

Evaluating the Nakajima et al. model for rectangular-aperture screens

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ABSTRACT

Nakajima and co-workers (1978) developed a method to measure the shape distribution of particles in terms of their size and thickness. This method was later extended to include models for square, rectangular, and circular aperture screens. In this project, the empirical equation for the probability of particles passing through the rectangular aperture screen in the Nakajima et al. model is replaced with a “Basic Probability” model and the results are compared. Next, regression equations relating the efficiency parameter of the model to selected screening operating variables are developed. Finally a “modified” Hatch and Mular model is introduced in the Nakajima et al. model and the results are compared with the experimental data. It is shown that both the new models fit the experimental results obtained from rectangular-aperture screens better than the original Nakajima et al. model.

Keywords: Screening; Particle size; Modelling

INTRODUCTION

A method for measuring the shape distribution of particles compatible with the common technique of size measurement with square-aperture sieves was developed by Nakajima et al. (1978). By measuring the particle dimensions and plotting them on a diagram with thickness and width as co-ordinates, they showed that particles can be divided into classes defined by their size and thickness. Figure 1 shows the typical size and thickness boundaries that are produced.

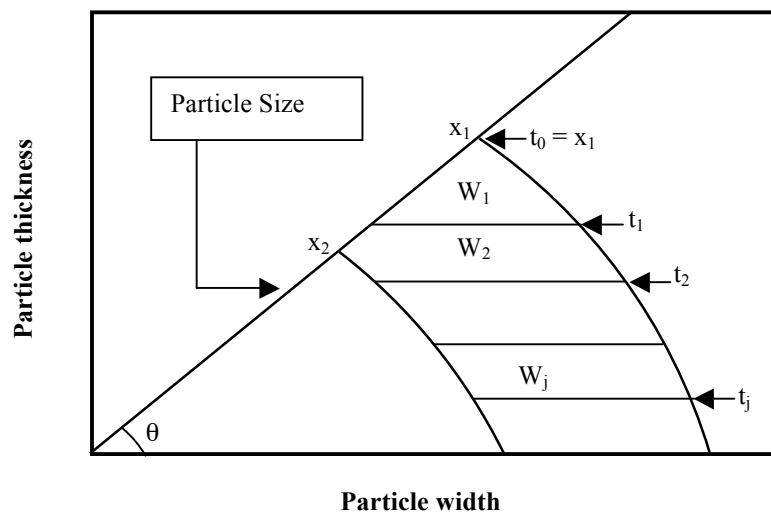


Fig. 1. Particle shape zones subdivided by square- and rectangular-aperture sieves (Nakajima et al., 1979)

The circular arcs are the loci of data obtained with square-aperture sieves and the horizontal lines are obtained with rectangular-aperture sieves (Nakajima et al., 1978). Changing to polar co-ordinates, the particle size (x) becomes the radius co-ordinate, which is defined as:

$$x = \sqrt{(t^2 + w^2)}/2 \quad (1)$$

where, t is the particle thickness and w is the width of the particle, and θ is the angle, which, by definition of w and t , must be between 0 and $\pi/4$ and is given by:

$$\theta = \tan^{-1}(t/w) = \sin^{-1}(t/\sqrt{2}x) \quad (2)$$

The weights of particles contained within the zones depicted in Fig. 1 are denoted by W_j , which contains the information on the shape distribution of the particles in this size range and is expressed by (Nakajima et al., 1978):

$$W_j = W_t \int_{x_2^*}^{x_1} f_x \int_{\theta_j}^{\theta_{j-1}} \bar{f}_s d\theta dx \quad (3)$$

where, W_j is the weight of particles in the j^{th} zone

W_t is the total weight of particles

x_1 and x_2^* are the maximum and minimum particle sizes in the j^{th} zone

θ_{j-1} and θ_j , are functions of the particle size, x , defined by the following equations :

$$\theta_{j-1} = (\sin^{-1}(t_{j-1}/\sqrt{2}x) ; \pi/4)_{\min} \quad (4)$$

$$\theta_j = \sin^{-1}(t_j/\sqrt{2}x) \quad (5)$$

t_j is the smallest slot width of the j^{th} zone .

f_x : is the size density distribution

$\bar{f}_s$: is the shape density function defined in terms of the shape factor θ and averaged over the size range.

