

A STUDY OF SMALL SCALE FEATURES IN THE ATMOSPHERE OF SATURN

by

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ABSTRACT

This thesis describes the analysis of some of the smaller scale features in Saturn's atmosphere, using data returned by the Voyager spacecraft. It is shown that despite the great differences in scale and composition, these features resemble flows which occur in both the Earth's atmosphere and the laboratory.

The first feature described is a small cluster of clouds in the northern hemisphere, the flow around which appears similar to the Karman vortex streets which occur when fluid flows past a solid obstacle. This similarity is emphasised by the vorticity measurements which show the expected structure for this type of feature. Due to the lack of solid objects in Saturn's atmosphere, this feature must be produced by a different process, and for this reason the vorticity budget is examined. It is suggested that the formation mechanism of the Saturnian feature could also be important for the features seen on the Earth.

The other feature considered in detail is a dark, oscillatory band, which occurs in the centre of a lighter zone, in Saturn's midlatitudes. The horizontal structure and wavelength of this are examined using two sets of images, whilst a third multispectral set is used to constrain its vertical structure. As a result of these measurements, it is suggested that the feature could be produced by a baroclinic instability, and a model of this type of flow is developed. This can produce disturbances with the observed wavelength which fit the vertical constraints supplied by the multispectral images.

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CHAPTER 1

INTRODUCTION

The existence of Saturn has been known for thousands of years, and, until the discovery in 1781 of Uranus by Herschel, it was the most distant planet known. It was not until 1610 that the first telescopic observations were made by Galileo. He first observed the rings, but interpreted them as being associated satellites, and was puzzled when some years later they had disappeared. It remained for Huygens to show that the observations could be explained by a ring which appeared to change as its orientation altered relative to the observer. It was discovered by Cassini in the 1670's that there were actually two rings (the A and B rings), the gap between these being known as the Cassini division. Over the years other rings and divisions have been reported with varying degrees of certainty. In 1973 the suggested ring structure contained the A, B, C and D rings with the possibility of additional material (outside the A ring). By 1980, using the Pioneer results, prior to the Voyager 1 Saturn encounter, three more rings (E, F, and G), had been added. However, with the arrival of Voyager I in late 1980 it was found that none of the ring systems were continuous, but that they all showed variations down to the smallest resolvable scales. This was demonstrated even better by the photopolarimeter instrument on Voyager 2 which showed that this structure was still present at scales of 100 m. It is thought that these local variations are due to resonances and spiral density waves set up through interactions with the satellites.

Other unexpected features were also observed in the rings, including the spokes in the B-ring and the 'braided' F ring, which are thought to be caused by electrostatic levitation and shepherding satellites respectively.

A similar dramatic increase was obtained in the information about the Saturnian satellites. The first to be discovered was Titan which was seen by Huygens in 1655. This was followed in 1671 by Cassini's discovery of Iapetus, and subsequently, in 1672 and 1674 of Rhea, Dione and Tethys. This list increased until in 1973 there were ten (Dione, Enceladus, Hyperion, Iapetus, Janus, Mimas, Phoebe, Rhea, Tethys and Titan). Since 1973, using a combination of Earth-bound, Pioneer and Voyager measurements, several more have been found. In particular, it was discovered that Janus is in fact two satellites sharing an orbit, which are expected to exchange positions about every four years. Small satellites also occupy Lagrange points of Dione (1980S6) and Tethys (198S13 leading and 1980S25 trailing). Whilst other satellites interact with the rings, in particular the F ring shepherds 1980S26 and 1980S27.

Of all the satellites discovered only one, Titan, has an atmosphere. This was known, from spectroscopic measurements, to contain methane (CH_4). Although Voyager measurements confirmed this, it was found to be only a minor constituent, the bulk of the atmosphere being composed of Nitrogen. Infrared and radio measurements showed that the surface pressure was ~ 1.6 bars and the temperature $\sim 93\text{K}$. Several other minor constituents of Titan's atmosphere have been discovered including Hydrogen Cyanide (HCN). This is interesting since it is known to be an important intermediary involved in the production of amino acids.

Many of the other major satellites of Saturn have been observed to have water ice or frost on their surfaces. When combined with their low densities this suggests that water or water ice may be the main constituents. These satellites show several different types of terrain including large craters and sinuous ridges. Although many of the surfaces are heavily cratered, parts of Enceladus appear geologically young, possibly because of heat produced through tidal forcing by Dione. Images of Tethys showed a large (4000 km) impact crater; there is also a trough approximately 100 km wide circling three quarters of the planet. This may be associated with the crater or could be caused by freezing of the interior producing expansion of the surface. The other satellites show a variety of other features including a spectacularly large impact crater on Mimas.

Because of its subtle colouring and lack of resolvable features, early observations of Saturn were often limited to the positions of the belt-zone interfaces. The first attempts to obtain a rotation period for the planet were made by Herschel, who found the time interval between similar appearances of the belts. This technique produced a rotation period of 10hr 16m 0.4s. Subsequent measurements were usually based upon the rotation period of spots, including one discovered by the comedian Will Hay in 1933. However, the lack of sufficiently distinguishable features meant that very few measurements were obtained in this way, and those that were, usually applied to the equatorial region. It is because of this lack of reliable measurements that the information supplied by the Voyager spacecraft is so valuable.

The great distance of the outer planets means that, even with the thrust provided by modern rockets, it takes a long time for a spacecraft to reach them. However this can be significantly reduced if the spacecraft can pass close to, and be accelerated by, an intermediate planet. An ideal situation for this type of trajectory

occurred in 1977 when the alignment of the planets was such that a spacecraft could be launched from the Earth to Neptune. This would be accelerated by Jupiter, Saturn and Uranus. In 1960 a mission called the Grand Tour was proposed which would accomplish this. Due to financial constraints this was turned down, however it was replaced by two simpler spacecraft (Voyager 1 and 2, described in more detail in chapter 2), which would fly by Jupiter and Saturn. In addition the trajectory of Voyager 2 was arranged so that it could be continued on to Uranus and Neptune.

The two Saturn encounters returned considerable amounts of data, including 35000 images, all of which will take years to analyse. However, it is possible that this will still not decisively answer such basic questions as the driving mechanism of the zonal flows. Although this thesis does not attempt to answer this, some of the possibilities are mentioned in chapter 4.

In addition to the horizontal structure derived from the images, the Voyager instruments provide details of the atmospheric composition and temperature structure, and some of the results are shown in chapter 5. These range from globally integrated albedo measurements to local temperature variations, and cover a pressure range of 10^3 to 25×10^{-7} mb. From the way in which the temperature varies with pressure, parameters such as the buoyancy frequency are calculated. The more detailed temperature measurements are used to determine the local vertical shear from the thermal wind equation.

Chapter 6 describes one of the small scale features studied in detail. This consists of a cluster of clouds to the east of a dark spot at approximately 40° N. The visual appearance of this resembles a Karman vortex street, and this similarity is also present in the derived vorticity distribution. This type of feature is normally only observed when fluid flows past a solid object. Since there are no local solid objects, this shows that some other type of flow must

be producing very similar results.

The other feature considered in detail is a wavelike disturbance centered at 47.5° N which is described in chapter 7. From the available images the mean zonal wavelength is estimated, whilst the way in which the features' appearance changes through different filters provides constraints upon the vertical structure. The thermal wind shear at this latitude is known from the calculations of chapter 5. These observations suggested that the feature could be produced by a baroclinic instability. This idea was tested by solving the differential equation associated with this process numerically, and comparing the results with the observed feature. It was found that using a vertical shear profile based upon that obtained from the thermal wind equation, disturbances with a dominant wavenumber close to that measured could be produced. These also fit the constraints imposed by the multispectral images.

A large part of the work described in this thesis was carried out on a computer. In addition to the standard routines used, other programs were specially written or modified. A brief description of these is given in appendix 1. The second appendix describes in greater detail the formulation of the numerical models used in chapter 7.

CHAPTER 2

VOYAGER MISSION AND INSTRUMENTATION

2.1 INTRODUCTION

As almost all the data used in this thesis originates from the Voyager mission it is necessary to describe the spacecraft and some of the instruments involved. This has been done because many of the features of the spacecraft, their trajectories, and instruments have a great influence on the nature of the data received, and hence restrict the number and type of images of any particular feature.

2.2 TRAJECTORY

The trajectories of the Voyager spacecraft were designed around certain objectives for the mission as a whole, (see Kohlase and Penzo 1977). Among these were the requirements that there should be a close encounter with Titan and an Earth occultation of the Saturnian rings. If, as happened, both these objectives were achieved by Voyager 1 then the Voyager 2 trajectory would be altered so that the gravity assist provided by Saturn would send the spacecraft on to a Uranus encounter in 1986.

The effect of these decisions on the Saturn encounter can be seen in fig. 2.1. The Voyager 1 spacecraft flew in below the ring plane, under the planet, and up out of the plane of the ecliptic. The Voyager 2 encounter was slower and more orientated towards the

northern hemisphere, flying down through the ring plane after closest approach.

2.3 VOYAGER - INTRODUCTION

A schematic diagram showing the main features of the Voyager spacecraft is given in fig. 2.2, whilst fig. 2.1 shows the trajectories.

The spacecraft are three axis stabilized which means that they attempt to remain in a stable attitude by maintaining their position relative to fixed objects such as the sun and stars rather than by spinning rapidly about their axis, like Pioneer. This does however have the disadvantage that the spacecraft has a small residual rocking motion which can lead to problems when trying to accurately 'navigate' images.

Most of the instruments used for the atmospheric observations are mounted on the scan platform, which has two degrees of freedom relative to the rest of the spacecraft. This can be pointed with an accuracy of $2.5 \text{ mr per axis } 3\sigma$, although the resultant position can be measured to an accuracy of $1.5 \text{ mr } 3\sigma$. The available slew rates range from $0.096 - 17.45 \text{ mrad/sec}$. Mounted upon the scan platform are the narrow and wide angle cameras comprising the imaging sub-system (ISS), the infra-red radiometer and spectrometer (IRIS), the photopolarimeter spectrometer (PPS) and the ultraviolet spectrometer (UVS). All these instruments are bore sighted so that they all observe the same part of the planet at one time, although at differing resolutions (see fig 2.3 for comparison). It was due to a problem in the scan platform that most of the high resolution coverage of Saturn's southern hemisphere by Voyager 2 was lost. For this reason most of the images considered will be of the Northern hemisphere.

One of the major requirements of the imaging experiment is a high bandwidth communications link. This is provided by the spacecrafts' X-band (8.4 GHz) transmitter which can cope with the maximum data rate of 115.2 kbps. This means that, despite the great distances involved, it is possible to send down an image every few minutes. Although it is not used for imaging data an S-band (2.1 GHz) transmitter is also available and can be used for the lower cruise science and engineering data rates. In addition to providing the telemetry and communications links necessary the signal can also be used to determine the spacecrafts' trajectory accurately, and to probe the structure of objects between the spacecraft and the Earth.

2.3.1 VOYAGER - IMAGING SUB-SYSTEM (ISS)

The Voyager imaging science subsystem was designed to satisfy a number of specific objectives as stated by Smith et. al. (1977). It consists of two bore sighted cameras one with a focal length of $\sim 1.5\text{m}$, and the other of $\sim 0.2\text{m}$. Both cameras have a selection of exposure times, filters and image readout times. In addition to using the cameras independently it is possible to shutter both simultaneously, leading to a pair of images with different scales but identical planet-camera transformations, this can be very useful as will be described later (see section 3.6.3).

Each camera has a filter wheel in front of it, giving a choice of clear, violet, blue, orange, green and ultraviolet for the narrow angle, with the wide angle having a Sodium and two methane filters instead of the UV. The spectral performance of the various camera-filter combinations and the radiometric performance of the cameras was examined by Benesh and Jepsen (1978) and Danielson et. al. (1981). In addition to this a grid of small dots called reseau marks were painted onto the faceplate of the cameras at precisely measured locations. This allows the geometric effects introduced in

the system to be estimated.

The outputs from the cameras are amplified and digitised to produce an image 800 pixels (see section 3.2) square and with a DN range of 0-255. Because of the difficulty of estimating the exposure time required for different geometries it sometimes happens that the image is either partly saturated or has a maximum DN of much less than 255.

2.3.1.1 VOYAGER IMAGING SEQUENCE - GENERAL -

In order to understand the imaging sequences that are used it is necessary to understand the constraints under which they are constructed.

The first and most important of these is the great distance separating the spacecraft from the ground. This defines the maximum data rate which can be used to obtain a given signal to noise ratio at the ground. This data rate has to be shared between all the instruments and telemetry signals, in particular the plasma wave and planetary radio astronomy equipment which make demands comparable to that of the imaging sub-system (ISS) on the available data rate (see Kohlhasse and Penzo 1977). Although it is possible for images to be recorded on the tape recorder for later playback this is usually only done around the near encounter period when the data acquisition rate nears its maximum.

There are two main ways in which the data rate requirements for the ISS may be reduced; changing the rate at which the images are read out of the camera or only reading out part of a frame. The second alternative means that more images can be returned but imposes more stringent pointing requirements.

A second important constraint imposed by the presence of the other instruments is the control of the spacecraft attitude and the scan platform. The spacecraft attitude is occasionally changed,

either when a trajectory correction manoeuvre is being carried out, or to enable one of the instruments to scan a particular part of the sky. Since all the instruments mounted on the scan platform (ISS,IRIS,PPS and UVS) are bore sighted, pointing requirements for the imaging system must be compatible with those of the other instruments.

2.3.1.2 VOYAGER IMAGING SEQUENCE - INTERNAL CONSTRAINTS -

Because of the time required to change the pointing of the cameras and the number of objects to be observed, the imaging sequence is divided into sets of images. For example a scan across one of the rings, or a high resolution mosaic of a satellite. Apart from the images specifically intended for atmospheric studies few are of relevance, an exception to this being the WA ring scans which sometimes include part of the planet, although the centre is usually over-exposed.

Most of the planetary imaging takes the form of general scans of a part of the planet, for instance the equatorial region. The exceptions to this are a few images targeted for a particular feature, such as Anne's spot in the southern hemisphere (see Smith et. al. 1981), or one of the brown spots.

Once a particular location is decided upon this still leaves the decision as to the particular mix of WA to NA images and the filters to be used. Whilst the NA images are at a higher resolution, they are less likely to contain much of the planets limb which may be required for pre-processing (see section 3.6.3). This problem can be avoided by taking simultaneous narrow/wide images, but this effectively halves the number of images obtained. These contradictory requirements frequently result in a mixture of NA only frames and simultaneous NA and WA frames. Around encounter when the resolution is highest, use is also made of WA only frames.

Another possible choice is which filter should be used. Some features appear differently through certain filters, whilst others seem unchanged. Also some of the filters, because of the spectral response of the camera need longer exposure times, and are therefore more liable to be smeared. All these considerations influence the possible choice of filters, and the range of filters through which each feature will be observed. As always a compromise is reached, some parts of the planet are imaged in several filters whilst other parts are only observed in one or two. For instance a large number of Voyager 2 Saturn observations were made using only green filter NA frames, but with accompanying WA frames taken through many different filters.

2.3.1.3 IMAGE SELECTION -

As might be expected the selection of images of a particular feature depends on the use to which they are to be put: the process is summarized in fig. 2.4. The major decision involved is whether or not the navigation information is required (this is needed if the images are to be map-projected or used for cloud tracking). If this is the case, then a curve must be fitted to the limb of the planet (see section 3.6.3) and for this to be done successfully a large section of the limb must be visible. If there is an image of the required feature where this condition is satisfied then there should be no problems. However, a week or so before encounter the planet fills the NA field of view so that only images with simultaneous WA images can be navigated. If this is not available, or if the planet also fills the WA then the only way the required information can be obtained is by attempting to fit a grid to the belt-zone interfaces, this is however an unreliable process. Because of these problems the images used are frequently not those at the highest resolution, but those which can be navigated.

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Other factors which influence the image selection process are; considerations about which filter is required, the time period in which an image must lie, and whether a single image or a set of images is needed. Examples of these type of requirements might be when one wishes to examine the evolution with time of a feature, or examine its appearance in several filters at one time. The overall effect of all these criteria along with the requirement for simultaneous navigation information, and the (sometimes unusual) imaging sequence can severely restrict the number of images available.

2.3.2 INFRA-RED INTERFEROMETER SPECTROMETER (IRIS)

The infra-red instrument is described by Hanel et. al. (1977). It consists of an IR interferometer covering the range $200-4000 \text{ cm}^{-1}$, and a radiometer operating over the range $5000-25000 \text{ cm}^{-1}$. Both these instruments share a telescope with a field of view of 0.25° (approx. 4.4 mrad , see fig. 2.3).

The spectra produced by this instrument can be used to identify some of the constituents of the atmosphere, and to produce a temperature - pressure profile for that part of the atmosphere responsible for the majority of the observed IR radiation (approx. $1-1000 \text{ mb}$) .

2.3.2.1 IRIS INSTRUMENT - CALIBRATION -

The calibration techniques used for the IRIS instrument are described in detail in Hanel et. al. (1980a), and Hanel et. al. (1980b). Briefly, the process can be divided into two parts. Firstly the data received is in the form of intensity against mirror displacement, and these must first be aligned with a reference spectrum and symmetrized, so that they may be cosine transformed to produce intensity as a function of wavenumber.

The second stage consists of calibrating the spectrum obtained. This is done with the use of deep space spectra, and the knowledge that the instrument is maintained at a constant temperature of 200.0 ± 0.1 K. The incident irradiance $I(\nu)$ at the wavenumber ν can be obtained from the measured irradiance $C_1(\nu)$ by

$$I(\nu) = \frac{C_2(\nu) - C_1(\nu)}{C_2(\nu)} \times B_\nu(T)$$

(from Hanel et. al. 1980a), where $C_2(\nu)$ is the measured deep space spectra, and $B_\nu(T)$ is the Planck function associated with the instrument temperature. In order to compensate for drift in the instrument the value $C_2(\nu)$ is estimated by interpolating between spectra taken before and after the observation $C_1(\nu)$. Several possible forms of error are considered by Hanel (1980a and 1980b), these are temporary temperature gradients set up in the instrument by differing sun angle effects, errors in B_ν due to drift in the thermistors, and noise in the detector. In addition there is an error associated with the uncertainty of the instrument pointing (1.5 mrad 3σ) when compared to the IRIS field of view so that it is not always certain which part of the planet is being observed.

2.3.2.2 IRIS - INVERSION TECHNIQUE

The inversion method used to obtain temperature measurements is described by Hanel et. al. (1977) and Gautier and Courtin (1979). This technique, whilst extremely useful, has certain characteristics which one should be aware of.

The method uses the parts of the brightness spectra corresponding to the absorption band of a suitable atmospheric constituent, which is assumed to have a known (assumed uniform) mixing ratio, and to be in thermodynamic equilibrium with its surroundings. If all the required information is available then the optical depth as a function

of frequency may be evaluated. Since the absorption coefficient changes with frequency the optical depth at a particular pressure level will also change, and from measurements of the intensity over a range of frequencies a profile of temperature as a function of pressure may be derived.

The main factors which limit the vertical resolution obtained by this method are listed by Gautier and Courtin (1979). They include the limited spectral resolution of the instrument, the signal to noise ratio of the spectrum used, and the change of absorption with pressure of the appropriate material.

During the Voyager 1 Saturn encounter the wavenumber range $200\text{--}600\text{ cm}^{-1}$ (where H_2 absorption is dominant) was used to construct the temperature profile between 70 and 700 mb (Hanel et. al. 1981), whilst the CH_4 band at approximately 1304 cm^{-1} was used to examine the region around 1mb. However, as noted by Hanel et. al. (1980b), the Voyager 2 IRLS instrument was slightly misaligned, and did not allow the retrieval of stratospheric temperatures.

One of the most worrying assumptions made for the IRLS analysis is that described by Conrath and Pirraglia (1983). In this it is assumed that the observed opacity is due only to hydrogen, and the possible effects of clouds and hazes have been ignored. Although the temperature measurements at the 150 mb level appear to correlate well with the observed velocity profile this is less apparent at lower levels, and it is possible that this is due to the omission of cloud effects from the model.

In addition to the inversion procedure, errors may arise in assigning the latitude to which a particular temperature measurement corresponds (see section 3.4.3). This effect is estimated by Pirraglia et. al. (1981) to give rise to a 3σ error of 1-3 degrees of latitude.

2.3.3 ULTRA-VIOLET SPECTROMETER (UVS)

Another instrument mounted on the scan platform is the ultra-violet spectrometer (UVS) operating over the wavelength range 500-1700 Å⁰. This instrument, which is described in detail by Broadfoot et. al. (1977), is used for upper atmosphere observations, and can be operated in two modes. In the first of these the instrument observes extended sources through a slit $\sim 0.1^\circ$ by 0.87° , this is mainly used to observe emissions from H atoms and molecules excited by solar radiation, or magnetospheric effects. From these spectra the composition of the atmosphere and its interactions with the magnetosphere can be inferred.

The second way in which the UVS can operate is in occultation mode when the UVS instrument is aimed at a point source (ie a star) at a time when the spacecrafts' trajectory causes the spacecraft-star line to be gradually obscured by the planetary atmosphere. While this is happening the received spectra are sampled rapidly (approximately every 0.3 s) so that when combined with the trajectory information the transmission as a function of frequency is known for a range of heights. Through comparison of this with the absorption due to a column of gas of known temperature and pressure, information about the composition and temperature over a range of heights can be determined. A more detailed explanation of the data analysis technique used is given by Festou et. al. (1981).

Because of the relatively low flux from the star being observed, and the rapid sampling rate, the signal to noise ratio for this type of observation is quite low. This leads to a rapid fall off of signal with increasing depth, and because of this the pressure levels sounded by this instrument are considerably less than those of the IRIS and radio experiments. For example the Saturn UVS temperature profile (see Festou and Atreya 1982) covers the 10^{-8} - 10^{-10} bar range.

2.3.4 PHOTOPOLARIMETER SPECTROMETER (PPS)

Another of the scan platform mounted instruments is the photopolarimeter spectrometer (PPS), which measures the intensity and polarization of the incident radiation at different wavelengths. The instrument is described in detail by Lillie et. al. (1977). It has a variety of filters covering the 2350 - 7500 Å range with a selection of aperture sizes and linearly polarized filters. Unfortunately the Voyager 1 PPS instrument was not working during the Saturn encounter (Stone and Miner 1981). The Voyager 2 instrument also had problems (Lane et. al. 1982) resulting in a restricted operational mode with observations only at 2640 Å and 7500 Å each through two orthogonal linear polarizations. Because of this the amount but not the direction of polarization could be determined.

The main use of the PPS, at least from the atmospheres point of view, is in the study of light scattered by the planetary atmosphere, and the way in which this information can be used to determine the structure of clouds and hazes. One of the most valuable contributions to this is the way in which the intensity of the scattered light varies as a function of phase angle (the angle between the incident and scattered radiation), which can constrain the types of particle used in cloud models.

2.4 RADIO SCIENCE

All the information returned by the spacecraft is encoded, and transmitted down the S-band and X-band (2.3 and 8.4 GHz respectively) radio links, the S-band link is also used to send commands up to the spacecraft. In addition however to its use as a communications link the transmissions also provide information about the spacecrafts' position, and about the medium between the spacecraft and the ground station (see Edelson et. al. 1979).

When the spacecraft passes behind the planet as observed from the Earth the radio signals are refracted through the atmosphere, and this affects the received signal. From this and the spacecraft trajectory information a profile of refractive index against height can be produced (see Eschleman et. al. 1977 for details). Using this in conjunction with an estimate of the atmospheric composition and the temperature at some level (from, for instance, the IRIS measurements) a temperature-pressure profile may be constructed.

The atmospheric levels for which this technique can be used extend from the minimum detectable depth of penetration down to the level at which absorption reduces the received signal to the noise level. This limits the temperature data to approximately the 1-1000 mb range, similar to that of the IRIS instrument.

FIGURE CAPTIONS - CHAPTER 2

Figure 2.1

This shows equatorial and polar views of the Voyager 1 and 2 encounters with Saturn. The times are in hours from closest approach to Saturn. The satellites are shown at their points of closest approach.

Figure 2.2

This figure shows some of the main components of the Voyager spacecraft. Most of the measurements described in this thesis are derived from the imaging and infrared instruments. These are both mounted on the scan platform shown at the top of the figure.

Figure 2.3

This figure shows the size of the field of view of the narrow-angle (NA) camera, the tick marks represent intervals of 100 pixels. The box labelled WA demonstrates the size of the central 100 pixels of the wide-angle camera. The field of view appropriate to a single infrared spectrum is indicated by circle labelled IRIS. The boxes labelled SK and SP indicate the 1.5 sigma limits of the azimuth and elevation axis within which the scan platform can be positioned (SP) and to which the resulting position is known (SK).

Figure 2.4

This figure demonstrates the way in which the various requirements for selecting an image interact. In particular it should be noted that the requirement that the resulting images be navigable, constrains both the choice of camera and the time period in which the images were taken.

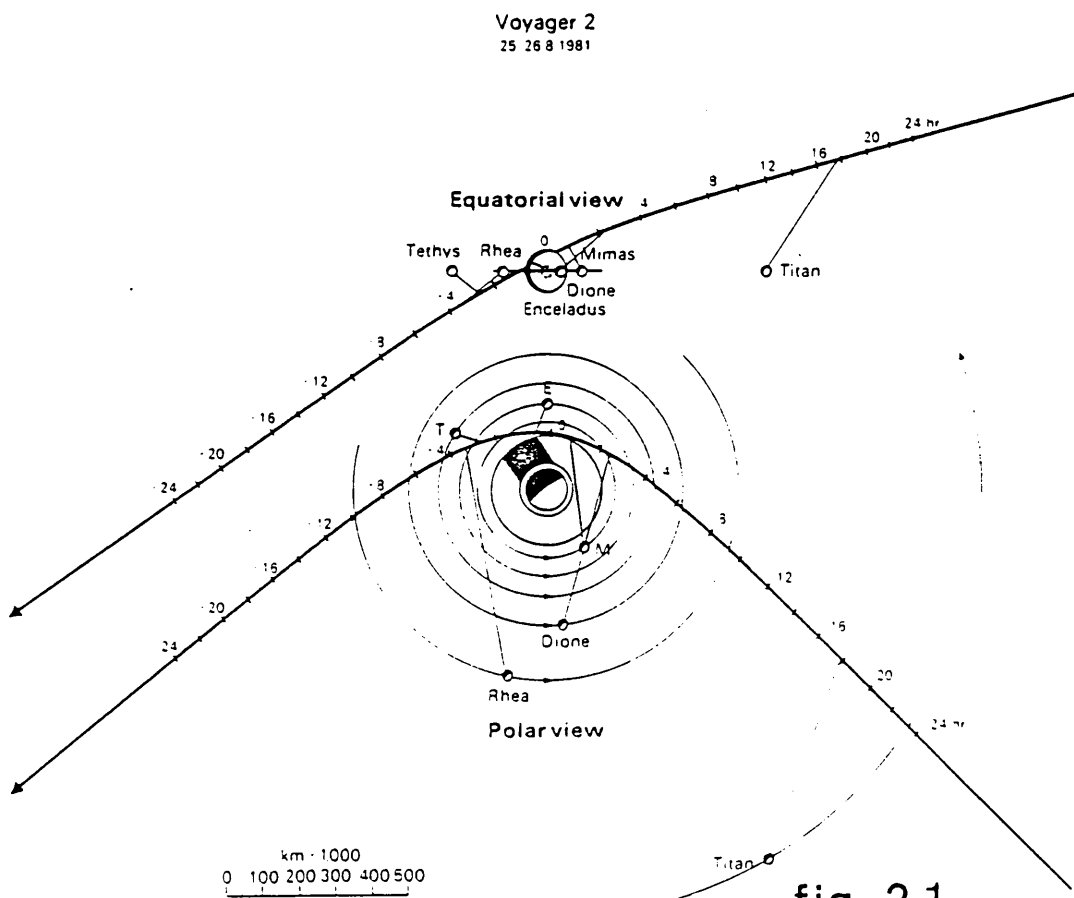
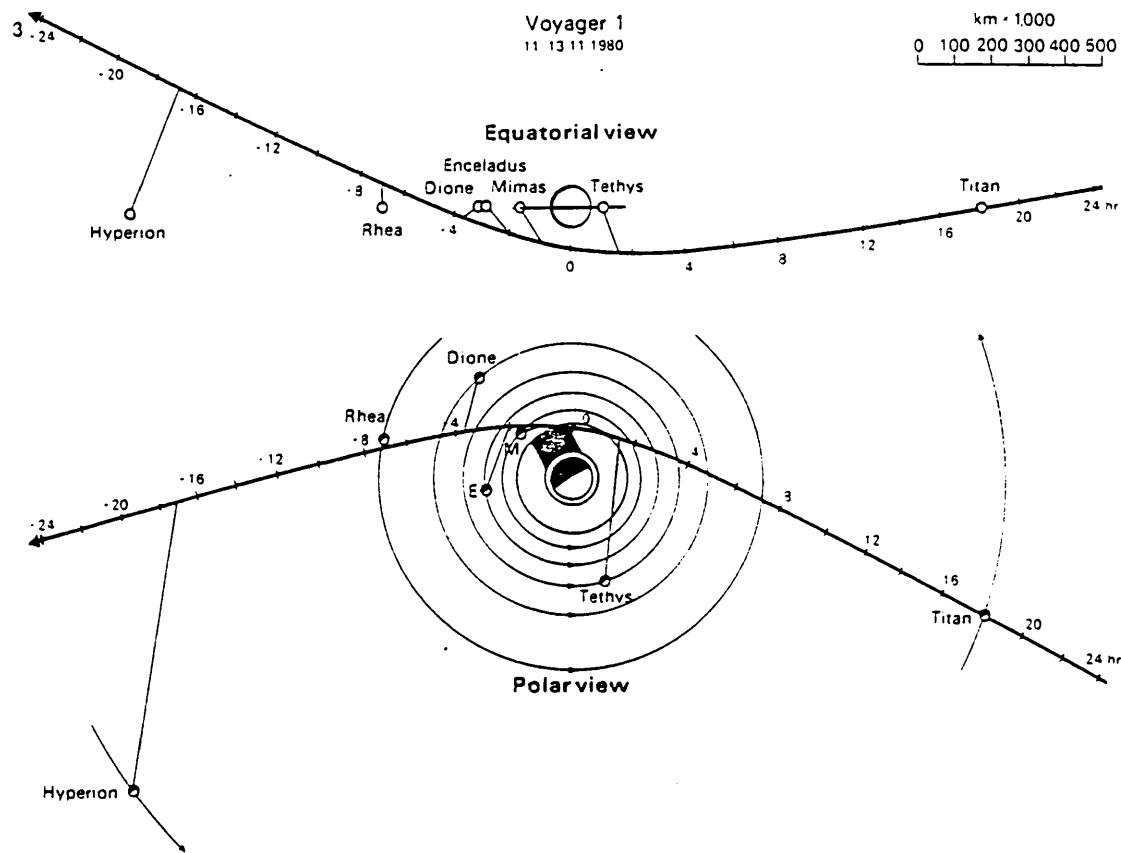


fig. 2.1

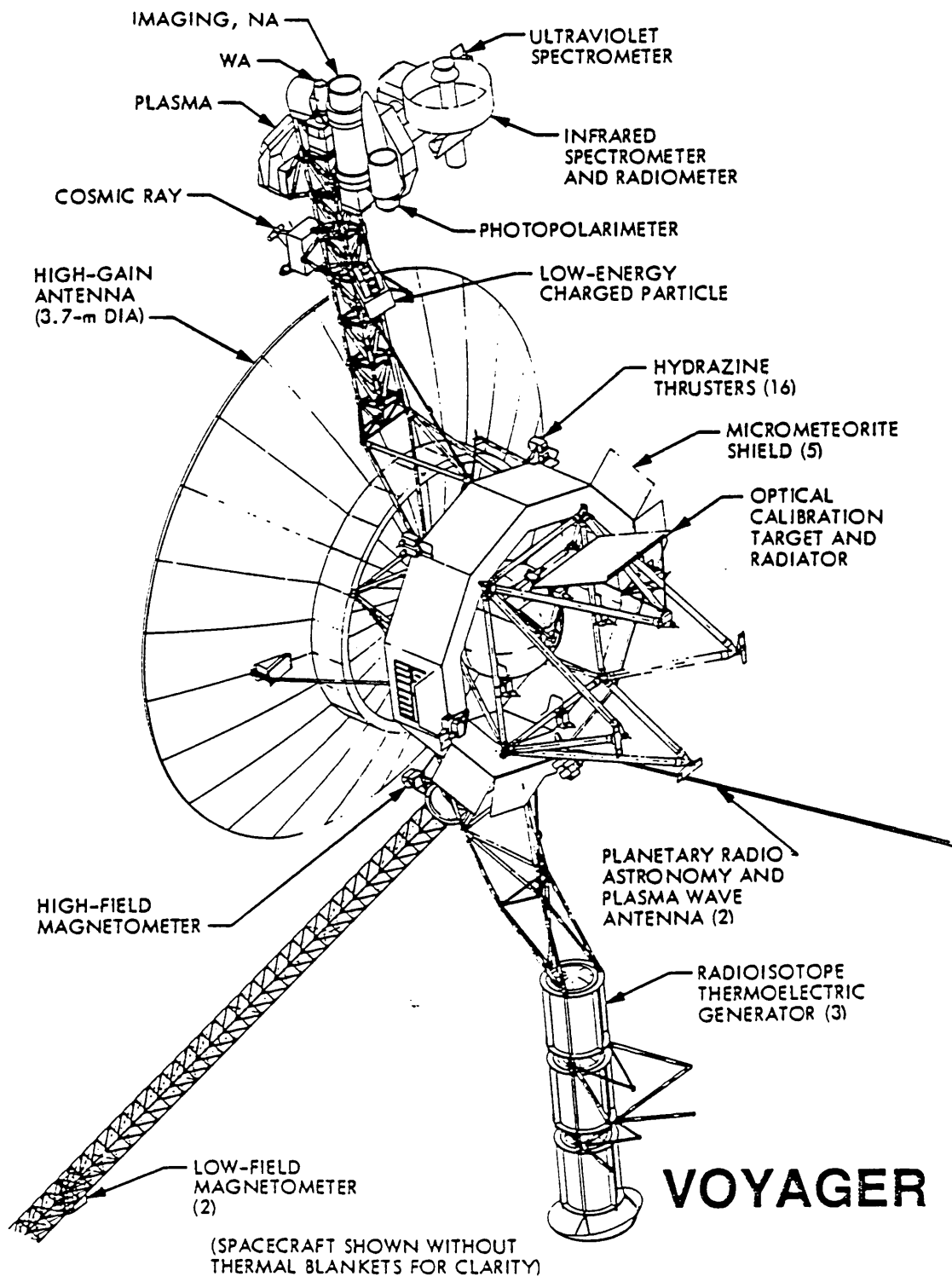


fig. 2.2

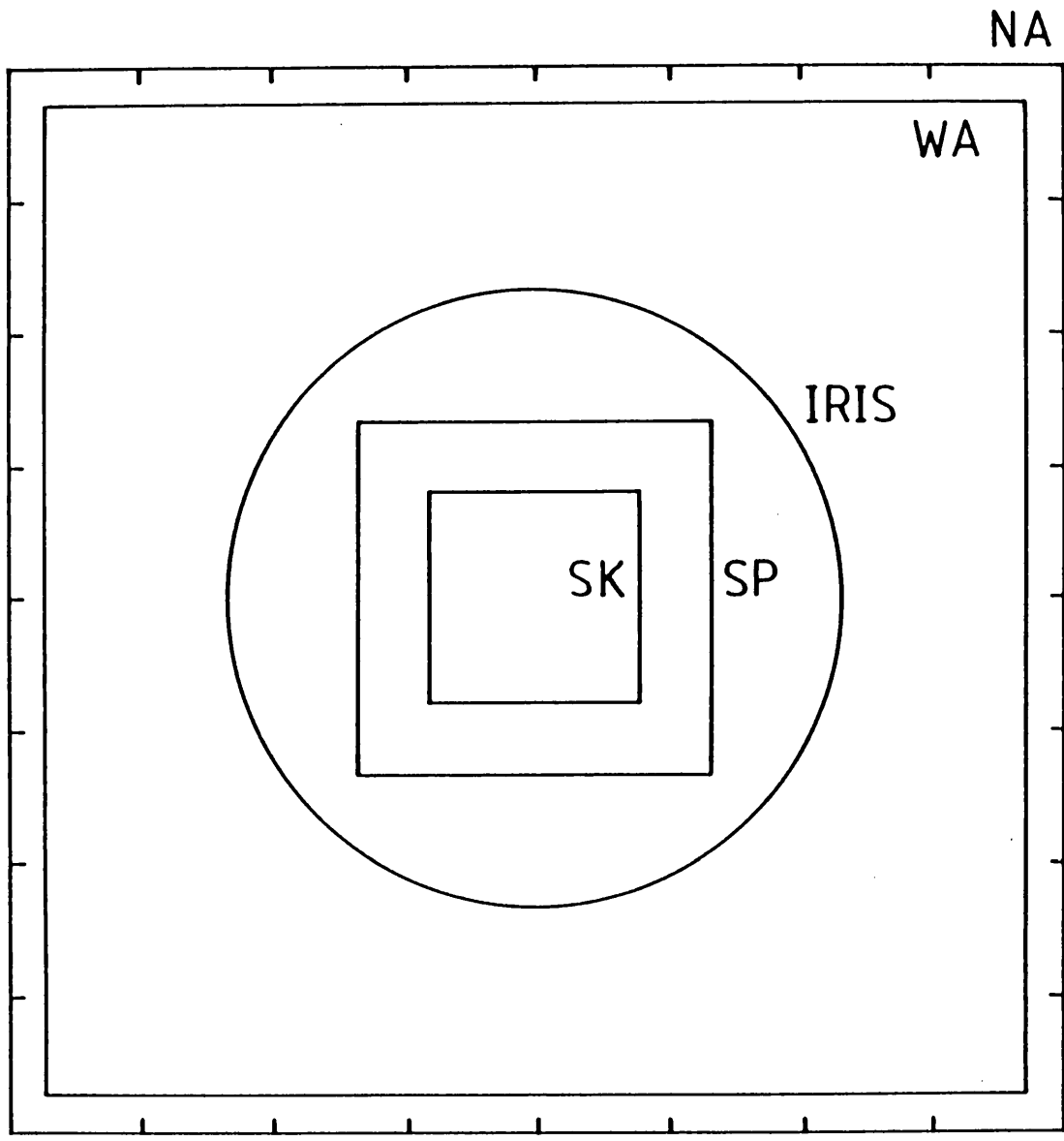


fig. 2.3

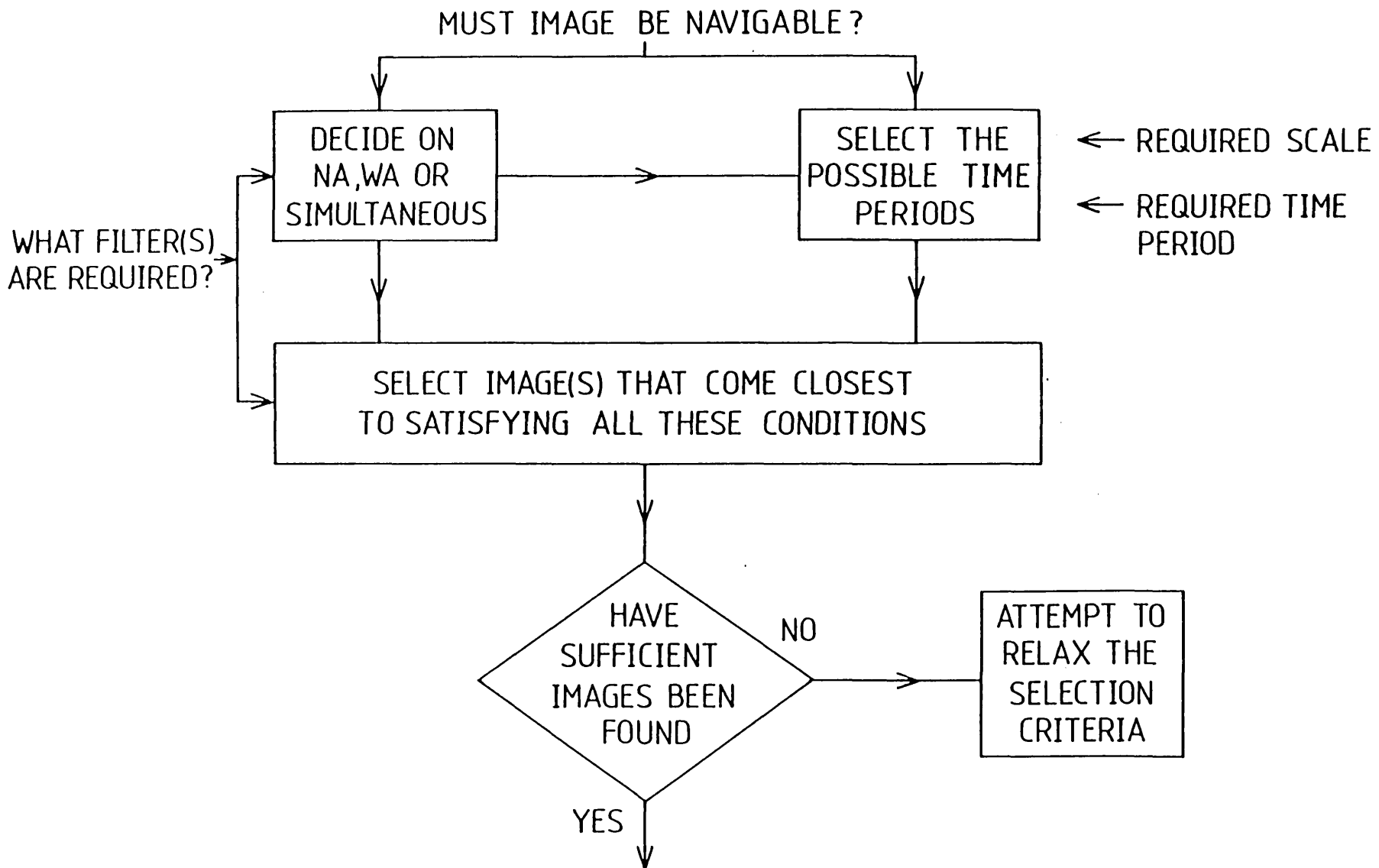


fig. 2.4

CHAPTER 3

IMAGE PROCESSING AND MEASUREMENT

3.1 INTRODUCTION

Most of the data used in this thesis has been derived from Voyager images, and the aim of this section is to describe in more detail the structure of these images, and the ways in which they are processed in order to make various types of measurements.

The first part considers the format of the images, and then describes in general terms the hardware and software of the IPIPS system upon which most of the processing was performed. I then describe the various measurement techniques used. Some of these require the images to be free of certain types of distortion, the methods used to achieve this, and to enhance the images, are considered in the final part.

3.2. IMAGE PROCESSING - DEFINITIONS

The pictures taken by the Voyager spacecraft are encoded and transmitted in the form of digital images. This is done by dividing each picture into a large number of identically sized square elements called pixels. The average intensity within each pixel is quantised and represented by a DN (Digital Number) level greater than or equal to zero. The pixels within each image are identified by a line and sample number measured downward, and to the right respectively,

with the top left pixel being at line one, sample one.

The images produced by the Voyager cameras are 800 pixels square, and are encoded in binary form as eight bits, giving a DN range of 0 to 255. Thus each image requires approximately 0.6 Mbytes (one byte is eight bits). So that each image produced by the spacecraft can be uniquely identified it has associated with it a reference number, this is generated by the Flight Data System on board the spacecraft, and is called the FDS number. There is also another reference number called the picture number, which is added at a later stage in the processing. In addition each image has attached to it a label which contains details of the processing which has been performed on it, and is updated every time it is altered in any way.

3.3.1 IPIPS - HARDWARE

Nearly all the processing described in this thesis, was performed on the Interactive Planetary Image Processing System (IPIPS) which now forms part of the Imperial College Atmospheric Physics group. This system is built around a VAX 11/780 (with associated magnetic tape and disk drives), and an interactive image processing device made by International Imaging Systems (the IIS).

The VAX is the centre of the system, it is used to copy images to and from tape and the IIS and for batch (as opposed to interactive) processing of images. It is also the host machine for a multiplicity of other tasks, including running the VICAR image processing system. the RSGL4 data-base system, and the CORE standard graphics system. The VAX has 3 Mbytes of main memory, and is a paged machine. This means that the operating system divides a requested amount of memory into 'pages', and arranges these so that the most frequently used are stored in main memory, with the

remainder located in disk. Because of this large images can, if scanned in an appropriate way, be processed nearly as efficiently as small images.

Most of the interactive image processing is carried out on the IIS. This consists of six image planes, each 512 pixels square, and each having a look up table allowing a mapping to be set up between the input and output DN values (see Adams and Driscoll 1979 for a more detailed description of the IIS architecture). The whole system is refreshed at standard video rates, and the outputs of one or more channels may be displayed on a colour monitor. The advantage of using the IIS is that it is possible to interact with it directly through a tracker ball, enabling the parameters associated with a program to be continuously altered until an optimal result is produced.

This produces more acceptable results than the trial and error approach which must be adopted when running batch processes on the Vax. One of the uses of the IIS is the production of movies, this can be done by displaying each channel in succession, giving a limit of six frames, or by splitting each frame into quarters giving a maximum of 24 frames, each 256 pixels square. One of the problems associated with the use of the IIS is that it fails to compensate for the shape of standard video monitors which have an aspect ratio of $\sim 3:2$, leading to pixels being imaged as rectangles. This effect can be avoided by previously contracting the image by the required amount in the sample direction, but this is not usually done unless there is some special reason.

3.3.2 IPIPS - SOFTWARE

The image processing system used is the VICAR (Video Image Communication And Retrieval) system. This was originally developed by J.P.L. (see Castleman 1979) to run on an IBM 360, and has been converted (Lawden and Pearce 1980) to enable it to be used on a VAX.

This system contains all the subroutines needed to read and write to a standard image format, and a large number of image processing programs, to these have been added routines associated with the IIS and PPL as well as numerous other programs (for instance the cloud tracking program).

Associated with some of the image processing is the requirement to store large tables of data (for example the wind velocity measurements), which are held in a format associated with the RSQL4 data-base system. This holds both numeric and character data in a compact form, and has programs suitable for sorting, searching, averaging, plotting etc.

The other major software packages are concerned with the graphics system, which comprises the Washington University implementation of the CORE graphics standard. This is used by application programs contained in the NCAR (National Centre for Atmospheric Research) graphics package.

3.4 IMAGE MEASUREMENT - INTRODUCTION

This section considers the various measurement techniques which are available for Voyager images, as well as the coordinate systems in which the results are expressed. Difficulties occur when making certain kinds of measurements which do not occur in the case of the Earth. These are mainly due to the planets significant oblateness, and the lack of a fixed visible surface. The main effect of this is the requirement that it must be possible to specify positions absolutely, rather than relative to some other visible feature, this in turn imposes requirements on the accuracy of the navigation information. The reliability of this data is also mentioned, as is the use of power spectra, cloud tracking and movies.

3.4.1 POSITION MEASUREMENTS

In order to define the position of a point on a planet it is necessary to specify a coordinate system, which should rotate about its axis with an appropriate period. It is then required that one be able to translate from image space into this system. This in turn calls for a knowledge of certain parameters which specify the orientation and position of the camera relative to the object being viewed.

3.4.1.1 LATITUDE-LONGITUDE COORDINATE SYSTEMS

For most purposes it is sufficient to represent Saturn as an oblate spheroid of polar radius 54456 km and oblateness 0.903 . This can lead to ambiguity in the definition of a latitude coordinate, the main alternatives being shown in fig 3.2 . Of these the most used are the planetocentric latitude θ_c corresponding to the angle subtended by a line passing through the planet centre, and the planetodetic (sometimes called the planetographic) latitude θ_d which refers to the angle subtended by the local normal to the surface. On a spheroid of equatorial radius R_e and polar radius R_p these are related by

$$\tan(\theta_d) = \left(\frac{R_e}{R_p} \right)^2 \times \tan(\theta_c) \quad (3.1)$$

Of these two forms the most convenient is θ_d which gives rise to simpler definitions of the Coriolis parameter, which depends on the angle between the axis of rotation and the local vertical.

The definition of a longitude system (for a given reference point) is straightforward except for the arbitrary decision as to whether it should increase in an easterly or westerly direction, the general convention is that it should increase towards the west, but this is not always used.

For measurement purposes it is often more convenient to transfer from the latitude-longitude system into a cartesian system, with the displacements measured in metres, and the z axis as the local normal. This tranformation can be written as :-

$$\delta x = -a \delta \lambda \quad (3.2) \quad \delta y = r \delta \theta \quad (3.3)$$

where a is the distance from the axis of rotation and r is the radius of curvature (note the minus sign showing that increasing x values correspond to decreasing west longitude). In the case of the Earth a and r can be replaced by the spherical approximations :-

$$a = R \cos(\theta) \quad (3.4) \quad r = R \quad (3.5)$$

(where R is the radius) but due to the oblateness of Saturn this can not be done and instead we use the formula for an elipsoid, where the distance a is given by :-

$$a = \frac{R_e^2}{(R_e^2 + R_p^2 \tan^2 \theta_d)^{\frac{1}{2}}} \quad (3.6)$$

and the local radius of curvature is given by (Jackson 1981)

$$r = R_e (1-e^2) (1-e^2 \sin^2 \theta_d)^{-\frac{3}{2}} \quad (3.7)$$

where e is the eccentricity defined by

$$e^2 = 1 - \left(\frac{R_p}{R_e} \right)^2 \quad (3.8)$$

Table 3.1 shows the values of some of these quantities for a range of latitudes, as well as the Coriolis parameter

$$f = 2\Omega \sin \theta_d \quad (3.9)$$

and the value of β , which is given by

$$\beta = \frac{df}{dy} = \frac{df}{d\theta_d} \times \frac{d\theta_d}{dy} = \frac{2\Omega \cos \theta_d}{r} \quad (3.10)$$

where Ω is the rotation rate.

PLANETOCENTRIC LATITUDE	PLANETOCENTRIC LATITUDE	LATITUDINAL SCALE	LONGITUDINAL SCALE	CORIOLIS PARAMETER f	BETA
degrees	degrees	km/deg	km/deg	rad/sec x 10 ⁻³	rad/sec/m x 10 ⁻¹¹
0.0	0.00	858.1	1052.4	0.000	0.666
5.0	4.08	860.0	1049.1	0.029	0.662
10.0	8.18	865.4	1039.3	0.057	0.651
15.0	12.32	874.3	1022.9	0.085	0.632
20.0	16.53	886.7	999.8	0.112	0.606
25.0	20.82	902.4	969.9	0.138	0.574
30.0	25.21	921.2	933.2	0.164	0.537
35.0	29.72	942.7	889.5	0.188	0.497
40.0	34.38	966.6	838.8	0.211	0.453
45.0	39.19	992.3	781.1	0.232	0.407
50.0	44.18	1019.2	716.4	0.251	0.361
55.0	49.35	1046.4	644.9	0.268	0.313
60.0	54.70	1073.1	566.9	0.284	0.266
65.0	60.24	1098.2	482.9	0.297	0.220
70.0	65.95	1120.7	393.4	0.308	0.174
75.0	71.81	1139.4	299.4	0.316	0.130
80.0	77.80	1153.6	201.7	0.323	0.086
85.0	83.88	1162.5	101.5	0.326	0.043
90.0	90.00	1165.5	0.0	0.328	0.000

Table 3.1

3.4.1.2 ROTATION PERIOD AND REFERENCE LONGITUDE -

In order to have a useful longitude system, it should co-rotate with some part of the planet, in the case of the Earth this is done by tying it to the surface. For the outer planets, where a surface (if present) is not visible, a solution is less apparent. Attempts have been made to use the rotation rate of particular feature (eg the Jovian red spot) but these can produce problems when this changes slowly with time, and have no obvious meteorological significance.

The solution usually adopted is to use the characteristic 'radio rotation period' of the planet. In the case of Saturn the radio emissions used are in the kilometre range, and are described by Warwick et. al., (1981) and Kaiser et. al. (1981). From observations of this radiation over a period of 267 days in 1980 Desch and Kaiser (1981) obtained a rotation period of 10hr 39min 24sec $\pm$ 7sec. Another determination of the rotation period was made by Carr et. al. (1981) over a range of frequencies, in two one month periods with a seven month separation, leading to a value of 10hr 39min 22 sec. The accuracy of this was questioned by Kaiser and Desch (1982), who also pointed out a discrepancy in the calculations, this led to a revised value of 10hr 39min 21sec (Carr et. al. 1982), although it was stated that the original value given by Desch and Kaiser was probably more accurate.

In addition to a rotation period the definition of a longitude system requires an arbitrary point to be defined as longitude zero, this was also provided by Desch and Kaiser. However as explained by Sergeyevsky and Van-Allen (1982), although the definition of longitude used by J.P.L. uses the Desch and Kaiser rotation period, there is a discrepancy of 222.2044 deg between the two systems owing to differing definitions of zero longitude.

3.4.1.3 IMAGE NAVIGATION -

Once the latitude - longitude system for the planet has been defined this leaves the problem of transforming the position of a point observed in an image into this system.

We will neglect for the moment effects introduced by deficiencies in the camera system, and assume that we have an undistorted image. These assumptions are justified by the preprocessing performed (see section 3.6).

Under these circumstances the situation is similar to that shown in fig 3.1; the requirement is to determine the position of the point P, observed at a certain line and sample within the image in terms of the latitude (θ) and longitude (λ) of the corresponding point P' on the planet. The transformation may be split into two parts : a rotational part, and a translational part.

The rotational part depends upon three angles: the sub-spacecraft latitude (this can be defined in several different ways which has led to confusion (see fig 3.2), however since all the possible definitions are inter-convertible the precise form used is not important as long as one is consistent); the sub-spacecraft longitude, which is the longitude of a line joining the centre of the planet to the spacecraft; and the north angle which is the angle between the projection of the rotation axis in the image and a reference direction (usually the direction of decreasing line number, but as with the sub-spacecraft latitude differences exist in the definition of this reference direction, and whether the angle is measured in a clockwise or anti-clockwise direction). Using these three angles a rotation matrix can be defined in which the spacecraft is positioned relative to the planet so that the sub-spacecraft latitude, longitude and the north angle are all zero.

The remaining part of the transformation depends only on the scale and orientation of the image plane relative to the centre of the planet. The size of the planet projected into the image plane can be determined from the range of the spacecraft from the planet, and some parameters characterizing the camera system (note that these parameters correspond to the 'ideal' image produced as a result of the preprocessing, which is not identical to the image as read out from the cameras). The two remaining pieces of information which are required are the sub-spacecraft line and sample, corresponding to the location of the centre of the planet in the image plane. Using all this information enables one to convert from a line and sample in the image to latitude and longitude on a spheroid with zero sub-spacecraft latitude, longitude and north angle.

Assuming that all the required information is available to sufficient accuracy a latitude and longitude can now be ascribed to any part of the planet appearing in the image. A more detailed description of this process and its inverse is given by Elliot and Schwartz (1977).

3.4.1.4 DERIVATION OF NAVIGATION PARAMETERS -

The sub-spacecraft latitude and longitude, north angle, and the range are fixed as a function of time once the trajectory and spacecraft attitude is defined. The values of these variables, along with other information are listed for each image, and are made available to experimenters as the SEDR (Supplementary Experimental Data Record) data. The first issue of this is based upon the expected trajectory, and is referred to as the predict SEDR. Due to inaccuracies in the knowledge of the precise spacecraft-planet geometry, the planetary mass and the perturbing effects of satellites, the actual trajectory will differ from that predicted. The position of the spacecraft however, can be accurately determined from an

analysis of the returned radio signal (Edelson et. al. 1979 claim the one sigma error in the range for the Voyager 1 Jupiter encounter to be 10 m), whilst other parameters, such as the north angle can be determined from the spacecraft orientation (obtained from the sun and star sensors). From these measurements a second, more appropriate, set of SEDR data is produced.

However inconsistencies occasionally appear, sometimes leading to a regeneration of the SEDR data, so that it is difficult to specify any particular set as being totally correct.

The two remaining parameters, the sub-spacecraft line and sample, present more of a problem, this is due partly to the spacecraft wobble, but mainly to the lack of accuracy in the measurement of the scan platform position (Jepsen et. al. 1980). The accuracy with which the scan platform elevation and azimuth angles can be determined is given (Kohlhase and Penzo 1977) as $1.5 \text{ mrad } 3\sigma$ per axis, so that there is a $\sim 99.75\%$ chance that the actual angle lies within 1.5 mrad of the measured angle (this is greater than the maximum accuracy of $2.5 \text{ mrad } 3\sigma$ per axis with which the scan platform can be commanded to move). The angle subtended by one scan line of the narrow angle camera is given by Smith et. al. (1977) as $9.25 \text{ } \mu\text{rad}$, so that the 3σ error in the scan platform corresponds to an uncertainty of about 162 pixels in the image. Because of these problems the values of the sub-spacecraft line and sample are obtained from the images themselves (see section 3.6.3) rather than as an independently measured quantity.

3.4.2 VELOCITY MEASUREMENTS - METHOD

Measurements are made of the velocity of features by observing the change in latitude and longitude of a point, and using equations 3.2 and 3.3 along with the time interval to compute the zonal and meridional velocity components.

Problems arose with this when it was not originally realized (Yagi 1981) that the variable r represented the local radius of curvature, not the distance from the centre of the planet, this was later corrected. Another difficulty arises, due to the change of r over finite latitude intervals (see equation 3.7). This was considered by Barrey 1982, who compared values of meridional velocity v calculated using the value of r at the midpoint with those produced using the first three terms of a series expansion of v in terms of latitude, and showed the differences to be unimportant.

3.4.2.1 VELOCITY MEASUREMENTS - INHERENT DIFFICULTIES -

The main assumption upon which cloud tracking is based is that one can accurately follow the motions of individual cloud targets, and that the motion of these will represent the velocity of the local flow.

The problem of accurate feature identification is exacerbated on Saturn by the lack of easily identifiable targets, as compared to the Jovian atmosphere. Also as observed by Ingersoll et. al. (1981) for the Jovian case, the difference in contrast and texture between different regions of the planet leads to measurements being much easier to make on some parts of the surface than on others.

The selection of trackable features from an image is a process which is very dependant on the individual, different people will pick different points and get slightly different results. In most cases the problem lies not in finding a definite feature in one frame, but in specifying the corresponding location in another frame. Because of these difficulties it has often been found helpful to have the cloud tracking performed by two people working together rather than just one individual.

Assuming that individual features can be tracked this leads to the discussion as to which types of features will yield the most pertinent results. The 'ideal' feature would occur at a known height, would not change shape or orientation, and would move passively along with the local flow. In practice the tracers used are clouds which are at an unknown height, and quite frequently disappear, or undergo considerable change of shape in only one rotation period. In some cases it is possible that a feature is only being advected by the local flow, however at other times there are so few available features, that the suspicion is raised that any object which stands out sufficiently from its surroundings to be trackable is probably interacting with them in some way.

Once the velocity at which objects appear to move has been determined, the question is raised as to what precisely has been measured. In general, because of the lack of any other reasonable assumptions, it is assumed that all these motions occur at the same height (or pressure) level. This may not be unreasonable since velocity profiles obtained by tracking images made through the green filter have been compared with, and found to be nearly identical to, similar profiles constructed from images made using the violet filter, and hence probably corresponding to a different height.

One of the other possibilities is that the measurements correspond not to a material velocity, but to the phase speed of a propagating wave. In this respect it is pointed out by Mitchell (1982) that, at least in the Jovian case, measurements have been made from features over a wide range of spatial scales, without observing the velocity obtained to be a function of size as would be expected from some kind of dispersive wave.

3.4.3 EFFECT OF ERRORS IN THE NAVIGATION DATA

Most of the errors in the past have been due to different interpretations of some of the quantities present in the SEDR data, it is hoped that these problems have all been corrected now. Apart from some form of systematic error, it appears unlikely that the SEDR data should be much out (these quantities change quite slowly with time, and any abrupt change would be noticed). The use of curve fitting (see section 3.6.3) to obtain the sub-spacecraft line and sample seems reliable, but produces results which vary of the order of 0.5 - 1 pixels, which, if the image is a wide angle and is used to navigate a simultaneous narrow angle can lead to errors of ~ 6 pixels. Other possible sources of error are in the camera parameters, although these have been carefully calibrated (Danielson et. al. 1981, Benesh and Jepsen 1978), and in the polar and equatorial radii, which is perhaps more likely.

The inverse perspective projection procedure is highly nonlinear, and the effect of an error in any one of the input parameters is hard to determine, and of doubtful use, since this would depend upon all the other parameters as well as the part of the image being considered. The reliability of the results can only really be tested by comparing results produced from different images with different navigation data. When this has been done by comparing velocity measurements, of the latitude of different features these seem to be self consistent. However these measurements are usually made when the relevant feature is near the centre of the planet in the image, and this is when the rate of change of latitude (or longitude) with displacement (measured in pixels), is a minimum. It is probable that discrepancies are more apparent nearer to the limb of the planet, and this may give rise to difficulties when points in this region are used.

The effect of errors on velocity measurements is further complicated by the subtraction of the two positions, which will tend to lessen the effects of some errors. This effect was considered by Ingersoll et. al. (1981) when he estimated the error in velocity measurements to be given by the resolution (defined as the distance in km between adjacent pixels) divided by the time separation.

The other major sources of error in velocity measurements are due to feature misidentification and to the upper limit imposed on position measurements by the finite size of each pixel. The first effect will tend to increase with increasing time difference, as slow changes occur in the shape of the feature, the second will become more important as the displacement, (measured in pixels) becomes smaller, which will happen when the time interval decreases. Thus there will come a point at which making measurements from image pairs with decreasing time separation will cease to yield an increasing velocity resolution.

3.4.4 PRODUCTION AND USE OF MOVIES

In many cases the amount of quantitative information in terms of velocity measurements which can be obtained from a set of images is small, due mainly to the difficulty of finding suitable cloud tracers.

It is partly because of this that movies are made, these consist of a sequence of at least two (although three is a much more acceptable minimum) images of about the same scale and orientation, displayed in succession. Raw images are not usually suitable for this due to the change of perspective. Because of this map-projected images are often used. With these it is sometimes advisable to shift the images in order to compensate for a uniform zonal velocity so that the, often more interesting, small scale motions are more easily visible.

The main objective in producing these movies is to enable one to obtain a qualitative idea of what is happening. They make it much easier to perceive cloud motions, even though these may not be sufficiently sharp for it to be possible to track them accurately, and determine their velocity. Thus while not directly producing any quantitative results they can be very useful in observing the general flow pattern, and deciding what, if any, other forms of analysis might prove useful.

3.4.5 OTHER MEASUREMENT TECHNIQUES

In addition to the methods already mentioned, there are other means by which information may be extracted from images.

Since the images can be radiometrically decalibrated (see section 3.6.1) the DN level of each pixel should be directly proportional to the energy flux incident on it. Because of this graphs of DN level against latitude or longitude may give some useful information about cloud heights or structure.

Information about spatially periodic structures, such as their dominant wavelength may be obtained by finding their power spectra. Although data in the low ($\sim <5$) order part of the spectrum is likely to be contaminated by sun-angle and mosaicing effects, whilst it is possible for the regular spacing of the reseaux to affect higher wavenumber oscillations.

Other techniques may also be used, for example the divergence method used by Hunt et. al. (1982) to estimate the vertical velocity.

3.5 IMAGES - PRODUCTION OF HARDCOPY

In order to produce photographic copies of the images, two devices were used, a Videoprint 5000 at Queen Elizabeth College, and an Optronics at the Rutherford Laboratory. The former displays the images on a c.r.t. screen and automatically takes a photo with a sufficiently long exposure to eliminate effects due to the raster. The latter works by scanning a variable intensity light source over a rotating drum covered with a piece of film.

The aim of both these devices is to produce a linear relationship between the dn level of each pixel, and the intensity of the corresponding location on the photograph produced. In addition, in order to correctly represent the shape of objects observed by the Voyager cameras, which produce square images, each pixel must be represented by a square.

3.6 IMAGE PRE-PROCESSING AND RESTORATION

This is separated from image enhancement, because it is concerned only with the removal of effects introduced at the various stages between the visible appearance of the cloud tops, and the received image. These can be split into different categories; distortions introduced by the camera, errors due to the transmission, and also included under this heading are effects due to the changing viewing geometry, and sun-angle effects. In addition the process by which the sub-spacecraft line and sample (see section 3.6.3) are derived from the image is explained. These processes are summarised in figs 3.3 and 3.4, with their effect on a sample image being shown in fig.3.5.

Before discussing the pre-processing in greater detail, it is perhaps useful to consider the different uses to which images are put, and the differing requirements which these impose. These can be

illustrated by three different cases, these are: when an image is to be examined to see if it holds any objects of interest; when one or more images are to be used for position or velocity measurements; and where a set of images are to be compared, perhaps in the form of a movie.

Just for observation purposes the raw images are often sufficient, sometimes coupled with the use of ISTRETCH* to enhance different sections of the image. However, the presence of noise spikes can be disturbing to the eye, and it is sometimes desirable to remove these.

For making measurements, it is necessary (as described in section 3.4.1.3) to be able to transform from image space to a position on the planet. To do this the geometric distortions introduced in the camera must be compensated for. In addition, it is necessary to determine the sub-spacecraft line and sample for each image. If any determinations of intensity variations across the image are to be made, then it is also required that the changes in the intensity scale across the image are compensated for. For the production of movies (and other image comparisons) small differences are easier to detect if the images have been map-projected to compensate for the changing viewing geometry. Also flashing effects due to one or more of the frames containing reseaux or data dropouts can be disconcerting, and should be removed if possible.

The various types of distortion will now be considered in more detail, along with the way in which they are compensated.

3.6.1 CAMERA DISTORTIONS

These can be divided into two types, those which alter the position of a pixel (geometric effects), and those which affect the DN value contained in a pixel (radiometric effects).

* see section 3.7

In order to help evaluate the geometric distortion introduced by the cameras, a field of small dots called *reseaux* have been painted on the camera faceplate, and their positions accurately measured (Danielson et. al. 1981). The relative position of these *reseaux* are changed by their path through the optics, and the electron optics of the camera, giving a slightly altered pattern in the resulting image. Using the expected locations the VICAR program RESLOC (see Jepsen et. al. 1980) searches for each *reseau*, using convolution techniques to determine the position accurately. From the shift in position of each *reseau* an estimate can be made of the amount by which each pixel has been moved. Using this information the program GEOMA is used to deliberately warp the image in order to compensate for the geometric distortion introduced. This is the process responsible for the small amount of pincushion distortion visible in fig. 3.5 4. Once all the *reseaux* have been located, they are no longer required, and are usually cosmetically removed by interpolating over adjacent pixels.

The radiometric effects can be approximately divided into two parts. There is a nonlinear relationship between the incident and observed DN level, which varies across the image, and is dependant on the optics, filter, and the camera used. There is also a dark current effect, this is cumulative with time, and depends upon the exposure period, and the rate at which the image is read out of the camera.

Thus the radiometric decalibration consists of subtracting an appropriate dark current image (these are taken at intervals), and then correcting the nonlinearities. This is done using calibration files (see Jepsen et. al. 1980, Danielson et. al. 1981, Benesh and Jepsen 1978), which for each camera-filter combination provide a table of actual DN values against those measured by the camera, for each pixel in the image. By interpolating between these values a continuous mapping between input, and output DN can be constructed,

and used to compensate for the nonlinearities. The complete radiometric decalibration process is performed by the VICAR program FICOR, when provided with the correct data files.

In addition to the geometric and radiometric effects, the cameras also give rise to other features in the images. For example the presence of errors in the vidicon gives rise to small black and white 'camera blemishes' in the images. The positions of these are known, and because of this they can easily be removed, this can be done by RESARR at the same time as the reseau removal. Another form of camera artefact is the appearance of dust rings in an image. These are caused by out of focus dust particles, and appear as dark haloes about 50 pixels across. It is possible to distinguish these from real image features by the fact that they remain approximately fixed in one position in the images, rather than moving with the planet.

3.6.2 TRANSMISSION ERRORS

Although the telecommunications from Voyager stand a good chance of being received (Edelson et. al. 1979 give a figure of 98% for the voyager 1 Jupiter encounter) errors still occur, one of the main problems being rain over the receiver in Spain. This appears in the images as either a random 'salt and pepper' pattern, or as missing lines, or parts of lines. This effect, while disturbing to look at is not difficult to detect, and can be removed by interpolating over adjacent pixels or lines using the VICAR program ADESPIKE. It is because of this inherent redundancy that the imaging data has an acceptable bit error rate of 5×10^{-3} (Edelson et. al. 1979), approximately two orders of magnitude greater than that of the other instruments, and equivalent to ~ 3000 single bit errors per frame

3.6.3 DERIVATION OF THE SUB-SPACECRAFT LINE AND SAMPLE

As explained in section 3.4.1.3 and 3.4.1.4, for navigation purposes, it is necessary to know the sub-spacecraft line and sample to a greater accuracy than is obtainable from the SEDR data. In order to determine these quantities to sufficient accuracy it is necessary to use the image itself.

Since the rest of the navigation data is known, the problem of determining the sub-spacecraft line and sample reduces to that of fitting an ellipse of known size and orientation to the limb of the planet (or that part of it which appears in the image). In order to do this accurately a reasonable amount of the limb (Jepsen et. al. 1980 estimate at least 20 degrees) must be in the field of view, better results will be obtained if more is visible. It is partly because of this that simultaneous narrow-wide exposures are made: more of the limb is normally visible in the wide angle, enabling a better fit to be made. From this, and knowing the fixed relationship between the cameras, the location of the planets centre relative to the narrow angle camera can be found.

This whole process is carried out by the VICAR program FARENC, which must be supplied with an image, and a list of parameters. The first thing the program does is to go through the image searching for edges, which may be limb points. The presence of the rings around Saturn can lead to erroneous results, and this is avoided by specifying the corners of a box which just surrounds the edge of the planet, and ignoring points outside this. Using the possible limb points obtained the program fits the calculated ellipse to them, it then throws out points more than a certain distance from this, and repeats this operation, until the required accuracy is obtained. In order to use only the limb points, and not the less accurate terminator points, the program also needs to know the sub-solar latitude and longitude, as given in the SEDR.

Many of the supplied parameters are used by the program to determine whether a particular change in DN level constitutes a possible limb point, while others specify the conditions under which points may be eliminated during the recursive curve fitting procedure, and the maximum permissible 'error' (defined in terms of the standard deviation of the distance separating the remaining points from the fitted curve) of the result. The effect of this plethora of free parameters on the result is difficult to estimate, tightening up the maximum acceptable error will tend to reduce the number of erroneous limb points included, but will also remove some of the genuine points. There comes a time, when it is more important to keep a range of points evenly distributed around the limb, than to delete one or two spurious points close to, but not on the limb. However the parameter settings needed to achieve this are unknown, and probably vary from image to image. Because of this there remains a small uncertainty of ~ 0.5 -1 pixels in the location of the planets centre. Since the location of features can seldom be specified to sub-pixel accuracy, this uncertainty is seldom important. An exception to this is when a wide angle image is being used to navigate a narrow angle image, which may lead to an error of ~ 6 pixels.

3.6.4 VIEWING GEOMETRY EFFECTS - MAP PROJECTION

As the planet rotates, features move across the camera's field of view, with an associated change in perspective. The main aim of map projection is to eliminate this problem by mapping each point on the three dimensional surface of a spheroid to some fixed point in a two dimensional image. This can obviously be done in an infinite number of ways, and different methods are usually developed in order to display some useful feature, or to preserve some specific property, For example to preserve distance and direction from one point, to maintain relative areas, or a special class called conformal

projections, in which the scale, although a function of position, is not dependant on direction. Hence in a conformal projection the angle between any two lines will be correct.

The normal method (called the direct method by Elliot and Schwartz 1977) of map-projection consists of defining the position in the projection corresponding to a given latitude and longitude. Using this information, a latitude - longitude grid can be drawn on the map. In the image processing case, however the technique used is somewhat different. The program scans the output image, and for each pixel it calculates the corresponding latitude and longitude. Using this, and the navigation information (see 3.4.1.3) the associated location in the input image can be found. The DN level at this position is estimated by taking either the DN level of the nearest pixel, or interpolating over the values of the four surrounding pixels.

Thus in order to use this method one needs to know the inverse (i.e. line and sample to latitude and longitude) equations for the desired projection, and the direct equations for the input perspective image (this is the process described in 3.6.1.3). In addition, unlike the Earth, in the Saturnian case the effects of the planets oblateness can not be ignored. Hence the problem becomes that of determining the inverse equations for the required, projection on a spheroid.

Although it would be possible to extend many of the standard projections to include the case of a spheroid, a more general method is obviously to be desired. This is provided by both Jackson (1981) and Elliot and Schwartz (1977), who consider the conformal projection of a spheroid onto a sphere. This can be performed by mapping the point (θ_d, λ) on the spheroid onto the point (θ_{co}, λ) on the sphere. The latitude θ_{co} , is called the conformal latitude, and is

defined (Elliot and Schwartz 1977) by :-

$$\tan \left(\frac{\pi}{4} + \frac{\theta_{co}}{2} \right) = \tan \left(\frac{\pi}{4} + \frac{\theta_d}{2} \right) \left(\frac{1 - \epsilon \sin \theta_d}{1 + \epsilon \sin \theta_d} \right)^{\epsilon/2} \quad (3.11)$$

Thus any conformal projection on a sphere can be used for the spheroidal case by replacing the latitude in the defining equation by the conformal latitude. Unfortunately this method requires the evaluation of the planetodetic latitude θ_d in terms of the conformal latitude θ_{co} , and it is not possible to use equation 3.11 to explicitly define θ_d . This problem is solved by Elliot and Schwartz 1977, who use a successive approximation technique, which allows θ_d to be determined iteratively to within a given tolerance.

One of the advantages of the map-projection process is the need to work temporarily in latitude longitude space, which is often convenient. For example due to the planets curvature the effect of limb darkening is present in the images (this may be less apparent at some wavelengths due to effects caused by the greater path length covered by rays near the limb of the planet), this can be partially compensated for by dividing the received intensity by $\cos(\theta_s)$, where θ_s is the angle between a line joining the sun to the planet's centre, and the local normal to the surface. This correction is most conveniently carried out during the map projection process, when θ_s may be easily calculated from the local latitude and longitude, and the sub-solar latitude and longitude (contained in the SEDR data).

The advantage of map-projected images is the simple geometry. However this is fixed to the radio-rotation period, and when producing movies the effect of this is often to produce a rapid drift of the relevant features across the screen, which frequently masks the effect of any changes in the local structure of the feature itself. The simple solution to this is to alter the 'special longitude' parameter of the map projection program by an amount dependent on the local zonal velocity and the time separation of the images, forcing

the feature to remain apparently stationary, although features to the north and south will still drift past.

