

FIRE SALES AND SYSTEMIC RISK IN FINANCIAL NETWORKS

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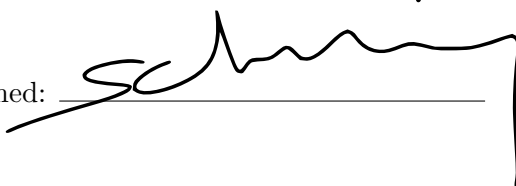


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Signed: _____

A handwritten signature in black ink, appearing to be 'Schmidt', written over a horizontal line. The signature is stylized with a large 'S' and a long, wavy tail that ends in a vertical stroke.

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Fire Sales and Systemic Risk in Financial Networks

ABSTRACT

This thesis studies systemic risk through direct and indirect contagion in financial networks.

The first and main part is devoted to modelling the phenomenon of fire sales in a network of financial institutions with common asset holdings, subject to leverage or capital constraints. In contrast to balance sheet contagion based on direct linkages, this price-mediated contagion occurs through common asset holdings, which we quantify through liquidity-weighted overlaps across portfolios. Exposure to price-mediated contagion leads to the concept of indirect exposure to an asset class, as a consequence of which the risk of a portfolio depends on the network of overlapping portfolios.

Our model provides an operational systemic stress testing framework for quantifying the exposure of the financial system to these effects. Using data from the European Banking Authority, we examine the exposure of the EU banking system to price-mediated contagion. Our results indicate that, even with optimistic estimates of market depth, moderately large macro-shocks may trigger fire sales which lead to substantial losses across bank portfolios, modifying the outcome of bank stress tests. Moreover, we show that price-mediated contagion leads to a heterogeneous cross-sectional loss distribution across banks, which cannot be replicated simply by applying a discount factor to portfolios.

We propose a bank-level indicator, based on liquidity-weighted overlaps of portfolios which can be used to quantify the contribution of a financial institution to price-mediated contagion. Using a granular supervisory dataset, we show that asset class aggregation has a considerable effect on estimated portfolio-level fire sales losses, an issue ignored in the literature so far.

The second part presents an in-depth sensitivity analysis of the Eisenberg & Noe model, a framework commonly used in systemic risk studies. We develop a Taylor series for the clearing vector and characterise it in terms of estimation errors in the liabilities matrix.

To my family, friends, mentors and colleagues.

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*It ain't what you don't know
That gets you into trouble.
It's what you know for sure
That just ain't so.*

Mark Twain



INTRODUCTION

In October 2008, the IMF's estimate of losses in US subprime mortgages was on the order of 500 bn USD (Hellwig (2009)) - a large but bearable loss one was thinking at the time - no cause for imminent global concern. After months of hard fought debates and warnings by the Bank of England, the IMF and other similar institutions, the population of the United Kingdom voted to leave the European Union on June 23rd 2016. The shock of this unexpected result caused end of day losses on the FTSE 100 around 75 bn USD, while intraday losses even exceeded 170 bn GBP ($\approx 200\text{-}220$ bn USD)¹. Although in comparison the “initial losses” triggered by the referendum outcome were smaller by a factor 7 roughly, more importantly the subsequent *amplification* of losses in both scenarios differs by several orders of magnitude! During the course of the global financial crisis, the initial losses of the US subprime market reached many other asset classes, economic sectors

¹<http://money.cnn.com/2016/06/24/investing/brexit-london-stocks-crashing/>

and countries, and ballooned into an estimated total loss in the high single digit trillion dollars (Hellwig (2009)). In contrast, in the months after the Brexit vote, the FTSE 100 hit an all-time high on October 11th 2016, as the pound weakened, and towards the end of 2016, the pound itself strengthened against the dollar and euro in spite of the UK Parliament having voted to back Theresa May’s plan to trigger article 50 and leave the EU by March 2017, with potentially severe consequences regarding the access to the single market and the UK’s trade relations in general. Thus, beyond other differing aspects, the two scenarios are also very different when comparing them solely on the basis of “initial losses” and subsequent “loss amplification”. Similarly, Hellwig (2009) makes the poignant observation that when the dot-com bubble burst, the initial losses in fact exceeded the 500 bn loss in the US subprime market of 2007.

The field of systemic risk studies channels and mechanisms of contagion that can lead to loss amplifications that can threaten the stability of the entire system, as observed under the crisis. Researchers in the field seek to understand, describe and model the very feedback effects that are at the heart of loss amplifications mechanisms as they have been observed in some past crises. *Systemic risk* itself is often defined as the risk that a minor or intermediately sized initial shock may be severely amplified – because of interdependencies and interactions of system components – and potentially bring down the entire financial system, thereby causing significant damage to other economic sectors and/or geographic regions². Fire sales, rapid forced sales by distressed portfolios, have been identified as one such loss amplifying channel through which systemic risk can emerge. One of the main goals of this thesis is to develop an operational framework for quantifying fire sales in regulatory stress tests.

²We will focus only on “systemic risk” in the financial sector.

0.1 THE NEED FOR MACROPRUDENTIAL STRESS TESTS

The financial crisis of 2007-2008 has underlined the need for better monitoring of systemic risk and led to the emergence of a “macroprudential” approach to financial regulation [Hanson et al. \(2011\)](#). One of the most important regulatory overhauls comes with the global (voluntary) Basel III Accords that introduce tighter capital regulations the supplementary leverage ratio (SLR), as well as new liquidity regulations such as the liquidity coverage ratio (LCR) and net stable funding ratio (NSFR). In the US, a significant part of these guidelines are poured into binding national legislation via the “Dodd-Frank Wall Street Reform and Consumer Protection Act”, while in the EU this is captured by regulatory pieces such as the Capital Regulation Directive (CRD IV), the European Market Infrastructure Regulation (EMIR), or Markets in Financial Instruments Directive (MIFIR II). An important part of these reforms is the conduction of *stress tests*: A scenario of severe economic downturn is designed by regulators, and, given their current balance sheets, banks are then required to project their losses under the assumptions of this scenario. The regulator in turn uses the responses and information provided by the banks to assess their stability and resilience to potential future stress scenarios.

More specifically, in the US, Article 165 of Dodd-Frank³ requires the Board of Governors of the Federal Reserve to conduct annual stress tests, the so-called “Comprehensive Capital Analysis and Review” (CCAR). For the EU, a requirement to conduct stress tests is laid down in the EU Regulation 1093/2010, establishing the European Banking Authority and specifying the coordination of a union-wide stress test in coordination with the European Systemic Risk Board (ESRB) and the European Central Bank (ECB)⁴:

³<https://www.congress.gov/bill/111th-congress/house-bill/4173/text>

⁴Regulation (EU) No 1093/2010 of the European Parliament and of the Council of 24 November 2010 establishing a European Supervisory Authority (European

*“The EBA Regulation gives the Authority powers to initiate and coordinate the EU-wide stress tests, in cooperation with the European Systemic Risk Board (ESRB). The aim of such tests is to assess the resilience of financial institutions to adverse market developments, as well as to contribute to the overall assessment of systemic risk in the EU financial system.”*⁵ Moreover, the ECB conducts in parallel a separate stress test of selected European banks under its direct supervision. This is an internal supervisory exercise conducted by the ECB, whose results are not publicly disclosed⁶.

Yet, as pointed out by the Basel Committee on Banking Supervision’s recent report, stress tests conducted by bank supervisors, which remain their primary monitoring tool, still lack a genuine macroprudential component (Basel Committee on Banking Supervision (2015)). Indeed, an important modelling paradigm in the above mentioned stress tests is the so-called “static balance sheet assumption”, which supposes that banks do *not* react to the stress scenario, but that their balance sheet is kept fixed for the duration of the stress test horizon. The report identifies the key missing ingredients towards genuine macroprudential stress tests as being “endogenous reactions and feedback effects to initial stress”. As noted by ECB Vice-President Vitor Constâncio (Anderson (2016)), in the current approach to bank stress tests *“no bank reaction is considered. It would be far more realistic to assume that market participants could react to adverse conditions, rather than assuming passive bank behaviour throughout the entire stress test period. Bank behaviour or reaction could take the form of deleveraging, straight capital increases or working out of non-performing loans.”*

There is ample empirical evidence that such deleveraging occurred on a large

Banking Authority), amending Decision No 716/2009/EC and repealing Commission Decision 2009/78/EC. See: <http://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:02010R1093-20140819>

⁵<https://www.eba.europa.eu/risk-analysis-and-data/eu-wide-stress-testing>

⁶https://www.bankingsupervision.europa.eu/about/ssmexplained/html/stress_test_FAQ.en.html

scale in 2008, leading to cross-asset contagion and amplification of losses in the financial system Kashyap et al. (2008), Brunnermeier and Pedersen (2009), Khandani and Lo (2011), Manconi et al. (2012), Cont and Wagalath (2016). The well-documented occurrence of fire sales during market downturns Ellul et al. (2011), Coval and Stafford (2007), Shleifer and Vishny (2011), Jotikasthira et al. (2012) is not a coincidence: portfolio constraints – capital, leverage or liquidity constraints – which financial institutions are subject to, force them to deleverage when these constraints are breached as a result of losses, leading to fire sales of assets Kyle and Xiong (2001), Cont and Wagalath (2013). Similar large-scale deleveraging is also foreseeable in future stress scenarios, and foreseen by financial institutions themselves: *“If we are unable to raise needed funds in the capital markets (including through offerings of equity and regulatory capital securities), we may need to liquidate unencumbered assets to meet our liabilities. In a time of reduced liquidity, we may be unable to sell some of our assets, or we may need to sell assets at depressed prices, which in either case could adversely affect our results of operations and financial condition”* Credit Suisse (2015).

Fire sales generate endogenous risk Shin (2010) and can also act as channel of loss contagion across asset classes and across financial institutions holding these assets Khandani and Lo (2011), Cont and Wagalath (2016), Caccioli et al. (2014). Unlike direct contagion through counterparty exposures Cont et al. (2013), fire sales spillovers are mediated by prices and thus defy limits on counterparty exposures and other institutional ring-fencing measures.

It is therefore important for supervisors to include in macro-stress testing frameworks used for assessing bank capital adequacy the impact of fire sales and the deleveraging of portfolios in stress scenarios. This is especially relevant given that, post-crisis in the US, supervisory stress tests have set the binding constraint for bank capital adequacy⁷.

⁷See the Section “Process and Requirements after CCAR 2016: <https://www.>

Fire sales and the resulting destabilizing feedback effects have been extensively studied in the literature [Kyle and Xiong \(2001\)](#), [Shleifer and Vishny \(2011\)](#) from a conceptual viewpoint. The challenge is to develop a quantitative framework versatile enough to be taken to empirical data and used in an operational macro-stress testing framework to quantify the endogenous risk and spillover effects arising from fire sales.

The goal of the present work is to address this challenge, by proposing a modelling framework for quantifying the exposure of the financial system to the endogenous losses and feedback effects resulting from fire sales in a macro-stress scenario. Previous attempts by regulators to account indirectly for the impact of fire sales in bank stress tests include the use of (exogenously specified) liquidation costs or increasing the severity of shocks in single-bank stress tests to account for possible loss amplification due to feedback from fire sales. These adjustments may mimic the severity of potential losses which may result from fire sales but fail to capture key cross-sectional features of fire sales spillovers, such as the contagion across asset classes and the heterogeneous distribution of fire sales losses across financial institutions and across asset classes of varying liquidity.

More recently, [Greenwood et al. \(2015\)](#), [Duarte and Eisenbach \(2013\)](#) have proposed a stress testing approach incorporating the impact on asset prices of deleveraging in bank portfolios, based on the assumption of *leverage-targeting* [Adrian and Shin \(2010\)](#) i.e. that, in response to a shock, financial institutions rebalance their portfolios to maintain a constant leverage, which leads to a linear deleveraging rule in reaction to market shocks. This approach was used to analyse fire sales spillovers in the EU banking system by [Greenwood et al. \(2015\)](#) and in the US banking system by [Duarte and Eisenbach \(2013\)](#). Both studies find evidence of potentially large exposures of the banking system to contagion via fire sales. This approach has been

[federalreserve.gov/newsevents/press/bcreg/bcreg20160629a1.pdf](https://www.federalreserve.gov/newsevents/press/bcreg/bcreg20160629a1.pdf).

explored by supervisors as a possible method for incorporating fire sales in macro-stress tests [Henry et al. \(2013\)](#), [Cappiello et al. \(2015\)](#), [Cappiello and Spuera \(2014\)](#). There is some empirical evidence that in the medium term large financial institutions maintain fairly stable levels of leverage [Adrian and Shin \(2010\)](#) but it is not clear why the same institutions would enforce such leverage targets in the short term, especially in stress scenarios where this could entail high liquidation costs.

We propose a different approach for modelling fire sales, based on the premise that deleveraging by financial institutions occurs in reaction to losses in their portfolios [Kyle and Xiong \(2001\)](#), [Cont and Wagalath \(2013\)](#). This deleveraging may be the result of investor redemptions for funds, as evidenced in [Ellul et al. \(2011\)](#), [Coval and Stafford \(2007\)](#), [Shleifer and Vishny \(2011\)](#), [Jotikasthira et al. \(2012\)](#); but for regulated financial institutions such as banks, large scale deleveraging is mainly driven by portfolio constraints -capital, leverage or liquidity constraints- which may be breached when large losses occur. We have focused for simplicity (and data availability) on leverage constraints, but the model is easily extendable to multiple constraints on portfolios. Given the one-sided nature of these constraints, such institutions react asymmetrically to large losses and large gains [Ang et al. \(2006\)](#). The asymmetry and the threshold nature of deleveraging differentiates our approach from models based on leverage targeting, leading to considerably different outcomes.

Deleveraging by financial institutions impacts market prices and, when portfolios are marked to market, leads to further losses which may in turn trigger further deleveraging, shown schematically in Figure 0.1.1. We quantify this impact and the resulting endogenous risk, paying attention to the estimation of market impact parameters; the magnitude of these parameters, and their heterogeneity across asset classes, is shown to greatly influence the results of the stress tests.

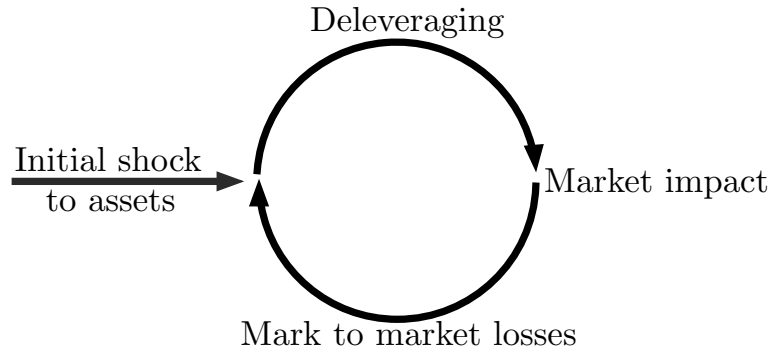


Figure 0.1.1: Schematic view of the model dynamics: A sufficiently large initial shock leads to the breach of portfolio constraints and triggers portfolio deleveraging in the form of forced liquidations. This impacts market prices, resulting in turn in new losses that may trigger further rounds of deleveraging.

We use these ingredients to design a systemic stress testing framework for banking systems. Application of the method to the EU banking system shows that this approach may lead to outcomes which are substantially different from single-bank stress tests. We emphasise in particular the concept of *indirect exposure*, which we show to be relevant for bank stress testing, and develop indicators for monitoring these as well as macroprudential policies to reduce the risk of large scale contagion through fire sales.

Before we turn to summarising the main contributions of the thesis, we present a short overview of the broader literature on systemic risk. This also allows us to situate our contributions to the academic research on systemic risk more precisely.

0.2 LITERATURE REVIEW: SYSTEMIC RISK

The “Handbook on Systemic Risk” (Fouque and Langsam (2013)) provides a recent broad overview of the research in systemic risk. It covers various aspects of the topic such as measuring systemic risk, regulating systemic risk, data requirements & collection for systemic risk analyses, and it reviews various types of models around systemic risk. Another broad review paper

on systemic risk and contagion in financial network is [Glasserman and Young \(2016\)](#). The “Systemic Risk Hub” is a website on systemic risk and regularly updated with new research papers and events in the field: <http://www.systemic-risk-hub.org/>.

Systemic risk is an intrinsically interdisciplinary field and spans multiple academic subjects, ranging from economics and finance via econometrics and statistics, to network theory and mathematics or dynamical- & complex systems theory. As such, it is challenging to write a few pages review of the systemic risk literature; over the past few years entire papers have been written that review specific subfields of systemic risk such as [Bisias et al. \(2012\)](#) on measures of systemic risk or [Hüser \(2016\)](#) on the interbank contagion literature.

The following selective presentation and grouping of the literature is reflective of my personal view on the field, and constitutes an attempt to give some structure to the vast amount of work that has been completed over the past decade(s). I will discuss the relation of my work to general equilibrium models in Chapter 5.

ECONOMICS AND FINANCE

CONCEPTUAL CONTRIBUTIONS. Important fundamental contributions to understanding systemic risk have been made in economics and finance. Large broker dealers stand at the heart of the intermediation chain in the financial system and infrastructure. [Duffie \(2010b\)](#) gives an excellent introduction to the mechanics at work inside large dealer banks and describes common modes of failure for these institutions. The failure of such dealer banks, as witnessed during the crisis in the cases of Lehman Brothers and Bear Stearns, is most often linked to the failure of the institution to obtain sufficient funding and therefore becoming *illiquid*, i.e. unable to perform

on due payments. The spiral towards such an illiquidity failure is usually driven by a loss of confidence of funding counterparties, and can therefore be viewed as a modern version of the classic depositor bank run. In the case of large broker dealers, the loss of funding can occur through OTC derivatives counterparties novating their positions to other dealer banks⁸, through short term creditors (such as overnight repo counterparties) not renewing their loans⁹, or the disappearance of prime brokerage clients¹⁰, all of which can put a serious liquidity strain on the dealer bank. [Duffie \(2010b\)](#) then proceeds to sketch some ways in which one may mitigate these issues, such as more stringent liquidity requirements for dealers and a reform of the tri-party repo market, as well as improved failure resolution mechanisms for broker dealers. Beyond the risk of “bank runs”, [French et al. \(2010\)](#), discuss important features of the plumbing of financial markets such conflicts of interest, agency problems and inadequate regulatory structures that allowed the build-up of the crisis, as well as inadequate bankruptcy and resolution that added oil to the fire during crisis. Concrete policy suggestions for tackling these are offered, such as the introduction of a systemic regulator, a regulation of executive pay in financial services, stricter capital requirements for financial institutions, and the introduction of contingent convertible bonds (CoCo’s) to recapitalize institutions via bail-in among others. A number of these suggestions have indeed developed into parts of the post crisis regulatory reform, as e.g. the establishment of the European Systemic Risk Board (ESRB) and the Financial Stability Oversight Council (FSOC), or the large regulatory reform launched through Basel III.

[Hellwig \(2009\)](#) gives a detailed account of how the losses in US subprime mortgages in 2006 - 2008 became systemic: Negligence, flaws in governance

⁸...by requiring the dealer to pay out balances in favour of the counterparty, or return cash collateral

⁹...and the dealer finding itself unable to transform its securities holdings into cash

¹⁰...because dealers may use excess cash that they obtain from rehypothecation of client collateral to finance themselves

and moral hazard led to the origination of loans in the US subprime market that never should have been granted. Similar flaws in the securitisation process led to these risks being obfuscated, packaged and then passed on to other investors. The combination of maturity mismatch, low capital buffers inappropriate internal risk controls then led to contagious failures as well as a large scale panic resulting in general market malfunctioning, which induced further failures and contagion. Beyond providing comparisons to past crises such as the Savings & Loans crisis, [Hellwig \(2009\)](#) also identifies three core contagion channels: (i) *Domino effects through contractual relations*, (ii) *Domino effects through asset prices*, and (iii) *Information contagion effects*. The model developed in the main part of this thesis seeks to quantify the second type of contagion. Similar accounts with different perspectives are offered for instance in [Brunnermeier \(2008\)](#) and [Blinder \(2013\)](#).

Important contributions on endogenous risk in the financial system have been made by [Adrian and Shin \(2014\)](#), [Brunnermeier and Pedersen \(2009\)](#), and [Danielsson et al. \(2012\)](#). As early as 2001, [Danielsson et al. \(2001\)](#), stressed the inherent procyclicality of some of the risk measurement practices introduced in Basel II, insisting that not attending to these issues may lead to large systemic crises in the future. Although there is no mention of “systemic risk” in [Granovetter \(1978\)](#), the concepts developed in the paper are of the utmost relevance for studies on systemic and endogenous risk. The author discusses phase transitions of outcomes in systems of interacting agents where each agent is subject to an individual threshold: Let there be two groups of potentially rioting people: in the first group, for the n^{th} individual the required number of people that must have already joined the riot for him/her to join as well is $n-1$; in the second group, the thresholds are the same except that for individual 3, say, the threshold is 3. Even though the two distributions of individual thresholds are remarkably similar, the dynamics in these two “systems” will differ fundamentally: in the first case

each rioter joining the mass will lead to another person joining such that the entire population riots in the end, while in the second case the thresholds are such that the “cascade stops” at the third person. We shall argue in this thesis that such thresholds are a natural way of describing forced liquidations by constrained portfolios.

An important contribution on fire sales was made in [Shleifer and Vishny \(1992\)](#), where the authors particularly emphasise the link between assets’ liquidation values and the set, or number of, potential buyers for these assets. [Duffie \(2010a\)](#) both documents empirically and models theoretically the price drop of an asset under such forced liquidation. The idea of “slow moving capital” precisely captures these inertly arriving buyers and the deviation of prices from “fundamental value” in the mean time. The later paper, [Shleifer and Vishny \(2011\)](#) provides a selective review of the fire sales literature, with a focus on fire sales of financial instruments and fire sales in macroeconomic models, as well as their role in the financial crisis.

A systemic trade-off between diversity and diversification is studied in [Beale et al. \(2011\)](#): The risk of an individual portfolio decreases with the degree of diversification. However, in the “maximal diversification” case, everybody will be diversified in the same manner - the system’s diversity is thus reduced, which is shown to lead to increased systemic risk. [Wagner \(2011\)](#) studies this problem from an asset pricing perspective.

EMPIRICAL CONTRIBUTIONS. In recent years there has been a growing empirical body of literature around various aspects of systemic risk. [Aguiar et al. \(2014\)](#) do an excellent cartographic job by mapping out the flow of funding, not between different nodes in a financial network, but by dissecting the flows *within* a single broker dealer, providing a description of what types of transformations occur at different layers of the dealer and how funding weakness at one such institution can spread to others with potential risks

to financial stability. Collateral flows in the opposite direction of cash in secured funding operations: the collateral network is analysed in depth in [Aguiar et al. \(2016\)](#). The networks are not entirely symmetric, as certain institutions play more important roles in different networks (e.g. Central Clearing Counterparties (CCPs) in the collateral network, and Money Market Funds in the funding network). Furthermore, there is a growing body of empirical studies by central banks on detailed and confidential regulatory data ([Ali et al. \(2016\)](#), [Langfield et al. \(2014\)](#), [Cont et al. \(2013\)](#), [Duffie et al. \(2014\)](#), [Abad et al. \(2016\)](#)) that nevertheless present numerous summary statistics, insights, and stylised facts that are useful for developing new models.

[Ellul et al. \(2011\)](#), [Coval and Stafford \(2007\)](#) offer evidence of fire sales by funds and insurance companies as well as compute estimates of the price impact of these forced liquidations. [Pierret \(2015\)](#) supplies empirical evidence for a “solvency-liquidity nexus”: less well-capitalised banks face higher funding costs and therefore a higher risk of suffering a run. Historically, liquidity and solvency have often been considered as two separate dimensions of financial stability, but work in this direction suggests that both aspects are tightly interconnected and should not be treated independently.

Short term funding, and the “run on repo” during the crisis are discussed in great detail in [Gorton and Metrick \(2012\)](#). Specifically related to the US tri-party repo market ¹¹, [Begalle et al. \(2016\)](#) provide a detailed analysis of the risk of fire sales in the US tri-party repo market, and how this market’s resilience can be increased; a number of policy suggestions have indeed been implemented as part of the post crisis tri-party repo market reform.

To this empirical academic literature, I would in fact add certain regulatory

¹¹The US tri-party repo market accounts roughly for half of the entire US repo market: https://www.newyorkfed.org/medialibrary/media/research/staff_reports/sr740.pdf.

documents and reports such as “Global systemically important banks: updated assessment methodology and the higher loss absorbency requirement” [Basel Committee on Banking Supervision \(2013a\)](#), “Basel III: The Liquidity Coverage Ratio and liquidity risk monitoring tools” [Basel Committee on Banking Supervision \(2013b\)](#), “Basel III: A global regulatory framework for more resilient banks and banking systems” [Basel Committee on Banking Supervision \(2011\)](#), “Standards: Minimum capital requirements for market risk” [Basel Committee on Banking Supervision \(2016\)](#), “Basel III leverage ratio framework and disclosure requirements” [Basel Committee on Banking Supervision \(2014a\)](#), “Basel III: the net stable funding ratio” [Basel Committee on Banking Supervision \(2014b\)](#), “Dodd-Frank Wall Street Reform and Consumer Protection Act” [Dodd-Frank \(2010\)](#), [EMIR \(2012\)](#), “Final Report of the National Commission the Causes of the Financial and Economic Crisis in the United States” [The Financial Crisis Inquiry Commission \(2011\)](#), [Valukas \(2010\)](#). Even though they are not academic papers in any classical sense, they are of a high relevance, as they set the legal and regulatory framework within which banks and other institutions operate, which should be reflected appropriately by models. I believe it is important to translate key regulatory and institutional features correctly into models.

ECONOMETRICS AND STATISTICS

There is a large econometric and statistical literature on the measurement of systemic risk. The majority of contributions in this stream try to extract signals from market data that allow to quantify systemic risk. [Bisias et al. \(2012\)](#) provides a comprehensive overview of the main measures of systemic risk in this literature. Among the most prominent measures to quantify systemic risk are the “Marginal Expected Shortfall (MES)” [Acharya et al. \(2017\)](#), “SRISK” [Brownlees and Engle \(2016\)](#) and “Conditional Value at Risk (CoVaR)” [Adrian and Brunnermeier \(2016\)](#). The Marginal Expected

Shortfall of an institution is defined as its expected short-term equity loss, conditional on the market taking a loss greater than its Value-at-Risk at a given level: $MES_i := \mathbb{E}[r_i | r_m < VaR_\alpha]$, where r_i, r_m denote the log returns of institution i and the market respectively, and VaR_α a constant denoting the tail risk in the market. Rather than the negative return r_i , *SRISK* considers the expected capital shortfall of a bank (its regulatory capital requirement minus the firm's equity) conditional on the system being under stress. Similarly, for a confidence level α an institution i 's Conditional Value at Risk, $CoVaR_{i,\alpha}$, is implicitly defined via $\mathbb{P}(X_i \leq CoVaR_{i,\alpha} | X_j = VaR_\alpha) = \alpha$, where X_i is the random variable (e.g. return) corresponding to $VaR_{i,\alpha}$. The authors then define the contribution of risk by institution j to the system as $\Delta CoVaR_j := CoVaR_\alpha^{system|X_j=VaR_\alpha} - CoVaR_\alpha^{system|X_j=Median_j}$ to measure. These measures are commonly estimated with GARCH models¹². Their primary goal is to quantify how much risk a particular institution adds to the entire system (and then make institutions internalise this risk by appropriate scaling of capital requirements).

A different method for quantifying systemic risk is suggested in [Kritzman et al. \(2011\)](#) via the “absorption ratio” (*AR*): The *AR* quantifies the amount of variation captured by the first n principal components as measure of systemic risk (capturing the intuition that during crises asset prices tend to co-move, and that a larger portion of the variance is therefore explained by the first few principal components). A high *AR* then indicates high correlations across asset class returns, as they are commonly observed during market downturns.

[Flood et al. \(2015\)](#) provide a comprehensive overview of econometric models for liquidity estimation, including the model suggested by [Kyle and Obizhaeva \(2016a\)](#). In [At-Sahalia et al. \(2015\)](#) contagion between asset classes is modelled via Hawkes processes. Similarly, [At-Sahalia et al. \(2014\)](#)

¹²See: <https://vlab.stern.nyu.edu/en/>.

uses self-exciting processes to model the endogenous risk in CDS markets.

We note that some of these measures have faced criticism however: For instance, [Mainik and Schaanning \(2014\)](#) show that systemic risk in the original definition of *CoVaR* is non-monotonic in the dependence of the two variables, and therefore suggest using a consistent version, as introduced in [Girardi and Ergün \(2013\)](#). Similarly, [Bernard and Czado \(2015\)](#) show that quantile regressions (as used in [Adrian and Brunnermeier \(2016\)](#)) can be problematic when faced with non-Gaussian data. By virtue of being based on market data, the above discussed measures can only pick up risk and stress that is currently being priced-in by (the wisdom and whims of) the market, and present thus more a “thermometer” of risk rather than a “barometer” of risk. Moreover, to the extent that asset prices are driven by price impact of large (or many small herded) trades, these measures can be misleading, as they cannot distinguish between such price moves and those due to a fundamental change in risk perception ([Anderson \(2016\)](#)). The ECB has expressed general scepticism in market data’s capability to properly reflect the *build-up* of systemic risk, which, as they argue, can only be captured ex-ante by monitoring exposure data ([Idier et al. \(2013\)](#), [Anderson \(2016\)](#)).

In general, even though these approaches provide statistical models for *what* happens, because they do not include an explicit model feature that generates the observed market dynamics but rather model the resulting statistical process directly, they often fall short of providing explanations to questions such as *why and how* it happens, and what to do about it.

NETWORK THEORY AND MATHEMATICS

Network models attempt to explain a greater part of the “why and how” questions. [Hüser \(2016\)](#) provides an excellent review of the interbank con-

tagion literature.

Important contributions to modelling the direct contagion via interbank exposures have been made in Cont et al. (2013) and Amini et al. (2016), where default cascades of bank failures were modelled directly.

This stream of work identified sparse subnetworks of “contagious exposures” (E_{ij} is a contagious exposure if the exposure of i to j exceeds the capital of i) that carry almost the same information as the entire network. Regulations such as the “large exposure limits¹³” for banks explicitly account for this, and have mitigated contagion via this channel significantly. This stream of work further lead to the development of metrics such as the bank specific “Contagion Index”, which quantifies the expected total loss (“Default impact”) triggered by the default of an institution, which can hence be used to quantify the level of systeminess of a bank.

“DebtRank” measures contagion through direct (interbank) exposures and is computed via the algorithm presented in Battiston et al. (2012). The concept differs in an important way from the Default Impact introduced by Cont et al. (2013): In the former framework, losses are assumed to be transmitted to counterparties even in absence of a default. This is based on the idea of the Merton model that the value of a bank’s debt decreases when its assets suffer a loss. In absence of perfect information, it is not clear however how the news of such losses would spread to counterparties and thereby affect the value of their debt holdings when the initial bank does *not* default.

In contrast, during deleveraging cascades, price-mediated contagion from mark-to-market losses *does* materialize before a default occurs. We shall quantify this in networks of overlapping portfolios, which gives rise to an “*Indirect* Contagion Index” (Chapter 4). More recently, several studies have

¹³See: <http://www.bis.org/publ/bcbs283.pdf>

focussed on such bipartite institution-asset networks for mutual funds and insurance sectors respectively (Guo et al. (2015), Getmansky et al. (2016)), and explored the concept of overlapping portfolios as a channel of contagion (Caccioli et al. (2015), Caccioli et al. (2014)). Caccioli et al. (2014) show that contagion in the institution-asset network can be modelled by a Galton-Watson process. On this note, we insist on a subtle but important difference of their modelling approach to ours: while they assume that liquidation of assets and the ensuing price-mediated contagion only occurs *upon* default, contagion through fire sales in our model occurs *before* default through banks precisely trying to avoid defaulting.

Another prominent model of direct contagion in the systemic risk literature is the Eisenberg and Noe model (Eisenberg and Noe (2001)). This model has been used in a host of studies on systemic risk. Important contributions to this stream of the literature have been made by Rogers and Veraart (2013), and more recently Kusnetsov and Veraart (2016) have made an important extension of the framework to multiple periods. We will review this literature in more detail in the introduction to Part II of the thesis and therefore postpone a deeper discussion of this literature until then.

An important problem within the network contagion literature, is the population of these networks under incomplete information, i.e. the estimation of bilateral exposures given marginal or aggregate information. Different methods have been developed from minimum entropy, to more sophisticated Bayesian Gibbs samplers that are shown to perform very well on actual data (Gandy and Veraart (2016)) for estimating such networks. Other noteworthy contributions to this literature are by Anand et al. (2015), Mistrulli (2011) and Di Gangi et al. (2015).

A classic recurring theme in network models is the “stable yet fragile” concept: The intuition is that stronger interconnectedness is beneficial as long

as shocks are small because the losses are spread more evenly across the system adding to its resilience; however, when losses are large, stronger interconnectedness is detrimental as it increases the extent of contagion. Contributions to this literature have for instance been made by [Acemoglu et al. \(2015\)](#) or [Caccioli et al. \(2015\)](#).

Finally, contributions of a more formal and mathematical type are made for instance in [Föllmer \(2014\)](#), [Carmona et al. \(2015\)](#) or [Chen et al. \(2013\)](#). While it may sometimes be difficult to find economic interpretations for some “systemic risk” parameters of these models, they can still provide deep insights about model features. For instance, [Föllmer \(2014\)](#) discusses existence and uniqueness issues of global risk measures that are consistent with local convex risk measures specified at individual nodes. Another important question that is addressed in [Föllmer \(2014\)](#) is how arbitrarily local conditional risk measures can be chosen to be, such that they still allow for a, at least local, consistent aggregation.

AGENT BASED MODELS

The last literature stream within our “grouping” of the systemic risk literature regards agent based models. A key contribution in this domain has been made by [Bookstaber et al. \(2014\)](#), who develop an ABM for a system of interacting hedge funds, cash providers and linkages occurs through funding, collateral and asset prices.

[Greenwood et al. \(2015\)](#) propose an ABM of banks following a leverage targeting rule to model fire sales, which we compare to our model in great detail in Sections 2.3 and 3.3. Another very interesting new contribution to fire sales triggered due to a funding shock, and the modelling of the interaction between solvency and liquidity is made in [Calimani et al. \(2016\)](#).

Bookstaber and Paddrik (2015) and Bookstaber et al. (2015) develop agent based models of market depth (liquidity) in crisis scenarios, and the resulting price dynamics. Leverage cycles have been studied in Aymanns et al. (2016) and references therein. Modelling systemic risk via Agent Based Models is still quite a young field, and I expect many exciting developments and findings in the years to come.

0.3 MAIN CONTRIBUTIONS OF THESIS

0.3.1 PART I: FIRE SALES AND SYSTEMIC STRESS TESTING

We present a framework for quantitative modelling of fire sales in a network of financial institutions with common asset holdings, subject to leverage or capital constraints. Asset sales are triggered in reaction to external shocks to asset values when portfolio constraints are breached; the market impact of these asset sales then leads to contagion of mark-to-market losses to other portfolios, which may in turn be led to deleverage if their constraints are breached. In contrast to balance sheet contagion which arises through direct bilateral exposures, this *price-mediated contagion* occurs through common asset holdings, even in absence of direct linkages between financial institutions. The resulting feedback loop may lead to loss amplification, *systemic risk* and large-scale instability of the financial system. Our model provides a method for quantifying the exposure of the financial system to these effects, which we apply to data on European banks. Overall, this leads to several interesting findings.

- **Existence of a tipping point:** We show the existence of a critical macro-shock level beyond which fire sales trigger large-scale contagion and lead to systemic risk. The level of this critical shock depends on the institutions' leverage, as well as the concentration and common-

ality of their asset holdings. In the European banking system, this critical shock size is found to correspond to large, but not extreme, losses in asset values.

- **Magnitude and heterogeneity of losses due to fire sales:** We find that fire sales contribute significantly to system-wide losses in stress scenarios, accounting for more than 35% of the total losses and between 20 to 40% of system bank equity. These results are significant enough to modify the outcome of bank stress tests.

Moreover, while the total system-wide loss can always be replicated in a stress test without fire sales by applying a larger shock to assets, the heterogeneous cross-sectional distribution of losses across banks cannot be reproduced in absence of fire sales by simply applying a larger macro-shock.

- **Second-round effects are significant:** fire sales may potentially lead to a feedback loop which generates market losses and further fire sales across other financial institutions. Ignoring these feedback effects and the corresponding second- and higher-round deleveraging may lead to a significant underestimation of system-wide losses.
- **Indirect exposures:** As a result of fire sales spillovers, a portfolio's exposure to an asset class in a stress scenario may be larger than its notional exposure. This naturally leads to the notion of *indirect exposure*, which is scenario-dependent, and can be quantified using our model. One striking finding is that a bank may have a non-zero (indirect) exposure to an asset class it does not hold. We find evidence that EU banks have substantial indirect exposures to real estate in EU countries where they do not issue mortgages.
- **Sensitivity to market depth across asset classes:** The level of fire sales losses in stress tests is found to be sensitive to the assumptions

on market depth across asset classes. Calibration to data on market prices, trading volumes and turnover reveals a significant dispersion in market depth across asset classes. We show that ignoring this heterogeneity of market depth across asset classes leads to a considerable bias in the estimation of fire sales losses. This underlines the importance of a rigorous sensitivity analysis of these modelling parameters when modelling fire sales.

- **Importance of gain-loss asymmetry and the threshold nature of fire sales:** We argue that fire sales arise when portfolio constraints such as leverage, liquidity or capital ratios are breached as a result of large portfolio losses. The *one-sided* nature of these constraints leads to an asymmetric reaction of institutions to gains and losses, which differentiates our model from the 'leverage targeting' model used in some previous studies of fire sales [Adrian and Shin \(2010\)](#), [Duarte and Eisenbach \(2013\)](#), [Greenwood et al. \(2015\)](#). Comparison with stress tests based on leverage targeting shows substantial differences in outcomes: leverage targeting models overestimate the magnitude of fire sales, especially at smaller shock levels, but underestimate the acceleration (convexity) of fire sales with increasing shock size present in the threshold model.
- **Distinction between insolvency and illiquidity:** unlike previous models of contagion which have mainly focused on modelling insolvency, our model distinguishes between failure due to insolvency and failure due to illiquidity.
- **Monitoring:** Based on the findings in our analysis, we design two indicators: First, the *Indirect Contagion Index* (ICI), which may be computed from bank portfolio holdings, measures a bank's exposure to price mediated contagion. Importantly, the computation of this

indicator does not require running a stress test and provides a simple proxy for monitoring exposure to fire sales contagion. Second, the *Systemic Vulnerability Indicator* (SVI) quantifies what initial shock is “required” to trigger large scale endogenous contagion through fire sales. As such it can be used to assess the resilience of the system against large scale fire sales contagion under a range of scenarios and assumptions.

- **Macroprudential policy tools against fire sales risk:** Several theoretical findings in the analysis of our model point towards using targeted capital charges for systemically important banks to mitigate the contagion emanating from fire sales losses. We use these findings to assess three different macroprudential policies, and find some of them to significantly outperform a blind benchmark policy where all banks are required to raise capital.

These findings have many implications for risk management in financial institutions and for the monitoring of systemic risk in the financial sector. In particular, they underline the need for a systemic approach to stress testing and the necessity of macroprudential tools for tackling the risks resulting from price-mediated contagion. In addition to these points, our work extends fire sales models in two technical directions: First, we introduce a level-dependent market impact function that captures the arrival of arbitrageurs *before* the price drops to zero. Second, using a supervisory dataset on portfolio holdings, we illustrate that choosing a different level of aggregation for the “asset classes” can lead to a difference of up to two orders of magnitude in estimated fire sales losses for individual banks. We believe the model presented here provides a useful tool for monitoring of system-wide and bank-level exposures to price-mediated contagion.

0.3.2 PART II. SENSITIVITY ANALYSIS OF THE EISENBERG AND NOE CLEARING VECTOR

In this part of the thesis, we analyse direct contagion in the Eisenberg & Noe model. The main contribution of this project is to perform a detailed sensitivity analysis of the clearing vector with respect to the relative liability matrix.

To this end, we define directional derivatives of order n of the clearing vector with respect to perturbation matrices that quantify the estimation errors in the relative liability matrix. This allows us to derive an exact Taylor series for the clearing vector. We study two optimisation problems that quantify the deviation of the clearing vector for estimation errors in the relative liabilities matrix, and obtain explicit solutions for both problems. Examples on a small network and European banks conclude the chapter.

0.4 STRUCTURE OF THESIS

The thesis is structured as follows.

Part I. The first part of the thesis develops a model for modelling the phenomenon of fire sales in a network of multiple portfolios and common asset holdings, with the goal of making it an operational and applicable framework for simulating fire sales contagion in next generation regulatory stress tests.

CHAPTER 1 presents our fire sales model and defines key concepts such as the liquidity-weighted overlap between portfolios, introduces the level-dependent market impact function and specifies the dynamics of a fire sales cascade. A simple numeric example with two banks illustrates some of the

key concepts.

CHAPTER 2 studies some properties of the fire sales cascade under different reaction functions of the banks: deleveraging as a response to the breach of one-sided portfolio constraints vs deleveraging from a leverage targeting behaviour. If portfolios need to liquidate assets, we allow for both proportional deleveraging across all marketable assets vs sequential deleveraging of assets, from the most liquid to the least liquid. We also analyse, on a theoretical level, the impact of aggregating different assets into an asset class on the fire sales loss estimates.

CHAPTER 3 applies our framework to a dataset of European banks, and a highly granular supervisory dataset of the Norwegian Central Bank. The main focus is the estimation of both bank level and system level fire sales losses, the quantification of indirect exposures in the European banking network, and illustrating the impossibility to replicate the cross sectional distribution of fire sales losses without a fire sales model. We discuss the calibration of the market impact function and conduct a rigorous sensitivity analysis of our results to this calibration. Moreover, we present empirical evidence of proportional deleveraging, and use the granular Norwegian supervisory dataset to analyse the effect of asset class aggregation on fire sales loss estimates in a realistic setting.

CHAPTER 4 discusses monitoring and macroprudential policy tools. We introduce the Indirect Contagion Index and the Systemic Vulnerability Indicator, and compute these for the European and Norwegian networks respectively. The chapter is concluded by a horse-race of different macroprudential policies with the goal of mitigating contagion from fire sales. We find that the predictive power of the Indirect Contagion Index can be used

to calibrate additional capital charges for systemically important institutions, which turns out to be the most effective macroprudential policy in our comparison.

CHAPTER 5 discusses implications of our findings and offers directions for future research.

Part II. The second part of the thesis is devoted to direct balance sheet contagion in the Eisenberg & Noe framework.

CHAPTER 6 presents a detailed sensitivity analysis of the Eisenberg & Noe clearing vector with respect to the relative liabilities matrix. To this end, we define and compute directional derivatives of the clearing vector, and derive a Taylor series for it. We then solve two optimisation problems that give explicit solutions for estimation errors of the clearing vector due to estimation errors in the relative liabilities matrix.

Part I

FIRE SALES, PRICE-MEDIATED CONTAGION AND SYSTEMIC STRESS TESTING

Dessine-moi un mouton.

Le Petit Prince

1

MODELLING FIRE SALES

This Chapter is based on two working papers that are joint work with Prof. Rama Cont: “Fire sales, price-mediated contagion and systemic risk” and “Fire sales, indirect contagion and systemic stress testing”.

1.1 A MODEL FOR FIRE SALES

1.1.1 FINANCIAL SYSTEM AND SYSTEMIC STRESS TEST DYNAMICS

We consider a stylised model of the financial sector, with multiple financial institutions whose portfolios may contain holdings across multiple asset classes. We refer to these institutions sometimes as ‘banks’. However, as we will explain below, the model is both applicable in a more general context to any constrained portfolio, and it is furthermore relevant to include non-banks in the scope of the model. Each institution holds two types of

assets:

1. **Illiquid assets:** These are portfolio holdings which are not easily marketable in general, and which would be subject to a deep discount if they are to be sold on short notice and/or during a stress scenario. We therefore assume that they are not subject to fire sales. This category includes non-securitised loans, commercial and residential mortgage exposures and retail exposures¹.
2. **Securities:** These are positions in marketable securities which may be liquidated over a short time scale if necessary. Sovereign bonds, corporate bonds and derivatives exposures are included in this category. Each asset class μ in this category is characterised by a market depth parameter D_μ , whose estimation we will describe in detail in Section 3.1.1. The market liquidity of assets within this group can vary strongly and our model accounts for this heterogeneity.

Financial institutions are labelled by $i = 1..N$, security (types) by $\mu = 1..M$ and illiquid asset classes by $\kappa = 1..K$. Securities holdings (in monetary units) of institution i are denoted $(\Pi^{i\mu}, \mu = 1..M)$ and holdings in illiquid assets are denoted $(\Theta^{i\kappa}, \kappa = 1..K)$. The total value of illiquid assets is denoted by $I_i = \sum_{\kappa=1}^K \Theta^{i\kappa}$. Finally, the capital (equity) of financial institution i is denoted C^i . The leverage ratio of the institution is then given by

$$\lambda^i(\Pi, C, I) = \frac{\sum_{\mu=1}^M \Pi^{i\mu} + I_i}{C^i} = \frac{\sum_{\mu=1}^M \Pi^{i\mu} + \sum_{\kappa=1}^K \Theta^{i\kappa}}{C^i} \leq \lambda_{\max}, \quad (1.1)$$

where the upper bound $\lambda_{\max} > 1$ corresponds to a regulatory leverage constraint, as for instance required by Basel III ($\lambda_{Basel} = 33.3$). For simplicity, we consider here that the leverage ratio will be the binding constraint for

¹This category further includes residual assets that do not suffer any direct or indirect losses in the stress scenario, and that are not available for deleveraging, but still contribute to balance sheet size and leverage (e.g. intangible assets).

financial institutions' capital, but the model may be easily adapted to include more than one constraint: for instance, if we introduce regulatory risk weights w_μ for the exposure in each asset class, the constraint on the ratio of capital to risk weighted assets may be expressed as

$$\frac{\sum_{\mu=1}^M w_\mu \Pi^{i\mu} + \sum_{\kappa=1}^K w_\kappa \Theta^{i\kappa}}{C^i} \leq R_{\max},$$

where $R_{\max}$ is the regulatory capital requirement. Other portfolio constraints, such as the liquidity regulation implemented via Basel III's Liquidity Coverage Ratio (LCR) and Net Stable Funding Ratio (NSFR), lead to similar one-sided linear inequalities².

The state of the financial system is summarised by the matrices of holdings Π, Θ and the vector of capital levels C . We will now describe how this system evolves when subject to macroeconomic stress.

1.1.2 STRESS SCENARIOS

We consider stress scenarios described through the percentage loss $\epsilon_\kappa \in [0, 100\%]$ in value for each asset class κ , as in the approach used in regulatory bank stress tests. For simplicity, and to emphasise contagion across asset classes, we consider shocks to illiquid assets only. The model perfectly accommodates stress scenarios that also include heterogeneous shocks to the marketable assets μ . The initial loss of bank i in the stress scenario $\epsilon = (\epsilon_\kappa, \kappa = 1..K)$ is given by

$$\langle \Theta^i, \epsilon \rangle = \sum_{\kappa=1}^K \epsilon_\kappa \Theta^{i\kappa}. \quad (1.2)$$

²When non-bank financial institutions are included in the model, similar constraints that are pertinent to the institution in question can be established, as e.g. self-imposed leverage constraints for hedge funds, or liquidity constraints for funds facing large scale investor redemptions.

This results in a loss in the value of illiquid holdings, reducing it to

$$I_i(\epsilon) := I_i - \langle \Theta^i, \epsilon \rangle = \sum_{\kappa=1}^K \Theta^{i\kappa} (1 - \epsilon_\kappa), \quad (1.3)$$

and a corresponding decrease in equity:

$$C_0^i(\epsilon) = (C^i - \langle \Theta^i, \epsilon \rangle)_+. \quad (1.4)$$

As we assumed that the initial shock is limited to illiquid assets, we have hence $\Pi_0 = \Pi$. The leverage of bank i then increases to

$$\lambda^i(\Pi_0, C_0(\epsilon), I(\epsilon)) = \frac{\sum_{\mu=1}^M \Pi^{i\mu} + I_i(\epsilon)}{C_0^i(\epsilon)} = \frac{\sum_{\mu=1}^M \Pi^{i\mu} + I_i - \langle \Theta^i, \epsilon \rangle}{(C^i - \langle \Theta^i, \epsilon \rangle)_+}. \quad (1.5)$$

We note that this ratio is well-defined: even if the combined effect of deleveraging and fire sales losses described in the next section result in $\sum_{\mu=1}^M \Pi^{i\mu} = 0$, the category of illiquid assets still includes, by assumption, balance sheet items that remain strictly positive for all stress scenarios (e.g. intangible goods), whence $I_i(\epsilon) > 0$.³

If this value exceeds the leverage constraint $\lambda_{\max}$, then the institution needs to deleverage, i.e. sell some of its assets. This occurs if the loss level exceeds the threshold

$$\langle \Theta^i, \epsilon \rangle \geq \frac{C^i (\lambda_{\max} - \lambda^i(\Pi, C, I))}{\lambda_{\max} - 1}, \quad (1.6)$$

which is proportional to the capital buffer of i or its distance from the leverage constraint prior to the shock. The amount of loss an institution can absorb before being led to deleverage is equal to its capital buffer *in excess* of regulatory requirements (Hellwig (2009)).

³In particular, for $C^i(\epsilon) = 0$, the leverage becomes infinite. Moreover, the model remains valid if one allows capital to become negative in (1.4). In this case the leverage becomes negative too, and one needs to treat this case separately. For simplicity, we therefore say that a bank has zero equity, even if the loss that it suffers strictly exceeds its capital.

In the case where the shock is applied to a single asset class κ , institution i needs to deleverage when the (percentage) loss ϵ_κ exceeds the level

$$\epsilon_{i\kappa}^*(\Pi, C, I) = \frac{C^i(\lambda_{\max} - \lambda^i(\Pi, C, I))}{(\lambda_{\max} - 1)\Theta^{i\kappa}}. \quad (1.7)$$

This threshold is clearly different across institutions, depending on their initial leverage, capital buffer, and holdings in the illiquid asset subject to losses. The model hence explicitly accounts for the heterogeneity in banks' individual resiliencies to losses.

Hence, any stress scenario ϵ which falls outside of the (convex) set

$$\mathcal{S}(\Pi, C, \Theta) = \left\{ \epsilon \in [0, 1]^K \mid \forall i = 1..N, \langle \Theta^i, \epsilon \rangle \leq \frac{C^i(\lambda_{\max} - \lambda^i(\Pi, C, I))}{(\lambda_{\max} - 1)} \right\}.$$

will lead to deleveraging by at least one financial institution. The convex set $\mathcal{S}(\Pi, C, \Theta)$ corresponds to a 'safety zone' of stress scenarios which all institutions can withstand; the larger this safety zone, the more resilient the network to losses. We assume that no distressed liquidations are triggered inside this zone. This is different from 'leverage targeting' models such as [Greenwood et al. \(2015\)](#), where portfolios react, and fire sales are triggered, in response to arbitrarily small shocks. The size of the safety zone thus depends the institutions capital buffers in excess of requirements. If the vector ϵ of shocks to assets lies outside the safety zone, then one or more institutions are led to deleverage their portfolios. We now describe how this deleveraging is modelled.

1.1.3 DELEVERAGING

If (and only if) the magnitude of losses in asset values is such that the leverage constraint is breached for institution i , it deleverages a proportion Γ^i of its portfolio in order to restore its leverage ratio to a leverage target

$\lambda_b \leq \lambda_{\max}$. By using the proceeds from the fire sale to pay off some short term debt, the reduction of the balance sheet's leverage is achieved. This leads to the following equation, for $\Gamma^i \in [0, 1]$:

$$\frac{(1 - \Gamma^i) \sum_{\mu=1}^M \Pi^{i\mu} + I_i(\epsilon)}{C_0^i(\epsilon)} = \lambda_b. \quad (1.8)$$

Next, we assume that after the initial shock, the illiquid assets' values do not depreciate further. In that case, $I(\epsilon)$ can be viewed as a static parameter that only depends on the initial illiquid asset holdings, Θ , and the initial shock ϵ .⁴ Given the parameter $I(\epsilon)$, solving (1.8) for Γ^i as a function of the remaining state variables (Π, C) yields

$$\begin{aligned} \Gamma^i(\Pi, C_0(\epsilon)) &= \left(\frac{\sum_{\mu=1}^M \Pi^{i\mu} + I_i(\epsilon) - \lambda_b C_0^i(\epsilon)}{\sum_{\mu=1}^M \Pi^{i\mu}} \wedge 1 \right) \mathbf{1}_{\frac{\sum_{\mu=1}^M \Pi^{i\mu} + I_i(\epsilon)}{C^i(\epsilon)} > \lambda_{\max}} \\ &= \left(\frac{C_0^i(\epsilon)(\lambda^i(\Pi, C_0(\epsilon), I(\epsilon)) - \lambda_b)}{\sum_{\mu=1}^M \Pi^{i\mu}} \wedge 1 \right) \mathbf{1}_{\lambda^i(\Pi, C_0(\epsilon), I(\epsilon)) > \lambda_{\max}}. \end{aligned} \quad (1.9)$$

For the first round of deleveraging, this can be easily written explicitly in terms of the parameters Θ and ϵ (and the *initial* capital C rather than $C_0^i(\epsilon)$):

$$\Gamma^i(\Pi, C) = \left(\frac{(\lambda_b - 1) \langle \Theta^i, \epsilon \rangle + C^i(\lambda^i(\Pi, C, I) - \lambda_b)}{\sum_{\mu=1}^M \Pi^{i\mu}} \wedge 1 \right) \mathbf{1}_{\lambda^i(\Pi, C_0(\epsilon), I(\epsilon)) > \lambda_{\max}}, \quad (1.10)$$

where we recall that $C_0^i(\epsilon) = (C^i - \langle \Theta^i, \epsilon \rangle)_+$ and $I_i(\epsilon) = I - \langle \Theta^i, \epsilon \rangle$ in $\mathbf{1}_{\lambda^i(\Pi, C_0(\epsilon), I(\epsilon)) > \lambda_{\max}}$. Even though ϵ is formally not a variable of the state space of our model, we will sometimes write by abuse of notation $\Gamma^i(\Pi, C(\epsilon), I(\epsilon))$ in order to highlight the dependence of Γ on ϵ .

⁴It is of course possible to let the value of the illiquid assets evolve as well. We chose to exclude this generalisation for the sake of clarity of exposition, and to keep the model more tractable.

For completeness, we note that in the leverage targeting model, one obtains

$$\Gamma_{L.T.}^i(\Pi, C_0(\epsilon)) = \left(\frac{C_0^i(\epsilon)(\lambda^i(\Pi, C_0(\epsilon), I(\epsilon)) - \lambda_{L.T.}^i)}{\sum_{\mu=1}^M \Pi^{i\mu}} \right) \wedge 1,$$

and in terms of the initial capital C

$$\Gamma_{L.T.}^i(\Pi, C) = \left(\frac{(\lambda_{L.T.}^i - 1) < \Theta^i, \epsilon >}{\sum_{\mu=1}^M \Pi^{i\mu}} \right) \wedge 1,$$

where the $\lambda_{L.T.}^i$ is bank i 's leverage target. We postpone a more thorough comparison of both models to Section 2.3, and study the implications of using one or the other type of response function for stress testing purposes in Section 3.3.

As in previous studies on fire sales [Greenwood et al. \(2015\)](#), [Duarte and Eisenbach \(2013\)](#) we assume that banks delever their marketable assets proportionally. An alternative would be to assume that banks optimise the order in which they liquidate assets as in [Braouezec and Wagalath \(2017\)](#), which we extend to the N institution and M assets case in Section 2.3. The proportional deleveraging assumption is supported by empirical studies on asset sales of large financial institutions [Getmansky et al. \(2016\)](#). Using a highly granular supervisory dataset from Norges Bank, the Norwegian Central Bank, we also find empirical support for this assumption, which is presented in Section 3.4.

In addition to the critical shock size ϵ_i^* in (1.7), we can define two further critical shocks: First, $\epsilon_{i\kappa}^{illiq} > \epsilon_{i\kappa}^*$ is the smallest shock to asset class κ which will lead to a *full* deleveraging of the portfolio in the first round, i.e. $\Gamma(\Pi_0, C_0(\epsilon_{i\kappa}^{illiq})) = 1$. This yields

$$\epsilon_{i\kappa}^{illiq} = \frac{\lambda_b C^i - I_i}{\Theta^{i\kappa}(\lambda_b - 1)}. \quad (1.11)$$

Second, $\epsilon_{i\kappa}^{insol}$, the initial shock that leads to an immediate insolvency in the first round, is given by

$$\epsilon_{i\kappa}^{insol} = \frac{C^i}{\Theta^{i\kappa}}.$$

We emphasise that these quantities are only reflective of the direct loss. As we shall see, in general the shock size at which an institution effectively becomes insolvent or illiquid may be smaller due to additional fire sales losses in higher rounds of the deleveraging cascade.

Figure 1.1.1 (circles) displays the dependence of the deleveraging ratio (1.10) on the shock level ϵ for a portfolio holding a single class of illiquid asset: this ratio is zero for shocks lower than the threshold ϵ_i^* corresponding to the breach of the leverage constraint, then increases linearly afterwards. There is a discontinuity at the onset of deleveraging, which depends on the size of the capital buffer which the bank intends to rebuild by deleveraging. In the case where the institution delevers so as to exactly return to compliance, i.e. $\lambda_b = \lambda_{\max}$ the dependence on the shock size is continuous and convex (solid line). In the example shown in Figure 1.1.1 we have assumed $\lambda_b = 0.95\lambda_{\max}$ which corresponds to a safety buffer 5% below the leverage constraint. Previous studies on fire sales have assumed instead a *linear* dependence of the volume of deleveraging with respect to the shock size (dashed line): this is the leverage targeting model, which we will discuss further in Sections 2.3 and 3.3.2.

Unlike the leverage targeting model, our model gives rise to deleveraging only if the shock level exceeds a (bank-specific) threshold ϵ_i^* . Once this threshold is reached, the volume of deleveraging increases linearly with $(\epsilon - \epsilon_i^*)_+$ until no more marketable assets are available for sale (i.e. $\Gamma^i = 1$). At this point, although the institution is still solvent⁵ ($C^i > 0$), it may become

⁵For simplicity, we consider default by insolvency to occur at $C = 0$. Alternatively, one could set this threshold at $C^i < C_{\min}$ with $C_{\min} > 0$ being some reasonable “minimum required survival capital”.

illiquid ($\sum_{\mu=1}^M \Pi^{i\mu} = 0$). The model thus leads to a natural distinction between failures due to insolvency, which may occur if the initial loss in asset values is large enough, and failures due to illiquidity, which may occur further down the road if the liquidation of marketable assets fails to raise enough liquidity.

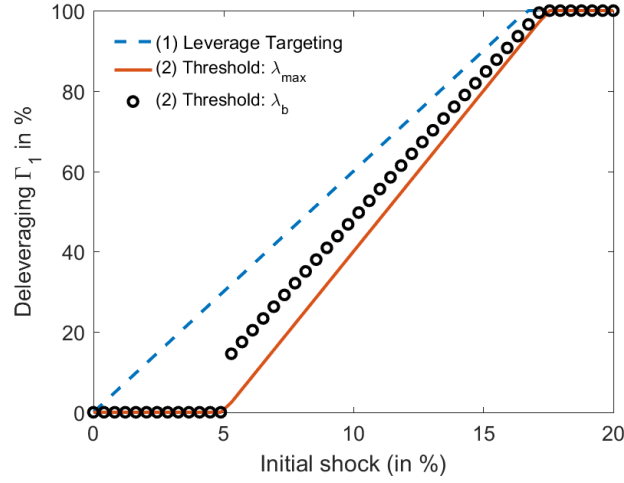


Figure 1.1.1: Volume of asset sales (% of marketable assets) as a function of the percentage loss in value of illiquid assets for a portfolio with 20% in illiquid assets and leverage of 25, and the Basel III leverage constraint of 33.3. Leverage targeting leads to a linear response (dashed line) with fire sales for arbitrarily small shock sizes. In contrast, our assumption of deleveraging leads to no asset sales for shocks smaller than a threshold and a linear increase above the threshold (solid line) until the entire portfolio is delevered. Finally, when the bank deleverages to restore a non-zero capital buffer ($\lambda_b < \lambda_{\max}$) we obtain the discontinuous response function (circles).

Summing across all institutions $j = 1..N$ in the system yields the total volume of asset sales (in monetary units) in the stress scenario ϵ :

$$q^\mu(\epsilon, \Pi, C) = \sum_{j=1}^N \Gamma^j(\Pi_0, C_0(\epsilon)) \Pi_0^{j\mu}$$

for the asset class μ . In the case $\lambda_b = \lambda_{\max}$, the functions $\Gamma^j(\Pi, C_0(\epsilon))$ are convex with respect to ϵ over a large range of values i.e. in the region $\max_i \Gamma^i \leq 1$. In this range, the aggregate volume of asset sales $q^\mu(\epsilon, \Pi, C)$ exhibits a convex dependence in ϵ , which leads to a ‘multiplier effect’: as

more severe stress scenarios (larger ϵ) are considered, the marginal response in terms of deleveraging also increases. An example, based on data from the EU banking system (see next section) is shown in Figure 1.1.2.

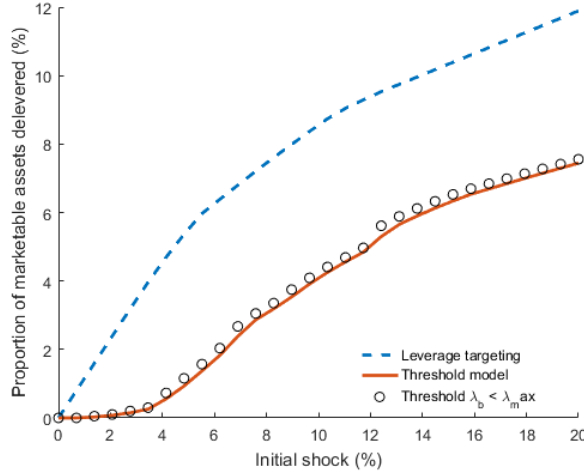


Figure 1.1.2: Volume of asset sales (% of marketable assets) across EU banks in reaction to a scenario in which the Spanish housing sector depreciates by ϵ (horizontal axis). The Basel III leverage constraint $\lambda_{\max} = 33.3$ is used in this example. The solid red line corresponds to $\lambda_b = \lambda_{\max}$, the circles to $\lambda_b = 0.95\lambda_{\max}$ and the dashed blue lines to the leverage targeting model.

