

MECHANICAL VIBRATIONS WITH COULOMB, VISCOUS AND
VISCOELASTIC DAMPING

By

HERBERT KWAKU BUKARI - D.I.C. (Accoust. & Vib.)

MAY 1972

A thesis submitted for the degree of Master of Philosophy
of the University of London and for the Diploma of Imperial
College.

UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE &
TECHNOLOGY -
CITY AND GUILDS COLLEGE
MECHANICAL ENGINEERING DEPARTMENT.

A C K N O W L E D G E M E N T S

First of all, I wish to express my sincerest thanks to my supervisor, Dr. P. Grootenhuys, for his advice and encouragement throughout the work on this thesis.

Secondly I wish heartily to thank our Laboratory Technician, Mr. Ray Gunn, for his help in the setting up of the experimental apparatus.

Thirdly I am indebted to Dr. C.F. Beards for his help through discussions during some difficult stages of this work.

Finally, I wish to thank Mr. J.E. Fleischer of the Ghana Universities Office, London, for his help in the typing and reproduction of this thesis.

	PAGE
Acknowledgement	(i)
Contents	(ii) to (viii)
ABSTRACT	1
The nature of coulomb force	3
Combination of different types of damping	4
<u>INTRODUCTION</u>	
VIBRATION AT LARGE	6
Mechanical vibrations	6
The effects of vibrations	7
Control of vibration levels	7
Shock and vibration control by damping	8
The nature of the damping force	9
List of symbols	11
CHAPTER 1	
1.1 THE NATURE OF DRY (COULOMB)FRICTION	12
1.2 Dry friction in a vibrating system	16
1.2.1 Accepted case	
1.3 Variation of the friction force with velocity in a vibrating system.	
1.4 Physical representation	23
1.5 Importance of the transition period	24
CHAPTER 2	
VIBRATIONAL COULOMB FORCE REGARDED AS A RETARDED FORCE.	26
2.1 An example in the use of d.d.e	28
2.2 Dry friction force considered as a retarded force.	29

	Page
2.3 Variation of friction force with frequency	30
CHAPTER 3	
ANALYSIS OF SOME SYSTEMS WITH COULOMB DAMPING	
3.1 Vibrating body in the presence of coulomb friction.	32
3.2 One degree of freedom system with dry friction, viscous and viscoelastic damping.	34
3.3 Special cases:	
(a) One degree of freedom system with viscoelastic and coulomb damping only.	35
3.4 One degree of freedom system having an external spring with coulomb and viscous damping.	36
3.5 One degree of freedom system having an external spring with coulomb damping only.	37
3.6 Variation of amplitude of vibrations with friction force for constant excitation force.	38
3.7 Variation of amplitude with friction force for constant excitation force and frequency with a spring in the system.	40
CHAPTER 4	
REVIEW OF PAST PAPERS	41
4.11 As Fourier series	42
4.2 Vibrating mass in the presenc of coulomb damping, without a spring.	43
4.2.1 Forced vibrations with coulomb damping.	44
4.3 The effect of coulomb damping on the resonant frequency.	45
4.4 The effect of dry friction on the phase difference.	46

<u>CHAPTER 5</u>	DESCRIPTION AND CALIBRATION OF THE APPARATUS	
5.1	The basic apparatus	48
5.1.1	The friction pad	50
5.1.2	The measuring Equipment	52
5.1.3	System paramaters	54
5.1.4	Theory of the electromagnetic damper.	57
5.2	Calibration	63
5.2.1	Calibration of the force measuring equipment.	63
5.2.2	Calibration of the accelerometer	66
5.2.3	Calibration of the displacement gauge	68
5.2.4	Static calibration of the friction block system.	72
<u>CHAPTER 6</u>	FORCED VIBRATIONS OF A BODY WITH COULOMB DAMPING.	
6.1	Introduction	76
6.2	Experimental work	77
6.2.1	Measuring of friction force at different frequencies.	77
6.2.2	Introduction of friction forces	78
6.2.3	Test of the friction blocks	84
6.2.4	Determination of parameters " α " and "h"	86
6.2.5	Curves for constant acceleration	88
6.2.6	Variation of friction force at constant acceleration.	92
6.3	Variation of coefficient of friction with frequency at constant amplitude.	93
6.3.1	Introduction of the displacement gauge.	93
6.3.2	Other data	95

	Page
6.4 Non-linear regression programme	95
6.4.1 Expansion of $\cos x$	96
6.5 Results	96
6.6 Variation of coefficient of friction with frequency at constant acceleration	108
6.7 Polar and Cartesian Plots.	112
6.8 Variation of amplitude with friction force at constant frequency	119
6.9 Introduction of a spring(restoring force) into the system.	122
6.10 Free Decay.	125
CHAPTER 7 FORCED VIBRATIONS WITH MIXED DAMPING	
7.1 Measuring Techniques	127
7.2 Stage one - with viscous damping only.	129
7.2.1 The Computer programme	129
7.2.2 Forced vibrations with coulomb and viscous damping.	135
7.2.3 Introduction of the viscous damping	136
7.2.4 Theoretical results	136
7.3 Stage two - with viscoelastic material	137
7.3.1 Viscoelastic and viscous damping.	146
7.3.2 Viesoelastic and coulomb damping.	151
7.4 Forced vibrations with the three types of damping.	157
CHAPTER 8 DISCUSSION OF RESULTS ON THE NATURE OF COULOMB FORCE.	162
8.1 Examination of coulomb force.	162
8.1.1 Relationship between h , C and R	163
8.1.2 Examination of the equation	166
8.1.3 The concept of optimum Coulomb damping ratio	167

8.1.4	With a spring in the system	168
8.2.	Friction force at constant amplitude of vibrations	169
8.2.1	Calculation of the delay time h from a given data	170
8.2.2	Determination of h from known values of η	171
8.2.3	Compatible order of magnitude	172
8.2.4	Examination of the friction force.	173
8.3	Vibrations induced by coulomb damping	173
8.4	Dependence of coefficient of friction on Amplitude.	174
CHAPTER 9	DISCUSSION ON VIBRATION WITH COMBINED DAMPING	176
9.1	Dissipation of Energy	177
9.1.1	In general	177
9.1.2	Viscoelastic damping	177
9.1.3	Coulomb damping	178
9.1.4	Comparision with normal presentation	178
9.2	The concept of equivalent viscous damping	178
9.2.1	Equivalent viscous damping for viscoelastic damping.	179
9.2.2	Equivalent viscous damping for coulomb damping.	179
9.3.	Experimental and theoretical comparison	180
9.3.1	Coulomb and viscous damping ratios	180
9.3.2	Comparison of equivalent damping coefficient C_{eq} .	181
9.3.3	Viscous and viscoelastic.	183
9.3.4	Viscoelastic and coulomb	
9.3.5	Combinations of all three types of damping	184
9.4	The effect of a combination of damping on the Resonant frequency.	186

9.4.1	The effect of combined coulomb and viscus damping on the resonant frequency	186
9.4.2	The effect of combined viscoelastic and viscous damping on f_r	187
9.4.2	Viscoelastic and coulomb damping	188
9.4.4	Combination of all three	188
9.5	The effect of a combination of damping on the phase.	189
9.5.1	Viscous and viscoelastic damping	189
9.5.2	Coulomb damping	189
9.5.3	Coulomb damping in the presence of viscous and or viscoelastic damping.	190
9.5.4	Cross points.	191
CHAPTER 10	CONCLUSION AND OBSERVATIONS	
10.1	Conclusions on the nature of coulomb force	192
10.2	Conclusions on vibration Response with combined Damping	194
10.2.1	The concept of equivalent viscous damping	194
10.2.2	Effect on the resonant frequency	194
10.2.3	The effect on the phase	195
REFERENCES		196
Appendix 1		
A.1	Determination of the static stiffness	203
A.2	Determination of the Dynamic Characteristics of Friction Block system.	207
A.3	Calculation of the Dynamic Stiffness	212
A.4	Theoretical Response Curves	213
A.5	Examination of the restoring forces ie the spring force.	214
A.6	Examination of the experimental curves	219

		Page
Appendix	2	
	Determination of delay time h from the measured phase angle with all 3 types of damping.	220
A.2.1	With coulomb and viscous damping only	220
A.2.2	With coulomb and viscoelastic damping only	221
A.2.3	Example with viscoelastic damping only.	222

ABSTRACT

The object of this thesis was primarily to determine the effect of a combination of different types of damping on a vibrating system experimentally for comparison with existing theoretical predictions.

In order to do this comparison effectively it was found necessary to examine individually the three different types of damping that are under consideration in this thesis (ie viscous, viscoelastic and coulomb damping).

Viscous damping has been examined thoroughly throughout the ages and found to produce theoretical results that are verifiable experimentally. Moreover because a one degree of freedom system with viscous damping results in a linear differential equation, viscous damping has become the standard upon which other types of damping are based.

Similarly viscoelastic damping has been examined by various researchers in different ways and there exists a vast literature on viscoelastic damping. The discrepancies inherent in the various theories evolved have been discussed and summarised, and different formulae suggested for free and forced vibrations. As in this thesis forced vibrations were envisaged and there are a lot of publications proving the validity of the theory and formula used for forced vibrations, it was deemed reasonable to accept the formula in use.

.....3.....

Coulomb damping however presented a problem, for the available literature is in some cases contradictory. Research Workers in lubrication have found evidence to support the fact that the static coefficient of friction is greater than the kinetic coefficient, and also that the kinetic coefficient decreases with increasing speed. Some researchers in mechanical vibrations have also observed a decrease in the kinetic coefficient of friction with frequency. Moreover it is pointed out that there is a possibility of an increase in the kinetic coefficient of friction with speed or frequency at higher speeds or frequencies.

A lot of the research work on coulomb damping have been based on the work of Den Hartog (5), who used a mean value for the kinetic coefficient of friction over a range of frequencies and so assumed that it was a constant. Assuming that the friction force is constant results in some contradictions, for in experimental work a varying phase difference exists between the excitation force and the response which cannot be satisfactorily explained. Further more some researchers have found evidence suggesting that the coefficient of friction is dependent upon the amplitude of vibrations. These contentions points and some others made it necessary that the nature of Coulomb force be investigated.

THE NATURE OF COULOMB FORCE

In this thesis a theory has been developed regarding Coulomb force as a retarded force, culminating in ~~the force~~ being represented by a complex force (ie a force having real and quadrature components).

.....

The dependence of coulomb force on frequency and amplitude have been established theoretically and experimentally. An attempt has also been made to explain self-excited vibrations induced by coulomb damping. The importance of the delay time was investigated and found to be dependent on the elastic stiffness in the system, the inertia of the moving body, the friction force and the nature of the surfaces in contact.

The concept of optimum damping has also been examined and explained, using this theory.

COMBINATION OF DIFFERENT TYPES OF DAMPING

The concept of equivalent viscous damping which is based on the theory of the energy dissipated per cycle has been examined and found to yield results that are verifiable experimentally. The normal expression used for coulomb damping was found to over estimate the equivalent viscous damping coefficient by a least 28% and therefore a new expression has been derived.

Viscoelastic damping was found to have no effect on the resonant frequency whilst viscous damping decreased the resonant frequency as expected. Coulomb damping however was found to be able to increase or decrease the resonant frequency depending on certain conditions.

An addition of viscoelastic damping and or viscous damping resulted in an advance of the phase before and a retardation after resonance. With coulomb damping however where there is a shift, in resonant frequency, a phase of 90° is obtained at the new resonant frequency.

When there is no noticeable change in resonant frequency an increment of coulomb damping produces the same effect as viscous or viscoelastic damping.

INTRODUCTION

Vibration at Large

The simplest natural vibrating systems have two attributes: inertia and stiffness about-an-equilibrium position. To a first approximation the force tending to restore the system to equilibrium is often proportional to the displacement and then simple harmonic motion occurs. Since all physical objects have inertia and do possess elasticity it is therefore understandable that we encounter vibrations in one form or another in our daily activities. Vibrations occur in acoustical, electrical, hydraulic, mechanical and many other physical systems. The study of vibrations therefore serves as a useful link between those different branches of physics and also between all the natural sciences. We would be unable to speak and hear without the vibrations of our larynges and eardrums. In fact, the beating of the heart and the oscillations of the lungs are forms of vibration; and chemical reactions would be impossible without the vibration of the molecules. Since vibration plays such an important role in all branches of science it is inevitable that the applied scientist encounters vibrations both as a friend and as a foe.

Mechanical Vibrations

In this thesis however we are concerned mainly with Mechanical Vibrations which involve the vibrations of

structures, for example, buildings and bridges; of machinery such as Lathes and air compressors and of vehicles as represented by aircrafts, trains and cars, etc. In general vibrations are inherent in all mechanical systems and it has become an established procedure through experience to perform theoretical analysis or experimental tests and sometimes both on these systems to ascertain whether the level of vibration is important or not. The criterion of acceptability may be based on the resonant frequency of the system, the fatigue limit of the structural members or the noise level of the system when in use.

The Effects of Vibrations

Whilst the phenomenon of mechanical vibrations has been utilised successfully in many mechanical devices such as washing machines, drills, mixers, massage machines, etc. the objectionable features of vibrations can be devastating. Locomotives, turbines, bridges, ships and aircrafts have bowed to the destructive power of vibration in the form of spectacular mechanical failures. The catastrophic collapse of the Tacoma Narrows Bridge in Washington State and the spectacular engine, failure of the ship, Queen Elizabeth II on its maiden voyage in 1969 are two well-publicised examples of the numerous victims of vibrations.

Control of Vibration Levels

To avoid these spectacular and sometimes fatal failures, and also to reduce the 'noise levels' of machines and vehicles

to alleviate human suffering, environmental engineers spend a lot of effort, time and money in an attempt to control vibrations. In, general, even in a well-designed machine, it is possible only to reduce rather than eliminate the dynamic forces that are responsible for shock and vibration. Then, even relatively small excitation forces can cause undesirable large responses should resonance or near resonance exist in a lightly damped system. A number of techniques are available for controlling the response: (1) proper control

of natural frequencies to prevent resonance with the excitation, (2) application of damping which is a mechanism by which energy is dissipated from a vibrating system. Damping is inherent in varying quantities in mechanical systems and thus constitutes a third attribute of vibrating systems, (3) use of isolators to reduce the transmission of the excitation forces from one part of the machine to another and (4) the addition of an auxilliary mass neutralizer to reduce the response.

These techniques are discussed in detail in many textbooks, see references (20) to (24). Therefore only the second method which is the main issue under consideration in this thesis will be discussed.

Shock and Vibration Control by Damping

Many methods have been used for introducing artificial damping into systems. It may be done electrically, for instance, though there are many purely mechanical devices available. A few of these are :

Viscous friction in a fluid: An example of this is a dashpot, in which a piston moves in a cylinder and the friction comes from the rush of fluid through the small gap between the side of the piston and the

cylinder wall.

- 2). High damping Materials: Examples are use of rubber in structural joints to increase damping, and fibre re-inforced blades in gas-turbine on account of their high internal damping.
- 3). Coating on Panels: Preparations are available for increasing the damping of metal panels.
- 4). Coulomb (dry) friction: In which surfaces are made to rub together during vibrations, examples are the use of friction in structural joints and frictional dampers. Also wads of knitted metal wire are placed in some springs to augment the friction.
- 5). Sandwich Construction: For example, panels made of thin metal sheets separated by a thin layer of visco-elastic material for reducing panel resonances; also used in engines to reduce noise.

The Nature of the Damping Forces

The examples that have just been cited illustrate the use of three types of damping. Method (1) involves Viscous damping, methods (2) (3) and (5) use visco-elastic damping and method (4) involves Coulomb damping.

Viscous damping force is known to be directly proportional to the velocity, especially at low speeds and because of the generality of its application and the ease in solving equations involving viscous damping, viscous damping has been given considerable attention in all textbooks on vibrations; see references (20) to (25).

Visco-elastic damping force is also known by other names such as hysteretic damping force and material damping force and is known to be proportional to the amplitude of vibrations but independent of the velocity. This has also been treated competently in the references cited above and the anomalies arising out of those books have been dealt with in reference (18).

Coulomb damping force has usually been assumed to be constant and independent of both speed and amplitude of vibrations and most textbooks on vibrations have relied upon this assumption in their presentation. Studies in lubrication have however exhibited a dependence of the Coefficient of Friction on speed and furthermore evidence can be produced from the experimental work of researchers in vibration work to support the dependence of friction force on frequency. As a result of this anomaly the earlier part of the work in this thesis is concerned with the examination of Coulomb damping forces on a simple mass system with and without a spring.

Further experiments involve combined effects:

- 1). Coulomb with Viscous,
- 2). Coulomb with Visco-elastic,
- 3). Viscous with Visco-elastic, and
- 4). Coulomb with Viscous and Visco-elastic.

LIST OF SYMBOLS

A = Accelerating force = mass x acceleration

a = Cross sectional area of conduction

α = Absorption coefficient

B = Magnetic flux density

β = Phase angle (w x h)

C = Constant

$\bar{C}$ = viscous damping coefficient

C = Constant (mass x (frequency w in rads.))

E = Example

η = Loss factor

e = e.m.f. induced in cylinder

F = Friction force

f = Frequency in Hz

G = Dynamic Magnification

g = Out of phase component of viscoelastic (elasticity) force.

γ = Coulomb damping ratio

H = Magnetic force

h = delay time

t = time in secs.

i = Current

u = velocity of damper cylinder

Magnetising current

v = sliding velocity

$j = \sqrt{-1}$

ω = frequency in rads/sec.

K = Constant

x = displacement

k = Stiffness

z = Complex parameter = $\alpha + i\omega$

L = Normal load (friction force)

M = Mass

η = frequency ratio

P = Excitation force

R = Friction force parameter

r = radius

$S = \frac{R}{k}$ - non-dimensional ratio

s = sliding speed

C H A P T E R 1

1.1. THE NATURE OF DRY (COULOMB) FRICTION

The two basic Laws of Friction have been known and accepted for a long time.

"Frictional resistance is proportional to the normal load and is independent of the area of contact". Leonardo da Vinci (1519) is credited with the earliest investigation of these Laws and Amontons (1699) re-interpreted them in more formal scientific terms (1). Later Coulomb (1785) verified the laws experimentally and made a clear distinction between Static and Kinetic friction, i.e. that the force required to maintain sliding was significantly lower than the force required to initiate sliding. Furthermore, he added a so-called third Law, that kinetic friction is nearly independent of the speed of sliding. As a result of Coulomb's contribution this type of dry friction is sometimes called "Coulomb" friction. Since then many theories supported by experimental work have been evolved to explain the mechanism of dry friction. The two main theories evolved are:

- (1) The "purely molecular theory" which maintains that the mechanism responsible for sliding friction is mainly due to the "shearing" of adhesive metal-to-metal junctions formed by the welding together of the tips of the surface irregularities where plastic deformation takes place under intense local stresses, giving rise to the frictional force. The deformation

leads to wear and to metal transfer between the surfaces.

- (2) The "Ploughing Theory" which advocates that the mechanism responsible for dry-friction is mainly due to the resistance encountered due to the high spots of one surface plastically ploughing through the other.

There is no absolute distinction between the two types even though they are fundamentally different if "shearing" is understood to mean a discontinuous process of breaking junctions whereas "ploughing" is understood to involve continuous applied pressure and continuous plastic flow whether or not a furrow be formed.

Bowden, Moore and Tabor have suggested that the two processes are jointly responsible for sliding friction. Sampson, Morgan, Reed and Muskat (2) found that this shearing-ploughing theory fitted in well with their experimental work and postulated that "shearing" occurs at the beginning of sliding and is responsible for the excess of static friction over kinetic friction, whereas "ploughing" contributes to both. The following qualitative explanation may then be made. Time is required for local adhesive junctions to form, probably because plastic deformation and rearrangement of surface molecules must first take place. After the junctions are sheared adhesion forces will be lower while the surface are in motion so that mainly ploughing takes place and the coefficient of friction drops to its kinetic value. Rabinowicz (3) emphasised the importance of the duration of the stationary contact of the surfaces, on which the strength of the metallic junctions

.....

are deemed to depend. The experimental work of references (2) and (3) show that the metallic junctions are weaker for the times of stationary contact of the order of milliseconds than for times of the order of seconds and that as sliding surfaces can be regarded as similar to surfaces in stationary contact for very short times (milliseconds) the coefficients of friction will be lower. It seems that the greater strength of junctions formed after long contact is due to a process analogous to creep; as a result of which the junctions increase in size with time, and the force to shear these larger junctions is greater than the force which would have been required initially.

The process of sliding is envisaged as follows: when the tangential load is applied to one of two surfaces in contact with another and gradually increased, at first, elastic deformation of the asperities and the substrate takes place and continues until the shear strength of the junctions is reached. Shearing of the junctions now takes place and the coefficient of friction falls off as the strong junctions which were formed during static loading are replaced by weaker ones. The influence of the strong junctions persists over a distance that is simply related to the average junction size.

With clean, like metals the junctions are very strong, and shearing of the junctions is accompanied by severe distortions of the surfaces. It seems plausible that, this distortion of the surfaces brings into contact parts of the surface which otherwise would have remained separate and thus by a "snow balling" mechanism, the true area of contact and therefore the resistance of the surfaces to motion become greater as sliding proceeds. This effect was noted in the case of copper by Ernst and Merchant (4).

Thus the value of the kinetic coefficient of friction depends upon the velocity, the bulk mechanical properties of the bodies involved, the plastic deformations and above all the past history of the motion.

The variation of the coefficient of friction with the speed of sliding may be explained using this theory. As the sliding speed is increased, the time of stationary contact decreases and as a result the strength of the junctions become weaker leading to lower values of the kinetic coefficient of friction.

Fig. 1.1 shows the variation of the coefficient of friction with sliding speed.

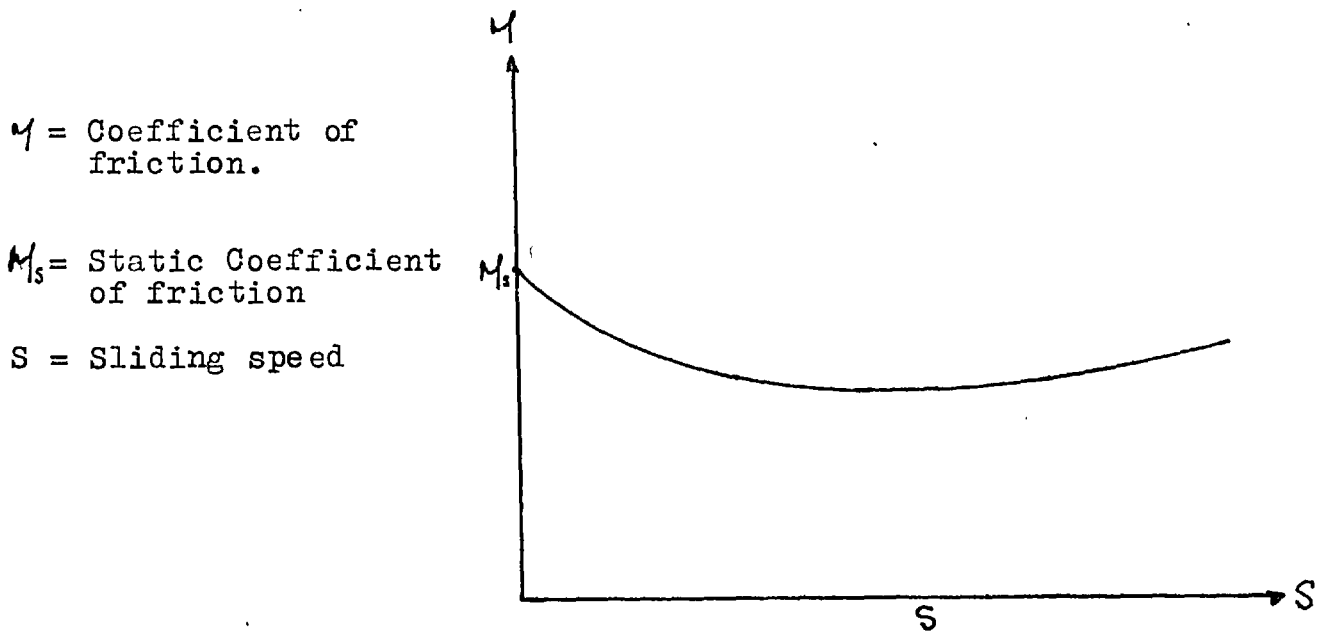


Fig. 1.1 Variation of μ with sliding speed.

DRY FRICTION IN A VIBRATING SYSTEM

1.2 The early studies which have been mentioned were for uni-directional motions (i.e. a drum or wheel rotating in one direction only or one body sliding on another in one direction). This is not the case in mechanical vibrations.

The variation of the friction force with motion in a forced vibratory situation can be represented in two ways. In one case it can be assumed that the motion is continuous and since the friction force always opposes the motion, it must undergo an abrupt sign change every half cycle. In the second case the motion is assumed to come to a stop every half cycle; in which case the change in sign of the friction force is gradual. These two cases are illustrated in Figs 1.2 to 1.4. Fig 1.2 shows a simple vibrating system

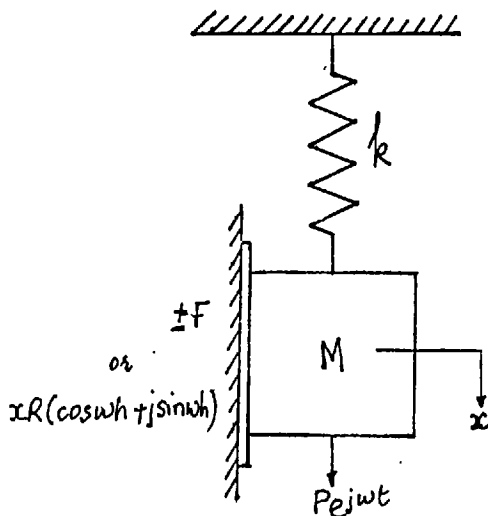


Fig.1.2 Diagram of Vibrating system.

which consists of a body of mass M suspended from a spring with a stiffness K and is acted upon by a periodic excitation force $Pe^{j\omega t}$. In moving, the body rubs against a solid wall which exerts a frictional force F on it opposing the motion. J.P.

Den Hartog (5) gave in his paper of 1926 solutions for both the non-stop

motion and motion with one stop in a cycle, ^{and} assumed that the frictional force F was constant. As Den Hartog's work has become the standard upon which subsequent work on Coulomb

(dry-friction) damping is based it is worthwhile to examine the resulting motions based on this assumption. Figs 1.3 and 1.4 are diagrams of the motion without and with one stop respectively as presented by Den Hartog. Fig 1.3 illustrates the wave forms for motion without stops

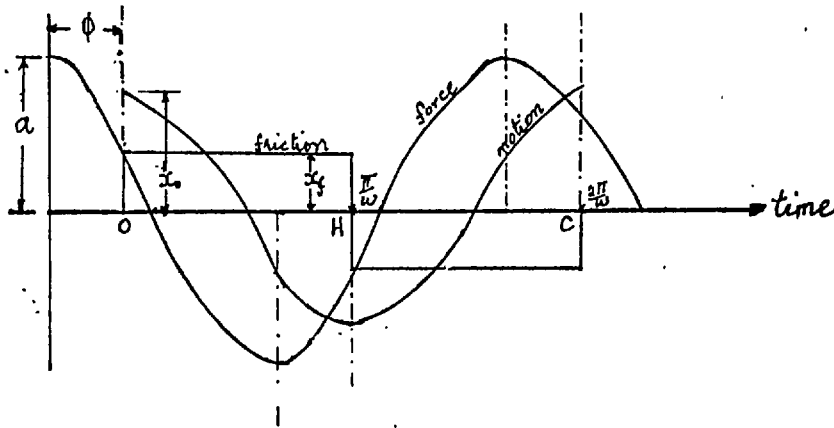


Fig. 1.3 Diagram of motion without stops

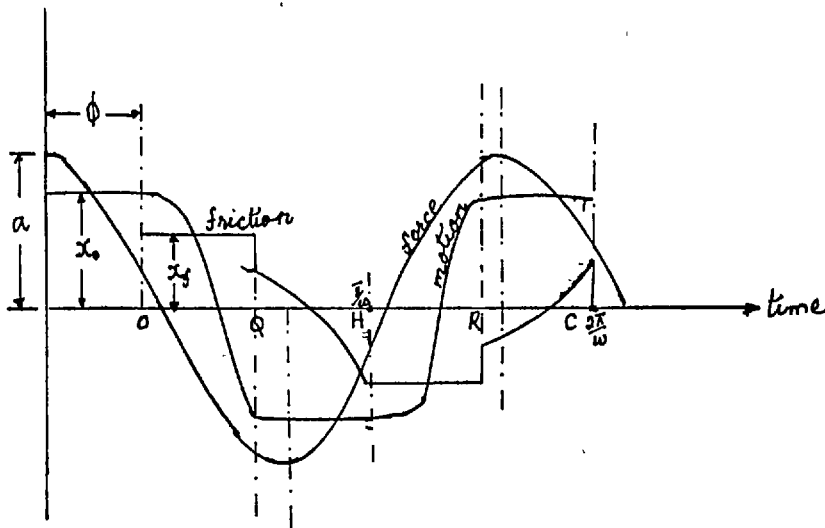


Fig. 1.4 Diagram of motion with one stop

:/.....

The excitation force and the motion are almost sinusoidal whereas the friction force undergoes an abrupt sign change at half a period at H, when the velocity is zero. Fig. 1.4 illustrates the case when there is one stop. In this case only the excitation force remains sinusoidal whereas the friction force remains constant for a quarter of a cycle (till the velocity is maximum) undergoes a change during the next quarter of a cycle (QH) and attains its maximum value but with a sign change at half period, H (when the velocity is zero). This process is repeated for the next half cycle. The resulting motion is clearly not sinusoidal and approximates to a square wave.

1.2.1 ACCEPTED CASE:

The first case illustrated in Fig 1.3 is not acceptable for in reciprocatory motions stops do occur. It is highly improbable that a body with inertia can be made to reverse its direction of motion without first coming to a momentary stop. All subsequent work on the system shown in Fig 1.2 have therefore been based on the assumption that there is a stop in the motion as illustrated in Fig 1.4. This view is also supported experimentally; for a display of the response (motion) of the system shown in Fig 1.2 either on an oscilloscope or by using a pen-recorder shows that the wave form of the motion is similar to that shown in Fig 1.4. The wave form approximates more and more to a square wave with increments in friction force. The frequency of the motion is the same as that of the disturbing force and the two halves of motion cycle follow the same law.

1.3 VARIATION OF THE FRICTION FORCE WITH VELOCITY IN A VIBRATING SYSTEM:

In a vibratory motion the velocity increases from zero to a maximum at quarter period, then decreases to zero within the next quarter period; so that at half period the velocity is zero. The motion is then reversed and the process is repeated for the next half cycle. The variation of the coefficient of friction with speed (i.e. velocity in one direction only) is shown in Fig 1.1 and using this relationship an analysis can be made for a whole cycle of vibrations. Fig 1.5 shows this relationship. OO' represents half a cycle of vibrations; V_p and V_n are the points of maximum velocity, positive and negative respectively. O , O' and O'' are stationary points.

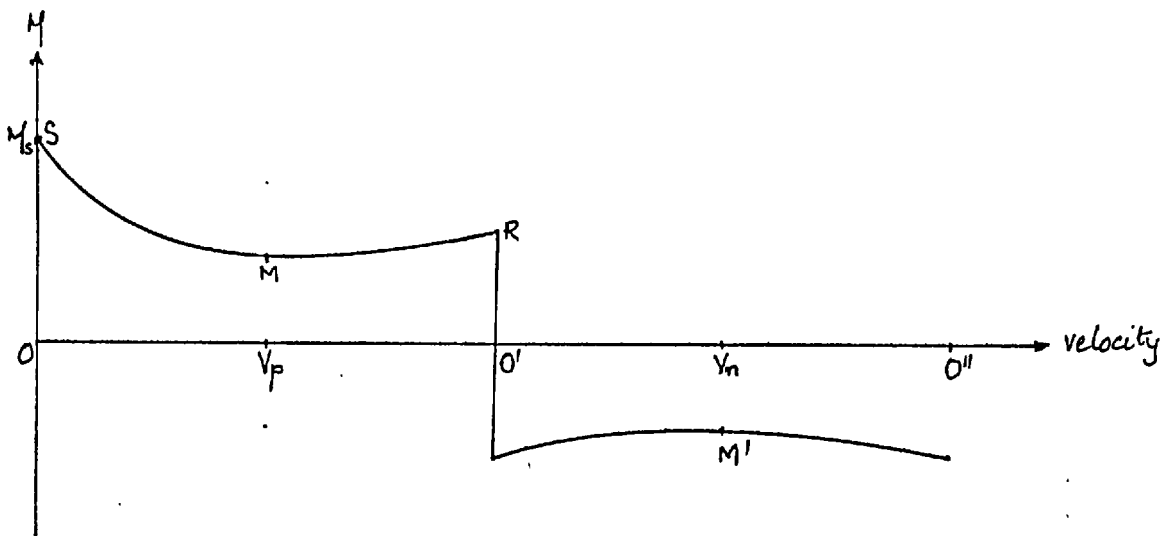


Fig. 1.5 Coefficient of friction as a function of speed

SM is a replica of Fig 1.1 during which the velocity increases from zero to maximum V_p whilst MR represents the reverse when the velocity decreases from V_p to zero. However, as there is insufficient time for the creep phenomenon to occur the coefficient of friction does not attain its static value. $O'O''$ represents the case when the friction force changes sig. $O'O''$ represents the case when the velocity is negative and therefore taking note of this Fig 1.5 can be simplified to Fig 1.6. In Fig 1.6 OO' represents the half cycle when the velocity is positive and OO'' represents the case when the velocity is negative.

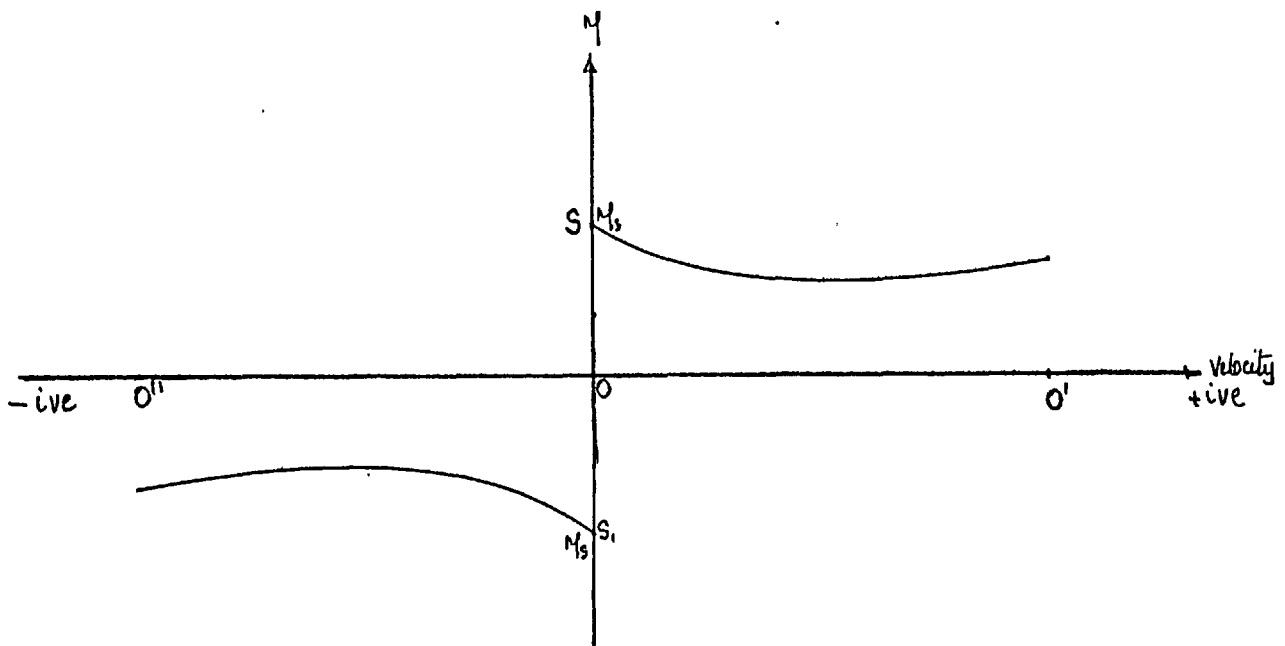


Fig 1.6 Coefficient of Friction as function of Velocity

.....

This relationship between the coefficient of friction and velocity as demonstrated in Fig 1.6 has been used by INOUE, MIYAURA AND MATSUSHITA (6) with satisfactory results. The relationship can be modified however if the discontinuity in the transition period, SOS, is analysed. Owing to insufficient time for creep to take place the static value of the coefficient of friction is not attained during the stop when the change of sign occurs so that SOS may be reduced to TOT,. Moreover as the friction velocity

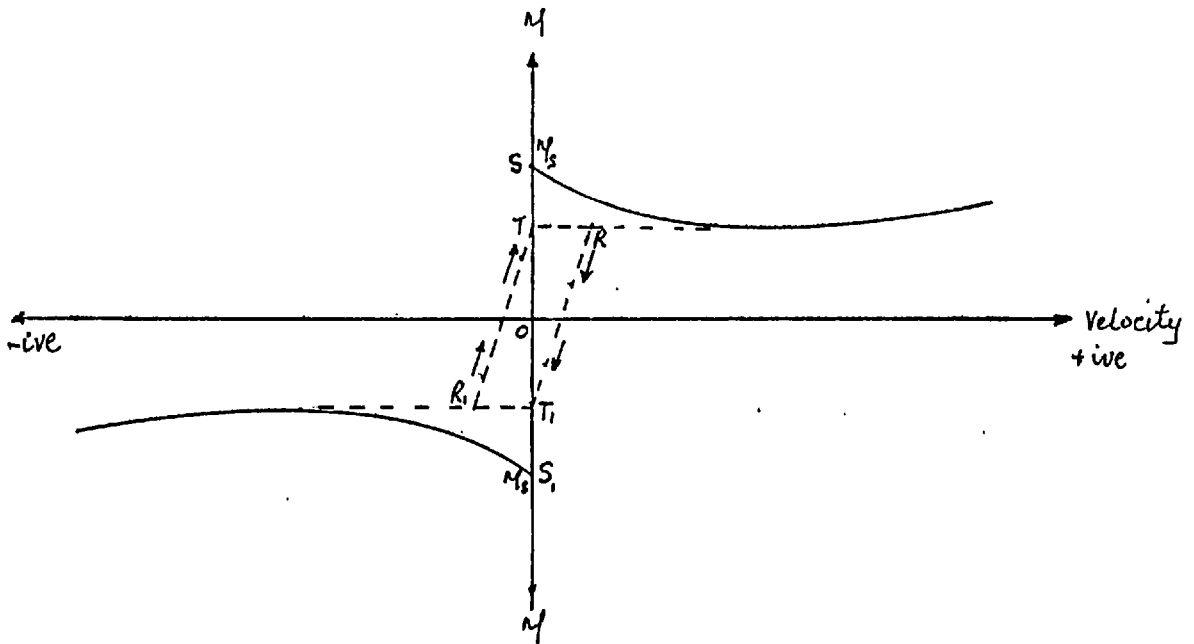


Fig. 1.7 Coefficient of friction as function of Velocity

relation is not reversible but depends upon the past history of the motion (2), the path taken during the transition is better represented by the rectangle TRT,R,T shown dotted in Fig. 1.7. The mean path during this transition period can

therefore be approximately represented by a line joining RR'. So that the modified coefficient of friction-velocity relation is as represented by Fig 1.8

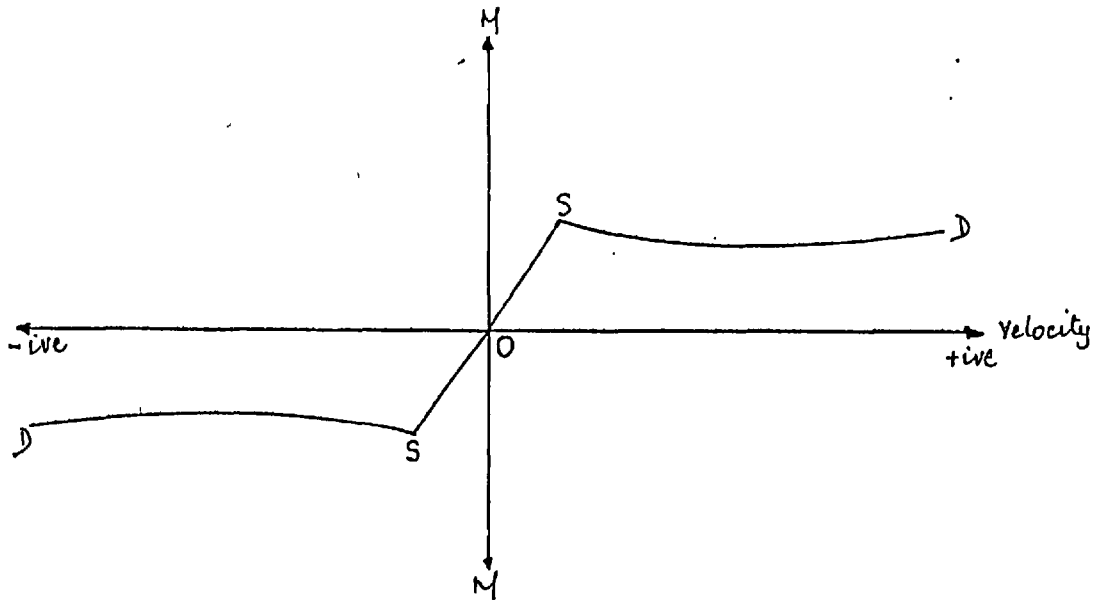


Fig 1.8 Coefficient of friction - Velocity Relation

A figure for the coefficient of friction-velocity relation similar to fig 1.8 was also obtained by Stefan Ziemba (7). In his analysis he suggested that elastic recoil takes place as a result of the difference between the static and kinetic coefficients of friction at every half-cycle. He also pointed out that the shear force had to decrease from the positive value, pass through zero and then attain the negative value. These points are acceptable as they reinforce the validity of the relationship shown in Fig. 1.8

1.4.

PHYSICAL REPRESENTATION

Theoretical models have been evolved to represent different relations between two different variables, thus there are models for Figs 1.5 to 1.8. Dry friction defined in the manner according to Fig 1.8 is represented by a system composed of a slider and a perfect (in the sense of energy, not necessarily linear) spring connected in series. Such a model is shown in Fig 1.9 in which the spring part is represented by K_f and the slider S_f .

Consider Fig 1.9. If a force is applied at one of the ends denoted E in the figure with the other end fixed and the whole model constrained to move vertically only, the

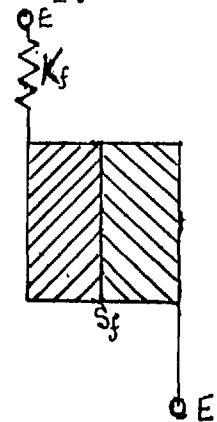


Fig. 1.9 Model for friction force

The extension will increase proportionally with an increment in the applied force until the limiting static force is attained, when sliding begins to take place between the surfaces of the slider. The point at which this occurs is denoted by S in Fig 1.10. Whilst OS represents the linear part. Once sliding takes place the force required to maintain the motion is reduced as the dynamic coefficient of friction is less than the static one and therefore the curve follows the path SD shown in the diagram. A comparison of Figs. 1.8 and 1.10 show that they are similar and therefore Fig 1.9 is a model for Fig 1.8.

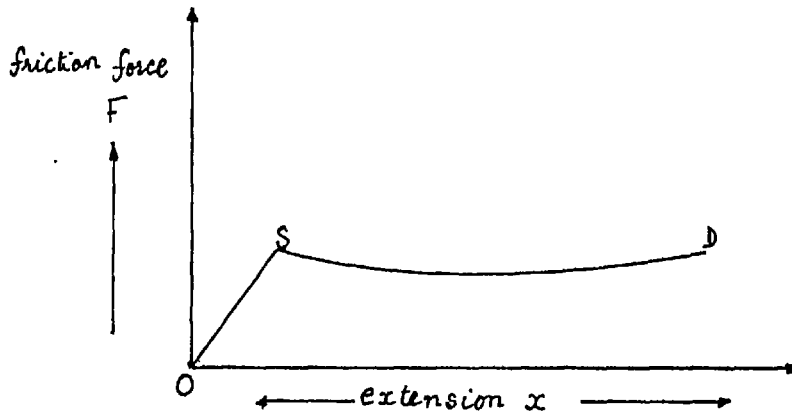


Fig 1.10. Force-extension curve for the model in Fig 1.9

A characteristic of the system represented by the model in Fig 1.9 is that when this type of friction force is used to dampen the vibrations of a system, the vibrations vanish for $t \longrightarrow \infty$. When the damping is weak oscillations are obtained which vanish for $t \longrightarrow \infty$.

1.5 IMPORTANCE OF THE TRANSITION PERIOD

In deriving the relationship shown in Fig 1.8 use was made of the transition period within which the friction force changes from a positive value say R to $-R$. To attain this sign change, the force has to decrease from positive R through zero to negative R . Time is required for this transition to take place, so that if one calls this transition period the delay time h , this delay time is seen to be very important. The model shown in Fig 1.9 is therefore very dependent on this delay-time. The effectiveness of the spring K_f and the slider S_f are dependent on the extra force produced during this delay-time as a result of the difference between static and kinetic coefficient of friction; and as already explained the value of the coefficient of friction is dependent upon the time of stationary contact of the surfaces (i.e. upon the delay-time).

Therefore a theory which takes this delay-time into consideration may produce results that are more realistic. One theory which involves delay-time is the theory of retardation which is described in the next chapter.

C H A P T E R 2

VIBRATIONAL COULOMB FORCE REGARDED AS A RETARDING FORCE

Retarded forces (8) are commonly encountered in control problems such as an Auto balance where a certain control action is produced artificially. In such a case, in view of inevitable time-lags in a control system, this action does not relate to the instant at which it is supposed to be exerted but to a past instant $t-h$, (that is, appears after a delay-time, h).

In modern control systems (involving electron-tube circuits) the control action is adjustable and this is emphasised by attaching the symbol λ , a variable parameter as a coefficient of retarded terms. Retarded actions can cause self-excited oscillations and to establish the regions of instability "Difference-differential equations" (d.d.e) are employed. The notations employed in this case are exemplified by the following two typical d.d.es which are encountered in antirolling stabilisation systems; one with retarded damping and the other with retarded restoring force.

$$\ddot{x} + a_1\dot{x} + \lambda\dot{x}_h + a_0x = 0 \quad (2-1)$$

$$\ddot{x} + a_1\dot{x} + a_0'x + \lambda x_h = 0 \quad (2.2)$$

The terms $a_1\dot{x}$ and a_0x represent the usual terms for damping and restoring forces respectively.

The terms λx_h and $\lambda\dot{x}_h$ are used to represent $x(t-h)$ and $\dot{x}(t-h)$ where "t" is the time under consideration and "h" is the delay-time. These terms can be expanded and simplified using Taylor's series. Thus for instance:

$$\begin{aligned} \dot{x}_h &= \dot{x}(t-h) = \dot{x}(t) - \frac{h}{1!} \ddot{x}(t) + \frac{h^2}{2!} \dddot{x}(t) \dots\dots\dots \left. \vphantom{\dot{x}_h} \right\} \dots\dots(2.3) \\ &= \dot{x} \left(1 - \frac{h}{1!} \left\{ \frac{\ddot{x}}{\dot{x}} \right\} + \frac{h^2}{2!} \left\{ \frac{\dddot{x}}{\dot{x}} \right\} \dots\dots\dots \right) \end{aligned}$$

If one tries to satisfy a d.d.e by a solution of the form

$$x = x_0 e^{zt} \quad \text{where } z = \alpha + j\omega$$

$$\text{then } \dot{x} = x_0 z e^{zt} \quad \ddot{x} = x_0 z^2 e^{zt} \quad \text{and } \ddot{x} = x_0 z^3 e^{zt} \dots\dots\dots(2.4)$$

and thus (2.3) becomes

$$\dot{x}_h = \dot{x} \left(1 - \frac{hz}{1!} + \frac{h^2 z^2}{2!} \dots\dots \right) = \dot{x} e^{-hz} \dots\dots\dots(2.5)$$

$$\text{similarly } x_h = x e^{-hz} \dots\dots\dots(2.6)$$

Substituting this expression into (2-1) one obtains the algebraico-transcendental characteristic equation:

$$f(z) = z^2 + a_1 z + \lambda z e^{-hz} + a_0 \dots\dots\dots(2.7)$$

The advantage in transforming the equation to a d.d.e is that a d.d.e characteristically is always of an infinite degree whatever its order may be. This means that a d.d.e (considered as an ordinary d.e.) may have an infinity of roots of its positive real parts, which leads to a self-excitation of oscillations with frequencies corresponding to the imaginary parts of these roots. It is precisely in this connection that the interest to these d.d.es. appeared in applied problems. When the equation is obtained in the form as in (2.7) the problem then consists in determining the zeros of the entire function $f(z)$. In considering the application to Coulomb dry-friction however the important things to note are the expressions for retarded damping and retarded restoring force.

$$\text{Retarded damping} = \dot{x}_h = \dot{x} e^{-hz}$$

$$\text{Retarded restoring force} = x_h = x e^{-hz}$$

.....

2.1 AN EXAMPLE IN THE USE OF d.d.e.

As an example of a transcendental problem we consider the d.d.e which may be regarded as a control problem involving a retarded restoring force.

Thus we have

$$\ddot{x} + p\dot{x} + \lambda x_h = 0 \quad \dots\dots\dots E.1$$

the characteristic equation with $x = x_0 e^{zt}$ as the solution

$$f(z) = z^2 + pz + e^{-hz} \quad \dots\dots\dots E.2$$

substituting for $z = \alpha + jw$ in this equation and separating the real and the imaginary parts one has

$$\begin{aligned} \alpha^2 - w^2 + p\alpha + \lambda e^{-h\alpha} \cos \beta &= 0 \\ 2\alpha w + pw - \lambda e^{-h\alpha} \sin \beta &= 0 \quad \dots\dots\dots E.3 \end{aligned}$$

where $\beta = wh$

$$\begin{aligned} \cos \beta &= (w^2 - p\alpha - \alpha^2) / \lambda e^{-h\alpha} \\ \sin \beta &= (pw + 2\alpha w) / \lambda e^{-h\alpha} \quad \dots\dots\dots E.4 \end{aligned}$$

from these two equations one obtains two derived equations

$$\begin{aligned} \cotan \beta &= (w^2 + p\alpha - \alpha^2) / w (p + 2\alpha) \quad - \quad E.5 \\ (w^2 - p\alpha - \alpha^2) + w^2(p + 2\alpha) &= \lambda^2 e^{-2h\alpha} \end{aligned}$$

If $\alpha = 0$, one has

$$\cos \beta_1 = w_1^2 / \lambda, \quad \sin \beta_1 = pw_1 / \lambda, \quad - \quad E6$$

$$\cotan \beta_1 = w_1 / p = (1/ph) \beta_1, \quad - \quad E7$$

$$\beta_1^4 + h^2 p^2 \beta_1^2 - h^4 = 0 \quad - \quad E8$$

The subscripts₁ are attached in order to indicate that these quantities correspond to the harmonic values ($\alpha = 0$)
Equation E7 is solved graphically and the roots $\beta'_1, \beta''_1, \beta'''_1$, etc which are infinite determined.

As equation (E7) does not contain the variable parameter λ , the roots $\beta'_1, \beta''_1, \beta'''_1, \dots$ may be regarded as "fixed roots" (depending only on the fixed parameters p and h). These roots

give, therefore, the harmonic frequencies w_1^I , w_1^{II} , w_1^{III} of the transcendental spectrum of the linear problem.

There remains yet the second equation (E8) whose positive root is:

$$\beta_{II}(\lambda) = \sqrt{-\frac{p^2 h^2}{2} + \frac{p^4 h^4}{4} + h^4} \quad \dots\dots(E.9)$$

This root $\beta_{II}(\lambda)$ is a function of λ and increases monotonically with λ starting with $\beta_{II}(0) = 0$. Hence as λ increases continuously from the value $\lambda = 0$ the root $\beta_{II}(\lambda)$ may coincide with one of the fixed roots, say β^I , for a certain value of $\lambda = \lambda^I$. At this point

$$\beta^I = \beta_{II}(\lambda^I) \quad \dots\dots\dots(E.10)$$

Similarly other "fixed roots β^{III} for other values λ^I , may be encountered.

Whenever such a coincidence (E.10) occurs, the two equations (E7) and (E8) have a common harmonic root $\beta_I = hw$, and the analytic function $f(z)$ has a purely imaginary root jw , so that $f(z) = 0$.

Thus by increasing λ other harmonic frequencies w_1^I , w_1^{II} , etc are encountered thus forming a "transcendental spectrum", the ratios $\frac{w_1^I}{w_1^{II}}$, being generally incommensurate.

2.2. DRY FRICTION FORCE CONSIDERED AS A RETARDED RESTORING FORCE

The analysis of dry friction in a vibrating system in the previous chapter showed that the coefficient of friction-velocity relationship can be represented as in fig. 1.8; the satisfactory model for this relationship being the one shown in fig. 1.9.

The importance of the time of stationary contact or "delay-time" has also been established, and therefore dry friction in a vibrating system can be represented as a retarded restoring force provided this representation satisfies the model in fig. 1.9.

A retarded restoring force is represented by (N.B. This is an assumption)

$$F = x_h = x e^{-hz} = x e^{-h(\alpha + jw)} \quad \text{as } z = \alpha + jw \quad \dots (2.8)$$

Thus

$$F = R' x e^{-h\alpha} (\cos wh + j \sin wh) \quad \dots \dots \dots (2.9)$$

where F is the friction force and R' is a constant.

α = absorption coefficient, h = delay-time

w = frequency in radians x = amplitude (displacement)

Conventional Coulomb force $F = \mu N$ where μ = Coefficient of friction and
N = Normal force acting on the moving body.

Comparing (2.9) and (2.10) R' has the units lbf/in,

$$\mu = e^{-h\alpha} (\cos wh + j \sin wh)$$

$$N \propto R' x \quad \therefore R' \propto \frac{F}{x}$$

2.3 VARIATION OF FRICTION FORCE WITH FREQUENCY

Friction force is represented as $F = x R' e^{-\alpha h} (\cos wh + j \sin wh)$

$$\left. \begin{array}{l} \text{The real part of the force is } x R' e^{-\alpha h} \cos wh \\ \text{The imaginary part of the force is } x R' e^{-\alpha h} \sin wh \end{array} \right\} \quad \dots (2.13)$$

The absolute value of the friction force at all frequencies is given by $F = x R' e^{-\alpha h} (\cos^2 wh + \sin^2 wh) = x R' e^{-\alpha h} \quad \dots (2.14)$

For a given Normal force N, the absorption or dissipation of energy must be constant if the amplitude of vibrations and the excitation force are kept constant.

This means that $R' e^{-\alpha h}$ is approximately a constant.

This is possible only if the product of " α " and " h " is approximately a constant at all frequencies.

Since R' is constant at a particular amplitude

$$\text{Thus } \alpha h = C \quad \dots \dots \dots (2.15)$$

Thus the delay-time " h " and the absorption or dissipation coefficient α are inversely proportional to each other.

This means that when the delay-time is small then the rate of absorption or dissipation of energy must be high and vice versa for energy to be conserved.

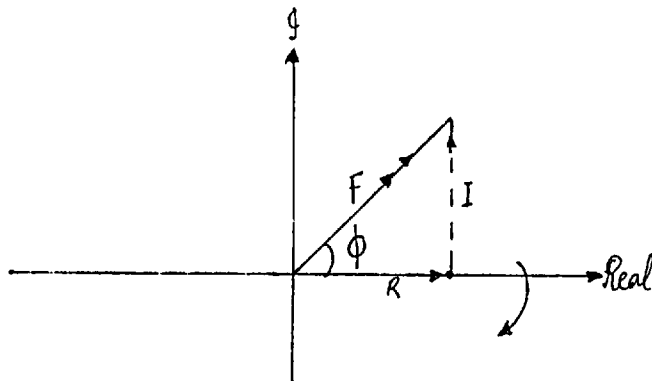
The friction F can therefore be represented as

$$F = xR (\cos wh + j \sin wh)$$

$$\text{where } R = R' e^{-\alpha h} = \text{Constant}$$

As represented the delay-time " h " now assumes great importance. The Real part $xR \cos wh$ represents the "spring" part of the force and the imaginary part $xR \sin wh$ represents the quadrature part. Depending on its sign it can either be a damping or an exciting force.

Friction force is thus shown to be a vector rotating clockwise (as the imaginary part is negative to start with) around the axes; the rate of rotation being dependent upon the variation of frequency " w " and the delay-time " h ".



R = Real part.

I = Imaginary part

F = Absolute value

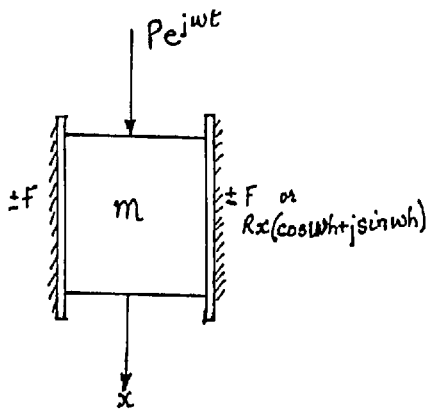
Fig 2.1. Complex representation of Coulomb force

As the imaginary part which may constitute the damping part does attain negative values within a certain frequency range at a particular delay-time, "self-excited" oscillations occur within this range if no other type of damping is present in the system. In fact this phenomenon of "stable" and non-stable" regions can be repeated as the frequency is increased. This fact is clearly demonstrated by and made use of in "Difference-differential equations" to determine the roots of such an equation and hence the stable and non-stable regions

CHAPTER 3

ANALYSIS OF SOME SYSTEMS WITH COULOMB DAMPING

3.1) Vibrating body in the presence of dry or Coulomb friction



Consider the simple system shown in fig. 3.1 where a body of mass M is acted upon by a sinusoidal excitation force $pe^{j\omega t}$. In moving the mass rubs against the solid walls which exert a total frictional force F (or $Rx(\cos\omega h + j \sin\omega h)$) on it opposing its motion. The equation of motion of this system is:

$$M\ddot{x} + R(\cos\omega h + j \sin\omega h) \dot{x} = Pe^{j\omega t}$$

fig. 3.1 vibrating body
without a spring

In order to determine the nature of the solution for this equation it is essential to examine the solution used for the sample of d.d.e. solved in Chapter 2.

The assumed solution was $x = x_0 e^{zt} = x_0 e^{(\alpha + j\omega)t} \dots (3.2)$

This solution was used for free vibrations.

In forced vibrations as in this case however, after the system has been in motion for some time and a "steady state" is attained, the following conditions must be satisfied:

- The frequency of the motion must be the same as that of the excitation force, and
- The downward half-cycle of the motion must follow the same law as the upward half-cycle.

In order that equation (3.2) should satisfy these two conditions $e^{\alpha t}$ must be a constant. As already shown in

equation (2.15) $\alpha h = \text{constant}$ and as the motion is periodic and has the same frequency as the excitation force, αt must also be constant.

Therefore equation (3.2) becomes:

$$x = x_0 e^{\alpha t} e^{j\omega t} = X e^{j\omega t} \dots\dots(3.3)$$

where $X = x_0 e^{\alpha t}$

The effect of $e^{\alpha t}$ in equation (3.2) therefore is to give the motion a permanent phase-lag ϕ as shown in figs 1.3 and 1.4. As this phase-lag and the nature of the wave shape of the motion are indirectly taken into consideration by representing the friction force as a complex quantity the motion can be approximately be represented (especially for small amplitudes of vibrations) by equation (3.3).

Using the solution $x = x e^{j\omega t}$ equation (3.1) becomes

$$(-M\omega^2 + R\cos\omega h + jR\sin\omega h) x = P e^{j\omega t} \dots\dots(3.4)$$

The absolute value of P is given by

$$P e^{j\omega t} = x \sqrt{(m\omega^2 + R\cos\omega h)^2 + (R\sin\omega h)^2} \dots(3.5)$$

The phase angle ϕ is given by

$$\tan \phi = \frac{R\sin\omega h}{m\omega^2 + R\cos\omega h} \dots\dots(3.6)$$

Vibrating body with constant peak acceleration

If the maximum acceleration of the system in fig. 3.1 is kept constant at all frequencies, then

$$m\omega^2 x = C' = \text{constant and}$$

equation (3.1) becomes

$$\left. \begin{aligned} C' + (R\cos\omega h + jR\sin\omega h)X &= P e^{j\omega t} \\ (C + R\cos\omega h + jR\sin\omega h)X &= P e^{j\omega t} \end{aligned} \right\} \dots(3.7)$$

where $C = \frac{C'}{X} = m\omega^2$ at any frequency
i.e. $C = F(\omega)$

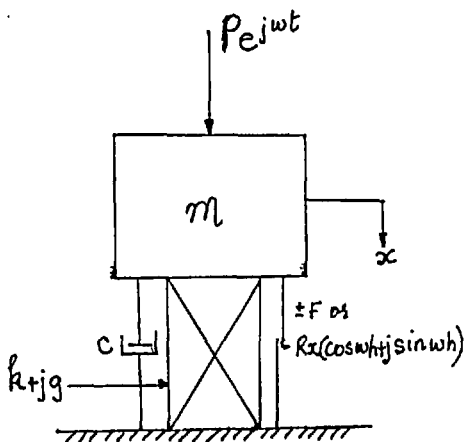
The absolute value of the excitation force P then becomes

$$Pe^{j\omega t} = X \sqrt{(C + R\cos\omega h)^2 + (R\sin\omega h)^2} \dots\dots\dots (3.8)$$

$$\text{and } \tan \phi = \frac{R\sin\omega h}{C + R\cos\omega h} \dots\dots\dots (3.9)$$

Therefore if in the system shown in fig. 3.1 the body is made to accelerate at a maximum constant value at all frequencies a phase difference should be detected between the excitation force and the response.

3.2 One degree of freedom system with dry friction, viscous and Visco-elastic damping:



Consider the system in fig. 3.2

in which the mass is subjected to periodic vibrations by the excitation force $Pe^{j\omega t}$ in the presence of viscous force (represented by $c\dot{x}$) Coulomb force (represented by $R(\cos\omega h + j\sin\omega h)$ and viscoelastic force. (represented by $(k + jg)x$). C is the coefficient of viscous damping, k and g being the in phase and quadrature components of the stiffness of the viscoelastic material.

fig. 3.2 Vibrating system with 3 types of damping.

