

LABORATORY SIMULATION OF TURBULENT-LIKE FLOWS

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Declaration

I hereby confirm that the work presented in this thesis is my own.

Prangchira Kewcharoenwong.

Abstract

Most turbulence studies up to the present are based on statistical modeling, however, the spatio-temporal flow structure of the turbulence is still largely unexplored. Turbulence has been established to have a multi-scale instantaneous streamline structure which influences the energy spectrum and other properties such as dissipation and mixing.

In an attempt to further understand the fundamental nature of turbulence and its consequences for efficient mixing, a new class of flows, so called “turbulent-like”, is introduced and its spatio-temporal structure of the flows characterised. These flows are generated in the laboratory using a shallow layer of brine and controlled by multi-scale electromagnetic forces resulting from a combination of electric current and a magnetic field created by a fractal permanent magnet distribution. These flows are laminar, yet turbulent-like, in that they have multi-scale streamline topology in the shape of “cat’s eyes” within “cat’s eyes” (or 8’s within 8’s) similar to the known schematic streamline structure of two-dimensional turbulence. Unsteadiness is introduced to the flows by means of time-dependent electrical current.

Particle Tracking Velocimetry (PTV) measurements are performed. The technique developed provides highly resolved Eulerian velocity fields in space and time. The analysis focuses on the impact of the forcing frequency, mean intensity and amplitude on various Eulerian and Lagrangian properties of the flows e.g. energy spectrum and fluid element dispersion statistics. Other statistics such as the integral length and time scales are also extracted to characterise the unsteady multi-scale flows.

The research outcome provides the analysis of laboratory generated unsteady multi-scale flows which are a tool for the controlled study of complex flow properties related to turbulence and mixing with potential applications as efficient mixers as well as in geophysical, environmental and industrial fields.

To my parents

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Chapter 1

Introduction

Turbulence is generally experienced and present in everyday life and in nature. The phenomenon has a broad range of applications, particularly in the area of mixing. For instance, turbulence is essential for combustion, chemical reactions and is natural mixer in many environmental, geophysical and industrial flows. In view of its great importance, it is therefore vital to understand turbulence. Such understanding will enable us to better predict and even control the phenomenon.

In contrast to its ubiquity, turbulence is highly complex. The subject has been studied over a century and although there have been significant contributions to the field, turbulence is not yet fully understood and remains one of the last unsolved problems in classical physics today. Most studies and predictions of turbulence are based on a statistical approach as the flow is sensitive to the initial conditions. Previous works in the field cover theoretical, numerical and experimental attempts to generate turbulence and find ways to predict it using certain closure models on a case-by-case basis.

An alternative approach to study turbulence has been proposed by Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a). Instead of generating turbulence to investigate its complex physics, the authors disassemble turbulence into its core characteristics and study the role of each characteristic independently as they reassemble it back, one by one, to turbulence. In this approach, a new class of flow, so-called “turbulent-like” is introduced. Such flow is reassembled from the laminar state where its physics is well established.

Recent works by Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a) have successfully reassembled the multi-scale schematic streamline structure of two-dimensional (2D) turbulence to fully-controlled laminar flows. Their work merely focuses on steady flows in order to investigate the role of multi-scale schematic streamline structure independently. It has been shown that such characteristic is accounted for a broad range of power law energy spectrum, $E(k) \sim k^{-p}$ where $5/3 < p < 3$, and Richardson-like pair separation, $\overline{\Delta^2} \sim t^\gamma$ where $\gamma \approx 2.45$, which is the diffusive property similar to those of turbulence. It is striking that the steady multi-scale flows exhibit such properties while essentially remaining laminar flows which require less power to generate and maintain than turbulence. Hence, the unique “multi-scale turbulent-like flow” concept bares the potential for efficient mixing at low Reynolds number.

A natural progression of the “turbulent-like” concept is to include that inherent unsteadiness present in turbulent flows and to examine the impact this has on both Eulerian and Lagrangian properties of the flows. In this work, unsteadiness, which is another key characteristic of turbulence, is reassembled to the multi-scale laminar flows. This is an important novelty of this work compared to Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a).

1.1 Objectives

The objectives of this thesis are twofold:

1. to extend the work by Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a) by incorporating unsteadiness in the reassembled multi-scale laminar flows to a much broader class of flows and more closely turbulent-like. Unsteadiness is a key characteristic of turbulence. We use these flows to study by analogy the phenomenon of turbulence and its mixing properties.
2. to investigate this new class of flow as a possible efficient stirring device at low Reynolds number.

To achieve these objectives, we simulate fully-controlled unsteady quasi-two-dimensional (Q2D) multi-scale laminar flows in the laboratory to study the relationship between the temporal and spatial characteristics of the flows. Specifically, flows are generated using time-dependent multi-scale electromagnetic forcing in a shallow layer of brine. We aim to investigate the effects of time-dependencies on the Eulerian properties of the flows i.e. flow features, topology and flow energy. For this, we examine the Eulerian velocity field, flow streamlines, Eulerian integral length scale, the stagnation point structure, as well as the energy spectrum and the Eulerian frequency spectrum.

Furthermore, we aim to investigate the Lagrangian properties, which serve as important measures for mixing. The stirring properties of the flows are explored qualitatively via stirring illustration and quantitatively via statistics of single fluid element and pair separation. The Lagrangian autocorrelation functions and their corresponding frequency spectra as well as the Lagrangian time scale are also examined.

Finally, we seek to determine any possible relationship between the Eulerian and Lagrangian properties of our unsteady multi-scale laminar flows and give comparison to the spatio-temporal structure found in fully developed turbulence. For instance, we test the validity of the Corrsin hypothesis, which relates the Lagrangian integral time scale to the velocity fluctuation variance and the Eulerian integral length scale.

1.2 Turbulent-like flows

As the name implies, we refer to turbulent-like flows as flows that exhibit characteristics of turbulence. Turbulent-like flows simulated in this work possess the following characteristics:

1. Unsteadiness, $\partial \mathbf{u} / \partial t \neq 0$.
2. Multi-scale schematic streamline structure in the shape of figure “8 in 8” similar to the structure of two-dimensional (2D) turbulence.
3. Broad range of power law of energy spectrum, $E(k) \sim k^{-p}$.

4. Diffusive with a Richardson-like pair separation statistics, $\overline{\Delta^2} \sim t^\gamma$ where $\gamma > 2$.

It is critical to emphasise that despite the presence of the characteristics mentioned above, the simulated flows remain essentially laminar. Characteristics of turbulence that are excluded in our flow simulation are three dimensionality and high Re. Due to the highly complex nature, lack of understanding, and difficulties to control 3D turbulence structure and topology, a quasi-two-dimensional (Q2D) flow, with more established and simpler topology is being simulated and investigated in this research.

1.3 Thesis outline

The present thesis is dedicated to the laboratory simulation and investigation of a new class of unsteady multi-scale flows. The thesis is organised as follows.

In Chapter 2, we present background on turbulence and its mixing properties as well as a brief review of previous work in the related field. Chapter 3 outlines the experimental apparatus and setup of the quasi-two-dimensional (Q2D) multi-scale flow simulated by a shallow layer of electrolyte (brine) driven by the electromagnetic forcing. The chapter also provides details of necessary preparation and flow measurement methodologies including particle tracking velocimetry (PTV) and dye visualisation (DV). The unsteadiness introduced by mean of time-dependent forcing is discussed in Chapter 4. We estimate the time scale associated to each characteristic length-scale in the flow by different methods. We then select to excite the flow scale individually or altogether by using different forcing intensity range and frequency in order to study their effects on the multi-scale flow and its mixing properties. In Chapter 5, the experimental data post-processing is detailed. The output of the PTV in the form of image series undergo various post-processing techniques and are finally interpreted into the Eulerian velocity fields. Their quality and accuracy are quantified.

Results are presented in two main sections. In Chapter 6, the Eulerian properties of the flow are studied. We demonstrate that the addition of unsteadiness via selected time-dependent forcing has little effect on the flow's schematic streamline topology i.e.

the number of stagnation points and the way they are connected to the streamlines remain persistent and the stagnation points only move slightly and slowly compared to the mean flow.

Chapter 7 is devoted to the Lagrangian statistics of the flow. The Lagrangian autocorrelation function and Lagrangian time scale are quantified. We show that the Corrsin hypothesis is valid in our unsteady multi-scale laminar flows. The time-dependent forcing improves the fluid element pair dispersion leading to a higher power law coefficient and a broader scaling range of dispersion before the random walk region occurs. Moreover, the time-dependent forcing also introduces the temporal (a.k.a. local) acceleration, which is generally present in turbulence but not in steady multi-scale flows in Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a), to our unsteady multi-scale flows. Lastly, conclusions and recommendations for future research are presented in Chapter 8.

Chapter 2

Literature Review

2.1 Turbulence

Most authors describe turbulence through its characteristics; turbulent flows are flows with random and unsteady motions with the presence of vortices and exhibit diffusive and dissipative behaviours (Tennekes & Lumley 1972). The origin of turbulence is rooted in the instabilities developed at high Reynolds number and the flow is sensitive to the initial conditions. Hence, predictions are mostly based on the statistical approach.

An important milestone in the statistical study of turbulence was marked by Kolmogorov (Kolmogorov 1941). His work was based on Richardson's idea on the energy cascade and the existence of vortices of all possible scales in turbulent flow (Richardson 1922). In Kolmogorov (Kolmogorov 1941), turbulence was distinguished in three regions, the energy injection region, the energy dissipation region and the inertial range, which is the intermediate region between the first two regions (Figure 2.1).

In the energy injection region, at scales larger than the integral length scale, L and the integral time scale, T_L , energy is injected into the flow and turbulence statistics are dependent on the boundary conditions such as the forcing and the flow geometry. In contrast, small scales turbulence statistics are isotropic (i.e. the geometrical and directional information is lost) and universal (i.e. statistics are the same in all turbulent flows given a sufficiently high Reynolds number). At scales smaller than the

Kolmogorov length scale, η , and time scale, τ_η , turbulence statistics depend only on the kinematic viscosity, ν , and the energy dissipation rate, ε . By dimensional analysis, Kolmogorov defined the length scale, time scale and velocity scale of the smallest eddies of turbulence as:

$$\eta = \left(\frac{\nu^3}{\varepsilon} \right)^{1/4}, \quad \tau_\eta = \left(\frac{\nu}{\varepsilon} \right)^{1/2} \quad \text{and} \quad v_\eta = (\nu\varepsilon)^{1/4}.$$

and these scales are often referred to as Kolmogorov microscales. The Reynolds number formed with η and v_η becomes unity, $\frac{\eta v_\eta}{\nu} = 1$ suggesting a balance between viscosity and the inertial forces. Hence, molecular viscosity is significant at these small scales.

In the inertial range where $\eta \ll r \ll L$, energy is cascaded between eddies of different sizes. At these scales, turbulence statistics are expected to be universal and depend on the local length scale, r , and the energy dissipation rate, ε . By dimensional analysis, Kolmogorov's hypothesis lead to a mathematical description of the energy cascade (Obukhov 1941):

$$E(k) \sim \varepsilon^{2/3} k^{-5/3},$$

where $E(k)$ is the energy spectrum and k is the wavenumber. It should be emphasised that Kolmogorov's model, which implies the direct energy cascade, was derived for intense three-dimensional (3D) turbulence with large Reynolds number.

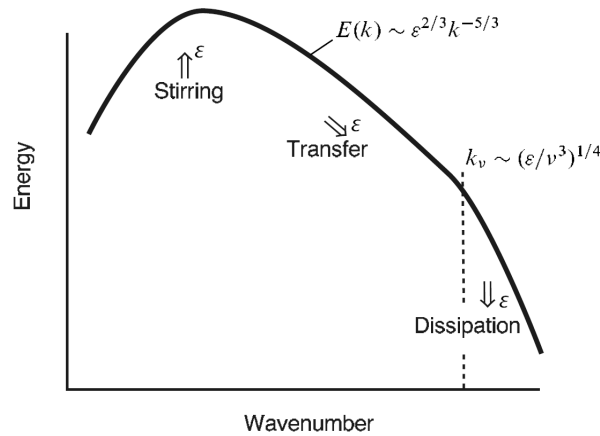


Figure 2.1: Simplified diagram of energy spectrum of three-dimensional turbulence (Vallis 2006).

The concept of two-dimensional (2D) turbulence, as observed in atmospheric and geophysical flows (Figure 2.4), was realised much later on. Two-dimensional turbulence have then been studied extensively as it is relatively simpler and easier to simulate, theoretically, numerically and experimentally, compared to 3D turbulence (see (Kraichnan & Montgomery 1980) and (Tabeling 2002) for a comprehensive review on 2D turbulence).

In 2D turbulence, there are two inertial cascade ranges (Kraichnan 1967). In an inverse energy cascade range, the energy spectrum scale as:

$$E(k) \sim \varepsilon^{2/3} k^{-5/3},$$

which has the same shape as in 3D turbulence but with an opposite direction of energy cascade i.e. energy is cascaded to larger scales in this range.

On the other hand, in a forward enstrophy cascade range, the energy spectrum has the shape of:

$$E(k) \sim \hat{\eta}^{2/3} k^{-3},$$

where $\hat{\eta}$ is the enstrophy cascade rate which is assumed to be constant and equal to the rate at which enstrophy is supplied to the flow e.g. by stirring. Such dual cascade process of 2D turbulence is illustrated in Figure 2.2 (Vallis 2006).

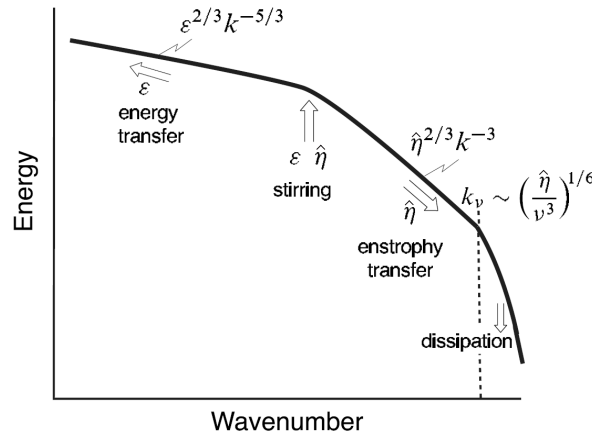


Figure 2.2: Simplified diagram of energy spectrum of two-dimensional turbulence (Vallis 2006)).

The microscales for 2D turbulence are described as follows:

$$\eta = \left(\frac{\nu^3}{\hat{\eta}} \right)^{1/6}, \quad \tau_\eta = \hat{\eta}^{-1/3} \quad \text{and} \quad v_\eta = (\nu^3 \hat{\eta})^{1/6}.$$

Pictures of 2D turbulence are illustrated in Figure 2.4. From the figure, the regions where all velocity components vanish are known as the stagnation points (also referred to as critical/saddle points in many literatures (Perry & Chong 1987)).

There are two types of stagnation points in 2D turbulence; 1) hyperbolic stagnation points which are associated to the straining regions in the flow and fluid elements are dispersed away from such points and 2) elliptic stagnation points which are associated to the centre of rotating vortices and fluid elements can only loop around without reaching into such points. The two types of stagnation points are shown in Figure 2.3.

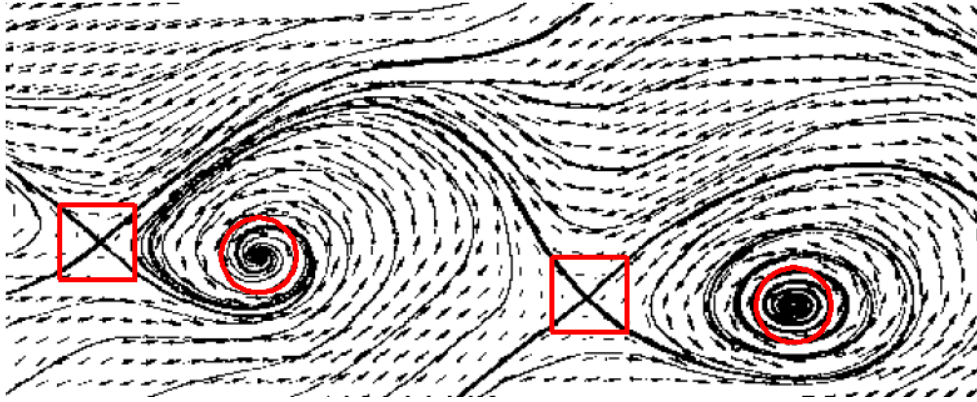


Figure 2.3: Typical instantaneous velocity vector field of 2D turbulence (Taken from Perry & Chong (1987)). The stagnation points are shown in red circle (elliptic) and square boxes (hyperbolic).

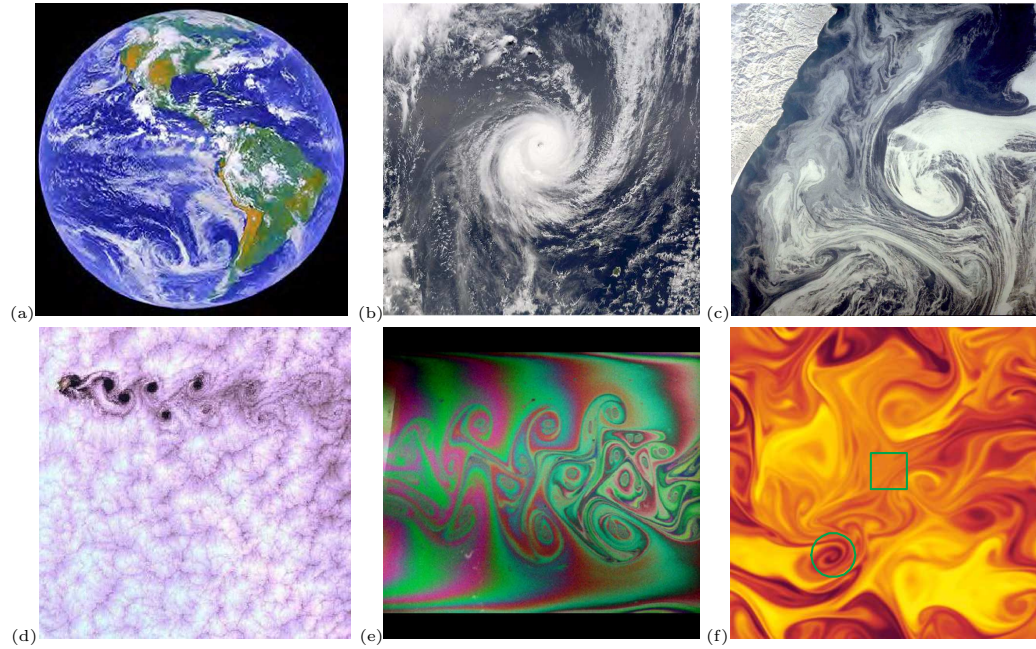


Figure 2.4: Examples of 2D turbulence:

- (a) Atmospheric turbulence on the earth's surface; the multi-scale feature as well as a hyperbolic stagnation point between two elliptical ones at the bottom of the figure can be observed (source: <http://rst.gsfc.nasa.gov/Sect16/Sect16.1.html>),
- (b) Cyclone Dina northeast of Mauritius and Reunion Islands in the Indian ocean; an elliptical stagnation point can be observed in the cyclone's eye (source: <http://veimages.gsfc.nasa.gov/2446/Dina.A2002020.0610.2km.jpg>),
- (c) Eddies in the Oyashio current in the Bering sea; sea-ice provides flow visualisation (source: http://www.efluids.com/efluids/gallery/gallery_pages/Vallis.jsp),
- (d) Karman vortex street formed by a layer of clouds above Alejandro Selkirk Island (source: http://www.efluids.com/efluids/gallery/gallery_pages/street_page.jsp),
- (e) KelvinHelmholtz instability obtained in soap film displaying transition to 2D turbulence downstream (source: <http://www.mae.buffalo.edu/research/laboratories/combustionlab/Flowing%20soap%20films/Flowing%20soap%20films.htm>),
- (f) Concentration of a passive scalar in isotropic turbulence obtained from Direction Numerical Simulation (DNS); a hyperbolic and an elliptical stagnation points are marked by the green square and circle respectively (source: http://www.efluids.com/efluids/gallery/gallery_pages/iso_turbulence_page.jsp).

According to Fung and Vassilicos, the 2D turbulent flows, with an energy spectrum $E(k) \sim k^{-p}$ where $p < 3$, have the multi-scale schematic streamline topology in the shape of cat's eyes within cat's eyes (8 in 8) as shown in Figure 2.5. A pair of cat eyes streamline structure consists of two co-rotating vortices, each with an elliptic stagnation point in the middle, and one straining hyperbolic stagnation point in between (Fung & Vassilicos 1998).

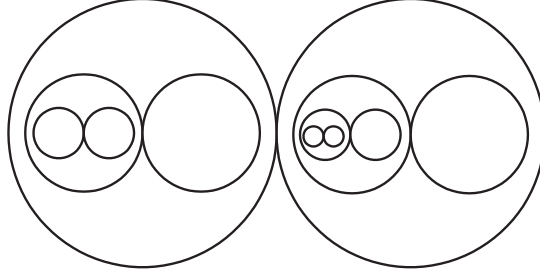


Figure 2.5: Multi-scale schematic streamline topology of 2D turbulence with an energy spectrum $E(k) \sim k^{-p}$ where $p < 3$ (Fung & Vassilicos 1998).

The schematic streamline topology in Figure 2.5 can be characterised by the number of stagnation points, n_s (Davila & Vassilicos 2003):

$$n_s \sim \left(\frac{L}{\eta} \right)^{D_s}, \quad (2.1)$$

where L and η are the outer and inner length-scales of the inertia range where $E(k) \sim k^{-p}$ holds and D_s is the fractal dimension of stagnation point distribution in space. The relation above suggests that the number of stagnation points increases as the fractal range of scales increases.

Attempts have been made to relate the power law exponent of the energy spectrum, p to the multi-scale nature of flow fields and characterised by the fractal dimensions. Early works along this line include those of Parisi, Mandelbrot Falconer and Frisch (Frisch 1995). The work by Davila and Vassilicos (Davila & Vassilicos 2003) establishes a relationship between the energy spectrum $E(k) \sim k^{-p}$ of the homogeneous isotropic turbulence and Gaussian models of it to the fractal dimension of stagnation point distribution, D_s , as:

$$p + \frac{2D_s}{d} = 3 \quad (2.2)$$

where d is the dimension of the embedding space i.e. 2 and 3 for two and three-dimensional respectively. Hence, $p + D_s = 3$ for 2D homogeneous isotropic turbulence. Direct numerical simulations (DNS) of inverse cascade of 2D turbulence by Goto and Vassilicos (Goto & Vassilicos 2004) reveals that $D_s = 4/3$ which leads to $p = 5/3$ in line with the theoretical value as shown in Figure 2.2. In addition, the DNS also confirms the geometry of the multi-scale schematic streamline structure of 2D turbulence in Figure 2.5.

Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a) have proposed that by generating a flow that has the multi-scale topology as shown in Figure 2.5, where the number of stagnation point is given by Eq. (2.1) and the relation in Eq. (2.2) is satisfied, such flow might exhibit some likeness to homogenous isotropic turbulence, which also follows Eq. (2.2). This hypothesis has been confirmed by their laboratory results (Rossi, Vassilicos, & Hardalupas 2006a), which will be discussed in more detail in Section 2.3.

2.2 Mixing

Researches have shown that turbulence is closely associated to mixing. The two phenomena share a number of similar characteristics such as multi-scale and vortical structure, unsteady motions, diffusive activities etc. Because of their similarities, it is often difficult to identify and distinguish the motions caused by mixing from those caused by turbulence. A useful remark to differentiate the two phenomena is that mixing is a process whereas turbulence is a state of flow. Beside, mixing can occur with or without the presence of turbulence.

As in the case with turbulence, mixing can be difficult to define, formally and mathematically. Mixing has been conceptually defined as “a transient process from initial segregation to ultimate homogeneity” (Chate, Villiermaux, & Chomaz 1999). In attempts to understand and optimise its characteristics, numerous authors have studied mixing from various perspectives and proposed a number of different measures for mixing.

The quality of mixing can be quantified by the striation thicknesses of the scalar tracer (e.g. dye) interfaces, $s = \frac{1}{2}(s_A + s_B)$ where s_A and s_B are the thicknesses of two entrained fluid layers labelled A and B respectively, thus s is smaller in better mixed fluid (Ottino 1989). The concentration variance, $c'^2 \equiv \langle (c - \langle c \rangle)^2 \rangle$ where c is the concentration scalar and $\langle c \rangle$ is the spatially averaged concentration, has also been proposed as a measure for mixing; in a mixing field, c'^2 is a decreasing function of time and c is well mixed when $c'^2 = 0$, i.e. $c = \langle c \rangle$, across the flow (Vassilicos 2002). Another mixing measure is the pair dispersion statistics, $\langle \Delta^2 \rangle$ where Δ is the separation between a pair of fluid elements, which is related to the concentration variance (Durbin 1980) and can be expressed mathematically for both turbulent (Antonsen, Fan, Ott, & Garcia-Lopez 1996) and non-turbulent flows (Thomson 1990). Other measures of mixing include using the entropy (DAlessandro & Dahleh 1999), the box counting method (Vassilicos 2002) and the ‘Mixed-Norm’ (Mathew, Mezic, & Petzold 2003). Amongst the proposed measures, the pair dispersion statistics remains one of the most common measures of mixing to date and is the measure being used in this research.

The study of pair dispersion statistics in fully developed turbulence is dated back to the work by Richardson that proposes the locality assumption which suggests that in the inertial range, $\langle \Delta^2(t) \rangle$ at time t are effectively dispersed by eddies comparable to the sizes of $\langle \Delta^2(t) \rangle^{1/2}$ (Richardson 1926). By applying Kolmogorov’s similarity argument, Obukhov and Batchelor obtained the Richardson’s pair dispersion law (Nicolleau & Yu 2007):

$$\langle \Delta^2(t) \rangle \sim \varepsilon t^3, \quad (2.3)$$

where ε is the kinetic energy rate of dissipation per unit mass, in the inertial range, $\eta^2 < \langle \Delta^2(t) \rangle < L^2$.

According to Batchelor, the pair dispersion statistics of fully developed turbulence has an exponential growth:

$$\langle \Delta^2(t) \rangle \sim e^t, \quad (2.4)$$

in the viscous range, $\Delta_0^2 < \langle \Delta^2(t) \rangle < \eta^2$ where Δ_0 is the initial pair separation and η is the Kolmogorov length-scale in the flow (Batchelor 1952). Furthermore, at time

$t \gg T_L$ where T_L is the integral time scale defined as $T_L = \int_0^\infty \langle \mathbf{u}_{L_i}(t) \cdot \mathbf{u}_{L_i}(t + \tau) \rangle d\tau$ where $\mathbf{u}_{L_i}$ is the Lagrangian velocities of fluid elements in the flow, the pair dispersion statistics in fully developed turbulence follow:

$$\langle \Delta^2(t) \rangle \sim t, \quad (2.5)$$

as a result of independent and uncorrelated motions of the dispersed fluid pairs (Fung & Vassilicos 1998).

On the other hand, as previously mentioned, mixing can occur without the presence of turbulence as in the cases of vortical and chaotic flows. These flows have different forms of pair dispersion; for instance, in steady vortical flows (i.e. steady shear flows), the pair dispersion grows linearly:

$$\Delta(t) \sim t, \quad (2.6)$$

by the spiral gradient structure imposed by 2D vortex (Vassilicos 2002). In the cases of globally chaotic flows (i.e. unsteady laminar flows with low Re), chaotic fluid pathlines can lead to exponential pair dispersion rates:

$$\Delta(t) \sim \Delta_0 e^{\zeta t}, \quad (2.7)$$

where ζ is the Lyapunov exponent and $\zeta(t) > 0$ (Antonsen, Fan, Ott, & Garcia-Lopez 1996). Nevertheless, due to the limited range of length and time scales of these non-turbulent flows, their rates of scalar mixing are much slower than that of fully developed turbulence.

Interestingly, in a new class of steady multi-scale flows studied by Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a), the pair dispersion statistics exhibits an approximate algebraic growth:

$$\langle \Delta^2 \rangle \sim t^\gamma, \quad (2.8)$$

where $\gamma > 2$, similar to that of fully developed turbulence, while essentially remain laminar flows. The algebraic pair dispersion obtained, so-called ‘‘Richardson-like’’, is a result on average of the underlying successive local exponential pair dispersion,

$\Delta(t) \sim \Delta_0 e^{\zeta t}$, in the vicinity of imposed multi-scale hyperbolic stagnation points in the flow. The fact that $\gamma > 2$ in the steady multi-scale flows implies that the pair dispersion in such flows is more effective than in the cases of quasi steady laminar shear flows where $\gamma = 2$.

It is striking to observe such steady multi-scale flows already exhibiting the pair dispersion statistics similar to that of turbulence while remaining steady laminar flows. In this work, time-dependent forcing is used to generate unsteady multi-scale flows and among many issues to be addressed, we aim to investigate the type of pair dispersion law that our unsteady multi-scale flows follow.

2.3 Previous experimental works

Two-dimensional flows have been studied extensively in the past few decades. Most previous experimental works and their generated flows fall under two distinct flow classes which were turbulence and laminar flows (both steady and unsteady i.e. shear, vortical and chaotic flows that do not inherit essential properties of turbulence, particularly the multi-scale feature). It was only recently that a new class of flows, where the schematic multi-scale topology of 2D turbulence was reassembled to laminar flows, had been introduced (Rossi, Vassilicos, & Hardalupas 2006a).

The techniques that have been used to generate these 2D flows in the past include the using of soap films, thin conducting liquids, rotating and stratification systems and cavity units with moving walls, for instance. Moreover, previous measurements performed ranges from using streak photographs, Dye Visualisation (DV), Particle Image Velocimetry (PIV), Particle Tracking Velocimetry (PTV) etc. In this section, some examples of previous experimental works on both 2D turbulent and non-turbulent flows are briefly reviewed.

A number of experimental works have been conducted to investigate unsteady laminar flows which can develop complex mixing patterns. Among them, Leong and Ottino presented a versatile cavity flow apparatus consisting of two moving walls and two

static walls immersed in a fluid bath to study mixing in low Reynolds number flows. They observed exponential growth of intermaterial area as expected in chaotic flows and the presence of a mixture of islands and chaotic regions as well as periodic points such as elliptic and hyperbolic point in the flow fields (Leong & Ottino 1989). Dolzhanskii et al. generated forced shear flows in a thin layer of an incompressible viscous fluid driven by electromagnetic forcing and obtained the stream function of vortical flows patterns arising once the primary shear flow loses stability from streak photographs (Dolzhanskii, Krymov, & Manin 1992).

Rothstein et. al investigated the development of persistent spatial patterns of chaotic advection in electromagnetically driven in 2D density stratified fluid layer. It was found that the stretching and folding process eventually led to structurally invariant but slowly decaying mixing patterns which was quantified by cross correlation between images separated by some time interval. When compared the time-periodic flows manifesting chaotic advection to the identically forced but weakly turbulent flows, the PDF of scalar concentration field variance revealed that the latter flows transported scalar and became homogeneous much more rapidly (Rothstein, Henry, & Gollub 1999).

Voth et al. generated 2D time-periodic flows by similar techniques and forced the flows by sinusoidal functions of electric current in attempts to determine the stretching field in the flows. DV and PTV measurements were performed and the stretching regions that coincide with the stable and unstable manifolds of hyperbolic fixed point of Poincaré map were captured (Voth, Haller, & Gollub 2002). This provides useful insight into the geometrical structure that underlines mixing. Further works by Voth et al. continued to determine the mixing rate of the flows generated from the same experimental setup (Voth, Saint, Dobler, & Gollub 2003). One interesting result obtained from the experiment was that the onset of non-periodicity did not lead to a change in either the mixing rate or the rate of growth of the mixing rate, hence, non-periodicity alone was not the primary feature that enhanced effective mixing. In addition, the mixing rate in their flows was more accurately predicted by an effective diffusion model than by the finite time Lyapunov exponents model.

Two-dimensional turbulence has also been previously simulated and studied in the laboratory. The first experimental measurement of the 2D inverse energy cascade, $E(k) \sim k^{-\frac{5}{3}}$, was obtained by Sommeria in a quasi-two-dimensional (Q2D) turbulence generated in electromagnetically driven liquid metal (i.e. mercury). The three-dimensional perturbation was suppressed by means of uniform magnetic field and it was demonstrated that turbulence can be generated by a hydrodynamic instability without any random forcing. The generated flows, however, did not achieve good spatial homogeneity and the influence of forcing anisotropy was not known (Sommeria 1986).

Cardoso et al. performed PIV measurement to obtain the instantaneous velocity field, the vorticity field and the stream function of freely decaying Q2D turbulence driven by electromagnetic forcing in a thin layer of electrolyte (Cardoso, Marteau, & Tabeling 1994). The authors investigate the global quantities (e.g. energy and enstrophy) and geometrical properties (e.g. number and size of eddies and their mean separation of particle) of the flows and noted the significant differences in the exponent of the entrophy over energy ratio from the predicted values by Batchelor (Batchelor 1969). William et al. also generated electromagnetically driven Q2D turbulent flows to study the mixing by investigating the time average spatial power spectrum, $E_\theta(k)$, of the dye concentration distribution and found the logarithmic slope of the spectrum to be steeper than the value predicted by Batchelor where such spectrum should scale as k^{-1} beyond the cutoff of the energy spectrum (Batchelor 1959). Furthermore, the magnitude of the slope was reported to increase with the wavenumber k (Williams, Marteau, & Gollub 1997).

Paret et al. revisited the experimental observation of the inverse energy cascade with an electromagnetically driven Q2D flow that satisfied the assumptions of stationarity, homogeneity and isotropy of turbulence (Paret & Tabeling 1997). The initial transient evolution towards the $k^{-\frac{5}{3}}$ stationary state was observed for the first time and was shown to be in good agreement with the scaling proposed by Kraichnan (Kraichnan 1967). Similarly, Rutgers simulated a Q2D decaying turbulence from soap films that

was reasonably isotropic and homogeneous and confirmed both power laws of the inverse energy cascade, $E(k) \sim k^{-5/3}$, and the forward enstrophy cascade, $E(k) \sim k^{-3}$ (Rutgers 1998). The effect of bottom friction in an electromagnetically driven Q2D flow, which causes the energy spectrum to deviate from the Kraichnan prediction of k^{-3} , was later on quantified by Boffetta et al (Boffetta, Cenedese, Espa, & Musacchio 2005).

An experiment that successfully simulates a new class of flows with the turbulent-like concept in the laboratory was first performed by Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a). The authors generated a fully controlled Q2D flows, with the schematic “8 in 8” streamline topology, by using a shallow layer of brine driven by multi-scale electromagnetic forcing. DV and PIV measurements were conducted and the work presented many interesting results. First, although the shape and size of the streamlines were sensitive to the forcing intensities, the multi-scale topology was found to be invariant over a broad range of Reynolds number, $600 < Re_{2D} < 9900$, i.e. the number of the stagnation points and the way they are connected to the streamlines remain the same as shown in Figure 2.6. The multi-scale laminar flows have a power-law energy spectrum, $E(k) \sim k^{-p}$ where $p \approx 2.5$ over $\frac{2\pi}{L} < k < \frac{2\pi}{\eta}$ where L and η are the outer and inner length-scales respectively. The finding also confirms the relation, $p + D_S = 3$, where the fractal dimension of the stagnation point distribution, D_s , for the experiment is 0.5.

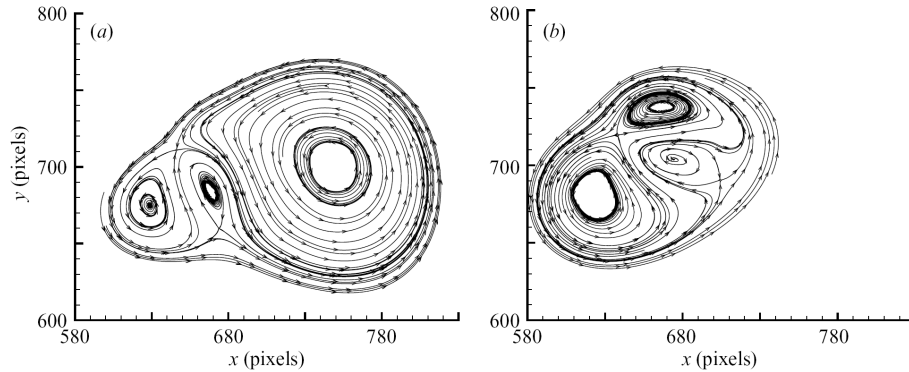


Figure 2.6: Streamlines at small scale extracted from PIV measurement (80 cm frame) for $Re_{2D} = 3600$ (left) and 9900(right). The physical length of 1 pixel is approximately 0.202 mm (Rossi et al. 2006a).

Furthermore, the single fluid element dispersion statistics of the multi-scale flows are in good agreement with Taylor’s ballistic and Brownian regimes (Taylor 1921) and the fluid pair dispersion statistics possess “Richardson-like” properties, $\langle \Delta^2(t) \rangle \sim t^\gamma$, where $\gamma > 2$. The Richardson-like behaviour of pair dispersion is a result of successive local exponential pair separation related to the smallest scale stagnation points, then the subsequent scale until the largest scale stagnation points have been reached. Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a) claims that such locality-in-scale process is directly related to the multi-scale topology of their flows.

In light of this turbulent-like flows concept, a new research direction is paved for the fundamental analysis of fully controllable flows, where properties of turbulence can be reassembled and investigated independently, which leads to a better understanding of turbulence.

2.4 Flow measurement techniques

Many flow measurement techniques have been invented to study flow dynamics, properties and behaviours. Examples of techniques that are widely used to measure two dimensional flows are particle image velocimetry (PIV), particle tracking velocimetry (PTV), dye visualisation and laser induce fluorescence (LIF) (see (Bernard & Wallace 2002) for measurement details and examples). Each measurement has different strengths and limitations, hence, selections are generally based on the quantities and properties of interest as well as the nature and constraints of the experiment.

PIV is widely used due to its direct space measurement of velocity field. The measurement uses an Eulerian approach with pattern matching techniques. PIV’s accuracy is determined by the size of each cell and the particle distribution within it. The spatial resolution is inversely proportional to the cell size and the measurement quality is a compromise between velocity and spatial resolution. Limitations of PIV are the inability to provide immediate Lagrangian statistics (the method does not track individual particles), inability to measure across the illuminated plane (parallel to the viewing direction) and the substantial computing time required to achieve optimal correlation.

However, when the spatial resolution is high (compared to flow scales) and the temporal resolution is sufficient, the velocity field obtained from PIV can be used for particle tracking in order to acquire the Lagrangian descriptions of the flow. In this case, particles are “virtually” injected into the velocity field for tracking and their Lagrangian statistics can be computed.

PTV measurement uses a Lagrangian approach to identify and follow individual particles. There are two main approaches for PTV measurement: streak method (the camera shutter is opened for a long time to capture streaklines) and time series method (multiple images containing information such as particle positions as a function of time are obtained). The latter method is capable of estimating particle positions, and hence velocities, more accurately provided that the sampling frequency is much higher than the highest frequency in the particle motion (Dalziel 2006). PTV’s accuracy depends on factors such as the particle size, the image quality, the acquisition frequency, the algorithms used to determine the particle positions etc. A number of algorithms for PTV have been developed by various researchers. Most algorithms are based on the assumption that the measurement has a small particle displacement and a smooth particle motion (Ruhnau, Guetter, Putze, & Schnorr 2005). The main disadvantage of PTV is that the spatial resolution is generally poorer than PIV as there is a limit to the number of particles to be tracked at a time. Furthermore, if the image particle density is too large, the false pairing probability becomes unacceptably large (Bernard & Wallace 2002).

For both PIV and PTV, the accuracy of the determined velocities is significantly affected by the time interval, Δt . The time interval should be small enough so that displacements in both x and y directions (Δx and Δy) are small when compared to the smallest scales in the flow but still large enough to be determined accurately. For example, if the interval and displacement are too large, small scale flow features might not be captured and PIV/PTV might mismatch particles whereas if the displacements are too small, particles might be recognised and reported to have no movements. In practice, Bernard and Wallace (Bernard & Wallace 2002) suggests that Δt is chosen so that the absolute displacement, $\Delta D = (\Delta x^2 + \Delta y^2)^{1/2}$ is approximately 20-30 times

larger than the particle diameter. However, this value is not always valid in all experiments. The selection of Δt also depends on the density of particles in the tracking area. In fact, this criterion is hardly applicable to multi-scale flows with a relatively high density of particle. To overcome this problem, an adaptive measurement has been developed and used as detailed in Chapter 5.

As our experiment in this work involves time-dependent forcing and multi-scale structure, particle displacements and velocities are expected to vary significantly both in spatial and temporal aspects. For instance, velocities near elliptic stagnation points in the smallest scales can be relatively small compared to the velocities around the hyperbolic stagnation points of the largest scales. Furthermore, the particle velocity at one instant might be of great difference to the velocity of the same particle at another instant due to the time-dependent forcing. Hence, ordinary or direct PIV is not suitable for the flow measurement in our particular experiment. PTV is considered a more favourable and practical choice due to its ability to track particles individually in a Lagrangian sense. Thus, PTV is capable of capturing fine details or resolution of velocities more accurately and measuring a rather complex pattern/behaviour of the flow. PTV's limitation on the spatial resolution is overcome by reproducibility and period combining techniques developed specifically for the measurement of our particular time-dependent multi-scale turbulent-like flow. The detail of the techniques is discussed in Chapter 5. Strict criteria apply that the flow is essentially laminar, periodic and reproducible, in other words, the measured quantities are identical within the measurement accuracy each time the experiment is repeated.

Chapter 3

Laboratory Experiments

3.1 Experimental concept

An experimental quasi-two-dimensional (Q2D) simulation of turbulent-like flows is performed in the laboratory. A shallow layer of electrolyte (brine) is activated by multi-scale electromagnetic (EM) forcing using the Lorentz body force $\mathbf{j} \times \mathbf{B}$, a vector product between the electric current density, $\mathbf{j}$ and the magnetic field, $\mathbf{B}$. Unsteadiness is introduced by time-dependent controlled forcing of $\mathbf{j}$.

3.1.1 Equation of motion

Our simulation can be quantified by the Navier Stokes equation in Eq. (3.1) which includes the gravity and extra Lorentz body forcing terms;

$$\frac{D\mathbf{u}}{Dt} + \frac{\nabla \mathbf{P}}{\rho} - \mathbf{g} = \nu \nabla^2 \mathbf{u} + \frac{\mathbf{j} \times \mathbf{B}}{\rho}, \quad (3.1)$$

where $\frac{D\mathbf{u}}{Dt}$ is the convective derivative of the velocity field $\mathbf{u}$, ∇ represents the operators $(\frac{\partial}{\partial x}, \frac{\partial}{\partial y})$, $\mathbf{P}$ denotes the pressure field and ρ and ν represent the fluid density and kinematic viscosity respectively. Incompressibility (Eq. (3.2)) also applies in our experiment.

$$\nabla \cdot \mathbf{u} = 0. \quad (3.2)$$

3.1.2 Time-dependent Lorentz body force

The Lorentz body force is assumed to be independent of the flow and mainly controlled by the electric current, I , which is equal to the uniform electric current density, $\mathbf{j}$, times the vertical cross section of the brine perpendicular to the direction of $\mathbf{j}$. The assumptions are justified in Eq. (3.3) and (3.4). The range of electric currents $I(t)$, which is directly related to the root mean square of horizontal flow velocity, u_{rms} , used in our experiment must satisfy a number of conditions:

1. The imposed electric field, $\mathbf{E}$, from electrodes is relatively large compared to the electric field, $\mathbf{u} \times \mathbf{B}$, induced by the flow velocities and the magnetic field, i.e. $|\mathbf{E}| \gg |\mathbf{u} \times \mathbf{B}|$. Hence, the electric current density, $\mathbf{j}$, is mainly controlled by the imposed electric field, $\mathbf{E}$:

$$\mathbf{j} = \sigma(\mathbf{E} + \mathbf{u} \times \mathbf{B}) \Rightarrow \mathbf{j} \approx \sigma \mathbf{E}, \quad (3.3)$$

where σ is the electrical conductivity.

2. The magnetic field, $\mathbf{B}$, generated by permanent magnets, is considered constant over time and the magnetic Reynolds number, $Re_m \equiv \mu \sigma u_{rms} l_0 \ll 1$, suggesting a negligible term $\text{curl}(\mathbf{u} \times \mathbf{B})$. Hence the magnetic induction equation is reduced to a Laplace equation (Thibault & Rossi 2003) and the Lorentz body force is independent of the flow:

$$\frac{\partial \mathbf{B}}{\partial t} = \text{curl}(\mathbf{u} \times \mathbf{B}) + \frac{1}{\mu \sigma} \nabla^2 \mathbf{B} \Rightarrow \nabla^2 \mathbf{B} \approx 0. \quad (3.4)$$

3. The root mean square of horizontal fluid velocity at the free surface of the brine, u_{rms} , which is directly related to I , satisfies the condition for the Hartman number:

$$H_a^2 \equiv \frac{f_{rms} H^2}{\nu u_{rms}} > 1, \quad (3.5)$$

where f_{rms} is the root mean square of a characteristic $\mathbf{f} \equiv \frac{(\mathbf{j} \times \mathbf{B})}{\rho}$, in order for the Lorentz body force to overcome the viscous forces including the bottom friction.

4. The Reynolds number, $Re_{3D} = \frac{u_{rms} H}{\nu}$, is not too large to ensure the laminar flow condition.

In this work, we use sinusoidal functions of current, $I(t)$, to generate the controlled time-dependent forcing. The time-dependent forcing function can be approximated as follows:

$$I(t) = \bar{I} + A \cos \frac{2\pi t}{T_f}, \quad (3.6)$$

where $\bar{I}$ is the mean current intensity, A is the amplitude of the forcing and T_f is the forcing period.

Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a) found the dependence of the steady forcing f_{rms} on u_{rms} to be $u_{rms} \sim f_{rms}^{0.85}$ for electrical currents I up to about 1A. The value, which satisfies all conditions required, is taken as the upper limit in this work. The mean horizontal flow velocity averaged over the flow field (horizontal 800×800mm square area with the centre at the stagnation point between the two largest magnets) is expected to be zero and proven to be so.

3.1.3 Multi-scale schematic streamline structure

The fractal streamline pattern of “cat’s eyes within cat’s eyes” (Figure 3.1(a)), observed in 2D turbulence with an energy spectrum $E(k) \sim k^{-p}$ where $p < 3$, is created by fractal EM forcing. The spatial distribution of the EM forcing is only determined by the size and the position of the magnets distributed just underneath the brine supporting plate as the electrical current density, $\mathbf{j}$, is uniform.

Three fractal scales are arranged; each scale is made of a magnet pair of the same size and of opposite poles, North and South as shown in Figure 3.1(b). The dimension of the square magnets that characterise the fractal scale are their horizontal sizes: $l_0 = 160\text{mm}$ (M160 magnets), $l_1 = 40\text{mm}$ (M40 magnets) and $l_2 = 10\text{mm}$ (M10 magnets). The horizontal spacing between the magnet pair is chosen to be the same size as the magnets’ horizontal dimension. The pairs are supported by iron plates thick enough to close the magnetic field. The size of the magnets is relatively small when compared to the dimension of the experimental tank (1700×1700mm²) and the EM forcing area is accounted for only 2.8% of the brine supporting plate.

The concept of the layout is to create and control stagnation points in each scale by mean of opposite forces generated by the magnet pairs. Hence, the number and positions of the stagnation points depend on the number and positions of the magnet pair in each scale. The total number of flow stagnation points depends on the fractal-like arrangement of the magnet pairs.

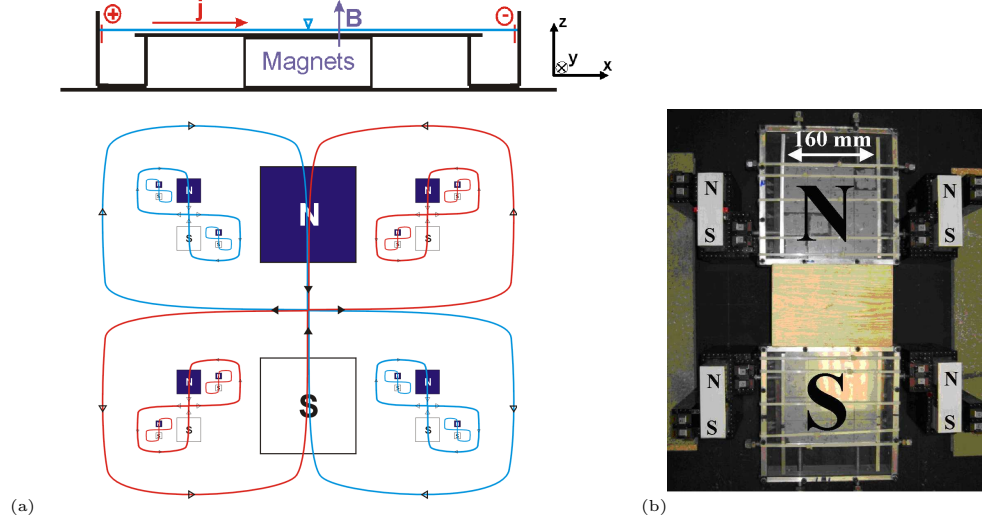


Figure 3.1: (a) schematic streamline structure of the multi-scale flow and (b) magnet arrangement underneath the experimental rig.

The horizontal layout of the magnets in Figure 3.1(a) can be described by iterative relations for the coordinates $(x_{\mathbf{N}n}, y_{\mathbf{N}n})$ and $(x_{\mathbf{S}n}, y_{\mathbf{S}n})$ of the **N**orth and **S**outh magnets in a pair and their centres (x_n, y_n) of the magnet size l_n where $(n = 0, 1, 2)$ as follows:

$$l_{n+1} = \left(\frac{1}{R_f} \right) l_n,$$

$$y_{n+1} = y_n \pm l_n,$$

$$x_{n+1} = x_n + \left(1 + \frac{1}{R_f} \right) l_n,$$

from which only points following $x_n = (1 + \frac{1}{R_f})y_n$ remain, and

$$y_{\mathbf{N}n} = y_n + l_n,$$

$$y_{\mathbf{S}n} = y_n - l_n,$$

$$x_{\mathbf{N}n} = x_{\mathbf{S}n} = x_n.$$

In this work, the aspect ratio between two consecutive scales of forcing is $R_f = 4$ and $x_0 = y_0 = 0$ lies in the centre of the tank.

The vertical distance between the magnet pair and the brine supporting plate is different for each magnet size. The distance is adjusted so as to obtain about the same Lorentz body forces above each magnet pair on the surface of the brine in the current experiment. The EM forces can be computed by the method detailed in Thibault and Rossi (Thibault & Rossi 2003). The method requires the knowledge of the brine thickness H value. The calculation result yields local mean values of the forcing above each magnet as $f_{40} = 1.025f_{160}$ and $f_{10} = 1.094f_{160}$ (Rossi, Vassilicos, & Hardalupas 2006a) when the vertical distances between the magnet pair and the brine supporting plate are adjusted at -40mm for M160, -11.2mm for M40 and -1mm for M10.

3.1.4 Quasi-two-dimensional (Q2D) electrolyte flow

The quasi-two-dimensional (Q2D) flow is simulated using a shallow layer of electrolyte (brine) as shown in Figure 3.1(a). The thickness of the brine, H must fulfil a numbers of conditions:

1. H must be very small compared to the size of the tank to obtain a Q2D flow and small enough to prevent the three dimensionality effect of flow in case of strong EM forcing.
2. H needs to be smaller than the size of the smallest magnets (10mm) as the magnetic field weakens exponentially with distance from the magnet according to the magnet size.
3. H must be large enough for the flow away from the magnets to overcome the bottom friction effect.

In the experiment, H is taken to be 5 mm, which is about half the size of the smallest magnets, to fulfil the conditions mentioned above. A single layer of brine is used in the experiment to achieve $H \cong 5\text{mm}$ as well as the stability of salt concentration, which

directly affects the brine's electrical conductivity and hence the EM forcing, over time.

The salt concentration of the brine is very high so as to minimise the vertical gradients of EM forcing within the stratified layer of brine. Conductivity dependence on salt concentration, $\frac{\partial\sigma}{\partial C}$, reduces in high concentration. We choose a salt concentration of about 158 g/l giving $\sigma \cong 16.6$ S/m, $\frac{\partial\sigma}{\partial C} \cong 0.0656$ [S/m]/[g/l] and $\rho \cong 1.1\rho_{water}$, as the salt reaches saturation at this concentration.

For quasi-two-dimensionality of a horizontal vortical flow, the vortex pressure pumping represented by the pressure gradient term in Eq. (3.1) scaled as $\frac{u_{rms}}{H}$ should be smaller than the viscous damping term scaled as $\frac{\nu u_{rms}}{H^2}$. The Reynolds number, $Re_{3D} = \frac{u_{rms}H}{\nu}$, should not be too large. Moreover, the Froude number, $Fr = \frac{u_{rms}}{\sqrt{gl}} \ll 1$, so as to avoid the pressure differences comparable to g across horizontal distances l ranging between H and l_0 (the largest magnet size) with corresponding characteristic velocities u . In our experiment, Fr lies between 0.004 and 0.073 (Rossi, Vassilicos, & Hardalupas 2006a).

Verifications of quasi-two-dimensionality for the same experimental setup as our experiment but with steady forcing were previously conducted and detailed in Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a). An initial optical approach used to test the Q2D condition over the range of forcing intensity $0A \leq I \leq 3A$ and brine thicknesses $2.5\text{mm} \leq H \leq 10.6\text{mm}$ shows no deforming effect during fluid motion suggesting a negligible vertical motion. A further posteriori verification using PIV measurement was also reported to fit well under the Q2D boundary set by the criteria which are based on the competition between the role of bottom friction and the vorticity of a single vortex in Satijn et al. (Satijn, Cense, Clercx, Van Heijst, & Verzicco 2001). The results from both verifications show that all the flows within the range considered are Q2D. This is also in good agreement with direct numerical simulation (DNS) of 2D NS with EM forcing and Rayleigh friction term simulating the bottom friction (Rossi, Hascoet, Vassilicos, & Hardalupas 2005) and 3D DNS (Lardeau & Rossi 2008).

3.2 Experimental apparatus, setup and preparation

3.2.1 Experimental apparatus and setup

The setup of the experimental simulation in the laboratory is illustrated in Figure 3.2(a). A square experimental tank is constructed with a brine supporting plate simulating the flow field in the middle. The brine supporting plate is located 150mm away from the left and right sides of the tank. The tank is filled with brine to obtain a shallow layer of a thickness $H \approx 5\text{mm}$ on the brine supporting plate. There are 43 electrodes with 40mm spacing on the left and right sides of the tank wall (Figure 3.2(b)); both sides have opposite polarities creating a uniform current running across the tank. Three different sizes of square Bonded NdFeB permanent magnets are arranged underneath the tank (Figure 3.1(b)) with the horizontal and vertical positions as described previously.



Figure 3.2: (a) Experimental rig setup and (b) electrodes at the side of the tank.

The camera is affixed to a rigid stand in the middle of the flow field above the tank and adjusted to capture the $1000 \times 1000 \text{mm}^2$ measuring flow field. Camera signals, both

exposure (when the camera captures the image) and busy (when the data is transferred), are recorded in time by the datalogger (Figure 3.3(a)) to provide the time step information. The datalogger also records the forcing by means of input current, I , and output voltage, V . Hence, the forcing intensity in each time step related to each image is monitored. The input current, I , was supplied by the power supply unit which was connected to the function generator. The function generator induced the sinusoidal waveform of the input forcing. The amplifier circuit box (Figure 3.3(c)) converted the high voltage input ($\pm 25\text{V}$) into low voltage output ($\pm 1.5\text{V}$) for the datalogger as well as adding the offset and adjusting the gain to the desirable range. As the datalogger can only measure the maximum voltage input of 2.5V , the Picolog adapter (Figure 3.3(b)) with interchangeable resistors was used to limit the amount of current flowing through the datalogger. The overall layout of the experiment equipment and apparatus is illustrated in Figure 3.4.