Nakajima and co-workers (1978) found that the following equation gave a good approximation to the observed shape distribution:

$$f_s(\theta) = (\alpha+1)(\beta+1)(4/\pi)(4\theta/\pi)^\alpha \left(1 - (4\theta/\pi)^{(\alpha+1)}\right)^\beta \quad (6)$$

where α and β are the parameters of the distribution.

In a subsequent paper, Nakajima and co-workers (1979) extended their model to include the effects of the shape distribution of particles, since this factor is known to have significant effects on the separation efficiencies of a screen. In preliminary experiments they showed that the probability of a particle passing through an aperture is dependent only on the particle thickness, t , and its width, w , provided the particle length, l , is less than twice t or w . The recovery of oversize particles in each zone is obtained by introducing an efficiency term, $E(x)$, into Eq. (3), i.e.

$$W_{oj} = W_t \int_{x_2^*}^{x_1} f_x \int_{\theta_j}^{\theta_{j-1}} E(x) \bar{f}_s d\theta dx \quad (7)$$

where

W_{oj} is the weight of particles recovered in the oversize product in the j^{th} zone,

$E(x)$ is screen efficiency (the fraction recovery of particles into the oversize product)

The equation for the screen efficiency, $E(x)$, used by Nakajima and co-workers followed that of Whiten et al. (1977), who showed that the fractional recovery into the coarse product of particles with probability P of passage is :

$$E(x) = \exp(-NP) \quad (8)$$

where, N is the number of trials for the particular operating conditions of the screen. The probability P varies with particle size and is easily calculated for spherical particles (Gaudin, 1939), but it was expressed in terms of the particle width, w , and thickness, t , to obtain a more general model. A single value of N is then calculated to give the best least square fit to all fractions (size and shape) of the screen products (Nakajima et al., 1979).

According to the model of Whiten et al (1977), P for a square aperture of size D is given by:

$$P = f_0 \left(1 - \frac{x}{D}\right)^2 \quad x \leq D \quad (9)$$

$$P = 0 \quad x > D$$

where, f_0 is the fraction open area of the screen cloth
 x is the particle size

Nakajima et al. (1979) showed that this definition for P adequately described the size distributions of the discharges from square-aperture screens, but there were systematic discrepancies for slotted-sieve data, where the model gave values slightly too small for the top subdivided zone in each size interval. In order to correct these discrepancies, Nakajima and co-workers used an extra term that was a function of θ . They found that the exponential term in Eq. (8) fitted their data closely when it took the form:

$$P = \cos^4(\theta - \pi/8) f_0 \left(1 - \frac{x}{D}\right)^2 \quad x \leq D \quad (10)$$

$$P = 0 \quad x > D$$

The empirical function of particle shape, $\cos^4(\theta - \pi/8)$, was introduced to account for the larger number of trials experienced by flatter particles, apparently because of their slower movement down the screen (Nakajima et al., 1979).

It was found that the Whiten et al. model (1977) was not capable of describing the behaviour of non-spherical particles on slotted screens and a totally empirical probability function was proposed for this situation by Nakajima et al. (1979):

$$P = \cos^4(\theta - \pi/8) f_0 \{1 - (x/D_0)((D_1/D_2)^2 \sin^2 \theta + \cos^2 \theta)^{1/2}\}^2 \quad (11)$$

$$P = 0 \quad \text{for } x > D_0 / \{(D_1/D_2)^2 \sin^2 \theta + \cos^2 \theta\}^{1/2}$$

$$\text{where } D_0 = \sqrt{(D_1^2 + D_2^2)}$$

D_1 = longer side of the rectangular aperture

D_2 = shorter side of the rectangular aperture

OBJECTIVES

The work described in the present paper was carried out to test and extend the theory of shape discrimination by screening outlined above. This work had three objectives:

- I. to develop a simpler theoretical equation for the recovery of non-spherical particles in the oversize on a rectangular-aperture screen to replace the empirical expression used by Nakajima et al. (Eq. (11)).
- II. to examine the effect of practical screen operating variables on the shape distribution of screen products. This was done by developing a regression equation that described the variation of the parameter N (Eq. (8)) with the screen operational variables.
- III. to extend the screening model to elongated particles. The Nakajima et al. model (1979) is restricted to particles whose length is less than twice their thickness or width and hence is effectively a two-dimensional model. An attempt is made here to develop a three-dimensional model to account for the behaviour of elongated particles.

