

A systems based approach for financial risk modelling and optimisation of the mineral processing and metal production industry

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Abstract: Large scale engineering process systems are subject to a variety of risks which affect the productivity and profitability of the industry in the long run. This paper outlines the short comings of the current methods of risk quantification and proposes a systems engineering framework to overcome these issues. The functionality of the developed model is illustrated for the case of mineral processing and metal production industries using a copper ore processing and refined metal production case study. The methodology provides a quantitative assessment of the risk factors and allows the opportunity to minimise financial losses, which would help investors, insurers and plant operators in these sectors to make appropriate risk hedging policies. The models developed can also be coupled with evolutionary or swarm based algorithms for optimising the systems. A numerical example is illustrated to demonstrate the validity of the proposition.

Keywords: *financial risk modelling; reliability based risk modelling; quantitative risk assessment; process systems optimisation; systems thinking;*

1. Introduction

Any large scale engineering operation is fraught with diverse risks which can disrupt the smooth operation of the business and result in significant monetary losses. For example, catastrophic events like fire, flood etc. can cause huge losses to an engineering production system. In many cases, events might be severe enough to disrupt operation or even result in the closure of the business. Other smaller events, like mechanical failures and breakdowns, can result in lowered production rates and the business might not be able to meet planned production targets. This might not only result in revenue losses, but also lower the credibility and reputation of the company for not being able to deliver promised goods in time to the downstream market. To counter the ill effects of these unforeseen risks, a company insures their operation especially for significant and catastrophic events. The idea is to pay a fixed premium to the insurance company, and the insurance company then compensates for some or all of the losses in the unfortunate case of business interruption.

Most financial risk modelling is based on statistical models of the insurance claims data (Embrechts et al., 1997) and as such a company which wants to insure a specific business operation against natural catastrophic events looks at the historical data of claims related to this and constructs a statistical probability distribution function. Based on this, a probability of the event actually occurring is calculated and then a price is set for the premium and the terms and conditions for the payout are agreed upon. However, this kind of statistical modelling does not take into account the underlying processes which govern the dynamics of the business interruption. The modelling approach presented in this paper aims to reflect the significant uncertainties in the process of a complex business operation chain or an engineering production system, thus providing a comprehensive assessment of the risks

involved. The tool presented would help the insurers to evaluate of the overall risk profile of the business; while the business management team would appreciate the root causes which give rise to specific risks and take appropriate counter measures or decide to transfer the risks to a third party. The focus of the paper is to describe the generic tool developed and illustrate its application through modelling the insurance risks for the mineral processing and metal production industry. A sample risk model for a copper ore processing and metal production system is developed. It contains the various physical process components like the crusher, mills, flotation cells, thickening and filtration unit, tailings dam, the smelter and the electro-refining unit. These physical processes are mapped into risk items and interconnected together in a systems framework, for estimating the overall risk profile of the operation. In the present context, risk is quantified by the shortfall in the planned yearly production target of the operation.

The rest of the paper is organised as follows. Section 2 presents a brief overview of the conventional quantitative techniques in financial risk forecasting. The literature on financial risk forecasting is discussed highlighting materials pertaining to the mineral processing and metals production industry. Section 3 describes the modelling philosophy and presents the software implementations of the various model components using hypothetical risk items to elucidate the various features of the modelling paradigm and present the simulation results. Section 4 applies the modelling concepts to build a risk model for copper ore processing, smelting and metal refining and presents the corresponding results. Section 5 explains the system optimisation using multi-objective evolutionary techniques. This is followed by discussions and conclusions in Section 6, followed by the appendix and the references.

2. Overview of financial risk forecasting:

2.1. Mathematical methods and techniques in risk modelling and forecasting

Forecasting in a risk context refers to the prediction of the expected outcome of a random variable. Data driven techniques are commonly used for quantitative forecasting of volatility. The jargon volatility is used in the genre of finance to represent the standard deviation of the returns and is a measure of the risk.

Classical time series based modelling techniques have been used widely in the field of empirical finance to forecast volatility. These range from the simple moving average (MA) models to the advanced GARCH (Generalised Autoregressive Conditional Heteroskedasticity) family of models (Danielsson, 2010). These techniques are suitable for modelling risk if a large quantity of past historical data is available. However in the present problem, it would be difficult to get a large amount of data for extreme events and, hence, these methods are generally not suitable.

Soft-computing methods like neural networks, fuzzy logic and evolutionary optimisation differ from conventional computing in the way that they are able to tolerate imprecision, uncertainty and vagueness in the problem. Fuzzy logic has found applications in quantifying risks and has been used in insurance classification, underwriting, projected liabilities, pricing, asset allocations etc. (Shapiro, 2004). Feed forward neural networks have been used as a pattern classification instrument in contractors risk assessment systems (Bakheet, 1995). Analysis of insolvency risk using Genetic Algorithms (GA) has been studied in (Varetto, 1998) and a Pareto frontier of the profitability and risk competitiveness using GA has been

documented in (Varetto, 1998). GAs have also been coupled with fuzzy logic for financial risk management in (Robinson and Yager, 1996). These soft computing techniques seem very promising and evolutionary optimisation techniques have been used in the present study to optimise the system. Due to the generic nature of the modelling philosophy proposed, it is envisaged that other soft computing techniques could be integrated in the modelling methodology depending on the specific problem at hand.

Many reliability based analysis for component failure have been investigated in large scale complex systems. Modelling systems based on the failure rates of individual components can give an idea of the overall risk involved in the complete process chain. A survey of probabilistic methods in reliability, risk and uncertainty analysis has been reported in Robinson (1998). Most of the methods related to this category try to characterise the uncertainty in system response due to uncertainty in internal or external system parameters. The mean value (MV) method (Kapur and Lamberson, 1977), Differential analysis methods (Zimmerman et al., 1990), other first order reliability methods like Hasofer-Lind (Hasofer et al., 1973), Rackwitz-Fiessler (Rackwitz and Flessler, 1978) etc. fall under this category. Some of the basic concepts from this field are introduced into the proposed modelling paradigm. The methodology proposed here takes a systems approach towards modelling and couples reliability data with discrete logic to arrive at a risk profile.

Most of the risk modelling techniques like GARCH and other methods based on past historical data is appropriate for events that occur with probabilities of around 1-5%. However for more rare catastrophic events whose probability of occurrence is of the order of ~0.1%, the Extreme Value Theory (EVT) is used. The EVT focusses on the tail zones of the probability distributions and does not require a-priori specification of the response distribution. EVT has been used for determining financial risk (Gençay et al., 2003; Gilli and Këllezi, 2006). The applications of EVT as a risk management tool for insurance and finance have been documented in (Embrechts, Resnick, and Samorodnitsky 1999). The EVT methods are a good alternative for risk modelling and can be used to compare the output predictions with the systems based model presented here.

Though a lot of literature exists on financial risk assessment in general, less work has been done in the area of financial risk modelling and analysis of chemical processes and specifically the mineral processing systems. Most studies focus on the perceptions and expert opinions of personnel associated with the mining sector and use a survey based technique to identify the risk vs. impact trade-off of specific components in the mining operation. However, such studies are qualitative, rather than quantitative, and lack a strong mathematical foundation. The objective of the present paper is to build and analyse models with some quantitative data from existing mineral processing operations.

For the chemical process systems, only specific components are modelled for risk analysis and appropriate countermeasures are proposed to mitigate the same. For example, Lavaja and Bagajewicz (2004) analyse the risks of individual component failures, like the risks associated with the heat exchanger etc. Most other risk modelling techniques use process dynamics equations of mass and energy balance to analyse the effects of uncertainty or failures in certain components and arrive at a risk profile (Podofillini and Dang, 2012). These methods are effective in a data rich environment but would fail, or give highly inaccurate results, if precise and sufficiently representative data is not available. The systems modelling philosophy introduced in this paper aims to handle the vagueness of

information and use dynamic models as a leverage for accuracy in circumstances where these are available.

In the present study, the risk related to the mineral processing activities is characterised predominantly by the loss of production. Any shortfall in the output expected, according to the production schedule, is considered a loss. This loss can be due to various causes, such as mechanical breakdowns, unforeseen human factors, earthquakes etc.

Operational failures refer to the breakdown of machinery like pressure vessel explosion, gear failure, generator breakdown, failure of drilling machines, trucks etc. Many publications look at specific mechanisms of failure and try to model these using different techniques. In Wu et al. (2012), corrosion based failure mechanisms for petrochemical plants are identified and a knowledge based reasoning model is developed for predicting the same. Structural safety assessments of building collapse in the context of reliability have been documented in Raphael et al. (2011). Risk evaluation of failure modes for turbine rotor blades based on Dempster Schafer's theory have been attempted in Yang et al. (2011). Stochastic and quantitative risk assessment techniques have been used in Marhavidas and Koulouriotis (2011) to model the worksites of an electric power provider. The use of Failure Mode Effect and Criticality Analysis (FMECA) and Failure Time Modelling (FTM) for the case of a processing industry is documented in Ahmad et al. (2012). In general, the approach towards reliability modelling using statistical methods is to model the mean time between failures (MTBF) using some probability distribution function. The outage duration (OD) and maintenance cost associated with the breakdown can be suitably approximated using some function, based on historical data. Similar statistical distributions are used to characterise our systems risk model which takes a more holistic view of the process and the interactions between its different components.

Significant risks can arise in the system operation if the human operator erroneously performs some activities. As a consequence of these errors, a loss of production can occur. Human Reliability Analysis (HRA) method (Dougherty, 1989) is a framework which is traditionally used as a probabilistic risk assessment methodology to quantify such risks. Many techniques for identifying hazards under a more general framework of quantitative risk assessment (QRA) have been used in process plants. Some of them are the Hazard and Operability (HAZOP) (Venkatasubramanian et al., 2000), Structured What-If Technique (SWIFT), Hazardous Scenario Analysis (HAZSCAN) (Lauridsen et al., 2001) etc. The systems risk modelling methodology, as proposed in this paper, can be extended to incorporate these effects. These are however not included as they are beyond the scope of the present paper.

3. Proposed risk modelling methodology

For large scale process systems, most analytical methods are intractable and hence a simulation based method must be resorted to. A new systems based modelling approach is proposed in this section and the risks associated with mineral processing are modelled along the lines of this new paradigm.

3.1. Overview of modelling philosophy

The general structure of the model is composed of several layers of nested subsystems as shown in Fig. 1. Each component in the practical engineering system can be abstracted

to represent a risk item with some inputs and outputs. The “in” and “out” flows can be manipulated suitably by some flow controls. Other associated components which affect the risk item are the lifeing engine, health of the specific risk item etc. The detail of abstraction would depend on the available data and the impact of each component on the overall risk profile. Most cases would include constituents where a large failure or breakdown would lead to significant financial losses. These failures might be due to unforeseen events like flooding, earthquake etc. or due to mechanical breakdowns. The risk modeller can apply his/her own discretion in more involved situations. For example some components might be known to fail regularly and hence the engineering team has a specific preventive maintenance strategy in place to mitigate the risk of major breakdowns. Then the modeller may or may not include this item in the risk calculation model.

Fig. 2 shows the inter-connections of the sub items in each risk item. In the context of a mineral processing plant, risk items can be the grinding process, flotation process, filtering process etc. Each would have a set of inputs which might affect the risk profile of the item. However, there need not be any specific mass balance for the processes and the inputs and outputs can be of different nature. This is specifically done to facilitate the modelling process in the event where there are unknown process variables or where the detailed knowledge of the processing plant at each stage is difficult to obtain. For the grinding process, a set of inputs can be the raw unground ore, water, energy etc. whereas, the outputs might be small sized ore, waste water etc.

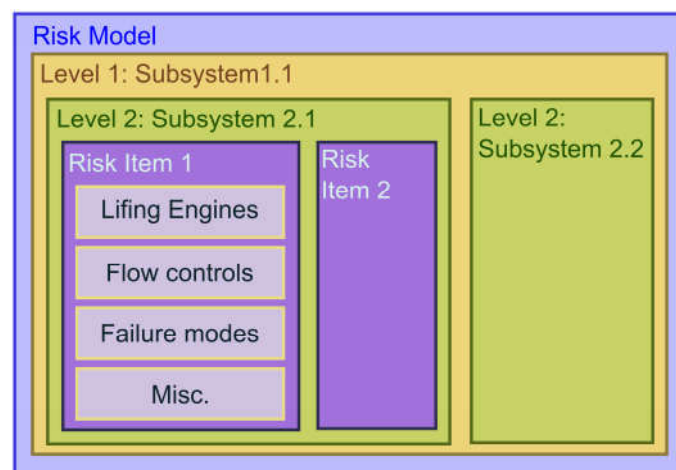


Fig. 1. Hierarchical scheme of the risk model.

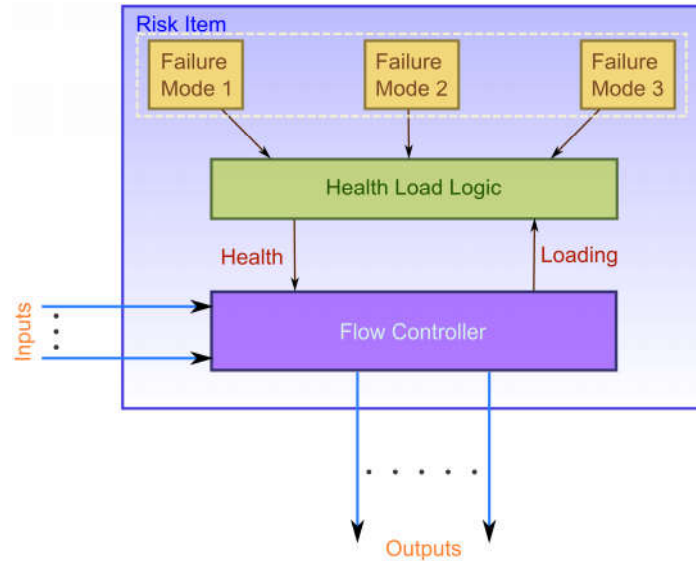


Fig. 2. Components of each risk item.

Each risk item can have multiple failure modes. For example the risk item for the grinding process can have a failure mode due to mechanical breakdown of the mill, severe earthquakes, accidental fire etc. For multiple inputs and outputs (MIMO) in each risk item, the flow controller is basically a set of discrete logical conditions which map the inputs to the corresponding outputs. The health of each risk item is a variable in the range $[0,1]$ for the duration of simulation and is affected by multiple factors. A value of 0 would indicate that the system is in a failed state and a value of 1 would indicate that the system is in its state of maximum health. Any value in between would represent other intermediate conditions for the health. Each failure mode can independently affect the health of the system. Thus, for example, if there is a fire in the milling plant, then one of the failure modes would become active and reduce the health of the risk item (grinding mill) to be 0. It would then be non-functional for a specific duration of time. In the same time interval, an unlikely event of flooding might occur and thus failure mode 2 would become active to make the health of the milling state to 0. The mill will come online, only when both the failure modes have reverted back from their failed states to their non-failed states.

A variable identified as the “Loading”, affects the health of a risk item and impacts the life consumption of the item. For example there might be more inflows than the rated capacity and the physical processing equipment might have an accelerated usage leading to a quicker mechanical breakdown. In such cases, depending on the incorporated logic, the loading variable affects the health of the system adversely over the simulated time period.

The health of the risk item as calculated from these complex interactions, manipulates the outputs from the flow controller. For the simplest case, the outputs are multiplied element-wise, by the health variable. This would imply that as the health of the risk item decreases, there would be a corresponding decrease in the output flows from the item. Thus, for the milling plant this might translate to lesser amount of finely ground ore coming out due to some faults in the ball mills and a corresponding decreased efficiency of processing the same amount of input.

Each risk item is characterised by 3 variables which are based on historical data of process operation by the industry and data of past insurance claims. These variables are the Time between Failures (TBF), Outage Duration (OD) and the Maintenance & Breakdown Costs (MB Costs). The TBF, MB Costs and the OD are represented by probability distribution curves with parametric representation.

