

Angular-Momentum-Based Sizing of Control Moment Gyro Cluster for an Agile Spacecraft

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The sizing of a control moment gyro (CMG) cluster is covered sparsely in literature, but it is of considerable interest to a practicing engineer. In this paper, the sizing of a cluster of four control moment gyros is investigated based on the mission angular momentum requirement of a typical agile spacecraft. The CMG skew angle β and individual CMG angular momentum h are determined as part of the sizing. The work focuses on the sizing of a minimally redundant cluster of four CMGs in a pyramid and roof-type arrangement considering their external angular momentum surfaces. Two approaches are discussed: the first considers maximum angular momentum about individual axis separately, and the second considers the momentum requirement about two axes simultaneously. The internal momentum surfaces of the CMG clusters are analyzed. Gimbal angle desaturation and the fuel consumed for it by the roof-type and pyramid configurations are also examined. Both the four CMG clusters are able to meet the maximum nonspherical angular momentum demanded by a typical mission about each axes. However, using method 1, the cluster is able to generate only 88% of the two-axis mission demand. The clusters meet the mission demand completely with method 2 using 20% more angular momentum per CMG and an increase in skew angle by 8–11° compared to method 1. The 20% increase in momentum increases power consumption by 11%. For a typical gimbal angle desaturation, the roof-type cluster uses 8.5% more fuel than the pyramid arrangement. However, the former has only one major internal singular surface in contrast with seven for the latter. This makes the roof-type arrangement very desirable to keep the singularity avoidance algorithm simple and amenable with the CMGs operating within the momentum capability, and achieve the required spacecraft agility.

Nomenclature

D	=	Jacobian matrix of the control moment gyro cluster
d	=	scalar that scales the null gimbal rate
F^b	=	body frame, attached to the spacecraft body
g	=	acceleration due to gravity, m/s^2
H	=	total angular momentum vector of control moment gyro cluster, $N \cdot m \cdot s$
H_t	=	angular momentum imparted by the thruster, $N \cdot m \cdot s$
h, h	=	magnitude and vector of the individual control moment gyro angular momentum, $N \cdot m \cdot s$
I_N	=	identity matrix of size $N \times N$
I_{sp}	=	specific impulse, s
L	=	moment arm of the thruster, m
M	=	mass of propellant, g
m	=	singularity index
$\hat{s}$	=	singular direction unit vector
t_i, g_i, h_i	=	unit vectors about the output torque, gimbal axis, and angular momentum axis
α	=	scalar correction term

β	=	skew angle of the control moment gyro cluster
γ, γ	=	gimbal angle and vector of gimbal angles, rad
ϵ_i	=	scalar representing the sign of the projection of $\hat{s}$ on h
θ_1, θ_2	=	angle for expressing the singular direction unit vector
ω	=	inertial body rates of the spacecraft, rad/s

I. Introduction

CONTROL moment gyros (CMGs) are capable of generating a very high torque compared to the reaction wheels (RWs), and yet consume only a fraction of electrical power demanded by RWs. RWs change the magnitude of angular momentum of a spinning wheel by a drive motor to generate output torque along the wheel spin axis. On the other hand, the CMGs have a wheel running at a constant speed that is fixed on a rotating gimbal. The gimbal rotates about an axis orthogonal to the wheel spin axis and produces an output torque orthogonal to both the spin axis and gimbal axis. This rotating gyric output torque has a magnitude given by the gimbal rate times the angular momentum. This help CMGs output a very high torque but in varying direction, in comparison to a low RW torque that is produced about a spacecraft body-fixed direction. If we have a $20 N \cdot m \cdot s$ wheel gimbaled at 1 rad/s gimbal rate, an instantaneous output torque of $20 N \cdot m$ is generated orthogonal to both the wheel spin axis and gimbal axis. In comparison, the maximum torque capability of RW is 1–2 N-m only [1]. A CMG consumes less than one-hundredth power the RW consumes per unit output torque. In addition, even though the absolute mass of the CMGs is more than the mass of RWs, the CMGs generate 15 times more torque per unit mass than the RWs do (updated from Tables 1 and 2 in [2]). This makes the CMGs an attractive choice for agile spacecraft that demand very high angular momentums to attain high body rates and large torque for rapid spot-to-spot imaging. However, due to the time-varying output torque, CMGs are affected by geometric singularities, which make their control laws and operation complicated. On the other hand, the

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RWs are operated with simple control law as they do not have internal geometric singularities, apart from saturation.

The sizing techniques for RWs are well known. Based on the mission requirements, they are typically arranged in a square pyramid configuration with some optimum azimuth and slant angles [3,4]. However, unlike RWs, sizing of CMGs are different as they change the direction of their spin axes but not their speeds (unless they are variable speed CMGs) to generate torque.

A. Literature Survey

The sizing of a CMG cluster has not been covered well in the literature. To the best of the authors' knowledge, there are only three papers that deal directly with the CMG sizing.

Lappas [5] and Richie et al. [6] addressed the sizing by considering a pyramid cluster and a single-axis maneuver in which two of the four CMGs participate. The single-axis torque and gimbal rates were assumed. The skew angle of the pyramid $\beta = 54.7^\circ$ was chosen and fixed at a constant value. The gimbal angle was assumed to be at zero degree at the instant of analysis and individual CMG angular momentum h was computed. However, the sizing was not linked to the momentum envelope of the CMGs and the three-axis momentum requirement of the spacecraft.

In the work of Sands [7], the skew angle of a cluster with three CMGs is varied to minimize the existence of internal singular momentum surfaces. It was reported that maximum singularity-free normalized momentum ($1h$) was obtained at $\beta = 90^\circ$. The corresponding value for a pyramid cluster with $\beta = 54.7^\circ$ was only $0.15h$ (where h is the magnitude of the individual CMG angular momentum). Sands also introduces singularity penetration with unit delay (SPUD). Here, whenever the singularity index of the CMG cluster falls below a set threshold, the current computed gimbal rates are discarded and the set of previous valid gimbal rates is given as the commanded rate [7,8].

In the CMG sizing study by Leve et al [9], the optimization of a 4-CMG cluster is done. The 4-CMG pyramid cluster is represented in terms of two Euler angles. Then, a cost function is defined in terms of the singularity index, torque error, and gimbal rate error. For a given set of initial gimbal angles, the CMG cluster is optimized for obtaining the minimum cost for a give maneuver or a set of maneuvers. The individual CMG angular momentum value was assumed in this case, and the pyramid cluster angles were computed.

B. Contribution

The sizing of a CMG cluster for a given spacecraft momentum demand is a very pertinent requirement for a practicing attitude control engineer. In this paper, a methodology to size a CMG cluster based on a general mission angular momentum requirement is presented. Both the individual CMG angular momentum h and the skew angle β are determined. The sizing is arrived at based on the spacecraft angular momentum demand and the saturation momentum envelope of the CMG cluster. The methodology is generic, and not based on a particular maneuver or initial gimbal angles.

The expressions for computing the momentum envelope of the CMG cluster are derived first. Then they are evaluated for two 4-CMG clusters, selected here for analysis because a real mission typically demands a 4-CMG cluster that is minimally redundant for attitude control of a spacecraft. A nonredundant cluster of three CMGs is analyzed in literature [7,10]. Lewis et al. have studied in detail the arrangement of a cluster of three CMGs (or a 3-CMG cluster) by varying the skew angles of each of the actuator independently [10]. The saturation angular momentum and maximum singularity-free momentum were taken as the base parameters for mixed skew angle selection. However, a cluster of three CMGs lacks redundancy. It also tends to occupy a larger footprint as at least one CMG gimbal axis will be aligned or near the horizontal plane. Because both these factors are critical for a real mission, the sizing of a 3-CMG cluster is not a part of this research.

For a cluster of four CMGs studied in this paper, two methodologies are proposed. The first one considers the maximum angular momentum requirement about each axis separately, whereas the

second method takes the 2-axis momentum demand simultaneously. The obtained momentum distributions are analyzed and compared with the mission angular momentum about the roll, pitch, and yaw axes. Finally, the type of the cluster, individual CMG angular momentum, and skew angle β are selected. The internal momentum surfaces are also studied to arrive and justify the final choice. The gimbal angle desaturation of CMG cluster is explored and illustrated with numerical simulations.

C. Organization

The rest of the paper is organized as follows. Section II explains about the derivation of CMG momentum surface based on the work by Margulies and Aubrun [11]. This is followed by their application to compute the saturation momentum surfaces of two widely used 4-CMG clusters: pyramid and roof type. In Sec. III, cluster sizing using method 1 is explained. In Sec. IV, cluster sizing based on method 2 is studied with relevant examples. In Sec. V, the internal momentum surfaces of the two clusters are analyzed, which is representative of the ease at which the steering laws can be effective in overcoming the internal singularities. In Sec. VI, the desaturation of CMG cluster using null motion is studied and illustrated with numerical simulation results. Section VII concludes the paper.

II. Momentum Envelope of CMG Cluster

There are three methods of studying the angular momentum envelope of CMGs. One is by projection of the angular momentum vectors along the direction of singularity [11]. The singular direction unit vector points in a direction about which the CMG cluster cannot generate an instantaneous torque. The second method is by using a cutting plane technique as suggested by Meffe and Stocking [12]. These methods do not use the gimbal angles explicitly. The third method developed by Wie uses Binet–Cauchy identity and uses the gimbal angles directly [13,14]. In this paper the method developed by Margulies and Aubrun using the singular direction unit vector is used for studying the momentum envelope [11].

A. Momentum Envelope Expression

For the i th CMG, let $\hat{h}_i$ be the unit vector along the angular momentum direction, $\hat{g}_i$ be the unit vector along the gimbal axis, and $\hat{t}_i$ be the unit vector along the instantaneous output torque direction. The unit vectors $\{\hat{t}_i, \hat{g}_i, \hat{h}_i\}$ thus form a set of orthogonal unit vectors rotating about the gimbal axis as shown in Fig. 1. The total angular momentum vector $\mathbf{H}$ is the sum of the individual angular momenta of CMG. As introduced by Margulies and Aubrun, let $\hat{s}$ be a unit vector of the punctured unit sphere defined

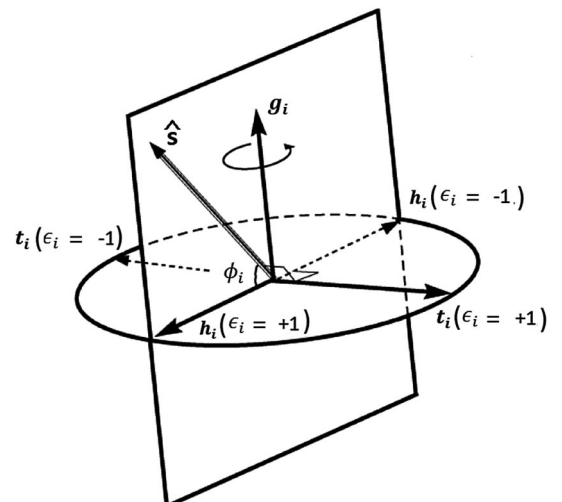


Fig. 1 CMG unit vectors and $\hat{s}$ used in momentum envelope calculation (adapted from [16]).

as $S = \{\hat{s} : \|\hat{s}\| = 1, \hat{s} \neq \pm \hat{g}_i, i = 1, 2, 3, \dots, n\}$. It is chosen this way so that $\hat{s}$ is not taken along the gimbal axis.

The vector $\hat{s}$ is the singular direction unit vector and defines a direction about which the CMG cluster cannot generate the commanded torque. This condition happens when the commanded torque aligns with $\hat{s}$. Thus, based on the recent work by Baker et al., the definition of $\hat{s}$ used in this study aligns with postulate 2, which states, "Singularities occur in instances of impossibility to generate the commanded torque" [15]. The expression for angular momentum envelope will be derived using the vectors $\hat{s}$ and $\hat{g}_i$. The unit vector $\hat{s}$ is defined as

$$\hat{s} = s_x \hat{i} + s_y \hat{j} + s_z \hat{k} \quad (1)$$

Two angles are used to parametrically represent $\hat{s}$ in the body frame F^b : θ_1 and θ_2 , which are successive rotations about the x axis and y axis, respectively. This form of representing $\hat{s}$ was reported to be numerically more suitable for computation of the momentum envelope [11,14].

$$\hat{s} = \sin(\theta_2) \hat{i} - \sin(\theta_1) \cos(\theta_2) \hat{j} + \cos(\theta_1) \cos(\theta_2) \hat{k} \quad (2)$$

Thus following [11,14] the expression for CMG angular momentum can be expressed as

$$\hat{h}_i = e_i \frac{\hat{s} - (\hat{g}_i \cdot \hat{s}) \hat{g}_i}{\sqrt{1 - (\hat{g}_i \cdot \hat{s})^2}} \quad (3)$$

where e_i is defined as

$$e_i = \text{sign}(\hat{h}_i \cdot \hat{s})$$

From Eq. (3), we have an expression for evaluating the angular momentum vector of the i th CMG in terms of the singular direction unit vector $\hat{s}$ and gimbal axis unit vector $\hat{g}_i$.

$$\begin{aligned} \therefore \text{The total angular momentum } \mathbf{H} &= \sum_{i=1}^N \hat{h}_i \\ \mathbf{H} &= \sum_{i=1}^N e_i \frac{\hat{s} - (\hat{g}_i \cdot \hat{s}) \hat{g}_i}{\sqrt{1 - (\hat{g}_i \cdot \hat{s})^2}} \end{aligned} \quad (4)$$

The unit vector along the gimbal axis will be a constant and it depends on the cluster skew angle β . The direction of the unit vector $\hat{s}$ is varied by changing θ_1 from 0 to 180° and θ_2 from 0 to 360° . So for a fixed value of β and for every value of θ_1 and θ_2 , we obtain the components of the angular momentum vector. The loci of all those points represent the momentum surface.

