LOCAL QUANTUM FIELD THEORY OF ELECTRIC CHARGES AND MAGNETIC MONOPOLES

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ABSTRACT

The local quantum field theory of electric charges and magnetic monopoles is considered. To arrive at the quantum field theory, we first review the relativistic classical particle theory of electric charges and magnetic monopoles, the introduction of magnetic charge into quantum mechanics, and the classical theory of electrically and magnetically charged fields. The Lorentz invariance of the bare quantum field theory is discussed. We then investigate the existence of a renormalized finite and Lorentz invariant quantum field theory. The abstract discussion of the general case is illustrated by the explicit application of the renormalization scheme that we propose to the quantum electromagnetodynamics of a single dyon field. We also investigate the possibility of incorporating point monopoles into the unified theory of electroweak interactions. Finally we give a supersymmetric extension of Zwanziger's quantum field theory.

TO THE GIRL WHO FORGOT HER NAME

FOREWORD

The present thesis is submitted by the author in fulfillment of the requirements for the Ph.D degree of the University of London and the DIC degree of Imperial College. It is mainly based on the content of three papers written by the author (references [6], [7], and [8]) and one paper (reference [9]) in collaboration with Abdus Salam and John Strathdee. I would like to take this opportunity in order to thank my advisor, Professor Abdus Salam, for his interest in my work and his continuous friendly advice (which unfortunately I did not follow all the time), and Professor John Strathdee for introducing me to the field theory of monopoles, for his continuous guidance and for the security that his presence provided to me.

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I INTRODUCTION

In a historic paper, in 1931, Dirac [1], attempting to determine the fine structure constant, introduced magnetic monopoles into quantum mechanics. It turned out that the consistency of the quantum theory required, instead of a complete determination of the fundamental electric charge, only a certain relation between the electric and magnetic charges. This relation implied that, if a single monopole of strength g existed, all electric charges should be integral multiples of a unit charge $\frac{2\pi}{g}$. However, in spite of numerous searches, no physical monopoles have been observed. It has been known for a long time that both the classical relativistic particle theory [2] and the nonrelativistic quantum mechanical theory [1] were consistent. It could be, however, that it is impossible to formulate a consistent relativistic quantum field theory of point-like electric and magnetic charges and therefore to explain, in this way, the absence of physical magnetic monopoles.

In 1947, Dirac [2] presented a relativistic quantum field theory of electric charges and magnetic monopoles with dynamical variables describing the motion of the "strings". In 1966 Schwinger [3] developed a non-local Hamiltonian density for the interaction of charges and monopoles with the electromagnetic field, free of string variables. Finally, a few years later, Zwanziger [4] presented his local Lagrangian field theory of charges and monopoles involving a pair of electric and magnetic potentials, interacting with matter fields, and depending on a fixed direction n_μ. This last dependence has been the source of many difficulties, because it violates explicitly Lorentz invariance, at the Lagrangian level at least, and complicates enormously any discussion concerning the renormalizability of Zwanziger's theory.

The next important development has been the formal demonstration, by Brandt, Neri and Zwanziger [5], of the restoration of Lorentz invariance in the full Green functions of the gauge invariant electric and magnetic currents. Based on this work the present author initiated a programme of regularization [6] and renormalization [7] of Zwanziger's theory, with emphasis on the Lorentz invariance of the renormalized theory. This is the subject of the present thesis. The discussion of the renormalization assumes the restoration of Lorentz invariance which does not occur perturbatively and therefore the theory is not expected to be renormalizable in the conventional purturbative sense. This unfortunate situation becomes less disturbing if one remembers that the theory necessarily contains large coupling constants due to the quantization condition imposed on the electric and magnetic charges.

Chapter II is a review whose purpose is to bring the reader to the point of knowledge at which the author's research started. It contains the classical relativistic particle theory of electric charges and magnetic monopoles, the introduction of magnetic monopoles in quantum mechanics, a brief description of the (inconsistent) classical field theory of electric and magnetic charges, and their local relativistic quantum field theory. Just before the fundamental formal proof of Lorentz invariance of the full Green functions of the electric and magnetic currents we give the derivation of some useful path integral representations of matrix elements of certain unitary operators.

Chapter III contains the regulators invented by the author [6] whose useful property is to regularize the quantum field theory of electric and magnetic charges in such a way that the proof of Lorentz invariance given in chapter II goes through in their presence.

