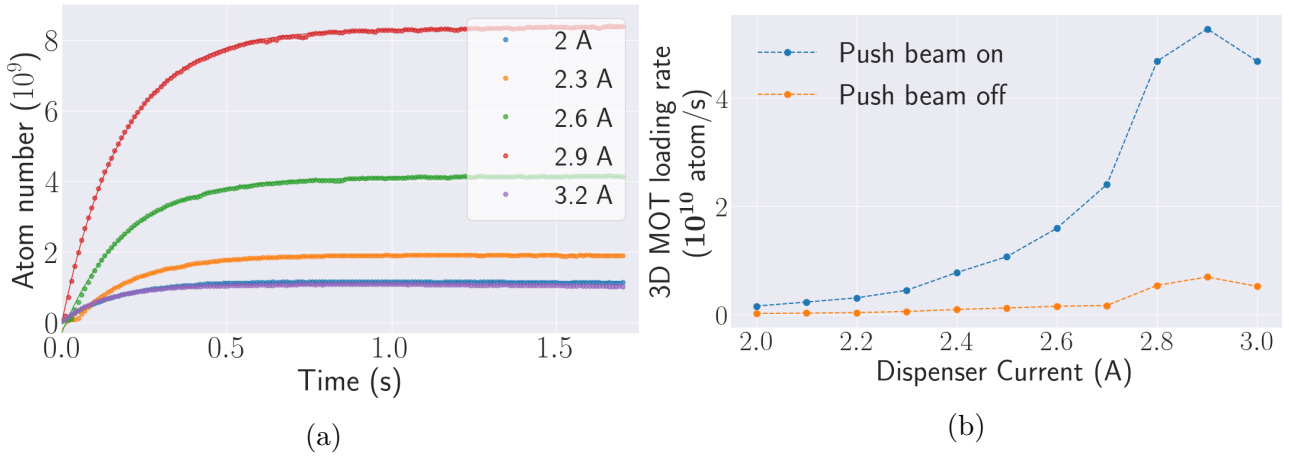


Figure 5.6: (a) Number of atoms in the 3D MOT versus time for a range of dispenser currents. (b) 3D MOT loading rates as a function of dispenser current. Shown are the loading rates with (blue) and without (orange) the push beam applied.

5.1.2 Magnetic field ramp

Having trapped and cooled approximately 10^9 atoms within the 3D MOT, we turn the quadrupole field off and cool the atoms further through an optical molasses. In order to allow Sisyphus cooling to reach the lowest temperature, we require the magnetic field to be low so that any Zeeman splitting is small. With a simple switch-off of the current, the field has a decay time of 13.7 ms, and it takes more than 31 ms before the field drops to 10%. In order to shorten this time we apply a large negative current to the coils followed by ramp back up to zero. Figure 5.7 shows the “B-field off” sequence step from fig. 5.1. We experiment with the length of the ramp down, then subsequently holding the large negative current, and then ramp back to zero which are represented by Δt_1 , Δt_2 , and Δt_3 respectively in fig. 5.7. We measure the field using a flux

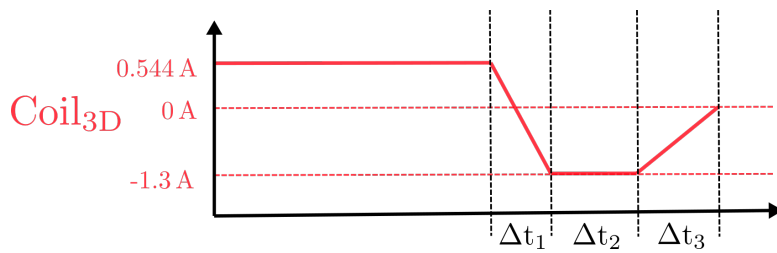


Figure 5.7: Sequence for the switching off the 3D MOT coil.

gate magnetometer with the probe positioned 13 cm below the bottom flange of the chamber. We vary the durations in fig. 5.7 whilst recording the field as a function of time.

Figure 5.8 shows how the magnetic field switch off is affected by Δt_1 , Δt_2 , and Δt_3 . Configuration 1 (blue) holds the negative current for too long (i.e. Δt_2 is too long). The signal quickly decays but undershoots the target value resulting in a negative field. Configuration 3 shows a situation where the negative current was not applied for long enough and a slow ramping of

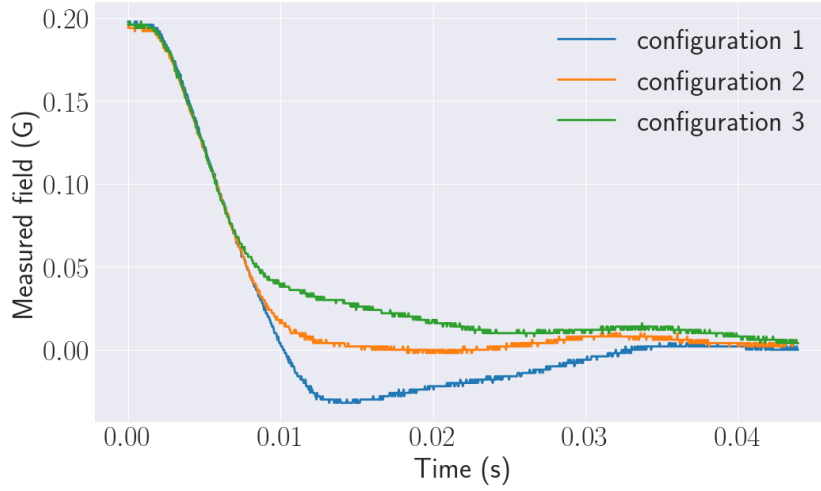


Figure 5.8: Magnetic field, measured outside of the vacuum chamber, as a function of time for three different configurations of Δt_1 , Δt_2 , and Δt_3 (defined in fig. 5.7).

the field down to zero still occurs. Finally, configuration 2 shows the case where the balance is just right resulting in a quick decay to zero. Guided by these plots we chose a Δt_1 of 0.1 ms ramping down to -1.3 A, then $\Delta t_2 = 2.2$ ms and finally $\Delta t_3 = 7.2$ ms. This brought the field down to 10% in only 10 ms, after which it remained smaller than that.

5.1.3 Temperature of 3D MOT

To measure the temperature of the 3D MOT I switch off both the 3D MOT beams and the magnetic field and allow the cloud of atoms to expand freely for some time. After that I flash on the MOT beams for 0.3 ms and record an image using a CCD camera. I then integrate the image along the y axis of the camera and fit a Gaussian of the form

$$c(x) = c_{\text{off}} + c_0 e^{-\frac{(x-x_0)^2}{2\sigma_x^2}}, \quad (5.1)$$

and similarly I integrate along the x axis of the camera and fit a Gaussian of the form

$$c(y) = c_{\text{off}} + c_0 e^{-\frac{(y-y_0)^2}{2\sigma_y^2}}. \quad (5.2)$$

Here $c(x)$ is the pixel counts at the horizontal position x of the image, $c(y)$ is the pixel count at the vertical position y of the image, c_{off} is the background pixel counts, c_0 is the amplitude, x_0 and y_0 are the horizontal and vertical position of the centre of mass of the cloud respectively, and σ_x and σ_y are the widths of the cloud along the x and y axes of the camera respectively.

The temperature is inferred from the change in cloud size with time after the MOT is switched

off though the relation [?],

$$\sigma^2(t) = \sigma^2(0) + \frac{k_B T}{m} t^2. \quad (5.3)$$

Here, k_B is the Boltzmann constant, T is the temperature of the cloud, m is the atomic mass of ^{87}Rb , and t is the time since the trapping beams and magnetic field were switched off.

We have imaged the atoms at times ranging from 11 ms to 30 ms after releasing them from the 3D MOT in increments of 1 ms. Figure 5.9 shows the fits of eq. (5.3) (dashed) to the fitted Gaussian widths of the atomic cloud (circles). The data and fit for the x axis of the camera is shown in blue whilst the y axis is shown in orange. The linear fits yield temperatures of:

$$T_x = 231 \pm 2 \mu\text{K} \quad (5.4)$$

$$T_y = 198 \pm 9 \mu\text{K}. \quad (5.5)$$

The poor fit along the y axis could be due to the atoms falling in gravity and experiencing a non-uniform magnetic field. This could cause the atomic cloud to expand inhomogeneously along its trajectory during imaging.

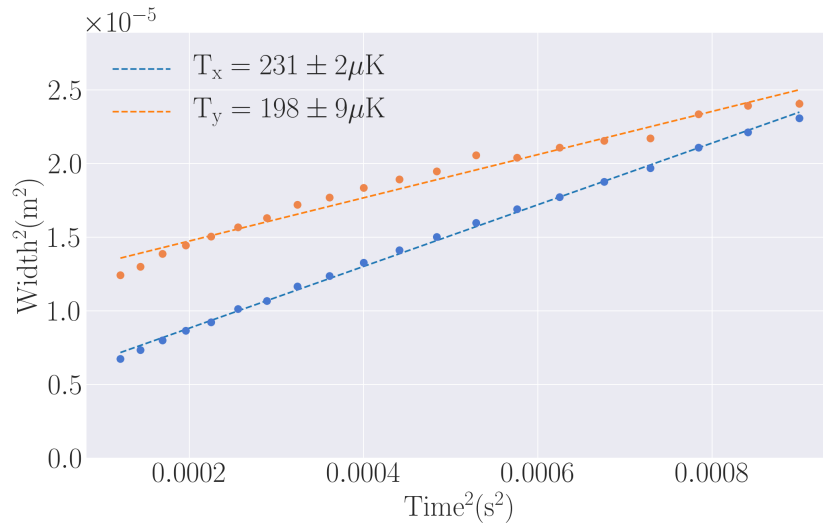


Figure 5.9: The squared cloud width as a function of the time since the atoms were released from the 3D MOT squared (circles) with the fits to the expansion of the cloud overlayed (dashed). The x axis data is shown in blue and the y axis data is shown in orange.

At this stage the thermal velocity distribution causes much more Doppler broadening of the Raman transitions than we wish to have for atom interferometry. We therefore further cool the atomic cloud using an optical molasses.

5.2 Molasses

We further cool the atomic cloud using Sisyphus cooling, which we refer to as the molasses stage. This cooling mechanism, which relies on a differential scattering rate between opposing beams that is a result of a changing local polarisation, leads to a final temperature $T \propto \Omega^2/|\delta|$ [?]. This allows temperatures much below the Doppler limit of $\sim 140 \mu\text{K}$ to be reached with an appropriate ramp of the detuning (δ) of the MOT beams followed by a ramp of the power (Ω^2). Starting at 10 ms into the switch-off of the magnetic field, we ramp the detuning of the cooling light by sweeping the frequency of the EOM in the Rb vapour cell lock loop (fig. 3.2). The repump light follows this, but we use the double-pass AOM (fig. 3.2) to compensate so that it stays resonant with the $|F = 1\rangle \longleftrightarrow |F' = 2\rangle$ transition.

Next, we ramp the power in each MOT beam from 23 mW to 0 mW over 0.2 ms. This ramp is linear for both the cooling light and repump light (though a slower ramp at the end might give a slightly lower temperature). Between the two ramps we allow 0.8 ms for the commands to be completed by the laser control system. Figure 5.10 summarises the sequence during this molasses ramp phase.

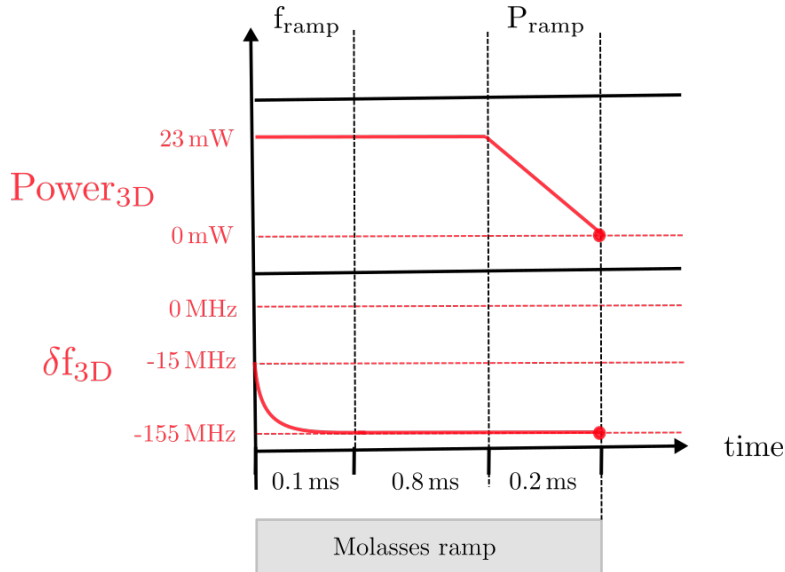


Figure 5.10: Sequence diagram for optical molasses.

I measure the temperature of the atom cloud after molasses using the same ballistic expansion method described in §5.1.3. Figure 5.11 shows the cloud size at times between 22 ms and 41 ms after the molasses is switched off. From eq. (5.3) we determine a molasses temperature of,

$$T_x = 7.8 \pm 0.4 \mu\text{K} \quad (5.6)$$

$$T_y = 7.5 \pm 0.8 \mu\text{K}. \quad (5.7)$$

I note that the initial size of the cloud is much smaller in fig. 5.11 than it was in fig. 5.9. That is because the alignment of the MOT beams was improved between the two measurements, resulting in a more compact MOT cloud. As will be discussed later, these final temperatures allow us to select a sufficiently large number of atoms to get us close to the theoretical quantum projection noise limit of the detection system. Thus these final temperatures are low enough for atom interferometry to work well.

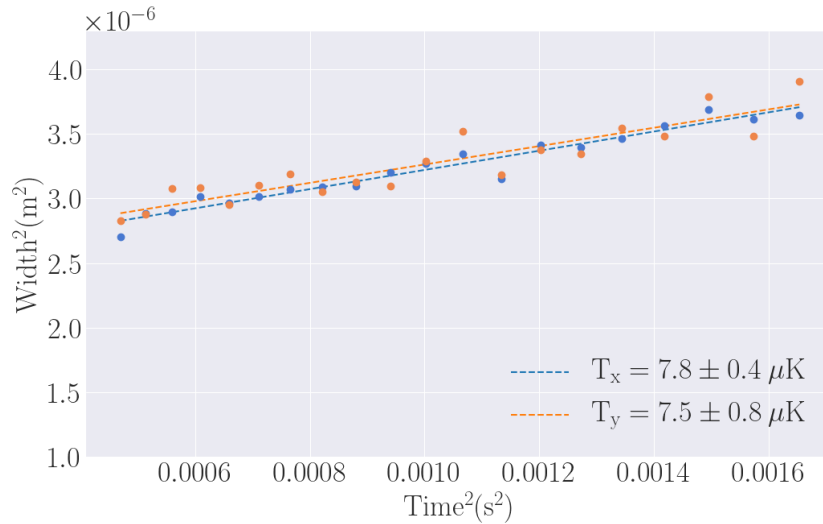


Figure 5.11: Ballistic expansion of the cloud after cooling in optical molasses. Circles: squared cloud width as a function of squared time since the atoms were released from the molasses. Dashed lines: fits to the expansion of the cloud. The x axis data is shown in blue and the y axis data is shown in orange.