As we aim to perform PTV measurements with passive scalar that simulated the fluid element motion without affecting the flow, the densities of particles must be close to that of the brine. We use Chemigum P83 (white powder) and small polystyrene particles, with diameter $< 1\text{mm}$ and densities compatible to the brine, in the measurement. To estimate the difference between the Lagrangian velocities of the particles, $\mathbf{u}_P$, and the actual fluid motion velocities, $\mathbf{u}$, we consider the relation below (Adrian 1991):

$$\|\mathbf{u}_P - \mathbf{u}\| = \frac{\rho_P D_P^2 \|\mathbf{a}_P\|}{36\rho\nu}, \quad (3.7)$$

where ρ_P and ρ are the densities of the particles and the fluid respectively, D_P is the particle diameter, $\|\mathbf{a}_P\|$ is the acceleration of the particles and ν denotes the fluid kinematic viscosity.

By quantifying the potential error of the particles following the fluid motion as a percentage of the fluid velocity, $\|\mathbf{u}\|$, we have:

$$error = \frac{\|\mathbf{a}_P\|}{0.36\|\mathbf{u}\|}, \quad (3.8)$$

as $\frac{\rho_P D_P^2}{\rho\nu} \cong 1$ holds in our experiment.

Ferrari (Ferrari 2007) had estimated the error given in Eq. (3.8) in our experiment with the steady forcing setting and quantified such error to be less than 1% for most part of the flow whilst the maximum error is less than 4%.

Small amount wetting agent (Kodak Photo FLO solution) is used to reduce the brine's surface tension and capillarity effect. We use two 500W spotlights (in fixed positions for all measurements) to enhance the light intensity of the image. The light positions must be carefully adjusted so as to avoid measurement noises from light reflection.

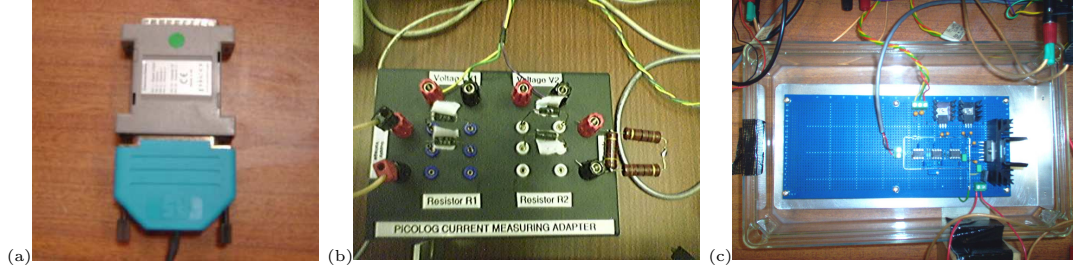


Figure 3.3: Experimental apparatus: (a) datalogger, (b) picolog adapter, (c) amplifier circuit box.

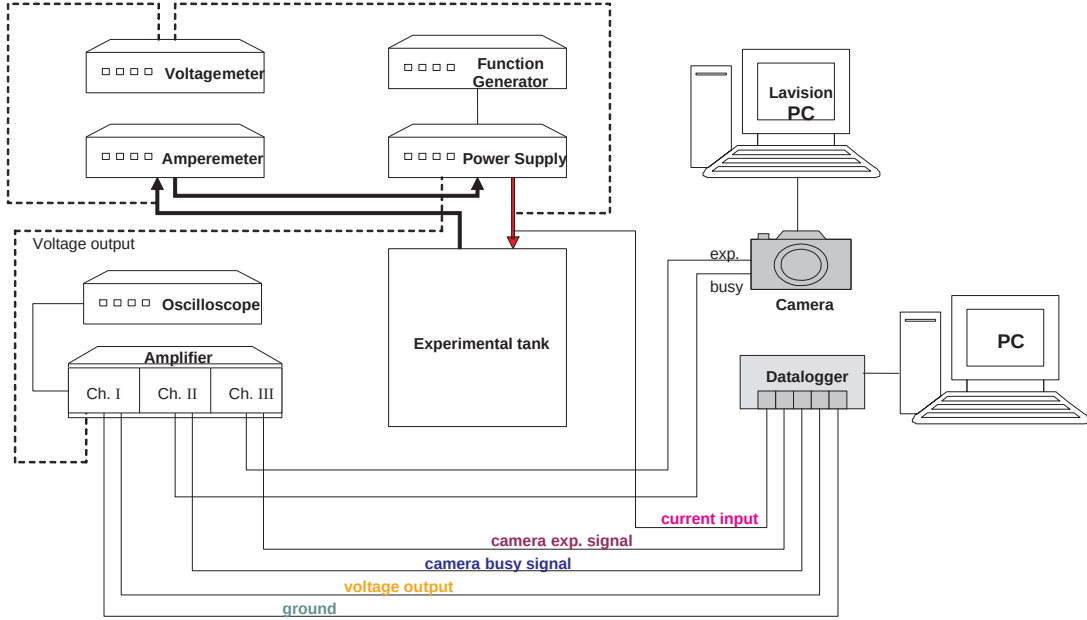


Figure 3.4: Block diagram showing the experimental circuit.

The experimental apparatus including their parameters and properties are summarised in Table 3.1 and Table 3.2.

Table 3.1: Experimental Apparatus.

Apparatus	Details
Experimental rig	Constructed in-house (See Table 3.2)
Camera	Model: Lavision ImageProPlus 4MCCD (for PTV and HDPTV measurements)
Camera	Model: Kodak MegaPlus ES 4.0 (6 mega pixel) (for dye visualisation)
LaVision computer	LaVision DaVis software (V.7 2004)
Personal computer	Picolog software (R.17 1992-2004)
Power supply*	Constructed in-house
Power Amplifier	
Amplifier circuit box*	Constructed in-house (Figure 3.3(c))
Function generator	Model: Feedback TWG501
Multimeter (2)	Model: TTI1705 (set as a voltagemeter and an amperemeter)
Oscilloscope	Model: Hameg Instrument HM303-6
Datalogger	Model: Picolog ADC-11 (Figure 3.3(a))
Picolog datalogger adapter	Constructed in-house (Figure 3.3(b))
Meter tool	Constructed in-house (Figure 3.5(a))
Calibrating plate	Constructed in-house (Figure 3.7(b))
Standing lamps (2)	500W each
Particles (Campaign I)	Polystyrene Diameter $\approx 800 \mu\text{m}$, Density ~ 1.05 compare to water
Particles (Campaign II and III)	Chemigum P83 Diameter $\approx 100\text{-}600 \mu\text{m}$, Density ~ 1.03 compare to water
Wetting agent	Kodak Photo FLO solution (Ethylene Glycol), soap
Dye colours	Orange, White and Green (Fluorescent)
Black ink	No metallic component
Bleach	Dilute

Note : * Courtesy of the Electrical and Electronics Services, Department of Mechanical-Engineering, Imperial College London.

Table 3.2: Experimental Rig components.

Component	Parameter	
Tank	Horizontal dimension	1700mm $\times$ 1700mm
Brine	Substance	NaCl
	Concentration, C	158 g/l
	Thickness, H	5 mm
	Kinematic viscosity, ν	1.326 mm ² /s
	Electrical conductivity, σ	$\cong$ 15.2 S/m
	$\partial\sigma/\partial C$	0.0656 [S/m]/[g/l]
	Density, ρ	$\sim$ 1.1 compared to water
Magnet	Type	Bonded NdFeB (permanent magnet)
	Residual induction, Br	$\cong$ 0.68T
	Horizontal dimension	160mm c 160mm (M160)
		40mm $\times$ 40mm (M40)
		10mm $\times$ 10mm (M10)
		60mm (M160)
	Vertical dimension	40mm (M40)
		10mm (M10)
Electrode	Material	Platinum (each electrode is made of a rope of 16 platinum wires)
	Diameter	11.5 μ m
	Length	40 mm

3.2.2 Pre-experimental preparation

To enhance accuracy, quality and consistency of the measurements, several pre-experimental preparations are carried out prior to each measurement campaign. The main aims are to achieve accurate experimental conditions, high quality output images and ensure that the conditions of the measurements remain as consistent and repeatable as possible throughout all realisations of the same experiment.

3.2.2.1 Experimental rig preparation

The brine supporting plate is cleaned to remove dust and impurities, and improve the smoothness. The plate is also painted in black with water based paint to provide a better contrast to particles and hence enhance the output image quality. It is crucial to check the electrodes along both sides of the tank and replace any broken ones to ensure a uniform current distribution across the flow field. We use a voltagermeter to check that the current runs through each individual electrode. The copper parts of the

electrodes are coated with water resistant paint to prevent corrosion. Moreover, we cover any equipment and apparatus that reflect light to the measuring flow field with black matt paper to reduce noise from light reflection in the measurements.

3.2.2.2 Brine layer adjustment

To ensure that the level of the brine is accurate at $H \approx 5\text{mm}$ with maximum deviation from H under 0.2mm , we conduct a brine thickness measurement using a meter tool (Figure 3.5) to measure the 7×7 points (every 180 mm apart) across the flow field. The supporters underneath the experimental rig are adjustable allowing corrections to the height of the bottom wall to be made locally which directly affect the brine level in particular areas. The water loss due to evaporation is measured to be approximately 0.3mm/day ; hence, the total change in H during a six-hour measurement is under 0.1mm and is considered negligible.

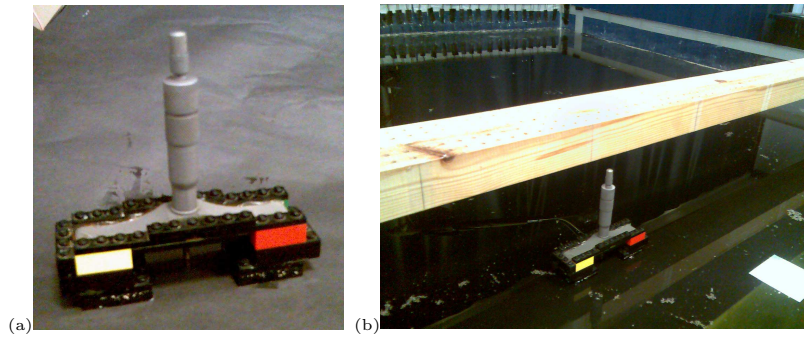


Figure 3.5: (a) Meter tool with a precision of $1/100$ inch (b) Meter tool being used to measure the brine layer thickness.

3.2.2.3 Resistance and conductivity of the brine layer

To measure the resistance of the brine layer, the experimental rig was setup normally as shown in Figure 3.2(a). The thickness of the brine layer H was adjusted to 5mm . We then switched off the function generator so that constant direct current was passed through the brine layer. By adjusting the power supply setting, a series of voltage readings was recorded and the corresponding values of current were measured with an amperemeter. Figure 3.6 shows the plot of voltage, V , against current, I . A straight

line was fitted in a least mean square sense and the gradient, which is equal to the resistance of the layer of brine was obtained.

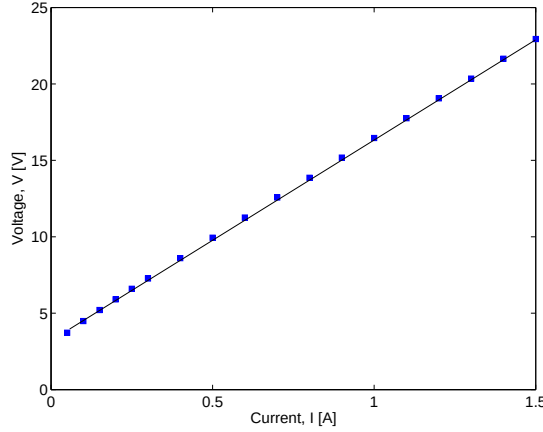


Figure 3.6: Approximation of the Ohm's law of the brine layer.

The measured Ohm's law for different experimental campaigns are as follows:

$$\text{Campaign I} \quad : \quad V = 13.294I + 3.421,$$

$$\text{Campaign II} \quad : \quad V = 13.102I + 3.445,$$

$$\text{Campaign III} \quad : \quad V = 13.176I + 3.265.$$

Hence, the resistance, R of the three campaigns are therefore 13.294Ω , 13.102Ω and 13.176Ω respectively.

The conductivity, σ , is given by $\sigma = l_c / RA_c$ where l_c and A_c denote the length and the cross sectional area of the conductor i.e. the brine layer. In the experiment, with $l_c = 1700\text{mm}$ and $A_c = 1700 \times 5 \text{ mm}^2$, the conductivity, σ , for the three experimental campaigns are 15.044S/m , 15.265S/m and 15.179S/m respectively.

3.2.2.4 Testing of the time-dependent forcing system

The function generator was used to generate the sinusoidal waveform for the time-dependent forcing system. We tested the accuracy and the quality (i.e. reproducibility and periodicity) of this sinusoidal forcing before applying it in the measurement. We recorded the voltage readings of the sinusoidal forcing over 20 cycles where the period

was set to 15 seconds with the voltage range of 0.25V-2.5V. The peak-to-peak amplitude and the forcing period as well as their standard deviations were identified. The test was repeated for seven times.

It was found that the standard deviation of the amplitude was under 1% whereas the forcing period reduced from 0.185% to 0.011% over the three hours of the testing period. Hence, before conducting the actual experiment, we connected a dummy resistor, which had the same resistance as the brine layer, to the forcing circuit and left the power supply on for at least three hours so as to allow the amplifier circuit box to heat up and the temperature stabilised. The resistor was then removed and the forcing circuit was connected to the tank, which was left on for further 10 minutes to allow any chemical reactions to take place and stabilise. The experimental measurement could then be conducted after this period. During the experiment, we checked the recorded forcing data at the end of each measurement to ensure consistent quality of the forcing throughout the entire campaign.

3.3 Experimental measurements

We performed two main types of flow measurements, Dye Visualisation (DV) and Particle Tracking Velocimetry (PTV) measurement. While dye visualisation provides an apparent and useful qualitative visual aid, PTV on the other hand provides quantitative information of the flow which is essential for flow analysis. Three experimental campaigns which differ in two main parameters, the forcing frequency and forcing intensity, are studied. The preliminary analysis for parameter selections in different experimental campaigns are detailed in Chapter 4. In the following sections, the measurement methodologies are present.

3.3.1 Particle tracking velocimetry (PTV)

In our particular experiment, we aim to simulate fully controlled Q2D flows that are periodic and reproducible in order to perform Particle Tracking Velocimetry (PTV) measurements and superimpose their particle statistics of the same flow (at the same

phase) to obtain good spatial resolution. To achieve such conditions, the flows are forced with approximated sinusoidal functions of current, $I(t) = \bar{I} + A \cos \frac{2\pi t}{T_f}$. Furthermore, we ensure that the forced flows essentially remains laminar with low Re_{3D} , low to moderate Re_{2D} and low velocity, hence, the mean features of the flow are considered reproducible or repeatable. With the flow reproducibility assumption, we conduct many realisations of PTV measurement of the same flow with the same forcing condition in order to obtain sufficient spatial particle statistics. The measurement of the same flow might extend over days therefore a number of calibration procedures are conducted aiming to sustain consistent experimental conditions.

To begin the PTV measurement, the brine level is measured at the beginning of the experiment each day. The brine level is then adjusted to 5 mm by adding some water to compensate the amount lost due to evaporation (approximately 0.3mm/day). We then capture the reference images i.e. the dark images, the calibration images, and the background images respectively. The dark images are taken with the camera's lenses cap on and with the light in the laboratory switched off. The calibration plate (Figure 3.11) is then used to calibrate the position of the camera. The plate is placed in the middle of the experimental tank at 350 mm away from each edge of the tank. The camera position is then adjusted according to the calibration plate and the calibration images are taken. The background images are images of the flow field without any particles. A few droplets of diluted wetting agent, which is used to reduce the surface tension of the brine, can be added in the middle of the tank. The substance effectively pushes particles outward to the sides of the tank leaving a clear particle-free flow field. The reference images, particularly the calibration and background images, are taken again every time that there are changes in the measurement conditions e.g. camera position and light level.

The current forcing system, which is composed of a power supplier, a signal generator and a signal amplifier, are switched on to warm up for approximately 1 hour prior to the measurement in order to allow the system to stabilise. Drops in frequency and forcing intensity are generally perceived during this time.

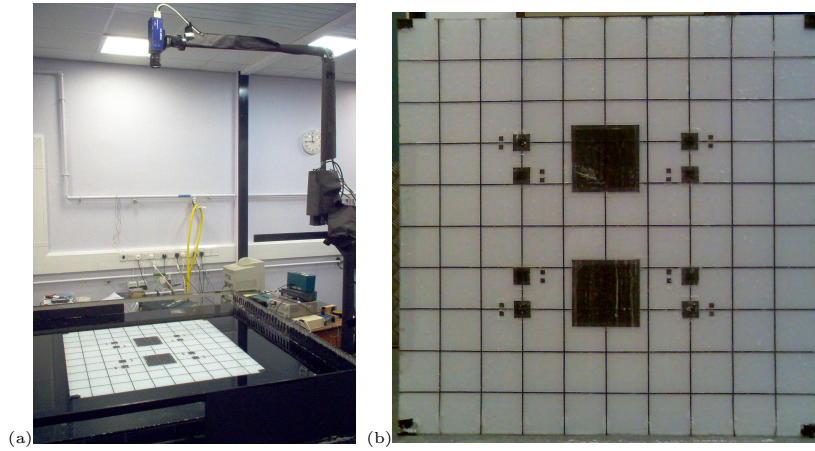


Figure 3.7: (a) Calibration (b) Calibrating plate.

In the first experimental campaign I, small white polystyrene particles with density compatible to that of the brine, are used. Polystyrene particles are quite uniform in size and bright hence they provide good contrast to the black background enabling particles to be identified in the PTV images. We prepare polystyrene particles by heating them in hot water until the particle expands to the size approximately ≤ 1 mm in diameter (by visual observation) then we add cold water to stop the expansion. Heating the particles also tend to help reduce particle static.

After the calibrations are complete and the forcing system stabilises, we add polystyrene particles to the tank while the forcing is on and manually mix them with a plastic rod trying to obtain a homogenous particle distribution across the flow field and breaking up any particle clusters. Polystyrene particles tend to attract each other when the distances between the particles are small ($<$ half of the particle diameter) as a result of the particle static and the capillarity effect. We add a small amount of soap to reduce the capillarity effect, however the effect cannot be totally eliminated. The extra forces from the particle static and the capillarity effect are considered negligible in our flow as they only become significant at the lengthscale comparable to half of the particle diameter, which is much smaller (> 25 times) than the smallest characteristic length-scale (M10) in the flow.

To avoid particle clusters, the density of the particle distribution in the flow is compromised. In general, there should be roughly at least one particle in every $30 \times 30 \text{ mm}^2$ in the flow field; this provides approximately 1,000 particle statistics per realisation. Some particles are deliberately put above the smallest magnets to ensure that small scale statistics are obtained. After mixing the particle manually, we leave the flow to settle to its own motion for approximately 2-3 periods, or the characteristic turnover times of the excited scale whichever one is longer, before starting to collect data for the flow measurement.

We start the data collection at the minimum current of the sinusoidal forcing. Great care has been taken in trying to start the measurement and collect data at the same instant of the forcing period. This is important in order for the data from all periods (at the same phase) to be superimposed in the post-processing stage. The Lavision ImageProPlus 4MCCD camera (resolution 2048×2048 pixel) captures output images at a frequency of 12-13Hz. The camera shutter speed is $8900 \mu\text{s}$ which is fast enough to catch up with the capturing intervals (0.0769-0.0833s) while allowing sufficient light to be captured. The datalogger is used to record the forcing (in form of current and voltage) and camera's trigger; hence, the exact moment each image is taken can be monitored. The camera stops the recording once the data collection is complete (~ 10 periods or 150s whichever one is longer). The duration from the start to the end of the recording is counted as one realisation. After each realisation, particles are manually mixed again before continuing to the next measurement.

In order to achieve high enough spatial resolution of the velocity field to resolve the smallest length scale in the flow, the velocity grid size must be smaller than the size of the smallest length scale which can be characterised by the M10 magnet size. Thus, we aim to obtain at least 250,000 particle statistics in each forcing case to allow for the velocity field with a grid size smaller than the M10 magnet size (≈ 20 pixel) to be computed (see Section 5.5 for details).

In experimental campaign I, one time-dependent forcing case, where $I(t) \approx 0.4 + 0.25 \cos \frac{2\pi t}{15}$, is performed. One realisation lasts for 10 forcing periods i.e. 150s. To

obtain sufficient particle statistics after superimposing data from all periods, at least 25 realisations are required. We perform 60 realisations of PTV measurement in Campaign I to achieve high particle statistics and take into the account of possible loss of statistics e.g. due to data post processing. Note that new particles are added to the PTV flow field every ten realisations as many particles tend to stick to the sides of the tank.

In experimental campaign II and II, we seek to increase the particle statistics and reduce the workload in the laboratory by performing PTV measurements with higher density of particles. We followed the same procedures as in Campaign I but the major difference is that Chemigum P83 (white powder) is used as particles instead of polystyrene. By using Chemigum, higher particle statistics per realisation can be obtained as Chemigum (diameter $\approx 100\text{-}600\ \mu\text{m}$) is much smaller than polystyrene (diameter $\approx 800\ \mu\text{m}$) and importantly, Chemigum exhibits less capillarity effect and its clusters can be separated by the flow; this also provides a result that is very close to flow measurement by dye. Small amount of diluted wetting agent is used to further reduce the capillarity effect.

Nevertheless, using Chemigum particles also have some disadvantages; Chemigum has less uniformity of the particle size, making image processing and particle tracking in the data post-processing stage more difficult; the trajectories obtained are noisy. Moreover, Chemigum's brightness reduces over time, possibly due to chemical reaction with the brine and bleach residue, resulting in low quality output images as time proceeds.

We use a fine colander to filter and distribute Chemigum into the measuring flow field to obtain particle sizes as regular as possible. As Chemigum particles exhibit much less static effect, they can be distributed denser with particles being much closer to each other. We aim to obtain at least 3-4 Chemigum particles in every $30\times 30\text{mm}^2$ in the flow field; this provides approximately 4,000 particle statistics per realisation. Every 10 realisations, we add fresh Chemigum powder so the particles remain bright throughout the measurement.

The motion of the flow in our experiment tends to advect particles away particularly from the small scale stagnation points leaving empty island areas in the flow causing difficulties in obtaining small scale statistics. Several techniques were tested in attempts to obtain particle statistics in small scale areas. We added small amount of Chemigum around the small scale areas and quickly but gently mix to separate clusters at the beginning of each realisation to ensure that there are particles in those areas at the start of the measurement. Moreover, we tried adding 2-3 small polystyrene particles around small scale stagnation points to enhance the statistics. It was also found during the experiment that polystyrene and Chemigum do not tend to attract. This finding can lead to an improvement in the experiment using a mixture of Chemigum and small amount of polystyrene particles together.

We repeated 25-40 realisations of PTV measurement with high density of particles in Campaign II and III. The PTV measurements were performed for at least 10 forcing periods or 150s (whichever one is longer) for all forcing cases. From a brief image post-processing of the high density PTV measurement result, it was found that at least 2,500 particle statistics can be obtained in each realisation with at least 1,500 particle statistics remain longer than 9 consecutive image sequences and approximately 800 particle statistics that remain traceable throughout the realisation. Hence, with the data superimposing technique which combines statistics together from all periods (at the same phase), it can be estimated that 25 realisations of high density PTV measurement can provide sufficient particle statistics ($\geq 250,000$) required for the construction of the Eulerian velocity field with sufficiently high spatial resolution.

3.3.2 Dye visualisation (DV)

Dye visualisation (DV) provides qualitative measurements which are useful as visual aids, references and simple validation for the PTV data. DV also clearly illustrates changes in the stirring mechanism of the flow as a result of the addition of unsteadiness (see Section 7.1). In this work, a standard 6 megapixel digital camera is used to take image series of dyes in the flow at a frequency of 2-3 Hz. Three water based dye colours, white, red and green (using fluorescence) are injected to the flow field, closed

to the small and medium flow scales, and the stirring evolution is captured. Very low concentration of bleach is used to remove the colours at the end of the measurement.

Chapter 4

Time-Dependent Parameters

One of the main focuses in this work is to study the effect of time dependency on the multi-scale flow characteristics and properties. In the experiment, a sinusoidal function of forcing is used to drive the flow causing it to be time-dependent. Two temporal parameters of the forcing are studied, the forcing period, which is directly related to the forcing frequency, and the forcing intensity i.e. the mean forcing intensity and the forcing magnitude. To select the range of interest of these temporal parameters, we perform a preliminary analysis based on the relationship between the length scale and the time scale of the flow at different forcing intensities. The length scale is referred to the physical size of the three imposed scales in the flow while the time scale is defined by the characteristic turnover time i.e. the time it takes for the flow to complete the associated length scale. The analysis outcome provides a “forcing map” that is used to select the ranges of forcing intensity and frequency for the experiments accordingly.

4.1 Preliminary analysis for parameter selection

To begin the analysis, we estimate the characteristic turnover times at different forcing intensities from the previous experimental steady forcing data in Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a), (Rossi, Vassilicos, & Hardalupas 2006b). The characteristic turnover times can be approximated by various methods. The general approach is to estimate the ratios between the characteristic length-scales and the flow velocity. The characteristic length scales can be taken as the size of the magnets, L_{M_i} , or the size of streamlines connected to the hyperbolic stagnation points in each flow

scale, L_{SP_i} where $i=10, 40, 100$ denotes the small, medium and large flow scale. The flow velocity used in the estimation can be the root mean square of the flow velocity (entire flow), u_{rms} , or the local flow velocity of the streamlines connected to the hyperbolic stagnation points, u_{SP_i} . Different methods for the characteristic turnover time approximation are summarised in the following sections:

4.1.1 Magnet size and root mean square of flow velocity ratio

In this method, the characteristic turnover time is taken as a ratio between the magnet size, L_{M_i} , and the root mean square of flow velocity, u_{rms} , given in Table 4.1:

$$T_{M_i} = \frac{L_{M_i}}{u_{rms}}.$$

Table 4.1: Typical scale of the steady multi-scale flow according to the forcing intensity; L_E is the integral length scale defined by the method in Section 6.4 (Source: courtesy of Rossi et al. (2006a)).

I [A]	u_{rms} [mm/s]	u_{max} [mm/s]	L_E [mm]
0.04	0.977	3.1	156.1
0.06	1.40	4.5	155.9
0.08	1.76	5.3	157.5
0.1	2.09	6.2	158.6
0.15	3.06	8.6	162
0.2	3.83	11.1	165.1
0.3	5.84	15.6	172.2
0.4	7.48	19.9	177.8
0.53	9.55	25	183.1
0.7	12.1	31	189.3
1	16.1	40	195.3

4.1.2 Stagnation point size and root mean square of flow velocity

The characteristic turnover time can also be estimated as a ratio between the size of the streamlines connected to the hyperbolic stagnation points and the root mean square of flow velocity:

$$T_{SP_i} = \frac{L_{SP_i}}{u_{rms}}.$$

The value L_{SP_i} is the measure of the distance from the stagnation point $\mathbf{x}_s$ where

the streamlines very close to the stagnation point take a turn (Rossi, Vassilicos, & Hardalupas 2006b). To calculate L_{SP_i} , Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006b) integrated the velocity $\mathbf{u}$ from PIV data along a streamline starting from a point $\mathbf{x}_s + \epsilon \mathbf{e}_\theta$ very close to a hyperbolic stagnation point $\mathbf{x}_s$ until it reaches a point $\mathbf{x}_l(\theta)$ when the quantity $\mathbf{u} \cdot \mathbf{e}_\theta$ changes sign suggesting a change in the direction of the streamline. The value ϵ is a very small distance (smaller than the PIV grid size) and $\mathbf{e}_\theta$ is a unit vector of angle θ . Hence, L_{SP_i} is defined as:

$$L_{SP_i} \equiv \langle |\mathbf{x}_l(\theta) - \mathbf{x}_s| \rangle_\theta,$$

where the average $\langle \dots \rangle_\theta$ is taken over θ . The data of L_{SP_i} sizes, as shown in Figure 4.1(a) is courtesy of Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006b).

4.1.3 Strain rate method

For an incompressible flow, stagnation points are characterised by a single positive strain rate or rotation rate, λ which is an eigenvalue of $\mathbf{J}_{ij} = \frac{\partial u_i}{\partial x_j}$ at the stagnation point ($\mathbf{J}\mathbf{X} = \pm\lambda\mathbf{X}$ for hyperbolic and $\mathbf{J}\mathbf{X} = i\pm\lambda\mathbf{X}$ for elliptic where $\mathbf{X}$ are eigenvectors). The strain rate, λ_i , is of unit 1/s and can be taken as an inverse of the characteristic turnover time:

$$T_{\lambda_i} = \frac{1}{\lambda_i}.$$

The strain rate values at different Re_{2D} are presented in Figure 4.1(b).

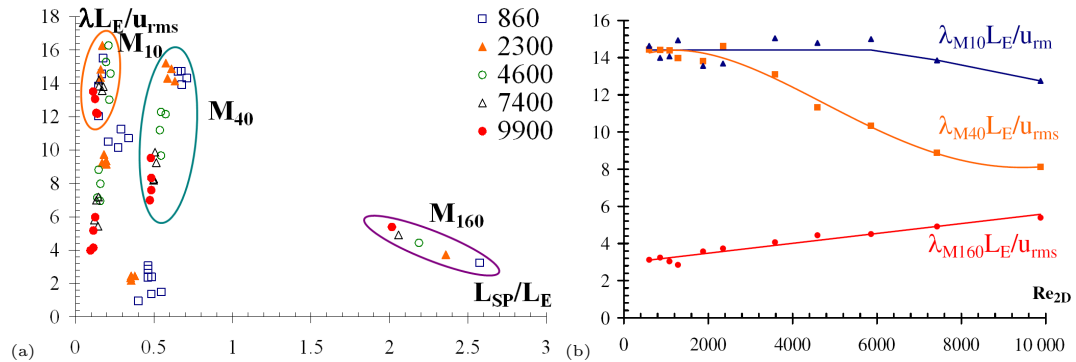


Figure 4.1: (a) Scatter plot of hyperbolic stagnation point strain rates, $\lambda L_E / u_{rms}$ and sizes, L_{SP} / L_E , (b) Dimensionless mean stagnation point strain rate versus Re_{2D} (Rossi et al. 2006b)).

4.1.4 Characteristic feeding time method

The characteristic feeding time is defined as:

$$T_{F_i} = \frac{L_{SP_i}}{u_{SP_i}},$$

where u_{SP_i} is the local flow velocity around the stagnation point.

For a steady flow, the characteristic feeding time, T_{F_i} , is thought of as the time for fluid elements to enter the dispersive area (hyperbolic stagnation points) and travel around a scale before repeating in loops. At $t < T_{F_i}$ fluid elements enter the dispersive area and for $t > T_{F_i}$, the fluid elements are looping around a scale. Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006b) has reported the characteristic feeding time, non-dimensionally as $T^* = \frac{T_{F_i} u_{rms}}{L_E}$ as presented in Figure 4.2. The typical feeding time scales are also reported to be:

$$\begin{aligned} T_{F_{10}} &= 0.27 \left(\frac{L_E}{u_{rms}} \right), \\ T_{F_{40}} &= 0.4 \left(\frac{L_E}{u_{rms}} \right), \\ 1.5 \left(\frac{L_E}{u_{rms}} \right) &\leq T_{F_{160}} \leq 2.5 \left(\frac{L_E}{u_{rms}} \right). \end{aligned}$$

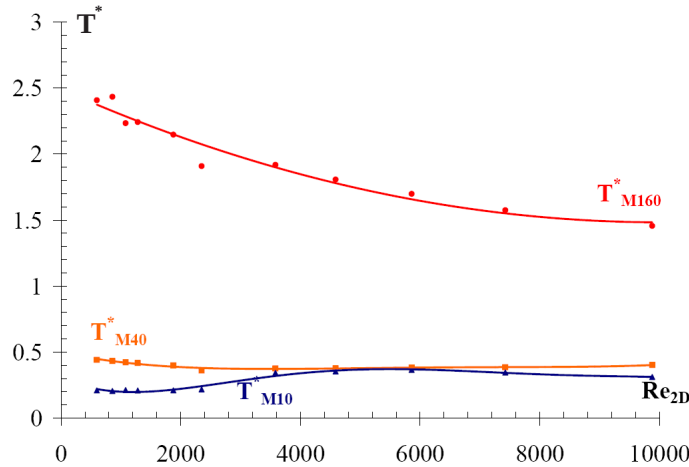


Figure 4.2: Dimensionless feeding time versus Re_{2D} (based on data in Rossi et al. (2006b)).

4.1.5 Spectral method

Local flow velocity associated to each length scale in the flow can also be estimated from the amount of energy contained in each specific length scale. The spectral method estimates the energy contained in each length scale by integrating the energy spectrum $\frac{E(k^*)}{E}$ (Figure 4.3), where E is the total energy in the flow i.e. $E = \int_{-\infty}^{\infty} E(k^*) dk^*$ and k^* is the wave number normalised by the wave number of the smallest M10 magnet, over the specific range of length scale. From the plot shown in Figure 4.3, in the region of interest, $\frac{E(k^*)}{E}$ can be approximated with the function $k^{*-2.5}$ i.e.:

$$\begin{aligned} \log_{10} \left(\frac{E(k^*)}{E} \right) &\approx \log_{10}(k^{*-2.5}) + C, \\ E(k^*) &\approx C_1 E k^{*-2.5}, \end{aligned}$$

where the constant C_1 can be found directly from Figure 4.3. Therefore, the amount of energy contained in each characteristic length scale can be approximated by:

$$\int_{k^* \in M_i} E(k^*) dk^* \approx C_1 E \int_{k^* \in M_i} k^{*-2.5} dk^*,$$

where M_i denotes the range of length-scale, $i = 10, 40$ and 160 .

The range M_{10} is taken as the length-scale from the size equivalent to the M10 magnet up to the M40 magnet. Similarly, the range M_{40} takes the value from the length-scale of the M40 magnet to the M160 magnet and the range M_{160} takes the value from the length-scale of the M160 magnet onward. By evaluating the above integral, the energy contained in each of the three flow scales, E_{M_i} , are obtained as follows:

$$\begin{aligned} E_{M_{10}} &= 0.023E, \\ E_{M_{40}} &= 0.208E, \\ E_{M_{160}} &= 0.768E. \end{aligned}$$

As $E \approx u_{rms}^2$, the local root mean square of flow velocity for each range of length-scale, u_{rms, M_i} , can be calculated from the energy relation given above. Finally, the characteristic turnover time approximated by the spectral method is defined as

$$T_{E(k)_i} = \frac{L_{SP_i}}{u_{rms, M_i}}.$$

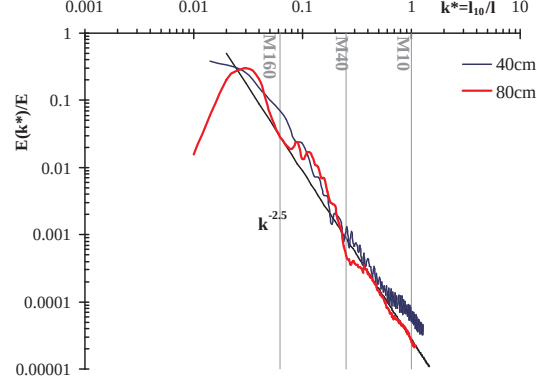


Figure 4.3: Average flow energy spectrum from PIV data (courtesy of Rossi et al. (2006a))

4.1.6 Parameter range of interests

The approximated characteristic turnover times obtained from the methods described are presented in Figure 4.4. Each method yields three time scales at various forcing as shown in Figure 4.4.

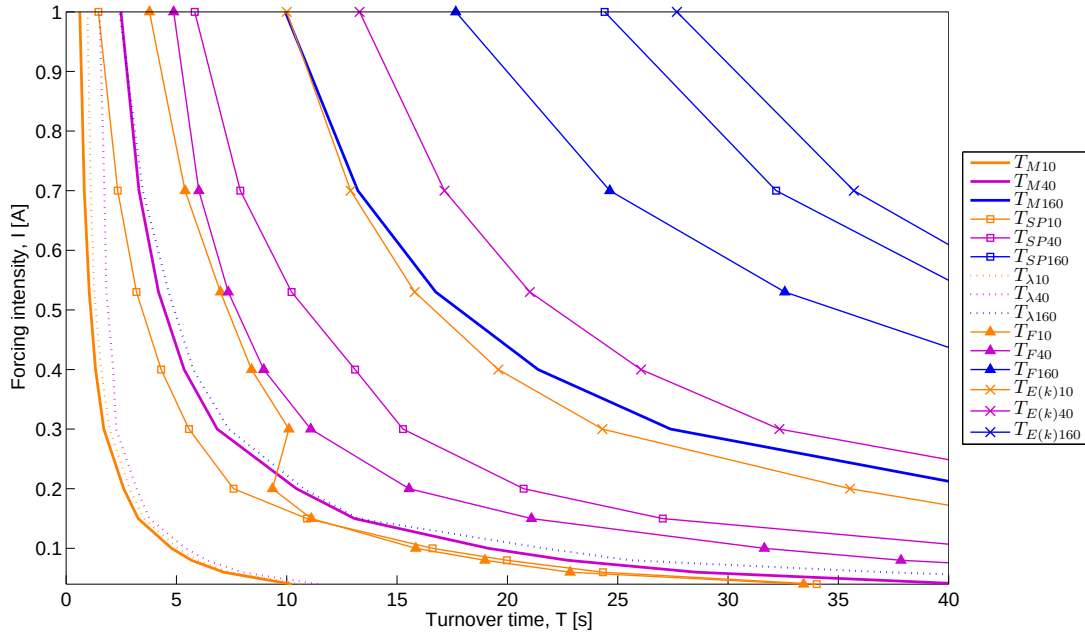


Figure 4.4: Approximation of the characteristic turnover times within the forcing range $0.1A \leq I \leq 1A$ of three characteristic length scales: yellow colour represents the small M10 scale, pink and blue represent the medium M40 and large M160 scales respectively.

From the approximated characteristic turnover times data, we can classify areas of interest by grouping together the turnover times of each length scale. The characteristic turnover times approximated by the strain rate and spectral method appeared to be scaled differently compared to other methods and therefore are not included in the grouping. Another important factor to consider in the classification is the boundary where the bottom friction has been overcome by the EM forcing at $I = 0.53A$ (Rossi et al. 2006a). In the steady multi-scale flows, the stagnation point strain rates become distinct and ordered according to the stagnation point length-scale after $I > 0.53A$. Beyond this limit, the power law exponent of the pair dispersion, γ , also becomes independent of Re_{2D} and is close to 3. Hence, flows below and above the limit at $I = 0.53A$ might have different trends and properties and are classified as separate families.

Figure 4.5 shows the classified areas of interest (I-VIII). The shading areas represent the boundaries of the approximated turnover times by various methods (yellow, pink and blue represent the boundary of the small (M10), medium (M40) and large (M160) scales respectively). The clear areas represent time scales in between two length scales. From Figure 4.5, we can select the forcing intensity and the forcing period so as to excite a particular individual scale or many scales altogether in the flow.

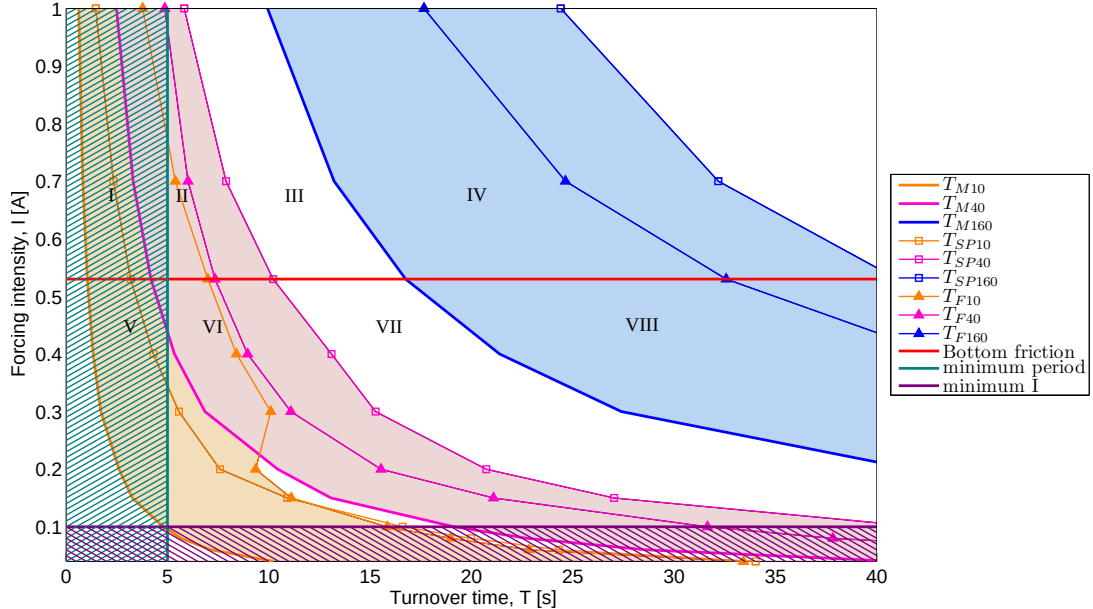


Figure 4.5: Classification of areas of interest shown with experimental constraints.

4.1.7 Experimental constraints

Our current experimental apparatus has several technical constraints:

1. The minimum forcing intensity is limited to 0.1A due to the accuracy of the power supply and datalogger that can supply and record the forcing intensity with limited significant figure. Moreover, the forcing error becomes relatively large when the forcing intensity are very small.
2. The maximum forcing intensity is limited to 1A due to constraints by the power supply, the experimental conditions and the maximum particle displacement per time step, d_{max} . High intensity of forcing above this level leads to significant increases in the temperature and chemical reactions of the brine in the experimental tank. High forcing intensity also leads to a faster flow and large d_{max} which is undesirable; d_{max} should be much smaller than the smallest M10 scale to be able to resolve it and prevent particle mismatching in PTV.
3. The minimum forcing amplitude is limited to $\Delta I \geq 0.115\text{A}$ also due to the limited accuracy of the power supply below this value.
4. The minimum forcing period is 5s restricted for the time resolution of the recorded data. The maximum frequency that the current camera can achieve is approximately 12-13Hz, hence only up to 60-65 images can be recorded in 1 period of 5s. The total number of images per period must remain large when compared to the number of images used for the polynomial fitting of trajectories for spatial quality of the velocity approximation in the data post-processing stage (details in section 5.3.3). In addition, previous PIV data show that 5s is far less than the turnover times for the largest scale and limits the effect of the time dependency on large scale motions. Visual observations in the experiment confirm that at the forcing periods less than 5s, we mostly observe oscillating motions resulting from high forcing frequency, rather than the desirable stirring and stretching/folding between scales. There is no limit for the maximum period, nevertheless, it should be noted that long forcing periods associating to low forcing frequencies lead to slow changes in the flow dynamics and characteristics.

From the technical constraints mentioned, the areas of interest are limited as shown in Figure 4.5. It can be observed that the areas labeled I is not accessible with the current experimental apparatus.

4.2 Experimental campaigns and flow families

As previously mentioned in Chapter 3, the general form of the sinusoidal function of forcing used in our experiment can be approximated as:

$$I(t) = n \left[\bar{I} + A \cos \left(2\pi \frac{t}{T_f} \right) \right],$$

where $\bar{I}$ is the mean current intensity, A is the forcing amplitude and T_f is the forcing period.

We carefully select different range of forcing parameters, n , $\bar{I}$, A and T_f for the experiment in order to cover all areas of interest shown in Figure 4.5 and allow cross comparison among different forcing cases. Three experimental campaigns, composed of 12 different forcing cases were performed. The details of the experimental methodologies were discussed in Section 3.3.1-3.3.2.

In the experimental campaign I and II, we aim to study the effect of the forcing period, hence we simulate flows with $T_f = 7.5\text{s}, 10, 12.5\text{s}, 15\text{s}$ and 18s (cases C1-C4 and C6) while keeping other forcing parameters constant. An additional forcing case C5, with $T_f = 12.5\text{s}$ and $\bar{I} = 0.3\text{A}$ was also simulated in order to investigate the effect of the mean forcing intensity $\bar{I}$ when compared the case C4. The forcing cases in the campaign I and II are chosen so as to excite more than one characteristic flow scale according to the forcing map in Figure 4.5.

In the experimental campaign III, we chose to collect sample data from each region according to Figure 4.5 and hence excited one characteristic flow scale at a time. We forced the flow with $T_f = 5\text{s}, 10\text{s}$ and 15s and $n=1, 1.26, 2.25$ and 2.505 while keeping either the forcing period, the relative forcing range (ratio between the forcing magnitude and the mean value, $\frac{\Delta I}{I_{mean}}$) or the forcing magnitude the same for cross comparison.

All different forcing cases are illustrated in Figure 4.6 and summarised in Table 4.2.

From the three experimental campaigns, we can categorise all considered flows into three main flow families, based on their forcing intensity ranges and the number of flow scales the forcing cover (Figure 4.6). The three flow families, which are expected to display different flow dynamics and stirring behaviours are listed as follows:

1. The flow family I includes all forcing cases whose forcing intensities cover more than one flow scale i.e. the forcing cases C1-C6.
2. The flow family II includes the forcing cases whose forcing intensities cover only one flow scale under the bottom friction effect threshold at $I = 0.53A$. The forcing cases in flow family II are C7, C9 and C11.
3. The flow family III includes the forcing cases whose forcing intensities cover only one flow scale above the bottom friction effect threshold at $I = 0.53A$. The forcing cases in flow family III are C8, C10 and C12.

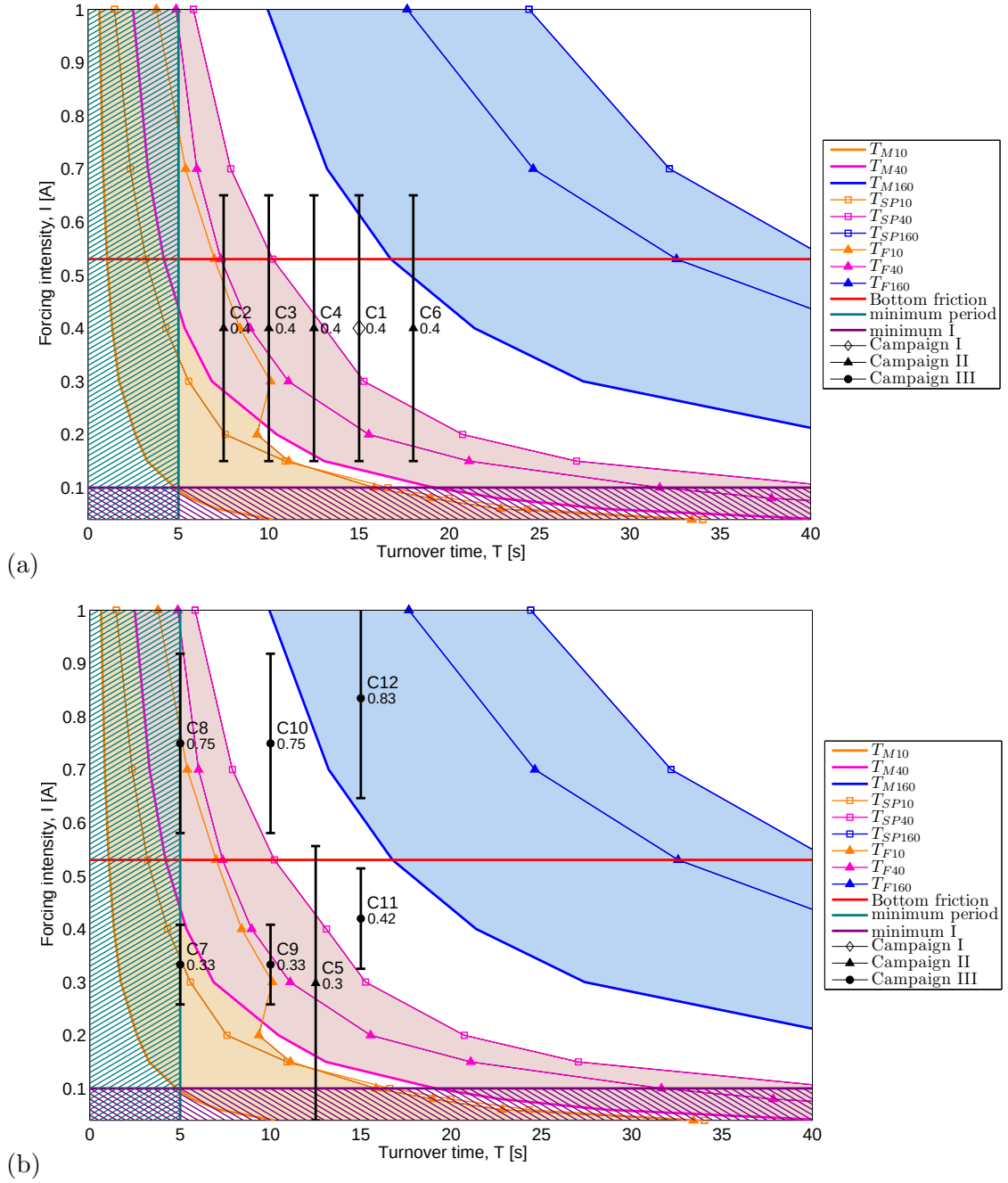


Figure 4.6: Different forcing cases in the three experimental campaigns.

Table 4.2: Summary of experimental campaigns.

Experiment	Case	Measurement conducted	Parameter studied $I(t) = n \left[\bar{I} + A \cos(2\pi \frac{t}{T_f}) \right]$				Number of realisations	Camera frequency	Brine thickness
			n	$I_{mean}, \bar{I}$ [A]	Amplitude, A [A]	Period, T_f [s]			
Campaign I	C1	PTV(LD), DV	1.000	0.400	0.250	15	60	13	5.014±0.157
	C2		1.000	0.400	0.250	7.5	40		
	C3		1.000	0.400	0.250	10	10		
Campaign II	C4	PTV(HD)	1.000	0.400	0.250	12.5	10	12	5.051±0.157
	C5		1.000	0.298	0.250	12.5	10		
	C6		1.000	0.400	0.250	18	40		
	C7		1.000	0.333	0.075	5	25		
Campaign III	C8	PTV(HD)	2.250	0.333	0.075	5	25		5.035±0.168
	C9		1.000	0.333	0.075	10	25	12	
	C10		2.250	0.333	0.075	10	25		
	C11		1.260	0.333	0.075	15	25		
	C12		2.505	0.333	0.075	15	25		

Chapter 5

Experimental Data Post-Processing

The outputs of the PTV measurements from the experiment are in the form of image sequences. Each image contains particles in the measured flow field at an associated time step. To interpret the image sequences into quantitative particle statistics, a number of data post-processing procedures are performed as outlined in Figure 5.1.

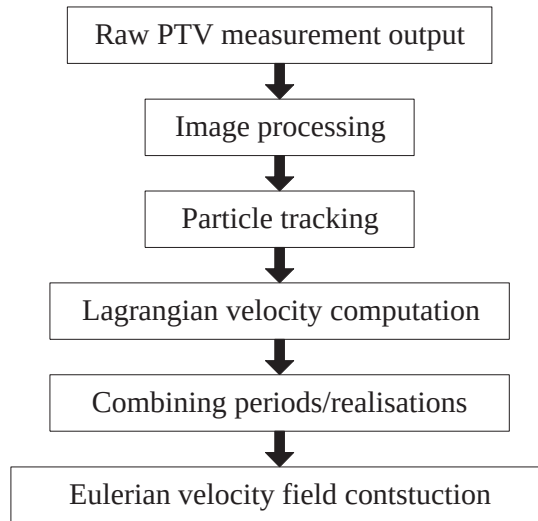


Figure 5.1: Experimental data post-processing.

The resolution of the output image is 2048×2048 pixels which represents the approximately 1000×1000 mm physical measured flow field. The physical length of one pixel is about 0.4883 mm.

5.1 Image processing

The raw image outputs from the experiment undergo a series of image processing procedures to improve the image quality and particle tracking efficiency. As shown in Figure 5.2, the raw image contains experimental noise and has a low contrast between the particles and the flow background. The histogram shows that the light intensity of the particles are very close to those of the background and this makes particle tracking difficult.

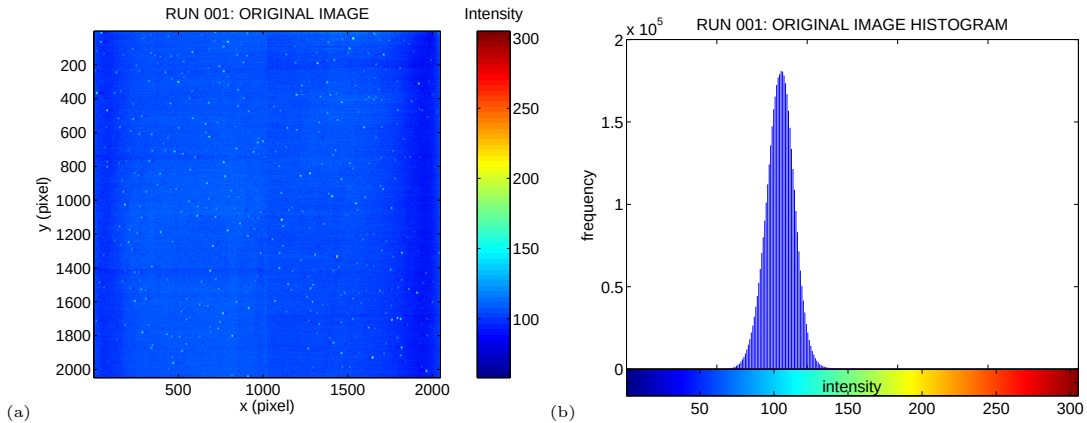


Figure 5.2: (a) A raw output image from the experiment (entire flow) (b) A raw output image histogram.

Image processing is performed to improve the images' quality for particle tracking. A number of techniques e.g. reference image comparison, smoothing, truncation, trimming and binary conversion are used to reduce noise and improve the particle contrast. Details of each image processing technique are given in Appendix 9.1. Figure 5.3 illustrates a comparison between a raw output image from the experiment and a processed output image in a black and white format which has a clear contrast between the particles and the flow field.

Although image processing is a powerful tool to prepare input images for particle tracking, the techniques must be performed carefully as they can also introduce some errors in the processed images. For instance, smoothing can cause diffusion of the image pixel intensity causing the particle sizes to be slightly enlarged. Some small or faint particles might be erased from the images and the dust at the rig's bottom

wall might still remain visible and become recognised as particles. Furthermore, some particles in the image tend to have “half-moon” shapes as a result of light projecting on particles on one side and shedding shadow on the other side during the experiment. Image processing can both enlarge and slightly change the shapes of these particles and cause a slight shift in the particle centroid, which is used to determine the particle position. We carefully processed the PTV images and tried to minimise the changes in the particles’ shape and size. The shift in particle centroid was observed to be smaller than 1 pixel on average after the image processing and this noise will be smoothen when the particle positions are fitted by our adaptive polynomial interpolation in Section 5.3.3.

In this work, image processing was carefully conducted and compared over a number of trials. The optimal image processing procedures as well as their associated parameters for each experimental campaign are summarised in Table 9.1 in Appendix 9.1.

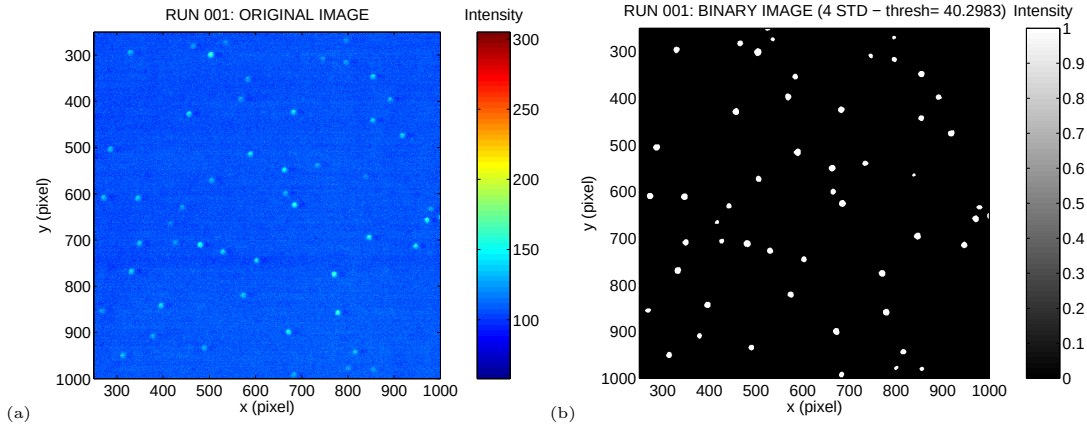


Figure 5.3: A comparison between: (a) a raw output image from the low density PTV measurement and (b) a processed output image (quarter of flow).

