arXiv:0707.0208v2 [astro-ph] 8 Oct 2007

Foam-like structure of the Universe

A. A. Kirillov
Institute for Applied Mathematics and Cybernetics, 10 Ulyanova str., Nizhny Novgorod, 603005, Russia

D. Turaev
Ben-Gurion University of the Negev, P.O.B. 653, Beer-Sheva 84105, Israel

Abstract

On the quantum stage spacetime had the foam-like structure. When the Universe cools, the foam structure tempers and does not disappear. We show that effects caused by the foamed structure mimic very well the observed Dark Matter phenomena. Moreover, we show that in a foamed space photons undergo a chaotic scattering and together with every discrete source of radiation we should observe a diffuse halo. We show that the distribution of the diffuse halo of radiation around a point-like source repeats exactly the distribution of dark matter around the same source, i.e. the DM halos are sources of the diffuse radiation.

1 Introduction

Wheeler pointed out that at Planck scales topology of spacetime undergoes quantum fluctuations [1]. In the present Universe such fluctuations carry virtual character and do not lead to detectable topology changes. However, in the past, the Universe went through the quantum stage when the temperature exceeded the Planckian value and the fluctuations were strong enough to form a non-trivial topological structure of space. In other words, on the very early, quantum stage Universe had to have a foam-like structure. During the cosmological expansion, the Universe cools, quantum gravity processes stop, and the topological structure of space freezes. There is no obvious reason why the resulting topology has to be exactly that of R^3 — relics of the quantum stage foam might very well survive, thus creating a certain distribution of wormholes in space. In the present paper we show that the whole variety of the observed Dark Matter (DM) phenomena admits a straightforward interpretation in terms of the foam-like topological structure of space. Moreover, the specific properties of the foam that are read from the observed DM distribution coincide with those that are derived theoretically from very basic physical principles: we show that the actual distribution of DM sources corresponds to the ground state of the linear field theory on the foamed space.

An arbitrary non-trivial topology of space can be described as follows. Given a Riemanian 3D manifold $\mathcal{M}$, we take a point O in it and issue geodesics from O in every direction. Then points in $\mathcal{M}$ can be labeled by the distance from O and by the direction of the corresponding geodesic. In other words, for an observer at O the space will look as R^3 (endowed with a metric lifted from $\mathcal{M}$). Given a point $P \in \mathcal{M}$, there may exist many homotopically non-equivalent geodesics connecting O and P. Thus, the point P will have many images in R^3. The observer might determine the topology of $\mathcal{M}$ by noticing that in the observed space R^3 there is a fundamental domain $\mathcal{D}$ such that every radiation or gravity source in $\mathcal{D}$ has a number of copies outside $\mathcal{D}$. The actual manifold $\mathcal{M}$ is then obtained by identifying the copies. In this way, we may describe the topology of space $\mathcal{M}$ by indicating for each point $r \in R^3$ the set of its copies $E(r)$, i.e. the set of points that are images of the same point in $\mathcal{M}$. Most of the time, we will simply speak about the images of points in R^3, without referring to $\mathcal{M}$.

Note that an observer ignorant of the actual topological structure of $\mathcal{M}$ will greatly overestimate the density of matter (as all the gravity sources outside the fundamental domain $\mathcal{D}$ are fictitious — each of them is just an image of some point in $\mathcal{D}$ seen from another direction). However, one cannot immediately apply the above picture to the explanation of DM effects: the Dark Matter emerges on galaxy scales while we do not see multiple images of galaxies densely filling the sky. Our idea that allows to link the observed DM effects with the topological structure of space is that the fundamental domain may be of such distorted shape that the direct recovery of the actual topology of space by detecting images of sources could be impossible. Indeed, the non-trivial topology at present is a remnant of quantum fluctuations at the very early Universe, and the randomness built in the structure of the original quantum foam can survive

the cosmological expansion. Namely, at the quantum stage the state of the Universe was described by a wave function defined on the space of Riemanian 3D manifolds. Once quantum gravity processes stop, the further evolution of the wave function was governed by the cosmological expansion only. It is highly unlikely that the expansion could led to a complete reduction of the wave function, i.e. to singling out one definite topological structure of the Universe. In other words, if at the end of quantum gravity era the Universe was not in a particular topological quantum eigenstate, it is not in such a state now. One cannot, therefore, speak about a definite topological structure of space, i.e. assign a definite set $E(r)$ of images to every point $r \in R^3$. A point $r' \in R^3$ can be an image of r with a certain probability only, hence instead of a discrete set of images, a smooth halo of images of every single point appears.

Even if we want to believe that a definite (classical) topological structure has happened to emerge out of the quantum foam, the randomness of this structure will persist: the wormholes which remained as the quantum foam tempered will be randomly cast in space. Moreover, we recall that a typical wormhole is obtained as follows: the interior of two remote spheres is removed from R^3 and then the surfaces of the spheres are glued together[1]. Such wormhole works like a conjugated couple of convex (spherical) mirrors, therefore a parallel beam of geodesics diverges after passing through the wormhole. Thus, if we place spherical wormholes randomly in R^3, the flow of geodesics that pass through a large number of the wormholes will have a mixing property (like the flow of Sinai billiard, or of Lorenz gas). For a point-like source for radiation or gravity, it means that some portion of photons/gravitons will be scattered by the spherical wormholes, which will create a specific smooth halo around every single source.