Chapter IV contains an abstract discussion of the renormalization of Zwanziger's theory.

Chapter V illustrates the discussion of the renormalization in the context of a special and simple case, where Lorentz invariance and renormalizability can be established purturbatively. This is the case of a single dyon field (i.e. a field carrying electric and magnetic charge) [8].

Chapter VI discusses the introduction of magnetic charge into the unified theory of electroweak interactions.

Finally, in chapter VII, we present a supersymmetric field theory of charges and monopoles [9].

The notation, often, suppresses Lorentz indices, i.e. the scalar product of two 4-vectors a_μ and b_μ is denoted as $a\cdot b$, $a\wedge b$ means $a_\mu b_\nu - a_\nu b_\mu$ and for a second rank tensor $F_{\mu\nu}$ the symbol $F\cdot F$ means $F_{\mu\nu}F^{\mu\nu}$.

II A REVIEW

1.Classical monopole theory

The generalized Maxwell equations describing the classical theory of electric and magnetic charges are

$$\partial \cdot F = J \qquad (II\ 1.1)$$

$$\partial \cdot F^{d} = K \qquad (II\ 1.2)$$

where F is the electromagnetic field strength tensor, F^{d} its dual, defined by

$$F^{d}_{\mu\nu} = \tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma} \qquad (II\ 1.3)$$

and J and K the electric and magnetic current 4-vectors respectively, defined by

$$J(x) = \sum_i e_i \int_{\Gamma_i} dz_i\, \delta^4(x - z_i) \qquad (II\ 1.4)$$

$$K(x) = \sum_i g_i \int_{\Gamma_i} dz_i\, \delta^4(x - z_i) \qquad (II\ 1.5)$$

Here $\epsilon_{\mu\nu\rho\sigma}$ is the Levi-Civita tensor, $\Gamma_i = \{z_{i\mu}(s) : -\infty \leq s \leq \infty\}$ is the one dimensional trajectory of the i-th particle corresponding to point-particle position vector $z_i(s)$, s being a proper-time scalar variable, and $e_i(g_i)$ is the electric(magnetic) charge of the i-th particle, having mass m_i.

The classical particle theory is completed with the generalized Lorentz force laws

$$m_i z_i'' = [e_i F(z_i) + g_i F^{d}(z_i)] \cdot z_i' \qquad (II\ 1.6)$$

where $z_i'(s) = \frac{dz_i}{ds}$ and $z_i'' = \frac{d^2 z_i}{ds^2}$.

This classical theory of electric charges and magnetic monopoles is complete by itself. Nevertheless, since we have in mind to quantize it eventually, we need an action involving some sort of

vector potentials, which will yield these equations upon variation of the defferent independent quantities entering it.

Following Zwanziger [4] we observe that eq.(II 1.2) has a particular solution of the form $F^d=(n\cdot\partial)^{-1}(n\wedge K)$, where n is an arbitrary fixed 4-vector and $(n\cdot\partial)^{-1}$ is an integral operator with kernel $(n\cdot\partial)^{-1}(x-y)$ satisfying the relation $n\cdot\partial(n\cdot\partial)^{-1}(x)=\delta^4(x)$. The general solution to eq.(II 1.2) may be written

$$F^d=(\partial\wedge A)^d+(n\cdot\partial)^{-1}(n\wedge K) \qquad (\text{II } 1.7)$$

or

$$F=(\partial\wedge A)-(n\cdot\partial)^{-1}(n\wedge K)^d \qquad (\text{II } 1.8)$$

where A_μ is a 4-potential which depends on the choice of gauge, the choice of n, and the specific form of the operator $(n\cdot\partial)^{-1}$. Similarly, the general solution to eq.(II 1.1) is

$$F=-(\partial\wedge B)+(n\cdot\partial)^{-1}(n\wedge J) \qquad (\text{II } 1.9)$$

or

$$F^d=(\partial\wedge B)+(n\cdot\partial)^{-1}(n\wedge J)^d \qquad (\text{II } 1.10)$$

where B_μ is another 4-potential. We will choose $(n\cdot\partial)^{-1}$ to be the integral kernel

$$(n\cdot\partial)^{-1}(x)=\tfrac{1}{2}\left[\int_0^\infty ds\,\delta^4(x-ns)-\int_0^\infty ds\,\delta^4(x+ns)\right] \qquad (\text{II } 1.11)$$

