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QUANTUM DYNAMICS OF SMALL
NUMBERS OF IONS IN A PENNING TRAP

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Declaraion of originality

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Abstract

Laser-cooled trapped ions are one of the main systems used in experiments in the fields of quantum optics and quantum information. There are two commonly-used types of ion trap, the radiofrequency trap and the Penning trap. In general, the RF trap has been more widely used in quantum information experiments, meaning that the set of experimental tools for conducting these types of experiments in a Penning trap is currently less complete. The aim of the work presented in this thesis is to increase this array of tools.

Our experiment uses $^{40}\text{Ca}^+$ ions in a Penning trap alongside lasers. Spectroscopy has been performed on single Doppler cooled ions, and on ions that have been prepared close to their ground state of motion using a multiple-stage sideband cooling technique resulting in an average phonon occupation number of $\bar{n}_z = 0.029 \pm 0.011$ from an initial $\bar{n}_z \approx 24$. This corresponds to a quantum ground state population of 97%, which is close to the sideband cooling limit in our system. An axial heating rate measurement of $\dot{\bar{n}}_z = 0.56 \pm 0.52 \text{ s}^{-1}$ has also been obtained, which is the lowest recorded to date, due in part to the large dimensions of our trap. Additionally improved control of the Penning trap radial motion has been demonstrated and the average phonon occupation number of the modified cyclotron motion has been reduced from $\bar{n}_+ \approx 150$ after Doppler cooling to $\bar{n}_+ = 0.6$ after sideband cooling. Some preliminary single-ion coherent operations have also been performed.

In the future we hope that this system of ground state cooled ions in a Penning trap can be used to generate highly-entangled states in two or more ions, with a view to performing simple quantum error-correcting codes and eventually an analogue quantum simulation of spin systems.

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Chapter 1

Introduction

1.1 Historical background

1.1.1 Development of the ion trap - two approaches

Since the advent of the theory of quantum mechanics in the first half of the 20th century, the prospect that a quantum system could potentially be studied in isolation from its environment has been an attractive one. This was demonstrated by Erwin Schrödinger in 1952 when he said

[We] never experiment with just one electron or atom or (small) molecule. In thought-experiments we sometimes assume that we do; this invariably entails ridiculous consequences... It is fair to state that we are not experimenting with single particles, any more than we can raise Ichthyosauria in the zoo. [1]

when discussing the existence of quantum jumps and the use of techniques such as studying cloud chamber tracks as verifications of quantum phenomena. By repeated measurement of a single quantum system we are often able to discover more about the underlying workings of the laws of physics than by performing experiments on an ensemble system.

An atomic ion is a good candidate to attempt to perform these kinds of experiments as their internal states can be manipulated with a high level of control, and their electric charge provides an easy way to apply forces to the particle to hold it in place. The problem is that despite this strong Coulomb interaction, the laws of electrostatics do not allow us to design a set of electrodes such that we can confine a charged particle in three dimensions simultaneously.

There are two main methods that have been developed in order to get around this fundamental issue. One, designed by Wolfgang Paul, takes its inspiration from the oscillating fields in a quadrupole mass filter and is now commonly known as the Paul trap after its creator, or as the radiofrequency (RF) trap. The other, developed by Hans

Georg Dehmelt, is based on the same principle as the Penning vacuum gauge developed by Frans Michel Penning, after whom the trap is named [2]. In a Penning gauge a static magnetic field is used in conjunction with charged plates in order to increase the mean free path of electrons inside the gauge. This increases the chance that the electrons will ionise a background gas molecule, allowing for greater sensitivity in pressure measurements. The same principle that increases the lifetime of the electrons in the gauge leads to confinement of ions.

As a testament to the utility of this technique one half of the Nobel Prize in Physics in 1989 was jointly awarded to Paul and Dehmelt ‘for the development of the ion trap technique’ (the other half was awarded to Norman Ramsey ‘for the invention of the separated oscillatory fields method and its use in the hydrogen maser and other atomic clocks’) [3].

1.1.2 Early demonstrations of laser cooling

The original rationale for attempting to reduce the temperature of trapped ions in the late 1970’s was to reduce the size of the Doppler effect in order to ‘improve fundamentally the resolution and accuracy of high-resolution spectroscopy’ [4], or simply as a valid goal in its own right, as articulated by Neuhauser et al. in the introduction to their paper:

Taking the attitude that the pursuit of as basic an ideal as a single particle at rest in space is a thoroughly worthwhile intellectual endeavor we are undertaking experiments along these lines [5].

The first demonstration of laser cooling was performed in an ion trap nearly simultaneously by the group of Peter Toschek in Heidelberg and by the group of David Wineland at the National Bureau of Standards (now NIST) by applying laser light close to the $S_{1/2} \longleftrightarrow P_{1/2}$ transitions of $^{138}\text{Ba}^+$ and $^{24}\text{Mg}^+$ respectively. In Heidelberg they used an RF trap and detected fluorescence light from the ion to verify that laser scattering and therefore cooling had occurred [5]. At NIST they used a Penning trap and an electronic detection method. This detection technique makes use of the moving image charges that appear on the electrodes due to the motion of the ion cloud inside the trap to produce a signal whose amplitude is proportional to the temperature of the ion cloud. The reduction in this signal level when the cooling laser is tuned to the red side of the transition is taken as evidence that the ion cloud has been cooled from initially above room temperature to close to 0 K [4].

The theoretical treatment in ref. [5] invokes the idea of motional sidebands, and this approach would later prove vital in reaching the ground state of motion for a single ion, and these motional sidebands were observed during spectroscopic measurements at National Institute of Standards and Technology (NIST) in 1987 [6]. The first resolved-sideband cooling to the ground state in one dimension was accomplished by NIST

in 1989 using the narrow-linewidth $S_{1/2} \longleftrightarrow D_{5/2}$ transition at 281.5 nm of a single $^{198}\text{Hg}^+$ ion [7]. They achieved this using an RF trap with a secular frequency of just under 3 MHz. Since the lifetime of the upper state is long, the rate of photon scattering is very low (and in the presence of external heating mechanisms it may not cool at all). To overcome this they artificially broadened the linewidth of their narrow-linewidth transition by coupling to a third auxiliary state (the fast decaying $P_{3/2}$ state) by shining some light at 398 nm. After the cooling a final phonon occupation number of $\bar{n} = 0.049 \pm 0.045$ was measured, using spectroscopic methods.

This result was later expanded upon in 1995 by Monroe et al., again at NIST, where a single $^9\text{Be}^+$ ion was cooled to the ground state in three dimensions by resolved-sideband cooling using a stimulated Raman transition between $S_{1/2}$ hyperfine sub-levels [8]. The three-dimensional ground state was reached with 92% probability. By this point the field of quantum information had begun to take shape and this system was quickly used to demonstrate spin-motion entanglement of a single ion [9].

1.1.3 Quantum information experiments

Quantum computation

At some point after the first demonstration of ground state cooling of a single ion, it was realised that cold trapped ions would be a good candidate for experiments in the relatively new field of quantum information. This was particularly true after Cirac and Zoller proposed a scheme in 1995, which uses the collective motion in a linear ion Coulomb crystal, a configuration where the Coulomb repulsion is equal and opposite to the trapping force, in order to transmit quantum information between different ions and create entangled states [10]. In quantum computation the quantum ‘qubit’ states $|0\rangle$ and $|1\rangle$, and superpositions thereof, are used in place of classical 0 and 1 bits. A universal set of conditional quantum gates can be used to perform quantum algorithms that result in significant speedup compared to classical computation (due to quantum effects, such as state superposition, interference and entanglement), for example Shor’s algorithm, which provides an efficient quantum computational method to factorise a large number [11].

Over the past 20 years many proof-of-principle experiments have been undertaken in the field of ion trap quantum computation, including the aforementioned Cirac-Zoller gate [12], a two-ion geometric phase gate, where a travelling wave produced by counter-propagating laser beams is used to coherently excite the collective ion motion [13], and a Mølmer-Sørensen gate where resonant ‘sideband’ interactions are used in order to produce collective state changes [14]. These basic building blocks have been used to create entangled states and in recent efforts to demonstrate small-scale error correcting codes [15, 16].

The ‘DiVincenzo criteria’ for quantum computing, which were established in 2000,

are:

- A scalable physical system with well characterised qubits,
- The ability to initialise the state of the qubits to a simple fiducial state,
- Long relevant decoherence times, much longer than the gate operation time,
- A “universal” set of quantum gates,
- A qubit-specific measurement capability [17].

Ion traps offer large levels of control compared with other quantum computing technologies, with reasonable decoherence times and single ion gate operations that can now be performed with fidelities on the level of 99.9999% [18]. However, the main issue for ion trap experiments is scaling up the number of qubits to the level where quantum computations become worthwhile. There is a lot of research being conducted in this direction, with surface traps with on-chip microwaves to perform operations instead of lasers [19], schemes to shuttle ions inside segmented traps [20–22], coupling the quantum state of ions in separate trapping regions [23] and reduction of heating rates in surface traps using laser cleaning [24, 25].

Quantum simulation

Aside from a fully-fledged quantum computer another form of quantum device is a ‘quantum simulator’, which is in some ways a simpler concept. The purpose of a quantum simulator is to replicate a Hamiltonian of interest, for example the Ising model Hamiltonian, given by

$$H = \sum_{i,j} J_{ij} \sigma_i^z \sigma_j^z \quad (1.1)$$

where i and j denote different ions and J_{ij} denotes the strength of the coupling between ions i and j , which is of particular interest because of its applicability to solid-state physics problems like quantum magnetism [26].

Broadly speaking there are two types of quantum simulator [27]; a ‘digital’ quantum simulation uses discrete gate pulses in a manner more akin to a normal quantum computer, such as that done by Lanyon et al. in ref. [28] where they simulate a number of different spin Hamiltonians, and an ‘analogue’ simulation where an interaction is produced that has a similar form to the Hamiltonian of interest, but with a higher level of control, as done by Islam et al. in ref. [29] where they produce a tunable-range Ising interaction in 16 ions using a travelling wave.

1.1.4 Penning traps

Since being used for the first demonstration of Doppler cooling as mentioned in section 1.1.2, Penning traps have been mainly used for precision measurement of the

masses and magnetic moments of subatomic particles [30, 31] and their antimatter equivalents [32], spectroscopy of highly charged ions [33] and work on non-neutral plasmas [34].

The most prominent use of a Penning trap for quantum information science are the experiments conducted by the group of John Bollinger at NIST in Boulder, Colorado. They use large, planar ion Coulomb crystals of approximately 300 ions and create a spin-motion coupling by applying an optical travelling wave, in the style of the geometric phase gate, with the frequency of the wave tuned close to that of one of the ‘drumhead’ axial modes of the crystal. The combination of this interaction and global state rotations realised using microwaves form the basis of an analogue quantum simulator of the Ising model Hamiltonian. They have currently benchmarked their tunable-range interaction [35]. In this experiment the axial drumhead modes are cooled only to the Doppler limit, meaning that there is significant population outside of the quantum ground state.

There is also an experiment being set up in PTB in Germany to measure the g -factor of the anti-proton using quantum logic spectroscopy in a double Penning trap, requiring a single sideband cooled ${}^9\text{Be}^+$ ion [36].

What these Penning trap experiments show is that there are situations in which the use of a Penning trap has significant advantages, but that the development of a full Penning trap quantum ‘toolkit’ is still ongoing. Having a reliable method of producing ground-state cooled ions in a Penning trap could potentially open up the possibility of performing experiments such as quantum logic spectroscopy, which are currently performed in an RF trap [37].

1.2 Motivation for this work

The field of ion trapping is a diverse and still-growing field. It now encompasses many different types of experiments. However, the tools used in these experiments are often similar; the electronic states of the ions are manipulated coherently and coupled to the motion of the ions inside the trap using a suitable interaction.

Any serious attempt to use a Penning trap to do quantum information science will require ions in or close to their motional ground state in at least one direction. Therefore the demonstration of axial sideband cooling as well as the development of methods to study the radial motion in the quantum regime represents a useful contribution to the field.

The general scheme of work for the ${}^{40}\text{Ca}^+$ Penning trap experiment in the Ion Trapping group at Imperial College is shown in Figure 1.1. By building on previous work done to perform single ion resolved-sideband spectroscopy [38], and to control the shape and rotation speed of ion Coulomb crystals using laser-induced torque [39] we aim to increase the Penning trap quantum information toolkit to the stage where we

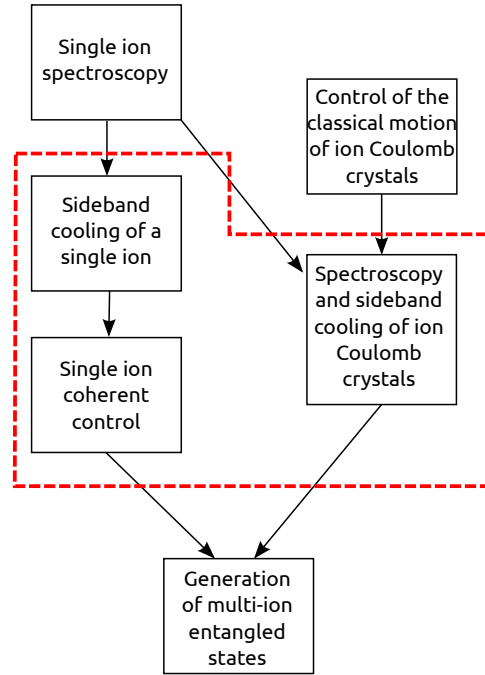


Figure 1.1: Schematic diagram showing the direction of the $^{40}\text{Ca}^+$ Penning trap experiment at Imperial College. The dashed box shows work that is covered in this thesis.

can produce multi-ion entangled states, and use them to perform small-scale quantum error-correcting codes [40].

The aim of the specific work presented in this thesis is to continue and expand upon previous axial sideband cooling results, to move them beyond proof-of-principle and ensure that we can reliably ensure $> 95\%$ ground state population. The other aims are to perform coherent operations on these sideband-cooled single ions, perform heating rate measurements, and to increase the level of control over the radial motion. In general the idea is to put together a set of techniques that work for one and two ions in a Penning trap that can then be scaled up to larger numbers of ions.

1.2.1 Structure of thesis

I begin first by presenting the relevant theory of $^{40}\text{Ca}^+$ ions in a Penning trap, and their interaction with monochromatic light. In Chapter 2 I describe the classical and quantum motion of single ions in a Penning trap, and then briefly explain some of the features of the dynamics of ion Coulomb crystals, in particular the rotation of these crystals in the laboratory frame of reference. Then in Chapter 3 I move on to cover the interaction of a single ion with monochromatic light and describe how this interaction is modified when the motion of the ion inside the trap is taken into account. I also present the background theory of laser cooling in a Penning trap in both the Doppler and resolved-sideband regimes. Chapter 4 contains the relevant atomic theory required

to understand the energy level structure of $^{40}\text{Ca}^+$ ions, including first and second-order shifts due to the applied magnetic field of the Penning trap. I also consider the geometric form of the strength of different transitions.

In Chapter 5 I describe the experimental equipment as it currently stands, paying particular attention to the Doppler cooling laser Pound-Drever-Hall locks and recent changes to the setup, both of which I was particularly responsible for.

In Chapter 6 I present the main results I obtained during my thesis, including narrow-linewidth spectroscopy of a single ion in both the axial and radial directions. I show examples of these spectra before and after resolved-sideband cooling, demonstrating significant ground state population in the case of the axial and the modified cyclotron motion and also a study of the axial heating rate of an ion in our trap at different motional frequencies, with a short discussion of the significance of this result when compared to heating rate measurements made by other research groups.

Finally, in Chapter 7 I discuss possible future directions for the experiment, including single ion coherence experiments and a scheme to perform a quantum error-correcting code in a Penning trap using only global pulses.

Chapter 2

Motion of a charged particle in a Penning trap

Understanding the motion undergone by ions in a Penning trap is clearly of paramount importance to us. The ways in which this motion departs from that of a particle in an ideal harmonic trap informs many of the decisions we make in our experiment. In this chapter I will introduce the properties of the motion of small numbers of ions in a Penning trap, in both the classical and quantum regimes. I will start first by describing the motion of a charged particle in crossed electric and magnetic fields, before looking at the classical equations of motion for a single ion in a Penning trap and their solutions. I will then give a short summary of the dynamics and structure of ion Coulomb crystals, including discussion of relevant rotating frames of reference. I will also give a description of the quantum dynamics of a single ion, including the form of the axial and radial wavefunctions. Finally, I will discuss two common types of small-amplitude radio frequency drive that are used in conjunction with a Penning trap to help control the radial motion: the rotating wall drive and axialisation.

2.1 Earnshaw's theorem

As mentioned in section 1.1, there is a fundamental limitation that prevents the trapping of a charged particle using only electrostatic potentials. The reason for this is that any electrostatic potential ϕ in vacuum must fulfil the Laplace equation

$$\nabla^2 \phi = 0. \quad (2.1)$$

We can define a potential that is quadratic in three dimensions in Cartesian coordinates of the form

$$\phi = \alpha x^2 + \beta y^2 + \gamma z^2 \quad (2.2)$$

where the constants α , β and γ denote the strength of the potential in each direction. In order for the force on a positively charged particle to point towards the trap centre (given by $\mathbf{F} = -q\nabla\phi$), and to therefore trap the charged particle, each constant should be positive. However, this requirement contradicts (2.1) which, if we substitute in a potential of the form given in (2.2), leads to the condition

$$2\alpha + 2\beta + 2\gamma = 0. \quad (2.3)$$

Therefore, if we create an electrode structure and apply voltages to it such that there is trapping potential in one dimension (in the x -direction for example, i.e. α is positive) then there must be a potential maximum in at least one of the other dimensions (i.e. one or both of β and γ must be negative in order to satisfy (2.3)). This will result in a force applied in the same direction as the displacement, and any positively charged particle will accelerate away from the centre of the trap in this direction. This application of basic electrostatics forms the background for ‘Earnshaw’s theorem’, which states that there is no stable stationary configuration of point charges held by their own electrostatic forces [41, 42].

The result of Earnshaw’s theorem is that if we wish to confine a charged particle in all three dimensions simultaneously, electrostatic potentials on their own will not suffice. The best we can do is to provide a trapping force along one or two of the trap axes, but ‘anti-trap’ along the others. There are two commonly-used solutions to this problem (the development of which are described in section 1.1), which are to either:

- add a magnetic field to confine ions in the plane perpendicular to the electrostatic trapping axis (the Penning trap),
- or oscillate between anti-trapping and trapping in such a way that there is a time-average trapping potential (radiofrequency or Paul trap).

I will only present the background theory to Penning traps in this thesis. There are many references that cover the theory of Paul traps in detail, for example refs. [2, 43, 44].

2.2 Charged particle in crossed E and B fields

In a Penning trap we use an electrostatic potential to confine the ions in one direction (we define this to be the z -axis), and add a magnetic field along this direction in order to confine ions radially. In order to gain some insight as to how this configuration leads to radial confinement it is useful to consider the motion of a charged particle in crossed electric and magnetic fields.

Consider a magnetic field applied along the z -axis, given by $\mathbf{B} = (0, 0, B_z)$, with an electric field applied at right angles to it. We can define the direction of the electric

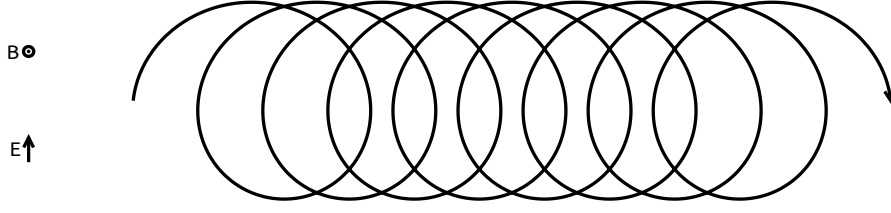


Figure 2.1: An example of the motion of a charged particle in crossed electric and magnetic fields, with parameters $E_y/B_z = 2/\pi$ and $q/m = 1$. The linear drift of the cyclotron loop centre to the right is the result of the E_y/B_z term in (2.7).

field as being the y -axis of the system without loss of generality, giving $\mathbf{E} = (0, E_y, 0)$. As a result of these fields, a particle with charge q and mass m will feel a Lorentz force given by

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}). \quad (2.4)$$

Since there is no component of the electric field in the z -direction and no magnetic field in the x or y -directions, there will be no component of the resultant Lorentz force in the z -direction. The particle will therefore continue to move with its initial velocity in this direction, which is not of particular interest, so we will ignore it. The equations of motion in the xy -plane are

$$x''(t) = \frac{qB_z}{m}y'(t) \quad (2.5)$$

$$y''(t) = \frac{q}{m}(E_y - x'(t)B_z). \quad (2.6)$$

which demonstrate that the x and y motions are coupled in this system. Despite this, we can find solutions for the motion of the form

$$x(t) = \frac{E_y}{B_z}t + a \sin\left(\frac{qB_z}{m}t\right) \quad (2.7)$$

$$y(t) = a \cos\left(\frac{qB_z}{m}t\right). \quad (2.8)$$

where a is an arbitrary amplitude of the oscillatory part of the motion. For $E_y = 0$ the motion is simply a circular orbit at the cyclotron frequency $\omega_c = \frac{qB_z}{m}$. However for $E_y \neq 0$, the centre of the cyclotron loop will drift linearly with time in a direction perpendicular to both of electric and magnetic fields (i.e. the $\mathbf{E} \times \mathbf{B}$ direction). An example trajectory of a particle in this electromagnetic field configuration is shown in Figure 2.1.

In a Penning trap the magnetic field is applied along the z -direction, and the component of the electric field in the xy -plane always points away from the centre of the trap. It follows that the $\mathbf{E} \times \mathbf{B}$ direction is always perpendicular to the radial direction, at any position in the trap. This means that we should expect the motion of a charged particle in a Penning trap to have cyclotron-like loops around a centre that drifts around the centre of the trap, as if we were to curl the linear drift shown in Figure 2.1 around into a loop.

2.3 The ideal Penning trap

2.3.1 Classical motion of a single ion

As we have established, the electromagnetic fields in a Penning trap are set up so that trapping occurs in the z (axial) direction due to the electrostatic potential, and a magnetic field is also applied along this direction in order to confine the ions in the xy (radial) plane. The ideal Penning trap has a cylindrical symmetry, so the coefficients in (2.2) obey $\alpha = \beta$ and $2\alpha = -\gamma$. These conditions lead to an electrostatic potential of the form

$$\phi = \frac{\gamma}{2}(2z^2 - r^2) \quad (2.9)$$

where $r = \sqrt{x^2 + y^2}$.

To produce an exact potential of this form the electrodes should be shaped such that they follow the equipotential surfaces, leading to ideal electrodes that follow the equations

$$r^2 - 2z^2 = r_0^2 \quad (2.10)$$

$$2z^2 - r^2 = 2z_0^2 \quad (2.11)$$

where z_0 and r_0 are constants. The surface that follows (2.10) is designated the ‘ring’ electrode, and the two surfaces that obey (2.11) are designated the ‘endcaps’. These ideal electrode shapes are shown in Figure 2.2. The electrode geometry of real Penning traps often departs significantly from the ideal, for considerations such as loading ions into the trap and optical access for laser beams. However, any departure from the ideal electrode shapes will lead to non-quadratic terms in the potential and so care must be taken that these terms are sufficiently small for the particular application.

If a positive voltage V is applied to each of the endcaps with respect to the ring electrode and we define the electrostatic potential to be zero at the centre of the trap,

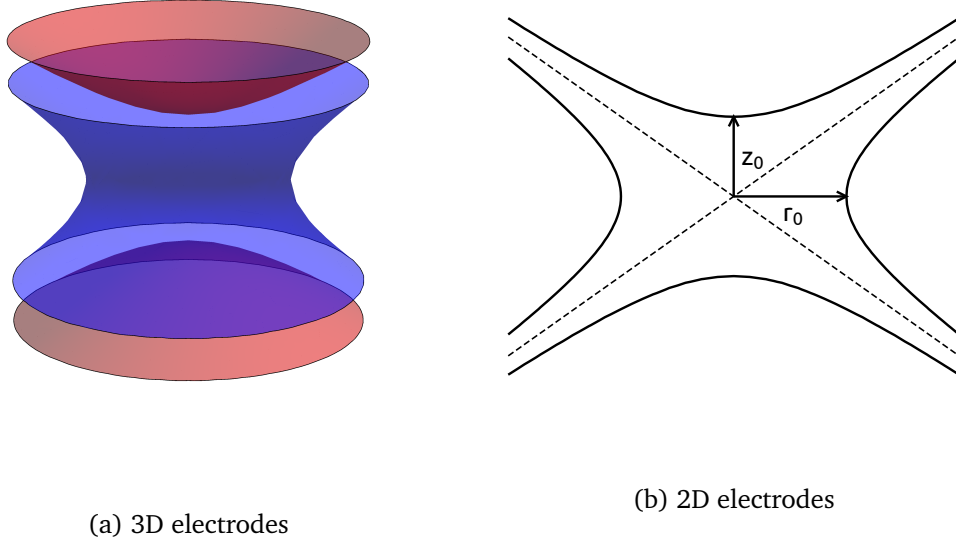


Figure 2.2: Ideal trap electrode structure for a Penning trap. (a) shows the three-dimensional electrode structure, with the two endcap electrodes in orange and single ring electrode in blue. (b) shows a two-dimensional cross section of the electrodes through the centre of the trap, with the characteristic electrode distances z_0 and r_0 labelled.

the potential is given by

$$\phi = \frac{V}{2d_0^2}(2z^2 - r^2) \quad (2.12)$$

where $d_0 = \sqrt{(r_0^2 + 2z_0^2)}/2$ is the characteristic distance between the centre of the trap and the electrodes. If we now substitute the form of these fields ($\mathbf{B} = (0, 0, B_z)$ and $\mathbf{E} = -\nabla\phi$) into (2.4) we can obtain the equations of motion for a singly-charged ion ($q = +e$) in Cartesian coordinates

$$x''(t) = \frac{1}{2}\omega_z^2 x + \omega_c y'(t) \quad (2.13)$$

$$y''(t) = \frac{1}{2}\omega_z^2 y - \omega_c x'(t) \quad (2.14)$$

$$z''(t) = -\omega_z^2 z \quad (2.15)$$

where $\omega_z = \sqrt{\frac{2eV}{md_0^2}}$ is the axial frequency and $\omega_c = \frac{eB}{m}$ is the cyclotron frequency. As in (2.5) and (2.6) we have coupled equations of motion in the x and y -directions. To decouple these equations we make a change of variables $u = x + iy$, motivated by the circular form of the motion in crossed $\mathbf{E}$ and $\mathbf{B}$ fields described in section 2.2, which leads us to expect that the radial motion should be circular. By use of an oscillatory

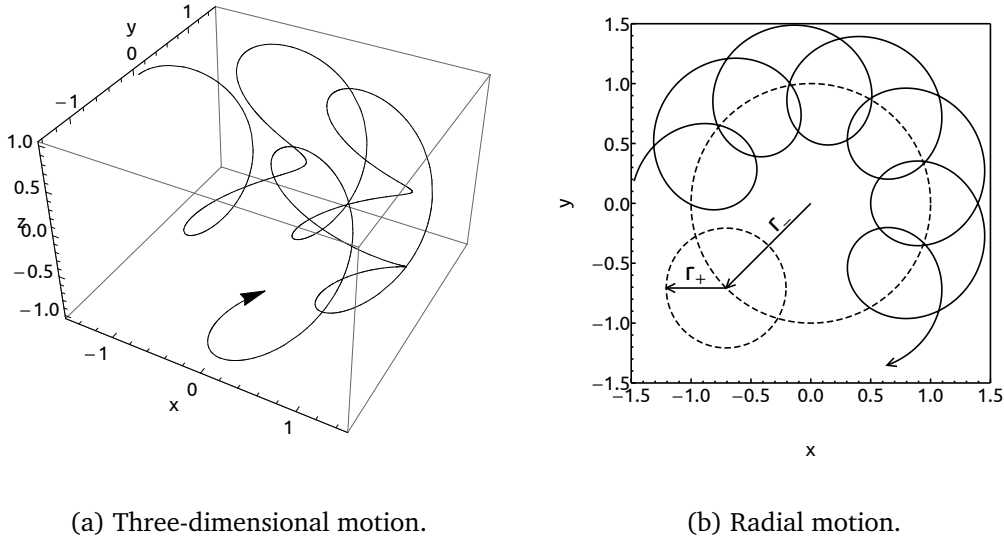


Figure 2.3: An example of the motion of a single, charged particle in a Penning trap. (a) shows the motion in three dimensions, and (b) shows the projection into the radial plane. In these examples $r_z = r_- = 2r_+$ and $\omega_z = \omega_+ = 10\omega_-$.

trial solution $u = re^{i(\omega t + \theta)}$, we find solutions for the radial mode frequencies given by

$$\begin{aligned}\omega_{\pm} &= \frac{1}{2} \left(\omega_c \pm \sqrt{\omega_c^2 - 2\omega_z^2} \right) \\ &= \frac{\omega_c}{2} \pm \omega_1\end{aligned}\tag{2.16}$$

with $\omega_1 \equiv \frac{1}{2}\sqrt{\omega_c^2 - 2\omega_z^2}$. The motional mode associated with the higher frequency solution ω_+ is called the ‘modified cyclotron’ motion and the motional mode associated with ω_- is called the ‘magnetron’ motion.

If there are solutions of the form $u(t) = r_{\pm}e^{i(\omega_{\pm}t + \theta_{\pm})}$, then $u(t) = r_+e^{i(\omega_+t + \theta_+)} + r_-e^{i(\omega_-t + \theta_-)}$ must also be a solution. By taking the real and imaginary parts of this expression we can obtain a general solution for the motion in Cartesian coordinates

$$x(t) = r_- \cos(\omega_-t + \theta_-) + r_+ \cos(\omega_+t + \theta_+)\tag{2.17}$$

$$y(t) = -r_- \sin(\omega_-t + \theta_-) - r_+ \sin(\omega_+t + \theta_+)\tag{2.18}$$

$$z(t) = r_z \cos(\omega_zt + \theta_z)\tag{2.19}$$

where the three modes of motion have amplitudes r_+, r_-, r_z and phases $\theta_+, \theta_-, \theta_z$ which are determined by initial conditions. An example of this kind of motion is shown in Figure 2.3, with Figure 2.3b showing the projection of the motion into the radial plane. The two radial modes form an epicyclic orbit around the trap centre.

Using the general solutions for the motion given in (2.17), (2.18) and (2.19) and their time derivatives we can obtain the time-averaged value of the potential and kinetic

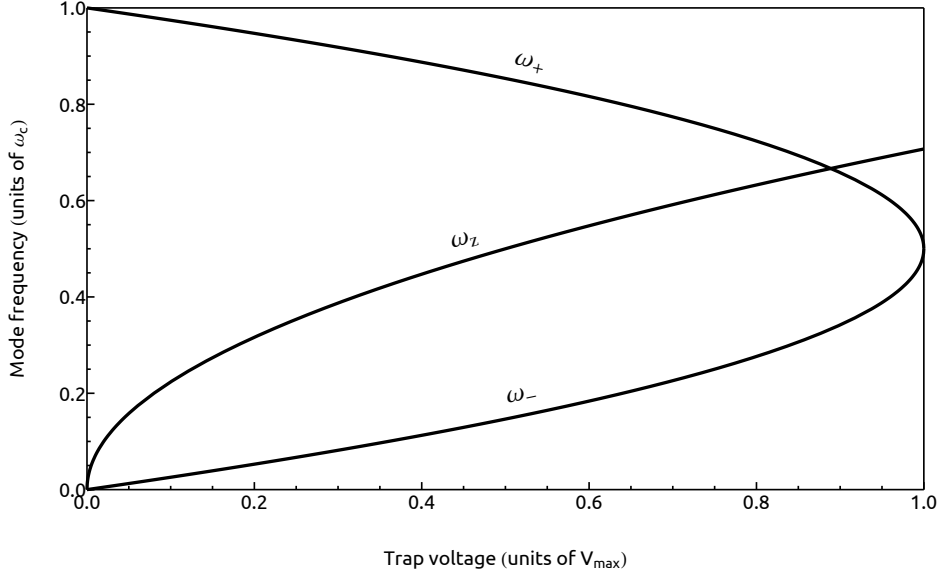


Figure 2.4: A plot of the frequency of each mode of motion in a Penning trap for changing endcap voltage.

energy associated with the motion, giving

$$\langle PE \rangle_t = \frac{1}{2}m \left(\frac{1}{2}r_z^2\omega_z^2 - \frac{1}{2}(r_-^2 + r_+^2)\omega_z^2 \right) \quad (2.20)$$

$$\langle KE \rangle_t = \frac{1}{2}m \left(\frac{1}{2}r_z^2\omega_z^2 + r_-^2\omega_-^2 + r_+^2\omega_+^2 \right) \quad (2.21)$$

and therefore the total energy in the motion

$$\begin{aligned} E &= \frac{1}{2}m \left(r_z^2\omega_z^2 + \frac{1}{2}r_+^2(2\omega_+^2 - \omega_z^2) + \frac{1}{2}r_-^2(2\omega_-^2 - \omega_z^2) \right) \\ &= \frac{1}{2}m(r_z^2\omega_z^2 + 2r_+^2\omega_+^2 - 2r_-^2\omega_-^2). \end{aligned} \quad (2.22)$$

The final term here associated with r_- , the amplitude of the magnetron motion gives a negative contribution to the total energy. From (2.21) we see that in order to reduce the kinetic energy of the magnetron motion, its amplitude must be reduced, but from (2.22) we see that in order to decrease the magnetron amplitude we must increase the total energy associated with this motion. This has implications for laser cooling as the normal method of using a red-detuned laser in order to remove energy via the Doppler effect will cause the magnetron motion to increase in amplitude. This complication will be discussed in section 3.4.1.

The existence of a stability criterion in a Penning trap can be seen from looking at the square root term in (2.16). For stable solutions we require that these mode

frequencies are real and therefore that $\omega_c > \sqrt{2}\omega_z$. This sets the maximum endcap voltage that can be applied for stable motion, given by

$$V_{\max} = \frac{B^2 d_0^2 e}{4m}. \quad (2.23)$$

Figure 2.4 shows the scaling of each of the mode frequencies with changing endcap voltage. Assuming that we can generate large enough DC voltages to reach the stability limit $V_{\max}$, the upper limit for mode frequencies that can be achieved in a Penning trap is set only by the strength of the applied magnetic field. For comparison, the secular frequency in a Paul trap (at zero DC bias) is given by

$$\omega_s = \frac{2\sqrt{2}eV_{\text{AC}}}{m\omega_{\text{AC}}r_0^2} \quad (2.24)$$

where V_{AC} and ω_{AC} are the amplitude and frequency of the RF trapping voltage. In general, generating large, stable RF voltages on the kV level can be problematic, and because of this (and for reasons of scalability) there is a drive to reduce the characteristic size (r_0) of RF traps. Conversely, there is no particular benefit to making smaller Penning traps when working with a single type of charged particle. One exception is in some proposed antiproton experiments where the form of the electrostatic potential must be customised in order to trap a laser-cooled, positively-charged ion close to the negatively-charged antiproton, and to shuttle them into different trapping regions [36].

2.3.2 Trapping multiple ions - ion Coulomb crystals

When multiple ions in a Penning trap are cooled simultaneously, the kinetic energy of each of the ions can become much lower than the Coulomb potential experienced due to the other ions in the trap. In this situation the ions form a crystalline structure with equilibrium positions where the force on any particular ion due to Coulomb repulsion from the other ions is equal and opposite to the trapping force. The Coulomb coupling constant Γ is given by

$$\Gamma = \frac{e^2}{4\pi\epsilon_0 a k_B T} \quad (2.25)$$

where a is the Wigner-Seitz radius, defined as $a^3 = 3/(4\pi n_0)$ for an ion number density n_0 . Crystallisation of a large, three-dimensional ion cloud occurs at roughly $\Gamma \gtrsim 170$ [45], although this depends strongly on the size and dimensionality of the cloud [46]. When in crystal form the ions exhibit collective motion, and this collective motion can be used to transmit quantum information between the ions (as in the Cirac-Zoller proposal [10], for example).

Rotating reference frames

In a Penning trap all of the applied potentials have cylindrical symmetry, and therefore Coulomb crystals will rotate with respect to the lab frame at some frequency ω_r . This means that for a crystal in a Penning trap there are two relevant frames of reference, the lab frame and the frame that rotates at the same frequency as the crystal.

In order to calculate the equilibrium ion positions and mode frequencies of a rotating crystal it is useful to first find the effective potential in the frame that rotates with the crystal. We can express the Lorentz force due to the trap in the lab frame, (2.4), in cylindrical coordinates (r, θ, z) and consider the radial component, F_r , which is given by

$$F_r = \left(\frac{1}{2} m \omega_z^2 + m \omega_c \dot{\theta} \right) r. \quad (2.26)$$

If we now transform into the stationary frame of the crystal, which rotates at a frequency of ω_r , we must also consider the centrifugal force, $F_{\text{cent}} = -m \boldsymbol{\omega}_r \times (\boldsymbol{\omega}_r \times \mathbf{r})$, resulting in a total radial force in the rotating frame of

$$F'_r = \left(\frac{1}{2} m \omega_z^2 - m \omega_c \omega_r + m \omega_r^2 \right) r'. \quad (2.27)$$

Now by considering the potential that leads to this radial force, and by adding the potential due to the Coulomb repulsion from the other ions, the apparent potential experienced by the j^{th} ion in the crystal frame in terms of the positions in the rotating frame r' and z' , is

$$\phi_j^{\text{eff}} = \frac{m}{2e} \omega_z^2 z'^2 + \frac{m}{2e} \left(\omega_c \omega_r - \omega_r^2 - \frac{1}{2} \omega_z^2 \right) r'^2 + \frac{e}{4\pi\epsilon_0} \sum_{i \neq j} \frac{1}{r'_{ij}} \quad (2.28)$$

where r_{ij} is the distance between the i^{th} and j^{th} ions. From this expression we can define an effective radial trapping frequency, ω_{eff} , given by

$$\omega_{\text{eff}} = \sqrt{\omega_c \omega_r - \omega_r^2 - \frac{1}{2} \omega_z^2}. \quad (2.29)$$

This frequency must be real for stable rotation frequencies, which leads to the condition $\omega_- < \omega_r < \omega_+$.

The equilibrium positions for each of the ions in a cold Coulomb crystal for a particular rotation frequency can now be found by performing a numerical simulation. We begin with a random distribution of ions in the trap and calculate the force on each ion using the potential given in (2.28). We can then move each ion a short distance proportional to this force. By repeating these steps of force calculation and movement many times we eventually reach the minimum energy configuration [39].