The value of $\epsilon = \pm 1$ determines whether the momentum vectors have a positive or negative projection along the singular direction unit vector. This means that, for an N -CMG cluster, there are 2^N momentum surfaces depending on the sign of the projections of their angular momentum along the singular direction. For example, for a 2-CMG cluster there will be 4 momentum surfaces for the sign combinations $[+ +]$, $[+ -]$, $[- +]$, and $[- -]$. For actuator sizing, the momentum surface obtained with maximal projection of the angular momentum $[+ +]$ (or $[- -]$) is studied. This is called the momentum envelope, and it is the maximum momentum of the CMG cluster in the three-dimensional space. This also represents the resultant angular momentum value attained at saturation singularity. Equation (4) can be used to study the momentum envelope of any CMG cluster.

B. Four CMGs in a Square Pyramid Cluster

This configuration is the most widely studied CMG configuration. It consists of four CMGs with their angular momentum vectors aligned with the four edges of a square pyramid such that the total angular momentum sums to zero as shown in Fig. 2a. The corresponding configuration with typical CMGs is shown in Fig. 2b. In this figure, all gimbal angles are at zero degrees. For each CMG the gimbal angle is taken positive counterclockwise.

By inspection of Fig. 2a, the gimbal axes unit vectors can be written as shown in Eq. (5).

$$\begin{aligned} \hat{g}_1 &= [\sin \beta \quad 0 \quad \cos \beta]^T \\ \hat{g}_2 &= [0 \quad \sin \beta \quad \cos \beta]^T \\ \hat{g}_3 &= [-\sin \beta \quad 0 \quad \cos \beta]^T \\ \hat{g}_4 &= [0 \quad -\sin \beta \quad \cos \beta]^T \end{aligned} \quad (5)$$

If the angular momentum vectors are rotated in the positive sense by $\gamma = [\gamma_1 \quad \gamma_2 \quad \gamma_3 \quad \gamma_4]^T$, the resultant angular momentum will be

$$\mathbf{H} = h \begin{bmatrix} -c\beta s\gamma_1 - c\gamma_2 + c\beta s\gamma_3 + c\gamma_4 \\ c\gamma_1 - c\beta s\gamma_2 - c\gamma_3 + c\beta s\gamma_4 \\ s\beta s\gamma_1 + s\beta s\gamma_2 + s\beta s\gamma_3 + s\beta s\gamma_4 \end{bmatrix} \quad (6)$$

$s\gamma_i = \sin \gamma_i$ and $c\gamma_i = \cos \gamma_i$, where $i = 1, 2, 3, 4$, and h = angular momentum per CMG.

The maximum angular momentum values of the cluster in the body frame F^b can be obtained by substituting the appropriate gimbal angles in Eq. (6) such that the individual CMG angular momenta are projected maximally about the corresponding body axes. The maximum momentum values for the pyramid cluster and the corresponding gimbal angles are given in Table 1.

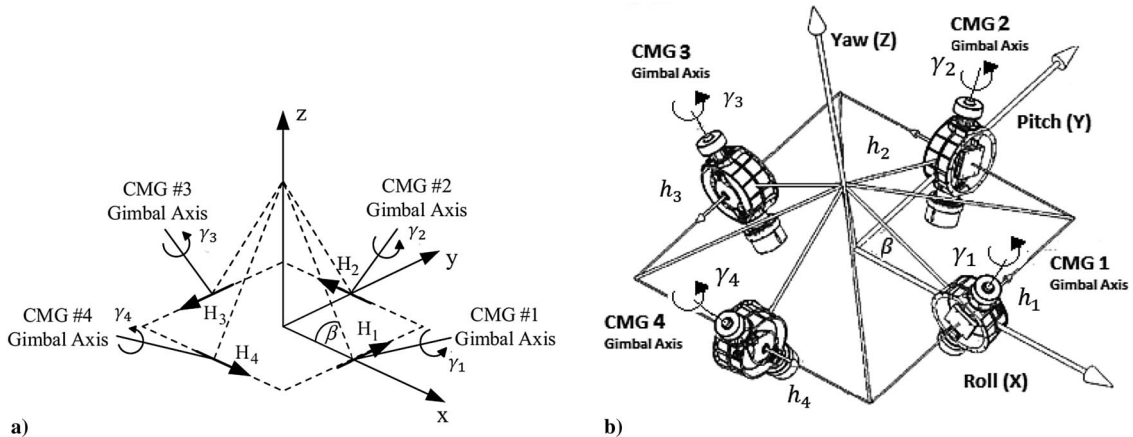


Fig. 2 Pyramid configuration. a) Schematic diagram (adapted from [17]). b) With typical CMGs (adapted from [18]).

Table 1 Maximum momentum values in pyramid cluster

Sl. No.	Gimbal angles	H
1	$[-90^\circ \ 180^\circ \ 90^\circ \ 0^\circ]^T$	$H_{\text{cmg}} = [2h(1 + \cos \beta) \ 0 \ 0]^T$
2	$[0^\circ \ -90^\circ \ 180^\circ \ 90^\circ]^T$	$H_{\text{cmg}} = [0 \ 2h(1 + \cos \beta) \ 0]^T$
3	$[90^\circ \ 90^\circ \ 90^\circ \ 90^\circ]^T$	$H_{\text{cmg}} = [0 \ 0 \ 4h \sin \beta]^T$

For equal angular momentum about the three axes, referring to Table 1, we have to solve for β in the equation $2(1 + \cos \beta) = 4 \sin \beta$. This can be simplified to get $\beta = 53.13^\circ$.

By expanding and numerically evaluating the expression in Eq. (4) with the gimbal axes unit vectors given in Eq. (5), the momentum envelope for four CMG cluster is plotted. The momentum distribution for $\beta = 53.13^\circ$ and $\beta = 30^\circ$ is given in Figs. 3a and 3b, respectively. All positive projections of the momentum are taken in this case (all $e_i/s = 1$).

From the angular momentum envelope plots, the $2N$ holes (circular loci) corresponding to an N -CMG cluster can be observed. All holes have unit diameter with center as the vector sum of the remaining $N-1$ angular momentum vectors. Here eight holes (circular loci) will be present in the momentum envelope (only four can be seen in Fig. 3a).

From the expression for angular momentum values given in Table 1, the maximum momentum value about each of the three axes is 3.2 units for $\beta = 53.13^\circ$. The components for $\beta = 30^\circ$ are X axis 3.73 units, Y axis 3.73 units, and Z axis 2 units. These are matching with the maximum values obtained along each coordinate axis from the angular momentum envelope plot. From Figs. 4a and 4b, we can observe that by decreasing the slant angle from $\beta = 53.13^\circ$ to $\beta = 30^\circ$, the angular momentum envelope becomes flatter. The maximum angular momentum about X axis and Y axis increases while the value about Z axis decreases. So, an appropriate β can be

chosen depending on the distribution required. An appropriate way of choosing β from a single-axis or two-axis maneuver will be explained subsequently.

C. Four CMGs in a Roof-Type Cluster

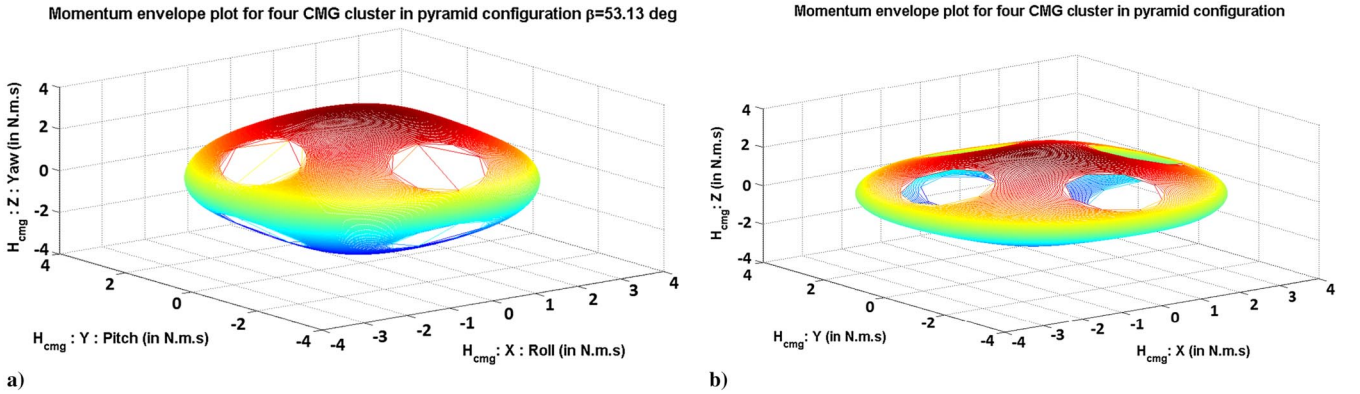
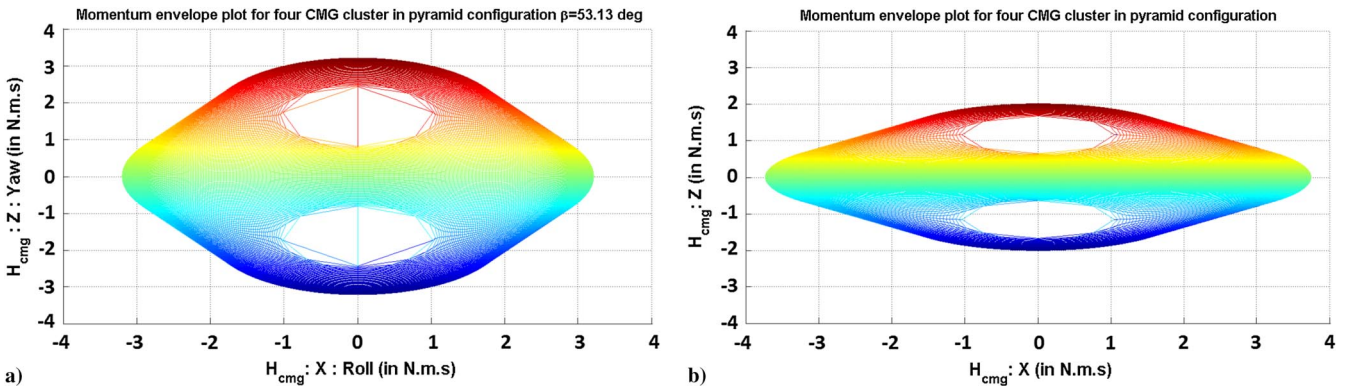
This arrangement has two pairs of CMGs with each pair having parallel gimbal axes located at each side of the roof as shown in Figs. 5a and 5b. The roof-type cluster has few advantages compared to the pyramid cluster. It has smaller number of internal momentum surfaces [19] and can be controlled in a decoupled manner to reduce the occurrence of singular gimbal configuration [20]. Here the angular momentum vectors are free to rotate on the plane of the roof, which is defined by the skew angle β . When all the angular momentums align along the edges, say, in the positive x axis, we get the maximum momentum value about this direction.

In Fig. 5a, the CMG cluster is shown at $\gamma = [0 \ 0 \ 0 \ 0]^T$. The resultant angular momentum is also zero in this configuration. The gimbal angles are taken as counterclockwise positive. At an instant when the gimbal angles are $\gamma = [\gamma_1 \ \gamma_2 \ \gamma_3 \ \gamma_4]^T$, the resultant angular momentum expression of the roof-type cluster in F^b can be written as

$$H = h \begin{bmatrix} c\gamma_1 + c\gamma_2 - c\gamma_3 - c\gamma_4 \\ c\beta s\gamma_1 + c\beta s\gamma_2 - c\beta s\gamma_3 - c\beta s\gamma_4 \\ s\beta s\gamma_1 + s\beta s\gamma_2 + s\beta s\gamma_3 + s\beta s\gamma_4 \end{bmatrix} \quad (7)$$

where $s\gamma_i = \sin \gamma_i$, $c\gamma_i = \cos \gamma_i$, $i = 1, 2, 3, 4$, and h = angular momentum per CMG.

As explained earlier, H reduces to zero for zero gimbal angles. Similar to the analysis done for the pyramid cluster, the maximum angular momentum values for the roof-type cluster are shown in Table 2. The unit vectors along the gimbal axis are given in Eq. (8).

**Fig. 3** Pyramid cluster momentum envelope (adapted from [18]). a) $\beta = 53.13^\circ$. b) $\beta = 30^\circ$.**Fig. 4** Pyramid cluster momentum envelope (adapted from [18]). a) X - Z view $\beta = 53.13^\circ$. b) X - Z view $\beta = 30^\circ$.

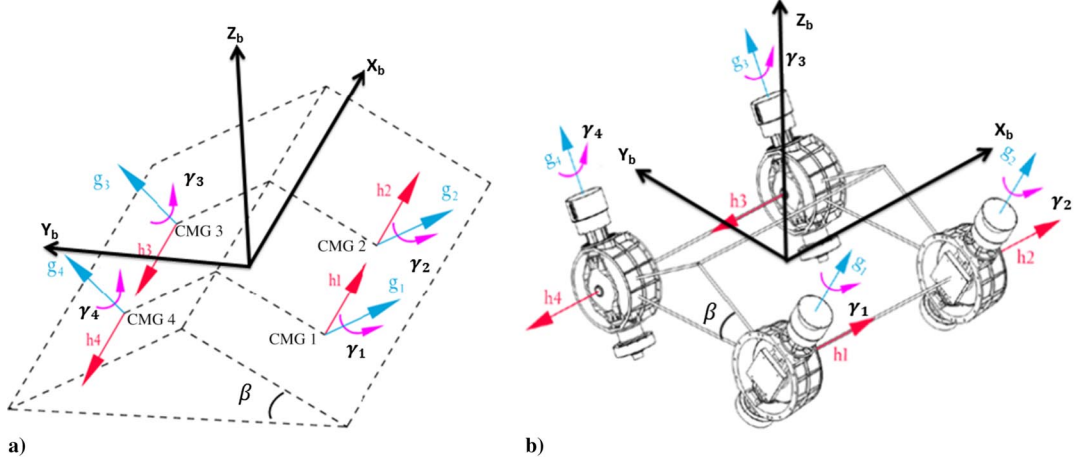


Fig. 5 CMGs in roof-type configuration (adapted from [18]). a) Schematic. b) With typical CMGs.