The processed images of the PTV measurements, with low and high density, are shown in Figure 5.4. The obvious difference between the outputs of the two measurements are the particle size and density in the images. It can be seen that the particle density in the high density PTV measurement is much higher (at least 5-6 times) than that of the low density one. Hence there are more particle statistics obtained within one realisation of the high density PTV measurement and the number of realisations repeated for

the same experiment can be reduced. Nevertheless, the image processing for the high density PTV output is also more difficult. This is because Chemigum particles vary in sizes, from very fine powder to large clusters. As some Chemigum particles are very small, it is sometimes difficult to distinguish them from the background and digital noise. As a consequence, processed images of the high density PTV measurement are generally noisier than those of the low density PTV measurement.

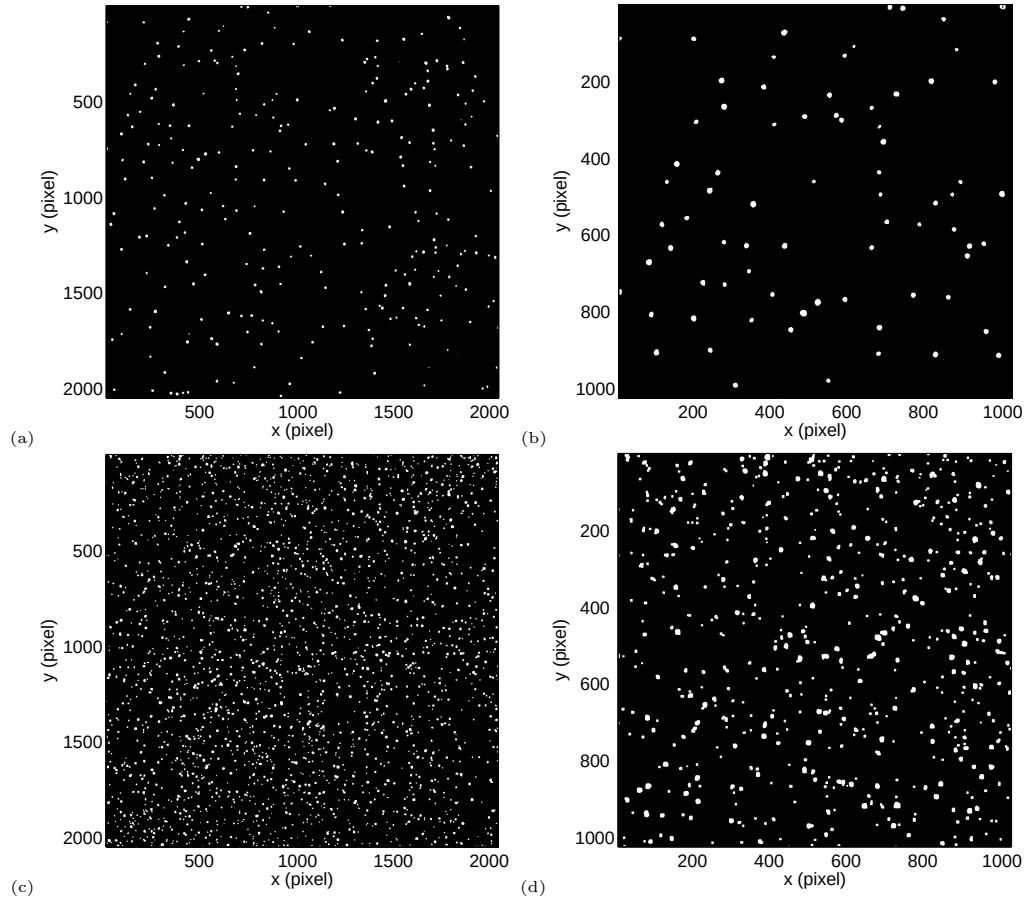


Figure 5.4: Processed images of the PTV measurement for low (first row) and high density (second row): (a,c) entire flow and (b,d) quarter of flow.

As stated previously, some remaining noises (i.e. dust) in the processed images might be recognised as particles, it is therefore important to carefully select the input parameters for particle tracking so as to reduce this noise. Furthermore, the input parameters are chosen so as to exclude particle clusters and very small or faint particles which disappear and reappear along the image sequences.

5.2 Particle tracking

DigiFlow v1.0.0 software (Dalziel 2006) has been used to perform particle tracking. The software first identifies the positions of the particles in each image and store them in the .dft format. The files are then used as an input to calculate the trajectories, which are then stored in the .dfd format files.

5.2.1 Input parameters for particle tracking

In DigiFlow, the input parameters required for particle tracking are particle dimensions in the image (i.e. area, $\text{edge}^2/\text{area}$ ratio and particle widths in x and y axes), maximum matching distance and changes allowed in size and shape of the particles. We analyzed the processed image from each realisation and plot the histograms of particle dimensions as shown in Figure 5.5.

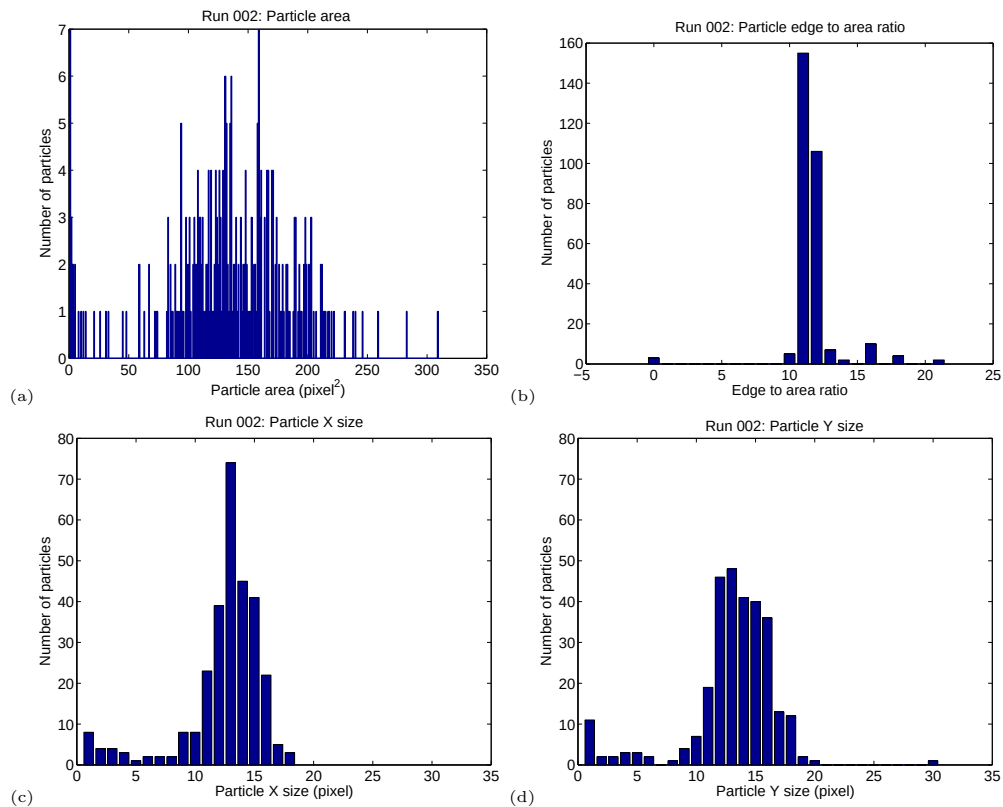


Figure 5.5: A example of particle parameter histograms: (a) Particle area (b) Edge to area ratio (c) Particle width in x axis (d) Particle width in y axis.

The histograms are used to determine the minimum and maximum dimensions of the “blobs” to be recognised as particles. By inspection, we determined the cut-off limits at the edges of the high occurrence range. Particles with values outside the cut-off limits are neglected and are not included in the particle tracking process. For example, very small values of widths and areas might be associated with noise whereas very large values might represent particle clusters in the image. Particle size and area should be much smaller than the size of the smallest M10 magnet (≈ 20 pixel, 400 pixel²).

The edge²/area ratio helps define the shape of the particles; highly distorted particles or noises will have very high edge²/area ratio values. The typical values of input parameters used in our particle tracking are given in Table 5.1. Note that the particle tracking is performed in the pixel unit.

Table 5.1: Typical input parameters for particle identification.

PTV Measurement	Area [pixel ²]		X size [pixel]		Y size [pixel]		Edge ² /Area
	min	max	min	max	min	max	
low density	50	250	7	18	7	18	30
high density	9	300	3	18	3	18	30

The maximum matching distance for particle tracking, ΔD_{max}^* , is very important and must be selected appropriately. The value can be estimated by the maximum particle displacement per time step, ΔD_{max} , and by the ratio between the maximum velocity, u_{max} , and the camera’s exposure frequency, F_c . In the latter method, we use the values of u_{max} from Table 4.1 (Rossi, Vassilicos, & Hardalupas 2006a) to estimate our u_{max} at the maximum forcing intensity in each time-dependent forcing case using a quadratic interpolation as shown in Figure 5.6. We expect the actual u_{max} for the time-dependent forcing cases to be less than the estimated u_{max} from the constant forcing cases due to the fact that the forcing is sinusoidal and the flow is never left to settle at the maximum forcing intensity. The ratio $\frac{u_{max}}{F_c}$ is presented in Table 5.2.

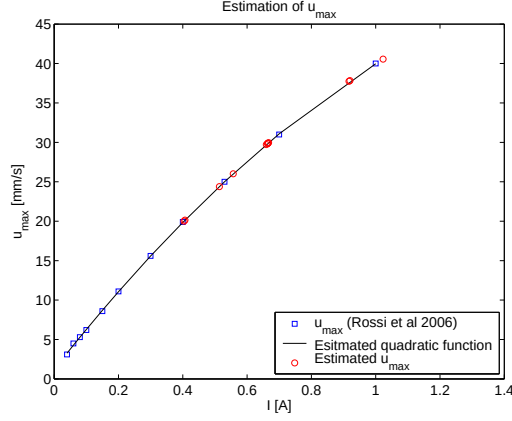


Figure 5.6: Estimation of u_{max} . The line corresponds to $u_{max} = -13.0I^2 + 51.8I + 1.22$.

Table 5.2: Maximum absolute displacement of particles per time step, ΔD_{max} , and maximum matching distance used as input for particle tracking, ΔD_{max}^* , for different forcing cases.

Experiment	Case	I_{max} [A]	u_{max} [mm/s]	F_c [Hz]	$\frac{u_{max}}{F_c}$ [pixel]	ΔD_{max} [pixel]	ΔD_{max}^* [pixel]
Campaign I	C1	0.660	29.7	13	4.68	4.95	5
	C2	0.667	30.0	12	5.12	5.00	6
	C3	0.664	30.0	12	5.12	4.87	6
Campaign II	C4	0.665	29.9	12	5.10	4.93	6
	C5	0.557	26.0	12	4.44	4.87	5
	C6	0.664	30.0	12	5.12	4.90	6
Campaign III	C7	0.404	20.0	12	3.41	4.96	5
	C8	0.917	37.7	12	6.43	6.86	7
	C9	0.407	20.1	12	3.43	4.95	5
	C10	0.920	37.8	12	6.45	6.98	8
	C11	0.514	24.4	12	4.16	4.79	5
	C12	1.023	40.6	12	6.93	7.92	8

The value ΔD_{max} is the actual maximum absolute displacement of all particles from the measurement. The absolute displacement is determined by the change in the particle position in any two consecutive frames of the PTV output image i.e. $\Delta D = (\Delta x^2 + \Delta y^2)^{1/2}$. From Table 5.2, ΔD_{max} in some forcing cases are larger than the estimated ratio $\frac{u_{max}}{F_c}$. This is due to the noises from the measurement and image processing which cause the particle positions to shift slightly resulting in additional displacements.

The examples of probability density function (PDF) and cumulative density function (CDF) of the particle displacement, ΔD , from the PTV measurement are shown in Figure 5.7. We select the maximum matching distance, ΔD_{max}^* , for the particle tracking input, to cover both ΔD_{max} and $\frac{u_{max}}{F_c}$ as shown in Table 5.2. In addition, we also select ΔD_{max}^* to be smaller than the particle size; this can help preventing particle mismatching significantly.

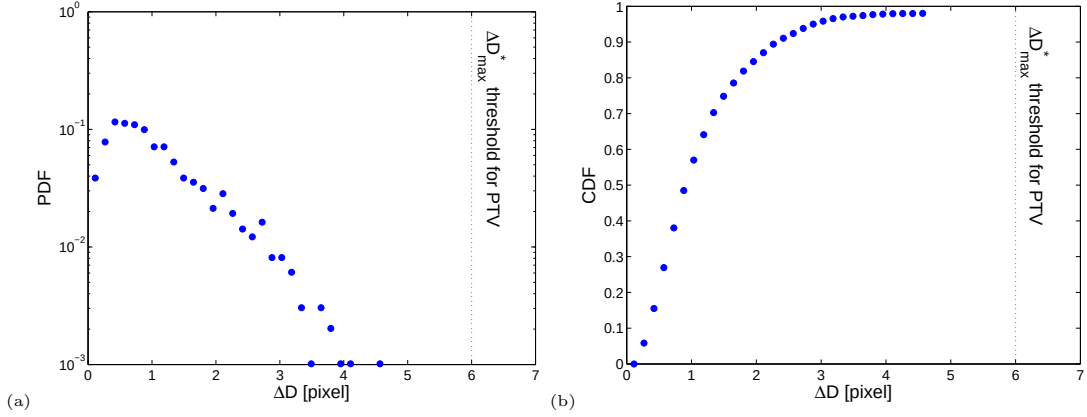


Figure 5.7: Examples of PDF(a) and CDF(b) of absolute particle displacement per time step, ΔD , of the case C6. The dot line indicates the maximum matching distance, ΔD_{max}^* , used as the input for the particle tracking.

Another important input parameter for particle tracking in DigiFlow is the pathlength, which is the number of consecutive time steps required to identify the particles. We select the pathlength to be 9 time steps i.e. the particles must have at least 9 time steps of history in order to be recognised as particles in the tracking. The selection of pathlength is a compromise between the trajectory being long enough to be certain that it belongs to a particle and not dust etc., but short enough to obtain a large number of trajectory sample size in the flow. This selection of pathlength allows for trajectories with incomplete histories (e.g. particles which enter and exit the measurement domain and trajectories which end so close to another particle trajectory that the two particles cannot be recognised) to be included, which help maintain a large number of samples for measurement statistics as shown in Table 5.3.

Table 5.3: Typical number of particles and trajectories recognised in particle tracking.

Particle tracking statistics from PTV	Low density	High density
Total particles recognised	300 - 450	2500 - 3000
Total trajectories with complete history	100 - 200	800 - 1200
Total trajectories with pathlength = 9	250 - 400	1500 - 1800

5.2.2 Trajectories reconstruction

Using the given input parameters, DigiFlow calculates the centroids of the particles and marks them as the particle positions. The software then calculates particle trajectories by relating particles in one image to the next images using a modified matching technique based on the transportation algorithm. Basically, once the particles are recognised, they are labelled and their characteristics are recorded. A set of associations between two sets of entities (e.g. at $t = t_n$ and $t = t_{n+1}$) is chosen such that it minimises some linear function of the associations it includes. Association functions generally included are particle positions, particle characteristics (e.g. size, shape, intensity etc.), temporal history and flow physics. Added well-chosen cost factors for association functions will aid correct matching, nevertheless they also increase the computing effort. In DigiFlow, the tracking results are relatively insensitive to the exact function used for the associating costs (Dalziel 2006). In addition, matching over a number of time step sequences prevents the mismatch from continuing.

We join short trajectories (each with a history of 9 time steps) from all time steps together and construct the complete trajectories in time. Each .dfd output file reports particle positions for all the 9 time steps for all particle detected which enables us to match the same particles together, i.e., we match the last 8 positions of one time step to the first 8 positions of the next consecutive time step as shown in Figure 5.8.

In the cases where some particle positions disappear from the measurement (e.g. leaving the measurement field or can no longer be recognised due to clustering with other

particles), their short trajectories (with 9 time steps) will not be recorded at the instance of the disappearance. On the other hand, when particles are entering the measurement field (or reappear from declustering), their short trajectories are recorded in the output file at the instance they emerge. Hence, once the trajectories reconstruction is complete, we obtain long particle trajectories, which vary in lengths as well as the starting points and the ending points, for each realisation of the PTV measurement as shown in Figure 5.9.

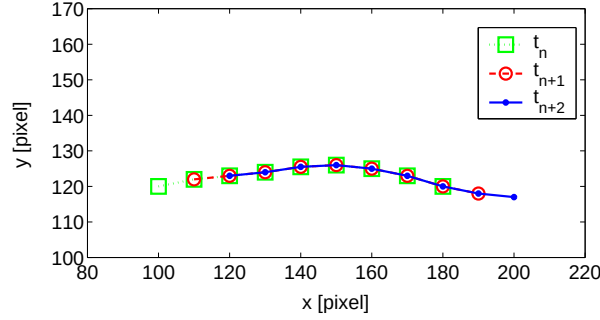


Figure 5.8: Schematic diagram of trajectory reconstruction.

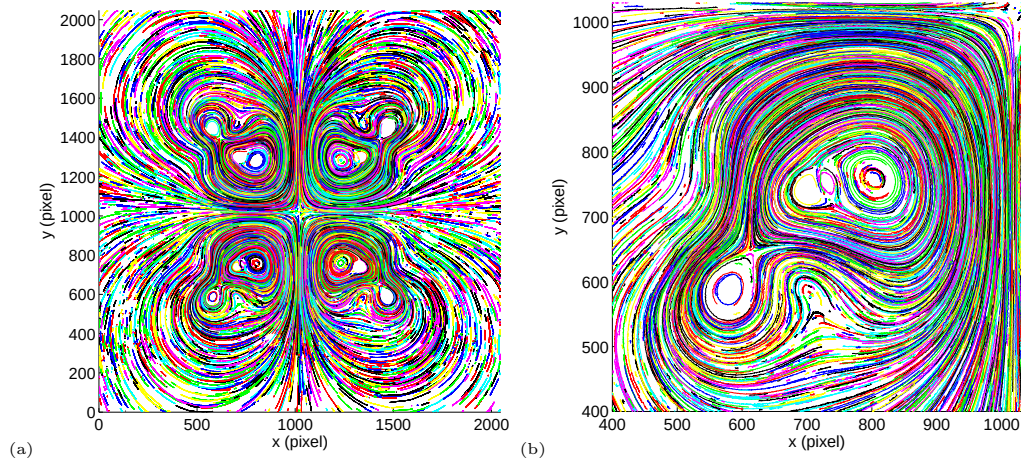


Figure 5.9: Example of reconstructed trajectories from 1 realisation of case C7 with approximately 22,500 trajectories: (a) entire flow, (b) quarter of the flow.

It is worth mentioning that DigiFlow also has an in-house option to construct the Eulerian velocity field. However, the option cannot be utilised directly for our experimental data as we need to perform time-rescaling as well as realisation and period combining to increase the particle statistics in the measurement field prior to constructing the Eulerian velocity field.

5.2.3 Particle tracking problems

In experimental campaigns I (PTV(LD)) and II (PTV(HD)), we experienced some technical difficulties with the PTV measurement system which caused up to 10% of the images to be randomly missing in the output image sequences. In addition, some images also have a problem of 1 pixel shift either upward or downward within the image. From observation during the data processing, the shifting problem only occurs in the vertical direction and always occurs following the missing images. These problems are thought to be due to failure in the fast storage mode of the measurement system.

To solve the missing image problem, we approximate the average relative displacement in each time step, $\overline{\Delta d}$. Only particles that are traceable throughout the realisation are included in the approximation to eliminate changes of $\overline{\Delta d}$ from other sources e.g. particles leaving or entering the measurement area. When an image is missing, $\overline{\Delta d}$ is approximately double at that instant as a result of double particle distances between the two consecutive images. We use a moving average with some threshold to identify the peaks for missing images. When an image is missing, we double the size of the time step for velocity calculation.

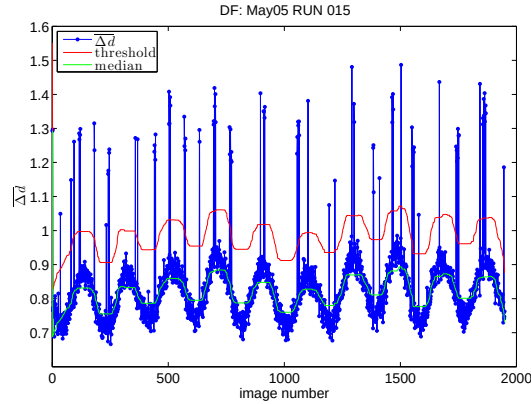


Figure 5.10: Estimation of $\overline{\Delta d}$.

The average relative displacement for the image sequences is shown in Figure 5.10. The value $\overline{\Delta d}$ approximated in this stage is only a rough estimation for the purpose of identifying missing and shifting images only. The irregular non periodic $\overline{\Delta d}$, as seen Figure 5.10, is observed for this reason.

Smaller peaks in $\overline{\Delta d}$ indicate the occurrences of the 1 pixel shift; an extra pixel is added to the displacement of all particles and causes $\overline{\Delta d}$ to increase. The problem of 1 pixel shift within the image is not corrected at this stage because the occurrences are infrequent ($\leq 10\%$ of the recorded output), the shift of 1 pixel is of the same order as the trajectory noise from the image processing and our adaptive polynomial interpolation scheme developed to calculate the Lagrangian velocity (Section 5.3.3) is not sensitive to this level of noises. In fact, our adaptive scheme effectively filters out these noises in the velocity interpolation.

The experimental campaign III (PTV(HD)) was conducted after the camera had been repaired and the problems of the missing images and shifting pixel can no longer be observed in the results.

5.3 Lagrangian velocity computation

As we seek to obtain good spatial resolution of the velocity field compared to normal PIV and PTV techniques, we combine (superimpose) particle statistics at the same forcing phase from all realisations and periods together. For this, the flows must be periodic and reproducible within tolerance and we require the Lagrangian particle statistics, particularly the velocities, at the same instants of forcing intensities and times of the period for the superimposition.

During the experiment, great care has been taken in trying to generate periodic and reproducible flows as well as collecting the PTV measurement data at the same instants of forcing intensities and times of the period. Nevertheless, there are still some differences due to the experimental nature of the measurements. In the following sections, we quantify the reproducibility and periodicity of the forcing. To tackle the slight differences of the instants when the PTV data were collected, we rescale the time and then compute the Lagrangian velocities using an adaptive polynomial interpolation.

5.3.1 Periodicity and reproducibility of the forcing

To investigate the forcing periodicity and reproducibility, the forcing intensity recorded by the datalogger from the measurement are compared; we compare all periods within each realisation and compare all realisations within one forcing case as shown in Figure 5.11.

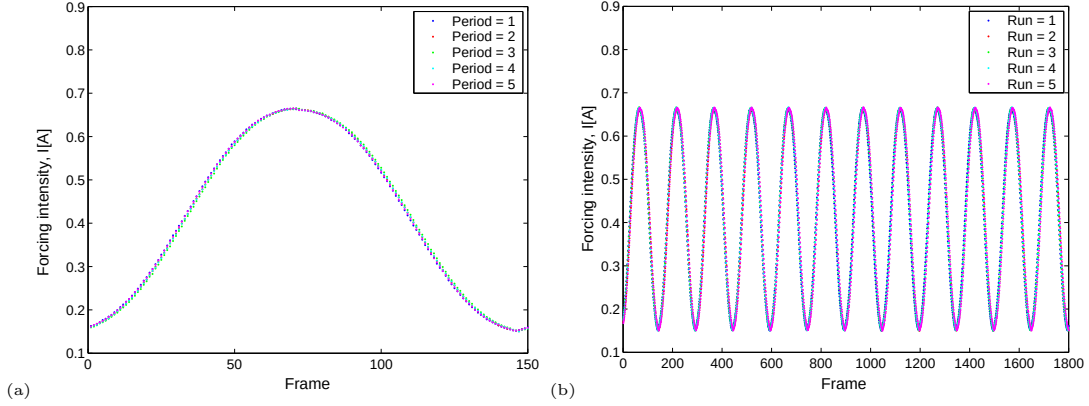


Figure 5.11: (a) Periodicity of the forcing and (b) Reproducibility of the forcing (only 5 periods and realisations (runs) are displayed for an illustrating purpose).

We then quantify the forcing periodicity and reproducibility by comparing the forcing period (T_f), the forcing intensities at the minimum, mean and maximum level (I_{min} , I_{max} and I_{mean} , where I_{mean} is the average of all values within one period) and the forcing amplitude ($A = I_{max} - I_{mean} = I_{mean} - I_{min}$) in all periods within each realisation and comparing the same parameters across all realisations as shown in Figure 5.12.

The standard deviations, σ , given as a percentage of the mean value of each quantity (T_f , A , I_{min} , I_{mean} and I_{max}) are reported in Table 5.4. From the table, the maximum standard deviation, σ_{max} , for all quantities are less than 1% within a realisation and also across all realisations. Hence, the forcing is considered periodic and reproducible. The only exception is in I_{min} where its σ_{max} can be up to 1.4-2.8% in the forcing cases C3 and C5 in Campaign II. This is due to the fact that I_{min} for both cases are very low, while the noise level (i.e. forcing intensity fluctuation) remains a consistent level making the relative standard deviation comparably larger. Nevertheless, the noise level for I_{min} is consistent for all realisations in the same forcing cases;

hence, the data should not be removed based on the justification of its relatively higher $\sigma_{\max}$ value.

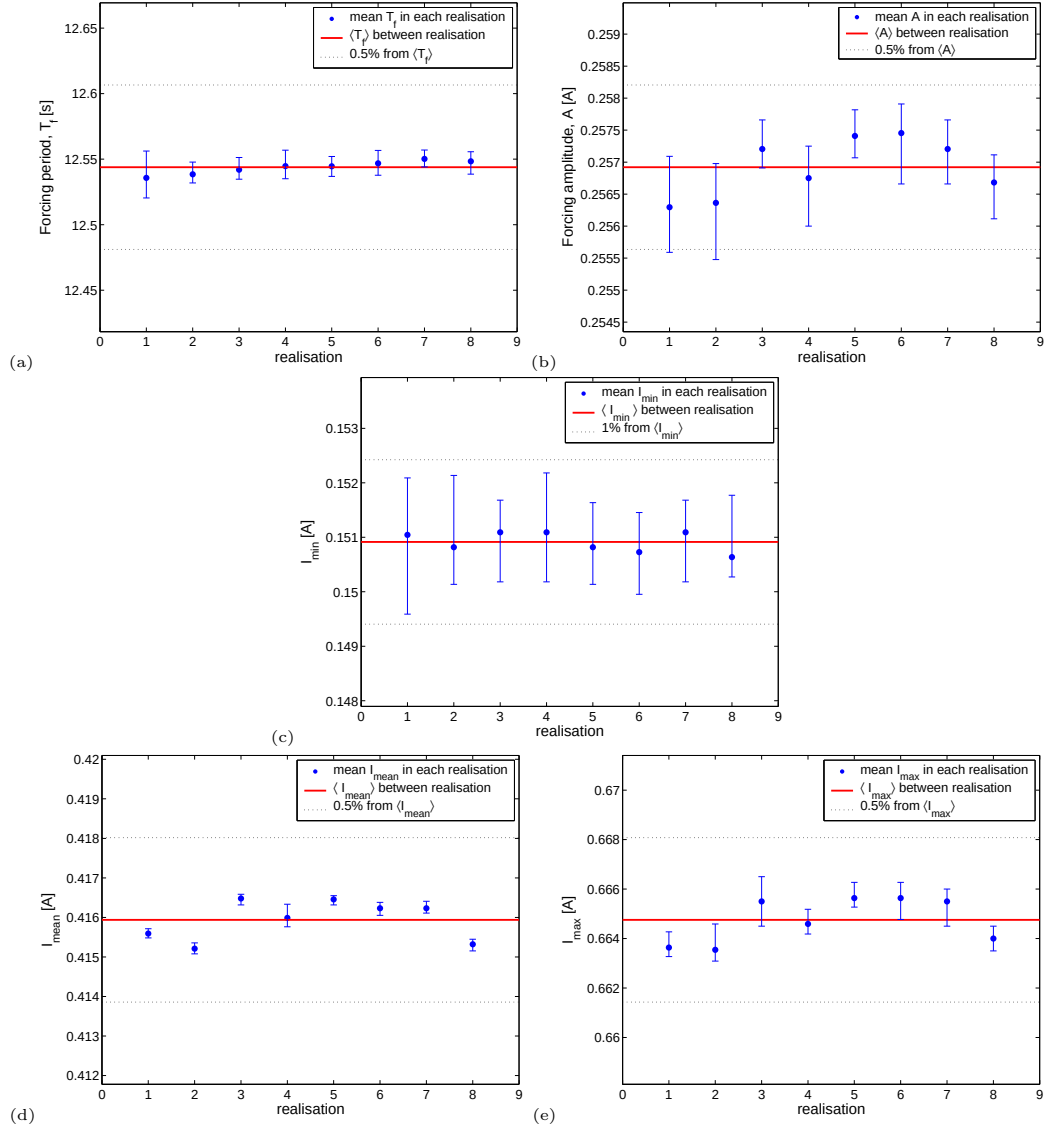


Figure 5.12: Examples of quality of forcing investigation: comparison of T_f , A , $I_{\min}$, I_{mean} and $I_{\max}$ ((a)-(e) respectively) with in each realisation and across all realisations. The blue dot indicates the mean value within each realisation whereas the blue bar indicates the minimum and maximum values. The red line is the average of the mean values across all realisations and the black dotted lines indicate $\pm 0.5\%$ variations from the average value (except in (c) where the variation is $\pm 1\%$).

Table 5.4: Quality of forcing: actual forcing obtained in the experiment and its reproducibility and periodicity measures.

Experiment	Case	Forcing parameter obtained $I(t) = \bar{I} + A \cos \frac{2\pi t}{T_f}$			Reproducibility $\sigma_{\max}$ across realisations (% of the mean value)					Periodicity $\sigma_{\max}$ within realisation (% of the mean value)				
		$I_{mean}, \bar{I}$ [A]	Amplitude, A [A]	Period, T_f [s]	$\sigma_{\bar{I}}$	$\sigma_{I_{\min}}$	$\sigma_{I_{\max}}$	σ_A	σ_{T_f}	$\sigma_{\bar{I}}$	$\sigma_{I_{\min}}$	$\sigma_{I_{\max}}$	σ_A	σ_{T_f}
Campaign I	C1	0.407	0.253	15.0	0.244	0.466	0.255	0.337	0.087	0.317	0.164	0.102	0.127	0.035
	C2	0.409	0.258	7.52	0.171	0.179	0.159	0.185	0.107	0.092	0.682	0.166	0.265	0.044
	C3	0.407	0.257	10.0	0.487	1.440	0.539	0.784	0.260	0.255	0.589	0.238	0.309	0.063
Campaign II	C4	0.408	0.257	12.5	0.122	0.123	0.139	0.178	0.038	0.122	0.501	0.139	0.184	0.057
	C5	0.298	0.259	12.5	0.230	2.851	0.188	0.327	0.187	0.169	2.027	0.148	0.222	0.053
	C6	0.408	0.256	18.0	0.596	1.464	0.416	0.253	0.186	0.323	0.531	0.122	0.202	0.070
Campaign III	C7	0.331	0.073	5.00	0.319	0.453	0.277	0.430	0.196	0.133	0.345	0.198	0.772	0.223
	C8	0.750	0.167	5.00	0.209	0.491	0.229	0.999	0.140	0.224	0.173	0.098	0.389	0.095
	C9	0.332	0.075	10.0	0.368	0.397	0.417	0.939	0.167	0.112	0.338	0.170	0.578	0.202
	C10	0.751	0.169	10.1	0.294	0.270	0.360	0.612	0.199	0.151	0.175	0.115	0.474	0.101
	C11	0.420	0.094	15.0	0.181	0.206	0.193	0.297	0.193	0.089	0.276	0.172	0.680	0.185
	C12	0.836	0.187	15.0	0.048	0.037	0.041	0.101	0.109	0.027	0.136	0.069	0.312	0.091

5.3.2 Time rescaling

As shown in the previous section, the experimental forcing is periodic and reproducible within tolerance and there are slight differences in the forcing intensities and periods. In addition, the instants at which the measurement data were recorded vary slightly between periods and realisations even though we have attempted to collect the measurement data at the same instants in the forcing by always starting the measurement when the forcing intensity reaches its minimum value at the start of the period during the experiment. These differences must be corrected in order to obtain the Lagrangian statistics at the same instants of forcing intensities and times of the period for the Lagrangian velocities superimposition.

We rescale the experimental time data recorded by the datalogger during the measurement to the reference time step as a fraction of the measured forcing period as follows:

$$t^* = \frac{t - t_0}{T_f}, \quad (5.1)$$

where t^* is the dimensionless and scaled reference time step, t is the experimental time step from the PTV measurement (i.e. when the images are taken as recorded by the datalogger), T_f is the measured forcing period (the mean values and standard deviations of all forcing cases are given in Table 5.4) and t_0 is the reference starting time step selected and set to the same value of forcing intensity for all realisations. We select t_0 to be the instant where $I(t) \approx I_{mean}$ on the ascending path of the sinusoidal function as such instant can be simply identified and is less noisy compared to the instants at the minimum or maximum forcing intensities where the sinusoidal forcing changes the direction. The rescaled time, t^* , is referred to as time, t , for the rest of this chapter.

5.3.3 Adaptive polynomial interpolation

As our flow has multi-scales and the forcing is time-dependent, we develop an adaptive polynomial interpolation technique, which can provide accuracy across a range of

length-scales and forcing intensities, to compute the Lagrangian statistics. We use cubic functions to fit a number of particle positions to approximate trajectories (Eq.(5.2)). The velocities are then extracted from quadratic functions (Eq.(5.3)). The orders of interpolation are selected so as to allow up to two changes in the direction of the trajectory fitting. Higher order of interpolation causes the trajectory approximation to be less effective to filter noises given the limited numbers of particle positions.

$$\mathbf{x}(t) = \mathbf{b}_3 t^3 + \mathbf{b}_2 t^2 + \mathbf{b}_1 t + \mathbf{b}_0, \quad (5.2)$$

$$\mathbf{u}(t) = 3\mathbf{b}_3 t^2 + 2\mathbf{b}_2 t + \mathbf{b}_1. \quad (5.3)$$

The technique has two main adaptive parameters; the number of positions used for interpolation, N_n and the total distance, d covered by the N_n positions. Thus at every time step, t , N_n is set to an initial value, N_i and is adapted so that $d_{min} \leq d \leq d_{max}$ and $N_{min} \leq N_n \leq N_{max}$. When $d < d_{min}$, N_n is increased until either d_{min} or N_{max} is reached. If d_{min} is reached, the trajectory position (x, y) as well as its velocity component (u, v) are interpolated. On the other hand, if N_{max} is reached and still $d < d_{min}$, the interpolation is discarded as the position fitting does not reach the minimum displacement to be recognised as a trajectory of a particle. This is often the case for the dust at the bottom of the experimental rig. When $d > d_{max}$, the process is reversed.

The accuracy of the adaptive technique is dominated largely by N_n and d therefore it is critical to select appropriate ranges for these parameters. For instance, N_n and d should be large enough to eliminate trajectory noises/errors while remaining small enough to capture important information such as local flow properties and small-scale features. More specifically, we select d_{min} to be 8-10 times higher than the trajectory noise level and d_{max} to be smaller than the perimeter of the smallest scale (M10 magnet). N_{min} is selected to allow enough positions to properly fit the trajectory while N_{max} is set to be less than the number of total positions in one period, N_{T_f} , to obtain local temporal quality. We conduct a number of trials to adjust the appropriate parameters for each forcing cases as presented in Table 5.5.

Table 5.5: Lagrangian velocity computation: Adaptive parameters.

Experiment	Case	N_{T_f}	N_{min}	N_i	N_{max}	d_{min}	d_{max}
Campaign I	C1	195	15	48	60	15	66
	C2	90	10	36	46	12	66
	C3	120	15	48	60	15	66
Campaign II	C4	150	15	48	60	15	66
	C5	150	15	48	60	15	66
	C6	216	15	48	60	15	66
Campaign III	C7	60	10	20	46	10	66
	C8	60	10	20	46	10	66
	C9	120	15	40	60	15	66
	C10	120	9	36	60	15	66
	C11	180	15	48	60	15	66
	C12	180	9	36	60	15	66

The adaptive technique allows local flow features to be captured as demonstrated in Figure 5.13. In high velocity regions where d_{max} is large, smaller N_n is required. On the other hand, larger N_n is needed in low velocity region particularly near the elliptical stagnation points to efficiently approximate the velocity while reducing the noise level.

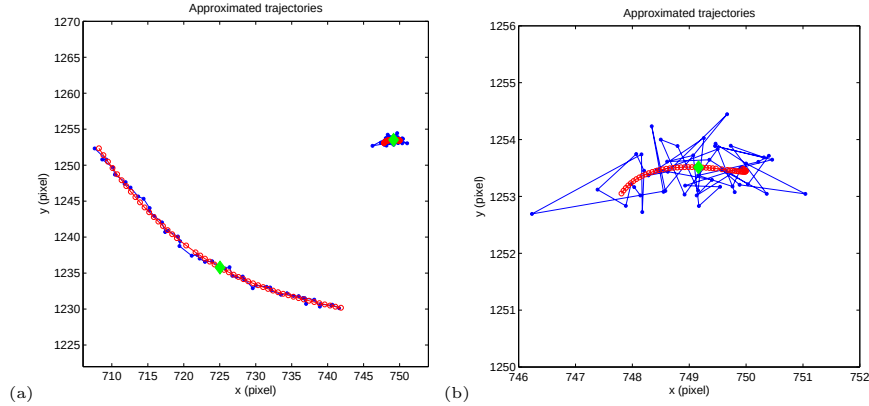


Figure 5.13: Polynomial fitting of trajectories and Lagrangian velocity approximation. The blue dots represent trajectories obtained from PTV measurement, the red circles shows the polynomial approximation of the trajectory using the adaptive technique and the green diamonds are the middle positions where Lagrangian velocities are extracted. (a) showing trajectories with high and low displacement near a small scale elliptical stagnation point (b) close-up of the low displacement trajectory.

5.3.3.1 Trajectory noises

Noises in the trajectories, as shown in Figure 5.14, arise from various sources. As previously mentioned, the image processing can cause the particle centroid to shift slightly and result in noisy trajectories. Moreover, trajectory mismatching during the particle tracking and the problem of random 1 pixel shift in the PTV output image also add noises to the trajectories.

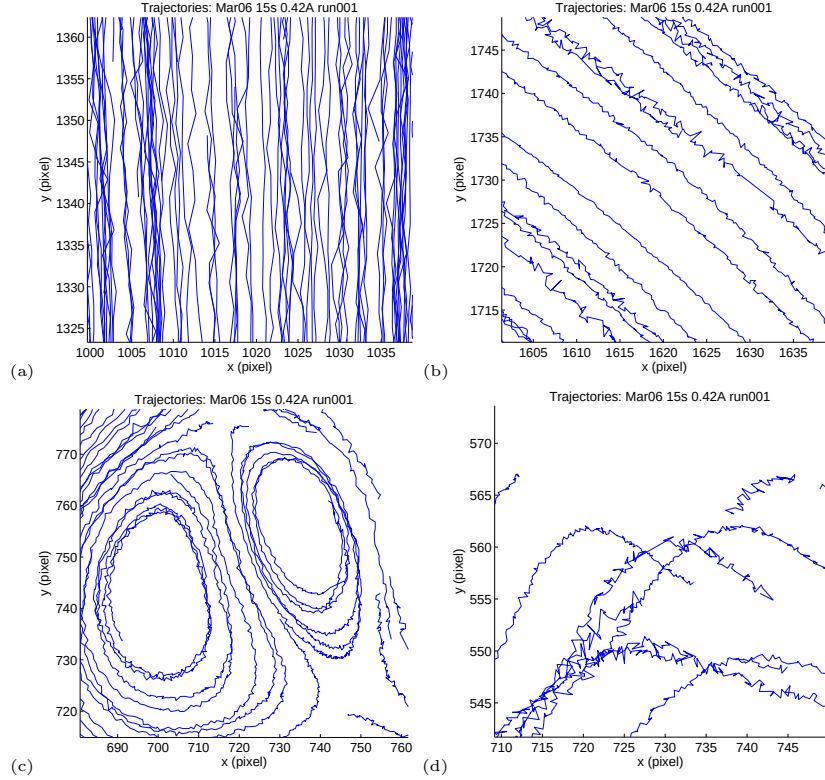


Figure 5.14: Trajectory noises in different regions of the flow: (a) high velocity region (above the large magnet M160), (b) low velocity region (near the edge of the flow), (c) around elliptical stagnation points and (d) around hyperbolic stagnation points.

From visual observation, the noise level in the trajectories varies with the location in the flow; trajectories in the areas where streamlines are simple (i.e. away from the stagnation points) generally contain noise of around 1-2 pixels whereas the noise is up to 1.5-3 pixels around the stagnation points as shown in Figure 5.14. These trajectory noises, which are filtered out by the adaptive polynomial interpolation, are quantified as the trajectory approximation errors in the next section.

5.3.3.2 Trajectory approximation error

The trajectory approximation error is quantified by the least mean square of the differences between the trajectory positions obtained from the PTV measurement and the polynomial approximation, which gives the average variation per position:

$$\delta_x = \sqrt{\frac{\sum_{i=1}^{N_n} \left(\mathbf{x}_{PTV_i} - \mathbf{x}_{approx_i} \right)^2}{N_n}}. \quad (5.4)$$

Examples of the trajectory approximation error in both x and y direction (δ_x , δ_y), as well as the error magnitude, δ are shown in Figure 5.15. Figure 5.16 presents examples of the distributions of the error magnitude as a percentage of the distance used for the trajectory approximation, δ/d .

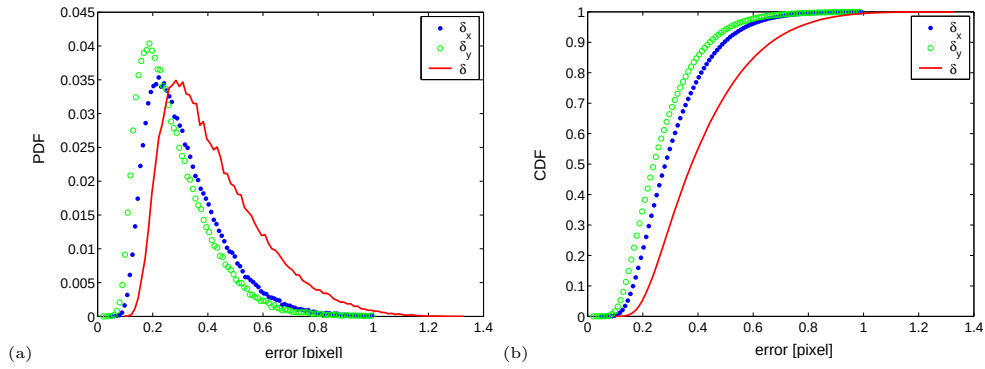


Figure 5.15: PDF and CDF of the trajectory approximation error in the x and y directions (δ_x , δ_y) and the error magnitude, δ .

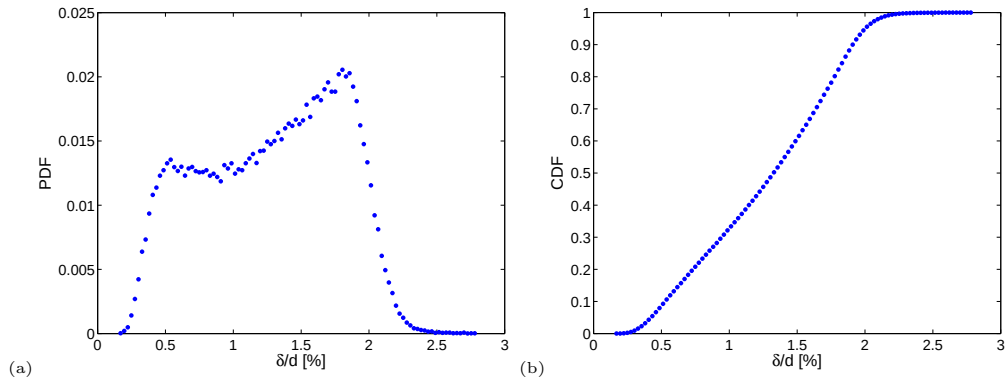


Figure 5.16: PDF and CDF of the error magnitude as a percentage of the distance used for the approximation, δ/d .

The trajectory approximation error (mean $\pm$ standard deviation) for all forcing cases are reported in Table 5.6. The values given are averaged over all approximated trajectories in each case. From Table 5.6, the polynomial approximation error on average is well within 1 pixel per position and 2.5% of the distance used for the approximation.

Table 5.6: Lagrangian velocity approximation error.

Experiment	Case	δ_x [pixel]	δ_y [pixel]	δ [pixel]	δ/d [%]
Campaign I	C1	0.370 $\pm$ 0.117	0.309 $\pm$ 0.107	0.485 $\pm$ 0.150	1.31 $\pm$ 0.376
	C2	0.300 $\pm$ 0.101	0.274 $\pm$ 0.106	0.410 $\pm$ 0.134	1.61 $\pm$ 0.518
	C3	0.417 $\pm$ 0.120	0.404 $\pm$ 0.133	0.584 $\pm$ 0.166	1.39 $\pm$ 0.349
Campaign II	C4	0.352 $\pm$ 0.118	0.329 $\pm$ 0.128	0.487 $\pm$ 0.161	1.36 $\pm$ 0.358
	C5	0.391 $\pm$ 0.147	0.371 $\pm$ 0.157	0.545 $\pm$ 0.200	1.42 $\pm$ 0.310
	C6	0.319 $\pm$ 0.109	0.290 $\pm$ 0.116	0.435 $\pm$ 0.147	1.29 $\pm$ 0.370
Campaign III	C7	0.273 $\pm$ 0.112	0.240 $\pm$ 0.106	0.368 $\pm$ 0.141	2.49 $\pm$ 0.901
	C8	0.304 $\pm$ 0.145	0.270 $\pm$ 0.136	0.412 $\pm$ 0.187	2.19 $\pm$ 0.891
	C9	0.292 $\pm$ 0.115	0.254 $\pm$ 0.108	0.391 $\pm$ 0.147	1.48 $\pm$ 0.454
	C10	0.286 $\pm$ 0.130	0.254 $\pm$ 0.123	0.387 $\pm$ 0.168	1.32 $\pm$ 0.500
	C11	0.313 $\pm$ 0.120	0.268 $\pm$ 0.113	0.417 $\pm$ 0.154	1.28 $\pm$ 0.377
	C12	0.314 $\pm$ 0.135	0.272 $\pm$ 0.126	0.420 $\pm$ 0.175	1.26 $\pm$ 0.509

5.4 Combining periods and realisations

At this stage, we combine the Lagrangian statistics at the same forcing phase from all periods and realisations together to increase the spatial resolution. As previously emphasised, we rely on two crucial hypothesis that the flows are periodic and reproducible and the main features of the flows at the same forcing phase are consistent within an acceptable tolerance. To satisfy such conditions, we have carefully conducted the experiments at low Re with periodic and reproducible forcing and rescaled data for the Lagrangian velocity extraction. In this section, we present the checking of the flows' reproducibility and periodicity as well as the procedures involved in combining the statistics.

5.4.1 Periodicity and reproducibility of the flows

In this section, the reproducibility and periodicity of the flow response to the forcing are checked using the Eulerian velocity field constructed by the method detailed in Section 5.5. At this stage, the Lagrangian statistics available to construct the Eulerian velocity field are low. As a result, the velocity field computation time is long (explanation in Section 5.5) and only moderate resolution of the velocity field can be obtained. We thus quantify the flow reproducibility and periodicity using the flow energy characterised by u_{rms} rather than the correlation. The energy of the entire flow (characterised by $u_{rms_{EF}}$) and above the large magnet M160 (characterised by $u_{rms_{M160}}$) at the same forcing phase from different periods and realisations are compared. The quantity $u_{rms_{M160}}$ is used in the checking as the moderate resolution of the velocity field can still capture the large scale of the flow where most of the energy resides.

To check the reproducibility, we first assume that the flow is periodic and combine all Lagrangian velocities at the same forcing phase from all periods together to obtain sufficient statistics and construct the Eulerian velocity field. We then compare $u_{rms_{EF}}$ and $u_{rms_{M160}}$ at the same t across all realisations. Similarly, to check the periodicity, we assume that the flow is reproducible and the statistics at the same forcing phase from all realisations are combined. The velocity field is then computed and compared across all periods.

Both reproducibility and periodicity are checked at the instant where $t/T_f = 0.00, 0.25, 0.50$ and 0.75 . The average standard deviations of u_{rms} across the 4 instants, $\bar{\sigma}$, are given in Table 5.7. The value $\bar{\sigma}$ is quoted as a in percentage of the mean value of u_{rms} for each instant. It can be seen that the flows differ only upto 1-2% and the flow is periodic upto differences of less than 0.5%. These checks confirm the validity (within tolerance) of the reproducibility and periodicity hypothesis for our flows.

Table 5.7: Reproducibility and periodicity of the flows.

Experiment	Case	Reproducibility			Periodicity		
		Grid size (pixel)	$\bar{\sigma}$		Grid size (pixel)	$\bar{\sigma}$	
			$\bar{\sigma}_{EF}$	$\bar{\sigma}_{M160}$		$\bar{\sigma}_{EF}$	$\bar{\sigma}_{M160}$
Campaign I	C1	113	1.558	1.647	44	0.242	0.342
	C2	36	0.164	0.230	44	0.152	0.157
	C3	66	2.199	1.285	37	0.432	0.336
Campaign II	C4	44	0.499	0.184	55	0.221	0.217
	C5	42	0.570	0.568	52	0.248	0.307
	C6	49	2.135	1.844	26	0.201	0.210
Campaign III	C7	25	2.340	1.866	31	0.192	0.166
	C8	28	1.229	0.824	40	0.323	0.468
	C9	33	1.474	1.051	28	0.114	0.160
	C10	40	1.083	0.627	33	0.230	0.177
	C11	40	1.068	0.788	33	0.101	0.092
	C12	49	0.478	0.500	44	0.193	0.221

5.4.2 Lagrangian statistics adjustment

A number of experimental cases were measured over several days, therefore it is necessary to adjust particle statistics from different days to take into account the alignment of the camera. The particle statistics from all periods and realisations within the same day are combined and the Eulerian velocity field is computed. We then identify the stagnation point positions (see Appendix 9.2 for detail) and compare them between different days. As the velocity field obtained at this stage has medium resolution, only the large (M160) and medium scale (M40) stagnation points are used in the comparison as shown in Figure 5.17. The small scale stagnation points are not resolved enough for this purpose. We compare the stagnation points positions of the 4 time instants where $t/T_f = 0.00, 0.25, 0.50$ and 0.75 between different days. The average differences of the stagnation point positions between days of measurement, which are used to correct the Lagrangian statistics to be on the same coordinate, are presented in Table 5.8.

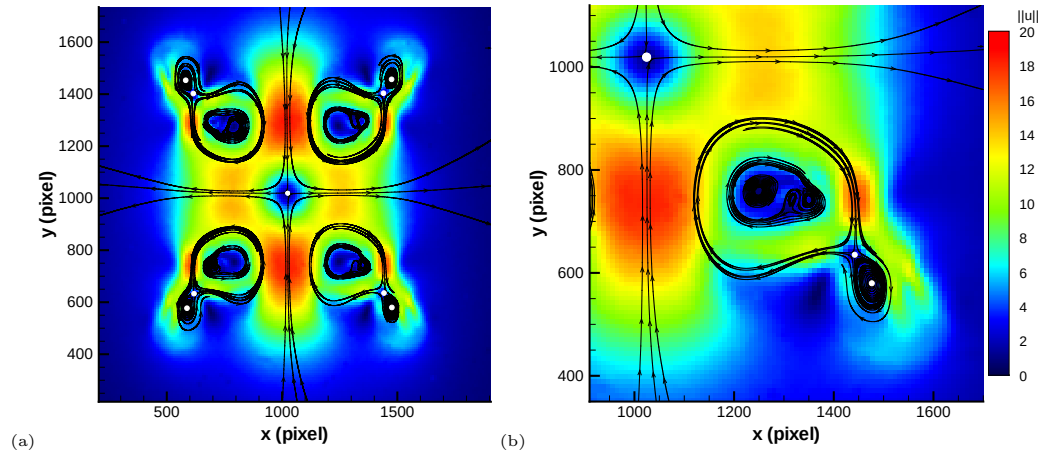


Figure 5.17: Large (M160) and medium scale (M40) stagnation point positions (white dots) in a medium resolution velocity field: (a) entire flow and (b) quarter of the flow. $\| \mathbf{u} \|$ is in mm/s.

Table 5.8: Position correction for data measured over different days.

Experiment	Case	Position correction		Note
		x (pixel)	y (pixel)	
Campaign I	C1	-1.466	-2.645	day n
		6.371	-3.242	day n+1
		-4.906	5.887	day n+2
Campaign III	C7	± 0.094	∓ 0.368	correction between day n and day n+1
	C8	∓ 0.357	± 1.185	
	C9	∓ 0.583	± 0.259	
	C10	∓ 0.028	∓ 3.376	

5.4.3 Flow symmetry

Despite previous post-processing procedures to increase the sample size for Lagrangian statistics to construct a higher resolution Eulerian velocity field, there are still a number of cases with insufficient Lagrangian statistics. Due to time constraints, it was not possible to obtain more data from the laboratory. Hence, we increase the sample size by using the flow symmetry condition.

The magnet configuration in the experiment (Figure 3.1) is set up to be symmetric. To validate whether such a set-up also generates symmetrical flows, we conduct a symmetry check. The case C7, which has the highest resolution (see Table 5.10), is used

in the symmetry validation. We combine all periods and realisations and compute the Eulerian velocity field of case C7 as described in Section 5.5. We then divide the flow into 4 quadrants using the central large scale hyperbolic stagnation point and compare their energy (characterised by u_{rms}) and autocorrelation, R_{ij} (Eq. (5.5)) where i and j denote the quadrant indices, N_k is the total number of grid points in the quadrant and k denotes the index of the grid point in each quadrant with $k \in \{1, \dots, N_k\}$.

$$R_{ij} = \left\| \sum_{k=1}^{N_k} \frac{\mathbf{u}_{i_k} \cdot \mathbf{u}_{j_k}}{u_{rms_i} u_{rms_j}} \right\|. \quad (5.5)$$

The standard deviation of u_{rms} among all four quadrants is found to be 0.612% of the mean values and the average R_{ij} compared between all quadrants is 0.997 with a standard deviation of 0.197%. As the standard deviation of u_{rms} is very small and R_{ij} is very close to 1, the flow generated by the fractal magnet configuration is considered symmetrical.

Using this symmetry, we increase the sample size for the Lagrangian statistics in 5 cases (Table 5.9). The velocity components are reflected about the large scale central stagnation point along the x or/and y axis. The sign of the velocity components changes according to the axis along which the data is reflected i.e. the sign of u components is reversed when the data is reflected along the y axis.

Table 5.9: Flow symmetry.

Experiment	Case	Symmetry centre		Symmetry axis
		x (pixel)	y (pixel)	
Campaign I	C1	1030.051	1036.838	x and y
Campaign II	C4	1024.761	1027.577	y
	C5	1023.959	1022.292	y
Campaign III	C11	1022.489	1018.605	y
	C12	1020.689	1018.298	x and y

5.5 Eulerian velocity field construction

The particle statistics obtained thus far are Lagrangian statistics. To construct an Eulerian velocity field, the Lagrangian particle velocities are mapped onto a regular grid using an in-house Grid_PTV software of the Department of Aeronautics, Imperial College London (courtesy of Rossi 2006).