If we additionally consider the Coriolis force in the rotating frame $F_{\text{Cor}} = -2m \boldsymbol{\omega}_r \times$

$\mathbf{r}'$ (which is mathematically similar to the force due to a magnetic field) we obtain an effective magnetic field in the crystal frame, given by

$$B^{\text{eff}}(\omega_r) = B_z - \frac{2\omega_r m}{e}. \quad (2.30)$$

$\omega_c/2$ frame Considering (2.30), there is a special case where $\omega_r = \omega_c/2$, which makes the effective magnetic field zero. This frame is often the natural choice of frame when considering Penning trap problems especially when considering single ions [47]. In this frame an ion experiences an effective potential that is harmonic in all three dimensions, with no coupling between the x' and y' motions. The axial motion remains unchanged, with frequency ω_z but the radial potential now has frequency $\omega_1 = \frac{1}{2}\sqrt{\omega_c^2 - 2\omega_z^2}$. In this frame the modified cyclotron and magnetron motions are transformed to clockwise and anticlockwise motion.

Modes of planar crystals

An important configuration is when the ions are entirely in the radial plane and form a two-dimensional crystal. In this case there are axial modes analogous to the modes of a drumhead. This occurs in the regime where the axial trapping strength is much stronger than the effective radial trapping strength, i.e. either when the endcap voltage is close to its upper limit, or the crystal rotation frequency is close to its upper or lower limits. This configuration utilises the strengths of the Penning trap, namely their lack of RF driven micromotion.

The modes of an ion Coulomb crystal are typically found by performing a Taylor expansion of the Lagrangian in the crystal frame around the equilibrium positions. For a two-dimensional crystal the axial modes are found using the stiffness matrix

$$K_{jk}^{zz} = \begin{cases} eV_0 - \sum_{l=1, l \neq j}^N \frac{k_e e^2}{R^3} & j = k \\ \frac{k_e e^2}{R^3} & j \neq k \end{cases} \quad (2.31)$$

that appears in the expansion. The modes and their associated frequencies are related to the eigenvectors and eigenvalues of this matrix [48].

2.3.3 Quantum dynamics

The Hamiltonian of a particle with charge e in a static electromagnetic field is given by

$$H = \frac{(\mathbf{p} - e\mathbf{A})^2}{2m} + e\phi, \quad (2.32)$$

where ϕ is the electrostatic potential, and $\mathbf{A}$ is the magnetic vector potential. If we choose the symmetric gauge, where $\mathbf{A} = \frac{1}{2}\mathbf{B} \times \mathbf{r} = \frac{1}{2}(-B_z y, B_z x, 0)$, the Hamiltonian

can be re-written as

$$H = \frac{1}{2m} \left(\left(p_x + \frac{1}{2}y(eB_z) \right)^2 + \left(p_y - \frac{1}{2}x(eB_z) \right)^2 + p_z^2 \right) + \frac{1}{4}m\omega_z^2 (2z^2 - r^2), \quad (2.33)$$

which can be separated into axial and radial parts $H = H_r + H_z$ where

$$H_z = \frac{p_z^2}{2m} + \frac{1}{2}mz^2\omega_z^2, \quad (2.34)$$

$$H_r = \frac{p_x^2 + p_y^2}{2m} - \frac{\omega_c L_z}{2} + \frac{1}{2}m\omega_1^2 r^2, \quad (2.35)$$

where $L_z = xp_y - yp_x$ is the angular momentum operator. The radial Hamiltonian is the same as an isotropic 2D quantum harmonic oscillator with frequency ω_1 , after the application of the unitary transformation $U(t) = e^{-i\frac{\omega_c L_z}{2\hbar}t}$, which equates to switching to a frame that is rotating with respect to the lab frame at a frequency of $\omega_c/2$ (this is the quantum mechanical equivalent of the classical rotating frame approach discussed in section 2.3.2).

If we assume a separable solution to the time-independent Schrödinger equation in cylindrical coordinates, we have

$$\psi(r, \theta, z) = R(r)Z(z)\Theta(\theta), \quad (2.36)$$

and we are able to solve the axial and radial Schrödinger equations separately.

The axial Hamiltonian is identical to that of the 1D quantum harmonic oscillator, and the energy eigenstates are found by solving the time-independent Schrödinger equation. The resulting wavefunctions and eigenenergies are given by the standard solutions

$$Z(z) = \frac{1}{\sqrt{2^{n_z} z_0 n_z! \sqrt{\pi}}} e^{-\frac{z^2}{2z_0^2}} H_{n_z} \left(\frac{z}{z_0} \right) \quad (2.37)$$

$$E_z = (n_z + \frac{1}{2})\hbar\omega_z \quad (2.38)$$

where $z_0 = \sqrt{\frac{\hbar}{m\omega_z}}$, H_{n_z} are Hermite polynomials and n_z is a positive integer.

To find the form of the angular part of the wavefunction, $\Theta(\theta)$ we recognise that ψ must be single-valued, and that we therefore have the boundary condition $\psi(r, \theta, z) = \psi(r, \theta + 2\pi, z)$. This leads to solutions of the form $\Theta(\theta) = e^{im_l\theta}$ where m_l is an integer. Since this is the only term in the wavefunction dependent on θ , if we apply the angular momentum operator we will obtain $L_z\psi = -i\hbar\frac{\partial}{\partial\theta}\psi = m_l\hbar\psi$.

In order to solve the radial part of the wavefunction we can substitute into (2.35) the explicit differential form of the operators. By performing a trick similar to one used when finding the Hydrogen atomic wavefunction, and re-expressing the radial part of

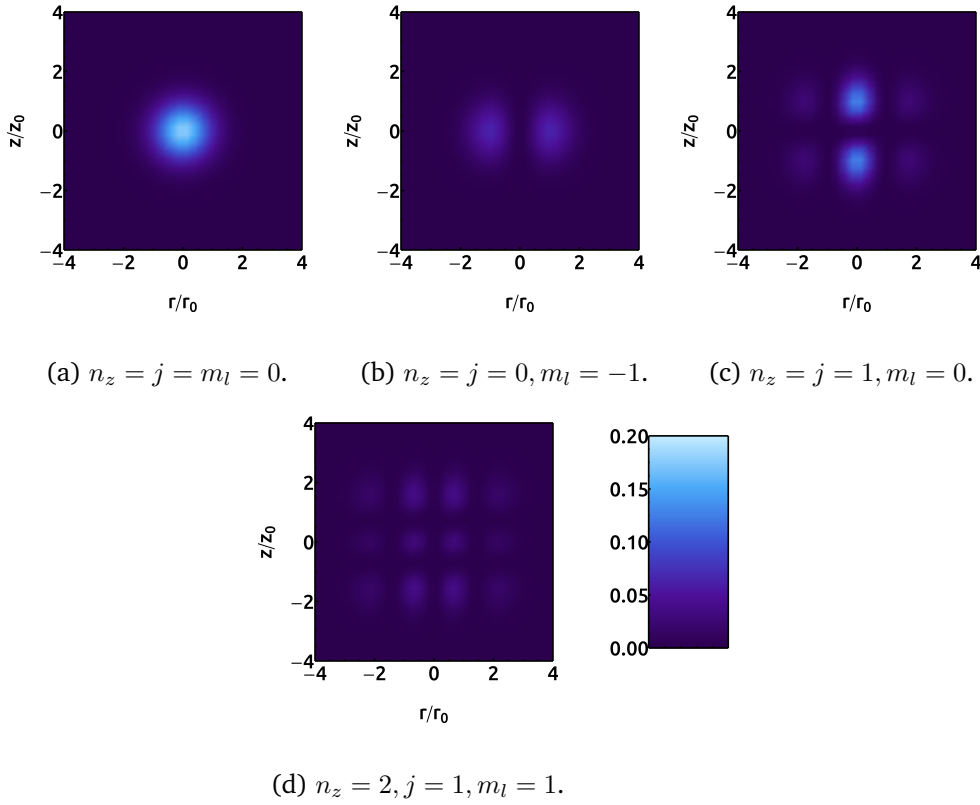


Figure 2.5: Examples of the spatial probability distributions for some low energy Fock states.

the wavefunction $R(r) = \frac{1}{\sqrt{r}}U(r)$ we obtain a standard partial differential equation that can be solved to obtain a full, three-dimensional wavefunction of the form

$$\psi_{j,m_l,n_z}(r, \theta, z) = \frac{1}{\pi^{3/4}r_0\sqrt{z_0}} \sqrt{\frac{j!}{2^{n_z}n_z!(j+|m_l|)!}} L_j^{|m_l|}\left(\frac{r^2}{r_0^2}\right) H_{n_z}\left(\frac{z}{z_0}\right) e^{-\frac{r^2}{2r_0^2} - \frac{z^2}{2z_0^2} + im_l\theta} \quad (2.39)$$

where j and n_z are non-negative integers and $r_0 = \sqrt{\frac{\hbar}{m\omega_1}}$. $L_j^{|m_l|}(x)$ are associated Laguerre polynomials.

The probability amplitude of the Fock (or ‘number’) states are given by $\psi_{j,m_l,n_z}^* \psi_{j,m_l,n_z}$, and some examples for low energy states are shown in Figure 2.5. The only angular dependence of the Fock state wavefunctions is in the $e^{im_l\theta}$ term, which drops out when the probability distributions are calculated, resulting in cylindrically symmetric solutions. However, if we have a superposition of Fock states this will not be the case. We expect that for a coherent state, which is the quantum analogue of classical motion, there should be some angular dependence that changes with time.

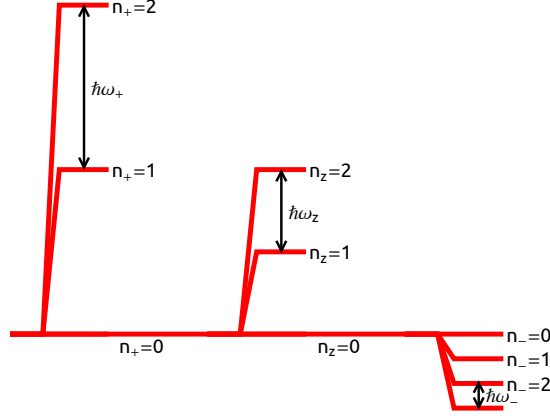


Figure 2.6: Diagram to show the the energy levels for quantum Penning trap motion $\omega_+ > \omega_z > \omega_-$, neglecting the zero-point energy

The eigenenergies of (2.39) in terms of the quantum numbers n_z , j and m_l are

$$E_{n_z, j, m_l} = \hbar\omega_z\left(n_z + \frac{1}{2}\right) + \frac{1}{2}\hbar\omega_1(2j + |m_l| + 1) - \frac{1}{2}m_l\hbar\omega_c. \quad (2.40)$$

By gathering the terms that are proportional to the classical mode frequencies ω_z , ω_+ and ω_- we obtain

$$E_{n_z, n_+, n_-} = \hbar\omega_z\left(n_z + \frac{1}{2}\right) + \hbar\omega_+\left(n_+ + \frac{1}{2}\right) - \hbar\omega_-\left(n_- + \frac{1}{2}\right) \quad (2.41)$$

where $n_+ = j + \frac{1}{2}(|m_l| - m_l)$, and $n_- = j + \frac{1}{2}(|m_l| + m_l)$ are the quantum numbers of the modified cyclotron and magnetron motion respectively. From the final term of this equation it is clear that if we add quanta to the magnetron motion we remove energy from the system, which is consistent with the classical form of the energy shown in (2.22).

We can rewrite the Hamiltonian with ladder operators for the three modes of motion

$$a_z = \frac{1}{\sqrt{2}z_0} \left(z + \frac{i}{m\omega_z} p_z \right) \quad (2.42)$$

$$a_{\pm} = \frac{1}{2r_0} \left(x \pm iy + \frac{2i}{m\omega_1} (p_x \pm ip_y) \right) \quad (2.43)$$

which gives

$$H = \hbar\omega_z \left(a_z^\dagger a_z + \frac{1}{2} \right) + \hbar\omega_+ \left(a_+^\dagger a_+ + \frac{1}{2} \right) - \hbar\omega_- \left(a_-^\dagger a_- + \frac{1}{2} \right). \quad (2.44)$$

We will need this formalism later when we come to look at the effect the motion of an ion in the trap has on its interaction with monochromatic light. We can also express the Cartesian coordinates in terms of these ladder operators

$$x = \frac{r_0}{2} \left(a_+ + a_+^\dagger + a_- + a_-^\dagger \right) \quad (2.45)$$

$$y = \frac{r_0}{2i} \left(\left(a_+ - a_+^\dagger \right) - \left(a_- - a_-^\dagger \right) \right) \quad (2.46)$$

$$z = \frac{z_0}{\sqrt{2}} \left(a_z + a_z^\dagger \right) \quad (2.47)$$

By comparing the quantum and classical equations for the total energy of the motion ((2.41) and (2.22)) we can obtain approximate expressions for the classical amplitudes of motion r_z , r_+ and r_- in terms of the motional quantum numbers n_z , n_+ and n_- , with

$$r_z^2 \approx 2z_0^2 \left(n_z + \frac{1}{2} \right) \quad (2.48)$$

$$r_+^2 \approx r_0^2 \left(n_+ + \frac{1}{2} \right) \quad (2.49)$$

$$r_-^2 \approx r_0^2 \left(n_- + \frac{1}{2} \right). \quad (2.50)$$

2.4 Addition of RF drives

While one of the main advantages of the Penning trap is that it is a ‘clean’ system without any oscillating drives, some small applied radio frequency drives can help to control the radial motion. Since the amplitude of these drives are much less than the typical amplitude of the applied RF voltage in a Paul trap the effect of micromotion should also be comparatively small. There are two main types of drive used in a Penning trap, the first is a ‘rotating wall’ drive used to control the rotation frequency of a crystal, and the other is a drive that couples the magnetron motion to either the axial or the modified cyclotron motion. In the case of coupling between the magnetron and modified cyclotron motion this is known as an ‘axialisation’ drive. In our case both of these drives are applied using a segmented ring electrode, which allows us to break the rotational symmetry of the trapping potential.

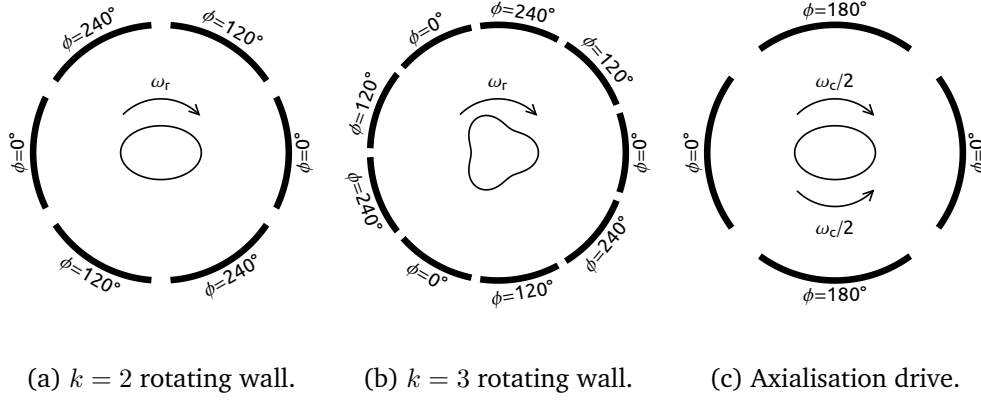


Figure 2.7: Electrode configurations for rotating wall and axialisation drives. A voltage of $V_{rw} = |V_{rw}| \sin(k\omega_{rw}t + \phi)$ is applied to each electrode to give rise to a potential of the required shape.

2.4.1 Rotating wall

As has been previously described in section 2.3.2, ion crystals in a Penning trap will rotate at a frequency ω_r with respect to the lab frame. To perform an experiment with these crystals it is often necessary to have a well-defined rotation frequency, and to have the phase of rotation locked to a reference signal. The idea behind a rotating wall drive is to create a spinning potential that breaks the cylindrical symmetry of the trap and forces the crystal to rotate at a known frequency. The simplest example of such a drive is the addition of a small quadrupolar potential which ‘squeezes’ slightly along one axis. The voltages on the electrode segments then oscillate with a fixed phase relationship in order to rotate this direction of squeezing.

Figure 2.7a shows the electrode configuration for a quadrupole ($k = 2$) rotating wall drive. In general, higher order rotating wall potentials may be applied to match the symmetry of the system in question. For example, large, 2D planar crystals have a hexagonal symmetry which is better suited to a $k = 3$ rotating wall [49].

To produce a rotating wall of order k , we require a number of ring electrode segments $\geq 2k + 1$. It is sufficient to have four ring electrodes to produce an oscillating quadrupole, but not a rotating one. The application of a rotating wall drive can modify the stability criteria for the trap [50].

2.4.2 Axialisation

Unlike the rotating wall drive, axialisation is a resonant effect that couples two of the modes of motion, namely the modified cyclotron and magnetron motion. This allows for both of the modes to be cooled simultaneously. As shown in (2.22) the magnetron motion contributes negatively to the energy of the motion. If, for example, resistive cooling is used to attempt to apply a damping force to the ion, the magnetron radius

will increase. To combat this we transfer energy from the magnetron motion to the cyclotron motion in such a way that we remove amplitude from both on average¹.

We apply the voltages shown in Figure 2.7c to the electrodes at the cyclotron frequency ω_c , which is the sum of the magnetron and modified cyclotron frequencies. The reasoning behind the application of this drive is that by applying an oscillating potential at ω_c we attempt to drive transitions $|n_-, n_+\rangle \longleftrightarrow |n_- - 1, n_+ + 1\rangle$. Classically it can be shown that a quadrupolar drive is required in order to couple the two motions [52]. Applying an oscillating voltage of amplitude V_a , but with the phase shown in Figure 2.7c, to each electrode produces an electric potential, ϕ_a , given by

$$\phi_a = V_a \frac{(x^2 - y^2)}{r_0^2} \cos \omega_c t. \quad (2.51)$$

By transforming to polar coordinates we can use some basic trigonometric identities to re-express the axialisation drive in terms of two counter-rotating quadrupole potentials, each rotating with a frequency of $\omega_c/2$.

$$\begin{aligned} \phi_a &= V_a \frac{\left((r \cos \theta)^2 - (r \sin \theta)^2\right)}{r_0^2} \cos \omega_c t \\ &= V_a \frac{r^2}{2r_0^2} (\cos(2\theta + \omega_c t) + \cos(2\theta - \omega_c t)) \end{aligned} \quad (2.52)$$

It is useful to see what this potential does in the $\omega_c/2$ rotating frame that is discussed in section 2.3.2. Here we have one non-rotating quadrupole potential, and one that rotates anticlockwise with respect to the magnetic field direction at an angular frequency of ω_c . In this frame we are attempting to drive zero frequency transitions between the degenerate clockwise and anticlockwise ion motions. This allows us to neglect the fast rotating term in this frame and consider the stationary quadrupole. This potential, $\phi'_a = V_a/r_0^2(x'^2 - y'^2)$, when added to the radial trapping potential creates one strong axis (which we can define as the x -axis) and one weak axis (the y -axis). If we have oscillatory x and y motions of the same amplitude, the frequency difference between the motion along the two axes will cause a time-dependent phase difference between the two motions. The motion will therefore transition from clockwise circular, through linear, to anticlockwise circular in a manner analogous to the change of polarisation of light from left-hand circular, to right-hand circular when passing through a birefringent material. The rate at which the motion transfers from clockwise to anticlockwise

¹In precision experiments this type of coupling is sometimes known as ‘sideband cooling’ [51]. To avoid confusion I will only use the term sideband cooling in reference to optical sideband cooling, as described in section 3.4.2.

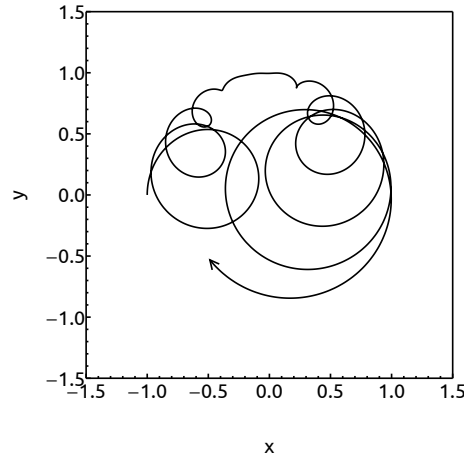


Figure 2.8: Motion of a single ion in a Penning trap with an axialisation drive, in the absence of cooling.

is given by

$$\begin{aligned}\Omega_a &= \omega_x - \omega_y \\ &= \sqrt{\omega_1^2 + \frac{eV_a}{mr_0^2}} - \sqrt{\omega_1^2 - \frac{eV_a}{mr_0^2}}.\end{aligned}\tag{2.53}$$

As mentioned in section 2.3.2, this exchange between clockwise to anticlockwise motion corresponds to an oscillation between magnetron and modified cyclotron motion in the lab frame. This motion is shown in Figure 2.8. As will be explained further in section 3.4.1 it is easier to cool the modified cyclotron motion, and therefore cooling of this motion along with the application of this drive allows for simultaneous reduction of the radii of both motions [53].

Chapter 3

Interaction of a trapped ion with monochromatic light

The interaction between trapped ions and electromagnetic radiation is the main means by which the quantum state of the ions is controlled. The combination of their quantised motion inside the trap and the ability to perform coherent operations using light contributes to the high level of control exhibited in quantum information experiments using trapped ions. In this chapter I will present the basic semiclassical theory of the light-ion interactions that we make use of in our experiment. This theory is semiclassical in the sense that we consider an ion with quantised energy levels (both due to the motion of the electrons around the nucleus, and due to the motion of the ion in the Penning trap), but consider a classical oscillating electric field.

3.1 Form of the interaction Hamiltonian

The first step towards establishing the form of the interaction Hamiltonian is to recognise our ability to separate the Hamiltonian of the system into independent parts, which allows the internal state of the ion, the ion motion in the trap and the light-ion interaction to be treated separately,

$$H = H_{\text{ion}} + H_{\text{trap}} + H_{\text{int}} \quad (3.1)$$

where H_{ion} is the Hamiltonian governing the internal electronic states of the atom, H_{trap} is the Hamiltonian of the ion's motion inside the trap (given in (2.44)) and H_{int} is the interaction Hamiltonian.

To demonstrate our ability to make this separation of the Hamiltonian, we can begin by working in the ion's rest frame, and therefore neglect the motion of the ion inside the trap, and then consider the Hamiltonian of the valence electron of the ion, with

charge $-e$, in an electromagnetic field

$$H_{\text{EM}} = \frac{(\mathbf{p} + e\mathbf{A})^2}{2m} - e\phi + \frac{e}{m}\mathbf{S} \cdot \mathbf{B}, \quad (3.2)$$

where $\mathbf{S}$ is the spin operator¹ and the other symbols are as in (2.32). By choice of the Coulomb gauge ($\nabla \cdot \mathbf{A} = 0$) the vector potential $\mathbf{A}$ becomes associated with the transverse electric field (i.e. electromagnetic radiation) and ϕ is associated with the ‘quasi-static’ longitudinal field (i.e. the field due to the charge of the nucleus and other electrons). If we expand the $(\mathbf{p} + e\mathbf{A})^2$ term and neglect the term that is non-linear in $\mathbf{A}$, we get

$$H_{\text{EM}} = \left(\frac{\mathbf{p}^2}{2m} - e\phi \right) + \frac{e}{m} (\mathbf{A} \cdot \mathbf{p} + \mathbf{S} \cdot \mathbf{B}) \quad (3.3)$$

$$= H_0 + H_{\text{int}}. \quad (3.4)$$

H_0 is the Hamiltonian that deals with the motion of the ion’s valence electron around the nucleus, equivalent to H_{ion} in (3.1). The second term is due to the interaction between the electron motion and the oscillating vector potential $\mathbf{A}$ and magnetic field $\mathbf{B}$. The details of H_{ion} and the energy levels of the particular ion species used in our experiment will be given in the next chapter, but in order to obtain the form of the interaction Hamiltonian it is sufficient to consider a two-level system, with known ground and excited wavefunctions $|g\rangle$ and $|e\rangle$ separated by $\hbar\omega_0$ in energy. By setting the energy to be zero halfway between the two levels we can then write H_{ion}

$$H_{\text{ion}} = -\frac{\hbar\omega_0}{2} |g\rangle \langle g| + \frac{\hbar\omega_0}{2} |e\rangle \langle e| \quad (3.5)$$

$$= \frac{\hbar\omega_0}{2} \sigma_z \quad (3.6)$$

where $\sigma_z = |e\rangle \langle e| - |g\rangle \langle g|$, which is equivalent to the Pauli- z matrix in the electronic state basis.

By substituting a plane-wave vector potential

$$\mathbf{A} = - \int \mathbf{E} dt = \frac{E_0}{\omega_L} \epsilon_L (e^{i(\mathbf{k}_L \cdot \mathbf{r} - \omega_L t)} + \text{c.c.}) \quad (3.7)$$

into (3.3) and dropping the $\mathbf{S} \cdot \mathbf{B}$ term, the interaction Hamiltonian becomes

$$H_{\text{int}} = \frac{e}{m} \frac{E_0}{2\omega_L} \epsilon_L \cdot \mathbf{p} (e^{i(\mathbf{k}_L \cdot \mathbf{r} - \omega_L t)} + \text{c.c.}) \quad (3.8)$$

where E_0 , ϵ_L , ω_L and $\mathbf{k}_L$ are the electric field amplitude, polarisation, frequency and

¹The $\mathbf{S} \cdot \mathbf{B}$ term is included for completeness as it is required in order to treat magnetic dipole transitions, which are commonly used in ion trap experiments to drive transitions between hyperfine ground states, but are not used in our experiment, so this term will be neglected later.

wavevector of the light. In order to simplify this expression a Taylor expansion of the exponential term is performed, $e^{i\mathbf{k}_L \cdot \mathbf{r}} \approx 1 + i\mathbf{k}_L \cdot \mathbf{r} + \dots$, where different orders in the expansion correspond to different types of transition. Taking only the constant term of the expansion is commonly known as ‘the dipole approximation’ and gives the Hamiltonian for electric dipole transitions. When making the dipole approximation we are assuming that the wavelength of the light is much greater than the size of the electron orbit. This is usually a very good assumption as the wavelength of light is typically on the order of 500 nm, and the electron orbital size is on the ångström (0.1 nm) level. We assume that if the transition is ‘dipole-allowed’ (by selection rules) that this term dominates, but if the transition is dipole-forbidden then we have to consider the term associated with electric quadrupole transitions, which is the symmetric part of the $i\mathbf{k}_L \cdot \mathbf{r}$ term [54]. Using the relationship $\mathbf{p} = \frac{1}{i\hbar}[\mathbf{r}, H_0]$ the electric dipole term of the Hamiltonian can be re-expressed as

$$H_{E1} = \frac{e}{i\hbar m} \frac{E_0}{2\omega_L} \boldsymbol{\epsilon}_L \cdot [\mathbf{r}, H_0] (e^{-i\omega_L t} + \text{c.c.}). \quad (3.9)$$

Evaluating the transition matrix elements $\langle g | H_{E1} | e \rangle$ and $\langle e | H_{E1} | g \rangle$ and substituting the form of H_0 given in (3.5) the interaction Hamiltonian becomes

$$H_{E1} = \hbar (\Omega_0 \sigma_- + \Omega_0^* \sigma_+) \cos(\omega_L t) \quad (3.10)$$

where

$$\Omega_0^{E1} = \frac{eE_0}{\hbar} \boldsymbol{\epsilon}_L \cdot \langle g | \mathbf{r} | e \rangle \quad (3.11)$$

is the dipole transition Rabi frequency, and $\sigma_+ = |e\rangle \langle g|$ and $\sigma_- = |g\rangle \langle e|$. If we follow the same steps for the electric quadrupole term we obtain a Hamiltonian of

$$H_{E2} = \hbar (\Omega_0 \sigma_- + \Omega_0^* \sigma_+) \sin(\omega_L t) \quad (3.12)$$

which is the same as (3.10) up to a global phase, but with a different Rabi frequency given by

$$\Omega_0^{E2} = \frac{eE_0\omega_0}{2\hbar c} \boldsymbol{\epsilon}_L \cdot \langle g | \mathbf{r} \mathbf{r} | e \rangle \cdot \hat{\mathbf{n}} \quad (3.13)$$

where $\hat{\mathbf{n}} = \mathbf{k}_L c / \omega_0$ is the unit vector in the direction of propagation of the light. Expressions for these Rabi frequencies can be found for transitions between specific states in a particular ion species. The method for calculating these frequencies, including their spatial dependence, will be discussed in the next chapter.

In order to study the dynamics associated with this Hamiltonian it is useful to move to the interaction picture. This representation lies somewhere between the Schrödinger and Heisenberg representations, where the ω_0 time dependence is absorbed into the wavefunctions, but the observables retain any time dependence caused by the interaction. To make this change in representation the unitary transformation

$H_{E1}^I = e^{\frac{iH_0t}{\hbar}} H_{E1} e^{-\frac{iH_0t}{\hbar}}$ is performed. At this point a ‘rotating wave approximation’ is also made. This assumes that the light is close to resonance ($\omega_0 \approx \omega_L$) and that we can average out the effects due to terms that oscillate quickly at $\omega_0 + \omega_L \approx 2\omega_0$, and only consider the effect of the slow varying terms in $\omega_0 - \omega_L = \delta_L$. This yields

$$H_{E1}^I = \frac{\hbar}{2} \left(\Omega_0 \sigma_- e^{-i\delta_L t} + \Omega_0^* \sigma_+ e^{i\delta_L t} \right). \quad (3.14)$$

For simplicity we will assume that the Rabi frequencies are real (i.e. $\Omega_0 = \Omega_0^*$).

To see what effect this Hamiltonian has on the quantum state of the ion, we need to consider the state probability amplitudes c_g and c_e in the state $|\psi\rangle = c_g |g\rangle + c_e |e\rangle$. By looking at the Schrödinger equation for this state we can obtain the following simultaneous equations

$$i\dot{c}_g = -\frac{1}{2i} \Omega_0 e^{i\delta_L t} c_e \quad (3.15)$$

$$i\dot{c}_e = \frac{1}{2i} \Omega_0 e^{-i\delta_L t} c_g \quad (3.16)$$

which can be solved for the initial conditions $|c_g(t=0)|^2 = 1$ and $|c_e(t=0)|^2 = 0$, to give the evolution of the excited state probability $|c_e(t)|^2$ with time,

$$|c_e(t)|^2 = \frac{\Omega_0^2}{\Omega^2} \sin^2 \left(\frac{\Omega t}{2} \right) \quad (3.17)$$

where $\Omega^2 = \Omega_0^2 + \delta_L^2$. This sinusoidal population transfer is commonly referred to as a Rabi oscillation. Figure 3.1 shows the form of these oscillations after an excitation of varying pulse length, and for different detunings from resonance. As the detuning from resonance δ_L is increased the amplitude of the oscillation decreases and the frequency increases. The inset shows the form of the lineshape of the interaction if the pulse time is kept constant and the frequency is varied across the resonance. The amplitude of the oscillations is given by $\frac{\Omega_0^2}{\Omega_0^2 + \delta_L^2}$ and results in a Lorentzian envelope to the lineshape, with a width given by $2\Omega_0$.

3.2 Axial sidebands

So far in this treatment we have neglected the motion of the ion inside the trap and implicitly assumed that our ion is stationary in free space. Classically, if we consider a trapped ion undergoing harmonic motion at some frequency ω , the Doppler effect will cause a modulation of the resonance frequency in a stationary ‘lab’ frame (or modulation of the apparent frequency of the light in the ion’s rest frame). This modulation will result in frequency modulation sidebands spaced by ω , with amplitudes dependent on the depth of the modulation, which is related to the amplitude of the motion. In order to find the form of these sidebands for quantum mechanical ion motion we need to

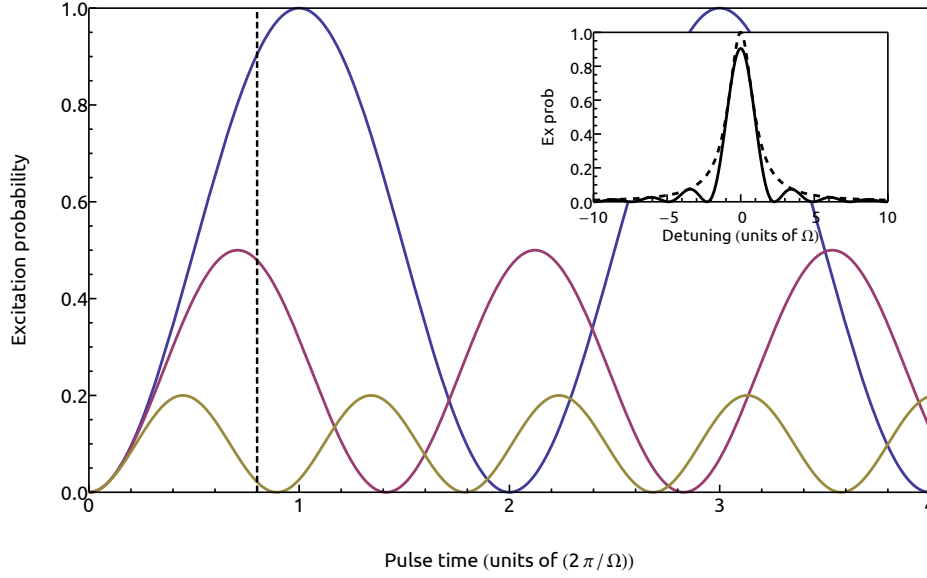


Figure 3.1: The excitation probability $|c_e(t)|^2$, plotted against pulse time. The curves shown are for light on resonance ($\delta_L=0$) (blue), and for detunings of $\delta_L = \Omega_0$ (magenta) and $\delta_L = 2\Omega_0$ (gold) from the transition frequency. The inset shows excitation probability against δ_L for a fixed pulse length of $2\pi \times 0.8/\Omega$ (this pulse time is marked on the main graph with a dashed line).

modify (3.14) to take into account the varying phase of the light in the ion's rest frame. Initially we will consider light travelling along the z -axis, and neglect any effects due to the radial motion, so this extra phase becomes $\mathbf{k}_L \cdot \mathbf{r} \rightarrow k_L z^2$. This gives

$$H^I = \frac{\hbar\Omega_0}{2} \left(e^{i(kz - \delta_L t)} \sigma_+ + e^{-i(kz - \delta_L t)} \sigma_- \right). \quad (3.18)$$

We can now substitute in (2.47), the quantum expression for z in terms of the creation and annihilation operators, to get

$$H^I = \frac{\hbar\Omega_0}{2} \left(e^{i(\eta_z(a_z + a_z^\dagger) - \delta_L t)} \sigma_+ + e^{-i(\eta_z(a_z + a_z^\dagger) - \delta_L t)} \sigma_- \right) \quad (3.19)$$

where the 'Lamb-Dicke parameter' $\eta_z = k_L z_0 / \sqrt{2} = \sqrt{\frac{2\hbar\pi^2}{m\omega_z \lambda^2}}$ has been introduced. This quantity gives some indication to the number of sidebands that will be produced for a given amplitude of motion. The Lamb-Dicke parameter can be re-expressed as $\eta_z = \sqrt{\frac{\omega_R}{\omega_z}}$ where $\hbar\omega_R$ is the recoil energy (the energy associated with an ion momentum change of $\hbar k_L$). An analogy can be drawn between this parameter and the classical frequency modulation index $h = \frac{\Delta f}{f}$, which gives the heights of the various sidebands produced. In the same way the Lamb-Dicke parameter sets the relative strengths of

²This position vector $\mathbf{r}$ corresponds to the ion's motion in the trap, as opposed to in e.g. (3.9) where it corresponds to the position of the electron with respect to the nucleus.

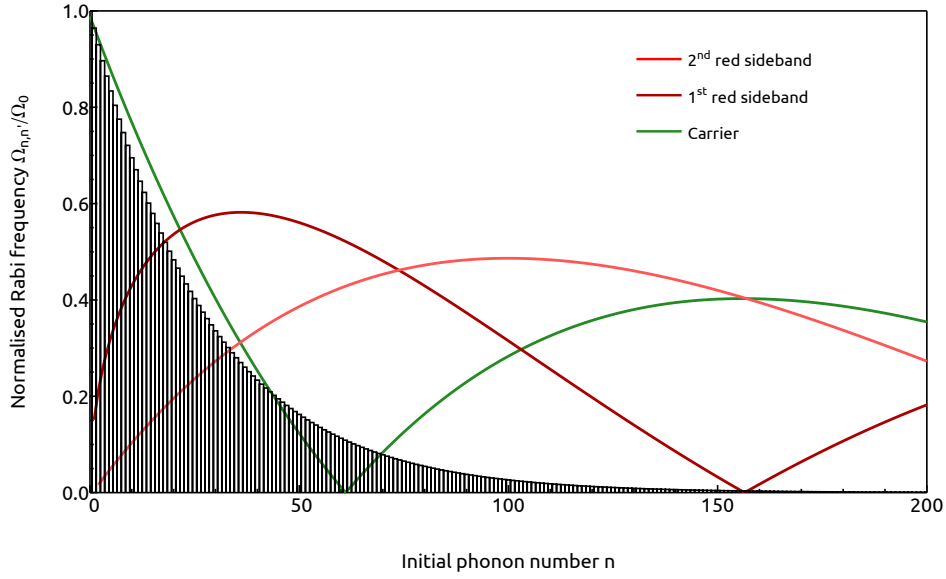


Figure 3.2: A plot of Rabi frequency on the carrier, first red sideband and second red sideband (green, dark red and red respectively) for $\omega_z = 2\pi \times 400$ kHz ($\eta = 0.153$ for the 729 nm transition in $^{40}\text{Ca}^+$). The probability distribution for a thermal state with $\bar{n} = 25$ is also shown for comparison.

different sidebands.

Excitation on a sideband results in a change in phonon number of the motion, the value of which is given by the order of the sideband. In order to evaluate the Rabi frequencies when driving transitions that change phonon number from $|g, n\rangle$ to $|e, n'\rangle$ we need to evaluate the matrix elements

$$\Omega_{n,n'} = \Omega_0 \langle n' | e^{i\eta_z(a_z + a_z^\dagger)} | n \rangle. \quad (3.20)$$

These elements can be evaluated to give the analytical form

$$\Omega_{n,n'} = \Omega_0 \sqrt{\frac{\min\{n, n'\}!}{\max\{n, n'\}!}} \eta_z^{|n'-n|} e^{-\eta_z^2/2} L_{\min\{n, n'\}}^{|n'-n|}(\eta_z^2) \quad (3.21)$$

where $L(\eta^2)$ is an associated Laguerre polynomial [55].