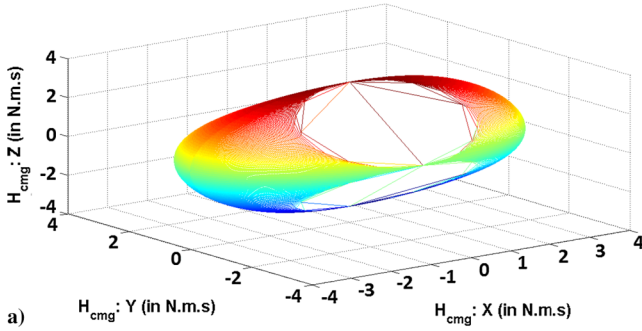
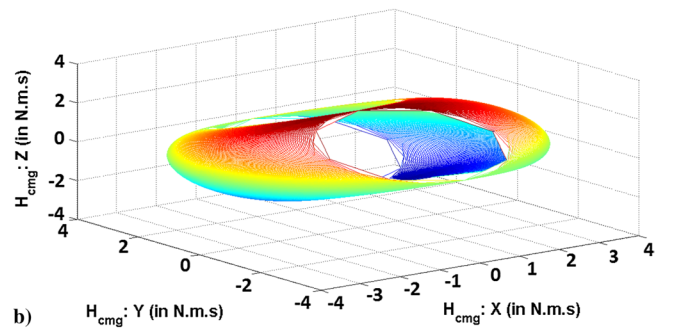
Table 2 Maximum momentum values in roof-type cluster

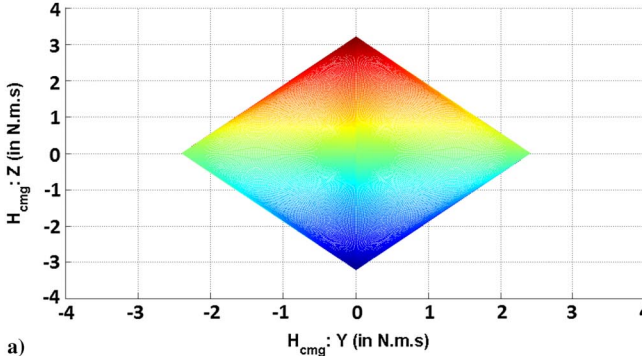
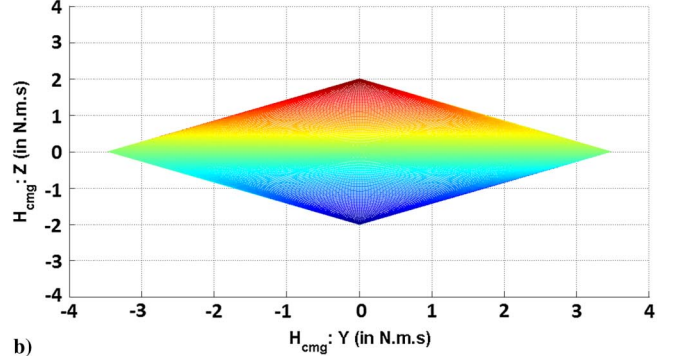
Sl. No.	Gimbal angle	H
1	$[0^\circ \ 0^\circ \ 180^\circ \ 180^\circ]^T$	$H_{\text{cmg}} = [4h \ 0 \ 0]^T$
2	$[90^\circ \ 90^\circ \ -90^\circ \ -90^\circ]^T$	$H_{\text{cmg}} = [0 \ 4h \cos \beta \ 0]^T$
3	$[90^\circ \ 90^\circ \ 90^\circ \ 90^\circ]^T$	$H_{\text{cmg}} = [0 \ 0 \ 4h \sin \beta]^T$

$$\begin{aligned}
 \hat{g}_1 &= [0 \ \sin \beta \ \cos \beta]^T \\
 \hat{g}_2 &= [0 \ \sin \beta \ \cos \beta]^T \\
 \hat{g}_3 &= [0 \ -\sin \beta \ \cos \beta]^T \\
 \hat{g}_4 &= [0 \ -\sin \beta \ \cos \beta]^T
 \end{aligned} \quad (8)$$

By using Eq. (4) the momentum envelope of the roof-type cluster is calculated and given in Figs. 7a and 7b by using the positive momentum projection. From the angular momentum envelope plots, the $2N$ holes corresponding to an N -CMG cluster can be observed. All holes have unit radius with center as the vector sum of the remaining $N-1$ angular momentum vectors. Here, the two holes of CMGs 1,2 and of CMGs 3,4 have merged and appear as a large hole with a radius of 2 units.

From the expressions given in Table 2, the maximum values of the angular momentum about the coordinate axes for $\beta = 53.13^\circ$ are 4, 2.4, and 3.2 units, respectively. For $\beta = 30^\circ$, the maximum values about the X, Y, and Z axes are 4, 3.46, and 2 units, respectively. These values are verified from momentum envelope. The shape of the momentum envelope is a function of the number of CMGs, the gimbal axis unit vector and the skew angle β . The momentum

 Momentum envelope plot for four CMG cluster in Roof-type Configuration $\beta=53.13$ deg

 Momentum envelope plot for four CMG cluster in Roof-type Configuration $\beta=30$ deg

 Fig. 6 Roof-type CMG cluster momentum envelope (adapted from [18]). a) $\beta = 53.13^\circ$. b) $\beta = 30^\circ$.

 Momentum envelope plot for four CMG cluster in Roof-type Configuration $\beta=53.13$ deg

 Momentum envelope plot for four CMG cluster in Roof-type Configuration $\beta=30$ deg

 Fig. 7 Roof-type CMG cluster momentum envelope (adapted from [18]). a) Y-Z view $\beta = 53.13^\circ$. b) Y-Z view $\beta = 30^\circ$.

surfaces are plotted for unit CMG angular momentum, which can be scaled appropriately to obtain the distribution of the momentum values about each body axis.

We observe here also that the lower the slant angle the flatter the envelope. The shape of the momentum envelope is used as a guideline for selecting the appropriate cluster configuration of CMGs to meet the mission requirement. For meeting the three axes' torque and momentum requirement with one CMG redundancy, 4-CMG cluster configurations are preferred [5]. In the next section, the 4-CMG cluster in pyramid and roof-type configurations are examined again and the individual CMG angular momentum and the slant angle β are calculated based on the mission demand.

III. Sizing of Pyramid and Roof-Type Cluster Configurations: Method 1

In this section the 4-CMG pyramid and roof-type CMG clusters will be sized for a typical mission requirement. This is proposed based on two approaches. In the first method, the maximum spacecraft angular momentum demand about the individual body axes will be compared with the maximum angular momentum expressions of the pyramid and roof-type cluster for CMG sizing. It should be noted that in this analysis, at one of the extreme values of the momentum envelope, the other components will be zero. However, these are taken as references for arriving at the skew angle β and h , which help us in obtaining the required envelope for a given mission requirement.

In the second method, two-axis mission angular momentum requirements will be simultaneously taken to arrive at the CMG sizing. The second method has the advantage of considering the two-axis spacecraft maneuver simultaneously into sizing. It should be mentioned that the momentum envelope is not a continuous surface but has circles or holes formed by the loci of the last momentum link. The values within this circle are not achievable. In addition, the internal singularity momentum surfaces are not explicitly considered for the sizing as they do not represent the maximum angular momentum condition.

A. Sizing of 4-CMG Pyramid Cluster

From Table 1, the extreme values of momentum for a 4-CMG pyramid cluster were derived. These are obtained for different sets of gimbal angles. Also, when x -axis momentum is maximum, y and z are zero and so on and so forth. However, these extreme values can be utilized for shaping the momentum envelope based on the mission requirement.

$$\frac{H_{\text{cmg}}:Z:\max}{H_{\text{cmg}}:X:\max} = \frac{4h \sin \beta}{2h(1 + \cos \beta)} = \frac{4 \sin(\beta/2) \cos(\beta/2)}{2 \cos^2(\beta/2)} = 2 \tan \frac{\beta}{2} \quad (9)$$

Let the spacecraft angular momentum about the body axes be $H_{\text{sat}} = [H_{\text{roll}} \ H_{\text{pitch}} \ H_{\text{yaw}}]^T$. The expressions for extreme values of the momentum envelope about the roll and pitch axes are same for the pyramid cluster (refer Table 1). Hence, for cluster sizing, the largest angular momentum value in the roll-pitch plane has to be selected. It should be noted that the relation in Eq. (9) remains unchanged in this condition.

$$\beta = 2 \tan^{-1} \left[\frac{H_{\text{yaw}}}{2 \max(H_{\text{roll}}, H_{\text{pitch}})} \right] \\ h = \frac{\max(H_{\text{roll}}, H_{\text{pitch}})}{2(1 + \cos \beta)} \quad (10)$$

Attaining the momentum distribution in the roll-pitch plane (x - y plane) is more critical while sizing CMG clusters. This is because, all agile maneuvers occur predominantly about the roll and pitch axes with a kinematic coupling alone about the yaw axis.

B. Sizing of CMGs in Roof-Type Cluster

The extreme values of the momentum envelope of the roof-type cluster are given in Table 2. Here only the maximum values of the pitch and yaw angular momentums are functions of the skew angle β . Hence we can write

$$\frac{H_{\text{cmg}}:Z:\max}{H_{\text{cmg}}:Y:\max} = \frac{4h \sin \beta}{4h \cos \beta} = \tan \beta \quad (11)$$

C. Numerical Example: Method 1

Consider the case of a typical Earth observation satellite. The roll, pitch, and yaw inertia values are 1200, 1300, and 1100 kg · m² with the peak expected rates as 4°/s, 3°/s, and 2°/s, respectively.[§] The total angular momentum requirement about each body axis can be computed as $H_{\text{sat}} = [83.8 \ 68.1 \ 38.4]^T$ N · m · s. Due to difference in inertia and maximum rates about each of the body axes, the momentum demand is unequal. The requirement is more in the roll-pitch (x - y) plane compared to the yaw (z) axis.

1. For Pyramid Cluster

Consider a numerical example with mission angular momentum requirement as computed above. If you consider a pyramid cluster, the skew angle and CMG angular momentum are

$$\beta = 2 \tan^{-1} \left[\frac{38.4}{2 \max(83.8, 68.1)} \right] = 25.8^\circ \\ h = \frac{\max(83.8, 68.1)}{2(1 + \cos \beta)} = 22 \text{ N} \cdot \text{m} \cdot \text{s}$$

2. For Roof-Type Cluster

For a given mission angular momentum, while using a 4-CMG roof-type cluster, the skew angle can be computed as given follows:

$$\beta = \tan^{-1} \frac{H_{\text{yaw}}}{H_{\text{pitch}}} = \tan^{-1} \frac{38.4}{68.1} = 29.4^\circ \quad (12)$$

Also by considering the roll axis (x axis) parallel to the roof axis (refer Fig. 5b), we can state that the extreme value of the roll angular momentum of the roof-type cluster is independent of the skew angle β and is given as $4h$ (refer Table 2). Hence from the total mission angular momentum about the roll axis the individual CMG angular momentum can be calculated as

$$h_r = \frac{H_{\text{roll}}}{4} = \frac{83.8}{4} = 20.95 \text{ N} \cdot \text{m} \cdot \text{s} \quad (13)$$

The angular momentum can also be computed from the mission pitch axes requirement and β as shown:

$$h_p = \frac{H_{\text{pitch}}}{4 \cos \beta} = \frac{68.1}{4 \cos \beta} = 19.5 \text{ N} \cdot \text{m} \cdot \text{s} \quad (14)$$

Hence the final wheel angular momentum should be chosen from the maximum among these two values: $h = \max(h_r, h_p)$. Here $h = 20.95 \text{ N} \cdot \text{m} \cdot \text{s}$ for the roof-type cluster.

D. Variation in the Maximum Momentum Based on Cluster Skew Angle

The values of maximum angular momentum for pyramid and roof-type cluster calculated from the expressions given in Tables 1 and 2 are listed in Table 3. The mission angular momentum requirement about the three body axes is also given. The variation of the

[§]From internal report of Indian Space Research Organisation.

Table 3 Angular momentum of CMG clusters about F^b for different β values with $h = 22.0 \text{ N} \cdot \text{m} \cdot \text{s}$ and $h = 20.95 \text{ N} \cdot \text{m} \cdot \text{s}$ for pyramid and roof-type clusters, respectively, based on the expressions given in Tables 1 and 2

Maximum angular momentum (in $\text{N} \cdot \text{m} \cdot \text{s}$)						
β	Pyramid, $h = 22.0$			Roof type, $h = 20.95$		
	H_x	H_y	H_z	H_x	H_y	H_z
65	62.7	62.7	79.9	83.8	35.4	75.9
60	66.1	66.1	76.3	83.8	41.9	72.6
54.73	69.5	69.5	72.0	83.8	48.4	68.4
53.13	70.5	70.5	70.5	83.8	50.3	67.0
45	75.3	75.3	62.3	83.8	59.3	59.3
40	77.8	77.8	56.7	83.8	64.2	53.9
35	80.2	80.2	50.6	83.8	68.6	48.1
29.4	82.5	82.5	43.3	83.8	73.0	41.1
25.8	83.8	83.8	38.4	83.8	75.4	36.5
22.5	84.8	84.8	33.7	83.8	77.4	32.1
20	85.5	85.5	30.2	83.8	78.7	28.7
Mission requirement	83.8	68.1	38.4	83.8	68.1	38.4

Here $x = \text{roll}$, $y = \text{pitch}$, and $z = \text{yaw}$ axes.

maximum angular momentum for the pyramid and roof-type cluster against the skew angle is shown in Fig. 8. The mission angular momentum requirements are shown as black horizontal lines. In both the table and the figure, the CMG angular momentums are taken as $h = 22.0 \text{ N} \cdot \text{m} \cdot \text{s}$ and $h = 20.95 \text{ N} \cdot \text{m} \cdot \text{s}$ for the pyramid and roof-type cluster, respectively, as obtained in the sizing calculations using method 1.