There might be situations where no information is supplied for a specific case. In such situations failure data from a standard component can be included in the model and modelling would not be hampered due to lack of data for a specific case. Another situation can arise where there is no information about some component and there are two or more likely candidate solutions. For example, the manufacturer of the ball mill might not be known, but in the market there might be three or four large conglomerates which manufacture this kind of product. Each make would have a different failure probability and repair time associated with it. So, assuming that the ball mill is manufactured by one of them, the composite risk profile of the overall system can be calculated by doing Monte Carlo (MC) runs taking each of these three or four different makes with equal probabilities in each simulation. If one of the makes is more commonly installed than the others, then it can be drawn more often (i.e. assigned higher probability than the others) from the set for the MC runs. This is an additional advantage of using this modelling methodology over other mass balance based methodologies.

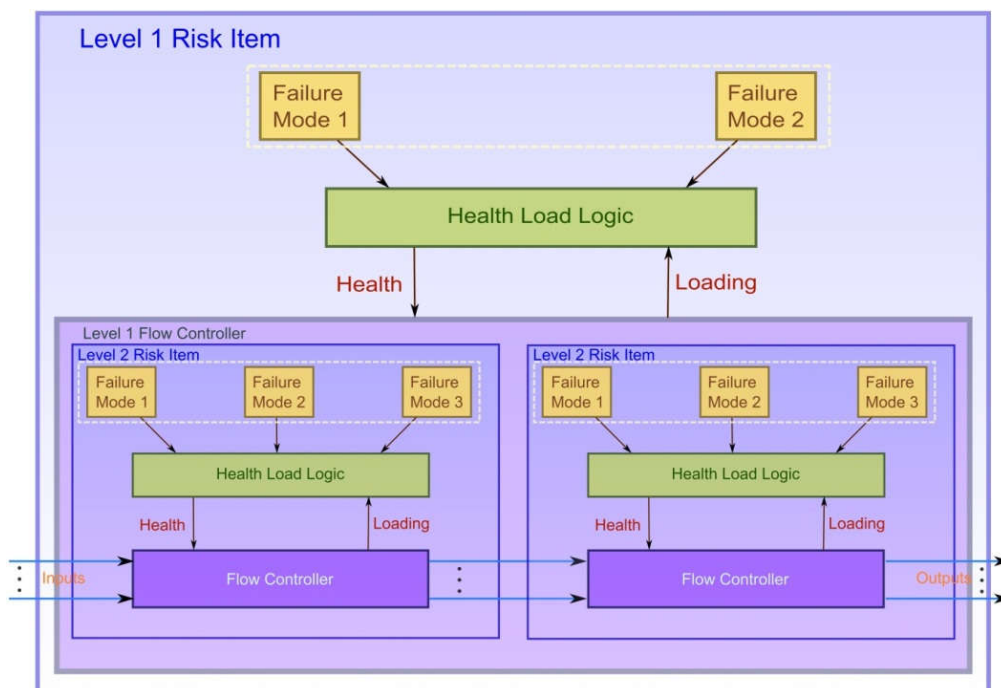


Fig. 3. Modelling risks in the present framework that can affect multiple risk items.

Due to the hierarchical structure of the model, certain risks which can affect a large number of equipment at once can be easily incorporated. Fig. 3 shows such a scheme within the present modelling framework. The Level 1 risk item subsumes the two smaller Level 2 risk items. Essentially the Level 2 risk items are placed in the flow controller of the Level 1 risk item with the inputs and the outputs providing the necessary link between the two levels. The Level 1 risk item might be catastrophic risks due to natural calamities which affect the

whole plant operation. For example, severe earthquakes would affect the whole operation and not any one equipment in particular.

3.2. Modelling of a single risk item

Fig. 4 shows the statechart diagram of a failure mode. It has two main states: the NoFailure State and the Failure State as shown by the enclosed boxes. The Failure State has two further sub-states LeadTime Sub-state and Maintenance Sub-state. The former indicates the time when the component sits idle after failure and before going for maintenance. This provision is made to account for additional costs incurred by the company during this period, if any. The arrows with a solid dot indicate a default transition. The arguments in the curly braces denote an assignment operation. Thus, when the model is simulated for the first time, it would be in the NoFailure State with Health=1. In the NoFailure State, a TBF (Time Between Failures) value is generated randomly from a pre-specified probability distribution. In a practical setting, the historical data of the failure of an equipment can be taken into account and then a probability distribution curve can be fitted into the data to be incorporated into the model. The transition from one state to the other is indicated by arrows again and these are governed by conditional statements in square braces, i.e. the system in Fig. 4 would shift from the NoFailure state to the Failure state after a random number of time steps as governed by the function $g_1(t)$ have elapsed. $g_1(t)$ should be defined as some function of TBF.

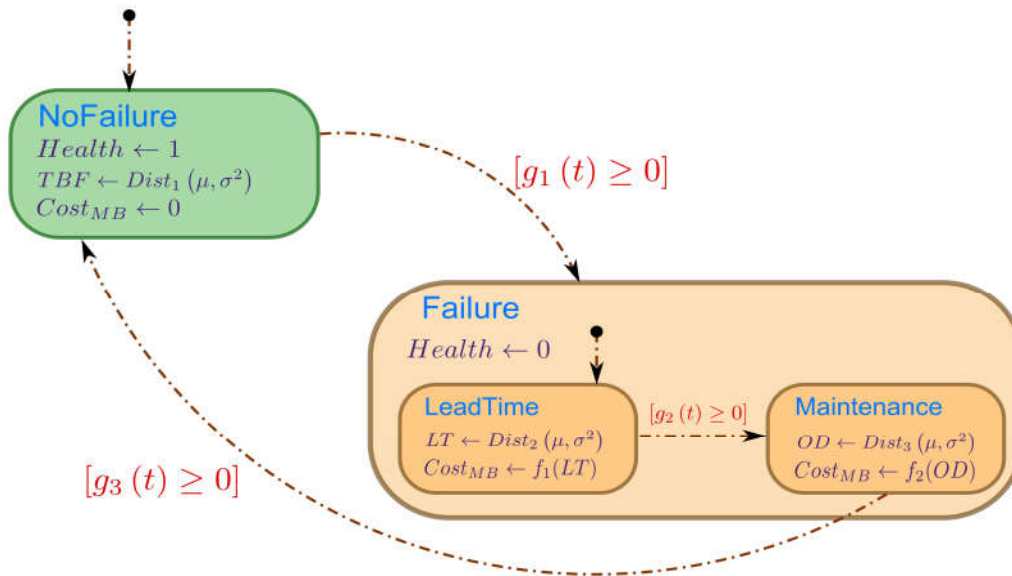


Fig. 4. State chart schematic of a failure mode.

On entering the failure state it would go into the LeadTime sub-state as this is the default transition. After a random number of time steps given by the function $g_2(t)$ (which depends on the randomly generated LT (Lead Time) variable) have elapsed, the system would transit to the Maintenance sub-state. The Maintenance sub-state generates an OD (Outage Duration) variable from another fitted probability distribution curve. Both in the LeadTime and the Maintenance sub-states, the associated costs are calculated through the functions f_1 and f_2 respectively. These functions depend on the time duration that the system is in that state. The system transits to the NoFailure state from the Maintenance sub-state after a

specified number of simulation time-steps that have elapsed, as dictated by the function $g_3(t)$ (which depends on the OD variable). The probability distributions are continuous and hence the number sampled from them is a real number. This essentially implies that the simulation is done in continuous time and the minimum time-step is limited by the capacity of the finite bit size of the computer, or the user can specify a small time step depending on the desired resolution of the model.

Fig. 5 shows the state chart schematic for the implementation of a Health-Load Logic. It has two states: Healthy and Faulty. The system transits from the Healthy state to the Faulty state if the input ($Health_{in}$) from the FailureMode is 0 and vice-versa, if the input is 1. In the Healthy state, the output Health is given as some linear/non-linear function (Ψ) of the input health and the loading from the flow controller.

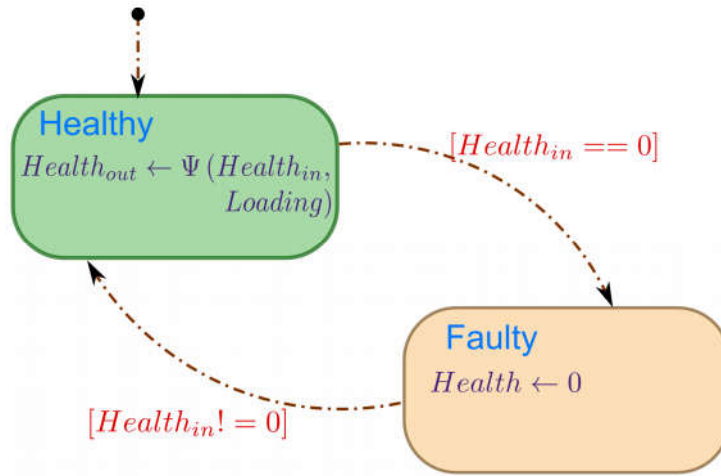


Fig. 5. State chart based schematic of Health-Loading logic.

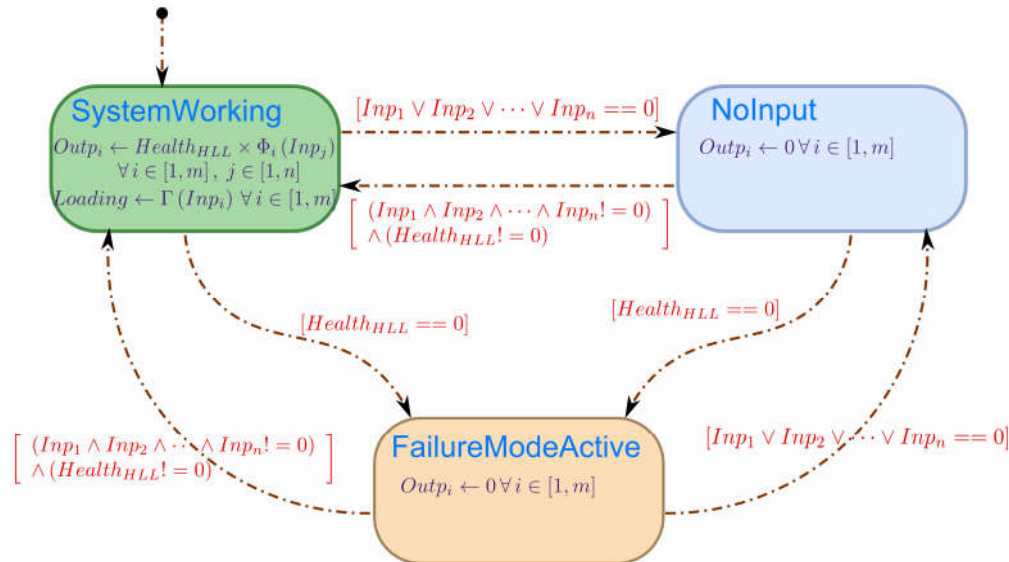


Fig. 6. State chart schematic for flow controller logic.

Fig. 6 shows the State chart based representation of the FlowController consisting of three states: SystemWorking, NoInput and FailureModeActive. The transitions from the different states are governed by the conditions of the system. The outputs ($Outp_i$) are some functions (Φ_i) of the inputs (Inp_i) multiplied by the Health of the system. The loading is

calculated in the SystemWorking state using some user defined function (Γ) which depends upon the inputs to the risk item.

3.2.1. Validation of the model for a risk item with a single failure mode

A hypothetical risk item with a single failure mode is simulated next to verify that the model is working. It has two inputs and one output. The simulation is done for 100 days and the resulting curves are reported in Fig. 7.

The following values are considered for the simulation. For the failure mode as in Fig. 4 the state transition functions are given by

$$g_1(t) := t_{state-entry} - TBF \quad (1)$$

$$g_2(t) := t_{state-entry} - LT \quad (2)$$

$$g_3(t) := t_{state-entry} - OD \quad (3)$$

where $t_{state-entry}$ is the time calculated from the instant that the system enters a particular state/sub-state. TBF, LT, OD refers to Time Between Failures, Lead Time and Outage Duration respectively. These are randomly drawn from the probability distributions given by Equations (4)-(6) respectively.

$$Dist_1 \sim 30 \ln \mathcal{N}(0.4, .25) \quad (4)$$

$$Dist_2 \sim \mathcal{U}(0, 5) \quad (5)$$

$$Dist_1 \sim 10 \ln \mathcal{N}(0.4, .25) \quad (6)$$

These types of curves are generally used to model failures of different components in reliability based designs. A typical characteristic of this curve is that it is asymmetric and has significant values in the right tail. The random numbers occasionally generated from the tail portion of the curve can thus model extreme events like large catastrophic failures.

The $Cost_{MB}$ of the system in the LeadTime and Maintenance sub-states in Fig. 4 is considered as

$$f_1 := LT \times MB_{costfactor_{LT}} \quad (7)$$

$$f_2 := OD \times MB_{costfactor_{OD}} \quad (8)$$

where $MB_{costfactor_{LT}}$ and $MB_{costfactor_{OD}}$ are constant values for the maintenance and breakdown cost expressed in units of £/day. The duration for which the equipment breaks down, multiplied by this factor gives the total costs incurred for the failure. These are taken as 10 and 5 £/day respectively.

The function (Ψ) in the Health-Loading logic in Fig. 5 is taken as

$$\Psi := Health_{in} - 0.1 \times Loading \quad (9)$$

The function (Φ) which relates the inputs to the outputs in Fig. 6 is taken as

$$\Phi_1 = 5 \times Inp_1 + 10 \times Inp_2 \quad (10)$$

The loading (Γ) is defined as the sum of the ratios of each input (Inp_i) to its rated input ($Inp_{i_{Rated}}$)

$$\Gamma = \sum_{i=1}^n \frac{Inp_i}{Inp_{i_{Rated}}} \quad (11)$$

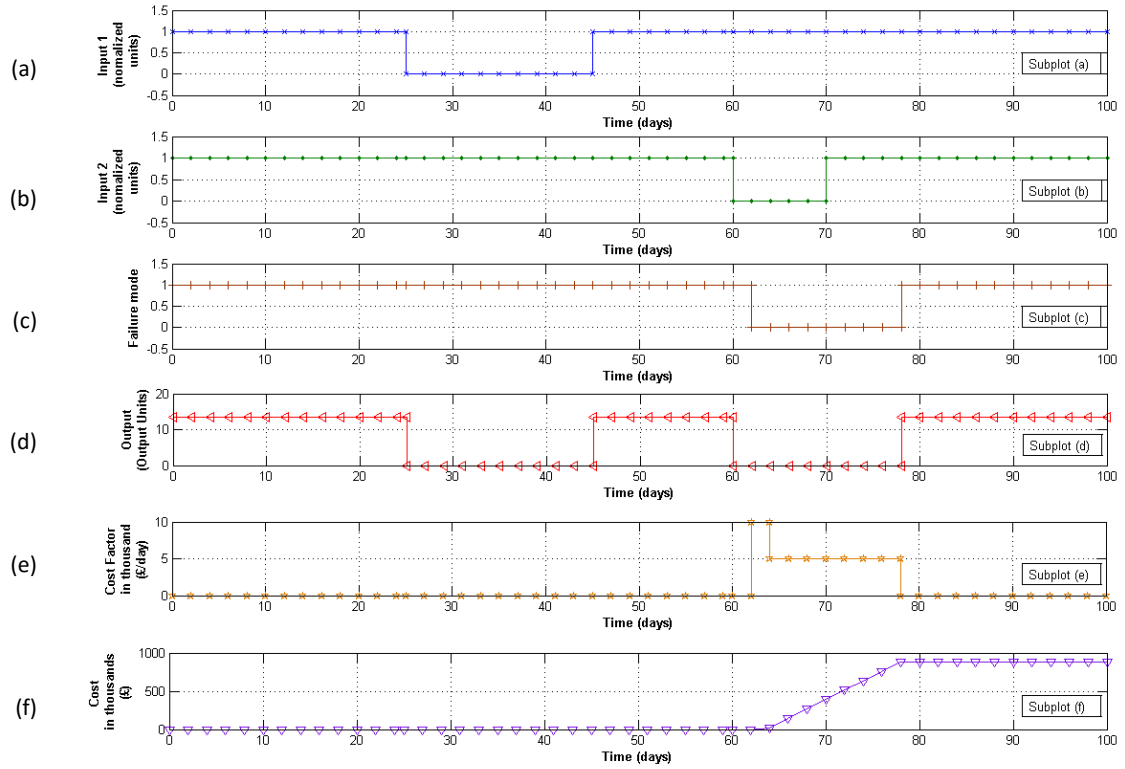


Fig. 7. Simulation of a sample risk item with one failure mode.