We may express F locally in terms of the potentials alone. Observe that any antisymmetric tensor G satisfies the identity

$$G=\frac{1}{n^2}\left\{[n\wedge(n\cdot G)]-[n\wedge(n\cdot G^d)]^d\right\}, \qquad (\text{II } 1.12)$$

since both left- and right-hand sides give the same value for $n\cdot G^d, n\cdot G$. From eqs.(II 1.8) and (II 1.10), we have

$$n\cdot F=n\cdot(\partial\wedge A) \qquad (\text{II } 1.13)$$

$$n\cdot F^d=n\cdot(\partial\wedge B) \qquad (\text{II } 1.14)$$

which yield the desired expression for F:

$$F=\frac{1}{\eta^2}\left(\left\{n\Lambda[n\cdot(\partial\Lambda A)]\right\}-\left\{n\Lambda[n\cdot(\partial\Lambda B)]\right\}^{d}\right) \qquad (\text{II } 1.15)$$

$$F^{d}=\frac{1}{\eta^2}\left(\left\{n\Lambda[n\cdot(\partial\Lambda A)]\right\}^{d}+\left\{n\Lambda[n\cdot(\partial\Lambda B)]\right\}\right) \qquad (\text{II } 1.16)$$

We substitute eqs.(II 1.15) and (II 1.16) into eqs.(II 1.1) and (II 1.2) to obtain Maxwell's equations in terms of the potentials:

$$\frac{1}{\eta^2}(n\cdot\partial\, n\cdot\partial A^{\mu}-n\cdot\partial\,\partial^{\mu}n\cdot A-n^{\mu}n\cdot\partial\,\partial\cdot A+n^{\mu}\partial^{2}n\cdot A-n\cdot\partial\,\epsilon^{\mu}{}_{\nu\kappa\lambda}n^{\nu}\partial^{\kappa}B^{\lambda}) = J^{\mu}, \qquad (\text{II } 1.17)$$

$$\frac{1}{\eta^2}(n\cdot\partial\, n\cdot\partial B^{\mu}-n\cdot\partial\,\partial^{\mu}n\cdot B-n^{\mu}n\cdot\partial\,\partial\cdot B+n^{\mu}\partial^{2}n\cdot B+n\cdot\partial\,\epsilon^{\mu}{}_{\nu\kappa\lambda}n^{\nu}\partial^{\kappa}A^{\lambda}) = K^{\mu}. \qquad (\text{II } 1.18)$$

Therefore corresponding to any solution F to Maxwell's equations there exist potentials A and B related to F by (II 1.15) and satisfying (II 1.17) and (II 1.18). Conversely, any pair of potentials satisfying (II 1.17) and (II 1.18) defines a unique solution F to Maxwell's equations given by (II 1.15). This correspondence is not one-to-one however.

The equations of motion (II 1.17) and (II 1.18) follow from the local classical action

$$S = S_1 + S_2 + S_3 \qquad (\text{II } 1.19)$$

$$S_1 = -\sum_i m_i \int_{\Gamma_i} ds \qquad (\text{II } 1.20)$$

$$S_2 = -\int d^4x \frac{1}{2\eta^2}\Big\{[n\cdot(\partial\Lambda A)]\cdot[n\cdot(\partial\Lambda B)^{d}] - [n\cdot(\partial\Lambda B)]\cdot[n\cdot(\partial\Lambda A)^{d}] + [n\cdot(\partial\Lambda A)]^2+[n\cdot(\partial\Lambda B)]^2\Big\} \qquad (\text{II } 1.21)$$

$$S_3 = -\int d^4x \ (J\cdot A + K\cdot B) \qquad (\text{II } 1.22)$$

upon independent variation of A and B. This manifestly local action is due to Zwanziger [4]. S_2 can be written in a slightly different form

$$S_2 = -\int d^4x \left\{ \frac{1}{8}[(\partial\wedge A)^2 + (\partial\wedge B)^2] + \frac{1}{4n^2}\{n\cdot[(\partial\wedge A)+(\partial\wedge B)]\}^2 \right.$$
$$\left. + \frac{1}{4n^2}\{n\cdot[(\partial\wedge B)-(\partial\wedge A)^d]\}^2 \right\} \qquad (\text{II } 1.23)$$

with use of the identity

$$G^2 = G_{\mu\nu}G^{\mu\nu} = \left(\frac{2}{n^2}\right)[(n\cdot G)^2 - (n\cdot G^d)^2] \qquad (\text{II } 1.24)$$