3.7 IMAGE ENHANCEMENT

This is a rather vague title, but is meant to refer to processes which in some way bring out various features of an image, rather than just remove the distortions introduced at another stage. Which of the various possible enhancement techniques is used will depend on what features one wishes to bring out.

In many cases the precise effect required is not known, one may just wish to check to see if there is anything interesting in the image. In this case one of the better methods is also one of the simplest. This is an IIS program called ISTRETCH in which the input DN level x is mapped to the output DN level y by a simple transformation of the type,

$$y = m x + c$$

with m and c being linked to the position of the tracker ball.

This is equivalent to simultaneously altering the brightness and the contrast. The main advantage of this method is that the cursor can be moved around, emphasizing different parts of the image, until some form of optimal result is obtained. Because of the simple nature of the transformation, its effect on other measurements, such as graphs of DN against line number, or power spectra can be easily visualized, this is not usually the case for more complex methods.

Another enhancement method is histogram equalization, the effect of which is to modify the image so that, over a specified area, a histogram of frequency of occurrence against DN level will be a horizontal line. This means that the contrast will be increased when there are many pixels containing similar DN values, and decreased where there are few. Whilst more complex than the ISTRETCH routine, the result bears a less apparent relationship to the original image,

and is generally less helpful. In addition, for Voyager images the routine attempts to compensate for the effects of limb darkening across the image which can tend to obliterate some of the more important detail.

More complex arithmetic, and logical operations can be performed on one or two images using the VICAR batch program (both the previous programs use the IIS) F2, which allows almost any FORTRAN expression to be entered. For example

$$"((IN1.GT.20).AND.(IN2.GT.20)) \times ((IN1+IN2)/2)"$$

will produce the mean of the two input images in the areas where both have a DN value greater than 20. Apart from its flexibility F2 is often used when several images need to be treated in an identical manner; For example when making movies or mosaics it is important that there are no rapid changes in intensity between images.

All the routines described so far have determined the resultant DN of a pixel solely from the original DN of that pixel, an exception to this is the program TFILT. This can be used both as a high-pass and low-pass filter, but has been found to be most useful in its capacity as a scene dependent filter. In this form it passes a box of a specified size across the image, and calculates the mean value of the pixels enclosed by this. It then alters the brightness and contrast, so that, in its simplest form the output DN corresponds to the deviation of the input DN from the mean, expressed as a fraction of the mean. For ordinary perspective images the effects produced by this process can be somewhat peculiar, especially near the limb of the planet, close to the rings, or the ring shadow. However, on map-projected images this program can be very useful and, for an appropriately sized box, will bring out details in the image at the expense of limb darkening, or the differing albedo of the belts and zones.

FIGURE CAPTIONS - CHAPTER 3

Figure 3.1

This figure (from Castleman 1979) shows the relationship between a point P' at latitude-longitude (λ, ϕ) on the surface, and its image P at line-sample $(-y, x)$. A useful intermediary between these two systems is the planet centred Cartesian coordinate system x', y', z' . This is related to the (x, y) system of the camera by a rotation matrix, and the position vector S .

Figure 3.2

The difference between some of the different definitions of latitude is illustrated by this figure. For a point P on the surface of an ellipsoid the two main alternatives are the angle θ_c (planetocentric latitude) subtended by a line passing through P and the planets centre C , and the angle θ_d (planetodetic latitude) of a line passing through P , and normal to the surface. The conformal latitude mentioned in section 3.6.4 has no simple geometric representation and is not shown. For a point S not on the surface (such as the spacecraft) the possible definitions of latitude are even greater. In addition to the planetocentric and planetodetic latitude of a line joining the planets centre C to the spacecraft, another form sometimes used (and occasionally confused with θ_d - see section 3.4.1.3) is the angle θ_j of a line passing through S , and normal to the surface.

Figure 3.3

This shows the sequence in which the various pre-processing operations are carried out. The only part of the process which varies is the radiometric decalibration which requires the selection of an appropriate calibration file and a dark current image. The choice of the dark current image is somewhat arbitrary, and usually the one which is separated by the smallest time interval from the image being pre-processed is used.

Figure 3.4

In addition to the SEDR data, for navigation purposes the sub-spacecraft line and sample are required. These are derived by the program FARENC which fits an ellipse of known size and orientation to the part of the planets limb visible in the image. Because of the difficulty in determining the position of the limb accurately it may be required that the program is used more than once with different parameters to obtain satisfactory results.

Figure 3.5

This figure demonstrates the effects of the various stages of preprocessing. The initial image (1) is radiometrically corrected to remove the nonlinear camera response (2). The reseaux marks are located and then removed (3) and the image is geometrically corrected (4). (this has introduced the apparent pincushion distortion). A similar image pre-processed by J.P.L. is shown in (5).

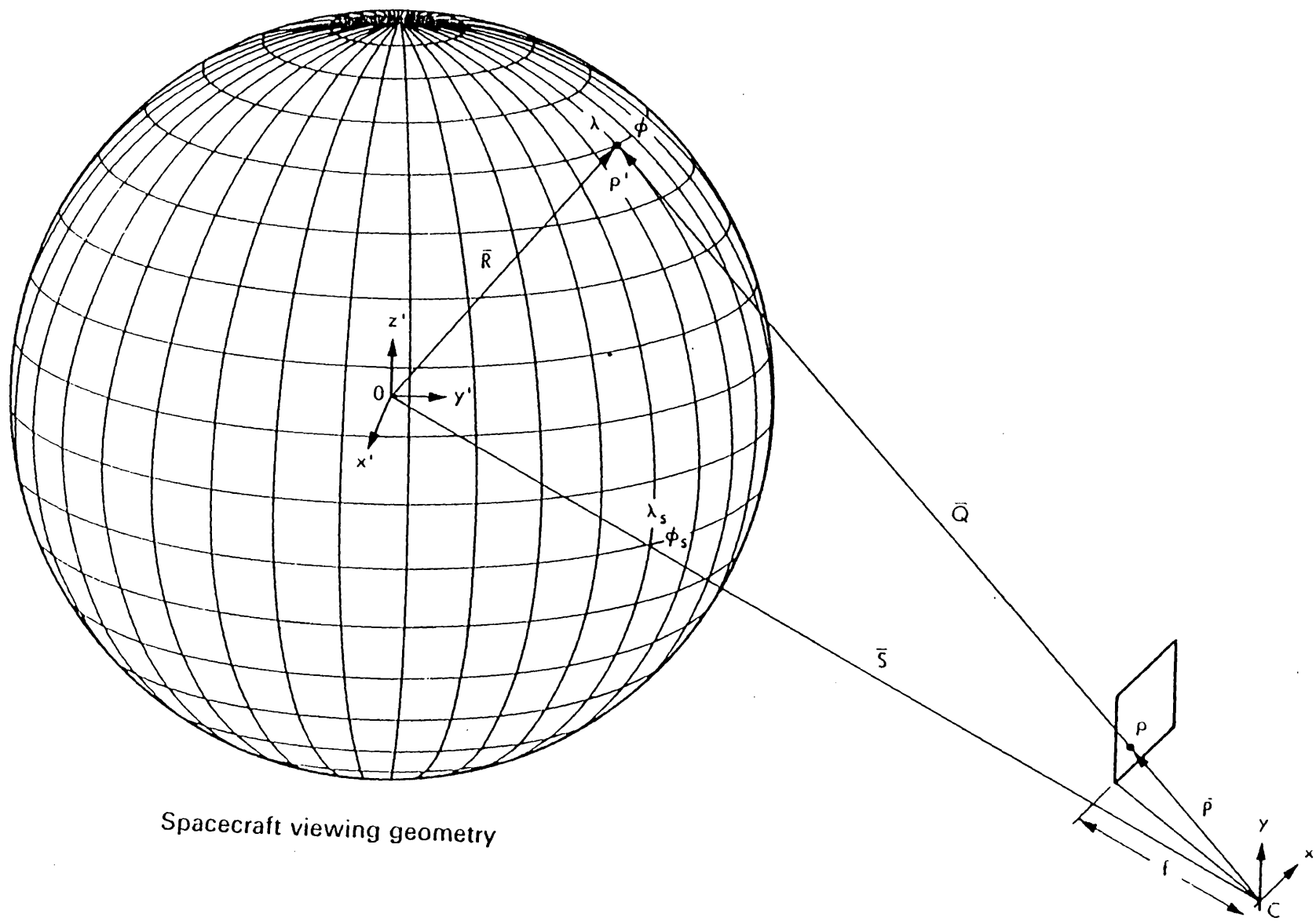


fig. 3.1

Spacecraft viewing geometry

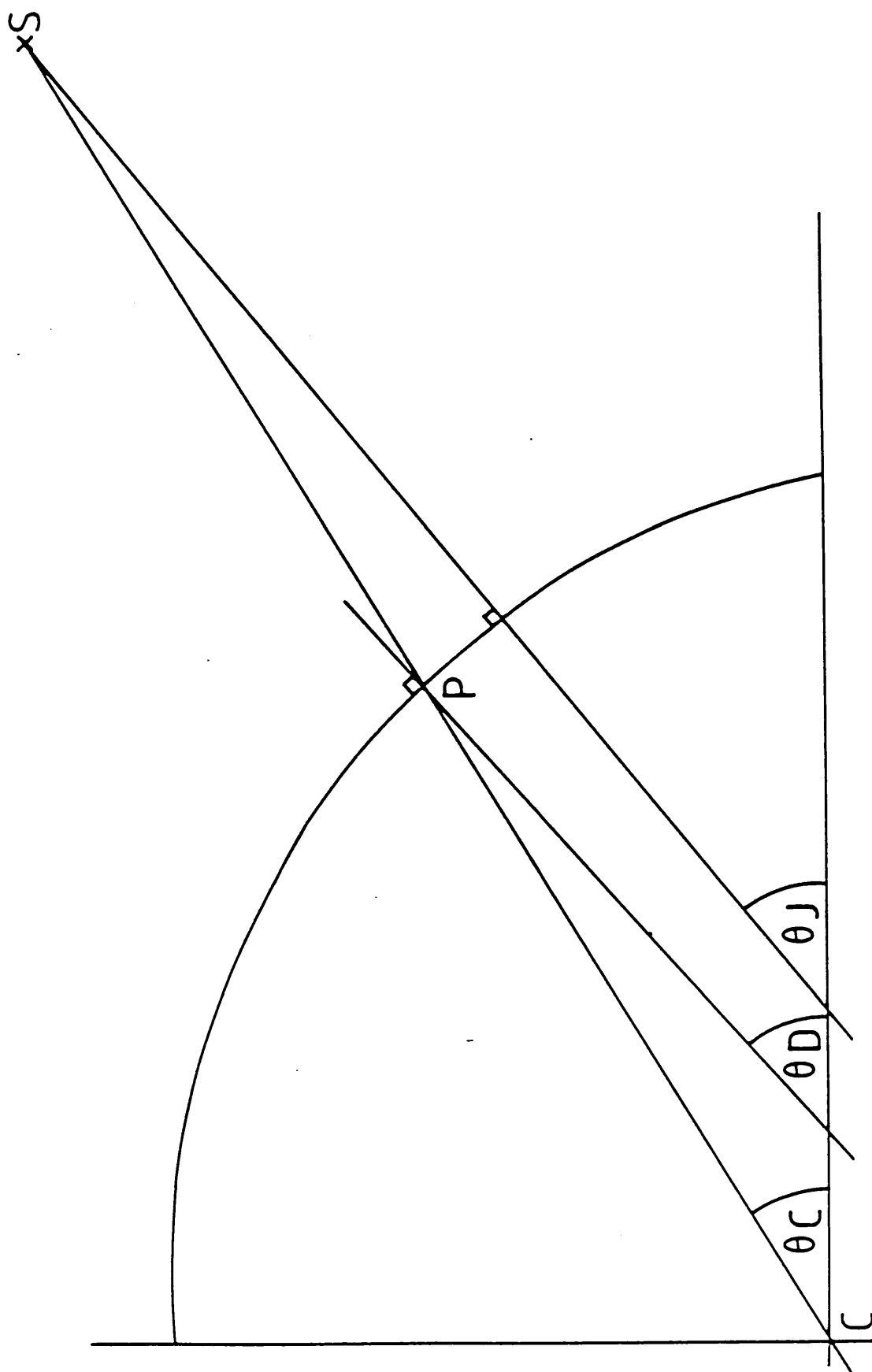


fig. 3.2

SCANRATE (1:1,3...)
OPERATING MODE (NAONLY,BOTSIM,...)
SPACECRAFT EVENT TIME
GAIN (LO,HI)

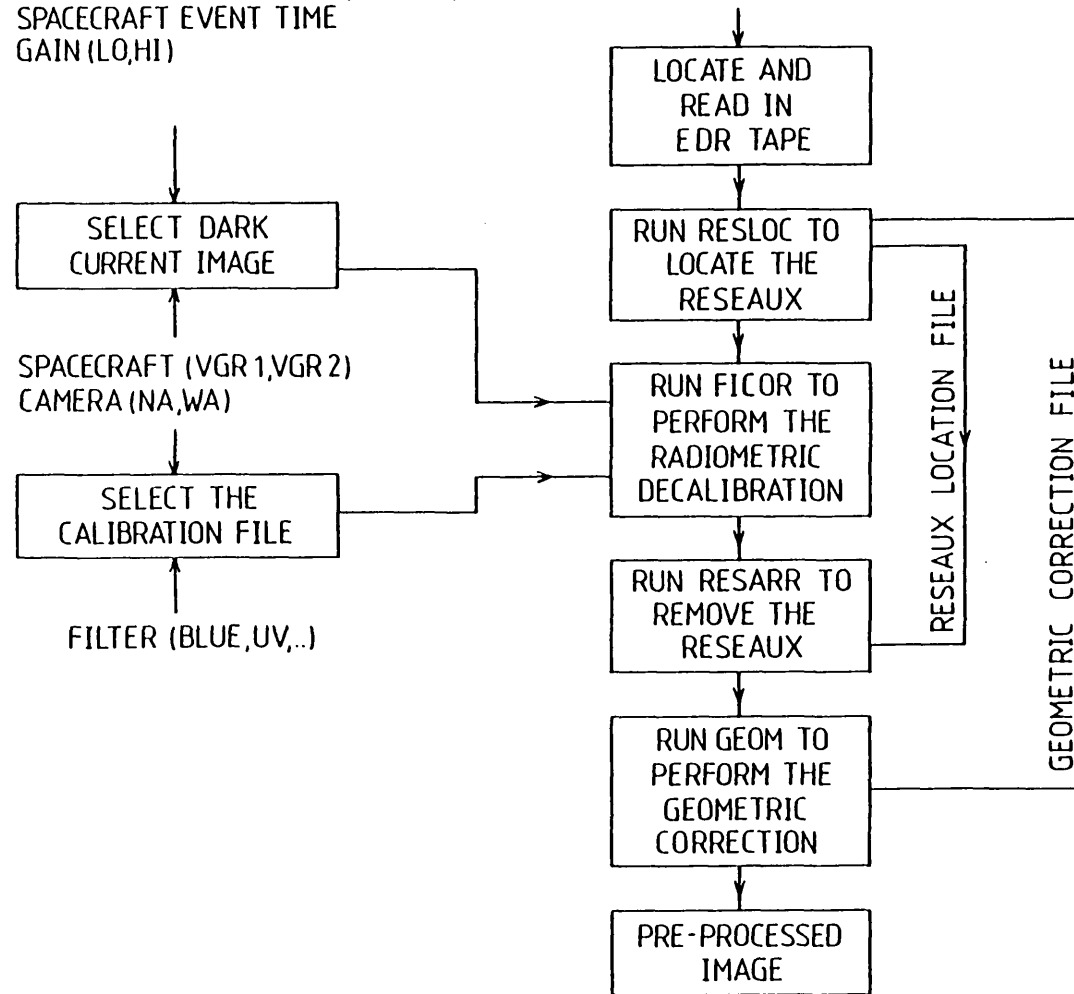


fig. 3.3

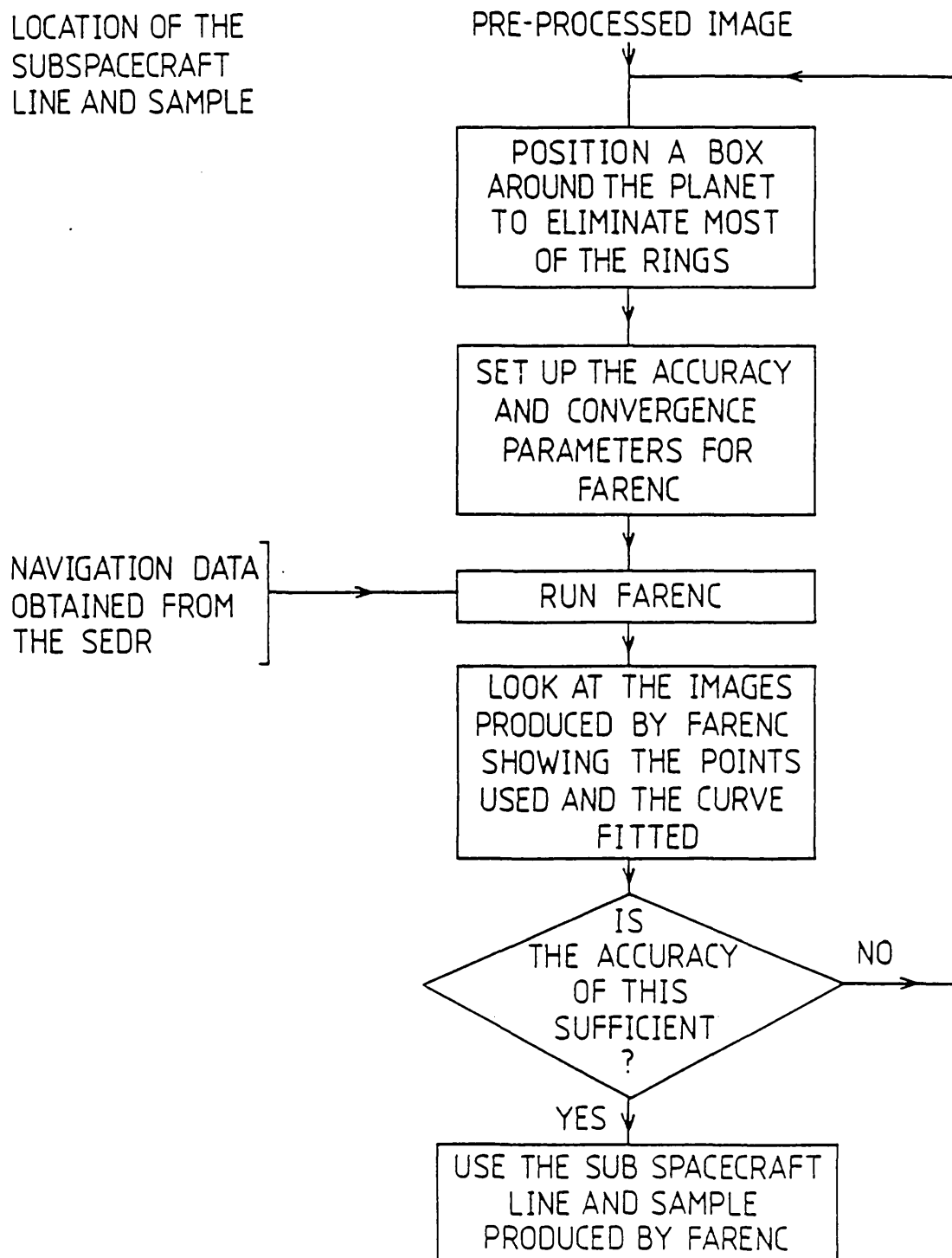
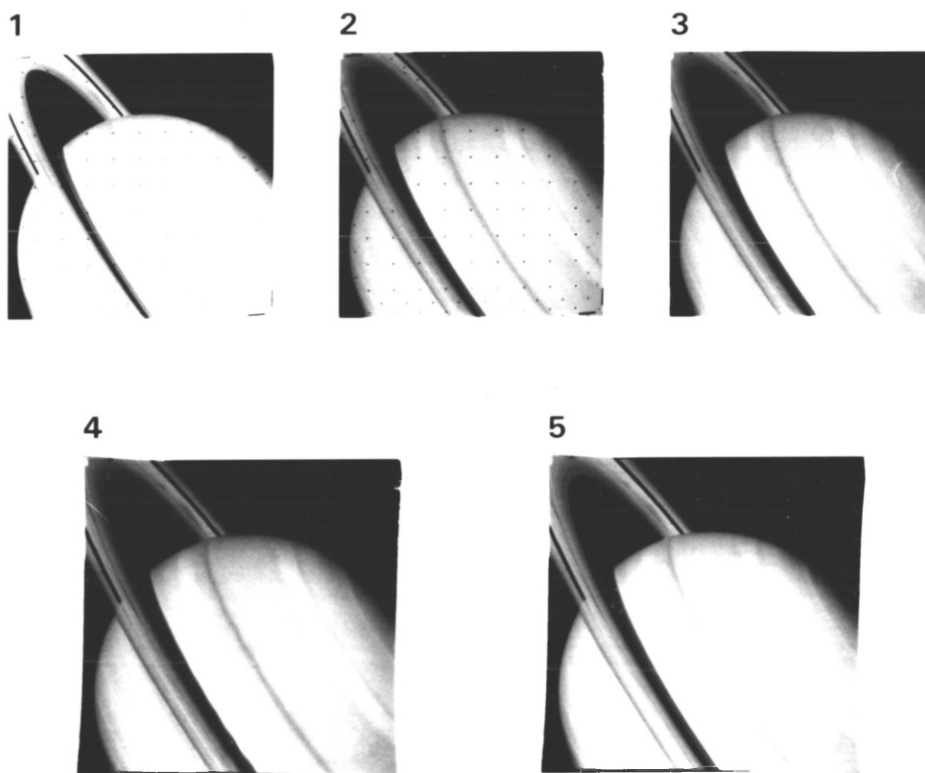


fig. 3.4



1. E.D.R. [Experimental Data Record] Image logged.

2. Radiometrically corrected Image.

3. Reseaux marks removed.

4. Geometrically corrected Image.

5. J.P.L. Processed Image.

↓
I.P.I.P.S.

fig. 3.5

CHAPTER 4

MODELS OF THE LARGE SCALE FLOW

4.1 INTRODUCTION

One of the basic challenges of the planetary atmospheres program is the development of a realistic large scale circulation model of the outer planets. The main requirement is that a model should be able to produce (or at least maintain) the observed multiple jet structure at the cloud tops, and the associated belt-zone pattern.

One of the main problems associated with the development of mean flow models lies in the lack of data concerning the intermediate regions of Jupiter and Saturn. From observations of the moment of inertia, gravity field, and radio emission, information about the structure of the deep interior can be derived. Surface observations give the cloud top velocity structure and temperature structure. However there are no direct measurements of the flow at intermediate levels.

Possibly because of this the models used can be divided into approximately two types; deep models which emphasize the role played by the deep internal heat source, and shallow models driven by the latitudinal temperature gradient maintained by solar heating. The relative importance of these two effects is expressed by Stone (1976) in terms of the Richardson number Ri , which can be defined as

$$Ri = \frac{g}{\theta} \frac{d\theta/dz}{(du/dz)^2} \quad (4.1)$$

For $Ri < 0$ the atmosphere is statically unstable and convection is expected to dominate, whilst for $Ri > 0$ the atmosphere is statically stable. However for $Ri > 0$ it is possible for instabilities to develop in the vertical shear with its associated horizontal temperature gradient. In addition to these two forms of instability there is the possibility of phenomena involving spatially dependent radiative and latent heat effects caused by the presence of clouds.

Unfortunately the value of Ri below the clouds is unknown, although, unless radiative transfer dominates, convection must surely take place somewhere in order to transfer the internally generated heat.

Although there are many different models of the zonal flow, there are certain dynamical constraints which limit the permissible flow patterns. A good example of this is the barotropic stability criterion for flow on the Earth's surface, although a different form of this condition may be important in deep atmospheres.

4.2 CLASSICAL MODELS

A simple model of the Jovian circulation is provided by Hess and Panofsky (1951). Using the limited velocity data available to them they found that the dark belts were regions of cyclonic shear, and that the lighter zones corresponded to anticyclonic shear. From this they surmised that the zones were regions of high pressure, giving rise to divergence and hence ascent, the lighter colour being due to ammonia condensing out.

A different approach was followed by Ingersoll and Cuzzi (1969). They assumed that the velocity measurements obtained were produced by the differing vertical shears brought about by latitudinal temperature gradients. This was modelled by assuming a fixed temperature difference between belts and zones and a fixed depth

between the observed cloud tops and a uniformly rotating lower boundary. By using the observed belt and zone widths they were able to estimate the velocity, obtaining values which agreed well with the observations.

The difficulty with both the Hess and Panofsky and Ingersoll and Cuzzi models is that they provide no explanation for the implied temperature structure, although the Ingersoll and Cuzzi paper does conclude that this must be produced either by internal heating or some form of preferential absorption by the zones.

4.3 CONVECTIVE MODELS

Some of the simplest types of model which are capable of producing the type of large scale motions observed on Jupiter and Saturn, are those driven by convection. Although convection can give rise to large scale flow patterns, for example Bénard convection, it is considerably restrained on the outer planets by the rapid rotation rate. This is demonstrated by Chandrasekhar (1981), where he shows that an ideal fluid will always be stable against thermal convection in the absence of viscosity.

The convective models used can be divided into two types, deep and shallow, depending on the relative importance of the underlying vertical structure and the rotation rate.

The case in which rotational effects dominate is considered by Williams and Robinson (1973). Their numerical model is of a shallow (20 - 500 km) layer bounded above and below by spherical surfaces at different temperatures, with induced motions initially being assumed to be axisymmetric. The vertical temperature gradient sets up a series of cells (analogous with Bénard convection), giving rise to alternately rising and falling motion as one moves away from the equator. The main effect of rotation is to restrict these cells to lower latitudes, and to induce a zonal motion through the

Coriolis force. The results produced by this model appear to agree with the observed Jovian velocity profile, with the characteristic pattern of alternating jets diminishing away from the equator.

One of the less satisfactory aspects of this model is the way in which the large scale flow depends upon the much smaller scale motions providing the eddy viscosity. This is parameterised using mixing length theory, in one case the mixing length corresponding to approximately half the width of the equatorial jet.

In order to test their assumption that the large scale motion is axisymmetric Williams and Robinson use a 3-D version of their model to which they add random perturbations. The effect of this is to superimpose a slowly growing three dimensional cell structure on the asymmetric flow. This suggests that the parameter range lies close to the transition from two to three dimensional motion.

The case of deep convection has been considered by Busse (1976), who argues that the presence of rotation will attempt to limit the fluid to two dimensional motion perpendicular to the axis of rotation in accordance with the Proudman - Taylor theorem (see, for example Tritton 1977). For a fluid filled sphere of radius R he states that convection will initially occur at a distance of approximately $R/2$ from the axis of rotation and will take the form of long rolls of fluid aligned parallel to the axis. As the applied temperature gradient increases these may grow until they occupy the whole sphere.

However for Jupiter and Saturn this process will be interrupted by the presence of the molecular hydrogen to metallic hydrogen transition, which will inhibit the formation of convective rolls passing nearer to the axis of rotation. The effect of this will be to decouple the Northern and Southern hemispheres, at (planetocentric) latitudes greater than that defined by

$$\cos(\theta) = r_i/R \quad (4.2)$$

where r_i is the radius of the metallic hydrogen transition.

For Jupiter this corresponds to latitude of ~ 45.0 deg (Busse 1976) which is approximately where the velocity profile ceases to be symmetric. For Saturn this should happen at ~ 65.0 deg, which also appears to be approximately correct.

In addition, as shown by Busse (1970) the change in depth of the system as one moves away from the axis of rotation should cause the rolls to drift in a manner corresponding to the motion of Rossby waves on a sphere.

The observed zonal motions at the surface are assumed to be driven by the eddy stresses set up by the convective rolls, but it is suggested that the non axisymmetric motions are not observed because of the smaller scale height near the surface.

The interaction between a zonal flow, and the convective rolls is considered by Ingersoll and Pollard (1982). They conclude that differential rotation could extend down into the planet without being disrupted by convection, although this would decrease the efficiency of the convection, giving an increased (superadiabatic) vertical temperature gradient. Because of this the zonal velocity will not be independent of the distance parallel to the rotation axis, as would be the case in a purely adiabatic atmosphere. Instead it may vary with a scale comparable with the planetary radius.

4.3.1 MAYR AND HARRIS'S MODEL

A different explanation of the differential rotation and equatorial superrotation observed on Jupiter and Saturn is given by Mayr and Harris (1981). They note that the zonal velocity at the equator is actually a minimum, the maximum value being attained, on Jupiter, at approximately ± 7 deg. Hence it is possible to drive the circulation at the equator by horizontal momentum diffusion from the north and south. The problem then becomes the supply of momentum to these near equatorial jets.

Their model provides this acceleration of the equatorial region through the interaction of a solar driven meridional circulation, with the vertical temperature profile. It is assumed that due to the presence of an internal heat source the troposphere is statically unstable, whilst higher up, especially near the equator, solar heating makes the atmosphere statically stable. The effect of this is that the vertical motion produced in the equatorial region by the mean meridional circulation will tend to heat up the middle atmosphere and cool the upper atmosphere. This in turn leads, through the thermal wind equation, to an acceleration of the equatorial region in the middle atmosphere, with a corresponding retardation at greater heights.

In order to produce a more detailed comparison with observations Mayr and Harris decompose the velocity and temperature fields into spherical harmonics, which are coupled through the presence of the Coriolis force and dissipative effects. By numerically solving the equations associated with this model, and using parameters representative of Jupiter they produce as results plots of velocity and temperature as a function of latitude and height. These show a pair of broad westerly jets centered at approximately 30 deg. N and S in the highest region replaced by a narrow equatorial jet limited to $\sim 15^\circ$ N and S lower down, at intermediate levels the zonal velocity takes on a multiple jet structure resembling the Jovian circulation. The main discrepancy between their results and the observed flow pattern is that the maximum velocity is about 5 ms^{-1} , but it is suggested that this indicates too large a value for the eddy diffusion coefficients.

This model is examined further by Mayr et. al. (1982) who consider the growth of hurricane type disturbances in an atmosphere which is statically stable at the top, but unstable lower down. Their disturbances consist of upward motion in the unstable regions

which gives rise to a pressure minimum, and hence cyclonic rotation at its base. However above this the local temperature increase caused by the higher temperature material moving up from below produces an increase in pressure and hence anticyclonic rotation. At higher levels where the atmosphere is statically stable a secondary circulation is set up with descending motion in the centre and cyclonic rotation.

This explanation is applied to the great red spot which IRIS measurements show to be at approximately the same temperature as its surroundings at the 1 bar level, but significantly cooler higher up. This is interpreted in terms of the model by placing the transition region between the primary (upward) and secondary (downward) circulation at the 1 bar level, with the higher measurements seeing the secondary circulations with its cool centre.

The connection between the individual hurricane like systems and the global circulation is provided through the static stability. The main effect of the large scale circulation is to produce ascent near the equator with regions of descent at higher latitudes, hence increasing the static stability near the equator and decreasing it to the north and south. This suggests that the hurricane type disturbances should be more common in mid latitudes (ie. regions of descent) than on the equator. The more detailed multiple cell circulation deduced by their numerical model (Mayr et. al. 1982) leads them to suggest that there should be latitudes in which hurricane type disturbances are common, separated by latitude bands in which hurricanes are rare, a fact which seems to be confirmed by the Voyager observations.

4.4 RADIATIVE - CONVECTIVE INSTABILITY

A possible explanation for the large scale banded structure in terms of the interaction between dynamical processes and the radiative flux is provided by Gierasch et. al. (1973) and Gierasch (1973). In the former this is modeled by a cloud layer with varying temperature and emissivity, whilst the latter uses a changing vapour concentration.

Gierasch et. al. 1973 consider the effect of a cloud layer of varying temperature and emissivity on both inertia-gravity and Rossby waves as well as an axisymmetric mode. They find that under particular circumstances it is possible for any of these modes to grow. By assuming that the growth of a particular mode will cease due to finite amplitude effects, when the vertical displacement exceeds a certain value they show that the axisymmetric mode is expected to dominate. Basing their parameters on the model produced by Lewis (1969) they obtain estimates for the width of the most rapidly growing mode which compare reasonably with the observed belt-zone separation.

The different relationship between the vertical motion and the heat generation terms used by Gierasch (1973) results in the loss of the unstable gravity waves, present in the model of Gierasch et. al. (1973). This also leads to Rossby wave type solutions being damped out, although it is suggested that in this case friction may be more important. However he shows that there is a mode, which is independent of wavenumber, which will grow at approximately the same rate as the axisymmetric mode. By assuming that the wave amplitude is limited not by finite amplitude effects, but by horizontal friction he produces an estimate for the band-zone wavelength, which is much less than that observed. A more realistic value is obtained by considering the solution on an equatorial β - plane. This also gives a solution for the non-symmetric mode which has a much smaller horizontal scale, because of this the effects of friction are

probably more important, and this is suggested as the reason why these modes are not apparent.

A model in which condensation provides the mechanism responsible for maintaining the belt-zone structure is given by Barcilon and Gierasch (1970). This consists of a single cloud layer (assumed to be water), with a free surface higher up. The lowest order form of this model simply relates the temperature gradient maintained by the clouds, to the thermal wind equation. However the presence of the Ekman layer provided by the free surface generates a higher order meridional flow term. Assuming that the clouds act so as to screen radiation from below (otherwise this type of oscillation will not occur), the outgoing energy flux can be approximated by a linear function of cloud top temperature, from this a differential equation for the potential temperature can be derived. It is then shown that this has solutions which are oscillatory functions of the northward distance y . The mechanism responsible for providing the initial (zonal) flow required by the model is not described, although it is suggested that the symmetric instabilities considered by Stone (1966 see section 4.5.4) could be responsible.

4.5 BAROCLINIC MODELS

4.5.1 THE RHINES PARAMETER

This corresponds to a wavenumber (and hence length) scale k_β given by:

$$k_\beta^2 = \frac{\beta}{2U} \quad (4.3)$$

where U is a representative velocity scale. It arises from a consideration of the wavenumber at which 2-d turbulence (dominant for $k > k_\beta$) and Rossby wave propagation (important for $k < k_\beta$)

are equally important.

The effect of 2-D turbulence on the energy spectrum $E(k)$ is constrained (in the absence of viscosity) because both the energy $E(k)$ and the enstrophy (see for example Pedlosky 1979) $k^2 E(k)$ must be conserved. This means that if all the energy was initially concentrated at wavenumber k_0 , no more than a quarter can ever be transferred to twice this wavenumber. Since viscosity will be effective over smaller scales, an initially random eddy field will evolve until most of its energy is concentrated in the lower wavenumber range (see fig. 4.1).

On the surface of a rotating sphere this behaviour is modified by the presence of the beta effect, which allows Rossby waves to propagate. It is possible for Rossby waves via non-linear interactions to transfer energy to smaller wavenumbers. However an investigation by Rhines (1975) of some of the various techniques by which this may occur led to the conclusion that this was not very efficient.

The net effect of these two mechanisms on an initially random field of eddies is that the eddies will gradually increase in size until their time scale is appropriate to Rossby wave propagation. These waves will have the effect of dispersing the energy over a larger area whilst, because of inefficiency of the non-linear interactions, concentrating it in wavenumber space, and giving rise to a peak in the energy spectrum, see fig. 4.1. The position of this peak was estimated by Rhines (1973) from the wavenumber k_β at which the two processes have the same time scale, to be that given by equation 4.3.

4.5.2 WILLIAMS'S MODEL 1

One of the shallow layer models is that of Williams (1978), which uses a primarily barotropic formulation. Allowance is made for baroclinic terms by forcing it in a way which simulates baroclinic effects. The components of this forcing field have a random phase, but are constrained to lie within a specified rectangle in wavenumber space. In the case of the Earth this forcing is further restricted to take place only in the mid-latitudes.

The evolution of the stream function and zonal velocity profile of one of Williams's Jovian simulations is shown in Fig. 4.2. It can be seen from this that the initial eddies rapidly combine to give rise to an almost axisymmetric state. This growth in the scale of the disturbance is thought to be halted by the Rhines effect (see section 4.5.1). This supposition is supported by the K.E. spectra (see Fig. 4.3) of the stages through which the model passes. These show that the energy whilst initially concentrated at the wavenumber associated with the forcing ($k_x = 7$) broadens out fairly rapidly until it reaches the wavenumber ($k_x = 5$) associated with the Rhines effect. At this point the transfer of energy to smaller wavenumbers is delayed, with the energy associated with the very low wavenumber range increasing slowly to an almost constant value.

As can be seen from Fig. 4.2. this model does not produce the equatorial jet characteristic of the Jovian circulation. This was however produced in another of Williams's models by increasing the amplitude of the forcing close to the equator, which gave rise to an equatorial jet whilst maintaining the pattern of zonal jets to the north and south.

4.5.3 WILLIAMS'S MODEL 2

Another, more realistic, model has been developed by Williams (1979) in his attempt to explain the observed zonal flow in terms of a shallow layer. This differs from the previous (Williams 1978) model in that the baroclinic eddies are created within the model from the imposed equator to pole temperature gradient, rather than being simulated statistically.

The main results of Williams's model are presented in terms of its evolution from an initial state over a period equivalent to several years; this development takes place in several stages.

In the initial stage the model possesses a weak, unstable axisymmetric flow resembling a Hadley cell. In this state baroclinic eddies start to appear, and give rise to a non-axisymmetric flow in the form of weak easterly and westerly jets. These increase in strength as the eddies begin to move towards latitudes with their own absolute vorticity (see Kuo 1951). Initially the zonal jets are quite narrow, but they coalesce into wider jets, until, at the end of a year, there are approximately five eastward, and five westward jets, each about 10^4 km in width, and with a velocity scale of 25 ms^{-1} . A size which suggests that the transition to larger scales may have been halted by the Rhines effect (see 4.5.1) which for $U=25 \text{ ms}^{-1}$ and the value used for beta gives a length scale of $\sim 1.2 \times 10^4$ km.

After this initial period of activity the model remains in an almost steady state for ~ 900 days without much change the flow being dominated by the alternating zonal jets. The eddy activity appears to be limited to neutral baroclinic waves forced by the smaller scale turbulence, and baroclinically unstable waves which grow within the westerly jets thus reducing the local temperature gradient. The lack of activity during this stage results in the mean zonal velocity decreasing whilst the pole to equator temperature increases due to the imposed heating.

This quiescent phase is abruptly terminated by the sudden breakup of the northernmost jet into smaller scale disturbances, closely followed by the other jets. The effect of this is to produce four zonal jets, each with a maximum velocity of the order of $\sim 130 \text{ ms}^{-1}$, once again in agreement with the Rhines effect.

After this period the flow begins to settle down, this time to a state in which there are four strong westerly jets and two major easterly ones. The calculation was terminated at this point due to the thermal imbalance resulting from the necessity of using a sufficiently high heating rate to avoid computational instability.

In an attempt to examine the final state of such a model Williams includes some results from similar models with a reduced resolution which were allowed to continue for a greater period. The results from these appeared to indicate that, certainly if a lower heating rate was used, the model would equilibrate after some years although possibly with some periodic behaviour with a period in the range 3 - 7 years. One of the other interesting points is the suggestion, based upon the similarity between Jupiter's observed structure and the models streamfunction that the clouds might be the result of horizontal pressure variations rather than convective clouds resulting from vertical motion.

4.5.4 STONE'S SYMMETRIC INSTABILITY

An interesting contrast to Williams baroclinic model is presented by Stone (1966), who considers the form of baroclinically unstable waves in a model similar to that used by Eady (1949). The important difference between these two is that whereas Eady assumed that the Richardson number $Ri \gg 1$, Stone assumed that $Ri \sim O(1)$, corresponding to a flow with a lower static stability or increased vertical shear. The effect of this is that although the conventional Eady type of baroclinic instability is still present it is dominated

by another form of instability.

This alternate form of baroclinic instability consists of long rolls of fluid, with their axis aligned with the zonal flow, and the sides of the rolls sloping in the meridional direction. Because of this symmetry about the axis they are termed symmetric instabilities. When the growth rate was examined as a function of λ (the meridional wavenumber) it was found that the most unstable waves were associated with $\lambda = \infty$, although all waves with λ greater than $\lambda = \pi/(1-Ri)^{\frac{1}{2}}$ were unstable.

A more detailed study of the variation of the least stable disturbance as a function of Ri was given by Stone (1970), who showed that the symmetric instability would dominate when $0.25 < Ri < 0.95$. The ranges of Ri over which various types of instability can occur are summarized below.

<u>Type of instability</u>	<u>Range of Ri</u>
Kelvin-Helmholtz	$Ri < 0.25$
symmetric baroclinic	$0 < Ri < 1$
Eady baroclinic	$0.84 < Ri$

It was suggested by Gierasch and Stone (1968) that the symmetric instability would transfer momentum equatorward, and hence could possibly explain both the axisymmetric banded structure and the equatorial jet observed on Jupiter. However this point was considered in more detail by Stone (1972), who showed that the equatorial acceleration could only occur if $Ri < 1/3$, which contradicts the observation of some asymmetric features (he also replies to Hide's 1970 query, agreeing that this model could not provide the required equatorial acceleration relative to the underlying surface). As a result of these difficulties they are led to conclude that it is unlikely that symmetric instabilities are responsible for the Jovian circulation.

A similar conclusion was reached by Walton (1975) who included the effects of friction, and found that the instability occurred at a finite wavenumber. However under these conditions the transfer of momentum is towards the pole, and hence would be unable to support an equatorial jet.

4.6 BAROTROPIC INSTABILITY CRITERIA

One of the conditions which limit the possible range of velocity profiles in a shallow atmosphere is the barotropic instability criterion, based upon a consideration of the rate of change of horizontal shear with latitude. This type of instability was first discussed by Rayleigh, who considered the flow in a cylindrical system where the angular velocity Ω was a function of the radius r . He showed that the system is stable if:

$$\frac{d}{dr} (r^2 \Omega)^2 > 0 \quad (4.4)$$

(this is called the Rayleigh criterion, see for example Chandrasekhar 1981), otherwise it is possible for small latitudinal displacements to grow, and extract energy from the basic flow.

The equivalent condition for flow in a shallow surface layer over a sphere is more complex. However a necessary (but not sufficient) condition for the flow on a β -plane to be unstable is that

$$\beta - \frac{d^2 u}{dy^2} = 0 \quad (4.5)$$

is satisfied somewhere (see Holton 1979), with β given by:

$$\beta = (df/dy)_{\theta_0} \quad (4.6)$$

A different form of equation (4.5) is developed by Ingersoll and Pollard (1982) who consider flows in a rotating adiabatic fluid filled sphere of the form $u = u(r)$, where r is the distance from the axis. They conclude that the flow could be unstable if $B - u_{rr} = 0$

somewhere, where

$$B(r) = \frac{2}{M} \frac{\Omega}{dr} \frac{dM}{dr} \quad M = \int_{-h}^{+h} \rho \, dz \quad (4.7)$$

with z being the distance parallel to the axis and $h(r)$ corresponding to the outer boundary of the sphere.

The possible relevance of these two forms of instability to the circulation of the outer planets has been considered by Ingersoll and Pollard (1982) from which fig. 4.4 was taken. This shows the value of d^2u/dy^2 for Saturn evaluated from the Voyager velocity profile compared with β (on the right) and $B \sin^2\theta$ (evaluated assuming a particular internal structure) on the left. It can be seen that the conventional condition is violated at a considerable number of latitudes in comparison with the second condition. This suggests that the limiting effect of B rather than β on d^2u/dy^2 could be more relevant in this case.

4.7 HIDE'S CONDITION FOR THE PRESENCE OF A SUPERROTATING EQUATORIAL JET

One of the problems associated with many of the models of the zonal flow is producing the equatorial jet. It has been pointed out by Hide (1970) that it is not possible to do this with an axisymmetric geostrophic model. This follows from the fact that to transfer momentum equatorward requires meridional motions which in turn imply a pressure field which varies with longitude. This is absent in axisymmetric motions.

The power of this limitation is somewhat restricted however, since some of the apparently axisymmetric models (such as that of Williams and Robinson 1973) in fact employ friction, giving rise to small scale non-axisymmetric motions.

4.8 OBSERVATIONAL EVIDENCE

An attempt to obtain information about the depth of the zonal flow was provided by Smith et. al. (1982), in the form of fig. 4.5. This was obtained using the geostrophic wind equation (Holton 1979) which relates the zonal velocity u to the geopotential height Φ (for a constant value of g this is proportional to the geometric height), and is given by

$$u = \frac{-1}{f} \frac{d\Phi}{dy} \quad (4.8)$$

From this equation, and the observed wind profiles Smith et. al. were able to determine the change with latitude of the thickness of a layer separating the cloud tops from a lower constant pressure surface rotating with the radio rotation period (see section 3.4.1.2). Now the thickness (in geopotential units) of a layer extending from a pressure level p_0 to a pressure level p_1 , and with a mean temperature T is given by (Holton 1979)

$$d\Phi = \int_{p_1}^{p_0} R T(p) d\ln(p) \quad (4.9)$$

Taking the observed equator to pole temperature difference to be 1- 2K, and assuming an approximately constant composition (i.e. constant R), they argue that if the observed change in Φ is not to be exceeded, the observed motions must penetrate down to at least the 2000 K level, corresponding to a pressure level of approximately $p = 10^4$ bars.

It has been pointed out by Allison and Stone (1983) that this analysis depends strongly on the condition of the assumed boundaries. In particular the lower boundary is assumed to comprise a constant pressure layer, rotating with the radio rotation period, and to be in hydrostatic equilibrium. It was also pointed out that the 1- 2K

temperature drop measured corresponded to a level well above the cloud deck, and may not be relevant. In view of these limitations it was suggested that it was still not possible to rule out models with shallow surface layers.

One of the other arguments used in favour of the deeper models is the apparent permanence of the zonal velocity. Although this is more apparent in the case of Jupiter, for which more historical data is available there also exist observations of Saturn, including some by Herschel in (1794). Some of these measurements are summarized by Lavega (1982), and appear to be approximately consistent with the Voyager observations, with an equatorial jet of $\sim 400 \text{ ms}^{-1}$. It should be noted that whilst the velocity profile may demonstrate long term stability, the pattern of belts and zones seems to undergo fairly rapid fluctuations. For example the lighter region observed between $23\text{--}36^\circ \text{ N}$. during the Voyager 1 encounter darkened at some time in the intervening months, until it was observed by Voyager 2 (Smith et. al. 1982) to be approximately the same albedo as the dark belt running from 36° N to 42° N .

4.8.1 CORRELATION ANALYSIS

In an attempt to model the different scale processes present in the atmosphere, one approach is to split the observed motions into two parts, a zonally averaged contribution and an eddy contribution. Thus the observed zonal velocity u is written

$$u = \bar{u} + u' \quad (4.10)$$

with the bar signifying the longitudinally averaged part and the dash the eddy component. Similar decompositions may be performed on the meridional velocity v , and the temperature field T . Using this parameterization the kinetic energy and the available potential energy can be broken up into mean and eddy parts, Beebe et. al. (1980),

give the conversion from eddy kinetic energy K' to mean kinetic energy $\overline{K}$ by :

$$\{K'\overline{K}\} = \frac{1}{N} \left[\frac{d\overline{u}}{dy} (\overline{u'v'}) \right] \quad (4.11)$$

where N is the number of velocity measurements considered. This correlation has been evaluated for Jupiter by Beebe et. al. 1980 and Ingersoll et. al. 1981. In both cases the value is positive, and it is estimated that if the results applied to a layer 2.5 bar in a thickness (equivalent to a mass of 10^4 kgm^{-2}) then the eddies would supply sufficient energy to replace the zonal mean kinetic energy in approximately 2-4 months.

This type of analysis requires a reasonably large number of velocity measurements, and for this reason is more difficult on Saturn. However it has been tried by Beebe et. al. (1982), who obtained a correlation coefficient of 0.5 (or 0.7 if the measurements were weighted by the number of measurements in each latitude bin), and concluded that as in the Jovian case the eddies were efficiently feeding energy into the zonal wind.

One of the most worrying features of this approach is the somewhat subjective process by which features are selected for measurement, and the way in which the global results vary depends upon the procedure used to remove 'incorrect' measurements, as well as being significantly influenced by certain large scale features. In addition, when this type of study was carried out on a numerical model of a stable eddy (P. Read, private communication), it was found that although it appeared that energy was being transferred between the various mean and eddy components this was being done in such a way as to leave the total mean and eddy parts the same. These difficulties, along with the fact that the velocity can only be determined at the cloud tops, gives rise to some doubt of the

significance of the results obtained using this method.

4.9 SUMMARY

It is unlikely that any of the models described precisely represent the actual flow. For example all the models either ignore the large scale albedo variations, or produce them as an integral part of the zonal velocity profile. Neither of these approaches explains how it is possible for whole latitude bands to become lighter or darker without corresponding changes in the zonal velocity profile. In this section I will attempt to compare some of the advantages and disadvantages of the different types of model.

Convective models (Busse, Williams and Robinson, Mayr et. al.) all suffer from the fact that convection is a small scale phenomena. This remains true even when individual convective cells combine to form large scale flow patterns, for example Bénard convection. For this reason convective models are very dependant upon the parameterization of the small scale viscosity. As was pointed out by Stone (1976) this is a particular difficulty in the model of Williams and Robinson with its large mixing length. He also doubted the dominance of their symmetric flow regime over cells aligned parallel to the rotation axis. This was explained for the model used by Mayr and Harris through the interaction of solar heating near the equator with the statically unstable atmosphere brought about by the internal heating. Although this model can produce a multiple jet structure, this is very dependent upon the cloud top height, and its change with altitude does not appear to agree with the Conrath and Pirraglia (1983) temperature measurements, which indicate a decrease of velocity with height outside the equatorial region. Because it is mainly confined to the lower levels, the deep convection model proposed by Busse (1976) is not expected to have any easily recognisable signature at the cloud tops, except

perhaps for the depth of the flow inferred by Smith et. al. (1982). It does however require that the flow be approximately identical in both hemispheres below a certain (approximately known) latitude, which does appear to be the case (R. Beebe, private communication).

The baroclinic instability models have the advantage that they have an intrinsic length scale (the Rossby radius of deformation, see for example Pedlosky 1979) and are therefore less dependant upon the unmeasurable small scale structure. The type of instabilities considered by Stone (1966) seem to be severely restricted by the limited range of Richardson number over which they operate. There also appears to be difficulties in providing the equatorial momentum transport needed to sustain the equatorial jet.

The baroclinic instability model provided Williams (1979) appears to agree reasonably well with the observed flow pattern. It is however unfortunate that in addition to the assumption of static stability it uses a rigid lower boundary. The importance of this assumption was demonstrated by Gierasch et. al. (1979) who considered the effect of an infinite adiabatic lower boundary. This model gave growth rates smaller than those for a rigid lower boundary. It would be interesting to know to what extent the results of Williams (1979) depend upon the presence of the rigid lower boundary.

The conditionally unstable models provided by Gierasch (1973) and Gierasch et. al. (1973) are made somewhat unsatisfactory by the way in which the dominance of the axisymmetric mode depends upon the importance of frictional or finite amplitude effects. For the model of Barcilon and Gierasch (1970) there is the additional difficulty that it requires an initial zonal flow for the mechanism to work. It has also been mentioned by Williams and Robinson (1973) that conditional instability models should have the upward branch of each cell considerably (a factor of about 5) narrower than the

downward branch, an asymmetry which is not observed on Jupiter and Saturn.

In conclusion whilst shallow conventional baroclinically unstable models such as those of Williams appear to reproduce the observed zonal flow pattern, and may not conflict with the observed wind and temperature measurements (see Allison and Stone 1983). The deep convective models of Busse could be important at lower levels and seem to account for the observed north-south asymmetry.

FIGURE CAPTIONS - CHAPTER 4

Figure 4.1

These figures, taken from Rhines (1975), illustrate the effect of beta upon the growth of two-dimensional turbulence.

Figures A and B show how the streamlines at a particular latitude change with time. It can be seen from fig A, which shows the pure turbulence ($\beta = 0$) case, that the eddies, which are initially small, grow with time. The effect of $\beta \neq 0$, is shown in fig B, where the same initial conditions give rise to westward propagating Rossby waves. It is clear that $\beta \neq 0$ reduces the rate at which the eddies merge. This is also apparent in fig C which shows how the energy spectra of A (continuous lines) and B change with time.

Figure 4.2

This figure shows the results obtained by Williams (1978) from his Barotropic model of Jovian turbulence. The plots show contours of the streamfunction (negative values are shaded) at (a) 4.6, (b) 23.0, (c) 46.0, (d) 73.3, (e) 161, and (f) 294.4 days. The graphs to the right hand side of each plot show the variation of the associated mean flow with latitude.

Figure 4.3

This shows the kinetic energy spectrum given by Williams (1978) for the model which produced the streamfunctions shown in fig 4.2. These correspond to the state of the model after (a) 4.6, (b) 13, (c) 23, (d) 46, (e) 74, and (f) 294 days.

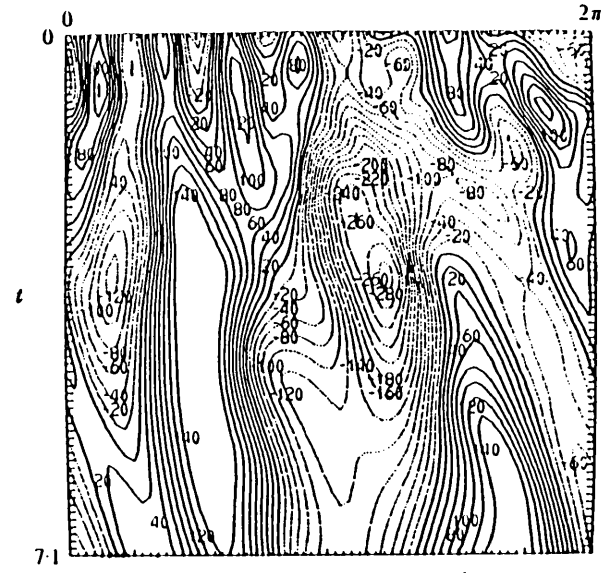
Figure 4.4

This figure, taken from Ingersoll and Pollard (1982), indicates the constraints imposed upon the zonal flow by the deep and shallow barotropic stability criteria. The left hand curves show the zonal velocity as a function of latitude (continuous line), and its reflection about the equator (dashed line). The graph on the right shows the second derivative of the zonal flow with respect to latitude, with the limiting effects of $B \sin^2 \theta$ on the left and β on the right. These are described in greater detail in section 4.6.

Figure 4.5

This figure is derived by Smith et. al. (1982) from measurements of the zonal flow at the cloud tops, and the observed equator to pole temperature gradient. Using these in the thermal wind equation they are able to estimate how the thickness of a layer separating the observed cloud tops, from a lower constant pressure layer of no motion, varies with latitude. This is expressed in fig 4.5 as a proportion of the depth of an adiabatic layer encompassing a temperature change of 200 K.

A



B

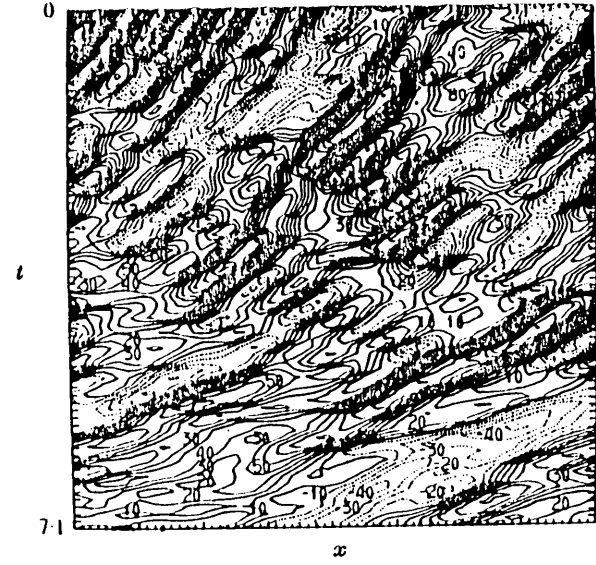
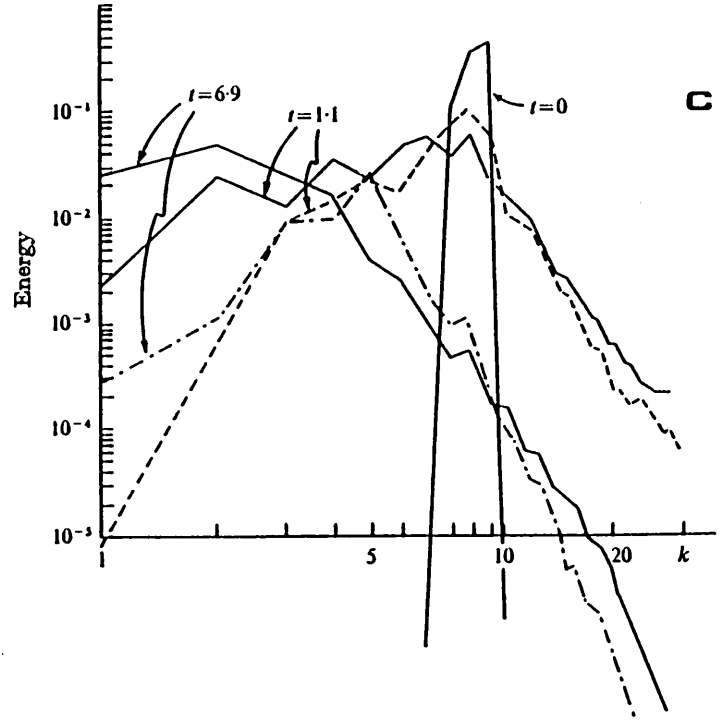


fig. 4.1

C



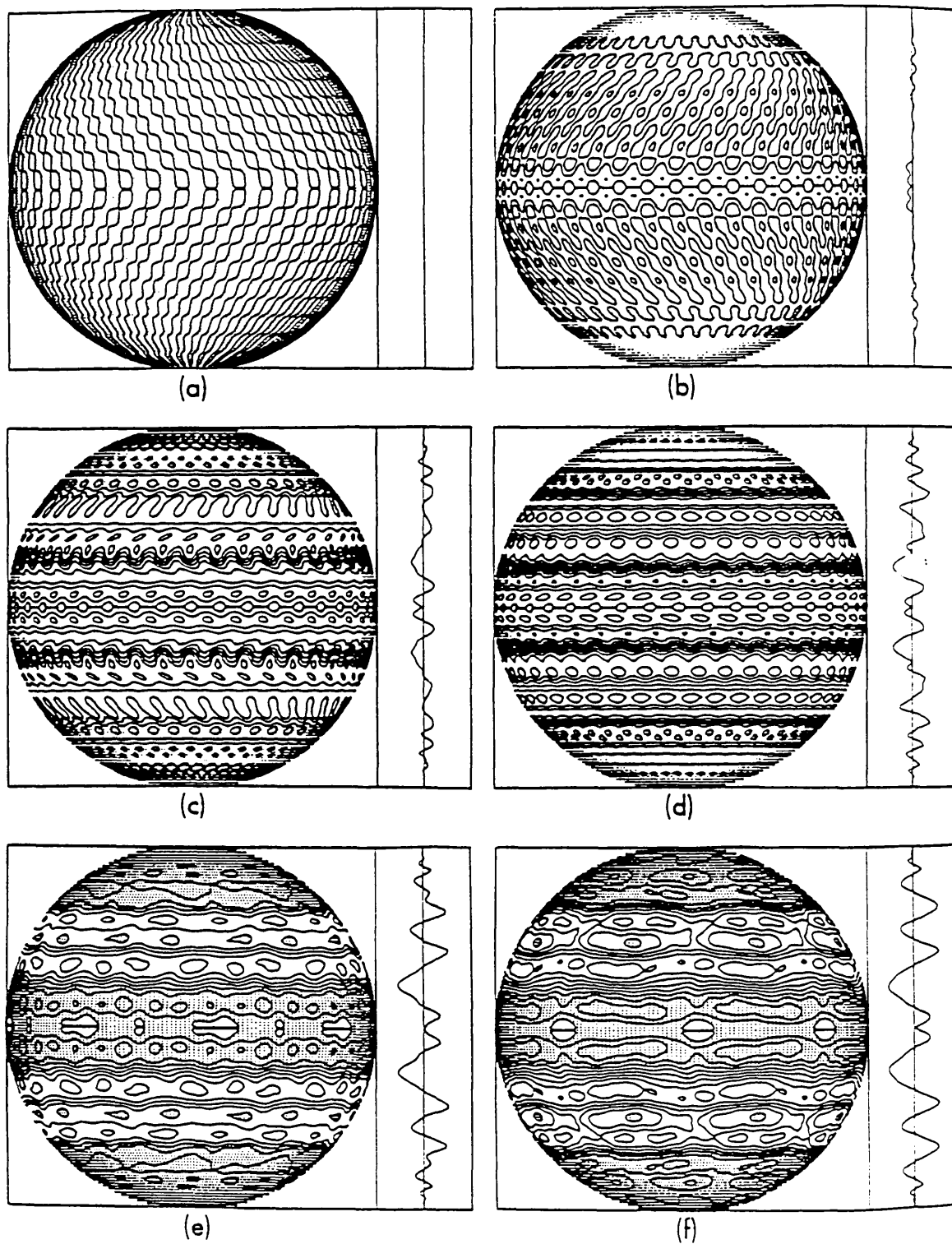


fig. 4.2

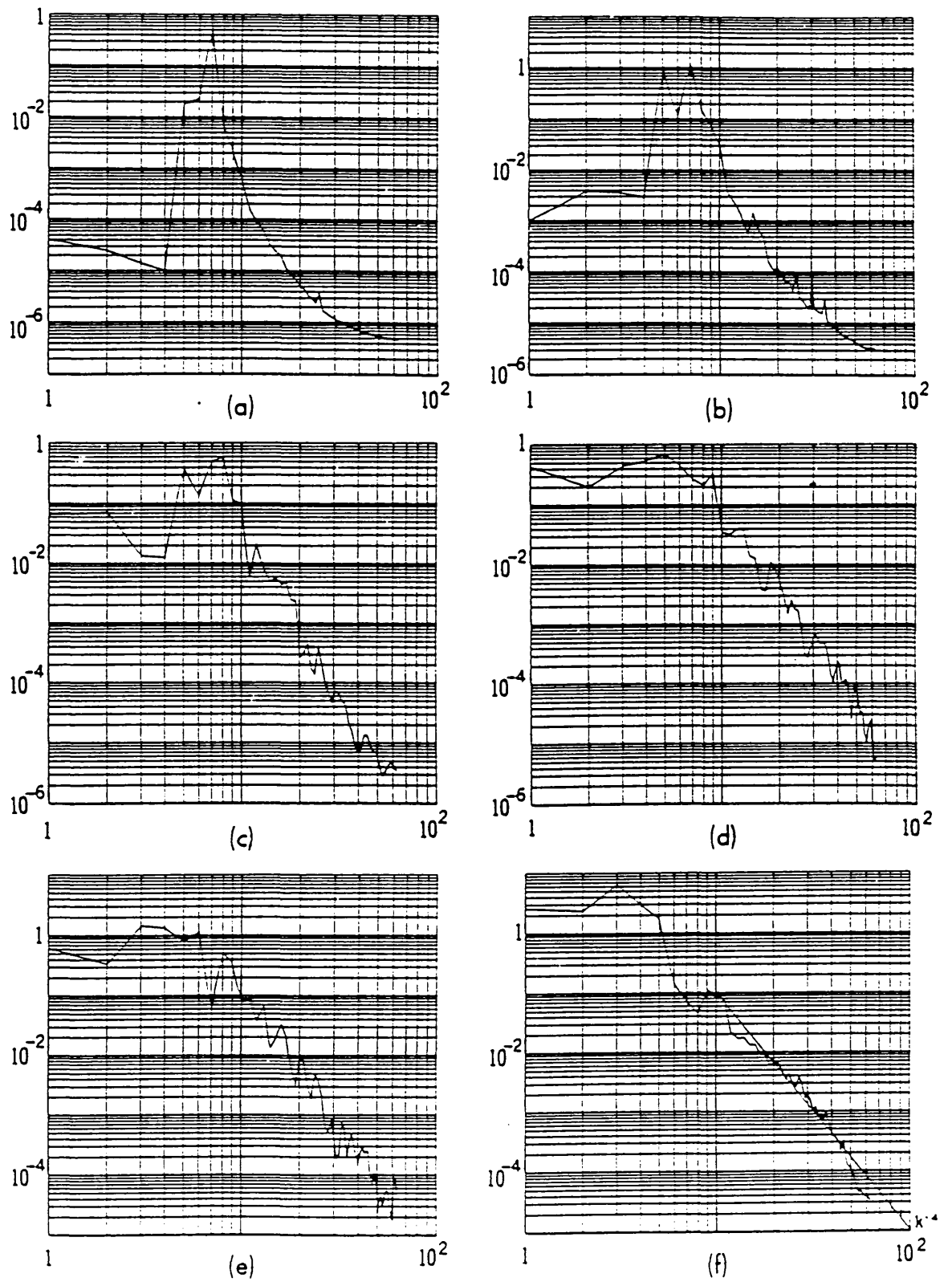


fig. 4.3

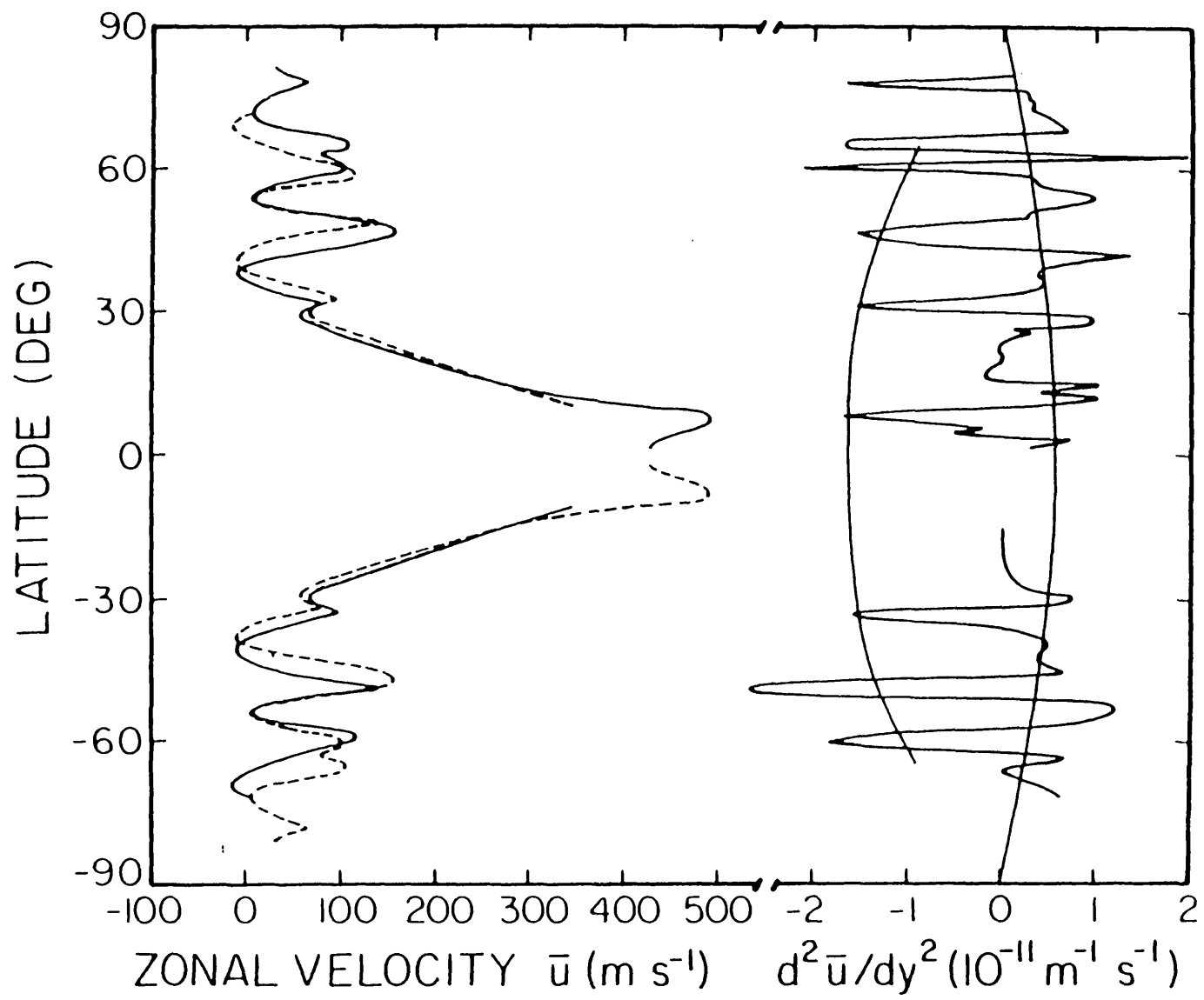


fig. 4.4

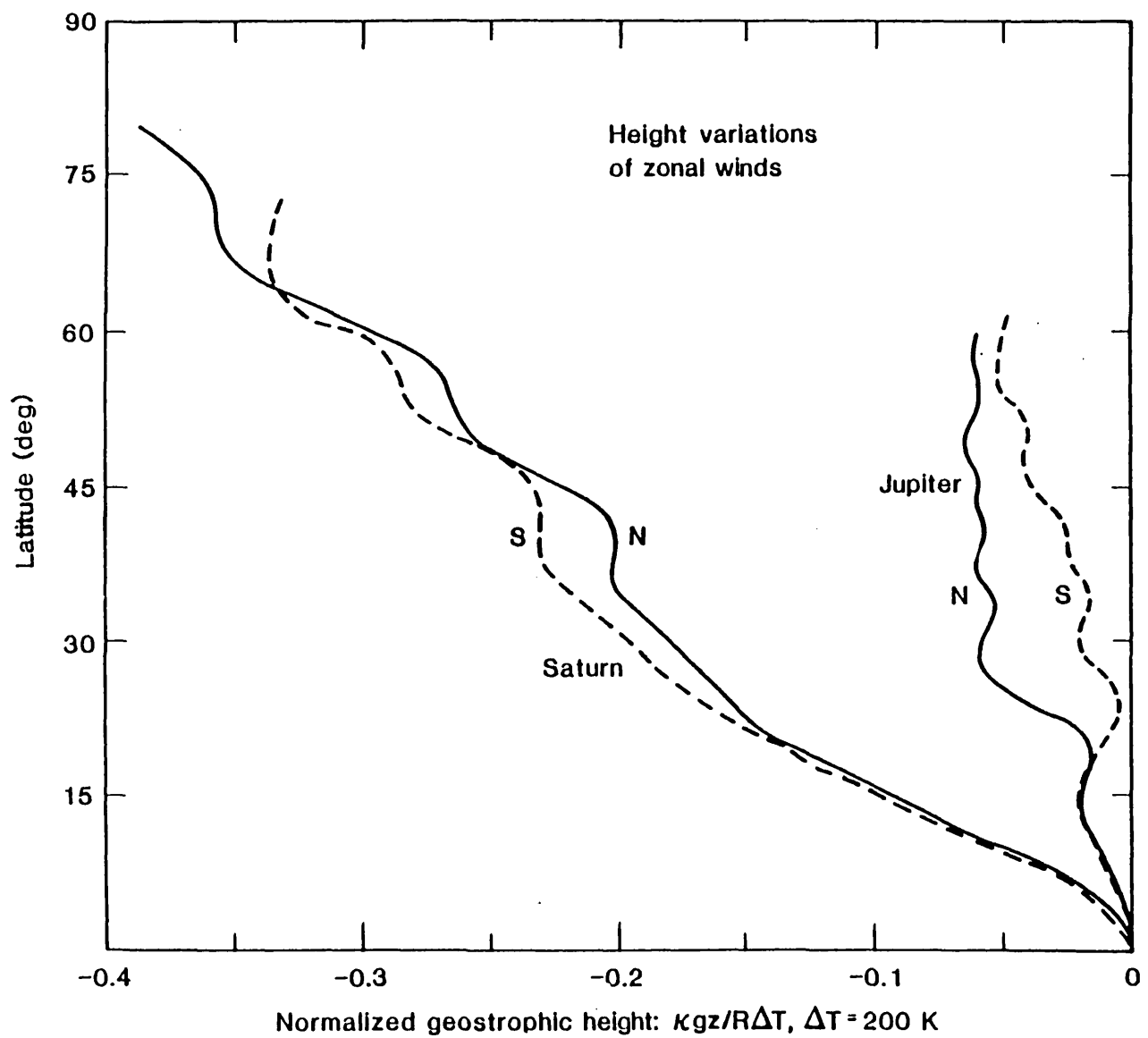


fig. 4.5

CHAPTER 5

TEMPERATURE AND COMPOSITION VARIATIONS

5.1 INTRODUCTION

Important parameters of any atmospheric model are likely to include estimates of the temperature and composition of the atmosphere.

This chapter explains the measurements available, and the way in which they constrain the various model atmospheres which have been produced.

The major constituents of Saturn have been inferred over the years until a reasonably detailed picture of the interior has been produced. The structure of some current models, and the constraints upon them are explained in the first part of this chapter. In addition to the major components there are a number of minor constituents, with those responsible for producing the various cloud layers being particularly important, and these are described next. Following this there is a discussion of the non-equilibrium components which are included to produce the observed spectra, particularly in the ultraviolet and infrared.

In addition to deriving the globally integrated equivalent temperature, the infrared (and radio science) experiments can also provide information about the way in which the temperature changes as a function of height. The available temperature-pressure profiles, as well as some of the quantities derived from them are described in

sections 5.4 - 5.6. After this the results from the UVS instrument, which determines the temperature at a much higher level, are described.

There are also available measurements of the latitudinal temperature gradient. These, along with the methods used to calculate the thermal wind from them, are described in the final sections.

5.1.1 INTERIORS

Some of the earlier models of the Jovian and Saturnian interiors are summarised by Hubbard and Smoluchowski (1973) and by Slattery (1977). These nearly all contain hydrogen as their main constituent, with differing amounts of other components. One of the major difficulties with these models is the uncertainty in the behaviour of hydrogen at high pressures. It is expected that, as the pressure increases, hydrogen will enter a state in which it behaves as an electrically conducting fluid. However the pressure at which this phase change takes place, and the precise nature of the transition are not well known. In addition to hydrogen as the major constituent, some of the later models mentioned by Hubbard and Smoluchowski include helium, and use a higher density central core in order to match the observed gravitational moments.