1. For Pyramid Cluster

From Table 3, we can see that for a pyramid cluster the angular momentum about the roll and the pitch axes is same as observed in the expressions given in Table 1. The distribution about the yaw axis increases as β becomes larger. The maximum angular momentum about the x and y axes is obtained as overlapping lines reducing from $88.2 \text{ N} \cdot \text{m} \cdot \text{s}$ at $\beta = 0^\circ$ (maximum) to $44.1 \text{ N} \cdot \text{m} \cdot \text{s}$ at $\beta = 90^\circ$ (refer Fig. 8). This trend is reversed about the yaw axis. As obtained in the sizing calculations, at $\beta = 25.8^\circ$, the maximum cluster angular momentum meets the maximum mission angular momentum demand. There is excess angular momentum about the pitch axis

while the roll and the yaw requirements are just met (refer Table 3 and Fig. 8).

2. For Roof-Type Cluster

For the roof-type cluster, the maximum angular momentum about the x axis is independent of the skew angle β (seen as a horizontal line in Fig. 8). The pitch and the yaw momentum distribution vary as β changes, with the former decreasing as the skew angle increases from 0 to 90° . As computed in the sizing calculation, at $\beta = 29.4^\circ$ and $h = 20.95 \text{ N} \cdot \text{m} \cdot \text{s}$, the maximum mission angular momentum demand is met. There is excess angular momentum about the pitch axis while the roll and yaw axes requirements are just met (refer Table 3 and Fig. 8). The excess angular momentum about the pitch is lower compared to the pyramid cluster. This is due to the larger β and lower h chosen for the roof-type cluster sizing. In addition, it should be noted that, for the same β and h , pyramid cluster gives equal maximum angular momentum about the roll and the pitch axes, whereas the roof-type cluster gives more about the roll axis. The maximum yaw axis contribution remains same for both the clusters.

E. Pyramid Cluster Sizing: Two-Axis Momentum Requirement

It should be remarked that the sizing using method 1 considers only the extreme values of the momentum envelope of the CMG clusters against the mission angular momentum requirement. However, the CMG clusters generally operate about the intermediate region. Geometrically, the spacecraft angular momentum demand can be considered as a cuboid with edge lengths corresponding to the mission angular momentum demand (83.8 , 68.1 , and $38.4 \text{ N} \cdot \text{m} \cdot \text{s}$, respectively, here). How well the entire angular momentum envelope of the CMG cluster meets the demands at all combinations of the spacecraft angular momentum is significant.

Consider Figs. 9 and 10. These show the spacecraft angular momentum requirement (red cuboid) overlapped onto the momentum envelope of the pyramid cluster with $\beta = 25.8^\circ$ as obtained from the sizing studies. Figs. 9a,9d,10a, and 10d represent $h = 22.04 \text{ N} \cdot \text{m} \cdot \text{s}$ case; Figs. 9b,9e,10b, and 10e represent the case with $h = 25 \text{ N} \cdot \text{m} \cdot \text{s}$. The third case is computed for $h = 30 \text{ N} \cdot \text{m} \cdot \text{s}$.

Ideally we would like the momentum envelope to be outside the satellite momentum cuboid. Since the sizing was done considering only the extreme values, it can be observed that $h = 22.04 \text{ N} \cdot \text{m} \cdot \text{s}$ just covers the extreme values about the x and z axes. It provides excess momentum about the y axis. However, the corner cases on the x - y plane, where there is demand for simultaneous x - y momentum

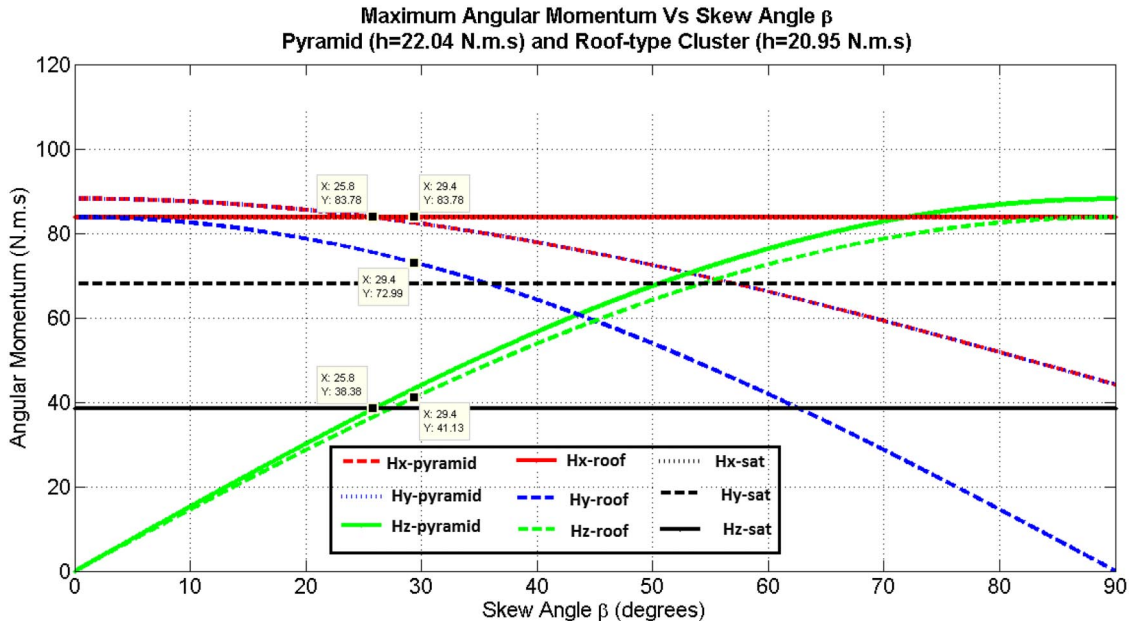


Fig. 8 Variation in the maximum CMG angular momentum for pyramid (with $h = 22.0 \text{ N} \cdot \text{m} \cdot \text{s}$) and roof-type cluster (with $h = 20.95 \text{ N} \cdot \text{m} \cdot \text{s}$) against skew angle β . Here $x = \text{roll}$, $y = \text{pitch}$, and $z = \text{yaw}$ axes.

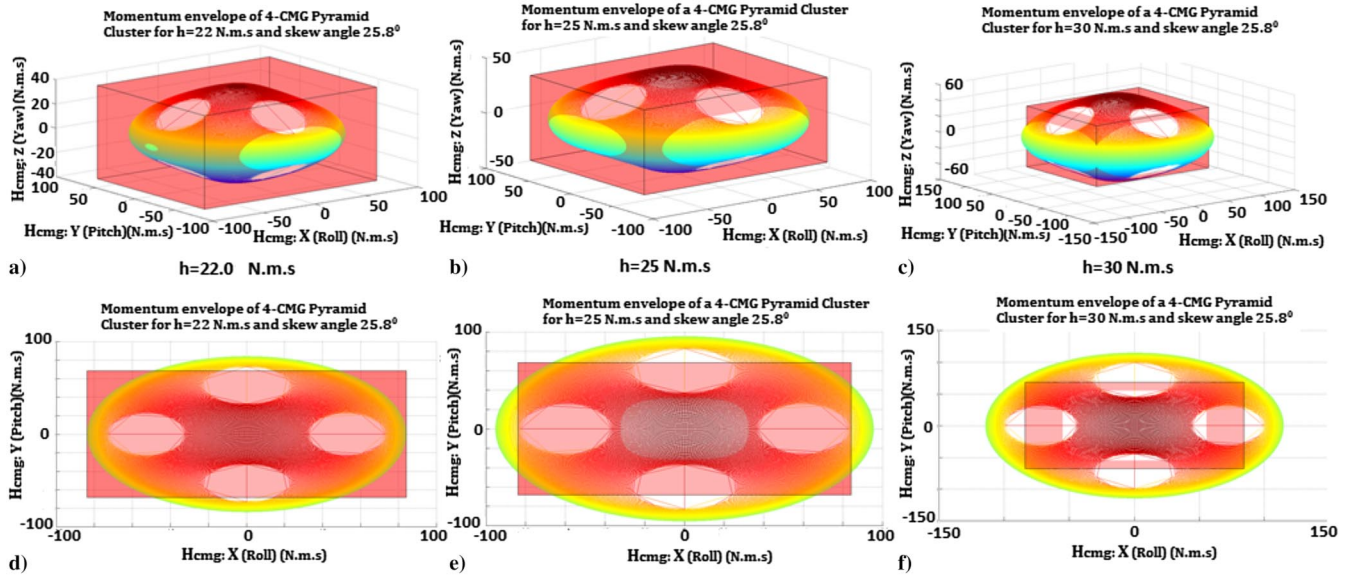


Fig. 9 CMG angular momentum envelope for pyramid cluster $h = 22.04, 25$, and $30 \text{ N} \cdot \text{m} \cdot \text{s}$ with spacecraft requirement shown by red cuboid for $\beta = 25.8^\circ$: isometric and X - Y views.

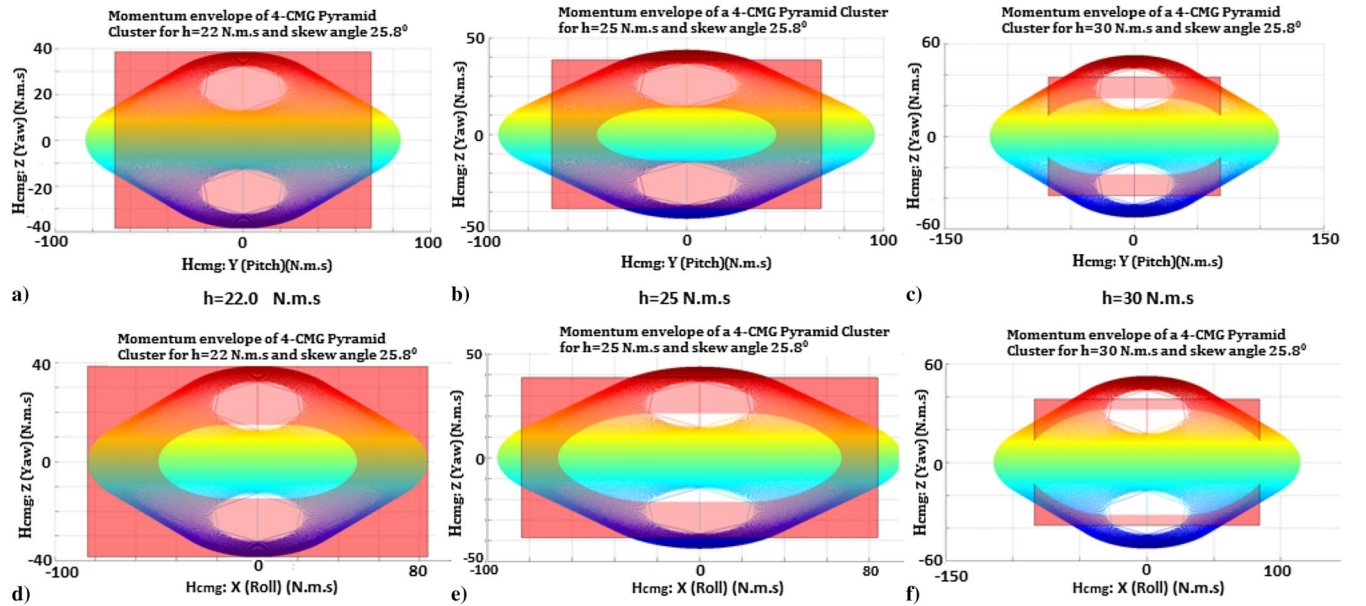


Fig. 10 CMG angular momentum envelope for pyramid cluster $h = 22.04, 25$, and $30 \text{ N} \cdot \text{m} \cdot \text{s}$ with spacecraft requirement shown by red cuboid for $\beta = 25.8^\circ$: Y - Z and X - Z views.

requirement, cannot be encompassed within the envelope as seen from Fig. 9d. It is decided to keep same β angle because reducing it further is not going to enlarge the momentum requirement. It will only result in enhancing the distribution about the x - y plane, which will not meet the two-axis momentum requirement.

Hence the individual angular momentum magnitude h is increased to $25 \text{ N} \cdot \text{m} \cdot \text{s}$. In Fig. 9e, for $h = 25 \text{ N} \cdot \text{m} \cdot \text{s}$, corner angular momentum (H_x, H_y) is $(78.1, 54.6) \text{ N} \cdot \text{m} \cdot \text{s}$ against $(70.6, 45.2)$ when $h = 22.04 \text{ N} \cdot \text{m} \cdot \text{s}$ [mission requirement is $(83.8, 68.1) \text{ N} \cdot \text{m} \cdot \text{s}$]. For the case where $h = 30 \text{ N} \cdot \text{m} \cdot \text{s}$, the cluster gives $(88.2, 72.8) \text{ N} \cdot \text{m} \cdot \text{s}$ for the corner case, thus encompassing the angular momentum demand. This case can be seen in Fig. 9f.

From Figs. 10a-10f, it can be noted that the y - z and x - z corner cases are not covered even with $h = 30 \text{ N} \cdot \text{m} \cdot \text{s}$. This is not critical, as for an agile spacecraft the simultaneous rate requirement occurs about the x and y axes and not about y - z or x - z . Hence if the sizing covers the corner case in the x - y plane, it is acceptable.