Grid_PTV calculates the Eulerian velocity field using two approaches, the Smoothed Particle Hydrodynamics, SPH (Monaghan 1992) and the biquadratic interpolation. The extrapolation window size (SW) is adjusted according to the number of samples available in the window. The SPH method is used in the regions where the sample number is high i.e. when there are more than 18 velocity samples in the window and SW is smaller than the size of smallest M10 magnet (~ 15 pixel). Grid_PTV switches to the biquadratic method where the samples in the window are low and SW is larger than the smallest M10 magnet for 24 velocity samples. The software increases SW until at least 24 velocity samples are within the window and extrapolates the velocity grid point via the biquadratic method. The grid approximation error is estimated from the standard deviation. It was observed during the Eulerian velocity field construction that the biquadratic method takes approximately 3-4 times longer to compute per grid point than the SPH method.

The number of grid points is directly related to the grid size. We extrapolated 350×350 grid points for cases C7-C11 and 300×300 grid points for the other cases giving the grid sizes of 5 and 6 pixels respectively. The selection is determined by two factors; the number of grid points must be high enough to resolve the smallest length-scale in the flow while being computationally viable. Thus, the grid size is chosen to be 5-6 pixels so as to allow at least 4-5 grid points to be fit in the smallest length-scale characterised by the smallest M10 magnet. In addition, given the number of total samples available for grid computation (Table 5.10), 300-350 grid points allow the velocity fields to be computed with ~ 50 -65% of sample overlap per grid point which is optimum for the grid resolution. Higher number of grid points resulting in higher data overlap do not provide any significantly higher resolution quality. Instead, it causes repetition and

increases the computation time considerably.

Incorrect grid points are then removed from the Grid_PTVA output. We compare the velocity components of each grid point to those of the 3×3 surrounding points. In the case where the values of the central grid point exceed the threshold (bounded by the average of the surround grid points and two standard deviations), the values of the central grid point are replaced by the average of the surrounding points. Finally, we use a 3×3 point averaging technique to remove some noise at the small scales and smoothen the velocity field.

5.6 Eulerian velocity field

An example of the constructed Eulerian velocity field is illustrated in Figure 5.18. Some streamlines are plotted showing the multi-scale schematic “8 in 8” structure. The stagnation point positions are shown in red dots (elliptic) and red square (hyperbolic). The detailed analysis of the Eulerian statistics is presented in Chapter 6.

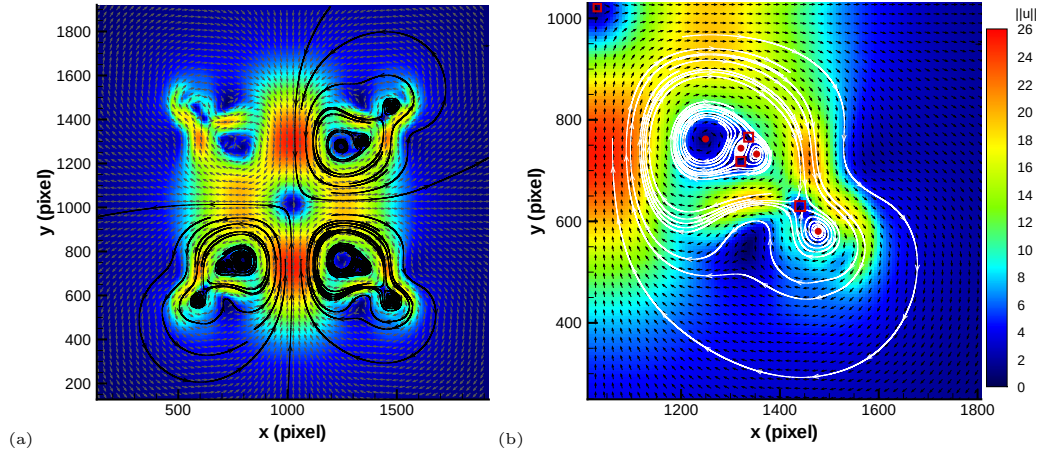


Figure 5.18: An example of a constructed Eulerian velocity field of case C6 at the instant $t/T_f = 0.25$ with 300×300 points of 6 pixels grid size. The velocity intensity $\|\mathbf{u}\|$ is shown in mm/s: (a) entire flow and (b) quarter of the flow. The elliptic stagnation points are shown in red dots and hyperbolic stagnation points are shown in red square.

Table 5.10 provides a summary of the quality of the constructed Eulerian velocity field. An example of the grid approximating SW is illustrated in Figure 5.19(a). There is a strong spatial dependence of SW on the flow features. In the area where streamlines are relatively steady (i.e. in the large flow scale), small SW as well as the SPH

method are used in the grid approximation. On the contrary, streamlines are very unsteady in the medium and small flow scales and particles are either dispersed vigorously away from the hyperbolic stagnation points or looping around the elliptical ones. As a result, there are fewer particles in the medium and small scales of the flow, SW is increased and the biquadratic method is used for the grid approximation when SW is large. Figure 5.19(a) shows that SW used for grid extrapolation in the medium flow scales are approximately 24-40 pixels which are still small enough when compared to the characteristic medium M40 magnet size ($\simeq 80$ pixels) to resolve the medium scales. The PDF and CDF of SW (Figure 5.20(a,b)) show that the majority of SW of the particular example ranges between 7-20 pixels which accounts for up to 85% of the total grid approximation. The mean extrapolation window, $\overline{SW}$, averaging over all points $\mathbf{x}$, is taken as the resolution of the PTV measurement and determines the smallest resolvable scale in our flows. From Table 5.10, $\overline{SW}$ for different experimental cases are approximately the same size as the smallest M10 magnet or smaller, providing fine enough resolution to capture the smallest characteristic scale of the flow.

The grid approximation noise (Figure 5.19(b)) provides the quality feedback on the biquadratic method, which is the method used when SW is large. The values represent the standard deviation given per mil (0.1%) of the points approximated by the biquadratic surface. The biquadratic approximation noise, $\overline{\sigma}_{noise}$, with an average taken over all points $\mathbf{x}_b$ (points which are approximated by the biquadratic method), is around 2-5% of the approximating surface when up to 40% of the biquadratic method is used in the grid approximation. The value $\overline{\sigma}_{noise}$ increases to 6-11% when only around 1-7% of the biquadratic method is used (see Table 5.10). High σ_{noise} are generally observed around the stagnation points. These regions are more sensitive due to the highly unsteady nature of the velocity components and fewer samples for grid extrapolation. Moreover, the velocity components in these regions are closed to zero causing the percentage of the standard deviation in these areas to be relatively high. Examples of PDF and CDF of the grid approximation noise of the case C6 are shown in Figure 5.20(c,d) and suggest that the level of most noises in the grid approximation of the particular forcing case is under 5%.

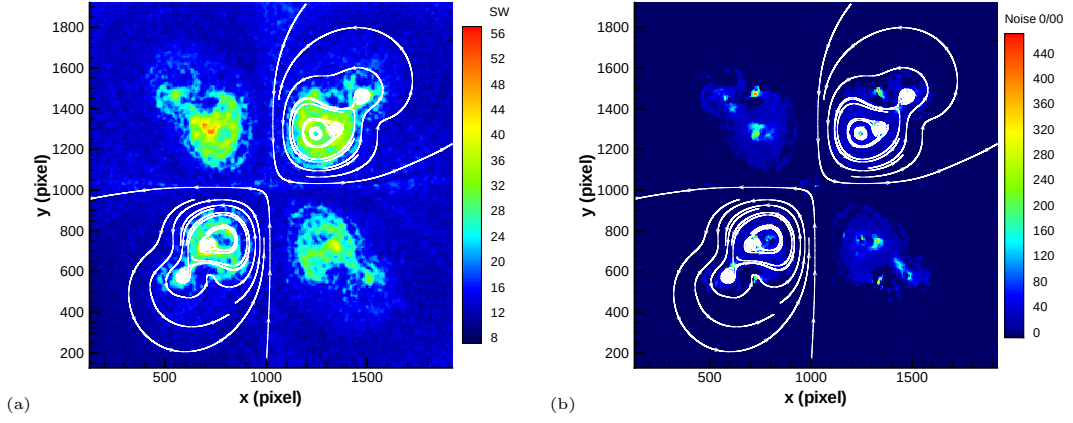


Figure 5.19: Examples of: (a) extrapolating window size (SW) shown in pixel and (b) grid approximating error of the constructed Eulerian velocity field; the values represent the standard deviation shown per mil (0.1%) of the approximated biquadratic surface. The quantities shown are of case C6 at the instant $t/T_f = 0.25$.

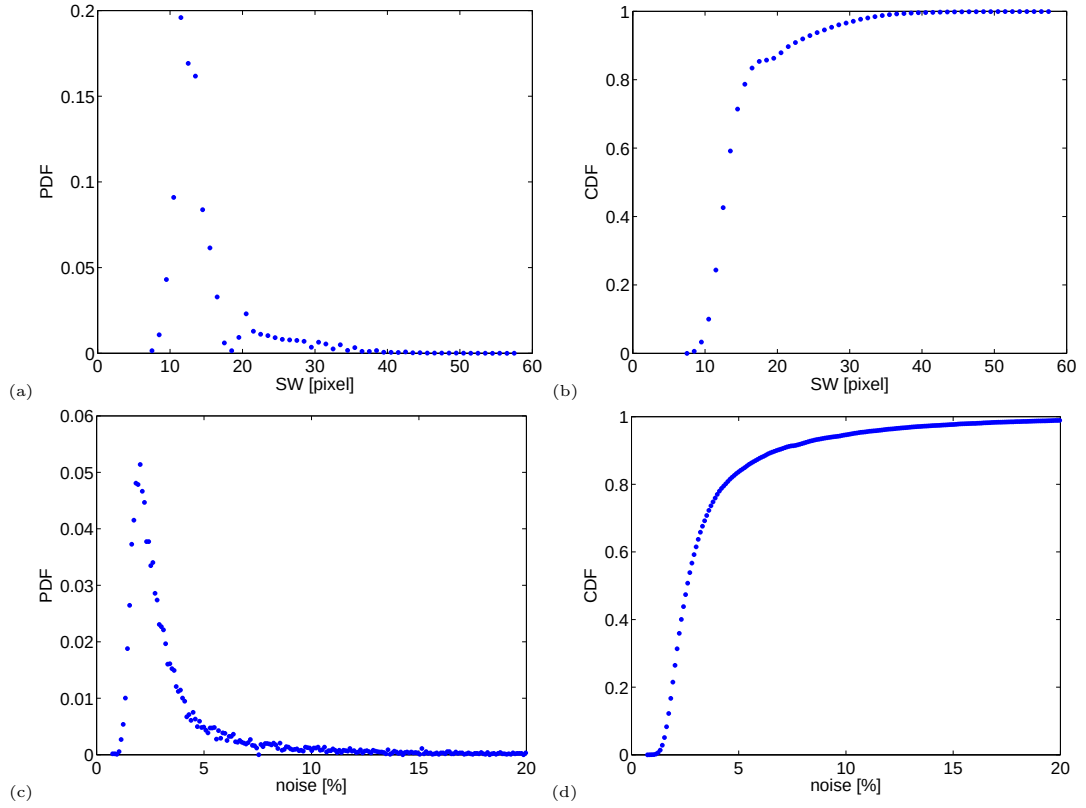


Figure 5.20: Probability Density Function (PDF) and Cumulative Density Function (CDF) of the extrapolating window size (a,b) and the grid approximation noise (c,d) of the case C6.

The confidence interval (CI) of the estimated velocity at each grid point is within $\pm \alpha \frac{\sigma_{noise}}{\sqrt{N_n}}$ from the actual value, where α is a constant and σ_{noise} and N_n are the standard deviation and the number of the velocity samples used in the estimation of the grid velocity respectively. The value of α is dependent on the size of the desired CI and the distribution of the estimated velocity at each grid point. In this work, $\alpha = 1.96$ assuming a normal distribution of the estimated grid velocity with 95% confidence interval (denoted by $CI_{95\%}$) and the number of samples (N_n) is 24. The value of $CI_{95\%}$ for each forcing case is presented in Table 5.10.

The PTV resolved Eulerian velocity field is two-dimensional (2D), however, it is extracted from the experimental flows which are in fact, quasi-two-dimensional (Q2D). Thus, we quantify the quasi-two-dimensionality by using the horizontal divergence, $\nabla_{2D} \cdot \mathbf{u}$ (where $\nabla_{2D} = \partial/\partial x + \partial/\partial y$) of the central 800×800 mm horizontal velocity field. All non-zeros values of $\nabla_{2D} \cdot \mathbf{u}$ are due to the third velocity components from the three-dimensionality effect of the flows and interpreted as error of the 2D Eulerian velocity field by $\Delta u/u_{max}$ as follows:

$$\frac{\Delta u}{u_{max}} = \frac{\Delta_{x/y} (\nabla_{2D} \cdot \mathbf{u})_{rms}}{u_{max}}, \quad (5.6)$$

where u_{max} is the maximum intensity of the velocity field, $\Delta_{x/y}$ is the length increment used to extract $\nabla_{2D} \cdot \mathbf{u}$ which is about the average extrapolating window size of PTV ($\overline{SW}$), $(\nabla_{2D} \cdot \mathbf{u})_{rms}$ is the root mean square of the divergence field.

The PDFs of $\nabla_{2D} \cdot \mathbf{u}$ for all forcing cases are shown in Figure 5.21 and $(\nabla_{2D} \cdot \mathbf{u})_{rms}$ are summarised in Table 5.10. From the figure and table, both $\nabla_{2D} \cdot \mathbf{u}$ and $(\nabla_{2D} \cdot \mathbf{u})_{rms}$ are close to zeros and $\Delta u/u_{max}$ for all cases range between 0.2-0.6% which is small compared to the noises from the experiment and Lagrangian trajectory approximation ($\approx 1\%$) and the grid approximation noise in Table 5.10. This implies that the three-dimensionality effect can be considered negligible in our flows.

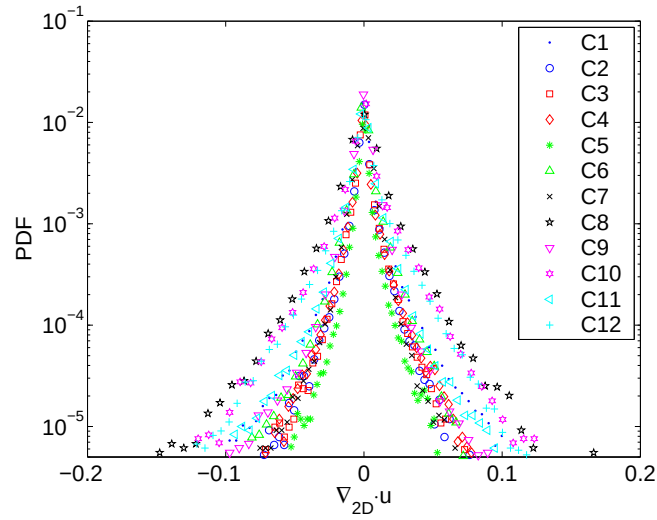


Figure 5.21: Probability Density Function of velocity field divergence, $\nabla_{2D} \cdot \mathbf{u}$.

Table 5.10: Eulerian velocity field quality: values are given by the mean $\pm$ the standard deviation of the quantities with the average taken over all time steps in each experimental case. The value $\bar{\sigma}_{noise}$ is in percentage of the mean value approximated by the biquadratic surface. $CI_{95\%}$ denotes the 95% confidence interval of the sampled mean (quoted in percentage of the mean value). %Biquad. denotes the percentage of the grid approximation by the biquadratic interpolation method.

Experiment	Case	No. of samples for Grid_PTV input	$\ u\ \pm \sigma_{\ u\ }$ [mm/s]	$\overline{SW} \pm \sigma_{SW}$ [pixel]	$\bar{\sigma}_{noise} \pm \sigma_{\sigma_{noise}}$ [%]	$CI_{95\%}$ [$\pm\%$]	%Biquad. [%]	$(\nabla_{2D} \cdot \mathbf{u})_{rms}$ [1/s]	$\frac{\Delta u}{u_{max}}$ [%]
Campaign I	C1	450,000	6.38 $\pm$ 0.90	13.6 $\pm$ 3.12	8.67 $\pm$ 9.76	3.47	7.42	0.013 $\pm$ 0.008	0.420
	C2	290,000	6.99 $\pm$ 0.47	18.5 $\pm$ 5.36	2.37 $\pm$ 2.66	0.95	38.3	0.009 $\pm$ 0.005	0.540
	C3	270,000	6.70 $\pm$ 0.67	19.9 $\pm$ 5.82	4.11 $\pm$ 3.53	1.64	33.9	0.010 $\pm$ 0.005	0.535
Campaign II	C4	240,000	7.12 $\pm$ 0.81	22.0 $\pm$ 6.40	4.10 $\pm$ 4.17	1.64	40.5	0.010 $\pm$ 0.006	0.600
	C5	280,000	5.26 $\pm$ 0.84	18.9 $\pm$ 4.19	4.39 $\pm$ 3.87	1.76	40.1	0.007 $\pm$ 0.004	0.530
	C6	450,000	7.08 $\pm$ 1.18	15.2 $\pm$ 5.32	4.37 $\pm$ 4.27	1.75	14.0	0.010 $\pm$ 0.005	0.301
Campaign III	C7	1,000,000	5.67 $\pm$ 0.08	11.4 $\pm$ 2.85	11.4 $\pm$ 9.21	4.56	1.36	0.010 $\pm$ 0.004	0.295
	C8	750,000	11.7 $\pm$ 0.18	13.3 $\pm$ 6.80	5.20 $\pm$ 5.64	2.08	12.0	0.020 $\pm$ 0.009	0.318
	C9	510,000	5.90 $\pm$ 0.19	13.3 $\pm$ 3.42	7.56 $\pm$ 6.15	3.02	4.24	0.011 $\pm$ 0.006	0.331
	C10	430,000	11.8 $\pm$ 0.41	17.0 $\pm$ 8.17	5.01 $\pm$ 5.33	2.00	20.4	0.018 $\pm$ 0.008	0.529
	C11	540,000	7.22 $\pm$ 0.36	13.3 $\pm$ 3.63	6.61 $\pm$ 6.31	2.64	7.28	0.013 $\pm$ 0.007	0.306
	C12	480,000	12.8 $\pm$ 0.68	13.5 $\pm$ 2.60	9.01 $\pm$ 9.94	3.60	5.06	0.018 $\pm$ 0.008	0.302

Chapter 6

Eulerian Statistics

Eulerian statistics provide a lot of critical information about the flow. Among those, the “flow structure” and the flow energy are the focus of this chapter. The flow structure refers to the schematic streamline geometry in the shape of the figure “8 in 8” with its topology characterised by the relation $n_s \sim (\frac{L}{\eta})^{D_s}$ where L and η are the outer and inner length-scales respectively, D_s is the fractal dimension and n_s is the number of stagnation points (Davila & Vassilicos 2003). According to Davila and Vassilicos (Davila & Vassilicos 2003), D_s relates to the power exponent p of the energy spectrum $E(k) \sim k^{-p}$ by $p + \frac{2D_s}{d} = 3$ where d is the dimension of the flow as described in Chapter 2. This relation holds for homogenous isotropic turbulence and gaussian models of it but not necessarily for the type of flows studied here.

Previous results of the same experimental setting but without the time-dependent forcing (Rossi, Vassilicos, & Hardalupas 2006a) and DNS (2D) (Hascoet, Rossi, & Vassilicos 2008) found the flow to have a power-law energy spectrum $E(k) \sim k^{-p}$ over $\frac{2\pi}{L} < k < \frac{2\pi}{\eta}$ and confirmed the relation $p + D_s = \text{const}$ when $d = 2$ which is invariant over a broad range of $600 < Re_{2D} < 9900$ (equivalent to the forcing range $0.04 < I[A] < 1$). Specifically, both experimental and DNS results found the p value to be close to 2.5 when D_s is approximately 0.5. In this chapter, we demonstrate that the addition of unsteadiness in the form of time-dependent forcing, which varies in forcing intensity (mean and amplitude) and forcing period, has little effect on the Eulerian statistics of the flow. This suggests that the multi-scale flow structure remains broadly invariant relative to the steady case.

In the following sections, the flow structure is observed from the Eulerian velocity field, its streamlines and its stagnation points characteristics. We investigate the flow's total energy characterised by u_{rms} , the energy spectra $E(k)$, which reflects how the energy resides in various scales and the Eulerian frequency spectra, $\Phi_E(\omega)$. Other flow properties such as the autocorrelation functions ($R_{2D}(r)$, $R_{1D}(r)$ and $R_E(\tau)$) as well as the Eulerian lengthscale, L_E are quantified. Furthermore, the effects of the time-dependent forcing on the flow's acceleration are also discussed.

6.1 Time-dependent schematic streamline topology

The constructed Eulerian velocity field of the case C6, which has the broadest range of forcing amplitude, longest forcing period and is expected to display the most variation of flow features with time-dependency, are shown in Figure 6.1. The four different instants in the forcing period, $\frac{t}{T_f}=0.00, 0.25, 0.50$ and 0.75 where $\frac{t}{T_f} = 0$ corresponds to the instant when $I(t) \approx I_{max}$ and $u_{rms}(t) \approx u_{rms,T_f}$ (Eq. (6.3)), are illustrated. Their streamlines are plotted using white lines and the multi-scale schematic pattern of the figure “8 in 8” can be clearly observed in each instantaneous velocity field.

From Figure 6.1, the time-dependent forcing causes the flow's energy intensity $\|\mathbf{u}\|$ to change and the streamlines to vary in shapes and sizes. Nevertheless, the schematic streamline topology, i.e. the number of stagnation points and the way they are connected to the streamlines, remains unchanged. Furthermore, the positions of the stagnation points are relatively persistent in that they only move slightly and slowly compared to the surrounding flow. The velocity field of other forcing cases also display similar behaviour with their time-dependent forcing, however, the variation are less obvious when the forcing period and forcing amplitude are smaller. The schematic streamline topology, particularly the number of stagnation points, are persistent for all time-dependent forcing cases studied in this work. This suggests that the fractal dimension of the stagnation point distribution, $D_s \approx 0.5$, is also constant in our flows.

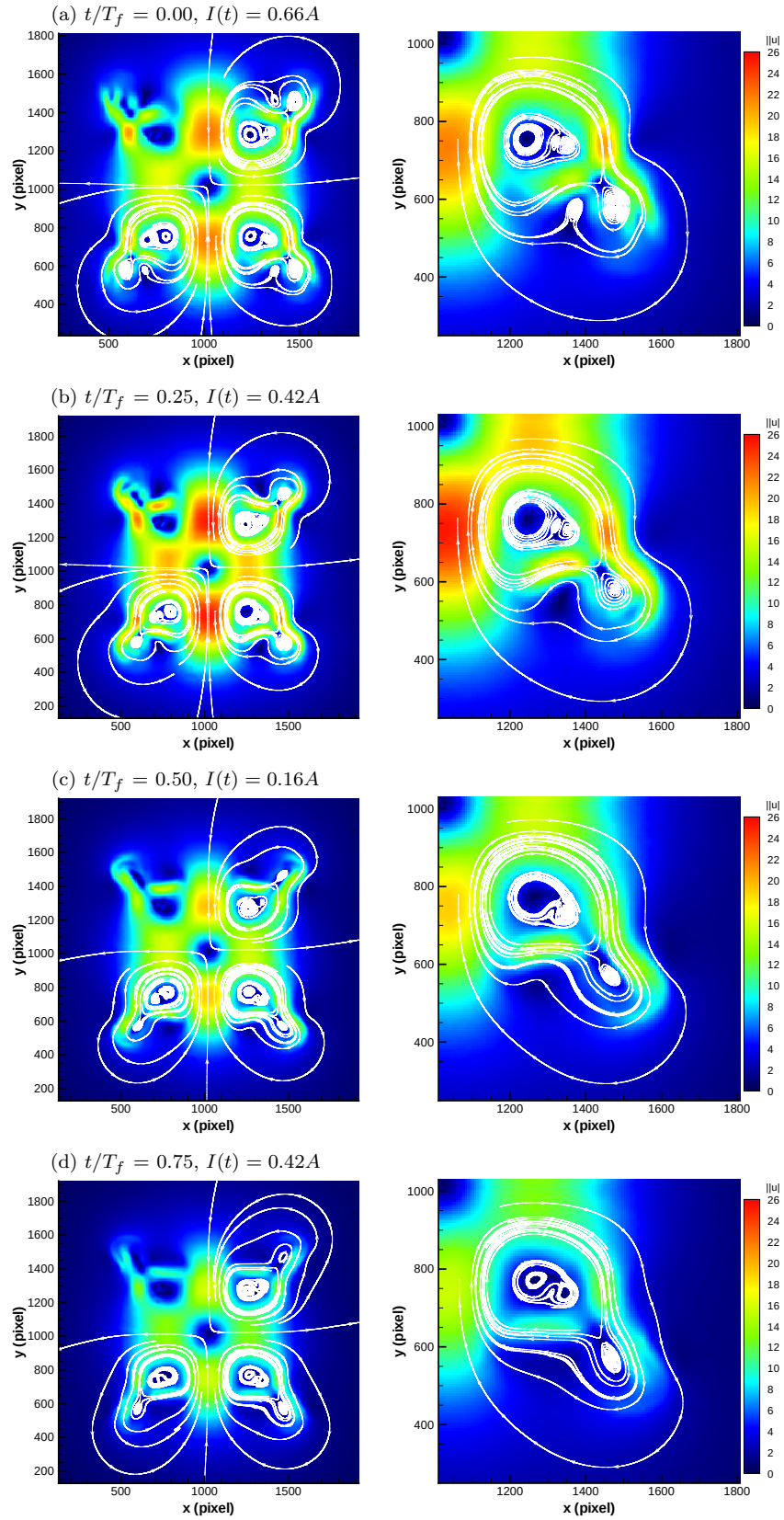


Figure 6.1: Instantaneous Eulerian velocity fields at four different instants $\frac{t}{T_f} = 0.00, 0.25, 0.50$ and 0.75 (left: entire flow, right: bottom right quarter of flow).

The difference in the velocity field between Figure 6.1(b) and (d) should be noted. Despite the same level of forcing intensity $I(\frac{t}{T_f} = 0.25) = I(\frac{t}{T_f} = 0.75) = 0.42\text{A}$, the flow's energy intensity at the instant $\frac{t}{T_f} = 0.75$ is noticeably lower than that of the instant $\frac{t}{T_f} = 0.25$. This suggests some delay in the flow response to the time-dependent forcing which is discussed in detail in Section 6.3.1.

6.1.1 Multi-scale analysis

We follow the procedure in Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a) and apply various sizes of averaging filters to the instantaneous velocity fields in order to illustrate their multi-scale topology. The filtered fields, shown in Figure 6.2 are averaged with a square window of size $L_w = 8.79, 43.95$ and 178.71 mm which are approximately compatible to the three different magnet sizes respectively. The filter operation reveals the flow scales of the same size and larger than L_w . Figure 6.2(a) shows the three fractal scales of the flow whereas the small M10 and medium M40 scales are filtered out from Figure 6.2(b) and (c) respectively. When subtracted the two filtered velocity fields of the two consecutive L_w i.e. Figure 6.2(c)-(b) and Figure 6.2(b)-(a), the flow pattern of eddies of the size ranging between the $43.95\text{-}178.71\text{mm}$ and $8.79\text{-}43.95\text{mm}$ can be seen (Figure 6.3). The energy of these multi-scale eddies is discussed in Section 6.4.2.

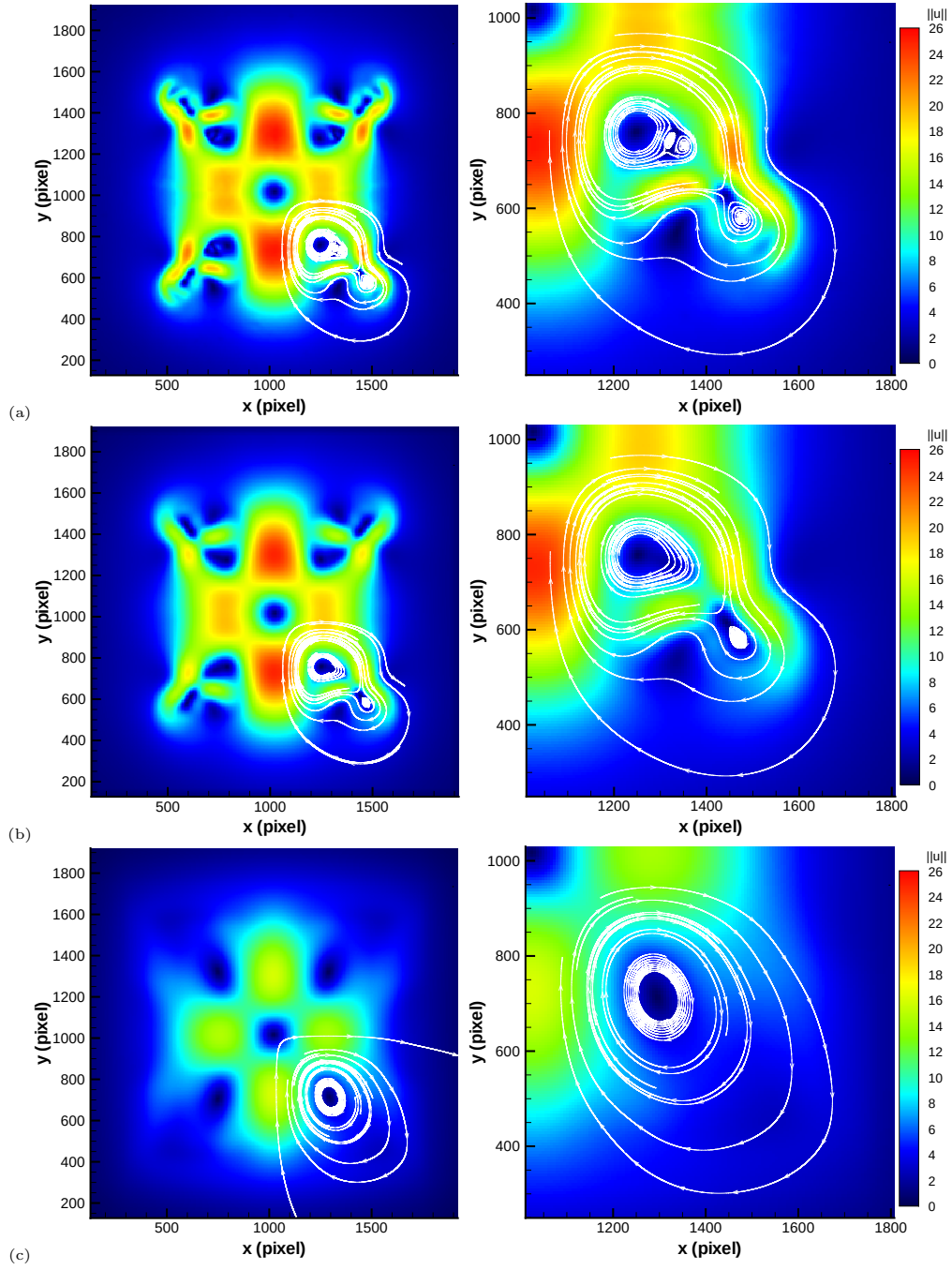


Figure 6.2: Eulerian velocity fields filtered by different averaging window sizes, L_w^2 (a) $L_w = 8.79$ mm (b) $L_w = 43.95$ mm (c) $L_w = 178.71$ mm. The velocity intensity $||\mathbf{u}||$ is shown in mm/s, 1 pixel ~ 0.488 mm) (left: entire flow, right: bottom right quarter of flow).

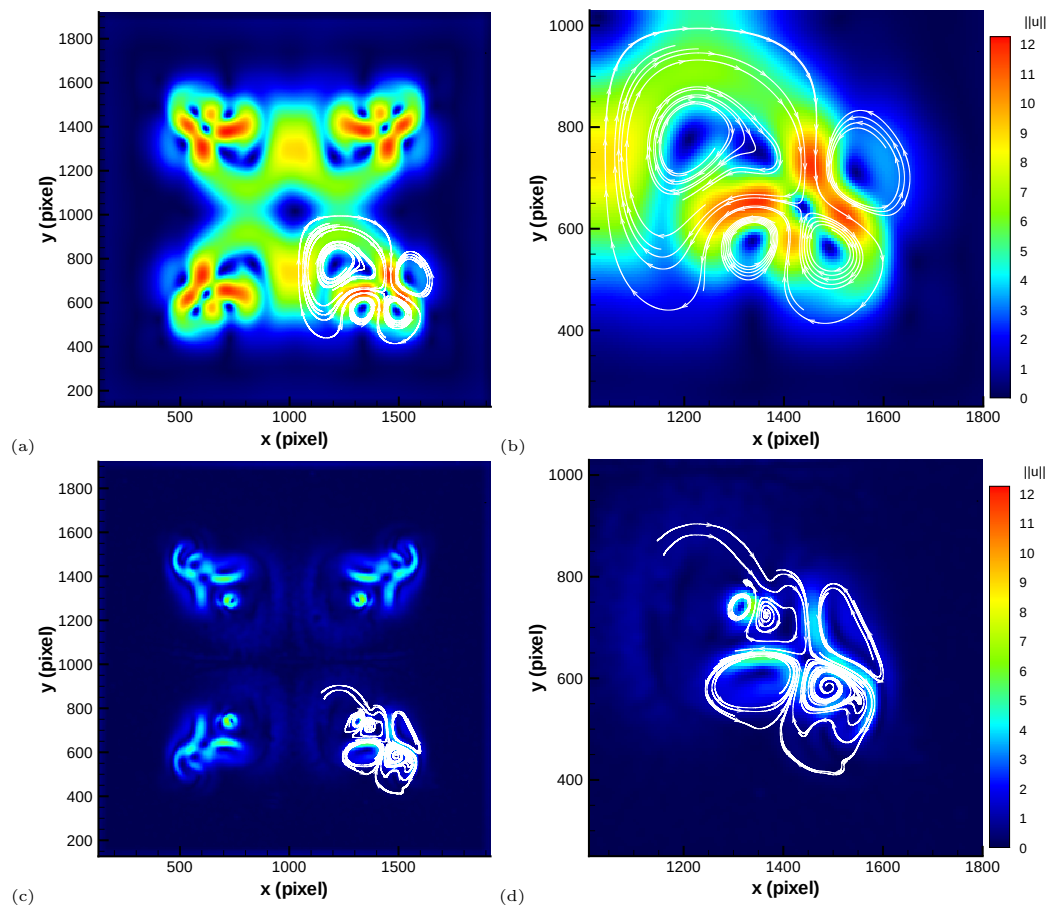


Figure 6.3: Different flow scales obtained by subtracting the filtered flows of Figure 6.2, (a) medium flow scale (Figure 6.2(b) - Figure 6.2(c)), (b) small flow scale (Figure 6.2(a) - Figure 6.2(b)). (left: entire flow, right: bottom right quarter of flow).

6.2 Stagnation point characteristics

The multi-scale schematic streamline structure is characterised by the number of stagnation points. Such structure has been claimed to be related to the statistics of fluid element diffusion. A new theory by (Fung & Vassilicos 1998), (Goto & Vassilicos 2004) and (Davila & Vassilicos 2003) proposed that the hyperbolic (straining) stagnation points are responsible for the vigorous exponential dispersion of fluid elements. The central argument is that a pair of fluid elements travel together for a long time until they reach the vicinity of hyperbolic stagnation points where the pair separate vigorously. The separation at the stagnation points follows an exponential growth of $\Delta(t) \sim \Delta_0 e^{\zeta t}$ which cannot occur elsewhere in the flow. The sequence of successive multi-scale exponential pair separations combine and lead to an algebraic separation

of the entire flow field on average overall (Goto & Vassilicos 2004), (Rossi, Vassilicos, & Hardalupas 2006a).

These hyperbolic stagnation points can be characterised by their positions, associated time scale, length scale, and area of direct influence. In the steady multi-scale flows studied in Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a), these measures are steady. In light of the importance of the hyperbolic stagnation points in flow's diffusivity, it is crucial to investigate their characteristics in response to the time-dependent forcing.

The position of the hyperbolic stagnation points, $\mathbf{x}_s$, are located by integrating trajectories in the Eulerian velocity field with its v -components reversed (See Appendix 9.2 for details). We then estimate the associated time scale by a single positive strain rate, λ , which is the eigenvalue, $\mathbf{J}\mathbf{X} = \pm\lambda\mathbf{X}$ of $\mathbf{J}_{ij} = \partial u_i / \partial x_j$ at the stagnation points where $\mathbf{X}$ is the eigenvector.

The associated length scale, L_s , can be thought of as the size of the streamlines passing very close to the hyperbolic stagnation points take a turn. The value can be identified by integrating the trajectories in the Eulerian velocity field along a streamline starting from a position very close to the hyperbolic stagnation point, $\mathbf{x}_s + \epsilon\mathbf{e}_\theta$, where $\mathbf{e}_\theta$ is a unit vector of angle θ and ϵ is a small displacement within the size of the Eulerian velocity grid. We have chosen ϵ to be half of the Eulerian velocity grid size and evenly distributed 30 positions around $\mathbf{x}_s$ with θ increment of 12 degree. The trajectories are integrated along the streamline until the value $\mathbf{u} \cdot \mathbf{e}_\theta$ change sign suggesting the instances when the trajectories turn and the position $\mathbf{x}_l(\theta)$ is recorded. The value of L_s is then defined as $L_s \equiv \langle |\mathbf{x}_l(\theta) - \mathbf{x}_s| \rangle_\theta$ where $\langle \dots \rangle_\theta$ denotes averaging over θ (Rossi, Vassilicos, & Hardalupas 2006b).

We calculated λ and L_s for all EM imposed hyperbolic stagnation points for all forcing cases. The scatter plots of the strain rate, λ normalised by u_{rms}/L_E versus t/T_f and versus the stagnation point sizes, L_s normalised by L_E , for a selection of cases from the three flow families are shown in Figure 6.4.

In the steady flows studied by Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006b), the strain rate of the small, medium and large hyperbolic stagnation points range as $\lambda_{10} > \lambda_{40} > \lambda_{160}$, and λ_{40} is close λ_{10} at low forcing intensity whereas λ_{40} is close λ_{160} at high forcing intensity. The fluid elements dispersion is more efficient when the three scales' strain rates are separate from each other.

Similar trends are also observed when the flows are forced by the time-dependent forcing where the intensity range covers only one flow scale. In the flow families II with the forcing intensity range below the bottom friction limit, λ_{40} is close to λ_{10} and in the flow families III with the forcing intensity range above the bottom friction limit, λ_{40} is close to λ_{160} (See Section 4.2 for flow family categorisation).

A distinctive trend of strain rates is observed in the flow family I where the forcing intensity range cover more than one flow scales. The flow family I shows much more significant response to the time-dependent forcing; λ_{10} and λ_{40} displays clear sinusoidal oscillations in line with the sinusoidal forcing. In contrast, λ_{160} displays very small oscillation suggesting that the time-dependent forcing has little effect on the large scale hyperbolic stagnation point strain rate. The magnitude of the strain rate oscillation increases with longer forcing period as the flow has more time to response to the time-dependent forcing. It is important to note that the oscillation of the strain rate of different scales are not necessarily synchronised. As a result, this allows for a cross over between λ_{40} and λ_{10} and the occurrences where $\lambda_{40} \geq \lambda_{10}$ as shown in Figure 6.4(a). Such occurrences have never been observed in the steady flows in Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006b).

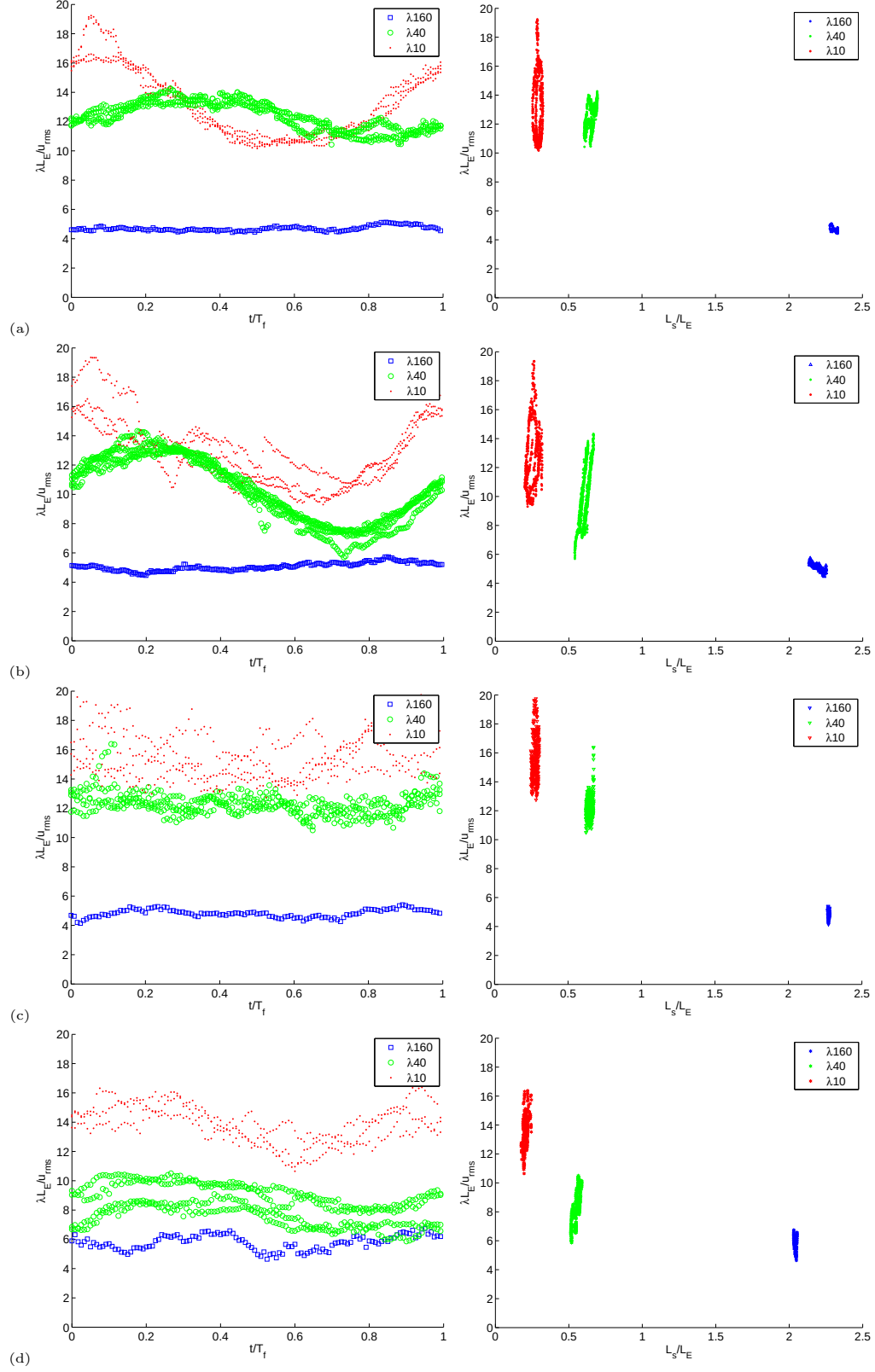


Figure 6.4: Strain rate and size of the EM imposed hyperbolic stagnation points of a selection of cases from the three flow families: (a) Family I (Case C5), (b) Family I (Case C6), (c) Family II (Case C9) and (d) Family III (Case C10) (Left: strain rate, λ , normalised by u_{rms}/L_E and Right: strain rate, λ normalised by u_{rms}/L_E versus the stagnation point sizes, L_s normalised by L_E).

The area of direct influence is referred to as the vicinity around the hyperbolic stagnation points, $\mathbf{x}_s$, where fluid element pairs with initial separation Δ_0 separate exponentially $\Delta(t) \sim \Delta_0 e^{\zeta t}$. We estimate the area of direct influence is by

$$Corr \equiv \frac{\mathbf{u} \cdot \mathbf{u}_s}{\max(\mathbf{u}^2, \mathbf{u}_s^2)}, \quad (6.1)$$

where $\mathbf{u}_s \equiv [\lambda(x - x_s), -\lambda(y - y_s)]$. In the vicinity around the hyperbolic stagnation points, the quantity $Corr$ is close to 1 and drop off steeply outside such area. The area of direct influence of the large (M160) and medium (M40) stagnation points for a selection of forcing cases from the three flow families are illustrated in Figure 6.5.

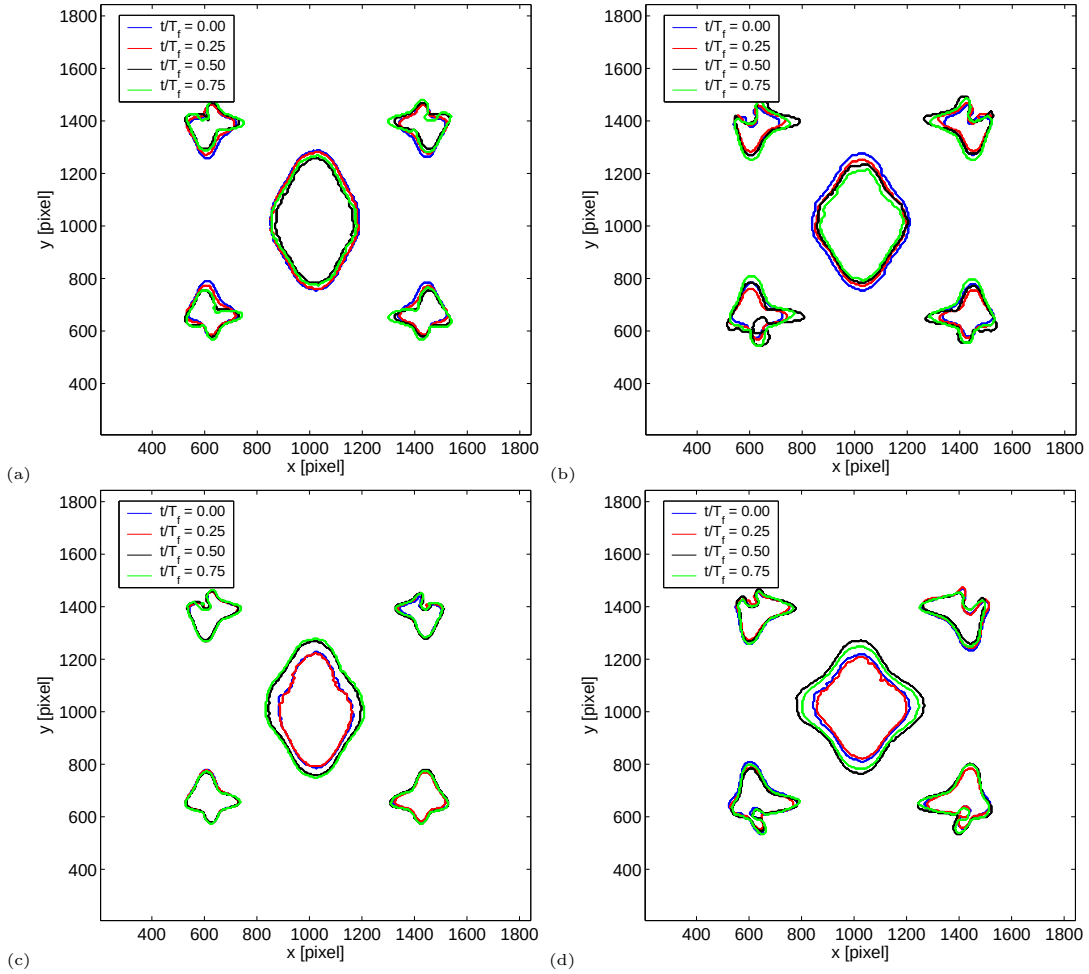


Figure 6.5: Areas of direct influence of the imposed M160 and M40 hyperbolic stagnation points: (a) Family I (Case C5), (b) Family I (Case C6), (c) Family II (Case C9) and (d) Family III (Case C10).

From Figure 6.5, the areas of direct influence vary slightly throughout the forcing period. The variation increases with higher forcing intensity, broader forcing intensity range and longer forcing period. Nevertheless, it can be seen that the imposed hyperbolic stagnation points and their areas of direct influence are relatively persistent in response to the time-dependent forcing overall.

6.3 Time-dependent flow energy

6.3.1 Mean flow energy

The mean flow energy can be measured via the root mean square velocity of the flow,

$$u_{rms}(t) = \sqrt{\frac{1}{N} \sum_{i=1}^N [\mathbf{u}(\mathbf{x}_i, t)]^2}, \quad (6.2)$$

where N is the total number of grid points in the velocity field. In the time-dependent flow, $u_{rms}(t)$ is a function of the forcing intensity $I(t)$, which is itself a function of the mean forcing intensity I_{mean} or $\bar{I}$, amplitude A and forcing period T_f .

The average flow energy throughout the forcing period, u_{rms,T_f} , is defined as:

$$u_{rms,T_f} = \sqrt{\frac{1}{N} \sum_{i=1}^N [u_{rms}(t)]^2}, \quad (6.3)$$

where N is the total number of all time steps in T_f . In fact, $u_{rms,T_f} \approx \langle u_{rms}(t) \rangle_{T_f}$ where $\langle \dots \rangle_{T_f}$ denotes the averaging by arithmetic mean for all time steps in the forcing period, T_f ; the difference between u_{rms,T_f} and $\langle u_{rms}(t) \rangle_{T_f}$ is within 1%. In our unsteady multi-scale flows, u_{rms,T_f} is only a function of I_{mean} , and is independent of A and T_f as $\frac{\partial}{\partial A} u_{rms,T_f} \simeq \frac{\partial}{\partial T_f} u_{rms,T_f} \simeq 0$ (Figure 6.6).

From Figure 6.6(a), the dependency of u_{rms,T_f} on I_{mean} of the time-dependent forcing data is well in line with that of the steady forcing (Rossi, Vassilicos, & Hardalupas 2006a) which suggests that the time dependent forcing does not affect the relationship between u_{rms} and I . In addition, the amplitude of the flow velocity, $A_{u_{rms}} = u_{\max} - u_{rms,T_f}$ has a linear dependency on both the Lorentz force, $\frac{\mathbf{A} \times \mathbf{B}}{\rho}$,

where A is the forcing amplitude, and the forcing period, T_f , with an arbitrary constant, C , as shown in Figure 6.6(b). The linear relationship confirms that the magnetic field, $\mathbf{B}$, is constant.

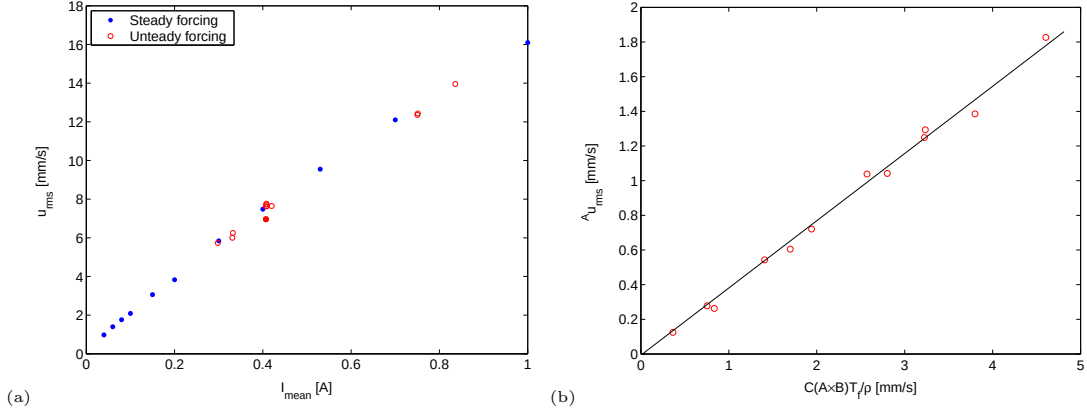


Figure 6.6: (a) Dependency of u_{rms} on I_{mean} : Blue dots indicate the steady forcing data (Rossi et al. 2006a). Red circle indicate the unsteady forcing data and the red solid dot denotes the unsteady forcing data of the case C1. The value of u_{rms} for the unsteady forcing is average u_{rms, T_f} . (b) Linear dependency of the flow velocity amplitude, $A_{u_{rms}}$, on the product of the Lorentz force and the forcing period, $\frac{A \times B}{\rho} T_f$ and C is an arbitrary constant. The linear relation can be represented by $A_{u_{rms}} = 0.384 A T_f$ confirming the constant magnetic field, $\mathbf{B}$.

From the data post-processing and instantaneous Eulerian velocity field shown in Section 6.1, it has been observed that the flow response has some delay to the time-dependent forcing; there is a time lag between the maximum forcing intensity and when the flow energy reaches its maximum value (Figure 6.7(a)). It has also been found that such delay seems to have a linear relationship with the forcing period as shown in Figure 6.8.

In Figure 6.8, the data of the case C1, displayed by a red solid dot, is the only case which does not fit the linear trend. When comparing the forcing intensity as a function of the dimensionless forcing period t/T_f (Figure 6.9(a)), we observed small differences for the case C1 from other cases (C2-C4 and C6) with the same time-dependent forcing intensity. The case C1 has slightly lower forcing intensity ($< 5\%$) in the range between $0.05-0.25t/T_f$ and $0.75-0.95t/T_f$. Nevertheless, the forcing of the case C1 reaches approximately the same level of I_{max} and I_{min} as in other cases C2-C4 and C6. Such small differences in the time-dependent forcing intensity are likely to have resulted from the experiment where the function generator generated slightly different

sine functions between the two campaigns.

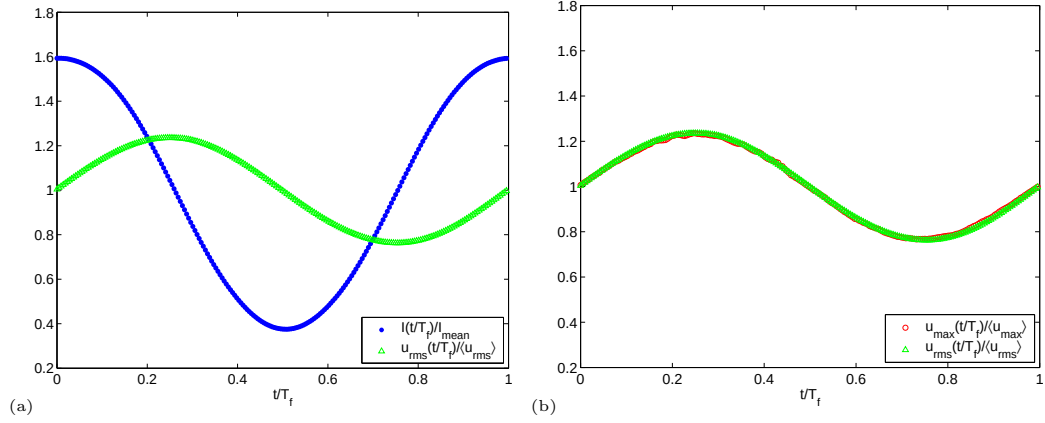


Figure 6.7: (a) Forcing intensity $I(t/T_f)$ normalised by I_{mean} and $u_{rms}(t/T_f)$ normalised by $\langle u_{rms}(t/T_f) \rangle$ versus non-dimensional time and (b) $u_{rms}(t/T_f)$ normalised by $\langle u_{rms}(t/T_f) \rangle$ and $u_{max}(t/T_f)$ normalised by $\langle u_{max}(t/T_f) \rangle$ versus non-dimensional time showing synchronisation.

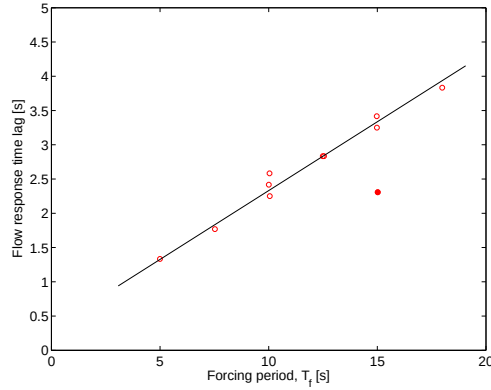


Figure 6.8: Linear dependency of the flow response time lag, t_{lag} , on the forcing period, T_f . The best fit linear function is approximately $t_{lag} = 0.18T_f + 0.50$. The red solid dot denotes the unsteady forcing data of Case C1.