In any case, no matter what is the exact origin of the randomness of the topological structure of space, one can take such random structure into account by introducing a certain measure on the space of all Riemanian 3D-manifolds $\mathcal{M}$. The observed topological or metric properties of space are then obtained by averaging over this measure. Thus, for example, an individual manifold $\mathcal{M}$ is defined by specifying, for any point $r' \in R^3$ the set $E(r')$ of its images (the points in R^3 that represent the same point of $\mathcal{M}$). Averaging over all manifolds $\mathcal{M}$, gives a distribution $\bar{K}(r, r')$ of the images of r':

$$\bar{K}(r, r') = \delta(r - r') + \bar{b}(r, r'), \tag{1}$$

where the first term corresponds to the point r' itself, while $\bar{b}(r, r')$ is a certain smooth distribution of additional images of r'; namely, in the neighborhood of a point r of volume d^3r there is (on average) $\bar{b}(r, r')d^3r$ images of r'.

It means that a single particle of matter at the point r' is always accompanied by a smooth density $\bar{b}(r, r')$ of exactly the same matter. This halo does not necessarily emit enough light to be identified, but it will always contribute to gravity. Thus, if the halo is not seen, it is detected by an anomalous behavior of the gravitation potential of the point-source. Such anomalous behavior is

[1] one can imagine a more general construction as well, where a pair of more complicated two-dimensional surfaces replaces the spheres

indeed universally observed starting with the galaxy scales, and constitutes the DM phenomenon. The existence of a quite rigid dependence between the density of luminous matter (LM) and the density of DM is a well-known observational fact. This fact allows us to interpret the DM phenomenon as an indication of the random topological structure of space, with formula (1) giving

$$\rho_{DM}(r) = \int \bar{b}(r, r')\rho_{LM}(r')d^3r'. \tag{2}$$

In fact, the simple law

$$\bar{b}(r, r') \sim |r - r'|^{-2} \quad \text{at} \quad |r - r'| \geq R_0 \tag{3}$$

(where R_0 is the galaxy scale) provides quite accurate description of all known DM effects. In particular, it allows to recover the whole variety of observed galaxy rotation curves [2]. It is also consistent with the observed fractal structure of the distribution of matter on large scales [3] -[8].

Note that relations (2),(3) give a good description for the observed DM phenomena, independently of a theoretical interpretation [3, 2]. We will, however, show that in our picture where $\bar{b}(r, r')$ is an averaged characteristic of the topological structure of space, empirical law (3) acquires a basic physical meaning.

It is also important that in our interpretation the DM halo is not actually dark. The image r of a point r' represents the same physical point, just seen from another direction. Therefore, if the source of gravity at r' is also a source of radiation, all its images in the halo will be luminous too. However, the halo radiation has a diffuse character and the brightness is very low (the halo radiates a reflected light, in a sense). In observations, relating the halo radiation to a particular point source could be a very difficult task[2]. In fact, the presence of a significant diffuse component in cosmic radiation, unidentified with any particular source, is well known [9]. Usually, the observed diffuse halos in galaxies are attributed to reflection from dust, and the general diffuse component is assumed to originate from very fade and remote galaxies, but it has never been related to DM halos. However, it was very convincingly demonstrated in [10] that the observed DM/LM ratio within the intracluster gas clouds is much less than that for galaxies. This observation gives a strong argument in support of our theory of DM effects: while for small and bright sources (galaxies) the luminosity of the halo is filtered out by the observer and the halo appears to be dark, for the extended radiation sources (cluster size plasma clouds) the diffuse halo radiation comes from the same region of space and is automatically accounted in the total luminosity of the cloud.

Indeed, we show below that the intensity of sources of radiation renormalizes according to the following law:

$$I_{total}(r) = I_{source}(r) + I_{halo}(r), \tag{4}$$

[2]We note that we neglect here the red shift of light. In the actual Universe every ghost image has its own red shift, depending on the value of the traversed optical path, which puts an additional problem in relating the halo radiation to the point source.

where

$$I_{halo}(r) = \int \bar{b}(r, r') I_{source}(r') d^3 r', \tag{5}$$

with the same $\bar{b}(r, r')$ as in (2). Therefore, in our picture, the luminosity of the DM is always proportional to its density. The gravitating halos of discrete light sources in the sky only appear to be dark, because of their diffuse character.