1. Selection of Wheel h Based on Two-Axis Momentum Demand and Power Consumption

As the angular momentum becomes larger for the same sized wheel, the running speed will be increased. The wheel power is proportional to the running speed. If variable speed or RW mode of operation is planned, the wheel power increases with increase in angular momentum.

For a typical wheel that gives $25 \text{ N} \cdot \text{m} \cdot \text{s}$ at 3250 rpm and $0.1 \text{ N} \cdot \text{m}$ peak torque ($0.02 \text{ N} \cdot \text{m}$ loss torque), the steady-state power (peak power) will be 6.2 W (39 W), 7 W (43 W), and 8.4 W (50 W) for wheels running at $22.04, 25$, and $30 \text{ N} \cdot \text{m} \cdot \text{s}$, respectively.[†] Here the corresponding wheel drive electronics power consumption is 6 W during steady-state operation and 22 W during peak condition. So

[†]Calculation of reaction wheel power is done based on internal wheel design report of Indian Space Research Organisation.

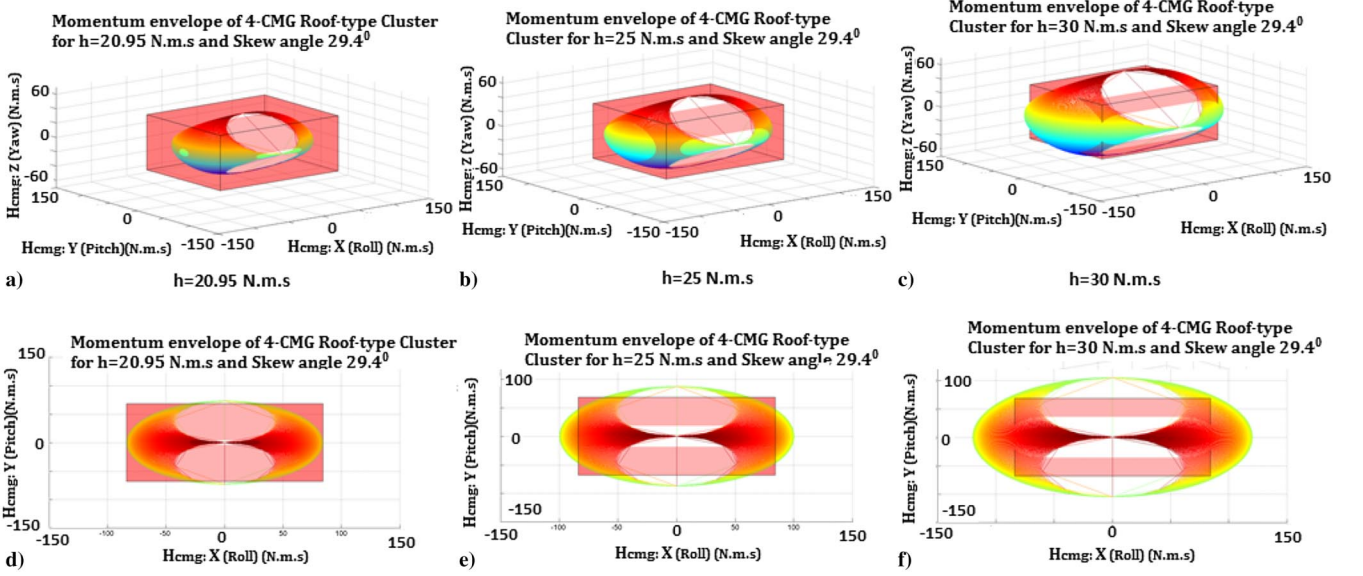


Fig. 11 CMG angular momentum envelope for roof-type cluster $h = 20.95, 25$, and $30 \text{ N} \cdot \text{m} \cdot \text{s}$ with spacecraft requirement shown by red cuboid for $\beta = 29.4^\circ$: isometric and X-Y views.

while selecting a wheel with higher angular momentum, the overheads due to high electrical power consumption must also be considered.

If we consider a 4-CMG cluster, the total power at steady-state (peak) condition at $25 \text{ N} \cdot \text{m} \cdot \text{s}$ for both wheel and wheel drive electronics together is 52 W (260 W) and for a $30 \text{ N} \cdot \text{m} \cdot \text{s}$ wheel it will be 57.6 W (288 W). An 11% increase in steady state and peak power is demanded by the $30 \text{ N} \cdot \text{m} \cdot \text{s}$ (with wheel drive electronics) compared to the $25 \text{ N} \cdot \text{m} \cdot \text{s}$ wheel throughout the mission life. Hence wheel angular momentum should not be fixed unilaterally based on meeting the two-axis angular momentum alone.

In this context, $h = 25 \text{ N} \cdot \text{m} \cdot \text{s}$ and $\beta = 25.8^\circ$ is appropriate as it consumes relatively less power and meets the 2-axis demand compared to the case with $h = 22.04 \text{ N} \cdot \text{m} \cdot \text{s}$. The maximum rate and angular momentum specifications are put forth to finally meet the agility requirement of the spacecraft. If the landmark acquisition and

tracking demand can be met by a $h = 25 \text{ N} \cdot \text{m} \cdot \text{s}$ wheel, the higher angular momentum option, with $h = 30 \text{ N} \cdot \text{m} \cdot \text{s}$ need not be considered.

F. Roof-Type Cluster: Two-Axis Momentum Demand

Similar to the sizing done on the pyramid cluster considering the simultaneous roll and pitch momentum demand from the spacecraft, the angular momentum envelope of the roof-type cluster is studied in this section. The momentum envelope overlapped on the spacecraft momentum requirement cuboid (in red) is shown in Figs. 11 and 12. For $h = 20.95, \beta = 29.4^\circ$, the maximum angular momentum values meet the mission requirement. However, for simultaneous roll-pitch requirement as shown in Fig. 11D, the (H_x, H_y) values are $(71.2, 38.5) \text{ N} \cdot \text{m} \cdot \text{s}$ against the mission demand of $(83.8, 68.1) \text{ N} \cdot \text{m} \cdot \text{s}$. Even though there is a larger shortfall in Y-Z and X-Z planes, large yaw rates are not expected with simultaneous roll and pitch requirement while performing agile maneuvers.

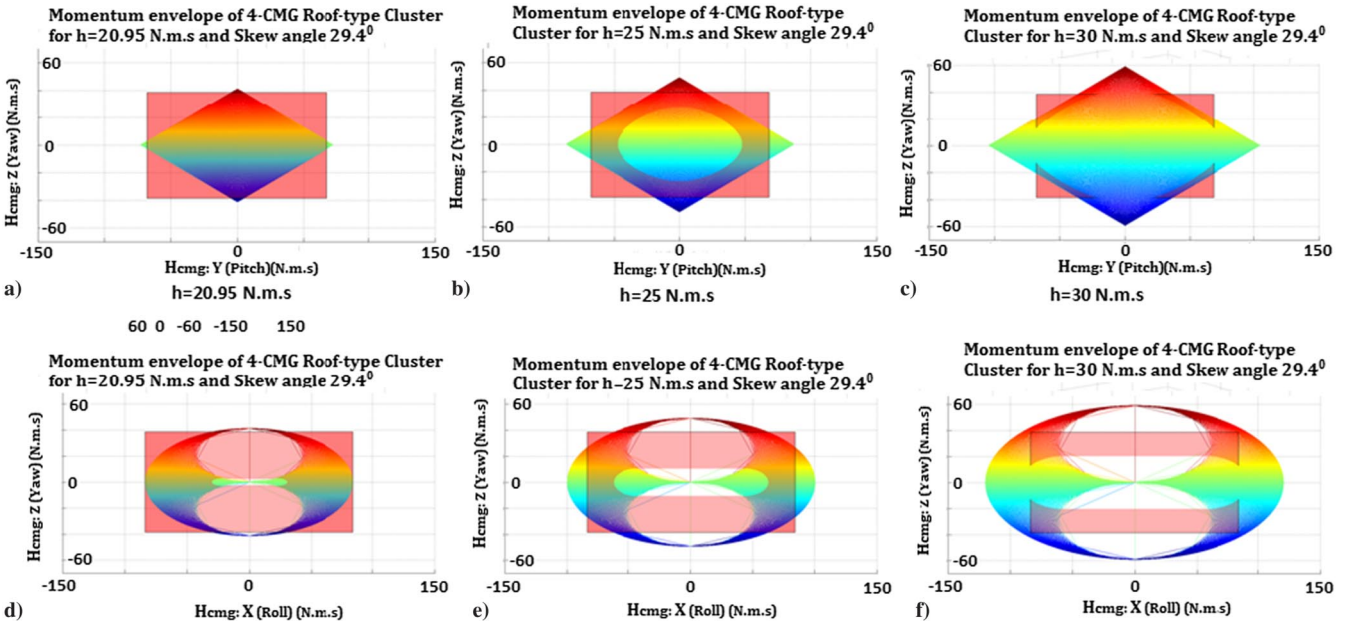


Fig. 12 CMG angular momentum envelope for roof-type cluster $h = 20.95, 25$, and $30 \text{ N} \cdot \text{m} \cdot \text{s}$ with spacecraft requirement shown by red cuboid for $\beta = 29.4^\circ$: Y-Z and X-Z views.

When the CMG momentum was increased to $h = 25 \text{ N} \cdot \text{m} \cdot \text{s}$, the momentum capability at the corner region increased to $(79.2, 53.2) \text{ N} \cdot \text{m} \cdot \text{s}$. The angular momentum requirements in the y - z and x - z planes also showed improvement. It should be noted that, because of the parallel gimbal axes for a pair of CMGs in the roof-type cluster, the y - z angular momentum values are lower than that seen for the pyramid cluster (compare Figs. 10b and 12b). But this trend is reversed for the momentum requirements in the x - z plane (compare Figs. 10e and 12e). Similar observations are seen when $h = 30 \text{ N} \cdot \text{m} \cdot \text{s}$, with the cluster able to encompass the simultaneous x - y momentum requirement. Though the pyramid cluster was unable to meet the two-axis requirements both in the y - z and x - z planes (Figs. 10c and 10f), the roof-type cluster is able to meet the momentum requirement in the x - z plane (Fig. 12f). However, it is unable to meet the two-axis requirement in the y - z plane as seen in Fig. 12c. As discussed earlier, the main criteria for cluster sizing should be the ability to meet the requirement in the x - y plane. However, the cluster should be capable of giving the required yaw axis momentum alone as demanded by the mission. Considering the power requirement of the wheel, $h = 25 \text{ N} \cdot \text{m} \cdot \text{s}$ and $\beta = 29.4^\circ$ is taken as the final configuration for the roof-type cluster.

1. Disadvantage of Method 1

Method 1 considers the maximum momentum of the CMG cluster for sizing as per the satellite momentum requirement. Though it gives a good starting point, the initial solution is unable to meet the simultaneous momentum requirement about the x and y axes. The designer will be able to size the cluster by varying the individual CMG angular momentum h for the obtained β as shown earlier. This was effective for both the pyramid and roof-type cluster. In the next section, method 2 is proposed, which considers the two-axis momentum requirement simultaneously for CMG sizing.

IV. CMG Sizing Method 2: Two-Axis Momentum Approach

In this section a procedure for sizing the pyramid and roof-type cluster using the two-axis momentum requirement is explained. The three-axis sizing is not attempted because, for an agile maneuver, it is more critical to meet the momentum requirement in the x - y plane. Only a small momentum requirement comes about the yaw (z) axis due to the kinematic coupling between the roll (x) and the pitch (y) axes. Though, occasionally, the mission can demand a large angle yaw maneuver, in this condition, the roll-pitch demand will be very small. It is noted and understood that the region inside the circular loci of the last momentum links that appear as holes on the envelope cannot be attained. These holes are not separately considered in the sizing procedure. Also, it is assumed that the projection of the angular momentum envelope on the x - y plane is a circle (for a pyramid cluster) or an ellipse (for a roof-type cluster). This approximation represents only the edge of the envelope and not the internal region. Yaw momentum demand is not considered in this sizing. The h and β obtained here will be checked to see whether the cluster configuration meets the yaw momentum requirement.