The equation of motion of the system in fig. 3.2 is

$$m\ddot{x} + c\dot{x} + kx + jgx + xR\cos\omega h + jxR\sin\omega h = Pe^{j\omega t} \dots\dots\dots (3.10)$$

assuming the solution $x = x_0 e^{j\omega t}$

$$(-m\omega^2 + jcw + k + jg + R\cos\omega h + jR\sin\omega h) x_0 = P$$

$$\text{i.e. } (k - m\omega^2 + R\cos\omega h + j(c\omega + g + R\sin\omega h)) x_0 = P \dots\dots\dots (3.11)$$

If substitution is made for various parameters as follows

$$w_o^2 = \frac{k}{m}, \quad \frac{R}{k} = S, \quad n = \frac{w}{w_o}, \quad C_o = 2 \sqrt{mk}, \quad \zeta = \frac{c}{C_o}$$

hence $c = 2\zeta \sqrt{mk}$, and $\eta = \zeta/k$ equation (3.11) becomes

$$(1 - n^2 + S \cos wh + j(2\zeta n + \eta + S \sin wh)) x_o = P/k \dots\dots(3.12)$$

The absolute value of P is given by

$$P/k = x_o \sqrt{(1 - n^2 + S \cos wh)^2 + (2\zeta n + \eta + S \sin wh)^2} \dots(3.13)$$

The phase angle ϕ is given by

$$\tan \phi = \frac{2\zeta n + \eta + S \sin wh}{1 - n^2 + S \cos wh} \dots\dots\dots(3.14)$$

3.3. SPECIAL CASES:

a). One degree of freedom system with visco-elastic and Coulomb damping only

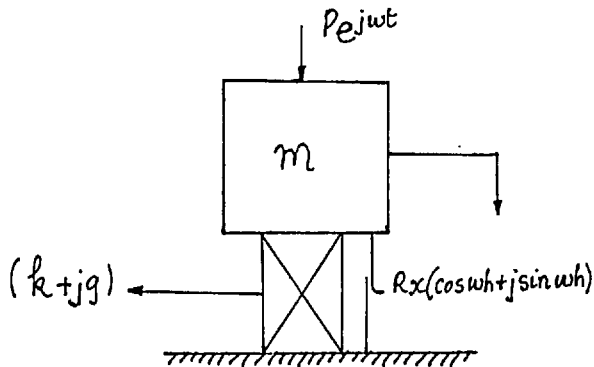


fig. 3.3 vibrating system with 2 types of damping

Fig. 3.3 shows a simple vibrating system with two types of damping only (Coulomb and viscoelastic). The equation of motion is:

$$m\ddot{x} + kx + Rxcoswh + j(\dot{g}x + Rxsinwh) = Pe^{j\omega t} \dots\dots\dots(3.15)$$

The absolute value of the excitation force $Pe^{j\omega t}$ is obtained by putting $\zeta = 0$ in equation (3.13) giving:

$$\frac{P}{kx_o} = \sqrt{(1 - n^2 + S \cos wh)^2 + (\eta + S \sin wh)^2} \dots\dots(3.16)$$

The phase angle ϕ is obtained from equation (3.14) by putting $r = 0$ giving

$$\tan \phi = \frac{\eta + S \sin wh}{1 - n^2 + S \cos wh} \dots\dots\dots(3.17)$$

3.4) One degree of freedom system having an external spring with Coulomb and Viscous damping:

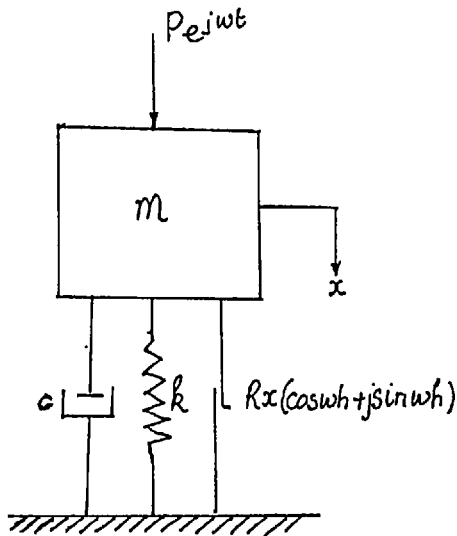


Fig 3.4 shows a vibrating system with an external spring that is subjected to forced periodic vibrations in the presence of Coulomb and viscous damping. In practice the spring has some internal damping but this can be neglected if it is small compared to the external damping. This assumption is nearly true for metallic springs (such as used in the experiment).

fig. 3.4 vibrating system with Coulomb and viscous damping

K is the stiffness of the spring. The equation of motion is:

$$m\ddot{x} + kx + c\dot{x} + Rx\cos\omega h + jRx\sin\omega h = Pe^{j\omega t} \quad \dots\dots(3.18)$$

making the same substitutions as was done for the general case, the absolute value of the force and the phase angle ϕ are obtained by putting $\eta = 0$ in equations (3.13) and (3.14) giving

$$P/kx_0 = \sqrt{\{(1 - n^2 + S\cos\omega h)^2 + (2\mathcal{J}n + S\sin\omega h)^2\}} \quad \dots\dots(3.19)$$

$$\text{and } \tan \phi = \frac{2\mathcal{J}n + S\sin\omega h}{1 - n^2 + S\cos\omega h} \quad \dots\dots\dots(3.20)$$

3.5) One degree of freedom system having an external spring with Coulomb damping only:

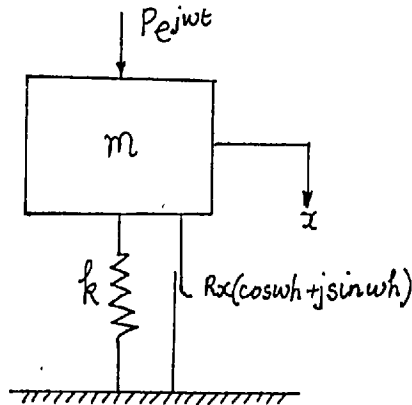


Fig 3.5 shows a vibrating system with Coulomb damping only.

fig. 3.5 Vibrating system with Coulomb damping only

The equation of motion of the system in fig. 3.5 is given by

$$mx + kx + xR(\cos \omega h + j \sin \omega h) = P e^{j\omega t} \quad \dots\dots 3.21$$

The absolute value of $P e^{j\omega t}$ and the phase angle ϕ are obtained by putting $\delta = 0$ in equations (3.19) and (3.20) giving

$$\frac{P}{kx_0} = \sqrt{\{(1 - n^2 + S \cos \omega h)^2 + (S \sin \omega h)^2\}} \quad \dots (3.22)$$

and $\tan \phi = \frac{S \sin \omega h}{1 - n^2 + S \cos \omega h} \quad \dots\dots\dots (3.23)$

3.6 Variation of amplitude of vibrations with friction force for constant excitation force

The equation of motion of a vibrating mass in the presence of 'dry' friction without an external spring is equation (3.1)

$$m\ddot{x} + R(\cos wh + j\sin wh)x = Pe^{j\omega t}$$

$$\therefore x = \frac{Pe^{j\omega t}}{-m\omega^2 + R(\cos wh + j\sin wh)} \dots\dots(3.24)$$

At a constant frequency, $Pe^{j\omega t} = \text{constant}$, $m\omega^2 = \text{constant}$ and $\cos wh$ and $\sin wh$ are constant.

Thus the only variable is R

The denominator has an absolute value

$$D = \sqrt{\{(m\omega^2 + R\cos wh)^2 + R^2\sin^2 wh\}}$$

Thus,

$$x = \frac{C_1}{C + R(C_2 + jC_3)} \quad (\text{i.e. } \frac{1}{x}) \dots\dots(3.25)$$

where, $C_1 = Pe^{j\omega t}$, $C = m\omega^2$, $C_2 = \cos wh$, $C_3 = \sin wh$, and C , C_1 , C_2 , and C_3 are constants.

If the friction force is considered constant (5) then,

$$\begin{aligned} m\ddot{x} + F &= Pe^{j\omega t} \\ m\omega^2 x &= Pe^{j\omega t} + F \\ x &= \frac{Pe^{j\omega t} + F}{m\omega^2} = \frac{C_1 + F}{C} \dots\dots(3.26) \end{aligned}$$

(i.e. a straight line)

Equation (3.25) produces a curve asymptotic to the R axis as the abscissa whereas equation (3.26) produces

a straight line which crosses the x axis at $\frac{c_1}{c}$.

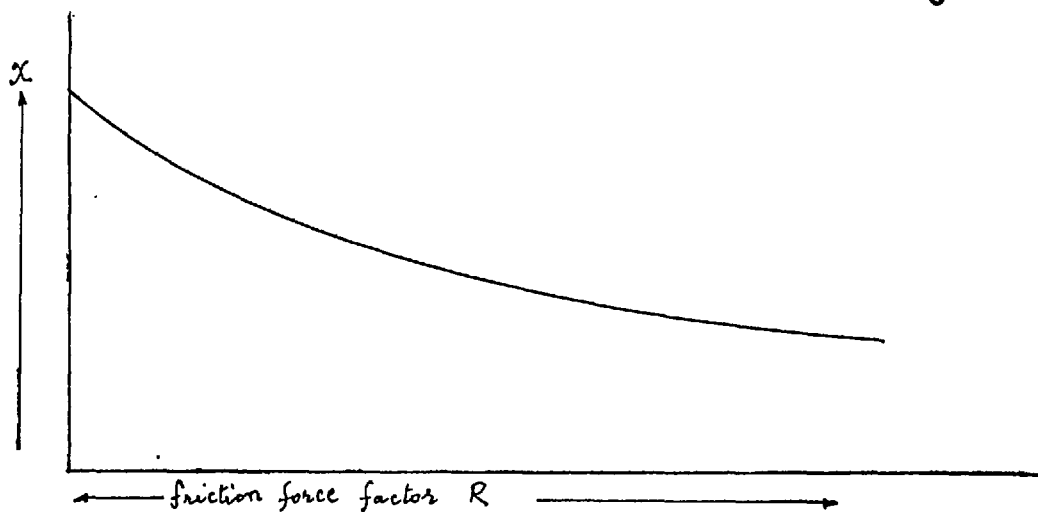


Fig.(3.6) Curve for equation (3.25)

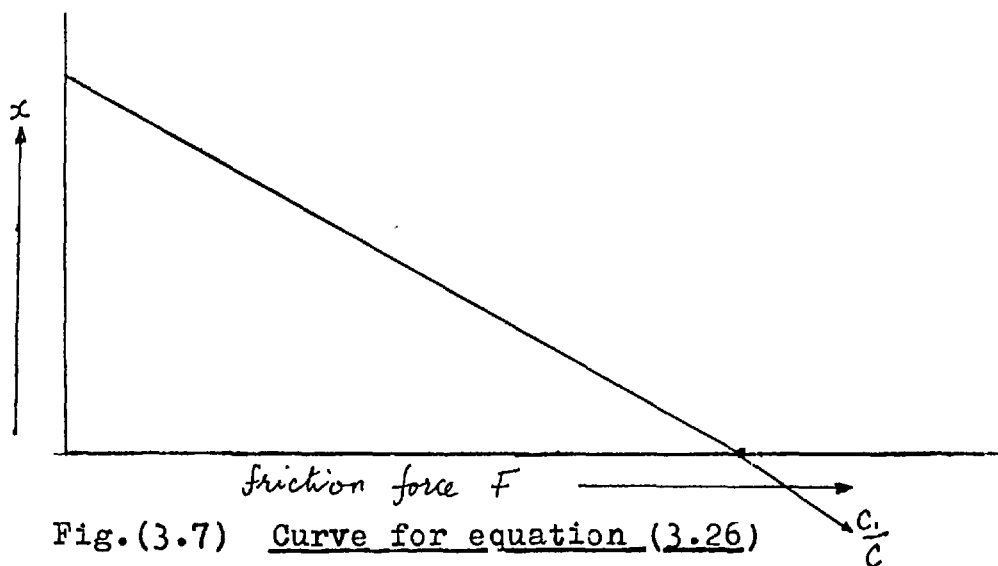


Fig.(3.7) Curve for equation (3.26)

$$\frac{c_1}{c} = \frac{Pe^{j\omega t}}{m\omega^2}$$

3.7 Variation of amplitude with friction force for constant excitation force and frequency with a spring in the system

With a spring in the system the equation of motion is

$$m\ddot{x} + kx + R(\cos wh + j\sin wh)x = Pe^{j\omega t}$$

therefore

$$x = \frac{Pe^{j\omega t}}{k - m\omega^2 + R\cos wh + jR\sin wh} \dots\dots(3.27)$$

which becomes

$$x = \frac{P/k e^{j\omega t}}{1 - n^2 + S\cos wh + jS\sin wh} \dots\dots(3.28)$$

At constant frequency $1 - n^2$ is constant as n is constant

$$\therefore x = \frac{P/k e^{j\omega t}}{C_4 + R(C_2 + jC_3)} = \frac{C_1/k}{C_4 + R(C_2 + jC_3)} \dots(3.29)$$

where $C_4 = 1 - n^2$

Thus the curve is similar to the one obtained for equation (3.25) in Fig.(3.6).

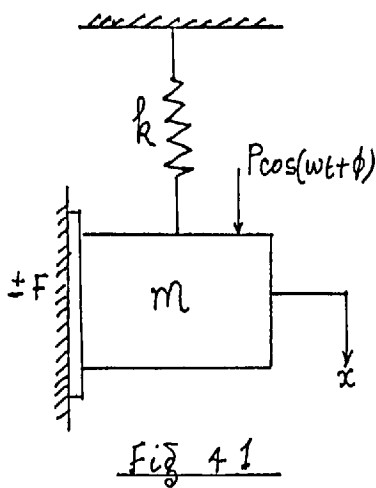
The phase ϕ is given by equation (3.23)

$$\text{i.e.} \quad \tan \phi = \frac{S\sin wh}{1 - n^2 + S\cos wh}$$

C H A P T E R 4

4-1 REVIEW OF PAST PAPERS

Most recent studies of vibrations involving Coulomb damping have been based on the work of J.P. Den Hartog (5) done in 1926. In this paper Den Hartog represents the equation of motion of a one degree of freedom system as shown in fig. 4.1 as follows:



$$m\ddot{x} + kx + F = P \cos (\omega t + \phi) \quad (4.1)$$

By introducing the "phase angle" ϕ into the excitation force Den Hartog was acknowledging the fact that there is some retardation involved in the motion. He however explained this by saying "The "Phase Angle" ϕ in the disturbing force has no meaning in a

differential equation and is only included for the purpose of subsequently writing the boundary conditions (4.3) in a simple form."

Using the abbreviations: $P/k = a$, $F/k = X_f$, $k/m =$ the differential equation for the interval of time can be written as:

$$\ddot{x} + \omega_n^2 (x - x_f) = a \omega_n^2 \cos (\omega t + \phi) \quad (4.2)$$

A steady-state solution for one-stop motion has to fulfil the following boundary conditions:

$$\left. \begin{array}{lll} t = 0 & x = X_0 & \dot{x} = 0 \\ t = \pi/\omega & x = -X_0 & \dot{x} = 0 \end{array} \right\} \quad (4.3)$$

.

A solution is obtained and deriving from it are the expressions for magnification factor and the phase angle ϕ .

$$\frac{X_o}{a} = \sqrt{V^2 - (F/P)^2 U^2} \dots\dots\dots (4.4)$$

$\frac{X_o}{a}$ is the "magnification factor",

$V = \frac{\beta^2}{\beta^2 - 1}$ the "response function"

$U = \frac{\beta \sin \beta \pi}{1 + \cos \beta \pi}$ the "Coulomb damping function" and

$\beta = \frac{\omega_n}{\omega}$ the "frequency ratio."

$$\cos \phi = \frac{X_o}{a} \cdot \frac{1}{V} \quad \text{and} \quad \sin \phi = \frac{-X_f}{a} \cdot \frac{U}{V}$$

4.1.1 AS FOURIER SERIES

Den Hartog stated in the same paper that the results quoted above can be obtained by representing the Coulomb damping term in the equation by a Fourier series. Though this was done independently by Den Hartog and Prof. E.O. Waters it was reproduced in Den Hartog's paper. Equation (4.1) thus becomes:

$$m\ddot{x} + kx - \frac{4F}{\pi} \sum \frac{1}{n} \sin n\omega t = P \cos(n\omega t + \phi) \dots\dots(4.5)$$

The derivation of the results are given in his paper and the author finds it unnecessary to repeat it here. However, an examination of the results will be made and compared with those obtained using the Theory of Retardation.

4.2 VIBRATING MASS IN THE PRESENCE OF COULOMB DAMPING WITHOUT A SPRING:

Equation of motion according to Den Hartog

$$m\ddot{x} + F = P \cos(\omega t + \phi) \dots \text{derived from (4.1) ..(4.6)}$$

OR

$$m\ddot{x} - \frac{4F}{\pi} \sum \frac{1}{n} \sin n\omega t = P \cos(\omega t + \phi) \text{ derived from (4.5) ..(4.7)}$$

Representing (4.6) Vectorially

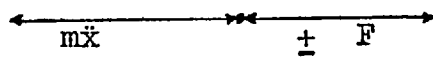


Fig. 4.2 vectorial representation of equation (4.6)

(4.7) cannot be represented vectorially until the value of the Sine and Cosine terms have been determined.

From Retardation Theory:

Equation is:

$$m\ddot{x} + xR\cos\omega h + j\sin\omega h = Pe^{j\omega t} \dots \dots \dots (4.8)$$

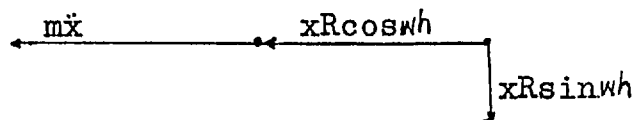


Fig. 4.3 Vectorial representation of equation (4.8)

Examination of the equations:

From Den Hartog's Theory

$$\text{Magnification factor} = \frac{X_o}{a} = \frac{\sqrt{V^2 - (\frac{xb}{a^2})^2}}{U^2} \dots \dots \dots (4.9)$$

when there is no spring $\omega_n = 0$, $\beta = 0$ hence $V = 0$

and $U = 0$ therefore $\frac{X_o}{a} = 0$

$$\cos \phi = \frac{X_o}{a} \cdot \frac{1}{V} = 0 \text{ and } \sin \phi = \frac{X_f}{a} \frac{U}{V} = 0$$

which is meaningless.

By Retardation theory there is a phase angle as well as a magnification factor.

4.2.1 Forced Vibrations with Coulomb damping:

System represented in fig. 4.1

equation of motion as presented by Den Hartog is

$$m\ddot{x} + kx + \underline{F} = P \cos (t + \phi)$$

by Retardation theory equation is (3.6):

$$\text{i.e. } m\ddot{x} + kx + xR\cos\omega h + jxR\sin\omega h = Pe^{j\omega t} \dots\dots$$

Vectorial Representation

All the forces in the L.H.S. of the equation (4.1) are real and so can be represented in a straight line as shown in fig 4.4

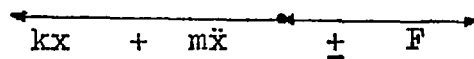


fig. 4.4 Vectorial representation of equation (4.1)

Retardation Theory: By this theory the vectorial representation is as shown in fig 4.5

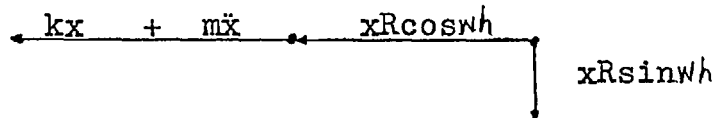


Fig 4.5 vectorial representation of equation (3.6)

From the vectorial representations there is no phase difference between the motion and the response if the friction force is considered as an alternating constant, whereas by the retardation theory there is a phase difference.

Den Hartog however, obtained an expression for the phase angle ϕ which he said had no meaning and proceeded to plot a graph for this theoretically. As regards the magnification his theory predicts that Coulomb damping decreases the resonant frequency whereas many experimenters have found evidence to the contrary. Among these are Jerome Ruzicka (9), Ruzicka and Derby (10) and E.S. Levitan (11).

4.3 THE EFFECT OF COULOMB DAMPING ON THE RESONANT FREQUENCY

Consider the equation of a one-degree of freedom system with a spring and Coulomb damping. The equation is represented by (3.6) i.e.

$$m\ddot{x} + kx + xR (\cos wh + j\sin wh) = Pe^{j\omega t}$$

The effective stiffness, $k_{eff}^2 = (k + R\cos wh)^2 + (R\sin wh)^2$

The damped resonant frequency, $w_d = \frac{k_{eff}}{m}$

The damped resonant frequency $w_d > w_0$ (the undamped resonant frequency) if $k_{eff} > k$.

Thus:

$$\begin{aligned} & (k + R\cos wh)^2 + (R\sin wh)^2 > k^2 \\ \text{i.e. } & k^2 + 2kR\cos wh + R^2(\cos^2 wh + \sin^2 wh) > k^2 \\ & k^2 + 2kR\cos wh + R^2 > k^2 \\ & 2k\cos wh > -R \end{aligned}$$

i.e.

$$\cos wh > -\frac{R}{2k}$$

Conversely $w_d < w_0$ if $\cos wh < -\frac{R}{2k}$

Thus:

$$\begin{aligned} & \text{Resonant frequency increases if } \cos wh > -\frac{R}{2k} \\ & \text{Resonant frequency decreases if } \cos wh < -\frac{R}{2k} \\ & \text{Resonant frequency is unchanged if } \cos wh = -\frac{R}{2k} \end{aligned} \quad (4.11)$$

Equation (4.11) shows that the change in resonant frequency is dependent upon the delay time which in turn depends upon the magnitude of the friction force constant R and the external restoring spring stiffness, k . The constant R is also dependent upon the amplitude of vibrations and therefore the delay time may change with a variation in stiffness, the

amplitude of vibrations or the magnitude of the friction force. This means that the change induced by the addition of Coulomb damping to a particular vibrating system may be either an increment or decrement depending upon the magnitude of the parameters mentioned above.

4.4 THE EFFECT OF DRY-FRICTION ON THE PHASE DIFFERENCE

Comparing equations (3.9); $\tan \phi = \frac{S \sin \omega h}{1 - n^2 + S \cos \omega h}$, and (3.12)

$$\tan \phi = \frac{2\gamma n + S \sin \omega h}{1 - n^2 + S \cos \omega h} \quad \text{one observes that the}$$

denominator is the same but the numerator is increased by the term ' $2\gamma n$ '. Thus provided $\sin \omega h$ is positive the phase ϕ is advanced. However if $\sin \omega h$ happens to be negative i.e. in the unstable region then the phase will be retarded. This means that when viscous damping is added to a system with dry friction, the phase is advanced before resonance but is retarded after resonance provided the system was stable in the first place. This conclusion is in agreement with the experimental results of (11). This is also true if Visco-elastic damping is added instead of viscous damping and the same principle applies when all three are mixed. The phase ϕ_v for viscous damping only is given by

$$\phi_v = \frac{2\gamma n}{1 - n^2} \quad \dots\dots\dots(4.12)$$

The phase ϕ_e for visco-elastic damping only is given by

$$\phi_e = \frac{\eta}{1 - n^2} \quad \dots\dots\dots(4.13)$$

Comparing (4.12) and (3.12) one observes that both the denominators and numerators are increased, thus whether the

.....

phase will be advanced or retarded by the addition of dry friction to the system depends upon whether

$$\frac{2\eta n}{1 - n^2} < \frac{2\eta n + S \sin wh}{1 - n^2 + S \cos wh} \quad \text{OR} \quad \frac{2\eta n}{1 - n^2} > \frac{2\eta n + S \sin wh}{1 - n^2 + S \cos wh} \quad \left. \vphantom{\frac{2\eta n}{1 - n^2}} \right\} \quad (4.14)$$

By comparing equations (4.13) and (3.16) one observes that this condition holds if the system contained visco-elastic damping before the addition of dry friction.

DESCRIPTION AND CALIBRATION OF THE APPARATUS5.1 THE BASIC APPARATUS:

The apparatus was set up as shown in fig. 5.1 on a lathe bed. The vibrator was a DERITRON VIBRATOR type 5B, the table of which was connected through a Force Guage to the vibrating body. The vibrating body was a rectangular hollow box 6" x 4" by 4" high (outside dimensions) and 1/4" thick. The choice of this type of vibrating body was influenced by two factors. Firstly the maximum vector force of the vibrator was given as 70lbf; but from practical experience it was considered safer not to exceed 56lbf if overheating was to be avoided. This means that for a fixed amplitude and frequency range the vibrating body must have a maximum mass. A body of weight 10lbf was considered adequate but a solid rectangular mild steel body of 10lbf weight would be too small because (a) a large side area was desirable in case it was necessary to increase the size of the friction pads, and (b) to enable the mass of the body to be changed if necessary without altering the dimensions and set up of the system. At the sides of the vibrating body are friction pads (described later) to introduce Coulomb damping into the system. The body was supported by a steel wire fixed unto a Salters-Spring balance to take off the dead weight. C_1 and C_3 (fig. 5.1) are guide spigots to whose faces (a restoring force system) either a metallic spring or a viscoelastic material which was a green PVC flexible hose are bounded, using two washers W to aid alignment. C_2 is

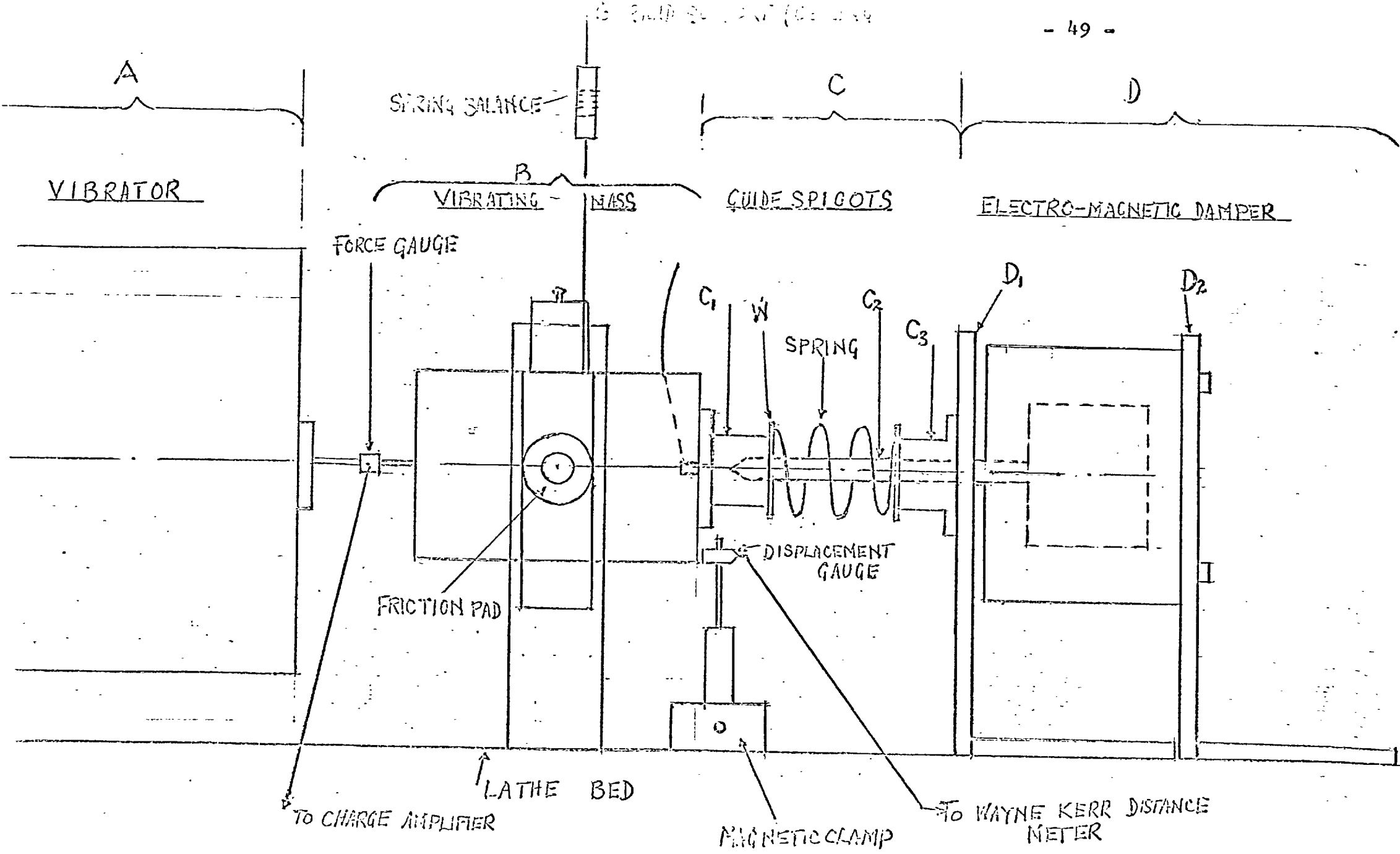


FIG. 1 - GENERAL VIEW OF THE VIBRATING SYSTEM

SCALE 1cm = 1 inch

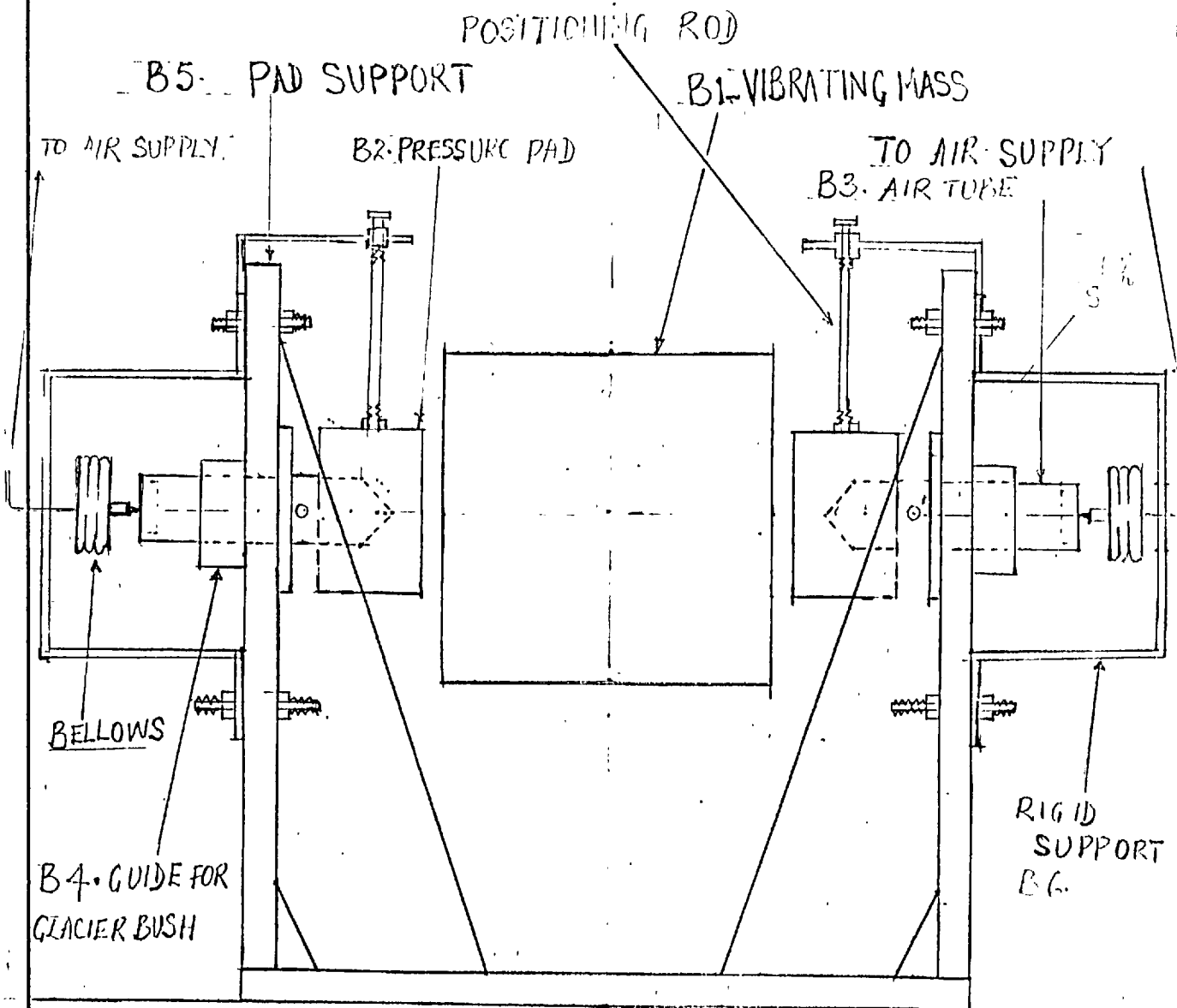
a connecting rod which screws into C_1 and passes through C_3 and D_1 and carries the cylinder of the electro magnetic damper at the end. The end has screw threads fitting the hole in this cylinder which is held tightly in place with a nut.

D_1 and D_2 are two brackets with holes at the base so that they can be firmly secured to the lathe-bed. D_1 supports a Glacier bearing Bush (08DU06) which is used to reduce the friction between the bearing and the air tube which passes through this bearing. The vibrating body, the guide spigots, the connecting rod and the brackets are all made of mild steel. The brackets are held unto the lathe-bed with bolts and nuts thus ensuring that the brackets are adjustable. This means that the distances between the brackets D_1 and D_2 and between the vibrator and vibrating mass can all be varied to suit a particular set up. By using a spirit-level, engineering squares and feeler gauges the vertical and horizontal alignment can easily be checked. The lathe bed of length 3' 9" had a long slot enabling the brackets to be attached onto it with nuts and bolts and also had holes drilled into the sides to support the pressure gauge. In order to obtain a tighter grip between the brackets and the lathe-bed a PVC $1/32$ sheet material was interposed between the lathe bed and the brackets. The bracket holding bolts were done up very tight: thus there was no noticeable loss in rigidity.

5.1.1 The Friction Pad (fig. 5.2) The friction pads were rectangular solid blocks of mild steel of dimensions $1\frac{1}{2}'' \times 1\frac{1}{4}'' \times 1\frac{1}{4}''$.

FRICITION PAD

B



SCALE 1/2" = 1"

FIG- 52- END VIEW OF FRICTION PAD ASSEMBLY

Screwed into the blocks are $\frac{3}{8}$ diameter mild steel tubes which pass through Glacier (11 DU14) bearing bushes supported by the brackets B5, labelled pad support. The ends of the tubes were later closed with mild steel bungs and centre marked so that the bellows could be held centrally. The bellows were held rigidly by a $\frac{1}{16}$ thick rectangular support fixed to the brackets B5. Air was supplied to the bellows through a pressure regulator (Morgreen pressure regulator), then through a pressure guage, (two were used; one reading $-30\text{lb}/\text{in}^2$ to $15\text{lb}/\text{in}^2$ and the later one reading 0 to $21\text{lb}/\text{in}^2$), and then into a T junction valve from which both the bellows were fed through plastic tubes. The air was obtained from the main supply in the departmental workshop through an air tap in the laboratory which was connected to the pressure regulator by a plastic hose. B6 is a mild steel bracket with a mild steel rod labelled positioning rod, of diameter $\frac{1}{4}$ which screws into a threaded hole in the pressure pad. After air has been introduced into the bellows and stable conditions attained the nuts are tightened to hold the friction pad rigidly in position to prevent movement. The screws labelled 'S' serve the same purpose as the positioning rods.

5.12

THE MEASURING EQUIPMENT

For the excitation force: An A.E.I Oscillator type R 2125, Dawe digital frequency meter (type 719A, serial No.55) and a Deritron amplifier E4433, and VP5B Vibrator.

Force measurement: Endevco Corp-Force Guage Model 2103-100 serial UA12, Fenlow Charge Amplifier and an oscilloscope, (Minirack) digital voltmeter (solatron LM 1420.2). B and K sound vibration analyser (1606) and Pre amplifier (B & K, 2105).

Pictorial views of the apparatus

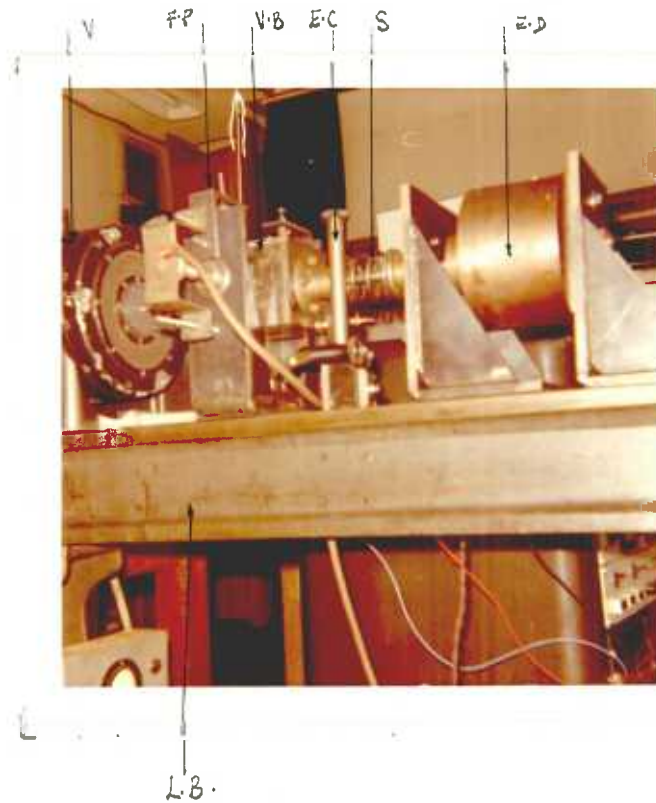


Fig 5.3 (left) Side view of the apparatus with a spring in the system.

- V = Vibrator.
 F.P = Friction pad bracket.
 V.B = Vibrating body.
 E.C. = electromagnetic clamp supporting displacement gauge.

- S = Spring
 E.D = electromagnetic damper (viscous damping)
 F.G = Force Gauge
 D.G. = Displacement Gauge
 V.M = Viscoelastic material (stiffness & viscoelastic damping)
 L.B = Lathe bed

Fig 5.4 (below) Side view of the apparatus with a viscoelastic material in the system.

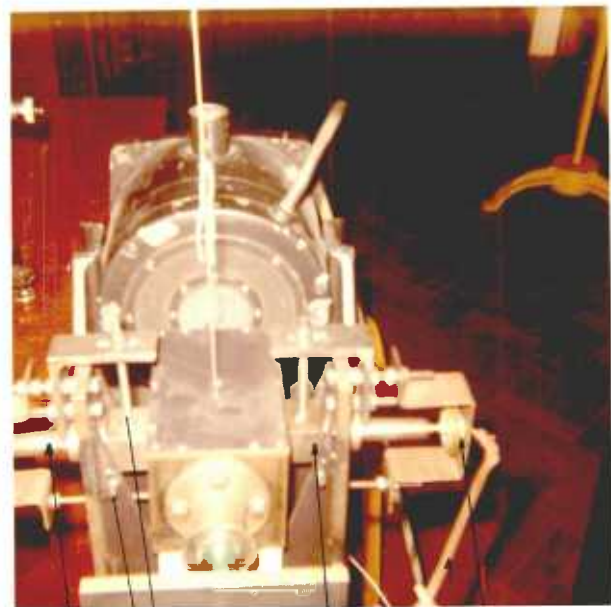
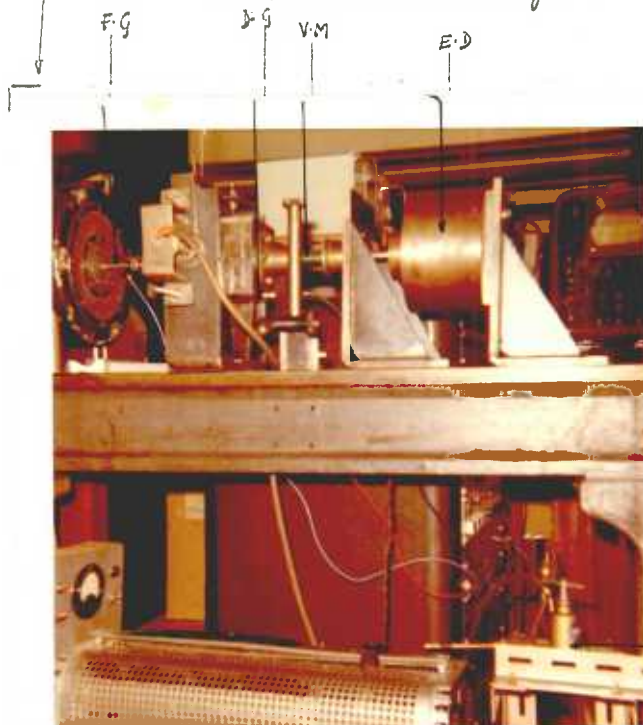


Fig 5.5 (above) Top view of the friction pad assembly.

- T.S = Tightening screw
 T.R = Tightening rod
 F.B = Friction Block
 A.P = Air Pipe
 B = Bellows
 P.R = Pressure regulator
 L.T = Linking Tube

Displacement Measurement: Wayne Kerr displacement guage (M.E.I 154) magnetic clamp, Wayne Kerr distance meter DM100B and a Wayne Kerr filter.

Acceleration Measurement: Environmental equipment accelerometer AQ 40 Resonant frequency 38KHZ. Transverse sensitivity 5% Frequency Response 70HZ, 0%. 700HZ, + 1%, 7KHZ, + 4% B and K sound and vibration analyser (1606), Fenlow charge amplifier B and K Pre-amplifier (2105)

Switch box:

Made from an insulated black metal box 6" x 4" x 4"
The switches were fixed into chosen positions and the connections soldered wherever this was convenient. The object of this was to insulate the wires and facilitate the change over from measuring different parameters.

PHASE MEASUREMENT

Dawe transistor phase serial No.2947 B & K Sound and Vibration Analyser.

5.1.3 SYSTEM PARAMETERS

SPRING: 3"/16 diameter mild steel coiled into a spring of length approximately $2\frac{2}{3}$ ".

Weight of vibrating body including the guide spigot = 9.331bf.

Weight of cylinder for electromagnetic damper = 0.6251bf.

Total mass = 9.9551bf.

Taking the washer and the P.V.C. or spring into consideration as part of the vibrating mass, Total vibrating mass 101bf.

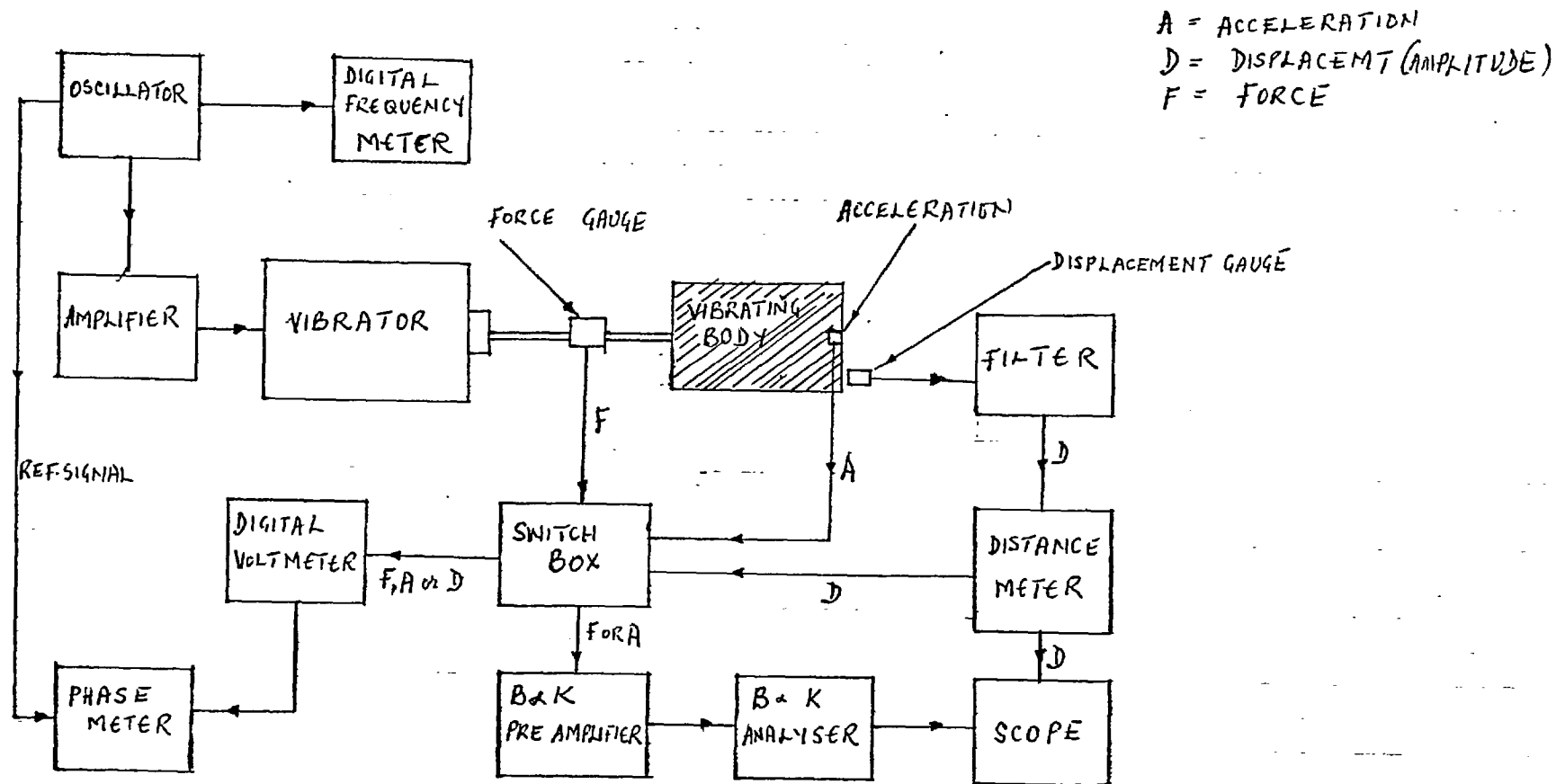


FIG. 5.6. BLOCK DIAGRAM OF THE APPARATUS

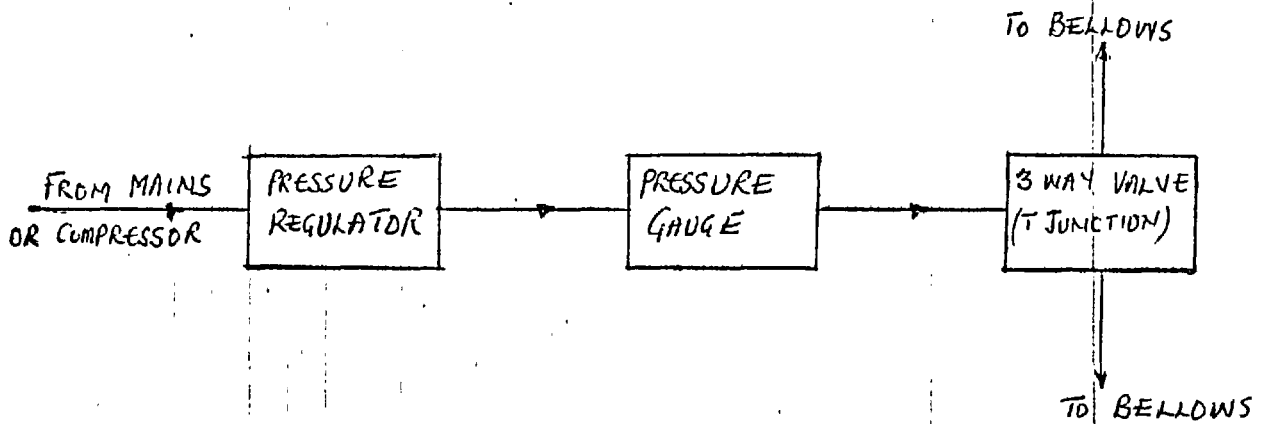


Fig 5.7 BLOCK DIAGRAM FOR THE AIR SUPPLY



Fig 5.8. BLOCK DIAGRAM FOR THE ELECTROMAGNETIC DAMPER.

THE VISCO-ELASTIC MATERIAL

This was a green P.V.C. flexible industrail hose.

Internal diameter about one inch

55B.S Softness. B.S. 2571

MAXIMUM TOLERANCE EFFICIENCY - 5% on internal and external dimensions and length - 10% on wall thickness .

THE SET UP

The general set-up of the apparatus and details of some parts are shown ^{by pictures and} block diagrams in Figs. 5.3 to 5.5.

5.1.4

THEORY OF THE ELECTRO-MAGNETIC DAMPER

F = Force required to move damper cylinder

U = Velocity of damper

B = Magnetic flux density

e = E.M.F. induced in cylinder

i = Current due to e

D = Diameter of Cylinder

l = Length of conductor in magnetic field
 $= \pi D$

L = Width of conductor

ρ = Resistivity of cylinder material

t = Thickness of Cylinder

a = Cross-sectional area of conductor Lt

R = Resistance of conductor

H = Magnetic force

I = Magnetising current.

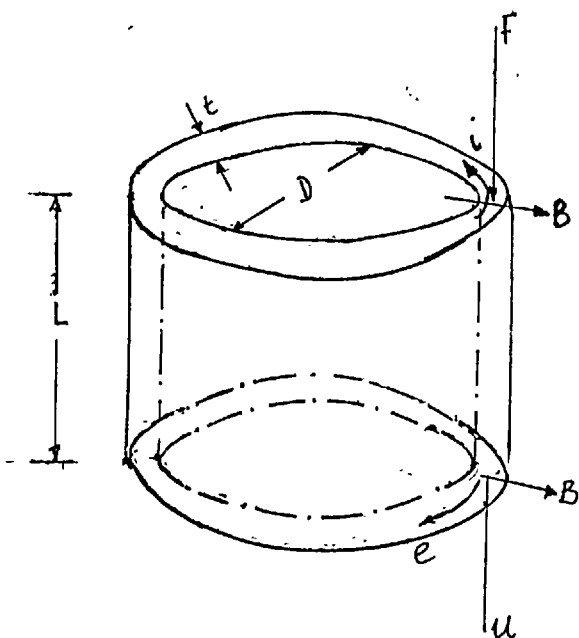


Fig. 5:9 cross section of the cylinder of the electromagnetic damper

For a conductor of length l , moving at a velocity u across a flux field B , the e.m.f. induced in the conductor is given by:

$$\underline{e} = u \underline{l} B$$

$$e = Bu l = Bu \pi D$$

$$\text{Resistance of conductor} = \rho / a = \rho \frac{\pi D}{L t}$$

Since the conductor is a loop, the e.m.f. causes a current i to flow through the conductor.

$$i = \frac{e}{R} = \frac{Bu \pi D}{\rho \frac{\pi}{L t}} = \frac{Bu l t}{\rho}$$

Now, to push a current carrying conductor through a flux field at a constant velocity, one must apply a force F , given by:

$$\underline{F} = i \underline{l} \underline{B}$$

$$\therefore F = B i l = B i \pi D$$

$$\begin{aligned} \text{Thus } F &= B \times \underline{Bu l t} \times \pi D \\ &= \underline{B^2 \pi D L t} \times u \end{aligned}$$

Now, $B = \sqrt{\mu} H$ where μ is the permeability of the magnetic material in the electromagnet.

also H is proportional to I

$\therefore B$ is proportional to I

Hence, for any constant magnetising current I passing through the coils of the electromagnetic field, the flux B remains constant

$\therefore F$ is proportional to $u = cu$

$$\text{Hence damping coefficient } c = \frac{B^2 \pi D L t}{\rho}$$

Thus demonstrating that a viscous type of damping is obtained with damping force proportional to velocity.

CALIBRATION OF VISCOUS DAMPER — (a)

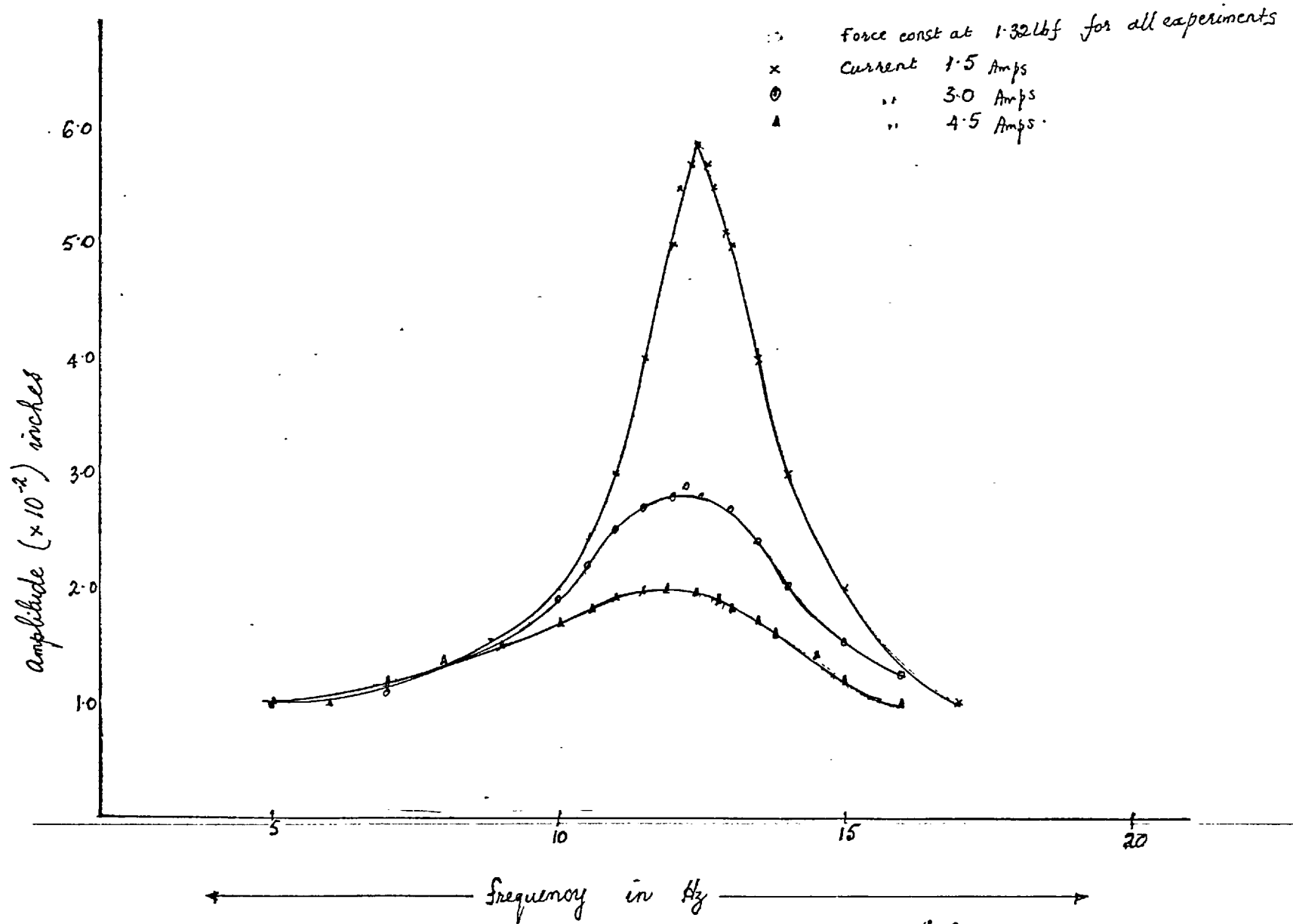


Fig 5.10 Frequency response with viscous damping only (at ^{the same} constant excitation force)

CALIBRATION OF VISCOUS DAMPER (b)

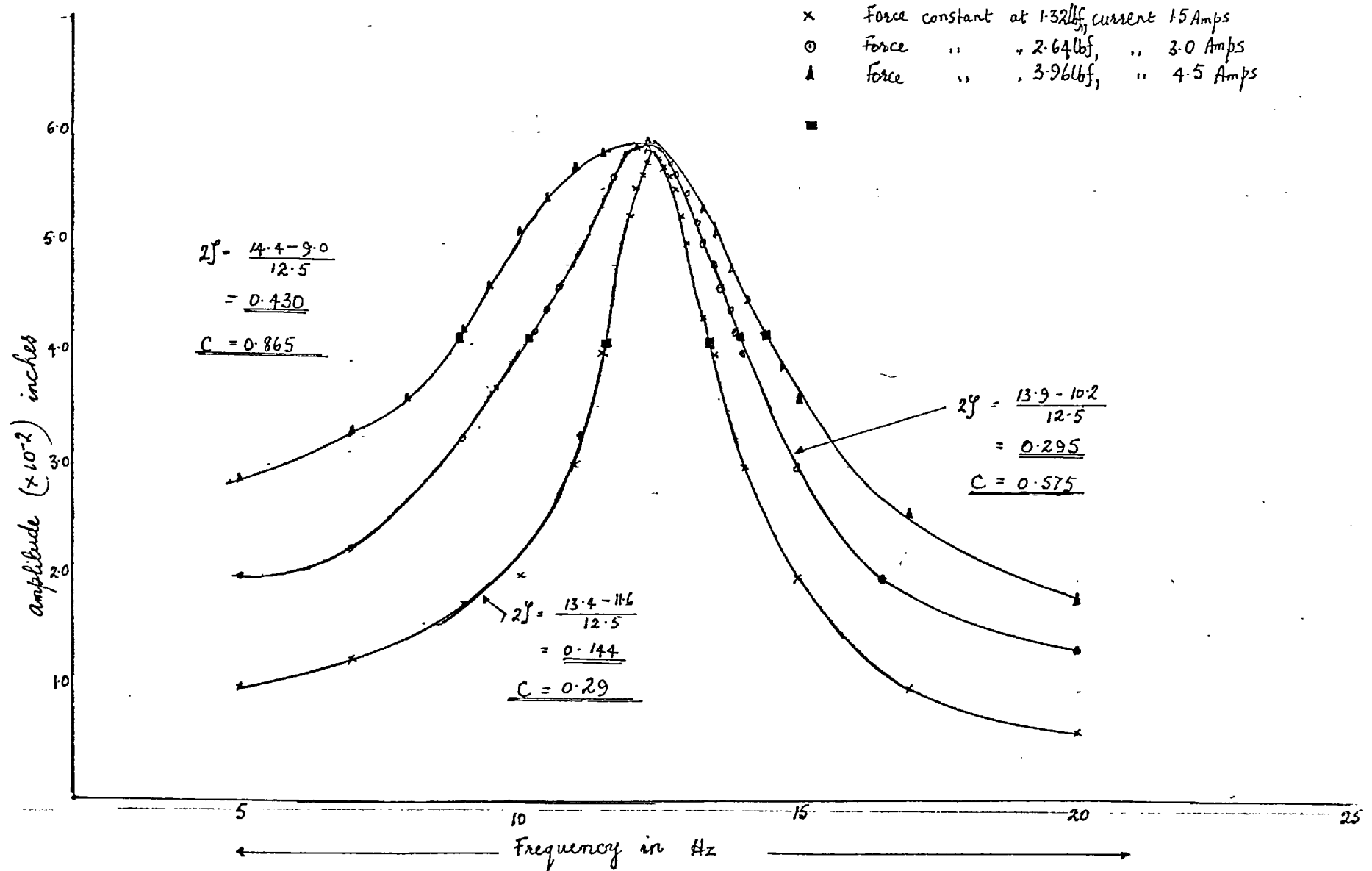


Fig 5.11. Frequency with viscous damping only (at different constant levels of excitation force)

CALIBRATION OF VISCOUS DAMPER

- 61 -

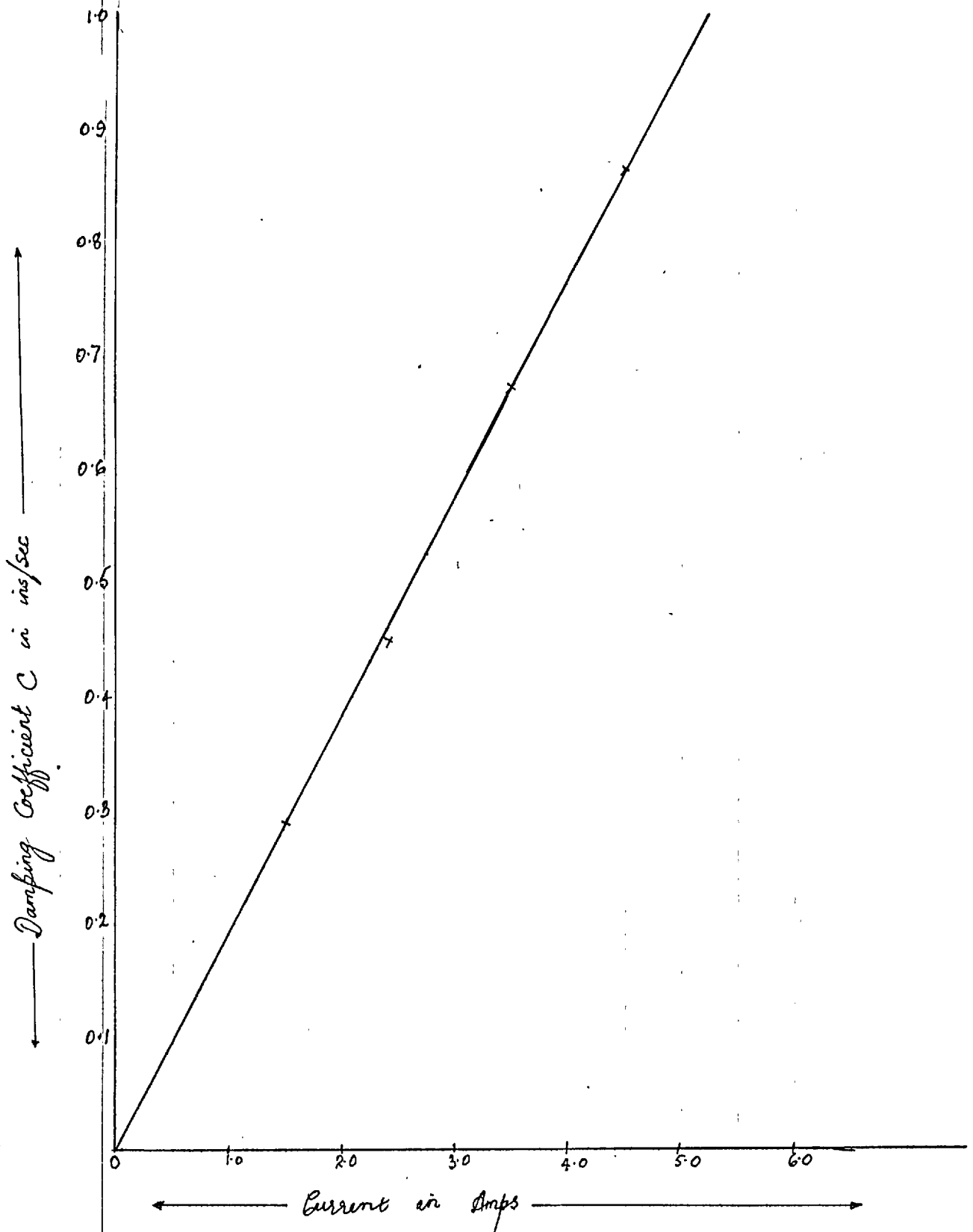


FIG 5.12. Variation of Damping coefficient, C with Current

If a rationalised system of units is used (eg. M.K.S.) this expression is correct as it stands. However, if C.G.S. units are used, a modification factor of 10^{-9} is required:

$$C = \frac{B^2 \pi D L T}{\rho} \times 10^{-9} \text{ mechanical ohms (dynes/cm)}$$

(in C.G.S. Units).

5.2 CALIBRATION:

The transducers requiring calibration are the Force guage, the Accelerometer and the Displacement Guage. The general set-up was as shown in the block diagram in fig. 5.6 By making use of the relation, $P = m\ddot{x} = mw^2x$(5.11) where P = excitation force in lbf, m = mass in lbs, x = displacement in inches. Calibrations for two of the transducers can be deduced if the third one is calibrated. For instance if the Force guage is calibrated and the voltage readings of the accelerometer and displacement guage are noted at a particular frequency the calibrations can be obtained directly as follows: the voltage reading for the accelerometer represents

$$\ddot{x} = P/m \text{ in/Sec}^2 \text{ --- (5.2)}$$

and from this, the displacement, $x = \frac{\ddot{x}}{w^2} = \frac{\ddot{x}}{(2\pi)^2 f^2} = \frac{P}{m(2\pi)^2 f^2} \text{ --- (5.3)}$

The relationships expressed in equations (5.2) and (5.3) are reversible therefore checks can be made by calibrating each transducer independently and thus obtaining calibrations of the others for comparison. The independent calibrations were done as follows:

5.2.1 CALIBRATION OF FORCE MEASURING EQUIPMENT

The calibration was done with an Environmental equipment charge calibrator (QMCC).

The system was set vibrating for some time to ensure that all components were working satisfactorily. The oscilloscope and digital Voltmeter were switched to D.C. and the lead of the Force guage was disconnected from the Force guage and connected to the Calibrator. Specified Charges were then introduced into the charge amplifier from the calibrator

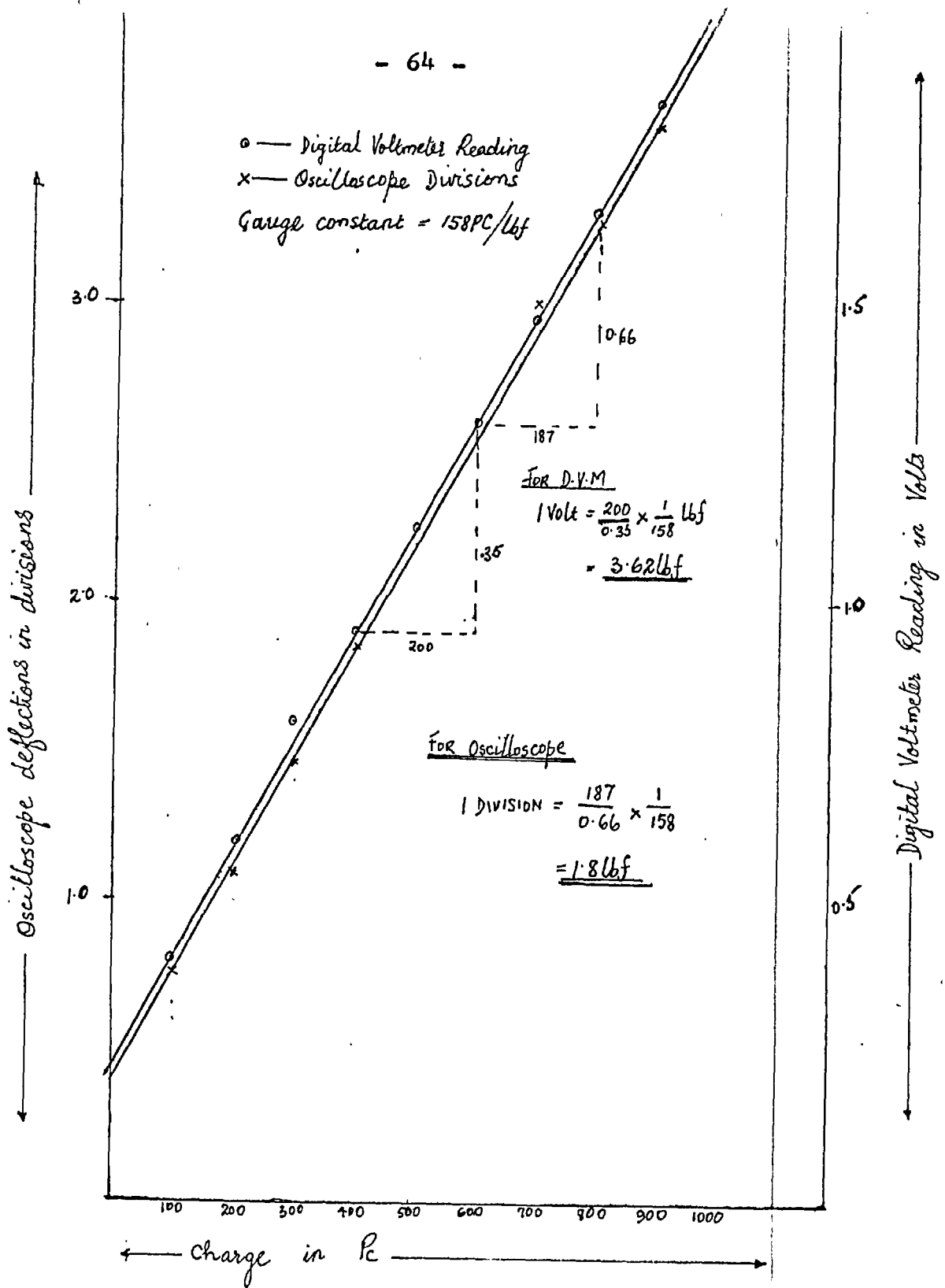


Fig. 5.13 Variation of D.V. Voltage and Oscilloscope Reading with Charge.

