

A STUDY OF ELECTRICAL BEHAVIOUR OF
THICK FERRITE RODS IN RECTANGULAR WAVEGUIDES

Thesis presented for the
Diploma of Imperial College

by

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ABSTRACT

This thesis is concerned with the experimental investigations of the so called Reggia-Spencer phase shifter, which consists of a thick longitudinally magnetised ferrite rod placed along the axis of a rectangular waveguide.

The experiments were aimed at establishing the validity or otherwise of the existing approximate theories dealing with the principle of operation of the structure. It was found that the phenomenological model proposed by Weiss describes the characteristics of the structure most accurately. The behaviour of the structure with variations of different parameters was also studied theoretically, with the help of the model. The results showed that the phase shifter fits in as the intermediate stage between the elliptical Faraday rotator and the anomalous mode waveguide.

Some aspects of the structure, noticed during the experimental investigations, were found to be very suitable for the development of useful devices. An electrically tunable band pass filter was developed, capable of being tuned over a range of 10% of the mid-band frequency. Another new device investigated was a very narrow band isolator, tunable over at least 200 Mc at x-band.

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ERRATUM

For "decibells" read "decibels".

CHAPTER ONE

INTRODUCTION

1.1 Outline of the Subject of Research.

The behaviour of a thick ferrite rod centrally placed along the axis of a rectangular H_{01} -mode waveguide and magnetised longitudinally, has excited the curiosity of microwave engineers since it was first reported by Bush⁽¹⁾ in 1956. The most important characteristic of such a configuration is the unaccountably large incremental phase delay of the propagating electromagnetic wave obtained as the magnetisation of the ferrite is increased from zero to saturation and beyond. The phenomenon starts when the ferrite loading of the waveguide cross-section exceeds a certain limit, typically about 10%. Up to a certain ferrite rod diameter, corresponding to about 20% loading of the cross-section, the insertion loss can be kept down to fairly low values if the ferrite rod ends are well matched to the empty waveguide. These limits of rod diameter are very much frequency dependent, decreasing as the frequency of the wave is increased. Within this range the device is reciprocal, the phase delay increasing with the application of the magnetising field, for both directions of propagation. The phase shift increases rapidly up to a value of the applied field corresponding to a little over saturation and then decreases slowly with further increase in the field. The maximum phase shift obtainable increases sharply with ferrite rod diameter in this range and is

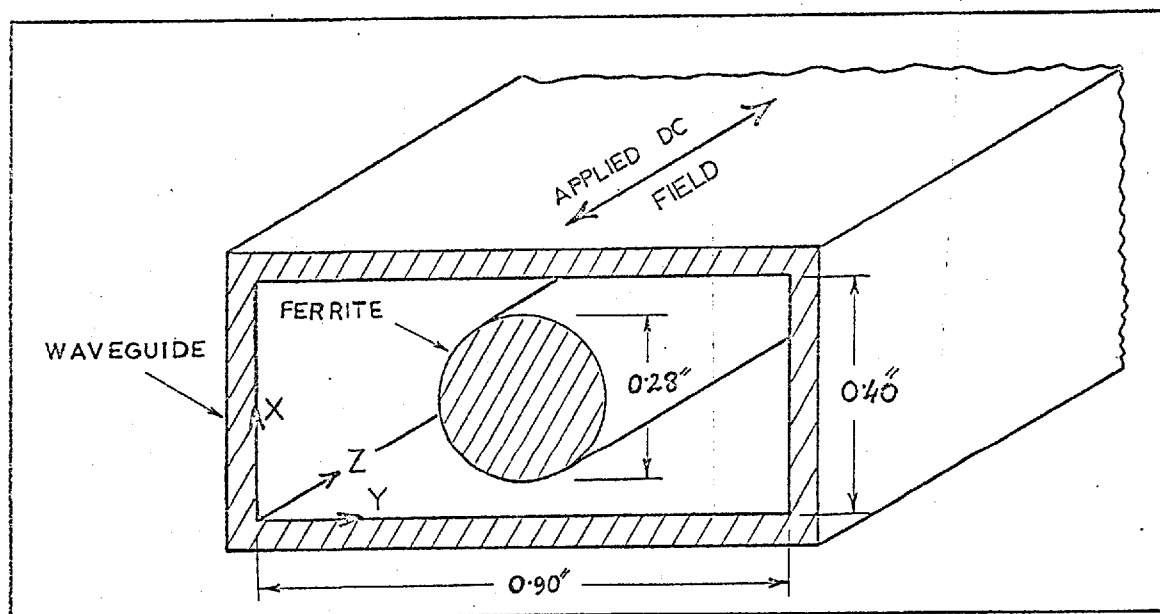


Figure 1.1. Geometry of the reciprocal ferrite phase shifter showing typical dimensions suitable for X-band.

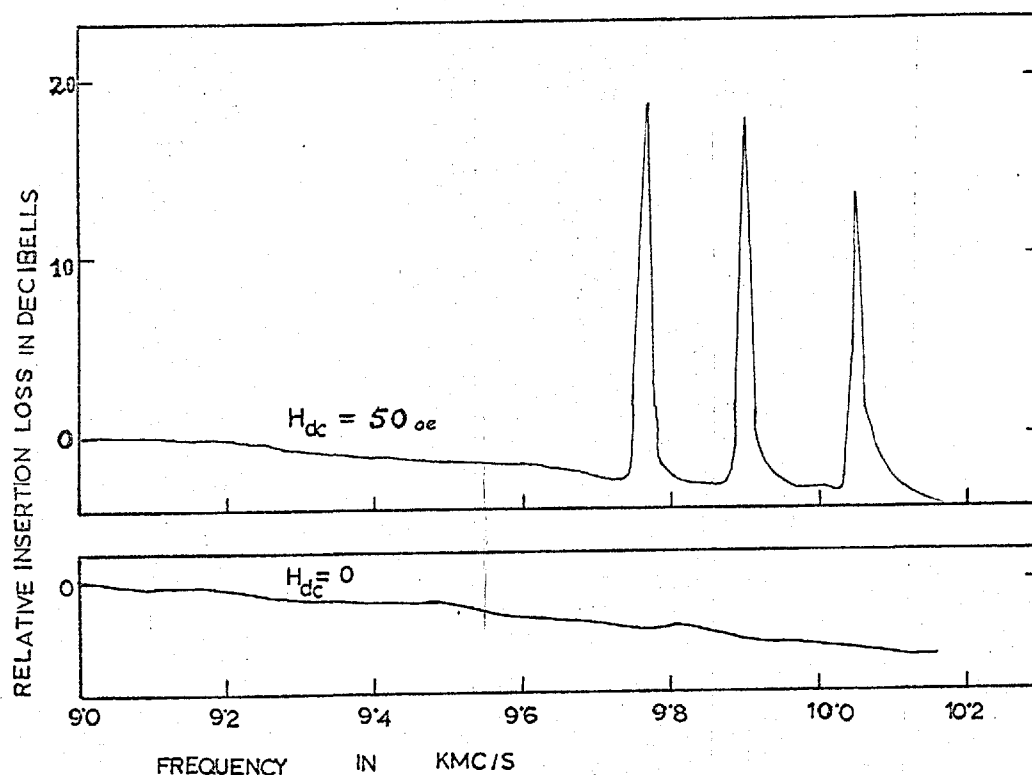


Figure 1.2. Insertion loss of the reciprocal phase shifter as a function of frequency. Mg-Mn ferrite rod, .276" dia., .551" long with 1.34" tapers used. (After Neckenbürger⁽³⁾; the decrease in insertion loss with frequency stated to be due to detector.)

approximately constant per unit length of the rod. If the ferrite rod diameter (or the frequency) is increased beyond this range, the increase in phase shift is markedly reduced and large fluctuations in the transmitted power, as would be associated with Faraday rotation, begin to occur.

Figure 1.1 shows the geometry of the reciprocal ferrite phase shifter. The arrangements for matching the ends of the ferrite rod to the empty waveguide (usually by tapering the rod) and for supporting the rod centrally in the waveguide (by polyfoam or teflon), have been omitted. A typical frequency response of the insertion loss of this device is given in figure 1.2, showing the high peaks above a certain frequency when the ferrite is magnetised. Figure 1.3 shows typical curves of phase shift, insertion loss and voltage standing wave ratio obtainable from this phase shifter as the applied dc magnetic field is varied.

The large reciprocal phase shift for small changes in the applied magnetic field and the small insertion loss of this configuration were reported by Bush at the "Convention on Ferrites" held under the auspices of the Institution of Electrical Engineers (London). He mentioned that best results were obtained for centrally placed cylindrical ferrite rods. Higher phase shifts could be obtained with the rod near the waveguide wall, but the voltage standing wave ratio was then high. At the same convention Bowness⁽¹¹⁾ mentioned the use of a similar arrangement for phase modulation and noted that it had a highly frequency dependent phase shift characteristic.

Reggia and Spencer⁽²⁾ (1957) made an exhaustive study of the properties of the structure at X-band. They noticed the highly

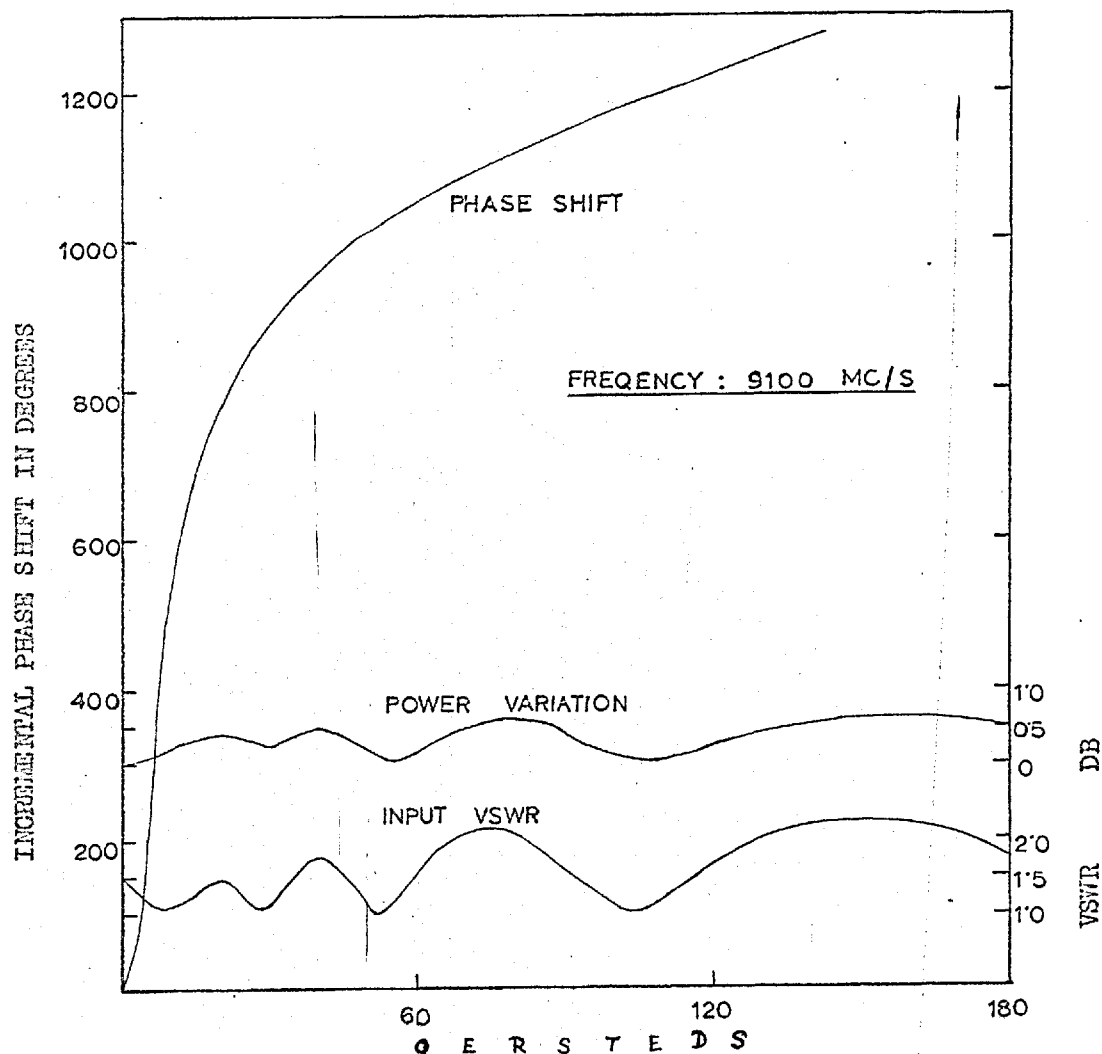


Figure 1.3. Typical performance curves of the reciprocal phase shifter. Data is for Ferramic R1 (Mg-Mn) ferrite rod, .270" in dia. and 5.5" long. (After Reggia and Spencer⁽²⁾.)

dependent character of the phase shift on ferrite rod diameter mentioned earlier. This can be clearly seen from figure 1.4, where the various ranges of the ferrite-rod-diameter are marked according to the phase shifting and insertion loss behaviour of the structure. The diameter at which the phase shift sharply increases has the same value at all intensities of the applied magnetic field. The maximum phase shift per unit length was found to be independent of the length of the specimen. Better matching and equivalent phase shift behaviour was observed with square rods of slightly greater cross-sectional area. More than 300° of phase shift with about 0.1 dB loss per inch was obtained. The phase shift was found to increase slowly with frequency in the operational range of the rod-diameter. They also demonstrated by various experiments that a high degree of energy concentration in the ferrite rod occurs, even without any magnetisation, when the rod diameter exceeds the lower limit of the phase-shifter region shown in figure 1.4. Weiss⁽⁶⁾ (1959) showed that the phase shift is much less in a reduced-height guide, and found that the amount of phase shift obtainable is roughly proportional to the saturation magnetisation of the ferrite specimen.

The propagation of electromagnetic waves in this structure has not as yet been rigorously analysed and there is considerable doubt about the mode or modes of propagation. The mechanism of the large phase delay, obtained without any accompanying Faraday rotation type fluctuation in the transmission, is also not completely understood. The work described in this thesis was undertaken with a view to obtaining a clearer insight into the nature of wave propagation in this system and to developing

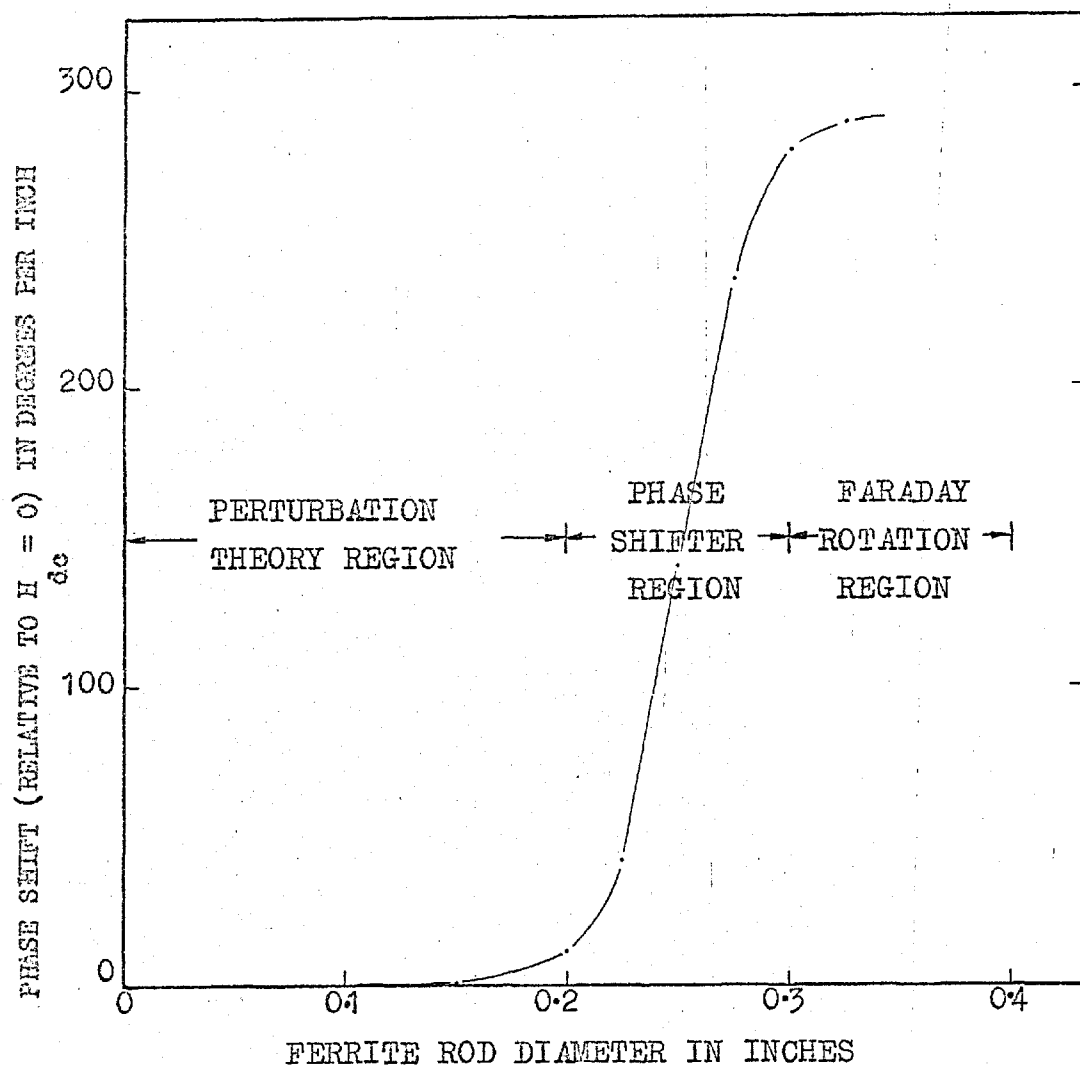


Figure 1.4. Phase shift versus ferrite rod diameter for the configuration of figure 1.1. Data taken at 9100 Mc/s for 4" long Ferramic R1 ferrite rods. Longitudinally applied dc magnetic field (H_{dc}) = 100 oersteds. (From the data of F. Reggia and E. G. Spencer⁽²⁾).

useful microwave devices, based on the properties of the configuration.

1.2 Devices Based on the Structure.

The first investigation of this configuration by Bush⁽¹⁾ was carried out with a view to the use of the phase shift properties of the structure to tune microwave valves. Bowmess⁽¹¹⁾ used the structure to phase modulate microwave signals at modulation frequencies up to 2500 c/s. He used slotted rectangular waveguides to prevent eddy-currents and pointed out the need for a narrow hysteresis-loop to reduce the temperature rise of the ferrite. Brown⁽¹⁷⁾ et al (1958) reported rapid temperature rise of the ferrite at 350 kc/s modulation frequency in a similar device. Reggia and Spencer⁽²⁾ made a thorough study of such phase modulators at X-band using Mg-Mn ferrite (R1) rods of round and square cross-section. Techniques for matching the ends of the rods to empty waveguide were also investigated. In 1961 Reggia⁽¹⁸⁾ reported the development of phase modulators over a frequency range covering 3 - 70 kMc/s. He presented the performance and design data of a modulator suitable for 23 - 25 kMc/s in which Ni-Zn ferrite (Saturation Magnetisation, $4\pi M_s \approx 5000$ gauss) rods were used. More than 3000° of phase shift was obtained per dB of maximum insertion loss. Mg-Mn ferrite ($4\pi M_s \approx 2000$) gave very much lower phase shifts at these frequencies. McCarter and Landry⁽¹⁹⁾ (1961) described a similar device for operation at $35 \text{ kMc/s} \pm 500 \text{ Mc/s}$.

The use of this configuration in microwave systems for electrical

switching, aerial beam scanning and changing radiation patterns was suggested by Reggia and Spencer⁽²⁾. They reported the use of modulation frequencies in the kilocycle range. Scanning angles as large as 40° were obtained by them, with variations of less than ± 2 dB in the main beam amplitude. Goodwin and Senf⁽²⁰⁾ (1959) used seventy two of these phase shifters to scan the beam of an array used for X-band radars through a solid angle of 42° by 56° . A total power of 25 watts only was necessary to control the phase shifters. A scan angle of 90° by 90° was considered feasible by them and the main problem in this use of the phase shifter was considered to be its temperature sensitivity. Clavin⁽³⁴⁾ (1960) was able to compensate for the temperature variation of phase, over -30°C to $+100^\circ\text{C}$ at 16.2 kMc/s, by providing a separate compensated bias coil.

An amplitude modulator utilising the fact that cross-polarized waves are excited in the system when the ferrite is magnetised was first developed by Bowness⁽¹⁵⁾ et al. They placed resistive cards by the side of the rod in a plane perpendicular to the input rf electric field to absorb this cross-polarized component. Tompkins⁽¹⁶⁾ et al mentioned the use of a resistive film between the sections of a split ferrite rod and reported isolation of over 40 dB with 0.5 dB zero-field insertion loss and less than 1 microsecond switching time. Reggia⁽²¹⁾ (1961) carried out a full investigation of this absorption switch at X-band. Ferrite rod length and cross-section and the resistivity of the film were optimised experimentally to obtain more than 60 dB isolation, 0.5 dB insertion loss and negligible phase shift

over a 30% bandwidth. The switch operated perfectly up to a temperature of about 150°C and remained well matched for all values of applied dc field. Bendix⁽²²⁾ obtained a switching time of 40 nanoseconds at X-band by removing the waveguide walls from the central region of the switch and winding the modulating coil directly on to the dielectric supporting the ferrite. Amplitude modulators for the range 21 to 28 kMc/s were reported by Reggia⁽²³⁾ (1964). Using Ni-Zn ferrite, isolation of over 75 dB was obtained with zero-field loss of 0.6 to 0.8 dB, and a switching time of 0.1 microsecond was achieved by techniques similar to Bendix's.

Maher⁽²⁴⁾ (1963) investigated the higher order modes at zero applied field which limit the bandwidth of operation of the amplitude modulator by causing reflection and absorption. From the fact that these losses disappeared in reduced-height guides, he concluded that this interfering mode is H_{10} in character rather than the expected H_{02} mode (E_{01} mode was rejected as being unable to exist in a rectangular waveguide). By reducing the waveguide height in the ferrite loaded region Maher developed a modulator to cover the frequency range of 8.2 to 12.4 kMc/s with a zero-field loss of less than 0.8 dB and an isolation of greater than 30 dB over the whole band. A description of the behaviour of the modulator, particularly the dependence of the isolation on optimum film resistivity was attempted by Helszajn⁽²⁵⁾ (1965), based on Miller's⁽²⁷⁾ theory of coupled transmission lines with unequal phase and attenuation constants.

A reflection type switch was proposed by Weiss^(28,29) (1960) in

which the input and output waveguides were mutually crossed, with a ferrite rod extending across the junction. Isolation as high as 60 dB in the demagnetised state, and less than 0.1 dB insertion loss over at least 5% bandwidth when the ferrite is magnetised, was observed, making the switch suitable for applications such as protection of masers. Suggestion for the development of switched isolators and circulators from this basic configuration was also made. This was based on the observation that at the centre plane of the junction the major axis of the elliptical wave-polarization, in transmitting state, is oriented at 45° with respect to the symmetry planes of the structure, and is non-reciprocal. This switch, named the "tetrahedral junction" by Weiss, was described by Hellszajn⁽³⁰⁾ (1965) in terms of the theory of the single tapered mode broad-band coupler, developed by Cook⁽³¹⁾ and Fox⁽³²⁾.

Various other devices, based on the reciprocal ferrite phase shifter geometry have been developed or proposed. Clavin⁽³³⁾ (1961) proposed a number of novel switches and a variable coupler based on the energy concentration effect and the excitation of cross-polarized waves in the structure. In one switch the energy was reflected by passing the ferrite rod through a narrow tube which was at cut-off when the ferrite was demagnetised. On the application of a large enough longitudinal dc magnetic field, the required intensity being inversely proportional to the frequency, propagation took place through the device. In another switch thin horizontal metal septa were placed by the side of the ferrite rod. The energy propagated freely when

the ferrite was demagnetised but was reflected when the ferrite became magnetised due to the excitation of cross-polarized waves. An absorption type switch was also proposed, in which lossy blocks placed by the side of the rod would absorb energy when the ferrite was not magnetised. On the application of a magnetising field the energy would be concentrated in the rod and would propagate without much loss. The variable coupler proposed by Clavin consisted of two waveguides coupled by conventional methods with a ferrite rod placed centrally in one of them. The field strength in the coupling region would be reduced when the ferrite was magnetised and it would thus be possible to vary the coupling coefficient continuously. Finally, a broad-band switchable circular polarizer has been proposed by Clarricoats⁽¹³⁾, based on the assumption that the mode propagating through the system when the ferrite is magnetised is a negative circularly polarized one. The ferrite rod would feed the energy into a circular waveguide and the sense of the output circular polarization could be switched by reversing the direction of the dc field.

1.3 Survey of Previous Investigations.

The novel characteristics exhibited by the reciprocal ferrite phase shifter have attracted many workers. An exact mathematical formulation of the problem is very complicated due to difficulties of the boundary conditions⁽⁴⁾⁽⁵⁾. This has led to the investigations of the problem by indirect and experimental methods.

In this connection Clavin⁽¹²⁾ (1958) found that ferrite slabs centered on either the narrow or the broad walls of the waveguide produce phase shifts of much smaller magnitude than a centrally placed rod of equivalent cross-section. The ferrite was magnetised parallel to the direction of propagation in all the cases.

Tompkins⁽⁴⁾ (1958) made a theoretical study of the propagation of circularly polarized waves along longitudinally magnetised ferrite rods in free space or when placed centrally inside a circular waveguide. It was found that modes with large phase constants, the ones which were negative circularly polarized with respect to the dc field, were very insensitive to the waveguide boundary. Tompkins pointed out that the propagation constant of the dominant mode (HE_{11} type hybrid mode^x) in this case is very similar to that of the reciprocal ferrite phase shifter for the same rod dimensions. It is to be noted, however, that the reciprocal phase shifter shows from 10 to 20% less phase shift than the systems studied by Tompkins propagating negative circular polarization.

Rizzi and Gatlin⁽⁷⁾ (1959) suggested that in the greater part of the structure the wave is negative circularly polarized and is confined to the vicinity of the ferrite. As a proof of their hypothesis they measured the insertion loss of a sharply twisted waveguide with the ferrite rod extending on both sides of the twist. They found the loss

^x In circular waveguides partially filled with longitudinally magnetised ferrite, longitudinal components of rf electric and magnetic fields must in general exist simultaneously.⁽¹³⁾

to be very small and independent of the angle of twist once the ferrite was magnetised to saturation. As to the mechanism of conversion to circular polarization, they suggested that the linearly polarized input wave is rotated by 45° when it meets the ferrite. This rotated wave can be considered as the combination of two waves with their polarizations parallel to the broad and narrow walls of the waveguide, respectively. The component wave polarized parallel to the broad walls has a higher phase velocity, so that, after some distance a phase difference of 90° occur between the two components, and the wave is converted to negative circular polarization. The two processes of 45° Faraday rotation and phase difference, occur simultaneously. The negative circularly polarized wave becomes concentrated in the ferrite and is not significantly affected by the waveguide boundary any more. Thus it remains circularly polarized up to nearly the output end where the reverse process takes place.

The high insertion loss peaks that occur in the phase shifter above a certain frequency were attributed by Clavin⁽¹⁴⁾ (1960) to the excitation of E_{01} mode in the ferrite loaded region. He measured the maximum diameter of ferrite rods at which loss peaks were just suppressed in such a phase shifter, using R1 ferrite at X-band, as a function of frequency. The diameters at which E_{01} mode can propagate in a circular waveguide filled with dielectric material of constant $\epsilon = 13$ (same as that of R1 ferrite) were also calculated. His calculated values of rod diameter, contrary to expectation, came out 5 to 9% lower than the measured values. He attributed this to the ferrite probably having a

lower dielectric constant and to the influence of the broad walls of the waveguide. Both of these arguments are rather questionable. Clavin also showed that the phase delay in such a structure increases appreciably with the increase in temperature. If used as a phase modulator, however, the peak-to-peak phase variation for a fixed change in magnetising current changes much more slowly in the same temperature range.

The excitation of cross-polarized components in the structure was first demonstrated by Bowness et al⁽¹⁵⁾ (1958) by placing resistance sheets parallel to the broad walls, by the sides of the ferrite rod. Attenuation of the propagating energy was found to increase sharply on magnetising the ferrite, when the rod diameter exceeded the lower limit of the phase-shifter region. A similar experiment was performed by Tompkins et al⁽¹⁶⁾ (1960) when they split the ferrite rod and put a resistive film inside it. It was found that when the ferrite was magnetised to saturation the insertion loss of the system was high and independent of the orientation of the film.

Neckenbürger⁽³⁾ (1962) carried out some important investigations on the structure. In one experiment he took the output from the phase shifter into a circular waveguide and measured the orientation of the polarization of the wave. He found the wave to be in general elliptically polarized, with very little rotation of the axes of the polarization-ellipse when the device operated in the phase-shifter range of frequencies. The axis of the ellipse parallel to the broad walls of the rectangular waveguide increased with increased magnetisation of the ferrite until the ellipse was converted into a circle. With

further increase of the dc field this axis became the major axis of the polarization-ellipse. When the frequency was increased beyond the point where insertion loss peaks begin to occur, large rotations of the axes were noticed. The output polarization in this case changed irregularly with the dc field, similar to that from an elliptical Faraday rotator. Neckenbürger also investigated the behaviour of the device while removing, one by one, the walls of the waveguide from the central section. Hardly any change in phase shift and attenuation characteristics was noticed when the narrow walls were removed. Phase shift increased slightly when one of the broad walls was also removed. On removal of all the four walls, large insertion loss peaks appeared and the phase shift was further increased, the total increase from the completely enclosed case being over 10%. With a resistive film in the middle of the ferrite rod parallel to the broad walls, high attenuation with applied field was observed for all the above cases. When all the four walls were removed sharp peaks were superimposed on the normal attenuation.

1.4 Theoretical Approaches.

The difficulties involved in a rigorous theoretical solution of the wave propagation in the system have led investigators to adopt various approximate methods to represent and analyse the behaviour of the structure. These have included the following:

- (a) Accounting for the waveguide boundary by images of the

"dielectric waveguide".

- (b) Representation of the electrical anisotropy of the system by an elliptical cross-section of the ferrite rod.
- (c) Application of the perturbation theory to a waveguide loaded with a full-height ~~central~~ ferrite slab.
- (d) Representation of the anisotropy of the geometry by a hypothetical, variable "tensor permittivity".

A brief account of these widely varying approaches is given below.

1.4.1 Analysis by the method of images.

Hsu Yan-sheng⁽⁵⁾ (1962) analysed the system with the assumption that the energy is carried by the ferrite rod in a negative circularly polarized HE_{11} type mode (dipole mode). The influence of the metallic walls was then accounted for by images of the rod-waveguide produced by reflections at the conducting surfaces. Only two images, the nearest two produced by the walls closest to the ferrite, were considered. The characteristic equation of the system was obtained by applying the boundary conditions at the rod surface to the combined field. The solution of this equation gave the propagation constant in the structure. The result quoted, however, showed 16% higher phase shift than the measured value. This discrepancy is similar to that noted in the comparison made by Tompkins⁽⁴⁾. No mechanism was suggested for the conversion of the H_{01} mode of the empty guide to negative circular polarization.

1.4.2 The elliptical ferrite rod waveguide.

Lyubimov⁽⁸⁾ (1962) analysed the case of a ferrite-rod-waveguide of

elliptical cross-section rigorously, with the idea that the ellipticity of the structure would be a fair representation of the anisotropy of the reciprocal phase shifter geometry. The two wave equations in E_z and H_z obtained by the application of Maxwell's equations to the ferrite medium were solved in the elliptic cylindrical coordinate system. His results showed two dominant modes of elliptical polarization in the magnetised ferrite, one with energy concentrated in the rod ("matched") and the other with energy mostly outside ("unmatched")^{*}. The ellipticity of the structure was found to add to the effect of the magnetised ferrite in splitting the phase constants of the two modes. On application of small dc magnetic fields these two modes separate out, the phase constant of the "matched" mode increasing and that of the "unmatched" mode decreasing rapidly. He stated that the "matched" wave would be excited by a transverse electric field along the major axis of the cross-section. In the reciprocal phase shifter the "matched" or "unmatched" character of the wave predominating at a particular excitation would be determined by the combination of the properties of the ferrite and the metallic screen.

1.4.3 Analysis by perturbation method.

Mikaelian and Teitel'baum⁽⁹⁾ (1959), in their application of the perturbation theory to the reciprocal ferrite phase shifter, took as

^{*} The statement that the "unmatched" wave has a direction of elliptical polarization opposite to the natural spin precession of the ferrite, made by Lyubimov, is probably an error.

the unperturbed system a rectangular waveguide, uniform along the longitudinal axis and loaded with a centrally located full-height unmagnetised ferrite slab of arbitrary thickness. The field components and the propagation constant for H_{01} mode in such a system were determined by standard methods. These components were then used in the well known perturbation relation for incremental propagation constant, to obtain an expression for the phase shift in the structure when the ferrite is magnetised. Assuming that the perturbed field is approximately the same as the unperturbed one, the incremental propagation constant is given by:

$$\Delta\gamma = \frac{j \omega \int_{S_F} (\Delta[\mu] \cdot \tilde{H}) \cdot \tilde{H}^* ds}{2 \int_{S_0} (\tilde{E} \times \tilde{H}^*) \cdot \tilde{u}_z ds} \quad (1.1)$$

Substituting values of the field components in this expression and neglecting cross-polarized waves generated by coupling due to off-diagonal components of the permeability tensor, Mikaelian and Teitel'baum obtained:

$$\Delta\gamma = \frac{A^2 \left[\gamma^2 (\mu - \mu_f) \left\{ t_1 + \frac{1}{K_d} \sin(K_d t_1) \right\} - K_d^2 (\mu_z - \mu_f) \left\{ t_1 - \frac{1}{K_d} \sin(K_d t_1) \right\} \right]}{2 \gamma \mu_f^2 \left[\frac{2}{\mu_0} \left\{ t_2 - \frac{1}{2K_0} \sin(2 K_0 t_2) \right\} + \frac{A^2}{\mu_f} \left\{ t_1 + \frac{1}{K_d} \sin(K_d t_1) \right\} \right]} \quad (1.2)$$

where,

$$[\mu] = \text{permeability tensor of ferrite} = \begin{vmatrix} \mu & -jk & 0 \\ jk & \mu & 0 \\ 0 & 0 & \mu_z \end{vmatrix}$$

- ω = Angular frequency.
 $\tilde{E}, \tilde{H}$ = RF field vectors.
 S_F = Area occupied by ferrite.
 S_O = Waveguide cross-sectional area.
 $\tilde{u}_z$ = Unit vector in the direction of propagation.
 ϵ_f, μ_f = Permeability and permittivity of unmagnetised ferrite.
 μ_O = Permeability of empty region of waveguide.
 K = $\frac{2\pi}{\lambda_O}$, where λ_O is the free space wavelength.
 K_O = $\sqrt{K^2 - \gamma^2}$, for the empty region of waveguide.
 K_d = $\sqrt{\mu_f \epsilon_f K^2 - \gamma^2}$, for the region occupied by ferrite.
 t = Width of waveguide.
 t_1 = Thickness of ferrite slab.
 t_2 = $\frac{t - t_1}{2}$.
 γ = Propagation constant of the unperturbed system,
 given by the following relation:

$$\begin{aligned}
 \frac{\mu_O}{\mu_f} K_d \tan \left(K_d \frac{t_1}{2} \right) &= K_O \cot (K_O t_2). \\
 A &= \frac{\sin (K_O t_2)}{\cos \left(K_d \frac{t_1}{2} \right)}
 \end{aligned}$$

Phase shifts calculated from equation (1.2) agreed well with experimental results using ferrite slabs, for slab thicknesses corresponding to over 25% loading of the waveguide cross-section. Agreement with experimental results using rods was, however, limited to very thin

rods, giving up to 10% loading of the guide cross-section. The range of applicability of the perturbation theory to the reciprocal phase shifter configuration is shown in figure 1.4. For larger rod cross-sections there was no agreement, the theoretical phase shift for slabs of same area being only about a third of that obtained with rods with diameter near the upper end of the phase-shifter region marked in figure 1.4. At first sight there does not seem to be any reason for such a large discrepancy, particularly in view of the fact that the dielectric loaded guide was taken as the "unperturbed" system. One is thus led to the conclusion that the neglect of the off-diagonal component of the permeability tensor of the ferrite, based on the assumption that a wave cannot exist in the system with its polarization parallel to the broad walls of the waveguide, is really not justified in the case of thick ferrite rods.

Button and Lax⁽¹⁰⁾ (1962) further simplified the expression for incremental phase shift by neglecting variations in μ_z with applied field. They also assumed that the field outside the ferrite slab can be neglected with respect to the field inside. The terms containing μ_z in the numerator and t_2 in the denominator of equation (1.2) were thus omitted and the equation reduced to:

$$\Delta\gamma = \frac{\gamma}{2} \cdot \frac{(\mu - \mu_f)}{\mu_f} \quad (2.3)$$

They stated that equation (1.3) shows the principle of operation of the

phase shifter. The variation in μ , which increases^x by about 20% due to the decline of the effective internal field as the ferrite saturates and the domains disappear, is magnified by the high phase constant of the slab loaded waveguide.

1.4.4 The phenomenological model.

A model to represent the reciprocal ferrite phase shifter, based on the various phenomena that affect wave propagation through the structure, was proposed by Weiss⁽⁶⁾ and later elaborated upon by Neckenb rger⁽³⁾. The model essentially consists of plane waves propagating through a medium that possesses hypothetical tensor permittivity and permeability. The plane wave assumption comes from the fact that H_{01} , H_{11} and HE_{11} modes, any one of which the mode propagating through the structure may resemble, are all similar to an uniform plane wave near the longitudinal axis of the system. The tensor permeability represents the anisotropy of the magnetised ferrite as modified by the rod and the waveguide geometry, and its transverse components are given by:

$$[\mu_T] = \begin{vmatrix} \mu & -jk \\ jk & \mu \end{vmatrix} \quad (1.4)$$

^x Button and Lax take the variation in μ as a decrease according to the behaviour of Polder's⁽³⁵⁾ form for μ . This would give the result as a decrease in phase shift, opposite to what is observed experimentally.

The hypothetical tensor permittivity was taken to be of the form:

$$[\epsilon_T] = \begin{vmatrix} \epsilon_x & 0 \\ 0 & \epsilon_y \end{vmatrix} \quad (1.5)$$

to account for the difference produced by the metal walls between the phase velocities of waves whose electric vectors are parallel to the x and y directions respectively. The component in the direction in which the basic H_{01} type mode is cut-off has a negative value.

Under these circumstances, the wave-equation in the system for propagation in the z-direction is given by:

$$\frac{d^2}{dz^2} \begin{vmatrix} E_x \\ E_y \end{vmatrix} = -\frac{\omega^2}{c^2} [\mu_T] [\epsilon_T] \begin{vmatrix} E_x \\ E_y \end{vmatrix} \quad (1.6)$$

where, C is the velocity of light in vacuum. The characteristic equation of the above has the following roots:

$$\lambda_{\pm} = \frac{1}{2} \mu (\epsilon_x + \epsilon_y) \pm \left[\frac{1}{4} \mu^2 (\epsilon_x - \epsilon_y)^2 + \epsilon_x \epsilon_y k^2 \right]^{\frac{1}{2}} \quad (1.7)$$

and a wave travelling in +z-direction has the form:

$$\begin{vmatrix} E_x \\ E_y \end{vmatrix} = \begin{vmatrix} (\lambda_+ - \mu \epsilon_y) & (\lambda_- - \mu \epsilon_y) \\ jk \epsilon_x & jk \epsilon_x \end{vmatrix} \begin{vmatrix} A_p \\ B_n \end{vmatrix} \quad (1.8)$$

where,

$$p, n = \exp \left(-j \frac{\omega}{c} \sqrt{\lambda_{\pm}} \cdot z + j\omega t \right) \quad (1.9)$$

A, B = Constants,

Equation (1.8) can be interpreted as the superposition of two elliptically polarized waves (the "p" and the "n" waves) of different ellipticities and phase velocities, with their axes parallel to the x and y coordinates of the system. The ratio of x and y components of the polarization ellipses are obtained from the respective column vectors as follows:

$$R_{p,n} = \frac{\lambda_{\pm} - \mu \xi_y}{k \xi_x} = \frac{(1-m)_{\pm} / \sqrt{(1-m)^2 + 4q^2 m}}{2q} \quad (1.10)$$

where, $m = \frac{\xi_y}{\xi_x}$ and $q = \frac{k}{\mu}$

When ξ_y is negative and $|k| \leq \mu$ (conditions of operation of the phase shifter) only a negative elliptically polarized mode (the "p-wave") can propagate in the system. The ellipse gradually changes into a circle as the ferrite is magnetised and q approaches unity, but its axes always remain parallel to the broad and narrow walls of the waveguide. If ξ_y becomes positive due to an increase of the rod diameter or the operating frequency, a positive elliptically polarized mode (the "n-wave") can also propagate and the system behaves like an elliptical Faraday Rotator. In the phase shifter range of operation ($\xi_y < 0$), if it is assumed that $|\xi_y| \ll \xi_x$, then $\lambda_{\pm}$ becomes approximately equal to $\mu \xi_x$, showing that the phase shift arises mainly from variations in the diagonal component μ of the permeability tensor.

Weiss also formulated qualitative expressions for scattering by the structure. He found that, under the condition that both modes can propagate, insertion loss peaks due to interference can occur for the right combination of phase shift and Faraday rotation. He said

that the mechanism for the occurrence of these loss peaks is similar to that obtained in a system treated earlier by him⁽²⁶⁾. This system consisted of a circular waveguide with a central ferrite rod and with rectangular waveguides at its input and output. Due to multiple reflections, large frequency dependent loss peaks were obtained for small Faraday rotations when the electrical length of the rotator section was a multiple of half guide-wavelength.

Quantitatively, Weiss was only able to make indirect comparison of the slope of the phase shift curves with the variation in the phase constant, for small changes in the permittivity, of a dielectric rod inside a circular waveguide. The experiments performed by Neckenb rger, however, seems to confirm the qualitative aspects of the theory by bringing out the following:

- (a) the "dielectric waveguide" behaviour of the ferrite rod,
- (b) the elliptical polarization of the dominant mode,
- (c) very little rotation of the axes of the ellipse when only one mode can propagate in the system, and
- (d) the gradual change to circular polarization with increased magnetisation of the ferrite.

1.5 Definition of Scope of Present Work.

It can be seen from the foregoing survey that the behaviour of electromagnetic waves in the reciprocal ferrite phase shifter is far from being completely understood. The perturbation analysis is

inappropriate by the basic assumption of negligible disturbance of the original field, as in the present case the perturbing process gives rise to a completely new orthogonal field of equivalent order of magnitude. This theory can account for only about 10% incremental phase shift, whereas Reggia and Spencer's results⁽³⁾ show that it can be more than 60%.

The normal modes of the system predicted by Wiess's model agree more readily with experimental evidence. However, the wholly non-physical properties attributed to the medium of propagation is unsatisfactory and cannot account for the complete reflection of energy by the conducting walls. To account for this reflection Hsu Yan-sheng had to make a priori assumption of the mode of propagation. The phenomenological properties also make it impossible to carry out direct quantitative comparison of the theory with experimental results. Lyubimov's analysis of the elliptical ferrite cylinder, though bringing out some characteristics of wave propagation subjected to two anisotropies, cannot be taken to bear directly upon the present problem.

The more obvious properties of the structure, like the large phase lag obtainable or the excitation of cross-polarized waves, have been extensively used to produce microwave devices. It is reasonable to expect that a greater understanding of the mode pattern in the configuration, besides being of purely academic interest, may lead to the fabrication of some other novel and useful devices. With this aim in view the first part of the work described in this thesis was devoted to experimentally ascertaining the modes of propagation and

as to which of the theories or parts thereof best fitted the characteristics of the structure. Once an understanding of the nature of the modes was achieved, the rest of the work was directed towards the development of new devices utilising some of the characteristics, for use in microwave communication systems.

CHAPTER TWO

PRELIMINARY INVESTIGATIONS

2.1 Introduction

This chapter deals with preliminary work carried out with a view to getting acquainted with the various aspects of the behaviour of electromagnetic waves propagating through the reciprocal ferrite phase shifter configuration. The propagating waves were subjected to different conditions designed to bring out the essential characteristics of the system. The two main theories existing on the subject, namely the perturbation analysis and the phenomenological model, were examined experimentally to ascertain as to which one was more appropriate and should be further investigated. In the course of these investigations it was found necessary to extend or repeat some of the work carried out previously on this problem. This brought to light some new features of the system which were then investigated in greater detail.