If this local classical action is the correct one, it should better give also, upon variation of the particle trajectories, the correct generalized Lorentz force laws (II 1.6). However, as we will see immediately, this is not the case. Independent variation of the particle trajectories gives

$$\delta S_1 = +\sum_i m_i \int_{\Gamma_i} ds \frac{d^2 z_\mu^i}{ds^2} \delta z^{i\mu} \qquad (\text{II } 1.25)$$

and

$$\delta S_3 = -\delta\left(\sum_i [e_i \int_{\Gamma_i} dz_\mu^i A^\mu(z^i) + g_i \int_{\Gamma_i} dz_\mu^i B^\mu(z^i)]\right) =$$
$$= -\sum_i \int_{\Gamma_i} ds \delta z_i \cdot [e_i (\partial\wedge A)(z_i) + g_i (\partial\wedge B)(z_i)] \frac{dz_i}{ds}. \qquad (\text{II } 1.26)$$

Requiring that $\delta S/\delta z^i = 0$ we obtain the equations of motion

$$m_i z_i'' = [e_i \partial\wedge A(z_i) + g_i \partial\wedge B(z_i)]\cdot z_i' \qquad (\text{II } 1.27)$$

instead of (II 1.6).

We observe that we would have obtained the correct generalized Lorentz force laws (II 1.6) if in eq.(II 1.26), instead of $\partial\wedge A$ and $\partial\wedge B$, we had $\partial\wedge A - (n\cdot\partial)^{-1}(n\wedge K)^d$ and $\partial\wedge B + (n\cdot\partial)^{-1}(n\wedge J)^d$ respectively. We will attempt therefore to find the modified form of S_3 whose variation will give the desired result. The replacement of $\partial\wedge A$

by $\partial\wedge A-(n\cdot\partial)^{-1}(n\wedge K)^d$ in (II 1.26) is equivalent to the replacement of A by $A+\partial\lambda$ where λ is a singular scalar function satisfying the constraint

$$\partial\wedge\partial\lambda(z_i)=-(n\cdot\partial)^{-1}(n\wedge K)^d=+\sum_j\frac{g_j}{2}\int_{-\infty}^{0}ds\int_{\Gamma_j}(n\wedge dz_j)^d\delta^4(z_i-ns-z_j)$$
$$-\sum_j\frac{g_j}{2}\int_0^{+\infty}ds\int_{\Gamma_j}(n\wedge dz_j)^d\delta^4(z_i-ns-z_j)\quad(\text{II }1.28)$$

Similarly the replacement of $\partial\wedge B$ by $\partial\wedge B+(n\cdot\partial)^{-1}(n\wedge J)^d$ in (II 1.26) is equivalent to the replacement of B by $B+\partial\bar{\lambda}$ with $\bar{\lambda}$ a singular scalar function satisfying the constraint

$$\partial\wedge\partial\bar{\lambda}(z_i)=(n\cdot\partial)^{-1}(n\wedge J)^d=+\sum_j\frac{e_j}{2}\int_0^{+\infty}ds\int_{\Gamma_j}(n\wedge dz_j)^d\delta^4(z_i-ns-z_j)$$
$$-\sum_j\frac{e_j}{2}\int_{-\infty}^{0}ds\int_{\Gamma_j}(n\wedge dz_j)^d\delta^4(z_i-ns-z_j)\quad(\text{II }1.29)$$

It is evident that $\partial\wedge\partial\lambda(\partial\wedge\partial\bar{\lambda})$ has its support on ''strings'' $x=z_j\pm ns$ emanating from the magnetically(electrically) charged particles. Dirac [2], in order to obtain in his formulation the correct equations of motion, imposed the "veto" that a charged-particle trajectory must never pass through a "string". In the following, instead of imposing the Dirac "veto", we will try to appropriately modify S_3 in order to get the correct equations of motion.