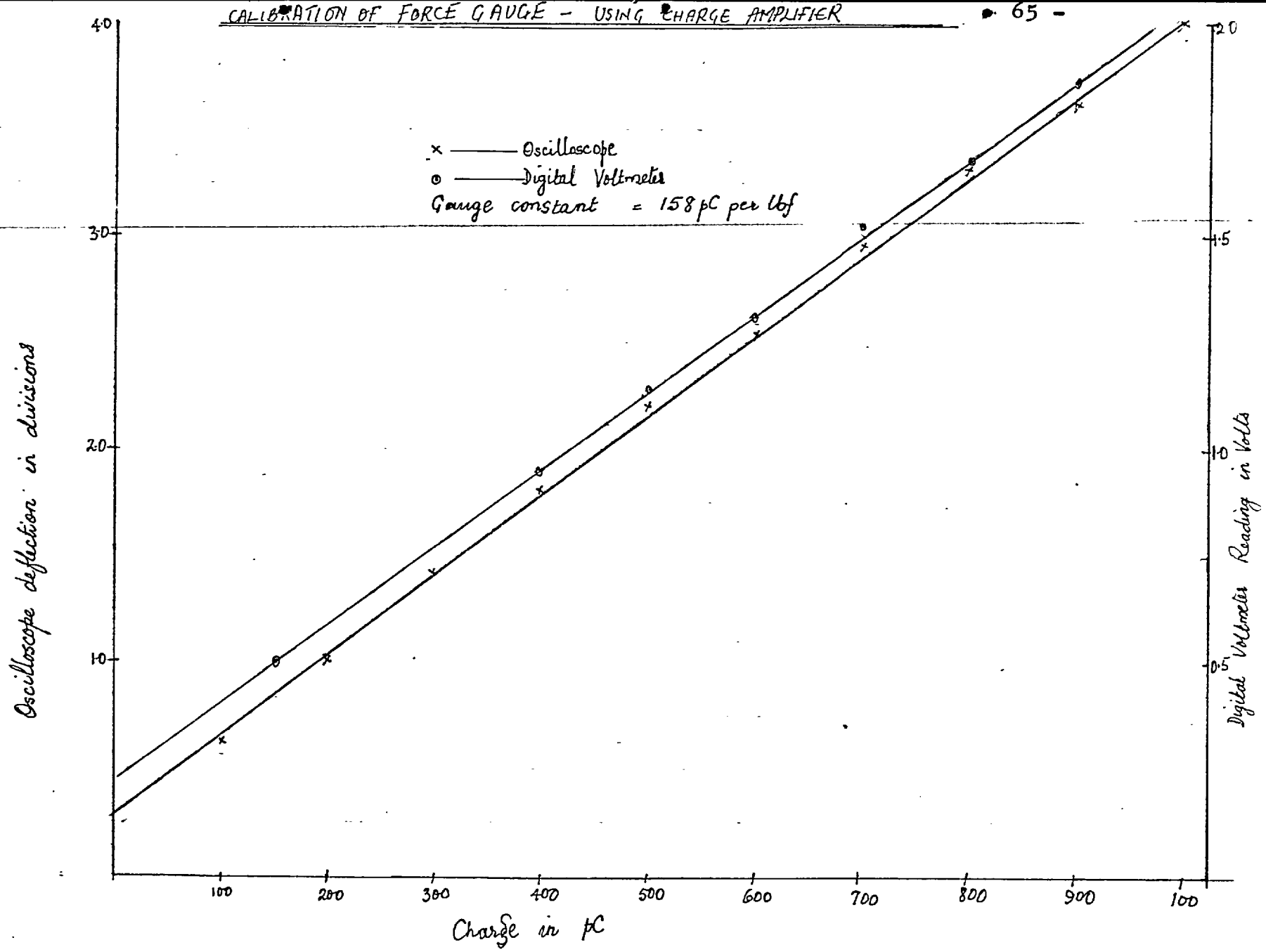


Fig 5.14 Variation of DV Voltage and Oscilloscope divisions with Charge.

and the corresponding voltage and deflection noted on the Digital Voltmeter and the Oscilloscope. This was repeated until sufficient points were obtained to plot ^{Fig 5-13 and 5-14} graphs. Readings were taken for minimum medium and maximum gain positions of the oscilloscope. Using the force guage calibration given by the manufacturer the calibrations of the Digital Voltmeter and Oscilloscope were thus obtained. The lead was reconnected to the force guage and some dynamic readings were taken bearing in mind that the reading was a peak to peak value as the Digital Voltmeter and Oscilloscope were reswitched to A.C.

5.2.2. CALIBRATION OF THE ACCELEROMETER:

The accelerometer was calibrated with set of B and K vibration measurement equipment already described and shown in the block diagram in fig. 5-6 . With the set up as shown in this diagram, the accelerometer was screwed onto the "vibrating table" of the vibration pick-up pre-amplifier with a $1/8$ Whitworth screw, and the pre-amplifier was then connected to the mains. The knobs marked on the pre-amplifier "Attenuation" and "Sensitivity Adjustment" switched to positions '1' and '0' respectively and the integration switch was switched to position "Acceleration". The analyser was then switched to the linear 100 mV range and the knob marked "Resonance Adjustment" turned until maximum deflection was obtained on the instrument meter. A small nut was then placed on the "vibrating table" and the Amplitude Adjustment knob turned until the nut just started rattling on the table. The acceleration peak value of the vibrations was now equals that of gravity. The

CALIBRATION OF ACCELEROMETER

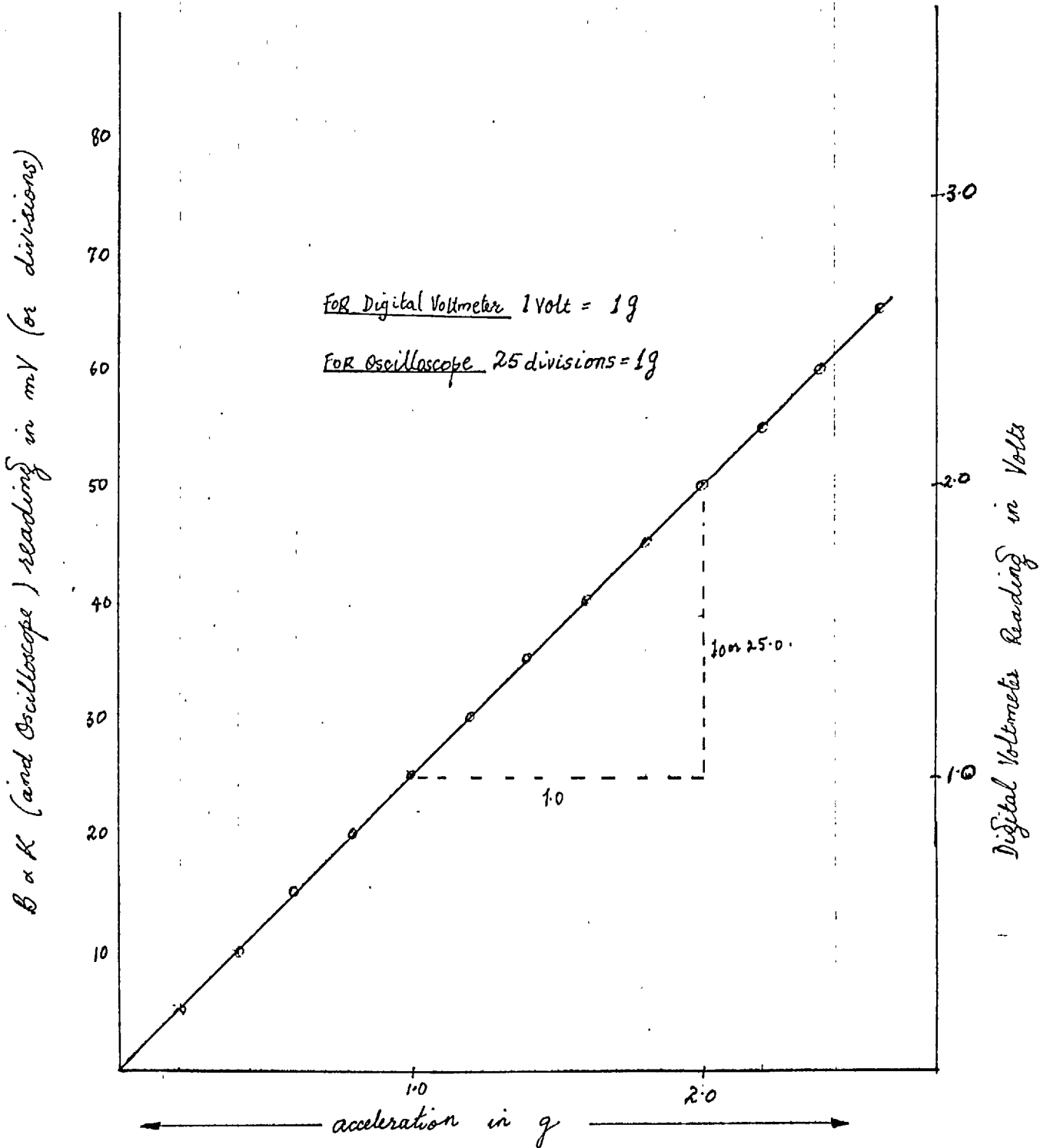


Fig. 5.15 Calibration of acceleration amplitude on Digital Voltmeter and Oscilloscope

manufacturers instructions also include adjustments that can be made so that the "Average Value" the "Peak Value" and the "Root mean square values" could be obtained directly on the meter. In this experiment only the peak voltage equivalent of vibrations equals to that of gravity was required. The experiment was therefore repeated three times and the main result was taken. The pre-amplifier was then disconnected from the mains, and the experiment was continued by fixing the accelerometer on the vibrating body and running at a "constant known acceleration" and measuring the force and displacement voltages to establish calibration constants for the force and displacement gauges.

5.2.3 CALIBRATION OF THE DISPLACEMENT GAUGE

The displacement gauge was calibrated in two different ways. It was calibrated statically with a dial gauge and dynamically with a travelling microscope. In the static calibration the set up was shown in fig. 5.16

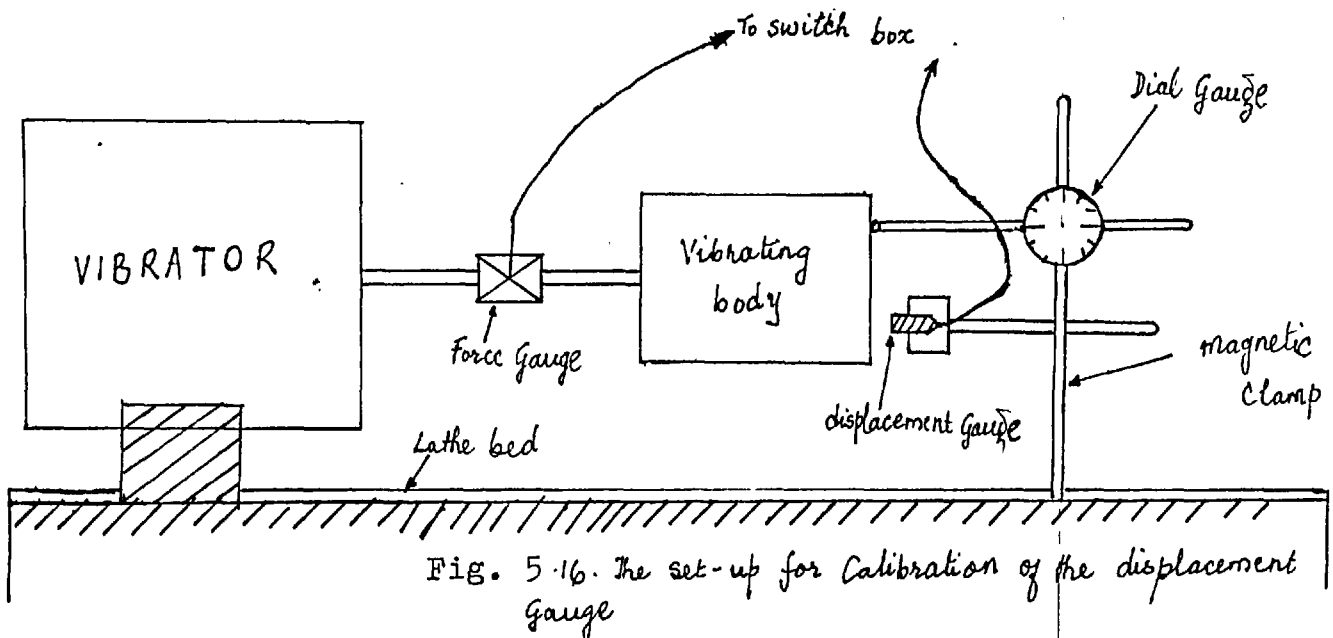


Fig. 5.16. The set-up for Calibration of the displacement Gauge

The vibrating body was pulled towards the dial gauge slowly until a specific reading, say 0.001" was attained, and the reading of the Digital Voltmeter and the Oscilloscope (which had both been switched to D.C.) were noted. The displacement was increased in steps and further readings were taken so that graphs could be plotted to obtain a mean static calibration.

In the second case where the travelling microscope was used, the dial gauge was removed and the system was set vibrating at a specific frequency. The amplitude of vibrations of the edge of the vibrating body nearest to the displacement gauge was measured and also the voltage readings on the Digital Voltmeter and Oscilloscope were noted. The excitation force was increased in stages and the measurements were repeated at a constant frequency. The force was then ^{kept} constant and the frequency varied whilst the parameters under consideration were measured again. After a mean value had been obtained the Excitation force and acceleration voltages were also read so that calibration constants could be established for the Force gauge and the accelerometer.

CALIBRATION OF DISPLACEMENT GAUGE -- (a) USING DIAL GAUGE (CONTD.)

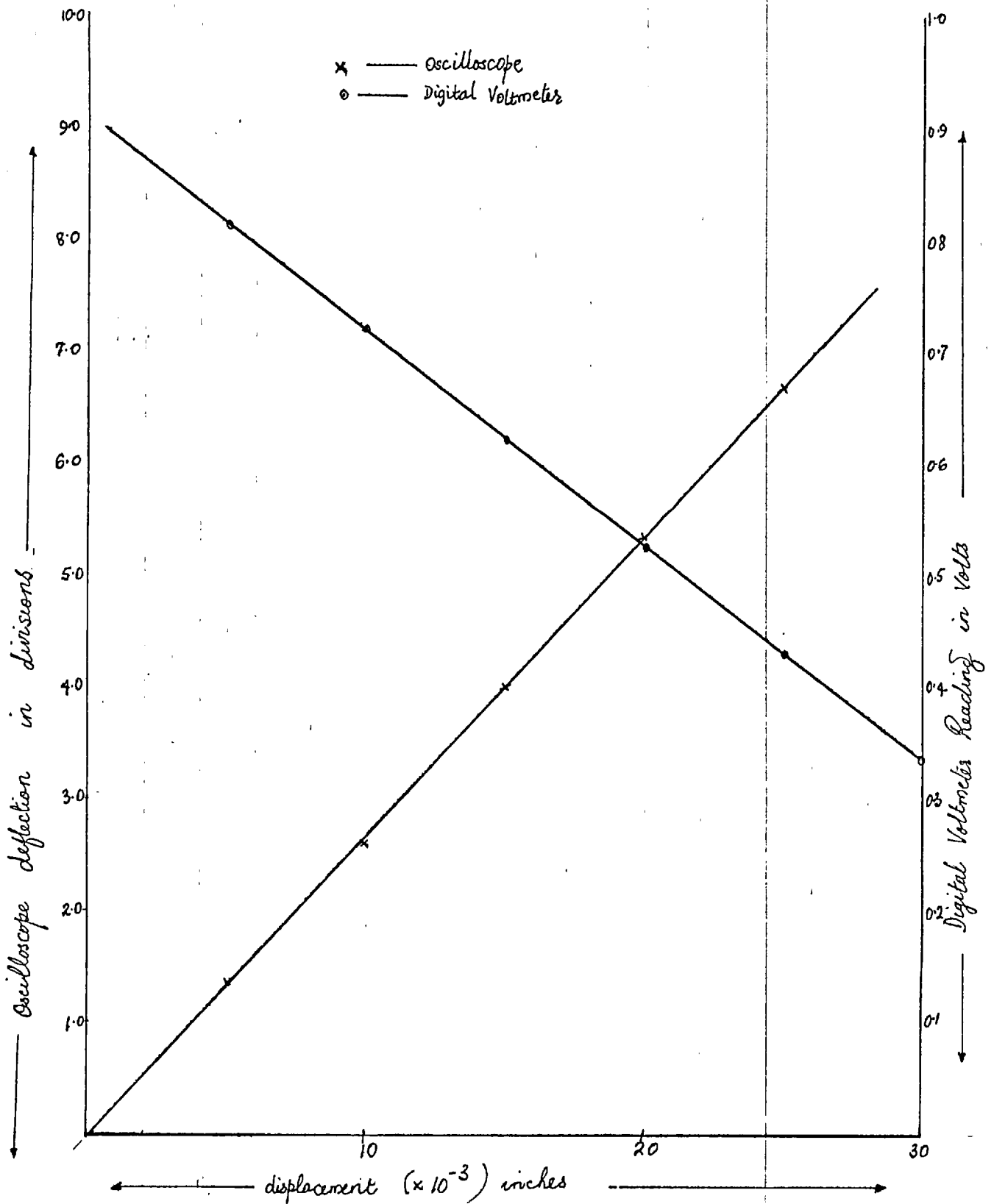


Fig 5.17. Variation of D.V. Reading and oscilloscope deflection with (amplitude) displacement

CALIBRATION OF DISPLACEMENT GAUGE -- (b) USING TRAVELLING MICROSCOPE (DYNAMIC TEST)

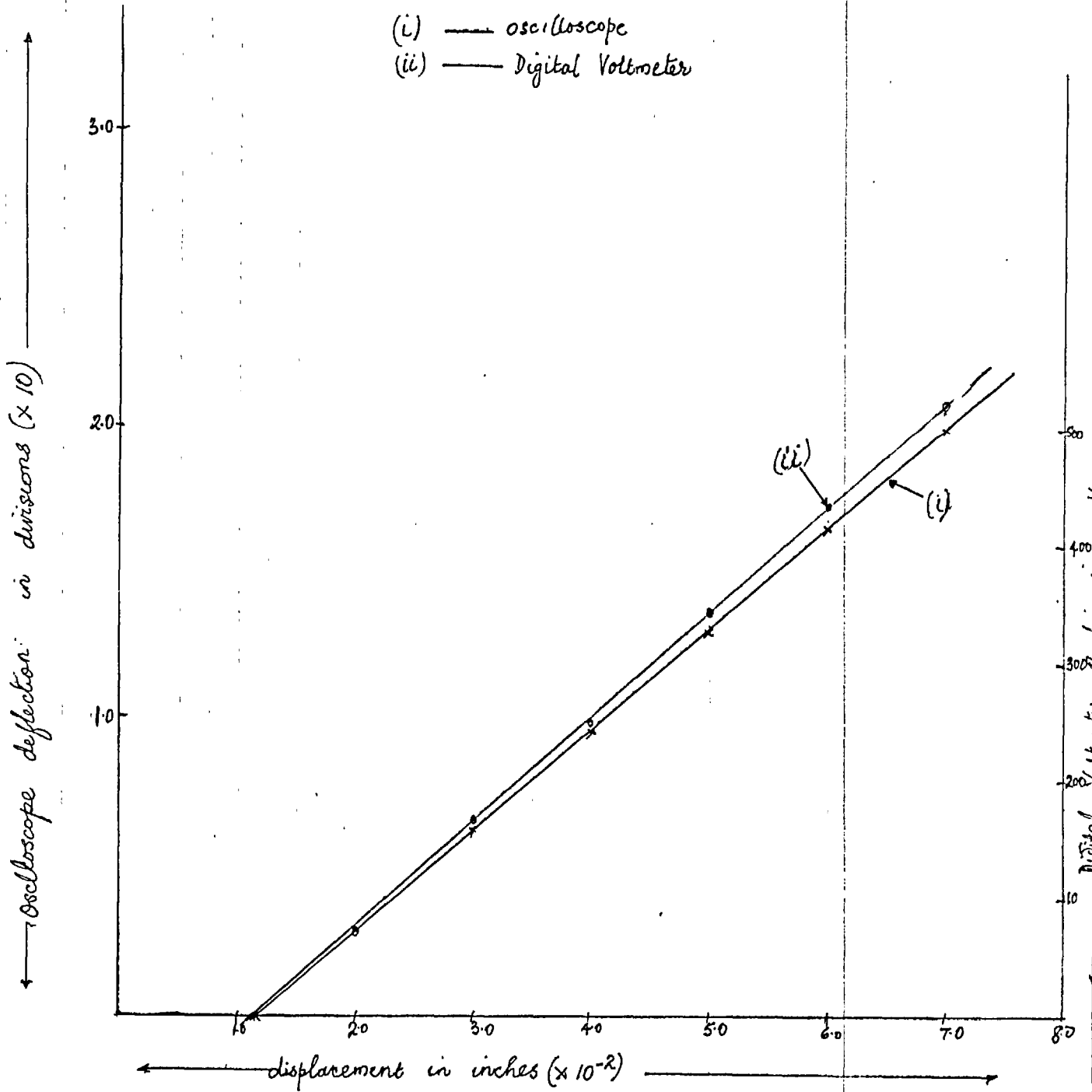
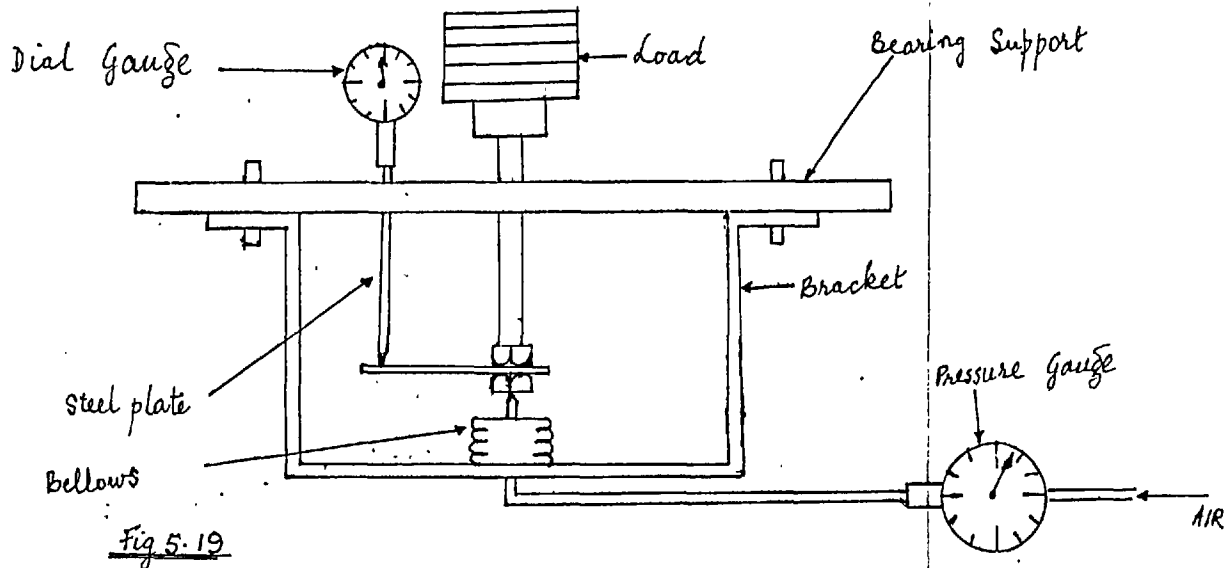


Fig 5.18 Variation of D.V. Voltage and oscilloscope deflections with amplitude

5.2.4 Static Calibration of the Friction Block system

In the friction block system, it was necessary to calibrate the pressure gauge while the system remained as set up. This meant that the brackets, blocks and bellows had to be assembled in place before the calibration could be done. The set-up for the calibration is as shown in fig. 5.19



The dial gauge was set to zero after a little pressure had been introduced into the bellows to counterbalance the inertia of the blocks and rods. A pressure of $1/2 \text{ lbf/in}^2$ was now introduced into the bellows and the reading on the dial gauge was noted. Load was now put on top of the friction block as shown, in increments until the dial gauge read zero again. (At each loading the dial gauge was tapped gently to relieve friction). The load was now removed and the reading on the dial gauge was noted again. This was found to be approximately the same as before. The pressure was now increased to $(1 \text{ lb per inch}^2)$. 1 lbf/in^2 and the process was repeated. This was done for pressures up to 6 lbf/in^2 .

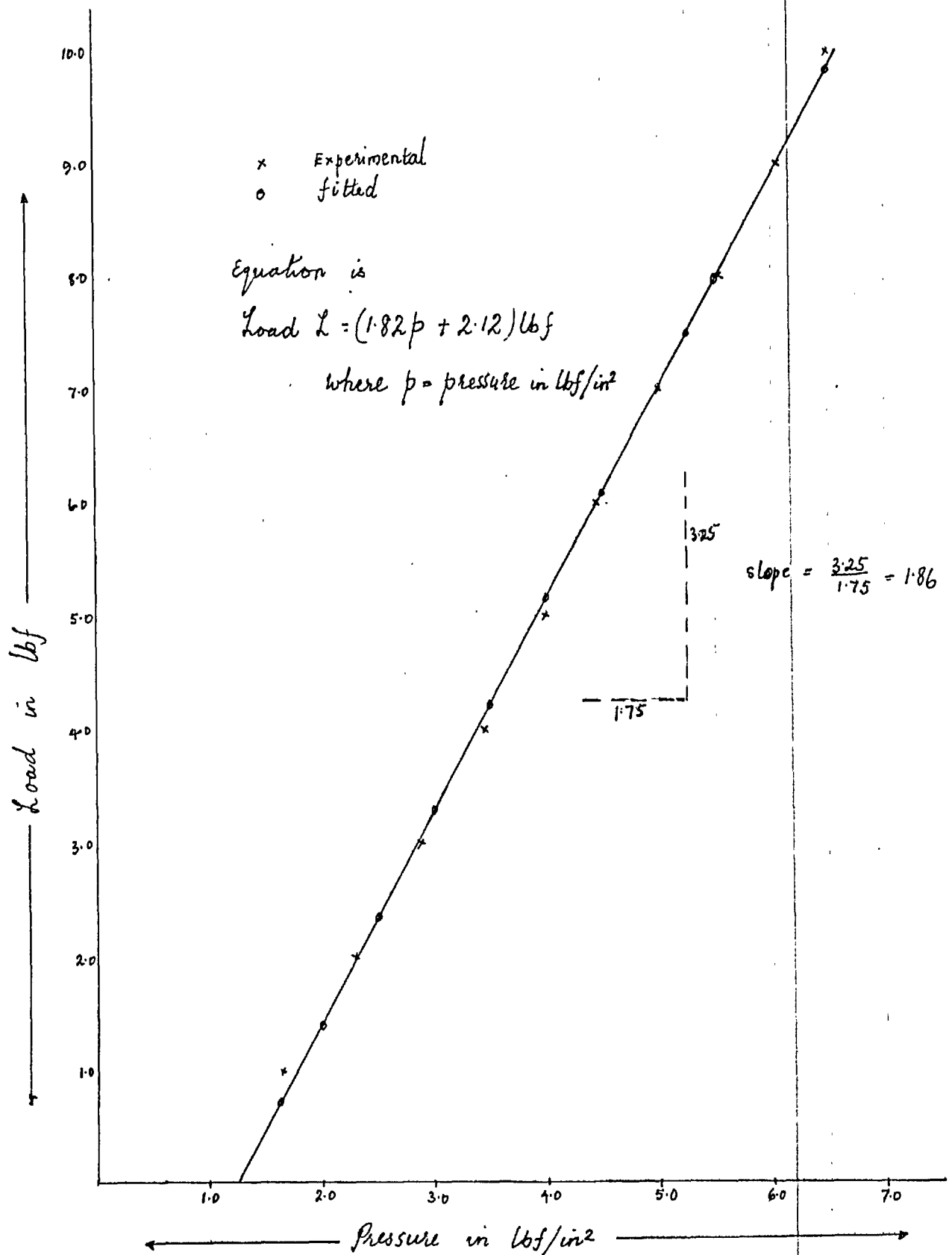
CALIBRATION OF PRESSURE GAUGE (4)

Fig 5.20 Variation of Load with pressure for zero deflection

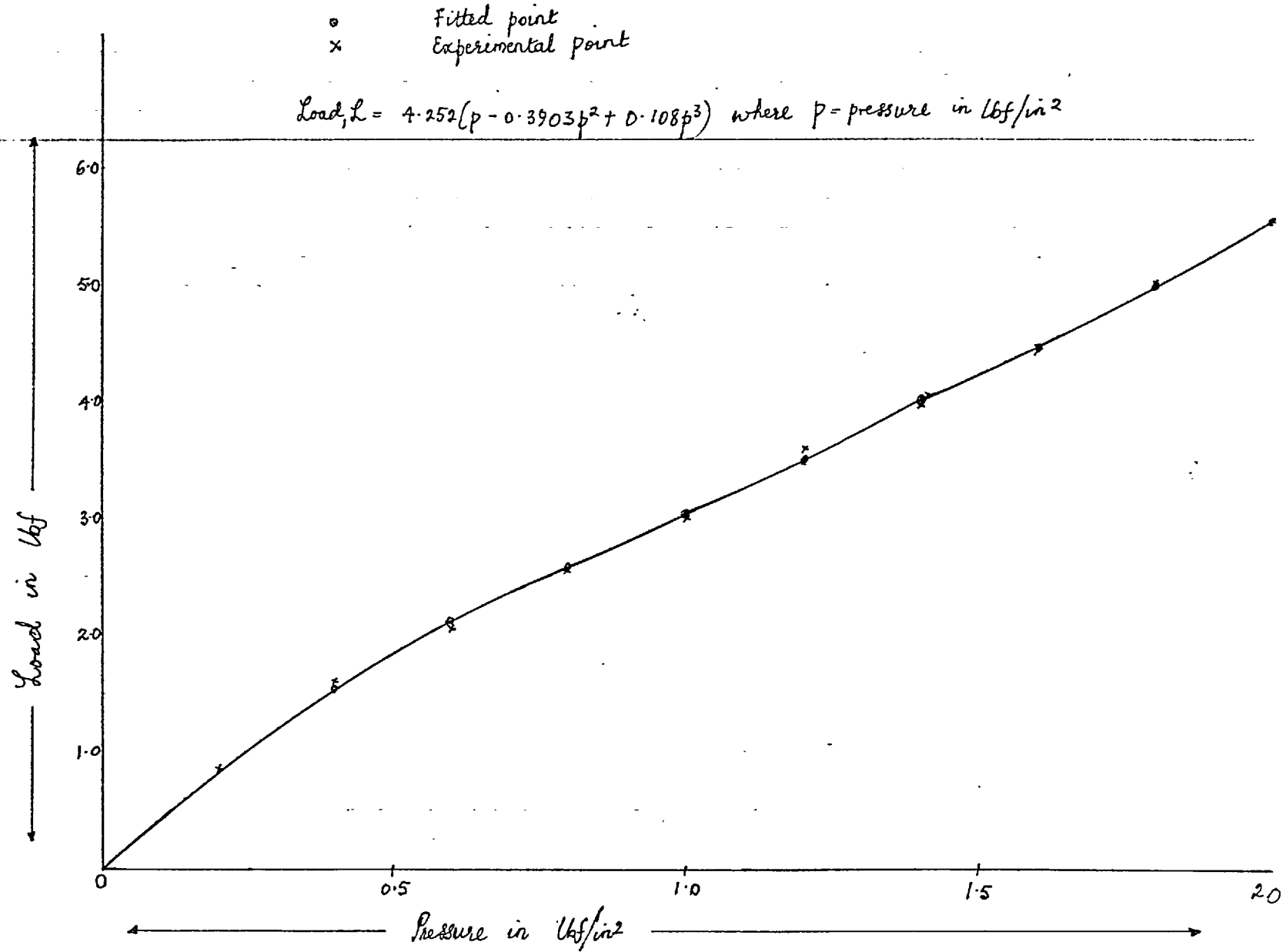


Fig 5.21 Variation of Load, L with pressure p for zero deflection

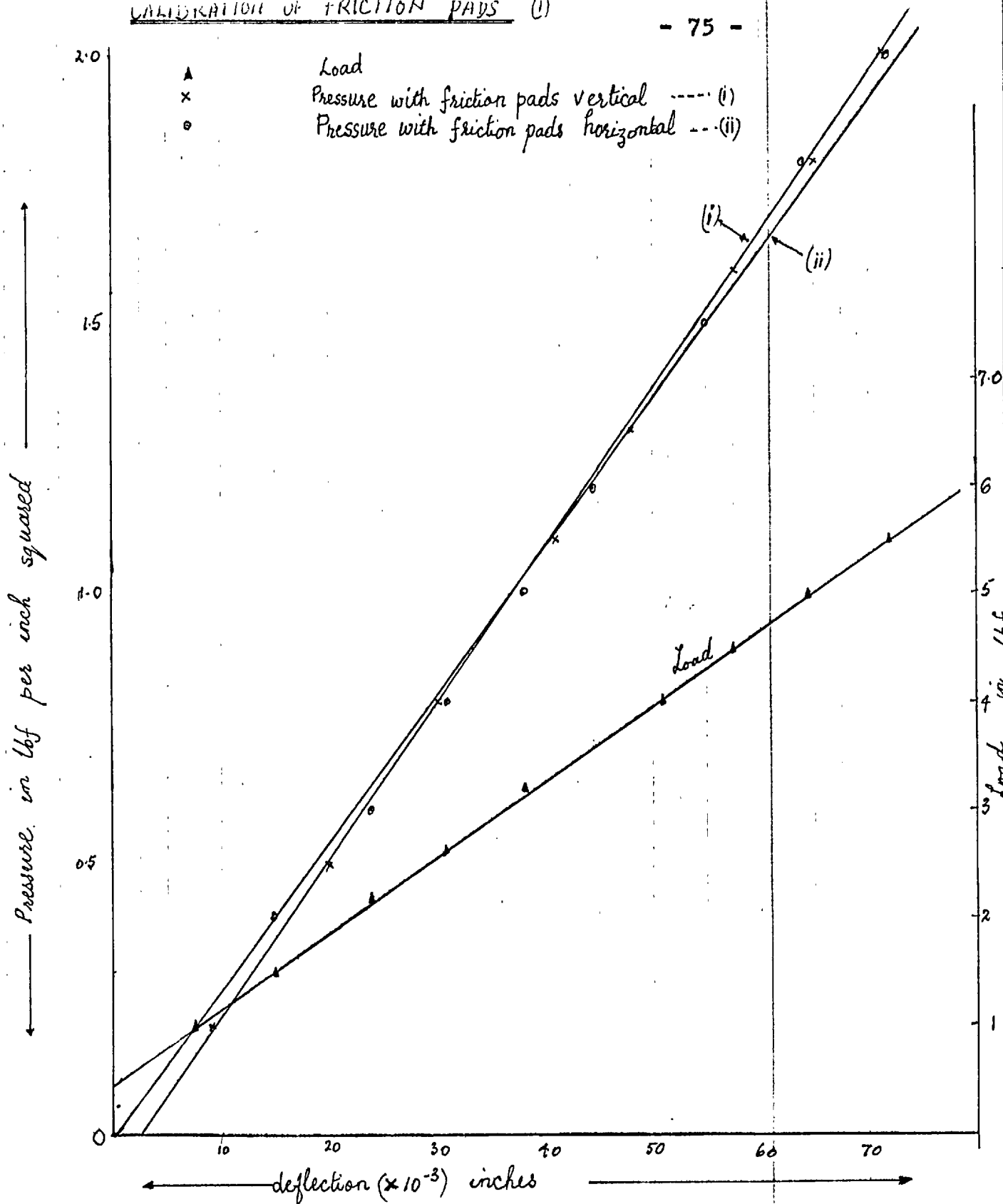


Fig 5.22 Variation of (Normal) Load and Pressure with deflection

CHAPTER 6

6. FORCED VIBRATIONS OF A BODY WITH COULOMB DAMPING

6.1 INTRODUCTION

The object of this work is to examine the Response and Phase of mixed damped systems (Coulomb, viscous and viscoelastic), both theoretically and experimentally. The Apparatus which is described in Chapter 5 is a single degree of freedom system which can be made to vibrate with any of the three types of damping mentioned either singly or together.

Viscous damping is well known and has been tested thoroughly experimentally and proved to agree with the theory. Because most damping mechanisms can be approximated to or are of a viscous nature and the equation of motion of a single degree of freedom system with viscous damping is linear, viscous damping has become the main subject of text books on vibration.

Viscoelastic damping has also been investigated by many researchers (12 to 19), and various equations have been tested and proved to agree with experimental work. The equation used in the case of forced vibrations does produce results that agree with experimental work, however if the same expressions are used for free oscillations the solution of the equation is meaningless. Neumark's report (18) makes this con-

tradiction clear and suggests that the equation be modified for free vibrations.

Very little work however has been done on Coulomb damping, therefore it was found necessary to investigate this thoroughly and either establish the validity of the assumptions and hence the expressions produced by other researchers (5,6,9,10 and 11) for coulomb damping or endeavour to establish an alternative theory that can be verified experimentally. As most previous work has been done based on the assumption that the coefficient of dry friction is constant, a test on the validity of this assumption was deemed to be of primary importance.

6.2 EXPERIMENTAL WORK

6.2.1 MEASUREMENT OF FRICTION FORCE AT DIFFERENT FREQUENCIES

As stated earlier the variation of the coefficient of friction with frequency was to be determined for a vibratory motion and it therefore meant that the apparatus had to be set up to satisfy equation (31). In this part of the experiment the displacement was not measured directly and the system was as shown in Fig.(6.1). Use was made of Newton's Second Law i.e. $P_0 e^{j\omega t} = m\ddot{x}$ (6.1) to test the linearity of the equipment and also enable the friction force to be determined. The frequency was varied within the range 3Hz and 1000Hz, and the acceleration was kept

constant by adjusting the input force.

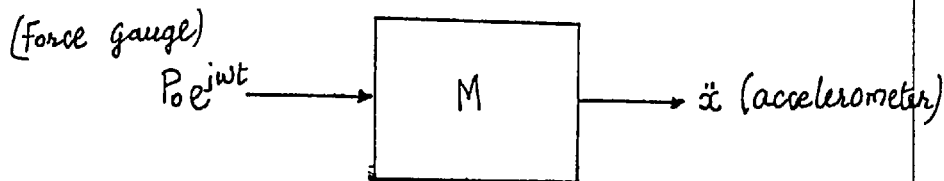


Fig. 6.1 Vibrating body with constant acceleration

This was repeated for different values of acceleration. As expected the measured force amplitude P_0 remained constant at all frequencies up to about 450Hz when a deviation from linearity was observed as shown in Fig.(6.3) because the mass M being a rectangular hollow box ceased to behave like a rigid body. Phase readings showed that there was no phase difference between the excitation and response forces.

6.2.2. INTRODUCTION OF FRICTION FORCES

Friction forces were now added to the above system. This was done by passing air into the bellows and thus putting pressure ^{on the} pads. As the friction pad system had been calibrated the Normal Force acting on the body was fixed at a particular value by using the regulator to produce the required equivalent pressure. The pressures at which readings were taken were $1\frac{1}{2}$, 2, 3, 4, and 5 lbf/in²

At each pressure setting a few minutes were

allowed for the pressure to become stable. The surface of contact of the friction blocks and the vibrating body were cleaned with trichloroethylene and dried before any contact was allowed to take place. The acceleration was kept constant at the same values as before and the excitation force, and the phase were measured with variation in frequency.

The system tested is thus represented in Fig.(6.2) which is a model for equation (3.1).

Without friction the equation of motion is:

$$m\ddot{x} = P_0 e^{j\omega t} = \text{Constant} \dots\dots\dots (6.1)$$

With friction the equation is

$$m\ddot{x} \pm F = P_1 e^{j\omega t}$$

$$\text{OR} \quad m\ddot{x} + xR(\cos\omega h + j\sin\omega h) = P_1 e^{j\omega t} \dots\dots (6.2)$$

hence from equation (6.1)

$$P_1 e^{j\omega t} = P_0 e^{j\omega t} \pm F \quad \dots\dots (6.2a)$$

$$\text{OR} \quad P_1 e^{j\omega t} = P_0 e^{j\omega t} + xR(\cos\omega h + j\sin\omega h) \quad \dots\dots (6.2b)$$

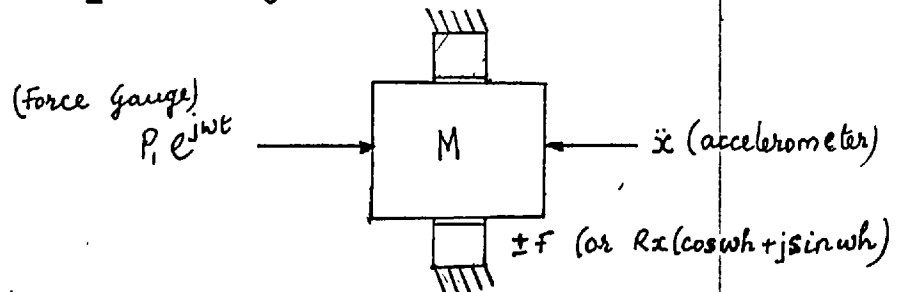


Fig. 6.2. Vibrating body on the presence of Coulomb
Force at constant acceleration

Fig 6.3. Variation of Force (or acceleration) with frequency at constant acceleration (or force) amplitude.

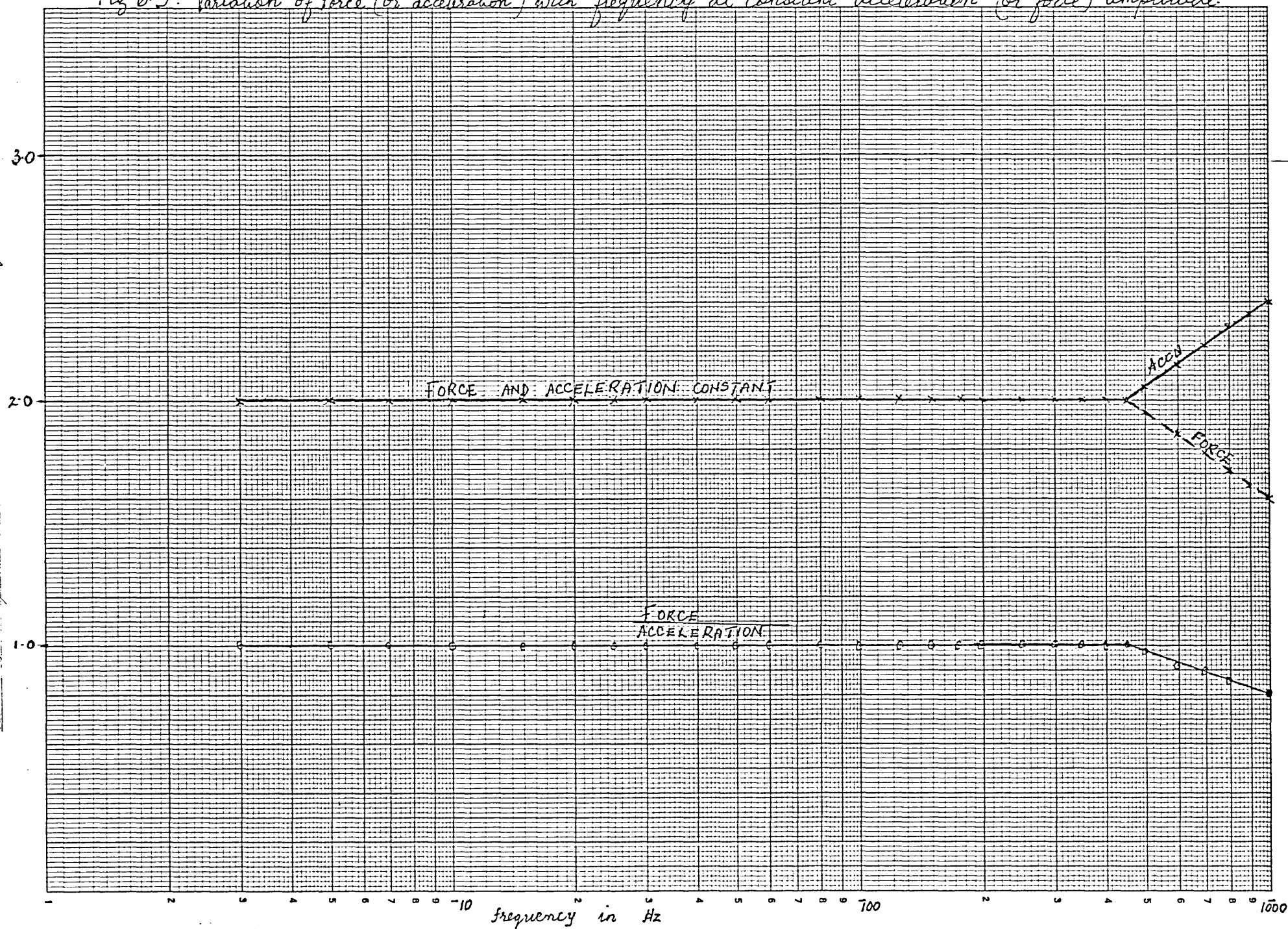


Fig 6.4. Variation of Excitation force with frequency for different normal clamping loads at constant acceleration amplitude

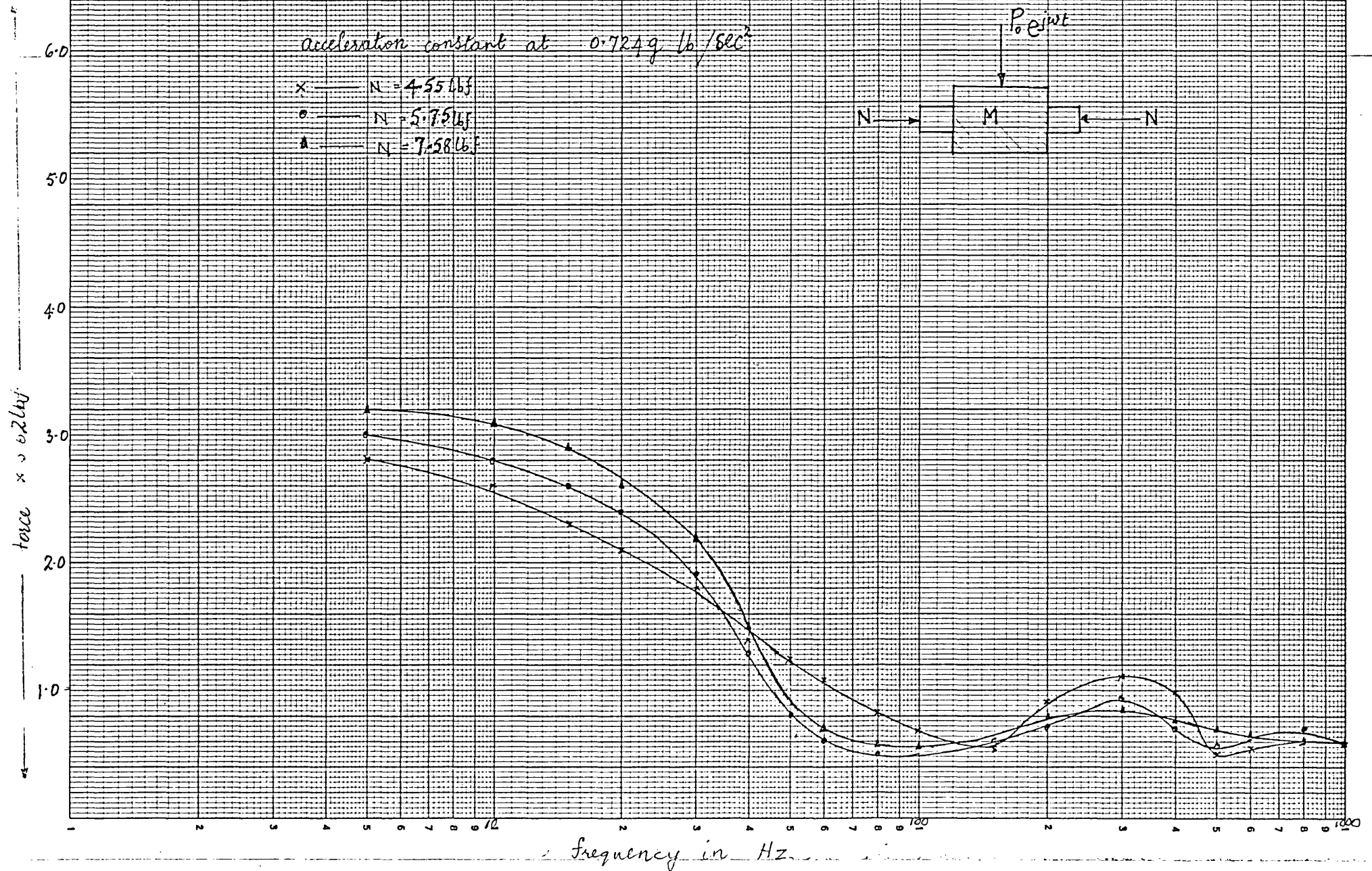


Fig 6.5. Variation of Excitation force with frequency at different normal clamping loads at constant acceleration ampli

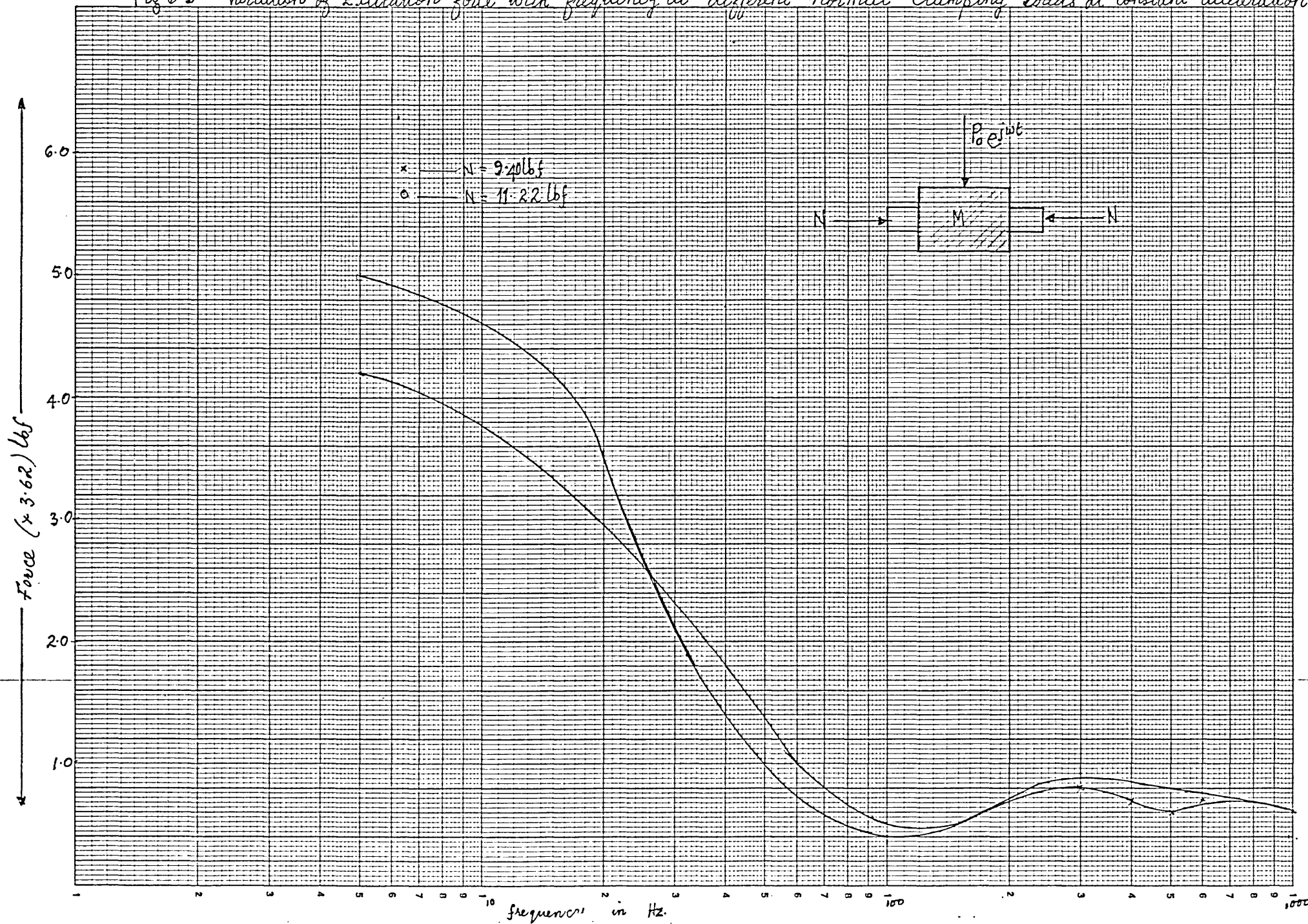
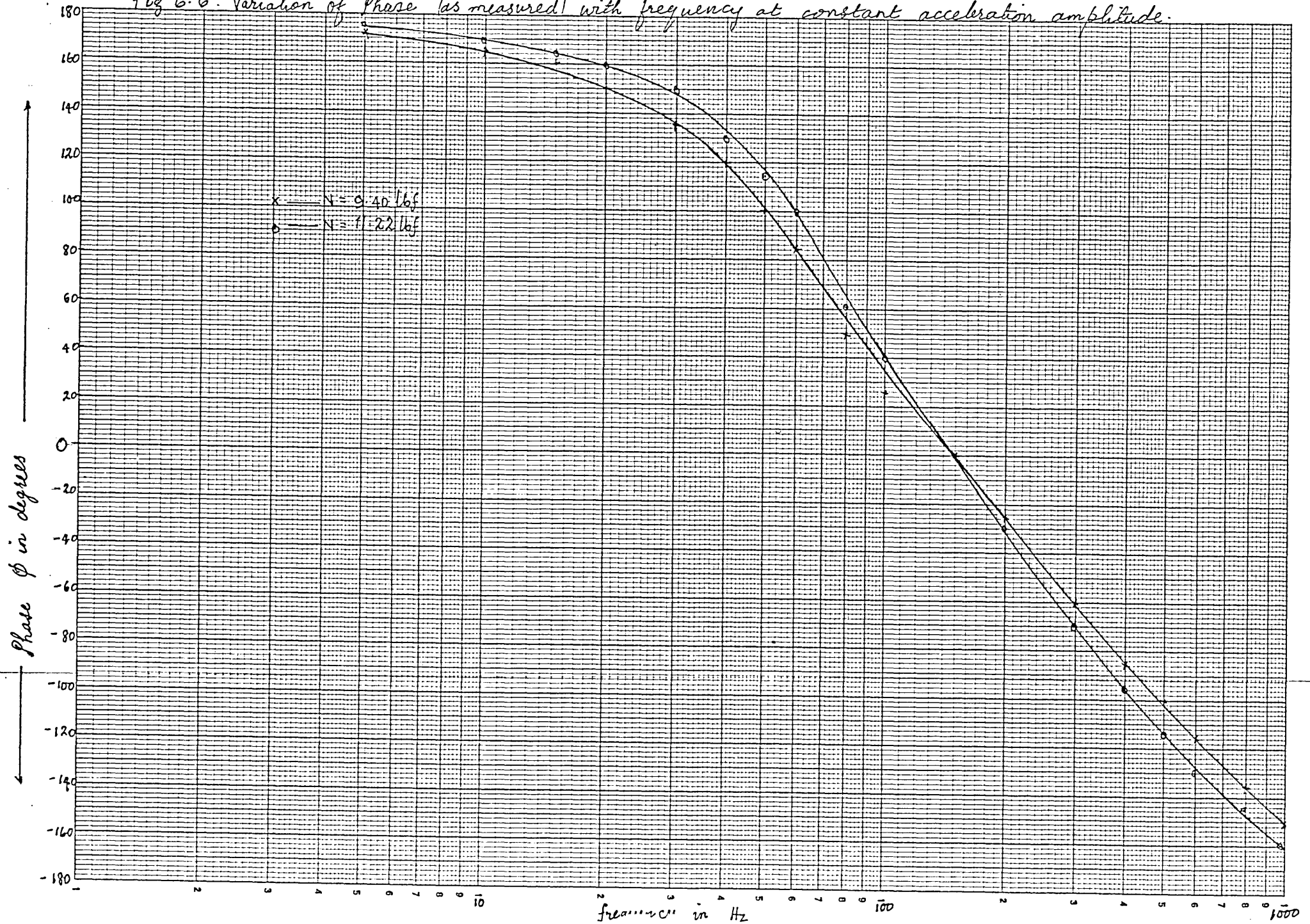


Fig 6.6. Variation of Phase (as measured) with frequency at constant acceleration amplitude.



Readings revealed that there was a phase difference between the excitation force and the acceleration; results are shown in Figs.(6.3) to (6.6).

As can be seen they do not support equation (4.6)

6.2.3 TEST OF FRICTION BLOCKS

To ensure that these unexpected results were not due to the friction block system a thorough test was made on the blocks separately. The bellows were designated "A" and "B" and tested statically and dynamically (Appendix 1). The static test was done by loading the bellows with known weights and measuring deflections with a dial gauge. The stiffness of the bellows was found to increase with the air pressure and therefore the static tests were repeated for specific cases with the same amount of air pressures in the bellows as was used in the previous dynamic experiments.

By feeding the results as data into a computer programme which uses the method of least squares to fit different types of curves, it was found that the bellows did not have a linear stiffness. The equations of the best fitting curves were found to be of the form

$$F = k(x + ax^2 + bx^3) \dots\dots\dots(6.4)$$

where F is force (lbf)

k is the spring rate (lbf/in)

a , b and c are constants

x is the displacement (in)

The spring rate or restoring force was examined for stability, since instability would lead to self-excited vibrations and so feed energy into the main vibrating system as a result of frequency entrainment. The methods by which the friction pads assembly were tested are described in Appendix 1; this includes the dynamic tests.

The dynamic test consisted of a forced-vibration in which the excitation force was kept constant and the displacement measured for one case. For the second case the displacement was kept constant and the force measured whilst the frequency was varied. This was done without and with different pressures of air in the bellows.

The results of both tests showed that the friction blocks could be unstable at some frequencies but as the resonant frequencies were so low, in the region of 10Hz as compared to the region of instability between 80Hz and 250Hz obtained in the main experiments the results obtained could not have been due to frequency entrainment.

The design of the friction pads was modified to include brackets carrying (positioning) rods and also to accomodate screws which when tightened prevent rolling of the friction blocks and transverse vibration.

The previous experiments were then repeated and though there was a slight change in the response at

low frequencies the basic curves remained unchanged, thus suggesting that the results obtained were the true experimental results for the system represented in Fig.(6.2). As these results clearly do not represent equation (6.2a) it was decided to fit the curves using equation (6.2b).

6.2.4 DETERMINATION OF PARAMETERS " α " AND "h"

In equation (6.2b), "h" is an unknown parameter and therefore values had to be assigned to "h" to obtain any curves. By making use of equation (2.9) where friction force

$$F = R'xe^{-\alpha h}(\cos wh + j\sin wh) , \text{ equation 6.2}$$

can be modified to

$$m\ddot{x} + R'xe^{-\alpha h}(\cos wh + j\sin wh)x = P_1e^{j\omega t} \dots(6.2c)$$

in which 'h' and ' α ' are the unknowns.

These unknown parameters are determined by feeding values of the measured excitation forces at different frequencies as data into into a special computer programme. This programme which is a "Non-linear Regression programme" is used to determine the best value of a variable (or variables) in an equation to give the best fit to a set of experimental results.

The use of this programme involves specifying the number of variable parameters in the assumed equation then partially differentiating the equation with respect to these variables. The theory of the programme can be summarised as follows:

$$\text{Let } Y_e = Y_{cal} + \frac{\partial F}{\partial a} \delta a + \frac{\partial F}{\partial b} \delta b + \frac{\partial F}{\partial c} \delta c + \frac{\partial F}{\partial d} \delta d + O(\delta a^2 \dots)$$

$$E = (Y_e - Y_{cal}) - \left(\frac{\partial F}{\partial a} \delta a + \frac{\partial F}{\partial b} \delta b + \frac{\partial F}{\partial c} \delta c + \frac{\partial F}{\partial d} \delta d \right)$$

where E is the error

$$E^2 = \frac{\partial F}{\partial a} (Y_e - Y_{cal}) - \left(\frac{\partial F}{\partial a} \delta a + \frac{\partial F}{\partial b} \delta b + \frac{\partial F}{\partial c} \delta c + \frac{\partial F}{\partial d} \delta d \right)$$

$$\text{Let } \frac{\partial F}{\partial (\delta a)} = 0$$

$$\text{then } \frac{\partial F}{\partial a} (Y_e - Y_{cal}) = \frac{\partial F}{\partial a} \left(\frac{\partial F}{\partial a} \delta a + \frac{\partial F}{\partial b} \delta b + \frac{\partial F}{\partial c} \delta c + \frac{\partial F}{\partial d} \delta d \right) \dots \dots \dots (A)$$

Similarly $\frac{\partial E^2}{\partial (\delta b)}$, $\frac{\partial E^2}{\partial (\delta c)}$, and $\frac{\partial E^2}{\partial (\delta d)}$ give 3 more equations like A.

Initially the variable parameters are given some values to enable the computer to obtain a calculated value to compare with the experimental value as in (A). The new values of the parameters are calculated from the results obtained and the process is repeated iteratively until the best results are obtained for the particular equation. As an example let the equation be (2.9) with 'h' and as the variables.

$$F = R'x e^{-\alpha h} (\cos wh + j \sin wh)$$

$$\frac{\partial F}{\partial \alpha} = -h R' e^{-\alpha h} (\cos wh + j \sin wh)$$

$$= -h R' x e^{-\alpha h} \quad (\text{as } \cos^2 wh + \sin^2 wh = 1) \quad \dots \dots (6.5)$$

$$\frac{\partial F}{\partial h} = R' e^{-\alpha h} x \{ \alpha (\cos wh + j \sin wh) + w (\sin wh + j \cos wh) \}$$

$$-\frac{F}{h} = R' e^{-\alpha h} x (\alpha + w) \quad \dots \dots \dots (6.6)$$

$$\text{Similarly } |F| = R' x e^{-\alpha h} \quad \dots \dots \dots (6.7)$$

Thus equation (6.5) to (6.7) are specified on the computer programme. The number of data points with the data (in which the excitation force was the ordinate (i.e. Y_e) and the frequency as the abscissa) are also

specified. It is also necessary to give the convergence criterion so that the iterative process can be curtailed. In order to estimate how close the theory fitted the results the convergence criterion of the sum of errors is specified. In addition the proportion of change of the parameters used when the change is fluctuating is specified.