1.1.4 PRICE IMPACT IN FIRE SALES MODELS

If the volume of deleveraging represents a sizeable fraction of the market depth of the assets, it may have a non-negligible impact on their market price. We now describe exactly this evolution of the asset price as a function of the liquidation size. The *market impact function* $\Psi_\mu : \mathbb{R} \rightarrow [-\infty, 1)$, specifies the correspondence between the size of the liquidation⁶ q_k^μ and the relative price change of the asset between rounds $k - 1$ and k

$$\frac{S_k^\mu - S_{k-1}^\mu}{S_k^\mu} = -\Psi_\mu(q_k^\mu).$$

⁶Deleveraging a portfolio by closing short positions ($q_k^\mu < 0$) pushes the price up. By excluding short positions from the model, one can restrict the market impact function to $\Psi_\mu : [0, +\infty) \rightarrow [0, 1]$

An adequate choice for Ψ_μ is continuous, increasing, concave, satisfies $\Psi(0) = 0$ and leads to non-negative prices. Ψ_μ may be thought of as an inverse demand function which we assume to be independent of the iteration step k . Assuming Ψ_μ is a smooth function, linearising for small volumes yields

$$\Psi_\mu(q) \simeq \frac{q}{D_\mu} \quad \text{where} \quad D_\mu = \frac{1}{\Psi'_\mu(0)}$$

is a measure of *market depth*⁷ for asset class μ . Naturally, this quantity depends on the asset μ , as the same dollar liquidation volume q will have a different price impact depending on the asset class. A simple one-parameter specification often used in practice is $\Psi_\mu(q) = \psi(q/D_\mu)$, where $\psi(\cdot)$ is an increasing function with $\psi'(0) = 1$. Classical examples of the price-impact literature are the linear and square root impact models. The linear model (Kyle (1985), Bertsimas and Lo (1998) Almgren and Chriss (2000), Obizhaeva (2012), Cont and Wagalath (2013)) is usually specified as

$$\Psi_\mu(q) = \frac{q}{D_\mu} \quad \text{where} \quad D_\mu = c \frac{ADV_\mu}{\sigma_\mu}, \quad (1.12)$$

where ADV_μ is the average daily volume in μ , σ_μ the daily volatility and c a coefficient that is estimated from transactions data and usually close to 1.⁸ The square-root model (Bouchaud (2010)) is usually specified as

$$\Psi_\mu(q) = c\sigma_\mu \sqrt{\frac{q}{ADV_\mu}}. \quad (1.13)$$

⁷In markets with a central limit order book, the market depth normally refers to the depth of the order book. Our definition of the market depth parameter is related to this notion. The larger our market depth parameter, the smaller the price impact for a given liquidation size. Similarly, if the limit order book has a large market depth, the order will be filled by resting limit orders close to the spread, resulting in a smaller price impact than for a thin order book.

⁸Interpreting the price impact relative to ADV in related studies (Obizhaeva (2012), Cont and Wagalath (2013), Ellul et al. (2011), Coval and Stafford (2007)) results in estimates of c that vary between 0.05 and about 5. The different datasets and episodes make a precise comparison difficult however. In essence, this uncertainty highlights the importance of a rigorous sensitivity analysis of the market depth.

Both the trading volume and the volatility are associated with a liquidation horizon τ during which the fire sale is assumed to take place. In most empirical studies on price impact, this horizon is taken to be daily by default, $\tau = 1$. If we assume the liquidation horizon τ to be longer than a day – arguably a plausible assumption for deleveraging by large bank or asset manager portfolios – then the market depth parameter needs to be adjusted accordingly. In the linear impact model, the adjustment is:

$$D_\mu(\tau) = c \frac{ADV_\mu \tau}{\sigma_\mu \sqrt{\tau}} = c \frac{ADV_\mu}{\sigma_\mu} \times \sqrt{\tau}. \quad (1.14)$$

This adjustment is important, and corresponds to the intuitive observation that liquidating the same portfolio over a longer horizon reduces impact. The liquidation horizon τ may be interpreted as the time window the banks dispose of to comply with portfolio constraints. In the case study below we will use $\tau = 20$ days.

By contrast, in the square root model, the impact of a transaction is invariant to a change in the liquidation horizon since the denominator and the numerator in (1.13) scale in the same way. This leads to the counter-intuitive (and, we believe, incorrect) conclusion that the price impact does not depend on the rate of liquidation. [Kyle and Obizhaeva \(2016b\)](#), [Kyle and Obizhaeva \(2016a\)](#) develop an elegant model to quantify the market impact of trades as a function of their size, and define in their model an asset specific illiquidity ℓ_μ as

$$\frac{1}{\ell_\mu} = \left(\frac{C \sigma_\mu^2}{0.25 ADV_\mu} \right)^{1/3}, \quad (1.15)$$

where $C = 2000$ USD (as the model is calibrated to US data). Unfortunately, it is easy to see that this quantity is invariant to scaling of τ as well. For this reason, we refrain from using these models in the sequel.

In a linear impact model, asset classes are differentiated according to their market depth D_μ . Greenwood et al. (2015) assume a uniform depth $D_\mu = 10^{13}$ (EUR) for all asset classes. This homogeneity assumption is not supported by empirical studies on market impact Bouchaud (2010), Obizhaeva (2012), Cont et al. (2014) which indicate that market impact varies widely across assets. Ignoring this heterogeneity may lead to biased results, overestimating losses in more liquid asset classes while underestimating losses in less liquid asset classes. In Section 3.1.1, we estimate the market depth both for individual Norwegian securities, as well as for larger European asset classes, and find a large degree of heterogeneity in the market depth of assets, ranging over several orders of magnitude from the most liquid to the least liquid.

EXTRAPOLATION TO LARGE VOLUMES. When applied to large transaction volumes, linear or square-root impact models may lead to negative prices. In Greenwood et al. (2015) this is addressed by capping the loss at 100%: $\Psi_\mu(q) = \min\left\{1, \frac{q}{D_\mu}\right\}$. Alternatively, an exponential specification, as used in Cifuentes et al. (2005)

$$\Psi_\mu(q) = 1 - \exp\left(-\frac{q}{D_\mu}\right) \quad (1.16)$$

also ensures that prices remain non-negative. Nevertheless, prices can get arbitrarily close to zero in both of these models. However, it is realistic to assume that long before the price level reaches zero, arbitrageurs will step in to purchase assets subject to fire sales at a discount. To capture this effect we introduce a price floor $B_\mu > 0$ and consider a two-parameter level-dependent price impact function, $\Psi_\mu : \mathbb{R} \times [0, +\infty) \rightarrow (-\infty, 1]$

$$\Psi_\mu(q, S) := \left(1 - \frac{B_\mu}{S}\right) \left(1 - \exp\left(-\frac{q}{\delta_\mu}\right)\right), \quad (1.17)$$

which ensures that the price stays above the level B_μ , whose calibration we also discuss in Section 3.1.1. Choosing

$$\delta_\mu = \left(1 - \frac{B_\mu}{S_0^\mu}\right) D_\mu \quad (1.18)$$

makes the specification (1.17) compatible with the linear specification (1.12) for small volumes in the *first* round of fire sales. From the definition of Ψ_μ , we immediately have

Proposition 1.1.1. *The market impact function in (1.17) has the properties for all time steps $k \geq 1$*

1. *For any $S_{k-1}^\mu \geq B_\mu$, Ψ_μ is continuous, increasing and concave in q_k^μ and fulfils $\Psi_\mu(0, S_k^\mu) = 0$.*
2. *$B_\mu \leq S_{k-1}^\mu$ acts as a lower bound on the price: for all q_k^μ , $\Psi_\mu(q_k^\mu, B_\mu) = 0$, and for all $S_k^\mu > B_\mu$: $\lim_{q_k^\mu \rightarrow +\infty} \Psi_\mu(q_k^\mu, S_{k-1}^\mu) = (1 - \frac{B_\mu}{S_\mu^\mu})$, whence $S_k = B_\mu$.*

As soon as we enter the fire sales cascade and the second round of deleveraging occurs, we are arguably no longer in the market conditions for which the market depth D_μ has been estimated, and for which both models should coincide for small volumes in order to be consistent. Hence, after the initial scaling applied in (1.18), we do *not* update δ_μ during the cascade by the iteration $\delta_{k+1}^\mu = \left(1 - \frac{B_\mu}{S_k^\mu}\right) \delta_k^\mu$. The discussion of the empirical calibration of δ_μ is also postponed to Section 3.1.1. Figure 1.1.3 shows the price

$$S_{k+1}^\mu = S_k^\mu \left(1 - \left(1 - \frac{B_\mu}{S_k^\mu}\right) \left(1 - \exp\left(-\frac{q_{k+1}^\mu}{\delta_\mu}\right)\right)\right)$$

for $k = 0$ as a function of the liquidated volume (in percent of ADV), and compares the price dynamics under our level-dependent market impact function to its evolution under the linear and exponential specifications. We

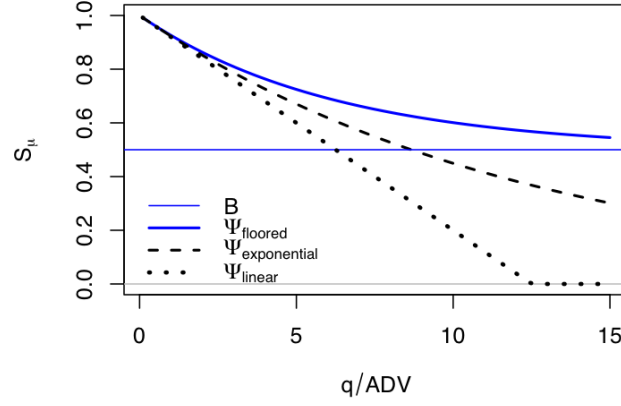


Figure 1.1.3: The relative change of the asset price as a function of the liquidation volume relative to market depth for three market impact functions. Prices can become negative in the linear model in absence of a cut-off (dotted), prices remain positive in the exponential model (dashed), prices level off at the floor in our proposed level-dependent specification (solid blue).

use the parameters $ADV = 100$, $\sigma = 0.02$, $c = 1$, $\tau = 1$, $B = 0.3$, $S_0 = 1$ and $q = 1..1500$ to generate the plot.

Overall, we have introduced a novel two-parameter market impact function, where the market depth δ_μ captures *how fast* the price of an asset (class) will deviate from its initial value at the onset of a fire sale, and the price floor B_μ quantifies *how far* it can fall at most. We believe that this function captures important characteristics of the price impact during large-volume forced liquidations that are not captured by previous impact functions in the literature. This increased realism, we believe, makes it particularly suitable for applications in systemic stress testing.

Finally, we note that there is a large literature on “(il)liquidity” in finance and economics, which for the sake of brevity, we chose not to review here. The reason for this choice is that we are concerned with a very precise aspect of liquidity, namely by how much the price of an asset changes given a liquidation of size q in this asset. Well-known measures such as [Amihud \(2002\)](#), or the corporate bond illiquidity measure of [Bao et al. \(2011\)](#) were not developed with this goal in mind and are therefore not appropriate for

our purposes.

1.1.5 FIRE SALES LOSSES AND PRICE-MEDIATED CONTAGION

We can now describe the processes which occur at the k -th round of deleveraging⁹. Denote by $\Pi_{k-1}, C_{k-1}, S_{k-1}$ respectively the holdings in marketable assets, the equity and the (vector of) asset prices after $k-1$ rounds of deleveraging. At round k :

1. Each institution j deleverages by selling a proportion $\Gamma^j(\Pi_{k-1}, C_{k-1})$ of its marketable assets, leading to an aggregate amount $q_k^\mu = \sum_{j=1}^N \Gamma^j(\Pi_{k-1}, C_{k-1}) \Pi_{k-1}^{j\mu}$ of sales in asset class μ .
2. The market impact of asset sales results in a decline in market prices, moving the market price to

$$S_k^\mu = S_{k-1}^\mu (1 - \Psi_\mu(q_k^\mu, S_{k-1}^\mu)). \quad (1.19)$$

For the sake of notational convenience, we will omit the second variable, S_k^μ , of Ψ_μ in the sequel and write $\Psi_\mu(q)$ as in the one variable case.

3. This decline in price changes the market value of holdings in asset class μ to

$$\begin{aligned} \Pi_k^{i\mu} &:= \Pi^{i\mu}(\Pi_{k-1}, C_{k-1}) \\ &= \underbrace{(1 - \Gamma^i(\Pi_{k-1}, C_{k-1}))}_{\text{Remainder after deleveraging by } i} \underbrace{\overbrace{\Pi_{k-1}^{i\mu}}^{\text{Previous value}} \left(1 - \Psi_\mu \left(\sum_{j=1}^N \Gamma^j(\Pi_{k-1}, C_{k-1}) \Pi_{k-1}^{j\mu} \right) \right)}_{\text{Price impact on remaining holdings}}. \end{aligned} \quad (1.20)$$

⁹We think the clarity of the exposition benefits from having specified concrete choices for Γ and Ψ_μ . However, the dynamics presented hereafter are equally applicable for different choices of Γ and Ψ_μ .

This generates two types of losses for portfolio i . The market impact of fire sales moves the price and causes mark-to-market losses on the remaining part of the portfolio given by

$$\begin{aligned} M^i(\Pi_{k-1}, C_{k-1}) &= \sum_{\mu=1}^M \left((1 - \Gamma^i(\Pi_{k-1}, C_{k-1})) \Pi_{k-1}^{i\mu} - \Pi_k^{i\mu} \right) \\ &= (1 - \Gamma^i(\Pi_{k-1}, C_{k-1})) \sum_{\mu=1}^M \Pi_{k-1}^{i\mu} \Psi_\mu \left(\sum_{j=1}^N \Gamma^j(\Pi_{k-1}, C_{k-1}) \Pi_{k-1}^{j\mu} \right). \end{aligned} \quad (1.21)$$

A second source of loss, not accounted for in previous studies, stems from the fact that assets are not liquidated at the current market price but at a discount: this ‘implementation shortfall’, as it is called in the literature on optimal trade execution ([Almgren and Chriss \(2000\)](#)) corresponds to the difference between the market price at the time of sale and the volume-weighted average price (VWAP) during liquidation. This VWAP lies somewhere between the pre- and post-fire-sales prices. We model it as a weighted average with weights $(1 - \alpha, \alpha)$, $\alpha \in [0, 1]$ of the pre-fire-sales price, S_{k-1} and post-fire-sales price, $S_k = S_{k-1}(1 - \Psi_\mu(q_k^\mu))$, where $\alpha = 0$ corresponds to zero implementation shortfall, and $\alpha = 1$ corresponds to full price slippage. This leads to the general formula for the implementation shortfall:

$$\begin{aligned} R^i(\Pi_{k-1}, C_{k-1}) &= \sum_{\mu=1}^M \left[\Gamma_k^i \Pi_{k-1}^{i\mu} - \left((1 - \alpha) \Gamma_k^i \Pi_{k-1}^{i\mu} + \alpha \Gamma_k^i \Pi_{k-1}^{i\mu} (1 - \Psi_\mu(q_k^\mu)) \right) \right] \\ &= \alpha \Gamma^i(\Pi_{k-1}, C_{k-1}) \sum_{\mu=1}^M \Pi_{k-1}^{i\mu} \Psi_\mu \left(\sum_{j=1}^N \Gamma^j(\Pi_{k-1}, C_{k-1}) \Pi_{k-1}^{j\mu} \right), \end{aligned} \quad (1.22)$$

where Γ_k^i is shorthand for $\Gamma^i(\Pi_{k-1}, C_{k-1})$. In the empirical examples below, we will use $\alpha = \frac{1}{2}$, which corresponds to a VWAP midway between the pre- and post-fire-sales prices. In Section 2.1, we argue that $\alpha \geq \frac{1}{2}$ is also a plausible assumption from a modelling perspective.

Summing (1.22) with (1.21) yields the total loss of portfolio i at the k -th

round of deleveraging:

$$\begin{aligned} L_k^i &:= L^i(\Pi_{k-1}, C_{k-1}) = M^i(\Pi_{k-1}, C_{k-1}) + R^i(\Pi_{k-1}, C_{k-1}) \\ &= (1 - (1 - \alpha)\Gamma^i(\Pi_{k-1}, C_{k-1})) \sum_{\mu=1}^M \Pi_{k-1}^{i\mu} \Psi_\mu \left(\sum_{j=1}^N \Gamma^j(\Pi_{k-1}, C_{k-1}) \Pi_{k-1}^{j\mu} \right). \end{aligned} \quad (1.23)$$

This loss reduces the equity of institution i by the same amount:

$$C_k^i = (C_{k-1}^i - L_k^i)_+.$$

Linearizing the market impact function Ψ_μ yields

$$L_k^i \approx (1 - (1 - \alpha)\Gamma_k^i) \underbrace{\sum_{j=1}^N \sum_{\mu=1}^M \frac{\Pi_{k-1}^{i\mu} \Pi_{k-1}^{j\mu}}{\delta_\mu} \Gamma_k^j}_{\Omega_{ij}(\Pi_{k-1})} = (1 - (1 - \alpha)\Gamma_k^i) \sum_{j=1}^N \Omega_{k-1}^{ij} \Gamma_k^j, \quad (1.24)$$

which shows that the magnitude of fire sales spillovers from institution i to institution j is proportional to the *liquidity-weighted overlap* Ω_{ij} between portfolios i and j [Cont and Wagalath \(2013\)](#):

$$\Omega_{ij}(\Pi) := \sum_{\mu=1}^M \frac{\Pi^{i\mu} \Pi^{j\mu}}{\delta_\mu}. \quad (1.25)$$

The matrix of portfolio overlaps

$$\Omega(\Pi) = \Pi D^{-1} \Pi^\top, \quad (1.26)$$

where D is the diagonal matrix of market depths D_μ , can be viewed as a weighted adjacency matrix of the underlying network linking portfolios through their common holdings. We further analyse the properties of this matrix in Section 4.1.1.

In summary, an initial loss in asset values may trigger a feedback loop in

which, at each iteration, deleveraging of portfolios leads to fire sales, causing prices to declines and generating mark-to-market losses which may in turn trigger further fire sales. Formally, the dynamics of such a “systemic stress test” can then be viewed as the composition of two functions: a first map that applies the macro shock scenario to bank portfolios, and a second map that takes the post-macro shock system state as input and produces the fire sales effects through a fixed point iteration. The first map is

$$f_0 : \mathbb{R}^{N \times M} \times \mathbb{R}^N \times \mathbb{R}^{N \times K} \times \mathbb{R}^K \rightarrow \mathbb{R}^{N \times M} \times \mathbb{R}^N;$$

$$(\Pi, C, \Theta, \epsilon) \xrightarrow{(a)} (\Pi_0, C_0(\epsilon), I(\epsilon)) \xrightarrow{(b)} (\Pi_0, C_0),$$

where f_0 , applies the macro shock scenario to the system in (a) and projects the resulting system state into the state space of the fire sales iteration in (b) by dropping variables that remain constant for the rest of the cascade, and solely incorporating them as parameters. For a given shock ϵ , in (a), the illiquid assets are initialised according to (1.3), and the state variables representing the matrix Π of portfolio holdings in marketable assets and the equity levels $C = (C_i, i = 1..N)$ are initialised by $\Pi_0 = \Pi$ and equation (1.4) respectively. For $k \geq 1$, at each iteration, the state variables are then updated according to the rule

$$f : \mathbb{R}^{N \times M} \times \mathbb{R}^N \rightarrow \mathbb{R}^{N \times M} \times \mathbb{R}^N; \tag{1.27}$$

$$(\Pi_{k-1}, C_{k-1}) \mapsto f(\Pi_{k-1}, C_{k-1}) = (\Pi_k, C_k)$$

where f is the map defined by

$$\begin{aligned}
\Pi_k^{i\mu} &= \Pi^{i\mu}(\Pi_{k-1}, C_{k-1}) \\
&= (1 - \Gamma^i(\Pi_{k-1}, C_{k-1})) \Pi_{k-1}^{i\mu} \left(1 - \Psi_\mu \left(\sum_{j=1}^N \Gamma^j(\Pi_{k-1}, C_{k-1}) \Pi_{k-1}^{j\mu} \right) \right) \\
C_k^i &= C_{k-1}^i - (1 - (1 - \alpha) \Gamma^i(\Pi_{k-1}, C_{k-1})) \sum_{\mu=1}^M \Pi_{k-1}^{i\mu} \Psi_\mu \left(\sum_{j=1}^N \Gamma^j(\Pi_{k-1}, C_{k-1}) \Pi_{k-1}^{j\mu} \right) .
\end{aligned}$$

with

$$\Gamma^i(\Pi, C) = \left(\frac{\sum_{\mu=1}^M \Pi^{i\mu} + I_i(\epsilon) - \lambda_b C^i}{\sum_{\mu=1}^M \Pi^{i\mu}} \wedge 1 \right) \mathbb{1}_{\frac{\sum_{\mu=1}^M \Pi^{i\mu} + I_i(\epsilon)}{C^i} > \lambda_{\max}} .$$

The fire sales map has thus only two state variables Π and C and a number of parameters: $I(\epsilon)$, α , λ_b , $\lambda_{\max}$ and Ψ_μ for $\mu = 1..M$.

1.1.6 FEEDBACK LOOPS, INSOLVENCY AND ILLIQUIDITY

The iteration described above continues in principle as long as at least one institution is in breach of its leverage/ capital constraint after losses due to deleveraging are accounted for. However, in the (realistic) situation where we assume that institutions build a non-zero buffer beyond the minimal capital requirements (i.e. $\lambda_b < \lambda_{\max}$ in the notation of Section 1.1.3), this *fire sales cascade* always terminates after a finite number T of iterations, as we will prove in the next section. This is not the case in leverage targeting models, which lead to infinite fire sales cascades (unless all banks become illiquid/insolvent during the cascade). During such a fire sales cascade, some institutions may become insolvent: this occurs if at any point in the iterations the loss L_k^i exceeds the capital C_{k-1}^i . If this happens institution i does not play any further role in subsequent rounds. Another type of failure

which may occur along the cascade is failure due to *illiquidity*: this occurs when an institution has sold all of its marketable assets and is left with no further liquid assets. This may occur even though the institution is still solvent¹⁰.

This distinction between failure due to insolvency and failure due to illiquidity is highly relevant in practice. In fact, one can note that this was precisely the scenario that occurred in the failure of Bear Stearns and Lehman Brothers.¹¹ In contrast to most default risk models and previous studies on fire sales, our model distinguishes between these two causes of failure and highlights the fact that institutions can fail even when they have positive equity. These concepts are illustrated in Figure 1.1.4.

Finally, differently from many models of default contagion (Caccioli et al. (2015), Awiszus and Weber (2015)), contagion of losses across institutions occurs not just at default but actually *before* the default of an institution, and its scope is not limited to counterparties. Deleveraging by distressed institutions, which is precisely aimed at *preventing* their default, is in fact what triggers this contagion.

We can now quantify the fire sales losses that arise during such a cascade. Denoting by T the length of the cascade, the fire sales loss for bank i is given by

$$FLoss(i, \epsilon) = \sum_{k=1}^T L^i(\Pi_{k-1}, C_{k-1}). \quad (1.28)$$

and the total system-wide fire sales loss is

$$SFLoss(\epsilon) = \sum_{i=1}^N FLoss(i, \epsilon). \quad (1.29)$$

¹⁰These points are still valid if the solvency criterion is set to $C^i \geq C_{\min}$, for some $C_{\min} > 0$.

¹¹See the letter by the then SEC chairman Christopher Cox to the Basel Committee on Banking Supervision <https://www.sec.gov/news/press/2008/2008-48.htm>.

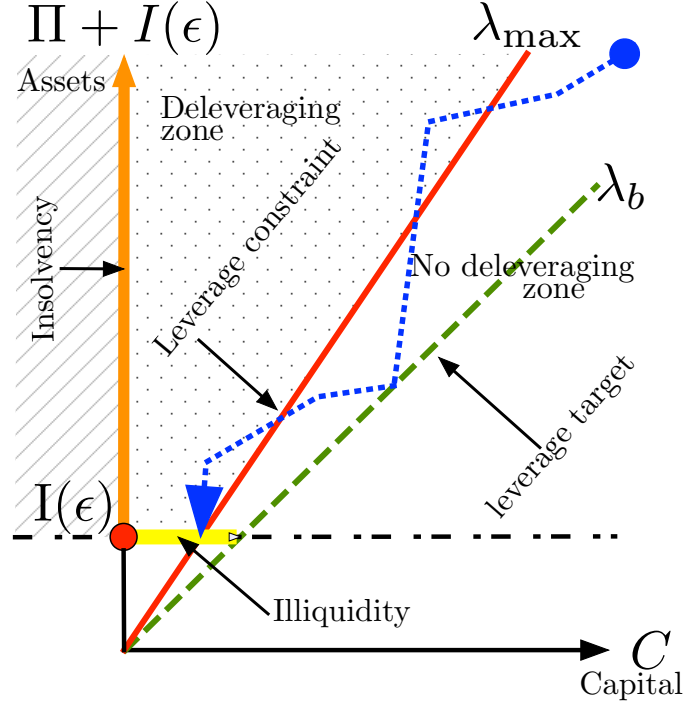


Figure 1.1.4: Evolution of asset values (vertical axis) and capital (horizontal axis) in a fire sales cascade. The solid red line corresponds to the leverage constraint, the dashed green line corresponds to a target leverage corresponding to an excess capital buffer, and the dotted blue line shows the path of a sample portfolio. Losses in asset values depreciate the equity, moving the portfolio closer to the origin; when the portfolio crosses the red line corresponding to the leverage constraint, deleveraging occurs. The institution tries to reconstitute a buffer by returning to the target leverage (green dashed line). The market impact of these asset sales lead to further losses and displace the state to the left; if it crosses the red line again, a new round of deleveraging follows etc. An institution becomes insolvent when it reaches the boundary $C = 0$ (vertical axis) and illiquid when it reaches the boundary $\Pi = 0$ (horizontal dashed line). If the institution is not yet illiquid when it becomes insolvent, the equations for Γ imply a final round of full deleveraging, sending the portfolio to the red dot $(C, \Pi + I(\epsilon)) = (0, I(\epsilon))$.

Note that the fire sales loss (1.28) does not include the initial loss which triggers the deleveraging: in absence of deleveraging and price-mediated contagion $Floss(i, \epsilon) = SLoss(\epsilon) = 0$.

The total loss for an individual institution and for the system are finally given by

$$Loss(i, \epsilon) = \langle \Theta^i, \epsilon \rangle + FLoss(i, \epsilon) \quad (1.30)$$

and

$$S\text{Loss}(\epsilon) = \sum_{i=1}^N \text{Loss}(i, \epsilon) \quad (1.31)$$

respectively. Table 1.1.1 summarises the notations of the model and references the equations where they are defined.

FIRE SALES LOSS AS A FUNCTION OF THE INITIAL SHOCK AND THE MARKET DEPTH. We remark that all values $\Pi_k^{i\mu}, C_k^i$ are well defined for any ϵ for each iteration k . Consequently, so are $F\text{Loss}(i, \epsilon), S\text{FLoss}(\epsilon), \text{Loss}(i, \epsilon)$ and $\text{Loss}(i, \epsilon)$ too. Indeed, this is a trivial consequence of monotone convergence, as each variable is either monotonically increasing (losses), or decreasing (portfolio value and capitals) and bounded from above (total system asset value) or below (zero) respectively.

What is more, this allows us to view $F\text{Loss}$ and $S\text{FLoss}$ as functions of the shock size ϵ and the market depth vector δ . In fact, this is precisely how we will conduct the systemic stress test in Chapter 3 as well as illustrate some concepts in the two bank setting in the next section: The portfolio holdings Π, Θ and capitals C will be parameters, calibrated directly from the data, and we will study fire sales losses for a variety of different shock scenarios and intensities ϵ , as well as conduct a sensitivity analysis by increasing and reducing the estimated market depths $\delta_\mu, \mu = 1..M$.

1.1.7 INDIRECT EXPOSURES

The starting point in portfolio risk analysis is the notion of exposure to an asset class, usually quantified by the notional volume of holdings, in monetary units, in that asset class. In absence of contagion, losses in a stress scenario for this asset class will be a linear function of the percentage shock to the asset value, the proportionality coefficient being this notional exposure.

Variable	Notation	Defined in
Financial institutions	$i, j = 1..N$	-
Asset class: illiquid assets	$\kappa = 1..K$	Section 2.1
Asset class: marketable assets	$\mu = 1..M$	Section 2.1
Number of iterations (rounds)	k	-
<i>State variables</i> (in EUR)		
Marketable assets	$\Pi^{i\mu}$	Section 2.1
Capital	C^i	Section 2.1
<i>Parameters</i>		
Illiquid asset (in EUR)	$\Theta^{i\kappa}$	Section 2.1
Initial shock (in %)	ϵ_κ	(1.5)
Market depth for asset class μ	D_μ and δ_μ	(1.20)
Price-slippage factor	α	(1.22)
<i>Key quantities</i>		
Deleveraging proportion at round k	Γ_k^i	(1.9)
Leverage of institution i	λ^i	(1.1)
Fire sales loss (k -th round)	$L^i(\Pi_{k-1}, C_{k-1})$	(1.23)
Fire sales loss for bank i (all rounds)	$FLoss(i, \epsilon)$	(1.28)
System-wide fire sales loss	$SFLoss(\epsilon)$	(1.29)
Total loss for bank i	$Loss(i, \epsilon)$	(1.30)
System-wide total loss	$SLoss(\epsilon)$	(1.31)

Table 1.1.1: Overview of model notations.

However, the endogenous risk arising from fire sales may amplify this initial loss in a non-linear way, leading to a total loss that is higher than the one implied by the notional exposure, leading to an *effective exposure* higher than the notional one. Furthermore, spillover effects from fire sales may further increase this loss through indirect contagion. The end result is that the (marginal) ratio of a portfolio's loss in an asset class to shocks affecting this asset class may be *higher* than its notional exposure. We capture this effect by defining the notion of *indirect exposure* and quantifying it using our model.

Denoting as above by $\Theta^{i\kappa}$ the holdings of institution i in the (illiquid) asset class κ , consider a stress scenario in which the asset κ depreciates by ϵ_κ . Then institution i has a direct loss $\epsilon_\kappa \Theta^{i\kappa}$ and, as long as the shock size is

small, no deleveraging occurs and the latter also represents the total loss, which increases linearly with ϵ_κ .

However, as stress levels increase, feedback effects and price-mediated contagion may amplify the initial losses and result in a loss $Loss(i, \epsilon_\kappa) = \epsilon_\kappa \Theta^{i\kappa} + FLoss(i, \epsilon) > \epsilon_\kappa \Theta^{i\kappa}$. The *effective exposure* to the asset class κ accounts for these additional losses and is defined as

$$E^{i\kappa}(\epsilon_\kappa) := \frac{Loss(i, \epsilon_\kappa)}{\epsilon_\kappa} = \underbrace{\Theta^{i\kappa}}_{\text{Notional exposure}} + \underbrace{\frac{FLoss(i, \epsilon_\kappa)}{\epsilon_\kappa}}_{\text{Indirect exposure}}. \quad (1.32)$$

Unlike the notional exposure, the *indirect exposure* depends on the shock size ϵ_κ and, more importantly, on the configuration –size, leverage– of *other* financial institutions with common asset holdings. Clearly, unlike the first term in (1.32), the second term cannot be computed by examining the portfolio of i alone as it depends on the entire network of overlapping portfolios which contribute to fire sales losses. Given the discussion in the previous section, this should not come as a surprise but it clearly departs from the common assumption that notional exposures of a portfolio are sufficient to quantify its risk. Here, the risk of a portfolio cannot be quantified in isolation, but depends on the network of overlapping portfolios and the constraints that these portfolios face.

Interestingly, a financial institution may even have a non-zero (indirect) exposure to an asset class which it does not hold in its portfolio i.e. the second term in (1.32) may be non-zero even if the first term is zero. Consider for example a bank B which does not hold any subprime asset-backed securities, but has common holdings with a bank A with large holdings in subprime ABS. A loss in subprime ABS entails no direct loss to B but, if large enough, may force A to deleverage by selling its marketable assets, also held by B, leading to mark-to-market losses for B. This means that B will have an in-

direct exposure to large losses in subprime ABS, an asset it does not hold! In this example, deleveraging by A only occurs if subprime losses are significant, so B has no exposure to a *small* depreciation in subprime ABS: the indirect exposure of B is clearly scenario-dependent. However, the threshold beyond which spillover from A to B occurs depends on the leverage, size and composition of A's portfolio, which is unknown to B. This example, far from being a curiosity, is in fact illustrative of the mechanism which led to amplification and contagion of losses from the subprime asset class to the entire global financial system in 2007-2008 [Longstaff \(2010\)](#).

1.2 A TWO BANK EXAMPLE

Before studying some properties of the fire sales cascade in more detail, we illustrate selected concepts in a concrete numerical example of the two banks described before. Let the bank balance sheets be given by

$$\Theta = \begin{pmatrix} 10 & 0 \\ 0 & 20 \end{pmatrix} \quad \Pi = \begin{pmatrix} 90 \\ 70 \end{pmatrix} \quad C = \begin{pmatrix} 4 \\ 4.5 \end{pmatrix}.$$

The two rows of Θ denote the holdings in ABS relating to the different geographic regions in which Bank A and B operate. Bank A is thus only (directly) exposed to the first of these asset classes, while Bank B only has a (direct) exposure to the second one. Furthermore, there is one marketable asset class Π , European sovereign debt, say, which both banks hold. The market depth of this asset class is assumed to be

$$D_0 = c \frac{ADV}{\sigma} \sqrt{\tau} = 0.4 \times \frac{50}{0.02} \sqrt{20} \approx 4500 \text{ with price floor } B = 0.5S_0.$$

The initial leverage of the banks is given by $\lambda^A = \frac{10+90}{4} = 25$ and $\lambda^B = 17.8$. The critical shock sizes to the illiquid asset classes that lead to deleveraging

for the respective banks (in the first round) are

$$\begin{aligned}\epsilon_{A,ABS_1}^* &= \frac{4(33-25)}{(33-1)10} = 10\%, & \epsilon_{B,ABS_2}^* &= 10.7\% \\ \epsilon_{A,ABS_2}^* &= \epsilon_{B,ABS_1}^* = +\infty.\end{aligned}$$

The initial shock sizes that lead to an illiquidity defaults (in the first round) are given by

$$\epsilon_{A,ABS_1}^{illiq} = \frac{33 \times 4 - 10}{10(33-1)} \approx 38.1\%, \quad \epsilon_{B,ABS_2}^{illiq} \approx 20\%.$$

Finally, the initial shock sizes that lead to an insolvency default (in the first round) are given by

$$\epsilon_{A,ABS_1}^{insol} = 40\%, \quad \epsilon_{B,ABS_2}^{insol} = 22.5\%.$$

1.2.1 DELEVERAGING DYNAMICS

We will now apply increasing shocks to the ABS exposures of Bank A and study the dynamics of the resulting fire sales cascade. The top panel of Figure 1.2.1 shows the deleveraging proportions of Bank A at different rounds as a function of the initial shock size. Obviously when $\Gamma_k^A = 1$ for some k , then $\Gamma_{k+1}^A = 0$ for all further rounds and higher shock sizes. The lower panel shows the losses in monetary units that Bank B suffers from indirect contagion because of the deleveraging by Bank A. The markers and line colours of both panels are matched: L_1 in the lower panel corresponds to the loss suffered by Bank B from the deleveraging Γ_1 of Bank A. In this case the maximum loss that Bank B suffers is approximately 1.4. This yields a new leverage $\lambda^2 = \frac{(70-1.4)+20}{4.5-1.4} \approx 31.6 < \lambda_{\max}$, such that Bank B will not engage in fire sales itself.

If the actual market depth D is lower than the estimate D_0 , the fire sales

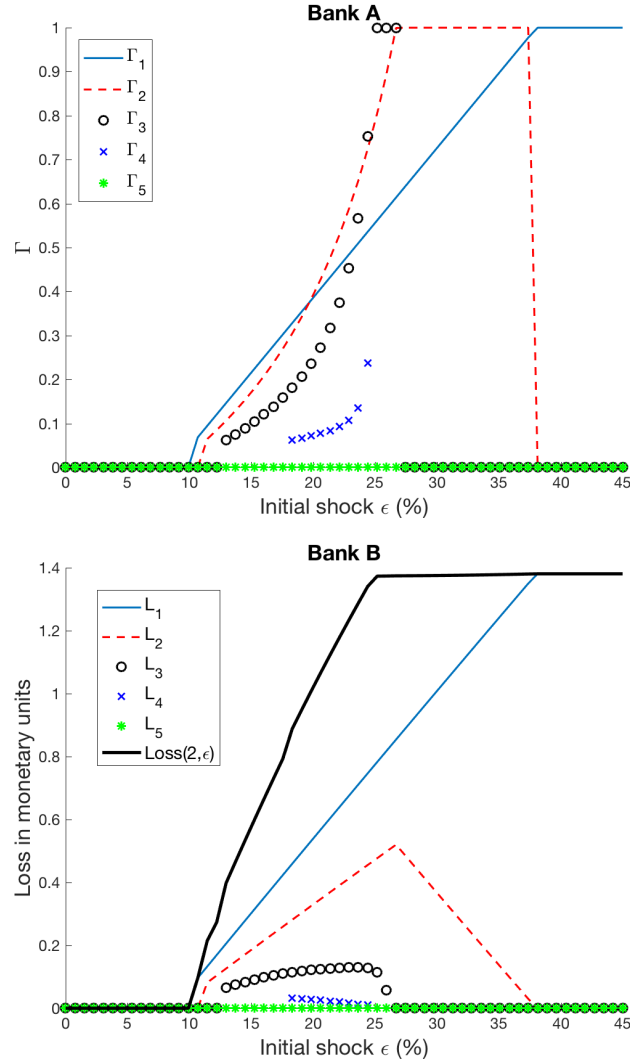


Figure 1.2.1: Top: Deleveraging proportions of Bank A in various rounds of the cascade as a function of the initial shock. Bottom: Resulting indirect contagion loss suffered by Bank B.

losses for Bank B will be higher, and potentially reach a point where Bank B is led to deleverage as well. For instance, a reduced market depth $D = 0.3 \times D_0$ causes fire sales losses that lead both banks to default at the end of the cascade, as shown in the two panels of Figure 1.2.2. As in practice there will be uncertainty around the market depth estimates D_0 , it is therefore important to assess the robustness of the fire sales losses and other results through a rigorous sensitivity analysis of this parameter. To perform this

sensitivity analysis, we scale the market depth estimate by a range of factors (both larger than 1 and smaller than 1) that we call *market depth scaling factors*. In the empirical application to EU data in Chapter 3 we discuss this at greater length.

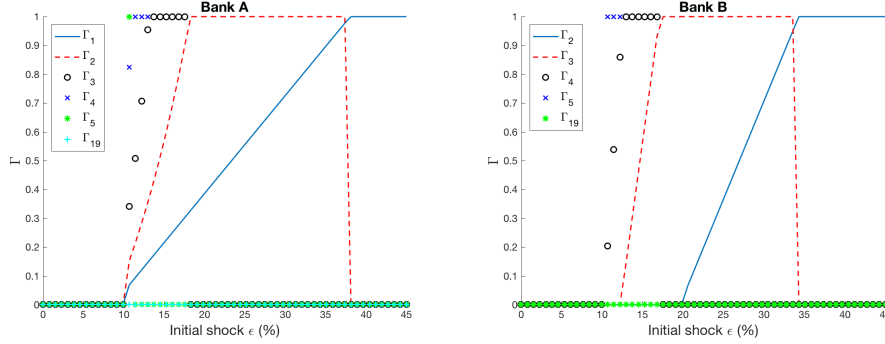


Figure 1.2.2: Deleveraging proportions of the two banks in various rounds of the cascade as a function of the initial shock to ABS_1 . As Bank B holds no direct exposure, $\Gamma_1^2 \equiv 0$, and it only starts to delever from the second round onwards.

Figure 1.2.3 shows the schematic dynamics of Figure 1.1.4 in a concrete example for the initial shock size $\epsilon = 16.8\%$ and the market depth $D = 0.5D_0$. We can read from the plot that Bank A defaults due to insolvency after 4 steps of the cascade, while Bank B defaults due to illiquidity after 6 iterations.

1.2.2 ILLIQUIDITY AND INSOLVENCY

The dynamics and the final state of the system at the end of the fire sales cascade shown in Figure 1.2.3 clearly depend on the choice of the initial shock size ϵ and D .¹² The heatmaps in the panels of Figure 1.2.4 show the final state of each bank at the end of the cascade as a function of the initial shock and the market depth scaling factor. The “origin” in the lower

¹²Of course, they also depend on the network itself, i.e. the holdings Θ, Π and C . However in practice, these will be given by data, whilst ϵ and D depend on the regulatory stress scenario and the market depth calibration and are hence exposed to some model risk.

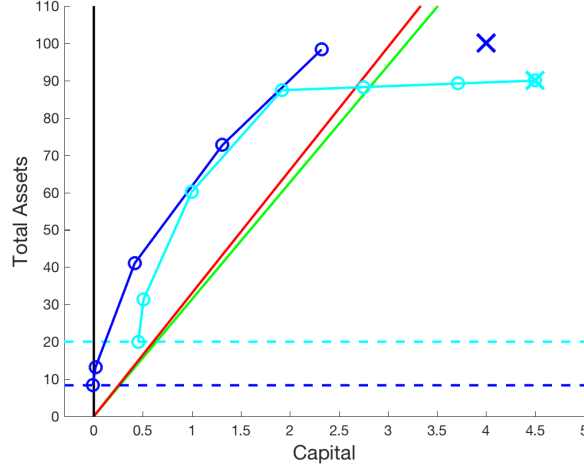


Figure 1.2.3: Example of the portfolio paths of banks 1 (dark blue) and 2 (cyan) in the state space for an initial shock to US subprime mortgages.

left-hand corner of the plots corresponds to the parameter values $(\epsilon, D) = (0, D_0)$. Reading the plots from left to right, the initial shock size increases, and from bottom to top the market depth estimate is *reduced* from its initial value by a market depth scaling factor, down to a minimum value of 30% of the initial estimate at the top of the plot. The red colour marks default due to insolvency¹³; the white colour corresponds to a default due to illiquidity; darker shades of green correspond to more marketable assets remaining in the portfolio at the end of the cascade.

There is a band of solid green colour to the left of $\epsilon = 0.1 = \epsilon_A^*$, because no deleveraging occurs below this critical shock size. At $D = 1$, when the shock size increases, Bank A becomes first illiquid and then insolvent. We note that the red “default” region is reached for initial shocks *smaller* than ϵ_A^{insol} ; this is due to the additional losses caused by fire sales in higher rounds of the cascade. Indeed, whenever a fire sales cascade is modelled, the default region would only start exactly at $\epsilon = \epsilon_A^{insol}$ if $D = \infty$. Interestingly, in this example, as the market depth decreases to about 45% of its initial estimate,

¹³The value of -100 USD at insolvency has been assigned purely for illustrative purposes so as to obtain a darker red.

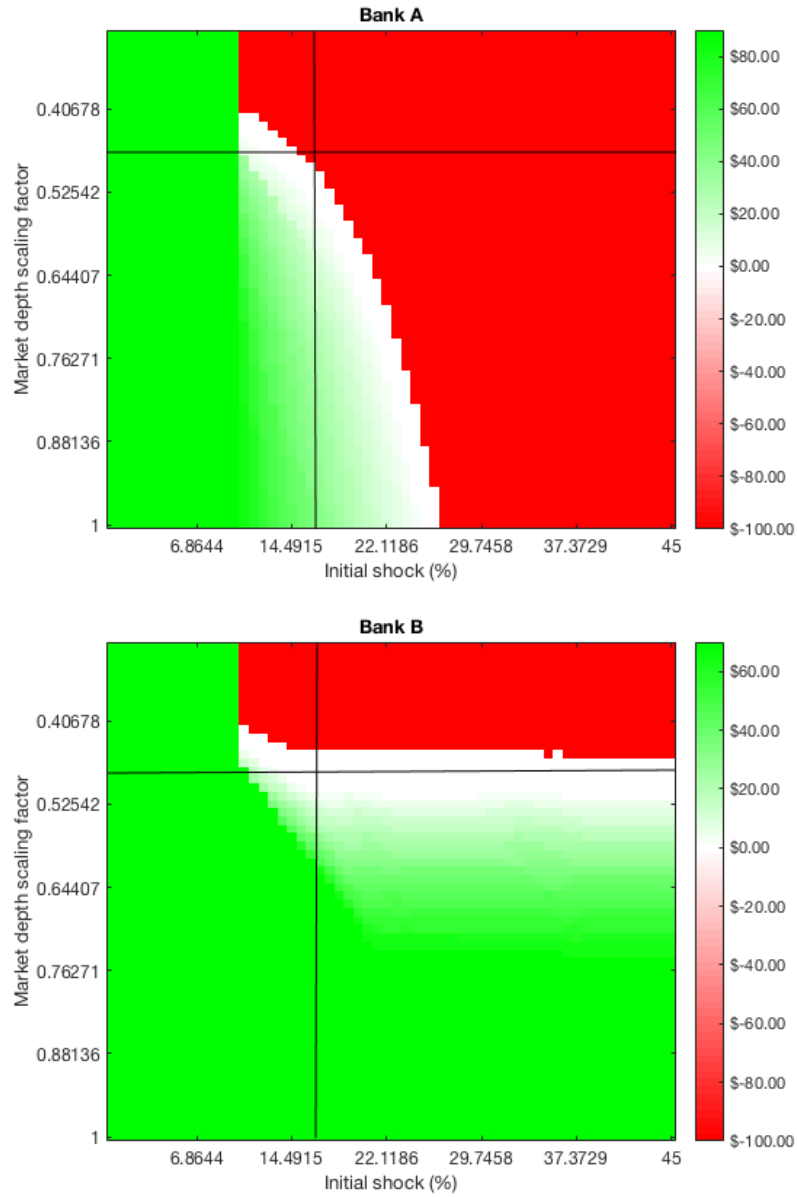


Figure 1.2.4: Red: Default due to insolvency. White: Default due to illiquidity. Shades of green: Value of marketable assets. The black lines indicate the approximate location of the parameter values for which the individual cascade is displayed in Figure 1.2.3.

we reach a region where the fire sales losses are sufficiently large, such that the bank will immediately default due to illiquidity as soon as a cascade is triggered. For market depths below 40% of the initial estimate, fire sales losses are even larger, and the bank defaults due to insolvency as soon as fire

sales are triggered. The heatmap for Bank B shows similar features, with contagion increasing rapidly with decreasing market depth.

1.2.3 NON-MONOTONICITY OF FIRE SALES LOSSES

The panels in Figure 1.2.4 suggest that with increasing shock size, there is an ordering of the states “no deleveraging”, “deleveraging”, “default by illiquidity”, and “default by insolvency”. While in our analyses we observed this often to be the case, it is not true in general. This effect becomes even more pronounced, when the VWAP parameter α is small: A larger shock leads to more deleveraging, but for small α losses from implementation shortfall are smaller relative to mark-to-market losses, whence the sum of losses can become decreasing as a function of the initial shock size. As we shall see in the next Section, $\alpha \geq \frac{1}{2}$ is sufficient to guarantee increasingness in the first round of fire sales. However, even for $\alpha = 1$, the fire sales loss is not monotonically increasing in ϵ in general. The panels in Figure 1.2.5 produce a counterexample in the case $\alpha = 1$ in this simple 2 bank network ($D = 0.3D_0$). It is unfortunate that our system does not have this monotonicity property, as this would be very useful for making statements on the ordering of the final states as a function of ϵ for arbitrary portfolio networks.

1.2.4 FIRE SALES LOSSES AND INDIRECT EXPOSURES

B’s notional exposure to ABS_1 is zero as it does not hold them, and conditional on small shocks, this is equal to its effective exposure. A’s notional exposure is given by $\Theta^{A1} = 10$, and conditional on small shocks this is also equal to A’s effective exposure. From the previous section, we know that A is resilient to shocks $\epsilon < \epsilon_A^* = \frac{C^A(\lambda_{\max} - \lambda^A(\Pi, C, I))}{(\lambda_{\max} - 1)\Theta^A}$. However, a larger depreciation in ABS that leads A to delever a part of its portfolio, generates

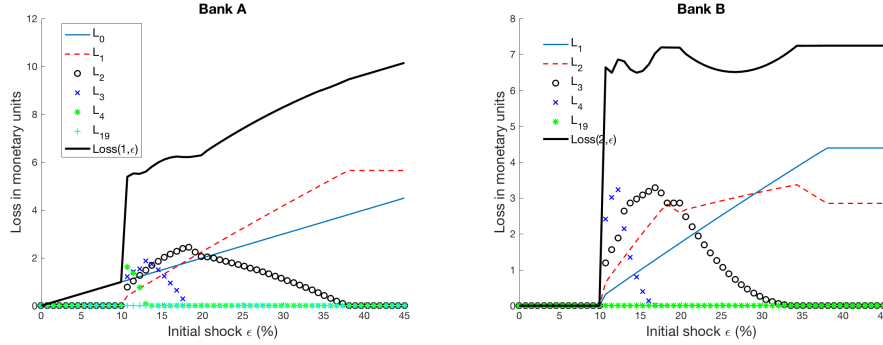


Figure 1.2.5: The fire sales loss L at various rounds, and the final $Loss$ suffered by banks 1 and 2 respectively as a function of the initial shock size. For Bank B the final $Loss$ is clearly non-monotonic in this example.

a price impact which in turn causes mark-to-market losses to B's portfolio.

When ϵ breaches this threshold, then A delevers the quantity

$$\begin{aligned}\Gamma^A(\Pi, C, \epsilon) &= \frac{(\lambda_b - 1)\Theta^{A1}\epsilon - C^A\lambda_b + \Pi^A + \Theta^{A1}}{\Pi^A} \wedge 1 \\ &= \frac{(\lambda_b - 1)\Theta^{A1}\epsilon + C^A(\lambda^A(\Pi, C, I_B) - \lambda_b)}{\Pi^A} \wedge 1.\end{aligned}$$

This causes a mark-to-market loss to B given by

$$\begin{aligned}FLoss_1^B(\Pi^B, \Pi^A, \Theta, C^A, \delta, \epsilon, A) &= \Pi^B \Psi\left(\frac{\Pi^A \Gamma_1^A}{\delta}\right) \\ &= \Pi^B \left(1 - \frac{P}{S}\right) \left(1 - \exp\left(-\frac{(\lambda_b - 1)\Theta^{A1}\epsilon + C^A(\lambda^A(\Pi, C, I_A) - \lambda_b)}{\delta} \mathbf{1}_{\epsilon > \epsilon_A^*}\right)\right) \\ &\approx \Pi^B \frac{(\lambda_b - 1)\Theta^{A1}\epsilon + C^A(\lambda^A(\Pi, C, I_A) - \lambda_b)}{D} \mathbf{1}_{\epsilon > \epsilon_A^*},\end{aligned}$$

where P denotes the price floor to avoid notational clashes. In turn, this corresponds to an indirect exposure given by

$$E^{B,\Theta}(\epsilon) = \begin{cases} 0, & \text{for } \epsilon \leq \epsilon_A^* \\ \frac{1}{\epsilon} \left[\Pi^B \left(1 - \frac{P}{S}\right) \left(1 - \exp\left(-\frac{(\lambda_b - 1)\Theta^{A1}\epsilon + C^A(\lambda^A(\Pi, C, I_A) - \lambda_b)}{\delta} \mathbf{1}_{\epsilon > \epsilon_A^*}\right)\right) \right], & \text{for } \epsilon > \epsilon_A^* \end{cases}.$$

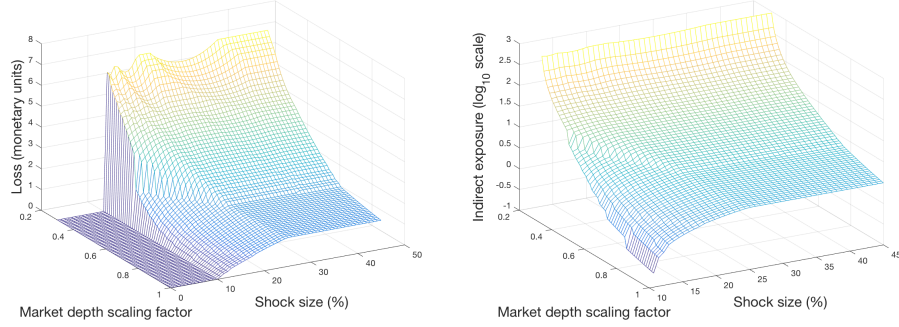


Figure 1.2.6: Fire sales loss of Bank B as a function of the initial shock and market depth scaling parameter (left), and the corresponding indirect exposure generating these losses (on log scale).

The left panel in Figure 1.2.6 shows the loss at the end of the fire sales cascade of Bank B as a function of the initial shock and the market depth scaling parameter. Reading the plot from front to back, the market depth is reduced, and losses increase rapidly. The right panel plots the indirect exposure that corresponds to these losses on a log scale. Note that this graph starts at $\epsilon = 10\%$, as indirect exposures are zero for smaller initial shocks.

Remark. This is not the case in the leverage targeting model, here we have for (a single round of fire sales) and *any* ϵ

$$E_{L.T.}^{B,\Theta^A} \approx \left[\Pi^A \left(\frac{(\lambda_b - 1)\Theta^{A1}}{D} \right) \right]. \quad (1.33)$$

Joy in looking and comprehending is nature's most beautiful gift.

Albert Einstein

2

A CLOSER LOOK AT FIRE SALES MODELLING: PROPERTIES AND ROBUSTNESS

This Chapter is based on the working papers: “Fire sales, price-mediated contagion and systemic risk”, and “Asset class granularity: Aggregation issues and liquidity estimates for stress testing”, both of which are joint work with Prof. Rama Cont.

2.1 GENERAL PROPERTIES

As we have seen in the previous section, our model leads to highly non-linear dynamics during the course of the cascade. In particular, the absence of a monotonicity property for the total loss of each bank, makes attempts at

proving general properties for arbitrary rounds or networks a rather interesting undertaking. Apart from not being entirely satisfactory on a mathematical level, we do not consider this to be a significant limitation in practice however. As we shall see in Chapter 3, in practically relevant examples, many results do indeed hold in the k -round case, and results are shown to be robust for a large range of realistic parameter values.

2.1.1 BASINS OF ATTRACTION AND DYNAMICS OF CASCADE

We return to the general case of N institutions with portfolio holdings in M marketable assets and K illiquid assets.

The cascade ends as soon as a fixed point is reached. Proposition 2.1.1 specifies the set of fixed points.

Proposition 2.1.1. *The set of fixed points $x = (\Pi, C)$ of the dynamical system given by the map f in (1.27) is given by the set $\mathcal{F}_{I(\epsilon)} := \prod_{i=1}^N \mathcal{S}_i \cup \mathcal{I}_i \cup \mathcal{J}_i$, where each component $(\Pi^i, C^i) \in \mathbb{R}^M \times \mathbb{R}$ lies in either the **survival region***

$$\mathcal{S}_i := \left\{ (\Pi^i, C^i) \in \mathbb{R}^M \times \mathbb{R} \left| \forall i : \sum_{\mu=1}^M \Pi^{i\mu} > 0, \quad C^i > 0, \quad \sum_{\mu=1}^M \Pi^{i\mu} + I_i(\epsilon) - \lambda_{\max} C^i \leq 0 \right. \right\}, \quad (2.1)$$

*the **illiquidity line***

$$\mathcal{I}_i := \left\{ (\Pi^i, C^i) \in \mathbb{R}^M \times \mathbb{R} \left| \forall i : \sum_{\mu=1}^M \Pi^{i\mu} = 0, \quad C^i > 0 \right. \right\}, \quad (2.2)$$

*or the **insolvency point***

$$\mathcal{J}_i := \left\{ (\Pi^i, C^i) \in \mathbb{R}^M \times \mathbb{R} \left| \forall i : \sum_{\mu=1}^M \Pi^{i\mu} = 0, \quad C^i = 0 \right. \right\}. \quad (2.3)$$

Proof. As capital and liquid assets are bound from below by 0, any point $(\Pi^i, C^i) \in \mathbb{R}^M \times \mathbb{R}$ such that $\sum_{\mu=1}^M \Pi^{i,\mu} < 0$ or $C^i < 0$ can never be reached. We will show that x is a fixed point of f if and only if each institution i 's state variable pair (Π^i, C^i) lies in one of the sets $\mathcal{S}_i, \mathcal{I}_i, \mathcal{J}_i$.

First assume that there is at least one i such that (Π_k^i, C_k^i) lies in the complement of $\mathcal{S}_i \cup \mathcal{I}_i \cup \mathcal{J}_i$. This means that for this institution i : $C_k^i \geq 0$, $\sum_{\mu=1}^M \Pi_k^{i\mu} > 0$ and $\sum_{\mu=1}^M \Pi_k^{i\mu} + I_i(\epsilon) - \lambda_{\max} C_k^i > 0$. It follows that $\Gamma^i(\Pi_k, C_k) > 0$, whence $\sum_{\mu=1}^M \Pi_{k+1}^{i\mu} < \sum_{\mu=1}^M \Pi_k^{i\mu}$. Consequently, (Π_k, C_k) was not a fixed point.

Now assume that all i lie in $\mathcal{S}_i \cup \mathcal{I}_i \cup \mathcal{J}_i$. First, the institutions that have defaulted due to insolvency, have exited the cascade and we have $C_{k+1}^i = C_k^i = 0$ and $\Pi_{k+1}^{i\mu} = \Pi_k^{i\mu} = 0$ for all μ . In this case $\Gamma^i(\Pi_k, C_k) = 0$ and there is no influence from these institutions on the others. Second, for the institutions that have defaulted due to illiquidity, it also follows from the update equations that $C_{k+1}^i = C_k^i > 0$ and $\Pi_{k+1}^{i\mu} = \Pi_k^{i\mu} = 0$ for all μ , such that they do not suffer any further mark-to-market losses, nor cause mark-to-market losses to other institutions. Finally, consider the institutions in the safety region $\mathcal{S}_i$. By definition of the deleveraging proportion, we have $\Gamma^i(\Pi_k, C_k) = 0$ and they do not delever, hence do not cause any mark-to-market losses, nor suffer mark to market losses from any other institution. (Π, C) is thus indeed a fixed point. $\square$

As there is an infinite number of fixed points, an immediate corollary from the Banach Fixed Point Theorem is that the map f is not a contraction. The set $\mathcal{F}_{I(\epsilon)}$ corresponds to the union of the “no deleveraging zone” in Figure 1.1.4, the illiquidity band at $\sum_{\mu=1}^M \Pi_k^{i\mu} = 0$ (yellow band in Figure 1.1.4) and the point $(\sum_{\mu=1}^M \Pi^{i\mu}, C^i) = (0, 0)$.

A natural question of interest is whether the threshold model reaches the set of fixed points in a finite number of steps. The following proposition shows

that under weak assumptions, this is always the case. We note that the illiquid holdings Θ also include illiquid balance sheet items such as intangible assets that are assumed not to suffer any losses in the stress scenario. Thus, for all practical purposes, $I_i(\epsilon) > 0$ for all i and $\epsilon > 0$ is a very reasonable assumption.

Proposition 2.1.2. *If $\lambda_b < \lambda_{\max}$ and $I_i(\epsilon) > 0$ for all i , then the fire sales cascade ends in a finite number of iterations.*

Proof. As the fire sales cascade ends as soon as a fixed point is reached, it is sufficient to show that for any configuration of parameters, there exists $T < \infty$ such that $\Gamma^i(\Pi_T, C_T) = 0$ for all i . Assume by contradiction that the cascade is infinite. This implies that there is at least one i such that $\forall k > 0, \exists l > k$ such that $\Gamma^i(\Pi_l, C_l) > 0$. Note that $\Gamma^i(\Pi_l, C_l)$ need not be positive for all l , as the infinite cascade could in principle be generated by cycles of different banks deleveraging in turns. Denote the set of banks i in such a cycle by $\mathcal{Z}$, with $|\mathcal{Z}| \geq 1$. We restrict our attention to the banks in $\mathcal{Z}$.

Let $\tilde{\Pi}_k^{i\mu}, \tilde{C}_k^i$ denote the securities holdings and the capital of i *after* deleveraging but *before* the price impact is accounted for, i.e.

$\tilde{\Pi}_k^{i\mu} := (1 - \Gamma^i(\Pi_{k-1}, C_{k-1}))\Pi_k^{i\mu}$, $\tilde{C}_k^i := C_{k-1}^i$. It then holds $\sum_{\mu=1}^M \tilde{\Pi}_k^{i\mu} + I_i(\epsilon) - \lambda_b \tilde{C}_k^i = 0$. However, as this bank is in the set $\mathcal{Z}$ and delevers again, at iteration l , say, we must have at l : $\sum_{\mu=1}^M \Pi_l^{i\mu} + I_i(\epsilon) - \lambda_{\max} C_l^i > 0$. This implies that between these two deleveraging rounds, i must have suffered a capital loss of *at least* (c.f. Fig. 2.1.1)

$$\tilde{C}_k^i - C_l^i \geq \frac{\sum_{\mu=1}^M \tilde{\Pi}_k^{i\mu} + I_i(\epsilon)}{\lambda_b} - \frac{\sum_{\mu=1}^M \tilde{\Pi}_l^{i\mu} + I_i(\epsilon)}{\lambda_{\max}} \geq \frac{(\lambda_{\max} - \lambda_b)I_i(\epsilon)}{\lambda_{\max}\lambda_b} > 0. \quad (2.4)$$

The total fire sales loss is trivially bounded by the finite value of initial total assets in the system, i.e. $\sum_{k=1}^{\infty} \sum_{i \in \mathcal{Z}} C_{k-1}^i - C_k^i \leq SLoss \leq \text{Total assets} < \infty$

∞ . Because we assumed that the cascade is infinite, this implies that the loss between two such steps in (2.4) cannot be bounded away from 0 in the limit. In turn, this either implies that $I(\epsilon)_i = 0$ for all i or $\lambda_b = \lambda_{\max}$. This is a contradiction. $\square$

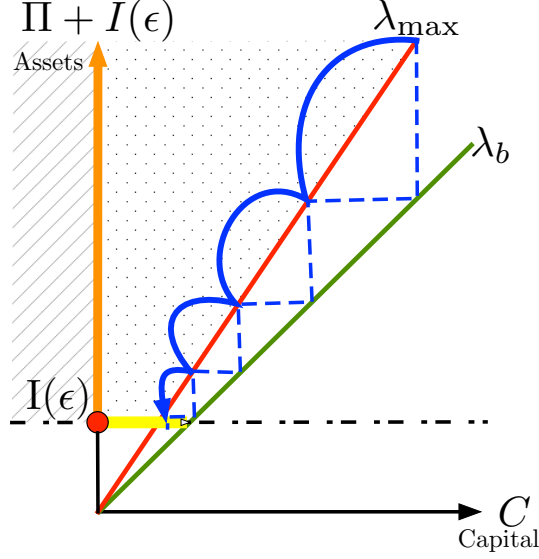


Figure 2.1.1: Because $\lambda_b < \lambda_{\max}$ there is, at each round, a minimum amount of deleveraging which is bounded away from zero, illustrated by the zig-zagging lines between the leverage constraint and the leverage target. Consequently the cascade will reach the illiquidity bound in a finite number of steps.