Figure 3.2 shows the variation in Rabi frequency with the initial Fock state n for low order sidebands. One important feature that can be seen in this plot is that in this example, Fock states close to $n = 155$ have a first red sideband Rabi frequency that is close to zero. This has implications for sideband cooling, as will be described in section 3.4.2. For a thermal ion, the probability of finding the ion in a particular Fock state is given by

$$P(n) = \frac{\bar{n}^n}{(\bar{n} + 1)^{n+1}} \quad (3.22)$$

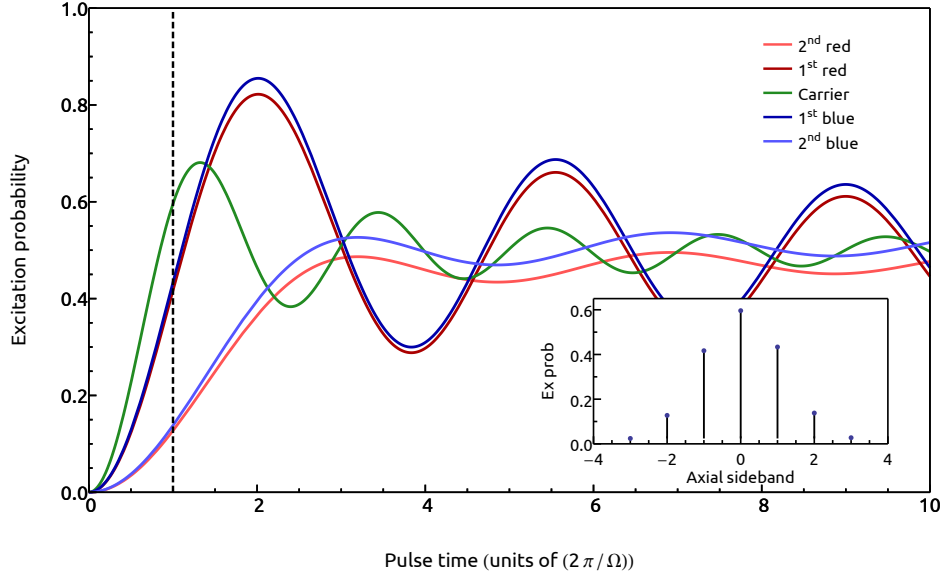


Figure 3.3: Excitation probability on resonance with each of the five central axial sidebands at an axial trap frequency of $\omega_z = 2\pi \times 400$ kHz ($\eta = 0.153$ for the 729 nm transition in $^{40}\text{Ca}^+$), with an average phonon number $\bar{n}_z = 25$ (close to the Doppler limit at this frequency, as shown in Figure 3.6). The inset shows the expected heights of the axial sidebands for a pulse time of $t = 2\pi/\Omega$, marked with the dashed line in the main panel.

where $\bar{n}$ is the average phonon occupation number [56]. In order to calculate the excitation probability $|c_e(t)|^2$ of the ion after a pulse of light of length t we need to add contributions from the Rabi oscillations of all the different Fock states, weighted by the probability that the ion is initially in that particular Fock state. The observed evolution of excitation probability on a sideband associated with a change of phonon number δn is therefore given by

$$|c_e(t)|^2 = \sum_{n=n'-\delta n}^{\infty} \frac{\bar{n}^n}{(\bar{n}+1)^{n+1}} \sin^2\left(\frac{\Omega_{n,n'}t}{2}\right) \quad (3.23)$$

Figure 3.3 shows an example of the sideband evolution for a thermal state with average phonon number $\bar{n}_z = 25$ at a trap frequency $\omega_z = 2\pi \times 400$ kHz, which is roughly the occupation we expect to reach after Doppler cooling for sensible experimental parameters (see section 3.4.1). The difference in frequency between terms in the sum in (3.23) leads to a decay in the visibility of the Rabi oscillations as the evolution time increases. The Rabi frequency scaling on different sidebands is such that in this example the decay in the carrier oscillation visibility occurs more quickly than that of the first order sidebands. There is also a small asymmetry between the red and blue first order sidebands. This is due to the population in the $n = 0$ Fock state, which has no red

sideband strength. Any measured asymmetry between the red and blue sidebands can be used to calculate the $\bar{n}$ of the thermal distribution. The inset shows an example of the sideband heights that would be obtained for the thermal state for the pulse length marked in the main figure panel.

3.2.1 The Lamb-Dicke Regime

If we look at the form of (3.20), the expression for the Rabi frequency for transitions between Fock states n and n' , we see that we can make an approximation similar in nature to the dipole approximation and perform a Taylor expansion on the exponential term. This gives

$$\Omega_{n,n'} = \Omega_0 \langle n' | e^{i\eta_z(a_z + a_z^\dagger)} | n \rangle \quad (3.24)$$

$$\approx \Omega_0 \langle n' | 1 + i\eta_z(a_z + a_z^\dagger) - \frac{1}{2}\eta_z^2(a_z + a_z^\dagger)^2 | n \rangle. \quad (3.25)$$

To make this step we have assumed that the amplitude of the motion of the ion is much smaller than the wavelength of the light $\eta_z(2\bar{n} + 1) \ll 1$. The regime in which this approximation holds is known as the ‘Lamb-Dicke regime’, and many schemes require that the ion is cooled into this regime.

This approximation leads to significantly simpler expressions for the Rabi frequencies, compared with the full expression given in (3.21). By collecting terms that correspond to particular sideband transitions, and therefore particular values of $\delta n = n' - n$, we can find the scaling of the sideband Rabi frequencies giving

$$\Omega_{n,n-1} = \Omega\eta\sqrt{n} \quad (3.26)$$

$$\Omega_{n,n} = \Omega(1 - \frac{1}{2}\eta^2(2n + 1)) \quad (3.27)$$

$$\Omega_{n,n+1} = \Omega\eta\sqrt{n+1} \quad (3.28)$$

where $\Omega_{n,n-1}$, $\Omega_{n,n}$ and $\Omega_{n,n+1}$ where these are the Rabi frequencies on the first red sideband, the carrier and the first blue sideband.

3.3 Radial sidebands

In order to find the form of the radial sidebands we use the same method as for the axial sidebands, but with a beam propagating in the x -direction rather than z . In the radial case both the modified cyclotron and magnetron motions have an effect on the interaction since the radial motion is coupled in the x and y directions. By adding a phase $\mathbf{k}_L \cdot \mathbf{r} \rightarrow k_L x$ and substituting in the creation and annihilation operator form of x from (2.45), the Rabi frequency for the transition between the motional state

$|g, n_+, n_- \rangle$ and $|e, n'_+, n'_- \rangle$ is given by

$$\Omega_{n_{\pm}, n'_{\pm}} = \Omega_0 \langle n'_{\pm}, n'_{\pm} | e^{i\eta_r(a_+ + a_+^\dagger + a_- + a_-^\dagger)} | n_+, n_- \rangle. \quad (3.29)$$

One thing to note is that both of the radial modes are associated with a single Lamb-Dicke parameter $\eta_r = kr_0/2$, which is characterised by the radial trapping strength in the $\frac{\omega_c}{2}$ rotating frame, ω_1 , rather than by the mode frequencies in the lab frame, $\omega_{\pm}$. The form of the Rabi frequency can be found in the same way as (3.21) to give

$$\begin{aligned} \Omega_{n_{\pm}, n'_{\pm}} = & \Omega_0 \sqrt{\frac{\min\{n_-, n'_-\}! \min\{n_+, n'_+\}!}{\max\{n_-, n'_-\}! \max\{n_+, n'_+\}!}} \eta_r^{|\delta n_-| + |\delta n_+|} e^{-\eta_r^2} \\ & \times L_{\min\{n_-, n'_-\}}^{|\delta n_-|}(\eta_r^2) L_{\min\{n_+, n'_+\}}^{|\delta n_+|}(\eta_r^2) \end{aligned} \quad (3.30)$$

where $\delta n_- = n'_- - n_-$ and $\delta n_+ = n'_+ - n_+$ are the changes in the radial quantum numbers.

Figure 3.4 shows an example of the radial sideband heights for realistic experimental parameters. The average phonon numbers used are $\bar{n}_- = 300$ for the magnetron motion and $\bar{n}_+ = 20$ for the modified cyclotron motion, and the mode frequencies are $\omega_+ = 2\pi \times 695 \text{ kHz}$ and $\omega_- = 2\pi \times 19.7 \text{ kHz}$. The resulting spectrum shows a comb of modified cyclotron sidebands separated by $\omega_+/2\pi$, with each of them surrounded by a comb of magnetron sidebands separated by $\omega_-/2\pi$. For these parameters there are many more magnetron sidebands visible around the carrier than there are cyclotron sidebands due to the larger average phonon number. However, due to the larger spacing between sidebands the overall width in frequency of the modified cyclotron sideband envelope is wider than that of the magnetron sidebands, meaning that the velocity associated with modified cyclotron motion is larger. The high value of $\bar{n}_-$ means that there is very little asymmetry between red and blue magnetron sidebands, and also means the general envelope of the sidebands is approximately Gaussian.

3.4 Laser cooling in a Penning trap

3.4.1 Doppler cooling with an intensity gradient

Doppler cooling is probably the most frequently used type of laser cooling in ion trap experiments, and it is often used as a preliminary cooling stage before other types of cooling, such as electromagnetically-induced transparency (EIT) [57] or sideband cooling [7, 8] are used. Doppler cooling is usually performed using dipole-allowed transitions, where the condition $\Gamma > \omega_z$ is met. This is referred to as the ‘weak-binding regime’, in which the underlying sideband structure caused by the ion motion cannot be resolved as the natural linewidth of the transition is too large. The basic principle of Doppler cooling is that if a laser beam is red-detuned, then it will predominantly

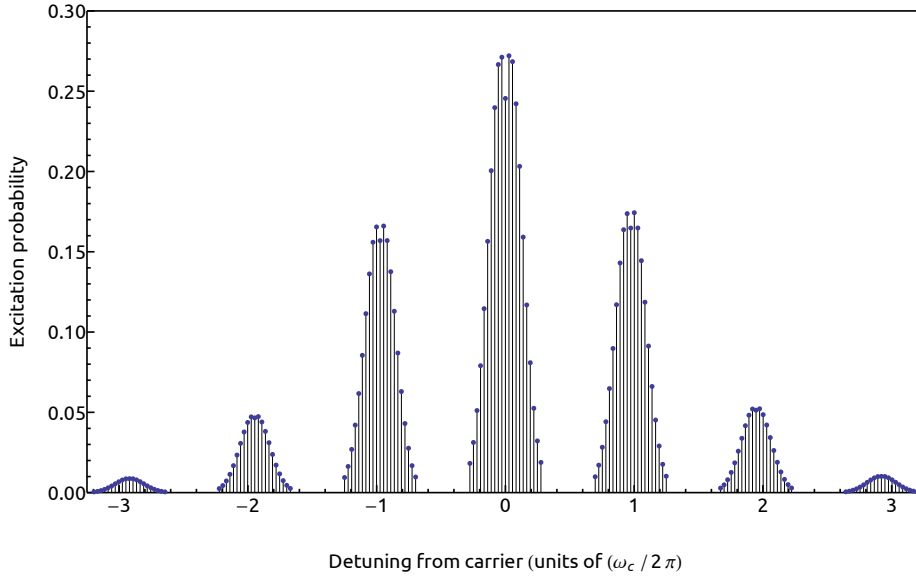


Figure 3.4: The calculated excitation probability sidebands for a beam propagating in the x -direction for a pulse time of $t_p = 1/\Omega_0$. The sidebands are spaced at the modified cyclotron frequency $\omega_+ = 2\pi \times 695.0$ kHz, and the smaller magnetron frequency $\omega_- = 2\pi \times 19.7$ kHz

interact with the ion when it is moving in the opposite direction to laser propagation. Since the re-emission of any absorbed photons should occur with equal probability towards and away from the laser there should be a removal of energy from the motion on average until a cooling limit is reached.

In order to model how laser cooling works in a Penning trap we can consider the effect of the absorption and re-emission of a photon on the motion of the ion. We assume that the photon scattering event results in an instantaneous change in ion velocity given by $\Delta \mathbf{v} = \frac{\hbar}{m}(\mathbf{k} - \mathbf{k}_s)$ and that it occurs at a time $t = t_0$. The ion motion in the Penning trap ((2.17), (2.18) and (2.19)) is initially characterised by mode amplitudes r_i , and phases θ_i and after the photon is scattered is characterised by amplitudes r'_i , and phases θ'_i . The change in square amplitude of each mode after a scattering event, $\Delta r_i^2 = r_i'^2 - r_i^2$, are given by

$$\Delta r_z^2 = \left(\frac{\Delta v_z}{\omega_z} \right)^2 - \frac{2r_z \Delta v_z}{\omega_z} \sin(\omega_z t_0 + \theta_z) \quad (3.31)$$

$$\Delta r_+^2 = \frac{((\Delta v_x)^2 + (\Delta v_y)^2)}{4\omega_1^2} - \frac{\Delta v_x}{\omega_1} r_+ \sin(\omega_+ t_0 + \theta_+) - \frac{\Delta v_y}{\omega_1} r_+ \cos(\omega_+ t_0 + \theta_+) \quad (3.32)$$

$$\Delta r_-^2 = \frac{((\Delta v_x)^2 + (\Delta v_y)^2)}{4\omega_1^2} + \frac{\Delta v_x}{\omega_1} r_- \sin(\omega_- t_0 + \theta_-) + \frac{\Delta v_y}{\omega_1} r_- \cos(\omega_- t_0 + \theta_-) \quad (3.33)$$

where all variables are as defined in Chapter 2. Equations (3.31) and (3.32) show

that for the amplitudes of the axial and modified cyclotron motion to be reduced, the instantaneous velocity prior to the scattering, $v_i(t_0)$, must be in the opposite direction to the propagation of the cooling light, $\mathbf{k}_i$. In (3.33) the changes in sign mean that to reduce the amplitude of the magnetron motion, the instantaneous velocity needs to be in the same direction as the propagation. This means that we have conflicting requirements to cool the two radial motions. By red detuning the cooling laser ($\omega_L < \omega_0$) we preferentially increase the rate of scattering when the velocity and laser beam propagation are in opposite directions due to the Doppler effect, which will result in cooling of the axial and modified cyclotron motion, but will result in heating of the magnetron motion. This kind of problem can be anticipated by looking at (2.22) and noting that the magnetron motion contributes negatively to the total energy.

The difficulty in simultaneously reducing the amplitude of all three modes of motion, is discussed in detail in ref. [58] by W. Itano and D. J. Wineland. One solution is to Doppler cool the axial motion as normal with a red-detuned beam propagating in the z -direction, and cool the radial motion with a separate red-detuned laser beam, propagating in the radial plane with a spatial offset from the trap centre, as shown in Figure 3.5. If we define the direction of propagation of this beam as the x -direction this means that there is an intensity gradient in the y -direction at the position of the ion. The combination of the red-detuning and the change of scattering rate at different points in the ion motion due to the intensity gradient results in a section of the parameter space in which we are able to simultaneously cool both of the radial modes.

In order to find the steady state limits for this type of cooling we first need to find expressions for the rate of change of the mode amplitudes. To find these rates, we multiply the expressions for the change in amplitude per scattering event, (3.31), (3.32) and (3.33), by the scattering rate. The scattering rate, including the Doppler effect, is given by

$$R = \frac{\Gamma \Omega^2}{(\Gamma^2 + 4(\mathbf{k} \cdot \mathbf{v} + \omega_0 - \omega_L)^2)} \approx \frac{\Gamma \Omega^2}{\Gamma^2 + 4\delta_L^2} \left(1 - \frac{8\delta_L \mathbf{k} \cdot \mathbf{v}}{(\Gamma^2 + 4\delta_L^2)} \right) \quad (3.34)$$

where a Taylor expansion has been performed, assuming that $\mathbf{k} \cdot \mathbf{v} \ll \Gamma$. If we average over all possible phases θ_i of the scattering event, and assume a linear intensity gradient across the ion motion of $I(y) = I_0 \frac{y}{y_0}$, the equations for the rate of change of the square

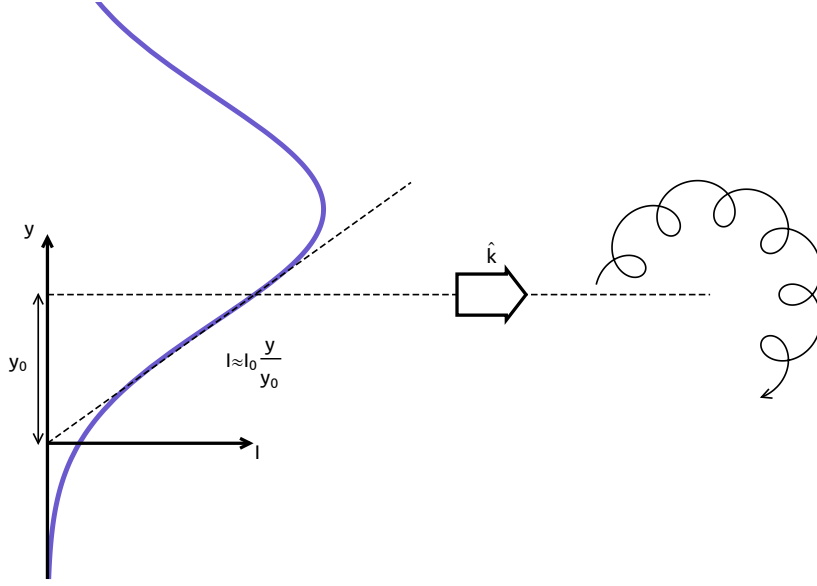


Figure 3.5: Offset beam method for cooling both radial modes simultaneously using a red-detuned laser beam. The size of the ion motion has been exaggerated; the calculation of single ion cooling limits assumes that this motion is much smaller than the cooling beam size, meaning that the intensity profile of the beam can be approximated to a linear gradient.

amplitudes become

$$\frac{dr_z^2}{dt} = \gamma_{sz} \left(\frac{\hbar^2 k_L^2 \left(1 + \left(1 + \frac{\gamma_{sr}}{\gamma_{sz}} \right) f_{sz} \right)}{m^2 \omega_z^2} - \frac{8 \hbar k_L^2 \delta_L r_z^2}{m(\Gamma^2 + 4\delta_L^2)} \right) \quad (3.35)$$

$$\frac{dr_+^2}{dt} = \gamma_{sr} \left(\frac{\hbar^2 k_L^2 \left(1 + \left(1 + \frac{\gamma_{sz}}{\gamma_{sr}} \right) (f_{sx} + f_{sy}) \right)}{4m^2 \omega_1^2} - \frac{4 \hbar k_L^2 \delta_L \omega_+ r_+^2}{m \omega_1 (\Gamma^2 + 4\delta_L^2)} + \frac{\hbar k_L r_+^2}{2m \omega_1 y_0} \right) \quad (3.36)$$

$$\frac{dr_-^2}{dt} = \gamma_{sr} \left(\frac{\hbar^2 k_L^2 \left(1 + \left(1 + \frac{\gamma_{sz}}{\gamma_{sr}} \right) (f_{sx} + f_{sy}) \right)}{4m^2 \omega_1^2} + \frac{4 \hbar k_L^2 \delta_L \omega_- r_-^2}{m \omega_1 (\Gamma^2 + 4\delta_L^2)} - \frac{\hbar k_L r_-^2}{2m \omega_1 y_0} \right) \quad (3.37)$$

where $f_{si} = \int P(\hat{k}_s) \hat{k}_{si}^2 d\Omega$ is a geometric factor which takes into account the emission pattern of the laser cooling transition, and γ_{sz} and γ_{sr} are the average scattering rates from the axial and radial beams respectively. By solving for the steady state we can evaluate the amplitudes of each of the modes after cooling, and then relate these to the phonon occupation numbers after Doppler cooling, $\bar{n}_i$, using (2.48), (2.49) and (2.50). Using (3.36) and (3.37), the condition for simultaneous reduction of both radial modes is

$$\omega_- < \frac{\Gamma^2 + 4\delta_L^2}{8y_0 k_L \delta_L} < \omega_+. \quad (3.38)$$

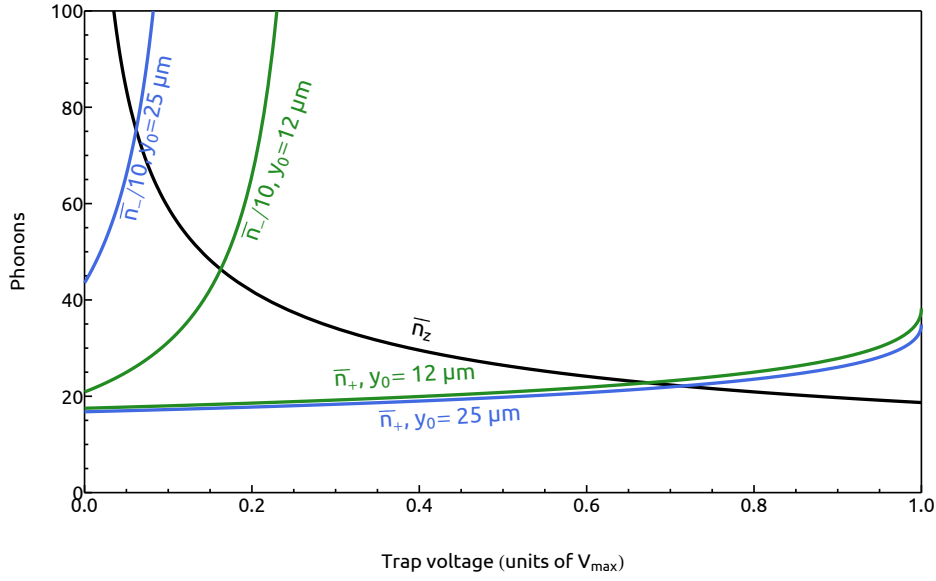


Figure 3.6: The theoretical limits for the average phonon numbers for the different modes after Doppler cooling. Note that the magnetron occupation number is divided by 10. Realistic beam gradient parameters of $y_0 = 12 \mu\text{m}$ and $25 \mu\text{m}$ have been used.

Provided that this condition is met, and by substituting the parameter values used in our experiment ($\delta_L = \Gamma/2$, $\gamma_{sz} = \gamma_{sr}$ and $f_{sz} = 0.4$ and $f_{sx} = f_{sy} = 0.3$) we obtain Doppler cooling limits for each mode in terms of the average phonon numbers of

$$\bar{n}_z \approx \frac{9}{20} \frac{\Gamma}{\omega_z} \quad (3.39)$$

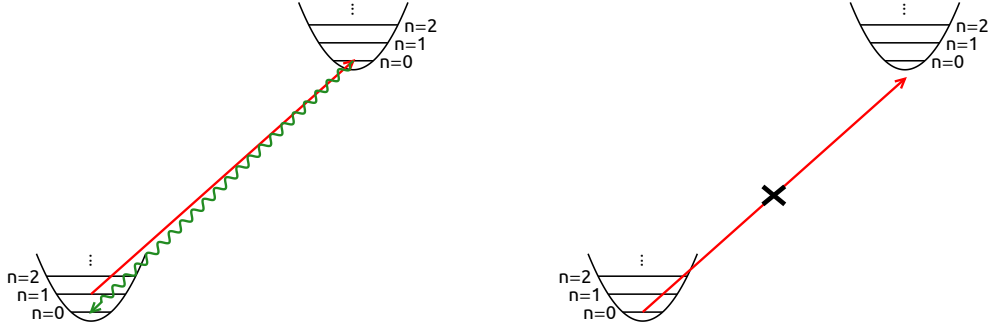
$$\bar{n}_+ \approx \frac{11}{20} \frac{\Gamma}{\omega_+} \frac{1}{\left(1 - \frac{\Gamma}{2y_0 k_l \omega_+}\right)} \quad (3.40)$$

$$\bar{n}_- \approx \frac{11}{20} \frac{\Gamma}{\omega_-} \frac{1}{\left(\frac{\Gamma}{2y_0 k_l \omega_-} - 1\right)}. \quad (3.41)$$

Figure 3.6 shows a plot of these limits for changing endcap voltage for a cyclotron frequency of $\omega_c = 2\pi \times 714.7 \text{ kHz}$ and realistic intensity gradient parameters of $y_0 = 12 \mu\text{m}$ and $25 \mu\text{m}$, showing that the endcap voltage at which the magnetron cooling ceases to work is dependent on the intensity gradient.

3.4.2 Sideband cooling

The necessity for cooling beyond the Doppler cooling limit can be seen in the sideband evolution for a thermal state shown in Figure 3.3. The decay in visibility of the oscillations indicates a lack of coherence in the system at the Doppler limit. In quantum information experiments we need to perform coherent operations on the ion and put



(a) Excitation on the red sideband and decay on the carrier.

(b) There is no transition on-resonance at the red sideband frequency from $|g, n = 0\rangle$.

Figure 3.7: In resolved-sideband cooling the laser is placed on resonance with the red sideband transition $|g, n\rangle \rightarrow |e, n - 1\rangle$. Once an excitation has taken place we rely on the fact that on average the decay occurs on the carrier transition in order to remove energy from the motion.

the ion in a known quantum state with high probability. For example we may need to perform a ‘ π -pulse’ on an ion to place it into the excited state. However, the excitation probability of the carrier in Figure 3.3 never exceeds 0.7. In order to increase the fidelity of these types of operations we must cool the ion into the Lamb-Dicke regime. One of the most common methods used in trapped ion experiments to cool beyond the Doppler limit is resolved-sideband cooling. As opposed to Doppler cooling, sideband cooling is performed in the strong-binding regime where $\Gamma < \omega_z$ and the sideband structure is therefore resolved. In a typical ion trap experiment the trap frequencies used are roughly an order of magnitude smaller than the linewidth of an electric dipole transition. Therefore in order to reach the strong-binding regime we must use a transition with a narrower linewidth, typically a stimulated Raman process or an electric quadrupole transition. Since our experiment uses an electric quadrupole transition for sideband cooling and spectroscopy, we will only consider this system.

The basic principle of sideband cooling is shown in Figure 3.7. The fact that the sidebands are resolved in the strong-binding regime means that we can specifically address the first red sideband transition. This drives the transition $|g, n\rangle \rightarrow |e, n - 1\rangle$. We use the fact that decay occurs predominantly on the carrier, especially as the ion is cooled into the Lamb-Dicke regime (a consequence of the (3.26), (3.27) and (3.28)), to remove one phonon from the motion on average per absorption. This process of absorption on the red sideband continues until we reach the ground state of the motion ($n = 0$), at which point there is no red sideband transition to excite.

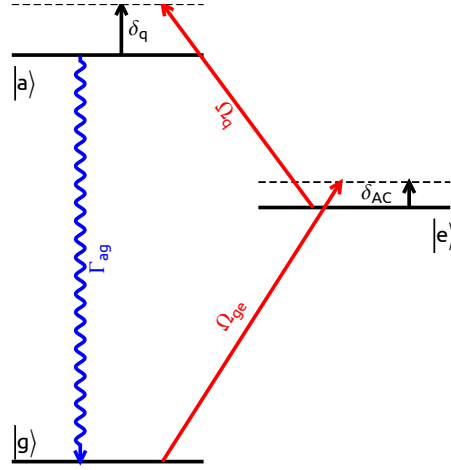


Figure 3.8: Cascade configuration for sideband cooling. One laser addresses narrow-linewidth, $|g\rangle \longleftrightarrow |e\rangle$ electric quadrupole transition and another ‘quenches’ this transition by addressing the $|e\rangle \longleftrightarrow |a\rangle$ electric dipole transition.

Artificial line broadening

When sideband cooling, the achievable cooling rate depends on the number of photons that can be scattered per second, and therefore on Γ , the decay rate of the excited state. Since the decay rate for a electric quadrupole transition is typically on the order of 1 s^{-1} , we have to artificially increase the rate of decay back to the ground state from the excited state. The theory behind a commonly-used broadening scheme is covered in detail by Marzoli et al [59]. The basic principle is to apply an extra laser frequency that couples the excited state to a third auxiliary state which has a large decay rate to the ground state, reducing the time before the ion can be re-excited on the red sideband to remove another phonon. The type of system used in our experiment is called a ‘cascade’ configuration, and is shown in Figure 3.8.

This coupling to the auxiliary state results in a new effective linewidth from the excited state to the ground state $\tilde{\Gamma}$ given by

$$\tilde{\Gamma} = \frac{\Omega_q^2}{(\Gamma_{ae} + \Gamma_{ag})^2 + 4\delta_q^2} \Gamma_{ae} \quad (3.42)$$

where Γ_{ag} and Γ_{ae} are the linewidths of the transitions between the auxiliary state and the ground and excited state respectively, and Ω_q and δ_q are the Rabi frequency and detuning of the coupling laser as labelled on Figure 3.8.

There is also an associated AC Stark shift of the sideband transition δ_{AC} , which changes the frequency at which the sideband cooling laser must be placed in order to

excite the first red sideband transition, given by

$$\delta_{AC} = -\frac{\Omega_q^2}{(\Gamma_{ae} + \Gamma_{ag})^2 + 4\delta_q^2} \delta_q. \quad (3.43)$$

This line broadening scheme can be used to broaden an electric quadrupole transition enough to reach the weak-binding regime, and to therefore Doppler cool using a dipole-forbidden transition [60].

Sideband cooling limits

After significant sideband cooling has occurred and there is significant population of the ground state, the cooling rate begins to slow as there is no possible red sideband transition from the ground state. Eventually a cooling limit is reached where the probability of the ion being in the $n = 1$ Fock state is sufficiently small that the rate of pumping into the $n = 0$ state by resonant driving of the red sideband is counter-balanced by the rate that population leaves the ground state due to off-resonant excitation and decay. To calculate this limit we consider the situation where the ion has already been cooled into the Lamb-Dicke regime the probability of excitation or decay on a second order sideband is negligible and that $P(n > 1)$ and only population in the $n = 0$ and $n = 1$ Fock states needs to be considered. We then assume that the sideband excitation on the electric quadrupole transition is the rate limiting step, and that excitation and decay on electric dipole transitions occur instantaneously, and that the sidebands are well resolved (i.e. $\tilde{\Gamma} \ll \omega$ and $\Omega_0 \ll \omega$). The total rate in this regime is given by

$$\begin{aligned} R &= \tilde{\Gamma} \sum_{n=0}^1 \sum_{m=-1}^1 P(n) \frac{\Omega_{n,n+m}^2}{2\Omega_{n,n+m}^2 + \tilde{\Gamma}^2 + 4\delta_m^2} \\ &= \frac{\tilde{\Gamma}\Omega_0^2\eta^2 P(n=1)}{2\Omega_0^2\eta^2 n_1 + \tilde{\Gamma}^2} + \frac{\tilde{\Gamma}\Omega_0^2}{4\omega_z^2} + \frac{\tilde{\Gamma}\Omega_0^2\eta^2 P(n=0)}{16\omega_z^2}. \end{aligned} \quad (3.44)$$

If we additionally consider the excitation by the quench laser at 854 nm, the subsequent decay at 393 nm and then a potential repumping stage at 397 nm if the decay leaves the ion in the wrong $S_{1/2}$ sublevel we can gather terms that change the phonon number of the ion and obtain rate equations given by

$$\frac{d}{dt} P(n=0) = \frac{\Omega_0^2\eta^2}{\tilde{\Gamma}} P(n=1) - \frac{\tilde{\Gamma}\Omega_0^2}{16\omega^2} (4(\tilde{\eta}^2 + \tilde{\eta}_r^2 + \eta_r^2 + \eta_q^2) + \eta^2) P(n=0) \quad (3.45)$$

$$\frac{d}{dt} P(n=1) = -\frac{d}{dt} P(n=0), \quad (3.46)$$

where are η_q , $\tilde{\eta}$, η_r and $\tilde{\eta}_r$ are Lamb-Dicke parameters associated with excitation at 854 nm, decay at 393 nm, excitation at 397 nm and decay excitation at 397 nm respectively. For non-isotropic decay (as in our case) there is an extra geometric factors

associated with $\tilde{\eta}$, $\tilde{\eta}_r$ and η_r , taking into account the likelihood that the emission is in a direction perpendicular to the motion of interest and has no effect.

Solving these equations for $P(n = 1) \approx \bar{n}$ results in a steady-state sideband cooling limit of

$$\bar{n}_{lim} = \left(\frac{\tilde{\Gamma}}{2\omega} \right)^2 \left(\frac{\tilde{\eta}^2 + \eta_q^2 + \eta_r^2 + \tilde{\eta}_r^2}{\eta^2} + \frac{1}{4} \right). \quad (3.47)$$

Using realistic experimental parameters of $\tilde{\Gamma} = 2\pi \times 50$ kHz and $\omega = 2\pi \times 400$ kHz gives a value of $\bar{n}_{lim} = 0.16$, which is well inside the Lamb-Dicke regime at this trap frequency.

Population trapping during sideband cooling

As previously noted in section 3.2, we can see in Figure 3.2 that the Rabi frequency of the first red sideband passes through zero at a Fock state, which we will call n_{zero} . The value of n_{zero} decreases as the value of the η_z increases (i.e. as ω_z decreases). In the example shown in Figure 3.2 for $\omega_z = 2\pi \times 400$ kHz this occurs at $n_{zero} \approx 155$. As we move to lower trap frequencies not only does the value of n_{zero} decrease, but the Doppler cooling limit in terms of $\bar{n}_z$ also increases, as can be seen in Figure 3.6. These two factors mean that for lower trap frequencies there is a significant probability that the ion is in a Fock state with $n > n_{zero}$ after Doppler cooling. When we attempt to sideband cool in this situation, population in low-lying Fock states will be sideband cooled close to the motional ground state, as normal. Population in the high-lying Fock states will also initially cool as normal, but as it reaches n_{zero} the red sideband strength is greatly reduced and population effectively becomes trapped in these states. This results in a non-thermal state where there is a probability that the ion is in a Fock state near n_{zero} , and a probability that the ion is in a thermal state close to the ground state, characterised by $\bar{n}_z$. We can model the state after this single stage sideband cooling as

$$P(n) = \begin{cases} P(\text{thermal}) \frac{\bar{n}_z^n}{(\bar{n}_z + 1)^{n+1}} & n < n_{zero} \\ (1 - P(\text{thermal})) & n = n_{zero}. \end{cases} \quad (3.48)$$

where $P(\text{thermal})$ is the probability that the ion is in the thermal state and therefore the probability that the ion is trapped in the high-lying Fock state is $1 - P(\text{thermal})$.

Figure 3.9 shows the excited state population evolution with probe time for a single ion after attempting to sideband cool on the first red sideband from outside the Lamb-Dicke regime, resulting population trapping. This example uses an axial trap frequency of $\omega_z = 2\pi \times 200$ kHz, which corresponds to a Lamb-Dicke parameter of $\eta = 0.22$ and a first red sideband dark state of $n_{zero} = 78$. A value of $P(\text{thermal}) = 0.8$, which roughly corresponds to the probability $P(n < n_{zero})$ at the Doppler limit, is used along with an average phonon number of $\bar{n}_z = 0.5$ for the thermal part of the population distribution.

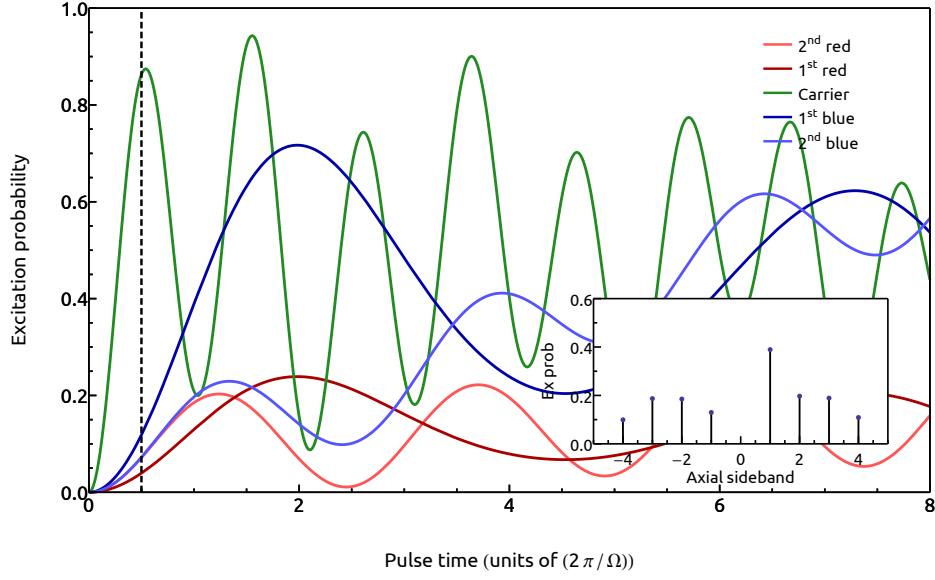


Figure 3.9: Time evolution of the excited state population on resonance carrier and first three sidebands for the non-thermal state caused by sideband cooling on the first red sideband from outside the Lamb-Dicke regime. Inset shows sideband heights for a pulse time of $t = \pi/\Omega$ (excluding the carrier for clarity).

The inset shows the sideband heights that would be obtained for the pulse length shown in the main panel. There is a clear asymmetry between the first order red and blue sidebands, which indicate a significant ground state population. However, there are also many visible higher-order sidebands that arise due to the trapped population that indicate that the state is non-thermal.

Chapter 4

Calcium ions

When planning an ion trap experiment, the choice of atomic species is clearly an important one. There are many factors that should be considered when making this decision, including:

- If we want a hydrogen-like level structure we require an ion species with a single valence electron and we are therefore limited to the alkali-earth elements, or some of the transition metals (e.g. mercury).
- There must be available laser sources at the ion's transition wavelengths. The availability of optical components, modulators, fibre optics, etc. at these wavelengths is an additional consideration.
- There should be a closed cycling transition for use in Doppler cooling. In a Penning trap the effect of the large applied magnetic field on any re-pumping scheme must also be considered.
- There should be a pair of long-lived, stable or meta-stable states that can act as the ground and excited states of a qubit, and we should be able to coherently move population between them.
- As can be seen in (2.23), the ion's charge-to-mass ratio has an effect on what trap frequencies can be obtained in a Penning trap. At a given magnetic field strength, a higher value of $\frac{e}{m}$ increases the maximum stable axial frequency. In general a larger trap frequency will result in lower values of $\bar{n}$ after Doppler cooling.
- The relative abundance of the chosen ion isotope has an effect on ion loading strategies. If a rare isotope is used then the loading scheme must preferentially create ions of this isotope in the trap rather than the more abundant ones.

Often a compromise must be made between these various considerations. In our experiment $^{40}\text{Ca}^+$ ions are used. The primary reason is the availability of diode laser sources at the transition wavelengths. In addition there is a suitable narrow linewidth

$S_{1/2} \longleftrightarrow D_{5/2}$ transition that can be used for coherent operations and there is no hyperfine structure that would complicate repumping schemes at large magnetic field strengths. $^{40}\text{Ca}^+$ has an average charge-to-mass ratio compared to the other possible species and it is by far the most common isotope of calcium, with a natural abundance of nearly 97% [61], meaning that we do not require an isotope-selective loading scheme. Another common choice of ion for use in a Penning trap is beryllium, which has a simpler energy level structure due to the lack of D states. Beryllium is less suited for experiments that require resolved-sideband cooling in a large magnetic field as they require Raman lasers as there is no electric quadrupole transition that can be used, and there are technical challenges associated with producing Raman lasers that bridge a large Zeeman splitting.

In this chapter I will present the atomic theory that informs many of the experimental decisions that we make in our experiment, including the energy levels, the first and second order Zeeman shifts and magnetically-induced shelving (or j -mixing).

4.1 Energy Levels

The relevant low-lying energy levels for $^{40}\text{Ca}^+$ ions, as well as the transitions that are used in the experiment are shown in Figure 4.1. The transition wavelengths and lifetimes are taken from refs. [62–64].