From the physical standpoint the foamed space is a porous system. It means that the coordinate volume, which comes out from the extrapolation of our local (solar) coordinate system, always exceeds the actual physical volume (due to the presence of wormholes). The ratio $V_{coord}/V_{phys} = Q$ defines the porosity coefficient of the foamed space. When we use the extrapolated coordinates we always overestimate (by the use of the Gauss divergence theorem) the actual intensity of a source of gravity or of an incoherent radiation. In gravity, the effect displays itself as the presence of Dark Matter. Hence, the porosity coefficient of the foamed space Q can be related to the ratio of Dark Matter density to the density of baryons in the Universe, i.e. $Q = \Omega_{DM}/\Omega_b$. Analogously, the same relation holds true for the ratio of two components of radiation (diffuse background and discrete sources), i.e. $Q = \Omega_{diffuse}/\Omega_{discrete}$. The relation

$$\Omega_{DM}/\Omega_b \approx \Omega_{diffuse}/\Omega_{discrete}$$

is the basic indication of a geometrical (topological) nature of DM effects.

We point out that certain models of the spacetime foam have already been considered in the literature (e.g., see Refs. [11, 12] and references therein). However the primary interest was there focused on setting observational bounds on the possible foam-like structure at extremely small scales (i.e., at very high energies) $\gtrsim 10^2 L_{pl}$ (where L_{pl} is the Planck length), while DM phenomena suggest that the characteristic scale of the spacetime foam L (and respectively of wormholes) should be of the galaxy scale, e.g., of the order of a few Kpc. The rigorous bounds obtained indicate that at small scales spacetime is extremely smooth up to the scales $\gtrsim 10^2 L_{pl}$, that was to be expected[3]. The common feature of such models is that photons, in addition to the chaotic scattering, undergo also modified dispersion relations, as it happens in all Lorentz violating theories with preferred frames (i.e., "Aether-like situations", e.g., see Refs. [13]) which should lead to a modification of the CMB spectrum acoustic peaks. The foam-like structure discussed in the present paper surely violates the Lorentz invariance and also leads to some modification of dispersion relations. However the Lorentz invariance and the standard dispersion relations violate only at galaxy scales ($L \sim$ of a few Kpc) which are unimaginably larger than any photon wave length $\lambda = c/\omega$ detected. We recall that in the Friedman Universe $\lambda, L \sim a(t)$, where $a(t)$ is the scale factor and the ratio $\lambda/L \ll 1$ remains

[3]Indeed, at those scales topology fluctuations have only virtual character and due to renormalizability of physical field theories they should not directly contribute to observable (already renormalized) effects. Topology fluctuations were strong enough only during the quantum stage of the evolution of the Universe, while the possible subsequent inflationary phase should considerably increase all characteristic scales of the foam. By other words, the relic foam - like structure of space may survive only on very large scales.

constant up to the quantum era. Therefore, such a modification cannot directly influence the CMB spectrum (though it surely influences via the DM effects discussed).

2 Random Topology of Space

In order to set a general frame for the study of a foamed space, let us start with a toy example where the space is a cylinder of radius R. The metric is the same as for the standard flat Friedman model

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2), \tag{6}$$

but one of the coordinates, say z, is periodic ($z + 2\pi R = z$). In what follows, for the sake of simplicity we neglect the dependence of the scale factor on time in (6), i.e. consider the Minkowsky space as the coordinate space. Thus the actual values of the coordinate z run through the fundamental region $z \in [0, 2\pi R]$. Such space can be equally viewed as a portion of the ordinary R^3 between two plane mirrors (at the positions $z = 0$ and $z = 2\pi R$). An observer, who lives in such space, may use the extrapolated reference system (i.e., $z \in (-\infty, \infty)$), however he/she easily notices that all physical fields are periodic in z. Consider the Newton's potential ϕ for a point mass M. In this space the exact expression can be easily found from the standard Newton's potential by means of the image method. Indeed, the periodicity in z means that instead of a single point mass $M\delta(r - r')$ at the point r' the observer will actually see an infinite series of images

$$\delta(r - r') \to K(r, r') = \sum_{n=-\infty}^{\infty} \delta(z - z' + 2\pi Rn)\delta(x - x')\,\delta(y - y'), \tag{7}$$

and the Newton's potential for a point source at $r' = 0$ takes the form

$$\phi = -GM \sum_{n=-\infty}^{\infty} 1/\sqrt{\rho^2 + (z + 2\pi Rn)^2}. \tag{8}$$

On scales $r \ll R$ we may retain only one term with $n = 0$ and obtain the standard Newton's potential for a point mass $\phi \sim -GM/r$, while for larger scales $r \gg R$ the compactification of one dimension will result in the crossover of the potential to $\phi \sim \frac{GM}{R} \ln r$ (note that this is indeed the shape of the potential that one reads from the observed galaxy rotation curves).

The anomalous behavior of gravity indicates that DM effects show up at this model on the scale of distances of order R. Indeed, let us consider a box of the size L and evaluate the total dynamical mass within the box

$$M_{tot}(L) = M \int_{L^3} K(r, 0)\, dV = M \left(1 + [\frac{L}{2\pi R}]\right). \tag{9}$$

Thus, if the observer is ignorant about the real topological structure of space he should conclude the presence of some extra (odd) matter. The reason is obvious, when we expand the coordinate volume it covers the physical (or fundamental) region of space many times and we respectively many times account for the same source (i.e., images of the actual source). Thus the increase of the total mass is fictitious. In the simplistic model under consideration the volume of the fundamental (physical) region behaves as $V_{phys} = L^3$ for $L < R$ and $V_{phys} = 2\pi RL^2$ for $L > R$. We note that at large distances $L \gg R$ the parameter $Q(L) = M_{tot}(L)/M - 1$ can be used to estimate the actual value of the physical volume: $V_{phys}(L) = L^3/Q(L)$, i.e., Q is the "porosity coefficient" of space at scales $L \sim R$.