6.2.5. CURVES FOR CONSTANT ACCELERATION

The equations which are applicable to the tested system that is represented by Fig.(6.2) are equations (3.5) and (3.8). However since the acceleration was kept constant it was found more convenient to use equation (3.8). In order to test the validity of equation (2.9), equation (6.2c) was also used. Thus the equations used for the computer were

$$Pe^{j\omega t} = X \sqrt{\{ (C + R \cos \omega h)^2 + (R \sin \omega h)^2 \}} \dots (3.8)$$

and

$$Pe^{j\omega t} = X \sqrt{\{ (C + R_0 e^{-\alpha h} \cos \omega h)^2 + (R_0 e^{-\alpha h} \sin \omega h)^2 \}}$$

where $C = m\omega^2$

As expected the results obtained by using the two equations above were identical for any set of experimental results: equation (3.8) has been used for all subsequent work. As the computer output included both the initial and final values of the variable parameters as well as the sum of the errors squared, and the percentage error in the calculated results good fits for the curves were obtained by varying the constants R and C, in addition

Fig 6.7 Vibration response of a body in the presence of Coulomb force at constant acceleration.

acceleration constant at 0.3125 lb/sec^2

x Experimental
o Theoretical

Equation of the form

$$\frac{P}{c} = C + R(\cos nh + j \sin nh) \text{ where } C = m\omega^2$$

Fitting values of parameters

page error - 14.15

$$h = 0.002403$$

$$R = 1.6951$$

$$C = 0.870$$

mass (x 4.0) lb

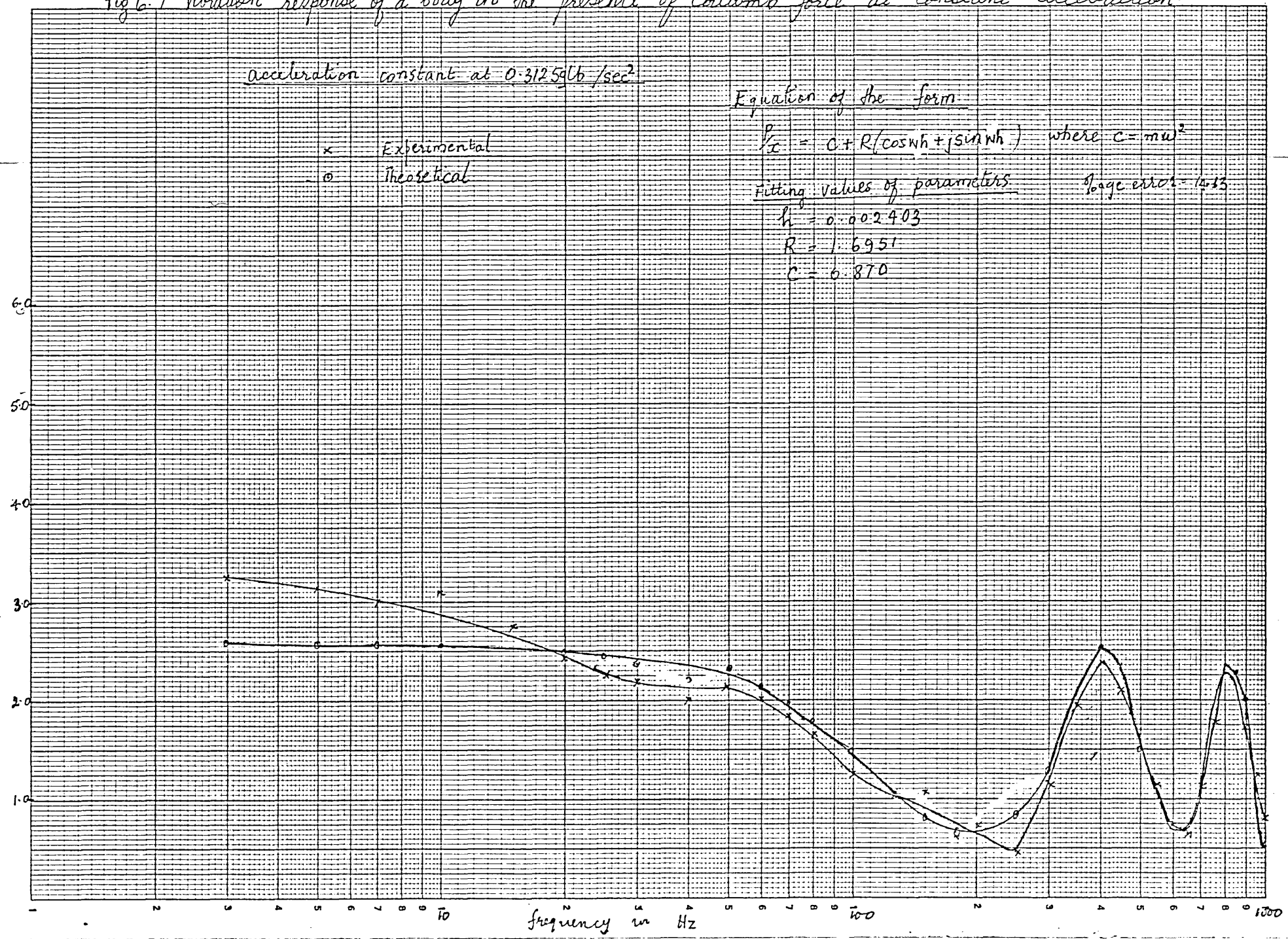


Fig 6.8 Vibration response of a body in the presence of Coulomb force at constant acceleration

acceleration constant at $19.16/\text{sec}^2$

x Experimental
o Theoretical

Equation of the form

$$P_x = C + R(\cos wh + j \sin wh) \text{ where } C = m\ddot{u}$$

Fitting values of parameters

%age = 6.29

$$h = 0.04191$$

$$R = 2.0005$$

$$C = 2.409$$

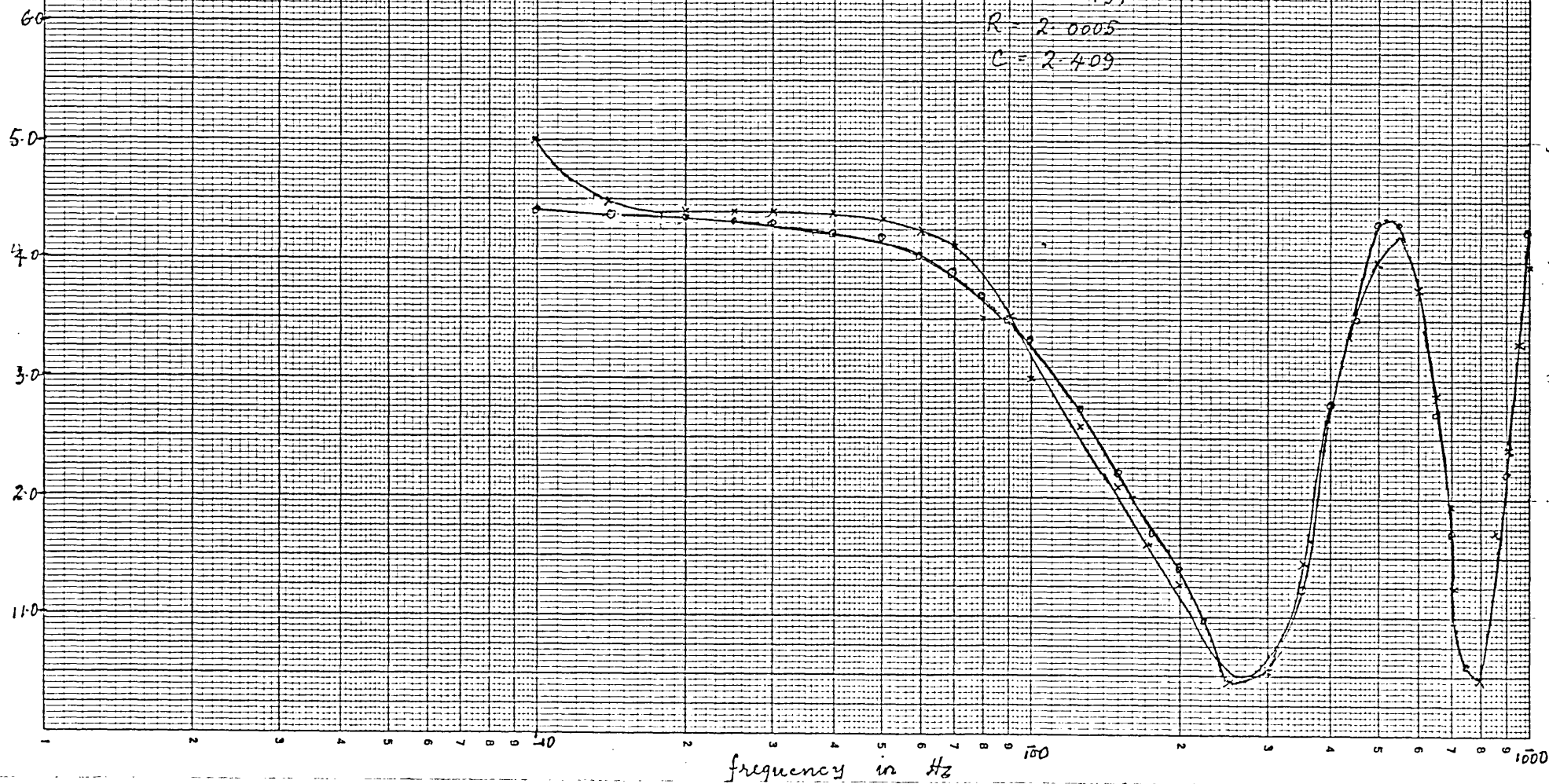


Fig 6.9 Vibration response of a body in the presence of Coulomb force at constant acceleration

Acceleration constant at 0.625 g lb/sec

Equation of form

$$P_x = C + R(\cos \omega h + f \sin \omega h) \quad \text{where } C = m \omega^2$$

%age error = 5.09

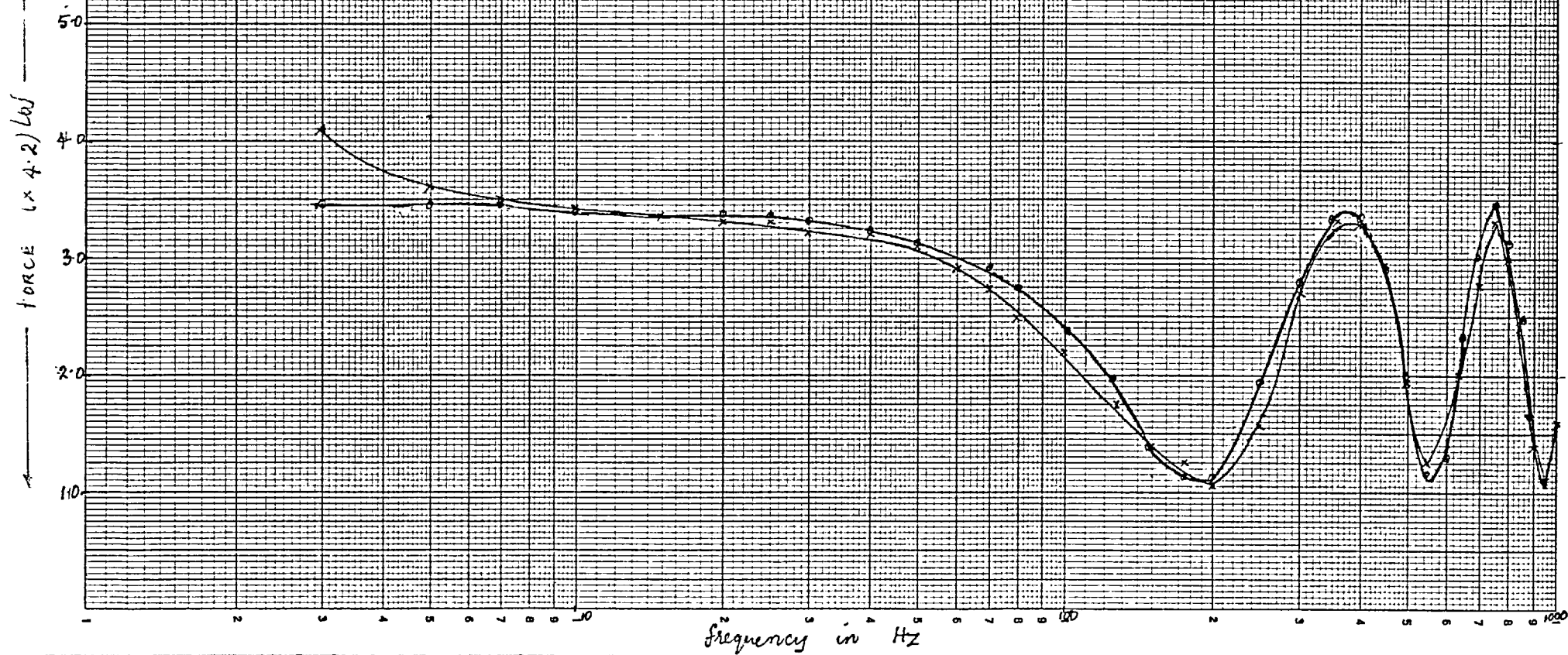
Fitting values of parameters

$$h = 0.00265$$

$$R = 1.7748$$

$$C = 2.278$$

x — Experimental
o — Theoretical



to h (the delay-time) in the equation. Figs.6.7 to 6.9 show the experimental and fitted curves for comparison. Also the values for the parameters R , C and h are given for each set of curves. The value of C and R were found to be proportional to the peak to peak value of the acceleration and the magnitude of the Normal force (i.e. the air pressure in the bellows).

6.2.6 VARIATION OF FRICTION FORCE AT CONSTANT ACCELERATION

In the experiments already described the acceleration was kept constant, thus $x = w^2 x = k$

$$\text{and displacement } x = \frac{k}{w^2} = \frac{k}{(2\pi)^2 f^2} \dots\dots\dots(6.9)$$

As the frequency range covered in this part of the experiments was from 3Hz to 1000Hz, it means that the ratio of the amplitude at the beginning to the amplitude at the experiment was very big.

$$\frac{\text{amplitude at beginning}}{\text{amplitude at end}} = \frac{1000^2}{3^2} = \frac{10^6}{9}$$

As the amplitude was small, of the order of 0.25" to start with, it is possible that slip might not have taken place at the higher frequencies (2). But as shown earlier the friction force is dependent on the amplitude of vibrations and therefore the friction cannot be expected to be constant over the frequency that was used. In order to determine the variation of the friction force with frequency it is necessary to keep the amplitude constant thus ensuring that R is constant. It was therefore

decided to repeat the experiment keeping the amplitude constant.

6.3 VARIATION OF COEFFICIENT OF FRICTION WITH FREQUENCY AT CONSTANT AMPLITUDE

6.3.1 INTRODUCTION OF THE DISPLACEMENT GAUGE

A Wayne Kerr displacement gauge was now added to the set-up. A fixed amplitude of 0.0165" peak to peak was chosen after due consideration had been given to the limitation of the equipment. The output of the vibrator was limited to about 70lbf (with cooling), and as the mass is constant, measurements could only be made from 3Hz to 60Hz in some cases and to only 50Hz in others; for with the amplitude constant the inertia force ($m\ddot{x}$) increases proportionally as the square of the frequency. Moreover the choice of amplitude ensured that the motion was approximately harmonic and also that this amplitude of vibrations was well within the linear range of the displacement gauge.

Having selected a suitable constant amplitude of vibrations the excitation force, the acceleration and the phase difference between them, were measured at different frequencies within the range specified above, without any friction force in the system. These measurements served as a check on the calibration of the force gauge and hence the accelerometer. A low pressure gauge reading from zero to 2lbf/in² pressure was introduced into the system, replacing the previous

pressure gauge which read from 30 lbf/in^2 to 15 lbf/in^2 . With the new pressure gauge in the air supply system the friction blocks were recalibrated in the same way as before. The low pressure gauge was introduced because the maximum friction force that could be introduced at the chosen fixed amplitude of vibrations at frequency range from 3 Hz to 60 Hz was about 30 lbf (i.e. about $2\frac{1}{2}\text{ lbf/in}^2$ pressure). It was therefore found necessary to take readings at low pressures thus requiring an accurate low pressure gauge. Pressures of 0.2 , 0.5 , 0.8 , 1.0 , 1.3 , 1.5 , 1.8 and 2.0 lbf/in^2 were introduced into the friction pads and at each pressure setting the experiment was repeated. The friction blocks were always cleaned with trichloroethylene and dried before the surfaces were allowed to come into contact. About 5 minutes were allowed for settling before any measurements were made. As the excitation force and the acceleration and the phase difference between them were measured it was therefore possible to calculate the value of the friction force at each frequency (see 6.3.7). In measuring the phase difference it was found necessary to measure the phase difference ϕ_f between the excitation force and the reference (taken from the oscillator) and also the phase difference ϕ_a between the acceleration and the reference. By subtracting one from the other ($\phi_f - \phi_a$), the phase difference between the excitation force and the acceleration was obtained.

As the normal force was known the coefficient of friction was determined at each frequency. The results obtained from this experimental work was used as data in a computer programme which uses the method of least squares to fit polynomial curves, to determine the best fitting equation for the experimental curves.

The best fitting equation was found to be of the form:

$$\mu = \mu_s(1 - af^2 + bf^4 \dots\dots\dots) \dots\dots\dots(6.10)$$

where μ = Coefficient of friction at any frequency

μ_s = Static coefficient of friction,

a and b are constants and f is the frequency in Hz.

6.3.2 OTHER DATA

Experimental results of some other researchers, Earles and Beards (28) was used as data in the same programme and found to produce similar results.

6.4 NON-LINEAR REGRESSION PROGRAMME

The same data as above was used in the non-linear regression programme to determine the parameters " " and "h" and also to test the equation (2.16)

$$\text{i.e. } F = XR(\cos wh + j \sin wh)$$

As shown in equation (6.7) the absolute value of F is constant (and therefore a constant value was

obtained); and this clearly did not fit the curve. The friction force was therefore split into real and imaginary components as shown in equation (2.13).

Using the real part

$$\text{i.e. } F_r = XR\cos wh = R\cos wh \text{ as } x \text{ is constant ... (6.11)}$$

good correlation was obtained with all the experimental curves and also fitting values were obtained for the parameters, (in this case R , and h). The curves obtained with this programme were similar to those obtained using equation (6.4).

6.4.1 EXPANSION OF COSX

An examination of (6.4) shows that ^{it} is similar to the expansion of $\cos x$ which is, $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$ (6.12)

6.5 RESULTS

The experimental results are shown in Fig.(6.10) and (6.11) in which the normal forces acting on the friction blocks is given (i.e 1.71bf to 6.11bf in Fig.(6.10) and 7.51bf to 11.101bf in Fig.(6.11)).

The equation used is $F = R\cos wh$ and for each loading force the values of the delay time "h" and the friction constant R , are given. Both the theoretical and experimental curves are shown. Having obtained the values of h and R , for different Normal loads

Equation: $\mu = \mu_0 \cos \pi h$

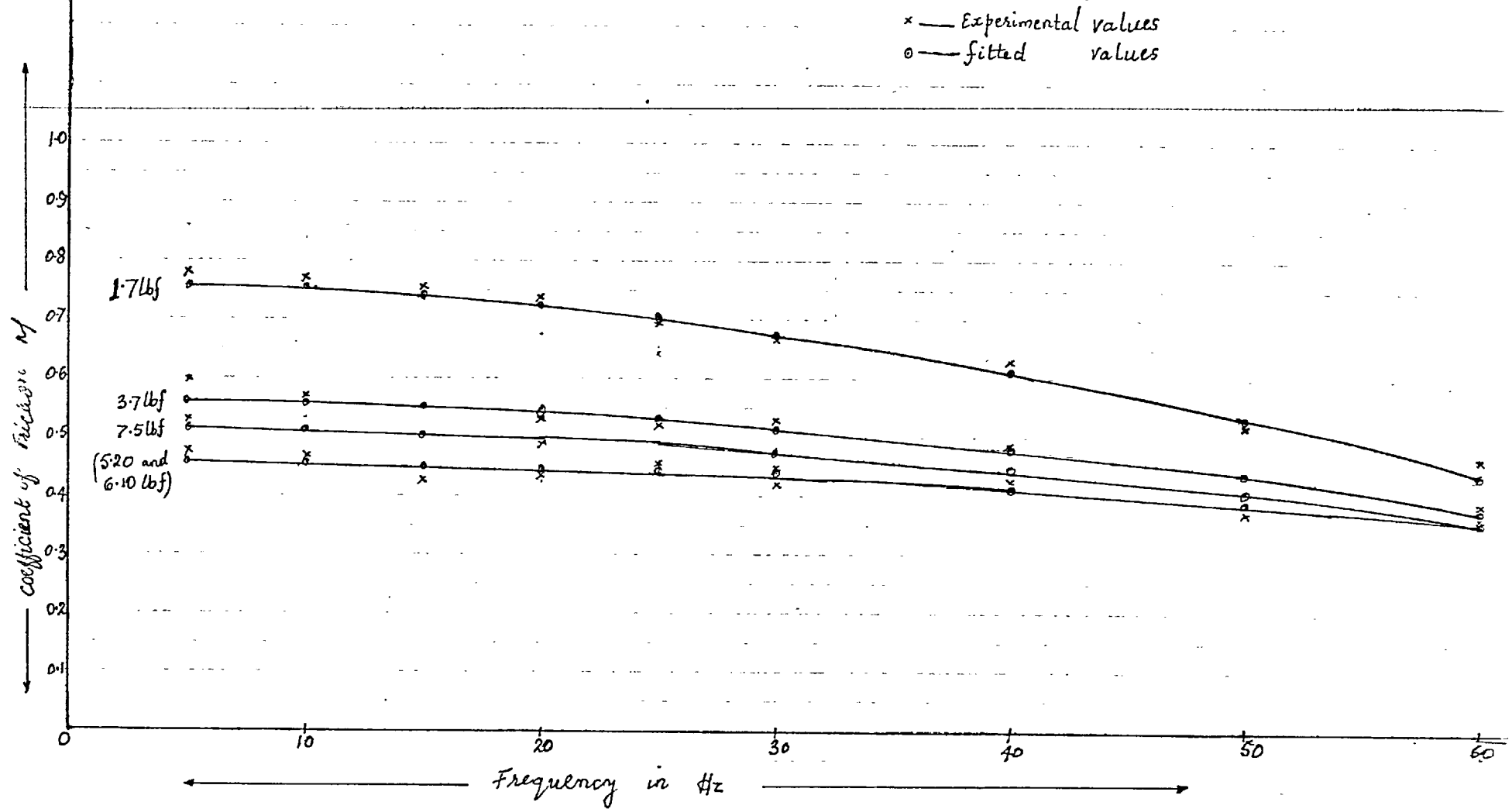


Fig 6.10 Variation of coefficient of friction with frequency at constant amplitude.

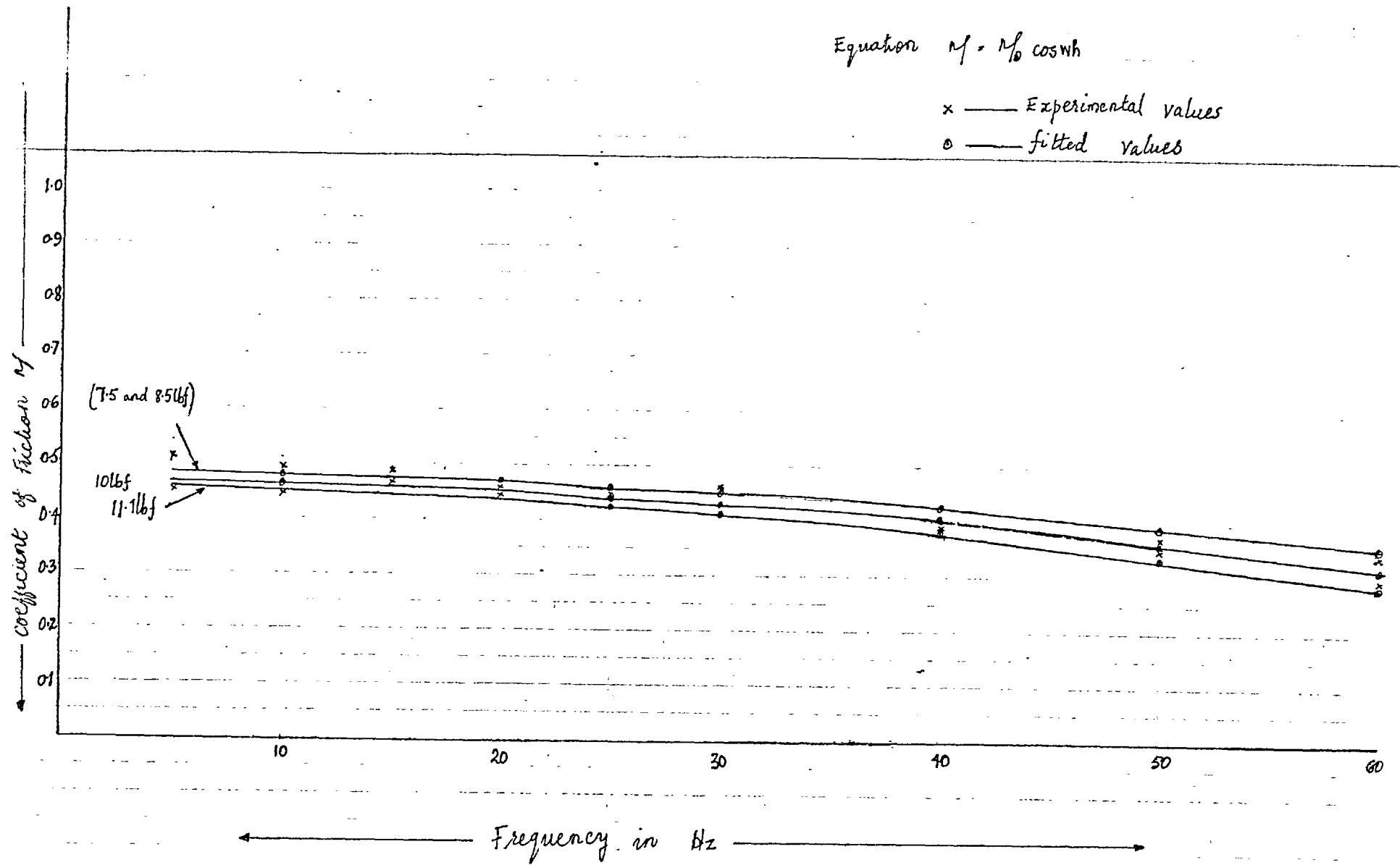


Fig 6.11 Variation of coefficient of Friction with frequency at constant amplitude.

Polynomial fit for the variation of coefficient of friction with frequency

x Experimental point
o Fitted point

Equations

$$\begin{aligned} N=3.7\text{lb}f \quad \mu_f &= 0.6295 - 0.0058f^2 + 0.0000365f^4 \\ N=1.7\text{lb}f \quad \mu_f &= 0.9445 - 0.0155f^2 + 0.000137f^4 \\ N=5.20\text{lb}f \quad \mu_f &= 0.481 - 0.00192f^2 + 0.00000027f^4 \\ N=5.90\text{lb}f \quad \mu_f &= 0.586 - 0.00585f^2 + 0.0000416f^4 \end{aligned}$$

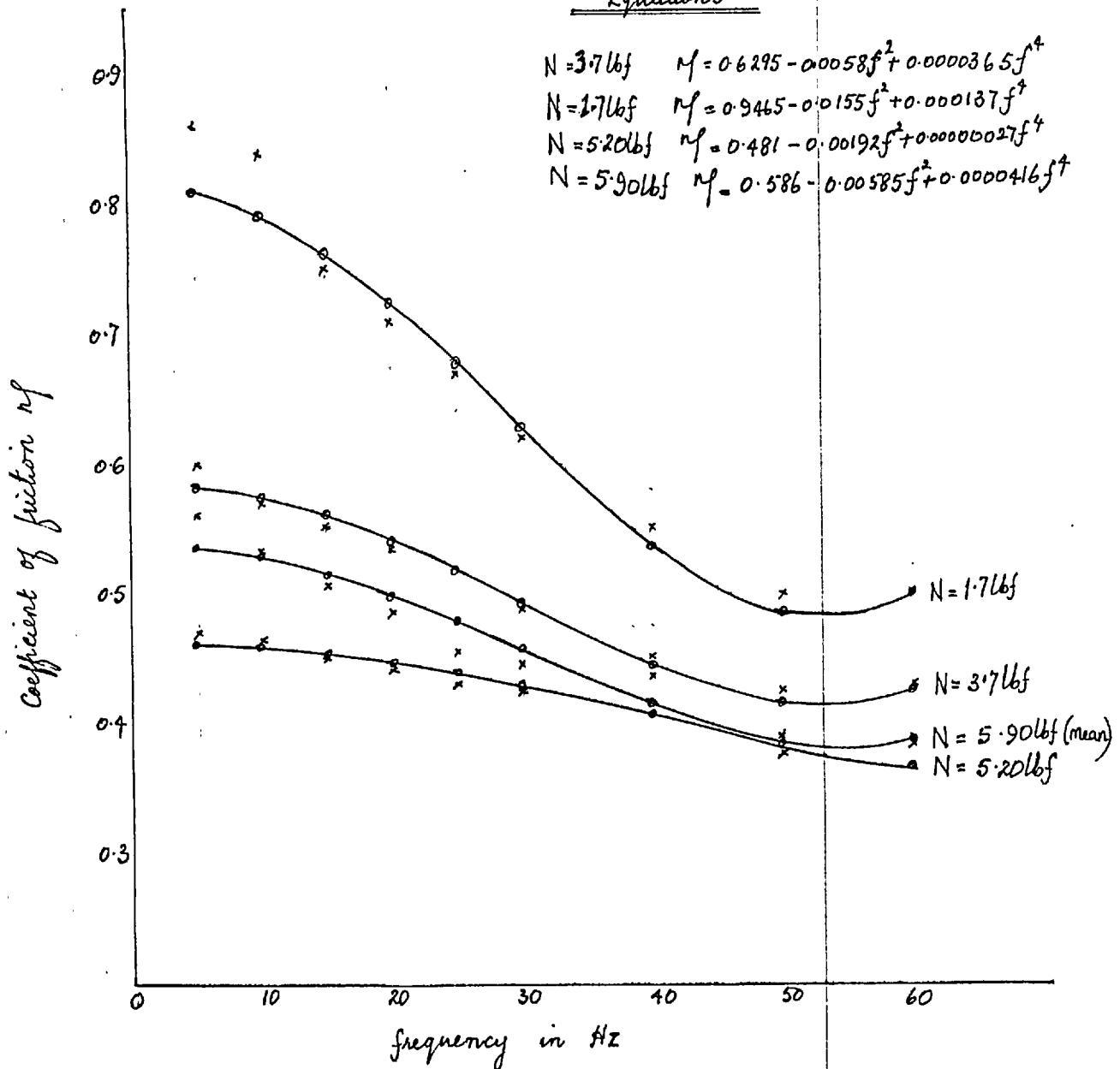


Fig 6.12: Variation of coefficient of friction μ_f with frequency

Polynomial fit for the variation of coefficient of friction with frequency

x Experimental points
o fitted points

Equations

N = 5.20 lbf (mean)	$\mu_f = 0.586 - 0.00585f^2 + 0.0000416f^4$
N = 6.10 lbf	$\mu_f = 0.46 - 0.000371f^2 + 0.0000140f^4$
N = 7.50 lbf	$\mu_f = 0.503 - 0.00171f^2 + 0.0000118f^4$
N = 8.50 lbf	$\mu_f = 0.537 - 0.00451f^2 + 0.0000272f^4$
N = 10.00 lbf	$\mu_f = 0.579 - 0.0101f^2 + 0.00018f^4$

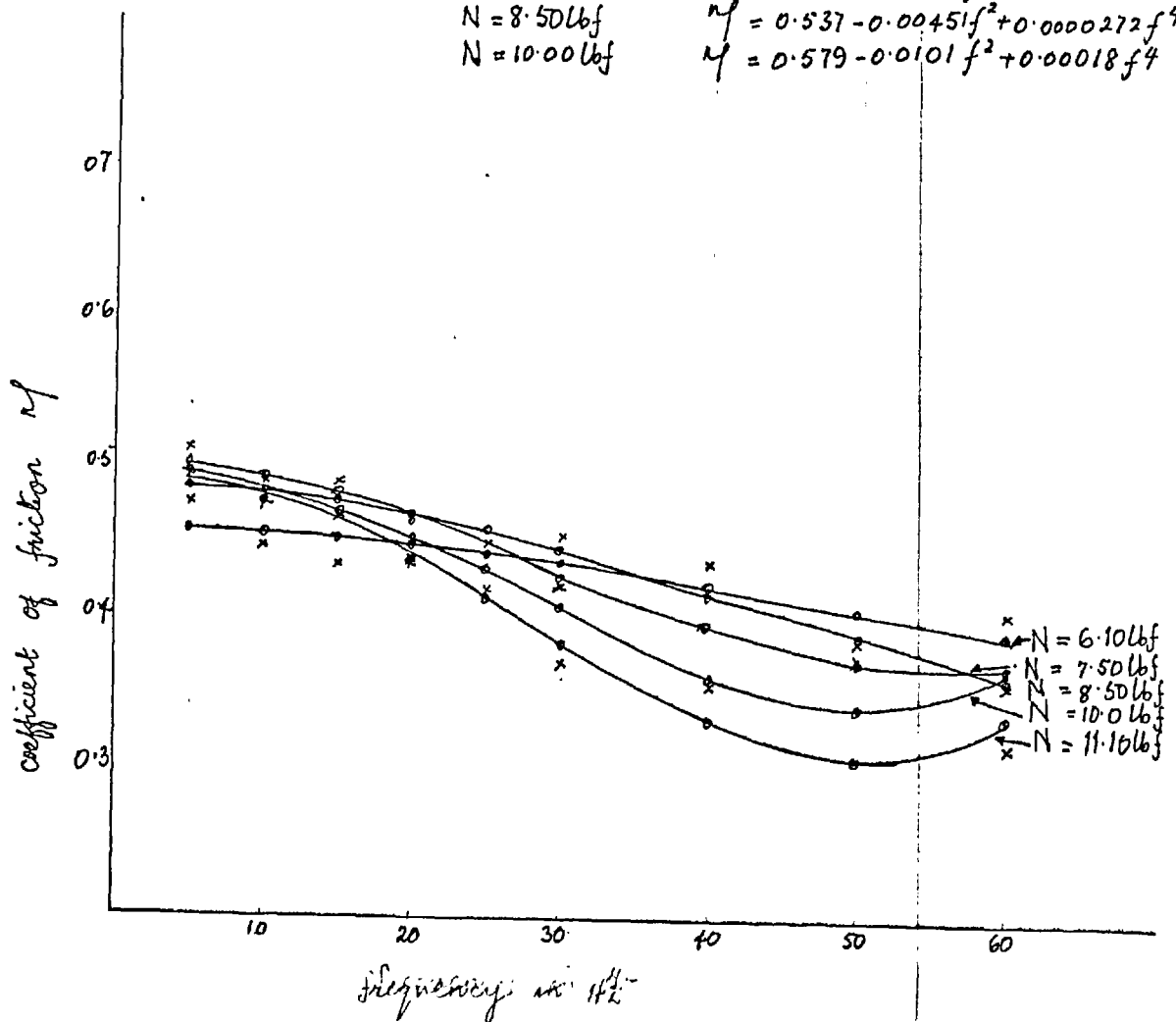


Fig 6.13 Variation of coefficient of friction μ_f with frequency

Comparison of fits for the exact and approximate equations.

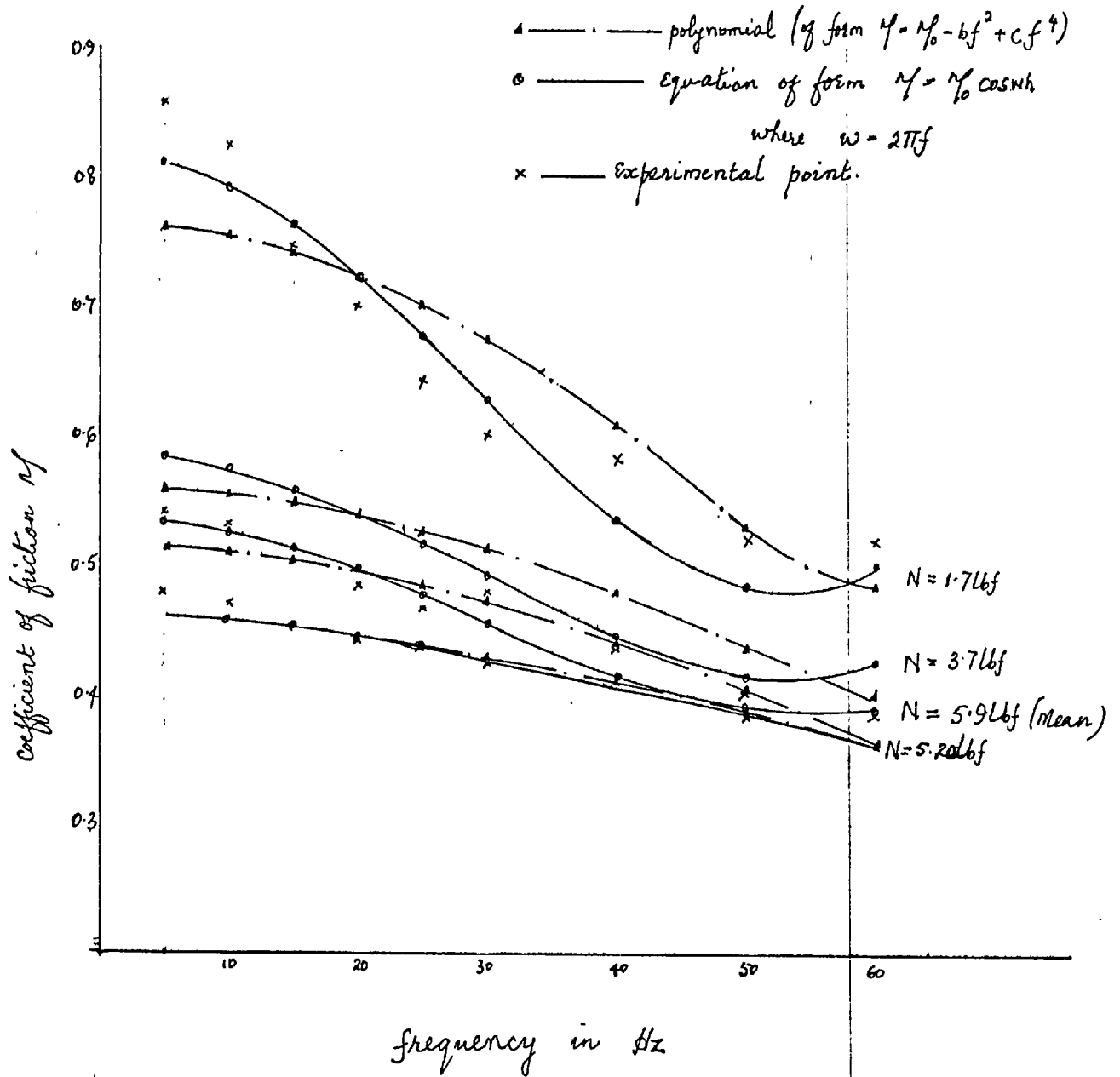


Fig 6.14 Variation of coefficient of friction with frequency

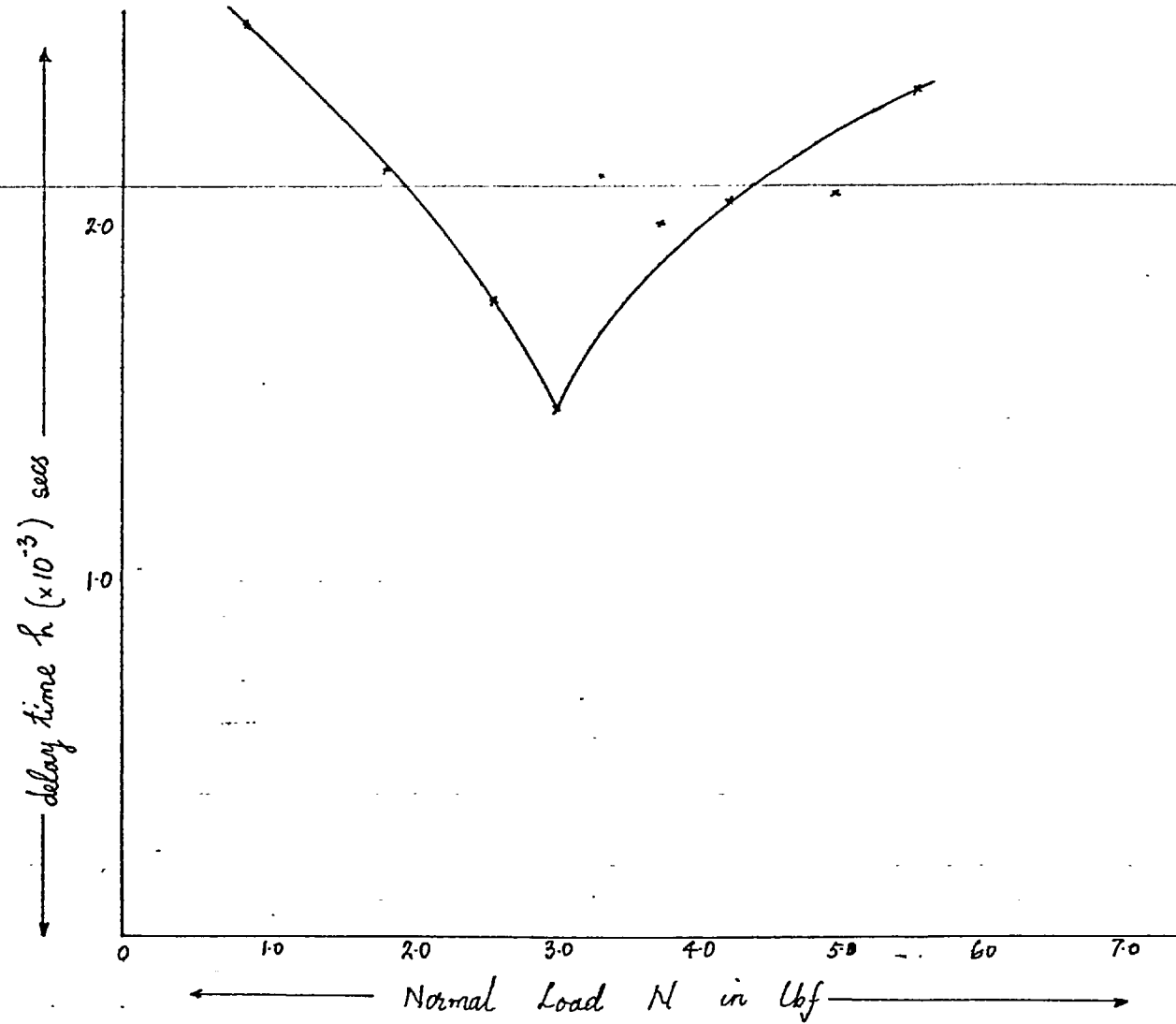


Fig 6.15 Variation of delay time h with normal load N

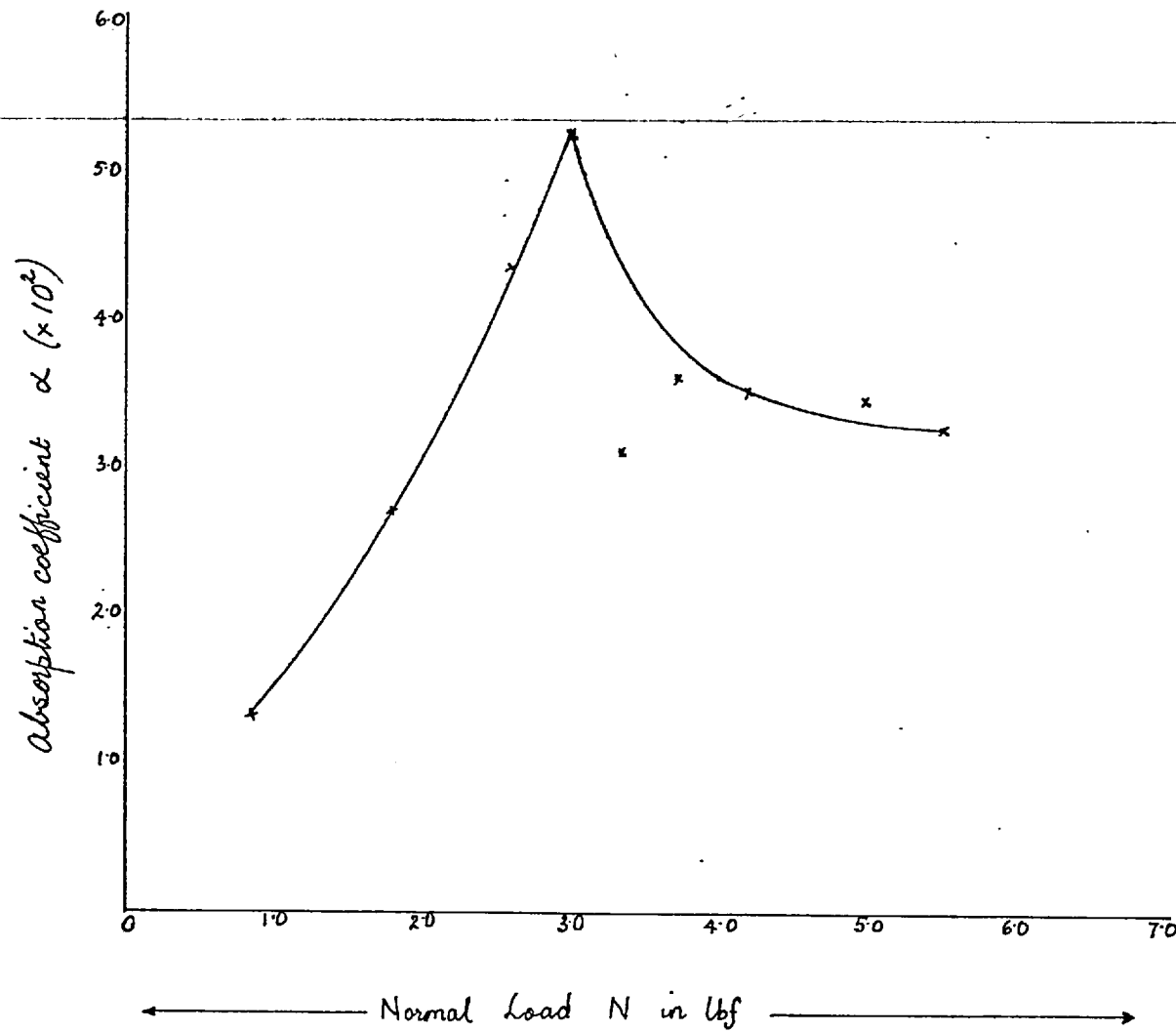


Fig 6.16 Variation of absorption coefficient α with normal load N .

it was easy to do a plot showing the variation of delay time with Normal pressure. This curve is shown in Fig.(6.15). As the constant $R = R_0 e^{-\alpha h}$, the value of α was obtained by substituting $R_0 e^{-\alpha h}$ for R in the programme and determining the value of α ; the results obtained for F were identical in both cases. Fig.(6.16) shows the variation of the absorption coefficient with Normal force. Figs.(6.12) and (6.13) show the fitted curves using the non-linear regression programme. The polynomial equation is of the form $M = M_s - bf^2 + cf^4 \dots$ and those are the first three terms of the expansion of $\cos x$.

Fig.(6.15) shows a comparison of the fits obtained using the exact equation and the first three terms of its expansion.

Figs.(6.17) to (6.19) are the curves obtained by using the experimental results of Earle and Beards. In this case the variation of the coefficient of friction with speed at constant amplitude is considered for different cases e.g. cast iron on steel. A least square programme was used in which the first three terms of the expansion of $\cos wh$ were used and the results show good agreements with the experimental ones. This result also demonstrates why electronic engineers in using oscillators to simulate Coulomb damping find that better agreement between theoretical and experimental work are obtained when the coefficient of

x — Experimental curve

• — Fitted curve

Equation of form

$$\mu = \mu_0 - bV^2 + cV^4$$

$$= 0.56 - 2.072V^2 + 0.596V^4$$

where V = peak slip velocity in inches.

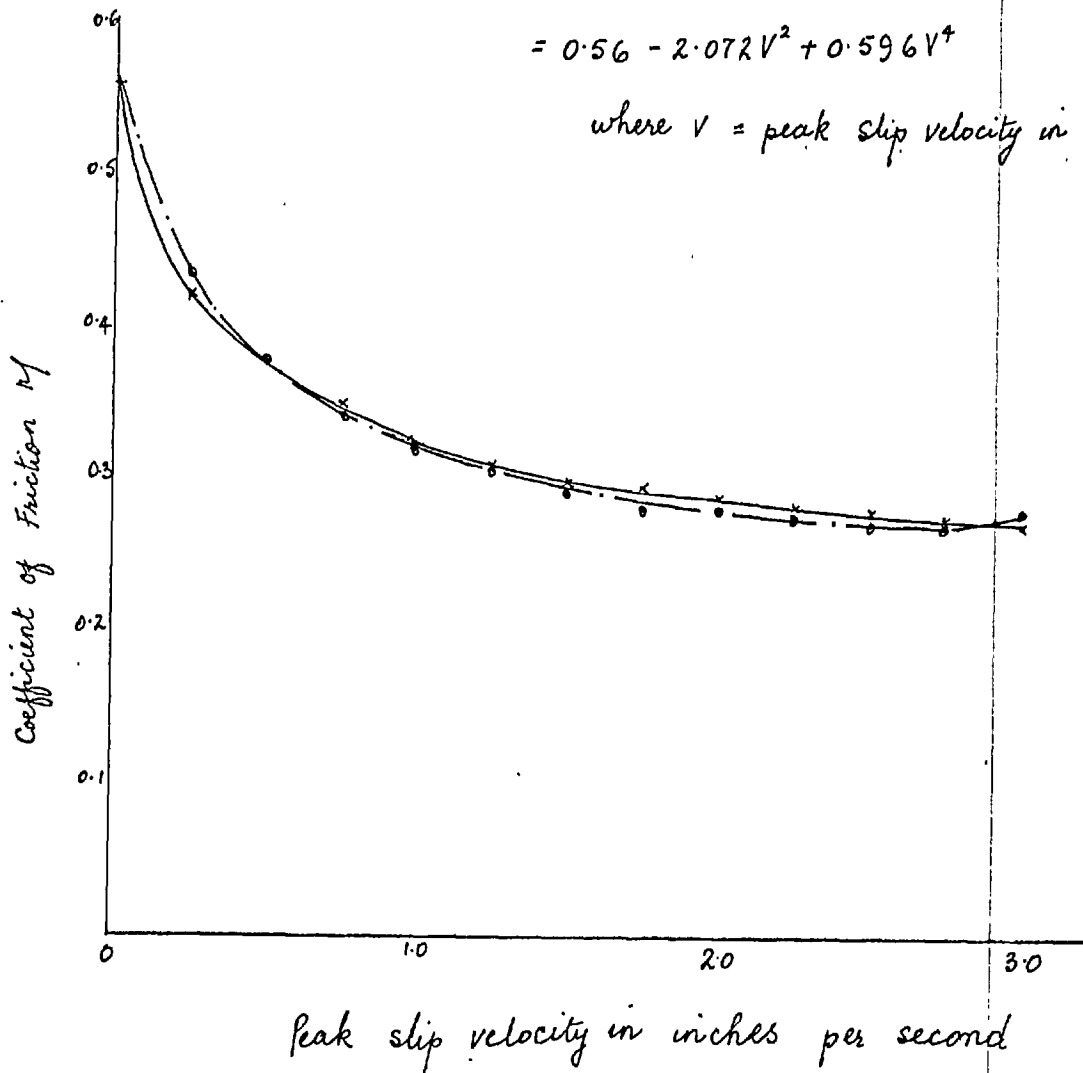


Fig 6.17. Variation of coefficient of friction with slip velocity

Earles and Beards Results

(Iron - steel)

x Experimental curve
o Fitted Curve

Equation of form

$$\mu = \mu_0 - bV^2 + cV^4$$

$$= 0.420 - 0.883V^2 + 0.173V^4$$

where V = peak slip velocity

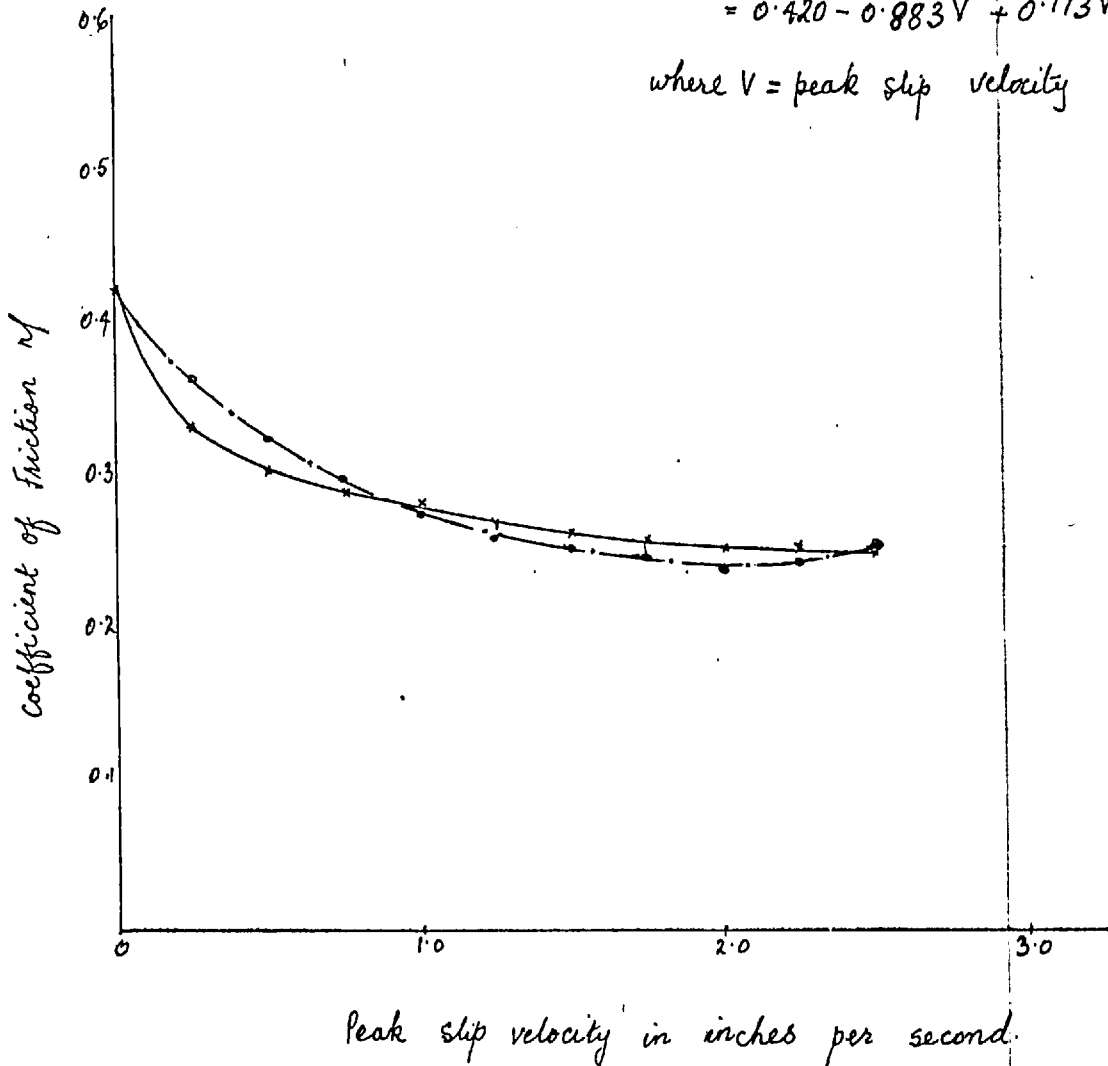


Fig 6.18 Variation of coefficient of Friction with slip velocity.

- 107 -
Earles and Beards results
 (Brass - steel)

x — Experimental curve

o — Fitted curve

Equation of form

$$\begin{aligned} \mu &= \mu_0 - bV^2 + cV^4 \\ &= 0.46 - 2.14V^2 + 1.4V^4 \end{aligned}$$

where V = peak slip velocity

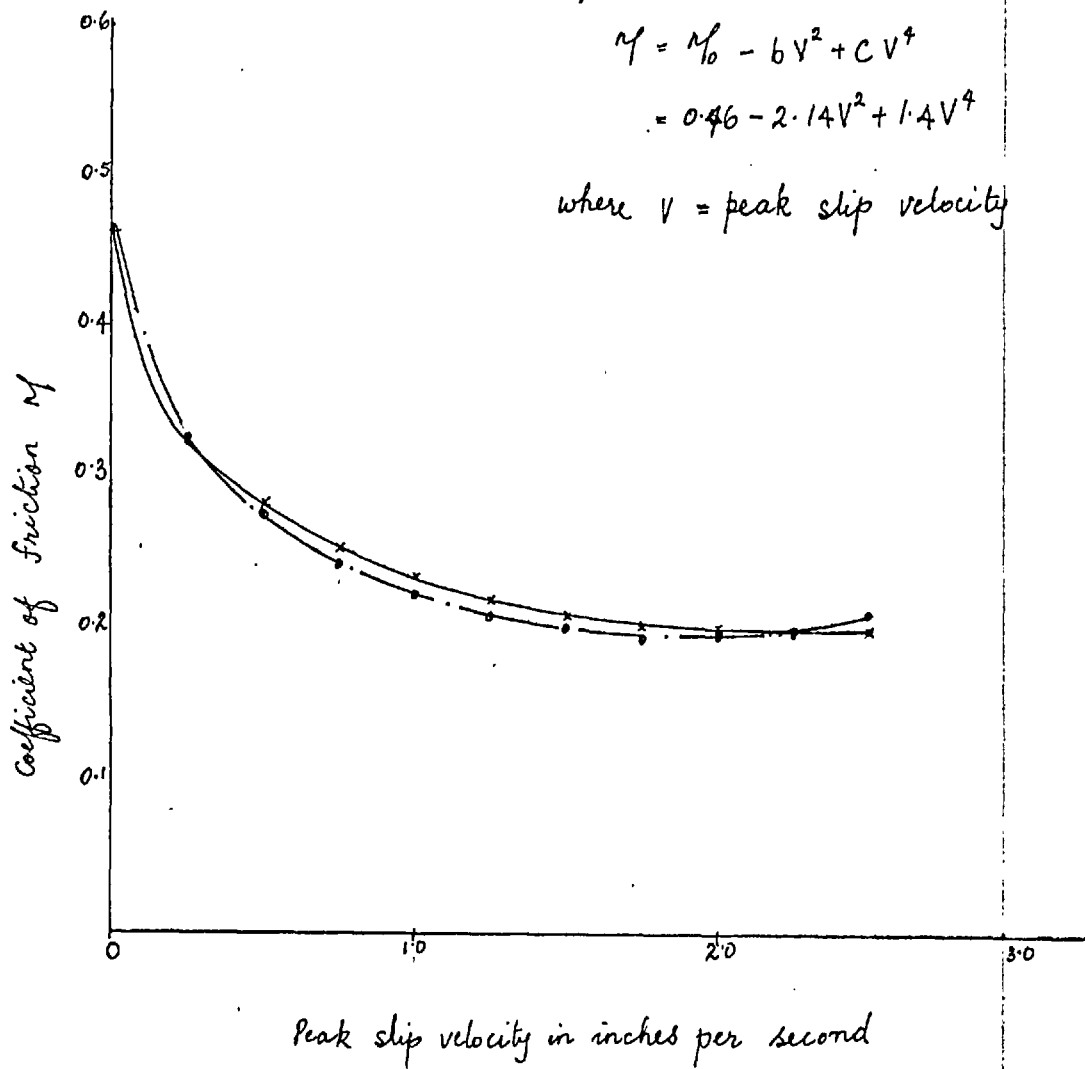


Fig 6.19 Variation of coefficient of friction with slip velocity

friction is represented by a term of the form $t(1 - x^2 + \gamma x^4)\dot{x}$ as demonstrated by Derek Lawden (43).

6.6 VARIATION OF COEFFICIENT OF FRICTION WITH FREQUENCY AT CONSTANT ACCELERATION

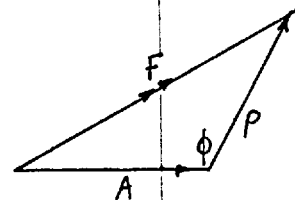
Having obtained satisfactory correlation between the theoretical analysis and the experimental results for the case when the amplitude was kept constant, it was decided to make use of the results obtained at constant acceleration. As stated already the friction force was calculated at each frequency and thence the coefficient of friction.

Let P = Excitation force (measured)

A = Accelerating force (measured i.e. $m\omega x^2$)

ϕ = Angle between P and A

F = Friction force



By use of the cosine rule F was easily obtained.

As shown in equation (6.7) the absolute value of F is a constant i.e. $F = R'e^{-\alpha h}x = R$ therefore this force was split up into real and imaginary parts. The real part is the (resisting) restoring force and therefore describes the resisting part of the friction force. This is demonstrated in Figs.(6.12) to (6.14)

Fig 6.20 Variation of Coulomb force (calculated) with frequency for constant acceleration

Coulomb force (N)

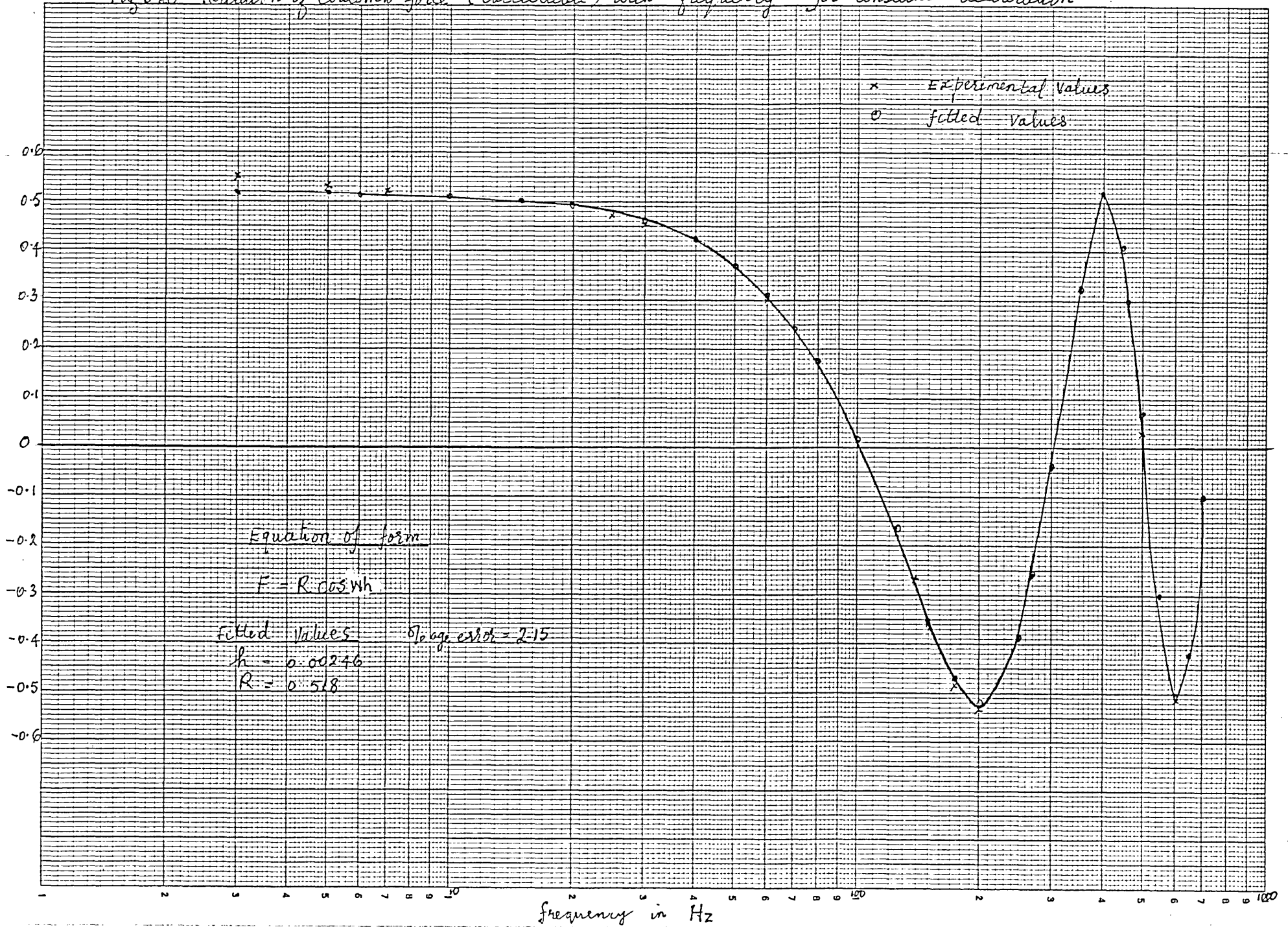


Fig 6.21 Variation of calculated Coulomb force with frequency for constant acceleration

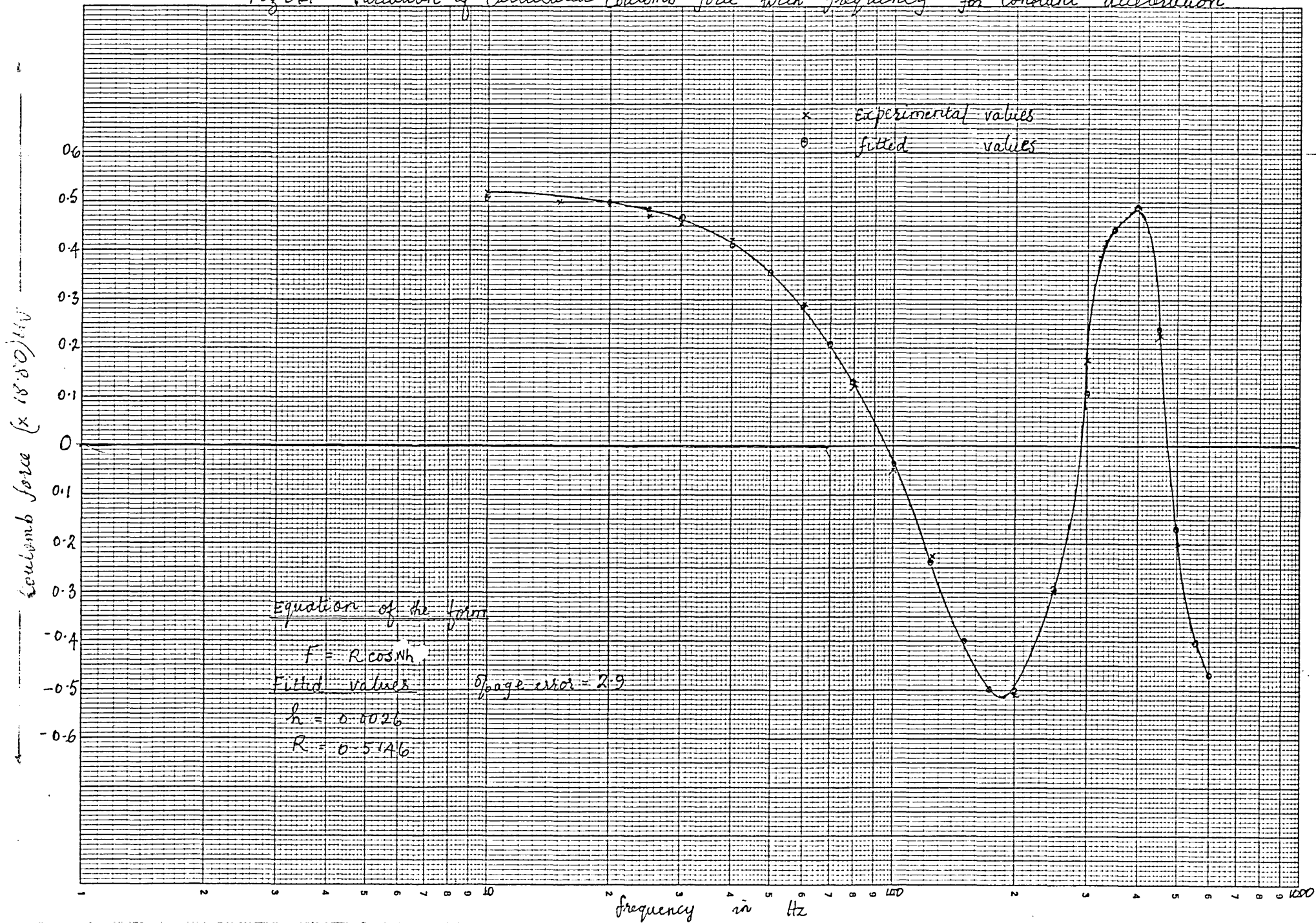
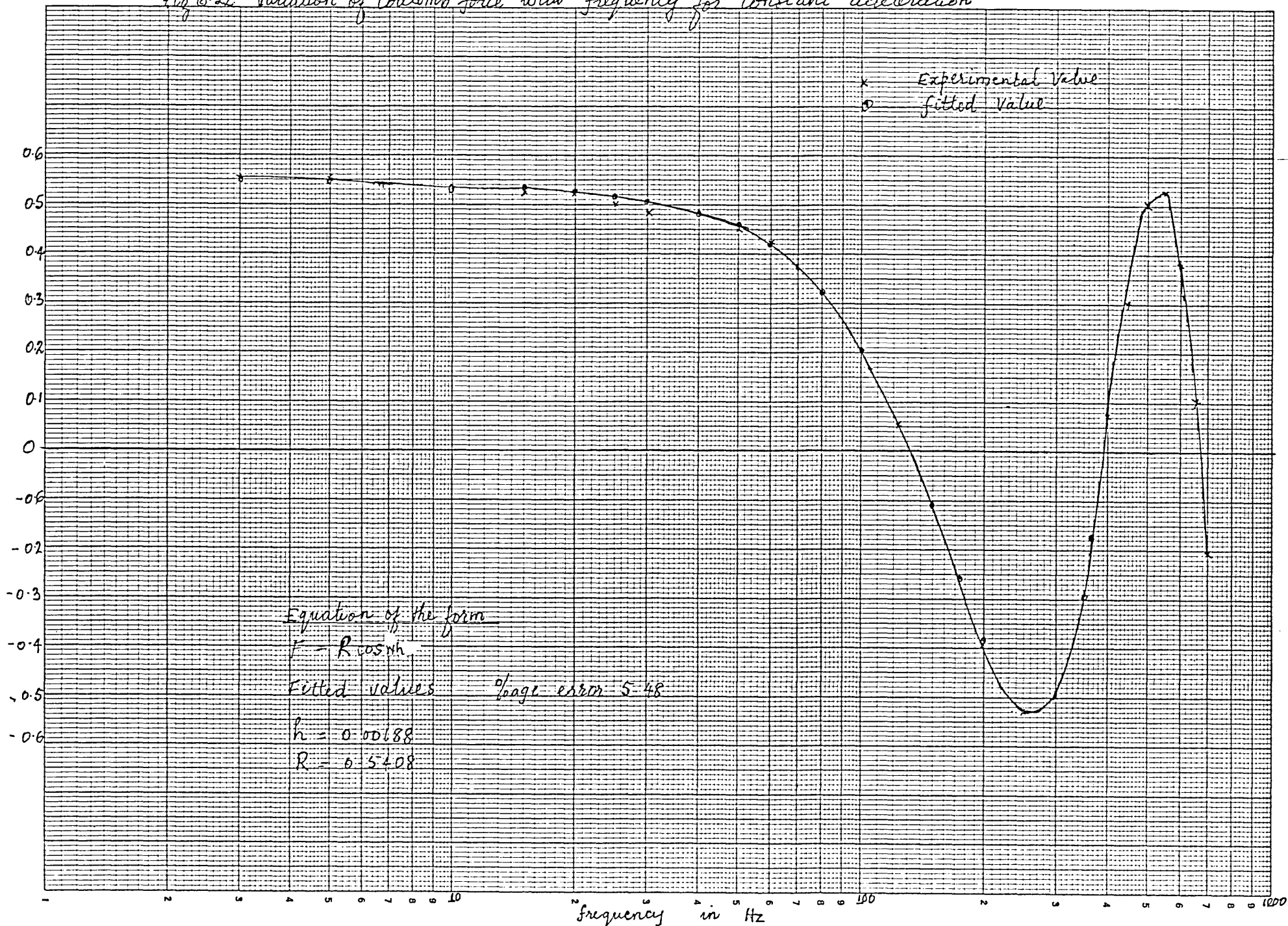


Fig 6.22 Variation of Coulomb force with frequency for constant acceleration

Coulomb force (N and g)



where good fits of the experimental results are obtained by using the expression $F = R \cos wh$. Using the same expression for the results obtained with constant acceleration very good agreement is obtained probably because there are more data points. These curves are shown in Figs (6.20) to (6.22)

6.7 POLAR AND CARTESIAN PLOTS

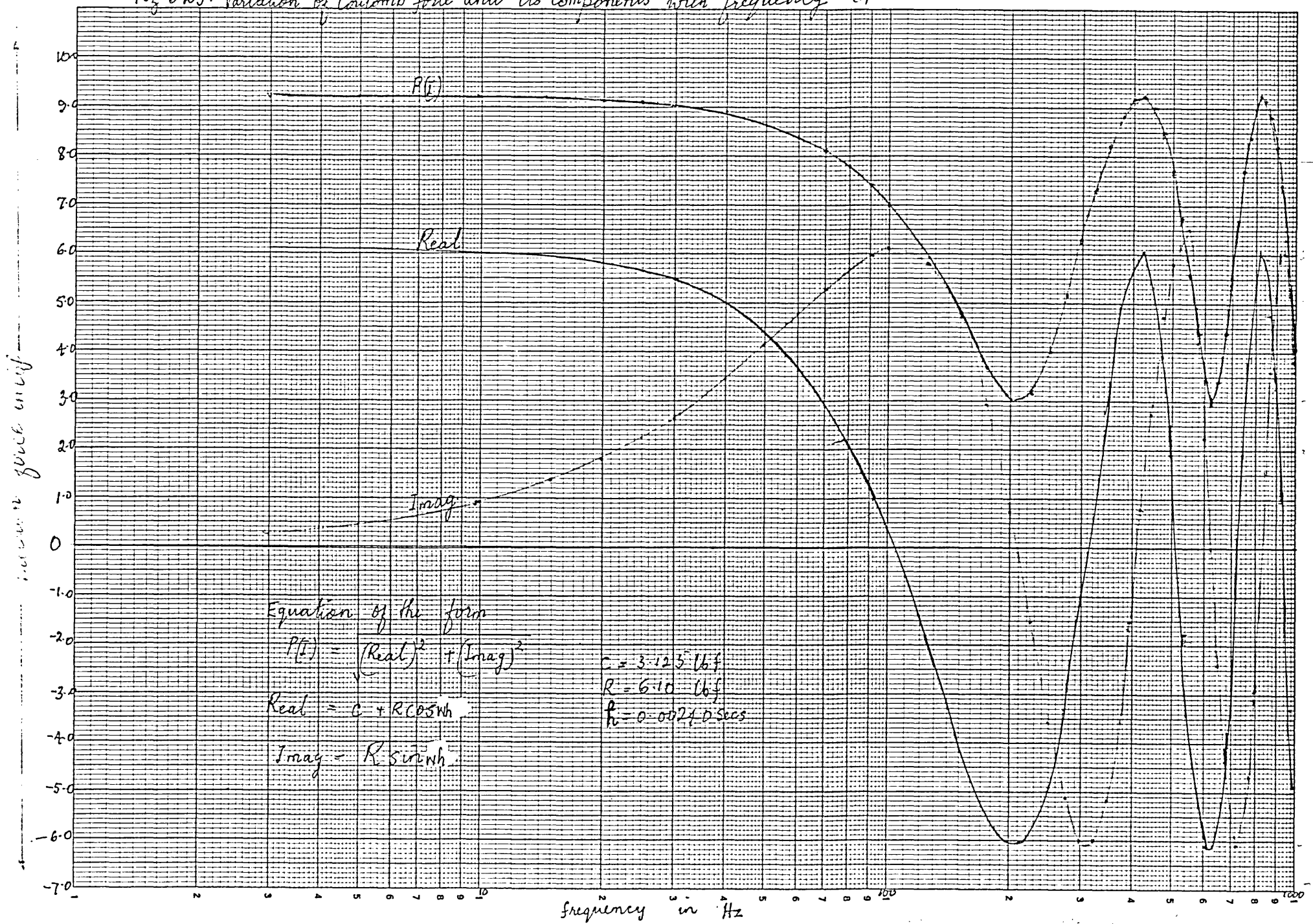
Using the values of R and h obtained from the computer programme a theoretical examination of the friction force was made. The real and imaginary parts taken on their own are expressions that are well known however in the context of the experimental work carried out equation (3.5) was used.