A. Method 2: Sizing of Pyramid Cluster

Consider the momentum requirement for the spacecraft about the x and y axes as (H_x, H_y) . The edges of the projection of angular momentum envelope on the x - y plane can be considered a circle. From Table 1, we can note the maximum value of pyramid cluster momentum about the x and y axes as $2h(1 + \cos \beta)$. The circle should be constructed in such a way that it passes through the point (H_x, H_y) . Consider the following relation:

$$\frac{H_x^2}{[2h(1 + \cos \beta)]^2} + \frac{H_y^2}{[2h(1 + \cos \beta)]^2} = 1 \quad (15)$$

We know (H_x, H_y) and thus have only one equation for the two unknowns h and β . Hence after simplification we obtain

$$\cos \beta = \pm \sqrt{\frac{H_x^2 + H_y^2}{4h^2}} - 1 \quad (16)$$

We know that the value of $\cos \beta$ will be between $[-1, 1]$. For a CMG cluster, $0 < \beta < 90$. Hence, $\cos \beta$ will be between $[0, 1]$ and only the positive root is to be considered. Then Eq. (16) can be converted into the following inequality relation:

$$\begin{aligned} \sqrt{\frac{H_x^2 + H_y^2}{4h^2}} - 1 &< 1 \\ \therefore h^2 &> \frac{H_x^2 + H_y^2}{16} \\ \Rightarrow h &> \frac{\sqrt{H_x^2 + H_y^2}}{4} \end{aligned} \quad (17)$$

B. Method 2: Sizing of Roof-Type Cluster

Similar to the pyramid cluster, for a roof-type cluster, the momentum envelope projection on the x - y plane can be considered as an ellipse and can be written as

$$\frac{H_x^2}{(4h)^2} + \frac{H_y^2}{(4h \cos \beta)^2} = 1 \quad (18)$$

Simplifying and rearranging Eq. (18), we get the following relation for $\cos \beta$.

$$\cos \beta = \pm \frac{H_y}{\sqrt{(4h)^2 - H_x^2}} \quad (19)$$

Putting forward the same assumptions as done for the pyramid cluster, since the skew angle $0 < \beta < 90^\circ$, only the positive square root is taken and the following relationship is obtained:

$$\begin{aligned} \frac{H_y}{\sqrt{(4h)^2 - H_x^2}} &< 1 \\ \therefore h &> \frac{\sqrt{H_x^2 + H_y^2}}{4} \end{aligned} \quad (20)$$

C. Method 2: Numerical Examples

Consider (H_x, H_y) requirement as $(83.8, 68.1) \text{ N} \cdot \text{m} \cdot \text{s}$. From Eqs. (17) and (20), the value for the individual CMG angular momentum is obtained as

$$h > \frac{\sqrt{H_x^2 + H_y^2}}{4} > \frac{\sqrt{83.8^2 + 68.1^2}}{4} > 26.995 \text{ N} \cdot \text{m} \cdot \text{s} \quad (21)$$

Thus for meeting the two-axis requirement, individual CMG momentum should be chosen more than $26.995 \text{ N} \cdot \text{m} \cdot \text{s}$ for both the pyramid and roof-type clusters. Suppose that we are selecting $h = 30 \text{ N} \cdot \text{m} \cdot \text{s}$; the skew angle for the pyramid and roof-type clusters is calculated as follows:

Pyramid Cluster:

$$\beta = \cos^{-1} \left[\sqrt{\frac{H_x^2 + H_y^2}{4h^2}} - 1 \right] = 36.9^\circ$$

Roof-type Cluster:

$$\beta = \cos^{-1} \left[\frac{H_y}{\sqrt{(4h)^2 - H_x^2}} \right] = 37.5^\circ \quad (22)$$

The corresponding yaw angular momentum is computed as follows:

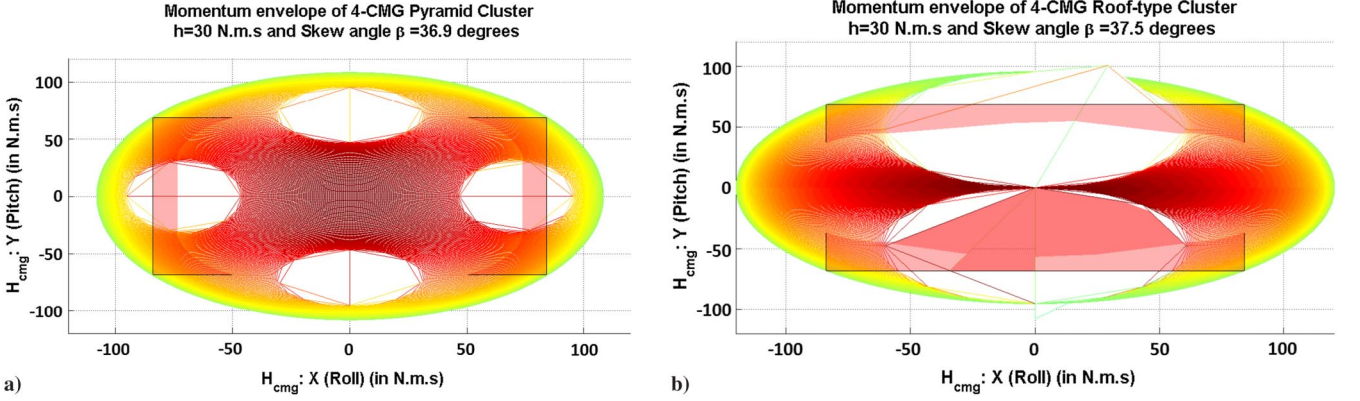


Fig. 13 CMG angular momentum envelope for pyramid and roof-type cluster with spacecraft requirement shown by red cuboid X - Y view (method 2). a) Pyramid cluster: $h = 30 \text{ N} \cdot \text{m} \cdot \text{s}$ and $\beta = 36.9^\circ$. b) Roof-type cluster: $h = 30 \text{ N} \cdot \text{m} \cdot \text{s}$ and $\beta = 37.5^\circ$.

Pyramid Cluster:

$$H_z = 4h \sin \beta = 4.30 \cdot \sin(36.9) = 72.1 \text{ N} \cdot \text{m} \cdot \text{s}$$

Roof-type Cluster:

$$H_z = 4h \sin \beta = 4.30 \cdot \sin(37.5) = 73.1 \text{ N} \cdot \text{m} \cdot \text{s} \quad (23)$$

The maximum value about the yaw axis is more than the mission requirement of $38.4 \text{ N} \cdot \text{m} \cdot \text{s}$ for both pyramid and roof-type clusters. The momentum envelope plots were similar to the ones observed for method 1; the only difference is that the momentum envelope circumference passes beyond the corner points as seen in Figs. 13a and 13b.

Method 2 considers the two-axis momentum requirement and sizes the pyramid and roof-type clusters. The inequality obtained for both the pyramid and roof-type clusters implies that individual CMG angular momentum h should be one-fourth of the hypotenuse determined by H_x, H_y to exactly meet the corner angular momentum requirement for a 4-CMG cluster.

V. Internal Momentum Surfaces

CMG sizing using both methods 1 and 2 was done based on the momentum envelope, which represents the maximum momentum capability of the cluster. However, the selection of the cluster (after sizing) should consider the internal momentum surfaces as well. The most difficult problem in CMG clusters is the existence of geometric singularities. The most challenging of these is the internal elliptic singularity, which is inescapable [19,21]. All internal singularities happen within the momentum envelope, i.e., at cluster configurations in which the total momentum is well below the saturation value. If we are getting simpler and less complicated internal momentum surfaces it makes the control law and thereby the CMG operation more simple.

Consider Figs. 14 and 15, which show the internal momentum surface of the pyramid and roof-type clusters. According to Margulies and Aubrun [11], the combinatorial arrangements of the e_i 's for a 4-CMG cluster is given. There are 2^N singular surfaces for N -CMG cluster. Out of these, only $2^N/2$ surfaces are distinct. In the given figures, seven distinct singular surfaces are plotted considering a cluster of four CMGs. The saturation momentum surface considered earlier for CMG sizing is not given here. For the pyramid cluster, all the seven surfaces exist. Their projections on the x - y , y - z , and z - x projections can be seen in Figs. 14 and 15, whereas for the roof-type cluster, only five surfaces exist. The internal momentum surfaces for $e_i = [-1 \ 1 \ -1 \ 1]$ and $e_i = [-1 \ 1 \ 1 \ 1]$ do not exist for the roof-type cluster. Except for the internal surface corresponding to $e_i = [-1 \ -1 \ 1 \ 1]$, the rest of the surfaces is simple circles. So, effectively the roof-type cluster has only one major internal singularity surface compared to seven for the pyramid cluster.

These internal surfaces represent internal elliptic and hyperbolic singularities in case of the pyramid cluster and only hyperbolic singularities in case of the roof-type cluster [19]. Though all internal hyperbolic singularities cannot be escaped using null motion, the simplicity of the internal momentum surfaces along with the absence of two out of the seven internal surfaces in a roof-type cluster makes it a preferred configuration for an agile spacecraft. The complicated geometry of the internal singular surfaces many of which are inescapable elliptic singularities weigh against the use of a pyramid cluster.

VI. Desaturation of CMGs

A. Momentum Dumping and CMG Null Motion

All momentum exchange actuators demand desaturation when they reach their maximum capability. It is well known that once the RW reaches its maximum allowable speed (angular momentum), it is brought back to its nominal operating speed by deceleration using the spin motor. The disturbance torques that act on the spacecraft body during this operation are canceled out by the attitude control thrusters. This is also termed as “momentum dumping” [22]. Analogously for CMGs, the gimbal angle changes and the cluster reaches its maximum momentum capability. In this condition, the gimbal angles should be brought back to a nominal configuration so that the cluster is desaturated.

An ideal gimbal angle set will have a high singularity index (meaning that the cluster is well away from singular geometry) and a symmetric gimbal angle combination to obtain zero resultant angular momentum in the body frame. Unlike RWs, the gimbaling causes very high output torque in CMGs. It is also well established that null motion (gimbal rates that cause zero torque output) can be utilized for rearrangement of CMG gimbals [19,21,23]. The gimbal angle rearrangement using null motion and preferred gimbal angles illustrated by Vadali et al. will be applied in this case [24]. The commanded gimbal angles for rearrangement can also be obtained from the random sampling technique as explained in [25].

B. Gimbal Angle Desaturation

Consider a cluster of four CMGs. As explained earlier, they can be arranged as a pyramid or a roof-type cluster. Consider a condition in which both clusters are saturated about the y axis. The gimbal angle combination for this are $\gamma = [0^\circ \ -90^\circ \ 180^\circ \ 90^\circ]$ and $\gamma = [90^\circ \ 90^\circ \ -90^\circ \ -90^\circ]$ for the pyramid and roof-type cluster, respectively. Based on the eigenanalysis explained in [19,21], both are classified as external elliptic singularities. The equation for computing null motion based on the Jacobian matrix and preferred gimbal angles is given in Eq. (24) [24].

$$\dot{\gamma}_{\text{null}} = d[I_N - D^T(DD^T + \alpha I_3)^{-1}D](\gamma_{\text{pref}} - \gamma) \quad (24)$$

Here, $\dot{\gamma}_{\text{null}}$ is the null gimbal rate, I_N and I_3 are $N \times N$ (N = number of CMGs) and 3×3 identity matrix, and D is the Jacobian matrix of

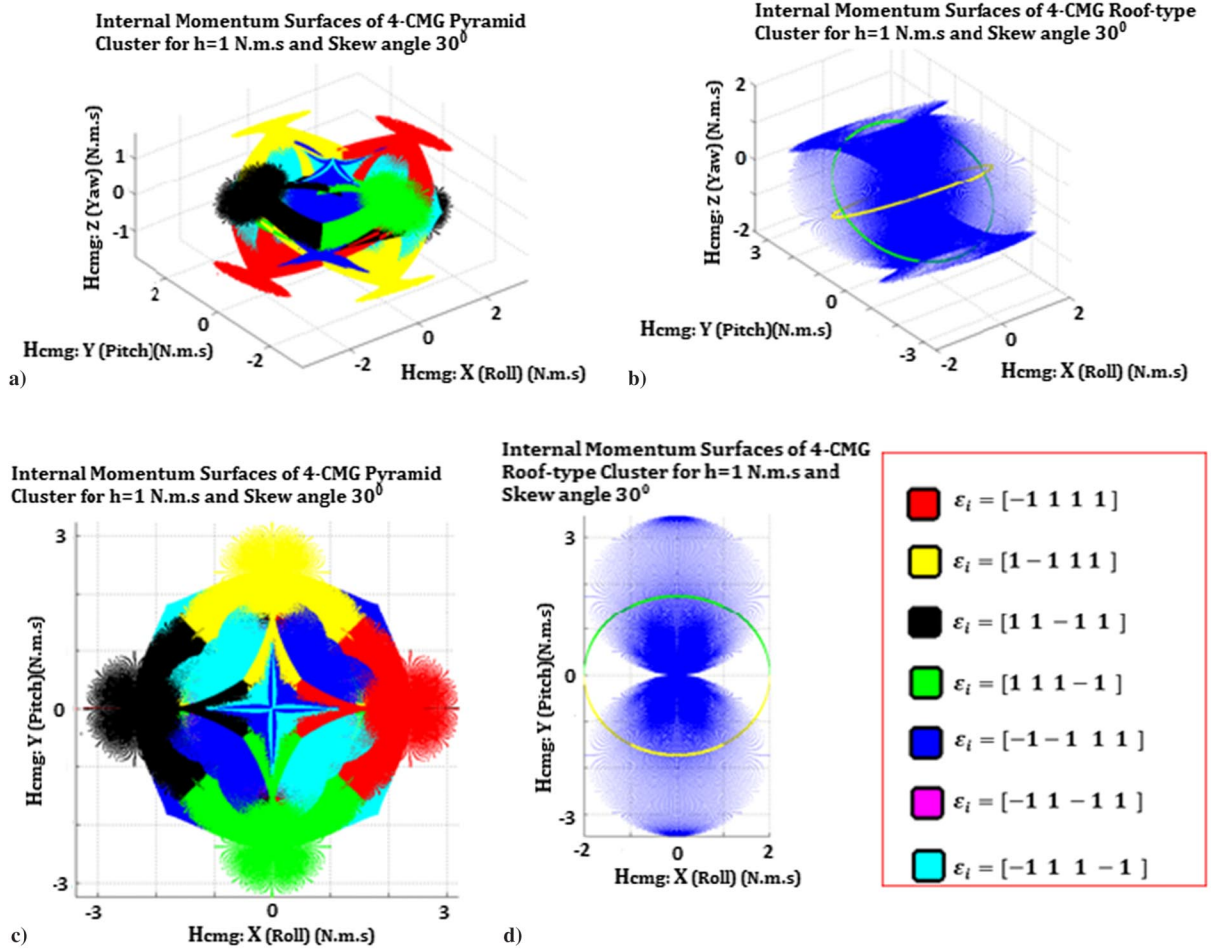


Fig. 14 Internal momentum surfaces for a 4-CMG pyramid and roof-type clusters with $h = 1$ and $\beta = 30^\circ$: isometric and X-Y view.