The space discussed above is rather simple: for an extended source we will see a countable set of its images without distortion. Therefore, one can easily detect the fundamental region of space and avoid consideration of fictitious sources. In the case of a general foamed topological structure this is hardly possible. Nevertheless, whatever the topological structure of the manifold is, we can apply the method of images: every topology can be achieved by introducing a certain equivalence relation in R^3 and gluing equivalent points together. Thus, a space of non-trivial topology is completely defined by indicating for every point $r' \in R^3$ the set $E(r') = \{f_1(r'), f_2(r'), \ldots\}$ of the points equivalent to it. In other words, a point source at a point r' in the fundamental region is accompanied by a countable set of images, or "ghost" sources:

$$\delta(r - r') \to K(r, r') = \delta(r - r') + \sum_{f_i(r') \in E(r')} \delta(r - f_i(r')) \tag{10}$$

where $f_i(r')$ is the position of the i-th image of the source.

For example, consider any source for radiation $J(r, t)$. Then according to (10) the electromagnetic potential $A(0, t)$ is described by the retarded potentials

$$A = \frac{1}{c} \int \frac{J_{t-|r|/c}}{|r|} dV + \frac{1}{c} \sum_i \int \frac{J_{t-|f_i(r)|/c}}{|f_i(r)|} dV_i. \tag{11}$$

The first term of this formula corresponds to the standard, "direct" signal from the source, while the sum describes the multiple scattering on the topological structure of space. A similar formula is obtained for the gravitational field.

It is clear that all physical Green functions for all particles acquire the same structure

$$G_{total}(0, r) = G_0(0, r) + \sum_{f_i(r) \in E(r)} G_i(0, f_i(r)). \tag{12}$$

Formally, one can use the standard Green functions, while the scattering will be described by the bias of sources

$$J_{total}(r, t) = J(r, t) + \int b(r, r') J(r', t) d^3 r', \tag{13}$$

where $b(r, r') = K(r, r') - \delta(r - r')$, i.e. we excluded the actual point source. In gravity the second term in (13) corresponds to the DM contribution (e.g., see

[3]). We note that in general the bias $b\,(r, r')$ is an arbitrary function of both arguments, which means that the nontrivial topological structure is capable of fitting an arbitrary distribution of Dark Matter.

The function $K(r, r')$ unambiguously defines the topological structure of the physical space. However, for a general foamed structure of space (a gas of wormholes) this function has a quite irregular character, i.e. it is not directly observable. One has to introduce a measure on the space of all 3D-manifolds and average the function K over this measure. The resulting function

$$\bar{K}(r, r') = \delta(r - r') + \bar{b}(r, r')$$

gives the (average) density, at the point r, of the images of the point r'.

Because of the averaging, the irregularities are smoothed out, hence the bias function $\bar{b}(r, r')$ is observable. Indeed, the averaging of (12) and (13) gives

$$G_{total}(0, r') = G\,(0, r') + \int \bar{b}(r, r') G\,(0, r))\, d^3 r \tag{14}$$

for Green functions, and

$$\rho_{total}\,(r, t) = \rho\,(r, t) + \int \bar{b}\,(r, r')\,\rho\,(r\prime, t)\, d^3 r' \tag{15}$$

for the density of matter. Therefore, when we can distinguish two components in the observed picture of the distribution of, say, gravity sources: discrete sources and a diffuse background, the discrete sources can be identified with the first term in the right-hand side of (15), i.e. with "actually existing" sources, while the diffuse halo can be identified with the second term, "the images". Then, by comparing the observed distribution $\rho(r')$ of actual (discrete) sources with the observed DM distribution

$$\rho_{halo}(r) = \int \bar{b}\,(r, r')\,\rho(r') d^3 r', \tag{16}$$

one can extract an information about the structure of the bias $\bar{b}$. In fact, the homogeneity of the Universe requires from $\bar{b}$ to be a function of $(r - r')$ only (which means that the form of DM halos does not, in general, depend on the position in space). In this case, the Fourier transform of (16) gives

$$\rho_{halo}(k) = \bar{b}(k)\rho(k), \tag{17}$$

which defines $\bar{b}$ uniquely. As we show in the next Section, the bias $\bar{b}$ extracted from the DM observations in this way has both a very simple form and a transparent theoretical meaning.

Note that being an averaged characteristics, the bias $\bar{b}$ does not determine the topology of space completely. Along with the one-point distribution $\bar{K}(r, r')$, one can consider joint distributions of images for several sources:

$$\bar{K}_n(r_1, \ldots, r_n;\ r'_1, \ldots, r'_n)$$

which is the averaged density of the images of the points $r'_1, \ldots, r'_n$ at the points $r_1, \ldots, r_n$. Only when all the functions K_n, $n = 1, 2, \ldots$, are determined, one will have a full description of the structure of the foamed physical space. However, the one-point bias functions $\bar{b}(r, r')$ carries the most important information.