2.2 Choice of Ferrite Material

All the work described in this thesis was carried out in the frequency range of 6.7 to 12.0 kMc/s (X-band). At these frequencies a commonly used ferrite material for low power phase shifter applications is the Ferramic R1 Mg-Mn Ferrite. For the purpose of the present investigations this material appeared eminently suitable and was chosen

because of its following properties:

- (a) Low magnetic and dielectric losses.
- (b) Moderately high Curie temperature (290°C).
- (c) Optimum saturation magnetisation (≈ 1700 gauss),
low enough to avoid low-field and resonance losses
on the one hand, and high enough to give sufficient
phase shift on the other.

Two 5" long R1 rods of diameters .313" and .281" respectively, were therefore obtained and used almost exclusively. Some Ferroxcube B2 Ni-Zn Ferrite material were also used during preliminary investigations for the purpose of comparison. These ferrites had a Curie temperature of 250°C and a saturation magnetisation of about 3000 gauss. The material was, however, very lossy and was not considered suitable for more sophisticated experiments. Common values ⁽¹³⁾ of dielectric constants for the R1 and B2 materials at X-band are as follows:

Ferramic R1 ferrite = 13.6

Ferroxcube B2 ferrite = 11.4

To make quantitative comparison with the results of previous workers, the saturation magnetisation of the ferrite samples to be used had to be known exactly. This was necessitated by the fact that saturation magnetisations show considerable variation from sample to sample for the same ferrite according to the exact conditions of manufacture. The following experiment was therefore performed to determine these quantities.

2.2.1 Measurement of saturation magnetisation.

Theory

The ferromagnetic resonance frequency of a finite ferrite sample depends on the value of the saturation magnetisation of the material. Kittel⁽³⁶⁾ has shown that, for an ellipsoidal specimen whose principal axes coincide with the co-ordinate axes, the frequency of resonance $\omega_r/2\pi$ can be obtained from

$$\frac{\omega_r}{|\gamma|} = \left[\left\{ H_a + (N_x - N_z)4\pi M_s \right\} \left\{ H_a + (N_y - N_z)4\pi M_s \right\} \right]^{\frac{1}{2}} \quad (2.1)$$

where,

H_a = Steady applied field in the z-directions, in oersteds.

γ = Gyromagnetic ratio = 2.8 Mc/(sec). (oersted) in c.g.s. units when Lande's splitting factor equals 2.

$4\pi M_s$ = Saturation magnetisation in gauss.

N_x, N_y, N_z = Demagnetising factors in the co-ordinate directions indicated by the suffices; $N_x + N_y + N_z = 1$.

If the specimen is a long rod of circular cross-section magnetised along the length (the z-direction), then N_z can be neglected. In that case $N_x = N_y = \frac{1}{2}$ and equation (2.1) reduces to

$$\frac{\omega_r}{|\gamma|} = H_a + \frac{(4\pi M_s)}{2} \quad (2.2)$$

The intensity of H_a required to produce resonance in a ferrite specimen can be located by the high absorption at that field of waves that are circularly polarized in the clockwise sense (positive) with

respect to the steady field. The approximate saturation magnetisation of the sample can then be calculated from equation (2.1) or (2.2), according to the shape of the specimen.

Measurements

Saturation magnetisation was measured for one sample each of the R1 and B2 ferrite rods used. The details of the magnetic field employed to magnetise the specimen longitudinally are given in section A1.3 in the appendix. Circularly polarized wave, modulated at 3 Kc/s, was fed into a circular waveguide with the specimen centrally placed along its axis. The output from this section was absorbed by a matched load after being sampled by a fixed probe. The rectified output from the probe was fed into a narrow band 3 Kc/s amplifier and output meter. The insertion loss due to the specimen was measured by varying the input signal strength by a precision attenuator, keeping the probe output constant.

Plots of the insertion loss versus the applied field strength are shown in figure 2.1. The resonance absorption of the specimens at 9100 Mc/s are very clearly defined over signal fluctuations from other causes. From the intensity of the applied field at resonance, the saturation magnetisations of the samples tested were calculated with the help of equation (2.1). The diameters of these rod samples were too large compared to their lengths for the demagnetising factor N_z to be neglected. The appropriate values of N_x , N_y and N_z were obtained from graphs given in reference (44) for an equivalent inscribed ellipsoid. Equation (2.1)

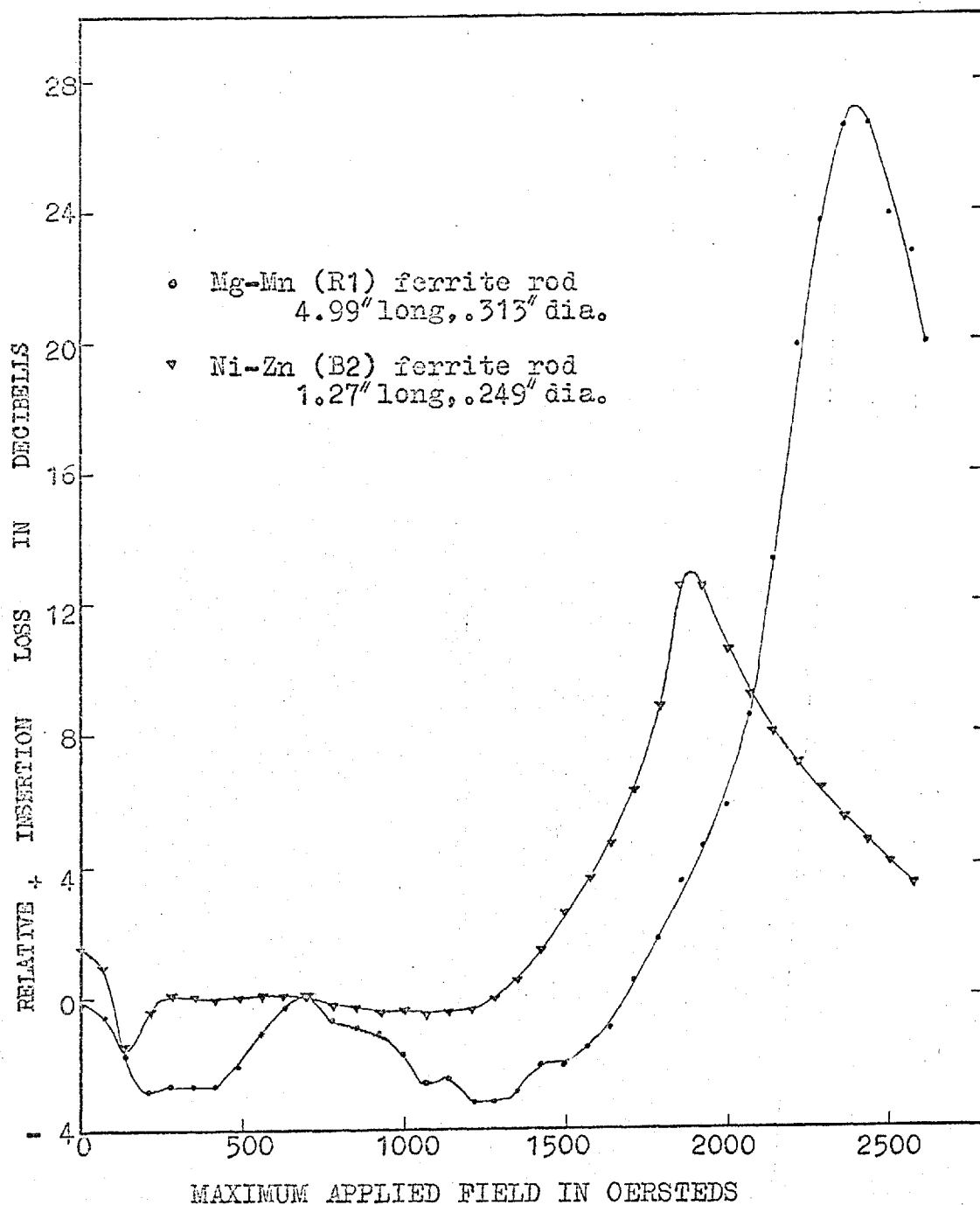


Figure 2.1. Resonance absorption of positive circularly polarized wave by ferrite specimens at 9100 Mc/s.

then gave the following results for the R1 and B2 specimens, respectively:

$$4\pi M_s \text{ for Mg-Mn (R1) ferrite} = 1790 \text{ gauss} \pm 3\%$$

$$4\pi M_s \text{ for Ni-Zn (B2) ferrite} = 3290 \text{ gauss} \pm 3\%$$

The saturation magnetisation of the R1 sample was also checked at 8000 Mc/s. The result agreed with the above value within the limits of experimental errors.

The maximum values of the applied field at the centre of the specimens were adopted for the calculations. The variation of the field over the length of the specimen was expected not to affect the results significantly. Total maximum error from this and other causes was estimated to be 3%. The temperature of the specimens were not checked during the experiment. However, as care was taken not to overheat the region containing the sample and a blower was used to remove any heat produced by the coils, the temperature of the specimens was most probably very near room temperature.

2.3 Phase Shift Measurements

Before investigating the mode structure in the reciprocal phase shifter configuration, it was decided to ascertain the phase shift performance of the ferrite specimens to be used. Dependence of the phase shift on the properties of the ferrite material and on frequency were studied. The insertion losses of the specimens were also measured as a function of the applied field and from this the absorption of energy by the specimens were calculated for the purpose of comparison.

A schematic diagram of the microwave bridge used for these measurements and the arrangement for supplying dc power to the magnetising coils is shown in figure 2.2. The bridge was in X-band rectangular waveguide of internal dimensions 0.9" x 0.4" in general. The microwave power used was square wave modulated at 3 Kc/s to go with the 3 Kc/s meter amplifiers. The calibrated phase shifter was of the line lengthening (trombone) type with very low input VSWR. The rotary attenuator used for the insertion loss measurements gave negligible phase shift when varied. The second set of phase shifter and attenuator was used for initial balancing of the bridge. The frequency meter shown could only indicate down to 8200 Mc/s. Below this range the oscillator frequency was checked by connecting it to a No. 14 waveguide system.

The elaborate arrangement for varying the coil current was necessary as current magnitudes from 0.001 amp to 1.5 amp, had to be adjusted equally accurately. All the investigations described in this thesis were carried out with the ferrite specimen magnetised parallel to the direction of propagation, and the coil arrangement was always as shown in the figure. The reciprocity of the systems under investigation were checked by reversing the applied steady field. It was found difficult to demagnetise the specimens completely after the application of the very high fields used. This was done for this and all further experiments by applying reversing fields of gradually decreasing amplitude to the specimen; the final demagnetisation was achieved by applying 50 c/s ac to the coils and slowly reducing that to zero. The coils used are described in sections A1.1 and A1.2 in the appendix.

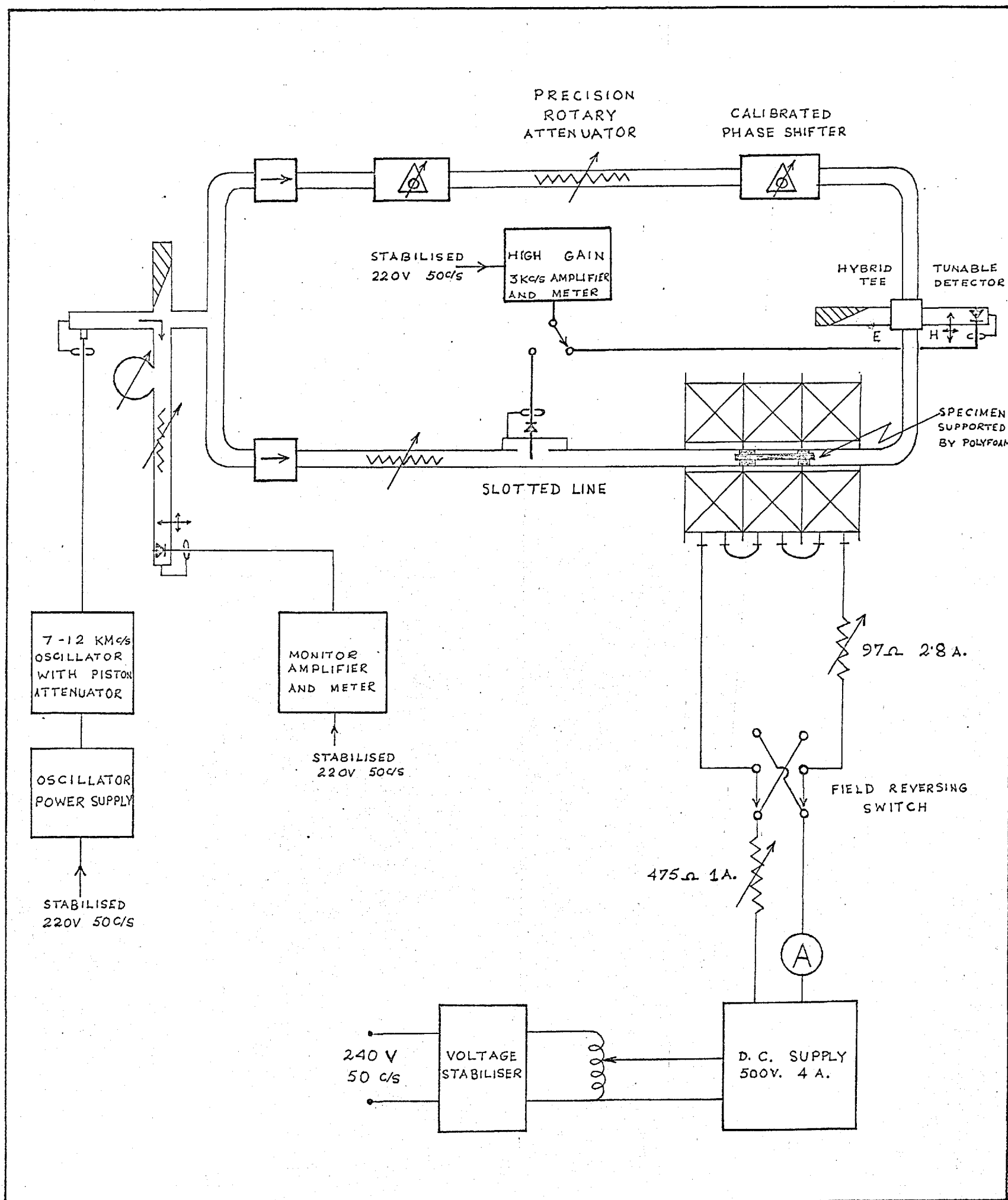


FIGURE 2.2. SCHEMATIC DIAGRAM OF MICROWAVE BRIDGE AND COIL POWER SUPPLY ARRANGEMENTS.

The bridge arrangement was chosen to reduce errors in the phase shift measurements due to reflections at the specimen, by isolating it from the reference circuit⁽³⁷⁾. This was essential as it was desired to study the intrinsic behaviour of the ferrite rods that is not masked by any end matching arrangements. The bridge also eliminated any error due to amplitude variation of the oscillator and was sensitive to small changes in the phase and the insertion loss of the specimen. The total possible error in the measured phase values is estimated to be of the order of $\pm 1\%$ except for those at 7000 Mc/s. This frequency was outside the range of the isolators used and the error is expected to be slightly higher.

The comparative incremental phase shifts of the B2 and RL samples whose saturation magnetisations had been determined before (section 2.2.1) are shown in figure 2.3. The inset in the figure shows a plot of the real part of the average effective permeability $(\bar{\mu}' + \bar{k}')$ for negative circularly polarized waves to be expected for these ferrites below saturation, obtained from reference (13). Figure 2.4 shows the frequency dependence of phase shift for the second RL rod specimen. Figure 2.5 shows the part of transmission losses due to the specimens from which the losses due to reflection have been subtracted.

Discussions

The maximum phase shift per inch for the B2 specimen in figure 2.3 is only about 14% lower than that for the RL. The difference to be expected due to the difference in the diameters of the two rods is

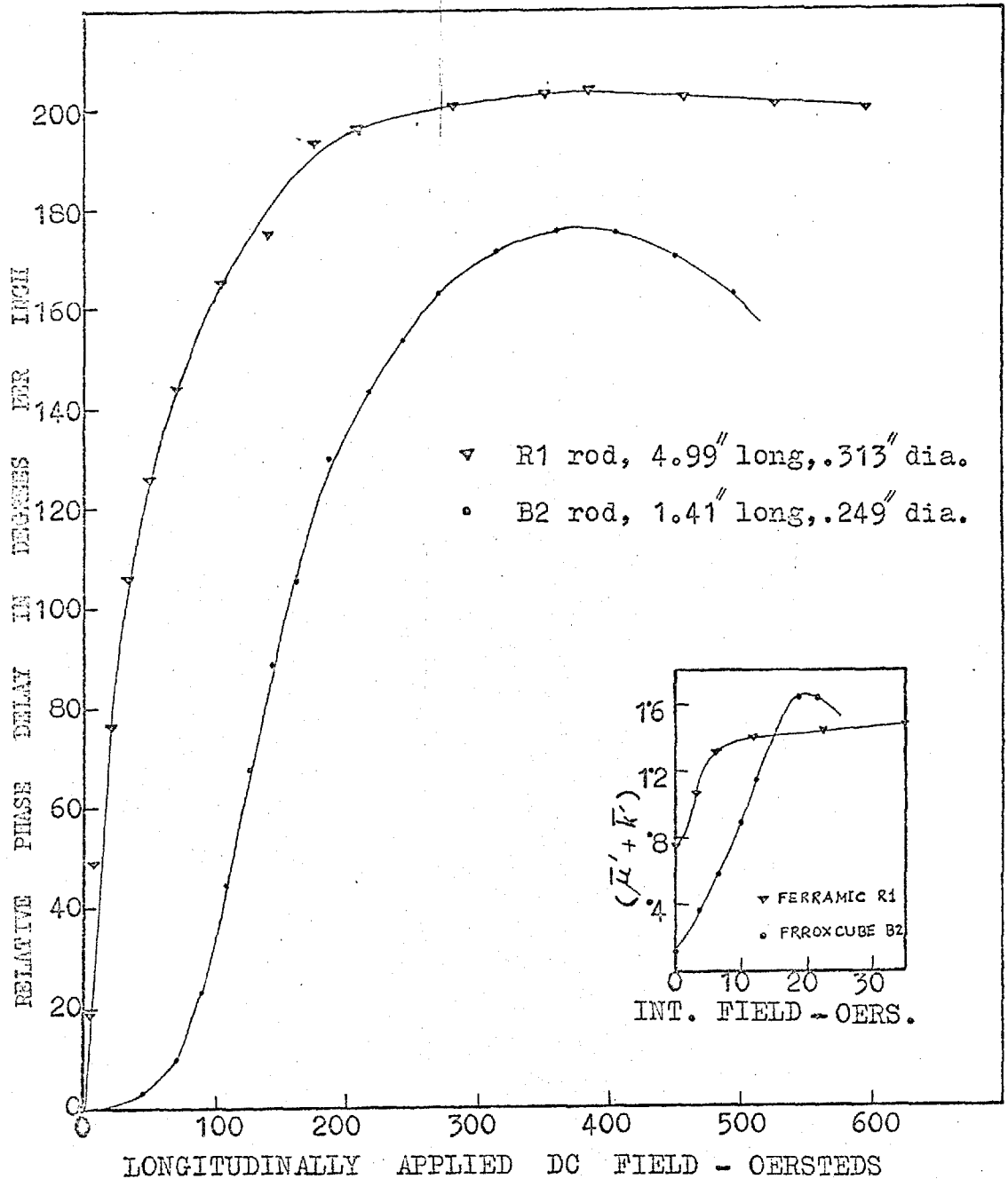


Figure 2.3. Phase shift characteristics of R1 and B2 ferrite specimens at 9100 Mc/s. Inset shows effective relative permeabilities for negative circular polarization below saturation, as obtained by Clarricoats⁽¹³⁾ for these materials.

about 50%, as can be seen from figure 1.4. It seems that this can be roughly accounted for by the higher saturation magnetisation value of the B2 specimen. This confirms the finding of previous workers that the phase shift is approximately proportional to the saturation magnetisation of the ferrite.

There appears to be a strong correlation between the shape of the phase shift curves and the respective $(\bar{\mu}' + \bar{k}')$ curves. ~~These results show that the phase shift is approximately proportional to the saturation magnetisation of the ferrite.~~

~~These results show that the phase shift is approximately proportional to the saturation magnetisation of the ferrite.~~ These results seem to suggest a resemblance of the propagating mode to a negative circularly polarized mode.

Figure 2.4 shows that the mode responsible for the phase shift persists to very low frequencies. This would suggest that the phase shift is due to a mode that is inherent to the system, rather than due to some secondary effects. The phase advance obtained for higher fields at the lower frequency merely means that the wavelength in the structure is slightly longer than at zero field. The variation of phase shift with applied field can still be accounted for by the variation in μ and k as stipulated by Weiss (see section 1.4.4). The maximum phase shift at the lower frequency is obtained at a lower applied field. This agrees with the manner in which μ and k would vary with frequency.

Comparing the maximum phase shifts of the two R1 samples at 9100 Mc/s, shown in figures 2.3 and 2.4, it can be seen that the phase shift due to the .281"-diameter rod is higher. With the help of figure 1.4 the phase shift of the .281"-diameter rod is estimated to be about 20% higher

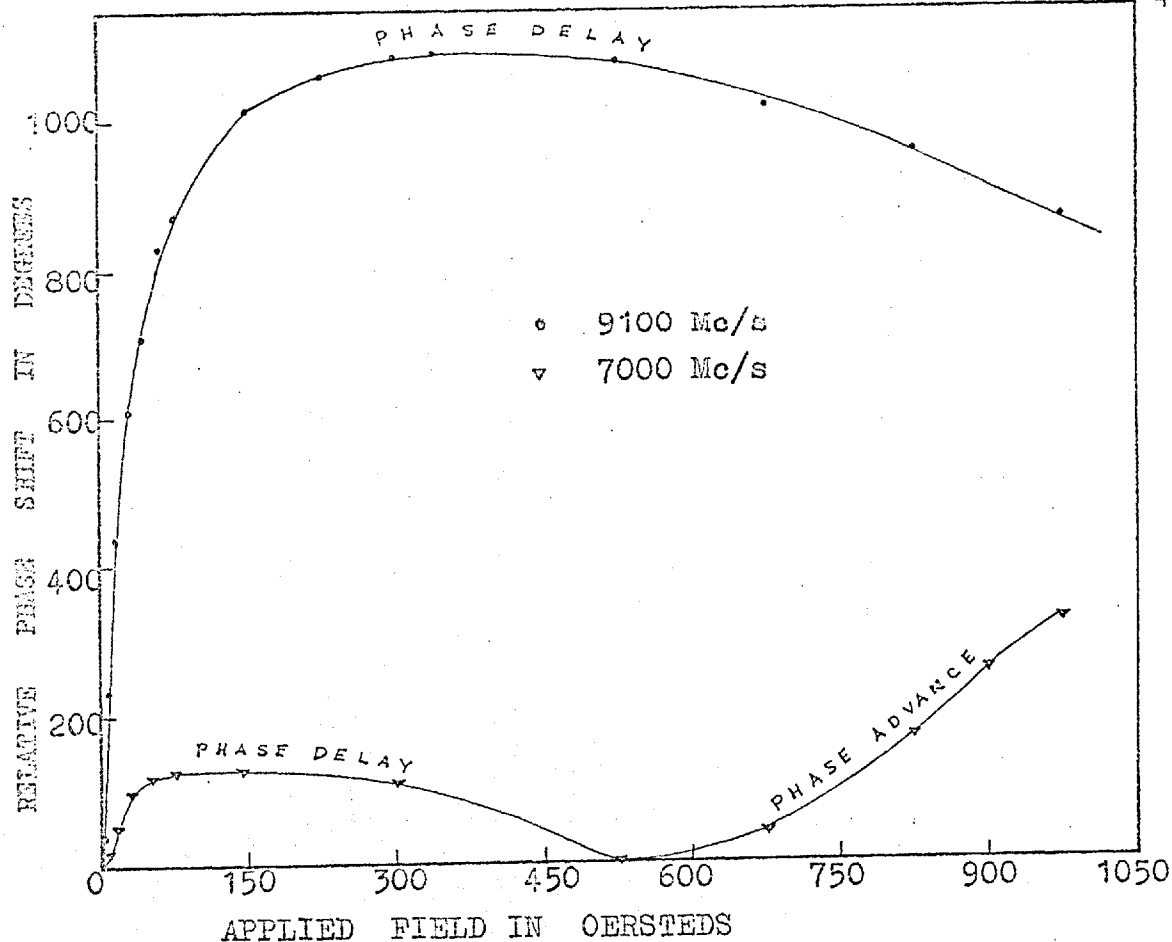


Figure 2.4. Frequency dependence of the phase shift characteristic of an R1 ferrite rod, 5" long and .281" in diameter.

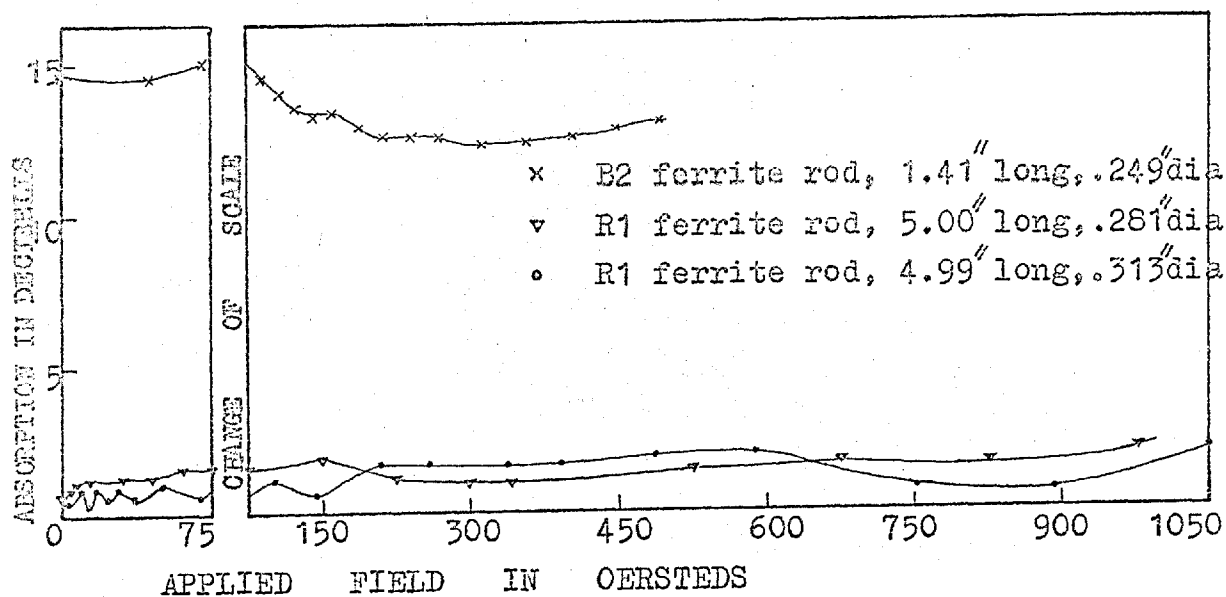


Figure 2.5. Absorption by ferrite specimens in the reciprocal phase shifter configuration at 9100 Mc/s.

than expected. This illustrates the difference in performance that may be expected from the same ferrite material manufactured at different times.

The first things that become obvious from figure 2.5 are the high loss of the B2 material and the fact that, of the two R1 specimens, the .281"-diameter rod is slightly more lossy. In addition, it can be seen that the .313"-diameter rod shows a periodic fluctuation of loss that is not accounted for by reflection. This may indicate that a second mode is on the verge of propagation in the case of this rod and that the fluctuations are reduced forms of the interference peaks noticed previously by other investigators, caused by energy trapping in the ferrite region.

2.4 Investigations of the Perturbation Theory

The approximations made in the perturbation analysis of the reciprocal phase shifter, described in section 1.4.3, suggest an obvious experiment to verify its validity. It would appear from the analysis that, a centrally located ferrite slab of cross-sectional area same as that of a rod of the same material, would give approximately equal maximum phase shifts. Measurements were therefore made on Ferroxcube B2 ferrite slab and rod specimens of the same cross-sectional area and length, with the following results.

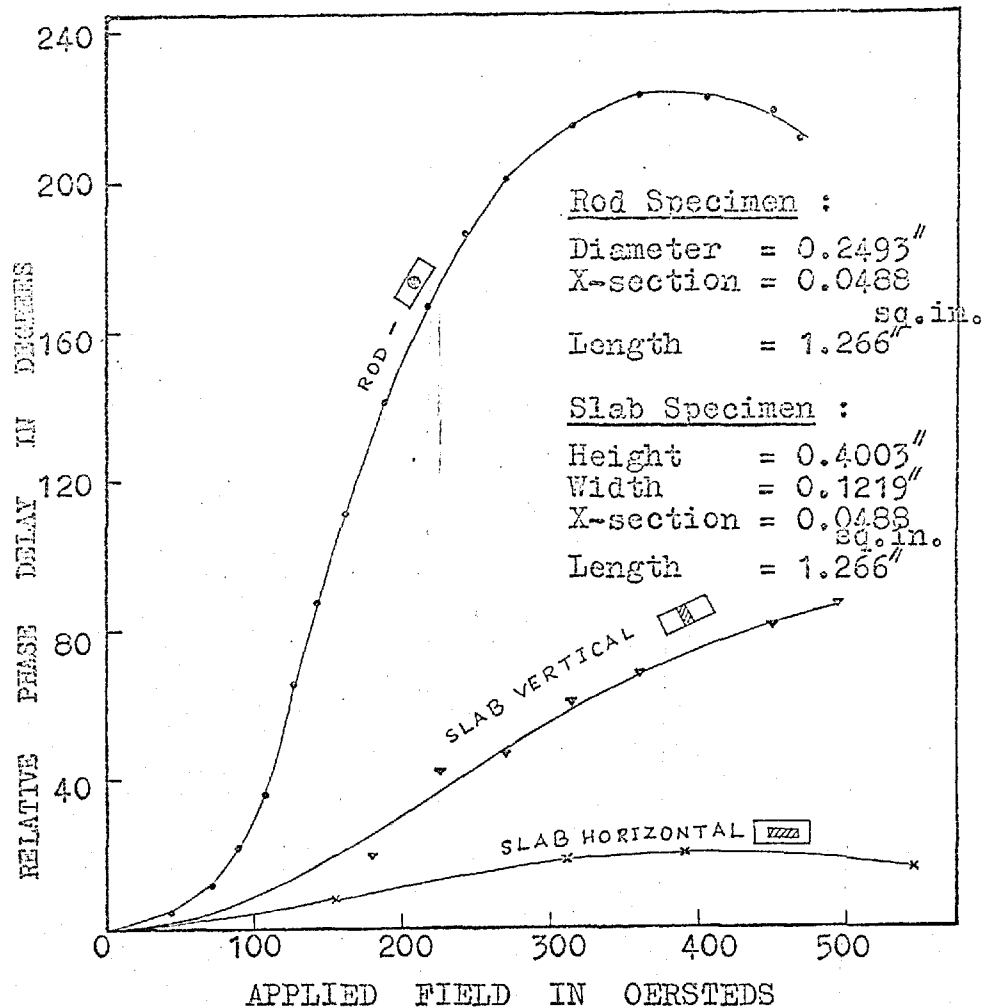
The measurements were made on the same bridge as shown in figure 2.2. One dimension of the slab specimen was taken as the narrow

dimension of the rectangular waveguide, for exact reproduction of the conditions of the analysis. The specimens were prepared by cutting with a diamond impregnated cutting wheel, the final dimensions of the slab being obtained by hand grinding with silicon-carbide powder. The results of the experiment are shown in figure 2.6(a) and (b).

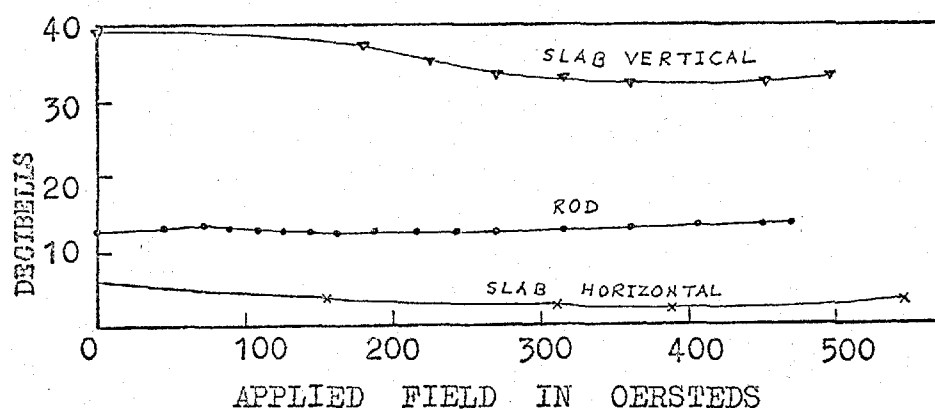
Discussions

The phase shift obtained with the slab placed parallel to the narrow dimensions, at the centre of the guide, is only about a third of that obtained with the rod. This agrees with the conclusions drawn from Mikaelian and Taitel'baum's calculations in section 1.4.3. It is to be noted that the reduced phase shift occurs in spite of the fact that there is greater concentration of rf energy in the slab than in the rod. This can be deduced from the much higher insertion loss for the vertical slab shown in figure 2.6(b). This suggests that the mechanism of phase shift in the rod is quite different from that in the slab. The horizontal orientation of the slab shows even less phase shift. This would be explained by the low rf energy concentration in the ferrite, as indicated by the low insertion loss.

The symmetry of the ferrite cross-section appears to be an important factor in determining the mode of propagation. This would seem to indicate a mode propagating in the z-direction, possessing x and y components of the electric field. Such a mode would require a symmetrical ferrite cross-section and would have to be very much concentrated in the ferrite to propagate through the system without change of form.



(a) Phase shift characteristics.



(b) Insertion loss characteristics.

Figure 2.6. Comparison of performance between rod and slab specimens of Ferroxcube B2 ferrite at 9100 Mc/s.

2.5 Investigations of the Insertion Loss Fluctuations

During the measurements described in section 2.3 it was found that with the variation in the intensity of the applied steady magnetic field, the insertion loss of the reciprocal ferrite phase shifter shows periodic fluctuations. These fluctuations were studied in detail with a view to understanding the causes that contribute towards them. The same equipment that was described in section 2.3 was used for these investigations. The arrangement of the equipment, however, was as shown in figure 2.7. This is the standard layout used for most of the work described in this thesis, with slight modifications.

The insertion loss was measured relative to an empty rectangular waveguide section (0.4" x 0.9" internal dimensions) of the same length as that containing the specimen. Measurements were taken by compensating for the loss due to the specimen by varying the precision rotary attenuator. The signal level at the detector was thus maintained constant and the crystal law did not affect the readings. With no specimen in the system the detector in the main branch was tuned by the adjustable probe to obtain voltage standing wave ratios below 1.01:1. This was done so that the detector would always present a matched load to the specimen and not add any complication to the results. The lower range of the wide-band isolator was 8200 Mc/s. This was considered adequate as most of the important work was carried out above this frequency.

It is estimated that the precision attenuator could measure loss values within 1% up to about 30 dB. Higher loss values were measured with the help of the attenuator of the output meter. The signal strength

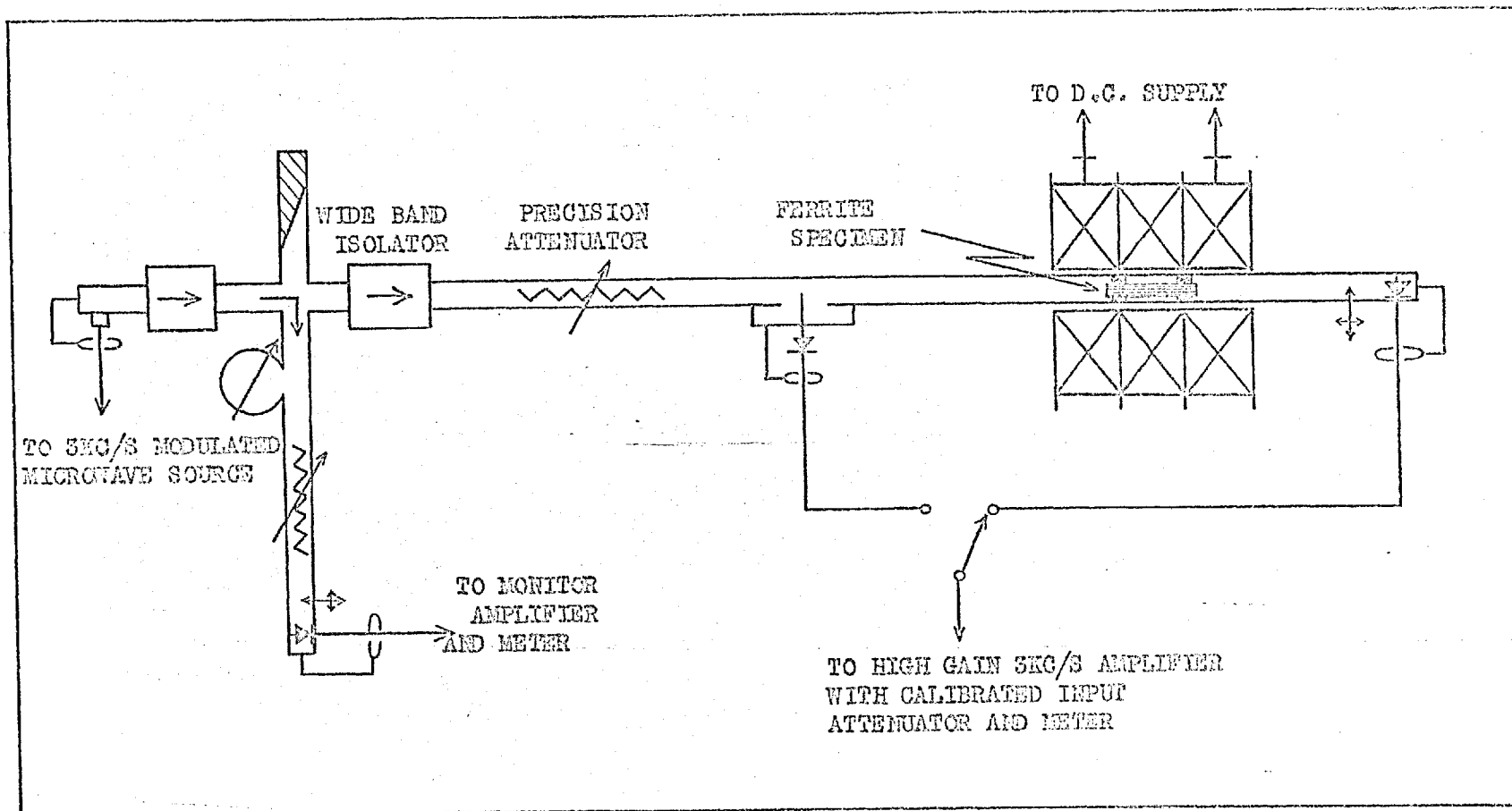


FIGURE 2.7. EQUIPMENT LAYOUT FOR INSERTION LOSS MEASUREMENTS.

at the detector was therefore not constant in these cases and the measurements were made assuming the detector crystal to obey a square law. However, spot calibration of the attenuator of the meter was carried out with the help of the precision attenuator, and the high insertion loss measurements are expected to be accurate to within 3%. For VSWR measurements square-law detection was in general assumed, as greater accuracy was not required. The monitor meter was used to maintain constant, the frequency and the magnitude of the microwave input. The procedure detailed here are not particular to the present investigations but was generally followed throughout the course of the work.

To record the form of the insertion loss curves accurately it was found necessary to follow the output of the detector^{as} the applied field was varied and note the maximum and the minimum points, as well as points in between when required. Insertion loss of the R1 ferrite rod of .313" diameter, centrally placed inside a rectangular waveguide, was measured at 9100 and 8200 Mc/s as a function of the steady longitudinal magnetic field. Insertion loss was also measured with the rods ended in .003" thick copper discs of the same diameter as the rod, in order to accentuate minor effects. The results of these measurements are shown in figure 2.8.

The rod was also placed centrally inside a circular waveguide of 1" internal diameter to permit Faraday rotation. The circular guide was connected to input and output rectangular waveguides through adapters. Transmission through this structure at 9100 Mc/s, as a

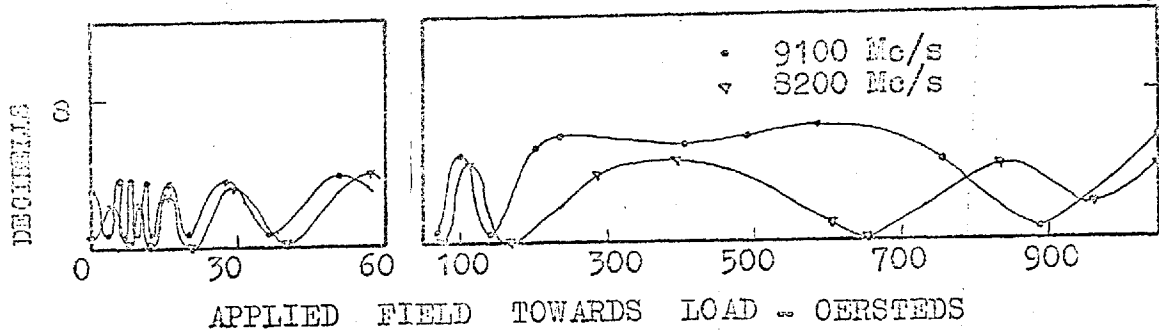
function of the applied steady field, was recorded with the input and output guides aligned, as well as orthogonal. These measurements, shown in figure 2.9, were taken for the purpose of comparison with the loss fluctuations in the reciprocal phase shifter configuration. Rotation of the plane of polarization was thought to be a possible cause of the fluctuations because of the similarity of the structure to a Faraday rotator.

Discussions

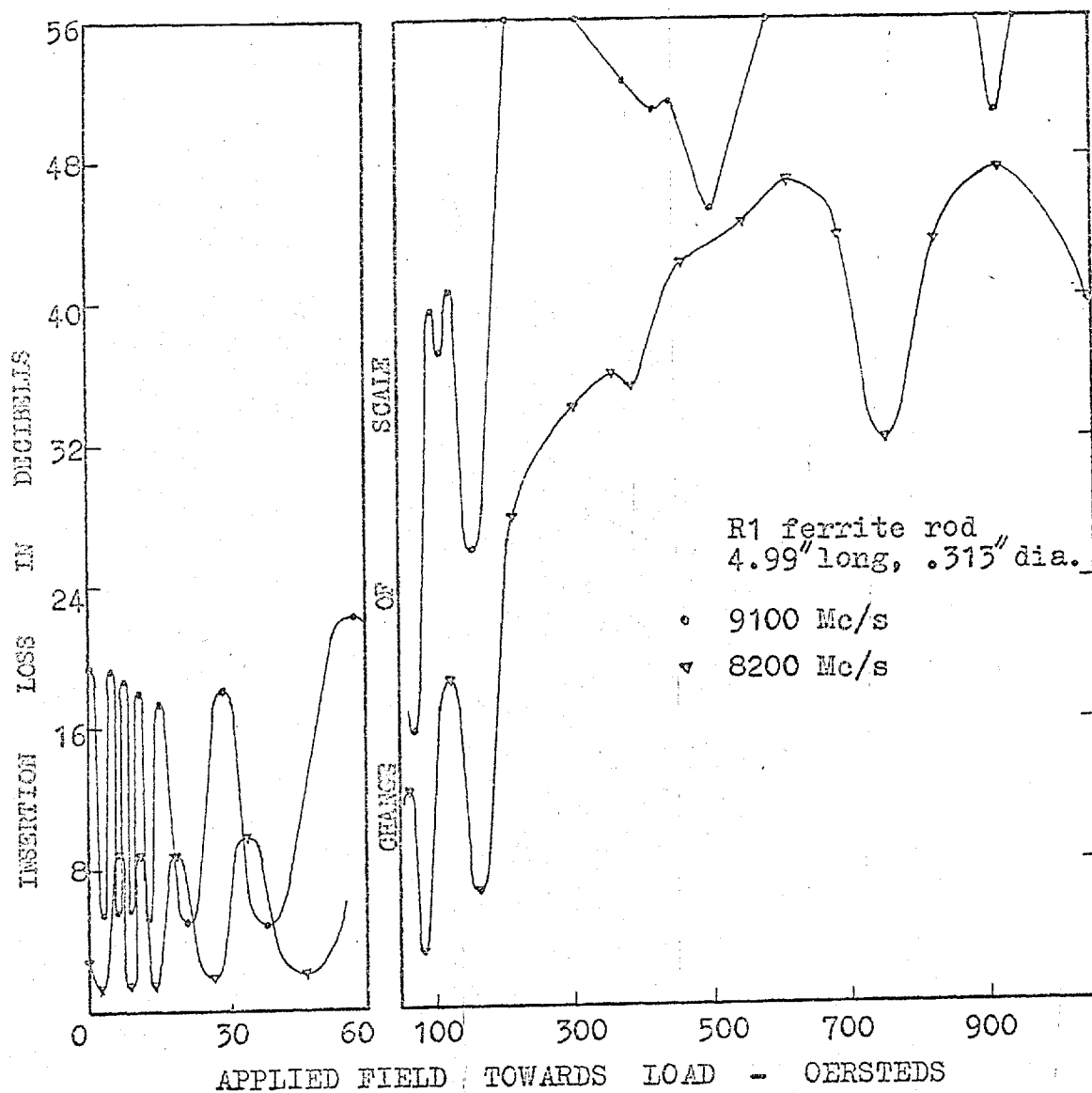
From figure 2.8 the loss fluctuations with the rod alone appears to be very similar to those with the rod ended in metal discs, except that in the later case the amplitudes of the fluctuations are larger. This seems to imply that the main cause of the loss is reflection from the ends of the rod. The increase in the amplitude of the fluctuations with increased magnetisation and at the higher frequency implies greater concentration of rf energy in the ferrite from these two causes. This is brought out very clearly in figure 2.8(b), where the very high losses noticed would be only possible if almost all the rf energy was in the ferrite.