The main points of interest in the energy level scheme are

- There is an electric dipole $S_{1/2} \longleftrightarrow P_{1/2}$ transition at 397 nm which provides the best possibility for a closed cooling cycle, and this is the transition that is used for Doppler cooling in our experiment.
- Decay from the $P_{1/2}$ to the $D_{3/2}$ states occurs with a branching ratio of about 1/16 [65], and these states therefore need to be repumped using the $D_{3/2} \longleftrightarrow P_{1/2}$ transition at 866 nm for continuous cooling to occur.
- In our experiment the electric dipole-forbidden decay from the $P_{1/2}$ to $D_{5/2}$ becomes weakly allowed due to first-order perturbations caused by the large magnetic field (see section 4.3), and therefore the $D_{5/2} \longleftrightarrow P_{3/2}$ transition at 854 nm also needs to be addressed to fully close the cooling cycle.
- The $S_{1/2} \longleftrightarrow D_{5/2}$ electric quadrupole transition at 729 nm, with an upper state lifetime of approximately 1 s [63], can be used for coherent manipulation of the state of the ion with a suitable narrow-linewidth laser.

4.1.1 Zeeman shifts

The large magnetic field required to operate the Penning trap has a significant effect on the internal energy levels of the ion. In order to calculate the magnitude of the Zeeman

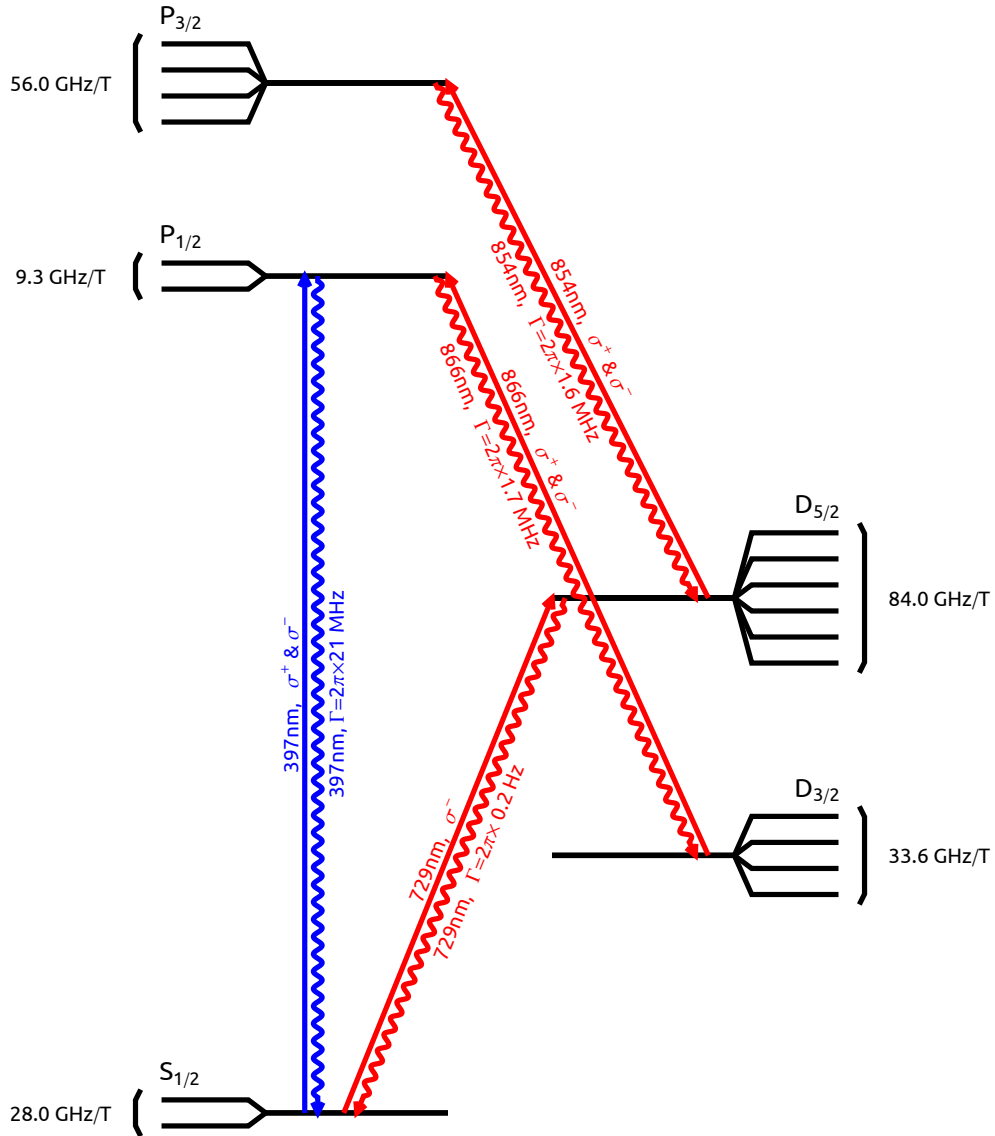


Figure 4.1: Energy level diagram of $^{40}\text{Ca}^+$ in a magnetic field. The relevant transition linewidths for our experiment and the values of the first-order Zeeman splitting are labelled. The lasers used in the experiment are also marked.

shifts we use perturbation theory. The Hamiltonian for the interaction between an electron in an axial magnetic field of strength B_z with z -components of the orbital and spin angular momentum of L_z and S_z is

$$H_{\text{Zeeman}} = \frac{\mu_B}{\hbar} B_z (g_l L_z + g_s S_z) \quad (4.1)$$

where μ_B is the Bohr magneton and $g_l = 1$ and $g_s \approx 2.002319 \approx 2$ are the orbital and spin g -factors. If we use first order perturbation theory, we can find the first order shift in the energy levels, which for an unperturbed spin-orbit wavefunction $|n^{(0)}\rangle$ is given by

$$\Delta E^{(1)} = \frac{\mu_B}{\hbar} \langle n^{(0)} | (g_l L_z + g_s S_z) | n^{(0)} \rangle = \mu_B g_j B_z m_j \quad (4.2)$$

where $g_j = \frac{j(j+1)-s(s+1)+l(l+1)}{2j(j+1)} + g_s \frac{j(j+1)+s(s+1)-l(l+1)}{2j(j+1)}$ is the Landé g -factor of the state [66]. The magnitude of this first-order energy splitting per tesla for each of the j -state manifolds is labelled in Figure 4.1.

In our experiments we use a magnetic field strength of order 2 T, and therefore have first-order Zeeman splitting of the various transitions between energy levels of order 30 GHz. Since the typical linewidth of these transitions (and the lasers we use to address them) are on the 10 MHz level we must also consider second order shifts. The second order shift of the energy of the unperturbed state $|n^{(0)}\rangle$ is given by

$$\Delta E^{(2)} = \sum_{m \neq n} \frac{\mu_B^2 B_z^2 |\langle m^{(0)} | L_z + g_s S_z | n^{(0)} \rangle|^2}{\hbar^2 (E_n^{(0)} - E_m^{(0)})}. \quad (4.3)$$

where we sum over contributions from all other states $|m^{(0)}\rangle$. By re-expressing the $|j, m_j\rangle$ states in terms of the $|l, m_l, j, m_j\rangle$ basis states and Clebsch-Gordan coefficients, we can find which terms contribute to the second order shifts.

Following this through for the example of the P states, the $P_{1/2}$ state can be decomposed to give:

$$\left| \frac{1}{2}, \pm \frac{1}{2} \right\rangle = \pm \sqrt{\frac{2}{3}} \left| 1, \pm 1, \frac{1}{2}, \mp \frac{1}{2} \right\rangle \mp \sqrt{\frac{1}{3}} \left| 1, 0, \frac{1}{2}, \pm \frac{1}{2} \right\rangle, \quad (4.4)$$

and the $P_{3/2}$ state can be re-expressed as:

$$\left| \frac{3}{2}, \pm \frac{1}{2} \right\rangle = \sqrt{\frac{1}{3}} \left| 1, \pm 1, \frac{1}{2}, \mp \frac{1}{2} \right\rangle + \sqrt{\frac{2}{3}} \left| 1, 0, \frac{1}{2}, \pm \frac{1}{2} \right\rangle \quad (4.5)$$

$$\left| \frac{3}{2}, \pm \frac{3}{2} \right\rangle = \left| 1, \pm 1, \frac{1}{2}, \pm \frac{1}{2} \right\rangle. \quad (4.6)$$

Expansions of the other $|j, m_j\rangle$ are given in Appendix A.1. From these expanded states

we can see that the only terms that contribute to (4.3) are ones that have the same values of l and m_j , but different values of j . At the magnetic field strength of 1.86 T that we use in our experiment, the second order shift for the $P_{1/2}$ states is -22 MHz, and the shift of the $P_{3/2}$ states with $m_j = \pm \frac{1}{2}$ is $+22$ MHz. The first and second-order Zeeman shifts of the levels used in our experiment are listed in Table A.1

4.2 Rabi frequency

The procedure to convert the expressions for the Rabi frequencies for E1 and E2 transitions given in (3.11) and (3.13) to a form in which we can calculate their values for typical experimental parameters is given in many places (e.g. ref. [62]). The basic steps are to first express the reducible Cartesian tensor operators in terms of irreducible tensors $\mathbf{C}$ (renormalised spherical harmonics) and rank 1 and rank 2 tensors $\mathbf{c}_i^{\Delta m_j}$ and $\mathbf{c}_{ij}^{\Delta m_j}$. This allows us to apply the Wigner-Eckart theorem, which separates the parts of the Rabi frequency that depend on m_j and m'_j and summarises them in terms of a $3j$ -symbol, and reduced matrix elements $\langle g \| r \mathbf{C}^{(1)} \| e \rangle$ and $\langle g \| r^2 \mathbf{C}^{(2)} \| e \rangle$ [67]. Using Fermi's golden rule we can express the Einstein A coefficient, A_{ge} , in terms of these reduced matrix elements to get

$$A_{ge}^{\text{E1}} = \frac{4c\alpha k_{ge}^3}{3(2j'+1)} \left| \langle g \| r \mathbf{C}^{(1)} \| e \rangle \right|^2 \quad (4.7)$$

$$A_{ge}^{\text{E2}} = \frac{c\alpha k_{ge}^5}{15(2j'+1)} \left| \langle g \| r^2 \mathbf{C}^{(2)} \| e \rangle \right|^2 \quad (4.8)$$

where α is the fine structure constant and k_{ge} is the wavenumber of the transition. We can then use this expression to find expressions for the Rabi frequencies of the transitions from $|j, m_j\rangle \longleftrightarrow |j', m'_j\rangle$ in terms of A_{ge}

$$\Omega_0^{\text{E1}} = \frac{eE_0}{\hbar} \sqrt{\frac{3(2j'+1)A_{ge}}{4\alpha c k_{12}^3}} \left(\begin{pmatrix} j & 1 & j' \\ -m_j & \Delta m_j & m'_j \end{pmatrix} \mathbf{c}^{\Delta m_j} \cdot \boldsymbol{\epsilon} \right) \quad (4.9)$$

$$\Omega_0^{\text{E2}} = \frac{eE_0}{\hbar} \sqrt{\frac{15(2j'+1)A_{ge}}{4\alpha c k_{12}^3}} \left(\begin{pmatrix} j & 2 & j' \\ -m_j & \Delta m_j & m'_j \end{pmatrix} \mathbf{c}^{\Delta m_j} : (\boldsymbol{\epsilon} \otimes \mathbf{n}) \right), \quad (4.10)$$

where $\mathbf{A} : \mathbf{B} = \sum_{i,j} A_{ij} B_{ij}$ is the Frobenius product. This process separates out the Rabi frequencies into the terms in brackets, which depend on the polarisation and propagation directions of the light, and a prefactor that depends on the lifetime of the state and electric field magnitude of the light.

4.2.1 Geometric considerations

One difference between the Penning trap and the radiofrequency trap that may not be immediately apparent is that in a Penning trap the axis of the trap and the atomic

quantisation axis are always in the same direction due to the magnetic field required to make the trap work. Conversely, in an rf trap there is a freedom to set the quantisation axis independently of the trap axis by adding a small magnetic field on the order of 1 G (e.g. [57]) or higher (~ 100 G) if hyperfine clock states are used (e.g. [18]).

By taking the parts of (4.9) and (4.10) that contain the geometric dependence, for transitions that involve a change in the magnetic quantum number of $\Delta m_j = m'_j - m_j$, we have geometric factors of

$$g^{\Delta m_j} = \left| \sum_i \mathbf{c}_i^{\Delta m_j} \boldsymbol{\epsilon}_i \right| \quad (4.11)$$

for electric dipole transitions, and

$$g^{\Delta m_j} = \left| \sum_{ij} \mathbf{c}_{ij}^{\Delta m_j} \boldsymbol{\epsilon}_i \mathbf{n}_j \right| \quad (4.12)$$

for quadrupole transitions, where $\mathbf{c}_i^{\Delta m_j}$ and $\mathbf{c}_{ij}^{\Delta m_j}$ are tensors (given in Appendix A.3). In our experiment we only use linearly polarised light due to the fact that we send light at 729, 854 and 866 nm down the same polarisation-maintaining (PM) fibre to the trap breadboard (the type of PM fibre only works for linear polarisation), and then along the same beam path, so there is no opportunity to convert a particular laser to circular polarisation (see section 5.5). If we define an angle ϕ between the beam propagation direction and the magnetic field and an angle γ between the linear polarisation vector and the magnetic field, we can use the exact form of the tensors $\mathbf{c}^{\Delta m_j}$ given in Appendix A.3 to find these geometric parts. This dependence of the transition strength on the angles γ and ϕ for the various transitions is plotted in Figure 4.2.

We want to Doppler cool the axial and radial motion in our trap simultaneously using an electric dipole transition. We therefore require that there is transition strength at both $\phi = 0^\circ$ and 90° for the particular transition that we use. It is possible to see from Figures 4.2a and 4.2b that $\Delta m_j = 0$ dipole transitions do not have any amplitude along the direction of the magnetic field, $\phi = 0^\circ$, and that therefore we must use $\Delta m_j = \pm 1$ transitions which do have amplitude at both angles.

Table 4.1 shows the parameters of the various Zeeman components of the $S_{1/2} \longleftrightarrow D_{5/2}$ electric quadrupole transition near to 729 nm that we use for spectroscopy and sideband cooling. The biggest atomic considerations as to which particular m_j states to use are the relative strength in the direction of interest and magnetic field sensitivity of the transition. A transition that is less sensitive to changes in magnetic field will show less frequency drift for a given drift in the field strength at the ion, resulting in longer coherence times and better reliability. However, the least field-sensitive states, the π transitions, have no strength in either the axial and radial directions and are therefore

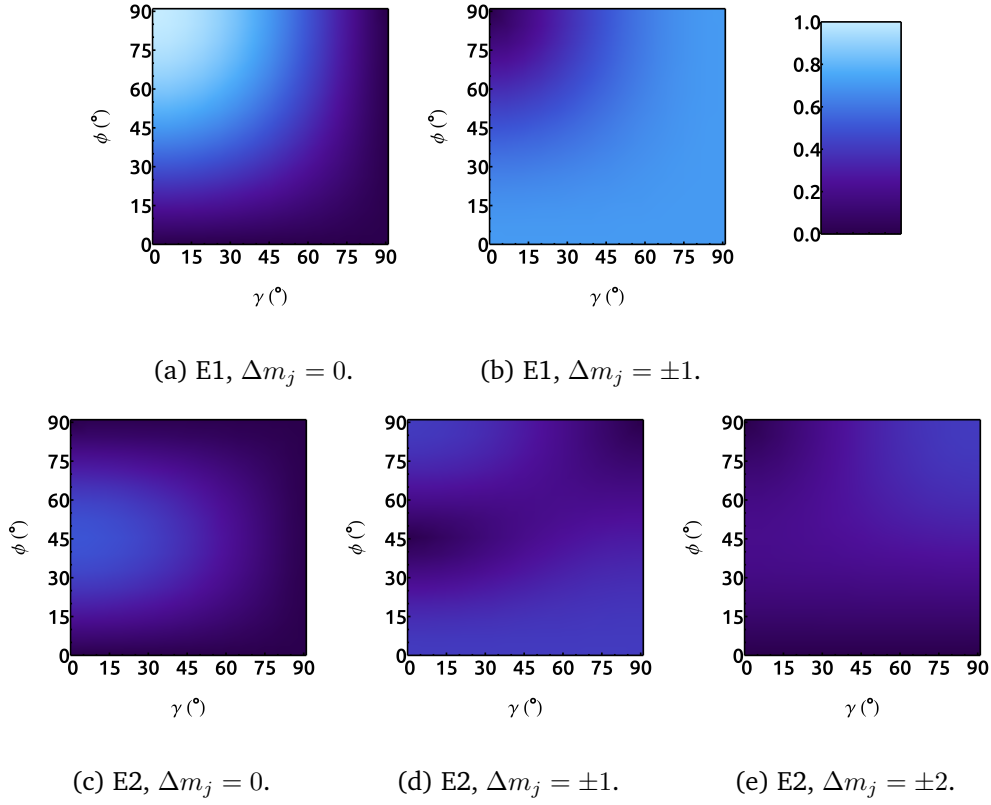


Figure 4.2: The geometric parts of electric dipole ((a)-(b)) and quadrupole transitions ((c)-(e)) for different values of Δm_j in terms of ϕ , the angle between the beam propagation direction and the magnetic field, and γ , the angle between the polarisation vector and the magnetic field.

$m_j (S_{1/2})$	$m'_j (D_{5/2})$	Type	$\Delta\nu_{\text{Zeeman}}(\text{GHz T}^{-1})$	Ax. Strength	Rad. Strength
-1/2	-5/2	δ^-	-28.0	0	0.408
-1/2	-3/2	σ^-	-11.2	0.365	0.365
-1/2	-1/2	π	5.6	0	0
-1/2	1/2	σ^+	22.4	0.258	0.258
-1/2	3/2	δ^+	39.2	0	0.183
1/2	-3/2	δ^-	-39.2	0	0.183
1/2	-1/2	σ^-	-22.4	0.258	0.258
1/2	1/2	π	-5.6	0	0
1/2	3/2	σ^+	11.2	0.365	0.365
1/2	5/2	δ^+	28.0	0	0.408

Table 4.1: Properties of the various $S_{1/2} \longleftrightarrow D_{5/2}$ transitions around 729 nm. The final column gives the maximum radial strength, meaning the angle of polarisation ϕ will not necessarily be the same for the different transitions.

inaccessible to us. For these reasons (and for other more technical reasons explained in section 5.4.6) the $S_{1/2}(m_j = -\frac{1}{2}) \longleftrightarrow D_{5/2}(m_j = -\frac{3}{2})$ transition is used in our experiment.

4.3 *j*-mixing

As explained in section 4.1.1, the size of our applied magnetic field means that second-order effects need to be considered. As well as the second-order energy shifts there are effects on the energy eigenstates of the system, representing the very beginning of the transition from the weak-field regime where the good quantum numbers are l, s, j and m_j to the strong-field regime where the good quantum numbers are l, m_l, s and m_s . Using perturbation theory, the first order correction to the atomic wavefunctions is given by

$$|n^{(1)}\rangle = \sum_{m \neq n} \frac{\mu_B B_z \langle m^{(0)} | (g_l L_z + g_s S_z) | n^{(0)} \rangle}{\hbar(E_n - E_m)} |m^{(0)}\rangle \quad (4.13)$$

meaning the eigenstates are an admixture of different j states. These extra corrections mean that additional decay routes become weakly allowed, specifically the $P_{1/2} \longleftrightarrow D_{5/2}$ decay which is dipole-forbidden ($\Delta j = 2$) at zero field [68]. Looking at the form of (4.13) and comparing it with the form of the second order Zeeman shifts in (4.3), we can see that the same states will contribute non-zero matrix elements i.e. states with the same l and m_j quantum numbers as the unperturbed state, but different values of j . For this reason this effect is known as ‘ j -state mixing’ or simply ‘ j -mixing’.

What we want to know is the number of decays that occur from the $P_{1/2}$ state to the $D_{5/2}$ levels, and therefore how often these decays due to j -mixing occur when we scatter photons on the $S_{1/2} \longleftrightarrow P_{1/2}$ transition. The average number of decays to $S_{1/2}$ before a decay to $D_{5/2}$ occurs n is

$$n = \frac{\Gamma(P_{1/2} \longleftrightarrow S_{1/2})}{\Gamma(P_{1/2} \longleftrightarrow D_{5/2})}. \quad (4.14)$$

Using Fermi’s golden rule in order to calculate these decay rates in terms of matrix elements we get¹

$$\begin{aligned} n^{-1} &\approx \left| \frac{\langle D_{5/2} || d || P_{1/2} \rangle}{\langle D_{3/2} || d || P_{1/2} \rangle} \right|^2 \\ &\approx 4.2 \times 10^{-7} B^2 \text{ T}^{-2} \end{aligned} \quad (4.15)$$

The matrix element $\langle D_{5/2} || d || P_{1/2} \rangle$ is zero at zero applied magnetic field, and in order to calculate this term at non-zero field we have to make the first-order perturbation to

¹A detailed calculation can be found in ref. [69].

the wavefunction given in (4.13). At the magnetic field strength of 1.862 T used in our experiment this means that roughly 0.7 million scattering events will occur on the $P_{1/2} \longleftrightarrow S_{1/2}$ before the ion decays to the $D_{5/2}$ state. At saturation intensity, this corresponds to 40 ms of photon scattering before the ion leaves the cooling cycle.

There are experimental implications of these j -mixing decays. Many of the experiments that we want to perform require that we are able to discriminate which state the ion is in after we perform some operation. Usually we need to be able to measure whether the ion is in a specific $S_{1/2}$ Zeeman substate or a specific $D_{5/2}$ Zeeman substate. This is typically done by addressing the $S_{1/2} \longleftrightarrow P_{1/2}$ transition and attempting to collect fluorescence photons, which should only be scattered if the ion was initially in the $S_{1/2}$ state (the ion is ‘bright’). However this additional j -mixing decay route means that each time we scatter a photon, there is some small probability that the ion will fall into the $D_{5/2}$ state and no more scattering will occur (the ion will become ‘dark’). This will limit the precision with which we are able to discriminate between ions in the ground and excited state. These implications will be discussed in more detail in section 5.6.

Chapter 5

Equipment

5.1 Introduction

The requirements for trapping, laser cooling and detecting a single ion or a cloud of ions in their most basic form can be stated as:

- A source of ions, which usually entails a source of neutral atoms and then some means to ionise them. As discussed in Chapter 4 the ion species should have a suitable energy level structure.
- An ion trap, consisting of a set of suitably shaped electrodes in order to produce a potential to trap the ions. In the case of a Penning trap we also require a constant and uniform magnetic field.
- Laser light to cool the ion in three dimensions. The light needs to have the correct frequency, polarisation and spatial properties to interact with the ions in order to cool all degrees of freedom.
- The ability to detect the ions, which usually means collecting fluorescence light, but ions can also be detected using electronic methods, where the oscillating ion causes a moving image charge on the trap electrodes, which can then be detected.

In this chapter I will describe the experimental apparatus that was used to achieve the results that I present in Chapter 6. As I am the third PhD student from the group to have used the same trap, laser systems and associated optics and experimental control system, much of what I describe here has been covered in detail in previous PhD theses in the group [70–72]. In order to avoid broad repetition in this chapter I will briefly summarise information that has been covered in previous theses where it is critical to understanding the way the experiment works, and will emphasise elements that have recently been changed, or parts of the experiment that have not been previously documented in detail.

5.1.1 My role in building the experiment

In the first year of my PhD I built the PDH locks for the two Doppler cooling lasers (section 5.4.4) and then began to run the main experiment alongside Sandeep Mavadia, and then alongside Joe Goodwin. During this period Joe and I improved the stability of the system, by first studying drifts in the measured $S_{1/2} \longleftrightarrow D_{5/2}$ transition frequency caused by reference cavity temperature changes, and trap movement inside the bore of the superconducting magnet (section 5.4.6 and 5.3.4). In my final year in the lab I did many maintenance tasks such as fixing the blue scanning cavity system (section 5.4.4), building a new computer control system for endcap pulsing (section 5.2) and implementing some improvements to the spectroscopy laser (section 5.4.6). Additionally I supervised the installation of new systems such as the direct digital synthesis (DDS) system (section 5.7.2), new camera imaging optics (section 5.6). Over the past three years I have been the main PhD student responsible for the day-to-day running and maintenance of the experiment.

5.2 Ions

The energy level structure and relevant atomic physics theory of $^{40}\text{Ca}^+$ ions are discussed in the previous chapter. I will now discuss how the trap is loaded with ions, using an atomic beam of calcium and a non-resonant photoionisation technique, and how we obtain the specific number of ions in the trap that we require by grounding the endcap electrodes for a short time.

Our source of ions is a resistive oven that produces a beam of calcium atoms. The oven consists of a 2 cm tantalum tube filled with calcium, and we pass a current of approximately 1.7 A through it in order to heat it up sufficiently to start producing an atomic beam, which is emitted through a small hole in the side of the tube. We then ionise some of the calcium atoms in this beam using a pulsed, frequency-doubled Nd:YAG laser at 532 nm. This is a non-resonant, three-photon process, and we therefore require a high intensity, using a peak pulse power of roughly 5 MW and a pulse duration of 5 ns. A resonant method for ionisation of calcium atoms using light at 423 nm has been demonstrated by other groups (e.g. [73]), which has the added advantage of being isotope-selective. As we currently only perform experiments with small numbers of ions it is easy for us to observe dark ions by viewing the ion crystal on an EMCCD or ICCD camera¹ and remove them using the endcap pulsing method described in the following paragraph. However, in the future we may require a way of producing larger pure crystals containing a specific number of ions (e.g. the quantum error-correcting code scheme described in section 7.4), and might need to use more precise resonant loading technique.

¹A video of a linear ion Coulomb crystal consisting of five $^{40}\text{Ca}^+$ ions and one dark ion of unknown species can be seen at <https://www.youtube.com/watch?v=08V1h6LqpY>.

Endcap pulsing

Our technique of loading ions using a non-resonant Nd:YAG laser is not particularly delicate, and as a result we often create more ions than we need for a particular experiment. These extra ions may include ‘dark’ (non- $^{40}\text{Ca}^+$) ions and they can be created over a large volume inside the trap due to the relatively large distance between the centre of the trap and the electrodes. This, combined with the lack of RF micromotion means that we can load ions in very large orbits that do not initially interact with the Doppler cooling beams, but can be slowly cooled into the centre and crystallise at some later time during an experiment. In order to remove any unwanted ions from the trap permanently, we have developed a system where we ground the endcap electrodes of the trap for a short amount of time ($\approx 10\ \mu\text{s}$). A pulse of the required length is sent to a simple transistor circuit, which ties the endcaps to ground for the duration of the pulse. A Stanford Research Systems DD535 delay generator was previously used to produce these pulses, but this has been replaced with an Arduino microcontroller that is controlled using serial communication. This allows the pulses to be controlled by a simple C# programme on the main experiment computer. In general, it is possible to reduce the size of a very large cloud containing thousands of ions to a small crystal of any specific number of ions using this system.

5.3 Ion trap

5.3.1 SPECtrap electrodes

SPECtrap was designed by Shailen Bharadia and the electrode design is described in his thesis (ref. [74]). It was intended to be used for work on large clouds and with highly-charged ions (e.g. [75]), rather than for work on small numbers of ions in the quantum regime. This explains why the electrode geometry is much larger than most other traps used for experiments on single ions, and also the presence of ‘capture’ electrodes which are designed to be used to trap externally-produced highly-charged ions. Figure 5.1 shows a cross section of the electrode structure. The trap consists of a stack of cylindrical electrodes made of oxygen-free copper. The compensation electrodes were originally designed to reduce trap anharmonicities experienced across the extent of a large cloud by applying voltages that cancel non-quadratic contributions to the potential from the endcap electrodes. The ring electrode is divided into four segments which allows us to apply an axialisation drive (Figure 2.4.2), or a dipole ($k = 1$) rotating wall drive that can be used to lock the rotation frequency of large clouds consisting of more than one type of ion [50, 76, 77]. Each ring segment has a 4 mm hole through it for optical access. In one opposing pair of segments these holes are used for the fluorescence collection and in the other pair the holes are used for entry and exit of the radial cooling beam.

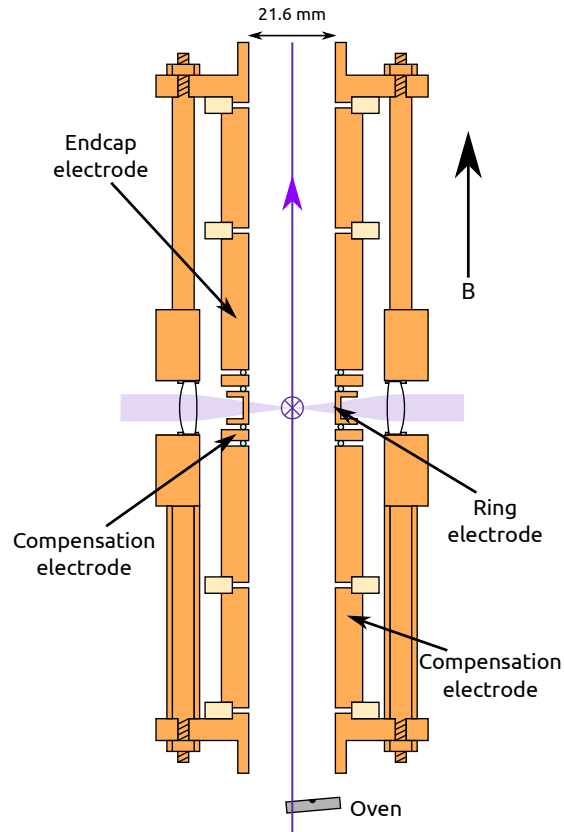


Figure 5.1: Schematic diagram showing the cross section of SPECTrap electrodes. The blue arrows show the propagation directions of the axial and radial cooling beams. The two $f = 20$ mm lenses used for fluorescence collection are also shown. Adapted from ref. [71].

In our experiment we keep the capture electrodes connected to ground, and connect a positive voltage to the endcaps. The ring and compensation electrodes are kept close to 0 V, but we apply small DC voltages to each of them of the order ~ 10 to 100 mV in order to compensate for stray electric fields. We are able to use the compensation electrodes for this purpose, rather than their originally intended purpose of cancelling the anharmonicity caused by the endcap electrodes, because we work with small numbers of ions and we can assume that higher-order terms in the potential are small across the extent of the ion crystals that we use.

The electrodes are mounted inside a custom-designed vacuum chamber, which is continuously pumped using an ion pump. The chamber is designed such that this pump is positioned approximately 1 m from the bore of our magnet, as it requires its own magnetic field to operate correctly. We obtain a pressure inside the chamber of approximately 10^{-10} mbar, which is sufficiently low to mean that background gas collisions that result in an observed loss of fluorescence signal from an ion are extremely rare (significantly less than one per hour).

5.3.2 DC voltage compensation

The stable positioning of the ion at the centre of the trap is especially important in a Penning trap, as our method of cooling the radial motion uses an intensity gradient, the size of which changes with ion position. Therefore we need to compensate for stray electric fields. The majority of these stray fields are due to the accumulation of calcium atoms on the electrode surfaces due to different work functions of calcium and copper. The work function of calcium is 2.87 eV and the work function of copper is ~ 5 eV [78], which means that a partial coating of calcium on a copper electrode will result in a negative voltage on the electrode surface that we intend to be at 0 V.

To compensate the effects of these patch potentials we use the hypothesis that their effect becomes small when the endcap voltage is increased. We therefore view a single ion on an ICCD camera and mark its position at high endcap voltage. We reduce the endcap voltage and observe the movement in the pixel position of the ion. Then we adjust the DC compensation voltages applied to the ring and compensation electrodes until the ion is returned to its high endcap voltage position. This allows us to change the endcap voltage and therefore adjust the trap frequencies without movement of the position of a single ion or the centre of a crystal.

Anecdotally, we have seen that if we run our atomic oven at a high current or for a long time we see a movement in the position of the trap centre on our ICCD that decays with a time constant of approximately one hour. This is consistent with measurements that have been made elsewhere [79]. Therefore we do not perform experiments or set the compensation voltage within an hour or two of a long period of running the oven.

5.3.3 Axialisation

As previously mentioned, our trap has a ring electrode that consists of four segments. This allows for the application of an axialisation drive of the form shown in Figure 2.7c. We produce signals of the correct phase using a signal generator tuned to the cyclotron frequency ω_c and two inverting and two non-inverting amplifiers. Additionally each of these amplifiers has a variable gain, and adds the DC compensation voltage to the output [72].

The variable gain on each electrode signal is used to move the null of the axialisation drive to the centre of the trap in order to minimise RF micromotion. To achieve this we adjust the amplitude of the signal on each electrode while viewing a single ion on a camera at high endcap voltage. When the amplitudes are set incorrectly the ion enters a very large radial orbit many pixels wide. We could also detect micromotion using RF-photon or photon-photon correlation [80] using a multi-channel analyser to measure the modulation of the scattering rate due to the driven micromotion.

A rotating wall drive system based around two AD9959 direct digital synthesis (DDS) evaluation boards is currently being developed by the group. Each of the channels will have an adjustable DC offset and AC amplitude as in the current setup to allow us to compensate for stray fields and RF micromotion. This system will have eight output channels with arbitrary phase relationship, which will allow us to apply an axialisation drive, or either a $k = 2$ or $k = 3$ rotating wall, shown in Figure 2.7.

5.3.4 Magnet

We produce the axial magnetic field that is required to operate our Penning trap using a vertical-bore superconducting magnet running in persistent mode. The magnet has a room temperature bore with a diameter of 10 cm. This bore diameter presents a significant barrier to optical access to the trap. We run the magnet to give a field strength of 1.862 T at the centre of the bore, although this could potentially be increased to approximately 2.5 T.

Trap positioning in magnet

Our magnet is essentially a single superconducting solenoid, and we attempt to place our trap in the centre of this solenoid. The magnetic field strength will not be constant over the whole of the bore volume, but it should be at its most homogeneous at its centre. This central point is the maximum of the magnetic field strength along the axis of the solenoid, but is the minimum in the perpendicular direction, as the field strength will be strongest near the solenoid wires. We position the trap inside the bore by initially mounting it on an optical breadboard, which sits on a metal plate, which in turn is bolted to a Genie lift. This holds the weight of the vacuum system and provides coarse positioning. For fine positioning of the height and angle of the trap we adjust

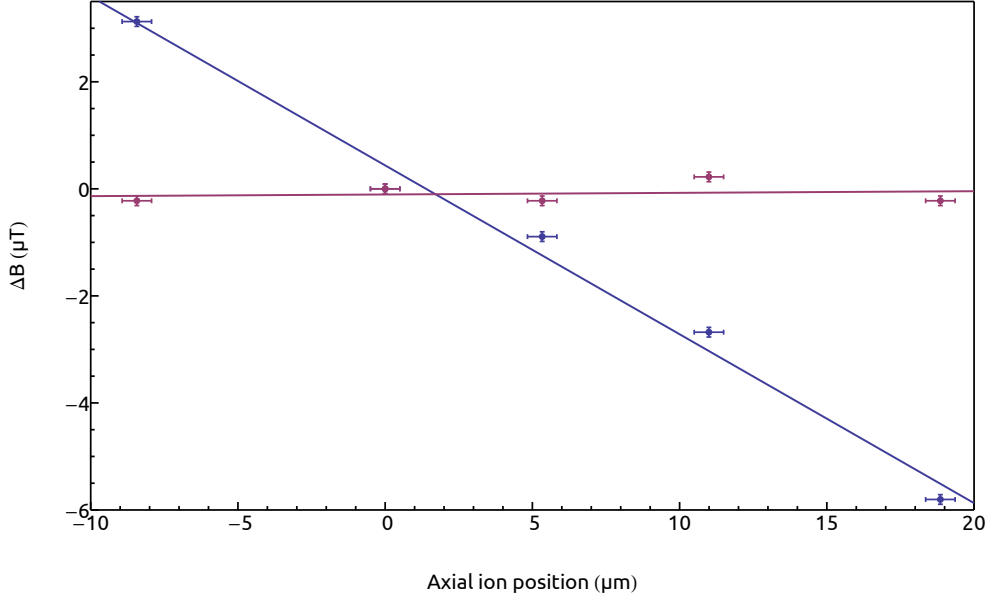


Figure 5.2: Measured magnetic field as a function of position along the axis of the magnet bore, before (blue) and after (magenta) the trap was moved up inside the bore by a distance of 3 mm.

three sharpened M6 bolts that pass through the breadboard and push onto the metal plate.

To make a precise measurement of the magnetic field gradient at the centre of the trap we measure the frequency of the $S_{1/2} \longleftrightarrow D_{5/2}$ spectroscopy transition of a Doppler-cooled ion. By changing the voltage on the compensation electrodes we can move the ion up and down inside the trap in a controlled way. We calibrate the movement by measuring how many pixels the ion image moves on an ICCD camera with changing compensation voltage and convert this to distance using a previously measured value for the magnification of the imaging optics. If we then perform spectroscopy on the transition for different ion positions we convert the measured change in transition frequency to a change in magnetic field strength to find the gradient.

Figure 5.2 shows the magnetic field gradient measured using this method, both before and after the trap was moved upwards inside the bore by approximately 3 mm. The movement of 3 mm was decided by comparing the measured field gradient to the old field measurement from many years ago shown in Appendix B. By performing a linear fit to both of these sets of data it shows that the process of moving the trap towards the magnetic field centre has reduced the field gradient at the position of the ion by approximately two orders of magnitude, from -0.31 T m^{-1} to 0.0031 T m^{-1} .

5.4 Laser sources and locks

A summary of the laser systems and the laser locking used in the experiment, including the current and temperature controllers, is shown in Table 5.1.

5.4.1 External Cavity Diode Lasers

All of the lasers that are used for resonantly driving atomic transitions in the experiment (i.e. not including the HeNe laser used in the scanning cavity transfer lock, and the Nd:YAG laser used for loading ions) are external cavity diode lasers (ECDLs). In the type of ECDL that we use the intrinsic linewidth and mode behaviour of the diode laser is stabilised by adding a diffraction grating inside the laser head [81]. The laser diode is essentially ‘self-seeded’ by sending first-order diffracted light back into the diode, which also creates an external cavity between the back facet of the diode and the grating. The main advantages of the ECDL over other types of laser are their low cost, relatively compact nature, and their relative simplicity to use². Linewidths of below 100 kHz are achievable with this configuration, without any active feedback from an external locking scheme[83].

The simplest ECDL configuration, is where the first-order diffracted light from the grating is directed back into the diode as shown in Figure 5.3. This is called the Littrow configuration and the grating is typically mounted so that its angle with respect to the diode can be adjusted, changing the centre wavelength of the light that is diffracted back into the diode. Another type of ECDL grating setup, called the Littman configuration, has a fixed grating angle and the wavelength selectivity is provided by an additional angle-adjustable mirror [84]. This arrangement can give a narrower linewidth due to the double pass of the grating, but lower output power due to higher losses in the external cavity.