The Saturn model described by Slattery (1977) involved the use of several different sources of data to estimate the H-He equation of state, and reduced the gap over which this was interpolated to an order of magnitude in the density. Using this, along with the observed gravitational moments, cloud-top temperature, and estimated rotation period, he describes two different models. These have a central core of heavy elements with a density of 10 or 15 g cm⁻³

extending out to a pressure of $\sim 9 - 10.5$ Mbar (corresponding to $11.25 - 13.4 \times 10^3$ km), this is surrounded by a metallic hydrogen region, extending out to about 3.2 Mbar (28×10^3 km from the centre), above which is a molecular H-He mixture extending up to the visible surface.

A slightly different model is suggested by Hubbard 1981, in which the central core is differentiated into an inner rock part, and an outer ice part in approximately solar proportions. This puts the core / H-He boundary at ~ 7 Mbar, with the metallic hydrogen region terminating nearer 4 Mbar.

An alternative method for estimating the size of the conducting core is provided by Hide (1981). By considering the number of magnetic flux lines N passing through a general surface S , he shows that, assuming that ohmic losses are small, N will be constant when S is a material surface rotating with the conducting core (ie at the radio rotation period). Thus the radius of the conducting core can be found by extrapolating the observed field components down until N is approximately constant. The problem with this method lies in the high spatial and temporal resolution of the magnetic field measurements required. However the magnetometer measurements obtained from the Pioneer 10 and 11 encounters with Jupiter enabled this technique to be used. This estimated the core radius to be 70% of the observed radius, in good agreement with other calculations.

Difficulties have arisen with models of the Saturnian interior, since it has become apparent from satellite observations (Hanel et. al. 1981, Ingersoll et. al. 1980, see section 5.1.2) that the heat emitted by Saturn seems to be in excess of that which would be expected purely by cooling from an initial high temperature state. A possible solution to this problem is considered by Stevenson (1980), who suggests that the deficiency might be supplied by gravitational separation of the helium from the hydrogen.

A mechanism for this is described by Stevenson and Salpeter (1977), which takes the form of the helium becoming insoluble in the surrounding hydrogen, and forming droplets at ~ 2 Mbar. These droplets fall, releasing energy, into the metallic hydrogen layer where they dissolve in the warmer fluid. The effect of this process on the atmosphere is to enrich the lower part of the metallic hydrogen layer in helium, while depleting the surface layers, the two being separated by the upper part of the metallic hydrogen region which is expected to be inhomogeneous.

Support for this idea is provided by the Voyager 1 IRIS observations of Saturn (Hanel et. al. 1981), which determined the He mass fraction Y of the atmosphere as $Y = 0.11$. This is significantly different from the value of $Y = 0.19 \pm 0.05$ given for Jupiter, and Hanel et. al. estimate that this difference could be accounted for by gravitational depletion of helium in the surface layers.

5.1.2 GLOBAL ENERGY BALANCE OF SATURN

Measurements of the albedo, and effective temperature of Saturn are useful because they can be combined to estimate the energy balance (radiated power / received power), which in turn is important for constructing models of the interior (see section 5.1.1).

Ground-based observations (see summary by Hanel et. al. 1983) give quite large values for the energy balance, for example Gautier and Courtin (1979) derived a value of 2.6 ± 0.9 . These appeared to be confirmed by the Pioneer results (Orton and Ingersoll 1980) which gave 2.8 ± 0.9 . However from an analysis of the Voyager 1 IRIS measurements, and including the effects of light scattered and absorbed by the rings, Hanel et. al. (1983) derived an energy balance value of 1.78 ± 0.09 .

5.2 ATMOSPHERIC COMPOSITION - EQUILIBRIUM MODELS

Since most observations, and in particular zonal velocity measurements, refer to the cloud tops, it is obviously important to know the height and composition of these clouds. Useful estimates of these are provided by the model of Weidenschilling and Lewis (1973) who assumed a solar composition atmosphere, including H_2O , CH_4 , NH_3 and H_2S . Starting from the bottom level their calculation proceeded in steps, moving up along a wet adiabat until the vapour pressure of one (or more) of the constituents exceeded its saturated vapour pressure. When this occurred the relevant constituent was assumed to condense out until the vapour pressure equaled the saturated vapour pressure, when the upwards calculation was resumed. Once a particular component had condensed it was assumed to remain at the same level. The results they obtained by applying this method to Saturn are shown in fig. 5.1. This shows an upper level solid NH_3 cloud above an ammonium hydrosulphide (NH_4SH) cloud, which in turn lies over a cloud composed of water ice at the top with aqueous NH solution lower down. One of the disadvantages of this type of model is the assumption that there are sufficient condensation nuclei present to avoid supersaturation, which will place the clouds at the lowest possible level.

A more detailed model of cloud growth and decay processes on Jupiter is provided by Rossow (1978), who considers the significance of various microphysical effects acting upon particles of different sizes. From this he concludes that it is probable that all three cloud layers could produce precipitation with a mean particle radius $> 10 \mu\text{m}$ ($> 100 \mu\text{m}$ for the ammonia-water cloud) in a time period of $\sim 10^4 \text{ s}$. The main processes responsible for the growth and removal of these medium sized particles ($10 - 100 \mu\text{m}$) are gravitationally induced collisions and precipitation. The smaller values of g for the Saturnian case will probably not make much difference, with the

medium sized particles just growing slightly slower but attaining marginally larger radii before removal.

The presence of ammonia gas in the Saturnian atmosphere has been confirmed by the Voyager 1 IRIS measurements (Hanel et. al. 1981).

However, owing partly to the absence of the expected strong absorption at $3\text{ }\mu\text{m}$, the detection of ammonia ice is less certain. Its presence has been inferred by Slobodkin et. al. (1978) who compared reflection spectra of ammonia frosts formed at different rates with those of gaseous ammonia. It was found that the difference in position of the absorption lines (shifting from ~ 3 to $\sim 3.3\text{ }\mu\text{m}$) could account for the difference between Jovian and Saturnian spectra provided the ammonia was gaseous in the former and solid in the latter.

A different approach was adopted by Orton (1983), who modeled the effect of ammonia ice particles (assumed to be spherical with radii based on the calculations of Rossow, 1978 for the Jovian clouds: this partially suppresses the strong ammonia absorption features) in an atmosphere with the Voyager 2 radio science temperature profile.

He then altered the particle scale height H_p and the top of the ammonia cloud deck, and compared the results with Pioneer infrared measurements made at 20 and $45\text{ }\mu\text{m}$ over the equator and at 15° S . The best equatorial results for the $45\text{ }\mu\text{m}$ measurements place the cloud top at 100 mb and with a scale height equal to the local scale height, while at 15° S the cloud must be lower (cloud top at 400 mb) or fall off more rapidly with height (scale height equal to 0.15 the local scale height). Unfortunately none of the cloud models considered fit the $20\text{ }\mu\text{m}$ observations which are consistently too bright (more so at 15° S than at the equator). It is suggested that this could be due to a stratospheric aerosol.

A different method was used by Apt and Singer (1982) who used Earth based observations of the equator and the south equatorial belt (SEB approximately $15-20^{\circ}$ S) at $0.6 - 2.5 \mu\text{m}$. By altering their model to fit the observations they obtained methane abundances of 37 and 30 m am over the SEB and equator respectively. For their methane mixing ratio of 7×10^{-4} this gave a pressure level of 425 mb for the tops of the equatorial clouds, and 530 mb for those over the SEB. Using different data and a simpler model West (1983) was able to deduce methane abundances over the equator and north temperate region (NTR, approx $27 - 34^{\circ}$ N) of 37 ± 3 and 39 m am respectively. Taking the methane mixing ratio to be 2×10^{-3} (derived by Courtin et. al. 1982 from the Voyager 1 IRIS spectrum) they convert these to a cloud-top pressure level of 170 - 190 mb.

5.3 ATMOSPHERIC COMPOSITION, NON-EQUILIBRIUM COMPONENTS

In addition to the atmospheric constituents mentioned so far, it is necessary, due to the failure of simple models with a layer of clear gas over an ammonia cloud deck, to include other components. The discrepancies include the excessive darkening observed during an eclipse of Titan by Saturn (D.A. Smith et. al. 1981a), the unusually low albedo of Jupiter in the UV (Axel 1972), and the problem of explaining the cloud colouration, since NH_3 , NH_4SH and H_2O are colourless (West, private communication).

For a simple model the albedo of Saturn in the UV would be expected to be high due to Rayleigh scattering by the hydrogen molecules. This should increase as λ^{-4} (where λ is the wavelength). However instead the observed albedo decreases (see, for example, Macy 1977). A similar problem exists for the Jovian case, and a possible solution to this was given by Axel (1972) who proposed the existence of a layer of small absorbing particles well above the ammonia clouds. This is consistent with the results

of D.A. Smith et. al. (1981a) for Saturn, who required an absorbing stratospheric aerosol (optical depth 0.4 ± 0.02 , at $2640 \text{ \AA}$) to extend up to at least 20 km and more probably 35 - 75 km above the 74 mb tropopause. An upper limit to the aerosol distribution is provided by the UV limb brightening which requires the presence of a layer of clear, Rayleigh scattering gas above the absorber.

Various types of non-equilibrium processes capable of producing aerosols at the required level in the Jovian atmosphere are described by Prinn and Owen (1976), including the effects of vertical advection and sunlight induced photochemical reactions. The relevance of the various materials produced to the UV scattering and cloud colour observations is discussed by Macy (1977). The suggested components include red phosphorus (derived from phosphine), hydrazine, methane and sulphur compounds (derived from Hydrogen Sulphide) as well as various organic products. Both methane and hydrazine compounds are considered by Macy (1977) to be unlikely. The possible range of hydrocarbons is large and has been investigated by Scattergood and Owen (1977) who found that the UV absorbing compounds could be produced from proton-irradiation of a CH_4 and H_2 mixture. However all these products are colourless unless nitrogen or sulphur are included. The main difficulty associated with the identification of the aerosol with the photochemical dissociation products of phosphine (PH_3) or hydrogen sulphide (H_2S) lies in the fact that both these chemicals are expected to be stable only at lower levels. It is suggested by Macy (1977) that this could be overcome by rapid ($< 10^6\text{s}$) vertical transport. That some such disequilibrium process must be at work is demonstrated by the infrared identification of PH_3 (Hanel et. al. 1981) the distribution of which appears, from models of the UV spectra, (Moore 1983), to be similar to that of the aerosol.

A model of Saturn's atmosphere containing Axel dust is provided by Podolak and Danielson (1977), who used a uniform distribution of particles with sizes 0 - 0.1 μm . By changing other parameters they found that the observations were most accurately reproduced by using 7 km-am of H_2 over 28 km-am of H_2 mixed with dust with an underlying, optically thick, cloud layer. A similar model is used by Macy (1977), who uses different parameters for the equatorial and temperate regions. At the equator he uses 19 km-am of H_2 extending down to 0.2 atm over a layer containing 23 km-am of H_2 with absorbing particles going down to 0.4 atm, with the lower haze layer being 32 km-am thick with the bottom at 1.1 atm. The temperate model is similar, with the abundances being 27, 15 and 52 km-am with the associated pressure levels at the bottom of each layer being 0.3, 0.4 and 1.1 atm.

Although the use of one type of aerosol to account for both the colouration and UV observations has the advantages of simplicity it may be incorrect. Bergstralh et. al. (1981) suggest the presence of a second aerosol with a maximum single scattering albedo at 4800 A tailing off to a minimum near 6000 A. From the PPS data West et. al. (1983) deduce the presence of, in addition to the stratospheric component, two additional aerosol layers one at high altitude (< 10 mb) over the poles, and another mixed in with the tropospheric cloud.

5.4 TEMPERATURE-PRESSURE PROFILES AVAILABLE

There are several temperature-pressure profiles available, derived from different instruments on different spacecraft, they are summarised in the table below.

INSTRUMENT	SPACECRAFT	LATITUDE	REFERENCE	PRESSURE RANGE (mb)
RS	VGR2	31 S	Tyler et. al. 1982	1-1000
RS	VGR2	36.5 N	Tyler et. al. 1982	1-1000
IRIS	VGR1	13 N	Hanel et. al. 1981	1-1000
IRIS	VGR1	22 N	Hanel et. al. 1981	1-1000
IRIS	VGR2	78 N	Hanel et. al. 1982	50-500
IRIS	VGR2	70 S	Hanel et. al. 1982	50-500
RS	VGR1	76 S	Tyler et. al. 1981	1-1000

IRIS - Infra-red imaging spectrometer

RS - Radio Science

There are however reasons for believing that some of these profiles are more accurate than others. Because of the differing geometry the radio occultation results from the Voyager 1 encounter are thought by Tyler et. al. (1982) to be less reliable than those from the Voyager 2 encounter. For this reason the Voyager 1 profiles were not used. The pressure range of the Voyager 2 IRIS profiles is restricted because of a misalignment in the instrument which caused a reduction in the signal to noise ratio at certain wavenumbers (see section 2.3.2). In figures 5.2 and 5.3 I show the six temperature pressure profiles considered. These are the result of fitting polynomials through points taken from the references given above. It can be seen from these figures that the temperatures derived from the IRIS measurements are distinctly

different from the radio occultation results, particularly around the tropopause. This could well be due to the neglect of cloud opacity effects (see section 2.3.2.3) in the IRIS temperature inversion, a possibility which was considered by Orton (1983) in his investigation of the structure of the ammonia clouds.

5.5 CALCULATION OF THE HEIGHT FROM A TEMPERATURE-PRESSURE PROFILE

We assume that the atmosphere is in balance, and use the hydrostatic equation

$$\frac{dp}{dz} = -\rho g \quad (5.1)$$

in conjunction with the ideal gas equation

$$\rho = \frac{p}{RT} \quad (5.2)$$

to obtain

$$\frac{dp}{dz} = -\frac{p g}{RT} \quad \text{ie} \quad dz = \frac{-R}{g} \times \frac{T}{p} dp \quad (5.3)$$

If we change the variable to $q = \ln(p)$, and integrate from a location with pressure p_0 and height z_0 to the point p_1, z_1 then we obtain

$$z_1 = z_0 - \frac{R}{g} \int_{\ln(p_0)}^{\ln(p_1)} T(q) dq \quad (5.4)$$

where we have assumed a constant composition, and acceleration due to gravity.

The method used is very simple. A polynomial of a specified order is fitted to the temperature-pressure points provided, using the NAG (National Algorithms Group) subroutines, and then integrated. The values at $\ln(p_1)$ are then subtracted from the values at $\ln(p_0)$ and multiplied by R/g to obtain the height. The change of variable to $\ln(p)$ is performed because this form is more amenable to the form of polynomial used for the curve fitting.

The constant R is the specific gas constant, and is given by the universal gas constant R^*

$$R^* = 8.3144 \text{ kJ mol}^{-1} \text{ K}^{-1}$$

divided by M , the harmonic mean of the molecular weights of the gases present. This is defined for a mixture of 2 gases (McIntosh and Thom 1978) as

$$M = \frac{m_1 + m_2}{\frac{m_1}{M_1} + \frac{m_2}{M_2}} = \frac{m_1/m_2 + 1}{\frac{m_1}{m_2} \times \frac{1}{M_1} + \frac{1}{M_2}} \quad (5.5)$$

where m_1 and m_2 are the masses of the two gases present, and M_1 and M_2 are their respective molecular weights. For the outer planets we can take H and He as being the major constituents, and for these we have

$$M_1 = M(\text{He}) = 4.003$$

$$M_2 = M(\text{H}_2) = 2.016$$

To determine the value of M we require the helium mass fraction $\gamma = m_1/m_2$. This is given by Hanel et. al. (1981) as

$$\gamma = 0.19 \pm 0.05$$

for Jupiter, and

$$\gamma = 0.11$$

for Saturn. Using these values along with M_1 and M_2 in equation 5.5 we obtain

$$M = 2.19 \text{ g mol}^{-1}$$

for Jupiter, and

$$M = 2.12 \text{ g mol}^{-1}$$

for the Saturnian case, the latter giving a value for the specific gas constant R , of

$$R = 3922 \text{ J K}^{-1} \text{ kg}^{-1}$$

The value of g to be used in equation 5.4 should be different for each of the profiles used, because of the different latitudes to which

they correspond. However for the purposes of comparison I have used an average value of 9.5 ms^{-2} .

5.6 PARAMETERS DERIVED FROM THE T-P PROFILES

By using the procedure described above the value of $Z(p)$ was evaluated for the six temperature-pressure profiles shown in figs 5.2 and 5.3, the results for four of these profiles are plotted in 5.4. The zero height level has been defined to be at 700 mb, the level below which an adiabat was assumed for the IRIS inversion (Hanel et. al. 1981). The stability of an atmosphere to vertical movements can be measured by the Brunt-Vaisala frequency (also called the buoyancy frequency) N , defined by

$$N^2 = \frac{g}{T} \left(\frac{dT}{dz} + \frac{g}{C_p} \right) \quad (5.6)$$

where C_p is the specific heat at constant pressure. The value of N is the frequency with which a parcel of gas would oscillate if displaced vertically from its equilibrium position. If N^2 is negative this indicates that the atmosphere is statically unstable, and that a parcel of gas will accelerate away from its initial position.

The value of N^2 as a function of pressure are shown in fig 5.5. It can be seen that the atmosphere is most stable at 20 - 40 mb, above the tropopause which is at 60 - 90 mb. Although, due to the difficulty in assigning an error to the T-P profiles, there are no errors shown in fig 5.5. Their effect can be estimated by comparing the different curves in figs 5.2 and 5.5. At higher pressure levels ($\sim 600\text{mb}$) the value of N^2 is determined by the small departures of $T(p)$ from an adiabat. Because of this the fact that $N^2 < 0$ for some of the profiles may be due to small errors in the temperature

profile, rather than a statically unstable atmosphere.

Another useful quantity is the density scale height h . This is defined as

$$h^{-1} = \frac{-1}{\rho} \frac{d\rho}{dz} = \frac{-d(\ln \rho)}{dz} \quad (5.7)$$

Using the ideal gas equation $\rho = p/RT$ this becomes

$$h^{-1} = -\frac{d}{dz} \ln \left(\frac{p}{RT} \right) \quad (5.8)$$

since, having $T(p)$, we have produced $Z(p)$ it is not difficult to evaluate h^{-1} . The value of h as a function of height is shown in fig 5.6.

Another quantity which is relevant when considering the possible flow regimes, and one of the few characteristic parameters which can be evaluated with any degree of reliability (at least for the 1-1000 mb region) is the Richardson number, Ri . This can be defined as

$$Ri = \frac{g}{\theta} \frac{\partial \theta / \partial z}{(\partial u / \partial z)^2} = \frac{N^2}{(\partial u / \partial z)^2} \quad (5.9)$$

where θ is the potential temperature.

The following table (taken from Stone 1966) describes the types of flow likely to be important for different values of Ri . Some of these regimes are described in more detail in chapter 4.

$Ri < -2$	Free convection
$-2 < Ri < 1/4$	Forced convection (Kelvin-Helmholtz instability)
$1/4 < Ri < 1$	Symmetric baroclinic instability
$1 < Ri < 1000$	Conventional baroclinic instability
$1000 < Ri$	Hadley cell

It should be noted that some of these transitions are more accurately known than others.

The only unknown quantity involved in the estimation of Ri is the vertical shear $\partial u / \partial z$. Although this can be evaluated via the thermal wind equation from the latitudinal temperature gradient, this will be a function of height and latitude. Instead, since the values of Ri closer to zero are of more interest, I have chosen to use an unnaturally high value of

$$\frac{\partial u}{\partial z} = -0.5 \text{ m s}^{-1} \text{ km}^{-1} = -0.5 \times 10^{-3} \text{ s}^{-1}$$

Using this to calculate Ri I obtained fig 5.7, from which it can be seen that $1000 > Ri > 1$, and hence that baroclinic instability may be important, at this level.

5.7 UPPER ATMOSPHERE TEMPERATURE

The temperature in the upper atmosphere can be estimated from the variation with time of the spectra returned by the UVS during a stellar occultation. The analysis of the occultation of delta Scorpii by Saturn, observed by Voyager 2 is described by Festou and Atreya (1982) whose results are shown in fig 5.8. Above ~ 1540 km (relative to the 1 bar level) they found that their data could be modelled reasonably well by assuming an isothermal atmosphere at a temperature of $800 \pm 120 \text{ K}$. Below 1540 km the isothermal assumption breaks down and is replaced by a model containing a linear increase in temperature with height. This gives a good fit to the 939 - 1023 Å observations providing a lapse rate of $1.25 \pm 0.05 \text{ K km}^{-1}$ is used. The lowest level probed by this technique, which relies on modelling the molecular hydrogen absorption, is ~ 1000 km where hydrocarbon absorption may become important. The temperature between this region and the highest level reached by the IRTS and radio occultation measurements is unavailable. However because the

temperature at each end of this gap is approximately the same the intervening region was modelled by Festou and Atreya (1983) as being isothermal.

5.8 EVALUATION OF u , $\partial u/\partial z$ AND N^2 AS FUNCTIONS OF LATITUDE AND HEIGHT

This section describes the calculation of various parameters as a function of height. The procedure is based upon the thermal wind equation which, for an ideal gas in hydrostatic equilibrium can be written (Gill, 1982).

$$\frac{\partial u}{\partial z} = \frac{-g}{fT} \left(\frac{\partial T}{\partial \theta} \frac{\partial \theta}{\partial y} \right) \quad (5.10)$$

The use of this to obtain $\partial u/\partial z$ as a function of height for a particular latitude requires: the variation of T with latitude as a function of pressure; estimates of f and g and $\partial \theta/\partial y$ at the appropriate latitude; and the height as a function of pressure.

The two Saturnian $T(p, \theta)$ profiles available for this type of analysis are those given by Pirraglia et. al. (1981), see fig 5.9 and Conrath and Pirraglia (1983), see fig 5.10. The first, derived from Voyager 1 IRIS observations, applies only to the northern hemisphere, covering latitudes $0 - 70^\circ$ N and pressure levels 150-730 mb. The second includes the Southern hemisphere, extending from 90° S to $\sim 80^\circ$ N (not continuously), but only at 150, 290, and 730 mb. Because of the better pressure range most of my calculations were performed using the Voyager 1 profile.

The temperature gradient data was obtained from figs 5.9 and 5.10 by drawing a grid over these, and estimating the temperature perturbation at each point. In addition to the errors generated by this process there is also a problem with the accuracy of the navigation, which was estimated by Pirraglia et. al. (1981) to give a 3σ error of $1 - 3^\circ$. For comparison between IRIS

measurements and velocity profiles Conrath and Pirraglia (1983) showed that, north of 30° , the thermal wind shear correlated with the velocity profile to within the pointing uncertainty.

It can be seen from fig 5.10 that the temperature structure at 150 mb is composed of a small scale part, approximately symmetric about the equator, superimposed upon a gradual increase in temperature towards the south pole. This large scale asymmetry is interpreted by Conrath and Pirraglia (1983) as being caused by seasonal effects. The large scale structure observed in the 730 mb profile may be due to absorption by intervening particles and hazes, rather than an actual temperature change.

In addition to the latitudinal temperature gradient, the temperature-pressure profile is also needed to determine the height. This is normally approximated by one of the profiles described in section 5.4 but using the value of g at the required latitude. The inconsistencies arising from this approximation can be estimated from figs 5.2 and 5.3, and are unlikely to be large compared with the errors in estimating $\partial T/\partial \theta$.

The other quantities required for the evaluation of $\partial u/\partial z$ (z) from equation 5.10, such as f and g are calculated as a function of latitude using the methods described in sections 3.4.1.1 and 5.8.1.

Assuming that the temperature data is available the thermal wind shear can be found using a combination of specially written routines and standard database programs. The procedure used is as follows. Given a $T(p)$ profile, and assuming $g=9.5 \text{ ms}^{-2}$ the program described in section 5.5 was run to produce a file containing temperature-pressure-height data. This is then differentiated to obtain $\partial T/\partial z$ and interpolated to give a pressure resolution identical to that of the temperature gradient data with which it is merged. After sorting this is then merged with the file produced by DIMEN (which produced the results of table 3.1) which has been interpolated to 0.5° intervals. Now that the value of g is known for each latitude the height and $\partial T/\partial z$

values are corrected by multiplying by $9.5/g$. The file is now sorted by pressure and subsorted by latitude and the value of $\partial u/\partial z$ computed from the thermal wind equation. After re-sorting on latitude and subsorting on pressure the value of $\partial^2 u/\partial z^2$ can also be obtained. The value of $\partial u/\partial z$ as a function of pressure for a range of latitudes, obtained using this method is shown in fig 5.11.

5.8.1 EVALUATION OF THE ACCELERATION DUE TO GRAVITY g

In order to produce, using the thermal wind equation, an estimate of the vertical shear from the latitudinal temperature gradient it is necessary to use an appropriate value for g . For the Earth the variation of g with latitude is usually ignored. However for Saturn which is larger and rotates more rapidly, g varies over approximately $9 - 11.8 \text{ ms}^{-2}$ which could produce significant errors in $\partial u/\partial z$. The formula which provides g as a function of latitude is normally expressed in terms of a series of spherical harmonics $P_n(\mu = \sin \theta)$, where P_n is the Legendre polynomial of order n and θ is the planetocentric latitude. If we let G be the gravitational constant, M the planetary mass, r the distance from the planets centre, a the equatorial radius and ω the rotation rate, and define

$$q = \frac{\omega^2 a^3}{G M} \quad (5.11)$$

then the radial and transverse components of g , including the effects of rotation, are given by (Anderson 1976)

$$g_r = \frac{-GM}{r^2} \left[1 - \sum_{i=0}^{\infty} (1+2i) J_{2i} \left(\frac{a}{r}\right)^{2i} P_{2i}(\mu) - \frac{2}{3} q \left(\frac{a}{r}\right)^3 (1-P_2(\mu)) \right] \quad (5.12)$$

and

$$g_\theta = \frac{-GM}{r^2} \cos(\theta) \left[\sum_{i=0}^{\infty} J_{2i} \left(\frac{a}{r}\right)^{2i} P'_{2i}(\mu) + \frac{1}{3} q \left(\frac{a}{r}\right)^3 P'_2(\mu) \right] \quad (5.13)$$

From these g can be evaluated using

$$g = (g_r^2 + g_\theta^2)^{\frac{1}{2}} \quad (5.14)$$

The coefficients J_i in equations 5.12 and 5.13 are called the gravitational harmonics, and must be determined for the object of interest. The values used for J_2 and J_4 are those determined by Null et. al. (1976) from an analysis of doppler tracking during the Pioneer Saturn encounter, combined with ground based observations of the motion of the Saturnian satellites. The values used were

$$J_2 = 16479 \times 10^{-6}$$

$$J_4 = -941 \times 10^{-6}$$

A value for J_6 is also provided, but this is probably significantly affected by the differential rotation, see Hubbard (1982). The mass M used is taken from Goody and Walker (1972), and is

$$M = 5690 \times 10^{23} \text{ kg}$$

The values of a and ω used were those given in chapter 3.

FIGURE CAPTIONS - CHAPTER 5

Figure 5.1

These figures show the nominal atmospheric models produced by Wiedenschilling and Lewis (1973) for the Jovian and Saturnian atmosphere. Figure A shows the model for Jupiter. The solid lines give the cloud density, whilst the dashed lines show how the amount of different compounds present in the overlying atmosphere changes with height. The results of similar calculations for Saturn are shown in B. The two parts of the lower cloud layer correspond to water ice, and aqueous NH_3 .

Figure 5.2

This figure shows some of the temperature-pressure profiles described in the text. They are:

—————	radio profile at 31 S
- - - - -	radio profile at 36 N
- - -	IRIS profile at 13 N
- + - + -	IRIS profile at 22 S

Figure 5.3

This figure shows some of the other temperature-pressure profiles mentioned in the text. They are:

—————	IRIS profile at 78 N
- - - - -	IRIS profile at 70 S

The reduced pressure range of these two profiles is caused by the restricted operation of the Voyager 2 IRIS instrument.

Figure 5.4 - 5.7

These figures show how various values derived from the temperature-pressure profiles shown in fig 5.2 vary with pressure.

5.4 - This shows the calculated height

5.5 - This shows the Brunt-Vaisaila

(bouyancy) frequency

5.6 - This shows the density scale height

5.7 - This shows the Richardson number

The four different curves shown in each of these figures correspond to the four temperature profiles of fig 5.2.

Figure 5.8

The two continuous curves above the 10^{-8} bar level in this figure, taken from Festou and Atreya (1982), show the temperature, and the number density of H_2 , retrieved from the UVS observations of the δ -Sco occultation. The values below 10^{-3} bars are derived from the infrared and radio measurements, the dashed line shows an interpolation between the two sets of data. Also shown is the UVS derived CH_4 number density, and the IRIS measurements of the He number density. The heights shown are relative to a zero level at 400 mb, approximately the location of the ammonia cloud tops.

Figure 5.9

These figures show the differences (in Kelvin) from the latitudinal mean temperature as a function of pressure, obtained by Pirraglia et. al. (1981). The shaded region along the top of each figure indicates the approximate position of the large scale (belt-zone) albedo variations. The Voyager 1 results for Saturn's northern hemisphere are shown in (a), whilst (b) and (c) show the Voyager 1 measurements for the northern and southern hemispheres of Jupiter.

Figure 5.10

This figure shows the latitudinal variation of the temperature at certain pressures levels, as derived by Conrath and Pirigalia (1983) from the Voyager 1 and 2 IRIS measurements. The dashed line shows the 150 mb results for the northern hemisphere reflected about the equator to emphasize the large scale assymetry.

Figure 5.11

Using the latitudinal temperature gradient derived from figure 5.9, in combination with a temperature-pressure profile, and estimating other quantities, I derived the thermal wind shear. This procedure is described in more detail in section 5.8. The figure shows how the derived values vary with latitude and pressure. The latitudes shown are planetodetic.

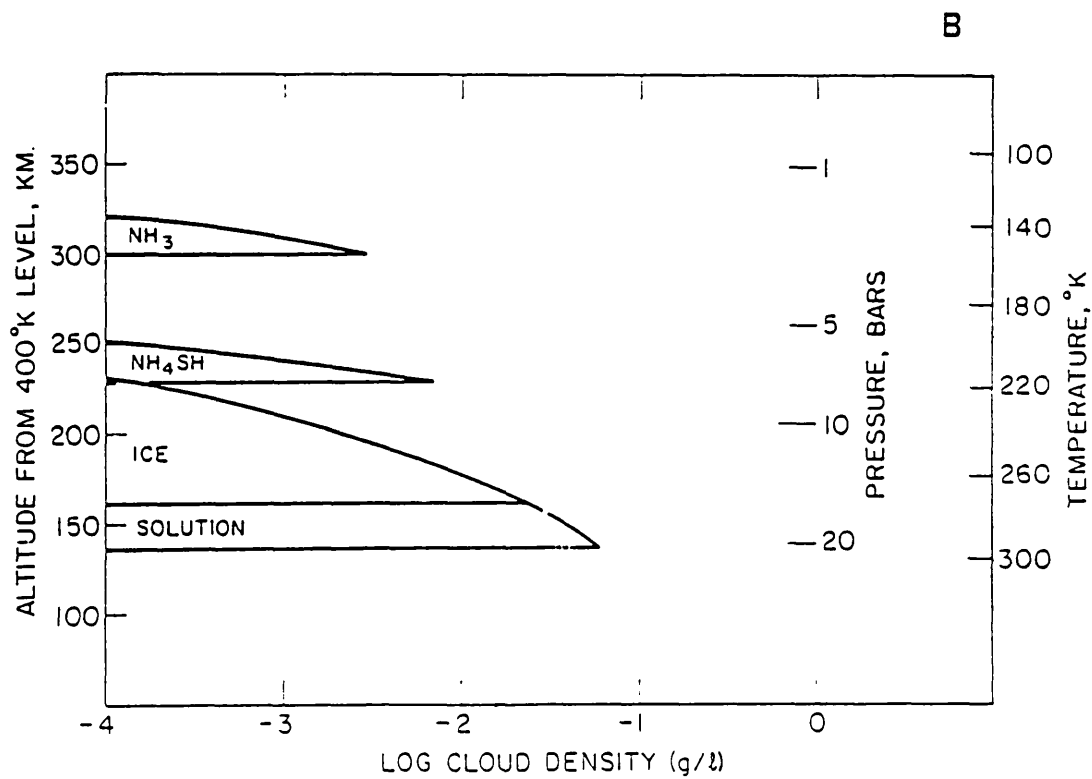
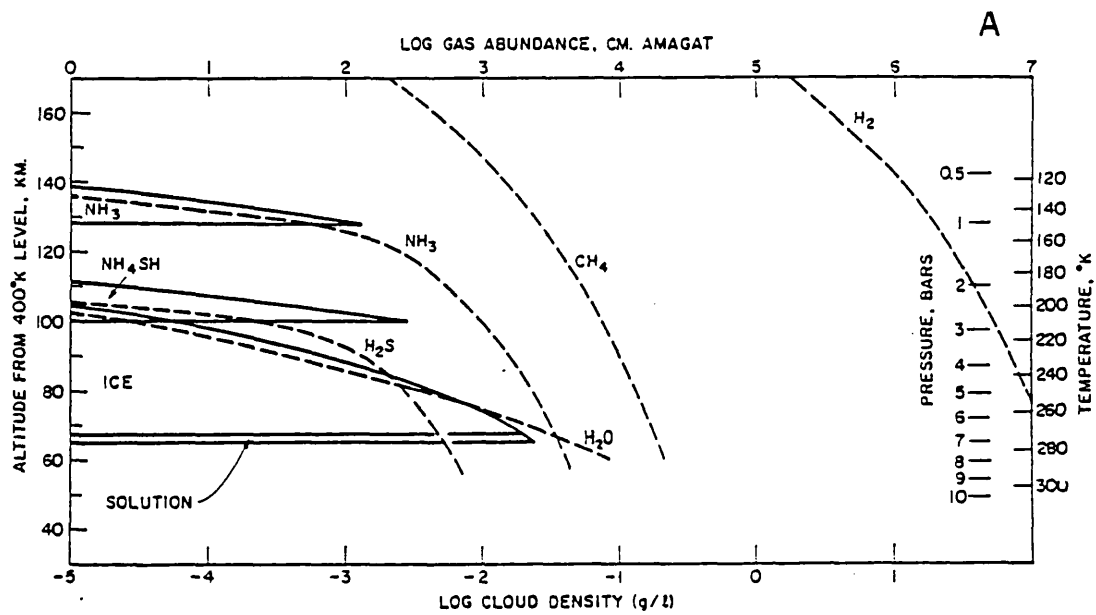


fig. 5.1

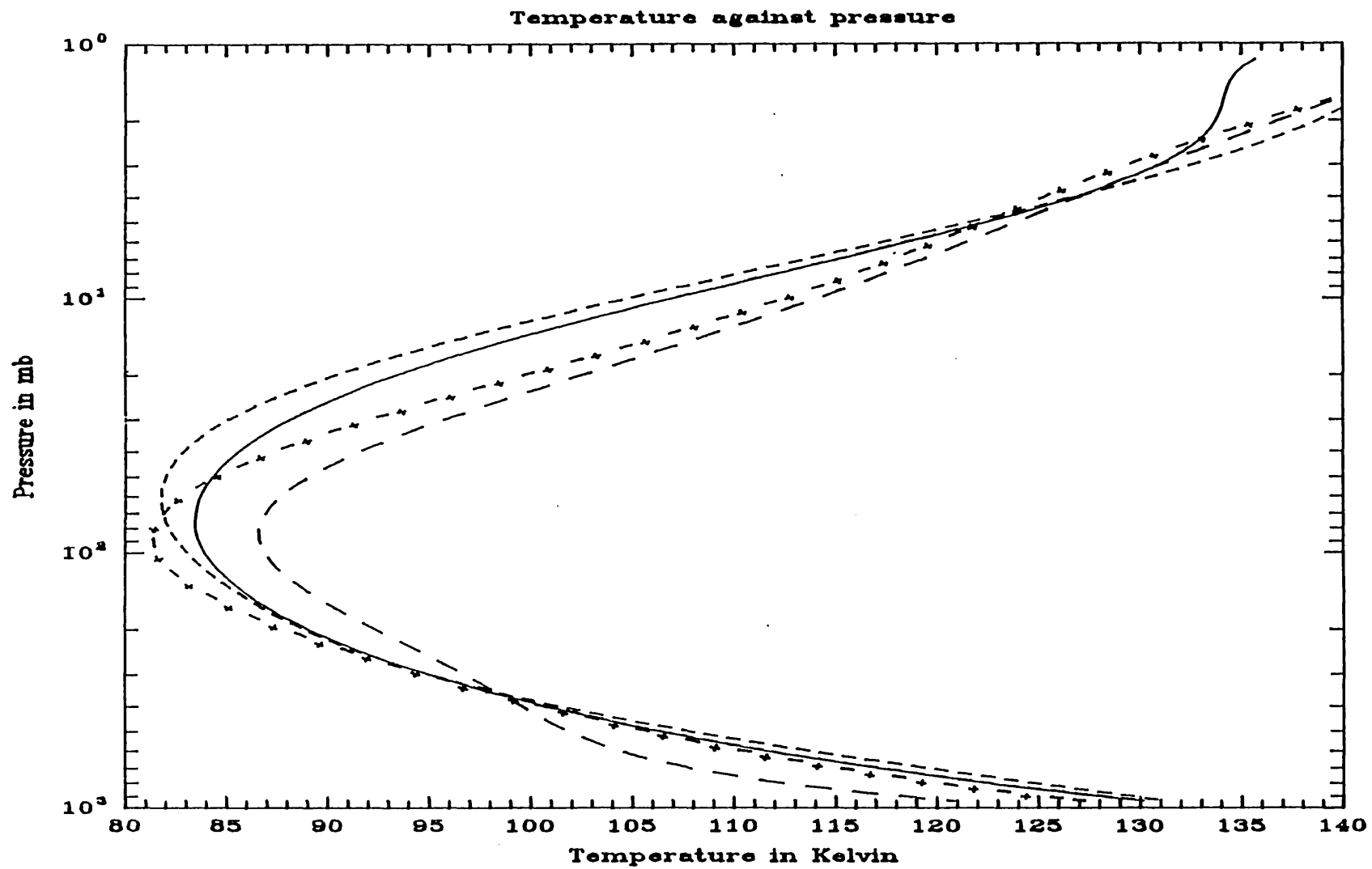
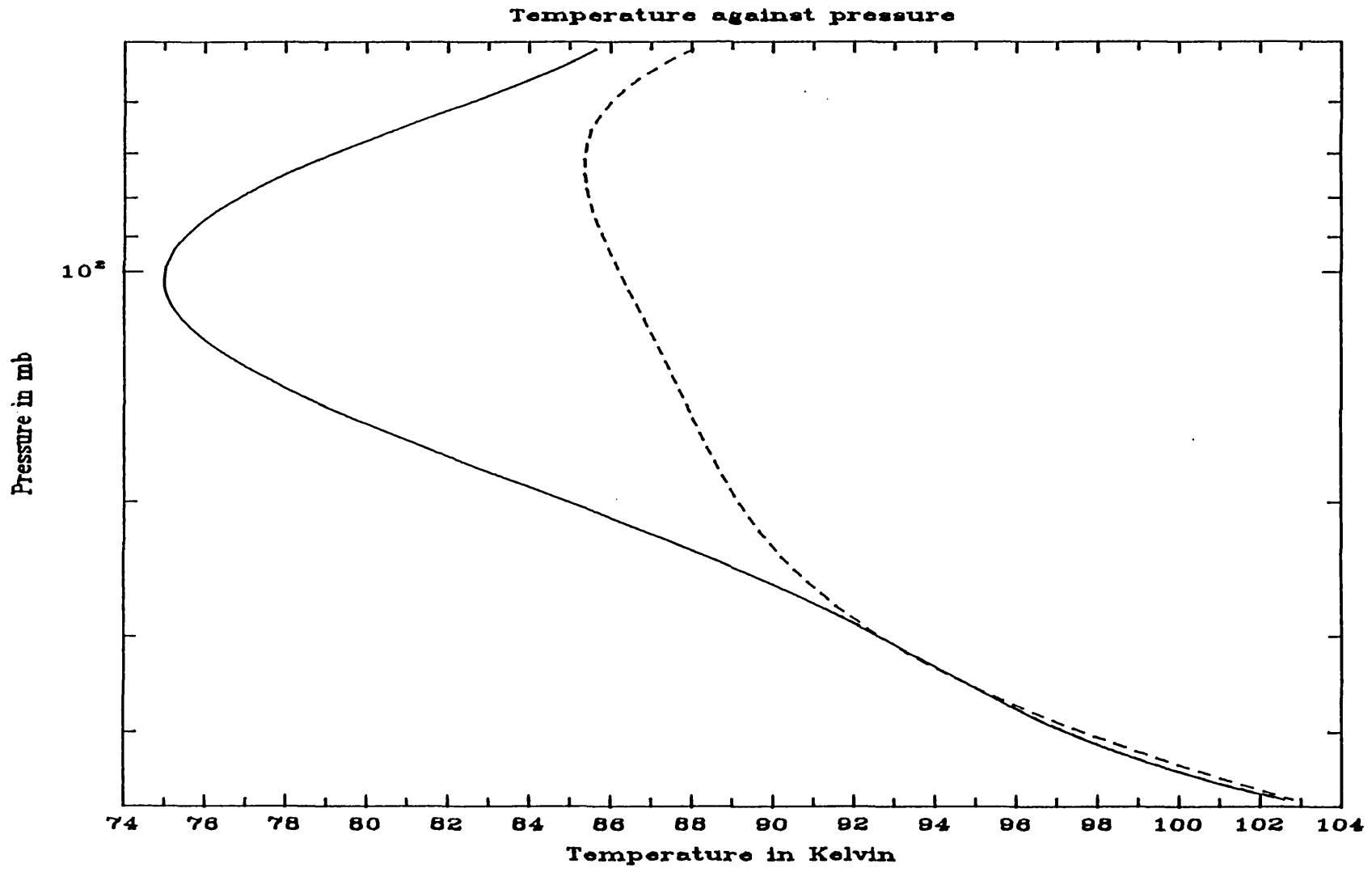


fig. 5.2

fig. 5.3



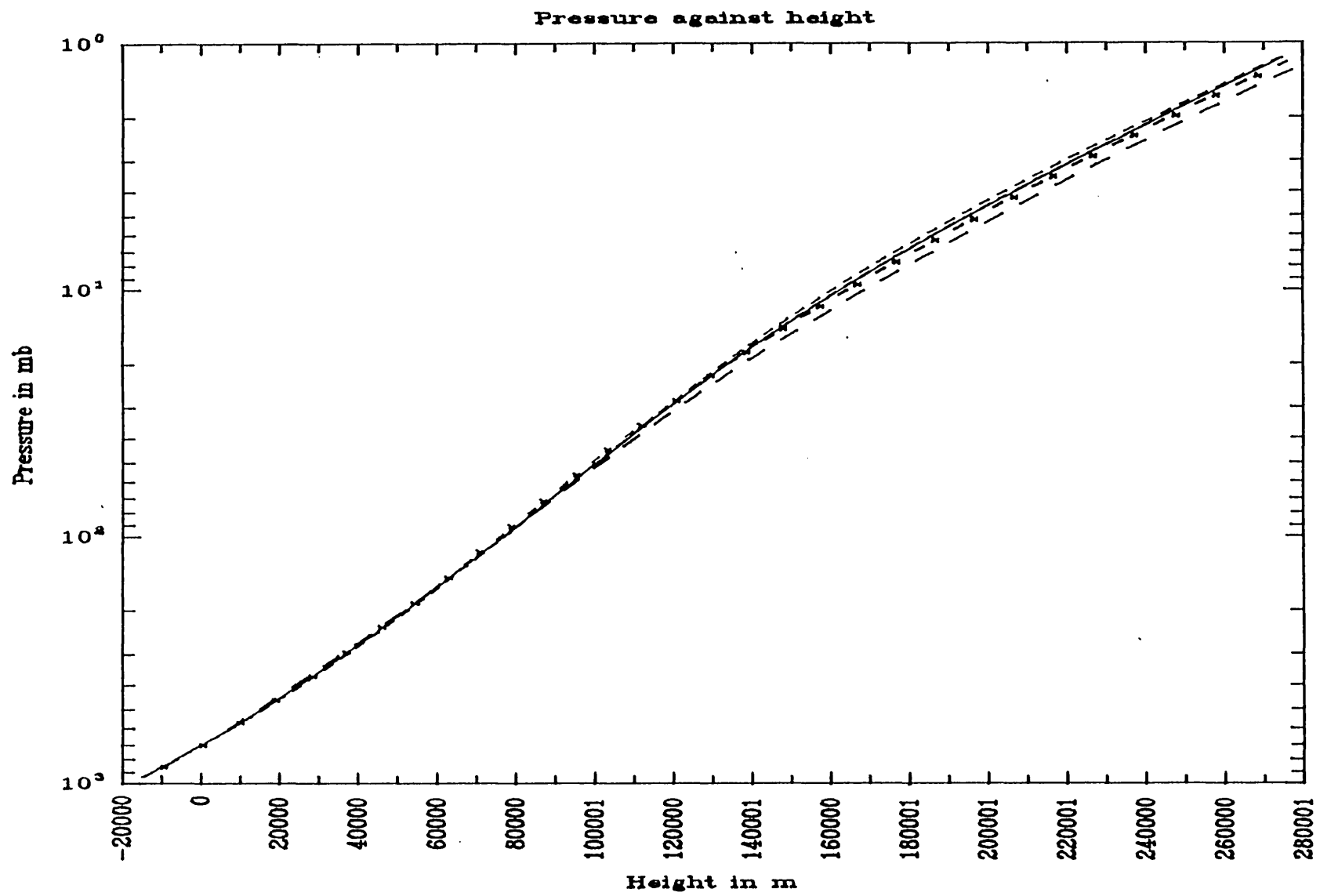
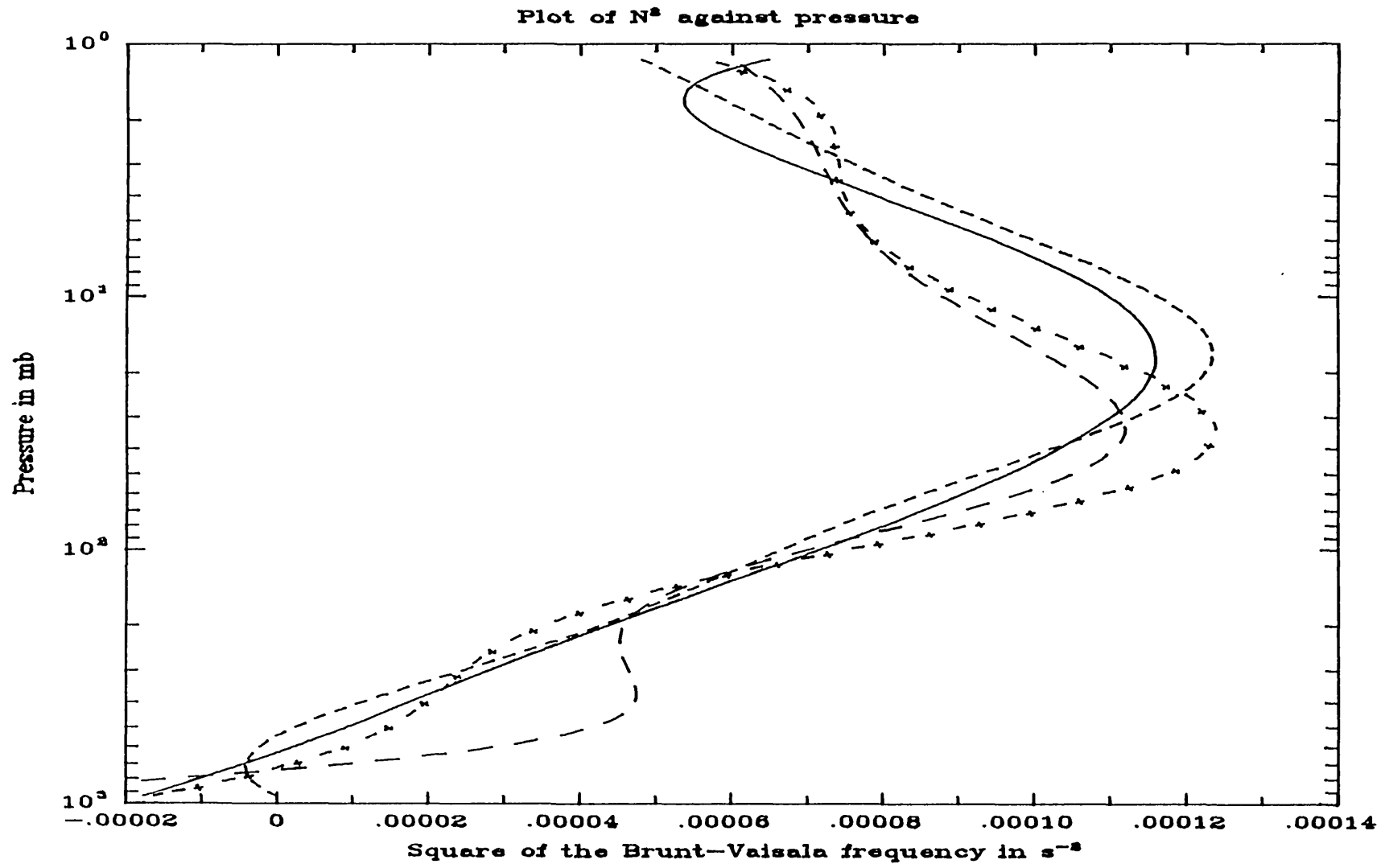


fig. 5.4

fig. 5.5



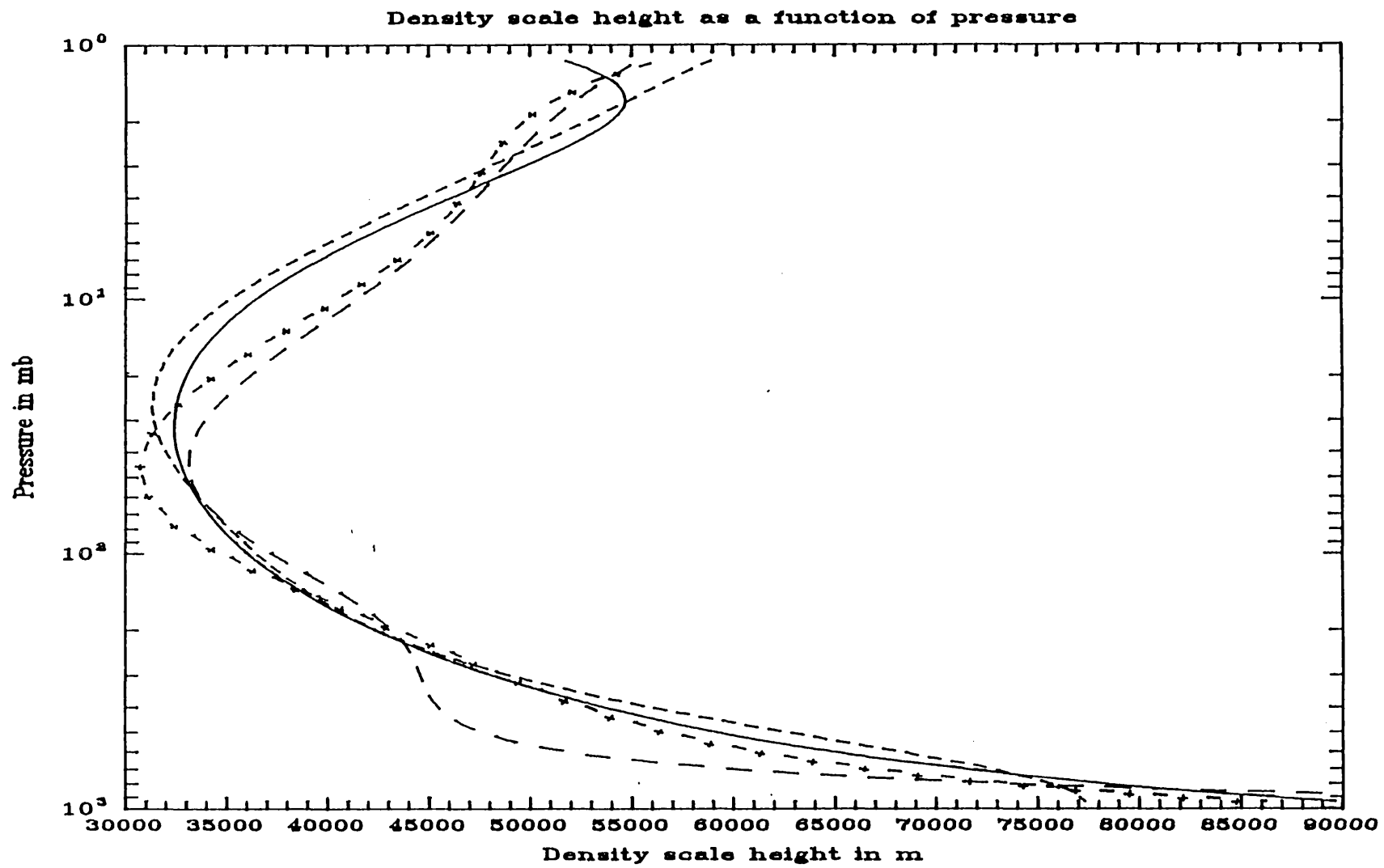


fig. 5.6

fig. 5.7

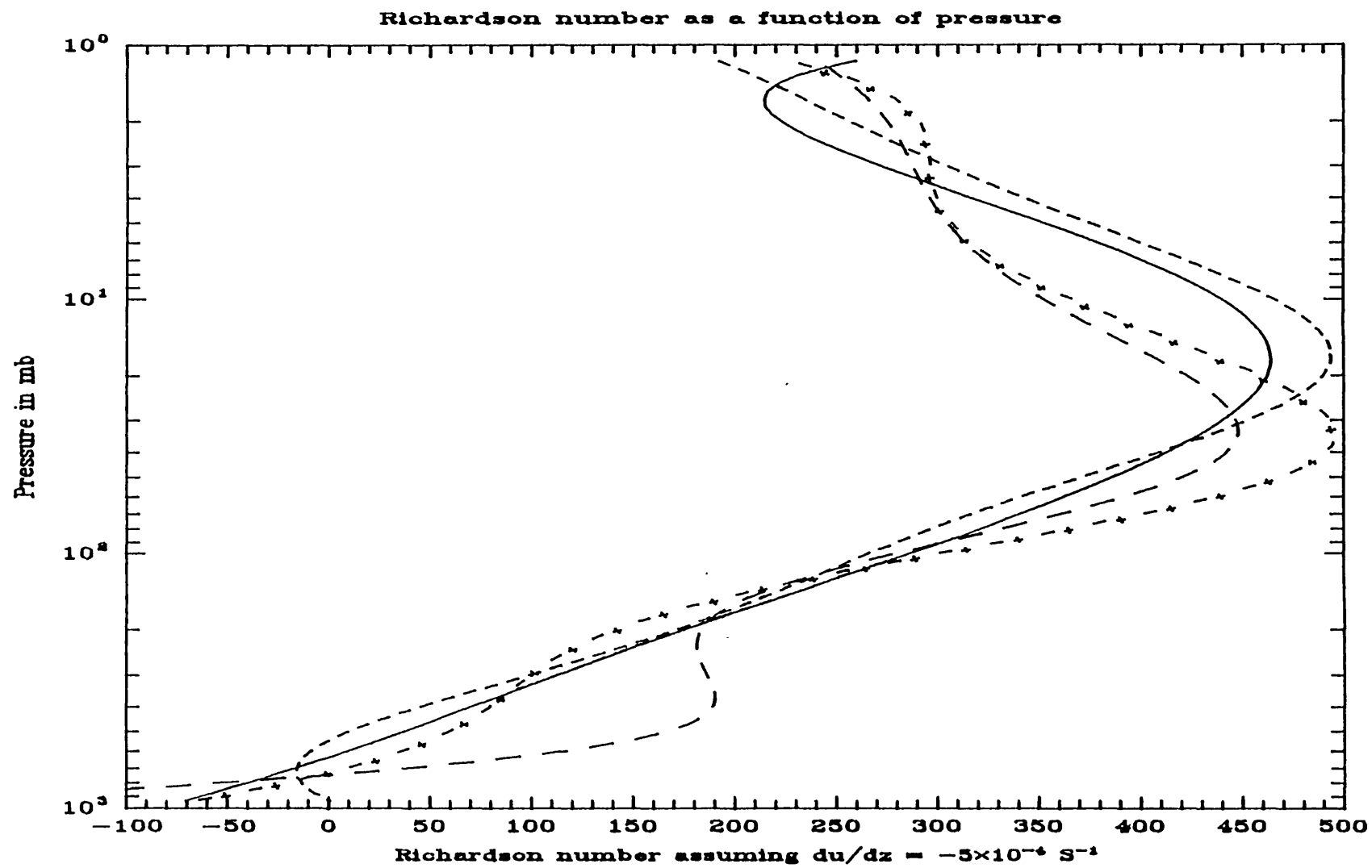
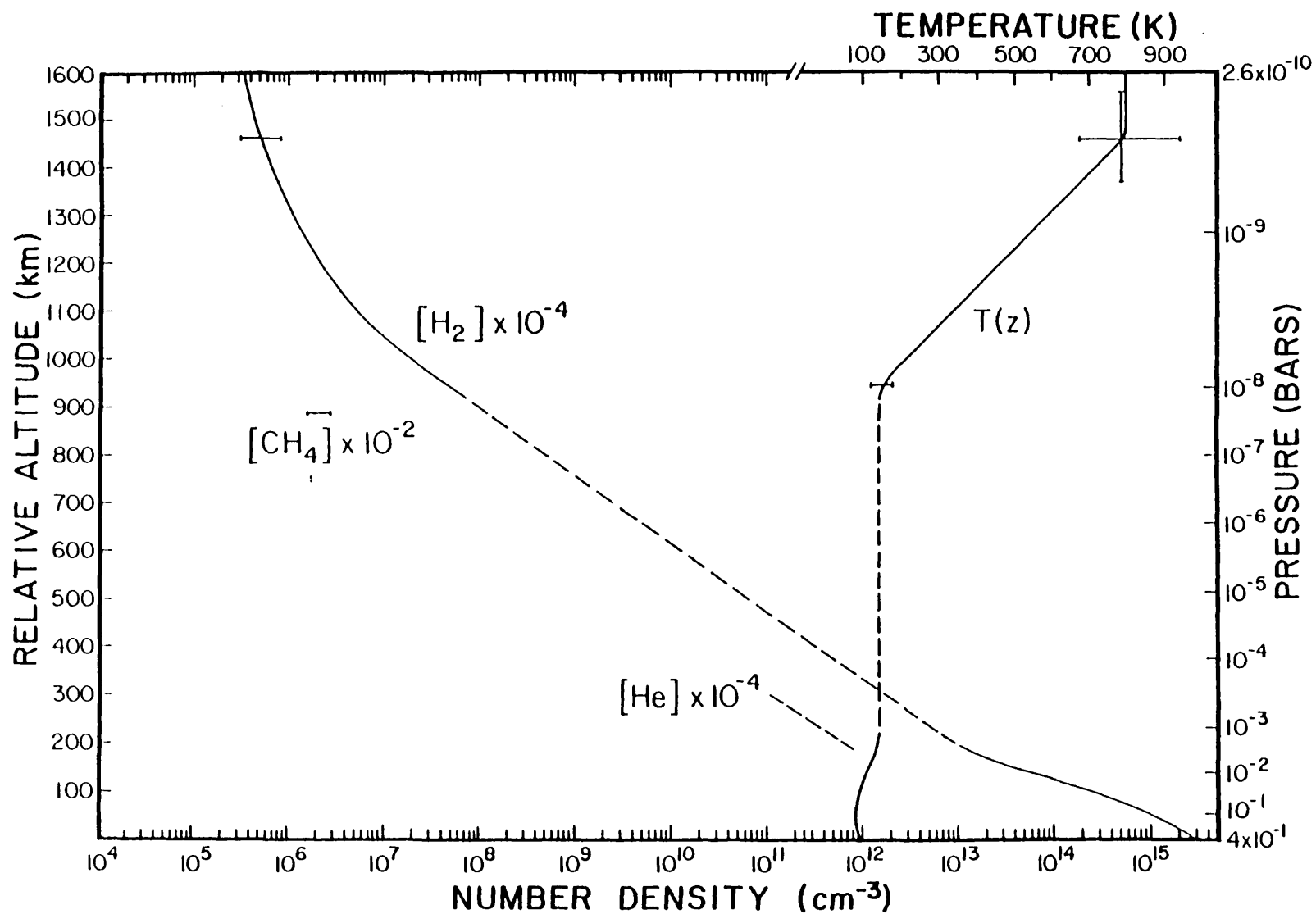


fig. 5.8



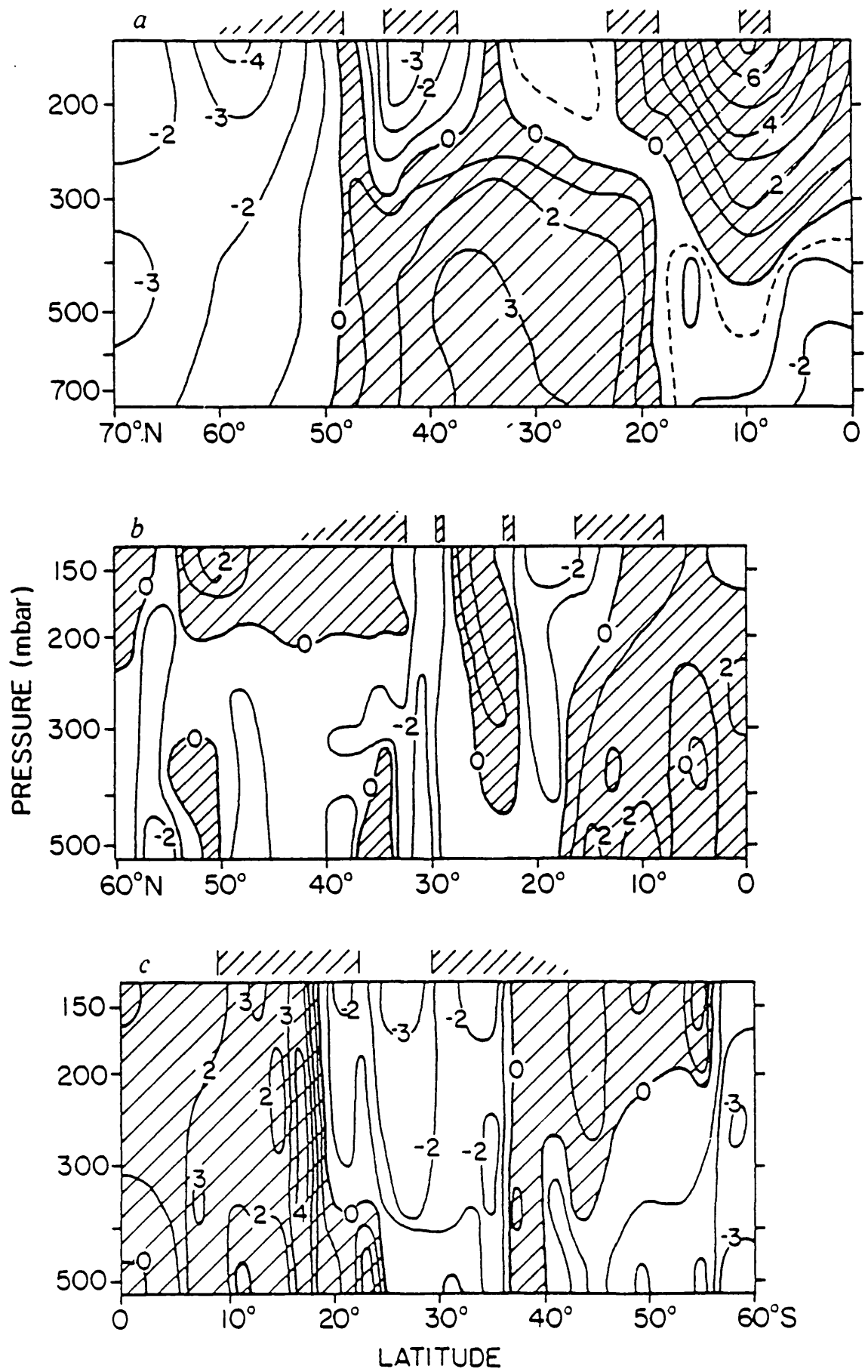


fig. 5.9

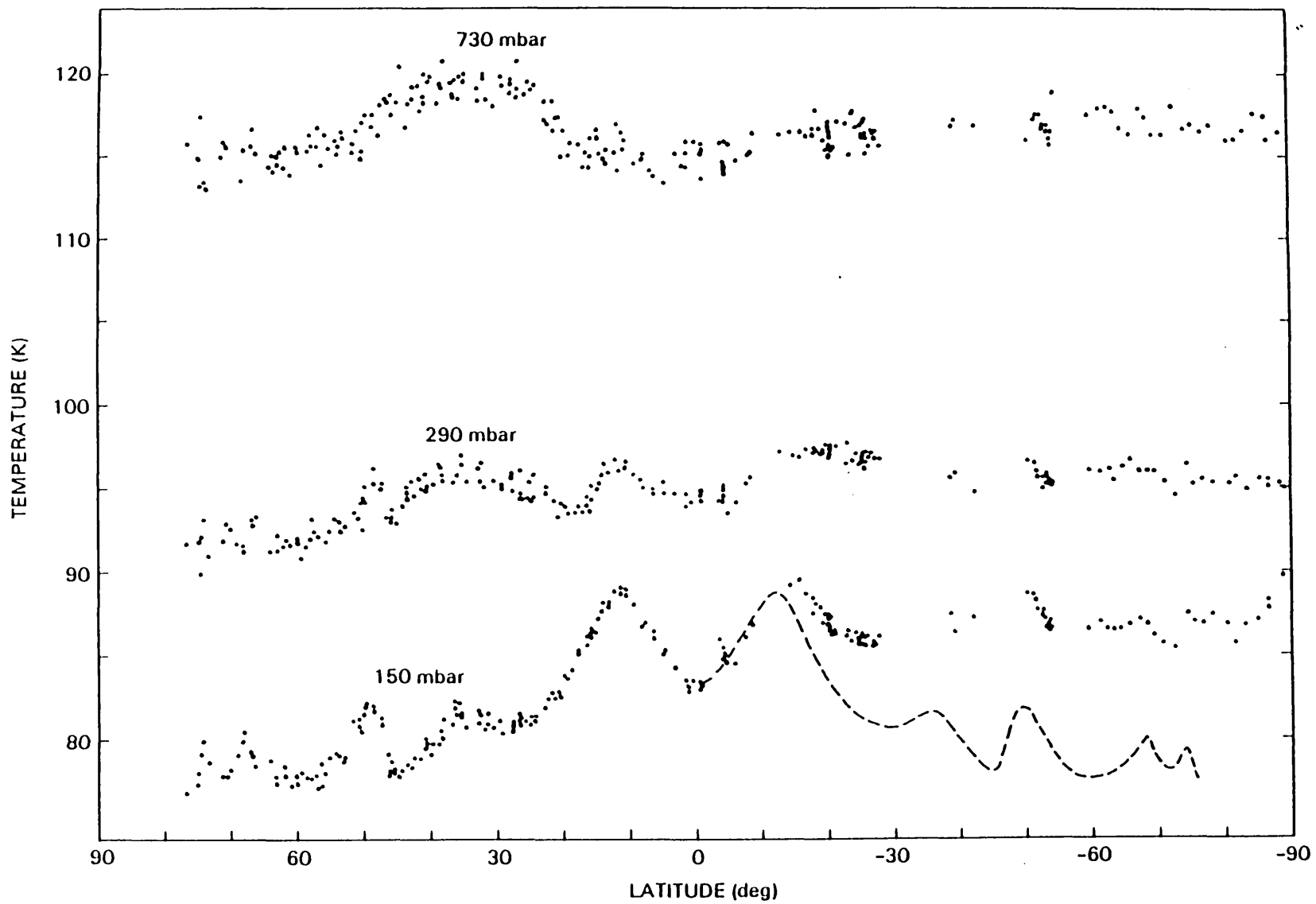


fig. 5.10

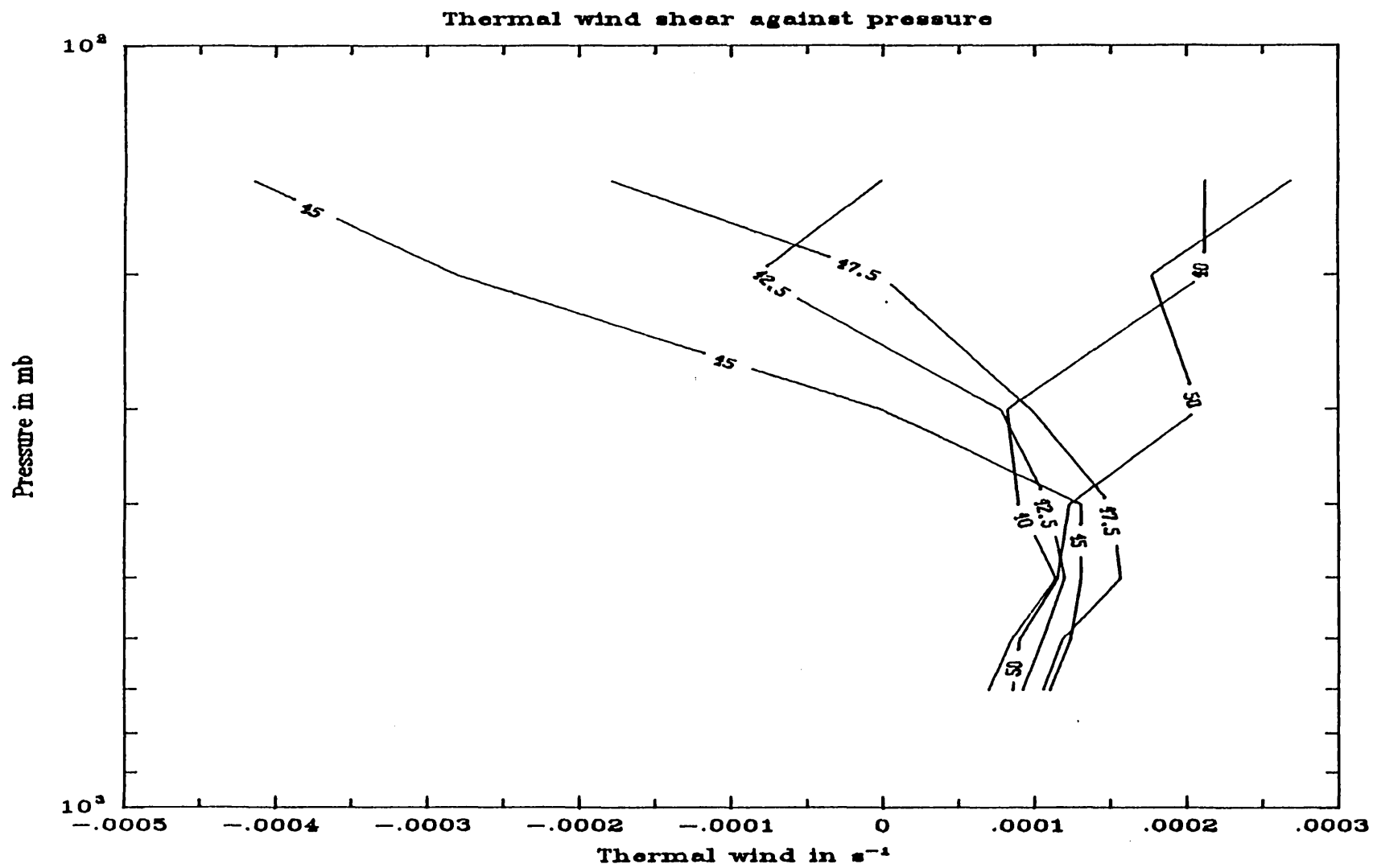


fig. 5.11

CHAPTER 6

A SATURNIAN FEATURE RESEMBLING A VORTEX STREET

6.1 INTRODUCTION

This chapter describes the analysis of a feature in the northern hemisphere of Saturn. The first section includes a description of the four images which were used for this study. This is followed by the results obtained from the measurement techniques used, including both the position and velocity of different parts of the feature. From these a streamline pattern has been obtained which is described subsequently along with the vorticity values derived from it, which show an apparent periodicity with longitude.

Various aspects of the observed feature closely resemble the flow patterns observed in a Karman vortex street, a type of flow observed in laboratory experiments, which is then described. This is followed by a consideration of the way in which this type of flow will be modified by the presence of other effects, such as stratification and rotation, which are likely to be important on Saturn. A possible intermediary between laboratory experiments and the Saturnian feature is seen in the Earth's atmosphere. This consists of a flow pattern, similar to that of a Karman Vortex street, which occurs in the lee of islands, and is described in the next section. The final part of this chapter discusses the relevance of Karman vortex streets to the flow observed on Saturn, and the necessary differences

between the Saturnian and laboratory features.

6.1.1 SET OF IMAGES USED FOR THIS STUDY

The measurements used in this study were derived from a set of four green images, separated by approximately one rotation period, taken 5-6 days before closest approach with a resolution of approximately 59 - 42 km/pixel. For ease of comparison these have all been map projected to cylindrical projections with a common scale of 25 km/pixel (at the equator) and with the longitude of the first sample increasing from 62.0 to 64.4 (corresponding to 18 m/s) to remove the overall drift. All four images are shown in fig 6.1a, while fig 6.2 shows an enlarged version of the first image incorporating a latitude-longitude grid.

When map projecting these images I used the subspacecraft latitude given by J.P.L. in the SEDR data. However, by comparison with subsequent issues of the SEDR it has become apparent that this was the planetodetic rather than planetocentric sub-spacecraft latitude, and hence the sub-spacecraft latitude used in the map projections was out by $\sim 2.5^\circ$. The main effect of this error is that the latitudes shown in figs 6.1 and 6.2 are $\sim 2.5^\circ$ larger than they should be, thus the centre of the feature should be at $\theta_c \sim 34^\circ$ or $\theta_d \sim 40^\circ$. For the sake of comparison all the figs use the incorrect latitude, but this has been taken into account in deriving the latitudinally dependent scale factors which are, at latitude $\theta_d = 40^\circ$.

latitudinal scale	0.9666×10^6 m/deg
longitudinal scale	0.8388×10^6 m/deg
degrees of longitude/hr	233.0 m/s

The longitudinally dependent latitudinal shift produced by the use of the incorrect subspacecraft latitude will also produce errors in the calculated meridional velocity. This effect has been estimated and

found to produce a maximum error of 2 m/s for all the image pairs, reducing to 0.4 m/s for the first pair. This error cannot therefore have produced the large meridional velocities observed in fig 6.1b.

It can be seen, from fig 6.2, that the feature consists of a dark spot $\sim 1^\circ$ in size, with bright patches of cloud to the east and southeast, leading to a large circular cloud $\sim 5^\circ$ to the east and finally several vague patches of cloud. The flow can be seen in more detail in a movie constructed using all four of the frames shown in fig 6.1a. From this it becomes clear that the feature centered at $\sim 56^\circ$ W is rotating in a clockwise direction. It also appears that the cloud to the south east of the dark spot is rotating anticlockwise, as is a patch of clouds at $\sim 53^\circ$ W. This is confirmed by the vorticity measurements given in section 6.1.4.