Figures 7(a) and 7(b) show the variation of the two inputs over time. It can be seen that the first input is unavailable from the 25th day to the 45th day and the second input is unavailable from the 60th to the 70th day. Figure 7(c) shows that a failure occurs sometime after 60 days and the system recovers from failure before the 80th day. Figure 7(d) in the red curve shows the output over time. As can be seen, for the periods during which there were no inputs, the output was zero. Also during the failure mode there is no output. Figures 7(e) and 7(f) show the cost factor and the total cumulative cost over time for the maintenance of the particular risk item. As can be seen, the cost factor curve takes non-zero values only during the period when the failure has actually occurred. Also, during the failure period it takes one value for the lead time and the other for the actual maintenance as discussed

previously. The last curve shows the cumulative cost of maintenance over time and the final value represents the total cost until that time.

3.3. Additional model features and illustrative examples

A chemical process operation has many different kinds of elements and it is not possible to model all of them using the basic risk item as presented in the previous section. The following additional feature improvisations are done which would help in modelling a variety of chemical process operations.

3.3.1. Modelling of a storage element

In most chemical processes there are storage elements at different stages. An example might be the stockpile of the input raw ore to a mineral processing plant. This kind of provision acts as a buffer against any upstream faults that might affect the production of the downstream components, albeit for a short time. Therefore, in a case where the mine fails to produce raw ore for a couple of days, the processing plant can still be kept running from the ore of the stockpile. Later when the mine recovers and comes online, it can accelerate its production over a certain period of time, to replenish the stockpile. This in effect acts as a natural hedge against some of the risks which might affect the process operation.

In the present modelling philosophy the storage element is modelled as a tank which has a maximum and minimum level. An analogy can be drawn with a physical water tank, which has a certain capacity and which can deliver a specified outflow rate without having any inflow as long as there is sufficient material in the tank (i.e. the level of the material does not fall below the lower level of the tank). The tank can be modelled on the same lines as the previous risk items with suitable modifications to the flow controller logic. In most cases the tank would not have failure modes due to mechanical breakdowns etc. But it would have failure modes due to large catastrophic events like floods (which can inundate the storage area) or earthquakes (which can damage the structural components of the storage space). The health associated with the tank would be influenced by these failure modes. Depending on the specific application area, there may be other events which affect the normal operation of these storage elements. For example, the tank might be physically a large open area of land used for storing intermediate products of the process. If they are precious metal ores or ones that can be easily consumed in the household (e.g. coal), there is a possibility of theft. Another example can be that of leakage of a tank, assuming that it contains liquid products. Technically speaking, these events cannot be classified as failure modes as they do not abruptly stop the operation of the tank. These would essentially affect the temporal health or instantaneous storage level of the tank. To incorporate these effects into the risk model, the functional mapping in the Health-Loading-Logic must be appropriately modified. A sample schematic of the tank element is shown in Fig. 8.

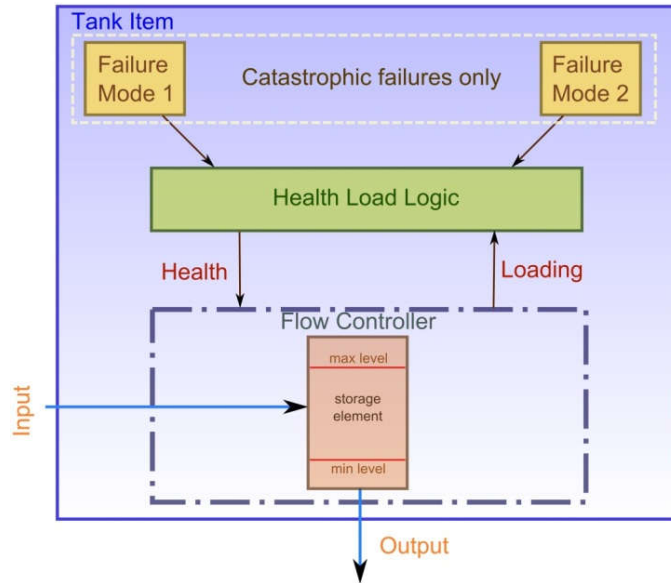


Fig. 8. Schematic of the tank element.

The state chart implementation of this is along the lines of the original modelling paradigm as described earlier. The flow controller is modified to include a minimum and a maximum tank level along with the current level of the tank. The current level of the tank is manipulated based on user specified logic. Another important difference of this item from the previous risk items is that the loading component which goes from the flow controller to the HLL is set to zero. This is because of the assumption that there are no moving mechanical components and therefore the degradation in the health of the tank is not dependent on the amount of loading that is impressed upon the tank at any given time.

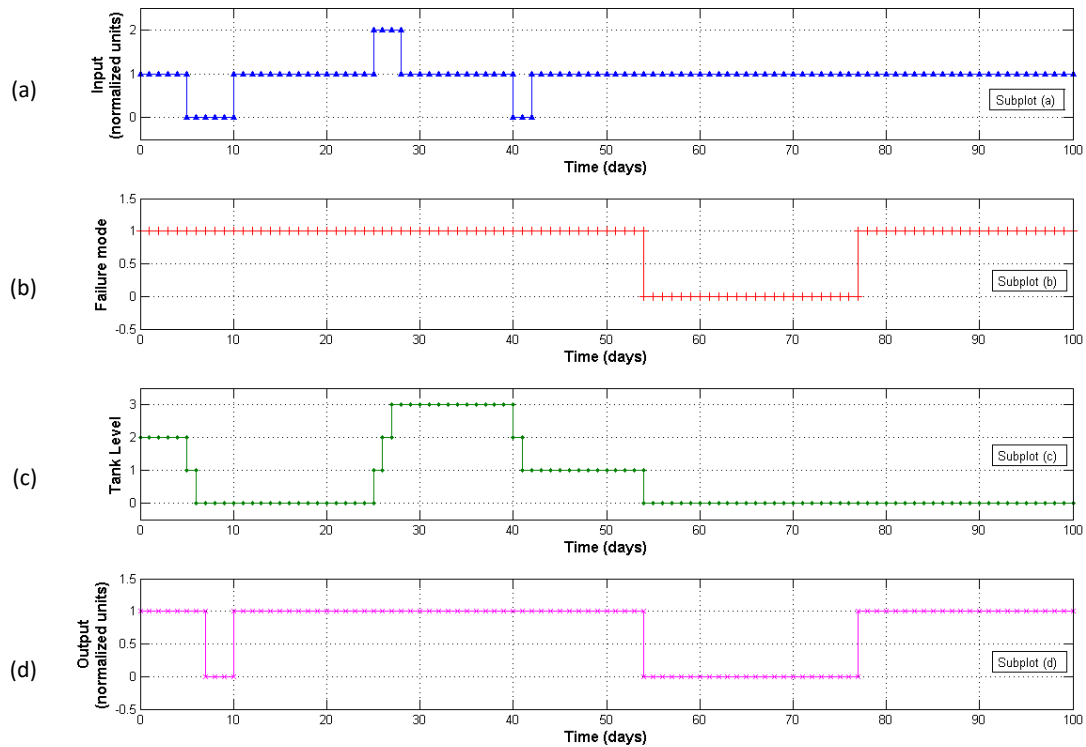


Fig. 9. Validation of the tank model.

Fig. 9 shows the time evolution of the various parameters during the simulation of one run of a hypothetical tank item. Figure 9(a) shows the input to the tank over a period of 100 days. The input falls from 1 to 0 on a couple of occasions, between the 5th and the 10th day and between the 40th and the 42nd day. The rated input is one, which is also the rated output. Between the 25th and the 28th day, the input is double than the rated input. This would go towards filling up of the tank in case its level is less than the maximum level. There is one catastrophic failure mode which affects the tank. The plot in Fig. 9(b) shows how the health is affected due to the single failure mode of the tank. Fig. 9(c) shows the instantaneous level of the tank. The maximum level of the tank is set to 5 and the minimum to 0. The initial level of the tank is set to 2. The simulation starts with this initial level of the tank. Then on the 5th day the input goes to zero. However, the tank still goes on producing the output for a couple of days from its own reserve until its level falls to the minimum level. After 2 days the tank capacity exhausts and there is still no input. At this time the output becomes zero. Again between the 25th and the 28th days when the output becomes 2, the tank supplies the rated output, but replenishes its reserves. As a consequence, the tank level rises to indicate that the tank is filling up. When the input goes to zero again between the 40th and the 42nd day, the output is supplied by the tank since the tank has enough capacity to provide output for this short interruption. The effect of the tank acting as a buffer mechanism to prevent any losses in the output is evident here. Finally when the failure mode becomes active, the tank is affected and it loses whatever level of material that it had. The output is also zero during this time as the tank is out of operation. For demonstrating the tank model, the simulation time step has been taken as 1 day and therefore the obtained tank levels seem to increase or decrease in a discrete, stepwise fashion. However, as discussed previously, the time step can be made smaller depending on the desired resolution of the model, to have a better approximation of a continuous time process.

3.3.2. Modelling of redundancy

Fig. 10 shows a supervisory logic and switching scheme, which can be used for modelling redundant components in the process flow model to improve system reliability. Many process systems have backup equipment which comes online if the main equipment fails. This does not hamper the process operation and gives sufficient time to the maintenance team to repair the failed component.

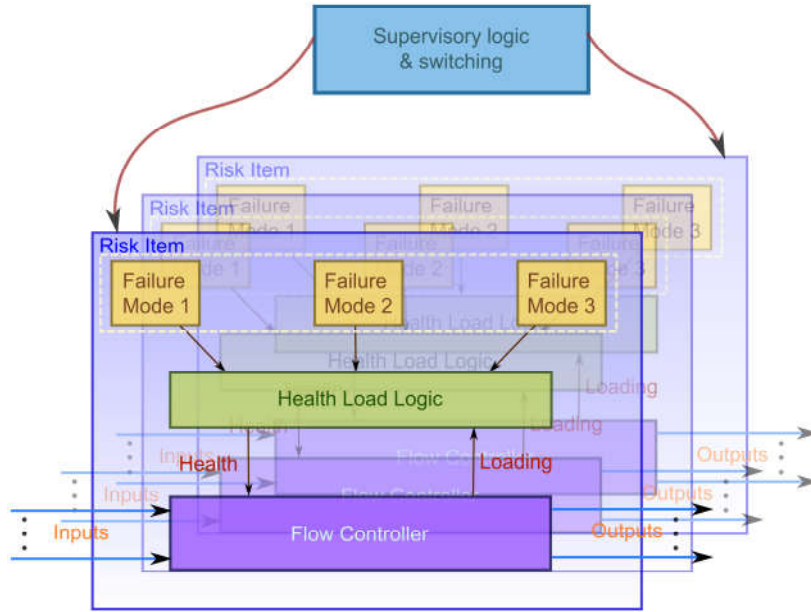


Fig. 10. Modelling a redundancy scheme.

The scheme employs two backup items in the event of failure of one item. The first backup item B comes online if the main item A fails. If both items A and B fail, then item C comes online. The actual logic of this switching between different backup components is achieved by the supervisory logic and switching block. Fig. 11 illustrates the state chart schematic.

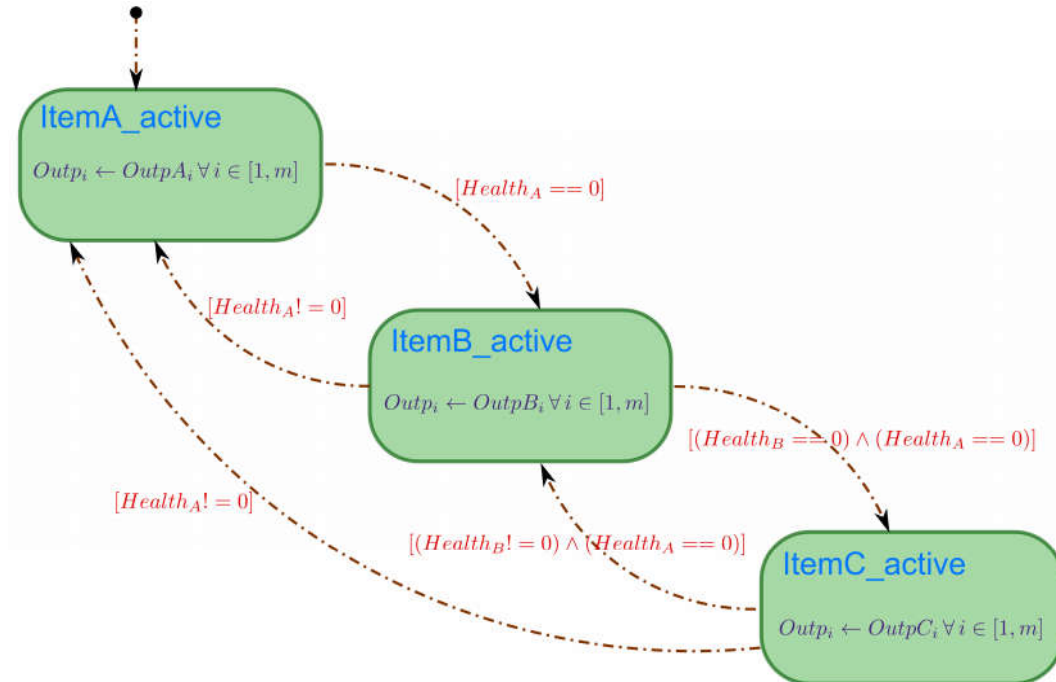


Fig. 11. State chart schematic of supervisory logic for the redundancy scheme.

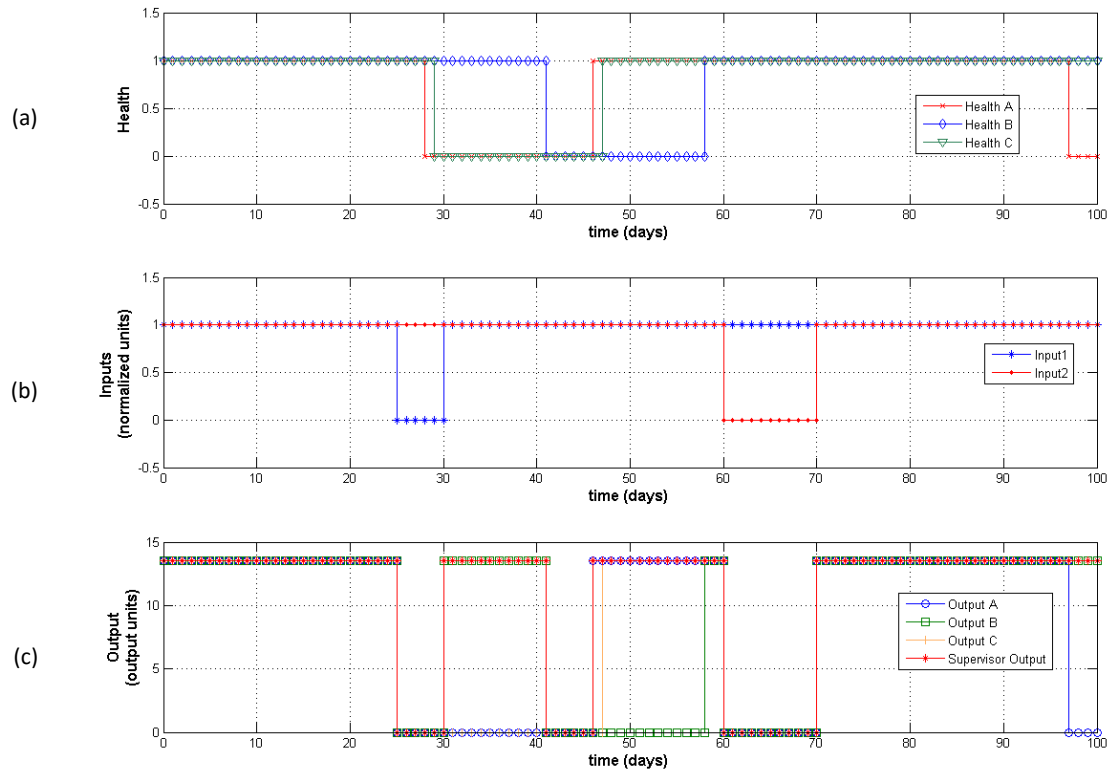


Fig. 12. Validation of the redundancy scheme modelling.