We investigate $u_{rms}(t/T_f)$ obtained from the forcing in Figure 6.9(a) and found the case C1 to have lower $u_{rms}(t/T_f)$ overall (Figure 6.9(b)). This could be the result from the fact that polystyrene particles ($\emptyset \approx 800\mu\text{m}$) were used in the PTV measurement of the case C1 whereas Chemigum P83 ($\emptyset \approx 100 - 600\mu\text{m}$) were used in other cases. As polystyrene particles have slightly larger diameter and higher density than Chemigum P83, they have higher inertia resulting in higher resistance to the change in motion of the particles by the flow. Consequently, polystyrene particles do not reach as high value of the velocity as the Chemigum particles. This also explains the difference in the time lag between the time-dependent forcing intensity and the flow response measured

by the two types of particles.

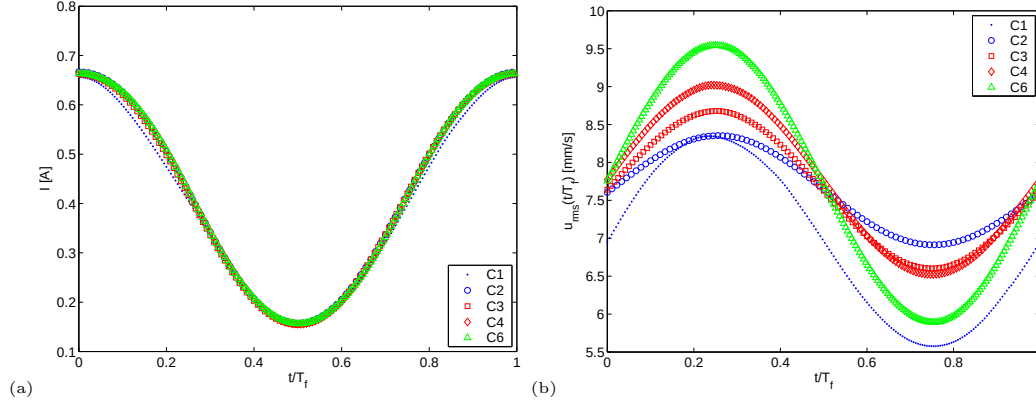


Figure 6.9: (a) Forcing intensity versus non-dimensional time and (b) flow velocity u_{rms} versus non-dimensional time.

The flow velocity u_{rms} also determines the dimensionless Reynolds number which is an important measure of turbulence. The value of two and three-dimensional Reynolds number, scaled as $Re_{2D} \equiv u_{rms} L_{PTV} / \nu$ and $Re_{3D} = u_{rms} H / \nu$ respectively, are given in Table 6.1. The time-dependent flows studied in this work have a range of $3000 < Re_{2D} < 9000$ and $20 < Re_{3D} < 55$. The fact that these flows have low values of Re_{3D} and low velocity (u_{rms} and u_{max} never exceed 15 and 40 mm/s respectively) are part of the main arguments that our turbulent-like flows are in fact laminar.

6.4 Eulerian spatial autocorrelations function and integral lengthscale

In general, energy spectra can be calculated directly from the Fourier transform of the velocity field. The important assumption for the process is that the field must be periodic. As our experimental flow is neither periodic, nor satisfies the zero boundary condition, the energy spectra is obtained from the Fourier transform of the velocity autocorrelation function, with its boundary condition approximated to be zero and hence periodic. This can be written in the two-dimensional discrete form as:

$$E_{2D}(\mathbf{k}_{ij}) = \sum_{r_i=0}^{N=1} \sum_{r_j=0}^{M=1} R_{2D}(\mathbf{r}_{ij}) e^{-i \frac{2\pi k_i r_i}{N}} e^{-i \frac{2\pi k_j r_j}{M}}, \quad (6.4)$$

where N and M are the number of grid points (rows and columns) in the i and j directions.

The Eulerian spatial autocorrelation function R_{2D} of the two-dimensional Eulerian velocity field, as shown in Figure 6.10, is defined as

$$R_{2D}(\mathbf{r}_{ij}) = \langle \mathbf{u}(\mathbf{x}_{ij}) \cdot \mathbf{u}(\mathbf{x}_{ij} + \mathbf{r}_{ij}) \rangle, \quad (6.5)$$

where the average $\langle \dots \rangle$ is taken over all points $\mathbf{x}_{ij}$.

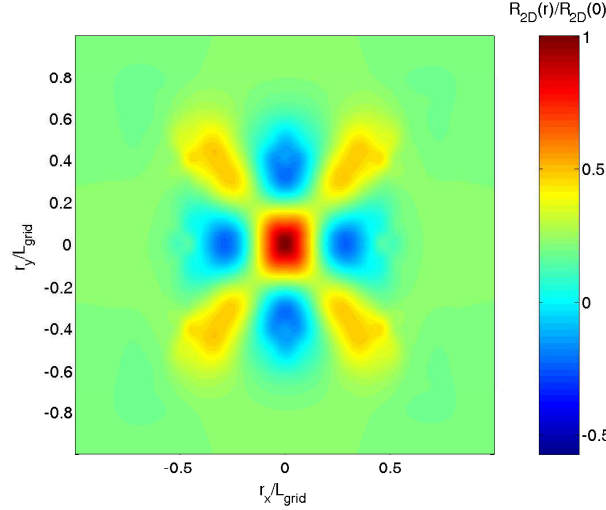


Figure 6.10: Two-dimensional Eulerian spatial correlation.

The one-dimensional spatial correlation $R_{1D}(r)$ (Figure 6.11) is then found by angular averaging of $R_{2D}(r)$ over all $\mathbf{r}_{ij}$ with $r - d_r/2 \leq |\mathbf{r}| < r + d_r/2$ binning where d_r is the spatial grid size of the Eulerian velocity field. The coordinate of r are chosen as $-0.94L_{grid} \leq \mathbf{r} \leq 0.94L_{grid}$ and $-0.91L_{grid} \leq \mathbf{r} \leq 0.91L_{grid}$ (L_{grid} is the size of the Eulerian velocity field), for the grid size of 5 and 6 pixels respectively, so as to avoid statistical problem at the velocity field edge and still large enough to cover the 800×800 mm central region. The values of $0.91L_{grid}$ (case C1-C6 and C12) and $0.94L_{grid}$ (case C7-C11) correspond to the physical lengths of and 800 and 803mm.

From Figure 6.11, $R_{1D}(r)$ of our unsteady multi-scale flows have a clearly defined negatively correlated region due to the deterministic looping of the large scale streamlines. The presence the negatively correlated region of the spatial correlation is different to that of homogeneous isotropic turbulence where the correlation decays to zero due to the random nature of the turbulent velocity field.

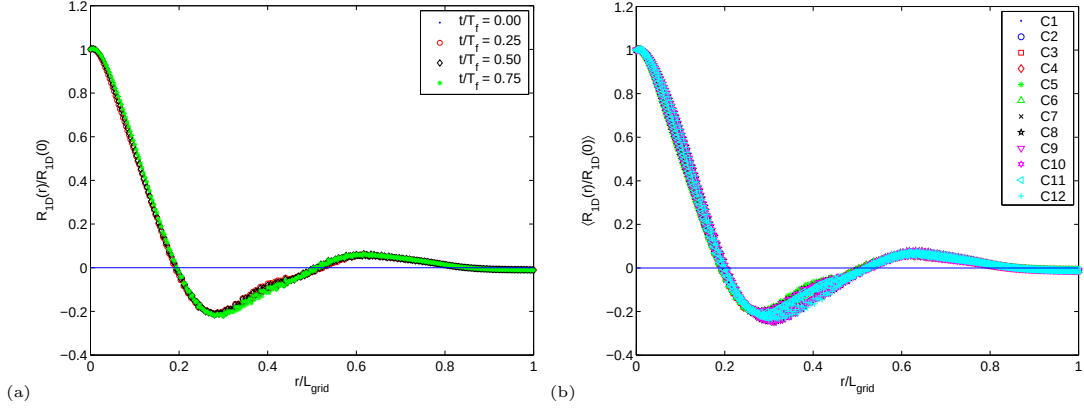


Figure 6.11: One-dimensional spatial correlation from angular averaging of R_{2D} : (a) $R_{1D}(r)$ at various t/T_f in the case C6 and (b) average $R_{1D}(r)$ of different experimental cases.

The Eulerian integral lengthscale L_E is then determined by

$$L_E(t) = \frac{1}{u_{rms}^2(t)} \int_0^\infty R_{1D}(l, t) dl. \quad (6.6)$$

The values of $L_E(t)$ for different forcing cases calculated from the conventional definition of Eq. (6.6) are in the range of 65-80mm as shown in Table 6.1. The values are equivalent to 1/3-1/2 of the largest M160 magnet size. In Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a), however, the Eulerian integral lengthscale is found by integrating the angular averaging of $|R_{2D}|$ which is the energy intensity of the velocity correlation function. The integral lengthscale calculated from the latter definition, $L_{E|R_{2D}|}$, is of the order of the largest M160 magnet size and approximately 2.55-2.65 times larger than L_E found from the definition in Eq. (6.6). To allow a direct comparison with the previous steady forcing result in Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a), $L_{E|R_{2D}|}$ are used in the scaling of the result presented in this work and will be referred to as L_E onward.

Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a) suggest that L_E is controlled by the multi-scale spatial forcing distribution and do not vary significantly with the forcing intensity. Figure 6.12 presents a comparison between L_E values obtained from the steady forcing (Rossi, Vassilicos, & Hardalupas 2006a) and the time-dependent (unsteady) forcing. The ranges of time-dependent $L_E(t)$ versus $u_{rms}(t)$ for each forcing case are shown in Figure 6.12(a) whereas the average $L_E = \langle L_E(t/T_f) \rangle_{T_f}$ is given

in Figure 6.12(b). It can be seen that L_E for our unsteady multi-scale flows are consistent with steady L_E when $u_{rms} \leq 8\text{mm/s}$ or $I \leq 0.4\text{A}$. At higher I or u_{rms} , L_E seems to carry on the linear trend and differs from the steady L_E by approximately 3-3.5%. The point of departure in the trend can possibly be at $I=0.53\text{A}$ where the bottom friction is overcome; beyond this point, unsteadiness seems to increase the L_E scale slightly. Nevertheless, L_E variations with both steady and unsteady forcing are of the same order overall and considered small compared to how I and u_{rms} vary.

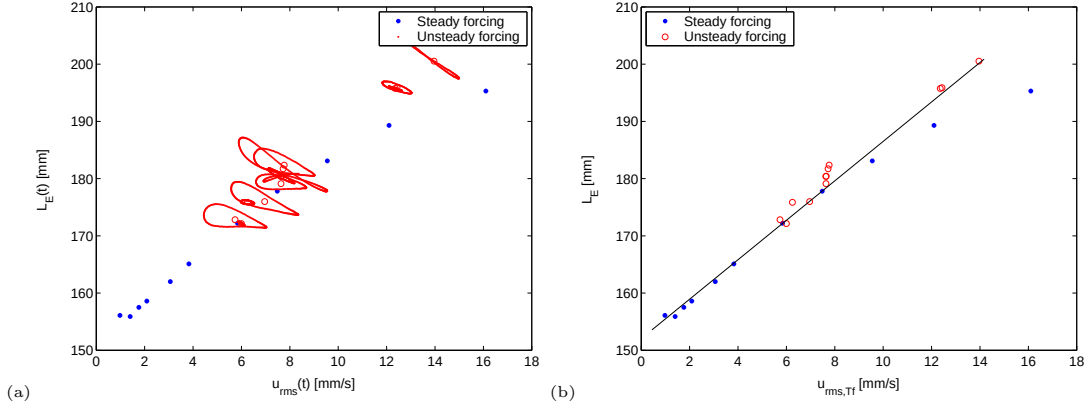


Figure 6.12: $L_E(t)$ versus $u_{rms}(t)$: blue dots indicate L_E of the steady forcing (Rossi et al. 2006a), red dots indicate L_E of the unsteady forcing cases (a) and red circles indicate the average L_E of each unsteady forcing case (b).

Table 6.1: Typical scales of the turbulent-like flow forced by different forcing cases. The values shown are the average within the forcing period of each case and their standard deviations. * denotes the “steady” cases; their velocity fields are obtained from a single snapshot of instantaneous Eulerian velocity field from the time-dependent forcing cases.

Experiment	Case	u_{rms,T_f} [mm/s]	$u_{\max}$ [mm/s]	$A_{u_{rms}}$ [mm/s]	L_E [mm]	$L_{E R_{2D} }$ [mm]	Re_{2D}	Re_{3D}	p
Campaign I	C1	7.03±0.99	19.07±2.73	1.39	68.71±0.64	175.98±1.98	4200±590	26.3±3.72	2.6
	C2	7.65±0.51	20.58±1.29	0.72	70.23±0.19	180.37±0.48	4600±310	28.8±1.93	2.6
	C3	7.67±0.73	20.67±1.95	1.04	69.79±0.38	179.12±0.95	4600±440	28.8±2.76	2.6
Campaign II	C4	7.82±0.88	20.71±2.46	1.25	70.91±0.58	182.35±1.65	4680±530	29.3±3.32	2.6
	C5	5.80±0.91	15.58±2.46	1.29	67.76±0.54	172.82±1.45	3460±550	21.6±3.44	2.6
	C6	7.83±1.29	20.80±3.41	1.83	69.67±1.02	181.72±3.12	4660±780	29.1±4.87	2.6
Campaign III	C7	6.00±0.09	16.71±0.24	0.12	66.61±0.02	172.13±0.26	3640 ±54	22.6±0.33	2.6
	C8	12.36±0.19	32.08±0.51	0.26	75.31±0.09	195.75±0.27	7500±115	46.6±0.71	2.54
	C9	6.25±0.20	17.39±0.53	0.28	67.87±0.10	175.85±0.26	3800±120	23.6±0.74	2.7
	C10	12.44±0.43	32.06±1.08	0.61	75.45±0.29	195.88±0.75	7530±260	46.9±0.71	2.6
	C11	7.65±0.38	20.67±1.06	0.54	68.32±0.28	180.43±0.91	4630±230	28.8±1.45	2.6
	C12	13.98±0.74	36.84±1.51	1.04	78.30±0.78	200.52±2.21	8420±450	52.6±2.78	2.54
Steady*	C5	5.79	15.88	-	68.37	174.46	3500	21.8	2.6
	C6	8.12	21.86	-	70.46	180.91	4900	30.6	2.6
	C9	6.25	17.25	-	67.98	176.18	3800	23.6	2.7
	C10	12.43	32.27	-	75.62	196.46	7500	46.9	2.6

6.4.1 PDF of Eulerian velocity field

The probability density function (PDF) of the instantaneous Eulerian velocity field at the four t/T_f instants of the case C6, which has the broadest forcing amplitude, longest forcing period and is expected to show the most variation in response to the forcing, is illustrated in Figure 6.13. The velocity intensity, $\|u(t)\|$, is defined as:

$$\|u(x_i, t)\| = \sqrt{[u^2(x_i, t) + v^2(x_i, t)]}, \quad (6.7)$$

where $u(t)$ and $v(t)$ are the velocity components. In Figure 6.13, the PDFs of $\|u(t)\|$, $u(t)$ and $v(t)$ normalised by $u_{rms}(t)$ are shown.

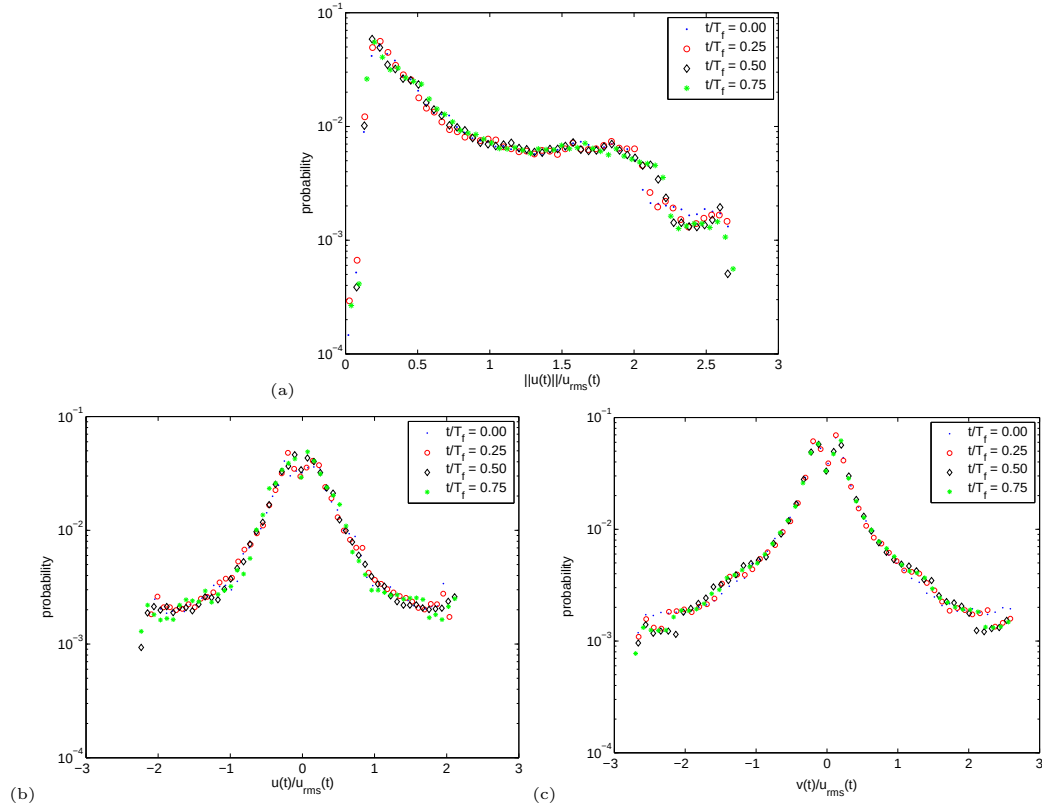


Figure 6.13: PDFs of the four instants of forcing $\frac{t}{T_f}=0.00, 0.25, 0.50$ and 0.75 of the case C6: (a) PDFs of velocity intensity, (b) PDFs of the u components of the velocity and (c) PDFs of the v components of the velocity.

From Figure 6.13(a), the PDFs of $\|u(t)\|$ have a broad range of values ranging from zero, which is associated with the stagnation points, to approximately $2.75u_{rms}(t)$, which occurs around the largest M160 magnets. The PDFs of $\|u(t)\|$ have a peak around $0.2u_{rms}(t)$ ($\simeq 1.54\text{mm/s}$), which covers the regions around the edges of the

multi-scale flow (Figure 6.14(b)). The regions of the velocity field associated with different ranges of $\|u(t)\|$ in the PDFs in Figure 6.13(a) are shown in Figure 6.14.

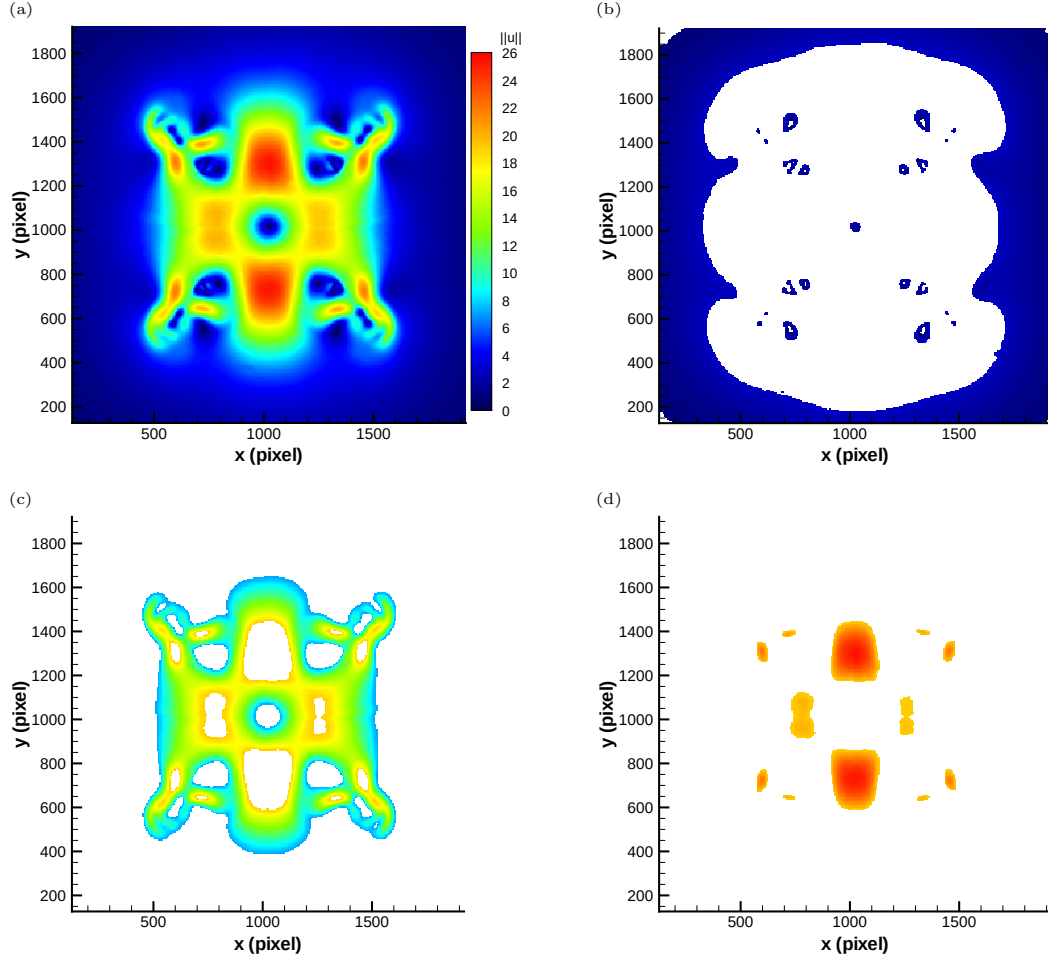


Figure 6.14: Regions in the velocity field covered by different ranges of the $\|u\|$ shown in Figure 6.13(a): (a) velocity field plotted with the entire range of $\|u\|$, (b) regions associating to the peak in the PDF with $0.1u_{rms}(t) \leq \|u\| \leq 0.3u_{rms}(t)$ ($0.95\text{mm/s} \leq \|u\| \leq 2.85\text{mm/s}$), (c) regions associating to the flat area in the PDF with $0.75u_{rms}(t) \leq \|u\| \leq 2u_{rms}(t)$ ($7.12\text{mm/s} \leq \|u\| \leq 19\text{mm/s}$) and (d) regions associated with the tail in the PDF with $\|u\| \geq 2u_{rms}(t)$ ($\|u\| \geq 19\text{mm/s}$).

The PDFs of the velocity components (Figure 6.13(b)-(c)) show a symmetry shape with the center at zero, which is corresponding to the stagnation points, and two peaks close to the zero value. Our flows have different u and v components due to the magnet arrangement that creates the stagnation points and such difference is evident in the PDFs. This is different to homogeneous isotropic turbulence where its PDFs of the velocity components are the same due to its isotropy.

Another difference between our unsteady multi-scale flows and homogeneous isotropic

turbulence is that the velocity PDFs of our flows have two symmetrical peaks at values close to zero rather than a single peak at zero as in the case of turbulence. As is known, the zero velocity regions in the flow field are associated with the stagnation points. Hence, our flows, which have less number of stagnation points than fully developed turbulence, also have less zero velocity regions and a lower distribution at the zero value in the PDFs.

From Figure 6.13, the normalised velocity PDFs of the four different instants in the same forcing case vary only slightly with the time-dependent forcing and they all have the same shape and trend. These PDFs not only agree with the relatively persistent flow structure as shown in Section 6.1, but also suggest the persistent statistical properties of our flows.

We investigate the statistical properties of our flows via the first, μ'_1 , and second moments, μ'_2 (mean and variance) of the normalised Eulerian velocity intensity, $||u(t)||/u_{rms}(t)$ for all time t in the forcing period T_f of each forcing case as shown in Table 6.2.

Table 6.2: First (μ'_1) and second moments (μ'_2) of the normalised Eulerian velocity intensity, $||u(t)||/u_{rms}(t)$. The values present are the average over all time t in the forcing period T_f and its standard deviation in percentage of the average values.

Experiment	Case	μ'_1	μ'_2
Campaign I	C1	0.767±0.044	0.411±0.126
	C2	0.776±0.274	0.398±0.829
	C3	0.775±0.117	0.400±0.352
Campaign II	C4	0.779±0.057	0.394±0.175
	C5	0.770±0.187	0.407±0.545
	C6	0.779±0.262	0.394±0.806
Campaign III	C7	0.764±0.053	0.416±0.150
	C8	0.792±0.049	0.373±0.165
	C9	0.767±0.049	0.411±0.179
	C10	0.792±0.034	0.372±0.115
	C11	0.771±0.042	0.406±0.123
	C12	0.804±0.282	0.354±1.031

The values of the moments presented in Table 6.2 are the average of all time t and its standard deviation as a percentage of the average value. The 1st moment for all cases ranges between 0.76-0.80 whereas the 2nd moment is between 0.35-0.41. Their standard deviation, σ are well under 1% except for the case C12 where σ of the 2nd moment is about 1% as a result of higher noise level in the Eulerian velocity field in this particular case. The low values of standard deviations for both the 1st and 2nd moments suggest that these moments (and the PDFs of $\|u(t)\|/u_{rms}(t)$) do not vary with respect to time. Hence, the Eulerian velocity intensity can be considered to be wide-sense stationary and confirms the persistency of the flow statistical properties.

Figure 6.15 and Figure 6.16 show the representative PDFs of $\|u\|$, u and v normalised by u_{rms,T_f} for all forcing cases, which are also categorised into three flow families. The PDFs are constructed from all of the Eulerian velocity statistics throughout the forcing period in each forcing case normalised by its u_{rms,T_f} .

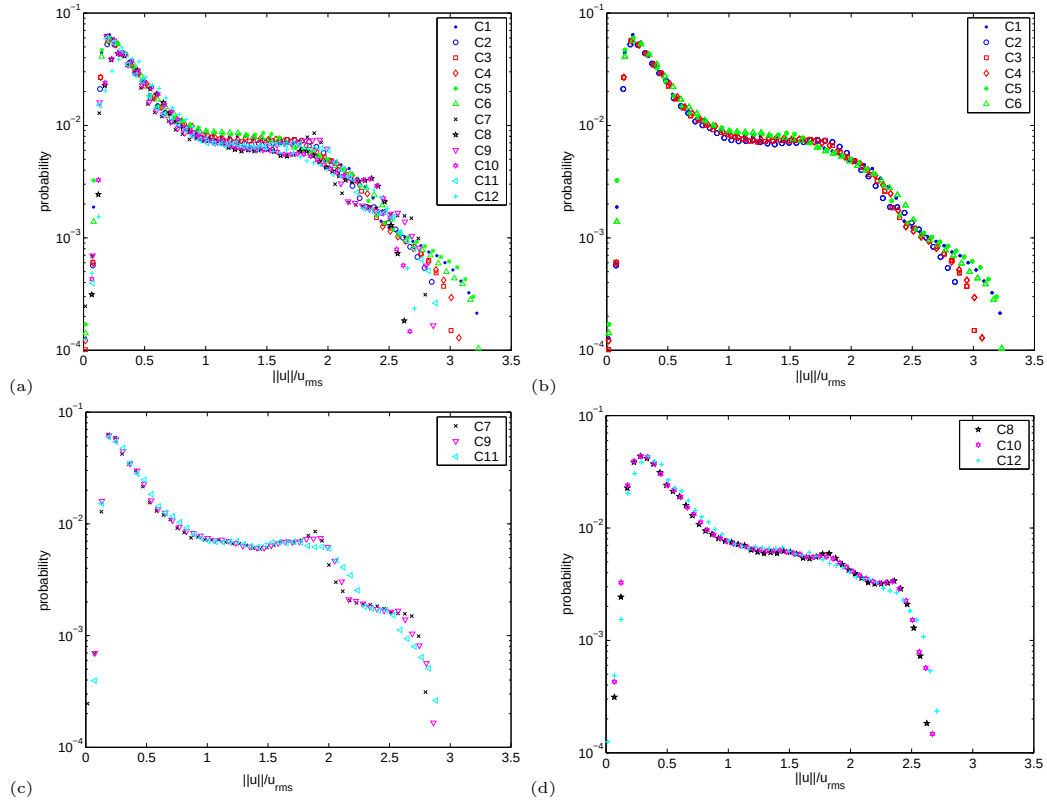


Figure 6.15: PDFs of normalised velocity intensity $\|u\|/u_{rms,T_f}$: (a) all different forcing cases, (b) flow family I, (c) flow family II and (d) flow family III.

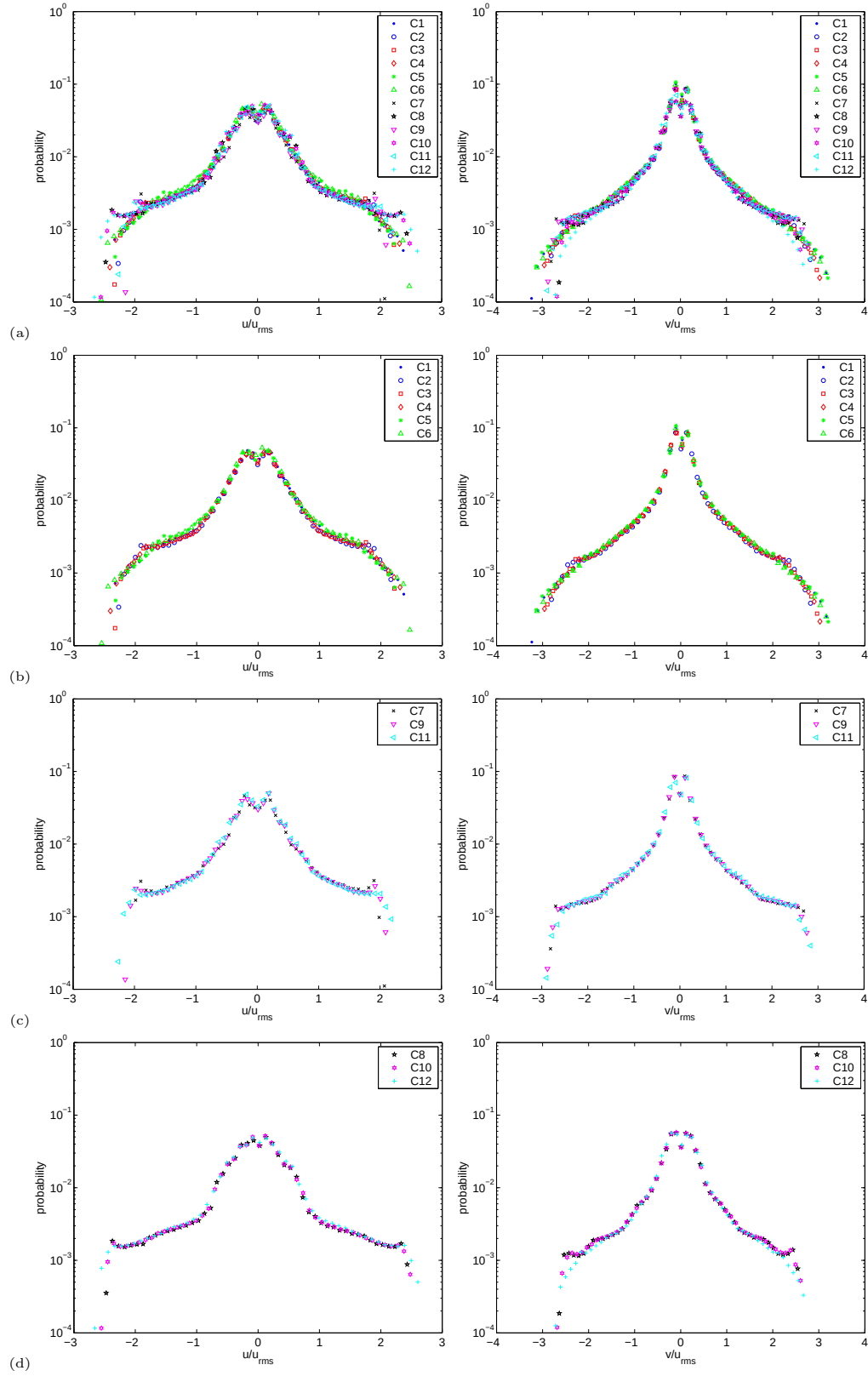


Figure 6.16: PDFs of velocity components (left: u component, right: v component): (a) all different forcing cases, (b) flow family I, (c) flow family II and (d) flow family III.

From the figures, the PDFs of each flow family display distinct peak values, degree of smoothness and the spread of the tails. In Figure 6.15, the PDF peaks are in the range of $0.2-0.3u_{rms,T_f}$ and shift towards higher values of $||u||$ as the mean forcing intensity increases. The PDFs of the cases C1-C4 and C6 (Figure 6.15(b)), which have the same forcing intensity and amplitude, reveal that the PDFs tail is broader as the forcing period increases. This is because as the forcing period increases, the flows have more time to respond to the time-dependent forcing and hence can reach higher velocity in agreement with Figure 6.9(b).

When comparing the PDFs of the velocity components between the different flow families, it can be seen that the flow family III (Figure 6.16(d)), which has higher forcing intensities than other flow families, displays the asymmetric peaks near the zero value that are less sharp. This can be explained by the fact that as the flows are forced with higher forcing intensity, the low velocity regions in the flow field reduces overall. Furthermore, it was previously shown in Figure 2.6 that the size of the stagnation points and the streamlines attached to them are smaller for higher forcing intensity. Hence, the low velocity regions around the stagnation points also reduces for this reason. Lastly, higher mean forcing intensity seems to reduce the range of $||u||/u_{rms,T_f}$ as a result of higher u_{rms,T_f} denominator in Figure 6.16(d).

6.4.2 Time-dependent energy spectrum

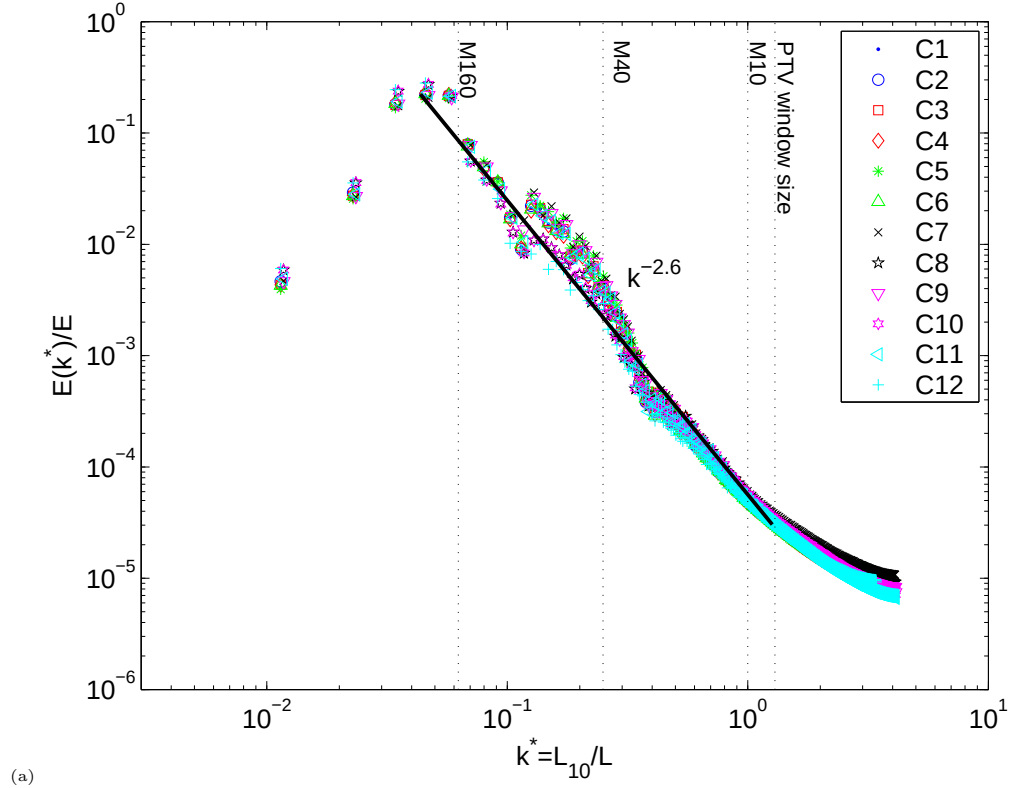
Fourier transform of R_{2D} of an instantaneous Eulerian velocity field provides two-dimensional Fourier coefficients which are also binned with $r - d_r/2 \leq |\mathbf{r}| < r + d_r/2$ to obtain the final one-dimensional energy spectra, $E(k)$ where $E = \int_0^\infty E(k)dk$ is the total energy in the flow.

The representative energy spectrum for all forcing cases, calculated by averaging all spectra from all time in the forcing period within each forcing case, are illustrated in Figure 6.17(a) and (b). The energy spectrum are also shown in three different families in Figure 6.18. From the figures, all energy spectra are very similar and exhibit slopes closer to a power law as the mean forcing intensity increase. The power exponent of

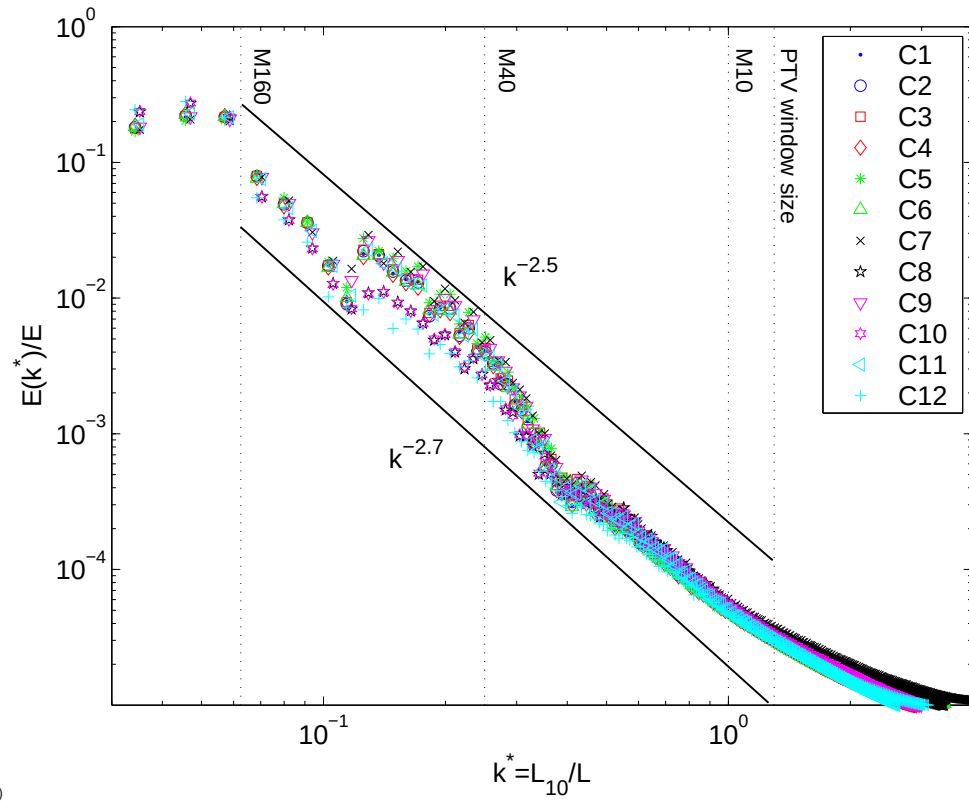
the energy spectrum, p , is estimated to be approximately $2.5 \leq p \leq 2.7$ in the range between the wavenumbers associated to the largest M160 magnets and smallest M10 magnets sizes. The values of the power exponent are in the same range as the results of the steady cases in Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a).

The cutoff range of the energy spectra is determined by the average window size used to map the Lagrangian velocities from PTV measurement on to a regular grid to construct the Eulerian velocity field. Any length-scales smaller than such window size are not resolved in our velocity field. In addition, the drops in the energy level at very low wavenumbers (lower than the wavenumber associated to the largest M160 characteristic lengthscale) are due to the boundary effect of the velocity field. The streamlines associated to these very low wavenumbers (i.e. of the size much larger than the largest M160 characteristic lengthscale) are incomplete, resulting in lower energy level in the energy spectrum. Such range of very low wavenumbers is beyond the wavenumber range of interest and is not considered in our analysis.

A selection of energy spectra from the three flow families at different forcing instants ($t/T_f = 0.00, 0.25, 0.50$ and 0.75) are shown in Figure 6.19-6.22. The energy spectra display oscillations at the scales that the forcing targets, in line with the forcing map as shown in Figure 4.6. For instance, the energy spectrum of the forcing case C9 (Figure 6.21(d)), which targets to excite only the medium flow scale below the bottom friction limit, display oscillations around the wave number associating to the M40 length scale while other scales remain relatively still. Likewise, the energy spectrum of the forcing case C10 (Figure 6.22(d)), which targets scales between the large and medium flow scales, display oscillations between the wave numbers associating to the M160 and M40 characteristic length scales. The same observations apply to the case C5 and C6 shown in Figure 6.19(d) and 6.20(d); these cases also display noticeably more oscillations due to broader forcing amplitudes and longer forcing periods.



(a)



(b)

Figure 6.17: Energy spectrum: (a) average $E(k^*)$ of different forcing cases and (c) ranges of the power exponent, p .

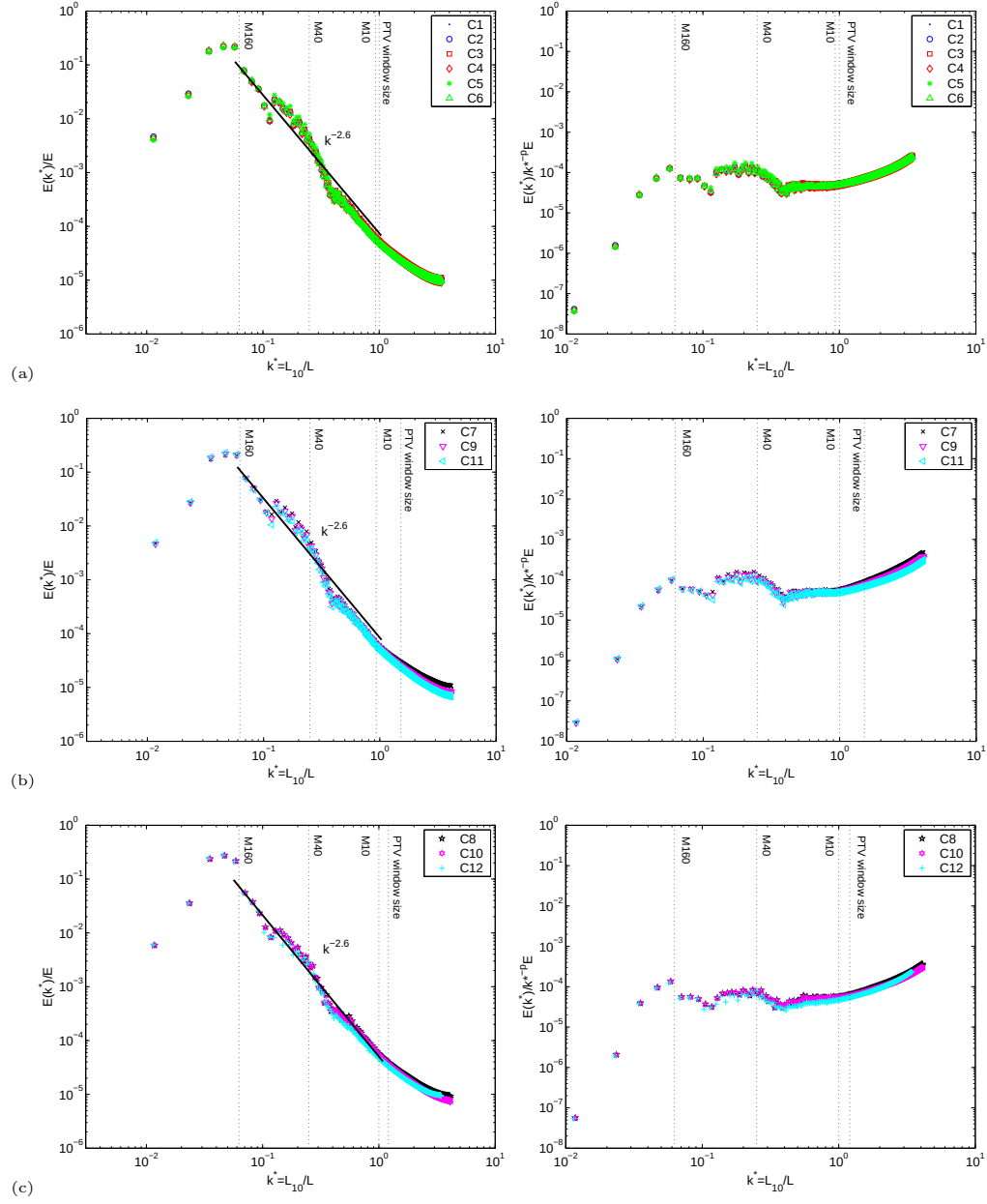


Figure 6.18: Energy spectrum of the three flow families: (a) flow family I, (b) flow family II and (c) flow family III.

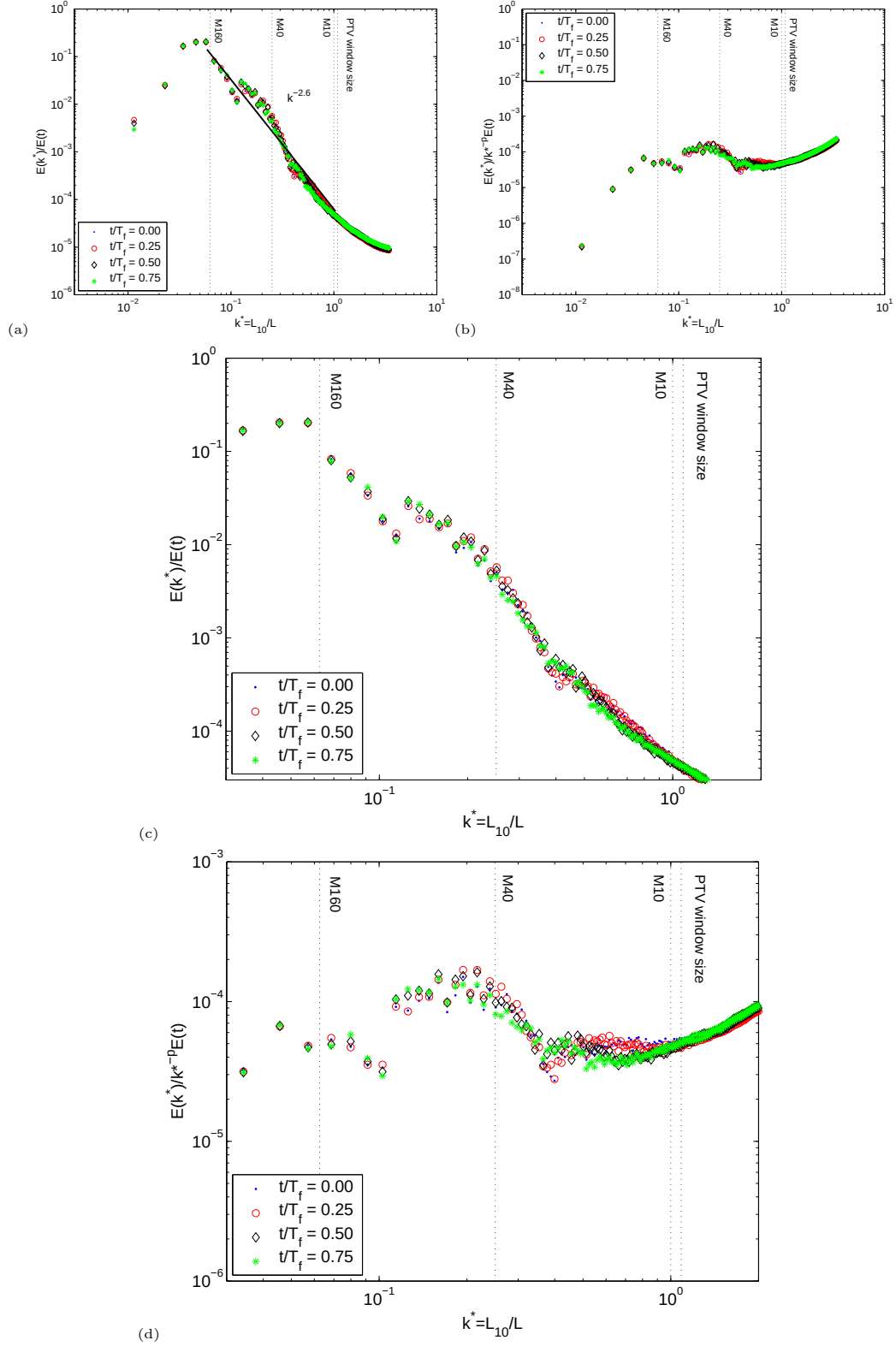


Figure 6.19: Energy spectra of the flow family I (Case: C5) at four different instants of forcing $\frac{t}{T_f} = 0.00, 0.25, 0.50$ and 0.75 .

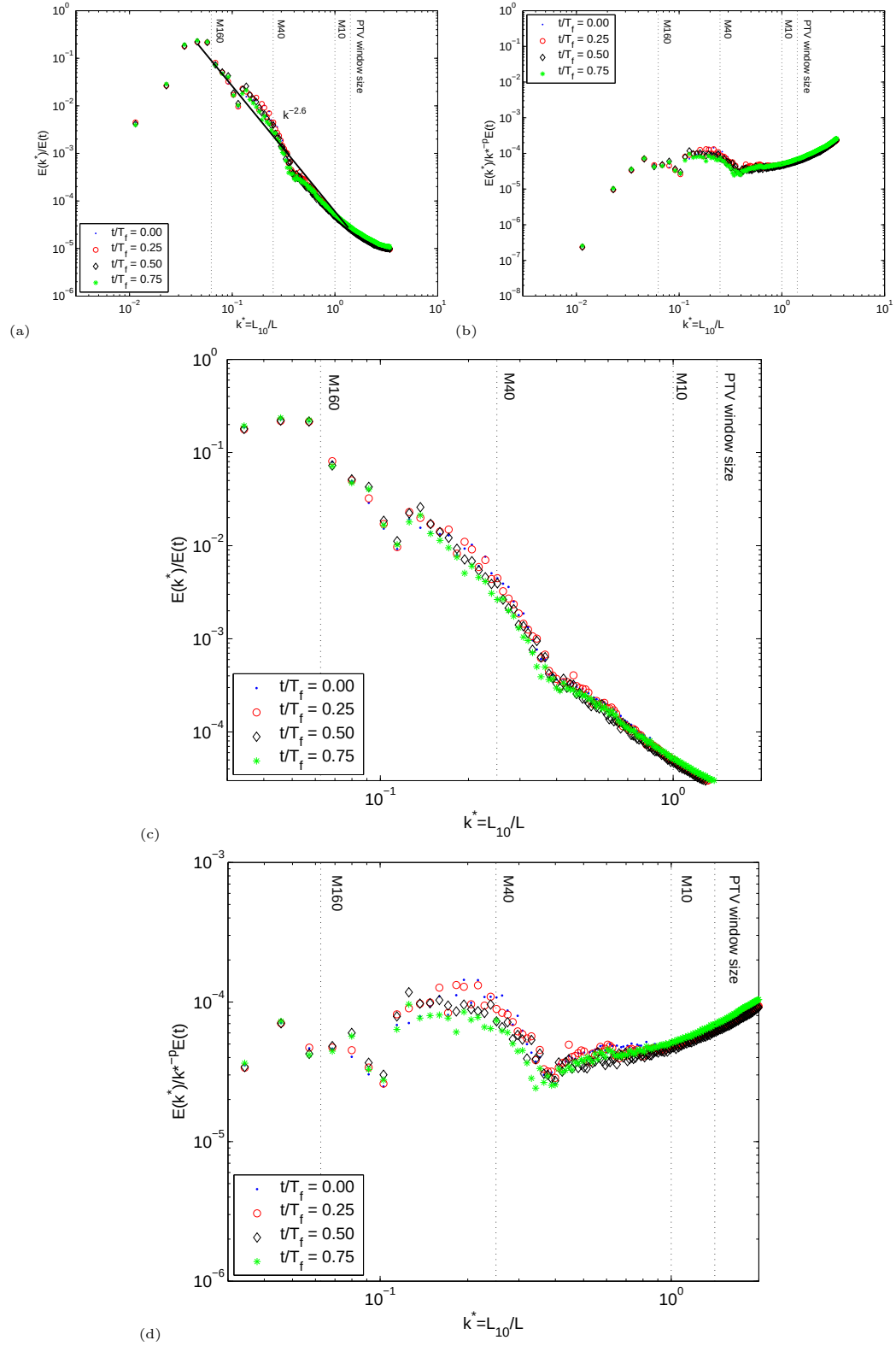


Figure 6.20: Energy spectra of the flow family I (Case: C6) at four different instants of forcing $\frac{t}{T_f} = 0.00, 0.25, 0.50$ and 0.75 .

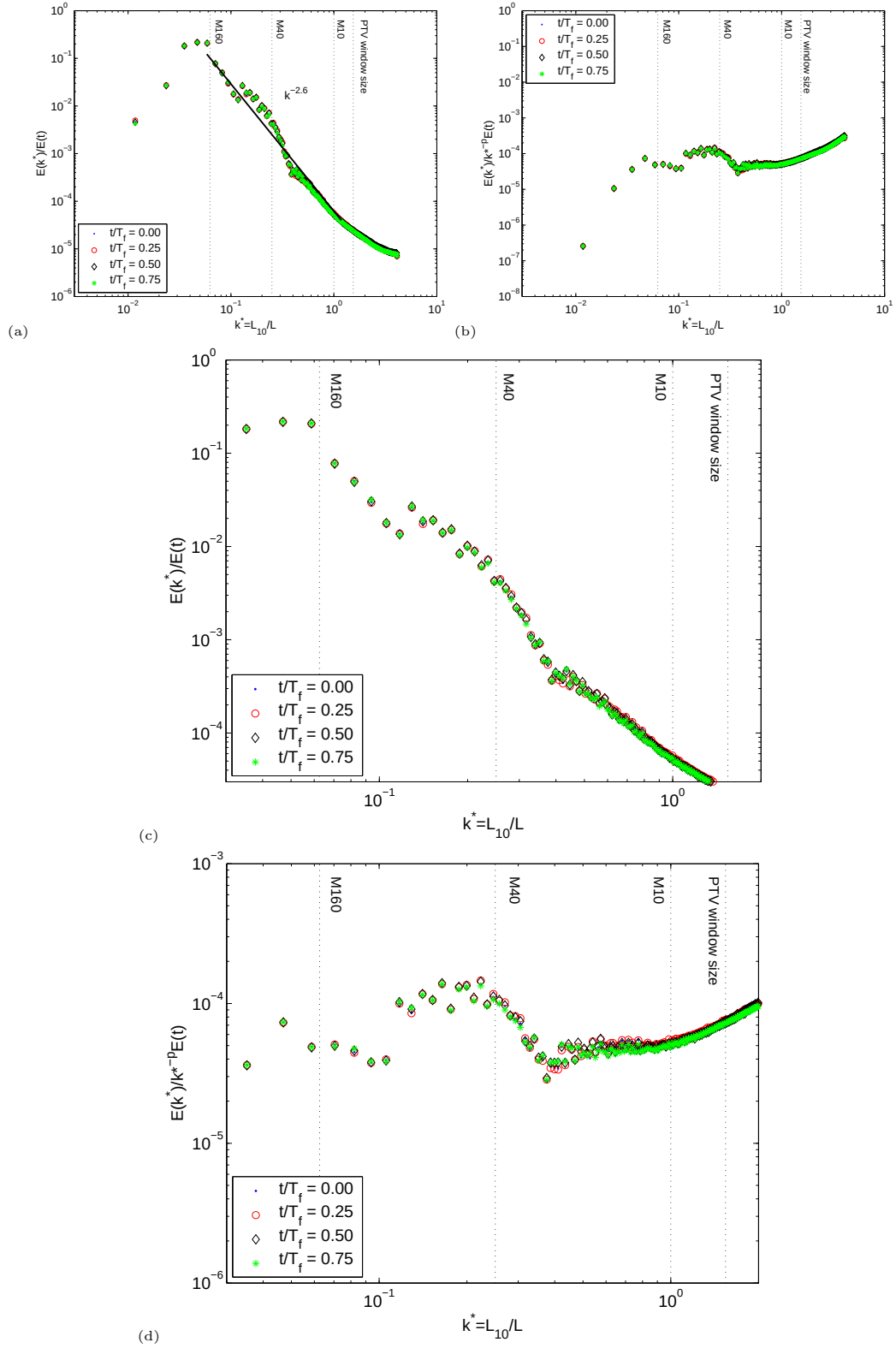


Figure 6.21: Energy spectra of the flow family II (Case: C9) at four different instants of forcing $\frac{t}{T_f} = 0.00, 0.25, 0.50$ and 0.75 .