From the images shown in figs 6.1a and 6.2 it appears that the dark spot and the white clouds are all part of the same feature. However this may not necessarily be correct. By using a sequence of images taken over about 40 rotation periods Sromovsky et. al. (1983) show that the bright patches of cloud form initially to the east of the dark spot, growing gradually until they appear as shown in fig 6.2. Because of the lower resolution of these earlier images it is difficult to see the evolution of the feature. However it seems that the fairly complex pattern seen in fig 6.2 only appears in the clouds close to the dark spot. Thus it is possible that the clouds are just acting as a tracer for the flow in the vicinity of the dark spot which would not otherwise be visible. An alternative, but perhaps less likely, explanation is that the pattern is entirely associated with the clouds, and that the positioning of the dark spot is coincidental.

6.1.2 POSITION MEASUREMENTS

In an attempt to elicit more information from the images, measurements were made of the position of several points. These included the edges of the dark spot and the approximate centres of the first three vortices which were estimated using the movie sequence. Using this technique for all four images produced the values given below for the various distances shown schematically in fig 6.3.

quantity	mean	standard error
d1	1750	105
d2	1560	150
h1	830	195
h2	680	115
a1	2430	105
a2	2640	150

There are certain reservations about these figures. The values of d1 and d2 correspond to the visible edge of the spot. However, since this is surrounded by clouds, and may be rotating, the significance of these estimates is uncertain except to provide a reasonable lower limit. The values for h and a depend upon a subjective estimate of the centres of rotation, and hence the errors given may be more representative of my ability to repeatedly locate a certain point, rather than a true estimate of the difference between the measured and actual values.

It is also possible to use these measurements to estimate the velocity of the vortices relative to the dark spot. Fig 6.4 shows the estimates of the longitude of the two edges of the spot (L and R) and the centres of the vortices (1, 2, and 3) as a function of time.

In each case the position in the first image has been subtracted so that all the curves pass through the origin.- By fitting straight

lines to these sets one obtains, for the left and right hand sides of the spot :-

$$U_l = 0.105 \text{ deg hr}^{-1} = 24.46 \text{ ms}^{-1}$$

and

$$U_r = 0.08 \text{ deg hr}^{-1} = 18.64 \text{ ms}^{-1}$$

while, averaging the values for the three vortices gives :-

$$U_v = 0.0773 \text{ deg hr}^{-1} = 18.01 \text{ ms}^{-1}$$

so that the drift rate away from the left hand side of the spot is :-

$$U_e = U_l - U_v = 6.43 \text{ ms}^{-1}$$

or, for the right hand side :-

$$U_e = U_r - U_v = 0.63 \text{ ms}^{-1}$$

There are a few points about this calculation which should be made clear. Although the correlation coefficients for the fitted lines are quite high (all above 0.96) it can be seen from fig 6.4 that the largest contribution to the drift rate U_e is due to the movement which takes place between the first and second images. If the first image was not used U_e would be smaller or, even perhaps, negative. This can not however be dismissed as navigation error which would tend to produce translations rather than changes of scale and is consequently unable to influence the relative velocity of two objects. It is perhaps safest to interpret the results of fig 6.4 as showing that a longitudinal drift rate of 0(1) m/s is possible but by no means certain. This could well be influenced by the small ~ 1 pixel displacements to which this velocity corresponds.

6.1.3 VELOCITY MEASUREMENTS

From the four images available it was possible, using the methods described in section 3.4.2, to produce three sets of velocity measurements. These are shown in vector form in fig 6.1b, with the origin of each vector corresponding to the location of the relevant feature in the earlier of the two frames used to determine the

velocity. It can be seen that the last two sets of measurements both show opposed large meridional drifts, probably caused by navigation errors, superimposed on the smaller scale structure. Partially because of this, but mainly due to the much better spatial coverage, the first set of measurements was used for all subsequent analysis. Fig 6.2 shows the first image of the pair of images from which these were obtained.

From the vectors shown in fig 6.1b it is difficult to observe much structure in the flow. However it is possible, using a technique described by Saucier (1955), to display this information in a more convenient form. The first stage of this requires the separation of each velocity measurement into a speed with an associated angle, each of which can be plotted on a latitude-longitude grid. From these it is possible to estimate the position of contours of equal speed (isotachs) and angle (isogons). These are shown in figs 6.5 and 6.6. By constructing lines which intersect each isogon at the appropriate angle it is possible to derive a streamline pattern, and this is shown in fig 6.7. It should be noted that although this type of analysis may sound unduly subjective it was carried out independently by Dr. Green who arrived at almost identical results. It can be clearly seen from fig 6.7 that the flow pattern shows an oscillatory behaviour, with a longitudinal wavelength of $\sim 5.2^\circ \pm 0.1^\circ$ corresponding to a distance of $4360 \pm 80\text{km}$, lower than the value of 5070 km obtained in section 6.1.2 .

It is possible, using figs 6.5, 6.6 and 6.7, to obtain an estimate of the vorticity. If we consider an area bounded upon two sides by streamlines and define s to be the co-ordinate parallel to the streamlines, n the co-ordinate normal to the streamlines, c the velocity at the centre (obtained from fig 6.7), and α the angle, then the vorticity ζ is approximated by

$$\zeta = \frac{\partial c}{\partial n} + c \frac{\partial \alpha}{\partial s} \quad (6.1)$$

as given by Saucier (1955), but with a change in the sign of the last term since I have assumed that positive angles are measured in a counter clockwise direction. By using this technique the vorticity was calculated for the boxes labelled A - G in fig 6.7, details of the calculation being given below.

box	shear term $\partial c / \partial n$	curvature term $c \partial \alpha / \partial s$	sum	vorticity ζ $\times 10^{-5} \text{ s}^{-1}$
A	-0.4	.9367	.5367	.977
B	-0.3895	-0.5387	-0.9282	-1.69
C	0.2667	-0.7208	-0.4541	-0.827
D	0.6154	0.3902	1.0056	1.83
E	0.0	-0.5485	-0.5485	-0.998
F	0.4706	-0.8685	-0.8685	-0.724
G	-0.4	0.2240	-0.76	-0.320

The results are shown as a function of longitude in fig 6.8 where, once again, some oscillatory behaviour can be seen. By comparing this with the original image shown in fig 6.2 it can be seen that, the large area of negative vorticity at a longitude of approximately 56.5° corresponds to the large, apparently rotating, cloud seen in the images. A graph of vorticity against latitude was also plotted, but did not show any obvious structure.

6.1.4 VORTICITY BUDGET

In this section I consider some of the possible sources for the vorticity shown in fig 6.8. The important features of the vorticity distribution observed are the magnitude, regular zonal separation, and the latitudinal displacement, with the cyclonic vortices to the south. The equation governing the vertical component of the relative vorticity ζ of a flow on the surface of a sphere is (Holton 1979).

$$\begin{aligned} \frac{\partial \zeta}{\partial t} + u \frac{\partial \zeta}{\partial x} + v \frac{\partial \zeta}{\partial y} + w \frac{\partial \zeta}{\partial z} + (\zeta + f) \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \\ + \left(\frac{\partial w}{\partial x} \frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} \frac{\partial u}{\partial z} \right) + v \frac{df}{dy} = \frac{1}{p^2} \left(\frac{\partial p}{\partial x} \frac{\partial p}{\partial y} - \frac{\partial p}{\partial y} \frac{\partial p}{\partial x} \right) \end{aligned} \quad (6.2)$$

The simplest way in which the vorticity can be changed is by horizontal or vertical advection. Unfortunately, because of the lack of observable features, no measurements of u directly upstream of the feature have been obtained. However from the zonal velocity profiles available it is apparent that $\partial u / \partial y \sim 0$ at this latitude, going from -1.6×10^{-5} at $\theta d = 37^\circ$ to 0.4×10^{-6} at $\theta d = 43^\circ$. This is of the same magnitude as the variations in fig 6.8, but varying as a function of y rather than x . Due to an insufficient knowledge of the quantities involved, the values of the other advection terms, the divergence term, and the term on the right representing the generation of vorticity by pressure-density solenoids cannot be estimated.

It is, however, possible to estimate the size of the term representing the generation of a vertical vorticity component through the interaction of the horizontal components. The value of $\partial u / \partial z$ is known from the thermal wind equation to be $\sim 0.15 \times 10^{-3} \text{ s}^{-1}$ at 40°N and 150-700 mb (see section 5.8). The value of $\partial w / \partial y$ is not known, but measurements of the divergence at this latitude have yielded, for certain features, a vertical velocity $w \sim 0 (1) \text{ ms}^{-1}$ (Hunt et. al. 1982) which, using a representative length scale $L \sim 0 (10^6) \text{ m}$ gives

$$\frac{\partial u}{\partial z} \frac{\partial w}{\partial y} \approx 0.15 \times 10^{-9} \text{ s}^{-2} \quad (6.3)$$

This appears to be very small, if however we assume that the vertical velocity is caused by the dark spot passing through the surrounding flow at $\approx 2 \text{ m s}^{-1}$ then this effect will act over $5 \times 10^5 \text{ s}$ producing a change in ζ of $7.5 \times 10^{-5} \text{ s}^{-1}$. One important point to remember when considering this term is that the presence of a non-zero vertical velocity will produce, at some level, an associated convergence or divergence. This in turn will alter the vorticity according to the neglected 5th term in Eqn 6.2. The final term in equation 6.2 which can be evaluated is that due to the beta effect, at $\theta_d = 40^\circ$, $df/dy = \beta \approx 0.45 \times 10^{-11} \text{ s}^{-1} \text{ m}^{-1}$, so that, using $v \approx 2 \text{ ms}^{-1}$ and $L \approx 10^6 \text{ m}$ we obtain $\nabla \zeta = -0.45 \times 10^{-5} \text{ s}^{-1}$.

We have considered some of the possible sources of the vorticity seen in fig 6.8. All the possibilities considered are capable of producing changes of at least the required magnitude, although the largest required the use of a value for w which is more probably a maximum rather than average value and, even if present, may not necessarily be upward. There is, however, no obvious way in which any of these mechanisms could produce the distinct periodicity observed in fig 6.8.

In addition to the longitudinal separation there is a distinct offset between the vortices of different signs. With the clockwise vortices appearing to the north of the anticlockwise ones. To produce this via the beta effect requires the meridional velocity to be positive to the north and negative to the south, as would happen if the flow was passing around the spot. For the tilting term to produce the same effect, the vertical velocity must reach a maximum at the latitude

midway between the rows of vortices. Although both these alternatives are possible it should be remembered that the meridional shear changes sign at approximately this latitude, and that this would tend to give rise to the type of latitudinal offsets observed through the vortex separation process described by Kuo (1969).

6.2 KARMAN VORTEX STREETS

It is apparent from the preceding discussion that the feature has a fairly well defined structure of alternating vortices. Associated with this there is an oscillatory cloud pattern to the east of a darker spot. These are also characteristic of Karman vortex streets, a type of flow which is observed in the laboratory when fluid flows past a solid obstacle. A comparison between the visual aspects of this type of flow and the Saturnian feature is shown in fig 6.9.

Between the flow observed in the laboratory and the features observed on Saturn there is a change in length scale of at least 10^6 (it would be more relevant, as described in section 6.2.1, to compare the Reynolds numbers but this cannot be done since the viscosity ν is unknown). However this type of flow regime is also thought to be responsible for the cloud pattern seen behind islands on the Earth at intermediate length scales of 10 km (the Reynolds number is also uncertain in this case because of the difficulty in estimating the value of the eddy viscosity). In the case of the Earth however other effects such as the presence of stratification and rotation can become important and may modify the flow observed.

The following sections describe both the atmospheric and laboratory observations of Karman vortex streets, including the changes introduced by rotation and stratification, both of which may be important on Saturn.

6.2.1 FLOW PAST AN INFINITE CYLINDER

If we consider a fluid with density ρ , viscosity ν , and undisturbed velocity U_0 , flowing past an infinite cylinder of diameter d , then it should be possible to characterize the results as a function of only the Reynolds number Re . This dimensionless parameter gives an idea of the relative importance of inertial and viscous effects and is given by :

$$Re = \frac{\rho U_0 d}{\nu} \quad (6.4)$$

For $Re \ll 1$ viscous effects predominate, and the flow is approximately symmetric with the flow upstream being a mirror image of the flow downstream. As Re increases it might be expected that the flow would tend towards an inviscid solution. It is however well known (see for example Batchelor 1967) that this is not necessarily true. The presence of the 'no slip' condition at the solid boundaries still produces viscous effects, and these manifest themselves in the form of boundary layers which become thinner with increasing values of Re . When $Re \sim 5$ the boundary layer separates from the cylinder to enclose two counter-rotating eddies. As the Reynolds number increases these eddies grow until, at $Re \sim 40$ the wake downstream of the eddies becomes unstable showing a regular oscillatory behaviour, which becomes more apparent with increasing distance from the obstacle. With increases in Re the wake becomes oscillatory soon after leaving the cylinder, developing into regularly spaced vortices until, at $Re \sim 100$, the two stationary eddies behind the obstacle have been replaced by vortices of alternating signs being shed regularly from each side of the cylinder. This leads to two, approximately parallel, rows of vortices. Those in one row having the opposite sign to, and being midway between, those in the other row. This type of flow is called a

Karman vortex street.

The evolution of this with increasing Reynolds number is considered by Berger and Wille (1972) and Tritton (1977). At $Re \sim 200$ the vortices are shed less regularly, but reappear at $Re \sim 400$ when the stability increases, due to the inclusion of turbulent fluid in the vortices.

At $Re \approx 3 \times 10^5$ the boundary layer, which has remained laminar prior to separation becomes turbulent soon after leaving the surface. Because of the greater ability of a turbulent boundary layer to withstand shear the boundary layer can now re-attach to the sides of the cylinder, to finally undergo turbulent separation. Since the final separation point is closer to the rear of the cylinder than the laminar separation point would be if the transition to turbulence had not occurred, the width of the wake is reduced. This reduces the drag, but the rather complex flow also decreases the regularity with which the vortices are shed. However at still higher values of Re ($> 7 \times 10^6$) the periodicity again becomes more apparent, and the drag coefficient increases (Roshko 1961). This is attributed to the boundary layer being turbulent when initial separation occurs, eliminating the possibility of boundary layer re-attachment.

6.2.2 THE VORTEX SPACING RATIO b/a

One of the values often measured when vortex streets are observed is the vortex spacing ratio. This is given by b/a , where b is the distance separating the two rows of vortices, and a is the distance between two adjacent vortices in the same row. The first attempt to evaluate this ratio analytically was that of Karman (see, for example, Milne-Thomson 1938). By considering the motion of two infinite, parallel, rows of point vortices it is shown that a necessary condition for this arrangement to be stable to small perturbations is that

$$\cosh \left(\frac{\pi b}{a} \right) = \sqrt{2} \quad \text{ie} \quad \frac{b}{a} = 0.281 \quad (6.5)$$

However, it has since been shown that, if higher order approximations are used, this configuration is also unstable. Another feature of this analysis was considered by Saffman and Schatzman (1982) who showed numerically that, for vortices of a finite size, this arrangement was stable (to first order) over a range of values for b/a centered around 0.281. The susceptibility of a vortex street to higher order disturbances causes Bearman (1967) to suspect the Karman condition. Instead he prefers the approach of Kronauer who suggests that the street will configure itself so as to minimise the drag coefficient C_d . That is, for a given undisturbed flow velocity U_0 and street velocity (relative to the undisturbed flow) U_s , the value of b/a will be that for which $C_d = C_{ds}$ where:-

$$\left(\frac{\partial C_{ds}}{\partial (b/a)} \right)_{U_s/U_0 = \text{const}} = 0 \quad (6.6)$$

By evaluating C_{ds} for various values of b/a and U_s/U_0 , Bearman (1967) produces a graph showing the expected variation, of b/a with U_s/U_0 . His measurements showed that a increased with distance from the obstacle but, as he was unable to measure b , the value of b/a is uncertain.

6.2.3 THE EFFECTS OF ROTATION

The presence of rotation can often have a strong modifying effect on flow regimes. These include the restrictions imposed on flow parallel to the axis by the Taylor Proudman theorem, and the occurrence of Ekman layers with the associated Ekman suction effects produced near surfaces. To characterise the importance of rotation and viscous effects the Rossby number and the Ekman number Ek are used.

These can be defined as

$$Ro = \frac{U}{2 \omega L} \quad (6.7) \quad \text{and} \quad Ek = \frac{\nu}{2 \omega L^2} \quad (6.8)$$

where ω is the local rotation rate. The Reynolds number Re is related to these by

$$Re = \frac{Ro}{Ek} \quad (6.9)$$

The change in the flow past a cylinder caused by rotation was examined by Boyer (1970). He found that the flow was basically similar to the non-rotating case described in section 6.2.1. As the Reynolds number increased an initially fully attached flow evolved to the form with two attached eddies, and finally to vortices being shed from the sides of the cylinder. However the main difference between this, and the non-rotating flow lies in the Reynolds numbers at which these transitions occur, for $Ro \sim 0.1$ the eddy shedding regime occurs only for $Re > 1000$, considerably larger than that required for the conventional case. Another effect observed was the increased size of eddies with the same sign as the rotation over those with the opposite sign. This was confirmed by Boyer and Kmetz (1982), who also considered the importance of the rotationally induced Ekman layers on the shed vortices. They found that, due to Ekman suction at the boundaries, the vortices would decay with an e folding time τ_0 given by:-

$$\tau_0 = \frac{Ro H}{U (2 Ek)^{\frac{1}{2}}} \quad (6.10)$$

where H is the height. It was also mentioned that these effects tended to concentrate the dye used into the cyclonic vortices, making them appear stronger.

The beta effect (Holton 1979) caused by the change in the Coriolis parameter with latitude can also have an effect upon large scale dynamics, introducing an asymmetry between prograde and retrograde flows. The effect of beta upon the separation of flow around a cylinder was considered by Merkin (1980) who found that although there was comparatively little change for the retrograde flows, prograde flows produced thinner boundary layers which separated closer to the rear stagnation point. The parameter restrictions used to obtain this result were criticised by Boyer and Davies (1982) who used a rotating water channel with sloping upper and lower surfaces to simulate $\beta \neq 0$. By measuring the length of the separation bubble they obtained similar results for the prograde flows, but found that retrograde flows separated earlier than flows with $\beta = 0$. In addition they found that the prograde $\beta \neq 0$ results included a slightly oscillatory flow downstream consistent with the structure of a stationary Rossby wave.

6.2.4 THE EFFECTS OF STRATIFICATION

The possible effects of the stratification on the formation of vortex streets behind islands has been considered by several people, see, for example, Berger and Wille (1972). Ruscher and Deardorff (1982) were unable to produce a vortex street in their numerical model unless they allowed the top of the mountain to penetrate the overlying inversion layer. The importance of stratification effects can be estimated using the Froude number Fr , defined by :-

$$Fr = \frac{U}{N H} \quad (6.11)$$

where N is the Brunt-Vaisala frequency and U and H are a characteristic velocity and height. For $Fr \gg 1$ the flow is a slightly perturbed version of the homogenous flow. The flow for $Fr < 1$

was examined by Brighton (1978) who, extending a previous analysis by Drazin, found that the flow was approximately two-dimensional further than U/N from the ends of the obstacle. This analysis also showed that rotational effects could be neglected unless $Ro < Fr$. These results were tested in the laboratory for the flow past various objects. It was found that within a distance U/N of the top of the obstacle the flow gave rise to lee waves, and that these tended to interact with the flow past the object to generate features called cowhorn eddies. Below U/N from the top the flow appeared to pass mainly around the obstacle, with separation occurring at the rear, producing a pattern of vortices resembling a classical vortex street.

6.3 ATMOSPHERIC WAKES IN THE LEE OF ISLANDS

Since the advent of space-borne cameras there have been several observations of periodic phenomena occurring in the wake of islands including Madeira and the Canary Islands (Hubert and Krueger 1962), Jan Mayen Island in the North Sea (Scorer 1978) and Cheju-do, Republic of Korea (Ruscher and Deardorff 1982). In all these cases the cloud patterns observed show tight spirals and/or sweeping circular wakes.

An early interpretation of these patterns by Hubert and Krueger (1962) suggested that their approximately cycloidal nature could be caused by inertial oscillations set up by the pressure difference between the wake and the undisturbed flow. For the more complex patterns observed it was suggested that vortex shedding caused by gravity waves in the lee of the island could be important.

However most of the more recent papers concentrate on the similarity between the observed features and the Karman vortex streets observed in the laboratory, and around man made objects in the

atmosphere and ocean. One of the obvious criticisms of such an identification lies in the clear difference between an infinite cylinder and an approximately conical island with an angle of $\sim 5-20^\circ$.

These difficulties were considered by Berger and Wille (1972) who discussed a previous analysis by Moll (1971) suggesting that the presence of stratification or possibly, Taylor columns could be important. Whilst his analysis demonstrated that the atmosphere was stably stratified when the vortex street was seen it is suggested that the effect of Taylor columns could be important, particularly if the length scale used in the definition of the Rossby and Ekman numbers corresponds to the width of the wake rather than that of the obstacle. They conclude that although it is possible that the feature is a classical vortex street, it is possible that other effects are also important.

An attempt to estimate the effect of the shape of the island upon the vortex shedding characteristics was carried out by Trischka (1980). In this study he assumed that the shedding period was independent of the Reynolds number, and compared, using different shaped cones, laboratory results with the observed periods of vortices shed from various volcanoes. This gave reasonable answers for three of the cases studied, but the others produced vortex shedding periods less than half those observed. This was attributed to the apparent size of the obstacle being increased by stratification.

A different approach was adopted by Ruscher and Deardorff (1982) who used a two dimensional numerical model of the planetary boundary layer to study the flow past Chejo-do. This model consisted of a mixed layer of variable depth and $Re \sim 390$ (caused by the eddy viscosity rather than molecular viscosity) passing over and around the island, and being confined below an inversion layer with a constant change in potential temperature. They found that if the

bottom of the inversion layer was at, or above, the top of the island (~ 2 km) then, although sinusoidal oscillations were observed in the wake, no vortices were shed. When the top of the mixed layer was reduced to 890 m in accordance with the observations vortices were shed from behind the island. The spacing ratio and the vortex shedding period derived from these results agreed well with the values observed, suggesting that their model contains all the processes vital to the formation of these features, and that vertical stratification, whilst it may be important in some cases, is not essential.

6.4 VALUES OF CHARACTERISTIC PARAMETERS FOR SATURN

An estimate of the relative importance of different effects can be obtained from the values of certain characteristic parameters. One of the most useful of these is the Reynolds number Re , but this cannot be calculated because no value for the viscosity (or even the relative importance of eddy and molecular viscosity) is available. However other parameters can be estimated, to do this we estimate a characteristic length of 10^6 m, a velocity U of 5 m s^{-1} and a depth D of 4×10^4 m (approximately one scale height). The buoyancy frequency N is estimated from fig 5.5 to be $N \sim 5 \times 10^{-3} \text{ s}^{-1}$ whilst values of the Coriolis parameter f and the beta parameter can be obtained from table 3.1 to give :-

$$f = 0.21 \times 10^{-3}$$

$$\text{beta} = 0.45 \times 10^{-11}$$

Using these values we obtain estimates for the Rossby and Froude numbers Ro and Fr as well as the dimensionless beta parameter Be (introduced by Merkin 1980) of

$$Ro = \frac{U}{fL} = 0.02$$

$$Fr = \frac{U}{ND} = 0.025$$

$$Be = \frac{\beta L^2}{U} = 0.9$$

This suggests that, for the Saturnian feature, the presence of rotation, stratification and the beta-effect may all play an important part, and should not be neglected.

The value of the vortex spacing ratio described in section 6.2.2 can be estimated, from the values given in 6.1.3 for a and the value given in 6.1.2 for b to be

$$b/a \approx 750/4160 \approx 0.16$$

This is smaller than the value of 0.281 normally assumed, see section 6.2.2, whilst the values measured for the Earth tend to be larger.

However b is difficult to estimate, and the values normally quoted for b usually correspond to the flow some way downstream rather than immediately behind the obstacle. Hence the value of b/a cannot reasonably be used to indicate that the flow is a Karman vortex street but shows that whatever the initial causes the characteristics are similar.

One interesting possibility is that the relationship given in section 6.2.3 for the decay rate of an individual vortex could be used to derive an effective viscosity. The relevance of applying this condition, which is derived from the Ekman suction at a solid surface is not clear, but it could provide a useful lower limit for the viscosity. If we assume that the vortices have decayed by a factor of e between the dark spot and the point at which they begin to become indistinguishable, i.e. over a length of :-

$$L \approx 10^7 \text{ m}$$

which has been covered at a velocity of

$$U \approx 2 \text{ ms}^{-1}$$

then the lifetime is approximately

$$\tau = L/U \approx 5 \times 10^6 \text{ s}$$

and the Rossby number is

$$Ro = \frac{U}{fL} \approx 10^{-3}$$

It is difficult to estimate an appropriate height, however if we use the value of the pressure scale height, and take

$$H \approx 5 \times 10^4 \text{ m}$$

then, using equation 6.10 we obtain

$$Ek = \frac{1}{2} \left(\frac{Ro H}{\tau U} \right)^2 \approx 10^{-11}$$

Using equation 6.9 we can derive a Reynolds number of

$$Re = \frac{Ro}{Ek} \approx 10^8$$

It should be noted that this estimate is only correct in the presence of horizontal boundaries. However it only describes the decay of individual vortices, and is not restricted to any particular formation process.

6.5 RELEVANCE OF KARMAN VORTEX STREETS TO SATURN

Since it is not clear whether the features observed in the Earth's atmosphere, in the lee of islands, are Karman vortex streets, it is not surprising that the case for Saturn is even less clear. The most obvious difference between the laboratory experiments and the Saturnian atmosphere lies in the absence of a solid object. Since the flow of an inviscid fluid is constrained by Kelvin's circulation theorem (see, for example, Batchelor 1967), which requires the circulation around a closed curve to remain constant, the inability to generate vorticity at a solid boundary is obviously important.

It appears, however, that it may be possible to produce the vorticity required for vortex street formation in the boundary layers associated with Taylor columns (see, for example, Tritton 1977).

These occur when a fluid is rotating sufficiently rapidly for the motion to become almost entirely two dimensional, and the effect of an obstruction at one level extends throughout the fluid parallel to the rotation axis. An experiment in which vortex shedding was observed to occur behind Taylor columns was described by Takematsu and Kita (1978). It is interesting to note that they observed symmetric pairs of vortices, similar to those seen by Brighton (1978), close to the top of an obstacle in a non-rotating flow, suggesting that the Taylor column may be only partially blocking the flow.

Although Taylor columns may be capable of producing vortex streets, they still require the presence of a solid surface at some level and are hence unlikely to occur. An alternative possibility is that, although the flow is similar to a Karman vortex street, the vorticity is being produced by some other mechanism. An example of this is an experiment by Ibbetson (private communication) in which a jet of fluid was introduced into a steady flow and produced a pattern similar to a vortex street. In this instance the vorticity was derived from the fluid entering the flow, and was larger than that which would have been produced if the incoming fluid had been acting like a solid surface. Since it is known that several other sources of vorticity are present, (see section 6.1.4), it is possible that the Saturnian feature is another example of this. And that the vorticity, although produced by a mechanism different to that associated with classical vortex streets, is supplied in such a way as to produce a similar effect. Since many of the results associated with Karman vortex streets apply to any flow containing parallel rows of opposed vortices, it should be possible to apply the same methods to the observed feature.

One of the most valuable results of this analysis is that the observations show that it is possible to produce a feature, with all the expected characteristics of a Karman vortex street, in the absence of an adjacent solid surface. The same mechanism may partially account for the flow observed on the Earth, where the measured vortex shedding period appears to indicate an obstacle larger than that present.

FIGURE CAPTIONS - CHAPTER 6

Figure 6.1

This figure shows the four images that were used to examine the structure of the 'vortex street' feature. Each of the images are separated by approximately one rotation period. From these, three sets of velocity vectors were obtained, and are shown below the appropriate image pair. The origin of each vector shows the position of the point being measured in the first image of each pair. The latitudes shown are planetocentric, whilst the longitudes increase towards the east.

Figure 6.2

This is an enlargement of the first image of fig 6.1 a, overlaid with a planetocentric latitude, west longitude grid.

Figure 6.3

This figure shows a schematic diagram of one of the images of fig 6.1 a to indicate the meaning of the measurements described in the text. The crosses correspond to the estimated centres of rotation of different parts of the feature.

Figure 6.4

This figure shows how the longitude of the left and right hand (L and R) sides of the dark spot, and the estimated position of the centres of rotation indicated in fig 6.3 (1,2 and 3) change with time. The lines join the position measurements for each feature in each of the four images.

Figure 6.5

This figure shows the isotachs (lines of equal speed) obtained from the first set of velocity vectors shown in fig 6.1 b. Those shown vary from 18 ms^{-1} at the north and south extremes, through 20 and 22 ms^{-1} to 25 ms^{-1} at the central contour on the left. The points indicate the positions at which the original velocity measurements were made.

Figure 6.6

This is similar to fig 6.5, but shows the isogons (lines of equal direction) obtained from the first set of velocity vectors shown in fig 6.1 b. The contours correspond to 10° intervals, varying from 200° (lat 41, long 54.5) to 140° (lat 40.5, long 57).

Figure 6.7

From fig 6.6 this figure was produced showing the streamlines, which indicate the direction of the flow at every point. The boxes A-G show the locations at which the velocity data was sufficiently detailed to enable the calculation of the vorticity.

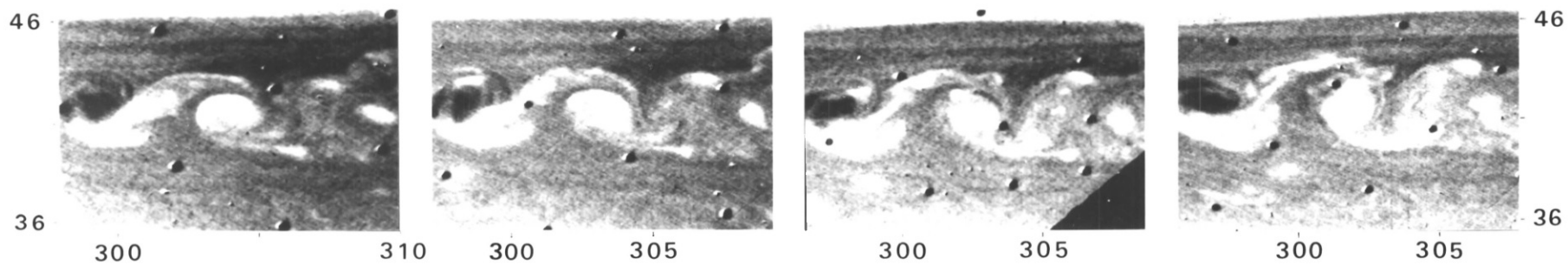
Figure 6.8

By using the streamlines shown in fig 6.7 in conjunction with the isotachs of fig 6.5 the vorticity at the locations A-G in fig 6.7 was calculated. This figure shows the results of the calculation as a function of longitude. The horizontal line shows the relative vorticity produced through the beta effect by a latitudinal displacement of 10^6 m .

Figure 6.9

This figure shows a comparison between the Saturnian feature, and a laboratory flow. The Saturnian image is one of those used in the production of fig 6.1. The lower image comes from A.M.Lippisch (1958), and shows a high speed picture of the development of a Karman vortex street behind a circular cylinder. The flow is visualized by means of smoke introduced into the flow upstream of the obstacle.

A



B

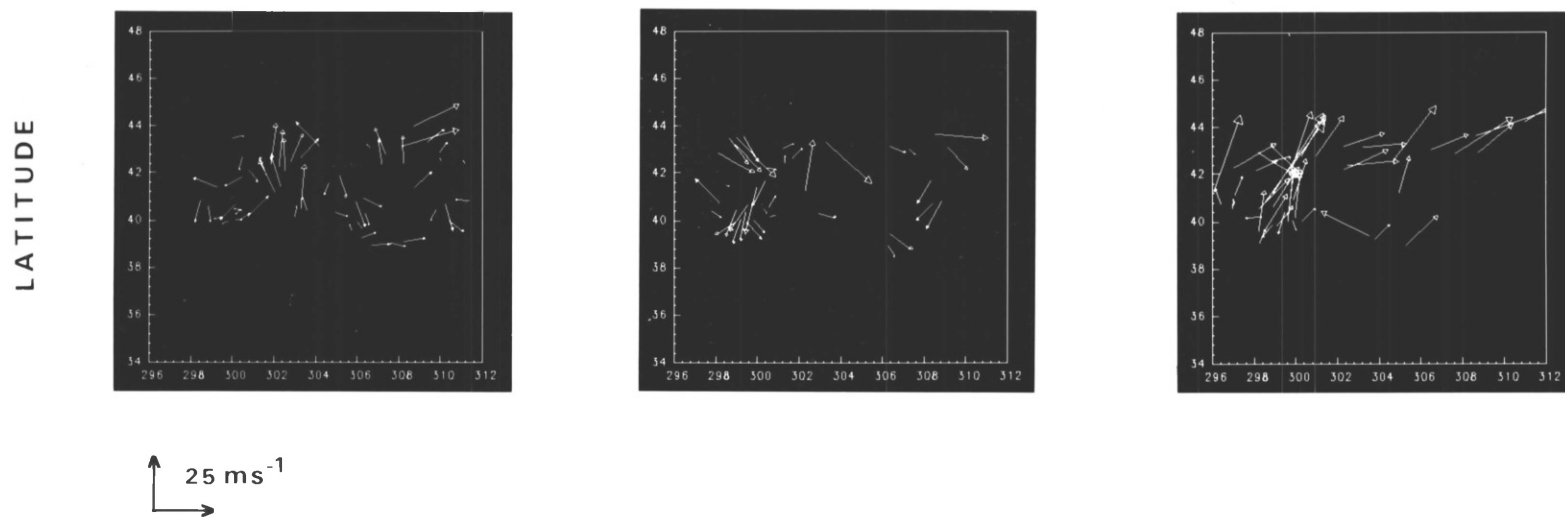


fig. 6.1

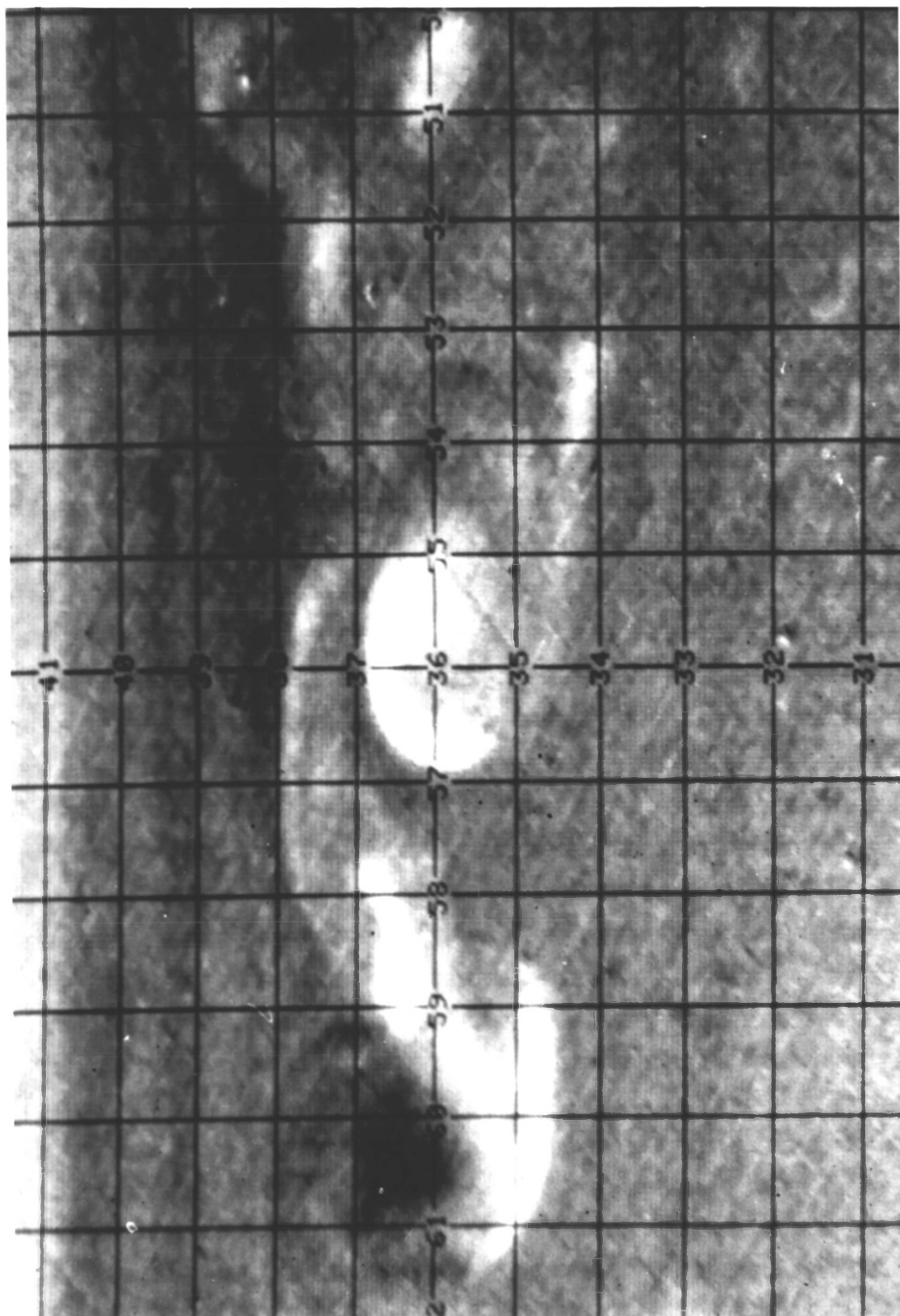


fig. 6.2

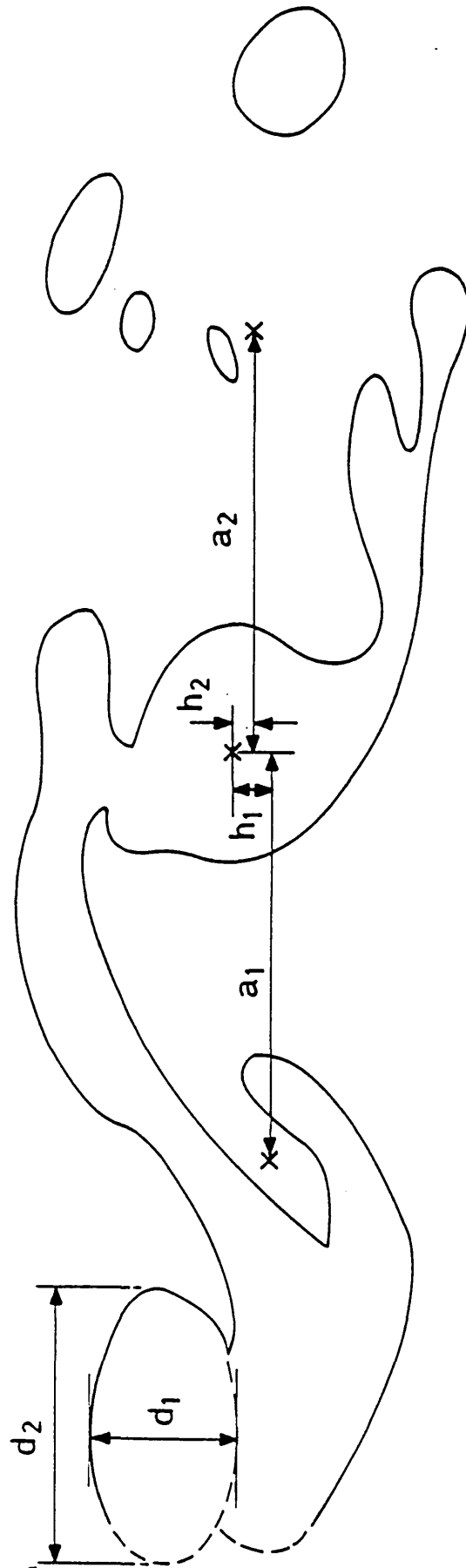
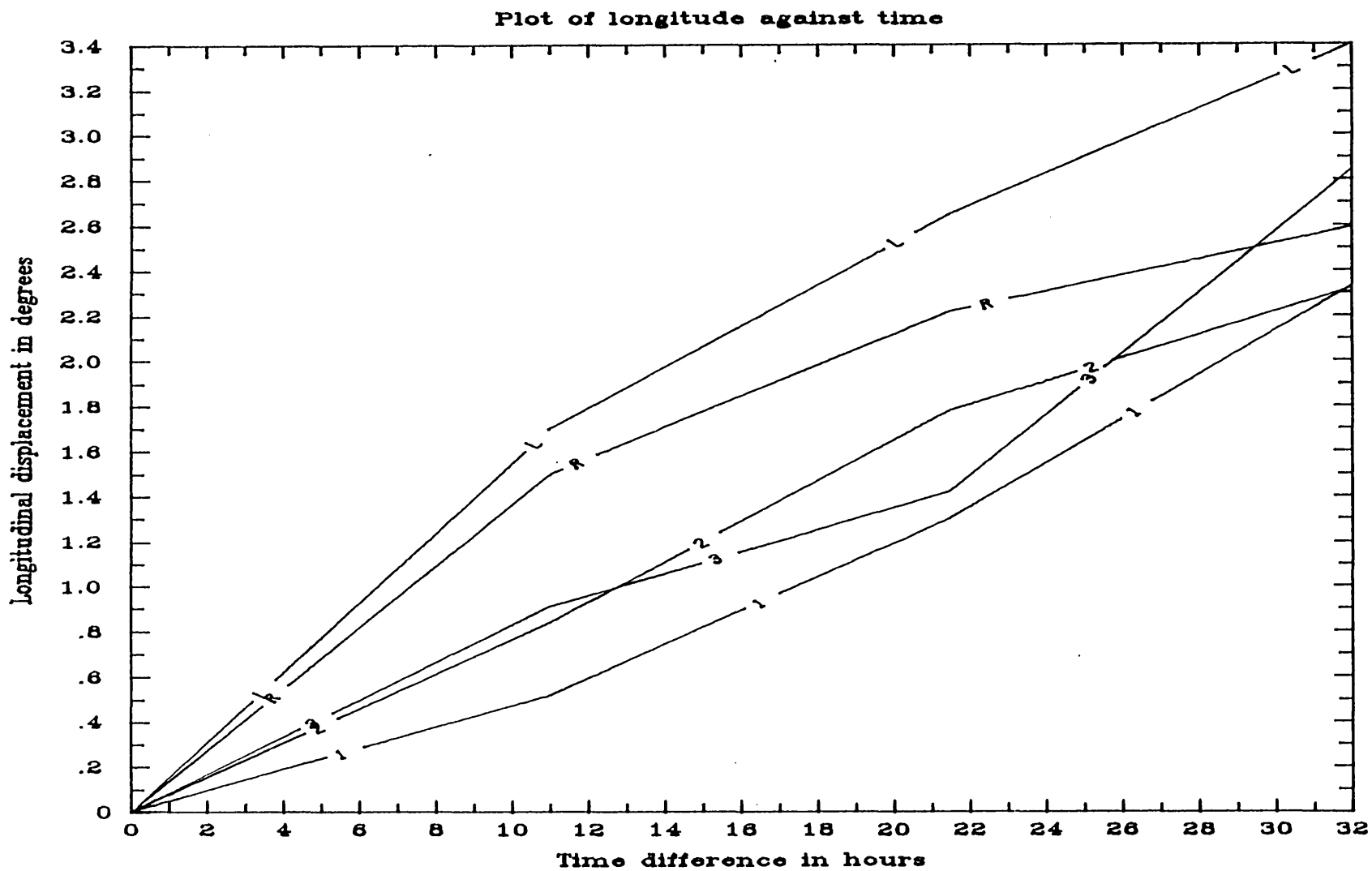


fig. 6.3

fig. 6.4



ISOTACHS

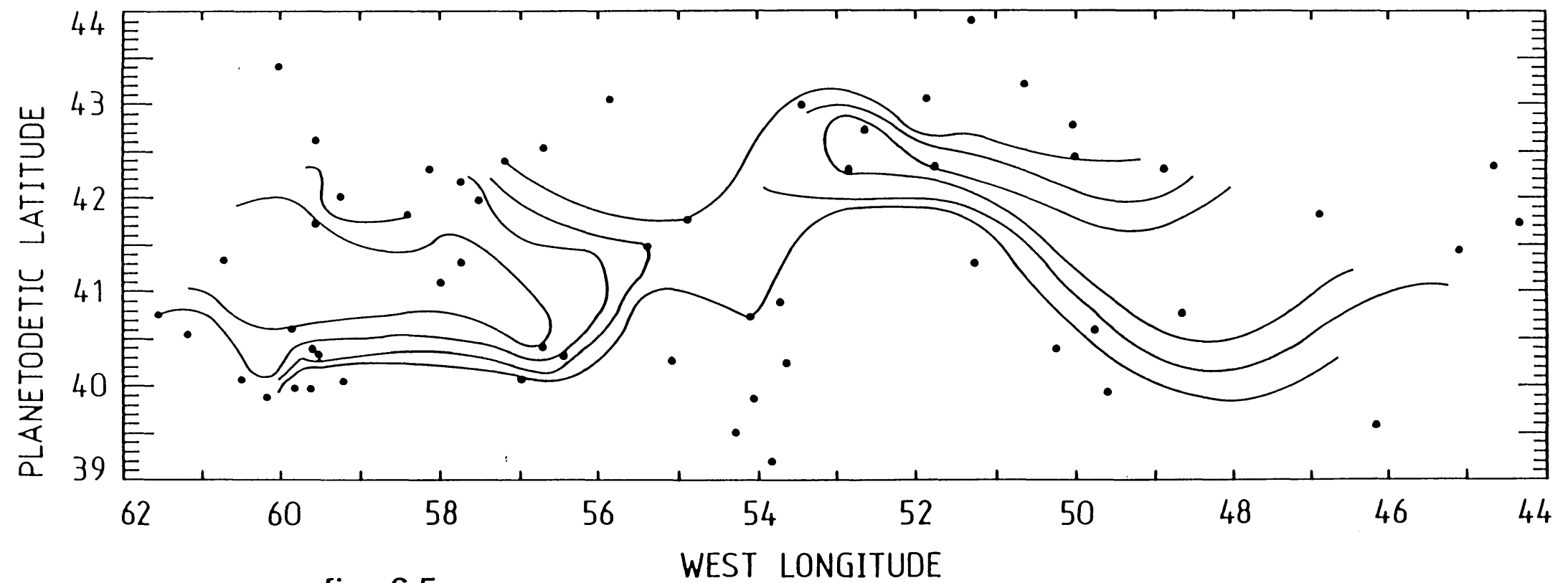


fig. 6.5

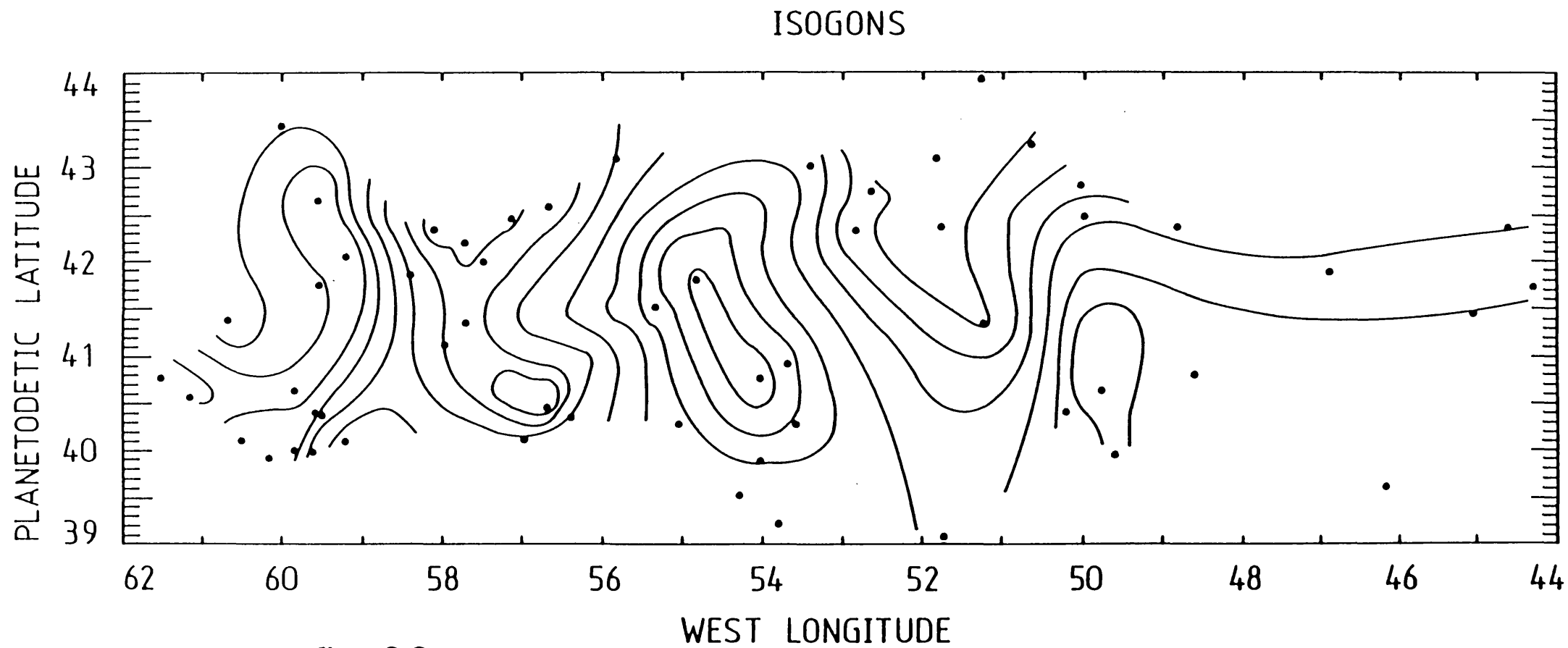


fig. 6.6

STREAMLINES

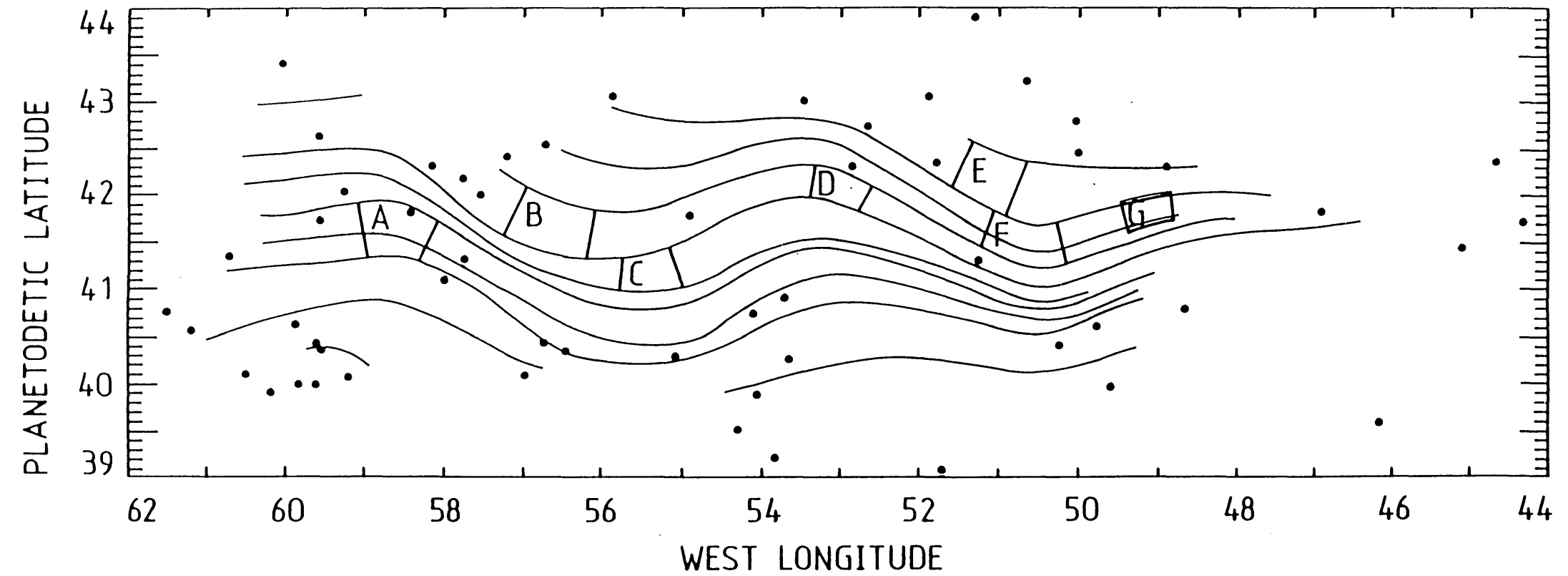
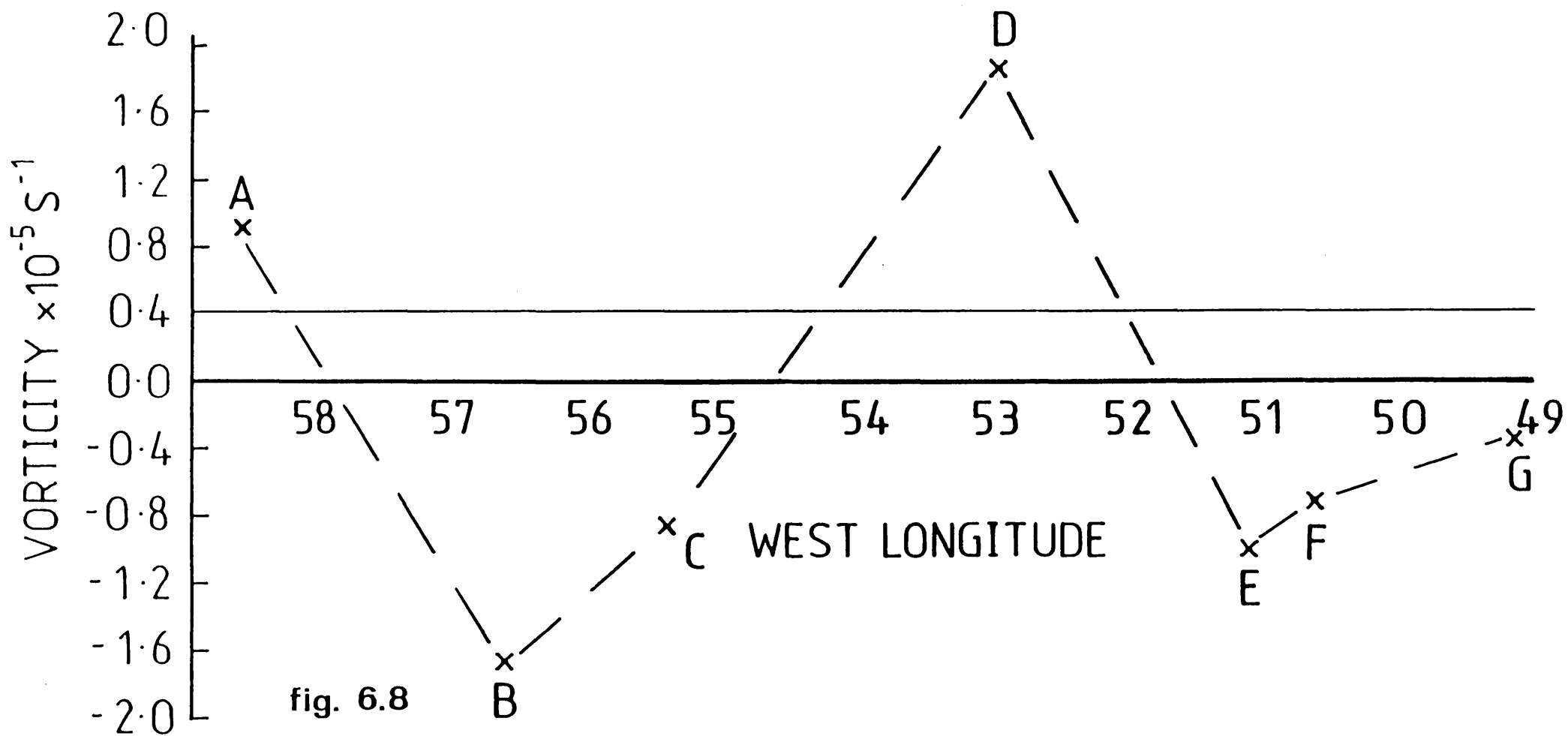


fig. 6.7



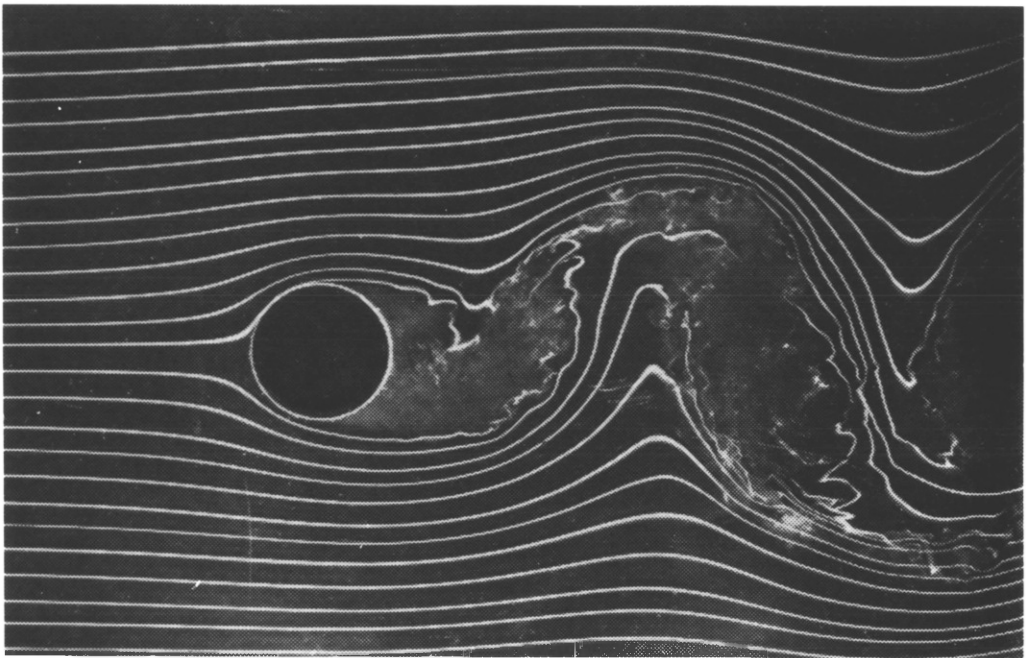
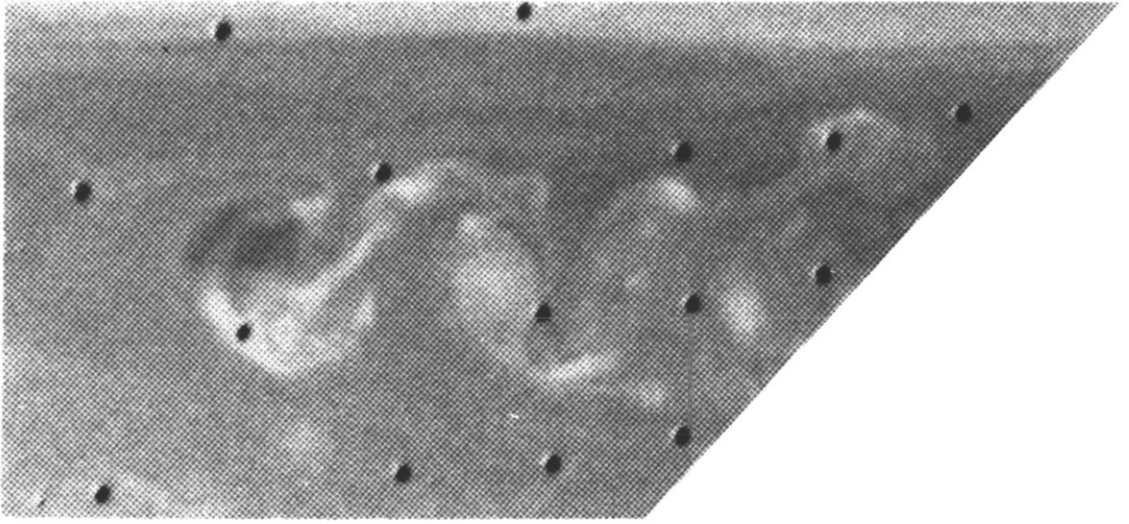


fig. 6.9

CHAPTER 7

RIBBON FEATURE

7.1 INTRODUCTION

This chapter describes the analysis and modelling of a feature which encircles the planet between 45 and 50 deg N. The feature consists of a light band, along the centre of which runs a darker stripe which varies periodically with longitude. Three sets of images were selected to study different aspects of this feature. The first was a high resolution set used to examine the detailed structure, and to make cloud tracked wind measurements. Because of the high resolution this showed only a small longitude range. In order to study the periodicity another set of images were used which covered the full 360 deg longitude range. Initially fourier transform techniques were used to find the wavelength from this. However, as explained in section 7.3.2, this proved difficult and an alternative method was used. A third set of five images with a small time separation, but taken through five different NA filters, was used to examine the appearance at different wavelengths. The use of these, particularly the UV image, to constrain the vertical structure is described in section 7.4.3. As is explained in section 7.5 the results of these investigations, in combination with the latitudinal temperature gradient obtained in chapter 5, suggest that the feature could be produced by a baroclinic instability.

The second part of this chapter considers this possibility in greater detail and attempts to model the process. The equations and boundary conditions used are described in section 7.6 and 7.6.1 followed by the necessary conditions for instability. As explained in section 7.6.2 these require the value of the vertical wind shear. This is available from the latitudinal temperature gradient, but only up to 150 mb. The value above this level is estimated by approximating the observations with different curves, and extrapolating these to higher levels. Using this technique it is found that all the models satisfy the necessary conditions for instability. In order to investigate the structure and growth rates of the disturbances, two numerical models were produced which attempted to solve the relevant equations for specified wind shear profiles. The results of the various different models are described in the final section of this chapter, and are compared with the measurements derived in the first part of the chapter. It is found that the model used agrees well with the observations. The differences can be accounted for by the omission of such possibilities as frontal effects, together with the lack of knowledge of the thermal wind shear at higher levels.

7.2 HIGH RESOLUTION IMAGES

This is a set of six green filter images, composed of three pairs taken on consecutive rotation periods three to four days before encounter. Since the WA image nearly fills the field of view these are almost the highest resolution, navigable, images of the ribbon feature.

The results of map-projecting, filtering, and joining the image pairs together is shown in figs 7.1a,b and c. It can be seen from these that the feature consists of a light zone containing a darker band showing a distinct north-south oscillation with longitude. This darker stripe appears fairly smooth except for one region where there is a local kink associated with some bright clouds. This, as will be

described in section 7.3, is one of several places in which this occurs. To the north of the dark stripe there are a few elongated white clouds which, when the images are displayed sequentially in the form of a movie, can be seen to be rotating anticlockwise. It also appears from the images as if the circulation to the south of the wave is clockwise, but this cannot be confirmed. These images were originally intended for cloud tracking but they were found to contain almost no suitable features. The results obtained only confirmed that the feature is located in an eastward (westerly) jet moving at $\sim 120 \text{ ms}^{-1}$, although it is not clear whether this is a material or phase speed. The meridional velocity also showed a lot of variation but had an average speed of $\sim 5 \text{ ms}^{-1}$.

However it was possible to determine from these images the width of the zone as $5^\circ \approx 5 \times 10^3 \text{ km}$ and an estimate of the meridional wavelength. If we assume that the feature extends from the elongated white clouds in the north to the bottom of the darker parts of the wave to the south, this gives a wavelength of $9 \times 10^3 \text{ km}$.

7.3 PRODUCTION OF THE MOSAIC USED TO DETERMINE THE WAVELENGTH

In order to produce a mosaic showing all 360° of longitude it is necessary to use several images, taken over at least one rotation period. Five images (taken about a week before closest approach) with a separation of about two hours were used to provide the full coverage at a reasonable resolution. Each of these images was map-projected to produce 5 overlapping cylindrical projections, all with a scale of 52 km/pix at the equator.

Several methods were tried to produce a single large mosaic from the five component parts. However it was found that the most acceptable results were obtained by simply taking the maximum brightness at each point. Finally, to show up the small scale structure better, the space variant filter program TFILT (see

section 3.7) was used.

Although the cylindrical projection mosaic was used for all the measurements, it is more convenient to present the resulting image in the form of a polar stereographic projection. This is shown in fig 7.2. This was produced in a similar way to the cylindrical projection, with the addition of a latitude-longitude grid (using the program DMAPGRID).

It should be mentioned that when producing the map-projection no allowance was made for the change in longitude produced by the zonal flow. However the longitudinal displacement produced by a 100 ms^{-1} jet in 10 hours would be ~ 72 pixels, approximately 1% of the total length.

It is clear from fig 7.2 that the feature extends around the whole planet and shows a distinct oscillatory structure. In a few places ($\sim 40^\circ, 175^\circ, 355^\circ$) white patches of cloud seem to be associated with local 'kinks' in the wavelike pattern. One of these was shown in the higher resolution images described in the previous section.

7.3.1 PRODUCTION OF THE LINE-SAMPLE DATASET

The next stage was to convert the structure of the feature into a quantitative form so that it could be used for further analysis. This was done by dividing the cylindrical projection into fourteen, 512 pixel long sections and displaying these, one at a time, on the IIS. Using the cursor a series of points lying along one side of the dark band were identified, and the 400 line-sample coordinates obtained were saved in the data-base. Although the data set produced is fairly reliable there are some locations at which the edges of the feature are not very clear, leading to possible spurious points. Another problem is caused by the occasional white patches of cloud which make the identification of the edges of the dark band less

reliable. However, bearing these limitations in mind, the resulting set of points is shown in fig 7.3. The sample number S in fig 7.3 can be related to the west longitude λ shown in fig 7.2 by

$$\lambda \approx 360 - \frac{S}{20.25} \quad (7.1)$$

7.3.2 FOURIER TRANSFORM ANALYSIS

Figure 7.4 shows the result of Fourier transforming (after interpolating to obtain a more convenient number of points) the data shown in fig 7.3. The difficulty with this technique lies in estimating the significance of the results obtained. Should the individual peaks be considered, or is only the large scale structure of significance?

Two of the terms which are almost certainly real are the components of wavenumbers zero and one (both too large to be shown in fig 7.4). The zero frequency part corresponds to the location of line zero in the measurement process. The wavenumber one component is more interesting and is almost certainly due to a navigation error. The effect of this error appears in fig 7.2 as a displacement between the centre of the grid and the location of the north pole. The effect of subtracting the wavenumber zero and one components from the original data-set is shown by the dotted line in fig 7.5.

The importance of the other components is more difficult to assess. However an attempt has been made to estimate the importance of the two components (wavenumbers 8 and 20) with the largest amplitude. The continuous line in fig 7.5 shows the result of adding these two terms with the appropriate amplitudes and phases. It can be seen that these components seem to be strongly influenced by the largest peaks, with 5 out of 7 of the largest peaks corresponding to extrema in the continuous curve. Since all four of the largest peaks correspond to longitudes ($\sim 59^\circ$, 173° , 350° and 356°) at which the anomalous white clouds are located (see fig 7.2) both the relevance

and measurement error of these peaks are questionable.

Because of this difficulty in assessing the importance of the smaller scale structure in fig 7.4 it is probably safer to interpret this as showing just a broad peak falling off at wavenumbers $\sim 40-55$.

7.3.3 ALTERNATE DERIVATION OF THE WAVELENGTH

Because of the difficulties experienced with the Fourier transform analysis, it was decided to use a different approach to estimate the wavelength. This consisted of finding all the extrema and taking the wavelength to be twice the average separation. This method had the advantage that it is fairly easy to check the results, and to see how they are affected by various parts of the data.

In order to use this method the data was interpolated to obtain a uniform 10 pixel spacing, and then the program MAXFIND was used to find the maximum and minimum values and their separation. This produced the distribution shown in fig 7.6 (the dashed line) which has a mean separation of 64.4 ± 4.1 pixels.

It can be seen from fig 7.6 that the distribution shows a distinct bimodal structure. From a comparison with fig 7.3 it appeared that the peak around 10-20 pixels could be caused by the small scale 'noise' in the data rather than the larger amplitude components. In an attempt to remove this effect the program was altered so that it ignored all the pairs of maxima and minima separated by less than three pixels in the line direction. This produced the distribution shown in fig 7.6 by the continuous line. It can be seen from this that most of the small sample separations were associated with small line separations and have been removed. This has slightly reduced the size of the 60-100 pixel peak and created a couple of subsidiary maxima at ~ 170 and 200 pixels. From an examination of the measurements it appeared that these were created when two (or three) peaks with a separation in the 60-100 pixel range had an intermediate peak removed. This produced

'echoes' of the main distribution occurring at two and three times the average spacing. For this reason all the values outside the main peak (30-130) pixels were rejected resulting in a mean separation of

$$76.5 \pm 3.5 \text{ pixels}$$

The longitudinal scale of the map projection is 20.25 pix/deg. Combining this with the longitudinal scale at this latitude (at $\theta_d = 47^\circ$ this is $756 \times 10^3 \text{ m/deg}$), and the factor of two needed to convert the separation between adjacent extrema to a wavelength gives

$$5710 \pm 260 \text{ km}$$

or alternatively, expressed as a longitudinal wavenumber we have

$$k_x = 1.1 \pm 0.05 \times 10^{-3} \text{ km}^{-1}$$

7.4 MULTISPECTRAL CHARACTERISTICS

This study is based upon a set of images taken through five different narrow angle filters with the effective wavelengths shown below (from Smith et. al. 1977).

filter	effective wavelength (Angstroms)
Orange	5910
Green	5660
Blue	4790
Violet	4160
Ultraviolet	3460

These pictures were taken sequentially with a time separation of about 5 minutes, 11 days before closest approach. They were then map-projected to a common scale of 50 km/pixel, and four of them are shown in figs. 7.7 a,b,c and d.

It can be seen from these figures that in most of the filters the feature appears approximately the same: a light band with an oscillatory dark stripe down the centre. The violet image is slightly different, with the southern part appearing a little brighter than the northern part. The filter which produces obviously different results is the ultraviolet (see fig 7.7d) in which the oscillatory pattern, instead of being superimposed upon the lighter band, acts as a division between the lighter (northern) and darker parts. This abrupt change between the violet and UV images (over a comparatively small wavelength range - not much larger than that between the blue and violet filters) is particularly interesting. This is because the UV is sensitive to the higher parts of the atmosphere and normally shows little variation over the planet, indicating that this feature extends unusually high into the atmosphere.

7.4.1 MULTISPECTRAL CHARACTERISTICS - QUANTITATIVE RESULTS.

Since the images have been radiometrically decalibrated (see section 3.6.1) the DN level is proportional to the incident flux. This means that the intensity change from north to south across the feature can be estimated from a graph of DN against line number produced from the images shown in fig 7.7. In order to remove some of the noise, the graph shown in fig 7.8a shows the mean DN value, averaged over 5 samples, as a function of line number for the violet image.

By using the same method for all five images it became apparent that all the filters (except the UV) produced similar results to those shown in fig 7.8a, with a fairly regular fall off of intensity with increasing latitude (ie decreasing line number). It was realized that 'limb-darkening' due to the changing sun-angle could partially account for this. The approximate size of this effect was investigated by altering the map-projection program (see section 3.6.4)

so that the DN value at each point was divided by the cosine of the angle between the incident solar radiation and the local normal to the surface. This produced the graph shown in fig 7.8b. It can be seen that this is closer to the structure seen in the original images, with a darker region between the two lighter regions.

Although this method indicates the importance of geometric effects it is difficult, without constructing a detailed model of the scattering processes involved, to compensate for this type of effect. Because of this a more empirical approach was adopted. The image taken through the green filter, as can be seen from fig 7.7, appears to be very nearly uniform apart from the dark band. It was decided to assume that this image was uniform, and to 'remove' the geometric effects from the other images by dividing by this image. It should be stressed that this is only an assumption, and that it fails to compensate for the changing spacecraft-planet geometry, or for the change in the scattering properties as a function of wavenumber.

When dividing some of the images by the green image it became apparent that there was a latitudinal offset between some of them, presumably caused by errors in the navigation parameters. These were compensated for by comparing graphs of DN against line number, like that shown in fig 7.8b, and introducing offsets in the line direction until the positions of corresponding features overlapped. Unfortunately, due to the very different appearance, it was impossible to carry out this procedure for the UV image which was assumed to be correctly aligned.

The result of shifting the images, dividing by the green image, multiplying by 128, and plotting graphs of DN against line number is shown in figs 7.9a, b, c, and d. In order to reduce the possibility of introducing spurious results during the division process, due to local features being more apparent in some filters than others, these graphs correspond to averages over a large number of samples

It can be seen from these figures that although variations exist between the north and south sides of the feature at different wavelengths they are small (approximately 3%) except in the ultraviolet which shows the north side to be significantly brighter than the south side.

Since the line numbers of the top and bottom part of the feature are known, fig 7.9d was used to find the change in UV intensity across the feature. The DN value derived for the south part of the feature is ~ 35.6 , whilst that for the north part is ~ 42.0 giving an increase of $\sim 18\%$. The error in this quantity is difficult to estimate but is likely to arise mainly from the division by the green image. An estimate of this error can be obtained by using instead the orange or blue images. This gives the ratio of the UV emission from the northern part of the feature to that of the southern part as $\sim 1.18 \pm 0.2$.

7.4.2 ESTIMATES OF THE HEIGHT OF THE FEATURE FROM THE UV FLUX

At UV wavelengths the λ^{-4} Rayleigh scattering behaviour of hydrogen molecules becomes important, because of this measurements made in the UV provide information about the vertical structure of the atmosphere. Although the Voyager UV filter is at the long wavelength end of the UV it is still possible to use the observed change in emission across the feature to place a lower limit on the height at which the albedo change observed in fig 7.7d can occur.

A simple model consists of a non-absorbing layer of hydrogen of optical depth τ over a base layer with a single scattering albedo of ω . Incoming radiation which passes through the hydrogen layer, is scattered from the bottom, and re-emerges from the top will have an intensity $\omega e^{-2\tau}$ times the incident intensity. As the thickness of the overlying hydrogen layer is increased there comes a point at which, even if ω changes from 0 to 1, the observed

intensity change across the feature cannot be reproduced. This will occur at an optical depth τ_c for which

$$e^{-2\tau} = \frac{42.0 - 35.6}{42.0} = 0.152 \quad (7.2)$$

Which gives $\tau_c = 0.942$. The optical depth due to Rayleigh scattering of an atmosphere with a 94% hydrogen - 6% helium mixture at a wavelength λ (in μm) corresponds to a pressure level P (in bars) given by (West et. al. 1983)

$$\tau = 0.0244(1+0.016\lambda^{-2}) P \lambda^{-4} \quad (7.3)$$

At the effective wavelength of the voyager UV filter, $\lambda = 0.346 \mu m$ this gives $\tau = 1.93 \times P$. Using $\tau = \tau_c$ this gives the pressure of the deepest layer at which the observed intensity change could be produced as ≈ 490 mb.

