

APPLICATIONS OF DIGITAL ELECTRONIC COMPUTERS
TO GRAVITY AND MAGNETIC INTERPRETATION

Thesis
submitted by

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ABSTRACT

Use has been made of a Digital Computer to develop a routine method of interpreting magnetic anomalies due to dykes and to square vertical prisms of infinite depth extent. In addition, a method is described for interpreting magnetic anomalies due to any three dimensional parallelepiped with horizontal top surface, and of infinite depth extent by reducing them to two two-dimensional dyke anomalies.

A computer method of fitting two dimensional and three dimensional rectangular vertical prisms to observed total field anomalies is developed. The method is later extended to the fitting of any two dimensional structure consisting of a number of two dimensional rectangular vertical prisms to observed total field anomalies. The principle of the method of fitting is easily extended to the fitting of other models and to the fitting of models and structures to observed gravity anomalies.

A computer method is described for calculating the total field anomalies due to a three dimensional structure for any assumed direction and intensity of magnetisation, and for determining the direction and intensity of magnetisation of a structure of known shape at a known depth from its total field

anomalies. The method is applied to investigate the nature of magnetisation of a seamount off the West Coast of the United States.

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CHAPTER 1

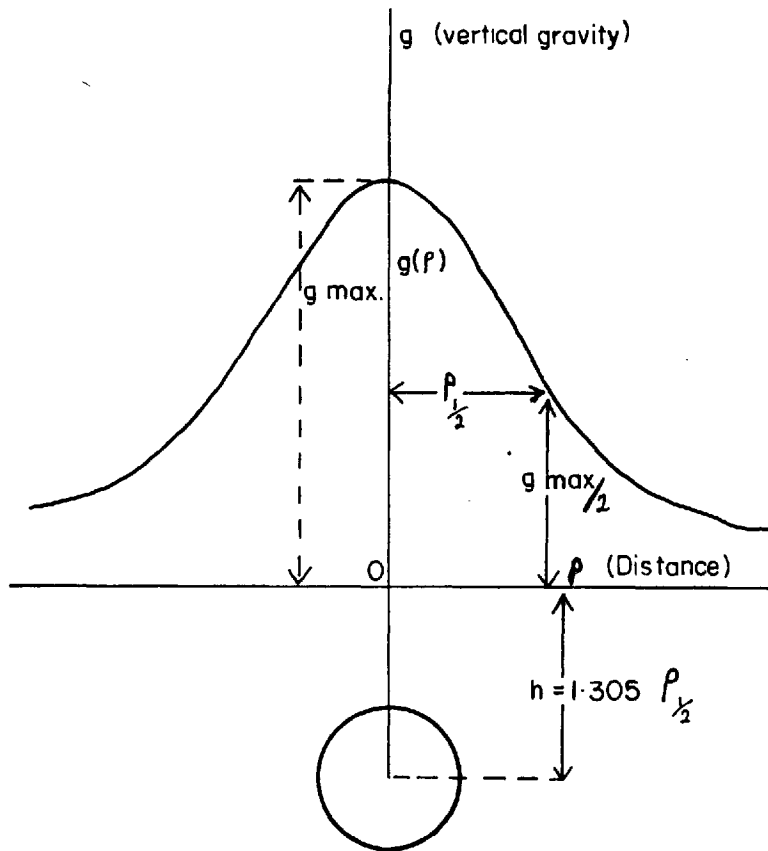
INTRODUCTION

The basic problem in gravity or magnetic interpretation is the deduction of geological information concerning the source of a gravity or magnetic anomaly. In the case of gravity surveys, the information for interpretation is usually in the form of a sufficient number of values of g , measured at a number of discrete points, to enable a gravity contour map to be made. In the case of magnetic surveys, it may be in the form of discrete values of magnetic field, or more commonly in the form of continuous profiles obtained from continuously recording airborne or ship-towed magnetometers. In either case the results are generally presented in the form of a map.

The first step in interpretation, irrespective of the quantity and quality of the field data, is a visual inspection of the map or profiles. Deductions and inferences are drawn in a qualitative manner from the sizes and configuration of the anomalies. This is based in part on a knowledge of the theoretical anomalies produced by certain types of structures, in part on the experience of the interpreter. The subsequent steps in the interpretation depend on a

number of factors, such as the quantity and quality of the data and the amount of effort that the economics of the situation justify. Most of the methods of interpretation in common use fall into one or other of the following categories. (1) Depth determinations based on simple half width or similar rules, (2) master curve matching and (3) structure fitting.

In the case of gravity surveys, the interpretation of anomalies characterised by nearly circular contours may be approached on the assumption of a spherical source, while for those with elongated contours a horizontal cylinder may be assumed. For these simple geometrical forms, direct relations can be ascertained between the half widths of the anomalies and the depths to their centre. Thus, the depth h to the centre of a sphere is obtained from the half width $P_{\frac{1}{2}}$ of its vertical gravity anomaly using the relation $h = 1.305 P_{\frac{1}{2}}$ (Fig. 1.1). For the cylinder $h = x_{\frac{1}{2}}$ where $x_{\frac{1}{2}}$ is the half width of a profile on the gravity anomaly perpendicular to its axis. The corresponding elementary approximations in magnetic interpretation are the point pole and point dipole for nearly circular contours and the line pole and line dipole for elongated contours. Depth rules similar to those for gravity interpretation can be derived for these models. The rules of course depend on the



Determining depth of sphere from half width $p_{1/2}$ on gravity profile

Fig. 1-1

direction along which the anomaly is measured. For total field anomalies, Smellie (1) has given charts from which factors can be read and multiplied by the maximum to half-maximum distance on the anomaly to yield the depth to the equivalent point pole, point dipole, line pole and line dipole. Jung (2,3) has developed depth rules for other simple geometrical forms, such as horizontal and dipping sheets, and has given some geometrical construction for rapid depth determination. As a rule such simple rules can be developed only for the simplest of models.

The simple methods described above suffer from two disadvantages, they make use of a limited number of discrete values and they are suitable only for very simple models. For more complicated models, a better approach is to compare the observed anomaly with master curves computed for various combinations of the parameters of the model. The preparation of these curves is made possible by the fact that for any particular body, the expression for its anomaly can be separated into an amplitude factor and a shape factor, which is independent of the absolute dimensions of the system. Consider, for example, the total field magnetic anomaly $T(x, y, 0)$ due to an arbitrary structure polarised along the direction of the Earth's field.

$$T(x,y,0) = J \iiint_V \left(l \frac{\partial}{\partial a} + m \frac{\partial}{\partial b} + n \frac{\partial}{\partial c} \right)^2 x$$

$$\frac{da \, db \, dc}{((x-a)^2 + (y-b)^2 + c^2)^{\frac{1}{2}}}$$

where J is the intensity of magnetisation, l, m, n the direction cosines of the Earth's field and a, b, c are the co-ordinates of a general point in the prism. This expression can be written in the form

$$T(x,y,0) = J \iiint_{V/h^3} \left(l \frac{\partial}{\partial \left(\frac{a}{h}\right)} + m \frac{\partial}{\partial \left(\frac{b}{h}\right)} + n \frac{\partial}{\partial \left(\frac{c}{h}\right)} \right)^2 x$$

$$\left[\left\{ \frac{x}{h} - \frac{a}{h} \right\}^2 + \left\{ \frac{y}{h} - \frac{b}{h} \right\}^2 + \left\{ \frac{c}{h} \right\}^2 \right]^{-\frac{1}{2}}$$

$$d\left(\frac{a}{h}\right) d\left(\frac{b}{h}\right) d\left(\frac{c}{h}\right)$$

$$= J f\left(\frac{x}{h}, \frac{y}{h}, \frac{p_j}{h}, i\right),$$

where h is a scaling factor, i the inclination of the Earth's field and p_j are the parameters defining the source. The function $f(x/h, y/h, p_j/h, i)$ defining the shape of the anomaly is independent of the absolute dimensions of the system, and depends for a given inclination i only on its geometry. The procedure actually employed in the interpretation, using any model, is to have a set of charts of f precomputed for various combinations of the parameters p_j , and to compare the observed anomaly with these charts.

A particular chart is then selected as showing the closest agreement in shape with the observed anomaly. Then if P_j are the parameters of the structure and p_j those of the model for which this chart was prepared, $P_j = H p_j$, where H is the ratio of the distance between any two convenient points on the observed anomaly to the distance between the corresponding points on the chart. The points usually used are the maximum, minimum, half quarter etc. of maximum and minimum. The amplitude factor which in the above case is J is obtained by taking the ratio of the maximum on the observed anomaly to the maximum of f on the chart.

As an example of the application of such charts, consider the interpretation of magnetic anomalies due to dykes. It can be shown that the shape of the magnetic anomaly due to a dyke depends only on two parameters, t/h and I , where t is the thickness of the dyke, h the depth to its top and I an angle depending on the direction of measurement of the field, the direction of magnetisation, and the dip and strike of the dyke. The procedure adopted in the interpretation is as outlined before, first computing master curves for various combinations of t/h and I and then

comparing the observed anomaly with them. A curve is then chosen as showing the closest agreement in shape with the observed anomaly and the depth to the top of the dyke is obtained by multiplying the h value of the selected master curve by the ratio of the distance between two convenient points on the observed anomaly to the distance between the corresponding points on the master curve. The thickness of the dyke is obtained by multiplying the t/h ratio for which the curve was computed by the depth. As another example where extensive use has been made of such charts mention may be made of those of Vacquier ⁽⁴⁾ for interpreting total field anomalies due to vertical rectangular prisms of infinite depth extent. All methods employing master charts have severe limitations in that with increasing number of charts the comparison of the observed anomaly with the charts becomes increasingly difficult and time consuming and with only a limited number of charts only crude estimates of the parameters can be made. In Chapters 2 and 3 of this thesis methods are developed which allow the total field anomalies due to dykes and vertical rectangular prisms to be interpreted in a simple and routine fashion.

The ideal in gravity or magnetic interpretation would be to determine from the gravity or magnetic anomaly the depth,

shape, size and density or susceptibility of the source producing it. Unfortunately, this is not possible because a given anomaly could be produced by anyone of an infinite number of possible sources^(5,6). Additional control information, such as might be obtained from drilling or from seismic surveys, is necessary if limitations are to be imposed on the number of solutions. Where such control data are available and are sufficient to make the solution unique or to limit the range of plausible solutions it should be possible to find a solution that is compatible with the available control data. The problem is usually approached in an indirect manner. From the results of preliminary analysis the assumption is made that the source has a certain shape depth and density, or susceptibility. The anomaly due to this body is calculated and the results of such calculations are compared with the observed anomaly. The assumed body is then adjusted by trial and error, until a reasonable agreement between the observed and computed anomalies is obtained. Obviously, the efficiency of such a method depends on the ability of the individual to make a close estimate of the solution and the speed with which the anomaly due to the structure is computed. A number of graphical⁽⁷⁾, mechanical⁽⁸⁾, and analogue⁽⁹⁾ methods have been designed for the rapid

computation of the anomaly. However, the method of fitting remained time consuming and laborious as a number of trial steps would be required in practice before the solution is achieved. For instance a case is mentioned⁽¹⁰⁾ where three months of steady calculations of this sort were required before a satisfactory picture of a salt dome was obtained.

The time factor and the labour of computation precluded any appreciable use of the above interpretation technique in the past. With the advent of the modern high speed digital computer, the situation is different. The ability of the computer to perform a series of simple numerical operations automatically and rapidly makes it possible to compute the anomaly due to an arbitrary structure very rapidly by dividing it into many elements and summing up their effects. Further, its ability to perform a cycle of operations repeatedly until a specific condition is reached, enables successive approximation methods to be developed for the fitting of a structure to an observed anomaly. This may be illustrated by considering a case described by Bott⁽¹¹⁾ where the electronic computer was used to determine the thickness of a sedimentary basin from the vertical gravity anomaly. The basin was divided into a number of strips (Fig. 1.2) and the thickness of the

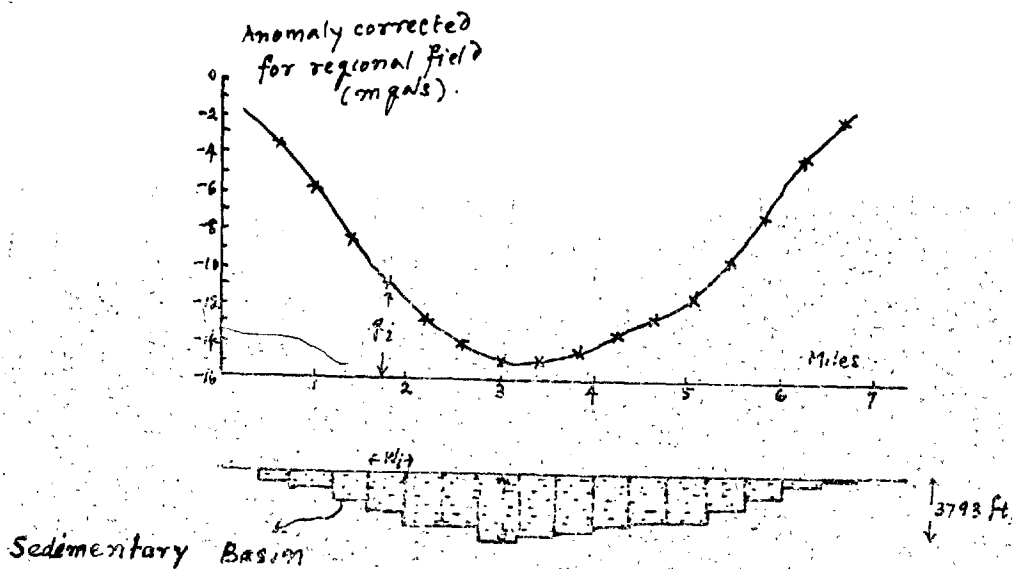


FIG. 1.2.

The interpretation of a gravity profile across the basin of the New Red Sandstone basin using a Digital Computer, for an assumed density contrast of 0.4 gm/cm^3 (After Bott).

sediments below each strip calculated. The first approximation to the thicknesses was obtained on the assumption that the observed anomaly was due to sheets at the surface. If g_i is the observed gravity anomaly over the i^{th} strip, the first approximation to the thickness of the i^{th} strip is $t_i = \frac{g_i}{2\pi G \sigma_i}$ where σ is the density contrast between the sediments and the surroundings, ~~w_i the width of the i^{th} strip~~ and G the gravitational constant. The anomaly due to the structure, consisting of blocks of thickness t_i , was calculated and subtracted from the observed anomaly. This results in a set of residuals R_{1i} which are smaller than the observed anomaly. In the next stage the thicknesses of the blocks were increased by amounts $t_i = \frac{R_{1i}}{2\pi G \sigma_i}$ and the anomaly due to the structure was recalculated and subtracted from the observed anomaly to leave a set of residuals R_{2i} which are smaller than R_{1i} . The process was repeated until the residuals were reduced to a sufficiently low value. Thus, each cycle of operations leads to a closer approximation. To go through this numerical process by any other means would be a most tedious proposition, yet the computer solved the problem in under three minutes.

Bott's method was developed for the solution of a specific problem. The method cannot be extended to the

general case, nor is it possible to extend the method to structure fitting in magnetic interpretation. In this thesis, a general technique is developed which allows both simple models and complex structures to be fitted to an observed anomaly. The method is illustrated with respect to its application to the interpretation of total field magnetic anomalies but the principle of the method is easily extended to the interpretation of gravity anomalies.

CHAPTER 2

ROUTINE METHODS FOR INTERPRETING THE MAGNETIC ANOMALIES DUE TO DYKES AND SQUARE VERTICAL PRISMS.

2.1 Introduction

A common method of interpreting a magnetic anomaly is to assume a simple model for the source producing it, and to evaluate the parameters of the model by comparing some characteristics of the observed anomaly with the corresponding and known characteristics of the anomaly due to the model. The characteristics generally used are the distances between convenient points on the anomaly, such as the maximum, minimum, half maximum and half minimum. A direct location of these points was possible in the past only for the simplest of models. For more complicated models, charts of anomalies due to the model for various values of its parameters were prepared and the interpretation was carried out by comparing the observed anomaly with these charts and selecting the one which showed the closest agreement between the shapes of the observed and computed anomalies. This was a time consuming process and as a result only crude estimates of the parameters were made using an inadequate number of charts. With the advent of

modern High Speed Digital Computers, it becomes possible to determine the points of maximum, minimum, half maximum, half minimum, etc. on the anomalies due to more complicated models and thus to develop a more direct method of interpreting their anomalies. Such a direct method has been developed in the present work for the interpretation of magnetic anomalies which may be assumed to be due to a two dimensional dyke and to a square vertical prism of infinite depth extent. In addition, a technique has also been developed for reducing the anomalies due to any three dimensional parallelopiped with rectangular horizontal top surface and of large depth extent, to two dimensional dyke anomalies. This enables the use of the method devised for the two dimensional dyke in the interpretation of the anomalies due to the parallelopipeds.

2.2 The Two Dimensional Dyke

A dyke can be approximated by a two dimensional parallelopiped with horizontal top surface and of infinite depth extent. This approximation is also applicable to other geological structures such as veins and contacts.

On a total magnetic anomaly map, the anomalies which are suitable for interpretation on a two dimensional parallelopiped approximation, reveal themselves as closed elongated

pattern of contours of a major feature, accompanied, in general, by a single minor feature of opposite sign running parallel to the major feature. Such a simple picture may be masked by the presence of strong regional gradients, in which case a correction for the regional gradient must be applied before the characteristic feature of the anomaly is revealed. When the length of the pattern is greater than four or five times its width, a profile perpendicular to the length near the centre of the pattern may be selected for interpretation on the assumption that it is due to a two dimensional dyke. The interpretation consists of determining from this profile, the depth to the top, the width, the geological dip and the intensity of magnetisation of the dyke, it being understood that the direction of orientation of the dyke is parallel to the direction of elongation of the contours. In the following sections, an expression for the anomaly due to a dyke is derived and the relative positions of the maximum, minimum, half, quarter and three quarter maximum and minimum are determined for a wide range of values of the parameters of the system. These results are later used to develop a method for determining the parameters of the dyke using the known distances between the points of maximum, minimum etc. on the observed profile.

2.21 Magnetic anomaly due to a two dimensional dyke

The anomaly due to a uniformly magnetised dyke can be evaluated easily by summing up the effects due to free poles on the surfaces of the dyke. Referring to Fig. 2.1, where the x-axis has been taken at right angles to the strike of the dyke, the horizontal and vertical magnetic fields due to a free pole distribution of μ_i per unit length on one surface are given by

$$H_i = - 2 \mu_i \int_{S_i}^{S_{i+1}} \frac{ds}{R} \cos \theta$$

$$\text{and } Z_i = - 2 \mu_i \int_{S_i}^{S_{i+1}} \frac{ds}{R} \sin \theta$$

$$\text{since } ds = - \frac{R d\theta}{\sin (\theta - D_i)},$$

$$H_i = 2 \mu_i \int_{\theta_i}^{\theta_{i+1}} \frac{\cos \theta d\theta}{\sin (\theta - D_i)}$$

$$\text{and } Z_i = 2 \mu_i \int_{\theta_i}^{\theta_{i+1}} \frac{\sin \theta d\theta}{\sin (\theta - D_i)}$$

Hence, it follows that,

$$\begin{aligned}
 H_i \cos D_i + Z_i \sin D_i &= \\
 2 \mu_i \int_{\theta_i}^{\theta_{i+1}} \frac{\cos(\theta - D_i)}{\sin(\theta - D_i)} d\theta &= \\
 2 \mu_i \left[\ln \sin(\theta - D_i) \right]_{\theta_i}^{\theta_{i+1}} &= \\
 = -2 \mu_i \ln \frac{R_{i+1}}{R_i}, & \quad (1)
 \end{aligned}$$

Since $R \sin(\theta - D_i) = \text{constant}$,

$$\begin{aligned}
 \text{and } H_i \sin D_i - Z_i \cos D_i &= -2 \mu_i \int_{\theta_i}^{\theta_{i+1}} \frac{\sin(\theta - D_i)}{\sin(\theta - D_i)} d\theta \\
 &= -2 \mu_i (\theta_{i+1} - \theta_i) \quad (2)
 \end{aligned}$$

Thus, from (1) and (2),

$$H_i = -2 \mu_i \left[\cos D_i \ln \frac{R_{i+1}}{R_i} + \sin D_i (\theta_{i+1} - \theta_i) \right] \quad (3)$$

and

$$Z_i = -2 \mu_i \left[\sin D_i \ln \frac{R_{i+1}}{R_i} - \cos D_i (\theta_{i+1} - \theta_i) \right] \quad (4)$$

If (p, r) are the direction cosines with respect to the x, z axis of the direction of magnetisation, and J, the intensity

of magnetisation, then

$$\begin{aligned} \mu_i &= J (p \sin D_i - r \cos D_i) \\ &= J (p^2 + r^2)^{\frac{1}{2}} \sin (D_i - U) \end{aligned} \quad (5)$$

where $U = \tan^{-1} \frac{r}{p}$.

The anomaly F_i due to the i^{th} surface, along a direction whose d.C's with respect to the x, z axis are l, n, is given by,

$$\begin{aligned} F &= l H_i + n Z_i \\ &= (l^2 + n^2)^{\frac{1}{2}} (H_i \cos V + Z_i \sin V) \\ &= -2J' \sin (D_i - U) \left[\cos (D_i - V) \ln \frac{R_{i+1}}{R_i} + \sin (D_i - V) (\theta_{i+1} - \theta_i) \right], \end{aligned}$$

where

$$J' = J (l^2 + n^2)^{\frac{1}{2}} (p^2 + r^2)^{\frac{1}{2}} \quad \text{and} \quad V = \tan^{-1} \frac{n}{l}.$$

The anomaly F due to the flat topped dyke shown in Fig. 2.2 is equal to $\sum_{i=1}^4 F_i$

With surface 3 at infinity, all terms at infinity cancel out and

$$\begin{aligned} F &= -2J' \sin (D - U) \left[\cos (D - V) \ln \frac{R_1}{R_2} + \sin (D - V) (\theta_1 - \theta_2) \right] \\ &\quad + 2J' \sin (-U) \left[\cos (-V) \ln \frac{R_1}{R_2} + \sin (-V) (\theta_1 - \theta_2) \right] \end{aligned}$$

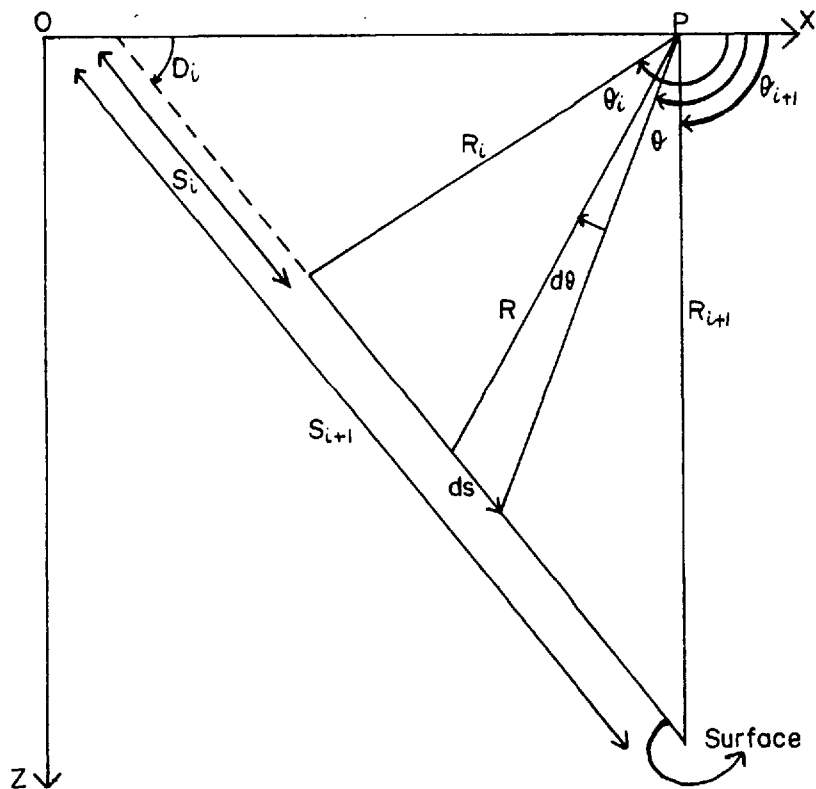


Fig. 2.1

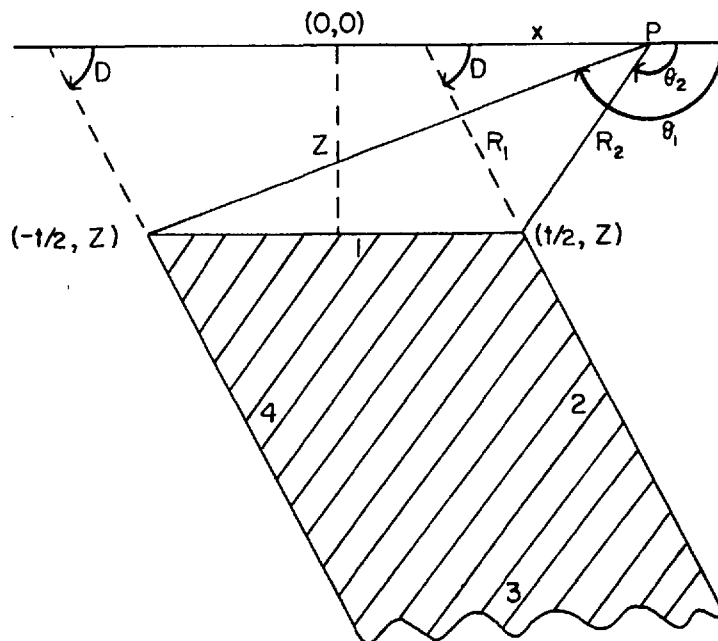


Fig. 2.2

$$= 2J' (\sin (D-U) \cos(D-V) - \sin(-U) \cos(-V)) \ln \frac{R_2}{R_1} \\ + 2J' (\sin(D-U) \sin(D-V) - \sin(-U) \sin(-V)) (\theta_2 - \theta_1)$$

Since, $\sin(D-U) \cos(D-V) - \sin(-U) \cos(-V) = \sin D \cos (D-U-V)$

and $\sin(D-U) \sin(D-V) - \sin(-U) \sin(-V) = \sin D \sin (D-U-V)$,

$$F = 2J' \sin D \left[\cos(D-U-V) \ln \frac{R_2}{R_1} + \sin (D-U-V) (\theta_2 - \theta_1) \right]$$

$$= 2J' \sin D \left[\cos I \ln \left\{ \frac{z^2 + (t' - x)^2}{z^2 + (t' + x)^2} \right\}^{\frac{1}{2}} + \right.$$

$$\left. \sin I \left\{ \tan^{-1} \frac{t' - x}{z} + \tan^{-1} \frac{t' + x}{z} \right\} \right]$$

where $t' = t/2$, the half width of the top surface of the dyke

and $I = U+V-D = \tan^{-1} \frac{p}{r} + \tan^{-1} \frac{n}{l} - D$.

The anomaly due to a dyke thus depends on the four parameters t' , z , I and J' , the latter being equal to $J(p^2 + r^2)^{\frac{1}{2}} (l^2 + n^2)^{\frac{1}{2}}$. If the function $\frac{F}{2J' \sin D}$ is

denoted by f and if all distances are measured in units of the depth z , then

$$f = \cos I \ln \left\{ \frac{1 + (t' - x)^2}{1 + (t' + x)^2} \right\}^{\frac{1}{2}} + \sin I (\tan^{-1} (t' - x) + \tan^{-1} (t' + x)) \quad (6)$$

where now t' and x are in units of the depth z . The function $f(x)$ defines the shape of the dyke anomaly, and depends only on the two parameters t' and I .

From (6), it follows that,

$$f(x, 180-I) = f(-x, I)$$

$$f(x, 180+I) = -f(x, I)$$

and $f(x, 360-I) = -f(-x, I)$

Thus, the shapes of f for I outside the range $0 \leq I \leq 90$ are obtained as the mirror images on either the f axis or the x -axis of the shapes in the range $0 \leq I \leq 90$. It is then sufficient to consider the range $0 \leq I \leq 90$ in all the following calculations.

2.22 The positions of the maximum and minimum of the anomaly.

At the maximum or minimum, $\frac{\partial f}{\partial x} = 0$.

i.e.

$$\cos I \left\{ \frac{-t'-x}{1+(t'-x)^2} - \frac{t'+x}{1+(t'+x)^2} \right\} + \sin I \left\{ \frac{1}{1+(t'-x)^2} + \frac{1}{1+(t'+x)^2} \right\} = 0$$

When multiplied throughout by $(1+(t'-x)^2)(1+(t'+x)^2)$

this equation reduces to

$$\cos I \left[-(1+(t'+x)^2)(t'-x) - (t'+x)(1+(t'-x)^2) \right] +$$

$$\sin I (-1 - (t'+x)^2 + 1 + (t'-x)^2) = 0$$

$$\text{i.e. } \cos I (x^2 - t'^2) 2t' - 2t' - 2 \sin I x 2t' = 0$$

$$\text{or } x^2 \cos I - 2 \sin I x - (t'^2 + 1) \cos I = 0$$

$$\text{Hence, } x = \tan I \pm (t'^2 + \frac{1}{\cos^2 I})^{\frac{1}{2}} \quad (7)$$

In this expression, the negative sign corresponds to the maximum and the positive sign to the minimum since, for the range $0 \leq I \leq 90$, f has the shape shown in Fig. 2.3.

2.23 The position of the centre of the top surface of the dyke

The distances of the maximum and the minimum given in (7) are measured from an origin chosen directly above the centre of the top surface of the dyke. It is evident from expression (7), that this origin is at a distance of $-\tan I$ (units of depth) from the point midway between the maximum and the minimum of the anomaly.

2.24 Numerical solution for half, quarter and three quarter maximum and minimum positions

Let f_M denote the maximum value of f , and $x_{SM/4}$ and $x'_{SM/4}$, the x co-ordinate of the points on the outside and inside limbs of the maximum at which $f = \frac{s}{4} f_M$ (Fig. 2.3). The corresponding quantities for the minimum are referred to as f_m , $x_{sm/4}$ and $x'_{sm/4}$.

The equations $f(x) = \frac{s}{4} f_M$ and $f(x) = \frac{s}{4} f_m$ were solved in the following manner.

1. The position x_M of the maximum was evaluated using expression (7) and the field f_M at this point calculated.

2. Two positions P_1 and P_2 were found where

$$f(P_2) < \frac{s}{4} f_M < f(P_1) \text{ by calculating } f(x_{(s+1)M/4} - j)$$

for $j = 1, 2, 3 \dots\dots$, until, for a particular value

$$\text{of } j, f(x_{(s+1)M/4} - j) < \frac{s}{4} f_M < f(x_{(s+1)M/4} - j + 1)$$

$x_{(s+1)M/4}$ was known from the previous calculations,

as the equations were solved in the order $s = 3, 2, 1$.

3. When two points P_1 and P_2 on either side of $\frac{s}{L} f_M$ were

found an inverse linear interpolation was carried out between $f(P_1)$ and $f(P_2)$ to obtain the first approximation Q_1 for the position of $\frac{s}{L} f_M$. (Fig. 2.4).

If $f(Q_1) < \frac{s}{L} f_M$, $f(Q_1)$ was used with $f(P_2)$ to obtain a

second approximation Q_2 by inverse linear interpolation.

If $f(Q_1) > \frac{s}{L} f_M$, $f(Q_1)$ was used with $f(P_1)$ for the same

purpose. This process of inverse linear interpolation was repeated, always using the two values of f nearest to $\frac{s}{L} f_M$ and lying on opposite sides of it, until a

position Q was reached at which
$$\left| \frac{f(Q) - \frac{s}{L} f_M}{\frac{s}{L} f_M} \right| < e$$

where e is the fractional error that is permitted in the solution of the equations. In the present work e was taken as 8×10^{-4} .

4. To obtain the values of $x'_{sm/L}$, steps (2) and (3) were repeated with j taking negative integral values from -1.
5. The positions $x'_{sm/L}$ and $x_{sm/L}$ for the minimum were obtained after inverting the anomaly by multiplying by -1 and carrying out steps (2) to (4) with x_m and f_m instead of x_M and f_M .

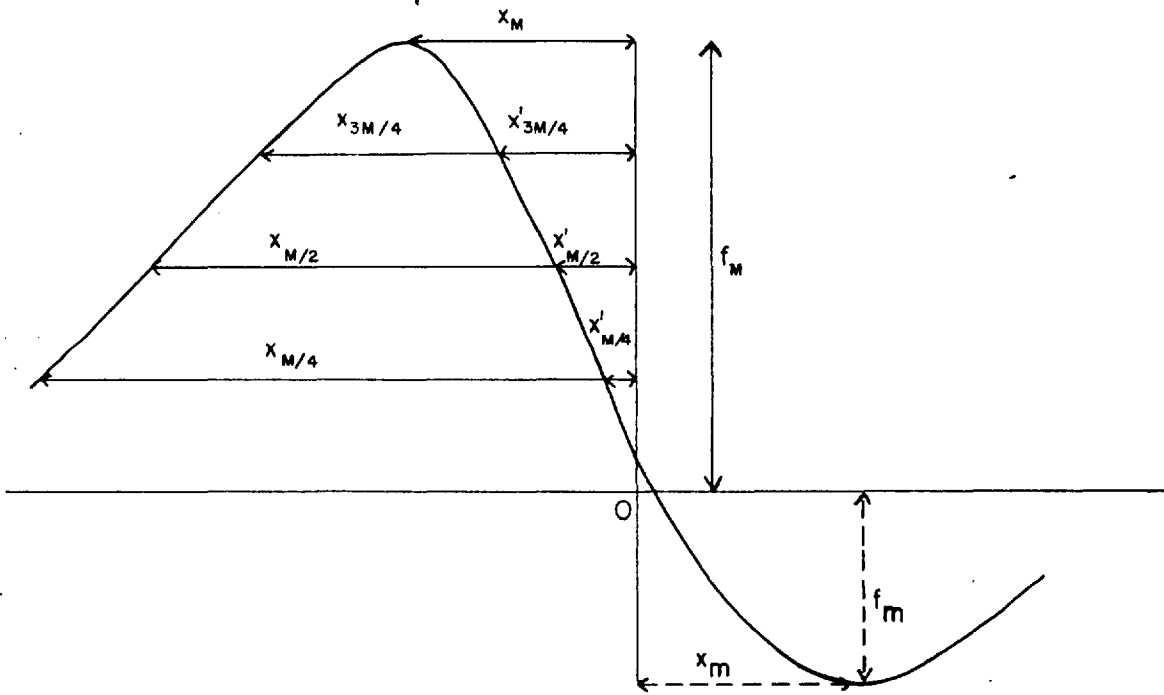


Fig. 2.3

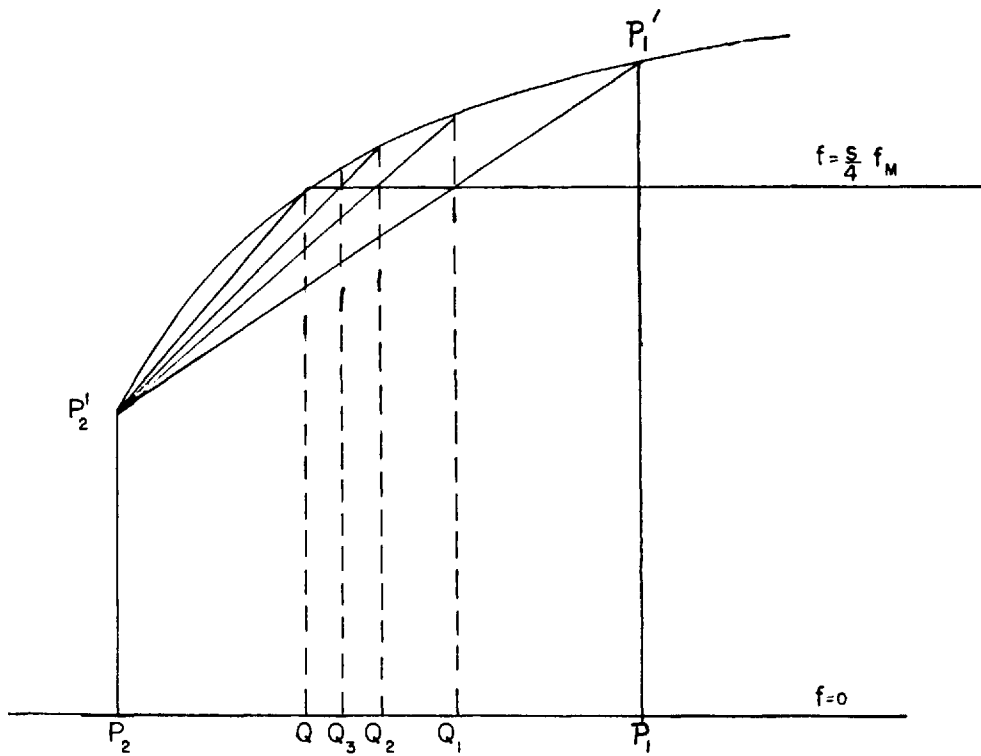


Fig. 2.4

2.25 The results of numerical calculations

The calculations were programmed for the Ferranti Mercury computer and the results corresponding to values of t from 0 to 15 at unit intervals, for each value of I from 0° to 90° in steps of 10° , were obtained. These results are given in Appendix I. In this Appendix, the values in the first and second lines corresponding to each value of t are the values of $x_{sm/4}$ and $x'_{sm/4}$ and those in the third and fourth lines (for I not equal to 0 or 90), the values of $x'_{sm/4}$ and $x_{sm/4}$. For $t = 0$, f_M is the maximum or the minimum value of f per unit thickness of a sheet i.e.

$$f_M(t = 0) = \frac{f_M(\Delta t)}{\Delta t}, \quad \Delta t \ll 1.$$

2.26 Charts for determining I , t , Z , J and D from the observed profile.

i). Determination of I and t :

From the results in Appendix I, the combinations of I and t giving a particular value for the ratio of either a pair of distances or the minimum to the maximum field can be determined and curves of constant ratios plotted on the I , t graph. By combining two sets of such curves, each set defining the ratios of different quantities, unique values for I and t can be obtained, if the two sets of curves intersect at

suitable angles. Unfortunately, it has not been found possible to obtain suitable intersections everywhere using the same two sets of curves for all values of I and t. Different sets of curves gave suitable intersections at different sections of the I, t plane and by combining all these curves, suitable intersections have been obtained for all values of I and t (Fig.2.5). The ratios used in the three sections shown in this figure are as follows.

Section A. (1)

$$\left| \frac{x_{3M/4} - x_{M/4}}{x_{M/2} - x'_{M/2}} \right|$$

(2)

$$\left| \frac{x'_{3M/4} - x'_{M/4}}{x_{M/2} - x'_{M/2}} \right|$$

Section B. (1)

$$\left| \frac{x_{3M/4} - x_{M/4}}{x_{M/2} - x'_{M/2}} \right|$$

(The same as (1) in section A).

(2)

$$\left| \frac{x_{M/2} - x_M}{x_{M/2} - x'_{M/2}} \right|$$

Section C. (1)

$$\left| \frac{f_m}{f_M} \right|$$

(2)

$$\left| \frac{x_{M/2} - x_M}{x_M - x_m} \right|$$

The curves of constant ratios were obtained in the following manner. A plot of ratio R as a function of I was first made for each value of t . Sixteen $R(I)$ curves were drawn corresponding to $t = 0, 1, \dots, 15$. The curve for a particular ratio R_1 was then obtained by reading off the values of I at which the line $R = R_1$ cuts the $R(I)$ curves and plotting these values as a function of t .

I and t can be simply determined from the observed profile using Fig. 2.5. Two ratios, appropriate to the relevant section in Fig. 2.5 are calculated from the observed profile and the point on the I, t plane corresponding to these ratios obtained by interpolation between the curves. The co-ordinates of this point are the values of I and t for the dyke which approximates the source producing the anomaly. The actual thickness of the dyke is obtained by multiplying t by the depth Z which is determined as follows.

ii. Evaluation of the depth:

Figs. 2.6 to 2.12 show various distances, in units of depth, plotted as a function of t for $I = 0$ to $I = 90$. An estimate of the depth is obtained by taking the ratio of a particular distance measured on the observed profile to the corresponding distance read from the appropriate figure, for the previously estimated values of I and t . Distances

most suitable for depth estimation are those that are little influenced by I and t , so that any error in the evaluation of I and t does not result in a substantial error in the estimation of the depth. It is evident from the figures that $x_{3M/4} - x_{M/2}$ for all I , $x_M - x_{M/2}$ for the range $0 \leq I \leq 70$ and the distances $(x_{3M/4} - x_{M/4})$ and $(x'_{3M/4} - x_{M/4})$ in the range $70 \leq I \leq 90$, satisfy this requirement, better than any other distances. These distances may therefore be used in preference to others for depth estimation. Other distances may be used to obtain independent checks on the depth estimate made from the above distances. If the different depth estimates agree, then the assumption that the anomaly is due to a dyke may be considered plausible.

iii. Intensity of magnetisation and geological dip.

The maximum value f_M of f is plotted in Fig. 2.13 as a function of t for $I = 0$ to $I = 90$. Since $f = \frac{F}{2J' \sin D}$

where F is the actual anomaly due to the dyke, the ratio of the maximum on the observed anomaly to the value of f_M corresponding to the calculated values of I and t , gives a value for $2J' \sin D$.

Now, $J' = J(1+n^2)^{\frac{1}{2}} (p^2+r^2)^{\frac{1}{2}}$, where J is the intensity of magnetisation, p , r are the d.c.s of the direction of

magnetisation and l, n those of the direction along which the anomaly is measured. J and D can be evaluated independently only if the direction of magnetisation is known. In particular, if the magnetisation is along the direction of the earth's field, and if the measured anomaly is the total anomaly, $p = l$ and $r = n$.

Thus,

$$D = I + \tan^{-1} \frac{r}{p} + \tan^{-1} \frac{n}{l}$$

$$= I + 2 \tan^{-1} \frac{n}{l}$$

and $J = \frac{1}{(p^2+r^2)(l^2+n^2)^{\frac{1}{2}}} \cdot J'$

$$= \frac{1}{l^2+n^2} \cdot \frac{1}{2 \sin D} \left| \frac{f_{\max}(\text{observed})}{f_M(I, t)} \right|$$

Let i be the inclination of the earth's field and s the angle made by the magnetic meridian with the strike direction.