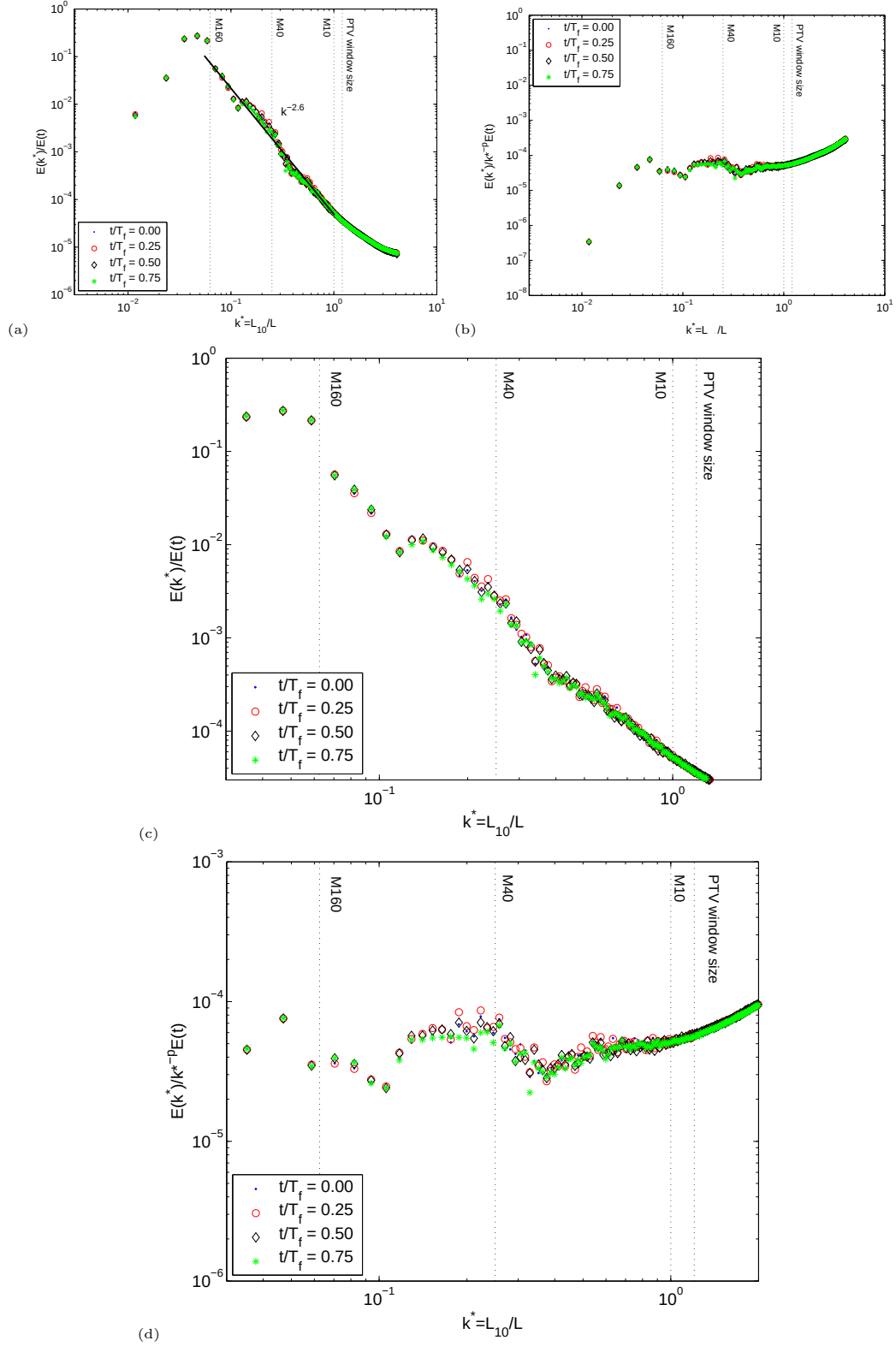
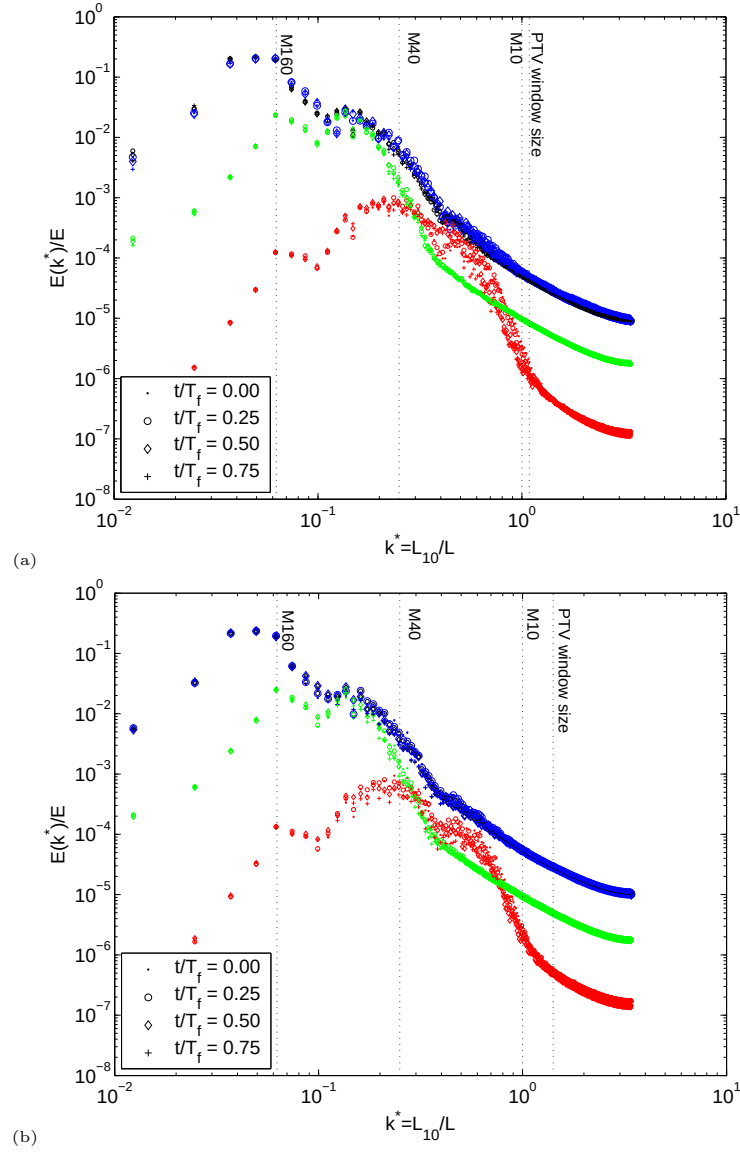


Figure 6.22: Energy spectra of the flow family III (Case: C10) at four different instants of forcing $\frac{t}{T_f} = 0.00, 0.25, 0.50$ and 0.75 .

The spectra of different scales in the flow in the case C6 is calculated by Fourier transform of the $R_{2D}(r)$ of the flow patterns of the small and medium scales eddies (Figure 6.3). These multi-scale spectrum, plotted at different forcing instants as shown in Figure 6.23, demonstrate the energy distribution in the particular scales in the flow. It can be seen that the multi-scale spectra contribute significantly to the entire flow's energy spectra in the regions of their characteristic scales. Furthermore, the oscillations of the spectrum at the scales that the time-dependent forcing targets can also be clearly observed.



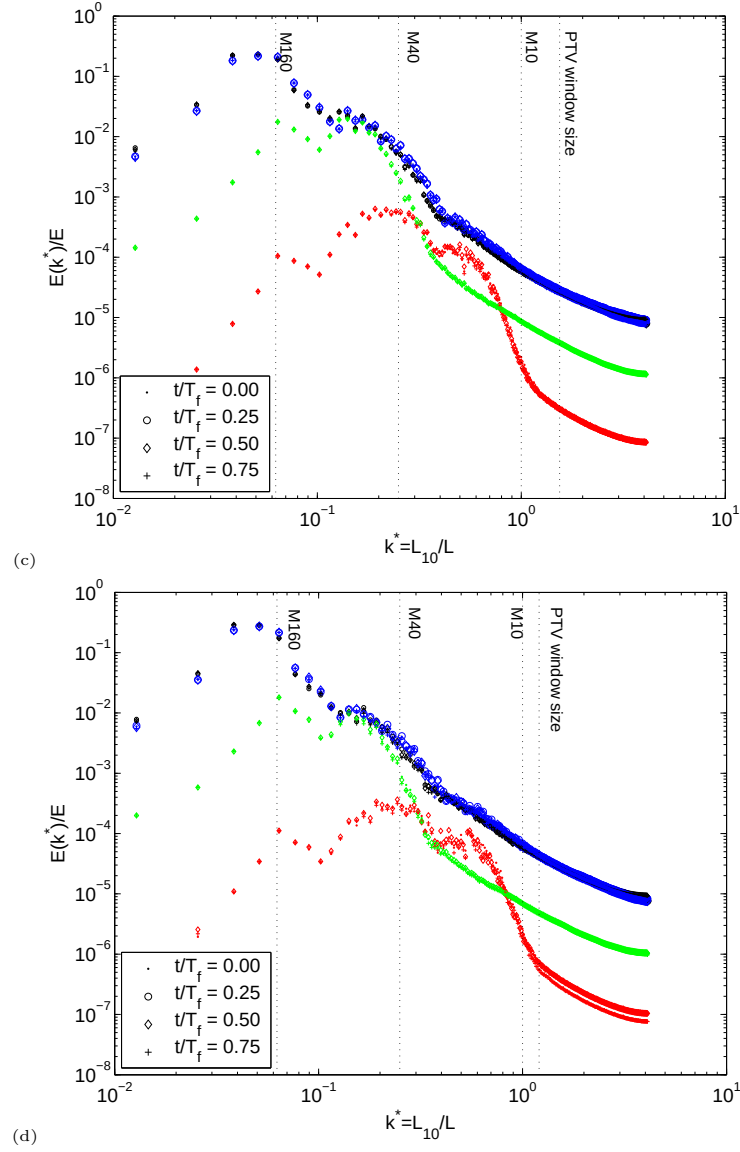


Figure 6.23: Energy spectrum associated to different flow scales of a selected forcing cases: (a) Family I: Case 5, (b) Family I: Case 6, (c) Family II: Case 9 and (d) Family III: Case 10. The energy spectrum are displayed using colour code: black colour corresponds to the actual flow; blue (coincides with black) corresponds to the flow filtered by $L_w \approx L_{M10}$ (Figure 6.3), green and red correspond to the medium and small flow scales respectively.

Despite oscillations in the energy spectrum at particular scales targeted by the forcing, both the entire energy spectrum and the multi scale energy spectrum are relatively persistent overall. This is expected and consistent with the structurally invariant flows observed in the laboratory and in the velocity fields in Figure 6.1. The different time-dependent forcing studied in this work do not vary the flow structure nor the energy distribution in the flow and the relation $p + D_s \approx 3$ still holds in our unsteady multi-scale flows. This finding is consistent with the argument of Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a) and Hascoet et al. (Hascoet, Rossi, & Vassilicos 2008) that the power exponent, p , of the energy spectra is controlled by the multi-scale topology of the flow.

6.5 Eulerian frequency spectra

The Eulerian frequency spectra, $\Phi_E(\omega)$, are obtained from Fourier transforms of the Eulerian temporal autocorrelation functions, $\mathbf{R}_E(\tau)$:

$$\Phi_E(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_E(\tau) e^{-i\omega\tau} d\tau, \quad (6.8)$$

where $\mathbf{R}_E(\tau)$ is defined as:

$$\mathbf{R}_E(\tau) = \langle \mathbf{u}_{E_i}(\mathbf{x}, \mathbf{t}) \cdot \mathbf{u}_{E_i}(\mathbf{x}, \mathbf{t} + \tau) \rangle, \quad (6.9)$$

with the average taken over all $\mathbf{x}$ and $\mathbf{t}$.

As the time-dependent flows are forced with an approximated sinusoidal function, where $I(t) \approx n \left[\bar{I} + A \cos \left(2\pi \frac{t}{T_f} \right) \right]$ (see Table 4.2), the velocity fields as well as their corresponding Eulerian temporal autocorrelation functions are also approximately sinusoidal with the same period, T_f , as shown in Figure 6.7 and Figure 6.24.

Thus, the Eulerian frequency spectra for each forcing case, $\Phi_E(\omega)$, consists of a single frequency component at the forcing frequency, $\frac{1}{T_f}$, with the total kinetic energy given by $\int_{\omega} \Phi_E(\omega) d\omega = u_{rms}^2$. The Eulerian frequency spectra of our unsteady multi-scale laminar flows are, therefore, discrete in nature. This is one of the major differences when compared to the homogeneous isotropic turbulence whose Eulerian frequency

spectra is continuous with $\Phi_E(\omega) = B\varepsilon^{2/3}u^{2/3}\omega^{-5/3}$ (Tennekes & Lumley 1972). In short, our currently studied unsteady multi-scale flows still lack the continuous multi-frequency forcing features.

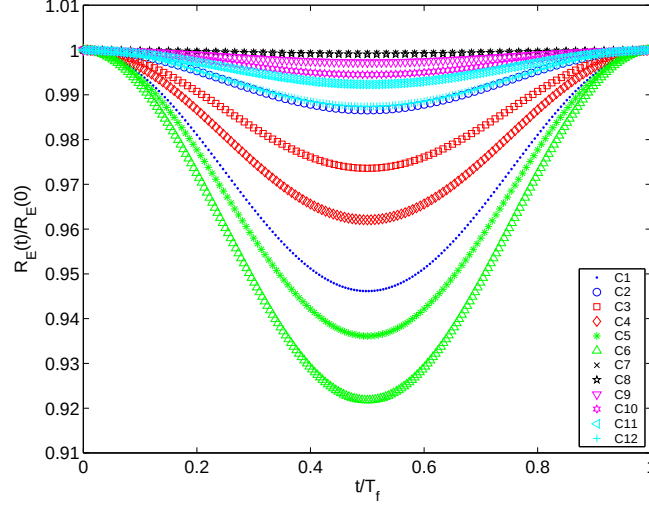


Figure 6.24: Eulerian temporal autocorrelation functions for all different forcing cases.

6.6 Acceleration

The addition of the time-dependent forcing, which causes our multi-scale flows to become unsteady, also introduces the temporal acceleration in our flow fields. Such temporal acceleration is generally present in turbulence but not in the steady multi-scale flows studied in Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a). Thus, the presence of the temporal acceleration is another important progression for our unsteady flows to be more turbulent-like. In this section, the acceleration of our unsteady multi-scale flows is investigated.

The acceleration can be written as the sum of the temporal (a.k.a. local) acceleration, $\mathbf{a}_L \equiv \partial \mathbf{u} / \partial t$ (the unsteady rate of velocity change at a fixed point) and the convective acceleration, $\mathbf{a}_C \equiv (\mathbf{u} \cdot \nabla) \mathbf{u}$ (the rate of velocity change due to the spatial derivatives):

$$\mathbf{a} \equiv \frac{D\mathbf{u}}{Dt} = \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u}. \quad (6.10)$$

In homogeneous isotropic turbulence, $\mathbf{a}_C$ and $\mathbf{a}_L$ are significantly and negatively correlated and mutually cancel one another leading the acceleration term, $\mathbf{a}$ to be small, hence $\|\mathbf{a}\| \ll \|\mathbf{a}_C\| \approx \|\mathbf{a}_L\|$ (Tsinober & Yeung 2001). On the contrary, in the steady multi-scale laminar flows studied in Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a), $\mathbf{a} = \mathbf{a}_C$ as $\mathbf{a}_L \equiv \partial \mathbf{u} / \partial t = 0$.

Table 6.3 shows the first, μ_1 , and second moments, μ_2 (mean and variance) of the acceleration intensity of our unsteady multi-scale flows.

Table 6.3: Acceleration: First (μ_1) and second moments (μ_2) of acceleration intensity [mm/s^{-2}]. The values given are the means and standard deviations of the moments.

Experiment	Case	$\ \mathbf{a}_C\ $		$\ \mathbf{a}_L\ $	
		μ_1	μ_2	μ_1	μ_2
Campaign I	C1	1.08±0.82	3.51±3.58	0.58±0.35	1.14±0.90
	C2	1.18±0.64	3.51±2.72	0.59±0.36	1.58±2.19
	C3	1.23±0.80	4.11±3.76	0.65±0.38	1.59±2.30
Campaign II	C4	1.25±0.88	4.27±4.10	0.65±0.36	1.33±1.00
	C5	0.79±0.63	2.02±2.12	0.55±0.36	0.73±0.54
	C6	1.30±1.05	4.89±5.05	0.59±0.36	1.10±1.30
Campaign III	C7	0.82±0.20	1.61±0.55	0.36±0.10	0.59±0.59
	C8	2.82±0.72	17.3±6.52	0.81±0.27	2.45±2.23
	C9	0.85±0.32	1.72±0.98	0.32±0.14	1.31±2.33
	C10	2.87±1.15	18.3±11.0	0.96±0.37	6.60±9.05
	C11	1.24±0.59	3.60±2.50	0.42±0.16	0.78±0.53
	C12	3.32±1.62	25.0±16.4	0.92±0.32	2.97±2.70

In our flows, $\mathbf{a}_L$ is small, $\|\mathbf{a}_C\| > \|\mathbf{a}_L\|$ (Figure 6.25) and the mutual cancelation between the two acceleration components is weak. This explains why our unsteady multi-scale flows still exhibit some Eulerian properties, including the shape of energy spectrum p , which are relatively persistent and very similar to the steady multi-scale flows, where $\mathbf{a}_L = 0$. The small $\mathbf{a}_L$ term, which implies weak temporal unsteadiness is due to the fact that our unsteady multi-scale flows are driven by simple sinusoidal functions of forcing, which are deterministic, smooth and periodic in nature while lacking multi-frequency features and randomness as in the case of fully developed turbulence.

By forcing the unsteady flows in the ways that produce a strong $\mathbf{a}_L$ term that is significantly and negatively correlated with $\mathbf{a}_C$, it is possible for the multi-scale structure of the current Eulerian velocity field as well as the energy spectrum shape p to change. With large $\mathbf{a}_L$ term, such unsteady flows are also expected to behave more like fully developed turbulence.

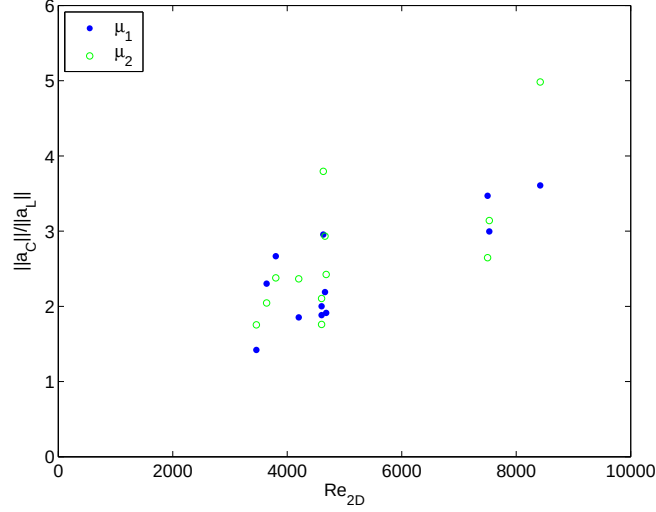


Figure 6.25: Ratio between the magnitude of convective acceleration, $\|\mathbf{a}_C\|$ and local acceleration, $\|\mathbf{a}_L\|$ over $3000 < Re < 9000$

In this chapter, we demonstrate the effects of the unsteadiness introduced by time-dependent forcing on the Eulerian properties of the multi-scale flows. The time-dependent forcing causes the flows' velocity and energy to be time-dependent where $u_{rms}(t) = f(\bar{I}, A, T_f, t)$ and $u_{rms, T_f} \sim \bar{I}$ (Eq.(6.3)), in line with the steady multi-scale flows data (Rossi, Vassilicos, & Hardalupas 2006a). The stagnation points' characteristics such as the strain rate, size and area of direct influence also become time-dependent. In response to the time-dependent forcing, the stagnation points move slightly and slowly compared to the surrounding flow; their sizes and areas of direct influence also vary slightly. Variations in the strain rate in the vicinity of hyperbolic stagnation points are more evident, particularly in the flow family I, which is forced by the time-dependent forcing that covers more than one flow scale.

Despite the time-dependencies in our unsteady flows, the multi-scale schematic streamline structure and topology, i.e. the number of the stagnation points and how they are

connected to the streamlines, remain persistent. The persistency is also reflected in the PDFs of the velocity components and intensity of the Eulerian velocity field as well as in the energy distribution within the flows as evident in the energy spectrum. Although the spectrum $E(k)$, normalised by the total energy E , oscillate at the wavenumbers, k , excited by the time-dependent forcing, the oscillations are subtle and do not change the overall shape of the energy spectrum where $E(k) \sim k^{-2.6}$. Such persistency is a result of a combination of the following two factors: first, the local acceleration, $\mathbf{a}_L$, that influences the multi-scale structure, is small in our unsteady multi-scale flows, which is similar to the steady multi-scale flows in Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a) where $\mathbf{a}_L = 0$; second, the fractal dimension of stagnation point distribution, D_s , which affects the energy spectrum shape, p , is kept constant i.e. $D_s \approx 0.5$ for all experiments performed. This finding confirms that the relation $p + D_s \approx 3$ (Davila & Vassilicos 2003), still holds in our unsteady multi-scale flows.

Chapter 7

Lagrangian Statistics

This chapter is devoted to the study of Lagrangian statistics of our unsteady multi-scale laminar flows. Among the questions to be addressed are: what is the effect of the time-dependent forcing on the stirring mechanism of the flows? what type of diffusion do our unsteady flows exhibit? do they follow the Richardson's law of pair separation, where $\langle \Delta^2(t) \rangle \sim t^3$ as in homogeneous isotropic turbulence (Obukhov 1941), or do they have the exponential growth, $\Delta(t) \approx \Delta_0 e^{\zeta t}$ where ζ is one Lyapunov exponent, as in chaotic advection observed in unsteady laminar flows (Antonsen, Fan, Ott, & Garcia-Lopez 1996)?

In an attempt to answer the above questions, we investigate the fluid element dispersion of the flows, which includes the statistics of single fluid element and pair dispersion as well as its corresponding PDFs. The Lagrangian correlation function and Lagrangian correlation time are quantified. To complete our analysis, the Lagrangian frequency spectra, which relates to temporal evolution of velocity fluctuations experienced by fluid elements in the flow field, are also examined.

7.1 Stirring illustration

Figure 7.1 shows a comparison of the stirring mechanism between the steady and time-dependent multi-scale flows. In the steady case, the flow exhibits a stretching mechanism dominated by the small(M10) and medium(M40) scales at short time

$tu_{rms}/L_E \leq 1$ and by large(M160) scale after $tu_{rms}/L_E > 1$. Although the initial multi-scale stretching provides good stirring to the flow, the trajectories eventually close up and loop around as a result of the steady streamline and limit the degree of “mixed-ness” at longer time. The time-dependent forcing, which causes the flow’s streamlines to be unsteady and the stagnation points to move slightly and slowly compared to the surrounding flow, can improve stirring significantly. The unsteady flow exhibits the “stretching and folding” mechanism similar to what is found in chaotic advection except for the clear distinction that ours has multi-scale. It can be seen from Figure 7.1(b) that the unsteady flow does not form a closed-loop but a much more complex pattern by stretching and folding which aids inter-scale and better stirring of the scalar(dye).

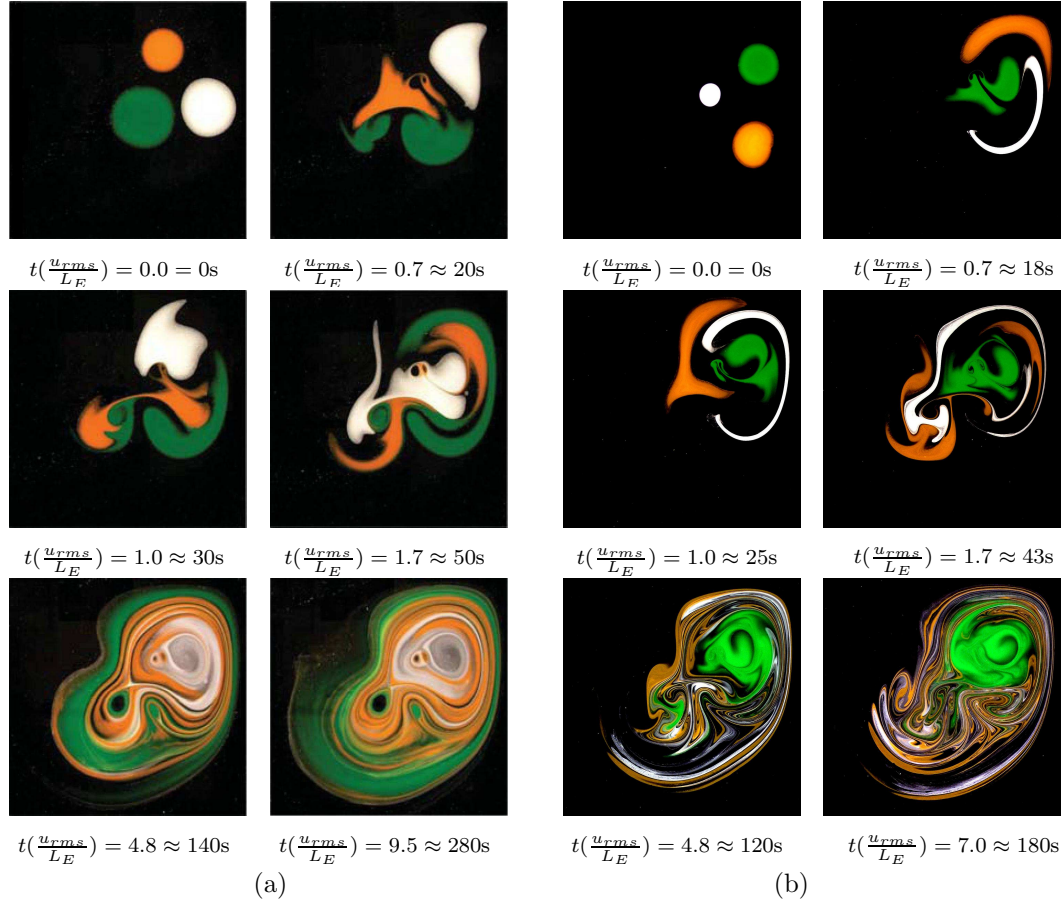


Figure 7.1: Stirring evolution of: (a) constant forcing flow $I = 0.3A$ (b) time-dependent flow forced with $I(t) = 0.4 + 0.25 \cos(2\pi t/T_f)$ where $T_f = 15s$.

7.2 Lagrangian statistics computation

7.2.1 Virtual particle tracking (VPT)

In order to obtain Lagrangian statistics, we perform the so-called “Virtual Particle Tracking (VPT)” which integrates trajectories of fluid elements (passive scalar tracer) from the PTV resolved Eulerian velocity field numerically. We focus on extracting the Lagrangian statistics in the central mixing region ($800 \times 800 \text{mm}$) of the flow. Nevertheless, some trajectories enter and exit the measurement field. Thus, we extend the flow outside the measurement field artificially to record these trajectories and keep the number of the fluid elements statistics constant. The original PTV resolved Eulerian velocity field ($L_{PTV} = 1000 \text{mm}$) is extended to the full size of the experimental tank (1700mm) using the 4^{th} order polynomial interpolation. The boundary condition at the edge of the tank is set to zero.

Fluid elements are then distributed in equilateral-triangular clusters, each of 3 tracers with an initial position of 1 pixel apart which is about 20 times smaller than the smallest M10 magnet size. These clusters are distributed throughout the entire $1700 \times 1700 \text{mm}$ region and are ‘uniformly random’ (Figure 7.2) i.e. they have random spatial distribution and orientation, however the amount of tracers in every spatial box of the size of the smallest M10 magnet (≈ 20 pixels) are uniform.

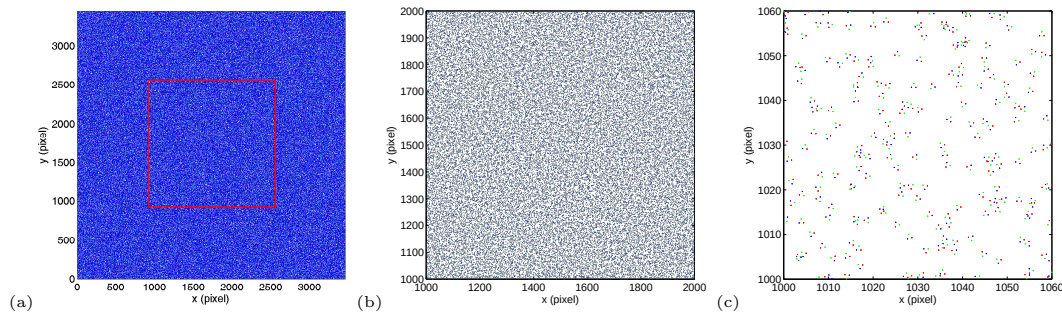


Figure 7.2: Initial distribution of scalar tracer: (a) Entire field (b,c) zoom of (a). The red box indicate the central $800 \times 800 \text{mm}^2$ mixing region.

Fluid elements are tracked using the 4^{th} order Runge Kutta method (RK4). A quadratic function is used to interpolate velocity grids in time between those available from the experiment (at every camera time step). A quadratic function is considered sufficient to

interpolate the velocity grid in time as it is the order of polynomial interpolation used in the velocity extraction imposed in Section 5.3.3. A cubic Lagrange interpolation function is used for spatial interpolation between grid points. We use the interpolating time step $t_{RK4} = 1/26 - 1/24 = 0.0385 - 0.0417s$ (See Section 7.2.2 for selection detail). Fluid elements are tracked until $\sim 540s$ or between $30T_f - 108T_f$, where T_f is the forcing period.

Fluid elements' positions and velocities at each time step are recorded and output is exported every four camera steps to save the computation storage. Only trajectories which cross the central mixing region are taken into the statistics. Figure 7.3, plotted with 0.05% and 0.5% of the taken statistics, shows the integrated fluid paths (streaklines/trajectories) crossing the central mixing region. From the figure, it can be observed that tracers remain approximately 10% away from the wall throughout the tracking time and do not loop around at the size of the tank. Most trajectories are within the 1000x1000mm PTV resolved Eulerian field and a relatively small number of statistics are integrated in the artificially extended region of the field. In addition, the velocity in the extended region is considered small ($< 0.1u_{rms}$) providing very large turnover time scale ($> 100L_E/u_{rms} \gg T_L$). Hence, it can be justified that the artificial extension of the field do not affect our integrated Lagrangian statistics.

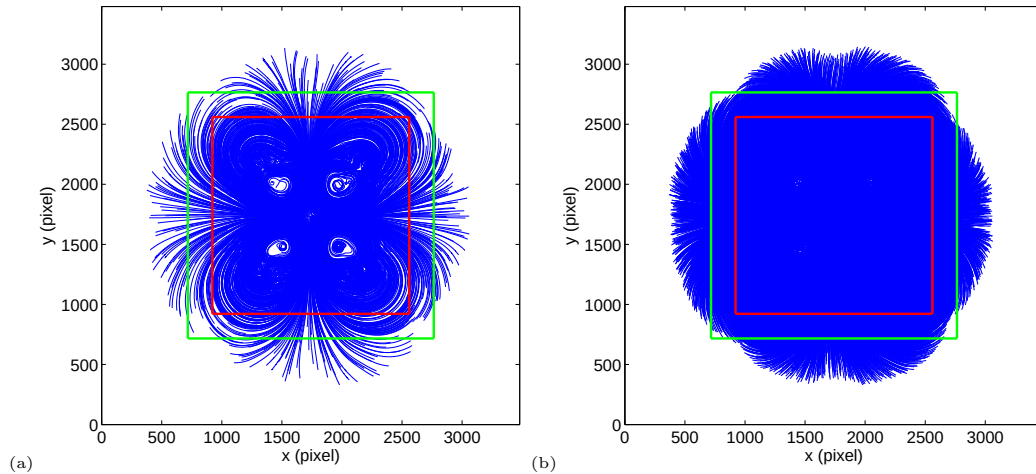


Figure 7.3: Tracked trajectories of the case C6 throughout the 540s with tracer density of: (a) 0.05% (≈ 580 trajectories) and (b) 0.5% (≈ 5800 trajectories). The red and green box respectively indicate the central 800x800mm region and the original 1000x1000mm (L_{PTV}) measurement field.

7.2.2 Determination of the Runge Kutta time step

The time step used in the Runge Kutta interpolation, t_{RK4} is determined by two main criteria:

1. t_{RK4} chosen should allow at least 10 positions (or time steps) to be fitted around the smallest scale in the flow i.e. the perimeter of the smallest L10 eddy, $\emptyset_{L10}$, for the velocity approximation, hence

$$t_{RK4} \leq \emptyset_{L10}/10u_{max_{T_f}}.$$

The smallest eddy in the flow is assumed to be about half of the smallest M10 magnet size, therefore $\emptyset_{L10}$ is estimated to be approximately 32.17 pixel. The value of $u_{max_{T_f}}$ used to calculate t_{RK4} is the maximum velocity of all time steps within each forcing case (Table 7.1). The value is used in the estimation to cover the most critical scenario even though the velocity around the smallest eddy might in fact be smaller than $u_{max_{T_f}}$.

2. t_{RK4} chosen should be small enough for the maximum displacement of the approximating tracer positions to remain within the size of the interpolating window per time step, thus

$$t_{RK4} \leq d_r/u_{max}.$$

This is to prevent the position of the tracers from exceeding the cubic Lagrange interpolating grids.

The results of the criteria are given in Table 7.1. We select t_{RK4} to be half of the camera step (0.0385-0.0417s) which fits both criteria for the virtual particle tracking. The value of $t_{RK4} = 0.0417$ s satisfies both criteria for all forcing cases except the case C12 where it is slightly larger than the first criterion of $\frac{\emptyset_{L10}}{10u_{max}}$. However, the value is still used as it is small enough to allow at least 9 positions to be fitted around $\emptyset_{L10}$ and the velocity around smallest eddy is expected to be smaller than $u_{max_{T_f}}$ used in the first criterion estimation.

Table 7.1: Selection of Runge Kutta time step, t_{RK4} .

Experiment	Case	u_{maxT_f} [pixel/s]	d_r [pixel]	$\frac{\emptyset_{L10}}{10u_{maxT_f}}$ [s]	$\frac{L_{PTV}}{u_{maxT_f}}$ [s]	t_{RK4} [s]
Campaign I	C1	47.05	6	0.0684	0.128	0.0385
	C2	46.24	6	0.0696	0.130	0.0417
	C3	47.96	6	0.0671	0.125	0.0417
Campaign II	C4	49.91	6	0.0645	0.120	0.0417
	C5	39.31	6	0.0818	0.153	0.0417
	C6	52.62	6	0.0611	0.114	0.0417
	C7	34.90	5	0.0922	0.143	0.0417
Campaign III	C8	67.44	5	0.0477	0.074	0.0417
	C9	37.20	5	0.0865	0.134	0.0417
	C10	69.08	5	0.0466	0.072	0.0417
	C11	45.86	5	0.0702	0.109	0.0417
	C12	78.68	6	0.0409	0.076	0.0417

7.2.3 Scalar tracer distribution optimisation

It has been observed that some particles, particularly those near the edge of the 1700mm tank never enter the mixing region during the tracking time (~ 540 s). In fact, only around 40-60% of particles distributed in the 1700x1700mm region entered the 800x800mm central region during the tracking (Figure 7.4). Therefore, optimisation can be done by reducing the area of initial particle distribution and excluding the inactive areas near the edge. This can save the computation time and storage significantly. For instance, reducing the initial particle distribution area to be 8.6 % from the tank can increase the number of particles entering the central region to 60%.

Note that the size of the initial area for particle distribution vary with the forcing period, mean forcing intensity and the length of the tracking time. Hence there is no universal initial area; the area must be pre-determined individually for each forcing case. It should also be noted that the area can be reduced more on the x-axis, where tracers are leaving the central region, than on the y-axis where tracers are entering (Figure 7.5).

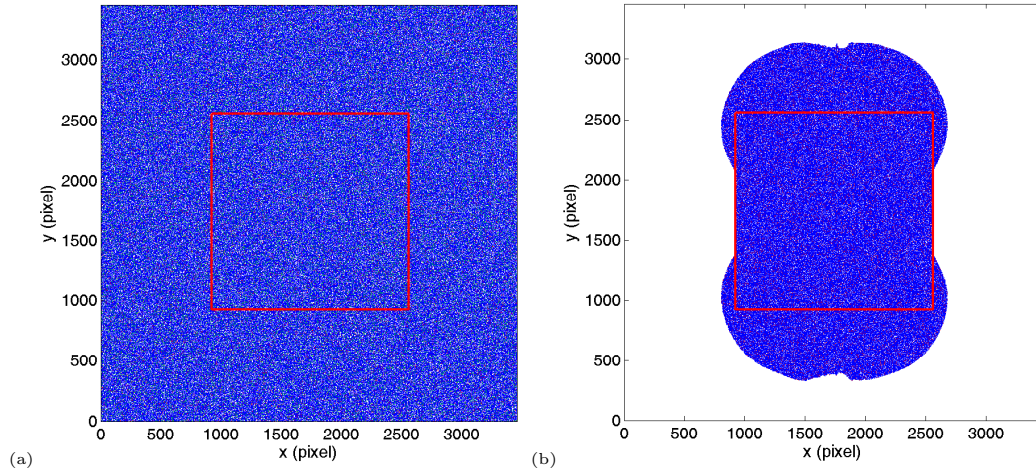


Figure 7.4: (a) Initial positions of scalar tracers distributed throughout the entire 1700x1700mm region and (b) initial positions of tracers which entered the 800x800mm central region during the tracking.

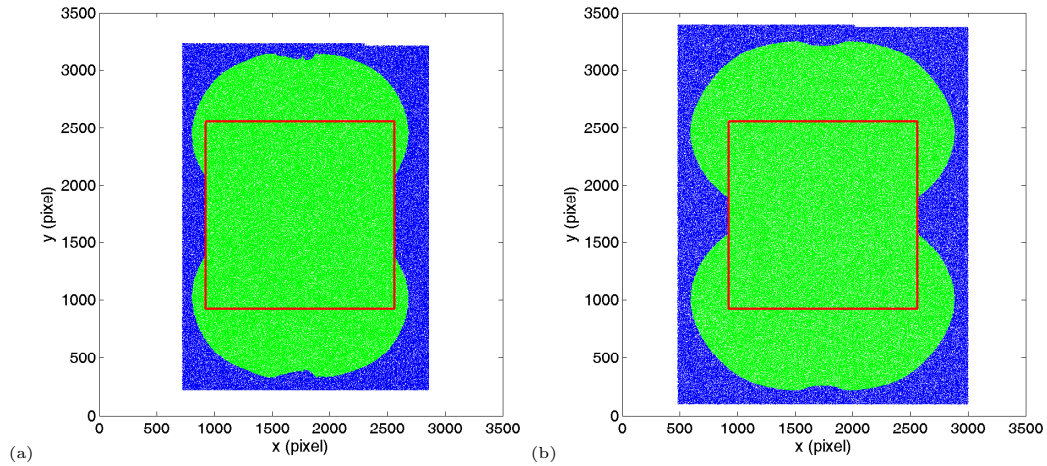


Figure 7.5: Examples of reduced area of initial distribution of scalar tracer of case C11 (a) and C12 (b). Blue indicates the initial distribution of tracers and green indicates tracers that enter the central 800x800mm² mixing region.

The numerical accuracy of the tracking algorithm has an approximated error of order 10^{-4} pixel per time step (see Appendix 9.3.1 for validation details). The numerical error is considered acceptable in our computation as it is much smaller than the measurement noise from the experiment and data post-processing, which is of order 1 pixel.

7.3 Lagrangian trajectories

Examples of Lagrangian trajectories integrated from the unsteady (time-dependent) flow are shown in Figure 7.6. The trajectories display chaotic and complex pattern. To demonstrate the effect of the time-dependent forcing, we compare the unsteady Lagrangian trajectories to steady ones. The steady trajectories are integrated from a single instantaneous Eulerian velocity field taken from the time-dependent velocity fields. We select the single instantaneous velocity field that has approximately the same mean flow energy, u_{rms} , as that of the time-dependent forcing case, u_{rms,T_f} , for the steady flow tracking.

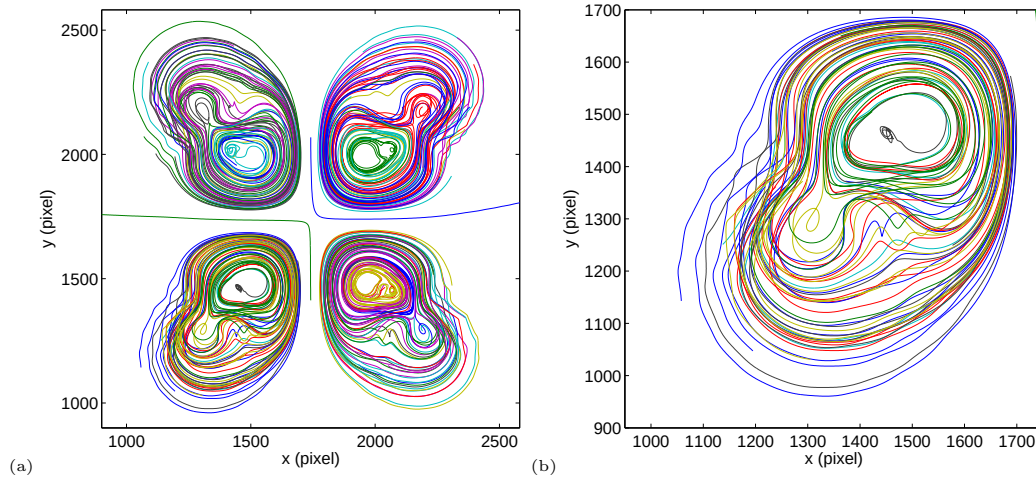


Figure 7.6: Selection of integrated fluid elements trajectories in unsteady flow over 540s: (a) entire flow and (b) quarter of the flow.

The trajectories of the steady and unsteady flow, originating from the same initial positions, are shown in comparison in Figure 7.7. While the steady flow trajectories follow its streamlines, the unsteady trajectories wiggle around without forming closed-loop patterns. Figure 7.7(c,d) and (e,f) display trajectories starting near the medium (M40) and small scale (M10) magnets respectively. It is evident that the unsteady flow trajectories originating in one scale can enter other adjacent scales (of both larger and smaller sizes) in short times whereas the steady flow trajectories tend to stay within their initial scales.

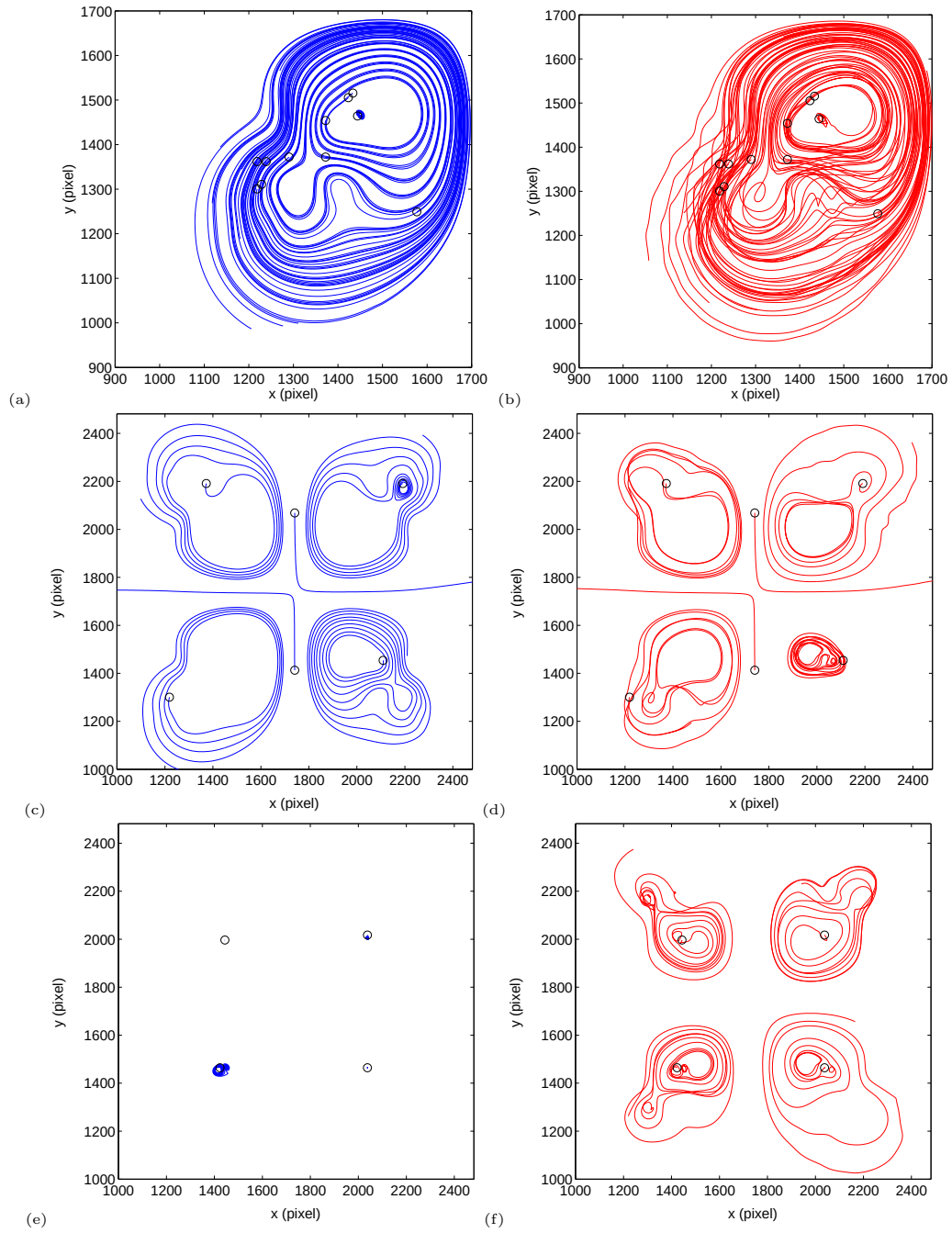


Figure 7.7: A comparison of fluid elements trajectories integrated from steady (left: blue lines) and unsteady flow (right: red lines) starting from the same initial positions displayed by black circles (o): (a,b) quarter of the flow. Trajectories with initial positions closed to the medium scale M40 magnets (c,d) and small scale M10 magnets (e,f).

7.4 Lagrangian autocorrelation function and time scale

The Lagrangian autocorrelation function is defined as:

$$R_L(\tau) = \langle \mathbf{u}_{L_i}(t) \cdot \mathbf{u}_{L_i}(t + \tau) \rangle \quad (7.1)$$

where the average $\langle \dots \rangle$ is taken over time t and over all trajectories $\frac{d}{dt}\mathbf{x}(\mathbf{x}_0, t) = \mathbf{u}_L(\mathbf{x}_0, t)$, whose paths cross the central mixing region, starting from uniformly random positions $\mathbf{x}_0$ and $\mathbf{u}_{L_i}(t)$ is the Lagrangian velocity of the fluid element at time t .

As the flows are unsteady but assumed statistically steady for long time, $R_L(\tau)$ is also sensitive to time t . Hence, we estimate $R_L(\tau)$ with $0 \leq t \leq 6L_E/u_{rms}$ which is considered a long time (i.e. up to six turnover time) in order for the Lagrangian statistics such as the fluid dispersion to establish different scaling regions and also be compatible to the previous steady data presented in Rossi et al. (Rossi, Vassilicos, & Hardalupas 2007).

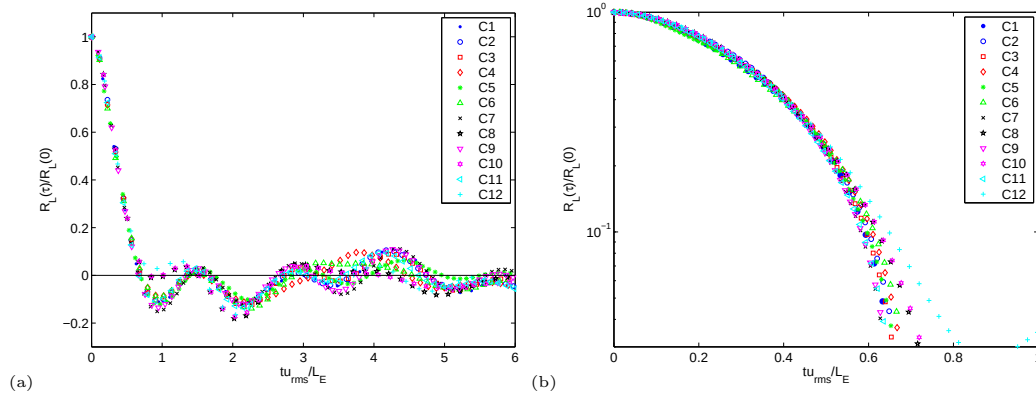


Figure 7.8: Lagrangian autocorrelation functions for all different time-dependent forcing cases: (a) linear plot and (b) semi-log plot. Data are plotted with 20% density, i.e. 1 every 5 points, to allow different forcing cases to be distinguished.

Figure 7.8 shows the Lagrangian autocorrelation functions for all time-dependent forcing cases. The Lagrangian autocorrelation functions are very similar up to the first zero-crossing where $R_L(\tau)/R(0) = 0$. Beyond this point, the Lagrangian autocorrelation functions tend to oscillate around the zero value. The oscillation of $R_L(\tau)$ is caused by the flow looping around the flow scales on average even though the trajectories are not perfectly forming closed loops. This is different to the Lagrangian autocorrelation

functions of fully developed turbulence where they decay to zero due to fluid elements being uncorrelated (in relative to their initial conditions) and random over time.

The Lagrangian autocorrelation functions of the three flow families are shown in Figure 7.9. It can be observed that $R_L(\tau)$ of the flows in Family II (Figure 7.9(c)), with forcing intensity range within one characteristic flow scale below the bottom friction constraint (see Figure 4.6), display deterministic harmonic oscillations around the zero value. In contrast, $R_L(\tau)$ of flows in Family III (Figure 7.9(d)), with high forcing intensity range above the bottom friction constraint, display intermittent oscillations around the zero value.

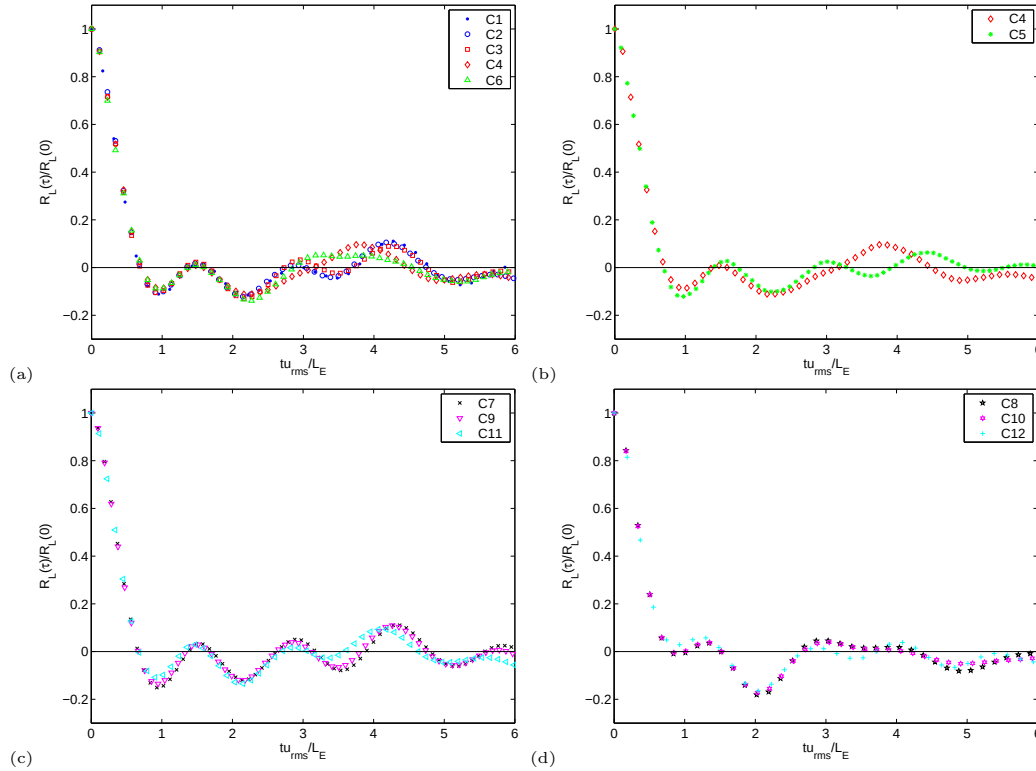


Figure 7.9: Lagrangian autocorrelation functions of the three flow families: (a) Family I with a broad range of forcing amplitude covering more than one characteristic flow scales ($I_{mean} = 0.4A$ and $A_F = 0.25A$), (b) Family I with $I_{mean} = 0.298A$ and $A_F = 0.25A$, (c) Family II with forcing range within one characteristic flow scale below the bottom friction constraint and (d) Family III with forcing range within one characteristic flow scale above the bottom friction constraint.

In flow Family I (Figure 7.9(a) and (b)), flows are forced with a broad range of forcing intensities which cover more than one characteristic flow scales. The flows in Family I, as shown in Figure 7.9(a), have the same values of the mean forcing intensity $I_{mean} =$

0.4A and forcing amplitude $A = 0.25A$ but with different forcing periods $T_f = 7.5, 10, 12.5, 15$ and $18s$. The values $R_L(\tau)$ in Figure 7.9(a) are similar up to approximately $2L_E/u_{rms}$. At $t > 2L_E/u_{rms}$, the values $R_L(\tau)$ of the flows with higher forcing periods tend to become intermittent and lose their deterministic harmonic oscillation. When compared to another flow in Family I (Figure 7.9(b)) with the same forcing amplitude $A = 0.25A$ and forcing period $T_f = 12.5s$ but with lower mean forcing intensity $I_{mean} = 0.298A$, it can be seen that the harmonic oscillation around the zero value remains clearer with lower mean forcing intensity. Overall, it can be summarised that higher I_{mean} and longer T_f cause the flow to decorrelate faster after $1L_E/u_{rms}$ whereas the forcing amplitude A has little effect on $R_L(\tau)$.

Figure 7.10 shows the Lagrangian autocorrelation functions of a selection of forcing cases from the three flow families. To demonstrate the effect of the time-dependent forcing on the Lagrangian autocorrelation function, we compare the values $R_L(\tau)$ of our time-dependent flow to the “steady” one. The values $R_L(\tau)$ of the steady flow is calculated from Lagrangian statistics that have been integrated from a single Eulerian velocity field taken from the time-dependent forcing case. From Figure 7.10, it can be seen that unsteadiness causes the flow to be less correlated; the oscillation around the zero value of $R_L(\tau)$ is smaller and closer to zero. Where the time-dependent forcing targets to excite a single flow scale, the difference in $R_L(\tau)$ between the steady and unsteady flow is less obvious and occurs only after $4L_E/u_{rms}$ for flows in Family II with low forcing intensity (Figure 7.10(c)) and after $3L_E/u_{rms}$ for flows in Family III with high forcing intensity (Figure 7.10(d)). The effect of unsteadiness on $R_L(\tau)$ is more apparent and is observable from $1L_E/u_{rms}$ for flow in Family I where the time-dependent forcing targets multiple flow scales instantaneously.