2.1.2 FIRE SALES LOSSES

Proposition 2.1.3. *The fire sales loss is decreasing in the market depth D_μ .*

Proof. Rather than a parameter, consider D_μ to be a variable of Ψ . We then get

$$\frac{d}{dD_\mu} \Psi_\mu = - \left(1 - \frac{B_\mu}{S}\right) \left(1 - \frac{B_\mu}{S_0^\mu}\right)^{-1} \frac{q}{D_\mu^2} \exp \left(\frac{-q}{\left(1 - \frac{B_\mu}{S_0^\mu}\right) D_\mu} \right) \leq 0.$$

Consequently, $\frac{d}{dD_\mu} L^i(\Pi_k, C_k) \leq 0$ for all $k \geq 0$ and i , whence

$$\frac{d}{dD_\mu} SF Loss \leq 0.$$

The consideration for $\frac{d}{d\delta_\mu}$ is exactly the same. $\square$

Proposition 2.1.4. *If $\alpha \geq \frac{1}{2}$, then for any κ , the first round fire sales loss with linear price impact is an increasing function of the initial shock size ϵ_κ .*

Proof. Fix any κ for the illiquid asset to be shocked. The expression of the deleveraging function in terms of the initial shock then becomes

$$\Gamma^i(\Pi, C, \Theta, \epsilon_\kappa) = \left(\frac{(\lambda_b - 1)\Theta^{i\kappa}\epsilon_\kappa - C^i(\lambda_b - \lambda^i(\Pi, C, I))}{\sum_{\mu=1}^M \Pi^{i\mu}} \mathbb{1}_{\{\lambda^i(\Pi, C, I(\epsilon_\kappa)) > \lambda_{\max}\}} \right) \wedge 1.$$

The linearised first round fire sales loss is equal to

$$L_1(\Pi_0, C_0, I(\epsilon_\kappa)) := \sum_{i=1}^N (1 - (1 - \alpha)\Gamma^i(\Pi_0, C_0, \Theta, \epsilon_\kappa)) \sum_{j=1}^N \Omega_{ij}(\Pi_0) \Gamma^j(\Pi_0, C_0, \Theta, \epsilon_\kappa)$$

where $\Omega_{ij}(\Pi) = \sum_{\mu=1}^M \Pi^{i\mu} \Pi^{j\mu} \delta_\mu^{-1} \geq 0$. It then follows that $\frac{d}{d\epsilon_\kappa} L_1(\Pi_0, C_0, I(\epsilon_\kappa)) \geq 0$ iff

$$\begin{aligned} & \sum_{i=1}^N \sum_{j=1}^N \left(\left(\frac{(\lambda_b - 1)\Theta^{j\kappa}}{\sum_{\mu=1}^M \Pi^{j\mu}} \mathbb{1}_{\lambda^j(\Pi, \Theta, I(\epsilon)) > \lambda_{\max}} \cdots \right. \right. \\ & \quad - (1 - \alpha)\Gamma^j(\Pi_0, C_0, \Theta, \epsilon_\kappa) \frac{(\lambda_b - 1)\Theta^{i\kappa}}{\sum_{\mu=1}^M \Pi^{i\mu}} \mathbb{1}_{\lambda^i(\Pi, \Theta, I(\epsilon)) > \lambda_{\max}} \cdots \\ & \quad \left. \left. - (1 - \alpha) \frac{(\lambda_b - 1)\Theta^{j\kappa}}{\sum_{\mu=1}^M \Pi^{j\mu}} \mathbb{1}_{\lambda^j(\Pi, \Theta, I(\epsilon)) > \lambda_{\max}} \Gamma^i(\Pi_0, C_0, \Theta, \epsilon_\kappa) \right) \right) \geq 0. \end{aligned}$$

This is equivalent to

$$N \sum_{j=1}^N \frac{(\lambda_b - 1)\Theta^{j\kappa}}{\sum_{\mu=1}^M \Pi^{j\mu}} \mathbb{1}_{\lambda^j(\Pi, \Theta, I(\epsilon)) > \lambda_{\max}} \geq (1 - \alpha) \sum_{i=1}^N \sum_{j=1}^M \left(\left(\Gamma^j(\dots) \frac{(\lambda_b - 1)\Theta^{i\kappa}}{\sum_{\mu=1}^M \Pi^{i\mu}} \mathbb{1}_{\lambda^i(\dots) > \lambda_{\max}} + \frac{(\lambda_b - 1)\Theta^{j\kappa}}{\sum_{\mu=1}^M \Pi^{j\mu}} \mathbb{1}_{\lambda^j(\dots) > \lambda_{\max}} \Gamma^i(\dots) \right) \right).$$

Now, as $\Gamma^i(\dots) \leq 1$ for all i , we have

$$\begin{aligned} & \sum_{i=1}^N \sum_{j=1}^M \left(\left(\Gamma^j(\dots) \frac{(\lambda_b - 1)\Theta^{i\kappa}}{\sum_{\mu=1}^M \Pi^{i\mu}} \mathbb{1}_{\lambda^i(\Pi, \Theta, I(\epsilon)) > \lambda_{\max}} + \frac{(\lambda_b - 1)\Theta^{j\kappa}}{\sum_{\mu=1}^M \Pi^{j\mu}} \mathbb{1}_{\lambda^j(\Pi, \Theta, I(\epsilon)) > \lambda_{\max}} \Gamma^i(\dots) \right) \right) \\ & \leq 2N \sum_{j=1}^N \frac{(\lambda_b - 1)\Theta^{j\kappa}}{\sum_{\mu=1}^M \Pi^{j\mu}} \mathbb{1}_{\lambda^j(\Pi, \Theta, I(\epsilon)) > \lambda_{\max}}, \end{aligned}$$

and the result follows from $\alpha \geq \frac{1}{2}$ by assumption. $\square$

We remark that $\alpha \geq \frac{1}{2}$ is not sufficient when the initial shock also affects the marketable assets. In fact, in this case the fire sales loss can be decreasing in ϵ_μ even when $\alpha = 1$. This is due to the fire sales loss taking the form

$$L_1^i(\Pi_0(\epsilon), C_0, I(\epsilon)) := \sum_{j=1}^N \sum_{\mu=1}^M \frac{(1 - \epsilon^\mu) \Pi_0^{i\mu} (1 - \epsilon^\mu) \Pi_0^{j\mu}}{\delta_\mu} \Gamma^j(\Pi_0, C_0, \Theta, \epsilon).$$

2.1.3 TIPPING POINT

Whether or not the critical shock $\min_i \epsilon_i^*$, where the first bank starts to delever, designates a tipping point to large scale contagion depends on the joint distribution of banks' sizes and leverages. Intuitively, if size and leverage are distributed countermonotonically, i.e. when the most leveraged banks are the smallest banks, then the system is very resilient: The fire sales losses caused by small banks are bearable by the larger and less levered banks; no second round contagion occurs. On the other hand, if size

and leverage are distributed co-monotonically and the most leveraged banks are also the largest, the picture changes radically: Fire sales losses caused by large banks will be sufficiently important to force smaller banks to delever as well, despite them having larger buffer from their smaller leverage; a deleveraging cascade can ensue in which the entire system spirals to default.

We illustrate this effect by anticipating slightly on the EBA data used in Chapter 3: Figure 2.1.2 shows in a scatter plot the leverage¹ of 90 European banks covered by the EBA dataset against their total assets (log-scale). The positive dependence of leverage on size clearly emanates from this graph, which suggests that large scale contagion is a possibility in the European banking system. The red asterisks and blue pluses show the counter-monotonic and co-monotonic cases respectively.

The left panel of Figure 2.1.3 shows the volume of first round deleveraging as a function of the percentage shock size to illiquid assets (assumed to be 10% of total assets in this example) for the EBA data, as well as a similar system with same marginal leverage and size distributions but perfect co-monotonic / counter-monotonic dependence between the two margins. The right panel shows the total volume of marketable assets delevered (over all rounds) as a function of the initial shock size. In the counter-monotonic case, there is no significant reaction for shock sizes below 20% (corresponding to a net loss of 2% of total assets). In contrast, in the co-monotonic case, shocks of 4% are sufficient to trigger wide-spread contagion. The response of the system with actual EBA data is remarkably close to the co-monotonic case.

¹We remark that to avoid fire sales in absence of a shock, as in Chapter 3, we have scaled the capitals of those banks with leverage above 33. The black crosses in Figure 2.1.2 denote the leverage from the raw data, the black circles correspond to the adjusted leverage for banks with unscaled leverage above 33. More detailed explanations about this procedure are provided in the Appendix.

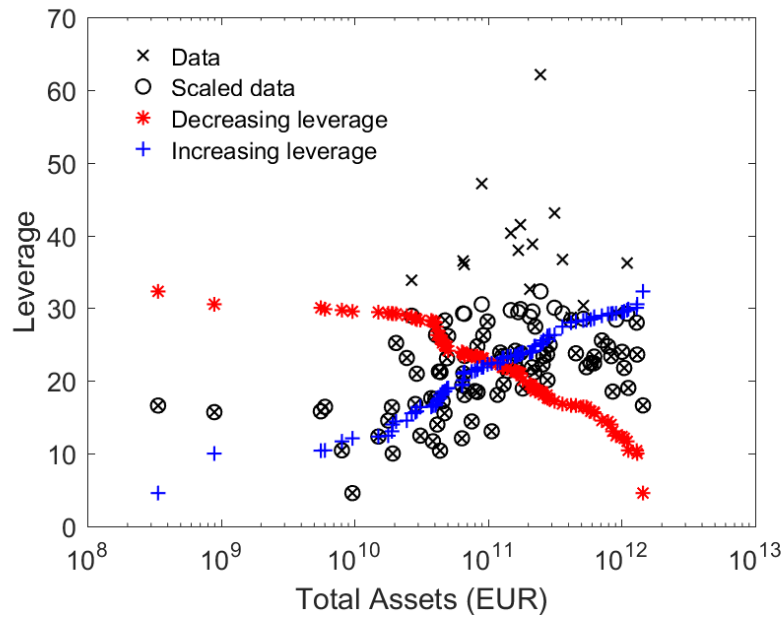


Figure 2.1.2: Leverage against total assets (logarithmic scale) for the actual data, and assuming a perfectly co-/counter-monotonic dependence between the two marginals.

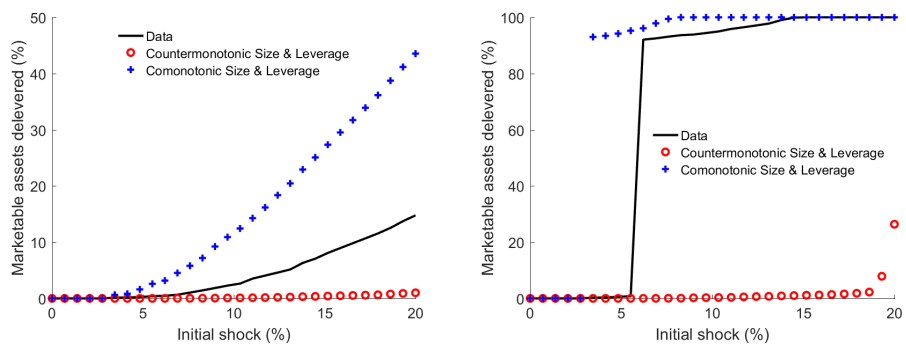


Figure 2.1.3: Volume of marketable assets delevered as a function of the initial percentage shock to illiquid assets in the first round (left) and all rounds (right).

2.1.4 BANK-LEVEL LOSS DISTRIBUTION

An argument sometimes advanced to avoid moving to a systemic stress testing framework is that, while fire sales effects are important and contribute to amplifying losses, their effect can be mimicked in bank-level stress tests by applying more severe shocks, without simulating fire sales in a detailed manner. Let us examine the validity of this point in general.

The common approach currently taken is to amplify individual bank losses by a pre-defined exogenous factor. Thus, without fire sales model, one scales the loss generated by the illiquid assets by a factor $c_1 > 0$ (or the liquid assets by a factor $c_2 > 0$), resulting in a total loss for Bank i given by

$$(1 + c_1) \sum_{\kappa=1}^K \Theta^{i\kappa} \epsilon_{\kappa} + c_2 \sum_{\mu=1}^M \Pi^{i\mu}. \quad (2.5)$$

First, we note that the total loss across all banks in the systemic stress test can obviously be reproduced in a macro stress test without fire sales by simply scaling the initial shock size by adequate factors as illustrated above. However, in absence of fire sales, bank losses are proportional to the banks notional exposure to the asset class subject to stress (assuming $c_2 = 0$ and only scaling c_1), whereas the loss in the systemic stress test is proportional to the *effective exposure*, so bank-level losses will not be reproduced correctly under this adjustment.

Let us compare the above expression to the unscaled total loss, using a single round of fire sales

$$\sum_{\kappa=1}^K \Theta^{i\kappa} \epsilon_{\kappa} + \sum_{i=1}^N (1 - (1 - \alpha) \Gamma^i(\Pi_0, C_0, \Theta, \epsilon_{\kappa})) \sum_{j=1}^N \Omega_{ij}(\Pi_0) \Gamma^j(\Pi_0, C_0, \Theta, \epsilon_{\kappa}). \quad (2.6)$$

Clearly, the second term in (2.5) does not depend on ϵ , while in (2.6), the second term is a quadratic function of ϵ . In general, it is hence impossible

that the simplified version in (2.5) can replicate the cross sectional distribution of fire sales losses obtained from a fire sales model. The bias increases as we account for higher order rounds of fire sales.

In the empirical section we shall see that this leads to a substantial and economic relevant bias, and that it is not possible to replicate the cross-sectional distribution of losses without a fire sales model, as suggested by our theoretical analysis here.

2.1.5 SENSITIVITY TO COMPOSITION OF SHOCK

Finally, Proposition 2.1.5 formalises the sensitivity of the threshold model to different initial stress scenarios. Denote by a^i the total initial assets of Bank i . As the total loss for each institution is naturally bounded by its total asset value, define $V = \prod_{i=1}^N [0, a^i]$, the set of initial losses. Further denote by $L_k^{i*} = \frac{C_{k-1}^i (\lambda_{\max} - \lambda^i(\Pi_{k-1}, I(\epsilon), C_{k-1}))}{\lambda_{\max} - 1} \geq 0$ the critical loss for institution i . Finally, denote by $\vec{F}L \in \mathbb{R}^N$ the vector of fire sales losses whose components are each institution's fire sales loss $FLoss(i, \epsilon) = \sum_{k=1}^T L^i(\Pi_{k-1}, C_{k-1})$.

Proposition 2.1.5. *If the graph corresponding to Ω has a diameter of at least 3, $L_0^{i*} > 0$ for all i , and the market depth is sufficiently large, then there exist sets $V_a, V_b \subset V$ of initial losses that have strictly positive Lebesgue measure and that lead to orthogonal fire sales losses, i.e.*

$$\forall (L_{0,a}, L_{0,b}) \in V_a \times V_b : \langle \vec{F}L(L_{0,a}), \vec{F}L(L_{0,b}) \rangle = 0. \quad (2.7)$$

Proof. We provide an explicit construction of two such sets V_a, V_b . Denote the sequence of four institutions that lie on a diameter of the indirect contagion graph Ω by i, j, p, q . Let scenarios of type a be the shocks $L_0^i = L_0^{i*} + \epsilon$ for $\epsilon > 0$, and $L_0^r = 0$ for all $r \neq i$. Let $\epsilon_1^* > 0$ be such that the fire sales losses to bank j fulfil $L_1^j = L_1^{j*}$. As no bank has a critical shock size of 0,

it follows that such an $\epsilon_1^* > 0$ exists for a sufficiently large market depth.²

Hence, we set $V_a := [0, L_0^{i*} + \epsilon_1^*] \times \prod_{r \in \{1, \dots, N\} \setminus \{i\}} [0, L_0^{r*}]$.

Let scenarios of type b be the shocks $L_0^q = L_0^{q*} + \epsilon$ for $\epsilon > 0$, and $L_0^r = 0$ for all $r \neq q$. Let ϵ_2^* be such that the fire sales losses to bank p fulfil $L_1^p = L_2^{p*}$.

As no bank has a critical shock size of 0, it follows that such an $\epsilon_2^* > 0$ exists for sufficiently large market depths. Hence, for any $\epsilon \leq \epsilon_2^* : L_k^i = L_k^j = 0$ for all k , and we set $V_b := [0, L_0^{q*} + \epsilon_2^*] \times \prod_{r \in \{1, \dots, N\} \setminus \{q\}} [0, L_0^{r*}]$.

The sets V_a and V_b have positive Lebesgue measure and fulfil by construction the property $\forall (L_{0,a}, L_{0,b}) \in V_a \times V_b : \langle \vec{F}L(L_{0,a}), \vec{F}L(L_{0,b}) \rangle = 0$. $\square$

2.1.6 POSITIVE SYSTEM EXTERNALITIES OF CAPITAL SURCHARGES

Let us now analyse how the system responds when banks are required to raise additional capital in order to increase their resilience. From (1.9), we get immediately

$$\frac{d}{dC^i} \Gamma^i(\Pi, C) = \begin{cases} 0 & \text{if } \epsilon \leq \epsilon_i^* \text{ or if } \epsilon \geq \epsilon^{illiq} \\ \frac{-\lambda_b}{\sum_{\mu=1}^M \Pi^{i\mu}} & \text{otherwise} \end{cases}, \quad (2.8)$$

where we recall that ϵ_i^* and ϵ^{illiq} are the critical deleveraging, and critical illiquidity shocks from (1.7) and (1.11). Recall that the first round loss is given by

$$L_1(\Pi_0, C_0, I(\epsilon_\kappa)) = \sum_{i=1}^N (1 - (1 - \alpha) \Gamma^i(\Pi_0, C_0, \Theta, \epsilon_\kappa)) \sum_{j=1}^N \Omega_{ij}(\Pi_0) \Gamma^j(\Pi_0, C_0, \Theta, \epsilon_\kappa),$$

²If the market depth is too shallow, it could happen that banks i and j spiral to default as soon as bank i begins to delever. In this case no such ϵ_1^* exists, as for all $\epsilon > 0 : L_1^j \gg L_1^{j*}$. Then for any $\epsilon \leq \epsilon_1^* : L_k^p = L_k^q = 0$ for all $k \geq 1$. At the same time, it could happen that the liquidity-weighted overlap of banks i and j is so small that $\epsilon_1^* > 100\%$. This does not affect our argument. The crucial point is that it is strictly larger than zero.

where $\Omega_{ij}(\Pi) = \sum_{\mu=1}^M \Pi^{i\mu} \Pi^{j\mu} \delta_{\mu}^{-1}$. Hence, if Bank i_0 is required to raise additional capital, the marginal reduction in its own fire sales loss per dollar raised is

$$\frac{d}{dC^{i_0}} L_1^{i_0} = \frac{\lambda_b}{\sum_{\mu=1}^M \Pi^{i\mu}} \left((-1 + 2(1 - \alpha)\Gamma^{i_0})\Omega_{i_0, i_0} + (1 - \alpha) \sum_{j \neq i_0}^N \Gamma^j \Omega_{i_0, j} \right).$$

Whenever $\frac{d}{dC^{i_0}} L_1^{i_0} < -1$ it is thus *in interest of the bank* to increase its capital, because by doing so, per dollar capital raised, the loss in a stress scenario is reduced by more than one dollar. On the other hand, if $\frac{d}{dC^{i_0}} L_1^{i_0} > -1$ it is not in the interest of the bank to raise capital. Let us now consider the response of the system-wide fire sales loss to capital increases

$$\frac{d}{dC^{i_0}} L_1 = \frac{d}{dC^{i_0}} L_1^{i_0} - \frac{\lambda_b}{\sum_{\mu=1}^M \Pi^{i_0, \mu}} \sum_{i \neq i_0} (1 - (1 - \alpha)\Gamma^i) \Omega_{ii_0}, \quad (2.9)$$

from which it is immediately clear that $\frac{d}{dC^{i_0}} L_1 < \frac{d}{dC^{i_0}} L_1^{i_0}$. So whenever it is in the interest to raise capital for a bank, this is beneficial for the system as a whole. An interesting tension arises when $L_1 < -1$ but $L_1^{i_0} > -1$. In this case, individual and systemic interests are not aligned: There is a positive externality to the system whereby the system-wide loss is reduced by more than one dollar per dollar raised by Bank i_0 . However, it is not in the interest of the bank to do so, because from its own perspective each dollar reduces its own loss by less than a dollar. In such cases, there is a need for a regulator to coordinate capital requirements, as banks themselves do not take into account the positive externality to the system of any additional capital they raise, and may end up with too low capital from a system perspective.

Figure 2.1.4 shows $\frac{d}{dC^i} L_1$ plotted against $\frac{d}{dC^i} L^i$. Clearly there are some banks for whom raising capital would be in their own interest, while for other banks raising capital is not in their own interest but has significant positive externalities for the system.

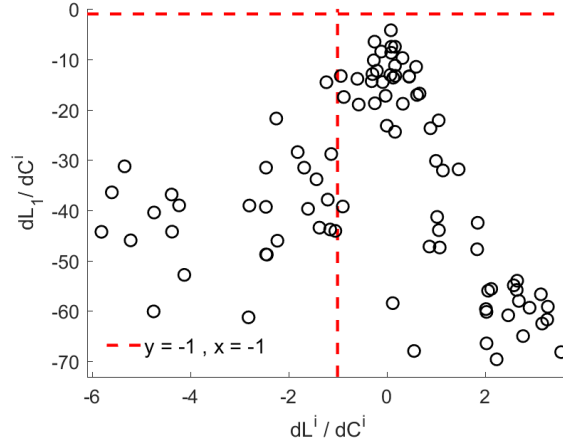


Figure 2.1.4: $\frac{d}{dC^{i_0}} L_1$ plotted against $\frac{d}{dC^{i_0}} L^i$. The red dashed lines indicate $\frac{d}{dC^{i_0}} L_1 = 1$ and $\frac{d}{dC^{i_0}} L^{i_0} = -1$ respectively.

We study the potential use introducing of such targeted capital requirements that have a positive system externality as a macroprudential policy in more detail in Chapter 4.

2.2 FROM ASSETS TO ASSET CLASSES: A NOTE ON GRANULARITY AND AGGREGATION ISSUES

So far we have taken the asset (class) μ as given without defining in greater detail what we exactly mean by an *asset class*. This is the focus of the next section. Specifically, we investigate the impact on our results when we choose different *levels of granularity* (or aggregation) for μ . Indeed, there is a wide range of possible levels of aggregation: At the most granular level there are the individual securities, identified by their International Security Identification Number (ISIN)³. These individual instruments can be aggregated along several dimensions such as their issuer, maturity, their issuer's industrial or institutional sectors, their type, or the geographical region of

³One could actually further distinguish a unique ISIN by the market it is traded on and Central Securities Depository (CSD) it is registered in, if there are several. We will not discuss this distinction.

their issuer, among others. At the other end of the granularity spectrum, we arrive at the “single marketable asset” (Cifuentes et al. (2005), Awiszus and Weber (2015)). It should be immediately clear, that any particular choice on the level of aggregation has an impact on the *path* of contagion during a fire sales cascade .

There is a rich literature on data aggregation dating back to the concept of “ecological fallacy” introduced by Robinson (1950), showing that it can be misleading to draw conclusions on individuals based on aggregate data (in his case on the literacy of immigrants). Within this literature, there are also more recent contributions that specifically relate to systemic risk – see for instance Chapter 2 in Fouque and Langsam (2013), Chinchalkar and Stein (2010), or Hale and Krainer (2015), and references therein. However, the main focus areas of this stream, have so far been (i) the level of granularity at which stress test scenarios should be defined and data be collected, (ii) computing credit risk and default estimates with loan-level vs. portfolio-level information, and (iii) at what level of aggregation an institution’s exposures to asset classes or to other institutions should be captured. To the best of our knowledge, the question of an “optimal level” of asset class granularity for modelling fire sales contagion has not received any attention in the literature so far.

AGGREGATING ASSET CLASSES AND THE PATH OF CONTAGION. Consider two local Italian and Spanish banks that hold disjoint portfolios at the ISIN-level, i.e. their portfolios hold no two assets with identical ISIN in common. Further assume that the Spanish bank is holding some off-the-run Italian government bonds, while the Italian bank holds on-the-run bonds. As the on-the-run and off-the-run securities carry different ISIN, this does not contradict the portfolios being disjoint at the ISIN level. However, when e.g. the Italian bank engages in a fire sale that is sufficiently large to temporarily

move the price of the on-the-run bonds, the price of the off-the-run treasuries would fall in tandem, causing mark-to-market losses for the Spanish bank. Therefore, when analysing these portfolios at the ISIN level, one could falsely conclude that they are orthogonal, when in fact, they are not. The “most granular level possible” is therefore not necessarily the most suitable one for modelling fire sales cascades.

Next, assume the two banks have “effectively orthogonal” portfolios in the sense that the fire sale of one institution can never affect the other one in a first⁴ round of fire sales. If one runs a systemic stress test and aggregates corporate bonds across all geographic regions into a single asset class (as in Greenwood et al. (2015)), then one might incorrectly conclude that these banks cause direct fire sales losses to each other and overestimate the amount of contagion. The observed overlap in this case is clearly artificial and only due to an aggregation level that is too coarse.

The level of (dis-)aggregation of μ directly affects the sparsity of the liquidity-weighted overlap matrix Ω , and thereby the possible paths that contagion can take across this network⁵. In the extreme case where all marketable assets are aggregated into a single asset Π , the overlap matrix Ω collapses into a scalar value, and *any* shock that causes liquidation by some financial institution, will immediately spread to *all* institutions in the network. This is clearly not realistic either.

Consequently, if one wants to model a realistic “path of contagion” during a fire sales cascade, the “right” choice of granularity of the asset class becomes an important modelling choice, next to the market impact and response

⁴Contagion in higher rounds can be possible because of other institutions having a portfolio overlap with *both* banks, (or because there is a chain of such overlapping portfolios from one bank to the other).

⁵Note that such an issue does not arise in models of *direct* contagion: an institution either does or does not have a balance sheet exposure to another institution. This exposure cannot be amplified or reduced by re-grouping the individual balance sheet items that make up this exposure.

functions. In particular, this raises two important questions: First, can we aggregate asset classes in such a way that the estimated fire sales loss is invariant? Second, does there exist an “optimal level of granularity”, which reflects the path that spillover losses would likely take in a crisis scenario, as realistically as possible? We will explore the first, more theoretical question in the sequel, and attack the second more empirical question in Chapter 4.

THE MARKET DEPTH OF AN “ASSET CLASS”. Let $\mu = 1..M$ denote individual assets, with average daily volumes ADV_μ and volatilities σ_μ . We wish to aggregate the assets $\mu = 1..m (m \leq M)$ to form a new asset class ν . Natural candidates for the definitions of the average daily volume and volatility are

$$ADV_\nu := \sum_{\mu=1}^m ADV_\mu \quad \text{and} \quad \sigma_\nu := \frac{1}{O} \sum_{\mu=1}^m \sigma_\mu O_\mu, \quad (2.10)$$

where O_μ is the amount outstanding (or market cap) and $O := \sum_{\mu=1}^m O_\mu$. The market depth of the aggregated asset class could then be defined as

$$D_\nu(\tau) := c \frac{ADV_\nu}{\sigma_\nu} \sqrt{\tau}, \quad (2.11)$$

with the analogous adaptation for δ_ν in our level-dependent market impact function. We will use this definition to aggregate asset classes in the rest of this section.

2.2.1 INVARIANCE OF FIRE SALES LOSSES UNDER ASSET CLASS AGGREGATION

Concretely, we ask whether we can aggregate or granularise the market impact function Ψ_μ such that the fire sales loss is *invariant* under the level of granularity at which it is estimated. For this exercise, we will consider

$\Gamma^{i\mu}(\Pi, C_0(\epsilon), \epsilon) = q_\mu^i(\epsilon)/\Pi_{i\mu}$ simply to be a function that describes the fraction of volume sold, without specifying how $q_\mu(\epsilon)$ is determined by Γ . Let us further assume that $\Psi_\mu(q) = \psi(q/D_\mu)$, which – except for our level-dependent impact function – encloses the previous examples of linear, exponential and square-root impact. Finally, we make the simplifying assumption that all the volatilities $\sigma_1.. \sigma_m$ are equal, whence we get that $\sigma_\nu = \sigma_1$, and market depths become additive, $D_\nu = \sum_{m=1}^m D_\mu$. When i holds $\Pi_{i\mu}$ and liquidates q_μ , with μ 's market depth being D_μ , then the fire sales loss that it suffers in the process, pertaining to asset class μ , is given by

$$L_1^i(\Pi_{i\mu}, q_\mu, D_\mu) = (\Pi_\mu - (1 - \alpha)q_\mu)\psi\left(\frac{q_\mu}{D_\mu}\right). \quad (2.12)$$

The total fire sales loss suffered by i in the first round is hence $\sum_{\mu=1}^M L_1^i(\Pi_{i\mu}, q_\mu, D_\mu)$. In order to study the impact of asset (class) aggregation, we introduce the following

Definition 1. *If for all $\Pi_\mu, \Pi_\eta, q_\mu, q_\eta, D_\mu, D_\eta \in \mathbb{R}$:*

$$L_1^i(\Pi_\mu, q_\mu, D_\mu) + L_1^i(\Pi_\eta, q_\eta, D_\eta) = L_1^i(\Pi_\mu + \Pi_\eta, q_\mu + q_\eta, D_\mu + D_\eta), \quad (2.13)$$

*then the fire sales loss function L_1^i is **aggregation invariant**.*

When asset holdings, liquidation volumes and market depths are all equal, then the first round fire sales loss is indeed aggregation invariant:

$$\begin{aligned} & L_1^i(\Pi_\mu, q_\mu, D_\mu) + L_1^i(\Pi_\eta, q_\eta, D_\eta) \\ &= (\Pi - (1 - \alpha)q)\psi\left(\frac{q}{D}\right) + (\Pi - (1 - \alpha)q)\psi\left(\frac{q}{D}\right) \\ &= (\Pi + \Pi - (1 - \alpha)(q + q))\psi\left(\frac{q + q}{D + D}\right) \\ &= L_1^i(\Pi_\mu + \Pi_\eta, q_\mu + q_\eta, D_\mu + D_\eta). \end{aligned}$$

However, this is merely a peculiarity of the singular case $\Pi_\mu = \Pi_\eta = \Pi$,

$D_\mu = D_\eta = D$ and $q_\mu = q_\eta = q$. In practice, the deviation can be substantial. Figure 2.2.1 uses Banks A and B from the introductory example with marketable holdings given by

$$\Pi = \begin{pmatrix} 70 & 0 \\ 0 & 90 \end{pmatrix}.$$

We then aggregate the two asset classes so as to retrieve the previous case of the single marketable asset. Figure 2.2.1 plots the ratio

$$\frac{L_1^i(\Pi_\mu + \Pi_\eta, q_\mu + q_\eta, D_\mu + D_\eta)}{L_1^i(\Pi_\mu, q_\mu, D_\mu) + L_1^i(\Pi_\eta, q_\eta, D_\eta)}$$

for a 20 % initial shock as a function of the ratio δ_μ/δ_η (with floor $B = 0.5$). After aggregation, fire sales loss estimates can be up to 30% smaller compared to the disaggregated case. In the same manner, if the disaggregated level were too granular, we could be underestimating the fire sales loss by 30% in this example.

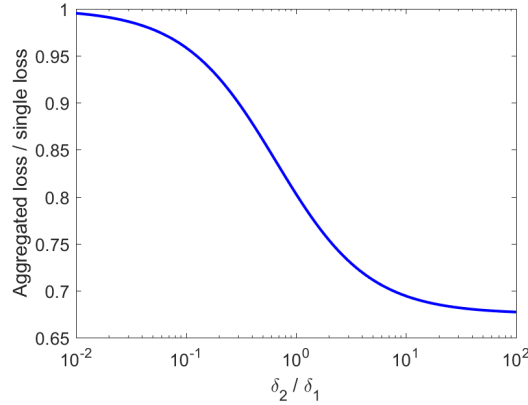


Figure 2.2.1: The plot shows the difference in fire sales losses for one bank when two assets of the same liquidity are merged into one single asset class. Losses are larger in the disaggregated case compared to the aggregated case. The liquidity of the underlying assets has little influence on the result.

As we shall see now, the general answer to our question on the existence of aggregation invariant fire sales loss functions is unfortunately negative for

any reasonable market impact function ψ .

Proposition 2.2.1. *There exists no aggregation invariant fire sales loss functions for multiplicative market impact functions of the form $\frac{\Delta S_\mu}{S_\mu} = \Psi_\mu(q_\mu) = \psi(\frac{q_\mu}{D_\mu})$, with ψ being continuous, increasing, concave, verifying $\psi(0) = 0$, and ψ not being uniformly zero.*

Proof. We show that there exists no aggregation invariant fire sales loss function for the single bank model. As for models with $n \geq 1$ banks, aggregation invariance must also hold for all one-bank subsystems, it follows a fortiori that there exists no aggregation invariant fire sales loss function for such systems either.

A fire sales function that is aggregation invariant is a function $L^i : \mathbb{R}^3 \rightarrow \mathbb{R}$ that fulfils the multidimensional Cauchy functional equation in three variables. It is well-known (Aczel and Dhombres (1989)) that if $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is a continuous function, and if

$$\begin{aligned} f(x_1 + y_1, x_2 + y_2, \dots, x_n + y_n) &= f(x_1, x_2, \dots, x_n) + f(y_1, y_2, \dots, y_n) \\ \text{for } x_k, y_k &\in \mathbb{R}, k = 1, \dots, n. \end{aligned}$$

Then, f is given by

$$f(x_1, x_2, \dots, x_n) = \sum_{k=1}^n c_k x_k. \quad (2.14)$$

This implies that

$$L^i(\Pi_\mu, q_\mu, D_\mu) = c_1 \Pi_\mu + c_2 q_\mu + c_3 D_\mu. \quad (2.15)$$

Comparing coefficients between (2.12) and (2.15), we see that clearly this is not compatible with our assumptions on ψ . $\square$

Knowing that, in general, we cannot aggregate asset classes without changing the resulting fire sales loss estimates, it becomes of interest to know more precisely how these estimates may change. From (2.10), it follows $\sigma_\nu \in [\min_\mu \{\sigma_\mu\}, \max_\mu \{\sigma_\mu\}]$, and $ADV_\nu \geq \max_\mu ADV_\mu$. On an intuitive level, it seems plausible that the liquidity of the aggregate asset class should be larger than the liquidity of each constituting asset (class), i.e. for all $\mu : D_\nu \geq D_\mu$. Technically however it is possible to find combinations of $ADV_\mu, \sigma_\mu, O_\mu > 0$ such that $D_\nu < D_\mu$. Indeed,

Proposition 2.2.2. *Consider the aggregation of $\mu = 1, 2$ as in (2.10). If $\sigma_1 < \sigma_2$ and ADV_1 is sufficiently large, or if σ_1 is sufficiently small, then $D_\nu < D_1$.*

Proof. We wish to know when

$$D_\nu = c \frac{ADV_1 + ADV_2}{\sigma_1 \frac{O_1}{O} + \sigma_2 \frac{O_2}{O}} \sqrt{\tau} \leq c \frac{ADV_1}{\sigma_1} \sqrt{\tau} = D_1. \quad (2.16)$$

WLOG, set $c = \tau = 1$. If we let $\sigma_1 \rightarrow 0$, then we get

$$\lim_{\sigma_1 \rightarrow 0} D_\nu = \frac{ADV_1 + ADV_2}{\sigma_2 \frac{O_2}{O}} < \infty \quad \text{while} \quad \lim_{\sigma_1 \rightarrow 0} D_1 = +\infty. \quad (2.17)$$

Hence there exists σ^* such that if $\sigma_1 < \sigma^*$, then $D_\nu < D_1$.

Next, assume simply that $\sigma_1 < \sigma_2$. It is easy to see that then $\sigma_1 \frac{O_1}{O} + \sigma_2 \frac{O_2}{O} > \sigma_1$. In that case, similarly, there exists an $ADV^* > 0$ such that if $ADV_1 \geq ADV^*$, then $D_\nu < D_1$. As $\delta_\nu = (1 - \frac{B}{S_0})D_\nu$, the inequalities of D also apply to δ . $\square$

As in practice a high ADV and low volatility are often associated with a large market capitalisation, we expect this not to happen too often in practically relevant cases. Nevertheless, aggregation of asset classes changes the cross-sectional distribution of fire sales losses, and thereby potentially the path

of contagion of a fire sales cascade. Depending on the state of the system and magnitude of the different parameters and variables, we expect three different effects to occur, potentially simultaneously, upon aggregation:

1. Fire sales losses of certain institutions decrease because the market depth of assets held has increased
2. Fire sales losses of certain institutions increase because the market depth of assets has decreased
3. Fire sales losses of certain institutions increase because an artificial overlap appears

In the extreme case this can lead to (unpredictable!) changes in the contagion dynamics: If the decrease in fire sales losses is sufficiently large, some institutions that were predicted to engage in fire sales pre-aggregation would not need to delever their portfolio, thereby further reducing fire sales losses to all institutions with whom they have overlapping portfolios. Second, if the increase in losses under effects (2) and (3) is sufficiently large some institutions may be led to deleverage their portfolio again, or for the first time, if they did not engage in fire sales at all pre-aggregation. Clearly, this additional deleveraging increases the fire sales losses to all institutions with whom these banks have overlapping portfolios and may therefore also change the dynamics of the subsequent iterations.

The important question is how the joint effect of these changes plays out in a concrete systemic stress test. We will attempt to offer a first answer to this empirical question by using ISIN level supervisory data from Norway and performing such an aggregation study in the next Chapter.

In summary, picking the right level of granularity for asset classes in systemic stress tests becomes both a heuristic and important task, as it may have a sizeable quantitative effect on the results.

2.3 ALTERNATIVE DELEVERAGING FUNCTIONS

Next we consider two alternatives for the banks' response function. First, we present the so-called “leverage targeting” model as introduced by [Greenwood et al. \(2015\)](#) and used thereafter in a number of different academic and regulatory studies on fire sales ([Duarte and Eisenbach \(2013\)](#), [Capponi and Larsson \(2015\)](#), [Cappiello and Spuera \(2014\)](#), [Cappiello et al. \(2015\)](#)). Thereafter, we will adapt the work of [Braouezec and Wagalath \(2017\)](#), in which banks seek to minimise the losses from price impact, to our framework and extend it to an N banks, M assets setting.

2.3.1 LEVERAGE TARGETING MODELS

The origins of the leverage targeting model of fire sales is often traced back to the empirical contribution of [Adrian and Shin \(2010\)](#). Using quarterly data, the authors show, convincingly, how US commercial banks adjust their balance sheets so as keep approximately a constant leverage over time⁶. Using this observation as starting point, [Greenwood et al. \(2015\)](#) build an elegant linear model of fire sales: In response to a shock that increases banks' leverage, they shed assets so as to return exactly to the leverage they previously had. The response function corresponding to this behaviour was shown as blue dashed lines in Figure 1.1.1.

As dynamical system, the leverage targeting model has the same representation, except that its deleveraging proportion is given by

$$\Gamma_{L.T.}^i(\Pi_k, C_k) = \frac{\left(\sum_{\mu=1}^M \Pi_k^{i\mu} + I_i(\epsilon) - C_k^i \lambda_{L.T.}^i\right)}{\sum_{\mu=1}^M \Pi_k^{i\mu}} \wedge 1, \quad (2.18)$$

⁶The authors also provide evidence that broker dealers overcompensate, i.e. increase their leverage in boom times and decrease it in downturns, resulting in “procyclical” leverage.

where constant $\lambda_{L.T.}^i$ is the institution i 's initial leverage and leverage target. The key difference between both models is thus the use of a different leverage target, and more importantly, a linear and symmetric reaction function which leads to fire sales for arbitrarily small shocks, as opposed to the threshold-based, antisymmetric reaction function which is zero for small shock sizes (and price increases), reflecting the *one-sided* nature of portfolio constraints. Technically, this is captured by the (non-)use of the indicator function $\mathbb{1}_{\lambda_t^i > \lambda_{\max}}$ which signals the breach of a portfolio constraint and the onset of deleveraging. We now discuss some properties of the fire sales cascade under a leverage targeting response function.

BASINS OF ATTRACTION. We begin by analysing the set of fixed points of the cascade.

Proposition 2.3.1. *The set of fixed points $x = (\Pi, C)$ of the dynamical system f with a leverage targeting response function is given by the set $\mathcal{F}_{I(\epsilon)} := \prod_{i=1}^N \mathcal{S}_i \cup \mathcal{I}_i \cup \mathcal{J}_i$, where each component $(\Pi^i, C^i) \in \mathbb{R}^M \times \mathbb{R}$ lies either on the **survival line***

$$\mathcal{S} := \left\{ (\Pi^i, C^i) \in \mathbb{R}^M \times \mathbb{R} \left| \forall i : \sum_{\mu=1}^M \Pi^{i\mu} > 0, \quad C^i > 0, \quad \sum_{\mu=1}^M \Pi^{i\mu} + I_i(\epsilon) - \lambda_{\max} C^i = 0 \right. \right\}, \quad (2.19)$$

*the **illiquidity line***

$$\mathcal{I} := \left\{ (\Pi^i, C^i) \in \mathbb{R}^M \times \mathbb{R} \left| \forall i : \sum_{\mu=1}^M \Pi^{i\mu} = 0, \quad C^i > 0 \right. \right\}, \quad (2.20)$$

*or the **insolvency point***

$$\mathcal{J} := \left\{ (\Pi^i, C^i) \in \mathbb{R}^M \times \mathbb{R} \left| \forall i : \sum_{\mu=1}^M \Pi^{i\mu} = 0, \quad C^i = 0 \right. \right\}. \quad (2.21)$$

Proof. The proof is analogous to Prop. 2.1.1 of the threshold model. $\square$

A consequence of this is the following

Proposition 2.3.2. *The fire sales cascade induced by the leverage targeting model is finite if and only if all banks default, i.e. for all i , $(\Pi^i, C^i) \in \mathcal{I}_i \cup \mathcal{J}_i$.*

The following proposition discusses orthogonal stress scenarios under leverage targeting dynamics.

Proposition 2.3.3. *Let $\Omega = \Pi D^{-1} \Pi^\top$ be the liquidity-weighted overlap matrix of a finite, long-only portfolio matrix Π , where D is the diagonal matrix of market depths, $D_\mu < \infty$ for all μ . If the undirected graph described by the weighted adjacency matrix Ω is connected, then, in the leverage targeting model, there exist no two shock scenarios that lead to orthogonal fire sale losses in the sense of (2.7).*

Proof. Let there be N banks and M asset classes. For any initially shocked bank i , fire sales losses spread to all neighbours of i , i.e. all institutions that hold any asset in common with i . As the graph induced by Ω is connected, its diameter is bounded by $\min\{N, M\}$ ($= N$, say). Therefore, for any scenario, after at most N steps, $L_N^j > 0$ for all j . Consequently, $\langle \vec{FL}(L_{0,a}), \vec{FL}(L_{0,b}) \rangle > 0$ for any two of arbitrary initial losses $L_{0,a}$ and $L_{0,b}$, and it is not possible to build two sets $V_a, V_b \subset V$ of orthogonal stress scenarios. $\square$

In Chapter 3, we will see that these theoretical differences lead to substantial differences in practical simulations.

2.3.2 DELEVERAGING BY LOSS MINIMISING BANKS.

Next we relax the proportional deleveraging assumption by allowing banks to optimise the order in which they liquidate assets within the marketable

asset class so as to minimise the resulting fire sales losses. To do so each bank solves in each period k

$$\min_{\Gamma^i \in [0,1]^M} \widehat{L}_k^i$$

under the constraint

$$\frac{\sum_{\mu=1}^M \widehat{\Pi}_k^{i\mu} + I^i(\epsilon)}{C_k^i} = \lambda_b \leq \lambda_{\max}.$$

The hat-variables highlight the fact that banks do not possess full information about the state of the network, and only anticipate their own impact. For notational convenience, we abbreviate $\Gamma^{i\mu}(\Pi_{k-1}, C_{k-1})$ to $\Gamma_k^{i\mu}$. Specifically the banks thus solve

$$\min_{\Gamma_k^{i\mu} \in [0,1]} \sum_{\mu=1}^M (1 - (1 - \alpha)\Gamma_k^{i\mu}) \Pi_{k-1}^{i\mu} \Psi_{\mu} \left(\Pi_{k-1}^{i\mu} \Gamma_k^{i\mu} \right) \quad (2.22)$$

under the constraint

$$\frac{\sum_{\mu=1}^M (1 - \Gamma_k^{i\mu}) \Pi_{k-1}^{i\mu} \left(1 - \Psi_{\mu} \left(\Pi_{k-1}^{i\mu} \Gamma_k^{i\mu} \right) \right) + I^i(\epsilon)}{\left(C_{k-1}^i - \sum_{\mu=1}^M (1 - (1 - \alpha)\Gamma_k^{i\mu}) \Pi_{k-1}^{i\mu} \Psi_{\mu} \left(\Pi_{k-1}^{i\mu} \Gamma_k^{i\mu} \right) \right)_+} = \lambda_b. \quad (2.23)$$

Using the linearisation

$$\Psi'_{\mu}(q) = \frac{\Gamma_k^{i\mu} \Pi_{k-1}^{i\mu}}{D_{\mu}},$$

and the additional assumptions that capitals and all prices remain positive the problem becomes

$$\min_{\Gamma_k^{i\mu} \in [0,1]} \sum_{\mu=1}^M (1 - (1 - \alpha)\Gamma_k^{i\mu}) \Pi_{k-1}^{i\mu} \frac{\Gamma_k^{i\mu} \Pi_{k-1}^{i\mu}}{D_{\mu}}$$

under the constraint

$$\sum_{\mu=1}^M (1 - \Gamma_k^{i\mu}) \Pi_{k-1}^{i\mu} \left(1 - \frac{\Gamma_k^{i\mu} \Pi_{k-1}^{i\mu}}{D_\mu} \right) + I^i(\epsilon) = \lambda_b \left(C_{k-1}^i - \sum_{\mu=1}^M (1 - (1 - \alpha) \Gamma_k^{i\mu}) \Pi_{k-1}^{i\mu} \frac{\Gamma_k^{i\mu} \Pi_{k-1}^{i\mu}}{D_\mu} \right),$$

which is again equivalent to

$$\min_{\Gamma_k^{i\mu} \in [0,1]} \sum_{\mu=1}^M -(\Gamma_k^{i\mu})^2 (1 - \alpha) \frac{(\Pi_{k-1}^{i\mu})^2}{D_\mu} + \Gamma_k^{i\mu} \frac{(\Pi_{k-1}^{i\mu})^2}{D_\mu}$$

under the constraint

$$\begin{aligned} & \sum_{\mu=1}^M \Gamma_k^{i\mu} \left((\lambda_b - 1) \frac{(\Pi_{k-1}^{i\mu})^2}{D_\mu} - \Pi_{k-1}^{i\mu} \right) + (\Gamma_k^{i\mu})^2 \left((1 - \lambda_b(1 - \alpha)) \frac{(\Pi_{k-1}^{i\mu})^2}{D_\mu} \right) \\ & - \lambda_b C_{k-1}^i + I^i(\epsilon) + \sum_{\mu=1}^M \Pi_{k-1}^{i\mu} = 0. \end{aligned}$$

This is a concave minimisation problem given by a quadratic objective function with quadratic constraints. As such it can be readily formulated as a “difference of convex functions program” (DC-programming [Horst and Thoai \(1999\)](#), [An and Tao \(2005\)](#)), for which there is a host of numerical algorithms available. We simply turn to MATLAB’s *fmincon* solver to study the solution of the problem in a simple 1 Bank, 2 asset case. Specifically, we apply a 15 % shock to Bank A, and plot in the left panel of Figure 2.3.1, as a function of the ratio of the two market depths, the amount of asset 1 and asset 2 respectively that the bank delevers in order to return to its target leverage for the full non-linear problem⁷. To vary the ratio of market depths, we simply keep the market depth of asset 1 fixed, and vary the market depth of asset 2 between $\frac{\delta_1}{3}$ and $3\delta_1$. For all practical purposes, the optimal solution boils down to selling assets sequentially from the most liquid to the least liquid (because as we shall see that the estimated market depth of assets differs

⁷In this range the linear approximation was close to identical. For this reason, we do not report it.

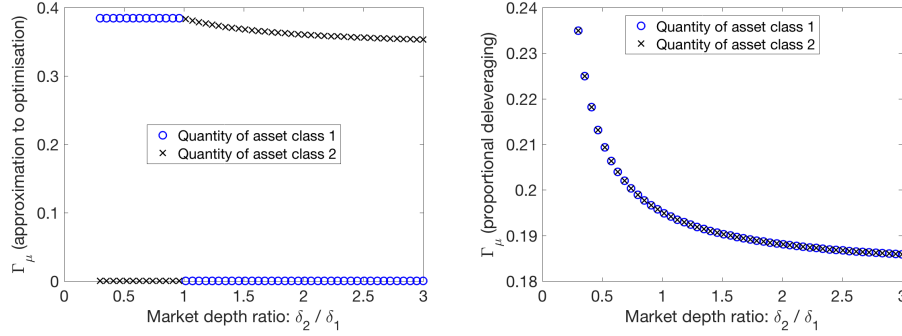


Figure 2.3.1: Left: Solution to the optimisation problem given by (2.22) and (2.23). Right solution to the problem with the additional constraint that $\Gamma^1 = \Gamma^2$.

by several orders of magnitudes for different asset classes). The right hand panel shows the solution of the problem under the proportional deleveraging assumption. This is implemented using the minimisation problem (2.22) with constraint (2.23) and the additional constraint that $\Gamma^{i,1} - \Gamma^{i,2} = 0$.

One may wonder how the picture changes if banks do not solve this exact optimisation problem, but rather follow some heuristic deleveraging rule. We consider two such heuristic rules: First the bank delevers assets proportionally to their market depth, second it sells proportionally to the fraction of marketable assets over market depth. We have implemented these deleveraging heuristics numerically as fixed point iterations. The panels in Figure 2.3.2 report the results of this numerical exercise. We see that the heuristics of selling assets proportionally to their market depth is reasonably close to the optimal solution. On the other hand, the heuristics of selling assets as ordered by the ratios $\frac{\pi_\mu}{\delta_\mu}$ is clearly not optimal, because the order is reversed.

Even though banks at each step optimise only under the information that is available to them, the system dynamics account for all bank deleveraging

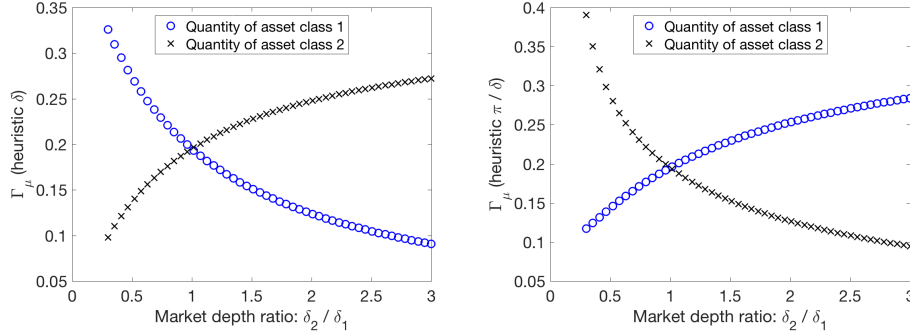


Figure 2.3.2: Heuristic solutions to the optimisation problem.

and are given by

$$\begin{aligned} \Pi_k^{i\mu}(\Pi_{k-1}, C_{k-1}) &:= \left(1 - \Gamma_k^{i\mu}\right) \Pi_{k-1}^{i\mu} \left(1 - \Psi_\mu \left(\sum_{j=1}^N \Gamma_k^{j\mu} \Pi_{k-1}^{j\mu}\right)\right) \\ C_k^i(\Pi_{k-1}, C_{k-1}) &:= \left(C_{k-1}^i - \left(1 - (1 - \alpha)\Gamma_k^{i\mu}\right) \sum_{\mu=1}^M \Pi_{k-1}^{i\mu} \Psi_\mu \left(\sum_{j=1}^N \Gamma_k^{j\mu} \Pi_{k-1}^{j\mu}\right)\right)_{+}, \end{aligned}$$

where $\Gamma_k^{i\mu}$ is the solution of the corresponding minimisation problem. Finally, we compare the fire sales loss suffered by the bank under these different deleveraging schemes. The left panel of Figure 2.3.3 shows the fire sales loss in monetary units of the bank under these different deleveraging schemes as a function of the ratio of market depths. The right plot shows the ratio of fire sales losses as a function of the market depth ratios. The larger the difference in market depths, the more beneficial it becomes for the bank to optimise the deleveraging order in this setting. We will conduct a larger empirical analysis on using these deleveraging function in Section 3.3.

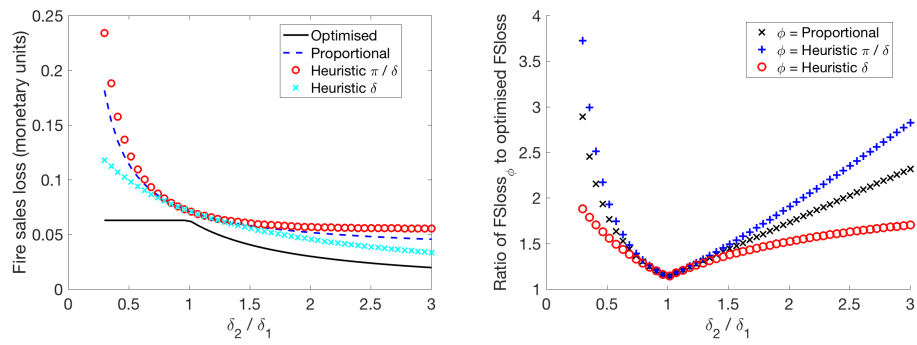


Figure 2.3.3: Left: Fire sales loss as a function of the ratio of the two market depths for different deleveraging schemes. Right: Ratio of fire sales loss under different deleveraging schemes relative to the optimal, sequential, deleveraging scheme.

Lady Bracknell: Good afternoon, dear Algernon, I hope you are behaving very well.
Algernon: I'm feeling very well, Aunt Augusta.
Lady Bracknell: That's not quite the same thing. In fact the two things rarely go together.

Oscar Wilde

3

SYSTEMIC STRESS TESTING

This Chapter is based on the working papers: “Fire sales, indirect contagion and systemic stress testing” and “Asset class granularity: Aggregation issues and liquidity estimates for stress testing” which are joint work with Prof. Rama Cont.

3.1 A SYSTEMIC STRESS TEST FOR THE EUROPEAN BANKING SYSTEM

We now use a dataset of European banks by the European Banking Authority to illustrate how the model may be calibrated to data and used to perform a systemic stress test.

3.1.1 DATA AND PARAMETER CALIBRATION

BANK BALANCE SHEETS

Our empirical study is based on data from the European Banking Authority (EBA), which provides information, collected in 2011, on notional exposures of 90 European banks across 148 asset classes.¹ Holdings are distinguished by geographical regions and asset classes, as specified in Table 3.1.1. [Greenwood et al. \(2015\)](#) assumed all assets to be available for liquidation; we only consider a subset to be marketable i.e. available for liquidation at short notice. We identify four classes of marketable assets (“securities”), which may be liquidated in a stress scenario; the other asset classes are classified as illiquid assets.² Assets are further labeled by 37 geographic regions, which correspond to the 27 countries of the EU (i.e. without Croatia at the time) plus the United States (US), Norway (NO), Iceland (IS), Liechtenstein (LI), Japan (JP), Asia (A1), Other non-EEA non-emerging countries (E3), Eastern Europe non-EEA (E5), Middle and South America (M1) and Rest of the world (R5). Hence, with the four marketable asset classes and 37 geographic regions, the matrix of marketable assets Π is given by a 90×148 matrix. The illiquid asset holdings are given by a 90×75 matrix Θ . This corresponds to 74 asset classes for commercial and residential mortgage exposures respectively in the 37 regions and a 75th asset class consisting of all remaining illiquid asset holdings.³

¹This data set was also used in studies by e.g. [Greenwood et al. \(2015\)](#), [Gandy and Veraart \(2016\)](#) and [Chen et al. \(2016\)](#), and hence facilitates comparison with the literature.

²In Table A.2.1 in the Appendix, we provide the data identifiers that allow correspondence with the EBA dataset.

³Our representation differs slightly from [Greenwood et al. \(2015\)](#) who considered 42 asset classes consisting of the 37 sovereign exposures by geographic region and five further classes, aggregated across all geographical regions: “commercial real estate”, “mortgages”, “corporate loans”, “small and medium enterprise loans” and “retail revolving credit lines”. This leads to a less granular model compared to ours as we distinguish assets both by type and country.

Illiquid assets
Residential mortgage exposures
Commercial real estate exposure
Retail exposures: Revolving credits, SME, other
Indirect sovereign exposures in the trading book
Defaulted exposures
Residual exposure (cf. Appendix Table A.2.1)
Securities / marketable assets
Corporate bonds
Sovereign debt
Direct sovereign exposures in derivatives
Institutional client exposures: interbank, CCPs,...

Table 3.1.1: Asset classes on the model balance sheets used in the systemic stress testing exercise.

ESTIMATING MARKET DEPTH

Let us recall the equations for the market impact function

$$\Psi_{\mu}(q, S) := \left(1 - \frac{B_{\mu}}{S}\right) \left(1 - \exp\left(-\frac{q}{\delta_{\mu}}\right)\right)$$

and market depth

$$\delta_{\mu} = \left(1 - \frac{B_{\mu}}{S_0^{\mu}}\right) D_{\mu},$$

respectively, with $D_{\mu} = c \frac{ADV_{\mu}}{\sigma_{\mu}} \sqrt{\tau}$, and proceed now to explain their calibration.

[Greenwood et al. \(2015\)](#) assume a uniform depth $D_{\mu} = 10^{13}$ (EUR) for all asset classes. This homogeneity assumptions is not supported by empirical studies on market impact [Bouchaud \(2010\)](#), [Obizhaeva \(2012\)](#), [Cont et al. \(2014\)](#) which indicate that market impact varies widely across assets. Ignoring this heterogeneity may lead to biased results, overestimating losses in more liquid asset classes while underestimating losses in less liquid asset classes.

Duarte and Eisenbach (2013) use haircuts and repo rates for determining the liquidity of different asset classes; this does introduce some heterogeneity across asset classes but the relation between these quantities and market impact is not entirely clear. For instance, haircuts may simply reflect the volatility of an asset, rather than be a predictor of an asset’s price resilience to large liquidations. We use a direct, data-driven approach to the modelling of market impact. To estimate the market depth parameters for each asset class using (1.14), we

- estimate volatility parameters σ_μ using daily returns of S&P sector indices⁴.
- obtain average daily volume estimates ADV_μ from annual volume data provided by the US Treasury and various central banks (Appendix A.1).

As noted above, we use $\tau = 20$ which corresponds to a liquidation horizon of 4 weeks, a fairly lenient assumption. Following the empirical findings in Obizhaeva (2012), Cont et al. (2014), we use $c = 0.4$ which corresponds to a one standard deviation price move for 40% of average daily trading volume (Obizhaeva (2012)). ADV for US corporate bonds was obtained from SIFMA (see Appendix A.1). For European bonds, we simply use the same sovereign-to-corporate ratio of ADV as observed in the US ($567.81bn/269.8bn \approx 0.48$) to estimate the ADV of European bonds in the “corporate” and “institutional” asset classes.

For some asset classes, data on trading volume are unavailable (or difficult to obtain). We work around this issue by estimating, based on OECD data, the following regression model for the relation between average daily volume

⁴<http://us.spindices.com/indices/fixed-income/sp-eurozone-sovereign-bond-index>
<http://us.spindices.com/index-family/us-treasury-and-us-agency/all>
<http://us.spindices.com/indices/fixed-income/sp-500-investment-grade-corporate-bond-index>

Asset class (sovereign unless specified)	ADV (bn EUR)	Market depth D_μ (10^{12} EUR)	Impact of 10 bn
US	567.8	428.8	0.00466
US (corp.)	269.8	70.2	0.02848
IT	28.82	21.8	0.09166
ES	28.0	21.1	0.09470
DE	24.7	18.7	0.10684
GB	24	18.1	0.11038
FR	10	7.58	0.26316
GR	8.17	6.18	0.32258
BE	6	5.53	0.36036
SE	3.92	2.96	1.67113
PT	2.27	1.71	1.15602

Table 3.1.2: Average daily trading volume and estimated market depth over $\tau = 20$ days, for the largest holdings in the EBA dataset. The impact in basis points are also given for a liquidation of 10 bn EUR over $\tau = 20$ days.

(ADV) and outstanding notional

$$\log ADV_\mu := c_1 \log(N_\mu) + c_0 + \varepsilon_\mu, \quad (3.1)$$

where N_μ denotes outstanding notional. We use the resulting estimates to calibrate the volumes for the remaining asset classes. Table 3.1.3 shows the results of this regression analysis for selected countries.

Coefficient	US treas	US corp	UK	DE	ES
c_1 :	0.53***	0.64***	0.56***	0.48	0.94***
std. dev.	0.13	0.15	0.05	0.30	0.14
c_0 :	0.63	-1.1**	-0.35**	0.4	-1.35***
std. dev.	0.55	0.55	0.14	0.99	0.37
adj. R^2	0.39	0.60	0.93	0.15	0.76
n	19	19	11	10	15

Table 3.1.3: Logarithmic regression of bond trading volume on outstanding notional (3.1).

Figure 3.1.1 shows the distribution of market depth estimates for all asset classes in the EBA data set, on a logarithmic scale. The histogram reveals considerable heterogeneity in the cross-sectional distribution of mar-

ket depth, with more than four orders of magnitude separating the most liquid from the least liquid assets. Table 3.1.2 provides values for average

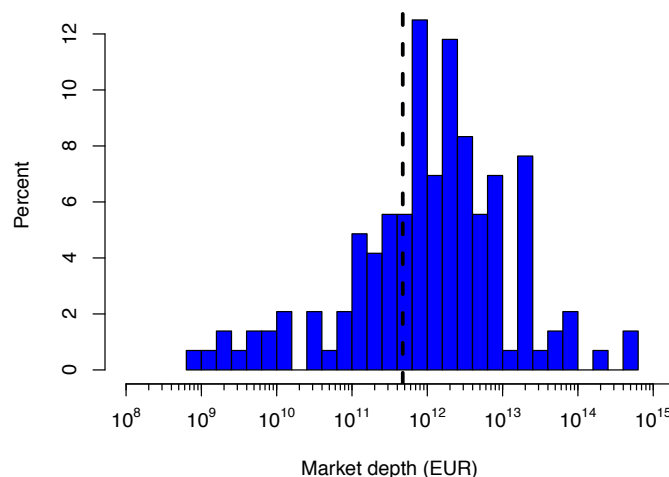


Figure 3.1.1: Histogram of estimated market depths for the 148 asset classes in the EBA dataset. The dashed vertical line indicates the holdings-weighted average depth given by (3.3).

daily volumes (ADV) and market depths for the asset classes representing the targets holdings. These values are used as base values; we will later perform a sensitivity analysis of our results with respect to changes in these values.

PRICE FLOOR B . In the results that we report below, we have assumed a simple uniform lower price floor at 50% of the initial value for all assets. It is of course possible to refine this approach, e.g. by allowing corporate bonds to fall further than sovereign debt, and by accounting for differences in credit ratings of the issuers for instance, or whether bonds are collateralised. To calibrate such a model in a realistic manner, one would however require more detailed information about the assets in each asset class, than what is provided by the EBA data. We have conducted a limited sensitivity

analysis allowing different price floors for different regions (differentiated by the credit rating of the issuing country) and did not find substantially different results. For the sake of brevity and simplicity, we therefore only report results using the uniform floor of 50%.

3.1.2 STRESS SCENARIOS

We define stress scenarios, similar to regulatory stress tests, in terms of percentage shocks across asset classes⁵, given by a vector $\epsilon = (\epsilon_\kappa, \kappa = 1..K) \in [0, 1]^K$ where ϵ_κ is the percentage loss (stress level) applied to asset class κ . We have chosen four scenarios in the examples below to illustrate the properties of the model:

1. Losses on residential and commercial real estate exposures in Spain (Region: ES);
2. Losses on residential real estate exposures in Northern and Western Europe (Regions: GB, BE, NO, SE);
3. Losses on commercial real estate exposures in Southern Europe (Regions: IT, GR, ES, PT);
4. Losses on commercial real estate exposures in Eastern Europe (Regions: CZ, EE, HU, LV, LT, PL, RO, SK, RU, BG, E3 (= other non-EEA non-emerging countries));

In each scenario, we increase the shock to the affected asset classes gradually from 0% to 20%; the other asset classes undergo no initial loss. To emphasise cross-asset contagion, we have chosen stress scenarios where initial losses

⁵The design of such scenarios is far from trivial and outside the scope of this presentation. On this occasion, we note that ironically the 2016 EBA stress test did not feature a “Brexit” scenario.

affect only illiquid assets in these examples, but scenarios can be extended to include initial shocks to marketable assets.