Thus, consider a source of radiation, constantly emitting light with the frequency ω, i.e. we have a density of the EM current $J(r')e^{i\omega t}$ such that

$$\langle J(r'_1)J^*(r'_2)\rangle = \delta(r'_1 - r'_2)I_{source}(r'_1), \tag{18}$$

where $I_{source}(r)$ is the spatial distribution of the intensity of the source. In order to take into account the effects of the non-trivial topology of space, $J(r)$ should be modified according to (13), i.e. $J(r_1)J^*(r_2)$ transforms into

$$\int K(r_1, r'_1)\, K(r_2, r'_2)\, J(r'_1)J^*(r'_2)d^3r'_1 d^3r'_2 = \int K(r_1, r')K(r_2, r')I_{source}(r')d^3r'.$$

Averaging over different topologies gives

$$(J(r_1)J^*(r_2))_{total} = \int \bar{K}_2(r_1, r_2;\ r', r')\, I_{source}(r')d^3r', \tag{19}$$

where $\bar{K}_2(r_1, r_2;\ r', r')$ is, by definition, the joint distribution of a pair of images of the point r'.

The points r_1 and r_2 can be images of the same point r' if and only if they are images of each other. Therefore, $\bar{K}_2(r_1, r_2;\ r', r')$ is proportional to $\bar{K}(r_1, r_2) = \delta(r_1 - r_2) + \bar{b}(r_1, r_2)$; more precisely

$$\bar{K}_2(r_1, r_2;\ r', r') = \delta(r_1 - r_2)\bar{K}(r_1, r') + \bar{b}(r_1, r_2)P(r_1, r_2, r') \tag{20}$$

where we denote as $P(r_1, r_2, r')$ the density at the point r_2 of the distribution of images of the point r' under the condition that the point $r_1 \neq r_2$ is an image of r_2.

As we see from (19),(20), while the phases of the source current $J(r')$ are delta-correlated (see (18)), there appear long-range correlations in the density of the total current — due to the term proportional to $\bar{b}(r_1, r_2)$ in the kernel $\bar{K}_2$. However, the characteristic wave length in $\bar{b}(r_1 - r_2)$ is of order of galaxy size, i.e. it is unimaginably larger than the wave length c/ω of the light emitted. Therefore, the contribution of the coherent part of the total current to the radiation is completely negligible: by (19),(20) we find

$$(J(r_1)J^*(r_2))_{total} = \delta(r_1 - r_2)\int \bar{K}(r_1, r')\, I_{source}(r')d^3r' + \quad \text{long wave terms},$$

which gives the following formula for the total intensity of sources (actual plus ghost ones)

$$I_{total}(r) = \int \bar{K}(r, r')\, I_{source}(r')d^3r' = I_{source}(r) + \int \bar{b}(r, r')\, I_{source}(r')d^3r'. \tag{21}$$

Comparing with (16), we see that the distribution of a diffuse radiation background associated to a luminous source coincides with the distribution of dark matter in the halo of the same source.

Note that for a non-stationary remote source of radiation the picture is more complicated. A momentary pulse at some point will create a spherical EM wave emanating from the point — and from its images. On the front of the wave only a small number of images will give an essential contribution, namely those which have comparable and shortest optical paths. This will lead to an interference picture on the front. We note that due to wormholes the signal from some images can reach an observer even earlier than the basic signal. Only with time elapsed, as the larger and larger number of images contribute, the interference picture disappears, and the diffuse radiation background given by (21) establishes.

In conclusion of this section, we recall that the observed homogeneity and isotropy of space require from the topological bias $\bar{b}(r, r')$ that defines both the DM distribution (16) and the distribution (21) of the sources of diffuse radiation to be the function of the distance $|r-r'|$ only: $\bar{b}(r, r') = \bar{b}(|r-r'|)$. The integral

$$Q\left(L\right) = 4\pi \int_0^L R^2 \overline{b}\left(R\right) dR \tag{22}$$

characterizes then the distortion of the coordinate volume or the porosity of space (i.e., $1/Q$ gives the portion of the fundamental region or the volume of the actual physical space in a coordinate ball of the radius L). In general there can be both a situation where $Q(L)$ tends to a finite limit as $L \to \infty$ and then $Q(\infty)$ defines the total amount of DM ($Q = \Omega_{DM}/\Omega_b = \Omega_{diffuse}/\Omega_{discrete}$), and the case where Q is unbounded. The last case indicates the presence of a certain dimension reduction of space at large distances (e.g. when $Q(L) \sim L^\alpha$ the dimension of the physical space reduces to $D = 3 - \alpha$ [5]).