In S_3 we replace A by $A+\partial\lambda$ and B by $B+\partial\bar{\lambda}$. Let us denote the change in S_3 by ΔS_3. We have

$$\Delta S_3=-\sum_i e_i\int_{\Gamma_i}dz_i\cdot\partial\lambda(z_i)+\sum_i g_i\int_{\Gamma_i}dz_i\cdot\partial\bar{\lambda}(z_i)\quad(\text{II }1.30)$$

If N_{ij} is the number of times the i-th particle hits the string emanating from the j-th particle, using the information concerning the discontinuity of λ and $\bar{\lambda}$ provided by eqs.(II 1.28) and (II 1.29) and assuming a closed trajectory, we can evaluate ΔS_3. The answer is

$$\Delta S_3=\sum_{ij}\frac{e_ig_j-e_jg_i}{2}N_{ij}\quad(\text{II }1.31)$$

This answer suggests the remedy to our problem. If we want to get from our action, upon variation of the particle trajectories, the correct generalized Lorentz force laws, we must make sure that the addition of ΔS_3, as given by eq.(II 1.31), to the action leads to the same physics no matter what the values of the integers N_{ij} are. This is equivalent to declaring that our action is defined only modulo V where

$$V \equiv \underset{i,j}{\text{minimum}} \left| \frac{e_i g_j - e_j g_i}{2} \right| . \qquad (\text{II } 1.32)$$

If we choose $(n\cdot\partial)^{-1}$ to be the integral kernel

$$(n\cdot\partial)^{-1}(x) = \int_0^\infty ds\, \delta^4(x-ns) \qquad (\text{II } 1.33)$$

instead of the definition in eq.(II 1.11), and no dyons are considered, the formula (II 1.32) is replaced by

$$V \equiv \underset{i,j}{\text{minimum}} \left| e_i g_j \right| \qquad (\text{II } 1.34)$$

This way the action maps into the circle of circumference V instead of into the real line.

The observation that S_3 should be defined only modulo V is due to Wu and Yang [10] who pointed out that this is the only way to make the action a continuous functional of the charged particle trajectories. In the framework of Dirac formulation it has been exploited by Brandt and Primack [11] in order to obtain the correct equations of motion variationally. Their prescription is to replace in S_3 the particle trajectories which intersect the string by a new trajectory which is the same as the old one except that it makes an infinitesimal contour around the string. The fact that such contours are equivalent modulo V necessitates the definition of S_3 modulo V.

A final remark concerning the classical local Zwanziger action is in order; it possesses the "duality" symmetry:

$$(F,F^d) \rightarrow (F\cos\vartheta - F^d\sin\vartheta, F\sin\vartheta + F^d\cos\vartheta) \qquad \text{(II 1.35)}$$

$$(J,K) \rightarrow (J\cos\vartheta - K\sin\vartheta, J\sin\vartheta + K\cos\vartheta) \ . \qquad \text{(II 1.36)}$$

Besides the classical local Zwanziger action we will describe another classical action which, although non local, it will be of use later. This action is due to Yan and is based on Schwinger's first-order non-local formulation of monopole theory [12].

The Schwinger-Yan action is essentially

$$I = I_1 + I_1 + I_3 \qquad \text{(II 1.37)}$$

$$I_1 = S_1 \qquad \text{(II 1.38)}$$

$$I_2 = -\int d^4x \left[\tfrac{1}{2} F \cdot (\partial \wedge A) - \tfrac{1}{4} F^2 \right] \qquad \text{(II 1.39)}$$

$$I_3 = -\int d^4x \left[J \cdot A + K \cdot \bar{B} \right] \qquad \text{(II 1.40)}$$

where

$$\bar{B}(x) = \int d^4x' F^d(x') \ f(x'-x) \qquad \text{(II 1.41)}$$

with $f_\mu(x)$ any vector function satisfying

$$\partial \cdot f(x) = \delta^4(x) \qquad \text{(II 1.42)}$$

We will make the choice

$$f_\mu(x) = n_\mu (n \cdot \partial)^{-1}(x) \qquad \text{(II 1.43)}$$

In this action A and F are independent variables. Variation of A gives (II 1.1) and variation of F gives

$$F(x) = \partial \wedge A(x) - \int d^4x' \left[f(x-x') \wedge K(x') \right]^d \qquad \text{(II 1.44)}$$

With the choice (II 1.43) for f(x) this equation is immediately seen to agree with equation (II 1.8). Using the form (II 1.44) for F and the conservation of K that follows from its definition, we readily derive eq.(II 1.2). However the independent variation of the particle trajectories, instead of giving the generalized