Let us recall the expression for the initial loss of bank i : $L_0^i = \sum_{\kappa=1}^K \Theta^{i,\kappa} \epsilon_{\kappa}$. Because banks have different nominal exposures in the illiquid assets, the stress scenario defined in terms of percentage shocks to asset classes leads to different losses in euro for different banks. As our stress scenarios focus on residential and commercial real estate exposures, it is important to note that in our model a “shock” of 5% to asset values means that banks suffer a loss equal to 5% of their notional exposure in these asset classes. This corresponds to an initial shock that is commensurate with the losses that banks have historically suffer during severe housing crises⁶.

STRESS SCENARIO SEVERITIES COMPARED TO OFFICIAL EBA STRESS-TEST SCENARIO. The final report of the EBA stress test states on p.10 that the net loss in the stress scenario reduces the average capital ratio from 8.9 % down to 7.7 %⁷. This net stress-test loss in the EBA scenario, is precisely the initial loss that our model uses as input to trigger a (potential) fire-sales cascade. Taking as an example the Spanish bank Santander, which had EUR 594 bn in risk-weighted assets (RWA), a 1.2% loss of RWA in capital corresponds exactly to an initial loss of EUR 7.13 bn. From the dataset we know that Santander holds EUR 82 bn direct exposures to Spanish residential and commercial mortgages. Hence an initial shock of 8.7% in our Scenario 1 corresponds exactly to the severity of the EBA stress-test loss for Santander. Our 20% shock in this scenario corresponds to an initial loss that is about 2.3 times the severity of the EBA stress test. Figure 3.1.2 shows that for different Spanish banks the initial shock size to Spanish residen-

⁶See a comparison of losses in percentage of gross lending across different countries’ housing crises in Chart 6 of the Norges Bank Staff Memo:http://static.norges-bank.no/pages/104012/Staff_Memo_5-2015_eng.pdf

⁷See http://www.eba.europa.eu/documents/10180/15935/EBA_ST_2011_Summary_Report_v6.pdf.

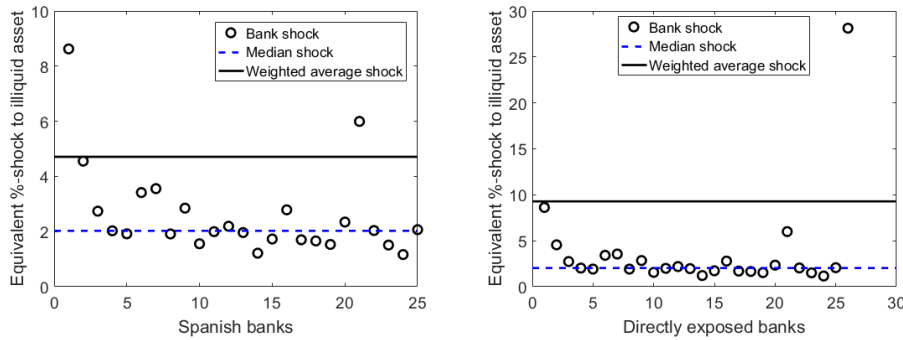


Figure 3.1.2: The initial shocks ϵ to the Spanish residential and commercial real estate exposures which, for each bank, generate an initial loss equal to 1.2 percent of RWA. Left: for Spanish banks. Right: For banks with direct exposures in excess of EUR 1 bn.

tial and commercial exposures which generates losses equal to 1.2 percent of RWA lies between 1.17 % and 8.7%. Taking the weighted (by RWA) average shock, one obtains an initial shock of 4.7%; the median shock is 2.03%. If instead of considering Spanish banks only, we consider banks with direct exposures in excess of 25% of their capital, we obtain the right plot. In this case, smaller Spanish banks with different business models have dropped out of the sample. The weighted average shock required to trigger losses equal to 1.2% of RWA in this case is equal to 9.3%. Overall, for this scenario, a reasonably severe regulatory stress test would thus fall into the range of 4 - 10%.

We perform the same computations for scenarios 2, 3 and 4, and report the results in Table 3.1.4 to provide a mapping of our scenario intensities to the official EBA stress scenario intensity.

Finally, for completeness we note that in [Greenwood et al. \(2015\)](#) and [Duarte and Eisenbach \(2013\)](#) the initial shocks are given by a 50% fall in GIIPS debt, and a 1% reduction of all assets, respectively. This corresponds to initial losses of EUR 343 bn and EUR 233 bn on a system wide level, and RWAs of 5.3 % and 2.07% for all affected banks, respectively. In comparison to our scenarios, as well as the EBA official scenario, these initial shocks are

<i>Correspondence of stress scenario intensities</i>	Scen. 1	Scen. 2	Scen. 3	Scen. 4
Weighted average shock size that creates EBA-equivalent initial losses (regional sample)	4.71%	6.38 %	36.5%	NA
Weighted average shock that creates EBA-equivalent initial losses (“direct exp. > 25% capital” sample)	9.28 %	9.14 %	24.0 %	33.5%**

Table 3.1.4: Asset class initial shock that needs to be applied in the four scenarios so as to generate an average initial loss in height of 1.2% of RWA, corresponding to the EBA stress scenario intensity. **Only six (small) banks have exposure in excess of 25% of their capital to these asset classes.

<i>Initial Losses</i>	EBA	Greenwood et al.	Duarte and Eisenbach ⁸
Net loss to average bank in % of RWA	1.2%	7.63% (GIIPS banks) 7.17% (large exp. banks) 5.25 % (all banks)	2.07% (all banks)

Table 3.1.5: Comparison: Magnitude of the initial loss in percent of RWA.

thus quite large. Both [Greenwood et al. \(2015\)](#) and [Duarte and Eisenbach \(2013\)](#) only consider one specific initial shock and do not vary the intensity of the stress.

COMPOSITION OF INITIAL SHOCK. The four scenarios that we consider do not only vary significantly in terms of intensity, but also in terms of their *composition*, i.e. which group of banks is affected by the initial losses. A desirable property of a fire sales model for systemic stress testing is that it respects the intuition that “dissimilar stress scenarios affect different institutions and evolve differently”. Consequently, one would expect that different initial loss distributions also lead to different fire sales loss distributions across banks. To this end, we wish to analyse to what degree the fire sales model is responsive to the *composition* of the initial losses. The “initial loss vector” $\vec{L}_0 \in \mathbb{R}^{90}$ records the initial loss for each bank in a given scenario.

⁸Duarte and Eisenbach use a different dataset, so we simply apply the same shock as they do: 1% loss to *all* assets.

We can then assess how similar or different two stress scenarios are by computing the pairwise sample correlations between the initial loss vectors of two such scenarios. Table 3.1.6 lists the pairwise sample correlations for all combinations of initial loss vectors resulting from the different scenarios.

Scenarios	1 & 2	1 & 3	1 & 4	2 & 3	2 & 4	3 & 4
$\rho_{a,b}^0$	0.0840	0.2130	-0.1449	-0.0509	0.0394	-0.0149

Table 3.1.6: Sample correlations between the initial loss vectors from the stress scenarios. The correlations are close to zero and indicate that the four stress scenarios are quite different in terms of which banks are hit by the initial shock. As one would expect, scenarios 1 & 3 are the most similar ones.

At the end of the fire sales cascade, we store the final bank-level fire sales losses in the vector $F\vec{Loss}(\epsilon) \in \mathbb{R}^{90} = (F\text{Loss}(1, \epsilon), \dots, F\text{Loss}(90, \epsilon))^\top$ and compute the sample correlation coefficients

$$\rho_{a,b}^{F\text{Loss}} = \frac{\text{Cov}(F\vec{Loss}(\epsilon_a), F\vec{Loss}(\epsilon_b))}{\sigma_{F\vec{Loss}(\epsilon_a)} \sigma_{F\vec{Loss}(\epsilon_b)}} \quad (3.2)$$

between the loss vectors obtained in different scenarios ϵ_a and ϵ_b . If the fire sales model is sensitive to the composition of the initial shock, then we expect that a small $\rho_{a,b}^0$ tends to lead to a low $\rho_{a,b}^{F\text{Loss}}$. As we shall see in the sequel, the (non-)existence of “orthogonal stress scenarios” (Prop. 2.1.5 and 2.3.3) results in drastically different results for the threshold and leverage targeting model in this respect.

3.2 RESULTS

We now report the results of our systemic stress test for the European banking system.

To perform the stress test, we use a leverage constraint $\lambda_{\max} = 33.3$, which corresponds to the Basel III leverage constraint. An alternative would be to use as constraint the ratio of capital to risk-weighted assets (RWA), or

both. Given the limitations of our data set, we have opted for the former, but the same exercise may be conducted with both, or further, constraints.

As discussed in Section 1.1.3, we assume a target leverage slightly below the constraint $\lambda_b = 0.95\lambda_{\max} = 31.3$ in order to have a buffer. We have conducted the stress test for various values $0.9\lambda_{\max} \leq \lambda_b \leq \lambda_{\max}$; results are similar across this range whence for brevity we only report them for $\lambda_b = 0.95\lambda_{\max}$. In most examples, fire sales die out after a few rounds; in all cases we have limited the iterations to 20 rounds and report losses for each round and the total losses for all rounds.

3.2.1 MAGNITUDE OF FIRE SALES LOSSES

Figure 3.2.1 shows the contribution of fire sales losses to total system-wide loss in the stress test in the four scenarios. In Scenario 1, we observe that when stress levels exceed 4-5%, fire sales losses cannot be neglected and may account for up to 70% of the total loss. This is definitely an argument for including fire sales and deleveraging effects in a stress test. In Scenario 4, no fire sales are triggered even for a 20% shock to the asset class. As seen in Table 3.1.4, this is due to EU banks' small exposure to these asset classes.

Figure 3.2.2 represents the magnitude of fire sales losses $SFLoss(\epsilon)$ as a percentage of bank equity for the four different stress scenarios. What is striking is the existence of a *tipping point* which separates a stability zone corresponding to small shocks from a systemic risk zone, where the magnitude of losses represents a considerable fraction of bank equity. The existence of such tipping points is characteristic of the collective behaviour of networks whose components are subject to thresholds [Granovetter \(1978\)](#). In Scenarios 1 and 2, we observe that the tipping point corresponds to large but not extreme losses in asset values. This provides an empirical confirmation of our previous observations on tipping points.

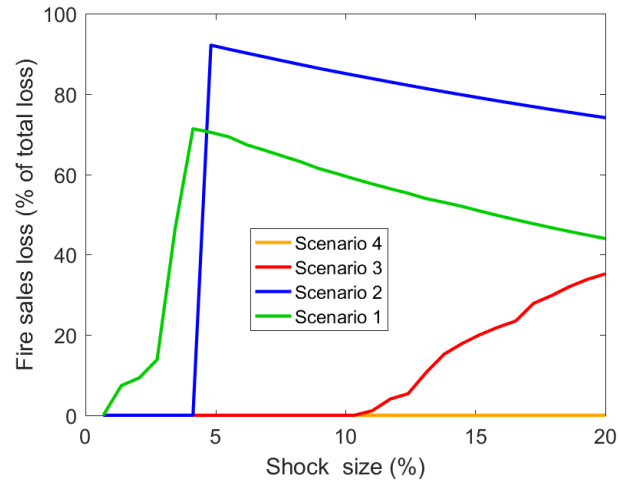


Figure 3.2.1: Contribution of fire sales losses to total system-wide loss, as a function of the initial stress level.

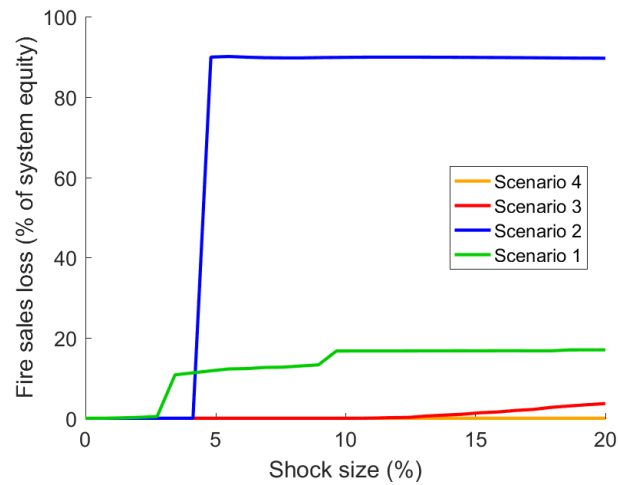


Figure 3.2.2: Fire sales losses (as % of bank equity) in the four stress scenarios as a function of the initial shock size. The fourth scenario does not generate fire sales in the threshold model.

The results of these plots are based on market depth parameters estimated as described in Section 3.1.1. In a market stress scenario, these market depths may however be different. This point may also be seen in terms of dependence on the liquidation horizon: as observed from (1.14), changing the liquidation horizon by a factor τ adjusts the depth parameter by a factor

$\sqrt{\tau}$. The sensitivity of results to both of these may be studied by adjusting the market depth parameters by a scaling factor; the value 1 corresponds to the values estimated in Section 3.1.1; values > 1 correspond to an increase in liquidation horizon, values < 1 correspond to a decrease in market depth.

Figure 3.2.3 analyses the dependence of system-wide fire sales losses $SFLoss(\epsilon)$ on this scaling factor. We observe the sensitivity of the result on the market depth level, which underlines the importance of a proper estimation procedure for these parameters. What is more, we can distinguish three regimes for $SFLoss(\epsilon, \delta_{scale})$ (where δ_{scale} is the market depth scaling factor and viewed as the second variable of $SFLoss$): (i) no fire sales (losses) for small shocks $\epsilon < \min_i \epsilon_i^*$, (ii) significant fire sales losses on the order of 10-20% of bank equity for sufficiently large stress levels in presence of large market depth, (iii) a region of *high systemic risk*, where fire sales losses deplete a large proportion of bank equity. The boundary between the regimes may be attained either by increasing the stress level ϵ or by decreasing the market depth scaling factor. In both cases, there is a tipping point above/below which large-scale deleveraging is triggered.

The “magnitude” of the meltdown depends on the market depth and varies roughly between 90% and 110% of system equity. Using a market depth scaling factor of 0.5 for instance further increases these losses to around 120% of system equity. We believe that any fire sales study should therefore by default include a rigorous sensitivity analysis with respect to the market depth parameter.

3.2.2 ILLIQUIDITY AND INSOLVENCY

Most theoretical models of financial contagion have focused exclusively either on insolvency or on illiquidity as the cause of bank failure, while bank stress tests have traditionally focused exclusively on solvency of banks. As

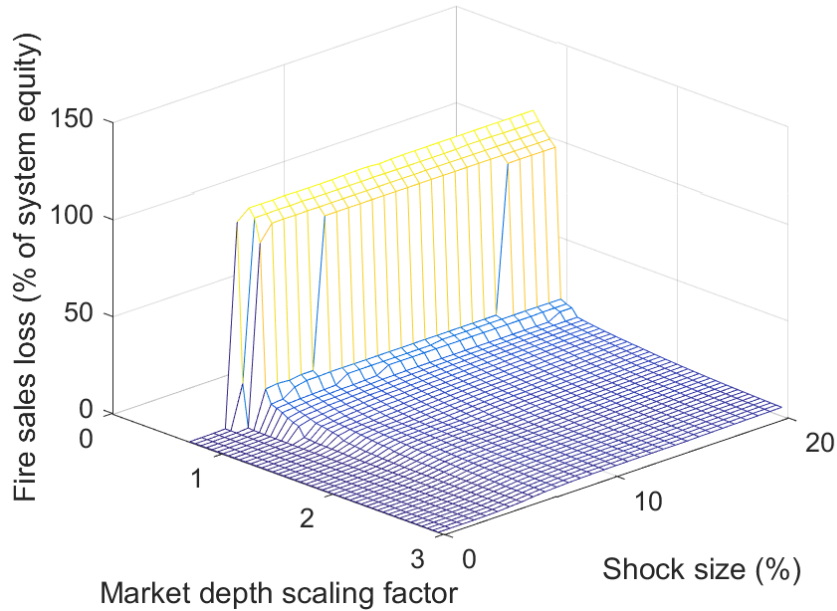


Figure 3.2.3: The fire sales losses as a function of the initial shock and the estimated market depth for the threshold model. We can clearly distinguish three regions: (i) no fire sales; (ii) fire sales contagion with losses between 10 - 20% of system equity, (iii) region of high systemic risk: a large portion of the banks default (i.e. absent regulatory intervention large scale contagion is to be expected). This shows that the market depth has a highly significant impact on the estimated losses and the extent of contagion.

discussed in Sections 1.1.6 and 1.2.2, our model allows for both possibilities: a bank may become insolvent due to asset losses, or become illiquid when all marketable assets have been sold. The model thus allows to examine which is the principal mode of failure in the stress test.

Note that this distinction is not available in [Greenwood et al. \(2015\)](#), [Duarte and Eisenbach \(2013\)](#): their assumption that all assets are available for liquidation entails that the only situation where an institution becomes illiquid is when there are no more assets available for liquidation, which implies that it is also insolvent.

We proceed now to analyse the number of failures respectively due to illiquidity and insolvency as a function of the initial stress level. The right (left) panel of Figure 3.2.4 displays the number of banks which become insolvent

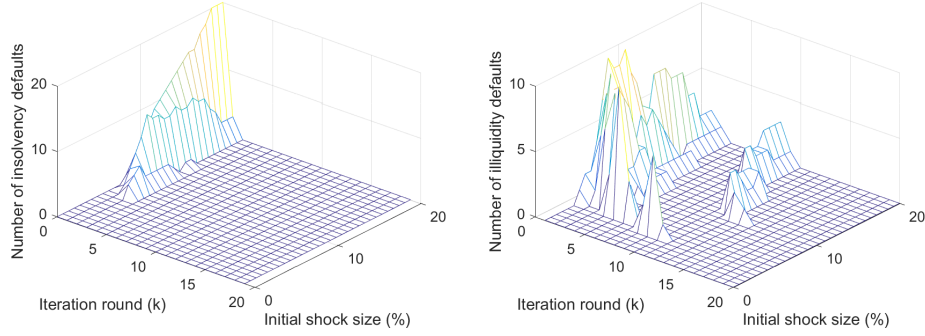


Figure 3.2.4: Left: Number of insolvent banks as a function of number of rounds of deleveraging and initial stress level. Right: Number of illiquid banks as a function of rounds of deleveraging and initial stress level.

(resp. illiquid) at a given number of rounds of deleveraging, as a function of the initial stress level. We observe that insolvency is far from being the only mode of failure: depending on stress levels, 10 to 20 banks are expected to default due to illiquidity, yet remain solvent. We also note that from the right panel, we can read that there are two waves of illiquidity defaults occurring: a first one in the few rounds immediately after the initial shock, and a second one after about 10 iterations of the cascade. This illustrates how contagion occurs between rounds five and ten *in absence* of banks defaulting, but precisely because of the actions that banks take to prevent their own default.

Figures 3.2.5 and 3.2.6 show respectively the number of failures due to insolvency and those due to illiquidity as a function of the market depth scaling factor and the initial shock size. We can see that, while the number of insolvent banks increases with the stress level, the number of failures due to illiquidity is non-monotone: it is maximal in the range of moderate shocks (between 5% and 10%) but then decreases as the shock level increases and insolvency becomes the main mode of default. This is particularly relevant, as we previously identified a shock of 4.9% to correspond precisely to the EBA stress scenario intensity. On this point we also note that our model

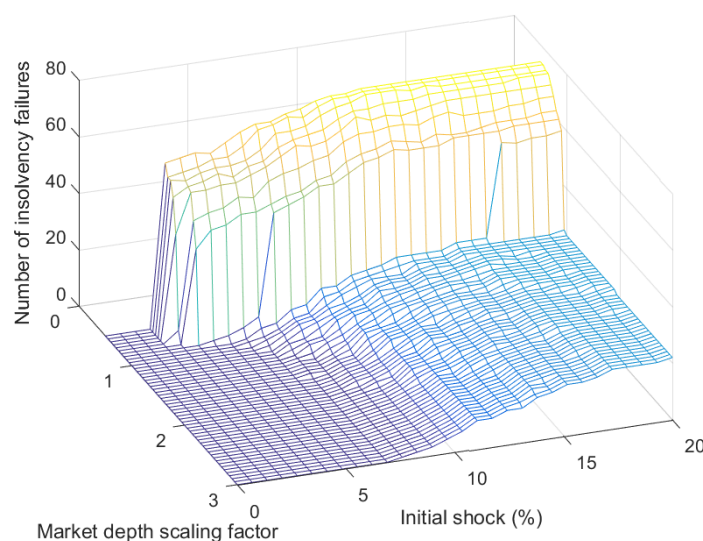


Figure 3.2.5: Number of banks failing due to insolvency in the systemic stress test.

does not include any bankruptcy costs. Inclusion of such losses would only amplify the effects we observe.

3.2.3 SECOND AND HIGHER ROUND EFFECTS

We end this section by discussing the impact of the number of rounds of the fire sales cascade. Although in theory deleveraging may continue for many rounds (and, in the leverage targeting model, for an infinite number of rounds), previous empirical studies have mostly focused on a single round of deleveraging. Findings in [Greenwood et al. \(2015\)](#) and [Duarte and Eisenbach \(2013\)](#) seem to suggest that simulating fire sales for several rounds compared to one or two rounds does not lead to significantly different results - the main effects of fire sales seem to be captured by one iteration. A possible explanation for these results is that, as noted above in Section 3.1.2, the initial shocks are quite large in these studies, and for this reason contagion in higher rounds plays less of a role. We nuance these findings and emphasise several effects which only become visible at second or higher rounds.

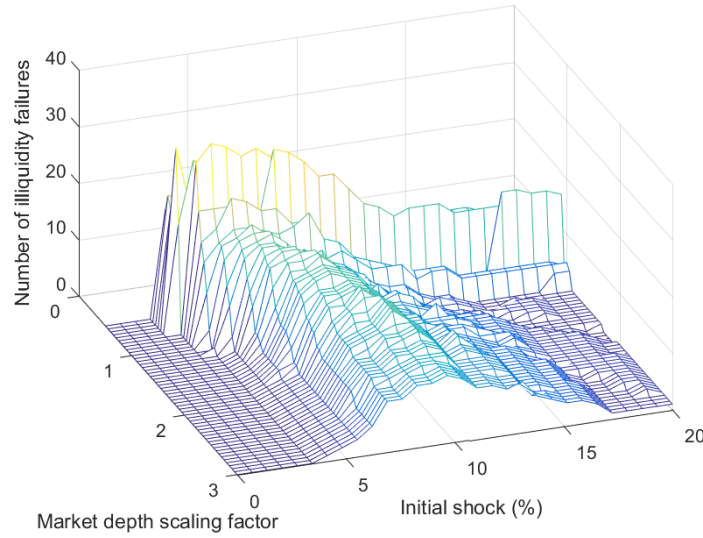


Figure 3.2.6: Number of banks failing due to illiquidity in the systemic stress test.

First, as observed in Figure 3.2.4, many bank failures occur only at higher rounds, especially failures due to illiquidity. Only examining a single round of the feedback loops thus leads to an underestimation of the number and severity of bank failures.

Second, the left panel in Figure 3.2.7 shows the fire sales loss at each round k on a log scale for the estimated market depth. The right panel in Figure 3.2.7 shows the ratio of total loss (20 rounds) to the first round loss, as a function of the initial stress level. The non-monotone feature of this dependence shows that fire sales contagion is most important for moderate stress levels, which trigger deleveraging but are not extreme enough to generate insolvency at the first round. In this case, second and higher rounds considerably change the outcome in terms of total loss level and number of bank failures.

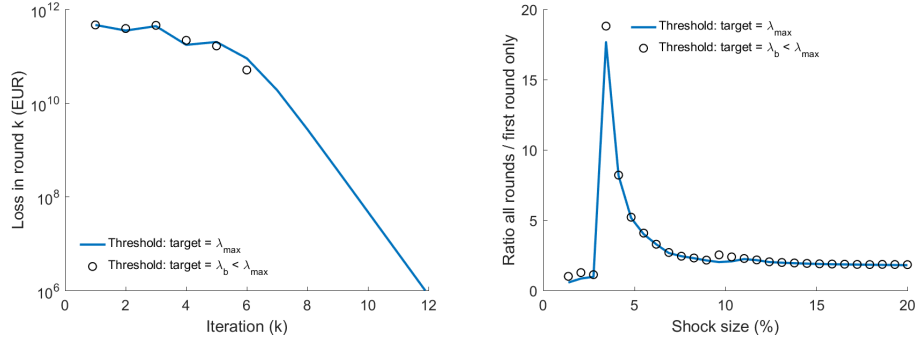


Figure 3.2.7: Left: Fire sales losses as a function of the iteration step at the estimated market depth for the leverage targeting model (dashed), and the threshold models with $\lambda_b = \lambda_{\max}$ (full) and $\lambda_b < \lambda_{\max}$ (circles). Right: Ratio of fire sales losses from full cascade to fire sales losses from first round only as a function of the initial shock size.

3.2.4 INDIRECT EXPOSURES

To assess how large indirect exposures may be in the European banking system, we use the results of the previous subsection to examine the magnitude of indirect exposures of European banks to residential and commercial mortgages. A first observation is that most European banks tend to have very few mortgages or loans outside of their own country, so direct exposure to foreign residential and commercial mortgage is zero in most cases, except for a few multinational banks. Given the large volume of mortgages on bank portfolios, it is generally assumed that the domestic housing market is the main risk factor for commercial bank portfolios. However, as we shall see now, European banks also have substantial indirect exposures to commercial and residential mortgages in *other* European countries where they do not issue loans.

The left panel in Figure 3.2.8 displays the (total) loss for two banks in our sample, HSBC and Santander, in the scenario where banks suffer losses on Spanish real estate exposures (Scenario 1). Santander, a major Spanish bank, holds a lot of Spanish mortgages on its portfolio, representing a notional exposure of 82.7 bn EUR, which corresponds to the slope of the

loss for shocks less than 3%, where no deleveraging occurs. HSBC, on the other hand, holds very few Spanish mortgages (less than 0.5 bn EUR) so its notional exposure to this asset class is much lower.

When losses on Spanish real-estate holdings exceed 3.5% of the notional value of the exposures, some Spanish banks start breaching their leverage constraints and deleverage their portfolios. At this point indirect losses lead to a sharp increase in the loss level. For a 5% shock level, the total loss for Santander lies somewhat over 25 bn EUR, which corresponds to an indirect exposure exceeding 500 bn EUR! In this regime, the indirect exposure is clearly the main source of losses. This deleveraging by Spanish banks then affects HSBC through price-mediated contagion: as the shock level increases from 2.75% to 3.45%, the fire sales loss for HSBC jumps from 36 M to 2.8 bn EUR. For a 4% shock level, HSBC's indirect exposure to Spanish real estate is close to 70 bn EUR, which corresponds to 2.8 bn EUR of losses due to indirect contagion. Note that in the case of HSBC, the indirect loss corresponds to 140 times its notional exposure to this asset class! This example suggests that in stress scenarios which are sufficiently severe to trigger fire sales, actual losses can be much higher than what is suggested by notional exposures. The difference corresponds to indirect exposures.

The right-hand graph in Figure 3.2.8 displays the magnitude of indirect exposures corresponding to these losses. We see that the indirect exposure of Santander, which mainly stems from its own deleveraging and deleveraging by other Spanish banks with similar assets, sharply increases when the shock size reaches the threshold of 3%, then peaks around 4%: as the stress level increases beyond a certain level, some banks become insolvent or illiquid, and cease to contribute to fire sales spillovers.

As illustrated in the right panel in Figure 3.2.8, these indirect exposures clearly depend on the severity of the stress scenario considered, the bank's

initial exposures, and the exposure and constraints of other portfolios holding similar marketable assets. An implication is that one cannot mimic the impact of indirect contagion by simply applying a higher stress level for all banks, as is implicitly done in current supervisory stress tests.

Table 3.2.1 reports indirect exposures to Spanish real estate for several other European banks. Comparing with their notional exposures, we observe that non-Spanish banks have a substantial indirect exposure to Spanish real estate which is not revealed by inspecting their notional exposures to this asset class, which are between ten and a hundred times smaller. This indirect exposure is due to the overlap in marketable assets with banks that are directly exposed to the Spanish housing market. The main asset classes contributing to this overlap are Spanish, UK, US, and Asian corporate bonds as well as Spanish, Asian, non-emerging EEA, and US government bonds.

These observations are not particular to the Spanish real estate sector or indirect exposures of UK banks. Other examples, which we do not report here for the sake of brevity, also show that in the European banking system a stress scenario affecting the commercial or residential real estate sector of a given country may generate significant losses arising from indirect exposures for foreign banks.

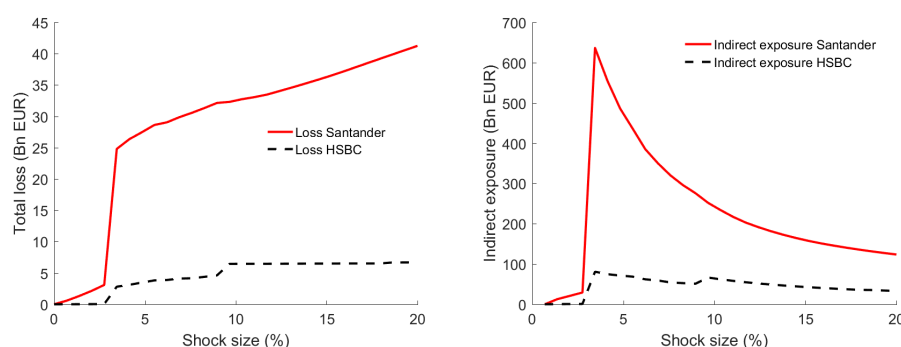


Figure 3.2.8: Left: Losses arising from a depreciation in Spanish real estate, for HSBC and Santander, as a function of the depreciation level (horizontal axis). Right: The indirect exposures corresponding to these losses.

Bank	Santand.	BBVA	Deutsche Bank	HSBC	RBS	Nordea
Notional exposure to Spanish real-estate	82.7	82.5	8.88	0.452	2.68	0.003
Indirect exposure to 2% shock	29.2	27.2	1.60	0.84	2.43	0.050
Indirect exposure to 5.5% shock	385	280	64.0	61.9	58.7	6.40
Indirect exposure to 10% shock	217	169	60.9	58.1	45.8	7.21
Capital	42.0	24.9	37.6	86.90	59.0	19.1

Table 3.2.1: Estimated direct and indirect exposures (billion euro) of selected banks to Spanish commercial and residential real estate.

The concept of indirect exposures may also be defined at the country level. Figure 3.2.9 shows the indirect exposures of European banks (aggregated by country) to the Spanish housing market (left panel: normal scale, right panel: logarithmic scale). Initial losses below 1% of the notional value of the exposures do not trigger fire sales, and do consequently not generate any indirect exposures.

As the stress level increases beyond 1%, fire sales are triggered and spillover losses materialise; when the initial shock size exceeds 3%, contagion increases fire sales losses by 2 - 3 orders of magnitude. What is striking is that these thresholds – 1%, 3% – are not large, suggesting that indirect exposures and losses arising from price-mediated contagion cannot be ignored in stress tests. The left panel of Figure 3.2.9 reveals that, after Spanish banks, British banks have the largest indirect exposure to the Spanish housing market. Table 3.2.2 reports estimated indirect exposures for different stress levels for the countries highlighted in Figure 3.2.9.

Overall, the direct exposure of European Banks to the Spanish residential and commercial real estate sector is 740 bn EUR. When averaged both across all banks and all shock sizes (between 0 and 20%), the indirect exposures amount in this example to 184% of the direct exposures, which is non-

negligible. To be clear, in the event of a meltdown of the Spanish housing sector, banks will, on average, be affected for up to 184% of their actual holdings in Spanish mortgages.

This clearly means that the housing market in one European country can strongly affect banks of other European countries, an issue which can only be tackled by macroprudential policies applied at a Europe-wide level.⁹

These indirect exposure figures, as all other results obtained in the stress tests, are sensitive to assumptions on market depth, or, alternatively, on the liquidation horizon. Figure 3.2.10 shows the estimated fire sales losses to the UK banking system as a function of the stress level to Spanish real estate and the market depth scaling factor. Similarly, Figure 3.2.11 shows the estimated indirect exposure of the UK banking system to the Spanish housing market as a function of the stress level and the market depth scaling factor. Reading the graph from front to back, we see that a contraction of market liquidity can greatly exacerbate fire sales losses: there is a tipping point at which losses jump from roughly 10 bn EUR to several hundred billion euro, and analogously indirect exposures increase from roughly 100 bn to several trillion EUR.

Indirect exposures and the losses arising from them provide a more palpable explanation of how an initial loss of about 500 bn USD in the US subprime sector was able to balloon into trillions of dollars of losses across multiple asset classes during the financial crisis.

3.2.5 RELEVANCE FOR MACROPRUDENTIAL STRESS TESTS

Given the magnitude of indirect contagion, it is not surprising that accounting for indirect losses can modify the outcome of bank stress tests. To show

⁹On this point, we note that Brexit does not act as a barrier for price-mediated contagion.

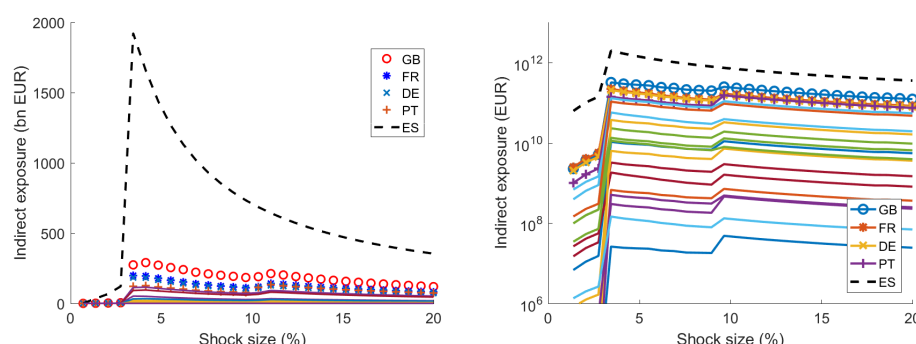


Figure 3.2.9: Indirect exposures of European banks, aggregated by country, to the Spanish housing market. The countries with the highest indirect exposures are listed in the legend. Three levels of “no indirect exposure”, “intermediate indirect exposure” (up to 10 bn EUR), and “large indirect exposure” (> 100 bn EUR) are visible as a function of the initial shock intensity.

Country	Direct exposure (bn EUR)	Indirect exposure (bn EUR)		Indirect exposure (bn EUR)		Indirect exposure (bn EUR)	
		Stress level: 1%	Stress level: 2%	Stress level: 3.5%	Stress level: 3.5%	Stress level: 3.5%	Stress level: 3.5%
ES	22.8	0	101.3	1662.4			
GB	7.9	0	3.7	293.2			
FR	15.9	0	4.1	196.4			
DE	2.4	0	3.2	183.9			
PT	697.0	0	1.6	127.4			

Table 3.2.2: Indirect exposures to Spanish mortgages of EU banks by country in billions of euros. For shocks below 1%, no fire sales occur and indirect exposures are zero. Shocks around 2% create significant indirect exposures within Spain, but negligible ones for other European countries. Initial shocks larger than 3.5% trigger large scale contagion that may result in cross-border spillovers. This is reflected by significant indirect exposures on the order of 100Bn EUR for other European countries.

this more clearly, we compare the outcomes of a bank-level stress test, where the (direct) loss resulting from initial stress is compared to its capital, with that of a systemic stress test, where we also account for fire sales spillovers which may follow from the initial loss.

Figure 3.2.12 plots, as a function of the shock size, the number of banks (out of 90 banks) in the sample which have enough capital to sustain the initial losses but become insolvent after a single round of fire sales is accounted for (at the estimated market depth). For moderate stress levels (5-10%) we

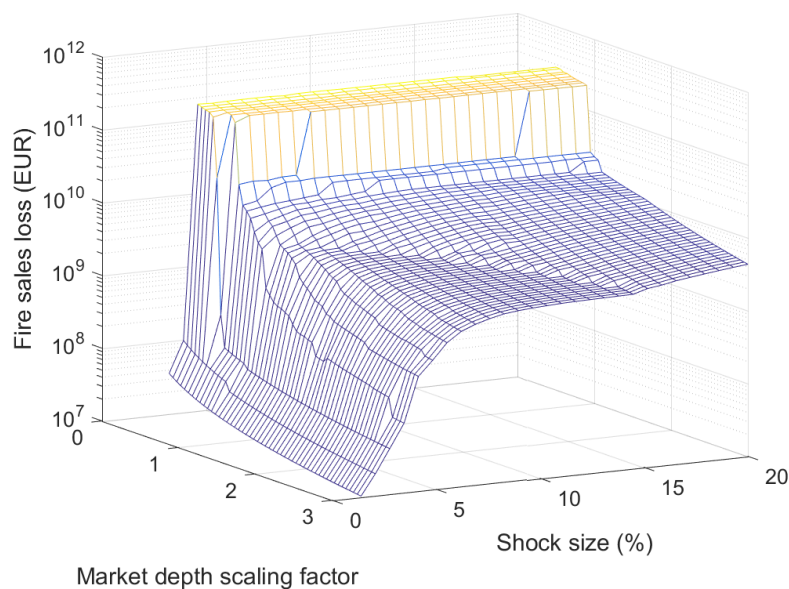


Figure 3.2.10: Fire sales losses (in EUR) suffered by the UK banking system after a shock to Spanish residential and commercial mortgage exposures as a function of the stress level and market depth. Even with optimistic market depth estimates, when the initial shock is above 3%, fire sales losses are non-negligible.

identify up to 10 banks (in Scenario 1), reported in Table 3.2.3, which pass the bank-level stress test but fail the systemic stress test.

Figure 3.2.13 shows the number of banks that fail the bank-level stress test but fail the systemic stress test as a function of the initial stress level and the market depth scaling factor for all rounds. We see that the discrepancy between the outcomes of the systemic and single-bank stress test is precisely largest in the regions corresponding to moderate to large stress levels typically used in supervisory stress tests.

These results illustrate that the contagious losses from fire sales can be of a sufficiently large magnitude to change the outcome of bank stress tests. We further note, that this comparison was limited to solvency defaults only; including illiquidity defaults, and accounting for bankruptcy costs which we ignore, will increase the number of banks that fail the systemic stress test

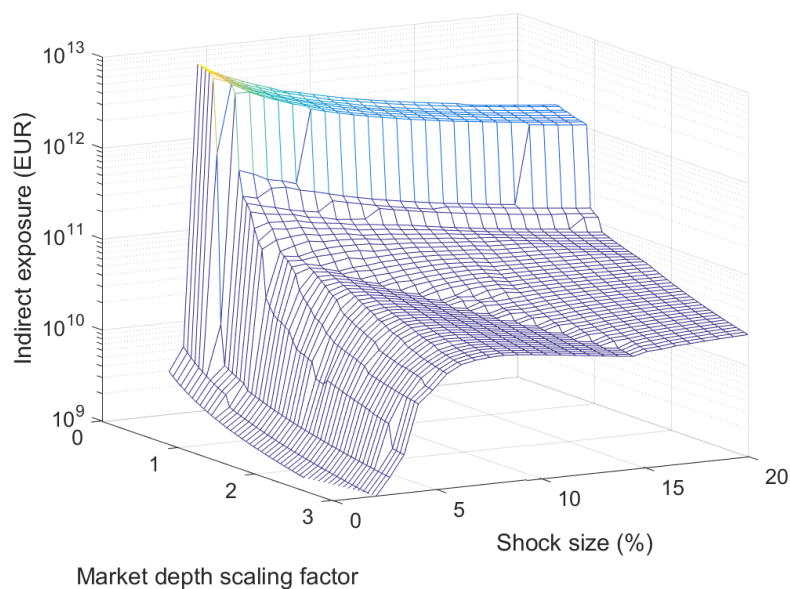


Figure 3.2.11: The indirect exposure (in EUR) of the UK banking system to the Spanish residential and commercial mortgage market as a function of the stress level and market depth. Even with optimistic market depth estimates, when the initial shock is above 3%, indirect exposures are non-negligible.

even though they passed the individual bank stress test.

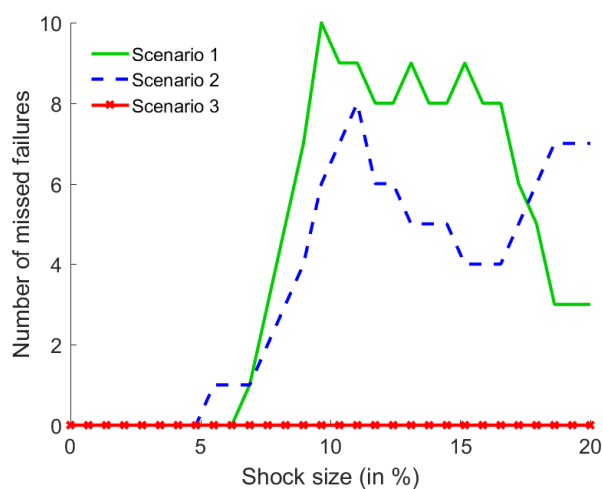


Figure 3.2.12: Number of banks with sufficient capital to withstand the initial stress, but failing to withstand the losses due to indirect contagion after a single round of deleveraging.

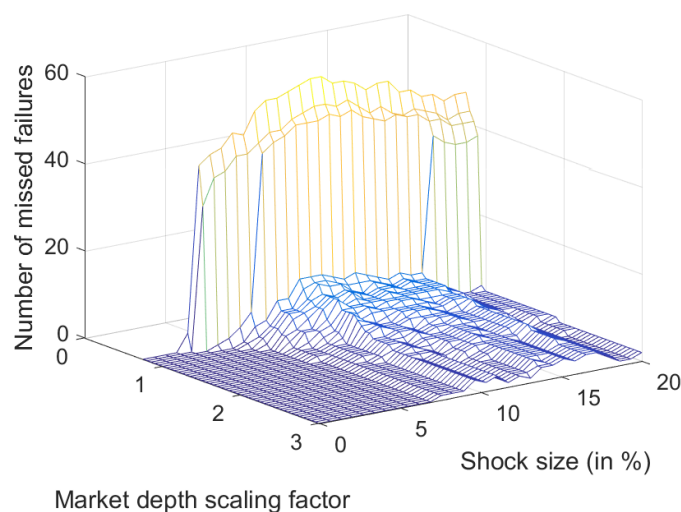


Figure 3.2.13: Number of banks with sufficient capital to withstand the initial stress, but failing to withstand the losses due to indirect contagion.

3.2.6 WHY INDIRECT EXPOSURES CANNOT BE REPLICATED IN SINGLE BANK STRESS TESTS

An argument sometimes advanced to avoid moving to a systemic stress testing framework is that, while fire sales effects are important and contribute to amplifying losses, their effect can be mimicked in bank-level stress tests by applying more severe shocks, without simulating fire sales in a detailed manner. We know from a comparison of coefficients in the theoretical analysis in Section 2.1.4 that this claim is not valid in general. Let us now examine this point quantitatively in more realistic empirical setting using the EBA dataset.

Figure 3.2.14 compares bank-level losses in the systemic stress test with fire sales (vertical axis) to losses of the same banks in a stress test without fire sales, but with severity of the shock chosen so that the total system-wide loss is equal in the two stress test configurations (horizontal axis). In this plot the scaling is solely done on the illiquid assets, c_1 from 2.1.4, and there is no scaling on liquid assets, $c_2 = 0$.

Scenario	Banks failing systemic stress test after 2 rounds of fire sales
1	BFA-BANKIA Caja de ahorros y pensiones de Barcelona Banco de Sabadell Banco popular Español Caixa d'Estivalis de Catalunya, Tarragona i Manresa Caixa de Aforros de Galicia, Vigo, Ourense e Pontevedra Groupe BMN Bankinter Caja de Ahorros y M.P. de Zaragoza, Aragon y Rioja Banco Pastor
2	Lloyds Banking Group Unicredit ING DnB NOR Nordea Bank SEB Svenska Handelsbanken Swedbank

Table 3.2.3: EU banks that pass the single-bank stress test for a stress level of 10% but fail the systemic stress test after a single round of fire sales.

As is clearly visible from this figure, the distribution of losses across banks is fundamentally different in these two cases. No amount of scaling at the level of macro-shocks defining the stress scenario can adjust for the cross-sectional heterogeneity of indirect exposures. Thus, even if the stress level is adjusted to make the total loss in the banking system equal in both stress tests, the allocation of losses across banks is essentially different once fire sales are accounted for. This is a strong argument for including a proper model of fire sales into any macroprudential stress testing framework.

In Figure 3.2.15, we set $c_1 = 0$ and instead try to account for the fire sales loss by scaling c_2 , which affects the marketable assets. Again, the scaling is chosen such that the total loss is exactly equal to the total loss computed with the fire sales model. Again, clearly, we are unable to reproduce the cross-sectional distribution of fire sales losses, with the bias being on the order of three orders of magnitude for some banks!

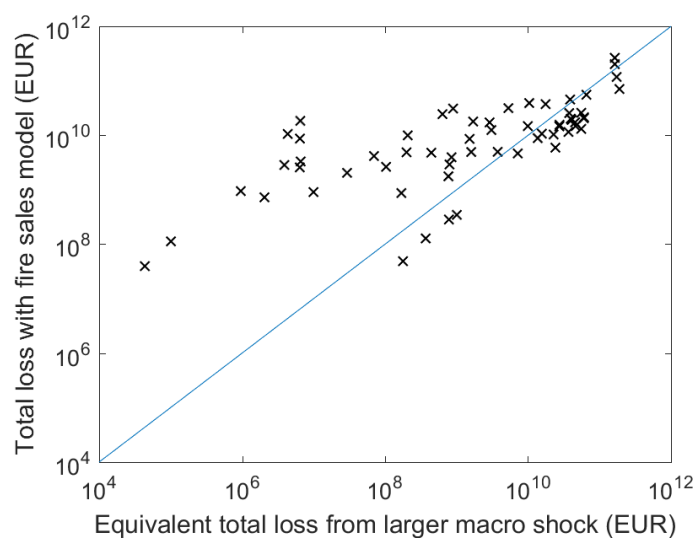


Figure 3.2.14: Scatter plot of the bank-level losses in a stress scenario with fire sales (vertical axis) against the bank-level losses estimated from an adequately scaled larger macro shock (horizontal axis). Clearly, it is not possible to account for fire sales in a stress scenario by using a larger initial macro shock. As the graph shows, even when the shock is scaled such that the same total loss is generated, the distribution of losses across banks will be fundamentally different.

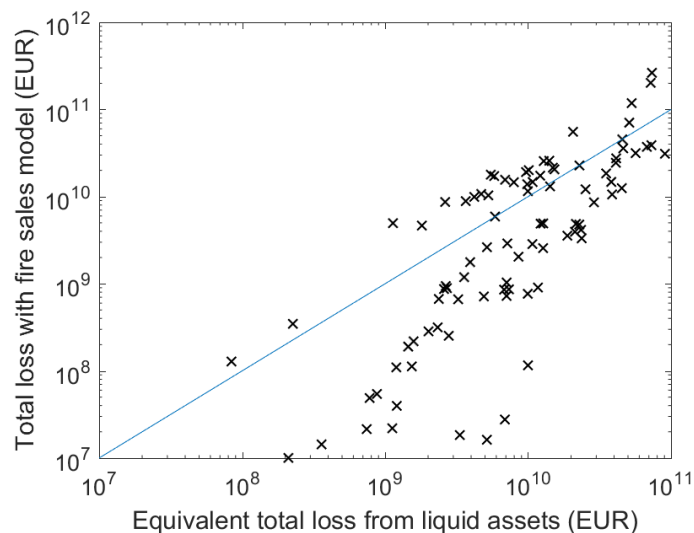


Figure 3.2.15: Scatter plot of the bank-level losses in a stress scenario with fire sales (vertical axis) against the bank-level losses estimated from an adequately scaled uniform shock to the liquid assets (horizontal axis). It is not possible to account for fire sales in this setting either. As the graph shows, even when the shock is scaled such that the same total loss is generated, the distribution of losses across banks will be fundamentally different.

3.3 ROBUSTNESS TESTS

In this section, we perform an important robustness check on the response function by letting banks optimise the order in which they liquidate the assets in their marketable portfolio, seeking to minimise the fire sales losses by selling more liquid assets first. Furthermore, we compare the results of the systemic stress test performed in the previous section to those obtained if a leverage targeting response function is used. We will argue that we believe the leverage targeting response function not to be adequate for modelling fire sales in systemic stress tests.

3.3.1 PRICE-IMPACT MINIMISING DELEVERAGING

The left panel in Figure 3.3.1 shows the fire sales loss as a function of the market depth scaling factor and initial shock size under proportional deleveraging, whereas the right panel shows the same plot under sequential deleveraging, i.e. selling the most liquid asset first. Quite astonishingly, the graphs look remarkably similar, the optimising behaviour of banks does not have a large effect on the system outcome, we only notice that the “high systemic risk region” only starts for lower market depth scaling factors.

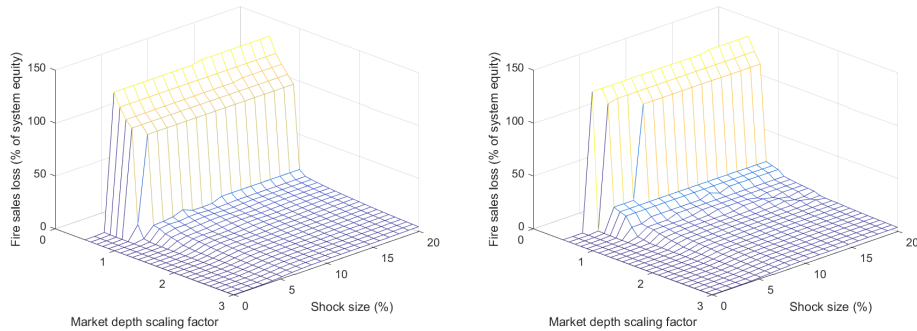


Figure 3.3.1: Fire sales losses as a function of the market depth scaling factor and the initial shock size under proportional deleveraging (left) and sequential deleveraging (right).

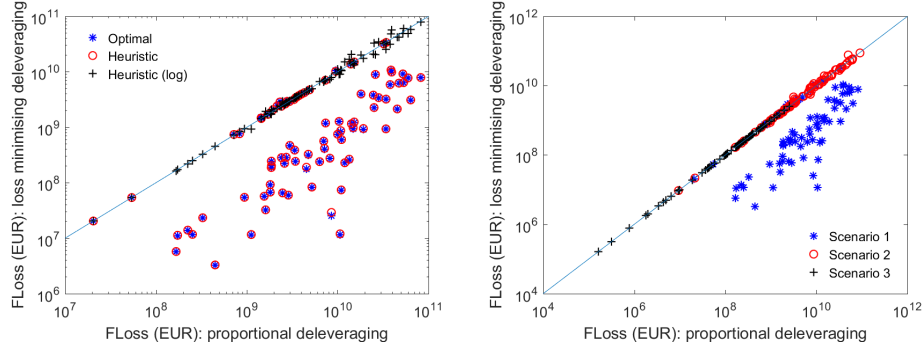


Figure 3.3.2: Left: Bank-level losses under different optimised deleveraging strategies vs proportional deleveraging for Scenario 1. Right: Bank-level losses under sequential deleveraging vs proportional deleveraging for the three scenarios in which fire sales are triggered.

The left panel in Figure 3.3.2 shows bank-level losses under different loss minimising strategies versus the bank-level losses under proportional deleveraging. The graph is produced on the boundary of the systemic risk region of scenario 1, showing that for some banks this has a large effect (using the optimal strategy, or the heuristic strategy of selling proportional to the market depth). The right panel shows bank-level losses under loss minimising deleveraging versus proportional deleveraging in the three different scenarios. Again, the market depth and initial shock size are chosen such that the scatter plot is produced for the boundary of the high systemic risk region of Scenario 1. In Scenarios 2 and 3, bank-level losses are almost identical under the different deleveraging schemes.

Overall, we see that an optimising behaviour by banks does not change the results in practice to any significant degree. Reasons for this may be twofold: First, when all banks “run for the exit” (Cont and Wagalath (2013), Pedersen (2009)), the asset class that everybody thought to be liquid turns out to be less liquid ex-post because ex-ante beliefs of liquidity lead to a disproportionately large selling volume. Second, it could be an artefact of the dataset’s level of granularity. Further analyses of this type on different datasets seem therefore warranted.

3.3.2 COMPARISON TO LEVERAGE TARGETING MODEL

We now turn to comparing the results obtained in the systemic stress test when using a leverage targeting response function rather than our proposed threshold model.

MAGNITUDE OF FIRE SALES LOSSES.

The left panel in Figure 3.3.3 shows the fire sales loss as a function of the initial shock size for the four scenarios, when using the threshold model (solid lines) and the leverage targeting model (dashed lines). A striking feature is that, while the 4 scenarios lead to different outcomes in our model, they lead to almost the same outcome in the leverage-targeting model. Moreover, in the leverage targeting model, fire sales losses are essentially independent of the initial shock size. This shows that losses in the leverage targeting model are not driven by the initial shock size but by the rebalancing generated by the leverage targeting rule. This leads, we believe, to a significant overestimation of the importance of fire sales, in particular for small initial shocks. Indeed, the right panel in Figure 3.3.3 shows the contributions of the fire sales loss to the total loss, which is largest for the smallest shocks.

Importantly, these findings are not a particularity of the estimated market depth level. As the right panel in Figure 3.3.4 shows, the leverage targeting model actually predicts large scale contagion across the entire financial system for all combinations of initial shock sizes and market depth scaling factors. Comparing the two panels in Figure 3.3.4, we see that the total loss in both models indeed only agrees in the “systemic risk region”. In all other regions, which are of importance for systemic stress testing, there is a substantial difference between the results of the two models.

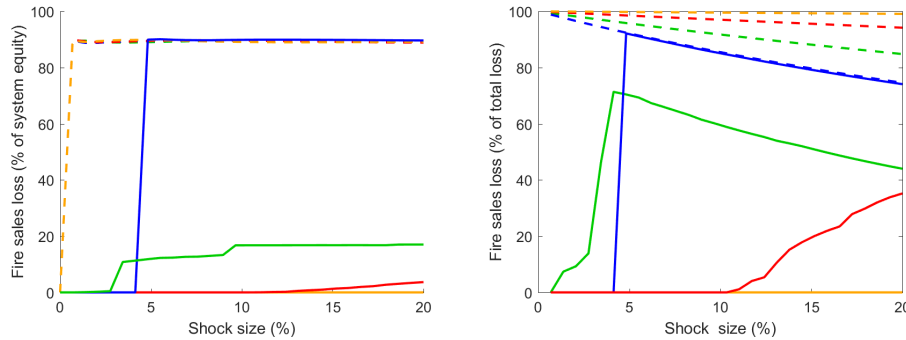


Figure 3.3.3: Left: The fire sales loss in each scenario as a function of the initial shock size at the estimated market depth under the threshold model (solid lines) and leverage targeting model (dashed lines). Right: The proportion of the total loss due to fire sales as a function of the initial shock size.

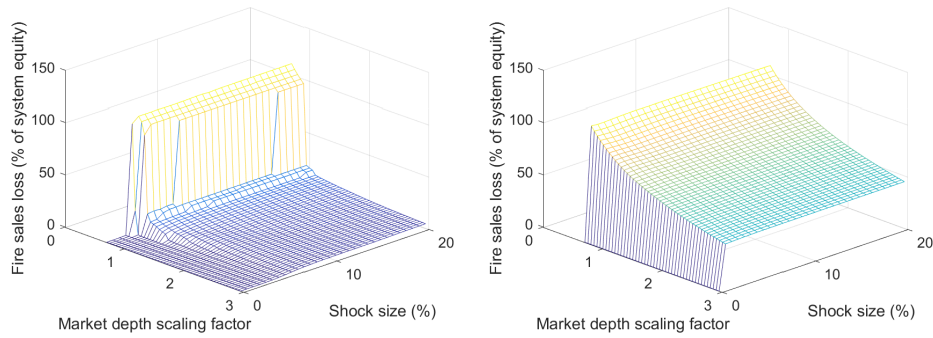


Figure 3.3.4: The fire sales loss in percent of total bank equity as a function of the initial shock size and the market depth scaling factor under the threshold model (left) and the leverage targeting model (right). The models disagree strongly in all regions, except for the "high systemic risk" region.

BANK-LEVEL LOSSES.

We now compare the fire sales loss estimates of the two models at the level of individual institutions. Figure 3.3.5 compares bank-level fire sales losses under the leverage targeting model, using the same market depth estimates. Almost all banks' fire sales losses are larger by several orders of magnitude in the leverage targeting model in comparison with the threshold model. This figure shows that fire sales losses are not only larger on a system-wide level in the leverage targeting model (as seen in 3.3.4), but are higher at the individual bank level. The initial shock and market depth estimate

parameter combination in scenario 2 lie in the “systemic risk region”. In this case, both models agree very well actually. However, given that almost all banks default in this case, this is a somewhat trivial observation.

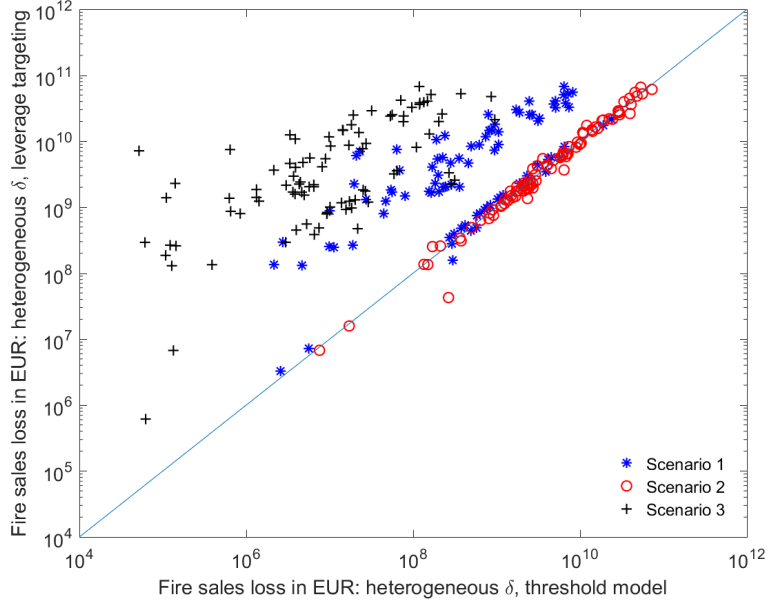


Figure 3.3.5: Bank-level fire sales losses under the leverage targeting model with heterogeneous market depths vs. the losses in the threshold model with heterogeneous market depths (on logarithmic scale).

IMPACT OF HETEROGENEITY IN MARKET DEPTH.

Greenwood et al. (2015) assume the same market depth parameter for all asset classes, whereas our estimation, shown in Figure 3.1.1, yields a heterogeneous distribution of market depth across asset classes. To investigate the influence of this heterogeneity, we compare bank-level losses with losses in a model with uniform market depth $\bar{D}$ computed as a holdings-weighted average:

$$\sum_{i,\mu} \frac{\Pi_{i,\mu}}{\bar{D}} = \sum_{i,\mu} \frac{\Pi_{i,\mu}}{D_\mu}. \quad (3.3)$$

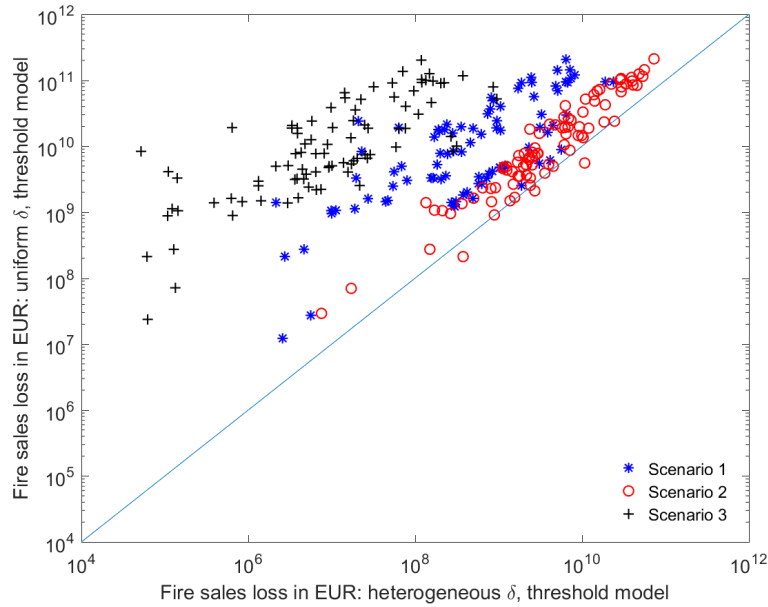


Figure 3.3.6: Scatter plot of bank-level fire sales losses under the threshold model with heterogeneous and uniform market depths respectively. The use of the equivalent uniform market depth increases fire sales loss estimates considerably compared to the case of heterogeneous market depth.

Figure 3.3.6 shows a huge impact of heterogeneity in market depth when estimating fire sales losses: when using the equivalent uniform market depth, the losses suffered by institutions increases - sometimes by more than three orders of magnitude. The reason is that the equivalent market depth reduces the liquidity of the most liquid asset classes under the heterogeneous depth estimates. As there is a positive correlation between the liquidity of an asset class and its market cap (or notional amount outstanding), the reduction of the estimated market of the most liquid asset classes has a significant effect on the results. In this case, the shift is sufficiently important for large scale contagion also to materialise in scenarios 1 and 3. Again, this clearly shows that bank stress tests with fire sales require a careful estimation of market impact/ market depth parameters for each asset class.

For completeness, we make two further comparisons in Figure 3.3.7: In the

top panel we compare bank-level fire sales losses using heterogeneous vs uniform market depths under leverage targeting dynamics. The difference is less striking than in Figure 3.3.6, the reason being that the dynamics of the leverage targeting model lead almost all banks to default regardless; hence accounting for heterogeneity in asset class liquidity becomes less important. In the lower panel, we compare bank-level losses when comparing the [Greenwood et al. \(2015\)](#) setting (leverage targeting & uniform market depth) to our setting (threshold model & heterogeneous depth). The results are qualitatively similar to those in Figure 3.3.5, but starker, as the effects of leverage targeting dynamics and uniform market depth choice get compounded.

SENSITIVITY TO COMPOSITION OF INITIAL SHOCK.

As final comparison between the threshold model and the leverage targeting model, we analyse their responsiveness to the composition of the initial shock. As shown in Chapter 1, under weak conditions, the threshold model guarantees the existence of “orthogonal stress scenarios”, whilst such scenarios do not exist under leverage targeting dynamics. We will now analyse how relevant these properties are in practice.