3 Topological bias: empirical and theoretical approach

In this Section we derive a formula for the bias function $\bar{b}(|r-r'|)$ and show that it fits the observed picture of DM distribution quite well. While in empirical considerations it is more convenient to view $\bar{b}(R)$ as a bias of sources (which means exploring the laws (16) and (21)), we achieve more theoretical insight when choose an equivalent description of the random topological structure of space by means of the bias of Green functions (see (14)). This means that instead of saying that each material point is accompanied by an infinite set of images, we say that each source excites an infinity of fields. Indeed, on a connected manifold of non-trivial topology there is an infinite number of geodesics connecting any two points. So the light emitted at a point P arrives at a point Q by an infinite number of non-homotopic ways. We may associate a separate EM field with each homotopy class: each of the fields propagates independently, but they sum up when interact with matter. When we describe things in R^3 by means of

the bias functions, we thus associate a separate field to each term in the right-hand side of (12). These terms differ by positions of the images $f_i(r)$. In our picture, where the topology is random, there is no preferred position for the i-th image, hence we have a system of an infinite number of fields $\{A_i\}$ which is symmetric with respect to any permutation of them (in other words, the fields are identical).

It is widely believed that the effects of quantum gravity should lead to a cut-off at large wave numbers. The cut-off at Λ means that the photons with wave numbers $|k| > \Lambda$ are never excited. We say that the field does not exist at such k. One can describe a cut-off of a more general form, by introducing a characteristic function $\chi(k)$: at $\chi(k) = 1$ the field with the wave number k exists, while at $\chi(k) = 0$ it does not. Because of the renormalizability of all physical field theories, the question of the determining exact form of the cut-off of a given field is of little importance. However, for the system of an infinite number of identical fields $\{A_i\}$ the cut-off function acquires a meaning.

Indeed, let us define $N(k) = \sum_i \chi_i(k)$ where the sum is taken over all the fields A_i. Thus, $N(k)$ is the number density of fields which exist (i.e. which are not forbidden to create particles) at the given wave number k. Here, the existence of the cut-off means that $N(k)$ can be finite for all k. As the fields sum up when interacting with the matter, the values of $N(k)$ greater than 1 lead to a stronger interaction than in the case of a single field. For example, consider a Newtonian potential[4]

$$\Delta\phi = 4\pi\gamma\rho.$$

In the Fourier representation we have

$$\phi(k) = \frac{-4\pi\gamma}{k^2}\rho(k). \tag{23}$$

If there exist $N(k)$ identical Newtonian gravity fields with the wave number k, each of them satisfies (23), while the effective potential (that which acts on matter) is given by $\phi_{eff}(k) = \sum_{i=1}^{N(k)} \phi_i(k)$ and satisfies, therefore,

$$\phi_{eff}(k) = \frac{-4\pi\gamma}{k^2}N(k)\rho(k).$$

This is equivalent to a renormalization of the source density

$$\rho(k) \to N(k)\rho(k),$$

and comparing with (17) gives

$$N(k) - 1 = \bar{b}(k).$$

Thus, the Fourier transform $\bar{b}(k)$ of the topological bias function can be interpreted as the excessive number density of fields (gravity or EM) at the wave number k, i.e. it is determined via a cut-off function.

[4]for the relativistic generalization see Sec. 2 in Ref. [3].

Although the problem of determining the exact shape of the cut-off is usually considered hopeless because the full quantum gravity theory has not been developed, an approach developed in [14] allows one to derive possible types of cut-off by means of simple thermodynamical models. For example, assume that the energy density and the total excessive number density of fields $\mathcal{N} = \int (N(k)-1) d^3k$ are finite. We also assume that $\mathcal{N}$ is a conserved quantity (along with the energy). Then the shape of the function $N(k)$ is determined uniquely by the condition that the system of the identical free fields is in the thermodynamical equilibrium (one should only choose the statistics for the fields and fix the values of thermodynamical parameters). Indeed, the state of the system with $N(k)$ identical free fields at the wave number k is determined by the numbers $n_i(k), i = 1, \ldots, N(k)$ of the particles with the wave number k for each field. In the case of Fermi statistics for the fields (that has nothing to do with the statistics for the particles which remains Bose), there cannot be more than one field in the given state, i.e. for every given k all the numbers $n_i(k)$ should be different. The energy density at the wave number k equals to $\omega_k \sum_{i=1}^{N(k)} n_i(k)$, where ω_k is the energy of a single particle; as we deal here with massless fields, we take $\omega_k = |k|$ (we put $h = c = 1$). In what follows we assume Fermi statistics for the fields (Bose statistics leads to a similar result [5, 15], however the computations in Fermi case are simpler). Then, the state of the lowest possible energy ("the ground state") corresponds to $\{n_1(k), \ldots, n_{N(k)}(k)\} = \{0, 1, \ldots, N(k) - 1\}$. This gives us the energy $|k|N(k)(N(k) - 1)/2$ at the wave number k. The total energy density is thus given by $\int \frac{|k|}{2} N(k)(N(k)-1) d^3k$. The ground state corresponds to the minimum of the total energy density. As the total excessive number density of fields $\mathcal{N} = \int (N(k) - 1) d^3k$ is assumed to be conserved, the problem of finding $N(k)$ reduces to minimizing $\int |k| N(k)(N(k) - 1) d^3k$ under the constraint $\int (N(k) - 1) d^3k = constant$. This gives us

$$N(k) = 1 + \left[\frac{\mu}{|k|} \right],$$

where the "chemical potential" μ is fixed by the value of $\mathcal{N}$. For the bias function $\overline{b}$ this gives

$$\overline{b}\,(k) = \begin{cases} \dfrac{\mu}{|k|} & \text{for} \quad |k| < \mu, \\ 0 & \text{for} \quad |k| > \mu. \end{cases} \tag{24}$$