5.3 Launching

5.3.1 Horizontal launch

As described in §2.3.4, we must launch the atoms along the Raman axis in order to lift the degeneracy between the two pairs of beams that can drive Raman transitions, but which impart opposite recoil momentum. In order to select an appropriate horizontal launch velocity I consider the velocity distribution of the atomic cloud which is given by the Maxwell-Boltzmann distribution

$$P_{MB}(v, v_R, T) = \sqrt{\frac{m}{2\pi k_B T}} e^{-\frac{m(v-v_R)^2}{2k_B T}}. \quad (5.8)$$

Here v is the component of an atom's velocity along the Raman axis, v_R is the same component of the cloud's centre of mass velocity, T is the temperature of the cloud, and m is the atom's mass. Figure 5.12a shows eq. (5.8) for an $8\mu\text{K}$ cloud with $v_R = 0$, plotted as a function of Doppler shift, $k_{\text{eff}}v$ of the Raman resonance. The full width at half maximum is approximately

160 kHz. By launching the cloud at 8 cm/s along the Raman axis, the beatnote frequency required to fulfil the Raman resonance condition[†] is shifted by +205 kHz for one counter-propagating beam pair, and -205 kHz for the other counter-propagating beam pair due to their oppositely signed k_{eff} . This situation is illustrated in fig. 5.12b where the launched atomic cloud sees a 205 kHz upward shift of the beatnote frequency between Raman beams 1 and 4. Conversely, the beatnote between beams 2 and 3 has been shifted down. The shift between the two resonance conditions is 410 kHz which is more the the width of the velocity distribution in fig. 5.12a. By adjusting the beatnote of the Raman beams by 205 kHz, only a single pair of counter-propagating beams can address the atoms. The other beam pair is now 410 kHz detuned from the launched atomic cloud.

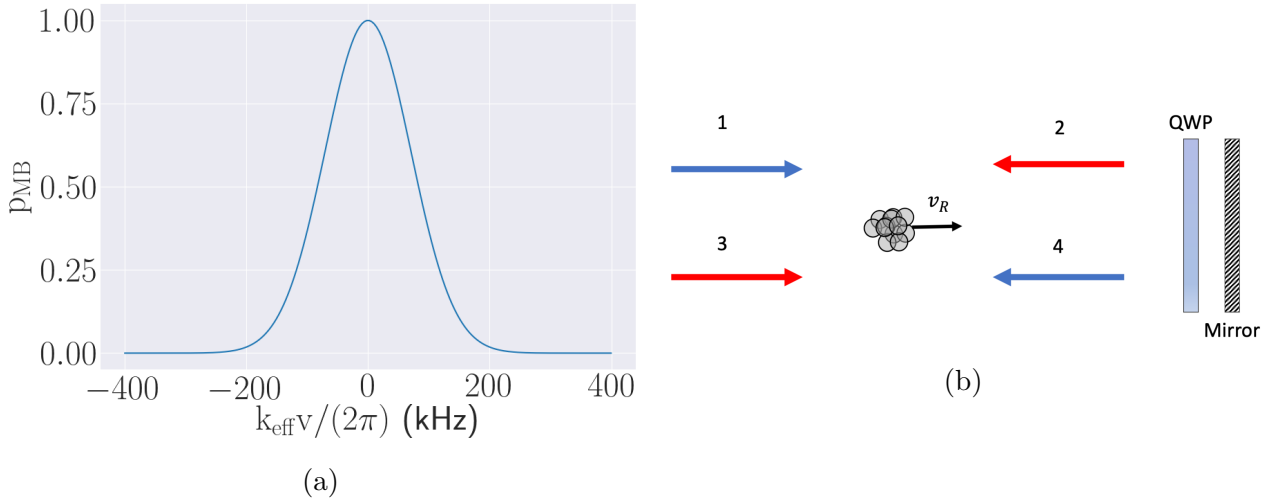


Figure 5.12: (a) Maxwell-Boltzmann distribution of a $8\mu\text{K}$ cloud plotted as a function of Doppler shift. (b) Schematic of the Raman beam retro-reflection configuration along with a launched atomic cloud.

5.3.2 Launch configurations

Figure 5.13 shows the orientation of the 4 equatorial MOT beams and the two Raman beams inside the chamber. Because the MOT beams are rotated 22.5° from the Raman beams, the frequency detunings required to launch the atoms with velocity $v_R^{(X)}$ along the X Raman axis are,

$$\delta f_{X\pm} = \pm \frac{v_R^{(X)}}{\lambda} \cos(22.5^\circ) \quad (5.9)$$

$$\delta f_{Y\pm} = \pm \frac{v_R^{(X)}}{\lambda} \sin(22.5^\circ). \quad (5.10)$$

[†]The Raman resonance condition is $\delta = \omega_{\text{eff}} - (\omega_0 + k_{\text{eff}}v_R + \frac{\hbar k_{\text{eff}}^2}{2m}) = 0$

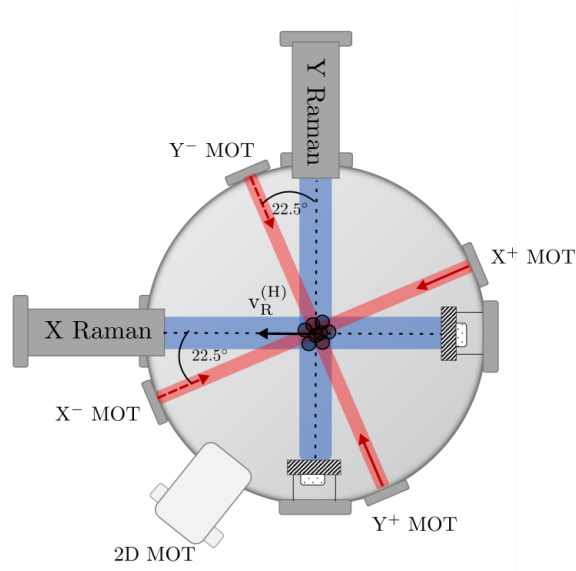


Figure 5.13: Configuration of the MOT beams relative to the Raman beams as viewed from above the chamber. The solid arrows show beams that have their frequency increased during the launching of the atoms, whilst dashed arrows show decreasing frequency. This schematic represents the setup for horizontal launching along the X Raman axis.

To achieve a launch velocity of $v_R^{(X)} = 8 \text{ cm/s}$, we need to increase the frequency of the X^+ MOT beam by 94.8 kHz and decrease the frequency of the X^- by the same amount. Similarly for Y^+ we need to increase the frequency by 39.2 kHz and decrease Y^- by the same amount.

For the vertical launch the situation is simplified because there is no angle between the Z MOT beams and the direction of launch. Hence a vertical launch at upward velocity $v_R^{(Z)}$ requires

$$\delta f_{Z\pm} = \pm \frac{v_R^{(Z)}}{\lambda}, \quad (5.11)$$

with Z^+ being the upward beam. Thus frequency shifts of $\pm 314 \text{ kHz}$ achieve the 24.5 cm/s vertical velocity described in §4.1.1. The configurations for launching along the two Raman axes are summarised in table 5.1.

MOT beam	X Raman	Y Raman
X^+	+94.8 kHz	-39.2 kHz
X^-	-94.8 kHz	+39.2 kHz
Y^+	+39.2 kHz	-94.8 kHz
Y^-	-39.2 kHz	+94.8 kHz
Z^+	+314 kHz	+314 kHz
Z^-	-314 kHz	-314 kHz

Table 5.1: Frequency shift required for launching at 8 cm/s along the two Raman axes and 24.5 cm/s vertically.

The sequence, including launching, is shown in fig. 5.14. We launch the atoms before cooling

them because the viscosity of the molasses is very low at the end point of the cooling sequence. We start the launching ramp 10 ms after the start of the magnetic field switch-off, and the molasses is fully up to speed in 1 ms. Following this, as before, the frequency of each molasses beam is swept down by 140 MHz over 0.1 ms, as indicated schematically in the figure. Finally, after a 0.8 ms delay to re-programme the laser system, the intensity of all the beams is swept to zero over 0.2 ms.

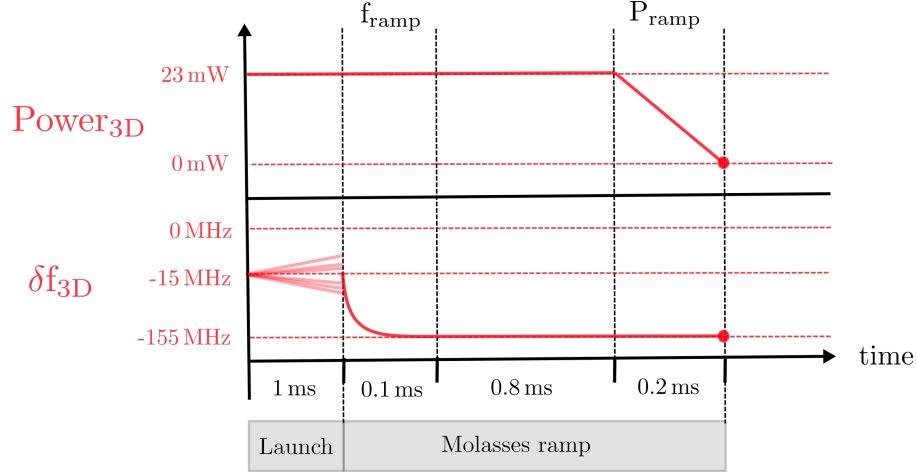


Figure 5.14: Atom launch and molasses ramp sequence. Not to scale.

5.3.3 Velocity & temperature after launch

Figure 5.15a shows the width of the atomic cloud after cooling with molasses launched at 8 cm/s along the X Raman axis and 24.5 cm/s vertically. Here the x axis of the camera is virtually identical to the X Raman axis, and the camera y axis is essentially vertical. The ballistic expansion gives temperatures of

$$T_x = 8.5 \pm 0.3 \mu\text{K}, \quad (5.12)$$

$$T_y = 10.2 \pm 0.3 \mu\text{K}. \quad (5.13)$$

We observe a possible increase in T_x of $0.7 \pm 0.5 \mu\text{K}$ compared with the $7.8 \mu\text{K}$ in fig. 5.11, and a larger increase of $2.7 \pm 0.9 \mu\text{K}$ in T_y . This suggests that the frequency ramps are not as smooth as they could be[†], but that has yet to be investigated. Similarly, fig. 5.15c shows the expansions

[†]Smooth frequency ramps allow all atoms to adiabatically follow the change in velocity required to maintain zero-velocity in their moving frame of reference. Too fast and not all atoms can follow the change in frequency and thus leads to effective heating of the cloud.

after launching vertically and along the Y Raman axis, which yields the temperatures

$$T_x = 8.6 \pm 0.3 \mu\text{K}, \quad (5.14)$$

$$T_y = 11.0 \pm 0.4 \mu\text{K}. \quad (5.15)$$

For the launch along the X Raman axis, fig. 5.15b shows the mean position of the cloud (circles) as a function of time since the molasses was switched off, as measured along the x (blue) and y (orange) axes of the CCD camera. Figure 5.15d shows the equivalent data for the launch along the Y Raman axis. The launch velocities are measured by fitting,

$$x_0(t) = x_0(0) + v_R^{(j)}t + \frac{1}{2}a_R^{(j)}t^2, \quad (5.16)$$

$$y_0(t) = y_0(0) + v^{(Z)}t + \frac{1}{2}a^{(Z)}t^2, \quad (5.17)$$

to the atomic cloud trajectories. Here, $x_0(0)$ and $y_0(0)$ are the vertical and horizontal positions of the cloud at time $t = 0$, $v_R^{(j)}$ and $a_R^{(j)}$ are the horizontal components of velocity and acceleration along Raman axis j , and $a^{(Z)}$ is the vertical component of acceleration. After fitting the data in fig. 5.15b to eqs. (5.16) and (5.17), I obtain launch velocities,

$$v_R^{(X)} = 7.9 \pm 0.3 \text{ cm/s}, \quad (5.18)$$

$$v^{(Z)} = 25.0 \pm 0.9 \text{ cm/s}, \quad (5.19)$$

and a horizontal acceleration of

$$a_R^{(X)} = 0.32 \pm 0.04 \text{ m/s}^2. \quad (5.20)$$

The vertical acceleration of -9.81 m/s^2 was used to calibrate the magnification of the imaging system. The fitted eqs. (5.16) and (5.17) are shown in fig. 5.15b as dashed lines in blue and orange respectively. Both measured launch velocities are consistent with the intended velocities of 8 cm/s and 24.5 cm/s . The horizontal acceleration differs from zero but this is surely explained by a 1.8° rotation between the vertical direction and the y axis of the camera. After fitting the data in fig. 5.15d to eqs. (5.16) and (5.17), I obtain launch velocities along the Y Raman axis,

$$v_R^{(Y)} = 8.2 \pm 0.2 \text{ cm/s}, \quad (5.21)$$

$$v^{(Z)} = 25.1 \pm 0.6 \text{ cm/s}, \quad (5.22)$$

and a horizontal acceleration of

$$a_R^{(Y)} = -0.33 \pm 0.04 \text{ m/s}^2. \quad (5.23)$$

Again, both measured launch velocities agree with the target velocities and the horizontal acceleration is consistent with a very small camera tilt.