Figure 3.3.4 illustrated that the fire sales losses under leverage targeting dynamics seem not to depend on the size of the initial shock. As we shall see now, they do not depend on the composition of the initial shock either, i.e. the fire sales loss distribution barely changes even when the stress scenario itself changes. The black crosses in Figure 3.3.8 mark the initial bank-level losses under Scenario 1 and 2 respectively. As noted in Table 3.1.6, the initial losses that individual banks suffer in these two scenarios are very different; this is reflected here by the black crosses not lying on the diagonal. The blue asterisks and red circles correspond to bank-level total losses when accounting for a single round of fire sales under the threshold model and

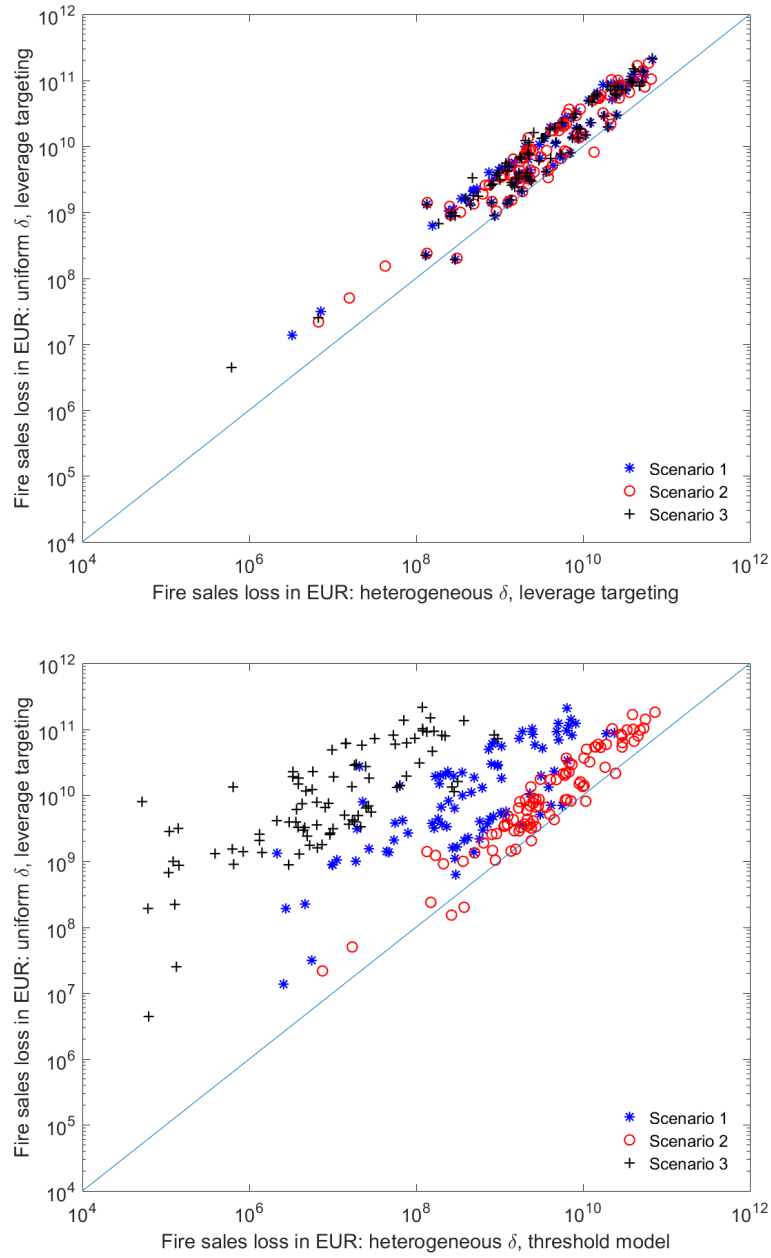


Figure 3.3.7: Top: Bank-level losses when comparing the use of heterogeneous vs uniform market depth with the leverage targeting response function. Bottom: Bank-level losses when comparing the [Greenwood et al. \(2015\)](#) setting (leverage targeting & uniform market depth) to our setting (threshold model & heterogeneous depth).

leverage targeting model respectively. Figure 3.3.9 shows the same bank-level losses after having iterated the fire sales cascade for 8 rounds. While, in the threshold model, the losses that some banks suffer in Scenario 1 differ by several orders of magnitude from the losses that they suffer in Scenario 2 (indicated by the blue asterisks lying significantly off the diagonal line), rather counter-intuitively, the losses suffered by banks in Scenario 1 and 2 are estimated to be almost the *same* in both scenarios by the leverage targeting model.

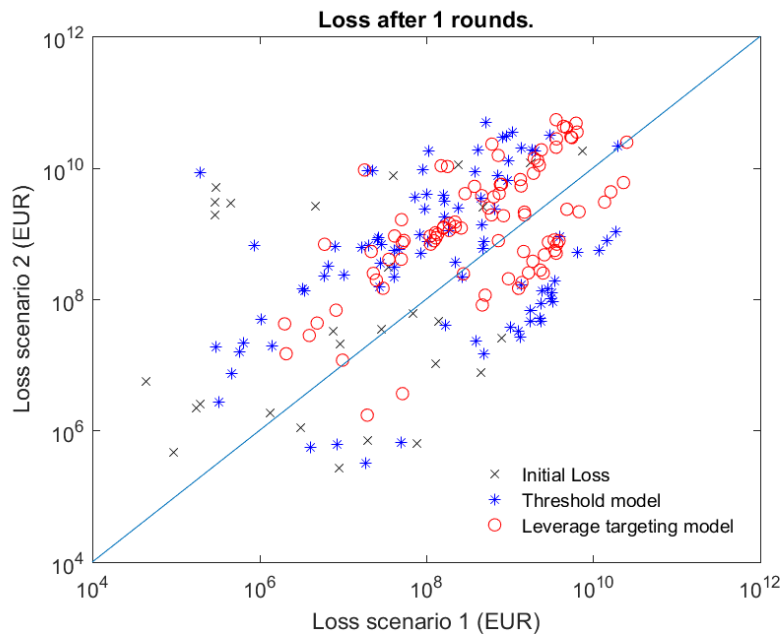


Figure 3.3.8: Bank-level initial losses (black crosses), and bank-level first round fire sales losses in Scenario 1 (horizontal axis) and scenario 2 (vertical axis) under the threshold model (blue asterisks) and leverage targeting model (red circles).

The left panel of Figure 3.3.10 shows that across a wide range of shocks, the use of the leverage targeting model leads to a very high correlation between the loss vectors (3.2) in different scenarios. In contrast, these correlations are quite moderate when using the threshold model, showing that the threshold model respects the intuition that different stress scenarios evolve differently. These results are particularly striking in light of the fact that the initial

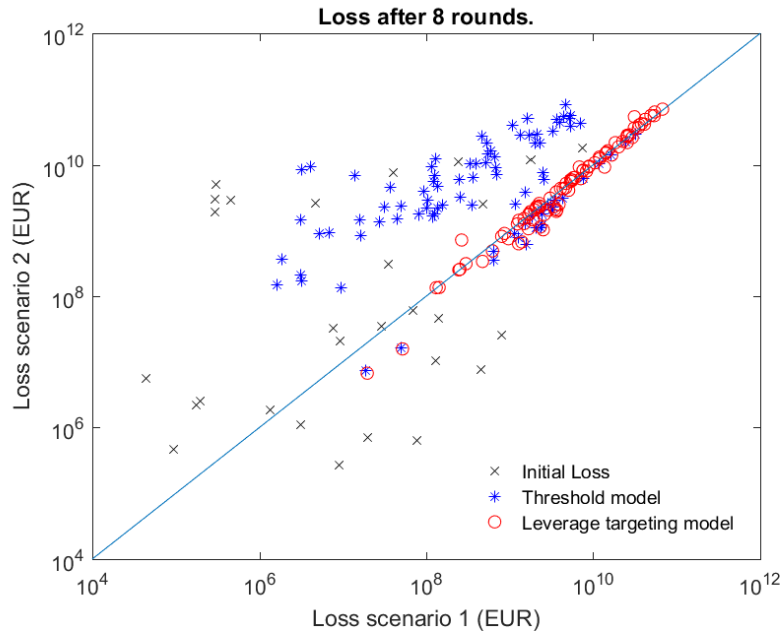


Figure 3.3.9: Bank-level initial losses (black crosses), and bank-level fire sales losses after eight rounds in Scenario 1 (horizontal axis) and scenario 2 (vertical axis) under the threshold model (blue asterisks) and leverage targeting model (red circles). While the heterogeneity of the initial losses leads to a heterogeneous cross-sectional distribution under the threshold model, individual bank losses have converged to essentially the same loss under the leverage targeting model.

loss vectors are close to being orthogonal. The right panel of Figure 3.3.10 shows the total loss vector correlations as a function of the cascade iteration round for an initial shock size of 15 % at the estimated market depth. In the first 3-4 iterations, through rebalancing of portfolios, the leverage targeting model “smooths out” fire sales losses evenly across banks, which results in the (I believe unrealistic) consequence that the final loss distribution across banks is almost independent of what the initial stress scenario was.

The panels in Figure 3.3.11 illustrate, for the Scenario-combination 1 & 2, that this is not a particularity of the market depth choice. Indeed, for a wide range of market depth scaling factors and shock sizes, the correlation ranges between 0.85 and 1 under the leverage targeting dynamics, while correlations range between 0.1 and 0.4 in all regions, except the systemic

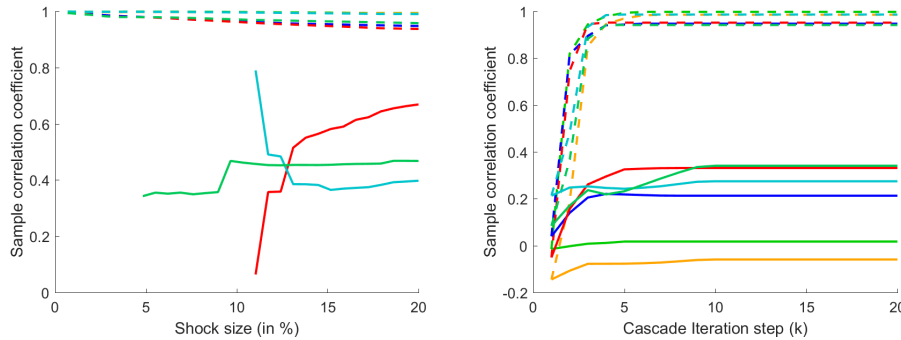


Figure 3.3.10: Left: Cross-sectional correlations of fire sales losses across banks as a function of the initial shock size. Right: Cross-sectional correlation of fire sales losses across banks as a function of the iteration step. The threshold model (full lines) retains the heterogeneity in the input scenario. This information is gradually lost in the leverage targeting model (dashed lines): Losses are smoothed out during the course of a fire sales cascade such that losses, despite arising from initially different scenarios, still converge to a nearly identical distribution.

risk region under threshold dynamics.

Finally, the left and right panels of Figure 3.3.12 show the correlation of the fire sales loss vectors between scenarios 1 and 2 as a function of the iteration step k and the market depth scaling factor for the leverage targeting model and the threshold model respectively, given a fixed initial shock of 13%. In the leverage targeting model, over the first 3 - 4 rounds, the correlation steeply increases from 0 to 1 for the entire range of market depth scaling factors. In contrast, the correlation is low and stable for a large range of market depth scaling parameters under the threshold model dynamics. Trivially, if both scenario specifications are in the systemic risk region, where large scale contagion occurs and most banks default, the correlation between the loss vectors of these scenarios is close to 1. Overall, this property makes the “number of rounds” an even more delicate parameter of the leverage targeting model.

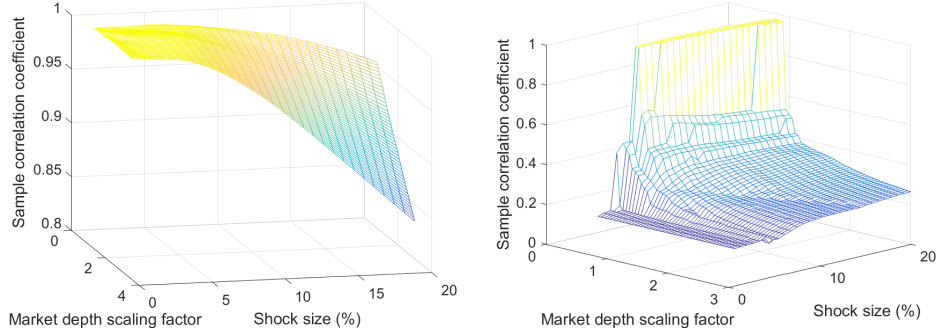


Figure 3.3.11: Sample correlation coefficient (as in the left panel of Figure 3.3.10) as a function of the shock size and the market depth scaling factor, under leverage targeting dynamics (left) and threshold dynamics (right). For visualisation purposes, the leverage targeting plot is slightly rotated. Note the difference in magnitude on the vertical z axis.

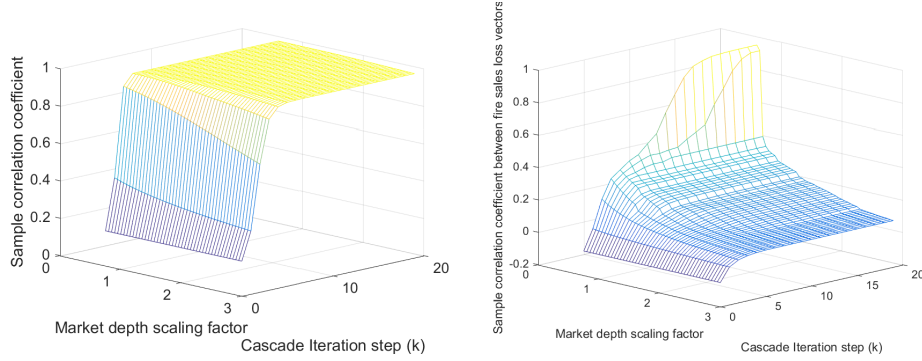


Figure 3.3.12: Correlation between the loss vectors in Scenarios 1 and 2 as a function of the market depth and cascade iteration step, for a fixed initial shock of 13% under leverage targeting dynamics (left) and threshold model dynamics (right).

3.4 FROM ASSETS TO ASSET CLASSES: A STUDY ON ISIN-LEVEL SUPERVISORY DATA FROM NORWAY

In the following section, we combine two proprietary and highly granular datasets of the Norwegian Central Bank, Norges Bank, to study the effect of choosing different levels of aggregation for the marketable asset classes on the magnitude and cross-sectional distribution of fire sales. In addition to that, we use the dataset to present some evidence in favour of the proportional deleveraging assumption. In Chapter 4, we will use the same data to propose a systemic vulnerability indicator for fire sales risk.

3.4.1 DATA: STYLISTED FACTS AND MODEL CALIBRATION

The first dataset (ORBOF) contains monthly bank balance sheet information for all Norwegian banks from 1986 - 2014. The second dataset from the Norwegian Central Securities Depository (CSD) “Verdipapirsentralen (VPS)” covers for the period 2006 - 2014, at a monthly frequency, the end-of-month portfolio holdings in all Norwegian registered securities by all Norwegian and foreign institutions at the ISIN level ¹⁰. The datasets have been merged by Statistics Norway (SSB) and each institution is identifiable via a unique anonymised numerical identifier. This allows us to reconstruct at a monthly frequency the (Norwegian registered) marketable assets of the banks’ portfolios for the period 2006 - 2014. Moreover, it gives us a full picture (at monthly frequency) of the trading of Norwegian assets by banks, funds, insurance companies and foreign investors. This latter point will be particularly helpful to calibrate the market depth of the assets.

SECURITIES HOLDING DATA. Table 3.4.1 lists the most important variables for each data point in the securities holding data.

Table 3.4.2 provides an overview of the market participants in the Norwegian securities market.

BANK BALANCE SHEETS. A core step is to combine the aggregate level bank balance sheet data with the granular portfolio data from the securities holdings data to reconstruct individual banks’ marketable assets portfolios. Table 3.4.4 shows a stylised balance sheet from our model. The category

¹⁰The dataset contains on average 2M observations per month, with 28 variables for each data point.

¹²Data only starts in 2011, and increases rapidly from 0 to 70, then jumps to above 200. We assume that this is due to the phasing in of the collection of this variable and therefore discard all entries below 200. By including this, we get an average of 178 and a standard deviation of 121.

Variable	Comment
ISIN	Unique security identifier: the “ μ ” at its most granular level
Unique owner ID	Institution i . If i is a bank, the ID allows matching with the balance sheet dataset.
Institutional sector of owner	cf. Table 3.4.2
Industrial sector of owner	ISO NACE-standard
Market value of holdings	This is precisely $\Pi^{i,\mu}$ in NOK
Gross amount sold over last month	In NOK
Gross bought over last month	In NOK
Security issuer ID	Unique (anonymised) security’s issuer ID
Security type	cf. Table 3.4.3
Industrial sector of security issuer	ISO NACE-standard
Institutional sector of security issuer	cf. Table 3.4.2
Security market price	End-of-month, in NOK

Table 3.4.1: The most important variables that are collected for each observation in the securities holding dataset.

Institutional Sector (SSB Code)	number (μ, σ)	% holdings	% volume
Private enterprises (2100)	23540 (2032)	14.68 (3.10)	1.37 (0.67)
Banks (3200)	143 (7)	11.78 (3.10)	55.75 (8.48)
Mortgage companies (3500)	14 (4)	1.60 (0.87)	0.71 (0.68)
Mutual funds (4300)	220 (12)	4.77 (1.21)	0.56 (0.39)
Life insurance & pension funds (5500)	120 (16)	11.84 (1.43)	1.00 (0.58)
Central government (6100)	10 (6)	21.90 (1.79)	9.03 (3.54)
Rest of the world (9000)	9178 (1176)	33.42 (4.46)	31.96 (10.67)

Table 3.4.2: Market participants by institutional sector and summary statistics on holding and trading.

Asset class	Number	% of wealth	% of trading
Equities	1423 (108)	56.44 (12.01)	31.03 (10.57)
Bonds	2100 (120)	33.40 (6.57)	35.67 (10.03)
Gov. bonds	22 (4) ¹¹	15.77 (4.08)	15.77 (4.08)
Covered bonds	274 (12) ¹²	5.98 (2.26)	5.98 (2.26)
Equity certificates	427 (77)	6.40 (3.16)	22.71 (9.80)
Warrants	594 (227)	2.26 (0.91)	0.24 (0.41)
Other	470 (67)	0.84 (0.14)	0.65 (0.39)

Table 3.4.3: Summary statistics of security type, aggregate value by type and trading volumes by type.

“foreign assets” captures the difference between the value recorded in the balance sheet data, which includes foreign holdings, and the total value

recorded in the securities holdings data, which only covers assets registered at the Norwegian CSD. We make the (optimistic) assumption that these assets are as liquid as US treasuries, and that they are not affected by the initial shock. This will thus only bias our results in the direction of understating the potential contagion from fire sales. Over the 8 years of monthly data, the average value of assets that we are unable to allocate is 4 percent of total assets. The foreign asset holdings tend to be larger for the larger banks, but even for the largest banks, this amount does not exceed 15-20 percent of total assets.

Illiquid assets
Net loans to clients (subdivision: commercial / residential real estate)
Net loans to Norwegian credit institutions
Net loans to foreign credit institutions
Intangible assets
Other claims and assets
Marketable assets (securities holdings data)
Certificates and Government bills
Bonds
Equities and equity certificates ¹³
Derivatives
Foreign assets

Table 3.4.4: Model balance sheet.

MARKET DEPTH OF NORWEGIAN SECURITIES. As the securities holdings data covers all market participants, we can easily compute the average daily trading volume of each asset μ for month t

$$ADV_{\mu,t} := \sum_{i=1}^{N_{system,t}} \frac{GAB_{\mu,t}^i + GAS_{\mu,t}^i}{2days_t}, \quad (3.4)$$

¹³These are instruments are specifically issued by Norwegian savings banks and similar to equities.

where $days_t$ is the number of trading days, $N_{system,t}$ is the total number of institutions in the system (which fluctuates as a result of mergers, new company creations and bankruptcies), and $GAB_{\mu,t}^i$ and $GAS_{\mu,t}^i$ are the gross amounts bought and sold respectively by institution i ¹⁴. Table 3.4.3 shows summary statistics for the main asset classes.

A significant portion of the assets do not trade every month. However, we find there is a very persistent relation between the amount outstanding and the turnover of the instruments that do trade. We employ the same workaround as previously used on the EBA data and regress the log average daily volume on the log amount outstanding for each period t :

$$\log_{10}(ADV_{\mu,t}) = c_{1,t} \log_{10}(N_{\mu,t}) + c_{0,t} + \varepsilon, \quad (3.5)$$

where $N_{\mu,t}$ is the amount outstanding, and $ADV_{\mu,t}$ the average daily trading volume. We fit the model for bonds, equities and all remaining asset types separately for each month. Table 3.4.5 displays the result averages over the 108 months. The panels in the left column of Figure 3.4.1 plot the regression lines for December 2013, while the panels in the right column show the estimates of the slope (red), intercept (black) and R^2 (green) as a function of the time period (i.e. the month that we run the regression for).

3.4.2 AN OPTIMAL LEVEL OF GRANULARITY FOR SYSTEMIC STRESS TESTING

We now analyse the magnitude and cross-sectional distribution of fire sales losses when we run the systemic stress test using different levels of granularity for the asset classes. To the best of our knowledge this is the first time

¹⁴Alternative definitions would be $\sum_i \frac{GAB_{\mu,t}^i}{days_t}$ or $\sum_i \frac{GAS_{\mu,t}^i}{days_t}$. However, we found our definition to be slightly superior in terms of data quality because the dataset does not cover sales and purchases by small Norwegian municipalities, which would be ignored by using these alternative definitions. The deviation is less than 0.5% however.

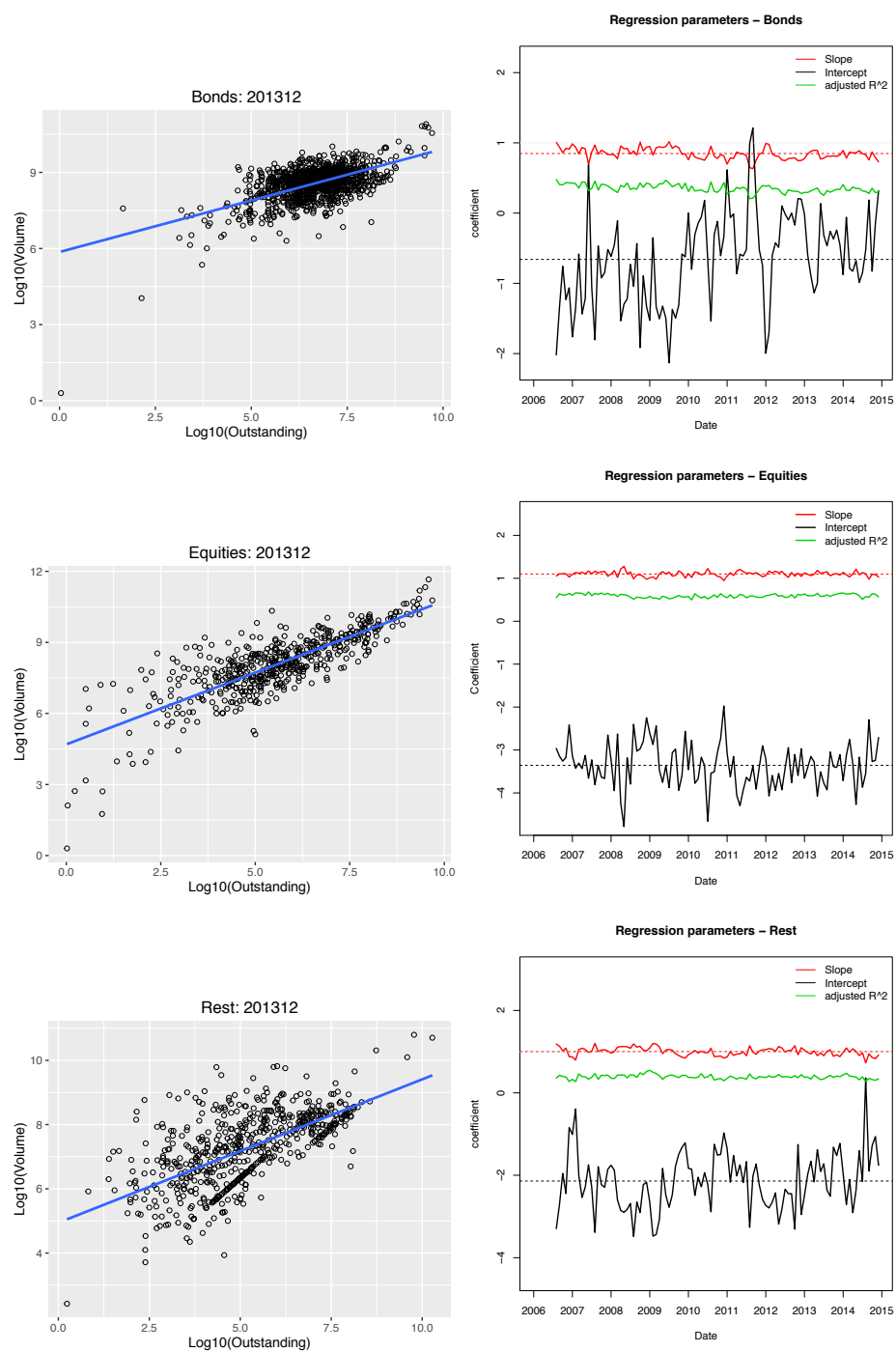


Figure 3.4.1: Left column: Scatter plots with OLS regression line for December 2013 for bonds, equities and all other assets grouped respectively. Right column: Regression estimates as a function of time.

	equities	bonds	rest
average c_1	1.10	0.85	1.00
std. dev. of estimates	0.059	0.089	0.095
average p-value	4.84e-69***	6.18e-39***	1.08e-30***
average c_0	-3.36	-0.66	-2.13
std. dev. of estimates	0.51	0.66	0.69
average p-value	3.12e-13***	0.21	0.01**
average adj. R^2	0.59	0.35	0.39
std. dev. of adj. R^2	0.04	0.06	0.05
average n per month	137238	37903	684

Table 3.4.5: Regression results of bond trading volume to issue size.

an analysis of this kind is performed. Given the information available in the securities holdings dataset, six levels of granularity present themselves naturally, from the most granular to the least granular:

1. *ISIN*: This is the single asset level, at this level bonds of different maturities by the same issuer are disaggregated.
2. *Issuer*: At this level all maturities and asset classes are grouped together by issuer.
3. *Industrial sector*: At this level, we group all securities belonging to issuers of the same industrial sector.
4. *Institutional sector*: At this level, we group all securities belonging to issuers of the same institutional sector.
5. *Asset class*: At this level, we aggregate all instruments of all issuers by type (i.e. bonds, covered bonds, equities...)
6. *Single asset*: At this level, we aggregate all assets into a single “marketable asset”.

Table 3.4.6 lists the number of “asset classes” at different levels of aggregation for December 2013. The numbers are roughly the same across all months.

Aggregation level	Held by all institutional sectors	Held by banks only
ISIN	5509	2930
Issuer	1871	912
NACE	214	139
Institutional Sector	18	17
Asset class	9	9
Single asset	1	1

Table 3.4.6: Number of “asset classes” for different layers of aggregation in December 2013.

Choosing a level of aggregation in a fire sales model is equivalent to making a statement on *cross asset class price impact*, i.e. the price impact that an asset class suffers because there is a fire sale in *another* asset class. Throughout this thesis, we work with an assumption of zero cross-asset class price impact. Thus aggregating two asset classes is similar to assuming a cross asset class price impact of 100%. In the sequel, we will therefore take the issuer level as a “benchmark” level of aggregation with respect to which we compare the other levels of aggregation.

MARKET DEPTHS. We begin by estimating the market depths at the ISIN level. To this end, we use the information on daily volume from the securities holdings data, and estimate volatilities from market price data from Oslo Børs¹⁵ As estimates turn out to be quite stable, we make a simplifying assumption on volatilities and use $\sigma = 0.03$ for stocks, $\sigma = 0.02$ for bonds, and $\sigma = 0.01$ for government bonds. By doing so, we can directly add trading volumes to compute the market depth at a higher level of granularity, as opposed to first computing market cap-weighted volatilities before aggregating¹⁶.

Figure 3.4.2 shows the distribution of market depths for the six levels of

¹⁵Bond indices: https://www.oslobors.no/ob_eng/markedssaktivitet/#/list/bondindices/quotelist/intraday. Share indices: https://www.oslobors.no/ob_eng/markedssaktivitet/#/list/shareindices/quotelist/intraday.

¹⁶This simplifies some computations considerably, as it avoids the trouble of computing volatilities for very irregularly traded, infrequently traded, and non-traded assets.

aggregation. The market depth of the single asset class is indicated by the vertical dashed line. As the level of granularity becomes coarser, there are less asset classes that remain with zero trading volume. The blue histograms also show the amount of non-traded assets; the red kernel density estimates include the estimated market depths of non-traded assets. Note that the histograms only show the *number of asset classes* for a specific market depth; they do not show the amount outstanding or market cap of these asset classes.

STRESS SCENARIO. As stress scenario we simply consider a housing crisis with banks making losses on their residential mortgage exposures. This shock is applied directly to the illiquid holdings, as obtained from the bank balance sheet data. We increase shock sizes from 0 to 10%.

FIRE SALES LOSSES. The panels in Figure 3.4.3 show the system-wide fire sales loss in percent of system equity as a function of the initial stress level and the market depth scaling factor, for the six different levels of aggregation. We see that due to the numerous non-traded assets, the loss estimates at the ISIN-level do not fall below a certain level, even when scaling the market depth estimates by two orders of magnitude! It is quite remarkable that for all other aggregation levels, the quantitative results agree quite well.

One striking feature when aggregating at asset-type and institutional-sector-level is the appearance of the “high systemic risk” tipping point at a market depth scaling factor around 10^{-1} and an initial shock of around 2%. The results of the systemic stress test performed on the EBA dataset previously showed similar features. This suggests, that if the systemic stress test on European banks were run at a significantly more granular level¹⁷ it is not

¹⁷We recall that we had only 148 asset classes for the entire European Banking system, while e.g. at an issuer level, we have 912 asset classes for Norway alone.

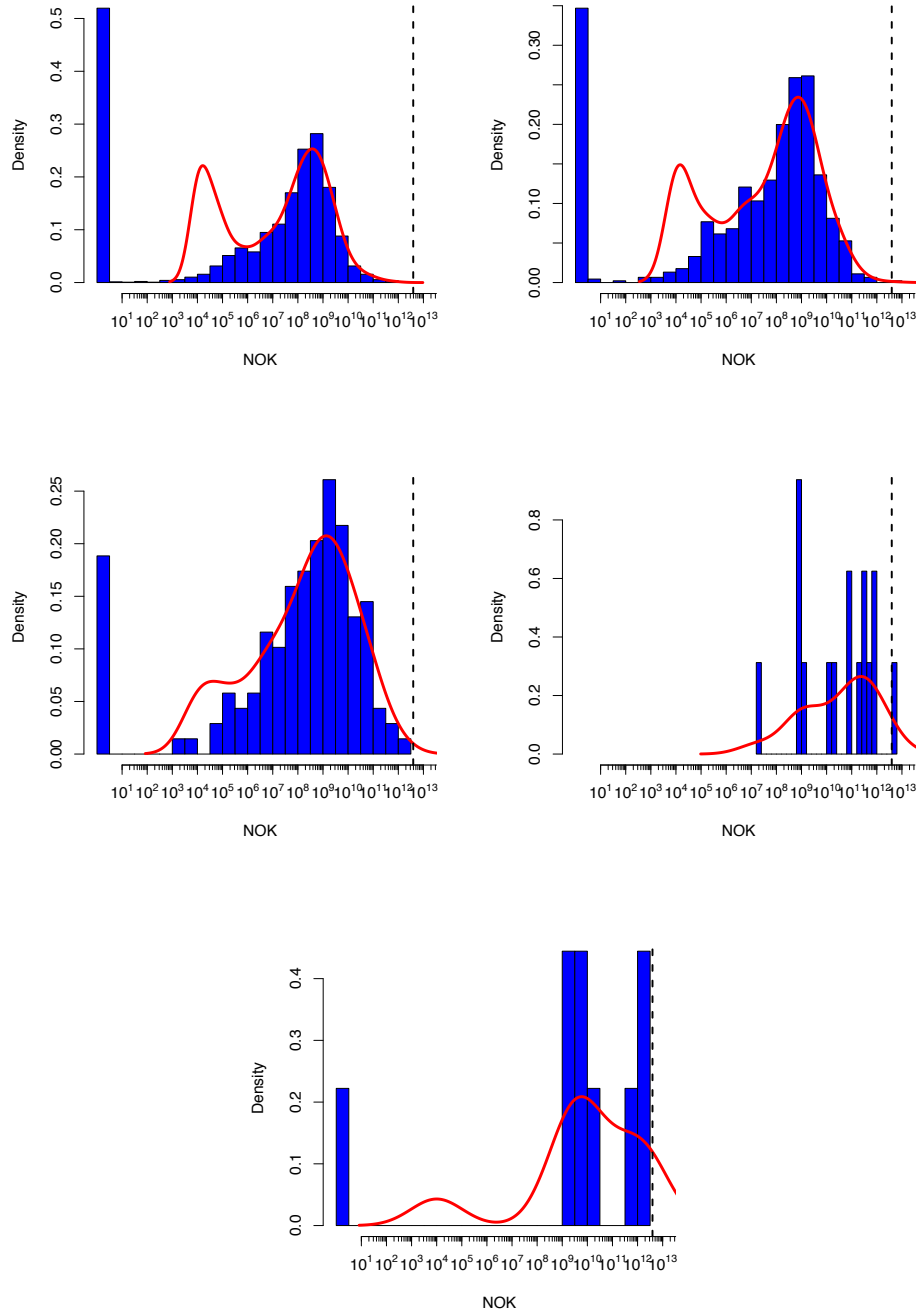


Figure 3.4.2: Histograms of estimated market depths under different levels of aggregation. The vertical dashed line indicates the estimated market depth of the single asset class. The red smooth line is the kernel density estimate, when including the estimated liquidity of non-traded assets. The areas under the curve and bars respectively add up to one.

unlikely that this cliff-effect may change both its location (wrt to the shock size and the market depth scaling parameter) and/or its steepness.

BANK-LEVEL LOSSES. The panels in Figure 3.4.4 show bank-level fire sales losses for the same stress scenario, intensity, and at the estimated market depth under different levels of aggregation for the asset classes. Although the total fire sales losses agree quite well in general, it is striking that the fire sales loss estimates for individual banks can differ by up to two orders of magnitude; note the logarithmic scale on the graph. What is more, for a less granular level of aggregation, it seems that we tend to underestimate the fire sales losses of the banks suffering the largest losses, while we tend to overestimate the losses of the banks that suffer the smallest losses. Arguably the most problematic bias here is the underestimation of the largest losses: A downward bias by a factor 10 - 100 of the largest bank losses, which is solely due to an increased market depth resulting from asset class aggregation, can lead to an underestimation of the actual risk and a false sense of security.

The four upper panels in Figure 3.4.5 show for the ISIN-level and industrial-sector granularity respectively the bank-level losses against the bank-level losses under issuer-level granularity for a market depth scaling factor less than 1 (top) and higher than 1 (middle). The two lower panels show the slope of the regression of fire sales loss vectors under these two aggregation levels against the fire sales loss vectors under the issuer aggregation-level. As the slope tends to be smaller than one, this means that the losses for large banks tend to be underestimated relative to the benchmark of using the issuer level of granularity, as suggested by the individual scatter plots in the upper panels.

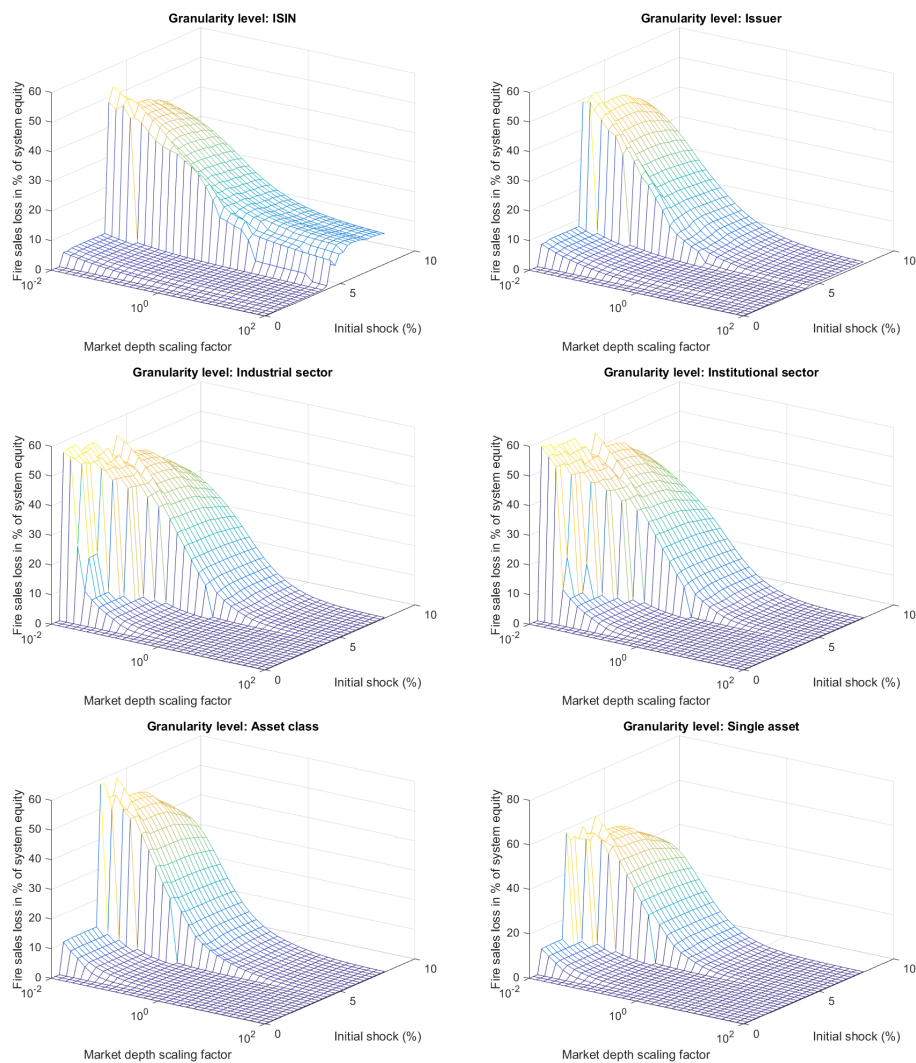


Figure 3.4.3: System-wide fire sales loss, estimated under different levels of aggregation as a function of the initial shock and the market depth scaling factor.

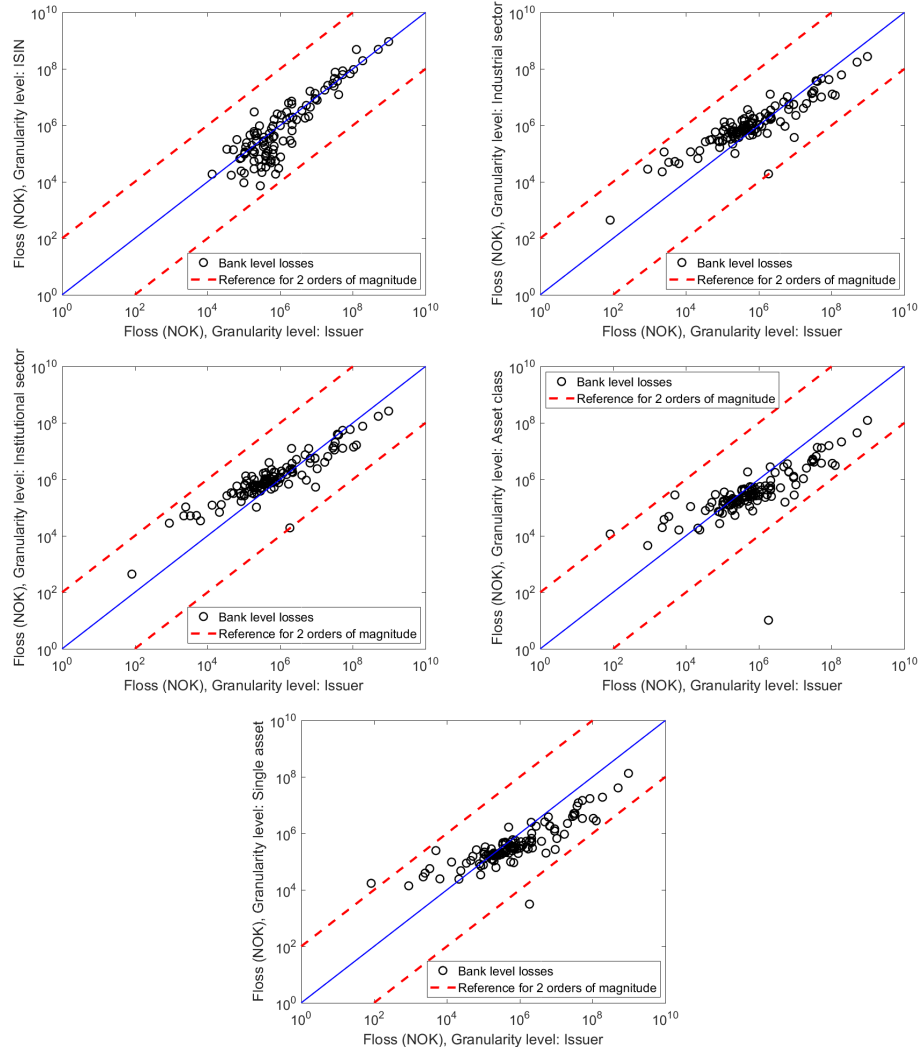


Figure 3.4.4: Bank-level fire sales losses, estimated under different levels of aggregation against the “issuer” aggregation level benchmark (horizontal axis).

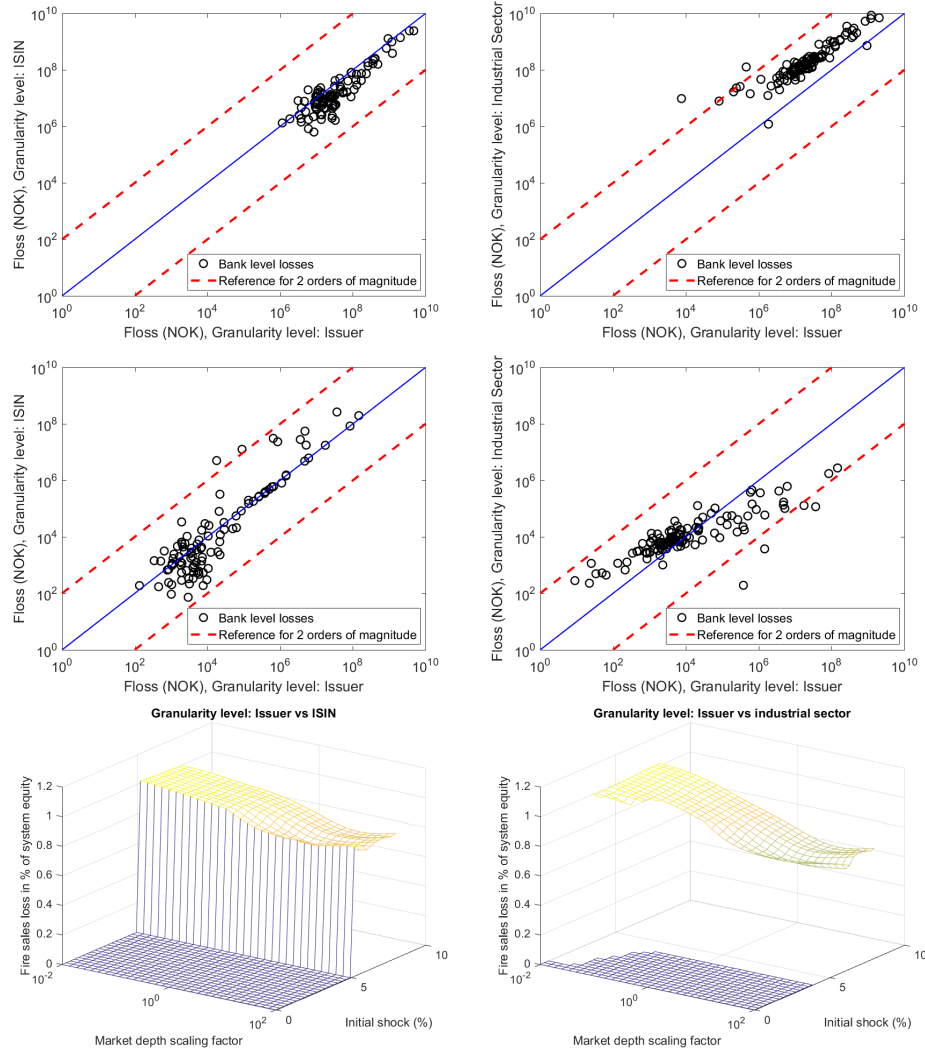


Figure 3.4.5: The left column panels show bank-level losses under ISIN-level granularity vs issuer-level granularity for (i) a lower market depth, (ii) a higher market depth, and (iii) show the slope of the regression of ISIN-level fire sales losses on issuer-level fire sales losses. The right column shows the same results for the comparison of industrial-sector-level granularity vs issuer-level granularity.

PRACTICAL IMPLICATIONS

In practice, several trade-offs regarding the level of granularity at which data should be collected arise. One advantage of more granular data is that the modeller can perform the aggregation into asset classes in a consistent way across the system, as well as perform a sensitivity analysis on this modelling choice. Indeed, when banks themselves perform the aggregation, and report aggregated data to the regulator, it becomes more challenging to ensure consistency of aggregation across banks. Secondly, more granular data also allows for a more robust estimation of the market depth, as well as for conducting a more rigorous sensitivity analysis. Third, as we have seen, a coarser level of aggregation may lead to an underestimation of the losses suffered by the most affected banks.

On the other hand, using a more granular level usually requires (substantially) more effort for data collection, -management and -storage, as well as for reconstructing bank balance sheets from the granular data, in particular when the exercise is being conducted at an international level. There are also confidentiality considerations that arise when positions are collected at a very granular level. As pointed out in the contribution by Stein in Chapter 2 of [Fouque and Langsam \(2013\)](#), there are further practical issues that concern the identifiability of institutions and assets. Indeed, reliably identifying different institutions, securities, and the institutions' holdings in assets is a practical issue from which we have almost entirely abstracted away. Legal Entity Identifiers (LEI) are internationally standardised alphanumeric character strings that, as the name suggests, allow the unique identification of legal entities. Another important issue in this regard is the grouping of branches and subsidiaries that belong to the same holding group. Work is under way to address these issues. Our main point is to illustrate how such “mundane” issues have a significant impact on results or risk assessments.

3.4.3 EMPIRICAL EVIDENCE OF PROPORTIONAL DELEVERAGING

Finally, we make use of the high granularity of the securities holding dataset to analyse whether we can find any evidence of fire sales in Norway in the period 2006 - 2014, and if so, whether the deleveraging tends to be proportional or whether institutions pick assets more selectively. Figure 3.4.6 shows the percentage of the wealth held by each institutional sector for all Norwegian registered assets. In the months after the Lehman default, a “bringing the money home” effect can be observed: foreign investors reduce their share of holdings in Norwegian assets from about 40 % to around 28%. For this reason we take the months of October - November 2008 to analyse the change in holdings. We note that banks and insurance companies are the first to absorb the shed assets of these investors. This illustrates an important point on the “availability of potential buyers” (Shleifer and Vishny (1992)) and the interaction of different regulations: It is beneficial for the system to have a diversity of differently constrained agents interacting with each other. In this case, if banks and insurance companies had had similar redemption profiles (or were constrained in a similar fashion by regulation), the amount of potential buyers in the scenario would have been severely reduced, probably leading to prices plummeting even further and causing further contagion.

Let Π_t^i denote the vector of holdings of institution i in month t : $(\Pi_t^{i,1}, ..\Pi_t^{i,M})^\top$.

The sale size

$$Size(i, t) := \frac{\sum_{\mu=1}^M \Pi_t^{i\mu} - \Pi_{t-1}^{i\mu}}{\sum_{\mu=1}^M \Pi_{t-1}^{i\mu}},$$

captures the size of sales and purchases in percent of last month’s total portfolio value of portfolio i . We can verify (or falsify) the proportional

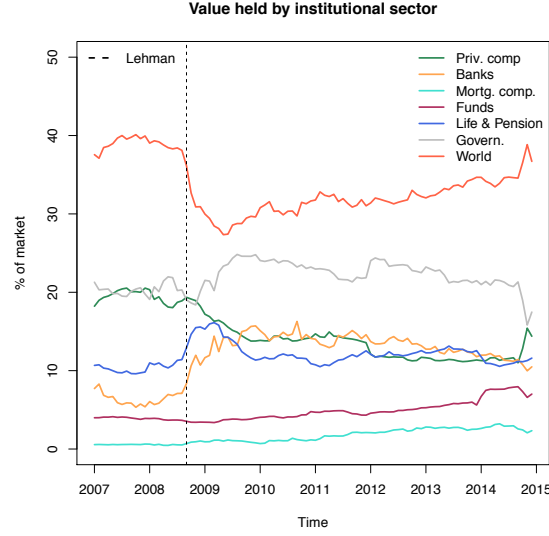


Figure 3.4.6: Percentage of the wealth held by each institutional sector for all Norwegian registered assets over time.

deleveraging assumption by analysing the colinearity

$$Col(i, t) := \frac{\langle \Delta \Pi_t^i, \Pi_{t-1}^i \rangle}{\|\Delta \Pi_t^i\| \|\Pi_{t-1}^i\|},$$

between the previous month's holdings and the portfolio change over the last month (with $\Delta \Pi_t^i = \Pi_t^i - \Pi_{t-1}^i$). Whenever i is deleveraging its portfolio proportionally, we have $Col(i, t) = -1$.

The histogram in the left panel of Figure 3.4.7 shows that indeed some investors deleverage their portfolio exactly proportionally (and some increase asset holdings exactly proportionally). There is also a number of investors who reduce the size of their portfolio by choosing a different deleveraging strategy. The peak at 0 reflects portfolio rebalancing, changing the composition of assets, but keeping roughly the same wealth invested in the market. The right panel plots individual asset sales and purchases for the 1000 largest portfolios with $Size(i, 2008/11) < -10\%$. The bulk of the data points lie,

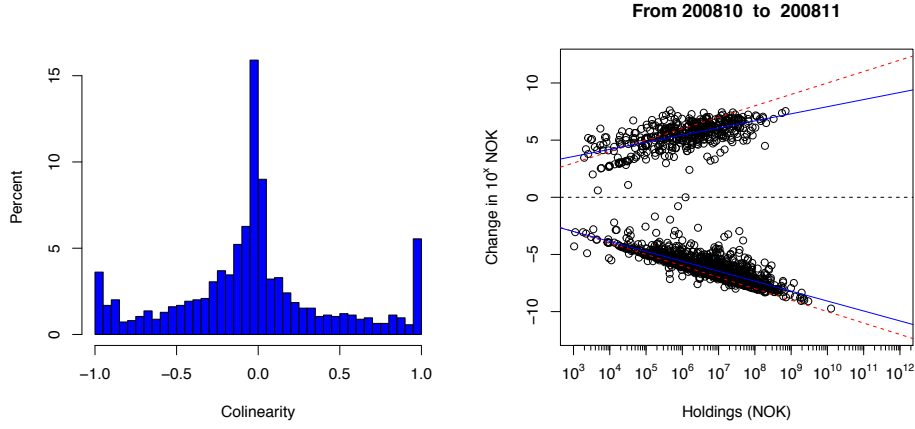


Figure 3.4.7: Left: Histogram of colinearity values. Right: Sales and purchases of single assets within the 100 largest portfolios reducing their size by more than 5%.

as expected, on and above the line $y = -x$ ¹⁸.

In contrast to the purchases, we can see that the sales cluster closer to the diagonal line. The slopes of the regressions (solid blue) for the sales and purchases are -0.87 and 0.62 respectively, with both coefficients having a p value below 10^{-10} . We interpret this as a further indication that portfolio changes are closer to proportional deleveraging for sales than for purchases.

Finally, Figure 3.4.8 analyses the dependence of proportional deleveraging, $Col(i, t)$ on the magnitude of the portfolio size reduction $Size(i, t)$. The plot shows that the larger the reduction in the portfolio size, the closer the deleveraging method is to proportional deleveraging (for a full portfolio liquidation, it is trivially exactly equal to proportional deleveraging). Hence, we can clearly see a number of investors who delever their portfolio proportionally, although they could choose to do so differently. The red curve

¹⁸Some marks lie below the $y = -x$ (red dashes) line, which means that on a net basis, an investor sells more assets than he holds in his account. This phenomenon can be due to short selling, aggregation issues (because assets are plotted at the issuer level) or data quality. As this only occurs for relatively small amounts (log-scale graph), we did not investigate this further.

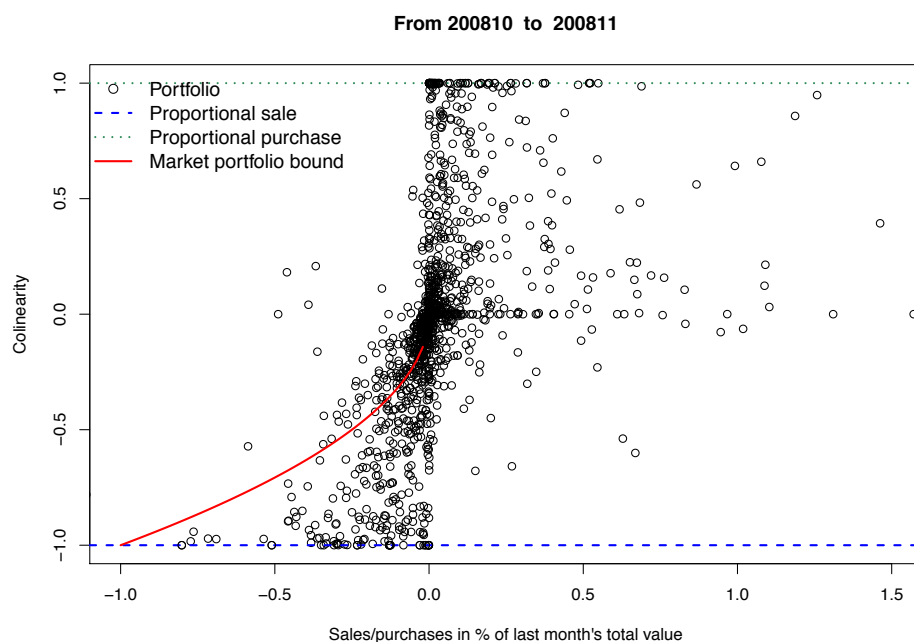


Figure 3.4.8: Proportional deleveraging: Colinearity of portfolio changes as a function of the size of the change.

provides a visual benchmark: One obtains the red curve when one starts with an equally weighted market portfolio of 2000 assets and reduces its size by selling one asset off at a time.

*The only good thing to do with good advice is pass
it on; it is never of any use to oneself.*

Oscar Wilde

4

MONITORING AND PRUDENTIAL TOOLS

This Chapter is based on the working papers “Fire sales, indirect contagion and systemic stress testing” which is joint work with Prof. Rama Cont, and “Fire sales and systemic risk: Monitoring and prudential policy”, which is joint work with Prof. Rama Cont and Prof. Artashes Karapetyan.

4.1 MONITORING

In this section, we propose two indicators for monitoring portfolio overlaps and the risk of fire sales in a network of overlapping and constrained portfolios.

4.1.1 LIQUIDITY-WEIGHTED OVERLAPS AND THE INDIRECT CONTAGION INDEX

PORTFOLIO OVERLAPS. We recall from (1.24) that the transmission of fire sales losses from portfolio i to j depends on the liquidity-weighted overlap between portfolio i and j :

$$\Omega_{ij}(\Pi) := \sum_{\mu=1}^M \frac{\Pi^{i\mu} \Pi^{j\mu}}{D_{\mu}}.$$

The matrix Ω of liquidity-weighted overlaps thus plays an important role in the transmission of fire sales losses, which may be viewed as a contagion process on a network of financial institutions in which the link from i to j is weighted according to the liquidity-weighted overlap Ω_{ij} . We call this network the *indirect contagion network*. Similar network structures were explored by [Braverman and Minca](#), [Guo et al. \(2015\)](#) for mutual funds. Figure 4.1.1 displays the indirect contagion network for European banks as implied by EBA data collected in 2011. The nodes correspond to different banks, with node size proportional to the logarithmic balance sheet size, and edges correspond to non-zero portfolio overlaps, with edge widths proportional to the logarithm of the liquidity-weighted overlap (and truncated at three orders of magnitude below the maximum value).

Figure 4.1.2 (left) shows the distribution of Ω_{ij} in this network. The peak at zero reflects the fact that many pairs of banks have no common asset holding (so, zero overlap) i.e. the network is sparse. On the other hand, the values are dispersed over five orders of magnitude, which illustrates the heterogeneity of the network.

The overlap matrix Ω also gives a glimpse of the nature of ‘second-round’ contagion effects in this network. The element (i, j) of the matrix Ω^2 may be interpreted as the (maximal) indirect contagion from i to j channelled

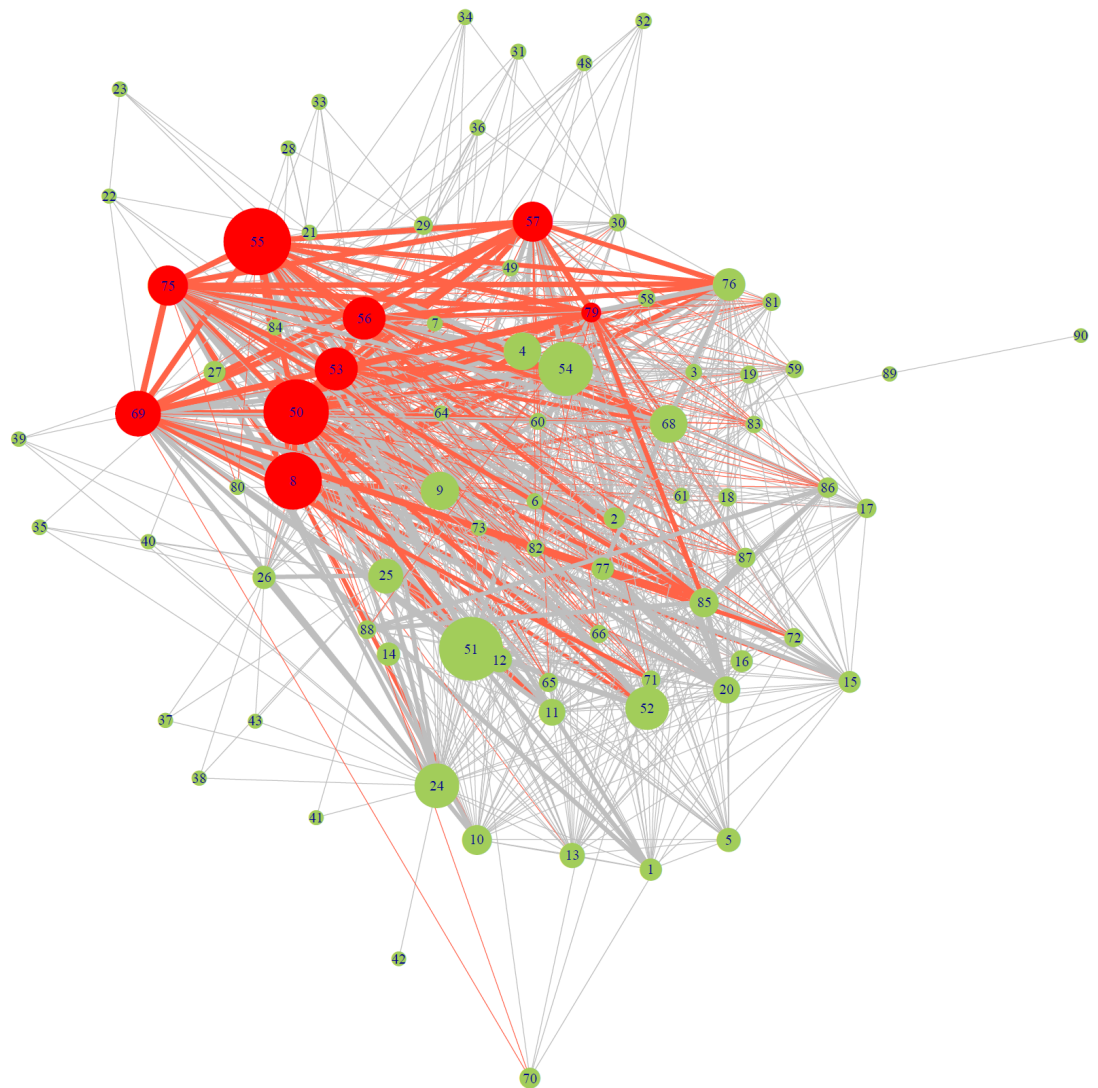


Figure 4.1.1: The European indirect contagion network. Node sizes are proportional to the log of total assets, edge widths to log liquidity-weighted overlaps.

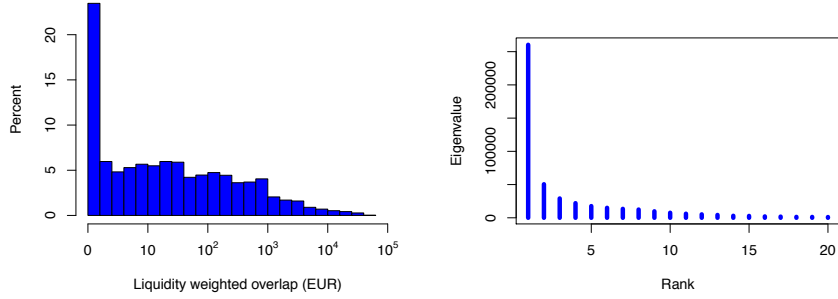


Figure 4.1.2: Left: histogram of liquidity weighted portfolio overlaps across EU banks (2011). Right: Ranked eigenvalues of the matrix Ω of liquidity-weighted portfolio overlaps.

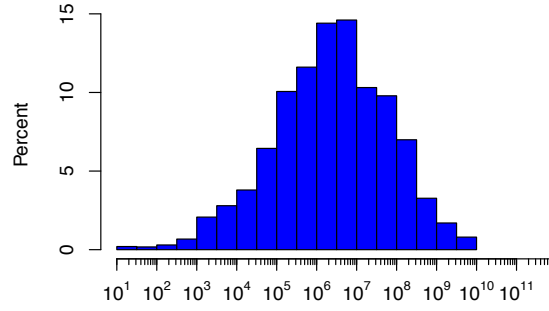


Figure 4.1.3: Histogram of 'second-order overlaps' (coefficients of the matrix Ω^2).

through other nodes:

$$\Omega^2(i, j) = \sum_k \Omega_{ik} \Omega_{kj}.$$

Even though the matrix Ω of portfolio overlaps is sparse, the matrix Ω^2 of second-order overlaps is dense. Figure 4.1.3 shows the histogram of these 'second-order overlaps'. Unlike Figure 4.1.2, where we see a large fraction of zero overlaps, here we see that all coefficients are strictly positive: this means that, although many pairs of portfolios have zero overlaps, second-round effects may potentially cause spillovers from *any* institution to any other! Moreover, by comparing the distribution of liquidity-weighted overlaps in

Figure 4.1.2 to second-round overlaps in Figure 4.1.3, we see that some shocks need *at least* two rounds to propagate from one bank to another. These observations show, as noted in previous chapters, via a simulation-free argument that second-round effects should not be ruled out a priori and may have a significant impact on the extent and magnitude of contagion across the network.