7.4.3 UV CONTRAST - MORE ADVANCED MODEL

Because of the inadequacies of the simple model, particularly the omission of absorption due to a stratospheric aerosol, a more complex model was used. This was one of the models used by Moore (1983) to model the globally averaged UV albedo observed by the (International Ultraviolet Explorer) IUE satellite. Her results for part of the longer wavelength range are shown in fig 7.10 a and b. The model used for this contained 3 km-am of molecular hydrogen over a mixed layer containing 15 km-am of hydrogen and a pure absorber which had an optical depth of 0.5 at 3100 Å. Below this, at a pressure of 172mb, there was a layer with a (frequency independent) single scattering albedo ω . The absorber used is presumed to be some form of Axel dust (see section 5.3) and to absorb as λ^{-1} on λ^{-2} , as shown in figs 7.10 a and b. The model preferred by Moore (1983) used the λ^{-2} absorber, with ω in the range 0.98-0.995.

The effect of using the same model to generate the geometric albedo at both 3100 Å and 3460 Å as a function of the albedo of the base layer is shown in fig 7.11, from which it can be seen that, due to the λ^{-4} behaviour of Rayleigh scattering, the value at 3460 Å is more sensitive changes in ω than that at 3100 Å. Although fig 7.11 shows that the change in albedo with ω this is not an appropriate quantity to compare with the change in intensity described in section 7.4.1. This is because the albedo calculated is integrated over the whole disk, whilst the measurements correspond to a small area. This is important since changes in the angle of incidence alter the optical path length between constant height surfaces, and hence the scattering properties. (This is different to the limb darkening effect described in section 7.4.1, which will also change the observed albedo with latitude, but only by a frequency independent scale factor.)

Figure 7.12 shows the log of the ratio of the ingoing and outgoing fluxes as a function of ω at a distance of ~ 0.75 radii from the centre of the disk. Also shown is the observed intensity change across the feature derived in section 7.4.1. It can be seen from this figure that if the maximum possible base albedo of 1.0 corresponds to the brighter (northern) side of the feature then the observed change could be reproduced by reducing ω to ~ 0.984 . However if the maximum value of ω used is 0.995 (the largest value that closely fits the globally averaged values shown in fig 7.10b) then a very large change of ω must occur across the feature. Since there are some parts of the planet which appear in the UV filter to show greater extremes of intensity than those found in the ribbon feature it seems unlikely (assuming that the value of ω can be taken to be approximately frequency independent) that the range of ω should much exceed that of 0.98-0.995 required by the globally averaged observations. These results suggest that, it may just be possible to reproduce the

observed intensity change across the feature by altering the albedo of the base layer. However to be certain of remaining within the constraints imposed by the globally integrated albedo measurements at least some of the latitudinal change in intensity must occur higher than the base layer, possibly caused by local variations in the optical depth of the aerosol present.

Since the pressure level at the base of this model is ~ 170 mb these results show that the process producing the observed feature must still have a reasonable amplitude at this height.

In addition to providing a lower limit to the vertical extension of this feature, the images shown in figs 7.7 a,b,c and d provide a further constraint upon the feature's structure. This arises from the observation that although the visual appearance changes between the green and UV images the variation with longitude is nearly identical with no apparent phase shift. This is interesting since the longer wavelengths are probably being reflected from the top of the ammonia clouds at around (see section 5.2) 400 mb. Hence the pattern at longer wavelengths will be caused by ammonia condensing out, probably due to vertical motion. The UV contrast is due to either the very top of the ammonia clouds or to vertical displacement of the stratospheric aerosol. So that to account for the longitudinal similarity of the UV and other images it is necessary to have a very small phase change in the vertical velocity field between ~ 170 and ~ 400 mb.

7.5 POSSIBLE TYPES OF INSTABILITY

The feature, as is apparent from fig 7.2 and was shown in section 7.3.3 has a fairly regular structure. However it seems unlikely that it is just a stationary cloud pattern since there is, as shown in section 7.4.3 little longitudinal variation with height while fig 5.11 shows that this region has a large vertical shear. Because of this it seems

reasonable to assume that the feature is some kind of wave. This could be either a response to some external forcing or might be generated internally by some form of instability. From the wavelength given above, assuming that the zonal flow is $\sim 120 \text{ ms}^{-1}$ and using the rotation period given in section 3.4.1.2 it is possible to estimate the period required for the entire pattern to rotate about the axis as 10hr 29 min. This is less than the rotation period of any of the satellites (see Smith et. al. 1981). It also seems unlikely that this external forcing could produce the feature over the small latitude range observed without producing similar effects elsewhere. Because of this it was decided to examine the possible types of instability which could occur.

There are many types of instability which can arise in a fluid. However it is possible to restrict those which need to be considered by an examination of various dimensionless parameters. For disturbances due to the latitudinal temperature gradient, or equivalently the vertical shear, a convenient parameter is the Richardson number Ri described in section 4.2. The value of Ri , which can be estimated from fig 5.7, (which uses $\partial u / \partial z = -5 \times 10^{-4} \text{ s}^{-1}$, a value close to the largest value observed, see fig 5.11) is significantly greater than one, suggesting that a conventional (rather than symmetric) baroclinic instability could occur.

Another possibility is a barotropic instability which derives its energy from the horizontal shear of the flow. There are, as described in section 4.6 two possible types of barotropic instability with the associated conditions for instability shown in fig 4.4. It can be seen from this that the jet at $\sim 47^\circ \text{ N}$ could be conventionally (shallow flow) unstable at the edges, or unstable at the centre due to the alternate (deep flow) condition produced by Ingersoll and Pollard (1982).

The structure of such an instability cannot be determined due to the lack of information about the flows in the deep interior. It is also not apparent for this type of disturbance what the length scale associated with the largest growth rate is, although it is expected to be quite small. And there is some doubt as to whether this type of disturbance would be seen at the surface or if, as suggested by Busse (1976) for the convective cells driving this model, it would be restricted to the lower levels. This is particularly relevant for this feature since, as shown by the abrupt change in the UV albedo (see section 7.4.3), it extends unusually high into the atmosphere.

The relative importance of (shallow) barotropic and baroclinic instability can be estimated by comparing the observed length scale L with the internal Rossby radius of deformation L_d , given by (Pedlosky 1979)

$$L_d = \frac{N D}{f} \quad (7.4)$$

where D is a representative depth scale. If we use

$$N^2 \approx 5 \times 10^{-5} \text{ s}^{-2} \quad (\text{see fig 5.5})$$

$$D \approx 4 \times 10^4 \text{ m} \quad (\text{the pressure scale height})$$

$$f \approx 0.25 \times 10^{-3} \text{ s}^{-1} \text{ rad/sec}$$

then we obtain

$$L_d \approx 1.1 \times 10^3 \text{ km}$$

In general length scales less than L_d will be associated with barotropic instability while length scales as large or greater than L_d correspond to baroclinic instability. The observed length scale is larger than L_d suggesting that barotropic instability is unlikely to be the cause. This seems reasonable since the horizontal shear from which this type of flow obtains its energy is smallest at the centre of the feature.

Another length scale of interest is the Rhines parameter (see section 4.5.1) which provides an approximate upper limit to the size to which baroclinic (and other) eddies can easily grow. This is given by

$$L\beta = \frac{2\pi}{k\beta} = 2\pi \left(\frac{2U}{\beta} \right)^{\frac{1}{2}} \quad (7.5)$$

At the appropriate latitude $\beta \approx 0.4 \times 10^{-11}$. However the value of U which should be used is less apparent. The meridional velocity associated with the eddies in the wave is $\sim 5 \text{ ms}^{-1}$ whilst the mean zonal flow at this latitude is greater than 100 ms^{-1} . However it is apparent from Rhines (1973) that the important quantity is the time scale associated with the eddy motion when compared with the frequency of a Rossby wave of the same scale. Because of this it seems more reasonable to use $U \sim 5 \text{ ms}^{-1}$ in equation 7.5. This gives a length scale of

$$L\beta \approx 10^4 \text{ km}$$

It is apparent from the preceding discussion that the feature observed has a wavelength between the size with which (baroclinic) eddies are expected to form and the maximum scale to which they are expected to grow. This should not perhaps be too surprising since the latitude at which the ribbon feature is seen shows the strongest latitudinal temperature gradient (see fig 5.10), and is consequently the latitude at which there is the maximum potential energy available to baroclinic effects.

It should be realized that the above list of possibilities is not conclusive. In particular there may be various types of radiative instabilities associated with the abrupt albedo change (particularly in the UV) across the feature.

7.6 BAROCLINIC INSTABILITY THEORY

The energy driving a baroclinic instability lies in the reservoir of available potential energy which must accompany a latitudinal temperature gradient. It can be shown (Hide and Mason 1975, Green 1981) that potential energy is released from this if particles of fluid are exchanged along paths which lie between the horizontal, and surfaces of constant potential temperature.

The equations used to examine this type of instability use the hydrostatic quasi-geostrophic beta-plane approximations, as derived, for example, by Holton (1979). The use of the quasi-geostrophic system can be justified by the size of the Rossby number which, taking $U \sim 100 \text{ ms}^{-1}$, $L \approx 5 \times 10^6 \text{ m}$, and, $f \approx 0.24 \times 10^{-3} \text{ s}^{-1}$ is:

$$Ro = U/fL \approx 0.08 \quad (7.6)$$

indicating the importance of geostrophic flow. In addition the Richardson number (see section 4.1 and fig 5.7) is large so that the direct effects of vertical shear and convection can be neglected.

Although vertical accelerations, neglected in the hydrostatic approximation, can not be directly measured they can be estimated from the vertical velocity derived by Hunt et. al. (1982). If we use the value $W \approx 1 \text{ ms}^{-1}$ and take one rotation period as an estimate of the characteristic time period then this gives a value considerably ($\sim 10^5$) smaller than the value of ρg due to the effects of gravity.

The various nonlinear curvature terms encountered in the development of the quasi-geostrophic system (see, for example, Holton chapter 2) are normally neglected. However the size of these terms was checked by estimating the values of u, v, f etc. at the latitude of the ribbon feature. It was found that all but one of the neglected terms were smaller than the retained terms. The exception was the $u^2 \tan(\theta)/a$ term in the equation containing the y component of the momentum, which was close to the size of the

meridional acceleration (dv/dy) term. However the terms containing u^2 are caused by the zonal flow producing a centrifugal force away from the axis. If the surface of the planet is assumed to lie on an equipotential, then the effect of this term will be to distort the surface shape. This effect was considered by Hubbard (1982) who showed that the presence of differential rotation produced a 'bulge' 8 km high at 45 deg N. Thus the main effect of this term will be to alter the value of g and also, due to the changed surface slope, the planetodetic latitude. However these effects will be small in comparison with the changes of g , f , and β which will occur over the finite latitudinal width of the feature being considered.

The appropriate equations expressed in isobaric coordinates, are given by Holton (1979) as

$$\frac{\partial}{\partial t} \left(\frac{-\partial \Phi}{\partial p} \right) = -\underline{V}_g \cdot \underline{\nabla} \left(\frac{-\partial \Phi}{\partial p} \right) + \sigma \omega \quad (7.7)$$

and

$$\frac{\partial \zeta}{\partial t} = -\underline{V}_g \cdot \underline{\nabla} (\zeta + f) - f_0 \underline{\nabla} \cdot \underline{V} \quad (7.8)$$

where σ is a static stability parameter defined by

$$\sigma = \frac{-1}{\rho \theta} \frac{\partial \theta}{\partial p} \quad (7.9)$$

and

$$\zeta = \underline{\nabla}^2 \Phi / f_0 \quad (7.10) \quad \underline{V}_g = \underline{k} \times \underline{\nabla} \Phi / f_0 \quad (7.11)$$

with

Φ	-	Geopotential
ω	-	Dp/Dt
f	-	coriolis parameter
θ	-	potential temperature

If we now indicate the dimensional variables with *'s, and estimate the characteristic length, velocity and pressure to be L , U , and P_0 respectively we can define non-dimensional variables by:

$$x^*, y^*, p^* = Lx, Ly, P_0 p$$

$$u^*, v^* = Uu, Uv$$

$$t^* = (L/U)t$$

$$\omega^* = (P_0 U/L)\omega$$

$$\phi^* = f_0 U L \phi$$

$$\zeta^* = (U/L)\zeta$$

$$\beta^* = (U/L^2)\beta$$

$$\sigma^* = \frac{ULf}{P_0^2} \sigma$$

after differentiating equation 7.7 , and using this to eliminate ω from equation 7.8 we obtain :

$$\frac{D\zeta}{Dt} + v_g \beta - \frac{\partial}{\partial p} \left[\frac{1}{Ro \sigma} \frac{D}{Dt} \left(\frac{-\partial \phi}{\partial p} \right) \right] = 0 \quad (7.12)$$

where Ro is the Rossby number. We now split ϕ into a mean part $\bar{\phi}$ and a perturbation part, where the perturbation part is assumed to vary sinusoidally with x, y and t , with the associated wavenumbers k_x and k_y and a phase velocity c , this can be written

$$\phi(x, y, p, t) = \bar{\phi}(y, p) + \phi(p) \exp(i(k_x(x-ct) + k_y y)) \quad (7.13)$$

By using equation 7.13 in 7.12 , and neglecting all terms of second order or less, we obtain:

$$(\bar{u}-c) \left(\frac{\partial}{\partial p} \left(\frac{\partial \phi / \partial p}{Ro \sigma} \right) - (k_x^2 + k_y^2) \phi \right) + \left(\beta \bar{u}_{yy} - \frac{\partial}{\partial p} \left(\frac{\partial \bar{u} / \partial p}{Ro \sigma} \right) \right) \phi = 0 \quad (7.14)$$

where $\bar{u}(y, p)$ is the zonal flow produced by the mean part of ϕ .

Although originally derived in pressure coordinates it was found that it was more convenient to work in height coordinates. This can be done by using the definition of σ , and the hydrostatic equation to produce (this time given, for convenience, in dimensional units)

$$(u-c) \left(\frac{f^2}{\rho} \frac{\partial}{\partial z} \left(\frac{\rho \partial \phi / \partial z}{N^2} \right) - (k_x^2 + k_y^2) \phi \right) + \left(\beta \bar{u}_{yy} - \frac{f^2}{\rho} \frac{\partial}{\partial z} \left(\frac{\rho \partial \bar{u} / \partial z}{N^2} \right) \right) \phi = 0 \quad (7.15)$$

and it was the equivalent, dimensionless form of this which was used in the numerical solutions.

7. 6. 1 BOUNDARY CONDITIONS

In addition to equation 7.15 it is necessary to specify appropriate boundary conditions to find a solution. The simplest type of baroclinic instability model is that of Eady (see, for example, Gill 1982) in which the flow is confined between horizontal boundaries. This is equivalent to setting the vertical velocity w to be zero at the boundaries, and requires that

$$(\bar{u}-c) \frac{\partial \phi}{\partial z} - \phi \frac{\partial \bar{u}}{\partial z} = 0 \quad (7.16)$$

Although the use of the $w=0$ condition has produced useful results for the Earth it seems unrealistic, (in the absence of a rigid boundary), to use it for the outer planets. This problem was considered by Gierash et al (1979) who used a two layer model. By altering the depth and static stability of the lower layer they showed that an infinite adiabatic region could be simulated by using $\phi = 0$ as the lower boundary condition.

Neither the $w = 0$ nor the $\phi = 0$ condition are entirely realistic representations of the Saturnian situation. However the $\phi = 0$ condition is probably much more applicable than the assumption of a rigid lower boundary in the atmosphere.

Because we assume that the temperature is constant at the boundary whilst the vertical shear is non-zero, the surface at which the lower boundary condition is applied will not be horizontal. However, as mentioned by Gierash et al (1979), this slope is of the order of the Rossby number, and is normally neglected.

7. 6. 2 NECESSARY CONDITIONS FOR INSTABILITY

Although there is no general solution to equation 7.15 it is possible to derive from it conditions which must be satisfied if the flow is to be unstable. This type of analysis was originally developed by

Charney and Stern (1962), however for the Saturnian situation it is more convenient to use it in the form derived by Gill^{*et.al.*}(1974) for oceanic flows. This is obtained from equation 7.15 by multiplying by ϕ^* (the complex conjugate of ϕ), integrating from a bottom layer at z_1 to the top at z_2 and then taking the complex part of the result. If, for simplicity, we assume $\bar{u}=u(z)$ and define

$$Qy = \beta - \frac{f^2}{\rho} \frac{\partial}{\partial z} \left(\frac{\rho}{N^2} \frac{\partial \bar{u}}{\partial z} \right) \quad (7.17)$$

then we obtain

$$\text{Im} \left[\frac{f^2}{N^2} \phi^* \frac{\partial \phi}{\partial z} \right]_{z_1}^{z_2} + c_i \int_{z_1}^{z_2} \frac{|\phi|^2}{|\bar{u}-c|^2} Qy \, dz = 0 \quad (7.18)$$

Now at the boundaries we use the conditions $w(z_2) = 0$ and $\phi(z_1) = 0$, this gives :-

$$\text{Im} \left[\frac{f^2}{N^2} \phi^* \frac{\partial \phi}{\partial z} \right]_{z_2} = c_i \frac{f^2}{N^2} \frac{|\phi|^2}{|\bar{u}-c|^2} \frac{\partial \bar{u}}{\partial z} \quad (7.19)$$

which, upon substitution in equation 7.18 gives

$$c_i \left\{ \left[\frac{f^2}{N^2} \frac{|\phi|^2}{|\bar{u}-c|^2} \frac{\partial \bar{u}}{\partial z} \right]_{z_2} + \int_{z_1}^{z_2} \frac{|\phi|^2}{|\bar{u}-c|^2} Qy \, dz \right\} = 0 \quad (7.20)$$

For the fluid to become baroclinically unstable we require that $c_i > 0$, and hence the quantity inside the brackets must be zero. Thus for instability to occur it is necessary that either $\partial \bar{u}(z_2)/\partial z$ and Qy have the opposite sign or that Qy changes sign between z_1 and z_2 .

The use of the $\phi = 0$ rather than the $w = 0$ condition at the lower boundary has removed a term containing $\partial \bar{u}(z_1)/\partial z$ from equation 7.20. If this is retained then equation 7.20 is identical to the result produced by Gill^{*et.al.*}(1974), and will allow instabilities to occur even if $Qy = 0$ (as originally considered by Eady).

The significant part played by the boundary conditions in determining whether the flow is to be unstable is important since the use of the $w = 0$ condition is unlikely to be realistic. The effect of horizontal boundaries was considered by Holton (1975) for the

stratosphere. He used $Q_y=0$ and showed that if the meridional heat flux was zero at the boundaries then it must be zero everywhere. This follows from the fact that, if the boundaries are parallel to the isentropes there is no requirement for meridional flows to cross the isentropes, so that meridional heat flux will be zero. However if the horizontal boundaries do not lie on an isentrope then the flow is constrained, at least close to the edges, to cross the isentropes. This gives the possibility of a non-zero meridional heat flux.

7.7 BAROCLINIC INSTABILITY - SUMMARY

As shown in the previous section the presence of available potential energy does not necessarily mean that baroclinic instabilities will grow. There have however, been many analyses of flows in which this does occur, with different assumptions and boundary conditions. This section describes some of these and the way in which they satisfy the necessary conditions for instability described in section 7.6.2.

The simplest models assume a constant vertical shear, constant static stability, and an infinite density scale height. One of these is that due to Eady, see, for example, Gill (1982). In this the mean flow is assumed to be confined between two horizontal boundaries on an f -plane ($\beta = 0$). In this case the flow is unstable because of the two boundaries, but has a short wave cut-off after which flow becomes stable. The amplitude of the temperature, pressure and vertical velocity fields produced reach a maximum at, and are symmetric about, the mid-level. This is the level at which the phase speed of the wave equals the velocity of the mean flow and is called the steering level. The structure of this disturbance has the lines of constant phase varying with height, with the pressure field at the top lagging 90 degrees behind that at the bottom surface.

A similar, but beta plane model, without the upper boundary was considered by Charney, and is described in Gill (1982). In order for this to be unstable it is necessary that $Q_y > 0$ (assuming a negative equator to pole temperature gradient), which will occur when beta is greater than zero. Unlike the Eady wave this has no shortwave cut-off. The flow produced by these conditions has the greatest meridional heat flux close to the boundary, with the amplitude of the disturbance decreasing with height.

A more complex case with both a rigid upper boundary and $\beta \neq 0$, was considered by Green (1960). In this, it was shown that, as beta increased, the instability became concentrated nearer the lower boundary. He also studied flows with a finite density scale height, and found that this lowered the steering level, and made the short waves unstable even if $\beta = 0$.

For flows in which the zonal velocity varies nonlinearly with height it is possible, as explained in section 7.6.2, for the flow to become unstable regardless of the conditions at the boundaries. This more general case was considered by Gill ^{et. al.} (1974) who evaluated the growth rates and energy conversion rates of oceanic flows with different vertical shear profiles. For the flows in which Q_y did not change sign it was found that, similar to the Charney case, the disturbance was associated with a boundary, at which the maximum energy conversion rate occurred. However, when Q_y was allowed to change sign, the rate of energy conversion developed a second maximum away from the boundary. An extreme example of this occurred when Q_y changed sign but $\partial \bar{u} / \partial z = 0$ at the top. This produced a disturbance which was associated only with the change in sign of Q_y , the energy conversion rate becoming zero at the upper boundary.

For the outer planets the nature of the lower boundary is uncertain. This problem was considered by Gierasch et. al. (1979) who used a two layer model with a constant static stability in each

layer and a constant value of $\partial \bar{u} / \partial z$ in the upper layer and $\bar{u} = 0$ in the lower layer. The boundary conditions used were that $w = 0$ at the top and bottom. In addition it was assumed that $\beta = 0$, ie $Q_y = 0$. They found that if the static stability of the lower layer was zero then, as the depth of the lower layer increased, the boundary condition at the interface approached $\phi = 0$. They also found that as the depth of the lower layer increased, the growth rate of the fastest growing disturbance decreased. This seems reasonable since, as shown in section 7.6.2, if $Q_y = 0$ instability cannot occur with the $\phi = 0$ lower boundary condition.

Since the disturbances they examined penetrated deep into the atmosphere, Gierash et. al. (1979) also considered the possible changes caused by a non-hydrostatic lower layer. They found that this led to similar growth rates, but with a different wavelength dependence. However, since they assumed a constant density lower layer, and because their model is only unstable because of the presence of the $w = 0$ condition at both the upper and lower boundaries, the significance of these results is questionable.

Another version of this problem was considered by Conrath et. al. (1981), who also used the $\phi = 0$ lower boundary condition. However, because of the particular way in which the Rossby radius of deformation and vertical shear were assumed to vary with height, their model had $Q_y = \beta$. Since they also assumed that the zonal velocity was zero at the bottom, and changed linearly with height they found that, using a form of the condition for instability given in section 7.6.2, only westward currents could be unstable.

However, as was suggested by Mitchell (1982), it can be seen from section 7.6.2 that the important quantity for determining instability is not $\bar{u}$ but $\partial \bar{u} / \partial z$ and if, as appears to be indicated by the IRIS results, u tends towards zero with increasing height, it will be the eastward jets which are unstable. Even with this alteration there is

uncertainty in the relevance of the results due to the restrictions placed by Conrath et. al. (1981) on the zonal velocity and the static stability. It is because of the particular form of these used that their results depend so strongly on the conditions at the boundaries. The only general way in which the stability of any particular case can be examined is by evaluating Q_y as a function of height and comparing this with $\partial\bar{u}/\partial z$ at the boundaries as described in section 7.6.2. This type of approach was used by Holton (1975) for the Earth's stratosphere and Gill^{et. al.} (1974) for the oceans.

The various analytical and numerical models discussed produce a detailed insight into the way in which baroclinic instabilities can occur. However they assume that the disturbances considered are of small amplitude and often neglect second order effects such as frontogenesis. It is partially because of this that the results of laboratory experiments are important. Fig 7.13 shows the results of some of the experiments described by Hide and Mason (1975). These show that the flow, although having the oscillatory structure that would be expected from the simple theory, also shows a distinct jet-stream structure. In addition to this it can be seen that at high rotation rates the flow breaks down, and becomes irregular, whilst at very low rotation rates the flow is axisymmetric. Another interesting result from this type of experiment concerns the existence of 'vacillations', in which the wave patterns undergo regular changes in amplitude, wavenumber, or shape, with a period approximately 20-100 times the rotation period.

7. 8 NUMERICAL SOLUTION OF THE EQUATION

Equation 7.15 along with the two boundary conditions constitutes a second order differential equation in ϕ containing a complex eigenvalue c . One approach to solving this is to rewrite the equation so that all the terms multiplied by c are on one side and then, use the

difference operators

$$\left(\frac{d\phi}{dz} \right)_{z_n} \approx \frac{\phi(z_n+dz) - \phi(z_n-dz)}{2 dz} \quad (7.21)$$

and

$$\left(\frac{d^2\phi}{dz^2} \right)_{z_n} \approx \frac{\phi(z_n+dz) - 2\phi(z_n) + \phi(z_n-dz)}{dz^2} \quad (7.22)$$

Equation 7.15 can then be approximated at the points $z=z_n$ $n=0,N$ by N simultaneous equations containing the N values of ϕ plus ϕ_0 and ϕ_{N+1} . Inclusion of the boundary conditions described in section 7.6.1 makes the set soluble, and equation 7.15 is reduced to the approximation

$$Ax = cx \quad (7.23)$$

where A is a non-symmetric tridiagonal matrix

, x is an $N \times 1$ column matrix containing the value of $\phi(z_n)$ and c is the complex velocity. This is a standard eigenvector/eigenvalue problem, and the NAG program F02AGP was used to find the N solutions. The solution of greatest interest is that with the maximum growth rate, ie the largest imaginary part of c .

This method worked well with the simpler, Eady type, models and had the advantage that no prior knowledge of the solution was required. However it was found that it produced discrepancies when trying to obtain solutions with a more detailed vertical structure, such as those produced by Gill *et al.* (1974). Although this could probably be solved by increasing the vertical resolution, this would be inefficient because the time required to solve the $N \times N$ system of equations increases approximately as N^3 (the largest value of N which it was convenient to use was 100 which took approximately 15 minutes of CPU time and generated $\sim 10^6$ page faults).

It was because of these difficulties that an alternative method was used. If we substitute

$$\theta = \frac{\partial \phi}{\partial z} \quad (7.24)$$

in equation 7.15 we obtain a system of four first order differential equations, with the associated boundary conditions being derived from those described in section 7.6.1, and the velocity c being an unknown eigenvalue. One of the ways in which this type of problem can be solved is by a technique known as the 'shooting method'. This assumes a value for the unknown parameters at one boundary and integrates (using for example Runge-Kutta or predictor corrector methods) to the other boundary. From the difference between the supplied boundary conditions and the values obtained a better estimate of the parameters can be made. If this process converges it can be used to obtain the values of the unknown parameters to a specified accuracy. This type of method is used by the NAG routine D02HBF which was used to solve this problem. In this case there were four unknown parameters, the real and complex parts of $\phi(z_1)$ and c . Initial estimates of these were obtained from the results of the original program. This technique of using two different methods, one to find the initial estimate and the other to refine the solution worked very well.

7. 8. 1 CALCULATION OF THE HORIZONTAL AND VERTICAL VELOCITY

Once c and $\phi(z)$ is known it is possible to calculate the values of u , v , and w as a function of height. The values of the geostrophic components u and v of the horizontal velocity follow from the definitions :-

$$u = \frac{-1}{f} \frac{\partial \phi}{\partial y} \quad (7.25) \quad v = \frac{1}{f} \frac{\partial \phi}{\partial x} \quad (7.26)$$

which, when combined with the definition of ϕ , and taking only the perturbation part gives

$$u = \frac{-iky}{f} \phi(z) \exp(i(k_x(x-ct) + k_y y)) \quad (7.27)$$

and

$$v = \frac{+ik_x}{f} \phi(z) \exp(i(k_x(x-ct) + k_y y)) \quad (7.28)$$

The derivation of the vertical velocity w is slightly more complex. However equation 7.7 gives w in terms of $\partial\phi/\partial p$ and using the definition of ϕ in this it can be shown that :-

$$w = \frac{-1}{N^2} \operatorname{Re} \left\{ (ik_x \exp(i(k_x(x-ct) + k_y y))) \left[(\bar{u}-c) \frac{\partial\phi}{\partial z} - \phi \frac{\partial\bar{u}}{\partial z} \right] \right\} \quad (7.29)$$

Since a baroclinic instability grows by releasing the available potential energy of the flow, it is obviously interesting to know how this conversion varies as a function of height. An expression for the rate of change of the perturbation energy can be derived from equation 7.12 by substituting

$$\phi = \bar{\phi}(y, p) + \phi'(x, y, p, t) \quad (7.30)$$

After neglecting all second order terms, multiplying by ϕ' , and integrating over a volume, a rather complex integral is obtained. This is simplified by the assumptions that, the disturbance is periodic in the x direction, the meridional velocity v is zero at the latitudinal boundaries, and that the vertical velocity w is zero at the top whilst $\phi = 0$ (and hence $\phi' = 0$), at the lower boundary. (Apart from the different lower boundary condition this case is equivalent to that considered by Pedlosky 1979). Now the energy E per unit mass of the perturbation is (see, for example, Gill 1974).

$$E = \frac{1}{2} \left[u^2 + v^2 + \frac{1}{N^2} \left(\frac{\partial\phi}{\partial z} \right)^2 \right] \quad (7.31)$$

Substitution of this into the result of the integration described above gives the average rate of change of E over a cycle to be the average value of

$$\frac{fv}{N^2} \frac{\partial\phi}{\partial z} \frac{\partial\bar{u}}{\partial z} - u v \frac{\partial\bar{u}}{\partial y} \quad (7.32)$$

where the first term represents the release of energy by baroclinic

processes while the second is due to barotropic instabilities (see section 4.6), obtaining energy from the horizontal shear of the mean flow. If we set $\partial u / \partial y = 0$ and use the expression for ϕ given in 7.13, then, writing ϕ in the form

$$\phi = A e^{i\theta} \quad (7.33)$$

we obtain, for the rate of change of energy over a cycle

$$\frac{1}{2N^2} k_x A^2(z) \frac{d\theta}{dz} \frac{\partial \bar{u}}{\partial z} \exp(2 k_x c_i t) \quad (7.34)$$

This is identical to the results obtained by Gill ^{et. al.} (1974) although he used a different lower boundary condition. (For computational convenience the density ρ in this equation was approximated in the calculation by using a constant density scale height).

7.8.2 VERTICAL SHEAR PROFILE

The profile of $\partial u / \partial z$, upon which the values used were based was that shown in fig 5.11 corresponding to the planetodetic latitude 45 deg N. This may appear unreasonable since the ribbon feature, as can be seen from fig 7.2, is centered at approximately 47.5 deg. N. However there are several reasons for believing this profile to be more relevant. Firstly the temperature values to derive fig 5.11 were taken from the Voyager 1 results shown in fig 5.9. These have a better height resolution than those of fig 5.11 which contains both the Voyager 1 and 2 results. However it can be seen from fig 5.11 that at 150 mb the greatest latitudinal temperature gradient is at approximately 47.5 deg, N. In addition it has been demonstrated by Piraggia et. al. (1981) that the thermal wind shear at 150 mb is closely correlated with the observed zonal velocity, which is greatest close to the centre of the ribbon feature.

If we assume that the profile at 47.5 deg. N is appropriate, there still remains the difficulty of the restricted pressure range over which this is available. It can be seen from fig 5.11 that $\partial u / \partial z$

varies linearly with $\log(p)$, (and hence height, see fig 5.4) over the 150-400 mb range whilst below 400 mb it is approximately constant. However it is at about the 300-400 mb level that the thermal wind ceases to be correlated with the zonal wind profile, and it is suggested by Conrath and Piragglia (1983) that this is due to the presence of clouds and hazes which are ignored in the IRIS temperature inversion (see section 2.3.2.2). For this reason a straightforward way of extending the profile of $\partial u/\partial z$ is to assume that the linear behaviour observed over the 150-400 mb range can be extrapolated to other pressure levels. A comparison between this and the measured values is shown in fig 7.14.

The assumption of a linear behaviour, whilst convenient for the initial estimates of the structure is unrealistic. An example of one of the somewhat extreme effects introduced by this is that if the zonal velocity was zero at the bottom (700 mb) then 200 km higher up there would be a 225 m/s westward flow. Another effect of these large vertical shears is that high in the atmosphere the Richardson number becomes smaller so that either symmetric instabilities or Kelvin-Helmholtz instabilities could become important.

Because of these difficulties a slightly more complex model of $\partial u/\partial z$ was adopted. The vertical wind shears derived by Conrath and Piragglia (1983) show that the zonal winds tend towards a state of uniform rotation with height. For this reason it was decided that a more realistic model of $\partial u/\partial z$ would decrease exponentially with height. In order to use this while still maintaining $\partial^2 u/\partial z^2$ decreasing close to the bottom (assuming that at least part of the structure below 400 mb is realistic) a quadratic rather than linear increase with height was used. This led to the model

$$\frac{\partial u}{\partial z} = (Az^2 - B)e^{-Cz} \quad (7.35)$$

where A, B and C are parameters.

In order to consider as wide a range of parameters as possible without departing too much from the observations, six profiles of $\partial u / \partial z$ were used. These are shown in fig 7.14 and are, the original measurements shown in fig 5.11 approximated by cubic splines, the linear model discussed earlier, and four profiles of the type described by equation 7.35.

The stability characteristics of the various profiles of $\partial u / \partial z$ can be examined using fig 7.15 which shows Q_y as a function of height (assuming the same values for N^2 and h^{-1} as described in section 7.8.3). It can be seen from this figure that all but one of the profiles used causes Q_y to change sign with height, and hence could be unstable without the presence of a horizontal boundary. The exception to this is the profile produced by fitting cubic splines to the points shown in fig 5.11, which gives $Q_y > 0$. However since $\partial u / \partial z < 0$ at the upper limit of the curve, it is still possible for instability to be associated with the boundary, in a manner similar to that suggested by Conrath et. al. (1981).

One of the interesting characteristics of the profiles of $\partial u / \partial z$ of the type described by equation 7.35 is the possibility of multiple sign changes in Q_y . It was particularly to study the effects introduced by this that the profiles C, E and F were used. As can be seen from figs 7.14 and 7.15 these have a similar shape, but the last two have a sufficiently rapid fall off with height to cause Q_y to change sign a second time at around 105 km. It is apparent from fig 7.14 that these profiles, although qualitatively similar, do not follow closely the measured values. This was done deliberately to avoid producing disturbances associated mainly with the upper boundary.

7.8.3 PARAMETERS USED IN MODEL

In order to use the profiles of $\partial u / \partial z$ considered in section 7.8.2 to produce solutions for ϕ and c it is necessary to specify various other quantities including the value of N^2 and the density scale height as a function of height. The value of N^2 used was that obtained from the radio science profile corresponding to 36°N , shown in fig 5.5, but a small value (less than the difference between the profiles) was added to make $N^2(Z_1) > 0$. This is not unreasonable considering the uncertainty in the value of N^2 at this level. The density scale height was assumed to be a constant $\sim 40 \text{ km}$.

In addition the program required the values of f and β which were obtained from the program which produced the values shown in table 3.1. The final piece of information required is the nature of the lower boundary condition, in all the cases considered this was the $\phi = 0$ condition discussed in section 7.6.1.

In order not to complicate unnecessarily the results, most of the calculations made assumed a meridional wavenumber of zero. This changes slightly the longitudinal wavenumber and structure of the least stable disturbance. This effect is considered in more detail for some of the profiles used, in the next section.

7.8.4 NUMERICAL RESULTS

Figure 7.16 shows the result of calculating the growth rate $k_x c_i$ as a function of wavenumber for the $\partial u / \partial z$ profile labelled A in fig 7.14. For convenience the wavenumber has been scaled by a factor of 10^7 , thus the measured wavelength of 5710 km derived in section 7.3.3 will correspond to a wavenumber of ~ 11 on this scale. The growth rate is the reciprocal of the time required for the amplitude of a perturbation to increase by a factor of e . Thus a feature with a growth rate 1.7×10^{-6} will increase by a factor of e in one week.

It can be seen from fig 7.16 that the most rapidly growing disturbance has a dimensionless wavenumber of 23.75 corresponding to a wavelength of ~ 2650 km.

It is interesting to examine the structure of this least stable disturbance in greater detail, and fig 7.17 shows the associated rate of energy conversion. It can be seen that this increases rapidly towards the upper boundary where it is greatest. This seems reasonable since, as mentioned in section 7.8.2, this configuration is only unstable because of the upper boundary condition.

Although useful it is difficult to estimate the significance of these results because of their dependence on the upper boundary condition. This leads to the model based upon the $\partial u / \partial z = \text{constant}$ profile (labelled B in figure 7.14) described in section 7.8.2. As can be seen from fig 7.15 this has Q_y changing sign at ~ 60 km, and can hence be unstable regardless of the flow at the upper boundaries. The growth rates as a function of wavenumber, with the upper boundary being placed at various levels between 75 and 200 km above the lower boundary, are shown in fig 7.18. From this it can be seen that increasing the height of the upper boundary reduces the growth rate of the least stable disturbance and increases its wavelength from 7×10^3 km to 25×10^3 km. Figure 7.19 shows the energy conversion rates associated with these disturbances (the x axis is not the same for all the curves, an arbitrary scaling factor has been used to make them all fit). It can be seen from this that initially the maximum is associated with the upper boundary but that when this is moved up to 200 km the maximum energy conversion rate occurs at 65 km. This is close to the height at which Q_y changes sign, suggesting that in this case the instability is being generated within the fluid rather than being influenced by the boundary conditions.

The visible appearance of the feature in the images is assumed to be due to the presence of ammonia clouds. Since these clouds are probably produced by condensation due to vertical displacements it is useful to consider the vertical velocity field generated by the model. The amplitude of the vertical velocity is shown in fig 7.20, from which it can be seen that the maximum seems to be associated with the upper boundary. In addition to the amplitude, the vertical velocity also has a phase. This is not shown, but the phase change between the top and the bottom levels alters from 13° to 8° as the location of the upper boundary is increased.

A more convincing demonstration that the disturbances can be independent of the upper boundary is provided by the profile of $\partial u / \partial z$ labelled C in fig 7.14. The curves of growth rate against wavenumber are shown in fig 7.21, whilst fig 7.22 shows the energy conversion rate as a function of height. It can be seen from this that the maximum conversion rate is initially associated with the upper boundary when this is at 75 km. However when the top is moved up to 95 km a secondary maximum appears at 34 km. As the height of the upper boundary is increased this grows, until, when the top is at 200 km, the maximum at 30 km dominates.

The suggestion that this indicates that the effect of the upper boundary has ceased to be important is further illustrated by fig 7.21. From this it can be seen that the growth rate drops by approximately a factor of 3 when the height of the upper boundary is increased from 100 to 200 km. This is considerably less than the change in the previous profile as can be seen from a comparison with fig 7.18.

The amplitude of the vertical velocity associated with this disturbance is shown in fig 7.23. From this it can be seen that the maximum vertical velocity still appears to be linked with the position of the upper boundary but that it is possible that this behaviour is becoming less pronounced. The phase information associated with fig

7.23 is not shown, but the phase change between the upper and lower boundaries of the models decreases from 4.5° when the top is at 75 km to 0.55° when the upper boundary is at 200 km.

Figure 7.24 shows the growth rates associated with the profile labelled D in fig 7.14. They show a considerable variation with the height of the upper boundary suggesting that, unlike the previous case, they are strongly dependent upon this. This conclusion is strengthened by fig 7.25 which shows that in all cases the maximum energy conversion rate occurs at the upper boundary. However when the top of the model is at 120 km a second maximum appears at ~ 55 km, approximately the same height as that at which $Q_y=0$, as can be seen from fig 7.15. The vertical velocity associated with this profile of $\partial u/\partial z$ is not shown but has a similar structure to that shown in fig 7.20

It is apparent that all the results discussed so far which appear to be independent of the upper boundary have very small growth rates (they increase by a factor of e in about three years). These are associated with wavenumbers approximately a factor of 4 or 5 larger than those observed. Disturbances with more realistic growth rates and wavenumbers have been produced, but only when the upper boundary is at ~ 100 km or below.

Although the presence of an upper boundary is unrealistic, it is possible that some other process is producing a similar effect. In addition to the boundary conditions, the necessary conditions for instability discussed in section 7.6.2 included the quantity Q_y . In particular instability could occur either when the boundary conditions were related in a particular way or when Q_y changed sign. Because of this it was thought possible that a second change in the sign of Q_y could produce similar results to the presence of an upper boundary. To test this possibility the profile labelled E in fig 7.14 was used. This is similar to profile C but produces two sign changes in Q_y , one

at ~ 30 km and the other around 100 km.

The way in which the growth rate varies as a function of wavenumber for this model (with the upper boundary at 200 km) is shown in fig 7.26. It can be seen from this that the most unstable disturbance has a growth rate of $\sim 2 \times 10^{-7} \text{ s}^{-1}$ for a wavenumber of ~ 5.5 . These values are similar to those obtained for other profiles of $\partial u / \partial z$ with an unnaturally low upper boundary. However fig 7.27 shows the energy conversion rate as a function of height for this disturbance. From this it is clear that the disturbance is most active at around 25 km, rather than close to the upper boundary. The modulus of the vertical velocity associated with this disturbance is shown in fig 7.28 where it can be seen that the maximum occurs at about 55 km. The variation of the phase of the vertical velocity with height is not shown, but alters by $\sim 3^\circ$ between the base and the 150 km level.

The second curve in fig 7.26 shows the growth rates associated with the profile of $\partial u / \partial z$ labelled F in fig 7.14. This has a structure similar to that of profile E. The growth rate of the least stable disturbance is slightly less, and is now associated with a wavenumber of ~ 8.5 .

One of the assumptions which has not been considered in any of the results discussed so far is the effect of a non-zero meridional wavenumber k_y . This is partially because of the difficulty in obtaining a reliable value for this. However fig 7.29 shows how the growth rates for profiles E and F alter if we use the value of k_y derived in section 7.2. It can be seen that the growth rates are slightly smaller (the minimum e folding times are 110 and 135 days). Also changed is the wavenumber of the least stable disturbance, which is now $k_x \sim 5$ and 11.

One of the quantities which has not been mentioned for the profile considered is the phase velocity c_r . In most cases this had an absolute value less than 1 ms^{-1} but was larger in the results

concerning profile B, reaching a maximum of 5 ms^{-1} when the upper boundary is at 75 km. These values are sufficiently small that they would not be distinguishable from the mean flow using the methods mentioned in section 7.2.

7.8.5 COMPARISON OF MODEL RESULTS WITH THE OBSERVATIONS

Before discussing the results of the model in greater detail the measurements of the feature made in the first part of this chapter will be briefly reviewed. One of the most reliable measurements is that of the longitudinal wavelength which was given in section 7.3.3 as $5710 \pm 260 \text{ km}$ corresponding to a wavenumber of $1.1 \times 10^{-3} \text{ km}^{-1}$. The latitudinal wavenumber is less certain but is estimated in section 7.2 to be $7 \times 10^{-7} \text{ m}^{-1}$.

Due to the lack of trackable features not associated with the wave pattern it is not possible to differentiate reliably between the phase speed of the wave and the velocity of the mean zonal flow. However the growth rate, and hence the imaginary part of the phase speed are bounded by the measurements of Sromovsky et. al. (1983) who found that the shape of the feature did not change appreciably over two days.

The vertical structure is also uncertain. However the results of section 7.4.3 suggest that the feature must penetrate up to at least the 170 mb level. In addition, if the visual pattern is caused by vertical motions then the vertical velocity field must have an almost constant phase between 170 mb and the level of the ammonia clouds.

These conditions are satisfied by all the vertical velocity fields produced by the model, which reached a maximum around or above the 170mb level. The phase change associated with this is sufficiently small (see previous section) to agree with the multispectral images.

The initial, simpler, profiles (A,B,C and D) considered were capable of producing unstable disturbances which appeared to be independent of the upper boundary. However these had a wavenumber of approximately 2-2.5 associated with the maximum growth rate of about 10^{-8} s^{-1} . These imply a wave of wavelength $\approx 25 - 31 \times 10^3 \text{ km}$ which will grow by a factor of e in 165 weeks, ie approximately three years.

However, as was shown in section 7.8.4, more realistic values for these quantities can be obtained if profiles of $\partial u / \partial z$ which produce a second sign change in Q_y are used. The profiles of this type considered had maximum growth rates of ≈ 13 and $21 \times 10^{-8} \text{ s}^{-1}$ at wavenumbers of 5.5 and 8.5 respectively. However these values assumed a meridional wavelength of zero. If instead the value derived in section 7.2 for k_y is used the growth rate changes to $\approx 10^{-7} \text{ s}^{-1}$ (115 days), and is associated with wavenumbers 5 and 11. This last value is identical to that derived in section 7.3.3 from the images.

It is apparent from fig 7.14 that the profile of $\partial u / \partial z$ used to obtain this result, although similar to the original measurements, was placed slightly lower. This was done to avoid producing disturbances which were associated mainly with the upper boundary. It appears from the results discussed in section 7.8.4 that the characteristics of the disturbances produced are strongly influenced by the changing sign of Q_y , rather than by the detailed structure of $\partial u / \partial z$. For this reason it is expected that the shift in height of the profile labelled F in fig 7.14 will not significantly alter the structure or wavenumber of the least stable disturbance.

One of the obvious differences between the observations and the model results lies in the change of intensity across the feature. The model assumes that all the fields vary sinusoidally, whilst the images (for example fig 7.1) show a more abrupt change. A similar discrepancy occurs on the Earth where quasi-geostrophic models of the

type used here can predict the large scale cyclones and anticyclones, but are unable to produce the smaller bands of cloud. However it has been shown by Hoskins (1982) that when certain ageostrophic effects are included in these models frontogenesis can occur, leading to an intensification of the local vertical velocity and the possibility of producing bands of cloud. Although the absence of horizontal boundaries on Saturn may restrict this process somewhat, it is still expected to be important. Thus the disturbances produced by the model, although initially smooth, would be expected to become more abrupt, similar to those observed.

The region around 45° N on Saturn is the only feature for which the possibility of baroclinic instability has been considered in detail. However there are several other locations on both Jupiter and Saturn where this process could also be important. A good example of one of these is provided by fig 7.30 which contains a mosaic showing the north polar region of Saturn. It is apparent from this that there is a feature with a strong wavenumber six component at 80° N. A comparison with fig 7.13 shows it to be very similar to features observed in the laboratory which are known to be baroclinically generated. Unfortunately, as is apparent from fig 5.10, measurements of the latitudinal temperature gradient are not available at this latitude, making the vertical wind shear uncertain.

This chapter has considered the use of a model of a baroclinically unstable disturbance to explain a wavelike feature in Saturn's atmosphere. Using a temperature structure based upon the IRIS measurements, it was shown that a feature with the observed wavelength could be produced. The vertical structure of this modelled wave satisfies the constraints imposed by the multispectral images. It is suggested that the same process may also occur on other parts of Jupiter and Saturn.

FIGURE CAPTIONS - CHAPTER 7

Figure 7.1a-c

These figures show the high resolution images of the ribbon feature. The three separate parts a, b and c were taken between 3 and 4 days before closest approach, and are each separated by one rotation period. Each image is derived from two original images which were map-projected to produce two cylindrical projections with a scale of 22 km/pix. These were then joined together, and filtered using the program TFILT to enhance the small scale structure. For convenience the resulting images were stretched in the line direction and divided into two parts in order to produce the figures shown. There is some overlap between the two parts.

The first two images (a and b) were pre-processed at JPL and still contain the original reseaux marks. These have been removed in c, but the camera blemishes (small black and white dots) still remain. All of the images show a pair of dust rings, which appear as dark haloes.

The rapid eastward motion of the feature is apparent from this sequence of images which are fixed to the radio-rotation period.

Figure 7.2

This shows a mosaic of the northern hemisphere with an associated latitude-longitude grid (planetocentric latitude, west longitude).

This was constructed from five images taken at approximately two hour intervals. These were individually pre-processed, and were then map-projected to produce five polar stereographic map-projections centered on the north pole, and with a scale of 164 km/pix. These were then combined by taking the maximum DN level at each point. The resulting image was then filtered using TFILT to enhance the smaller scale features, and the program ADESPIKE was used to remove the bright streaks associated with the edge of the image. After this the grid was generated and added in.

The filtering has not completely removed the shading across the images, resulting in the starfish pattern radiating from the centre. Also apparent are the dust rings which were seen in fig 7.1, and the camera blemishes. The slight mis-centering apparent is probably due to errors in the navigation parameters, and is mentioned further in section 7.3.2.

Figure 7.3

By producing an equivalent cylindrical version of the mosaic shown in fig 7.2, and tracing along one side of the ribbon feature, the 400 measurements upon which this figure is based were produced. The line direction corresponds to the latitudinal displacement while the sample number varies with longitude. The relationship between the sample number of fig 7.3 and the west longitude of fig 7.2 is:-

$$\lambda = 360 - \frac{S}{20.25}$$

Figure 7.4

This was produced by interpolating the data shown in fig 7.3 to produce a regular spacing, and then Fourier transforming the result. The wavenumbers shown are dimensionless, and correspond to the interval of fig 7.3. In order to show up the higher wavenumber components the zero and one wavenumber components have been allowed to go off the scale.

Figure 7.5

The dotted line shows the same data as fig 7.3, but after the removal of the wavenumber zero and one components. The continuous line demonstrates the sum of the two components with the greatest amplitude (wavenumbers 8 and 20, see fig 7.4).

Figure 7.6

The dashed line shows a histogram of the separation between adjacent maxima and minima of the curve in fig 7.3. The continuous line is similar but shows the effect of removing all the peaks separated by less than 3 pixels in the line direction.

Figure 7.7 a-d

These show the varying appearance of the ribbon feature in different filters. They were taken within a few minutes of each other, 11 days before encounter, and have been map projected to a common scale of 50 km/pix at the equator. The appropriate filters are a) green, b) blue, c) violet and d) UV. The orange image is not shown but is very similar to the green image. The aspect ratio of the images has been altered to enable them to fit the page size. The boxes in 7.7 a) show the areas from which some of the graphs below were obtained. In order to preserve the large scale albedo variations these figures have not been filtered. This is responsible for the lack of contrast in figs 7.7 a and b.

Figure 7.8a

This shows how the intensity seen through the violet filter varies with line number (averaged over 5 samples, the area of the smaller box shown in fig 7.7a). Fig 7.8b shows how this changes when an attempt is made to remove the limb darkening.

Figure 7.9 a-d

These show how the intensity of the ribbon feature varies with line number (and hence latitude) at different wavelengths. The images concerned were all previously divided by the green image as explained in section 7.4.1 to reduce the effects of the changing sun-angle. The relevant filters are a) orange, b) blue, c) violet, and d) UV. In order to avoid being unduly influenced by local features, these graphs correspond to averages over 695 samples, the size of the larger box in fig 7.7 a.

Figure 7.10

This figure from Moore (1983) shows how various forms of the atmospheric model described in the text compare with observations of the UV albedo. The albedo measurements shown are globally integrated IUE results. The model uses 3 km-am of H₂ over a 15 km-am layer containing an absorbing aerosol with an optical depth of 0.5 at 3100 Å. Below this is a single scattering base layer of albedo ω . The curves in a) use the aerosol with λ^{-1} behaviour, whilst those in b) show the preferred λ^{-2} behaviour. The different curves in a) and b) demonstrate the effect of changing the base albedo of the model. The values shown are :-

A	ω	=	1.0
B	ω	=	0.98
C	ω	=	0.99
D	ω	=	0.995

The discrepancy between the model and the observations shortward of 2300 Å is accounted for by Moore (1983) using phosphine absorption. However this has a negligible effect at the wavelengths of the Voyager UV filter.

Figure 7.11

This shows how the results of the model which produced fig 7.10 change between 3100 and 3460 Å (the wavelength of the Voyager UV filter). This is shown as a function of the albedo of the base layer.

Figure 7.12

This is similar to fig 7.11, but shows the log of the ratio of the ingoing and outgoing fluxes at 0.75 radii from the planets centre.

Figure 7.13

This figure, taken from Hide and Mason (1975), shows the flow near the top of a rotating fluid annulus. The inner boundary is being maintained at a temperature 9 K cooler than the outer boundary. The only parameter changing is the rotation rate, which was (in rads/sec) a) 0.41, b) 1.07, c) 1.21, d) 3.22, e) 3.91 and f) 6.4. It can be seen that the flow is initially axisymmetric, but with increasing rotation rate an oscillatory behaviour becomes apparent. The dominant wavenumber of this increases with the rotation rate until, eventually, the flow becomes irregular.

Figure 7.14

This shows the various models used for the variation of the thermal wind shear with height. The dashed line shows a cubic spline fit to the values given for 45° N in fig 5.11. The curve B is a straight line fit to the observations. The other curves represent different approximations to the measurements of the type given by equation 7.35, and are described further in the text.

Figure 7.15

This shows how the function Q_y , defined in section 7.6.2 varies with height. This has been evaluated using the curves shown in fig 7.11 along with estimates of N^2 and the density scale height.

Figure 7.16

This figure shows how the growth rates calculated by the model vary with wavenumber, for the profile of $\partial u / \partial z$ labelled A in fig 7.14. The x axis represents a dimensionless form of the wavenumber, which is a factor of 10^7 larger than the value in m^{-1} . The crosses represent the results of the model, while the lines show the result of interpolating between these using cubic splines. The dashed line shows the results produced by the finite difference model (INSTAB3, see section 7.8), whilst the continuous line was derived from the iterative model (INSTAB4) which is known to produce more accurate results, see section 7.8.

Figure 7.17

This shows the average rate at which the least stable disturbance (i.e. the one with the greatest growth rate in fig 7.16) releases available potential energy. It can be seen that this is greatest at the upper boundary at 58 km.

Figure 7.18

This is similar to fig 7.16, but shows instead the growth rate as a function of wavenumber for the profile labelled B in fig 7.14. The different curves indicate how this alters as the height of the upper boundary is changed from 75 to 200 km. Unlike fig 7.16, only the results of the iterative program (INSTAB4), are shown.

Figure 7.19

This shows how the energy conversion rate of the least stable disturbances of fig 7.18 vary with height. Again the numbers indicate the height of the upper boundary. Because the amplitude of the disturbances changes with time there is no absolute scaling, the x axes of the various graphs being scaled differently to make them fit.

Figure 7.20

This is similar to fig 7.19, but shows instead how the vertical velocity of the various disturbances varies with height. Again the x axis is shown for convenience, only the zero position being common to all the plots.

Figure 7.21

This is the same as fig 7.18, but for the profile labelled C in fig 7.14.

Figure 7.22

This is the same as fig 7.19, but for the profile labelled C in fig 7.14.

Figure 7.23

This is the same as fig 7.20, but for the profile labelled C in fig 7.14.

Figure 7.24

This is the same as fig 7.18, but for the profile labelled D in fig 7.14.

Figure 7.25

This is the same as fig 7.19, but for the profile labelled D in fig 7.14.

Figure 7.26

This shows how the growth rates of the profiles labelled E and F in fig 7.14 vary with wavenumber.

Figure 7.27

This is similar to fig 7.19, but for the profile labelled E in fig 7.14.

Figure 7.28

This is similar to fig 7.20, but for the profile labelled E in fig 7.14.

Figure 7.29

This shows how the growth rates of fig 7.26 change for a non zero meridional wavenumber. The value used was $k_y = 7 \times 10^{-7}$ in section 7.2. On the dimensionless wavenumber scale used here, this corresponds to $K_y=7$.

Figure 7.30

This shows a mosaic similar to that of fig 7.2, but for higher latitudes. This is derived from four images which were projected to produce a polar orthographic projection. The superimposed grid shows the planetodetic latitude and the west longitude.

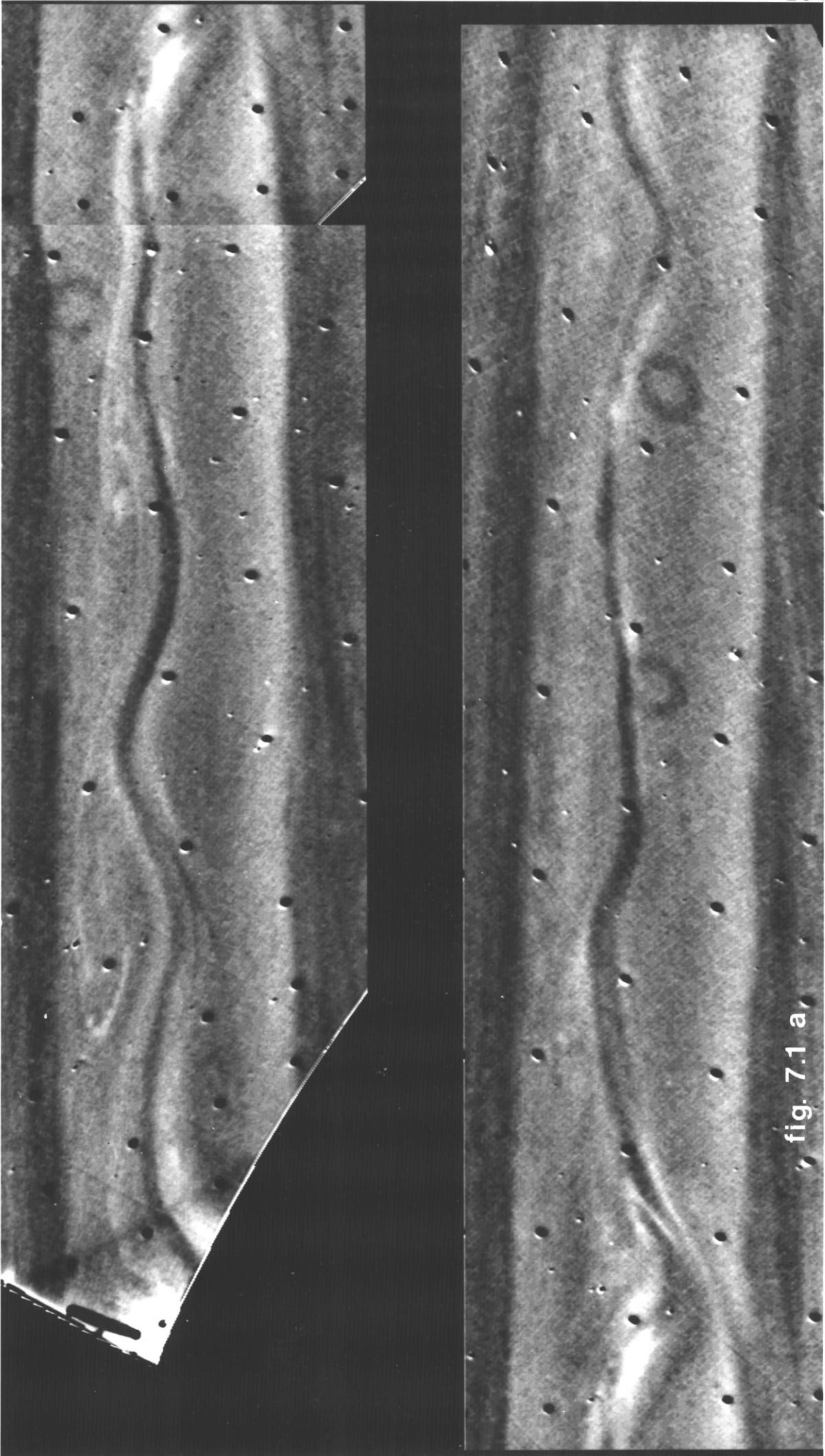


fig. 7.1 a,

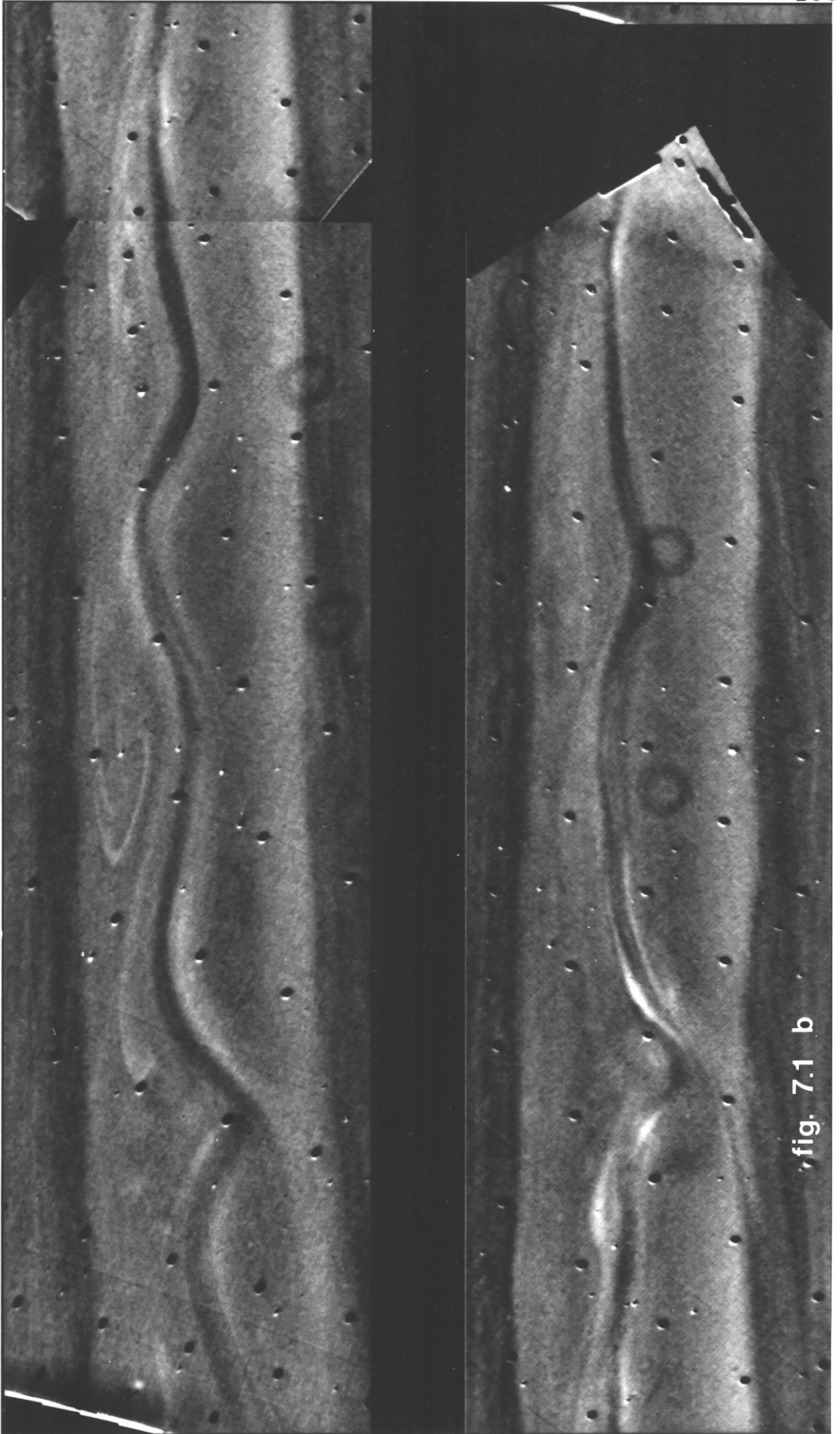


fig. 7.1 b



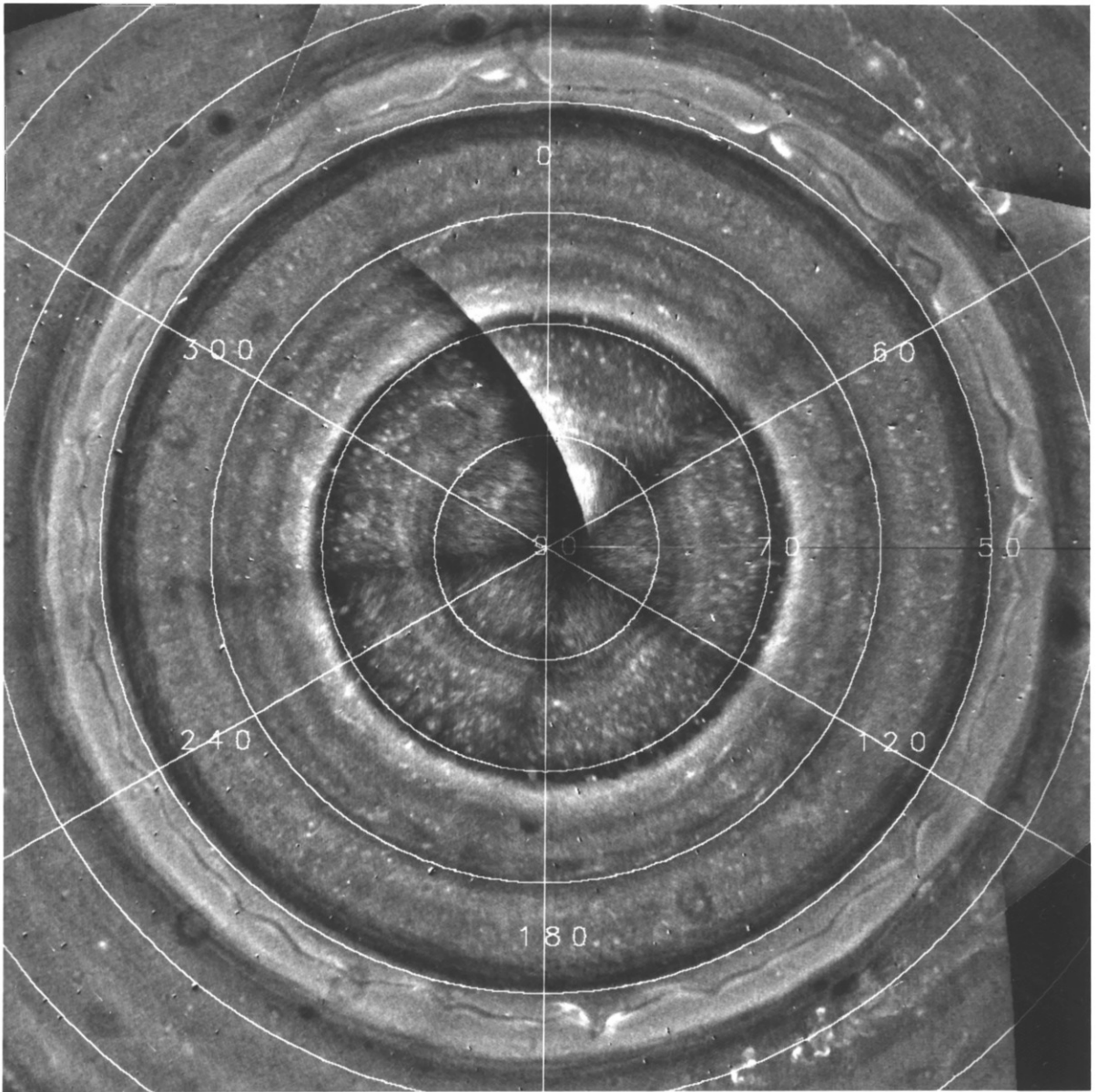


fig. 7.2

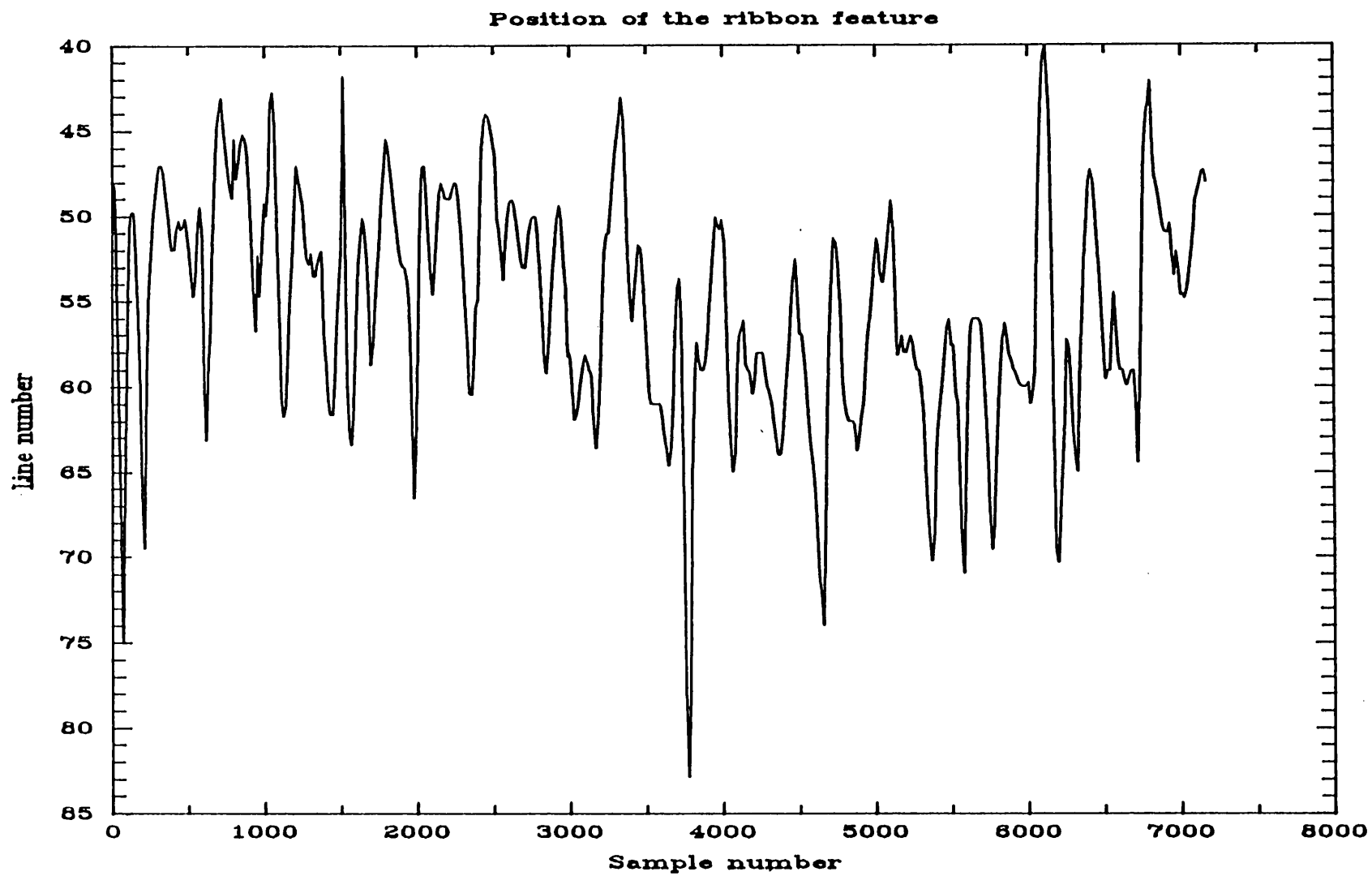
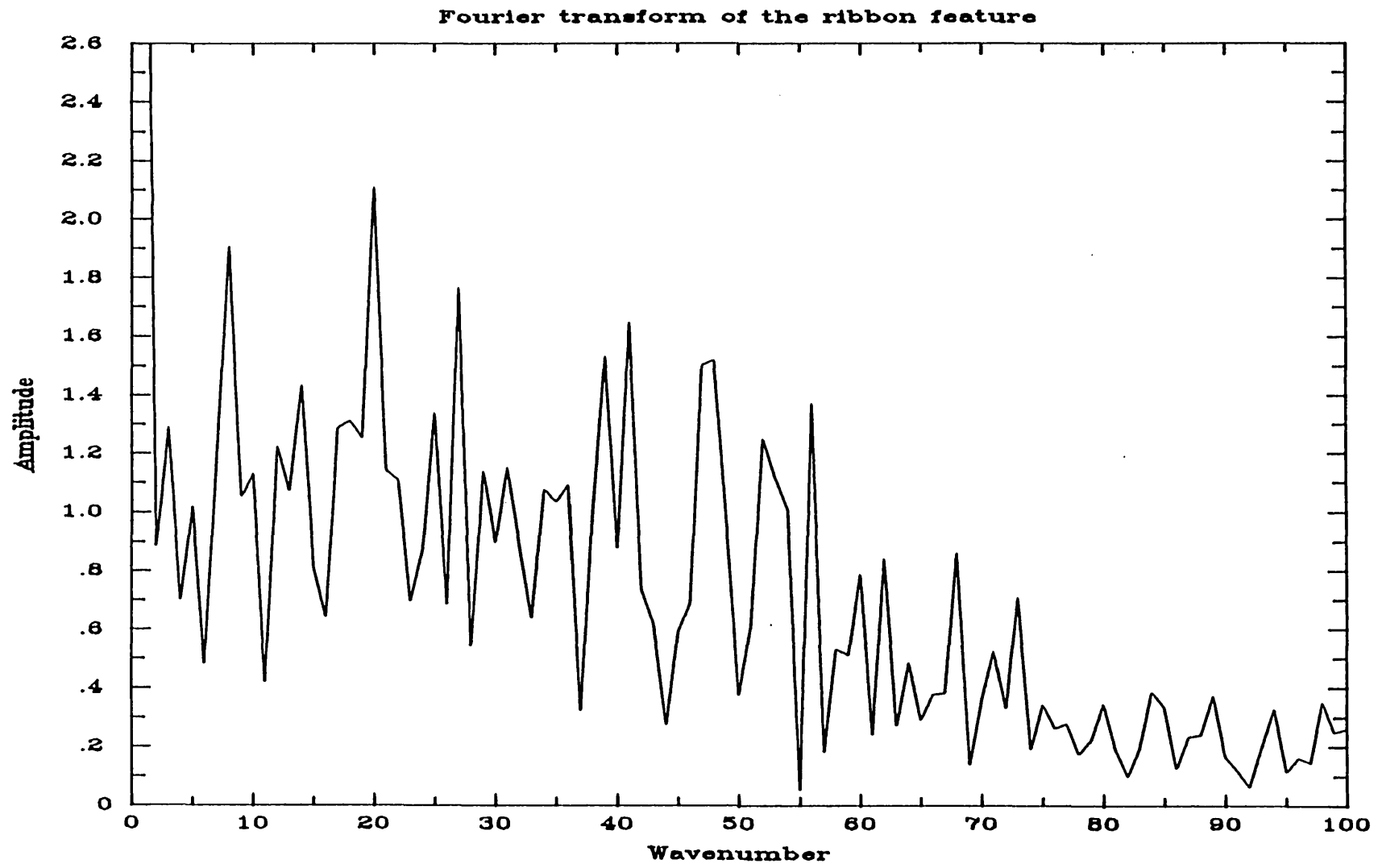