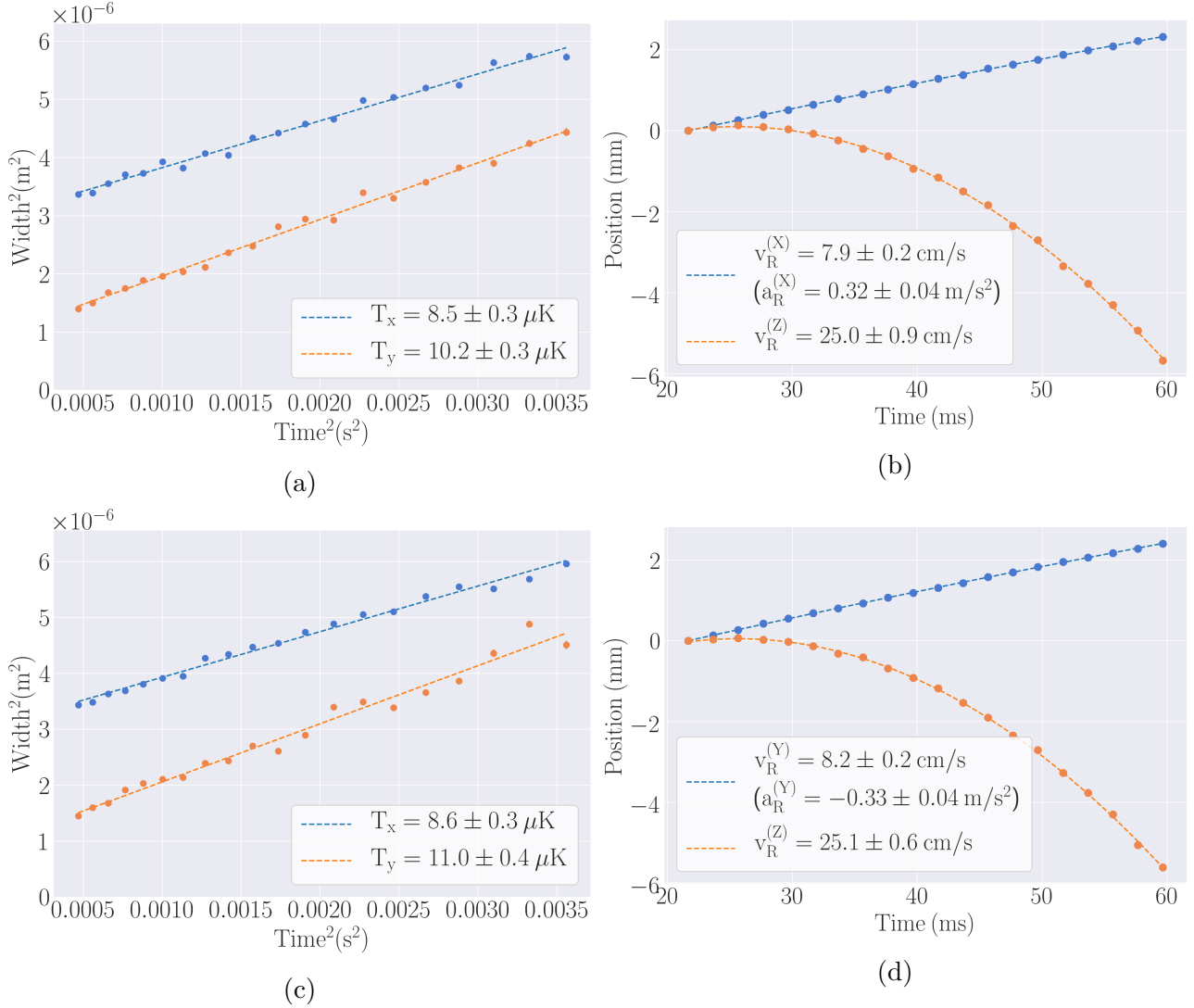


Figure 5.15: Squared cloud width as a function of the time since the atoms were released from the molasses (circles) with fits overlayed (dashed) for launching along the (a) X Raman axis and (c) Y Raman axis. Position of the cloud as a function of time since the atoms were released from the molasses (circles) with fits overlayed (dashed) for launching along the (b) X Raman axis and (d) Y Raman axis. In all four graphs, data for the X and Y axis of the camera are shown as blue and orange respectively.

After launching and cooling the atomic cloud in an optical molasses, we have approximately 10^8 atoms ready for state preparation.

5.4 State Preparation

The sequence of steps that we call state preparation has two goals. (i) We need to prepare all the atoms in the same starting state, $|F = 1, m_F = 0\rangle$ and (ii) We need to narrow the velocity spread along the Raman beam so that the π and $\pi/2$ pulses have the desired effect on all the atoms. These are both important for achieving good fringe visibility. Each step of our state preparation sequence is shown in fig. 5.16. The distinct coloured backgrounds each represent a different configuration of the laser system where blue is configuration I, orange is configuration II, and green is configuration III. In the first part of state preparation we apply a 0.5 G magnetic field along Z and use the two Z MOT beams in laser configuration III to optically pump atoms from $|F = 2\rangle$ into $|F = 1\rangle$ via $|F' = 2\rangle$. Both beams are right-hand polarised, making one of them σ^+ and the other σ^- . To ensure we do not recycle atoms out of $|F = 1\rangle$ during this process we switch the repump beam off. In step 2 we switch on the repump beam and switch off the cooling beam, to transfer atoms from $|F = 1, m_F = \pm 1\rangle$ to $|F = 1, m_F = 0\rangle$ via $|F' = 0, m_F = 0\rangle$. These first two steps move all the population into the $|F = 1, m_F = 0\rangle$ state. After this we rotate the magnetic field to align with the axis of the microwave horns. For step 3 we apply a microwave pulse that transfers atoms from $|F = 1, m_F = 0\rangle \rightarrow |F = 2, m_F = 0\rangle$. The polarisation of the microwave field is not well defined, but this transition is well separated from the others by the Zeeman shift. Following this we tune the laser system to configuration I, whilst also rotating the magnetic field to align with one of the two Raman axes. In step 4 we use the Raman beams to apply a long pulse to transfer a narrow velocity class of atoms back into $|F = 1, m_F = 0\rangle$. Finally, for step 5 the laser is tuned back to configuration II, and we use a resonant pulse to blow away the atoms remaining in $|F = 2\rangle$. This 5 step sequence allows us to prepare a narrow velocity class of atoms in $|F = 1, m_F = 0\rangle$. Imperfections in the various steps leads to either atoms prepared in the wrong magnetic sub-state or atoms in the correct state but the wrong velocity class. Both situations lead to atoms not taking part in the interferometer as they cannot be resonantly driven by the Raman beam. Thus they become background atoms which are still detected at the end of the sequence leading to a loss of contrast.

Figure 5.17 shows the sequence for the state preparation described above. In comparison to the previous sequence diagrams where I represented the frequency output of individual components such as δf_{3D} for the output of the 3D MOT beams, I now combine them all into the parameter δf_{output} . This parameter now represents the detuning of the system at various points in the sequence. When the line representing δf_{output} is red, this indicates that the output of the system during that time step is coming from the MOT beams, and when it is blue from the Raman beams. The three colours of the detuning values for δf_{output} , indicate the configuration of the laser system required to reach that detuning. Blue for configuration I, orange for configuration II, and green for configuration III as in fig. 5.16. To further emphasise when the laser configuration is changed, I have shaded the diagram with the colour representing

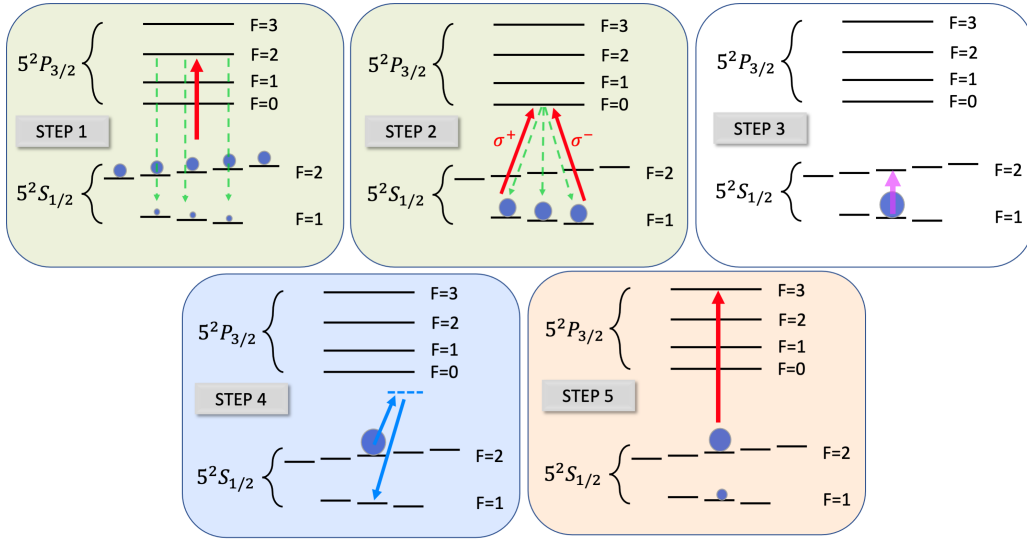


Figure 5.16: Energy level diagrams showing the process for each step of state preparation in sequential order from left to right and top to bottom. The coloured backgrounds each represent a unique configuration of the laser as shown in fig. 3.3. Blue corresponds to configuration I, orange is configuration II, and green is configuration III.

the laser configuration at that time step. For example, the time blocks ‘step 1’ and ‘step 2’ in the diagram are green to show that the laser is in configuration III. This is then followed by a wait of 3 ms in which the laser is releasing its frequency and phase lock, adjusting the frequency driving the EOM, and re-engaging both locks to go to configuration I for step 4 of state preparation.

The rest of this section will detail how we detected the state of the atoms, and then how we optimised this state preparation sequence.

5.4.1 Detection

Our state sensitive detection involves applying a resonant optical pulse to the atoms and measuring the induced fluorescence using the detection system described in §3.4. We first apply a magnetic field along the Z and use the Z MOT beams (without repump light) to drive the $|F = 2\rangle \longleftrightarrow |F' = 3\rangle$ transition. The fluorescence emitted by the atoms is captured by the photodiode and converted to a voltage V_2 . Next, atoms in $|F = 1\rangle$ are recycled into $|F = 2\rangle$ by turning on the repump beam which drives the $|F = 1\rangle \longrightarrow |F' = 2\rangle$ transition. Simultaneously, the $|F = 2\rangle \longrightarrow |F' = 3\rangle$ transitions is still being driven, so the voltage measured by the photodiode, V_{tot} , is now a representation of the total atom number. Then all atoms are blown away from the interaction region using the Z^+ and Y^+ MOT beams tuned resonantly to the $|F = 2\rangle \longleftrightarrow |F' = 3\rangle$ transition. The sequence is now repeated to get a measurement of the background signals B_2 and B_{tot} which we subtract from the raw voltages. When it comes to interferometry we are interested in the population of $|F = 2\rangle$ as a fraction of the total which

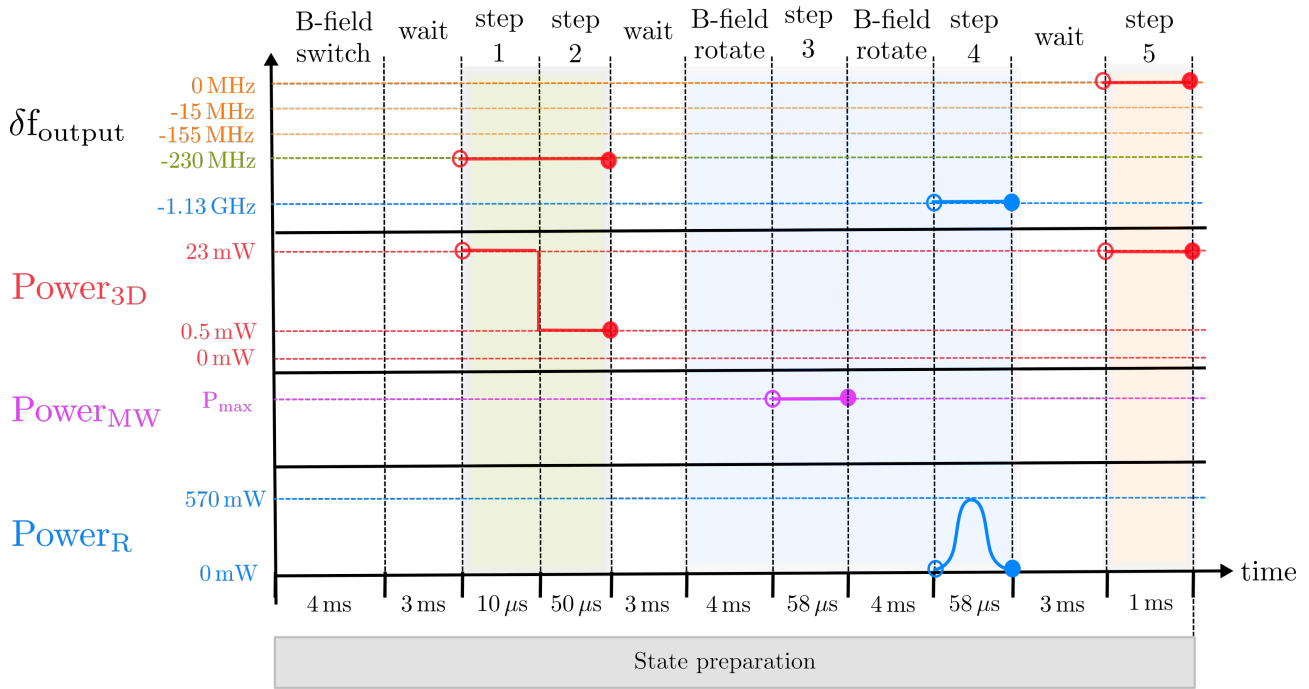


Figure 5.17: Sequence diagram up until the end of state preparation. In comparison to the previous sequence diagrams, δf_{output} represents the detuning from $|F' = 3\rangle$ of the light applied to the atoms at various times in the sequence. When the δf_{output} line is red, the light is being applied from the MOT beams, when the δf_{output} line is blue, the light is being applied by the Raman beams. In addition, the 3 colours, orange, green, and blue, of the detuning values correspond to which configuration of the laser system is needed to achieve that detuning. For example, to achieve a detuning of -15 MHz, the laser system must be in configuration II. To show when the laser is in the different configurations, the diagram's shaded regions represent when the laser is in configuration I, II, and III by blue, green and orange. Lastly, a hollow circle represents when a parameter's output is switched on, and a filled circle represents when the parameters output is switched off.

we construct using

$$P_2 = \frac{N_2}{N_{\text{tot}}} = \frac{V_2 - B_2}{V_{\text{tot}} - B_{\text{tot}}}. \quad (5.24)$$

Figure 5.18 depicts the power and frequency during the detection sequence. It should be noted that we drop the initial and final $\delta\tau = 100 \mu\text{s}$ from each detection sequence step to allow the voltage measured to stabilise due to transient behaviour of the detection light and/or the photodiode amplifier.

5.4.2 Step 1 & 2 - Optical pumping

At the end of the molasses, approximately 90% of atoms are distributed amongst the Zeeman magnetic sublevels of $F=2$. The remaining atoms are distributed amongst the sublevels of $F=1$. In the first step of our state preparation scheme we use the Z MOT beams to pump atoms from $F=2$ into $F=1$ via $F' = 2$. For any polarisation of the light, there are dark states in $F=2$ that

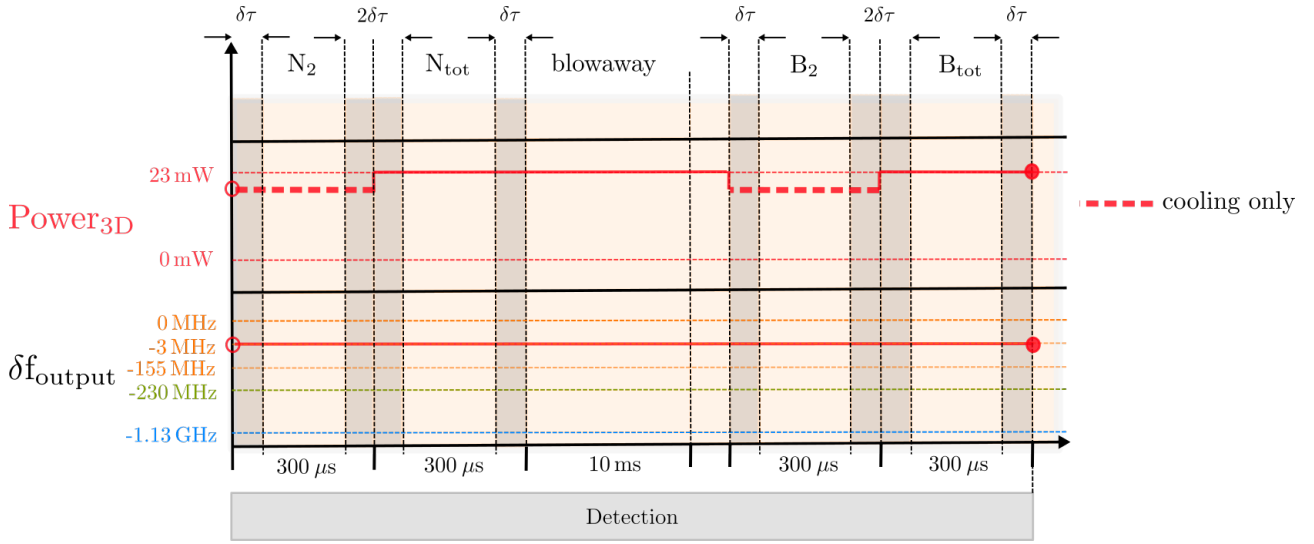


Figure 5.18: State sensitive detection. During detection of the atoms and background of state $|F = 2\rangle$ only the cooling beam is used to drive $|F = 2\rangle \longleftrightarrow |F' = 3\rangle$ transitions. During detection of the total number of atoms and the total background, both cooling and repump beams are used.

are not excited to $F' = 2$, but these evolve into bright states through the Larmor precession induced by the magnetic field that we apply along Z [?]. The time needed to pump atoms out of $F=2$ depends on the strength of the applied magnetic field. Figure 5.19a shows the fraction of atoms that we measure in $F=1$ as a function of the step 1 pulse length for three different value of the magnetic field along Z. One can see that as the field increases, the time taken to pump atoms out of $F=2$ decreases. With a field of 0.49 G, 99% of the atoms occupy $F=1$ after only $10 \mu\text{s}$.