The fluctuations are approximately accounted for by the phase shift values recorded in figure 2.3, allowing for the fact that the junction impedance between the empty region and the ferrite rod must change as the energy concentrates in the rod with increased magnetisation. From the fluctuations the phase shift at 8200 Mc/s for this rod is estimated to be about 25% less than at 9100 Mc/s.

An additional variation of the loss, superimposed on the main



(a) Rod alone.



(b) Rod with metal discs at ends.

Figure 2.8. Insertion loss of R1 ferrite rod in rectangular waveguide of internal dimensions 0.4" x 0.9".

fluctuations, can be seen at the higher frequency. This was remarked upon before in section 2.3, and can be visualised to be due to a Faraday rotation type of behaviour of the system causing energy trapping.

The loss fluctuations recorded in figure 2.9 are in a system where Faraday rotation is known to occur, and the fluctuations are due to a combination of this and phase shift effects. The peaks due to Faraday rotation can be recognised by comparing the two curves of figure 2.9. The patterns are seen to bear no resemblance with the fluctuations of insertion loss recorded in figure 2.8(a). Much greater variation in transmission occurs when normal Faraday rotation is permitted, though the phase shift would be expected to be less in this system, due to less rf energy concentration in the ferrite. Thus Faraday rotation can most probably be assumed to be suppressed in the normal operation of the phase shifter.

2.6 Effects of Discontinuities

The extent of rf energy concentration in the ferrite and its dependence on the steady magnetic field would appear to be an important criterion of distinction between the two main contending theories on the reciprocal ferrite phase shifter. In the assumptions of the perturbation analysis the influence of the waveguide walls is very important, whereas, in Weiss's model the energy has to be so much concentrated in the ferrite, that the walls play only a secondary role

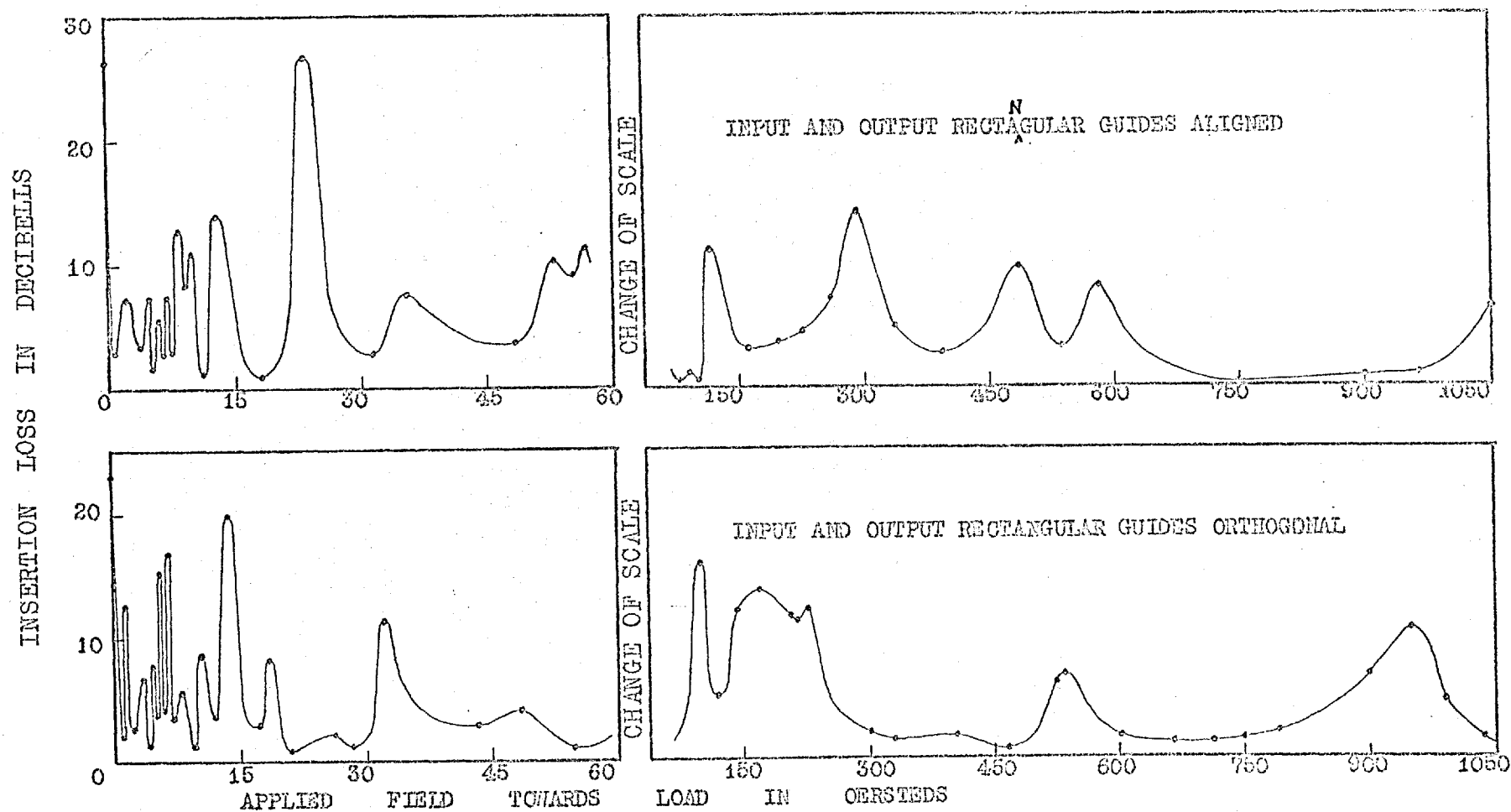


Figure 2.9. Insertion loss of R1 ferrite rod, .313" dia. & 4.99" long, in a circular waveguide, at 9100 Mc/s.

in determining the modes of propagation. Neckenbürgers experiments, described in section 1.3, brought out the rod-waveguide behaviour of the structure. It was however felt that a more extensive knowledge of the energy concentration effect was necessary for a clearer appraisal of the situation. With a view to obtaining this information, some discontinuities were introduced into the phase shifter arrangement and their effects on propagation through the structure were studied.

The arrangement of equipment for this experiment and the procedure of taking measurements were the same as described in section 2.5. Quite a few measurements had to be taken below the frequency limits of the isolators shown in figure 2.7. To reduce the effects of reflection at these frequencies a high attenuation was generally maintained with the rotary attenuator and insertion loss values were measured as far as possible with the help of the input attenuator of the meter. In the second part of this experiment, very high loss values had to be measured. In these cases it was noticed that scattering from the probe in the slotted line introduced a variation in the readings according to whether the probe happened to be at a voltage maximum or a minimum, although the probe penetration was kept quite small (< 2 mm.). Though the effect was small, for consistent results the probe was always placed at a voltage minimum before taking a reading. This procedure was followed for all high insertion loss measurements throughout this work.

One discontinuity consisted of a metal sheath covering the central part of the ferrite rod. Insertion loss for this arrangement as a

function of the steady magnetic field intensity was measured at various frequencies for two sizes of ferrite rods, a B2 rod of .249"-diameter and an R1 rod of .313"-diameter. Representative results and experimental arrangements are shown in figures 2.10 and 2.11. The other discontinuity studied was very similar to the one above except that the rectangular waveguide outside the metal tube was completely blocked off by a diaphragm. Rf energy could, therefore, only propagate through the ferrite loaded metal tube. The insertion losses for this arrangement are plotted in figures 2.12 and 2.13.

As shown in figure 2.13, the ends of the R1 rod were in this case matched to the empty waveguide by dielectric tapers of approximately the same dielectric constant as the ferrite. The tapers, each $1\frac{3}{4}$ " long, were of an artificial dielectric of constant $\epsilon \approx 15$, composed of BaTiO_3 suspended in polystyrene. The material was found to be too brittle to be turned on a lathe-machine, and the tapers were prepared by rubbing on glass paper.

Discussions

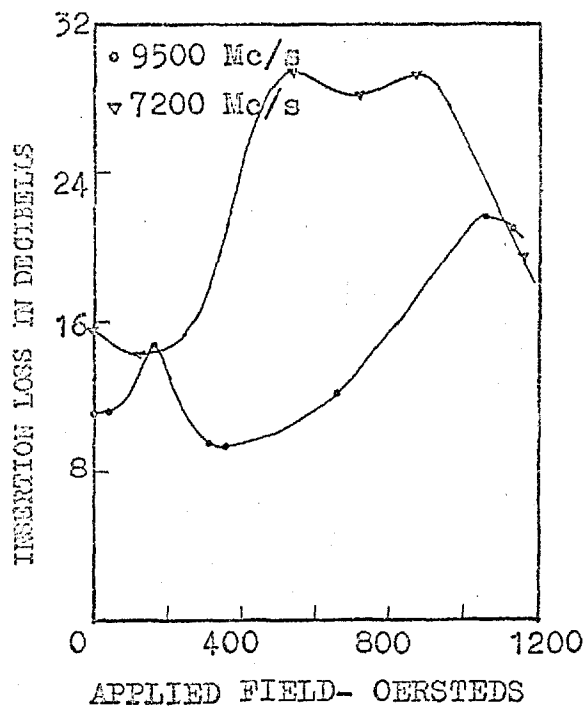
Figures 2.10 and 2.11 show that the concentration of microwave energy in the ferrite rods is very high indeed. The high loss of B2 material shown in figure 2.10(a) dropped sharply when part of the rod was covered by the copper tube as in figure 2.10(b). This effect is brought out more clearly at 7200 Mc/s when the tube containing the rod

would be at cut-off for a normal H_{11} limit-mode.* Most of the energy at 7200 Mc/s would therefore be excluded from the region of the ferrite rod inside the metal tube. A comparison of the curves of figure 2.10(a) and (b) for this frequency shows the high degree of rf energy concentrated in the ferrite rod in the reciprocal phase shifter configuration.

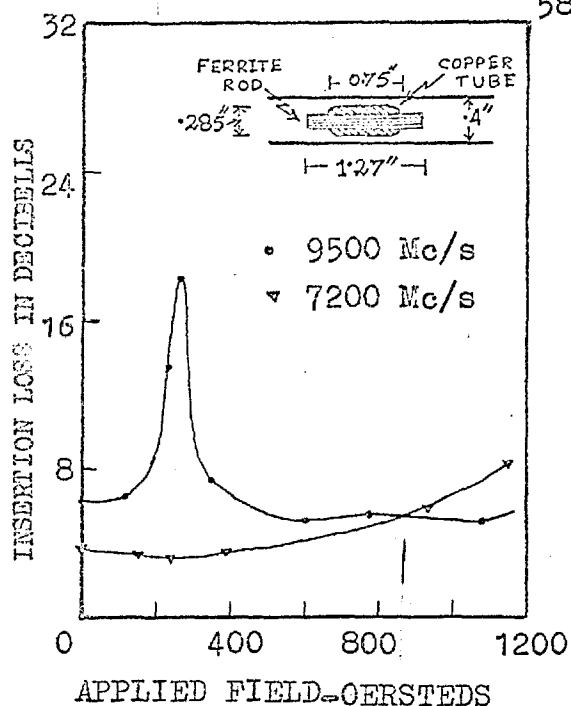
At 9500 Mc/s the drop in loss is still apparent, though the tube is now well above cut-off. Another effect now becomes noticeable in the form of a high peak of insertion loss, as shown in 2.10(b). This is thought to be due to interference between the waves travelling through the ferrite and through the gap between the metal tube and the waveguide. The wave travelling through the ferrite suffers phase shift relative to the other wave and for a particular value of the applied field is exactly out of phase with it. A substantial part of the rf energy must now be passing through the rod to produce the large peak measured. This effect appears to be suitable for the production of a cheap and compact electrically tunable band-stop filter.

The case of the R1 rod of .313" diameter is shown in figure 2.11.

* For normal H-modes the cut-off points are not affected by the state of magnetisation of the ferrite, the material behaving like a non-magnetic dielectric at the point of cut-off.⁽³⁸⁾ Through thinner tubes, anomalous propagation can occur; but as such propagation is very lossy, the wave would be expected to follow any alternative path provided. The term "limit-mode" indicate the behaviour of the mode at cut-off, according to which the quasi-H and the quasi-E modes of the ferrite loaded guide are classified.



(a) Rod alone.



(b) Partially sheathed rod.

Figure 2.10. Insertion loss of a .249" diameter B2 ferrite rod in rectangular waveguide.

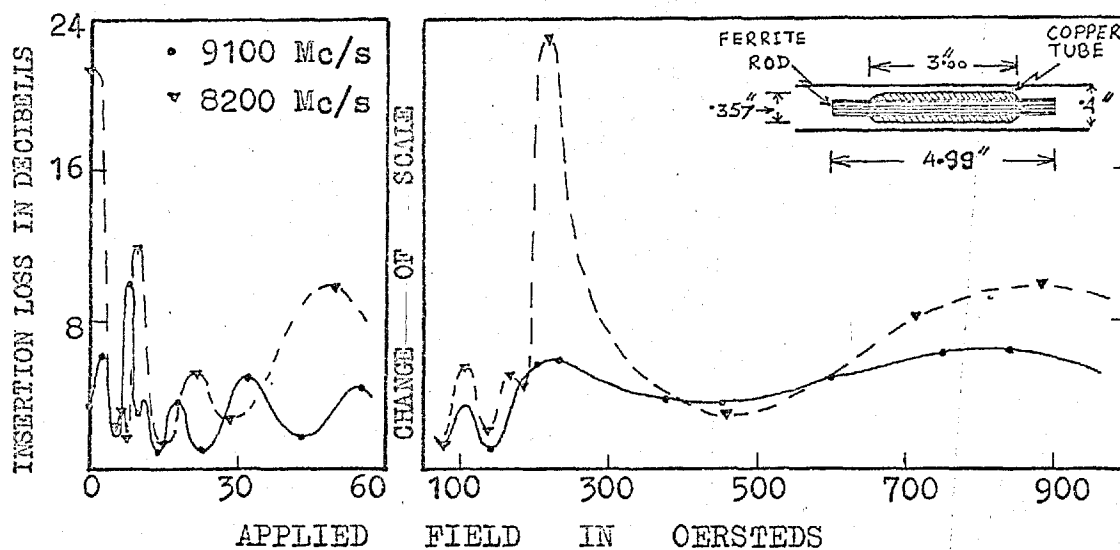


Figure 2.11. Insertion loss of a .313" diameter sheathed R1 rod in rectangular waveguide.

The interference peaks for 8200 Mc/s are larger than for the thinner B2 rod at 9500 Mc/s. More interesting, however, is the behaviour of the insertion loss at 9100 Mc/s. At this frequency no high insertion loss peaks occur except at small applied fields. The insertion loss curve is very similar to that for the rod alone, shown in figure 2.8(a). Thus at this frequency the rf energy seems to be so much concentrated in the ferrite that even a closely fitting metal tube has little influence on the propagation, particularly at the higher applied field strengths. This effect appears to be a signal for the beginning of Faraday rotator type behaviour of the structure, which was noticed at this frequency for this rod in sections 2.3 and 2.5.

Figures 2.12 and 2.13 show propagation through the ferrite loaded metal tube as a function of the steady magnetic field intensity. The calculated cut-off frequencies for the normal H_{11} limit-mode in these tubes are as follows:

.249" dia. tube filled with B2 ($\epsilon_r \approx 11.4$) = 8200 Mc/s.

.313" dia. tube filled with R1 ($\epsilon_r \approx 13.6$) = 6000 Mc/s.

The isotropic permeability of the unmagnetised ferrite has been assumed the same as for free space. For the R1 rod, therefore, all the curves represent conditions well above cut-off for the normal dominant mode. In the case of the B2 rod, behaviour below cut-off was also recorded.

Figure 2.13 shows that increasing propagation takes place through the R1 rod as it is magnetised. At 8500 Mc/s there is little loss except at very low fields. At lower frequencies the insertion loss,

which begins at a very high value, falls off quickly as the ferrite saturates. This variation in insertion loss can, in this case, be only due to a high degree of increasing rf energy concentration in the rod as the applied field strength is increased. That this effect is due to energy concentration, is brought out by the fact that the loss falls off quicker the higher the frequency of the wave. Energy concentration is an accepted result of increasing frequency in dielectric-rod-waveguide theory. The possibility that the cut-off point of the propagating mode depends on the magnitude of the steady field was rejected because of the absence of any sharp point of low-loss propagation, as the applied field was increased. Also, for the unmagnetised ferrite the mode would have to be the normal one, with no cut-off at this tube-diameter. Low-loss propagation would thus have resulted at zero applied field, for all the frequencies.

It is to be noted that the mode responsible for the effect is the same as that occurs in the reciprocal phase shifter configuration, set up in the uncovered part of the rod at the input. This energy concentration would account for the large incremental phase delay obtained in the structure. Such a high degree of energy concentration agrees with Weiss's prediction of the negative elliptically polarized character of the propagating mode. Only for an anticlockwise polarized mode would the effective permeability increase with increased steady field intensity (see inset in figure 2.3) and energy concentration occur.

The large loss peak at 8500 Mc/s is most probably due to the fact

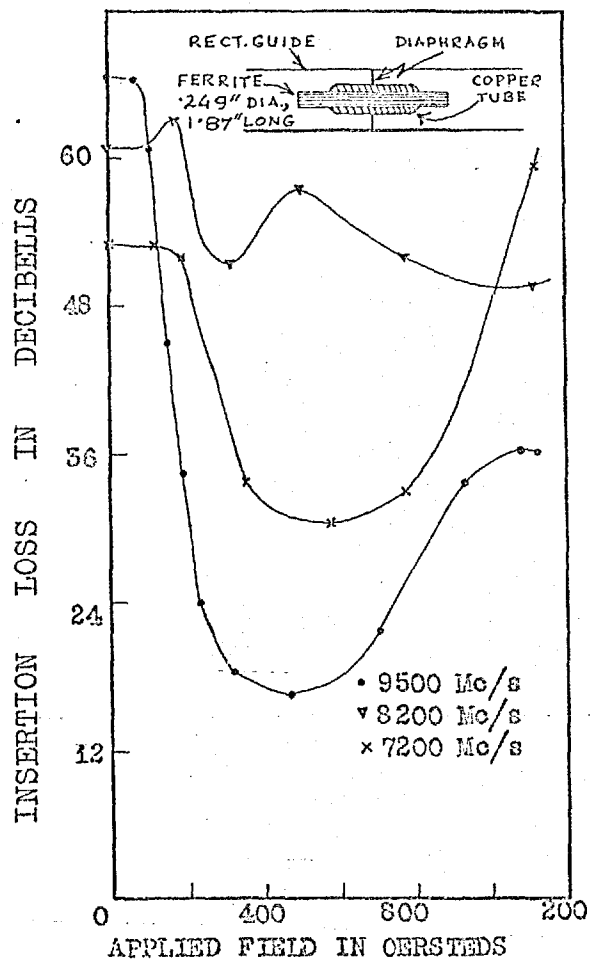


Figure 2.12. Propagation through a sheathed B2 ferrite rod in a blocked-off rectangular waveguide.

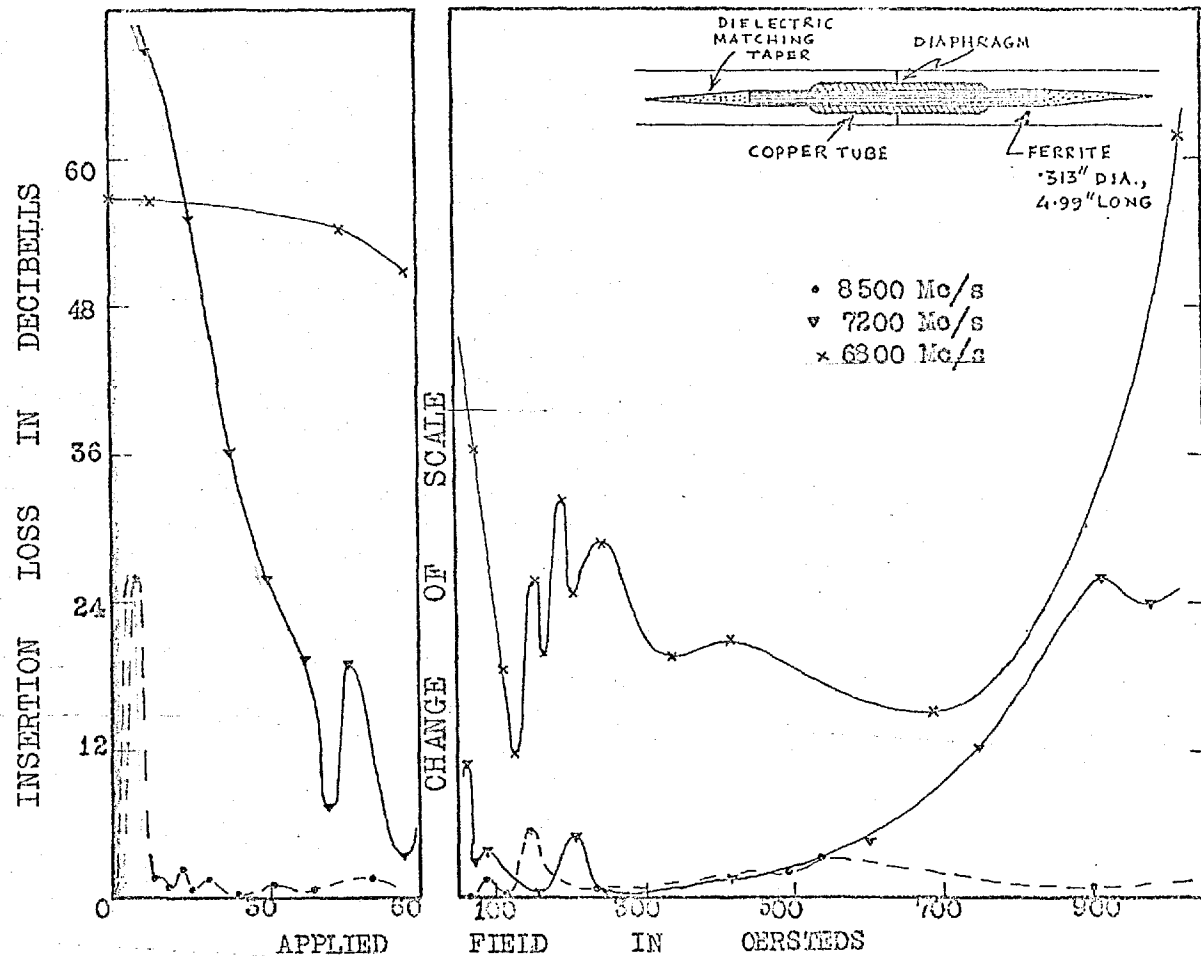


Figure 2.13. Propagation through a sheathed R ferrite rod in a blocked-off rectangular waveguide.

that the ferrite was initially slightly magnetised in the reverse direction. The application of a small field demagnetised it completely, when the large loss occurred. The fluctuation at the lower frequencies are probably due to phase change of the portion of the rod inside the metal tube. At 8500 Mc/s, however, the wave hardly "sees" the tube at all, as noted earlier for 9100 Mc/s. The curves of figure 2.13 suggest the fabrication of a high speed, high-power, wide band, reflecting switch. A helical magnetising winding could replace the copper tube and a biasing current could be provided for changing the operating frequency of the switch.

Figure 2.12 shows similar behaviour for the B2 rod. As expected, there is very little propagation when the frequency goes down to 8200 Mc/s, due to cut-off condition of the tube. Surprisingly, however, the insertion loss comes down to relatively low values again at 7200 Mc/s for high magnetising field values. This is attributed to the anomalous propagation of positive circularly polarized waves through narrow tubes, studied by many authors^(39,40). The propagation takes place when the effective permeability of the ferrite for positive circular polarization becomes negative. This effect was investigated further, as described in the next chapter, and eventually utilised in the recognition of the sense of polarization in the reciprocal phase shifter configuration.

It may be pointed out that such relatively low-loss propagation of the anomalous mode as shown in figure 2.12 indicate that the mode set up in the uncovered portion of the ferrite rod at these values of

the steady field is well matched to the anomalous mode. A further deduction would be that the phase shift in the reciprocal phase shifter and the anomalous propagation are due to similar modes under different operating conditions. This point will be discussed in greater detail in Chapter Four.

2.7 Ellipticity of Polarization

From the preliminary investigations described so far it would appear that the phenomenological model proposed by Weiss possibly represents the various characteristics of the reciprocal phase shifter most closely. Before settling this question finally, however, it was decided to investigate the elliptical polarization aspect of the model in greater detail. As described in section 1.3, various authors have dealt with this point before. It was felt necessary to supplement their conclusion with more detailed results and the following experiments were therefore performed.

2.7.1 The sharply twisted junction

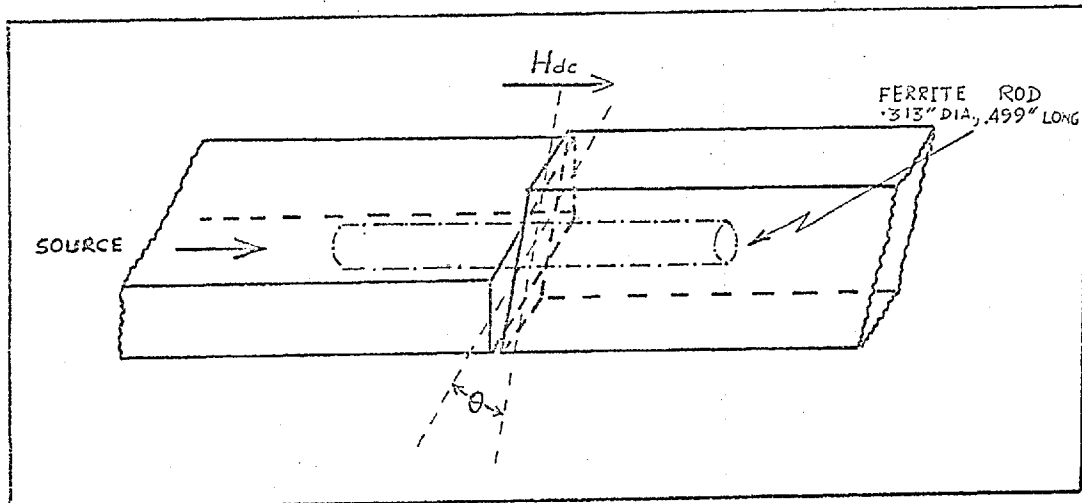
The arrangement for this experiment is shown schematically in figure 2.14(a). Rizzi and Gatlin⁽⁷⁾ had previously measured the insertion loss of such an arrangement as a function of θ for one particular intensity of H_{dc} . In the present experiment, as distinct from their work, a relatively large rod-diameter for the operating frequency was used and the measurements were taken over an extended

range of applied field strength.

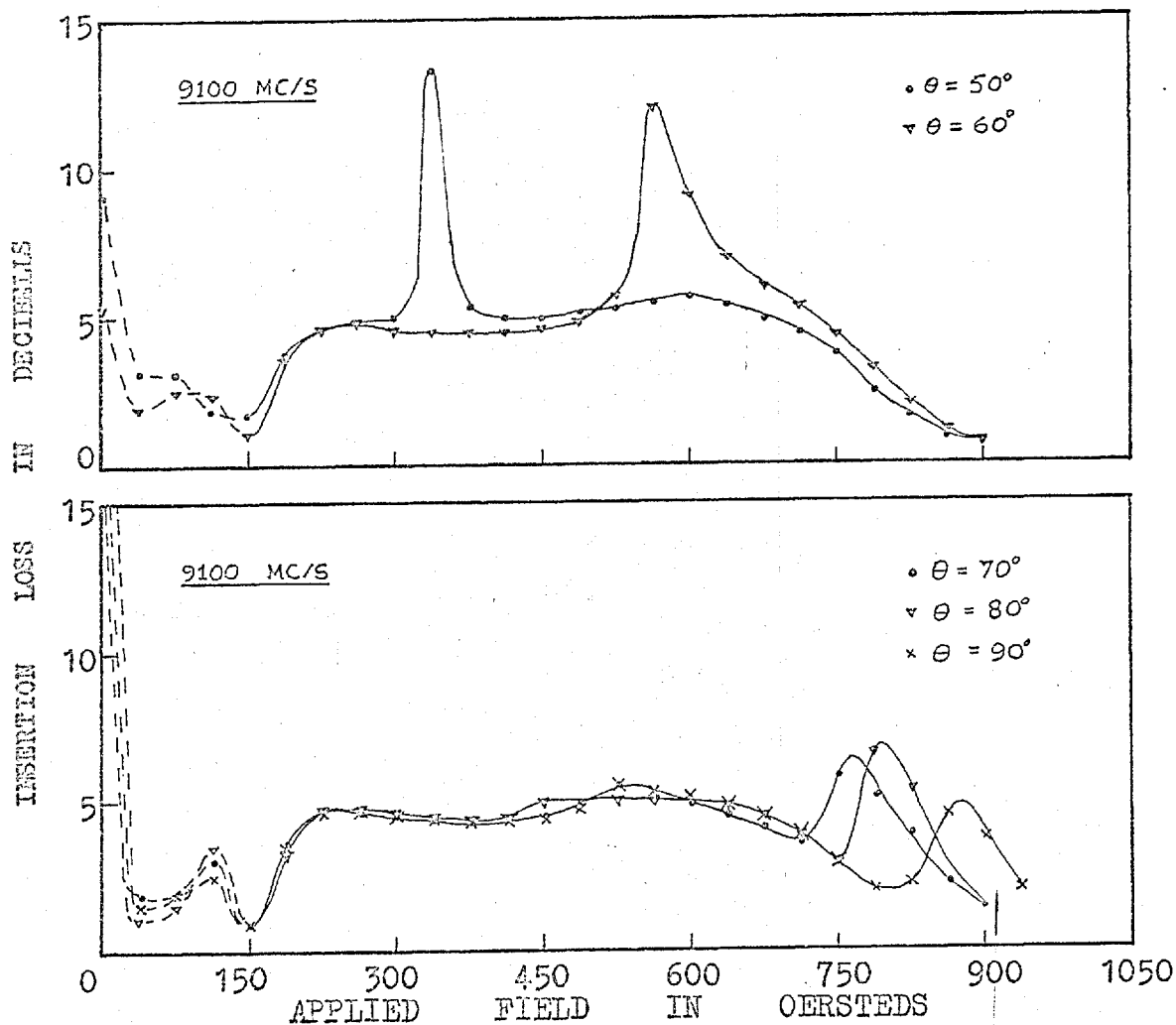
The equipment layout and the measurement procedure were the same as described in section 2.5. The rotating joint shown in figure 2.14(a) was made by removing the keys from the locking ring of the waveguide flanges. The rod was centrally placed in the junction and protruded equally on both sides. The angles were measured by noting the inclinations of the input and output rectangular sections with respect to a reference surface. These values are estimated to be accurate to $\pm 2^\circ$. Up to a twist of 40° the insertion loss pattern was very similar to the straight guide case shown in figure 2.8(a). This is not unexpected, as even without the rod the insertion loss of the junction for $\theta = 40^\circ$ was measured to be only 0.48 dB. The results for $\theta = 50^\circ$ onwards, however, showed some interesting features and are plotted in figure 2.14(b). The curves have been drawn discontinuously at low field strengths as they do not show all the small fluctuations that occur there.

Discussions

From the curves of figure 2.14(b) the insertion loss appears to be independent of the angle of twist for all values of the applied field, except for one peak of loss corresponding to each value of θ for which the measurements were taken. It appears to be necessary for the ferrite to be only very slightly magnetised for the full propagation to take place. The constant coupling from the input guide to the output under such extensively varying conditions seem to be only



(a) Experimental arrangement.



(b) Insertion loss as a function of H_{dc} with the angle of twist as a parameter.

Figure 2.14. Propagation through an abruptly twisted junction of rectangular waveguides containing a thick ferrite (R1) rod.

possible if the propagating mode is elliptically polarized.

Residual magnetisation of the ferrite was found to introduce considerable uncertainty in the values of the insertion loss at zero applied field. Very careful demagnetisation of the specimen was found to be necessary each time. This would appear to be a serious problem in the practical utilisation of Weiss's "Tetrahedral Junction" described in section 1.2, which is based on a similar arrangement.

The insertion loss peaks shown in figure 2.14(b) seemed to be related to the peaks shown in figure 1.2. It must be noted, however, that at this frequency the peaks do not occur in a straight guide for this rod (see figure 2.8(a)). In fact the peaks were very sensitive to the angle of the twist, occurring at different values of H_{dc} and in different magnitudes, as θ was varied. They were also sensitive to reflections from anywhere in the waveguide system. The positions of the peak varied if slight changes occurred at the junction, or even far away in the output section. Repeating of results, that is, getting the peaks at the same field strength every time, proved very difficult due to the number of variables affecting the phenomenon. The sudden appearance of the peaks at a particular field strength and for a particular angle of twist could not be accounted for by normal Faraday rotation. The peculiar characteristics exhibited by the peaks appeared intriguing enough to warrant their further investigation. This is described in section 2.8.

2.7.2 The discontinuous waveguide system

To get an idea of the degree of independence from the waveguide boundary of the propagating mode, experiments were performed on a system where the rectangular guide was discontinuous in the central region. A sketch of the arrangement used is shown in figure 2.15.

As described in section 1.3, Neckenb rger carried out investigations on a similar arrangement. In the present experiments the intention was to verify the ellipticity of the mode under such conditions. The transmission through the structure was therefore measured for various twists of the output section with respect to the input. Another difference with Neckenb rger's arrangement was that the open ends of the waveguides were covered by metal diaphragms to introduce further impediment to the wave propagation, and so that there was no stray pick-up by the output section.

The equipment layout of figure 2.7 was used for these measurements. The metal bobbin of the magnetising coil (see section A1.2) was not expected to influence the propagation by reflections, as previous results showed that the rf energy sticks very close to the ferrite rod. Insertion loss as a function of the steady field strength was measured for 0° , 60° and 90° twist of the output section. The results are shown in figure 2.15.

Discussions

The curves show that, except at zero applied field, the insertion loss patterns are very similar to the continuous guide case of figure

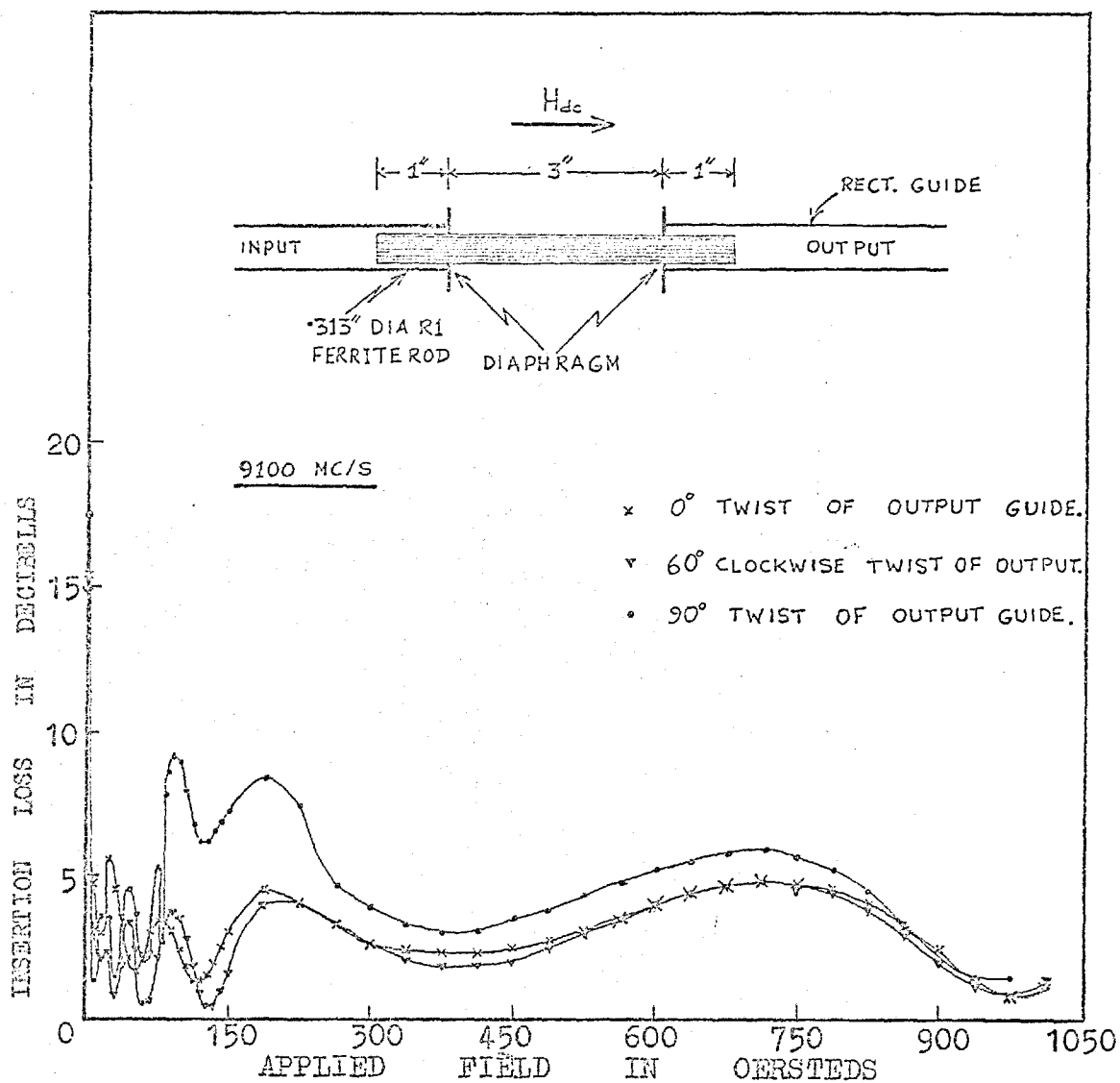


Figure 2.15. Propagation through a .313" dia. R1 ferrite rod between a discontinuous rectangular waveguide system.

2.8(a). If anything, the losses are appreciably lower in this case, particularly for 0° and 60° twist. It should be noted that no sharp peaks of loss are present for the twisted guides.

These results emphasize the degree of rf energy concentration in the ferrite rod and the ellipticity of the modes of propagation. The remarkable ability of the modes to couple into the empty waveguide without much reflection under such conditions, shows that the H_{01} mode of the empty waveguide converts naturally into the mode propagating in the loaded region without much modification. It also suggests a single mode of propagation in the ferrite loaded region.

2.8 Investigations of the Insertion Loss Peaks

The large peaks of insertion loss in sharply twisted guides discovered during the experiments described in section 2.7.1, at first appeared to be a further complication in an already puzzling phenomenon. They were so prominent that a definite mechanism was thought to be needed to produce them. As this mechanism could be an important aspect of the phase shifter configuration, these insertion loss peaks were studied in detail using the same specimen and the same experimental set up as described in that section.

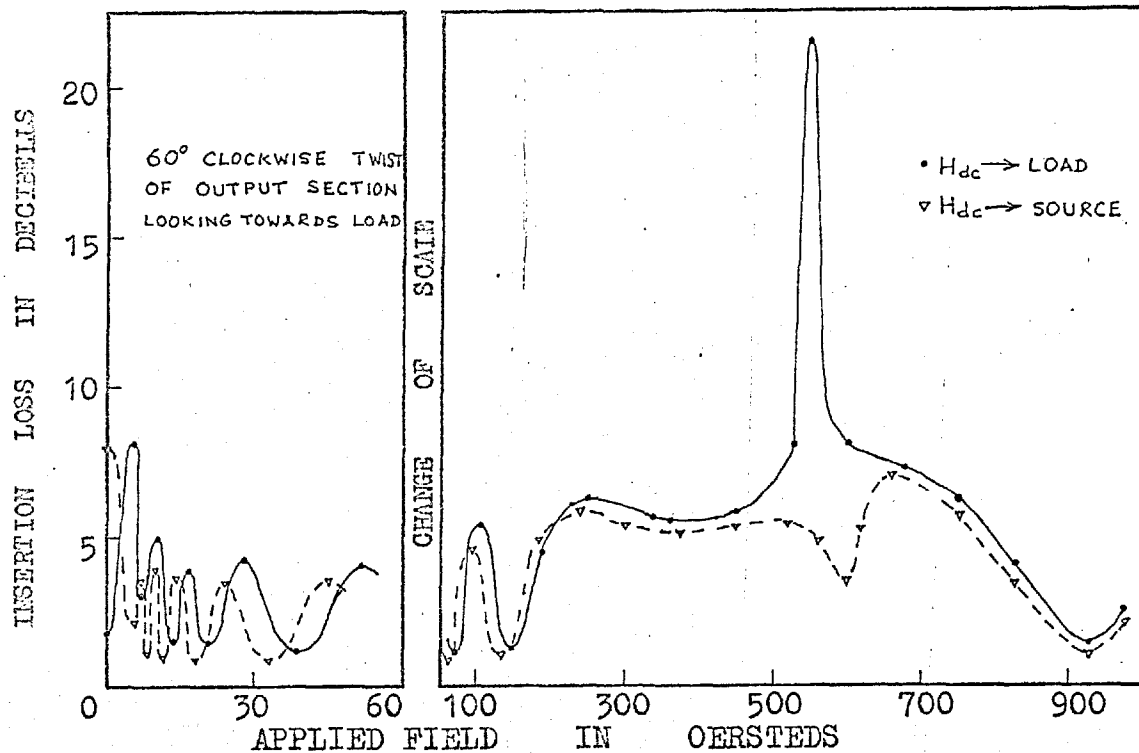
2.8.1 Direction of the steady magnetic field

As a first step, the direction of the steady field applied to a 60° clockwise twist was reversed to see if the phenomenon resembled

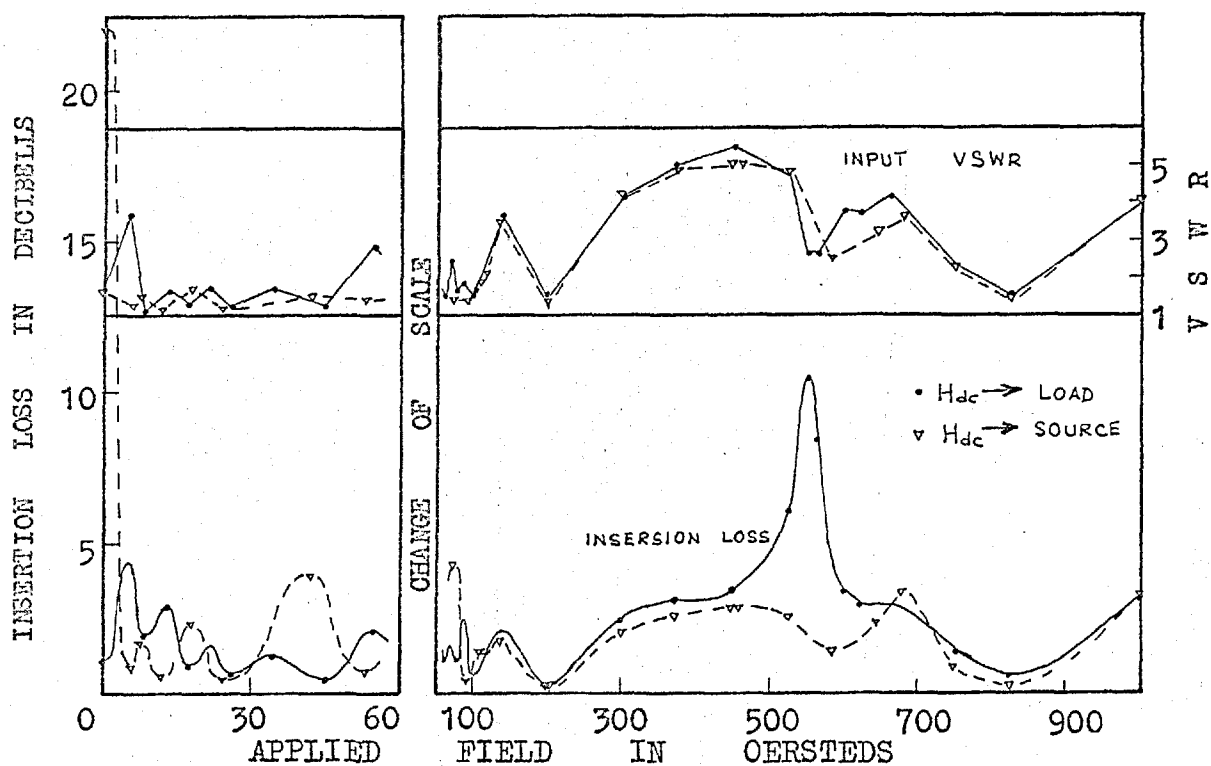
Faraday rotation in being non-reciprocal. This was found to be the case, as shown in figure 2.16(a). Instead of a peak, a dip in the loss was noticed around the same steady field strength for the reversed direction. Measurements were also taken with the ferrite rod ended in the matching tapers described in section 2.6, to cut down the reflection due to impedance variation. The results, plotted in figure 2.16(b), show the loss peak to be still present, though reduced in magnitude. The reflections at the ends of the rod of the normal H_{01} wave can therefore be taken to play only a very small part in the phenomenon.