fig. 7.3

fig. 7.4



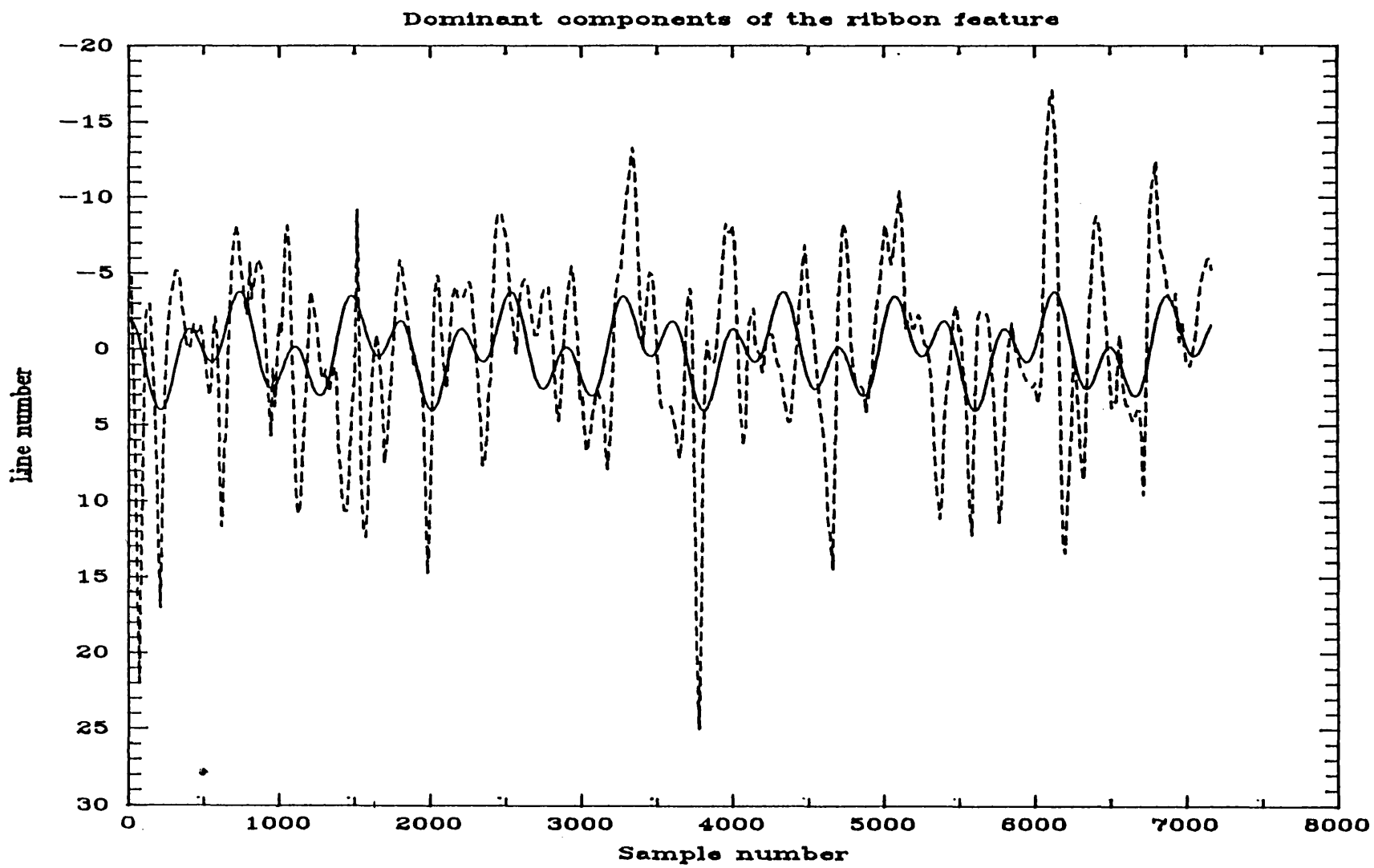
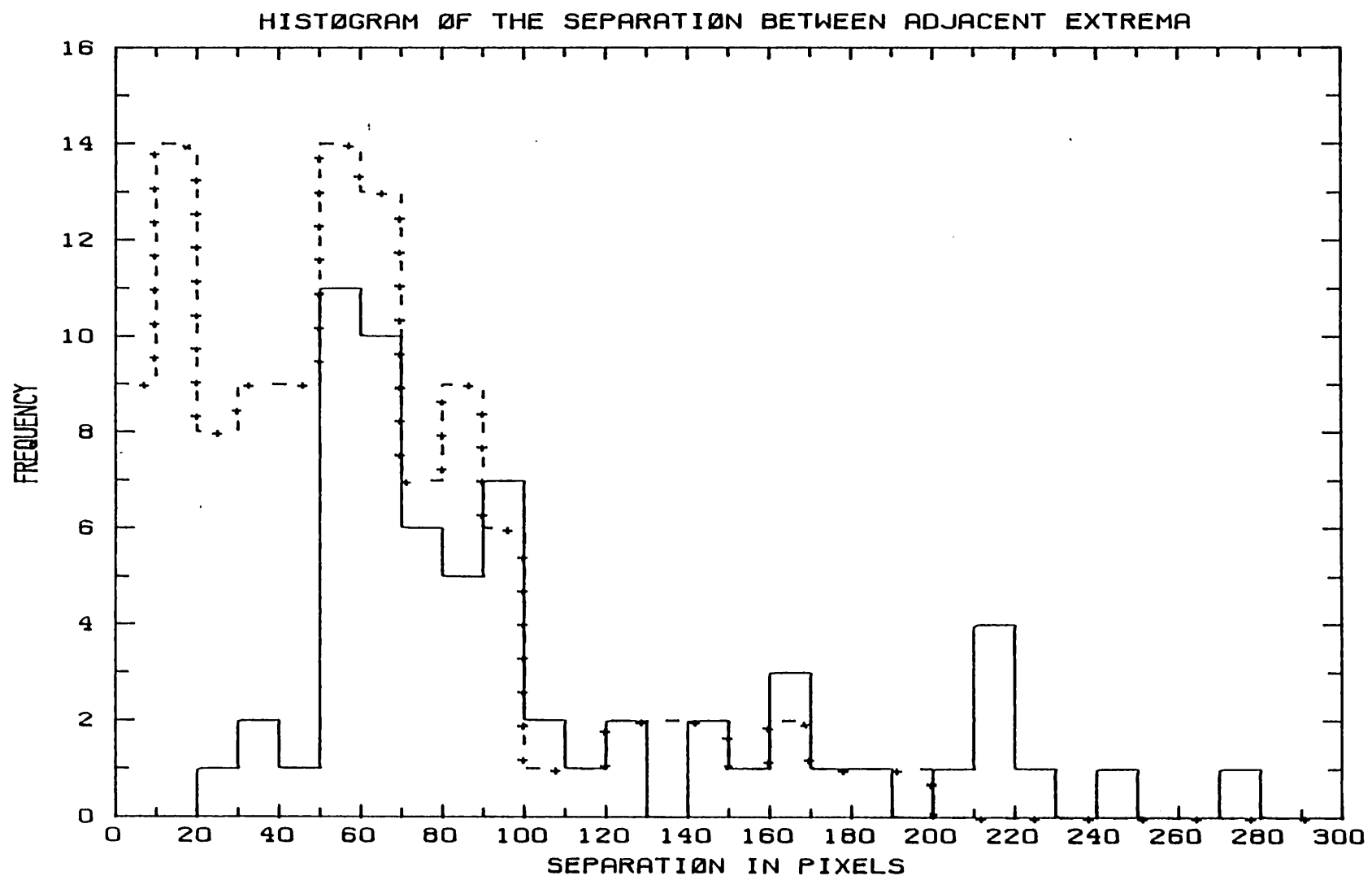


fig. 7.5

fig. 7.6



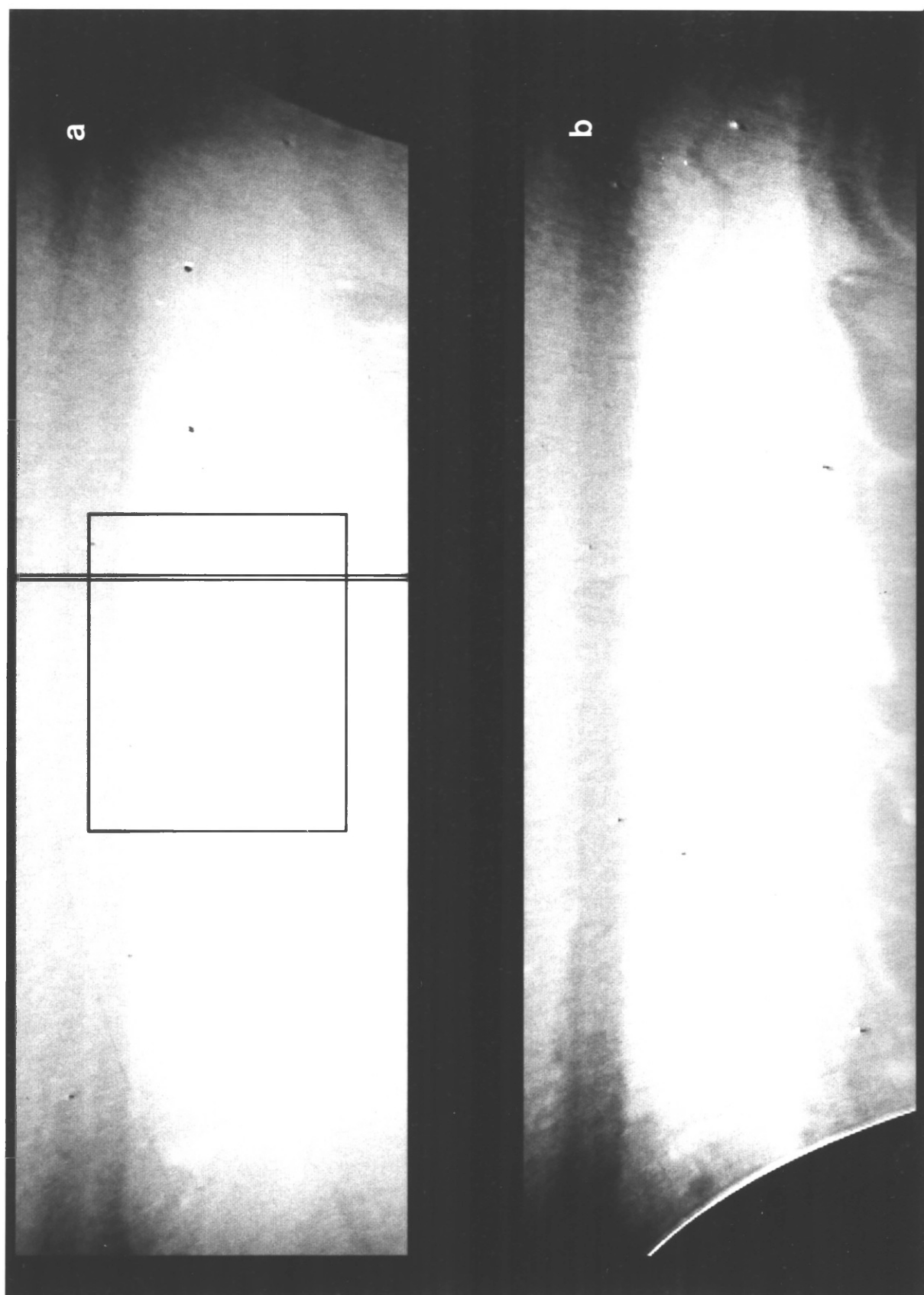


fig. 7.7 a,b

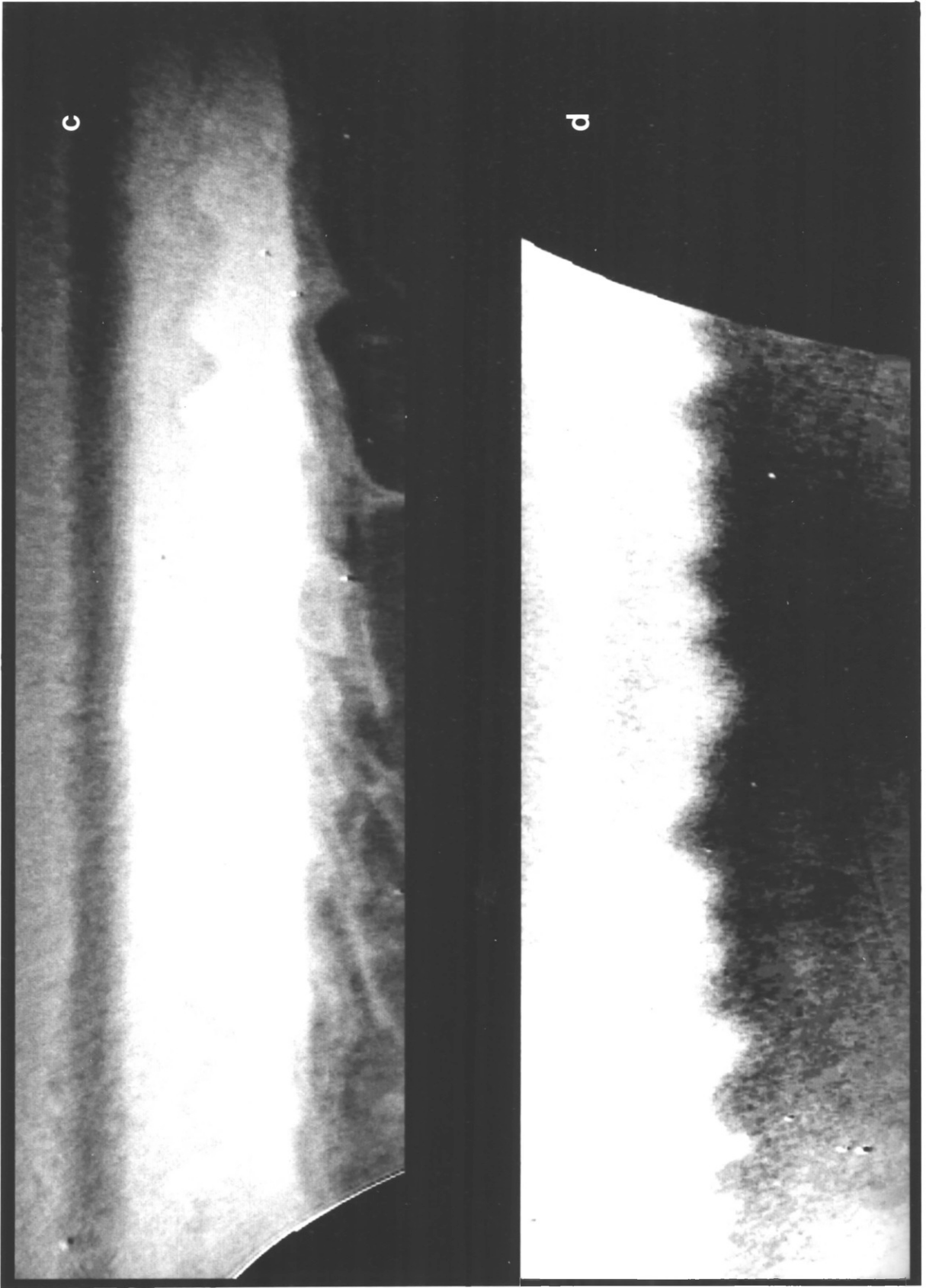
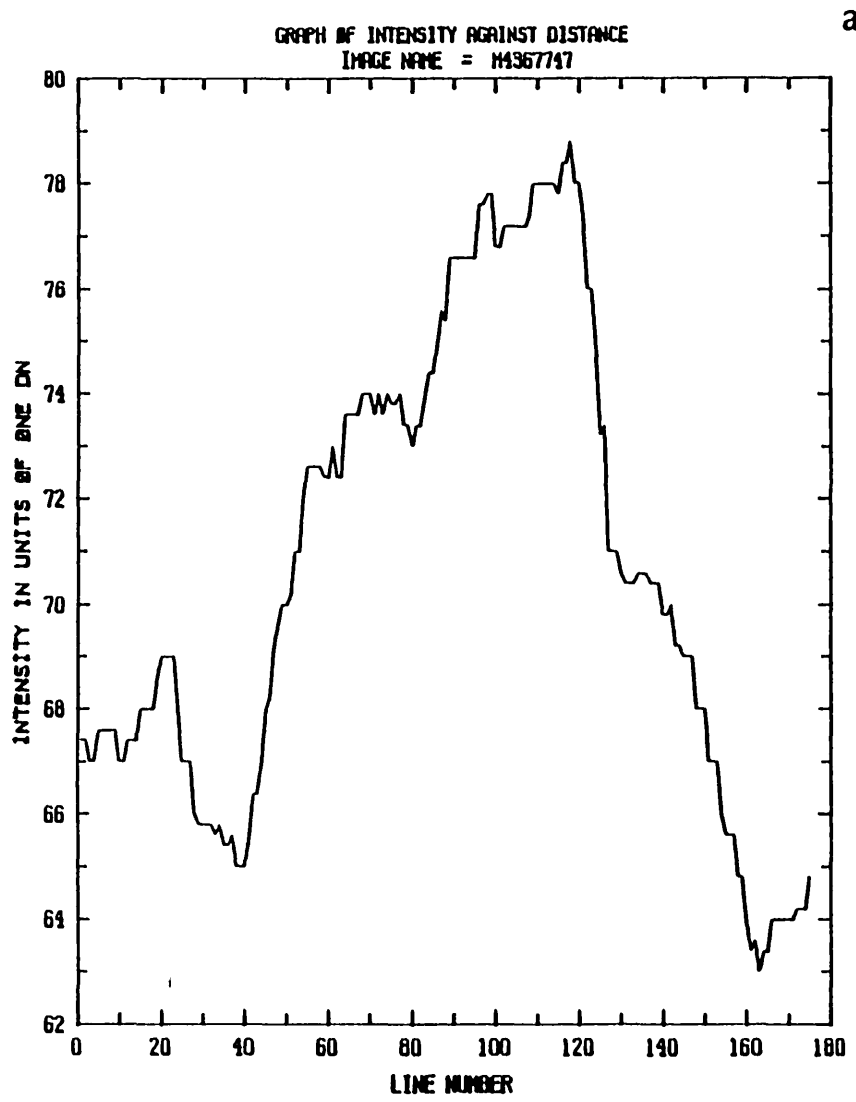
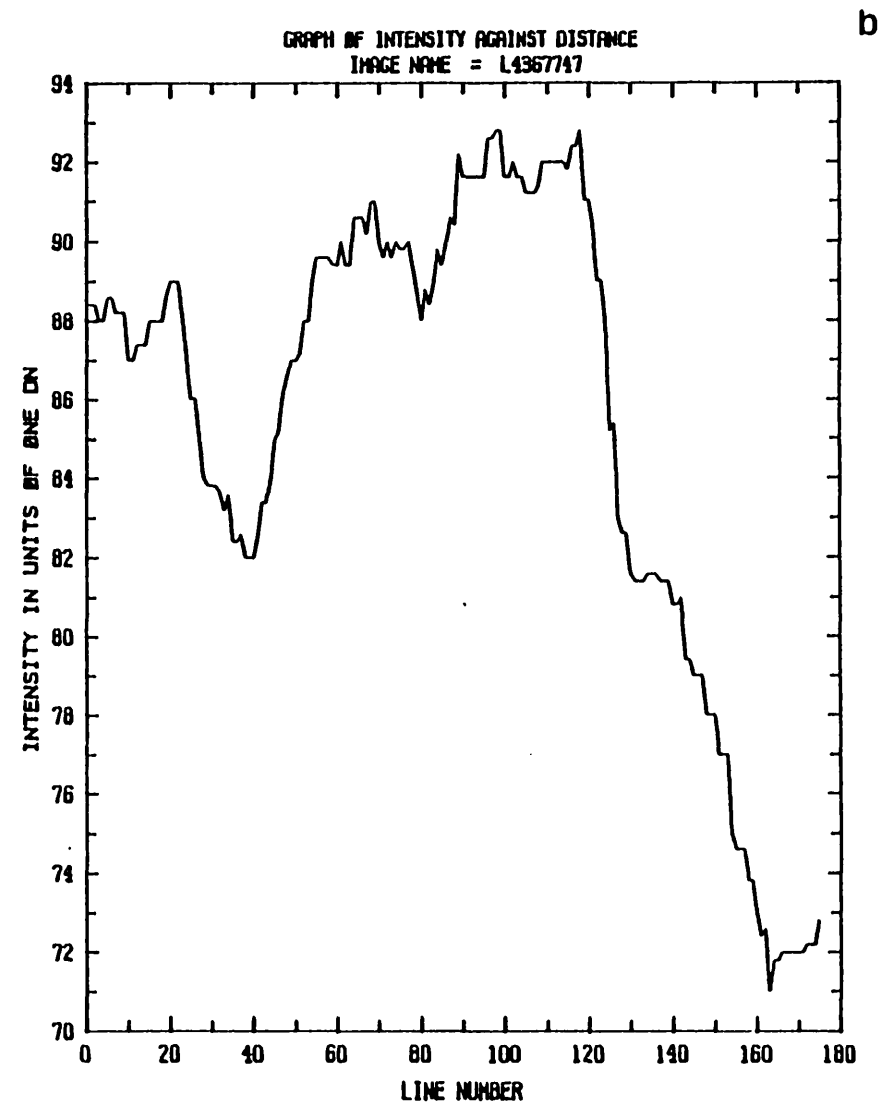


fig. 7.7 c,d

fig. 7.8

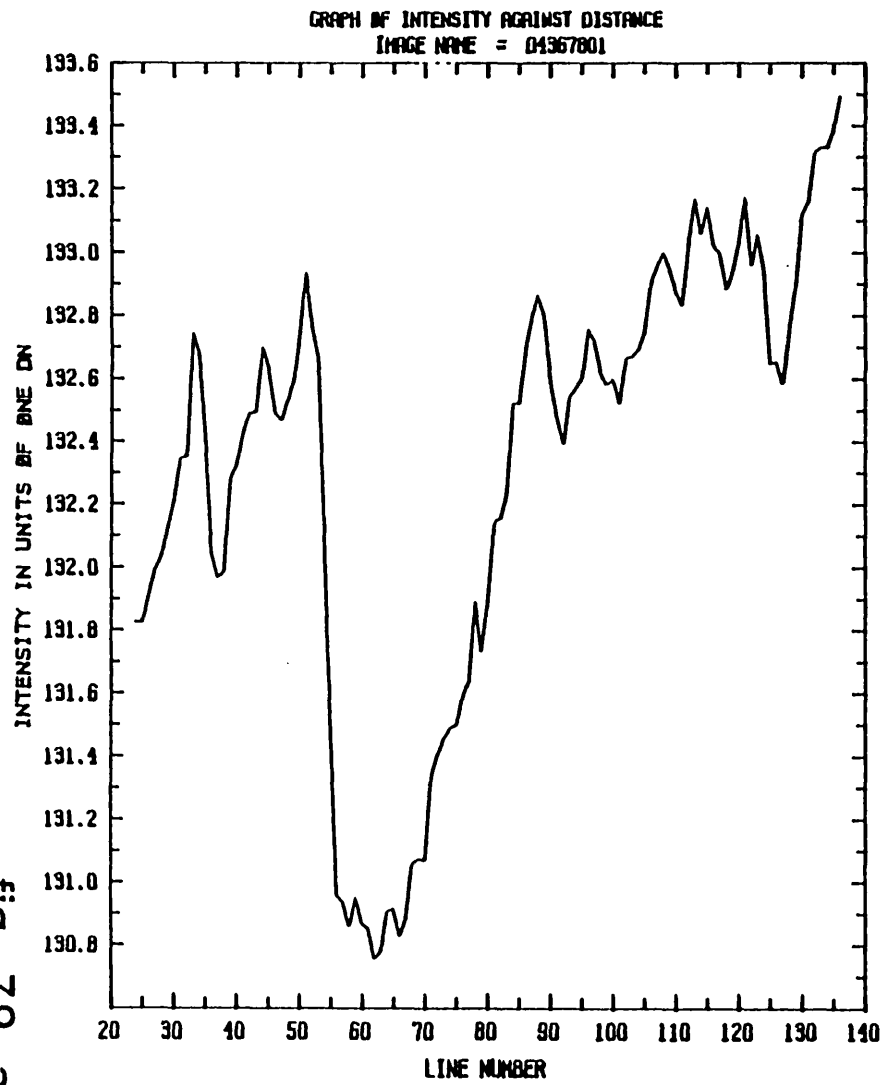


a

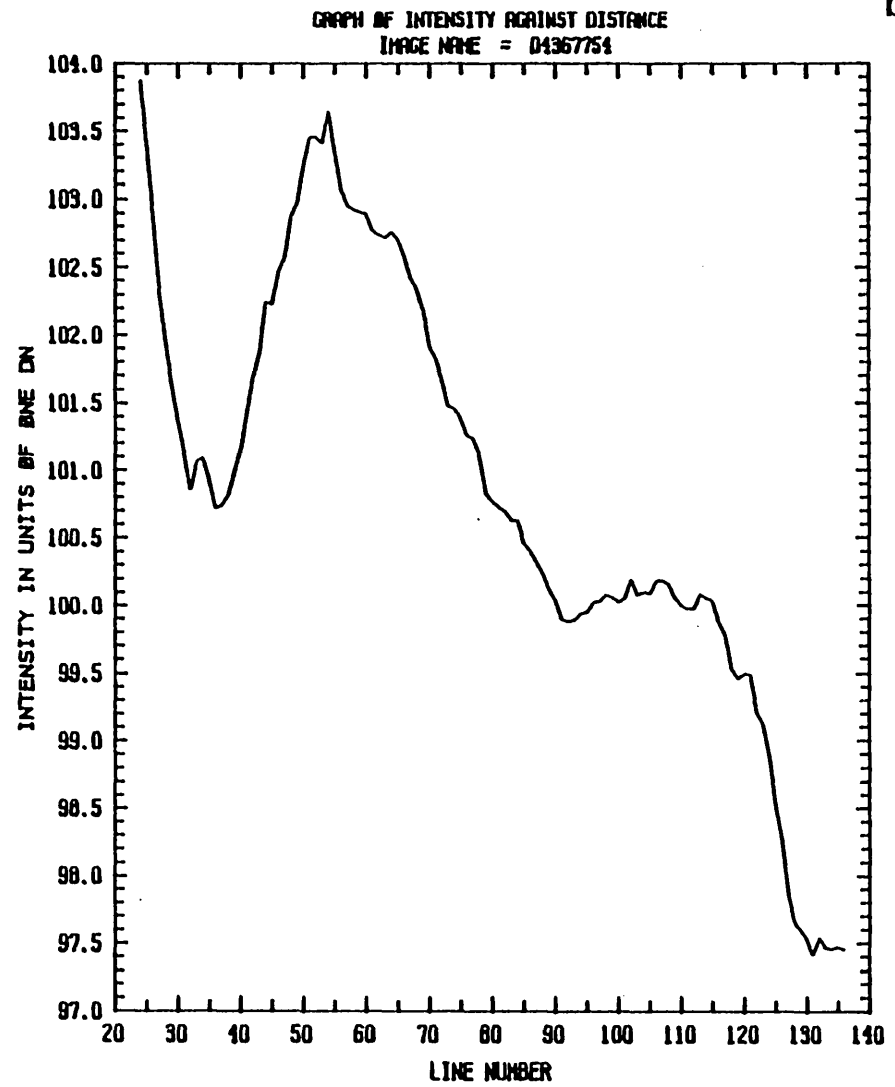


b

fig. 7.9 a,b

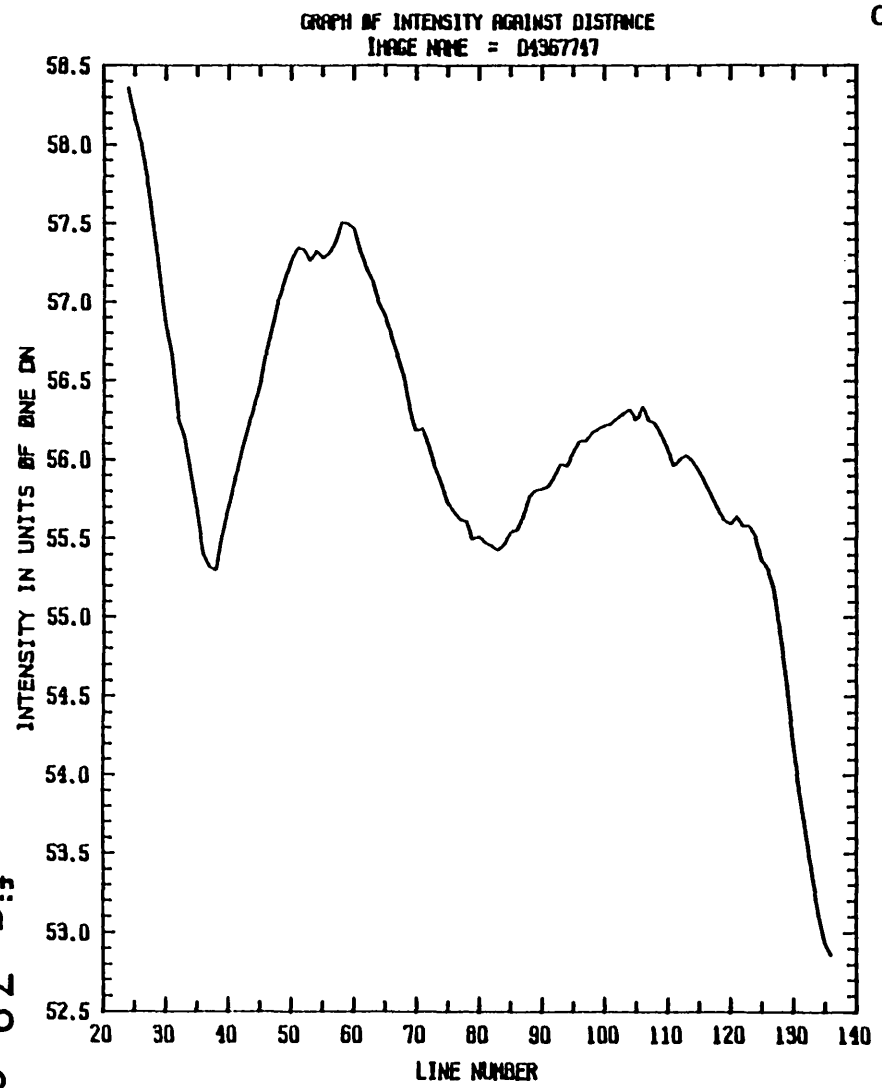


a

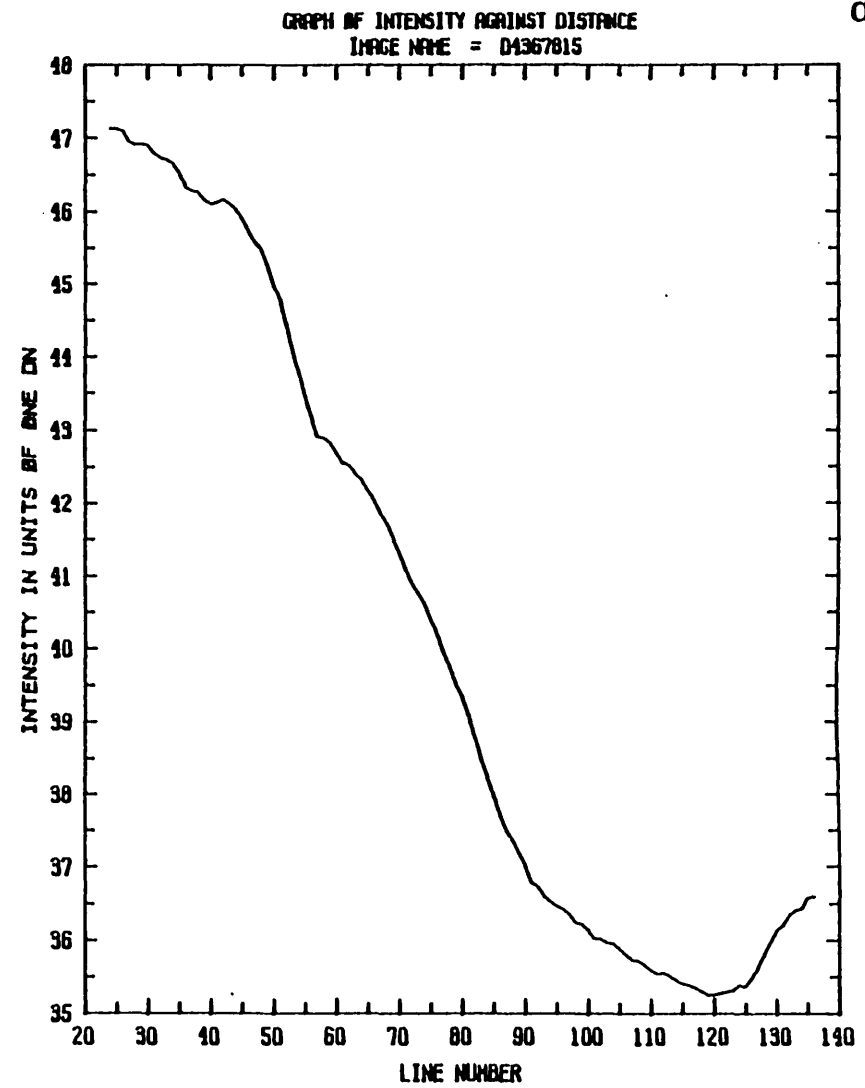


b

fig. 7.9 c,d



c



d

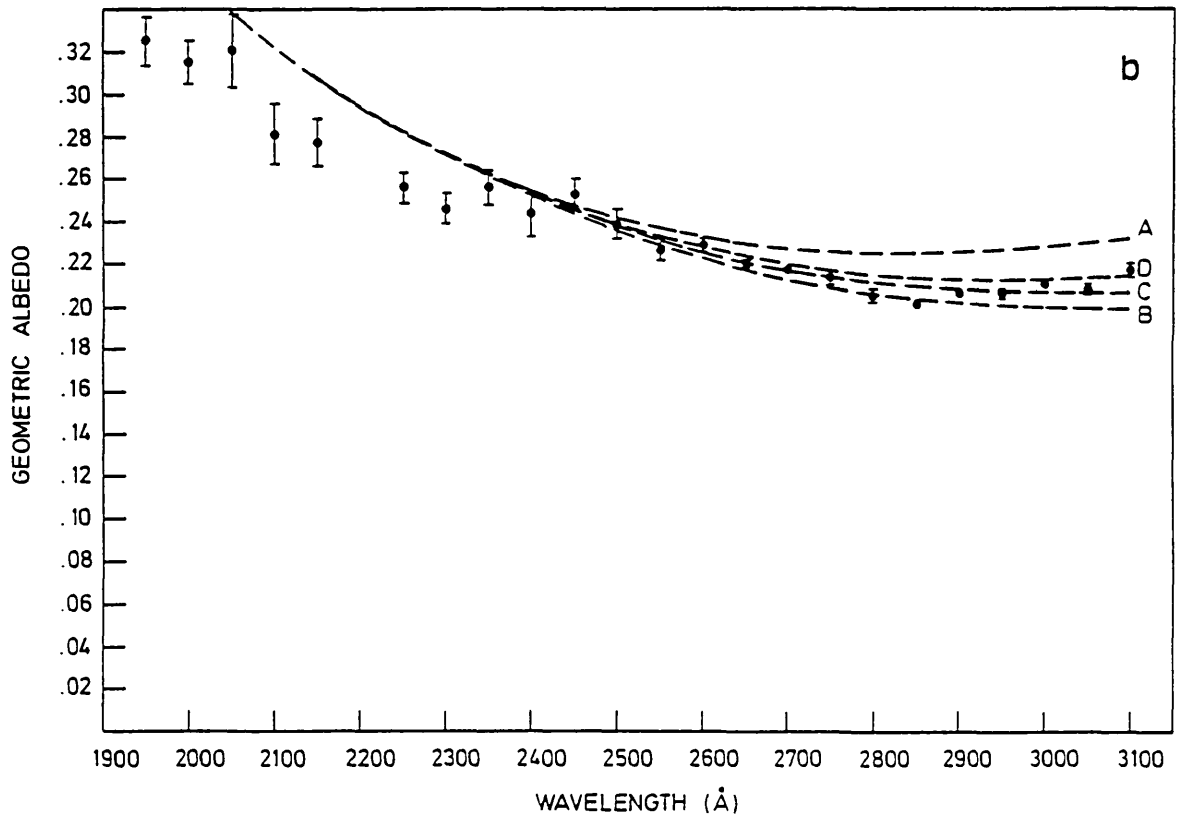
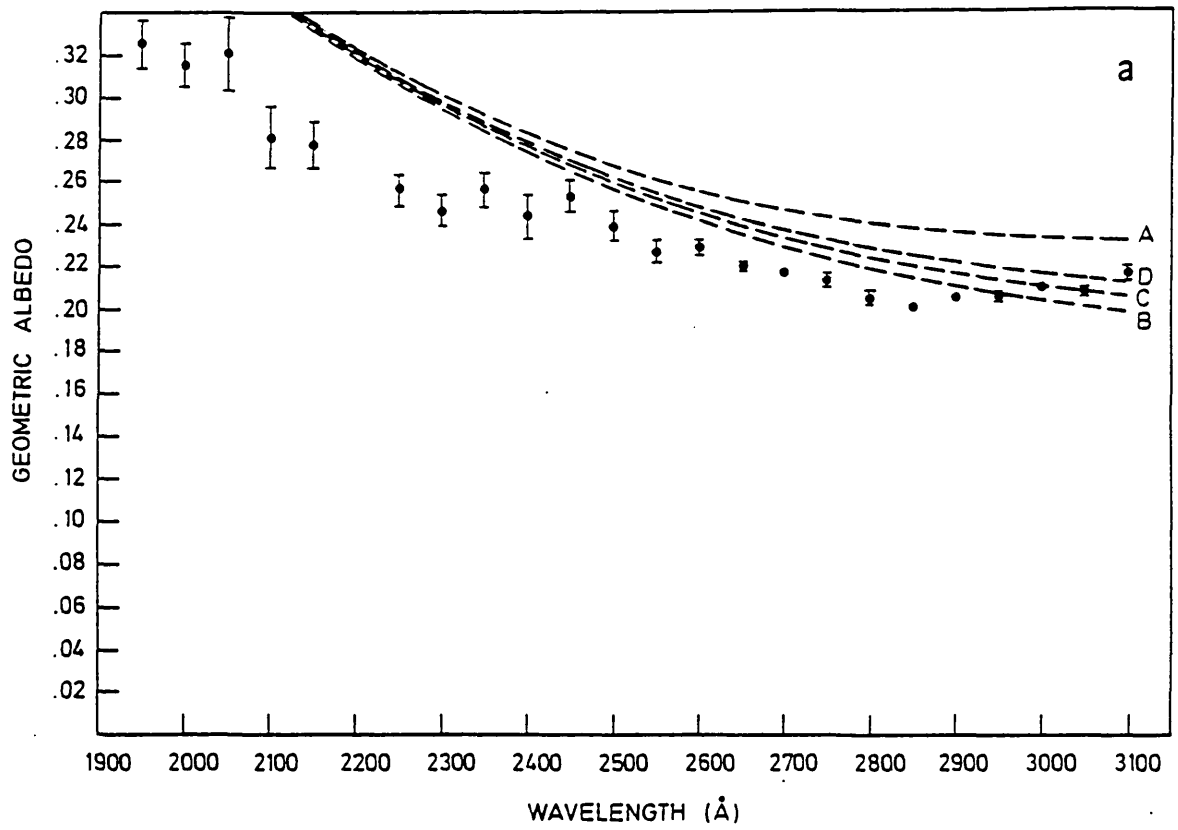


fig. 7.10

fig. 7.11

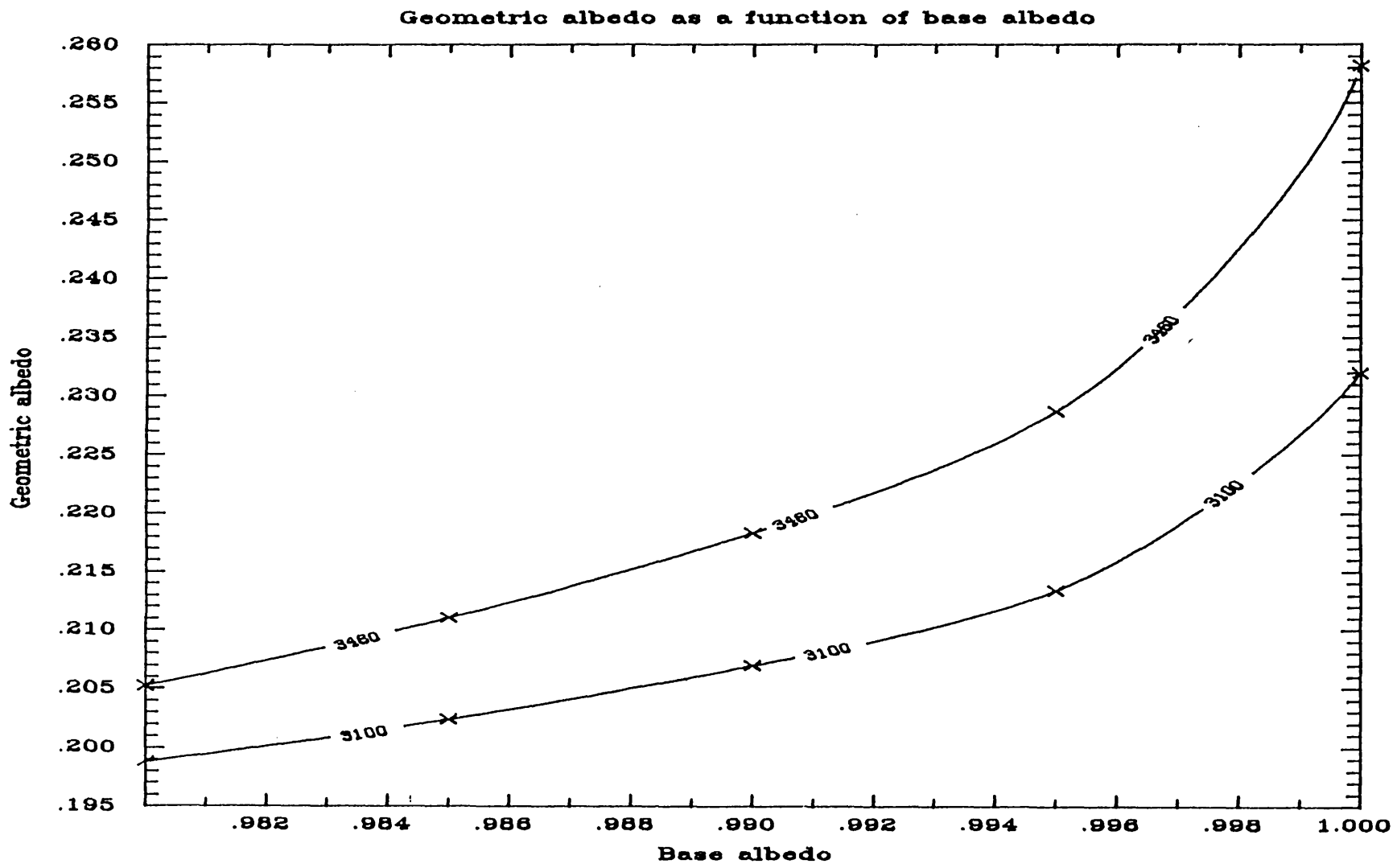
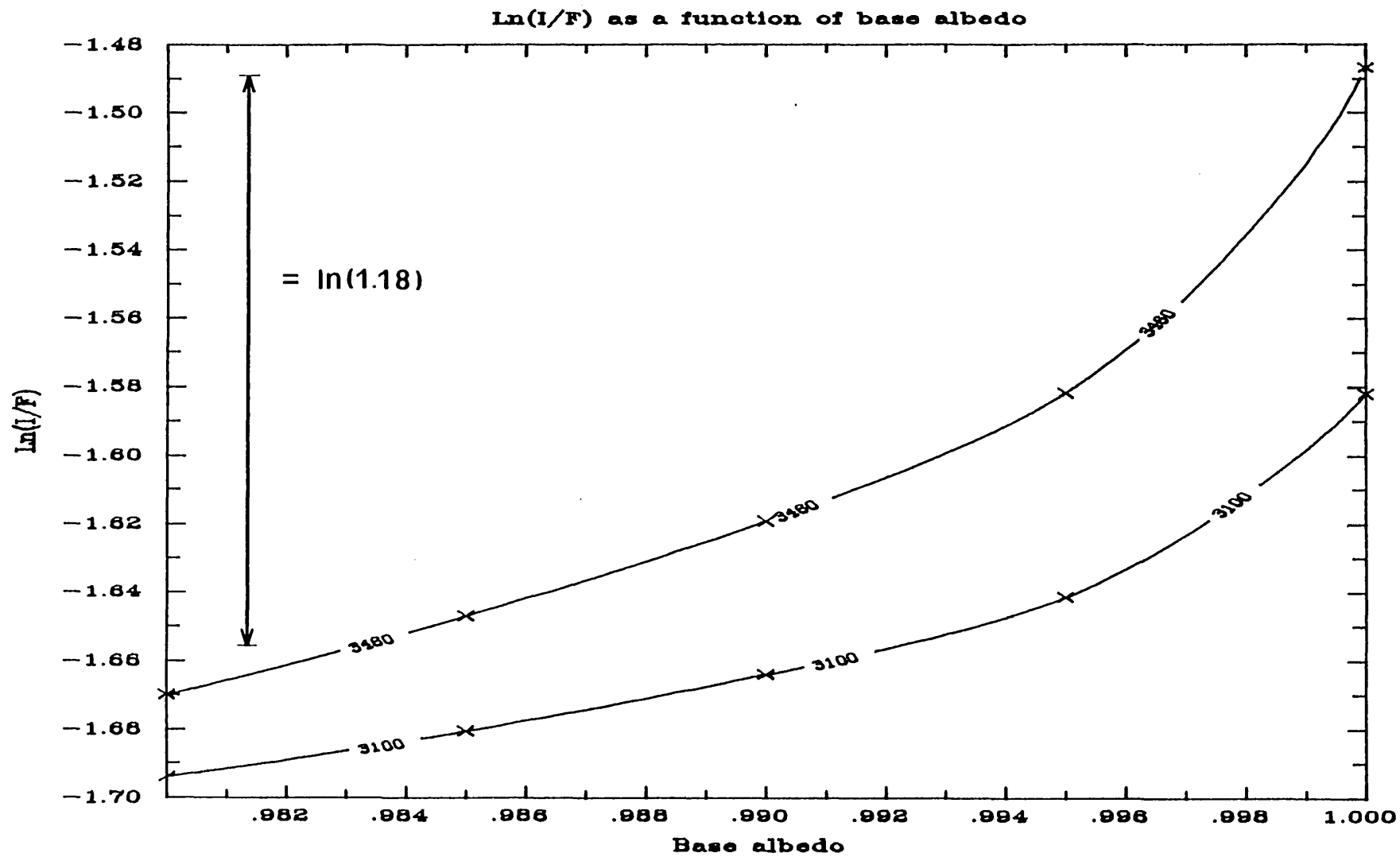


fig. 7.12



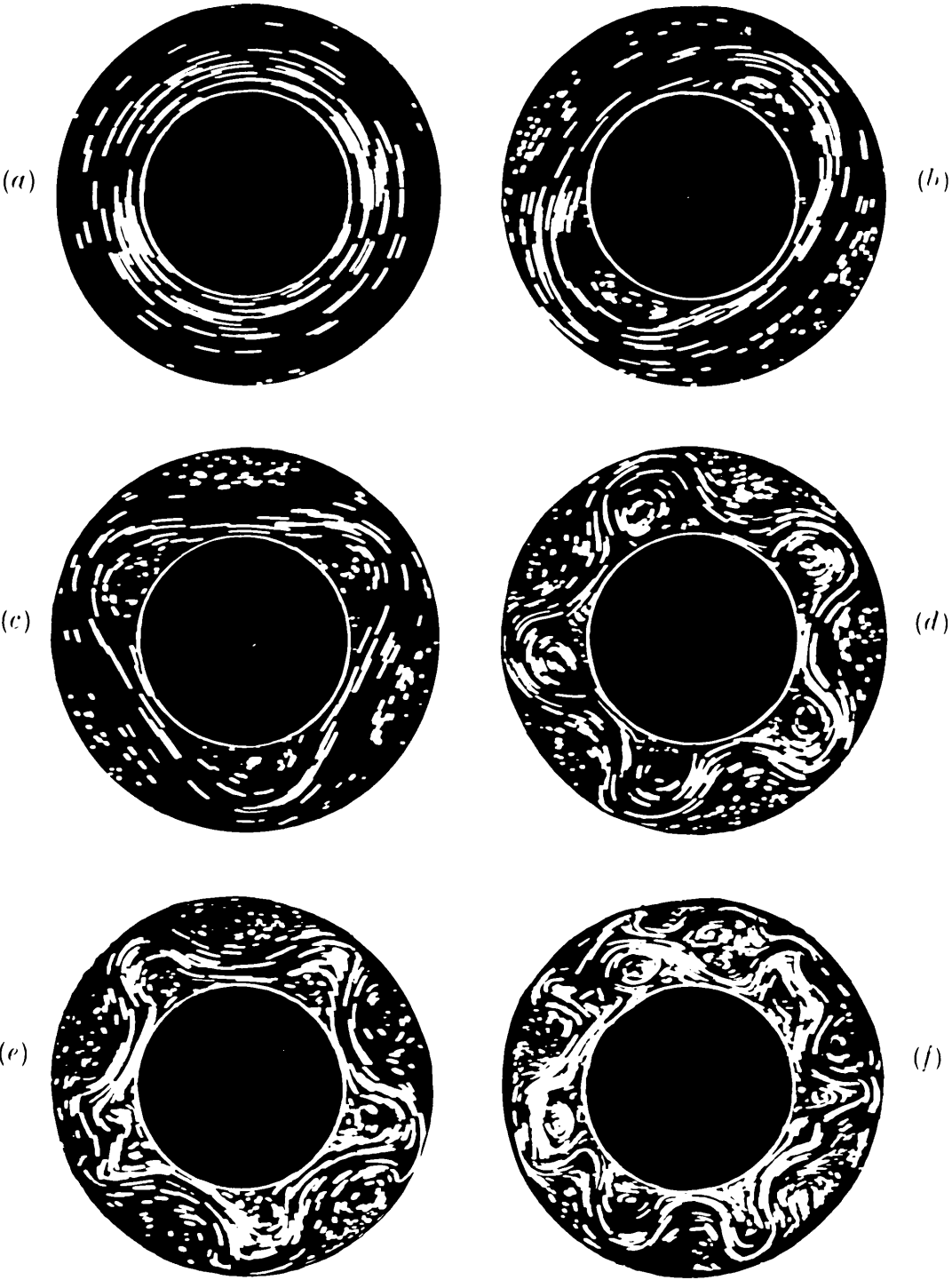


fig. 7.13

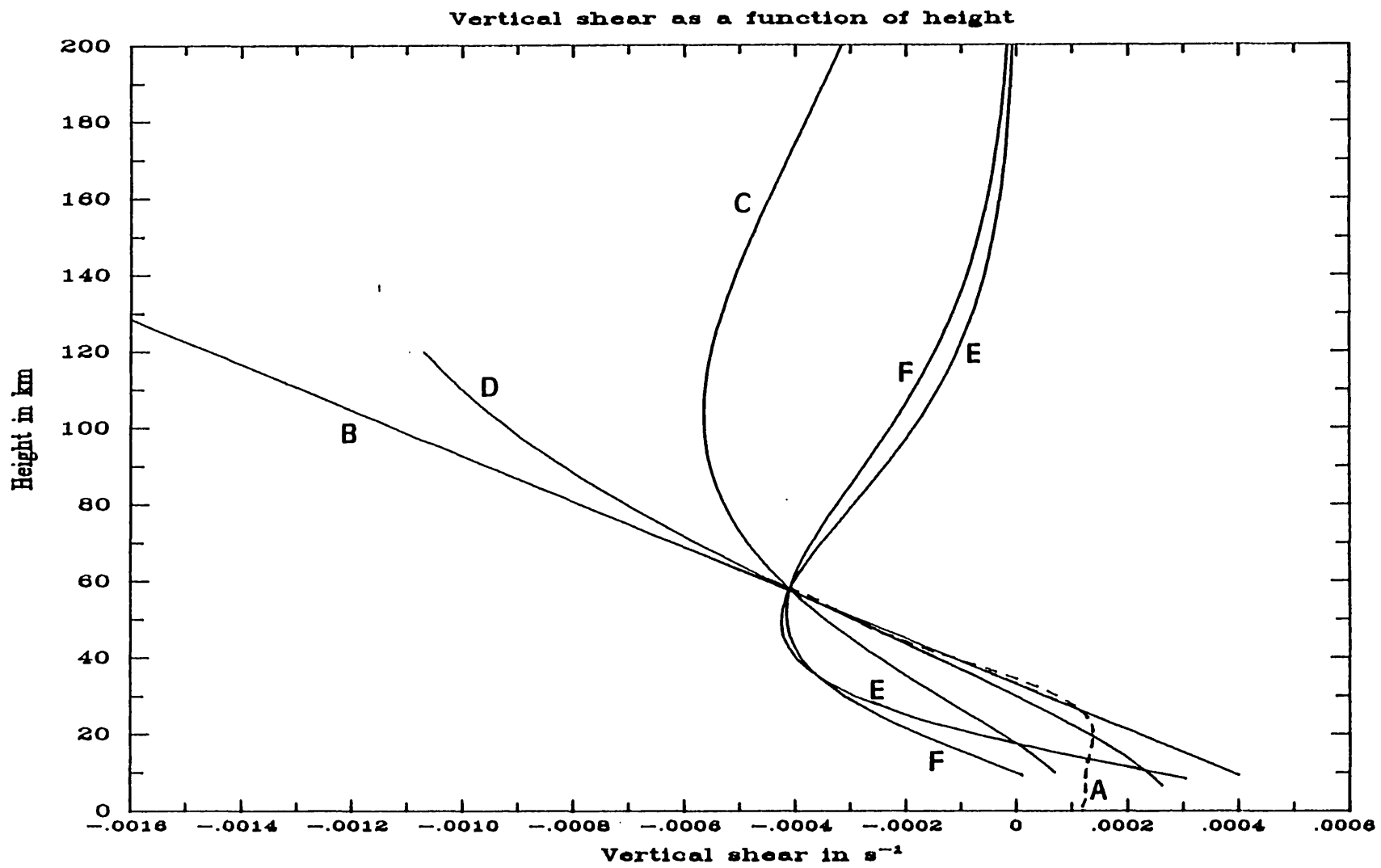


fig. 7.14

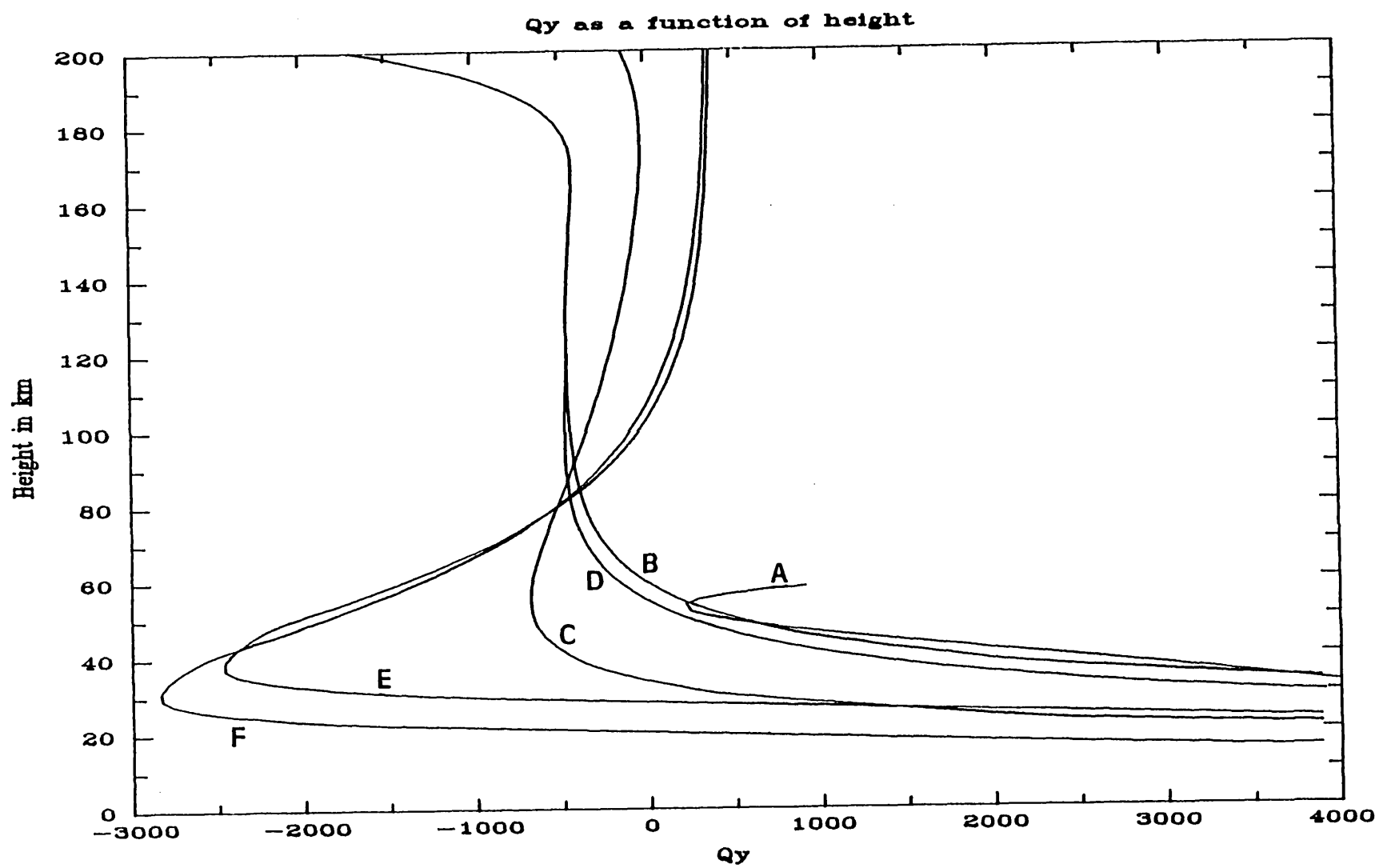


fig. 7.15

fig. 7.16

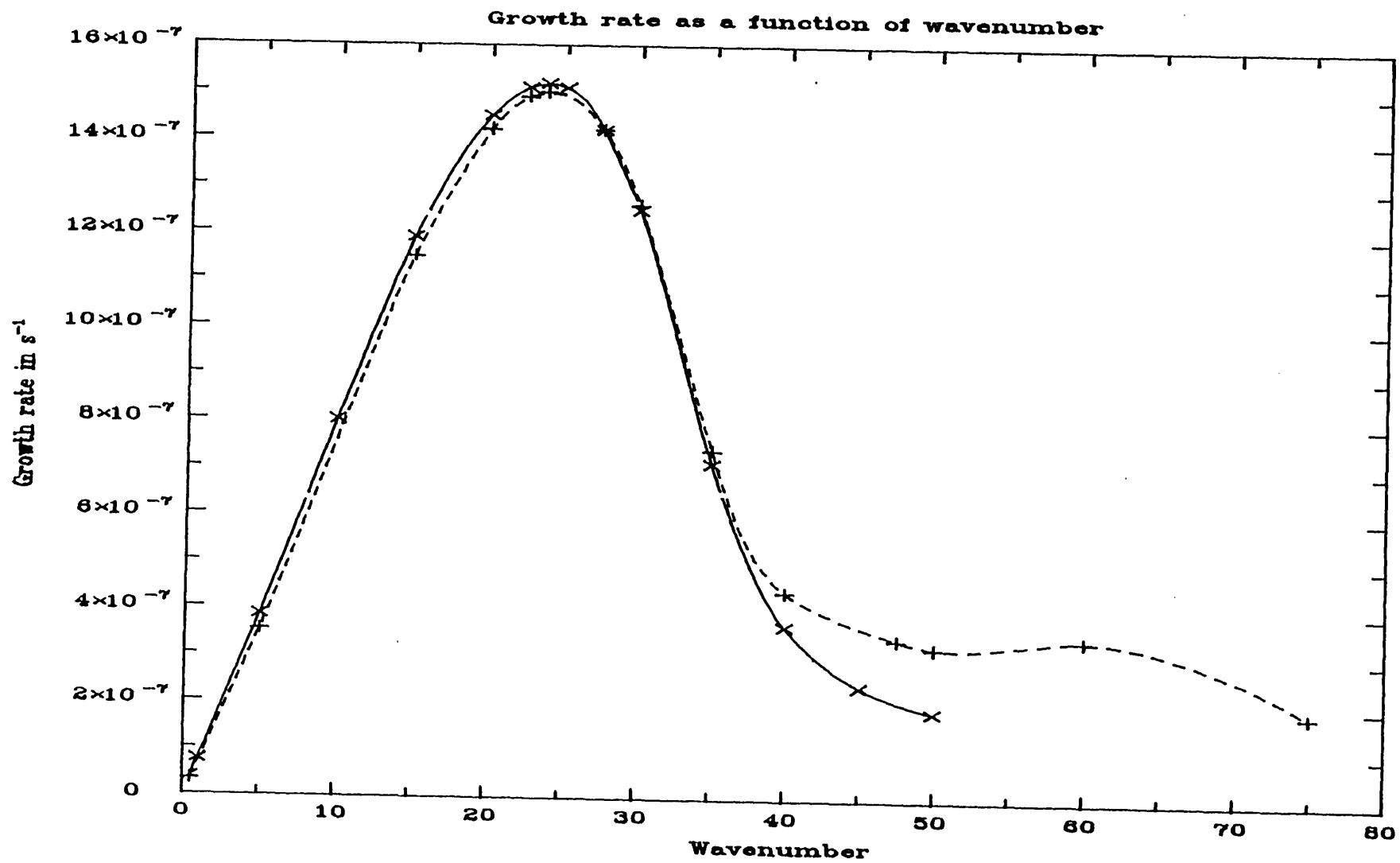


fig. 7.17

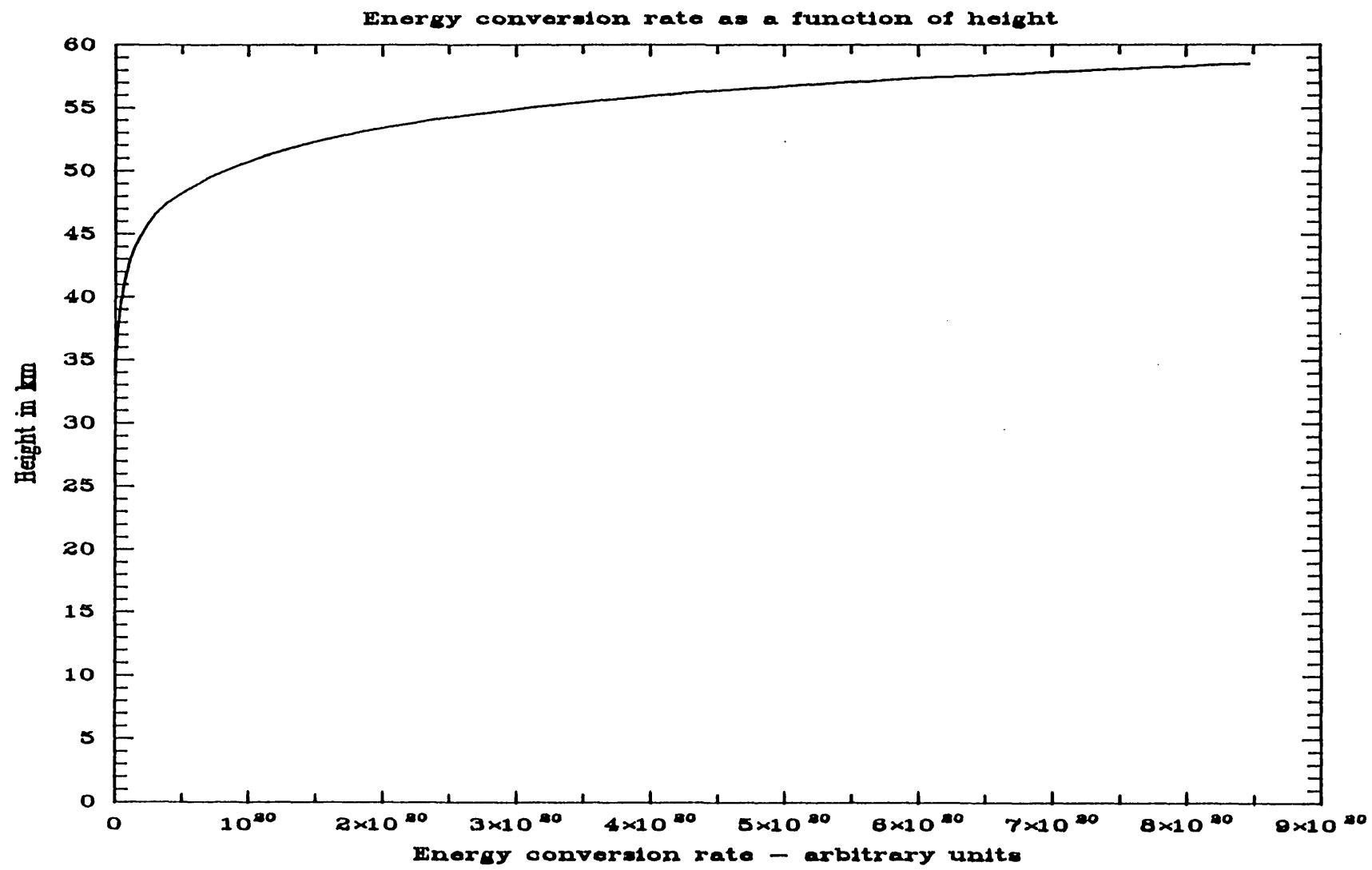
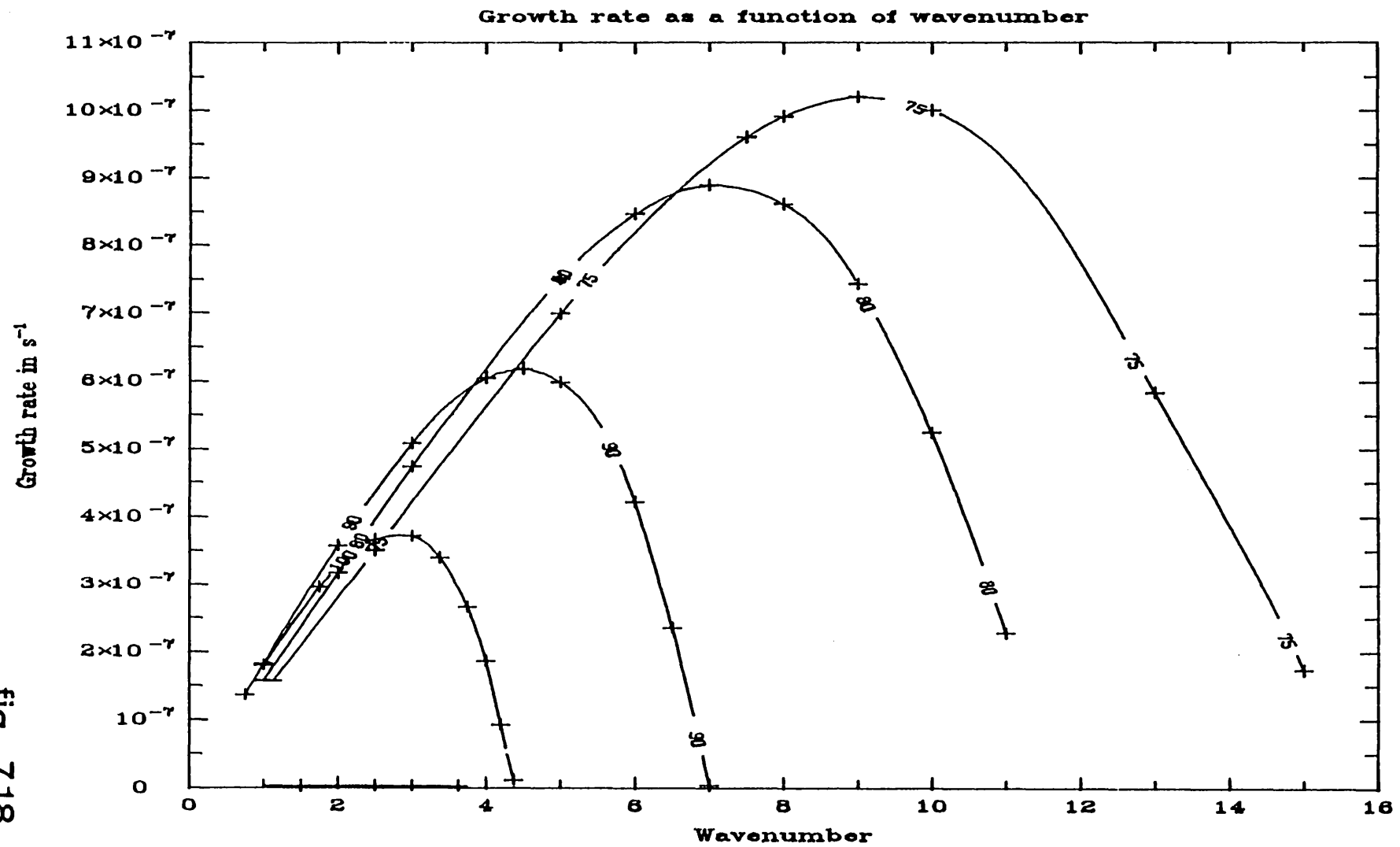