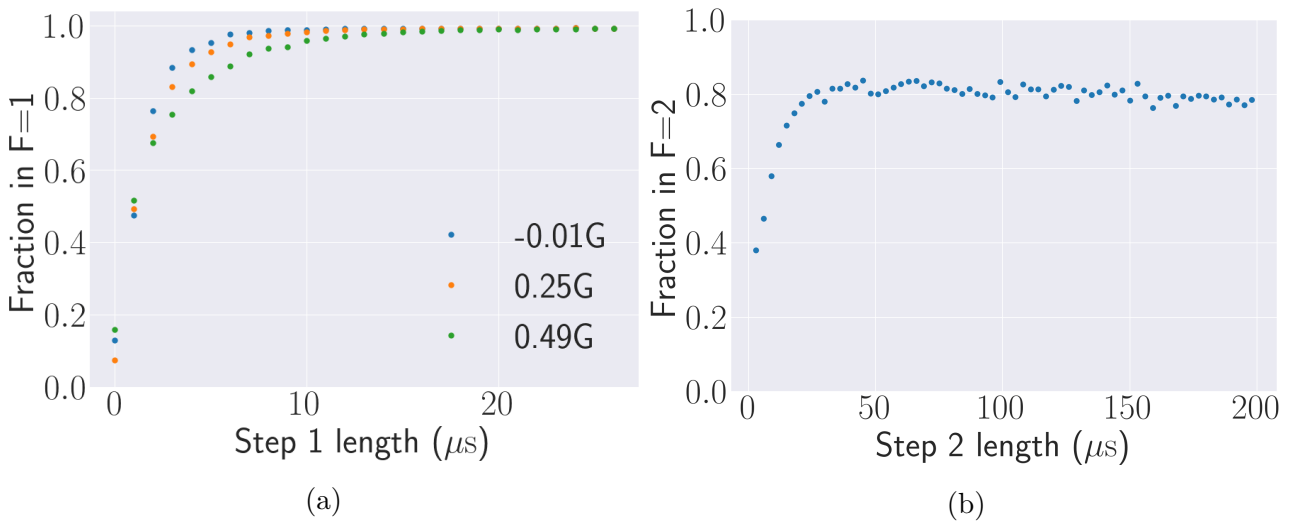


Figure 5.19: (a) Fraction of atoms in $F=1$ as a function of the step 1 pulse length. Shown are three different bias field settings - -0.01 G (blue), 0.25 G (orange) and 0.49 G (green). (b) Fraction of atoms in $F=2$ after step 3 as a function of the step 2 pulse length.

This optical pumping leaves the atoms distributed amongst the three Zeeman sublevels of $|F = 1\rangle$. In step 2 we pump atoms out of $|F = 1, m_F = \pm 1\rangle$ and into the dark state $|F = 1, m_F = 0\rangle$ via $|F' = 0, m_{F'} = 0\rangle$ using the repump light of the MOT Z beams. To determine how quickly this pumping happens, we apply step two for a range of times and after each of these times we transfer the $|F = 1, m_F = 0\rangle$ population to $|F = 2, m_F = 0\rangle$ using a resonant microwave pulse (step 3 described in §5.4.3). The fraction of atoms in $F=2$ after this transfer is plotted in fig. 5.19b, which shows that the pumping is complete after $50\mu\text{s}$. In order to determine how well states $|F = 1, m_F = \pm 1\rangle$ are pumped into state $|F = 1, m_F = 0\rangle$ by step 2, we make use of the microwave spectrum described in the next section.

5.4.3 The microwave transitions and step 3

For step 3 we use a microwave pulse to resonantly drive atoms in $|F = 1, m_F = 0\rangle$ to $|F = 2, m_F = 0\rangle$. To drive this $\Delta m_F = 0$ transition most effectively we rotate the magnetic field over 4 ms until it is parallel to the microwave field. This rotation rate, set by the relaxation of eddy currents, is very much slower than the Larmor frequency so the atoms remain in the same states.

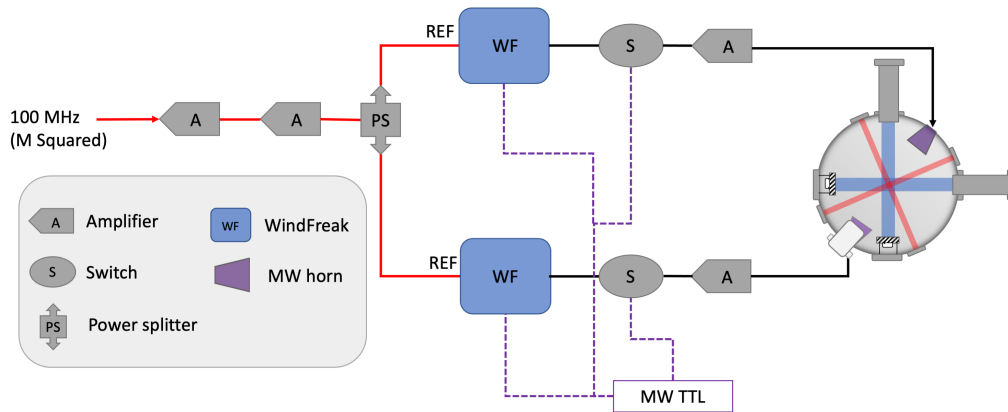


Figure 5.20: Microwave electronics along with the two horns mounted inside of the chamber.

Two Pasternack PE9859/SF-10 C band horns (see fig. 3.5) launch the microwaves into the chamber. Figure 5.20 shows the orientation of these horns inside the chamber, together with the radio frequency electronics used to control them. One horn is mounted to a flange below the equator and the other is above the equator on the flange directly opposite. Each horn is driven by a dedicated dual-channel WindFreak SynthHD microwave synthesizer, referenced to the same 100 MHz clock used by the phase lock of the Raman laser system. The raw clock signal is amplified by 22 dB before being split and supplied to each WindFreak. A digital line (not shown) controls the frequency of each synthesiser and TTL levels control the internal on/off switches. Because the interferometer would be compromised by even a low level of

microwave leakage, each synthesiser output is fitted with a second on/off switch in front of the final amplifier driving the horn.

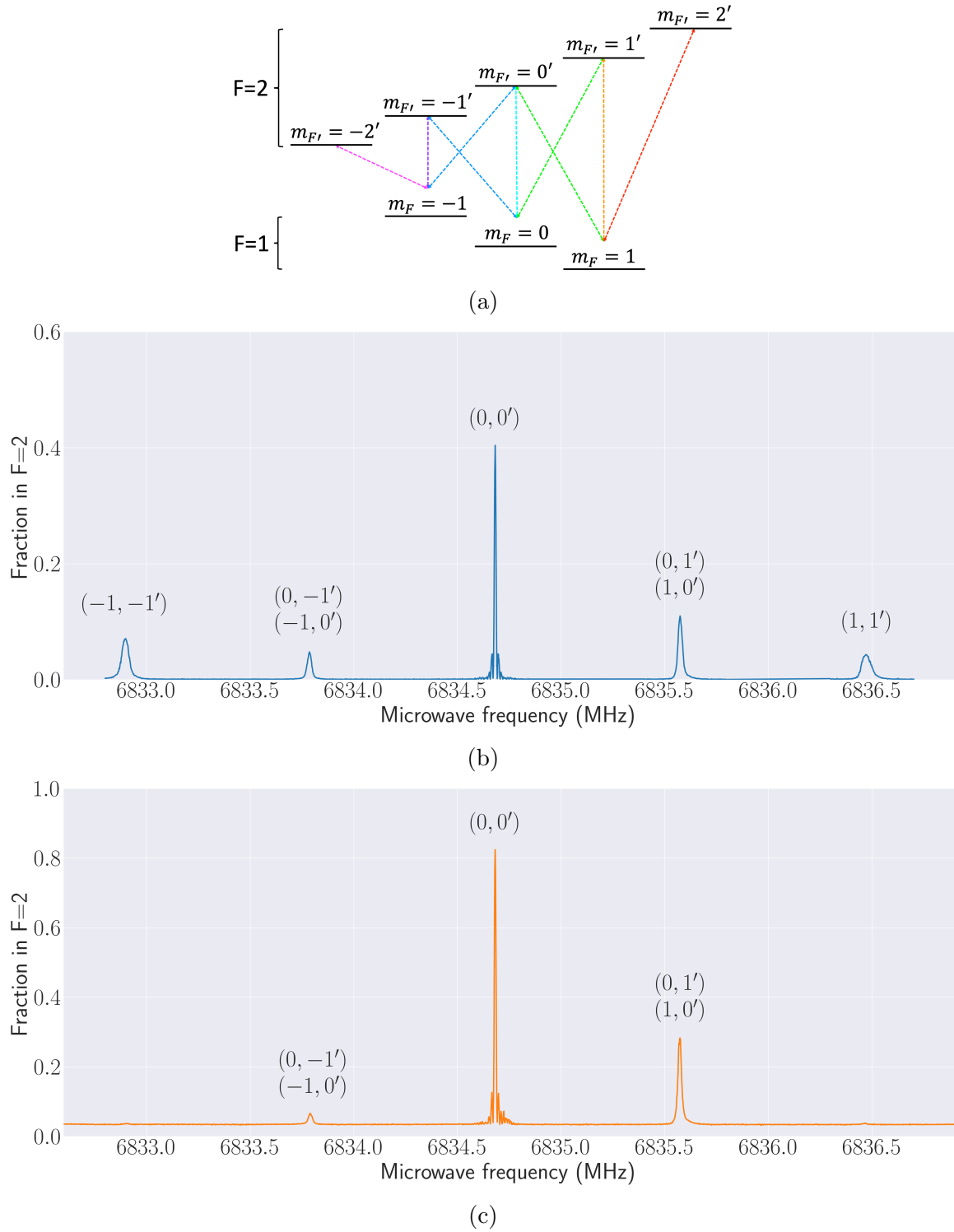


Figure 5.21: (a) Energy level diagram of available magnetic dipole transitions for the $5^2S_{1/2}$ energy level manifold in ^{87}Rb . Microwave spectrum after (b) step 1 and (c) step 2.

Figure 5.21a shows the relevant energy levels in the presence of the magnetic field. The Zeeman shifted frequencies required to resonantly address each of the allowed transitions are colour

coded. We are interested here in the $\Delta m_F = 0$ transitions, of which there are three. We observe these by applying the microwave field for $58 \mu\text{s}$ after the end of step 1, then measuring the $F=2$ fluorescence signal. On scanning the microwave frequency we obtain the spectrum shown in fig. 5.21b, where the labels (i, j') are a shorthand for $(F = 1, m_F = i) \longleftrightarrow (F = 2, m_{F'} = j')$. The $(0, 0')$ peak is the clock transition that we want to use. The presence of the $(-1, -1')$ and $(1, 1')$ peaks, shows that step 1 leaves population in the states $|F = 1, m_F = \pm 1\rangle$. The spectrum shown in fig. 5.21c is taken in the same way, but after the application of step 2. The disappearance of the $(-1, -1')$ and $(1, 1')$ peaks shows that step 2 has succeeded in pumping atoms that were in $|F = 1, m_F = \pm 1\rangle$ into $|F = 1, m_F = 0\rangle$. This is accompanied by a corresponding increase in the height of the clock transition peak. I note that a small (3.5%) background appears in the spectrum taken after step 2. That is due to off resonant pumping from $F=1$ to $F=2$ via the $F' = 1$ state in the $5P_{3/2}$ manifold, which lies only 72 MHz above the $F' = 0$ state (see fig. 2.5).

5.4.4 The Raman spectrum

Before moving on to step 3 (velocity selection) I look again at the hyperfine spectrum, this time obtained using Raman pulses. I take advantage of the fine resolution of a long Raman pulse to study various aspects of our system. I prepare the atoms in $|F = 1, m_F = 0\rangle$ using the first two steps of state preparation and blow away any atoms remaining in $F=2$. I then apply a Raman probe pulse to transfer atoms to $|F = 2, m_F = 0\rangle$ and measure the resulting fraction of atoms in $F=2$ using our detection sequence. By scanning the Raman beat-note frequency, ω_{eff} , I obtain the two spectra shown in fig. 5.22. The broad peaks on either side come from the velocity-sensitive transitions, driven by beams 1 & 2 and 3 & 4 in fig. 5.12b. These are the transitions we can use for atom interferometry. They are shifted from the clock transition by an angular frequency difference of $\hbar k_{\text{eff}}^2/(2m) \pm k_{\text{eff}} v_R$, where the first term is the recoil shift and the second is the Doppler shift. The narrow peak comes from the co-propagating beams 1 & 3 and 2 & 4 (fig. 5.12b) and has negligible recoil and Doppler shifts. This peak would not appear if the beams were perfectly circularly polarised, as intended. The duration of the Raman probe pulse was set to maximise the broad peaks. We find that we must apply a longer probe pulse along the Y Raman axis, which indicates a lower intensity in that beam.

To fit the Doppler-broadened line profiles, P_D , I use,

$$P_D = \alpha \int_{-\infty}^{+\infty} p_{\text{MB}}(v, v_R, T) P_2(\Omega_D, \delta(v, \omega_{\text{eff}}), \tau) dv. \quad (5.25)$$

Here α is the fraction of atoms prepared in $|F = 1, m_F = 0\rangle$ after step 2, p_{MB} is the Maxwell-Boltzmann distribution, and Ω_D is the Rabi frequency for the Doppler-broadened transition.