From figure 2.16(b) the non-reciprocal behaviour of the structure can be seen not to be limited to the one high peak. Non-reciprocal peaks occur from the lowest applied field strengths. The high initial losses at zero applied field in the curves for reversed direction of H_{dc} in figures 2.16(a) and (b) were most probably due to the remnant magnetisation of the ferrite slightly shifting the point of occurrence of the peak.

Figure 2.16(b) also shows a plot of the input voltage standing wave ratio of the configuration. Corresponding to the loss peak the VSWR shows a dip, indicating that the energy is trapped and absorbed rather than reflected. This, of course, has to be the case from thermodynamic considerations, due to the non-reciprocal nature of the structure. A combination of high non-reciprocal peak of loss with a low VSWR therefore appears to be the trade mark of this phenomenon. According to this criterion, the structure experimented with can be seen from figure 2.16(b) to behave in a similar manner also near zero and at 42 and 74



(a) Rod ends unmatched.



(b) Matching tapers at the ends of the rod.

Figure 2.16. Effect on the insertion loss peaks of the direction of the steady magnetic field and of matching the ends of the ferrite rod. .313" dia., 4.99" long R1 rod used at 9100 Mc/s.

oersteds intensities of the steady field, applied in the direction of the source.

It was also found that for the reversed direction of the applied field the loss peak reappears for an equal twist of the output guide in the opposite direction. Corresponding angles of twist for the two directions of H_{dc} were measured to be within $\pm 1^\circ$ of each other. In the present case the loss peaks at the high field strengths occurred for a clockwise twist of the output section with respect to the direction of H_{dc} . This would appear to be opposite to what would be expected from the direction of normal Faraday rotation in an infinite medium.

The peak of loss was reciprocal with respect to the direction of the applied field for a twist angle of 90° . No large peaks were noticed below 49° of twist, perhaps because the discontinuity became negligible at such low values of twist with the ferrite rod in the junction.

The dependence of the peaks on the direction of the applied field would seem to indicate that some form of Faraday rotation must be mainly responsible for the occurrence of the phenomenon.

2.8.2 Importance of the junction discontinuity

Some experiments were carried out at 9100 Mc/s to find out how important the exact shape of the junction was to the production of the insertion loss peaks. In one experiment a metal diaphragm was introduced into the rotating junction, fitting the ferrite rod closely

and blocking-off the rest of the guide. All other conditions were the same as before. With this arrangement the peak was not detected for any angle of twist of the output section. The insertion loss pattern was always very similar to the case of the straight guide without obstruction, shown in figure 2.8(a) for 9100 Mc/s.

This result shows that the exact nature of the discontinuity and thus the form and phase of the reflected wave is very important. It also shows that the effect is not due to the wave becoming linear and cross-polarized with respect to the output guide. If this was the case, the peaks would not disappear when the diaphragm was introduced. It would thus appear that some effect in addition to Faraday rotation is necessary for the peaks. This effect could be the exact phase relationship between the incident and the scattered waves.

The waveguide was also opened up at the rotating junction to see the effect on the insertion loss. Various values of the gap up to 1" was checked for a twist angle of 60° . No peaks were noticed and the loss pattern remained the same as in a straight continuous guide.

To prove conclusively that the loss peaks were not due to the wave becoming linear and cross-polarized at a particular point in the structure, the ferrite rod was put into a rectangular waveguide gradually twisted by 90° over a 6" length. This structure was checked over the whole range of applied magnetic field for various lengths of the rod in the twisted region. A twist of the output plane with respect to the input from 0° to 75° (the rod being 5" long) was thus checked. No high peaks were detected.

The importance of the polarization of the wave at the output end of the rod was investigated by putting the rod completely in the input section of the rotating junction with its output end flush with the junction. The output guide was twisted 90° with respect to the input. The insertion loss of this arrangement as a function of the applied field was found to decrease as the steady field intensity increased and at the same time show the familiar oscillatory form. It finally settled down to the same form as shown in figure 2.8(a), though the loss magnitude was about double. There was a large peak of insertion loss again, showing that the output end polarization is not the important consideration in this phenomenon. This experiment also indicated the gradual increase of cross-polarized component up to the limit of the magnetising field strength (1100 oersted) used. This point was confirmed in later experiments described in chapter three.

2.8.3 Effects of rod displacement

The two basic phenomena likely to be responsible for the insertion loss peaks, phase change and Faraday rotation, are both proportional to the length of the specimen. It was therefore decided to determine the effect on the insertion loss peaks of displacing the ferrite rod longitudinally with respect to the twist-junction.

Measurements were taken for some spot displacements of the rod for a twist angle of 60° at 9100 Mc/s. The insertion loss peak was obtained with slight variations, in all the cases.

These results, however, could not be taken to be conclusive, and a

continuous variation of rod position was attempted. The rod was supported centrally by two thin paxylin spiders near the two ends. Two pieces of silk threads (.006" diameter) were tied at the two ends of the rod and taken out of the waveguide through the slots in the VSWR indicator and the output detector tuning probe assembly (figure 2.7). These threads were used to displace the rod in either direction by up to 2.6 cm. The limit of 2.6 cm was imposed by considerations of uniformity of the applied magnetic field, the ferrite rod being 12.65 cm long (see figure A1.3). The insertion loss peak was located with the centre of the rod coinciding with the junction plane. Then, keeping the applied field constant the insertion loss was recorded as a function of rod displacement. Slight variations of the applied field, however, did occur during the course of the experiment.

The results for a twist angle of 52.4° are shown in figure 2.17. It can be seen that though the peak magnitude fluctuates with rod position, the essential characters of it, that is, the non-reciprocity and the low VSWR, are maintained for the total displacement of the rod by 5.2 cm. Only when the displacement became too large, so that the average H_{dc} over the specimen was no longer constant, did the insertion loss fall off. Even then the peak was found to reappear when the intensity of H_{dc} was increased. Also, peaks at other values of H_{dc} were noticed for the largely displaced rod.

These results show clearly that the whole length of the rod is involved in producing the phenomenon rather than one part of it on either side of the junction. If either the input or the output section

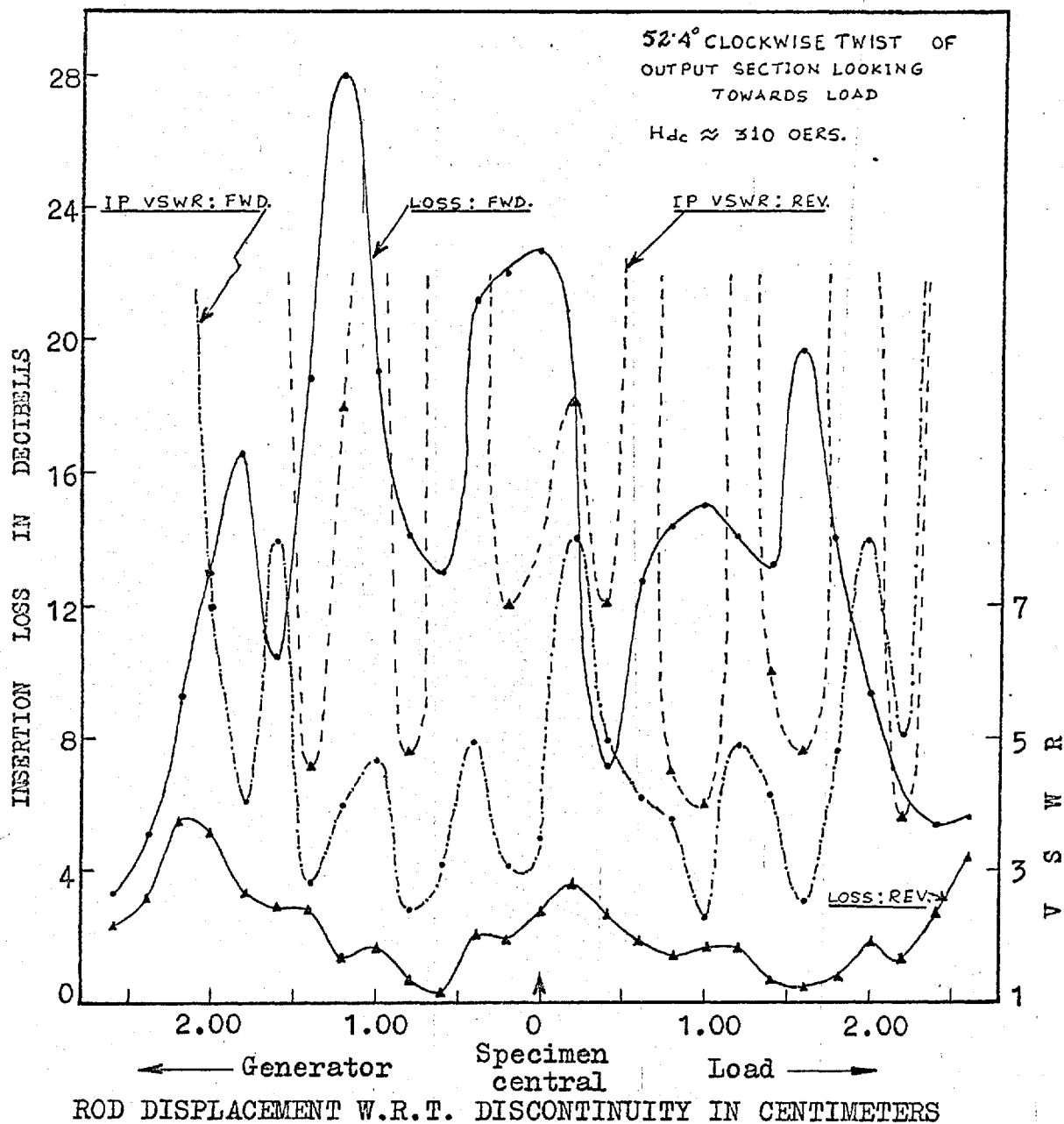


Figure 2.17. Effect of rod displacement on the insertion loss peak. .313" dia., 4.99" long (12.65cm.) R1 ferrite rod used at 9100 Mc/s. FWD indicates direction of H_{dc} towards load.

was only responsible, and assuming that either phase shift or Faraday rotation or some combination of these two must be causing the effect, then the phenomenon would have had to disappear and reappear with the rod displacement. It would seem that the part played by the abruptly twisted junction is confined to a very small region around the junction. The results rule out conventional length dependent Faraday rotation, which was strongly indicated as the main cause of the loss peaks by the results of section 2.8.1. The phenomenon appears to be an end-to-end effect involving the whole rod, on which the non-reciprocal character of the magnetised ferrite is impressed through the abrupt junction.

2.8.4 Frequency dependence

The effect of variation of the wave-frequency on the insertion loss peaks was also investigated. The equipment used was the same as shown in figure 2.7. For large variations of the operating frequency all the equipment and the output detector in particular, were carefully retuned. However, for small frequency variations, such as when recording the frequency response of one peak, this was not considered necessary.

From measurements at spot frequencies, high peaks for a 60° twist angle were detected from 9000 Mc/s upwards for the R1 rod of .313" diameter. In later experiments, to be detailed in chapter Five, the peaks were detected down to 8850 Mc/s for a shorter length of rod passing through two twists. The lower frequencies, however, were not

thoroughly investigated and a lower limit cannot be set for the occurrence of these peaks.

The steady field strength required to produce a loss peak, as a function of frequency, is shown in figure 2.18 for a twist angle of 64° . The field requirement can be seen to decrease with decreasing frequency. In subsequent experiments with the double twist the field requirement was found to increase with decreasing frequency. The field intensity in that case was one order lower. This would indicate a correlation with the phase shift behaviour of the ferrite, which decreases with increasing field strength in the former case and increases in the later.

The frequency response of a 50.5° twisted junction, for a value of H_{dc} for which the loss peak occurs, is shown in figure 2.19 for both directions of the applied field. It can be seen that, under the conditions of the loss peak, the system behaves like a highly selective, non-reciprocal, absorption cavity. The bandwidth at points where the loss is down by 20 dB is less than 9 Mc for a centre frequency of 9099 Mc/s. It was found in later experiments that the selectivity is lowered for shorter lengths of the ferrite rod.

Of all the causes that may be contributing to the production of the loss peaks, only the phase shift of the wave can be responsible for the degree of frequency sensitivity indicated above. Such a high Q is possible if the wave is suffering repeated reflections in the system in the process of getting trapped and if exact phase relationships

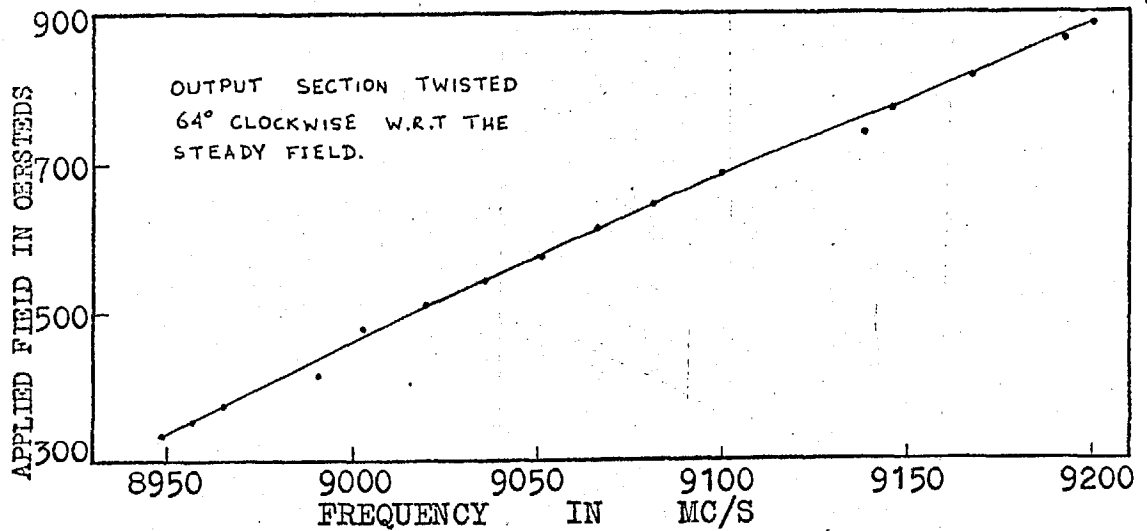


Figure 2.18. Frequency dependence of the steady magnetic field required to produce a loss peak.

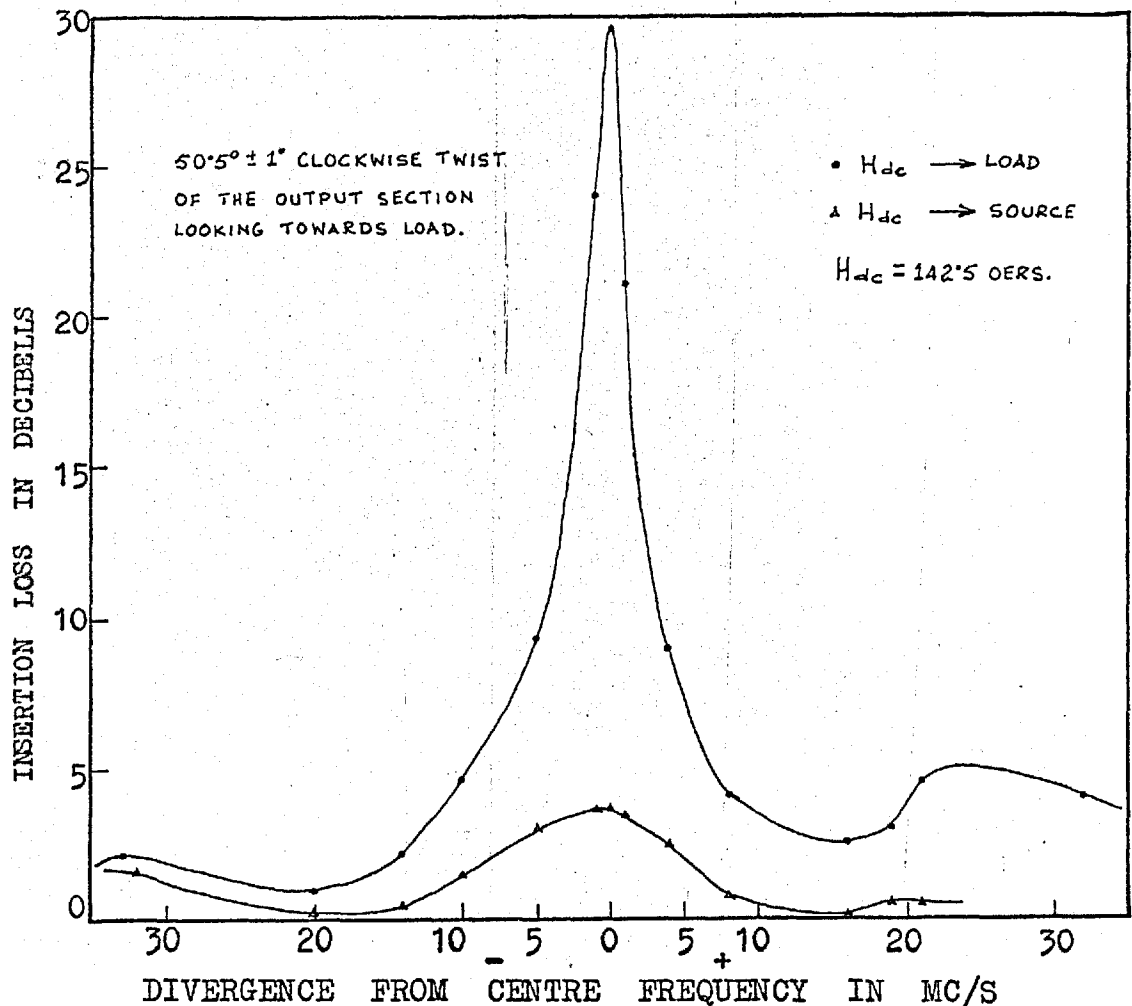


Figure 2.19. Frequency response of a sharply twisted waveguide junction containing a .313" dia., 4.99" long R1 ferrite rod along the longitudinal axis, around a centre frequency of 9099 Mc/s.

are necessary for the reflections to occur. These points will be discussed further in Chapter Four.

The characteristics of the twisted junction revealed during these investigations suggested the development of a novel device. It was realised that a very narrow band tunable isolator could be fabricated based on this structure. Some work was carried out in this direction and will be described in Chapter Five.

CHAPTER THREE

THE MODE PATTERN

3.1 Introduction

The investigations described in Chapter Two confirmed that the perturbation analysis, as detailed in section 1.4.3, is not applicable to the case of a thick ferrite rod inside a rectangular waveguide. It was also found that none of the behaviours of the system violates the predictions of the phenomenological model of Weiss. In this chapter the detailed work carried out to recognise the modes of propagation of electromagnetic waves in the configuration is described. It was found by direct measurements that, in the phase shifter range of rod-diameter or frequency, the behaviour of the system differs considerably from that of an elliptical Faraday rotator, which it resembles. The dominant and the first higher order modes of the system predicted by the phenomenological theory were identified. The results led us to the conclusion that the phenomenological model qualitatively describes the reciprocal ferrite phase shifter very well indeed.

3.2 Wave Polarization

3.2.1 The structure as an elliptical Faraday rotator

The phenomenological model predicts that, in the useful region of operation, the reciprocal ferrite phase shifter configuration has a

single elliptically polarized normal mode,^x with its sense of polarization anticlockwise with respect to the longitudinally applied steady magnetic field. The mode experiences a scalar permittivity and permeability in the structure (see section 1.4.4) and propagates without rotation or change of form. The picture differs from an ordinary Faraday rotator, which has two, in general elliptically polarized, oppositely running normal modes, possessing different phase velocities. Faraday rotation effects can be thought to be due to the beating between these two modes, as they progress along the structure.

At first sight, the single normal mode prediction of the phenomenological theory would appear to be wrong. As pointed out by Rizzi and Gatlin⁽⁷⁾, in the range of frequency and rod diameter of interest, the ferrite rod would propagate energy in the H_{11} limit-mode, even if completely enclosed by a metal sheath. It would seem, therefore, entirely reasonable to assume that when such a rod is placed inside the rectangular waveguide, a H_{11} type limit-mode would be able to exist in any orientation in the system. Variations of the steady magnetic field would not affect this conclusion, as the cut-off point

^x A "normal mode" is defined⁽³²⁾ as having a unique distribution of transverse electric and magnetic field components, independent of frequency and position along the waveguide. For a given frequency, normal modes have unique phase velocities, except for degenerate pairs, which have the same phase velocity. The normal modes of a circularly symmetric Faraday rotator are the positive and negative circularly polarized modes. If two waveguides, each of which allow propagation in a certain mode, are coupled together continuously, then the combined system will have two normal modes, in general different from the original modes.

of this mode is independent of the magnetisation of the ferrite.⁽³⁸⁾
 In that case the structure would appear to be analogous to an elliptical Faraday rotator.

Perhaps the best way to visualize the behaviour of a Faraday rotation device in a non-circular waveguide is through the coupled mode approach.^x This method is based on the theory of coupled waves developed by Miller⁽²⁷⁾, and uses two orthogonal linearly polarized modes continuously coupled through the off-diagonal component of the permeability tensor. Power cycles back and forth between these two modes as the wave travels along the waveguide. The form of polarization at any transverse section is dependent on the respective amplitudes and phases of the two modes at that point, and is always linear in the case of a circular waveguide. In a non-circular guide complete transfer of power from one mode to the other does not take place and the phase of one mode changes with respect to the other as they travel along the guide. The wave polarization at a transverse plane is therefore elliptical in general. The normal mode and the coupled mode explanations of Faraday rotation are completely equivalent⁽⁴²⁾.

With the help of Miller's results it can be shown that the wave polarization in a Faraday rotator, composed of a non-circular metal waveguide and an axial ferrite rod, changes from ^{linear} to elliptical and back to linear again, as the wave progresses along the waveguide. The ellipticity is due to the difference in the phase velocities of the two

^x

See, for example, reference (41).

orthogonal linear coupled modes. Simultaneously, the axes of the polarization ellipse will suffer rotation. The amount and the direction of the rotation at any transverse plane will depend upon the amplitudes and the phases of the two coupled waves at that plane. In general the rotation would first be clockwise with respect to the steady field direction. At some point along the waveguide the major axis will turn back and return to zero rotation when the coupled waves are 90° out of phase with respect to each other. Only if at this plane the two waves are also of equal amplitude, will the wave-polarization be circular. As the wave travels further, the rotation will become anticlockwise with respect to the steady field direction.

If the linearly polarized input wave is initially launched in the orientation which has the lesser phase velocity (parallel to the narrower dimension of the waveguide), then the sense of the elliptical polarization will be anticlockwise with respect to the steady field direction, as long as the phase difference between the two coupled waves do not exceed 180° . The phase difference seldom exceeds 180° in the cases when the phase velocities in the two orthogonal directions are near enough for appreciable transfer of power from one mode to the other to take place (i.e., for appreciable ellipticity).^{*}

^{*}

According to Miller's theory of distributed wave coupling, complete transfer of power can only take place if the phase velocities of the two coupled modes are equal. The proportion of maximum power transferred decreases as the ratio between the phase constant difference and the coupling constant increases from zero. When this ratio, $\frac{\beta_1 - \beta_2}{C}$, equals 2, only half the power can be transferred. The maximum phase difference between the waves in this case is 180° .

Some of the behaviour of the elliptical Faraday rotator described above had already been noticed in the reciprocal phase shifter structure. Thus it had been seen that the ferrite rod concentrates most of the rf energy within itself and behaves like a waveguide. That the propagating mode is elliptically polarized in an anticlockwise sense with respect to the steady field, had been suggested by previous workers and indications found in the experiments described in Chapter Two. The non-reciprocal character of the system, noticed during the investigations on the insertion loss peaks described in section 2.8.1, also suggest a strong resemblance to a Faraday rotation device.

Neckenbürger's measurements of the output wave-polarization from the reciprocal phase shifter configuration, as mentioned in section 1.3, would seem to provide a direct answer to this question of whether the structure behaves like a Faraday rotator or not. His results, however, contained some uncertainties. The results showed considerable rotation of the major axis of the polarization ellipse. Also, circular polarization was found at very low applied field intensities (18 to 43 oersteds). According to the phenomenological model circular polarization in a non-circular waveguide would only occur when $\mu = k$ in the hypothetical tensor permeability (1.4). Though these hypothetical components are modified forms of the actual tensor components of the ferrite material, they cannot be imagined to satisfy the $\mu = k$ relation at such low fields for the frequencies used by him.

The anomalies in Neckenbürger's results could be due to the fact that he had a part of his ferrite rod specimen projecting into the

circular waveguide used to check the output polarization. It was, however, necessary to be certain about this matter in view of the considerable arguments in favour of a Faraday rotation explanation. Moreover, Neckenb rger had not ascertained whether the output polarization depends on the length of the specimen or not.* The single normal mode of the phenomenological theory would necessitate length-independence, whereas, Faraday rotation would make the polarization vary with the length of the specimen. For these reasons measurement of the output polarization was carried out in a manner similar to Neckenb rger, but with some modifications.

3.2.2 Experimental investigations

The changes made for this experiment to the equipment described in section 2.5, is shown in figure 3.1(a). The only difference in the arrangement shown there with Neckenb rger's are the dielectric tapers used for end-matching instead of the rod itself being tapered to a point. The use of dielectric taper to match into the circular guide prevented any Faraday rotation in that portion from masking the actual results.

The rotating junction^{consisted} of ordinary waveguide flanges rotating with respect to each other. An annular shim of .003" thick phosphor-bronze with serrated inner edge (internal diameter 1", same as the circular guide) was used to maintain good contact as the circular guide was rotated. The angle of rotation was measured by a fixed protractor

* Private correspondence.

on the circular guide.

Without the specimen in and with a rectangular-to-circular transformer at the rotating junction the difference between signal strengths with the probe parallel to the narrow dimension of the rectangular guide (E_v) and with it parallel to the broad dimension (E_H), was always greater than 35 dB. The input VSWR in this case for all rotations of the circular guide was between 1.12:1 and 1.13:1.

E_v and E_H were measured for different specimen lengths and diameters. The positions of the major axes of the polarization-ellipses were also recorded. The rotations of the axes were found to be very small (maximum 12°). A bracketing technique had to be adopted to measure them exactly. The angles on either side, when the probe response was down by about 10% from the maximum, were noted and their mean taken as the inclination of the major axis. Some measurements were made along the complete azimuth to make sure that the wave polarization was really elliptical and not just linear with an inclination to the vertical. By changing the direction of H_{dc} it was ascertained that these rotations of the axes were non-reciprocal. The variations in the length of the specimens were achieved by sticking small additional lengths of ferrites, by a polystyrene-base glue, to the original length. All readings were taken relative to the case of the unmagnetised ferrite. The results are shown in figures 3.1 and 3.2.

Uncertainties in the measured values were introduced due to the following causes:

- (a) Need to place the specimen exactly central everytime.

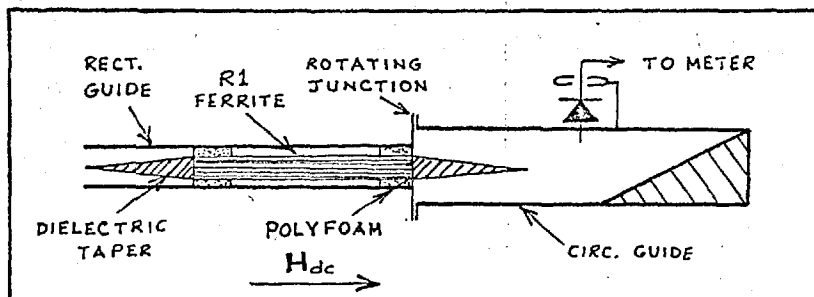
- (b) The dielectric taper at the output was slightly eccentric.
- (c) Angle indicating mechanism could be read only up to $\pm 0.5^\circ$.
- (d) Variations in E_v and E_H due to remaining uncertainty of contact at the rotating junction.
- (e) The rotational axis of the junction may have been eccentric by a small amount.
- (f) Finite thickness of the probe.
- (g) Reflection from discontinuities.

It is estimated that the signal strength readings can be taken to be correct to $\pm 5\%$ and the angle measurements to $\pm 2^\circ$.

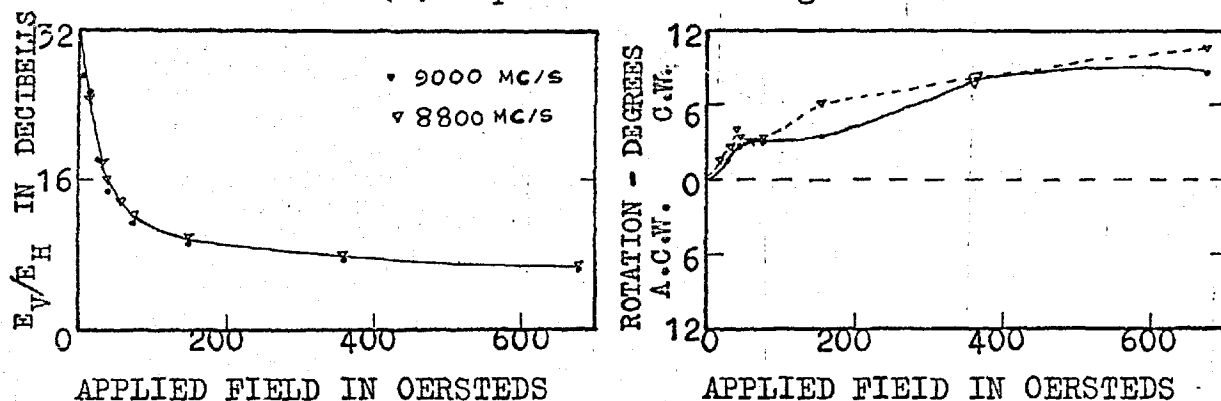
Discussions

The results of figures 3.1(b) and (c) are for the .281"-diameter R1 rod, which at the frequencies used, do not show the energy trapping effects discussed in section 2.5, when used in the reciprocal phase shifter configuration. Figure 3.1(d) is for the .313"-diameter rod, which shows some fluctuating absorption of energy at 9000 Mc/s.

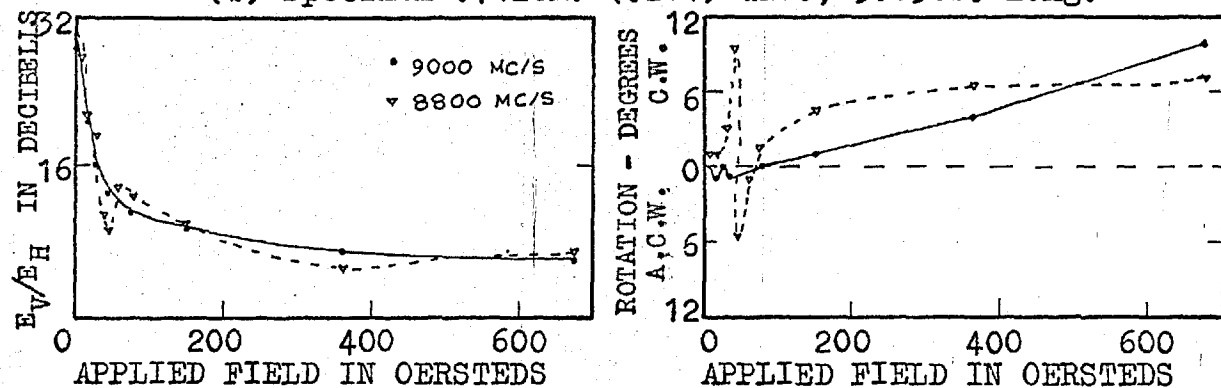
It can be seen from the figure that some rotation of the polarization ellipse occur for both the specimens and at all frequencies. The amounts of rotation are in general compatible with the magnitude of E_H excited. Both clockwise and anticlockwise rotations are obtained, as predicted by the theory of coupled modes in elliptical Faraday rotators. The frequency dependence of the rotation found, can also be explained by this theory. The phase velocity difference (and so $\frac{\beta_1 - \beta_2}{c}$)



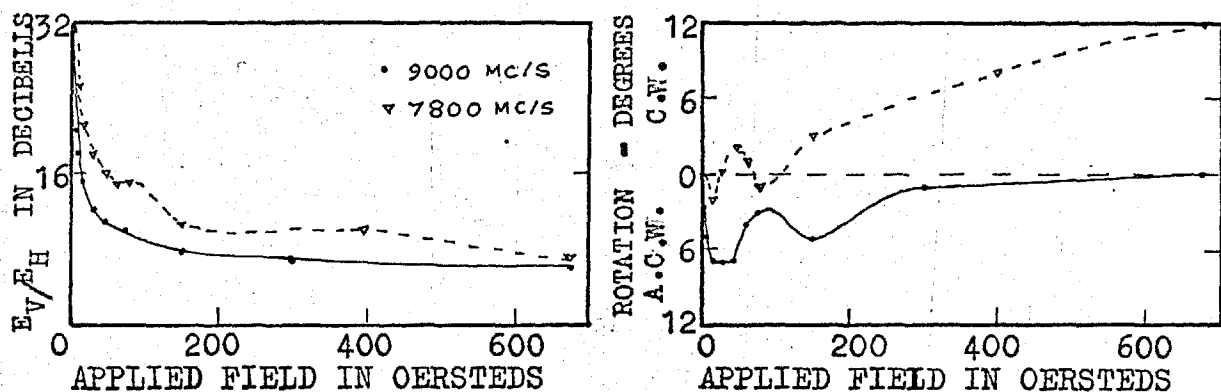
(a) Experimental arrangement.



(b) Specimen .712cm. (.281") dia., 5.65cm. long.



(c) Specimen .712cm. (.281") dia., 7.08cm. long.



(d) Specimen .795cm. (.313") dia., 11.25cm. long.

Figure 3.1. Polarization of output wave from the phase shifter.

would be expected to vary as the frequency changes. This will result in variations in the phases and amplitudes of E_v and E_H and thus variation in the rotation.

The E_H excited increases nearly monotonically as the applied steady magnetic field intensity is increased. It increases quicker for the thicker diameter specimen, although this was seen earlier in section 2.3 to have lesser phase shift. From this it is concluded that the phase shift is not directly related to the magnitude of the cross-polarized component excited, but is governed more by the properties of the material. It can be seen from figure 3.1 and 3.2 that at no point is the wave completely circularly polarized, as opposed to Neckenb rger's results. This is contrary also to conjectures of many previous workers, as surveyed in Chapter One. The frequency dependence of the cross-polarized components excited can be seen to be small for the wide range of frequencies checked. The elliptical wave wave polarization of figure 3.1 would agree well with the explanation of this system both by the phenomenological model and as an elliptical Faraday rotator. Hence, to distinguish between these two theories, the length-dependence of the polarization also had to be investigated.

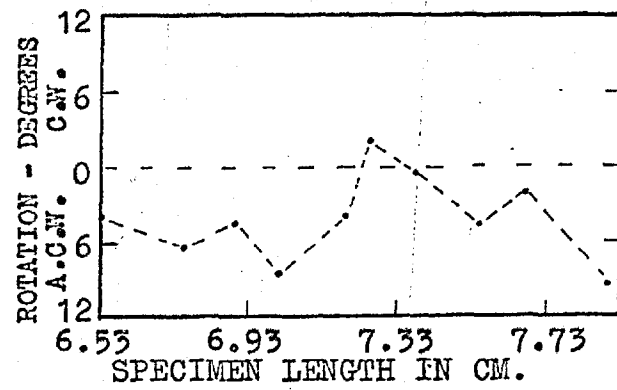
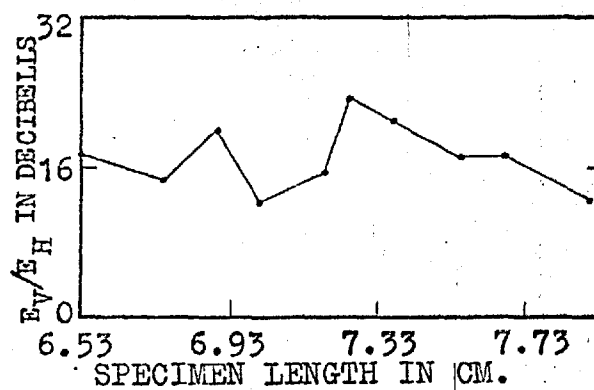
The specimen more likely to show Faraday rotation, the .313"-diameter R1 rod at 9000 Mc/s, was used for this part of the experiment. The amount of Faraday rotation to be expected in a saturated infinite medium of this material is about $115^\circ/\text{cm}$, for a saturation magnetisation of 1790 gauss. From the view point of the theory of coupled modes this means that the cross-polarized mode would increase from zero to

a maximum and back to zero in a distance of about 1.57 cm. In the present case, where the phase velocities of the two coupled modes would be unequal, the theory predicts the cycling of E_H to occur over a much shorter distance, if the magnitude of the coupling between the waves remain the same. The output polarization form was therefore recorded for a variation of the length of the specimen up to 1.4 cm.

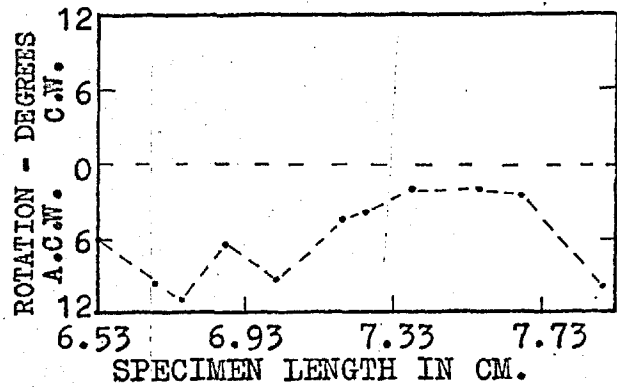
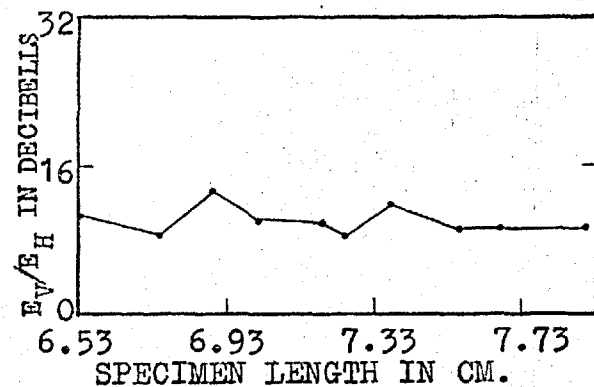
The results are shown in figure 3.2. It can be seen that the expected fluctuations in E_H could not be detected over the length investigated. There are some fluctuations at very low fields, which could partly be due to experimental errors, in view of the smallness of the value of E_H measured. At higher applied fields there is hardly any fluctuation that cannot be accounted for by experimental errors. The greater the value of H_{dc} , the greater is E_H and the less is the fluctuation in it. This is completely contrary to the behaviour expected from a Faraday rotator.

The rotations of the major axis, in figure 3.2, can perhaps be said to be fluctuating around the zero value. In this respect the structure may be behaving like an elliptical Faraday rotator. These small rotations probably cannot be accounted for by the approximate theories on the system at present in existence.

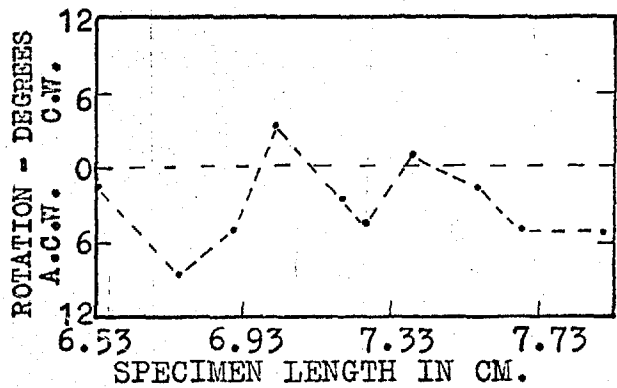
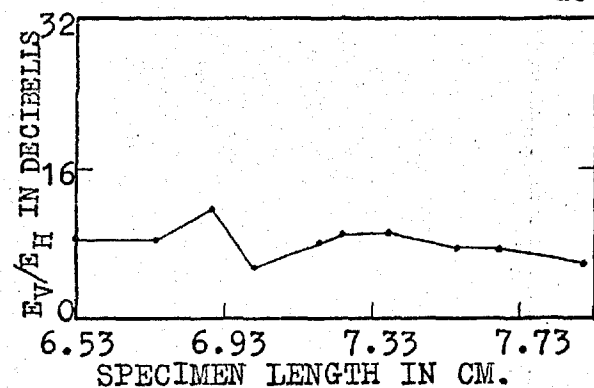
One is thus led to the conclusion that the system has only one elliptically polarized normal mode, as predicted by the phenomenological model. The ellipse does not alter significantly as the wave progresses along the system, but the axes of the ellipse possibly suffer limited rotation. The rotation most probably decreases for thinner rods and



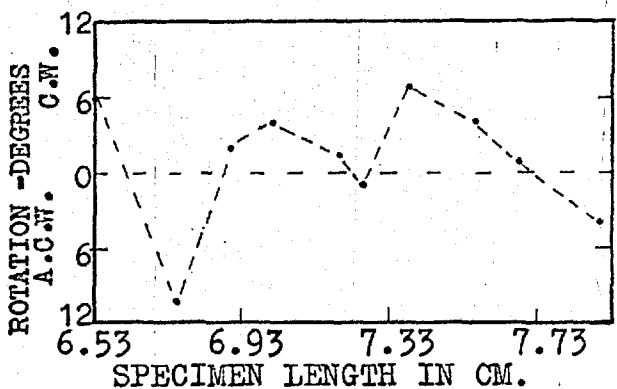
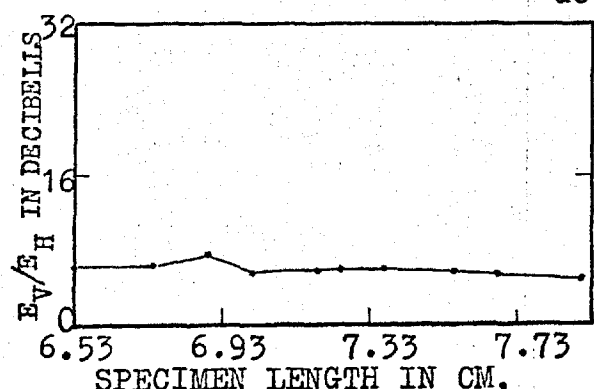
(a) $H_{dc} = 15$ oersteds.



(b) $H_{dc} = 75$ oersteds.



(c) $H_{dc} = 150$ oersteds.



(d) $H_{dc} = 675$ oersteds.