Then, $n = \sin i$

$$l = \cos i \sin s$$

Therefore, $D = I + 2 \tan^{-1} \left(\frac{\tan i}{\sin s} \right)$

and $J = \frac{1}{2(1 - \cos^2 i \cos^2 s)} \cdot \frac{1}{\sin D} \cdot \frac{f_{\max}(\text{observed})}{f_M(I, t)} \left| \right|$

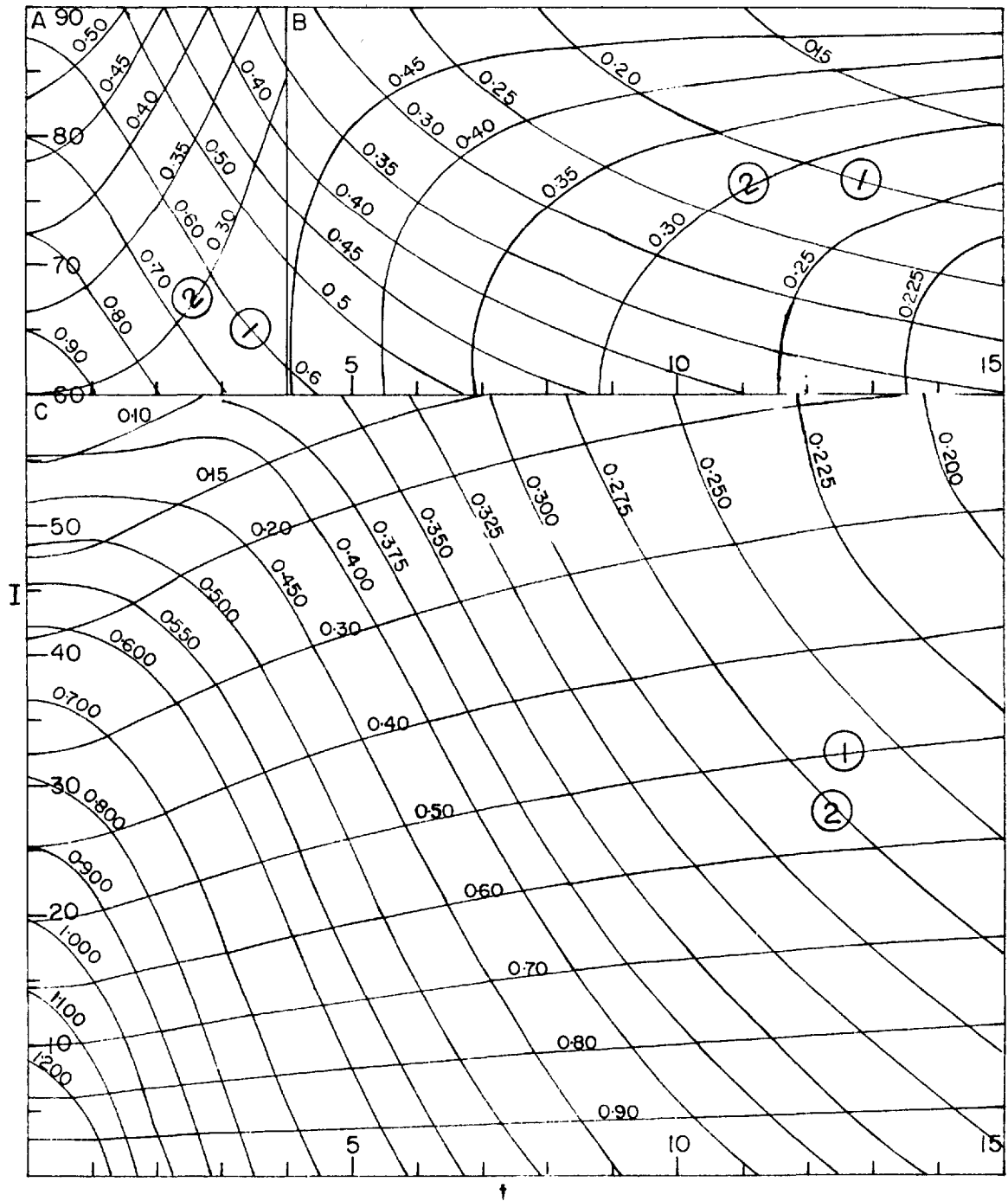


FIG 2.5

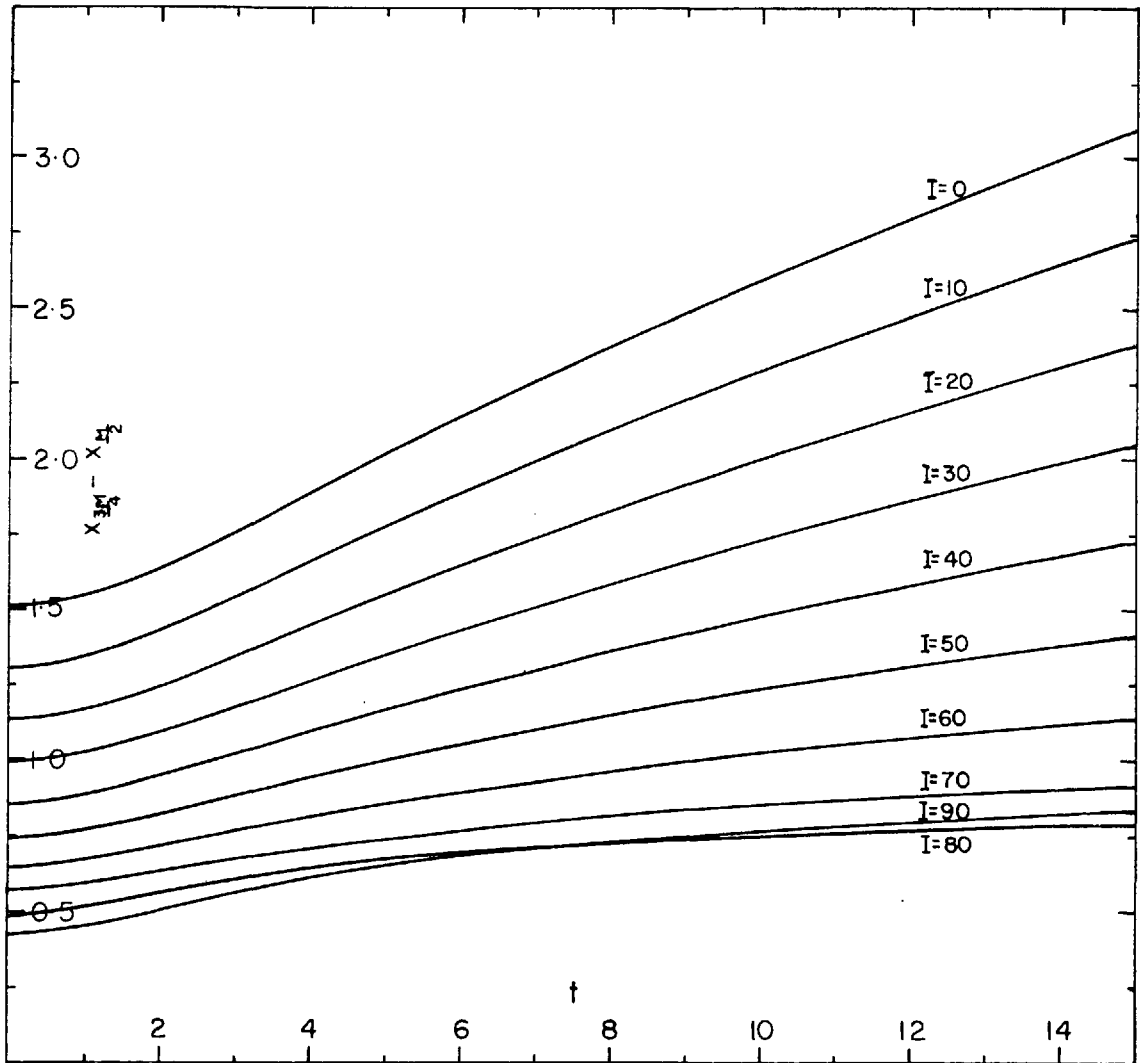


Fig. 2.6

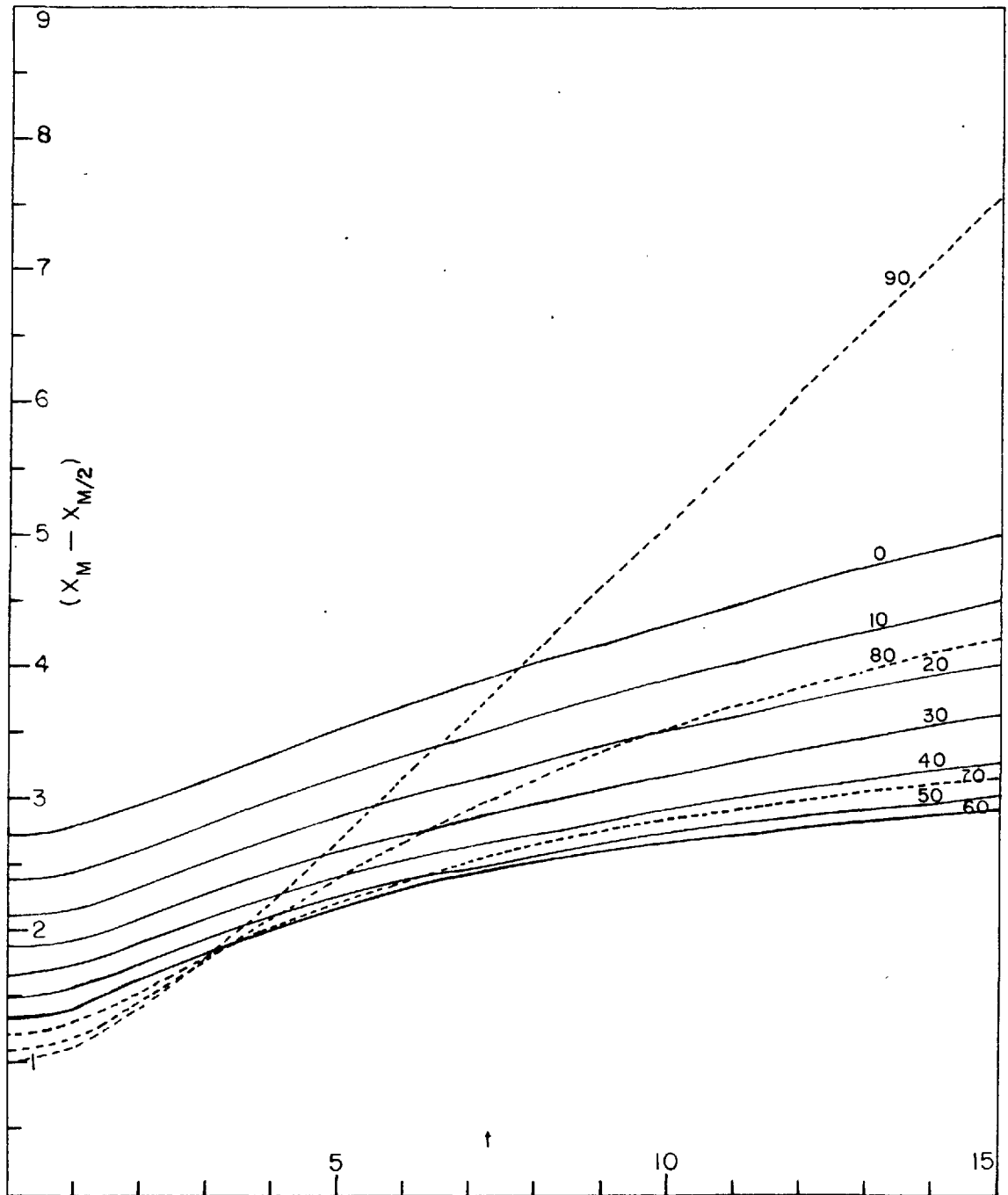


Fig. 2.7

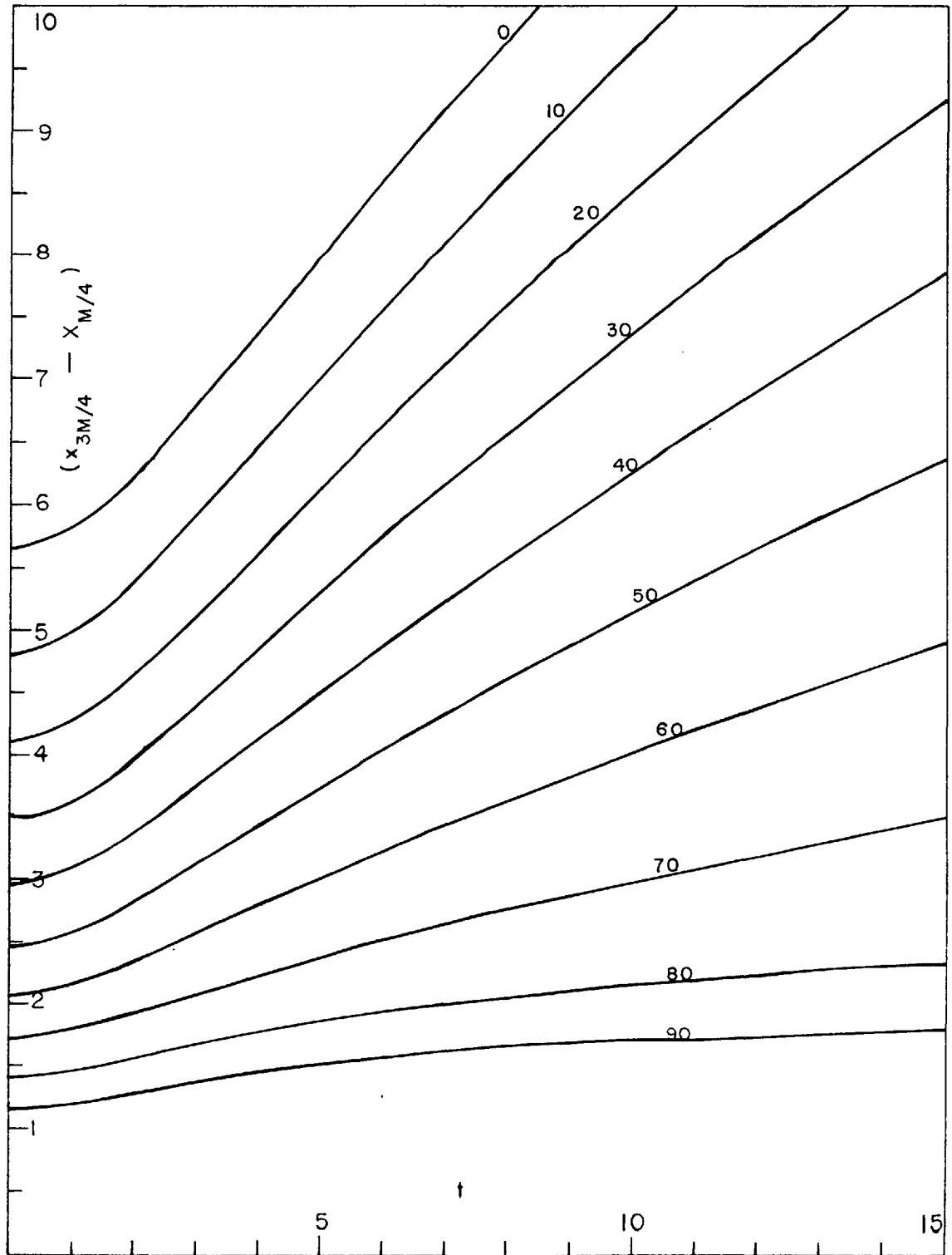


Fig. 2-8

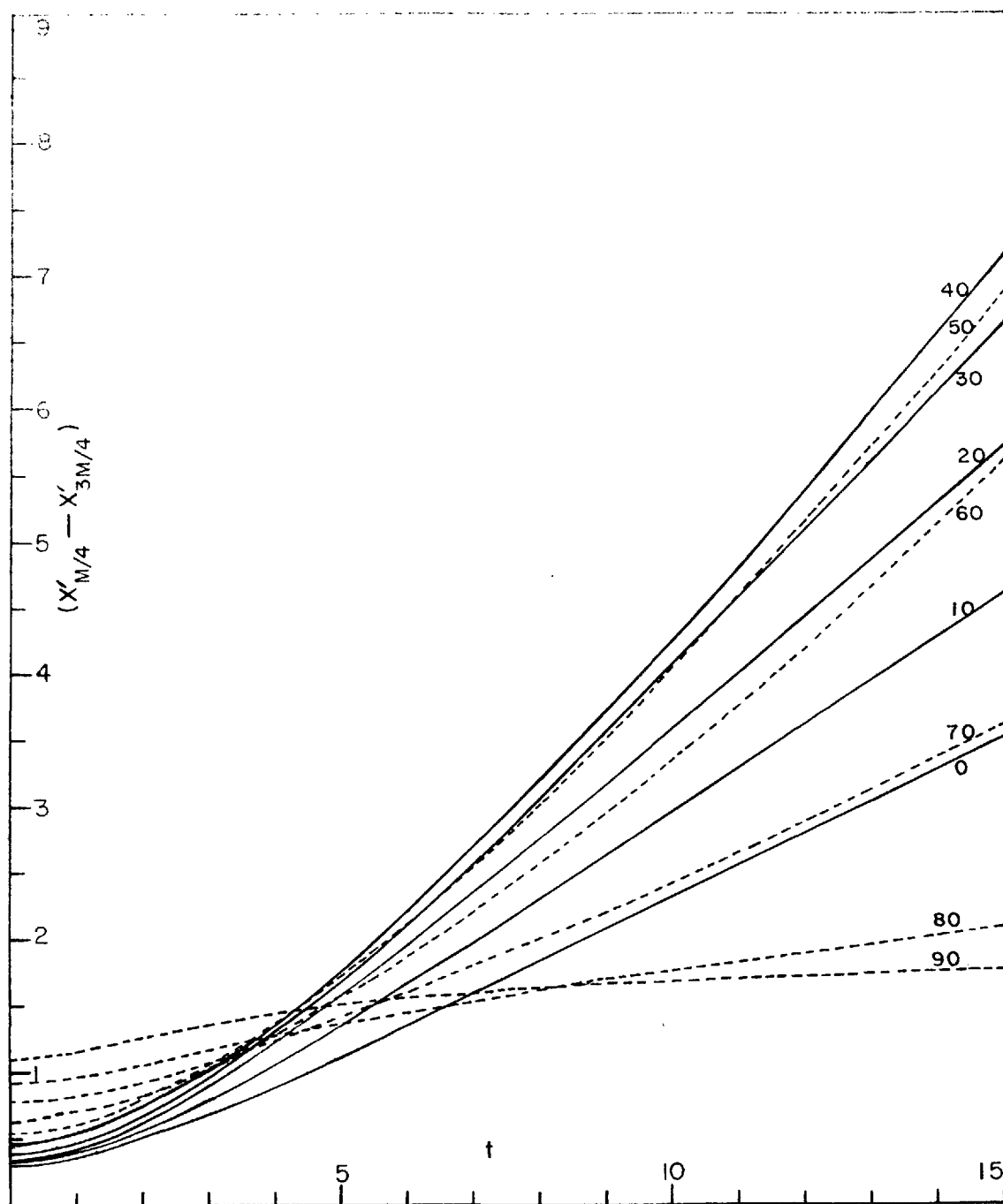


Fig. 2.9

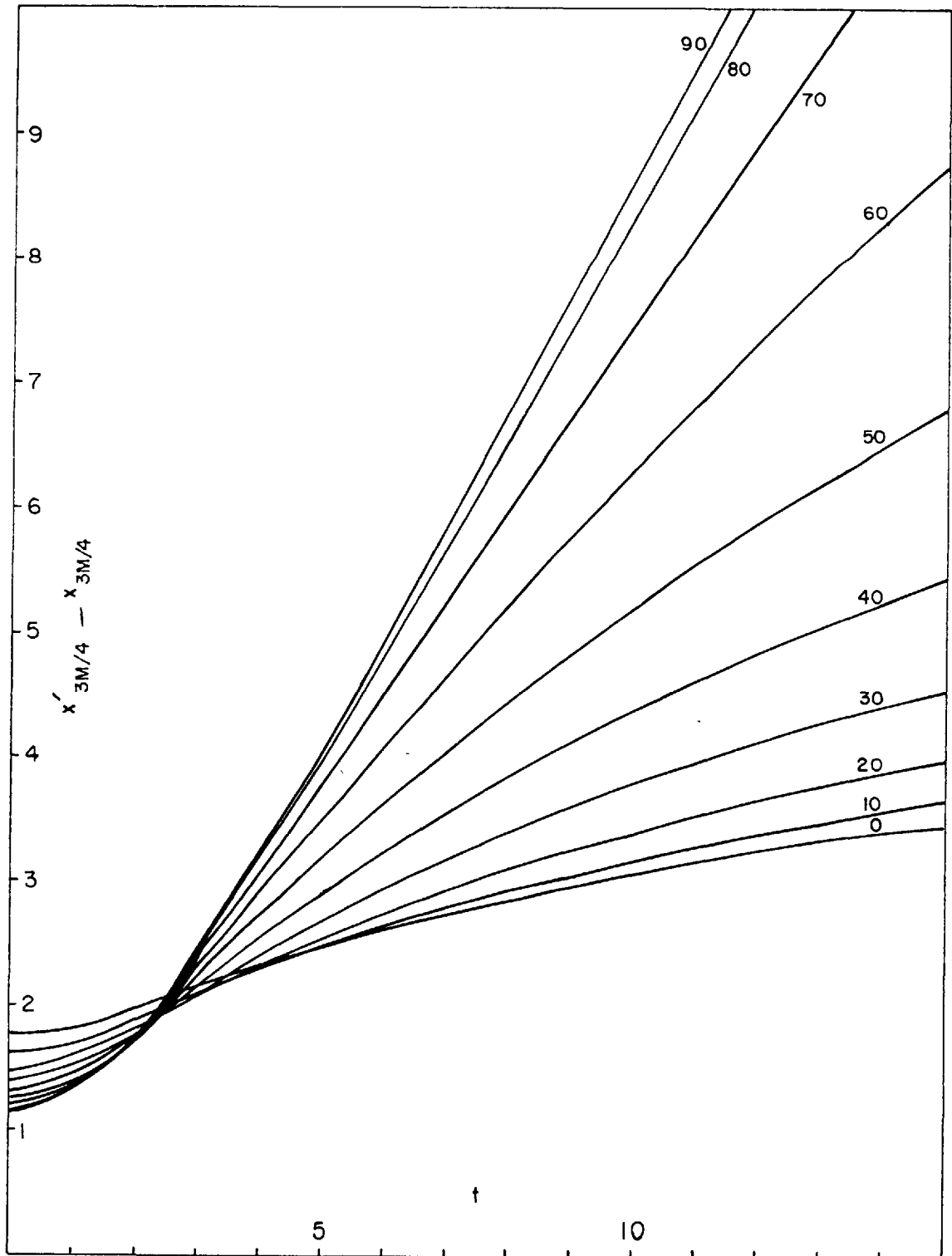


Fig. 2-10

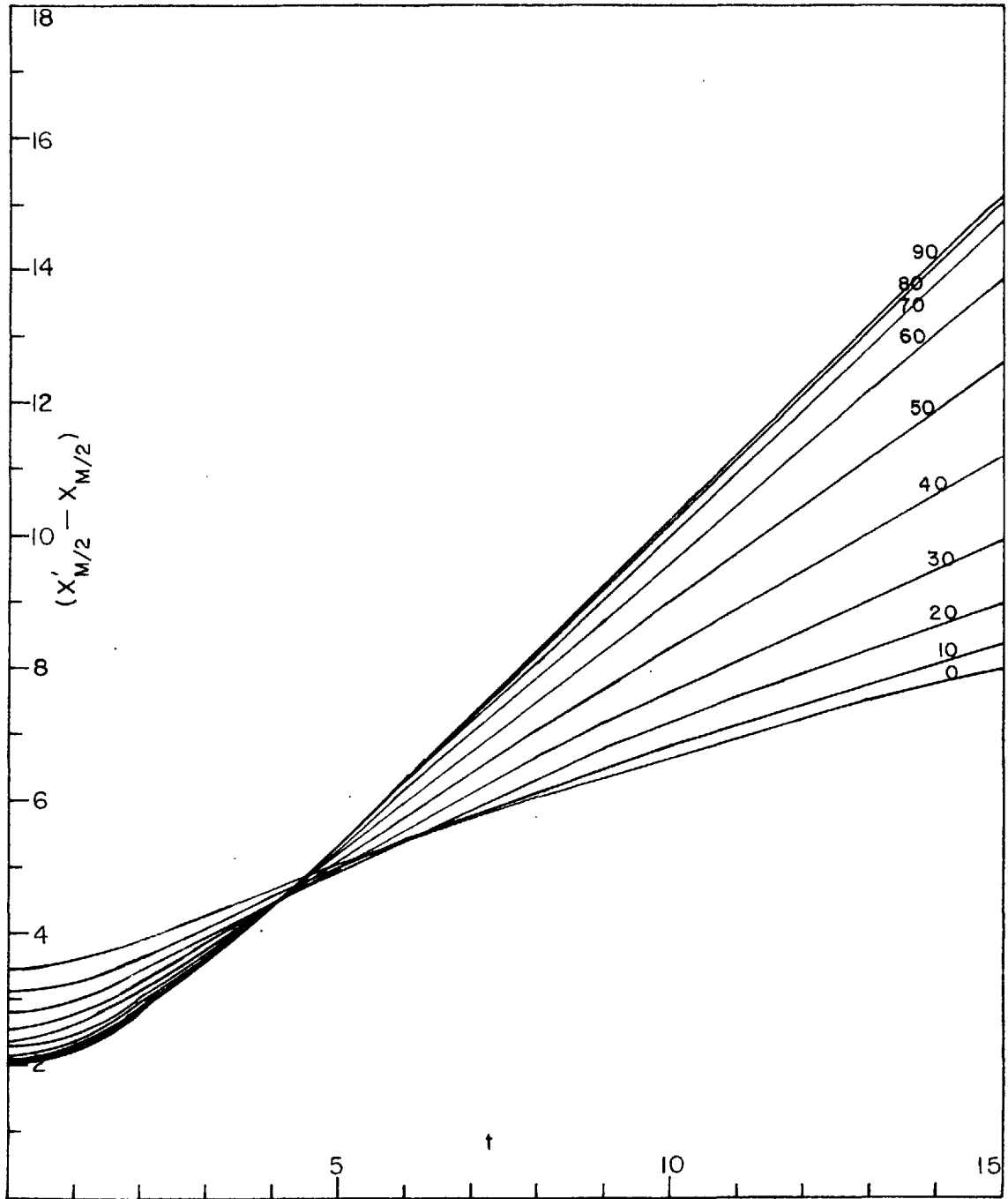


Fig. 2-11

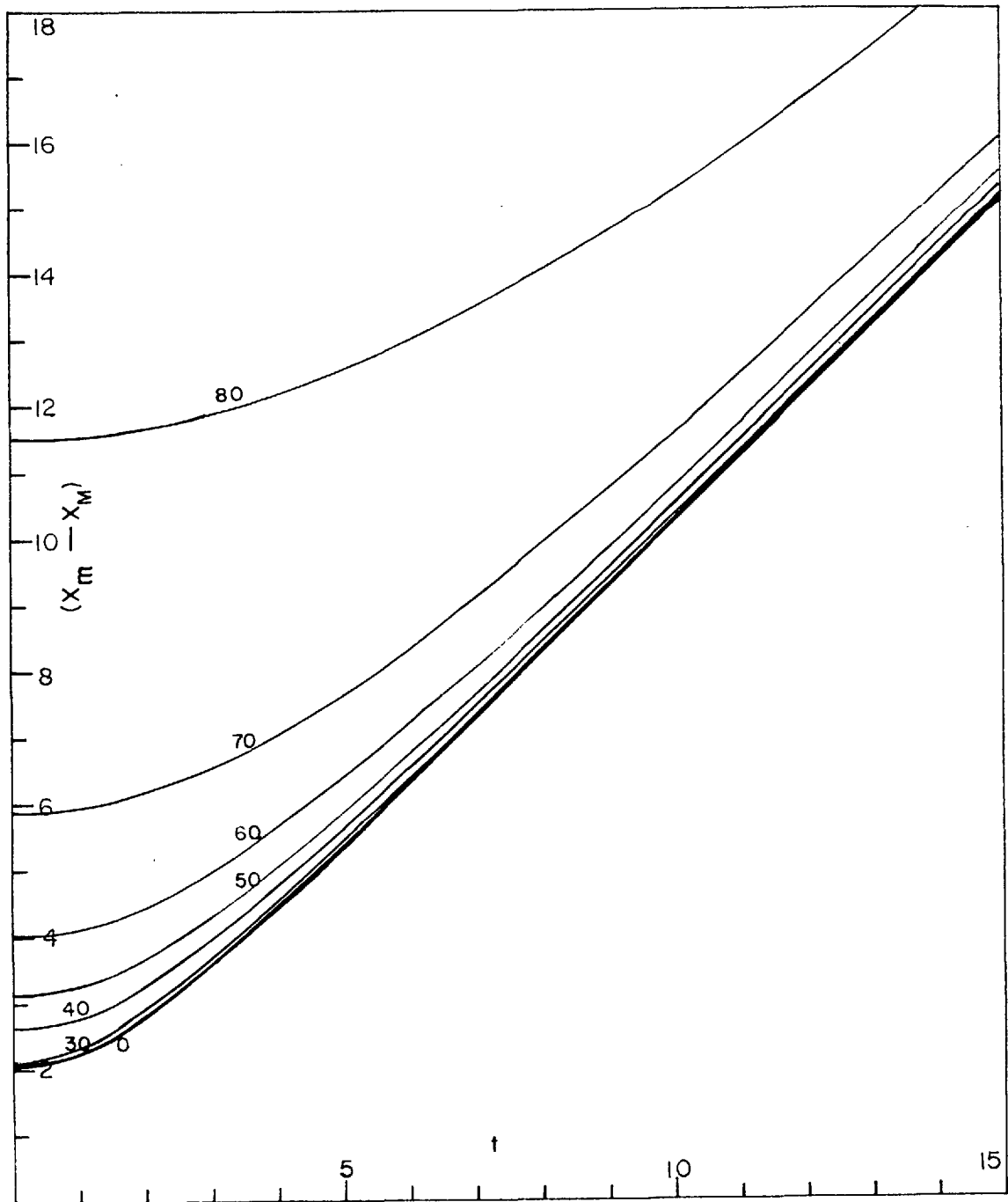


Fig. 2.12

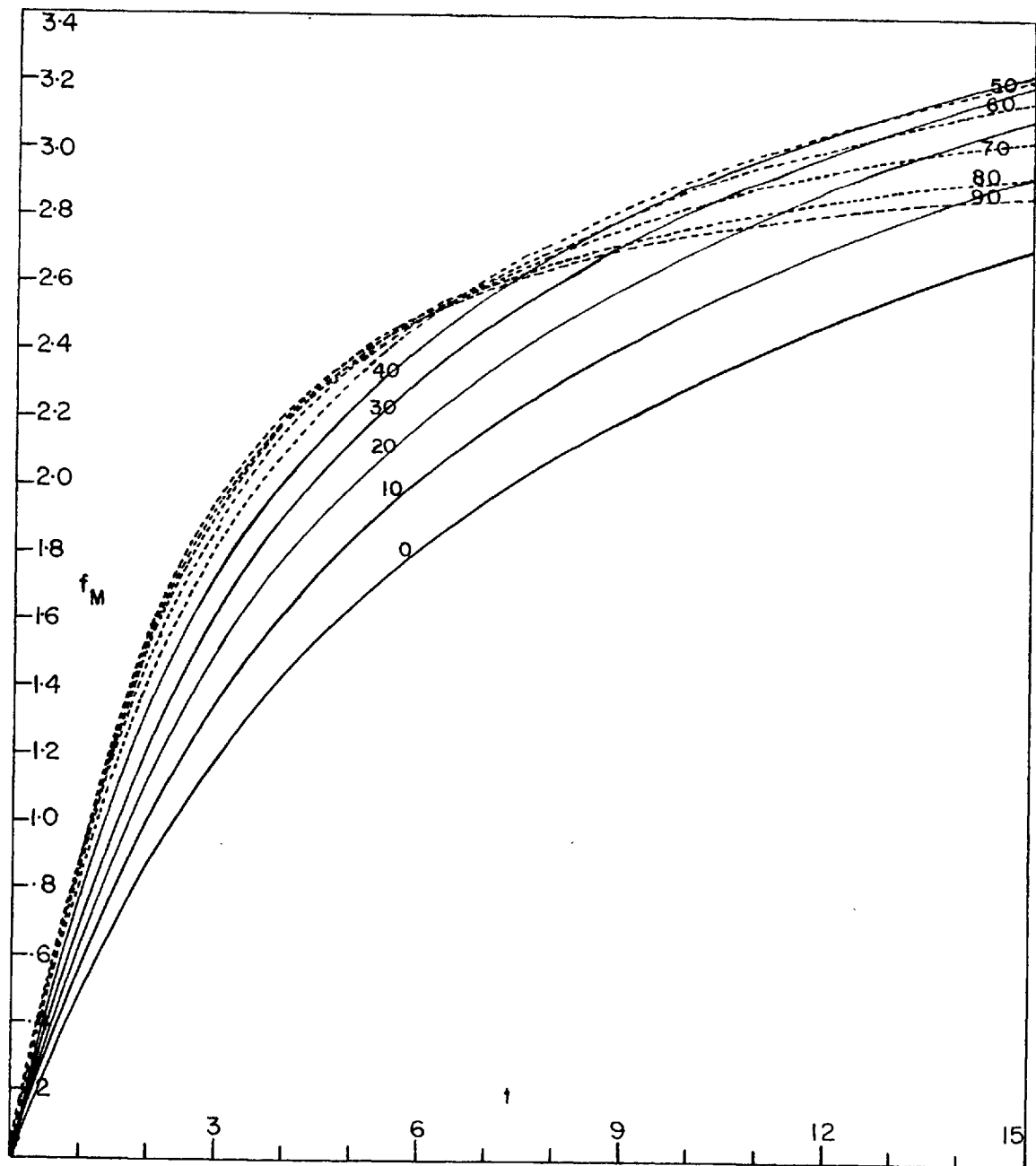


Fig. 2-13

2.3 Vertical Prism with Square Horizontal Section

Anomalies which are suitable for interpretation on a square vertical prism approximation have the general appearance shown in Fig. 2.29. Igneous plugs and many intrabasement features produce anomalies of this type. In the northern hemisphere, they are characterised by a positive anomaly consisting of nearly circular contours with a broader negative anomaly to the north of the positive anomaly.

In the following sections, an interpretational scheme is developed, which uses the distances between the points of maximum, minimum, half maximum, half minimum etc. on some selected profiles on the anomaly.

2.31 Magnetic anomaly due to a general vertical rectangular prism.

Let a set of rectangular axes a, b, h be chosen parallel to the sides of the prism with origin at the point at which the anomaly is to be evaluated. If $a = a_1, a = a_2; b = b_1, b = b_2; h = h_1, h = h_2;$ are the six planes defining the sides of the prism, the total anomaly T at the origin due to the prism is given by

$$T(0,0,0) = \int_{h_1}^{h_2} \int_{b_1}^{b_2} \int_{a_1}^{a_2} \left\{ \left(1 \frac{\partial}{\partial a} + m \frac{\partial}{\partial b} + n \frac{\partial}{\partial h} \right) \times \right. \\ \left. \left(J_1 \frac{\partial}{\partial a} + J_2 \frac{\partial}{\partial b} + J_3 \frac{\partial}{\partial h} \right) \right\} \cdot \frac{1}{R} dh db da$$

where (a, b, h) is a general point in the prism, l, m and n , the d.c.'s of the earth's field with respect to the three axes, J_1, J_2 and J_3 , the three components of magnetisation and $R = (a^2 + b^2 + h^2)^{\frac{1}{2}}$.

$$\text{Since } \frac{\partial^2}{\partial h^2} \left(\frac{1}{R} \right) = - \left(\frac{\partial^2}{\partial a^2} + \frac{\partial^2}{\partial b^2} \right) \frac{1}{R},$$

$$T(0,0,0) = (J_3n - J_1l) \int_{h_1}^{h_2} \int_{b_1}^{b_2} \int_{a_1}^{a_2} - \frac{\partial^2}{\partial a^2} \left(\frac{1}{R} \right) dh. db. da.$$

$$+ (J_3n - J_2m) \int_{h_1}^{h_2} \int_{b_1}^{b_2} \int_{a_1}^{a_2} - \frac{\partial^2}{\partial b^2} \left(\frac{1}{R} \right) dh. db. da.$$

$$+ (J_1m + J_2l) \int_{h_1}^{h_2} \int_{b_1}^{b_2} \int_{a_1}^{a_2} \frac{\partial^2}{\partial a \partial b} \left(\frac{1}{R} \right) dh. db. da.$$

$$+ (J_1n + J_3l) \int_{h_1}^{h_2} \int_{b_1}^{b_2} \int_{a_1}^{a_2} \frac{\partial^2}{\partial h \partial a} \left(\frac{1}{R} \right) dh. db. da.$$

$$+ (J_2n + J_3m) \int_{h_1}^{h_2} \int_{b_1}^{b_2} \int_{a_1}^{a_2} \frac{\partial^2}{\partial b \partial h} \left(\frac{1}{R} \right) dh. db. da.$$

$$\begin{aligned}
 = & \left[\left[(J_3n - J_1l) \tan^{-1} \frac{bh}{Ra} + (J_3n - J_2m) \tan^{-1} \frac{ah}{Rb} \right. \right. \\
 & + (J_1m + J_2l) \ln(R+h) + (J_1n + J_3l) \ln(R+b) \\
 & \left. \left. + (J_2n + J_3m) \ln(R+a) \right] \frac{h_2}{h_1} \frac{b_2}{b_1} \frac{a_2}{a_1} \right] \quad (8)
 \end{aligned}$$

2.32 The anomaly on the north-south profile passing through the axis of a square prism of infinite depth, oriented north south, when the magnetisation is entirely induced.

Consider the case of a square prism of infinite depth ($h_2 = \infty$), oriented with one side parallel to the meridian ($m = 0$) and with the direction of magnetisation along the direction of the earth's field. ($J_1 = lJ$, $J_2 = 0$, $J_3 = nJ$). In this case, the maximum and the minimum of the anomaly will lie on the north-south line passing through the axis of the prism i.e. the line for which $b_1 = -b_2 = -t'$ where t' is half the side of the square prism.

If the anomaly on this line is denoted by $F(x)$, where x is the distance measured along the meridian from the axis of the prism, then, from (8),

$$\begin{aligned}
 F(x) = & 2J \left[(n^2 - l^2) \left(\tan^{-1} \frac{h_1 t'}{Ra} - \tan^{-1} \frac{t'}{a} \right) + \right. \\
 & \left. + n^2 \left(-\tan^{-1} \frac{ah_1}{Rt'} - \tan^{-1} \frac{a}{t'} \right) + nl \ln \frac{R-t'}{R+t'} \right] \frac{a_2}{a_1}
 \end{aligned}$$

where $a_1 = -t' - x$

and $a_2 = t' - x$

Collecting terms in n^2 and noting that

$$- \left[\tan^{-1} \frac{h_1 t'}{Ra} + \tan^{-1} \frac{ah_1}{Rt'} \right]_{a_1}^{a_2} = \left[\tan^{-1} \frac{at'}{Rh_1} \right]_{a_1}^{a_2},$$

$$\text{and } \left[\tan^{-1} \frac{t'}{a} + \tan^{-1} \frac{a}{t'} \right]_{a_1}^{a_2} = 0,$$

$$F(x) = 2J \left[n^2 \tan^{-1} \frac{at'}{Rh_1} + l^2 \tan^{-1} \frac{t'h_1}{Ra} + nl \ln \frac{R-t'}{R+t'} \right]_{a_1}^{a_2}$$

If all the distances are measured in units of the depth h_1 ,
and if $f(x) = \frac{F(x)}{J}$, then

$$f(x) = 2 \left[n^2 \tan^{-1} \frac{at'}{R} + l^2 \tan^{-1} \frac{t'}{Ra} + nl \ln \frac{R-t'}{R+t'} \right]_{a_1}^{a_2} \quad (9)$$

where now x , a_1 , a_2 and t' are in units of the depth and
 $R^2 = 1 + a^2 + t'^2$.

2.33 The derivative of $f(x)$ with respect to x .

Let $g(a)$ denote the function inside the brackets in
(9). Then, the derivative $f'(x)$ of $f(x)$ is given by the

relation

$$\begin{aligned}
 f'(x) &= 2 \left[-\frac{2g(a)}{2a} \right]_{a_1}^{a_2} \\
 &= -2 \left[n^2 \frac{t'}{R(a^2+1)} - \frac{1^2 t'}{R} \left\{ \frac{1}{a^2+1} + \frac{1}{a^2+t'^2} \right\} + n1 \frac{2t'a}{R(a^2+1)} \right]_{a_1}^{a_2} \\
 &= -2t' \left[-\frac{1^2}{R(a^2+t'^2)} + \frac{(n^2-1^2) + 2n1a}{R(a^2+1)} \right]_{a_1}^{a_2} \quad (10)
 \end{aligned}$$

The equation $f'(x) = 0$ can be solved numerically and the position x_m of the maximum or the minimum determined. Once this is done, the solution of the equations $f(x, y, 0) = \frac{s}{4} f(x_m)$, for any profile passing through the maximum or the minimum can be accomplished in the same manner as for the two dimensional dyke.

3.34 The solution of the equation $f'(x_m) = 0$

The functions $f(x)$ and $f'(x)$ have the general forms shown in Fig. 2.14 when the inclination i of the earth's field is in the range $0 \leq i \leq 90$. To solve the equation $f'(x_m) = 0$ an initial value x_i for x_m , given by $x_i = \tan I \mp (t'^2 + \frac{1}{\cos^2 I})^{\frac{1}{2}}$, was assumed. In this

expression $I = 2i - \pi/2$. This initial value is in fact the position of the maximum or minimum on the total anomaly

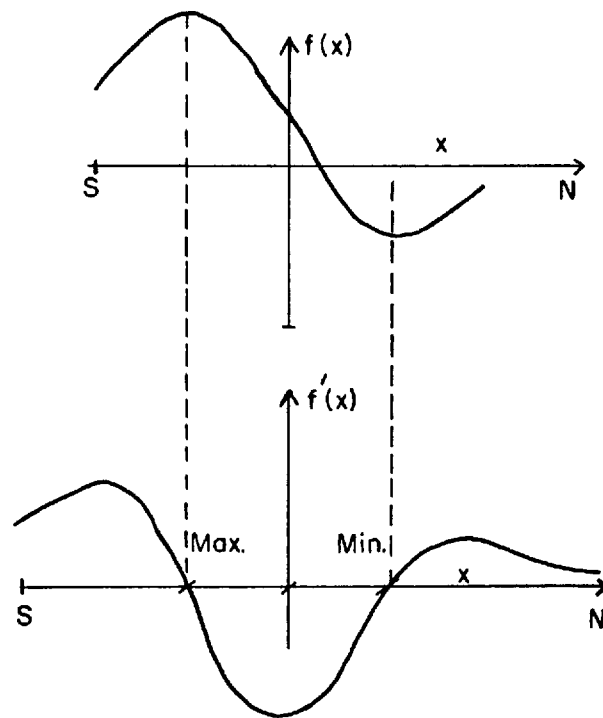


Fig.2-14

due to the two dimensional block considered previously, whose vertical section in the north-south direction is identical with the vertical section of the square prism. Consider first the solution for the maximum. The derivative $f'(x_i)$ for $x_i = \tan I - (t'^2 + \frac{1}{\cos^2 I})^{\frac{1}{2}}$ was evaluated using expression (10). A point Q_1 , where $f'(Q_1)$ has opposite sign to $f'(x_i)$ was then found by calculating $f'(x_i + j)$ or $f'(x_i - j)$, depending on whether $f'(x_i) > 0$ or $f'(x_i) < 0$, for $j = 1, 2, 3 \dots$. Successive inverse linear interpolation between two values of $f'(x)$ nearest to zero and of opposite sign was carried out until, for a value of x , $|f'(x)| < e$, where e is the number that is used as an approximation to zero. In the present work e was taken as 5×10^{-5} . This value was considered accurate enough since the maximum value of $f'(x)$ is of the order of unity.

For the minimum, $f'(x)$ was inverted by multiplying by -1 and the above operations repeated with $x_i = \tan I + (t'^2 + \frac{1}{\cos^2 I})^{\frac{1}{2}}$.

2.35 Solution for half, quarter and three quarter maximum and minimum positions.

The solution for the positions of half, quarter and

three quarter maximum and minimum on three profiles were carried out in exactly the same way as for the two dimensional dyke. The three profiles were:

1. the North-South profile passing through the maximum and the minimum,
2. the East-West profile passing through the maximum, and
3. the East-West profile passing through the minimum.

The expression for the anomaly on the North-South profile has been given already. For the East-West profile passing through the maximum or the minimum, the formula becomes, on putting $h_2 = \infty$, $J_1 = lJ$, $J_3 = nJ$ and $J_2 = m = 0$ in (8), and measuring distances in units of h_1 ,

$$\begin{aligned}
 T(x_m, y, 0) &= T(x_m, -y, 0) \\
 &= J \left[\left[n^2 \tan^{-1} \frac{ab}{R} + l^2 \tan^{-1} \frac{b}{Ra} \right. \right. \\
 &\quad \left. \left. + 2nl \ln(R+b) \right] \begin{matrix} b_2 & a_2 \\ b_1 & a_1 \end{matrix} \right] \quad (11)
 \end{aligned}$$

where $a_1 = -t' - x_m$

$a_2 = t' - x_m$

$b_1 = -t' - y$

$b_2 = t' - y.$

2.36 The computed results for the square vertical prism

The results of numerical calculations are given in Appendix II. These results cover the range $t = 2t' = 0.1, 1, 2 \dots 10$ for each value of i from 0° to 90° in steps of 10° . The six rows of numbers in this appendix corresponding to each value of t represent the following cases.