EXPERIMENTAL

A sample, 205 kg, of crushed limestone aggregate of less than 20 mm in size, produced by a cone crusher, was provided. The sample was subjected to size analysis using square-aperture sieves of 19, 14, 9.5, 6.7, 4.75 and 3 mm. The cumulative oversize distribution of the sample was approximated by a Rosin-Rammler distribution of:

$$W_r = 100 \exp \left[- \left(\frac{x}{11} \right)^{3.8} \right] \quad (12)$$

where, W_r is the weight percent retained and x is the size (Nakajima et al., 1978).

The size density distribution of the sample (f_x) was obtained by differentiating Eq. (12). The size range of 9.5 to 6.7 mm, 58.5 kg, was used as the screen feed in this experiment.

A sample of this material was sieved on the slotted-sieves of 8, 7.2, 6, 4.9, 4, 3.55 and 3 mm width. The best fit values of α and β of Eq. (3) fitted to these sieving results were 2.47 and 2.27 respectively.

A small-scale industrial-type "Mogensen" vibrating screen was used for the screening experiments. The screen was 0.45m wide, 0.90m long and vibrated with an in-flow circular motion. The screen feed rate could be controlled by means of a hopper and a vibrating feeder. The frequency of vibration of the screen could be varied between 0 and 1440 rpm by a controller, and the amplitude of vibration between 0 to 4 mm by changing the position of two balance weights. The screen surface inclination could be varied manually between 0° and 30°. The screens used were perforated steel plates with rectangular apertures of 6×20, 7×22 and 8×25 mm. The rectangular apertures were oriented perpendicular to the flow of material.

Appropriate combinations of five independent operational variables, each at three different levels were studied (Table 1). The experimental design used was a central composite design, consisting of a 2^{5-1} fractional factorial design, 10 axial runs and 5 centre point replications (31 experiments in total). The variables and their values are shown in Table 1.

The screen oversize from each experiment was sieved through slotted apertures of 7.2, 4.9 and 3.55 mm aperture width. The experimental design, the oversize recovery and the slotted-sieve data of screen oversize for each experiment are shown in Table 3 (Appendix).

Table 1 Values of the operational screening variables used in the experiments

Variable	Name	Levels		
		Low	Medium	High
		–	0	+
Fe	Feed rate (tph)	9.0	13.5	18.0
In	Inclination (degree)	10.0	12.5	15.0
Fr	Frequency of vibration (rpm)	1000	1220	1440
Am	Amplitude of vibration (mm)	2.4	3.2	4.0
Aw	Aperture width (mm)	6.0	7.0	8.0

RESULTS / DISCUSSION

The experimental data were fitted to Eq. (7) to obtain values for the screen efficiency parameter N (Eq. (8)). The fit of these data to the Nakajima et al. model for non-spherical particles on slotted screens was examined by using the empirical probability function (Eq. (11)) to calculate values of P . From these, values for N were obtained for each of the 31 data sets. The analysis of variance for all the data sets showed that the Nakajima et al. model (1979) was significant at the 5% level. The “adjusted R^2 statistics (R^2_A) of the model, averaged over all data sets, was 0.965.

In an attempt to improve the modelling results and to develop a slotted screen model compatible with the Nakajima et al. model (1979) for square-aperture screens, the empirical probability function for the recovery of particles in the screen oversize (Eq. (11)) was replaced with a conventional probability model for rectangular-aperture screens originally developed by Gaudin (1939):

$$P = (D_1 - x)(D_2 - x) / ((D_1 + d)(D_2 + d)) \quad (13)$$

where D_1 is the length of the aperture, D_2 is the width of the aperture, x is the particle size and d is the screen wire diameter. Using the fraction open area definition, f_0 , (Whiten et al., 1977), Eq. (13) becomes:

$$P = f_0 (D_1 - x)(D_2 - x) / (D_1 D_2) \quad (14)$$

Assuming that irregularly shaped particles arrive on a line parallel to the screen surface, with their greatest cross section presented to the apertures, the probability of passing through a rectangular opening perpendicular to the flow of material may be defined as:

$$\begin{aligned} P &= f_0 (D_1 - w)(D_2 - t) / (D_1 D_2) & t < D_2 \\ P &= 0 & t \geq D_2 \end{aligned} \quad (15)$$

where w is the particle width and t is the particle thickness as defined in the Nakajima et al. model (1978). The length of the screen opening is more than three times its width, so if the thickness of a particle is less than the aperture width it has a chance to pass through,

otherwise the probability of passing is zero. Substituting for w and t from Eq. (2) into Eq. (15) gives:

$$P = f_0 (D_1 - \sqrt{2} x \cos \theta)(D_2 - \sqrt{2} x \sin \theta) / (D_1 D_2) \quad (16)$$

Equation (16) was introduced into the Nakajima et al. model (1979) in place of Eq. (11) and was fitted to the experimental data. The “adjusted R^2 statistic” (R^2_A) of this “Probability” model, averaged over 31 data sets, was 0.985 and thus the “Probability” model is statistically more significant than the original Nakajima et al. model (1979). Comparison of the two models, averaged over the 31 data sets, is shown in Table 2. The proposed “Probability” model was simple and produced results closer to the experimental data compared to the original Nakajima et al. model (1979).

A full second-order polynomial model (Eq. (17)) relating N to the operational variables of the vibrating screen (see Table 1), was fitted to the N 's obtained from 31 sets of data. The R -square (R^2) of the model was 0.98.

$$N = 45.7 + 0.73 \text{ Fe} - 1.6 \text{ In} - 0.02 \text{ Fr} - 1.2 \text{ Am} - 5.6 \text{ Aw} + 0.03 \text{ Fe}^2 - 0.04 \text{ Fe*In} + 0.0001 \text{ Fe*Fr} + 0.05 \text{ Fe*Am} - 0.2 \text{ Fe*Aw} - 0.004 \text{ In}^2 + 0.001 \text{ In*Fr} + 0.2 \text{ In*Am} - 0.12 \text{ In*Aw} + 0.0001 \text{ Fr*Aw} + 0.8 \text{ Am*Am} - 1.1 \text{ Am*Aw} + 1.2 \text{ Aw*Aw} \quad (17)$$

Nakajima and co-workers (1979) had shown that the probability of a particle passing through a square aperture sieve is largely independent of its length, unless it is exceptionally elongated. To overcome this shortcoming of the Nakajima et al. model to represent particle recovery in three-dimensional space, an attempt was made to include particle elongation by introducing an efficiency function, originally proposed by Hatch and Mular (1979), into the model.

The dimensions of some 750 aggregate particles in the size range of 6.7 – 9.5mm were measured with callipers. It was assumed that the mass of each particle was proportional to the product of its dimensions. The size of each particle, x , was obtained (Eq. (1)) and the ratio of length, l , to the size (elongation) was calculated for each particle. A Rosin-Rammler equation was fitted to these data:

$$R(E) = \exp(-(El/1.8)^7) \quad (18)$$

where $R(E)$ is the cumulative fraction larger than El and

El is the elongation of each particle:

$$El = l / x \quad (19)$$

The “elongation density distribution” of the sample, obtained by differentiating Eq. (18), was introduced into Eq. (7):

$$W_{oj} = W_t \int_{x_2^*}^{x_1} f_x \int_{El_2}^{El_1} f_l \int_{\theta_j}^{\theta_{j-1}} E(x) \bar{f}_s d\theta d(El) dx \quad (20)$$

where,

W_{oj} is the weight of particles recovered in the oversize product in the j^{th} zone,

El is the particle elongation,

f_l is the elongation density distribution of a size fraction

The formulation of the Hatch and Mular model (1979) was initially in terms of the number of possible combinations in which a particle could pass through a screen aperture and then the efficiency curve equation was derived from that (Hess, 1983):

$$E_i = 1/[1 + \exp\{b(V_{50} - V_i)\}] \quad (21)$$

where E_i is the weight fraction of the feed reporting to the oversize product

V_i is the average volume of particle in the i^{th} size fraction

V_{50} the particle volume at which $E_i = 0.5$

b is a constant

Assuming that the shape factor of the particles is constant and that the particle volume is proportional to the cube of the geometric mean particle size, Eq. (21) becomes:

$$E_i = 1/(1 + \exp((x_{50}^3 - x_i^3)/k)) \quad (22)$$

where x_i is the geometric mean particle size

x_{50} is the particle size at which the $E_i = 0.5$

k is a constant

The parameters x_{50} and k were found to depend on the operational screening variables (Hatch and Mular, 1979).