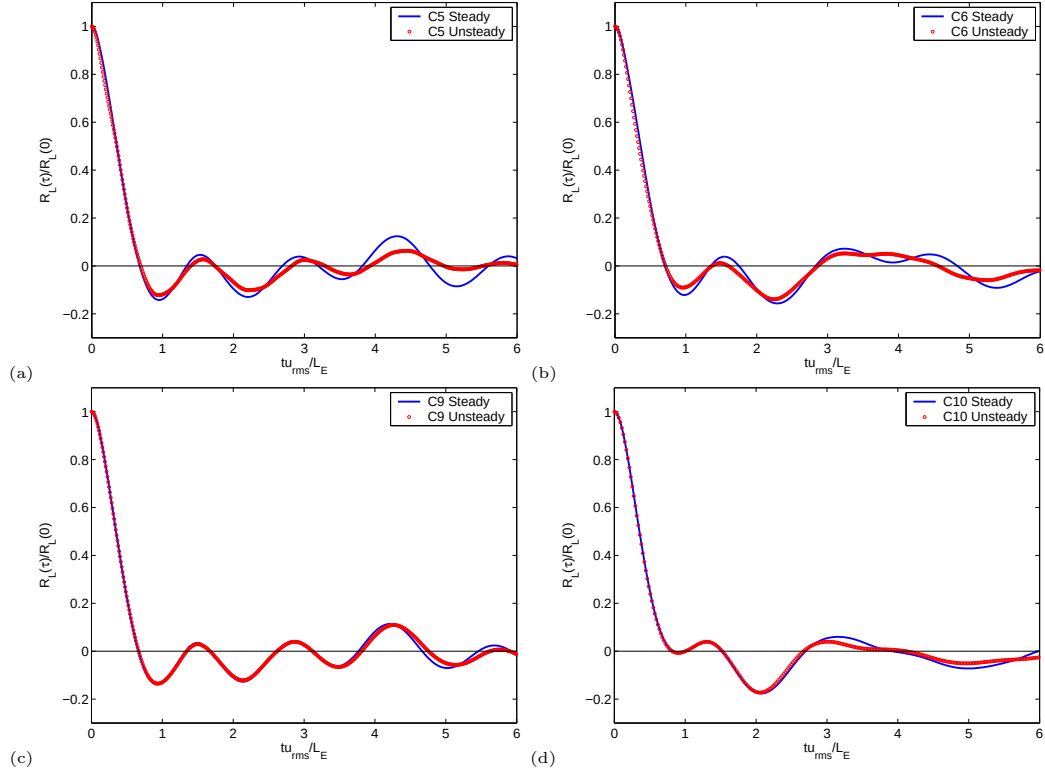


Figure 7.10: A comparison of $R_L(\tau)$ between the steady and unsteady flows for a selection of forcing cases from the three flow families: (a) and (b) Family I with forcing range covering more than one characteristic flow scales, (c) Family II with forcing range within one characteristic flow scale below the bottom friction constraint and (d) Family III with forcing range within one characteristic flow scale above the bottom friction constraint.

Another important Lagrangian property that is used to characterise our flows is the Lagrangian correlation time, T_L . The value T_L is interpreted as being the time it takes on average for the fluid elements to forget their histories or initial conditions. The conventional definition of the Lagrangian correlation time is

$$T_L = \int_0^\infty R_L(\tau)/R_L(0) d\tau. \quad (7.2)$$

As the computed Lagrangian data are in fact finite, we define the Lagrangian correlation time function as

$$T_L(t) = \int_0^t R_L(\tau)/R_L(0) d\tau, \quad (7.3)$$

where $0 \leq t \leq 6L_E/u_{rms}$.

Figure 7.11 shows the function $T_L(t)$ for different forcing cases. After $t > L_E/u_{rms}$, the

value $T_L(t)$ oscillates as a result of the oscillation in $R_L(\tau)$. It has been observed that T_L is highly sensitive to the limit of the integration due to the indeterministic trend of R_τ over long time. Thus, it is not possible to establish a universal integrating limit for all forcing cases at a long time. We, therefore, estimate the Lagrangian correlation time by $T'_L = \overline{T_L(t)}$ with averaging over the range $L_E/u_{rms} \leq t \leq 6L_E/u_{rms}$ as shown by the black solid lines in Figure 7.11.

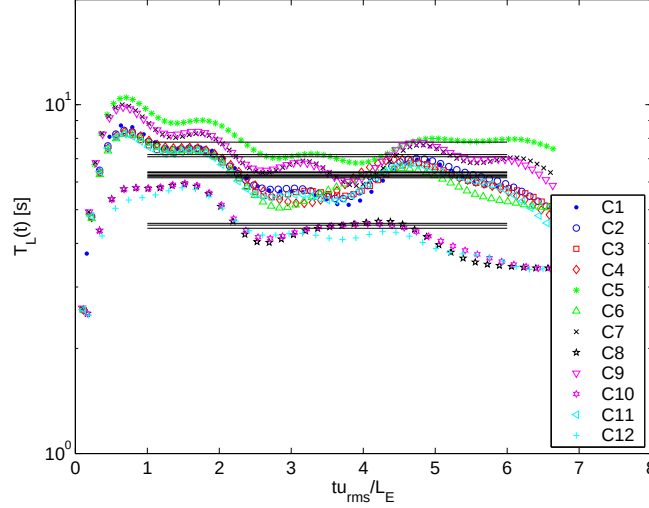


Figure 7.11: Lagrangian correlation time functions for all different unsteady forcing cases. The black solid lines are the averages of the functions over $L_E/u_{rms} \leq t \leq 6L_E/u_{rms}$.

Another method to estimate the Lagrangian correlation time is proposed as an alternative to the averaging method. We define the integrating limit, z , up to the first zero crossing of R_τ i.e. when the function is first uncorrelated, hence

$$T_L^* = \int_0^z R_L(\tau)/R_L(0) d\tau. \quad (7.4)$$

The definition is based on the assumption that the negative and positive values in the oscillation after the first zero crossing will eventually cancel out over a very long time. Moreover, the magnitude of the correlation function up to the first zero crossing is more than 80% compared to the magnitude of the oscillation and can be regarded as the significant range.

The Lagrangian correlation time calculated by the two definitions mentioned above are shown in comparison in Table 7.2. From the table, the Lagrangian correlation time for the unsteady flows are slightly lower ($\leq 2\%$) than those of the steady flows of the same forcing case. This finding is as expected as the addition of unsteadiness causes the flow to be more chaotic hence the fluid elements decorrelate from their original state faster. Other timescales, such as $T_L(t = \frac{6L_E}{u_{rms}})$ and $t_{R(\tau)=0}$, are also given as a reference in Table 7.2.

Table 7.2: Lagrangian time scales: $T_L^* = \int_0^z R_L(\tau)/R_L(0)d\tau$ where z is the time at the first zero crossing of R_τ , $T_L' = \overline{T_L(t)}$ are the average of $T_L(t) = 1/R(0) \int_0^t R(\tau)d\tau$ over the time range $L_E/u_{rms} \leq t \leq 6L_E/u_{rms}$.

Experiment	Case	u_{rms} [mm/s]	L_E [mm]	T_L' [s]	T_L^* [s]	$T_L(t = \frac{6L_E}{u_{rms}})$ [s]	$t_{R(\tau)=0}$ [s]	Re_{2D}
Campaign I	C1	6.96	176.0	6.32	8.74	5.83	17.2	4200
	C2	7.63	180.4	6.42	8.44	5.92	16.4	4600
	C3	7.64	179.1	6.26	8.27	5.78	16.2	4600
Campaign II	C4	7.77	182.4	6.39	8.37	5.65	16.7	4680
	C5	5.74	172.8	7.81	10.5	7.96	20.8	3460
	C6	7.72	181.7	6.17	8.24	5.28	16.8	4660
Campaign III	C7	6.00	172.1	7.09	9.99	7.03	19.0	3640
	C8	12.4	195.8	4.51	5.78	3.44	12.6	7500
	C9	6.25	175.9	7.19	9.83	6.96	18.7	3800
	C10	12.4	195.9	4.57	5.77	3.52	12.9	7530
	C11	7.64	180.4	6.22	8.23	5.67	15.9	4630
	C12	14.0	200.5	4.42	5.78	3.52	22.0	8420
Steady*	C5	5.79	174.5	8.01	10.7	8.00	20.4	3500
	C6	8.12	180.9	6.19	8.41	5.15	15.7	4900
	C9	6.25	176.2	7.22	9.84	7.08	18.7	3800
	C10	12.4	196.5	4.60	5.83	3.40	13.1	7500

Figure 7.12 illustrates the ratio between the Lagrangian time scale T_L and the Eulerian time scale L_E/u_{rms} over the considered range of $3400 < Re_{2D} < 8500$. It is found that the Lagrangian time scale estimated by the integration of R_τ up to the first zero crossing follows $T_L^* \sim 0.357L_E/u_{rms}$. On the other hand, the Lagrangian time scale obtained by averaging $T_L(t)$ over $L_E/u_{rms} \leq t \leq 6L_E/u_{rms}$ is given by

$T'_L \sim 0.270L_E/u_{rms}$. The values estimated by the two methods differ by approximately a factor of 1.32.

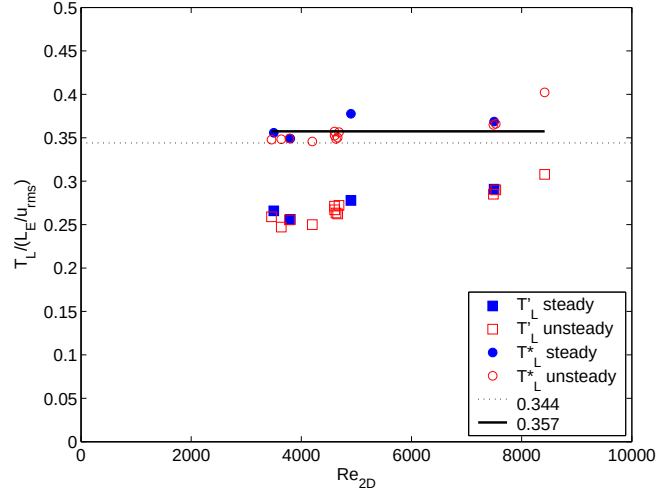


Figure 7.12: Lagrangian integral time scale T_L scaled with L_E/u_{rms} as a function of Re_{2D} . The blue dots indicate the value of the steady flow with the statistics integrated from a single instantaneous Eulerian velocity field from the time-dependent forcing case. The values are in good agreement with the result of the steady flow $T_L \sim 0.344L_E/u_{rms}$ in Rossi et al. (2007); the black dotted line indicates $T_L/(L_E/u_{rms})$ at 0.344 and the black solid line indicates the average value of T_L^* for all different unsteady forcing cases. The red circles indicate $T_L/(L_E/u_{rms})$ of the time-dependent forcing cases.

The Lagrangian time scale $T_L^* \sim 0.357L_E/u_{rms}$ is in good agreement with the result of the steady flow in Rossi et al. (Rossi, Vassilicos, & Hardalupas 2007) where $T_L \sim 0.344L_E/u_{rms}$. For compatibility, T_L^* is used to scale our results and is referred to as T_L in this work. Note that T_L in this work is the inner time scale whereas the outer time scale is at least of an order of magnitude larger.

According to Corrsin (Corrsin 1963), the ratio between the Eulerian timescale L_E/u_{rms} and the Lagrangian timescale T_L should be constant in a turbulent flow with a broad range of power law energy spectrum. In this section, we show that our turbulent-like flows, which also possess a broad range of power law energy spectrum, exhibit a ratio between the Eulerian and Lagrangian timescales, $T_L/(L_E/u_{rms})$, that is close to a constant in line with Corrsin's hypothesis.

7.5 Lagrangian frequency spectra

The Lagrangian frequency spectra (a.k.a. the Lagrangian time spectra), $\Phi_L(\omega)$, reveals the temporal evolution of the velocity fluctuations experienced by fluid elements in the measurement field. The Lagrangian frequency spectra is obtained from Fourier transform of $R_L(\tau)$ as follows:

$$\Phi_L(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_L(\tau) e^{-i\omega\tau} d\tau. \quad (7.5)$$

In homogeneous isotropic turbulence, $\Phi_L(\omega)$ exhibits a plateau for $\omega < \omega_L$ followed by $\Phi_L(\omega) \sim \varepsilon \omega^{-2}$ for $\omega > \omega_L$ (Inoue 1951). Similar trends have previously been observed in the steady multi-scale laminar flows (Rossi, Vassilicos, & Hardalupas 2007). Their spectrum display a plateau for approximately a decade of frequency followed by an approximate power-law, $\Phi_L(\omega) \sim (\omega T_L)^{-\alpha}$ where $2.5 < \alpha < 3.5$ and α varies with the Reynolds number. The steeper slope of the spectrum suggests that less energy is distributed towards high frequencies which are associated to the small scale motions in the flow.

The Lagrangian frequency spectrum of our unsteady multi-scale laminar flows for all time-dependent forcing cases are illustrated in Figure 7.13 and are distinguished into three flow families in Figure 7.14. A comparison of $\Phi_L(\omega)$ between the steady and unsteady flows of a selection of different forcing cases from the three flow families is also shown in Figure 7.15. From the figures, the spectrum exhibit a plateau over approximately 0.6-1 decade of frequency for $\omega T_L < 0.1$. At $\omega T_L > 0.1$, two distinct trends of $\Phi_L(\omega)$ can be observed between the flow family I (Figure 7.14(a) and (b)), with the forcing intensity range covering more than one flow scales, and the families II and III (Figure 7.14(c) and (d)), with the forcing intensity range covering just one flow scale.

In the flow family II and III, $\Phi_L(\omega)$ follow the same trend as those of the steady multi-scale laminar flows for $\omega T_L > 0.1$. The Lagrangian frequency spectrum display an approximate power-law, $\Phi_L(\omega) \sim (\omega T_L)^{-\alpha}$ for a decade of frequency where $\alpha \approx 3.3$ and 3.6 for the flow family II and III respectively. When the forcing intensity range

covers just one flow scale, it appears that the forcing period have little effect on α is dependent only on the mean forcing intensity which is directly related to the Reynolds number.

A different trend of $\Phi_L(\omega)$ is observed for the flow family I with the forcing intensity range covering more than one flow scales. The Lagrangian frequency spectrum display significant peaks at particular frequencies between $0.2 < \omega T_L < 1$. Such peaks can be clearly observed in Figure 7.15(a) and (b). The frequency at which the peak occurs is influenced by the forcing period; as the forcing period increases, the frequency of the peak in $\Phi_L(\omega)$ decreases (Figure 7.14(a)). It appears that the range of forcing intensity, i.e. how it covers different flow scales, also affects the shape of $\Phi_L(\omega)$. The peak in $\Phi_L(\omega)$ in the case C5 extends to higher frequencies when compared to that of case C4 with the same forcing amplitude and forcing period (Figure 7.14(b)). This is because the forcing range of the case C5 also covers the small flow scale (M10) whereas the forcing range of the case C4 does not (Figure 4.6). Hence, the forcing of the case C5 also excites the small flow scale resulting in higher energy being distributed in the higher frequencies which are associated to the small flow scale.

The peaks in $\Phi_L(\omega)$ for $0.1 < \omega T_L < 1$ are desirable as they suggest that more energy is being distributed in these frequencies leading to more chaotic flows. Furthermore, the peaks in $\Phi_L(\omega)$ cause the power-law slope to be shallower bringing α closer to 2 and the shape of $\Phi_L(\omega)$ closer towards that of turbulence. It is therefore justified to state that the forcing that targets multiple scales at once leads to better stirring efficiency than the forcing that target individual flow scale.

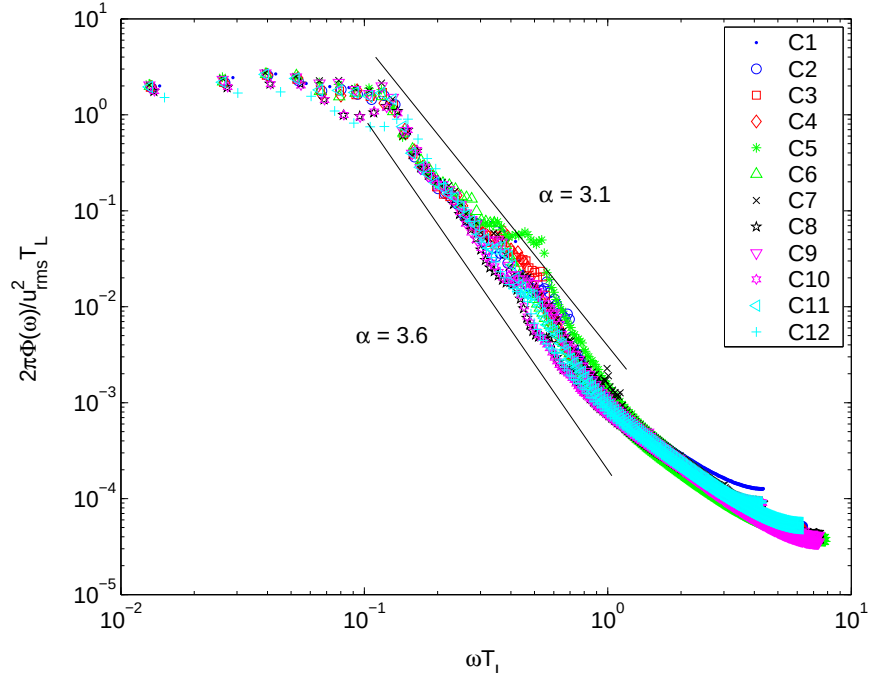


Figure 7.13: Lagrangian frequency spectrum for all time-dependent forcing cases

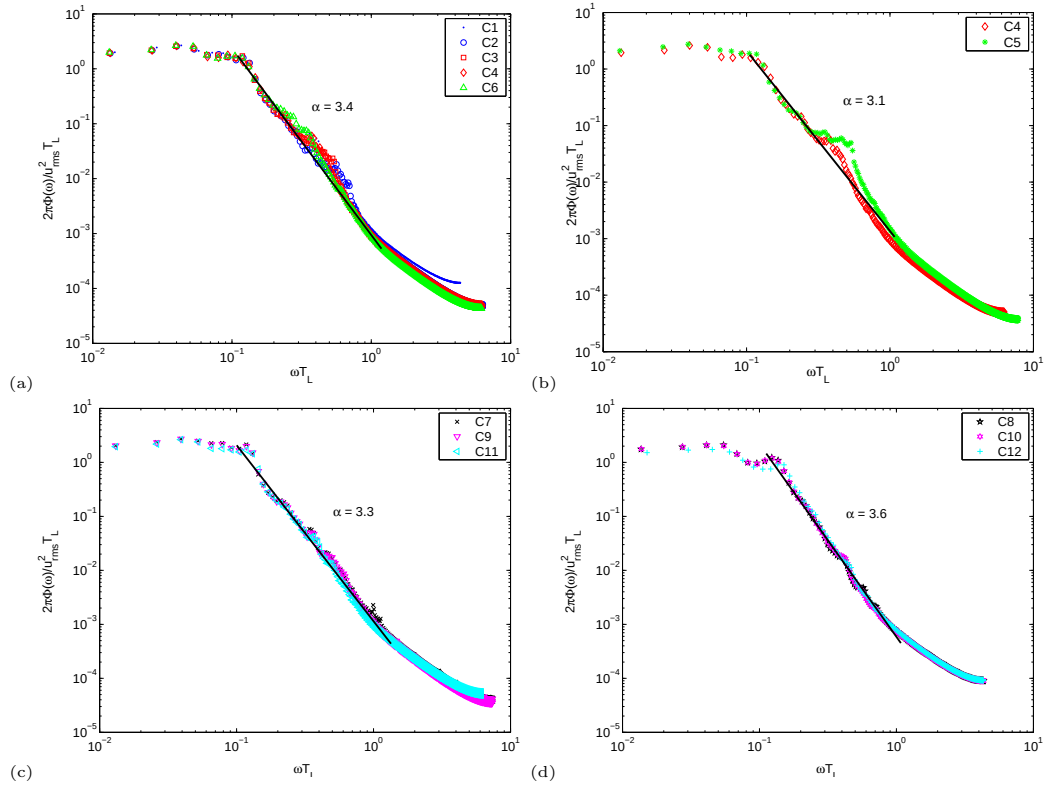


Figure 7.14: Lagrangian frequency spectrum shown in three flow families: (a) and (b) Family I, (c) Family II and (d) Family III

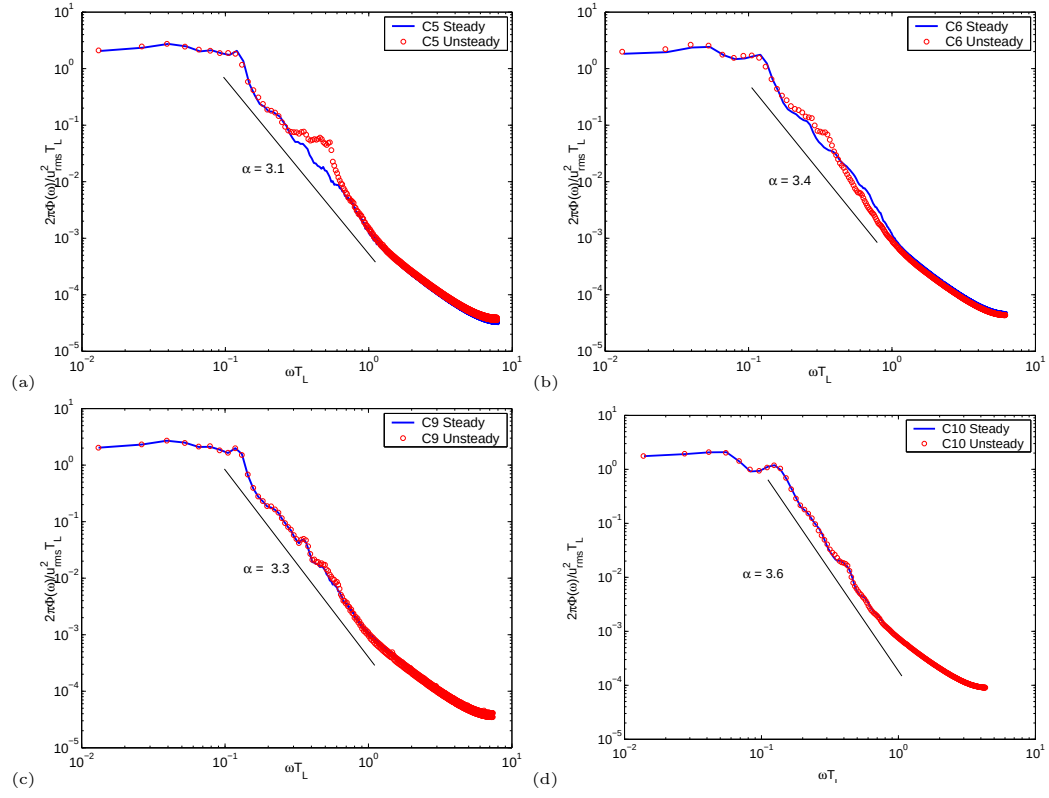


Figure 7.15: A selection of forcing cases from the three flow families showing a comparison of the Lagrangian frequency spectrum between the steady and unsteady flows: (a) Family I: Case C5, (b) Family I: Case C6, (c) Family II: Case C9 and (d) Family III: Case C10

7.6 Fluid element dispersion

In this work, we quantify mixing through statistics of fluid element dispersion. In our unsteady multi-scale flows, mixing is dominated by stirring and diffusion at molecular level, being much smaller, is assumed negligible. We extract statistics of single fluid dispersion and pair dispersion from the Lagrangian trajectories starting from random initial positions at time $t = 0$. The initial separation is 1 pixel which is much smaller (~ 20 times) than the smallest length scale in our flows. As the fluid dispersion statistics is sensitive to the initial separation (Nicolleau & Vassilicos 2003), (Bourgoin, Ouellette, Xu, Berg, & Bodenschatz 2006), the value is kept constant for all forcing cases. The fluid dispersion results are presented as follows:

7.6.1 Single fluid element dispersion

The statistics of single fluid element dispersion, $\langle (\mathbf{x}(t) - \mathbf{x}_0)^2 \rangle$ normalised by L_E^2 for all forcing cases, are shown in Figure 7.16. The dispersion trend is very similar for all flow families. At $t < T_L$ we observe ballistic dispersion where $\langle (\mathbf{x}(t) - \mathbf{x}_0)^2 \rangle \sim t^2$. For $t > T_L$ where T_L is finite, $\langle (\mathbf{x}(t) - \mathbf{x}_0)^2 \rangle \sim t$, in agreement with an analytical relation by Taylor (1921).

Figure 7.17 shows the dispersion statistics of a selection of forcing cases from the three flow families in comparison to the statistics of the steady flows. From the figure, both the steady and unsteady flows have very similar single fluid element dispersion statistics. The flow family I, with the forcing intensity range covering more than one flow scales, has a slightly higher power exponent value of $2.05 \leq \gamma \leq 2.1$ for the initial ballistic regime suggesting slightly stronger initial dispersion than the steady flows where $\gamma = 2$. Nevertheless, it can be seen that the addition of unsteadiness to the flows has little effect on $\langle (\mathbf{x}(t) - \mathbf{x}_0)^2 \rangle$ overall.

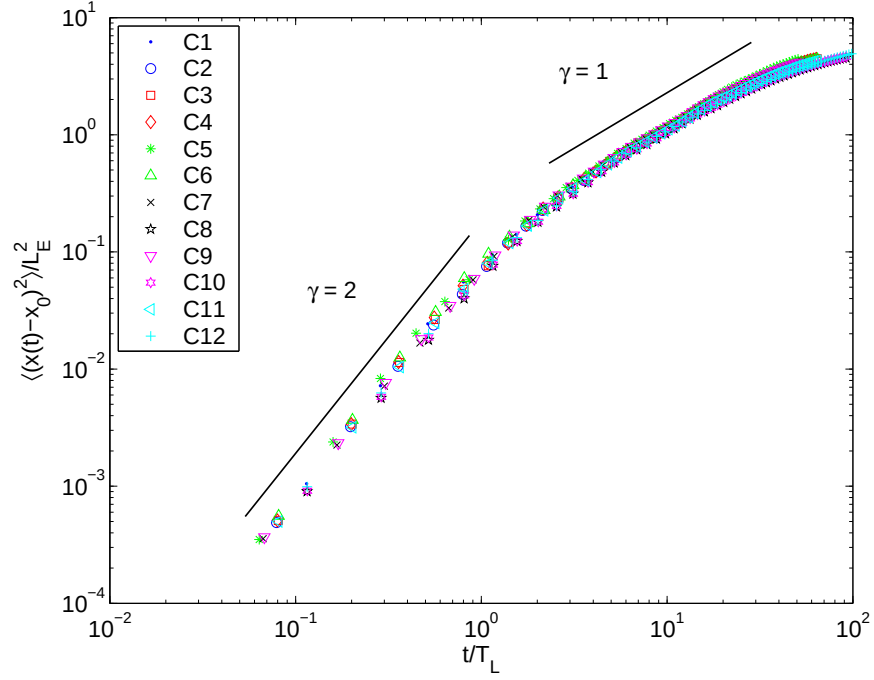


Figure 7.16: Fluid element dispersion: single particle statistics.

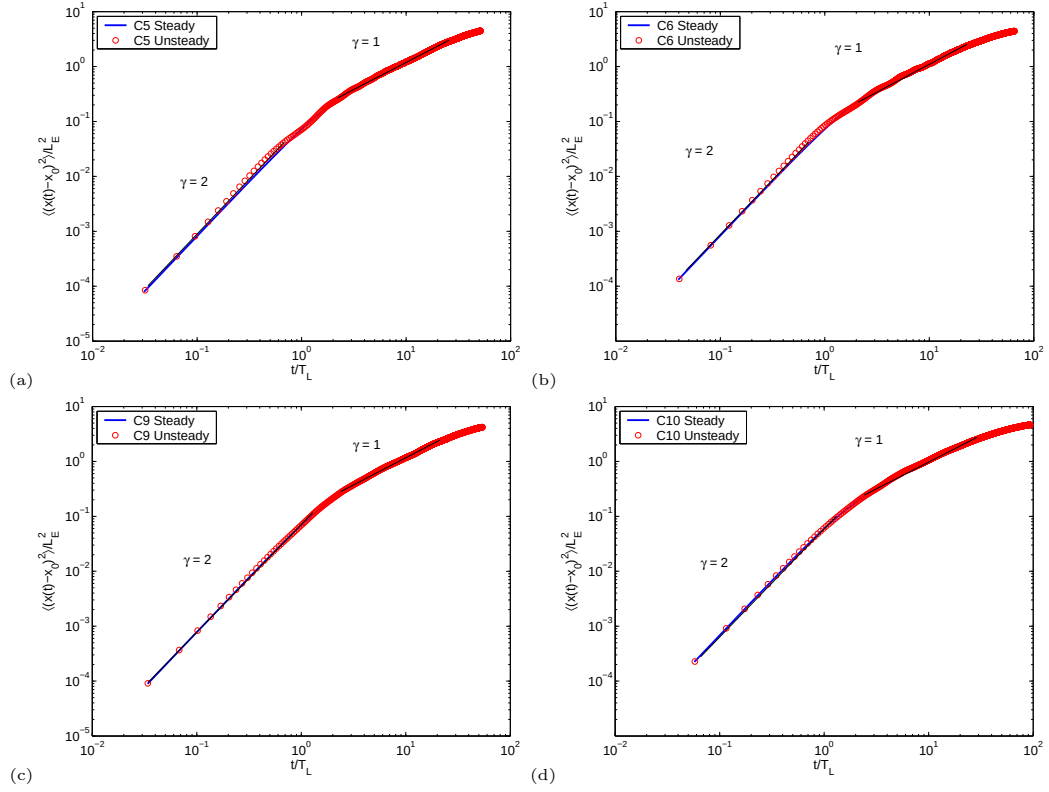


Figure 7.17: Single fluid element dispersion statistics of a selection of forcing cases from the three flow families: (a) and (b) Family I, (c) Family II and (d) Family III.

7.6.2 Fluid element pair dispersion

The statistics of fluid element pair dispersion, $\langle \Delta^2 \rangle$, is defined as the mean square of the fluid pair separations, Δ , where $\Delta = (\mathbf{x}_2 - \mathbf{x}_1)$ with $\mathbf{x}_1(t)$ and $\mathbf{x}_2(t)$ denoting the displacement vectors from a fixed point $[0,0]$ of the two fluid elements in the pair at time t (the notation (t) is dropped for brevity in the following analysis).

The pair dispersion statistics, as shown in Figure 7.19-7.22 for our unsteady multi-scale flows, have three main distinctive regimes;

1. the initial ballistic regime, $(\langle \Delta^2 \rangle - \Delta_0^2) \sim t^2$, at $t < T_L$,
2. the intermediate dispersive regime, which extends for more than a decade of T_L ,
3. the final random walk regime where $\langle \Delta^2 \rangle \sim t$.

At $t < T_L$, fluid element pairs travel together and separate only by the motion of the mean surrounding flow. Besides, for time t being small, fluid elements travel approximately in straight lines giving $\mathbf{x}_1 = \mathbf{x}_1(0) + \mathbf{u}_1 t$ and similarly for $\mathbf{x}_2$, which gives:

$$\langle \Delta^2 \rangle = \langle [\Delta_0 + (\mathbf{u}_2 - \mathbf{u}_1)t]^2 \rangle,$$

where $\Delta_0 = \mathbf{x}_2(0) - \mathbf{x}_1(0)$ is the initial fluid pair separation at $t = 0$. Hence,

$$\langle \Delta^2 \rangle = \langle \Delta_0^2 \rangle + \langle (\mathbf{u}_2 - \mathbf{u}_1)^2 t^2 \rangle - \langle 2\Delta_0 \cdot (\mathbf{u}_2 - \mathbf{u}_1)t \rangle.$$

Due to homogeneity and isotropy of fluid element and Δ_0 distribution in the field and that Δ_0 is independent of $(\mathbf{u}_2 - \mathbf{u}_1)$, $\langle 2\Delta_0 \cdot (\mathbf{u}_2 - \mathbf{u}_1)t \rangle \approx 0$ (Figure 7.18(a)) and $\langle 2\Delta_0 \cdot (\mathbf{u}_2 - \mathbf{u}_1)t \rangle \ll \langle \Delta_0^2 \rangle + \langle (\mathbf{u}_2 - \mathbf{u}_1)^2 t^2 \rangle$ (approximately 2-3 order of magnitude smaller as shown in Figure 7.18(c)), which can be considered negligible for $t < T_L$. This leads to the initial ballistic regime:

$$(\langle \Delta^2 \rangle - \Delta_0^2) \sim t^2$$

for $t < T_L$ in our flows as shown in Figure 7.19 and 7.20. Note also that as t and $\langle \Delta^2 \rangle$ are small, Δ_0^2 is significant and cannot be assumed negligible in this regime; this explains why such regime is not observable in $\langle \Delta^2 \rangle$ statistics in Figure 7.21 and 7.22.

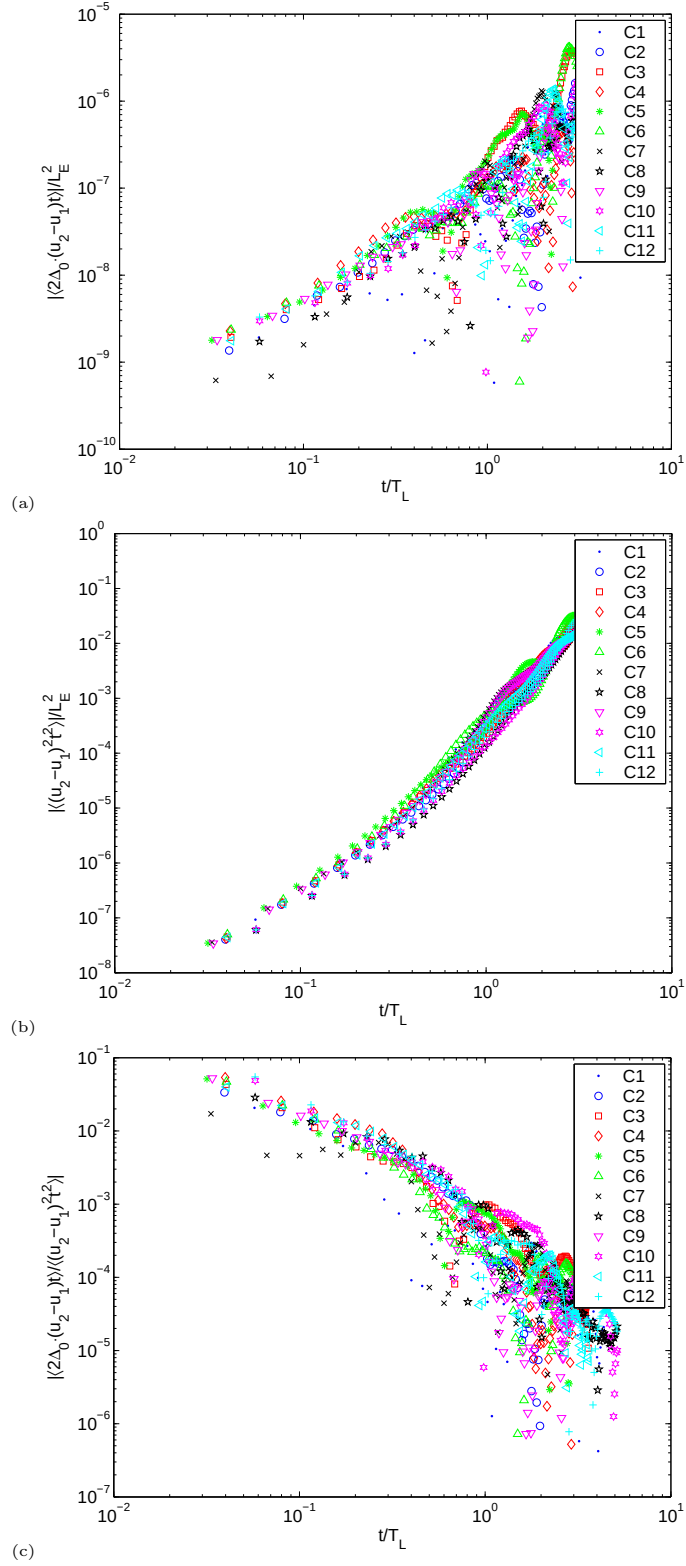


Figure 7.18: Components of pair dispersion statistics in the initial ballistic regime: (a) $|\langle 2\Delta_0 \cdot (\mathbf{u}_2 - \mathbf{u}_1)t \rangle|/L_E^2$, (b) $|\langle \Delta_0^2 \rangle + \langle (\mathbf{u}_2 - \mathbf{u}_1)^2 t^2 \rangle|/L_E^2$ and (c) ratio between $\frac{|\langle 2\Delta_0 \cdot (\mathbf{u}_2 - \mathbf{u}_1)t \rangle|}{|\langle \Delta_0^2 \rangle + \langle (\mathbf{u}_2 - \mathbf{u}_1)^2 t^2 \rangle|}$.

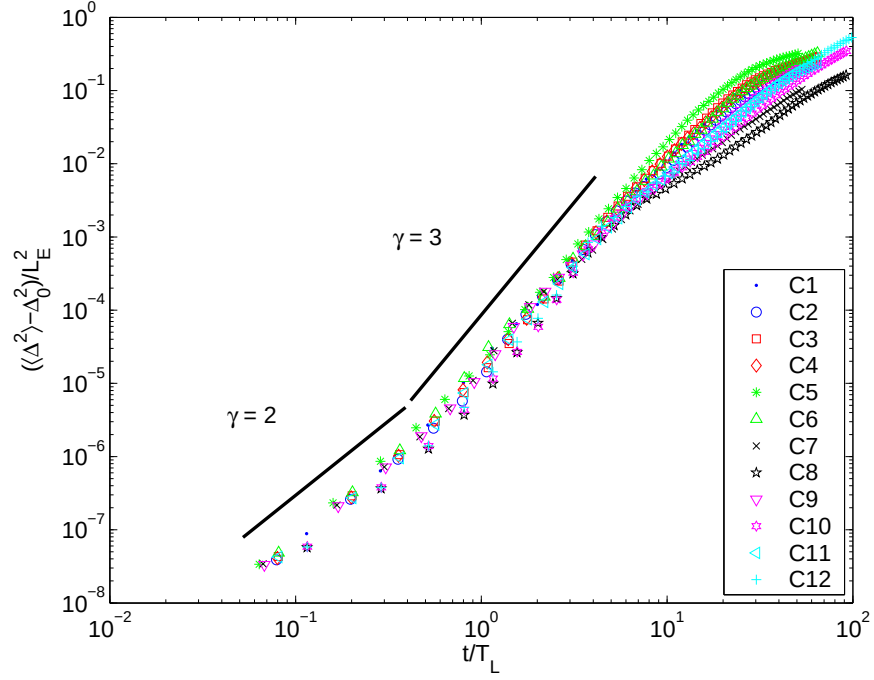


Figure 7.19: Fluid element pair dispersion statistics, $(\langle \Delta^2 \rangle - \Delta_0^2)/L_E^2$, for all forcing cases.

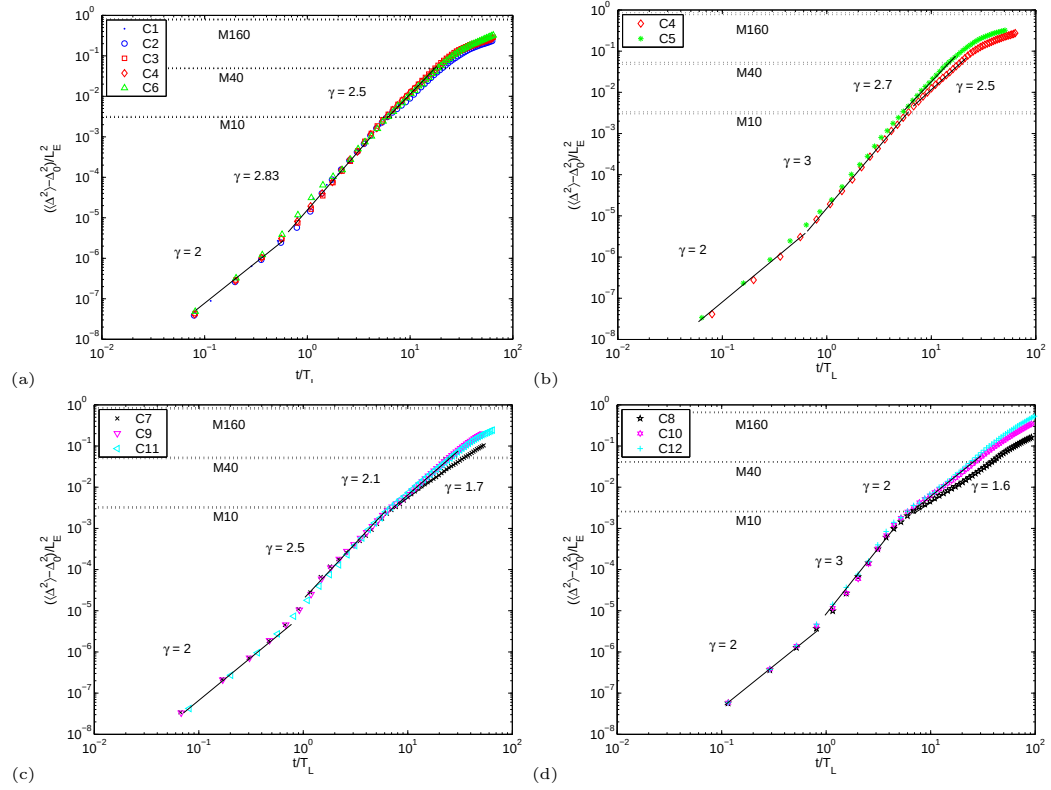


Figure 7.20: Fluid element pair dispersion statistics, $(\langle \Delta^2 \rangle - \Delta_0^2)/L_E^2$ for different flow families: (a) and (b) Family I, (c) Family II and (d) Family III.

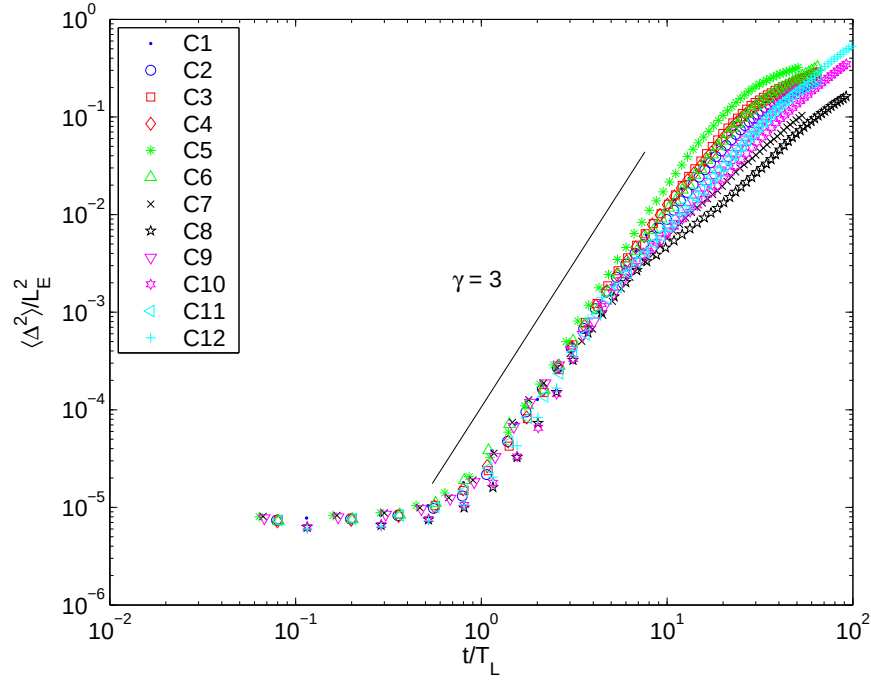


Figure 7.21: Fluid element pair dispersion statistics, $\langle \Delta^2 \rangle / L_E^2$, for all forcing cases.

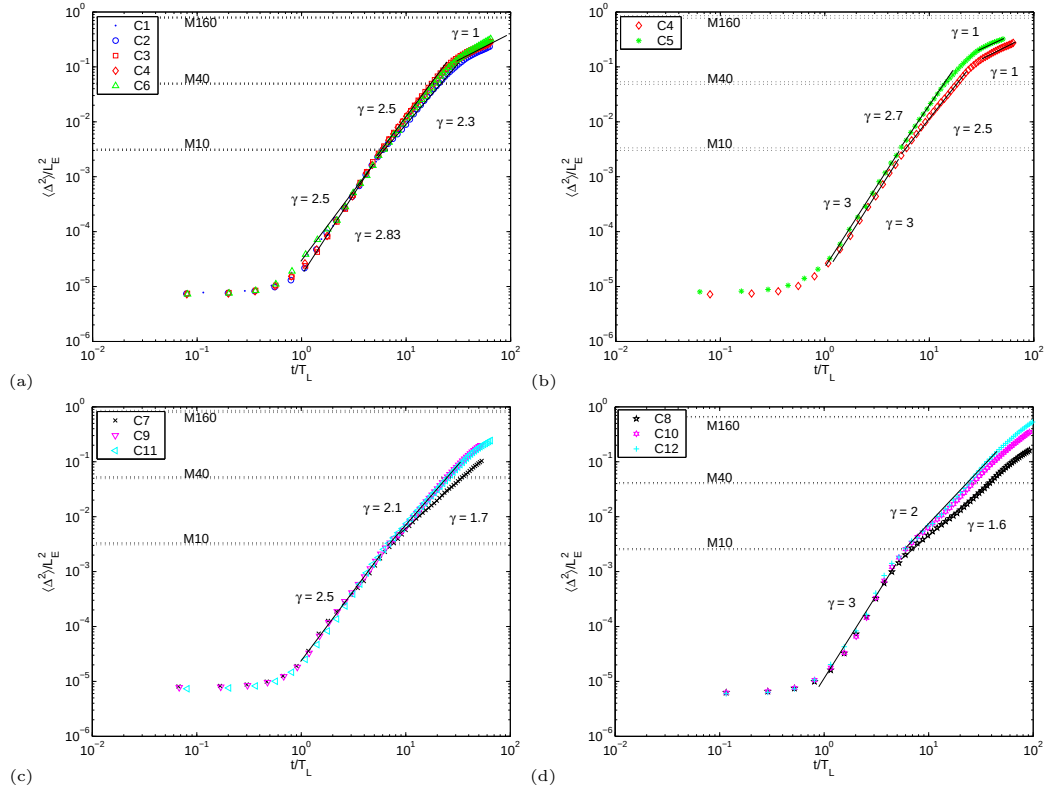


Figure 7.22: Fluid element pair dispersion statistics, $\langle \Delta^2 \rangle / L_E^2$, for different flow families: (a) and (b) Family I, (c) Family II and (d) Family III.

Following the initial ballistic regime, the pair dispersion statistics follow a “Richardson-like” law, $\langle \Delta^2 \rangle \sim t^\gamma$ where $\gamma > 2$. In this intermediate dispersive regime, Δ_0^2 becomes relatively small and can be assumed negligible as $(\langle \Delta^2 \rangle - \Delta_0^2) \approx \langle \Delta^2 \rangle$ in this regime. Once $\langle \Delta^2 \rangle$ exceed the medium M40 length scale on average, the fluid pairs continue traveling apart until the separation between the fluid pairs are large enough and fluid pair motions become uncorrelated from each other. This takes place in the last regime where fluid pair motions start exhibiting random walk behavior, $\langle \Delta^2 \rangle \sim t$.

From Figure 7.20 and 7.22, the three flow families exhibit different intermediate dispersive regime of which two sub-ranges can be observed from the differences in the value of γ . The first range is when $t \geq T_L$ and the second range starts when $\langle \Delta^2 \rangle$ exceed the length scale of the smallest characteristic flow scale L_{M10}^2 ; a drop in γ can be seen at this point.

In the flow family I, with the forcing covering multiple flow scales, both sub-ranges of the intermediate regime exhibit an approximated “Richardson-like” power-law of $\langle \Delta^2 \rangle \sim t^\gamma$, where $\gamma > 2$; in the first sub-range, $2.5 \leq \gamma \leq 3$ whereas in the second sub-range, $2.3 \leq \gamma \leq 2.7$. In the flow family II and III (with the forcing targeting only one flow scale), while the first sub-range displays the Richardson-like power law where $\gamma > 2$ ($\gamma \approx 2.7$ and 3 for Family II and III respectively), the second sub-range displays pair dispersion closer to the ballistic motion where $\gamma = 2$. Exceptions are in the case C7 and C8 in the flow Family II and III respectively; these two cases exhibit $\gamma < 2$ in the second sub-range, hence, they have lower stirring efficiency. This is due to their high forcing frequency leading to oscillation in the flow motions rather than the stretching and folding motions which are relatively more effective for fluid dispersion.

The finding of the “Richardson-like” power-law in the dispersive regime is in agreement with the result of the steady multi-scale flows (Rossi, Vassilicos, & Hardalupas 2006a). The authors of (Rossi, Vassilicos, & Hardalupas 2006a) proposed an explanation for the power-law where $\gamma > 2$ in such regime to be a result of successive exponential straining actions by the directly imposed three scales stagnation points and fluid pair dispersion is first imposed by the smallest (M10) stagnation point, then by the medium

(M40) and large (M160) stagnation points respectively. Such Richardson-like locality concept may also explain the change in slope γ when $\langle \Delta^2 \rangle$ exceeds each of L_i (i denotes the three M10, M40 and M160 scales). The fluid pair dispersion is first imposed by the M10 stagnation points and when the separation between the fluid pairs are larger than L_{10} , the fluid pairs are no longer being dispersed by the M10 stagnation points. Hence, the stirring contribution from the M10 stagnation points reduces after $\langle \Delta^2 \rangle > L_{10}^2$ on average and decreases the overall slope γ beyond such point. The fluid pair dispersion is then imposed by the medium M40 stagnation points and the process is repeated. The fluid pair dispersion statistics $\langle \Delta^2 \rangle$ never reach the order of the largest M160 flow scale, L_{160}^2 , which is also approximately L_E^2 . This is because, unlike in the other two smaller flow scales, the flow motion in the largest M160 flow scale tend to trap fluid elements within each quarter of the tank.

When compare to the steady flows (Figure 7.23), the addition of unsteadiness gives two significant improvements on the pair dispersion statistics; the intermediate regime extends for at least a further decade of T_L before the random walk regime begins and the power law exponents γ also increases. The latter are less obvious in some forcing cases where the time-dependent forcing is relatively weak i.e. in the flow family II with small forcing intensity range.

Turbulent diffusivity, $\partial(\Delta/L_E)^2/\partial(t/T_L)$, are also presented to complement the pair dispersion statistics in Figure 7.23. The turbulent diffusivity, which is much less sensitive to Δ_0 (Nicolleau & Yu 2007), confirms the initial ballistic regime ($\partial(\Delta/L_E)^2/\partial(t/T_L) \sim t$), the final random walk regime ($\partial(\Delta/L_E)^2/\partial(t/T_L)$ presents a plateau) and the approximated power-laws in intermediate dispersive regime in our flows. The derivative $\partial(\Delta/L_E)^2/\partial(t/T_L)$ also reveals the underlying exponential dispersions which are less visible in $\langle \Delta^2 \rangle$ statistics. Such exponential dispersions are more observable in the cases where the forcing target only one flow scale (Figure 7.23(c),(d)). The fact that these forcing cases target only one flow scale and have weaker time-dependencies might be the reason for their $\partial(\Delta/L_E)^2/\partial(t/T_L)$ to display relatively more exponential behaviours (similar to chaotic advection) than other forcing cases that target multiple-scales and display better approximated power-laws.

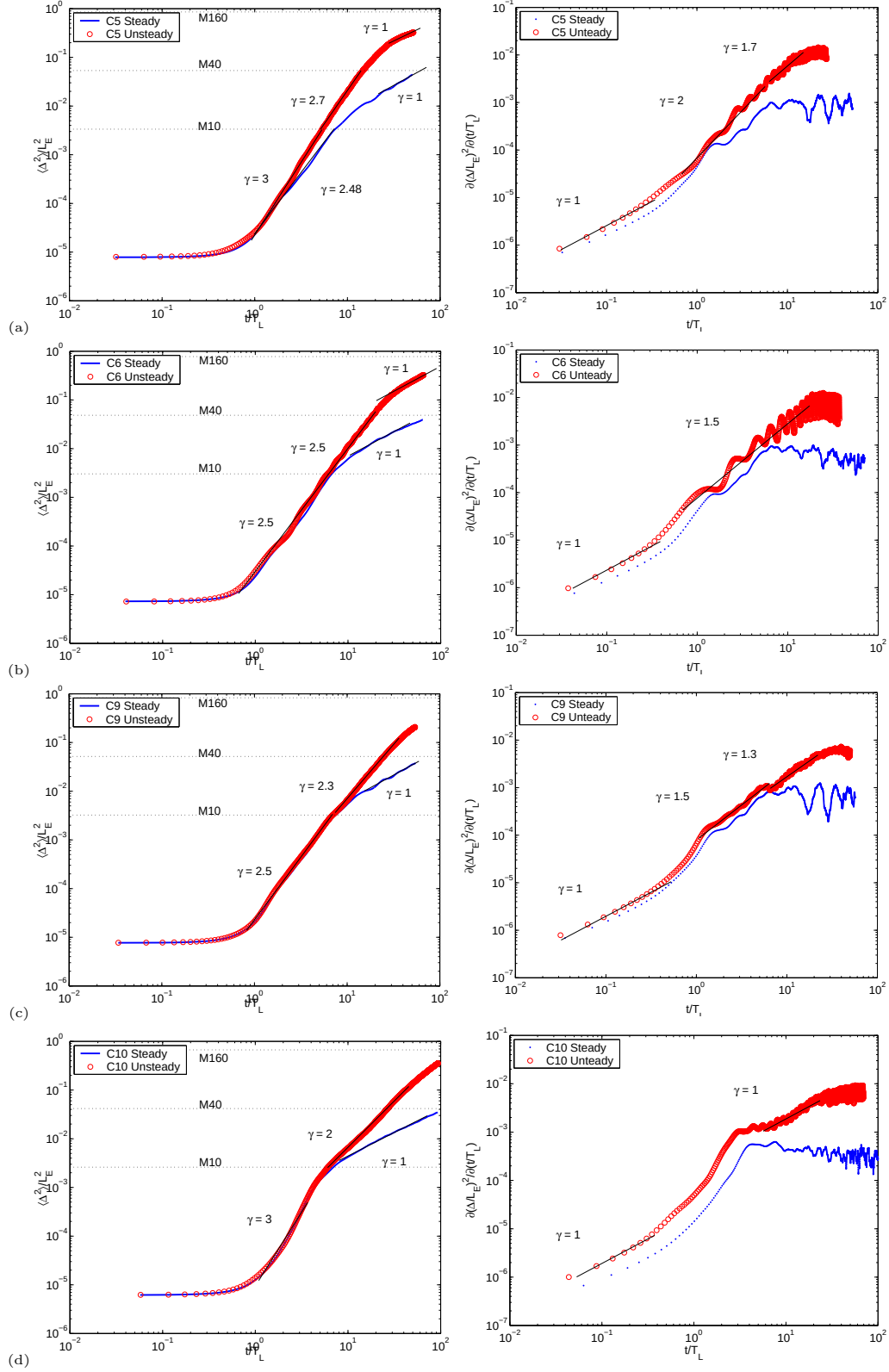


Figure 7.23: Comparison of fluid pair dispersion, $\langle \Delta^2 \rangle / L_E^2$, and turbulent diffusivity, $\partial(\langle \Delta^2 \rangle / L_E^2) / \partial(t/t_L)$, between steady and unsteady forcing of the cases C5, C6, C9 and C10. As the turbulent diffusivity is the first order derivative of the fluid pair dispersion statistics, the power exponent of the turbulent diffusivity is less than the power exponent of the fluid pair dispersion by one; this confirms the approximated power law of the fluid pair dispersion statistics.