No knowledge of the semiconductor physics behind how laser diodes work is really required in order to use an ECDL, but it is useful to know the effect of changing each parameter on the diode output. Typically, there are four main parameters that can be changed in an ECDL system to produce light at the correct frequency:

- By increasing the current that is passed through the laser diode, the number of charge carriers injected into the junction is increased. This increases the number of recombination events and therefore the output power of the laser, but also changes the centre frequency of the diode emission due to changes in the refractive index of the gain material. A rule-of-thumb value for the current dependence of the diode laser centre frequency is $-3 \text{ MHz } \mu\text{A}^{-1}$ [85]. Changing the current passing through the diode will also have some effect on its temperature.

²Despite this, regular users of ECDLs will not be surprised to hear that they are studied for their tendency to exhibit chaotic behaviour [82].

Transition	Wavelength	Source type	Locking system	Performance
$S_{1/2} \longleftrightarrow P_{1/2}$	397 nm	$2 \times$ Tui-Optics ^a DL100 heads and controllers. Nichia NDV4814T laser diode.	PDH lock to two $\mathcal{F} \approx 40$ cavities to reduce linewidths and scanning cavity transfer lock to remove long-term drift of PDH cavities (section 5.4.4).	Linewidth ≈ 1 MHz, drift ≈ 1 MHz h ⁻¹
$D_{3/2} \longleftrightarrow P_{1/2}$	866 nm	Homemade ECDL with homemade current supply and Thorlabs temperature controller. Eagleyard EYP-RWE-0860-06010-1500-SOT02-0000 laser diode.	Red scanning cavity transfer lock (section 5.4.5).	Drift ≈ 1 MHz h ⁻¹
$D_{5/2} \longleftrightarrow P_{3/2}$	854 nm	Homemade ECDL with homemade current supply and Wavelength electronics temperature controller. Eagleyard EYP-RWE-0870-06010-0750-SOT01-0000 laser diode.	Red scanning cavity transfer lock (section 5.4.5).	Drift ≈ 1 MHz h ⁻¹
$S_{1/2} \longleftrightarrow D_{5/2}$	729 nm	Homemade ECDL with homemade current supply and Thorlabs temperature controller. Toptica TA-0735-0500-4 tapered amplifier.	PDH lock to low drift, high-finesse cavity (section 5.4.6).	Linewidth < 1 kHz ^b

Table 5.1: Summary of the different laser systems, including sources, controllers and frequency locks.

^aTui-Optics is now Toptica^bRecent linewidth measurements using low-power spectroscopy suggest a linewidth of 300 Hz.

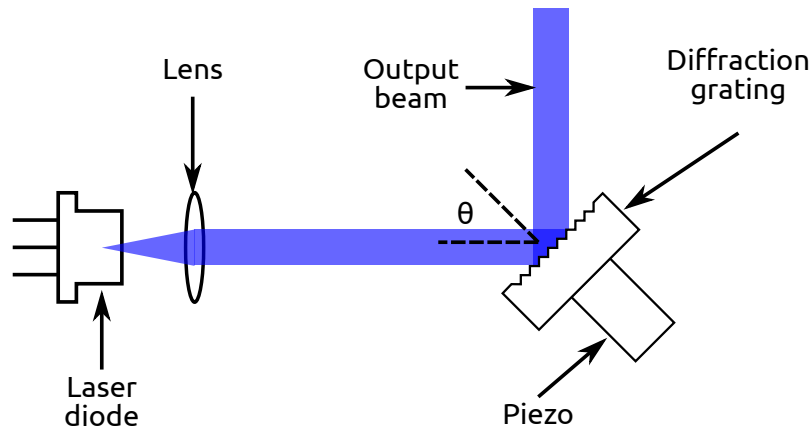


Figure 5.3: Schematic diagram of the Littrow configuration for an extended cavity diode laser.

- An increase of the temperature of the laser diode will cause thermal expansion inside the diode. There will be a change in the optical path length of the gain material and therefore a change in the centre frequency of the diode emission. The rough figure often quoted for the frequency dependence with temperature is 0.3 MHz mK^{-1} , but this will depend on the particular diode [85]. This thermal expansion will also change the free spectral range of the internal cavity and the position of the internal modes.
- The angle of the diffraction grating with respect to the laser diode sets the centre wavelength of the light diffracted back into the diode, and therefore the stable wavelengths of the external cavity.
- As shown in Figure 5.3, the diffraction grating inside an ECDL is usually mounted on a piezoelectric device. By changing the voltage applied across this device we can adjust the length of the external cavity. This voltage is the main parameter used for making small adjustments to the laser frequency, and by optimisation of the other three parameters a mode-hop-free tuning range of many GHz can be achieved [81]. If the grating is mounted correctly this should not change the angle of the grating with respect to the laser diode.

As has been alluded to, there are several different mode structures that contribute to the behaviour of an ECDL. The mode structure of an ECDL is shown in Figure 5.4, and the overall gain profile is the product of all of these effects [81]. What this shows is that there are many competing modes with broadly similar gain, and so hopping between

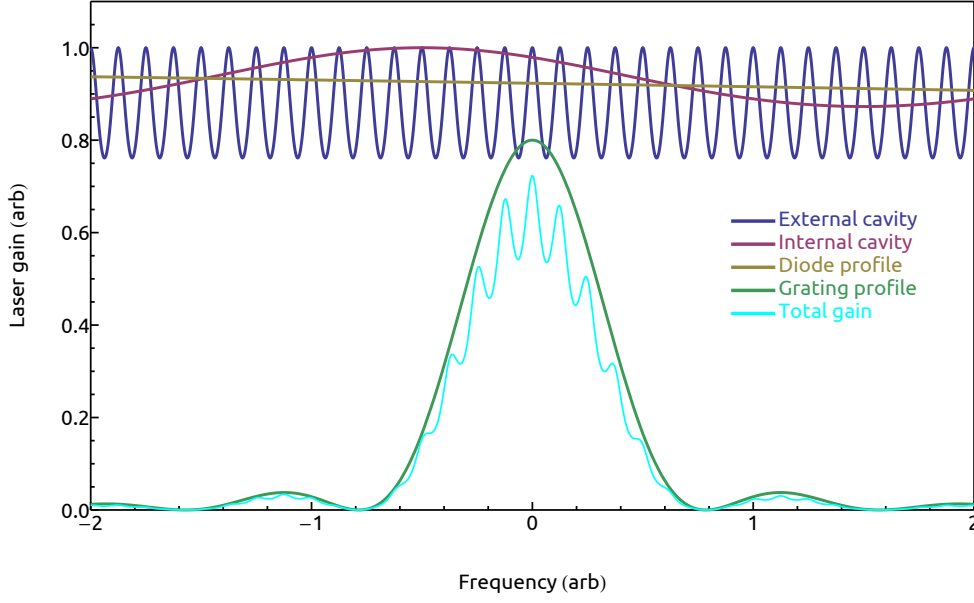


Figure 5.4: The modes of an extended cavity diode laser and the overall gain. Adapted from ref. [81].

different modes can occur for small changes in the parameters of the laser. Some laser diodes come with anti-reflection coated front facets for use in ECDLs, as this means they do not support internal modes and do not lase without feedback. This simplifies the mode structure and allows for a larger mode-hop free tuning range as the competing modes are further apart in frequency.

The fundamental quantum limit for the linewidth that can be achieved using a diode laser is the Schawlow–Townes linewidth, $\Delta\nu_{ST}$,³ which is given by

$$\Delta\nu_{ST} = \frac{\pi h \nu (\Delta\nu_c)^2 (1 + \alpha^2)}{P_{out}} \quad (5.1)$$

where $\Delta\nu_c$ is the full width at half maximum frequency linewidth of the laser cavity, P_{out} is the laser output power, ν is the centre frequency and $(1 + \alpha^2)$ is an enhancement factor present in diode lasers due to a coupling between the intensity and phase [87]. For typical experimental parameters this linewidth is approximately 30 kHz [81]. However, in practice the linewidth of a laser will typically be much higher than this limit due to technical noise. A commonly-used estimate for the linewidth of an ECDL is given by

$$\Delta\nu_{ECDL} = \Delta\nu_0 \left(\frac{\tau_d}{\tau_d + \tau_e} \right)^2 \quad (5.2)$$

³This quantum limit comes from spontaneous emission causing amplitude noise inside the laser cavity. A derivation can be found in ref. [86].

where $\Delta\nu_0$ is the diode chip linewidth, and τ_d and τ_e are the round-trip times of the diode and external cavities [88]. This estimate does not take the grating parameters into account, but it has been shown that these properties (such as diffraction efficiency and line spacing) have a significant effect on the linewidth obtained from the ECDL [88].

5.4.2 Optical cavities

All laser frequency locking schemes involve some sort of stable optical frequency reference, whether that is an absolute reference, such as another laser light source that has inherently low drift (e.g. a gas laser) or an atomic transition, or relative reference such as an optical cavity. Optical cavities provide a very good reference as they can be produced with spacers that have very low drift and thermal expansion. The resonance frequency is extremely sensitive to the cavity length. In the words of Bergquist et al [89]

if the spacer for an optical cavity were the Earth, a human hair added to the diameter would cause a frequency shift of about 300 Hz!

and for the kind of stability that we require for our 729 nm spectroscopy laser this means that the average movement of the mirror surface will be substantially less than the size of an atom.

Probably the most common type of cavity used as a frequency reference consists of two spherical mirrors with radii of curvature R_1 and R_2 placed a distance L apart. All of our cavities are symmetric, which means that $R_1 = R_2 = R$. This mirror arrangement can support Hermite-Gaussian modes. The mode frequencies are given by

$$\nu_{l,m,q} = \frac{c}{2L} \left(q + \frac{l+m+1}{\pi} \arccos(g) \right) \quad (5.3)$$

where q is an integer denoting the longitudinal mode and l and m are integers denoting the particular TEM_{lm} transverse mode, and $g = 1 - \frac{L}{R}$ is called the g parameter, which must satisfy $-1 \leq g \leq 1$ for a stable symmetric resonator [90, 91]. In order to couple light into the cavity in a particular transverse mode, the incoming beam should match the intensity distribution of the mode as closely as possible. For incoming beams that have approximately Gaussian intensity profile, it is easiest to couple into the TEM_{00} mode, followed by other low order modes. The process of closely matching the intensity profile of one of the transverse modes of the cavity is called ‘mode matching’

The frequency difference between different longitudinal modes (i.e. q and $q+1$) is called the ‘free spectral range’ of the cavity and is given by

$$\nu_{\text{FSR}} = \frac{c}{2L}. \quad (5.4)$$

For light mode-matched to a particular transverse mode of the cavity then this will be the frequency spacing between different transmission peaks. The linewidth $\Delta\nu$ of a cavity depends on the reflectivity of the mirrors

$$\Delta\nu = \frac{1 - r_1 r_2}{\pi \sqrt{r_1 r_2}} \nu_{\text{FSR}} \quad (5.5)$$

where r_1 and r_2 are the field amplitude reflection coefficient of the mirrors. Higher reflection coefficients mean that more light can build up in the cavity and therefore interference effects mean that the frequency selectivity is stronger.

The finesse of a cavity, $\mathcal{F}$ is given by

$$\mathcal{F} = \frac{\nu_{\text{FSR}}}{\Delta\nu} = \frac{\pi \sqrt{r_1 r_2}}{1 - r_1 r_2} \quad (5.6)$$

The finesse of an optical cavity can be measured using the ‘ringdown’ technique. Resonant light is injected into a cavity and the transmitted light is measured using a fast photodetector. If the light into the cavity is quickly turned off, the amount of light stored in the cavity, and therefore the transmitted light intensity, will decay with a characteristic time constant related to the finesse of the cavity. Any losses in the cavity will reduce g_{rt} and therefore reduce the finesse of the cavity, and will also reduce the transmitted intensity.

The type of cavities we use consist of a horizontal spacer made of glass or metal. More complicated rugby ball types that are suspended to cancel out the effect of gravitational sagging of the spacer are now also commonly used [92].

For a cavity where the light passes through air, changes in temperature and in pressure result in a change in the refractive index of the air, which changes the effective length of the cavity. This has an identical effect on the resonance frequencies of the cavity as a change in the physical length. The magnitude of these effects can be calculated using the Edlén equation, an empirical formula for the refractive index of air in terms of its pressure and temperature [93]. These effects are large enough that ideally a reference cavity should be either evacuated or sealed and temperature-controlled to make it sufficiently stable.

5.4.3 Pound-Drever-Hall scheme

One widely-used type of laser frequency lock is the Pound-Drever-Hall (PDH) method. This technique generates an error signal using an optical cavity as a reference. The frequency of the laser we want to lock is modulated at some frequency ω_{mod} , which produces sidebands. When we then measure the frequency-dependent interference when these sidebands are reflected back from the front of the cavity we obtain a high-bandwidth error signal that is used to stabilise the laser.

The light reflected back from the front face of a cavity is made up of the prompt

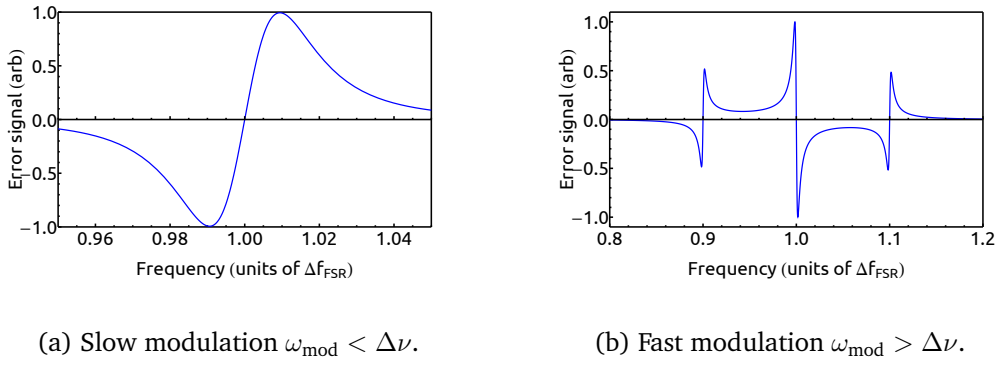


Figure 5.5: The form of the error signals produced using the Pound-Drever-Hall locking scheme.

reflection and light that leaks out from inside the cavity. On resonance these two interfere destructively and the reflected power will be minimum. An advantage of using this light to produce an error signal rather than the light transmitted through the cavity is that it provides an immediate comparison of the incoming laser frequency with that of the light built up inside the cavity. This will give a higher bandwidth lock than using the transmitted light, where any fast fluctuating components will be averaged out.

The frequency modulation sidebands can be produced by directly modulating a diode current or by externally modulating, often by using an electro-optic modulator. The disadvantage to diode current modulation is that the sidebands are produced at the laser source, and will therefore be sent to the experiment, whereas if an external EOM is used the light sent to the experiment can be picked off at a point before the sidebands are added. This is particularly important for a spectroscopy laser as power in frequency sidebands could interact with transitions other than the one intended. The laser is frequency modulated at a frequency ω_{mod} , and the frequency sidebands that are produced have different phases [94].

There are two main regimes of the PDH lock, one where the modulation frequency is smaller than the linewidth of the reference cavity ($\omega_{\text{mod}} < \Delta\nu$) which are sometimes referred to as ‘dither’ locks, and the other where the modulation frequency is larger than the cavity linewidth ($\omega_{\text{mod}} > \Delta\nu$). In the dither lock regime we can think of the frequency as being slowly changed in order to measure the gradient of the reflected signal and therefore which side of the cavity resonance the laser frequency is on. The exact form of the error signal is different in each of these regimes, as the sidebands are predominately transmitted in the dither lock regime because they are inside the cavity linewidth, but are predominately reflected in the case where they lie outside the bandwidth of the cavity.

Following the treatment in ref. [95], for a lossless cavity consisting of two mirrors,

both with amplitude reflection coefficient r , the frequency-dependent reflection coefficient $F(\omega)$ is given by

$$F(\omega) = \frac{r (e^{i\omega/\nu_{\text{FSR}}} - 1)}{1 - r^2 e^{i\omega/\nu_{\text{FSR}}}}. \quad (5.7)$$

If we measure the reflected intensity and then pick out the frequency component at ω_{mod} (achieved using a mixer) we obtain an error signal given by $F(\omega)F^*(\omega + \omega_{\text{mod}}) - F^*(\omega)F(\omega - \omega_{\text{mod}})$. In the dither lock regime the error signal is

$$\epsilon_{\text{dither}} = 2\sqrt{P_c P_s} \frac{d|F(\omega)|^2}{d\omega} \omega_{\text{mod}} \quad (5.8)$$

where P_c and P_s are the powers in the carrier and in each of the sidebands. The form of this error signal is shown in Figure 5.5a. In the fast modulation regime the

$$\epsilon_{\text{PDH}} = 2\sqrt{P_c P_s} \text{Im} (F(\omega)F^*(\omega + \omega_{\text{mod}}) - F^*(\omega)F(\omega - \omega_{\text{mod}})) \quad (5.9)$$

As shown in Figure 5.5b, the position of the sidebands is visible outside the bandwidth of the cavity. When the laser is locked we expect that the frequency should remain inside the bandwidth of the cavity, therefore we are interested in the linear slope at the centre of the error signal, which is given by

$$\epsilon_{\text{PDH}} = -\frac{8\sqrt{P_c P_s}}{\Delta\nu} \delta f \quad (5.10)$$

where the quantity $\frac{8\sqrt{P_c P_s}}{\Delta\nu}$ is called the ‘discriminant’.

Whether the error signal is generated in either the fast or slow regime, once it has been produced it is used as a feedback signal to one or more of the laser parameters in order to keep the laser frequency on resonance with the reference cavity.

5.4.4 Doppler cooling laser system

As can be seen in Table 5.1, the Doppler cooling laser system consists of two independent ECDLs that we use to address the $\sigma^\pm$ transitions in the $S_{1/2} \longleftrightarrow P_{1/2}$ manifold at 397 nm. Both of these lasers have a double-passed AOM in the beam path in order to turn the light to the experiment on and off with a rise/fall time of 40 ns, controlled by the FPGA control system (see section 5.7.1). After the AOMs the two beams are combined on a 50 : 50 beamsplitter and are coupled into two polarisation-maintaining fibres. These fibres send the light to the trap breadboard where one is used as the axial cooling beam and the other as the radial cooling beam.

For many years the only laser stabilisation of the Doppler cooling lasers was a side-of-fringe lock to a pair of optical cavities that had a high level of drift (many 10’s of MHz) over a timescale of minutes. A scanning cavity transfer locking system was designed in order to improve on this performance, but during its development it was

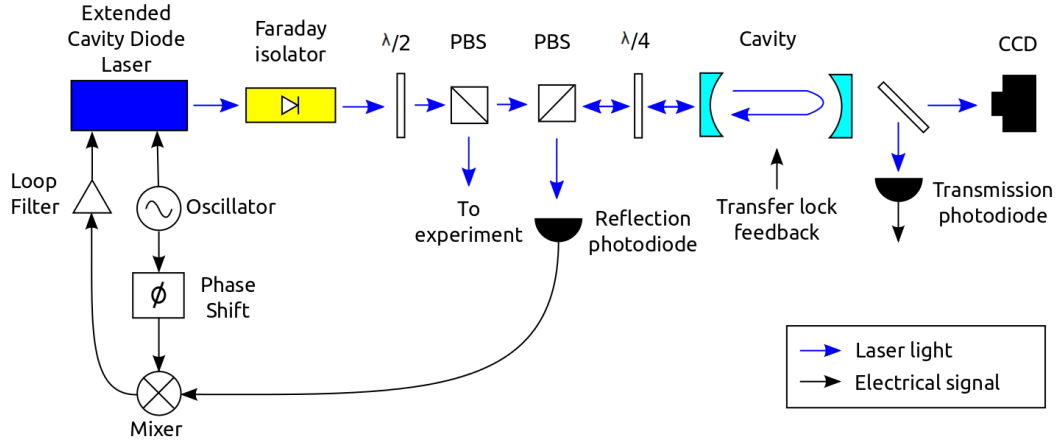


Figure 5.6: Simplified schematic diagram of the PDH setup used for each of the Doppler cooling lasers.

discovered that the linewidth of each of the lasers was much larger than anticipated (roughly 10 MHz). This is comparable with the transition linewidth, and therefore we designed and built a PDH lock to reduce the laser linewidth, using the original reference cavities. The transfer lock is now used to eliminate the drift in the PDH cavities by feeding back to a piezoelectric transducer controlling the lengths of the PDH cavities.

Blue PDH locks

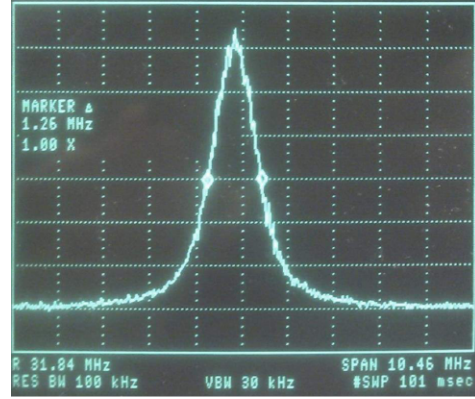
A schematic of the equipment used to PDH lock each of the Doppler cooling lasers is shown in Figure 5.6. Each of the lasers has its own lock to an individual optical cavity. The design of the reference cavities is described in ref. [96] and a schematic diagram of the spacer is given in Appendix C.1. Each cavity has a length of roughly $L = 100$ mm, which is adjusted using a piezoelectric transducer, and each of the mirrors has a radius of curvature $R = 300$ mm. The long term stability of these cavities is relatively poor, and their drift over a few minutes is often larger than the linewidth of the $S_{1/2} \longleftrightarrow P_{1/2}$ transition. This high drift could be due to a faulty piezoelectric transducer, as the drift on one cavity in particular is especially bad, and is worse if its length has been changed a significant amount. The finesse of these cavities is $\mathcal{F} \approx 40$, which equates to a cavity linewidth of $\Delta\nu \approx 38$ MHz.

The diode current is modulated using a voltage controlled oscillator (VCO) at a frequency of $\omega_{\text{mod}} = 2\pi \times 11$ MHz to produce sidebands in the slow modulation regime. We modulate the current of the diode rather than using an external EOM both because it is relatively simple, and that modulators are not cheaply available at near-ultraviolet wavelengths.

The part of the loop filter that deals with the fast feedback to the laser diode current is closely based on the design in [97] and a circuit diagram is given in Appendix C.2.



(a) PDH locks off.



(b) PDH locks on.

Figure 5.7: Heterodyne beat note signal produced by combining the two 397 nm lasers on a fast photodetector, as viewed on a spectrum analyser. Note that the spectrum analyser sweep time is 20 ms in (a) and 101 ms in (b).

There is also slower, integral feedback to the ECDL piezoelectric transducer which is controlled by an Arduino microcontroller. The rationale for this combination of analogue and digital feedback systems is that we are able to pause the value of the integral feedback very easily if the laser becomes unlocked using digital feedback.. The transmission photodiode signal is used to verify that light is passing through the cavity which we take as evidence that the laser is locked. By comparing the transmission intensity to a threshold value, we produce a digital lock logic signal which is used by the FPGA control system so that an experiment can be paused if one of the lasers becomes unlocked.

We measure the linewidth of the two lasers by performing a heterodyne beat note measurement. We do this by tuning the lasers to close to the same frequency and then combine them on a fast photodetector. We view this beat note signal in the frequency domain using a spectrum analyser as shown in Figure 5.7. The laser locks narrow the linewidth on the timescale of the spectrum analyser frequency sweep, and also reduce drift on slower timescales. The linewidth of the lasers is reduced from roughly 10 MHz to 1 MHz.

Scanning cavity transfer locks

The scanning cavity transfer lock that we use to stabilise the PDH reference cavities is based on the design described in ref. [98] and the initial implementation and characterisation of this system was carried out by Joe Goodwin and is described in detail in refs. [72, 99]. The basic principle of this transfer lock is to use a HeNe ‘master’ laser, stabilised using the technique described in ref. [100], and to transfer this stability to the ‘slave’ lasers. We do this by scanning the length of a cavity, into which we couple

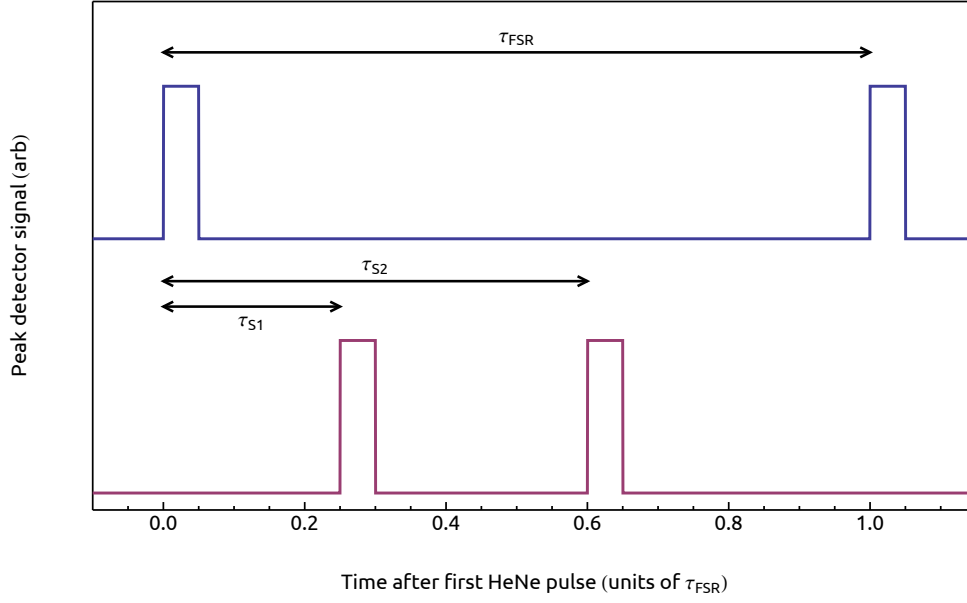


Figure 5.8: Schematic diagram to show the principle of operation of the scanning cavity transfer locks. The ‘master’ peaks from the stabilised HeNe laser are in blue and the ‘slave’ peaks from the two Doppler cooling lasers are shown in magenta.

the master and slave lasers. We detect the peaks in transmitted intensity and convert to TTL pulses of fixed length using a peak detector circuit, resulting in peaks like those shown in Figure 5.8. These peaks are sent to an Arduino microcontroller, which measures the relative arrival time of these peaks. The wavelengths of the master and slave lasers are far enough apart that we can separate the transmission from each using a dichroic mirror. In Figure 5.8 I have shown just two slave peaks for clarity, but as the Doppler cooling laser wavelength is less than that of the HeNe laser, there are three or four slave peaks between two master peaks. dichro

We scan the length of the cavity over one free spectral range and measure the time between master pulses τ_{FSR} . This provides a time reference and we then feed back to the two slave lasers, to ensure that their arrival times, τ_{S1} and τ_{S2} , remain the same.

The cavities are run in a near confocal arrangement ($g \approx 0$ in (5.3)), which means that the transverse modes overlap, which simplifies the mode structure so that other transverse peaks are less likely to appear and affect the quality of the lock for small changes in the mode matching into the cavity due to mirror drifts.

Recent Changes to the Blue Scanning Cavity The original scanning cavity mirrors never produced the finesse that was expected, and due to high losses the transmission through the cavity was very low. This affected the performance of the lock and also meant that we had to use a PMT to detect the slave peaks rather than a more sim-

ple photodiode. When one of the piezoelectric transducer stacks broke in November 2014 the decision was taken to replace these mirrors with two off-the-shelf broadband dielectric spherical mirrors (Thorlabs CM254-075-E02) each with a 150 mm radius of curvature to maintain near-confocal arrangement. During the same process vacuum-compatible SMA feedthrough connectors were added to the system to allow the cavity to be evacuated in future. This was a substantial operation that took several months but resulted in an increase in both the finesse and the transmitted light intensity, which has improved the performance of the lock.

Performance and possible improvements to the blue locking system

The performance of our Doppler cooling laser system is critical to the running of our experiment, as we need to be able to reliably cool ions to close to the Doppler limit. Our current system works sufficiently well to do this, and is reasonably robust, allowing us to leave the lasers locked for many hours while maintaining a constant fluorescence level from an ion. However it is not ideal that we require two separate locking systems for each laser. This problem essentially arises due to the large 10 MHz linewidth that we measure out of our ECDL heads. It is surprising that a commercial ECDL system with associated temperature, current and piezo drivers should exhibit such a large linewidth. Usually technical (i.e. current supply) noise is the primary cause of linewidth broadening, but the current supply is specified at $< 10 \mu\text{A}$ noise at frequencies 0 to 1 MHz [101] which does not account for the linewidth we measure. Poor optical feedback into the diode from the diffraction could be a factor, due to the non-AR coated front facet of our diodes and the design wavelength of the aspheric lenses inside the head. With appropriate investment a linewidth of $< 1 \text{ MHz}$ should be realistic, even at a near-ultraviolet wavelengths.

5.4.5 Repump laser system

The laser systems used to address the $D_{3/2} \longleftrightarrow P_{1/2}$ transition at 866 nm, and the $D_{5/2} \longleftrightarrow P_{3/2}$ transition at 854 nm, required to repump the ion from dark states, are described in refs. [70, 71]. The 854 nm laser is turned on and off using a double-passed AOM, which is controlled using the FPGA experimental control system. The 866 nm laser remains on throughout our experiments. The light from the two lasers is combined and coupled through a fibre EOM, after which they are sent to the trap breadboard using polarisation-maintaining fibre.

EOM repumping scheme

Due to the large Zeeman shifts present in a Penning trap (discussed in section 4.1.1), the various transitions that must be addressed in order to repump the ion from the $D_{3/2}$ states are separated by $> 1 \text{ GHz}$. Additionally, as described in section 4.3 the $D_{5/2}$

states also need to be repumped during Doppler cooling. This means that we need to address transitions at a total of ten different frequencies. The first-order Zeeman shifts of the states results in a simple ratio between the splittings of the $D_{5/2} \longleftrightarrow P_{3/2}$ transition and the $D_{3/2} \longleftrightarrow P_{1/2}$ transition. This means that we can use a single source at 854 nm and one at 866 nm and still hit all of the repumping transitions that we need to hit. We apply frequencies of 1.7 GHz and 10.4 GHz to an electro-optic modulator to produce sidebands that are near to resonant with the $\sigma^\pm$ transitions that we need to address. The second-order Zeeman shifts described in section 4.1.1 are small enough that they can be compensated for by adjusting the centre and modulation frequencies.

The two beams are combined before being coupled into the fibre EOM. They are combined on a dichroic mirror which also filters any amplified spontaneous emission at close to 854 nm present in the 866 nm laser.

Red scanning cavity

The two repump lasers are frequency-stabilised against long term drifts using a lock very similar to the 397 nm scanning cavity transfer lock described in section 5.4.4. This lock is described in refs. [72, 99] and uses the same cavity design and light from the same stabilised HeNe as the blue scanning cavity system.

5.4.6 729 nm spectroscopy laser lock

The PDH lock of our 729 nm spectroscopy laser was originally built by Dan Crick and a detailed description can be found in refs. [70–72]. The fundamentals of the locking are based on ref. [97]. The cavity finesse has been measured to be $\mathcal{F} \approx 60000$ using a ringdown technique. This corresponds to a linewidth of 25 kHz. The spacer of the reference cavity for this lock is made of ultra-low expansion (ULE) glass, with a mirror spacing of length $L = 10$ cm. ULE glass has a thermal expansion coefficient that passes through zero at a temperature referred to as the ‘zero-crossing point’. If the temperature of the cavity is close to this point, the length of the cavity will become insensitive to small changes in temperature. To find the zero crossing point of our particular cavity we built a control system that is capable of reducing the temperature of the cavity and its large vacuum chamber to as low as 10 °C and then measure the position of the $S_{1/2} \longleftrightarrow D_{5/2}$ transition at different values of cavity temperature. We measure the relative frequency of the cavity resonance by locking to a particular mode and finding the frequency applied to the AOM at the centre of the carrier. This data is shown in Figure 5.9 and a quadratic fit, giving a zero-crossing temperature of 17.4 °C.

The bandwidth of the AOM that we use to scan the laser frequency is roughly 30 MHz, which is much less than the cavity free spectral range of 1.5 GHz. Therefore we do not necessarily have the freedom to lock to the $\text{TEM}_{0,0}$ or other low-order mode of the cavity when we address a particular $S_{1/2} \longleftrightarrow D_{5/2}$ transition. In general,

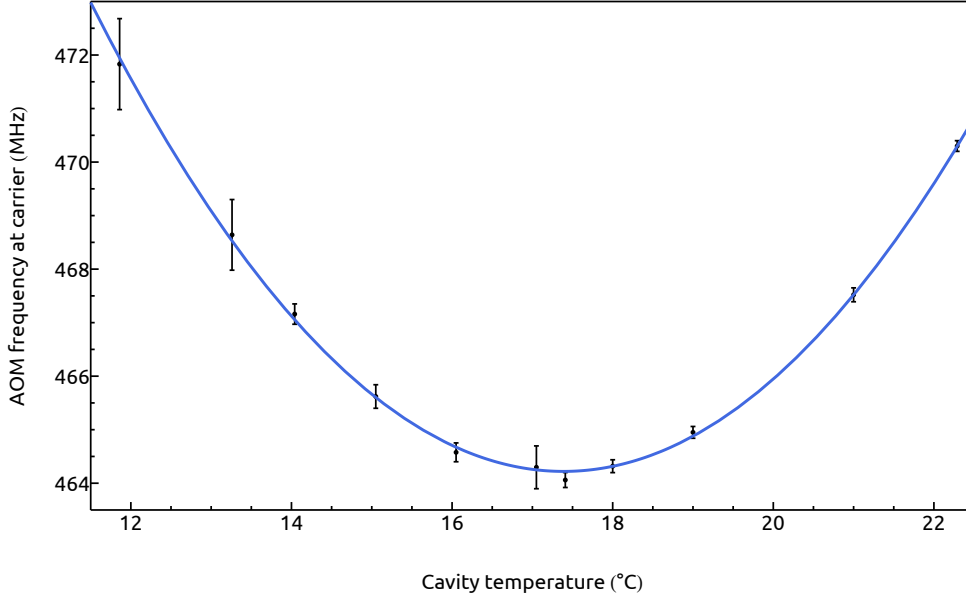


Figure 5.9: Position of the $S_{1/2} \longleftrightarrow D_{5/2}$ transition at different cavity temperatures in order to find the zero-crossing point of our ULE cavity.

the lock performs best when using a low-order mode of the cavity due to better mode-matching, so alongside the magnetic field sensitivity, we must also consider the nearest cavity mode to a particular transition when deciding which Zeeman sublevels to use as our qubit states.

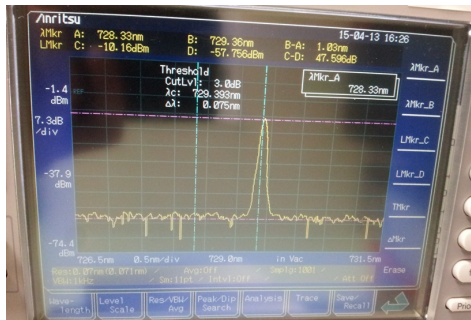
The most significant change to this system since previous PhD theses [71, 72] has been the removal of the injection lock that was previously used to increase the power into the tapered amplifier system. The use of an optical spectrum analyser⁴ showed us that the injection lock was very sensitive to the slave current. Small changes in this current resulted in additional spectral features appearing on our laser, separated by a wavelength of approximately 0.5 nm. This revealed to us that our diagnostic readout for this injection lock, a heterodyne beat note measurement, did not give us full information about the spectral properties of the injection lock. By rearranging the optics we found that we were able to remove the injection lock system and still have sufficient power to saturate the tapered amplifier. This has significantly simplified the system and has reduced drifts of the effective power of the laser into extra spectral components.

5.5 Trap breadboard optics

We use polarisation-maintaining fibre to send the laser light from the optical table to the trap breadboard that is positioned underneath the superconducting magnet. Fig-

⁴Kindly lent to us by Robert Murray and the Femtosecond Optics Group.

5.5. TRAP BREADBOARD OPTICS



(a) Output of PDH-locked master laser before injection into slave.



(b) Output of slave laser with imperfect injection lock.

Figure 5.10: Photographs of the output of an optical spectrum analyser showing the spectral properties of our 729nm laser. (a) is the light from the master laser with the PDH lock on, showing a single peak. (b) is the output of the injection locked slave laser with imperfectly-set slave current. there are two clear peaks of almost equal power.

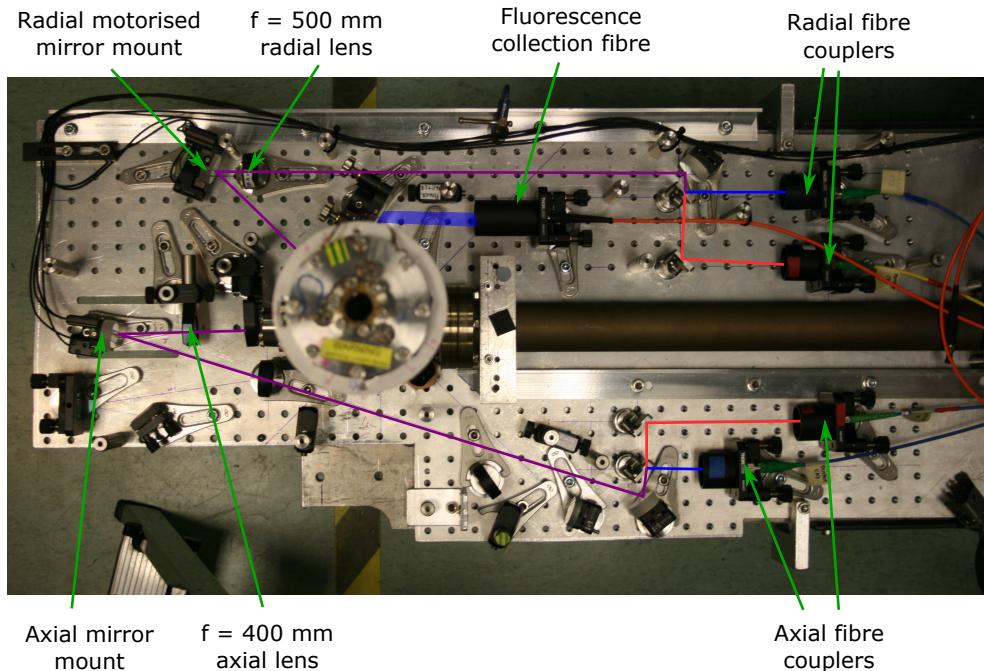


Figure 5.11: Labelled photograph of the trap breadboard optics.

ure 5.11 shows the optics that are on the trap breadboard. The Doppler cooling and repump laser light arrive on the trap breadboard from separate fibres and are then combined using dichroic mirrors. The two lenses that are used to focus light to the centre of the trap are situated on the breadboard outside the vacuum chamber, and have focal lengths of $f = 400$ mm for the axial beam, and $f = 500$ mm for the radial beam. These lenses give beam waists of $45\text{ }\mu\text{m}$ and $55\text{ }\mu\text{m}$ respectively at the trap centre. The axial beam enters the chamber at breadboard level and hits a mirror inside to send it vertically through the trap. The radial beam is directed vertically up the bore outside the trap before entering the chamber via a mirror glued to the side of the chamber. We use a computer-controlled motorised mirror mount in the radial beam path in order to move it in a controlled manner. After the beams have passed through the trap they hit quadrant photodiode detectors which aid with beam alignment.