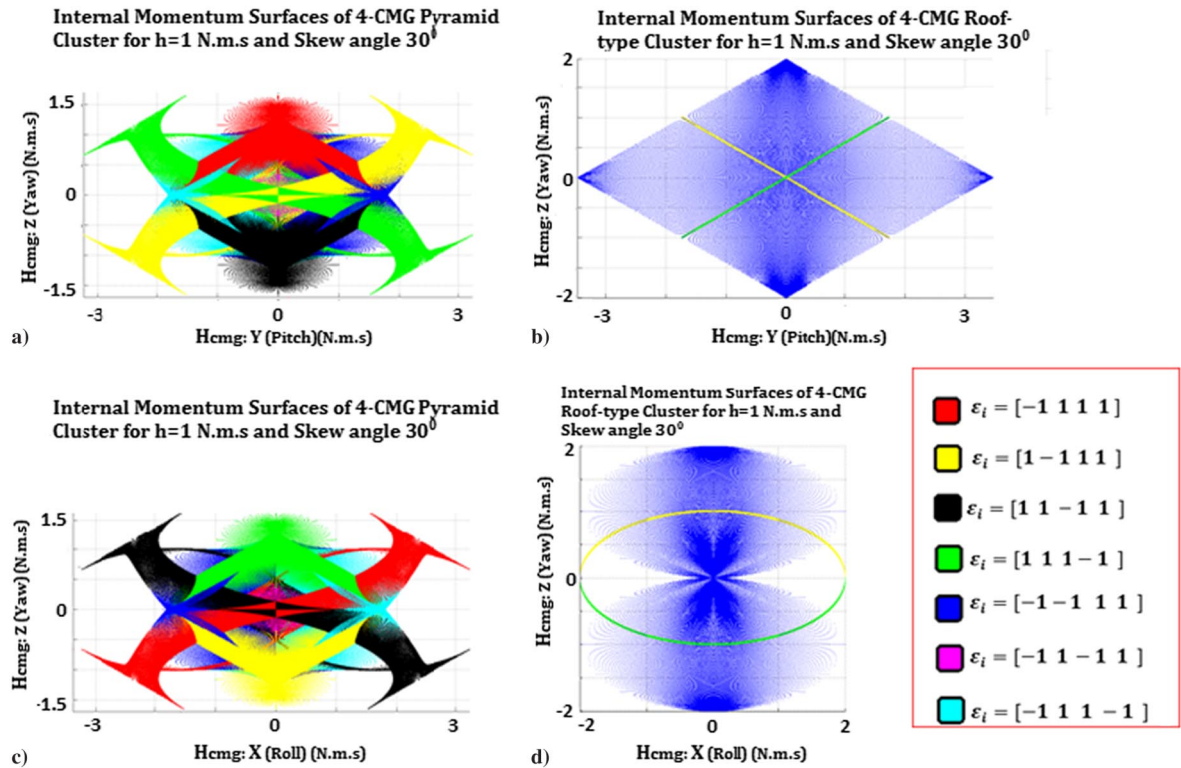


Fig. 15 Internal momentum surfaces for a 4-CMG pyramid and roof-type clusters with $h = 1$ and $\beta = 30^\circ$: Y-Z and X-Z view.

the CMG cluster. The Jacobian matrix is obtained by differentiating the total CMG angular momentum with respect to the gimbal angles γ . In this case, it is obtained as the derivative of Eqs. (6) and (7) for the pyramid and roof-type cluster, respectively [19,21]. The scalar term α is the correction factor that makes the DD^T matrix invertible at singular gimbal angle configurations. The parameters of α are defined in [23] in terms of the singularity index $m = (1/h)\sqrt{DD^T}$ and is called singularity robust inverse steering law. As explained in [24], α is used here to invert the DD^T in the null motion steering law. The singularity index m is a metric that represents how far the CMG cluster is away from a singular geometry. When the cluster is near singular, m is close to zero and it increases close to 1 when the cluster is away from the singular gimbal angle configuration.

The instantaneous gimbal angles are given by γ and γ_{pref} is the preferred gimbal angle (a constant) chosen before executing the gimbal angle desaturation. The difference between the preferred and current gimbal angles is projected onto the null space of the Jacobian matrix D and its scaling is determined by the scalar variable d . Hence during the start of desaturation, the angle difference will be high and the steering law will compute a large gimbal rate, which becomes less as the angle difference decreases. The parameters and conditions used for the numerical simulation are listed in Table 4.

The simulation results of the gimbal angle saturation are illustrated from Figs. 16–21. The performance of a cluster of four CMGs in

pyramid and roof-type arrangement on a rigid spacecraft is compared. Both the clusters are assumed to be at a saturation condition, with maximum momentum about the y axis. In this situation, they are desaturated by commanding the CMGs to go to a preferred set of gimbal angles using null motion [refer to Eq. (24)]. Ideally the null motion should not give any torque output while rearranging the gimbals. However, this is valid only in case of an internal hyperbolic singularity [14,19,21]. But CMG saturation represents an external elliptic singularity that cannot be overcome using null motion. Consequently, the null motion will result in change in CMG angular momentum and will impart torque onto the spacecraft. To stabilize the spacecraft attitude, thrusters are used for attitude control. It is assumed that the spacecraft has N_2 -based cold-gas thrusters (specific impulse is 70 s [26]), capable of giving a maximum torque of 1 N · m (2 N thrusters at a moment arm of 0.5 m) about each of the three body axes.

The time history of gimbal angles during the CMG cluster desaturation is shown in Fig. 16. “Pyrmid” and “Roof” are assigned for the cluster of four CMGs in pyramid and roof-type arrangement, respectively. The initial gimbal angle of the pyramid cluster is $\gamma_{\text{init,pyrmid}} = [0^\circ \ -90^\circ \ 180^\circ \ 90^\circ]^T$, which is rearranged to $\gamma_{f,\text{pyrmid}} = [17.8^\circ \ -62.3^\circ \ 166.2^\circ \ 50.1^\circ]^T$. For the roof-type cluster the corresponding values are $\gamma_{\text{init,roof}} = [90^\circ \ 90^\circ \ -90^\circ \ -90^\circ]^T$ and $\gamma_{f,\text{roof}} = [74^\circ \ 38^\circ \ -38^\circ \ -74^\circ]^T$.

Recall that the preferred gimbal angles for this rearrangement were chosen as $\gamma_{\text{pref}} = [45^\circ \ -45^\circ \ 45^\circ \ -45^\circ]^T$. The gimbal rate computed by the null motion law [refer to Eq. (24)] decreases as the difference between the gimbal angles decreases. It also depends on the evolution of the null projection matrix $([I_N - D^T(DD^T + \alpha I_3)^{-1}D])$, which itself is a function of the gimbal angles. The gimbal angles reach a steady-state value when the null motion law gives near-zero gimbal rates. Hence the final angles attained are different from γ_{pref} for both the pyramid and roof-type clusters. However, our aim here is to rearrange the CMG cluster from saturation singularity. Hence the absolute values of the final angles are not important.

The evolution of gimbal rates during the CMG cluster desaturation is given in Fig. 17. For sake of clarity, only the first 5 s are shown because the maximum value of gimbal rate is below $1^\circ/\text{s}$ from 5 to 500 s. The peak rates are attained between $t = 0.1$ s and $t = 0.3$ s for both the pyramid and roof-type clusters. For the pyramid cluster, CMGs 2 and 4 attain a peak rate of $40.7^\circ/\text{s}$ and $-39.3^\circ/\text{s}$ at $t = 0.16$ s. CMGs 1 and 3 gimbal rates remain below $3^\circ/\text{s}$. For the roof-type cluster, the peak gimbal rates are $\dot{\gamma}_{\text{roof}} = [-15.3 \ -45.9 \ 45.8 \ 15.3]^T \text{ deg/s}$ at $t = 0.22$ s.

The variation in the singularity index m during the CMG cluster desaturation is given in Fig. 18. As explained earlier, this will be zero

Table 4 Parameters used for CMG desaturation simulation

Sl. No.	Parameter	Value
1	CMG cluster	Pyramid and roof-type
2	CMGs	4
3	Skew angle, β	30°
4	CMG momentum, h	$25 \text{ N} \cdot \text{m} \cdot \text{s}$
5	Initial angle: pyramid	$[0^\circ \ -90^\circ \ 180^\circ \ 90^\circ]^T$
6	Initial angle: roof	$[90^\circ \ 90^\circ \ -90^\circ \ -90^\circ]^T$
7	Preferred angle, γ_{pref}	$[45^\circ \ -45^\circ \ 45^\circ \ -45^\circ]^T$
8	Gimbal rate (max.)	$45.8^\circ/\text{s}$
9	Null motion weight, d	1
10	PID gains: c, k, T	$5.78 \text{ N} \cdot \text{m}/(\text{rad/s}), 11.3 \text{ N} \cdot \text{m}/\text{rad}, 2.5 \text{ s}$
11	Thruster torque (max.)	$1 \text{ N} \cdot \text{m}$ (per axis)
12	Thruster moment arm	0.5 m
13	Specific impulse	70 s (N_2 cold gas)
14	Total time	500 s
15	Integration time step	0.01 s

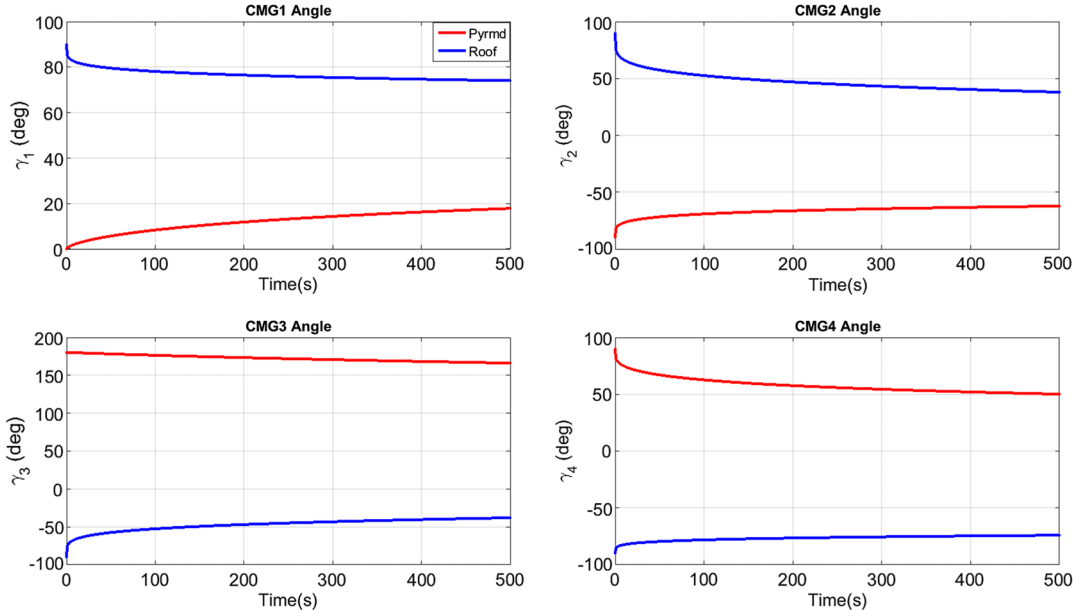


Fig. 16 Time history of CMG gimbal angles during desaturation of pyramid and roof-type clusters using null motion.

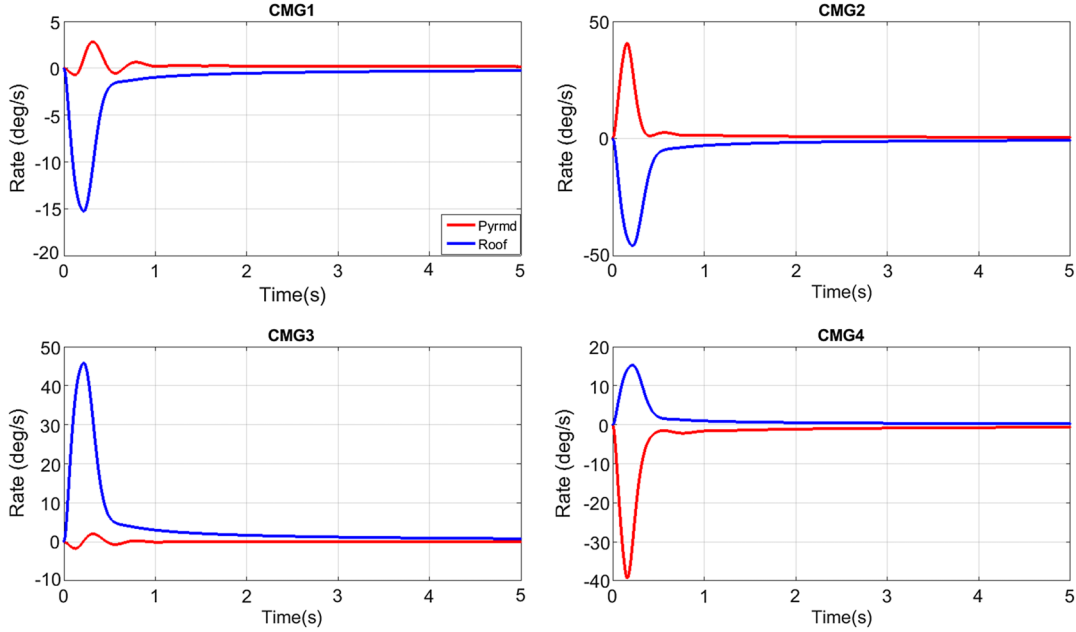


Fig. 17 Time history of CMG gimbal rates during desaturation of pyramid and roof-type clusters using null motion (first 5 s are shown).