Figure 3.2. Output wave-polarization as a function of length for the .795cm. dia. specimen at 9000 Mc/s.

lower frequencies. The cross-polarized mode of the coupled-mode theory appears from the results not to be able to exist in the system, except perhaps at very low fields for this thick rod.

That normal Faraday rotation does not take place in the structure, would seem to mean that the normal mode of this system is basically different from a combination of two cross-polarized H_{11} limit-modes that would exist with such a rod in space or when completely enclosed by a metal sheath. With this rod in a square waveguide, whose sides are equal to the narrow dimension of the rectangular guide, complete Faraday rotation of the linearly polarized wave would obviously take place. It is surprising that increasing one of the dimensions of the square waveguide causes the positive circularly polarized normal mode of the system to go to cut-off and warps the negative to elliptical polarization. This cut-off behaviour is reminiscent of anomalous cut-off of positive circularly polarized waves in very thin ferrite filled metal tubes, studied by Thompson⁽³⁹⁾ and others. The first branch of this anomalous mode⁽⁴⁰⁾ propagates through infinitely thin tubes, but goes to cut-off when the tube diameter exceeds the value where the normal dominant mode can propagate. There would thus appear to be some connection of the present system with a system propagating the anomalous mode. This was also pointed out earlier in section 2.6 and will be further discussed in the next chapter.

The above results combined with Neckenb rger's⁽³⁾, show that a switchable circular polarizer as suggested by Clarricoats⁽¹³⁾ can be easily fabricated using this configuration. The ferrite rod will have

to be tapered into a circular guide to permit some coupling into the cross-polarized component. A variable magnetising current will enable exact circular polarization to be obtained over a large bandwidth. This polarizer will be easier to manufacture than similar devices⁽³²⁾ already available.

3.3 Modal Regions

With the validity of the phenomenological model established, it was necessary to investigate the predictions of the theory in detail. As a first step it was decided to investigate the regions of occurrence of the different modes of the system in the frequency domain.

Weiss⁽⁶⁾ and Neckenb rger⁽³⁾ had taken the insertion loss peaks shown in figure 1,2 as the indication of the breaking out of the higher order mode. This procedure was considered unsatisfactory in view of the fact that similar peaks can be induced at much lower frequencies by certain discontinuities, as has been seen in sections 2.7 and 2.8. It was thought that a cavity formed by a section of the reciprocal phase shifter configuration would perhaps give a clearer indication of the division between the regions.

The cavity was formed by attaching by polystyrene glue, two .003" thick copper discs of the same diameter, to the ends of the .313" diameter R1 ferrite rod. The coupling by this arrangement had previously been found to be small enough (see section 2.5) for the system to behave like a cavity. It had also been found that this

discontinuity does not alter the essential characteristics of the system. The insertion loss and the input VSWR of this system were measured for the frequency range of 8,000 to 10,100 Mc/s for a longitudinally applied steady magnetic field intensity of 112.5 oersteds. The field strength was a convenient value, chosen to approximately saturate the ferrite material, but still be low enough to be within the rising portion of the phase shift curve for this rod.

The experimental set up was the same as shown in figure 2.7. To cover such a wide frequency band accurately, the equipment had to be retuned and a reference reading taken for each frequency of measurement. Readings were taken at every 25 Mc in general to accurately record the sharp fluctuations in the insertion loss. The results are shown plotted in figure 3.3.

Discussions

Up to about 8700 Mc/s the system can be seen to behave like a very long cavity, with resonance at approximately every 170 Mc. Up to this point the system can be taken to possess a single mode only.

Beyond 9200 Mc/s the loss curve seems to have lost any regular pattern. The VSWR was found to be generally low in this region. At these frequencies, most probably, the positive elliptically polarized mode can also propagate through the system. The system thus shows spurious responses as well as effects of Faraday rotation. The gradual rise in the level of insertion loss in this region is most probably due

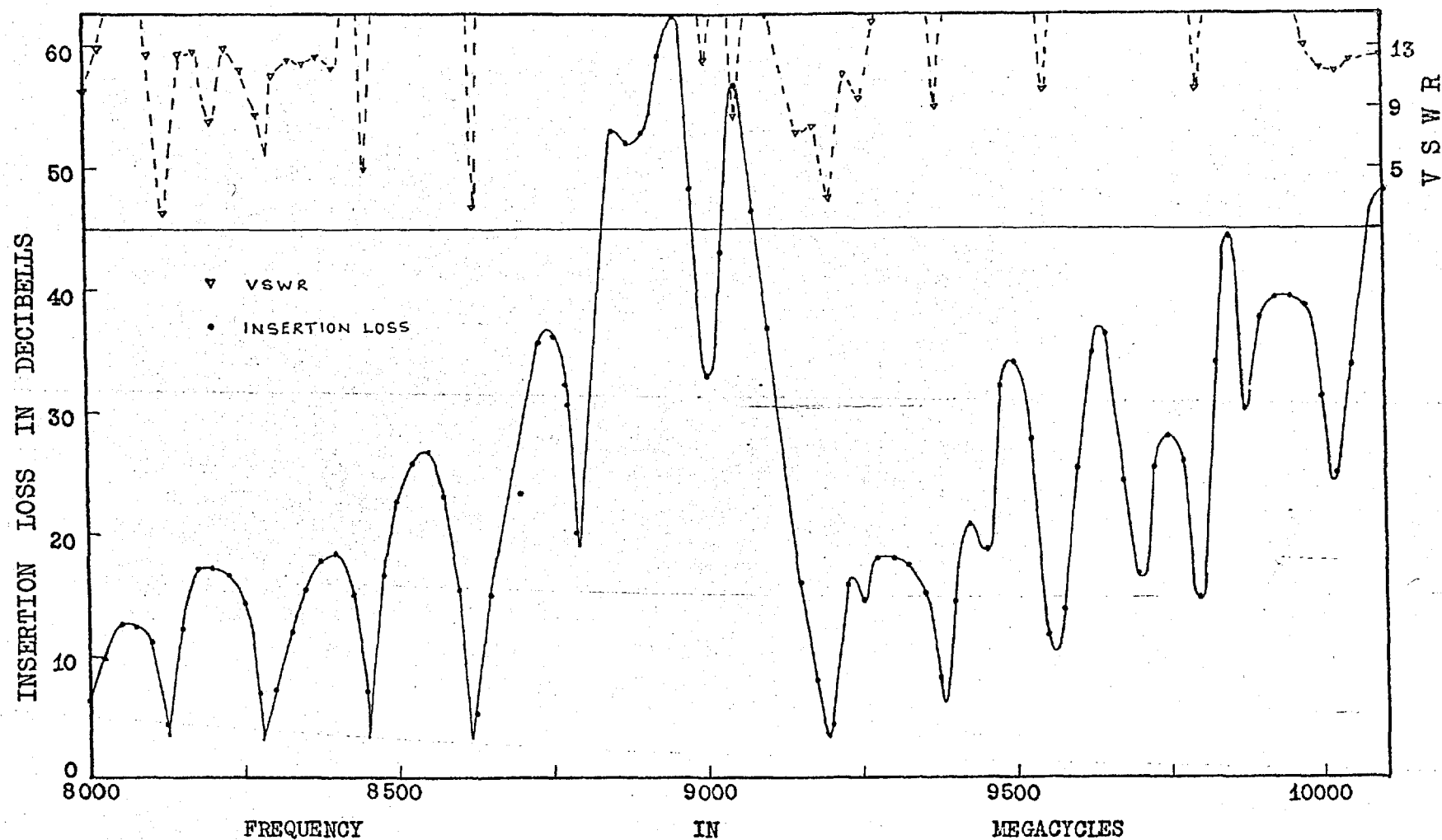


Figure 3.3. Frequency response of a 12.65cm.(4.99') long, .795cm.(.313') dia. R1 ferrite rod, with .795cm dia. copper discs at the ends, in a rectangular waveguide, for an applied field intensity of 112.5 oersteds.

to variations in coupling into and out of the ferrite loaded region.

In between 8700 and 9200 Mc/s the insertion loss of the system rises to very high level. The VSWR was not found to be correspondingly high in general and came down to fairly low values at some points. The character of the insertion loss curve in this region does not indicate that a second mode is in existence. Increased energy concentration in the ferrite with increasing frequency, and variation in the coupling must be playing a part to produce this high loss. Even below 8700 Mc/s the maximum loss can be seen to increase with frequency. However, the main cause of the high loss is thought to be energy trapping effects that occur at these frequencies even for the rod alone, as seen previously in section 2.5. The rotation of the polarization ellipse noticed in the previous section combined with the right amount of phase shift could cause this effect in a similar manner to Weiss's explanation of the insertion loss peaks of figure 1.2.

The energy trapping effect can also be seen around 8200 Mc/s, but to a much lesser magnitude. From this and from the results of the previous section it is deduced that for thinner rods and lower frequencies the loss peaks, when they occur, will progressively decrease in magnitude, due to less rf energy concentration and less rotation.

3.4 The Dominant Mode

After obtaining an approximate knowledge of the regions of operation of the different modes of the structure, the next step was to identify these modes and compare them with the modes of propagation of the phenomenological model. From the experiments described so far, the elliptically polarized character of the mode can be taken to have been well established. As to the sense of the polarization, no direct experimental verification had been attempted by previous workers to establish that the dominant mode is polarized in an anticlockwise sense with respect to the longitudinal steady magnetic field. The investigations of Chapter Two, and particularly of sections 2.3 and 2.6, brought out characteristics of the structure that suggested a negatively polarized mode. However, this point is perhaps the most important aspect of the phenomenological theory and needed direct experimental confirmation. The following experiments were performed with a view to establishing the sense of polarization of the dominant mode in the reciprocal phase shifter configuration.

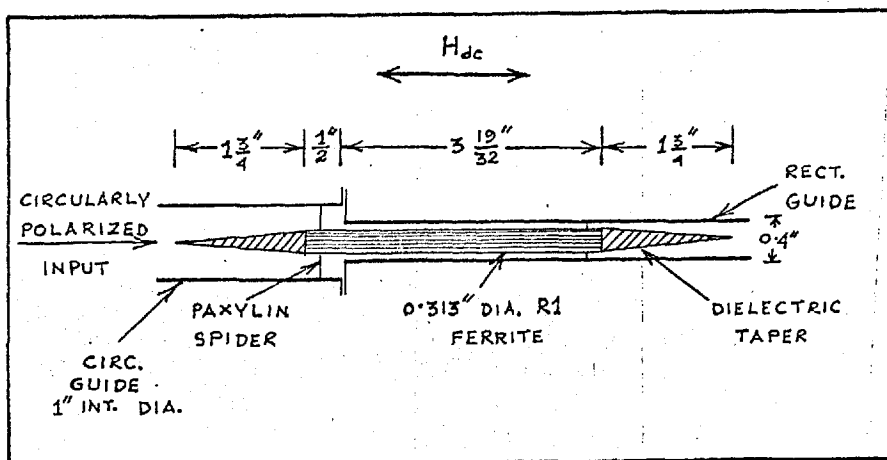
3.4.1 Selective propagation of circularly polarized waves

If the dominant normal mode had a negative sense of polarization, then it was thought that the system should show a marked preference for a negative circularly polarized input as compared to a positive one. So a circularly polarized H_{11} - mode wave was fed into the system as shown in figure 3.4(a). The sense of the polarization was altered by changing the direction of the steady field H_{dc} . The insertion loss

of this system relative to an empty section of H_{01} -mode rectangular waveguide was measured as a function of the intensity of H_{dc} .

The arrangement of the rest of the equipment was the same as described in section 2.5, except that a rectangular to circular adapter and a quarter-wave plate was used immediately before the arrangement shown in figure 3.4(a) to circularly polarize the wave. The measurements were taken at 8200 Mc/s. At this frequency, the results of section 3.3 showed that propagation in only the dominant mode can be expected for the .313"-diameter R1 rod. The circular polarizer had been originally designed for higher frequencies and had to be modified to obtain a good circularity of polarization. This was done by introducing a thin polyethylene slab ($1" \times 1" \times \frac{1}{8}"$; dielectric constant ≈ 2.3) into the polarizer, which consisted of a section of circular waveguide with a metal vane loading at an angle of 45° with the vertical. The position of polyethylene slab was adjusted by trial and error for the best result. The output from this polarizer was tested by a rotatable probe, as used in section 3.2. The maximum variation of signal strength for all azimuthal positions of the probe was 1.85 dB and the input VSWR to the polarizer was less than 1.9:1.

In the experimental arrangement a portion of the R1 rod was left projecting into the circular waveguide to obtain a good match. This small length was not expected to introduce any errors due to Faraday rotation from the ~~remnant~~ linearity of the input wave. Without the specimen put in, the abrupt junction gave an insertion loss of



(a) Experimental arrangement.

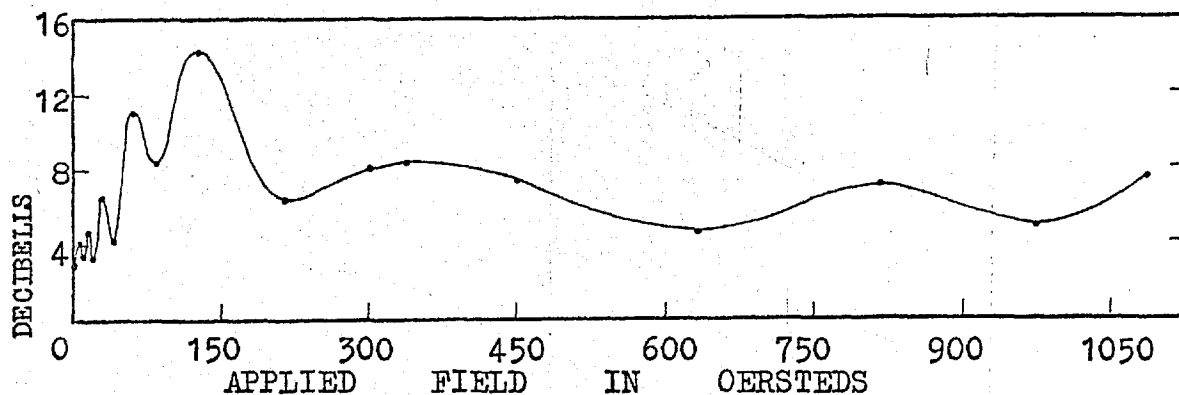
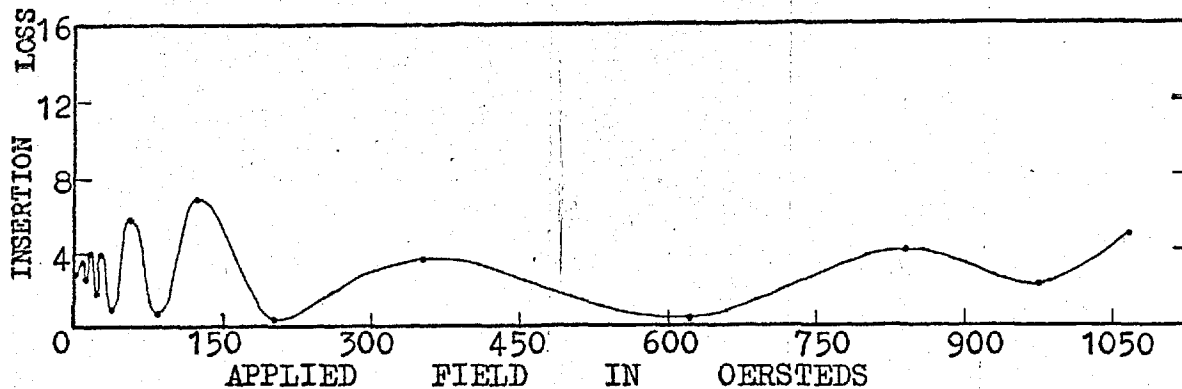
(b) Input wave positive circularly polarized with respect to H_{dc} .(c) Input wave negative circularly polarized with respect to H_{dc} .

Figure 3.4. Propagation of circularly polarized H_{11} mode at 8200 Mc/s through an abrupt circular to rectangular waveguide junction containing an axial thick ferrite rod.

1.97 dB for circularly polarized input. With the specimen, the insertion losses for the two directions of H_{dc} were as shown in figure 3.4(b) and (c). The input VSWR's were found to be generally low - never greater than 3.5:1. This is attributed to energy trapping by the circular polarizer which would convert the reflected energy to a cross-polarized wave.

Discussions

From figure 3.4(b) and (c) it is obvious that the system accepts negative circular polarization much more readily than it does the positive one. The insertion loss for the positive circular polarization can be seen to show a marked upward trend as the intensity of the steady field is increased from zero, whereas, that for the negative circular polarization goes down. The difference in the insertion losses for the two directions of H_{dc} increases to over 7.5 dB at a field strength where the specimen is approximately saturated. With further increase in H_{dc} the difference decreases slowly, but remains appreciable up to quite high field strengths.

The superimposed fluctuations in the curves are very similar to that obtained with the rod in a rectangular waveguide, without the matching tapers. They are attributed to variations in the phases of the waves scattered from different points in the system. The fluctuations are the same for both the senses of polarization, which would indicate that the mode propagating in the structure is the same for both the inputs. The negative circular polarization is able to

excite this mode more readily and is thus better matched to the system.

These results suggest a strong correlation of the dominant mode, with negative circular polarization. However, there is a point of doubt here in that, the negative circular polarization is inherently better able to propagate through the abrupt junction. This arises from the fact that, for negative circular polarization, the energy is mostly concentrated inside the magnetised ferrite rod⁽⁴⁾ and passes through the junction without much reflection. For the positive circularly polarized wave the energy is mostly outside the rod and suffers much greater reflection at the junction.

3.4.2 Recognition by the anomalous mode waveguide

The previous experiment still left some ambiguity in the interpretation of the dominant mode, and some experiment had to be devised for a positive recognition of the sense of the polarization of the wave. In section 2.6 an anomalous propagation had been noticed through a ferrite filled metal tube, too narrow to allow normal propagation. It is well known^(39,40) that such anomalous propagations occur only in the positive circularly polarized mode. It was thought that this anomalous-mode-waveguide could be used to recognise the polarization of the wave at the output of the reciprocal phase shifter.

At first it was necessary to construct a suitable polarization detector. It was decided to use a small length of the same B2 ferrite rod that showed the anomalous propagation at 7200 Mc/s, recorded in figure 2.12. The length of the detector had to be kept small so that

enough power would be available at the output for measurements, as the propagation in this mode is very lossy. A 0.47" (1.2 cm) length of the rod was introduced into a closely fitting hole at the centre of a copper block of the same length. This block was then fitted inside a circular waveguide of 1" internal diameter, conducting silver paint being applied at the joints for good contact.

This device was then tested by feeding it with a linear H_{11} wave at 7200 Mc/s. The output from the polarization-detector was tested for circularity by a probe rotatable in the azimuth. The probe assembly was similar to that shown in figure 3.1(a). Measurements were taken with the matched load after the probe as shown, and also with a short circuit replacing the matched load. This was done to increase the signal strength at the probe by creating a standing wave. The energy transmitted through the polarization detector was too small for the short circuit to have any effect at the input end.

Figure 3.5 shows the output from the polarization detector for the linear input. The zero level in figure 3.5 indicates the limit of detectability. Compared to this level, the probe response with the polarization-detector removed and with the matched load at the end, was 68 dB with the probe parallel to the E-vector and 44 dB with it perpendicular. The difference is rather low and is attributed to the fact that the probe used had to be very long, reaching nearly to the centre of the waveguide, to detect the small output of the polarization detector. The probe may also not have been perfectly radial. Appreciable propagation through the polarization detector took place only for a longitudinally applied steady magnetic field (H_2) of 970 oers.

At this field strength the anomalous-mode guide was most probably a multiple of half-guide-wavelength.⁽⁴⁰⁾

From the curves of figure 3.5 the device would appear to be a very efficient detector of positive circular polarization. For the linear input the output appears to be perfectly circularly polarized with very little difference in transmission for the two directions of H_2 . All these measurements were taken with the help of the rotary attenuator of figure 2.7. It was found that though the signal strength was very low the measurement could be done very accurately, a variation of $\pm 1\%$ in the attenuation setting being appreciable. The output from the device could perhaps be increased by designing it so that it was a multiple of half wavelength at a field strength where the transmission loss, as shown by figure 2.12, is a minimum. This was not attempted as the present one was found quite suitable for the purpose.

This polarization-detector was then used in the arrangement shown in figure 3.6 to measure the proportion of negative circular polarization in the output wave from the reciprocal ferrite phase shifter. The rest of the equipment was the same as shown in figure 2.7, except that a separate dc supply was used for H_2 . For this experiment the output detector (figure 2.7) was tuned for maximum sensitivity and not for the best VSWR. The section of coil used to magnetise the polarization detector (H_2) required 1.75 amp to produce the necessary 970 oers. (see Table A1.1). As the coil was not designed for such a high current, its supply had to be switched off after every reading to prevent overheating.

A magnetic shield, made of iron sheets, was placed between the two coils as shown, so that H_2 would not affect the magnetisation of the specimens. The minimum distance between the coil centres was $13\frac{1}{2}$ " (34.3 cm), for the shortest specimen. From Table A1.1 in the Appendix it can be seen that, a field variation of considerably less than 28 oers. would occur at the centre of the specimen as the direction of H_2 was reversed, without the magnetic shield. With the shield, the effect of the detector coil on the specimen was considered negligible.

Various RI specimens, noted on figure 3.6, were tested. The procedure was to keep the direction of H_1 fixed and reverse the direction of H_2 , to obtain the differential transmission. If the proportion of negative circular polarization was more in the wave reaching the polarization detector, then the output detector response would be greater when the directions of H_1 and H_2 were opposite. The results are plotted in figure 3.6 as a function of the intensity of H_1 . Investigations were also carried out with H_1 reversed, when similar results were obtained. The individual output from the polarization detector, for H_1 and H_2 opposite, actually increased as H_1 was increased; while, that for H_1 and H_2 in the same direction decreased, but less sharply. Beyond $H_1 \approx 400$ oers. both outputs decreased but their difference continued to increase for much higher values of H_1 , as shown in figure 3.6.

Discussions

From the curves of figure 3.6 it can be seen that the proportion of negative circular polarization is always greater in the output from

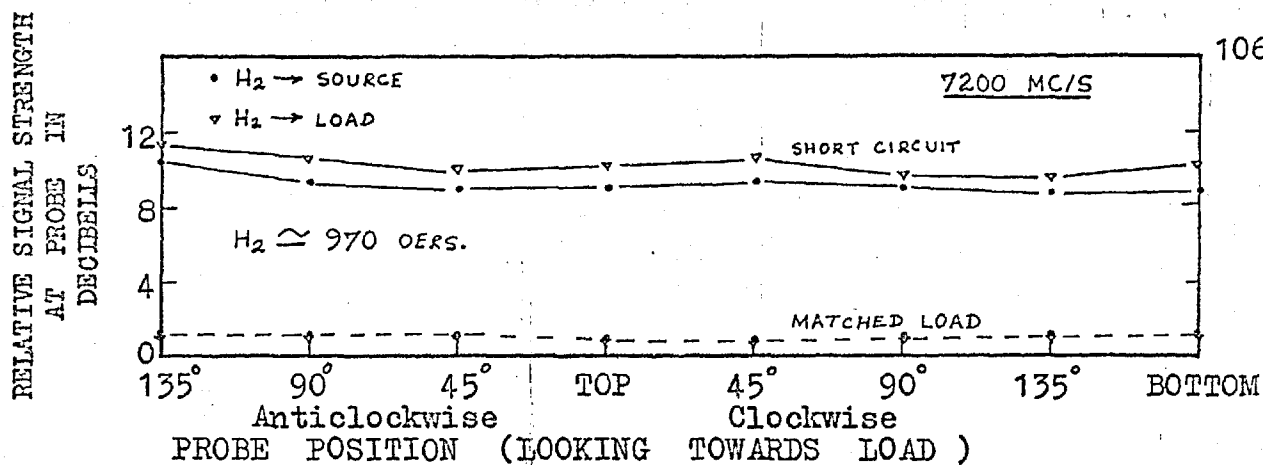


Figure 3.5. Output from the anomalous mode waveguide with linear H_{11} mode at its input.

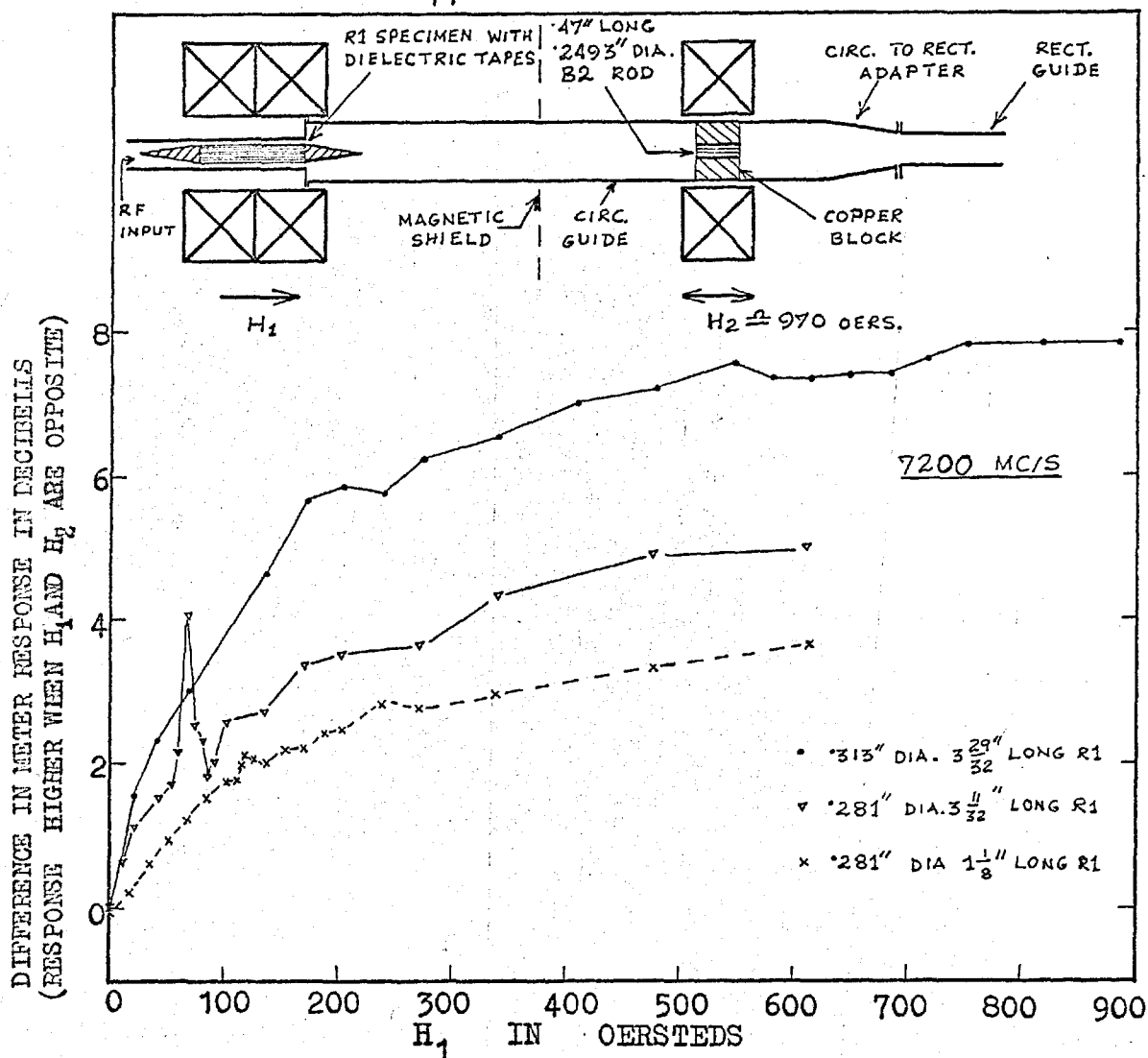


Figure 3.6. Differential transmission through the anomalous mode waveguide when fed with the output from a reciprocal phase shifter.

the reciprocal phase shifter when the ferrite is magnetised. This means that the output and thus the mode of propagation at 7200 Mc/s is negative elliptically polarized for the cases studied. For $H_1 = 0$ the mode can be seen to be linear, and becomes increasingly negative elliptical in character as H_1 is increased. Even at such a low frequency, considerable ellipticity is achieved for the thicker rod at large values of H_1 . For the .281"-diameter rod the ellipticity is much less at this frequency, which is as expected according to relation (1.10) for ellipticity discussed in section 1.4.4. The negative ϵ_y in the expression would be expected to increase and positive ϵ_x to decrease for the thinner rod, whereas, the ratio $\frac{\mu}{k}$ would change very little. This would result in a more linear wave. The length dependence of the ellipticity in the case of the .281"-diameter specimens can most probably be attributed to greater demagnetisation for the shorter specimen, in view of the large difference in their lengths.

The mode identified here must be the same as that responsible for the large phase shift. It was seen in section 2.3 that the same form of incremental phase delay persists down to 7000 Mc/s at least, even for the thinner (.281"-diameter) rod. That this is the dominant mode of the structure is evident from the low frequency at which the measurements were taken. More extensive studies of the structure at other frequencies can be carried out with anomalous-mode waveguides of different diameters, if required.

The sudden peak in ellipticity in the case of the $3\frac{11}{32}$ " long,

.281"-diameter rod was checked several times and the result is an average of all the readings. The peak is most probably a result of the way the different properties of the material, on which the ellipticity depends, vary with the magnetisation of the ferrite. Similar sharp fluctuations of the cross-polarized component was noticed for this rod (figure 3.1(c)) in the experiments described in section 3.2.2. That intrinsic properties of the material are responsible for the peak is also borne out by the fact that similar variations, though much less in amplitude, can be seen in the other two curves also. If we examine expression (1.10) for the ellipticity given by the phenomenological theory, it can be seen that variations of inter-dependent terms k , μ , ϵ_x and ϵ_y with applied field may combine to give such sharp fluctuations in the ellipticity. Quantitative comparison is, however, not possible due to the hypothetical nature of the components.

3.5 The Higher Order Mode

The definite identification of the dominant mode was very convincing as to the qualitative applicability of the phenomenological model to the reciprocal ferrite phase shifter. This led us to the investigation of another important aspect of the theory. As described in section 1.4.4, the model predicts a positive elliptically polarized higher order mode for the system, whose cut-off point depends on the ferrite rod diameter and the frequency of the wave. It can propagate

when these parameters make ξ_y positive. That the mode has a cut-off point different from the negative polarization is the main distinction of this system from a Faraday rotator. The present section describes the identification of this higher order mode by cavity resonance techniques,

The cavity was formed of a small length of the .313"-diameter ferrite rod, with copper diaphragms at both ends. The arrangement and the dimensions are shown as an inset in figure 3.7(b). From the experiment described in section 3.3 the frequency at which the higher order mode was expected to begin to propagate for this specimen was known. The length of the ferrite was so chosen that the highest frequency at which a cavity formed of it would be $\lambda_g/2$ long (at zero applied field) was slightly below the frequency at which the higher order mode could propagate. The shortest possible length was chosen so that other resonances might not complicate the situation. In choosing this length, help was taken of results of Reggia and Spencer⁽²⁾, who had determined the guide-wavelengths with unmagnetised ferrite rods, as a function of the rod diameter. The final length was arrived at by trial and error. The diaphragms consisted of .4" x .9" rectangular pieces with .210" dia. central coupling holes. The process of determining the optimum method of coupling into and out of the cavity will be described in Chapter Five. The joints between the diaphragms and the waveguide walls were painted over by conducting silver paint for good contact. The diaphragms were stuck to the ferrite by polystyrene glue.

The responses of this cavity at different frequencies were obtained

by keeping the frequency fixed and varying the longitudinal steady magnetic field. The equipment layout and the procedure for the measurements were the same as described in section 2.5. The results are shown in figure 3.7, where the insertion loss of the cavity is plotted as a function of the steady field intensity, with the frequency as a parameter.

Discussions

Figure 3.7(a) shows the resonance of the cavity in the dominant mode. The resonance frequency decreases as the applied field is increased, as would be expected from the phase shift measurements obtained in section 2.3. The phenomenological theory predicts a similar behaviour for the negative elliptically polarized mode (see (1.7) section 1.4.4) if we remember that the diagonal component μ of the permeability tensor increases as the ferrite gets saturated.

Figure 3.7(b) shows a range of frequencies, just above the highest frequency of figure 3.7(a), where the cavity shows no resonance. In this range the cavity is too long for resonance in the negatively polarized mode and the positive elliptical polarization, expected in a Faraday rotator, is cut-off in this system. The reason for the very high isolation obtained just after the cavity stops resonating in the dominant mode, is not clearly understood. It could be an energy trapping effect, similar to that noticed in the frequency response of the long rod, in section 3.3. Indeed, the VSWR's corresponding to these high peaks were quite low and were unable to account for the

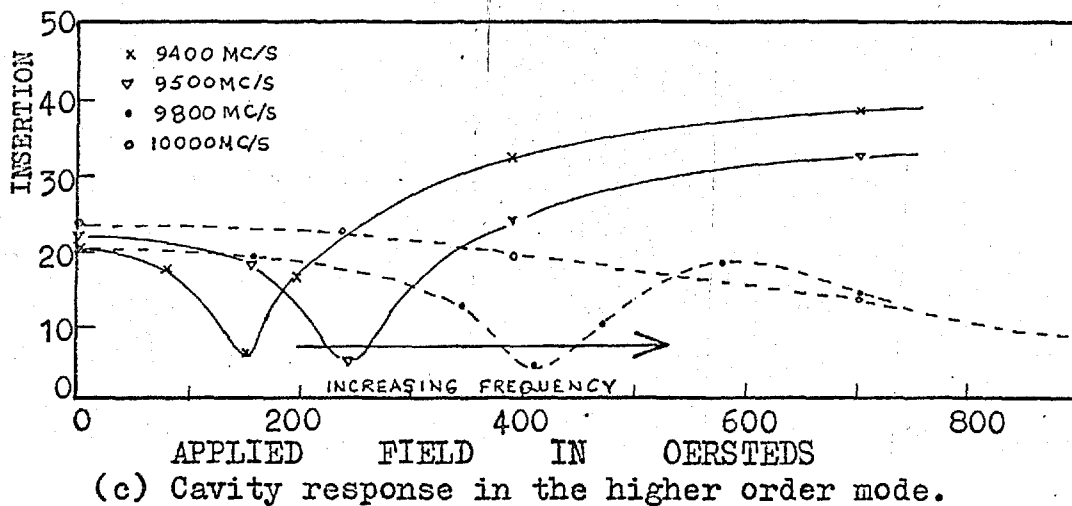
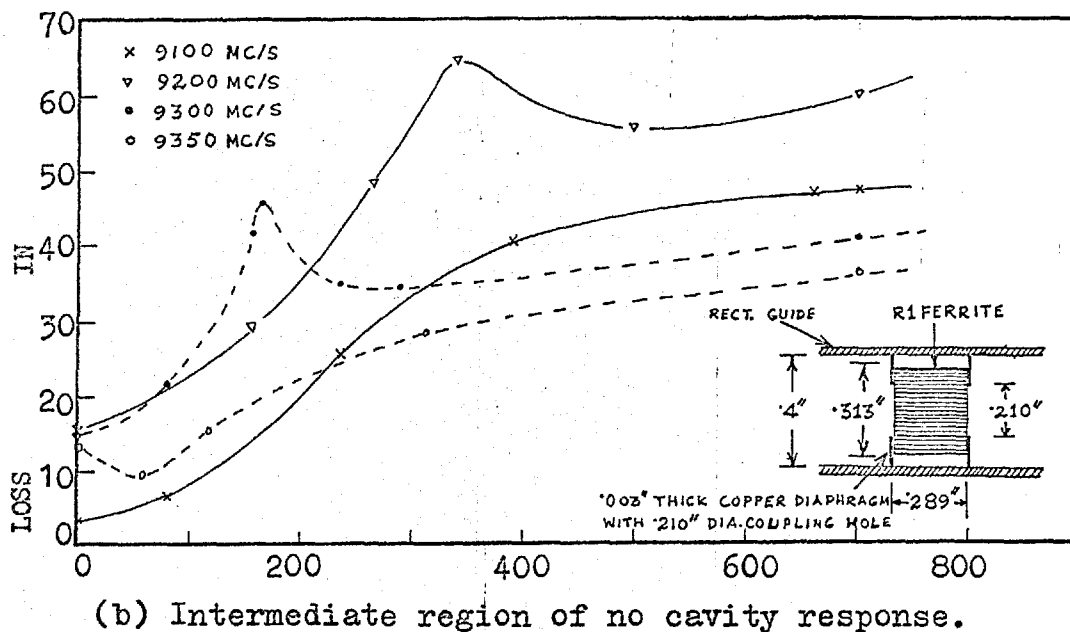
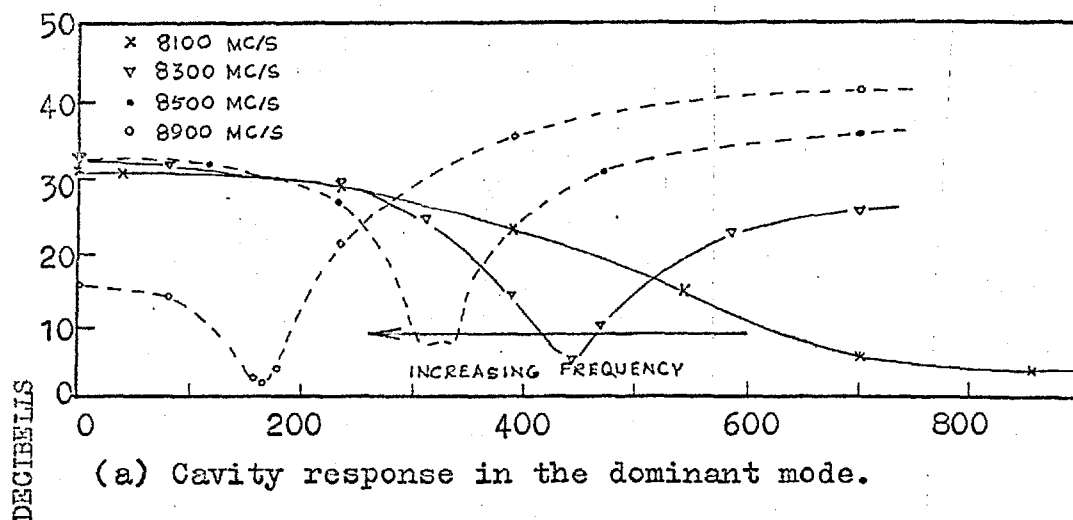


Figure 3.7. Resonance splitting in a cavity formed of a section of the reciprocal ferrite phase shifter geometry.

amount of loss obtained.

The response of the cavity in the mode of present interest is shown in figure 3.7(c). The cavity begins to resonate again as the frequency is increased above that of figure 3.7(b), at a comparatively high applied field. As the frequency is further increased the steady field strength has to be increased to obtain the resonance. The movement of the resonance point with increasing frequency is marked in figure 3.7(a) and (c). The behaviour of the cavity is very similar to the resonance splitting noticed by van Trier⁽⁴³⁾ in a cavity formed of a circular waveguide containing a ferrite rod along the axis. van Trier used the phenomenon to accurately measure Faraday rotation in the structure.

In the present system the difference from van Trier's case is that, the mode whose resonance frequency increases with increased magnetisation of the ferrite, cannot propagate below a certain frequency, and there is a discontinuity in the splitting. The mode of figure 3.7(c) is therefore identified as one containing predominantly positive circular polarization. The phase advance obtained for this mode with increased steady field intensity is as predicted by expression (1.7) of Weiss's theory, for the positive elliptically polarized higher order mode. With the advent of this higher order mode the system begins to behave just like an elliptical Faraday rotator.

It would thus appear that the main characteristics of the reciprocal ferrite phase shifter configuration are quite correctly described by the phenomenological model. Some minor features, like

the small rotation of the polarization ellipse found in section 3.2 does not seem to be covered by the theory. This will have to await a more rigorous analysis of the system for its explanation. However, the surprising accuracy of the very approximate model of Weiss justifies some investigations into the extent of its applicability. A limited investigation was carried out and is described in the next chapter.

CHAPTER FOUR

THEORETICAL CONSIDERATIONS

4.1 Introduction

In this chapter the conclusions drawn on the experimental findings of Chapter Two and Three are summarised. Some suggestions are put forward on aspects of the reciprocal ferrite phase shifter characteristics which are not completely understood. The phenomenological model of Weiss is investigated for various assumed boundary conditions, with the help of the theory detailed in Chapter One. A clearer picture is thus obtained of the way in which the novel modes predicted by the model fit in with modes which are already well established. Some new features that came to light during these investigations are also discussed.

4.2 Synopsis of Conclusions

4.2.1 The phase shift

The previous two chapters detailed the experimental verification of the mode pattern in the reciprocal phase shifter configuration. It was seen that under certain conditions the structure supports only one elliptical mode with its sense of polarization anticlockwise with respect to the direction of the steady magnetic field. It was found that a high proportion of the rf energy concentrates increasingly into

the ferrite as it gets saturated. This energy concentration was concluded to be mainly responsible for the large phase shift obtainable from this device, in combination with the increase in the component μ of the permeability tensor that occurs as the ferrite material saturates.

In the phenomenological theory (detailed in section 1.4.4), the energy concentration could be represented by an increase of the component ϵ_x of the hypothetical permittivity tensor. Weiss in his interpretation of the theory assumed that the only quantity that physically increases is the component μ . It is felt that this is wrong, and that ϵ_x also increases separately with the application of the steady field. The percentage increase in μ obtained in practice cannot alone account for the degree of increasing energy concentration noticed in section 2.6. Neither can it completely account for the incremental phase delays observed experimentally, if we use expression (1.7) of the phenomenological theory to obtain this phase shift. This is true even if we neglect the term containing k , which tends to reduce the phase shift when ϵ_y is negative. In fact, increase of up to 80% is needed in ϵ_x to account for the experimentally observed incremental phase shifts.

It is felt that the energy concentration that causes ϵ_x to increase, is due to the resemblance of the propagating mode to a negative circularly polarized mode.^{*} This is brought out by the resemblance

^{*} Energy concentration in the ferrite is a well known effect of negative circular polarization. See, for example, reference (4).

of the phase shift curves shown in figure 2.3 to the respective $(\bar{\mu}' + \bar{k}')$ curves. The experiments of sections 3.2 and 3.4 show that the resemblance to negative circular polarization increases with increased steady field intensity. This increasing resemblance is most probably the main factor that causes the large percentage increase in ϵ_x , necessary to account for the phase shift.

4.2.2 The modes of the structure

The dominant negative elliptically polarized mode of the structure has been well confirmed by the experiments of sections 3.2 and 3.4. The single-mode propagation of the structure in the phase shifter region of operation enables very good match with empty waveguides to be obtained, if the ferrite rod is gradually tapered to a point. The H_{01} mode of the empty waveguide is slowly warped into the normal mode of the ferrite loaded region. A single permitted mode at each cross-section enables the taper length to be comparatively short.

~~The small fluctuations in the insertion loss of the system remaining even with the tapered rod can most probably be attributed to the small rotations of the axes of the polarization ellipse found during the measurements of section 3.2. The phenomenological model appears not to be able to account for these rotations.~~

The small fluctuations in the insertion loss of the system remaining even with the tapered rod⁽²⁾ can most probably be attributed to the small rotations of the axes of the polarization ellipse found during the measurements of section 3.2. The phenomenological model appears not to be able to account for these rotations.

The other mode of the system has been shown in section 3.5 to be cut-off below a certain frequency, as predicted by the phenomenological theory. When propagating, this mode has been shown to possess the characteristics of the positive circular polarization. From the similarity of the structure to an elliptical Faraday rotator when both modes can propagate, it is deduced that the higher order mode is elliptically rather than circularly polarized.

4.2.3 The negative permittivity

The phenomenological theory assumes that in the structure a wave cannot propagate with its E-vector parallel to the broad walls of the waveguide, and attributes a negative permittivity in this direction to the hypothetical medium of propagation. This means that any wave excited with its polarization parallel to the broad walls is exponentially attenuated. However, there is also a coupling mechanism through the off-diagonal component k of the permeability tensor, which causes a wave to be continuously excited in this orientation for an H_{01} input. The combination of these two effects can be thought to produce the component of the elliptically polarized wave parallel to the broad walls.

It is difficult to see why a wave polarized parallel to the broad walls will not be able to propagate in the system when the same wave can propagate through the ferrite rod enclosed by a closely fitting metal sheath. The constraint is obviously imposed by the anisotropy of the metal boundary. The degree of this constraint, represented by

the negative magnitude of ϵ_y in the phenomenological model, must depend on

- (a) the aspect ratio of the metal waveguide,
- (b) the ratio of the ferrite rod cross-section to the metal guide cross-section, and
- (c) the frequency of the wave.