5.6 Fluorescence collection and ion detection

The fluorescence light produced by scattering on the $S_{1/2} \longleftrightarrow D_{5/2}$ transition at 397 nm is collected into two multimode fibres of diameter $200\text{ }\mu\text{m}$. Each of these fibres sends the light into a separate photomultiplier tube (PMT). By removing one of these fibres we have the ability to send the light to an imaging system so that we can view the ions on an Andor ICCD camera. We require that this imaging system relays the image a significant distance from the superconducting magnet, as the electronics inside the ICCD can be damaged if it is placed too close to the magnetic field. This relay imaging system consists of two 50.8 mm diameter achromatic doublet lenses of focal lengths of 150 mm and 500 mm which gives us a further magnification of 3.3 , resulting in a total magnification of 10 . The reason for using achromatic doublet lenses even though we are only interested in light emitted at 397 nm is that their design results in fewer aberrations than singlet lenses, which was confirmed in simulations done on Zemax by Jieyi Liu [102]. An image of a single ion on an Andor ICCD camera with a pixel size $13\text{ }\mu\text{m} \times 13\text{ }\mu\text{m}$ is shown Figure 5.12.

In order to measure the qubit state of the ion we use standard fluorescence detection techniques associated with electron shelving spectroscopy [56, 103, 104]. By applying our Doppler cooling lasers (except for the 854 nm , which would empty the $D_{5/2}$ state) we scatter photons conditional on the initial electronic state of the ion. These lasers couple an ion in the $S_{1/2}$ electronic state to the Doppler cooling cycle meaning that photons are scattered by the ion and can be detected, but an ion ‘shelved’ in the $D_{5/2}$ state will not be coupled to this cycle and will scatter no photons. If the timing of these scattering events is random and independent, the number of photons n that we detect in a period t_d should follow a Poisson probability distribution, given by

$$P(n) = \frac{e^{-\lambda} \lambda^n}{n!}, \quad (5.11)$$

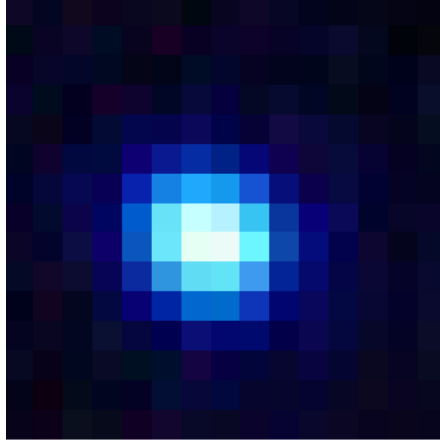


Figure 5.12: Image of a single ion on our ICCD camera using the new imaging system. Each pixel is $13 \mu\text{m} \times 13 \mu\text{m}$.

where λ is the mean number of counts in the detection period.

For an ion prepared in the $D_{5/2}$ state this photon count distribution will be solely due to background photons that reach the PMTs, and will have a mean of $\lambda = R_d t_d$, where R_d is the background photon detection rate. In our experiment R_d is dominated by laser scatter of the radial 397 nm beam from surfaces such as vacuum windows and trap electrodes. For an ion prepared in the $S_{1/2}$ state the photon count distribution will be due to photons scattered by the ion as well as the background counts, resulting in a Poisson distribution with mean $\lambda = (R_d + R_b)t_d$, where R_b is the scattering rate from the ion, multiplied by the photon detection efficiency. In order to differentiate between these two states, we must count for a sufficiently long t_d that there is little overlap between the photon count distributions associated with an ion prepared in the two different states. We can then set some threshold value of photons which discriminates between the states with low error.

However in the high magnetic field associated with the Penning trap, decay from the $P_{1/2}$ state into the $D_{5/2}$ state can occur due to the j -mixing effect described in section 4.3. This means that the Doppler cooling cycle in the absence of the 854 nm laser is not completely closed and an ion prepared in the ‘bright’ state ($S_{1/2}$) can decay at some point during the detection period and become ‘dark’. When this occurs the ion will stop scattering, and only background photons will be detected for the rest of the period. This leads to a photon count probability distribution given by

$$P(n) = e^{-t_d/\tau} \frac{e^{-R_t t_d} (R_t t_d)^n}{n!} + \frac{1}{R_b \tau} \int_{R_d t_d}^{R_t t_d} e^{-(\lambda - R_d t_d)/R_b \tau} \frac{e^{-\lambda} \lambda^n}{n!} d\lambda \quad (5.12)$$

with $R_t = R_d + R_b$ is the total detected photon rate from a bright ion including background, and τ is the characteristic lifetime associated with j -mixing decay. The first

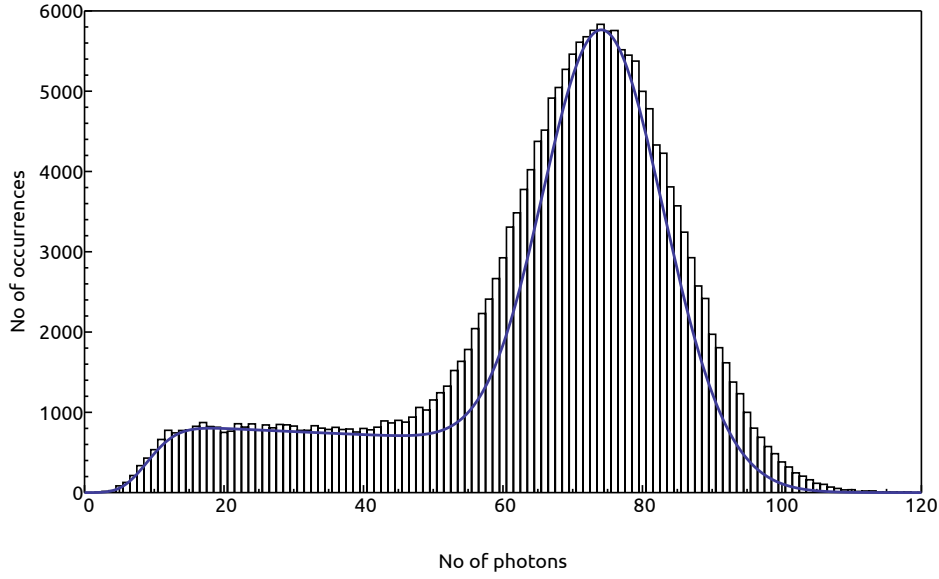


Figure 5.13: Histogram of the number of 397 nm photons obtained from a single ion in a 42 ms count period. The ion is initially prepared in the $m_j = +\frac{1}{2}$ state in the $S_{1/2}$ manifold. The blue line is a fit to (5.12), giving a characteristic j -mixing lifetime of $\tau = 134$ ms.

term is the expected Poisson distribution in the absence of decay, weighted by the probability that the ion does not decay during the detection period, $e^{-t_d/\tau}$. The second term gives the averaged distribution over all the times at which the ion can decay during the detection period.

Figure 5.13 shows a histogram of the number of photons detected in our experiment in a 42 ms detection period from an ion prepared in the $S_{1/2}$ state. The roughly Poissonian distribution centred at a photon count of $n \approx 75$ is predominately made up of occurrences where the ion remains bright for the whole of the detection period (i.e. the first term in (5.12)). The additional, roughly flat distribution for photon counts of $15 \lesssim n \lesssim 60$ appears due to j -mixing decays during the detection period (the second term in (5.12)).

To demonstrate how these j -mixing decays effect our state determination a histogram of the photon counts detected in a 8 ms detection period from a typical experiment is shown in Figure 5.14. In this particular case the ion has been prepared in the $S_{1/2}$ and $D_{5/2}$ states roughly half the time each. In order to distinguish between the two states we set a threshold of $n_{\text{thresh}} \approx 6$, where the two Poisson distributions intersect. However if a decay occurs in the detection period we will obtain less photons than anticipated and we are less able to distinguish between the two states using a simple threshold method.

In order to reduce the error associated with state detection in our experiment we

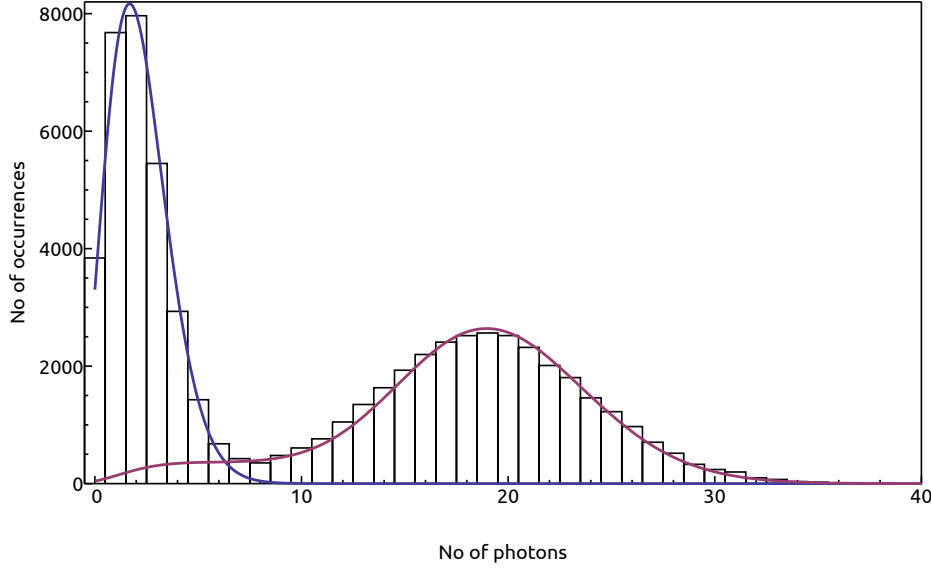


Figure 5.14: Histogram of photons detected during an 8 ms count period from an ion prepared $S_{1/2}$ and $D_{5/2}$ roughly half of the time each. The solid lines shows the expected photon distribution for occasions where the ion was prepared ‘dark’ (blue) and ‘bright’ (magenta).

need to reduce the overlap between the distributions. However, since j -mixing decay is an event that occurs with a certain probability with each scattering event, it is not sufficient to count for longer or increase the scattering rate as the overall probability of a decay during the detection period will increase. Therefore we need to either reduce R_d , by using anti-reflection optics or reducing background scatter by other means, or increase the value of R_b by increasing the detection efficiency of the system, by using a detector with higher quantum efficiency or using collection optics with higher numerical aperture.

One way we could potentially improve our single ion state measurement without any change to the rates R_d and R_b would be to implement a maximum likelihood estimation scheme. This involves splitting the detection period of length t_d into smaller sub-bins of length t_b . This additional time-dependent fluorescence information allows for easier discrimination between ions that were prepared in the $D_{5/2}$ and those ions that were prepared in the $S_{1/2}$ state but decay into the $D_{5/2}$ state during the detection period [104].

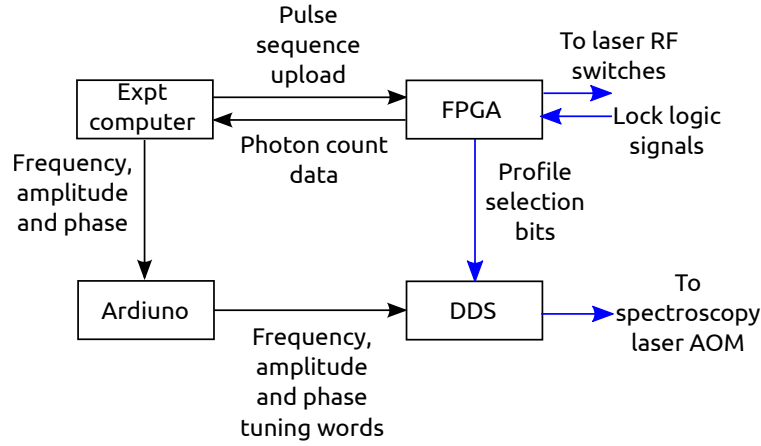


Figure 5.15: Schematic of computer control system used in the experiment.

5.7 Experimental control

5.7.1 FPGA pulse sequence control

In order to perform the pulse sequences necessary to perform our spectroscopic experiments, we use a system based around a field programmable gate array (FPGA) with a C[#] user interface. The original version of this system was built by Shamim Patel and Stefan Zeeman [105, 106], and is still fundamentally the same with some changes to the data structure and modifications to allow for different types of experiments to be performed.

The C[#] interface is used for designing the pulse sequences which are saved as .xml files. These files are then converted to FPGA-readable .hex files that tell the FPGA the timing and on/off state of the various lasers in the pulse sequence. A signal to start the experiment is sent from the computer to the FPGA, at which point the FPGA repeats the pulse sequence either until it has completed the specified number of sequences or it receives a pause or stop signal from the computer. The FPGA controls the on/off state of the lasers using TTL signals sent to various RF switches that control the voltage to the double-passed AOMs in the laser beams.

Additionally there are logical inputs sent from the various locking systems to the FPGA in order to ensure that all of the lasers are locked during the experiment. If one or more of the lasers becomes unlocked the data is flagged as containing a laser error and can be omitted from the dataset. The box contains a mains-phase trigger that allows us to control when the pulse sequences are started to avoid line-broadening due to stray magnetic fields.

At specified points in the pulse sequence the FPGA sends data to the computer which is saved in a file along with metadata about the experiment and the C[#] program

analyses this data and plots the histograms and spectra for live viewing.

5.7.2 DDS signal generator for spectroscopy laser

A recent and very important addition is the integration of a direct digital synthesis (DDS) system designed and built by Vince Jarlaurd to generate the oscillating voltage for the spectroscopy laser AOM. This system produces the signals necessary to switch between multiple sidebands for complicated sideband cooling sequences, which is especially key for cooling the radial motion, where there are two sets of sidebands, both of which are associated with high average phonon numbers. Prior to this we used an individual HP signal generators to address each sideband, controlled via GPIB. This system took roughly ~ 1 s to update the frequency, which meant that we needed a separate signal generator for each frequency we needed to apply during a pulse sequence

A basic DDS system consists of a frequency tuning word, a phase accumulator, a digital-to-analog converter (DAC) and an output filter. At each clock cycle the frequency tuning word tells the DDS how much phase to add to the phase accumulator, and then a lookup table is used to see what the sine wave output voltage should be at this phase. The output is changed to this digital value, and is fed into a DAC which converts the signal to an analogue output. This output will have unwanted components at the clock frequency due to digitisation noise, so a low-pass filter is used to remove these components.

The Analog Devices AD9910 is used in ‘single-tone’ mode and can provide up to eight ‘profiles’ with their own independent frequency, amplitude and phase tuning words, which can be selected using 3 high-low logical inputs to the DDS board. This has been the main development that has allowed us to do multi-stage sideband cooling, and should allow us in future to perform arbitrary single ion state rotations.

The C[‡] code communicates serially with an Arduino which is used to write the amplitude, phase and frequency tuning words to the DDS memory. This transfer is fast enough (< 100 μ s) that it could be done during a pulse sequence if more than eight profiles were required, but we don’t need to do this currently

Chapter 6

Results

6.1 Introduction

In this chapter I will present results obtained mainly during the last year of my PhD, which build on work done in earlier years in which I was also an active participant. In particular, these results use techniques developed to perform the first optical resolved-sideband spectroscopy in a Penning trap [38], and the first demonstration of optical sideband cooling in a Penning trap [72, 107]. The former is expanded upon in section 6.2, where I present improved sideband spectroscopy of the axial and radial motions of a single ion, and in section 6.3.3 I report on significant extensions to our initial sideband cooling results, showing ground state cooling of the axial motion for ions at lower trap frequency, from temperatures initially well outside the Lamb-Dicke regime, and also present an example of a coherent operation in the form of a Rabi oscillation. To a lesser extent the expertise gained in a study of the conformations of ion Coulomb crystals in our trap [39] has also been used in order to perform spectroscopy of a two-ion crystal after Doppler cooling, which is shown in section 6.2.4.

I will also report the first demonstration of sideband cooling of the modified cyclotron motion of a single ion, showing a significant reduction in temperature in comparison with Doppler cooling. While the axial mode structure in a Penning trap more readily lends itself to use in quantum information science (as in refs. [35, 108]), the technical difficulty associated with reliably cooling the radial motion presents a significant barrier to the use of this type of trap in quantum information experiments. Therefore any improvements in the ability to control this motion are of importance. I will then present a study of the axial heating rate in our trap at different trap frequencies and also on a significant reduction of these rates to the extent that our current measurements are the lowest reported to date. Heating rates are of interest in ion trap experiments as they place a practical limit on how long coherent operations can be allowed to take. Finally I present a comparison with other heating rates in section 6.4.2. Our heating measurements are unique amongst ion trapping experiments as cooling to

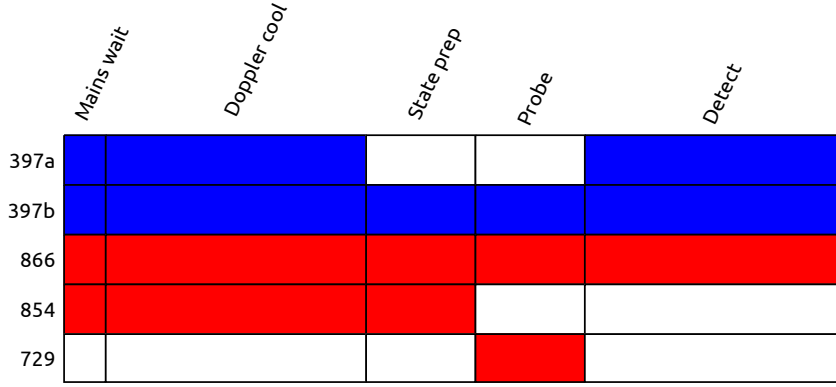


Figure 6.1: The laser pulse sequence for spectroscopy of Doppler-cooled ions. Colour indicates that the laser light is turned on during the pulse period, white indicates that it is turned off. Typical pulse lengths are 5 to 8 ms for Doppler cooling and count periods, 100 μ s for state preparation and ~ 30 μ s for the probe.

the ground state of motion has never previously been achieved in a Penning trap, but has also never been demonstrated in a trap as large as ours before.

6.2 Spectroscopy of Doppler-cooled ions

6.2.1 Spectroscopy pulse sequence

The first optical sideband spectroscopy of ions in a Penning trap was completed in the first year of my PhD and is published in ref. [38]. The technique used to perform narrow-linewidth spectroscopy on the $S_{1/2} \longleftrightarrow D_{5/2}$ transition in trapped $^{40}\text{Ca}^+$ ions underpins most of the experiments that we perform in the group, and the ability to measure the temperature of the motion of a single ion has allowed us to improve the system to the level where we can reliably cool the axial motion to close to the Doppler limit and therefore to extend to more complicated work.

We use the pulse sequence shown in Figure 6.1 controlled by the FPGA system described in section 5.7.1 for spectroscopy of Doppler-cooled ions. All of our experiments are synchronised to the mains cycle to prevent line broadening due to any stray fields caused by electrical equipment in the lab. Once the experiment has been triggered by the mains phase we Doppler cool for between 6 and 8 ms, a long enough period to reach the Doppler limit and also long enough to collect enough photons to verify that the ion is scattering light from the lasers. If insufficient photons are collected during this period then the subsequent measurement is rejected to avoid including experiments where the laser had become unlocked, or the ion was temporarily heated away from the trap centre due to a background gas collision. We then prepare the ion in the correct $S_{1/2}$ ground state for the particular $S_{1/2} \longleftrightarrow D_{5/2}$ transition we are using by turning off

one of the 397 nm Doppler cooling lasers. If the ion is in the incorrect $S_{1/2}$ state it will be optically pumped into the correct state after two scattering events on average. We can leave the other 397 nm laser on throughout the experiment as it should have no effect once we are prepared in the correct $S_{1/2}$ state. After this short state preparation period we apply light of frequency close to the narrow linewidth $S_{1/2} \longleftrightarrow D_{5/2}$ transition for a known pulse time, which will excite the ion with some probability. Once the probe pulse has finished we turn on the Doppler cooling lasers again and collect fluorescence, performing a projective measurement of the state of the ion in the manner described in section 5.6. The only exception is that the 854 nm laser remains off during the detection period, otherwise we would repump the $D_{5/2}$ state in which we are attempting to shelve, and the ion would always appear bright. The 866 nm laser remains on throughout the experiment. The exact lengths of the cool, count and probe period can be changed for different types of experiments depending on factors such as the rate of fluorescence photon detection and the on-resonance Rabi frequency of the $S_{1/2} \longleftrightarrow D_{5/2}$ transition (which scales with the root of the 729 nm laser intensity at the ion).

This pulse sequence is then repeated a number of times (typically 100-400) in order to build up sufficient statistics to allow us to determine the probability that the ion has been excited by the probe pulse with sufficiently small error. In statistics, this kind of two-outcome experiment is referred to as a ‘Bernoulli trial’. After each of these sets of repeated measurements the frequency of the spectroscopy laser is increased so as to scan across the transition, and then the readings from the most recent set of repeats are sent from the FPGA to the experiment computer, on which the probabilities are displayed in real-time (section 5.7.1).

6.2.2 Axial motion

The theory behind the appearance of resolved sidebands, due to the combination of the Doppler effect and the ion’s motion, is given in section 3.2. The probability of excitation of an ion with average phonon number $\bar{n}$, with light detuned an angular frequency δ from the i^{th} -order sideband after a pulse length t is found by combining (3.23) and (3.17) to give

$$|c_e(t)|^2 = \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(\bar{n} + 1)^{n+1}} \frac{\Omega_{n,n+i}^2}{\Omega_{n,n+i}^2 + \delta^2} \sin^2 \left(\frac{\sqrt{\Omega_{n,n+i}^2 + \delta^2}}{2} t \right), \quad (6.1)$$

where $\Omega_{n,n+i}$ is the Rabi frequency of the transition $|g, n\rangle \longleftrightarrow |e, n+i\rangle$, given in (3.21).

To perform axial spectroscopy on a single ion we first load the trap and verify that we have a single ion, either by viewing it on an ICCD camera, or blocking the 854 nm light and observing the discrete fluorescence levels caused by using quantum jumps. We

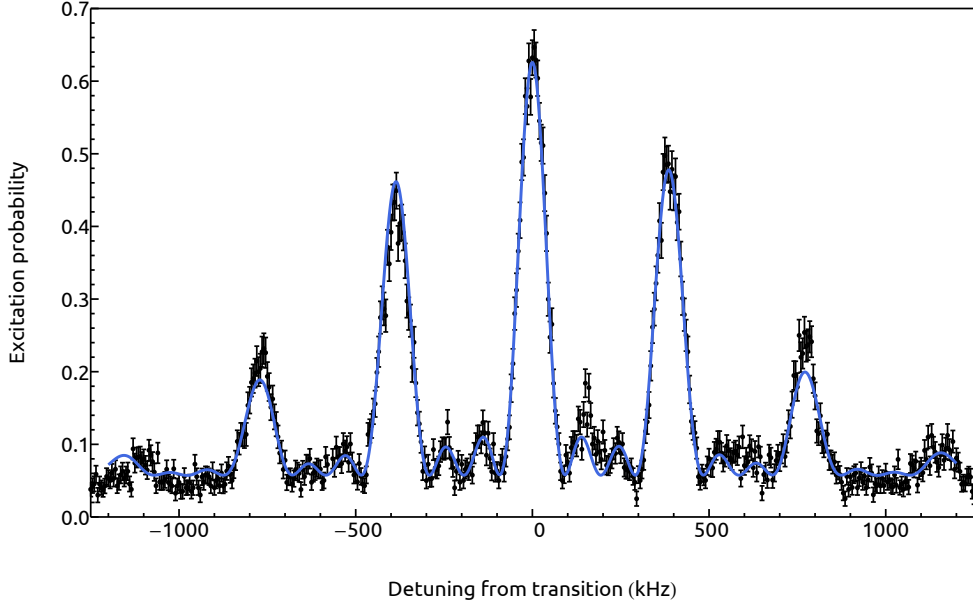


Figure 6.2: Spectrum of the axial motion after Doppler cooling with a probe pulse length of $10\ \mu\text{s}$. The axial frequency is $\omega_z = 2\pi \times 386\ \text{kHz}$. Each point represents 400 measurements. The fitted curve is to the Rabi dynamics of a thermal state.

then couple light from the 729 nm laser through the fibre that carries the 866 and 854 nm repump laser light used. The output beam from this fibre travels vertically through the trap centre, along the direction of the magnetic field. We adjust the frequency of the two Doppler cooling lasers to locate the resonance position and then attempt to red-detune them symmetrically from the resonance by approximately 9 to 12 MHz. Once the lasers are locked and the fluorescence level is stable we run the pre-programmed pulse sequence shown in Figure 6.1 by sending a start signal to the FPGA control system. The total length of this type of spectrum is roughly 20 min.

Figure 6.2 shows the axial spectrum obtained after Doppler cooling for 7 ms at an axial trap frequency of $\omega_z = 2\pi \times 386\ \text{kHz}$. The probe pulse time used in this spectrum is $10\ \mu\text{s}$, which is short enough to ensure that the sidebands are not saturated. By fitting these data to the model in (6.1) we can extract the average phonon occupation number of the ion motion and the Rabi frequency of the interaction. This gives an average phonon number of $\bar{n}_z = 24.6 \pm 2.0$ and a Rabi frequency of $\Omega = 2\pi \times 47 \pm 14\ \text{kHz}$. This occupation number is in good agreement with the Doppler cooling limit, which is $\bar{n}_z = 24.5$ at this trap frequency, using (3.39). However even at the Doppler limit, for a Lamb-Dicke parameter of $\eta = 0.1558$ we have $\eta\sqrt{2\bar{n}_z + 1} = 1.1$, which means that the ion motion is still significantly outside the Lamb-Dicke regime.

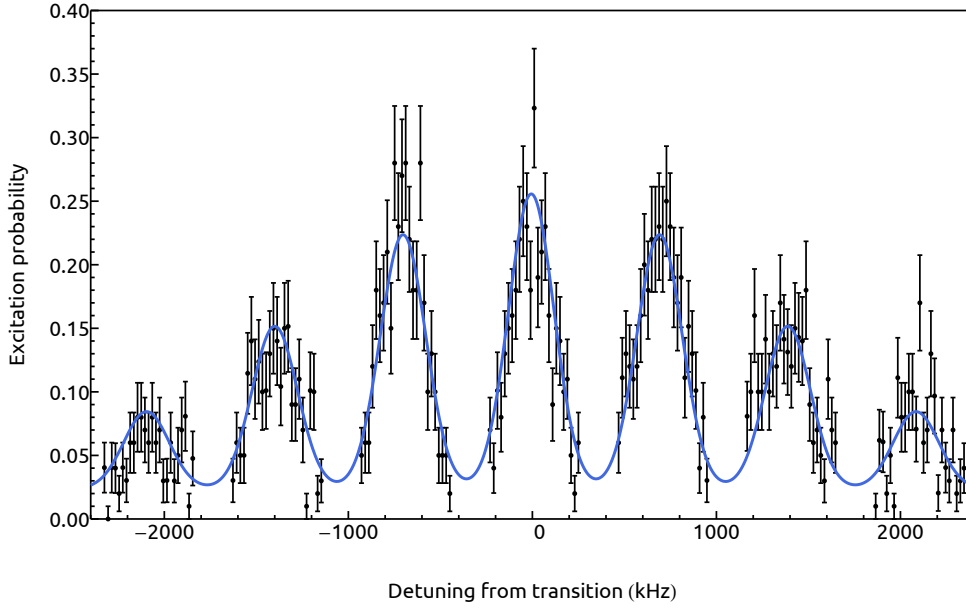


Figure 6.3: Spectroscopy of a single ion with a beam propagating in the radial direction. The mode frequencies are $\omega_+ = 2\pi \times 703$ kHz and $\omega_- = 2\pi \times 12$ kHz. Each point represents 100 experiments. The fit is to a comb of Gaussian distributions with heights given by another Gaussian distribution.

6.2.3 Radial motion

By coupling the 729 nm laser through the radial fibre shown in Figure 5.11, we are able to send the light through the trap in a direction perpendicular to the magnetic field and therefore probe the radial motion. We expect sidebands of the form described in section 3.3 (shown in Figure 3.4), with convolved combs of sidebands due to the magnetron and modified cyclotron motions.

The procedure we use to prepare a single ion before taking a radial spectrum is the same as that described in section 6.2.2 for an axial spectrum. The only difference is that more care is taken to ensure that the radial cooling beam is moved to the correct position. By moving this beam in the radial plane, we change the value of the intensity gradient parameter y_0 and find the beam positions at which the radial cooling fails and we lose fluorescence and then try to position the beam roughly halfway between these two points. We ensure that this position results in a stable level of fluorescence, and that the size of the ion orbit is not visibly changing on an ICCD camera.

Figure 6.3 shows the spectrum obtained after probing the radial motion for 100 μ s. The modified cyclotron frequency and the magnetron frequency used are $\omega_+ = 2\pi \times 703$ kHz and $\omega_- = 2\pi \times 12$ kHz respectively. At this low value of magnetron frequency we are unable to clearly resolve the different magnetron sidebands and the spectrum therefore consists of several broad sidebands separated at the modified cyclotron fre-

quency. This spectrum is ‘windowed’ around each modified cyclotron sideband so that we only take points at laser frequencies where we expect sidebands to appear. The fit shown is to a comb of Gaussian distributions, spaced at the modified cyclotron frequency, with heights given by another Gaussian distribution. From this we obtain a FWHM for the modified cyclotron motion of 2.97 ± 0.15 MHz, which equates to an average phonon occupation number of $\bar{n}_+ = 123 \pm 6$. The FWHM of the magnetron motion is 292 kHz, which gives to $\bar{n}_- = 4430 \pm 200$. From (3.40) and (3.40) the theoretical limit for the final phonon occupation numbers for a value of $y_0 = 25 \mu\text{m}$ are $\bar{n}_+ = 17$ and $\bar{n}_- = 795$, both of which are almost an order of magnitude smaller than we have been able to achieve so far with intensity gradient cooling of the radial motion. Anecdotaly, we have been able achieve some level of trade-off between these two occupation numbers (i.e. decreasing the $\bar{n}_+$ and increasing $\bar{n}_-$ and vice versa) by moving the radial beam and therefore changing the intensity gradient. However, we have been unable to decrease either of these occupation numbers to close to the theoretical cooling limits shown in Figure 3.6 for reasons that are currently unknown.

6.2.4 Preliminary two-ion axial spectroscopy

The effective radial trapping potential for a Coulomb crystal in a Penning trap is a function of the rotation speed of the crystal and is maximum when the crystal rotates in the lab frame at $\omega_c/2$, when the radial frequency is given by $\omega_1 = \sqrt{\omega_c^2/4 - \omega_z^2/2}$. If we increase the axial frequency of the trap there is a point at which the axial trapping strength is equal to the radial trapping strength, $\omega_z = \omega_1$. For two ions this is the point at which the crystal transitions from lying along the trap axis to lying in the radial plane. This means that there is a limit on the axial frequency that can be reached for a two-ion ion Coulomb crystal in an axial string given by $\omega_z < \omega_c/\sqrt{6}$. Since ω_c is proportional to the magnitude of $\mathbf{B}$, the axial frequency of these two-ion axial strings is fundamentally limited by the magnetic field strength.

To produce a two-ion axial crystal, we load two or more ions into the trap and then ground the endcaps for a short amount of time, as described in section 5.2, until only two ions remain. The resulting ion crystal is viewed on an ICCD camera and the laser frequencies and radial beam position are adjusted to ensure that the crystal is stably aligned along the trap axis. The mirror sending the fluorescence light to the camera optics is then removed, so that this light is detected on the second PMT (see section 5.6 for details on the fluorescence collection and imaging system).

The same pulse sequence is used as for single ion axial spectroscopy, with the only difference being that a longer count period of 13 ms is used in an attempt to improve the discrimination of the number of shelved ions. The histogram of the number of photons collected during 13 ms count periods shown in Figure 6.4 illustrates the problem of trying to determine whether zero, one or two ions have been shelved during the probe period in our experiment. The combination of the low collection efficiency, relatively

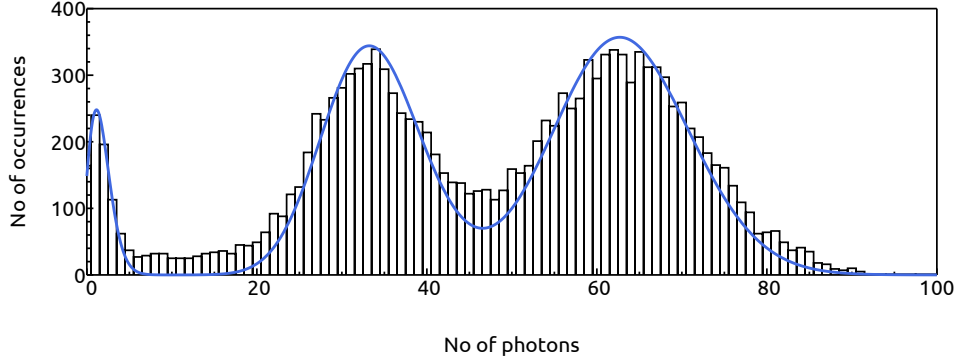


Figure 6.4: Histogram of photon counts from a two ion axial string for many 13 ms count periods.

high background scatter and j -mixing means that it is not easy to set threshold values of photon counts that determine the state of the two ions and therefore to measure the excitation probability. A fit to three Poisson distributions is shown to demonstrate the underlying behaviour. We have used a long enough probe pulse that there is substantial probability that both ions are excited.

Figure 6.5 shows the obtained excitation spectrum for an axial two-ion crystal, with a centre of mass axial frequency of $\omega_z = 2\pi \times 234$ kHz after a $100 \mu\text{s}$ probe pulse. Here a single threshold has been set to measure the probability that both of the ions are shelved by the spectroscopy laser.

The first thing to note is that no resolved sidebands are visible in this spectrum (although there is some hint of structure in the centre of the spectrum where the excitation probability drops to 5%). We expect that for a two-ion axial crystal in the strong-binding regime we should see two convolved combs of sidebands, one set separated by the centre-of-mass motion frequency $f_z = \omega_z/2\pi$ (the same as the axial frequency of a single ion) and the other set spaced by the frequency of the breathing mode $f_{\text{breathe}} = \sqrt{3}f_z$, but there is no indication of this sideband structure.

If we assume a roughly Gaussian lineshape due to Doppler broadening and account for the fact that we measure the double probability of excitation by fitting the square of a Gaussian (also a Gaussian with different height and width) we obtain a FWHM Doppler width of 2.3 MHz. This width corresponds to a temperature of 2.4 mK and average phonon occupation numbers of $n_{\text{COM}} = 218$ and $n_{\text{breathe}} = 126$.

The spectrum is likely unresolved due to a combination of high saturation due to the long probe time used and also a difference in carrier frequency for the two ions. The separation of the two ions in the crystal is of order $20 \mu\text{m}$ and from our measurements of the axial magnetic field gradient (shown in Figure 5.2), a difference in carrier position for the two ions of the order ≈ 5 to 10 kHz is realistic.

A spatially resolved system using an Electron-Multiplying Charge Coupled Device

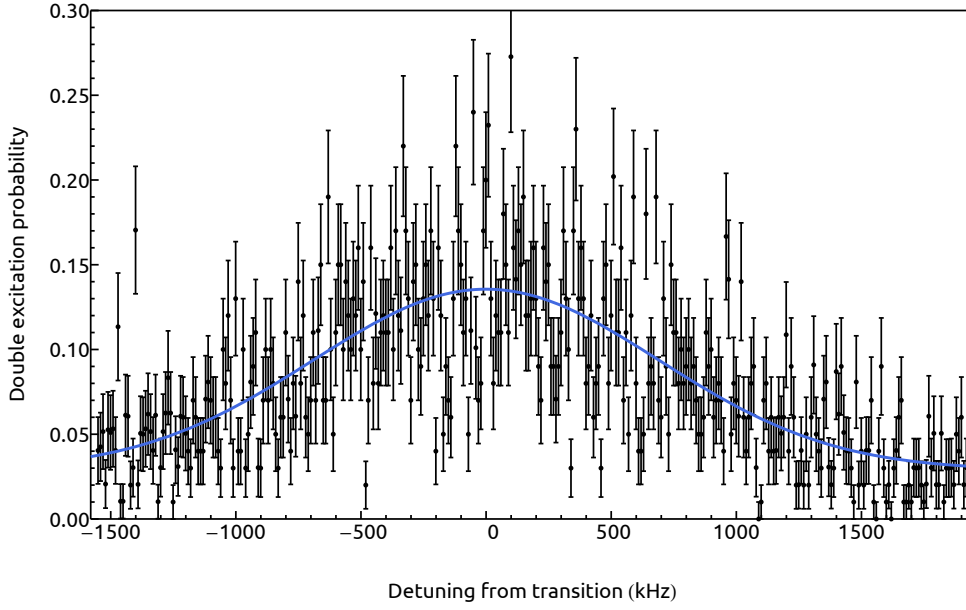


Figure 6.5: Double excitation probability spectrum for an axial two-ion crystal with a fit to a simple Gaussian model, which gives a Doppler FWHM of 2.3 MHz.

(EMCCD) would allow us to see both ions individually and exact knowledge of the pixel positions of the two ions would mean that less background scatter would be included in the readings. This would allow us to verify which of the two ions has been shelved with far less error, and would also mean that the effect of any difference in the carrier frequency of the two ions due to inhomogeneities in the magnetic field strength will be resolved.

6.3 Sideband cooling of a single ion

Once we have Doppler cooled the ions we need to cool them further until we enter the Lamb-Dicke regime, for reasons outlined in section 3.2.1. Our relatively low trap frequencies mean that even when we reach the Doppler limit we still sit outside the Lamb-Dicke regime. All the future experiments we want to do, including two-ion entangling gates and more complicated schemes with planar ion Coulomb crystal (see sections 7.3 and 7.4) will require that we are inside the Lamb-Dicke regime. An overview of the theory of sideband cooling, including discussion of population trapping and broadening the transition using light at 854 nm is given in section 3.4.2.

The Rabi frequencies on the red, carrier and blue sidebands in the Lamb-Dicke regime are given in (3.26), (3.27) and (3.28). If we substitute these scalings into (6.1), we obtain much simpler expressions than when using the full expression for the Rabi frequencies. Furthermore, for $\bar{n} \ll 1$ the occupation of the n^{th} Fock state quickly

becomes very small and so we need only include the first few terms of the sum in (6.1).

At a trap frequency of $\omega_z = 2\pi \times 400$ kHz the Lamb-Dicke parameter is $\eta = 0.153$, and the $\sqrt{n}$ scaling of Rabi frequency of the first-order sidebands only holds to within 5 percent for the first few Fock states, so care must be taken that the ion is sufficiently close to the ground state that the approximations above hold. We expect that after sideband cooling $\gtrsim 90\%$ of the population will be in the ground state, which means that $P(n \geq 4) = 0.001\%$, and it is therefore valid to only include the first 4 terms of the sum.