First of all the real part of the equation i.e. $F_{\text{real}} = (mw^2 + R \cos wh)x$ (6.13) was considered separately, mw^2x being derived from the value at which the acceleration was kept constant. Then the imaginary part in which $F_{\text{imag}} = R \sin wh$ (6.14) where F_{imag} = Imaginary part of the friction force was considered.

The absolute value of the excitation force was given by equation (3.5). The results for three different cases are shown in Figs. 6.23 to 6.28, in which mw^2 represented by C i.e. $mw^2 = C$ (6.15).

Using equation (3.9) the phase angles were also obtained. These calculations were done by the computer and the range covered was the same (i.e. 3Hz to 1000Hz)

Fig 6.23. Variation of Coulomb force and its components with frequency (1)



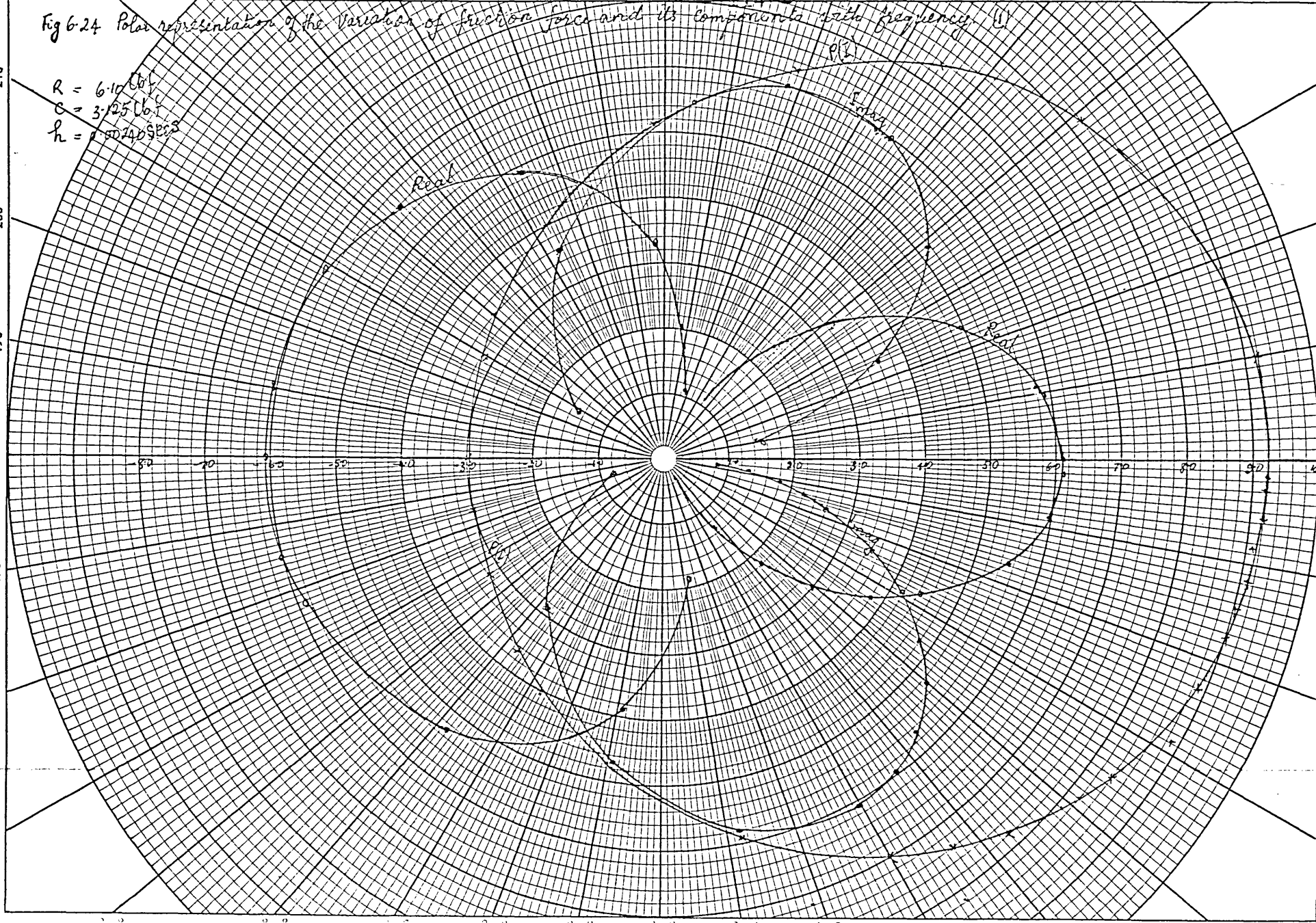


Fig 6-24 Polar representation of the Variation of friction force and its components with frequency (1)

$$R = 6.10 \text{ lb}_f$$

$$C = 3.25 \text{ lb}_f$$

$$h = 0.00489 \text{ sec}$$

Fig. 6.25 Polar representation of the Variation of Coulomb force and its components with frequency (2)

Equation of the form

$$P(f) = \sqrt{(\text{Real})^2 + (\text{Imag})^2}$$

$\text{Real} = C + R \cos \omega h$

$\text{Imag} = R \sin \omega h$

$C = 6.250 \text{ lb}$
 $R = 3.24 \text{ lb}$
 $h = 0.00265 \text{ sec}$

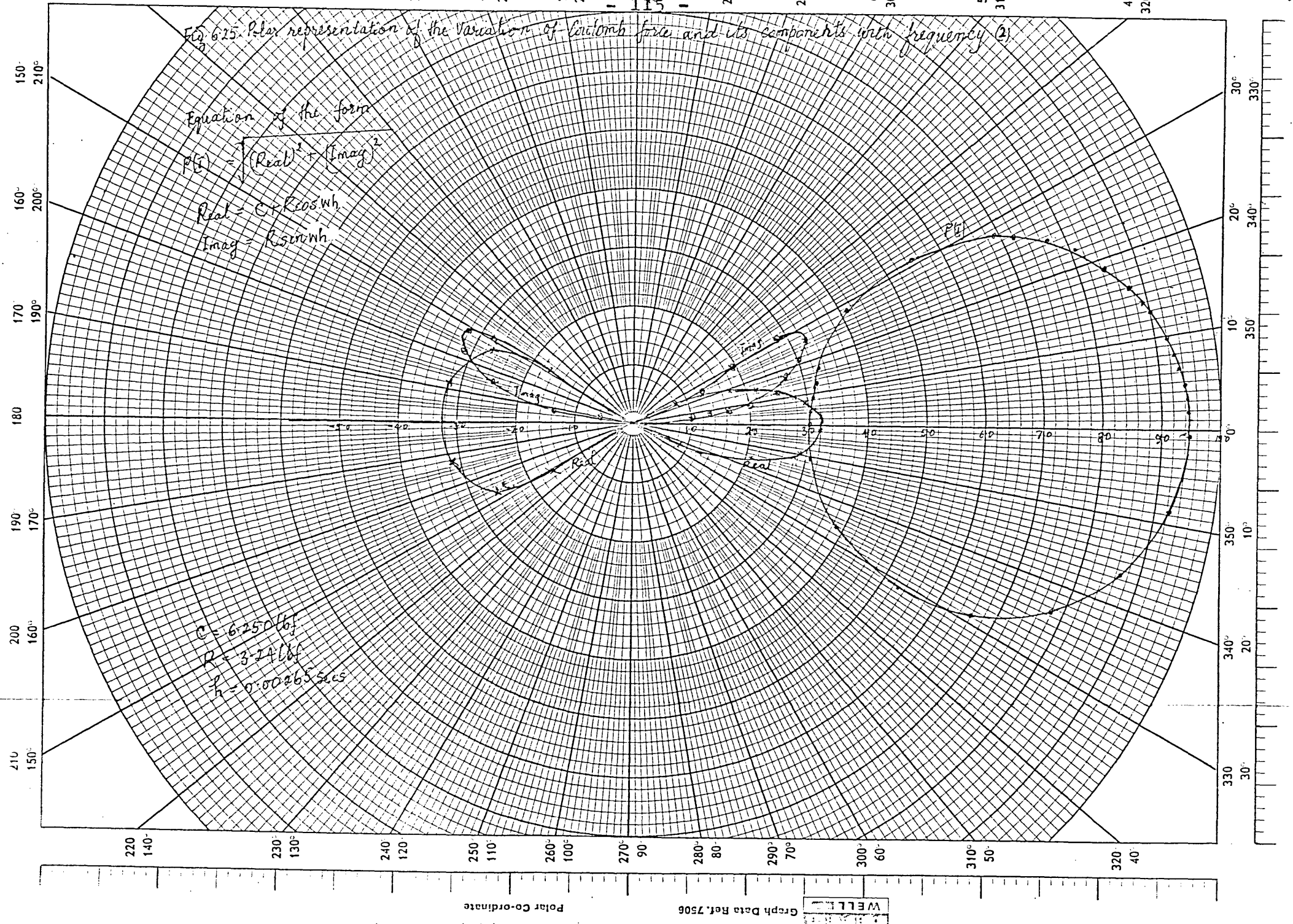


Fig 6.26 Variation of Coulomb force and its components with frequency (2)

FORCE IN LB

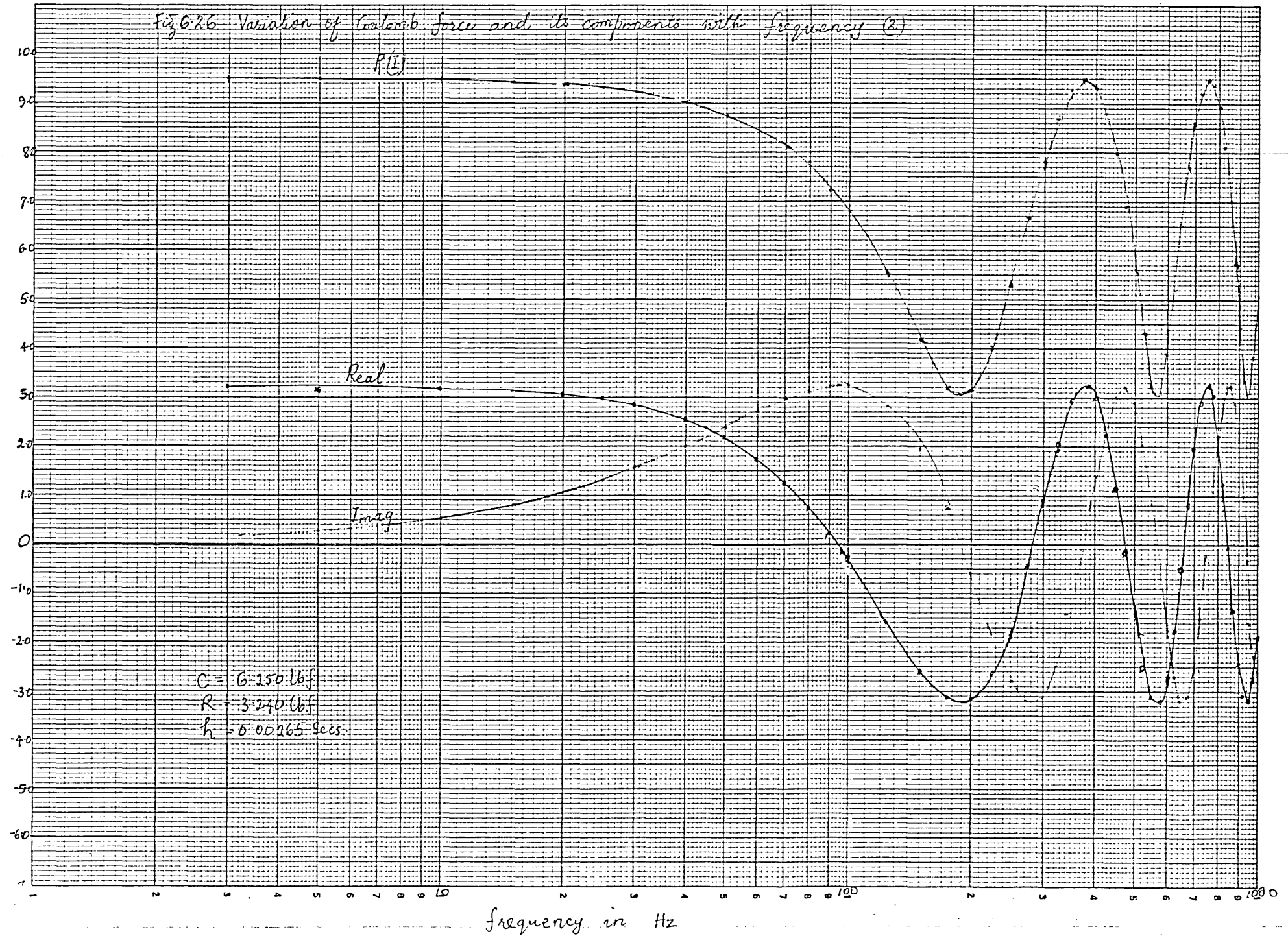
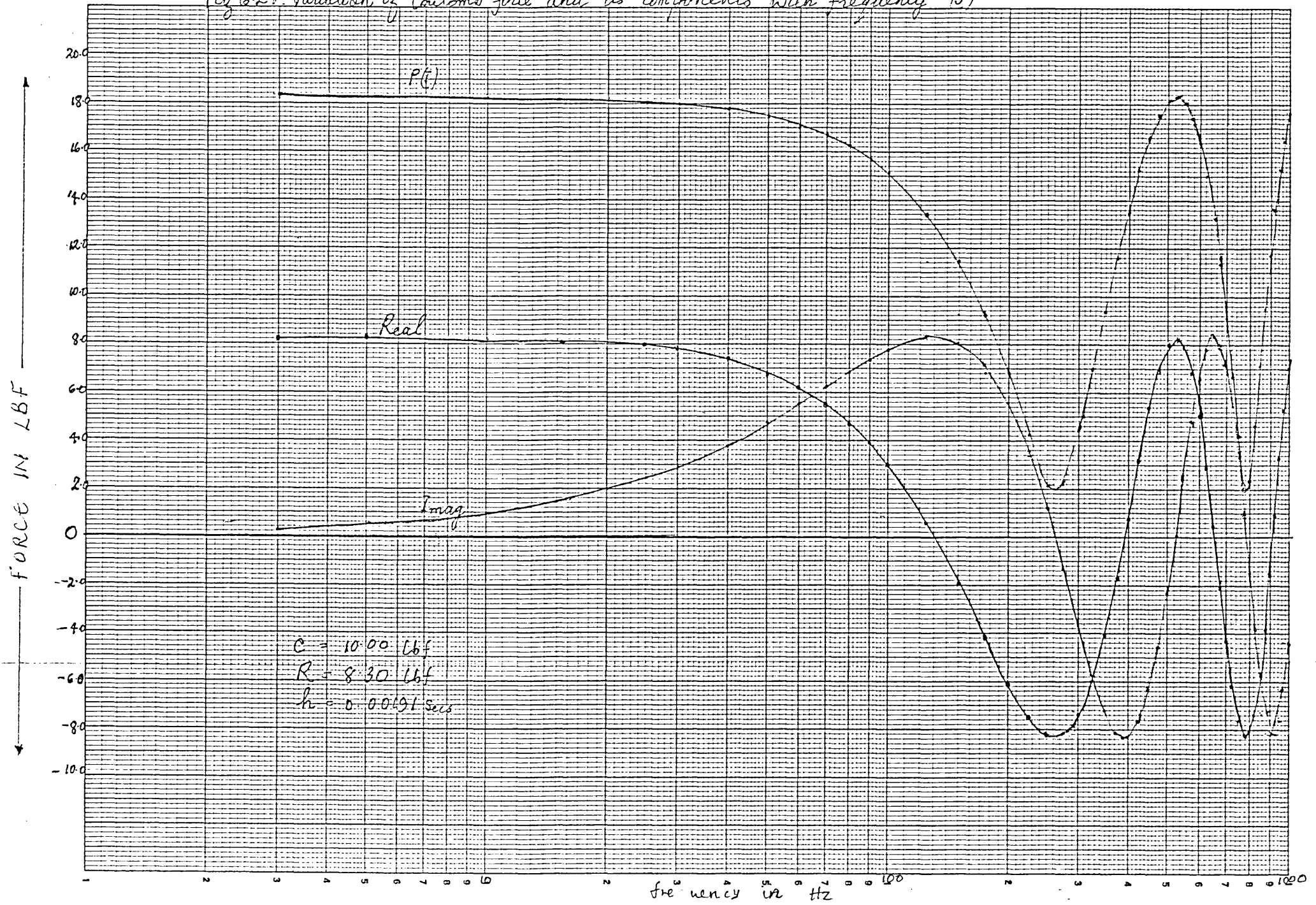
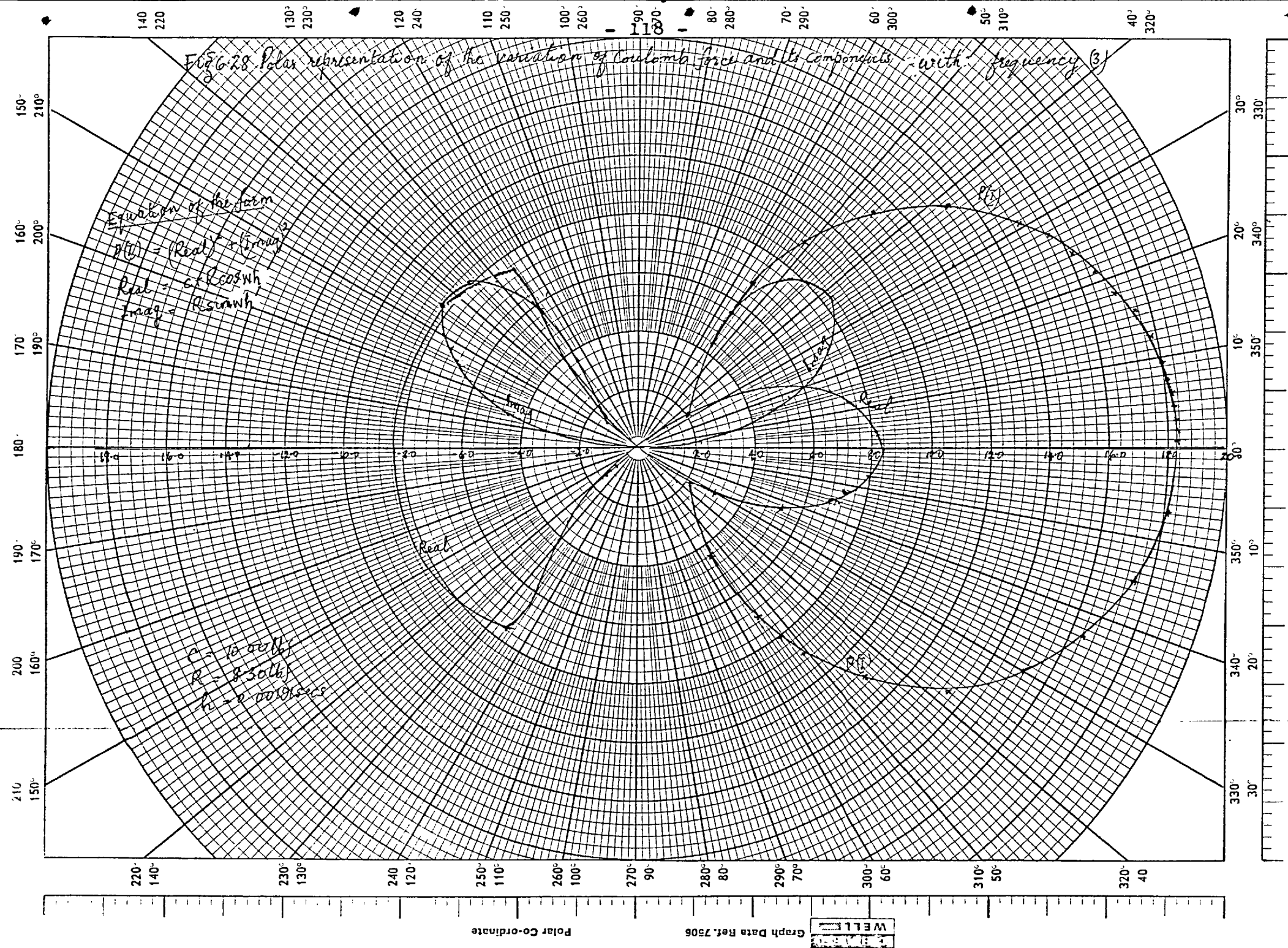


Fig 6.27. Variation of Coulomb force and its components with frequency (3)





as in the experimental work. Thus polar plots were made of the three variables represented by equations (6.13), (6.14) and (3.5) which was represented as

$$X \sqrt{(C + R \cos wh)^2 + (R \sin wh)^2} = P e^{j \omega t} \dots (6.16)$$

6.8 VARIATION OF AMPLITUDE WITH FRICTION FORCE AT CONSTANT FREQUENCY

To test equations (3.25) and (3.26) a series of experiments were performed. At a chosen frequency the force required to accelerate the mass at a certain amplitude was measured; the acceleration and phase difference were also recorded. Friction was now introduced in stages, keeping the excitation force constant and the new amplitude of vibrations, acceleration and phase difference were measured. The readings were taken only after stable conditions had been attained. The friction force was increased in stages up to about 20 lbf and the readings taken. This procedure was repeated for different frequencies so that the variation of amplitude of vibrations with friction force at constant frequency was obtained, and also the friction force was calculated as explained above. The coefficient of friction was thus calculated for the various normal forces (i.e. friction forces) at various frequencies and the results are included in Figs. (6.7) to (6.12) The curves showing the variation of displacement with friction force are

Experimental results. (No spring)

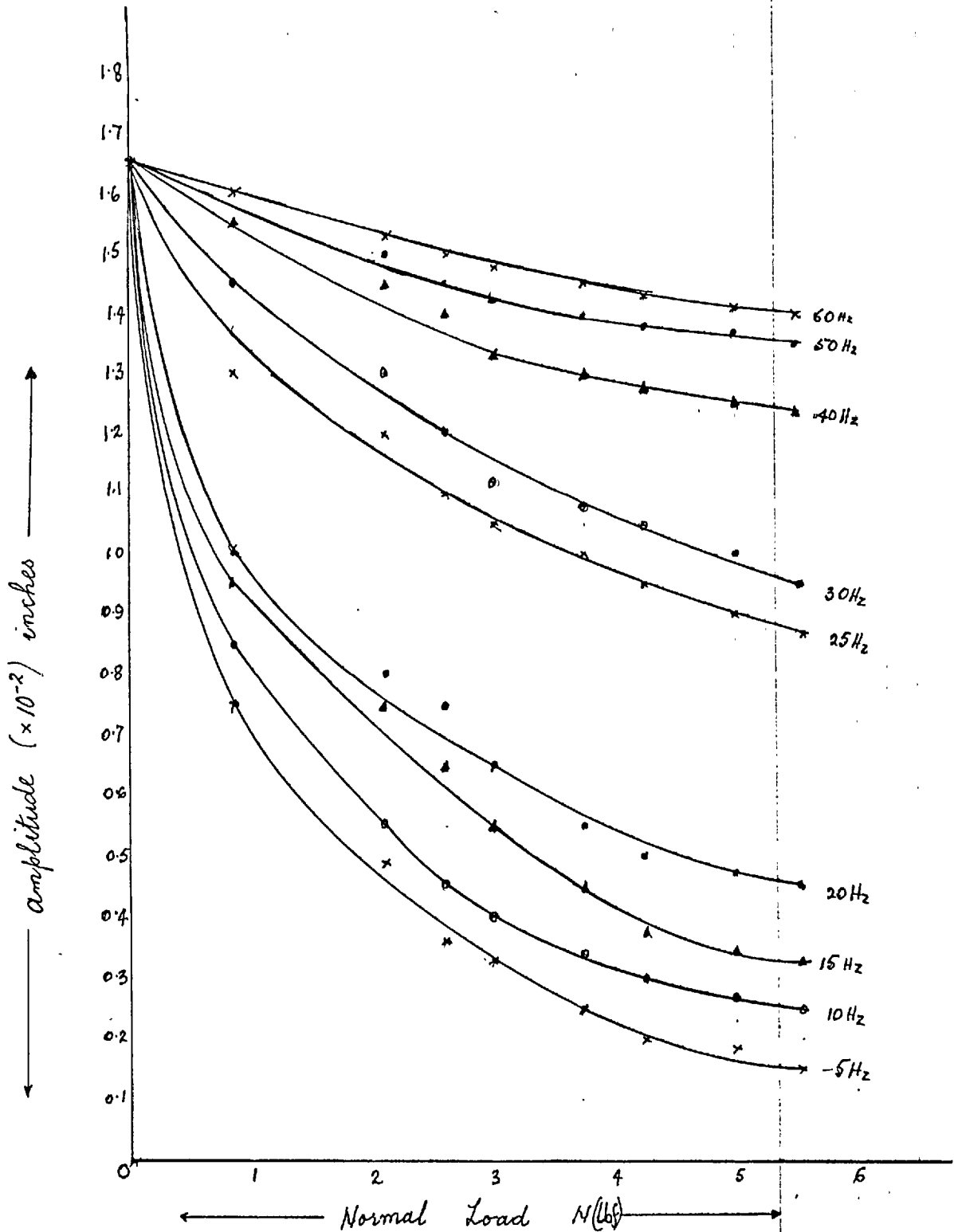


Fig 6.29. Variation of amplitude at constant frequency, with Normal Load.

Theoretical results. (no-spring)

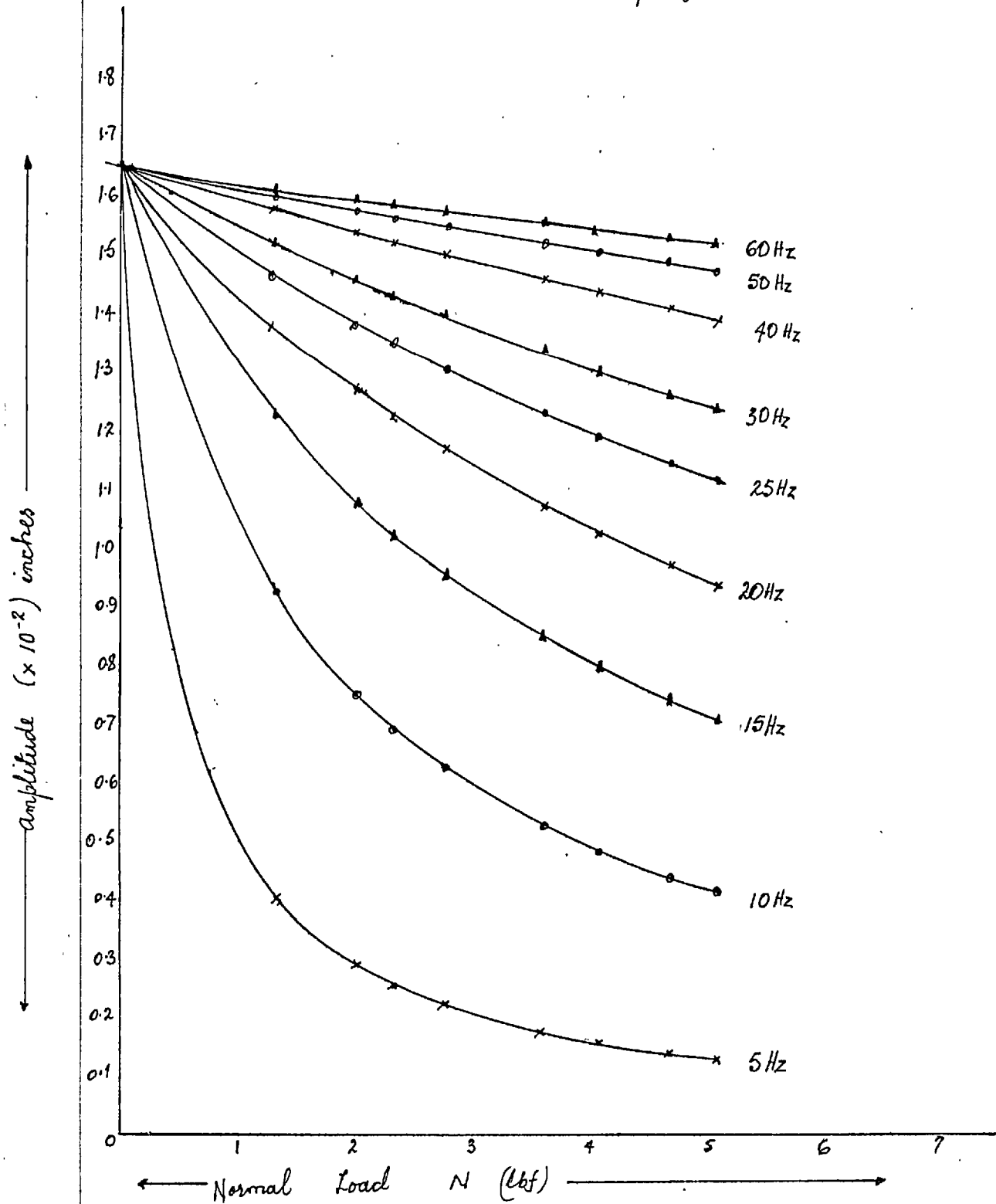


Fig 6.30 Variation of amplitude at constant frequency with Normal Load.

similar to that in Fig.(3.6). Theoretical results for equation (3.25) were obtained for comparison with the experimental results and these do agree closely with the experimental results. These are shown in Figs.(6.29) and (6.30). Moreover, the phase relationship was calculated theoretically and compared with the measured phase difference. The results show a reasonable agreement considering the limits of the equipment used. Since the excitation force was kept constant and the mass and frequency known it was therefore possible to calculate the value of R using equations (3.9) and (3.25) as x and ϕ were measured and therefore known. This was done as a check on the results obtained by the method described earlier (i.e. 2 forces and included angle).

6.9 INTRODUCTION OF A SPRING (RESTORING FORCE) INTO THE SYSTEM

A spring was now introduced thus giving rise to a system that is represented by equation (3.21). The experiments described in 6.1 were repeated i.e. the variation of displacement with the friction force at constant force and frequency were determined for all the cases considered in the previous experiment. It was not deemed necessary to calculate the coefficient of friction from these results as the other two calculations yielded similar values. The phase was however measured at each stage with the other results for

Experimental results (With Spring)

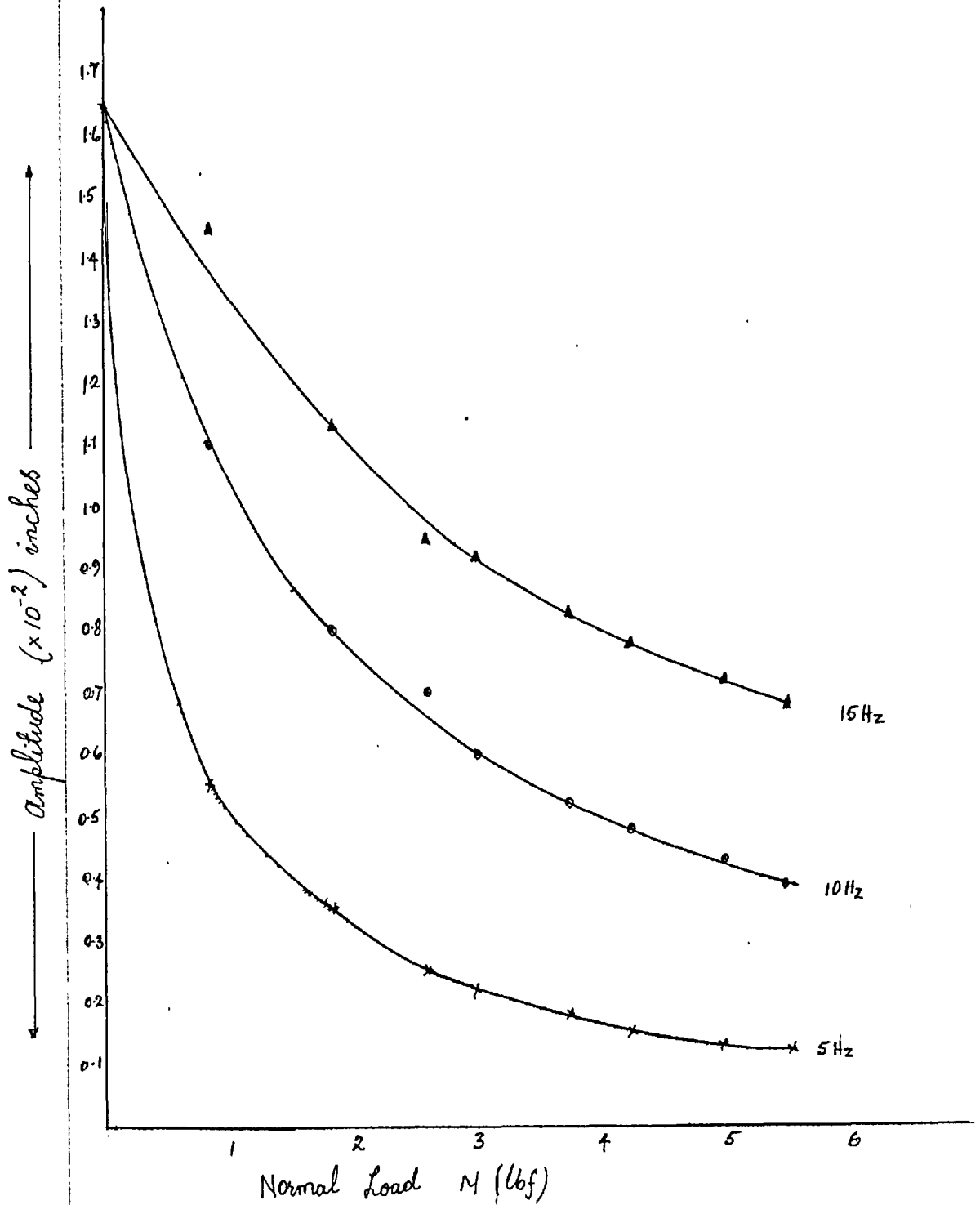


Fig. 6.35. Variation of amplitude with Normal Load at constant frequency

Theoretical results (with spring)

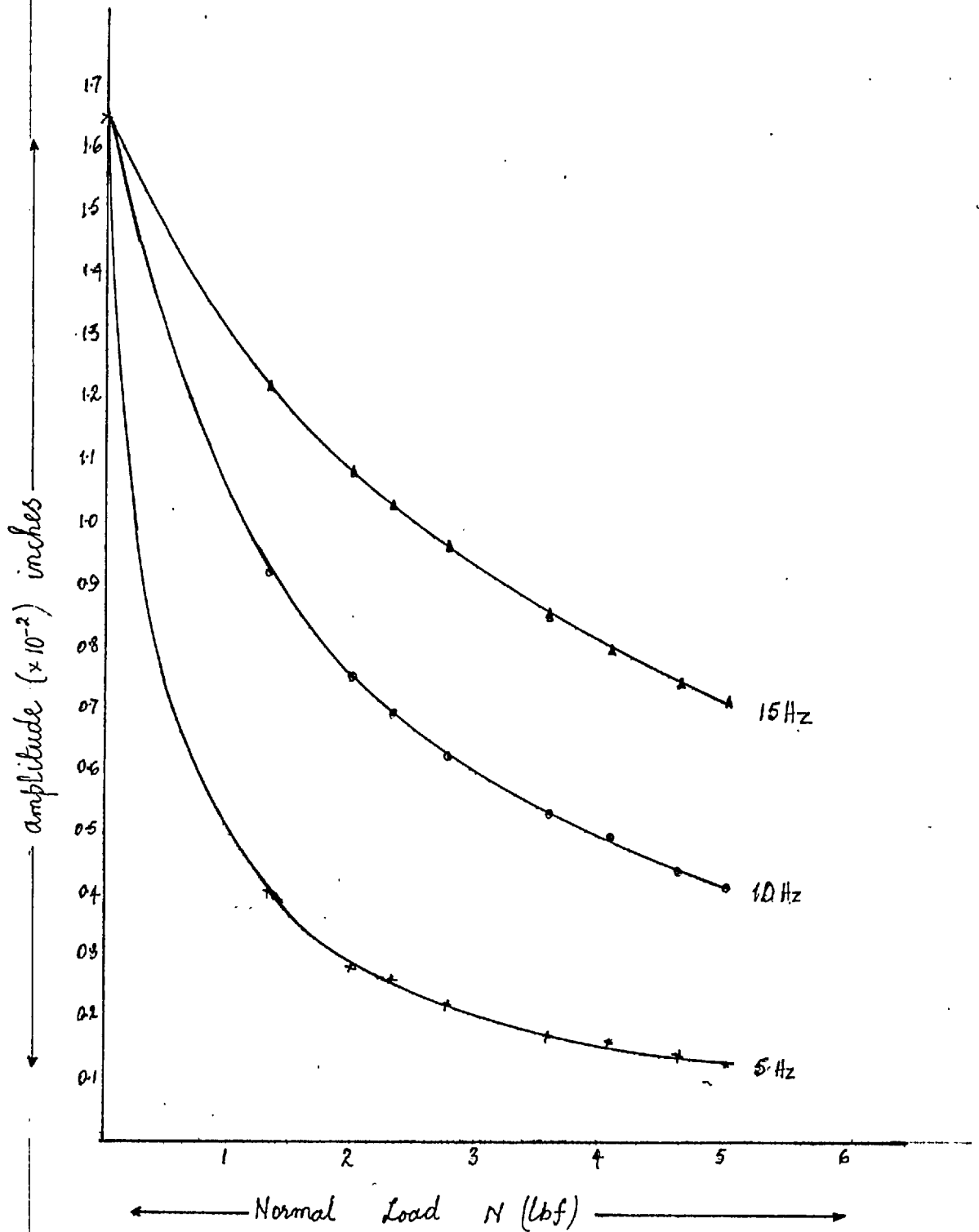
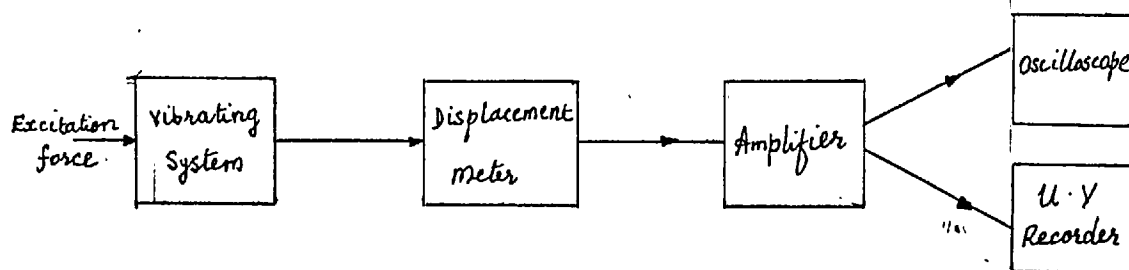


Fig 6.32. Variation of amplitude with Normal Load at constant frequency

comparison with theoretical results. The results are shown in Figs.(6.31) and (6.32).

6.10 FREE DECAY

By feeding the displacement into a U.V. recorder attempts were made to obtain free decay traces for some of the cases considered above. A block diagram for the set up is shown below:



The chart speed and motor speed of the U.V. recorder were chosen and tested. When a suitable combination had been obtained the vibrating system was allowed to run for some time to obtain stability. The motor of the U.V. recorder was started and the excitation force was cut off immediately after thus enabling a free decay trace to be obtained. The results are shown in Figs.(6.33a) to (6.33e) though these have not been analysed.

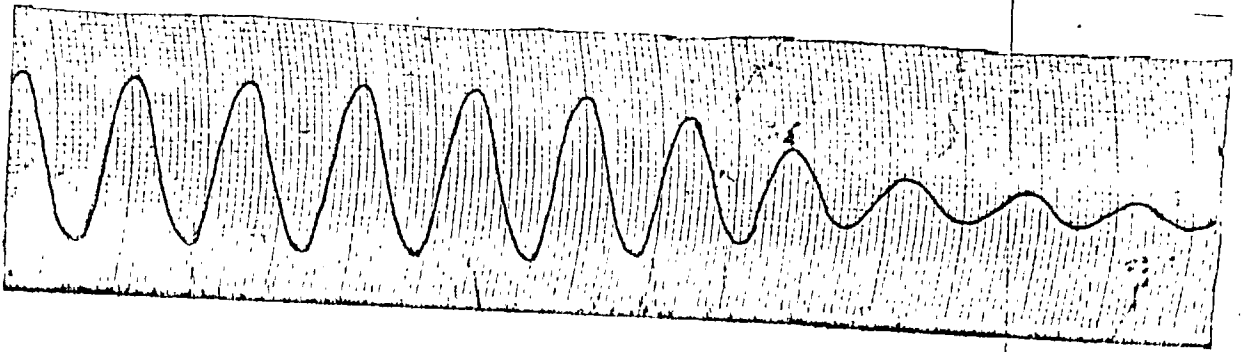


Fig 6.33a. Zero normal loading, $N(0 \text{ lbf/in}^2)$ at 13.4 Hz

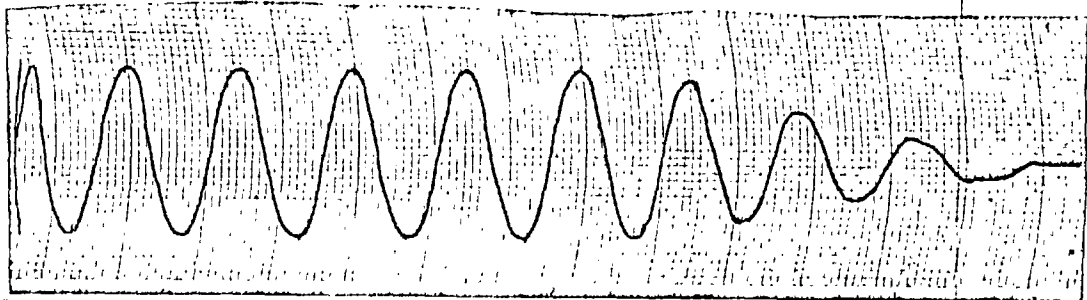


Fig 6.33b. $N = 0.86 \text{ lbf}$, (0.2 lbf/in^2) at 13.4 Hz

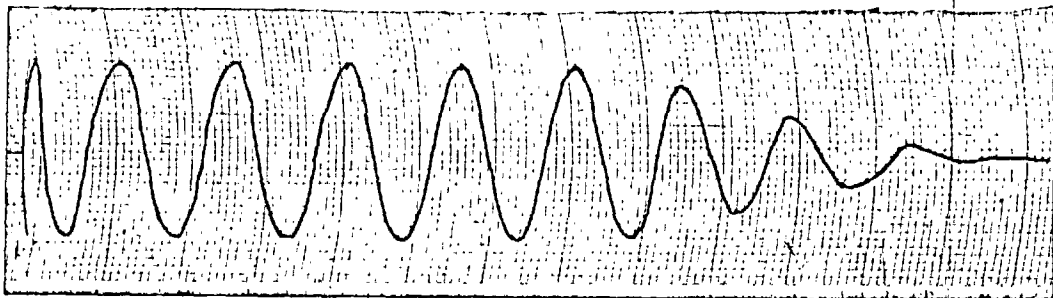


Fig 6.33c. $N = 1.85 \text{ lbf}$ (0.5 lbf/in^2) at 13.4 Hz



Fig 6.33d. $N = 0.86 \text{ lbf}$ (0.2 lbf/in^2) at 30 Hz

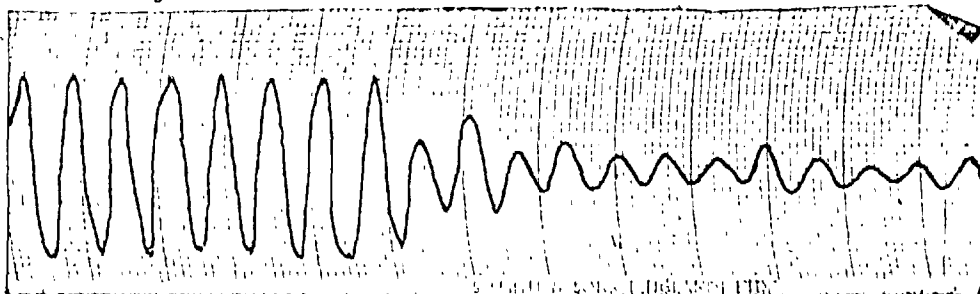


Fig 6.33e. $N = 2.60 \text{ lbf}$ (1.0 lbf/in^2) at 30 Hz

CHAPTER 7

FORCED VIBRATIONS WITH MIXED DAMPING

The experimental and theoretical results in the previous tests showed good agreement and therefore experimental work was began on the forced vibration-Response tests with the three types of damping discussed (Coulomb, Viscous and Viscoelastic) taking the phase relationship into consideration.

7.1 MEASURING TECHNIQUES

The measurements could be made in four different ways:

- 1). Forced resonance tests, keeping the excitation force constant and measuring the amplitude and phase angle giving a response curve (from which the required quantities can be calculated).
- 2). Forced resonance tests, keeping the amplitude constant and measuring the excitation force and phase angle, giving a response curve (from which the required quantities can be obtained by calculation).
- 3). Forced non-resonance oscillations with measurements of the force, displacement and the phase angle between them.
- 4). Forced resonance tests, keeping the amplitude

constant and measuring the excitation and accelerating forces and the phase angle between them, giving response curves from which the required quantities can be calculated.

Of the methods described above (1) and (3) were found to be unsuitable. The friction force is somewhat dependent upon the amplitude of vibrations and therefore method (1) which involves measuring a variable amplitude may lead to spurious results. Moreover, it is more difficult to measure the displacement near resonance than it is to measure the force. Method (3) will not show the effect of damping, whatever type it may be, on the resonant frequency. Methods (2) and (4) are similar and were found acceptable, but in terms of experimental work it was found that the magnitude of the accelerating and excitation forces were comparable and therefore liable to give better phase measurements. For this reason method (4) was used even though the response curves are obtained by plotting the variation of the excitation force with frequency.

In order to test different combinations the experiments were done in two stages. In the first stage the metallic spring was used as the restoring force spring in the system whilst in the second stage the visco-elastic material provided the restoring force spring.

7.2. STAGE ONE WITH VISCOUS DAMPING ONLY

Viscous damping was supplied by an electro-magnetic damper. However, two resonance response-tests were made without any external damping to determine the hysteretic damping in the spring. In all the tests the amplitude of vibrations was kept constant at 0.00825" and the force required to maintain this value of amplitude at each frequency measured. The acceleration was also measured and the phase angle between the force and acceleration at each frequency was also determined making use of a Reference Signal as in the earlier experiments. From the results obtained without external damping the amount of hysteretic damping in the spring was determined. Readings were taken for three values of current and from the calibration curves the coefficients of viscous damping, and the damping ratios were obtained by calculation.

These values and other known constants were now fed as data into a computer programme to obtain theoretical curves for each set of data.

7.2.1 THE COMPUTER PROGRAMME

The programme uses "The Ritz-Galerkin method" to solve ordinary second order linear and non-linear equations with harmonic forcing. The non-linearity is provided by a cubic polynomial in X , with either viscous or hysteretic damping. The solution is of the

form $X = X_0 + X_1 \sin(\omega t - \phi)$ and values of X_0 and frequency ω were computed for a range of values of X_1 . Since no higher harmonics are included in the solution, the solution is a first approximation to the exact solution for equations with non-linear springs.

For linear springs such as were used in this experimental work the solution is exact. The equation in this case is the well known one for a single degree of freedom system with one mass and one spring.

$$m\ddot{x} + c\dot{x} + kx = Pe^{j\omega t} = P\sin(\omega t - \phi) \dots (7.1)$$

from which

$$x = \frac{P_0}{k} \sqrt{\frac{1}{(1 - (\frac{\omega^2}{\omega_0^2})^2 + (2 \frac{c}{c_0} \frac{P}{\omega})^2)}$$

$$= \frac{P_0}{k} \sqrt{\{(1 - n^2)^2 + (2\eta n)^2\}}$$

$$\omega_0^2 = \frac{k}{m} \quad \text{and} \quad c_0 = 2\sqrt{km} \quad \eta = \frac{c}{c_0}$$

$$\text{and } \tan \phi = \frac{2\eta n}{1 - n^2}$$

Since in the experimental work the displacement was kept constant it was found necessary to obtain the theoretical solution in terms of P , keeping x constant.

Thus as $\frac{1}{\sqrt{(1 - n^2)^2 + (2\eta n)^2}}$ is constant.
eqn. becomes

Fig 7.1 Forced Vibration response for viscous damping only. (Experimental results)

- 131 -

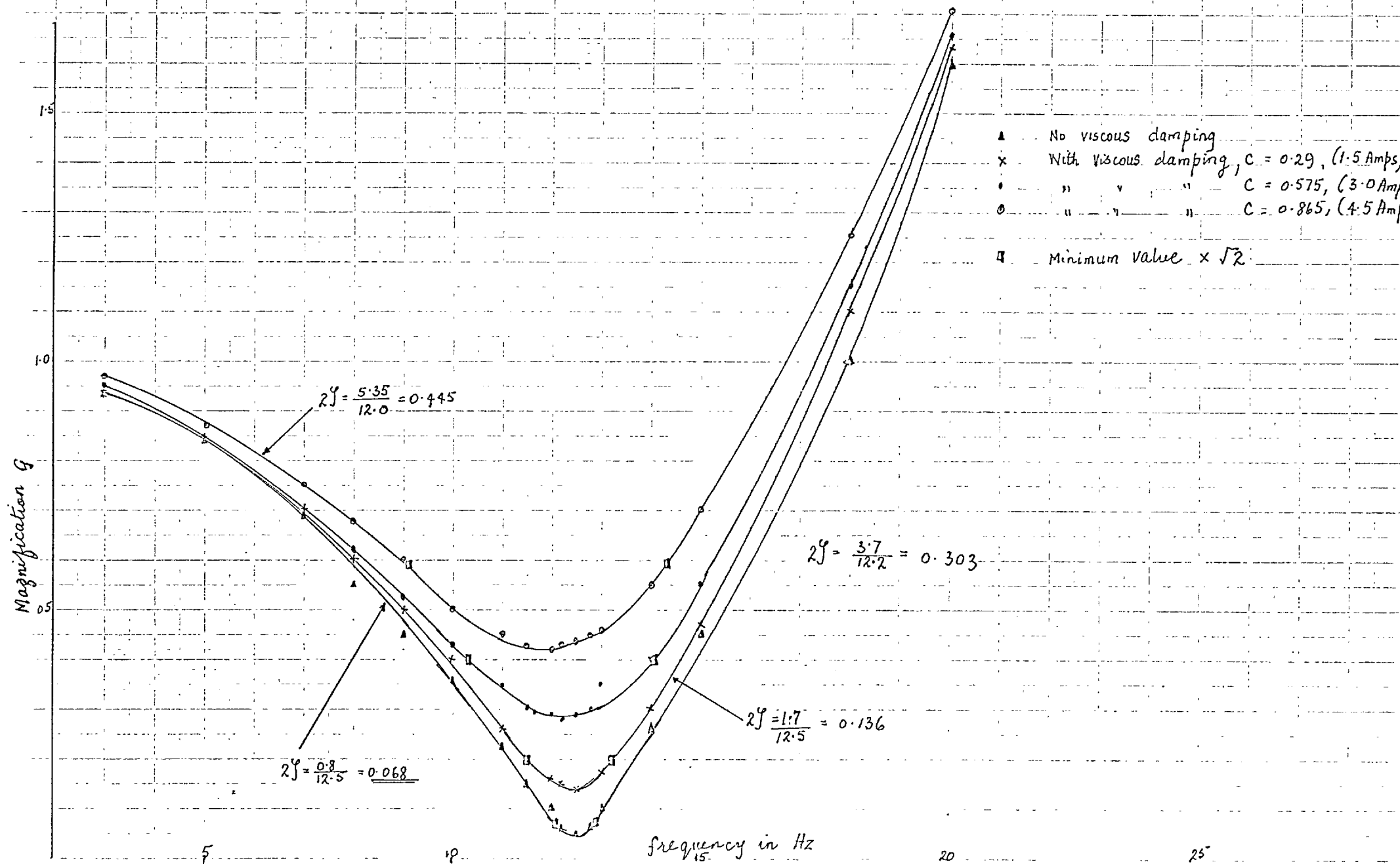


Fig 7.2. Phase response for viscous damping only (Experimental results)

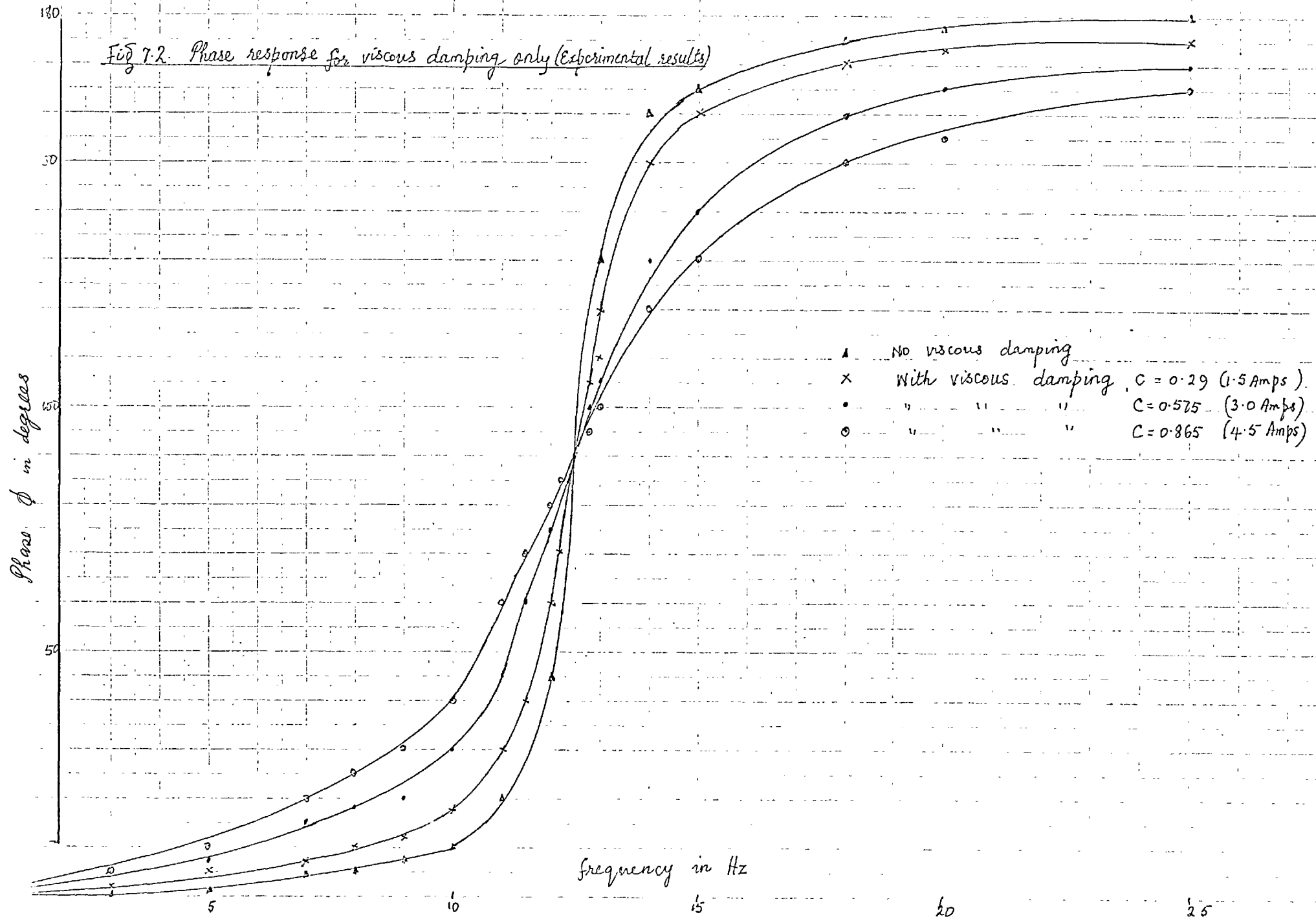


Fig 7.3 Forced vibration response for viscous damping only (Theoretical curves)

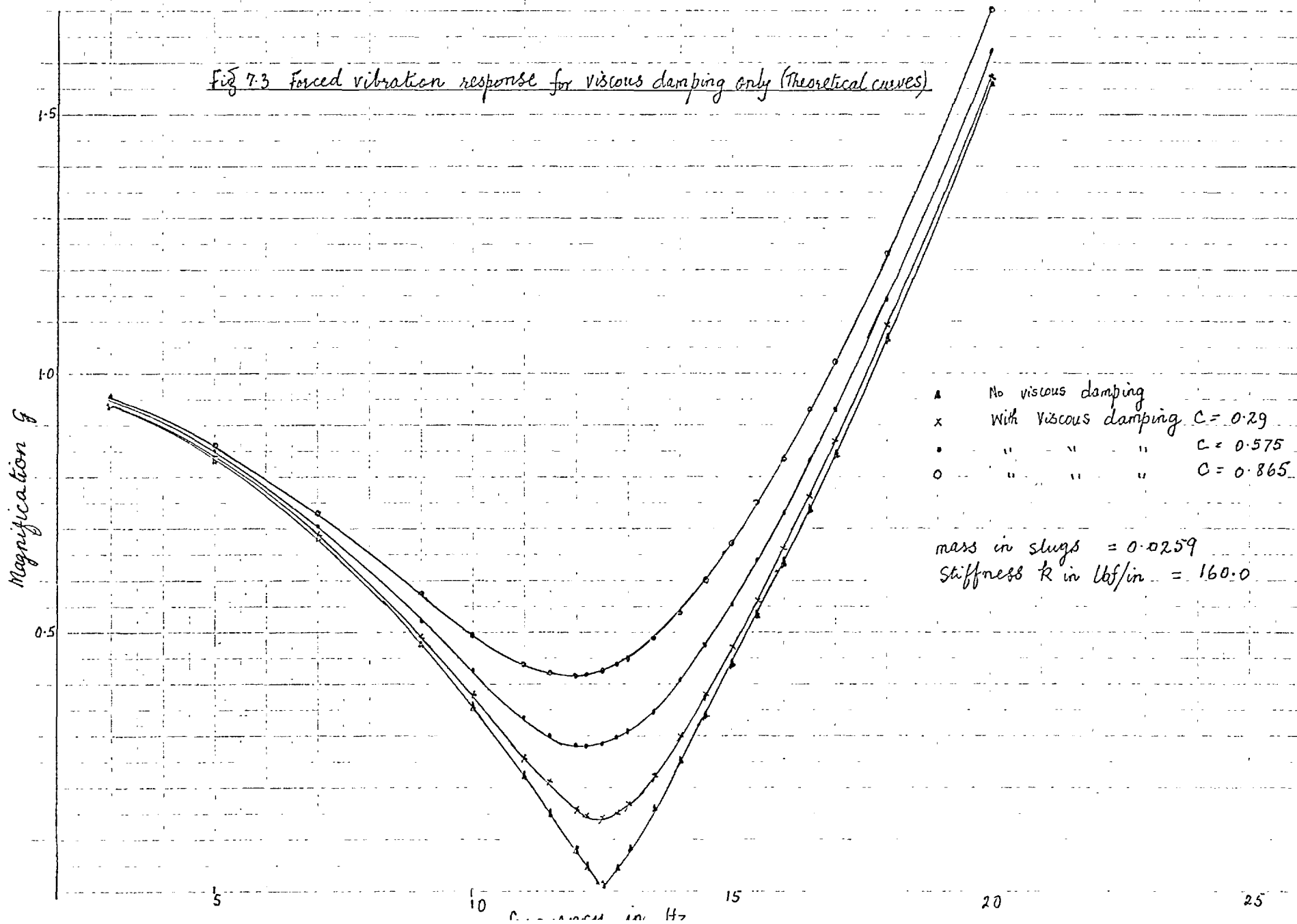


Fig. 7.4 Phase response for viscous damping only (Theoretical curves)

x with viscous damping $C = 0.29$
 • " " " $C = 0.575$
 o " " " $C = 0.865$

Phase ϕ in degrees

Phase ϕ in degrees

frequency in 20 Hz

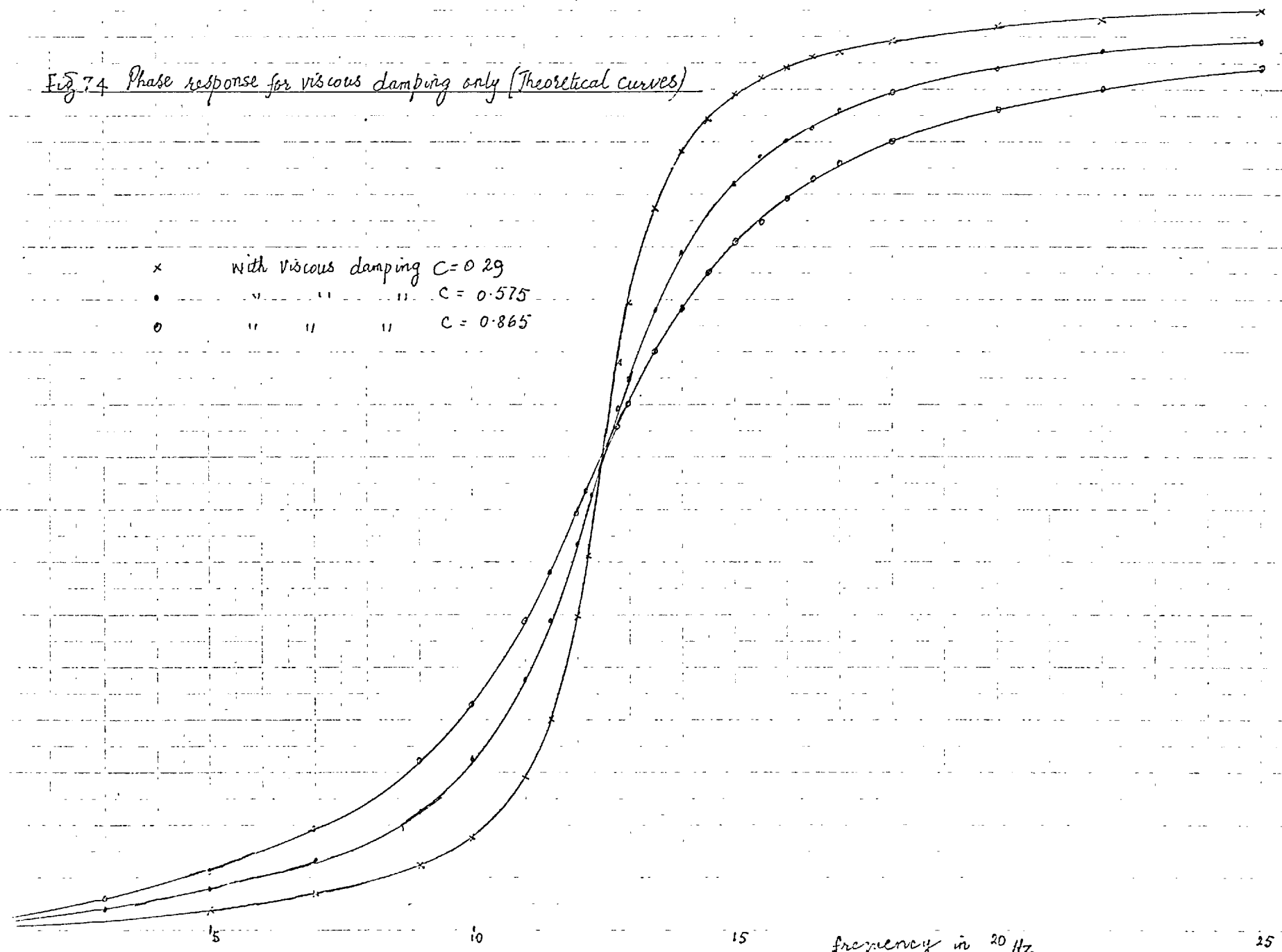
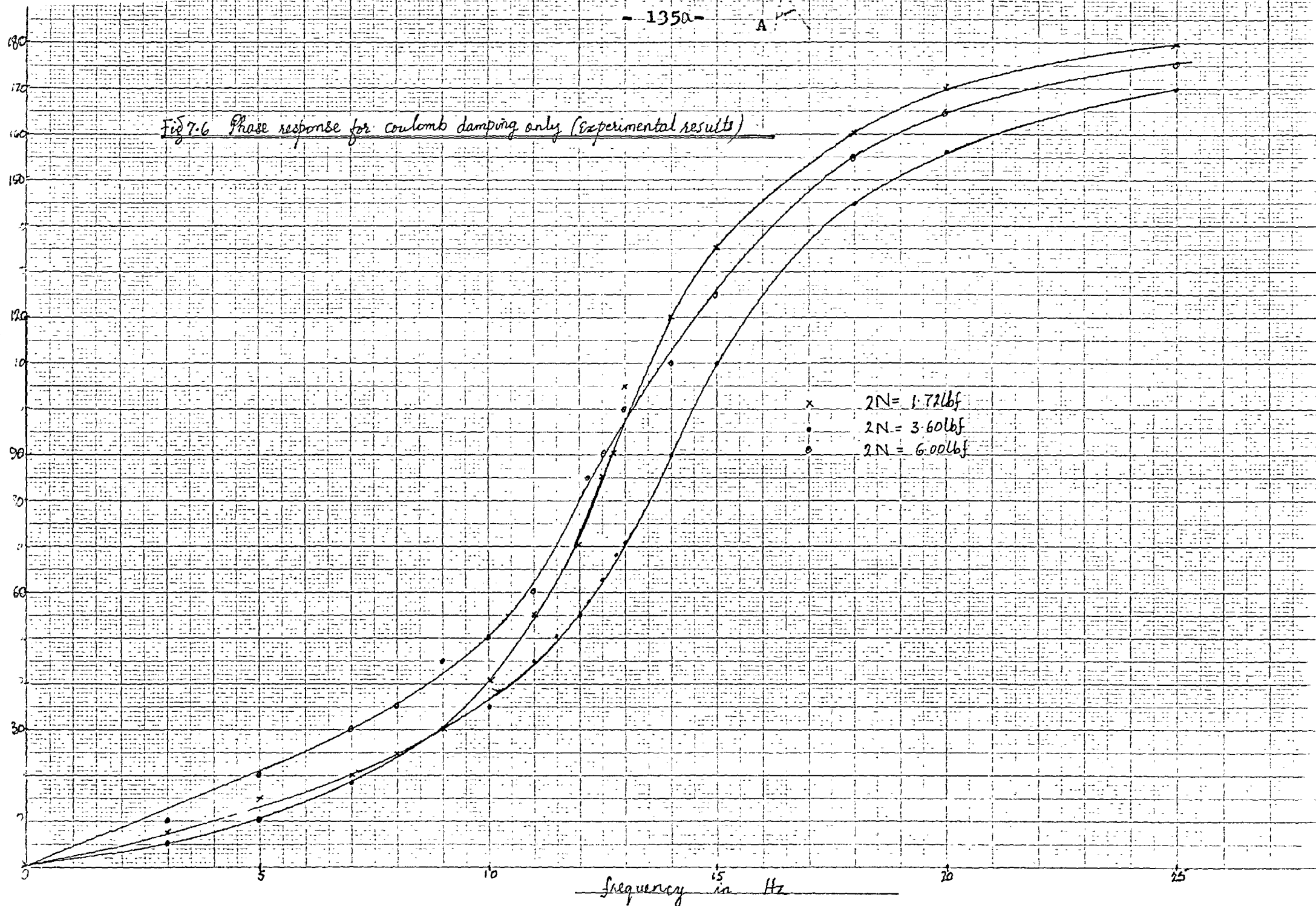


Fig 7-6 Phase response for coulomb damping only (experimental results)Phase ϕ in degrees

$$X_0 = \frac{P_0}{k} \quad X \quad G \quad \text{where } G = \frac{1}{\sqrt{(1 - n^2)^2 + (2n\gamma)^2}}$$

$$X_1 = \frac{P_1}{k} \quad X \quad G$$

$$\frac{X_1}{X_0} = \frac{P_1}{P_0} \quad X \quad \frac{k}{P_0}$$

$$\therefore P = \frac{P_0}{X_0} \times X_1 = G_1 X_1 \quad \text{where } G_1 = \frac{P}{X_0}$$

The phase remains unchanged.