Fig. 12 illustrates the effective working of the redundancy scheme. Fig. 12(a) shows how the health of the three systems evolves over time. All the three systems are connected in parallel driven by the same input and evolve in time simultaneously. The supervisory logic decides which of the three systems should be chosen for the output. All three systems are out of operation (zero health) for specified time intervals. Between the 41st and the 46th day, all of the three systems are concurrently out of operation. Therefore, not only the main equipment, but also both the backup equipment fail during this time. As expected the output curve as shown in Fig. 12(c) is zero during this time. The only other two times when the output curve is zero is when either of the two inputs become zero. Apart from these three time periods, the system is able to maintain some output as at least one of the three risk items in the redundancy scheme is working.

3.3.3. Maintenance schedules

Most risk or reliability models consider constant maintenance and repair schedules which make it easier to calculate the reliability in an analytical or a semi-analytical framework. However, this grossly oversimplifies reality, as different maintenance schedules may be followed. Therefore, it is important to capture the effects of these schemes in the risk modelling methodology as these would significantly affect the end result. Since there is a health parameter in the proposed modelling scheme and a corresponding loading logic to manipulate it, a wide variety of maintenance scenarios can be modelled by the tool and their effects on the overall reliability of production can be studied. An interesting trade-off that can be inferred from these is whether it is more efficient and cost effective to have storage units after different stages of operation or whether a carefully designed maintenance plan is better in the long run. This trade-off is not always trivial and would depend on the type of intermediate products in the process, availability of cheap labour etc. For example, if the

intermediate products are solids, then some space can be allocated for their storage and the capital costs needed for this will not be too high. However, if the intermediate products are in the form of emulsions or other fluids then a proper leak-proof pressure vessel needs to be constructed and the capital costs might turn out to be prohibitive. In such cases availability of cheap labour costs would mitigate the reliability issue to some extent, by enforcing strong maintenance policies in the process plant.

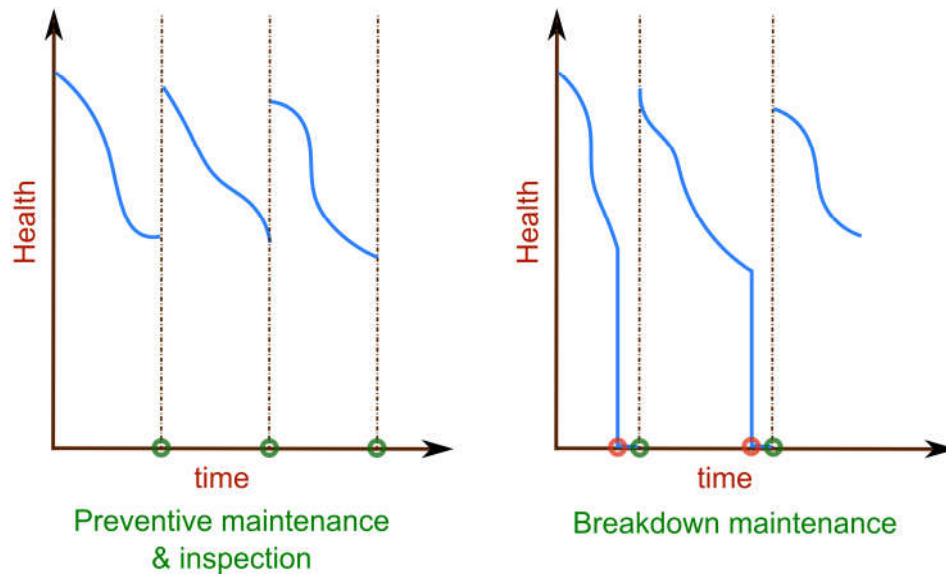


Fig. 13. Different maintenance strategies.

Fig. 13 shows the schematic of a couple of commonly used maintenance strategies which are incorporated in the present modelling framework. Due to the generic nature of the tool, other more complicated maintenance strategies can also be simulated and their effect on the overall system reliability with the associated expenses can be studied. In the first figure, the maintenance team does a periodic maintenance at a short specified constant time interval. The green circles on the time axis indicate the instants where the maintenance has taken place. Due to maintenance, the health of the system goes up every time. In the second figure, maintenance is done only after there is a breakdown. The red circles on the time axis indicate a breakdown where the health of the system goes to zero. The system regains its health after there is maintenance as indicated by the green circle.

Fig. 14 and Fig. 15 show two sample cases where there is a periodic maintenance at a constant interval of 50 and 20 days respectively. Due to this maintenance schedule the probability of failure of the equipment is delayed in time. Therefore, even though the equipment has the same failure probability, it does not fail immediately after maintenance. However, as is clear from Fig. 14 and Fig. 15, a longer maintenance period might result in the equipment failing in between the maintenance intervals, whereas a shorter maintenance period would prevent the equipment from going into the failure states. In both the figures, the output goes to zero on two instances when there are no inputs. Additionally, in the case of longer in-between maintenance times in Fig. 14, the health goes down to zero during the length of time there is no maintenance. In case the equipment would not have had a breakdown, the preventive maintenance policy would have been applied on the 50th day and the next failure would have been delayed in time. However, since the breakdown has

already taken place, the usual maintenance schedule is implemented to bring the equipment back online.

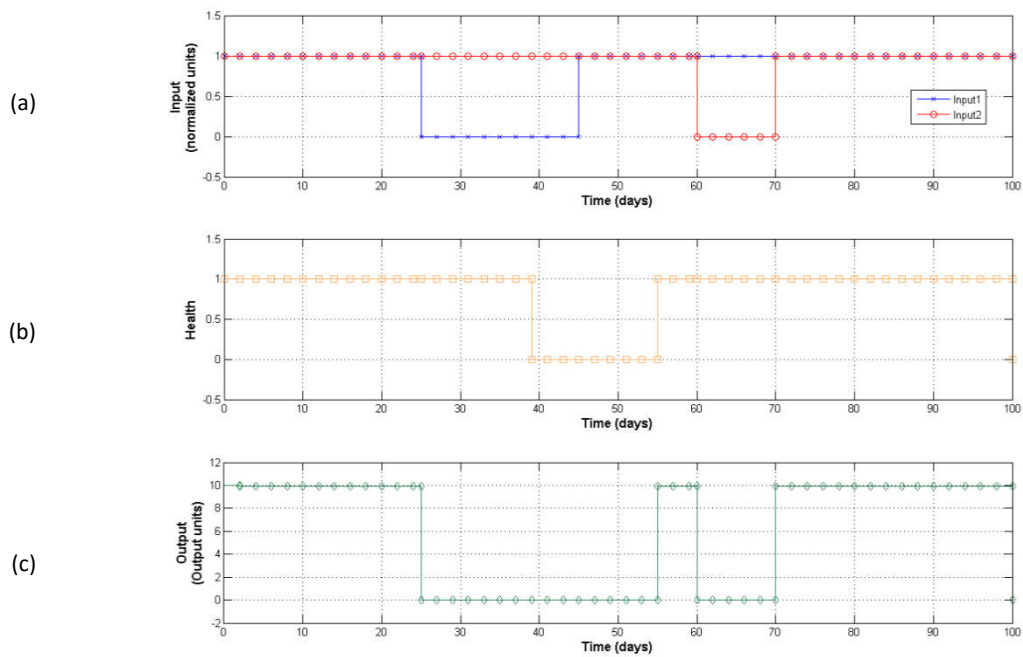


Fig. 14. Constant interval maintenance with a period of 50 days.

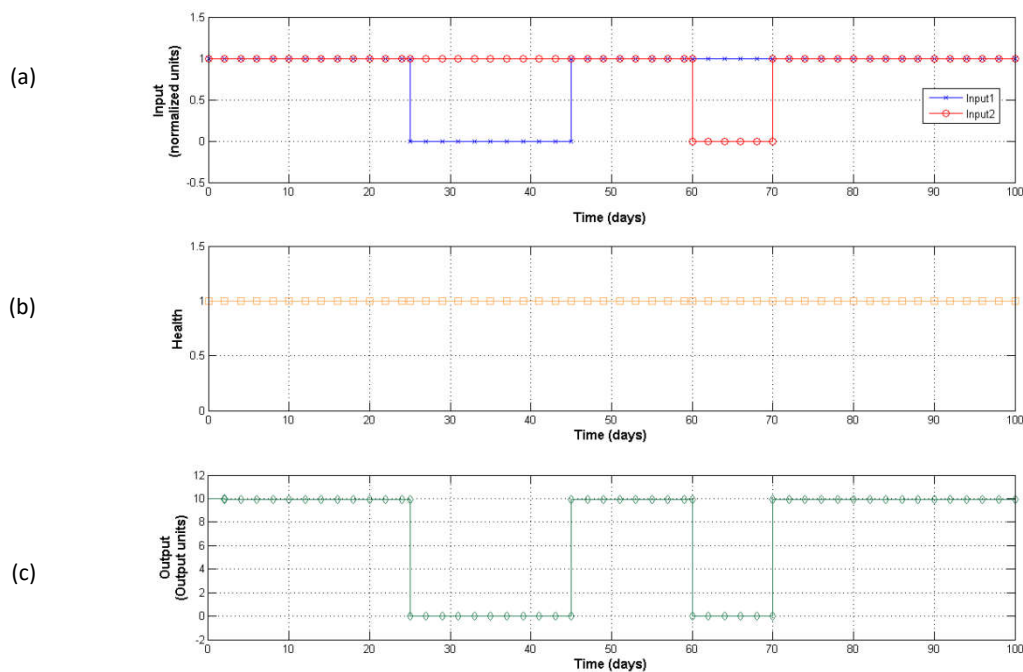


Fig. 15. Constant interval maintenance with a period of 20 days.

3.3.4. Modelling hybrid systems

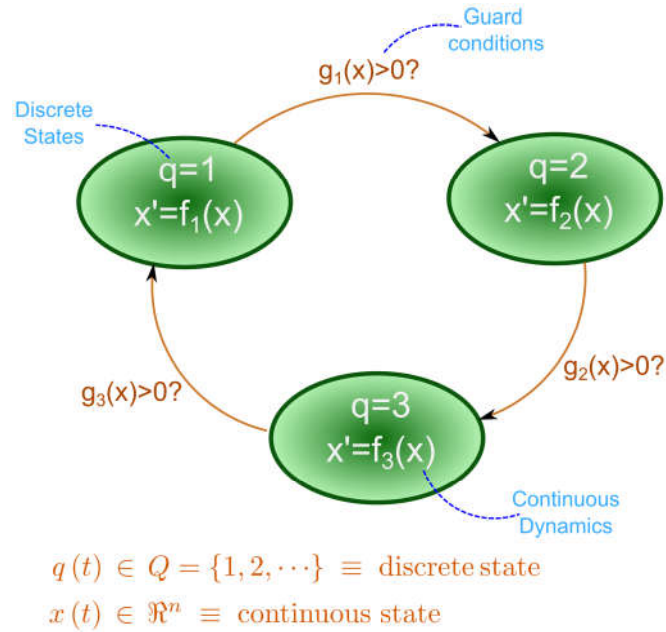


Fig. 16. Schematic of a Hybrid automaton.

Hybrid systems are those systems which include both continuous and discrete dynamics. These are useful in capturing a wide range of dynamic phenomena, which have instantaneous transitions in between smooth evolution of state variables. A schematic of a hybrid automaton is shown in Fig. 16. The green shapes represent the discrete states and the arrows represent the transition between them. These transitions are dictated by the guard conditions as shown on the arrows. When the system is in any of these states, the state variables dynamically evolve according to the dynamical equation which is specific to that state.

Fig. 17 shows a schematic for the evolution of a switched system's state variable over time. The blue curves represent the evolution of the state trajectory due to some underlying set of differential equations. The red circles and the associated arrows indicate the switching of the state from one point to the other due to some switching logic. The switching is assumed to be instantaneous, i.e. the underlying temporal dynamics of the discrete switching logic are not considered.

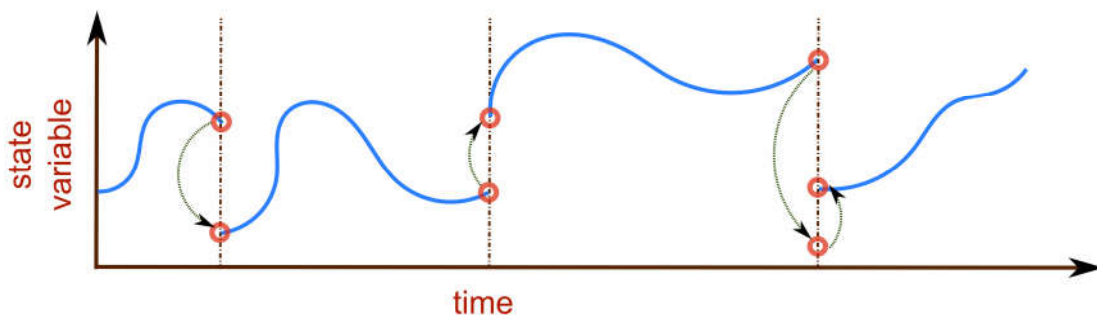


Fig. 17. Schematic of continuous evolution of state variables with discrete switching instants.

In the model presented here, the health of a risk item is modelled by a differential equation and coupled with the switching logic of the failure mode to form a hybrid system. A first order differential equation model of the health is considered as in Equation (12). The value of the constant k can be suitably chosen to fit some empirical data and it controls the rate at which the health deteriorates over time.

$$\frac{dH(t)}{dt} = -kH(t) \quad (12)$$

Fig. 18 shows a state chart schematic of the hybrid system in the failure mode block of a risk item. The encircled red portion is the code for the continuous time differential equation of the health variable. This coupling between the discrete and the continuous dynamics is achieved by declaring continuous variables in Matlab Stateflow (Matlab Inc. 2010) so that it tracks the derivative of the state. Also a zero crossing needs to be enabled, to accurately calculate the exact values of the variables at points of sudden discrete switching transition. This zero crossing method enables the solver to ‘go back in time’ in the event of a discrete transition and march forward in time again with smaller time steps, to capture the dynamics at the transition point.