6.3.1 Broadening the sideband cooling transition

Two of the most important parameters to adjust to achieve the required sideband cooling rate and final temperature are the frequency and power of the 854 nm light used to change the effective decay rate of the 729 nm transition. As described in section 3.4.2 the electric quadrupole transition that we use for sideband cooling has a decay rate of ~ 1 s, and we therefore need to artificially broaden this transition to achieve a fast enough cooling rate to allow us to cool to the motional ground state.

There is a complication in tuning our laser close to the resonant frequency of the particular 854 nm transition used for broadening, as we produce all of the frequencies that we require to repump the ion after decays due to j -mixing from a single source (see section 5.4.5). The ion can decay into any one of four $D_{5/2}$ states due to j -mixing, whereas in order to broaden the linewidth of the particular $S_{1/2} \longleftrightarrow D_{5/2}$ transition we need to address one particular excited state. The fact that we use our 854 nm light for two different things means that we do not have total freedom to change the parameters. In practice we coarsely set the 854 nm parameters to ensure that we repump decays due to j -mixing and then fine tune the frequency and power of the light in order to obtain sufficient broadening of the transition for sideband cooling. To optimise the amount of broadening, and also to quantify the AC Stark shift caused by the 854 nm light, we take interleaved spectra with the 854 nm laser on and off during the probe pulse.

An example of an interleaved set of spectra with different amounts of 854 nm laser power applied during the probe pulse is shown in Figure 6.6. Longer pulse times are used when the 854 nm light is on, as the linewidth we observe becomes limited by the effective linewidth of the transition, rather than by the Rabi frequency. The peak excitation probability for a given probe pulse time becomes smaller, and therefore a longer probe pulse time is required to increase the peak above the background level. The increase in the 854 nm power by a factor of 2 between the two broadened spectra results in an increase in AC Stark shift from roughly 14 to 26 MHz (consistent with the doubling we expect) and an increase in the measured Lorentzian linewidth from 11 to 17 kHz.

Once we have optimised the broadening we optimise the frequency of the sideband cooling laser by repeated measurement of the red sideband height after sideband cool-

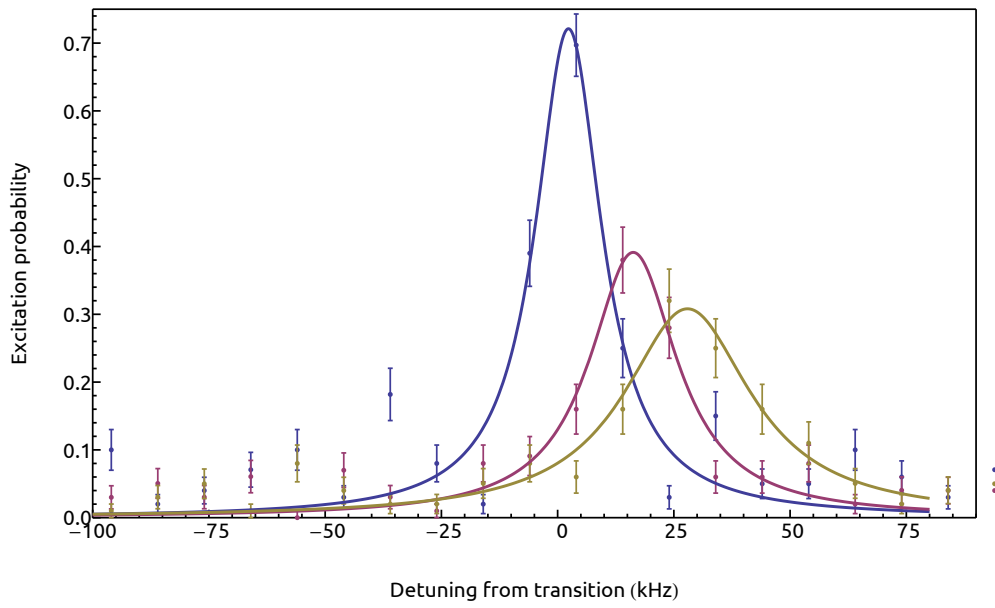


Figure 6.6: Demonstration of artificial broadening of the $S_{1/2} \longleftrightarrow D_{5/2}$ transition by application of 854 nm light. The unbroadened spectrum (blue) is shown along with broadened spectra with 50% (magenta) and maximum (gold) 854 nm power. The probe pulse length is 40 μ s for the unbroadened spectrum, and 2 ms for both broadened spectra.

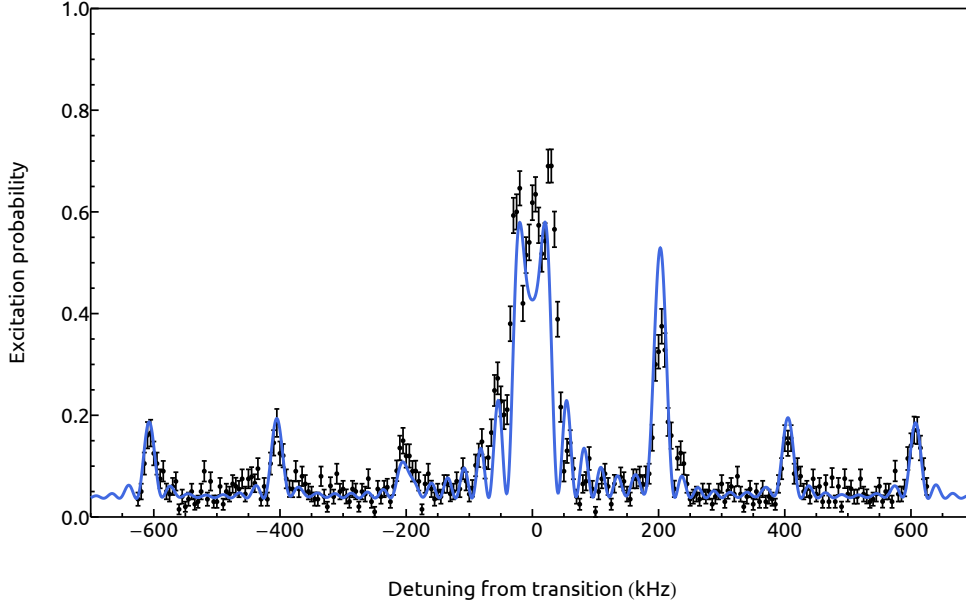


Figure 6.7: Spectrum after 21 ms of sideband cooling on the first axial red sideband. Trapping of population in a dark state of the first red sideband is indicated by the appearance of significant second and third order sidebands.

ing. We scan the sideband cooling frequency around the AC Stark-shifted red sideband transition until we find the point at which the red sideband height is minimised.

6.3.2 Population trapping in high-lying Fock states

In section 3.4.2 we saw that complications can arise if we naively attempt to sideband cool an ion on the first red sideband from outside the Lamb-Dicke regime. At higher values of $\bar{n}_z$ significantly more scattering events have to occur on the first sideband in order to remove the phonons in order to reach the ground state, additionally there are Fock states where first red sideband Rabi frequency is very close to zero (Figure 3.2) and optical pumping into these dark states can occur.

To demonstrate this population trapping we first prepare a single ion close to the Doppler limit at an axial trap frequency of $\omega_z = 2\pi \times 200$ kHz. We then add a single 10 ms period of sideband cooling on the first red sideband using the pulse sequence shown in Figure 6.1 directly after the Doppler cooling period and perform spectroscopy.

Figure 6.7 shows the excitation probability after 10 ms of sideband cooling. At $\omega_z = 2\pi \times 200$ kHz the Fock state at which the Rabi frequency of the first red sideband passes through zero is $n_{\text{zero}} = 78$. At the Doppler limit ($\bar{n}_z \approx 45$), the probability that the ion is in a Fock state $n \geq n_{\text{zero}}$ is approximately 0.18. The clear asymmetry between the strength of the first order red and blue sidebands makes it clear that there is a significant amount of population in the ground state, but the appearance

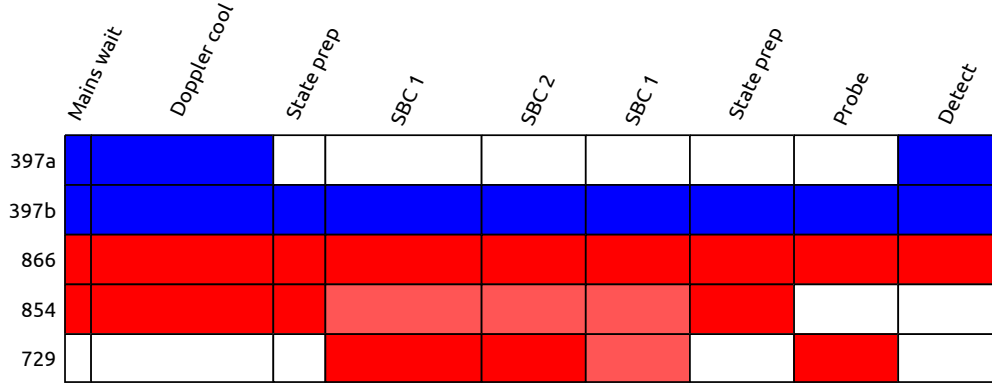


Figure 6.8: The pulse sequence used for multiple-stage sideband cooling. Lighter fill indicates that reduced power is used during the particular pulse.

of high order sidebands is not consistent with a thermal state with this ground state probability, as the height of high order sidebands should generally be smaller than the first order sidebands. The data is shown along with a fit to the model (3.48). The fit gives a probability of sideband cooling of $P(\text{thermal}) = 0.84$, and this sideband cooled population is characterised by an average phonon occupation number of $\bar{n}_z = 0.15$.

6.3.3 Multiple-stage sideband cooling of the axial motion

At a trap frequency of $\omega_z = 2\pi \times 400$ kHz the Rabi frequency for the first red sideband goes through zero at $n_z = 156$. At the temperature we reach after Doppler cooling there is a probability of approximately 3×10^{-3} that the ion is in a state with phonon number higher than before sideband cooling. Therefore at this increased trap frequency we still will have a small amount of population stuck in the dark state after a single stage of sideband cooling, meaning we cannot infer the ground state population from the heights of the first-order sidebands. In order to overcome these problems associated with cooling from outside the Lamb-Dicke regime, we must use a sideband cooling scheme that empties these high Fock states. As the zero Rabi frequency points are different for each sideband we can cool sequentially on multiple different red sidebands. Higher order sidebands also have the advantage that each scattering event removes more phonons from the motion. However, as the ion gets colder the strength of high order sidebands quickly diminishes. Theoretical work done by Franziska Beck into optimising sideband cooling protocols (ref. [109]) suggests that using three sideband cooling stages should be sufficient. This sequence is shown in the pulse sequence in Figure 6.8, with cooling initially on the first red sideband, then on the second red sideband and finally returning to the first red sideband with lower intensity. The reduced intensity allows us to get close to the sideband cooling limit derived in section 3.4.2 by reducing the off-resonant excitation of the carrier transition.

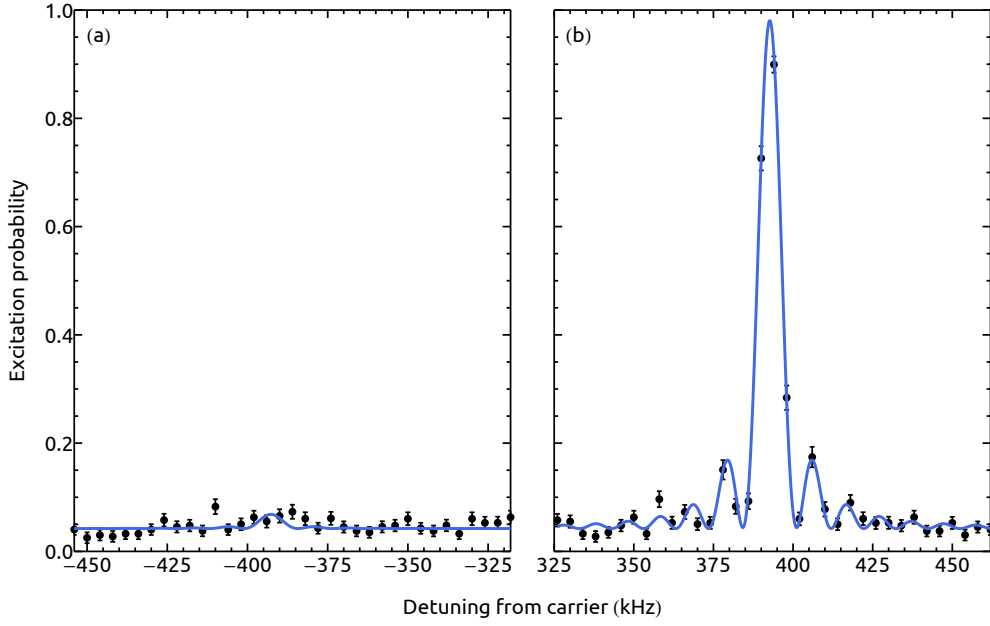


Figure 6.9: (a) Red and (b) blue sidebands after multiple-stage sideband cooling. Each point represents 400 repeated measurements. The fit is to the Rabi dynamics of an ion in the Lamb-Dicke regime, and gives $\bar{n}_z = 0.029 \pm 0.011$

To perform sideband cooling experiments, we first prepare a Doppler-cooled single ion using the method described in section 6.2.2 and then begin the sideband cooling pulse sequence. The lengths of the sideband cooling pulses used for these experiments are 10 ms of initial cooling on the first red sideband, then 5 ms on the second, and then 5 ms on the first at 25% laser intensity.

Figure 6.9 shows the first red and first blue axial sidebands after multiple-stage sideband cooling. The absence of a clear red sideband, along with the large amplitude of the blue sideband, indicates that there is high population in the quantum ground state. The fit gives an average phonon number of $\bar{n}_z = 0.029 \pm 0.011$.

Figure 6.10 shows the detailed shape of carrier associated with the sidebands shown in Figure 6.9. This high visibility of the fringes associated with the Rabi dynamics indicates the increased coherence of the system after sideband cooling. The probe pulse time is 50 μ s, which for the fitted Rabi frequency of $\Omega = 2\pi \times 44.5$ kHz means that on-resonance we are performing a 1.11π -pulse.

Rabi oscillation

Any conceivable quantum information experiment that we would hope to conduct in the future will require the ability to perform coherent manipulation of the state of single and multiple ions. One of the simplest ways to demonstrate coherence as well as to measure the timescale over which the system decoheres is to perform a Rabi

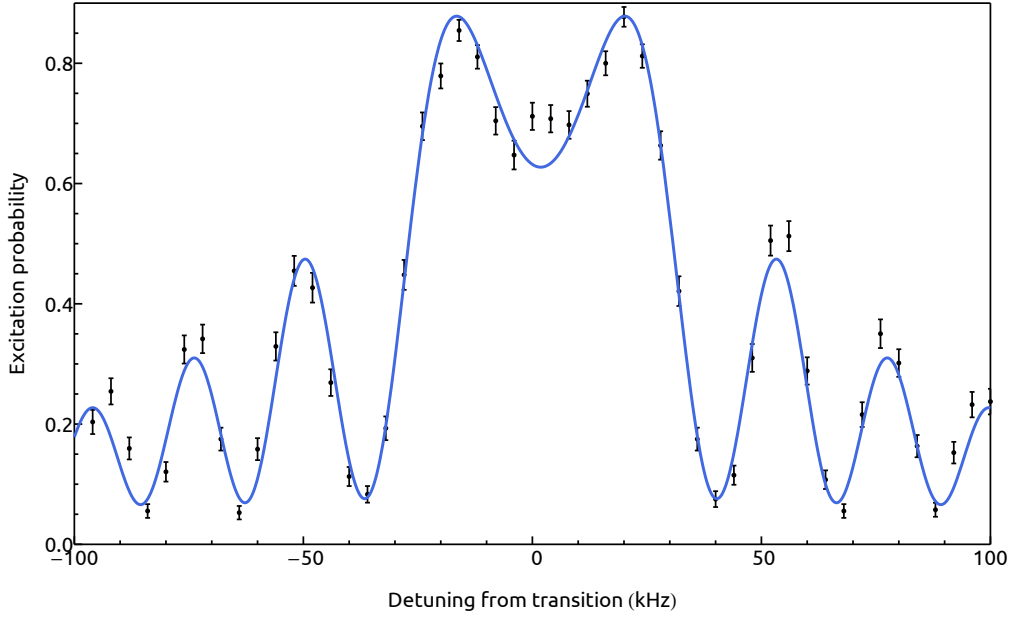


Figure 6.10: Scan over the carrier after multiple stage sideband cooling. Each point represents 400 repeated measurements.

oscillation. To perform a Rabi oscillation we increase the pulse time after each set of experiments rather than the laser frequency as we would to perform spectroscopy. In order for us to obtain a full amplitude Rabi oscillation the ion must be in the ground state with high probability, otherwise contributions from other Fock states will result in a loss of contrast. We therefore sideband cool the axial motion of a single ion using the multiple-stage method described in section 6.3.3 and then probe.

Figure 6.11 shows the Rabi oscillation obtained after multiple-stage sideband cooling. The intensity of the 729 nm was at roughly a third of its maximum value during the probe period. The fit gives a Rabi frequency of $\Omega = 2\pi \times 28$ kHz and a characteristic decay time of the visibility of the oscillation of 789 ± 40 μ s.

The Rabi oscillation method of measuring the coherence time does not differentiate between different sources of decoherence. Noise in the laser frequency and intensity, changes in the transition frequency due to magnetic field noise all contribute to the loss of visibility of the oscillation. As the total amount of phase accumulated by the Rabi oscillation increases the experiment becomes increasingly sensitive to changes in Rabi frequency laser intensity noise, as a change in Rabi frequency will have a cumulative effect after many rotations.

6.3.4 Sideband cooling of the radial motion

Despite the two coupled motions present in the radial plane resulting in a more complicated sideband structure, the principle for cooling the modified cyclotron motion

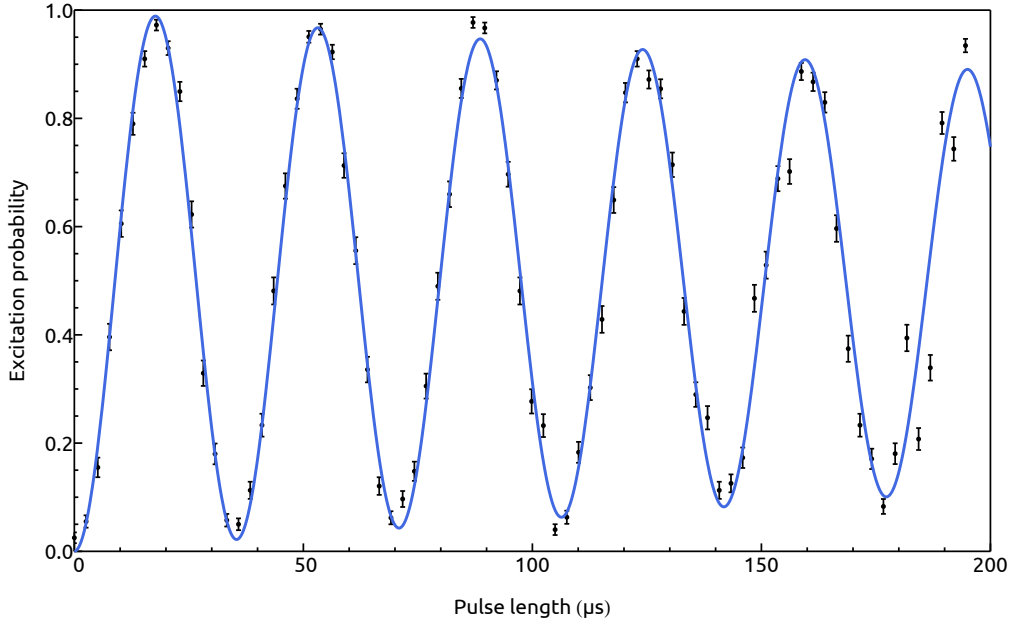


Figure 6.11: Excitation probability against probe pulse time on-resonance with the axial carrier transition after multi-stage sideband cooling. The fit is to a sinusoid with exponentially decaying visibility.

is similar to the axial motion. However, as each of the cyclotron sidebands consist of many closely spaced magnetron sidebands it is harder to pinpoint the exact position of each cyclotron sideband. In our experiments the sideband cooling laser was placed slightly above the centre frequency of the modified cyclotron sideband in order to ensure that we do not heat the magnetron motion, and to attempt to achieve some simultaneous cooling of both radial modes. The pulse sequence used is the same as for axial multiple-stage sideband cooling Figure 6.8.

Figure 6.12 shows the spectrum over the central three sidebands after sideband cooling of the modified cyclotron motion. Here the radial frequencies are $\omega_+ = 2\pi \times 695$ kHz and $\omega_- = 19$ kHz. The asymmetry in the first order sidebands is clear, and as for the axial motion this indicates that there is significant population in the ground state. The fit is to three equally spaced Gaussian lineshapes of the same width with independent amplitudes. The relative heights of the red and blue sidebands gives an average occupation number in the modified cyclotron motion (due to the magnetron motion) of $\bar{n}_+ = 0.59$. The width of each of the modified cyclotron sidebands is 300 kHz, which indicates that the $\bar{n}_- = 6000$, which is very similar to the result after Doppler cooling, shown in section 6.2.3.

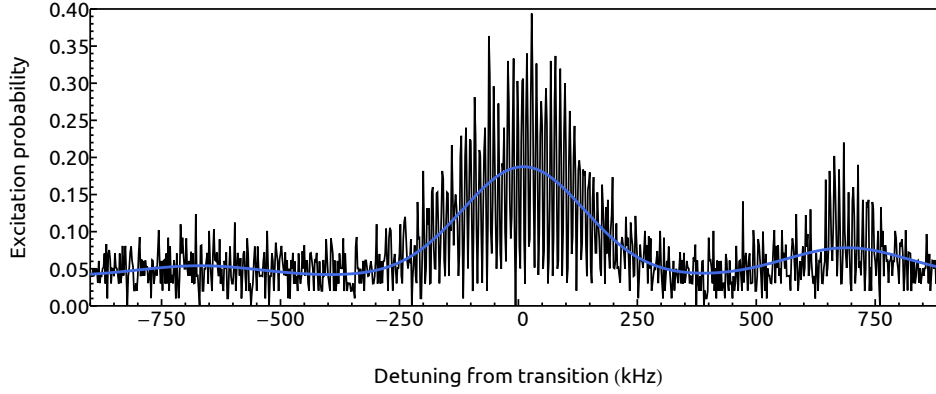


Figure 6.12: Spectrum after sideband cooling of the modified cyclotron motion. The sideband cooling protocol is 10 ms sideband cooling on the first red sideband, 5 ms on the second red sideband and then 10 ms on the first red sideband. The probe time is 200 μ s.

Effect of axialisation

From Figure 6.12 it is clear we have not been able to achieve any significant reduction in the phonon occupation number of the magnetron motion using sideband cooling. The high value of $\bar{n}_-$ and small frequency spacing between the magnetron sidebands suggests that it will be very challenging to reach the ground state of both motions solely using resolved-sideband cooling. By using axialisation alongside optical sideband cooling the aim is to remove phonons from the the magnetron and modified cyclotron motions simultaneously.

Figure 6.13 shows the height of the centre of the modified cyclotron sidebands, from high order red sidebands to the third blue sideband with and without modified cyclotron sideband cooling in the presence of axialisation. The first thing to note from the data is that the modified cyclotron sideband cooling has clearly reduced the width of the central part of the spectrum. However, what is surprising is that for high order sidebands (i.e. 4th to 10th red sidebands) the sideband amplitude is actually increased after sideband cooling, which is inconsistent with a thermal state. This lump at high order is instead consistent with a relatively large orbit driven motion, the kind we would expect from undamped micromotion caused by the oscillating axialisation drive we have applied. When the Doppler cooling lasers are turned off for the sideband cooling period this motion is no longer damped and the amplitude increases.

This result shows that while axialisation is still a very useful tool that allows us to reach high axial frequencies without a large increase in the size of the radial orbit, it causes some heating in the radial motion that we are as yet unable to control. A DDS-based system for producing the axialisation drive is currently under development which should allow full control over the amplitude, frequency and phase of the axi-

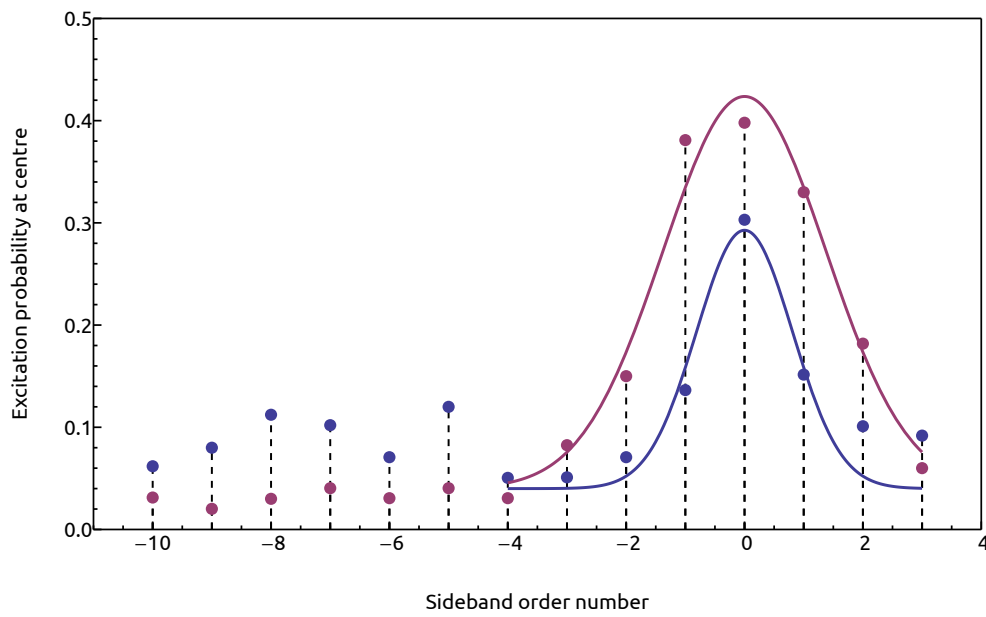


Figure 6.13: Measured excitation probability at the centres of modified cyclotron sidebands with (blue) and without (magenta) modified cyclotron sideband cooling with axialisation present. A fit of the central sidebands of both spectra to a Gaussian is shown to demonstrate the simultaneous narrowing and reduction in amplitude of the central sideband distribution.

sation drive applied to each electrode, and should allow for reduction of this effect in a systematic way. It is also clear that since we have optimised the compensation using non-spectroscopic methods, we might be able to use sideband spectroscopy measurements to improve our axialisation parameters in future.

6.4 Study of axial heating rates

6.4.1 Heating rate measurements and comparison with previous results

The heating rate of a sideband cooled single ion gives a very good indication of the degree to which the quantum system is isolated from the surrounding environment, and the time frame in which quantum logic operations must be completed before coherence is reduced. To measure the heating rate in our trap we sideband cool the axial motion of a single ion close to the ground state and measure the temperature by performing spectroscopy in the manner described in section 6.3.3. We also perform the same experiment, but with a wait period of tens of milliseconds before the probe pulse. In this time the ion heats from the ground state and we then compare this temperature after the wait with the one obtained immediately after sideband cooling, which allows us to establish the rate at which the ion has heated from the ground state. These heating rate measurements are done in an interleaved fashion, so that we can be sure the initial temperature is the same before each of the experiments.

Figure 6.14 shows an example of one of these sets of spectra taken at an axial frequency of 195 kHz. The blue points are taken immediately after sideband cooling and the magenta and gold points are taken with heating periods of 50 and 100 ms respectively. The height of the red sidebands in this example indicates that we have not ground-state cooled the motion, but since we are attempting to measure a change in $\bar{n}_z$ with time, any clear asymmetry between the first order red and blue sidebands is sufficient. By taking the $\bar{n}_z$ values from the fits we can then plot the increase in phonon number with time. These plots are shown in Figure 6.15 for trap frequencies of 195 and 405 kHz. From the gradients of the fitted lines gives heating rates of $\dot{\bar{n}}_z = 0.56 \pm 0.52 \text{ s}^{-1}$ at 195 kHz and $0.67 \pm 0.60 \text{ s}^{-1}$ at 405 kHz.

Figure 6.16 shows the heating rate measurements obtained at several different trap frequencies. Recent results are shown in blue and older results are shown in magenta. The newer data is taken after a more accurate measurement of the correct axialisation frequency was made, which meant that a significantly lower amplitude was applied to the ring electrodes. It is clear from the data that this has substantially reduced the heating rate that we measure. There is evidence of a peak in the heating rate near to an axial frequency of 350 kHz, which is close to half of the applied axialisation frequency of 714.7 kHz. This could be due to direct excitation of the axial motion due to a misalignment of the trap electrodes or to coupling via the radial motion. It is clear

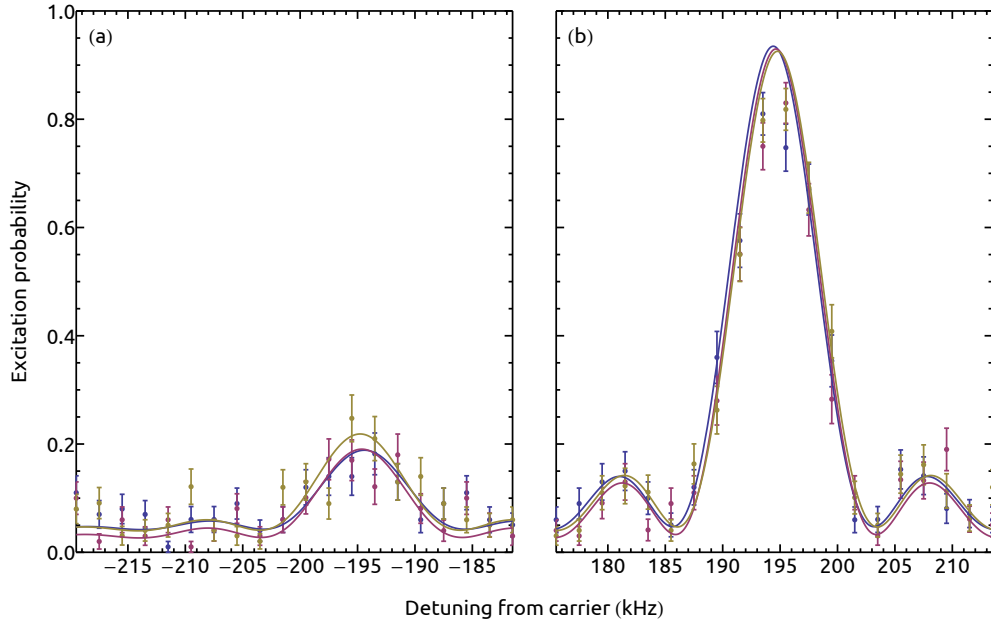


Figure 6.14: Spectroscopy of the first red and blue sidebands after sideband cooling at a trap frequency $\omega_z = 2\pi \times 195$ kHz. The data is for no heating period (blue), and 50 ms (magenta) and 100 ms (gold) heating periods. Each point represents 100 repeated measurements.

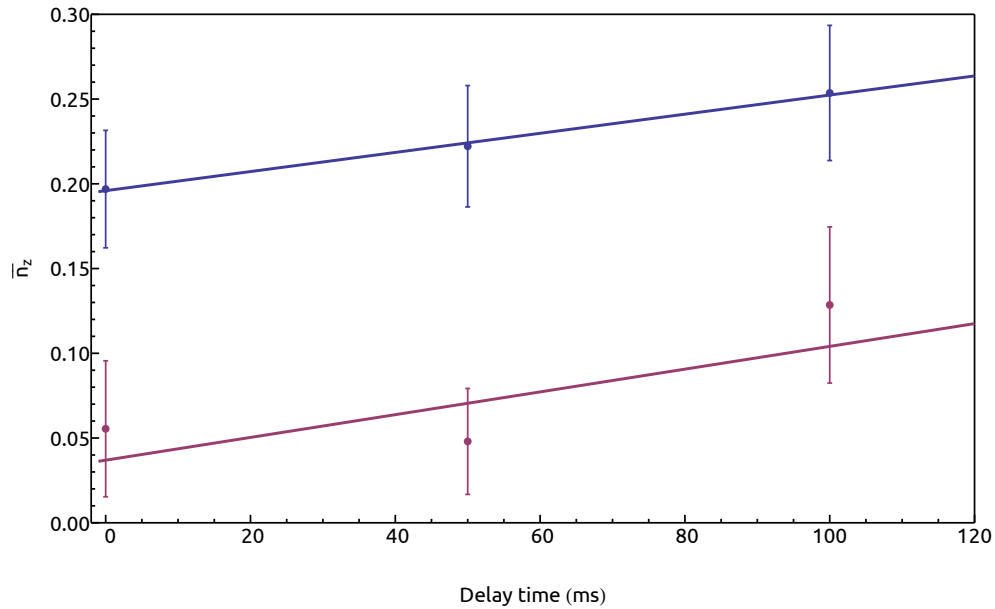


Figure 6.15: Average phonon occupation number of the axial motion against the time between the end of the sideband cooling period and the probe pulse. Taken for an axial frequency of 195 kHz (blue) and 405 kHz (magenta).

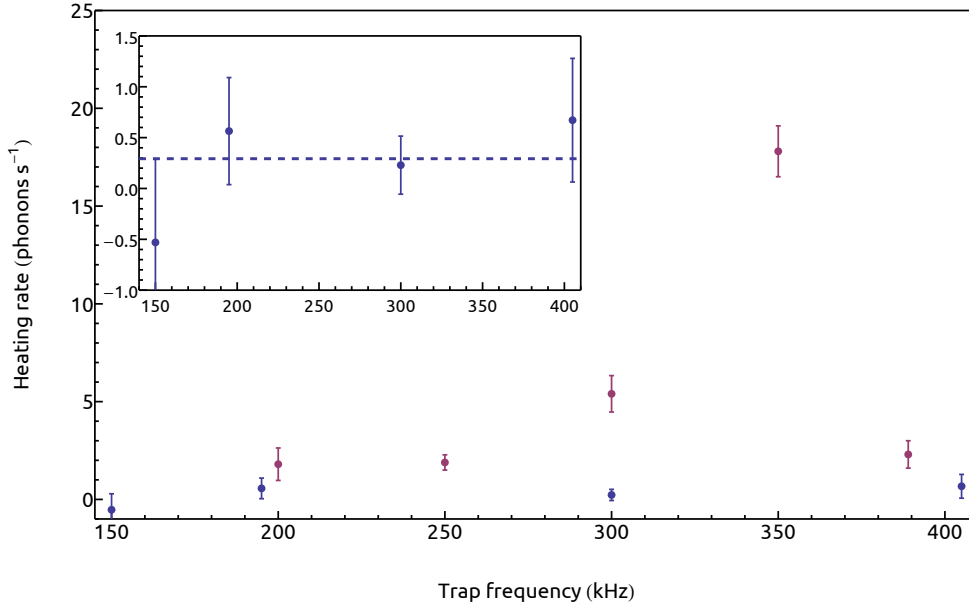


Figure 6.16: Heating rate measurements for different axial trap frequencies. New data (blue) after improvements to axialisation meant that we could apply lower amplitude drive compared with the old data (magenta). The inset shows a zoomed-in plot of the new data.

from the effect of axialisation on the radial motion (section 6.3.4) that there is still room for improvement in the alignment of axialisation, so we may be able to further reduce our heating rate.

The inset of Figure 6.16 shows a zoomed-in view of the more recent data. These results exhibit no clear scaling with trap frequency, although the error on each point is large compared with its absolute value. The main source of error in these heating rate measurement is from the error on each individual $\bar{n}_z$ measurement, which in turn comes from the relatively low number of repeated measurements used for each data-point (100 rather than the 400 used in Figure 6.9, for example). This means that there is clear scope for reducing the error on this data, either by increasing the number of repeats per point or by including a longer heating period in order to greater constrain the linear fit of the heating rate. However this presents a considerable challenge in practice, as the time taken for each heating rate spectrum is currently ~ 1 h and an increase in the number of repeats would extend this time and make it difficult to perform a controlled experiment due to laser frequency drifts. Additionally long heating times increase the probability of a background gas collision when the Doppler cooling lasers are off which can result in the ion being lost entirely from the trap.

6.4.2 Comparison with heating rates from other groups

It is of interest to compare our measured heating rate with others in different traps from ion trapping groups around the world. We have made a measurement in a very large trap in comparison with others, which means our measurement can be used to help understand the way heating rates scale with ion-electrode distance. We have also made the first measurement of a heating rate of a single ion in a Penning trap, whereas all others have been performed in Paul traps in the presence of large amplitude oscillating voltages. The differences in methodology, trap structure, other equipment etc. make it very difficult to draw conclusions from the data.

The quantity that is usually used to compare heating rates from different systems, in order to take into account the different ion masses and trap frequencies used in different experiments is the inferred spectral noise density $S_E(\omega)$ rather than $\dot{n}$ in phonon s^{-1} . The spectral noise density is related to $\dot{n}$ using the formula

$$S_E(\omega) = \frac{4m\hbar\omega}{e^2}\dot{n}. \quad (6.2)$$

Figure 6.17 shows heating rate measurements obtained in many other ion trap experiments from different research groups. The rationale for plotting scaled spectral noise as in Figure 6.17 is that the phonon energy is $\hbar\omega$ and therefore it requires more energy to add a phonon to a motion at higher trap frequency. Different heating mechanisms have different associated power law scaling with the trap frequency and electrode distance $S_E(\omega) \propto \omega^{-\alpha}d^{-\beta}$, as summarised in ref. [111].

One point of interest is that the two measurements made in the largest traps, (ours and Poulsen et al. in ref. [112]) show no visible scaling with trap frequency of the heating rate in units of phonon s^{-1} . This is unusual, as most traps have increased heating at low trap frequencies, and could suggest that the heating rate mechanism is different at these large trap dimensions. Specifically, the spectral noise density associated with Johnson noise cause by a resistance R at a temperature T

$$S_E = \frac{4k_B T R}{d^2}, \quad (6.3)$$

which is frequency-independent, $\alpha = 0$, and has a distance scaling of $\beta = 2$. It is proposed that anomalous heating due to trap potentials scales strongly with distance, with $\beta \approx 4$ and therefore will dominate in small traps. This type of heating is usually associated with a frequency scaling of $1 < \alpha < 2$, which is not consistent with what we have measured.