Lorentz force laws (eq.(II 1.6)) gives the equation

$$m_i z_i'' = [e_i(\partial \wedge A) + g_i(\partial \wedge \bar{B})] \cdot z_i' \qquad \text{(II 1.45)}$$

which differs from eq.(II 1.6) when a particle hits a string. The situation is similar to the one in the Zwanziger case and the solution is the same. Redefining the Schwinger-Yan action in such a way that it maps into a circumference of length V, we get the correct generalized Lorentz force laws

The two actions given above are completely equivalent, at the classical level at least, since they lead to the same Maxwell equations and generalized Lorentz force laws, which provide a complete description of the classical system. The equivalence between them can be made more explicit [13] by giving the transformation that takes the one into the other, with the specific choice of the operator $(n\cdot\partial)^{-1}(x)$ and the function f(x) made above. Substituting the expression for F given in (II 1.15) into I_2 we have

$$I_2 \rightarrow -\int d^4x \left[\tfrac{1}{2} \left\{ n \wedge [n \cdot (\partial \wedge A)] - \left\{ n \wedge [n \cdot (\partial \wedge B)] \right\}^d \right\} \cdot (\partial \wedge A) \right.$$
$$\left. - \tfrac{1}{2} \left\{ n \wedge [n \cdot (\partial \wedge A)] - \left\{ n \wedge [n \cdot (\partial \wedge B)] \right\}^d \right\}^2 \right] \qquad \text{(II 1.46)}$$

We will evaluate this expression taking $n^2=1$. The first term in the integrand is equal to

$$-[n \cdot (\partial \wedge A)]^2 + [n \cdot (\partial \wedge B)] \cdot [n \cdot (\partial \wedge A)^d] \qquad \text{(II 1.47)}$$

The second term, with the use of the identity (II 1.24) can be easily seen to be equal to

$$\tfrac{1}{2}[n \cdot (\partial \wedge A)] - \tfrac{1}{2}[n \cdot (\partial \wedge B)]^2 \qquad \text{(II 1.48)}$$

Consider now the integral

$$\int d^4x [n \cdot (\partial \wedge B)] \cdot [n \cdot (\partial \wedge A)^d] \qquad \text{(II 1.49)}$$

We have

$$\int d^4x\,[n\cdot(\partial\wedge B)]\cdot[n\cdot(\partial\wedge A)^d] = \int d^4x\,\tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}n_\rho[n_\kappa(\partial\wedge B)_{\kappa\sigma}](\partial\wedge A)_{\mu\nu}=$$

$$= \int d^4x\,\tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}n_\rho(n\cdot\partial)B_\sigma(\partial\wedge A)_{\mu\nu} = -\int d^4x\,\tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}n_\rho B_\sigma(n\cdot\partial)(\partial\wedge A)_{\mu\nu}=$$

$$=-\int d^4x\,\tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}n_\rho B_\sigma(n\cdot\partial)\partial_\mu A_\nu=\int d^4x\,\tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}n_\rho\partial_\mu B_\sigma(n\cdot\partial)A_\nu=$$

$$= \int d^4x\,\tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}n_\rho\partial_\mu B_\sigma n_\kappa(\partial_\kappa A_\nu-\partial_\nu A_\kappa) =$$

$$=-\int d^4x\,\tfrac{1}{2}n_\rho\epsilon_{\rho\nu\mu\sigma}(\partial_\mu B_\sigma-\partial_\sigma B_\mu)n_\kappa(\partial_\kappa A_\nu-\partial_\nu A_\kappa)=$$

$$= -\int d^4x\,[n\cdot(\partial\wedge A)]\cdot[n\cdot(\partial\wedge B)^d] \qquad \text{(II 1.50)}$$