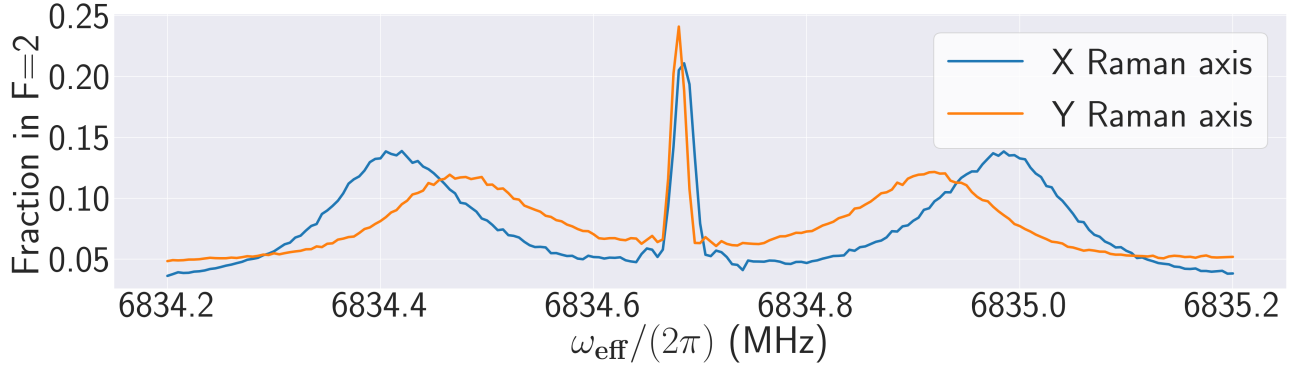


Figure 5.22: Raman spectra obtained by applying a $42\ \mu\text{s}$ probe pulse using the X Raman beam (blue) and a $58\ \mu\text{s}$ probe pulse using the Y Raman beam (orange).

As before, P_2 is given by eq. (2.21). To fit the narrow line profiles, P_N , I modify eq. (2.21),

$$P_N = \alpha P_2(\Omega_N, \delta(v=0, \omega_{\text{eff}}), \tau), \quad (5.26)$$

where Ω_N is the Rabi frequency for the narrow transition. I set $v=0$ here because the narrow line has no Doppler shift.

These line shapes have been fitted to the spectra shown in fig. 5.22 including a constant background B , with α set to 0.8 and τ set to the pulse duration used. The results of the fitting are plotted in fig. 5.23a for the X axis and in fig. 5.23b for the Y axis. We see that the line shapes describe the spectrum quite well, though some small systematic discrepancies are evident. For the present purposes we do not worry about these as the main goal is to check that the apparatus is working properly, not to make a high-precision measurement.

Parameter	X Raman	Y Raman
B	0.0447(5)	0.0532(4)
v_R	10.93(2) cm/s	8.35(3) cm/s
T	5.9(1) μK	6.9(2) μK
Ω_D	$2\pi \times 9.4(1)$ kHz	$2\pi \times 6.3(1)$ kHz
Ω_N	$2\pi \times 3.80(3)$ kHz	$2\pi \times 2.89(1)$ kHz
$\omega_0/(2\pi)$	6834.6840(1) MHz	6834.67964(9) MHz

Table 5.2: Fit parameters for the Raman spectra.

The fit parameters are listed in table 5.2. The backgrounds are both a few percent. The velocity along the X axis is a little higher than the 8 cm/s launch velocity. Given that the spectrum was taken 63 ms after launch, this would be consistent with a tilt of $\sim 2^\circ$ away from the horizontal. The tilt along Y is smaller. The temperatures are both slightly below the 8 μK measured by ballistic expansion (see figs. 5.15a and 5.15c). At the time of taking these data, there was some drift in the polarisation of the MOT beams which caused the temperature to vary from day to day, so we think this was a real difference. In the next iteration of this experiment we will be

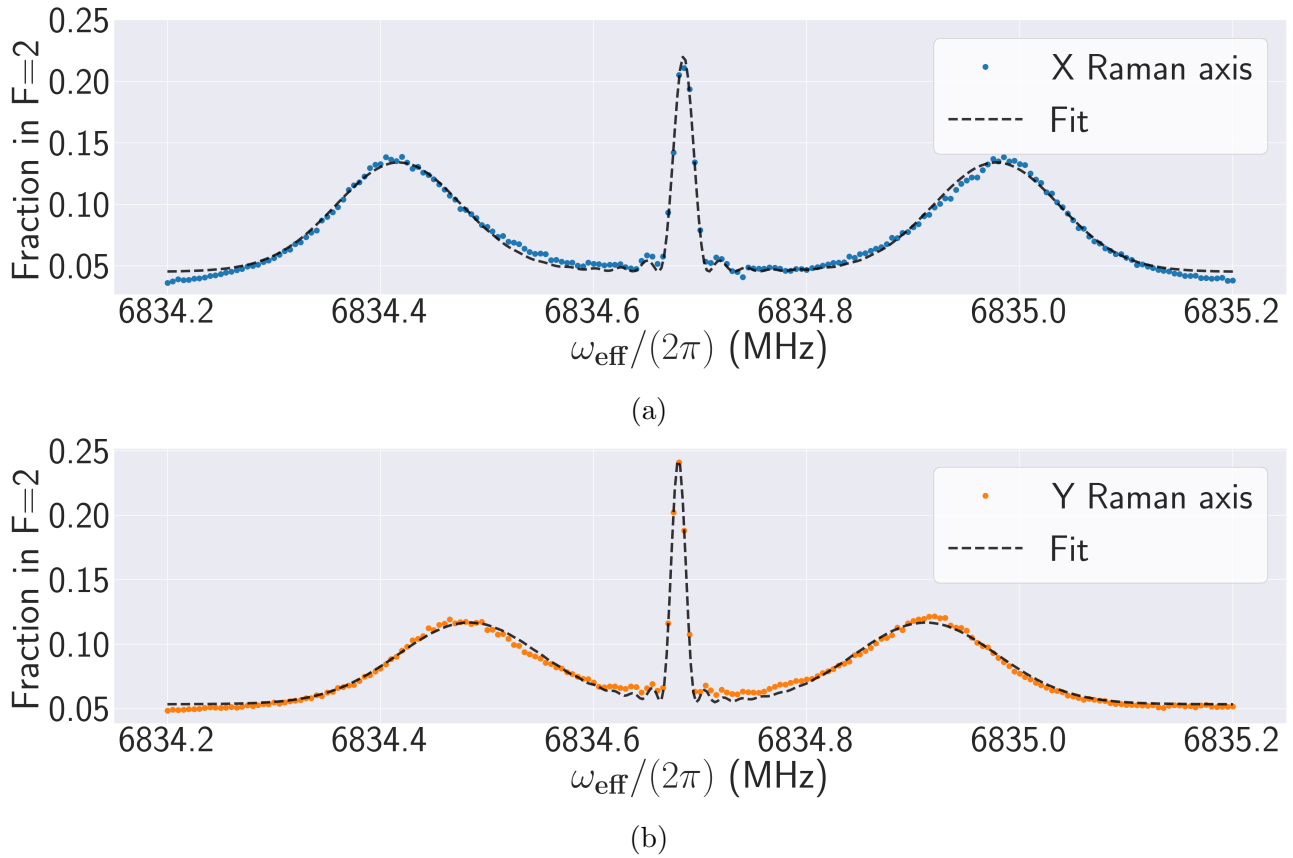


Figure 5.23: (a) Raman spectrum from fig. 5.22 for the X Raman axis (circles, blue) together with a fit to a linear combination of eqs. (5.25) and (5.26) (black, dashed). (b) Same as (a) but for the spectrum obtained on the Y Raman axis (circles, orange).

implementing linear polarisers in each of the MOT beams to remedy this. On the X axis, the value of Ω_D corresponds to a total intensity in the incident Raman beam of 8.8 mW/cm^2 at the site of the atoms and to an incident power of 242 mW at the fibre output. On the Y axis these are 5.8 mW/cm^2 and 161 mW . These are both lower than the 330 mW that I measured when first setting up the Raman fibres. I suspect that the efficiency of coupling into the fibres at the laser end has dropped from 80% on both fibres to 60% on X and 40% on Y. This cannot now be checked without breaking vacuum, but it certainly suggests the need to stabilise the input coupling in the future.

The ratio of Ω_N to Ω_D allows us to quantify the imperfect polarisation, ϵ , as follows. The electric field of the Raman beam at the location of the atoms has two frequency components, labelled as 1 and 3 in fig. 5.12b. Let us write them as $\mathcal{E}_1(\omega_1)(\sigma^+ + \epsilon\sigma^-)$ and $\mathcal{E}_3(\omega_2)(\sigma^- + \epsilon\sigma^+)$. In the ideal case $\epsilon = 0$, but the presence of the narrow peaks in fig. 5.22 suggest $\epsilon \neq 0$. I assume the same amount of ellipticity in both fields because this is presumably due to a misalignment between the Raman fibre axis and the QWP axis that converts the linear light to circular. After the retro-reflecting module the two fields have the polarisations reversed and are therefore $\mathcal{E}_2(\omega_1)(\sigma^- + \epsilon\sigma^+)$ and $\mathcal{E}_4(\omega_2)(\sigma^+ + \epsilon\sigma^-)$. The Rabi frequencies Ω_D of the two

Doppler-broadened lines come from the products $\mathcal{E}_1(\omega_1)\mathcal{E}_4(\omega_2)\sigma^+\sigma^+$ and $\mathcal{E}_2(\omega_1)\mathcal{E}_3(\omega_2)\sigma^-\sigma^-$ (neglecting the small ϵ^2 terms). These are equal in magnitude. By contrast, the Rabi frequency of the narrow peak has four contributions. There are two cross terms from the incident beams, $\mathcal{E}_1(\omega_1)\mathcal{E}_3(\omega_2)(\sigma^+\epsilon\sigma^+ + \epsilon\sigma^-\sigma^-)$, and two more from the reflected beams $\mathcal{E}_2(\omega_1)\mathcal{E}_4(\omega_2)(\sigma^-\epsilon\sigma^- + \epsilon\sigma^+\sigma^+)$. Because there is negligible loss in the retro-reflecting optics all four of these terms are equal and thus we can write[†] $\Omega_N = 4\epsilon\Omega_D$. For both Raman axes $\epsilon = \Omega_N/(4\Omega_D) \sim 0.1$ which corresponds to having 1% of the intensity in the unwanted polarisation. This may have been due to a systematic error in the initial setup of the fibre orientation, but it does not cause any problem for the atom interferometry and therefore does not need to be fixed.

The fits return values for the central frequency of 6 834 684.0(1) kHz along the X axis and 6 834 679.64(9) kHz along the Y axis. These are to be compared with the known value [?] of the clock transition which is 6 834 682.610 kHz. The 0.7 G magnetic field that was applied along the Raman axis will have produced an upward shift in the transition frequency of 0.282 kHz because of the Zeeman shift [?]. Hence the discrepancy is +1.1(1) kHz along the X axis and -3.3(1) kHz along Y. It is possible that these differences result from light shifts because the magic intensity ratio was only approximately set, but it is not particularly useful to pursue this here, given the small but evident differences between the fitting function and the actual spectrum.

5.4.5 Step 4 & 5 - Velocity selection & blowaway

After the first three state preparation steps most of the atoms are prepared in the state $|F = 2, m_F = 0\rangle$ with a temperature along the Raman beams of 6-8 μK . The blue dashed line in fig. 5.24 illustrates the velocity distribution given by eq. (5.8), expressed in terms of the Doppler shift of the Raman transition. The orange line in fig. 5.24 is the Raman resonance lineshape given by eq. (2.21) for a square π -pulse lasting 20 μs , roughly the shortest time for which there is enough power to make a π -pulse. One can see that even at 20 μs , only a small fraction of the thermal distribution is addressed by the pulse.

In order for the interferometer to work, we need to select a subset of the atoms within a comparable, or even narrower, width so that the contrast of the interferometer will not be too badly compromised by the velocity spread. We do this in step 4 by driving the atoms down from $F=2$ to $F=1$ using a Raman pulse with a Blackman intensity profile, given by

$$\Omega(t) = \Omega_{\text{eff}} \left(\frac{21}{50} + \frac{1}{2} \cos\left(\frac{2\pi}{\tau}t\right) + \frac{2}{25} \cos\left(\frac{4\pi}{\tau}t\right) \right), \quad (5.27)$$

[†]More generally, the two contributions interfere and $\Omega_N = 4\epsilon\Omega_D \cos(2\omega_{\text{eff}}L/c)$, where L is the optical path length between the atoms and the retro-reflecting mirror. In my experiment, the mirror placement was such that the two contributions interfered constructively.

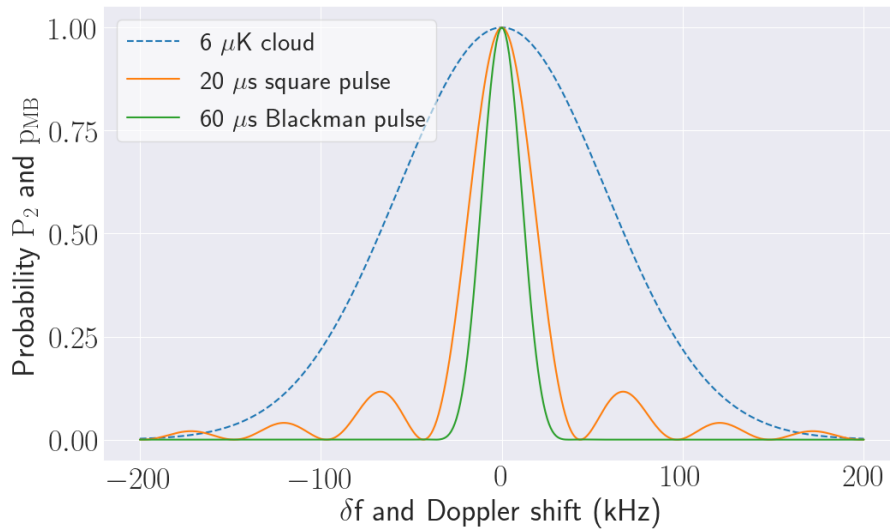


Figure 5.24: Probability distributions. Blue line: Thermal distribution of Doppler shifts for a $6\ \mu\text{K}$ cloud. Orange line: transition line shape for a square π -pulse lasting $20\ \mu\text{s}$. Green line: transition line shape for a Blackman pulse having $\tau = 60\ \mu\text{s}$ and peak Rabi frequency $2.23\ \pi/\tau$.

over the time interval $-\frac{1}{2}\tau \leq t \leq \frac{1}{2}\tau$ and zero outside that interval. The green line in fig. 5.24 shows the transition line shape for a Blackman pulse having $\tau = 60\ \mu\text{s}$ calculated by numerical integration of the Schrödinger equation. The peak Rabi frequency $2.23\ \pi/\tau$, is chosen to give a peak transition probability of 1. This pulse shape is used for selecting a narrow group of velocities because it does not have the side lobes seen in the orange line, which are characteristic of a square pulse. After applying the Blackman pulse in step 4, tuned to the centre of the velocity distribution, step 5 is to blow away the atoms that remain in the $F=2$ manifold using a push beam that is resonant with the cycling transition. This completes the state preparation.