1. The first two lines give the positions on the north-south line passing through the maximum.
2. The third line gives the positions on the east-west line passing through the maximum.
3. The fourth and fifth lines give the positions on the north-south line passing through the minimum.
4. The sixth line gives the positions on the east-west line passing through the minimum.

For the east-west lines, the distances are measured in the east-west direction from the maximum or minimum. For $i = 0$, the calculations have been done only for the negative feature. For $i = 90$, there is no negative feature, and as such only three rows of numbers are given.

2.37 Charts for determining thickness, depth and intensity of magnetisation from the observed anomaly.

Fig. 2.15 shows the i, t plane divided into different sections, each section containing curves of constant ratios

of a pair of distances. The choice of the particular ratio of characteristics for each section was based on the following two considerations.

1. The distances used in the ratio shall be measured on the major feature, and
2. The resulting curves shall intersect at suitably large angles with the $i = \text{constant}$ lines

Unfortunately it has not been found possible to satisfy these conditions using a single ratio of characteristics for all i .

To determine t from the observed anomaly the ratio of the relevant distances on the north-south profile is measured and, knowing the inclination of the earth's field, the value of t corresponding to this ratio is read from Fig. 2.15. When t , in units of depth, is thus determined the chart corresponding to the particular inclination is selected from one of the Figs. 2.16 to 2.25. The ratio of a distance on one of the three profiles on the anomaly to the corresponding distance read from this chart for the already evaluated t , gives a value for the depth. A number of independent estimates of the depth can be made using the various distances given in the charts. Distances most suitable for depth estimations are those that

show little variation with t . Fortunately, for each value of the inclination, there is at least one distance which satisfies this condition.

The notations used in the figures when referring to distances are as follows. $x_s/4$ is the distance of the position on the outer limb of the maximum or the minimum at which the anomaly is $s/4$ times the maximum or minimum, $x'_s/4$ is the corresponding distance on the inner limbs and $Y_s/4$ is the distance of position on the east-west profile passing through the maximum or minimum. In addition the symbols S and N, meaning south and north, help to distinguish between the maximum and the minimum. For the northern hemisphere i.e. for $i = 0$ to $i = 90$, N refers to the negative part of the anomaly and S to the positive part. The order must be reversed for use in the southern hemisphere ($-90^\circ \leq i < 0$).

The maximum f_M and the minimum f_m are plotted in Figs 2.26 and 2.27 as functions of t for each value of i from 0 to 90. Since the actual maximum and minimum are J times f_M and f_m respectively, values for J can be obtained by taking the ratio of the maximum or the minimum on the observed anomaly to the value of f_M or f_m corresponding to the previously evaluated t .

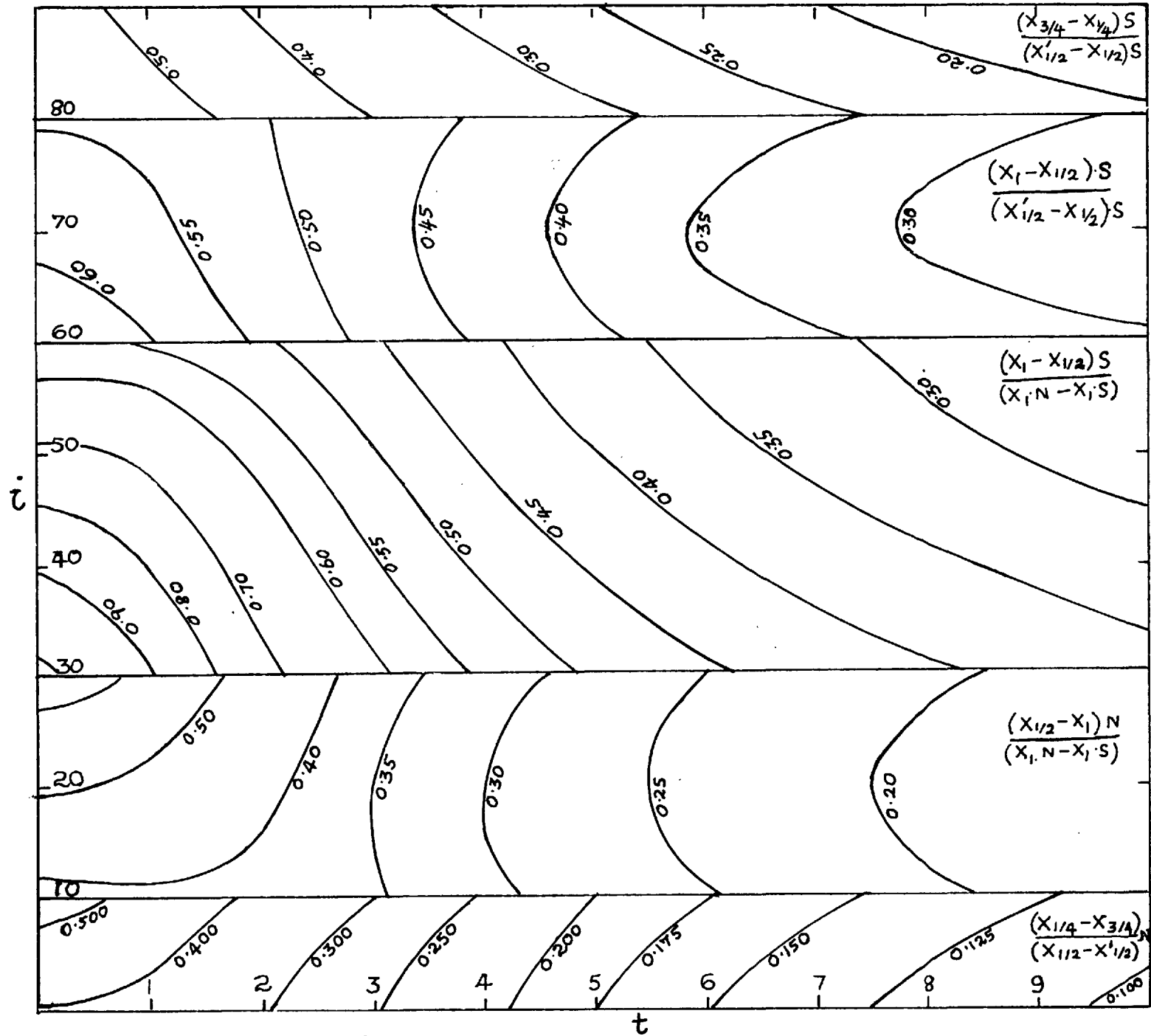


FIG. 2.15

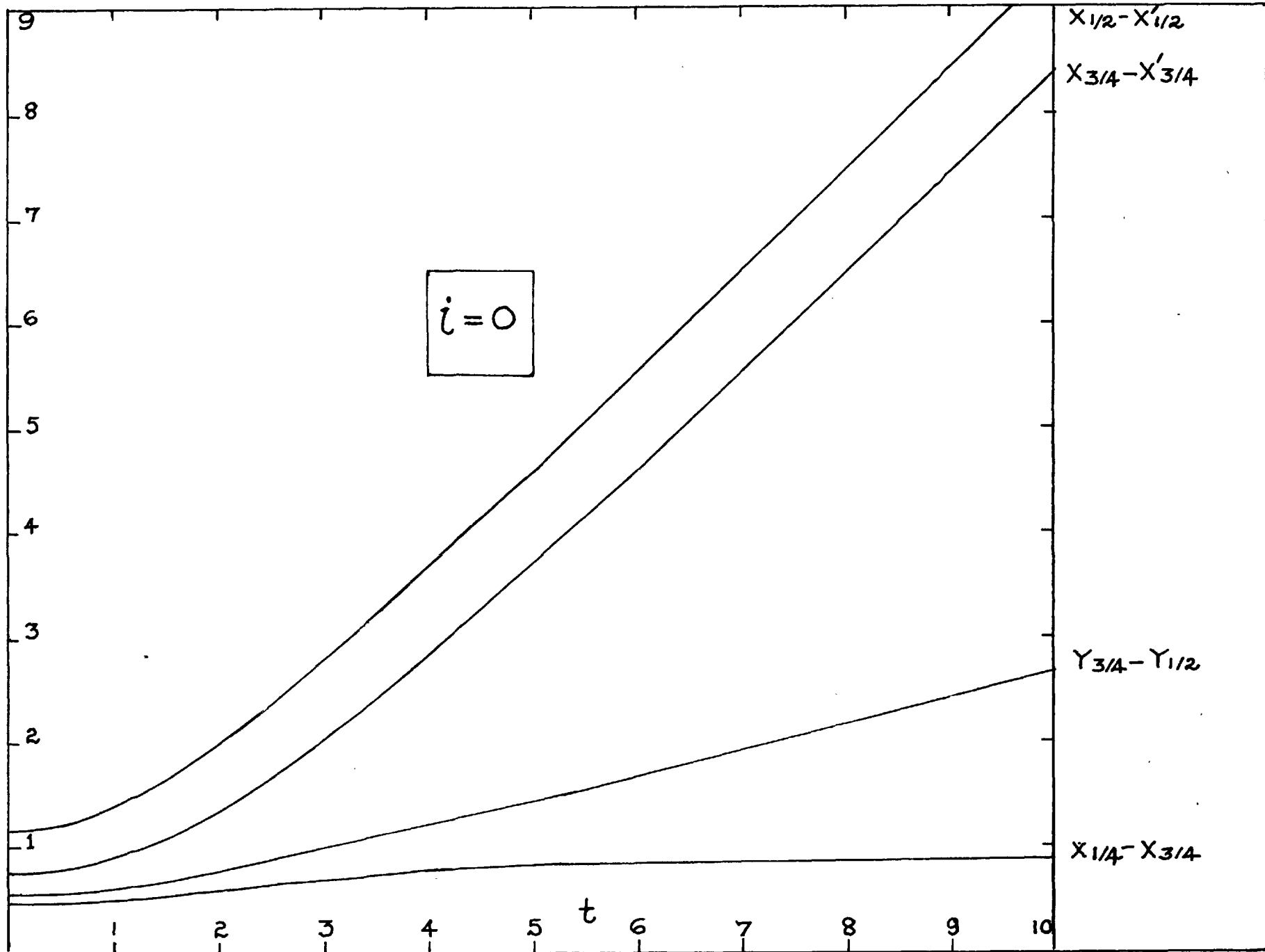


FIG 2.16.

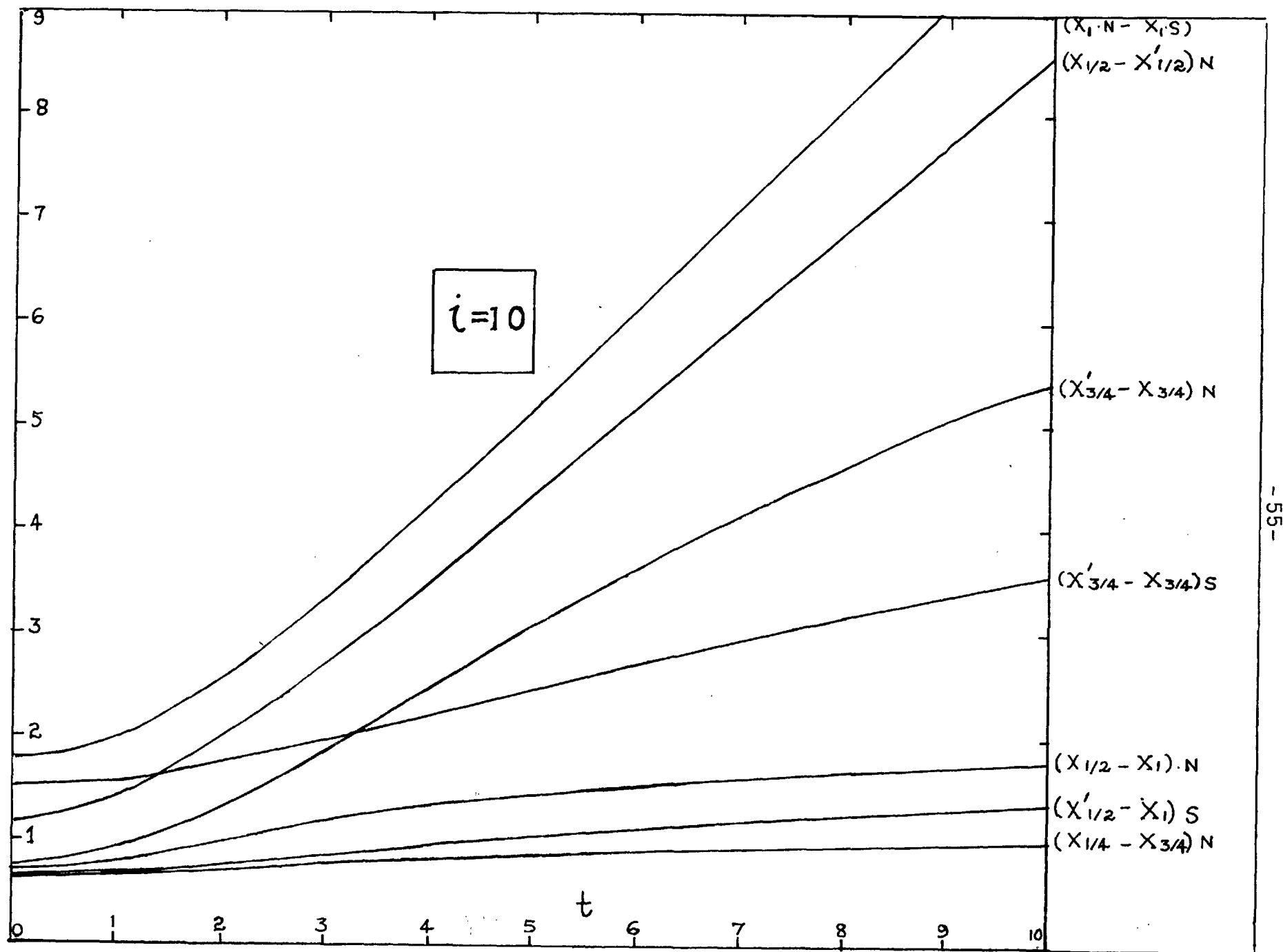


Fig 2-17

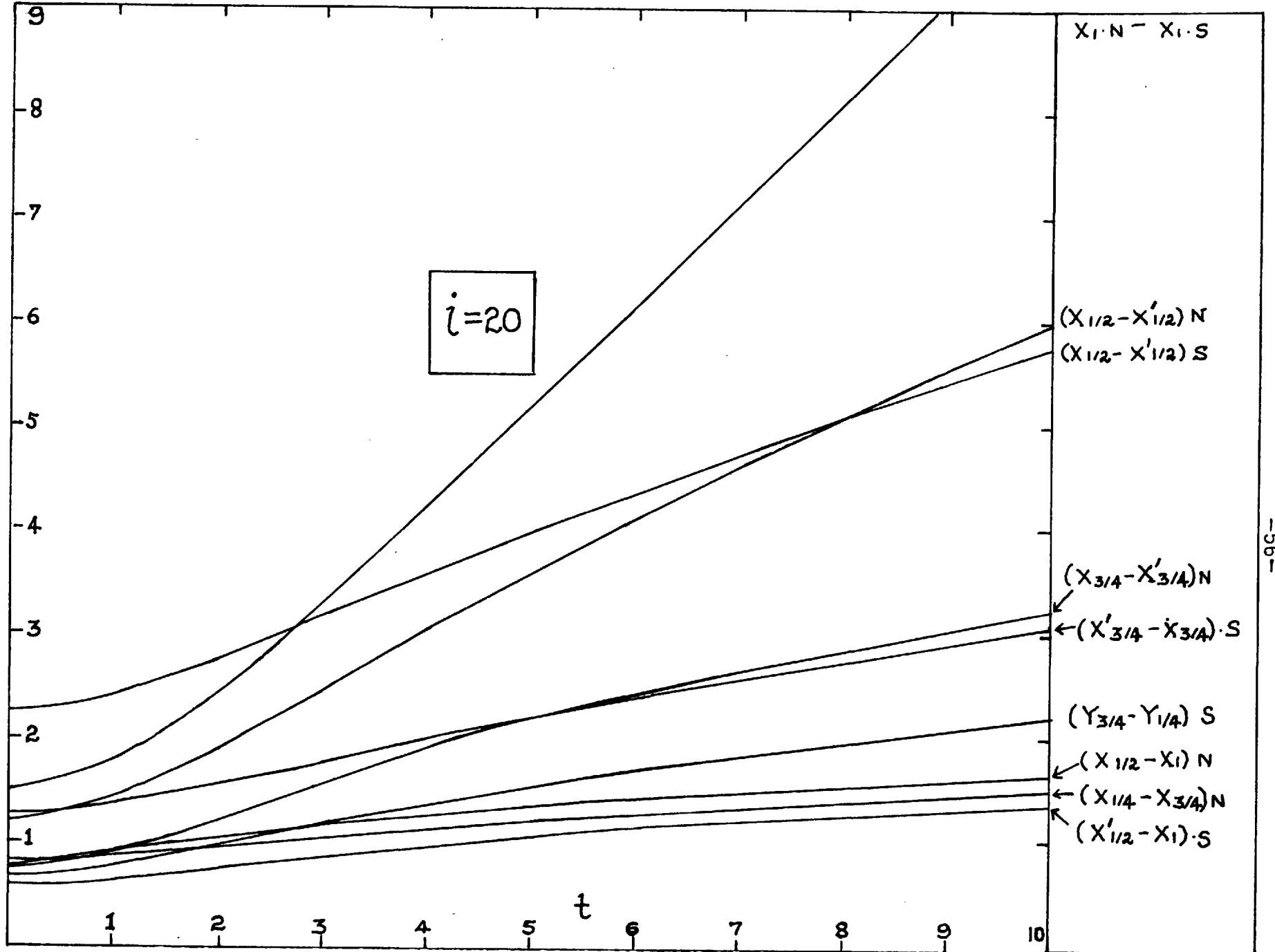


FIG 2.18

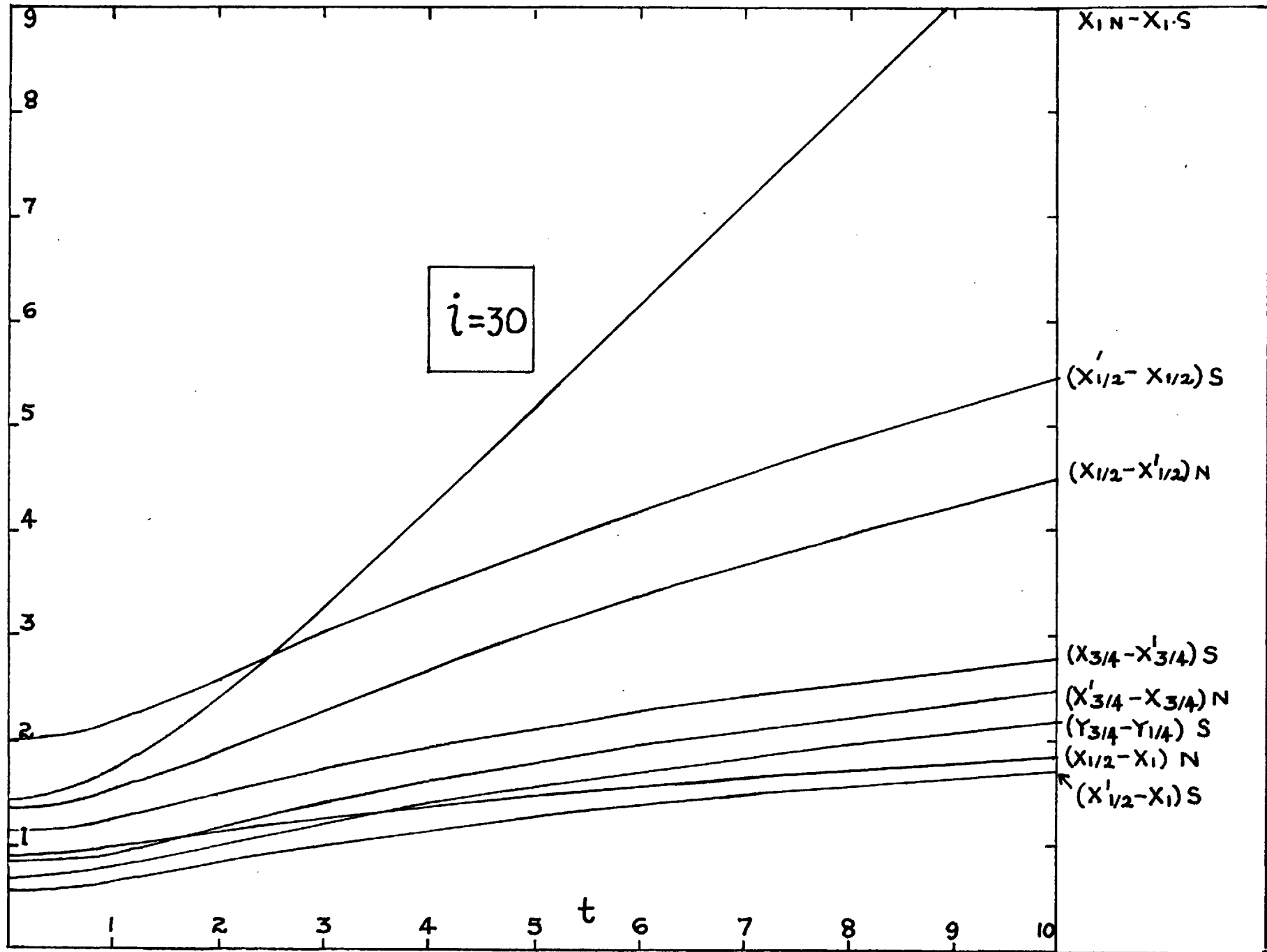


FIG 2-19.

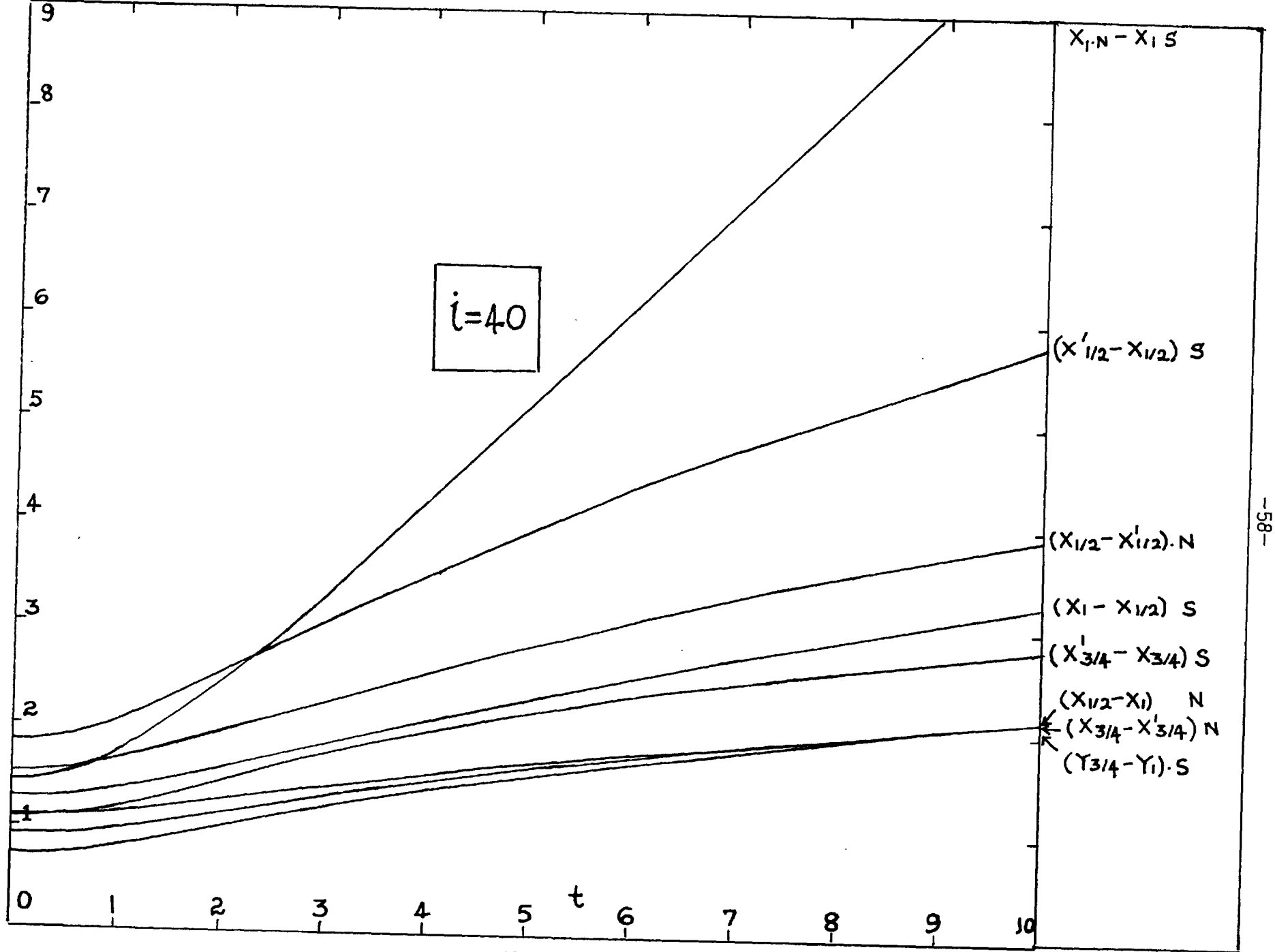


FIG 2-20

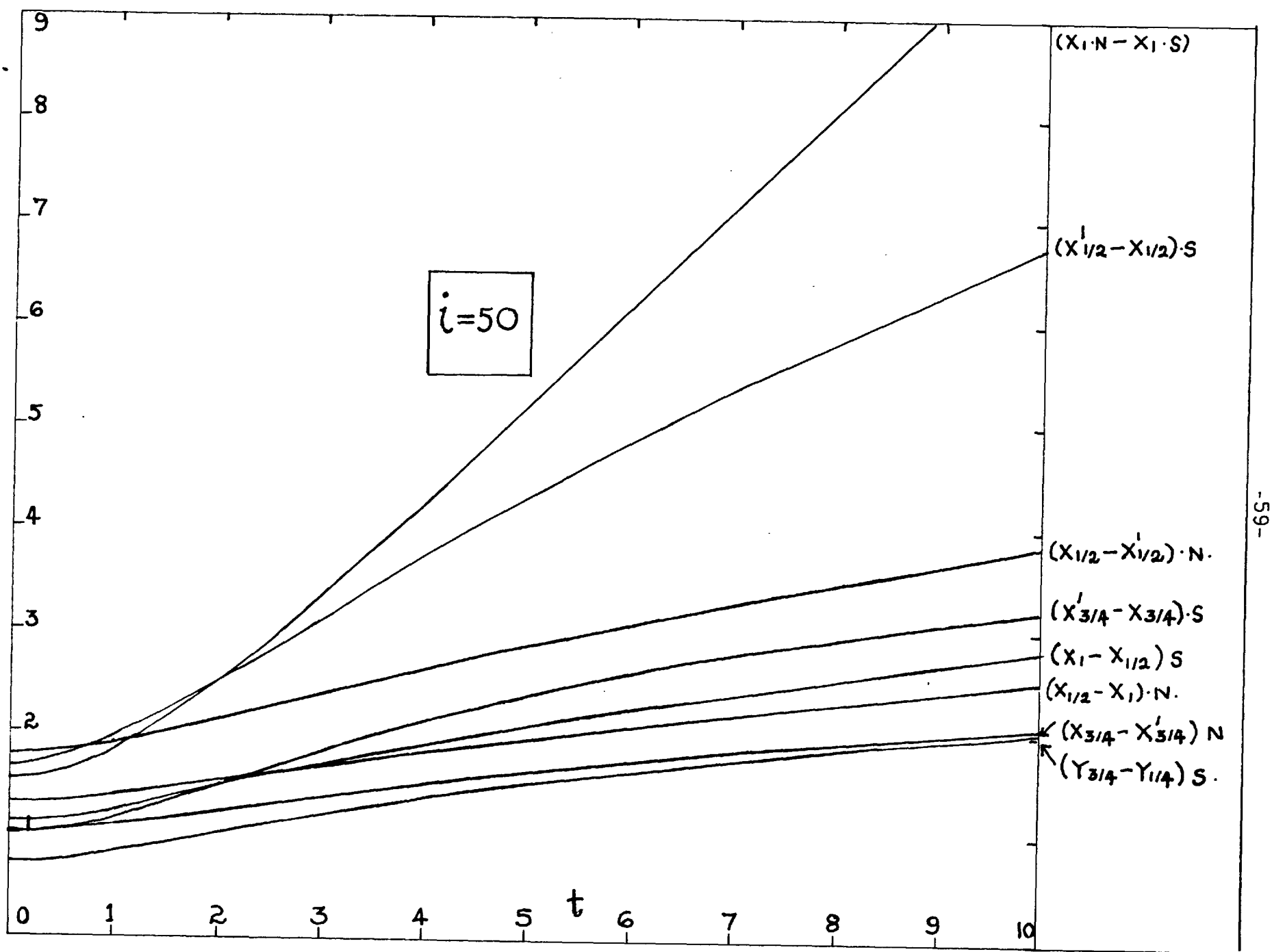


FIG 2-21

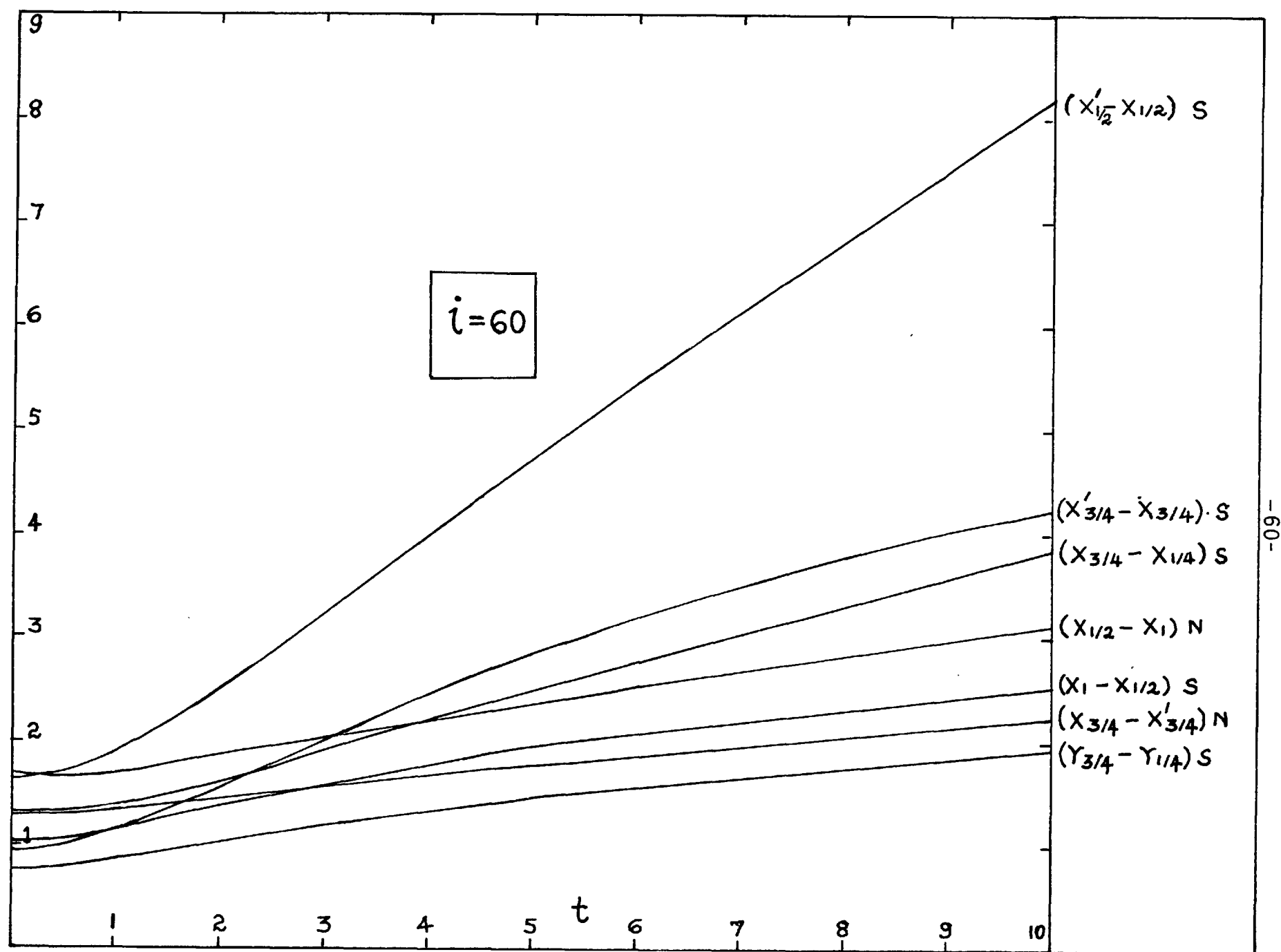


FIG 2.22

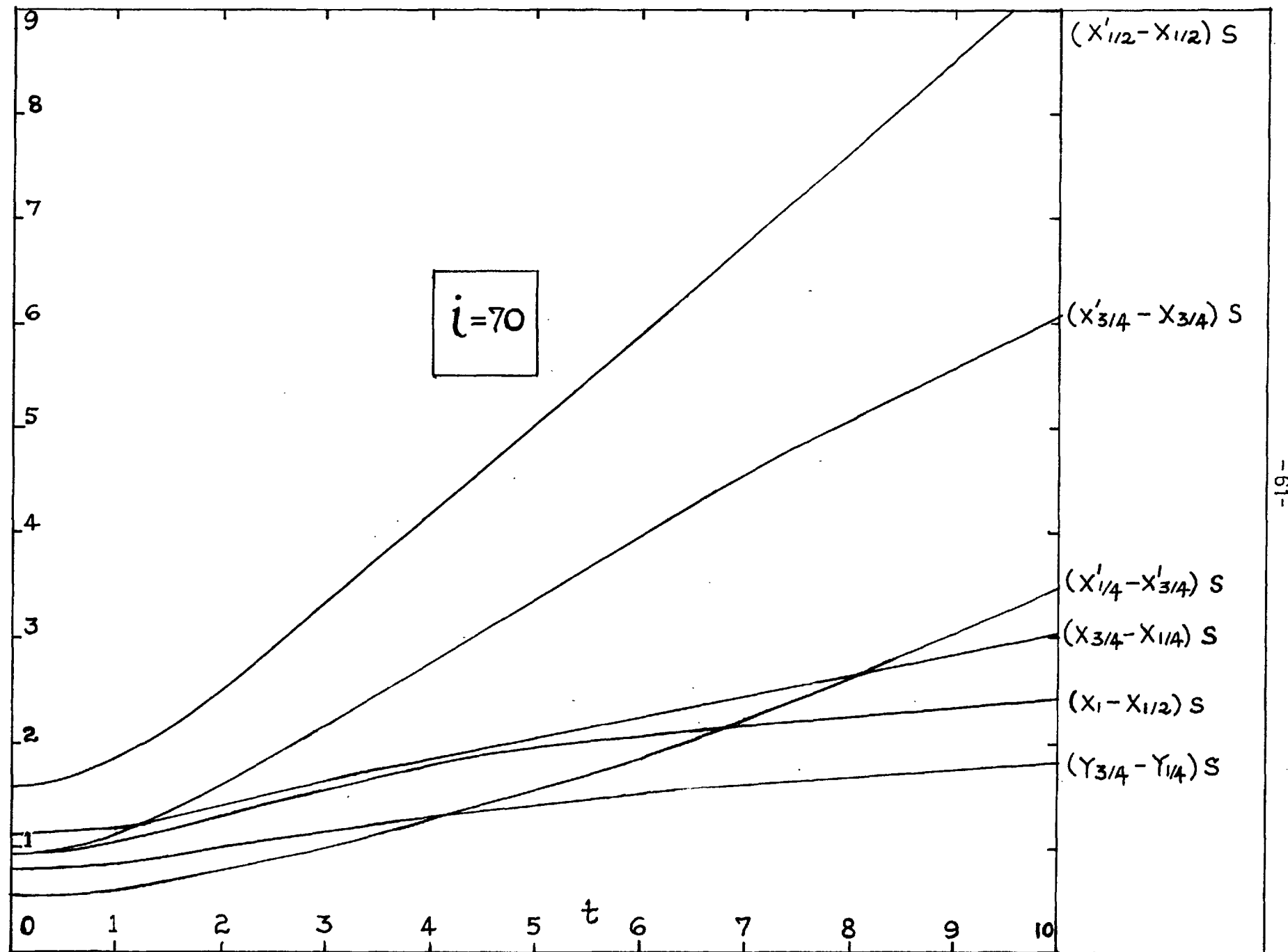


FIG. 2.23

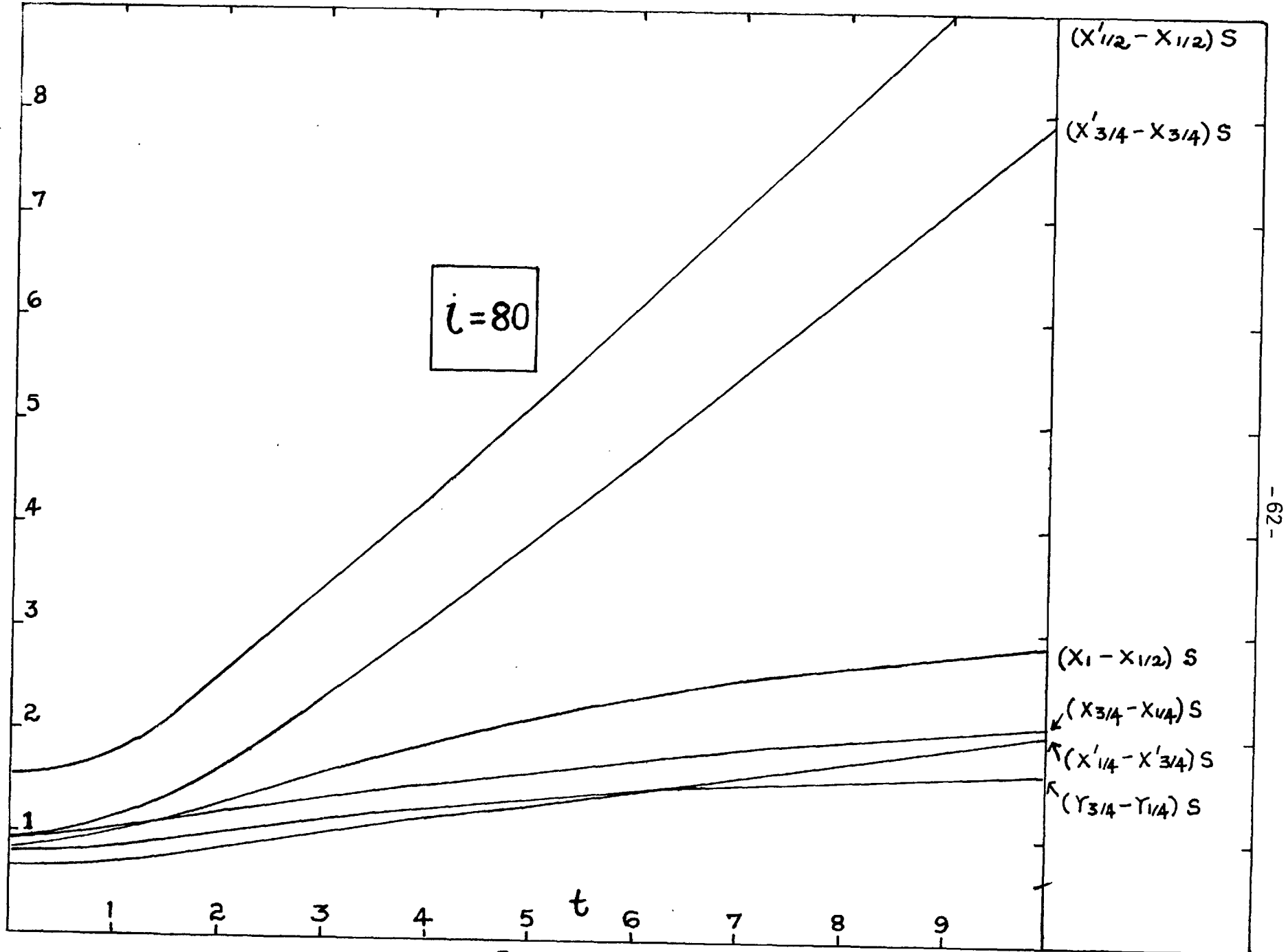


FIG 2.24.