The constraint also depends on the state of magnetisation of the ferrite as can be seen from the results of section 3.2. The decrease in fluctuations of the cross-polarized field with large increase of steady field intensity (when μ decreases) may indicate that the influence of the walls is increasing. This is inspite of the fact that the coupling to the cross-polarization must be quite large due to increase in k .

Perhaps the constraint could be physically looked at from an electrostatic point of view. The electric lines of force naturally like to line up parallel to the narrower dimension. The presence of the ferrite rod accentuates this bias. By concentrating the H_{01} input within itself it further reduces the influence of the far walls and in effect increases the influence of the nearer walls. A cross-polarized wave under such circumstances would probably have its fields too distorted by the nearer walls to remain a H_{01} -type mode any more and would thus be unable to propagate.

4.3 The Insertion Loss Peaks

The sharp dips in transmission investigated in section 2.8 can

most probably be attributed to, comparatively small, scattering suffered by the elliptically polarized wave, enhanced by re-reflections due to favourable phasing conditions. The process ultimately traps most of the rf energy in the ferrite loaded section and large, sharp peaks of insertion loss occur. The mechanism of the formation of the peaks is thought to be very similar to the peaks observed in the straight guide at higher frequencies and to the interference effect of Weiss, mentioned in section 1.4.4. Suhl and Walker⁽⁴⁵⁾ analytically predicted similar interference and enhanced Faraday rotation for the case of a semi-infinite ferrite slab magnetised in the direction of propagation.

In the simple system treated by Weiss⁽²⁶⁾ (see section 1.4.4) the interference effect that occurs even for small rotations can be explained physically as follows. The vertically polarized (parallel to narrow dimensions of input/output rectangular guides) input wave suffers some rotation in the ferrite loaded circular waveguide. At the output of the circular section it can be resolved into a vertically and a horizontally polarized component. The horizontal component is totally reflected back. This reflected wave suffers further rotation in the same direction and may be resolved again into vertical and horizontal components at the input end.

Now, if the electrical length of the ferrite loaded section is $\frac{n\lambda_g^x}{2}$ then the backward travelling horizontal component after re-reflection

* This includes the phase change introduced by reflection at the end. Note that the case being considered here is for H_{dc} applied in the direction of propagation.

will be in phase with the vertical input wave. The new input will be continuously decreased up to the output and due to

- (a) Faraday rotation, its horizontally polarized component going to strengthen the trapped horizontal wave, and
- (b) interference, caused by the horizontal wave suffering rotation and producing a vertical component in anti-phase.

This process will continue until, for small amounts of Faraday rotation, most of the input energy is trapped in the ferrite loaded section as a horizontally polarized wave. Under these circumstances this section would behave just like a resonant absorption cavity.^{*}

The present case is complicated by the fact that the input and output to the system cannot be treated as simple polarizer and analyser, and also because the wave is elliptically polarized. Scattering occurs at three points instead of two of the simpler system - the two ends and the abruptly twisted junction. However, the results of investigations of section 2.8 suggest that the phenomenon depends on the same factors responsible for the simpler case discussed above.

Thus, many evidences were found to the effect that correct phasing of the scattered waves was very important for the phenomenon. The very high-Q cavity like behaviour of the structure also suggested phase shift dependence and re-reflection of energy. The energy trapping effect is brought out by the low VSWR. That the abrupt twist

* Weiss's results ⁽²⁶⁾, however, do not show the input VSWR's for his device.

plays the most important part in the trapping of the energy, is brought out by the absence of any major effect of matching the ends of the ferrite rod. This also suggests that the trapped wave must have such a polarization that it cannot couple to the empty guide at the end of the loaded section even if the rod ends are well matched. The non-reciprocal nature of the system with respect to steady field direction, except for a twist angle of 90° , also suggests that the scattering at the twist junction is mainly responsible for trapping the wave. The non-reciprocity connects the phenomenon to some sort of Faraday rotation. That the ferrite rod as a whole is behaving like a resonant length is brought out by the absence of any effect of displacing the rod with respect to the junction. It was found in later experiments that the effect depends very much on the total length of the ferrite rod.

Figure 4.1 gives the end views of some arrangements of systems in which the insertion loss peaks occur. The arrangement (a) shows a system in which the output guide is twisted in a clockwise direction by 60° with respect to the input. Looking at the system as a whole it can be seen that there is a plane AA in which maximum energy can be stored. Waves with their E-vector polarized along this plane will suffer maximum reflection at both ends of the rod. Similarly a wave with its E-vector along the plane BB will encounter least discontinuity in the system and will propagate most freely through it. It is believed that when conditions are suitable, energy is scattered into and trapped in the plane AA. The small non-reciprocal rotations of the polarization ellipse noticed in section 3.2 helps in the trapping process. It

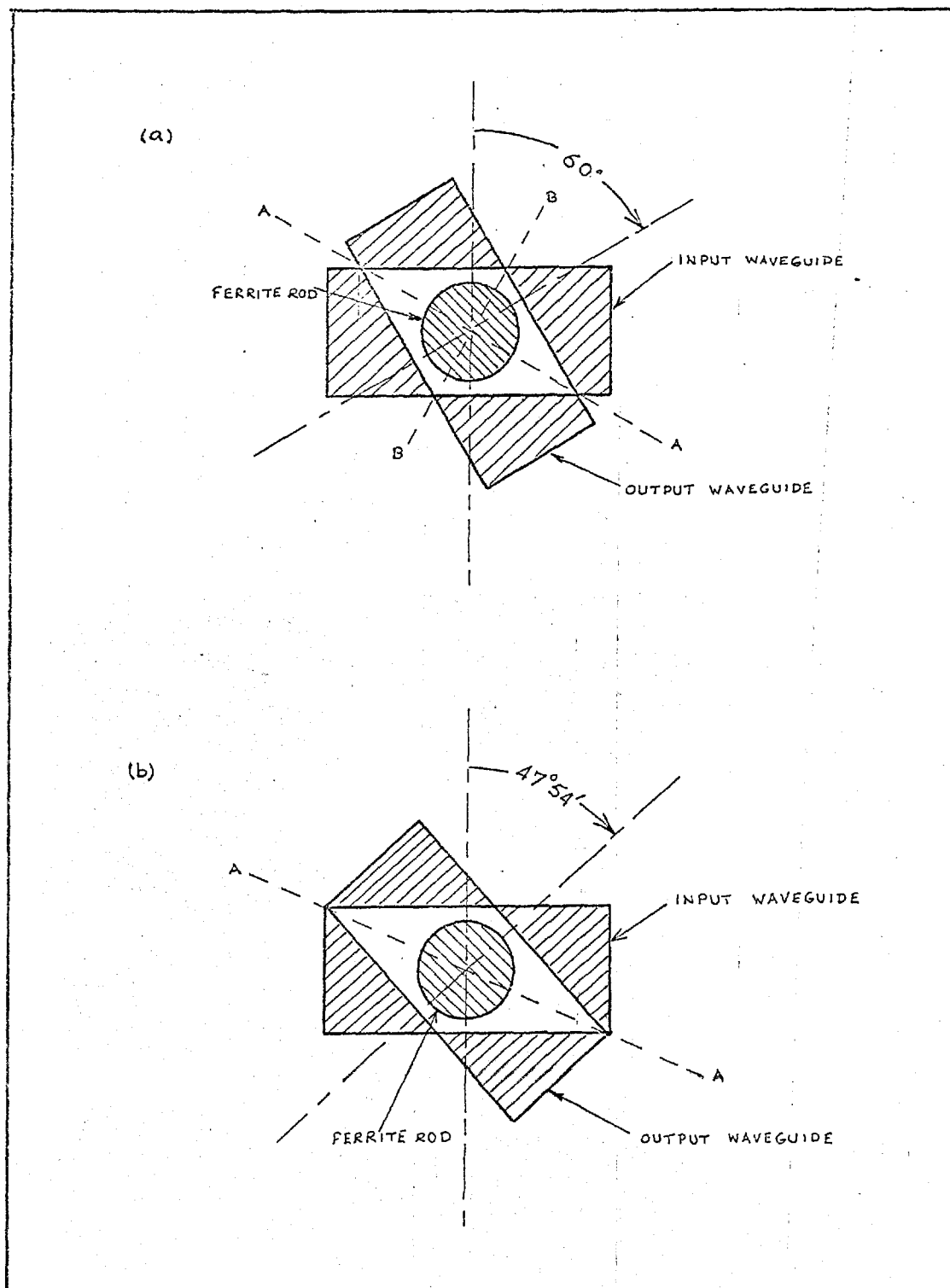


Figure 4.1. Arrangements of systems in which insertion loss peaks may occur. Waveguide aspect ratio 9:4.

should be noted that the rotations of the major axes are in a direction as to be accentuated by the twist of the output section (see figure 3.2). As these rotations have been noticed for thinner rods and at much lower frequencies, it is felt that this phenomenon may occur more extensively than is brought out by the present investigations.

Considering the two orthogonal planes of symmetry of the system, AA and BB, we can identify them with the horizontal and vertical planes of the simpler system discussed earlier. The non-reciprocity of the present system arises from the fact that the input polarization is not along the plane BB. At a twist angle of 90° both the planes are equally capable of trapping the rf wave and the system is reciprocal. If under the condition of an insertion loss peak the steady field direction is reversed, the wave should be scattered along a plane which makes an equal angle in the opposite direction with the input polarization. The non-coincidence of this plane with BB is thought to be the reason why the dips in the insertion loss in figure 2.16 do not occur at the same field strength as the peaks.

The resonant length of the structure gives the inherently small VSWR's noticed. The magnitude of waves scattered by the abrupt junction into the plane AA depends on the wave polarization at the junction. This perhaps causes the fluctuations in the insertion loss shown in figure 2.17 as the rod is displaced with respect to the junction. The fluctuations are comparatively small, as the curves of figure 3.2 show that the change of rotation along the length is small.

Figure 4.1(b) shows the condition that is considered best for

the trapping effect. In this case the diagonals of the two waveguides coincide and they are twisted at an angle $47^{\circ}54'$ with respect to each other. The dimension of the rhombic system along AA is now the largest possible. This interpretation is borne out by the fact that the loss peaks are largest and strongest at the smaller angles, as can be seen from figure 2.14, though the discontinuity is smaller for small twists. Below $47^{\circ}54'$ the system does not possess two orthogonal planes of symmetry any more (except at 0°) and the mechanism of energy trapping discussed above does not apply. The twist discontinuity also becomes too small for appreciable power to be scattered by it for the small rotations. It should be noted that during the investigations of section 2.8, the loss peaks have never been noticed below 49° of twist.

4.4 Mode Spectrum of the Phenomenological Model

The results of the work described so far show that the phenomenological theory described in section 1.4.4 very accurately predicts the novel negative elliptically polarized mode in which the rf wave propagates through the reciprocal ferrite phase shifter. Neckenb rger⁽³⁾ has shown that if the boundary conditions are changed by giving an equal positive value to ϵ_x and ϵ_y , then the theory predicts the two oppositely running circularly polarized modes of the infinite (or, circularly symmetric) ferrite medium. It would be interesting to see what other modes are predicted by the model for an extensive

variation of boundary conditions and how the mode of the phase shifter fits into this spectrum of modes.

With this idea, propagation constants and field components were calculated with the help of expressions (1.7) and (1.10) after assigning appropriate values to ϵ_x and ϵ_y to represent different boundary conditions. These values accounted for the different phase velocities of the wave produced by the metal boundary in the two coordinate directions. In general an anisotropic boundary was considered and values of ϵ_x and ϵ_y were gradually taken towards negative direction to stimulate decreasing frequency. Alternatively, the decreasing values of ϵ_x and ϵ_y simulated the decrease in the dimensions of the metal boundary.

Dependence on the steady magnetic field intensity was studied by assigning various values to the ratio k/μ . It must be remembered that the k and μ considered here are components of the tensor permeability as modified by the ratio of the ferrite cross-section to the waveguide boundary. They can, however, be taken to vary in a similar manner with applied field as does the intrinsic μ and k of the ferrite material.⁽³⁾ Only $\mu > 0$ has been considered, as for $\mu < 0$ an infinite number of incipient modes can exist in the ferrite medium. The results are tabulated in Appendix Two. The propagation constants for the various boundary conditions investigated have been plotted in figure 4.2, and are discussed in the following paragraphs. The values of figure 4.2 are in units of $\frac{\omega}{c} \sqrt{\mu \epsilon}$ and, in addition to the

variations shown, they also vary according to the magnitude of μ .

Case 1. (Figure 4.2(a)): $\epsilon_x = +15\epsilon$, $\epsilon_y = +\epsilon$.

In this case the metal boundary allows wave propagation in both orientations. It is thus representative of the elliptical Faraday rotator, or the phase shifter configuration under conditions such that the higher order mode discussed in section 3.5 can propagate. For this case, and for all others considered, the phenomenological model predicts two normal modes. The phase constants of the two modes can be seen to start from separate points even at zero intensity of the steady field, H_{dc} (though, at zero H_{dc} the n-mode does not exist). This separation agrees with the prediction of more rigorous analysis of wave propagation subjected to two anisotropies, carried out by Lyubimov⁽⁸⁾. For all the cases of anisotropic boundaries considered here, the propagation constants of the two modes of the model were found to be separated out at zero H_{dc} .

With an increase of H_{dc} the phase constant of the p-mode increases^{*} and that of the n-mode goes down towards 0. The p-mode does not show any interesting behaviour and keeps on propagating until $\mu = 0$. The n-mode however goes to cut-off at $k = \mu$. With further increase of H_{dc} an exponentially attenuating solution is obtained for this mode, the attenuation constant increasing with an increase of H_{dc} . The behaviour of the two modes with variations of H_{dc} is therefore

* Note that this "increase" does not take into account variations in μ .

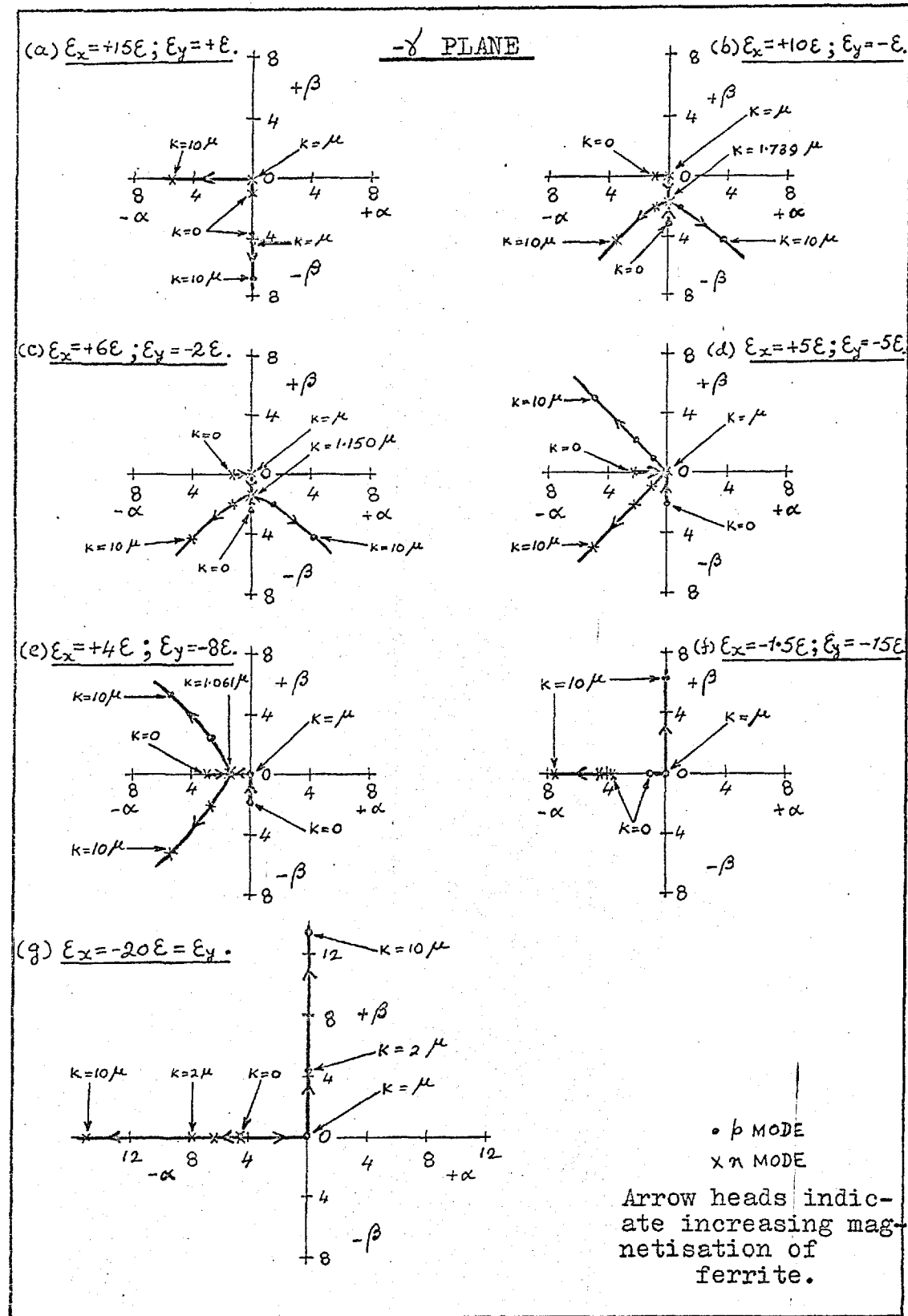


Figure 4.2. Propagation constants in a bounded ferrite rod for various boundary conditions, derived from the phenomenological model. Magnitudes are in units of $\frac{\omega}{c} \sqrt{\mu \epsilon}$.

similar to the circularly polarized normal modes of the infinite ferrite medium.

The form of the modes, however, change with the applied field, unlike those of the infinite medium. The p-mode which begins linear (parallel to the smaller dimension of the metal boundary) at $H_{dc} = 0$, becomes negative elliptically polarized at small values of H_{dc} . It gradually changes to circular polarization at $k = \mu$. Beyond this point the polarization again becomes elliptical, but with the major axis now parallel to the larger dimension of the metal boundary. The n-mode is positive elliptically polarized and begins as a very thin ellipse parallel to the larger dimension. With an increase of H_{dc} the minor axis parallel to the narrow dimension grows, but the ellipse remains oriented parallel to the larger dimension for the highest values of H_{dc} considered,

$$\text{Case 2: } \underline{\epsilon_x = +12 \epsilon, \epsilon_y = 0.}$$

With a decrease of the permittivity components, the n-mode goes nearer and nearer to cut-off but still cuts off at the same value of H_{dc} (i.e. $k = \mu$). The phase constant of the p-mode also decreases to some extent. When $\epsilon_y = 0$ the n-mode remains at the cut-off point for all values of H_{dc} . The negative elliptically polarized p-mode has the same phase constant for all H_{dc} , but changes from linear to circular and back to elliptical as before.

Case 3: (Figure 4.2(b)): $\epsilon_x = +10 \epsilon$, $\epsilon_y = -\epsilon$.

With further decrease in the components, ϵ_y becomes negative. This is the condition of operation of the reciprocal phase shifter. The n-mode now begins below cut-off and moves towards the cut-off point as H_{dc} is increased. Beyond $k = \mu$ its propagation constant becomes imaginary and the mode begins to propagate. Its phase constant now increases with further increase of H_{dc} . The phase constant of the p-mode decreases* with the increase of H_{dc} and at a certain value of H_{dc} , appropriate to the ascribed boundary conditions, the phase constants of the p and the n modes become equal. Then onwards the model again has only a single mode of propagation.

Another point of interest is that both the p and the n modes are now negative elliptically polarized, as pointed out by Weiss⁽⁶⁾. The p-mode begins with its major axis parallel to the narrow dimension and the n-mode has its major axis parallel to the broad dimension. Beyond the point $k = \mu$ both modes have their major axes parallel to the broad dimension, as in case 1.

At the point $k = 1.739 \mu$ for this case the two modes coincide. Beyond this point the theory predicts the same negative elliptical polarization for both the modes. It also predicts a linear component for each mode. The linear components of the two modes are, however, 180° out of phase with respect to each other and can be thought to

* The decrease is due to contributions from k_z . This is negligible at low fields, where the large phase delays are obtained from this system.

cancel out. Two complex propagation constants are predicted as shown in the figure, one exponentially increasing and the other attenuating at the same rate. ~~As a result, the modes in this case are degenerate~~

~~propagating constants only and the solutions for a steady state have the~~

~~straight perpendicular lines~~

Case 4 (Figure 4.2(c)): $\epsilon_x = +6\epsilon$, $\epsilon_y = -2\epsilon$.

As ϵ_x and ϵ_y goes further towards the negative direction, the behaviour of the modes remain much the same as in case 3, as long as $|\epsilon_x| > |\epsilon_y|$. For the present case the phase constant of the p-mode begins from a smaller value. The two modes coincide for a value of H_{dc} when $k = 1.150 \mu$.

Case 5 (Figure 4.2(d)): $\epsilon_x = +5\epsilon$, $\epsilon_y = -5\epsilon$.

This is the case when the absolute magnitudes of the permittivity components are equal. Only the p-mode can now propagate in the system. At the transition point $k = \mu$ the p-mode also goes to cut-off and the two modes coincide, being both negative circularly polarized at that point. Beyond this point there is no propagating mode for the system.

The solutions for the p-mode in the quadrant $(-\alpha, +\beta)$, for values of H_{dc} greater than required for $k = \mu$, have been adopted. This has been done to fit in with the other cases and particularly with figure 4.2(e), which had to be drawn as shown to avoid a solution giving

exponential gain in the passive system. The solutions beyond $k = \mu$ indicate an exponential attenuation, ~~the first minimum and subsequent minima~~. The system thus behaves like a conventional cut-off waveguide beyond this point. The mode remains negative elliptically polarized, with the major axis parallel to the broad dimension.

Case 6 (Figure 4.2(e)): $\mathcal{E}_x = +4\mathcal{E}$, $\mathcal{E}_y = -8\mathcal{E}$.

As the permittivity components go further down the point of coincidence occur on the $-\alpha$ axis at a value of H_{dc} greater than that required for $k = \mu$. For this particular boundary condition the coincidence occur at $k = 1.061 \mu$. The p-mode still cuts off at $k = \mu$ though it starts from a lower phase coefficient value at $H_{dc} = 0$.

At cut-off, the p-mode is still negative elliptically polarized with the major axis along the narrow dimension. The cut-off n-mode is now negative circularly polarized at $k = \mu$.

Case 7 : $\mathcal{E}_x = 0$, $\mathcal{E}_y = -12\mathcal{E}$.

With $\mathcal{E}_x$ going to zero the p-mode also cuts off, remaining linear and at the cut-off point (0) for all values of H_{dc} . The n-mode does not exist under this condition.

Case 8 (Figure 4.2(f)): $\mathcal{E}_x = -1.5\mathcal{E}$, $\mathcal{E}_y = -15\mathcal{E}$.

If negative values are assigned to both $\mathcal{E}_x$ and $\mathcal{E}_y$ the propagation constants for both the modes begin on the $-\alpha$ axis at $H_{dc} = 0$. The

p-mode now changes its sense of polarization and becomes positive while the n-mode remains negative elliptically polarized.

With the increase of H_{dc} the n-mode moves further along the $-\alpha$ axis. The p-mode, however, moves towards the cut-off point and reaches it at $k = \mu$. Beyond this value of H_{dc} this mode possesses an imaginary propagation constant. The solutions on the $+\beta$ axis have been adopted for the forward travelling mode, again to fit in with the previous cases. This phase advance solution is also confirmed by the backward wave nature^(39,40) of the anomalous mode in a thin guide discussed in Chapter Three.

The p-mode now is very similar to the anomalous mode except that it is elliptically polarized. The phase constant increases with H_{dc} until the mode is again cut-off at $\mu = 0$. The major axis of the ellipse remains parallel to the narrow dimension of the boundary for all values of H_{dc} , but the minor axis grows as H_{dc} is increased. The cut-off n-mode begins with its major axis parallel to the larger dimension, becomes circular at $k = \mu$ and then on has its major axis parallel to the narrow dimension.

$$\text{Case 9 (Figure 4.2(g)) : } \underline{\mathcal{E}_x = -20 \mathcal{E}_y = \mathcal{E}_y.}$$

This is the case of the circularly symmetric guide, too narrow to allow ordinary wave propagation, filled completely or partially with longitudinally magnetised ferrite. We see that the phenomenological model predicts all the characteristics of anomalous propagation under such circumstances very accurately.

Two oppositely running circularly polarized modes are predicted which, due to the symmetry, start at the same point on the $-\alpha$ axis for $H_{dc} = 0$. With the increase of H_{dc} the modes behave similarly to the previous case. Beyond $k = \mu$ the positive circularly polarized p-mode begins to propagate, the phase constant increasing with the increase of H_{dc} . The corresponding phase constants are higher than the previous case. They become very high for infinitely thin tubes, represented by high negative values of the permittivity components. This behaviour of the anomalous mode is also predicted by more exact analysis.

The present case was also investigated for $\mu < 0$. It appears that there is not much change in the behaviour of the p-mode, but the n-mode becomes able to propagate beyond the point $k = -\mu$. Thus a linearly polarized incipient H_{11} -type mode is predicted.

Conclusions

From the foregoing discussion of the mode spectrum of the phenomenological model it would appear that the negative elliptically polarized normal mode of the elliptical Faraday rotator converts into the phase shifter mode under the constraint that wave propagation cannot take place oriented in one of the transverse coordinate directions. When normal wave propagation is prevented along both transverse coordinates, the mode converts into the anomalous mode and propagates when $k > \mu > 0$.

For the phase shifter arrangement, the theory predicts single

mode operation except for a small range of H_{dc} that satisfies the following conditions, derived from relation (1.7):

$$\frac{1}{4}\mu^2(\mathcal{E}_x + \mathcal{E}_y)^2 > \left(\frac{1}{4}\mu^2(\mathcal{E}_x - \mathcal{E}_y)^2 + \mathcal{E}_x \mathcal{E}_y k^2\right) > 0 \quad (4.1)$$

Under the conditions of expression (4.1) two negative elliptically polarized propagating modes are predicted with their major axes parallel to the larger dimension of the metal boundary. In that case the wave-polarization would again vary with distance and some rotation of the major axis occur, as these two modes beat with each other.

At a lower frequency or for a smaller waveguide (but still large enough to permit normal propagation along one transverse coordinate) cases 5 and 6 predict that wave propagation will take place only for values of H_{dc} below that for which $k = \mu$. The negative elliptically polarized mode in these cases behave exactly like the positive circular mode of the infinite ferrite medium.

The anomalous propagation predicted by the model appears not to be limited to completely filled narrow circular waveguides. The model, as we have seen, applies equally well to waveguides filled partially with gyromagnetic semiconductors. Case 8 predicts anomalous elliptically polarized mode in non-circularly symmetric waveguides, completely or partially filled with longitudinally magnetised ferrite. The occurrence of this type of propagation thus appears to be more extensive than have been investigated up to the present.

For the constrained propagations, the phase delay of the p-mode

is decreased by contributions from the k component of the permeability tensor. Thus the k component advances the phase of the phase shifter mode, as well as the anomalous mode, same as it does for the positive circular mode in the infinite medium. This would appear to indicate a fundamental connection of the phase shifter mode with the normal positive circular polarization. The connection appears to increase as the wave is more and more constrained, as has been seen from Cases 5 and 6. It seems that at moderately high fields (when k is important) the interaction of these two modes with the spinning electrons of the gyromagnetic medium may be similar.

The above investigation with the help of the phenomenological model has given us a better understanding of the mode pattern in small waveguides containing longitudinally magnetised ferrite. Future attempts at solution of the problem by numerical methods may possibly be helped by the mode spectrum presented here. There appears to be many novel and interesting features of these modes to justify further investigations both theoretically as well as experimentally.

CHAPTER FIVE

DEVICES

5.1 Introduction

One aim of the present investigations was the development of useful microwave devices based on the phase shifter configuration. This aim was always kept in view when carrying out experimental investigations on the structure. During these investigations some behaviours of the structure were noticed which were thought suitable for the development of devices. Notable among these are;

- (a) the capability of a cavity based on this structure of being tuned by the variation of the steady magnetic field, and
- (b) the non-reciprocal insertion loss peaks.

The use of these characteristics in developing a tunable in-line band-pass filter and a tunable narrow-band isolator are described in this chapter. Some other characteristics, also suitable for the development of new devices, have already been remarked upon in relevant sections of Chapters Two and Three.

5.2 Development of a Tunable Cavity

The large phase shift properties of the present structure automatically suggests its use in resonant lengths to produce a tunable filter. Such a filter would be expected to provide a large tuning range, as we have seen that the decrease in maximum phase shift with the decrease in frequency is small. Results of measurements of wavelengths in the

ferrite loaded region (not included in this thesis) indicate that with the .313" dia. specimen an incremental phase delay of over 50% is obtained at 9100 Mc/s. Experiments of section 2.5 indicate that for the same specimen, if the frequency is lowered by 9.9 % the phase shift drops by about 25%. Thus a tuning range of at least 10% could be expected from a cavity using this sample, assuming that the initial phase constant also drops by the same amount. Insertion loss at resonance should not be too high if a low-loss ferrite like R1 is used, provided that the number of resonant sections can be kept small.

A simple in-line electrically tunable filter of moderate bandwidth would be very useful in microwave communication systems. The tunable YIG filters at present available are inherently narrow band devices. Filters tuned by transversely magnetised ferrite have been developed by Chitre⁽⁴⁶⁾ and Reggia⁽⁴⁷⁾, which are suitable for system use. The resonant structure in Chitre's filter consists of a thick iris loaded at one side by a sliver of ferrite. Reggia used a cavity formed by silvering the sides of a rectangular block of ferrite. Both these structures suffer from the defect of not being easily adjustable, thus imposing severe manufacturing and tuning field tolerances if the filter is composed of more than one section. The promise of an easily manufacturable and adjustable filter of good performance and small size was therefore felt worthy of further investigations.

5.2.1 Form of Obstacle

The first step in obtaining a resonant cavity from the phase shifter structure was to determine the best form of obstacle to be used and the

optimum way of coupling into and out of the cavity. The full-length .313" dia. rod was used for the first part of this experiment. The behaviour of this rod with same diameter copper discs at ends has already been shown in Figure 2.8(b). Results for some other end pieces are plotted in Figure 5.1. None of the obstacles shown made electrical contact with the surrounding metal walls.

It can be seen that a small hole in the upper disc reduces the maximum and minimum losses considerably. An increase of the disc diameter together with a simultaneous increase of the central hole appears to keep the minimum losses more or less the same, while increasing the maximum losses achievable. With the big rectangular end pieces, the minimum losses are much higher, as would be expected. However, the fluctuations are more regular and do not show the large increase with field intensity as do the disc cases. This is more like the behaviour one would expect from a resonating cavity. It was therefore concluded that the conventional diaphragm with a central coupling hole would probably be the most suitable form of obstacle to place at the ends of the rod to form the cavity.

The next step was to determine the optimum size of the hole that would give critical coupling. This was again done by trial and error for a small length of the .313" dia. R1 rod. The length was chosen in a manner similar to that described in section 3.5, to form a $\lambda_g/2$ cavity resonant in the dominant mode. The arrangement of the cavity was similar to that shown in figure 3.7(b), except that this cavity was slightly longer. Various coupling hole diameters were tried and the results are shown in figure 5.2.

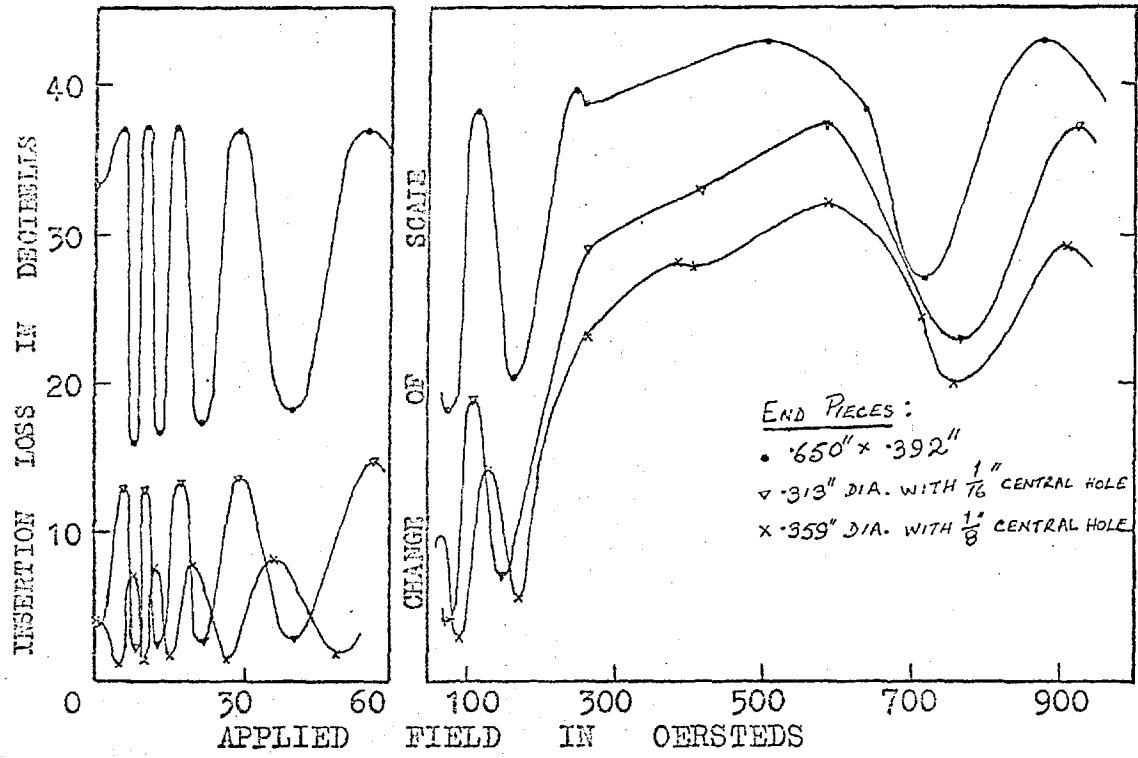


Figure 5.1. Insertion loss of the 4.99" long, .313" dia. R1 specimen with various .003" thick copper end pieces, at 8200 Mc/s.

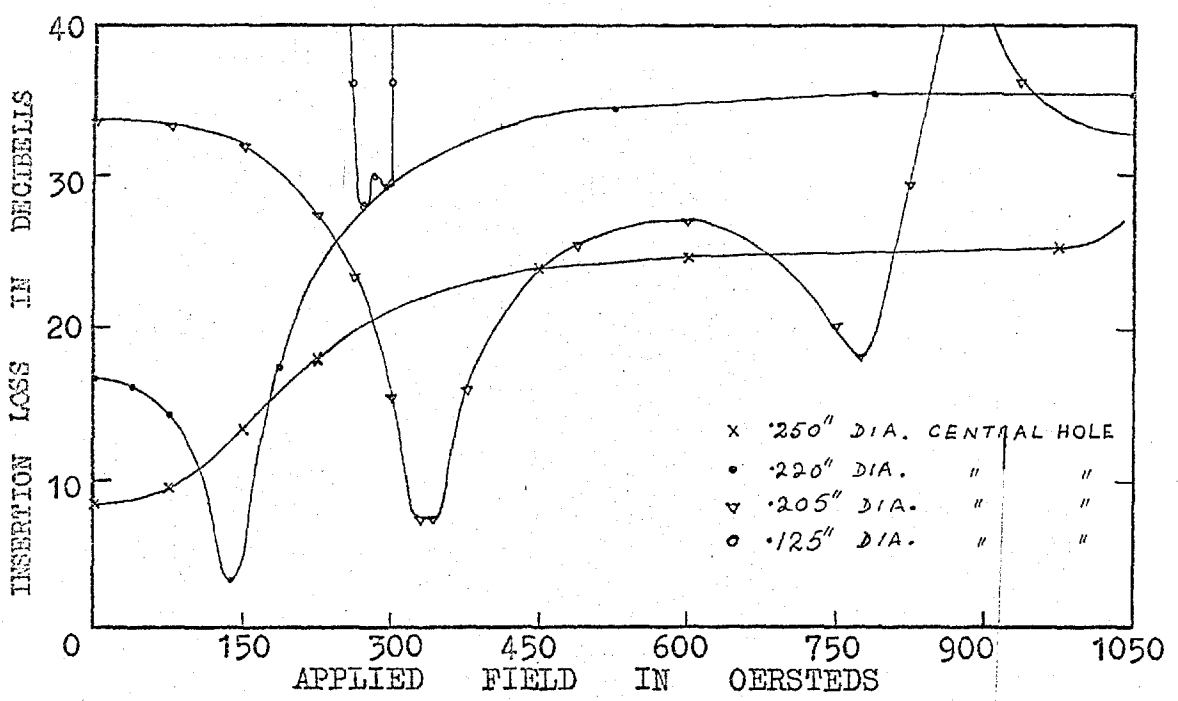


Figure 5.2. Effect of variation of coupling hole size on a cavity formed of a .313" dia., .392" long R1 ferrite cylinder with .003" thick copper diaphragms at its ends blocking off the waveguide. Frequency 8200 Mc/s.

Best results were obtained for a coupling hole diameter of .220".

This result formed the basis of calculations for determining the sizes of coupling holes for other cavities, in later experiments. The calculations were based on relations given by Wheeler (48) of the coupling co-efficient in terms of the ratio of effective volumes of the coupling hole, the cavity and the waveguide.* By proportionately varying the size of the hole with the help of these relations it was found that optimum coupling could be obtained in all cases to a first approximation.

The cavity shown in figure 3.7 was then formed and its behaviour at various frequencies checked, as recorded in the figure. Over 10 percent tuning range was obtained with this cavity resonating in the dominant mode. However, the insertion losses and input VSWR's at resonance varied considerably with frequency. Another was attributed to the variation of coupling with frequency. Another disadvantage of this arrangement was the large d.c. magnetic field required for tuning.

5.2.2 Arrangement to reduce coupling variations

By tuning the cavity described in the previous sub-section the resonator volume could be thought to be kept approximately

* The coupling between a resonant cavity and a waveguide has been expressed by Wheeler for end wall coupling as :

$$\text{The loading power factor} = \frac{V_c^2}{16 V_r V_g} \quad (5.1)$$

where V_c = Effective volume of coupling hole ($\propto$ (hole dia.)³).
 V_r = Effective volume of resonator ($\propto$ (physical volume)).
 V_g = Effective volume of waveguide ($\propto$ (guide wavelength)).

constant over the whole tuning range. However, it could not change the effective volume of the waveguide coupling into the cavity. Thus, from expression (5.1) the coupling coefficient would be expected to vary with frequency. It was thought that if the electrical length of the waveguide was also made to vary as H_{dc} was varied, then a more constant coupling should result.

The arrangement to accomplish this is shown in figure 5.3, where the ferrite rod extends on both sides of the cavity. The length of the input and the output sections ^{were} determined for this experiment from considerations of coupling alone. Wheeler ⁽⁴⁸⁾ shows that the waveguide length affecting the coupling can be taken to be $\frac{\lambda_g}{2\pi}$ (one radian length). The input and output sections were made considerably longer than this. The length of the cavity was kept the same as in figure 5.2.

The coupling hole diameter was obtained with the help of expression (5.1) and the results of figure 5.2. After allowing for the fact that the coupling hole is now loaded by dielectric on both sides instead of only one, the result for coupling hole diameter obtained was .152". The results recorded in figure 5.3 indicate that this gave nearly optimum coupling, especially at the higher end of the tuning range. The junctions of the diaphragms and the guide walls were again painted over by conducting paint.

The results of figure 5.3 show that a large tuning range has been achieved by this arrangement and that the insertion losses at resonance are substantially lower. This cavity resonated at about 8700 Mc/s for $H_{dc} = 0$. The tuning range was thus approximately

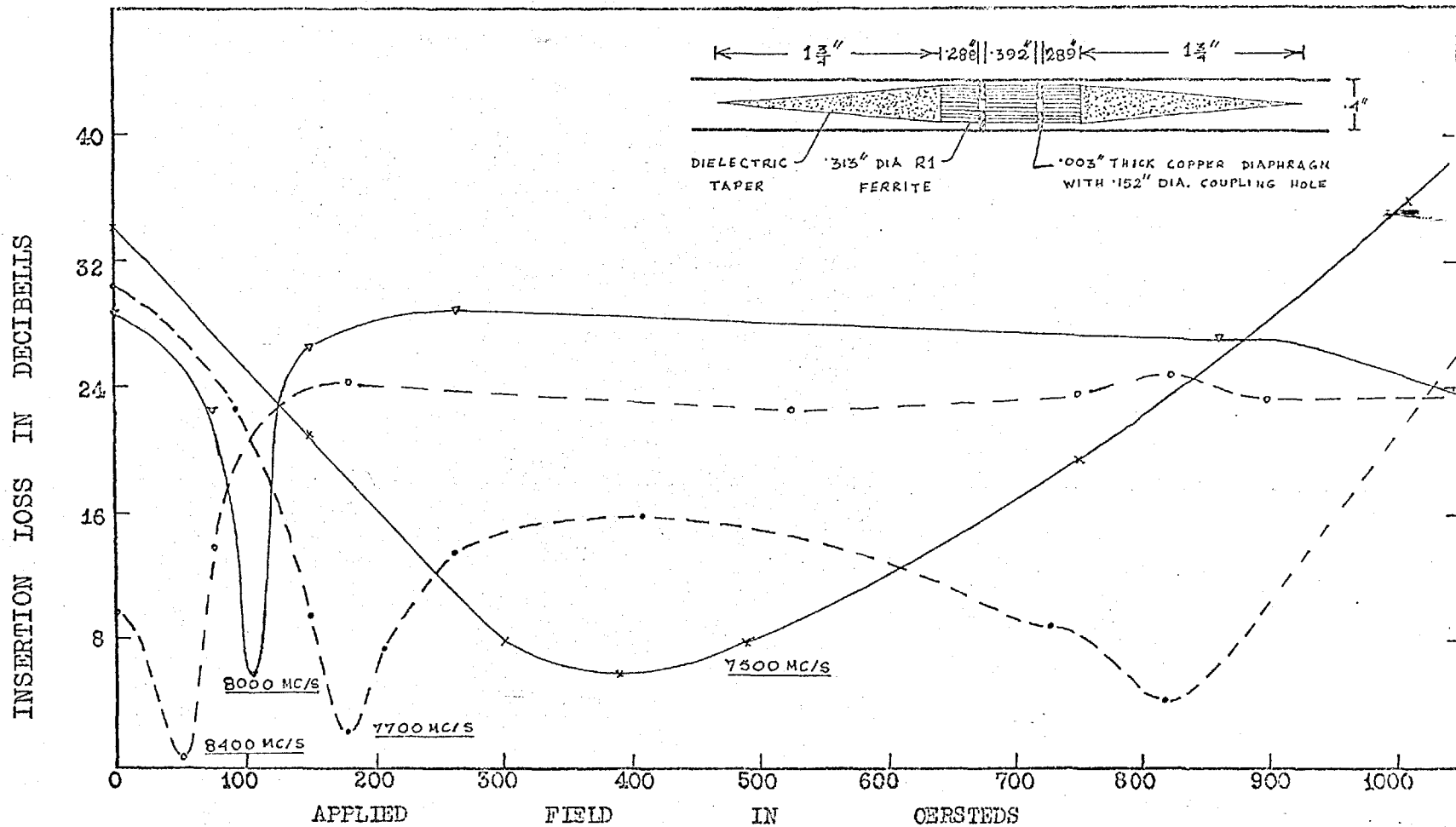


Figure 5.3. Insertion loss of a single section filter using $.313" \text{ dia. R1}$ specimen.

15 percent of the mid-range frequency. At the lower end of the tuning range large VSWR's at resonance (about 10:1 at 7500 Mc/s) indicated that the coupling was less than optimum. This is perhaps because of the fact that in the expression (5.1) both V_g and V_r must have increased due to the energy not being as effectively bound to the ferrite rod as at higher frequencies. The relatively high insertion loss at resonance for 8,000 Mc/s could have been due to errors of measurement. The input VSWR was low for this case.

It can be seen that the whole tuning range was covered for H_{dc} intensities less than 400 oers. This was a considerable improvement on the case of figure 3.7 where a maximum of over 800 oers was required.