The orange line in fig. 5.25 shows a Raman spectrum recorded after step 2, in the same way as fig. 5.23. The frequency is shown relative to the clock transition. The Doppler broadened line is well described by a Gaussian having a width of $184\ \text{kHz}$ (FWHM), corresponding to a temperature of $9.7\ \mu\text{K}$. After steps 4 and 5, the atoms, now in state $|F=1, m_F=0\rangle$, should have a much narrower velocity distribution. In order to demonstrate that, we probe their velocity distribution with a $45\ \mu\text{s}$ square pulse that drives them back up to $F=2$ and we measure the resulting $F=2$ population as a function of the probe laser frequency, as plotted by the green line in fig. 5.25. The peak at the clock frequency is unchanged because this transition is insensitive to the velocity, but the Doppler-shifted peak is both narrower and larger because the new velocity distribution is much narrower. The width of this peak is $31\ \text{kHz}$ (FWHM), corresponding to a “temperature” of $270\ \text{nK}$, which is narrow enough to proceed with interferometry.

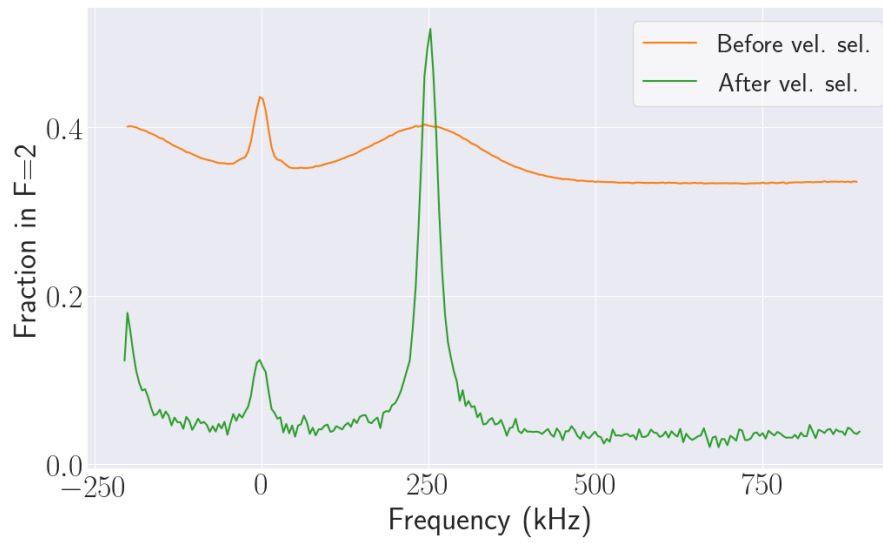


Figure 5.25: Raman spectra. The zero of frequency is the clock frequency. Orange line: spectrum (with a vertical offset) taken after step 2 with a $45\,\mu\text{s}$ square pulse having a Rabi frequency of $2\pi \times 10\,\text{kHz}$. Green line: spectrum taken after steps 4 and 5 using the same square pulse. The Blackman pulse in step 4 had a width of $85\,\mu\text{s}$ and the same peak Rabi frequency.

Chapter 6

Interferometry

After state preparation we now have a narrow velocity distribution of approximately 10^7 atoms ready for interferometry. In this chapter I describe the initial fringes obtained along each Raman axis, the subsequent optimisation to improve the fringe contrast, a comparison between the two axes, and finally measurement of the fringe noise distribution.

6.1 Fringe Optimisation

6.1.1 Initial fringes

To set the appropriate pulse areas we set the power of the Raman beams to be maximum and scan the pulse duration to find the π -time - when the Rabi oscillation reaches it's maximum. To set the $\pi/2$ pulses, we then lower the power until the Rabi oscillation reaches half of its maximum. In this way we have a $\pi/2 - \pi - \pi/2$ pulse sequence with pulses of equal duration. We then scan the fringe pattern by varying $\Phi_{\text{off}}^{(2)}$ - the phase of the middle pulse (see eq. (2.52)). Figure 6.1 shows an interferometer fringe obtained in this way for $2T = 1$ ms. This choice of short T makes the interferometer comparatively insensitive to accelerations, with a fringe period of 1.56 m/s^2 . Consequently it is not much affected by the vibration noise, which was measured in §4.4.2 to be 3 mm/s^2 along X and 2 mm/s^2 along Y. To fit the data I use,

$$P_2 = P_{\text{off}} - \frac{C}{2} \cos(-2\Phi_{\text{off}}^{(2)} + \Phi_0). \quad (6.1)$$

Here, P_{off} is the offset probability, C is the contrast, and Φ_0 is any additional interferometer phase. From fits of eq. (6.1) to X and Y fringes I measure contrasts of 0.146 ± 0.009 and 0.143 ± 0.005 respectively. Both contrasts are much less than the ideal of 1. In addition, neither

fringe exhibits a minimum at $\Phi^{(2)} = 0$. This is indicative of an acceleration of the atoms, as shown in eq. (2.52).

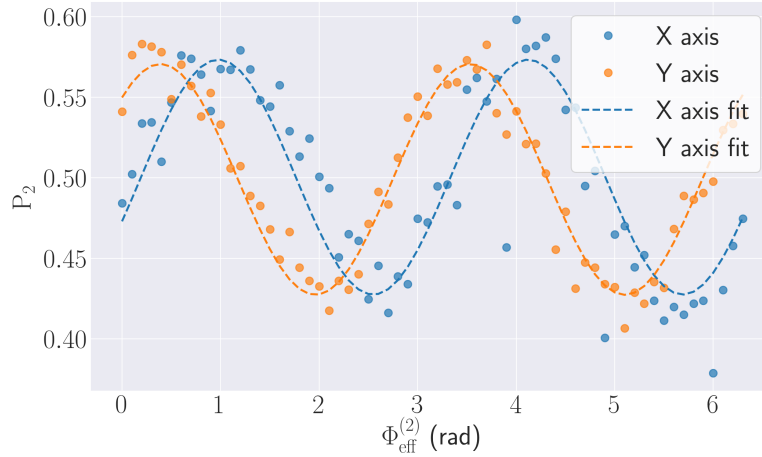


Figure 6.1: Interferometer fringe along X (blue) and Y (orange) Raman axes for an interferometer time $2T = 1$ ms. Fits for the X and Y axes (dashed lines) give contrasts of 0.146 ± 0.009 and 0.143 ± 0.005 respectively.

6.1.2 Chirp rate

We have seen in §5.4.4 that there is a small component of gravity along each Raman axis as shown schematically in Figure 6.2. The atomic cloud therefore accelerates along the Raman axis which, through the Doppler shift, chirps the Raman detuning, δ . To stay resonant with the atoms throughout the interferometer sequence we make a compensating chirp of the beat-note frequency, ω_{eff} . We set a start frequency which we use during our velocity selection pulse and then linearly vary ω_{eff} until the end of the third interferometer pulse.

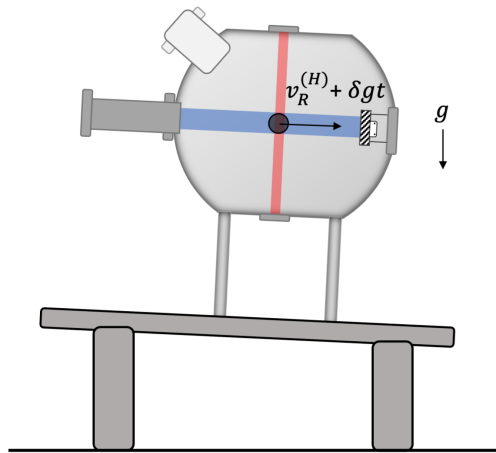


Figure 6.2: An accelerating atomic cloud due to a small angle between the horizontal and Raman axis.

I optimised the chirp rate by looking at the contrast of the Rabi oscillations, which is maximised when we maintain the detuning $\delta(\omega_{\text{eff}}) = 0$. In fig. 6.3 the dashed blue line shows the Rabi oscillation measured on the Y axis, 23 ms after launch, without any chirp of ω_{eff} . The oscillation is barely visible, and much the same result is seen 63 ms after launch. However, there is a huge improvement when a chirp of 0.43 MHz/s is applied, as shown by the solid blue and orange lines in fig. 6.3. In this way I am able to tune the chirp rate such that all three interferometer pulses will be resonant with the accelerating atomic cloud. Table 6.1 details the optimal chirp

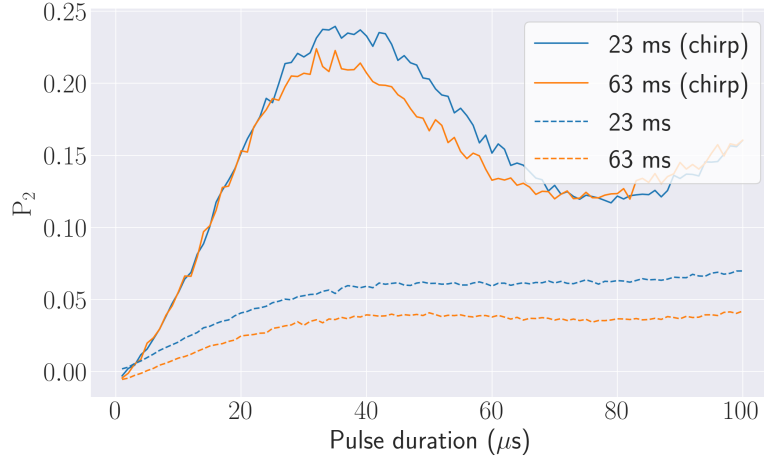


Figure 6.3: Rabi oscillations for the Y Raman axis with no chirp (dashed) and with a chirp (solid). Oscillations taken 23 ms (blue) and 63 ms (orange) after launch are shown.

parameters determined by this process. Both these chirp rates convert to an acceleration of $\sim 0.2 \text{ m/s}^2$ or a tilt angle of $\sim 1^\circ$.

Parameter	X Raman	Y Raman
Start frequency	6.834 920 GHz	6.834 412 GHz
Chirp rate	0.53 MHz/s	0.43 MHz/s

Table 6.1: Optimised chirp parameters.

Figure 6.4 shows the interferometer fringes (dots) together with fits to eq. (6.1) (dashed) for the X Raman axis (blue) and the Y Raman axis (orange) after chirping the laser to compensate for the tilt of the experiment with respect to the vertical. One can see that the contrast of both fringes has increased significantly when compared to the initial fringes of fig. 6.1. The fits give contrast values of 0.217 ± 0.003 and 0.327 ± 0.004 for the X and Y Raman axes respectively. In addition, both fringes have their minima at $\Phi_{\text{off}}^{(2)} = 0$ as one would expect after compensating for the acceleration with the chirp. The difference in contrast values between the two axis is likely a result of the coarse nature of this step of optimisation. With more refinement, detailed below, we were able to reduce this difference, although not completely eradicate it.

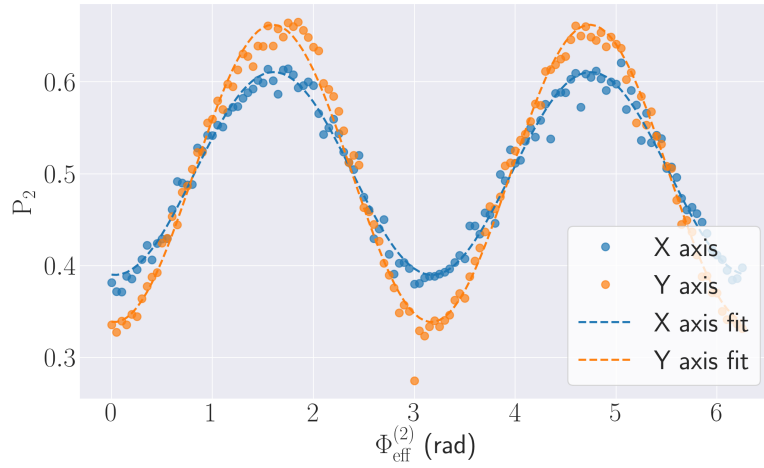


Figure 6.4: Interferometer fringe for $2T = 1$ ms for the X (blue) and Y (orange) Raman axes after optimising the chirp rate. Fits for the X and Y axes (dashed blue and dashed orange respectively) give contrasts of 0.217 ± 0.003 and 0.327 ± 0.004 respectively.

6.1.3 Retro-reflection

During construction of the chamber, the retro-reflecting module was installed and coarsely aligned to retro-reflect the Raman beam for each axis. Now that we have interference fringes, I can use them to tune the module's alignment. Actuators on each retro-reflecting module allow fine in-situ control of the angle of the mirror (see fig. 4.16b). As before, I empirically optimise the contrast of the Rabi oscillations. After each adjustment to the retro-reflecting module the chirp rate and start frequency must also be adjusted to account for the changing k_{eff} . Following such an adjustment I take an interference fringe to observe the change in contrast. Figure 6.5a show fringes after making adjustments to the vertical tilt of the retro-reflecting mirror on the Y Raman axis. One can see that the contrast increases significantly between the blue and

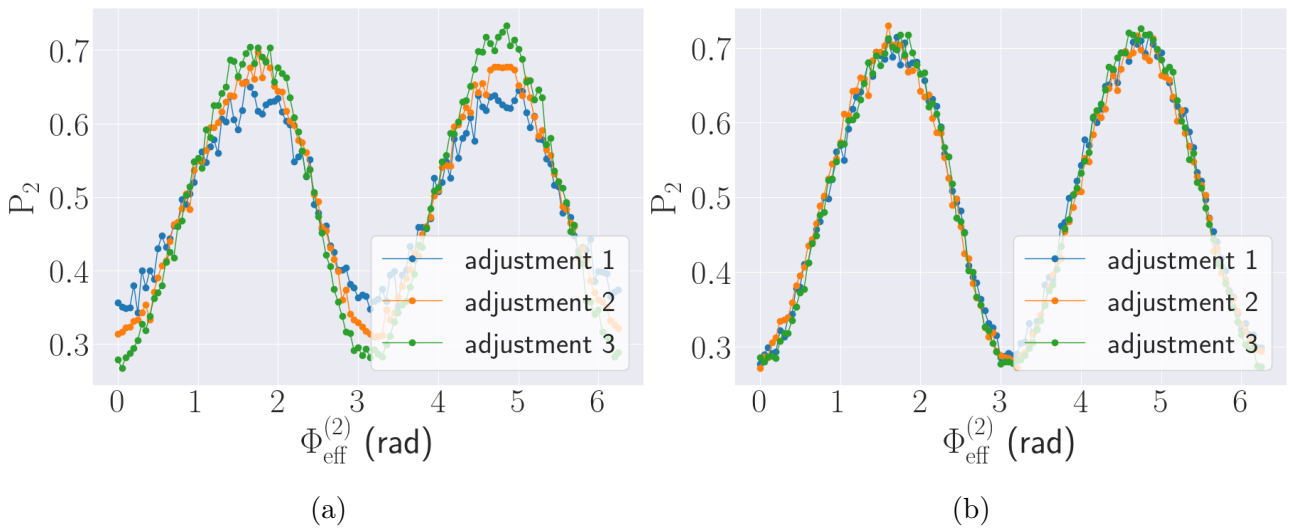


Figure 6.5: Interference fringes for $2T = 1$ ms after adjusting the (a) vertical actuator and (b) horizontal actuator on the Y Raman axis.

green curve. Only a small increase in contrast of the fringes is observed for adjustment of the horizontal actuator, as shown in fig. 6.5b. This tilt adjustment is repeated in a similar manner for the X Raman axis, after which we find the parameters in table 6.2 to be optimal. These final chirp rates indicate angles of 1.5° and 0.6° between the horizontal and the X and Y Raman beams respectively.