The results are shown in Figs.(7.1) to (7.4).

Figs.(7.1) and (7.2) are experimental results whilst

Figs.(7.3) and (7.4) are theoretical results.

Both the experimental and theoretical response curves show the variation of G, the magnification factor with frequency.

7.2.2. FORCED VIBRATIONS WITH COULOMB AND VISCOUS DAMPING

Experiments were now performed to verify equations (3.22) and (3.23).

With the apparatus set up as described the viscous damping was switched off. Coulomb damping was now introduced into the system pressurising the bellows to give a specific normal loading. At this stage the pressure gauge with low pressure scale was used. Forced vibrations were performed as before except that the alignment of the blocks was checked and sufficient

time was allowed to obtain stable conditions before the readings were taken. The parameters measured were the same as those considered with viscous damping in the system. The pressure was changed and the experiment was repeated. It was found necessary to repeat the experiments if there was an interruption during the test for a fairly long time (half an hour). After (satisfactory results i.e.) repeatable results were obtained viscous damping was now introduced, the current being at the same level as was used in the case when , there was no coulomb damping.

7.2.3 INTRODUCTION OF THE VISCOUS DAMPING

By introducing viscous damping into a system with Coulomb damping the model was now that in Fig.(3.4) and equations (3.19) and (3.20) applied.

To ensure that the level of Coulomb damping was the same as before readings were taken to check with the previous results obtained and when this showed agreement the viscous damping was then introduced into the system and the readings taken. The readings were taken at the same frequencies in all cases described.

7.4 THEORETICAL RESULTS

A special computer programme was written for the solution of equations (3.22) and (3.23).

All the parameters in these equations were either

known or calculable except the delay time "h".

The data included the mass (m in lbf), the stiffness of the spring (k) in lbf/in, displacement (x) in inches, the friction force constant (R), the viscous damping coefficient (c) and the viscoelastic coefficient (g). The value of "h" could be guessed as an initial value and from the results obtained better values could be used until the best fit was obtained. The method was found to be laborious and not verifiable.

Instead equation (3.23) was used

$$\text{i.e.} \quad \tan \phi = \frac{R \sin \omega h}{R \cos \omega h} \quad \text{as} \quad k - m\omega^2 = 0 \\ \text{and } S = \frac{R}{k}$$

Thus $\phi = \omega h$

and thus "h" is obtained.

This experimental value was now fed as data together with the other parameters and theoretical solutions for (3.22) and (3.23) were obtained for comparison with the experimental results.

The experimental results are shown in Figs.(7.5), (7.6), (7.9) and (7.10). The theoretical results are shown in Figs.(7.7), (7.8), (7.11) and (7.12).

7.3. STAGE TWO WITH THE VISCOELASTIC MATERIAL

The viscoelastic material was now substituted for the metallic spring. It was bonded to the spigots with EVO-STICK (an impact adhesive) and when satisfactory bonding had been obtained forced frequency response tests were performed as before. The displacement

Fig 7.5 Forced frequency response for coulomb damping only (Experimental results)

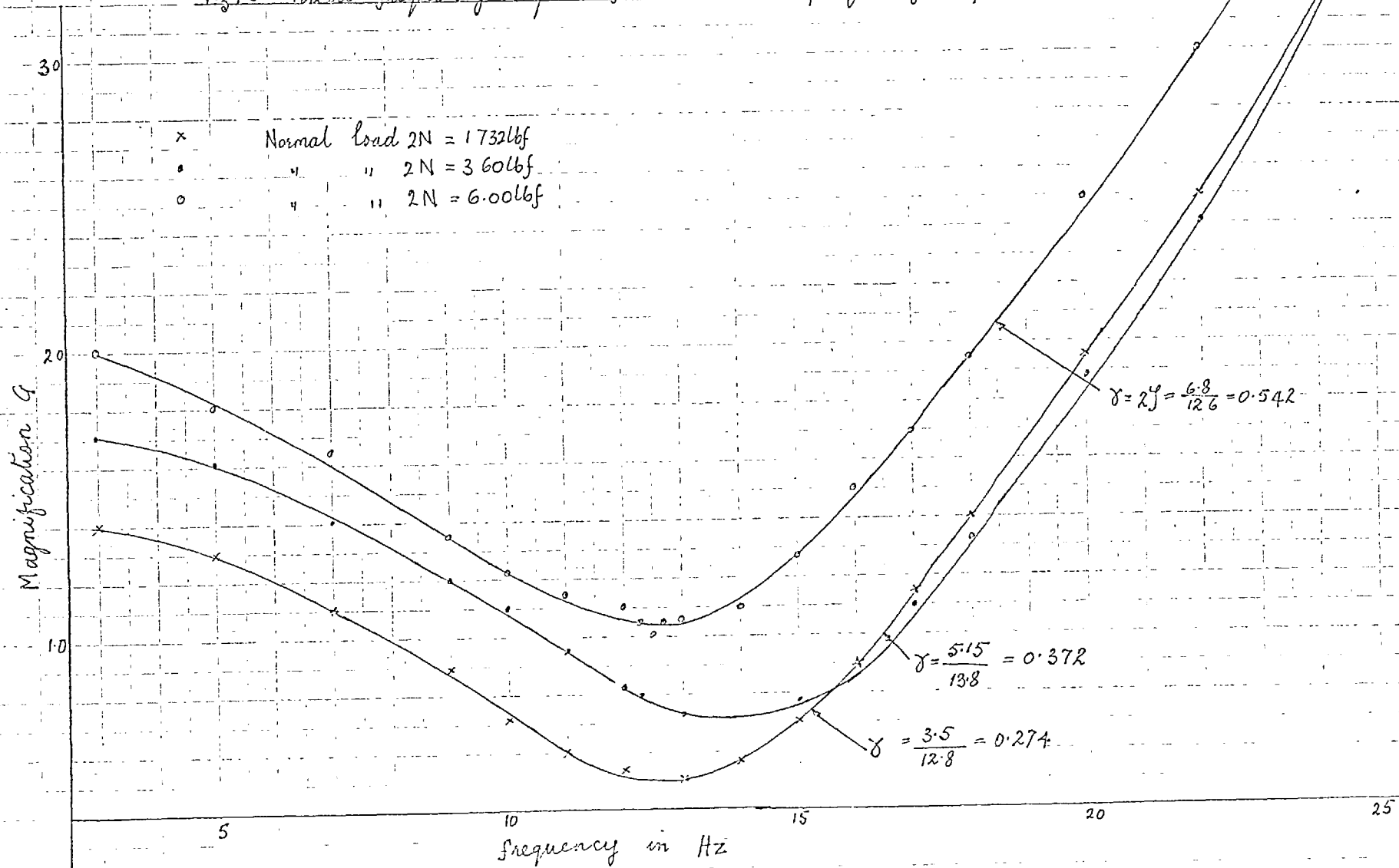
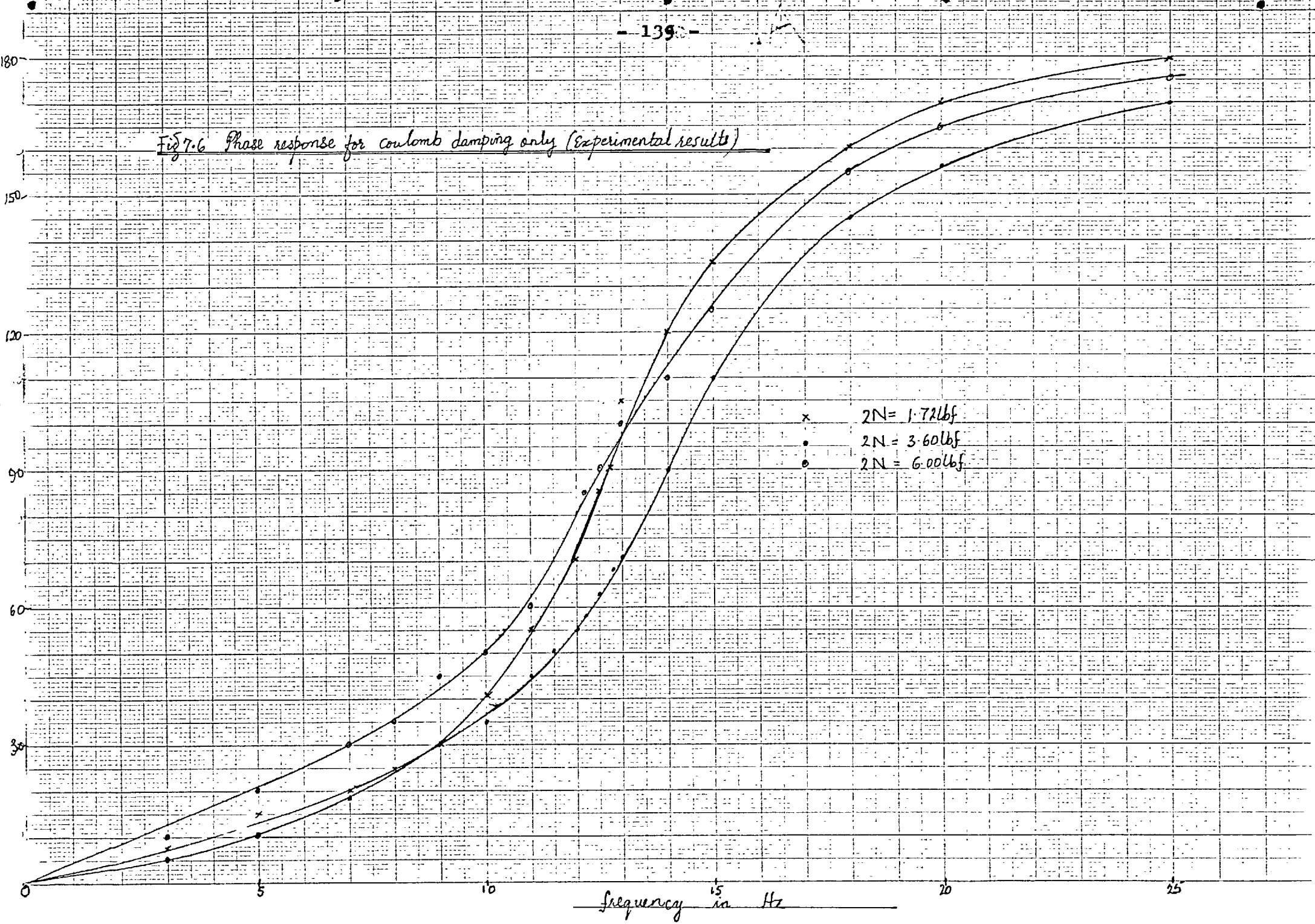


Fig 7.6 Phase response for coulomb damping only (Experimental results)

Phase ϕ in degrees



frequency in Hz

Fig 7.7 Forced Vibration response for Coulomb damping only (Theoretical curves)

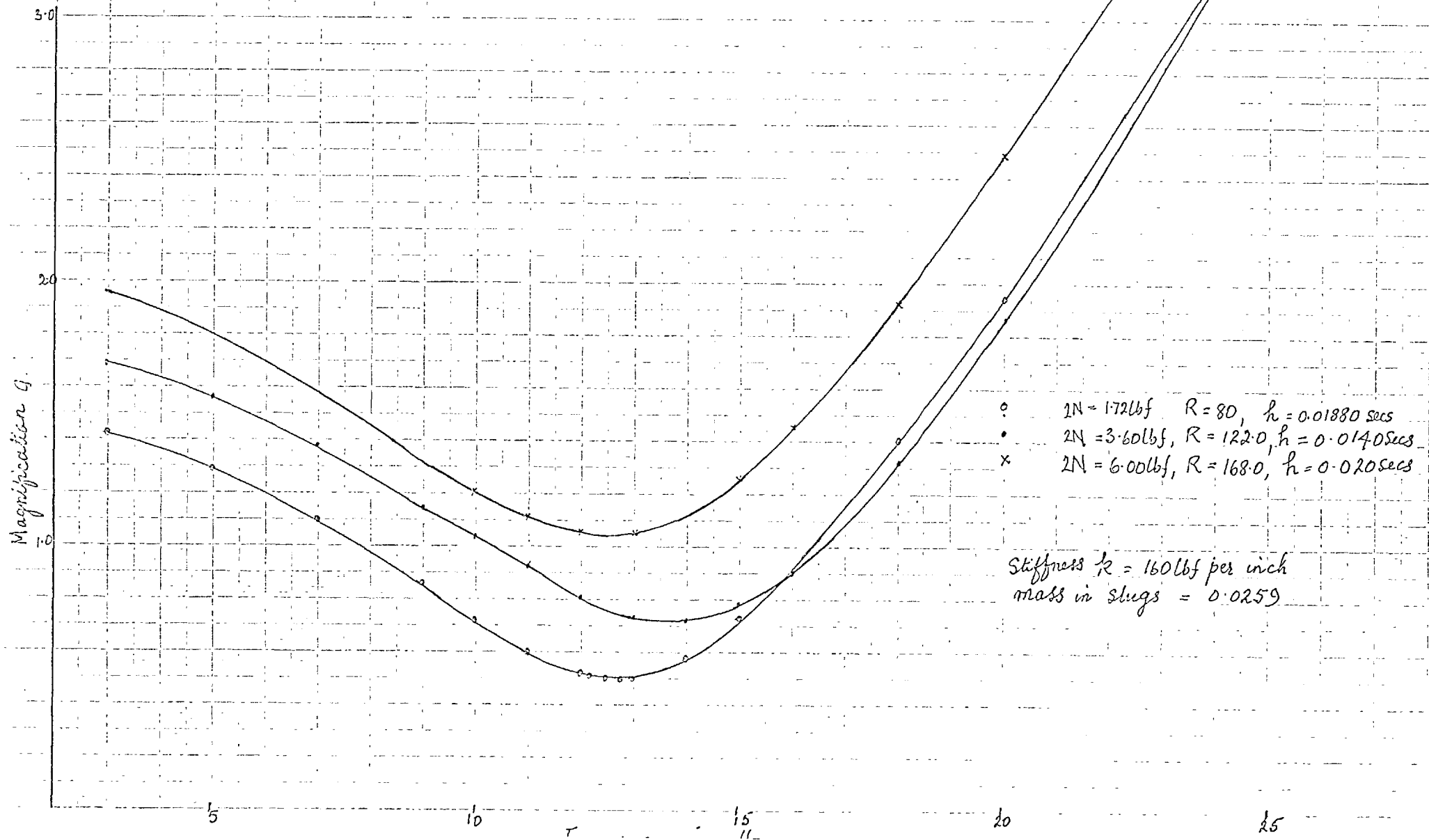


Fig 7.8 Phase response for coulomb damping only (Theoretical curves)

Phase ϕ in degrees

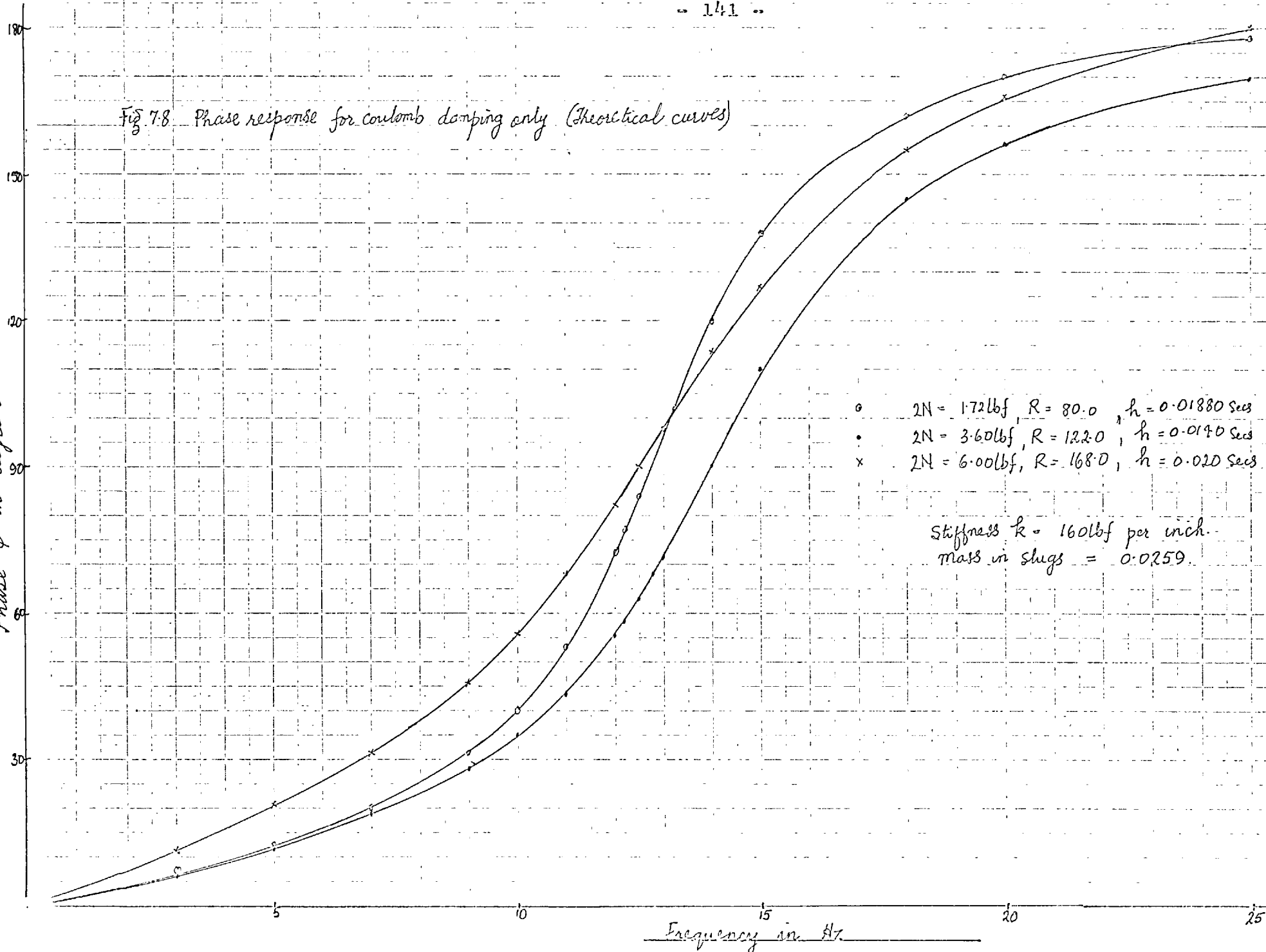


Fig 7.9 Forced vibration response with coulomb and viscous damping (Experimental results)

Magnification g

30

20

10

- x Load $2N = 1.72 \text{ lbf} + 1.5 \text{ Amps}$ ($C = 0.29$)
- Load $2N = 3.60 \text{ lbf} + 3.0 \text{ "}$ ($C = 0.575$)
- o Load $2N = 6.00 \text{ lbf} + 4.5 \text{ "}$ ($C = 0.865$)
- $\sqrt{2} \times$ minimum force point

$$2\zeta = \frac{14.5}{12.3} = 1.18$$

$$2\zeta = \frac{7.4}{13} = 0.57$$

$$2\zeta = \frac{4.8}{12.5} = 0.385$$

Frequency in Hz

5

10

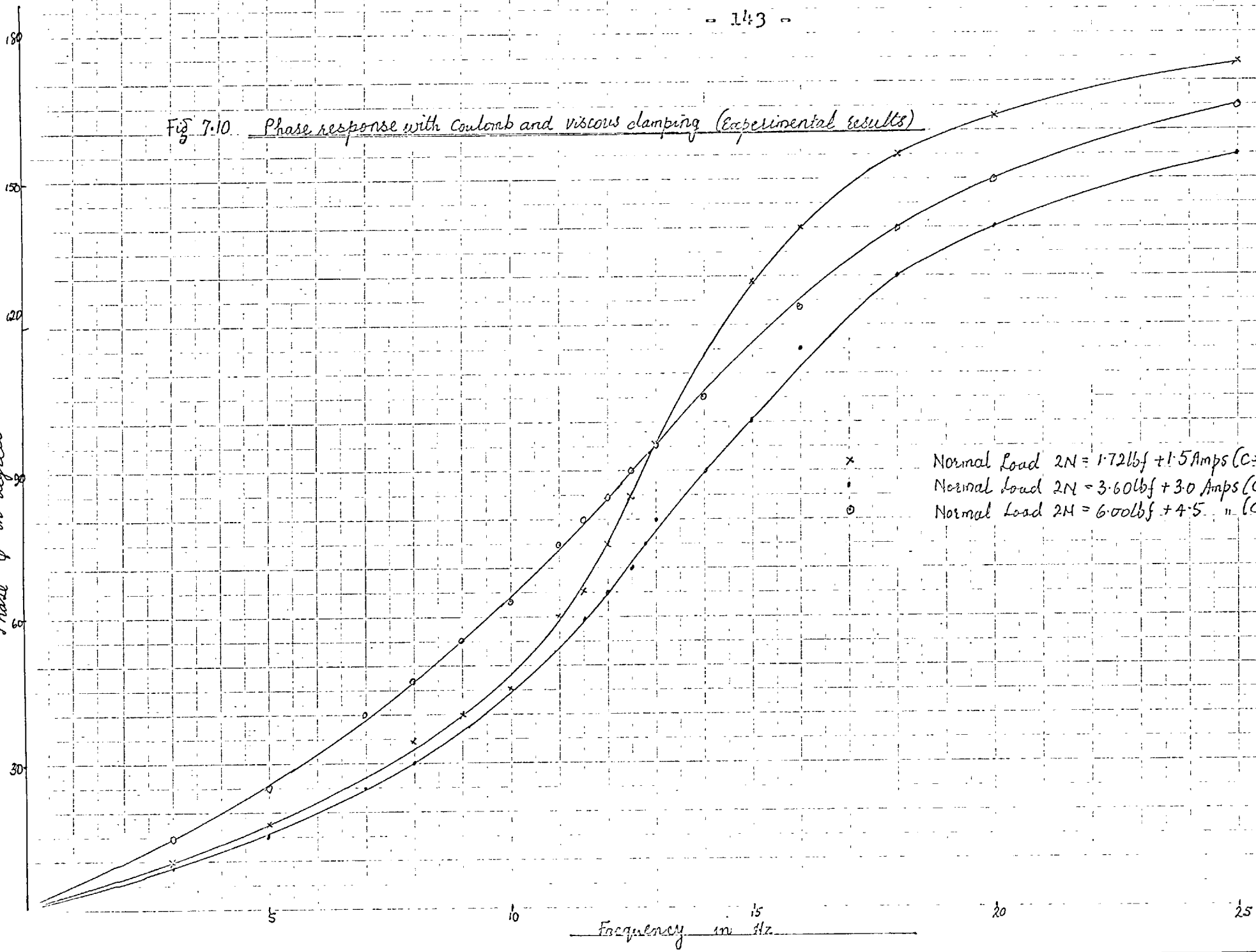
15

20

25

Fig 7.10. Phase response with Coulomb and viscous clamping (Experimental results)

Phase ϕ in degrees



x Normal Load $2N = 1.72 \text{ lbf} + 1.5 \text{ Amps}$ ($C = 0.29$)
 • Normal Load $2N = 3.60 \text{ lbf} + 3.0 \text{ Amps}$ ($C = 0.575$)
 o Normal Load $2N = 6.00 \text{ lbf} + 4.5 \text{ "}$ ($C = 0.865$)

Frequency in Hz

Fig 7.11. Forced frequency response with Coulomb and viscous damping.

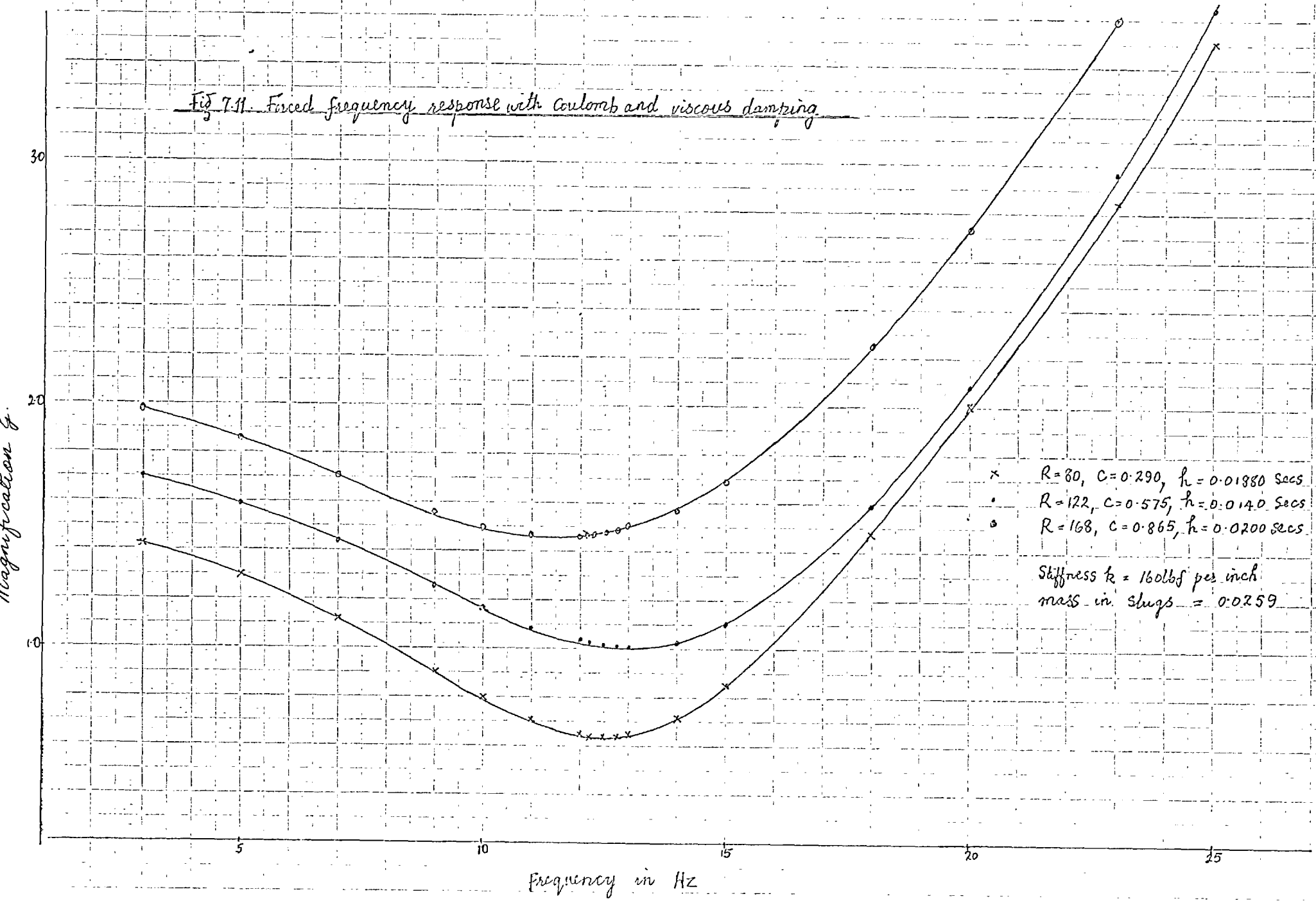
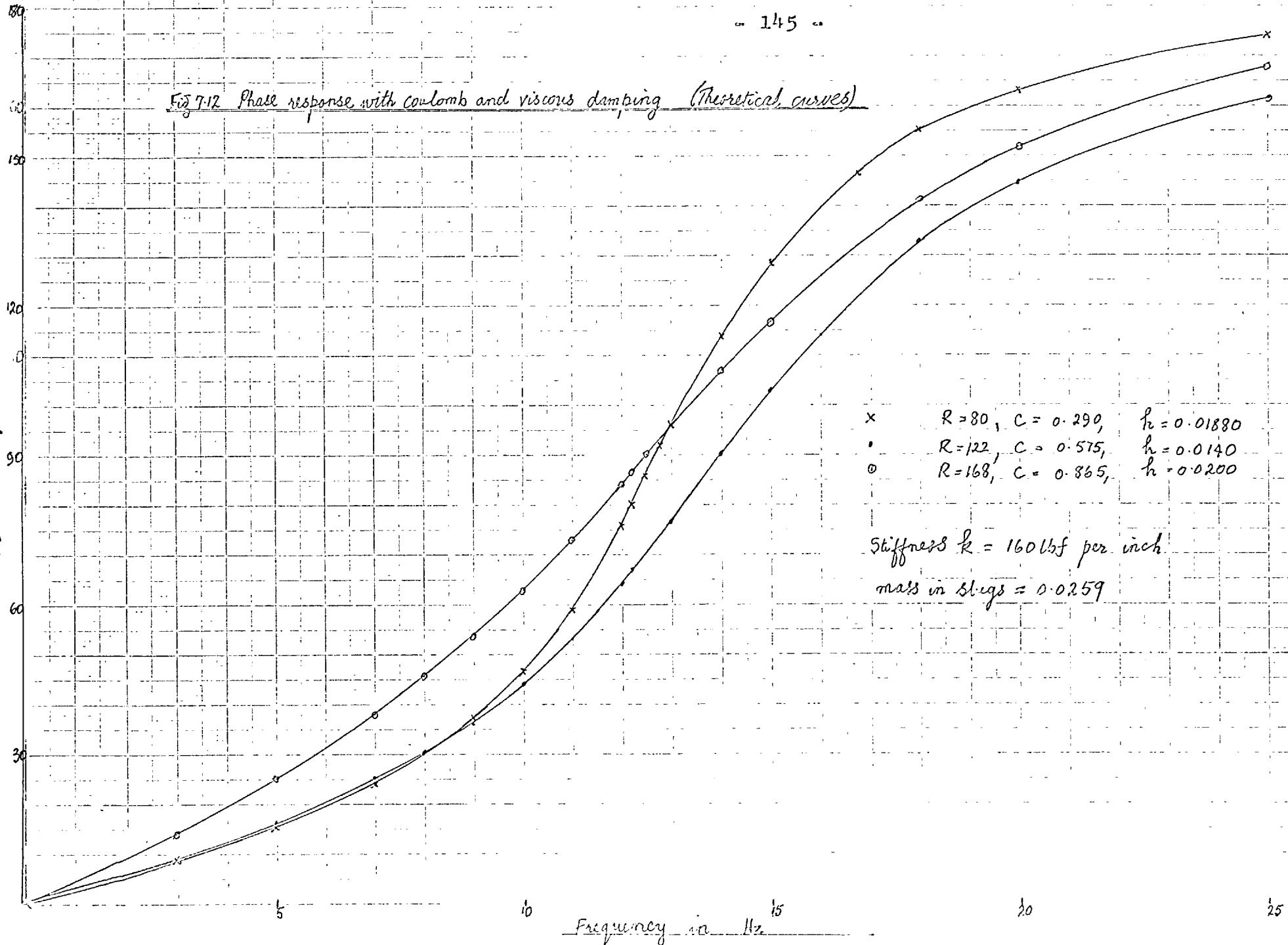


Fig 7.12 Phase response with coulomb and viscous damping (Theoretical curves)

Phase ϕ in degrees



was kept constant as before and the excitation force, the acceleration and the phase difference between them were measured. From the graph obtained, the loss factor, η and the stiffness k of the viscoelastic material were determined. The values obtained for η and k were now used as data in the computer programme which uses the "Ritz-Galerkin method" to solve differential equations, to obtain theoretical results for comparison with the experimental results.

The equation of motion in this case is

$$m\ddot{x} + kx + i\eta\dot{x} = Pe^{j\omega t}$$

which simplifies to

$$P/kx_0 = \sqrt{(1-n^2)^2 + \eta^2} \quad \dots\dots\dots(7.7)$$

and

$$\tan \phi = \frac{\eta}{1 - n^2} \quad \dots\dots\dots(7.8)$$

The equations used were therefore equations (7.7) and (7.8).

7.3.1. VISCOELASTIC AND VISCOUS DAMPING

Viscous damping was now introduced into the system so that the experimental model was that in Fig.(3.5) with the friction force $F = 0$ and the equation is obtained from equations (3.13) and (3.14) when $S = 0$. The amount of viscous damping introduced was the same as was the case with the spring in the system. The amount of current passed in the three different cases were 1.5

Fig 7.13 Forced frequency response with viscoelastic and viscous damping (Experimental results)

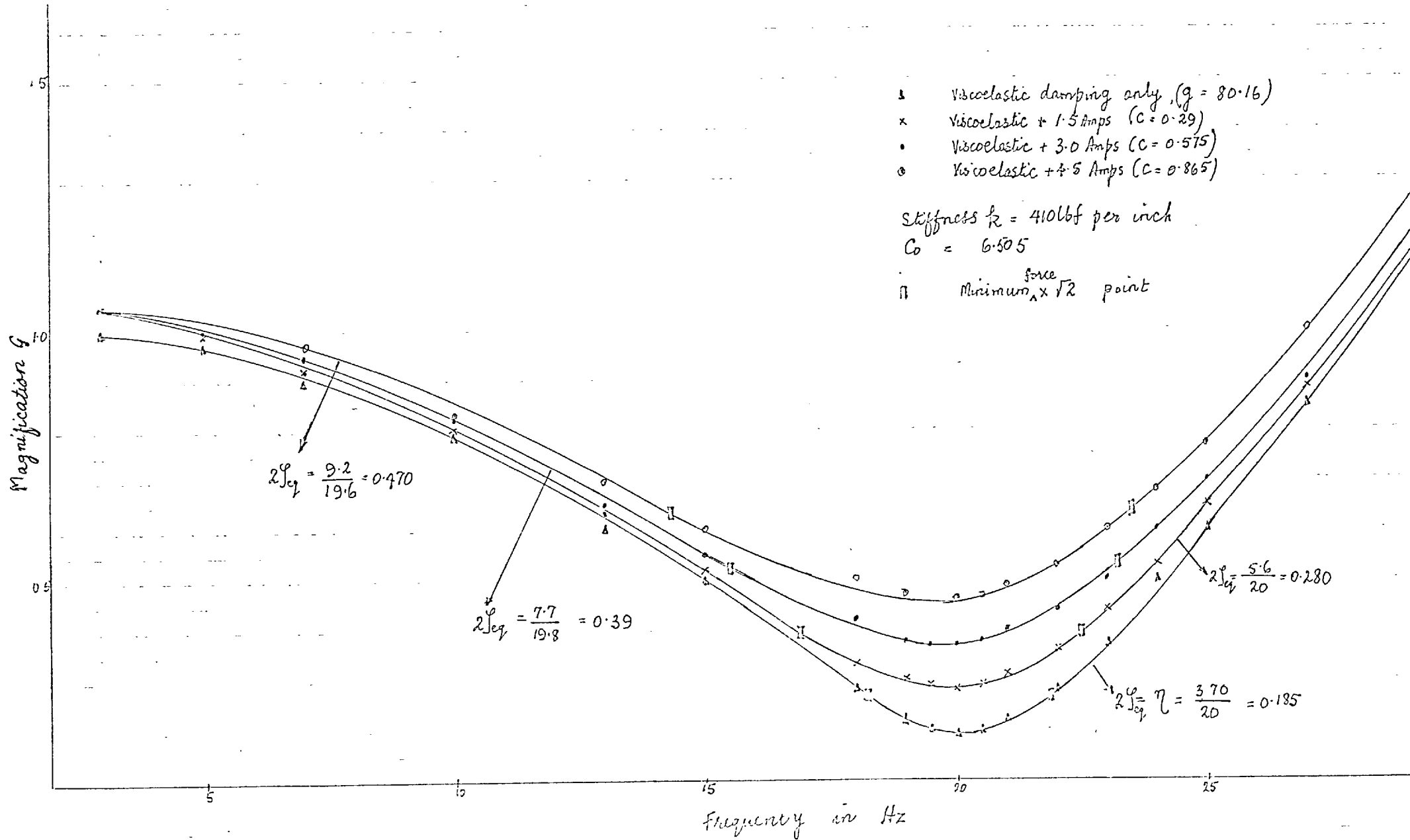


Fig 7.14 Phase response with viscoelastic and viscous damping (Experimental results)

Phase ϕ in degrees

- Δ viscoelastic damping only
- $\times$ viscoelastic + 1.5 Amps ($C = 0.290$)
- $\bullet$ viscoelastic + 3.0 Amps ($C = 0.575$)
- $\circ$ viscoelastic + 4.5 Amps ($C = 0.865$)

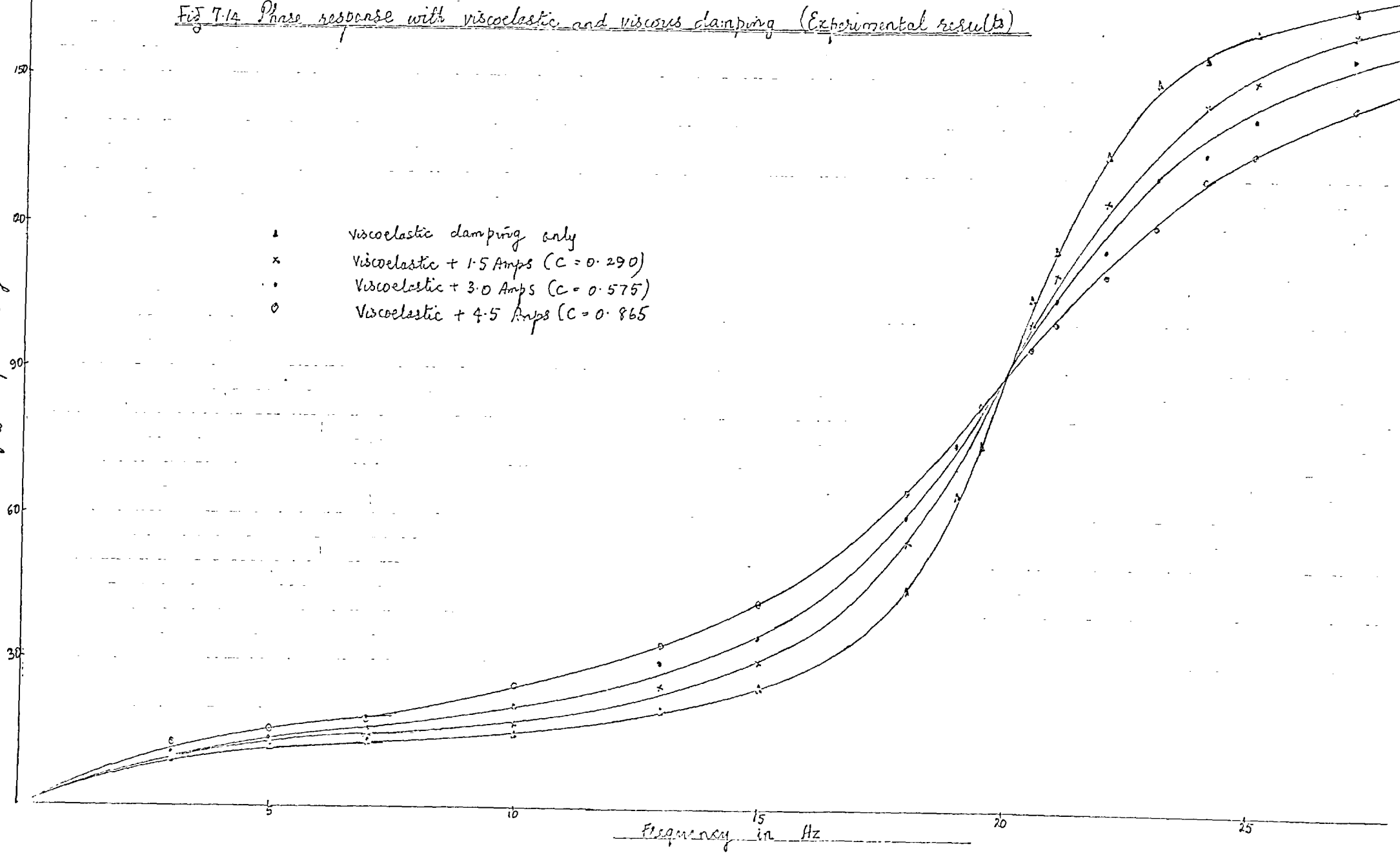


Fig 7.15 Forced frequency response with viscoelastic and viscous damping (Theoretical curves)

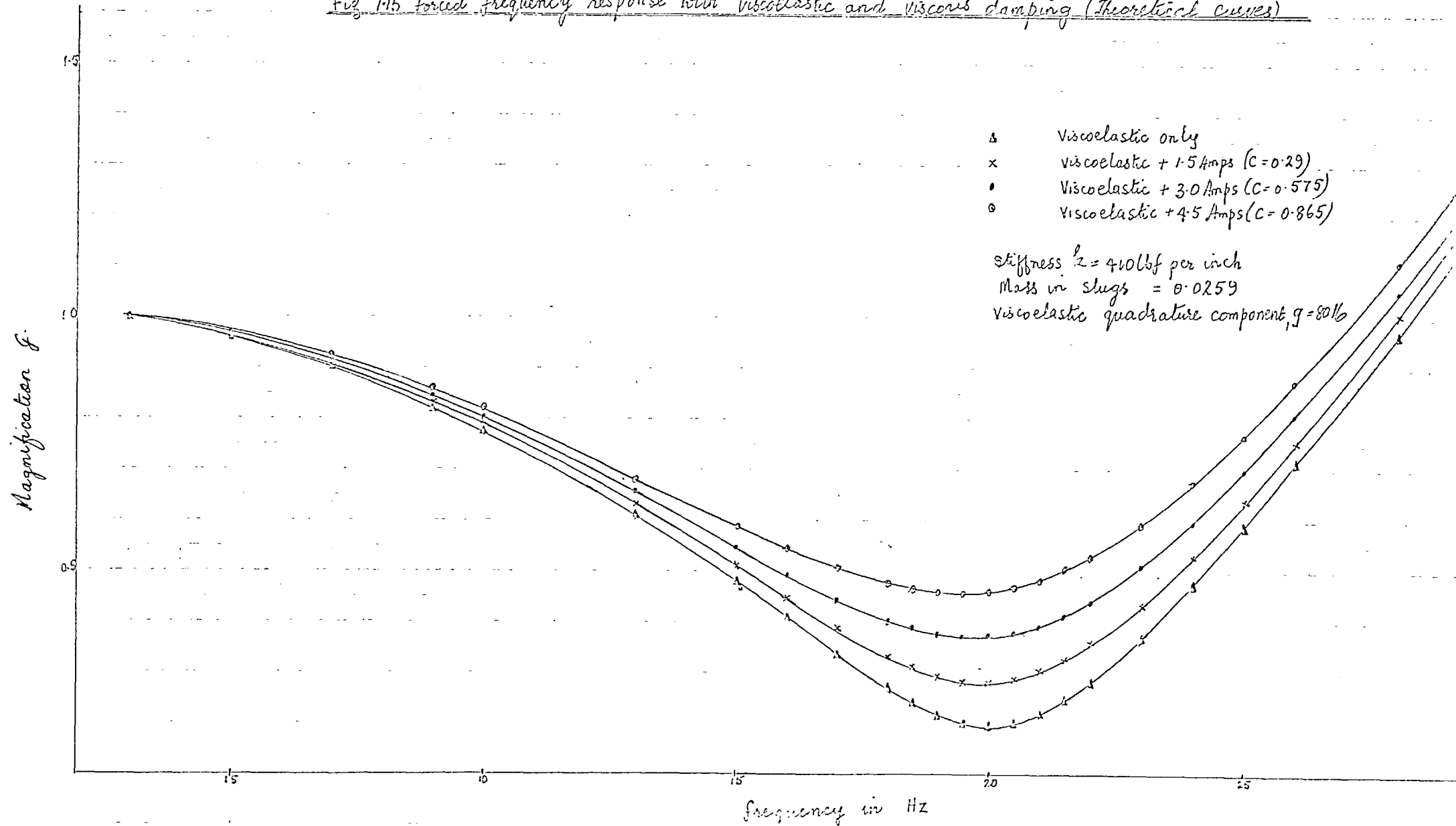
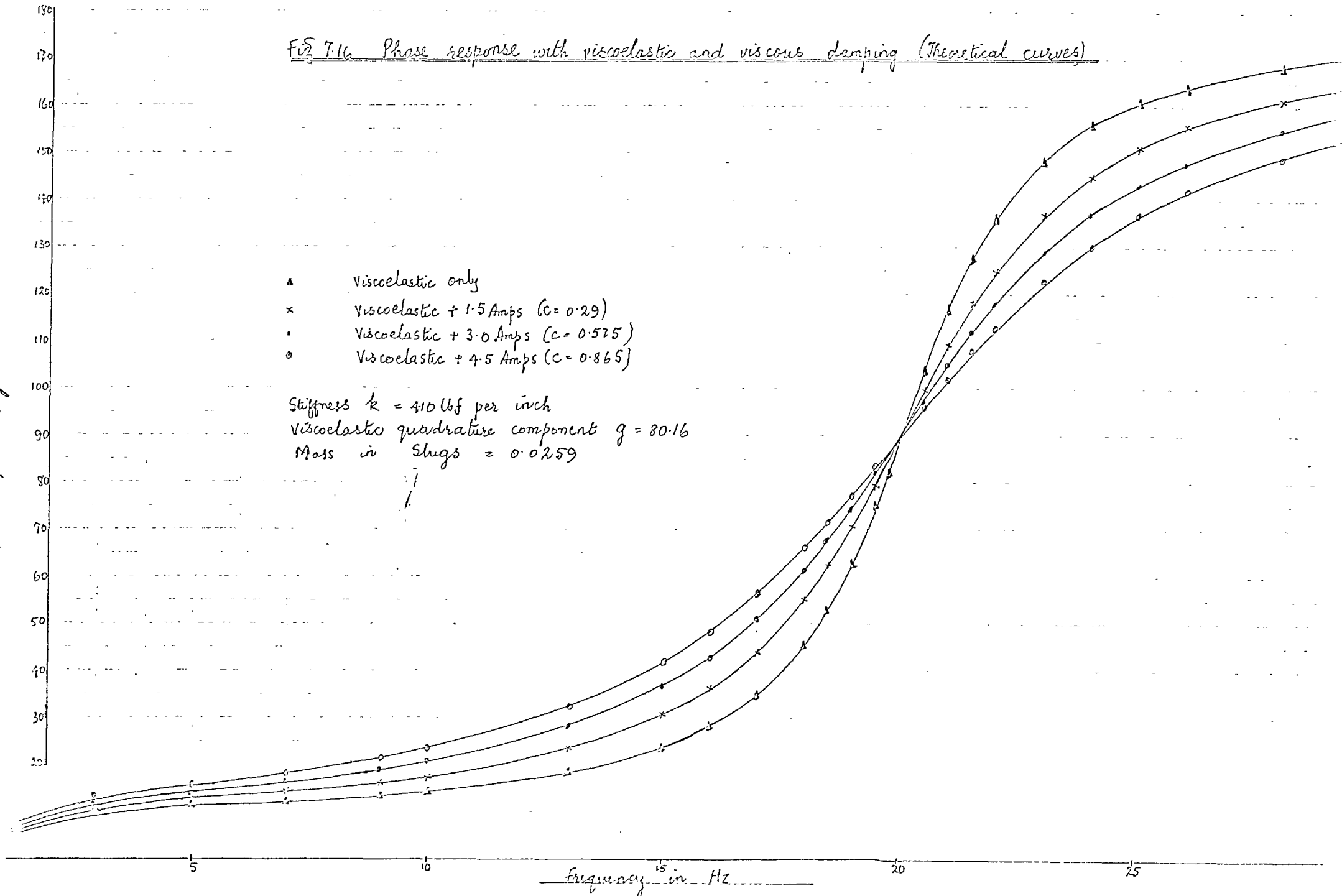


Fig 7.16 Phase response with viscoelastic and viscous damping (Theoretical curves)

Phase ϕ in degrees

- ▲ Viscoelastic only
- × Viscoelastic + 1.5 Amps ($C = 0.29$)
- Viscoelastic + 3.0 Amps ($C = 0.575$)
- Viscoelastic + 4.5 Amps ($C = 0.865$)

Stiffness $k = 410$ lbf per inch
 Viscoelastic quadrature component $q = 80.16$
 Mass in Slugs $= 0.0259$



Amps, 3.0 Amps and 4.5 Amps, respectively.

From the experimental curves obtained the equivalent damping ratio, ζ_{eq} , and the equivalent loss factor η_{eq} were calculated and used as data in the Ritz-Galerkin method computer programme. The results obtained by using an equivalent loss factor η_{eq} were in general less agreeable with the experimental results than when an equivalent damping ratio ζ_{eq} was used. The degree of agreement depends upon the ratio of the two types of damping in the system. Better results were obtained by using equations (3.13) and (3.14) directly in the special computer programme. The experimental results as well as the theoretical ones using equations (3.13) and (3.14) are shown in Figs. 7.13 to 7.16.

Figs. (7.13) and (7.14) show that the experimental results and Figs. (7.15) and (7.16) show the theoretical results. In the results shown the equivalent viscous damping coefficient c is given as well as the amount of current used in Amps. All the response curves portray the variation of the magnification factor G , with frequency.

7.3.2. VISCOELASTIC AND COULOMB DAMPING

With the loss factor η of the viscoelastic material and its stiffness known, it was now appropriate to introduce Coulomb damping into the system and note its effects on the resonant frequency, the phase

and the magnification factor G . Coulomb damping was introduced in three stages by the same amount as before when the metallic spring was in place of the visco-elastic material. The same parameters (i.e. the excitation force, the acceleration and the phase) were measured at a constant amplitude of vibration. The model for this experiment was Fig.(3.3) and the equations were (3.16) and (3.17).

In order to obtain theoretical solutions for equations (3.16) and (3.17) the results obtained from the experiment were used as data in the Non-linear Regression programme to obtain values for " h " (the delay-time) for the different combinations of visco-elastic and coulomb damping. As a check the delay time " h ", was also calculated from the measured phase angle ϕ and compared with the results obtained from the non-linear regression programme. Theoretical solutions for equations (3.16) and (3.17) were thus obtained and Figs.(7.17) to (7.20) show the results both experimental and theoretical.

The experimental results are shown in Figs.(7.17) and (7.18) and the theoretical results are shown in Figs.(7.19) and (7.20).

The method by which the delay time " h " was determined from the measured phase angle is described in Appendix 2.