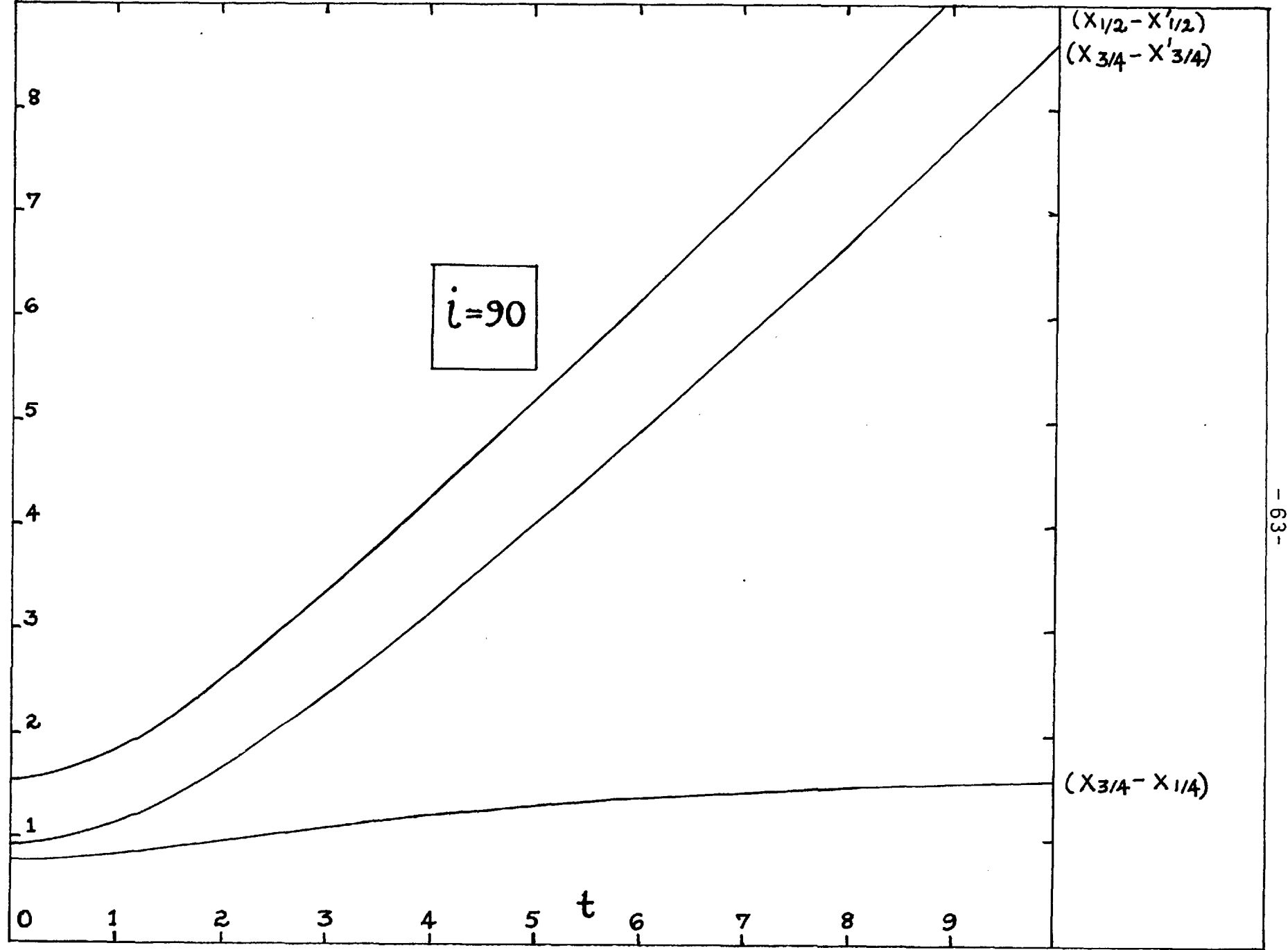


FIG 2.25

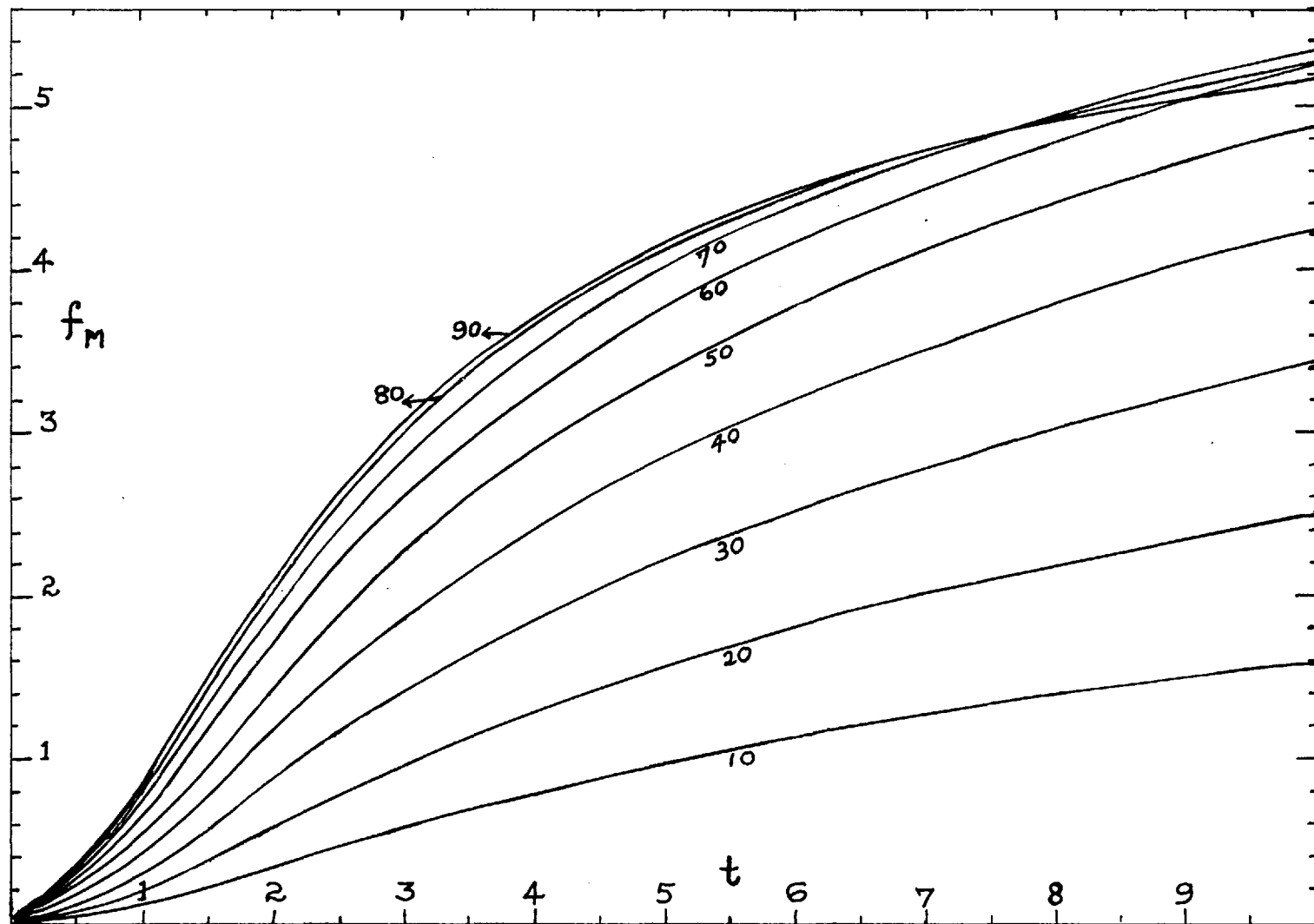


FIG 2.26

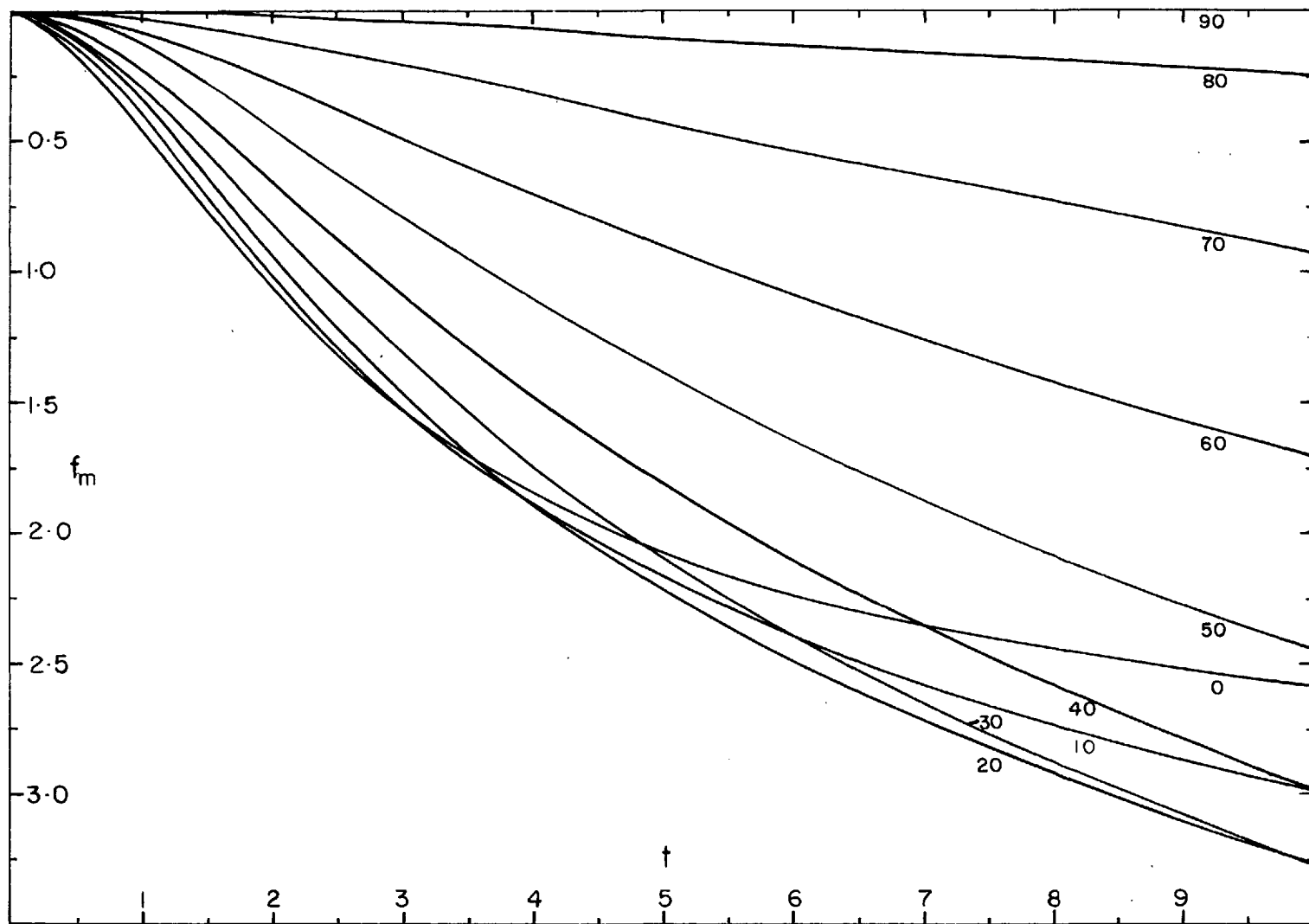


Fig. 2-27

2.4 Reduction of anomalies due to certain types of three dimensional structures to two dimensional anomalies.

Consider a structure (Fig. 2.28) bounded by a horizontal cylinder $f(yz) = 0$ and two similar surfaces $\phi(x, y, z) = 0$ and $\phi(x-t, y, z) = 0$. Such a structure has a constant horizontal thickness t parallel to the x -axis. Let $T(x, y, 0)$ be the magnetic or gravity anomaly due to this structure at an arbitrary point $P(x, y, 0)$. If the structure is displaced parallel to itself by an amount $n\Delta x$, then, the anomaly at P due to the displaced structure is $T(x - n\Delta x, y, 0)$. Consider an infinite number of displacements, each separated by an amount Δx , in either sense along the x axis. The sum of the anomalies at P due to all the displaced structures is

$$\sum_{n=-\infty}^{\infty} T(x - n\Delta x, y, 0)$$

If the intensity of magnetisation or density of the original structure is uniform, the displaced structures form a two dimensional structure with intensity of magnetisation or density $t/\Delta x$ times that of the original structure. Thus, if the anomaly produced by this two dimensional structure for the same intensity of magnetisation or density as the original structure is denoted by $T'(y, 0)$,

$$T'(y, 0) = \frac{\Delta x}{t} \sum_{n=-\infty}^{\infty} T(x - n\Delta x, y, 0)$$

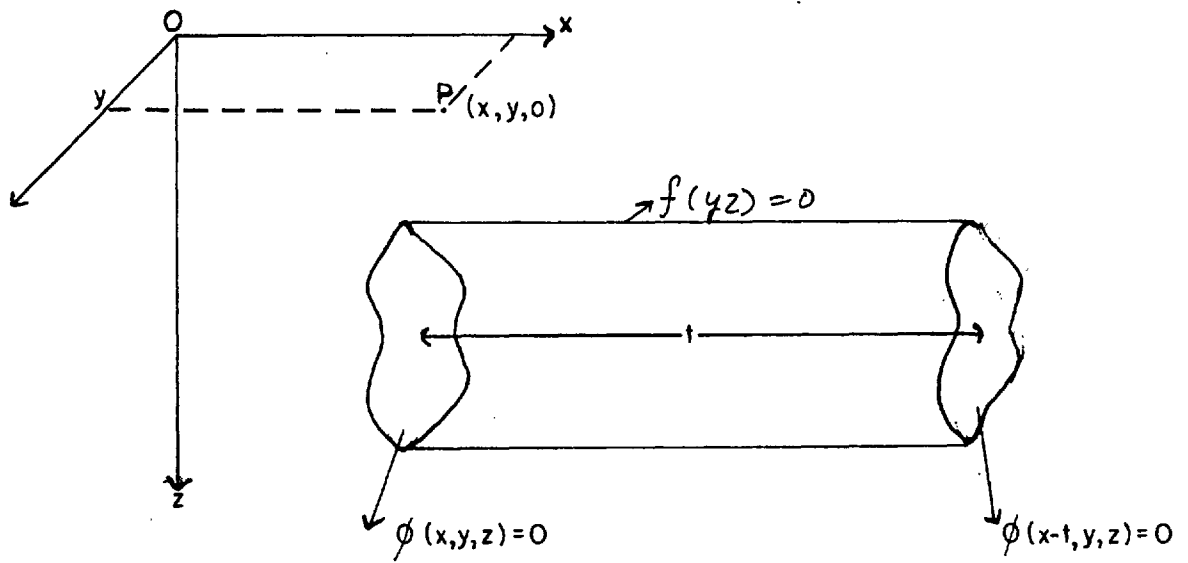


Fig. 2.28

and in the limit

$$t T'(y,0) = \int_{-\infty}^{\infty} T(x, y, 0) dx.$$

Thus an integration of the anomaly of the original structure parallel to the direction of constant thickness results in t times the anomaly of a two dimensional structure with vertical section, coinciding with the vertical section of the original structure. If the direction of constant thickness can be indentified from the observed anomaly, an integration (in practice a summation of grid values) of the anomaly parallel to this direction can be carried out for various values of y and a profile across the hypothetical two dimensional structure obtained. This profile may be used for a two dimensional interpretation of the anomaly.

If the structure producing the anomaly is considered to be a parallelopiped bounded by a horizontal rectangular top surface with sides of length t_1 and t_2 and inclined surfaces extending to a great depth and dipping at angles D_1 and D_2 then, the integration of the anomaly along directions parallel to the two sides of the rectangle will result in two two dimensional dyke anomalies. These will refer to:

1. the anomaly due to the dyke with a top surface of width t_1 at the depth z , and with its walls dipping at an angle D_1 , for an intensity of magnetisation $t_2 \bar{J}$, and

2. the anomaly due to the dyke with a top surface of width t_2 at the depth z , and with its walls dipping at an angle D_2 , for an intensity of magnetisation $t_1 \bar{J}$. $\bar{J}$, in magnitude and direction, defines the intensity of magnetisation of the parallelopiped producing the observed anomaly. From the first anomaly, t_1 , the depth z and the combined parameter $I_1 = U_1 + V_1 - D_1$ can be evaluated using the charts prepared for the two dimensional dyke. U_1 and V_1 are respectively the angles made with the horizontal by the projections of the direction of magnetisation and the direction along which the anomaly is measured, on the vertical plane perpendicular to the side of width t_2 . Similarly, from the second anomaly t_2 , z , and $U_2 + V_2 - D_2$ can be evaluated. U_2 and V_2 have meanings similar to those of U_1 and V_1 . If the direction of magnetisation is known, for instance if the magnetism is induced, the values of J , D_1 , and D_2 can also be evaluated.

An example.

The total field anomaly due to a vertical square prism, dimensions $2 \times 2 \times \infty$ in grid units, was calculated on the following assumptions:

1. Depth to top = 2 grid units
2. The inclination of the earth's field is 60°

3. The magnetisation is induced and its intensity is 10^{-2} e.m.u./c.c.
4. The prism is oriented parallel to the magnetic meridian. This anomaly is shown in Fig. 2.29. The aim is to proceed in the reverse direction and find the parameters of the prism producing this anomaly. In this case, the obvious direction for orienting the approximating parallelopiped is the magnetic north-south. The field values extrapolated at the corners of the grid shown in this figure, are then summed up along directions perpendicular to the meridian and the sums plotted as a function of distance along the meridian. The resulting profile is shown in Fig. 2.30. The charts prepared for the two dimensional dyke can be used now to interpret this profile. On this profile,

$$x_M - x_{M/2} = 3.9,$$

$$x_M - x_m = 5.5,$$

$$f_M = 2860$$

$$\text{and } f_m = -800$$

Therefore

$$\frac{x_M - x_{M/2}}{x_M - x_m} = 0.709$$

$$\text{and } \frac{f_m}{f_M} = 0.280$$

Referring back to section C of Fig. 2.5, the values of I and t at which these measured ratios are taken, are obtained by interpolation between the curves. Thus, $t = 0.9$ and $I = 35^\circ$.

If the magnetisation is assumed to be induced, then $U = V = 60^\circ$ and $I = 60 + 60 - D$ where D is the geological dip of the structure in the meridian plane.

Thus $D = 120 - 35 = 85^\circ$ (True value, 90°). The following table gives estimates of the depth using four distances.

Distance	From reduced profile (fig.2.29)	From charts (for $I = 35^\circ$, $t = 0.9$)	z
1. $x_M - x_{M/2}$	3.9	1.82	2.14
2. $x_M - x_m$	5.5	2.50	2.20
3. $x'_{3M/4} - x_{3M/4}$	3.2	1.45	2.21
4. $x'_{M/2} - x_{M/2}$	5.8	2.65	2.19

The mean of the four depth estimates is 2.19. (True value 2.0).

The width of the top surface of the prism along the magnetic north-south is $2.19 \times 0.9 = 1.97$ (True value 2.0).

Since the four depth estimates are consistent, the assumption that the reduced field is due to a two dimensional dyke can be considered justified.

The lack of exact agreement between the estimated depth and the true depth is probably due to the doubtful accuracy

with which the anomaly is prepared by interpolation from values computed from only a few points, and the finite interval over which the summation is carried out in the reduction process.

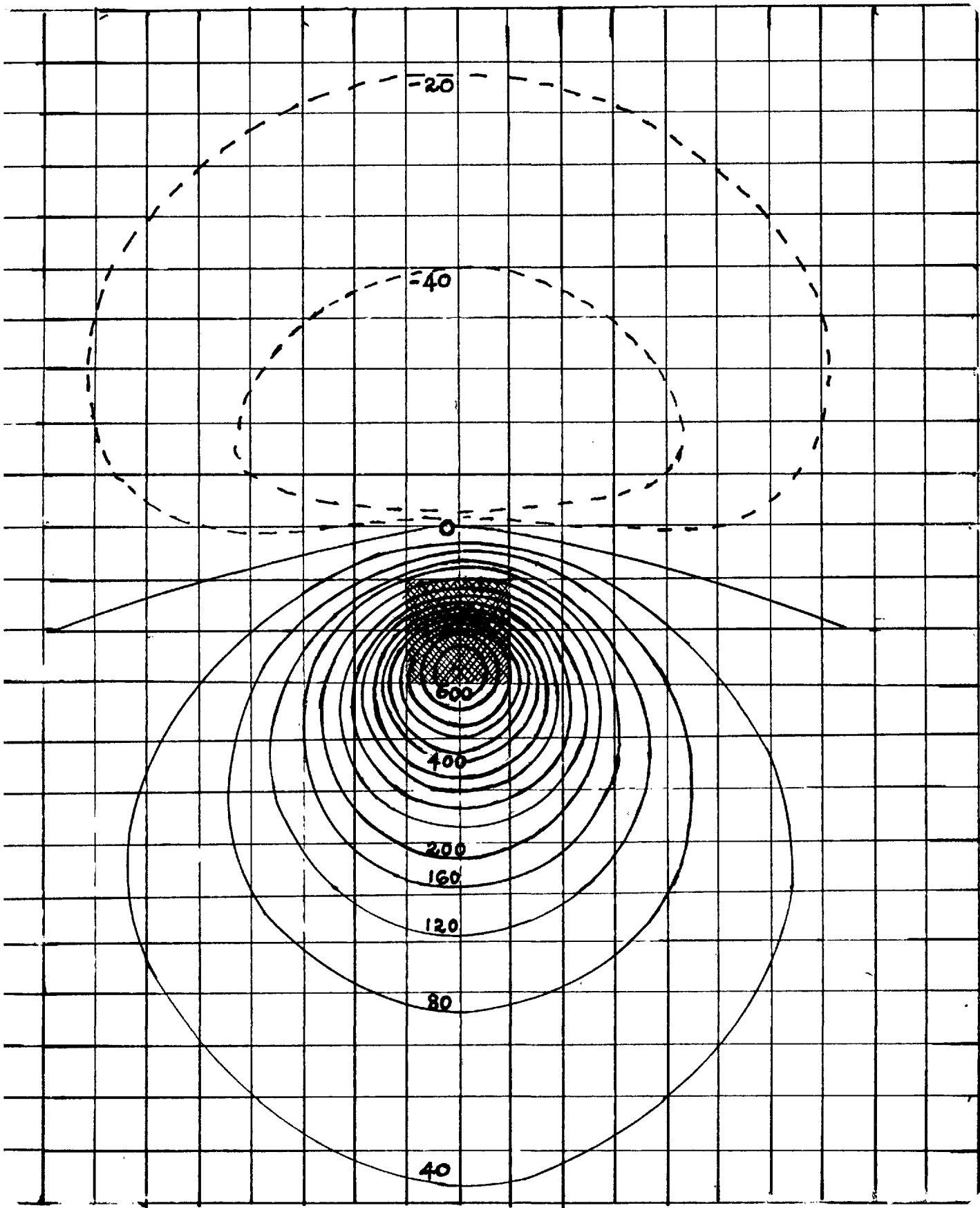


FIG. 2.29

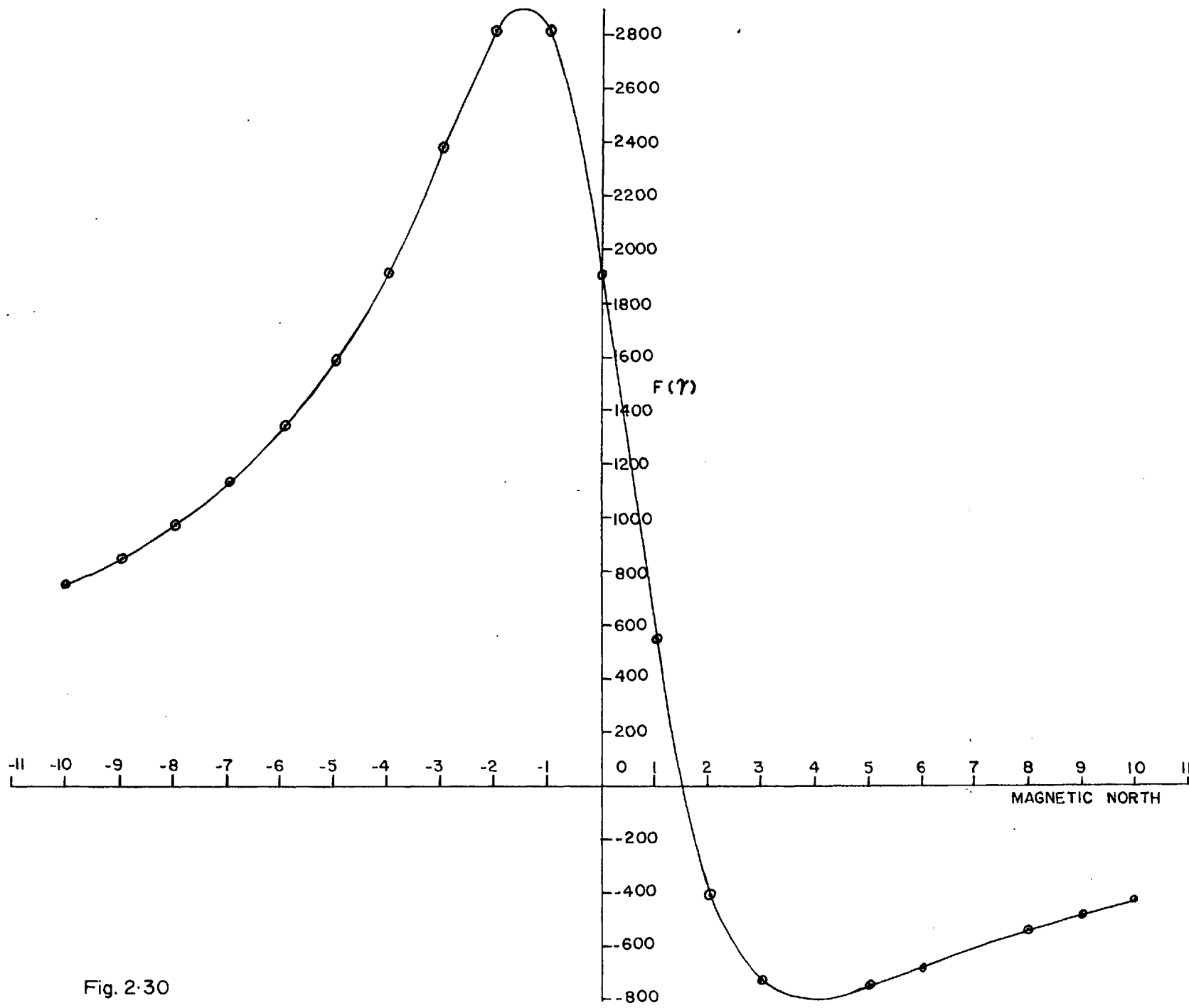


Fig. 2.30

CHAPTER 3

AUTOMATIC MODEL FITTING

3.1 Introduction

The method described in the previous chapter for evaluating the parameters of vertical and inclined parallelopipeds has the advantage that it is simple, rapid and easy to apply. However, it may lack precision since it uses only a few points on the observed anomaly in the evaluation. Where a more accurate estimate of the parameters is required it is necessary to use a larger number of points on the anomaly and to adjust the model until the best fit between the observed anomaly and the anomaly due to the model is obtained. Such a process was carried out in the past by the laborious process of trial and error. Now, however, the fitting can be done more easily using a High Speed Digital Computer. In the present work, computer programmes have been devised for fitting two dimensional and three dimensional vertical rectangular

prisms to observed magnetic anomalies. Since many geological features can be approximated by these models, the programmes can be used for the routine interpretation of a large number of naturally occurring anomalies.

3.2 The two dimensional vertical rectangular prism

When the observed anomaly consists of closed elongated pattern of contours, with the length of the pattern greater than about four or five times its width, a central profile perpendicular to the direction of elongation may be chosen and interpreted on the assumption that it is due to a two dimensional prism. Alternatively, the field values may be integrated along directions parallel to the length of the pattern, and the resulting two dimensional anomaly may be used for interpretation on the same assumption. Since all the dimensions of the approximating prism and the direction of magnetisation are considered unknown, the anomalies used for interpretation may be due to sources resembling a line dipole, a vertical or a horizontal sheet, a vertical or an inclined dyke, and structures associated with a vertical fault. The reason for including an inclined dyke is that its anomaly is the same as that due to a vertical dyke with the same top surface, for a different direction and intensity of magnetisation.

3.21. Magnetic anomaly due to a two dimensional vertical rectangular prism.

Let the x-axis be chosen perpendicular to the direction of strike of the prism. With p , r , l , n , and J having the meanings given previously, the anomaly, $T_m(x_i)$ due to the prism at the point $P(x_i)$ is given by

$$T_m(x_i) = -2J \int_{a_1}^{a_2} \int_{h_1}^{h_2} (1 \frac{\partial}{\partial a} + n \frac{\partial}{\partial h}) (p \frac{\partial}{\partial a} + r \frac{\partial}{\partial h}) \ln R da dh$$

Where (a, h) is the co-ordinate of a general point in the prism with respect to P as origin, and $h_1 = C_3$, $h_2 = C_4$, $a_1 = C_1 - x_i$, $a_2 = C_2 - x_i$ and $R^2 = a^2 + h^2$ (Fig. 3.1).

Since $\frac{\partial^2}{\partial a^2} (\ln R) = -\frac{\partial^2}{\partial h^2} (\ln R)$,

$$\begin{aligned} T_m(x_i) &= -2J \int_{a_1}^{a_2} \int_{h_1}^{h_2} ((-pl+nr) \frac{\partial^2}{\partial h^2} + (pn+lr) \frac{\partial^2}{\partial a \partial h}) \ln R da dh \\ &= -2J (nr-pl) \left[\left[\tan^{-1} \frac{a}{h} \right]_{h_1}^{h_2} \int_{a_1}^{a_2} \frac{a^2}{a_1} - 2J(pn+lr) \left[\int \ln R \cdot \int_{h_1}^{h_2} \int_{a_1}^{a_2} \right] \right] \end{aligned} \quad (1)$$

Now, $2J(nr-pl) = 2J' \sin I$ and $2J(pn+lr) = 2J' \cos I$,

where $J' = J(p^2+r^2)^{\frac{1}{2}} (l^2+n^2)^{\frac{1}{2}}$ and $I = \tan^{-1} \frac{r}{p} + \tan^{-1} \frac{n}{l} - \pi/2$

refer to quantities used in Chapter 2.

Also, since $\tan^{-1} \frac{a}{h} (IP) \ln (h+ja)$ and $\ln R = (RP) \ln(h+ja)$,

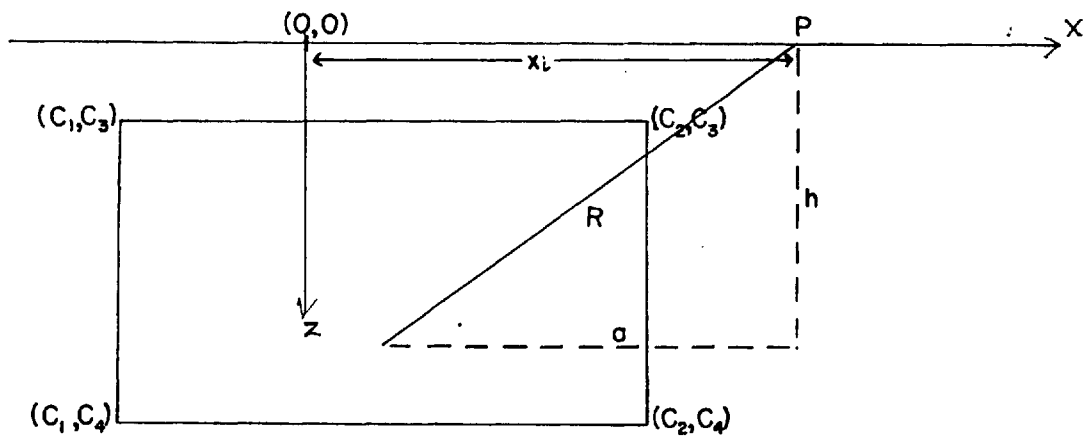


Fig. 3·0

where $j = \sqrt{-1}$ and (IP) and (RP) are the real and imaginary parts of the complex number,

$$\left[\left[\tan^{-1} \frac{a}{h} \right]_{h_1}^2 \right]_{a_1}^2 = (\text{IP}) \ln \frac{(h_1 + ja_1)}{(h_1 + ja_2)} \frac{(h_2 + ja_2)}{(h_2 + ja_1)} \quad \text{and}$$

$$\left[\left[\ln R \right]_{h_1}^2 \right]_{a_1}^2 = (\text{RP}) \ln \frac{(h_1 + ja_1)}{(h_1 + ja_2)} \frac{(h_2 + ja_2)}{(h_2 + ja_1)}$$

If $Y_i + jZ_i = \ln \frac{(h_1 + ja_1)}{(h_1 + ja_2)} \frac{(h_2 + ja_2)}{(h_2 + ja_1)}$, then

$$\begin{aligned} T_m(x_i) &= -2J' \cos I. Y_i - 2J' \sin I. Z_i \\ &= -J_2 Y_i - J_1 Z_i \end{aligned} \quad (2)$$

where $J_1 = 2J' \sin I$ and $J_2 = 2J' \cos I$.

3.22. Method of fitting.

Let $T_0(x_i)$ be the observed anomaly. Then the sum of squares of the residuals given by $V = \sum_i (T_0(x_i) - T_m(x_i))^2$ is a convenient factor for use as an estimate of the closeness of fit between the observed and computed anomalies. The smaller this sum the better the fit and vice versa. The values of $C_1 \dots C_4$ and J_1 and J_2 which make this sum a minimum are required. An algebraic solution for these values is not possible since V is a non-linear function in the parameters $C_1 - C_4$. However, V can be minimized if a start

is made with a set of assumed values for the parameters and the parameters adjusted in steps leading to smaller values of V . A stage will be reached eventually, when no further decrease in V is possible. The main problem in minimising V is therefore finding adjustments that lead to smaller values of V . Two alternative methods of doing this are considered below.

Method 1.

Assuming a set of initial values for C_1, C_2, C_3 and C_4 ,
1) The values of J_1 and J_2 which give the best fit between $T_0(x_i)$ and $T_m(x_i) = -J_2 Y_i - J_1 Z_i$, are calculated. These values are,

$$J_1 = - \frac{\sum Y_i^2 \times \sum Z_i T_0(x_i) + \sum Y_i Z_i \times \sum Y_i T_0(x_i)}{\sum Z_i^2 \times \sum Y_i^2 - (\sum Y_i Z_i)^2} \quad \text{and}$$

$$J_2 = - \frac{\sum Z_i^2 \times \sum Y_i T_0(x_i) + \sum Y_i Z_i \times \sum Z_i T_0(x_i)}{\sum Z_i^2 \times \sum Y_i^2 - (\sum Y_i Z_i)^2}$$

The anomaly due to the prism corresponding to these values of J_1 and J_2 is subtracted from the observed anomaly to obtain the residual anomaly $R(x_i) = T_0(x_i) - T_m(x_i)$. In addition, $V(C_k)$, the sum of squares of the residuals, is evaluated.

2). The anomaly per unit thickness of each of the sheets which form the sides of the prism are then correlated with the residual anomaly. These anomalies are $\frac{\partial T_m(x_i)}{\partial C_k}$,

and the correlation functions are $b_k = \sum_i R(x_i) \frac{\partial T_m}{\partial C_k}$,

$k = 1 \dots 4$. If the correlation function b_k is positive, C_k has to be increased and vice versa.

3). The values of C_k are increased by amounts U_k , proportional to their respective correlation functions i.e. C_k are made $C_k + U_k$ where $U_k = \mu b_k$, μ being a proportional constant. μ is determined on the assumption that the residual anomaly is due to sheets of thickness (μb_k) placed along the sides of the prism. That is,

$$\sum_i \left\{ R(x_i) - \sum_{k=1}^4 (\mu b_k) \frac{\partial T_m(x_i)}{\partial C_k} \right\}^2,$$

considered as a function of μ , is a minimum.

$$\text{Thus, } \mu = \frac{\sum_i \left\{ R(x_i) - \sum_{k=1}^4 b_k \frac{\partial T_m(x_i)}{\partial C_k} \right\}}{\sum_i \left\{ \sum_{k=1}^4 b_k \frac{\partial T_m(x_i)}{\partial C_k} \right\}^2}$$

4). Step 1 is repeated with the adjusted values of C_k . If the sum of squares of the new residuals happens to be larger than the sum of squares of the residuals calculated in the earlier

stage, that is if $V(C_k + U_k) > V(C_k)$, then $V(C_k + 2^{-s} U_k)$ is evaluated for $s = 1, 2, 3 \dots$ until, for a value of s , $V(C_k + 2^{-s} U_k) < V(C_k)$. When a better fit is obtained, the operations are carried through Step 2, 3 and 4 repeatedly using the new residuals, until one of the following conditions is satisfied, either,

$$(i) \sum_i R^2(x_i) < e_1 \sum_i T_0^2(x_i), \text{ or}$$

$$(ii) R_q^2(x_i) - R_{q+1}^2(x_i) < e_2 R_q^2(x_i).$$

In these, R_q and R_{q+1} are the values of the residuals at the end of the q^{th} and $(q+1)^{\text{th}}$ stage of adjustment and e_1 and e_2 are small fractions. The fulfillment of the first condition corresponds to the stage when a major part of the anomaly has been accounted for by the model and that of the second corresponds to the stage when the rate of minimisation is considered small.

Method 2.

This method consists of the following steps.

Step 1. As in method 1.

Step 2. The residual anomaly is assumed as due to sheets of thickness U_k placed along the sides of the prism and the values of U_k which make

$$\sum_i \left\{ R(x_i) - \sum_{k=1}^4 U_k \frac{\partial T_m}{\partial C_k} \right\}^2$$

a minimum are calculated. These values are obtained by solving the equations

$$\sum_{k=1}^4 \left\{ U_k \sum_i \frac{\partial T_m}{\partial C_k} \frac{\partial T_m}{\partial C_s} \right\} = \sum_i R(x_i) \frac{\partial T_m}{\partial C_s}, \quad s = 1 \dots 4$$

Step 3. The values of C_k are increased by amounts U_k .

Step 4. As in method 1.

While in both methods the residual anomaly at any stage is assumed as due to sheets stacked along the sides of the prism, in the first method the thickness of these sheets are assumed proportional to the respective correlation functions and only one unknown, namely the proportional constant μ is determined, whereas in the second method the thickness of the sheets are assumed unknown and the four thicknesses determined by a least square process. The second method was found to yield better results, but the principle of the first method is utilised in a later chapter for the fitting of a structure of arbitrary shape to an observed anomaly.

Both methods require that the derivatives $\frac{\partial T_m}{\partial C_k}$ be evaluated. These are obtained easily from equation (1). Thus,

$$\begin{aligned}
 \frac{\partial T}{\partial C_1^m} &= \frac{\partial T}{\partial a_1} = \left[\frac{-J_1 h + J_2 a}{a^2 + h^2} \right]_{h_1}^{h_2}, \quad a = a_1 \\
 \times \frac{\partial T}{\partial C_2} &= \frac{\partial T}{\partial a_2} = - \left[\frac{J_1 a + J_2 h}{a^2 + h^2} \right]_{h_1}^{h_2}, \quad a = a_2 \\
 \frac{\partial T}{\partial C_3} &= \frac{\partial T}{\partial h_1} = \left[\frac{J_1 a + J_2 h}{a^2 + h^2} \right]_{a_1}^{a_2}, \quad h = h_1 \\
 \frac{\partial T}{\partial C_4} &= \frac{\partial T}{\partial h_2} = - \left[\frac{J_1 a + J_2 h}{a^2 + h^2} \right]_{a_1}^{a_2}, \quad h = h_2
 \end{aligned}
 \tag{3}$$

$$\tag{4}$$

In carrying out step 4 it has been found useful to depart from the previous procedure and to adopt the following alternative procedure with a view to making full use of the simple scheme given in equation (2) for evaluating the anomaly. This alternative procedure is as follows. The initial increments U_k are calculated as before and $V(C_k + U_k)$ evaluated. One of two situations may occur, either $V(C_k + U_k) > V(C_k)$ or $V(C_k + U_k) \leq V(C_k)$. If $V(C_k + U_k) > V(C_k)$ then the sequence of values $V(C_k + 2^{-s} U_k)$ are calculated for $s = 1, 2, \dots$ until for a value of s , $V(C_k) > V(C_k + 2^{-s} U_k) < V(C_k + 2^{-s+1} U_k)$. The minimum of V in the chosen direction is then assumed to be the minimum of the parabola passing through the above three

values of V and the values of the parameters corresponding to this minimum are evaluated. These values are $C_k + \lambda U_k$, where

$$\lambda = \frac{1}{2^{s+1}} \frac{V(C_k + 2^{-s+1} U_k) - 4V(C_k + 2^{-s} U_k) + 3V(C_k)}{V(C_k + 2^{-s+1} U_k) - 2V(C_k + 2^{-s} U_k) + V(C_k)}$$

If $V(C_k + U_k) \leq V(C_k)$, an improvement has already been obtained. Nevertheless, the sequence of values $V(C_k + s U_k)$ are calculated for $s = 1, 2, \dots$ until for a value of s , $V(C_k + s U_k) > V(C_k + \overline{s-1} U_k) - V(C_k + \overline{s-2} U_k)$. As before, the minimum value in the chosen direction is assumed to be the minimum of the parabola passing through the three values of V and the values of the parameters corresponding to this minimum are evaluated. These values are $C_k + \lambda U_k$ where,

$$\lambda = \frac{s-2}{2} + \frac{1}{2} \frac{V(C_k + s U_k) - 4V(C_k + \overline{s-1} U_k) + 3V(C_k + \overline{s-2} U_k)}{V(C_k + s U_k) - 2V(C_k + \overline{s-1} U_k) + V(C_k + \overline{s-2} U_k)}$$

In both cases after determining λ , steps 2, 3 and 4 are carried out many times until one of the conditions for terminating the adjustments is satisfied.

The above scheme ensures that once a direction leading

to smaller values of V has been found, the maximum possible decrease in V along the direction is obtained.

The Flow Chart for least square fitting is given in appendix III. In this Flow Chart, a device has been introduced for keeping any of the four parameters C_1 , C_2 , C_3 and C_4 fixed during adjustments. This is done by including with the initial values of C_k a set of weighting factors d_k taking a value 1 or 0, depending on whether or not the corresponding C_k is to be adjusted. At any stage of the adjustments the C_k are altered by amounts $d_k \Delta U_k$.

3.23 An illustration of the method.

The method, programmed for the Ferranti Mercury Computer has been tested using the calculated anomalies of a number of theoretical models. In every case a satisfactory result was obtained even though the initial model assumed for computation was far different from the actual model. As an example, two cases of solutions using the same anomaly but with different initial models are illustrated in Fig. 3.1. In this figure the true initial and adjusted models are shown. A representative rate of convergence may be seen from the following tables.

CASE 1

Input: $C_1 = -5 \text{ km}$ $d_1 = 1$)
 $C_2 = 4 \text{ "}$ $d_2 = 1$)
 $C_3 = 1 \text{ "}$ $d_3 = 1$)
 $C_4 = 6 \text{ "}$ $d_4 = 1$)

Initial assumed model
and weighting factors.

Distance -11 -10 -9 -8 -7 -6

$x_i \text{ (km)}$

Anomaly 112 119 126 134 141 147

$T_0 \text{ (s)}$

Distance 1 2 3 4 5 6

$x_i \text{ (km)}$

Anomaly -188 -237 -252 -245 -229 -210

$T_0 \text{ (s)}$

Distance -5 -4 -3 -2 -1 0

$x_i \text{ (km)}$

Anomaly 148 141 119 72 -7 -103

$T_0 \text{ (s)}$

Distance 7 8 9 10 11 12

$x_i \text{ (km)}$

Anomaly -192 -175 -160 -148 -136 -

$T_0 \text{ (s)}$

Output: q C_1 C_2 C_3 C_4 J_1 J_2

 1 -4.15 3.59 4.09 19.46 -72.6 269.9

Successive
stages of
approximation. 2 -0.46 0.56 3.51 53.64 -434.1 1400.4

 3 -0.42 0.68 4.06 86.49 -434.2 1418.5

$$\begin{array}{lcl}
 \text{Parameters of fitted model} & \left(\begin{array}{l} C_1 = -0.421 \\ C_2 = 0.681 \\ C_3 = 4.058 \\ C_4 = 86.488 \\ 2J' = 1484 \times 10^{-5} \text{ e.m.u./c.c.} \\ I = -17^\circ \end{array} \right. & \begin{array}{l} (0) \\ (0) \\ (4.0) \\ (20) \\ (0) \\ (-15^\circ) \end{array} \\
 2J'(C_2 - C_1) & = & 1635 \quad (1600)
 \end{array}$$

The values given in the brackets are the values of the parameters of the actual model producing the anomaly.

For the fitted model, the residual anomaly $R(x_i)$ was as follows:

Distance x_i (km)	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
Residual R (γ)	-1	-1	-1	-1	0	1	2	2	3	2	-3	-3
Distance x_i (km)	1	2	3	4	5	6	7	8	9	10	11	-
Residual R (γ)	-3	-1	2	3	3	2	1	0	-1	-2	-3	-

CASE II

Input: The same as in Case I except that for the initial model $C_3 = 6$ and $C_4 = 9$.