One can make different assumptions and, perhaps, arrive at different formulas for the bias. However, this simplest bias function provides a very good description of the observed distribution of DM. Indeed, in the coordinate representation bias (24) takes the form

$$\begin{aligned} \overline{b}\,(\vec{r}) &= \frac{1}{2\pi^2} \int_0^{\mu} \left(\bar{b}\,(k)\, k^3 \right) \frac{\sin(kr)}{kr} \frac{dk}{k} = \\ &= \frac{\mu}{2\pi^2 r^2} \left(1 - \cos(\mu r) \right). \end{aligned} \tag{25}$$

As it was shown in [2], by choosing $\mu = \pi/(2R_0)$ where R_0 is of order of a galaxy size (i.e. a few Kpc), bias (25) applied to spiral galaxies produces the pseudo-isothermal DM halo[5] $\rho = \rho_0 R_C^2/(R_C^2 + r^2)$, where R_C is the core radius which has the order of the optical disk radius $R_C \sim R_{opt}$. We note that this is in a very good agreement with the observations (see [16]). In fact, by fitting one parameter μ in accordance to Tully-Fisher law [17], relations (16) and (25) quite accurately represent the whole variety of the observed galaxy rotation curves [16, 2] (we recall that bias (25) is derived from thermodynamical considerations, so it is quite natural to allow the chemical potential μ fluctuate in space; exact mechanisms governing these fluctuations are described in [3, 2]).

From (22),(25) one can find that starting with the galaxy scale the porosity of space behaves as $Q(r) \sim r/R_0$. Thus the total dynamical mass for a point source within the radius r increases also as

$$M(r) \sim M(1 + r/R_0). \tag{26}$$

Importantly (see the previous Section), the same conclusion holds for the luminosity of the point source (i.e., for a galaxy or an X-ray source). Therefore, one can not immediately conclude from (26) a linear growth of the ratio $M_{tot}(r)/M_b(r)$ of gravitational (dynamical or lensing) to the barionic mass: the result depends on how much of diffuse radiation is discarded at the observations.

Observations suggest that the number of baryons within the radius r behaves as $N_b(r) \sim r^D$ with $D \simeq 2$ (see e.g. Refs. [6, 7, 8] where the $\simeq r^2$ behavior was reported up to at least 200 Mpc). Thus, the observed baryonic density Ω_b falls inverse proportionally to the deepness of the observations and is well below 1. In the standard picture the total gravitational mass grows as $\sim R^3$, as it should be in a homogeneous Universe, so the linear growth of $M_{tot}(r)/M_b(r)$ predicted by bias (25) is indeed consistent with observations. However, the linear growth starts to show up with the scales larger than cluster size, while the reported mass to luminosity ratio remains approximately the same on the galaxy scale and on the cluster scale. To resolve the problem, we invoke the results of [10] where there was demonstrated that the intracluster gas clouds may not carry dark matter. In our picture this is indeed the case, as the intracluster cloud is an extended source of X-ray radiation, of size much larger than R_0. Thus, the associated diffuse background sums up with the "direct" signal, so all the ghost sources of gravity that lie within the cloud are visible as well. This means the absence of "dark" matter in the cloud or, in other words, that the number of baryons in the cloud is greatly overestimated — most of the contribution to the cloud luminosity is given by the diffuse halo, i.e. by fictitious sources due to the non-trivial topology of space. It is easy to check that correcting the baryon density of the intracluster gas in accordance with (21), (25) provides indeed the linear growth of $M_{tot}(r)/M_b(r)$ starting right from the galaxy scale.

[5]This result is valid for a single galaxy, while in the presence of a distribution of galaxies the resulting halo acquires the Burket - type form $\rho = \rho_0 R_C^3/(R_C^2 + r^2)(R_* + r) + \rho_H$, where ρ_H is a homogeneous background formed by all galaxies and R_* is the scale at which DM halo merges to the homogeneous DM background (e.g., see for discussions Ref. [3]).

Note that at very large scales the diffuse radiation can hardly be separated from the very faint sources. Therefore, the picture of the homogeneous distribution of matter (i.e., of the Friedman Universe) is restored. In fact, an arbitrary foam-like structure of space (i.e., any choice of the bias $\overline{b}(r)$) agrees perfectly with the observational large-scale homogeneity and isotropy of the Friedman Universe provided that the actual physical volume $V_{phys}(r) = 4/3\pi r^3/Q(r)$ (the volume of the fundamental region of the coordinate space) is homogeneously filled with matter. Indeed, in this case the number of actual sources within the radius r behaves as the physical volume $N_b(r) \sim V_{phys}(r) \sim r^3/Q(r)$. Along with the actual sources we always observe images (DM and diffuse radiation) and every source produces $\Delta N \sim Q(r)$ additional images. Thus the total number of images behaves always as $N_b(r)\,Q(r) \sim r^3$, i.e., produces a homogeneous distribution.