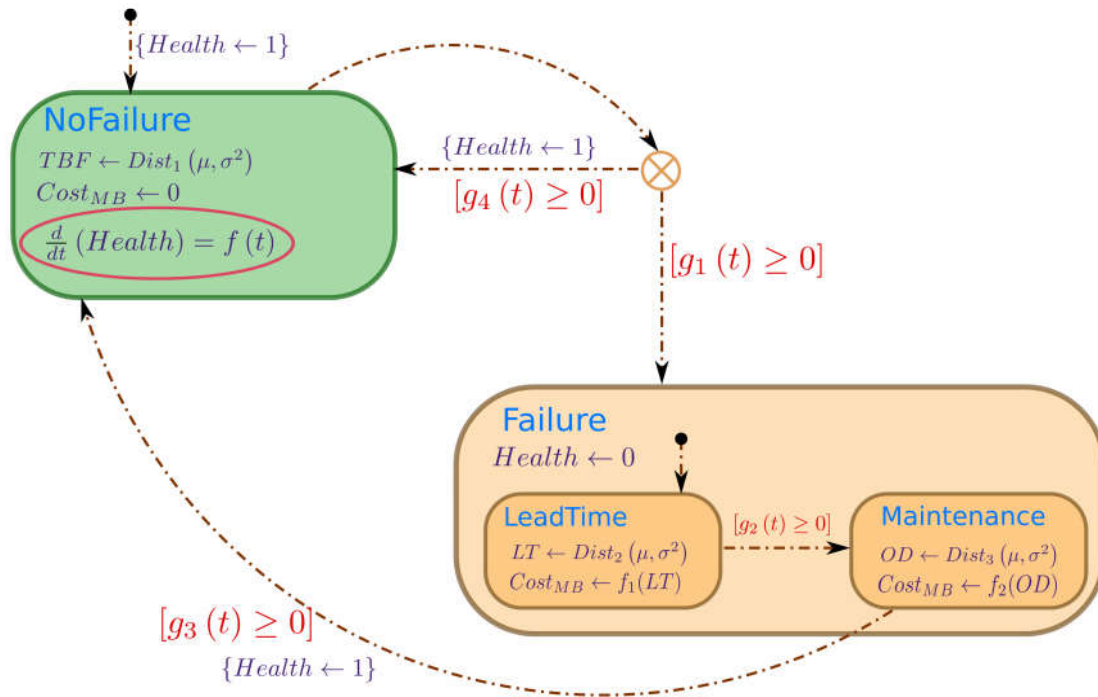


Fig. 18. State chart schematic of a hybrid system.

The additional function $g_4(t)$ is defined as

$$g_4(t) := T_{pm} - t_{state-entry} \quad (13)$$

where T_{pm} is the periodic maintenance time.

Fig. 19 and Fig. 20 show the simulation results with this kind of hybrid modelling. In Fig. 19 the maintenance interval is 50 days and in Fig. 20 the maintenance interval is 20 days. As can be seen, there are no inputs during two time intervals. During these two intervals the system output is zero. At the other time periods, the health decreases from its maximum value. Since the output is affected by the health, the nature of the output curve is also similar to that of a scaled version of the health curve. This in effect models the decreasing efficiency of the equipment over time. After maintenance, the health variable shoots up up instantaneously and then again follows Equation (12) to represent the degradation from the time after maintenance. In Fig. 19, there is a failure of the system between the 49th and the 67th day. However due to the shorter maintenance time in Fig. 20, the system does not go into the failure mode and its health is propped up after every maintenance. Hence, apart from the time that there is no input, the system produces some output depending on its health and the cumulative output quantity is higher than that in the previous simulation. This also matches qualitatively with our understanding that some sort of preventive maintenance at short regular intervals can check the loss of production from the equipment.

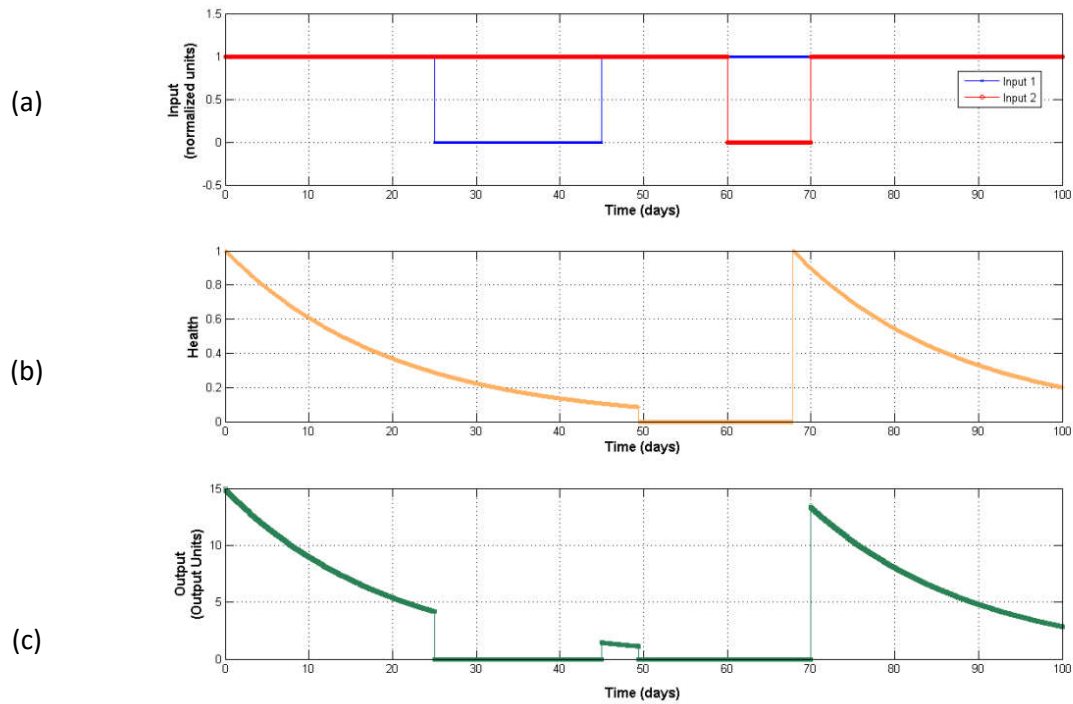


Fig. 19. Risk item with health modelled by differential equation and switching logic with long maintenance interval.

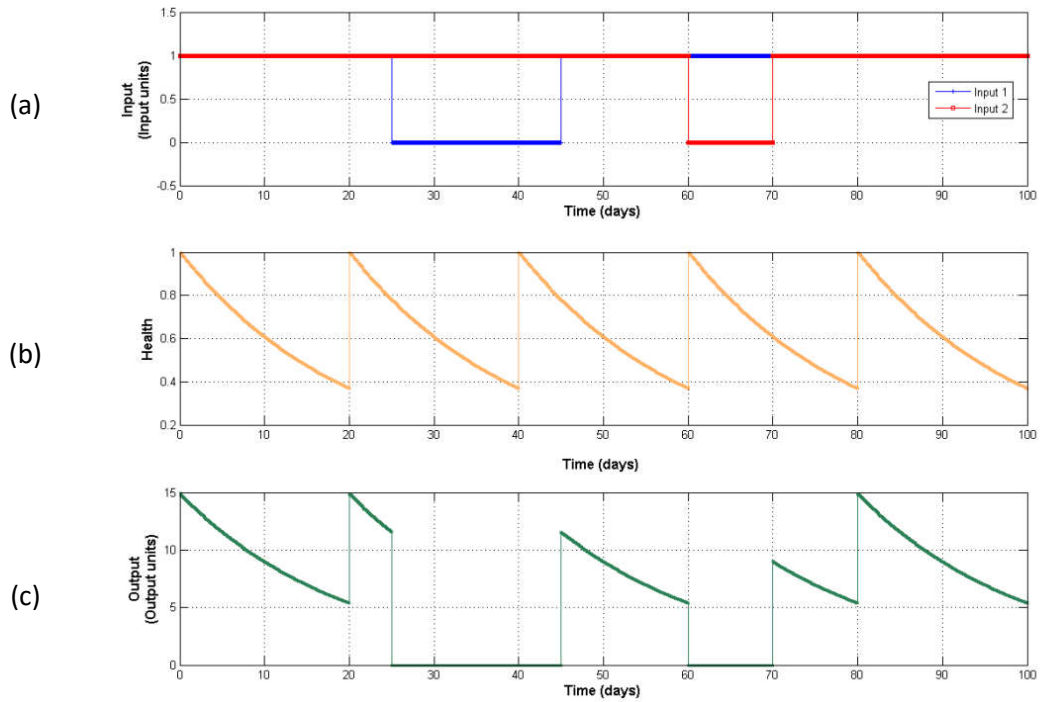


Fig. 20. Risk item with health modelled by differential equation and switching logic with long maintenance interval.

Fig. 19 and Fig. 20 assume that the health of the equipment decreases at a constant rate over a specified interval of time. This might be true in most cases. But in some cases, the decrease in health would be linked to whether the system has any input or not. To understand the practical setting of these two different cases, a couple of examples can be cited. An example of the former case might be a flotation cell which may always contain some material irrespective of whether it is in operation or not. Therefore it has to withstand the stress of the contained fluid even if there is no input. This might result in leakage of the unit even if it is not used actively for flotation operation for a prolonged period of time. The second case might be that of a mechanical crusher for example. The wear and tear of the crusher components can only occur if there is an input to process. Therefore, it is logical to conclude that the health of such a mechanical system would be linked to the availability of input. Fig. 21 shows the results of modelling an equipment governed by a health variable as discussed above. From Fig. 21 it is clear that the health remains constant during periods where there are no inputs and resumes from that value when the inputs are available. Fig. 22 shows a schematic of this implementation in state chart logic. A flag is used which triggers if there are no inputs to the system. This flag dictates the change of the health states when the NoFailure mode is active.

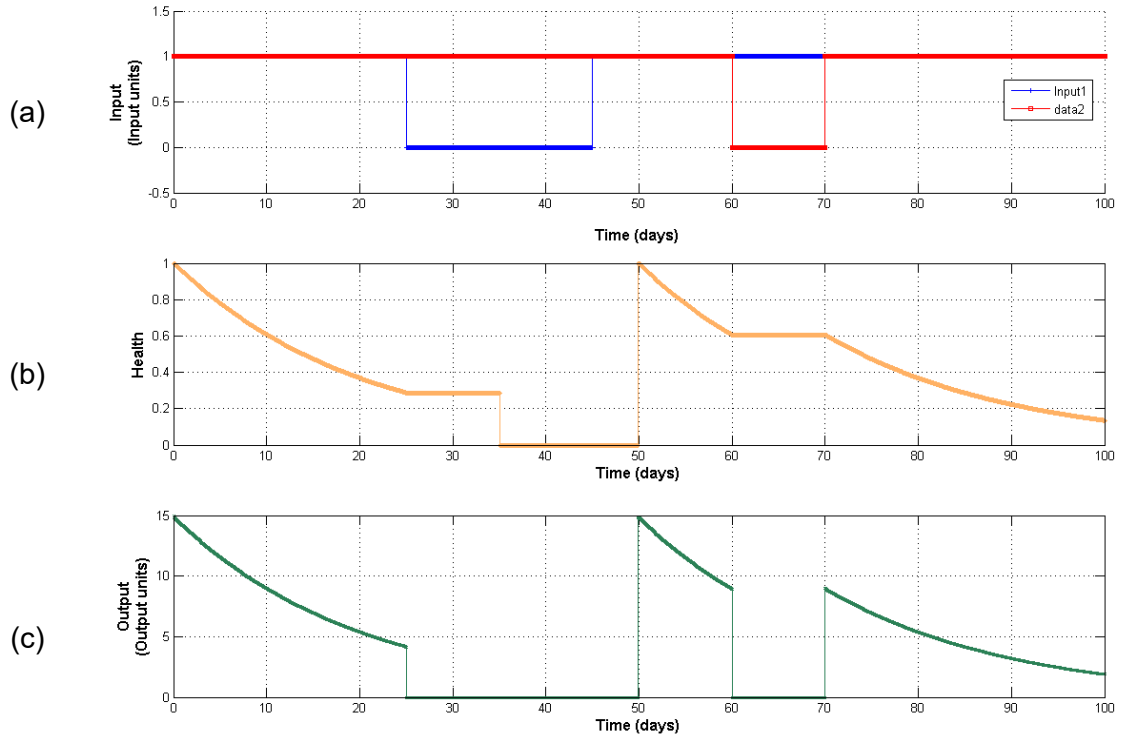


Fig. 21. Risk item with health modelled by differential equation which stays constant during periods of no input.

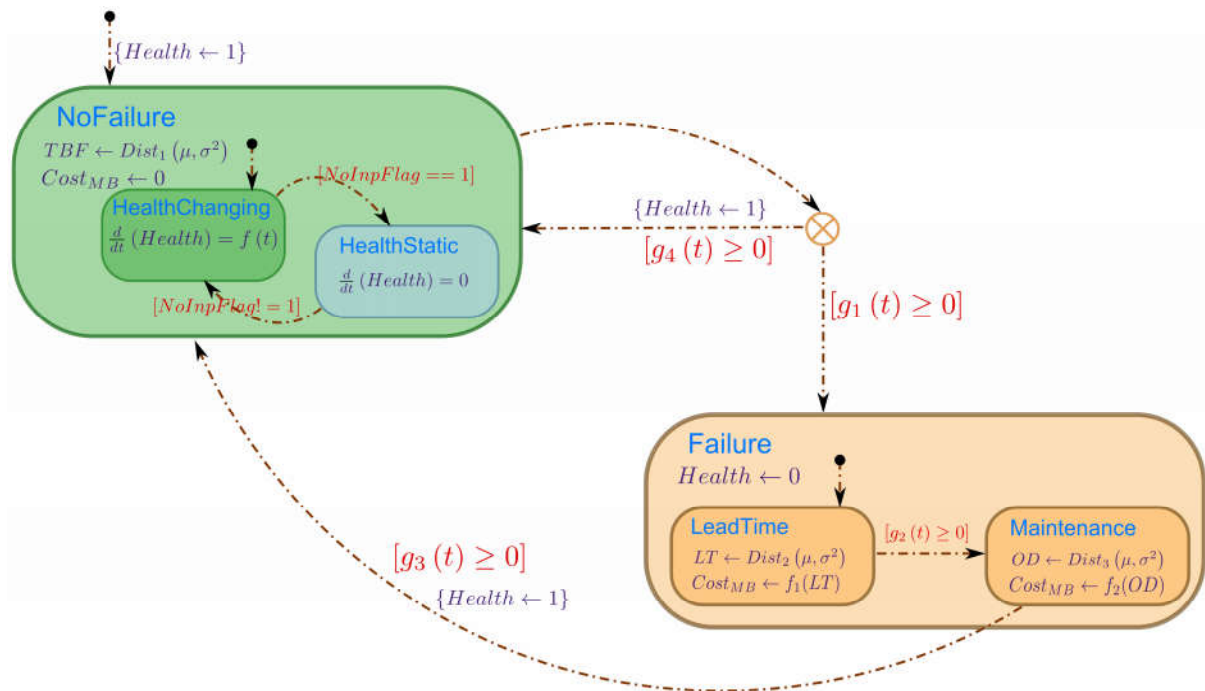


Fig. 22. Implementation of the above scheme in State chart based logic.

4. Application to the case of copper ore processing and metal production industry

Mineral processing and metal production are wide disciplines and numerous chemical processes are coupled in the conversion of the ore to the final product. Each of these chemical processes is specific to the particular mineral and hence there is no generic template for all the mineral processing operations.

4.1. Modelling of financial risks due to operational hazards in copper ore processing and metal production

Fig. 23 shows a sample schematic of a copper processing and metal production system. The flow of each component through the process is indicated by different coloured arrows and corresponding labels. Operations which have a similar nature or function are identified by similar coloured blocks in the figure. The red dotted lines encircling one or more blocks indicate that they are grouped together and are considered as a single composite unit for the risk model case study carried out in Matlab.