A quantitative way of examining the heterogeneity of this network is to compute the eigenvalues and eigenvectors of Ω . The right-hand plot of Figure 4.1.2 shows the ranked eigenvalues of Ω . The first few eigenvalues clearly dominate the others by an order of magnitude. This means that the overlap in the portfolio network can be characterised quite well by a low-dimensional factor model for portfolio holdings. We obtain similar results on the more granular Norwegian data. For brevity, we only report results on the EBA data.

INDIRECT CONTAGION INDEX. The results above indicate that the contribution of fire sales to system-wide losses is mainly driven by liquidity weighted portfolio overlaps (1.25). As observed above, the principal component analysis (eigenvalue decomposition) of this matrix reveals that its first two eigenvectors capture the main structure of the network of portfolio overlaps. Similar to [Capponi and Larsson \(2015\)](#), we use this insight to design a simple indicator, based on institutional portfolio holdings, which quantifies the exposure of a bank to price-mediated contagion without resorting to a large-scale stress test.

The idea is to use an eigenvalue decomposition of the matrix of portfolio overlaps to identify the main institutions which contribute to price-mediated contagion, by computing their eigenvector centrality in the network defined by portfolio overlaps. To compute this, we proceed as follows:

1. Collect portfolio holdings $\Pi^{i\mu}$ by asset class for each financial institution in the network, at the granularity level corresponding to bank stress tests.
2. Estimate a market depth parameter D_μ for each asset class.
3. Compute the liquidity-weighted overlaps $\Omega_{ij}(\Pi) := \sum_{\mu=1}^M \frac{\Pi^{i\mu}\Pi^{j\mu}}{D_\mu}$ of portfolios.
4. Compute the principal (largest) eigenvalue and the corresponding eigenvector $U = (U_i, i = 1 \dots N)$ of the matrix of liquidity-weighted overlaps $\Omega(\Pi) = \Pi D^{-1} \Pi^\top$.¹
5. The component U_i of the principal eigenvector U then provides a measure of centrality of the node i in the network whose links are weighted by the overlap matrix (1.26).

We define the **Indirect Contagion Index (ICI)** of a financial institution as its component in the normalised principal eigenvector U_i of the matrix (1.26) of liquidity weighted portfolio overlaps:

$$ICI(i) = U_i. \quad (4.1)$$

Figure 4.1.4 shows the principal eigenvector of Ω . From a numerical perspective, this is most robustly computed as the the left singular vector of the singular value decomposition of $\Pi\sqrt{D^{-1}}$. The Indirect Contagion Index can be directly read from Figure 4.1.4. The banks with highest ICI are labelled. All banks that are identified are either Global Systemically Important Banks (GSIBs) or Domestic Systemically Important Banks (DSIB). It

¹We note that absent large short positions, all U_i will be non-negative. Indeed, a sufficient condition for the Perron-Frobenius Theorem to assert the non-negativity of all components of the first eigenvector is that the corresponding matrix has non-negative entries and is irreducible. Irreducibility of the liquidity-weighted overlap matrix is equivalent to the indirect contagion network being strongly connected, which is the case for all practically relevant cases.

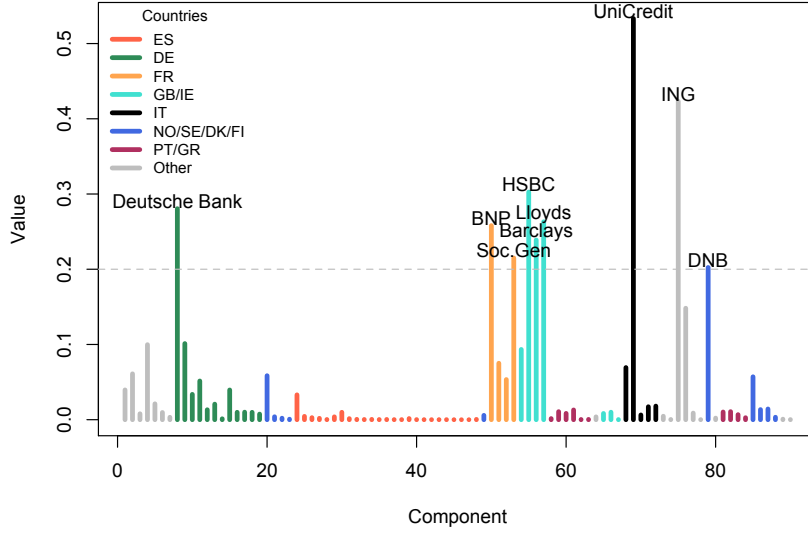


Figure 4.1.4: The principal eigenvector of the the matrix of liquidity-weighted portfolio overlaps is concentrated on a few banks, all of which are observed to be GSIBs (DNB is a DSIB only). ICI_i corresponds to the value of bank i in this graph. The eigenvector is normalised to have norm 1, such that this provides a relative ranking of the banks.

is furthermore important to note that the banks are not ranked according to their size; rather, they are ranked by the amount of contagion they could trigger, should they engage in a fire sale. Figure 4.1.5 displays the eigenvector relating to the second largest eigenvalue of Ω . The second eigenvector loads heavily on French banks and the Italian Unicredit.

The normalisation by the market depth in steps 2 and 3 in the description of the ICI construction is very relevant. Leaving this step aside, i.e. computing the eigenvector centrality of institutions in the *nominal overlap* network $\Omega_{nominal} := \mathbf{III}^\top$ leads to the left panel of Figure 4.1.6. For ease of comparison, we have reproduced the first eigenvector of the liquidity-weighted overlap matrix in the right panel. Clearly, the set of banks with highest ICI changes: The contribution of large banks with liquid holdings is overstated in comparison to medium-sized banks with less liquid holdings².

²We note that for instance in 2007 Lehman had total assets equalling 690 Bn USD,

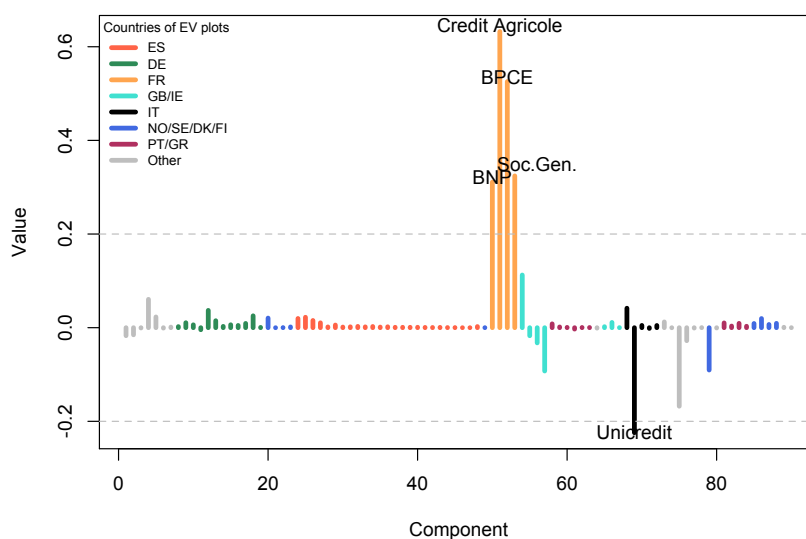


Figure 4.1.5: The eigenvector corresponding to the second largest eigenvalue of the the matrix of liquidity-weighted portfolio overlaps is concentrated on French and Italian banks.

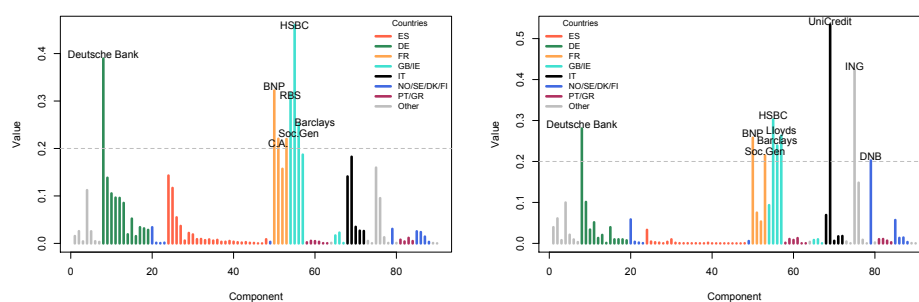


Figure 4.1.6: Left: ICI computed on nominal overlap matrix. Right: ICI on liquidity-weighted overlap matrix.

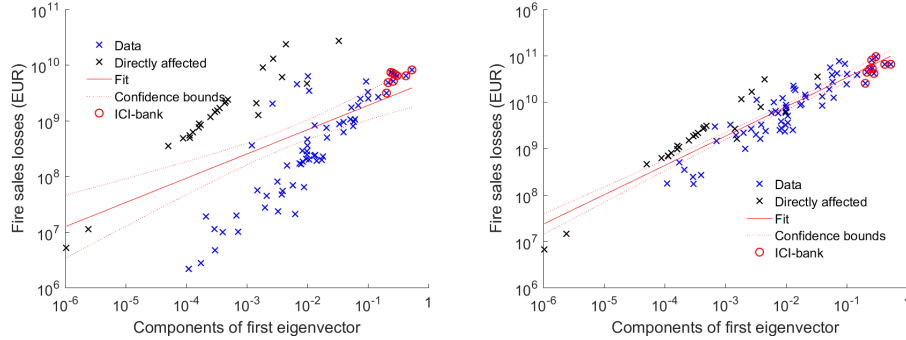
ICI AND FIRE SALES LOSSES. The (logarithmic) cross-sectional regression of fire sales losses on the Indirect Contagion Index (Figure 4.1.7 and Table 4.1.1) reveals a significant positive relation between ICI and fire sales losses.

while JP Morgan and Citibank had assets of 1.5 and 2.1 tn USD (https://www.sec.gov/Archives/edgar/data/806085/000110465908005476/a08-3530_110k.htm, <https://www.sec.gov/Archives/edgar/data/19617/000119312508043536/d10k.htm>, <https://www.sec.gov/Archives/edgar/data/831001/000119312508036445/d10k.htm>).

	Model 1: $L^i = \beta U_i + \varepsilon_i$	Model 2: $\log(L^i) = b \log(U_i) + \varepsilon_i$
Slope	$1.42 \cdot 10^{10***}$ ($4.56 \cdot 10^9$)	$0.44***$ (0.07)
Intercept	$1.67 \cdot 10^9***$ ($0.48 \cdot 10^9$)	$3.71***$ (0.19)
n	90	90
R^2	0.10	0.28
Slope	$1.76 \cdot 10^{11***}$ ($1.45 \cdot 10^{10}$)	$0.63***$ (0.03)
Intercept	$7.40 \cdot 10^9***$ ($1.53 \cdot 10^9$)	$5.16***$ (0.08)
n	90	90
R^2	0.63	0.85

Table 4.1.1: Regression of bank losses on the Indirect Contagion Index for all banks.

In Figure 4.1.7, the red circles indicate the banks with largest ICI highlighted in Figure 4.1.4; the black crosses indicate banks that are subject to the initial shock scenario 1; the blue crosses indicate all other banks. The regression R^2 for the left panel however is relatively low at 0.28. In the systemic risk region (right panel), the regression is much better with an $R^2 = 0.85$.

**Figure 4.1.7:** Left: Logarithmic regression of fire sales losses on Indirect Contagion Index (4.1) at the estimated market depth. Right: The same regression inside the systemic risk region.

Taking a closer look at the left panel of 4.1.7, we can actually identify two clusters of banks: those that are affected by the initial losses, and all other banks only suffering losses from price-mediated contagion. Figure 4.1.8 and Table 4.1.2 provide a clearer picture: When running separate regressions

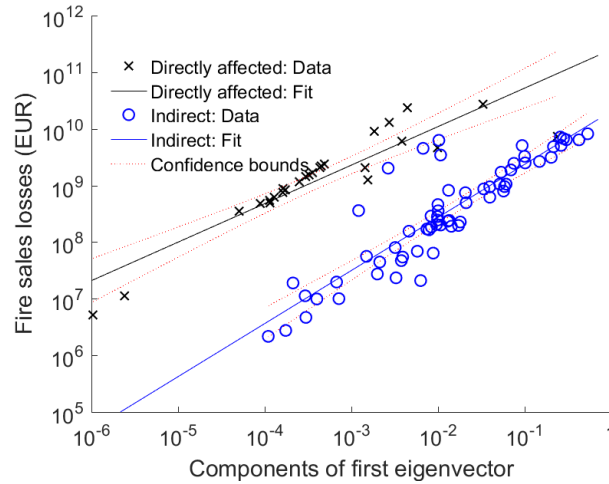


Figure 4.1.8: Separate regressions for banks that were initially shocked and all other banks.

	Affected: $\log(L^i) = b \log(U_i) + \varepsilon_i$	Indirect: $\log(L^i) = b \log(U_i) + \varepsilon_i$
Slope	0.68*** (0.07)	0.94*** (0.06)
Intercept	5.40*** (0.23)	4.31*** (0.13)
n	26	64
R^2	0.82	0.79

Table 4.1.2: Separate regressions of bank losses on the Indirect Contagion Index for initially shocked, and initially non-shocked banks respectively.

on these two sets of banks, the R^2 increases to 0.8 in the non-systemic risk region!

In the panels of the left column of Figure 4.1.9, we plot as a function of the initial shock size and the market depth scaling factor from top to bottom (i) the regression slope when including all banks, (ii) the regression slope for affected banks only, (iii) the regression slope for initially unaffected banks only. Note the different vertical axis labels: Clearly, running separate regressions for initially affected and unaffected banks increases their quality a lot. The panels in the right column present, in a similar fashion, the regression R^2 as a function of the initial shock size and market depth scaling factor for these three regressions. Again we note the different vertical axes,

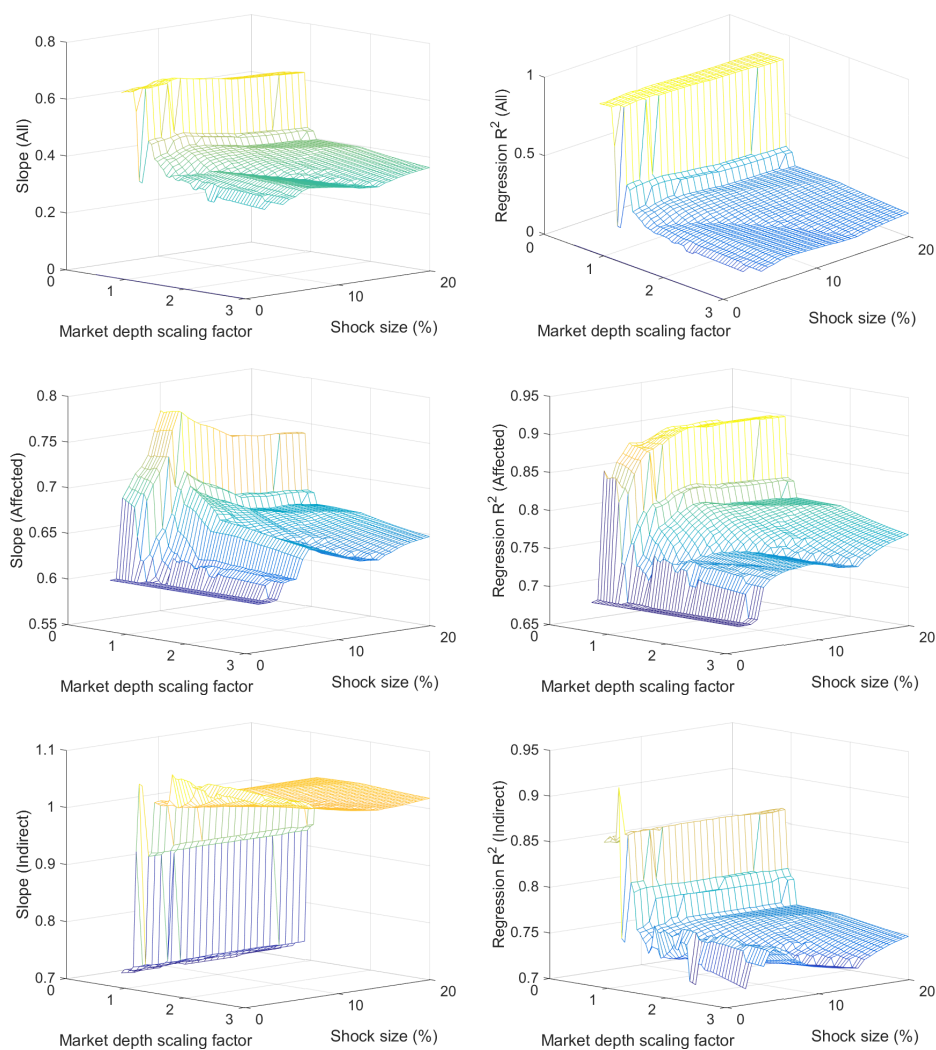


Figure 4.1.9: Left column: Regression slope as a function of the shock size and market depth for all banks, shocked banks only, and non-shocked banks only. Right column: R^2 as a function of the shock size and market depth for all banks, shocked banks only, and non-shocked banks only.

and observe a stark improvement in the quality by separating the groups of banks.

Hence, for any shock scenario, the ICI is a good predictor of bank-level losses should fire sales be triggered. These results suggest that the Indirect Contagion Index may be used as a quantitative indicator of ‘interconnect-
edness’ in the assessment methodology for Global Systemically Important Banks [Basel Committee on Banking Supervision \(2013a\)](#). The information contained in this index complements those currently used for GSIB identification and identifies banks which may contribute to the indirect contagion channel.

4.1.2 SYSTEMIC VULNERABILITY INDICATOR

Finally, we use the threshold model to build an indicator of systemic vulnerability. For the remainder of this section, we return to the Norwegian data using the issuer-level granularity level. The key idea is simply to track the boundary of the “high systemic risk” region. More precisely, we seek to quantify at which point, as a function of estimated market depth and initial shock size, we should expect large-scale contagion to occur. Consequently, if for optimistic parameters (small shock size / high market depth) large-scale contagion is observed, then the system is currently in a fragile state. On the other hand, if for conservative values (larger shock size / low market depth) no large-scale contagion is observed, the system can be assumed to be resilient. We define the **systemic vulnerability indicator**

$$SVI(x, \delta_{scale}, t) := \operatorname{argmin}_{\epsilon} \left\{ \frac{SFLoss(\epsilon, \delta_{scale}, t)}{\sum_{i=1}^N C_i(t)} \geq x \right\}, \quad (4.2)$$

to be the initial shock size that, for a given market depth scaling factor δ_{scale} and period t , triggers fire sales losses in excess of at least x percent of total

bank equity. The indicator is displayed in Figure 4.1.10. We then take a weighted average of this quantity over a set of realistic market depths and define the **average systemic vulnerability indicator**

$$ASVI(x, t) := \sum_{i=1}^n w_i SVI(x, \delta_{scale}^i, t), \quad (4.3)$$

where w_i are weights that sum to one, specifying the plausibility of different market depth scaling factors. Figure 4.1.10 shows $SVI(0.20, \delta_{scale}, t)$ for $\delta_{scale} \in [10^{-2}, 1]$ and $t = 2007/01, \dots, 2014/05$. Reading the plot from front to back, the market depth scaling factor increases, and so does the initial shock size required to trigger a fire sales cascade causing losses in excess of 20% of total bank equity. Over time the behaviour is less regular, but overall the minimum shock required to trigger fire sales loss in excess of 20 % is larger towards the end 2013 - 2014 compared to during the financial crisis and European sovereign debt crisis. The system has thus become more resilient during the period.

Figure 4.1.11 shows the average systemic vulnerability indicator (4.3) for $x = 0.2$ and $x = 0.3$ with $w_{i,t} \equiv \frac{1}{8}$, because $SVI(0.2, t)$ and $SVI(0.3, t)$ are only defined for the eight lowest market depth scaling factors over all time periods. In Figure 4.1.10, this can be seen as the depth of the surface when reading the plot from front to back. E.g. for $t = 2014/01$, the surface is also defined for larger market depth scaling factors, however including these values in the averages would make comparison between different time periods difficult, as the averages are not taken over the same sets of market depths. To ensure consistency when plotting $ASVI(x, t)$ for several different system-wide percentage losses x in the same graph, it is important to take averages over the *same* set of market depths scaling factors. Figure 4.1.11 clearly suggests that the Norwegian banking system has grown more resilient to leverage-triggered contagion from fire sales since 2013.



Figure 4.1.10: Minimum shock required to trigger large fire sales cascades causing fire sales losses in excess of 20% of total bank equity as a function of time and market depth and time: $SVI(0.20, \delta_{scale}, t)$. Z AXIS: SVI

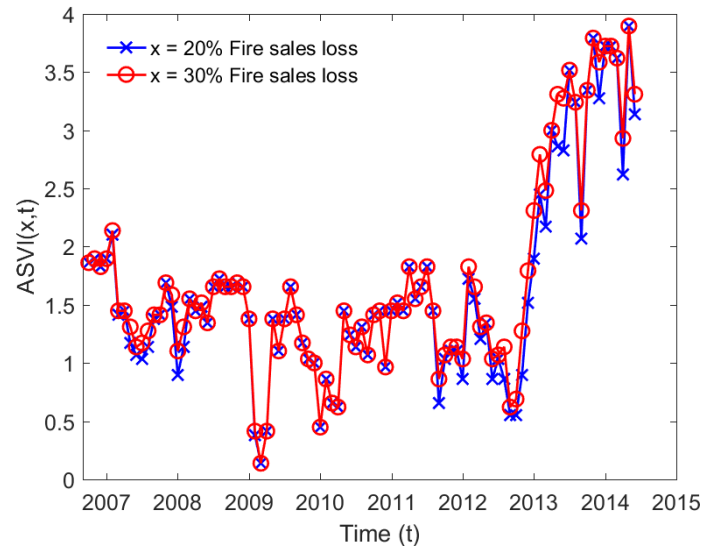


Figure 4.1.11: Average minimum shock over all market depths: $SVI(x, t)$ for $t = 2006, \dots, 2014$ and $x = 0.2, 0.3, 0.4, 0.5$.

4.2 MACROPRUDENTIAL POLICIES

One of the novel features of the post-crisis regulatory reform is an additional capital charge for “Systemically Important Banks” (GSIBs). These additional charges increase via a point system that quantifies the systemicness of the bank, as measured in the Basel framework³.

Our final object of study in this Part on fire sales and systemic stress testing is to analyse, within our framework, what type of prudential policies can help mitigate the risk and contagion arising from fire sales. Similarly to the SIFI-capital charges, we propose additional capital charges that are allocated in a targeted way throughout the system so as to reduce fire sales losses as effectively as possible.

Let C_+^i be the *additional capital* that bank i needs to raise. Define the *total additional capital*

$$C_+^{system} = \sum_{i=1}^N C_+^i. \quad (4.4)$$

First, we introduce a benchmark policy in which we require all banks to raise capital uniformly. Then, based on findings of the previous sections, we introduce three macroprudential policies where the same total amount C_+^{system} is raised across the system, but the individual capital charges C_+^i differ so as to minimise the risk of, or contagion from, fire sales. We can then evaluate the policies by comparing how much more efficiently they reduce fire sales losses relative to the benchmark and to each other.

We note that several non-trivial pre-conditions need to be fulfilled for a successful application of this methodology in practice:

- The regulator needs to be in possession of precise data on portfolio holdings;

³See Table 2: <http://www.bis.org/publ/bcbs255.pdf>

- One needs to have reliable estimates of the market depth of the various asset classes under the given shock scenarios;
- One needs to have a plausible model on the constraints of different market participants that triggers them to engage in fire sales.

In the sequel, we will assume all this to be given, and illustrate the methodology within our setting.

4.2.1 BENCHMARK POLICY: UNIFORM INCREASE IN CAPITAL RATIOS

We require all banks to raise capital uniformly under the benchmark policy. This corresponds to scaling the individual capitals, leading under the benchmark B to the additional capital requirement

$$C_{B,add}^i(C, C_+^{system}) = C_+^{system} \frac{C^i}{\sum_{j=1}^N C^j},$$

and the final capital requirement

$$C_B^i = C^i + C_{B,add}^i.$$

4.2.2 SIFI CAPITAL SURCHARGE USING THE ICI

The first macroprudential policy uses the predictive power of the Indirect Contagion Index and requires banks with higher ICI to raise more capital. As regression results on the logarithm of the ICI were the most convincing in the previous section, we set the additional capital under this policy to be

$$C_{ICI,add}^i(ICI, C_+^{system}) = \frac{\log(ICI_i)}{\sum_{j=1}^N \log(ICI_j)} C_+^{system}. \quad (4.5)$$

4.2.3 SIFI CAPITAL SURCHARGE WITH POSITIVE SYSTEM EXTERNALITY

The second prudential policy uses our results from Section 2.1.6: Requiring certain banks to raise capital reduces system-wide fire sales losses significantly by forcing these banks to internalise the systemic risk that they contribute to the system. However, some of these banks have no incentive to do so voluntarily because the outcome of the cost-benefit analysis per euro of capital raised is unfavourable for them.

Therefore, we require precisely those banks with the largest positive system externality (PSE) to the system to raise most capital, even if this is not in their own interest. For simplicity, we chose a proportional way of allocating the capital surcharges under the positive system externality policy:

$$C_{PSE,add.}^i \left(\frac{dL_1}{dC^i}, C_+^{system} \right) = C_+^{system} \frac{w_i}{\sum_{j=1}^N w_j},$$

where

$$w_i = \min \left(0, -\frac{dL_1}{dC^i} \right).$$

A calibration issue with this policy is that $\frac{dL_1}{dC^i}$ depends on ϵ and is in fact scenario dependent. As a workaround, we use a simplifying assumption to compute $\frac{dL_1}{dC^i}$ and postulate a uniform shock that leads all institutions to delever 10% of their portfolio. In spirit, this assumption is close to the information in the indirect contagion network: the network shows the liquidity-weighted overlaps of portfolios, which in fact only become relevant as soon as one of the institutions in the network starts to delever.

4.2.4 TARGETED RISK WEIGHTS FOR SYSTEMICALLY IMPORTANT ASSETS

The final macroprudential policy is inspired by the concept of “Systemically Important Assets” introduced in [Acharya and Öncüb \(2013\)](#). We imple-

ment this idea quantitatively by modifying the risk weights of systemically important assets, which we identify as those with largest contribution to the principal **right** eigenvector of the singular value decomposition of $\Pi\sqrt{D^{-1}}$. Adjusting risk weights is not a novel idea at all; in April 2014 for instance, the National Bank of Belgium informed the EBA about its intention “*to modify capital requirements in order to account for an increase in macroprudential or systemic risk that could have a severe impact on the financial system and the Belgian real economy*”⁴. The proposed measure was an increase of 5% on the risk weights for retail exposures secured by Belgian residential immovable property for Belgian IRB banks. However, when determining the risk weight of an assets, usually only the intrinsic risk to the fundamental value of the asset, and hence its price is taken into account. The proposal here is to add a risk weight surcharge that quantifies the endogenous risk of the asset class in terms of its contribution to the propagation of losses in the financial market during a deleveraging cascade. The targeted risk-weights are calibrated as follows:

1. Compute the singular value decomposition of $\Pi\sqrt{D^{-1}}$. Denote its first **right** eigenvector by $v \in \mathbb{R}^M$.
2. Define the normalised risk weight of asset class μ $\rho_\mu^+ = \frac{\log(v_\mu \mathbb{1}_{v_\mu > 0})}{\sum_{\nu=1}^M \log(v_\nu \mathbb{1}_{v_\nu > 0})}$.
If $v_\nu = 0$, set $\rho_\mu^+ = 0$.⁵
3. Adjust the risk-weight of an asset class as

$$\rho_\mu^{new} = \rho_\mu^0 + \eta \rho_\mu^+,$$

where ρ_μ^0 is the Basel risk weight, and η is a scaling parameter.

⁴<https://www.eba.europa.eu/-/eba-publishes-opinion-on-measures-to-address-macroprudential-or-systemic-risk>

⁵As previously discussed, under mild assumptions, which hold in practice, the Perron Frobenius theorem guarantees that the risk weights are positive. In networks with severe short positions, it is however possible that some risk weights become negative.

4. Compute the capital increase for each bank that corresponds to the updated risk weights of the systemically important assets as described below.

A surcharge ρ_μ^+ leads to an increase of the risk-weighted assets of an institution that holds this asset, given by

$$RWA_+^i(\Pi, \rho, \rho^+) = \sum_{\mu=1}^M (\rho_\mu^0 + \eta \rho_\mu^+) |\Pi_{i\mu}| = RWA_{init}^i + \eta \Delta RWA_i, \quad (4.6)$$

where $RWA^i(\Pi, \rho) = \sum_{\mu=1}^M \rho_\mu^0 |\Pi_{i\mu}|$, are the risk-weighted assets, $\Delta RWA_i = \sum_{\mu=1}^M \rho_\mu^+ |\Pi_{i\mu}|$ the increase in risk-weighted assets due to the risk weight surcharges, and $\eta > 0$ is a normalization constant which assures that $\sum_{i=1}^N C_{TRW}^i = C_+^{system}$ for comparability. After the banks' risk-weighted assets have increased, they raise capital C_{TRW}^i so as to return to their initial capital ratio, i.e. $\frac{C^i + C_{TRW}^i}{RWA_+^i} = R^i$. This yields $C_{TRW}^i = \eta R^i \Delta RWA_i$. Because $C_+^{system} = \eta \sum_{i=1}^N R^i \Delta RWA_i$, the normalization constant is given by $\eta = \frac{C_+^{system}}{\sum_{i=1}^N R^i \Delta RWA_i}$. Therefore, institution i receives an additional capital requirement

$$C_{TRW,add}^i(\Delta RWA_i, C_+^{system}) = C_+^{system} \frac{R^i \Delta RWA_i}{\sum_{i=1}^N R^i \Delta RWA_i}.$$

REMARKS. Note that the benchmark policy as defined above is equivalent to increasing the risk weights of *all* asset classes uniformly by a scaling factor η and assuming that banks raise capital so as to maintain their current capital ratio with the increased RWA, i.e. find C_U^i and $\eta > 1$ such that, given (4.4),

$$R^i = \frac{C^i + C_U^i}{\eta RWA^i}. \quad (4.7)$$

As the benchmark requires all banks to raise capital, this may lead to even very well-capitalised banks being required to raise additional capital. As

this is inefficient, we set a “minimum leverage” $\lambda_{\min} = 10$, such that banks at, or below, this leverage do not raise any capital. Specifically, instead of $\sum_{i=1}^N C_+^i = C_+^{system}$, we therefore require that $\sum_{i=1}^N C_+^i \mathbb{1}_{\{\lambda_+^i > \lambda_{\min}\}} = C_+^{system}$, where λ_+^i is the leverage of bank i after it has raised an additional capital in height of C_+^i .

4.2.5 MACROPRUDENTIAL POLICIES ADDRESSING FIRE SALES EXTERNALITIES: COMPARATIVE STATICS

To compare the effectiveness of the different policies, we compute the reduction of fire sales losses, RoF , under a given policy, P , compared to the benchmark, B , in percentage of initial total bank equity:

$$RoF_P(\delta, \epsilon, C_+^{syst.}) = \frac{SFLoss_B(\epsilon, C_+^{syst.}) - SFLoss_P(\epsilon, C_+^{syst.})}{\sum_{i=1}^N C_i}.$$

If RoF_P is positive, this means that for a given initial shock size, market depth, and total additional capital, fire sales losses have been reduced more efficiently by the policy P than by the benchmark. For a given total amount of capital raised by the banking system, a successful policy P is thus an allocation of additional capital requirements to those banks that need capital most from a systemic perspective.

The four panels in Figure 4.2.1 show (from left to right, by row) the system fire sales loss after a 15% increase in system capital according to the uniform, Indirect Contagion Index (ICI), Positive System Externality (PSE), and Targeted Risk Weights (TRW) policies. We note that the systemic risk region of Figure 3.2.3 in which large-scale contagion occurs has disappeared and the maximal loss has been reduced to around 20% of total bank equity.

The panels in Figure 4.2.2 show the reduction of fire sales losses, RoF , as a function of the shock size and the market depth. From these graphs,

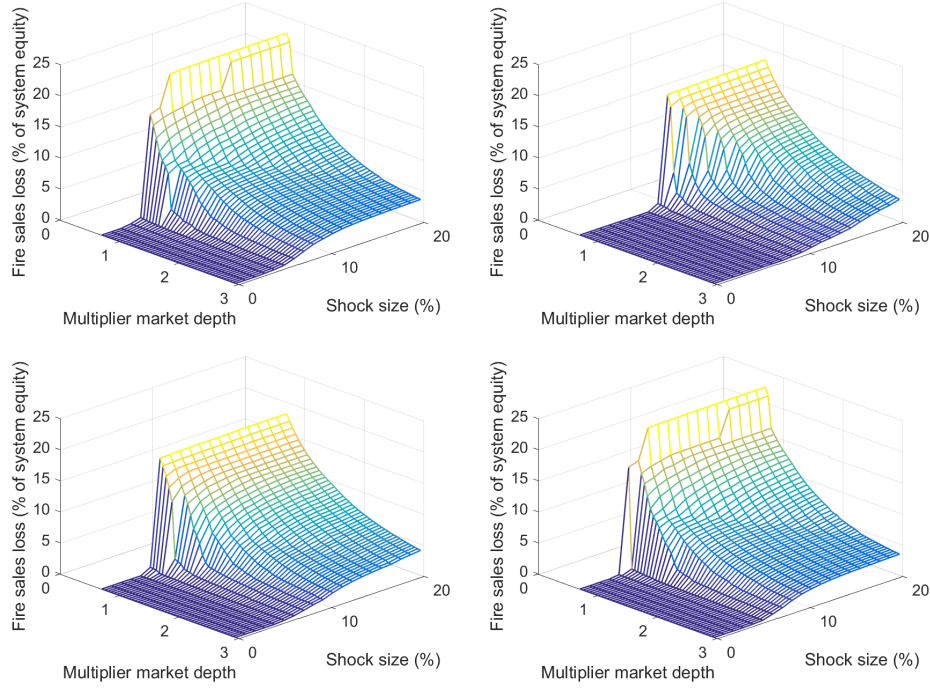


Figure 4.2.1: Fire sales losses under the benchmark, *ICI*, *PSE* and *TRW* policies as a function of the initial shock size and market depth scaling factor (from left to right, by row).

we see that although, averaged across shock sizes and market depths, all macroprudential policies beat the benchmark, the policy using the indirect contagion index in the top panel is most successful in reducing fire sales losses. For this reason, we focus on the *ICI* policy only to perform a final cost-benefit analysis by computing the reduction in fire sales losses per euro of capital added:

$$\frac{SF\text{Loss}(\epsilon, 0) - SF\text{Loss}_{ICI}(\epsilon, C_+^{system})}{C_+^{system}}.$$

On the one hand, in the systemic risk region, each euro of capital added reduces fire sales losses by 6 - 7 euro. On the other hand, in the non-systemic risk region, additional capital only reduces the fire sales loss by up to 80 cent per euro. In this narrow sense, the added capital is only beneficial on a net basis in the systemic risk region, where it helps to prevent a system

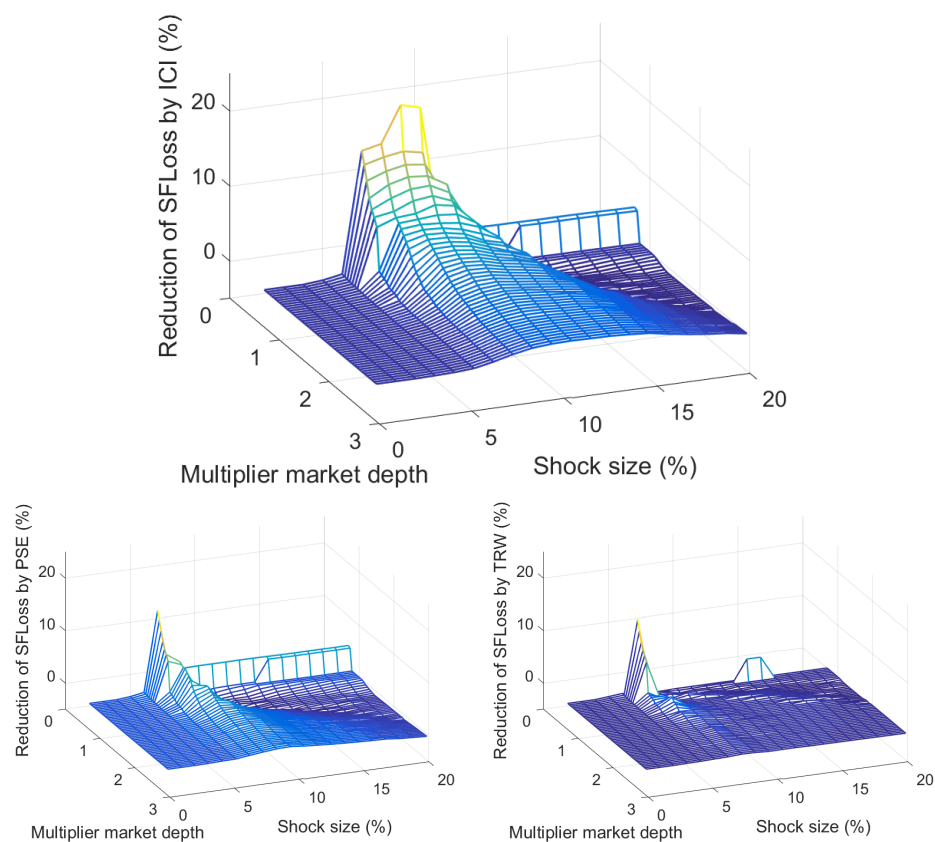


Figure 4.2.2: Reduction of fire sales losses, RoF , under the three macroprudential policies as a function of the initial stress level and the market depth scaling factor.

meltdown as illustrated in the panels of Figure 4.2.1. In the regions of no fire sales, or intermediate fire sales, the costs outweigh the benefits in this narrow sense.

4.3 ROBUSTNESS

We conclude this chapter by briefly discussing the robustness of our results under different response functions.

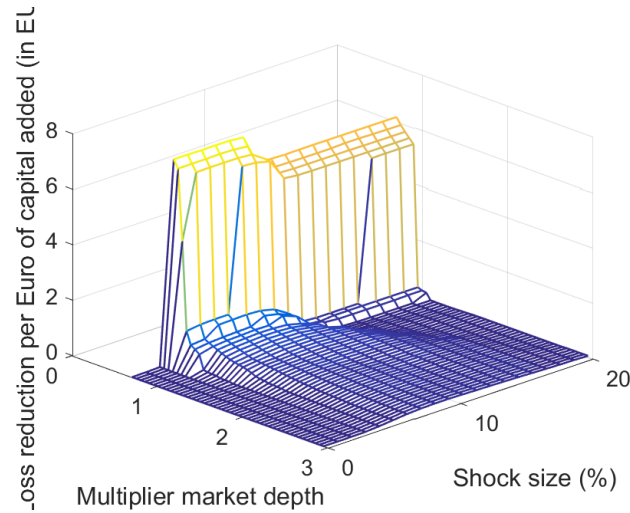


Figure 4.2.3: Cost-benefit analysis of the *ICI* policy: Reduction of fire sales losses in EUR per euro of capital added.

4.3.1 DELEVERAGING WITH MINIMAL PRICE IMPACT

As we observed in Section 3.3, bank-level losses are close to identical under the proportional deleveraging assumption and an optimised deleveraging by banks. This suggests that the indirect contagion network is reasonably robust to such a modification to the assumption on bank behaviour under stress, and that the Indirect Contagion Index retains most of its predictive power.

The same reasoning applies to macroprudential policies. Indeed, the left panel of Figure 4.3.1 shows the outperformance of the *ICI* policy relative to the benchmark. The right panel shows the loss reduction per euro added. Under optimised deleveraging, the systemic risk region is a bit smaller, leading to a somewhat reduced benefit, but overall the policy's effectiveness is comparable under these dynamics and proportional deleveraging.

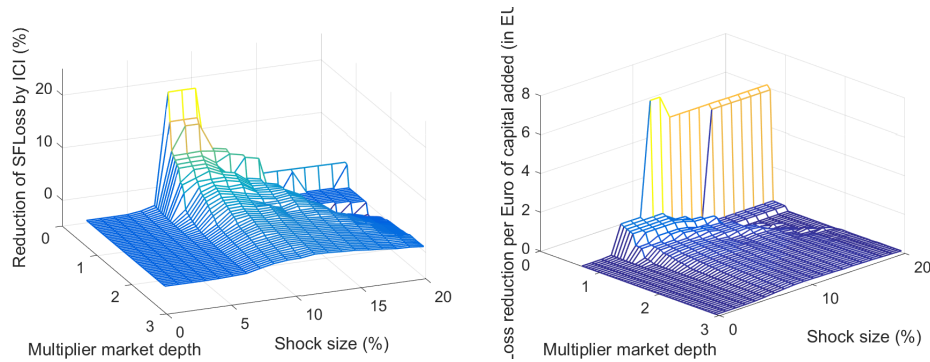


Figure 4.3.1: Left: Reduction of fire sales losses under the *ICI* policy with sequential deleveraging dynamics. Right: Cost-benefit analysis of the *ICI* policy under sequential deleveraging dynamics.

4.3.2 LEVERAGE TARGETING

On the other hand, when a leverage targeting model is used, we conclude somewhat differently:

- **Macroprudential policies:** As we saw in Section 3.3, bank-level losses under a leverage targeting model do not depend in any relevant magnitude on the composition of the initial shock; different initial shocks lead to the same cross-sectional distribution of losses across banks. Similarly, one can show that the cross-sectional distribution across assets (i.e. which assets cause the fire sales losses) is also almost identical under different initial shocks. Hence, one may be lead to thinking that one can calibrate risk weight surcharges proportional to an asset's contribution to fire sales losses. We performed this calculation, and found this policy indeed to be very effective. However, we believe that this result is both misleading and overstating the efficiency of such a policy, because in reality the cross-sectional distribution across asset classes is **not** independent of the initial shock composition. Hence, the policy's effectiveness mainly arises as a result of the endogenous dynamics under a leverage targeting model, rather

than an inherent superiority of the policy.

- **Systemic Vulnerability Indicator:** It is simply not possible to compute the SVI under leverage targeting dynamics, because any combination of initial stress and market depth scaling leads to eventually most of the banks defaulting. One could say that under leverage targeting dynamics, we have $SVI \equiv 0$, but this information is clearly not of any use to policy makers. Alternatively, one may choose to stop the fire sales cascade after just a few rounds, however in this case the number of rounds has a significant impact on the SVI , and we do not see a clear line of argumentation for why the cascade should be stopped after just 1,2,3,4 or 5 rounds.
- **Indirect Contagion Index:** The regressions of fire sales losses on ICI are very good, as the fire sales losses under leverage targeting dynamics correspond (at bank level) quite closely to the bank-level losses of the high systemic risk region.

*We should not overestimate our ability to think of
everything that could go wrong.*

David Murphy, Norges Bank Financial Stability Day
2015.

5

CONCLUSION AND THE ROAD AHEAD

We have presented a stress-testing framework for quantifying the endogenous risk exposure of the banking system to fire sales and feedback effects which may arise from macro-shocks to the financial system. Our findings provide quantitative evidence for the importance of *endogenous risk* (Danielsson et al. (2004), Shin (2010), Cont and Wagalath (2016)) in the financial system and have implications for systemic stress testing, macroprudential policy and risk management in financial institutions.

5.1 NEED FOR A SYSTEMIC STRESS TESTING FRAMEWORK FOR CAPITAL ADEQUACY

Supervisory stress tests, especially in the US, have become the binding constraint for bank capital adequacy. Theoretical studies Danielsson et al. (2004), Shin (2010), Pedersen (2009), Cont and Wagalath (2013) have re-

peatedly pointed out the importance of *endogenous risk* for financial stability. Fire sales are arguably an extreme example of endogenous risk and procyclical behaviour, yet they are not systematically integrated into bank stress testing methodologies.

Our quantitative findings join a list of previous studies in pointing out the importance and magnitude of fire sales and price-mediated contagion as a risk amplification mechanism for systemic risk. Even under benign assumptions on market liquidity, the magnitude of exposures arising from this channel is too large to ignore; losses arising from this channel can dominate other types of risk exposures in certain stress scenarios. More importantly, we have shown that risk exposures of financial institutions arising from fire sales cannot be replicated in single-institution stress tests, even after scaling to extreme stress levels. This pleads against a widespread approach which consists in simply applying a (constant) discount to asset values in stress tests to account for liquidation costs: we have argued that this discount is endogenous and strongly depends on the degree of leverage, concentration and overlap of asset holdings across financial institutions.

These observations, which are based on public data and may be readily replicated by regulators, plead for a systemic approach to bank stress testing that accounts for fire sales contagion in a proper way. We have presented the building blocks of an operational framework for estimating such endogenous effects and incorporating them into a macroprudential stress testing framework.

5.2 ROBUST MODELLING OF FIRE SALES

The main building blocks of a fire sales model consist in characterising the triggers that can lead different institutional agents to delever their portfolios, and specifying how asset prices react to distressed liquidations as a function

of their size. Our work extends the current literature in several directions.

PRICE IMPACT AND ASSET CLASSES. First, we have introduced a level-dependent price impact function that is consistent with the empirically observed price impact for small liquidation volumes, but takes into account the arrival of arbitrageurs or other deep-pocketed investors that will step in as buyers before the price falls to zero. We believe this type of market impact function to be more suitable for quantifying the price impact under large volume liquidations, as needed in systemic stress testing. This increases, we believe, the realism and reliability of the quantitative estimates for fire sales losses.

Furthermore, we have introduced a previously ignored “variable” to the modelling debate: the level of aggregation of asset classes. Our theoretical results show that there exists no reasonable market impact function under which fire sales losses are invariant under aggregation. As we have shown that deviations of fire sales loss estimates can be substantial under different levels of aggregation, this becomes a non-negligible issue. In our empirical analysis, we find that bank-level losses can indeed diverge by up to two orders of magnitude for individual banks when computing fire sales losses under different levels of granularity. Importantly, when the aggregation level is too coarse the bias tends to underestimate the losses of the most affected institutions, potentially leading to a false sense of security. Based on heuristics and a limited study on Norwegian data, we have argued that an issuer-level aggregation level seems reasonable. Furthermore, we argue that – data management allowing – it is advantageous to collect and use more granular data in order to perform the aggregation one self, and to conduct a sensitivity analysis in a more robust manner.

Overall, we have highlighted the importance of the “level of granularity” as important “modelling choice” in practical systemic stress testing. Clearly,

further studies on defining and estimating the market depth of asset classes, and finding an optimal level of granularity in general, are warranted.

RESPONSE FUNCTION. Second, we have argued in favour of modelling fire sales as distressed deleveraging of portfolios subject to one-sided constraints, leading us to develop our threshold model of fire sales. Recent empirical studies on fire sales spillovers [Duarte and Eisenbach \(2013\)](#), [Greenwood et al. \(2015\)](#) have explored a different mechanism for fire sales, based on the idea that banks maintain a ‘leverage target’, which leads them to rebalance their portfolios in a procyclical manner following changes in asset values. An oft-cited argument to support this model is the empirical correlation between quarterly changes in asset size and debt size for banks [Adrian and Shin \(2010, 2014\)](#).

While both models incorporate the idea of market impact of deleveraging and the resulting endogenous portfolio losses and contagion effects, they differ in some important ways:

1. *Gain-loss asymmetry*: in the leverage targeting model, banks respond in a symmetric way to gains and losses, deleveraging in the face of losses but also increasing leverage by buying assets when asset values increase. This may be a reasonable description of the medium-term (e.g. quarterly) dynamics of bank or fund portfolios [Coval and Stafford \(2007\)](#), but not necessarily a realistic description for the short-term reaction of portfolios under stress. When performing stress tests, we are not concerned with the gradual rebalancing of portfolios which may be conducted in an orderly manner, with little market impact, but primarily concerned with fire sales, i.e. rapid deleveraging triggered when large losses saturate one-sided portfolio constraints. Due to the one-sided nature of these (leverage, capital, liquidity) constraints, fire

sales do not occur in reaction to gains, but only in reaction to *sufficiently* large losses. We believe this is a better description of the reaction of portfolios in stress scenarios.

2. *The threshold nature of fire sales:* in the leverage targeting model, bank (de)leveraging occurs in response to arbitrarily small changes in asset values, regardless of their capital or liquidity buffers. In our model, deleveraging only occurs when losses are large enough to trigger portfolio constraints: for shocks below this critical level, there is no deleveraging. As a result, the leverage targeting model overestimates the magnitude of deleveraging in response to small shocks, which is zero in our model.
3. *Dependence of deleveraging on magnitude of losses:* Leverage targeting implies a (first round) volume of deleveraging linear in the size of the portfolio loss. By contrast, in our threshold model the volume of (first round) deleveraging has a (piecewise) convex dependence on the loss size: deleveraging accelerates as we increase the shock size to more extreme levels, leading to a ‘multiplier effect’ absent in the leverage targeting model. An example of this convex dependence in the shock size was displayed in Figure 1.1.2.
4. *Finite length of fire sales cascades:* The assumption of leverage targeting leads to an infinite sequence of iterations, which never cease since at each round further mark-to-market losses are generated endogenously, which leads to a deviation from the target leverage and generates new asset sales or purchases. In stress tests, one then needs to choose an ad-hoc number of iterations to compute the loss. The choice of this number was shown to influence the results significantly. In contrast, in a threshold model with a capital buffer $\lambda_b < \lambda_{\max}$, the fire sales cascade always terminates after a finite number of iterations,

typically found to below 15 in our empirical examples. Table 5.4.1 summarises a few of the main properties of the dynamics that result under these different deleveraging reactions.

5. *(In-)sensitivity to the composition of the initial shock:* Our results indicate that the leverage targeting model, contrary to the threshold model, is not responsive to the composition of the initial shock. This leads to the counter-intuitive, we believe, conclusion that different stress scenarios lead to almost identical cross-sectional distributions of fire sales losses across banks. This effect can be dampened by reducing the number of rounds for which the fire sales cascade is run to below 4, but this complicates the choice of the “number of iterations” parameter by adding a further consideration.
6. *Indirect exposures:* Because under leverage targeting dynamics, the fire sales loss is essentially independent of the shock size, indirect exposures are not well defined for small shock sizes. This difficulty is avoided in the threshold model, because by definition indirect exposures are zero for shocks that are smaller than the minimum critical shock size $\min_i \epsilon_i^*$.

Beyond the comparison with the leverage targeting model, we have also studied the effect of simple optimising and heuristic behaviours by banks on the cross-sectional distribution of fire sales losses. Astonishingly, the effect of this change in response function was rather small. In the same context, using a granular data on portfolio holdings and portfolio changes from Norway, we were able to find both evidence of proportional deleveraging and of a more selective type of deleveraging. Overall, these findings confirm, as in [Getmansky et al. \(2016\)](#), that the proportional deleveraging assumption is not an unreasonable first approximation, and that our conclusions should furthermore be relatively robust to the changes in the order of deleveraging.

5.3 MONITORING AND PRUDENTIAL POLICY

Our model also has some implications for monitoring and prudential policies aimed at mitigating the risk from fire sales

1. *Bank-level exposure to price-mediated contagion may be quantified using the Indirect Contagion Index:*

Conducting a system-wide stress test can be difficult and costly. We have identified a simple proxy, the Indirect Contagion Index (Section 4.1.1), for assessing the vulnerability of banks to fire sales, in absence of a system-wide stress test. We have argued that this indicator may be used alongside other measures of interconnectedness currently used for identifying global systemically important banks (GSIBs) [Basel Committee on Banking Supervision \(2013a\)](#).

2. *Implications for the interaction between banks and non-banks:*

Price-mediated contagion is not limited to banks; any institution exposed to fire sales risk and having common asset holdings with banks may become a source of price-mediated contagion for the banking sector. There is ample empirical evidence of fire sales by asset managers [Coval and Stafford \(2007\)](#), [Jotikasthira et al. \(2012\)](#) and insurance companies [Ellul et al. \(2011\)](#).

Regulators have been concerned with monitoring the interaction between the banking sector and non bank financial institutions (‘shadow banks’); current attempts to monitor these exposures have focused on balance sheet linkages between shadow banks and the regular banking system measured solely in terms of direct exposures and liabilities [Grillet-Aubert et al. \(2016\)](#). Given that a large fraction of financial assets are held by non-banks – large asset managers, pensions funds, and insurance companies – the scope for indirect contagion from ‘shadow

banks' to the banking sector through the fire sales channel exists and its proper assessment calls for a *system-wide* stress test for quantifying these inter-sector exposures. Extending the present model to include non-banks would require a careful analysis of mechanisms which would lead such institutions to shed assets: redemption risk for asset managers and asset-liability mismatch for pension funds are plausible avenues to consider (Calimani et al. (2016)).

3. *Indirect exposures as tool for risk management:*

As pointed out by Ellul et al. (2014) “forward-looking institutions that rationally internalize the probability of fire sales are incentivized to adopt a more prudent investment strategy during normal times which leads to a safer portfolio entering the crisis.” One of the obstacles to internalizing the risk of fire sales is that its proper assessment requires some knowledge of the concentration of asset holdings across financial institutions, which is typically only available to regulators.

One of the by-products of our stress testing approach is the ability to compute the *indirect exposures* of an institution to various asset classes (Section 1.1.7). Communicating to an institution the magnitude of its indirect exposures to various asset classes and risk factors, as evaluated in the systemic stress test, allows the institution to have a better assessment of its risk and provides incentives to reduce such exposures.

4. *Need for transnational coordination on stress testing and macroprudential policy:*

As revealed by our analysis of common asset holdings in Section 4.1.1, the European indirect contagion network is completely connected and banks in different countries may have strongly overlapping portfolios, which lead to substantial indirect exposures across EU countries, as

shown in Table 3.2.1. Unlike direct exposures, which may be limited through various capital restrictions, portfolio overlaps and indirect exposures are difficult to put limits on: any such restrictions would amount to limiting international diversification of bank portfolios. Any meaningful systemic stress test should therefore account for the magnitude of these cross-country indirect exposures, cannot be conducted at the level of a single country, and consequently calls for transnational coordination of macroprudential policies.

We note that privacy laws which limit regulators' ability to share data with other regulators or jurisdictions do not present an insurmountable obstacle to this exercise. An international authority, such as the Bank for International Settlements for instance, could coordinate the systemic stress test on a secured server such that national data is not accessible to regulators of other jurisdictions, but the full network is available to run the algorithm of the systemic stress test. A possible implementation could follow the outline sketched below:

- (a) National regulators upload balance sheet data and portfolio holdings of institutions in their jurisdiction to the server via a secure web-form;
- (b) On the server, this data is automatically matched across jurisdictions using unique legal entity identifiers (LEI), resulting in the reconstruction of the full "institutions - assets" network. Furthermore, having estimated market depths of all relevant asset classes, this also allows the computation of portfolio overlaps and the indirect contagion index.
- (c) The stress test scenario and other assumptions are fed into the model, and the systemic stress test is run on the server;
- (d) The results are communicated back in appropriately anonymised form to national regulators and institutions. The server is wiped,

deleting all data of the full network.

Importantly, it should be possible to implement the above steps in such a way that at no point in the process any regulator has access to any data that does not pertain to their own jurisdiction. Indeed, it is not necessary that anybody have access to the full network that is required to run the systemic stress test and to compute the indirect contagion index; with appropriate standardisation and matching procedures in the data collection process, this reconstruction can be performed automatically on the server without granting anybody access to the full network.

5. *Systemic vulnerability indicator and capital surcharges:*

By quantifying what magnitude of a shock is required to trigger a systemic fire sales cascade, the systemic vulnerability indicator can be used as a proxy for quantifying the resilience of the financial system to fire sales risk.

Knowing for what combinations of initial shock sizes and market depths a system meltdown may be expected, a regulator can “move” this boundary of the high systemic risk region to market depth and shock level combinations, where she feels the system is safe. We have shown that this can be done in our framework via additional capital charges for systemically important banks, calibrated using the indirect contagion index. In that way, the banks that contribute most to fire sales spillovers are targeted and forced to internalise this externality.

6. *The role of mark-to-market accounting rules:*

Previous studies [Allen and Carletti \(2008\)](#), [Ellul et al. \(2014\)](#) have focused on mark-to-market accounting as the channel of transmission of losses in fire sales contagion.

Allen and Carletti (2008) present arguments against the use of mark-to-market accounting for determining solvency during crisis periods; indeed, such temporary suspensions were used in 2009 by US banks. Jotikasthira et al. (2015) use the example of the insurance sector to caution against this recommendation.

Our model offers a perspective on this debate: having distinguished between marketable ('Level I') assets and illiquid assets, we can note that a suspension of mark-to-market accounting rules is only likely to affect the latter. Suspension of mark-to-market accounting for illiquid assets may indeed affect the first step in our iteration, in which losses to illiquid assets trigger the initial deleveraging. But once deleveraging by a set of institutions takes place, the subsequent losses are not accounting losses, but market losses in marketable securities whose magnitude is not affected by accounting conventions. Thus, temporary suspension of market accounting rules may reduce the scope of institutions affected by an initial stress to some illiquid asset class, but once fire sales affect liquid 'Level I' securities, all institutions holding them will be affected by market losses.

5.4 FUTURE RESEARCH

Our project has raised a series of new questions and problems for future research.

1. *Interaction of solvency and liquidity:*

When focussing on banks only, the leverage constraint may not be the most relevant trigger for fire sales. Indeed, in reaction to a solvency shock, banks may have other, more preferable, options than a fire sale to comply with regulations, such as e.g. only gradually accounting for

the losses in their balance sheets via a suspension of mark-to-market accounting. On the other hand, if banks are constrained by liquidity, time will be of the essence and turning to a fire sale to raise liquidity may be the only viable option¹.

It is instructive at this point to briefly return to the case of Bear Stearns: Bear Stearns was in full compliance with its regulatory capital, but lost access to market funding, leading it to default due to illiquidity in the matter of a week (Duffie (2010b)). The loss of market funding is however not an idiosyncratic event: It is a consequence of the market losing confidence in the bank, and thereby directly linked to its *perceived* solvency.

A more realistic mechanism for banks would thus include an additional step and take the form:

- (a) An initial shock hits some ABS or other illiquid assets on the bank's balance sheet (and pushes up its leverage);
- (b) Based on available information, the market players make a judgement on whether or not to roll-over funding to the weakened and potentially troubled bank. At this stage *information contagion* plays a crucial role².
- (c) If the bank is unable to obtain sufficient funding through regular credit and repo lines, it engages in a fire sale of marketable assets which it can liquidate at short notice. If this is still insufficient, the bank fails by *defaulting on a payment* that is due, and thereupon, or when such a failure to pay is imminent to happen, files for bankruptcy.

¹I am particularly grateful to Martin Hellwig for very insightful comments on this point.

²In Bear Stearn's case other large broker dealers were flooded by "novation requests" from clients wishing to move a part of their derivatives business away from Bear Stearns to reduce exposure. Usually such novation requests are granted, and other dealer banks starting to refuse new novation requests intensified market concerns sharply.

- (d) Regardless of the failure of this bank, the prices of liquidated assets have been temporarily depressed, causing mark-to-market losses to other banks, which raises concerns about their (perceived) solvency (return to step 2).

In this sense, our model leaps over this intermediate mechanism of perceived solvency and takes the leverage constraint itself to act as trigger for fire sales. [Pierret \(2015\)](#) documents empirically this link between banks' solvency and their liquidity via their cost of funding. However, in order to use this mechanism as a trigger for fire sales in a systemic stress test it is important to be able to go even further than [Pierret \(2015\)](#). Indeed, one needs to quantify, for a given shock, the amount of funding a bank *can* obtain in the market and compare this to its short term liquidity outflows³. Further research needs to explore this avenue, and there is a dire need for (i) empirical studies describing the effects on a quantitative level, (ii) models capturing this type of mechanism (as in [Calimani et al. \(2016\)](#)), and (iii) data to populate these models (which is usually confidential supervisory data).

I believe this issue to be one of the most important current challenges in advancing our understanding of systemic risk and improving models aimed at systemic stress testing.

2. *Confirm, or nuance and refine, some of the presented findings:*

PROPORTIONAL DELEVERAGING. We were able to present evidence in favour of proportional deleveraging based on a Norwegian supervisory dataset. It would be of great value if similar studies could be performed in larger markets to confirm (or reject) the limited empiri-

³Rama Cont has made this important point in numerous debates and policy panels: the critical quantity that needs to be compared to the outflows is not the amount of liquid assets a bank currently *has*, but the amount it *can raise* in a stress scenario.

cal evidence we have gathered so far. Moreover, it would be useful to study the behaviour of different institutional agent types.

Regarding optimising behaviour by banks, it seems rather implausible that in a fire sales scenario portfolio managers will solve a concave minimisation problem and then use the solution of this algorithm to deleverage their portfolios. It appears far more likely that portfolio managers have an (good) idea of the liquidity of the assets in their portfolio and choose to delever them in rough buckets depending on the market conditions for the various assets.

Finally, the fact that we only found a very limited impact on changing from proportional deleveraging to sequential deleveraging may be due to our dataset and the level of aggregation in it - a reproduction of this analysis on different datasets should be performed.

ASSET GRANULARITY. The question of at what level of granularity data should be collected should not be dismissed by modellers or policy makers as minor technical issues and left to IT and data managers to resolve. Both the questions that policy makers wish to answer, as well as the different types of models that economists/modellers use to address these questions, put clear constraints and demands on what an optimal “level of aggregation” should be. We have clearly illustrated the importance of the level of aggregation as a modelling choice in fire sales models, and shown how fire sales loss estimates change when the level of granularity that is used in the model changes. Hence, there is much scope for further work in this direction.

3. *Inclusion of different institutional agents and sectors into the model:*

As mentioned above a *system-wide* perspective does not only include all relevant banks, but takes a holistic view of the system which includes important other institutional players such as large insurance

companies, asset managers, hedge funds and pension- or sovereign wealth funds. These agents interact with banks in the same asset markets and contagion from and to these institutions is therefore a genuine risk.

Taking such a holistic view also has implications for market depth: In such a setting the market depth of an asset is truly endogenous and depends on the other institutions' ability to step in as buyers when some agents are forced to delever. This would also extend the price floor B , which we simply postulated to be a sufficiently low level, to become an endogenous variable of the system.

Finally, (unintended) consequences and interactions of different types of regulations can be analysed in such a setting. Bank regulation such as Basel III and regulations for the insurance sector as introduced via Solvency II have the clear goal of making each institutional sector safer. However, if these lead to a homogenisation of rules (and thereby a homogenisation of potentially binding portfolio constraints) we may be limiting the set of potential buyers in a stress scenario, resulting in further price declines and more intense contagion. The natural complementarity of banks' and insurance companies' balance sheets and asset-liability mixes can be used as a positive feature; in this sense a systemic stress test that includes other sectors can be in the interest of all agents, as their interactions may in some instances reduce rather than intensify the stress.

4. *Policy responses to fire sales*: The policy goal of reducing fire sales losses can be achieved through either making institutions more resilient and ensuring fire sales do not occur in the first place, or by increasing the market depth of assets, such that the price impact of fire sales becomes less important. One possibility to increase the market depth is to increase the liquidation horizon τ , because as previously discussed,

scaling the liquidation horizon, τ , increases the market depth by $\sqrt{\tau}$. Hence, instead of interpreting the market depth scaling factor solely as a sensitivity analysis parameter, one can also view it as a policy parameter. In our model, such an “increase of τ ” can be achieved by giving institutions longer time to return to compliance with the leverage constraint. A second possibility would be a targeted “Quantitative Easing” scheme, where the central bank buys assets that are currently under fire sales pressure. This reduces the price impact on the assets and it moreover provides an opportunity to potentially earn liquidity premia on such assets. Quantitative easing has already been used widely as an unconventional monetary policy tool - our model suggests that it can also be used as an unconventional financial stability tool, adding a “market maker of last resort” role to central banks. Also, the (temporary) inclusion of an asset in a central bank’s collateral pool increases the market depth of an asset because it allows institutions (with access to central banks’ secured funding) to swap this part of their marketable assets into cash at a given ratio defined by the haircut.