fig. 7.18



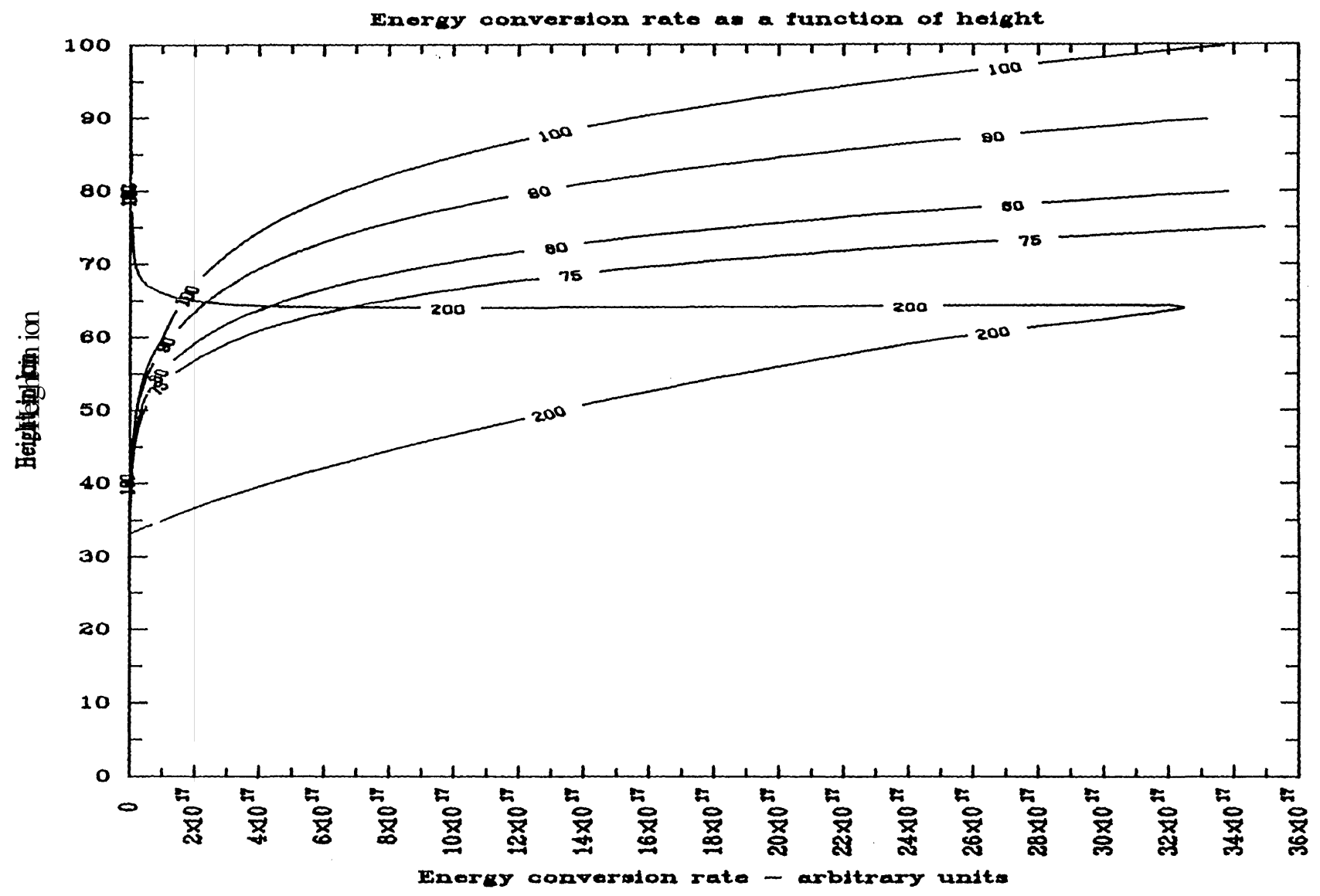


fig-7.19

fig. 7.20

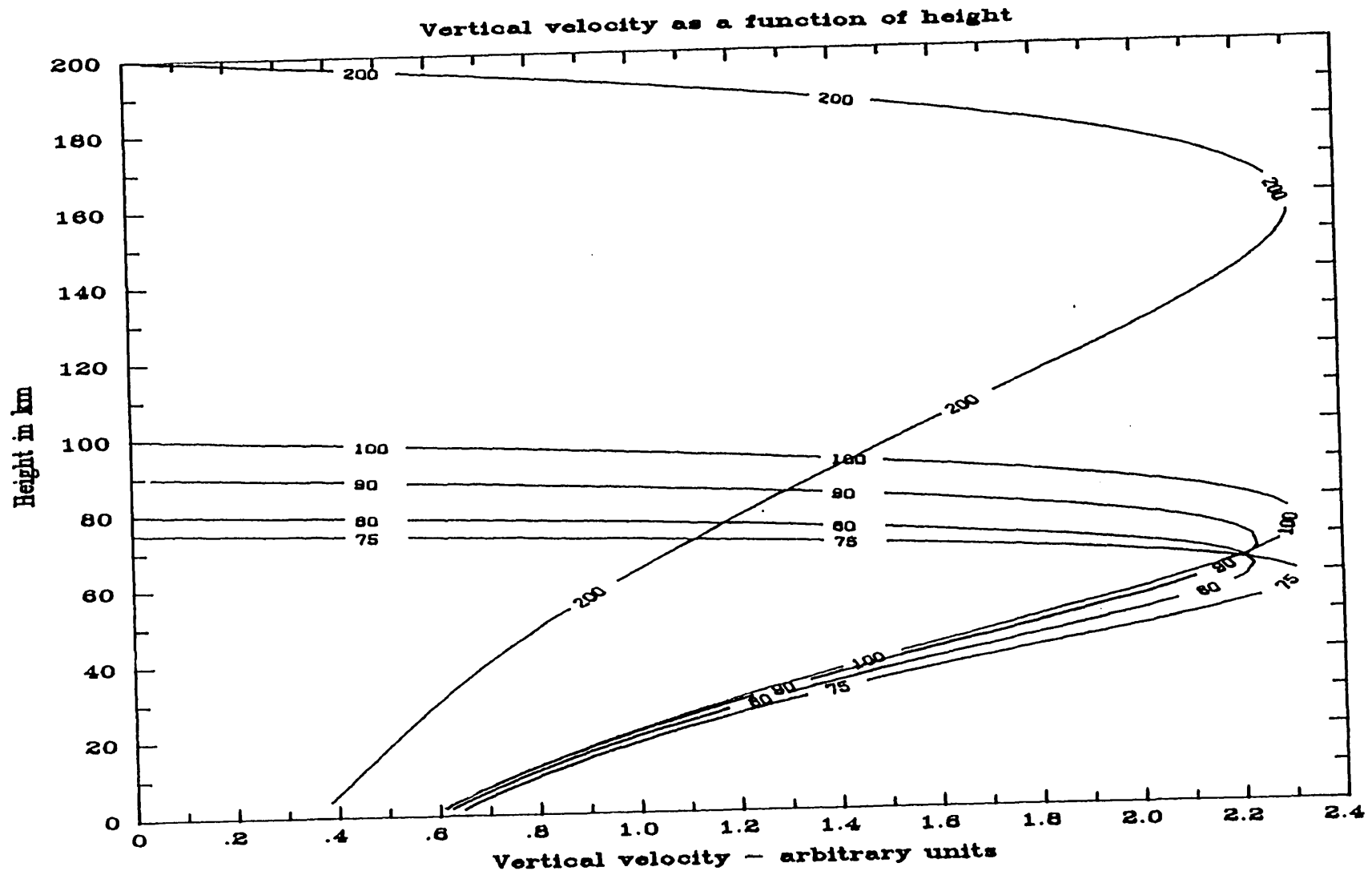
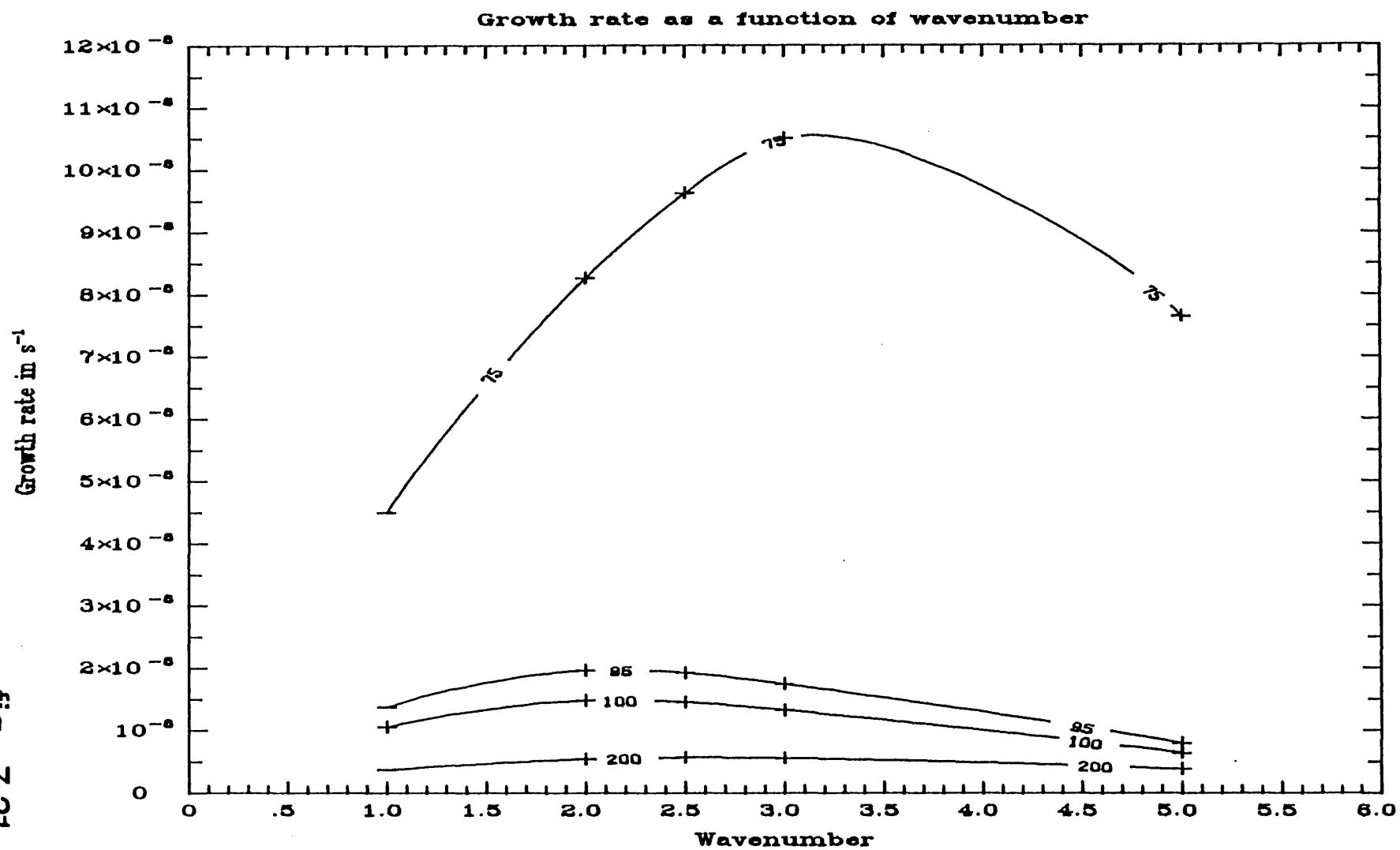


fig. 7.21



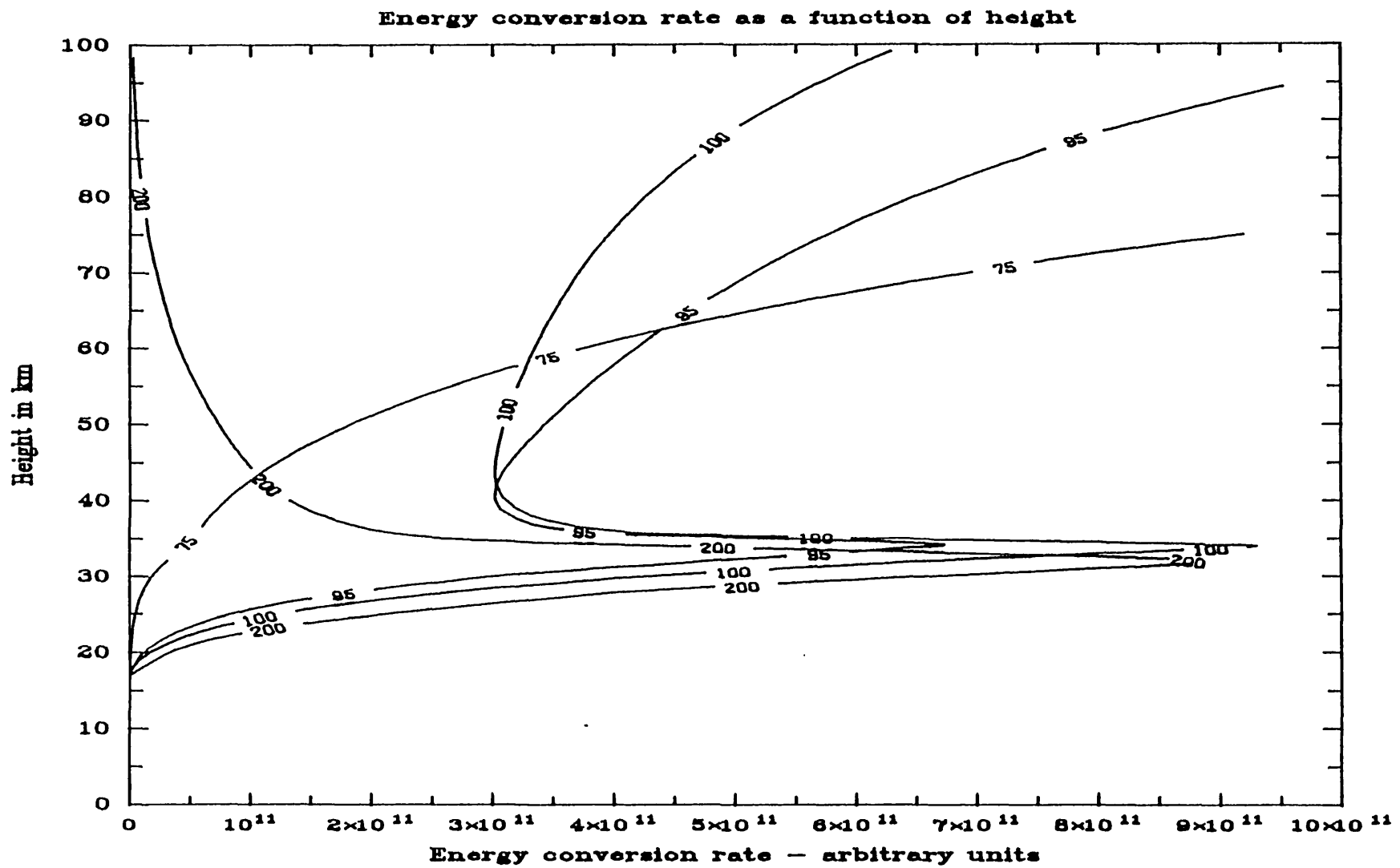


fig. 7.22

fig. 7.23

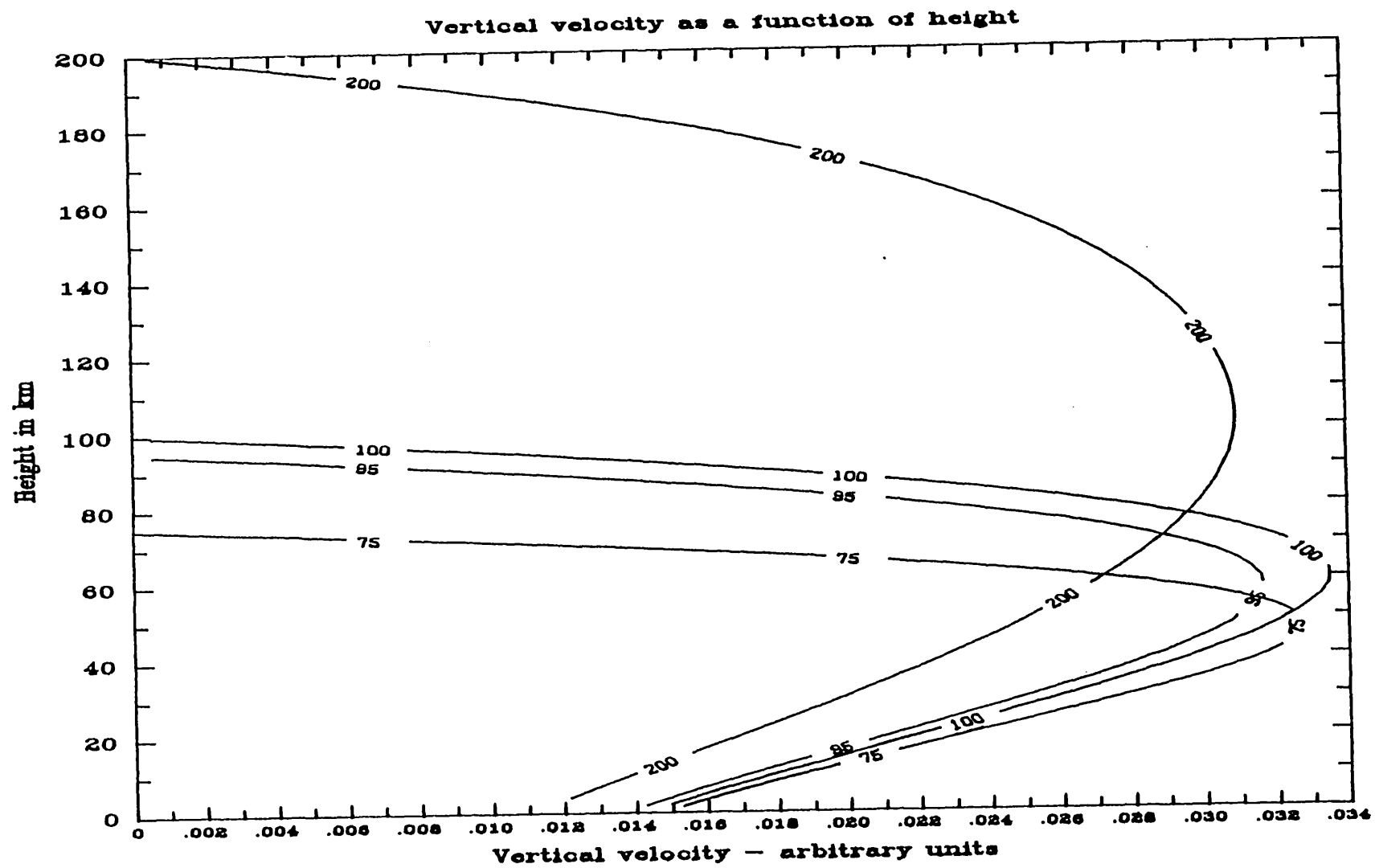


fig. 7.24

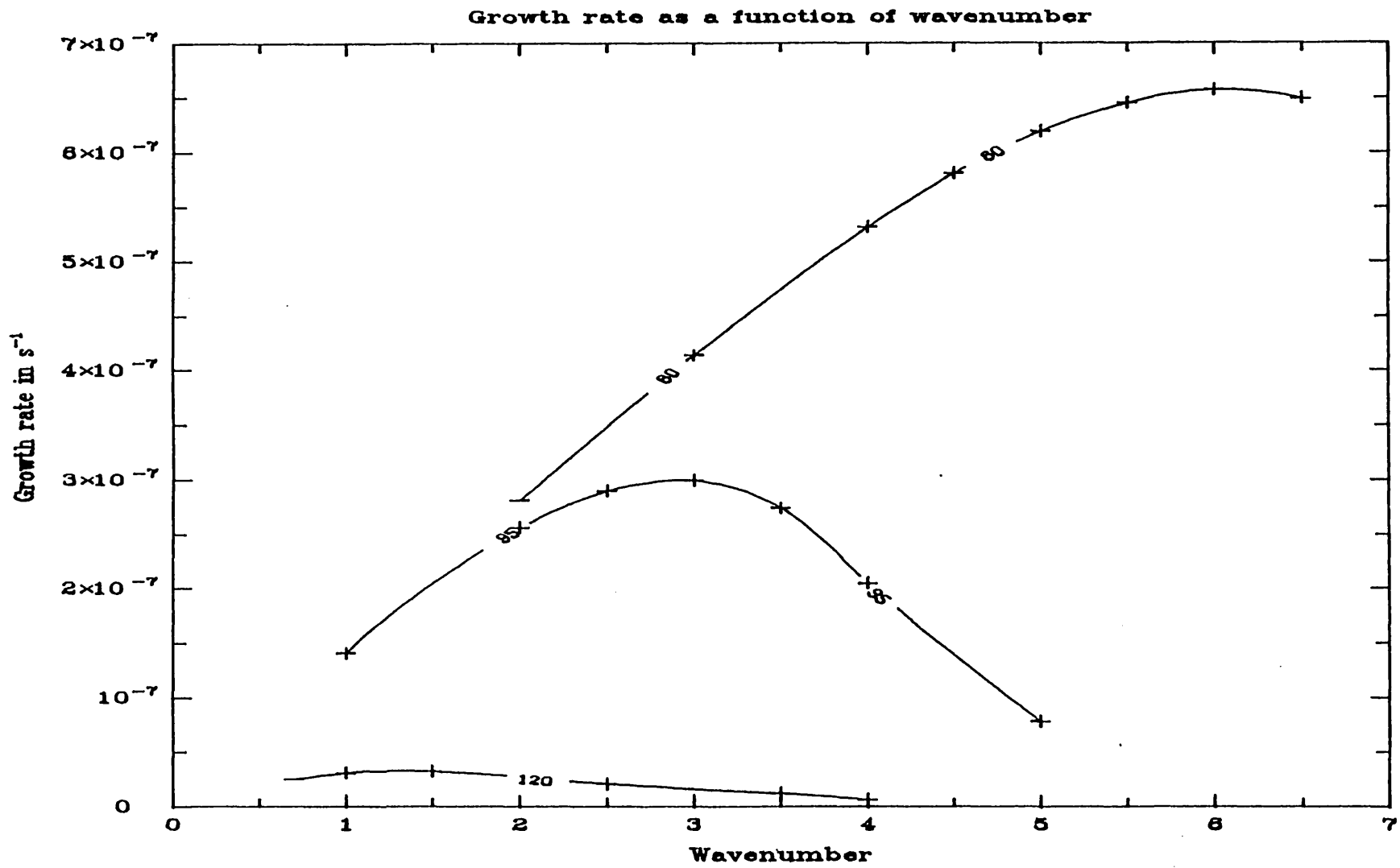


fig. 7.25

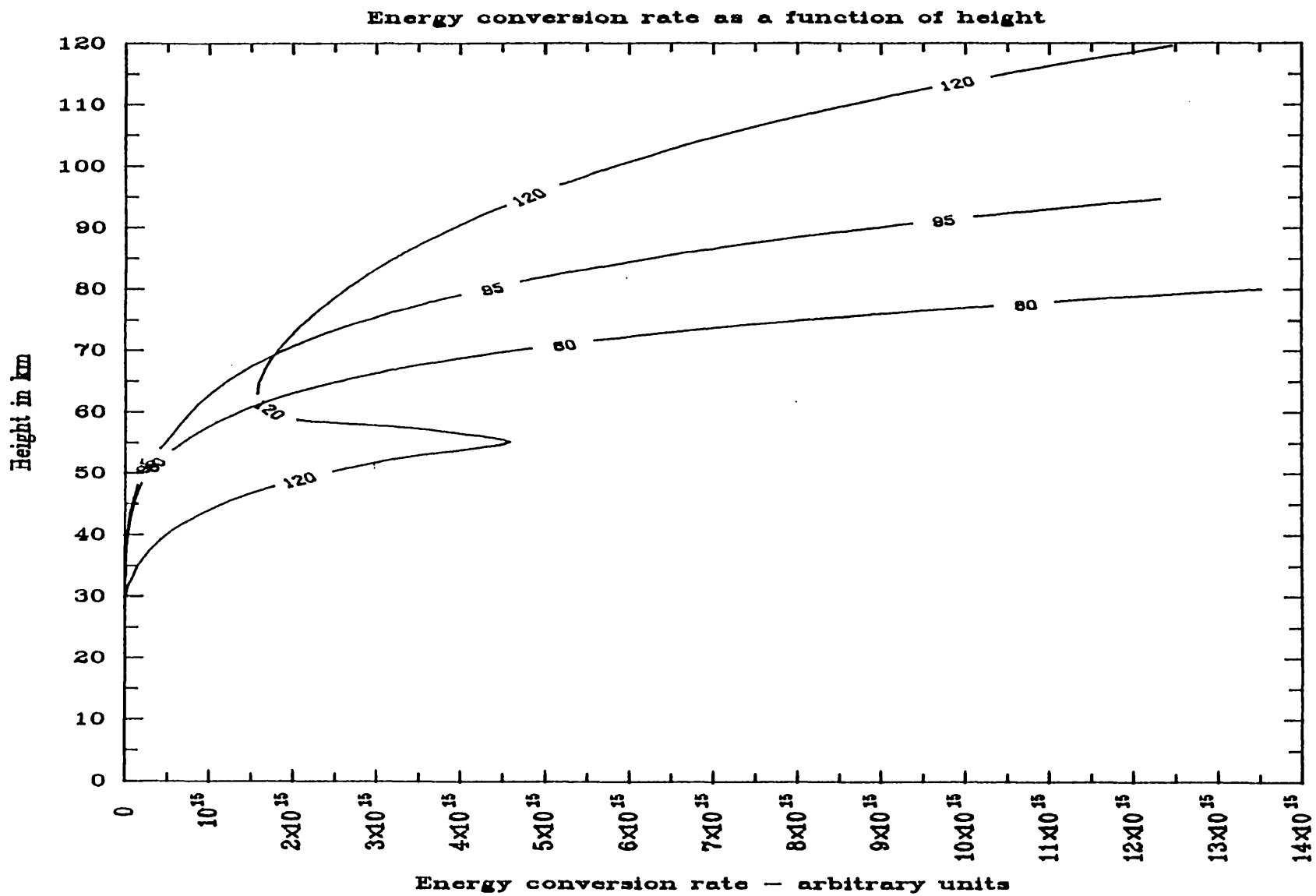
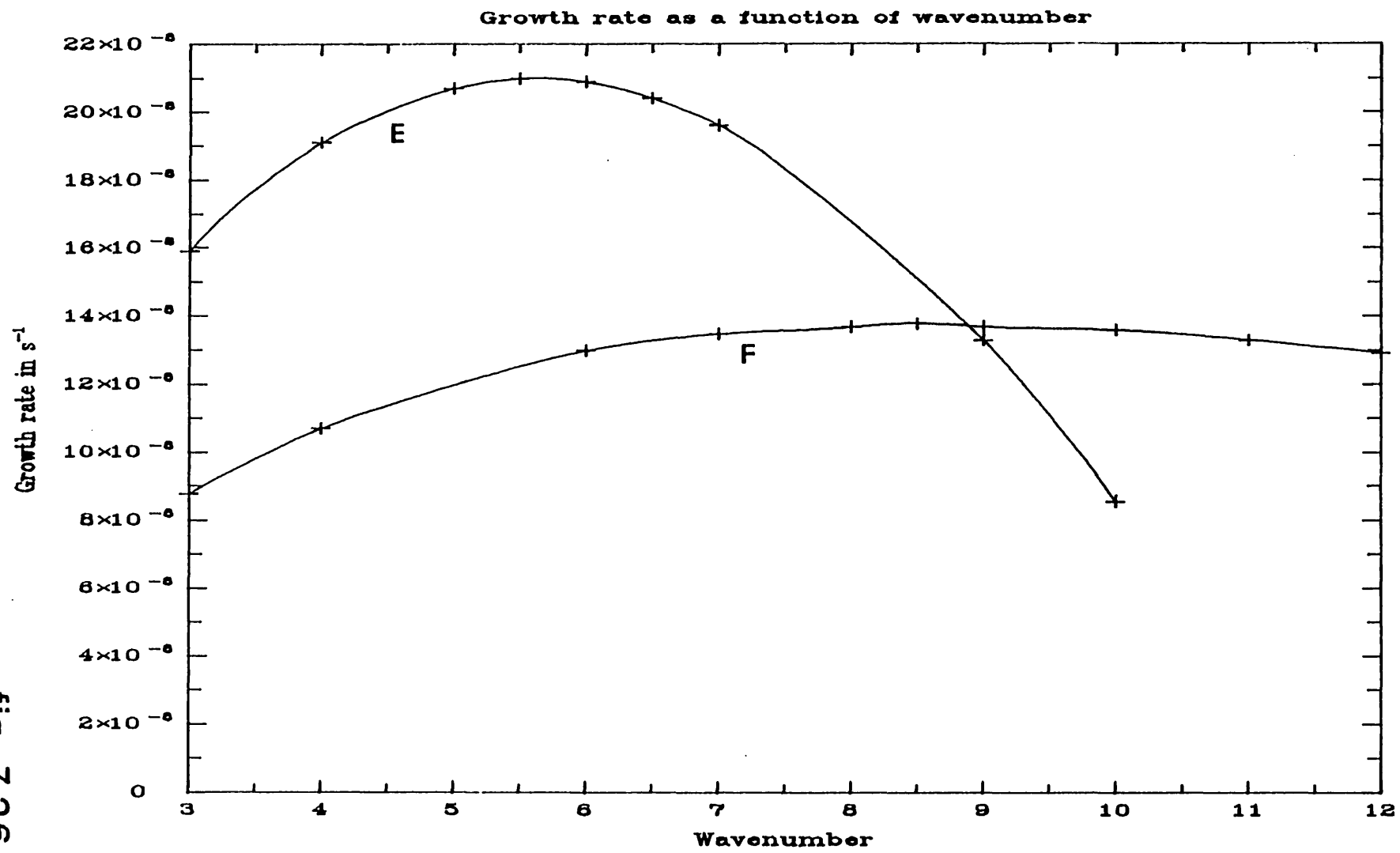


fig. 7.26



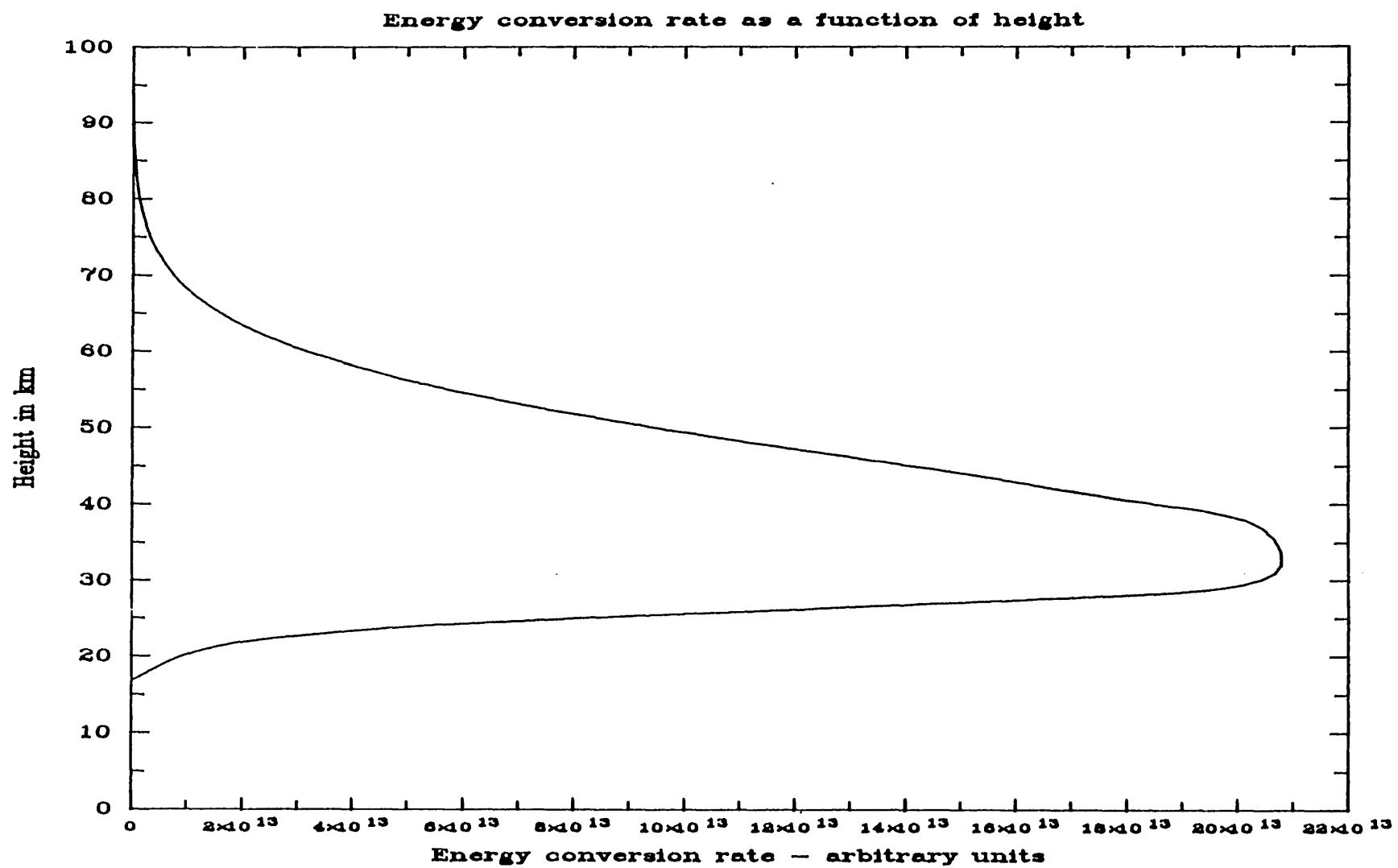


fig. 7.27

fig. 7.28

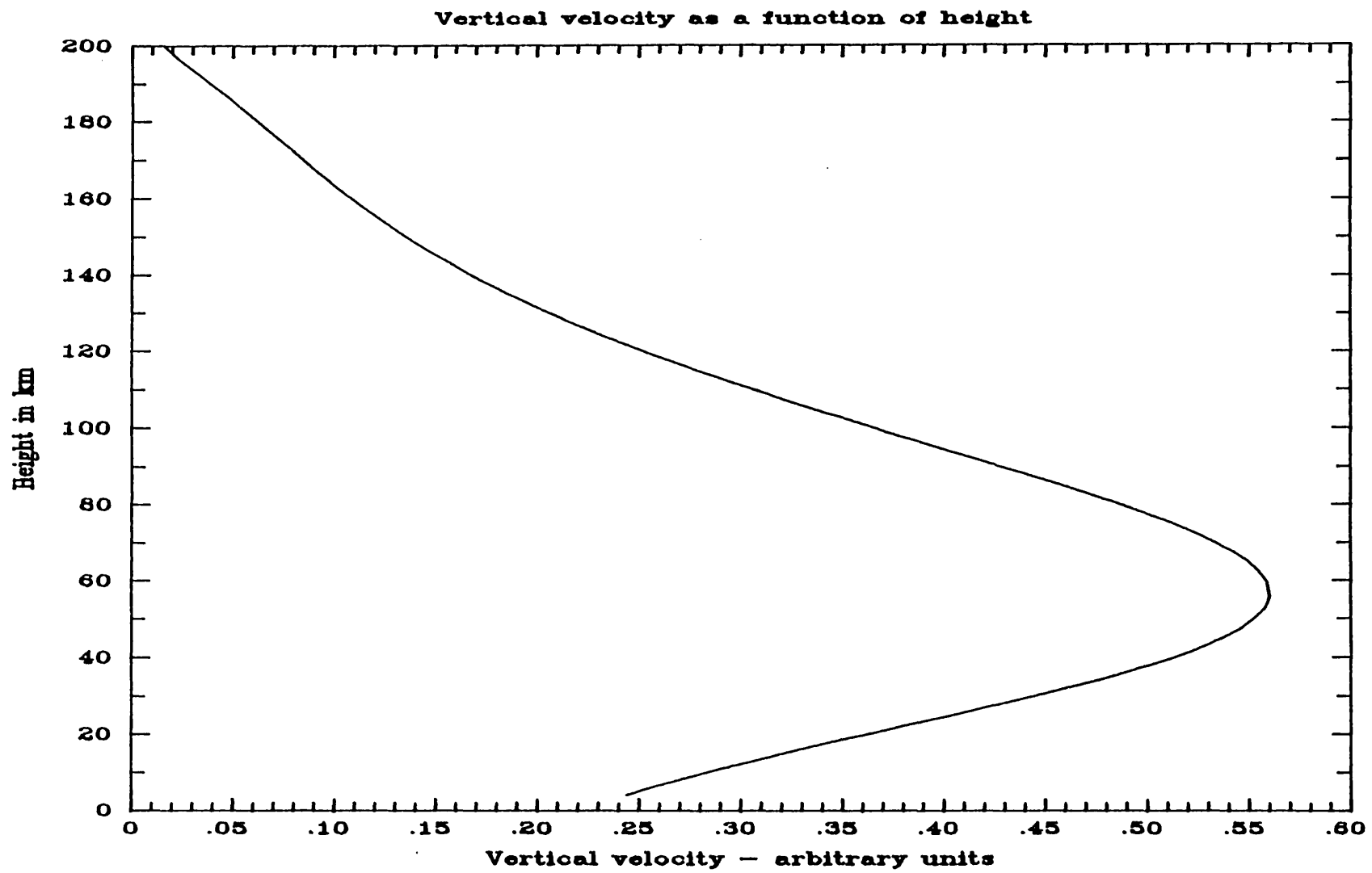
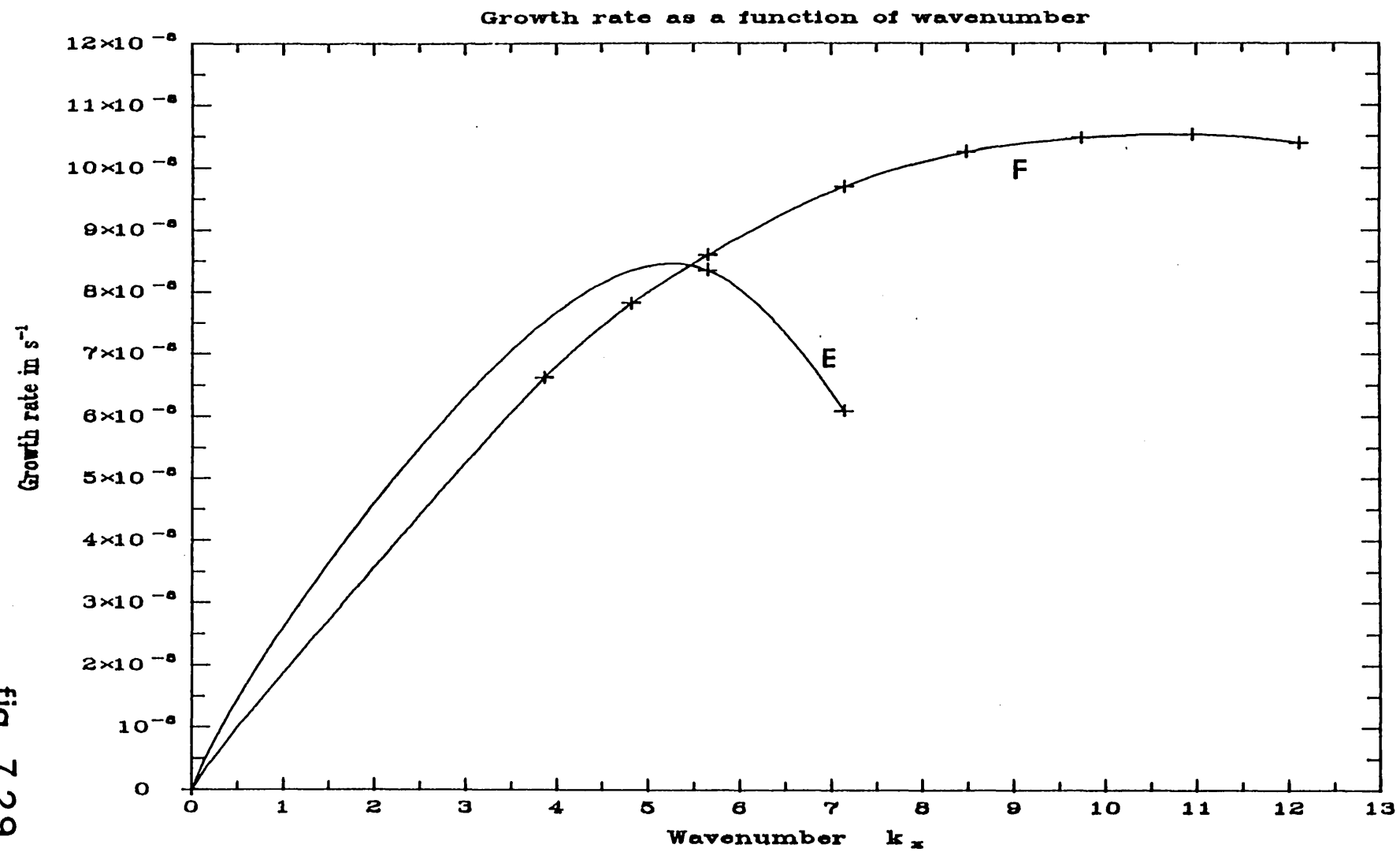


fig. 7.29



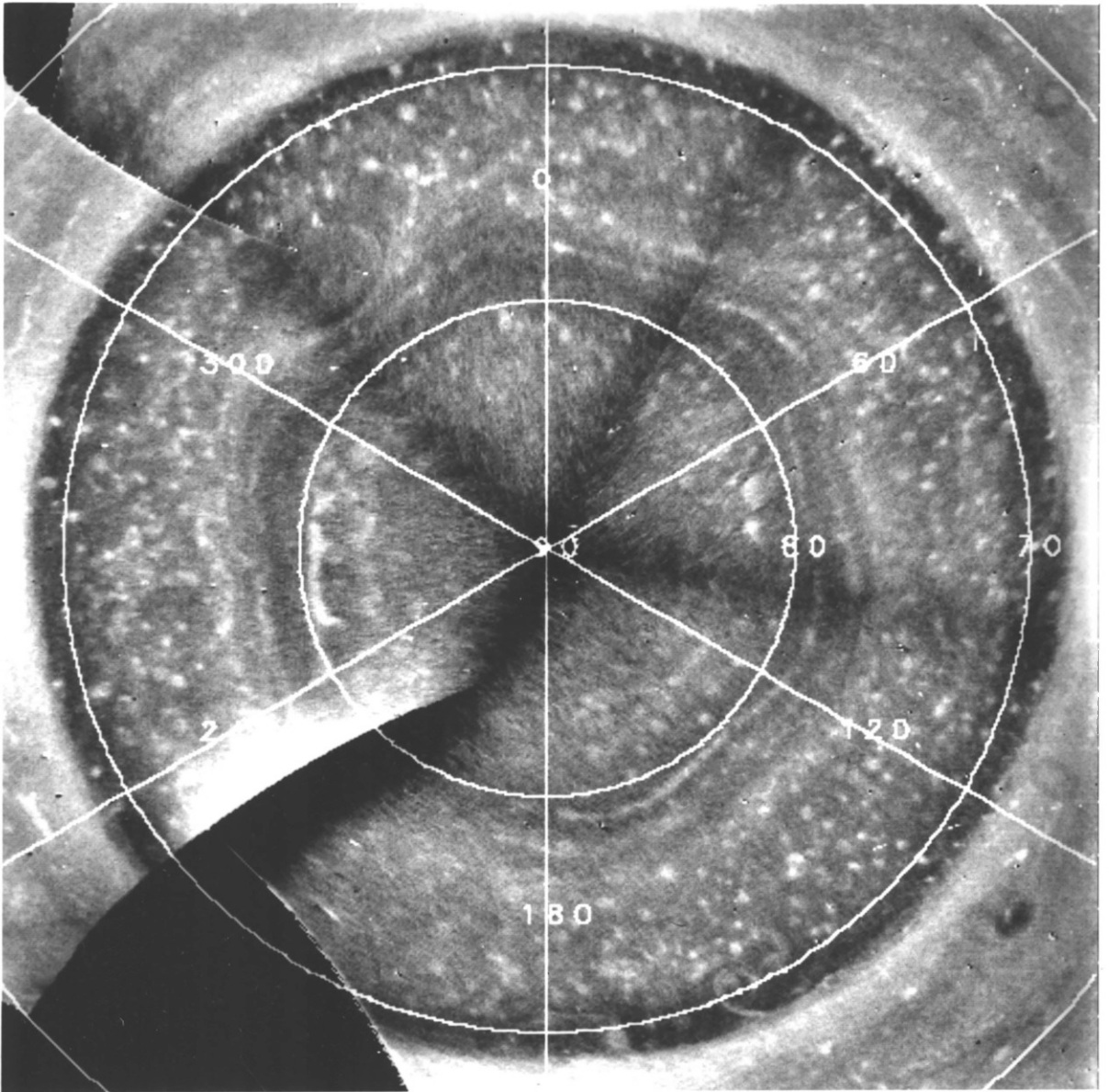


fig. 7.30

CHAPTER 8

DISCUSSION

The aim of this thesis is, as indicated in the introduction, to investigate the structure and causes of a sample of the small scale features observed in Saturn's atmosphere by the Voyager spacecraft. In particular a localised cloud feature at 40° N, and a wavelike feature encircling the planet at 47° N, were discussed in detail in chapters 6 and 7. However, in order to understand the limitations of the data it is necessary to discuss the constraints imposed by the spacecraft and its instruments, which were described in chapter 2, and the effects of the various image processing and restoration techniques, which were considered in chapter 3. The small scale features considered exist in a large scale flow, which has been measured by cloud tracking but is not fully understood, and some of the ideas about its formation and structure were described in chapter 4. It is also useful to have an estimate of the atmospheric temperature and its variation with height. These are mainly derived from infrared measurements and were discussed in chapter 5. From these other useful quantities were calculated, in particular the buoyancy (or Brunt-Vaisala) frequency, which is shown in fig 5.5. In addition to the average vertical temperature profiles there are also, over a limited pressure range, measurements of the structure of the horizontal temperature field. From these, using the thermal wind equation, the vertical shear of the zonal flow was calculated

and the results obtained for a range of latitudes in the Northern hemisphere are given at the end of chapter 5.

Chapter 6 contains the analysis of the first of the two features considered in detail, which is a 10^4 km long, cluster of white clouds to the east of a dark, $\approx 10^3$ km diameter spot. The feature is located at the maximum of a westward (easterly) jet centred at approximately 40° N North. For this analysis four images were selected and measured to produce streamlines. From these the vorticity was calculated at several locations, and when plotted as a function of longitude, showed a distinctly periodic behaviour (fig. 6.8). Both the appearance and the derived structure of the flow show a strong resemblance to the Karman vortex streets observed in the laboratory, and to similar features seen in the Earth's atmosphere. Karman vortex streets are periodic features produced when fluid passes around a solid obstacle. Although their characteristics are altered by the presence of such effects as stratification and rotation, this type of flow still requires the presence of a solid body. This is unlikely to occur upon Saturn.

This requirement for a solid surface follows from Kelvin's theorem, which states that in an inviscid, barotropic fluid the absolute circulation of a fluid element is conserved. In the laboratory case of a uniform incompressible flow this implies that the vorticity observed is produced in the boundary layer surrounding the solid surface.

Since regions of alternating vorticity were measured for the Saturnian feature, some other vorticity generation mechanism must be mimicing the production of vorticity, by boundary layers, in the laboratory studies. The feature seen on Saturn is located in a baroclinic fluid which is known (from cloud tracked wind measurements and the thermal wind shear results) to possess both horizontal and vertical shear; also the beta-effect will, in the presence of

meridional motion, produce relative vorticity (see section 6.1.4).

One example of a mechanism which is capable of generating the observed horizontal vorticity is the twisting or tilting term in the potential vorticity equation. As shown in section 6.1.4 this can produce a significant horizontal vorticity component through the combination of the vertical shear and the vertical motion expected to be associated with the dark spot.

The production of a feature which resembles a vortex street, but is formed in a different way is not unknown. Examples have been reported in the laboratory behind water jets introduced into channels (private communication, see section 6.4). The observation of a Saturnian feature similar to a Karman vortex street, but without the associated solid object, may be relevant to the features seen in the Earth's atmosphere. In particular it could explain why the features observed can appear larger than would be expected from the size of the obstacle alone.

The second feature considered in detail is a 5° wide zone of clouds centred at 47.5° N, moving east at 120 ms^{-1} , and containing a darker 'ribbon' feature 10^3 km wide with an approximately periodic longitudinal structure. A mosaic showing the full 360° of the feature was produced from a set of low resolution images so that an estimate of its longitudinal wavelength could be made (figure 7.2). Initially this was attempted using a Fourier transform technique, but since this (as was shown in section 7.3.2) was unduly influenced by some of the larger, and less reliable, peaks a different approach was adopted. A program was written which removed all the peaks separated by less than three pixels in the line direction and then found the mean separation of adjacent peaks. This gave a longitudinal wavelength of $5710 \pm 260 \text{ km}$. A second set of higher resolution images of this feature were made into a movie loop which demonstrated that it appears to have a cyclonic circulation to the

north and an anticyclonic circulation to the south.

A third set of images was used to examine the multispectral characteristics. These show that in most of the filters the feature takes the form of a dark band near the centre of a bright zone. However in the UV (3460Å) filter this is replaced by an abrupt change in the relative intensity between the northern (lighter) and southern parts of the feature. At these wavelengths it is (apart from a few small spots) the only non-axisymmetric feature seen in the northern hemisphere. Because of this sudden transition the vertical structure of the feature was examined with a model normally used for globally averaged albedo measurements. By altering this until the observed albedo variation was produced it was shown in section 7.4.3 that, unless one goes considerably outside the parameter range required by the globally integrated shorter wavelength data, the feature must extend up to at least 170 mb. An additional constraint imposed by the longitudinal similarity of the UV and other images is that, assuming the observed brightness variations are the result of vertical displacements, the phase of the vertical velocity cannot change significantly between 170 mb and the top of the ammonia clouds (≈ 400 mb, see section 5.2).

The observed structure resembles that of a baroclinic instability (sometimes referred to as sloping convection) which, like the vortex street feature, occurs in both the Earth's atmosphere and laboratory experiments. This type of flow takes place when the imposed conditions are such that it is possible for a disturbance to extract energy from a latitudinal temperature gradient. When this happens fluid on the warmer side of the flow is exchanged with higher fluid on the cool side. This process is known to be important in the Earth's atmosphere and has been the subject of many analytical and numerical studies. However the conditions for instability involve the boundary conditions and often require the presence of a rigid

lower boundary which is not present on Saturn. This difficulty was considered by Gierash et. al. (1979) who produced a more appropriate lower boundary condition. Using this I solved the quasi-geostrophic potential vorticity equation for a sinusoidal perturbation numerically, to determine the structure of the Saturnian disturbance. Two different methods of doing this were developed, and these are described in section 7.8. When supplied with the static stability and the vertical wind shear (both evaluated using the methods described in chapter 5) they provided the geopotential as a function of height, as well as the growth rate and phase speed.

The main difficulty in using this model to predict the structure of a Saturnian instability was the limited (150 - 700 mb) range over which the latitudinal temperature gradient, and hence the thermal wind shear, was known. This is unfortunate since the types of solution obtained depend upon this, particularly through the sign of the quantity Q_y . In order to obtain useful solutions curves were fitted to that part of the data available and then extrapolated to greater heights. The parameters used in these curves were chosen to bracket the likely behaviour, and can be divided into groups depending upon the way in which they satisfy the necessary conditions for instability. The simplest of these used a cubic spline fit to the thermal wind values available, and was only unstable because of the presence of an upper boundary. The second used a straight line fit to the data and satisfied the conditions for instability by a change in the sign of Q_y . The other profiles used were all based upon equation 7.35 but with differing parameters. These all produced at least one sign change in Q_y , and hence satisfied the necessary conditions for instability. Some altered sufficiently rapidly to cause Q_y to change sign a second time.

Since all the profiles of $\partial u / \partial z$ tested were found to be potentially unstable the numerical models were used to examine the structure and growth rates of the disturbances produced. These were then compared with the observations of the Saturnian feature described earlier. It was found that (as described in section 7.8.4) the vertical velocity fields produced extended high into the atmosphere, above the 170 mb level required by the UV measurements, and with a sufficiently small phase change to account for the similarity between the UV and the other images. The simpler profiles of $\partial u / \partial z$ (ie excluding those with multiple sign changes in Q_y) can also reproduce the observed longitudinal wavelength and a plausible growth rate ($\approx$ one month) but only with an unreasonably low upper boundary. When this is raised to a level at which it ceases to be very important, the growth rates become very small, whilst the longitudinal wavelength increases to about five times that observed.

However the effect of an artificially low upper boundary appears to be similar to that of an additional sign change in Q_y . This supposition was examined using the profiles labelled E and F in fig 7.14. It was shown in section 7.8.4 that using these it was possible to produce a disturbance which, when the measured meridional wavelength was taken into account, produced a least stable disturbance with a wavelength very close to that observed.

A major difference between the model results and the observed feature lies in the cloud patterns, which rather than varying sinusoidally like those of the model, show an abrupt change in intensity across the feature. However a similar effect occurs in models of the Earth's atmosphere, where simple quasi-geostrophic models can predict the observed cyclones and anticyclones but are unable to reproduce the various frontal and jet stream features associated with them. It is expected that the inclusion of more complex effects in the model (in particular advection by the

ageostrophic wind) would lead to the growth of fronts. The large gradients of vertical velocity associated with these are likely to intensify the changes in cloud density, producing abrupt intensity changes similar to those observed.

In conclusion it is clear that the model used is capable of producing disturbances similar to those seen. In particular, using profiles of the vertical shear based upon the measured values, perturbations with the observed wavenumber can be produced. Unfortunately, due to the absence of measurements of the latitudinal temperature gradient above 150 mb, the detailed structure of the disturbances cannot be safely predicted. However they lie high enough in the atmosphere to satisfy the observed UV variations, and have a sufficiently small variation with height to account for the longitudinal similarity of the UV and longer wavelength images.

Thus the only differences between the model and the observed feature are those which can be ascribed to either known deficiencies of this type of model, or insufficient knowledge of the horizontal temperature gradient. It has been shown that for all the extensions of the temperature gradient profile considered this region was baroclinically unstable. It seems unlikely that any other model of this feature could produce the observed results with the same accuracy.

Also shown, near the end of chapter 7, is another feature at approximately 80° N. Unfortunately no thermal wind data was available at this latitude, and so it could not be modelled reliably. However it can be seen that it is very similar to the laboratory flows observed by Hide and Mason (1975) which are known to be caused by baroclinic instability.

In conclusion it can be stated that in this thesis two small scale features have been examined in some detail. Both of them are similar to laboratory and atmospheric flows but differ in important ways. The lack of a solid obstacle in one and a rigid lower boundary

in the other. However both seem to contain, as far as can be told, all the important features of their Earthbound counterparts. It is probable that a further examination of these and other features, along with a comparison with those seen on the Earth, will produce a better understanding of the conditions needed for their formation. In particular this should provide information about the effects of the Earth's surface upon atmospheric flows.

One of the interesting aspects of this study is the way in which the data from different wavelength instruments is complimentary. A good example of this is provided by the feature examined in chapter 7. The visual images provided details of the horizontal flow, whilst the change with wavelength (particularly in the UV) constrained the vertical structure. The longer wavelength IR results provided information about the local temperature distribution, and hence possible driving mechanisms. However one of the points demonstrated by this study is the way in which both good spatial and spectral resolution is required. Higher spatial resolution of the temperature field would have made the estimates of the thermal wind shear more accurate, whilst if the images had covered a greater wavelength range (even at reduced spatial resolution) a more reliable determination of the vertical structure could have been made. It is hoped that future planetary atmospheres missions will be able to provide this more detailed coverage.

APPENDIX 1 - PROGRAMS USED

Many different programs and subroutines were used in the studies described in this thesis. In addition to the standard software packages mentioned in section 3.3.2, there were the programs referred to in section 3.6.1, required to pre-process the images. The list below gives a brief description of those programs which I wrote or modified, and which were used in the production of this thesis. Some of these, like F2, were only modified, whilst others, such as INSTAB3 and 4, were written from scratch. However the majority were mainly purpose written, but include sections of code taken from other programs where this was more convenient, as mentioned in the individual cases. Next to the names, the length of each program is given in lines (including comments).

DGMAPGRID - 361 lines

This uses the projection routines of the subroutine TRANSFORM in conjunction with the CORE graphics package to produce grids complete with latitude-longitude numbering. The use of TRANSFORM means that perspective, polar orthographic and stereographic, and cylindrical grids can be produced. Whilst the use of CORE implies that these can be displayed on the IIS, overlaid on images, or plotted, without altering the program.

DIMEN - 191 lines

Because of the significant oblateness it is unreasonable to approximate Saturn by a sphere. This makes the calculation of various quantities non-trivial. This program calculates the planetocentric and planetodetic latitude, the latitudinal and longitudinal scales and the values of $f\beta$, and g at 1° intervals. The value of g is obtained from the subroutine GRAVIT.

F2 - 901 lines

This is a VICAR program written by JPL to run on an IBM machine. In order to use it on the VAX it was necessary to reverse the order in which it was assumed that bytes were packed in a 16 bit halfword, and replace the sections of IBM assembler with FORTRAN function calls. In the process parts of the program were unravelled and commented.

The program has been used many times, in particular it has been used to mosaic together figs 7.2 and 7.30, by finding the maximum DN at each point.

FINDIN - 559 lines

This subroutine was used by INSTAB3 and INSTAB4 to provide values, and derivatives with respect to height, of N^2 and $\bar{u}$. The values for N^2 were obtained by using a NAG routine to perform a cubic spline fit to the values supplied. This was then differentiated to provide the rate of change with height. The problem for $\bar{u}$ was more complex, with several different approximations being used. There was a similar version of the cubic spline method used for N^2 . Another set of routines was written which could evaluate different polynomial approximations, whilst a third used the form of equation 7.35. In addition to these there were routines capable of evaluating the profiles used by Green (1960), Gill et. al. (1974), Conrath et. al. (1981), and others which were used for testing INSTAB3 and INSTAB4.

GRAVIT - 85 lines

This subroutine was written to determine the effective (including centrifugal terms), acceleration due to gravity g , at different latitudes. It uses the value of the planetary mass, the rotation rate, and the mean gravitational harmonics J_2 , J_4

and J6 to evaluate g at a given latitude, from the expressions given in section 5.8.1.

INSTAB3 - 328 lines

This is the program referred to in section 7.8, which was used to estimate the solutions of the eigenvalue-eigenvector problem of equation 7.23. The solutions are found by the NAG program F02AGP, most of the code in INSTAB3 being associated with reading in the values to be used, and using these to set up the associated matrices.

Once the solutions have been found, the one with the largest growth rate is selected and the details of this are printed out. One of the largest sections of the code is associated with the formula used for the zonal flow $\bar{u}$ and its derivatives with respect to height. This is done using the subroutine FINDIN described earlier.

INSTAB4 - 540 lines

This attempts to solve the same problem as INSTAB3 but in a different way. This uses the NAG routine D02HBF, which attempts to solve a set of differential equations with unknown parameters subject to specified boundary conditions. As with INSTAB3 most of the code is associated with the input of various values, and using these to formulate the problem in the manner required by D02HBF. Again the subroutine FINDIN is used to evaluate the formula for the zonal wind and its derivatives.

LINEIN - 172 lines

In order to quantify the position of the ribbon feature, as described in section 7.3.1, it was necessary to be able to locate the edge of the feature, and save the results in the database. This program allows the user to draw a line joining a sequence of points on the IIS, and saves the resulting locations in an RSG14 database file.

MAP - 613 lines

This program uses the subroutine TRANSFORM to produce map-projections. It can either perform the operation internally, (one pixel at a time), or can produce a dataset approximating the transformations in a format suitable for the program GEOM. This routine also has a third mode of operation in which it can be used to test the projection routines present in TRANSFORM. Some of the I/O routines contained in this program are slight modifications of those written by R. Gorley as part of a registration program.

The map-projection program, with its associated subroutine TRANSFORM, was essential to many of the studies described in this thesis, and was used in the production of all the images shown.

PTOZ - 211 lines

This program carries out the procedure described in section 5.5 to derive the height from a temperature-pressure profile. The procedure used is to call one of the NAG routines to fit a curve to the data and integrate it. The program also requires as input parameters, the specific gas constant, and a value for the acceleration due to gravity.

RFFT1 - 306 lines

This is derived from a Fourier transform which forms part of the IPIPS VICAR software. It reads in a file in the RSG14 database format, and produces an output file containing the amplitude and phase of its Fourier components. This program was used to obtain the Fourier transform of the ribbon feature described in section 7.3.2.

TRANSFORM - 1491 lines

This subroutine, which was originally part of the program MAP, contains the calculations necessary to produce map-projections. The perspective transformation is based upon code originating at JPL, but was altered to improve the speed of operation. The subroutine is capable of producing Mercator, Polar orthographic and Stereographic projections, and can read in as inputs, perspective, polar orthographic and stereographic projections. The addition of code written by M. Dalton has also made available the possibility of reading and writing various types of cylindrical projections.

VGRAPH - 215 lines

This program will produce a graph of DN against line or sample number. It works on a subset of an image and can, if required, average the result in the sample (or line) direction. The graph is produced by calling the NCAR graphics package. It was this program which produced figures 7.8 and 7.9.

APPENDIX 2 DERIVATION OF THE NUMERICAL MODELS

This appendix explains in more detail the derivation of the numerical model used in chapter 7. As described in section 7.6, a hydrostatic quasi-geostrophic form of the potential vorticity equation was used, based upon that given by Holton (1979, chapter 6). The use of these approximations is justified by the small Rossby number, and the large Richardson number. The coordinate system used is the normal x, y, z system, with the z axis normal to the 'surface'. However, due to the oblateness of Saturn the angle between the z axis and the equatorial plane (the planetodetic latitude), is significantly larger than the angle subtended by a line joining the surface to the planets centre (the planetocentric latitude). Thus the formula for f, β etc., require the planetodetic latitude (this is discussed in section 3.4.1.1.). The situation is further complicated by the presence of the mean zonal flow which, at the latitude of the ribbon feature, is sufficiently great to produce a meridional acceleration which will slightly distort the surface. However, this effect is small in comparison with the use of the beta-plane approximation, which assumes that f and β are the same over the whole 5° latitude span.

We define various non-dimensional variables (without an *) by using a characteristic length, velocity and pressure (L , U and P_0), along with the local coriolis parameter f_0 to give

$$\begin{aligned} x^*, y^*, p^* &= Lx, Ly, P_0 p \\ u^*, v^* &= Uu, Uv \\ t^* &= (L/U) t \\ \omega^* &= (P_0 U / L) \omega \\ \Phi^* &= f_0 U L \Phi \end{aligned}$$

we define a geostrophic velocity by

$$\underline{v}_g^* = u_g^* \underline{i} + v_g^* \underline{j} = \frac{1}{f_0} \left(-\frac{\partial \Phi^*}{\partial y^*} \underline{i} + \frac{\partial \Phi^*}{\partial x^*} \underline{j} \right)$$

and a geostrophic vorticity by

$$\zeta_g^* = \frac{\nabla^2 \Phi^*}{f_0} = \frac{1}{f_0} \left(\frac{\partial^2 \Phi^*}{\partial x^{*2}} + \frac{\partial^2 \Phi^*}{\partial y^{*2}} \right)$$

Now from Holton (1979) we have

$$\frac{\partial \zeta_g^*}{\partial t^*} + \underline{v}_g^* \cdot \nabla (\zeta_g^* + f) = f_0 \frac{\partial \omega^*}{\partial p^*} \quad (\text{A2.1})$$

if we use

$$\frac{D}{Dt^*} = \frac{\partial}{\partial t^*} + u_g^* \frac{\partial}{\partial x^*} + v_g^* \frac{\partial}{\partial y^*}$$

then A2.1 can be written

$$\frac{D \zeta_g^*}{Dt^*} + v_g^* \frac{\partial f}{\partial y} = f_0 \frac{\partial \omega^*}{\partial p^*} \quad (\text{A2.2})$$

To convert to the dimensionless system we use

$$\zeta_g^* = \frac{U}{L} \zeta_g \quad \frac{D}{Dt} = \frac{\partial}{\partial t} + u_g \frac{\partial}{\partial x} + v_g \frac{\partial}{\partial y}$$

$$\frac{D\zeta_g^*}{Dt^*} = \left(\frac{U}{L}\right)^2 \frac{D\zeta_g}{Dt}$$

we also have

$$f_0 \frac{\partial \omega^*}{\partial p^*} = \frac{f_0 U}{L} \frac{\partial \omega}{\partial p}$$

and

$$v_g^* \frac{\partial f}{\partial y^*} \simeq v_g^* \frac{\partial}{\partial y^*} (f_0 + \beta^* y^*) = v_g^* \beta^*$$

where we have defined

$$\beta^* = \frac{U}{L^2} \beta$$

Using these results the dimensionless form of A2.2 is

$$\frac{D\zeta_g}{Dt} + v_g \beta - \frac{1}{Ro} \frac{\partial \omega}{\partial p} = 0 \quad (\text{A2.3})$$

where $Ro = U/f_0 L$ is the Rossby number.

Now also from Holton (1979) we obtain

$$\frac{\partial}{\partial t^*} \left(\frac{-\partial \Phi^*}{\partial p^*} \right) + v_g^* \cdot \nabla \left(\frac{-\partial \Phi^*}{\partial p^*} \right) = \sigma^* \omega^* \quad (\text{A2.4})$$

where σ is a static stability parameter defined by

$$\sigma^* = \frac{-1}{\rho^* \theta^*} \frac{\partial \theta^*}{\partial p^*}$$

or, in a dimensionless form

$$\sigma = \frac{p_0^2}{U L f_0} \sigma^*$$

using the same technique that was used for A2.1, A2.4 becomes

$$\frac{D}{Dt} \left(\frac{-\partial \Phi}{\partial p} \right) = \sigma \omega \quad (\text{A2.5})$$

dividing this by R_0 , and differentiating with respect to pressure this becomes

$$\frac{\partial}{\partial p} \left(\frac{1}{R_0 \sigma} \frac{D}{Dt} \left(-\frac{\partial \Phi}{\partial p} \right) \right) = \frac{1}{R_0} \frac{\partial \omega}{\partial p}$$

which can be used to eliminate ω from A2.3, to give

$$\frac{D \zeta_g}{Dt} + v_g \beta - \frac{\partial}{\partial p} \left(\frac{1}{R_0 \sigma} \frac{D}{Dt} \left(-\frac{\partial \Phi}{\partial p} \right) \right) = 0 \quad (\text{A2.6})$$

Now we split the geopotential Φ into mean and perturbation parts of the form

$$\Phi(x, y, p, t) = \bar{\Phi}(y, p) + \phi(p) e^{i[k_x(x-ct) + k_y y]} \quad (\text{A2.7})$$

where $\bar{\Phi}$ describes a mean flow given by

$$\bar{u}(y, p) = \frac{-\partial \bar{\Phi}}{\partial y}$$

using A2.7 we obtain

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} + i \phi e^{i[k_x(x-ct) + k_y y]} \left(k_x \frac{\partial}{\partial y} - k_y \frac{\partial}{\partial x} \right) \quad (\text{A2.8})$$

after substituting A2.7 and A2.8 in A2.6, using the definitions of ζ_g and v_g , and neglecting second order terms, we obtain

$$(\bar{u} - c) \left[\left(\frac{\phi}{R_0 \sigma} \right)_p - (k_x^2 + k_y^2) \phi \right] + \left[\beta - \bar{u}_{yy} - \left(\frac{\bar{u}_p}{R_0 \sigma} \right)_p \right] \phi = 0 \quad (\text{A2.9})$$

Although this is given in pressure coordinates, it was more convenient to work in height coordinates. Equation A2.9 can be converted to this system by using the hydrostatic equation :

$$\frac{\partial}{\partial p^*} = \frac{-1}{\rho g} \frac{\partial}{\partial z^*}$$

whilst the static stability parameter σ can be defined in terms of the buoyancy frequency N , to give

$$R. \sigma = \left(\frac{\rho \cdot N^2}{L \rho g} \right)^2$$

using these equations A2.9 becomes

$$(\bar{u} - c) \left[\frac{L^2 f^2}{D^2} \frac{1}{\rho} \left(\frac{\rho \phi_z}{N^2} \right) - (k_x^2 + k_y^2) \phi \right] + \left[\beta - \bar{u}_{yy} - \frac{L^2 f^2}{D^2} \frac{1}{\rho} \left(\frac{\rho \bar{u}_z}{N^2} \right) \right] \phi = 0 \quad (\text{A2.10})$$

In addition to this we require the boundary conditions.

For the $w=0$ condition we obtain (by substituting A2.7 and A2.8 in A2.5, and converting to height coordinates)

$$(\bar{u} - c) \frac{\partial \phi}{\partial z} - \phi \frac{\partial \bar{u}}{\partial z} = 0 \quad (\text{A2.11})$$

One approach to solving A2.10 and A2.11 is to use the finite difference approximations

$$\frac{\partial \phi}{\partial z} \approx \frac{\phi(z + \delta z) - \phi(z - \delta z)}{2 \delta z}$$

and

$$\frac{\partial^2 \phi}{\partial z^2} \approx \frac{\phi(z + \delta z) - 2\phi(z) + \phi(z - \delta z)}{\delta z^2}$$

substituting these into A2.10 and A2.11, and separating out all the terms involving c , gives (using the definition of the density scale height h , given in section 5.6)

$$\begin{aligned}
& \left(\frac{\bar{u} L^2 f^2}{D^2 N^2 \delta_z^2} + \frac{\bar{u} L^2 f^2}{2 \delta_z D^2 N^2} \left(\frac{-1}{h} - \frac{1}{N^2} \frac{\partial N^2}{\partial z} \right) \right) \phi(z + \delta z) + \left(\frac{\bar{u} L^2 f^2}{D^2 N^2 \delta_z^2} - \frac{\bar{u} L^2 f^2}{2 \delta_z D^2 N^2} \left(\frac{-1}{h} - \frac{1}{N^2} \frac{\partial N^2}{\partial z} \right) \right) \phi(z - \delta z) \\
& + \left(\beta - \bar{u}_{yy} - \frac{L^2 f^2}{D^2 N^2} \left(\bar{u}_{zz} + \bar{u}_z \left(\frac{-1}{h} - \frac{1}{N^2} \frac{\partial N^2}{\partial z} \right) \right) - \bar{u} (k_x^2 + k_y^2) - \frac{2 \bar{u} L^2 f^2}{D^2 N^2 \delta_z^2} \right) \phi(z) = \\
& c \left\{ \left(\frac{L^2 f^2}{D^2 N^2 \delta_z^2} + \frac{L^2 f^2}{2 \delta_z D^2 N^2} \left(\frac{-1}{h} - \frac{1}{N^2} \frac{\partial N^2}{\partial z} \right) \right) \phi(z + \delta z) + \left(\frac{L^2 f^2}{D^2 N^2 \delta_z^2} - \frac{L^2 f^2}{2 \delta_z D^2 N^2} \left(\frac{-1}{h} - \frac{1}{N^2} \frac{\partial N^2}{\partial z} \right) \right) \phi(z - \delta z) \right. \\
& \quad \left. - \left[\frac{2 L^2 f^2}{D^2 N^2 \delta_z^2} + (k_x^2 + k_y^2) \right] \phi(z) \right\}
\end{aligned} \tag{A2.12}$$

for A2.10, whilst A2.11 becomes

$$\begin{aligned}
& \left(\frac{2 u L^2 f^2}{D^2 N^2 \delta_z^2} \right) \phi(z - \delta z) + \left\{ \beta - u_{yy} - \frac{L^2 f^2}{D^2 N^2} \left(u_{zz} + u_z \left(\frac{-1}{h} - \frac{1}{N^2} \frac{\partial N^2}{\partial z} \right) \right) - u (k_x^2 + k_y^2) - \frac{2 u L^2 f^2}{D^2 N^2 \delta_z^2} \right. \\
& \quad \left. + 2 \delta_z u_z \left(\frac{L^2 f^2}{D^2 N^2 \delta_z^2} + \frac{L^2 f^2}{2 \delta_z D^2 N^2} \left(\frac{-1}{h} - \frac{1}{N^2} \frac{\partial N^2}{\partial z} \right) \right) \right\} \phi(z) = \\
& c \left\{ \left(\frac{2 L^2 f^2}{D^2 N^2 \delta_z^2} \right) \phi(z - \delta z) - \left(\frac{2 L^2 f^2}{D^2 N^2 \delta_z^2} + (k_x^2 + k_y^2) \right) \phi(z) \right\}
\end{aligned} \tag{A2.13}$$

The $\phi=0$ boundary condition follows trivially from A2.12

An alternate approach to solving A2.10 is to split ϕ and c into real and imaginary parts, ie

$$\phi = \phi_r + i\phi_i \quad c = c_r + i c_i$$

using these A2.10 can be written

$$\begin{aligned} & \phi_{zz} + \left(\frac{-1}{h} - \frac{1}{N^2} \frac{\partial N^2}{\partial z} \right) \phi_z - \frac{D^2 N^2}{L^2 f^2} (k_x^2 + k_y^2) \phi \\ & + \left[\frac{D^2 N^2}{L^2 f^2} (\beta - u_{yy}) - \left(u_{zz} + \left(\frac{-1}{h} - \frac{\partial N^2}{\partial z} \right) u_z \right) \right] \times \quad (A2.14) \\ & \frac{(\phi_r (u - c_r) - \phi_i c_i) + i ((u - c_r) \phi_i + c_i \phi_r)}{(u - c_r)^2 + c_i^2} = 0 \end{aligned}$$

Now if we define

$$\theta = \frac{\partial \phi}{\partial z} \quad (A2.15) \quad \text{then} \quad \frac{\partial \theta}{\partial z} = \frac{\partial^2 \phi}{\partial z^2}$$

using these, the real part of A2.14 becomes

$$\begin{aligned} & R_e(\theta_z) + \left(\frac{-1}{h} - \frac{1}{N^2} \frac{\partial N^2}{\partial z} \right) R_e(\theta) - \frac{D^2 N^2}{L^2 f^2} (k_x^2 + k_y^2) R_e(\phi) \\ & + \left[\frac{D^2 N^2}{L^2 f^2} (\beta - u_{yy}) - \left(u_{zz} + \left(\frac{-1}{h} - \frac{\partial N^2}{\partial z} \right) u_z \right) \right] \frac{((u - c_r) R_e(\phi) - c_i I_m(\phi))}{(u - c_r)^2 + c_i^2} = 0 \quad (A2.16) \end{aligned}$$

whilst the imaginary part gives

$$\begin{aligned} & I_m(\theta_z) + \left(\frac{-1}{h} - \frac{1}{N^2} \frac{\partial N^2}{\partial z} \right) I_m(\theta) - \frac{D^2 N^2}{L^2 f^2} (k_x^2 + k_y^2) I_m(\phi) \\ & + \left[\frac{D^2 N^2}{L^2 f^2} (\beta - u_{yy}) - \left(u_{zz} + \left(\frac{-1}{h} - \frac{\partial N^2}{\partial z} \right) u_z \right) \right] \frac{((u - c_r) I_m(\phi) + c_i R_e(\phi))}{(u - c_r)^2 + c_i^2} = 0 \quad (A2.17) \end{aligned}$$

Equations A2.15, A2.16 and A2.17 are four, first order differential equations for the real and imaginary parts of ϕ and θ , and can be approximated using the NAG routine D02HBF, as described in section 7.8.

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Dynamical features in the northern hemisphere of Saturn from Voyager 1 images

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Results of the analysis of Voyager measurements of divergence and inferred vertical velocity of convective cloud features in the northern hemisphere of Saturn are presented here. The winds in this region have been measured by tracking the cloud elements observed in sequences of images. The derived zonal winds imply that these convective features reside at the maximum of strong easterly jets centred at 41° N planetocentric latitude and that the jet is barotropically unstable. The divergences and local dynamics of the Saturn features are found to differ from those of the jovian equatorial plumes.

During the encounter of Saturn by Voyager 1, Smith *et al.*¹ reported observations of distinct cloud features occurring in the northern hemisphere of the planet's atmosphere. In the region bounded by 35 – 42° N, several bright V-shaped features have been seen, while at $\sim 29^\circ$ N, a distinct cloud oval is apparent (Fig. 1, feature 3.) (All latitudes in this article are planetocentric.) The large spot is particularly evident in the UV images and was seen to persist throughout the Voyager 1 observational period from 24 August to 10 November 1980. The V-shaped features varied greatly in shape during the observation period but persisted in their longitudinal distribution as they drifted in an easterly jet centred at 41° N. The changes in the visible appearance of these features suggests that they are convective clouds where the rapid variations in the shape and structure of these cloud systems is associated with local vertical motions. There is no evidence from the IR observation of Hanel *et al.*² that these features affect the temperature structure of the upper troposphere. However, the IRIS field of view is large compared with the visible cloud structures observed by the imaging system³. The IRIS instrument essen-

tially measures the average temperature for the total region seen in the imaging frame. Indeed, any upper atmosphere aerosol layers present would further reduce any possible temperature contrasts.

In the atmosphere of Jupiter, we have shown that the equatorial plumes are convective cloud structures^{3,4} and their meteorological characteristics have been estimated by procedures developed using the University College London (UCL) Interactive Planetary Image Processing System⁵. We apply these techniques for the analysis of the Saturn observations and present here the first measurements of divergence of saturnian cloud systems. Using this information, we then calculate the vertical velocities which relate to the observed changes in the cloud structures. In addition, we measure the wind speeds of the clouds in the latitude band 30 – 45° N and estimate the stability of the flow for the period of the observations. These new measurements provide valuable insight into the variations of the cloud structures and through comparison with our jovian studies⁴ this will provide information on the differences in the observed meteorologies.

For this study, a series of blue filter narrow angle images of Saturn was used. The observations were made for the period 3–8 November 1980 when the spatial resolution varied between 120 and 60 km pixel⁻¹.

The images were first radiometrically decalibrated at the Jet Propulsion Laboratory (JPL) using the standard procedures⁶. To provide an accurate navigation of the images that transforms the line and sample system into a planetocentric latitude/longitude coordinate system, two independent procedures were used. At JPL, the method used simultaneous wide and narrow angle frames to locate the centre of the planet from which the subsequent transformation is then determined⁶. An interactive procedure, developed at UCL, uses a 'least-squares' minimization method to fit an ellipse to the image, constrained by the spacecraft navigation and camera data. We have shown elsewhere⁶ that both of these methods produce similar results although the error in determining the position of the centre of the planet becomes greater than 1 pixel when 20% or less of the planet's limb is available (see ref. 7).