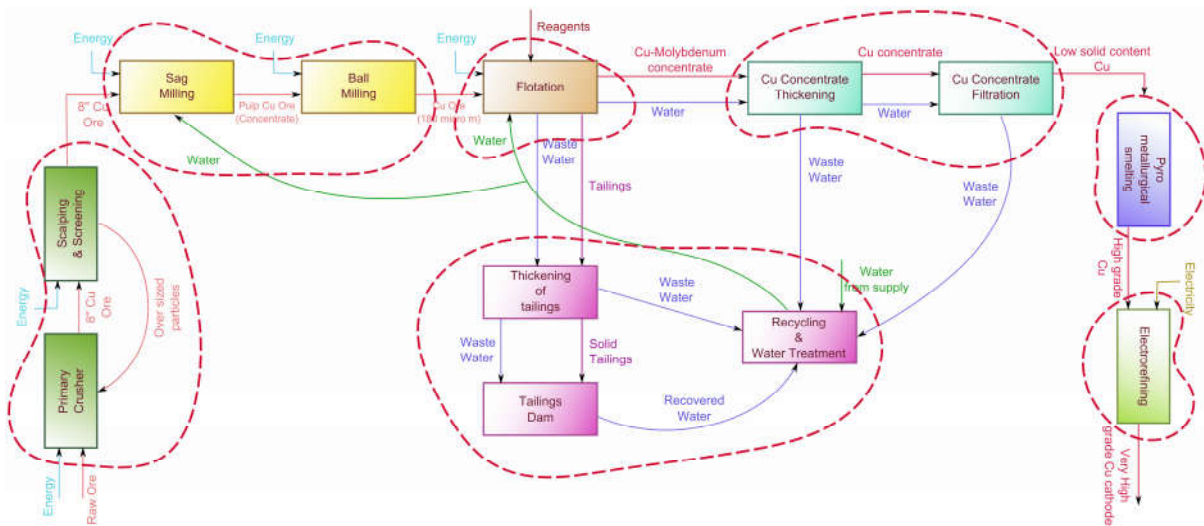


Fig. 23. Schematic for a copper ore processing and metal production system.

In the present simulation example, the financial losses only due to mechanical breakdowns of the copper processing and metal production operations are characterised. This is a simplified example where maintenance data from a processing and metal production operation is used. The information is suitably scaled and shifted to maintain anonymity.

4.1.1. Failure modes of the different components

There are different ways in which the individual components can fail thus leading to a disruption of the overall copper production process. Assuming that there is no redundancy scheme in the process flow, i.e. no backup machinery for individual components, it can be seen that the failure of one component would hamper the whole down-stream process. For the simulation model presented here, the various sub-systems of the copper production operation along with the possible failure modes and their statistical distributions are outlined in Table A 1. Most of the failures would disrupt only the downstream operations (i.e. all the downstream risk items would not have any input and they would be in the NoInput state as in Fig. 6). However, failure of the water pumping station would also affect other upstream

processes like grinding and flotation, which use the recycled water as input. This is automatically taken care of by the model structure and the corresponding processes go into the NoInput state as shown in Fig. 6.

These failure modes represent a broad class of failures lumped together and grouped under a specific category. In cases where the company knows that a certain critical component is more likely to fail, it would enforce a proper maintenance strategy in place so that the outages can be addressed in a short time or preventive action may be taken to decrease the possibility of an outage.

The failure data represents a wide class of smaller failures which disrupt operation of the particular equipment. Therefore, the maintenance team would have detailed data of all the faults that occurred, the time taken to clear the fault and the actual maintenance costs associated with it. For example, the failure mode of the mechanical breakdown of a crusher would include the breakdowns associated with the electric motors which drive the unit, the wear and tear of the mechanical equipment involved in the crushing operation, failure of the control system etc. All these failures eventually affect the output of the particular equipment and disrupt the operation. These are clubbed under one umbrella term representing the generic failure mode i.e. failure of the crushing operation. Table A 1 (in Appendix) shows the various failure modes with the corresponding fitted data of the TBF and OD probability curves. The TBF is fitted with an exponential probability distribution curve and the OD is fitted with a lognormal probability distribution. The data is shifted and scaled to maintain anonymity. The scaling is also done to discern the failures easily in the final output curves.

4.1.2. Matlab based risk model development for copper ore processing and metal production

A Matlab tool for operational risk modelling of the copper processing unit is developed using the proposed modelling methodology. The various subsystems are modelled as risk items with multiple failure modes. The distributions of the TBF, OD and MBCost are given in Table A 1. The state transition functions for the failure modes are taken as in Equations (1)-(3). The functions for $Cost_{MB}$ calculations are shown in Equations (7)-(8) and the function (Ψ) in the Health-Loading logic is as in Equation (9). The loading (Γ) is follows Equation (11) and the function Φ in Equation (10) is:

$$\Phi_i = LF_c \times Outp_{i_{Rated}} \quad \forall i \in \{1, 2, \dots, m\} \quad (14)$$

where LF_c is the composite load factor calculated as

$$LF_c = \prod_{j=1}^n \left(\frac{Inp_j}{Inp_{j_{Rated}}} \right) \quad (15)$$

The rationale for this is that, depending on the current value of the input variable at any specific time, a load factor is calculated as a fraction of the rated input values. The output is suitably multiplied by this load factor. Hence, if there is a lower value of the input than the rated value, then the output is proportionally lowered with respect to the input. The inputs

and outputs of each risk item with their corresponding rated values as used in the model are given in Table A 2.

4.1.3. System characterisation and simulation results

The system layout shown Fig. 23 and the data presented in Table A 1 are used to simulate the risks associated with the operation for a period of one year. Fig. 24 shows one realisation of the refined copper output. Since the original data is scaled up, the failures are magnified in the simulation and the plant remains operational for intermittent periods. Fig. 25 shows the time evolution of the cumulative maintenance and breakdown costs of the process operation. The first subplot in Fig. 25 shows the total cumulative cost over time and the second sub plot shows the time evolution of the cumulative costs of each failure mode. A comparison between Fig. 24 and Fig. 25 shows that during the periods where there is output of copper from the system, the maintenance and breakdown curves are flat. At times when there is no output, some components of the MB costs associated with some failure mode increases. The other failure modes are not active during those time instants. The curve of the total cumulative costs also increases every time there is a failure (due to the failure modes that are active during the corresponding period of time). This agrees with the intuitive understanding of the problem. It is also shown that no MB costs are associated with periods when the system is running smoothly, whereas MB costs are incurred during breakdowns. Another important consideration in this model is that there are no parallel processes with redundancy schemes or storage elements in the system. The consideration of parallel processes would make the system modular and its output could take discrete values in between the maximum and the minimum range. This is because even if one component of the parallel path fails, others can process a part of the input and produce a lower level of output, instead of the plant coming to a complete standstill. In the next section, redundancy schemes and storage elements are introduced and the system is optimised for performance.

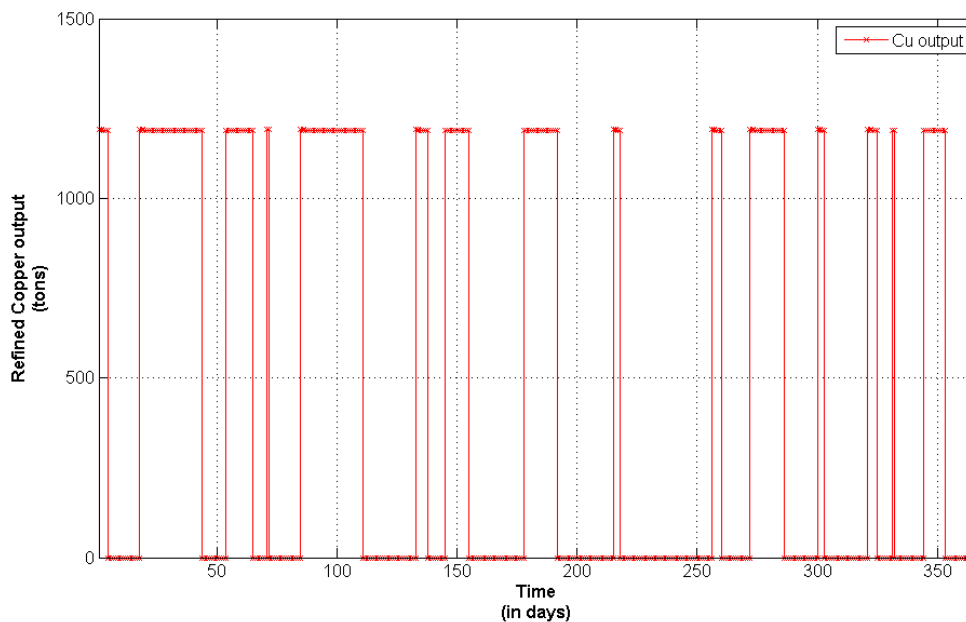


Fig. 24. Refined copper produced over a period of 1 year.

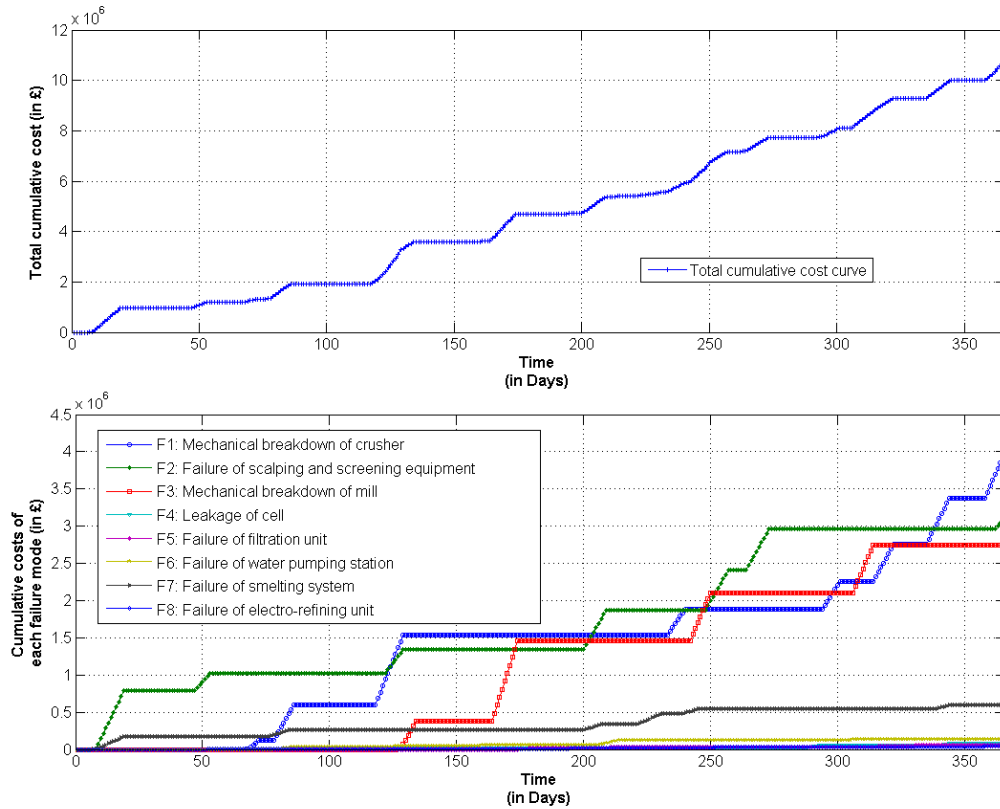


Fig. 25. Total cumulative costs due to maintenance and breakdown and its individual components over time.

Another consideration in the model is about the event start and end points. A breakdown event can occur which starts near the end of the year and carries over to the next year. It can even be that the system is in a state of partial breakdown at the start of the year and recovers fully during the year. For the former, the model calculates the costs associated with the event till the end of the year, i.e. the period of interest. For the latter, the model assumes that the system is in a fully healthy state and none of the components have failed at the start of the year. However these can easily be changed to include other scenarios depending on the specific problem at hand.

The system is run in a Monte Carlo (MC) fashion for a hundred times and the total cumulative cost due to maintenance and breakdown is plotted over time. Since each of the individual failure modes fail at different times, the state trajectories of these curves are different. Also as is evident from Fig. 26 there is a large variance of the total cumulative cost at the end of one year.

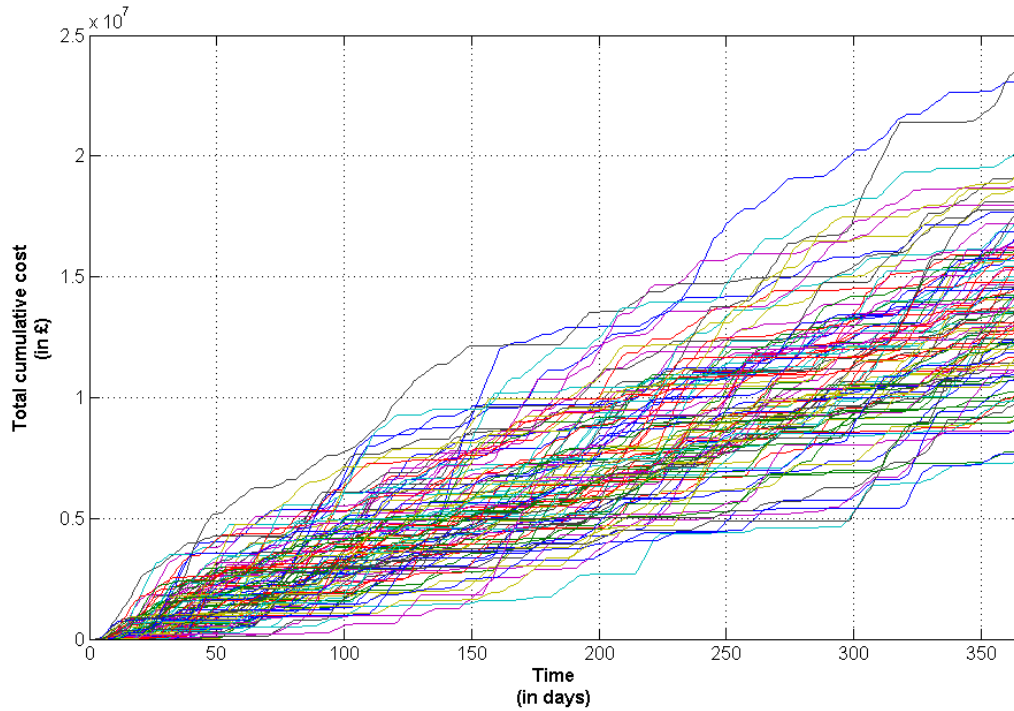


Fig. 26. One hundred Monte Carlo runs of the evolution of total cost over time.

To characterise the system effectively, one thousand MC simulations are carried out and histograms of the total copper output and the total MB costs are plotted in Fig. 27. The mean (μ) and variance (σ) of the data from the two histograms are plotted in Table 1. The histograms show that the curves have a slightly lingering tail towards the right. This indicates that there are a few rare incidents where the MB costs incurred are very large. However, in most cases the costs lie within a certain specified band near the centre of the histogram.

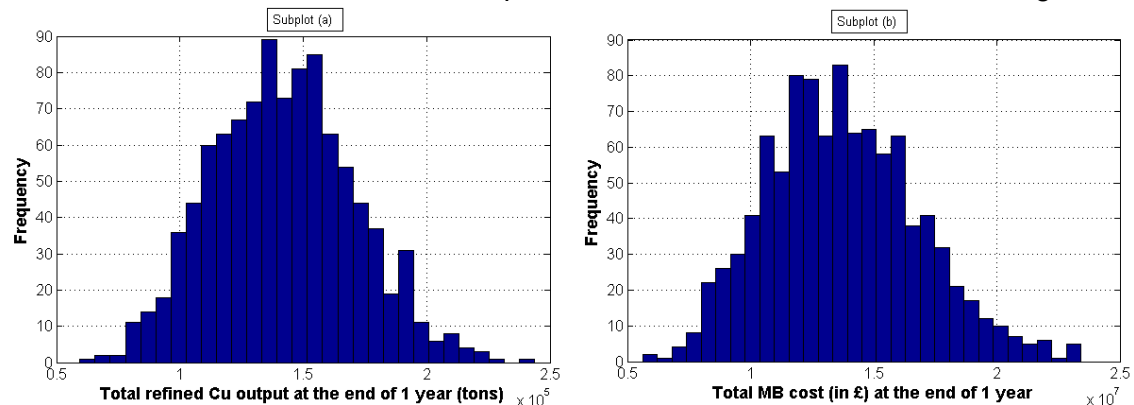


Fig. 27. Histogram of the a) total refined Cu output and b) the total MB cost at the end of 1 year for 1,000 Monte Carlo simulations.