Finally, similar to the countercyclical capital regulation that has been introduced by Basel III, one can foresee a countercyclical liquidity- and leverage regulation: By tightening leverage and liquidity constraints in benign times one builds a buffer that can be released during stressed conditions, allowing more institutions to step in as buyers of distressed assets.

In the process of the money market fund reform, the Securities and Exchange Commission (SEC) has recently adopted several rules that shall reduce fire sales externalities in mutual funds: Examples of such rules are the right for funds to introduce redemption gates when they face a large amount of redemptions in distressed market conditions, as

well as “swing pricing” which allows funds to pass an anticipated fire sales discount on to investors⁴.

5. *Relation to general equilibrium models:* I end the conclusion by discussing similarities and differences between the model developed in this thesis and general equilibrium models. Indeed, general equilibrium models that include fire sales have been developed as in Caballero and Simsek (2013). One could argue that the indirect contagion network arising from overlapping portfolios is a concrete example of the “complexity” described in their model. Although the initial shock is similar in nature, the propagation of the shock differs quite substantially: the authors postulate a circular chain of direct exposures between the N banks in their model, as well as a single asset that suffers fire sales externalities. This very stylized setup allows for a precise general equilibrium analysis, however the restrictive assumption on interbank exposures as well as the assumption of a single asset class for fire sales, make it challenging to calibrate the model to data. In the sequel, I will discuss three main points: (i) equilibria in the fire sales model developed in this thesis, (ii) the notion of dynamics and time, and (iii) market clearing for asset prices.

On the existence and type of equilibria: First, we note that the fixed points to which the fire sales cascade converges correspond to equilibrium states within the model. As such our model allows for multiple types of equilibria (illiquidity, insolvency and survival) for each bank, and furthermore multiple equilibria within the illiquidity and survival categories. By specifying the reaction functions of the banks, we are in essence specifying *how* banks seek to return to the equilibrium state of survival. Contagion occurs in this process through negative spillovers

⁴Although the efficiency of swing pricing remains to be tested in practice, as it is limited to 2 %: <https://www.sec.gov/rules/final/2016/33-10234.pdf>

(price impact) to other banks.

This leads us to the second consideration on time and dynamics: A key notion in this process is the speed at which banks deleverage their portfolios: If a bank delevers its portfolio very slowly, the price impact of this action will be negligible; in contrast, if the bank liquidates its portfolio by dumping its assets on the market within a very short time frame, the price dislocation will be significant. Therefore, the speed at (or equivalently the time horizon over) which the liquidation takes place, has an effect on what equilibrium point(s) the bank(s) will reach. This concept, to the best of my knowledge, is not captured by classical general equilibrium models, which often put a stronger emphasis on describing the equilibrium state itself, than the manner in which the agents get there. Consequently, although general equilibrium models provide an elegant framework to study welfare effects, the fact that in our model fire sales losses (and hence contagion) depend on how agents get to an equilibrium complicates such an approach.

However, this opens the door for new research, which we shall illustrate by discussing the market clearing condition: [Shleifer and Vishny \(1992\)](#) study the equilibrium price under fire sales as a function of the availability (“debt capacity”) of potential buyers. In this respect, we note that our model departs from key features of equilibrium models as it does not feature a market clearing condition. Indeed, the price is not derived endogenously by matching supply and demand, but by calibrating an exogenous market depth that enters the market impact function and thereby creates an inverse demand function that quantifies the new price, given a certain liquidation size.

[Chen et al. \(2014\)](#) analyse contagion between banks in an equilibrium setting enforcing a market clearing condition between the banks in their model. However, as Figure 3.4.6 shows, assuming the market to

clear between banks is clearly not realistic in general. In particular, when a particular institutional sector is under stress and liquidates assets, it will in all likelihood be *another* institutional sector stepping in as buyer.

Hence, in order to derive the equilibrium price from supply and demand, it seems that a *conditio sine qua non* involves the modelling of *all* relevant other institutional sectors: large asset managers, insurance companies, pension funds, the central bank, and mutual- or hedge funds to the extent pertinent. This brings us back to agent based models, as I believe that this type of model currently present the best shot at developing an operational methodology that allows to quantify contagion from fire sales and similar feedback effects. Indeed, the considerable heterogeneity in institutional constraints and portfolios (which we showed to be relevant even when only focussing on heterogeneity *within* the banking sector) will in all likelihood be quite difficult to square with the stylised format of classical general equilibrium models. A successful breakthrough in this direction would however pave the way towards a more general cost-benefit analysis of macroprudential policies. Indeed, by requiring higher capital ratios to mitigate the risk from fire sales, we ignored the fact that banks may reduce lending during this period, leading to potentially slower economic growth. In our current model, we cannot quantify this effect and weigh it against the benefit of an increased resilience to fire sales. At the same time, when including such effects from reduced lending, I believe that a more precise estimation bankruptcy costs also needs to be included in order for the cost-benefit analysis to remain balanced.

Property	Leverage targeting model	Threshold model: $\lambda_b = \lambda_{\max}$	Threshold model: $\lambda < \lambda_b < \lambda_{\max}$
Length of cascade.	Infinite. Geometric convergence	Infinite. Geometric convergence	Finite
First round volume of fire sales as function of initial shock.	Linear.	Convex.	Increasing and discontinuous.
First round fire sales loss as function of initial shock.	Linear, assuming full realized loss. Concave otherwise.	Convex, assuming full realized loss.	Increasing and discontinuous.
Total fire sales loss as function of initial shock.	Increasing and strictly concave.	Generally increasing and initially convex, non-monotonicities are possible though.	Non-linear, discontinuous (and potentially non-monotonic for certain configurations of λ_b and δ .)
Indirect exposures.	Ill-defined.	Well-defined.	Well-defined.
Sensitivity to composition of stress scenario.	Weak: Orthogonal stress scenarios do not exist. Different scenarios lead to similar loss distributions.	Strong: Orthogonal stress scenarios exist. Different scenarios lead to different loss distributions.	Strong: Orthogonal stress scenarios exist. Different scenarios lead to different loss distributions.
Sensitivity to simulation parameters.	Large: number of iterations of fire sales cascades has strong influence on results.	Moderate: existence of critical shock size makes convergence faster in practice.	Small: fire sales cascade ends after a finite and endogenous number of iterations.
Magnitude of fire sales loss (on EBA dataset).	Large.	Moderate.	Moderate.
Suggested main modelling application.	Medium and long term modelling of bank balance sheets.	Short term system-wide stress testing.	Short term system-wide stress testing.

Table 5.4.1: Summary of comparison between linear leverage targeting model and two threshold models.

Part II

A SENSITIVITY ANALYSIS OF THE EISENBERG & NOE CLEARING VECTOR

In the end, a theory is accepted not because it is confirmed by conventional empirical tests, but because researchers persuade one another that the theory is correct and relevant.

Fischer Black (1986)

6

ON THE ROBUSTNESS OF THE EISENBERG & NOE CLEARING VECTOR

This Chapter is based on the working paper “Sensitivity of the Eisenberg & Noe clearing vector to individual interbank liabilities”, which is joint work with Zachary Feinstein, Weijie Pang, Birgit Rudloff, Stephan Sturm and Mackenzie Wildman. This material is the result of an AMS workshop on financial mathematics in Utah in June 2015, and is based on work supported by the National Science Foundation under Grant No.1321794.

6.1 INTRODUCTION

In the second Part of the thesis, we turn to a commonly used model of direct contagion in the systemic risk literature by Eisenberg & Noe.

LITERATURE REVIEW. An important stream of the network contagion literature has focused on interbank contagion, building on the network model of Eisenberg and Noe (2001). Extensions of this model have been developed to include effects such as fire sales or cross-ownership, different maturities or bankruptcy costs (Kusnetsov and Veraart (2016), Awiszus and Weber (2015), Elliott et al. (2014), Cifuentes et al. (2005), Feinstein (2015), Feinstein and El-Masri (2016), Rogers and Veraart (2013), Di Gangi et al. (2015)). Furthermore, central banks have applied the model to study default cascades in the banking systems in their jurisdiction (Anand et al. (2014), Halaj and Kok (2015), Boss et al. (2004), Elsinger et al. (2013), Upper (2011), Gai et al. (2011)). See Hüser (2016) for a comprehensive review of the interbank contagion literature.

A key ingredient required to estimate contagion in these models is the so-called liabilities matrix L , where L_{ij} is the nominal liability of bank i to bank j . Often, the exact bilateral exposures are not known and thus need to be estimated (Halaj and Kok (2013), Anand et al. (2015), Elsinger et al. (2013), Halaj and Kok (2015)). Despite considerable efforts after the crisis to improve data collection, data gaps have not been closed yet and the estimation of specific bilateral exposures remains an important issue (Langfield et al. (2014), Anand et al. (2015), Anand (2015), Financial Stability Board and International Monetary Fund (2015)). Beyond logistical issues such as the standardization of reporting formats and the creation of unique institution identifiers, legal hurdles, which restrict regulators' access only to data pertinent to their respective jurisdictions, remain. In such cases, bilateral

exposures between home and foreign institutions still need to be estimated or collected from the home institutions. The early literature often used entropy maximizing techniques to “fill in the blanks” in the liabilities matrix given the total assets and liabilities of banks (viz. the row and column sums of L). However, a growing empirical literature has shown that real-world interbank networks look quite different from the homogeneous networks that are obtained with such techniques (Bech and Atalay (2010), Mistrulli (2011), Cont et al. (2013), Soramäki et al. (2007)). A recent Bayesian method to estimate the bilateral liabilities, given the total liabilities, is proposed in Gandy and Veraart (2016). In particular, Mistrulli (2011) and Gandy and Veraart (2016) show how wide estimates of systemic risk may fluctuate when estimating contagion on real world and heterogeneous networks versus uniform networks. This highlights the pivotal role that the matrix of bilateral exposures plays in quantifying the extent of contagion when computing default cascades. To the best of our knowledge, Liu and Staum (2010) is so far the only paper that performs a sensitivity analysis of the Eisenberg & Noe model. Their analysis is however limited to the sensitivity of the clearing vector with respect to the initial net worth of each bank.

MAIN CONTRIBUTIONS. The main contribution of this project is to perform a detailed sensitivity analysis of the clearing vector with respect to the relative liability matrix in the standard Eisenberg and Noe framework.

To this end, we define directional derivatives of order n of the clearing vector with respect to “perturbation matrices”, which quantify the estimation errors in the relative liability matrix. This allows us to derive an exact Taylor series for the clearing vector. Moreover, we introduce a set of “basis matrices”, which specify a notion of fundamental directions for the directional derivative, and it turns out that the directional derivative of the clearing vector can be written as a linear combination of these basis matrices. We

proceed to use this result to study two optimisation problems that quantify deviations of the clearing vector given estimation errors in the liabilities matrix, and obtain explicit solutions for both problems.

Finally, we illustrate our results in two example cases. First in a small four bank network, and second in a the case of European banks using the same dataset as in Part I. Our results suggest that even though the clearing vector can deviate by as much as 15% for moderate estimation errors, when there is large heterogeneity in bilateral interbank liabilities, these deviations seem to be small. For this reason it is difficult to say ex-ante whether the errors will be large or not and it underlines the importance of feeding systemic risk models with accurate data.

More importantly, we find that for the same shock scenario, the intensity and extent of price-mediated contagion in the European banking network dominates the direct contagion from the Eisenberg and Noe model.

6.2 SENSITIVITY ANALYSIS OF EISENBERG & NOE CLEARING VECTOR

We consider a financial system consisting of n banks, $\mathcal{N} = \{1, \dots, n\}$. For $i, j \in \mathcal{N}$, $L_{ij} \geq 0$ is the **nominal liability** of bank i to bank j . Equivalently, L_{ij} is the exposure of bank j to bank i . $L \in \mathbb{R}^{n \times n}$ is called the **liabilities matrix** of the financial network, and we assume that no bank has an exposure to itself, i.e. $L_{ii} = 0$ for all $i \in \mathcal{N}$. The **total liability** of bank i is given by $\bar{p}_i = \sum_{j=1}^n L_{ij}$. The **relative liability** of bank i to bank j is denoted $\Pi_{ij} \in [0, 1]$, where $\Pi_{ij} = \frac{L_{ij}}{\bar{p}_i}$ when $\bar{p}_i > 0$. We allow $\Pi_{ij} \in [0, 1]$ to be arbitrary when $\bar{p}_i = 0$ and only require $\sum_{j=1}^n \Pi_{ij} = 1$.¹ We denote the **relative liability matrix** $\Pi \in \mathbb{R}^{n \times n}$. Any relative liability matrix Π must

¹Note that the arbitrary choice of Π_{ij} in the case $\bar{p}_i = 0$ has no impact on the outcome of the Eisenberg-Noe model since the transpose of the relative liability matrix Π is multiplied by the incoming payment vector $p(\Pi)$, whose j^{th} entry is 0 when $\bar{p}_j = 0$ (cf. (6.2)).

Assets	variable	Liabilities	variable
Interbank	$a_i^{IB} = \sum_{j=1}^n L_{ji}$	Interbank	$l_i^{IB} = \sum_{j=1}^n L_{ij}$
External assets	x	External liabilities	l_i^0
	$(a_i^0 \text{ in Section 6.3})$	Capital	c_i

Table 6.2.1: Stylised bank balance sheet.

belong to the set of admissible matrices $\mathbb{A}$, defined as the set of all right stochastic matrices with entries in $[0, 1]$ and all diagonal entries 0:

$$\mathbb{A} = \left\{ \Pi \in [0, 1]^{n \times n} \mid \forall i : \Pi_{ii} = 0, \sum_{j=1}^n \Pi_{ij} = 1 \right\}. \quad (6.1)$$

Finally, denote the **net assets** of bank i from outside the banking system by x_i . A bank balance sheet then takes the simplified form of Table 6.2.1 and a financial system is given by the triplet $(\Pi, x, \bar{p}) \in \mathbb{A} \times \mathbb{R}_+^n \times \mathbb{R}_+^n$.

A bank is solvent when it has positive capital, i.e. when the sum of its net external assets and performing interbank assets exceeds its total liabilities. In this case, the bank honours all of its obligations. However, if the value of its obligations is greater than the bank's net assets plus performing interbank assets, then the bank will default² and repay its obligations pro-rata.³ These rules yield a *clearing vector* as the solution of the fixed point problem

$$p(\Pi) = \bar{p} \wedge (x + \Pi^\top p(\Pi)). \quad (6.2)$$

Let $p : \mathbb{A} \times \mathbb{R}_+^n \times \mathbb{R}_+^n \rightarrow \mathbb{R}_+^n; (\Pi, x, \bar{p}) \mapsto p(\Pi)$ be the fixed point function. As proved in (Eisenberg and Noe, 2001, Thm. 2), the clearing vector is unique if a system of banks is **regular**, which is defined as follows: A **surplus set** $S \subseteq \mathcal{N}$ is a set of banks in which no bank in the set has any obligations

²In this variant of the model the bank defaults due to insolvency. There is no direct concept of default due to illiquidity.

³This corresponds to assuming that all interbank claims can be aggregated to a single figure per bank and that all creditors of a defaulting bank are paid *pari passu*.

to a bank outside of the set and the sum over all banks' external net asset values in the set is positive, i.e. $\forall (i, j) \in S \times S^c : \Pi_{ij} = 0$ and $\sum_{i \in S} x_i > 0$. Next, consider the financial system as a directed graph in which there is a directed link from bank i to bank j if $L_{ij} > 0$. Denote the **risk orbit** of bank i as $o(i) = \{j \in \mathcal{N} \mid \text{there exists a directed path from } i \text{ to } j\}$. This means that the risk orbit of bank i is the set of all banks which may be affected by the default of bank i . A system is **regular** if every $o(i)$ is a surplus set. Uniqueness of the clearing vector has important consequences in terms of the continuity of the function p , which in turn is important for our sensitivity analysis. For this reason we will assume our financial network to be regular in the sequel.

Proposition 6.2.1. *Consider a regular financial network $(\Pi, x, \bar{p})$ in which x and $\bar{p}$ are fixed. The function p , defined in (6.2), is continuous with respect to $\Pi \in \mathbb{A}$.*

Proof. This proof follows the logic of (Feinstein et al., 2015, Lem. 5.2) and Ren et al. (2014). Fix the net assets x and total obligation $\bar{p}$. Let $\phi : [0, \bar{p}] \times \mathbb{A} \rightarrow [0, \bar{p}]$ be the function defined by $\phi(p, \Pi) = (\phi_1, \dots, \phi_n)^\top$, where

$$\phi_i(p, \Pi) = \bar{p}_i \wedge \left(x_i + \sum_{j=1}^n \Pi_{ji} p_j \right), \quad i \in \mathcal{N}.$$

The function ϕ is continuous and non-decreasing with respect to p and Π_{ij} for $i, j \in \mathcal{N}$. Because the network is regular and has a unique fixed point, it follows from (Feinstein et al., 2015, Prop. A.2) that the graph

$$\text{graph}(p) = \left\{ (\Pi, p) \in \mathbb{A} \times [0, \bar{p}] \mid \phi(p, \Pi) = p \right\}$$

is closed. Define the projection $\Psi : \mathbb{A} \times [0, \bar{p}] \rightarrow \mathbb{A}$ as $\Psi(\Pi, p) = \Pi$. By (Feinstein et al., 2015, Prop. A.3), Ψ is a closed mapping in the product topology. Then, in order to show that p is continuous, take $U \subset [0, \bar{p}]$ closed.

Then

$$p^{-1}[U] = \left\{ \Pi \in \mathbb{A} \mid p(\Pi) \in U \right\} = \Psi(\text{graph}(p) \cap (\mathbb{A} \times U)).$$

The graph of p is closed and $\mathbb{A}$ is closed by definition. Hence $p^{-1}[U]$ is closed and the function p is continuous with respect to Π . $\square$

6.2.1 QUANTIFYING ESTIMATION ERRORS IN (RELATIVE) LIABILITIES MATRICES

We assume that some estimation error is included in the entries of the relative liability matrix, which leads to a deviation of the clearing vector from the “true” clearing vector $p(\Pi)$. Denote the “true” relative liabilities matrix by Π and let $\Pi + hB$ denote the liabilities matrix that includes some estimation error, for a perturbation matrix B and $h \in \mathbb{R}$. We define two classes of perturbation matrices: In the first class, $\mathbb{B}$, we assume that the existence or non-existence of a link between two banks is known to the regulator and hence, the error is limited to a misspecification of the size of that link. This set is defined as follows:

Definition 6.2.2. For a fixed $\bar{p} \in \mathbb{R}_+^n$,

$$\mathbb{B}^n(\Pi) = \left\{ B \in \mathbb{R}^{n \times n} \mid \forall i : b_{ii} = 0, \sum_{j=1}^n b_{ij} = 0, \sum_{j=1}^n b_{ji} \bar{p}_j = 0, \text{ and } (\Pi_{ij} = 0) \Rightarrow (b_{ij} = 0) \forall j \right\}. \quad (6.3)$$

The summation conditions ensure that the total liabilities and total assets respectively of each bank are left unchanged by the perturbation. Of course it is not possible to have $\Pi + hB \in \mathbb{A}$ for any $h \in \mathbb{R}$. We merely require that there exists an $h^* > 0$ such that for all $h < h^*$, we have $\Pi + hB \in \mathbb{A}$. A possible choice is for instance given by

$$h^* = \min \left\{ \min_{b_{ij} < 0, \bar{p}_i > 0} \frac{-\Pi_{ij}}{b_{ij}}, \min_{b_{ij} > 0, \bar{p}_i > 0} \frac{1 - \Pi_{ij}}{b_{ij}} \right\}.$$

We exclude from this calculation of h^* any i where $\bar{p}_i = 0$ since this has no impact on the results. We will consider in particular a bounded set of perturbations

$$\mathbb{B}_F^n(\Pi) := \mathbb{B}^n(\Pi) \cap \{B \in \mathbb{R}^{n \times n} \mid \|B\|_F \leq 1\}. \quad (6.4)$$

AN ORTHONORMAL BASIS FOR PERTURBATION MATRICES. We will now construct an orthonormal basis for the matrices in $\mathbb{B}$. For example, in the case $n = 4$, the general form of a matrix $B \in \mathbb{B}^n$ for a fully connected network Π_K can be written as

$$\mathbb{B}^4(\Pi_K) = \left\{ \text{diag}(\bar{p})^{-1} \begin{pmatrix} 0 & z_1 & z_2 & -z_1 - z_2 \\ z_3 & 0 & z_4 & -z_3 - z_4 \\ z_5 & -\sum_{i=1}^5 z_i & 0 & \sum_{i=1}^4 z_i \\ -z_3 - z_5 & \sum_{i=2}^5 z_i & -z_2 - z_4 & 0 \end{pmatrix} \mid z \in \mathbb{R}^5 \right\},$$

from which it is clear that there are 5 degrees of freedom. It is easy to see that in general one has $df = n^2 - 3n + 1$. In the case $n = 4$, the first two basis elements E_1 and E_2 are given by

$$E_1 = \text{diag}(\bar{p})^{-1} \begin{pmatrix} 0 & \frac{1}{\bar{p}_1} & 0 & \frac{-1}{\bar{p}_1} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{\bar{p}_3} & 0 & \frac{1}{\bar{p}_3} \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad E_2 = \text{diag}(\bar{p})^{-1} \begin{pmatrix} 0 & 0 & \frac{1}{\bar{p}_1} & \frac{-1}{\bar{p}_1} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{\bar{p}_3} & 0 & \frac{1}{\bar{p}_3} \\ 0 & \frac{1}{\bar{p}_4} & \frac{-1}{\bar{p}_4} & 0 \end{pmatrix}.$$

In general we note that $\mathbb{B}^n(\Pi)$ is a closed, convex polyhedral set; we will take advantage of this fact in order to generate a general method for constructing basis matrices for $\mathbb{B}^n(\Pi)$.

1. Define

$$\vec{\mathbb{B}}^n(\Pi) := \left\{ b \in \mathbb{R}^{n^2} \left| \sum_{j=1}^n b_{i+n(j-1)} = 0, \quad \sum_{j=1}^n \bar{p}_j b_{n(i-1)+1+j} = 0, \right. \right. \\ \left. \left. \mathbb{1}_{\{\Pi_{ij}=0\}} b_{i+n(j-1)} = 0 \quad \forall i, j \right\},$$

to be a vectorised version of $\mathbb{B}^n(\Pi)$.

2. Construct a matrix $A(\Pi) \in \mathbb{R}^{(n^2+2n) \times (n^2)}$ such that

$\vec{\mathbb{B}}^n(\Pi) = \{b \in \mathbb{R}^{n^2} \mid A(\Pi)b = 0\}$. (The total degrees of freedom for $\vec{\mathbb{B}}^n(\Pi)$ (and therefore also for $\mathbb{B}^n(\Pi)$) is given by the rank of the matrix $A(\Pi)$.)

3. An orthonormal basis of $\vec{\mathbb{B}}^n(\Pi)$ can be found by generating the orthonormal basis $\{e_1, \dots, e_{df}\}$ of the null space of $A(\Pi)$.

4. Finally our basis matrices $\{E_1, \dots, E_{df}\}$ can be generated by reshaping the basis of the null space of $A(\Pi)$ by setting $E_{k;i,j} := e_{k;i+n(j-1)}$ for any $k = 1..df$ and $i, j = 1..n$.

Definition 6.2.3. *The set*

$$\mathbb{O}(\Pi) := \{E_1, \dots, E_{df}\} \tag{6.5}$$

*is an **orthonormal basis of perturbation matrices (batrices)** for the relative liability matrix Π .*

We define two matrices to be orthogonal when their vectorised forms are orthogonal in $\mathbb{R}^{n^2}$, and note that, by construction, the batrices E_k have unit Frobenius norm.

REMARK. A more general case can be considered in which we allow for errors that create links where there are none, or remove links that do exist.

This set is defined as follows: For a fixed $\bar{p} \in \mathbb{R}_+^n$,

$$\left\{ B \in \mathbb{R}^{n \times n} \mid \forall i : b_{ii} = 0, \sum_{j=1}^n b_{ij} = 0, \sum_{j=1}^n b_{ji} \bar{p}_j = 0, \text{ and } (\Pi_{ij} = 0) \Rightarrow (b_{ij} \geq 0) \forall j \right\}. \quad (6.6)$$

For simplicity, we will mainly focus on the first case $\mathbb{B}^n(\Pi)$.

6.2.2 DIRECTIONAL DERIVATIVES OF THE EISENBERG & NOE CLEARING VECTOR

Next, we analyse the error between the clearing vector of a perturbed liability matrix, $p(\Pi + hB)$, and the clearing vector of the original liability matrix, $p(\Pi)$ for small perturbations hB , with B in $\mathbb{B}^n(\Pi)$. Let $B \in \mathbb{B}^n(\Pi)$. In the case that the following limit exists, we define the directional derivative of the clearing vector $p(\Pi)$ in the direction of a perturbation matrix B as

Definition 6.2.4.

$$\mathcal{D}_B(p(\Pi)) = \lim_{h \rightarrow 0} \frac{p(\Pi + hB) - p(\Pi)}{h}. \quad (6.7)$$

The first-order Taylor expansion of p about Π gives

$$p(\Pi + hB) - p(\Pi) = h\mathcal{D}_B(p(\Pi)) + O(h^2).$$

The following theorem provides an explicit formula for the directional derivative of the clearing vector for a fixed financial network.

Theorem 6.2.5. *Let $(\Pi, x, \bar{p})$ be a regular financial network. The directional derivative of the clearing vector $p(\Pi)$ in the direction of a perturbation matrix $B \in \mathbb{B}^n(\Pi)$ is given almost everywhere by*

$$\mathcal{D}_B(p(\Pi)) = \left(I - \text{diag}(\delta)\Pi^\top \right)^{-1} \text{diag}(\delta)B^\top p(\Pi), \quad (6.8)$$

where $\text{diag}(\delta)$ is the diagonal matrix defined as $\text{diag}(\delta) = \text{diag}(\delta_1, \dots, \delta_n)$, where

$$\delta_i = \mathbb{1}_{\{x_i + \sum_{j=1}^n \Pi_{ji} p_j(\Pi) < \bar{p}_i\}}.$$

Here, (6.8) holds outside of the measure-zero set $\{x \in \mathbb{R}_+^n \mid \exists i \in \mathcal{N} \text{ s.t. } x_i + (\Pi^\top p(\Pi))_i = \bar{p}_i\}$.

Proof. We assume that the net external assets lie in the set

$$\{x \in \mathbb{R}_+^n \mid \nexists i \in \mathcal{N} \text{ s.t. } x_i + (\Pi^\top p(\Pi))_i = \bar{p}_i\}. \text{ Denote } \alpha^{(1)} = x + \Pi^\top p(\Pi) = (\alpha_1^{(1)}, \dots, \alpha_n^{(1)})^\top \text{ and } \alpha^{(2)} = x + (\Pi + hB)^\top p(\Pi + hB) = (\alpha_1^{(2)}, \dots, \alpha_n^{(2)})^\top.$$

By continuity of p with respect to Π , (Prop. 6.2.1), we have for all $i \in \mathcal{N}$, $\alpha_i^{(2)} \rightarrow \alpha_i^{(1)}$ as $h \rightarrow 0$ and thus $\mathbb{1}_{\{\{\alpha_i^{(1)} < \bar{p}_i\} \cap \{\alpha_i^{(2)} > \bar{p}_i\}\}} \rightarrow 0$, $\mathbb{1}_{\{\{\alpha_i^{(1)} > \bar{p}_i\} \cap \{\alpha_i^{(2)} < \bar{p}_i\}\}} \rightarrow 0$ and $\mathbb{1}_{\{\{\alpha_i^{(1)} < \bar{p}_i\} \cap \{\alpha_i^{(2)} < \bar{p}_i\}\}} \rightarrow \mathbb{1}_{\{\alpha_i^{(1)} < \bar{p}_i\}}$. To prove the existence of $\mathcal{D}_B(p(\Pi))$, we will show that the following two limits,

$$\begin{aligned} (\overline{\mathcal{D}_B(p(\Pi))})_i &= \limsup_{h \rightarrow 0} \frac{p_i(\Pi + hB) - p_i(\Pi)}{h} \text{ and} \\ (\underline{\mathcal{D}_B(p(\Pi))})_i &= \liminf_{h \rightarrow 0} \frac{p_i(\Pi + hB) - p_i(\Pi)}{h}, \end{aligned}$$

are equal for each component. Consider the upper limit

$$\begin{aligned}
 (\overline{\mathcal{D}_B(p(\Pi))})_i &= \limsup_{h \rightarrow 0} \frac{p_i(\Pi + hB) - p_i(\Pi)}{h} \\
 &= \limsup_{h \rightarrow 0} \frac{1}{h} \left(\left(\bar{p}_i \wedge \left(x_i + \sum_{j=1}^n (\Pi_{ji} + hb_{ji}) p_j(\Pi + hB) \right) \right) - \left(\bar{p}_i \wedge \left(x_i + \sum_{j=1}^n \Pi_{ji} p_j(\Pi) \right) \right) \right) \\
 &= \limsup_{h \rightarrow 0} \left(0 \times \mathbb{1}_{\{\{\alpha_i^{(1)} > \bar{p}_i\} \cap \{\alpha_i^{(2)} > \bar{p}_i\}\}} + \frac{\bar{p}_i - (x_i + \sum_{j=1}^n \Pi_{ji} p_j(\Pi))}{h} \mathbb{1}_{\{\{\alpha_i^{(1)} < \bar{p}_i\} \cap \{\alpha_i^{(2)} > \bar{p}_i\}\}} \right. \\
 &\quad + \frac{x_i + \sum_{j=1}^n \Pi_{ji} p_j(\Pi + hB) + h \sum_{j=1}^n b_{ji} p_j(\Pi + hB) - \bar{p}_i}{h} \mathbb{1}_{\{\{\alpha_i^{(1)} > \bar{p}_i\} \cap \{\alpha_i^{(2)} < \bar{p}_i\}\}} \\
 &\quad \left. + \frac{\sum_{j=1}^n \Pi_{ji} (p_j(\Pi + hB) - p_j(\Pi)) + h \sum_{j=1}^n b_{ji} p_j(\Pi + hB)}{h} \mathbb{1}_{\{\{\alpha_i^{(1)} < \bar{p}_i\} \cap \{\alpha_i^{(2)} < \bar{p}_i\}\}} \right) \\
 &= \left(\sum_{j=1}^n \Pi_{ji} (\overline{\mathcal{D}_B(p(\Pi))})_j + \sum_{j=1}^n b_{ji} p_j(\Pi) \right) \mathbb{1}_{\{\alpha_i^{(1)} < \bar{p}_i\}} \\
 &= \delta_i \sum_{j=1}^n \Pi_{ji} (\overline{\mathcal{D}_B(p(\Pi))})_j + \delta_i \sum_{j=1}^n b_{ji} p_j(\Pi).
 \end{aligned}$$

Similarly, we get

$$(\underline{\mathcal{D}_B(p(\Pi))})_i = \delta_i \sum_{j=1}^n \Pi_{ji} (\underline{\mathcal{D}_B(p(\Pi))})_j + \delta_i \sum_{j=1}^n b_{ji} p_j(\Pi).$$

Hence, each component $(\mathcal{D}_B(p(\Pi)))_i$ is again a fixed point problem. Assuming that this fixed point problem has a unique solution it follows

$$(\overline{\mathcal{D}_B(p(\Pi))})_i = (\underline{\mathcal{D}_B(p(\Pi))})_i,$$

for all $i = 1, \dots, n$. Therefore, under such an assumption, $\mathcal{D}_B(p(\Pi))$ is well defined and it is the solution to the fixed point equation

$$\mathcal{D}_B(p(\Pi)) = \text{diag}(\delta) \Pi^\top \mathcal{D}_B(p(\Pi)) + \text{diag}(\delta) B^\top p(\Pi).$$

Next, we proceed to show that $(I - \text{diag}(\delta) \Pi^\top)$ is invertible, which establishes uniqueness of the fixed point and the directional derivative (6.8) to

conclude the proof.

First, assume that $\text{diag}(\delta)\Pi^\top$ is irreducible. Then by the Perron-Frobenius Theorem, $\text{diag}(\delta)\Pi^\top$ has an eigenvector $\vec{p} > \vec{0}$ with $\|\vec{p}\|_1 = 1$ corresponding to the eigenvalue $r = \rho(\text{diag}(\delta)\Pi^\top)$, where ρ is the spectral radius of a matrix. Under the assumption of a regular network, at least one bank must be solvent, i.e. there exists i so that $\text{diag}(\delta)_{ii} = 0$. This implies that the i -th column sum of $\text{diag}(\delta)\Pi^\top$ is strictly less than 1. Thus there is some matrix $M \geq 0$, $M \neq 0$ so that each column sum of $\text{diag}(\delta)\Pi^\top + M$ is 1, i.e.

$$\mathbf{1}^\top (\text{diag}(\delta)\Pi^\top + M) = \mathbf{1}^\top.$$

Note that the column sums of $\text{diag}(\delta)\Pi^\top$ are at most 1 since each row sum of Π is 1. Therefore the spectral radius of $\text{diag}(\delta)\Pi^\top$ must satisfy $\rho(\text{diag}(\delta)\Pi^\top) \leq \|\text{diag}(\delta)\Pi^\top\|_1 \leq 1$. Moreover, we must have $\rho(\text{diag}(\delta)\Pi^\top) < 1$. Otherwise, $r = \rho(\text{diag}(\delta)\Pi^\top) = 1$, which implies

$$1 = \mathbf{1}^\top \vec{p} = \mathbf{1}^\top (\text{diag}(\delta)\Pi^\top + M) \vec{p} = \mathbf{1}^\top (\vec{p} + M\vec{p}) = 1 + \mathbf{1}^\top M\vec{p} > 1.$$

Therefore, we can conclude that in the case $\text{diag}(\delta)\Pi^\top$ is irreducible, $\rho(\text{diag}(\delta)\Pi^\top) < 1$.

Now suppose that $\text{diag}(\delta)\Pi^\top$ is reducible, i.e. $\text{diag}(\delta)\Pi^\top$ is similar to a block upper triangular matrix D , with irreducible diagonal blocks D_i , $i = 1, \dots, m$ for some $m < n$. Under the assumption of a regular network, each D_i has at least one column whose sum is strictly less than 1. As in the preceding case, this implies that $\rho(D_i) < 1$ for each i and therefore

$$\rho(\text{diag}(\delta)\Pi^\top) = \rho(D) < 1.$$

Since the maximum eigenvalue of $\text{diag}(\delta)\Pi^\top$ is strictly less than 1 it follows immediately from the characteristic polynomial that 0 must not be an

eigenvalue of $I - \text{diag}(\delta)\Pi^\top$. This suffices to show that $I - \text{diag}(\delta)\Pi^\top$ is invertible.

□

6.2.3 A TAYLOR SERIES FOR THE EISENBERG & NOE FIXED POINT

In the same manner, we can define higher order directional derivatives.

Definition 6.2.6. *For $n \geq 1$, we define the n^{th} order directional derivative of the clearing vector with respect to a perturbation matrix B as*

$$\mathcal{D}_B^{(n)} p(\Pi) = \lim_{h \rightarrow 0} \frac{\mathcal{D}_B^{(n-1)} p(\Pi + hB) - \mathcal{D}_B^{(n-1)} p(\Pi)}{h}, \quad (6.9)$$

when the limit exists, and

$$\mathcal{D}_B^{(0)} p(\Pi) = p(\Pi). \quad (6.10)$$

Remarkably, as Theorem 6.2.7 shows, all higher order derivatives also have an explicit formula, which allows us to obtain an exact Taylor series for the clearing vector. We impose an additional assumption on allowable perturbations hB so that the matrix $\text{diag}(\delta)$ (as defined in Theorem 6.2.5) as a function of $\Pi + hB$ is fixed with respect to h , i.e. we require h sufficiently small so that the same subset of banks is in default when the liability matrix is $\Pi + hB$ as when the liability matrix is Π . Let

$$h^{**} = \sup \left\{ h > 0 \left| x_i + \sum_{j=1}^n \Pi_{ji} p_j(\Pi) < \bar{p}_i \Leftrightarrow x_i + \sum_{j=1}^n (\Pi_{ji} + h b_{ji}) p_j(\Pi + hB) < \bar{p}_i \text{ for } i = 1, \dots, n \right. \right\}.$$

We necessarily have $h^{**} > 0$ because we exclude the measure-zero set $\{x \in \mathbb{R}_+^n \mid \exists i \in \mathcal{N} \text{ s.t. } x_i + (\Pi^\top p(\Pi))_i = \bar{p}_i\}$ in which a bank is exactly at the brink of default.

Theorem 6.2.7. *Let $(\Pi, x, \bar{p})$ be a regular financial network. Then, for*

$B \in \mathbb{B}^n(\Pi)$, and for all $n \geq 1$:

$$\mathcal{D}_B^{(n)} p(\Pi) = n(I - \text{diag}(\delta)\Pi^\top)^{-1} \text{diag}(\delta)B^\top \mathcal{D}_B^{(n-1)} p(\Pi) \quad (6.11)$$

$$= n! \left((I - \text{diag}(\delta)\Pi^\top)^{-1} \text{diag}(\delta)B^\top \right)^n p(\Pi), \quad (6.12)$$

where $\mathcal{D}_B^{(0)} p(\Pi) = p(\Pi)$. Moreover, for $h < \min \left\{ h^*, h^{**}, \frac{1}{\rho((I - \text{diag}(\delta)\Pi^\top)^{-1} \text{diag}(\delta)B^\top)} \right\}$, where h^* and h^{**} are defined as in (6.2.1) and (6.2.3), respectively, we have the representation

$$p(\Pi + hB) = \left(I - h(I - \text{diag}(\delta)\Pi^\top)^{-1} \text{diag}(\delta)B^\top \right)^{-1} p(\Pi). \quad (6.13)$$

This representation holds outside of the measure-zero set $\{x \in \mathbb{R}_+^n \mid \exists i \in \mathcal{N} \text{ s.t. } x_i + (\Pi^\top p(\Pi))_i = \bar{p}_i\}$.

Proof. We prove the result by induction. Theorem 6.2.5 shows the result for $n = 1$. We now assume that equation (6.11) holds for $n = k$ and we proceed to show that it holds for $n = k + 1$. We show the existence of (6.9) by computing the two limits:

$$\begin{aligned} \overline{(\mathcal{D}_B^{(k+1)} p(\Pi))}_i &= \limsup_{h \rightarrow 0} \frac{(\mathcal{D}_B^{(k)} p(\Pi + hB))_i - (\mathcal{D}_B^{(k)} p(\Pi))_i}{h} \\ \underline{(\mathcal{D}_B^{(k+1)} p(\Pi))}_i &= \liminf_{h \rightarrow 0} \frac{(\mathcal{D}_B^{(k)} p(\Pi + hB))_i - (\mathcal{D}_B^{(k)} p(\Pi))_i}{h}. \end{aligned}$$

The first-order Taylor approximation for matrix inverses gives for $X, Y \in \mathbb{R}^{N \times N}$ and ϵ small enough: $(X + \epsilon Y)^{-1} \approx X^{-1} - X^{-1} \epsilon Y X^{-1}$ (Gentle, 2007, p. 152). We thus have that

$$(I - \text{diag}(\delta)(\Pi + hB)^\top)^{-1} \approx (I - \text{diag}(\delta)\Pi^\top)^{-1} + h(I - \text{diag}(\delta)\Pi^\top)^{-1} \text{diag}(\delta)B^\top (I - \text{diag}(\delta)\Pi^\top)^{-1}. \quad (6.14)$$

This allows us to compute

$$\begin{aligned}
 \Delta \mathcal{D}_B^{(k)} &:= \mathcal{D}_B^{(k)} p(\Pi + hB) - \mathcal{D}_B^{(k)} p(\Pi) \\
 &= k(I - \text{diag}(\delta)(\Pi + hB)^\top)^{-1} \text{diag}(\delta) B^\top \mathcal{D}_B^{(k-1)} p(\Pi + hB) - \dots \\
 &\quad k(I - \text{diag}(\delta)\Pi^\top)^{-1} \text{diag}(\delta) B^\top \mathcal{D}_B^{(k-1)} p(\Pi) \\
 &= k((I - \text{diag}(\delta)\Pi^\top) - h \text{diag}(\delta) B^\top)^{-1} \text{diag}(\delta) B^\top \mathcal{D}_B^{(k-1)} p(\Pi + hB) - \dots \\
 &\quad k(I - \text{diag}(\delta)\Pi^\top)^{-1} \text{diag}(\delta) B^\top \mathcal{D}_B^{(k-1)} p(\Pi) \\
 &\approx k \left[(I - \text{diag}(\delta)\Pi^\top)^{-1} + (I - \text{diag}(\delta)\Pi^\top)^{-1} h \text{diag}(\delta) B^\top (I - \text{diag}(\delta)\Pi^\top) \right]^{-1} \times \dots \\
 &\quad \text{diag}(\delta) B^\top \mathcal{D}_B^{(k-1)} p(\Pi + hB) - k(I - \text{diag}(\delta)\Pi^\top)^{-1} \text{diag}(\delta) B^\top \mathcal{D}_B^{(k-1)} p(\Pi) \\
 &= k(I - \text{diag}(\delta)\Pi^\top)^{-1} \text{diag}(\delta) B^\top \left(\mathcal{D}_B^{(k-1)} p(\Pi + hB) - \mathcal{D}_B^{(k-1)} p(\Pi) \right) + \dots \\
 &\quad k(I - \text{diag}(\delta)\Pi^\top)^{-1} h \text{diag}(\delta) B^\top (I - \text{diag}(\delta)\Pi^\top)^{-1} \text{diag}(\delta) B^\top \mathcal{D}_B^{(k-1)} p(\Pi + hB).
 \end{aligned}$$

Next, we note that since $\mathcal{D}_B^{(k)} p(\Pi) = k! \left((I - \text{diag}(\delta)\Pi^\top)^{-1} \text{diag}(\delta) B^\top \right)^k p(\Pi)$, where $p(\Pi)$ and $(I - \text{diag}(\delta)\Pi^\top)^{-1}$ are both continuous with respect to Π , $\mathcal{D}_B^{(k)} p(\Pi)$ is also continuous with respect to Π . Consider now the upper limit

$$\begin{aligned}
 \overline{\mathcal{D}_B^{(k+1)} p(\Pi)} &= \limsup_{h \rightarrow 0} \frac{(\mathcal{D}_B^{(k)} p(\Pi + hB)) - (\mathcal{D}_B^{(k)} p(\Pi))}{h} \\
 &= \limsup_{h \rightarrow 0} \frac{k(I - \text{diag}(\delta)\Pi^\top)^{-1} \text{diag}(\delta) B^\top \left(\mathcal{D}_B^{(k-1)} p(\Pi + hB) - \mathcal{D}_B^{(k-1)} p(\Pi) \right)}{h} + \dots \\
 &\quad \limsup_{h \rightarrow 0} \frac{k(I - \text{diag}(\delta)\Pi^\top)^{-1} h \text{diag}(\delta) B^\top (I - \text{diag}(\delta)\Pi^\top)^{-1} \text{diag}(\delta) B^\top \mathcal{D}_B^{(k-1)} p(\Pi + hB)}{h} \\
 &= k(I - \text{diag}(\delta)\Pi^\top)^{-1} \text{diag}(\delta) B^\top \mathcal{D}_B^{(k)} p(\Pi) + \dots \\
 &\quad k(I - \text{diag}(\delta)\Pi^\top)^{-1} \text{diag}(\delta) B^\top (I - \text{diag}(\delta)\Pi^\top)^{-1} \text{diag}(\delta) B^\top \mathcal{D}_B^{(k-1)} p(\Pi) \\
 &= k(I - \text{diag}(\delta)\Pi^\top)^{-1} \text{diag}(\delta) B^\top \mathcal{D}_B^{(k)} p(\Pi) + (I - \text{diag}(\delta)\Pi^\top)^{-1} \text{diag}(\delta) B^\top \mathcal{D}_B^{(k)} p(\Pi) \\
 &= (k+1)(I - \text{diag}(\delta)\Pi^\top)^{-1} \text{diag}(\delta) B^\top \mathcal{D}_B^{(k)} p(\Pi).
 \end{aligned}$$

Similarly, we obtain $\underline{\mathcal{D}_B^{(k+1)} p(\Pi)} = (k+1)(I - \text{diag}(\delta)\Pi^\top)^{-1} \text{diag}(\delta) B^\top \mathcal{D}_B^{(k)} p(\Pi)$.

The existence of the limit and the result follow.

Within the neighbourhood

$$\mathcal{H}_1 := \left\{ h > 0 \mid h < \min \left\{ h^*, h^{**}, \frac{1}{\rho((I - \text{diag}(\delta)\Pi^\top)^{-1}\text{diag}(\delta)B^\top)} \right\} \right\},$$

it further immediately follows that

$$p(\Pi + hB) = \sum_{n=0}^{\infty} \frac{n!}{n!} h^n \left((I - \text{diag}(\delta)\Pi^\top)^{-1}\text{diag}(\delta)B^\top \right)^n p(\Pi).$$

Since $\rho(h(I - \text{diag}(\delta)\Pi^\top)^{-1}\text{diag}(\delta)B^\top) = h\rho((I - \text{diag}(\delta)\Pi^\top)^{-1}\text{diag}(\delta)B^\top)$ and by assumption, $h < \frac{1}{\rho((I - \text{diag}(\delta)\Pi^\top)^{-1}\text{diag}(\delta)B^\top)}$, the spectral radius of $h(I - \text{diag}(\delta)\Pi^\top)^{-1}\text{diag}(\delta)B^\top$ is strictly less than 1. Therefore, its Leontief inverse exists (see, for example (Meyer, 2000, p. 618)) and the Taylor series evaluates to

$$p(\Pi + hB) = \left(I - h(I - \text{diag}(\delta)\Pi^\top)^{-1}\text{diag}(\delta)B^\top \right)^{-1} p(\Pi).$$

To see that this Taylor series representation for $p(\Pi + hB)$ is exact, we use the condition that $\text{diag}(\delta)$ is fixed for $h < h^{**}$, for h^{**} defined as in (6.2.3). By the definition of p (6.2) and $\text{diag}(\delta)$ (defined in Theorem 6.2.5), along with the fact that $I - \text{diag}(\delta)\Pi^\top$ is invertible, as shown in the proof of Theorem 6.2.5, we have

$$p(\Pi + hB) = \left(I - \text{diag}(\delta)(\Pi + hB)^\top \right)^{-1} [\text{diag}(\delta)x + (I - \text{diag}(\delta))\bar{p}].$$

Then since $\rho(h(I - \text{diag}(\delta)\Pi^\top)^{-1}\text{diag}(\delta)B^\top) < 1$ by assumption, we have

$$\lim_{n \rightarrow \infty} \left[\left(I - \text{diag}(\delta)(\Pi + hB)^\top \right)^{-1} h \text{diag}(\delta)B^\top \right]^n = 0,$$

using a property of the spectral radius (see (Meyer, 2000, p. 617)). Now we can apply the following fact about Neumann Series (see (Meyer, 2000, p.

126))): $(X+Y)^{-1} = \sum_{k=0}^{\infty} (-X^{-1}Y)^k X^{-1}$ if X^{-1} exists and $\lim_{n \rightarrow \infty} (X^{-1}Y)^n = 0$. Thus we have

$$\begin{aligned} p(\Pi + hB) &= \left(I - \text{diag}(\delta)(\Pi + hB)^\top \right)^{-1} [\text{diag}(\delta)x + (I - \text{diag}(\delta))\bar{p}] \\ &= \sum_{k=0}^{\infty} \left(h(I - \text{diag}(\delta)\Pi^\top)^{-1} \text{diag}(\delta)B^\top \right)^k (I - \text{diag}(\delta)\Pi^\top)^{-1} [\text{diag}(\delta)x + (I - \text{diag}(\delta))\bar{p}] \\ &= \sum_{k=0}^{\infty} \left(h(I - \text{diag}(\delta)\Pi^\top)^{-1} \text{diag}(\delta)B^\top \right)^k p(\Pi), \end{aligned}$$

which is equal to $(I - h(I - \text{diag}(\delta)\Pi^\top)^{-1} \text{diag}(\delta)B^\top)^{-1} p(\Pi)$, as above. $\square$

As the terms in B are no longer linear, we cannot express the Taylor series as a linear combination of basis matrices.

6.2.4 CLEARING VECTOR DEVIATION FOR LIABILITIES ESTIMATION ERRORS

We return to the first order directional derivative to quantify the error in the clearing vector given estimation errors of the relative liability matrix in a particular case. Let $B \in \mathbb{B}^n(\Pi)$ and assume that for a given $h \in \mathbb{R}$: $\Pi + hB \in \mathbb{A}$. We consider the estimation error

$$\min_{B \in \mathbb{B}^n(\Pi)} \|p(\Pi + hB) - p(\Pi)\|_2.$$

In order to remove the dependence on h and the magnitude of B , we consider instead

$$\min_{B \in \mathbb{B}_F^n(\Pi)} \lim_{h \rightarrow 0} \frac{\|p(\Pi + hB) - p(\Pi)\|_2}{h} = \min_{B \in \mathbb{B}_F^n(\Pi)} \|\mathcal{D}_B(p(\Pi))\|_2. \quad (6.15)$$

Because B appears via a linear term in (6.8), this allows us to use the batrices in an elegant way to quantify the estimation error in the Eisenberg

and Noe clearing vector.

Proposition 6.2.8. *Let $(\Pi_0, x, \bar{p})$ be a regular banking network. The estimation error is given by*

$$\max_{B \in \mathbb{B}_F^n(\Pi)} \|D_B p(\Pi)\|_2 = \|\mathcal{D}_{\bar{E}} p(\Pi)\|_2^o.$$

where $\mathcal{D}_{\bar{E}} p(\Pi) := (\mathcal{D}_{E_1} p(\Pi), \dots, \mathcal{D}_{E_{df}} p(\Pi)) \in \mathbb{R}^{n \times df}$ and $(E_1, \dots, E_{df})$ is an orthonormal basis, $\mathbb{O}(\Pi)$ of the space $\mathbb{B}^n(\Pi)$, and $\|\cdot\|_2^o$ denotes the spectral norm of a matrix. Furthermore, the clearing vector deviation corresponding to the solution of the maximisation problem is achieved by

$$B^*(\Pi) := \sum_k z_k E_k, \quad (6.16)$$

where z_k are the components of the (normalised) eigenvector corresponding to the maximum eigenvalue of $\mathcal{D}_{\bar{E}} p(\Pi)^\top \mathcal{D}_{\bar{E}} p(\Pi)$.

Proof. Note first that for any perturbation matrix $B \in \mathbb{B}^n(\Pi)$ with formulation $B = \sum_k z_k E_k$:

$$\|\mathcal{D}_B p(\Pi)\|_2^2 = \left\| \sum_k z_k \mathcal{D}_{E_k} p(\Pi) \right\|_2^2 = z^\top \mathcal{D}_{\bar{E}} p(\Pi)^\top \mathcal{D}_{\bar{E}} p(\Pi) z. \quad (6.17)$$

Immediately this implies:

$$\begin{aligned} \max_{B \in \mathbb{B}_F^n(\Pi)} \|\mathcal{D}_B p(\Pi)\|_2^2 &= \max_{\|z\|_2 \leq 1} z^\top \mathcal{D}_{\bar{E}} p(\Pi)^\top \mathcal{D}_{\bar{E}} p(\Pi) z \\ &= \lambda_{\max}(\mathcal{D}_{\bar{E}} p(\Pi)^\top \mathcal{D}_{\bar{E}} p(\Pi)) \\ &= (\|\mathcal{D}_{\bar{E}} p(\Pi)\|_2^o)^2 \end{aligned}$$

□

6.2.5 A MODEL WITH SOCIETY

In this section, we assume that in addition to their interbank liabilities, banks also have a liability to society. From a modelling perspective, one can view this as moving a subset of the external liabilities included in x_i into a zero-node of the interbank liability matrix. Hence, the set of institutions becomes $\mathcal{N}_0 = \{0\} \cup \mathcal{N}$. Without loss of generality, we assume that all banks $i \in \mathcal{N}$ owe money to at least one counterparty $j \in \mathcal{N}$ within the system. Otherwise, a bank who owes no money can be absorbed by the society node as it plays the same role within the model structure. The question of interest is then how the payout to society may be mis-estimated given estimation errors in the relative liabilities matrix.

The interbank liability matrix L of the previous section is expanded to $L_0 \in \mathbb{R}^{(n+1) \times (n+1)}$ given by

$$L_0 = \left[\begin{array}{ccc|c} 0 & \cdots & L_{1n} & L_{10} \\ \vdots & \ddots & \vdots & \vdots \\ L_{n1} & \cdots & 0 & L_{n0} \\ \hline 0 & \cdots & 0 & 0 \end{array} \right] = \left[\begin{array}{c|c} L & l_0 \\ \hline 0 & 0 \end{array} \right],$$

where $l_0 = (l_{10}, \dots, l_{n0})^\top$ is the society liability vector. We require that at least one bank have an obligation to society, i.e. $L_{i0} > 0$ for some $1 \leq i \leq n$. The total liability of bank i is now given by $\bar{p}_i = \sum_{j=0}^n L_{ij}$ and the relative liability matrix Π_0 is transformed accordingly, i.e. $\Pi_{ij} \in [0, 1]$ and in particular when $\bar{p}_i > 0$, $\Pi_{ij} = \frac{L_{ij}}{\bar{p}_i}$. As in Section 6.2, we allow $\Pi_{ij} \in [0, 1]$ to be arbitrary when $\bar{p}_i = 0$. An admissible relative liability matrix Π_0 thus belongs to the set of all right stochastic matrices with entries in $[0, 1]$, all

diagonal entries 0, and at least one $\Pi_{i0} > 0$:

$$\mathbb{A}_0 = \left\{ \Pi_0 \in [0, 1]^{(n+1) \times (n+1)} \mid \forall i : \Pi_{ii} = 0, \sum_{j=0}^n \Pi_{ij} = 1 \text{ and } \exists i \text{ s.t. } \Pi_{i0} > 0 \right\}. \quad (6.18)$$

An admissible *interbank* relative liability matrix Π thus belongs to the set

$$\mathbb{A}_I = \left\{ \Pi \in [0, 1]^{n \times n} \mid \forall i : \Pi_{ii} = 0, \sum_{j=1}^n \Pi_{ij} \leq 1 \text{ and } \exists i \text{ s.t. } \sum_{j=1}^n \Pi_{ij} < 1 \right\}, \quad (6.19)$$

which has the same properties as the original interbank relative liability matrix $\mathbb{A}$ defined in (6.1), except that at least one bank has a liability to society.

6.2.6 OVERESTIMATION IN THE PAYOUT TO SOCIETY

Next, we use the directional derivative in order to quantify how the payout to society could be overestimated due to errors in the estimated interbank relative liability matrix. As it turns out, this problem also has an elegant solution using the batrices introduced above. As before, we assume that $(\Pi_0, x, \bar{p})$ is a regular network and that the relative liabilities to society $\pi_0 = (\Pi_{10}, \dots, \Pi_{n0})^T$ and the total liabilities $\bar{p}$ are exactly known.

Definition 6.2.9. *Let $(\Pi_0, x, \bar{p})$ be a regular banking network. The payout to society is defined as the quantity $\pi_0^\top p(\Pi)$.*

To study this optimization problem, we assume that at least one bank, but not all banks default. The following remark shows that this assumption is meaningful to exclude trivial cases.

REMARK. Let $(\Pi_0, x, \bar{p})$ be a regular system with the interbank relative liability matrix $\Pi \in \mathbb{A}_I$ and $B \in \mathbb{B}^n(\Pi)$. If all banks default, or if no bank

defaults, then the payout to society remains unchanged for an arbitrary admissible perturbation B .

Proof. Let B be an arbitrary perturbation matrix. We show that in both cases $\pi_0^\top \mathcal{D}_B(p(\Pi)) = 0$.

1. Assume that no bank defaults. Then $\text{diag}(\delta) = 0$, and the result holds trivially as $\mathcal{D}_B(p(\Pi)) = 0$.

2. Assume all banks default. Then $\text{diag}(\delta) = I$. Hence, $\pi_0^\top \mathcal{D}_B(p(\Pi)) = \pi_0^\top (I - \Pi^\top)^{-1} B^\top p(\Pi)$. Note that $\pi_0^\top (I - \Pi^\top)^{-1} = \mathbf{1}^\top$, because by definition $\pi_0^\top = \mathbf{1}^\top (I - \Pi^\top)$. Using this and the definitions of $\mathcal{D}_B(p(\Pi))$ and B , it follows $\pi_0^\top \mathcal{D}_B(p(\Pi)) = \sum_{i=1}^n \sum_{j=1}^n b_{ji} p_j(\Pi) = 0$. $\square$

Let $B \in \mathbb{B}^n(\Pi)$ and assume that for a given $h \in \mathbb{R} : \Pi + hB \in \mathbb{A}_I$. Then the minimum payout to society is

$$\min_{B \in \mathbb{B}^n(\Pi)} \pi_0^\top p(\Pi + hB).$$

In order to remove the dependence on h and the magnitude of B , we subtract the constant term $p(\Pi)$ and consider instead

$$\min_{B \in \mathbb{B}_F^n(\Pi)} \lim_{h \rightarrow 0} \pi_0^\top \frac{p(\Pi + hB) - p(\Pi)}{h} = \min_{B \in \mathbb{B}_F^n(\Pi)} \pi_0^\top \mathcal{D}_B(p(\Pi)). \quad (6.20)$$

Use this with the basis elements to get:

Proposition 6.2.10. *Let $(\Pi_0, x, \bar{p})$ be a regular banking network. The shortfall to society due to estimation errors in the liability matrix is given by*

$$\min_{B \in \mathbb{B}_F^n(\Pi)} \pi_0^\top \mathcal{D}_B(p(\Pi)) = -\|\pi_0^\top \mathcal{D}_{\bar{E}} p(\Pi)\|_2,$$

where $\mathcal{D}_{\bar{E}} p(\Pi) := (\mathcal{D}_{E_1} p(\Pi), \dots, \mathcal{D}_{E_{df}} p(\Pi)) \in \mathbb{R}^{n \times df}$ and $(E_1, \dots, E_{df})$ is an orthonormal basis, $\mathbb{O}(\Pi)$ of the space $\mathbb{B}^n(\Pi)$. Furthermore, the shortfall is

achieved by

$$B_0^*(\Pi) := - \sum_{k=1}^{df} \frac{\pi_0^\top \mathcal{D}_{E_k} p(\Pi)}{\|\pi_0^\top \mathcal{D}_{\bar{E}} p(\Pi)\|_2} E_k \quad (6.21)$$

Proof. Since the problem

$$\min \pi_0^\top \mathcal{D}_{\bar{E}} p(\Pi) z \quad \text{s.t.} \quad \|z\|_2 \leq 1,$$

has is a linear objective it is equivalent to

$$\max \pi_0^\top \mathcal{D}_{\bar{E}} p(\Pi) z \quad \text{s.t.} \quad z^\top z = 1.$$

By KKT conditions we know that

$$\begin{aligned} (\mathcal{D}_{\bar{E}} p(\Pi))^\top \pi_0 - 2\mu z &= 0 \\ z^\top z &= 1. \end{aligned}$$

The first condition implies $z^* = \frac{(\mathcal{D}_{\bar{E}} p(\Pi))^\top \pi_0}{2\mu}$. Plugging this into the second implies $\mu \in \left\{ \pm \frac{\|\pi_0^\top \mathcal{D}_{\bar{E}} p(\Pi)\|_2}{2} \right\}$. With two possible solutions we plug these back into the original objective to find that the minimum is attained at $\mu = -\frac{\|\pi_0^\top \mathcal{D}_{\bar{E}} p(\Pi)\|_2}{2}$ for an optimal value of:

$$\pi_0^\top \mathcal{D}_{\bar{E}} p(\Pi) z^* = - \frac{(\pi_0^\top \mathcal{D}_{\bar{E}} p(\Pi)) (\pi_0^\top \mathcal{D}_{\bar{E}} p(\Pi))^\top}{\|\pi_0^\top \mathcal{D}_{\bar{E}} p(\Pi)\|_2} = -\|\pi_0^\top \mathcal{D}_{\bar{E}} p(\Pi)\|_2. \quad (6.22)$$

Therefore, the solution is

$$B_0^*(\Pi) = \sum_{k=1}^{df} z_k^* E_k = - \sum_{k=1}^{df} \frac{\pi_0^\top \mathcal{D}_{E_k} p(\Pi)}{\|\pi_0^\top \mathcal{D}_{\bar{E}} p(\Pi)\|_2} E_k. \quad (6.23)$$

□

6.3 EMPIRICAL APPLICATION: ASSESSING THE ROBUSTNESS OF SYSTEMIC RISK ANALYSES

In this section we assess the robustness of conclusions drawn from systemic risk studies that use the Eisenberg & Noe algorithm to model direct contagion. Before performing a larger study on a dataset of European banks, we present an illustrative example in a small network of four banks.

6.3.1 SMALL NETWORK EXAMPLE

Let the liabilities matrix and the net worth be given by respectively

$$L = \begin{pmatrix} 0 & 7 & 1 & 1 \\ 3 & 0 & 3 & 3 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}, \quad x = \begin{pmatrix} 0 \\ 2 \\ 2 \\ 2 \end{pmatrix}.$$

The total liabilities vector is consequently given by $\bar{p} = (9, 9, 3, 3)^\top$. With 0 net worth, Bank 1 defaults initially. The clearing vector can be easily computed to be $p = (4.5, 7.5, 3, 3)^\top$, showing that Bank 2 also defaults through contagion. The left panel in Figure 6.3.1 shows the network of four banks, with edge widths proportional to liability size; defaulting banks are coloured in red. The right panel illustrates the matrix

$$B = \begin{pmatrix} 0 & 0.3230 & -0.1615 & -0.1615 \\ -0.0381 & 0 & 0.0190 & 0.0190 \\ 0.0571 & -0.4845 & 0 & 0.4274 \\ 0.0571 & -0.4845 & 0.4274 & 0 \end{pmatrix},$$

which is the solution to the maximisation problem $\max_{B \in \mathbb{B}_F^n(\Pi)} \|D_B p(\Pi)\|_2$. Edge widths are proportional to the absolute value of the entries in B , red

colouring denote negative entries, the green colour denotes a positive entry. The lower panel shows the relative difference $\frac{\|p-\hat{p}\|}{\|p\|}$, where $\hat{p}$ is the clearing vector of $\hat{\Pi} = \Pi + hB$, for $0 \leq h \leq h_{\max} \approx 0.68$. When $h = h_{\max}$, we get

$$L^* = \begin{pmatrix} 0 & 9 & 0 & 0 \\ 2.76 & 0 & 3.12 & 3.12 \\ 1.12 & 0 & 0 & 1.88 \\ 1.12 & 0 & 1.88 & 0 \end{pmatrix}.$$

which has the clearing vector

$$\hat{p} \approx (4.11, 6.11, 3, 3)^\top.$$

One can immediately verify that L^* has indeed the same total interbank assets and liabilities for each bank, but they are distributed in a different manner. Hence, in this example, there can be a deviation of up to 15% in the relative norm of the clearing vector for a network that is still consistent with the total assets and total liabilities.