The assumption made by Hatch and Mular that the shape factors of the particles is constant and that the particle volume is proportional to the cube of the geometric mean particle size, which allows the use of Eq. (22) instead of Eq. (21), is not always valid as the particles in a size fraction may have a wide range of different shapes. The volume of each particle in a size range, (V_i), may be better represented by the product of the particle length, (l), width (w), and thickness, (t):

$$E_i = 1/(1 + \exp((x_{50}^3 - (lwt))/k)) \quad (23)$$

Substituting l from Eq. (19) and w and t from Eq. (2) into Eq. (23), the fractional recovery of particles of size x becomes:

$$E(x) = 1/(1 + \exp((x_{50}^3 - (El.x\sqrt{2}.x.\cos\theta\sqrt{2}.x.\sin\theta))/k)) \quad (24)$$

Equation (24) was introduced into the Nakajima et al. model (1979) in place of Eq. (11) and was fitted to the experimental data. This modification gave too small a value for the top zone (+7.2 mm) in the size intervals investigated. In a manner similar to Nakajima et al. (1979) a parameter, θ , was introduced into the model (Eq. (24)):

$$E(x) = 1/(1 + \exp(\theta(x_{50}^3 - (El.x\sqrt{2}.x.\cos\theta\sqrt{2}.x.\sin\theta))/k)) \quad (25)$$

The result of fitting this model (Eq. (25)) is shown in Table 2 under the heading, "revised" Hatch and Mular model. The "adjusted R^2 statistic" (R_A^2) of this model, averaged over 31 data sets, was 0.987, which is statistically more significant than the original Nakajima et al. model (1979). The results of fitting the Nakajima et al. model, the "Probability" model and the "revised" Hatch and Mular model, averaged over the 31 experimental data sets for

the rectangular aperture screens is shown Table 2. The last row of Table 2, shows “the root mean square error”, ε , which is defined by:

$$\varepsilon = \sqrt{\sum (W_j^* - W_j)^2 / Z / \sum W_j^*} \quad (26)$$

where W_j^* and W_j are respectively, the experimental and the best fit values from the models for the weight of particles in the j^{th} zone and Z is the total number of zones in the size range (Nakajima et al., 1978). Using this criterion, the “revised” Hatch and Mular model gives the closest results to the experimental data. Therefore by introducing particle elongation and a “revised” Hatch and Mular efficiency function, a better model was produced.

Table 2 Comparison of the results of fitting the Nakajima et al. model, the “Probability” model and the “revised” Hatch&Mular model for rectangular-aperture screens

Slot intervals (mm)	Feed (kg)	Recovery to oversize (kg)			
		Experimental	Nakajima et al. model	Probability model	Revised Hatch &Mular model
+7.2	6.13	5.29	5.73	5.77	5.14
-7.2 +4.9	29.66	21.71	24.14	24.05	22.44
-4.9 +3.6	19.66	9.08	6.44	7.76	7.67
-3.6	3.04	0.94	1.77	3.02	2.08
<i>The root mean square error*</i>			<i>0.050</i>	<i>0.046</i>	<i>0.026</i>

*see Eq. (26)

Two second-order polynomial models, in the light of the equations proposed by Hatch and Mular (1979), relating parameters x_{50} and k to the operational screening variables (see Table 1) were produced. Both regression models were statistically significant and their corresponding R-square (R^2) values were 0.91 and 0.96, respectively.

$$x_{50} = 35.19 + 1.62 \ln -10.44 Aw - 0.004 Fe^2 + 0.0003 \ln^* Fr + 0.13 \ln^* Am - 0.34 \ln^* Aw - 0.000002 Fr^2 + 0.54 Am^2 - 0.75 Am^* Aw + 1.24 Aw^2 \quad (27)$$

$$k = -226.2 - 1.03 Fe^2 + 5.66 Fe^* Aw + 3.71 \ln^2 - 0.09 \ln^* Fr + 0.31 Fr^* Am - 60.6 Aw^* Am + 18.56 Aw^2 \quad (28)$$