Output:

Successive stages of approximation.	1	-3.06	2.25	7.18	12.18	-270.8	1074
	2	-0.35	-0.29	6.44	17.63	-13022	50278
	3	-0.27	-0.23	4.99	24.82	-19141	72211
	4	-0.17	-0.15	4.28	37.19	-21724	79016
	5	-0.04	-0.02	3.95	65.13	-23695	82651
	6	-0.08	-0.10	4.01	127.60	-24687	83991

$$C_1 = 0.0777 \text{ km} \quad (0)$$

$$C_2 = 0.0961 \text{ " } \quad (0)$$

$$C_3 = 4.01 \text{ " } \quad (4.0)$$

$$C_4 = 127.60 \text{ " } \quad (127)$$

$$2J' = 87540 \times 10^{-5} \text{ e.m.u./cc } (10)$$

$$I = -16^\circ \quad (-15^\circ)$$

$$2J'(C_2 - C_1) = 1602 \quad (1600)$$

Distance	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
x_i (km)												
Residual $R(x)$	-1	0	-1	0	0	1	1	1	1	0	-2	-2
Distance	1	2	3	4	5	6	7	8	9	10	11	-
x_i (km)												
Residual $R(x)$	-1	1	2	2	1	1	0	-1	-1	-2	-2	-

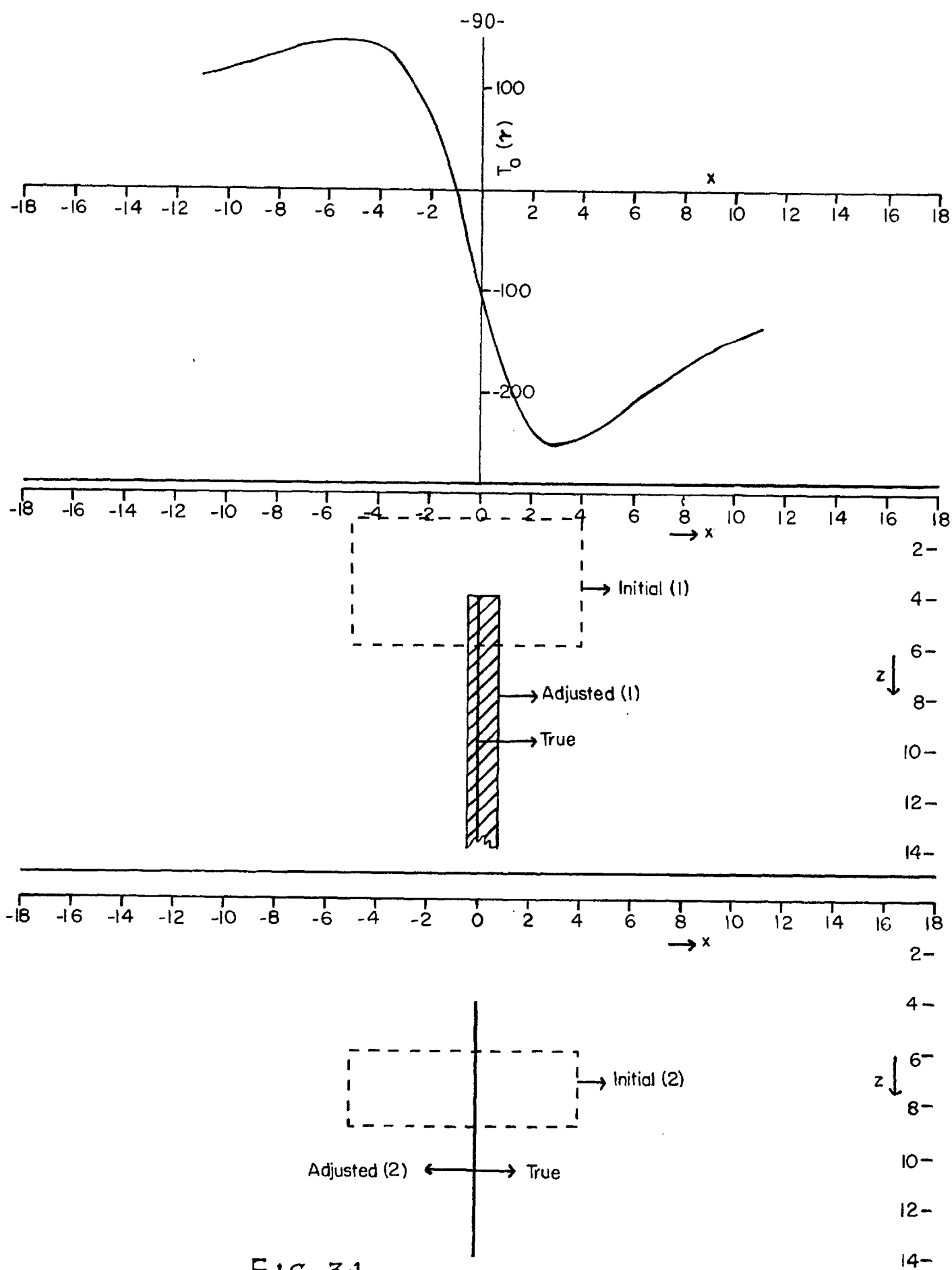


FIG 3.1

The solution took about 25 secs. in the first case and about 40 secs. in the second case. The time for solution on any particular computer depends on the initial values assumed for the parameters and the number of field points used. In the above cases, the initial models were chosen to be greatly different from the actual model. In practice, it would be possible to make a closer initial estimate from a visual picture of the anomaly. The charts given in Chapter 2 for the two dimensional dyke may be used with advantage for the purpose of initial estimation, in which case, the depth to bottom may be chosen arbitrarily as twice the depth to top. In addition, a smaller number of points could be used in the fitting without much loss in accuracy and the solution achieved in less time.

Since the time for solution is small, the computer programme can be used in the routine two dimensional interpretation of observed anomalies.

3.24. Application of the method for testing the accuracy of depth estimation based on rectangular prismatic approximation.

An obvious use to which the programme can be put is to test how far the depth estimates based on a rectangular prismatic approximation represent the actual depth to top of the structure producing the anomaly. The test can be

For the adjusted structure $J' = 1074.8 \times 10^{-5} \text{ e.m.u./c}$

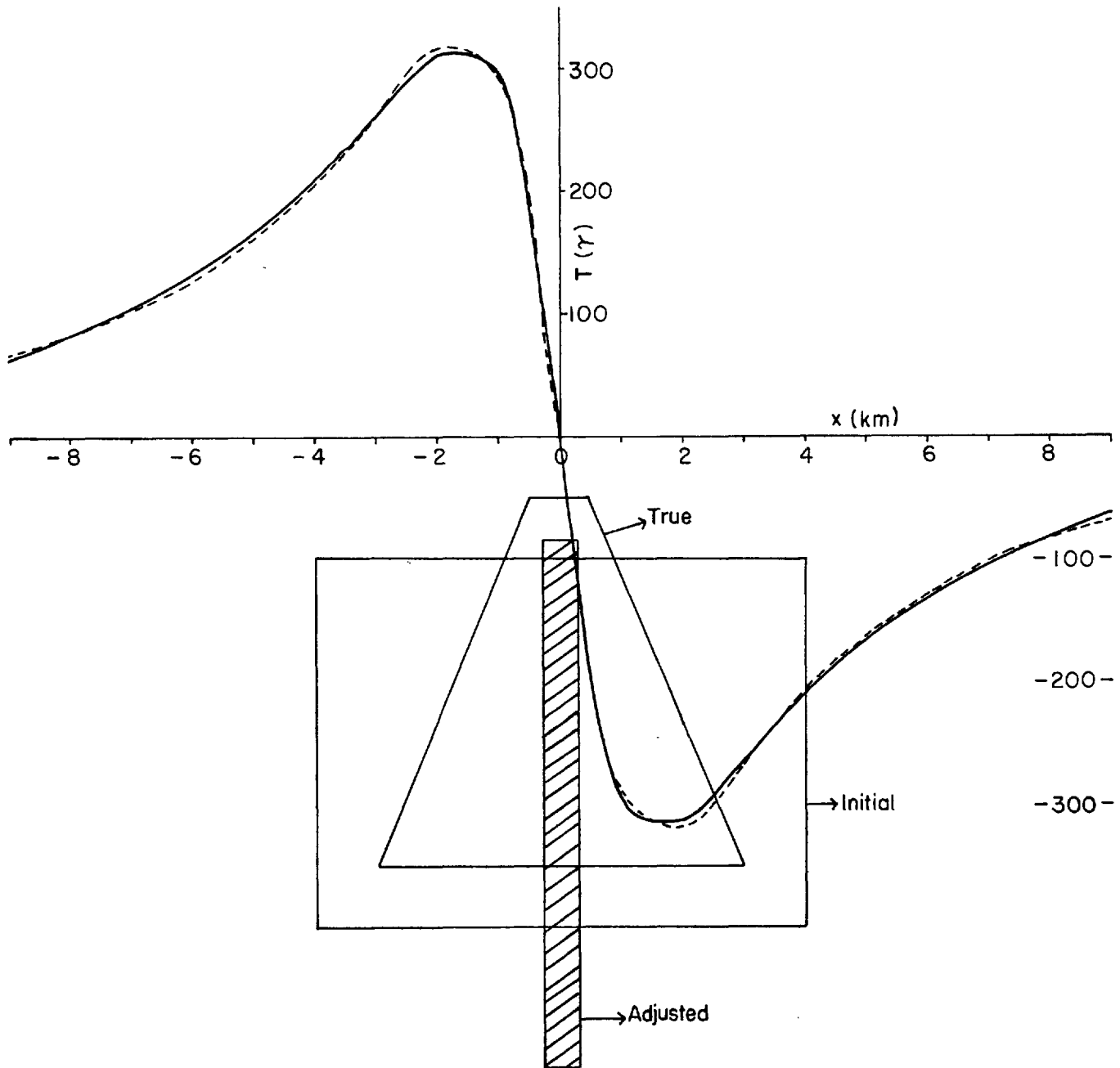


Fig. 3.2

For the adjusted structure $J' = 1483.5 \times 10^{-5}$
e.m.u/c.c

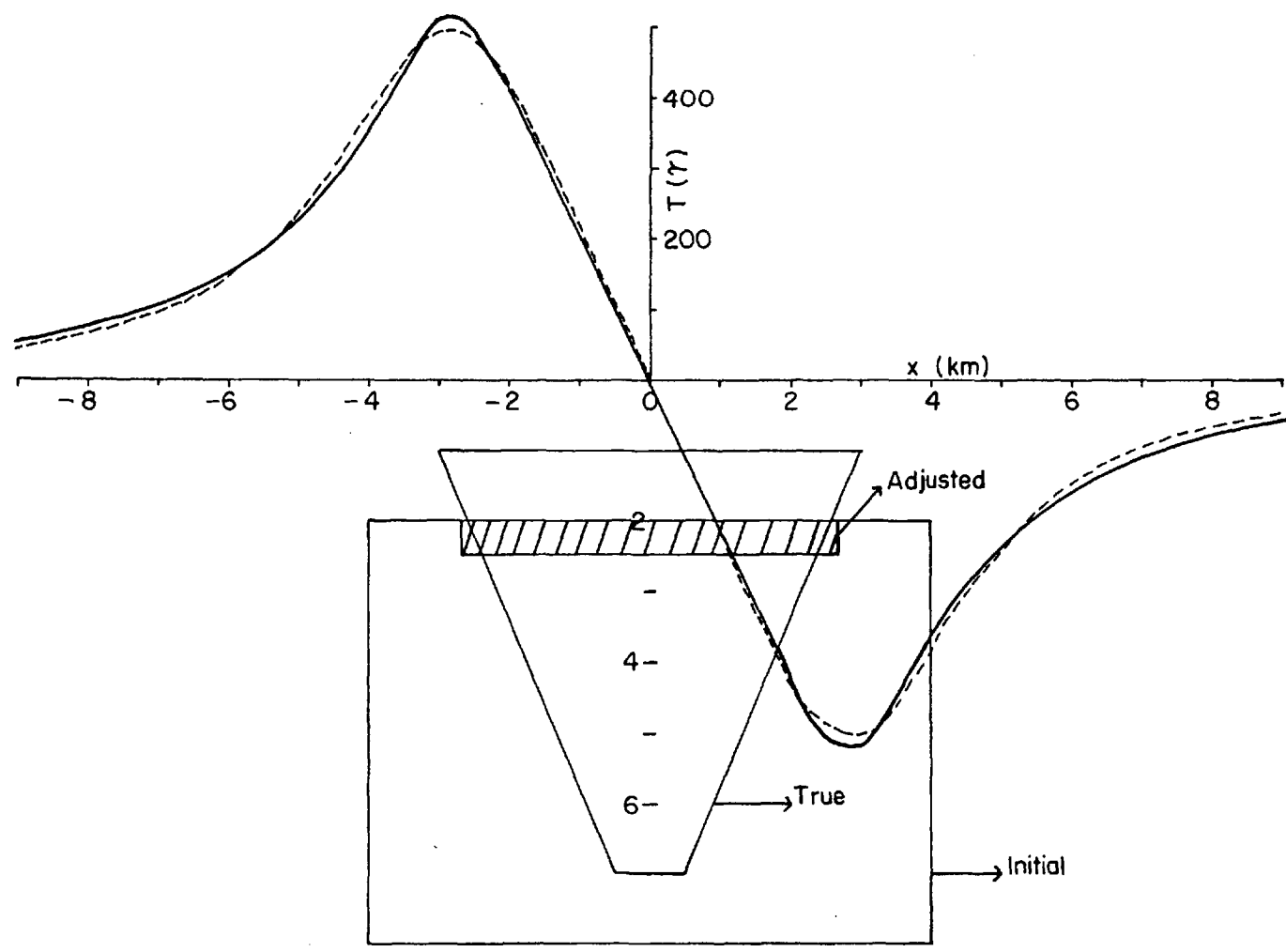


Fig.3-3

For the adjusted structure $J' = 199.7 \times 10^{-5}$
e.m.u./c.c

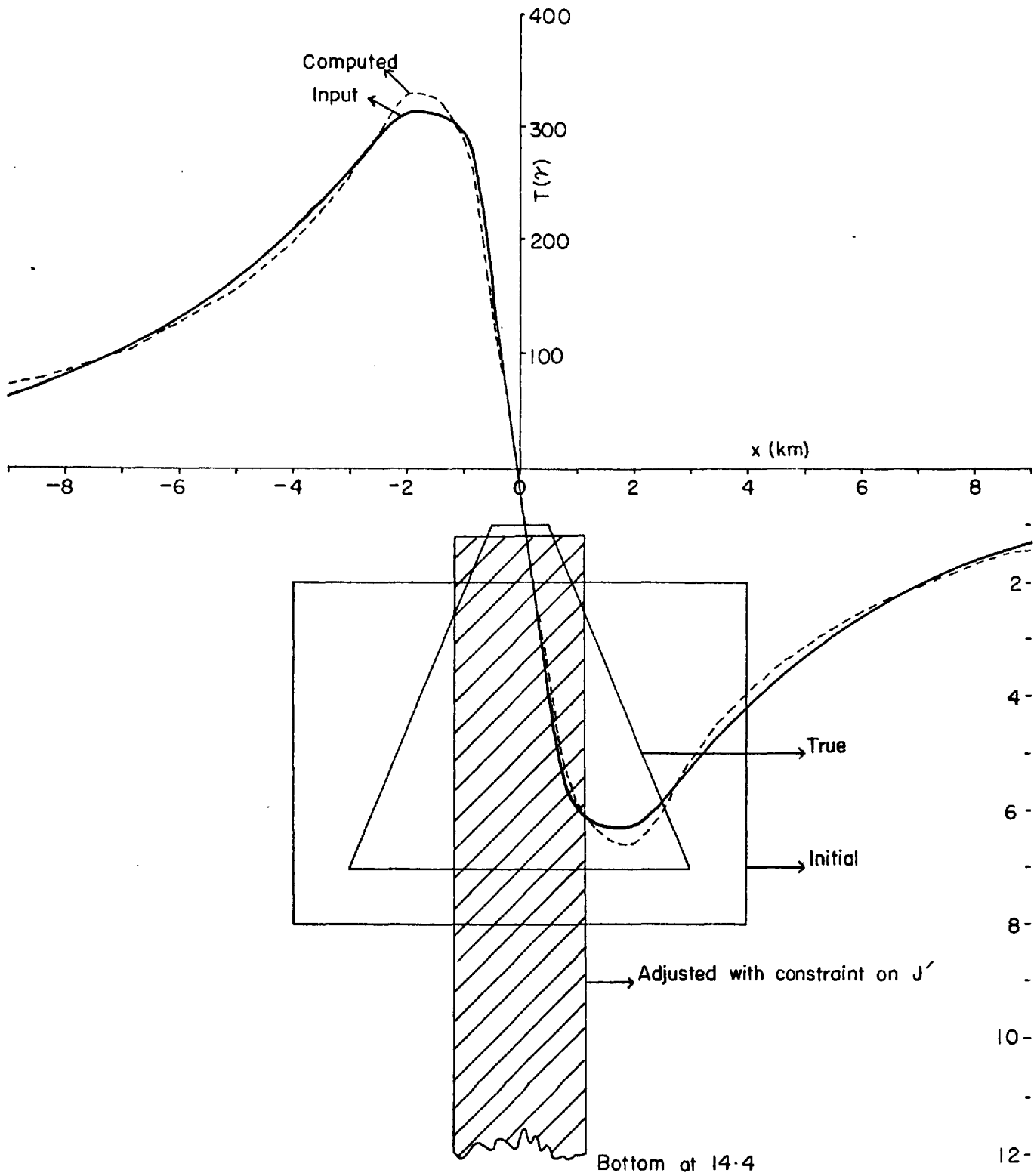


Fig. 3-4

For the adjusted structure $J' = 199.6 \times 10^5 \text{ e.m.u./ce}$

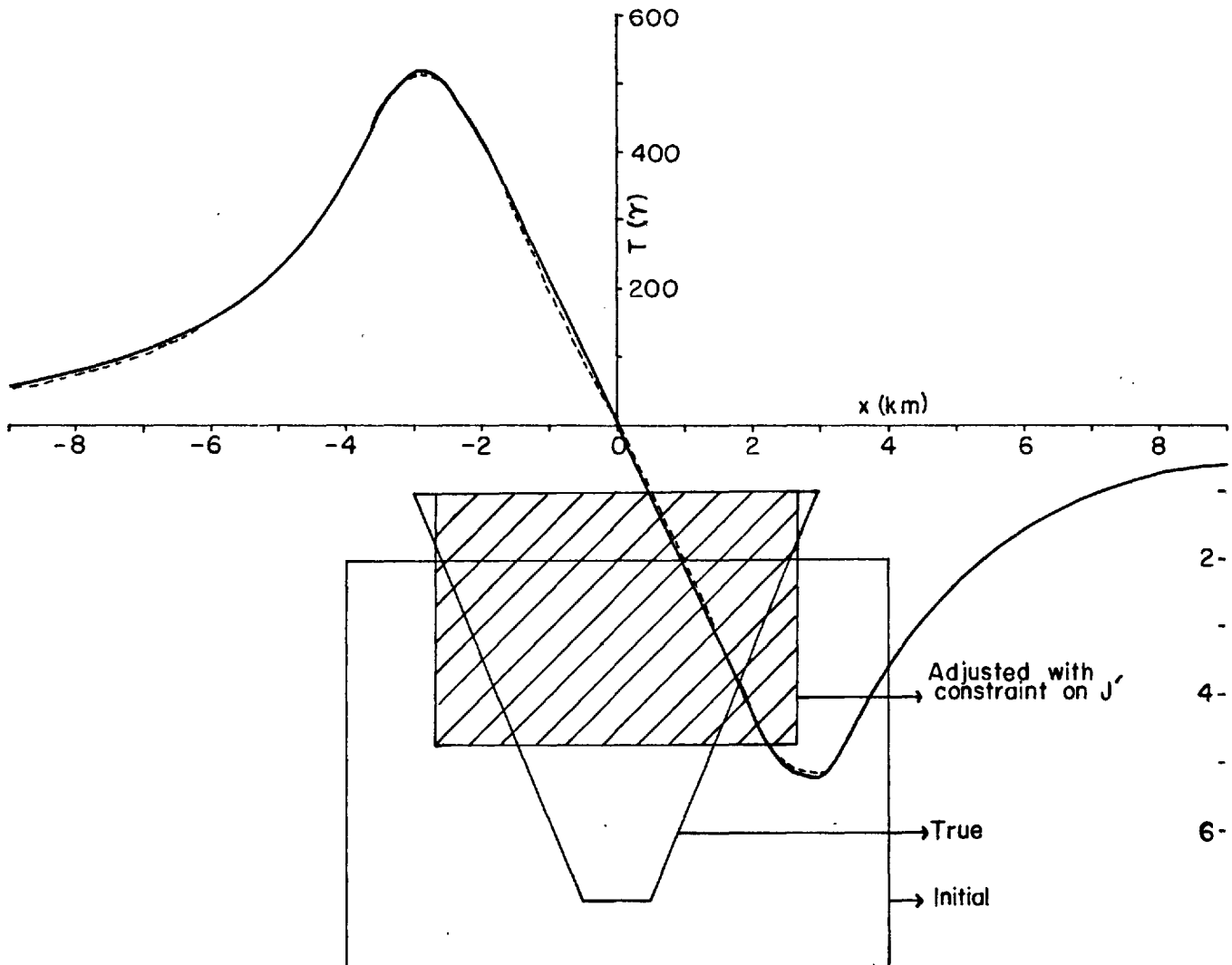


Fig.3-5

performed by fitting a rectangular prism to anomalies due to models which are not rectangular prisms. As an example, the fitting of a rectangular prism to the anomalies due to two trapezoidal prisms is illustrated in Figs. 3.2 and 3.3. The anomalies were calculated for $J' = 200 \times 10^{-5} \frac{c \cdot m \cdot u}{c \cdot c}$ and $I = 0$. It is interesting to note that rectangular prisms with dimensions quite different from those of the chosen trapezoidal prisms, produce almost the same anomaly as these prisms. In practice it would be impossible to distinguish between the two structures from the anomaly alone.

When, for the sake of interest, the intensity $J' = \frac{1}{2} (J_1^2 + J_2^2)^{\frac{1}{2}}$ was reset to the true value $200 \times 10^{-5} \frac{c \cdot m \cdot u}{c \cdot c}$ during adjustments whenever J' exceeded this value, a better estimate of the parameters, especially that of the depth to top, was obtained. (Figs. 3.4 and 3.5).

3.3 The three dimensional vertical rectangular prism

3.31 Method of fitting

The principle of the method of fitting used for the two dimensional prism can be easily extended to the three dimensional case. As before, the only requirement is that the anomaly of the prism and the anomaly per unit thickness of the sheets which form the sides of the prism be evaluated.

3.32 The anomaly due to the prism

An expression for the total magnetic anomaly due to a rectangular prism was derived in Chapter 2. It was shown that

$$T_m(x_i, y_i, 0) = \left[\left[\left[E_1 \tan^{-1} \frac{bh}{Ra} + E_2 \tan^{-1} \frac{ah}{Rb} + E_3 \ln(R+a) \right. \right. \right. \\ \left. \left. + E_4 \ln(R+b) + E_5 \ln(R+h) \right] \int_{h_1}^{h_2} \int_{b_1}^{b_2} \int_{a_1}^{a_2} \right] \quad (1)$$

where $E_1 = J_3^n - J_1^l$, $E_2 = J_3^h - J_2^m$,
 $E_3 = J_2^n + J_3^m$, $E_4 = J_1^n + J_3^l$,
 $E_5 = J_1^m + J_2^l$, and referring to Fig. 3.6.
 $a_1 = C_1 - x_i$, $a_2 = C_2 - x_i$
 $b_1 = C_3 - y_i$, $b_2 = C_4 - y_i$
 $h_1 = C_5$, $h_2 = C_6$
 $R = (a^2 + b^2 + h^2)^{\frac{1}{2}}$

The co-ordinates of the corners of the prism and the field point P, are with respect to a set of axes chosen parallel to the sides of the prism. In the following it is assumed that the orientation of the prism is known and that x, y, z have been chosen parallel and perpendicular to the sides of the prism.

It should be noted that the expression (1) for the anomaly of the prism can be re-written in the

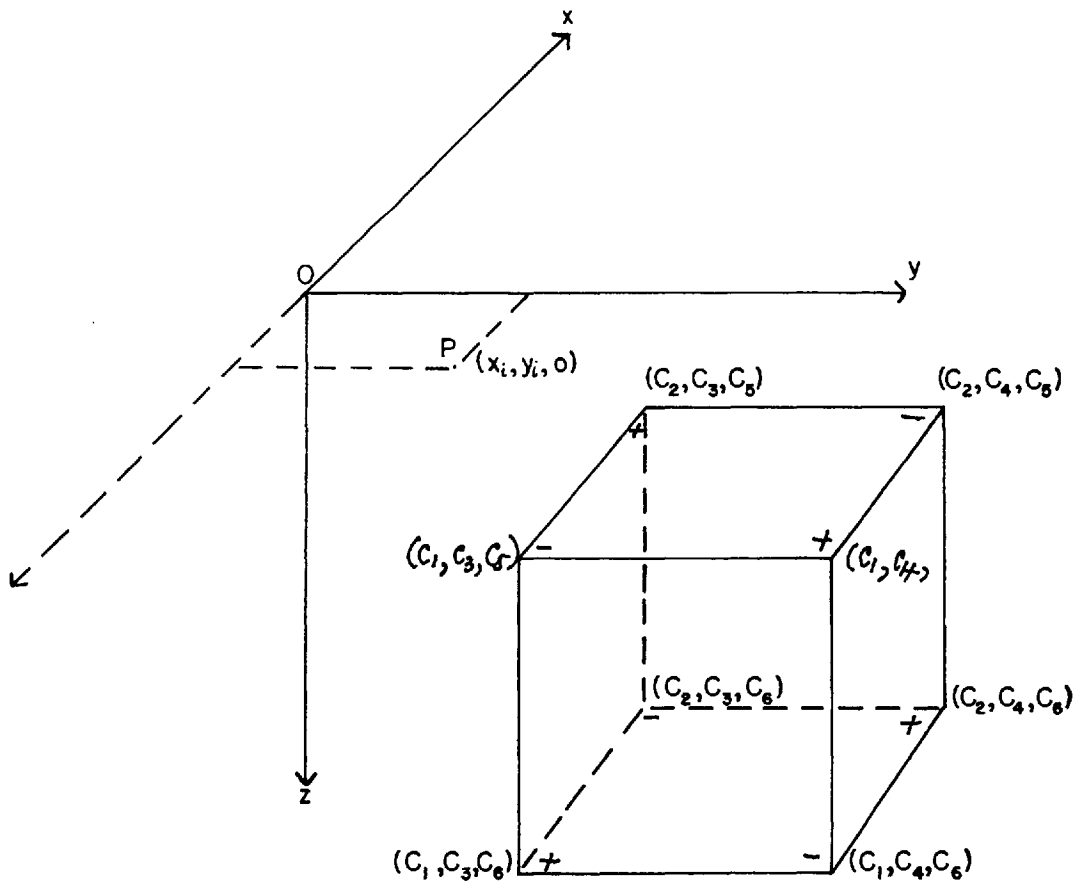


Fig.3.6

simple form

$$\begin{aligned}
 T_m(x_i, y_i, 0) = & E_1 \tan^{-1} \left\{ \frac{\text{I.P.}}{\text{R.P.}} \text{ of } \prod (Ra + jsbh) \right\} + \\
 & E_2 \tan^{-1} \left\{ \frac{\text{I.P.}}{\text{R.P.}} \text{ of } \prod (Rb + jsh) \right\} + \\
 & E_3 \ln \prod (R+a)^s + E_4 \ln \prod (R+b)^s + \\
 & E_5 \ln \prod (R+h)^s \quad (2)
 \end{aligned}$$

where the product $\prod$ is the product of the appropriate functions evaluated over the eight corners of the prism. s is +1 for corners marked + and is -1 for corners marked -.

3.33 Expression for the anomaly per unit thickness of the sheets which form the sides of the prism.

These anomalies are given by $\frac{\partial T_m}{\partial C_j}$, $j = 1 \dots 6$.

Denoting the expression inside the brackets in (1) by $F(a, b, h)$, we get after some algebraic manipulation,

$$\begin{aligned}
 \frac{\partial F}{\partial a} &= \frac{b(E_1 - E_2) + aE_5}{R(R+h)} + \frac{hE_1 + aE_4}{R(R+b)} + \frac{E_3}{R} + \frac{E_1 - E_2}{a^2 + b^2} \cdot b. \\
 \frac{\partial F}{\partial b} &= \frac{a(E_2 - E_1) + bE_5}{R(R+h)} + \frac{hE_2 + bE_3}{R(R+a)} + \frac{E_4}{R} + \frac{E_2 - E_1}{a^2 + b^2} \cdot a. \\
 \frac{\partial F}{\partial h} &= \frac{hE_4 - aE_1}{R(R+b)} + \frac{hE_3 - bE_2}{R(R+a)} + \frac{E_5}{R} + \frac{aE_1}{a^2 + h^2} + \frac{bE_2}{b^2 + h^2}
 \end{aligned}$$

$$\text{Now } \frac{\partial T_m}{\partial C_1} = \frac{\partial T_m}{\partial a_1} = - \left[\left[\frac{\partial F}{\partial a} \right] \begin{matrix} h_2 & b_2 \\ \int & \int \\ h_1 & b_1 \end{matrix} \right], \quad a = a_1 \quad (3)$$

$$\frac{\partial T_m}{\partial C_2} = \frac{\partial T_m}{\partial a_2} = \left[\left[\frac{\partial F}{\partial a} \right] \begin{matrix} h_2 & b_2 \\ \int & \int \\ h_1 & b_1 \end{matrix} \right], \quad a = a_2$$

$$\frac{\partial T_m}{\partial C_3} = \frac{\partial T_m}{\partial b_1} = - \left[\left[\frac{\partial F}{\partial b} \right] \begin{matrix} h_2 & a_2 \\ \int & \int \\ h_1 & a_1 \end{matrix} \right], \quad b = b_1 \quad (4)$$

$$\frac{\partial T_m}{\partial C_4} = \frac{\partial T_m}{\partial b_2} = \left[\left[\frac{\partial F}{\partial b} \right] \begin{matrix} h_2 & a_2 \\ \int & \int \\ h_1 & a_1 \end{matrix} \right], \quad b = b_2$$

$$\frac{\partial T_m}{\partial C_5} = \frac{\partial T_m}{\partial h_1} = - \left[\left[\frac{\partial F}{\partial h} \right] \begin{matrix} b_2 & a_2 \\ \int & \int \\ b_1 & a_1 \end{matrix} \right], \quad h = h_1 \quad (5)$$

$$\frac{\partial T_m}{\partial C_6} = \frac{\partial T_m}{\partial h_2} = \left[\left[\frac{\partial F}{\partial h} \right] \begin{matrix} b_2 & a_2 \\ \int & \int \\ b_1 & a_1 \end{matrix} \right], \quad h = h_2$$

For the sake of simplicity, the direction of magnetisation was assumed to be along the direction of the earth's field, i.e. $J_1 = lJ$, $J_2 = mJ$ and $J_3 = nJ$ where J is the intensity of magnetisation. The procedure for determining the parameters $C_1 \dots C_6$ and J is then similar to that outlined for the two dimensional case. The residuals at any stage of adjustment are calculated for that particular value of J which gives the best fit between the anomaly of the model

and the observed anomaly.

The calculations have been programmed for the Ferranti-Mercury Computer. The programme was tested using the calculated anomalies of a number of models. As in the two dimensional case, satisfactory results were obtained, starting from arbitrary initial values. As an example, the solution for the parameters of the prism which produces the anomaly given in Fig. 2.29, is illustrated in Figs. 3.7(a) and 3.7(b). Three profiles on this anomaly were used in the fitting. These were the North South profile passing through the maximum and the minimum, the East-West profile passing through the maximum and the East-West profile passing through the minimum. Altogether thirty-five field points were used. The time taken for solution on the Ferranti-Mercury Computer was $2\frac{3}{4}$ minutes. This time is much more than the time mentioned previously for the two dimensional prism, and is due to the more complicated nature of the expression for the anomaly.

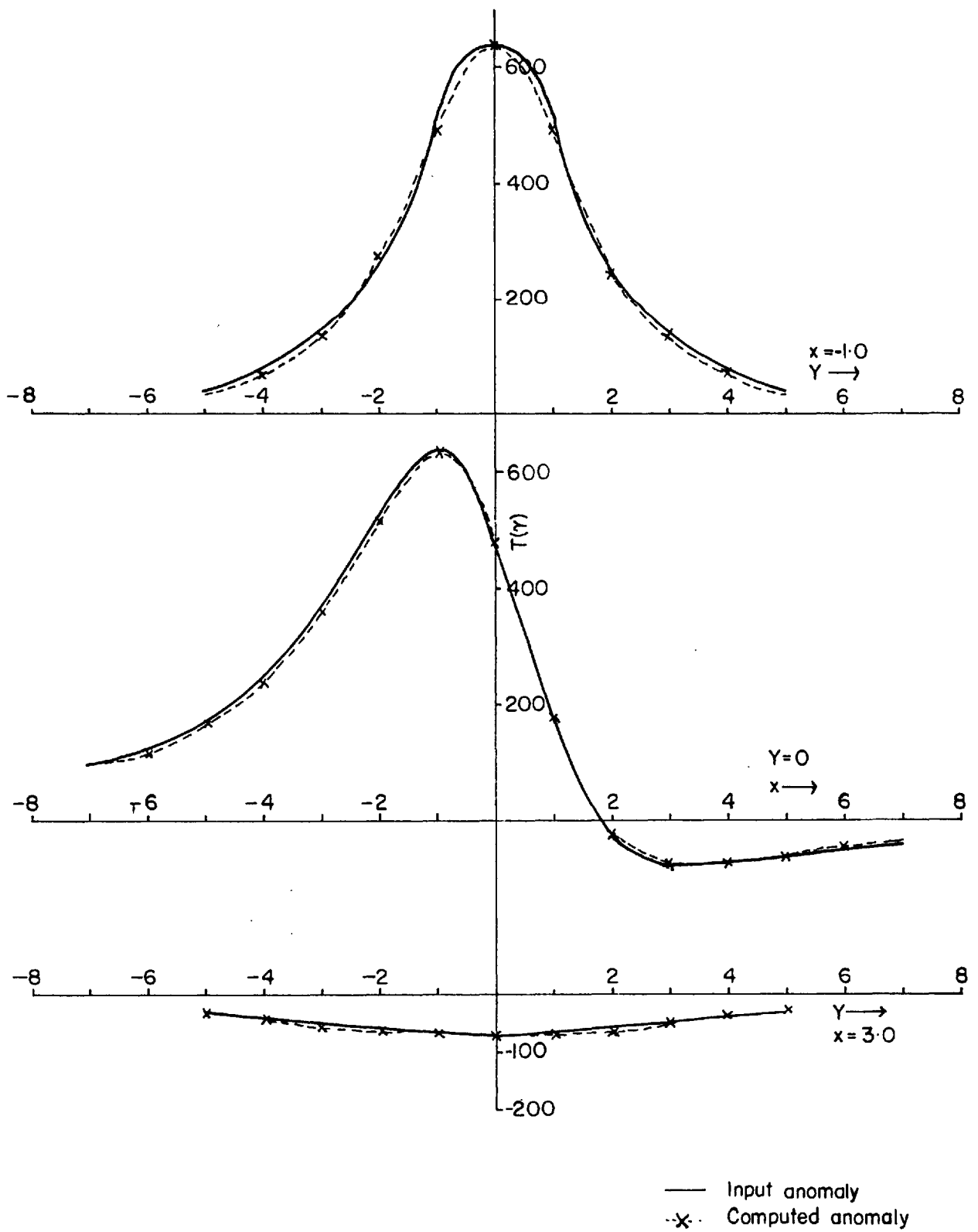


Fig. 3.7 (a)

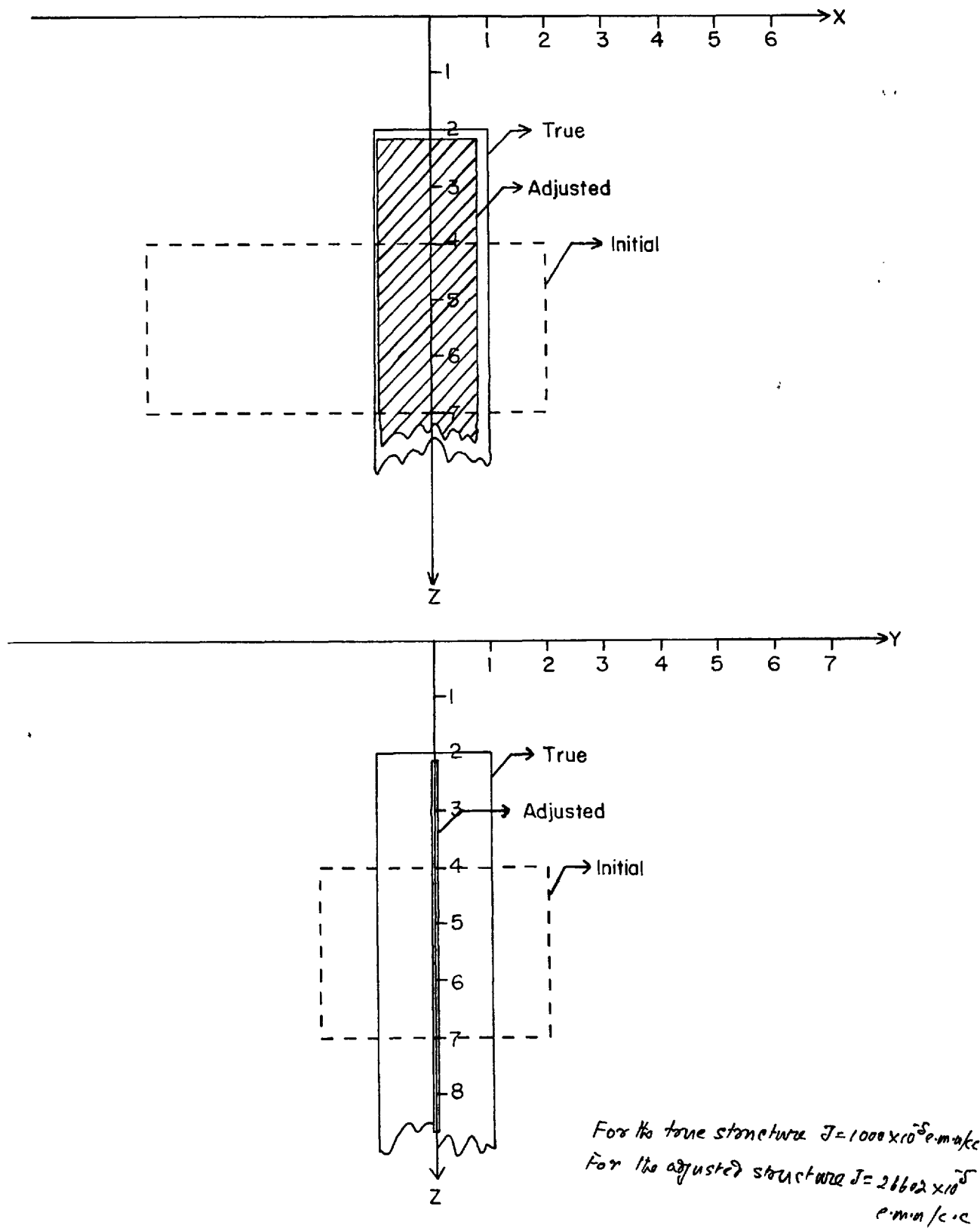


FIG 3.7 (b)

CHAPTER 4

AUTOMATIC STRUCTURE FITTING

4.1 Introduction

As mentioned in the introductory chapter, it would be possible, in theory, to find an infinite number of structures which would produce a given observed anomaly. However, in certain cases, limitations can be imposed so that the structure becomes unique, or is constrained to lie in a certain range. The limitations may be, for example, a given density or susceptibility contrast between the structure and the surroundings or a given depth to the top as revealed by drilling. Reference was cited to a case of automatic structure fitting for gravity interpretation with the imposition of such limitations. The method was based on the fact that the vertical gravity anomaly appears directly above the anomaly in the distribution of mass causing it.

A direct extension of this method to magnetic interpretation is not possible as, in general, the magnetic anomaly will be displaced relative to the anomaly in the dipole distribution. An obvious way to overcome this difficulty is to combine the method devised for structure fitting in gravity with a technique devised by Baranov⁽¹³⁾ for transforming the magnetic

anomaly into pseudogravimetric anomalies. In this chapter other alternative procedures are considered.

4.2 Formulation of the general problem for the two dimensional case.

Let the structure producing the observed anomaly be considered to be composed of a number of vertical rectangular prisms, not necessarily of equal width, arranged along the x-axis in a predetermined way. By increasing the number of such rectangular prisms the model can be made to approximate to any real structure with any desired accuracy. Let h_{1p} and h_{2p} be the depths to the top and bottom of the p^{th} block. The problem is to find some of the h_{1p} and h_{2p} , for an assumed direction and intensity of magnetisation, leaving the remaining h_{1p} and h_{2p} at known values, and thus get one structure, whether unique or not, which will account for the observed anomaly.

4.21 Method of solution.

The principle of the method is the same as that of the first method described in the previous chapter. Initial values for h_{1p} and h_{2p} are assumed, and

1. The anomaly $T_m(x_i)$ of the resulting structure is subtracted from the observed anomaly, to obtain the

residual anomaly $R(x_i)$. The sum of squares of these residuals, denoted by $V(h_{1p}, h_{2p})$, is also evaluated.

2. The anomalies per unit thickness of the sheets at the depths h_{1p} and h_{2p} and of width equal to the width of the p^{th} block are then correlated to the residual anomaly.

The anomalies of the sheets are $\frac{\partial T_m}{\partial h_{1p}}(x_i)$ and $\frac{\partial T_m}{\partial h_{2p}}(x_i)$

and the correlation functions are

$$b_{1p} = \sum_i R(x_i) \frac{\partial T_m(x_i)}{\partial h_{1p}} \quad \text{and}$$

$$b_{2p} = \sum_i R(x_i) \frac{\partial T_m(x_i)}{\partial h_{2p}}$$

If any of h_{1p} or h_{2p} is to be kept constant, the correlation function corresponding to it is made zero.

3. h_{1p} and h_{2p} are increased by amounts $U_{1p} = \mu b_{1p}$ and $U_{2p} = \mu b_{2p}$, where

$$\mu = \frac{\sum_i \left\{ R(x_i) \sum_p \left(b_{1p} \frac{\partial T_m(x_i)}{\partial h_{1p}} + b_{2p} \frac{\partial T_m(x_i)}{\partial h_{2p}} \right) \right\}}{\sum_i \left\{ \sum_p \left(b_{1p} \frac{\partial T_m(x_i)}{\partial h_{1p}} + b_{2p} \frac{\partial T_m(x_i)}{\partial h_{2p}} \right)^2 \right\}^2}$$

4. If $V(h_{1p} + U_{1p}, h_{2p} + U_{2p}) > V(h_{1p}, h_{2p})$, the sequence of values $V(h_{1p} + 2^{-r} U_{1p}, h_{2p} + 2^{-r} U_{2p})$

are calculated for $r = 1, 2, \dots$ until for a value of r , $V(h_{1p} + 2^{-r} U_{1p}, h_{2p} + 2^{-r} U_{2p}) < V(h_{1p}, h_{2p})$. When a better fit is obtained, steps 2, 3 and 4 are carried out repeatedly with the new residuals until $\sum_j R^2(x_i) < \epsilon \sum_j T_0^2(x_i)$, that is, until a major part of the observed anomaly has been accounted for by the structure.

4.22 Expression for the anomaly of the structure

Let a_p and a_{p+1} be the distances of the vertical sides of the p^{th} block from the point at which the anomaly is measured. Then, the anomaly due to the p^{th} block is equal to $-J_2 Y_{ip} - J_1 Z_{ip}$, where $J_2 = 2J' \cos I$, $J_1 = 2J' \sin I$ and

$$Y_{ip} + jZ_{ip} = \ln \frac{(h_{1p} + ja_p)(h_{2p} + ja_{p+1})}{(h_{1p} + ja_{p+1})(h_{2p} + ja_p)}$$

(Refer Chapter 3, equation (2)).

The anomaly due to the entire structure is given by

$$\begin{aligned} T_m(x_i) &= -J_2 \sum_p Y_{ip} - J_1 \sum_p Z_{ip} \\ &= -J_2 Y_i - J_1 Z_i \end{aligned} \quad (1)$$

$$\text{where } Y_i + jZ_i = \sum_p (Y_{ip} + jZ_{ip})$$

$$\begin{aligned} &= \sum_p \ln \frac{(h_{1p} + ja_p)(h_{2p} + ja_{p+1})}{(h_{1p} + ja_{p+1})(h_{2p} + ja_p)} \\ &= \ln \prod_p \frac{(h_{1p} + ja_p)(h_{2p} + ja_{p+1})}{(h_{1p} + ja_{p+1})(h_{2p} + ja_p)} \end{aligned} \quad (2)$$

Thus, the anomaly of a two dimensional structure which is assumed to consist of a number of vertical rectangular prisms, can be evaluated in a very simple way. This is an advantage as in the method outlined for the structure fitting the anomaly due to the structure has to be evaluated many times.

4.23 Expression for the derivatives of the anomaly with respect to the depths.

The derivatives $\frac{\partial T_m}{\partial h_{1p}}$ and $\frac{\partial T_m}{\partial h_{2p}}$ are the same as the derivatives with respect to h_{1p} and h_{2p} of the anomaly due to the p^{th} block. They are obtained by substituting the appropriate values of a_1 , a_2 , h_1 and h_2 in equation (4) Chapter 3.