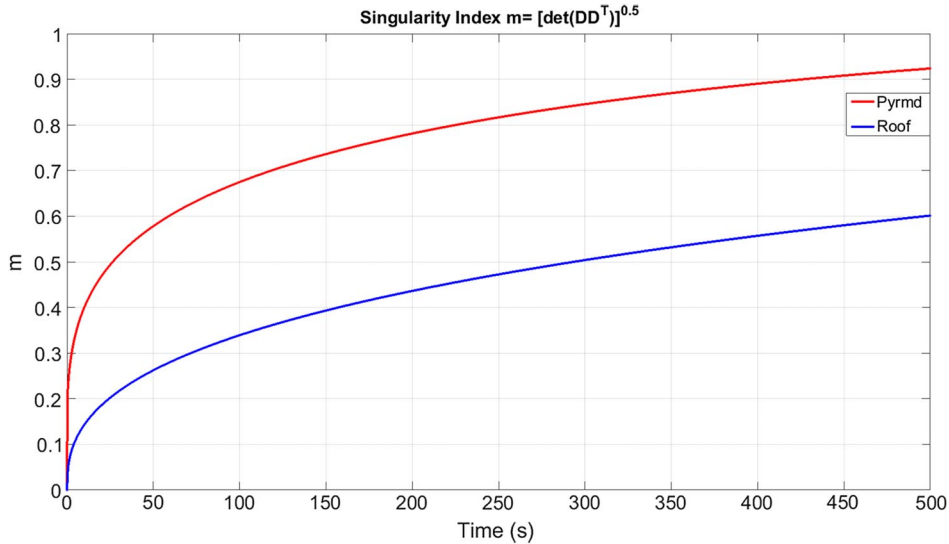


Fig. 18 Time history of singularity index m during desaturation of pyramid and roof-type clusters using null motion.

at the singular condition and increases near one as the cluster moves away from singularity. Both the pyramid and roof-type clusters start from saturation singularity condition where $m = 0$. At the end of 500 s, the final values are $m = 0.9$ and $m = 0.6$ for the pyramid and roof-type clusters, respectively. This means that both the clusters are desaturated with the pyramid cluster moving further away from singularity than the roof-type arrangement.

The null motion will not result in a zero disturbance torque when used at or near the vicinity of a saturation singularity. Thus the CMG disturbance torque acting on the spacecraft body during the gimbal angle rearrangement is illustrated in Fig. 19. Since the gimbal rates decrease after 5 s, only the first 10 s are shown. The CMGs in pyramid cluster imparts a disturbance torques about all three axes, with a maximum value of $2.3 \text{ N} \cdot \text{m}$ about the pitch axis. It then decreases to less than $0.2 \text{ N} \cdot \text{m}$ after 5 s. The roof-type cluster causes disturbance torques about the pitch axis only with a maximum value of $5.1 \text{ N} \cdot \text{m}$, which also decreases to $0.24 \text{ N} \cdot \text{m}$ after 5 s. The width of the peak disturbance torque is marginally higher for the roof-type cluster (by 0.2 s) compared to pyramid arrangement.

The torque demanded from spacecraft thrusters for stabilizing its attitude against the CMG disturbance torque is given in Fig. 20. The thrusters have a maximum capability of $1 \text{ N} \cdot \text{m}$ per axis. Corresponding to the disturbance torque imparted by the CMG cluster, torque is demanded about all three axes for pyramid arrangement with peak requirement about the pitch (y axis). Recall that the maximum disturbance torque imparted by the pyramid cluster about the pitch axis was $2.3 \text{ N} \cdot \text{m}$. Hence the thruster torque gets saturated about this axis. The peak thruster torque about the roll (x axis) and yaw (z axis) are 0.05 and $0.17 \text{ N} \cdot \text{m}$, respectively. For the roof-type arrangement, the peak torque is demanded about the pitch axis alone. The thruster gets saturated in the negative direction from $t = 0.2 \text{ s}$ to $t = 4 \text{ s}$. Then it changes sign and remains saturated in the positive direction for 0.2 s . It reduces to $-0.2 \text{ N} \cdot \text{m}$ after $t = 6 \text{ s}$.

The N_2 -based cold-gas thrusters have 2 N capability and a moment arm of 0.5 m . The total fuel consumed for this maneuver can be calculated by computing the total impulse imparted from the thruster. The expression for fuel consumption is given in [26] as

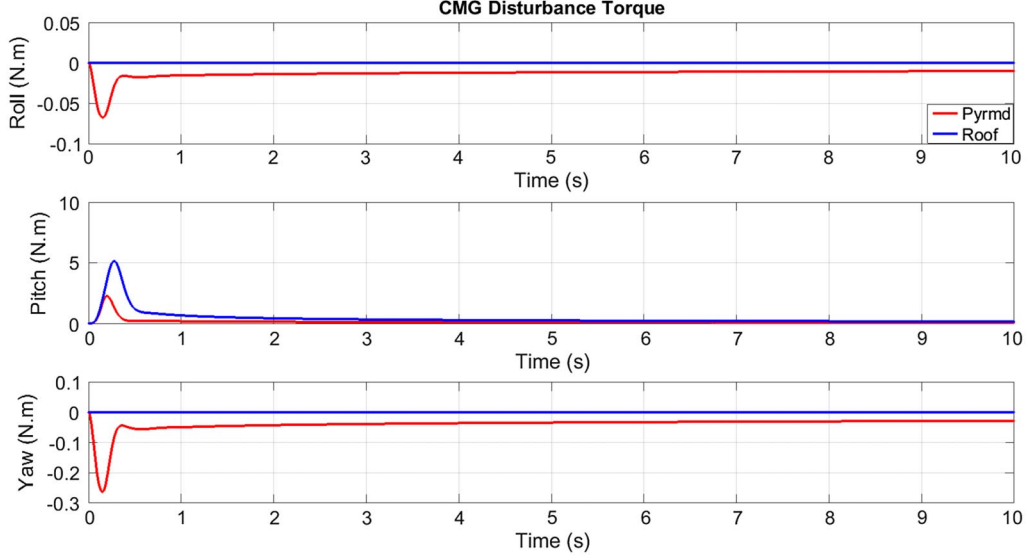


Fig. 19 Time history of disturbance torque imparted onto the spacecraft by CMGs during desaturation of pyramid and roof-type clusters using null motion (first 10 s are shown).

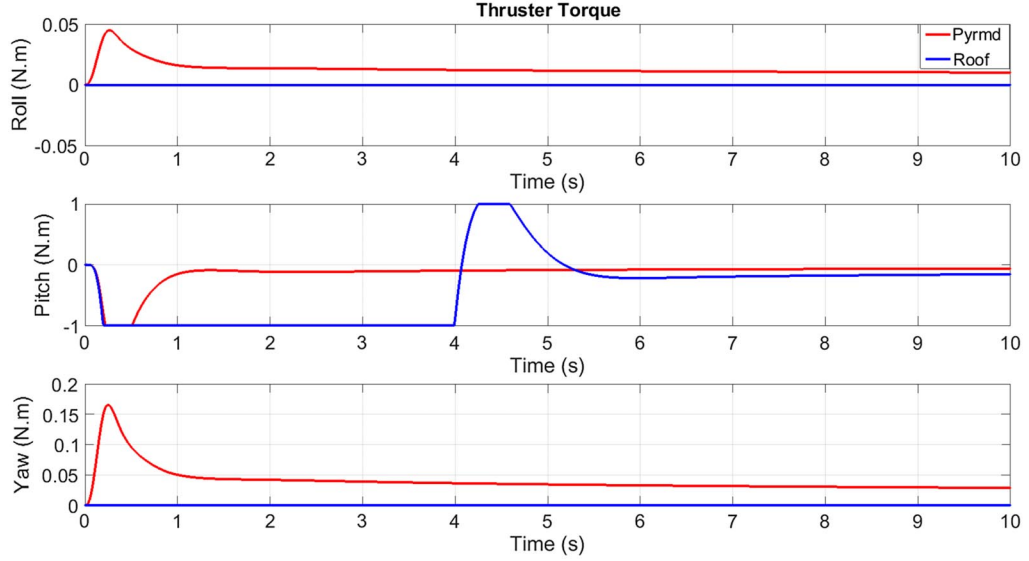


Fig. 20 Time history of thruster torque demanded to maintain the spacecraft attitude during CMG desaturation of pyramid and roof-type clusters using null motion (first 10 s are shown).

$$M = \frac{|H_t|}{g I_{sp} L} \times (1000) \quad (25)$$

Here M is the mass of the propellant in grams, $|H_t|$ is the absolute value of the angular momentum imparted from thruster in $\text{N} \cdot \text{m} \cdot \text{s}$, g is the acceleration due to gravity in m/s^2 , I_{sp} is the specific impulse in seconds, and L is the moment arm in meters. The I_{sp} for N_2 -based cold-gas propulsion is given as 70 s in [26]. This can be calculated for each axis and summed to get the total propellant mass in grams. Figure 21 shows the propellant consumption in grams while executing the gimbal angle desaturation. At the end of 500 s, the total mass consumed for the pyramid cluster desaturation is 49.2 g against 53.4 g for the roof-type arrangement. The propellant mass demanded by roof-type arrangement is higher than that of the pyramid arrangement by 8.5%. These results cannot be generalized as it is highly dependent on the initial gimbal angles and the preferred gimbal angles chosen for rearrangement. It is worth noting that choosing an appropriate preferred gimbal angle that results in lower disturbance torque, higher singularity index, and lower

propellant consumption based on the saturation geometry is an open research problem.

VII. Conclusions

The sizing of a minimally redundant cluster of four CMGs in pyramid and roof-type arrangement is done based on a typical mission requirement. Spacecraft momentum demand is unequal about the body axes—determined by the difference in inertia and peak rate requirement. Consequently, a near-spherical momentum envelope for a 4-CMG pyramid cluster is of little practical use. Agile spacecraft have large momentum requirement in the roll–pitch (x – y) plane with a yaw demand only to counter the kinematic coupling during a 2-axis maneuver.

Two sizing methodologies are proposed for finding h and β for a 4-CMG cluster based on the spacecraft momentum requirement. In method 1, the maximum values from the momentum envelopes were used to size the cluster. This gave a set of h and β . Then to meet the two-axis momentum requirement in the roll–pitch plane, the envelope was scaled by changing h alone retaining the same β .

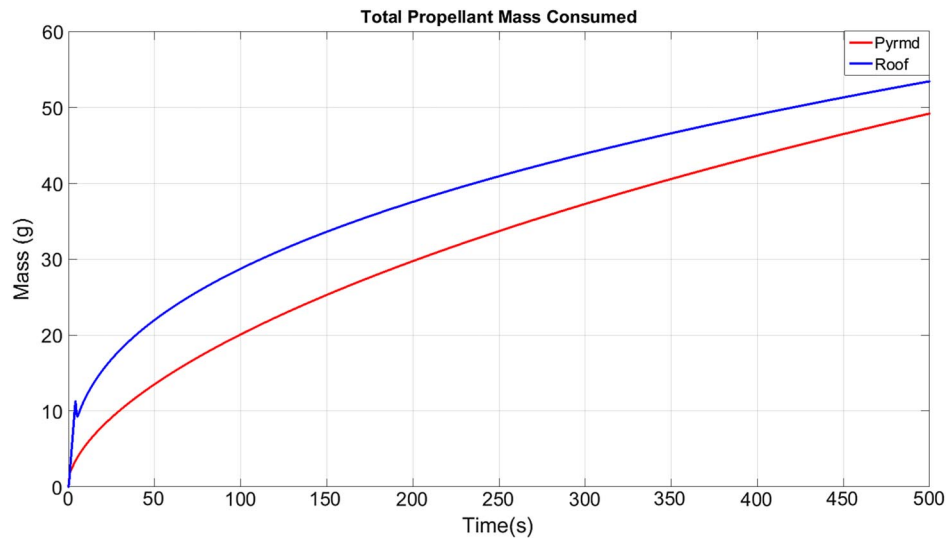


Fig. 21 Time history of propellant mass consumed during the thruster firing for maintaining the spacecraft attitude during gimbal angle desaturation of pyramid and roof-type clusters using null motion.

This arrangement was able to fulfill 88% of the two-axis mission angular momentum requirement.

In method 2, the two-axis momentum requirements were taken together to size the cluster by considering the momentum envelope projection on the x - y plane as a circle or an ellipse. This could fulfill the mission angular momentum requirement with 20% more angular momentum per CMG and an increase in skew angle by 8 – 11° compared to method 1. The 20% increase in momentum increases the electrical power consumption by 11%. In addition, a study of the internal momentum surfaces inside the roof-type and pyramid clusters was done. The former has only one major internal singular surface compared to seven inescapable internal momentum surfaces for the latter.

Lastly, the gimbal angle desaturation is done using CMG null motion and preferred gimbal angles. For a typical case of y -axis saturation and same set of preferred gimbal angles, the pyramid cluster gave better singularity index and lower propellant consumption (8.5% less). But this result is not generic and is highly dependent on the initial condition and the selection of the preferred gimbal angles.

Hence various aspects have to be considered while sizing a CMG cluster. Though primarily done on the basis of the spacecraft momentum requirement and the momentum envelope geometry, the importance of the internal momentum surfaces, electrical power, and fuel consumption should be considered to select and finalize a cluster configuration. The roof-type cluster meets the nonspherical mission momentum demand. Though it consumes more fuel for a typical gimbal angle desaturation, it has only one major escapable internal singular surface. This helps in easier singularity avoidance during CMG operation. A $h = 25 \text{ N} \cdot \text{m} \cdot \text{s}$ wheel consumes 11% less power compared to a wheel operating at $h = 30 \text{ N} \cdot \text{m} \cdot \text{s}$. Hence a 4-CMG roof-type cluster with $h = 25 \text{ N} \cdot \text{m} \cdot \text{s}$ and $\beta = 30^\circ$ is suitable for the agile spacecraft considered in this study. The effectiveness of this 4-CMG roof-type cluster can be further studied in future by using it as the main attitude control actuator to perform multiple landmark acquisition and tracking maneuvers on a typical agile spacecraft.

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