4 Conclusion

In conclusion, we briefly repeat basic results. First of all the concept of space-time foam introduced by Wheeler can be crucial in explaining properties of the present day Universe. The random ("foamed") topological structure leads to the fact that every discrete source in the sky should be surrounded with a specific halo (a random distribution of images). We call this phenomenon a topological bias of sources. In gravity such halo modifies the standard Newton's law and appears as the Dark Matter phenomenon. In particular, the Universal rotation curve (URC) constructed in [2] on the basis of the topological bias shows a very good fit to the empirical URC [16]. We stress that in a general foamed space the bias $b(r, r')$ is a random function of both arguments which means that the form of the DM halo can arbitrary vary in space. By other words any observed distribution of DM can be easily fitted by a proper choice of the foamed structure. However, the simplest bias function which we derived theoretically from a basic physical (thermodynamical) considerations seems to give a quite accurate account of the DM effects in a huge range of spatial scales.

As it was demonstrated in this paper, in the foamed space the halos around discrete sources are actually not dark, but form the diffuse background of radiation. Moreover, the ratio of the two components (the diffuse background and discrete sources) is exactly the same as the ratio of DM and baryons ($\Omega_{DM}/\Omega_b = \Omega_{diffuse}/\Omega_{discrete}$).

We note that the foamed picture of our Universe allows to explain the problem of missing baryons. Recall that the direct count of the number of baryons gives a very small value $\Omega_b \sim 0.003$ for the whole nearby Universe out to the radius $\sim 300 h_{50}^{-1} Mpc$ e.g., see [18]. In our picture, this means only that at the radius $\sim 300 h_{50}^{-1} Mpc$ the actual volume is ten times smaller, than in the Friedman space ($V_{phys} \simeq 0.1 V_F$), i.e. the actual density is ten times bigger which reconciles the observed small baryon density with the primordial nucleosynthesis constraints.

We stress that any homogeneously filled with matter foamed space (i.e., an

arbitrary choice of the bias function $b\,(r, r'))$ agrees perfectly with homogeneity and isotropy of the Universe and does not contradict to the standard Friedman model. The general foamed Universe can be viewed as the standard Friedman space filled with a gas of wormholes. In such a picture the Large Scale Structure has an equilibrium character, for it reflects the foamed topological structure of space (i.e., the distribution of wormholes) formed during the quantum period of the evolution of the Universe.

Finally, we have demonstrated that in a foamed space any non-stationary and sufficiently remote signal is accompanied with a formation of a specific interference picture at the front of the wave (stochastic interference) which rapidly decays.

5 Acknowledgment

This research was supported in part by the joint Russian-Israeli grant 06-01-72023.

References

[1] Wheeler J.A. , (1964) in: *Relativity, Groups, and Topology,* B.S. and C.M. DeWitt (eds.), Gordan and Breach, New York.

[2] Kirillov, A.A., Turaev, D., 2006, MNRAS, 371, L31.

[3] Kirillov, A.A., 2006, Phys. Lett. B, 632, 453.

[4] Kirillov, A.A., 2002 Phys. Lett. B, 535, 22.

[5] Kirillov, A.A., 2003 Phys. Lett. B, 555, 13.

[6] Pietronero L., 1987, Physica, A144, 257.

[7] Ruffini R., Song D.J., and Taraglio S., 1988 A&A, 190, 1.

[8] Labini, S.F., Montuori, M., Pietronero, L., 1998 Phys. Rep. 293, 66.

[9] Sarazin C.L., X-ray Emissions from Clusters of Galaxies, Cambridge University Press (1988).

[10] Clowe, D., et al., 2006, arXiv:astro-ph/0608407v1, to appear in ApJL.

[11] F.R. Klinkhamer, Nuclear Phys., **B578** 277 (2000); F.R. Klinkhamer and C. Rupp, Phys. Rev. **D70**, 045020 (2004); Phys. Rev. **D72**, 017901 (2005); Bernadotte S., Klinkhamer F.R. Phys. Rev. **D 75**, 024028 (2007).

[12] N. Mavromatos and M.˜Sakellariadou, "Relativistic modified Newtonian dynamics from string theory?, arXiv:hep-th/0703156

[13] T. Jacobson, S. Liberati and D. Mattingly, Annals Phys. **321**, 150 (2006); C. Eling, T. Jacobson and D. Mattingly, "Einstein-aether theory," arXiv:gr-qc/0410001.

[14] Kirillov, A.A., 1999 Sov.Phys.JETP, 88, 1051.

[15] Kirillov, A.A., Turaev, D., 2002 Phys. Lett. B, 532,185.

[16] Persic, M., Salucci, P., Stel, F., 1996, MNRAS, 281, 27.

[17] Tully, R.B., Fisher, J.R. 1977, A&A, 54, 661.

[18] Persic, M., Salucci, P., 1992, MNRAS, 258, 14.