Thus,

$$\frac{\partial T_m}{\partial h_{1p}} = \left[\frac{J_1 a + J_2 h}{a^2 + h^2} \right]_{a_p}^{a_{p+1}}, \quad h = h_{1p}$$

and

$$\frac{\partial T_m}{\partial h_{2p}} = - \left[\frac{J_1 a + J_2 h}{a^2 + h^2} \right]_{a_p}^{a_{p+1}}, \quad h = h_{2p}$$

4.24 The flow chart for a special case.

The flow chart, given in appendix IV, has been prepared for the special case when all the h_{1p} are to be kept constant at the same value h say. Further, each

block is assumed to have the same width d and the number of blocks is equal to the number of observation points, the observation points being directly above the centre of each block. The initial values for h_{2p} are built in the programme and are equal to h . This eliminates the need for making an initial guess about the depth to the bottom. In cases where the mean depth to bottom may be ascertained roughly this depth may be used instead of h as the initial values for h_{2p} and the solution achieved in less time. It will be shown later that the programme written for finding h_{2p} when all the h_{1p} are kept constant can also be used for the case when all the h_{2p} are kept constant and the h_{1p} are to be determined.

A simplification of the expression for the anomaly is possible, when the top or bottom surface of the structure is flat. Assuming that $h_{1p} = h$, $p = 1 \dots N$, where N is the number of blocks.

$$\prod_{p=1}^N \frac{(h_{1p} + j a_p)}{(h_{1p} + j a_{p+1})} = \frac{h + j a_1}{h + j a_2} \cdot \frac{h + j a_2}{h + j a_3} \dots \frac{h + j a_N}{h + j a_{N+1}} = \frac{h + j a_1}{h + j a_{N+1}}$$

Thus, from (2)

$$Y_i + j Z_i = \ln \left\{ \frac{h + j a_1}{h + j a_{N+1}} \prod_{p=1}^N \frac{h_{2p} + j a_{p+1}}{h_{2p} + j a_p} \right\} \quad (3)$$

$$\text{and } T_m(x_i) = -J_2 Y_i - J_1 Z_i.$$

4.25 An example of application

A profile across a north-south trending total magnetic anomaly is shown in Fig. 4.1. This anomaly was obtained by Mason R.G., in his magnetic survey off the west coast of the United States. The intensity, inclination and declination of the Earth's field at the location of the anomaly are $50,000\gamma$, 59° and 17°E respectively. A two dimensional interpretation of the anomaly by Mason, using the classical approach of trial and error solution, has appeared elsewhere (12). In the following, the applicability of the method developed in this section for such an interpretation is demonstrated.

A flat top surface at the ocean floor was assumed for the structure producing this anomaly. Assuming that the magnetism is entirely induced, the shape of the bottom surface was calculated for two values of susceptibility contrast. These are shown in Figs. 4.2(a) and 4.2(b). For the first case, the bottom of the calculated structure penetrates into the mantle and therefore the assumed susceptibility contrast of 4×10^{-3} cgs is too low. For the second case for a susceptibility contrast of 8×10^{-3} cgs a plausible solution was obtained. A third case, illustrated in Fig. 4.2(c) represents the solution for a susceptibility contrast of 12×10^{-3} on the assumption of a flat bottom for the structure at 6.3 km (base of the volcanics). The programme

was made to carry on the adjustments upwards by assigning to the susceptibility the negative of the value intended. The correlation functions then have opposite sign and the adjustments will proceed in the opposite direction. The anomaly of the structure, however, will be evaluated correctly, as the change in sign of the susceptibility and hence of J_1 and J_2 will be annulled by the change in sign of the function given in equation (2) when $h_{2p} < h_{1p}$.

The solutions for the three cases were accomplished in about ten stages of approximation, each stage taking about 20 seconds on the Ferranti Mercury Computer. In all the three cases, after satisfactory reduction of the residuals at the initial stages, a stage was eventually reached when further reduction of the residuals was very slow. The results, printed out at the appropriate stages when the method became slow, are shown in the following table. It is seen that the residual anomaly in gammas denoted by $R_p(r)$ has a maximum value of 35 in the first case and 7.8 in the second case. Thus, from a consideration of the closeness of fit alone, the second case represents a more probable solution than the first case.

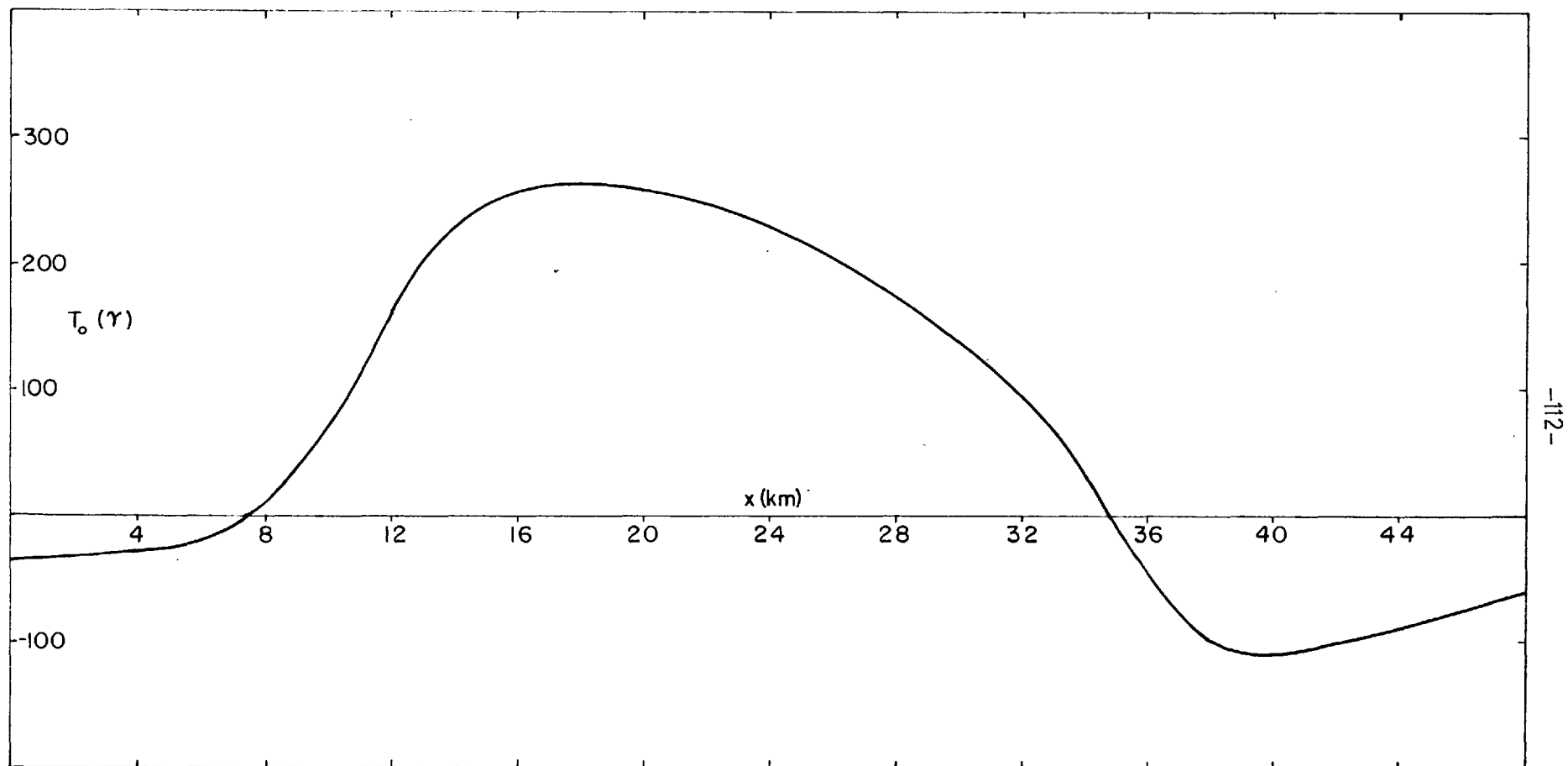


Fig. 4.1

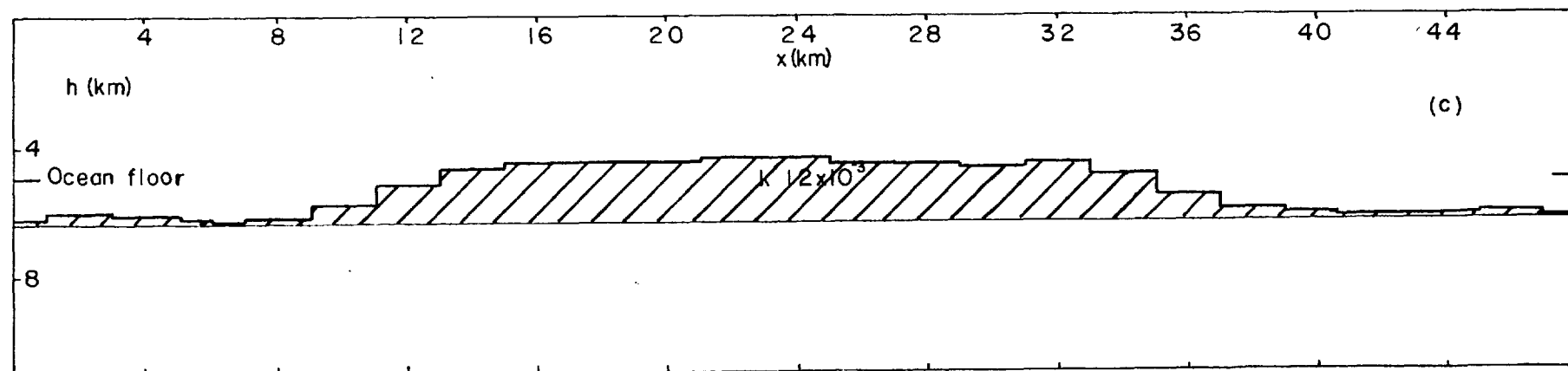
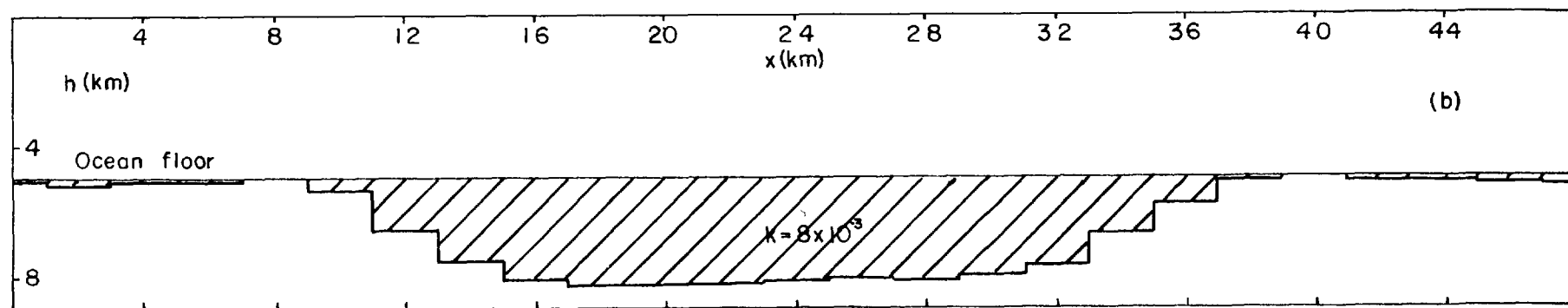
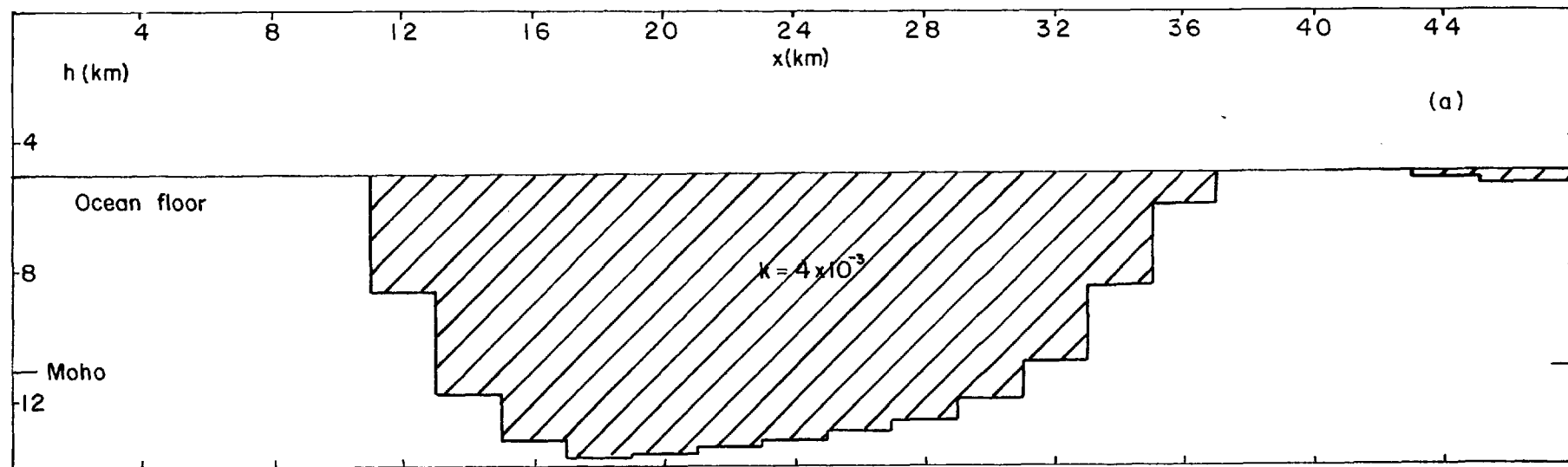


Fig. 4.2

Case I: $K=4 \times 10^{-3} \text{ cgs}$

$h_1 p = 5 \text{ km}$

Case II: $K = 8 \times 10^{-3} \text{ cgs}$

$h_1 p = 5 \text{ km}$

p	$T_0(r)$	$h_{ap}(\text{km})$	$R_p(r)$	$h_{ap}(\text{km})$	$R_p(r)$
1	-35	5.01	1.2	5.10	-2.3
2	-31	5.00	1.3	5.20	0.1
3	-27	5.00	-5.2	5.17	5.0
4	-20	5.00	-2.1	5.06	3.9
5	11	5.00	-32.8	5.00	0.9
6	75	5.00	-35.0	5.27	-6.8
7	162	8.64	-21.4	6.58	-2.1
8	230	11.80	-6.9	7.59	3.9
9	257	13.20	-6.4	8.11	0.0
10	265	13.70	-5.3	8.28	0.9
11	259	13.60	-6.1	8.28	1.7
12	248	13.40	-4.1	8.24	4.0
13	230	13.20	-3.4	8.19	2.4
14	206	12.90	-3.1	8.17	-2.3
15	175	12.60	-2.9	8.16	-7.8
16	140	11.90	2.5	8.02	-6.1
17	98	10.70	12.7	7.66	4.4
18	34	8.48	12.2	6.77	7.8
19	-41	5.99	2.4	5.80	3.5
20	-100	5.00	-8.7	5.13	-2.9
21	-112	5.00	-3.2	5.01	4.2
22	-103	5.00	-0.7	5.09	5.2
23	-89	5.23	-2.9	5.18	1.7
24	-75	5.36	-1.9	5.22	2.3
25	-63	5.29	5.3	5.14	8.4

4.26 An alternative method

When the structure has a flat top or bottom at a known depth and its vertical extent is known to be small compared to its depth to the top, another more direct method can be developed. In this case, the structure can be assumed, as a first approximation, to be composed of a number of

horizontal sheets of varying strengths, the strength being defined as the effective intensity of magnetisation J' times the thickness of the sheet. These strengths can be obtained by solving a set of linear equations. Thus, if U_p denotes the strength of the p^{th} sheet and A_{ip} its anomaly per unit strength at the point where the observed anomaly is T_i ,

$$T_i = \sum_{p=1}^N A_{ip} U_p$$

Using vector notations, $\bar{T} = \bar{A} \bar{U}$ or

$$U = \bar{A}^{-1} \bar{T} \quad (1)$$

where $\bar{T}$ is the vector $(T_1 \dots T_N)$, $\bar{U}$ the vector $(U_1 \dots U_N)$ and $\bar{A}$ the matrix whose elements are A_{ip} .

Noting that the anomaly per unit thickness of the sheet for an effective intensity of magnetisation J' is

$$= -2J' \left[\frac{a \sin I + h \cos I}{a^2 + h^2} \right]_{a_p}^{a_{p+1}},$$

where the symbols have the meanings given previously,

$$A_{ip} = -2 \left[\frac{a \sin I + h \cos I}{a^2 + h^2} \right]_{a_p}^{a_{p+1}}$$

After obtaining the strengths of the sheets using the relation (1), they are converted into effective thicknesses by dividing by J' . The anomaly due to the resulting structure, which is composed of blocks of thickness

$\frac{U}{J_p}$, $p = 1 \dots N$, is then calculated and subtracted from the observed anomaly. If the sheet approximation is correct, the resulting residual anomaly R_1 will be small and the solution may be considered complete. Otherwise, the method aims at reduction of these residuals by successive approximations. At each stage of approximation, the thicknesses of sheets at the depth h which will account for the residual anomaly are calculated and added to the bottom of the blocks. The new residuals are calculated and the process repeated many times until a stage is reached when either the residuals are considered small or the rate of reduction of the residuals is slow. If $\bar{R}$ denotes the vector $(R_1 \dots R_N)$ where the R_1 are the residuals at a particular stage, then the thicknesses of the sheets added to the bottom of the blocks at this stage is $\frac{1}{J}$, $\bar{A}^{-1} \bar{R}$.

The method has the advantage that once the matrix $\bar{A}$ is inverted for a particular value of intensity of magnetisation, the inverted matrix can be used for calculations using other values of intensity. In addition, in problems like basement relief calculations where the relief of the basement is small compared to its depth, the solution would be accomplished in one step.

The method was used in the solution of the problem considered in 4.25 for two more values of susceptibility. The solutions are illustrated in Figs. 4.3a and 4.3b and the numerical results corresponding to these figures are given in the table below.

The most time consuming part of the method was the inversion of the matrix $\bar{A}$ which took about 2 minutes on the Ferranti Mercury Computer for the 25 x 25 matrix considered in the example. However, once this matrix was inverted, the rest of the calculations took only a minute and a half or less for each of the two cases.

Case I: $K = 6 \times 10^{-3}$

$h = 5$ km.

Case II: $K = 12 \times 10^{-3}$

$h = 5$ km

p	h_{2p} (km)	$R_p(r)$
1	5.00	0.8
2	5.23	0.8
3	5.18	2.2
4	5.00	-1.6
5	5.00	-6.5
6	5.00	-10.9
7	7.09	-6.0
8	9.49	2.0
9	8.86	0.0
10	9.57	2.0
11	9.62	1.0
12	9.89	2.0
13	9.93	3.0
14	9.63	2.0
15	8.67	-2.0
16	8.69	-4.0
17	9.30	1.5
18	8.22	3.0
19	5.86	0.8
20	5.00	-6.3
21	5.06	-4.0
22	5.39	-2.0
23	5.07	0.6
24	5.04	-0.3
25	5.59	0.8

h_{2p} (km)	$R_p(r)$
5.04	1.1
5.10	-0.4
5.23	0.1
5.03	-1.2
5.23	-0.4
5.00	-0.6
6.07	-1.0
7.01	1.0
6.66	-1.0
7.12	1.0
7.03	0.0
7.12	-1.0
7.19	-1.0
7.25	1.0
6.78	1.0
6.59	0.0
6.80	0.5
6.49	-1.0
5.78	0.3
5.00	-1.7
5.13	0.0
5.22	-1.0
5.12	0.2
5.01	-0.6
5.35	+0.5

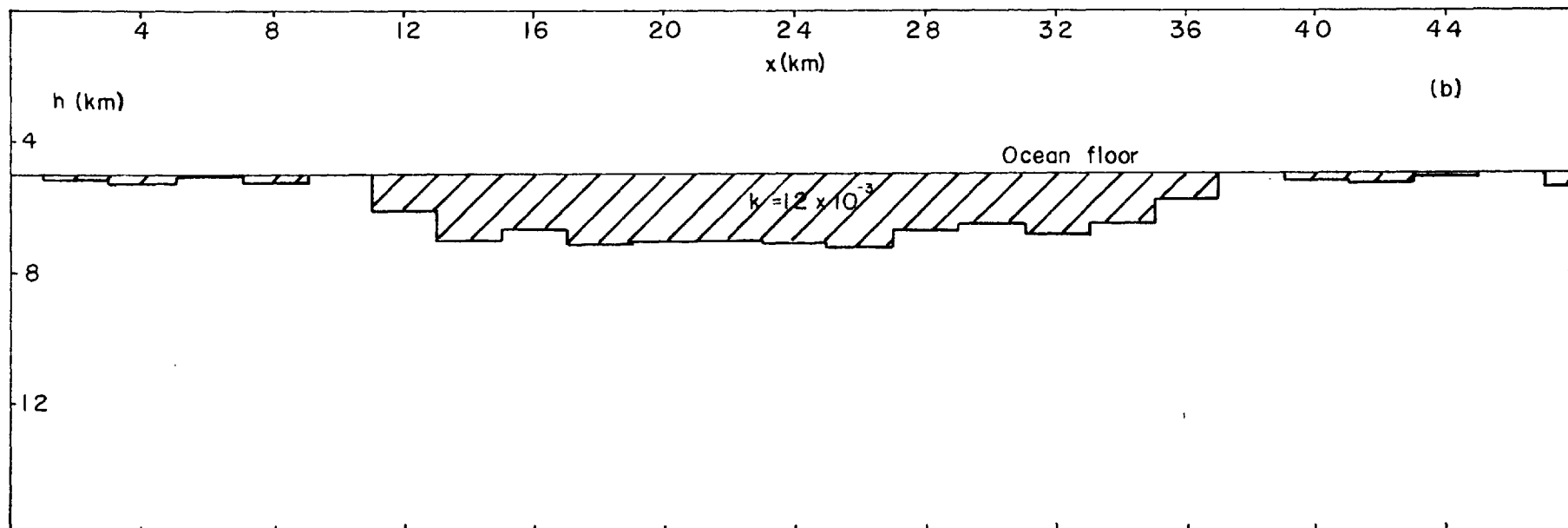
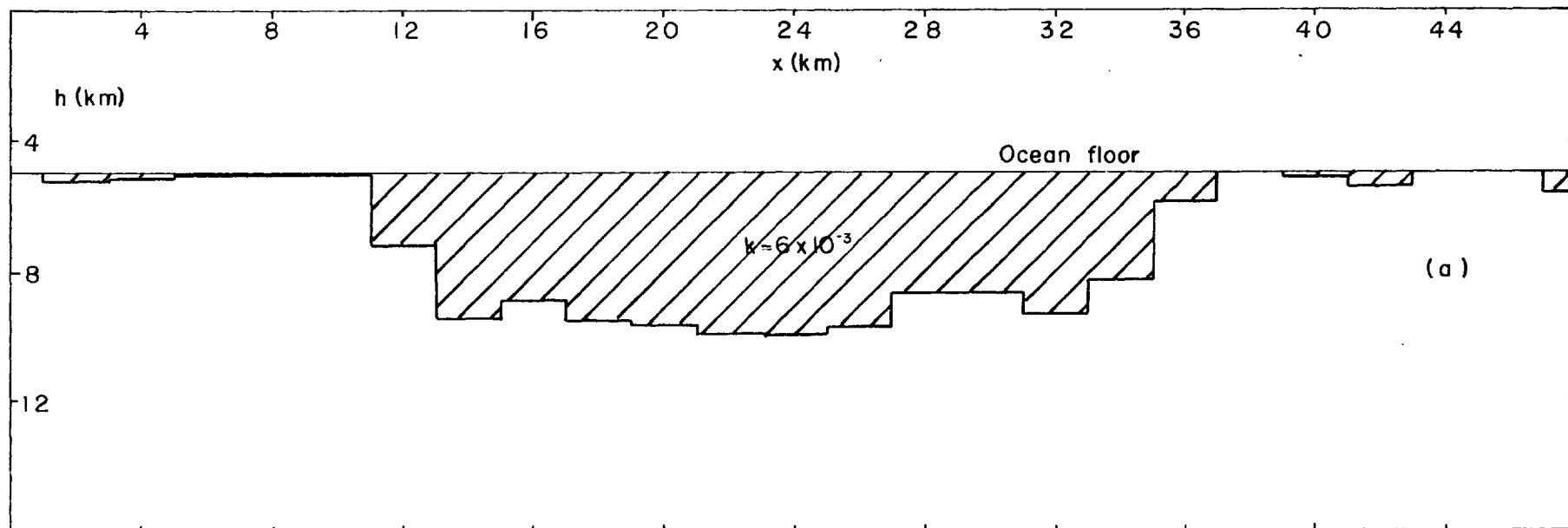


Fig. 4.3

CHAPTER 5

DETERMINATION OF DIRECTION AND INTENSITY OF MAGNETISATION OF UNIFORMLY MAGNETISED BODIES OF KNOWN TOPOGRAPHY

5.1 Introduction

The calculation of structures which produce a given anomaly, for an assumed direction and intensity of magnetisation, was considered in the previous chapter. The reverse problem, that is the determination of the direction and intensity of magnetisation from the observed anomaly when the structure producing the anomaly is known also deserves consideration as it has many useful applications. For instance, in marine magnetic surveys one encounters volcanic mountains at the bottom of the ocean which reveal themselves by the magnetic anomaly they produce. From the results of the magnetic and topographic surveys, the best fitting direction and intensity of magnetisation of these seamounts can be computed making the simple assumptions that they are uniformly magnetised and their bottom is at the ocean floor. The validity of these assumptions can be tested later by computing the anomaly due to the seamount for the determined direction and intensity of magnetisation and comparing it with the observed anomaly. If close agreement occurs between

the observed and computed anomalies the assumptions can be considered valid and the determined direction and intensity of magnetisation then give the true direction and intensity of magnetisation of the seamount.

5.2 Method of solution

Consider a rectangular co-ordinate system x, y, z with origin at the surface and with the z axis directed vertically downwards. Let $f_1(x_i, y_i, 0)$, $f_2(x_i, y_i, 0)$ and $f_3(x_i, y_i, 0)$ be the total field anomalies due to the structure for unit magnetisation along the x , y and z axes respectively. Then, if $T_s(x_i, y_i)$ is the total field anomaly due to the structure for an intensity of magnetisation $\bar{J}$ with components J_1 , J_2 and J_3 ,

$$T_s = J_1 f_1 + J_2 f_2 + J_3 f_3.$$

If the observed anomaly $T_0(x_i, y_i)$ is measured with respect to an arbitrary datum, then the values of J_1 , J_2 and J_3 which give the best fit between $T_0(x_i, y_i) - f_0$, where f_0 is a constant, and the anomaly $T_s(x_i, y_i)$ of the structure are required. For best fit in the least square sense,

$$\sum_{i=1}^N (T_0 - f_0 - T_s)^2 \quad \text{is a minimum}$$

i.e. $\sum_{i=1}^N (T_0 - f_0 - J_1 f_1 - J_2 f_2 - J_3 f_3)^2$ is a minimum.

Considering f_0 also as an unknown, the conditions of minimum are obtained by setting the derivatives of this sum with respect to J_1 , J_2 , J_3 and f_0 equal to zero. Thus

$$f_0 \sum f_1 + J_1 \sum f_1 f_1 + J_2 \sum f_2 f_1 + J_3 \sum f_3 f_1 = \sum T_0 f_1$$

$$f_0 \sum f_2 + J_1 \sum f_1 f_2 + J_2 \sum f_2 f_2 + J_3 \sum f_3 f_2 = \sum T_0 f_2$$

$$f_0 \sum f_3 + J_1 \sum f_1 f_3 + J_2 \sum f_2 f_3 + J_3 \sum f_3 f_3 = \sum T_0 f_3$$

$$f_0 \sum 1 + J_1 \sum f_1 + J_2 \sum f_2 + J_3 \sum f_3 = \sum T_0$$

If the functions f_1 , f_2 and f_3 are calculated for all the points at which T_0 is given the above equations can be set up and solved for J_1 , J_2 , J_3 and f_0 .

5.21 The calculation of f_1 , f_2 and f_3 for a three dimensional body.

These functions can be obtained by integrating the effects of free poles on the surface of the structure. In the following an alternative procedure is considered as it is easier to programme for a computer and seems to take less computer time. The structure is divided into horizontal sheets bounded by the topographic contours and its anomaly obtained by integrating the anomalies of these sheets. Each topographic contour is first approximated by a number of sections parallel and perpendicular to the x axis. The

anomaly due to the horizontal sheet bounded by these sections is obtained in the following manner.

In Chapter 3, expressions for the total anomaly per unit thickness of the sheets which form the sides of a vertical rectangular prism were derived. Considering only the horizontal sheet at the depth h , the following expression for its anomaly per unit thickness $T'(x_i, y_i, h)$, is obtained.

$$T'(x_i, y_i, h) = \left[\left[\frac{hE_4 - aE_1}{R(R+b)} + \frac{hE_3 - bE_2}{R(R+a)} + \frac{E_5}{R} + \frac{aE_1}{a^2+h^2} + \frac{bE_2}{b^2+h^2} \right] \right]_{b_1}^{b_2} \left[\right]_{a_1}^{a_2}$$

where the symbols in the right hand expression have the meanings defined in Chapter 3. The term $\frac{aE_1}{a^2+h^2}$ is independent of b and therefore it vanishes when the difference of its values at $b = b_1$ and $b = b_2$ is evaluated. Similarly the term $\frac{bE_2}{b^2+h^2}$ vanishes when the difference of its values at $a = a_1$ and $a = a_2$ is evaluated.

$$\begin{aligned} \text{Thus, } T'(x_i, y_i, h) &= \left[\left[\frac{hE_4 - aE_1}{R(R+b)} + \frac{hE_3 - bE_2}{R(R+a)} + \frac{E_5}{R} \right] \right]_{b_1}^{b_2} \left[\right]_{a_1}^{a_2} \\ &= \sum_i S_i \left\{ \frac{hE_4 - aE_1}{R(R+b)} + \frac{hE_3 - bE_2}{R(R+a)} + \frac{E_5}{R} \right\} \quad (2) \end{aligned}$$

where the summation is over the four corners of the rectangle. s is +1 for corners marked + and -1 for corners marked -

(fig. 5.0a). The horizontal sheet bounded by the sections drawn parallel and perpendicular to the x-axis, (Fig. 5.0b) can be divided into a number of rectangular sheets and its anomaly per unit thickness obtained by summing up the anomalies due to all these rectangular sheets. It is evident from the figure, that this sum reduces to the sum of the function given in (2) over only the corners of the sections approximating the contour.

At other corners the functions are summed up twice, once with positive weight and again with negative weight and therefore these corners do not contribute to the sum. Thus, if $T'(x_i, y_i, h)$ is the anomaly per unit thickness of the horizontal sheet bounded by the contour,

$$T'(x_i, y_i, h) = \sum_i' s \left\{ \frac{hE_4 - aE_1}{R(R+b)} + \frac{hE_3 - bE_2}{R(R+a)} + \frac{E_5}{R} \right\}$$

over the corners of the sections.

Since $E_1 = J_3n - J_1l$, $E_2 = J_3n - J_2m$, $E_3 = J_2n + J_3m$, $E_4 = J_1n + J_3l$, and $E_5 = J_1m + J_2l$, the above expression reduces to

$$\begin{aligned} T'(x_i, y_i, h) = & J_1 \sum_i' s \left\{ \frac{nh + la}{R(R+b)} + \frac{m}{R} \right\} \\ & + J_2 \sum_i' s \left\{ \frac{nh + mb}{R(R+a)} + \frac{l}{R} \right\} \\ & + J_3 \sum_i' s \left\{ \frac{lh - na}{R(R+b)} + \frac{mh - nb}{R(R+a)} \right\} \end{aligned}$$

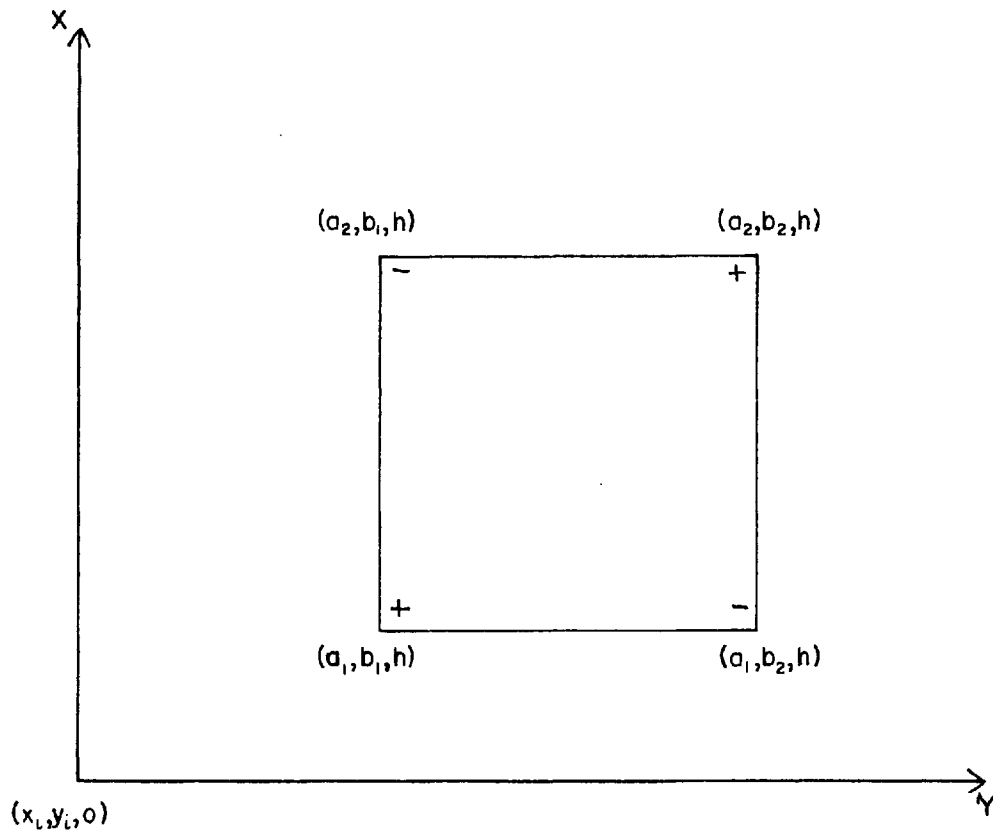


Fig. 5-0a

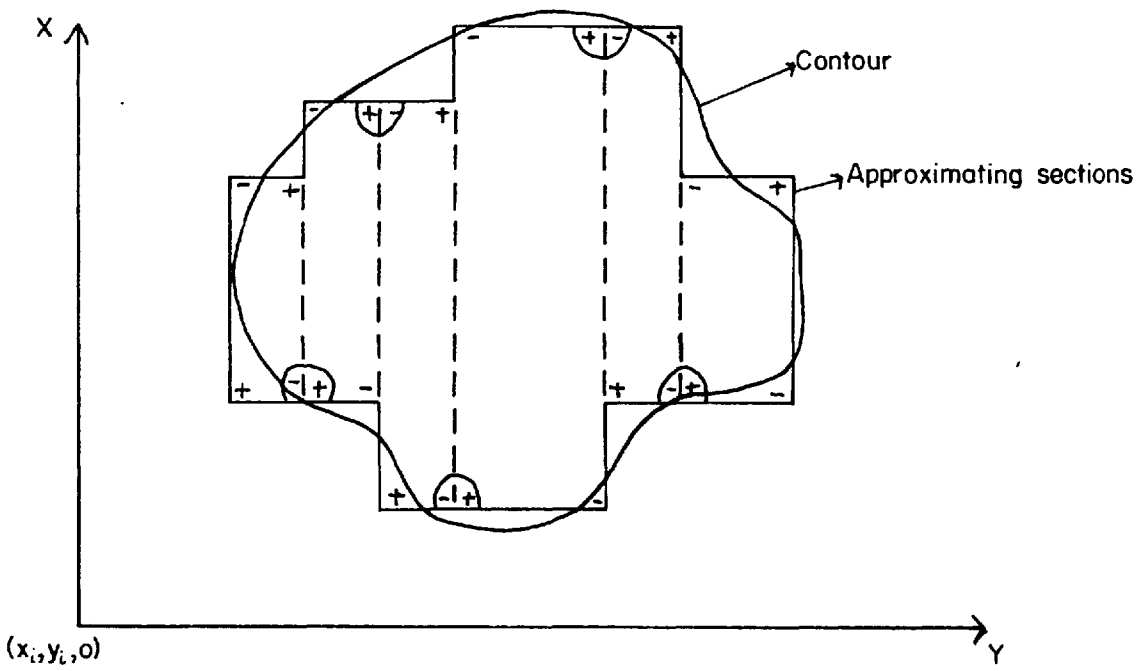


Fig. 5-0b

The anomaly $T_s(x_i, y_i, 0)$ due to the entire structure, is given by the relation

$$T_s(x_i, y_i, 0) = \int_{h_T}^{h_B} T'(x_i, y_i, h) dh,$$

where h_T and h_B are respectively the depths to the top and bottom of the structure.

But $T_s(x_i, y_i, 0) = J_1 f_1 + J_2 f_2 + J_3 f_3$, and so

$$f_1 = \int_{h_T}^{h_B} \left\{ \sum \frac{s}{R} \left\{ \frac{nh + la}{R + b} + m \right\} \right\} dh \quad (3)$$

$$f_2 = \int_{h_T}^{h_B} \left\{ \sum \frac{s}{R} \left\{ \frac{nh + mb}{R + a} + 1 \right\} \right\} dh \quad (4)$$

$$\text{and } f_3 = \int_{h_T}^{h_B} \left\{ \sum \frac{s}{R} \left\{ \frac{lh - na}{R + b} + \frac{mh - nb}{R + a} \right\} \right\} dh \quad (5)$$

The integration with respect to depth is performed numerically. Denoting the integrands in (3), (4) and (5) for the contour at depth h_j by U_j , V_j and W_j , a simple scheme for numerical integration, using the trapezoidal rule, is as follows:

$$f_1 = \sum_{j=1}^p \frac{1}{2} U_j (h_{j+1} - h_{j-1})$$

$$f_2 = \sum_{j=1}^p \frac{1}{2} V_j (h_{j+1} - h_{j-1})$$

$$\text{and } f_3 = \sum_{j=1}^p \frac{1}{2} W_j (h_{j+1} - h_{j-1}), \text{ with}$$

$h_0 = h_1$ (the depth to the top h_T)

and $h_{p+1} = h_p$ (the depth to the bottom h_B).

p is the number of contours used in the calculations.

The input data for the computer is thus the depth of each contour and the x, y co-ordinates of the corners of the approximating sections. When the co-ordinates of the negative corners ($s = -1$) are given the co-ordinates of the positive corners ($s = 1$) are defined and vice versa. Thus, it is necessary to use the co-ordinates of the negative corners only or the co-ordinates of the positive corners only, as input data, if the programme is properly designed. In the Flow Chart given in appendix V, it is assumed that the co-ordinates of the negative corners only, in the counter clockwise order, are used as input data. With the observer facing the x -axis, the lowest right hand corner for each contour, is a negative corner for the usual right hand

co-ordinates system. Thereafter, the signs for other corners are alternatively positive and negative.

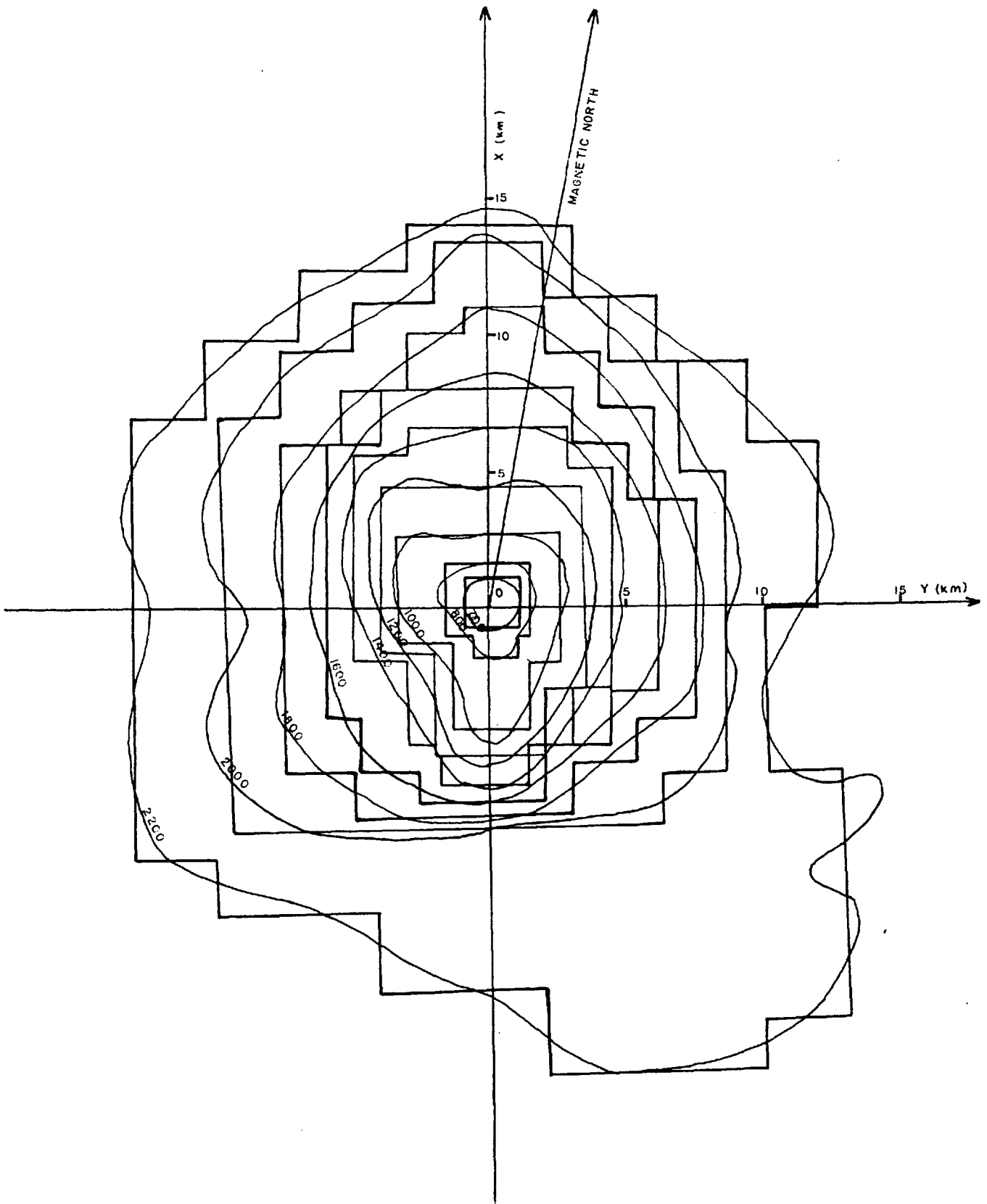
The additional information required for the calculations are the orientation of the Earth's field with respect to the x, y and z axes, the co-ordinates of the points at which the anomaly is observed and the magnitude of the anomaly at these points.

5.3 An example of application

The method has been used to investigate the nature of the magnetisation of a seamount off the West Coast of the United States. The geographical location of this seamount is $32^{\circ}15'N$, $127^{\circ}15'W$. The intensity of the Earth's magnetic field at this location is 50,000 γ and its inclination and declination are respectively 56° and $13^{\circ}30'$.

The total magnetic anomaly map of the seamount is shown in Fig. 5.1. The map was prepared by the interpolation of the values of the anomaly measured on the tracks shown in the figure. Only the values of the anomaly at the points marked by the crosses on these tracks were used in the calculation.