Fig 717 Forced frequency response with viscoelastic and coulomb damping (Experimental results)

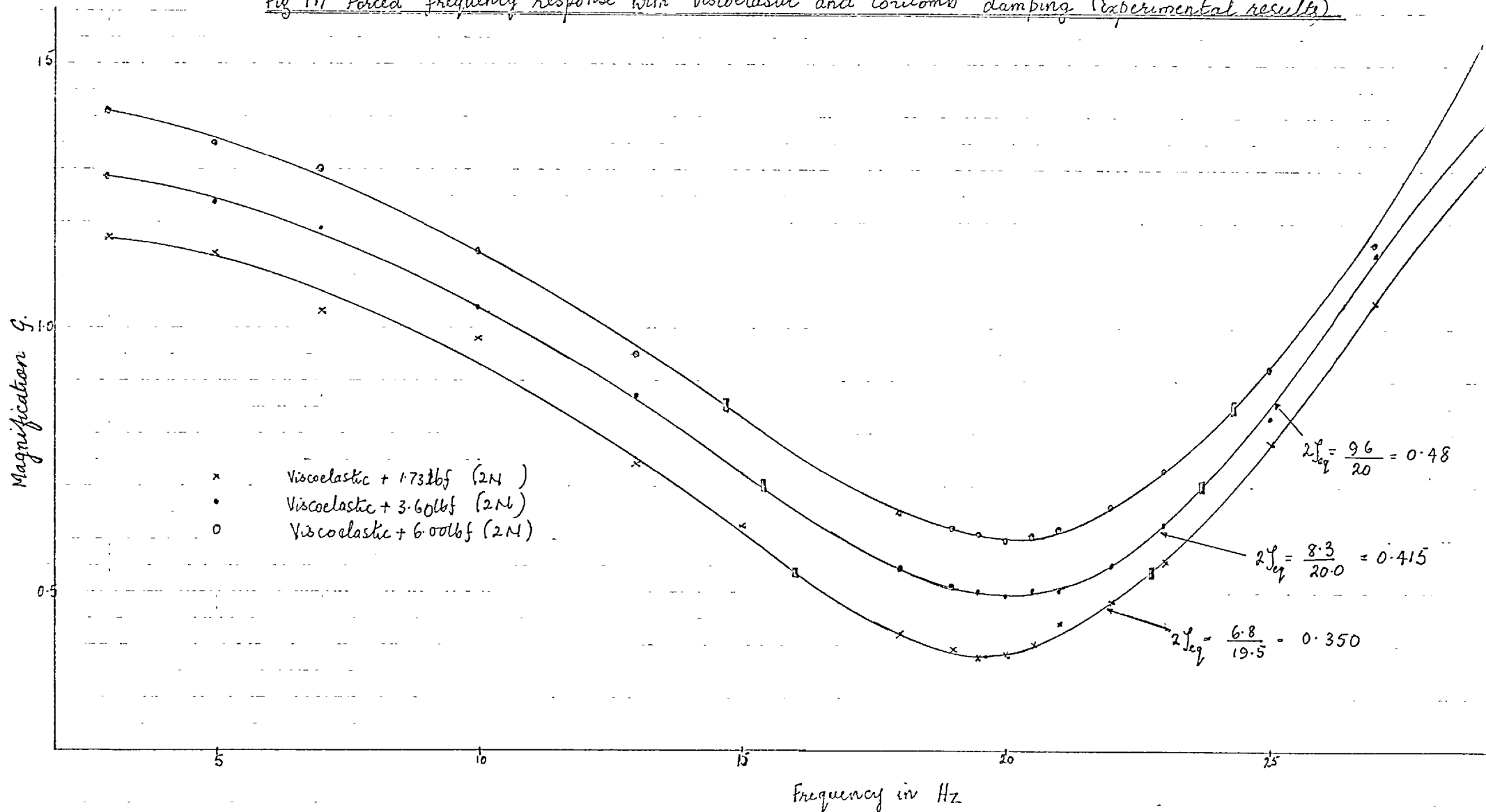


Fig 7.18 Phase response with viscoelastic and Coulomb damping (Experimental results)

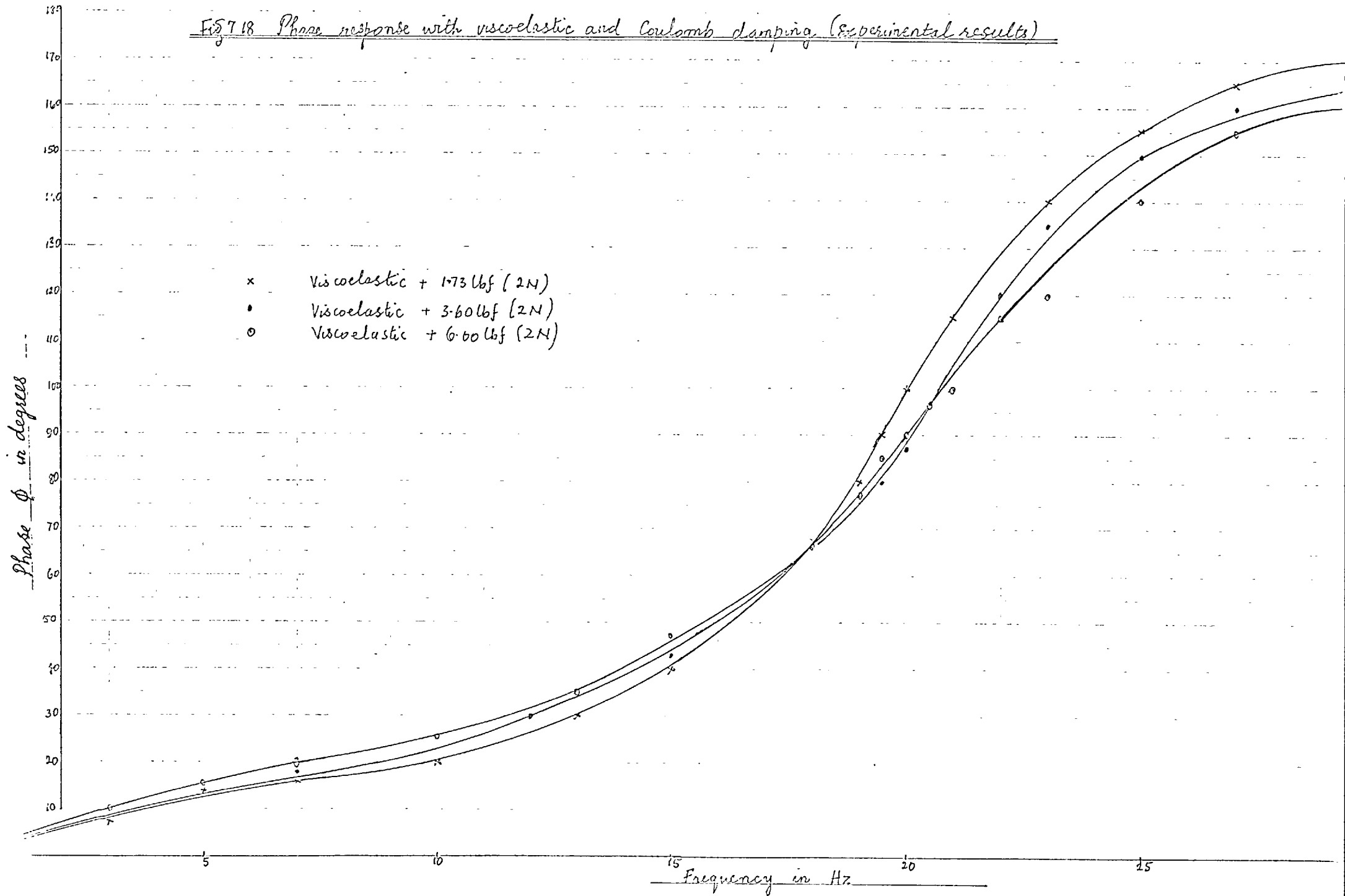


Fig 719. Forced frequency response with viscoelastic and Coulomb damping (theoretical curves)

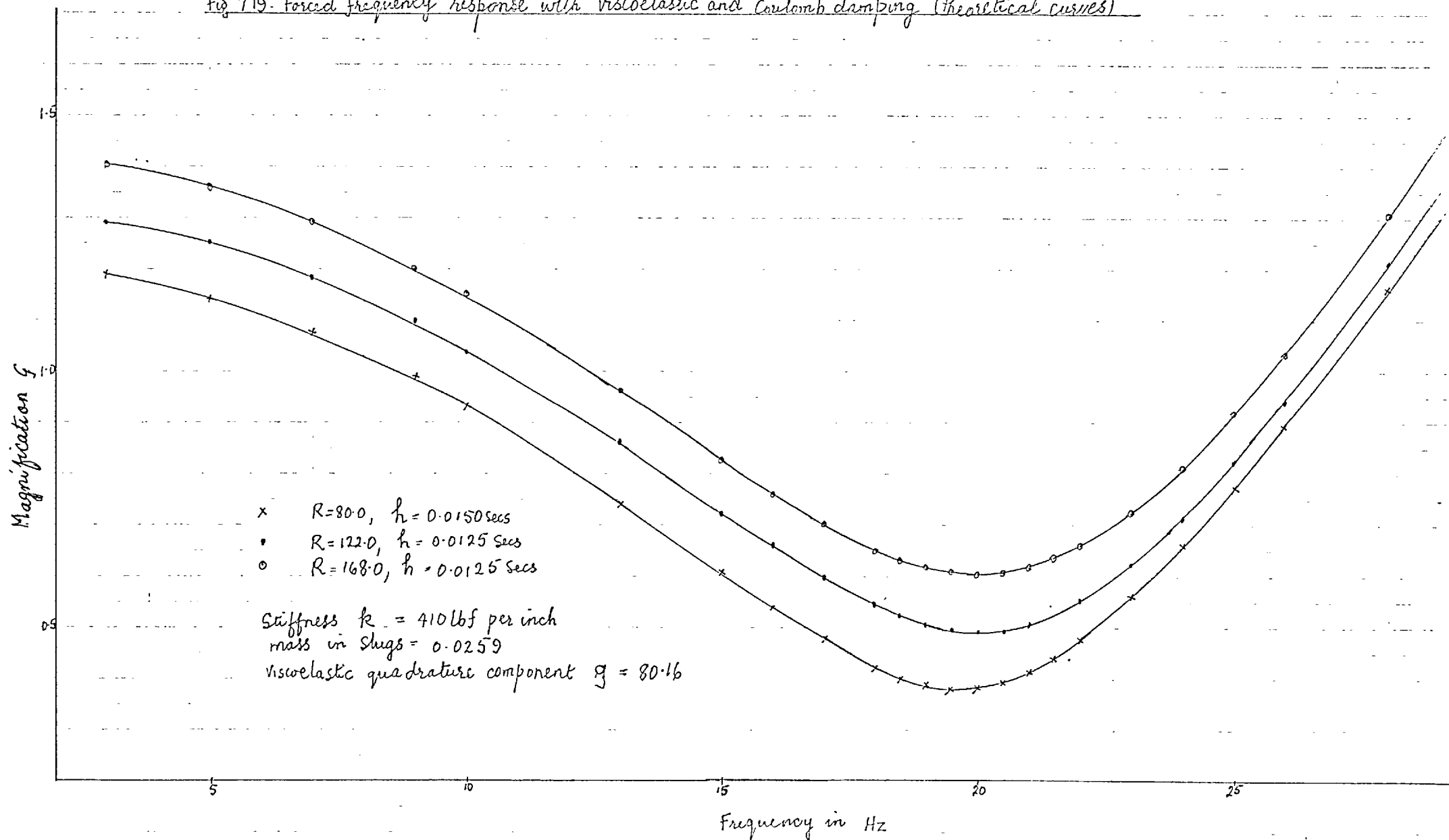
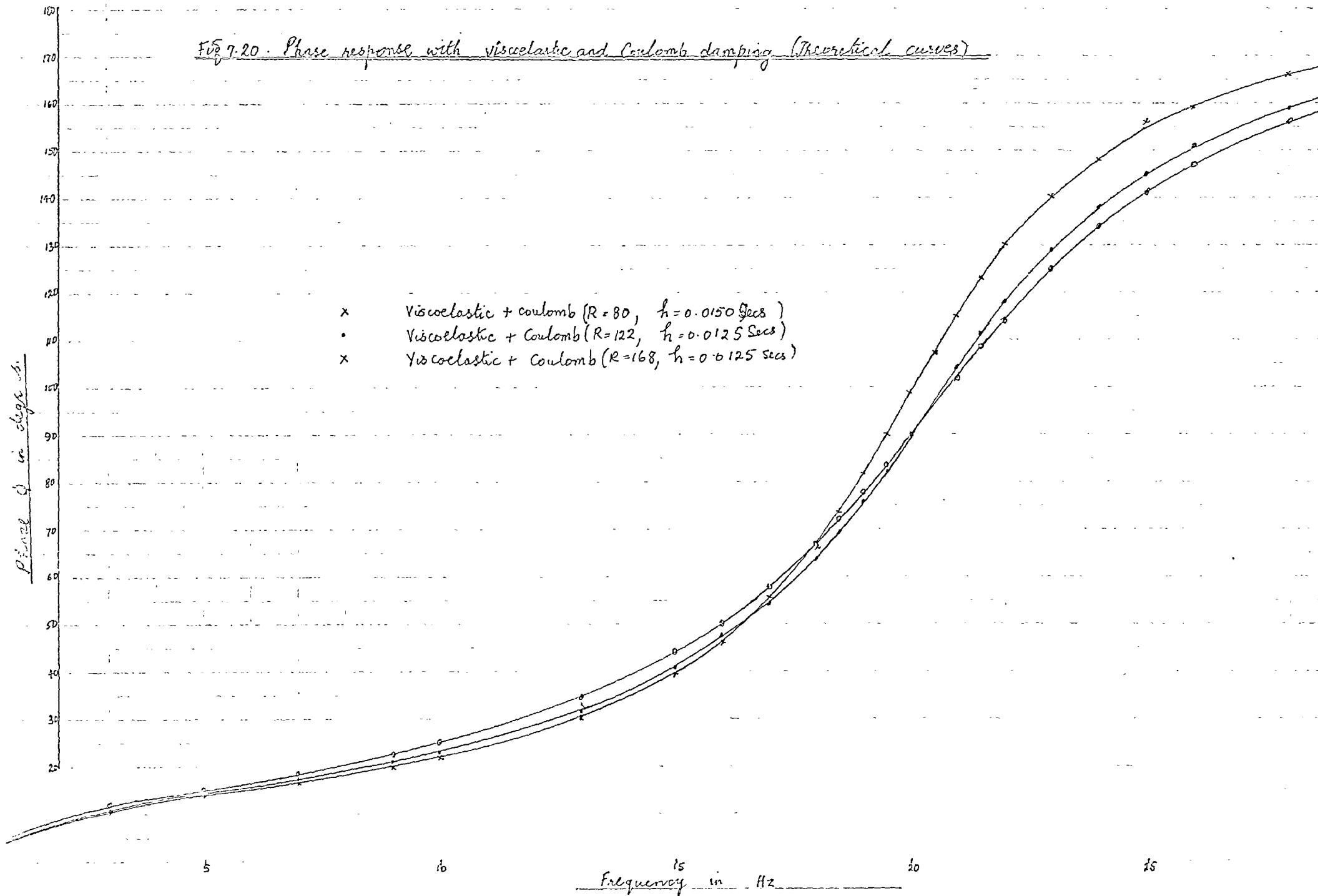


Fig 7.20. Phase response with viscoelastic and Coulomb damping (Theoretical curves)



7.5 FORCED VIBRATIONS WITH THE THREE TYPES OF DAMPING

The experimental work for this case is demonstrated by Fig.(3.2) and hence equations (3.13) and (3.14) were found to be rather tedious. Whilst the amount of viscous damping could be accurately assessed it was found that the amount of Coulomb damping introduced was not so readily estimatable owing to the fact that different results could be obtained if the conditions (i.e. the alignment of the blocks, the state of the contact surfaces and the tightness of the positioning rods) were not the same in all cases.

For this reason the experimental work became rather repetitive. First of all a forced-response test was made with the visco-elastic material only and measurements made as before; a specific amount of Coulomb damping was now added and a force response test made again and the parameters remeasured.

Maintaining this set-up a specified amount of viscous damping was now added and the test repeated and the parameters remeasured.

The procedure was repeated for different combinations of Coulomb and viscous damping. The results for the different cases and combinations are shown in figs 7.21 to 7.24. Figs 7.21 and 7.22 show the experimental results and Figs 7.23 and 7.24 show the theoretical results.

Fig 7.21 Frequency response with viscoelastic, coulumb and viscous damping (Experimental results)

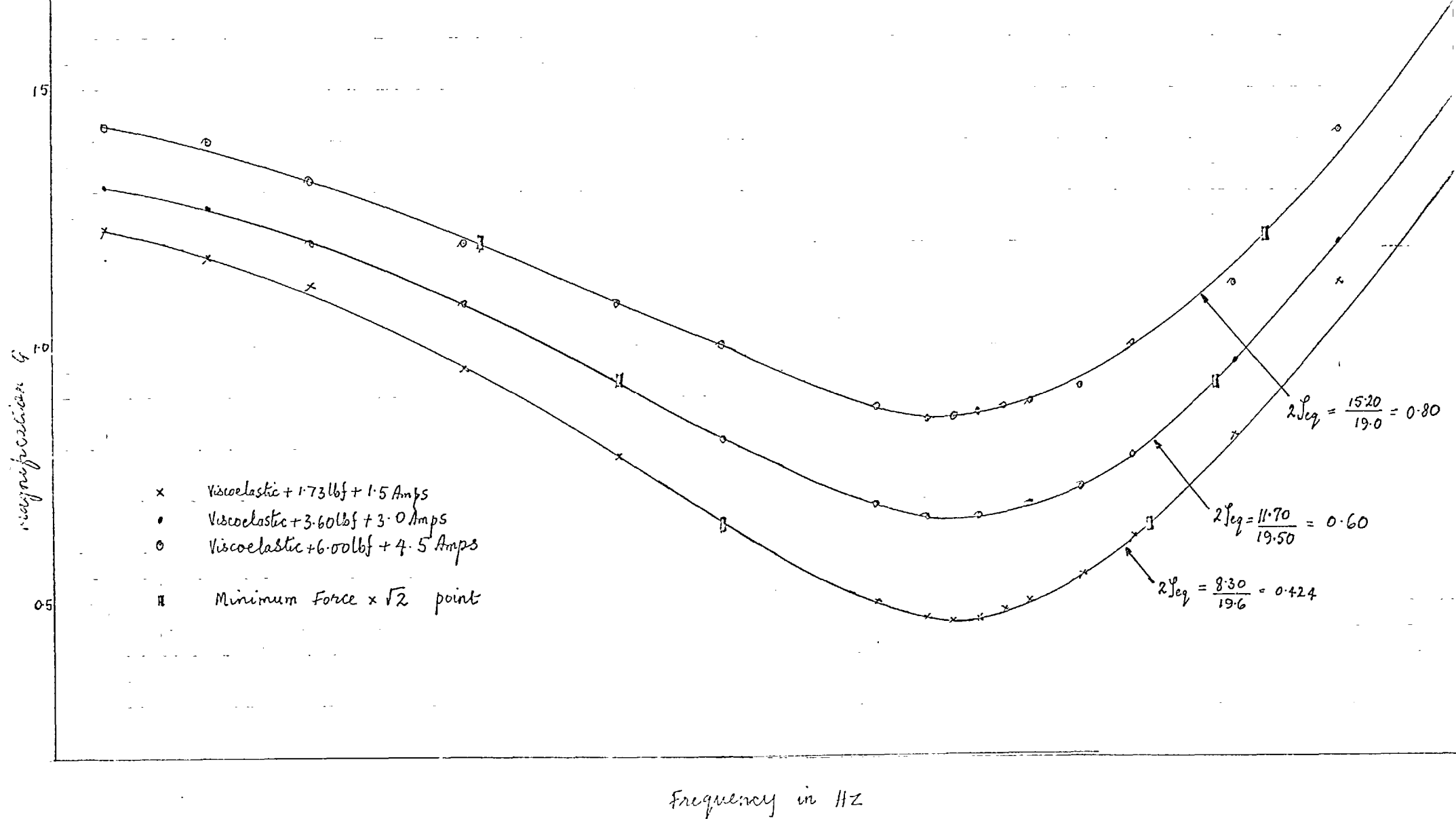


Fig 7.2.2 Phase response with viscoelastic, coulomb and viscous damping (Experimental results)

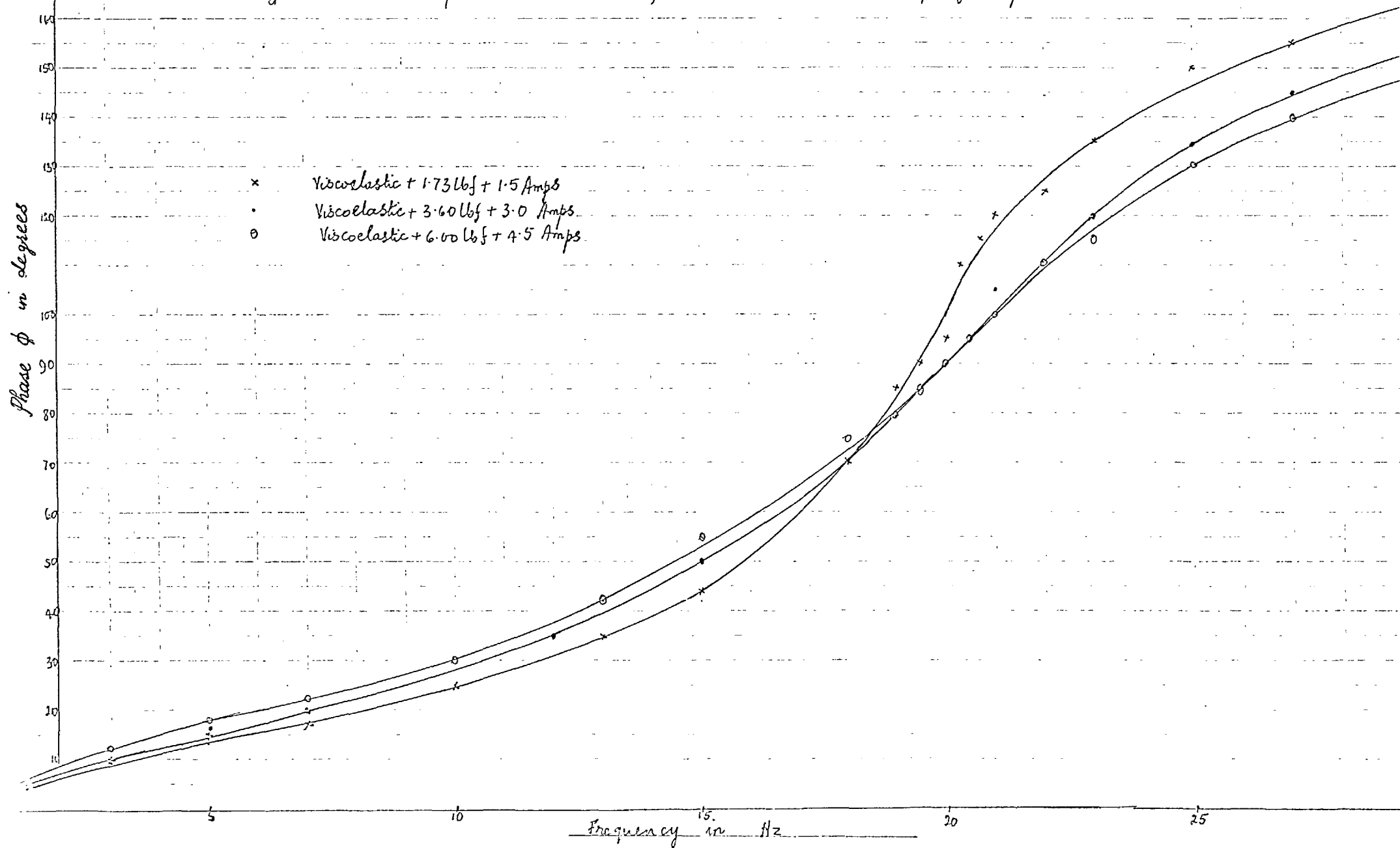


Fig 7.23 Frequency response with viscoelastic, coulomb and viscous damping (Theoretical curves)

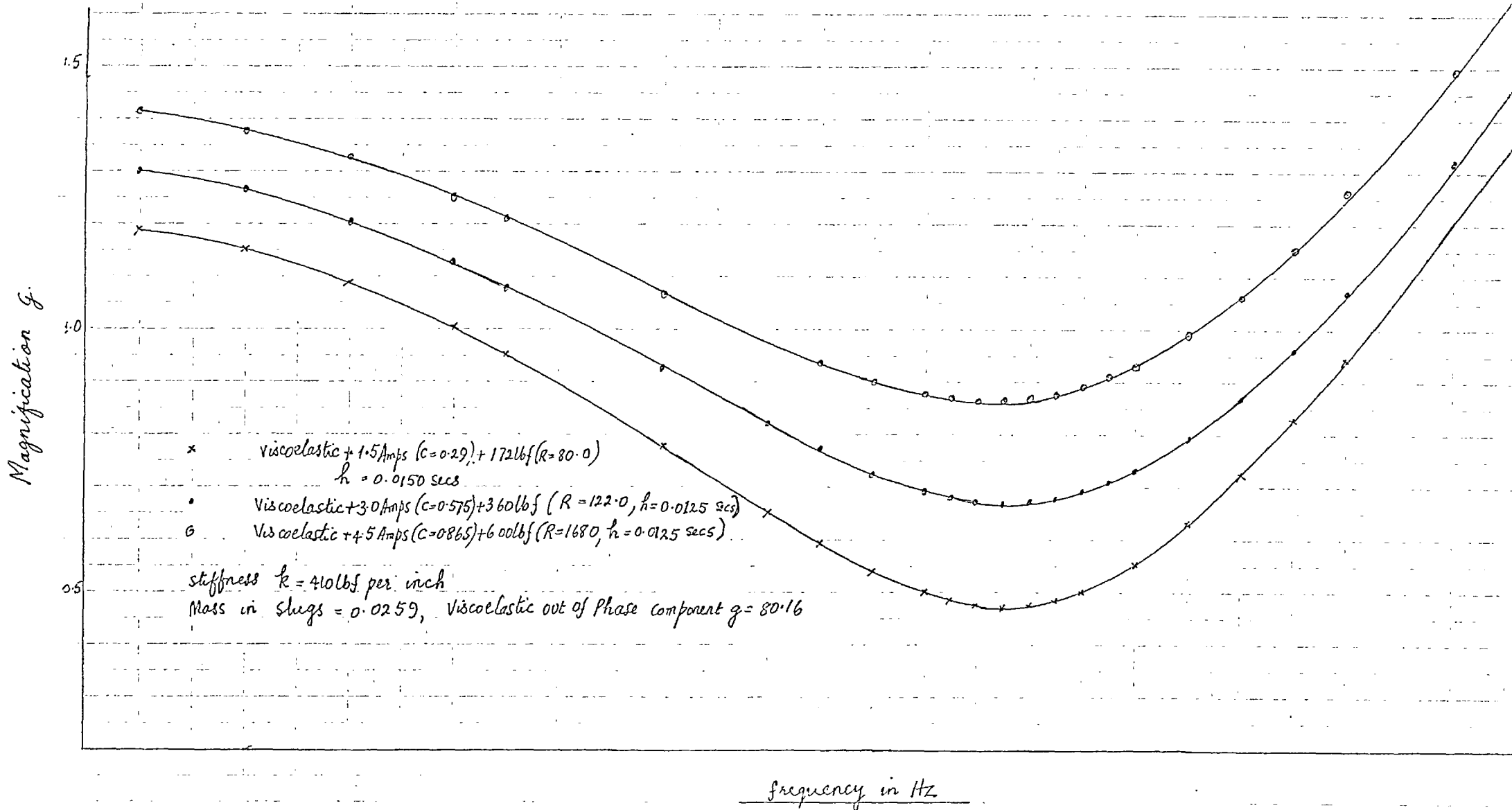
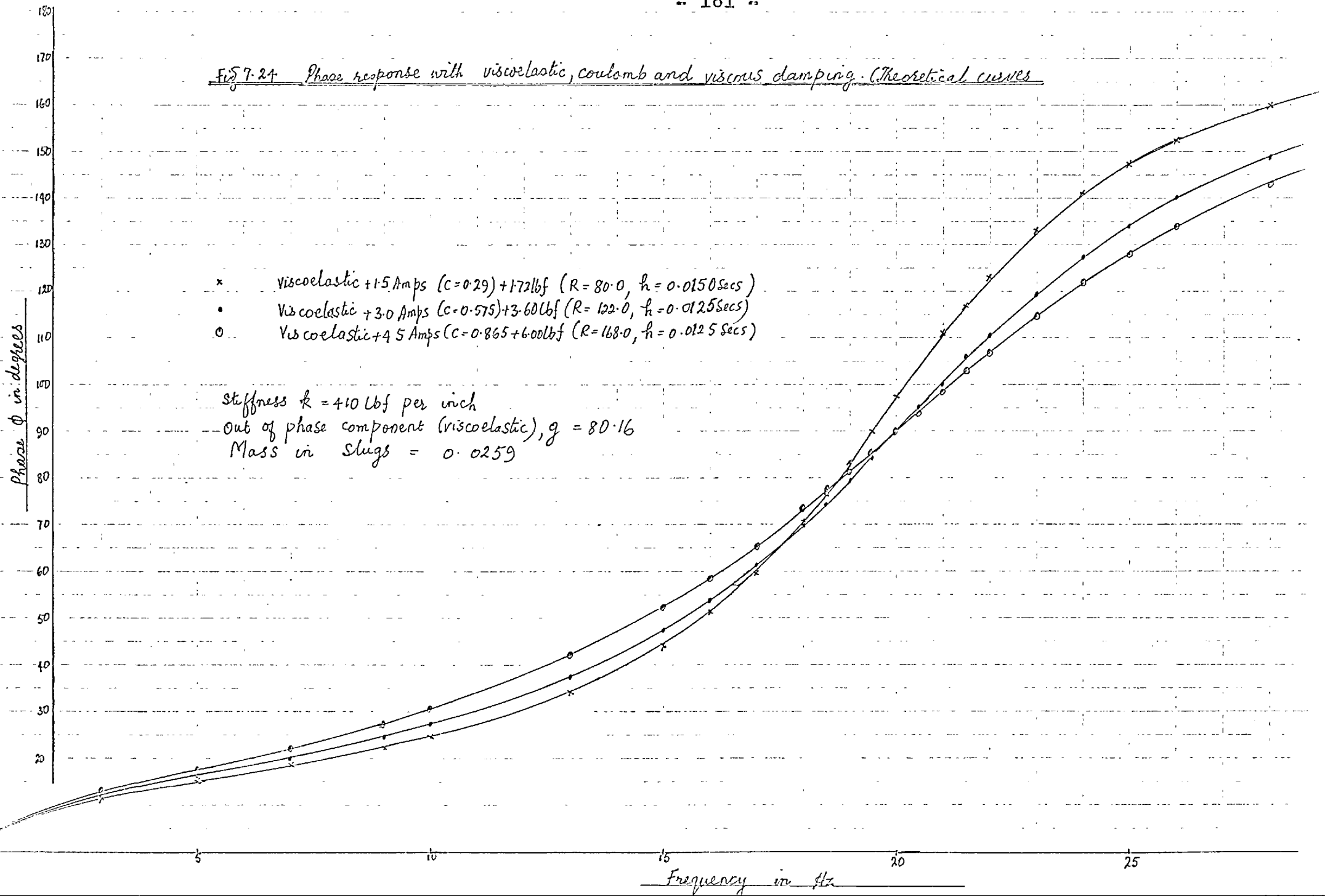


Fig 7.24 Phase response with viscoelastic, coulomb and viscous damping. (Theoretical curves)



C H A P T E R 8

DISCUSSION OF RESULTS ON THE NATURE OF COULOMB FORCE

8.1 EXAMINATION OF COULOMB FORCE

Figures 6.3 to 6.9 are response curves for fig. 6.2 and clearly demonstrate that coulomb force is better represented as a complex force rather than as a constant. These curves are experimental and theoretical solutions of equation (6.2b) which is presented in the form

$$P/x = C + R(\cos wh + j \sin wh)$$

where $C = mw^2$

In figures 6.7 to 6.9 the acceleration was kept constant at $5g/16$, g and $5g/8$ which are in the ratio 1:3.2:2. The fitting values of the constant C , are 0.87, 2.41 and 2.28 giving a ratio of 1:2.8:2.6, as compared with the ratio of acceleration levels used just quoted above. While the order of magnitude of the two set of ratios are compatible there exists a discrepancy in the exact values obtained. The constant C has to be multiplied by 4.2 and the amplitude of vibrations, x to obtain the actual values of the acceleration forces, and therefore exact agreement of the two set of ratios can only be expected if the amplitude of vibrations was kept constant too. As the amplitude was not deliberately kept constant the slight discrepancy in the two set of ratios is inevitable.

In general the fitted and experimental curves do show good agreement in all the three cases shown.

.....

8.1.1 RATIONSHIP BETWEEN h, C and R

Table 8.1 shows the values of acceleration, the constant C and R, and the ratio C/R corresponding to different values of delay time, h as obtained from the fitted curves including two cases that are not shown in the figures.

Acceleration in g.	C	R	$\frac{C}{R}$	h
5/16	0.87	1.695	0.51	0.00240
5/8	2.28	1.175	1.94	0.00260
g	2.45	2.00	1.23	0.00191
5/4g	3.00	2.75	1.10	0.00220
25/16g	3.50	3.55	0.99	0.00225

Table 8.1

VALUES OF CONSTANTS AS OBTAINED FROM CURVES

The relationship between C and h, R and h, and C/R and h are shown in fig. 8.1. The common feature of the three curves is that there is a minimum delay time for some values of C, R or C/R. As $C = mw^2$, it means that for a particular frequency of oscillations the delay-time is minimum which is understandable. The friction force F is a complex quantity and therefore with the inertia force mw^2x , being kept constant it is reasonable to expect that at a particular frequency and amplitude of vibrations, there is an optimum value of friction force constant R that will result in a minimum delay-time. The ratio C/R, must

/.....

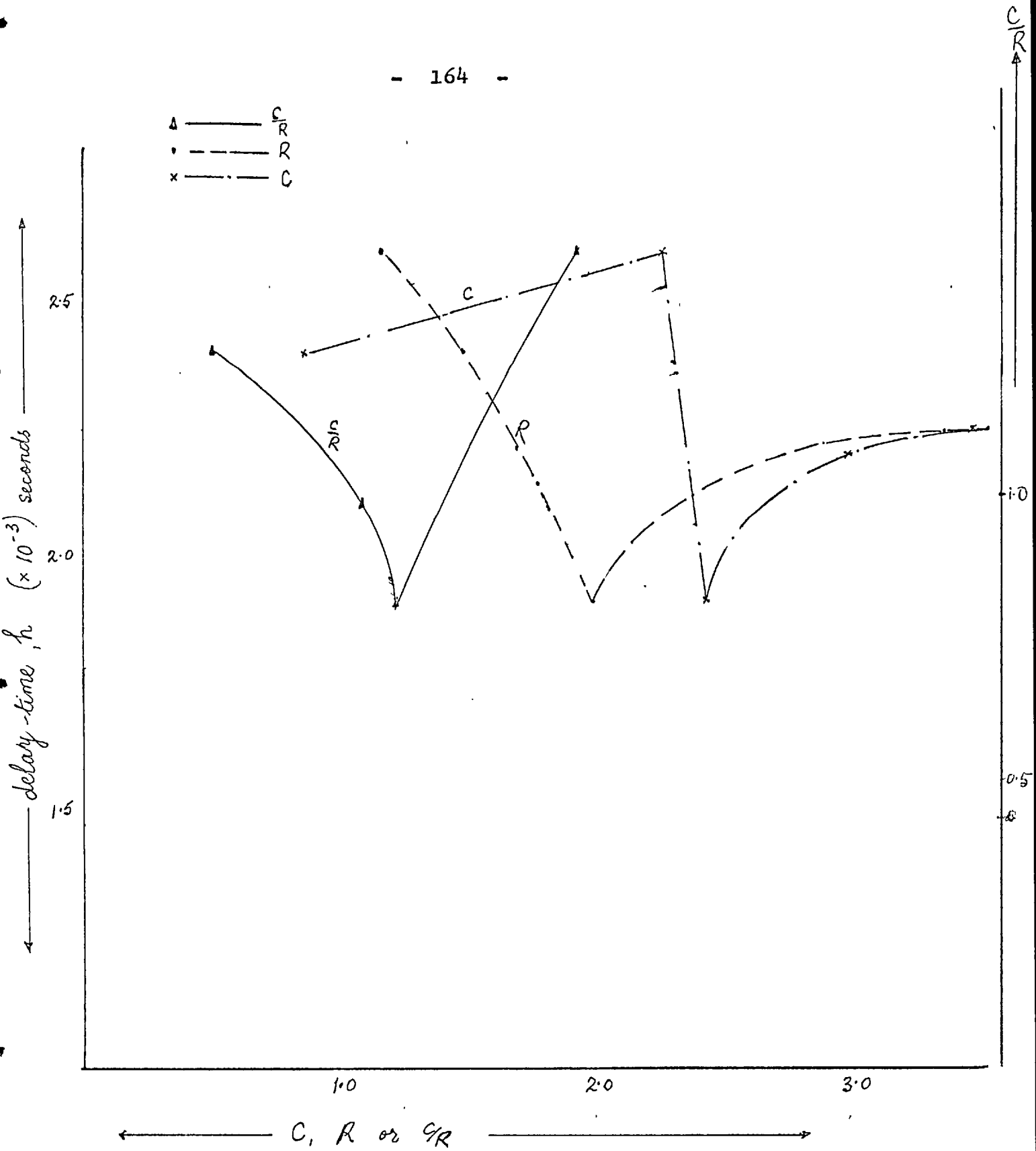


Fig 8.1. Variation of delay-time, h with C, R and $\frac{C}{R}$.

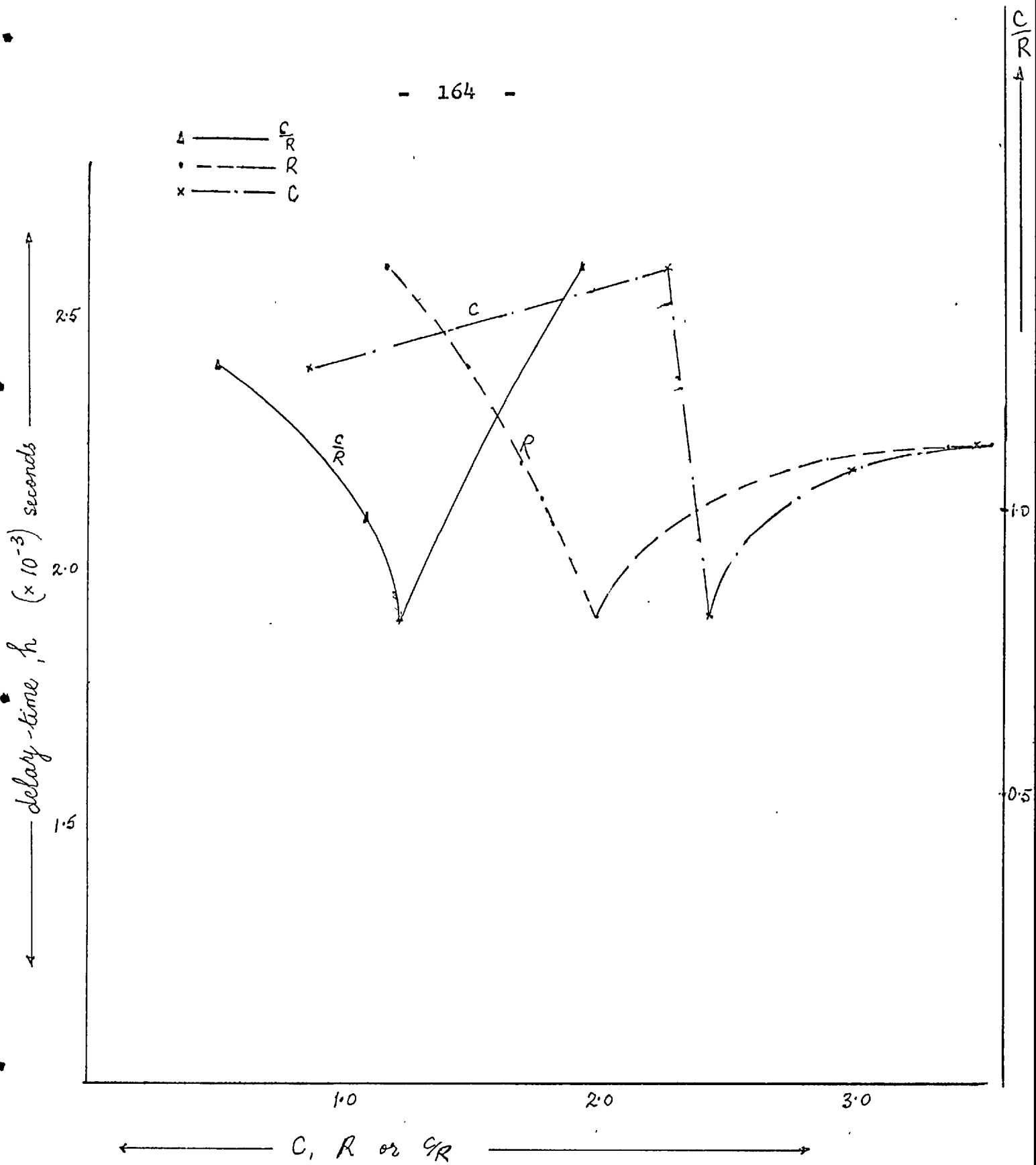


Fig 8.1. Variation of delay-time, h with C, R and $\frac{C}{R}$.

be expected to show a minimum value for the delay time in view of the fact that both C, and R exhibit this phenomenon. For the particular system used in the experimental work, the delay-time is minimum when the acceleration is g, and the inertia force 10.3lbf, (2.45×4.2) , the friction force $F = 8.41\text{lbf}$ and $C/R = 1.23$.

* Curves from which these were obtained are not given in the results.

8.1.2 EXAMINATION OF THE EQUATION

Equation (6.2b) can be represented in the form:

$$C + Re^{jwh} = P/X \dots (8.1) \text{ where } e^{jwh} = \cos wh + j \sin wh$$

The acceleration $mw^2x = A/K = \text{constant}$ in this experiment

$$\therefore X = \frac{K}{mw^2} = \frac{K}{C}$$

$$\text{thus } e^{jwh} = \frac{PC}{RK} - \frac{C}{R} \text{ substituting for } x \dots (8.2)$$

Squaring both sides

$$e^{-w^2h^2} = \left(\frac{PC}{RK} - \frac{C}{R} \right)^2 = \left(\frac{PC}{RK} \right)^2 - \frac{2PC^2}{R^2K} + \frac{C^2}{R^2}$$

$$-w^2h^2 = \log \left(\frac{P^2C^2}{R^2K^2} - \frac{2PC^2}{R^2K} + \frac{C^2}{R^2} \right) = \log_e \left(\frac{P^2C^2 - 2PC^2K + C^2K^2}{R^2K^2} \right)$$

$$-2w^2h \frac{dh}{dP} = \frac{P^2C^2 - 2PC^2K + C^2K^2}{P^2C^2 - 2PC^2K + C^2K^2} \times \frac{2C^2P - 2C^2K}{P^2C^2 - 2PC^2K + C^2K^2}$$

$$\text{For max or min } \frac{dh}{dP} = 0 \therefore 2C^2P - 2C^2K = 0$$

$$\text{hence } P = K$$

If equation (8.3) is now differentiated with respect to C one obtains:

$$-w^2h^2 = \log \left(\frac{P^2C^2 - 2PC^2K + C^2K^2}{R^2K^2} \right)$$

$$-2w^2h \frac{dh}{dC} = \frac{2P^2 - 4PK + 2K^2}{C(P^2 - 2PK + K^2)}$$

$$\text{For max or min } \frac{dh}{dC} = 0 \therefore P^2 - 2PK + K^2 = 0$$

$$\text{i.e. } (P - K)^2 = 0 \text{ i.e. } P = K$$

.....

A similar result is obtained when the equation is differentiated with respect to R. To obtain corresponding values for the other variables when $P = K$, and there is minimum delay, a substitution is made in equation (3.5)

$$\text{i.e. } P/X = \sqrt{(C + R\cos wh)^2 + (R\sin wh)^2} \dots$$

thus

$$\begin{aligned} K/X &= \sqrt{(C + R\cos wh)^2 + (R\sin wh)^2} \\ C^2 &= C^2 + 2CR\cos wh + R^2\cos^2 wh + R^2\sin^2 wh \\ &\text{as } K/X = C. \end{aligned}$$

$$R^2 = -2CR\cos wh$$

$$\text{i.e. } \cos wh = \frac{-R}{2C} \dots\dots\dots(8.4)$$

Thus minimum values of h are obtained when

$$\left. \begin{aligned} R &= -2C\cos wh \\ C &= \frac{-R}{2\cos wh} \\ h &= -\frac{1}{w} \cos \frac{-1R}{2C} \end{aligned} \right\} \dots\dots\dots(8.5)$$

8.1.3 THE CONCEPT OF OPTIMUM COULOMB DAMPING RATIO

Some research workers (5), (9) have shown that for forced vibrations with Coulomb damping, the Coulomb damping ratio can be expressed in the form F/P where F is the friction force and P the excitation force amplitude. They have stressed the importance of the fact that, for a certain **value** of Coulomb damping in a particular system optimum damping is obtained. Some researchers for example, (9) to (11) however express the damping ratio in the form F/K , where K is the effective stiffness of the elastic members in the system. As there is no spring in the system for the experimental results being discussed only the ratio F/P will be examined here using the theory evolved in this thesis.

Equation (8.4)

$\cos wh = \frac{-R}{2C}$ gives the relationship between R and C for a minimum value of $\cos wh$.

As the minimum condition is that $P = K = CX$ equation (8.4) can be modified to

$$\cos wh = \frac{-RX}{2F} = \frac{-F}{2P} \dots\dots\dots(8.6)$$

when $F = 2P$ no motion is possible and $w = 0$

hence infinite delay time i.e. $wh = 0$

Clearly, when $\cos wh$ is minimum, $\sin wh$ must be maximum.

The friction force F is represented by two parts.

The real part (i.e. the in phase part) = $RX \cos wh$ and

the imaginary part (i.e. the damping part) = $RX \sin wh$

Thus when conditions for minimum delay time h, are approached the in-phase part of the friction force approaches a minimum value whilst the damping part (i.e. out of phase part) approaches maximum.

8.1.4 WITH A SPRING IN THE SYSTEM

The researchers just quoted in connection with the concept of damping ratio worked on systems having springs in them. In 4.4 it was shown that the addition of Coulomb force to a one degree of freedom system with or without damping, may increase or decrease the resonant frequency dependent on one factor, that is the value of $\frac{-R}{2K}$ (represented by F/K) by the researchers quoted).

If $\cos wh$ is minimum then $\frac{-R}{2K}$ is minimum and this results in a small change of resonant frequency. $\sin wh$ however is maximum at this stage and therefore optimum damping is obtained. The analysis in 8.1.3 shows that optimum damping is obtained for a certain ratio of F/P when $\cos wh$ is minimum thus explaining why similar results are obtained by expressing the damping ratio in

either of the two forms. The results obtained here do confirm the results of the quoted researchers, and implies that optimum damping can be obtained in a Coulomb-damped system by simply manipulating the ratio's F/P or F/K . This means changing the value of F or P as in most cases the stiffness K has a fixed value.

8.2 FRICTION FORCE AT CONSTANT AMPLITUDE OF VIBRATIONS

Figures 6.10 and 6.11 show the variation of the coefficient of friction with frequency. These curves are similar to those obtained by researchers in lubrication (1) to (4) whose work was based on frictional force behavior in one direction only. However, similar results were obtained by vibration of the coefficient of friction with sliding velocity in a vibrating medium.

Figures 6.12 and 6.13 show curves obtained by using the formulae $\mu = \mu_s - bf^2 + Cf^4$, which is the first three terms of the expansion of $\cos wh$, to fit the experimental results and while the fall in coefficient of friction with frequency is expected, there is also a decrease in the value of the coefficient of friction with increasing normal loads as illustrated by Dokos (33). As the friction blocks get stiffer with increasing normal loads, the increasing rigidity of the blocks would reduce the amount of rolling (rocking of the friction device) taking place if rolling was taking place at all. This implies that the coefficient of friction as measured could be a combination of the coefficient of sliding and rolling friction.

Since specific attempts were made to eliminate rolling of the blocks and while this might not have been totally efficient it makes it unlikely that this could be

the sole cause of this anomaly. A more likely reason for the results obtained is the dependence of the damping force on the ratio F/P . As the optimum damping ratio of F/P is approached a corresponding minimum real part $XR\cos\omega h$ of the friction force is obtained, and this results in a decrease in the coefficient of friction. Fig 6.14 shows a comparison of the curves obtained using the exact equation $F = R x \cos\omega h$ with that obtained using its expansion together with the experimental results. The variation of the delay-time with the Normal force at constant amplitude shown in fig 6.15 is similar to the one obtained for vibrations at constant acceleration shown in fig 8.1 and the analysis in 8.1.2 do apply here. The variation of the absorption coefficient α with the normal pressure shows it to be inversely proportional to the delay-time as already established in the theory in chapter 2. The values of the delay times obtained at constant amplitude (0.00186 secs. to 0.00252 secs.) are compatible with those obtained at constant acceleration (0.00191 secs to 0.00260 secs).

8.2.1 CALCULATION OF THE DELAY-TIME h FROM A GIVEN DATA

In order to ascertain whether the magnitude of the delay-time obtained is consistent, the delay-time was obtained by using data published by other researchers (28) and (33) in the non-regressive computer programme to obtain values for h . Table 8.2 shows the results:

COMBINATION OF MATERIALS		DELAY TIME h	
		Computed Value (a)	Calculated from (b) mean values
Ref	{ Steel on Steel	0.0025	0.0033
(9)	{ Brass on Steel	0.00335	0.0034
	{ Cast iron on Steel	0.00262	0.00253
(3)	Mild Steel on Mild Steel	0.00199	
	(Doko's)		

Table 8.2

The amplitude used by Earles and Beards (28) was 0.0025" whilst the amplitude used by the author was 0.00825". In the calculations for table 8.2 an amplitude of 0.005" (i.e. mean of 0025" and 00825") was assumed for the first three combinations whilst an amplitude of 0.00825" was used for the last case where the given amplitude of 0.053" was considered too big. The choice of amplitude enabled a fair comparison to be made for two cases involving mild steel on mild steel. The results shown in the second column are obtained by using the quoted static and dynamic coefficient of friction as given by Earles and Beards (28) and the mean speed was determined from the graphs.

8.2.2 DETERMINATION OF h FROM KNOWN VALUES OF M

In order to determine the delay-time h for a particular combination therefore it is necessary that (1) the amplitude of vibrations be known and (2) the coefficient of static friction be known and the dynamic coefficient of sliding friction be known for a mean speed:

Thus if M_s = Static coefficient of sliding friction

M = Dynamic " " " at speed v

Then $w = \frac{v}{x}$ where w = frequency in radians, x = amplitude of vibrations.

Hence $M = M_s \cos wh$

$\cos wh = \frac{M}{M_s}$ hence $wh = \phi$ radians say

Then $h = \frac{\phi}{w}$ secs.(8.7)

.....

8.2.3. COMPATIBLE ORDER OF MAGNITUDE

The order of magnitude of h obtained from other results are compatible with those obtained by the author, all being of the order of milliseconds. The values obtained by using only two values of the coefficient of friction show very good agreement with the computed ones and therefore good results can be obtained by using the method shown in 8.2.2.

The value of 0.00199 secs. obtained by using Dokos' results for mild steel on mild steel falls within the range of 0.00186 to 0.0026 secs. as obtained by the author thus proving the consistency of the results.

8.2.4. EXAMINATION OF THE FRICTION FORCE

Figs 6.20 to 6.28 show the variation of friction force with frequency. Figs. 6.20 to 6.22 are similar and do show a shift in the frequency at which minimum force is obtained with a change in normal load. This further illustrates the dependence of the delay-time on the normal load. Figs 6.23, 6.26 and 6.27 show curves for the variation of the friction force and its components, Real and Imaginary with frequency and do show that negative damping is possible at some range of frequencies. The polar plots in figs 6.24, 6.25 and 6.28 confirm that friction force is a vector which rotates with increasing frequency, as deduced in 2.3, the rate of rotation being governed by the other forces (in this case, inertia force) in the system as the curves representing $P(I)$ show.

8.3 VIBRATIONS INDUCED BY COULOMB DAMPING

The ability of Coulomb force to induce vibrations instead of dampen them has been encountered by many research workers and as such some experiments have been designed using Coulomb force to illustrate self-excited vibrations. Most text books on vibrations (20) to (24) do use some models involving Coulomb forces as experimental aids to illustrate self-excited vibrations.

The discussion in 8.2.4 and also the deduction in 2.3 show that the damping part of friction force can become an exciting force within certain ranges of frequency giving rise to stable and non-stable regions. Researchers (35) established stable and non-stable regions for their project (which involved a disc rotating in contact with the end of a canti-lever held by clamps) with the radius of the disc, and

and the length and slope of the canti-lever as parameters. An examination of the three parameters involved shows that the velocity was dependent upon the radius if the frequency was constant; the slope of the clamp controlled the magnitude of the Coulomb force acting on the disc, and the length of the canti-lever determines the stiffness in the system. As established in 8.1.1 the delay-time h is dependent upon R and C and when there is a spring in the system, on the stiffness. This suggests that vibrations induced by Coulomb force can be controlled by changing the relevant parameters in the system. The effect of Coulomb force on Frequency entrainment is illustrated by reference (6).

8.4 DEPENDENCE OF COEFFICIENT OF FRICTION ON AMPLITUDE

Fig 6.29 to 6.32 show the relationship between the normal pressure and the amplitude of vibrations at constant frequency. Most of the work (1) to (4) on Coulomb damping especially involving the coefficient of friction has been done at constant amplitude and on the occasions when the response was measured more attention was paid to the magnification rather than the effect of the amplitude of vibrations on the magnitude of the friction force. However in Stefan Ziembas (7) analysis, the dependence of the friction force on the amplitude of vibrations was implied when he concluded that the system represented by fig 1.9 in the presence of strong damping would tend asymptotically to the equilibrium position for t approaching infinity. Earles and Beards (28) also encountered this in their research. Whilst theoretically, the variation of slip amplitude at constant frequency with normal forces across the damper should be a straight line according to their representation of the damping force, the

experimental results were all more of curves, than straight lines. Earles and Beards concluded from these results in conjunction with others that the coefficient of friction decreases with increasing slip amplitude. Similarly Dokos (33) found it necessary to express the slip frequency f_s in the manner

$$f_s = C_1 e^{C_2 L} \dots\dots\dots(8.8)$$

Also $s = \frac{V}{f_s}$ where V = velocity in in/sec.
and f_s = frequency in Hz

Thus $s = \frac{V}{C_1 e^{C_2 L}}$

where S = slip amplitude and C_1 , and C_2 are constant and
 L = Normal Load.

Equation (8.9) shows a kind of dependence of the slip amplitude on the normal Load even though this equation was evolved to suit a particular set of experimental results. As the theoretical curves in figs 6.29 to 6.32 show good agreement with the experimental curves the relationship as expressed by the equation:

$$F = Rx(\cos wh + j \sin wh)$$

is shown to be valid.

The constant $R = \frac{F}{X}$ therefore R decreases with increasing X , thus confirming the conclusion of Earles and Beards that the Coefficient of Friction decreases with increasing slip amplitude.

DISCUSSION ON VIBRATION RESPONSE WITH COMBINED DAMPING

In practical mechanical systems there is often more than one type of damping, even though one type may predominate. It may be found necessary to increase the damping in the system if resonance conditions are attained in the system or the amplitude of vibrations is deemed too large as to be dangerous. The construction of the system may be such that a particular type of damping can be introduced conveniently to augment the damping.

The quantity of damping that may be introduced depends upon the effect desired by the vibration engineer. The objective may be to shift the resonant frequency either up or down to a safer value, or simply to decrease the amplitude of vibrations or as is often the case to do both. In order to do this effectively different concepts have been developed throughout the years to enable the engineer to make reasonable substitutions in mathematical equations and hence predict the effect of a damping combination in a system safely. Among these is the "concept of equivalent viscous damping" which is used extensively. This concept is based upon the theory of the Dissipation of energy. In this chapter therefore the energy dissipated by the different damping forces per cycle will be determined, so that the "equivalent viscous damping" theory can be examined.

The effect of the combination of damping on the resonant frequency and the phase angle will also be examined.

9.1 Dissipation of Energy

9.1.1 In General

Work done per cycle = $X_0 \int_0^T F(x) \omega \cos \omega t \, dt$
 where $F(x)$ is the damping force.

Viscous damping

Damping force = $c \dot{x}$

Let the motion be sinusoidal,

$$x = X_0 \sin \omega t$$

$$\dot{x} = \omega X_0 \cos \omega t$$

thus viscous damping force = $c \omega X_0 \cos \omega t$

$$\begin{aligned} \text{Hence W.D./cycle} &= X_0^2 \int_0^T c \omega^2 \cos^2 \omega t \, dt = \frac{c \omega^2 X_0^2}{2} \int_0^T (1 + \cos 2\omega t) \, dt \\ &= 4c X_0^2 \omega^2 \int_0^{\frac{T}{2}} (1 + \cos 2\omega t) \, dt \\ &= 2c X_0^2 \omega^2 \left(t + \frac{\sin 2\omega t}{2\omega} \right)_0^{\frac{T}{2}} \\ &= 2c X_0^2 \omega^2 \left(\frac{\pi}{2\omega} + \frac{\sin 2\omega \frac{2\pi}{\omega}}{2\omega} \right) \\ &= \frac{2c X_0^2 \omega \pi}{2} = \pi c \omega X_0^2 \end{aligned}$$

9.1.2 Viscoelastic damping

For viscoelastic damping, W.D./cycle = $\pi K_0 g$ (9.3)

9.1.3 Coulomb Damping

Coulomb damping force = $j k x \sin \omega t$

$$\begin{aligned}
 j k x &= X_0 \cos \omega t \\
 \text{W.D./cycle} &= X_0^2 \int_0^{2\pi/\omega} R \sin \omega t \cos^2 \omega t \, dt \\
 &= \frac{4 X_0^2 R \sin \omega h}{2} \int_0^{2\pi/\omega} (1 + \cos \omega t) \, dt \\
 &= 2 X_0^2 R \sin \omega h \left[t + \frac{\sin 2 \omega t}{2 \omega} \right]_0^{2\pi/\omega} \\
 &= 2 X_0^2 R \sin \omega h \frac{\pi}{2 \omega} \\
 * &= \pi R X_0^2 \sin \omega h \quad (9.4)
 \end{aligned}$$

9.1.4 COMPARISON WITH NORMAL PRESENTATION.

With friction force F represented as a constant

$$\text{W.D./cycle} = 4 F X_0 \quad (9.5)$$

where $F = \mu N$

$$\text{Now } R = \frac{\mu N}{x} \quad \therefore R = \frac{F}{x}$$

Thus (9.4) can be presented as $\pi F X_0 \sin \omega h$ (9.6)

A comparison equations (9.5) and (9.6) show that by assuming that the friction force is constant the energy dissipated per cycle is overestimated.

The overestimation is minimum when $\sin \omega h = 1$ and even then the ratio of the as used value to the actual one is $4:\pi$ ie 1.26 : 1

9.2 THE CONCEPT OF EQUIVALENT VISCOUS DAMPING

The forces applied to vibrating systems by many actual damping mechanisms are not only difficult to define mathematically but lead to non-linear differential equations of motion.

* This relationship does not apply when $\omega \rightarrow 0$

For these reasons the damping forces in many systems are often represented in mathematical models by a viscous damping force of equivalent energy dissipating capabilities; Viscous damping is easily defined and its inclusion leads to a linear differential equation of motion. Equations (9.2) (9.3) and (9.4) are the expressions for the energy dissipated per cycle for the three types of damping dealt with in this thesis.

9.2.1 EQUIVALENT VISCOUS DAMPING FOR VISCOELASTIC DAMPING

Let C_{eq} = equivalent viscous damping coefficient
then

$$\begin{aligned} \pi C_{eq} \omega X_0^2 &= \pi X_0^2 \zeta \\ C_{eq} &= \frac{\pi}{\omega} = \frac{R}{\omega} \end{aligned} \quad (9.7)$$

9.2.2 EQUIVALENT VISCOUS DAMPING FOR COULOMB DAMPING

$$\begin{aligned} \pi C_{eq} \omega X_0^2 &= \pi R X_0^2 \sin \omega h \\ C_{eq} &= \frac{R \sin \omega h}{\omega} \end{aligned} \quad (9.8)$$

Let γ be the equivalent coulomb damping ratio, then $\gamma = \frac{C_{eq}}{J_0} =$

$$\frac{R \sin \omega h}{\omega \times 2k} \times \omega_0 = \frac{R \sin \omega h}{2kn} = \frac{S \sin \omega h}{2n} \quad (9.9)$$

where $S = \frac{R}{k}$ and $n = \frac{\omega}{\omega_0}$

Using the conventional expression

$$\begin{aligned} \pi C_{eq} \omega X_0^2 &= 4 \pi F X_0 \\ C_{eq} &= \frac{4F}{\pi \omega X_0} \end{aligned} \quad (9.10)$$

9.3

EXPERIMENTAL AND THEORETICAL COMPARISON

As established in the vectorial representation in fig 4.5 the damping component of the coulomb force is in quadrature with the in-phase component and since the viscoelastic and viscous damping forces are also in quadrature with the real force it means that the damping forces are additive. This implies that the damping ratio, or loss factor or equivalent damping ratios can be added to obtain the resultant equivalent damping ratio or loss factor.

In order to establish the validity of this relationship, the damping ratios and their equivalents were obtained from the experimental response curves obtained for the individual types of damping, and also those obtained for the different combinations of damping. These values were also determined by direct calculation using equations (9.7) to (9.10) and hence obtained values for the equivalent damping coefficients for comparison with those derived from the experimental results.

9.3.1

COULOMB AND VISCOUS DAMPING RATIOS

Table 9.1 shows the values of the equivalent damping ratio γ for coulomb damping and viscous damping, 2ζ for different normal loads as well as the values for combinations of the two types of damping.

Coulomb only		Viscous only		Coulomb + Viscous		Error	
Applied load		Current		Theoretical	Exp.	Error	page
N	δ	Amps	2c				Exp.
1.72	0.274	1.5	0.142	0.416	0.335	0.031	3.0
3.60	0.372	3.0	0.282	0.658	0.57	0.038	15.4
6.00	0.442	4.5	0.425	0.967	1.07	0.103	9.6

Table 9.1 COMPARISON OF EXPERIMENTAL AND THEORETICAL
DAMPING RATIOS

As can be observed the percentage error between the experimental and theoretical results ranges from 3 to 15.4. The theoretical results were obtained by assuming that the conditions were the same for all the cases but as already pointed out, a slight difference in alignment, in the cleanliness of the surfaces of contact and the ratio $\frac{R}{K}$ could result in different values of coefficient of friction being obtained for a particular set of conditions. Taking this into consideration, the percentage difference is an expected.

9.3.2 COMPARISON OF EQUIVALENT DAMPING COEFFICIENTS C_{eq}

The equivalent damping coefficient C_{eq} was calculated using equations (9.8) and (9.10) for comparison with the values obtained from the experimental results. Table 9.2 shows the comparable values for the two types of damping and some combinations of the two.

Coulomb only			Viscous only			Combinations		
(i) normal load L	(ii) $4\pi\%$	(iii) no shown	(iv) Exp. current	(v) current	(vi) Exptl.	(vii) (ii)+(vi)	(viii) (ii)+(vi)	(ix) Exptl.
.72	1.30	1.01	1.12	1.5	0.20	0.94	0.90	0.79
3.60	1.9	1.36	1.54	3.0	0.575	1.525	1.255	1.18
6.0	2.34	2.24	2.22	4.5	0.865	2.25	1.935	2.20

Tab. 9.2. COMPARISON OF THE CALCULATED EQUIVALENT COEFFICIENT DAMPING WITH THOSE OBTAINED EXPERIMENTALLY.

As $\gamma = 25$, C_{eq} obtained for columns (ii) and (iii) had to be halved to give the viscous equivalent before additions were made.

The results obtained by using equation (9.8) shown in column (iii) are more comparable with the experimental ones shown in column (iv) than the results obtained by using equation (9.10) shown in column (ii). In general the use of equation (9.10) results in an overestimation of the damping as already predicted in 9.1.4.

The results for the combinations shown in columns (vii) to (ix) show a similar trend to the cases discussed, except for the case when the normal load is 6.0 lbf. In this case (9.8) slightly underestimates the equivalent damping coefficient C_{eq} but this is probably due to the fact that the delay time is near minimum, (see Fig 6) at this loading, thus producing optimum damping. Even then the difference is acceptable and equation (9.8) as shown produces better results.

Viscoelastic		Viscous			Expected		Experimental		Difference		Percentage	
Loss factor (i) η	C_{eq} (ii)	Current (iii)	2ζ (iv)	C (v)	$2\zeta_{eq}$ (vi)	C_{eq} (vii)	$2\zeta_{exp}$ (viii)	C_{eq} (ix)	$2\zeta_{exp}$ (x)	C_{eq} (xi)	$2\zeta_{exp}$ (xii)	C_{eq} (xiii)
0.185	0.636	1.5Amps	0.089	0.29	0.284	0.94	0.26	0.91	0.004	0.03	1.4	3.2
0.185	0.636	3.0	0.177	0.575	0.362	1.21	0.39	1.23	0.028	0.02	7.7	1.7
0.185	0.636	4.5	0.265	0.865	0.450	1.50	0.47	1.57	0.020	0.03	4.4	2.0

Table 9.3 Comparison of the calculated and experimental loss factors and equivalent damping coefficients.

Combination											
viscoelastic		Coulomb		Expected		Experimental		Difference		Percentage	
(i) η	(ii) C_{eq}	(iii) γ	(iv) C_{eq}	(v) η_{eq}	(vi) C_{eq}	(vii) η_{eq}	(viii) C_{eq}	(ix) η_{eq}	(x) C_{eq}	(xi) η_{eq}	(xii) C_{eq}
0.185	0.636	0.171	0.59	0.356	1.15	0.350	1.14	0.006	0.01	1.7	1.0
0.185	0.636	0.245	0.845	0.420	1.36	0.415	1.34	0.005	0.02	1.2	1.5
0.185	0.636	0.338	1.17	0.523	1.70	0.48	1.56	0.043	0.14	8.2	8.2

Table 9.4 Comparison of the estimated γ_{eq} and C_{eq} with the measured values.

9.3.3

VISCOUS AND VISCOELASTIC

Table 9.3 shows the results for viscous and viscoelastic damping. The expected and experimental results for the combined damping show good agreement and thus confirms the works of other research workers e.g Neumarks report (18).

9.3.4

VISCOELASTIC AND COULOMB

Table 9.4 shows the results for viscoelastic and coulomb damping together with combination of the two types of damping. The parameters under consideration are the equivalent loss factors and equivalent damping coefficients.

The experimental and theoretical results compare favourably with a maximum difference percentage of 8.2. With friction in the system more heat would be generated, thus resulting in a rise in temperature, and as the loss factor of a viscoelastic material decreases with rise in temperature as shown by reference (16), it is understandable that the measured values are consistently less than the expected ones.

9.3.5

COMBINATIONS OF ALL THREE TYPES OF DAMPING

The comparison of equivalent damping ratios, for a combination of two different types of damping has been discussed and found to be agreeable. Therefore a comparison of all the three types of damping together is shown in table 9.5 in which as before the expected value is the sum of the three individual damping ratios and the experimental values are the measured ones.

Types of Combination (i)	Coulomb (ii) $\gamma = 2\%$	Viscous (iii) 2%	Viscoelastic (iv)
1.5Amps + 1.72lbf + Viscoelastic	0.171	0.089	0.185
3.0 " 3.60lbf + "	0.245	0.177	0.185
4.5 " 6.00lbf + "	0.338	0.265	0.185

Types of Combination (i)	Expected (v)	Experi. l. (vi)	Difference (vii)	(viii)
1.5Amps + 1.72lbf + Viscoelastic	0.445	0.424	0.021	4.7
3.0 Amps + 3.60lbf " Viscoelastic	0.60	0.60	0.007	1.2
4.5 Amps 6.00lbf + Viscoelastic	0.788	0.80	0.012	...

ζ_{eq}
Table 9.5 Comparison of measured ζ_{eq} with the expected values.

The maximum percentage difference between the expected and measured equivalent damping ratios is 4.7 and this is exceptionally good. Thus in practical cases where possible, the equivalent damping ratios for different types of damping can be determined experimentally separately and the equivalent damping ratio for the combined cases obtained by simply adding the individual damping ratios. Where it is impossible to do separate tests good results can be obtained by calculations based on the equivalent damping coefficient C_{eq} , by using the appropriate equations. In the case of viscous and viscoelastic damping the normal expressions yield good results and are reliable. With coulomb damping however the normal and accepted expression overestimates the equivalent damping coefficient and therefore equation (9.8) which yields better results should preferably be used.

9.4 The effect of a Combination of Damping on the Resonant frequency

9.4.1 The effect of combined Coulomb + Viscous damping on the Resonant frequency

Fig 7.1 shows the experimental response curves for three different amounts of viscous damping while fig 7.5 shows the response for three different amounts of coulomb damping. In the case of viscous damping, the resonant frequency is reduced with incrementⁱⁿ damping as expected. With Coulomb damping the resonant frequency f_r increased from 12.5HZ to :-

12.6	for a normal load of 1.72lbf
13.8	for a " " of 3.60lbf
12.8	for a " " of 6.00lbf.

These results show that the change in resonant frequency is not dependent on the normal load only but also on other parameters which as shows in 4.3 are the delay time h , the frequency of vibrations and the spring stiffness, k . The condition for increment or decrement in resonant frequency is given in equation (4.11).

Resonant frequency increases if	$\cos w_0 h > -\frac{R}{2k}$
" " decreases " "	$< -\frac{R}{2k}$
" " is unchanged " "	$= -\frac{R}{2k}$

Table 9.6 shows the values for $\cos w_0 h$ and $-\frac{R}{2k}$

R	Cosw ₀ h	$-\frac{R}{2k}$
80	0.094	-0.25
122	0.455	-0.38
168	0.0	-0.52

Table 9.6 Examination of the conditions for change of resonant frequency.

Clearly $\cos w_0 h > -\frac{R}{2k}$ in all the cases considered and the largest change in resonant frequency f_r occurred as expected in the second case (ie $R=122.0$) when the difference between $\cos w_0 h$ and $-\frac{R}{2k}$ was greatest.

The Combined effect

The conditions as presented in this system with coulomb damping favour an increase in the resonant frequency f_r whilst viscous damping results in a decrease in f_r . The effect of a combination of viscous and coulomb damping therefore depends upon the relative magnitude of each type of damping. If coulomb damping predominates in this particular system then f_r will increase, where as if viscous damping predominates f_r will decrease in value. The changes in f_r for viscous damping as well as coulomb damping and some combinations of the two are shown in table 9.7. The undamped resonant frequency is 12.5 Hz.

	Coulomb only	Viscous only	Combination
Type of Combination	f_r	f_r	f_r
1.721bf + 1.5 Amps	12.6	12.5	12.5
3.601bf + 3.0 "	13.8	12.2	13.0
6.001bf + 4.5 "	12.8	12.0	12.3

Table 9.7 Comparison of resonant frequencies f_r for coulomb and viscous damping with a combination of the two.

9.4.2 The effect of combined viscoelastic and viscous damping on f_r

Viscoelastic damping has no effect on the resonant frequency f_r and therefore the combined effect is that due to viscous damping only and this means that the resonant frequency decreases. The experimental results do confirm this; for the resonant frequency decreased from 20 Hz to 19.8 Hz for $C=0.575$ and to 19.6 Hz for $C= 0.865$ even though it remained constant for $C=0.29$.

9.4.3

VISCOELASTIC AND COULOMB DAMPING

The effect on f_r is totally dependent on the coulomb damping, for viscoelastic has no effect on f_r . Thus whether f_r increases or decreases when there is a combination of viscoelastic and coulomb damping in a particular system depends on equation (4.11) as already pointed out. Table 9.8 gives the values of $\cos w_0 h$, and $-\frac{R}{2k}$ for different normal loads with the corresponding damped resonant frequencies. The undamped resonant frequency being 20 Hz.

R	f_r	$\cos w_0 h$	$-\frac{R}{2k}$
80.0	19.50	-0.31	-0.0980
122.0	20.10	0	-0.149
168.0	20.30	0	-0.205

Table 9.8 Examination of the condition for a change in resonant frequency.