5.2.3 Frequency response

Figure 5.4 shows the frequency response of an arrangement similar to that of section 5.2.2 using .281" dia. R1 rod. The length of the cavity was chosen to give a similar performance to that shown in figure 5.3. Some adjustment to the length was made so that the cavity resonated at a slightly higher frequency at $H_{dc} = 0$. This was done, as it was intended to incorporate this cavity in the final design where it was wished to cover a tuning range of about 8200 to 9200 Mc/s. Due to the use of the smaller diameter rod it was expected that the unwanted response of the cavity (see figure 3.7) in the higher order mode would be quite a bit removed from the upper end of this tuning range.

The length of the input and output feeders was made long enough so that in the final 2-section filter the cavities and the

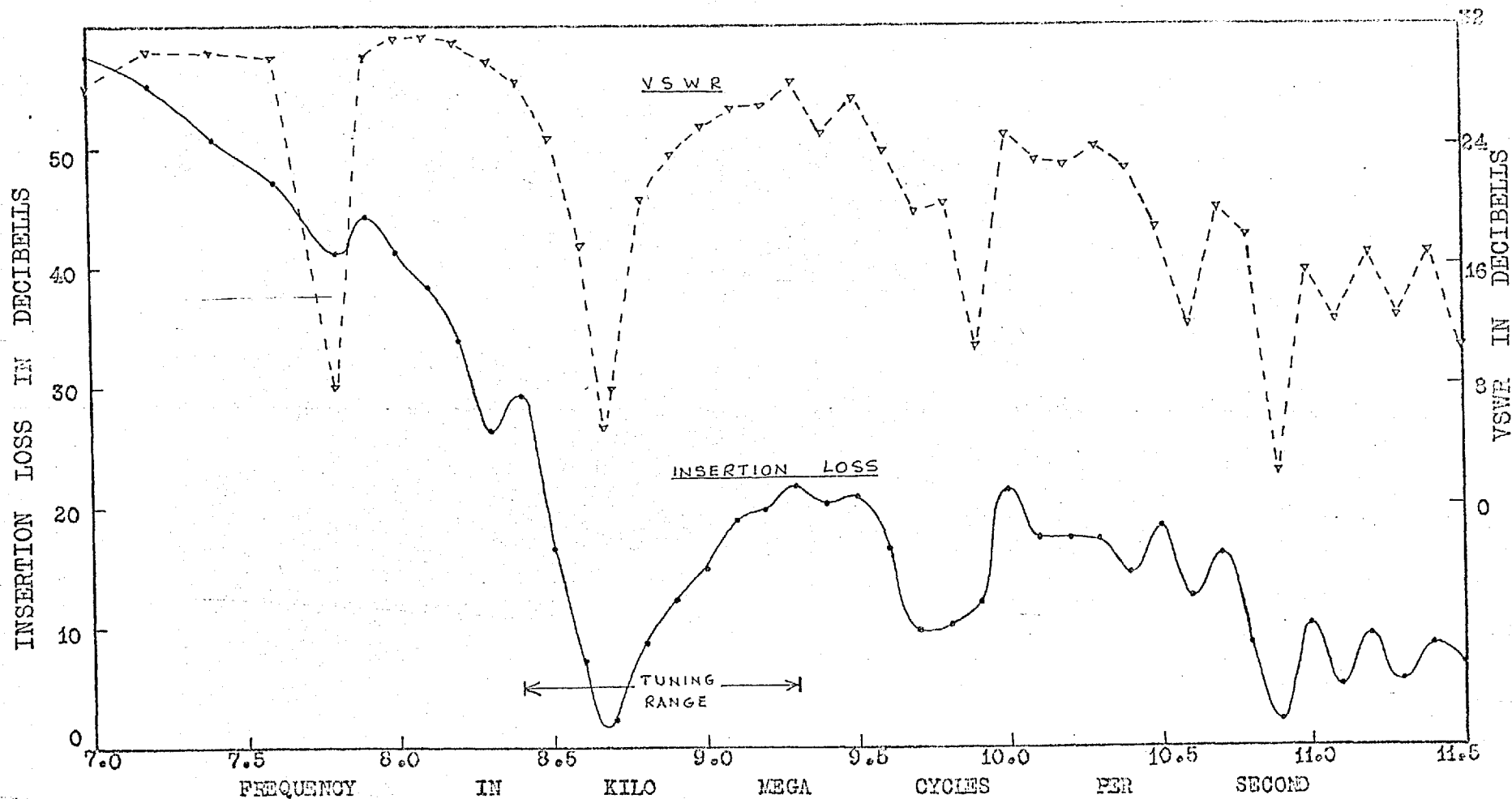


Figure 5.4. Frequency response of a single section filter using .281" dia. R1 ferrite (.376" long cavity, .564" long feeder sections) at 75 oers. applied field.

$\lambda g/4$ coupling section would be uniformly magnetised. This should reduce the steady field requirement and give a better tuning characteristic and performance. The limit on the length was imposed by the requirement of providing a uniform H_{dc} over the whole filter, and by the consideration of making the final length as small as possible.

The coupling hole diameter was calculated as before to be about .140". This size was used for this preliminary experiment to get an idea of the adjustment, if any, that would be required for the final design.

The response of this cavity was recorded over a frequency range of 7,000 to 11,500 Mc/s at a steady field intensity of 75 oers. The insertion loss at resonance was found to be quite low for this approximately calculated size of the coupling hole. It was found that the cavity could be tuned over a range of about 10 percent of the mid-band frequency. This was lower than that obtained with the .313" dia. rod in the previous section, though the measurements of section 2.3 showed that the .281" dia. specimen gives a higher phase shift. From this it is concluded that the rod diameter is the most important factor for determining the tuning range.

The curves of figure 5.4 show that the nearest spurious response for this arrangement is around 9700 Mc/s. This is thought to be due to the response of the cavity in the higher order mode. The low insertion loss at 10,900 Mc/s is most probably due to the cavity becoming equal to λg . The cavity is very much over coupled

at these frequencies due to the dielectric effect of the ferrite rod. The deterioration of performance at the high frequency end of this cavity indicates that filters based on this structure will have to be used in conjunction with a low pass filter if this range is of interest.

In the low frequency end, progressively high isolation was obtained. However, an anomalous absorption was noticed at 7,800 Mc/s as indicated by the low VSWR in the figure. Although there was an accompanying dip in the insertion loss, it remained quite high. This absorption phenomenon was investigated in greater detail and is described in the next sub-section.

5.2.4 The anomalous absorption

The absorption effect of the arrangement of the previous section was investigated at neighbouring frequencies to determine its characteristics. The results are shown in figure 5.5. Measurements of the insertion losses and the VSWR's were taken separately as there was little correlation between their fluctuations. The variations in the VSWR's as H_{dc} was varied were followed continuously in this case and the minimum points were located accurately. The effect was found to occur for this cavity from 7800 Mc/s up to 8200 Mc/s, as shown. Frequencies below 7800 Mc/s were also checked, but no evidence of the absorption was noticed.

Figure 5.5 shows that the absorption occurs over a wide range of H_{dc} intensity, the required magnitude increasing as the frequency is raised. Corresponding to the minima in the VSWR the insertion loss showed peaks or dips but remained quite high. The steady field

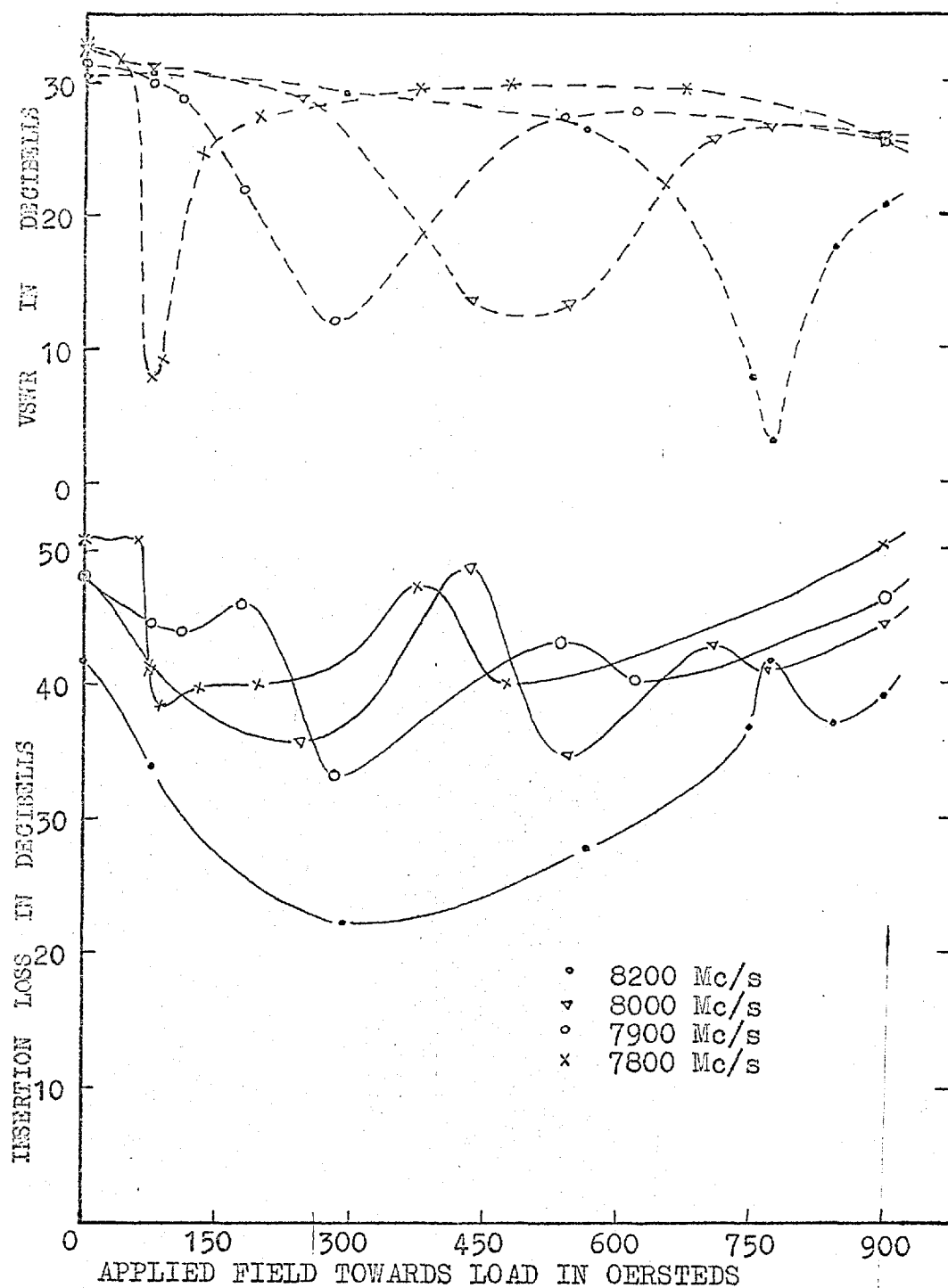


Figure 5.5. Anomalous absorption behaviour of the filter section of figure 5.4.

had to be varied by a large amount (over 800 oers), to retain the effect, for a relatively small variation of frequency (400 Mc). The system was also checked at 7800 Mc/s after removing the input and output sections. The cavity by itself did not show the absorption effect. It should be noted, however, that the coupling was much decreased in this case due to the removal of the ferrite feeder sections.

One explanation of the phenomenon would be that the cavity was resonating in a mode with a high phase constant, which decreased with the increase of H_{dc} . The absorption is due to the dielectric and magnetic losses of the ferrite in the resonator. At the frequencies concerned, however, the R1 specimen was not found to be very lossy. Also, the mode spectrum discussed in section 4.4 does not predict such a mode under these conditions. However, the phenomenological model does predict two negative elliptically polarized modes for this system, for values of H_{dc} greater than that for which $k = \mu$. This critical value of H_{dc} may be quite low for these frequencies due to the transverse demagnetisation of the long rod. If the two modes are able to propagate they could cause heating and energy trapping effects in the same way as happens due to Faraday rotation.

5.2.5 Arrangements for adjusting the cavity

In order to facilitate lining up of the assembled filter it was necessary to provide an easy means of adjusting each resonant element. The use of resonant cavities in this design was fortunate in that, conventional tuning screws could be used for these

adjustments. Holes were therefore tapped for 10 BA ($\frac{1}{16}$ " dia.) screws at the centre of the two narrow sides and one broad side of the cavity of section 5.2.3. It was thought that screws projecting from the narrow walls should have the same effect on the resonant frequency as the capacitive screws projecting from the broad walls, because we were dealing with elliptically polarized waves. The screws from the sides were expected to provide a finer control over the resonant frequency.

The results of introducing the screws into the cavity are shown in figure 5.6. All the screws appear to affect the cavity in the same way, except that much more penetration is necessary for the screws from the narrow sides, as expected. The results shown are for a frequency below the tuning range of the cavity. The minimum insertion loss can be seen to be gradually reduced as more screws are introduced. At frequencies where the original cavity could resonate, the screws decreased the value of H_{dc} required for resonance. Effect of introducing more than one screw from the sides was also checked. It was found that this disturbed the fields too much and increased the insertion loss at resonance. In fact it was found that the maximum allowable penetration for the single pair of side screws was .225". Even for this penetration the insertion loss at resonance for H_{dc} values slightly above zero was found to have increased considerably. * The top screw (on the broadwall) was found not to affect the resonance insertion loss

* Note that the resonance insertion losses of all the filters described here were found to be maximum when the ferrites were very slightly magnetised.

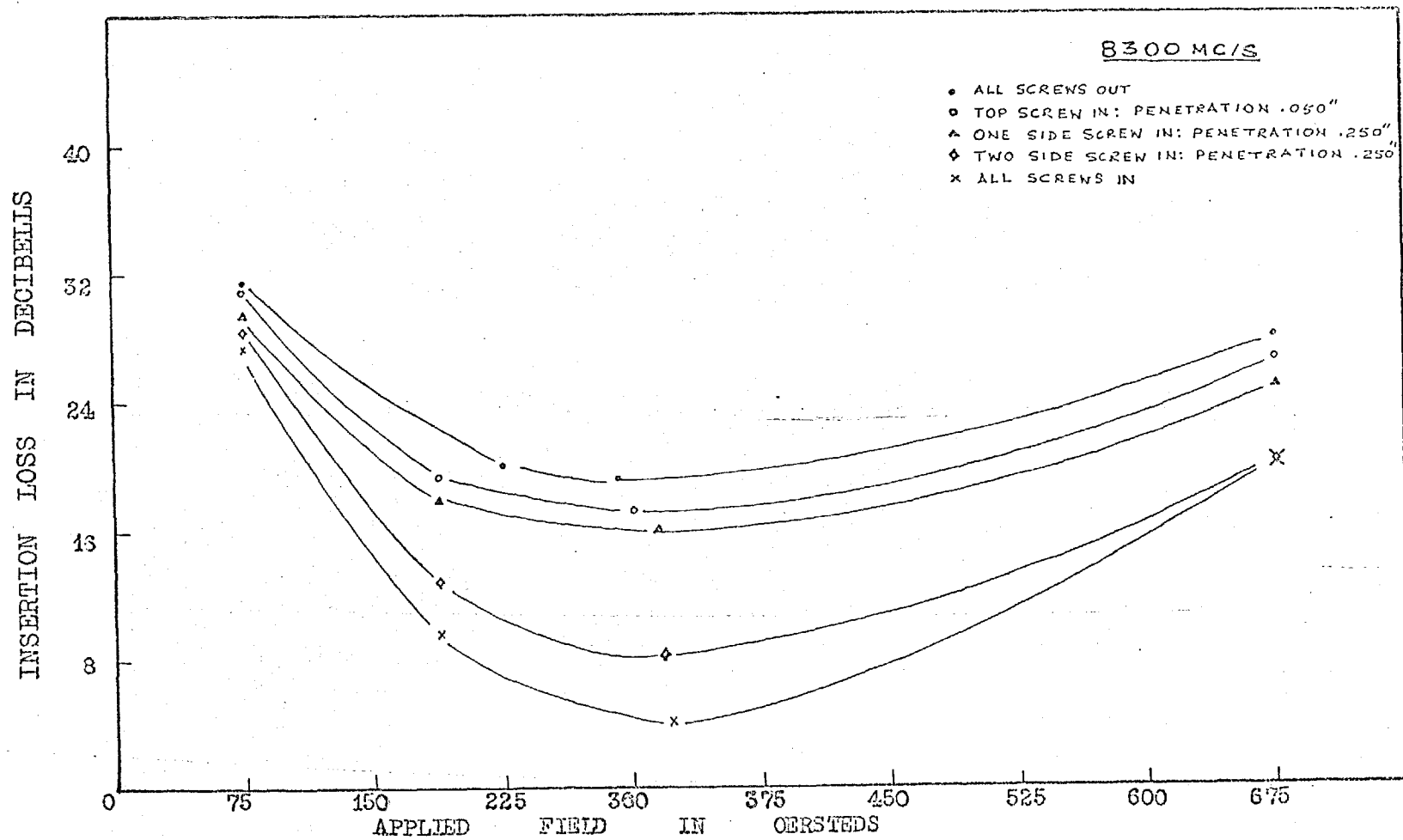


Figure 5.6. Effect of tuning screws on the filter section of figure 5.4.

appreciably even when nearly touching the ferrite rod in the cavity.

The resonant frequencies at or near $H_{dc} = 0$ were not very much affected by the side screws, because of the linear nature of the mode at low fields. This feature appeared to be very suitable for the tracking of the multi-section filter. The cavities would be lined up at low H_{dc} by the side screws. The side screws were also expected to increase the tuning range by extending the low frequency end of it without affecting the high frequency end. However, as we have seen above, the amount of increase would be small due to the limit imposed on screw penetration.

5.3 Electrically Tunable Two-Section Bandpass Filters.

5.3.1 Untunable quarter-wavelength coupling.

The first attempt at a two-section filter was to combine two cavities similar to the one shown in figure 3.7, through an empty rectangular waveguide quarter-wavelength section.

The quarter-wave coupling was chosen because of its suitability for this, comparatively narrow band, filter.

Each cavity could thus be individually made and tested, and the final adjustment of the filter proved much easier.

The length of the quarter-wavelength section was chosen to correspond to approximately the middle of the tuning range. A section of the filter through the narrow sides of the rectangular guide is shown in figure 5.7. The cavities were individually assembled in sections of rectangular waveguide and then finally put together inside a closely fitting sleeve of internal dimensions 1" x ½" (not shown here). The sleeve

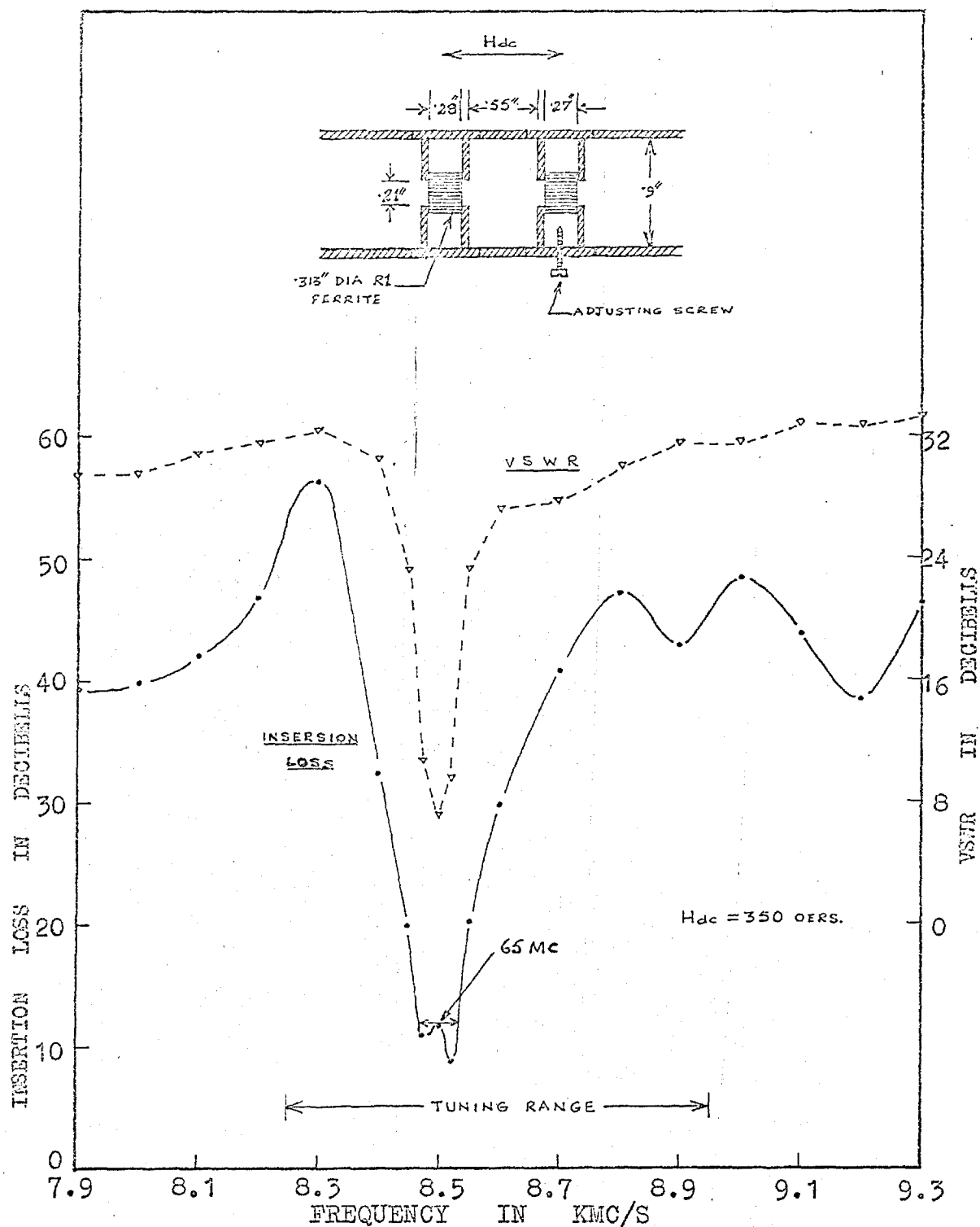


Figure 5.7. Frequency response of a two-section filter using untunable quarter-wavelength coupling.

was made from conveniently available British X-band guide which just fits on the outside of the WG-16 used. This method of assembly is similar to that used by Chitre ⁽⁴⁶⁾. Conducting silver paint was used to provide good connection between the diaphragms forming the cavities and the guide-walls. Only one adjusting screw, as shown, was used to line up the cavities.

Figure 5.7 shows the frequency response of this filter at approximately the middle of the tuning range. The insertion loss in the pass band is rather high. It was found to be even higher (≈ 12 dB) at the low frequency end of the tuning range and for very small values of H_{dc} . This is attributed to manufacturing defects and to the use of lossy silver paint in the cavities. The asymmetry of the insertion loss in the pass band is attributed to non-optimum length of the coupling section ⁽⁴⁹⁾. At the high frequency end of the tuning range the pass band was symmetrical. The tuning range of 700 Mc is lower than expected. This also could most probably be improved by carefully manufacturing the cavities to have nearly equal resonant frequencies independent of the adjusting screws.

It is thought that with better manufacturing techniques, reasonable performance could be obtainable from the filter. The advantage of this design over that of figure 5.3 is that, this filter is not limited, by Faraday rotation considerations, to operation in the dominant mode only. If both modes of figure 3.7 were used, the filter could be tuned over a much larger frequency range. This would of course be at the cost of having

two responses for the same value of H_{dc} , and a low/high pass filter would have to be used in conjunction. The axial steady magnetic field intensity required for tuning this filter is high compared to the alternative design mentioned earlier.

5.3.2 Tunable quarter-wavelength coupling.

The high field strength required for tuning, and the non-optimum length of the coupling section of the filter described in the previous section prompted us to investigate the possibility of using a quarter-wavelength section of the same ferrite loaded waveguide as formed the cavities.

Ferrite rods would also be used at the input and output of the cavities, as described in section 5.2.2. It was thought that such an arrangement should have the following advantages:-

- a. Variation of coupling with frequency will be less.
- b. Much **lower** steady field intensity would be required.
- c. The coupling section will always be of optimum length; non-symmetric pass bands should not occur.
- d. Due to the arrangement being like a continuous rod, the ferrite will be more uniformly magnetised.

Tracking the two cavities should thus be easier.

A sectional view of the arrangement, based on that of section 5.2.3, is shown in figure 5.8. The coupling holes were slightly longer in this case to compensate for the effect of the adjusting screws. Screws were also provided

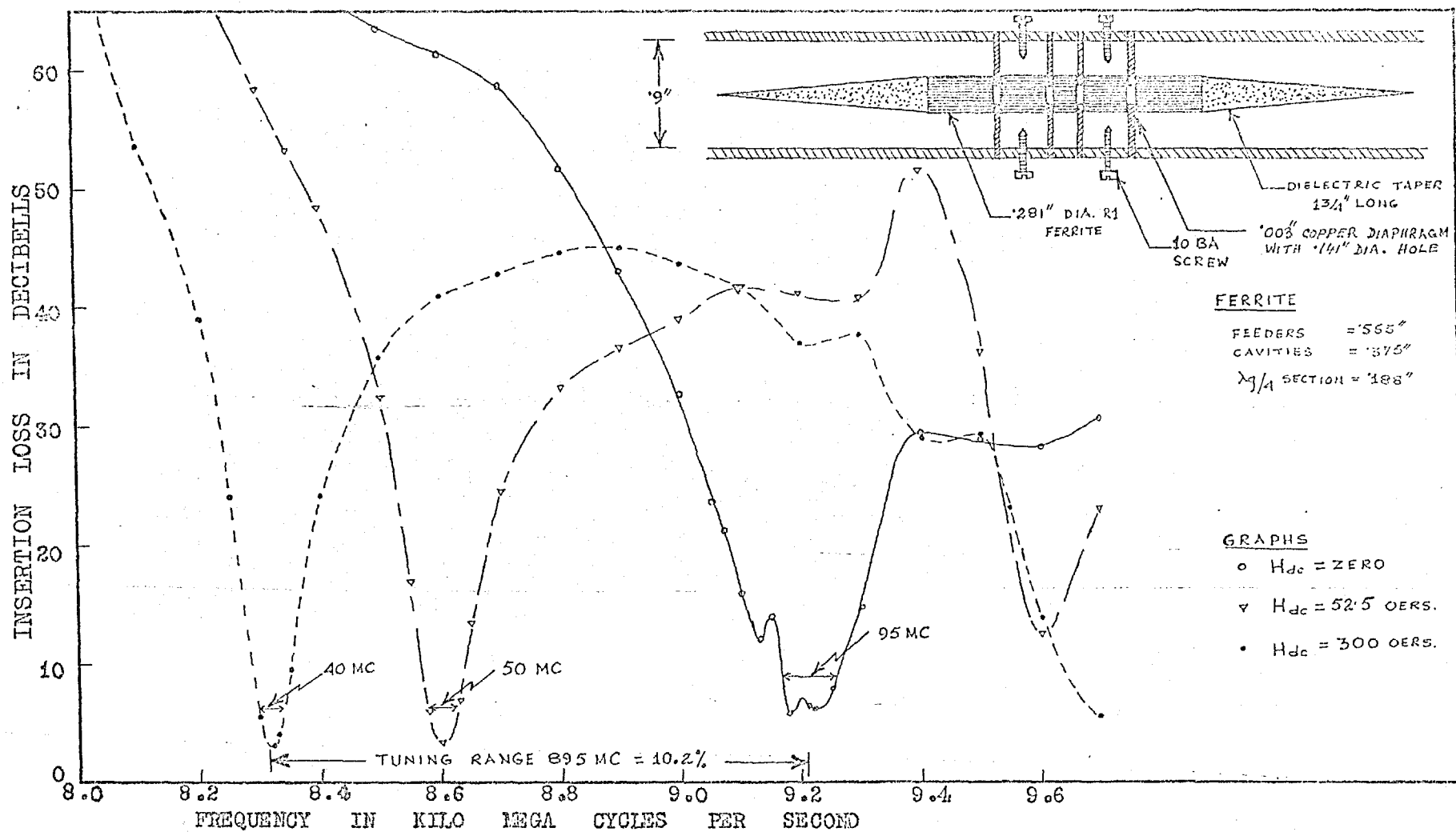


Figure 5.3. Frequency response of a two section filter using tunable quarter-wavelength coupling.

at the centres of one broad side for the $\lambda_g/4$ - section, as well as for the cavities. The advantage of the dielectric matching tapers used, compared to tapering the ferrite rods themselves, is that the steady magnetic field is required over a much shorter length in the former case.

The filter was assembled in sections of WG-16 waveguide of appropriate lengths. The use of silver paint was avoided by making the diaphragms extend up to the outer edges of the guide-walls (their size was 1" x 1/2"). The whole assembly was then pressed together to make good contact. All the waveguide sections had to be very carefully manufactured and ground flat by hand so that good connection was made all round. The ferrite sections were also cut and ground very carefully, so that the two cavities were equal in length up to the figures shown on the diagram.

An external view of the assembled filter and the mechanism for pressing the sections together are shown in figure 5.9. The square flanges shown were hard-soldered on to the waveguide so that they would not slip when the nuts were tightened. Slots were cut on the sleeve holding the filter sections together, for the introduction of the adjusting screws. Though six side screws per cavity are shown, only the central one on each side was used for adjusting the filter. One central screw on the far side cannot be seen because it has been screwed in. Of the top screws, the central one was for the adjustment of the $\lambda_g/4$ -section.

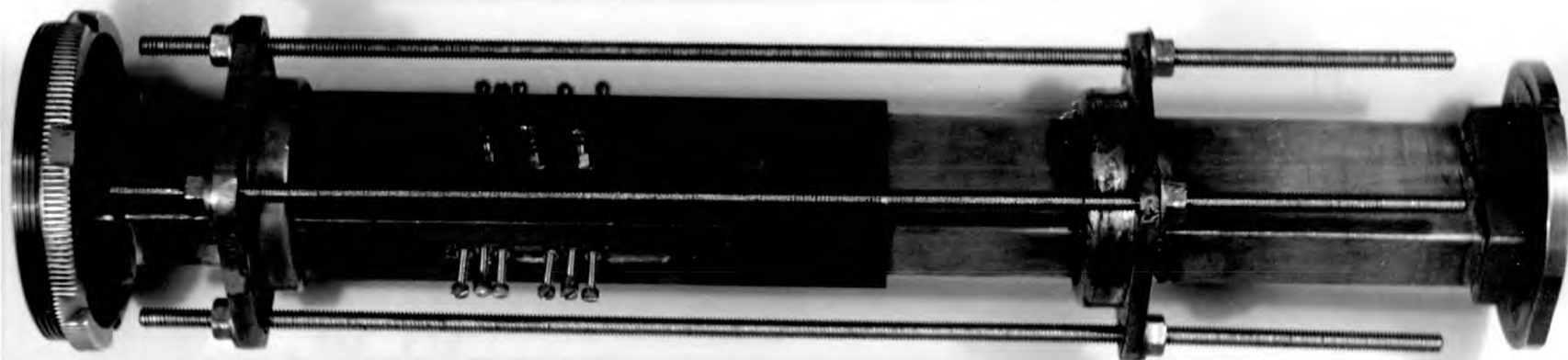


FIGURE 5.9. EXTERNAL VIEW OF THE ASSEMBLED FILTER.

It was attempted as far as possible to make the resonant frequencies of the two cavities same, without the aid of the screws. The procedure was to slightly grind down the rod of the cavity that had the lower frequency of resonance. This was found difficult as dismantling and reassembling the filter changed the resonant frequencies of the cavities in an unpredictable manner. By carefully removing only the section that required grinding, leaving all the others in position the resonant frequencies were brought fairly close to each other at the end.

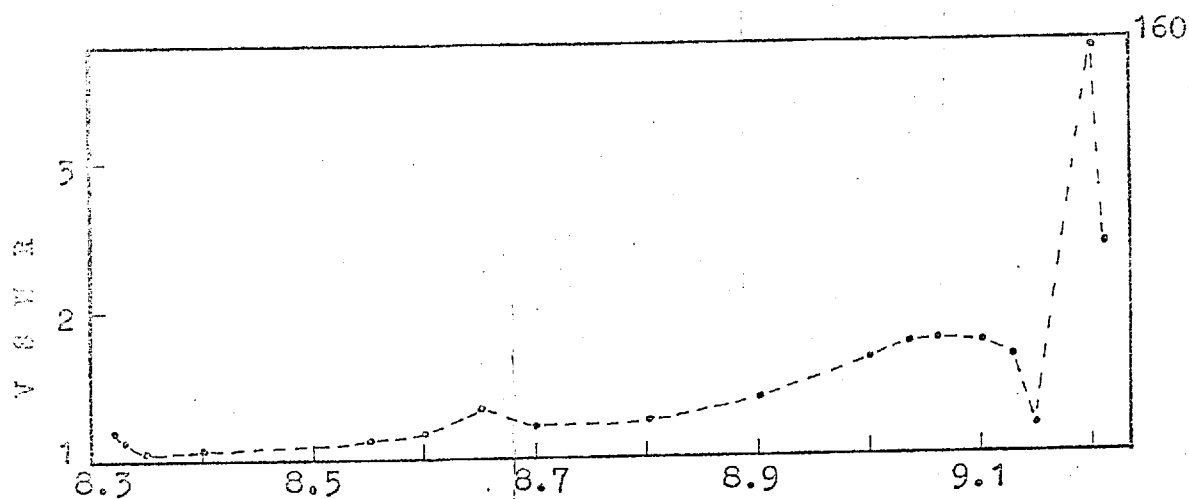
This enabled the screws from the narrow sides to be used mainly to increase the tuning range, as discussed in section 5.2.5. The alignment procedure for the filter was as follows. The central side screws were introduced to the maximum permissible limit and the filter was tuned by them for maximum transmission, at the lower frequency end of the tuning range. Then it was readjusted at the higher frequency end by the top screws of the cavities. The procedure was repeated until no further improvement was obtained in the transmission. Then the $\lambda g/4$ was adjusted at the mid-tuning-range frequency. The filter was then checked over the whole tuning range and slightly readjusted for best performance. Results indicated that better characteristics could probably be obtained by making the coupling section slightly longer.

The frequency response of the filter at the two extremities of the tunings band and at about mid-band are shown in figure 5.8. On the low frequency side the curves did not come down again, and isolations of over 70 dB were always obtained.

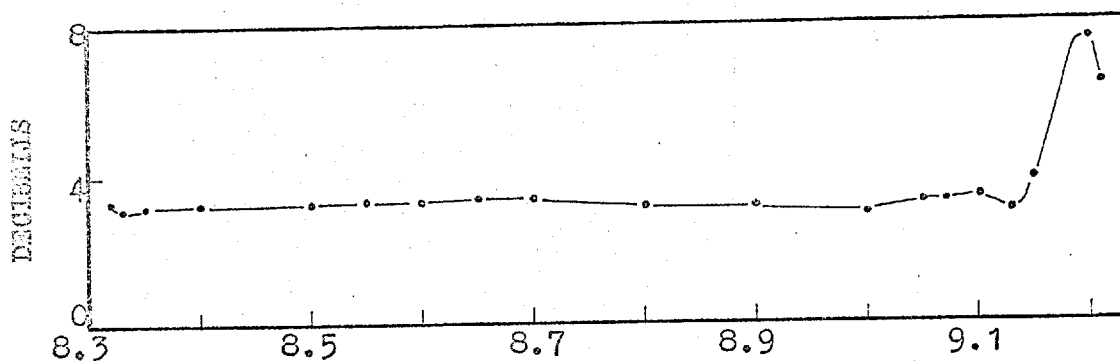
On the high frequency side the performance can be seen to have deteriorated beyond 9500 Mc/s, as expected from the frequency response of figure 5.4. The anomolous absorption discussed in section 5.2.4 was also noticed here for the response of $H_{dc} = 300$ oers, at 8000 Mc/s. The insertion loss of the device was higher at and near $H_{dc} = 0$. The performance of the filter over the whole tuning range is recorded in figure 5.10. It can be seen that a very uniform performance was obtained except at the extreme top end. The maximum H_{dc} required was 300 oers., and about 80 percent of the tuning range could be covered by only about a quarter of that **field** strength.

Figure 5.8 also indicates the 3 dB bandwidths of the response curves. As can be seen, there was a rather large variation in the bandwidth. It was thought that this could partly be due to the adjusting screws. At high H_{dc} values the frequency sensitivity of the effect of the side screws was added to the normal Q of the cavity. The 3 dB bandwidth thus became narrow. At low values of H_{dc} the linearly polarized mode made the side screws ineffective and the bandwidth was broader. To check this hypothesis, frequency responses of the filter **were** measured with the side and top screws mostly removed. The results are shown in figure 5.11.

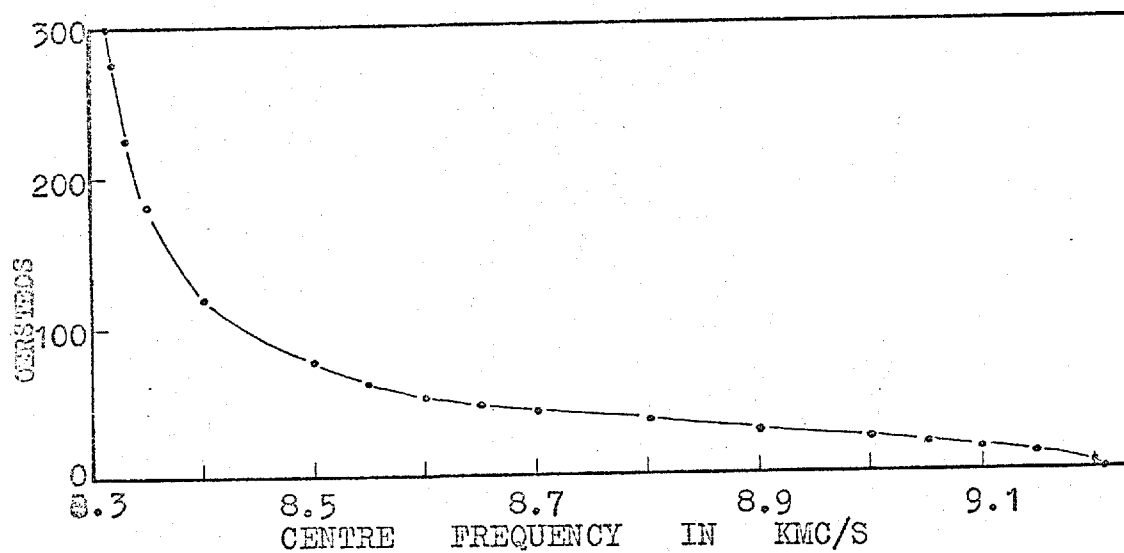
The variation in the bandwidth was **substantially** reduced by the withdrawal of the screws. Insertion losses in the pass-band were also found to have been lowered, the most marked improvement being at the high frequency end. With the screws



(a) Input VSWR at resonance.



(b) Mid-band insertion loss.



(c) Tuning field intensity.

Figure 5.10. Performance curves for the filter shown in figures 5.8 and 5.9.

in, the worst response was that at $H_{dc} = 5.25$ oers., where the insertion loss was high and the 3 dB bandwidth was as large as 135 MC. In the case of figure 5.11 the responses for $H_{dc} = 0$ and 5.25 oers. are much more regular. One disadvantage in this case is that the tuning range has been slightly reduced.

The performance of this filter is quite suitable for system applications. Used in conjunction with a low pass filter, isolation in the stopbands of over 70 dB can be easily obtained. The insertion loss in the pass band is moderate and can be further decreased by using better ferrite materials. The tuning range could be increased by using a thicker diameter rod, as shown by the results of section 5.2.2. Problems of Faraday rotation in the input and output members in this case could be overcome by slightly reducing the waveguide height in those regions.

5.4 Narrow-band Tunable Isolator.

As pointed out in section 2.8, the characteristics of the insertion loss peaks obtained with an abruptly twisted junction of rectangular waveguides containing an axial ferrite rod, are suitable for the development of a very narrow band isolator. In this section results of attempts at obtaining a higher forward to reverse loss ratio are described. Performance data of a prototype isolator are also presented.

A $3 \frac{7}{8}$ " length of the .313" dia. R1 specimen was used for this investigation. The insertion loss of a 52°

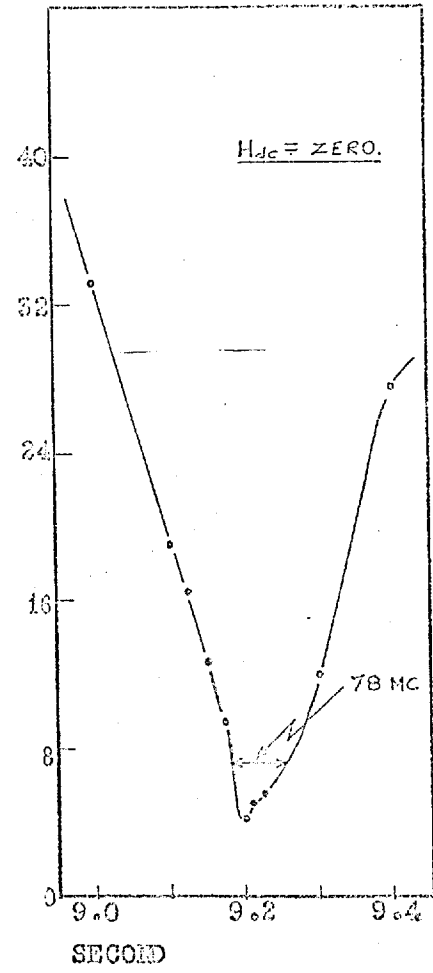
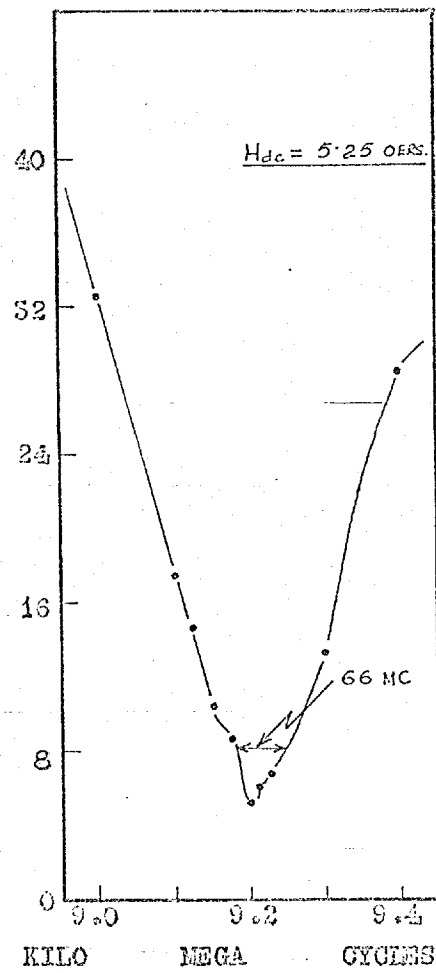
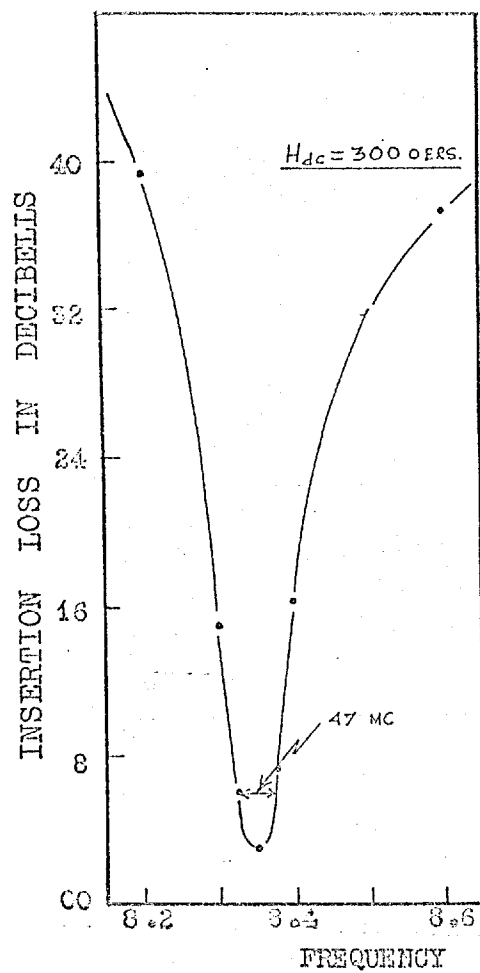


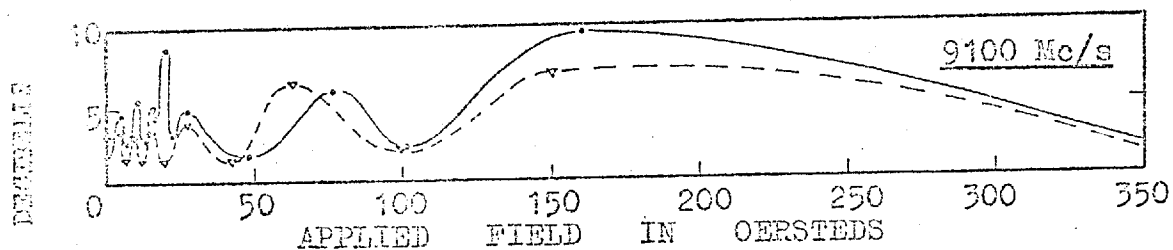
Figure 5.11. Effect of withdrawal of side screws from the cavities of the filter of figure 5.8. Tuning range was reduced to 869 Mc (9.9%) in this case.

twisted junction with this rod placed centrally along the axis is shown in figure 5.12 (a). The large peaks obtained with the longer rod (see section 2.8) seems to have disappeared. The best non-reciprocal behaviour can be seen to be around $H_{dc} = 20$ oers. The rod was then passed through two twists in the same direction, of 52° each. The central waveguide section was $1 \frac{7}{8}$ " long and the rod protruded 1" on each side of this section. The insertion loss of this arrangement as a function of steady field intensity is shown in figure 5.12(b). A very high non reciprocal peak can be seen to have been obtained at $H_{dc} = 18$ oers.