Table 1: Mean and variance of the important outputs from the model

Item	μ	σ
Copper Output (tonnes)	141,856.5	28,881.7
Total MB cost (£)	13,714,990	3,086,199

The information from these simulations can help the management to allocate a certain amount of fund each year to cover these losses. The concepts of Value at Risk and Tail Value at Risk, as discussed previously, can be used here to identify possible risk mitigation mechanisms.

5. System Optimisation with multi-objective evolutionary algorithms

The advantage of abstracting the process knowledge in the framework of such a tool is that it can be used not only to characterise the system, but also to optimise the system and decide the appropriate policies to employ for risk mitigation. The optimisation problem might be posed in many different ways. One way might be to look at an optimal maintenance schedule for all the equipment, so that there is least amount of downtime of the system. The same problem can be framed in a multi-objective fashion as well with two or more conflicting objectives. Frequent maintenance schedule might make implementation financially prohibitive or labour intensive. So, a trade-off can be made between the associated costs and the benefits achieved thorough such implementation. This can be done though multi-objective optimisation. Another way to frame the optimisation problem is to look for the effect of redundancy schemes on the system throughput. It is obvious that keeping backup equipment would ensure less system downtime. But the key question is to identify the number and type of equipment and more importantly to quantify the effect of reduced downtime versus the initial capital costs for installation. The system can also be simulated to look at how long it takes to recover the initial capital investments due to the improvements it offers and the resulting steady supply of production volume. A third way to frame the optimisation problem is to have some form of stockpiles at each intermediate stage, so that even if some operation fails in the process chain, the downstream production can continue unaffected for a short time until the upstream processes resume again. There is of course a trade-off between the capital investments required in such a case versus the improvement in reliability achieved by putting this equipment in place. A more complicated optimisation problem might be framed by looking at the effect of all three options coupled together.

Intelligent evolutionary or swarm based algorithms are useful for optimising these kinds of models. It is difficult to incorporate the standard methods of convex optimisation or MINLP (Mixed Integer Non Linear Programming) due to the inclusion of finite state machines, stochastic time dependent phenomena and other constraints which might be non-convex. It is therefore expedient to use intelligent techniques like genetic algorithms, differential evolution, particle swarm optimisation etc. in such cases. In the following sections, the risk model of the copper processing and metal production system is optimised using Multi-Objective Genetic Algorithms (MOGA).

There are many multi-objective evolutionary algorithm methods like the Non-dominated Sorting Genetic Algorithm (NSGA) (Deb et al., 2002), Niched-Pareto Genetic Algorithm (NPGA) (Horn et al., 1994), Pareto Archived Evolution Strategy (PAES) (Knowles and Corne, 2000), Strength Pareto Evolutionary Algorithm (SPEA) (Zitzler et al., 1998) MOEA/D (Q. Zhang and Li, 2007) etc. The generic principles of all these algorithms are the same in that they use the concepts of selection, crossover and mutation to evolve future generations. They differ mainly in the way the ranking of the different individuals are done and the nature of selection pressure that is applied for evolving newer generations. Each algorithm has its own advantages and disadvantages. The NSGA II algorithm (Deb et al., 2002) is one of the

popular ones which work in a wide range of scenarios. It is also employed for simulation in this paper.

In some optimisation problems, like the case considered here, the objective function does not provide a unique value at a particular point. Every time it is evaluated, a slightly different value comes up due to the stochastic nature of the underlying problem. These kinds of problems cannot be handled by conventional gradient descent methods which rely on derivative information at a point. However evolutionary algorithms are expedient in optimising these kinds of functions. In evolutionary computation literature these kinds of problems are known as noisy or uncertain function optimisation problems (Jin and Branke, 2005). There are many methods that offer improvements over existing evolutionary algorithms (EAs) to deal with noisy or uncertain function optimisation problems (Beyer, 2000). The simplest technique is to evaluate the function multiple times at the same point and calculate the expected value of the objective function at that point. The EA ranks the solutions based on this expected value and subsequently performs selection, crossover and mutation. The self-averaging nature of EAs (Tsutsui and Ghosh, 1997) helps in the convergence of solutions, i.e. the solution vectors which have good fitness values survive through the generations and remain in subsequent populations. However, the convergence of the algorithm is more time consuming than the simple GA. This method is adopted in the present simulation for its simplicity.

5.1. System description for optimisation and performance objectives

The basic process flow of the copper processing and metal production system as described in Section 4.1 is taken and a few modifications are made to it for process optimisation. It is noted from the simulations and also from the system data in Table A 1, that the failures associated with the crushing, scalping and screening processes are much higher (since the TBF multiplication factors are much lower). Hence to mitigate the effect of the failures to some extent, a backup system with the same configuration and failure properties is introduced in the system. This acts as a standby and allows the system to be online in the case of failure of one of the units. This is implemented in the model in a similar way as described in Section 3.3.2. Along with the redundancy scheme, two stockpiles are introduced in the process flow. One of them is introduced before the SAG and ball milling process and the other one is introduced before the pyro-metallurgical smelting unit. This is done from process knowledge. The rationale behind stockpiling before the SAG grinding operation is to keep the processing plant running at a specified feed rate in case there is variability in the production of the ores in the upstream mining process. This also helps to safeguard against the failures associated with the grinding process. In most cases, the output of the thickening and filtration unit in a concentrator is shipped to a distant pyro-metallurgical smelter. Therefore, the rationale behind putting another stockpile before the pyro-metallurgical unit is that it might represent material brought from other locations as well. Also, it is desired to keep the smelting unit in constant operation throughout and a storage unit of the feed material helps in the smooth operation of the system.

In the present simulation study, the idea is to use an optimisation algorithm and find out the values of the capacities of the stockpiles at each point in the process flow chain. The choice of the number of redundant units and the location of the stockpiles could also be taken as decision variables in the optimisation algorithm and optimised together with the stockpiling capacities. But in cases where process knowledge exists, it is wiser to design

some parts of the system by taking the practical design constraints into consideration. This is because there are a lot of factors in terms of ease of use and practicability which can be easily deduced by intuitive judgement but is hard to frame down in terms of mathematical equations.

It is evident that a higher capacity stockpile would give the system high reliability since in case of upstream failures, the stockpile would supply the required materials to keep the operation running. However, the cost associated with the land requirement of a stockpile and the maintenance of the same shoots up in proportion to its capacity. Hence, a judicious trade-off is required to find out the optimal values. This is achieved by framing the problem as a multi-objective optimisation one and solving it using MOGA.

Also, in the present case the upstream production from the mine is considered to be constant for a period of 1 year, which is also the period under study. Therefore, there is no provision of replenishing the stockpiles in the case they get exhausted. This in turn implies that the initial capacity of the stockpile at the beginning of the year must be sufficient enough to take care of the losses in production during that year. In a more practical scenario, the process systems manager can ask the mine to ramp up the production to a certain level for a few weeks to reach the capacities of the stockpiles, once they are empty after supplying to the process subsequent to a breakdown. This would result in a lower capacity requirement for the stockpile and thus lower allocation of capital for the same. This, however, requires heuristic inputs from the management and is also constrained by mine planning, the operating life of the mine, labour availability etc. and hence is not included in the present simulation scenario.

For system optimisation the following two cost functions are considered:

$$J_1 = \int_{t=0}^{365} (P_r - P_a) dt \quad (16)$$

$$J_2 = \int_{t=0}^{365} (MB_{tot}) dt + \sum_{i=1}^N H_{\text{tank}_i} \quad (17)$$

where, P_r is the rated production per day, P_a is the actual production per day, MB_{tot} is the total maintenance and breakdown cost associated with the whole operation per day. The first objective function J_1 in Equation (16), represents the loss of production due to the various failures in the system. The second objective function J_2 in Equation (17) represents the total cumulative cost due to maintenance and breakdown along with the holding and capital costs associated with the stockpiles. N represents the maximum number of stockpiles. This is proportional to the size of the tanks. The values of these are calculated as in Equations (18) and (19). It is obvious that due to the underlying stochastic dynamics of the different failure modes, both J_1 and J_2 would evaluate to different values each time and, hence, a boot strapping method is required to calculate the expected values of these integrals.

$$H_{\text{tank}1} = 10000 + 50 * Cap_{\text{tank}1} \quad (18)$$

$$H_{\text{tank2}} = 15000 + 80 * Cap_{\text{tank2}} \quad (19)$$

where Cap_{tank1} and Cap_{tank2} are the capacities of tanks (stockpiles) 1 and 2 in tons respectively.

5.2. Optimisation results and discussions

The system is optimised through the NSGA II algorithm which is discussed previously. The number of individuals in each population is taken as 30 and the NSGA II algorithm is run for 50 generations. The expected value of Equations (16) and (17) is calculated by evaluating the objective functions for 10 times at each point and averaging the result. A penalty function method is adopted for solutions which fall in the infeasible regions of the search space. A tournament selection is adopted with a tournament size of 2. An intermediate crossover function is chosen which creates children by random weighted average of the parent genes. The crossover fraction is chosen as 0.8. A Gaussian mutation function is chosen for the mutation operation. The Pareto front population fraction is chosen as 0.7 of the total population. The range of optimisation variables (tank capacities) are taken as $[40000, 400000]$ and $[5000, 70000]$ for Tank1 and Tank2 respectively. The NSGA II implementation of Matlab's optimisation toolbox is used in the present study. The risk models developed in Simulink are coupled with the optimiser using Matlab scripts. Fig. 28 shows the solutions in the final generation of the algorithm. The blue points represent the dominated solutions and the red ones indicate the non-dominated or the Pareto solutions. The Pareto solutions indicate the best trade-off that has been found by the NSGA II algorithm. A reduction in one of the objective functions would invariably result in an increase in the other objective function. From the Pareto front, three representative solutions are chosen and their function values along with the optimisation variables (capacities of the stockpile) are reported in Table 2.

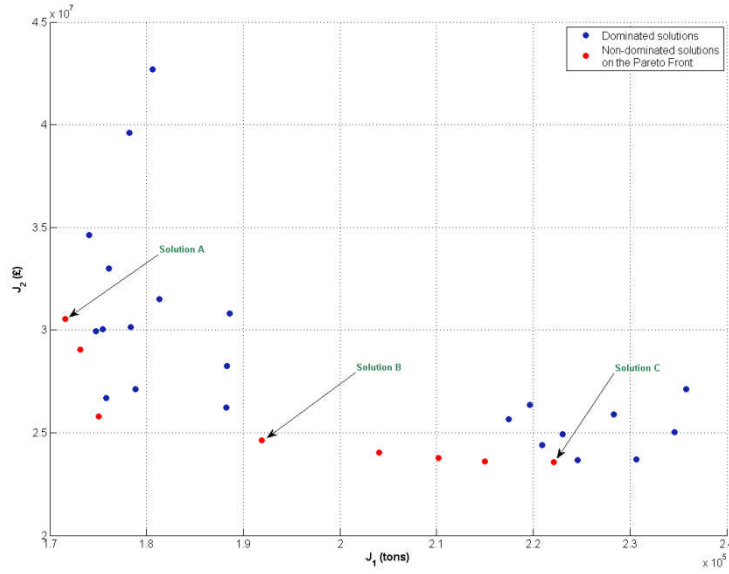


Fig. 28. Solutions in the final generation of MOGA showing the Pareto front.

Table 2: Representative solutions on the Pareto front.

Solution	J1 (£)	J2 (tonnes)	Stockpile 2 capacity (tonnes)	Stockpile 1 capacity (tonnes)
A	171,574	30,548,128	54,532	123,774
B	191,906	24,637,750	31,785	54,981
C	222,106	23,579,387	10,000	40,002

The system is simulated with these values of stockpile capacities and the cumulative losses in Cu production and the cumulative MB and other costs are plotted in Fig. 29 and Fig. 30 respectively. The values of J_1 and J_2 are the final values of these curves at the end of one year. It is to be noted that since the system is stochastic in nature, the same value of J_1 and J_2 is not obtained in this independent simulation run. However, the general trend and the nature of the solutions are preserved. For example, in Fig. 29, the total cumulative losses for copper production is the smallest for Solution A, while it is the largest for Solution C. Solution B lies in between these two extremes. However this comes at a cost. In Fig. 30 this is exemplified where it can be seen that the Solution A has the highest cumulative costs for a year while Solution C has the lowest. Solution B again lies in between these two extremes. Therefore, this clearly illustrates the trade-off between reliability and escalating costs of the system.

Another noticeable effect is that the cost curves start from different initial values in Fig. 30 for the different representative solutions. This is explained by the fact that the tank capacities for Solution A are higher than that of solutions B and C as can be seen from Table 2. The initial

values in Fig. 30 represent the total capital costs and running costs which increase in proportion with the size of the storage unit. Another observation from Fig. 29 is that from the start of the simulation time, the solution A does not incur any cumulative losses in production for a large period of time. This trend is followed by B and C for shorter periods of time. This is due to the fact that the capacities of the stockpiles in Solution A are higher than those of Solutions B and C. Therefore, any shortfall in production is supplied by the stockpiles as long as there is material available in it. In this simulation no provision is made for replenishing the stockpiles during the period of operation of the whole year. Hence, the stockpiles become ineffective after they run out of material to supply.

The two figures, Fig. 31 and Fig. 32 illustrate this point more effectively. Fig. 31 shows the production of copper metal over time for the three cases. Fig. 32 shows how the level of materials in one of the stockpiles fall with time, as it tries to keep up the rated supply when there is a failure. In Fig. 32, the initial levels are different with Solution A having the highest due to largest capacity and Solution C having the lowest.

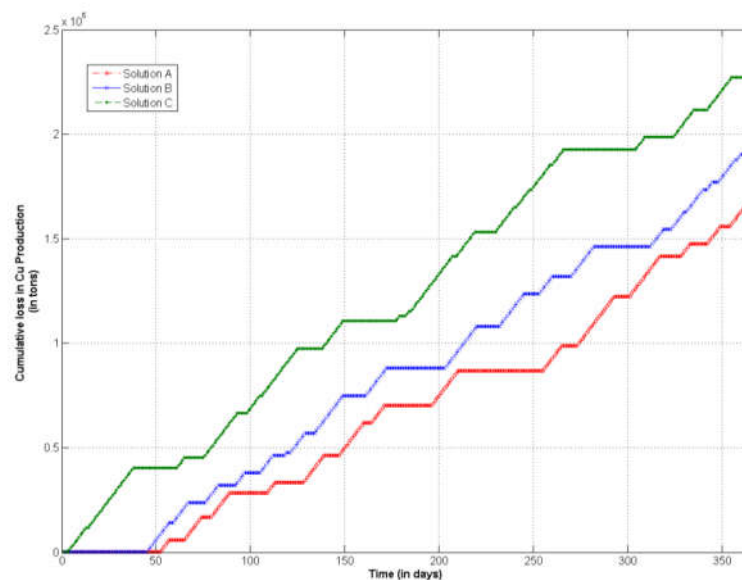


Fig. 29. Cumulative losses in copper metal production for the three representative solutions on the Pareto front.