For $R = 80$ a decrease in resonant frequency f_r is expected as $\cos w_0 h < -\frac{R}{2k}$. In the other two cases however $\cos w_0 h > -\frac{R}{2k}$ and therefore an increase in f_r is expected. The results confirm this prediction.

9.4.4

COMBINATIONS OF ALL THREE

The change in frequency depends on the relative magnitude of coulomb and viscous damping. If conditions are such that an addition of coulomb damping reduces resonant frequency f_r then an addition of viscous damping will reduce it further. Table 9.9 shows a comparison of resonant frequencies for different types of damping and for the combination of the three.

Combination of damping	viscoelastic only f_r	Coulomb only f_r	Viscous only f_r	Combina- tion only f_r
viscoelastic+ 1.72lbf + 1.5Amps	20.0	19.5	20	19.5
" 3.60lbf + 3.0 "	20.0	20.10	19.8	20.0
" 6.00 " + 4.5 "	20.0	20.30	19.6	20.0

Table 9.9 Comparison of resonant frequencies for different amounts of damping.

VALIDITY OF EQUATION

In general Equation (4.11) gives a valid condition for the change in resonant frequency and can be safely used to predict the effect on a system when coulomb damping is added.

9.5 The effect of a Combination of damping on the Phase

9.5.1 Viscous and Viscoelastic Damping

In general an increment in the quantity of damping results in an acceleration of the phase before the natural resonant frequency, but a retardation after the natural resonant frequency. This result is in agreement with both the theoretical representation in test books for example, reference (20) to (24) and the experimental results obtained by researchers who have investigated viscous (5) or viscoelastic damping (13) to 17 singly, or a combination of the two (13).

9.5.2

COULOMB DAMPING

Coulomb damping produces the same effect on the phase as viscous and viscoelastic damping if conditions are such that the resonant frequency remains unchanged. If however there is a shift in the resonant frequency as a result of introducing coulomb damping the phase curve is also shifted in such a way as to obtain 90° phase shift at the damped resonant frequency. Fig 7.6 shows this clearly where the phase is 90° at 12.5 Hz, 12.8Hz and 14 Hz respectively instead of all being at 12.5 Hz as would be the case for viscous or viscoelastic damping. The effect on the phase at frequencies not too close to the resonant frequency is to accelerate the phase before and retard it after resonance.

9.5.3 COULOMB DAMPING IN THE PRESENCE OF VISCOUS AND OR VISCOELASTIC DAMPING

Figs 7.10, 7.18 and 7.22 show the effect on the phase when coulomb damping is introduced into a system containing viscous, or viscoelastic damping or a combination of the two. Fig 7.10 shows that the phase is 90° at frequencies of 12.5 and 14Hz respectively which is identical with the frequencies at which the phase was 90° for coulomb damping only shown in fig. 7.6. An examination of Figs 7.18 and 7.22 show that the phase is 90° at 20 Hz for the cases where the resonant frequency did not shift but otherwise the phase curve underwent a shift in accordance with the shift in frequency.

9.5.4 CROSS POINTS:

In accordance with the condition established in equation (4.14), it can be observed in the figures discussed involving phase curves having coulomb damping that the cross points of the curves (i.e. the frequencies at which the phase has the same value) for different curves of coulomb damping are not the same in all cases. In fig 7.6 the phase is the same at 103.0° for normal loads of 1.71bf and 3.61bf at a frequency of 13.3 Hz whilst in fig 7.10 it is 95° at 13 Hz for the same normal loads with differing amounts of viscous damping added.

With viscoelastic damping in the system fig.7.18 shows that all the three curves cross when the phase is 65° at a frequency of 18Hz for the normal loads of 1.71bf, 3.601bf and 6.01bf respectively.

Fig 7.22 shows three cross points the curves for

- (a) 1.7 and 3.61bf cross for a phase of 60° at 18Hz
- (b) 1.7 and 6.01bf cross for a phase of 75° at 19.4Hz
- (c) 3.6 and 901bf cross for a phase of 90° at 20Hz.

The figures used in this discussion are all experimental results and the theoretical curves also confirm the existence of cross points even though the phase angles and frequencies may differ slightly from the experimental ones.

CHAPTER 10

CONCLUSIONS AND OBSERVATIONS

The conclusions shall be made in two parts as follows:-

1. Conclusions on the nature of coulomb force.
2. Conclusions on the effect of a combination of Coulomb, Viscous and Viscoelastic damping in pairs or all together on a vibrating system.

10.1 CONCLUSIONS ON THE NATURE OF COULOMB FORCE

1. Coulomb force is not a constant at all frequencies as is normally assumed, but is dependent upon the frequency of vibrations or the speed of the motion.

This conclusion is supported by the experimental results of both lubrication and vibration research workers quoted in this thesis.

2. Coulomb force is dependent on the amplitude of vibrations or the slip amplitude in the case of a unidirectional motion. This dependence of Coulomb force on amplitude remained unobserved for a long time because most of the early researchers performed their experiments at constant amplitude of vibrations or slip amplitude. The research workers (28) and (33) who varied the slip amplitude or vibration amplitude did observe this dependence.
3. Coulomb force is a retarding force and can be represented in a vibratory motion in accordance with the theory derived in this thesis from the Retardation theory by a complex force of the form $F = xR (\cos wh + j \sin wh)$.

CHAPTER 10 Continued

This presentation explains the existence of the phase difference between the excitation force and the response of a system having only coulomb damping. It also helps to establish the regions of stability (positive damping) and non stability (negative damping).

4. Coulomb force is dependent on the delay time "h" which itself is dependent upon the parameters of the vibrating system. The parameters are the inertia of the moving body, the elastic stiffness, and the ratio of the friction force, F to the elastic force, KX_0 in the system.
5. In a particular vibrating system, there is a ratio of $\frac{F}{KX_0}$ or $\frac{F}{P}$ (where P is excitation force), for which at a particular frequency of vibrations optimum damping is obtained.
6. Given the static coefficient and the kinetic coefficient of friction at a particular frequency or speed, the delay time "h" can easily be obtained if the amplitude of vibrations or slip amplitude is also known. The delay time h , being of the order of mS e.g 0.0026S, for system without external elastic systems (springs) and of the order of 0.015.Seconds for systems with springs in them. This compares favourably with the delay-times of 0.010 to 0.015 Secs that were found to give satisfactory results by I.Finnie and B.Y. Lim (31) in their paper dealing with the effect of delay terms on vibration excitation.
7. Coulomb force may induce vibrations instead of damping them if the conditions are such that the quadrature component of the force is negative.

N.B. The conclusions have been verified by experiment over a frequency range of 3-30 Hz.

10.2 Conclusions on Vibration Response With Combined Damping

10.2.1 THE CONCEPT OF EQUIVALENT VISCOUS DAMPING

1. The concept of equivalent viscous damping yields good results and can be used to predict the effect of a combination of damping provided the right expressions are used to represent the energy dissipated by the different types of damping.

The expressions used for viscous and viscoelastic damping yield results that are verifiable experimentally. The normal expression used for Coulomb damping (ie $W.D = 4FX_0$) overestimates the equivalent viscous damping coefficient by a minimum of 28% when $\sin wh = 1$, in the new expression in this thesis (ie $W.D = \pi FX_0 \sin wh$), which produced comparable results with those obtained experimentally.

2. The Coulomb damping ratio $\chi = 2\zeta = \eta$ where ζ is the viscous damping ratio and η is the viscoelastic loss factor.

10.2.2 EFFECT ON THE RESONANT FREQUENCY

3. Viscoelastic damping has no effect on the resonant frequency, in the absence of a different elastic medium in the system, but viscous damping decreases the resonant frequency whilst coulomb damping may or may not change the resonant frequency depending on this identity.

$$(\text{decrease}). \quad -\frac{R}{2K} \geq \cos w_0 h \geq -\frac{R}{2K} \quad (\text{increase})$$

In the presence of another elastic medium in the system, an addition of viscoelastic damping results in an increase in the effective stiffness and thus an increase in the resonant frequency.

4. The effect of a combination of viscoelastic and viscous or coulomb damping on the resonant frequency f_r is the same as that due to the viscous or coulomb damping only.
5. The effect of a combination of coulomb and viscous damping on the resonant frequency is a compromise of the two effects.

Thus if conditions favour an increase in resonant frequency f_r , for coulomb damping only, since viscous damping tends to decrease f_r , the net effect depends upon the relative magnitude of the two types of damping.

However if conditions favour a decrease in f_r for coulomb damping only, then since viscous damping tends to decrease f_r , the net effect will be a decrease in f_r greater than that due to either of the two types of damping acting alone.

10.2.3 THE EFFECT ON THE PHASE

6. For viscoelastic and viscous damping, an increment in the amount of damping results in an acceleration of the Phase before resonance but in a retardation of the phase after resonance.

7. An increment in the amount of Coulomb damping in a system has the same effect as viscous and viscoelastic damping in general. However if there is a shift in f_r due to the increased damping, then the phase curve is shifted accordingly to give a phase of 90° at the new f_r . It is therefore possible that the phase ^{of} a particular ^{system} $\wedge$ may decrease with increment in the amount of damping.

REFERENCES

1. Lubrication and friction by Peter Freeman Applied Mechanics Monographs (Pitman).
2. Friction behaviour During the slip Portion of the Stickslip process - by J.B. Sampson, F. Morgan, D.W. Reed and M. Muskat (Journal of Applied Physics Vol 14, 1943).
3. The nature of the static and kinetic coefficients of friction - Ernest Rabinowicz (Journal of Applied Physics - Vol 22, number 11, November 1951).
4. Proceedings of the Summer Conference on friction and Surface Finish - by Hans Merchant and M. Eugene Ernst (M.J.T. June 1940 Page 76,).
5. Forced vibrations with combined Coulomb and viscous Friction - by J.P. Den hartog, (Transactions of the American Society of Mechanical Engineers 1926).
6. On the Self-Synchronization of Mechanical Vibrators (Part 3, Forced Frictional Vibrations) - by Junkichi INOVE, SUGA MIYAURA and Osami Matsushita. (Bulletin of the JSME Vol 13, No. 56, 1970).
7. Dry friction vibration Damping - Stefan Ziemba (Arch Mech. Stos 2 (Dept. of Mechanics of Continuous media, Polish academy of Sciences June 1956).
8. Non-linear Oscillations - by Nicholas Minorsky
(Van Nostrand)

9. Resonance characteristics of unidirectional viscous and coulomb damped vibration isolation system - by Jerome E. Ruzika (Transactions of ASME : Journal of Engineering for Industry Paper No.67 Vibr.21).
10. Vibration with Nonlinear Damping- by Jerome E. Ruzicka and Thomas F. Derby. (Journal of Engineering for Industry May,1971).
11. Forced oscillations of a spring - mass system having combined Coulomb and viscous Damping - by Edwin S. Levitan (The journal of the Acoustical Society of America Vol.32 No.10 October 1960).
12. Representation of the mechanical damping possessed by rubber-like materials and structures - by J.C. Snowden (J.Acoustical Soc. of America 35, 6 June 1963).
13. Vibration of dynamic systems with linear hysteretic damping by T.K. Caughey. (Nat. Congr. App. Mech. University of California Berkeley Vol. 1. June 1962 The American Society of Mech. Eng.).
14. Note on the relations between viscous and structural damping coefficients by W.W. Soroka. (Journal of the Aeronautical Sciences Vol.16, No.7 July, 1949).
15. The Concept of Complex damping by N.O. Myklestad (Journal of App. Mechanics Vol.19 No.3 September 1954).
16. Measurement of the dynamic properties of damping materials by P. Grotenhuis - (Proceedings of the international Symposium on the Damping of vibrations of plates by means of Layers. Association Belge des Acousticiens - Leuven Belgium September, 1967).
17. The general theory of "hysteretic damping" by R.E.D. Bishop. (Aeronautical Quarterly Vol.V11 Part 1 February 1956).

18. Concept of Complex stiffness applied to problems of Oscillation with viscous and hysteretic damping by S. Neumark.
(Royal Aircraft establishment report No. Aero 2592 September 1957).
19. Vibration control with viscoelastic materials by P. Grotenhuis
(Journal Society of the Envi. Eng. No.38, May, 1969).
20. Vibration response with different types of damping by H.K. Bukari D.I.C. thesis Mech. Eng. Dept. Imperial College London 1969).
21. Shock & Vibration concept in Eng. (Design by Charles Crede Prentice Hall (1965)).
22. Vibration and Impact by Ralph Burton (Dover Publications (1968)).
23. Mechanical vibrations - by J.P. Den Hartog (McGram Hill 4th edition 1956).
24. Shock & vibration handbook Vol.2 C.M. Harris & C.E. Crede (McGram Hill).
25. Mechanical and Electrical vibrations (Methuen's Monographs on physical subjects (1941)).
26. Acoustics & vibrations physics R.W.B. Stephens and A.E. Bate (Edward Arnold Ltd.).
27. Mechanical vibrations. An introduction to Matrix methods (Longmans) by J.M. Prentis and F.A. Leckie.
28. Some aspects of frictional damping as applied to vibrating beams by S.W. Earles & C.F. Beards (Inst. Mech. tool Des. Res. Vol.10 1970).
29. Frictional vibrations - by David Sinclair (A.S.M.E. July 31, 1953).
30. Theory of vibration Isolation - Shock & vibration Handwork by C.E. Crede and J.E. Ruzicka.

31. A simple model to illustrate some aspects of vibration excitation by Delayed Terms - by I. Finnie and B.Y. Lim
Experimental mechanics - December 1964.
32. Forced vibrations of a two degree of freedom system with combined Coulomb and viscous damping by G.C.K. Yeh.
33. Sliding friction under extreme pressures by S.J. Dokos. J. App. Mech. Vol.13.
34. Steady forced vibrations as influenced by Damping. by L. Jacobsen, - A.S.M.E. (1930). Ap. M. 52 - 15.
35. Vibrations induced dry friction by R.P. Javis and B. Mills, (Proceeding of I.M.E. 1963-1964 Vol.178 Page 1 No.32.)
36. Free vibrations of non-linear cubic mass system in the presence of coulomb damping by V.A. Bapat and P. Srinivasam (J. Sonia vib (1970) 11.11, 121 - 137).
37. A Study of amplitude frequency plots with non-linear damping by (Brown Eng. Comp. Inc. Huntsville, Alabama).
38. A direct method for estimating the effect of coulomb friction on free oscillations - by M.D. Frost (Aerodynamics note 182 Dept. of Aeronaut. Eng. University of Sydney).
39. Behavior of one degree of freedom system under vibrating solid friction of high frequency - by G. Nishimura, Y. Jimbo and M. Takano (Journal of Faculty of Engineering - University of Takyo Vol. XXVlll No.3 (1966)).
40. Determination of the Complex Young's modulus of polymers at audio frequencies in a wide termperature range by P. Heydemann and A Zosel. (Austica Vol.12 (1962)).
41. Dry friction damping with force proportional to displacement - by Leon Wallerstein Jr. (Lord Manufacturing Company Eric Pennsylvania.

42. Exeural vibration of symmetrical multi-layer beams with viscoelastic damping. by J.A. Agbasiere and P. Grootenhuis (J.M. Eng. Science Vol. 10 No.3 1968).
43. Mathematics of Eng. systems - by Dereck F. Lawden (University paper backs).
44. Advanced Eng. Mathematics - 2nd Edition by Erwin Kreyszig (John Willey and Sons, inc.)
45. Technology of Instrumentation by Eric Pearson (English Universities Press Limited).
46. Measurements in mechanical dynamics - by D.N. Keast, B. Beranek, and Newman inc.)
47. Forced vibrations in a mechanical system having a Non-linear Restoring force. by C.A. Ludeke Journ. App. Physics (July 1946).
48. On one term approximations of forced Non-harmonic vibrations by G. Schwesinger Journ. App. Mechs. June 1950.
49. Non-Linear vibrations - By J.J. Stoker Interscience Publishers.
50. Introduction to Non-linear Mechanics by M. Kryloff and M Bogoliuboff (Princeton University Press).
51. The mechanics of vibrations - R.E.D. Bishop and D.C. Johnson Cambridge University Press 1960).
52. Engineering Design with Rubber by A.R. Payne and J.R. Scott. McLaren & Sons Ltd. 1960.
53. The Design of Electro-magnetic vibration generators. Proceedings of conference on Machines for Materials and Environmental Testing. Inst. of Mech. Eng. and S.E.E. 1965.
54. Damping of Materials and members in structural mechanics.
55. "Molecular Theory of Friction" by G.A. Tomlinson Philosophical mag. Vol. 7 Series 7. 1929.

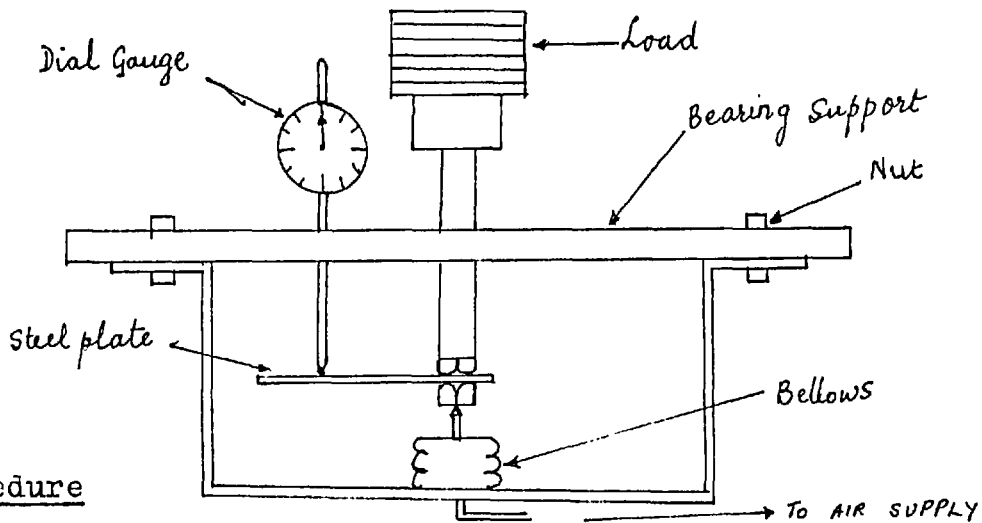
56. The sliding of metals. Frictional Fluctuations and vibrations of moving parts by F.B. Bowden, L. Leben and D. Tabor. The Engineer Vol.168 1939.
57. Studies in lubrication. Friction Phenomena and the slip process by F. Morgan, M. Muskat and D.W. Reed. Journal of App. Phys. Vol.12 1941.
58. "Surface Roughness and sliding Friction" by J.J. Bikerman Reviews of Modern Physics Vol.16 1944.
59. The attenuation of noise and ground vibrations from railways by P. Grootenhuis. Proc. Env. and Human factors in Eng. Soc. Env. Eng. 1967.
60. Forced vibrations in systems with elastically Supported Dampers by J.E. Ruzicka. Masters thesis, M.I.T. Cambridge Mass 1957.
61. The design of electromagnetic vibration generators - by P. Grootenhuis and J.W. Gearing. Proc. Conf. Machine for materials and envtl. testing I. Mach./ S.E.E. Manchester, 1965.
62. On the magnitude of the Mechanical Component of Solid friction by C.D. Strong and C.R. Lewis. Journ. of App. Phys. Vol.20 1949.
63. Dry Friction Damped Isolators by G.W. Painter. Product Eng. 1959.
64. The friction and Lubrication of Solids (by F.P. Bowden and D. Tabor (Oxford University Press 1950).
65. Friction of clean metals and the influence of adsorbed film by F.P. Bowden and J.E. Young. Proc. Roy. Soc. A 208 P.311 1951).
66. On the Oscillations of "Summed up and differential Types" under Parametric Excitation (Vibration system with Damping) by Toshio Yamamoto and Akihiko Saito. J.S.M.E. Bulletin of 1968. Vol.11 No.43.
67. Friction of Solid films on Steel at high sliding velocities. by R.L. Johnson, D. Godfrey & E.E. Bisson. N.A.C.A., T.N. 1578 (1948).

A P P E N D I X 1

A1. Determination of the Static Stiffness

The Static Stiffness of the friction block system was determined in two ways. Block B was tested while the friction block remained in the bearings and block A was taken out of the bearings before testing.

The set up for block B was as shown:



Procedure

The dial gauge was set at zero and readings were taken as the Load was increased and also when loading was decreased. This was done for various air pressures acting on the bellows from No Air to 6lb/in^2 gauge pressure. The results are shown in fig. A.1.....

Fitting Curves

By feeding the readings obtained as data for a least Square Computer programme with variable Laws the best fitting curves were obtained as well as the Laws governing the spring system of the bellows. The experimental curves showed that the bellows were non-linear. The results are shown in Fig. A.2

.....

Fig. A.2

BLOCK. B

- 204 -

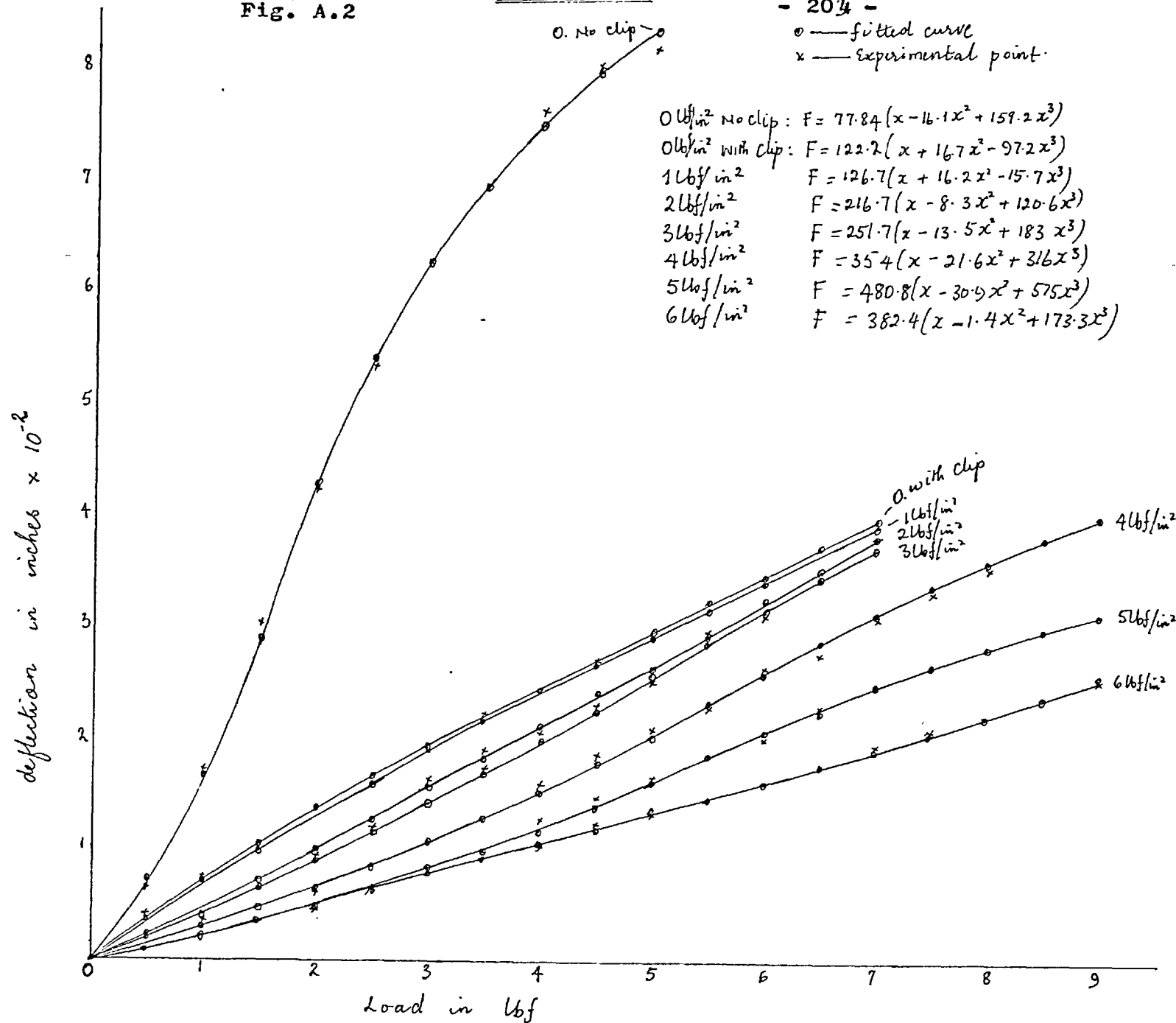
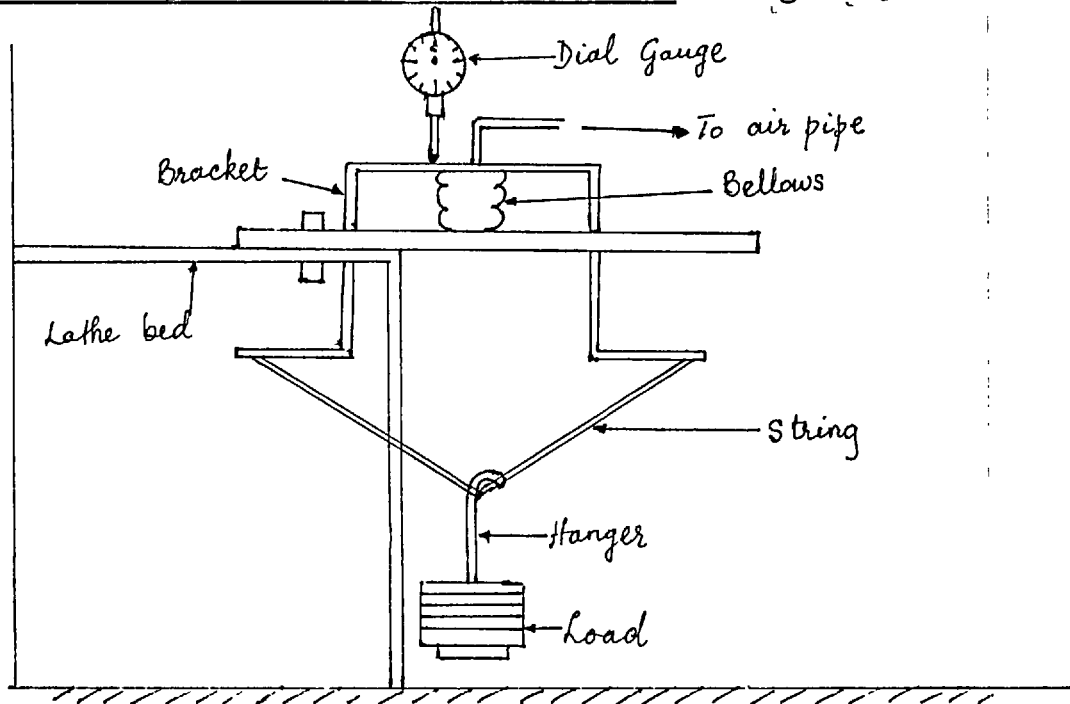


FIG. A.2. Variation of deflection with Load.

This procedure was repeated for Block A even though in the case of Block A the result is not directly comparable or applicable to the main experiment; for the test on A was done excluding the bearings

The Set up for Block A was as shown **Fig. A.3.**



The static deflections were noted with increasing Load as well as decreasing load and the mean taken. The pressure range was zero to 6lb/in^2 on the bellows. The results are shown in fig. A.4.

$$\begin{aligned}
 0 \text{ lbf/in}^2 \text{ No clip} & F = 67.4(x - 21.4x^2 + 467.3x^3) \\
 0 \text{ lbf/in}^2 \text{ with clip} & F = 1193.8(x - 62.744x^2 + 8450.20x^3) \\
 1 \text{ lbf/in}^2 & F = 158.5(x - 2.69x^2 + 47.12x^3) \\
 2 \text{ lbf/in}^2 & F = 134.23(x + 6.66x^2 - 69.03x^3)
 \end{aligned}$$

$$\begin{aligned}
 3 \text{ lbf/in}^2 & F = 197.2(x - 4.90x^2 + 48.99x^3) \\
 4 \text{ lbf/in}^2 & F = 234.6(x - 6.89x^2 + 57.77x^3) \\
 5 \text{ lbf/in}^2 & F = 185.76(x + 8.65x^2 - 140.75x^3) \\
 6 \text{ lbf/in}^2 & F = 167.87(x + 18.00x^2 - 257.46x^3)
 \end{aligned}$$

o — fitted curve
 x — Experimental point

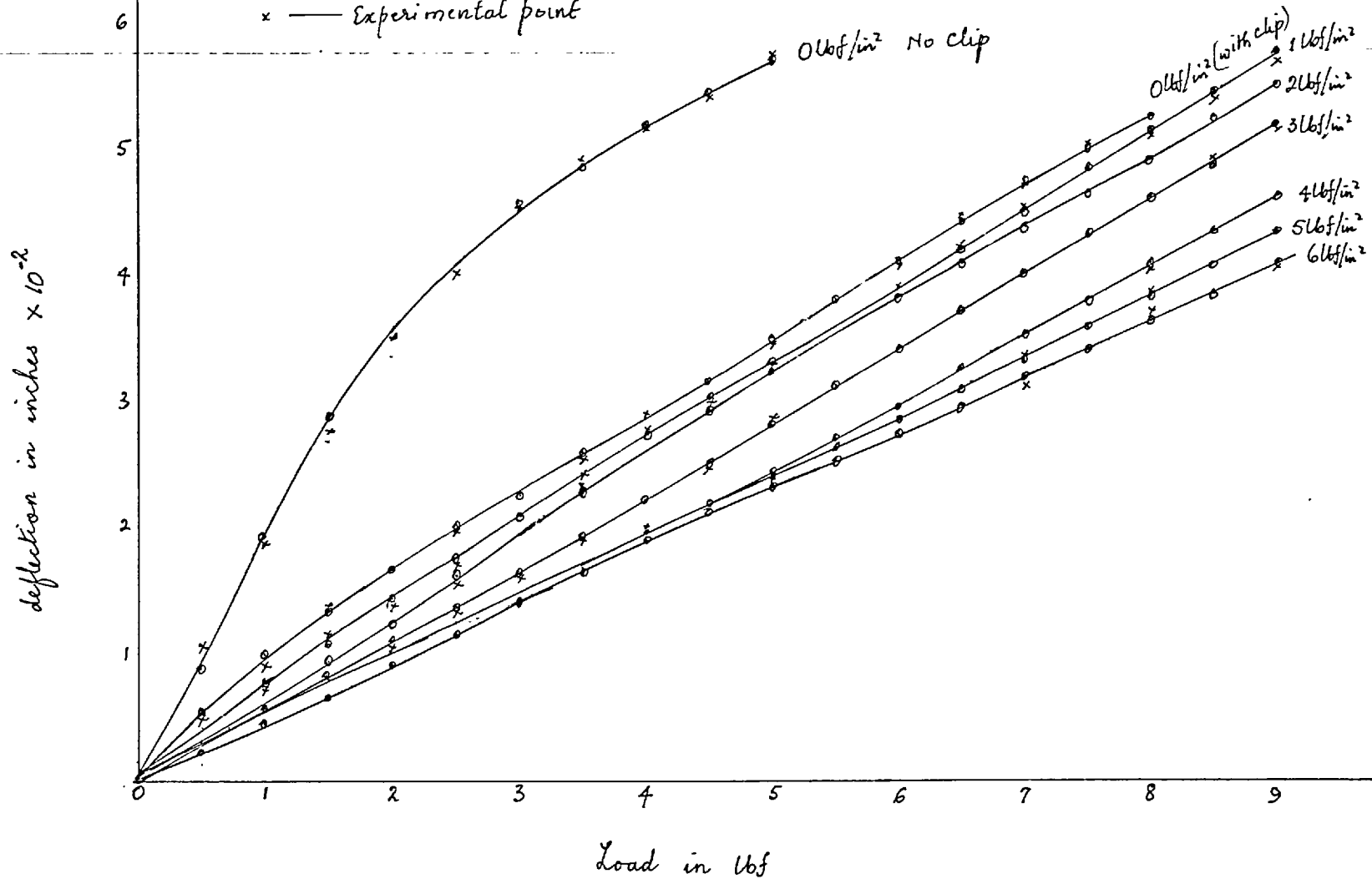
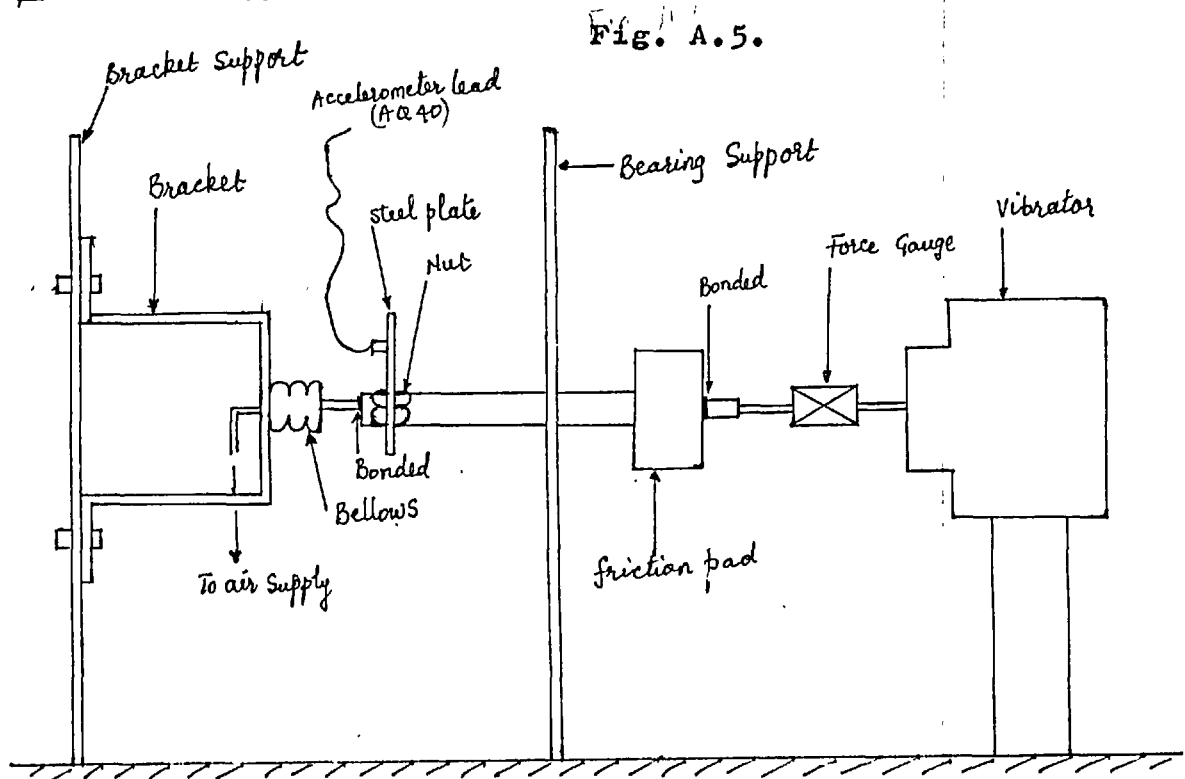


Fig. A.4. Variation of deflection with Load.

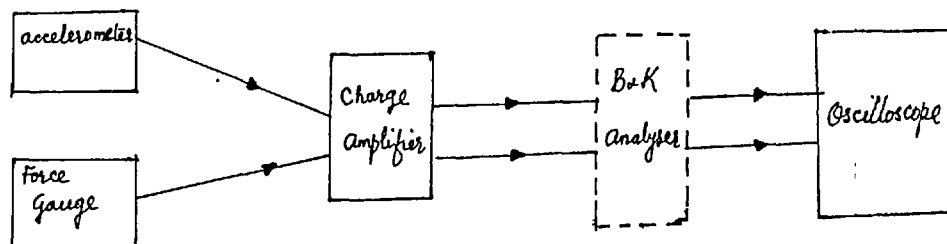
A2. Determination of the Dynamic characteristics of Friction Block System:

This was done by a simple forced vibration frequency response test. Each block was tested separately. The set up was ^{as} shown in fig. A.5 below:



The measuring system was set up as in the previous cases (but omitting the analyser after it was determined that its use did not alter the results).

For Block B the set up was as shown in the block diagram

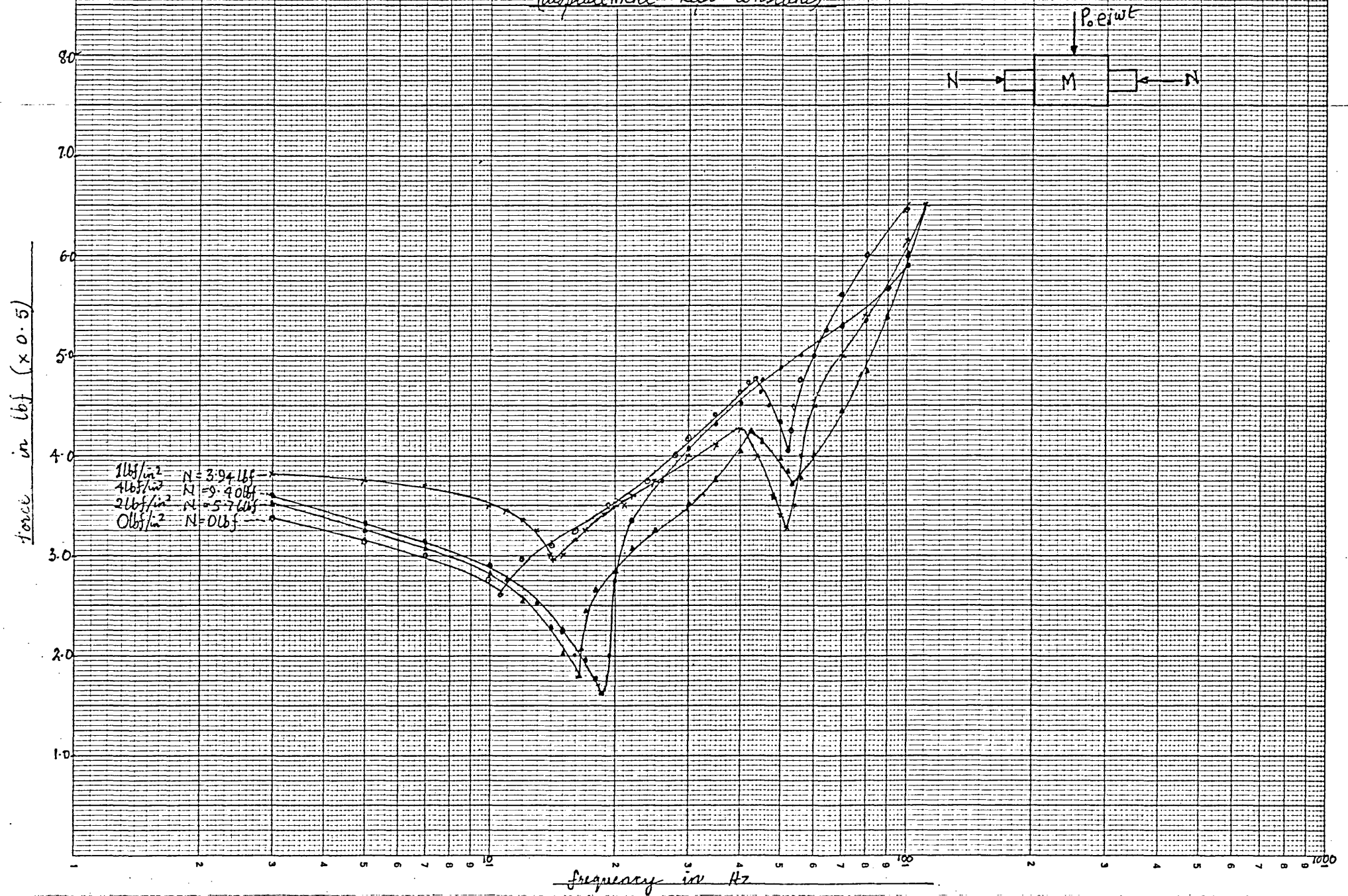


For Block A a Wayne-Kerr displacement gauge (MD 1-76) was used instead of the accelerometer.

In both cases the force required to keep a constant acceleration or displacement was measured at each frequency.

Fig. 1 - Frequency response for Block B for different normal loads N .

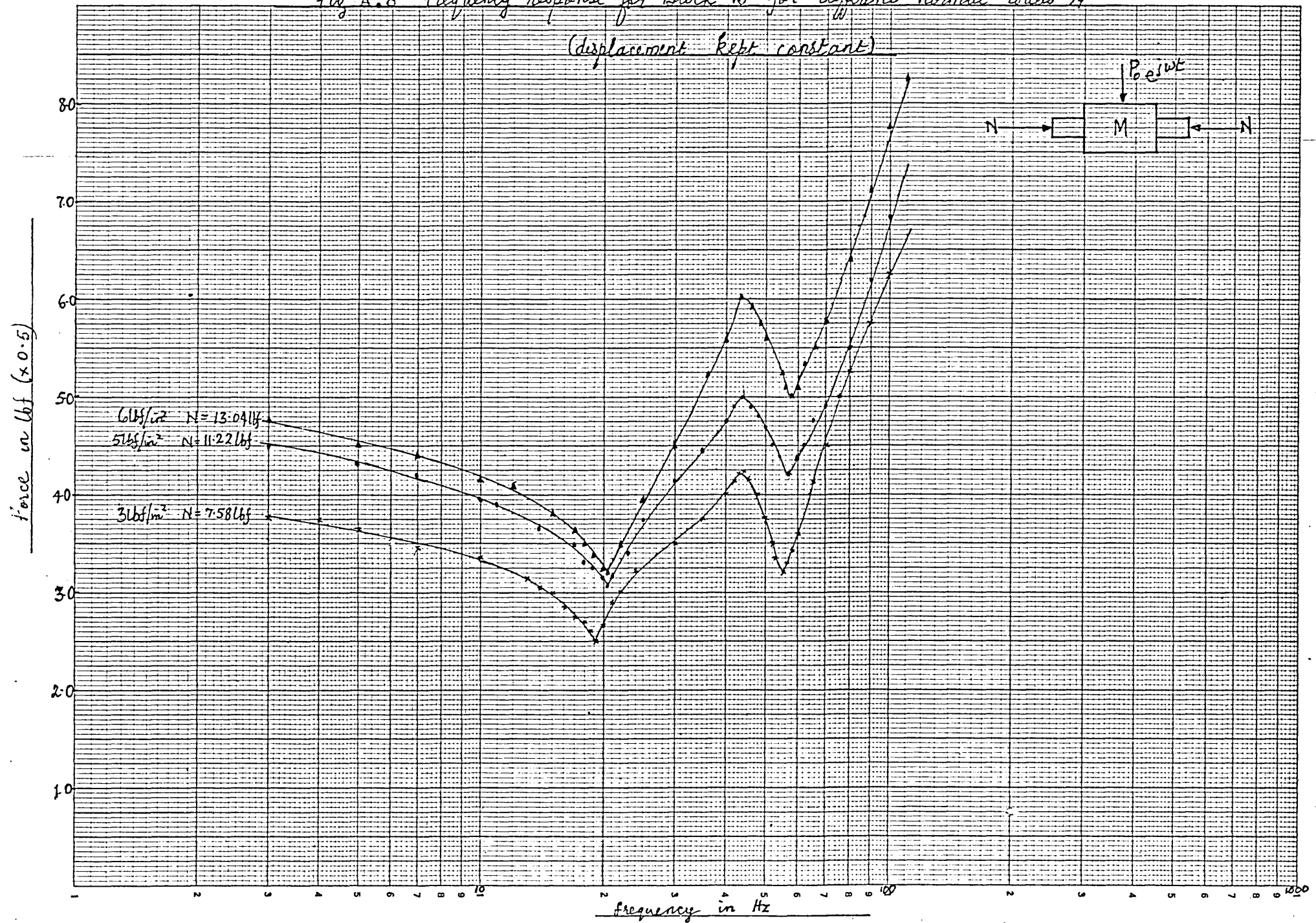
(displacement kept constant)



- 208 - B

Fig. A.8 Frequency response for Block B for different normal loads N

(displacement kept constant)



- 209 -

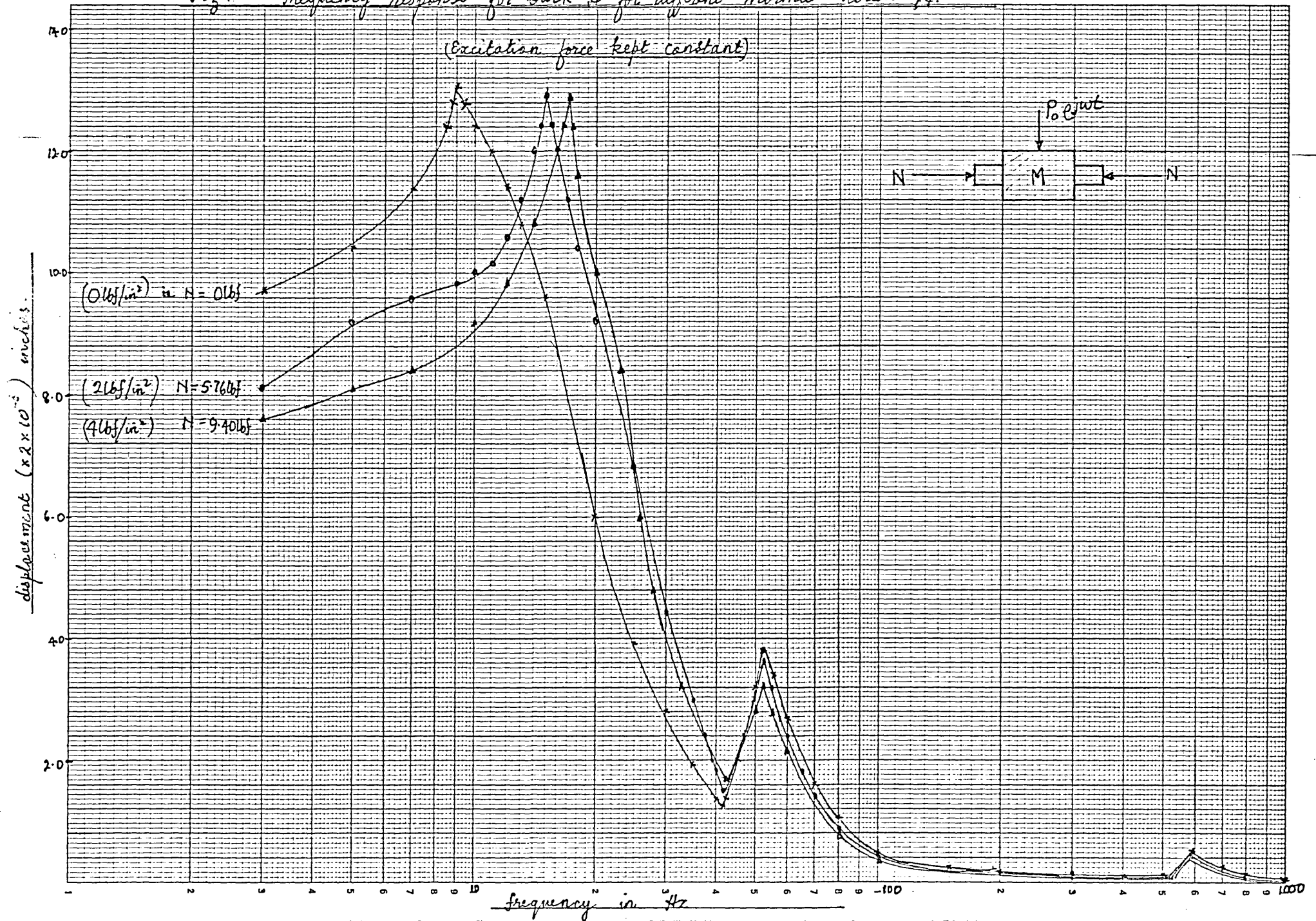
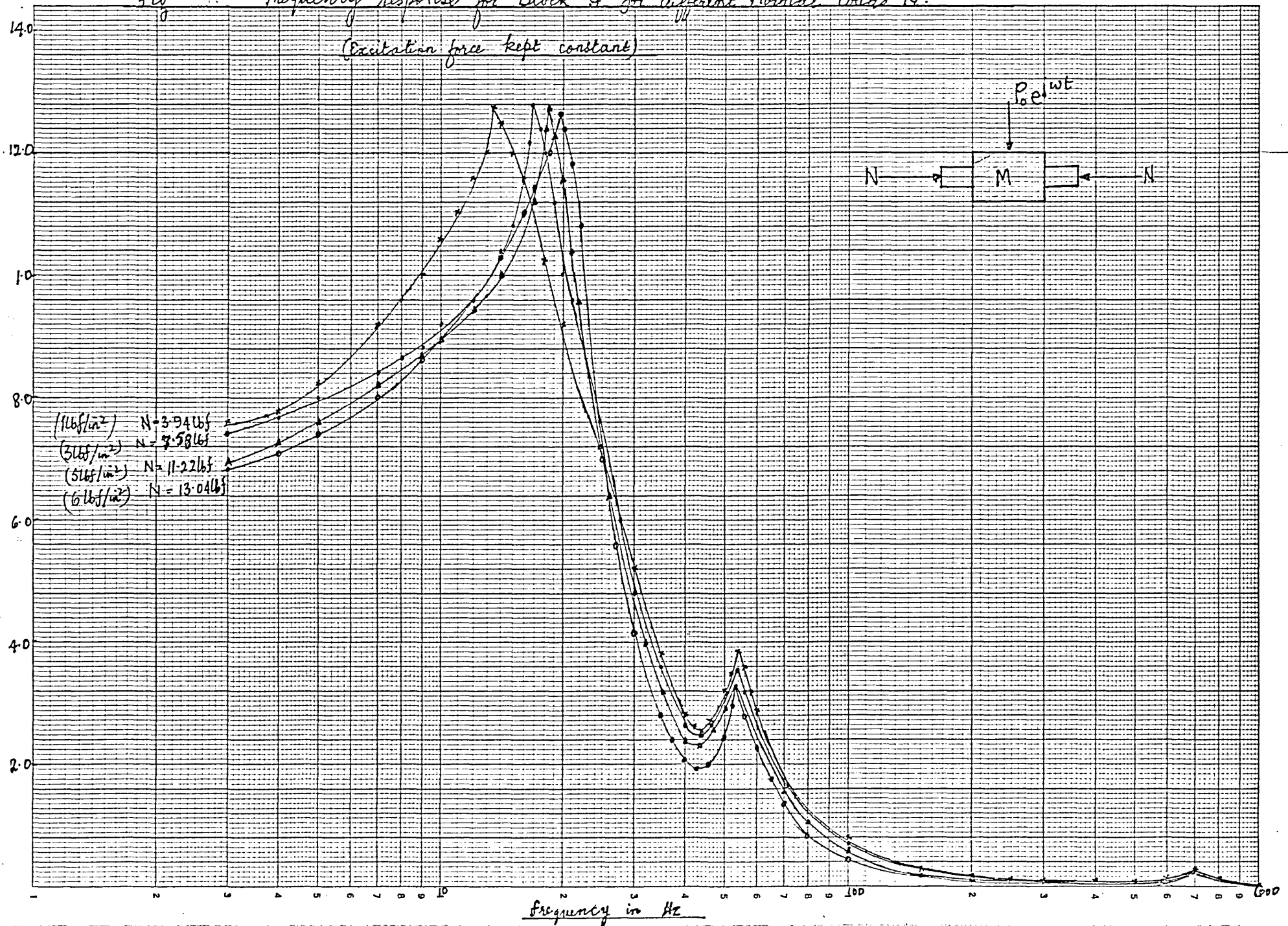
Fig. 1. Frequency response for Block A for different normal loads N .

Fig A.10 Frequency response for Block A for different Normal loads N .

(Excitation force kept constant)

displacement in inches



The total vibrating mass was weighed, and Frequency-Response curves were plotted and the resonant frequencies estimated for zero air pressure on the friction blocks up to 6lb/in² pressures.

Since the vibrating mass was known it was possible to determine the dynamic stiffness of the system at each pressure setting.

Graphs of the pressure against dynamic and spring rate are shown in figs ^{A.9}K_a & ^{A.10}K_b.

Fig. ^{A.9}K_a is clearly non-linear, and by using the least square method using a computer, the law governing the stiffness and pressure was determined.

$$(K = K_0 + 10.23p - 0.713p^2); \quad K_0 = 10.57\text{lb/in}$$

A.3. Calculation of the Dynamic Stiffness

If W = frequency in radians per second

K = stiffness in lb/in

m = mass in lb

g = gravitational acceleration.

$$W^2 = \frac{Kg}{m} \quad K = \frac{W \times m}{g} = \frac{(2\pi)^2 f^2 \times m}{386.2}$$

For Block A mass = 1.105lbs

For Block B mass = 0.820lbs

$$K = \frac{39.3 \times 1.105}{386.2} f^2 \quad \text{or} \quad \frac{31.2}{386} \times f^2 \quad \text{for B}$$

K_A = Stiffness of Block A

K_B = Stiffness of Block B.

K_m = Mean stiffness of Blocks A & B together.

PRESSURE	BLOCK A		BLOCK B		MEAN
	Frequency c/s	K_A lb/in	Frequency c/s	K_B (lb/in)	K_m (lb/in)
0	9.2	9.5	10.6	9.4	9.45
1	13.0	19.0	14.4	17.3	18.18
2	15.0	25.1	16.5	23.2	24.15
3	16.6	30.8	18.0	27.2	29.0
4	17.0	32.5	18.5	28.7	30.6
5	17.6	34.7	19.2	31.0	32.85
6	19.4	42.1	19.5	32.1	37.10

Laws were used to determine the one which fitted the experimental curves best. The Laws were:

- (1) Linear plus square Law i.e. $K(x + bx^2)$
- (2) Linear plus cubic Law i.e. $K(x + cx^3)$
- (3) Linear plus square + Cubic Law i.e. $K(x + bx^2 + cx^3)$

~~The results are shown in figs.....~~

The best fitting Law was Linear plus Square plus Cubic

A.4. Theoretical Response Curves:

Having obtained the Law governing the non-linear spring theoretical response curves were plotted by using a computer. The programme solves an ordinary second order non-linear differential equation with harmonic forcing; the non-linearity being provided by a cubic polynomial in x . Damping may be viscous or hysteretic. The solution takes the form $X = X_0 + X_1 \sin(\omega t - \phi)$ and values of X_0 and frequency ω are computed for a range of values of X_1 .

Since no higher harmonics are included in the solution, the solution is a first approximation to the exact solution. Two types of non-dimensional X may be obtained. Different codes are used for the different spring laws and the appropriate data fed in e.g. for spring Law $F = K(Y + BY_2 + UY_3)$ code number is 1003 and the data fed in is M, K, B, U , and F .

A.5 EXAMINATION OF THE RESTORING FORCES i.e. THE SPRING FORCE

From the computer results it was determined that the restoring force is given by the Law $F = K(x + bx^2 + cx^3)$. As the equation is cubic there should be three values of x i.e. displacement which yield zero restoring force.

Thus:

$$0 = K(x + bx^2 + cx^3) = Kx (1 + bx + cx^2)$$

substituting the values for the constants for example for Zero pressure in block A, the equation becomes

$$58.358x (1 - 21.191x + 337.50x^2) = 0$$

$$x^2 - \frac{21.191}{337.50}x + \frac{1}{337.50} = 0 \quad \text{i.e. } x^2 - 10^{-1}x + \frac{0.63x + 10^{-2}}{0.296} = 0$$

$$x = 1/2 (0.063 \pm \sqrt{(0.063)^2 - 4 \times 10^{-2} \times 0.296})$$

$$= 1/2(0.063 \pm \sqrt{0.004 - 0.01184}) = 1/2 (0.063 \pm \sqrt{-78.4 \times 10^{-4}})$$

$$= 1/2(0.063 \pm j 8.86 \times 10^{-2}) = 0.0315 \pm j 0.0443$$

$$\text{Thus } x = Ae^{(0.0315 + j 0.0443)t} + Be^{(0.0315 + j 0.0443)t}$$

OR

$$x = e^{0.315t} (\underline{\underline{G \cos 0.0443t + D \sin 0.0443t}})$$

$$\text{and } x = 0$$

The solutions for the other cases are shown in the table.

BLOCK A

Pressure (lb/in ²)	Spring Law	Solution
0	$58.358(x-21.19x^2+337.50x^3)$	$Ae^{(0.0315+j0.0443)t}$ $+ Be^{(0.0315-j0.0443)t}$ or $e^{0.0315t}(C\cos 0.0443t + D\sin 0.0443t)$
1	$48.734(x-7.698x^2+45.088x^3)$	$Ae^{(0.086+j0.121)t}$ $+ Be^{(0.086-j0.121)t}$ or $e^{0.086t}(C\cos 0.121t + D\sin 0.121t)$
2	$65.327(x-6.111x^2+23.659x^3)$	$Ae^{(0.125+j1.63)t}$ $+ Be^{(0.125-j1.63)t}$ or $e^{0.125t}(C\cos 1.63t + D\sin 1.63t)$
3	$97.252(x-7.383x^2+25.368x^3)$	$Ae^{(0.146+j0.135)t}$ $+ Be^{(0.146-j0.135)t}$ or $e^{0.146t}(C\cos 0.135t + D\sin 0.135t)$
4	$130.12(x-7.349x^2+23.874x^3)$	$Ae^{(0.155+j0.182)t}$ $+ Be^{(0.155-j0.182)t}$ or $e^{0.155t}(C\cos 0.182t + D\sin 0.182t)$

BLOCK A (cont.)

Pressure (lb/in ²)	Spring Law	Solution
5	$145.06(x - 4.721x^2 + 0.855x^3)$	$(x - 5.25)(x - 0.25)x =$ 5.25 or 0.25
6	$160.48(x - 6.181x^2 + 15.26x^3)$	$\Lambda e^{(0.203 + j0.158)t}$ + $B e^{(0.203 - j0.158)t}$ or $e^{0.203t}(C \cos 0.158t + D \sin 0.158t)$

BLOCK B

Pressure (lb/in ²)	Spring Law	Solution
0	$41.979(x-5.076x^2 + 42.293x^3)$	$Ae^{(0.0595+j0.142)t}$ $+ Be^{(0.0595-j0.142)t}$ or $e^{0.0595t}(C\cos 0.142t - D\sin 0.142t)$
1	$60.77(x-5.374x^2 + 24.378x^3)$	$Ae^{(0.11+j0.17)t}$ $+ Be^{(0.11-j0.17)t}$ or $e^{0.11t}(C\cos 0.17t + D\sin 0.17t)$
2	$109.54(x-8.17x^2 + 31.61x^3)$	$Ae^{(0.129+j0.128)t}$ $+ Be^{(0.129-j0.128)t}$ or $e^{0.129t}(C\cos 0.128t + D\sin 0.128t)$
3	$138.78(x-8.366x^2 + 302.56x^3)$	$Ae^{(0.138+j0.0558)t}$ $+ Be^{(0.138-j0.0558)t}$ or $e^{0.138t}(C\cos 0.0558t + D\sin 0.0558t)$
4	$152.1(x-5.608x^2 + 628.40x^3)$	$Ae^{(0.0046+j0.04)t}$ $+ Be^{(0.0046-j0.04)t}$ or $e^{0.0046t}(C\cos 0.04t + D\sin 0.04t)$

BLOCK B

Pressure (lb/in ²)	Spring Law	Solution
5	$125.7(x - 2.304x^2 + 11.59x^3)$	$(x - 0.21)(x + 0.41)x = 0$ 0.21 or -0.41
6	$143.49(x - 1.805x^2 - 17.969x^3)$	$Ae^{(0.05 + j0.176)t}$ + $Be^{(0.05 - 0.176)t}$ or $e^{0.05t}(C\cos 0.176t + D\sin 0.176t)$

A.6 EXAMINATION OF THE EXPERIMENTAL CURVES

In the experiment the acceleration was kept constant.

$$\text{Thus if } W^2x = K \dots\dots\dots(1)$$

where W = frequency in radians per second

x = displacement in inches

K = constant.

$$-MW^2 = P, \dots\dots\dots(2)$$

with friction

$$\underline{MW^2x \pm |F| = P_2 \dots\dots\dots(3) \text{ where } |F| = Rx(\cos wh + j \sin wh)}$$

At the cross points (a) and (b)

$$P_2 = P, \text{ i.e. } |F| = 0$$

from (1)

$$x = \frac{K}{W^2}$$

At (a) and (b)

$$W_1^2x_1 = W_2^2x_2 = K$$

Thus

$$\frac{W_1^2}{W_2^2} = \frac{x_2}{x_1}$$

Examining the solutions the only real values obtained were
at 51b/in² for both blocks

$$\underline{\text{For Block B: } x_1 = 0.21 ; x_2 = -0.41}$$

$$\therefore \frac{x_2}{x_1} = \frac{-0.41}{0.21} = \left\{ \frac{W_1}{W_2} \right\} \text{ i.e. imaginary}$$

$$\underline{\text{For Block A: } x_1 = 5.25, \text{ and } x_2 = 0.25}$$

$$\text{Thus: } \frac{5.25}{0.25} = \frac{W_2^2}{W_1^2} = 21$$

$$\therefore \frac{W_2^2}{W_1^2} = \underline{\underline{4.6}}$$

From Experimental results f₁ and f₂ are 52.5c/s and 240c/s.

$$\text{Ration } \frac{240}{52.5} = 4.57$$

52.5 This agrees with the expected value.

APPENDIX 2

A.2

DETERMINATION OF DELAY TIME "h" FROM THE MEASURED PHASE ANGLE
WITH ALL 3 TYPES OF DAMPING

In the presence of Coulomb, viscoelastic and viscous damping the phase angle θ is given by equation (3.14).

$$\tan \theta = \frac{2 \zeta n + S \sin wh}{1 - n^2 + S^2 \cos wh}$$

at frequency $w = w_0$, $n = 1$

$$\tan \theta = \frac{2 \zeta n + S \sin wh}{S \cos wh} \quad \text{--- (A2.2) in this equation}$$

ζ , the viscous damping ratio, η , the loss factor, S the friction force parameter and W are known, the only unknown being "h".

Squaring both sides of equation (A2.2) one obtains

$$\tan^2 \theta (S^2 \cos^2 wh) = 4\zeta^2 + 4S \sin wh + 4\zeta\eta + S^2 \sin^2 wh + 2S\eta \sin wh + \eta^2 S^2 \sin^2 wh \sec^2 \theta + 4\zeta + 2\eta) S \sin wh + 4\zeta^2 + \eta^2 - S^2 \tan^2 \theta = 0$$

$$\sin wh = \frac{1}{2S^2 \sec^2 \theta} \left\{ - (4\zeta + 2\eta) S \pm \sqrt{(16\zeta^2 + 16\zeta\eta + 4\eta^2) S^2 - 4S^2 \sec^2 \theta (4\zeta^2 + \eta^2 - S^2 \tan^2 \theta)} \right\}$$

$$\sin wh = \frac{1}{S \sec^2 \theta} \left\{ - 2\zeta + \eta \pm \sqrt{[\tan^2 \theta (S^2 \sec^2 \theta - 4\zeta^2 - \eta^2) + 4\zeta\eta]} \right\} \quad \text{A.2.3}$$

A2.1

WITH COULOMB AND VISCOUS DAMPING ONLY

By putting $\eta = 0$ in equation (A2.3) an expression for $\sin wh$ is obtained for coulomb and viscous damping only.

Thus

$$\begin{aligned} \sin wh &= \frac{1}{S \sec^2 \theta} \left\{ 2\zeta \pm \tan \theta \sqrt{(S^2 \sec^2 \theta - 4\zeta^2)} \right\} \\ &= \frac{1}{S} \left\{ -2\zeta \cos^2 \theta \pm \sin \theta \cos \theta \sqrt{(S^2 \sec^2 \theta - 4\zeta^2)} \right\} \quad \text{A2.4} \end{aligned}$$

A2.2

WITH COULOMB AND VISCO-ELASTIC DAMPING ONLY

In this case $s_c = 0$ therefore by putting $s = 0$ in equation A2.3 the expression becomes.

$$\begin{aligned} \sin wh &= \frac{1}{S \sec^2 \theta} \left\{ -\eta \pm \sqrt{(\tan^2 \theta - (\sec^2 \theta s^2 - \eta^2))} \right\} \\ &= \frac{1}{S} \left\{ -\eta \cos^2 \theta \pm \sin \theta \cos \theta \sqrt{(s^2 \sec^2 \theta - \eta^2)} \right\} \quad (A2.5) \end{aligned}$$

From equation (A2.3), (A2.4) and (A2.5) $\sin wh$ is evaluated and then the angle is obtained in radians and "h" determined.

If the angle = ϕ radians say ----(A2.6) then $hw = \phi$ and $h = \frac{\phi}{w}$

The value of the delay time "h" obtained this way was found to be of the same order as the values obtained by using the Non-linear regression programme. With all the parameters known, theoretical solutions of equations (3.16) and (3.17) were obtained by using the computer.

A.2.3 EXAMPLE

WITH VISCOELASTIC DAMPING ONLY

$$K = 408, \eta = 0.3456 \text{ and } S = 0.196 : \frac{1}{S} = 5.1$$

$$\begin{aligned} \sin wh &= \frac{1}{S} \left\{ -\eta \cos^2 \theta \pm \sin \theta \cos \theta \sqrt{(S^2 \sec^2 \theta - \eta^2)} \right\} \\ &= 5.1 \left\{ -0.3456 \cos^2 \theta \pm \sin \theta \cos \theta \sqrt{(0.196^2 \sec^2 \theta - 0.3456^2)} \right\} \\ &= 5.1 \left\{ -0.00263 \pm 0.086 \times 0.0382 \times 130 - 0.120 \right\} \\ &= 5.1 \left\{ -0.00265 \pm 0.086 \times 2.2 \right\} = 5.1 \left\{ 0.00265 \pm 0.189 \right\} \\ &= 5.1 \left\{ -0.19165 \text{ or } + 0.18635 \right\} = 0.978 \text{ or } 0.95 \\ &= 78^\circ \qquad 72^\circ \end{aligned}$$

$$wh = 72^\circ = \frac{72 \times \pi}{80} \text{ rads.}$$

$$h = \frac{72^\circ}{180 \times 2f} = 0.018 \text{ For } 78^\circ$$

and 0.015 for 72°