Measurements of velocities of the upper clouds in the Saturn atmosphere have been made by interactively tracking individual features observed in the images. The techniques have been shown⁶ to be accurate to within ± 3 m s⁻¹ in zonal and meridional winds at a spatial resolution of 60 km pixel⁻¹ and

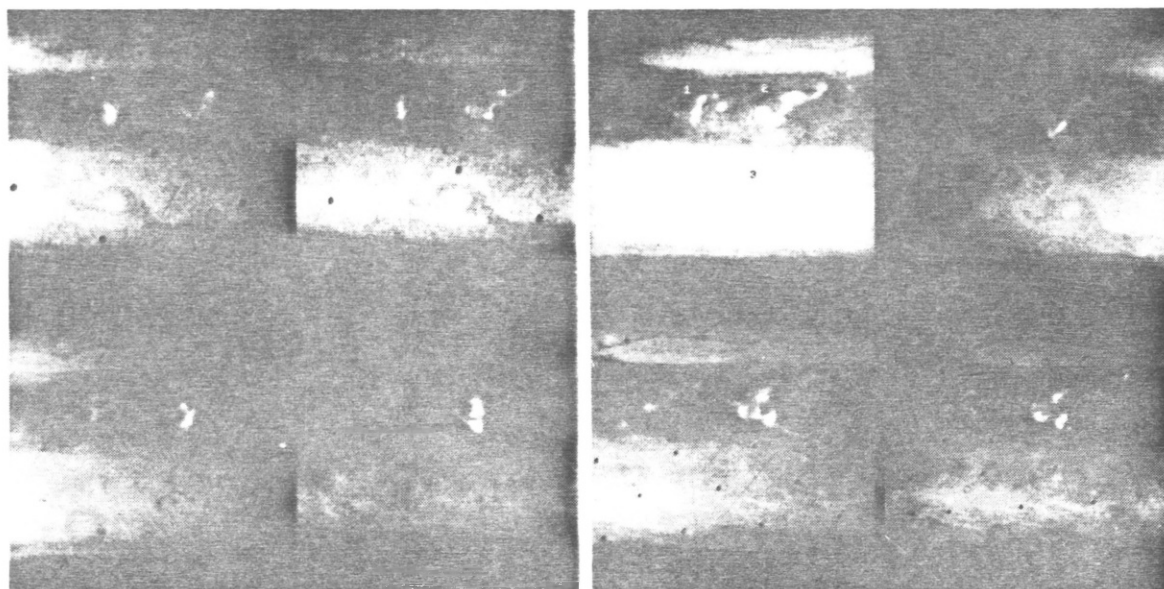


Fig. 1 Sequence of eight blue Mercator narrow angle images of the mid-latitudes of Saturn. The dates and times of the images are: 15:05:22, 3 November 1980; 11:57:22, 4 November 1980; 22:11:46, 4 November 1980; 08:25:22, 5 November 1980; 20:46:58, 5 November 1980; 17:22:10, 6 November 1980; 12:08:34, 8 November 1980; 22:24:34, 8 November 1980.

produce results that compare favourably with other schemes¹. The velocities are measured relative to the SYSTEM III radio period of 10 h 39.4 min (ref. 8) using images separated by time intervals of 10–20 h. Considerable contrast enhancement of the image was first necessary to show clearly the cloud elements to be tracked.

The measurements of the divergence of the V-shaped clouds were made by the procedures set out in ref. 4. The sequence of 10 blue narrow angle images were first map projected into a Mercator format centred at 41° N using a single pixel interpolation scheme developed at UCL⁷. Then, the divergence of these features was estimated from the changing cloud areas. We estimate the area of the convectively active region from the assumption that the brightness in the cloud system, which we have previously shown^{4,9,10} to be a reliable approach.

If we now assume that the vertical motion in the layer of outflow takes place over a vertical scale D , then the vertical velocity required to produce the observed divergence field is:

$$W \leq D \operatorname{div} \mathbf{V} \quad (1)$$

We estimate D as the scale height of the atmosphere at the cloud top, which will then enable an estimate to be made of the vertical motions associated with these cloud systems.

Figure 1 shows the variation in the V-shaped cloud structures for the period 15:05:22, 3 November to 22:24:34, 8 November 1980. All times are GMT at the spacecraft. This time sequence clearly indicates the large morphological change in the convective cloud structures. At the edge of feature 2, considerable developments in the clouds become apparent in the final three images of the sequence. Figure 1 shows less noticeable changes during this time.

The third main cloud feature seen in Fig. 1 is the UV spot. The prominence of this feature at short wavelengths is consistent with it being higher than its surrounding clouds. A well defined flow pattern can be seen to the west of this feature in Fig. 1, which is morphologically similar to the 'turbulent' regions to the west of the Great Red Spot and White Ovals on Jupiter.

In Fig. 2, we summarize the basic characteristics of the motions in the mid-latitude region bounded by 30–50° N. The mean zonal velocity is shown in Fig. 2a; a strong easterly jet is situated at 41° N which corresponds to the apex of the V features. We find also that the strongest shear occurs along the southern edge of this jet. The small inflection in the zonal velocity profile at 42–43° N is associated with small changes in the cloud morphologies.

To discuss the significance of these results and the stability of the flow, we estimate $\bar{u}$, $d^2\bar{u}/dy^2$, and their associated errors, using the procedures set out in ref. 11. The measured zonal flow is composed of contributions from the mean flow, the eddies and errors which arise from effects of spacecraft navigation, preprocessing of the data and uncertainties in the identification of individual features. Following the procedures of Ingersoll *et al.*¹¹, we estimate the error in $\bar{u}$, $\sigma(\bar{u})$ as $\pm 5 \text{ m s}^{-1}$. We compute $d^2\bar{u}/dy^2$ at latitudinal position y_n by:

$$(d^2\bar{u}/dy^2)_n = (\bar{u}_{n+2} - 2\bar{u}_n + \bar{u}_{n-2})/\Delta y_n^2$$

where $\Delta y_n = y_{n+1} - y_{n-1}$.

Then the error in $d^2\bar{u}/dy^2$ is $\sim \pm \sqrt{6}\sigma(\bar{u})/\Delta y^2$ (ref. 11).

The stability of the retrograde jet may be investigated by examining the barotropic stability condition for a zonal velocity $\bar{u}$ (see, for example, ref. 12), given by:

$$\beta - d^2\bar{u}/dy^2 \geq 0$$

where $\beta = (2\Omega/r) \cdot \cos \theta$, Ω is the planetary rotation, r the planetary radius (local radius or curvature), θ the planetocentric latitude and y the northerly direction. In Fig. 2b, we have superimposed the value of β with $d^2\bar{u}/dy^2$, while the additional curves illustrate the errors in $d^2\bar{u}/dy^2$ estimated by the procedures set out above. We see that the flow between 40 and 41.5° N is barotropically unstable in more stringent conditions. The figure further suggests that the region extending to 37° N

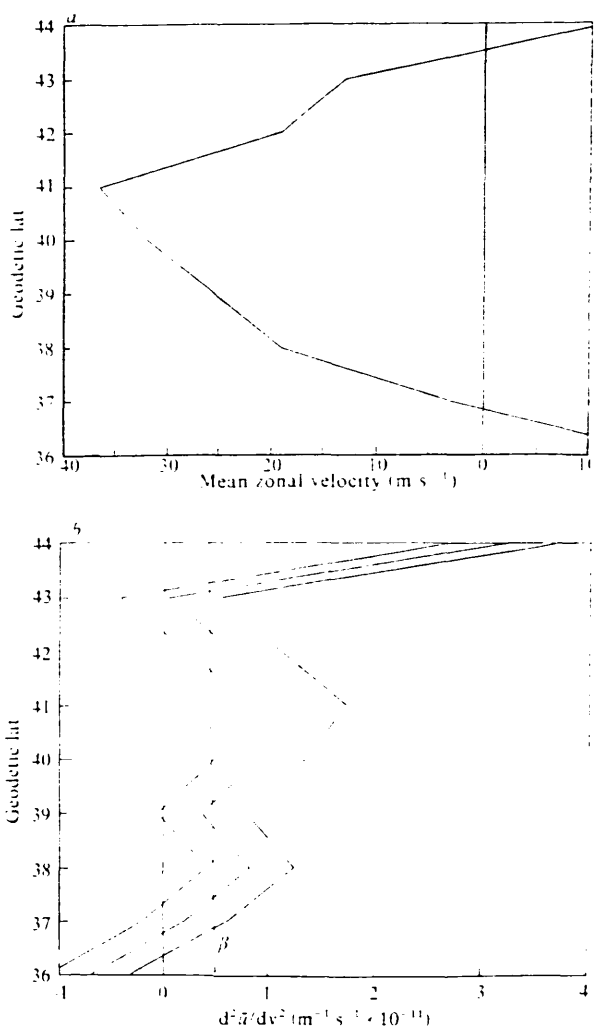


Fig. 2 The dynamical properties of the region 30–50° N derived in this study. a, The zonal wind $\bar{u}$ is shown with 1σ r.m.s. values. b, The curve for $d^2\bar{u}/dy^2$, together with the value of β at 41° N. The central curve is the variation of $d^2\bar{u}/dy^2$. The curves at the left and right show the variation in $d^2\bar{u}/dy^2$ when the errors are taken into account.

is close to being unstable in these conditions. The V-cloud features reside in the centre of the unstable region, which is also the maximum in the easterly jet. The stable region of the flow lies outside the latitudes of 37–42° N.

Figure 3 shows the divergence of these active regions. There is little change in the characteristics of the UV spot, which is apparent from the image sequence seen in Fig. 1. The changes in area shown in Fig. 2 for the convective features were used to determine the 90% of maximum brightness threshold contour using the procedures described in ref. 4. These variations in area of the active clouds may then be used to estimate the divergence of the features which are shown in Fig. 3. As expected, feature 2 is the most active, with values reaching $\sim 5 \times 10^{-5} \text{ s}^{-1}$. The other convective feature is less active, with a maximum value of $3 \times 10^{-5} \text{ s}^{-1}$. A negative divergence implies the dissipation of the cloud feature.

Comparing the results with those of our study¹ of the jovian equatorial plumes, we find that the divergences are much stronger for the saturnian features. The largest Saturn values are more than three times greater than those associated with the most active jovian plume. Assuming that the anvil expansion we are measuring takes place at $\sim 500 \text{ mbar}$ level² over a depth of a density scale height of 40 km (ref. 13), then the divergence maxima imply vertical motions of $\sim 2 \text{ m s}^{-1}$.

On Jupiter, these large scale motions are propagated throughout the troposphere¹. Although the saturnian vertical

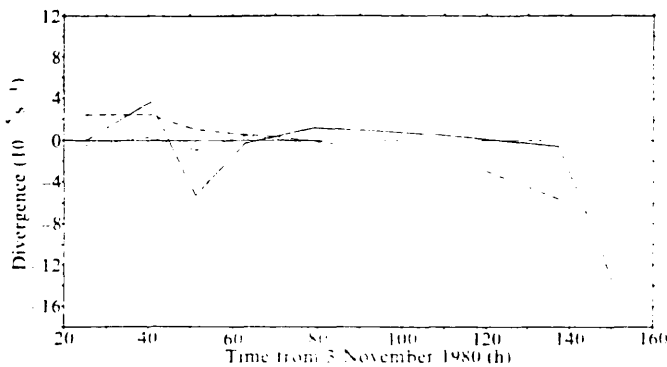


Fig. 3 Measurements of divergence of cloud features in the northern hemisphere of Saturn for 150 h, commencing at 00 h, 3 November 1980. The convective clouds are features 1 (—); feature 2 (---); the UV spot.

motions are apparently stronger, the IR observations do not suggest any large scale perturbations in the temperature structure in a horizontal or vertical direction. It is possible that the larger aerosol layers in the troposphere of Saturn provide sufficient opacity to diminish the temperature contrasts. Furthermore, the IRIS observations are essentially the average response for the narrow angle field of view, so that these small scale convective clouds may not produce a sufficiently large signature to be noticeable.

The present estimates of larger vertical motions in the atmosphere of Saturn compared with Jupiter are consistent with the differences in the zonal velocity and internal structure of the two planets. The internal heat source on Saturn supplies approximately twice as much energy to the atmosphere as solar heating⁷. This situation, together with the smaller gravity, which

then leads to a larger atmospheric scale height on Saturn, is consistent with the larger convective motions discussed above.

From these first estimates of divergence and vertical motions of cloud features observed in the northern hemisphere of Saturn we find that the associated flows of the convective clouds are stronger than the similar cloud features observed in the atmosphere of Jupiter. The vertical velocities we estimate for these motions are of a similar magnitude to large scale terrestrial convective storms (see, for example, ref. 9), although on Saturn these storms occupy a considerably greater spatial scale.

We find that the Saturn convective cloud features are situated in the maximum of the easterly jet which corresponds to a barotropically unstable region of the atmosphere (Fig. 2). It is possible that these mesoscale cloud features are generated by large scale convergence in the deep cloud layers beneath the visible surface of the planet. These stronger motions are consistent with the differences in the structures of Saturn and Jupiter.

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SOME DYNAMICAL PROPERTIES OF VORTEX STREETS IN SATURN'S ATMOSPHERE FROM ANALYSES OF VOYAGER IMAGES

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Abstract. We present the first observations of a vortex street in an atmosphere other than that of the Earth made from a sequence of images of Saturn taken by Voyager 2 in August 1981. The analysis of these images shows that the feature sits at the maximum of the westward jet and suggests that it may be produced by material rising up from below the level of the visible clouds.

Introduction

The recent Voyager encounters with Saturn (see, for example, Smith et al., 1981, 1982) have revealed a considerable number of features which are not visible from the Earth. One of the most variable areas of the planet is the westward jet at about 40°N. It was from rapidly changing cloud features in this region that Hunt et al. (1982) were able to estimate a vertical velocity of the order of 1 m sec^{-1} , which is considerably larger than those found for Jupiter (Hunt et al., 1982).

There is a considerable difference between the observations made by Voyager 1 in 1980 and those made by Voyager 2 in 1981. In the latter, many more cloud features were visible, especially in the region around 41°N. One of these features appeared to consist of a dark spot with a series of clouds to its east, resembling a vortex street, which is a flow pattern observed in the laboratory when fluid is moving around an obstacle (Batchelor, 1977). This type of flow has also been observed in the Earth's atmosphere in the wake of islands (Chopra and Hubert, 1965; Berlin, 1981). However, the presence of this type of feature on Saturn is made more interesting by the lack of solid obstacles which are normally associated with its production.

In this paper we attempt to measure some of the properties of the Saturnian vortex street and to compare it with those observed on Earth and in the laboratory. In addition, we suggest how it may have formed and its effect upon the local flow.

Initial Processing of Data

Only four images were suitable for this study and they were obtained using the Voyager 2 narrow angle camera (with the green filter) on 20 and 21 August 1981, with each pair being separated by approximately one rotation period of 10h 39.4 km (Desch et al., 1980). Due to the decreasing distance between the spacecraft and the planet, the

resolution of these images varies from about 59 to 42 km per pixel.

Because of the changes in perspective between the images, all four were map projected to a common scale of 25 km per pixel. Each map projection was centred 0.8° to the left of the previous one, in order to remove the features' westward drift of about 18 m sec^{-1} . This produced the images shown in figure 1a.

Measurements

Measurements of both the spatial structure and the velocity of the cloud features may be determined from the images.

The velocity measurements were obtained from the unprojected images by identifying a feature in one frame and then locating the same feature in a subsequent image. From the known time separation the velocity of the feature may be calculated. Because of the rapid changes in the shape of a cloud between images, it was not practical to track features in frames separated by more than one rotation period, so that from the four frames, three sets of measurements were obtained. These are the velocity vectors shown in figure 1b. Due to the uniformity of this latitude band it proved impossible to obtain any measurements to the west (upstream) of this feature.

It is possible to measure the positions of various parts of the vortex street, by identifying each part of the feature in the map projected images and then calculating the latitude and longitude. Using this method the distances shown in figure 2a as a_1 , a_2 , h_1 , h_2 , d_1 and d_2 were obtained for each image, and from these values of D , H , and A were calculated, giving $D = 1685 \pm 90 \text{ km}$; $H = 800 \pm 120 \text{ km}$; $A = 4980 \pm 180 \text{ km}$, where $D = (d_1 + d_2)/2$, $H = (h_1 + h_2)/2$, and $A = a_1 + a_2$.

In addition to these values, from a longitude-time graph of the position measurements, it was possible to obtain an approximate value of $U_e = 1.1 \text{ m s}^{-1}$ for the drift rate of the street relative to the dark spot on the left. However, since this value corresponds to a movement of only 5 pixels between the first and last images, its interpretation is very doubtful and it may well be incorrect by a factor of, say, 2 or 3.

Another parameter associated with vortex streets is the vortex strength, κ (see Wilson, 1980), which is defined as

$$\kappa = qr \quad (1)$$

where r is the distance of a point from the centre, C , of a vortex, and q is the velocity tangential to a circle of radius r about C .

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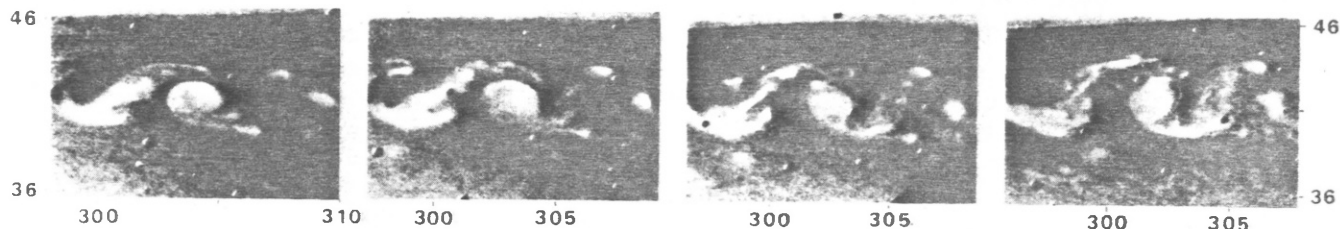


Fig. 1a. The sequence of map projected green images of the mid-latitudes of Saturn, which shows the structure of the vortex street. From left to right, the images were obtained on 20 August 1981 14.24.47, 21 August 1981 01.02.23, 11.29.35, 22.01.35.

The method used was similar to that used for the position measurements. For each pair of images an estimate was made of the position of the centre of the second vortex, and of a feature which appeared to have anticyclonic circulation around it. From these values, and the time separation between the images, values for q and r (and hence κ) were obtained. This procedure was repeated for each pair of images to give

$$\kappa = 13.9 \pm 4.0 \times 10^6 \text{ m}^2 \text{ s}^{-1}$$

Time lapse sequences of these images have shown anticyclonic vortices to the north of the obstacle and cyclonic vortices to the south, in the classical manner.

Description of Vortex Streets

A Karman vortex street is a type of instability which can arise in the wake of a barrier. It consists of vortices which are shed regularly from alternate sides of the obstacle (usually a cylinder) and form into two rows, the vortices in one row having the opposite vorticity and being situated midway between the vortices in the other row.

This behavior is shown schematically in figure 2b. In figure 1a, we show four images of the feature which is thought to be a vortex street. It appears to be created by the zonal flow (which is westward at this latitude) passing the dark cloud (also moving towards the west, but at a higher velocity) of diameter 1700 km, centred at

41°N, and to be giving rise to the clouds which extend at least 9000 km to the east. The vortices in the street have been seen to possess anticyclonic vorticity to the north of the obstacle and cyclonic vorticity to the south. Vortex streets have been seen in the Earth's atmosphere in the wake of Gran Canaria (Chopra and Hubert, 1965) and Madeira (Berlin, 1981), and some of the properties of these, as well as the results for Saturn and theoretical values, are summarised in table 1.

Properties of Vortex Streets

Due to the paucity of reliable velocity measurements, and especially to the lack of a value for the undisturbed flow, it is difficult to obtain much quantitative information from this feature. In this section the measured value of h/a is used to show that the feature has approximately the same 'shape' as a vortex street. In addition, the vortex shedding rate N is estimated both from the measured value of U_e and, by assuming that the feature is a classical vortex street, from the vortex strength κ . The two values, which agree reasonably well, are then used to obtain an estimate for lifetime of the vortices.

While figure 1a resembles a vortex street, another clue is provided by the ratio h/a . A theoretical consideration of the stability of an infinite vortex street in a uniform flow (Lamb, 1932), gives the condition that

$$\frac{h}{A} = \frac{\text{arcosh}(\sqrt{2})}{\pi} = 0.281 \quad (2)$$

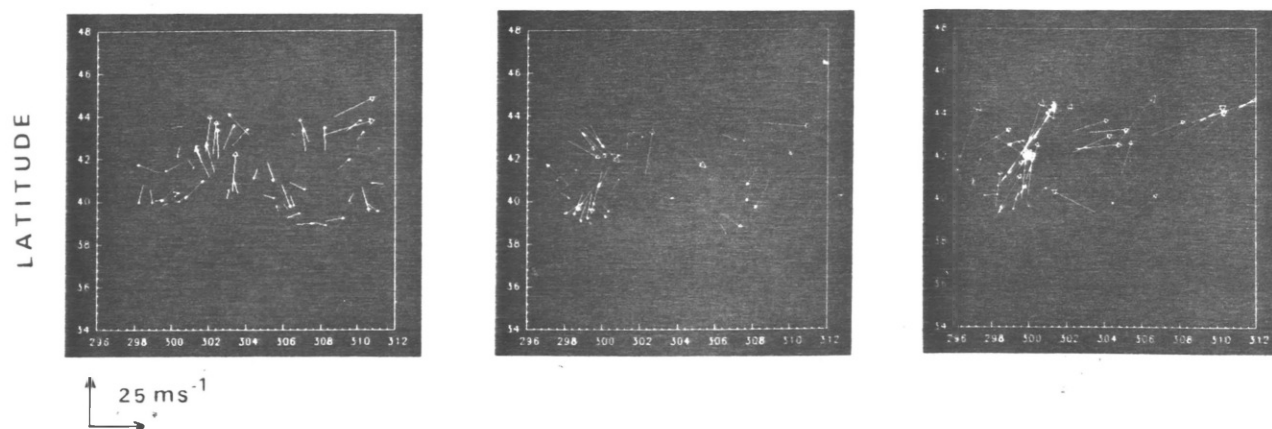


Fig. 1b. The velocity measurements illustrating the flow in the vortex street. The origin of each vector corresponds to the position of the relevant feature, in the first of two frames from which the velocity was calculated.

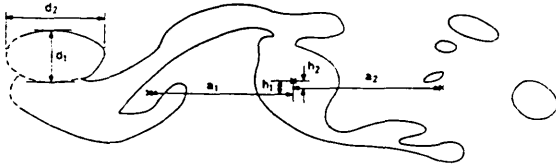


Fig. 2a. A schematic diagram of the cloud elements seen in fig. 1a which shows the way in which values for a and h are derived from estimated centres of rotation, indicated with an X.

The ratio H/A for the Saturnian feature is 0.16 ± 0.025 while the values for the Earth are 0.35 to 0.43 (see Chopra and Hubert, 1965; Berlin, 1981). The value obtained for the Saturnian feature does not differ greatly from either the theoretical or observed values of H/A . Part of the difference in these values may be attributed to our being able to estimate only the positions of the first three vortices whereas it appears from laboratory experiments that the characteristic street configuration of the vortices does not appear until they are some way down-stream.

If we assume that the observed feature is a vortex street, then some of its properties may be calculated. For convenience, the flow will be considered to be moving in a system in which the 'cloud obstacle' is at rest.

It is possible to obtain a value for the rate, N , at which vortex pairs are shed from the obstacle by using

$$N = \frac{U_e}{A} \quad (3)$$

In conjunction with the measured values for U_e and A , this gives

$$N = 2.2 \times 10^{-7} \text{ s}^{-1}$$

The relationship between D , N and U_o , the velocity of the undisturbed flow to the west of the obstacle, is

$$S = \frac{ND}{U_o} \quad (4)$$

where S is the Strouhal number. This has been found to be about 0.2 for both a cylinder in the laboratory and Jan Mayen Island in the North Sea (see Scorer, 1978). Using $S = 0.2$ and the values given above for N and D , we obtain

$$U_o = 1.85 \text{ m s}^{-1}$$

This can be compared with the drift rate of the vortices away from the obstacle of

$$U_e = 1.1 \text{ m s}^{-1}$$

Although, as mentioned above, the value for N is not very accurate, the ratio

$$\frac{U_e}{U_o} = \frac{NA}{ND/S} = 0.59 \quad (5)$$

assuming that the value of $S = 0.2$ is relevant, is independent of the value used for N .

An alternative value for U_e may be obtained using the values for K , A , and D , from a consid-

eration of the velocity, v , induced by one row of vortices in the other. Using an expression given by Milne-Thomson (1962), this gives:

$$v = \frac{K\pi}{A} \tanh \frac{\pi H}{A} = 4.07 \text{ m s}^{-1} \quad (6)$$

But approximately

$$U_e = U_o - v \quad (7)$$

Hence from (5), we have

$$\frac{U_e}{U_o} = \frac{U_o - 4.07}{U_o} = 0.59 \quad (8)$$

giving $U_o = 9.9 \text{ m s}^{-1}$ and $U_e = 5.8 \text{ m s}^{-1}$. From these, using (1), we obtain

$$N = 1.16 \times 10^{-6} \text{ s}^{-1}$$

The first value of N yields a vortex production rate of one every twenty six days while the latter gives one every four days. These are 75 times greater than the values obtained for the Earth (table 1).

The length L of the vortex street is about 9000 km which means that the age ($t = L/U_e$) of the eastmost part of it is probably between 17 and 95 days (from the two values given above). This suggests that the eddy dissipation is extremely slow in the Saturnian atmosphere.

From an analysis of the streamlines of the first pair of images in figure 1b, we find evidence of an oscillatory behaviour with wavelength $\sim 4.8 \times 10^6 \text{ km}$. When streamlines are used in conjunction with the velocity measurements we obtain a vorticity of $1 \times 10^{-5} \text{ s}^{-1}$ for the single point considered. This is similar in magnitude to the change in coriolis effect of $\sim 3.8 \times 10^{-6} \text{ s}^{-1}$ created by a latitudinal displacement of 10^3 km (approximately the size of the dark spot), which suggests that the various β -plane effects considered by Boyer and Davies (1982) may be relevant in this case.

Discussion

The Saturnian feature is particularly interesting, not simply for the properties of the vortex street itself, but because Karman vortex

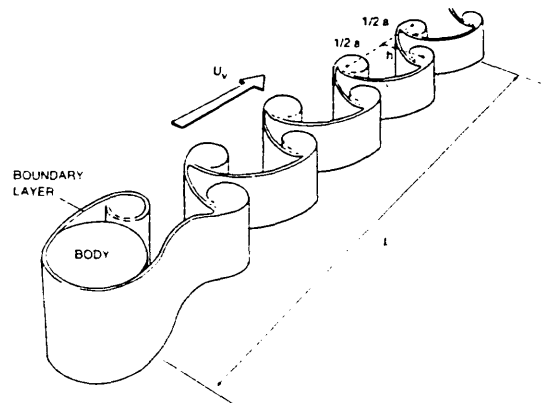


Fig. 2b. A schematic diagram of a vortex street (after Berlin, 1981) showing the meaning of some of the parameters used in this paper.

TABLE 1. A Summary of the properties of Vortex Streets

Parameter	Symbol	Saturn	Gran Canaria (Berlin, 1981)	Madeira (Chopra and Hubert, 1965)	Predicted Values
Diameter of obstacle	D	1.7×10^3 km	45 km	43 $\pm$ 3 km	
Bandwidth of street	H	800 $\pm$ 120 km	55 $\pm$ 10 km	83 km	
Longitudinal spacing	A	5×10^3 km	130-165 km	190 km	
Undisturbed zonal flow	U_0	(1.8-9.9) m s ⁻¹	7.5-10 m s ⁻¹	10 m s ⁻¹	
Zonal flow of vortices	U_e	(1.1-5.8) m s ⁻¹	5 m s ⁻¹	7.5 m s ⁻¹	
Length of vortex street	L	9×10^3 km	800 km	540 km	
Von Karman ratio	h/a	0.16 $\pm$ 0.025	0.35-0.4	0.43	0.28
Vortex production rate	N	(2.2-11) $\times 10^{-7}$ s ⁻¹	3.8×10^{-5} s ⁻¹	3.95×10^{-5} s ⁻¹	
Vortex shedding period	1/N	(52-10) days	7.2 hr	7.15 hr	
Vortex lifetime	t	(95-17) days	45 hr	20 hr	
Strouhal number	S	0.2 (assumed)	0.22-0.17	0.17 $\pm$ 0.1	0.2 (cylinder)

streets are usually observed only when fluid flows past a solid body. However, we consider a solid object in the atmosphere to be unlikely. The barrier around which the flow is passing appears to be a large, dark, approximately oval feature, which is on the extreme left of the images in figure 1a.

The size of this feature is difficult to determine, as its shape appears to change from frame to frame, which would not be expected if the feature were behaving like a solid obstacle. This suggests that either the dark spot is not the actual obstacle but is an unassociated dark cloud feature, or the dark spot is seen through a gap in the clouds, which are moving, and hence changing the amount of the feature which is observed. It seems likely that the latter suggestion is correct, and that the dark feature is in fact an integral part of the flow pattern because it appears centred in approximately the same position in all the images of the vortex street feature (including some taken a few days before those in figure 1a). Alternatively, the idea that the dark spot is being seen through a gap in the clouds is substantiated by the observation that not only is it surrounded by clouds, but as can be seen from figure 1a, one wisp of cloud actually appears to pass over it.

If the dark spot is located below the visible cloud tops, then it is necessary that the westerly speed increase with decreasing height. This idea is supported by the thermal wind shear derived from temperature measurements (Pirraglia et al., 1981) of $du/dz \approx +0.26$ m s⁻¹ km⁻¹, although this measurement corresponds to the 150 mb level, well above the visible clouds.

There appears to be no theoretical reason why a solid obstacle is required to give rise to a vortex street, since, as shown by Batchelor (1977), it is possible to produce a free boundary layer between two parallel streams of fluid moving at different speeds. It is therefore possible that the obstacle responsible for producing the vortex street is a large 'blob' of fluid moving at a different (high westward) velocity to the unperturbed flow in the location of the vortex street. This then suggests that the 'blob' may be produced by material rising up from below the level of the visible clouds. The

presence of the vortex street must exert a drag on the obstacle so that the street must be associated with a source of westward momentum.

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APPENDIX 3

SENSITIVITY OF THE RESULTS OF CHAPTER 7

1.0 INTRODUCTION

This appendix describes the ability of a model of a baroclinic instability to reproduce the characteristics of a wave-like feature observed in the atmosphere of Saturn . The only detailed observations of this feature are provided by Voyager images , and these have been used to constrain its structure . In particular the average zonal wavelength of the feature was found , in chapter 7 , to be 5710 ± 260 km . It was also shown that the vertical velocity should be significant up to at least the 170 mb level , with a small vertical phase change.

The model used was described in chapter 7 , where the results demonstrated the variation of the structure and growth rates with different vertical profiles of $\partial u / \partial z$. In particular it was found that some of the profiles could produce disturbances independent of the boundaries , and with a wavenumber close to that observed . This section considers these in greater detail , and shows how they change for different profiles of $\partial u / \partial z$ and N^2 .

2.0 RESULTS - VARIATION WITH $\partial u / \partial z$

The profiles of $\partial u / \partial z$ described in section 7.8.2 were restricted to being symmetric about the origin . In order to increase the variety of $\partial u / \partial z$ profiles tested a more complex form is used here . They are given by

$$\frac{\partial u}{\partial z} = (Az^2 + Bz + C) e^{-Dz} \quad (1)$$

which , for $C=0$, are equivalent to the profiles of section 7.8.2 . In order to avoid producing profiles with a maximum in $\partial u/\partial z$ above the bottom level (and hence the possibility of an additional sign change in Q_y) , eqn (1) was replaced by

$$\frac{\partial u}{\partial z} = \left(\frac{\partial u}{\partial z} \right)_z , \quad \text{for } z_0 < z < z'$$

where z_0 is the bottom level , and z' is the height at which $\partial u/\partial z$ is a maximum .

As explained in section 5.8 the vertical wind shear values are derived from IRIS temperature measurements . However these are altered by the presence of clouds ; an effect which is not taken into account by the inversion process . Because of this the measurements become less reliable with increasing depth . Thus the highest available measurements , which are at 150 mb , should be the most reliable .

Fig 1a shows the various profiles used , all of these are derived from eqn 1 and pass through the common 150 mb measurement mentioned earlier . They were all used in combination with the N^2 profile labelled 't' in fig 5b . The value of the parameter Q_y (defined in section 7.6.2) produced by these curves is shown in fig 1b .

Fig 2a shows how the computed growth rate varies with wavenumber for the various $\partial u/\partial z$ profiles of fig 1a . It is clear that these curves have a common character ; the profiles with greater growth rates being associated with disturbances of higher wavenumber . However the variation in growth rate is considerably larger than the variation in wavelength, with all the disturbances having a least stable wavenumber between 5 and 11 . Fig 2b shows how these growth rate curves alter when a meridional wavenumber $k_y=7$ is considered . It can be seen that this

reduces the growth rates , whilst slightly increasing the least stable wavenumber .

The profiles of $\partial u / \partial z$ shown in fig 3a were obtained by multiplying the profile labelled j in fig 1a by 0.5 , 1.0 and 1.5 . They were used to examine how the growth rate curves vary when the profiles of $\partial u / \partial z$ do not pass through a common point. Fig 3b shows the values of Q_y produced by these profiles , whilst the resulting growth rate curves are shown in fig 4a . It is apparent that the main effect of changing $\partial u / \partial z$ is to alter the growth rates - again with a tendency for larger growth rates to be associated with greater wavenumbers . From a comparison of figs 2a and 4a it is clear that removing the constraint that all the $\partial u / \partial z$ profiles pass through a common point has not greatly altered the structure of the growth rate curves : increasing the value of $\partial u / \partial z$ by a factor of 1.5 increased the maximum growth rate by a factor of 1.6 , while the associated wavenumber changed by approximately 7 % . Fig 4b shows how the growth rates of fig 4a change when $k_y=7$ is used .

3.0 RESULTS - N^2 VARIATION

The various N^2 profiles available , from both infra-red and radio science experiments , are shown in fig 5a . As explained in section 5.8 the IRIS results are expected to be affected by the neglect of cloud opacity in the temperature retrieval . They also correspond to the near equatorial region , whilst the radio science values are nearer to the midlatitude region being considered . For these reasons the radio science profiles are expected to be more appropriate .

The values of N^2 used in the program were produced by fitting cubic splines (with three equally spaced internal knots) to the radio science measurements for the region at 36.5 N . This produced the '+ - + - ' curve shown in fig 5b It was this N^2 profile which was used for the

comparison of the various $\partial u / \partial z$ profiles described in section 2.0 .

In order to study the effects of altering N^2 it was necessary to extrapolate the curve shown in fig 5b below the level at which $N^2 = 0$. This slightly altered the shape of the fitted profile , producing the dashed curve shown in fig 5b . The effects of changes in N^2 were then estimated by comparing the results from the dashed curve with those of the remaining four curves of fig 5b . These were obtained by adding $\pm 0.4 \times 10^{-5}$ at the top and bottom . The range covered by this set of curves is approximately the same as the difference between the two radio profiles at 31 S and 36.5 N. For all the profiles used the lower boundary was placed at the level where $N^2 \approx 0$. Below this level it was assumed to be adiabatic .

Using the curves of fig 5b , along with the vertical shear profile labelled j in fig 1a produced the variation of Q_y with height shown in fig 6 .

The growth-rate curves derived from the different N^2 profiles are shown in fig 7a . The effect has been mainly to alter the wavenumber of the least stable disturbance (7-10) without greatly reducing the growth rate .

4.0 CONCLUSION

Chapter 7 described the development and use of a model of a baroclinic instability to model a feature observed in the Saturnian atmosphere . The structure of the disturbances produced by several different vertical shear profiles was examined , and it was shown that some of these had both the observed wavelength , and a vertical velocity structure which satisfied the constraints imposed by the images .

The aim of this appendix was to see to what extent the wavelength of the least stable wave depended upon the $\partial u / \partial z$ and N^2 profiles used . The results of section 2.0 demonstrated the range of profiles of $\partial u / \partial z$ used produced a factor of four change in the growth rate , whilst the associated least stable wavenumbers changed from 5 to 11 (this changed to 4-11 when a meridional wavenumber of 7 was considered) . This appendix also showed that the results were not greatly restricted by forcing all the $\partial u / \partial z$ profiles to pass through a common point .

Section 3.0 considered the sensitivity of the results to changes in the N^2 profile used . For realistic changes in N^2 it was found that the maximum growth rates did not change much . However for the $\partial u / \partial z$ profile used the wavenumber of the most dominant wave changed from 7 to 10 .

It has been shown that the temperature structure observed in the midlatitudes of Saturn is capable of producing a wavelike feature similar to that observed . The main difference is that , for most of the profiles tried , the wavelength produced by the model is about 40% larger than the observed value . However this may be because the model results apply only to small perturbations , whilst the observations correspond to a fully developed feature , probably limited by finite amplitude or dissipative effects .

FIGURE CAPTIONS - APPENDIX 3

Figure 1a

This shows some of the various models used for the variation of the thermal wind shear with height . These are all of the form given by equation 1 in section 2.0 . All the curves pass through a common point which is derived from the 150 mb IRIS measurement .

Figure 1b

This shows how the function Q_y , defined in section 7.6.2 varies with height . This has been evaluated using the curves shown in fig 1a along with the N^2 profile labelled '+ - + -' in fig 5a

Figure 2a

This figure shows how the growth rates calculated by the model vary with wavenumber for the profiles of $\partial u / \partial z$ given in fig 1a . The x axis represents a dimensionless form of the wavenumber , which is a factor of 10^7 larger than the value in m^{-1} . The crosses represent the results of the model , while the lines show the result of interpolating between these using cubic splines .

Figure 2b

This shows how the growth rates of fig 2b change for a non zero meridional wavenumber . The value used was $7 \times 10^{-7} m^{-1}$ as derived in section 7.2 . On the dimensionless wavenumber scale used here this corresponds to $K_y = 7$.

Figure 3a

This shows some of the other profiles used to model the variation of the thermal wind shear with height , these are again all of the form given by equation 1 in section 2.0 . The curve labelled 'o' is identical to that labelled 'j' in fig 1a , the curve labelled 'n' is half this value , while the curve labelled 'p' was obtained by multiplying the curve 'o' by 1.5 . These profiles were used to examine the effect of forcing all the $\partial u / \partial z$ profiles to pass through a common point .

Figure 3b

This figure is similar to figure 1b , but shows how the function Q_y , varies with height for the $\partial u / \partial z$ profiles of fig 1a .

Figure 4a

This figure is similar to fig 2a , and shows how the growth rates calculated by the model vary with wavenumber for the profiles of $\partial u / \partial z$ given in fig 3a .

Figure 4b

This shows how the growth rates of fig 4b change when a (dimensionless) meridional wavenumber of $k_y=7$ is used .

Figure 5a

This figure shows some of the temperature-pressure profiles described in the text , they are :

-----	radio profile at 31 S
- - - - -	radio profile at 36 N
— — —	IRIS profile at 13 N
- + - + -	IRIS profile at 22 S

Figure 5b

This figure shows the various N^2 profiles considered in this appendix . The ' + - + - ' profile (curve t) was derived from the radio-science measurements at 36.5 N as described in the text . In order to examine the effect of changes in N^2 it was necessary to extrapolate this curve below the point where $N^2 \approx 0$, this produced the dashed curve (curve 's')

The other four curves were generated by adding or subtracting 0.4×10^{-5} at the top and bottom . Reading from left to right at the 50 km level these are curves q,r,u and v respectively

Figure 6

This figure is similar to figure 1b . It shows how the parameter Q_y for the profile labelled 'j' in fig 1a varies when the different N^2 profiles shown in fig 5b are used

Figure 7a

This figure is similar to fig 2a . It shows how the growth rates for the $\partial u / \partial z$ profile labelled 'j' in fig 1a vary when the different profiles of N^2 given in fig 5b are used .

Figure 7b

This shows how the growth rates of fig 7b change when a (dimensionless) meridional wavenumber $k_y=7$ is used .

fig 1a

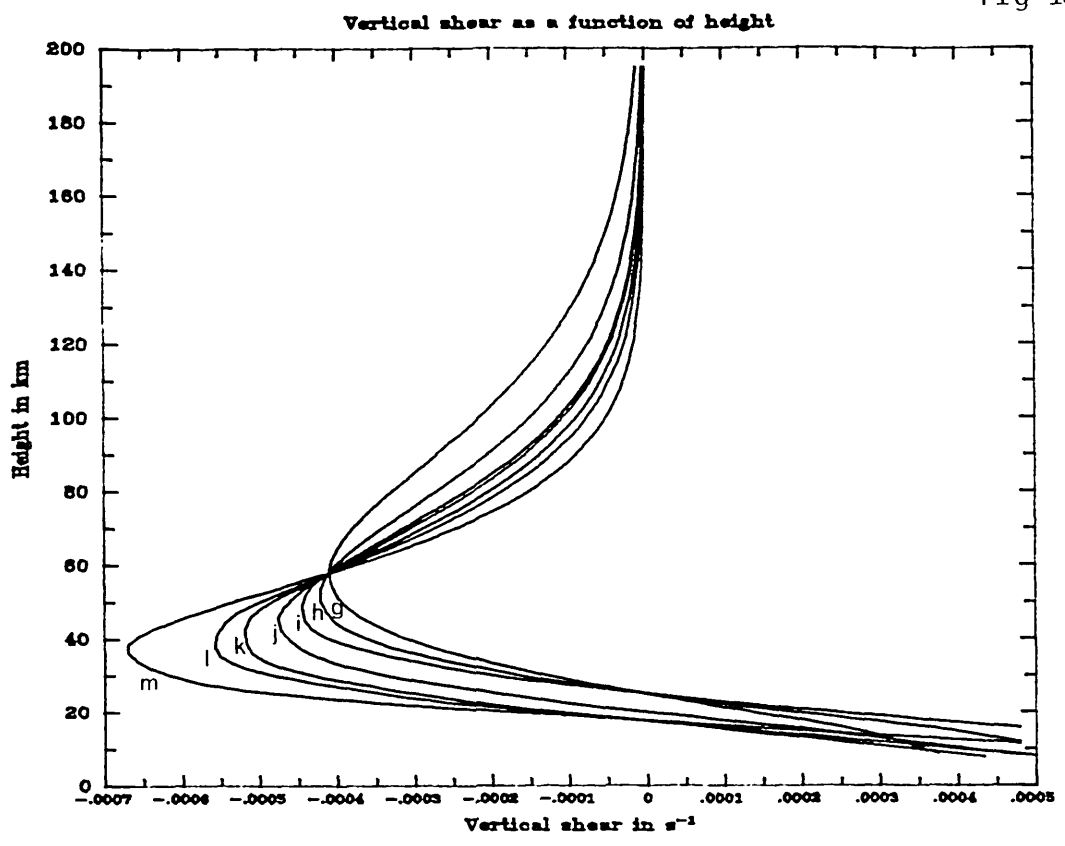


fig 1b

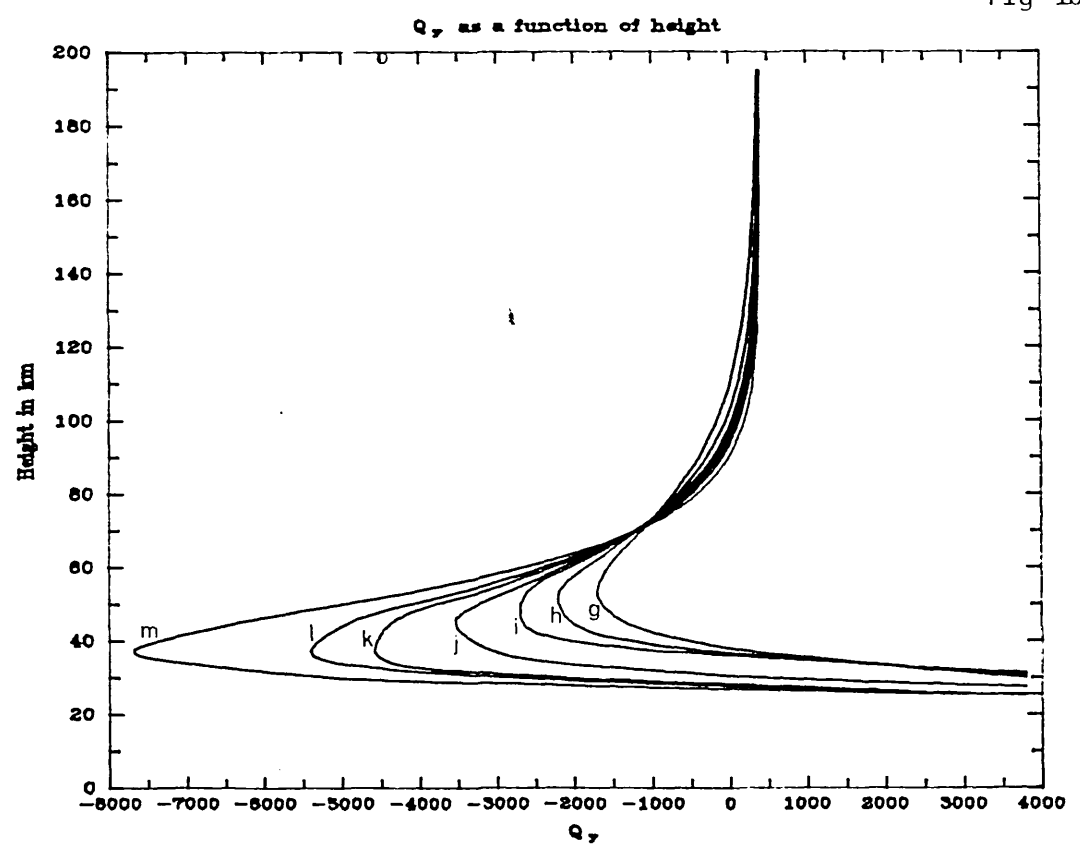


fig 2a

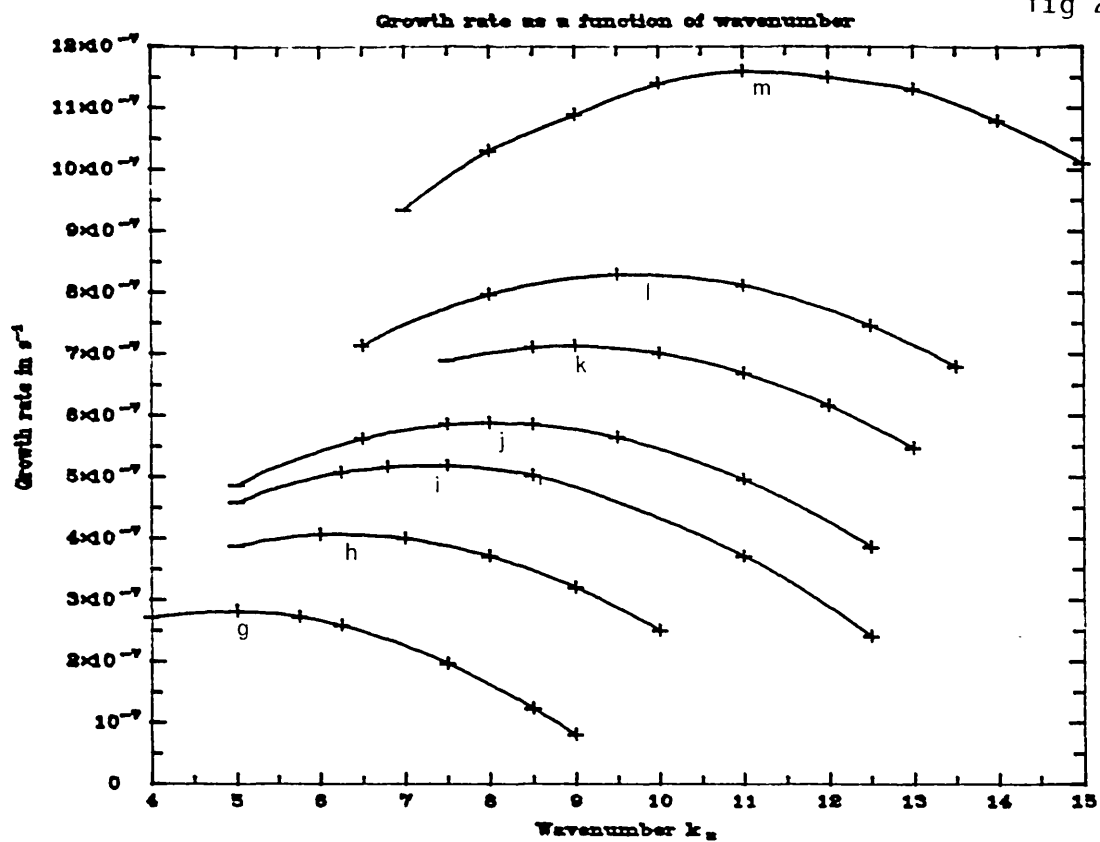


fig 2b

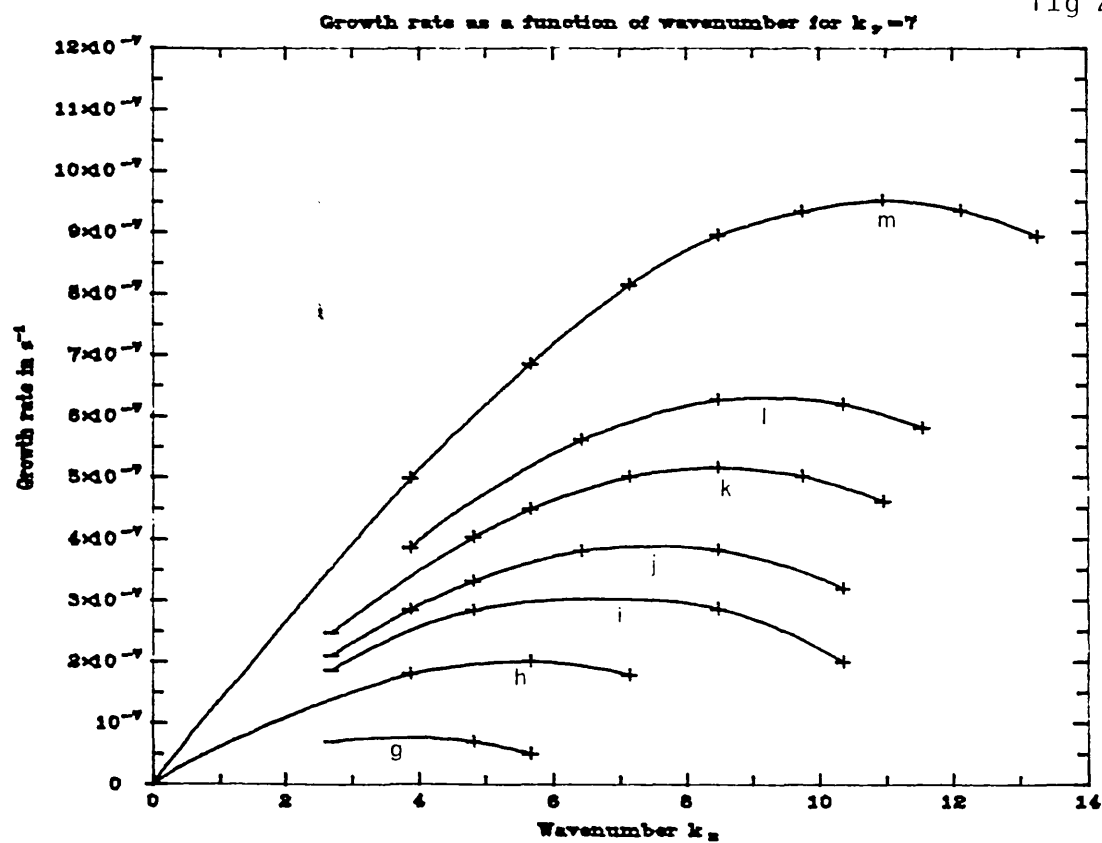


fig 3a

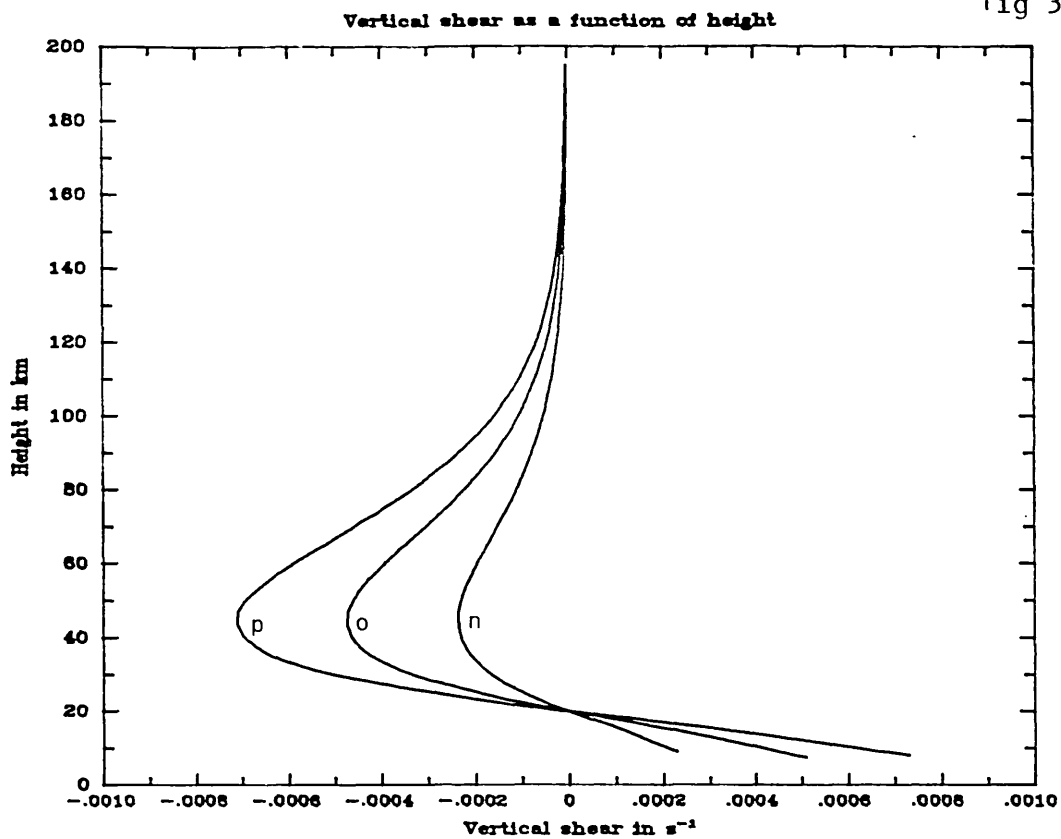


fig 3b

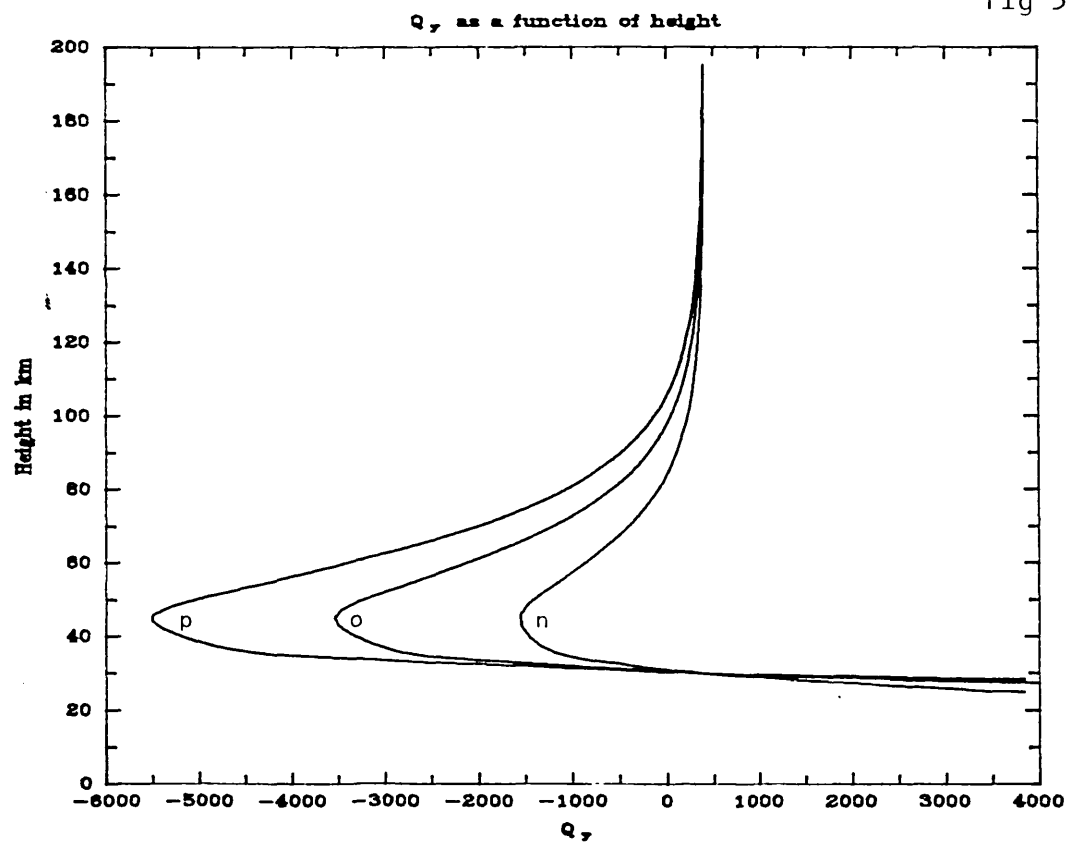


fig 4a

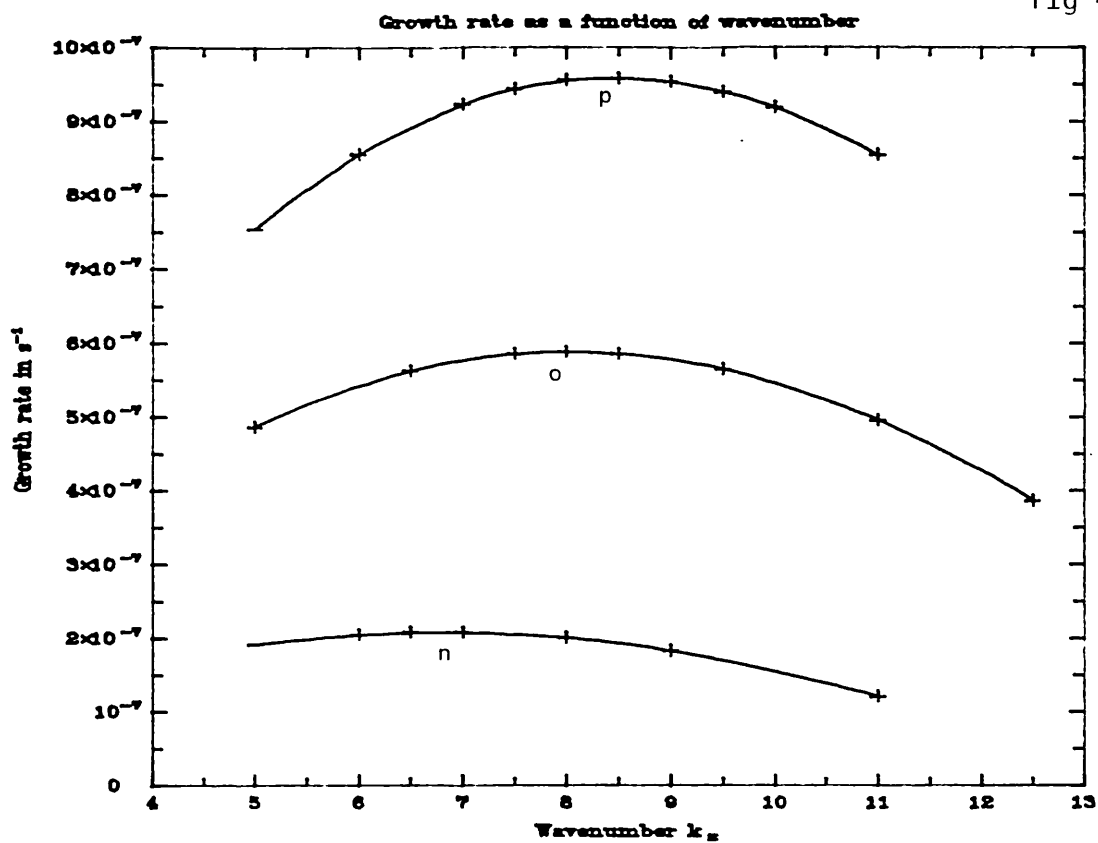


fig 4b

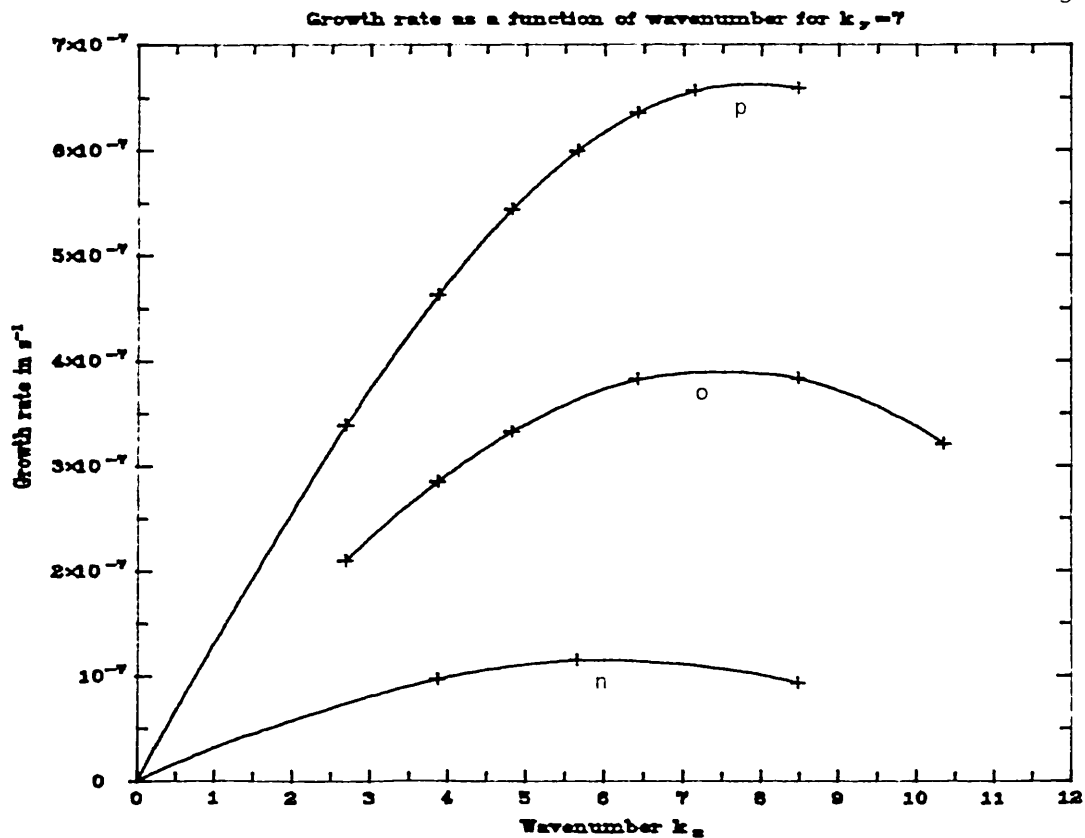


fig 5a

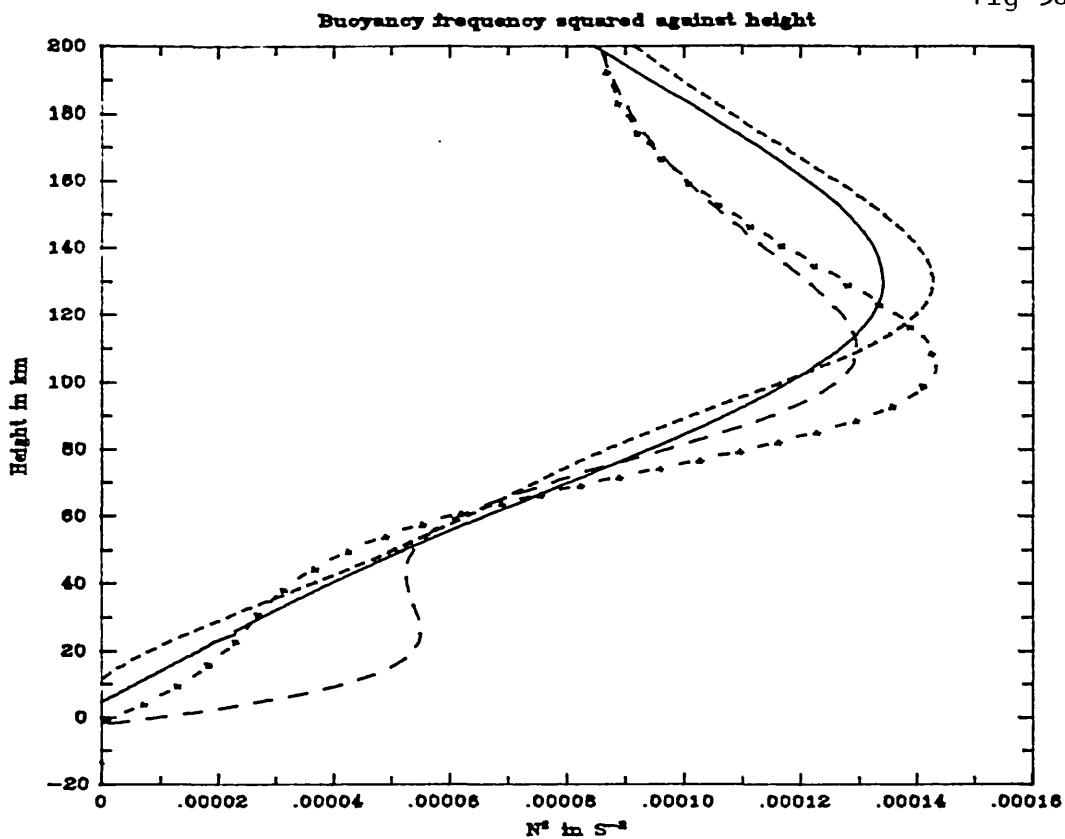
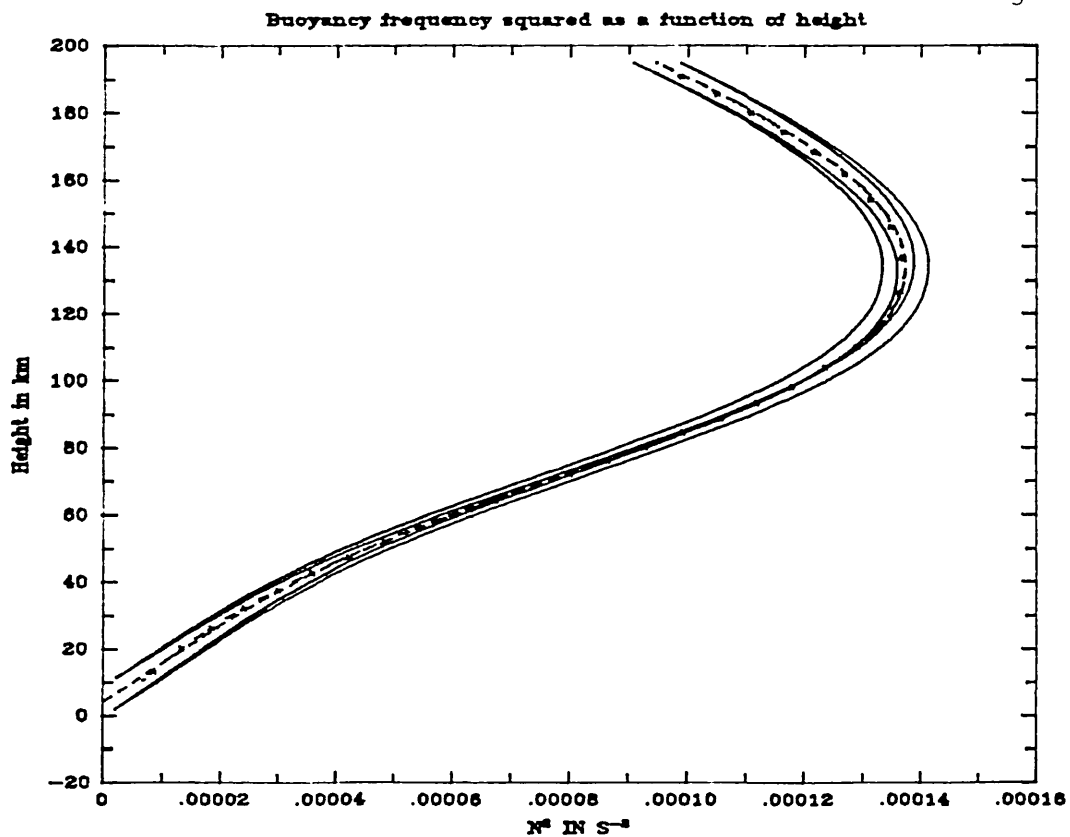


fig 5b



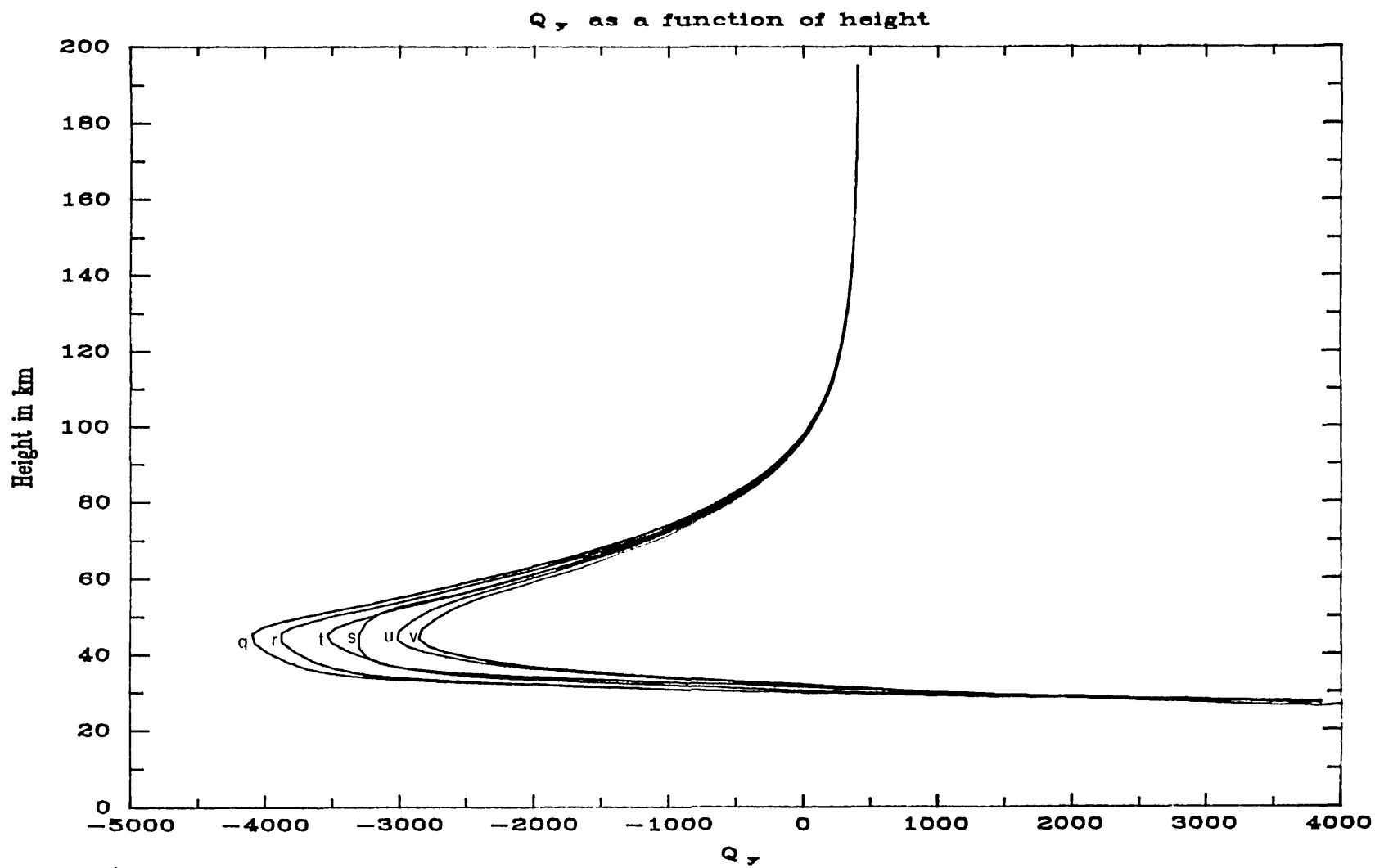


fig 6

fig 7a

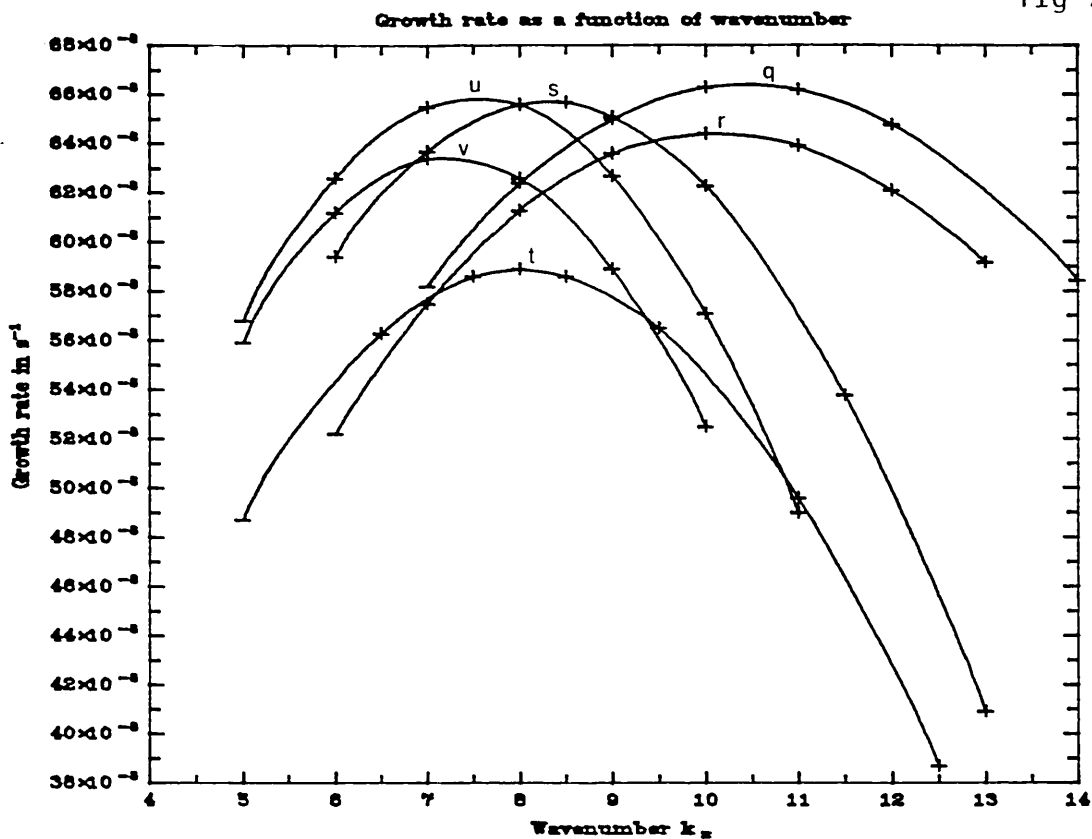


fig 7b