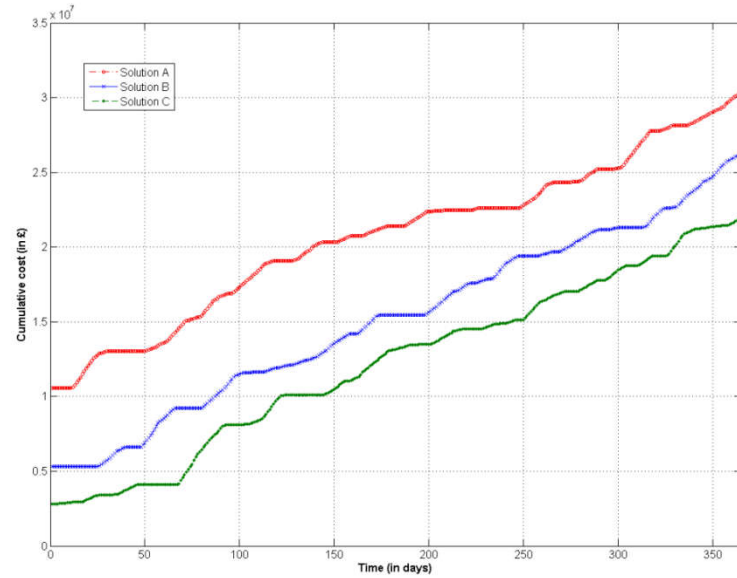


Fig. 30. Cumulative costs for the three representative solutions on the Pareto front.

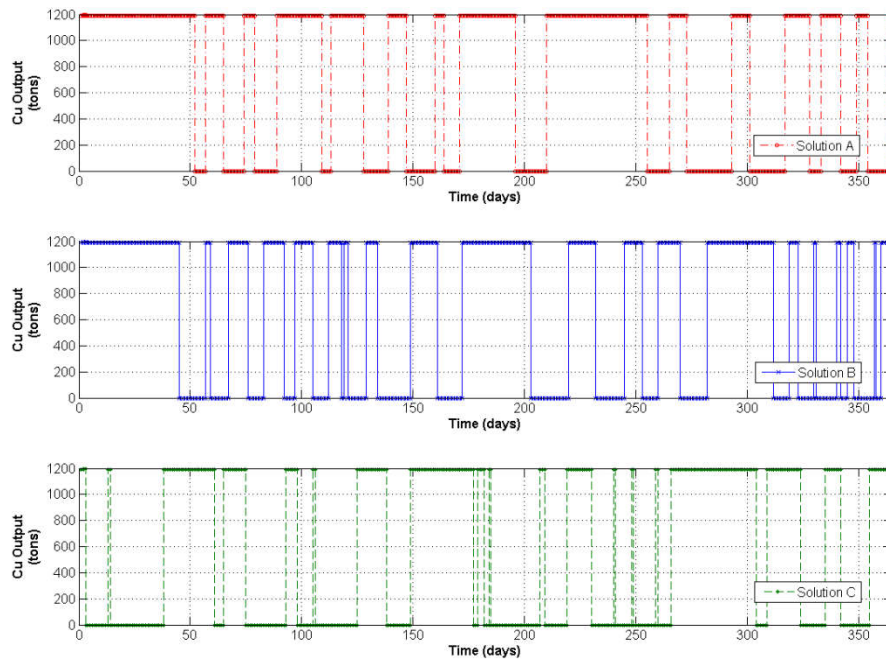


Fig. 31. Production of copper metal over time for the three representative Pareto solutions.

In Fig. 31 another noticeable effect is that there is a steady output for a little more than 50 days for Solution A and a little less than 50 days for Solution B. This implies that the stockpiles supply the rated output during that period. This is also clear from Fig. 32 where the level of the tanks fall during the initial period. For Solution C, there seems to be an anomaly, that there are periods of failure in the initial days but the level of the stockpiles does not fall during that period to provide for the failure. This is not an anomaly, as the stockpiles are situated in-between the process chain and cater to the upstream failures only and not the downstream ones. This implies that the failures have occurred in the

downstream components of the stockpile, i.e. in the pyro-metallurgical smelter or the electro refining equipment.

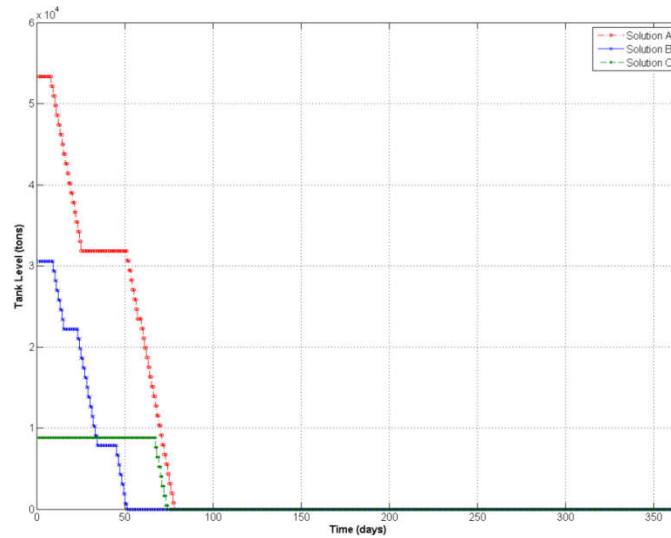


Fig. 32. Amount of materials over time in the stockpile before the smelter.

It can be intuitively understood that if a stockpile is kept at the end of the process chain, after the electro refining unit, then it would have the maximum effect on maintaining a rated product output. However, the material at this stage is already marketable and it does not make much financial sense to keep reserves of already finished goods in anticipation of a loss in production. Therefore, this is not considered in the present study. Any in-between storage units would not be able to exert any control over downstream failures, as also exemplified in this case.

6. Relative merits and de-merits of the modelling paradigm

The ability of the model to handle incomplete information in the process chain is especially useful from a practical standpoint. The systems level model also gives flexibility to the designer, to refine models in places where necessary, and use coarser top level models where appropriate. The modelling philosophy is not limited to any particular class of systems and can be used to model risks for other large scale systems as well. Unlike other risk modelling methods presented by contemporary researchers, this is a new and different view of risk modelling, which is accomplished by abstracting all the process system components in a systems level modelling framework. The user should be aware of both the advantages and the short comings of the model to get a better insight into the risk profile for his particular application domain. The next few paragraphs provide a summary of these.

One of the advantages of using this method is that it can be easily extended to incorporate a hybrid systems based modelling (where both continuous and discrete dynamics co-exist). In the present simulation study this is illustrated by considering a differential equation model for the health of a system. However, this need not be limited to the abstractions of the model only. It might well extend to modelling the physical dynamics of the process equipment. These might be useful in circumstances where a more detailed modelling of one of the components is desired and the other items can be modelled at a

much coarser level of abstraction. For example, the smelter is very expensive and it generally does not have a standby. On the other hand, the grinding mills are comparatively less expensive and might have a standby. Therefore, the failures associated with the smelter would act as a bottleneck and disrupt the production schedule. In such cases it might be advantageous to capture the dynamics of the smelter using underlying physics based differential equations and integrate them into the risk model in this kind of systems framework. This also opens up possibilities of using Dynamic Risk Assessment methodologies as proposed in recent literatures (Podofillini and Dang, 2012; Podofillini et al., 2010).

A few important pitfalls with the use of hybrid systems are that, under some conditions, they might not be well posed and might have issues with uniqueness and existence of solutions (Goebel et al., 2009). Hybrid verification techniques are often used to track whether the system goes into unsafe states during execution (Tomlin et al., 2003). Another issue with hybrid systems is the problem of zeno executions (Goebel et al., 2009). This undesirable phenomenon occurs due to the system having infinitely many discrete transitions in a finite time (J. Zhang et al., 2001). Zeno hybrid systems bring in imprecision in the simulation procedure and make the computation time consuming (J. Zhang et al., 2001).

Special mention needs to be made for integrating the effects of catastrophic events in the model. Though the model is able to handle the probability distribution of a catastrophic risk and affect multiple risk items at once, there exist some computational difficulties in this process. This is mainly due to the fact that these are extremely rare events and the probability would be in the order of one in a million. Since this is a simulation based method, getting an appropriate statistic for these extreme failures would require a large number of Monte-Carlo (MC) simulation runs which would be computationally prohibitive. This problem can be circumvented in two ways. One is to use the present philosophy and parallelize the code so that each MC run can be evaluated independently. This would effectively reduce the simulation time to manageable limits if run on a high performance computing cluster. The other is to use a combination of analytical or semi-analytical methods with this simulation based method for predicting the times of occurrence of these events based on prior historical data.

The optimisation problem as in the present paper, where the objective function is computationally expensive and stochastic (i.e. different objective function values are obtained in multiple runs with the same input decision variables) makes it almost intractable due to huge computational requirements. A sample size greater than ten would be required in each function evaluation, to properly approximate the expected value of the resulting distribution. This would be a more critical issue if the underlying reliability parameters (outage durations and time between failures) of each component have heavy tailed distributions.

One way to address such problems is to use surrogate models or proxies in two stages - one surrogate for approximating the distribution of the objective function in each evaluation and another for approximating the objective function itself, as we have pursued in our recent papers (Babaei et al., 2015a, 2015b). Both these surrogate models are dynamically updated during the optimisation process for better fidelity of the surrogates.

However, using such proxies are not expedient in cases where the distribution to be approximated is not uni-modal and has multiple peaks. We have described the issue in (Babaei et. al., 2015a) where we found that simple Monte Carlo simulation was better at approximating such distributions as opposed to proxies like polynomial chaos expansions, non-intrusive spectral projections etc. and finding efficient solution techniques for such optimisation problems is still an open question. Hence, choosing the best method of optimisation for such problems depend a lot on the objective function itself (i.e. its underlying non-linearities, nature of the stochastic distributions etc.) and varies for each individual case.

Such intricate optimisation schemes have not been used in the present paper and the standard multi-objective GA has been resorted to, since the motive is to show that such developed risk models can be easily cast in an optimisation framework and different trade-off designs might be constructed which would help decision makers. Of course, application of MOGA on this problem requires huge computational resources and therefore it was required to limit the number of expensive function evaluations to complete it in a realistic time frame. Therefore, the reported solution suffers from convergence issues. Nevertheless, since the focus of the optimisation section (in 5.2) is to compare the solutions at different regions in the Pareto front and not the absolute values of $J1$ and $J2$ themselves, it is believed that this does not hinder the qualitative discussions and the consequent insights detailed in the paper.

In order to take into account the failures due to human behaviour, management decisions, environmental risks etc. a composite risk index needs to be developed. It is recognised that the present model needs to be augmented with these risks to arrive at a better position for forecasting the failure risks. Some methodologies to quantify these risk indices are exemplified in (Abhulimen, 2009; Reniers et al., 2011). In Abhulimen (2009) various hazard data have been classified into fuzzy sets which correspond to the failure outcomes of the risk components. A Monte-Carlo and Markov chain algorithm is then used for training and simulation to give a weighted risk index. In Straub (2005), a Bayesian network model has been implemented for risk assessment of natural hazards. In such a framework the geological and metrological parameters for evaluating catastrophic environmental risks can be given as input. A risk rating system can then be developed and the causal connections can be modelled as edges between the different nodes of the Bayesian network. After appropriate training, the output of the network can be used to predict the composite risk due to these environmental factors. This can be extended for the risks related to human behaviour and management decisions as well. Future work can focus on the integration of these methods with the reliability based models that have already been developed.

7. Conclusions

In this paper a novel risk modelling methodology has been proposed for large scale engineering process systems. It employs a systems approach towards modelling and is generic enough to be coupled with other modelling techniques. A few customisations along the lines of the original modelling paradigm have been proposed whereby other paradigms like hybrid systems, evolutionary optimisation etc. can be coupled to the basic risk model. A Matlab based implementation of the concepts has been illustrated. This might serve as a starting point for those who want to apply this methodology to other applications or customise the model themselves for their own use. Further extensions can be done on the

same lines and the model can be coupled to Bayesian networks, Markov chains, hidden Markov models, probabilistic neural networks or similar techniques to predict environmental or management risks for example. A numerical simulation has been presented to elucidate the validity of the present modelling approach.

Appendix

Table A 1: Shifted, scaled and anonymised data of a mineral processing operation, used for the simulation.

Components	Failure modes						
	Name	TBF (exponential) ($Dist_1$)		OD (lognormal) ($Dist_3$)			$MB_{\text{costfactor}_{OD}}$
		Multiplication factor (days)	μ	Multiplication factor (days)	μ	σ	(£ / day)
Crushing, Scalping & Screening process	Mechanical breakdown of crusher (F1)	43	0.75	5	0.13	0.24	8179
	Failure of scalping and screening equipment (F2)	32	1.21	8	0.18	0.29	6728
Sag & ball milling	Mechanical breakdown of mill (F3)	81	0.8	6	0.36	0.31	10872
Flotation	Leakage of cell (F4)	64	1.3	4	0.11	0.16	1190
Thickening and filtration unit	Failure of filtration unit (F5)	87	1.16	3	0.14	0.22	912
Recycling, Storage & water treatment	Failure of water pumping station (F6)	96	1.45	5	0.15	0.43	807
Pyro metallurgical smelting	Failure of smelting system (F7)	73	0.7	4	0.59	0.3	1537
Electro refining	Failure of electro-refining unit (F8)	110	0.53	3	0.05	0.2	534

Table A 2: Inputs and outputs of each risk item with their corresponding rated values.

Components	Inputs	Rated Inputs	Outputs	Rated Outputs
Crushing, Scalping & Screening process	Energy (kWh)	17,200	8" Cu Ore (tonnes/day)	40,000
	Raw Ore (tons/day)	40,000		
Sag & ball milling	Energy (kWh)	865,200	180 µm Cu Ore (tonnes/day)	40,000
	8" Cu Ore (tonnes/day)	40,000		
	Recycled water (tonnes/day)	16,800		
Floatation	Energy (kWh)	64,400	Cu Concentrate (tons/day)	1,196
	180 µm Cu Cu Ore (tonnes/day)	40,000	Process water (tonnes/day)	113
	Recycled water (tonnes/day)	57,200	Tailings (tons/day)	38,760
			Waste water (tonnes/day)	69,600
Thickening and filtration unit	Cu Concentrate (tonnes/day)	1,196	Low Solid Content Cu (tonnes/day)	1,196
	Process water (tonnes/day)	113	Waste water (tonnes/day)	67
Recycling, Storage & water treatment	Tailings (tonnes/day)	38,760	Recycled water (tonnes/day)	74,000
	Waste water (tonnes/day)	69,667		
	Makeup water (tonnes/day)	30,120		
Pyro metallurgical smelting	Low Solid Content Cu (tonnes/day)	1,196	High grade Cu cathode (99.6%)(tonnes/day)	1,196
Electro refining	High grade Cu cathode (99.6%) (tonnes/day)	1,196	Very High grade Cu (99.99%)(tonnes/day)	1,196

Abbreviations

EAs: Evolutionary Algorithms, 31	MINLP: Mixed Integer Non Linear Programming, 29
EVT: Extreme Value Theory, 3	MOEA/D: Multi Objective Evolutionary Algorithm with Decomposition, 30
FMECA: Failure Mode Effect and Criticality Analysis, 4	MTBF: Mean Time Between Failures, 4
FTM: Failure Time Modelling, 4	MV: Mean Value, 3
GARCH: Generalised Autoregressive Conditional Heteroskedasticity, 2; Generalized Autoregressive Conditional Heteroskedasticity, 3	NPGA: Niche-Pareto Genetic Algorithm, 30
HAZOP: HAZard and OPerability, 4	NSGA II: Non-dominated Sorting Genetic Algorithm II, 30, 33
HAZSCAN: HAZardous SCenario ANALysis, 4	OD: Outage Duration, 4, 7
HRA: Human Reliability Analysis, 4	PAES: Pareto Archived Evolution Strategy, 30
MA: Moving Average, 2	QRA: Quantitative Risk Assessment, 4
MB: Maintenance & Breakdown, 7	SAG: Semi Autogenous Grinding, 31
MBCost: Maintenance and Breakdown Cost, 25	SPEA: Strength Pareto Evolutionary Algorithm, 30
MC: MOnTe Carlo, 27	SWIFT: Structured What-If Technique, 4
MIMO: Multiple Inputs and Multiple Outputs, 6	TBF: Time Between Failures, 7

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