CONCLUSION

Nakajima et al. (1979) extended the screening efficiency averaging technique to include the effect of the shape distribution of particles, but they did not include screening operating variables in their model and they used an empirical function to describe the fraction recovery of particles in the screen oversize for rectangular-aperture screens. In this paper, a probability equation, using the width and thickness of particles, was used in place of the empirical function. Comparison of the averaged “adjusted R^2 statistic” values showed that the “Probability” model was statistically more significant than the original Nakajima et al. model. This model is simple and is compatible with the Nakajima et al. model (1979) for square-aperture screen. A regression function of operating variables was derived for rectangular-aperture screens, which enables the user to predict the efficiency parameter of the model, N , and the shape distribution of the screen oversize. The interactions of variables are also shown to be significant, which have to be considered when locating the optimum operating conditions.

To investigate the effect of particle length, a “revised” Hatch and Mular model (1979) was incorporated in the Nakajima et al. model (1979). Comparison of the averaged “adjusted R^2 statistic” values of the “revised” Hatch and Mular model with the original Nakajima et al. model for rectangular aperture screens showed that this model was statistically more significant and gave a closer reproduction of the experimental data. The two parameters of the “revised” Hatch and Mular model were fitted to the screening variables with regression equations.

APPENDIX

Table 3 The experimental design, the oversize recovery and the slotted-sieve data

Run	Feed rate* (tph)	Inclination (degree)*	Frequency (rpm)*	Amplitude (mm)*	Aperture width* (mm)	Oversize Recovery (%)	Weight % retained on the slotted-sieves				
							+ 7.2 mm	-7.2 + 4.9 mm	- 4.9 +3.55 mm	- 3.55 mm	
1	-	-	-	-	+	10.14	47.47	41.14	10.40	0.99	
2	+	-	-	-	-	75.00	13.95	59.33	24.06	2.66	
3	-	+	-	-	-	85.29	11.60	56.63	28.11	3.66	
4	+	+	-	-	+	49.28	16.28	53.45	27.83	2.43	
5	-	-	+	-	-	82.86	10.89	57.78	27.60	3.74	
6	+	-	+	-	+	38.24	19.98	51.89	25.36	2.77	
7	-	+	+	-	+	20.00	25.32	61.54	12.18	0.96	
8	+	+	+	-	-	85.29	9.89	56.44	30.58	3.08	
9	-	-	-	+	-	82.61	11.50	58.98	26.26	3.25	
10	+	-	-	+	+	45.59	18.85	59.40	19.64	2.10	
11	-	+	-	+	+	47.06	20.74	60.34	17.24	1.86	
12	+	+	-	+	-	89.71	10.79	55.05	30.94	3.67	
13	-	-	+	+	+	50.00	18.87	59.83	19.33	1.98	
14	+	-	+	+	-	91.04	12.19	54.51	30.04	3.27	
15	-	+	+	+	-	92.54	10.81	54.95	30.78	3.46	
16	+	+	+	+	+	70.59	12.44	56.29	28.27	2.55	
17	0	0	0	0	0	76.50	12.12	59.58	25.23	3.07	
18	0	0	0	0	0	73.53	12.75	60.00	24.93	2.32	
19	0	0	0	0	0	74.63	11.51	59.59	26.03	2.88	
20	0	0	0	0	0	75.00	13.05	56.62	27.97	2.36	
21	0	0	0	0	0	76.12	11.59	58.63	26.85	2.93	
22	-	0	0	0	0	67.65	12.92	61.85	21.72	3.51	
23	+	0	0	0	0	76.80	12.13	56.94	27.62	3.31	
24	0	-	0	0	0	71.01	13.30	55.69	27.77	3.24	
25	0	+	0	0	0	76.81	12.67	57.87	26.45	3.01	
26	0	0	-	0	0	70.59	15.51	58.51	23.38	2.60	
27	0	0	+	0	0	76.47	10.74	58.37	28.49	2.39	
28	0	0	0	-	0	69.12	13.73	61.05	22.53	2.69	
29	0	0	0	+	0	76.95	12.85	60.42	24.11	2.62	
30	0	0	0	0	-	87.14	11.39	56.43	28.35	3.82	
31	0	0	0	0	+	47.83	18.50	56.43	22.88	2.19	

*See Table 1

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