The topographic map and the rectangular sections used as an approximation to the topographic contours are shown in Fig. 5.2. The x-axis was chosen along a direction



FIEBERLING SEAMOUNT MODEL

DEPTH IN FATHOMS

Fig 51

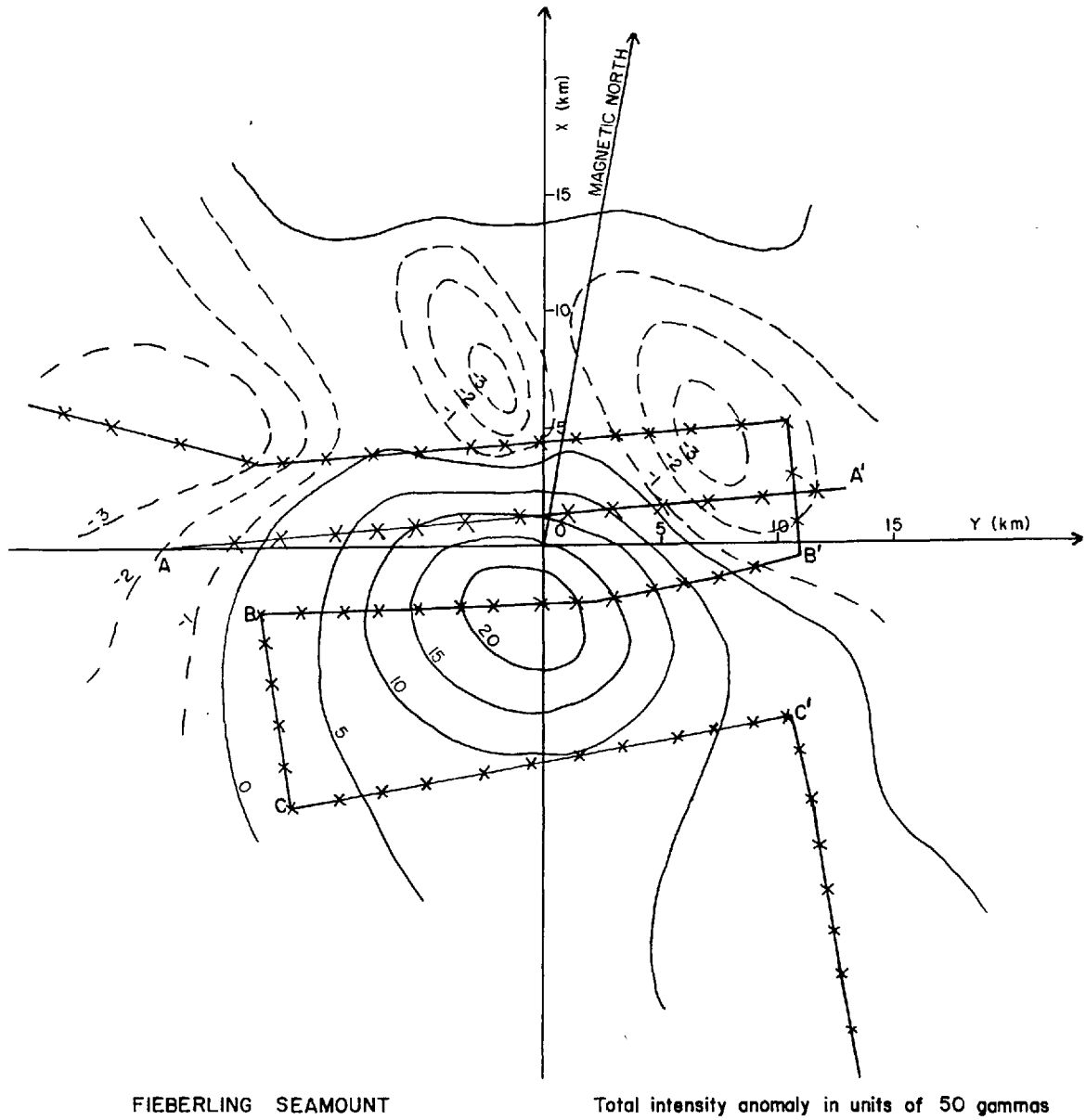


Fig. 5.2

making an angle of $10^{\circ}30'W$ with the magnetic north, for, the contours can be represented by a minimum of rectangular sections parallel and perpendicular to this direction.

The observed anomaly was assumed to be due to that part of the seamount projecting above the ocean floor which is at a depth of 2200 fathoms, and the intensity J , inclination U and declination E of the magnetisation vector were determined. The following values were obtained.

$$J = 8 \times 10^{-3} \text{ e.m.u./cc.}$$

$$U = 48^{\circ}24'$$

$$E = 11^{\circ}48'$$

The observed and best fitting anomalies on the three tracks AA', BB' and CC' are shown in Fig. 5.3. In this figure, the computed anomaly has been plotted above a base value of 42 r, the latter being the constant field, as determined by the computer, that had to be subtracted from the observed anomaly. The agreement between the observed and computed anomalies is close enough to make the following general conclusions.

1. A major part of the observed anomaly is due to the seamount for uniform magnetisation.
2. The intensity of magnetisation is large being equivalent to a susceptibility of 16.0×10^{-3} c.g.s. suggesting the presence of strong remanent magnetism.

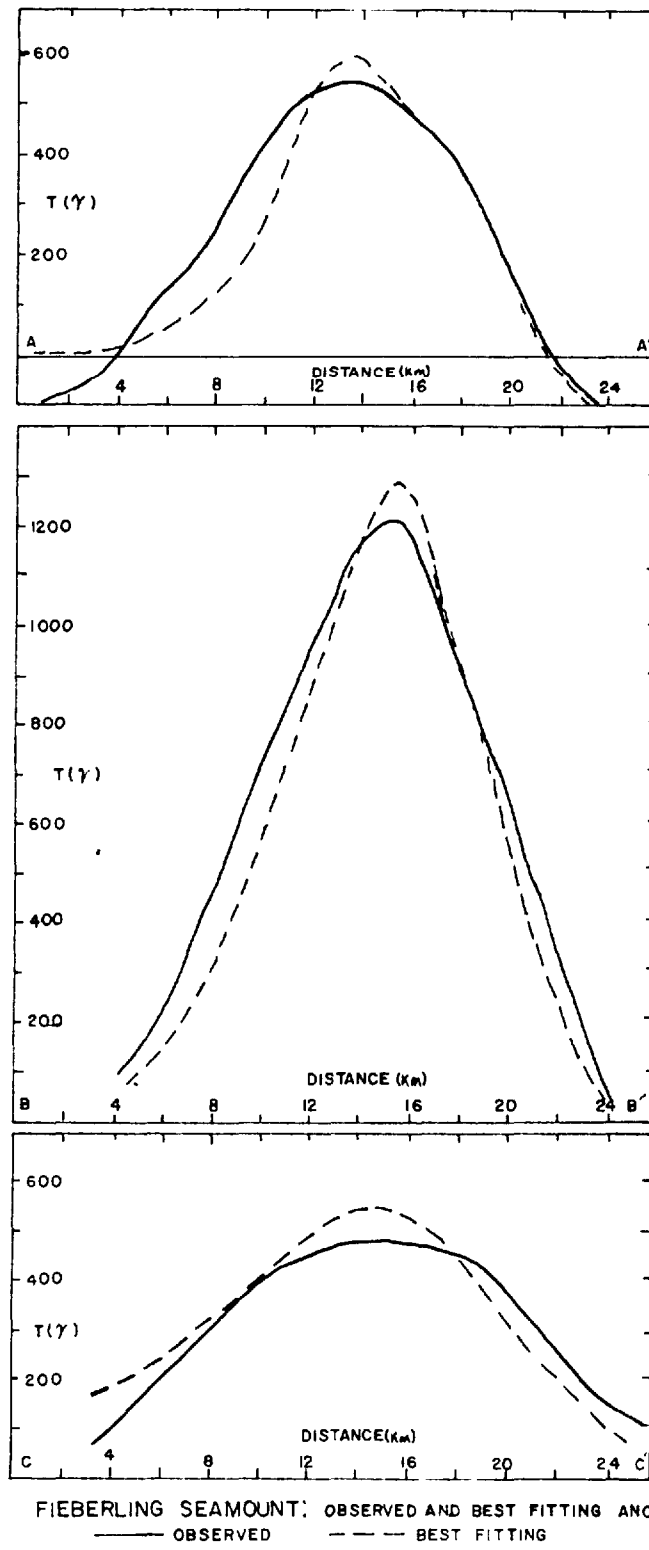


Fig. 5-3

3. The magnetisation vector is contained roughly in the vertical plane containing the magnetic meridian since the declination of the magnetisation and the Earth's field are approximately equal.
4. The inclination of the magnetisation vector ($48^{\circ}24'$) is different from the inclination of the Earth's field ($56^{\circ}30'$).

The total intensity of magnetisation is the vector sum of two components, the induced and the remanent. The remanent magnetisation vector must therefore lie in the plane containing the direction of the total intensity of magnetisation and the direction of the Earth's field. In the absence of a knowledge of the susceptibility value, the direction and magnitude of the remanent vector cannot be determined. However, a lower limit to the magnitude of this quantity can be set. This lower limit is the magnitude of the remanent vector when its direction is at right angles to the direction of the Earth's field. If l, m, n are the direction cosines of the direction of the Earth's field and p, q, r those of the total intensity of magnetisation, the minimum value of the intensity of remanent magnetisation is $J \sin \left\{ \cos^{-1} (pl + qm + rn) \right\}$, and the value of the induced component corresponding to this minimum remanence is $J(pl + qn + rn)$. Using the

computed results,

$$p = \cos 48.4 \cos 11.8 = 0.6500$$

$$q = \cos 48.4 \sin 11.8 = 0.1360$$

$$r = \sin 48.4 = 0.7480 \quad \text{and}$$

$$J = 8 \times 10^{-3}.$$

Also, $l = \cos 56.5 \cos 13.5 = 0.5365$

$$m = \cos 56.5 \sin 13.5 = 0.1286$$

$$n = \sin 56.5 = 0.8340$$

Hence, the minimum values of remanence

$$= 8 \times 10^{-3} \sin \left\{ \cos^{-1} (0.5365 \times 0.650 + 0.1286 + 0.834 \times 0.748) \right\}$$

$$= 8 \times 10^{-3} \sin \left\{ \cos^{-1} 0.990 \right\}$$

$$= 8 \times 10^{-3} \times 0.141 = 1.128 \times 10^{-3} \text{ e.m.u./cc.}$$

The induced component $= 8 \times 0.99 \times 10^{-3} = 7.92 \times 10^{-3} \text{ emu/cc.}$

This is equivalent to a susceptibility of $15.84 \times 10^{-3} \text{ c.g.s.}$

In the following table, the intensity, inclination and declination of the remanent magnetisation vector calculated for various assumed values of the susceptibility are given

$K \times 10^3$ (susceptibility)	$J_R \times 10^3$ (intensity)	I_R (inclination)	E_R (declination)
0	8.00	48°24'	11°48'
2	7.01	47°15'	11°37'
4	6.03	45°43'	11°23'
6	5.05	43°39'	11°03'
8	4.08	40°26'	10°39'
10	3.14	35°22'	10°01'
12	2.23	26°12'	9°06'
14	1.45	0°12'	7°24'
16	1.14	-37°12'	3°22'

It is seen that for values of susceptibility which would normally be expected for oceanic lavas (about 4×10^{-3} cgs or less) most of the magnetisation is remanent.

It has been suggested that the seamounts in the North Pacific Ocean might have drifted away from an East-West oceanic rise prevailing in the south of the equator. If so, the direction of remanent magnetisation will indicate the geographical location of the seamount when it was formed, assuming that the direction of the Earth's field has not changed since the formation of the seamount. The calculated values of the inclination of the remanent vector are shallower than the inclination of the Earth's field at the present location of the seamount and are consistent with the idea of a drift away from the oceanic rise in the south. If susceptibility values greater than about 4×10^{-3} are ruled out, the seamount could have originated only a few degrees of latitude south of its present location.

Time of computation

The time of computation, for any particular computer, depends on the total number of sections used to approximate the contour and the number of observed field values used in the fitting. On the Ferranti-Mercury Computer, the time taken was roughly 0.10 seconds per negative corner per

field value. The solution of the above problem was achieved in about 8 minutes.

CHAPTER 6

SUMMARY AND CONCLUSIONS

The existing methods of interpreting gravity and magnetic measurements are laborious and inadequate to handle the vast amount of data now obtained using improved techniques of prospecting. In this thesis an attempt has been made to show that by a direct or indirect use of Digital Computers a more rapid, more detailed and less laborious interpretation of the data is possible. The problems discussed are:

- i. The interpretation of magnetic anomalies due to dykes, square vertical prisms and parallelopipeds of infinite depth extent:

The use of a computer has made it possible to determine the maximum, minimum, half, quarter and three quarter maximum and minimum characteristics of the anomalies due to dykes and to square vertical prisms for a wide range of values of the parameters defining the shapes of the anomalies. It has been possible from the results thus obtained to devise a simple method of identifying the parameters from an observed anomaly and thence of determining the depth dimensions and intensity of magnetisation of these bodies.

Further, a simple method given for reducing the anomalies due to three dimensional parallelopipeds to two two dimensional dyke anomalies has enabled the use of the method devised for the dyke in the interpretation of the anomalies due to the parallelopipeds.

Collectively, these methods form a routine basis for the interpretation of aeromagnetic surveys over sedimentary basins, the interpretation of anomalies consisting of nearly circular contours being approached using a square prism, those with elongated shape using the dyke and those of the intermediate shape using the parallelopiped. These methods are far superior to the chart method of Vacquier et al⁽⁴⁾ for interpreting such anomalies. The charts of Vacquier have been prepared for a limited number of discrete values of the parameters and as such the depth estimates made using them are necessarily crude. Further, the comparison of the observed anomaly with these charts is a tedious and time consuming process. The present methods are more direct, rapid and easy to apply. Also, they are more accurate since a virtually continuous range of values of the parameters are covered in the graphs given for the interpretation.

ii. Automatic model fitting:

It has been possible to devise computer programmes which enable the automatic fitting of two dimensional and three

dimensional vertical rectangular prisms to observed total field magnetic anomalies. The programmes have been tested using the anomalies due to a number of theoretical models as input data for the computer. The parameters determined by the computer have been found to be in satisfactory agreement with the known parameters of the model. The solutions were achieved on the Ferranti Mercury Computer in about half a minute for the two dimensional case and about $2\frac{1}{2}$ minutes for the three dimensional case. These times contrast strikingly with the several hours that would be required to achieve such a fit by the conventional methods.

The advantage of this method over the previous method is that it uses a larger number of points in the anomaly. The results of interpretations are therefore less likely to be affected by small irregularities in the anomaly due to the effect of neighbouring sources and/or due to the source being not of the assumed shape. Further, the depth extent of the source is considered unknown, the computer being allowed to decide for itself whether this is large or small.

One disadvantage of the method is that it requires a set of initial values for the parameters. It has been shown that even if the initial values are far from the best fitting values of the parameters a satisfactory solution would be achieved. However, the time of computation and hence the

cost depends on the accuracy of the initial values. The closer the initial values are to the actual values of the parameters the smaller is the time required for solution. Use may be made of the graphs given for the dykes and square prisms for the initial estimation of the parameters, in which case the depth to the bottom may be assumed arbitrarily as equal to twice the depth to the top.

The programmes enable a more detailed, more rapid and more reliable interpretation of aeromagnetic data than has been possible hitherto. It is suggested that the programmes be put to a number of practical tests. This may point the need for any improvement in numerical technique or modification to the programme.

iii. Automatic structure fitting:

The method devised for the fitting of two dimensional vertical rectangular prisms has been extended to the fitting of any structure which may be considered to consist of a number of such prisms. The usefulness of the method has been demonstrated by applying it to the solution of a problem considered by Mason⁽¹²⁾. A number of different solutions corresponding to various assumed values of susceptibility were obtained in a matter of a few minutes. This time contrasts with the several hours taken to achieve similar

solutions by a trial and error method.

- iv. Determination of the direction and intensity of magnetisation of a uniformly magnetised body of known topography from its total field magnetic anomaly:

A method has been devised for the solution of this problem and in the process a method of computing the total field magnetic anomaly due to a three dimensional structure of arbitrary shape has also been devised. The method will have applications in establishing the direction and intensity of magnetisation of a number of terrestrial features and seamounts over which magnetic and topographic surveys have been made. In the present work it has been used to investigate the nature of magnetisation of the Fieberling seamount, off the West Coast of the United States. It was concluded that;

- a. A major part of the anomaly observed over this seamount is due to uniform magnetisation of the seamount.
- b. The intensity of magnetisation is large, being equivalent to a susceptibility of 16.0×10^{-3} c.g.s. suggesting the presence of a strong remanent magnetisation.
- c. The magnetisation vector is contained approximately in the vertical plane containing the magnetic meridian.

- d. The inclination of the magnetisation vector ($48^{\circ}.24'$) differs from the inclination of the Earth's field at this location ($56^{\circ}.30'$). This difference could be due to the direction of the remanent vector being different from the direction of the Earth's field. Alternatively, the difference might arise as a result of ignoring the effects due to the root of the seamount.

Conclusions:

The limited number of applications of Digital computers to the interpretation of potential field data described in this thesis amply demonstrate the advantages of computer method over conventional method of interpretation. However, in practice, these advantages must be weighed against the high computer cost. If simple standard programmes such as those described above are developed, the advantages gained by using the computer might far outweigh the cost of computer time.

APPENDIX I

Characteristics of Magnetic Anomalies due to Dykes.

<u>Notations used in the following</u> <u>table.</u>	<u>Notations in the text.</u>
I Angle involving the direction of measurement of field, direction of magnetisation and angle of strike and dip of the dyke.	I
T Width of top surface of dyke in units of the depth	t
FM Maximum/minimum value of anomaly	f_M, f_m
XM Position of maximum/minimum	x_M, x_m
X(sM/4) Position of sM/4 times maximum/minimum	$x_{sM/4}^*, x_{XM/4}'$ $x_{sM/4}', x_{sm/4}$

I = 0

T	XM	FM	$X(3M/4)-XM$	$X(M/2)-XM$	$X(M/4)-X(3M/4)$
0	-1.000	0.500	-1.217 0.548	-2.733 0.732	-5.657 0.325
1	-1.118	0.481	-1.244 0.588	-2.800 0.799	-5.839 0.377
2	-1.414	0.881	-1.306 0.678	-2.957 0.956	-6.283 0.514
3	-1.803	1.195	-1.376 0.779	-3.147 1.146	-6.844 0.700
4	-2.236	1.444	-1.443 0.876	-3.342 1.339	-7.432 0.910
5	-2.693	1.647	-1.506 0.964	-3.530 1.527	-8.025 1.136
6	-3.162	1.818	-1.562 1.044	-3.707 1.707	-8.622 1.369
7	-3.640	1.966	-1.615 1.117	-3.879 1.879	-9.206 1.607
8	-4.123	2.095	-1.663 1.183	-4.043 2.041	-9.770 1.848
9	-4.610	2.209	-1.707 1.244	-4.199 2.198	-10.334 2.089
10	-5.099	2.312	-1.749 1.301	-4.349 2.346	-10.880 2.331
11	-5.590	2.406	-1.788 1.353	-4.492 2.490	-11.411 2.573
12	-6.083	2.492	-1.824 1.402	-4.630 2.629	-11.925 2.815
13	-6.576	2.571	-1.859 1.448	-4.763 2.763	-12.443 3.056
14	-7.071	2.644	-1.892 1.492	-4.892 2.892	-12.934 3.296
15	-7.566	2.712	-1.923 1.533	-5.017 3.018	-13.434 3.536

l = 10

T	XM	FM	$X(3M/4)-XM$	$X(M/2)-XM$	$X(M/4)-X(3M/4)$
0	-0.839	0.587	-1.084	-2.391	-4.820
			0.524	0.713	0.342
	1.192	0.413	-0.584	-0.768	-0.317
			1.382	3.152	6.671
1	-0.956	0.561	-1.117	-2.462	-4.995
			0.576	0.798	0.406
	1.308	0.400	-0.613	-0.816	-0.358
			1.404	3.218	6.857
2	-1.249	1.015	-1.187	-2.623	-5.417
			0.695	1.002	0.573
	1.601	0.742	-0.677	-0.931	-0.460
			1.459	3.369	7.306
3	-1.635	1.360	-1.262	-2.809	-5.939
			0.830	1.253	0.806
	1.988	1.019	-0.750	-1.067	-0.597
			1.524	3.559	7.885
4	-2.067	1.628	-1.331	-2.993	-6.492
			0.961	1.512	1.077
	2.419	1.243	-0.820	-1.206	-0.750
			1.591	3.760	8.509
5	-2.522	1.843	-1.392	-3.168	-7.046
			1.080	1.768	1.372
	2.875	1.430	-0.885	-1.340	-0.910
			1.654	3.960	9.154
6	-2.991	2.022	-1.444	-3.335	-7.596
			1.189	2.015	1.681
	3.344	1.588	-0.943	-1.468	-1.076
			1.713	4.154	9.776
7	-3.468	2.175	-1.493	-3.489	-8.140
			1.289	2.252	1.996
	3.821	1.726	-0.996	-1.589	-1.243
			1.768	4.340	10.406

I = 10 (CONTD)

8	-3.951	2.308	-1.538	-3.636	-8.654
			1.379	2.478	2.319
	4.303	1.847	-1.045	-1.703	-1.411
			1.820	4.518	11.019
9	-4.437	2.425	-1.579	-3.777	-9.166
			1.463	2.697	2.646
	4.789	1.956	-1.090	-1.812	-1.579
			1.868	4.689	11.613
10	-4.926	2.531	-1.617	-3.911	-9.662
			1.541	2.907	2.974
	5.278	2.054	-1.131	-1.917	-1.746
			1.914	4.859	12.197
11	-5.417	2.626	-1.652	-4.040	-10.161
			1.613	3.110	3.303
	5.769	2.143	-1.169	-2.017	-1.913
			1.957	5.017	12.771
12	-5.909	2.713	-1.685	-4.164	-10.634
			1.681	3.307	3.633
	6.262	2.225	-1.205	-2.112	-2.079
			2.000	5.165	13.325
13	-6.403	2.793	-1.720	-4.282	-11.091
			1.744	3.496	3.965
	6.755	2.301	-1.239	-2.205	-2.245
			2.036	5.314	13.883
14	-6.897	2.867	-1.749	-4.397	-11.555
			1.804	3.680	4.294
	7.250	2.371	-1.271	-2.293	-2.410
			2.074	5.459	14.412
15	-7.392	2.936	-1.777	-4.507	-12.011
			1.861	3.861	4.624
	7.745	2.437	-1.301	-2.379	-2.573
			2.110	5.599	14.952

I = 20

T	XM	FM	$X(3M/4)-XM$	$X(M/2)-XM$	$X(M/4)-X(3M/4)$
0	-0.700	0.671	-0.976	-2.109	-4.110
			0.509	0.707	0.371
	1.428	0.329	-0.637	-0.825	-0.320
			1.592	3.682	7.935
1	-0.812	0.638	-1.014	-2.184	-4.269
			0.573	0.809	0.442
	1.540	0.321	-0.655	-0.857	-0.348
			1.610	3.740	8.113
2	-1.096	1.140	-1.094	-2.346	-4.662
			0.724	1.061	0.631
	1.824	0.602	-0.697	-0.933	-0.418
			1.658	3.886	8.562
3	-1.475	1.509	-1.175	-2.527	-5.143
			0.901	1.379	0.903
	2.203	0.837	-0.747	-1.026	-0.511
			1.718	4.078	9.151
4	-1.902	1.790	-1.247	-2.702	-5.645
			1.074	1.716	1.227
	2.629	1.032	-0.796	-1.121	-0.613
			1.783	4.284	9.796
5	-2.353	2.011	-1.309	-2.864	-6.150
			1.236	2.054	1.584
	3.081	1.197	-0.842	-1.213	-0.721
			1.847	4.493	10.470
6	-2.819	2.194	-1.360	-3.015	-6.647
			1.384	2.384	1.964
	3.547	1.339	-0.884	-1.300	-0.829
			1.909	4.697	11.138
7	-3.294	2.348	-1.407	-3.155	-7.131
			1.521	2.703	2.358
	4.022	1.464	-0.923	-1.383	-0.938
			1.970	4.896	11.786

1 = 20(CONTD)

8	-3.775	2.481	-1.449	-3.290	-7.604
			1.646	3.016	2.764
	4.503	1.574	-0.958	-1.462	-1.048
			2.026	5.094	12.438
9	-4.260	2.598	-1.487	-3.413	-8.073
			1.762	3.315	3.178
	4.988	1.673	-0.991	-1.536	-1.156
			2.076	5.275	13.070
10	-4.748	3.702	-1.521	-3.530	-8.514
			1.870	3.606	3.597
	5.476	1.763	-1.022	-1.607	-1.263
			2.126	5.456	13.697
11	-5.238	2.797	-1.553	-3.641	-8.962
			1.971	3.891	4.019
	5.966	1.845	-1.049	-1.675	-1.372
			2.175	5.631	14.303
12	-5.730	2.882	-1.583	-3.749	-9.395
			2.068	4.164	4.442
	6.458	1.921	-1.075	-1.739	-1.478
			2.221	5.800	14.906
13	-6.223	2.961	-1.611	-3.853	-9.806
			2.153	4.428	4.874
	6.951	1.991	-1.100	-1.802	-1.584
			2.265	5.964	15.492
14	-6.716	3.034	-1.637	-3.953	-10.224
			2.237	4.687	5.304
	7.444	3.056	-1.123	-1.862	-1.689
			2.307	6.123	16.071
15	-7.211	3.101	-1.661	-4.049	-10.636
			2.316	4.940	5.731
	7.939	2.117	-1.145	-1.920	-1.792
			2.348	6.283	16.640

I = 30

T	XM	FM	$X(3M/4)-XM$	$X(M/2)-XM$	$X(M/4)-X(3M/4)$
0	-0.577	0.750	-0.887	-1.870	-3.492
			0.501	0.714	0.412
	1.732	0.250	-0.713	-0.914	-0.336
			1.869	4.380	9.576
1	-0.681	0.709	-0.931	-1.946	-3.640
			0.577	0.830	0.486
	1.836	0.245	-0.722	-0.930	-0.352
			1.884	4.432	9.749
2	-0.950	1.252	-1.023	-2.114	-3.992
			0.761	1.126	0.685
	2.105	0.466	-0.745	-0.974	-0.395
			1.924	4.568	10.172
3	-1.316	1.639	-1.113	-2.294	-4.418
			0.987	1.513	0.977
	2.470	0.657	-0.776	-1.030	-0.451
			1.981	4.752	10.755
4	-1.732	1.926	-1.190	-2.459	-4.867
			1.215	1.934	1.333
	2.887	0.819	-0.808	-1.091	-0.514
			2.042	4.959	11.411
5	-2.176	2.149	-1.255	-2.609	-5.315
			1.432	2.368	1.734
	3.331	0.958	-0.839	-1.150	-0.579
			2.107	5.179	12.094
6	-2.637	2.329	-1.309	-2.746	-5.747
			1.636	2.797	2.165
	3.792	1.080	-0.868	-1.208	-0.646
			2.171	5.388	12.795
7	-3.108	2.480	-1.355	-2.872	-6.173
			1.822	3.222	2.625
	4.263	1.188	-0.896	-1.263	-0.711
			2.234	5.600	13.474

1 = 30 (CONTD)

8	-3.586	2.610	, -1.394	-2.988	-6.595
			2.000	3.637	3.097
	4.741	1.284	-0.921	-1.314	-0.777
			2.294	5.807	14.160
9	-4.068	2.723	-1.430	-3.097	-6.996
			2.166	4.041	3.582
	5.223	1.371	-0.945	-1.363	-0.842
			2.352	6.008	14.825
10	-4.554	2.824	, -1.462	-3.199	-7.392
			2.316	4.436	4.084
	5.709	1.450	, -0.966	-1.410	-0.906
			, 2.407	6.203	15.488
11	-5.043	2.914	-1.492	-3.296	-7.779
			2.459	4.821	4.591
	6.197	1.523	, -0.987	-1.456	-0.969
			2.460	6.398	16.133
12	-5.533	2.996	-1.519	-3.391	-8.156
			2.593	5.201	5.106
	6.687	1.590	-1.006	-1.498	-1.032
			2.511	6.582	16.772
13	-6.024	3.071	-1.544	-3.477	-8.527
			2.720	5.572	5.627
	7.179	1.653	-1.025	-1.538	-1.094
			2.560	6.757	17.397
14	-6.517	3.140	-1.567	-3.561	-8.897
			2.840	5.932	6.152
	7.672	1.711	-1.042	-1.578	-1.154
			2.607	6.932	18.015
15	-7.011	3.204	-1.589	-3.641	-9.258
			2.957	6.286	6.679
	8.166	1.766	-1.058	-1.615	-1.216
			, 2.652	7.103	18.619

I = 40

T	XM	FM	$X(3M/4)-XM$	$X(M/2)-XM$	$X(M/4)-X(3M/4)$
0	-0.466	0.821	-0.812	-1.665	-3.956
			0.500	0.732	0.470
	2.145	0.179	-0.827	-1.049	-0.368
			2.257	5.338	11.816
1	-0.559	0.772	-0.864	-1.747	-3.085
			0.585	0.859	0.541
	2.237	0.176	-0.829	-1.054	-0.375
			2.269	5.383	11.962
2	-0.805	1.349	-0.970	-1.921	-3.386
			0.802	1.191	0.735
	2.484	0.339	-0.836	-1.071	-0.395
			2.302	5.505	12.862
3	-1.149	1.747	-1.075	-2.100	-3.753
			1.081	1.640	1.021
	2.828	0.484	-0.850	-1.097	-0.424
			2.351	5.680	13.928
4	-1.549	2.034	-1.162	-2.263	-4.132
			1.377	2.147	1.375
	3.287	0.611	-0.867	-1.130	-0.458
			2.410	5.883	13.565
5	-1.981	2.252	-1.233	-2.406	-4.512
			1.668	2.679	1.782
	3.659	0.722	-0.886	-1.164	-0.493
			2.474	6.099	14.260
6	-2.433	2.427	-1.291	-2.529	-4.883
			1.949	3.221	2.228
	4.111	0.821	-0.904	-1.199	-0.530
			2.539	6.320	14.961
7	-2.896	2.571	-1.339	-2.642	-5.242
			2.213	3.763	2.707
	4.575	0.909	-0.923	-1.232	-0.565
			2.604	6.545	15.673

1 = 40 (CONTD)

8	-3.369	2.693	-1.381	-2.744	-5.592
	5.047	0.989	2.462	4.303	3.213
9	-3.846	2.800	-1.417	-2.839	-5.936
	5.525	1.064	2.695	4.836	3.741
10	-4.329	2.894	-1.449	-2.926	-6.270
	6.007	1.128	2.918	5.362	4.283
11	-4.814	2.977	-1.478	-3.008	-6.601
	6.492	1.189	3.127	5.881	4.842
12	-5.301	3.053	-1.504	-3.086	-6.915
	6.979	1.246	3.328	6.368	5.412
13	-5.791	3.122	-1.527	-3.159	-7.228
	7.469	1.299	3.518	6.894	5.992
14	-6.282	3.186	-1.549	-3.229	-7.535
	7.960	1.349	3.697	7.390	6.584
15	-6.774	3.243	-1.569	-3.296	-7.844
	8.432	1.396	3.865	7.877	7.186
			-1.038	-1.456	-0.837
			3.064	8.168	21.104

I = 50

T	XM	FM	$X(3M/4)-XM$	$X(M/2)-XM$	$X(M/4)-X(3M/4)$
0	-0.364	0.883	-0.748	-1.489	-2.489
			0.505	0.761	0.546
	2.747	0.117	-1.004	-1.264	-0.427
			2.837	6.766	15.085
1	-0.442	0.826	-0.808	-1.576	-2.595
			0.598	0.894	0.610
	2.826	0.116	-1.001	-1.262	-0.426
			2.846	6.800	15.215
2	-0.658	1.429	-0.935	-1.762	-2.841
			0.842	1.249	0.785
	3.041	0.226	-0.998	-1.259	-0.431
			2.871	6.903	15.570
3	-0.969	1.832	-1.061	-1.951	-3.140
			1.174	1.745	1.040
	3.353	0.327	-0.998	-1.261	-0.439
			2.918	7.066	16.071
4	-1.343	2.114	-1.167	-2.117	-3.445
			1.547	2.323	1.354
	3.726	0.418	-1.001	-1.270	-0.453
			2.966	7.247	16.695
5	-1.753	2.323	-1.251	-2.258	-3.747
			1.931	2.945	1.717
	4.136	0.500	-1.008	-1.283	-0.466
			3.025	7.457	17.361
6	-2.188	2.487	-1.319	-2.377	-4.045
			2.309	3.592	2.125
	4.571	0.573	-1.017	-1.299	-0.482
			3.093	7.677	18.066
7	-2.638	2.621	-1.375	-2.482	-4.330
			2.681	4.254	2.563
	5.022	0.641	-1.026	-1.316	-0.499
			3.151	7.902	18.785

I = 50 (CONTD)

8	-3.100	2.733	-1.422	-2.576	-4.610
			3.036	4.922	3.036
	5.424	0.702	-1.037	-1.333	-0.515
			3.217	8.133	19.518
9	-3.570	2.830	-1.461	-2.660	-4.882
			3.381	5.592	3.531
	5.953	0.758	-1.047	-1.350	-0.532
			3.282	8.357	20.239
10	-4.045	2.914	-1.496	-2.733	-5.145
			3.704	6.260	4.057
	6.428	0.810	-1.057	-1.367	-0.548
			3.346	8.587	20.964
11	-4.524	2.989	-1.526	-2.803	-5.402
			4.025	6.930	4.593
	6.908	0.858	-1.067	-1.384	-0.564
			3.408	8.796	21.680
12	-5.007	3.057	-1.553	-2.868	-5.656
			4.323	7.592	5.155
	7.390	0.903	-1.076	-1.400	-0.580
			3.470	9.012	22.388
13	-5.492	3.118	-1.577	-2.928	-5.904
			4.615	8.254	5.728
	7.875	0.945	-1.085	-1.416	-0.596
			3.529	9.236	23.093
14	-5.979	3.174	-1.599	-2.985	-6.146
			4.895	8.909	6.316
	8.363	0.985	-1.094	-1.431	-0.611
			3.588	9.439	23.782
15	-6.468	3.226	-1.619	-3.039	-6.386
			5.162	9.561	6.919
	8.851	1.023	-1.102	-1.446	-0.626
			3.645	9.645	24.473

I = 60

T	XH	PH	$X(3H/4) - XH$	$X(H/2) - XH$	$X(H/4) - X(3H/4)$
0	-0.268	0.933	-0.694	-1.338	-2.077
			0.515	0.808	0.646
	3.732	0.067	-1.310	-1.640	-0.538
			3.799	9.104	20.445
1	-0.330	0.870	-0.763	-1.431	-2.162
			0.612	0.936	0.703
	3.794	0.067	-1.305	-1.635	-0.535
			3.805	9.133	20.547
2	-0.504	1.492	-0.914	-1.634	-2.353
			0.876	1.299	0.848
	3.968	0.131	-1.294	-1.617	-0.528
			3.832	9.215	20.828
3	-0.768	1.895	-1.073	-1.846	-2.580
			1.252	1.815	1.051
	4.232	0.193	-1.282	-1.601	-0.531
			3.861	9.344	21.263
4	-1.096	3.167	-1.211	-2.031	-2.807
			1.695	2.432	1.295
	4.560	0.249	-1.273	-1.589	-0.517
			3.901	9.509	21.810
5	-1.470	2.364	-1.323	-2.187	-3.029
			2.174	3.112	1.573
	4.934	0.302	-1.268	-1.583	-0.516
			3.949	9.709	22.436
6	-1.874	2.514	-1.415	-2.316	-3.245
			2.570	3.833	1.881
	5.330	0.351	-1.266	-1.581	-0.517
			4.000	9.911	23.110
7	-2.299	2.634	-1.491	-2.425	-3.448
			3.172	4.585	2.215
	5.763	0.396	-1.257	-1.582	-0.520
			4.068	10.127	23.806

1 = 60 (CONTD)

8	-2.740	2.734	-1.553	-2.519	-3.646
			3.677	5.354	2.570
	6.204	0.438	-1.269	-1.585	-0.524
9			4.135	10.359	24.524
	-3.192	2.818	-1.606	-2.601	-3.842
			4.173	6.137	2.952
	6.656	0.476	-1.272	-1.590	-0.528
			4.203	10.590	25.250
10	-3.653	2.892	-1.650	-2.672	-4.021
			4.663	6.927	3.354
	7.117	0.513	-1.277	-1.596	-0.533
			4.259	10.821	26.000
11	-4.120	2.956	-1.689	-2.736	-4.209
			5.142	7.727	3.776
	7.584	0.547	-1.282	-1.603	-0.537
			4.325	11.048	26.724
12	-4.593	3.013	-1.722	-2.795	-4.388
			5.610	8.530	4.216
	8.057	0.579	-1.287	-1.610	-0.543
			4.390	11.272	27.475
13	-5.069	3.065	-1.752	-2.849	-4.563
			6.064	9.335	4.679
	8.533	0.609	-1.291	-1.618	-0.548
			4.455	11.495	28.206
14	-5.548	3.112	-1.778	-2.898	-4.734
			6.514	10.144	5.149
	9.012	0.637	-1.297	-1.625	-0.554
			4.519	11.725	28.938
15	-6.030	3.156	-1.802	-2.944	-4.900
			6.948	10.951	5.640
	9.494	0.664	-1.302	-1.633	-0.559
			4.581	11.950	29.662

I = 70

T	XM	FM	X(3M/4)-XM	X(M/2)-XM	X(M/4)-X(3M/4)
0	-0.176	0.970	-0.648	-1.208	-1.718
			0.530	0.855	0.778
	5.671	0.030	-1.934	-2.411	-0.774
			5.724	13.755	31.031
1	-0.219	0.908	-0.725	-1.309	-1.783
			0.629	0.987	0.825
	5.714	0.030	-1.929	-2.404	-0.770
			5.728	13.775	31.047
2	-0.343	1.536	-0.906	-1.536	-1.927
			0.899	1.339	0.941
	5.838	0.060	-1.917	-2.385	-0.759
			5.739	13.834	31.328
3	-0.539	1.936	-1.112	-1.789	-2.087
			1.294	1.845	1.068
	6.034	0.089	-1.900	-2.359	-0.744
			5.758	13.928	31.614
4	-0.795	2.197	-1.306	-2.021	-2.337
			1.777	2.451	1.251
	6.390	0.116	-1.881	-2.333	-0.729
			5.786	14.054	32.039
5	-1.099	2.380	-1.475	-2.222	-2.360
			2.322	3.134	1.425
	6.594	0.143	-1.864	-2.308	-0.716
			5.821	14.206	32.547
6	-1.442	2.516	-1.617	-2.392	-2.518
			2.912	3.069	1.608
	6.937	0.168	-1.850	-2.287	-0.705
			5.863	14.379	33.123
7	-1.813	2.621	-1.738	-2.536	-2.644
			3.532	4.644	1.800
	7.308	0.192	-1.838	-2.270	-0.695
			5.911	14.567	33.744

I = 70 (CONTD)

8	-2.207	2.707	-1.340	-2.659	-2.762
			4.174	5.449	2.001
	7.702	0.215	-1.831	-2.256	-0.686
			5.963	14.767	34.409
9	-2.619	2.778	-1.928	-2.764	-2.875
			4.832	6.279	2.210
	8.114	0.237	-1.824	-2.245	-0.680
			6.020	14.975	35.095
10	-3.045	2.838	-2.003	-2.855	-2.984
			5.494	7.125	2.432
	8.540	0.257	-1.819	-2.237	-0.675
			6.083	15.197	35.802
11	-3.481	2.891	-2.067	-2.935	-3.091
			6.169	7.988	2.657
	8.976	0.277	-1.816	-2.232	-0.671
			6.148	15.426	36.522
12	-3.927	2.937	-2.123	-3.006	-3.194
			6.850	8.861	2.827
	9.422	0.295	-1.814	-2.228	-0.668
			6.214	15.656	37.253
13	-4.380	2.978	-2.173	-3.068	-3.290
			7.525	9.743	3.132
	9.875	0.313	-1.813	-2.225	-0.665
			6.280	15.887	37.992
14	-4.839	3.015	-2.217	-3.124	-3.386
			8.203	10.634	3.383
	10.334	0.330	-1.813	-2.224	-0.663
			6.347	16.117	38.770
15	-5.302	3.049	-2.256	-3.175	-3.479
			8.682	11.531	3.639
	10.797	0.346	-1.814	-2.223	-0.661
			6.414	16.346	39.481

I = 80

T	XM	FM	X(3M/4)-XM	X(M/2)-XM	X(M/4)-X(3M/4)
0	-0.087	0.992	-0.609	-1.095	-1.412
			0.551	0.921	0.945
	11.430	0.008	-3.829	-4.765	-1.513
			11.457	27.628	62.468
1	-0.109	0.921	-0.695	-1.205	-1.463
			0.648	1.046	0.988
	11.452	0.008	-3.826	-4.761	-1.510
			11.459	27.638	62.501
2	-0.174	1.562	-0.906	-1.465	-1.569
			0.910	1.376	1.082
	11.516	0.015	-3.818	-4.748	-1.503
			11.465	27.668	62.608
3	-0.280	1.958	-1.173	-1.779	-1.679
			1.288	1.834	1.187
	11.622	0.023	-3.807	-4.727	-1.487
			11.474	27.717	62.783
4	-0.425	2.211	-1.453	-2.096	-1.775
			1.752	2.378	1.287
	11.767	0.030	-3.790	-4.701	-1.471
			11.488	27.784	63.022
5	-0.607	2.382	-1.723	-2.396	-1.858
			2.282	2.984	1.381
	11.949	0.037	-3.771	-4.672	-1.453
			11.505	27.870	63.365
6	-0.822	2.505	-1.977	-2.673	-1.931
			2.864	3.638	1.467
	12.165	0.045	-3.750	-4.640	-1.434
			11.526	27.972	63.679
7	-1.068	2.597	-2.209	-2.925	-1.995
			3.489	4.334	1.549
	12.410	0.052	-3.728	-4.607	-1.414
			11.552	28.089	64.088

I = 80 (CONTD)

8	-1.340	2.670	-2.422	-3.152	-2.052
			4.151	5.064	1.628
	12.683	0.059	-3.706	-4.574	-1.394
			11.582	28.221	64.541
9	-1.637	2.729	-2.615	-3.358	-2.103
			4.844	5.824	1.704
	12.980	0.065	-3.685	-4.543	-1.375
			11.616	28.366	65.038
10	-1.955	2.777	-2.790	-3.543	-2.150
			5.566	6.609	1.776
	13.298	0.072	-3.665	-4.512	-1.358
			11.653	28.522	65.571
11	-2.292	2.816	-2.947	-3.710	-2.194
			6.312	7.416	1.844
	13.635	0.078	-3.646	-4.484	-1.341
			11.694	28.694	66.136
12	-2.645	2.853	-3.090	-3.861	-2.233
			7.076	8.242	1.914
	13.988	0.085	-3.629	-4.458	-1.325
			11.738	28.869	66.731
13	-3.013	2.884	-3.220	-3.998	-2.271
			7.859	9.083	1.981
	14.355	0.091	-3.613	-4.434	-1.311
			11.785	29.058	67.273
14	-3.393	2.911	-3.337	-4.122	-2.308
			8.657	9.941	2.046
	14.736	0.097	-3.599	-4.412	-1.297
			11.835	29.254	67.991
15	-3.725	2.935	-3.444	-4.235	-2.342
			9.467	10.810	2.113
	15.127	0.103	-3.586	-4.392	-1.285
			11.887	29.455	68.568

1 = 90

T	XM	FM	X(3M/4)-XM	X(M/2)-XM	X(M/4)-X(3M/4)
0	0.000	1.000	-0.577 0.577	-1.000 1.000	-1.155 1.155
1	0.000	0.927	-0.669 0.669	-1.118 1.118	-1.198 1.198
2	0.000	1.571	-0.910 0.910	-1.415 1.415	-1.288 1.288
3	0.000	1.966	-1.240 1.240	-1.803 1.803	-1.379 1.379
4	0.000	2.214	-1.624 1.624	-2.236 2.236	-1.455 1.455
5	0.000	2.381	-2.039 2.039	-2.692 2.692	-1.517 1.517
6	0.000	2.498	-2.475 2.475	-3.162 3.162	-1.567 1.567
7	0.000	2.585	-2.923 2.923	-3.640 3.640	-1.609 1.609
8	0.000	2.652	-3.383 3.383	-4.123 4.123	-1.643 1.643
9	0.000	2.704	-3.849 3.849	-4.610 4.610	-1.671 1.671
10	0.000	2.747	-4.320 4.320	-5.099 5.099	-1.697 1.697
11	0.000	2.782	-4.797 4.797	-5.591 5.591	-1.718 1.718
12	0.000	2.811	-5.276 5.276	-6.083 6.083	-1.737 1.737
13	0.000	2.836	-5.758 5.758	-6.577 6.577	-1.753 1.753
14	0.000	2.858	-6.242 6.242	-7.071 7.071	-1.767 1.767
15	0.000	2.876	-6.728 6.728	-7.567 7.567	-1.781 1.781

APPENDIX II

Characteristics of total field anomalies due to square vertical prisms of infinite depth extent

Notations used in the following tables.