6.3.2 ROBUSTNESS ANALYSIS OF DIRECT CONTAGION IN THE EUROPEAN BANKING SYSTEM

As empirical application, we use the same dataset of European banks from the EBA that has been used in previous studies using the Eisenberg & Noe framework ([Gandy and Veraart \(2016\)](#), [Chen et al. \(2016\)](#)) as well as in Part I of this thesis. As in these papers, given the heuristic approach to the dataset, our exercise should be considered to be an illustration of our results and methodology, rather than a realistic full fledged empirical analysis. We will comment further on the use of the Eisenberg & Noe model in systemic risk analyses and supervisory stress tests in the Discussion 6.4.

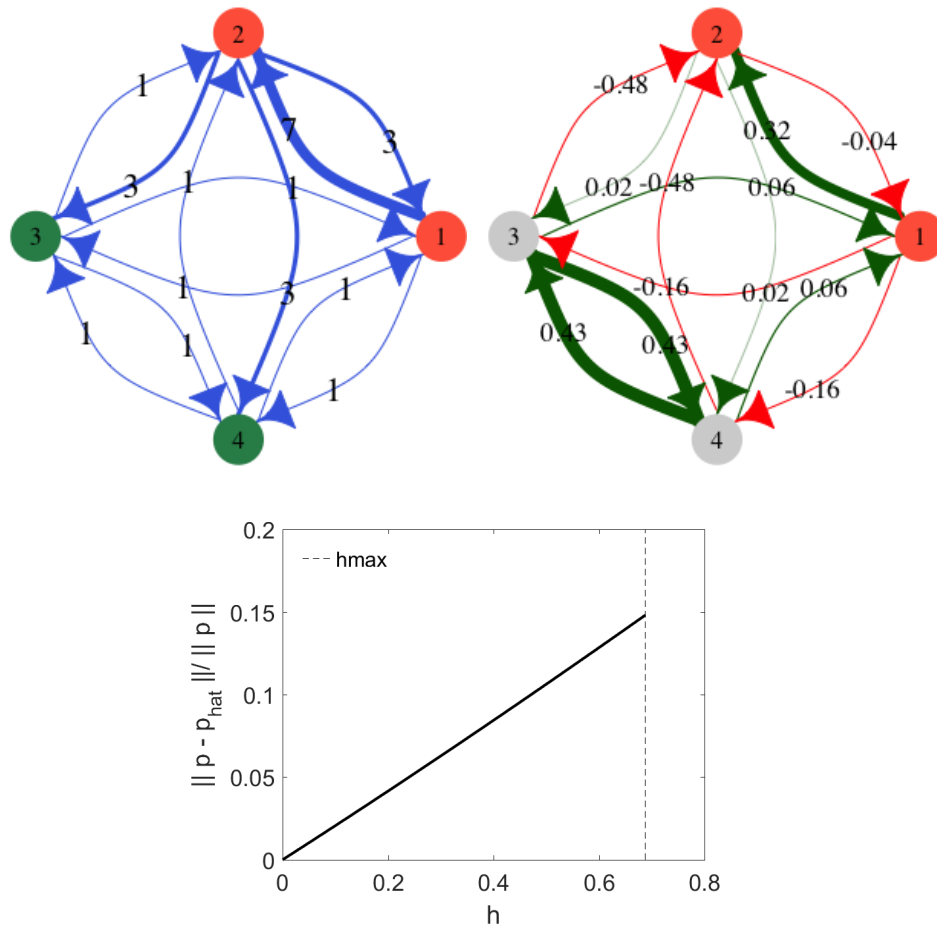


Figure 6.3.1: Top-left: Initial network. Top-right: Perturbation corresponding to the solution of the optimisation problem. Bottom: Relative error as a function of the perturbation size h .

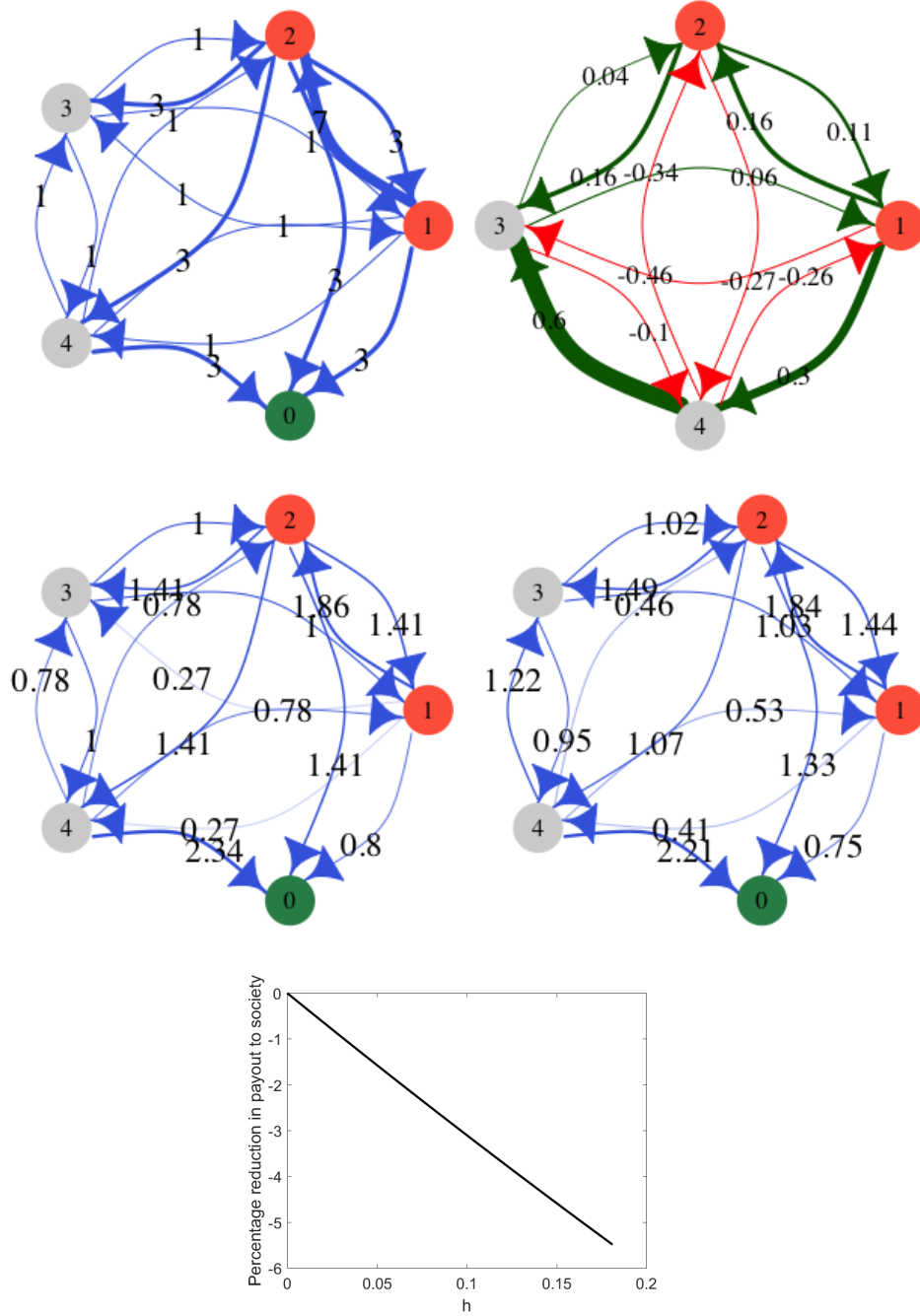


Figure 6.3.2: Top-left: Liabilities network. Top-right: Perturbation corresponding to the solution of the optimisation problem. Middle-left: Clearing payments under Π . Middle-right: Clearing payments under $\Pi + h_{\max}B$. Bottom: Percentage reduction to society for $h \in [0, h_{\max}]$.

6.3.3 DATA, BANK BALANCE SHEETS, INTERBANK LIABILITIES AND INITIAL STRESS.

BANK BALANCE SHEETS. With respect to this model's data requirements, the EBA dataset only provides information on some of the asset side quantities of Table 6.2.1. To be precise, information is only available on the total assets TA_i , the capital c_i and a proxy for interbank exposures, a_i^{IB} . To populate the remaining key variables of the Eisenberg Noe model, we therefore first assume, as in [Chen et al. \(2016\)](#), that for each bank the interbank liabilities are equal to the interbank assets. Furthermore, we assume that all non-interbank assets are external assets, and the non-interbank liabilities are liabilities to a society sink-node. Hence,

$$\begin{aligned} l_i^{IB} &:= a_i^{IB} \\ l_i^0 &:= TA_i - l_i^{IB} - c_i \\ a_i^0 &:= TA_i - a_i^{IB}, \end{aligned}$$

and the Eisenberg-Noe model variables are

$$\begin{aligned} \text{Total liabilities: } \bar{p}_i &= l_i^0 + l_i^{IB} \\ \text{Total external assets: } x_i &= a_i^0. \end{aligned}$$

Note that the net worth hence exactly corresponds to the book value of equity, or the banks' capitals: $TA_i - \bar{p}_i = a_i^0 + a_i^{IB} - l_i^{IB} - l_i^0 = c_i$.

GENERATING THE INTERBANK LIABILITY MATRIX. The final key ingredient to the model is the (relative) liabilities matrix. This is usually highly confidential data, and is not provided in the EBA data set. [Gandy and Vaart \(2016\)](#) propose an elegant Bayesian sampling methodology to generate individual interbank liabilities, given information on the total interbank lia-

bilities and total interbank assets of each bank. The authors have developed an *R*-package called “systemicrisk” that implements a Gibbs sampler to generate samples from this conditional distribution. As our analysis requires an initial liability matrix, we use the European Banking Authority (EBA) data as input to their code in order to generate such a liability matrix. As suggested by the authors, from the starting point of having assumed the total interbank assets and liabilities to be the equal for each bank, we perturb these slightly to fulfil the condition that L be connected along rows and columns Gandy and Veraart (2016). We then run the Gandy & Veraart algorithm, with parameters $p = 0.5$, $\text{thinning} = 10^4$, $n_{\text{burn-in}} = 10^9$, $\lambda = \frac{pn(n-1)}{\sum_{i=1}^N a_i^{IB}} \approx 1.217810^{-3}$, to create one realisation of a 87×87 network of banks from the data⁴. The estimated interbank network, as generated via the Gandy & Veraart algorithm, is depicted in Figure 6.3.3. Node sizes are proportional to balance sheet size, edge widths are proportional to the logarithm of the liabilities L_{ij} .

STRESS SCENARIO. For simplicity and to compare the magnitudes of direct and indirect contagion, we use Scenario 1 from Part I. We assume the initial shock size to be 15% (for small shock sizes there is no contagion in the Eisenberg & Noe model). This causes three initial defaults and one contagious default. In Figure 6.3.3, the banks that default initially are depicted by black squares, the contagious failure is depicted as a red square. For comparison, we also show the final failures as estimated in the fire sales model of Part I for the same scenario and initial shock size, with the maximal market depth multiplier that we considered. It is striking that the fire sales model estimates additional 13 failures due to insolvency and five failures due to illiquidity to occur. For smaller shock sizes (between 5% - 15%)

⁴We needed to exclude banks DE029, LU45 and SI058 because the mapping of the data to the model as described above created violations of the conditions for the algorithm and resulted in an error message.

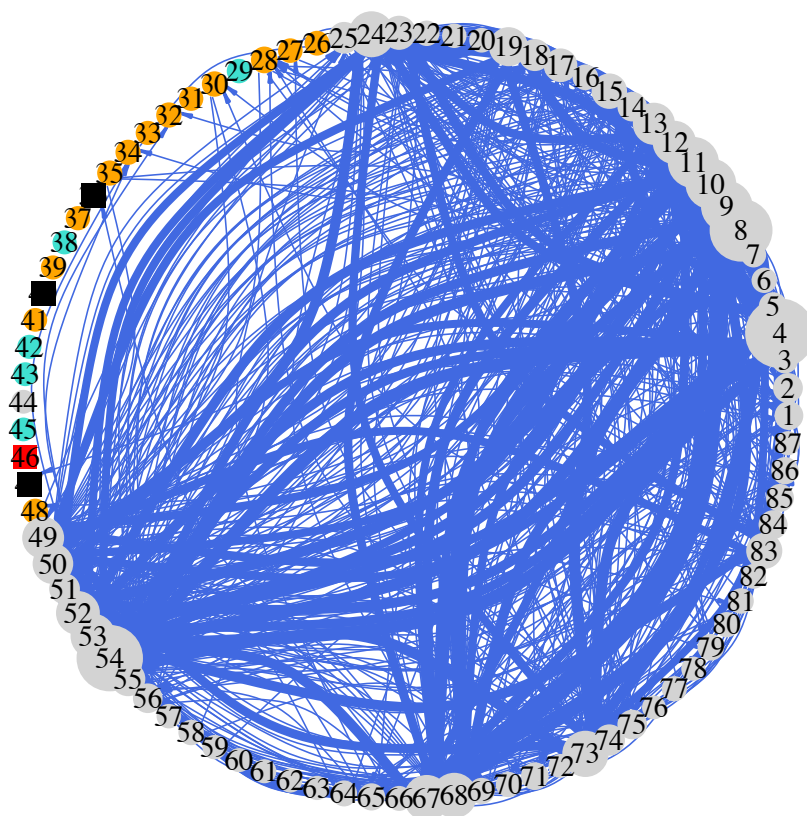


Figure 6.3.3: The EBA interbank network as estimated using the Gandy & Veraart algorithm. The black squares are the initial failures, the red square is the contagious failure. For comparison, we have included the insolvency defaults (orange circles) and illiquidity defaults (turquoise circles) as estimated in the fire sales model of Part I for the same stress scenario, initial shock size and largest market depth scaling factor.

the number of insolvency defaults decreases while the number of illiquidity defaults increases, leading to roughly the same number of additional total failures, see Figures 3.2.5 and 3.2.6.

6.3.4 CLEARING VECTOR DEVIATIONS

Figure 6.3.4 depicts the relative difference $\frac{\|p - \hat{p}\|}{\|p\|}$, where $\hat{p}$ is the clearing vector of $\hat{\Pi} = \Pi + hB$. The large difference in magnitude in the entries of the liability matrix from the EBA data compared to our previous toy example of four banks, restricts the maximal h to be very small. Over the entire range of possible h , the difference in the clearing vectors computed from Π and $\Pi + hB$ is negligible (10^{-13} !).

For this reason, we refrain from presenting the results on the shortfall to society for the EBA data.

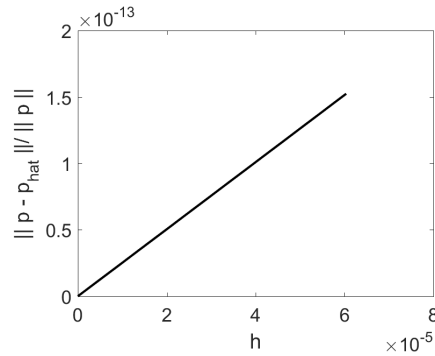


Figure 6.3.4: Relative error of the clearing vector as a function of the perturbation size h .

6.4 DISCUSSION

We conclude this Part by discussing our findings, as well as the Eisenberg & Noe model in a broader context.

ROBUSTNESS. Even though we saw in our simple four bank example that clearing vectors may change significantly (up to 15%), when there are estimation errors in the relative liabilities matrix, our analysis of the EBA data suggests that this is not necessarily the case for larger networks. Indeed, in this case the difference in magnitude (factor 10 - 20) between different banks and their interbank exposures, constrains the network quite heavily, and this also severely constrains the range over which we can conduct our sensitivity analysis, $\Pi + hB$.

A priori it is therefore not clear how sensitive the clearing vector estimate is to estimation errors of the relative liability matrix. An option for future research may be to “change” directional derivative, once $h_{\max}$ is reached, such that larger perturbations may be considered. A further alternative may be to relax the problem, and investigate “bad” directions for the directional derivative rather than the one we considered.

MAGNITUDE OF CONTAGION. From Figure 6.3.3, it sticks out that even with optimistic estimates of market depth, the indirect contagion via fire sales clearly dominates the direct contagion, as estimated via the Eisenberg & Noe model. Indeed, the number of contagious failures jumps from 1 to 19 in the same scenario!

This stark difference in contagion can be retraced to three important mechanisms. First, in the fire sales model banks react *before* they default, hence price-mediated contagion already occurs before any bank has gone bankrupt. Second, in the fire sales model banks can default both due to illiquidity and to insolvency. Finally, the “incoming liabilities” in the Eisenberg & Noe model “reduce” losses to some extent - we will comment on this in more detail in the next paragraph when we discuss the Eisenberg & Noe model in a broader context.

An interesting exercise for the future may be to compare the indirect contagion in the fire sales model to the direct contagion in a “default cascade” model as in [Cont et al. \(2013\)](#), [Amini et al. \(2016\)](#), or in other extensions of the classical Eisenberg & Noe model such as the extensions incorporating bankruptcy costs [Rogers and Veraart \(2013\)](#), or different maturities [Kusnetsov and Veraart \(2016\)](#).

EISENBERG & NOE MODELS OF SYSTEMIC RISK. It is worth noting that the original paper, [Eisenberg and Noe \(2001\)](#), studies systemic risk in a *payment system* or clearing network. In this setting, a bank defaults through illiquidity when it has insufficient cash to settle its total payment obligation with counterparties. The variables in the original framework are specifically given as the *exogenous operating cash flow*, x , the total payment obligation $\bar{p}$, and the relative liability matrix Π of incoming and out-going payments. In case of such a default, it is assumed that the bank makes pro-rata payments to all of its creditors, and that all payments are *pari-passu*.

Since then, the model has been used quite extensively in systemic risk models to model a different type of mechanism, default by insolvency. In this setting, the variable x is usually identified as “net external worth” of the bank, or sometimes the “capital”, and the liabilities matrix captures the net present value of all liabilities between institutions in the network. The work presented in this part of the thesis is equally applicable to the payment systems interpretation as the capital variant of the model. In general though, we believe that a deeper analysis on the actual applicability of the Eisenberg & Noe algorithm to “classical” systemic risk studies is warranted. The most important aspects that need to be addressed are balance sheet structure, maturity and seniority of claims:

BALANCE SHEETS. In the payment systems version, x is the net *cash* that institutions have at their disposal, this is an *asset* of a bank. In the systemic risk version, x often denotes *capital*⁵. This item is however located on the *liability* side of the balance sheet. From this point of view the identification of x as net external assets of the bank seems the most convincing to us.

TIMING AND THE NATURE OF LIABILITIES. In my view, the most important restrictive feature of the Eisenberg & Noe model for applications in systemic risk studies is the model's one period nature. While this is a natural assumption in a payment system, it certainly is not so when considering networks of general financial liabilities. On the one hand, during market turbulence, a large derivatives portfolio may face a sudden payment obligation in the form of a margin call.⁶ On the other hand, an issuer of zero-coupon bonds faces no payment obligation until maturity, which may lie several decades into the future. As discussed in the introduction and Part I of the thesis, the difference between insolvency and illiquidity is crucial for modes of failures of large banks. Citigroup is commonly said to have been effectively insolvent in November 2008. However, due to government support, and because financial markets were still willing to provide funding because they believed the bank would not be allowed to fail, Citi never defaulted on any payments.⁷

An important extension of the model in this direction has been recently made by Kusnetsov and Veraart (2016). Importantly, their contribution shows that by ignoring the maturity of liabilities systemic risk may be significantly

⁵There is no discussion of capital in the original setting. In fact, the only use of the word "capital" in the original paper occurs in the name of LTCM.

⁶As a matter of fact, AIG got distressed during the financial crisis because of large margin calls by its derivatives counterparties, Goldman Sachs and Société Générale; the difference between its collateral posted and collateral called peaked at 12.4 bn USD on September 15th 2008. See: <https://www.chicagofed.org/publications/working-papers/2014/wp-07>.

⁷https://en.wikipedia.org/wiki/Citigroup#November_2008.2C_Collapse_.26_US_Government_Intervention_.28part_of_the_Global_Financial_Crisis.29

underestimated: If A owes to B today, B owes to C tomorrow and C owes to A the day after tomorrow, then if A is unable to pay today, it will default, weakening the position of B which may default as well, and in turn weaken the position of C , which also may end up defaulting. If however all maturities are collapsed into a single point of time, the payment that A receives from C in three days may be sufficient to “cover” its payment to B , in which case no defaults occur.

Furthermore, how a default of Bank A will affect Bank B and B ’s liabilities towards Bank C depend on the type of liability: If Bank B holds bonds of Bank A , a default of Bank A will reduce the value of these assets of Bank B to the bonds’ recovery value if any. This affects the solvency position of Bank B , but if these bonds were zero coupon bonds, say, with a remaining maturity of more than a year, the impact on Bank B ’s liquidity position will be negligible. On the other hand, if B was expecting a large variation margin payment on derivatives, which itself will need to pass on to C in a very short interval of time, not receiving the cash due, may have a severe impact on B ’s liquidity position and its ability to perform on its contractual obligations towards C . Adding to the point we made on balance sheets above, this again highlights the importance of being specific and differentiating liquidity and solvency channels for contagion analysis.

Finally, another important feature that is usually not incorporated in Eisenberg & Noe models of bank contagion are netting agreements. When comparing the top-left and middle-left panels of Figure 6.3.2, we see that initially both parties have a liability of 1 to each other. After the default of Banks 1 and 2, bank 3 still pays 1 unit to Bank 4, but Bank 4 only returns 0.78. This follows the legal framework that non-defaulting banks need to fulfil their obligations to defaulting banks, but the defaulting banks repay their creditors pro-rata pari passu. However, in case of derivatives contracts for

instance, it is standard market practice to use “ISDA Master Agreements”⁸, which enclose sections on “close-out netting”. Concretely, if Banks 3 and 4 had signed such a master agreement, and these exposures in our example were all covered by the agreement, the close-out netting would result in a net 0 exposure, and no bank would pay the other anything. In this case, Bank 3 would not lose the 0.22 units of money, and Bank 4 would not “obtain” them for pro-rata distribution among all other creditors, thereby diminishing their recovery values.

INTERACTION WITH OTHER MODELS. The final point I wish to make is on the interaction with other models of systemic risk.

Currently, the academic literature on systemic risk offers an array of models that focus on different channels of contagion: direct exposures, fire sales, funding squeezes through liquidity hoarding... In reality, during a crisis all of these channels will be at work simultaneously and interact. In order to move closer towards a genuine systemic stress testing framework it will be important to develop models in a modular fashion such that they can be combined in a consistent manner.

For instance, to allow a consistent combination of models, it is important that the timing $t = 0, 1, 2..T$ is consistent between modules. Agent Based Models as in Bookstaber et al. (2014) present a natural candidate for combining different institutions, asset markets, contagion channels and constraints/optimising behaviours in a unified framework.

⁸See: https://en.wikipedia.org/wiki/ISDA_Master_Agreement#Close_out_and_netting.

Bibliography

- Jorge Abad, Iaki Aldasoro, Christoph Aymanns, Marco D’Errico, Linda Fache Rousov, Peter Hoffmann, Sam Langfield, Martin Neychev, and Tarik Roukny. Shedding light on dark markets: First insights from the new EU-wide OTC derivatives dataset. Technical report, 2016. URL https://www.esrb.europa.eu/pub/pdf/occasional/20160922_occasional_paper_11.en.pdf.
- Daron Acemoglu, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. Systemic risk and stability in financial networks. *American Economic Review*, 105(2):564–608, February 2015. URL <https://doi.org/10.1257/aer.20130456>.
- Viral V. Acharya and T. Sabri Öncüb. A proposal for the resolution of systemically important assets and liabilities: The case of the repo market. *International Journal of Central Banking*, 9(1):291–350, 2013. URL <http://www.ijcb.org/journal/ijcb13q0a14.pdf>.
- Viral V. Acharya, Lasse H. Pedersen, Thomas Philippon, and Matthew Richardson. Measuring systemic risk. *Review of Financial Studies*, 30(1):2–47, 2017. URL <https://doi.org/10.1093/rfs/hhw088>.
- J. Aczel and J. Dhombres. *Functional Equations in Several Variables*. Cambridge University Press, 1989. ISBN 9781139086578. URL <https://doi.org/10.1017/CB09781139086578>. Cambridge Books Online.
- Tobias Adrian and Markus K. Brunnermeier. CoVaR. *American Economic Review*, 106(7):1705–41, July 2016. URL <https://doi.org/10.1257/aer.20120555>.
- Tobias Adrian and Hyun Song Shin. Liquidity and leverage. *Journal of Financial Intermediation*, 19(3):418–437, 2010. URL <https://doi.org/10.1016/j.jfi.2008.12.002>.
- Tobias Adrian and Hyun Song Shin. Procyclical leverage and value-at-risk. *Review of Financial Studies*, 27(2):373–403, 2014. URL <https://doi.org/10.1093/rfs/hht068>.

- Andrea Aguiar, Richard Bookstaber, and Thomas Wipf. A map of funding durability and risk. Technical report, 2014. URL https://www.financialresearch.gov/working-papers/files/OFRwp2014-03_AguiarBookstaberWipf_MapofFundingDurabilityandRisk.pdf.
- Andrea Aguiar, Dror Y. Kenett, Richard Bookstaber, and Thomas Wipf. A map of collateral uses and flows. *OFR Working Paper*, 2016. URL https://www.financialresearch.gov/working-papers/files/OFRwp-2016-06_Map-of-Collateral-Uses.pdf.
- Robleh Ali, Nick Vause, and Filip Zikes. Systemic risk in derivatives markets: a pilot study using cds data. *Bank of England Financial Stability Paper*, 38, 2016. URL http://www.bankofengland.co.uk/financialstability/Documents/fpc/fspapers/fs_paper38.pdf.
- Franklin Allen and Elena Carletti. Mark-to-market accounting and liquidity pricing. *Journal of Accounting and Economics*, 45(2–3):358–378, 8 2008. URL <https://doi.org/10.1016/j.jacceco.2007.02.005>.
- Robert Almgren and Neil Chriss. Optimal execution of portfolio transactions. *Journal of Risk*, 3:5–39, 2000. URL <https://doi.org/10.21314/JOR.2001.041>.
- Yakov Amihud. Illiquidity and stock returns: cross-section and time-series effects. *Journal of Financial Markets*, 5(1):31 – 56, 2002. URL [https://doi.org/10.1016/S1386-4181\(01\)00024-6](https://doi.org/10.1016/S1386-4181(01)00024-6).
- Hamed Amini, Rama Cont, and Andreea Minca. Resilience to contagion in financial networks. *Mathematical Finance*, 26(2):329–365, 2016. URL <https://doi.org/10.1111/mafi.12051>.
- Le Thi Hoai An and Pham Dinh Tao. The dc (difference of convex functions) programming and dca revisited with dc models of real world nonconvex optimization problems. *Annals of Operations Research*, 133(1):23–46, 2005. URL <https://doi.org/10.1007/s10479-004-5022-1>.
- K. Anand, G. Bédard-Pagé, and V. Traclet. Stress testing the Canadian banking system: A system-wide approach. *Bank of Canada Financial Stability Review*, 2014. URL <http://www.bankofcanada.ca/wp-content/uploads/2014/06/fsr-june2014-anand.pdf>.
- Kartik Anand. The missing links - a global study on uncovering financial network structure from partial data. *Presentation slides*, 2015.

- Kartik Anand, Ben Craig, and Goetz von Peter. Filling in the blanks: network structure and interbank contagion. *Quantitative Finance*, 15(4):625–636, 2015. URL <https://doi.org/10.1080/14697688.2014.968195>.
- Ronald W. Anderson. Stress testing and macroprudential regulation: A transatlantic assessment. *Systemic Risk Center, Financial Markets Group & CEPR Press*, 2016. URL http://voxeu.org/sites/default/files/Stress_testing_eBook.pdf.
- Andrew Ang, Joseph Chen, and Yuhang Xing. Downside risk. *Review of Financial Studies*, 19(4):1191–1239, 2006. URL <https://doi.org/10.1093/rfs/hhj035>.
- Yacine At-Sahalia, Roger J.A. Laeven, and Liorana Pelizzon. Mutual excitation in eurozone sovereign {CDS}. *Journal of Econometrics*, 183(2):151 – 167, 2014. URL <https://doi.org/10.1016/j.jeconom.2014.05.006>. Analysis of Financial Data.
- Yacine At-Sahalia, Julio Cacho-Diaz, and Roger J.A. Laeven. Modeling financial contagion using mutually exciting jump processes. *Journal of Financial Economics*, 117(3):585 – 606, 2015. URL <https://doi.org/10.1016/j.jfineco.2015.03.002>.
- Kerstin Awiszus and Stefan Weber. The joint impact of bankruptcy costs, fire sales and cross-holdings on systemic risk in financial networks. *Working Paper*, 2015. URL <http://www.stochastik.uni-hannover.de/fileadmin/institut/pdf/JointImpactPaper.pdf>.
- Christoph Aymanns, Fabio Caccioli, J. Doyne Farmer, and Vincent W.C. Tan. Taming the basel leverage cycle. *Journal of Financial Stability*, 27: 263 – 277, 2016. URL <https://doi.org/10.1016/j.jfs.2016.02.004>.
- Jack Bao, Jun Pan, and Jiang Wang. The illiquidity of corporate bonds. *The Journal of Finance*, 66(3):911–946, 2011. URL <https://doi.org/10.1111/j.1540-6261.2011.01655.x>.
- Basel Committee on Banking Supervision. Basel III: A global regulatory framework for more resilient banks and banking systems. Technical report, 2011. URL <http://www.bis.org/publ/bcbs189.pdf>.
- Basel Committee on Banking Supervision. Global systemically important banks: updated assessment methodology and the higher loss absorbency requirement. *BIS*, 2013a. URL <http://www.bis.org/publ/bcbs255.pdf>.

- Basel Committee on Banking Supervision. Basel III: The liquidity coverage ratio and liquidity risk monitoring tools. Technical report, 2013b. URL <http://www.bis.org/publ/bcbs238.pdf>.
- Basel Committee on Banking Supervision. Basel III: leverage ratio framework and disclosure requirements. Technical report, 2014a. URL <http://www.bis.org/publ/bcbs270.pdf>.
- Basel Committee on Banking Supervision. Basel III: the net stable funding ratio. Technical report, 2014b. URL <http://www.bis.org/bcbs/publ/d295.pdf>.
- Basel Committee on Banking Supervision. Making supervisory stress tests more macroprudential: Considering liquidity and solvency interactions and systemic risk. *BIS Working Paper*, 2015. URL <http://www.bis.org/bcbs/publ/wp29.pdf>.
- Basel Committee on Banking Supervision. Standards: Minimum capital requirements for market risk. Technical report, 2016. URL <http://www.bis.org/bcbs/publ/d352.pdf>.
- Stefano Battiston, Michelangelo Puliga, Rahul Kaushik, Paolo Tasca, and Guido Caldarelli. Debtrank: Too central to fail? financial networks, the Fed and systemic risk. *Scientific reports*, 2, 2012. URL <https://doi.org/10.1038/srep00541>.
- Nicholas Beale, David G. Rand, Heather Battey, Karen Croxson, Robert M. May, and Martin A. Nowak. Individual versus systemic risk and the regulator’s dilemma. *Proceedings of the National Academy of Sciences*, 108(31):12647–12652, 2011. URL <https://doi.org/10.1073/pnas.1105882108>.
- Morten L. Bech and Enghin Atalay. The topology of the federal funds market. *Physica A: Statistical Mechanics and its Applications*, 389(22): 5223 – 5246, 2010. URL <https://doi.org/10.1016/j.physa.2010.05.058>.
- Brian Begalle, Antoine Martin, James McAndrews, and Susan McLaughlin. The risk of fire sales in the tri-party repo market. *Contemporary Economic Policy*, 34(3):513–530, 2016. URL <https://doi.org/10.1111/coep.12126>. COEP-Jul-2014-00072.R2.

- Carole Bernard and Claudia Czado. Conditional quantiles and tail dependence. *Journal of Multivariate Analysis*, 138:104 – 126, 2015. URL <http://dx.doi.org/10.1016/j.jmva.2015.01.011>.
- Dimitris Bertsimas and Andrew W. Lo. Optimal control of execution costs. *Journal of Financial Markets*, 1(1):1–50, 1998. URL <https://doi.org/10.1016/j.jfineco.2011.03.020>.
- Dimitrios Bisias, Mark Flood, Andrew W. Lo, and Stavros Valavanis. A survey of systemic risk analytics. *Annual Review of Financial Economics*, 4(1):255–296, 2012. URL <https://doi.org/10.1146/annurev-financial-110311-101754>.
- Alan S. Blinder. *After The Music Stopped: The Financial Crisis, the Response, and the Work Ahead*. The Penguin Press, 2013.
- Richard Bookstaber and Mark Paddrik. An agent-based model for crisis liquidity dynamics. *OFR Working Paper*, 2015. URL https://www.financialresearch.gov/working-papers/files/OFRwp-2015-18_Agent-based-Model-for-Crisis-Liquidity-Dynamics.pdf.
- Richard Bookstaber, Michael D. Foley, and Brian F. Tivnan. Market liquidity and heterogeneity in the investor decision cycle. *OFR Working Paper*, 2015. URL https://www.financialresearch.gov/working-papers/files/OFRwp-2015-03_Market-Liquidity-and-Heterogeneity-in-Investor-Decision-Cycle.pdf.
- Rick Bookstaber, Mark Paddrik, and Brian Tivnan. An agent-based model for financial vulnerability. *Office for Financial Research Working Paper*, 2014. URL https://www.financialresearch.gov/working-papers/files/OFRwp2014-05_BookstaberPaddrikTivnan_Agent-basedModelforFinancialVulnerability_revised.pdf.
- Michael Boss, Helmut Elsinger, Martin Summer, and Stefan Thurner. Network topology of the interbank market. *Quantitative Finance*, 4(6):677–684, 2004. URL <https://doi.org/10.1080/14697680400020325>.
- Jean-Philippe Bouchaud. Price impact. *Encyclopedia of Quantitative Finance*, 2010. URL <https://doi.org/10.1002/9780470061602.eqf18006>.

- Yann Braouezec and Lakshithe Wagalath. Risk-based capital requirements and optimal liquidation in a stress scenario. *Review of Finance*, to appear, 2017. URL <https://ssrn.com/abstract=2781964>.
- Anton Braverman and Andreea Minca. Networks of common asset holdings: Aggregation and measures of vulnerability. *Statistics and Risk Modeling, Forthcoming*. URL www.ssrn.com/abstract=2379669.
- Christian Brownlees and Robert F. Engle. Srisk: A conditional capital shortfall measure of systemic risk. *Review of Financial Studies*, 2016. URL <https://doi.org/10.1093/rfs/hhw060>.
- Markus K. Brunnermeier. Deciphering the liquidity crunch 2007 - 2008. *Journal of Economic Perspectives*, 23(1):77–100, 2008. URL <https://doi.org/10.1257/jep.23.1.77>.
- Markus K. Brunnermeier and Lasse Heje Pedersen. Market liquidity and funding liquidity. *Review of Financial Studies*, 22(6):2201–2238, 2009. URL <https://doi.org/10.1093/rfs/hhn098>.
- Ricardo J. Caballero and Alp Simsek. Fire sales in a model of complexity. *The Journal of Finance*, 68(6):2549–2587, 2013. ISSN 1540-6261. doi: 10.1111/jofi.12087. URL <http://dx.doi.org/10.1111/jofi.12087>.
- Fabio Caccioli, Munik Shrestha, Cristopher Moore, and J. Doyne Farmer. Stability analysis of financial contagion due to overlapping portfolios. *Journal of Banking and Finance*, 46:233 – 245, 2014. URL <https://doi.org/10.1016/j.jedc.2014.09.041>.
- Fabio Caccioli, J. Doyne Farmer, Nick Foti, and Daniel Rockmore. Overlapping portfolios, contagion, and financial stability. *Journal of Economic Dynamics and Control*, 51(0):50 – 63, 2015. URL <https://doi.org/10.1016/j.jedc.2014.09.041>.
- Susanna Calimani, Grzegorz Halaj, and Dawid Zochowski. Simulating fire-sales in banking and shadow banking system. *Mimeo*, 2016.
- Lorenzo Cappiello and Dominik Spuera. Fire-sale externalities in the Euro area banking sector. *European Central Bank Financial Stability Review*, pages 147–158, 2014. URL <https://www.ecb.europa.eu/pub/pdf/other/financialstabilityreview201411.en.pdf>.
- Lorenzo Cappiello, Linda Fache Rousova, and Mattia Montagna. Systemic risk, contagion and financial networks. *European Central Bank Special*

- Feature*, pages 147–158, 2015. URL <https://www.ecb.europa.eu/pub/fsr/shared/pdf/sfcfinancialstabilityreview201511.en.pdf>.
- Agostino Capponi and Martin Larsson. Price contagion through balance sheet linkages *. *Review of Asset Pricing Studies*, 2015. URL <https://doi.org/10.1093/rapstu/rav006>.
- Rene Carmona, Jean-Pierre Fouque, and Li-Hsien Sun. Mean field games and systemic risk. *Communications in Mathematical Sciences*, (4): 911 – 933, 2015. URL <http://www.pstat.ucsb.edu/faculty/fouque/PubliFM/Carmona-Fouque-Sun-George70-revised-1-28-14.pdf>.
- Chen Chen, Garud Iyengar, and Ciamac C Moallemi. An axiomatic approach to systemic risk. *Management Science*, 59(6):1373–1388, 2013. URL <https://doi.org/10.1287/mnsc.1120.1631>.
- Chen Chen, Garud Iyengar, and Ciamac C. Moallemi. Asset-based contagion models for systemic risk. *Working Paper*, 2014. URL <https://moallemi.com/ciamac/papers/asset-contagion-2014.pdf>.
- Nan Chen, Xin Liu, and David D. Yao. An optimization view of financial systemic risk modeling: Network effect and market liquidity effect. *Operations Research*, 64(5):1089–1108, 2016. URL <https://doi.org/10.1287/opre.2016.1497>.
- Shirish Chinchalkar and Roger M. Stein. Comparing loan-level and pool-level mortgage portfolio analysis. *Moody’s White Paper*, 2010. URL http://www.rogermstein.com/wp-content/uploads/Comparing_loan-level_and_pool-level_mortgage_portfolio_White_Paper.pdf.
- Rodrigo Cifuentes, Gianluigi Ferrucci, and Hyun Song Shin. Liquidity risk and contagion. *Journal of the European Economic Association*, 3(2-3): 556–566, 2005. URL <https://doi.org/10.1162/jeea.2005.3.2-3.556>.
- Rama Cont and Lakshithe Wagalath. Running for the exit: Distressed selling and endogenous correlation in financial markets. *Mathematical Finance*, 23:718–741, 2013. URL <https://doi.org/10.1111/j.1467-9965.2011.00510.x>.
- Rama Cont and Lakshithe Wagalath. Fire sales forensics: Measuring endogenous risk. *Mathematical Finance*, 26:835–866, 2016. URL <https://doi.org/10.1111/mafi.12071>.

- Rama Cont, Amal Moussa, and Edson Bastos e Santos. Network structure and systemic risk in banking systems. *Handbook of Systemic Risk*, pages 327 – 367, 2013. URL <https://doi.org/10.1017/CB09781139151184.018>.
- Rama Cont, Arseniy Kukanov, and Sasha Stoikov. The price impact of order book events. *Journal of Financial Econometrics*, 12:47–88, 2014. URL <https://doi.org/10.1093/jjfinec/nbt003>.
- Joshua Coval and Erik Stafford. Asset fire sales (and purchases) in equity markets. *Journal of Financial Economics*, 86(2):479 – 512, 2007. URL <https://doi.org/10.1016/j.jfineco.2006.09.007>.
- Credit Suisse. Credit Suisse annual report. 2015. URL <https://www.credit-suisse.com/media/assets/corporate/docs/about-us/investor-relations/financial-disclosures/financial-reports/csg-ar-2015-en.pdf>.
- J. Danielsson, P. Embrechts, C. Goodhart, C. Keating, F. Muennich, O. Renault, and H.S. Shin. An academic response to Basel II. *FMG*, 2001. URL <http://www.bis.org/bcbs/ca/fmg.pdf>.
- Jon Danielsson, Hyun-Song Shin, and Jean-Pierre Zigrand. The impact of risk regulation on price dynamics. *Journal of Banking & Finance*, 28(5): 1069–1087, 5 2004. URL [https://doi.org/10.1016/S0378-4266\(03\)00113-4](https://doi.org/10.1016/S0378-4266(03)00113-4).
- Jon Danielsson, Hyun S. Shin, and Jean-Pierre Zigrand. Procyclical leverage and endogenous risk. *Working Paper*, 2012. URL http://www.princeton.edu/~hsshin/www/Procyclical_Leverage_Endogenous_Risk.pdf.
- Domenico Di Gangi, Fabrizio Lillo, and Davide Pirino. Assessing systemic risk due to fire sales spillover through maximum entropy network reconstruction. *Working Paper*, 2015. URL http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2639178.
- Dodd-Frank. Dodd-frank wall street reform and consumer protection act. Technical report, 2010. URL <https://www.congress.gov/111/plaws/publ203/PLAW-111publ203.pdf>.
- Fernando Duarte and Thomas M. Eisenbach. Fire sale spillovers and systemic risk. *Federal Reserve Bank of New York Staff Report*,

- 645, 2013. URL https://www.newyorkfed.org/medialibrary/media/research/staff_reports/sr645.pdf.
- Darrell Duffie. Presidential address: Asset price dynamics with slow-moving capital. *The Journal of Finance*, 65(4):1237–1267, 2010a. URL <https://doi.org/10.1111/j.1540-6261.2010.01569.x>.
- Darrell Duffie. *How Big Banks Fail and What to Do About It?* Princeton University Press, 2010b.
- Darrell Duffie, Martin Scheicher, and Guillaume Vuillemeys. Central clearing and collateral demand. *ECB Working Paper Series*, 1638, 2014. URL <https://www.ecb.europa.eu/pub/pdf/scpwps/ecbwp1638.pdf>.
- Larry Eisenberg and Thomas H. Noe. Systemic risk in financial systems. *Management Science*, 47(2):236–249, 2001. URL <https://doi.org/10.1287/mnsc.47.2.236.9835>.
- M. Elliott, B. Golub, and M. O. Jackson. Financial networks and contagion. *American Economic Review*, 104(10):3115–53, 2014. URL <https://doi.org/10.1257/aer.104.10.3115>.
- Andrew Ellul, Chotibhak Jotikasthira, and Christian T. Lundblad. Regulatory pressure and fire sales in the corporate bond market. *Journal of Financial Economics*, 101(3):596 – 620, 2011. URL <https://doi.org/10.1016/j.jfineco.2011.03.020>.
- Andrew Ellul, Chotibhak Jotikasthira, Christian T. Lundblad, and Yihui Wang. Mark-to-market accounting and systemic risk: evidence from the insurance industry. *Economic Policy*, 29(78):596 – 620, 2014. URL <https://doi.org/10.1111/1468-0327.12030297-341>.
- H. Elsinger, A. Lehar, and M. Summer. Network models and systemic risk assessment. *Handbook on Systemic Risk*, pages 287 – 305, 2013. URL <https://doi.org/10.1017/CB09781139151184.016>.
- EMIR. European market infrastructure regulation. Technical report, 2012. URL <http://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32012R0648&from=EN>.
- Zachary Feinstein. Financial contagion and asset liquidation strategies. 2015. URL <https://arxiv.org/pdf/1506.00937v2.pdf>. Working paper.

- Zachary Feinstein and Fatena El-Masri. The effects of leverage requirements and fire sales on financial contagion via asset liquidation strategies in financial networks. 2016. URL <https://arxiv.org/pdf/1507.01847v2.pdf>. Working paper.
- Zachary Feinstein, Birgit Rudloff, and Stefan Weber. Measures of systemic risk. *Working Paper*, 2015. URL <https://arxiv.org/pdf/1502.07961v5.pdf>.
- Financial Stability Board and International Monetary Fund. The financial crisis and information gaps - sixth progress report on the implementation of the G-20 data gaps initiative. 2015. URL <http://www.imf.org/external/np/g20/pdf/2015/6thprogressrep.pdf>.
- Mark D. Flood, John C. Liechty, and Thomas Piontek. Systemwide commonalities in market liquidity. *OFR Working Paper*, 2015. URL https://www.financialresearch.gov/working-papers/files/OFRwp-2015-11_Systemwide-Commonalities-in-Market-Liquidity.pdf.
- Hans Föllmer. Spatial risk measures and their local specification: The locally law-invariant case. *Statistics & Risk Modeling: Special Issue on Systemic Risk*, 31:79–101, 2014. URL https://www.math.hu-berlin.de/~foellmer/papers/Spatial_Risk_Measures.pdf.
- Jean-Pierre Fouque and Joseph A. Langsam, editors. *Handbook on Systemic Risk*. Cambridge University Press, 2013. ISBN 9781139151184. URL <https://doi.org/10.1017/CB09781139151184>. Cambridge Books Online.
- Kenneth French, Martin Baily, John Campbell, John Cochrane, Douglas Diamond, Darrell Duffie, Anil Kashyap, Frederic Mishkin, Raghuram Rajan, David Scharfstein, Robert Shiller, Hyun Song Shin, Matthew Slaughter, Jeremy Stein, and Ren Stulz. *The Squam Lake Report: Fixing the Financial System**. Princeton University Press, 2010. URL <http://press.princeton.edu/titles/9261.html>.
- Prasanna Gai, Andrew Haldane, and Sujit Kapadia. Complexity, concentration and contagion. *Journal of Monetary Economics*, 58(5):453–470, 2011. URL <https://doi.org/10.1016/j.jmoneco.2011.05.005>.
- Axel Gandy and Luitgard A. M. Veraart. A Bayesian methodology for systemic risk assessment in financial networks. *Management Science*, 2016. URL <http://papers.ssrn.com/abstract=2580869>.

- James E. Gentle. *Matrix Algebra*. Springer-Verlag New York, 2007. ISBN 978-0-387-70872-0.
- Mila Getmansky, Giulio Girardi, Kathleen Weiss Hanley, Stanislava Nikolova, and Loriana Pelizzon. Portfolio similarity and asset liquidation in the insurance industry. *SSRN Working Paper*, 2016. URL <https://ssrn.com/abstract=2749691>.
- Giulio Girardi and A Tolga Ergün. Systemic risk measurement: Multivariate GARCH estimation of CoVaR. *Journal of Banking & Finance*, 37(8): 3169–3180, 2013. URL <https://doi.org/10.1016/j.jbankfin.2013.02.027>.
- Paul Glasserman and H.P. Young. Contagion in financial networks. *Journal of Economic Literature*, to appear, 2016. URL <http://dx.doi.org/10.1257/jel.20151228>.
- Gary Gorton and Andrew Metrick. Securitized banking and the run on repo. *Journal of Financial Economics*, 104(3):425 – 451, 2012. URL <https://doi.org/10.1016/j.jfineco.2011.03.016>. Market Institutions, Financial Market Risks and Financial Crisis.
- Mark Granovetter. Threshold models of collective behavior. *American Journal of Sociology*, 83:489–515, 1978. URL <http://www.jstor.org/stable/2778111>.
- Robin Greenwood, Augustin Landier, and David Thesmar. Vulnerable banks. *Journal of Financial Economics*, 115(3):471 – 485, 2015. URL <https://doi.org/10.1016/j.jfineco.2014.11.006>.
- Laurent Grillet-Aubert, Jean-Baptiste Haquin, Clive Jackson, Neill Killeen, and Christian Weistroffer. Assessing shadow banking – non-bank financial intermediation in Europe. ESRB Occasional Paper 10, European Systemic Risk Board, July 2016. URL https://www.esrb.europa.eu/pub/pdf/occasional/20160727_occasional_paper_10.en.pdf.
- Weilong Guo, Andreea Minca, and Li Wang. The topology of overlapping portfolio networks. *Statistics and Risk Modeling*, 33(3-4):139–155, 2015. URL <https://doi.org/10.1515/strm-2015-0020>.
- Grzegorz Hałaj and Christoffer Kok. Assessing interbank contagion using simulated networks. *Computational Management Science*, 10(2):157–186, 2013. URL <https://doi.org/10.1007/s10287-013-0168-4>.

- Grzegorz Halaj and Christoffer Kok. Modelling the emergence of the interbank networks. *Quantitative Finance*, 15(4):653–671, 2015. URL <http://dx.doi.org/10.1080/14697688.2014.968357>.
- Galina Hale and John Krainer. Aggregation level in stress testing models. *Federal Reserve Bank of San Francisco Working Paper*, 2015. URL <http://www.frbsf.org/economic-research/files/wp2015-14.pdf>.
- Samuel G. Hanson, Anil K Kashyap, and Jeremy C. Stein. A macroprudential approach to financial regulation. *Journal of Economic Perspectives*, 25(1):3–28, 2011. URL <http://www.ingentaconnect.com/content/aea/jep/2011/00000025/00000001/art00001>.
- Martin F. Hellwig. Systemic Risk in the Financial Sector: An Analysis of the Subprime-Mortgage Financial Crisis. *De Economist*, 157(2):129–207, 2009. URL <https://doi.org/10.1007/s10645-009-9110-0>.
- J. Henry, C. Kok, A. Amzallag, P. Baudino, I. Cabral, M. Grodzicki, M. Gross, G. Halaj, M. Kolb, M. Leber, C. Pancaro, M. Sydow, A. Vouldis, M. Zimmermann, and D. Zochowski. A macro stress testing framework for assessing systemic risks in the banking sector. Occasional Paper Series 152, European Central Bank, 2013. URL <https://www.ecb.europa.eu/pub/pdf/scpops/ecbocp152.pdf?d6abeaccc88f250577caff6f51130ecd>.
- R. Horst and N. V. Thoai. Dc programming: Overview. *Journal of Optimization Theory and Applications*, 103(1):1–43, 1999. URL <https://doi.org/10.1023/A:1021765131316>.
- Anne-Caroline Hüser. Too interconnected to fail: A survey of the interbank networks literature. *SAFE Working Paper Series*, 2016. URL <https://ssrn.com/abstract=2577241>.
- Julian Idier, Gildas Lamé, and Jean-Stéphane Mésonnier. How useful is the Marginal Expected Shortfall for the measurement of systemic exposure? A practical assessment. *ECB Working Paper Series*, 1546, 2013. URL <https://www.ecb.europa.eu/pub/pdf/scpwps/ecbwp1546.pdf>.
- Chotibhak Jotikasthira, Christian T. Lundblad, and Tarun Ramadorai. Asset fire sales and purchases and the international transmission of funding shocks. *Journal of Finance*, 67(6):2015–2050, 2012. URL <https://doi.org/10.1111/j.1540-6261.2012.01780.x>.

- Chotibhak Jotikasthira, Christian T. Lundblad, and Yihui Wang. Is historical cost accounting a panacea? Market stress, incentive distortions, and gains trading. *Journal of Finance*, 70(6):2489–2538, 2015. URL <https://doi.org/10.1111/jofi.12357>.
- Anil Kashyap, Raghuram Rajan, and Jeremy C. Stein. Rethinking capital regulation. In *Maintaining Stability in a Changing Financial System*, pages 431–71. Federal Reserve Bank of Kansas City, 2008. URL <https://www.kansascityfed.org/~media/files/publicat/sympos/2008/kashyaprajanstein031209.pdf>.
- Amir E. Khandani and Andrew W. Lo. What happened to the quants in august 2007? evidence from factors and transactions data. *Journal of Financial Markets*, 14(1):1–46, 2011. URL <https://doi.org/10.1016/j.finmar.2010.07.005>.
- Mark Kritzman, Yuanzhen Li, Sébastien Page, and Roberto Rigobon. Principal components as a measure of systemic risk. *The Journal of Portfolio Management*, 37(4):112–126, 2011. URL <https://doi.org/10.3905/jpm.2011.37.4.112>.
- Michael Kusnetsov and Luitgard A. M. Veraart. Interbank clearing in financial networks with multiple maturities. *Working Paper*, 2016. URL <https://ssrn.com/abstract=2854733>.
- Albert Kyle and Anna Obizhaeva. Market microstructure invariance: Empirical hypotheses. *Econometrica*, 84:p.1345–1404, 2016a. URL <https://doi.org/10.3982/ECTA10486>.
- Albert Kyle and Anna Obizhaeva. Trading liquidity and funding liquidity in fixed income markets: Implications of market microstructure invariance. *Working paper*, 2016b. URL <https://www.frbatlanta.org/-/media/Documents/news/conferences/2016/0501-financial-markets-conference/papers/kyle-trading-liquidity-and-funding-liquidity.pdf>.
- Albert S. Kyle. Continuous auctions and insider trading. *Econometrica*, 53(6):pp. 1315–1335, 1985. URL <http://www.jstor.org/stable/1913210>.
- Albert S. Kyle and Wei Xiong. Contagion as a wealth effect. *The Journal of Finance*, 56(4):1401–1440, 2001. URL <https://doi.org/10.1111/0022-1082.00373>.

- S. Langfield, Z. Liu, and T. Ota. Mapping the UK interbank system. *Journal of Banking and Finance*, 45:288–303, 2014. URL <http://dx.doi.org/10.1016/j.jbankfin.2014.03.031>.
- Ming Liu and Jeremy Staum. Sensitivity analysis of the Eisenberg and Noe model of contagion. *Operations Research Letters*, 38(5):489–491, 2010. URL <https://doi.org/10.1016/j.orl.2010.07.007>.
- Francis Longstaff. The subprime credit crisis and contagion in financial markets. *Journal of Financial Economics*, 97(3):436–450, September 2010. URL <https://doi.org/10.1016/j.jfineco.2010.01.002>.
- G. Mainik and E. Schaanning. On dependence consistency of CoVaR and some other systemic risk measures. *Statistics and Risk Modeling*, 31:49–77, 2014. URL <https://doi.org/10.1515/strm-2013-1164>.
- Alberto Manconi, Massimo Massa, and Ayako Yasuda. The role of institutional investors in propagating the crisis of 2007–2008. *Journal of Financial Economics*, 104(3):491 – 518, 2012. URL <https://doi.org/10.1016/j.jfineco.2011.05.011>.
- Carl D. Meyer. *Matrix Analysis and Applied Linear Algebra*. SIAM, 2000. ISBN 978-0-898714-54-8.
- P.E. Mistrulli. Assessing financial contagion in the interbank market: Maximum entropy versus observed interbank lending patterns. *Journal of Banking and Finance*, 35:1114 – 1127, 2011. URL <https://doi.org/10.1016/j.jbankfin.2010.09.018>.
- Anna Obizhaeva. Liquidity estimates and selection bias. *Working paper*, 2012. URL <http://ssrn.com/abstract=1178722>.
- Lasse H. Pedersen. When Everyone Runs for the Exit. *The International Journal of Central Banking*, 5:177–199, 2009. URL <http://www.ijcb.org/journal/ijcb09q4a10.pdf>.
- Diane Pierret. Systemic risk and the solvency-liquidity nexus of banks. *International Journal of Central Banking*, 11(3):193–227, 2015. URL <http://www.ijcb.org/journal/ijcb15q3a5.pdf>.
- Xuemin Ren, George X. Yuan, and Lishang Jiang. The framework of systemic risk related to contagion, recovery rate and capital requirement in an interbank network. *Journal of Financial Engineering*, 01(01):1450004, 2014. URL <https://doi.org/10.1142/S2345768614500044>.

- WS Robinson. Ecological correlations and the behavior of individuals. *American Sociological Review*, 15(3), 1950. URL <http://www.jstor.org/stable/2087176>.
- L. C. G. Rogers and L. A. M. Veraart. Failure and rescue in an interbank network. *Management Science*, 59(4):882–898, 2013. URL <https://doi.org/10.1287/mnsc.1120.1569>.
- Hyun S. Shin. *Risk and Liquidity*. Oxford University Press, 2010.
- Andrei Shleifer and Robert Vishny. Fire sales in finance and macroeconomics. *Journal of Economic Perspectives*, 25(1):29–48, 2011. URL <https://doi.org/10.1257/jep.25.1.29>.
- Andrei Shleifer and Robert W. Vishny. Liquidation values and debt capacity: A market equilibrium approach. *The Journal of Finance*, 47(4):1343–1366, 1992. URL <https://doi.org/10.1111/j.1540-6261.1992.tb04661.x>.
- K. Soramäki, M. L. Bech, J. Arnold, R. J. Glass, and W. E. Beyeler. The topology of interbank payment flows. *Physica A: Statistical Mechanics and its Applications*, 2007. URL <https://doi.org/10.1016/j.physa.2006.11.093>.
- The Financial Crisis Inquiry Commission. Final report of the national commission on the causes of the financial and economic crisis in the united states. Technical report, 2011. URL http://fcic-static.law.stanford.edu/cdn_media/fcic-reports/fcic_final_report_full.pdf.
- Christian Upper. Simulation methods to assess the danger of contagion in interbank markets. *Journal of Financial Stability*, 7(3):111–125, 2011. URL <https://doi.org/10.1016/j.jfs.2010.12.001>.
- Anton R. Valukas. Report of Anton R. Valukas, Examiner. Technical report, 2010. URL <https://jenner.com/lehman>.
- Wolf Wagner. Systemic liquidation risk and the Diversity – Diversification trade-off. *The Journal of Finance*, 66(4):1141–1175, 2011. URL <https://doi.org/10.1111/j.1540-6261.2011.01666.x>.

*I don't like novels that end happily. They depress me
so much.*

Oscar Wilde



APPENDIX

A.1 SOURCES FOR MARKET DATA

US treasuries: <http://www.sifma.org/uploadedFiles/Research/Statistics/StatisticsFiles/SF-US-SF-Trading-Volume-SIFMA.xls>

US corporate bonds: <http://www.sifma.org/uploadedFiles/Research/Statistics/StatisticsFiles/Corporate-US-Corporate-Trading-Volume-SIFMA.xls>

DE treasuries: <http://www.deutsche-finanzagentur.de/en/institutional-investors/secondary-market/>

UK treasuries: <http://www.dmo.gov.uk/index.aspx?page=Gilts/Turnover>

ES treasuries: <http://www.tesoro.es/sites/default/files/estadisticas/18.pdf>

FR treasuries: http://www.aft.gouv.fr/rubriques/trading-volume_109.html

IT treasuries: http://www.dt.tesoro.it/export/sites/sitodt/modules/documenti_en/debito_pubblico/presentazioni_studi_relazioni/3_3_2000_13_43_The-Italian-Treasury-.pdf BE treasuries: http://www.debtagency.be/fr_products_olo_volume.htm

SE treasuries: http://www.riksbank.se/Documents/Rapporter/Finansmarknaden/2014/rap_finansm_140829_eng.pdf

PT treasuries: http://www.igcp.pt/fotos/editor2/2015/Estatisticas/12_Transacacies_medias_diarias_OT_e_BT_Dez15_1.pdf

GR treasuries: <http://www.bankofgreece.gr/Pages/en/Markets/HDAT/statistics.aspx>

A.2 ADDITIONAL TABLES

Balance sheet item	EBA dataset identifier
Assets	
<i>Illiquid assets Θ</i>	
Residential mortgage exposures ($\epsilon_\kappa \geq 0$)	33013
Commercial real estate exposures ($\epsilon_\kappa \geq 0$)	33018
Retail: Revolving exposures ($\epsilon_\kappa \equiv 0$)	33015
Retail: SME exposures ($\epsilon_\kappa \equiv 0$)	33016
Retail: other exposures ($\epsilon_\kappa \equiv 0$)	33017
Indirect sovereign exp. in the trading book ($\epsilon_\kappa \equiv 0$)	34017
Defaulted exposures ($\epsilon_\kappa \equiv 0$)	33020
Remaining exposures* ($\epsilon_\kappa \equiv 0$)	-
<i>Securities II</i>	
Institutional client exposures	33010
Corporate exposures	33011
Sovereign exposures	34013 , 34014 , 34015
Direct sovereign exposures in derivatives	34016
Liabilities	
Tier 1 capital	30014
Debt	-

Table A.2.1: Variables mapping: EBA data identifiers to model balance sheets.

THRESHOLD CALIBRATION: BASEL 3 LEVERAGE RATIO At the time the data was collected, banks were following Basel II guidelines, which corresponds to 8% ratio of capital to risk-weighted assets (RWA), and the Basel 3 leverage constraint was not yet in place. At the time, some banks had a significantly higher leverage than the cap of 33 to be introduced by Basel 3. In order to avoid spurious fire sales that would occur in absence of an exogenous shock, we therefore scale these banks' capital levels linearly to a buffer slightly below 33, as shown in Figure A.2.1. Similarly, Greenwood et al. (2015) cap the leverage at 30 in their analysis.

CORRECTING FOR DATA INCONSISTENCIES. The EBA data provides information on notional exposures of each bank to 148 asset classes.¹ Bank

¹For more details on the regulatory definition of "exposure" cf. the EBA methodological note:

https://www.eba.europa.eu/documents/10180/15932/EBA-ST-2011-004-Detailed-Methodological-Note_

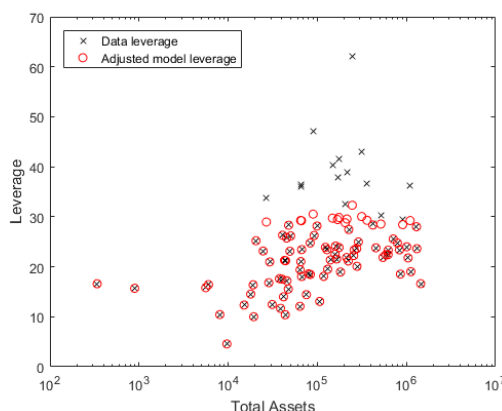


Figure A.2.1: Leverage of banks in the EBA dataset and adjusted leverage used as model input: By adding capital to the banks with leverage above 33, the leverage of these banks is brought slightly below 33.

BE005 records a zero value for total exposures despite positive exposures in the individual asset classes. As a proxy for this bank, we use information from the “Total assets after the effects of mandatory restructuring plans”², which is usually quite close to the values of “total exposures” across the dataset.

Another consistency issue is that the individual exposures in the dataset do not always sum up to the total exposure figure. Indeed, EBA explains in a footnote of the “total exposure” data that: “*Total exposures is the total EAD according to the CRD definition based on which the bank computes RWA for credit risk. Total exposures, in addition to the exposures broken down by regulatory portfolios in this table* [“this table” corresponds to the four asset classes in the portfolio II above], *include EAD for securitisation transactions, counterparty credit risk, sovereigns, guaranteed by sovereigns, public sector entities and central banks.*” (*emphasis added*). Due to this, the sum over the individual balance sheet items deviates in our model (both positively and negatively) from the “total exposures” information recorded in the dataset. In order to correct for this deviation and enforce that the sum of our individual balance sheet items is equal to the “total exposure” in the data, we add/subtract a “remaining exposures” term to the illiquid assets category. The average size of the negative correction terms is 5.9% of the corresponding balance sheet sizes. Double counting is thus a minor issue. The average size of the positive correction terms is

1.pdf as well as the identifier 33021 in the dataset.

²This information is recorded under identifier 30029 in the EBA dataset.

13.4%. This average is inflated by five small banks that are outliers and have correction terms above 50%. Excluding these outliers, the average of the correction terms reduces to 9%. The average size of the correction term over the entire data set is 8.9%. On average, we thus underestimate the balance sheet sizes of the banks. This may only bias results in the sense of underestimating the impact of fire sales.

An alternative approach to achieve a consistent matching of our model balance sheets with the “total exposure” data would have been to scale all assets on each balance sheet up / down by the adequate factor. However, we preferred adding/subtracting an amount to the “other illiquid assets” category, as they play no role in the fire sale cascade and we thereby avoid distorting individual exposures in the dataset by scaling them.