Parameter	X Raman	Y Raman
Start frequency	6.834 900 GHz	6.834 417 GHz
Chirp rate	0.65 MHz/s	0.27 MHz/s

Table 6.2: Parameters after optimising for retro-reflection.

Figure 6.6 shows the interferometer fringes obtained after optimising the retro-reflection along each axis. Fitting each of the fringes gives contrasts of 0.524 ± 0.006 and 0.434 ± 0.004 for the X and Y axes respectively. These are now three to four times larger than the contrasts originally observed and plotted in fig. 6.1. These contrast values are to be compared to the theoretical maximum of approximately 0.95 when one considers the Raman beam spot size and the possibility of off-resonantly exciting the atoms during the interferometry pulses. Further fine-tune optimisation would likely increase the contrast values but we can at least attribute some of the loss of contrast to the higher than desired vacuum pressure discussed in §3.2.

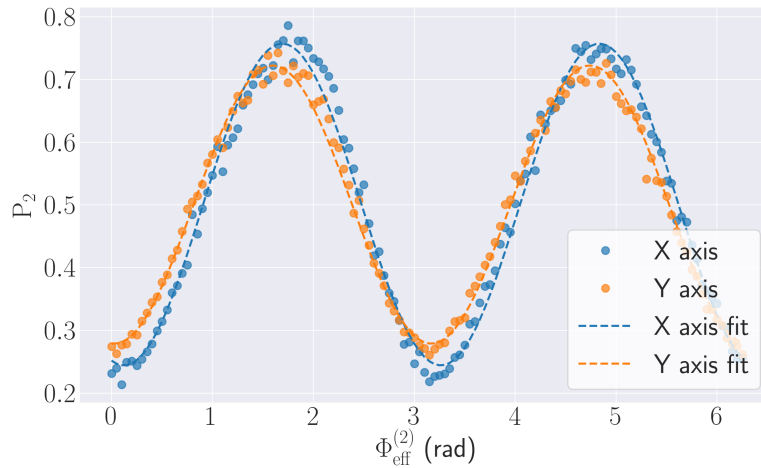


Figure 6.6: Interferometer fringe for $2T = 1 \text{ ms}$ for the X (blue) and Y (orange) Raman axes after optimising the retro-reflection. Fits for the X and Y axes (dashed) give contrasts of 0.524 ± 0.006 and 0.434 ± 0.004 respectively. The residual relative phase shift is likely from one of the axis' chirp rate and/or start frequency not being perfectly optimal for the atoms.

6.2 Decoherence

6.2.1 Rabi frequency inhomogeneity

The contrast of the interferometer fringes can be compromised by spatial variation of the Raman Rabi frequency, as discussed in §4.1.1. To measure this variation, I measure Rabi oscillations with the atoms at various positions. I prepare atoms in $|F = 1, m_F = 0\rangle$ using the state preparation steps 1 and 2, as described in §5.4.2, then I apply a single Raman probe pulse of duration τ . Figure 6.7a shows the Rabi oscillations measured on the X Raman axis as a function of τ , taken 23 ms (blue), 43 ms (orange), and 58 ms (green) after launching the atoms. I then fit the first maximum in each curve to find the position of the peak and take this as an estimate of the π -pulse time, τ_π . Figure 6.7b shows the measured τ_π values as a function of height above the optic axis for the X (blue) and Y (orange) Raman axes. I use the vertical launch velocity to convert the time since launch to a height where 0 mm corresponds to the initial height of the atomic cloud. As expected from the discussion in §5.4.4, τ_π is roughly 1.5 times longer on the Y Raman axis than it is on the X Raman axis because the two axes have different Raman beam intensities. There is very little variation of the π -time to be seen on either axis, confirming that the Raman beam hot spots have now been fixed, as discussed in chapter 4. There does seem to be a general trend on the Y axis for the π -time to be shorter at lower height. This may indicate that the initial height of the cloud lies above the optic axis. On the X axis, there seems to be a minimum near -2 mm, so the cloud may also start slightly above that axis. However, these possible offsets are close to the limit of the noise so require further investigation to be sure.

6.2.2 Contrast vs 2T

After optimisation with $2T = 1$ ms, the interferometers gave contrasts of 52% along X and 43% along Y, as shown in fig. 6.6. I now measure at longer $2T$ in order to assess how quickly coherence is lost. Figure 6.8a shows the fringes observed using a $2T = 10$ ms interferometer whose first pulse is applied 24 ms after launching the atoms. The orange dots measured on the Y axis show a fringe that is visible but rather noisy. That is because the vibration of the apparatus produces an rms interferometer phase noise of 0.8 rad. The contrast of the fringe is $C \simeq 38\%$. The fringe is harder to see in the blue dots measured on the X axis because there is a larger (1.2 rad.) phase noise from vibrations and because there has been a jump resulting in two fringes almost exactly out of phase with each other, as seen most clearly around $\Phi_{\text{eff}}^{(2)} = 3.6$ rad and again around $\Phi_{\text{eff}}^{(2)} = 5.6$ rad. We can nevertheless see from the range of P_2 that $C \simeq 30\%$. The data points in fig. 6.8b are for an interferometer with $2T = 20$ ms. With rms phase noise of 3.2 rad on the Y axis and 4.8 rad on X, the fringes are now very indistinct. The

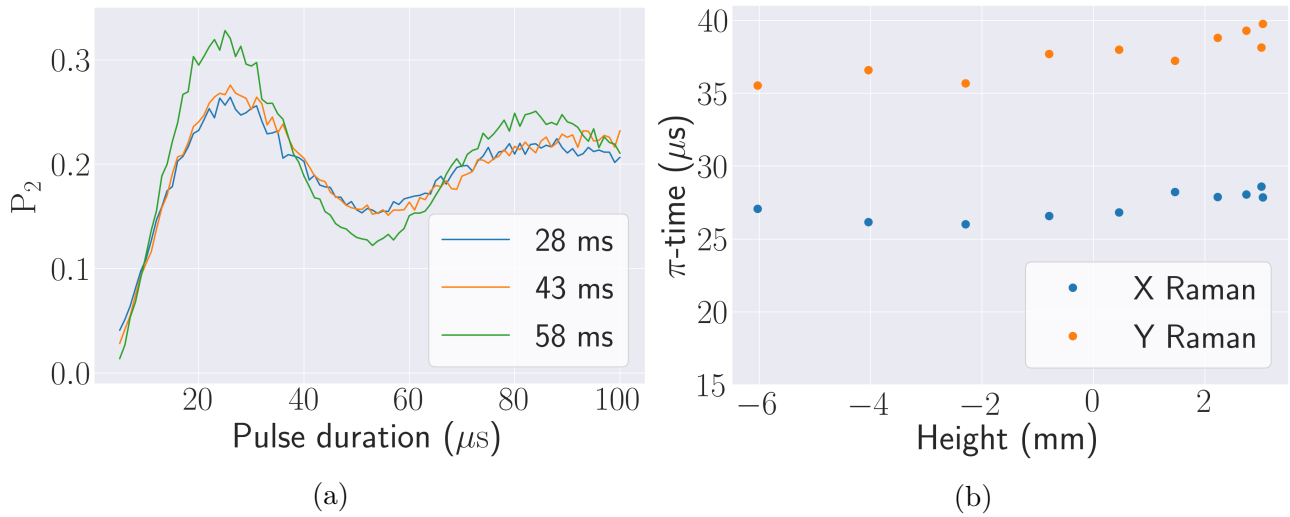


Figure 6.7: (a) Transition probability measured on the X Raman axis as a function of Raman probe pulse duration. The Rabi oscillations are shown for the probe pulse applied at three different times after launching, corresponding to three different positions of the atoms. (b) Time of maximum P_2 versus height of the atom cloud. A height of 0 mm corresponds to the initial height. Data for the X and Y Raman axes are shown in blue and orange respectively.

contrast on the Y axis is $\simeq 34\%$, but along the X axis it has declined further to $C \simeq 16\%$. In fig. 6.8c $2T$ has been increased to 40 ms and now the phase noise due to vibration is large enough to scan over the fringes without needing to scan $\Phi_{\text{eff}}^{(2)}$. In this figure we plot the signal of the MEMS accelerometer, and this allows us to recover the fringe. The orange points show the fringe obtained along Y, with a contrast of $C \simeq 23\%$. However, the contrast on the X axis is so low ($\sim 5\%$) that the fringe cannot really be discerned.

In fig. 6.8d I plot how the contrast varies as a function of T . The dashed lines show a least squares fit to the exponential form

$$C = A \exp^{-T/\tau}, \quad (6.2)$$

where τ is a time constant. For the X axis $\tau = 8.15 \pm 0.08$ ms and for the Y axis $\tau = 33 \pm 3$ ms. This indicates that the rate of decoherence is approximately four times more on the X axis compared to the Y axis. I am not sure what is causing the rapid loss of contrast along the X axis. One possibility is that the “magic” ratio for the X Raman beam was not set properly and this allowed a difference in light shift to occur between the two halves of the interferometer due to the fluctuating power of the laser. This would lead to pulses that are out of resonance with the atomic cloud, leading to a reduction in the contrast. Clearly this is a topic to be pursued by the next student to confirm if this is indeed the case.

I do not know what is causing the loss of contrast, and that is clearly a topic to be pursued, however the decoherence is slow enough that there is adequate contrast to operate the interferometers on both axes at least up to $T = 10$ ms.

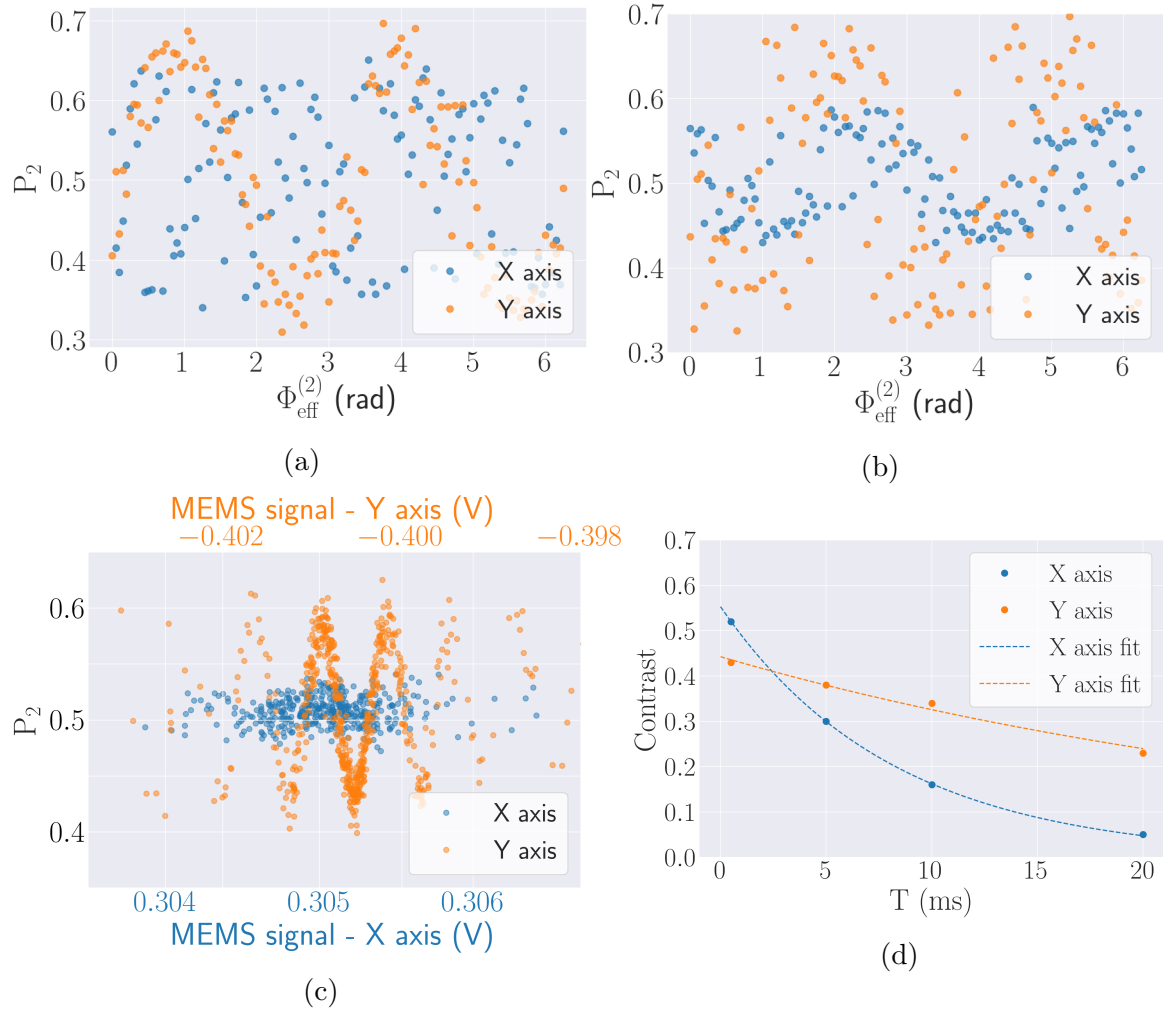


Figure 6.8: Interferometer fringes for (a) $2T = 10$ ms, (b) $2T = 20$ ms, and (c) $2T = 40$ ms along the X (blue) and Y (orange) Raman axes. For (a) and (b) the fringe is plotted against $\Phi_{\text{eff}}^{(2)}$, whereas for (c) it is plotted as a function of the MEMS output voltage (see text). (d) Contrast plotted as a function of T . Dots: measured data. Dashed: Exponential fit to the data.

6.3 Fringe Noise Distribution

I turn now to the noise in the interferometer fringes. For a fringe swept out by the ambient vibrations and plotted against the reading of the MEMS accelerometer there are three contributions to consider. (i) noise in the interferometer phase, $\sigma_{\Phi_{\text{int}}}$, which is a combination of the noise in the MEMS accelerometer reading and the phase noise in the beat note of the Raman light, (ii) the quantum projection noise in P_2 , σ_{QPN} , and (iii) the detection noise in P_2 , σ_{DET} . Using eqs. (2.53), (3.5) and (3.8) I add these three in quadrature to express the noise in P_2 as

$$\sigma_{P_2} = \sqrt{\frac{P_2}{N_{\text{tot}}} (1 - P_2) + \frac{\sigma_V^2}{V_{\text{tot}}^2} (1 + P_2^2) + \frac{C^2}{4} \sin^2(\Phi_{\text{int}}) \sigma_{\Phi_{\text{int}}}^2}, \quad (6.3)$$

We see that the noise in P_2 depends on P_2 itself and on Φ_{int} , and therefore will vary across a fringe. To observe this variation I operate a $2T = 25$ ms interferometer along both axes[†]. I keep the Raman beat-note phase constant and let the vibrational noise scan the fringe pattern. Figure 6.9a shows the fringes obtained for the X (blue) and Y (orange) Raman axes. To fit the data in fig. 6.9a I use

$$P_2 = P_{\text{off}} - \frac{C}{2} \cos(S[V + V_0]) \quad (6.4)$$

with four fitting parameters, P_{off} , C , S , and V_0 . Here V is the MEMS accelerometer voltage, V_0 is its bias (zero error) and the scale factor S converts that voltage to interferometer phase. In principle S can be known using the manufacturers conversion from output current to acceleration together with the gain of our transimpedance amplifier, but at the time of this measurement the latter was not known accurately enough. The fitted fringes give contrasts of 0.067 ± 0.003 for X and 0.292 ± 0.004 for Y.