It should also be noted that the turbulence diffusivity, which is a measure of mixing, increases significantly at long time ($t > 10T_L$) in the unsteady multi-scale flows when compared to the steady ones. This implies higher mixing rate in our unsteady multi-scale flows. To quantify such property, we present the average value of turbulence diffusivity, $\langle \partial(\Delta/L_E)^2 / \partial(t/T_L) \rangle$, for each regime of the pair dispersion statistics i.e. the initial ballistic regime where $t < T_L$, the intermediate dispersive regime $T_L \leq t < 20T_L$ and the final random walk regime $20T_L \leq t < 30T_L$ in Table 7.3.

Table 7.3: Average turbulence diffusivity in different pair dispersion regimes.

Experiment	Case	$\langle \partial(\Delta/L_E)^2 / \partial(t/T_L) \rangle$		
		$t < T_L$ [10^{-5}]	$T_L \leq t < 20T_L$ [10^{-3}]	$20T_L \leq t < 30T_L$ [10^{-3}]
Campaign I	C1	2.89	4.04	8.18
	C2	1.86	3.21	6.25
	C3	1.77	4.38	7.71
Campaign II	C4	2.29	4.10	7.11
	C5	2.21	6.07	10.3
	C6	3.09	3.61	7.52
Campaign III	C7	1.97	1.20	2.27
	C8	1.52	1.11	2.32
	C9	1.80	1.98	5.11
	C10	1.70	2.16	5.03
	C11	1.97	2.06	5.17
	C12	1.89	2.26	6.17
Steady*	C5	1.33	0.71	0.91
	C6	1.18	0.68	0.60
	C9	1.20	0.61	0.69
	C10	0.49	0.42	0.38

From the table, the forcing case C5, of which the forcing intensity range covers the most number of flow scales according to Figure 4.6, provides the highest $\langle \partial(\Delta/L_E)^2 / \partial(t/T_L) \rangle$ after $t > T_L$, suggesting the most effective mixing among all forcing cases. In contrast, the forcing cases C7 and C8, which have the highest forcing frequency, display the smallest values of $\langle \partial(\Delta/L_E)^2 / \partial(t/T_L) \rangle$, and imply the least effective forcing cases for mixing.

The scaling laws found in our flows apply to pair dispersion of fluid elements whose initial pair separation is much smaller than the smallest flow scale. It has been observed that $\gamma = \gamma(\Delta_0)$ for $\Delta_0 < L_{M10}$ (Figure 7.24). If Δ_0 increases to approximately half order of the smallest scale L_{M10} , $\langle \Delta^2 \rangle \sim t^2$ for $t > T_L$. The power-law exponent, γ , continues dropping as initial pair separation, Δ_0 , increases. This can be explained by the fact that as Δ_0 increases toward the size of the M10 stagnation point, the fluid element pairs feel weaker effect of the dispersion by such stagnation points. In addition, as previously mentioned, the current configuration of the experimental flow has limited large scale dispersion; the large flow scale still exhibit strong closed loops and fluid elements do not cross out of the large scale loops. Hence, with large Δ_0 , the fluid element pairs are dispersed mainly by the medium scale (M140) stagnation points. The multi-scale features of the flow is essentially ‘lost’ and the stirring efficiency of the flow significantly decreases as shown in Figure 7.24.

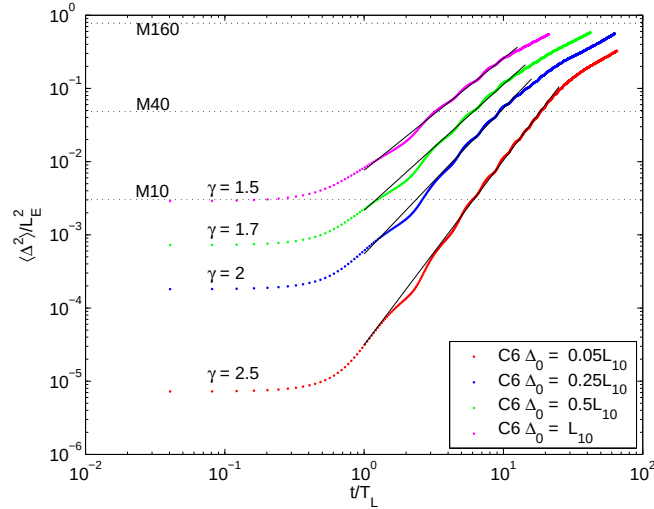


Figure 7.24: Fluid pair dispersion of the cases C6 for different initial pair separation Δ_0

We further relate the statistics of single fluid element dispersion to the fluid pair dispersion. By assuming $\Delta_0^2 \ll \langle \Delta^2 \rangle$, i.e. negligible, at a time t that is sufficiently large, it follows that;

$$\langle \Delta^2 \rangle \approx \langle \|\mathbf{x}_1\|^2 \rangle + \langle \|\mathbf{x}_2\|^2 \rangle - 2\langle \mathbf{x}_1 \cdot \mathbf{x}_2 \rangle$$

and if assumed isotropy, $\langle \|\mathbf{x}_1\|^2 \rangle \cong \langle \|\mathbf{x}_2\|^2 \rangle$, hence;

$$\langle \Delta^2 \rangle \approx 2\langle \|\mathbf{x}_1\|^2 \rangle - 2\langle \mathbf{x}_1 \cdot \mathbf{x}_2 \rangle.$$

In homogeneous isotropic turbulence, the quantity $2\langle \mathbf{x}_1 \cdot \mathbf{x}_2 \rangle \approx 0$ at long time leading to $\langle \Delta^2 \rangle \approx 2\langle \|\mathbf{x}_1\|^2 \rangle$. However, the quantity $2\langle \mathbf{x}_1 \cdot \mathbf{x}_2 \rangle$ remains significantly large in our unsteady multi-scale laminar flow. In fact, it appears that $\langle \mathbf{x}_1 \cdot \mathbf{x}_2 \rangle \approx \langle \|\mathbf{x}_1\|^2 \rangle$ as shown in Figure 7.25.

We therefore have;

$$\langle \|\mathbf{x}_1\|^2 \rangle - \langle \mathbf{x}_1 \cdot \mathbf{x}_2 \rangle = \langle \mathbf{x}_1 \cdot (\mathbf{x}_1 - \mathbf{x}_2) \rangle = \langle \mathbf{x}_1 \cdot \Delta \rangle \approx 0,$$

suggesting that the single fluid element dispersion statistics and the fluid pair dispersion statistics are uncorrelated. Since $\langle \|\mathbf{x}_1\|^2 \rangle \approx \langle \mathbf{x}_1 \cdot \mathbf{x}_2 \rangle$, the fluid pairs travel in parallel to each other on average. This can be interpreted as our unsteady multi-scale flows being deterministic. In contrast, $\langle \mathbf{x}_1 \cdot \mathbf{x}_2 \rangle \approx 0$ in homogeneous isotropic turbulence due to its random nature which leads to a strong correlation between the single fluid element dispersion statistics and the fluid pair dispersion statistics i.e.

$$\langle \mathbf{x}_1 \cdot \Delta \rangle \approx \langle \|\mathbf{x}_1\|^2 \rangle.$$

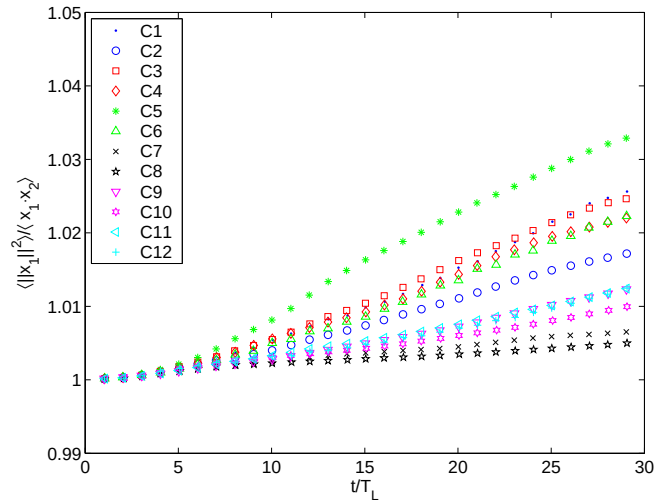


Figure 7.25: Ratio between $\langle \|\mathbf{x}_1\|^2 \rangle$ and $\langle \mathbf{x}_1 \cdot \mathbf{x}_2 \rangle$

The ratio $\frac{\langle \|\mathbf{x}_1\|^2 \rangle}{\langle \mathbf{x}_1 \cdot \mathbf{x}_2 \rangle}$ in Figure 7.25 suggests the effectiveness of stirring of each forcing case; smaller $\langle \mathbf{x}_1 \cdot \mathbf{x}_2 \rangle$, which implies less correlation between the two fluid elements in the pair and gives higher $\frac{\langle \|\mathbf{x}_1\|^2 \rangle}{\langle \mathbf{x}_1 \cdot \mathbf{x}_2 \rangle}$ ratio, is desirable. From the Figure, differences in $\frac{\langle \|\mathbf{x}_1\|^2 \rangle}{\langle \mathbf{x}_1 \cdot \mathbf{x}_2 \rangle}$ are observable after $t > 5T_L$. The flow Family I, with the forcing intensity range covering more than one flow scales, has shown to stir the flow more effectively than Families II and III with the forcing intensity ranges covering just one flow scale. The forcing of the case C5, which covers the most number of flow scales has the highest stirring effectiveness according to Figure 7.25. When compared among Families II and III, it appears that longer forcing periods, T_f , also provide relatively better stirring. This finding is in line with our previous argument in Section 6.3 that longer T_f allows more time for the flows to response to the time-dependent forcing.

7.7 PDF of relative pair dispersion statistics

The PDFs of the relative pair dispersion $\Delta_r(t) = \Delta(t) - \Delta_0$ normalised by $\Delta_{rms}(t)$, of the selected forcing cases from the three flow families are shown in comparison between the steady and unsteady flows in Figure 7.26. The PDFs of the steady and unsteady flows can be estimated by the following exponential function which is the same form as that of turbulence as predicted by Richardson, Batchelor and Kraichnan (Nicolleau & Vassilicos 2003):

$$p(\Delta_r(t), t) \simeq \kappa e^{-C(\Delta_r(t))^\beta} \quad (7.6)$$

where $\Delta_r(t) = \frac{(\Delta(t) - \Delta_0)}{\Delta_{rms}(t)}$.

In the steady flows, the PDFs of the relative pair dispersion collapse into the distribution given in Eq. 7.6 after $t \geq 3T_L$. As expected, the addition of unsteadiness causes PDFs of the relative pair dispersion of the unsteady flows to be time-dependent where the coefficients κ and C decrease with time. Hence, as time increases, the probability of small $\Delta_r(t)$ drops while the probability of large $\Delta_r(t)$ rises, resulting in a broader distribution and a larger variance. Such trend is similar to the Richardson pair dispersion distribution observed in isotropic turbulence:

$$p(\Delta_r(t), t) \simeq t^{-9/2} e^{-\frac{C'}{t}(\Delta_r(t))^{2/3}}, \quad (7.7)$$

where the coefficients $t^{-9/2}$ and $\frac{C'}{t}$ also decrease with time. Nevertheless, $\beta \neq 2/3$ in our unsteady multi-scale flows; the coefficient is found to be in the range $0.25 < \beta < 0.4$.

The PDFs of the unsteady multi-scale flows shows improvement in the fluid pair dispersion statistics when compared to those of the steady flows. When compared among different unsteady flow families, the flows family I, with the forcing targeting multiple flow scales, shows better improvement of pair dispersion distribution than the flow family II and III, with the forcing targeting only a single flow scale. This supports the argument in Section 7.6.2 that the time-dependent forcing that covers multiple flow scales provides stronger fluid pair dispersion leading to better stirring efficiency than the forcing that targets individual flow scale.

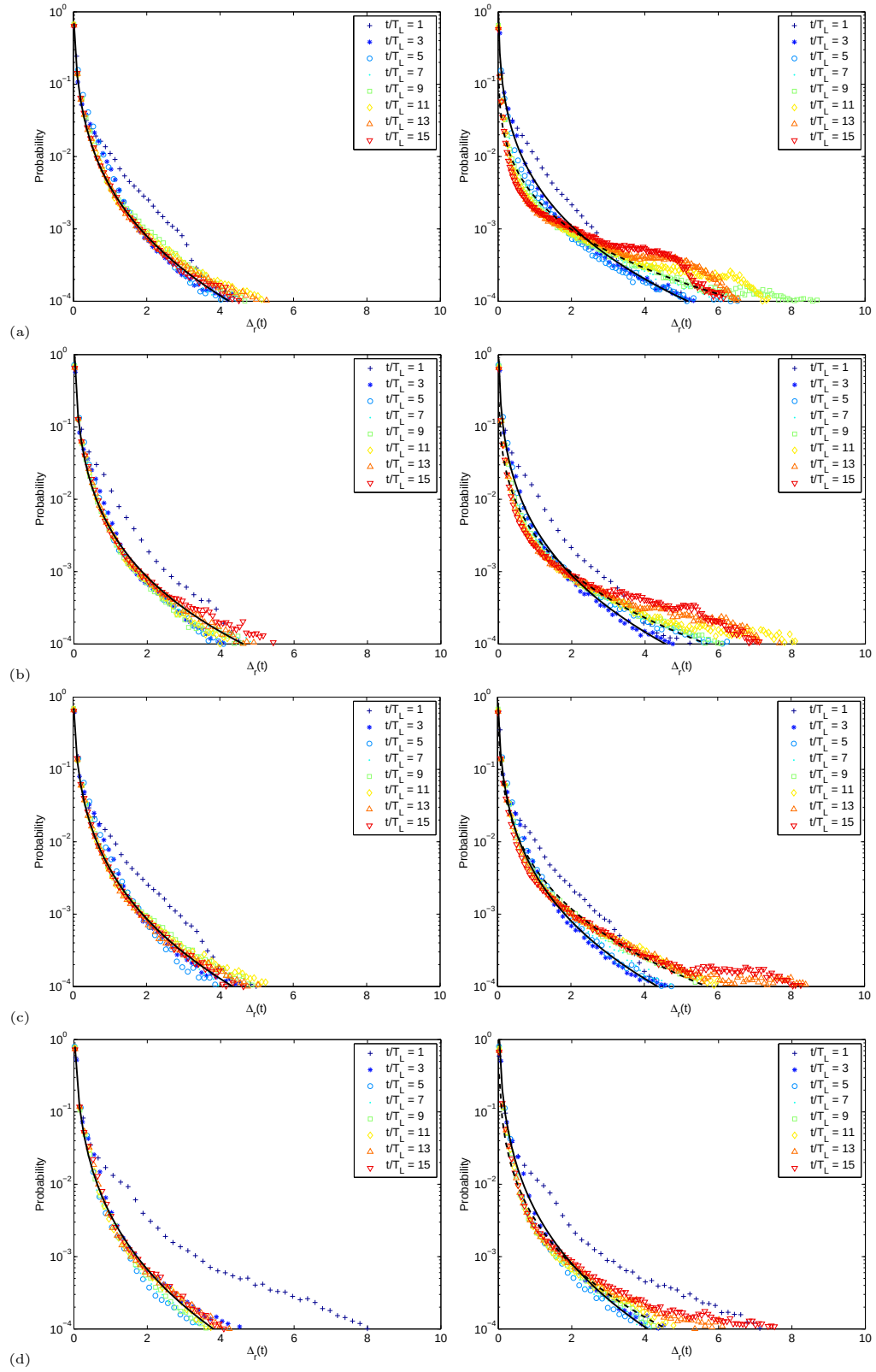


Figure 7.26: PDFs of relative pair separation of a selection of cases from the three flow families (top to bottom: case C5, C6, C9 and C10) of the steady (left) and unsteady flows (right). The solid line and the dotted line are the approximated fitting $p(\Delta_r(t), t) \simeq t^{-9/2} e^{-\frac{C'}{t} (\Delta_r(t))^{2/3}}$ at $t/T_L = 3$ and 11 respectively.

In this chapter, the effects of the unsteadiness introduced by time-dependent forcing on the Lagrangian properties of the multi-scale flows are demonstrated. The Lagrangian trajectories of the unsteady multi-scale flows develop chaotic pathlines rather than following and repeating the flow streamlines as in the case of steady multi-scale flows. As a result, trajectories originating in one flow scale can enter and exit the next consecutive scale and are no longer being trapped in their initial flow scales. DV reveals that our unsteady flows display clear “stretching and folding” processes when compared to the steady multi-scale flows where only the stretching process dominates.

In the unsteady multi-scale flows, $\Phi_L(\omega)$, bares some similarities to that of homogeneous isotropic turbulence; $\Phi_L(\omega)$ exhibits a plateau region followed by an approximate power-law, $\Phi_L(\omega) \sim (\omega T_L)^{-\alpha}$, but with a steeper slope of $3.1 < \alpha < 3.6$ rather than $\alpha = 2$ as in the case of 2D turbulent flows. Besides, the ratio between the Eulerian time-scale, $\frac{L_E}{u_{rms}}$, and the Lagrangian correlation time, T_L , of the unsteady multi-scale flows is close to a constant in line with Corrsin’s hypothesis which is valid for turbulent flows with a broad range of power law energy spectrum.

Our unsteady multi-scale flows display “Richardson-like” pair dispersion statistics, $\langle \Delta^2 \rangle \sim t^\gamma$ where $\gamma > 2$ in the intermediate dispersive regime between the initial ballistic and final random walk regimes. While the addition of unsteadiness in the form of time-dependent forcing has little effect on the single fluid element dispersion statistics, it has been shown to extend the Richardson-like scaling range at long time compared to the steady multi-scale flows and suggests better stirring features. It has also been observed that γ is influenced by the initial pair separation, Δ_0 and the characteristic length-scale, L_i^2 , imposed in our flows; γ reduces as Δ_0 increases and γ also reduces when $\langle \Delta^2 \rangle$ exceeds each L_i^2 .

The statistics of single fluid element dispersion and pair dispersion are further related and it appears that $\langle \mathbf{x}_1 \cdot \mathbf{x}_2 \rangle \approx \langle ||\mathbf{x}_1^2|| \rangle$ leading to $\langle \mathbf{x}_1 \cdot \Delta \rangle \approx 0$ in our unsteady multi-scale flows. This suggests that most fluid element pairs travel together in parallel on average and that our unsteady multi-scale flows are, in fact, deterministic unlike turbulence which are random.

Chapter 8

Conclusion

8.1 Discussion of results

In summary, we have demonstrated how unsteadiness can be reassembled to the multi-scale laminar flows (with the schematic streamline structure) by means of sinusoidal functions of forcing. We have also shown that the time-dependent forcing can be designed so as to excite individual or multiple flow scales altogether. The generated unsteady multi-scale flows are closer to being “turbulent-like” than the steady ones in many aspects which are summarised as follows:

As expected, the flows’ velocity and energy become time-dependent; $u_{rms}(t)$ is a function of $\bar{I}$, A , and T_f whereas u_{rms,T_f} (Eq.(6.3)) is only a function of $\bar{I}$ and is well in line with the constant forcing data in (Rossi, Vassilicos, & Hardalupas 2006a). The stagnation points’ characteristics such as the strain rate, size and area of direct influence also become time-dependent. The stagnation points move slightly and slowly compared to the surrounding flow whereas the multi-scale schematic streamline structure and topology i.e. the number of the stagnation points and how they are connected to the streamlines remains persistent. This is because the multi-scale topology is controlled by the fractal dimension of the stagnation point distribution, D_s , which is kept constant for all experiments performed. The energy spectrum, $E(k)$, is also found to be relatively persistent; the spectrum $E(k)$, normalised by the total energy E , oscillate slightly at the wavenumbers, k , excited by the time-dependent forcing. Nevertheless, the oscillations are subtle and do not change the overall shape of the energy spectrum

where $E(k) \sim k^{-2.6}$. Hence, the relation $p + D_s \approx 3$, as suggested by Davila and Vassilicos (Davila & Vassilicos 2003), also holds in our unsteady multi-scale flows.

The unsteady stagnation points, even with very slight movements, result in the “stretching and folding” mechanism which stirs the flows more effectively when compared to the steady multi-scale flows where only the stretching mechanism dominates. This is evident qualitatively in dye visualisation (DV), Lagrangian trajectories and quantitatively in the improvements in the pair dispersion statistics. The unsteady Lagrangian trajectories originating in one scale can enter other adjacent scales unlike in the steady flows where trajectories only exhibit closed loops following the streamlines. In the pair dispersion statistics, the Richardson-like scaling range where $\langle \Delta^2 \rangle \sim t^\gamma$ and $\gamma > 2$, is extended at long time and the power-law exponent, γ , even increases in one of the forcing cases where more than one flow scale is excited.

The pair dispersion statistics clearly display different regions; the change in the pair dispersion shape, γ , occurs when the fluid element pair separation size on average, $\langle \Delta^2 \rangle$, exceeds each of the characteristic length-scale, L_i^2 , imposed in our flows. Furthermore, the pair dispersion shape is also influenced by the initial pair separation, Δ_0 ; in fact γ reduces as Δ_0 increases, which suggests that the pair dispersion is less effective as Δ_0 approaches the characteristic size of the stagnation point. These imply that our unsteady multi-scale flows possess a property similar to Richardson’s “locality assumption” in that the turbulent diffusivity $\frac{d}{dt} \langle \Delta^2(t) \rangle$ at time t is dominated by the eddies of size $\langle \Delta^2(t) \rangle^{1/2}$; in our flows, the concept of eddies is replaced by the straining region around the hyperbolic stagnation points.

The Lagrangian frequency spectrum, $\Phi_L(\omega)$, of our unsteady multi-scale flows also bare some similarities to that of homogeneous isotropic turbulence in that it exhibits certain plateau region followed by an approximate power-law, $\Phi_L(\omega) \sim (\omega T_L)^{-\alpha}$, but with a steeper slope of $2.5 < \alpha < 3.5$ rather than $\alpha = 2$ as in the case of turbulent flows. The addition of unsteadiness, when forced efficiently at the effective scales, increases the energy at certain frequencies and brings the overall spectrum shape closer to that of 2D turbulence with $\alpha = 2$.

Last but not least, we found that the ratio between the Eulerian time-scale, $\frac{L_E}{u_{rms}}$, and the Lagrangian correlation time, T_L , of our unsteady multi-scale flows is close to a constant in line with Corrsin's hypothesis which is valid for turbulent flows with a broad range of power law energy spectrum.

It is striking to observe that the addition of time-dependent forcing in the form of simple sinusoidal functions has led to considerable improvements in the stirring properties of the flows mentioned above, despite the fact that these flows require approximately the same power input as in the case of the steady flows with the same level of mean forcing intensities. Hence, it is fair to claim that our unsteady multi-scale flows have better stirring efficiency and are more turbulent-like.

We emphasise on our unsteady multi-scale flows being turbulent-like as they are still clearly distinct from turbulence. Although properties such as the energy spectrum, Lagrangian frequency spectrum, pair dispersion statistics etc. begin to exhibit trends similar to turbulence, they still differ from the exact forms of the fully developed turbulence properties. Other main differences between our unsteady multi-scale flows and turbulence include the acceleration, the Eulerian frequency spectra and the deterministic feature of the flows. Our multi-scale flows also have a limited number of scales (i.e. 3) in the flows and lack of the advection or sweeping effect by the large scales as the unsteady stagnation point are relatively persistent and only move slightly and slowly compared to the mean flow.

The addition of the time-dependent forcing introduces the local acceleration, $\mathbf{a}_L \equiv \frac{\partial \mathbf{u}}{\partial t}$, in our multi-scale unsteady flows. Nevertheless, such term is considered small when compared to the convective acceleration, $\mathbf{a}_C \equiv (\mathbf{u} \cdot \nabla)\mathbf{u}$, and the mutual cancelation between the two acceleration components are weak, unlike in turbulence where the two terms are significantly and negatively correlated and mutually cancel one another resulting in a small acceleration term, $\mathbf{a}$, overall. In fact, the term $\mathbf{a}_L$ being small in our unsteady flows is expected as the time-dependency of the sinusoidal functions of forcing is smooth and hence the temporal unsteadiness is weak.

The sinusoidal functions of forcing is also responsible for the lack of continuous broad range of power law of the Eulerian frequency spectra, $\Phi_E(\omega)$. In our unsteady multi-scale flows, $\Phi_E(\omega)$, reveal a single peak in Fourier space as a result of only one temporal scale, i.e. one frequency, being present. To achieve a broad range of power law of Eulerian frequency spectra similar to what turbulence possesses, we must produce the multi-temporal scale feature in the flows. This can be obtained by adding the time-dependent forcing with many different frequencies.

Finally, it is worth noting the difference in the correlation between the fluid element single and pair dispersion statistics. Unlike in turbulence where there are some degree of correlation between the two quantities i.e. $\langle \mathbf{x} \cdot \Delta \rangle \approx \langle \|\mathbf{x}^2\| \rangle$, we observe an uncorrelated relationship, $\langle \mathbf{x} \cdot \Delta \rangle \approx 0$, in our unsteady multi-scale flows as Δ is very small and $\mathbf{x} \gg \Delta$. This implies that the growth of pair separation is not comparable to that of single fluid dispersion and most fluid pairs remain together in the flows. Hence, our flows are considered deterministic as opposed to turbulence which is random.

8.2 Application perspectives

The outcome of this research bares potential applications in the area of energy efficient mixers. Our generated unsteady multi-scale flows require very little power (≤ 1 watt) to run and achieve a broad range of continuous power law of energy spectrum as well as other turbulent-like properties in the flows. Nevertheless, the current particular set-up of our experiment might not allow direct applications as a mixing device as yet, as it is generally not practical nor desirable for substance to be mixed or stirred in brine solution with the presence of electrical current and magnetic field.

New designs of a mixing device (preferably 3D), which create the stagnation points by other methods that are non-reacting and non-contaminating to the substance to be mixed, will be required. This might prove to be more complicated than if we were to apply the multi-scale concept to generate turbulence for mixing as in the case of the fractal grid, for instance. Nevertheless, the great importance of our experiment compared to other approaches that generate turbulence to mix, is that our work allows full

control and direct access to the individual properties of turbulence to be investigated and applied. This cannot be achieved by any other methods that directly generate turbulence. Hence, our flows are very valuable for the fundamental analysis and understanding of turbulence even though they might be of less direct applications to the industries at present.

8.3 Possible improvements

8.3.1 Experimental improvements

The light level in the experiment can be increased so as to increase the intensity of the particles in the PTV output images. For this, two additional 500W spotlights can be added to the other two corners of the experimental rig on top of the two spotlights which already exist in the current experimental setup. The function generator connected to the power supply can be replaced by a computer controlled forcing unit for higher accuracy and more complex forcing signals.

All realisations of the same flow should be measured within the same day to minimise changes in the experimental conditions, essential calibration and corrections required both in the laboratory and during data post-processing. Lastly, at the forcing intensities higher than 0.5A, the size of the medium and small scale streamlines get considerably smaller, hence, when performing measurements with high forcing intensities, the camera can be lowered to focus on the smaller area in the measurement field. This can help enhancing the quality of the measurement output.

8.3.2 Data processing improvements

In the image post-processing, the size of the window for smoothing can be readjusted; a smaller window size can be used to eliminate the increase in the output particle size hence, reduce the noise due to the shift in the particle centroids. Furthermore, the output image with grey level particles can be used instead of the binary output for the particle tracking. This can be obtained from the dot product between the grey level

and binary output images. More specific criteria for particle recognition can then be added including smaller change allowance for particle properties in each frame and a higher number of threshold levels to identify the particle positions.

As previously mentioned in Section 5.3.3, a more accurate adaptive method for constructing the Eulerian velocity field from Lagrangian velocities is available as detailed in Ferrari and Rossi (Ferrari & Rossi 2008). In the method mentioned, the order of interpolation is also adaptive therefore it is more robust and can improve the level of noise further by 2-5% compared to the current method used in this work.

8.4 Recommendations for future work

Thus far, we have achieved our objectives in integrating the unsteadiness feature of turbulence to the multi-scale laminar flows and investigating the effects of time-dependency on both the Eulerian and Lagrangian properties of the flows. In order to obtain a more complete analysis and progress further towards being more turbulent-like, a number of future works can be undertaken. Further research on this work are recommended as follows:

More variations of the time-dependent forcing can be investigated. In this work, simple sinusoidal functions of forcing were used to generate the unsteady flows. As a result, although the flows have shown significant improvements in the pair separation statistics, $\langle \Delta^2 \rangle$, at long time and suggest better stirring features, the time-dependency of the sinusoidal forcing was rather limited. This is evident in the small local acceleration, $\mathbf{a}_L \approx \frac{\partial \mathbf{u}}{\partial t}$ (Section 6.6), little changes to the Lagrangian autocorrelation functions, $R_L(\tau)$, when compared to the steady forcing cases (Section 7.4) and steep slopes of Lagrangian frequency spectra, $\Phi_L(\omega)$ with the power law exponents $\alpha > 2$ after the plateau region (Section 7.5). To enhance the time-dependency of the flows, some variations to the time-dependent forcing are suggested below:

1. The shape of the forcing signal can be designed to be non-periodic, more irregular and change more abruptly. For instance, multi forcing frequencies can be

combined with various forcing signal shapes (e.g. triangle, step, trapezoidal, etc.) to enhance the forcing acceleration and irregularity. The former, which is related to the energy input rate, directly affects the local acceleration while the latter helps inhibiting flows from repeating the same path as in the case of the time periodic forcing.

2. The forcing polarity can also be varied. All experiments in the current work were performed only with the positive forcing intensity. The negative forcing intensity would force the flow in the opposite direction. Hence, more chaotic behaviours are expected for the flows generated by the forcing that fluctuates between the positive and negative polarities.

Other measures of stirring/mixing can also be considered, for instance, the material line i.e. the rate of growth of the mixing length and the concentration variance, c'^2 . The material line can be investigated numerically by integrating the boundary of the scalar blobs from the Eulerian velocity field and experimentally using dye visualisation. Besides, the concentration variance can be measured by the Laser Induced Fluorescent (LIF) technique.

Another main parameter that can be of interest is the fractal property of the flows; this includes the fractal dimension of the stagnation point distribution, D_s , quantified as the ratio between the magnet sizes in two consecutive scales and the number of iterative scales in the flows. These can be varied by changing the sizes, numbers, positions and distributions of the magnets underneath the experimental rig and, hence, the spatial properties of the forcing change accordingly.

Unsteadiness in the flows can also be generated by other methods such as changing the angles and positions of the magnets. The latter method, in particular, not only provides unsteadiness but also introduces the advection, which is another feature similar to what is observed in homogeneous isotropic turbulence, to our unsteady multi-scale flows. Hence, by moving the magnets underneath the experimental tank, the effect of the smaller scales being advected by the larger scales can be simulated and studied.

Chapter 9

Appendix

9.1 Image processing

The output images from the PTV measurements are in a built-in compressed format (.im7) files which can be directly accessed in Matlab for image processing. In this section, the details of different image processing techniques are discussed.

9.1.1 Reference image comparison

To subtract the background noise, each raw output image is compared to the reference image. Any light intensity values in the output image that are lower than those in the reference image are set to zero while all higher values remain the same. Figure 9.1 shows the output from the reference image comparison.

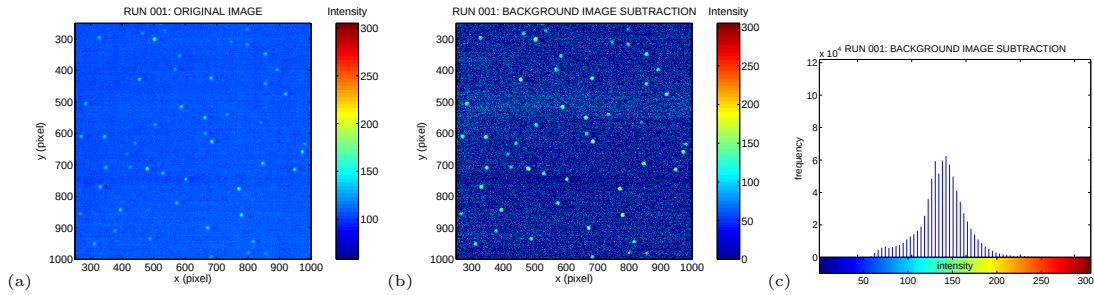


Figure 9.1: Background subtraction process (a) raw image (b) image after background subtraction (c) histogram of the output from background subtraction.

There are two main types of reference images. Background images (Figure 9.2(a)) are images of the flow field taken without any particles. The images contain visible

noises from the flow field imperfection (e.g. scratches, dust, uneven paint) and light reflection. Dark images (Figure 9.2(b)) are images taken with the camera's cap on and light in the laboratory switched off. They contain invisible noises from electronics equipment and surrounding. The reference images used are averaged over 400 images.

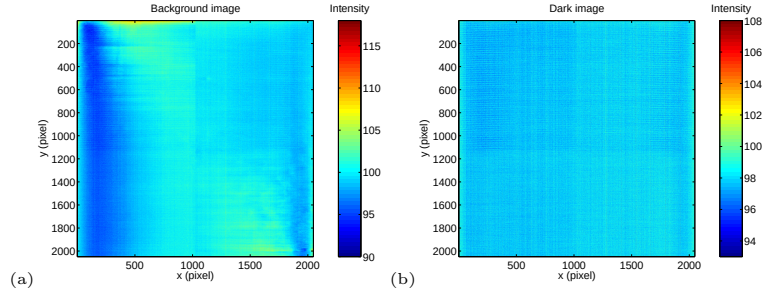


Figure 9.2: (a) Background image (b) Dark image.

In the case where there is no reference image from the experiment, it can be constructed by several techniques. Figure 9.3a shows an example of a constructed background image from averaging over 1950 raw output images. This method can be optimised by removing the particles from the raw output images prior to averaging (Figure 9.3(b)). The reference image can also be constructed and used for processing, image by image, by averaging over rows or columns as shown in Figure 9.3(c). However, it is limited to reduce noises with simple and consistent patterns (e.g. low light intensity on the edges of the image). In the case where there are noises with irregular and random patterns, local noise reduction technique can be used instead. The technique identifies any values exceeding the mean value (with some threshold) of the local area specified by the a window size and replaced them by the mean value of the local window.

9.1.2 Smoothing/Filtering

Smoothing uses a two dimensional convolution with the filter matrix rotated 180 degrees. The process helps focusing the areas of high light intensities and diffusing the area of low light intensity values. The convolution equation in spatial form is written

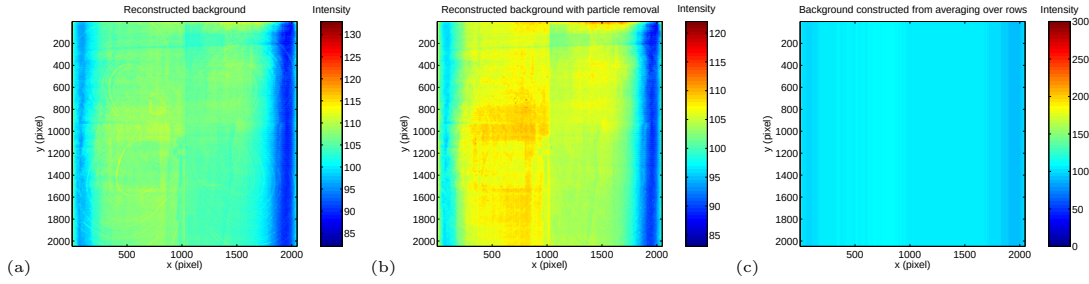


Figure 9.3: Constructed background: (a) from averaging every 10 images over 1950 raw output images, (b) from averaging over raw output images with prior particle removal and (c) from averaging over rows.

as:

$$c(n_1, n_2) = \sum_{k_1=-p}^p \sum_{k_2=-p}^p a(k_1, k_2) b(n_1 - k_1, n_2 - k_2) \quad (9.1)$$

where $p = 1.5$ and 2.5 for a smoothing window sizes 3 and 5 respectively.

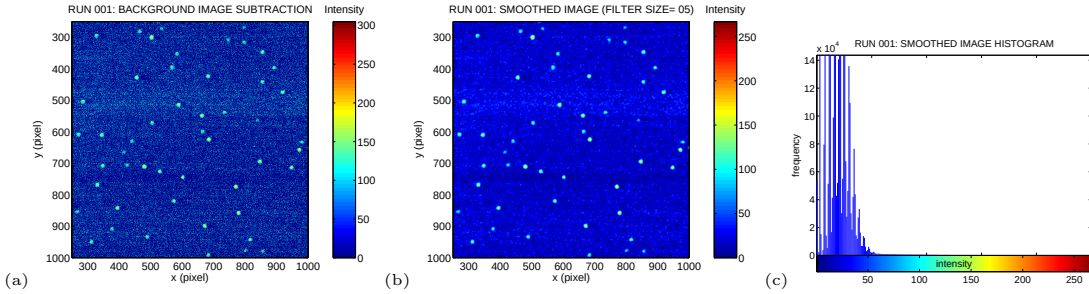


Figure 9.4: Smoothing process output: (a) image before smoothing, (b) image after smoothing and (c) histogram of the output from smoothing process.

9.1.3 Trimming

Trimming is optionally used as one of the final steps to remove the image edges where there are large amounts of noises which cannot be effectively eliminated by other image processing techniques.

9.1.4 Truncation

Low level noises and some faint particles are eliminated by truncation method (Figure 9.5). The mean and standard deviation values of the image are calculated. Any light

intensity values in the image which are less than a threshold value, given by the mean with a selected factored standard deviation are put to zero. All values higher than the threshold remain the same.

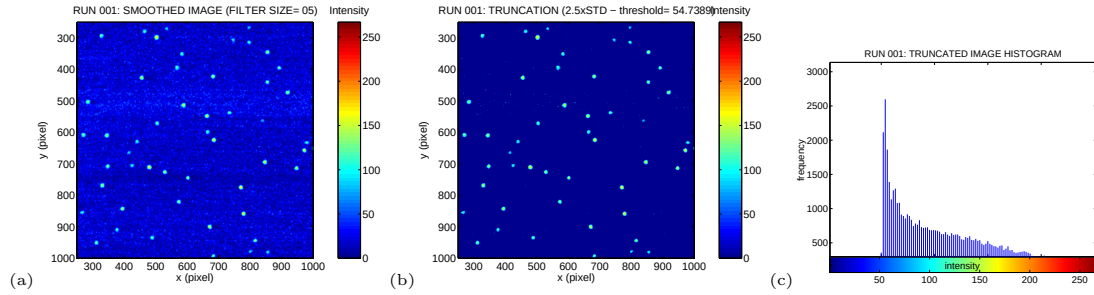


Figure 9.5: Truncation process output: (a) image before truncation, (b) image after truncation and (c) histogram of the output from truncation process.

9.1.5 Binary conversion

Binary conversion is similar to the truncation technique where values below the threshold are put to zero. Any values higher than the threshold are replaced by one. As a result, a binary black (0) and white (1) image is obtained as shown in Figure 9.6b. Binary format images are more favourable for particle tracking as they help simplify the tracking input parameters and reduce the tracking time.

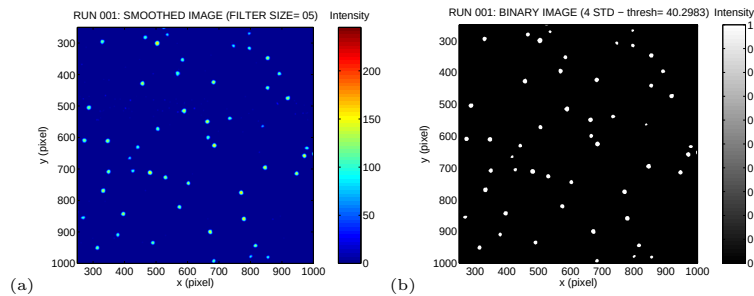


Figure 9.6: Binary conversion process: (a) image before binary conversion and (b) binary format image

Typical image processing procedures and their values used to post-process raw PTV output data are summarised in Table 9.1

Table 9.1: Summary of image processing procedures for different campaigns

Experiment	Image processing procedures	Typical values used	
Campaign I PTV May 2005	1. Background subtraction (Background constructed by averaging along rows)	threshold	= 1.07-1.09
	2. Smoothing/Filtering	window size	= 5
	3. Truncation	STD threshold	= 2.5
	4. Smoothing/Filtering	window size	= 5
	5. Binary conversion	STD threshold	= 3-4
Campaign II PTV Dec 2005	1. Smoothing/Filtering	window size	= 3
	2. Background subtraction (Background image taken from the experiment)	threshold	= 1.12-1.15
	3. Smoothing/Filtering	window size	= 3
	4. Truncation	STD threshold	= 2-3
	5. Smoothing/Filtering	window size	= 5
	6. Binary conversion	STD threshold	= 0.5-2.5
Campaign III PTV Mar 2006	1. Smoothing/Filtering	window size	= 5
	2. Background subtraction (Background image taken from the experiment)	threshold	= 1.12-1.25
	3. Smoothing/Filtering	window size	= 3-5
	4. Truncation	STD threshold	= 1-2
	5. Smoothing/Filtering	window size	= 3
	6. Binary conversion	STD threshold	= 1-2

9.2 Stagnation point identification

There are various methods of locating the stagnation points which are also referred to as saddle or critical points in a flow. The Newton-Raphson method, for instance, is widely used. However, it requires a good guess of the starting point in order for it to converge to the solution. In this work, we determine the positions of the stagnation points using two methods, 1) using the flow visualisation software and 2) linear transformation of the velocity field (Diakonos & Schmelcher 1998).

In the first method, the positions of the stagnation points are determined manually by plotting flow streamlines using a flow visualisation software. We then identify the zero velocity points in the flow where streamlines cannot reach such regions. Streamlines either loop around the elliptic stagnation points or disperse away from the hyperbolic stagnation points. We use Tecplot software, which has a built-in function that can integrate streamlines in a flow, to plot the streamlines and manually locate the stagnation points in Section 5.4.2. Tecplot software integrates streamlines using a second-order Runge-Kutta integration of the velocity field. The integrating step size is set to a small value of 0.005 s. This method serves the purpose well when only a few stagnation points are to be identified as in the case in Section 5.4.2.

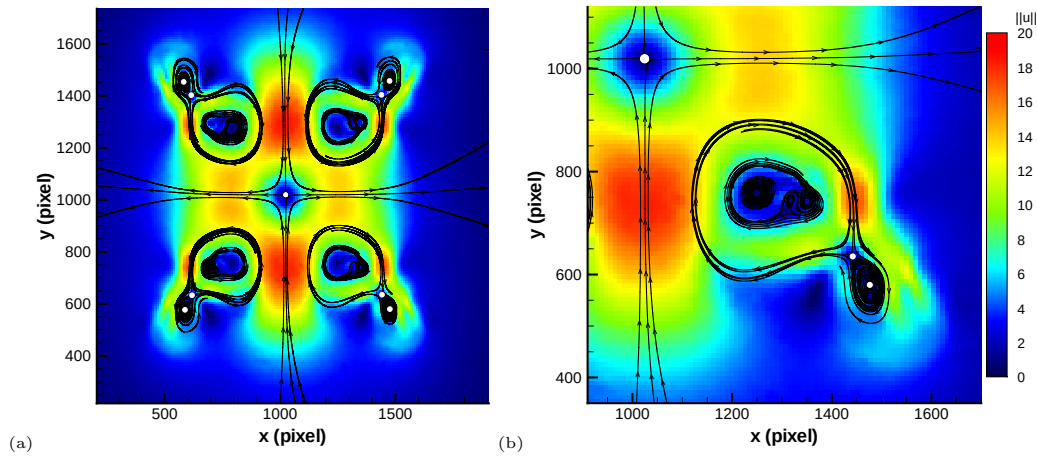


Figure 9.7: Stagnation point identification by manual plotting of streamlines. Large (M160) and medium scale (M40) stagnation point positions are shown in white dots: (a) entire flow and (b) quarter of the flow. $||u||$ is in mm/s.

The second method, which is used in Section 6.2, identifies the positions of the hyperbolic stagnation points by integrating trajectories in a reversed velocity field. By applying the linear transform ($u = -u$, $v = v$), the u -component of the velocity field is reversed (Figure 9.8). As a result, the overall flow streamlines are partially reversed and drawn towards the hyperbolic stagnation points rather than dispersing away from them as shown in (Figure 9.9(b)). The trajectories are integrated using a fourth-order Runge-Kutta method with an integrating step size of half the camera time step.

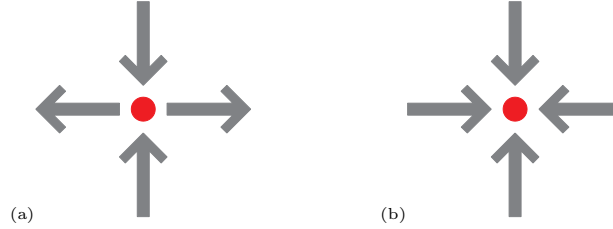


Figure 9.8: Linear transformation ($u = -u$, $v = v$). Simplified diagrams showing velocity components near a hyperbolic stagnation point: (a) original velocity component, (b) velocity component with linear transform on the u -components causing the velocity field to be partially reversed and streamlines are directed toward the hyperbolic stagnation points instead.

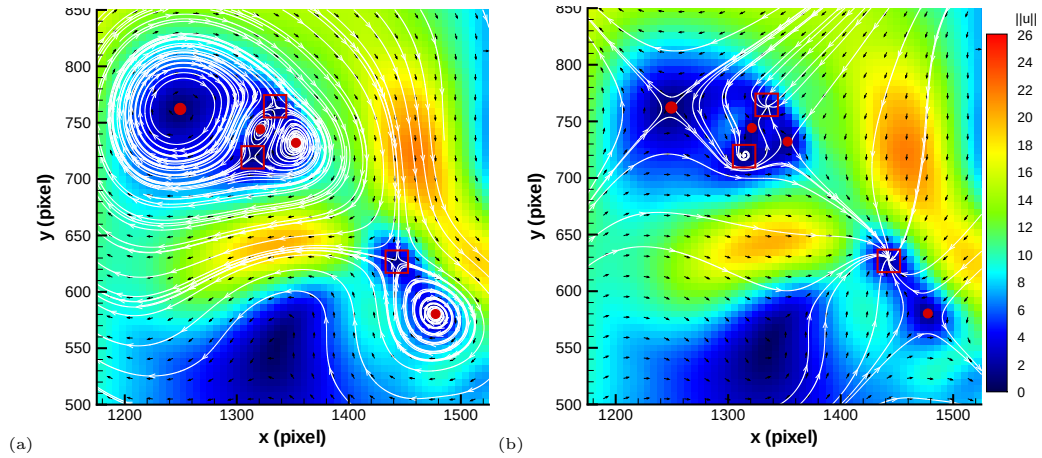


Figure 9.9: Stagnation point identification by reversing u -component method: (a) original velocity field, (b) velocity field with reversed u -components. The elliptic stagnation points are shown in red dots and the hyperbolic stagnation points are shown in red square boxes. Note that the originally elliptic stagnation points in (a) become the hyperbolic ones in the reserved velocity field (b).

9.3 Validation of virtual particle tracking

The virtual particle tracking code was validated using two approaches: 1) validated against the functions with known exact solutions and 2) validated against the previous data presented in Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a).

9.3.1 Validation against known functions

9.3.1.1 Spatial validation

To validate the particle tracking algorithm spatially, a simple constant circular field is used to verify whether the tracking algorithm returns expected values with an acceptable level of error. The analytical form of such circular field is given by the following differential equations:

$$u = \frac{\partial x}{\partial t} = -y, \quad v = \frac{\partial y}{\partial t} = x$$

and the solution is given by:

$$x = -r \sin t, \quad y = r \cos t,$$

where $x^2 + y^2 = r^2$, which is the equation of a circle with radius $r = \sqrt{x^2 + y^2}$.

In the validation, the circular field used is scaled to $0.3u = -y$ and $0.3v = x$ to be compatible to the same order as u_{max} and u_{rms} in our experiment. Hence, the analytical expression for the trajectory path is $x = -r \sin(0.3t)$ and $y = r \cos(0.3t)$ respectively.

We track the trajectory starting from the initial position $x_0 = 0$, $y_0 = 10$. Thus, the expected trajectory path should follow $x^2 + y^2 = 10^2$. Note, however, that it is very difficult to achieve an approximation of a perfect circle due to the discrete nature of the data. This is because, over time, slight errors from numerical roundup can cause the approximation to diverge, resulting in a spiral feature rather than a close loop circle.

The tracking algorithm returns a circular path with radius $r \approx 10$ pixels as expected.

The numerical error per time step (Figure 9.10) is quantified as:

$$\text{error per time step} = \left(\frac{1}{N} \right) \sum_{i=1}^N ((x_{\text{track}} - x_{\text{exact}})_i^2 + (y_{\text{track}} - y_{\text{exact}})_i^2)^{0.5} \quad (9.2)$$

where i denotes the i^{th} time step and N is the total number of time step. The potential error per time step is in the order of $1 \times 10^{-4} - 2 \times 10^{-4}$ pixel, which is considered acceptable within the tracking time of $t \approx 270$ s and the trajectory loops around a circle up to 27 times.

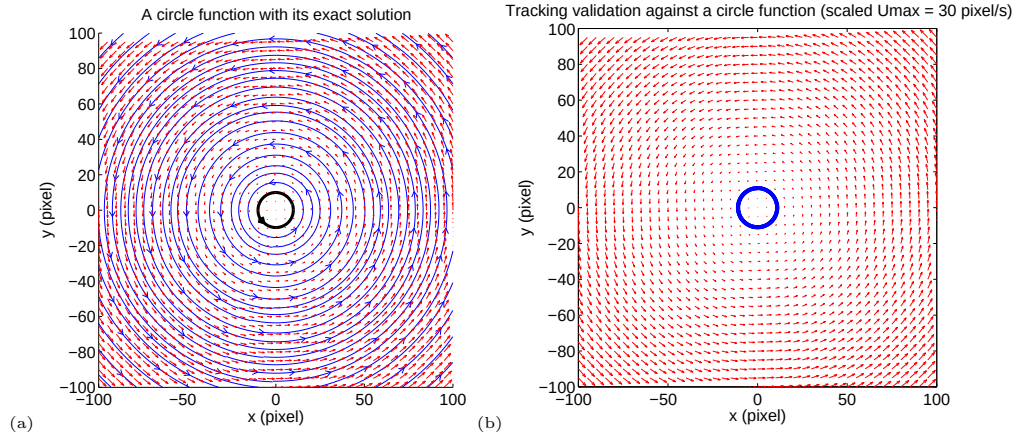


Figure 9.10: Tracking validation using a circular field: (a) Exact solution $x^2 + y^2 = 10^2$, (b) Tracked trajectory path with radius $r \approx 10$ pixels.

9.3.1.2 Temporal validation

It is also crucial to validate the temporal performance of the tracking algorithm. In this case a simple time dependent field is used where:

$$u(t) = \frac{\partial x}{\partial t} = \frac{2\pi A}{10} \cos\left(\frac{2\pi t}{10}\right), \quad v(t) = \frac{\partial y}{\partial t} = 0$$

and the solution for the trajectory path is therefore:

$$x(t) = A \sin\left(\frac{2\pi t}{10}\right) + x_0, \quad y(t) = y_0$$

with x_0 and y_0 denoting the initial position.

The amplitude A was set to 120 with the initial position was $x_0 = 150$, $y_0 = 150$

in our validation and the sampling period was $T_s = 1/24$ seconds. The trajectory path returned by the tracking algorithm is as expected where:

$$x_{track}(i) = 120 \sin\left(\frac{2\pi}{10} \frac{i}{24}\right) + 150, \quad y_{track}(i) = 150$$

with i denoting the i^{th} time step. Figure 9.11 shows the plots of the tracking results $x_{track}(t)$ and $y_{track}(t)$. Note that the algorithm outputs the location once every eight time steps.

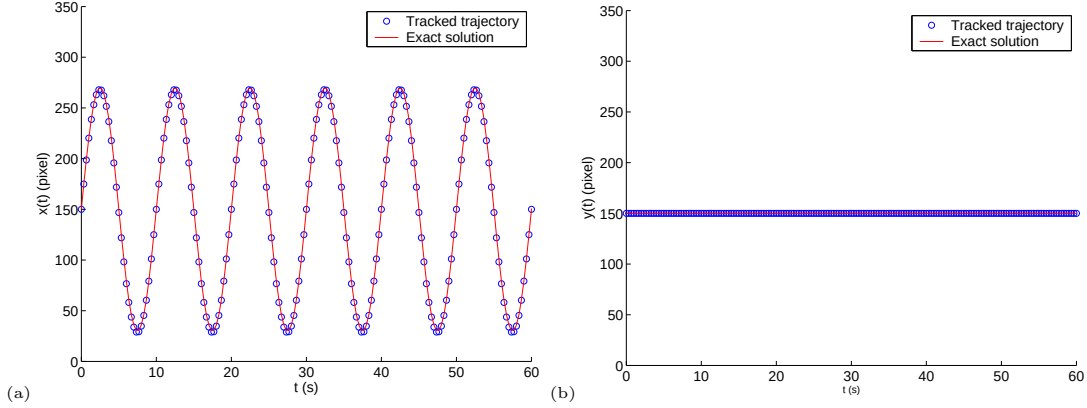


Figure 9.11: Results of tracking validation of a time dependent field where $u(t) = \frac{\partial x}{\partial t} = 120 \frac{2\pi}{10} \cos\left(\frac{2\pi t}{10}\right)$, $v(t) = \frac{\partial y}{\partial t} = 0$: (a) obtained trajectory path along $x(t) = 120 \sin\left(\frac{2\pi t}{10}\right) + 150$, (b) obtained trajectory path along $y(t) = 150$.

From the tracking result, the mean temporal error in each period is approximately 0.654% of the peak-to-peak amplitude of the validating function. The temporal error seems to be constant for all period and does not accumulate over time. The validation was performed for 270s allowing 27 sinusoidal cycles to be tested.

9.3.2 Validation against previous data

The virtual particle tracking was also performed with a steady forcing velocity field obtained by taking a single snapshot from the time dependent velocity fields of our unsteady multi-scale flow. The velocity field chosen was that of the case C5 at the instance where $I(t) \approx 0.1A$ which has the closest forcing intensity to the result presented in Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a) with $I = 0.1A$.

We integrated the velocity field using the 4th order Runge Kutta scheme with spatial

cubic Lagrange interpolation until $t \approx 35T_L$ (270s). We tracked 1,500,000 uniformly random distributed particles with initial pair separation, $\Delta_0 = 1\text{pixel}$ and the integration time step, $t_{RK4} = 1/24\text{s}$. Only 1,190,523 particles enter the central $800 \times 800 \text{ mm}^2$ region and taken into the statistics of the particle dispersion.

The result of the VPT using a single snapshot of the velocity field from the unsteady multi-scale flow provides the fluid pair dispersion statistics that is in good agreement and consistent with the steady result in Rossi et al. (Rossi, Vassilicos, & Hardalupas 2006a) as shown in Figure 9.12.

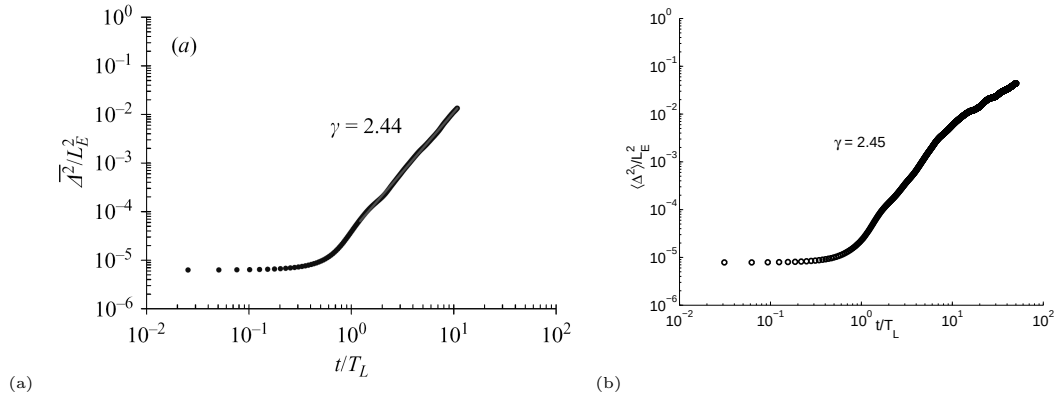


Figure 9.12: Fluid pair dispersion statistics of: (a) a steady multi-scale flow with $I = 0.1A$ taken from Rossi et al. (2006a) and (b) a steady flow with its velocity field taken from a single snapshot of an unsteady multi-scale flow at the instance where $I(t) \approx 0.1A$.

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