Notation in the text.

I	Inclination of the Earth's field	i
T	Thickness of the prism in units of the depth	t
XM	Position of maximum/minimum	x_m^S, x_m^N
FM	Value of maximum/minimum	f_M, f_m
$X_{(kM/4)}$	Positions of $kM/4$ times maximum/minimum	$x_{k/4}, x'_{k/4}, y_{k/4}^S$ and $x_{k/4}, \bar{x}_{k/4}, y_{k/4}^N$

l = 0

T	XM	FM	$X(3M/4)-XM$	$X(M/2)-XM$	$X(M/4)-XM$
1	0.000	-0.005	-0.364 0.364 -0.679	-0.575 0.575 -1.203	-0.822 0.822 -2.156
2	0.000	-0.403	-0.454 0.454 -0.828	-0.697 0.697 -1.423	-0.965 0.965 -2.482
3	0.000	-1.047	-0.691 0.691 -1.166	-1.001 1.001 -1.932	-1.305 1.305 -3.245
4	0.000	-1.529	-1.024 1.024 -1.571	-1.397 1.397 -2.549	-1.728 1.728 -4.183
5	0.000	-1.855	-1.418 1.418 -2.000	-1.837 1.837 -3.208	-2.187 2.187 -5.191
6	0.000	-2.079	-1.849 1.849 -2.440	-2.299 2.299 -3.886	-2.662 2.662 -6.336
7	0.000	-2.240	-2.303 2.303 -2.886	-2.774 2.774 -4.573	-3.145 3.145 -7.295
8	0.000	-2.360	-2.771 2.771 -3.334	-3.256 3.256 -5.265	-3.634 3.634 -8.367
9	0.000	-2.452	-3.247 3.247 -3.784	-3.743 3.743 -5.961	-4.125 4.125 -9.447
10	0.000	-2.526	-3.731 3.731 -4.229	-4.233 4.233 -6.661	-4.618 4.618 -10.529
11	0.000	-2.585	-4.217 4.217 -4.687	-4.726 4.726 -7.362	-5.113 5.113 -11.615

l = 10

T	XN	FM	X(3M/4)-XM	X(M/2)-XM	X(M/4)-XM
.1	-1.610	0.001	-0.988	-2.019	-4.095
			0.516	0.665	0.767
			-0.593	-0.913	-1.256
	0.158	-0.005	-0.342	-0.514	-0.690
			0.395	0.655	1.010
			-0.605	-1.213	-2.172
1	-1.765	0.107	-1.062	-2.139	-4.321
			0.517	0.677	0.790
			-0.660	-1.017	-1.397
	0.207	-0.390	-0.433	-0.644	-0.846
			0.475	0.762	1.134
			-0.833	-1.432	-2.496
2	-2.196	0.335	-1.179	-2.301	-4.825
			0.582	0.769	0.903
			-0.863	-1.313	-1.791
	0.360	-1.026	-0.689	-0.999	-1.261
			0.646	0.981	1.382
			-1.162	-1.927	-3.241
3	-2.719	0.574	-1.332	-2.693	-5.480
			0.658	0.872	1.029
			-1.136	-1.694	-2.276
	0.614	-1.526	-1.073	-1.522	-1.855
			0.812	1.184	1.611
			-1.543	-2.497	-4.120
4	-3.270	0.788	-1.499	-3.038	-6.194
			0.732	0.971	1.149
			-1.452	-2.115	-2.794
	0.949	-1.893	-1.537	-2.163	-2.570
			0.939	1.335	1.784
			-1.934	-3.066	-5.011
5	-3.829	0.973	-1.671	-3.400	-6.940
			0.801	1.061	1.257
			-1.794	-2.557	-3.324
	1.335	-2.172	-2.029	-2.875	-3.356
			1.032	1.448	1.916
			-2.327	-3.620	-5.878

I = 10 (CONTD)

6	-4.386	1.132	-1.845	-3.762	-7.695
			0.865	1.143	1.354
			-2.154	-3.011	-3.657
			-2.516	-3.626	-4.185
	1.747	-2.394	1.106	1.538	2.024
			-2.724	-4.154	-6.713
7	-4.941	1.271	-2.019	-4.125	-6.455
			0.924	1.218	1.441
			-2.529	-3.473	-4.391
			-2.974	-4.400	-5.036
	2.176	-2.578	1.168	1.613	2.116
			-3.128	-4.680	-7.525
8	-5.492	1.392	-2.188	-4.490	-9.214
			0.978	1.285	1.520
			-2.915	-3.941	-4.925
			-3.397	-5.188	-5.909
	2.615	-2.734	1.223	1.680	2.197
			-3.537	-5.202	-8.309
9	-6.039	1.501	-2.357	-4.852	-9.970
			1.028	1.347	1.592
			-3.309	-4.414	-5.458
			-3.776	-5.987	-6.791
	3.061	-2.868	1.272	1.740	2.271
			-3.952	-5.716	-9.077
10	-6.583	1.597	-2.523	-5.211	-10.725
			1.075	1.404	1.658
			-3.709	-4.689	-5.991
			-4.131	-6.792	-7.685
	3.513	-2.986	1.316	1.794	2.339
			-4.372	-6.227	-9.827

I = 20

T	XM	FM	$X(3M/4)-XM$	$X(M/2)-XM$	$X(M/4)-XM$
1	-1.186	0.002	-0.841	-1.674	-3.336
			0.441	0.582	0.684
			-0.542	-0.853	-1.212
	0.324	-0.004	-0.327	-0.473	-0.603
			0.437	0.761	1.263
			-0.704	-1.244	-2.221
1	-1.360	0.194	-0.897	-1.779	-3.543
			0.474	0.634	0.754
			-0.630	-0.986	-1.396
	0.417	-0.352	-0.410	-0.595	-0.756
			0.502	0.850	1.375
			-0.850	-1.461	-2.542
2	-1.777	0.580	-1.014	-2.014	-4.028
			0.555	0.756	0.916
			-0.859	-1.313	-1.830
	0.669	-0.955	-0.618	-0.912	-1.158
			0.623	1.014	1.584
			-1.163	-1.929	-3.250
3	-2.276	0.962	-1.155	-2.301	-4.628
			0.635	0.879	1.081
			-1.161	-1.721	-2.346
	1.022	-1.471	-0.870	-1.334	-1.706
			0.727	1.155	1.774
			-1.519	-2.448	-4.052
4	-2.796	1.291	-1.301	-2.609	-5.268
			0.707	0.985	1.228
			-1.506	-2.164	-2.886
	1.428	-1.881	-1.112	-1.792	-2.330
			0.806	1.267	1.934
			-1.890	-2.967	-4.849
5	-3.322	1.569	-1.446	-2.917	-5.924
			0.767	1.076	1.356
			-1.876	-2.625	-3.431
	1.862	-2.211	-1.334	-2.252	-2.992
			0.870	1.359	2.072
			-2.274	-3.481	-5.625

1 = 20 (CONTD)

6	-3.848	1.808	-1.586	-3.224	-6.581
			0.819	1.153	1.468
			-2.266	-3.096	-3.977
	2.311	-2.486	-1.530	-2.701	-3.674
			0.921	1.436	2.196
			-2.667	-3.990	-6.379
7	-4.371	2.014	-1.722	-3.529	-7.233
			0.863	1.220	1.568
			-2.668	-3.573	-4.522
	2.770	-2.719	-1.714	-3.133	-4.368
			0.966	1.503	2.309
			-3.068	-4.493	-7.109
8	-4.892	2.195	-1.852	-3.828	-7.882
			0.903	1.280	1.658
			-3.077	-4.055	-5.064
	3.236	-2.920	-1.880	-3.546	-5.068
			1.006	1.563	2.412
			-3.482	-4.996	-7.827
9	-5.411	2.356	-1.979	-4.116	-8.517
			0.938	1.333	1.739
			-3.499	-4.539	-5.606
	3.707	-3.098	-2.035	-3.947	-5.772
			1.040	1.617	2.509
			-3.902	-5.498	-8.530
10	-5.927	2.501	-2.099	-4.402	-9.148
			0.969	1.381	1.814
			-3.926	-5.027	-6.146
	4.182	-3.256	-2.179	-4.326	-6.477
			1.071	1.666	2.600
			-4.326	-6.002	-9.216

I = 30

T	XM	FM	X(3M/4)-XM	X(M/2)-XM	X(M/4)-XM
1	-0.906	0.004	-0.731	-1.434	-2.810
			0.414	0.558	0.669
			-0.514	0.822	-1.205
	0.511	-0.004	-0.323	-0.450	-0.553
			0.492	0.895	1.587
			-0.740	-1.302	-2.311
1	-1.071	0.301	-0.794	-1.541	-3.014
			0.462	0.636	0.774
			-0.611	-0.966	-1.404
	0.636	-0.293	-0.390	-0.551	-0.683
			0.544	0.971	1.698
			-0.881	-1.514	-2.625
2	-1.467	0.869	-0.911	-1.767	-3.473
			0.569	0.809	1.016
			-0.857	-1.311	-1.859
	0.944	-0.829	-0.535	-0.785	-0.998
			0.638	1.112	1.913
			-1.178	-1.958	-3.298
3	-1.943	1.401	-1.040	-2.031	-4.022
			0.670	0.982	1.276
			-1.179	-1.734	-2.390
	1.338	-1.325	-0.688	-1.055	-1.337
			0.721	1.243	2.131
			-1.519	-2.449	-4.048
4	-2.443	1.845	-1.166	-2.300	-4.595
			0.752	1.133	1.521
			-1.544	-2.190	-2.940
	1.771	-1.743	-0.828	-1.323	-1.801
			0.789	1.356	2.333
			-1.881	-2.946	-4.795
5	-2.950	2.215	-1.285	-2.566	-5.168
			0.820	1.261	1.746
			-1.933	-2.661	-3.494
	2.224	-2.094	-0.953	-1.580	-2.219
			0.845	1.455	2.521
			-2.259	-3.445	-5.526

I = 30(CONTD)

6	-3.458	2.527	-1.396	-2.829	-5.738
			0.877	1.372	1.952
			-2.338	-3.140	-4.048
	3.689	-2.394	-1.069	-1.832	-2.636
			0.893	1.544	3.696
			-3.649	-3.944	-6.344
7	-3.967	2.796	-1.500	-3.069	-6.298
			0.924	1.470	2.144
			-2.757	-3.625	-4.600
	3.161	-2.653	-1.173	-2.054	-3.047
			0.935	1.623	2.802
			-3.049	-4.440	-6.943
8	-4.475	3.031	-1.598	-3.312	-6.650
			0.965	1.558	2.323
			-3.182	-4.113	-5.149
	3.639	-2.882	-1.270	-2.274	-3.451
			0.971	1.695	3.020
			-3.460	-4.938	-7.631
9	-4.982	3.240	-1.610	-3.543	-7.391
			1.002	1.636	2.492
			-3.616	-4.602	-5.696
	4.121	-3.086	-1.362	-2.486	-3.850
			1.005	1.762	3.171
			-3.279	-5.436	-8.310
10	-5.488	3.422	-1.777	-3.768	-7.922
			1.034	1.708	2.653
			-4.053	-5.095	-6.240
	4.606	-3.269	-1.447	-2.690	-4.244
			1.033	1.824	3.316
			-4.302	-5.936	-8.975

l = 40

T	XH	FM	X(3M/4)-XM	X(M/2)-XM	X(M/4)-XM
.1	-0.690	0.005	-0.659	-1.263	-2.423
			0.399	0.554	0.684
			-0.494	-0.800	-1.207
	0.737	-0.003	-0.332	-0.448	-0.536
			0.566	1.071	2.001
			-0.799	-1.398	-2.463
1	-0.843	0.419	-0.722	-1.369	-2.619
			0.469	0.664	0.830
			-0.597	-0.951	-1.412
	0.881	-0.222	-0.379	-0.521	-0.632
			0.608	1.138	2.116
			-0.932	-1.601	-2.764
2	-1.210	1.175	-0.845	-1.592	-3.052
			0.616	0.910	1.173
			-0.854	-1.304	-1.872
	1.217	-0.656	-0.476	-0.677	-0.843
			0.687	1.271	2.357
			-1.215	-2.024	-3.404
3	-1.663	1.848	-0.966	-1.834	-3.550
			0.761	1.179	1.580
			-1.189	-1.736	-2.408
	1.630	-1.090	-0.575	-0.844	-1.084
			00.764	1.410	2.620
			-1.543	-2.497	-4.118
4	-2.145	2.393	-1.075	-2.067	-4.058
			0.881	1.431	1.999
			-1.567	-2.198	-2.963
	2.075	-1.475	-0.664	-1.005	-1.328
			0.833	1.539	2.881
			-1.896	-2.981	-4.836
5	-2.639	2.837	-1.172	-2.290	-4.559
			0.981	1.657	2.410
			-1.970	-2.675	-3.521
	2.537	-1.810	-0.743	-1.157	-1.570
			0.892	1.658	3.132
			-2.267	-3.471	-5.545

1 = 40 (CONTD)

6	-3.138	3.207	-1.260	-2.503	-5.048
			1.062	1.861	2.810
			-2.385	-3.159	-4.078
			-0.816	-1.301	-1.807
			0.943	1.766	3.373
7	-3.638	3.522	-2.651	-3.964	-6.244
			4		
			-2.103		
			3.009		
8	-4.140	3.796	-1.340	-2.703	-5.530
			1.132	2.045	3.198
			-2.815	-3.647	-4.633
			-0.882	-1.437	-2.039
			0.989	1.868	3.603
9	-4.641	4.038	-3.046	-4.459	-6.929
			3.488		
			-2.362		
			3.971		
			-2.593		
10	-5.143	4.255	-1.414	-2.894	-5.997
			1.192	2.212	3.575
			-3.249	-4.139	-5.185
			-0.943	-1.566	-2.267
			1.030	1.962	3.825
11	-4.457	-2.801	-3.452	-4.954	-7.605
			-1.481	-3.078	-6.457
			1.246	2.366	3.941
			-3.692	-4.632	-5.735
			-1.000	-1.691	-2.491
12	4.946	-2.990	1.068	2.051	4.042
			-3.866	-5.450	-8.275
			-1.544	-3.258	-6.907
			1.294	2.510	4.297
			-4.138	-5.125	-6.281
13	4.946	-2.990	-1.053	-1.809	-2.710
			1.103	2.133	4.248
			-4.284	-5.946	-8.934

I = 50

T	MI	PH	$X(3H/4)-MI$	$X(H/2)-ME$	$X(H/4)-ME$
1	-0.515	0.000	-0.000	-1.130	-2.118
			0.390	0.560	0.732
			-0.480	-0.705	-1.219
	1.040	-0.000	-0.300	-0.460	-0.564
			0.097	1.310	2.551
			-0.096	-1.559	-2.716
2	-0.640	0.538	-0.678	-1.140	-0.305
			0.400	0.700	0.900
			-0.586	-0.739	-1.417
	1.185	-0.147	-0.280	-0.520	-0.610
			0.709	1.370	2.076
			-1.017	-1.745	-2.093
3	-0.977	1.469	-0.806	-1.460	-2.712
			0.056	1.030	1.336
			-0.650	-1.205	-1.872
	1.531	-0.458	-0.458	-0.619	-0.747
			0.781	1.510	2.940
			-1.263	-2.146	-3.599
3	-1.390	2.060	-0.926	-1.600	-3.164
			0.905	1.486	1.000
			-1.194	-1.730	-2.405
	2.049	-0.794	-0.520	-0.725	-0.898
			0.857	1.655	3.247
			-1.597	-2.602	-4.263
4	-1.861	2.880	-1.026	-1.805	-3.610
			1.106	1.900	2.473
			-1.501	-2.195	-3.050
	2.390	-1.100	-0.580	-0.825	-1.035
			0.930	1.809	3.565
			-1.939	-1.675	-4.982
5	-2.340	3.370	-1.100	-2.061	-4.043
			1.277	2.220	3.003
			-1.992	-2.674	-3.515
	2.863	-1.390	-0.624	-0.910	-1.173
			0.995	1.951	3.377
			-2.099	-3.550	-5.077

I = 30 (CONTD)

6	-2.631	3.771	-1.180	-2.252	-4.450
			1.424	3.604	3.696
			-3.416	-3.161	-4.071
			-0.603	-1.005	-1.305
			1.056	0.005	4.161
7	-3.225	4.109	-2.674	-4.040	-0.367
			1.550	0.950	4.500
			-2.854	-3.651	-4.624
			-0.727	-1.035	-1.434
			1.111	0.212	4.472
8	-3.812	4.400	-3.060	-4.530	-7.043
			1.650	5.005	4.019
			-3.297	-4.144	-5.174
			-0.786	-1.101	-1.519
			1.100	2.334	4.703
9	-4.310	4.055	-3.438	-5.020	-7.217
			1.755	3.623	5.324
			-3.740	-4.630	-5.723
			-0.803	-1.033	-1.000
			1.209	0.442	5.043
10	-4.810	4.001	-3.854	-5.321	-0.383
			1.305	3.000	2.123
			-4.197	-5.134	-6.267
			-0.837	-1.302	-1.796
			1.212	0.500	5.310
11	-5.260	-3.445	-4.475	-6.035	-0.041

I = 60

T	XH	FH	$X(3H/4)-XM$	$X(H/2)-XM$	$X(H/4)-XM$
.1	-0.372	0.008	-0.555	-1.016	-1.854
			0.408	0.606	0.816
			-0.471	-0.778	-1.223
	1.507	-0.001	-0.45	-0.571	-0.656
			0.828	1.681	3.382
			-1.068	-1.844	-3.167
1	-0.475	0.645	-0.636	-1.137	-2.041
			0.508	0.755	1.003
			-0.579	-0.929	-1.419
	1.622	-0.080	-0.434	-0.568	-0.660
			0.880	1.760	3.520
			-1.167	-1.998	-3.398
2	-0.747	1.727	-0.790	-1.371	-2.430
			0.760	1.137	1.474
			-0.845	-1.284	-1.861
	1.959	-0.264	-0.476	-0.629	-0.738
			0.942	1.885	3.788
			-1.410	-2.369	-3.960
3	-1.121	2.607	-0.925	-1.595	-2.843
			1.078	1.640	2.096
			-1.195	-1.718	-2.385
	2.371	-0.481	-0.524	-0.698	-0.827
			1.020	2.043	4.126
			-1.706	-2.804	-4.615
4	-1.551	3.264	-1.029	-1.784	-3.234
			1.407	2.203	2.798
			-1.586	-2.184	-2.929
	2.816	-0.697	-0.569	-0.765	-0.913
			1.100	2.211	4.488
			-2.031	-3.265	-5.294
5	-2.011	3.768	-1.110	-1.946	-3.601
			1.720	2.786	3.542
			-2.004	-2.663	-3.479
	3.278	-0.902	-0.609	-0.825	-0.993
			1.177	2.378	4.852
			-2.376	-3.738	-5.980

1 = 60 (00070)

5	-2.486	4.172	-1.174	-2.037	-3.046
			2.003	3.374	4.300
			-2.430	-3.151	-4.026
			-0.644	-0.873	-1.066
			1.049	2.041	5.213
	3.751	-1.091	-0.727	-4.017	-0.600
7	-2.970	4.504	-1.007	-2.212	-4.275
			2.265	3.057	5.082
			-2.801	-3.642	-4.375
			-0.676	-0.940	-1.136
			1.317	2.695	5.574
	4.009	-1.265	-3.111	-4.701	-7.336
8	-3.459	4.786	-1.275	-2.327	-4.594
			2.490	4.533	5.875
			-3.531	-4.126	-5.119
			-0.704	-0.975	-1.204
			1.380	2.845	5.921
	6.713	-1.425	-3.495	-5.102	-8.009
9	-3.951	5.031	-1.312	-2.434	-4.899
			2.711	5.055	6.606
			-1.707	-4.031	-5.661
			-0.730	-1.010	-1.267
			1.439	2.940	6.255
	5.199	-1.574	-3.033	-5.677	-8.672
20	-4.446	5.245	-1.343	-2.534	-5.197
			2.898	5.649	7.450
			-4.240	-5.127	-6.200
			-0.753	-1.057	-1.307
			1.465	3.129	6.601
	5.686	-1.712	-4.206	-6.168	-9.331

I = 70

T	XH	PH	X(3H/4)-XH	X(H/2)-XH	X(H/4)-XH
1	-0.240	0.000	-0.518	-0.932	-1.526
			0.220	0.550	0.928
			-0.465	-0.772	-1.230
	2.400	-0.000	-0.744	-0.390	-1.001
			1.064	2.316	4.202
			-1.464	-2.407	-4.208
2	-0.311	0.731	-0.600	-1.051	-1.610
			0.530	0.006	1.117
			-0.573	-0.923	-1.410
	2.607	-0.030	-0.562	-0.723	-0.831
			1.224	2.406	5.002
			-1.480	-2.530	-4.240
3	-0.510	1.900	-0.993	-1.351	-0.103
			0.216	1.008	1.587
			-0.840	-1.273	-1.347
	2.706	-0.105	-0.535	-0.755	-0.870
			1.276	2.606	5.331
			-1.695	-2.242	-4.727
3	-0.804	2.850	-0.971	-1.561	-2.588
			1.215	1.740	2.016
			-1.191	-1.704	-2.354
	3.094	-0.209	-0.619	-0.802	-0.926
			1.348	2.757	5.664
			-1.220	-3.244	-5.335
4	-1.100	3.524	-1.112	-1.760	-2.346
			1.607	2.406	2.043
			-1.526	-2.167	-2.885
	3.523	-0.319	-0.657	-0.851	-0.984
			1.439	2.951	6.042
			-2.246	-3.081	-5.989
5	-1.377	4.014	-1.216	-1.920	-3.264
			2.173	3.112	3.733
			-2.008	-2.645	-3.422
	3.975	-0.430	-0.692	-0.698	-1.039
			2.513	3.115	6.444
			-2.560	-4.135	-6.050

I = 70 (CONTD)

6	-2.015	4.392	-1.299	-2.063	-3.550
			2.676	3.853	4.564
			-2.448	-3.132	-3.959
			-0.723	-0.939	-1.089
			1.601	3.305	6.854
7	-2.472	4.695	-2.905	-4.600	-7.331
			-1.362	-2.173	-3.811
			3.176	4.616	5.423
			-2.898	-3.623	-4.494
			-0.752	-0.977	-1.134
8	-2.942	4.946	1.683	3.489	7.264
			-3.256	-5.072	-8.003
			-1.412	-2.268	-4.054
			3.668	5.394	6.300
			-3.354	-4.116	-5.028
9	-3.420	5.160	-0.777	-1.010	-1.175
			1.763	3.670	7.670
			-3.619	-5.549	-8.673
			-1.454	-2.350	-4.282
			4.141	6.176	7.190
10	-3.903	5.345	-3.817	-4.611	-5.560
			-0.800	-1.041	-1.212
			1.840	3.848	8.075
			-3.991	-6.030	-9.342
			-1.487	-2.423	-4.500
10	-3.903	5.345	4.597	6.964	8.089
			-4.283	-5.108	-6.090
			-0.821	-1.068	-1.247
			1.913	4.021	8.475
			-4.372	-6.515	-10.007

I = 80

T	XH	FM	$X(3M/4)-XH$	$X(M/2)-XH$	$X(M/4)-XH$
.1	-0.118	0.010	-0.487	-0.840	-1.421
			0.439	0.705	1.070
			-0.462	-0.769	-1.234
	5.672	-0.000	-2.223	-2.441	-2.595
			1.728	4.159	9.282
			-2.886	-4.871	-8.008
1	-0.154	0.786	-0.587	-0.979	-1.603
			0.550	0.860	1.255
			-0.569	-0.918	-1.418
	4.646	-0.005	-1.027	-1.303	-1.483
			2.233	4.629	9.562
			-2.456	-4.157	-6.869
2	-0.260	2.052	-0.813	-1.277	-1.992
			0.841	1.244	1.698
			-0.836	-1.264	-1.834
	4.825	-0.019	-1.009	-1.287	-1.468
			2.288	4.724	9.748
			-2.583	-4.355	-7.177
3	-0.427	3.009	-1.067	-1.597	-2.411
			1.254	1.764	2.270
			-1.187	-1.691	-2.326
	5.119	-0.039	-1.019	-1.302	-1.486
			2.343	4.841	10.006
			-2.777	-4.661	-7.649
4	-0.652	3.666	-1.306	-1.887	-2.798
			1.756	2.375	2.924
			-1.583	-2.150	-2.841
	5.481	-0.065	-1.044	-1.333	-1.522
			2.407	4.983	10.326
			-3.014	-5.030	-8.212
5	-0.926	4.128	-1.516	-2.138	-3.138
			2.328	3.056	3.640
			-2.007	-2.625	-3.361
	5.883	-0.094	-1.075	-1.373	-1.566
			2.482	5.147	10.692
			-3.277	-5.432	-8.824

I = 80 (CONTD)

6	-1.242	4.469	-1.694	-2.348	-3.435
			2.956	3.792	4.407
			-2.449	-3.109	-3.883
			-1.107	-1.413	-1.611
			2.567	5.330	11.091
	6.308	-0.124	-3.565	-5.856	-9.461
7	-1.594	4.731	-1.842	-2.523	-3.691
			3.623	4.571	5.216
			-2.903	-3.598	-4.404
			-1.141	-1.454	-1.657
			2.656	5.522	11.511
	6.750	-0.155	-3.862	-6.295	-10.113
8	-1.974	4.939	-1.964	-2.669	-3.916
			4.325	5.385	6.056
			-3.364	-4.090	-4.924
			-1.172	-1.493	-1.700
			2.749	5.724	11.945
	7.203	-0.187	-4.180	-6.744	-10.773
9	-2.377	5.110	-2.067	-2.790	-4.113
			5.050	6.227	6.924
			-3.833	-4.584	-5.442
			-1.204	-1.530	-1.741
			2.843	5.925	12.389
	7.665	-0.218	-4.510	-7.201	-11.435
10	-2.798	5.254	-2.150	-2.893	-4.288
			5.793	7.090	7.812
			-4.304	-5.079	-5.959
			-1.233	-1.565	-1.778
			2.939	6.131	12.837
	8.133	-0.248	-4.840	-7.664	-12.101

I = 90

T	XM	FM	$X(3M/4)-XM$	$X(M/2)-XM$	$X(M/4)-XM$
.1	0.000	0.010	-0.461 0.461 -0.461	-0.766 0.768 -0.768	-1.235 1.235 -1.235
1	0.000	0.805	-0.568 0.568 -0.568	-0.916 0.916 -0.916	-1.417 1.417 -1.417
2	0.000	2.094	-0.835 0.835 -0.835	-1.261 1.261 -1.261	-1.829 1.829 -1.829
3	0.000	3.059	-1.185 1.185 -1.185	-1.686 1.686 -1.686	-2.315 2.315 -2.315
4	0.000	3.709	-1.581 1.581 -1.581	-2.142 2.142 -2.142	-2.821 2.821 -2.821
5	0.000	4.157	-2.005 2.005 -2.005	-2.615 2.615 -2.615	-3.333 3.333 -3.333
6	0.000	4.479	-2.447 2.447 -2.447	-3.097 3.097 -3.097	-3.846 3.846 -3.846
7	0.000 v	4.719	-2.901 2.901 -2.901	-3.584 3.584 -3.584	-4.357 4.357 -4.357
8	0.000	4.904	-3.363 3.363 -3.363	-4.074 4.074 -4.074	-4.868 4.868 -4.868
9	0.000	5.051	-3.833 3.833 -3.833	-4.566 4.566 -4.566	-5.377 5.377 -5.377
10	0.000	5.170	-4.306 4.306 -4.306	-5.059 5.059 -5.059	-5.885 5.885 -5.885

I = 10

APPENDIX III

Flow Chart for Automatic Model fitting (Two Dimensional
Vertical Rectangular Prism)

```
( Read Cj          Initial values of parameters
( Read dj,  j = 1 .... 4  Weighting factors.
  Read N            Number of field points used
                     for fitting
```

Read x_i	x coordinate of field point
------------	-----------------------------

Read T_i , $i = 1 \dots N$ Observed field.

$$f_0 = \sum_i T_i^2$$

$S = 1$

(1) $V_3 = V_2$) Successive values of the sum
 $V_2 = V_1$) of squares of the residuals.

$$V_1 = 0$$
$$W_j = 0, j = 1 \dots 5$$
$$i = 1$$
$$(5) \quad a_1 = c_1 - x_1$$
$$a_2 = c_2 - x_i$$

$$Y_1 + \sqrt{-1}Z_1 = \ln \frac{(C_3 + \sqrt{-1}a_1)(C_4 + \sqrt{-1}a_2)}{(C_3 + \sqrt{-1}a_2)(C_4 + \sqrt{-1}a_1)} \quad \text{Anomaly due to the model}$$

$$W_1 = W_1 + Z_i Z_i$$

$$W_2 = W_2 + Z_i Y_i$$

$$W_3 = W_3 + Z_i T_i$$

$$W_4 = W_4 + Y_i Y_i$$

$$W_5 = W_5 + Y_i T_i$$

is $i = N$? No. $i = i + 1$ (5)

Yes

$$\begin{aligned} J_1 &= (W_2W_5 - W_3W_4)/(W_1W_4 - W_2W_2) \\ J_2 &= (W_2W_3 - W_1W_5)/(W_1W_4 - W_2W_2) \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \begin{array}{l} \text{Best fitting} \\ \text{value of } J_1 \text{ and} \\ J_2 \end{array}$$

$$V_1 = f_0 + J_1W_3 + J_2W_5 \quad \text{Sum of squares of the residuals}$$

is $s = 1$? Yes $\rightarrow (14)$

No.

is $V_1 > V_2$? Yes $\rightarrow (4)$

No.

is $q = 1$? yes. $t = 2$, (3)

No.

$$\lambda = \lambda/2$$

$\downarrow$

(3)

$$(4) \quad \text{is } t = 1? \text{ No. } \lambda = \lambda - 2\lambda' + \frac{\lambda'}{2} \frac{(V_1 - 4V_2 + 3V_3)}{(V_1 - 2V_2 + V_3)} s = 1 \rightarrow (3)$$

Yes

$q = 2$

$$\text{is } V_0 > V_1? \text{ Yes } \lambda = \frac{V_2 - 4V_1 + 3V_0}{V_2 - 2V_1 + V_0} \cdot \frac{\lambda}{2}, \quad s = 1 \rightarrow (3)$$

Inverse parabolic interpolation to find the approximate minimum on the chosen direction.

No.

$$\lambda = \lambda/2$$

$$(3) \quad c_j = h_j + \lambda a_j U_j, \quad j = 1 \dots 4.$$

$\downarrow$

(1)

$$(14) \quad \text{is } C_1 > C_2 \quad \text{Yes } x = C_1, C_1 = C_2, C_2 = x, \quad \text{Interchange } C_1 \text{ and } C_2 \text{ if } C_1 > C_2$$

$$J_1 = -J_1, J_2 = -J_2 \rightarrow (15)$$

No.

$$(15) \quad \text{is } C_3 > C_4 \quad \text{Yes } x = C_3, C_3 = C_4, C_4 = x, \quad \text{Interchange } C_3 \text{ and } C_4 \text{ if } C_3 > C_4$$

$$J_1 = -J_1, J_2 = -J_2 \rightarrow (16)$$

No.

$$(16) \quad \text{is } \frac{V_1}{f_0} < e_1? \quad \text{yes} \rightarrow (30)$$

No

$$\text{is } \frac{V_0 - V_1}{V_1} < e_2? \quad \text{yes} \rightarrow (30)$$

No

$$h_j = C_j, \quad j = 1 \dots 4 \rightarrow (17)$$

$$(17) \quad W_j = 0, \quad j = 1 \dots 12$$

$$b_j = 0, \quad j = 1 \dots 4$$

$$i = 1$$

$$(18) \quad g_1 = 0, \quad j = 1 \dots 4$$

$$j = 1$$

$$a = C_j - x_i$$

$$k = 3$$

$$(20) \quad R^2 = a^2 + C_k^2$$

$$g_j = g_j - \frac{(-J_1 C_k + J_2 a)}{R^2} \quad)$$

Anomaly due to the sheets

$$g_{(k+2)} = g_{(k+2)} - \frac{(J_1 a + J_2 C_k)}{R^2} \quad)$$

is $K = 4$? No. $k = k + 1 \rightarrow (20)$

Yes

is $j = 2$? No. $j = j + 1 \rightarrow (19)$

Yes

$$g_2 = -g_2$$

$$g_4 = -g_4$$

$$g_5 = T_i + J_1 Y_i + J_2 Z_i$$

$$j = 1$$

$$l = 1$$

$$(21) \quad k = l$$

$$(22) \quad W_j = W_j + g_k g_k \quad)$$

$$j = j + 1 \quad)$$

is $k = 4$? No. $k = k + 1 \rightarrow (22) \quad)$

yes $)$

$$b_i = b_i + g_i g_5 \quad)$$

is $i = 4$? No $i = i + 1 \rightarrow (21) \quad)$

Yes $)$

is $i = N$? No $i = i + 1 \rightarrow (18) \quad)$

Yes $\rightarrow (23) \quad)$

Setup the
coefficients of the
equations for
finding out the
thickness of each
sheet

(23) Solve the following equations for U_1, U_2, U_3 and U_4

$$W_1 U_1 + W_2 U_2 + W_3 U_3 + W_4 U_4 = b_1 \quad)$$

$$W_2 U_1 + W_5 U_2 + W_6 U_3 + W_7 U_4 = b_2 \quad)$$

$$W_3 U_1 + W_6 U_2 + W_8 U_3 + W_9 U_4 = b_3 \quad)$$

$$W_4 U_1 + W_7 U_2 + W_9 U_3 + W_{10} U_4 = b_4 \quad)$$

Solve for the
thicknesses of
the sheets

(Choleski Method)

$$s = 0, t = 1, q = 1, \lambda' = 1, \lambda = \lambda', v_0 = v_1 \rightarrow (2)$$

(30) Print C_j , $j = 1 \dots 4$

$$\text{Print } J' = \frac{1}{2}(J_1^2 + J_2^2)^{\frac{1}{2}}$$

$$\text{Print } I = \tan^{-1} \frac{J_1}{J_2} \qquad \text{Print results}$$

Print x_i

$$\text{Print } R(\mathbf{x}_i) = T_i + J_1 Y_i + J_2 Z_i, \quad i = 1 \dots N$$

Stop.

APPENDIX IV

Flow Chart for Two Dimensional Structure Fitting

Read N.	No. of field points
Read T_i , $i = 1 \dots N$	Field values
Read d	Distance between successive field points.
Read h	Depth to top or bottom.
Read s	s is +1 or -1 according as the top or the bottom of the structure is at the depth h.
Read Z	Inclination of direction of measurement of field.
Read E	Angle made by the magnetic meridian with the direction of strike.
Read Z'	Inclination to the horizontal of permanent magnetism.
Read E'	Angle made by the vertical plane containing the permanent magnetism with the strike.
Read J	Intensity of induced magnetisation
Read Q.	Q ratio i.e. ratio of permanent to induced magnetisation.
$h = h/d$	
$\mathcal{P} = \cos Z \sin E$	
$n = \sin Z$	
$l' = \cos Z' \sin E'$	
$n' = \sin Z'$	

$$J_1 = 2 sJ (2 n^2 + Q (l'n + n'l))$$

$$J_2 = 2 sJ (l^2 - n^2 + Q(l'l - n'n))$$

$$h_p = h$$

$$R_p = T_p \quad p = 1 \dots N$$

$$V_0 = \sum_{p=1}^N R_p^2$$

$$V_1 = V_0$$

$$(1) \quad V_3 = V_1$$

$$g_p = h_p, \quad p = 1 \dots N$$

$$(2) \quad p = 1$$

$$b_p = 0$$

$$i = 1$$

$$(4) \quad a_1 = p - i - 0.5$$

$$a_2 = p - i + 0.5$$

$$W_{ip} = \frac{J_1 a_2 + J_2 h_p}{a_2^2 + h_p^2} - \frac{J_1 a_1 + J_2 h_p}{a_1^2 + h_p^2}$$

$$b_p = b_p + W_{ip} R_i$$

$$\text{is } i = N? \quad \text{No. } i = i + 1 \rightarrow (4)$$

Yes

$$\text{is } p = N? \quad \text{No } p = p + 1 \rightarrow (3)$$

$$= \frac{(\bar{b} \times \bar{W})^T \times \bar{R}}{(\bar{b} \times \bar{W})^T \times (\bar{b} \times \bar{W})}$$

$$(5) \quad p = 1$$

$$(6) \quad h_p = g_p + \mu b_p$$

$$\text{is } s = 1? \quad \text{Yes. is } h_p > h? \quad \text{yes}$$

$$\rightarrow (7)$$

No.

No

is $h > h_p$? yes $h_p = h$

No

↓

↓

$h_p = h$

(7)

(7)

is $p = N$? No. $p = p + 1 \rightarrow (6)$

Yes.

(7) $h_0 = h$

$h_{N+1} = h$

$V_1 = 0$

$i = 1$

(8) $Y = 1$

$Z = 0$

$p = 1$

(9) $a = p - i - 0.5$

$\times \quad (Y + \sqrt{-1}Z) = (Y + \sqrt{-1}Z) \times \frac{h_{p-1} + \sqrt{-1}a}{h_p + \sqrt{-1}a}$

is $p = N + 1$? No. $p = p + 1 \rightarrow (9)$

Yes.

$(Y + \sqrt{-1}Z) = \log_e (Y + \sqrt{-1}Z)$

$R_i = T_i + J_2 Y + J_1 Z$

$V_1 = V_1 + R_i R_i$

is $i = N$? No. $i = i + 1 \rightarrow (8)$

Yes

is $V_1 > V_2$? yes $\rightarrow (10)$

is $V_1 > V_3$? yes $\rightarrow (10)$

is $V_1 < e_1 V_0$? yes $\rightarrow$ (11)

is $V_3 - V_1 < e_2 V_1$? yes $\rightarrow$ (11)

(10) $\mu = \mu/2$
↓
(5)

(11) {Print (dh_p) Depths to the top or bottom of
 { the structure.
 (Print R_p , $p = 1 \dots N$ Residual anomaly.
 Stop.

APPENDIX V

Flow Chart for Determining the Direction and Intensity of
Magnetisation of a Body of known Topography from its Total
Magnetic Anomaly.

Read Z	Inclination of the earth's field.
Read E	Angle made by the magnetic meridian with the x-axis.
$e = \cos Z \cos E$	) Direction cosines of the earth's field.
$m = \cos Z \sin E$	
$n = \sin Z$	
$k = 1$	
Read p.	Number of contours.
$i = 1$	
(1) Read h_i	Depth of the i^{th} contour
Read N_i	Number of negative corners on the i^{th} contour.
$j = 1$	
$L = k.$	
(2) Read a_k	) x, y coordinates of successive negative corners in the i^{th} contour.
Read b_k	
$k = k + 1$	
is $j = N_i$? No.	$j = j + 1 \rightarrow (2)$
Yes	
$a_k = a_L$	

$b_k = b_L$

$k = k + 1$

is $i = p$? No. $i = i + 1 \rightarrow (1)$

Yes

$h_0 = h_1$

$h_{p+1} = h_p$

$i = 1$

(3) $d_i = 0$) Clear stores for
 is $i = 14$? No. $i = i + 1 \rightarrow (3)$) setting up the
 coefficients of the
 equations

Yes

$s = 1$

(4) $f_1 = 0$)
 $f_2 = 0$) Clear stores for numerical integration
 $f_3 = 0$)

Read x)
 Read y) Coordinates of observed field points.

Read f_5 . $f_5 = T_0$, the observed field at (x,y)

is ~~the~~ spurious

character^{signifying end of data.}_{encountered}

during read? Yes $\rightarrow (10)$

No.

$k = 1$

$i = 1$

(5) $u_i = 0$)
 $v_i = 0$) Clear stores for evaluating the
 $w_i = 0$) contribution due to the i^{th} contour

$$q = 2$$

$$j = 1$$

$$a = a_k - x$$

(6) is $q = 1$? No. $b = b_k - y$, $q = 1$, $k = k + 1 \rightarrow (7)$

Yes

$$a = a_k - x$$

$$q = 2$$

(7) $R = (a^2 + b^2 + h_i^2)^{\frac{1}{2}}$

$$U_i = \frac{1}{R} \left(\frac{ma + nh_i}{R + b} \right) - U_i$$

$$V_i = \frac{1}{R} \left(\frac{mb + nh_i}{R + a} \right) - V_i$$

$$W_i = \frac{1}{R} \left(\frac{nh_i - na}{R + b} + \frac{mh_i - nb}{R + a} \right) - W_i$$

is $j = 2N_i$? No. $j = j + 1 \rightarrow (6)$

Yes

$$k = k + 1$$

$$f_1 = f_1 + \frac{1}{2} U_i (h_{i+1} - h_{i-1})$$

$$f_2 = f_2 + \frac{1}{2} V_i (h_{i+1} - h_{i-1})$$

$$f_3 = f_3 + \frac{1}{2} W_i (h_{i+1} - h_{i-1})$$

is $i = p$? No. $i = i + 1 \rightarrow (5)$

Yes

Print x

Print y

Print f_1

Print f_2

Print f_3

$f_4 = 1$

$k = 1$

$i = 1$

(8) $j = i$

(9) $d_k = d_k + f_i f_j$

$k = k + 1$

is $j = 5$? No. $j = j + 1 \longrightarrow (9)$

Yes

is $i = 4$? No. $i = i + 1 \longrightarrow (8)$

Yes

$g(3s-2) = f_1$

$g(3s-1) = f_2$

$g_{3s} = f_3$

$h_s = f_5$

$s = s + 1$

↓

(4)

- (10) Solve the following four equations for J_1, J_2, J_3 and f_0

$$d_1 J_1 + d_2 J_2 + d_3 J_3 + d_4 f_0 = d_5$$

$$d_2 J_1 + d_6 J_2 + d_7 J_3 + d_8 f_0 = d_9$$

$$d_3 J_1 + d_7 J_2 + d_{10} J_3 + d_{11} f_0 = d_{12}$$

$$d_4 J_1 + d_8 J_2 + d_{11} J_3 + d_{13} f_0 = d_{14}$$

Print $J = (J_1^2 + J_2^2 + J_3^2)^{\frac{1}{2}}$ Intensity of magnetisation

Print $U = \tan^{-1} \frac{J_3}{(J_1^2 + J_2^2)^{\frac{1}{2}}}$ Inclination of the magnetisation vector.

Print $E = \tan^{-1} \frac{J_2}{J_1}$ Angle made by the vertical plane containing the magnetisation vector with the x-axis.

Print f_0

$r = 0$

- (13) Print hr observed field

Print $J_1 g_{r+1} + J_2 g_{r+2} + J_3 g_{r+3} + f_0$ best fitting field

is $r = s$? No. $r = r + 3 \longrightarrow (13)$

Yes

Stop.

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