In order to determine the noise σ_{P_2} I partition the residuals of the fit into bins 0.2 rad wide. I only keep bins that contain more than 5 data points to reduce sparse sampling contributions. Within each bin I calculate the standard deviation of the residuals and these are shown as blue and orange circles in figs. 6.9b and 6.9c for X and Y respectively. For each data point in fig. 6.9b I use the fitted fringe for the X Raman axis in fig. 6.9a to convert the MEMS accelerometer voltage to a value of Φ_{int} and to find the corresponding value of P_2 . I also do this for the data points in fig. 6.9c using the fit for the Y Raman axis. I now have a series of tuples (P_2, Φ_{int}) along with the measured values of σ_{P_2} . I fit eq. (6.3) to this data and plot it in figs. 6.9b and 6.9c as a function of MEMS accelerometer voltage (red, dashed). In making this fit I use the known values of C and V_{tot} (1.08 V and 0.61 V for X and Y respectively) and the corresponding values of N_{tot} (5×10^7 and 3×10^7). The fit parameters are σ_V and $\sigma_{\Phi_{\text{int}}}$.

The fit gives $\sigma_V = \{12 \pm 1, 7 \pm 1\}$ mV for $\{X, Y\}$, and these are almost a thousand times larger than the intrinsic noise of $\{58, 13\}$ μ V that was measured in §3.4.3 with the detectors in the dark. In comparison with V_{tot} , σ_V is 1%. Since the intensity noise of the laser light is only 0.1%, I have to conclude that the laser frequency is unstable. Given the 6 MHz natural line width of the transition, this instability must be at the level of a few hundred kHz. This should be checked with a Fabry-Pérot cavity to confirm if this indeed the case.

The fit also gives $\sigma_{\Phi_{\text{int}}} = \{350 \pm 90, 180 \pm 10\}$ mrad for $\{X, Y\}$. The phase noise in the beat note of the laser light is only 31 ± 1 mrad, as presented in §3.1.3, so the dominant part of the phase noise here is coming from the MEMS accelerometer noise. Of course, that is as it should be since the whole idea is to make an accelerometer that is better than the MEMS accelerometer. With $2T = 25$ ms, the measured phase noise corresponds to acceleration noise of $\{14 \pm 4, 7.3 \pm 0.4\}$ μ g. The noise figures given in the data sheet for the Innalabs AI-Q-2010 accelerometer are $< 7 \mu$ g (0.1 - 10 Hz) and $< 70 \mu$ g (10 - 500 Hz). Since the relevant bandwidth here is $\sim 1/T$, or

[†]After further optimisation, fringes were resolvable for $2T = 25$ ms along both Raman axes.

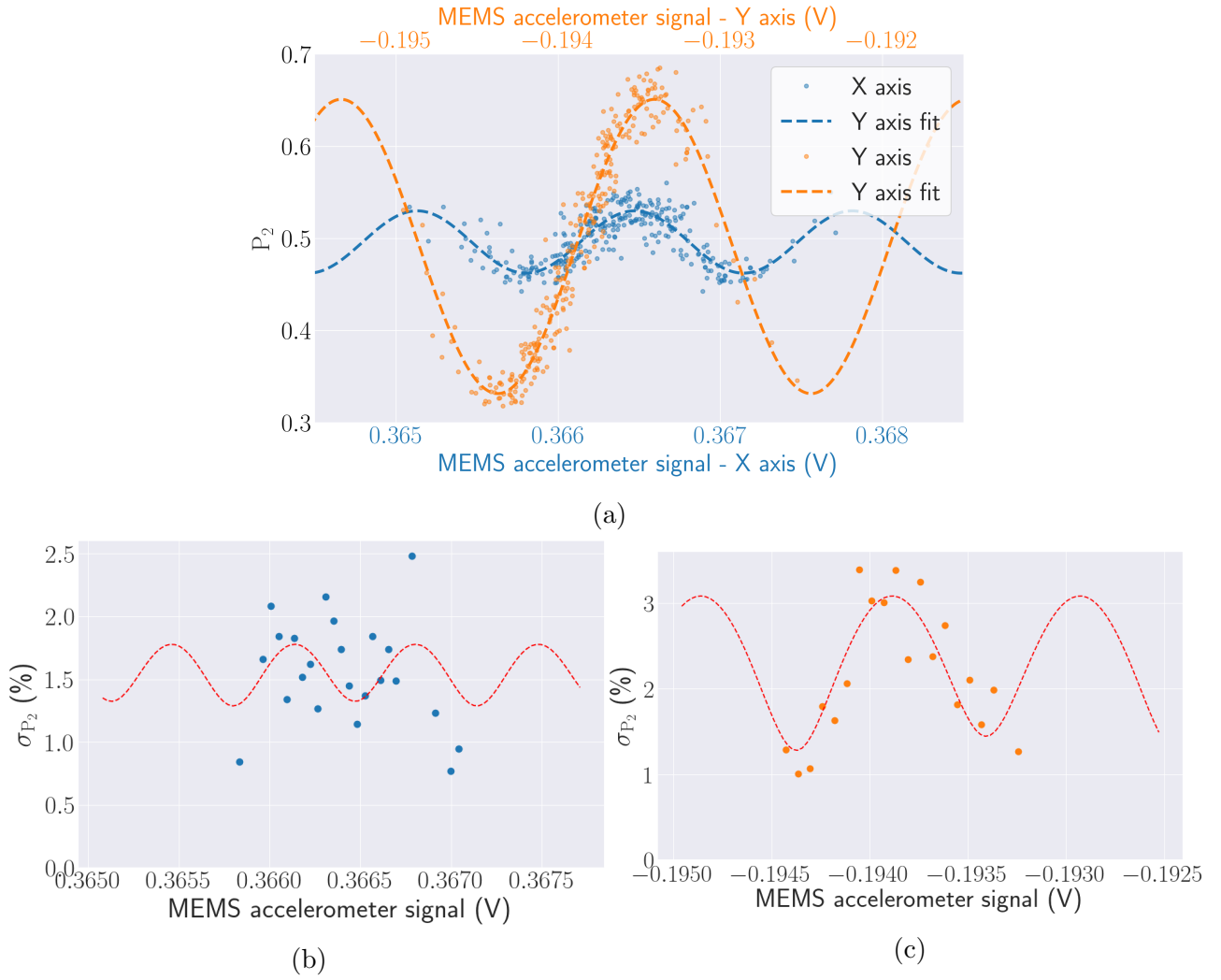


Figure 6.9: (a) Interferometer fringe obtained along the X Raman axis (blue) and Y Raman axis (orange) together with a fit to the fringes (dashed). (b) Blue dots: Standard deviation of binned residuals measured along the X Raman axis. Dashed line: fit of the blue points to eq. (6.3). (c) Same as (b) but for the fringe taken along the Y Raman axis.

80 Hz, the noise I have measured is entirely consistent with that.

To summarise, let us consider the interferometer on the Y axis, operating with $2T = 25$ ms, $N_{\text{tot}} = 3 \times 10^7$ and $C = 0.3$. At $\Phi_{\text{int}} = \pi/2$, where the interferometer has the greatest sensitivity to acceleration, $P_2 \simeq 0.5$. The quantum projection noise, expressed as an uncertainty in the acceleration per shot is

$$(\sigma_a)_{\text{QPN}} = \sqrt{\frac{P_2}{N_{\text{tot}}}(1 - P_2)} \times \frac{2}{C k_{\text{eff}} T^2} = 24 \text{ ng} \quad (6.5)$$

where k_{eff} is defined below eq. (2.6). The detection noise σ_{DET} gives an acceleration noise of

$$(\sigma_a)_{\text{DET}} = \frac{\sigma_V}{V_{\text{tot}}} \sqrt{1 + P_2^2} \times \frac{2}{C k_{\text{eff}} T^2} = 3 \mu\text{g}. \quad (6.6)$$

However, we expect that this can be greatly reduced by stabilising the frequency of the laser to 50 kHz or better. Then σ_V/V_{tot} will be reduced to 0.1% due to the laser intensity noise, reducing $(\sigma_a)_{\text{DET}}$ to 350 ng. The 31 mrad phase noise arising from the laser light gives an uncertainty in acceleration of

$$(\sigma_a)_\Phi = \sigma_{\Phi_{\text{int}}} \times \frac{1}{k_{\text{eff}} T^2} = 1.3 \mu\text{g} \quad (6.7)$$

For the interferometer on the Raman X axis, the acceleration noise arising from the detection noise is made worse by the lower contrast, and becomes comparable with the uncertainty due to laser phase noise. I expect however that the contrast on the Raman X axis can be improved at least to the level of the other arm.

6.4 Conclusion

In §6.1 I presented the fringes obtained for a $2T = 1$ ms interferometer before optimisation was performed. I then detailed how I was able to increase the contrast of the fringes by tuning the chirp rate of the laser to compensate for a small angle between the Raman axes and the horizontal. Next, I showed how I used the fringes to adjust the retro-reflection of the Raman beams. After these improvements I was able to achieve contrasts of 0.524 ± 0.006 and 0.434 ± 0.004 for the X and Y Raman axes respectively. Following this, in §6.2, I investigated the decoherence of the fringes on both axes. By mapping the Rabi frequency of the Raman transition over a range of positions, I showed that the telescopes I designed and built were providing uniform illumination of the atoms. A lack of uniformity had limited the coherence of an earlier, 1-axis interferometer. By increasing the time T between interferometer pulses I was able to observe the loss of coherence in the interferometer over time. I saw that this was well described as an exponential decay and I was able to measure the decoherence time on each axis. Finally, in §6.3 I investigated how the various sources of noise affect the measurement of acceleration using the interferometer. I found that the dominant noise comes from the uncertainty in measuring P_2 . I argued that this is due to inadequate stability of the detection laser frequency, and that it should be possible to reduce that noise by at least a factor of 10. Once that is done, I showed that the uncertainty in acceleration will be limited by the phase noise in the Raman beat note. For an interferometer time of $2T = 25$ ms that uncertainty is $1 \mu\text{g}$ and scales as $1/T^2$. The next phase of this project will be to reduce the decoherence so that both axes can operate at $2T = 50$ ms and to reduce the detection noise so that the laser phase noise is the limiting factor. That will reduce the noise level to 300 ng per shot or, with 4 shots per second, to 150 ng at 1 second. This is close to the original design goal of 100 ng at 1 second.

Chapter 7

Conclusion

7.1 Summary

The aim of the on-going project here at Imperial College London is to develop a field-deployable 3-axis inertial navigation system that takes advantage of a high bandwidth MEMS accelerometer coupled with the long-term stability of a cold atom interferometer. This second stage of the project has been to upgrade our previous 1-axis system to 2-axes capable of measuring accelerations perpendicular to gravity with a sensitivity close to 100 ng at 1 second. The focus of my PhD has been to build and characterise the first iteration of the new 2-axis system.

With our current setup we are able to load over 10^9 atoms in the 3D MOT in 100 ms by utilising a $2D^+$ MOT. The 3D MOT is formed by overlapping three pairs of beams at the centre of a quadrupole field. With control of the frequency and power of each beam we have been able to launch the atoms, perform sub-Doppler cooling, manipulate the internal state of the atoms, and implement a state sensitive detection. I have shown that using these beams in combination with microwave fields and the Raman beams we are able to cool the atomic ensemble to temperatures as low as $6\text{ }\mu\text{K}$ and prepare a narrow velocity class in $|F = 1, m_F = 0\rangle$.

I designed a novel Raman optics setup that has been chosen to minimise wavefront curvature whilst achieving the 40 mm spot size we require for measuring horizontal accelerations. I have constructed two identical Raman tubes and installed them inside of the chamber to further minimise any distortions that would be introduced by viewports. The superior intensity profile of these optics compared to our previous setup was confirmed using a measurement of the π -time as a function of position within the Raman beam. The first interference fringes obtained with these new Raman optics showed low contrast values of approximately 14% along both axes for $2T = 1\text{ ms}$. Through a combination of chirp rate and retro-reflection adjustment I have been able to improve the contrasts to 52% and 43% along the X and Y axes respectively.

I then studied how the fringe contrast decreased as a function of $2T$. This study revealed decoherence times of 8 ms on the X axis and 33 ms on the Y axis. As of the time of writing we do not know why the X axis is not as good as the Y axis, and it is left for the next student to diagnose and fix this. Finally I measured the noise in the fringes on both axes with $2T=25$ ms. At the maximum slope, which gives the highest sensitivity to acceleration, I measured a noise along the Y axis of $3\mu\text{g}$ per shot. With 4 shots per second, this is almost four times higher than our ultimate design goal of 400 ng in 1 second with $2T = 25$ ms. This was limited by the detection noise, which was much higher than expected. We should be able to reduce this detection noise well below the design noise level by improving the frequency stability of the detection light. The noise will then be $1.3\mu\text{g}$ per shot due to phase noise in the Raman beat note. With optimisation of the underlying computer control programme we expect to achieve data rates of close to 8 Hz in the near future, allowing us to come close to our ultimate design goal in the next stage of the project.

Finally it should be noted that the system was operated in an urban environment in London without the use of vibration isolation. This demonstrates the robustness with which we are able to build such inertially sensitive instruments and shows that such a device is mature enough to be produced and operated in the commercial sector.

7.2 Outlook

My demonstration and characterisation of this atom interferometer constitutes the first phase in the development of a 2-axis device. To increase the sensitivity I suggest several improvements. The first is to understand the source of the decoherence, which currently limits the useful interferometer time T to about 10 ms, so that we can increase T to 25 ms. Second, we need to correct the frequency fluctuation responsible for the excess detection noise. Third, the cycling rate of the experiments is currently limited to 4s^{-1} by dead time in the software. If this is minimised, I see cycling rates up to 8s^{-1} achievable in the near-future. Fourth, it would help to implement direct digital synthesisers and linear polarisers into the control train of each MOT beam. A feedback loop could then allow us to actively control the frequency and intensity of each beam, as well as purifying the polarisation. In addition to stabilising the temperature of the atomic ensemble this would also give the atoms a more accurate launch velocity which will be crucial in the next stage of this project where we aim to measure rotation.

The ultimate value of such a device comes from operation in the field. In parallel to this project, another project in collaboration with M Squared lasers is currently on-going. The aim is to develop a compact, robust, 1-axis cold atom interferometer. Three such interferometers will be tested by operating them aboard a moving vehicle. The long term stability will be measured by running all 3 devices simultaneously and measuring the relative drift of the output signals

over time. These represent important steps in demonstrating the usefulness of such quantum technology as well as moving the device towards a more user-friendly size and complexity. Finally, much engineering work will need to be done to miniaturise much of the apparatus, in addition to making it more robust, in order to achieve a truly field-deployable device that is capable of withstanding operation aboard a moving vehicle.