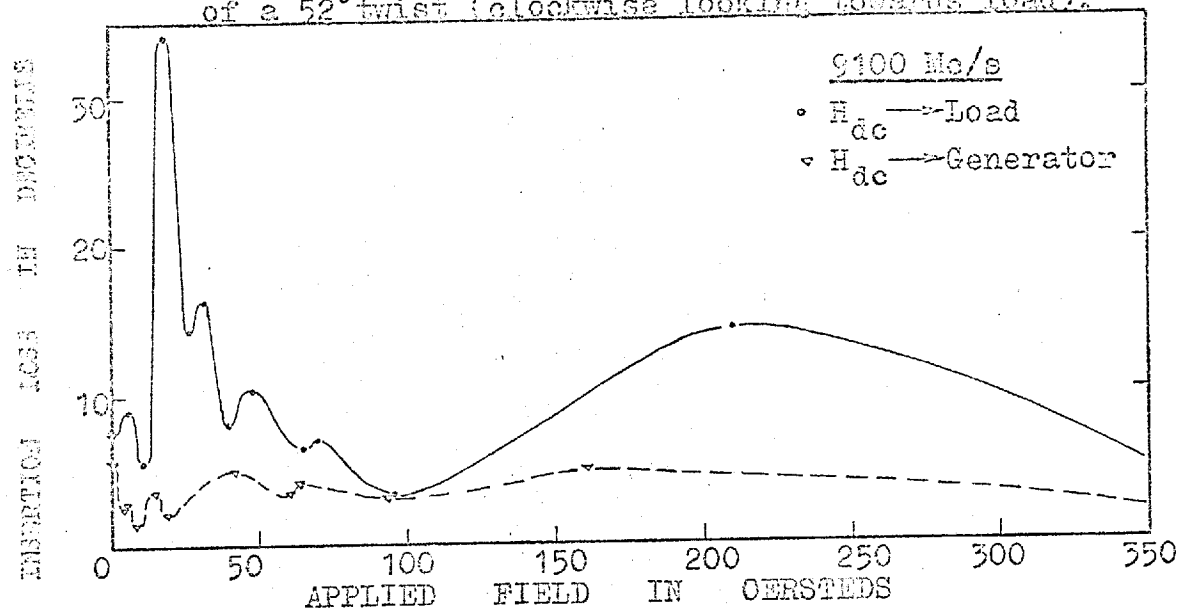
The frequency response of this peak is shown in figure 5.12 (c). The bandwidth at points where the loss falls 20 dB below the peak is 12.25 MC. This is about 30 percent larger than that obtained with the 5" long rod in section 2.8.4. So it appears that the frequency selectivity of the arrangement could be increased if needed by increasing the length of the rod.

It was found that, by varying the twist of the output section and the steady field intensity, the loss peak could be tuned over more than 200 Mc in the X-band. These results are shown in figure 5.13. It is expected that if both twist angles as well the steady field are varied a larger tuning range can be obtained.

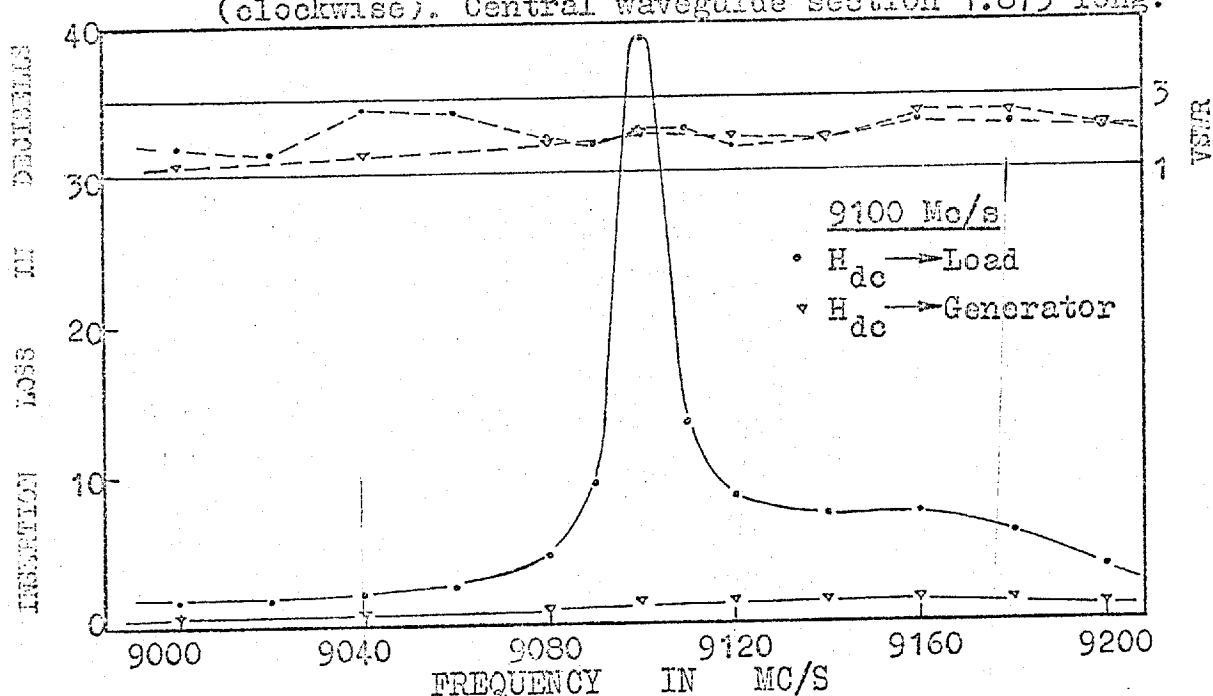
~~This~~ device could be used in communication systems to tune out unwanted signals. It could also be used as a high-speed, matched input switch in narrow band applications.



(a). .313" dia., 3.875" long R1 rod centrally placed along the axis of a 52° twist (clockwise looking towards load).

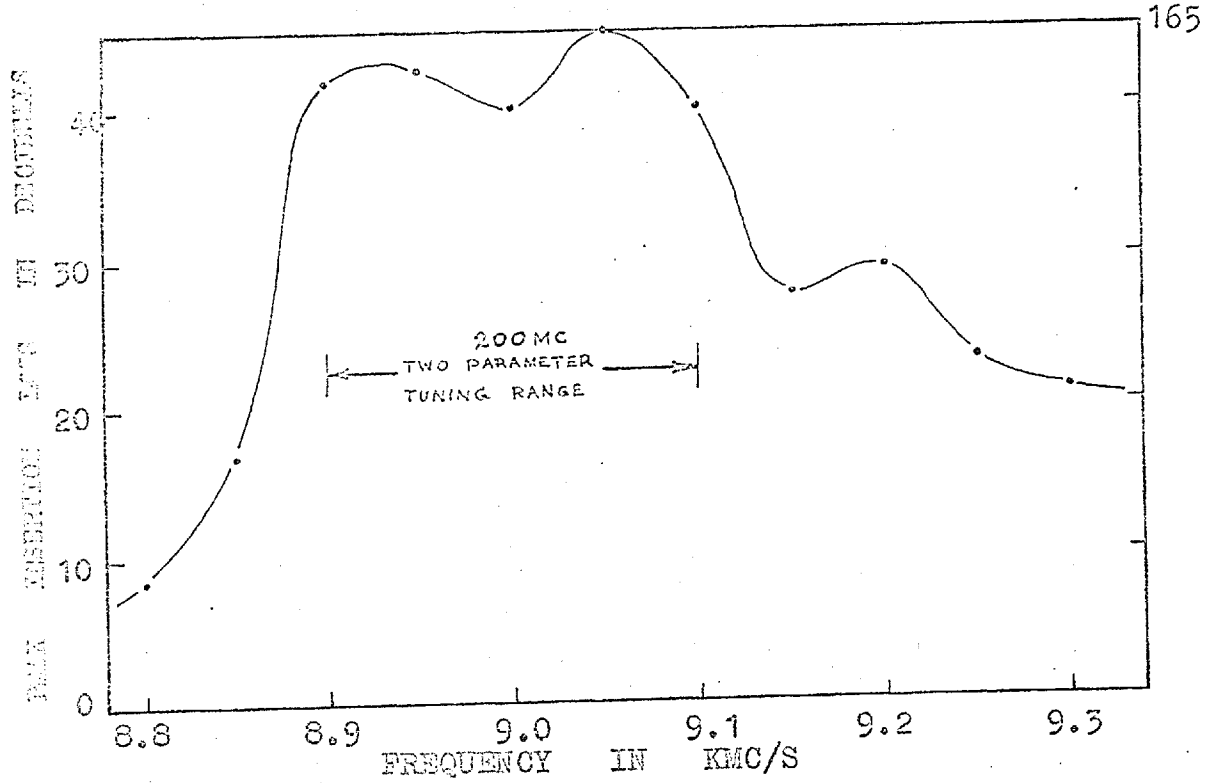


(b) Same rod centrally placed in two consecutive 52° twists (clockwise). Central waveguide section 1.875" long.

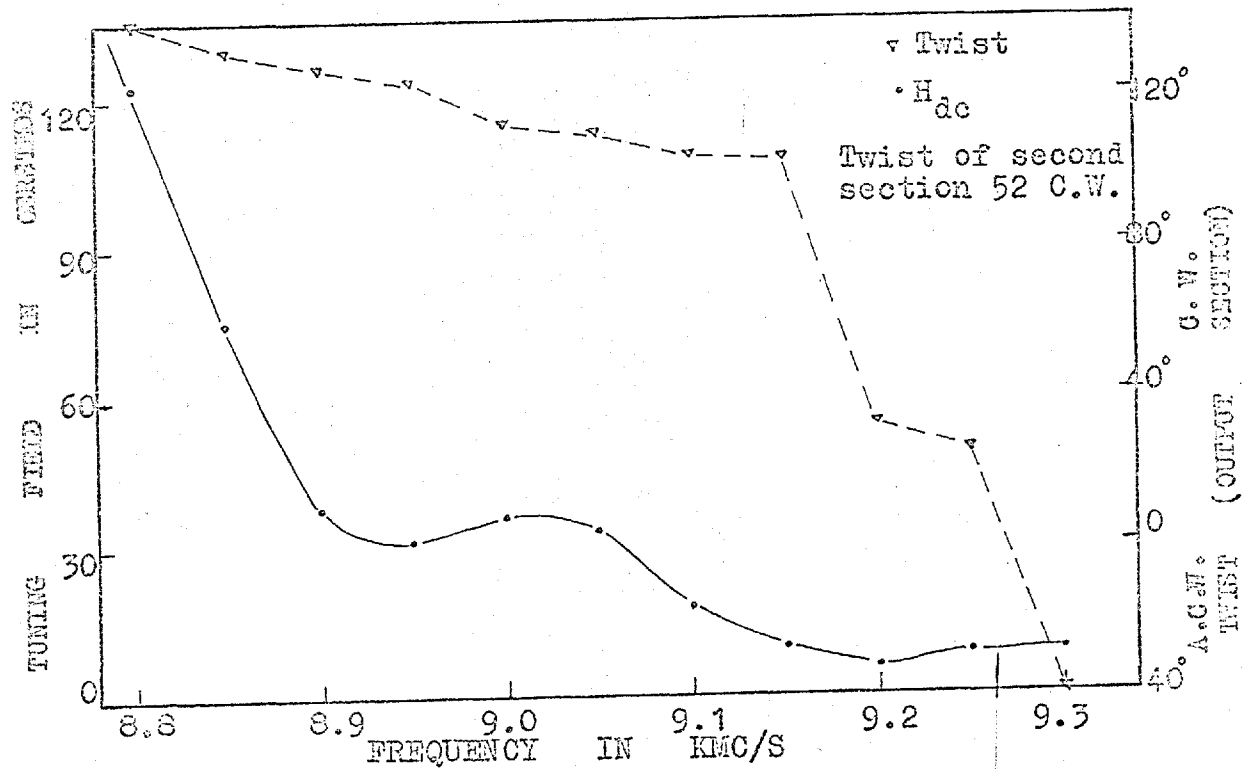


(c) Frequency response of the arrangement of figure (b).

Figure 5.12. Development of a double-twist isolator.



(a) Tuning range of the double-twist isolator of figure 5.12.



(b) Variation of parameters necessary to obtain the tuning.

Figure 5.15. Performance curves of the double-twist isolator.

However, since no important applications for the device could be envisaged, further efforts ~~were~~ not exerted in perfecting it for system use.

APPENDIX ONESTATIC MAGNETIC FIELDS

During the course of this work various dc magnetic fields had to be used to magnetise the ferrite specimens. Field intensity requirements varied from over 2000 oersteds to a fraction of an oersted. Arrangements for obtaining these fields are described in the following sections.

A1.1 Magnetic Field used for Preliminary Experiments

Two coils, each of length 11.2 cm., internal diameter 13.0 cm. and external diameter 28.3 cm. were available. These were used side by side, thus giving an effective coil length of 22.5 cm. The field strength along the axis of this system for a current of 0.5 amp (maximum allowable) flowing through each coil is shown in Figure A1.1. The field values were obtained with the help of a calibrated fluxmeter and a search coil and are estimated to be accurate to within 1%. From the graph following data were calculated for future use:

Average field intensity over 12.65 cm.

for 0.5 amp. coil current = 700 oers. $\pm$ 1%

Variation of field over 12.65 cm. = 13.9%

The length 12.65 was considered, as this was the maximum length of the ferrite specimens used. For shorter samples one coil only was

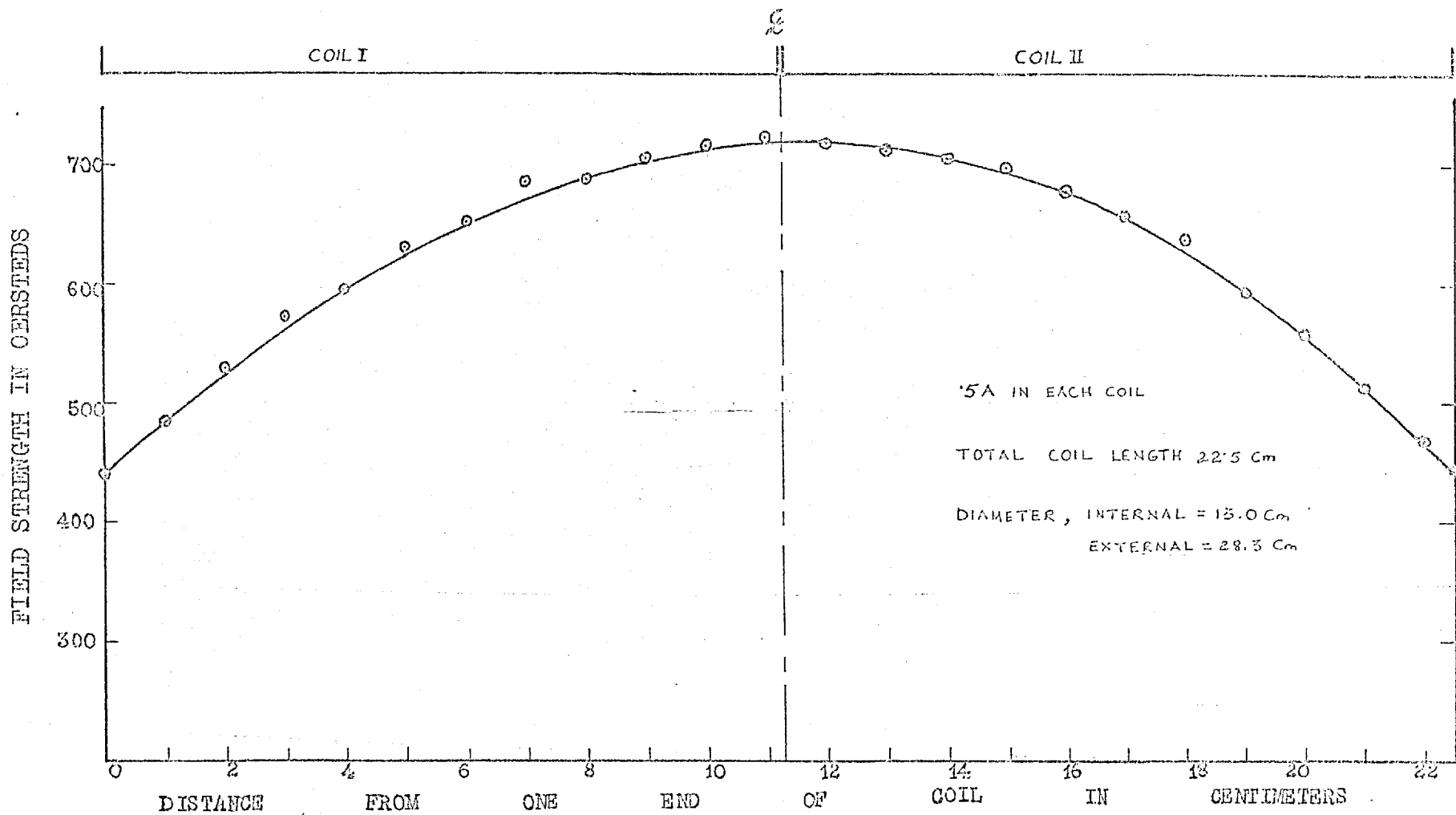


Figure A1.1. Field strength along the axis of the coil combination for 0.5 ampere flowing in each coil.

sometimes used. The maximum length of ferrite samples in this case was 3.57 cm., and over this length the average magnetic force was estimated to be 450 oers. $\pm$ 1% for a coil current of 0.5 amp.

Al.2 Magnetic Field used for Later Investigations

The dc field described in section Al.1 was judged to be unsuitable for more sophisticated experiments where it was necessary to be sure of a constant field over the whole length of the specimen. It was also necessary to extend the investigations to higher fields than achievable by these coils. A 28 cm. long coil was therefore designed, with a continuous current rating of 1 amp., suitable for use with a 500 volt, 4 amp. dc power supply. The coil was divided into three separate sections and wound on brass bobbins for better heat dissipation and easier handling. The inner diameter of the bobbin was determined by the flange-dimension of standard X-band waveguides. This in its turn, in combination with the maximum length over which a steady field was required, determined the total length of the coil.

A plot of the field strength along the longitudinal axis of the coil is shown in figure Al.2. With a current of 1 amp. flowing through the three sections, connected in series and bolted together, the following results were obtained:

Average field intensity over 14 cm. = 750 oers. $\pm$ 1%

Variation of field over 14 cm. = 7.7%

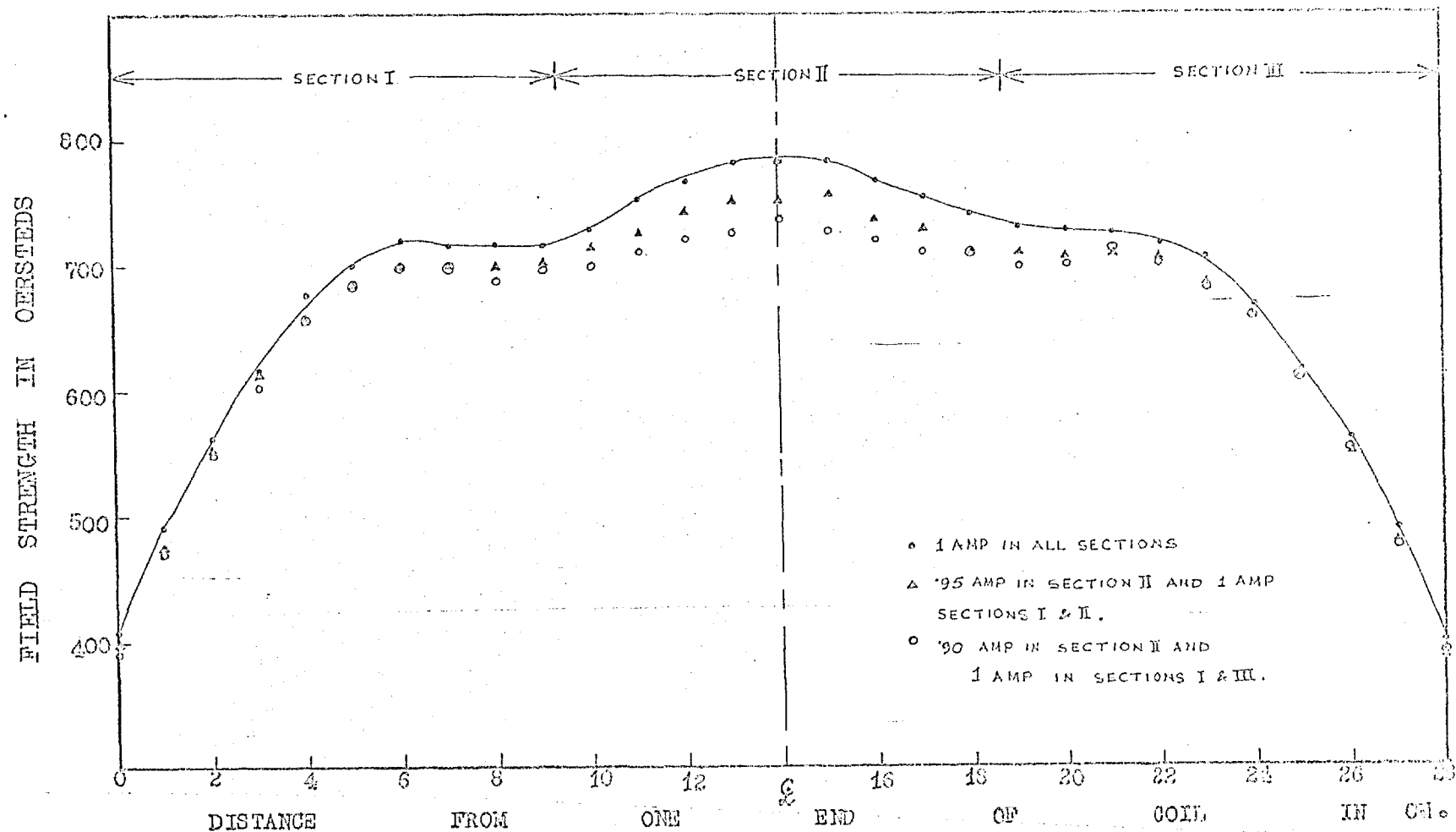


Figure A1.2. Field strength along the axis of the three-section brass-bobbin coil. Inner dia. of bobbin 5.9cm.

To obtain a more uniform field, measurements were made with reduced current flowing through the central section, with the following results:

Field variation with 0.95 amp. through central section = 6.3%

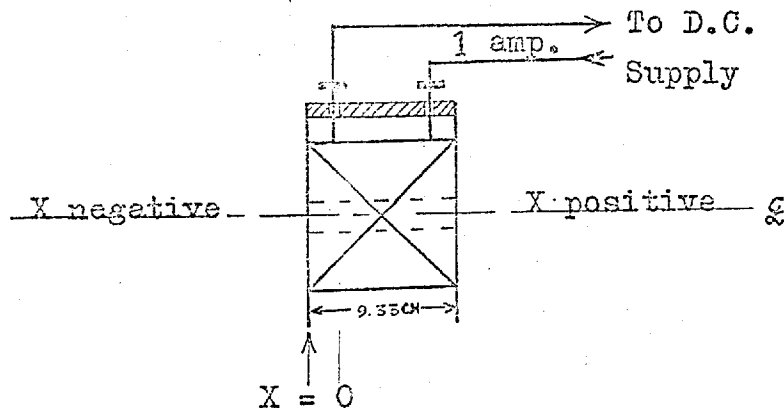
Field variation with 0.90 amp. through central section = 5.8%

The field strength, however, dropped considerably as can be seen from figure A1.2.

Attempt at getting a more uniform field was also made by putting air-gaps between the sections. Extended plotting of field strength along the axis of one of the sections was carried out. The result is shown tabulated in Table A1.1. Field distributions for the combination were then calculated for various air-gaps, assuming the sections to be identical. Minimum field variation was obtained with air-gaps of 1 mm., but this was only 0.5% lower than that obtained with no gaps. The improvement is thus even less than by the previous method. These improvements were not considered worthwhile and it was decided to use the sections bolted together with the same current flowing through all of them.

A1.3 Magnetic Field used for Resonance Investigations

Field strengths of up to 2500 oers. were required for the determination of saturation magnetisation from the resonance absorption of the ferrite specimens. This was achieved by a coil wound on a circular waveguide of 1" internal diameter, which was then placed



X in cm.	Av. axial field in oers. $\pm 1\%$
- 21	8.37
- 20	8.65
- 19	9.49
- 18	10.60
- 17	11.44
- 16	13.67
- 15	16.18
- 14	18.69
- 13	22.60
- 12	27.62
- 11	32.09
- 10	38.50
- 9	44.64
- 8	53.85
- 7	65.01
- 6	80.91
- 5	103.23
- 4	131.69
- 3	170.19
- 2	223.20
- 1	277.33
0	340.66
+ 1	408.74
+ 2	475.14
+ 3	522.57
+ 4	549.63

X in cm.	Av. axial field in oers. $\pm 1\%$
+4.66	555.21
+ 5	553.54
+ 6	534.29
+ 7	495.23
+ 8	438.59
+ 9	373.86
+ 10	304.67
+ 11	243.85
+ 12	188.88
+ 13	145.08
+ 14	112.16
+ 15	88.44
+ 16	70.31
+ 17	57.75
+ 18	47.99
+ 19	40.46
+ 20	33.48
+ 21	27.90
+ 22	22.88
+ 23	19.53
+ 24	16.74
+ 25	14.51
+ 26	13.11
+ 27	11.44
+ 28	10.32
+ 29	8.93

Table A1.1. Average axial field intensity of one section of the coil of figure A1.2 for a current of one ampere.

inside the coils described in section A1.1. The arrangement and the dimensions of the coil combination are shown in figure A.1.3. Plots of field intensities available at the centre and $2\frac{1}{2}$ " away from the centre along the longitudinal axis of the system are also shown, for a constant current of 0.5 amp. flowing through each of the outer coils.

The lower line in figure A1.3 represents the minimum field intensities on a 12.65 cm. long specimen. It can be seen that over such a length the field variation changes from 14% at low fields to over 30% for the highest field strengths. This was considered good enough for the requirements of the experiments carried out. For current values higher than 1 amp. through the inner coil it was found necessary to switch off immediately after taking a reading, to prevent overheating.

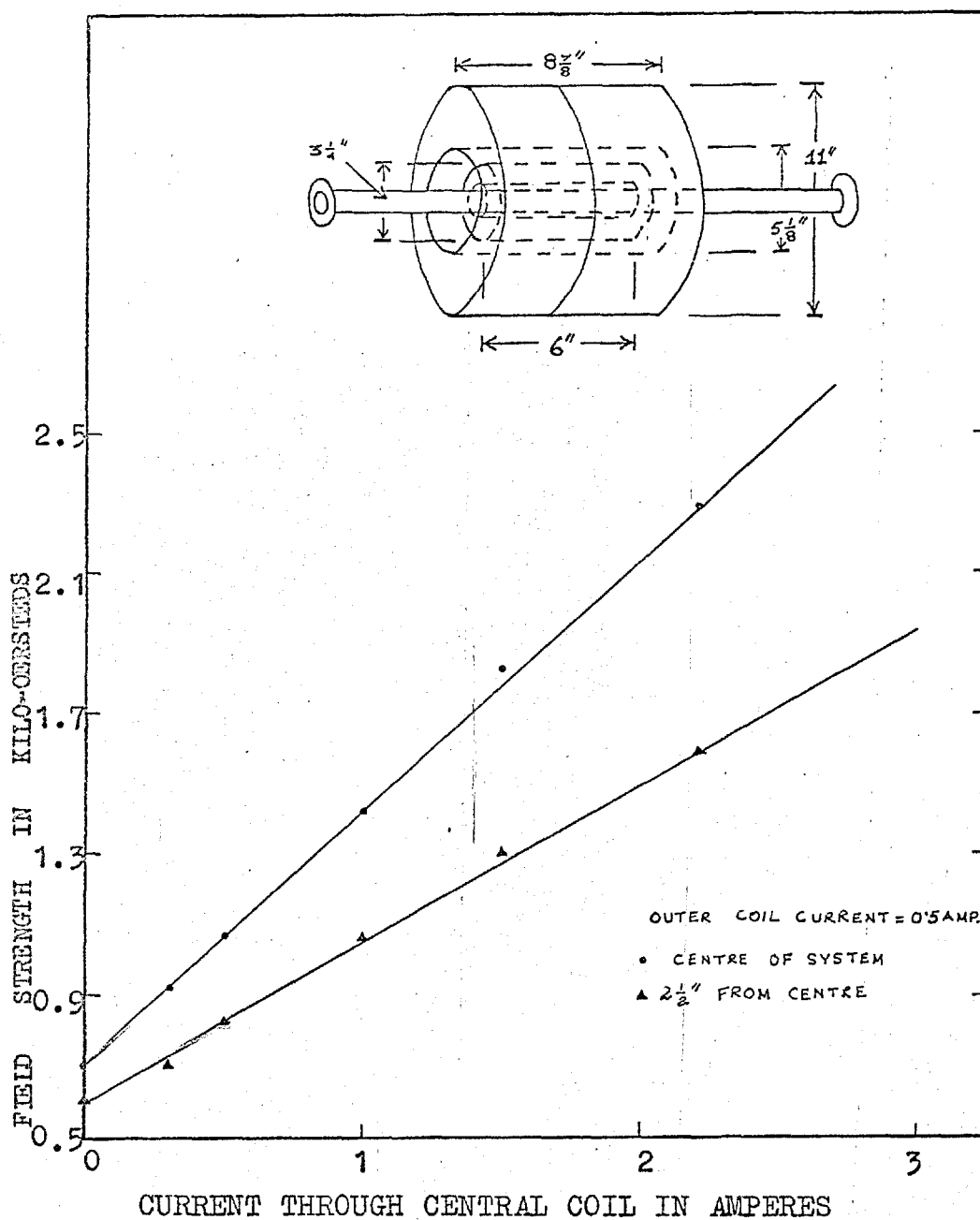


Figure A1.3. Field strength of coil combination used for resonance experiments.

APPENDIX TWO

CALCULATION OF PROPAGATION CONSTANTS

The propagation constants and the ratios of field components in a bounded ferrite rod, as a function of the magnetisation of the ferrite, were calculated with the help of expressions (1.7) and (1.10), respectively, of the phenomenological theory. Various boundary conditions were represented by varying the components ϵ_x and ϵ_y of the permittivity tensor. The results are tabulated below.

In these tables the propagation constant γ is obtained by the relation

$$\gamma_{p,n} = j \frac{\omega}{c} \sqrt{\lambda_{\pm}} \quad (\text{A2.1})$$

Only positive values of μ and k have generally been considered, and ϵ_x and ϵ_y have been represented as multiples of a small scalar quantity ϵ . The term NE in the columns for ratio of the field components indicate that both components are non-existent.

Case 1

Assumed parameters: $\epsilon_x = +15\epsilon$ and $\epsilon_y = +\epsilon$.

$$\therefore \lambda_{\pm} = 8\mu\epsilon \pm \sqrt{49\mu^2\epsilon^2 + 15k^2\epsilon^2}$$

k $\text{in } \mu $	p-mode			n-mode		
	$\lambda_{+ \text{in}} \mu \mathcal{E} $	$\gamma_p \text{in} \frac{\omega}{c} / \mu \mathcal{E} $	E_x/E_y	$\lambda_{- \text{in}} \mu \mathcal{E} $	$\gamma_n \text{in} \frac{\omega}{c} / \mu \mathcal{E} $	E_x/E_y
0	+15.00	+j 3.87	∞	+1.00	+j 1.00	NE
0.5	+15.52	+j 3.94	-j 1.94	+0.48	+j 0.69	+j 0.069
0.9	+15.82	+j 3.98	-j 1.10	+0.18	+j 0.42	+j 0.061
1.0	+16.00	+j 4.00	-j 1.00	0	0	+j 0.067
1.1	+16.19	+j 4.02	-j 0.92	-0.19	+ 0.44	+j 0.072
2.0	+18.44	+j 4.29	-j 0.58	-2.44	+ 1.56	+j 0.12
10.0	+47.36	+j 6.88	-j 0.31	-31.36	+ 5.60	+j 0.22

Case 2

Assumed parameters: $\mathcal{E}_x = +12\mathcal{E}$ and $\mathcal{E}_y = 0$.

$$\therefore \lambda_{\pm} = 6\mu \pm \sqrt{36\mu^2 \mathcal{E}^2}$$

k $\text{in } \mu $	p-mode			n-mode		
	$\lambda_{+ \text{in}} \mu \mathcal{E} $	$\gamma_p \text{in} \frac{\omega}{c} / \mu \mathcal{E} $	E_x/E_y	$\lambda_{- \text{in}} \mu \mathcal{E} $	$\gamma_n \text{in} \frac{\omega}{c} / \mu \mathcal{E} $	E_x/E_y
0	+12.00	+j 3.46	∞	0	0	NE
0.9	+12.00	+j 3.46	-j 1.11	0	0	0
1.0	+12.00	+j 3.46	-j 1.00	0	0	0
10.0	+12.00	+j 3.46	-j 0.10	0	0	0

Case 3

Assumed parameters: $\epsilon_x = +10\epsilon$ and $\epsilon_y = -\epsilon$

$$\therefore \lambda_{\pm} = 4.5 \mu \epsilon \pm \sqrt{30.25 \mu^2 \epsilon^2 - 10 k^2 \epsilon^2}$$

k in $ \mu $	p-mode			n-mode		
	$\lambda_{+in} \mu \epsilon $	$\gamma_p \sin \frac{\omega}{c} / \mu \epsilon $	E_x/E_y	$\lambda_{-in} \mu \epsilon $	$\gamma_n \sin \frac{\omega}{c} / \mu \epsilon $	E_x/E_y
0	+10.00	+j 3.16	∞	-1.00	+1.00	NE
0.5	+9.77	+j 3.13	-j 2.15	-0.77	+0.88	-j 0.046
0.9	+9.21	+j 3.03	-j 1.13	-0.21	+0.46	-j 0.088
1.0	+9.00	+j 3.00	-j 1.00	0	0	-j 0.100
1.1	+8.76	+j 2.96	-j 0.89	+0.24	+j 0.49	-j 0.11
1.5	+7.28	+j 2.70	-j 0.55	+1.72	+j 1.31	-j 0.18
1.739	+4.50	+j 2.12	-j 0.32	+4.50	+j 2.12	-j 0.32
2.0	+4.50 + j 3.12	-0.70 + j 2.23	+5.50 + j 3.12 +j 20.00	+4.50 - j 3.12	+0.70 + j 2.23	+5.50 - j 3.12 +j 20.00
10.0	+4.50 + j 31.14	-3.67 + j 4.24	+5.50 + j 31.14 +j 100.00	+4.50 - j 31.14	+3.67 + j 4.24	+5.50 - j 31.14 +j 100.00

Case 4

Assumed parameters: $\epsilon_x = +6\epsilon$ and $\epsilon_y = -2\epsilon$

$$\therefore \lambda_{\pm} = 2 \mu \epsilon \pm \sqrt{16 \mu^2 \epsilon^2 - 12 k^2 \epsilon^2}$$

k in $ \mu $	p-mode			n-mode		
	$\lambda_{+in} \mu \mathcal{E} $	$\gamma_{pin} \frac{\omega}{c} \sqrt{ \mu \mathcal{E} }$	E_x/E_y	$\lambda_{-in} \mu \mathcal{E} $	$\gamma_{nin} \frac{\omega}{c} \sqrt{ \mu \mathcal{E} }$	E_x/E_y
0	+6.00	+j 2.45	∞	-2.00	+1.41	NE
0.9	+4.50	+j 2.12	-j 1.20	-0.50	+0.71	-j 0.28
1.0	+4.00	+j 2.00	-j 1.00	0	0	-j 0.33
1.1	+3.22	+j 1.79	-j 0.79	+0.78	+j 0.88	-j 0.42
1.150	+2.00	+j 1.41	-j 0.58	+2.00	+j 1.41	-j 0.58
2.0	+2.00 + j 5.66	-1.41 + j 2.00	$\frac{+4.00 + j 5.66}{+j 12.00}$	+2.00 - j 5.66	+1.41 + j 2.00	$\frac{+4.00 - j 5.66}{+j 12.00}$
10.0	+2.00 + j 34.42	-4.03 + j 4.27	$\frac{+4.00 + j 34.42}{+j 60.00}$	+2.00 - j 34.42	+4.03 + j 4.27	$\frac{+4.00 - j 34.42}{+j 60.00}$

Case 5

Assumed parameters: $\mathcal{E}_x = +5\mathcal{E}$ and $\mathcal{E}_y = -5\mathcal{E}$.

$$\therefore \lambda_{\pm} = \pm \sqrt{25 \mu^2 \mathcal{E}^2 - 25 k^2 \mathcal{E}^2}.$$

k in $ \mu $	p-mode			n-mode		
	$\lambda_+ \text{ in } \mu\mathcal{E} $	$\gamma_p \text{ in } \frac{\omega}{c} \sqrt{ \mu\mathcal{E} }$	E_x/E_y	$\lambda_- \text{ in } \mu\mathcal{E} $	$\gamma_n \text{ in } \frac{\omega}{c} \sqrt{ \mu\mathcal{E} }$	E_x/E_y
0	+5.00	+j 2.24	∞	-5.00	+2.24	NE
0.9	+2.18	+j 1.48	-j 1.60	-2.18	+1.48	-j 0.63
1.0	0	0	-j 1.00	0	0	-j 1.00
1.1	+j 2.29	+1.07 - j 1.07	+5.00 + j 2.29 j 5.50	-j 2.29	+1.07 + j 1.07	+5.00 - j 2.29 j 5.50
2.0	+j 8.66	+2.06 - j 2.06	+5.00 + j 8.66 j 10.00	-j 8.66	+2.06 + j 2.06	+5.00 - j 8.66 j 10.00
10.0	+j 49.75	+4.98 - j 4.98	+5.00 + j 49.75 j 50.00	-j 49.75	+4.98 + j 4.98	+5.00 - j 49.75 j 50.00

Case 6

Assumed parameters: $\mathcal{E}_x = +4\mathcal{E}$ and $\mathcal{E}_y = -8\mathcal{E}$.

$$\therefore \lambda_{\pm} = -2\mu\mathcal{E} \pm \sqrt{36\mu^2\mathcal{E}^2 - 32k^2\mathcal{E}^2}$$

k in $ \mu $	p-mode			n-mode		
	$\lambda_{+in} \mu \mathcal{E} $	$\gamma_p \text{in} \frac{\omega}{c} \sqrt{ \mu \mathcal{E} }$	E_x/E_y	$\lambda_{-in} \mu \mathcal{E} $	$\gamma_n \text{in} \frac{\omega}{c} \sqrt{ \mu \mathcal{E} }$	E_x/E_y
0	+4.00	+j 2.00	∞	-8.00	+2.83	NE
0.9	+1.18	+j 1.08	-j 2.55	-5.176	+2.27	-j 0.78
1.0	0	0	-j 2.00	-4.00	+2.00	-j 1.00
1.061	-2.00	+1.41	-j 1.42	-2.00	+1.41	-j 1.42
2.0	-2.00 + j 9.60	+2.68 - j 2.18	$\frac{+6.00 +}{j 9.60}$ +j 8.00	-2.00 - j 9.60	+2.68 + j 2.18	$\frac{+6.00 -}{j 9.60}$ + j 8.00
10.0	-2.00 + j 56.24	+5.40 - j 5.20	$\frac{+6.00 +}{j 56.24}$ +j 40.00	-2.00 - j 56.24	+5.40 + j 5.20	$\frac{+6.00 -}{j 56.24}$ +j 40.00

Case 7

Assumed parameters: $\mathcal{E}_x = 0$ and $\mathcal{E}_y = -12\mathcal{E}$.

$$\therefore \lambda_{\pm} = -6 \mu \mathcal{E} \pm \sqrt{36 \mu^2 \mathcal{E}^2}$$

k in $ \mu $	p-mode			n-mode		
	$\lambda_{+in} \mu \mathcal{E} $	$\gamma_p \text{in} \frac{\omega}{c} \sqrt{ \mu \mathcal{E} }$	E_x/E_y	$\lambda_{-in} \mu \mathcal{E} $	$\gamma_n \text{in} \frac{\omega}{c} \sqrt{ \mu \mathcal{E} }$	E_x/E_y
0	0	0	∞	-12.0	+3.46	NE
0.9	0	0	∞	-12.0	+3.46	NE
1.0	0	0	∞	-12.0	+3.46	NE
10.0	0	0	∞	-12.0	+3.46	NE

Case 8

Assumed parameters: $\epsilon_x = -1.5\epsilon$ and $\epsilon_y = -15\epsilon$.

$$\therefore \lambda_{\pm} = -8.25 \mu \epsilon \pm \sqrt{45.56 \epsilon^2 \mu^2 + 22.5 \epsilon^2 k^2}$$

k in μ	p-mode			n-mode		
	$\lambda_{+ \text{ in } \mu \epsilon}$	$\gamma_p \text{ in } \frac{\omega}{c} / \mu \epsilon $	E_x/E_y	$\lambda_{- \text{ in } \mu \epsilon}$	$\gamma_n \text{ in } \frac{\omega}{c} / \mu \epsilon $	E_x/E_y
0	-1.50	+1.23	∞	-15.00	+3.87	NE
0.9	-0.30	+0.55	+j 10.89	-16.20	+4.03	-j 0.89
1.0	0	0	+j 10.00	-16.50	+4.06	-j 1.00
1.1	+0.29	-j 0.53	+j 9.26	-16.80	+4.10	-j 1.09
2.0	+3.39	-j 1.84	+j 6.13	-19.90	+4.46	-j 1.65
10.0	+39.66	-j 6.30	+j 3.64	-56.20	+7.50	-j 2.74

Case 9

Assumed parameters: $\epsilon_x = -20\epsilon$ and $\epsilon_y = -20\epsilon$.

$$\therefore \lambda_{\pm} = -20 \mu \epsilon \pm \sqrt{400 k^2 \epsilon^2}$$

k in $ \mu $	p-mode			n-mode		
	$\lambda_+ \text{ in } \mu E $	$\gamma_p \text{ in } \frac{\omega}{c} / \mu E $	E_x/E_y	$\lambda_- \text{ in } \mu E $	$\gamma_n \text{ in } \frac{\omega}{c} / \mu E $	E_x/E_y
0	-20.00	+4.47	NE	-20.00	+4.47	NE
0.5	-10.00	+3.16	+j 1.00	-30.00	+5.48	-j 1.00
0.9	-2.00	+1.41	+j 1.00	-38.00	+6.16	-j 1.00
1.0	0	0	+j 1.00	-40.00	+6.32	-j 1.00
1.1	+2.00	-j 1.41	+j 1.00	-44.00	+6.63	-j 1.00
2.0	+20.00	-j 4.47	+j 1.00	-60.00	+7.75	-j 1.00
10.0	+180.00	-j 13.42	+j 1.00	-220.00	+14.83	-j 1.00
-10.0	+220.00	-j 14.83	+j 1.00	-180.00	+13.42	-j 1.00
-2.0	+60.00	-j 7.75	+j 1.00	-20.00	+4.47	-j 1.00
-1.0	+40.00	-j 6.32	+j 1.00	0	0	-j 1.00
-0.5	+30.00	-j 5.48	+j 1.00	+10.00	-j 3.16	-j 1.00

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