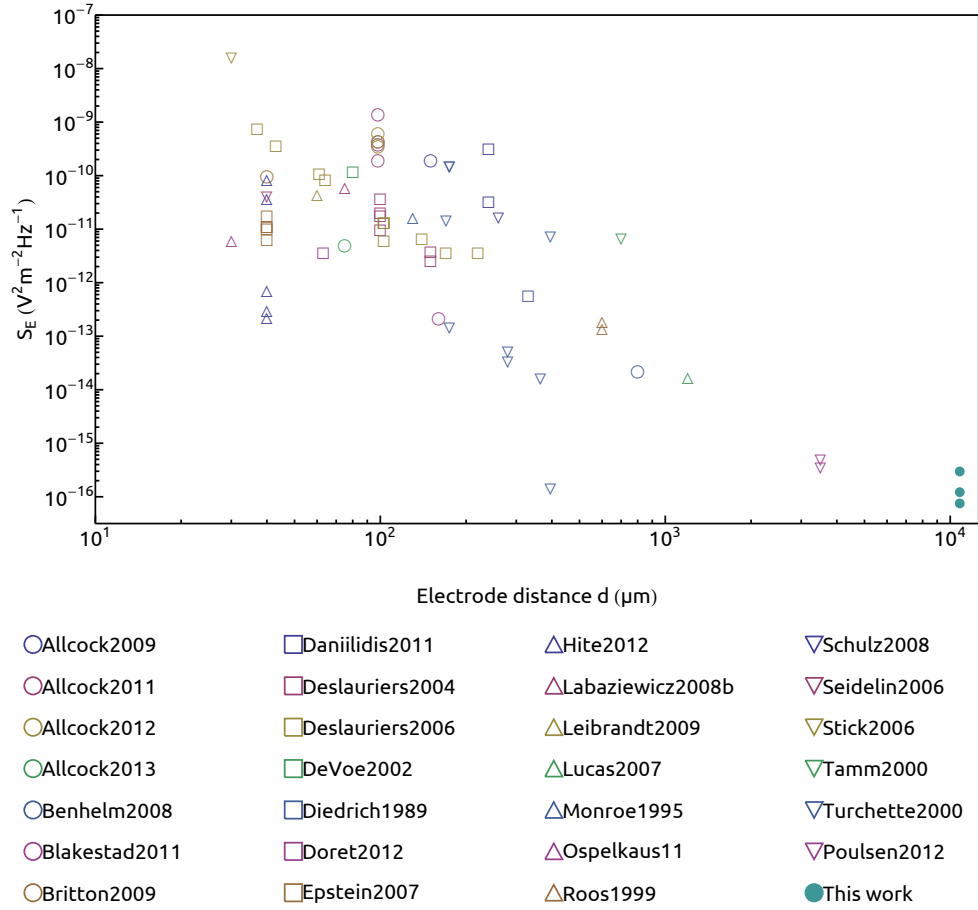


Figure 6.17: Heating rates in spectral noise against trap dimension for room temperature ion traps. The main source for this data is ref. [110]. Additionally, ref. [111] has been used to provide references to the primary sources.

6.5 Conclusion

The results presented in this thesis represent a significant step forward in the capabilities of the group and greatly expand the scope of experiments we will be able to perform (and move us towards our long-term goals, described in the next chapter), but are also of significant interest in their own right. In summary

- As an extension to our previous work in which we performed sideband cooling of the axial motion in a Penning trap for the first time [72], we have ground-state cooled the axial motion of a single ion in a Penning trap using a multiple-stage sideband cooling protocol. This has allowed us to cool to the ground state of motion with 97% probability. This protocol has also allowed us to cool at lower axial trap frequencies than previous results, ensuring we have no trapping in dark states. For lower trap frequencies (higher η), the strength of the first order sidebands is increased, and it is these sideband couplings that are commonly used in quantum information processing, as is discussed in section 7.3.
- We have performed preliminary spectroscopy on an axial ion coulomb crystal of two ions. With some improvements to our state detection we should be able to resolve the underlying sideband structure of a two-ion axial spectrum and therefore sideband cool the collective motion of a two ion crystal. The measurement we have made has shown that the phonon occupation number of both modes is low enough ($n \lesssim 200$) after Doppler cooling that we should be able to achieve substantial reduction in $\bar{n}_z$ using multiple-stage sideband cooling. This is an important initial step towards creating multi-ion entangled states.
- We have improved the laser intensity stability, enabling us to perform an improved Rabi oscillation and measure a coherence time of $\tau \approx 790 \mu\text{s}$.
- We have performed optical sideband cooling of the modified cyclotron motion of a single ion for the first time. We have reached an occupation number of $\bar{n}_+ < 1$, but have as yet been unable to get any significant reduction in the phonon occupation number of the magnetron motion.
- We have measured the heating rate of our trap to be roughly $0.5 \text{ phonon } s^{-1}$. This is the lowest value that has been recorded to date. We have observed no clear scaling of our heating rate with trap frequency, which suggests we have a different heating mechanism to other, smaller traps.

Chapter 7

Outlook

In Chapter 6 I presented results of axial and modified cyclotron sideband cooling in a Penning trap. Using the techniques that have been developed during my PhD we have the unique ability to study the motion of single ions in a Penning trap in the quantum regime. Additionally, the results I have shown are a prerequisite for future experiments at Imperial (which I will summarise in this chapter) and also in other research groups.

7.1 Single-ion state rotations and coherence measurement

The Rabi experiment shown in Figure 6.11 is the first step towards implementing single-qubit state manipulation, corresponding to rotation about the x and y axes of the Bloch sphere. The coherence time we have measured using the Rabi method does not discriminate between different noise sources, with frequency and intensity noise of the 729 nm spectroscopy laser both contributing to the loss of visibility in the oscillation at longer probe times.

In order to separate the contribution between these effects a Ramsey-type experiment can be performed on a sideband-cooled single ion. A Ramsey-type experiment consists of two $\frac{\pi}{2}$ pulses separated by a delay period of time T . If we define the phase of the interaction such that $\phi = 0$ at the start of the first pulse, for a detuning of $\delta = \omega_0 - \omega_L$ a phase of

$$\phi = \delta(\tau_p + T) \quad (7.1)$$

will be accumulated between the laser field and the ion's state amplitudes by the time the second pulse, where τ_p is the duration of a $\frac{\pi}{2}$ pulse. On the Bloch sphere this means that the second $\frac{\pi}{2}$ pulse will cause a state rotation about a different axis in the xy -plane to the first pulse. This results in fringes corresponding to places where the two $\frac{\pi}{2}$ pulses occur in and out of phase. By evaluating the excited state population amplitude using (3.15) and (3.16) with the phase, in the absence of decoherence we obtain a fringe

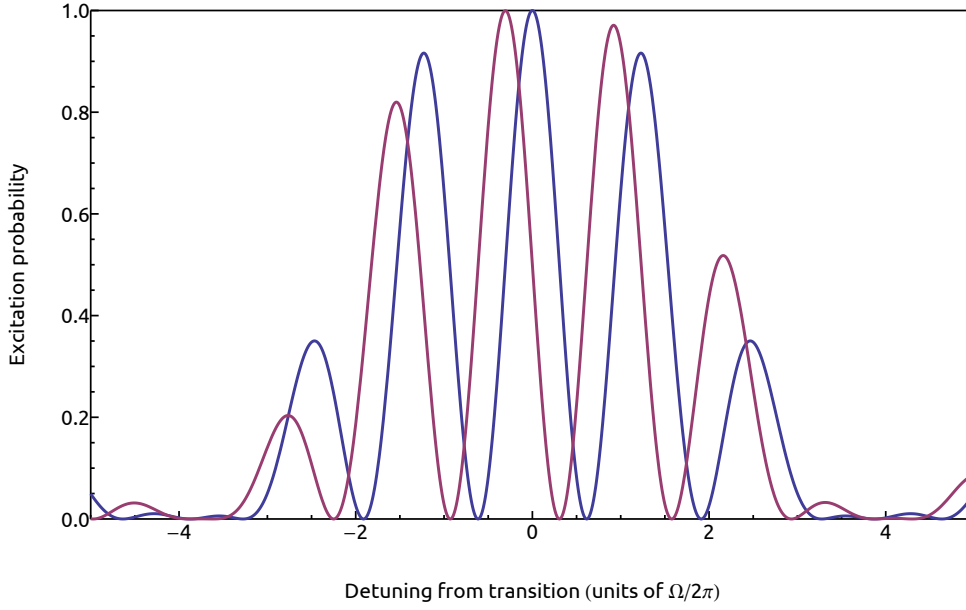


Figure 7.1: Ramsey fringes for a delay period $T = 1/\Omega$. the blue curve shows free evolution, the magenta curve shows the effect of applying an AC Stark shift during the delay period.

pattern given by

$$|c_e(T + 2\tau_p)|^2 = 4 \frac{\Omega_0^2}{\Omega^2} \sin^2 \left(\frac{\Omega \tau_p}{2} \right) \left(\cos \left(\Omega \frac{\tau_p}{2} \right) \cos(-\delta T) + \frac{\delta}{\Omega} \sin \left(\Omega \frac{\tau_p}{2} \right) \sin(-\delta T) \right) \quad (7.2)$$

where $\Omega = \sqrt{\delta^2 + \Omega_0^2}$, and the form of these fringes is shown in blue on Figure 7.1. A relatively short delay period of $T = 1/\Omega$ has been used to keep the figure clear, but in practice longer delay periods can be used, which results in more fringes inside the same envelope.

The maximum phase accumulated during each of the Ramsey pulses is $\frac{\pi}{2}$ so the process is less sensitive to intensity noise of the 729 nm laser than the Rabi method. However, if there is noise in either the laser frequency or in the transition frequency on the timescale of the experiment the visibility of the fringes will be reduced. By measuring the loss in contrast of the fringes with increasing delay period we can calculate the coherence time caused by these kinds of noise. This will give us an indication of what steps we need to take in order to improve our coherence time, i.e. whether our main source of decoherence is due to frequency or intensity noise at the ion.

With a small modification to the current experimental system we could implement rotations about the z -axis using the AC stark shift [113]. In order to achieve this we can detune from the particular $S_{1/2} \longleftrightarrow D_{5/2}$ transition that we are using in the experiment

by more than a sideband in order to avoid scattering photons. In our experiment the other $S_{1/2} \longleftrightarrow D_{5/2}$ transitions are many GHz away and can be ignored. If we perform a Ramsey experiment where we apply the AC stark shift during the delay period, the shift will change the phase difference between the first and second pulses and therefore change the form of the Ramsey fringes that we obtain. The change in the transition frequency caused by the AC Stark shift is given by

$$\Delta\omega_{\text{AC}} = \frac{\Omega_0^2}{2\delta}, \quad (7.3)$$

and for our current Rabi frequency of roughly $\Omega_0 = 2\pi \times 35$ kHz and a detuning of $\delta = -2\pi \times 500$ kHz we can achieve a shift of $\Delta\omega_{\text{AC}} = 2\pi \times 1.23$ kHz. In order to obtain a π phase change we would therefore need to interact for $410 \mu\text{s}$, which is a long time for a single ion gate, but is achievable given our low heating rate. The form of the Ramsey fringes that would be obtained in this experiment for an AC Stark shift induced phase change of $\frac{\pi}{2}$ is shown in magenta on Figure 7.1.

7.2 Two-ion spectroscopy and sideband cooling

In section 6.2.4 preliminary spectroscopy of an axial two-ion crystal was presented. There were no clearly-resolved sidebands, in part due to the poor state discrimination from the PMT photon-counting method used (Figure 6.4). An improved detection system based on an Andor EMCCD camera is currently being developed. The quantum efficiency of this camera is $\sim 50\%$ at 400 nm, which is a significant improvement on the $\approx 20\%$ quantum efficiency of each of our PMTs. Additionally as we will only collect counts from a small area of the sensor the amount of background-scatter collected should be significantly reduced. The ability to spatially resolve the ions means that we can measure the state of each ion independently, not just the total number of bright ions and that the detection scheme should be scalable to larger numbers of ions. For certain experiments this extra information is important, for example when studying quantum magnetism the state $|\uparrow\downarrow\uparrow\downarrow\rangle$ would indicate an anti-ferromagnetic interaction and $|\uparrow\uparrow\downarrow\downarrow\rangle$ would indicate a ferromagnetic interaction [29].

Once we are able to perform spectroscopy and clearly resolve the different centre-of-mass and breathing mode sidebands and assuming that our two-ion heating rate is comparable with that of a single ion, we should be able to closely follow the method used to sideband cool a single ion in order to reach the Lamb-Dicke regime of both modes.

It would also be of particular interest to perform axial spectroscopy on a two-ion Coulomb crystal aligned in the radial plane. This is the smallest example of the kind of two-dimensional structure that we would like to study because of its suitability to the Penning trap due to the lack of RF micromotion (see section 7.4). However, in order

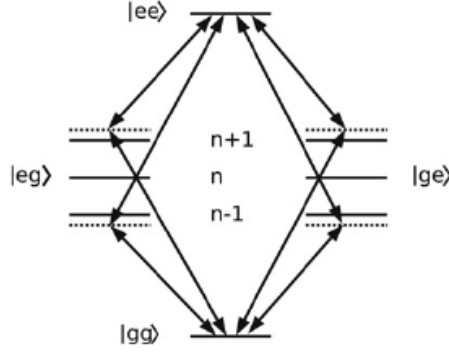


Figure 7.2: Energy levels and laser frequencies required to perform a two-ion Mølmer-Sørensen gate [113].

to perform this experiment it is likely that we would need to implement a rotating wall drive (section 2.4.1), because any change in the crystal rotation frequency would cause the ion spacing to change. The addition of a rotating wall drive would necessitate a redesign of our trap to include a larger number of ring electrode segments. A locked crystal rotation frequency would also allow us to perform stroboscopic top-down imaging of rotating crystals.

7.3 Mølmer-Sørensen gates

Once we have a two-ion crystal that has been sideband-cooled to close to the motional ground state in both of its axial modes of motion we can apply a Mølmer-Sørensen interaction in order to produce an entangled state [114]. In order to apply this interaction a bichromatic laser beam is used, with frequencies tuned close to the red and the blue sideband frequencies of one of the collective modes of motion of the crystal, as shown in Figure 7.2.

Using the interaction Hamiltonian given in (3.19) in the Lamb-Dicke regime, the Hamiltonian due to this bichromatic field can be expressed as

$$H_{\text{MS}} = -i\hbar\eta\Omega_0\sigma_x \left(a^\dagger e^{-i\delta_\nu t} + a e^{i\delta_\nu t} \right) \quad (7.4)$$

where δ_ν is the detuning from the sideband transition [113]. This interaction causes collective changes in the internal state of the ions and results in entangled states in the $|\pm\rangle$ basis.

As this is an effect caused by near-resonant excitation on sideband transitions the strength of the interaction is proportional to η , the Lamb-Dicke parameter. This means that the interaction is stronger at smaller trap frequency where the first sideband strengths are stronger, which would lead to faster gates [115], and since we have currently seen no frequency-scaling of our heating rate with trap frequency (Figure 6.16)

we are able to decrease our trap frequency until off-resonant excitation on the carrier becomes a problem. However, off-resonant excitations can be minimised by turning the gate on and off adiabatically [116].

In our experiment the bichromatic beam could be produced either by using a combiner to combine the red and blue sideband frequencies, produced by two separate DDS systems, before they are fed into the AOM, or by amplitude modulation of a single voltage close to the trap frequency.

This type of interaction has been used to perform a two-ion quantum gate with a fidelity of 99% [14], and also in more complicated experiments where it has been used to produce a tunable range interaction [117].

7.4 Quantum error-correcting codes

One potential avenue for research is using small, two-dimensional Coulomb crystals to perform quantum information experiments. The possibility of using this system in order to perform a small quantum error-correcting code is described in ref. [40]. As with other quantum information schemes in a Penning trap we use the lack of RF micromotion to our advantage and work with a two-dimensional system, and use global entangling pulses

In the quantum error-correcting codes under consideration, the five-qubit code and the five-qubit repetition code, the quantum information of one ion, the ‘logic’ qubit, is distributed amongst the five code ions, such that if the state of these code ions is measured at some later time, the resulting ‘syndrome’ measurement gives information as to what error, if any, has occurred to the logic qubit state, and what local operation can be used to correct for this error. This increases the effective coherence time of the system against certain types of errors, depending on the specific code. The five-qubit code protects against single-qubit X , Y or Z errors [118] and the five-qubit repetition code protects against dephasing effects on up to two qubits.

The particular system that we can use in order to implement these codes is a six-ion crystal with one central ion (the logic qubit) surrounded by the other five ions which sit at the vertices of a pentagon (the code qubits), as shown in Figure 7.3. We can obtain this configuration and phase-lock the crystal rotation frequency using a rotating wall drive. This scheme uses the fact that the motional modes of some crystals mirrors the symmetry of the target states that we need in order to perform the error correcting codes. We can find the form of axial drumhead modes and their frequencies using the stiffness matrix K_{jk}^{zz} in the manner described in section 2.3.2 and use the fact that there are frequency degenerate modes to perform global entangling pulses. The crystal will be rotating so the fact that we can generate these states with global pulses makes it much simpler.

The global entangling pulses are produced in a similar manner to the two-ion phase

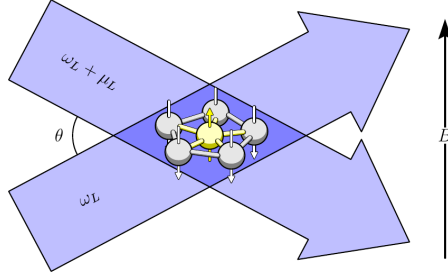


Figure 7.3: Schematic of the laser beams required to produce the optical dipole force required to perform the quantum error-correcting code. Figure from [40].

gate described in ref. [13] and the tunable-range Ising interaction in ref. [35]. Two angled laser beams are incident on the crystal with frequencies ω_L and $\omega_L + \mu_L$, as shown in Figure 7.3 in order to produce an optical dipole force. The light creates a travelling wave which causes a differential AC Stark shift between the $|\uparrow\rangle$ and $|\downarrow\rangle$ states, and if μ_L is tuned close to one of the motional mode frequencies then we can coherently drive the motion so that a geometric phase is accumulated. If the code ions are initialised in the state $|+\rangle^{\otimes 5}$, then different phases are introduced in the different terms, depending on how well the terms match the particular mode that is being driven. For example, if the optical dipole force is applied close to the centre-of-mass mode of the crystal, this has a particularly strong effect on the states $|\uparrow, \uparrow, \uparrow, \uparrow, \uparrow\rangle$ and $|\downarrow, \downarrow, \downarrow, \downarrow, \downarrow\rangle$ and the force on all of the ions is in the same direction, which most closely matches the form of the centre-of-mass motion, where all of the ions move in phase with one another. Figure 7.4 shows the steps required to generate the error-correcting code states and to perform the syndrome measurement.

There are many additions to the current set of experimental methods that might be required in order to implement the codes, such as installing a top-down imaging system that would allow for individual-ion state detection, or a single-ion addressing system that would be able to track the crystal rotation using either acousto-optical modulators [119] or a MEMS device [120].

7.5 Summary

The Penning trap is a widely used method of confining charged particles for use in precision measurement of fundamental constants, tests of fundamental theories such as quantum electrodynamics and non-neutral plasma physics. However, their use in quantum information experiments has been relatively infrequent compared with the radiofrequency (RF) trap. As a result some of the common techniques that have been used in RF traps for many years have not previously been demonstrated in a Penning trap. This work extends many of these methods to the Penning trap and has demon-

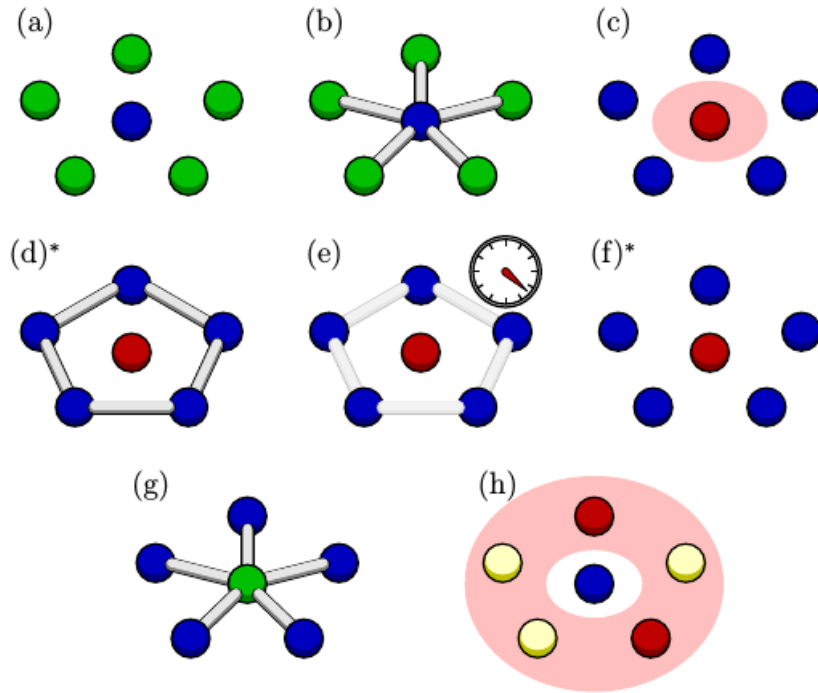


Figure 7.4: (a) State preparation of the ring ions in the $|+\rangle^{\otimes 5}$ product state. (b) Application of the entangling gate to entangle each of the ring qubits to the central logic qubit. (c) Measurement of the central qubit in order to teleport the quantum information of the central qubit to the ring ions. (d) The five-qubit code requires an additional entanglement operation to entangle the ring qubits. (e) Storage of the quantum information. (f) For the five-qubit code the entangling operation (d) is reversed. (g) Re-entanglement of the ring qubits with logic qubits. (h) The syndrome measurement performed on the ring ions in order to establish if an error has occurred to the logic qubit. Figure from ref. [40].

strated ground state cooling of the axial and modified cyclotron modes, measurement of a low heating rate, and coherent single ion state manipulation. This work will pave the way for further experiments in the field of quantum information, particularly using two-dimensional systems where the benefits of the Penning trap are more immediately apparent.

Appendix A

Atomic physics miscellany

A.1 Expansion of the $|j, m_j\rangle$ states using Clebsch-Gordan coefficients

For reference, here are the expansions of the $|j, m_j\rangle$ states in the $|l, m_l, s, m_s\rangle$

For the $S_{1/2}$ states, with $j = \frac{1}{2}$ and $l = 0$

$$\left|\frac{1}{2}, -\frac{1}{2}\right\rangle = \left|0, 0, \frac{1}{2}, -\frac{1}{2}\right\rangle \quad (\text{A.1})$$

$$\left|\frac{1}{2}, +\frac{1}{2}\right\rangle = \left|0, 0, \frac{1}{2}, +\frac{1}{2}\right\rangle \quad (\text{A.2})$$

For the $P_{1/2}$ states, with $j = \frac{1}{2}$ and $l = 1$:

$$\left|\frac{1}{2}, -\frac{1}{2}\right\rangle = -\sqrt{\frac{2}{3}}\left|1, -1, \frac{1}{2}, +\frac{1}{2}\right\rangle + \sqrt{\frac{1}{3}}\left|1, 0, \frac{1}{2}, -\frac{1}{2}\right\rangle \quad (\text{A.3})$$

$$\left|\frac{1}{2}, +\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\left|1, +1, \frac{1}{2}, -\frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}\left|1, 0, \frac{1}{2}, +\frac{1}{2}\right\rangle \quad (\text{A.4})$$

For the $P_{3/2}$ states, with $j = \frac{3}{2}$ and $l = 1$:

$$\left|\frac{3}{2}, -\frac{3}{2}\right\rangle = \left|1, -1, \frac{1}{2}, -\frac{1}{2}\right\rangle \quad (\text{A.5})$$

$$\left|\frac{3}{2}, -\frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}\left|1, -1, \frac{1}{2}, +\frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|1, 0, \frac{1}{2}, -\frac{1}{2}\right\rangle \quad (\text{A.6})$$

$$\left|\frac{3}{2}, +\frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}\left|1, +1, \frac{1}{2}, -\frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|1, 0, \frac{1}{2}, +\frac{1}{2}\right\rangle \quad (\text{A.7})$$

$$\left|\frac{3}{2}, +\frac{3}{2}\right\rangle = \left|1, +1, \frac{1}{2}, +\frac{1}{2}\right\rangle \quad (\text{A.8})$$

For the $D_{3/2}$ states, with $j = \frac{3}{2}$ and $l = 1$:

$$\left| \frac{3}{2}, -\frac{3}{2} \right\rangle = -\sqrt{\frac{4}{5}} \left| 2, -2, \frac{1}{2}, +\frac{1}{2} \right\rangle + \sqrt{\frac{1}{5}} \left| 2, -1, \frac{1}{2}, -\frac{1}{2} \right\rangle \quad (\text{A.9})$$

$$\left| \frac{3}{2}, -\frac{1}{2} \right\rangle = -\sqrt{\frac{3}{5}} \left| 2, -1, \frac{1}{2}, +\frac{1}{2} \right\rangle + \sqrt{\frac{2}{5}} \left| 2, 0, \frac{1}{2}, -\frac{1}{2} \right\rangle \quad (\text{A.10})$$

$$\left| \frac{3}{2}, +\frac{1}{2} \right\rangle = \sqrt{\frac{3}{5}} \left| 2, +1, \frac{1}{2}, -\frac{1}{2} \right\rangle - \sqrt{\frac{2}{5}} \left| 2, 0, \frac{1}{2}, +\frac{1}{2} \right\rangle \quad (\text{A.11})$$

$$\left| \frac{3}{2}, +\frac{3}{2} \right\rangle = \sqrt{\frac{4}{5}} \left| 2, +2, \frac{1}{2}, -\frac{1}{2} \right\rangle - \sqrt{\frac{1}{5}} \left| 2, +1, \frac{1}{2}, +\frac{1}{2} \right\rangle \quad (\text{A.12})$$

For the $D_{5/2}$ states, with $j = \frac{5}{2}$ and $l = 1$:

$$\left| \frac{5}{2}, -\frac{5}{2} \right\rangle = \left| 2, -2, \frac{1}{2}, -\frac{1}{2} \right\rangle \quad (\text{A.13})$$

$$\left| \frac{5}{2}, -\frac{3}{2} \right\rangle = \sqrt{\frac{1}{5}} \left| 2, -2, \frac{1}{2}, +\frac{1}{2} \right\rangle + \sqrt{\frac{4}{5}} \left| 2, -1, \frac{1}{2}, -\frac{1}{2} \right\rangle \quad (\text{A.14})$$

$$\left| \frac{5}{2}, -\frac{1}{2} \right\rangle = \sqrt{\frac{2}{5}} \left| 2, -1, \frac{1}{2}, +\frac{1}{2} \right\rangle + \sqrt{\frac{3}{5}} \left| 2, 0, \frac{1}{2}, -\frac{1}{2} \right\rangle \quad (\text{A.15})$$

$$\left| \frac{5}{2}, +\frac{1}{2} \right\rangle = \sqrt{\frac{2}{5}} \left| 2, +1, \frac{1}{2}, -\frac{1}{2} \right\rangle + \sqrt{\frac{3}{5}} \left| 2, 0, \frac{1}{2}, +\frac{1}{2} \right\rangle \quad (\text{A.16})$$

$$\left| \frac{5}{2}, +\frac{3}{2} \right\rangle = \sqrt{\frac{1}{5}} \left| 2, +2, \frac{1}{2}, -\frac{1}{2} \right\rangle + \sqrt{\frac{4}{5}} \left| 2, +1, \frac{1}{2}, +\frac{1}{2} \right\rangle \quad (\text{A.17})$$

$$\left| \frac{5}{2}, +\frac{5}{2} \right\rangle = \left| 2, +2, \frac{1}{2}, +\frac{1}{2} \right\rangle \quad (\text{A.18})$$

A.2 Zeeman shifts

Table A.1 shows the first and second-order shifts of the low-lying energy levels in $^{40}\text{Ca}^+$ that we use in our experiment. These values are calculated in the manner described in section 4.1.1.

l_j	m_j	$\Delta\nu_Z^{(1)}(\text{GHz T}^{-1})$	$\Delta\nu_Z^{(2)}(\text{MHz T}^{-2})$
$S_{1/2}$	$\pm 1/2$	± 14.0	0
$D_{3/2}$	$\pm 3/2$	± 16.8	-17.2
	$\pm 1/2$	± 5.60	-25.8
$D_{5/2}$	$\pm 5/2$	42.0	0
	$\pm 3/2$	± 25.2	$+17.2$
	$\pm 1/2$	± 8.40	$+25.8$
$P_{1/2}$	$\pm 1/2$	± 4.66	-6.51
$P_{3/2}$	$\pm 3/2$	± 28.0	0
	$\pm 1/2$	± 9.34	$+6.51$

Table A.1: The first and second-order Zeeman shifts $\Delta\nu_Z = \Delta E_Z/h$ of the relevant low-lying $^{40}\text{Ca}^+$ energy levels.

A.3 Spherical tensors

The tensors $\mathbf{c}^{(\Delta m_j)}$ that appear in (4.9) are given by

$$\mathbf{c}^{(-1)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} \quad (\text{A.19})$$

$$\mathbf{c}^{(0)} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (\text{A.20})$$

$$\mathbf{c}^{(+1)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix} \quad (\text{A.21})$$

$$(\text{A.22})$$

The tensors $\mathbf{c}^{\Delta m_j}$ that appear in (4.10) are given by

$$\mathbf{c}^{(-2)} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & i & 0 \\ i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (\text{A.23})$$

$$\mathbf{c}^{(-1)} = \frac{1}{\sqrt{6}} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & i \\ 1 & i & 0 \end{pmatrix} \quad (\text{A.24})$$

$$\mathbf{c}^{(0)} = \frac{1}{3} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \quad (\text{A.25})$$

$$\mathbf{c}^{(+1)} = \frac{1}{\sqrt{6}} \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & i \\ -1 & i & 0 \end{pmatrix} \quad (\text{A.26})$$

$$\mathbf{c}^{(+2)} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & -i & 0 \\ -i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (\text{A.27})$$

Appendix B

Magnet

These are old measurements of the variation of the magnetic field strength along the axis of the bore of our superconducting magnet, made with a Hall probe. These measurements were performed by Shailen Bharadia and only exist in hard copy form, so I have included them here for reference.

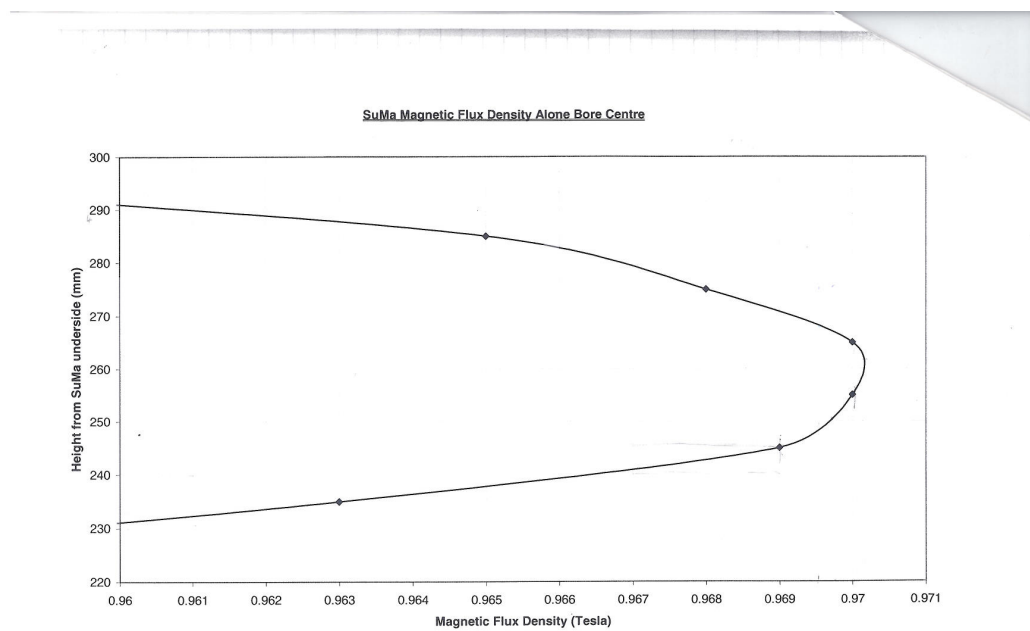


Figure B.1: Measured magnetic field strength against height from top of the superconducting magnet near the field centre.

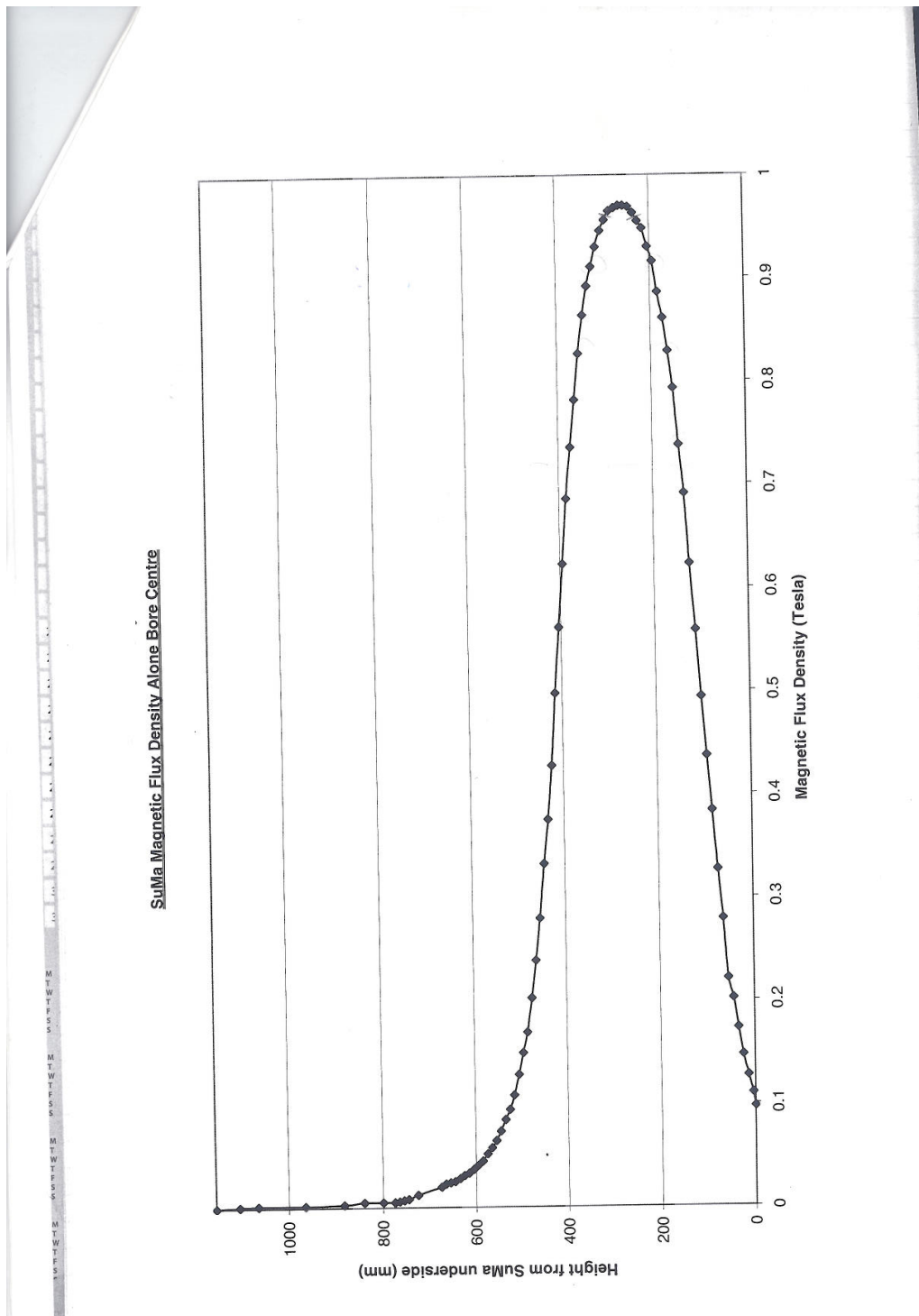


Figure B.2: Measured magnetic field against height from top of superconducting magnet bore.

Appendix C

Blue PDH lock documentation

C.1 Cavity spacer design

The cavity spacers are made out of Zerudor glass. The mirror spacing is $L = 10$ cm. The mirrors are attached to piece (a) and piece (c).

C.2 Circuit diagram of the PDH loop filter

The circuit shown in Figure C.2 is used to provide feedback to the laser diode current.

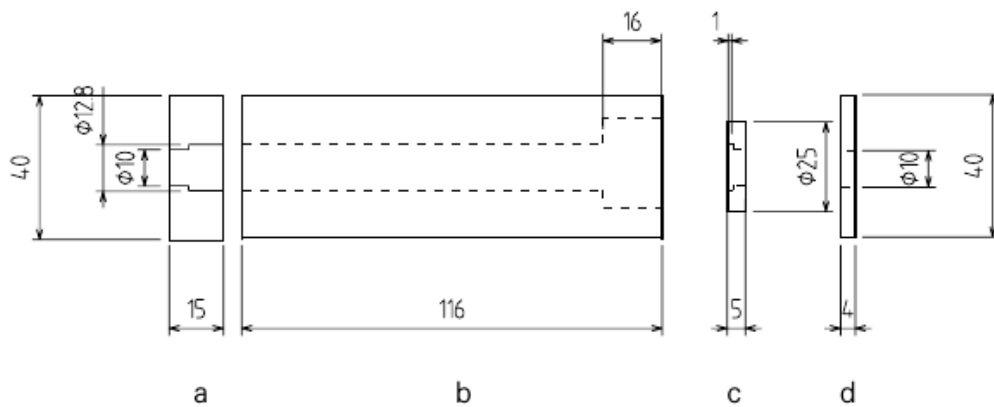


Figure C.1: Diagram of the design of the blue PDH cavities, taken from ref. [96].

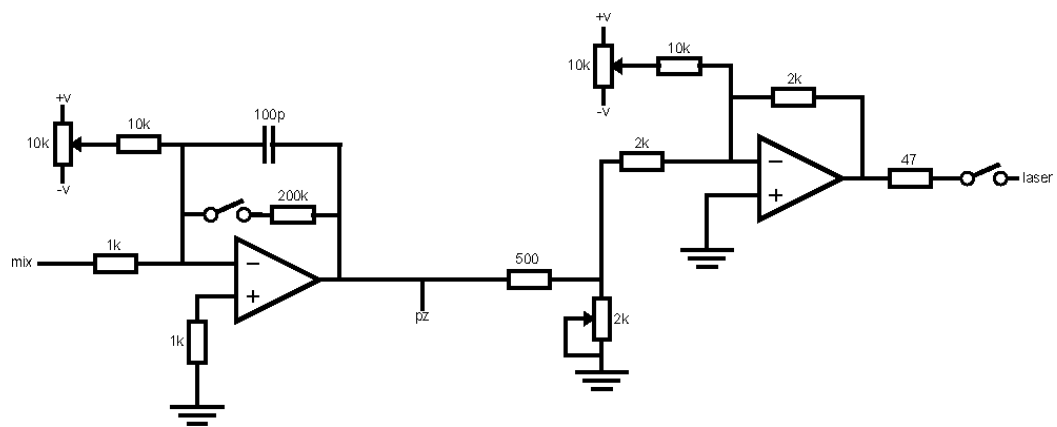


Figure C.2: Circuit diagram of the loop filter used for feedback to the laser diode current in the blue PDH locks.

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