Combining eqs.(II 1.47),(II 1.48) and (II 1.50) we get

$$-\int d^4x\,[\tfrac{1}{2}\{n\wedge[n\cdot(\partial\wedge A)]-\{n\wedge[n\cdot(\partial\wedge B)]\}^d\cdot(\partial\wedge A)\}$$

$$-\tfrac{1}{2}\{n\wedge[n\cdot(\partial\wedge A)]-\{n\wedge[n\cdot(\partial\wedge B)]\}^d\}^2]=$$

$$=-\tfrac{1}{2}\int d^4x\,\{[n\cdot(\partial\wedge A)]^2+[n\cdot(\partial\wedge B)]^2+[n\cdot(\partial\wedge A)]\cdot[n\cdot(\partial\wedge B)^d]-[n(\partial\wedge B)\cdot]\cdot[n(\partial\wedge A)^d]\} \qquad \text{(II 1.51)}$$

which is exactly S_2 in eq.(II 1.21). Also substituting the expression for F given in (II 1.15) in eq.(II 1.41) we get

$$\bar{B}_\mu=(n\cdot\partial)^{-1}(n\cdot F^d)_\mu=(n\cdot\partial)^{-1}n\cdot(\partial\wedge B)_\mu=(n\partial)^{-1}[(n\cdot\partial)B_\mu-\partial_\mu(nB)]=$$

$$=B_\mu-\partial_\mu(n\cdot\partial)^{-1}(n\cdot B) \qquad \text{(II 1.52)}$$

which is equal to B_μ up to a gauge transformation.

An interesting property of the Schwinger-Yan action, for closed trajectories, is its invariance under a combined string rotation and a singular gauge transformation [13]. More specifically consider the transformation

$$A\to A+\partial\lambda\,,\quad \bar{B}\to\bar{B}'=(n'\cdot\partial)^{-1}n'\cdot F^d,\quad F\to F,\ \Gamma_i\to\Gamma_i \qquad \text{(II 1.53)}$$

with the function $\lambda(x)$ satisfying the condition

$$\partial\wedge\partial\lambda=\left\{[(n'\cdot\partial)^{-1}n'-(n\cdot\partial)^{-1}n]\wedge K\right\}^{d} \qquad \text{(II 1.54)}$$

The change in I_2 induced by this transformation is

$$\delta I_2=-\int d^4x\,\tfrac{1}{2}F\cdot(\partial\wedge\partial\lambda)=-\int d^4x\,\tfrac{1}{2}F^{d}\left\{[(n'\cdot\partial)^{-1}n'-(n\cdot\partial)^{-1}n]\wedge K\right.=$$

$$=\int d^4x\left\{[(n'\cdot\partial)^{-1}n'\cdot F^{d}]\cdot K-[(n\cdot\partial)^{-1}n\cdot F^{d}]\cdot K\right\}=$$

$$=\int d^4x\,\bar{B}'\cdot K-\int d^4x\,\bar{B}\cdot K \qquad \text{(II 1.55)}$$

The change in I_3 with the choice (II 1.11) for $(n\cdot\partial)^{-1}$ is

$$\delta I_3=\sum_{i,j}\frac{e_i\vartheta_j-e_j\vartheta_i}{2}N_{ij}-\int d^4x\,\bar{B}'\cdot K+\int d^4x\,\bar{B}\cdot K \qquad \text{(II 1.56)}$$

with N_{ij} integers, by an argument analogous to the one employed in the evaluation of ΔS_3 in eq.(II 1.30), using the condition (II 1.54). Therefore

$$\delta I=\sum_{i,j}\frac{e_i\vartheta_j-e_j\vartheta_i}{2}N_{ij} \qquad \text{(II 1.57)}$$

which proves the invariance of the action under the transformation (II 1.53), since I is defined only modulo V. The transformation (II 1.54) is equivalent to a singular gauge transformation combined with a Lorentz transformation, namely the inverse of the transformation that moves the "string" n_μ.

For historical reasons we will also give the classical action introduced by Dirac [2]. It involves the electromagnetic vector potential A_μ and strings $y_\mu^j(s,\tau)$ $(-\infty\le s\le\infty\ ,0\le\tau\le\infty)$ extending from each monopole trajectory ($\tau=0$) to infinity ($\tau=\infty$). The electromagnetic field is given as a sum of two terms

$$F_{\mu\nu}=(\partial\wedge A)_{\mu\nu}+\sum_j G^{dj}_{\mu\nu} \qquad \text{(II 1.58)}$$

The first term which is the curl of the vector potential, effectively describes the electromagnetic field of the electric charges together with a number of infinitely thin solenoids, lying along the

strings. The second term, given in terms of

$$G^{j}_{\mu\nu} = g_j \iint ds\, d\tau\, \delta^4(x-y^j)\left(\frac{\partial y^j}{\partial s} \wedge \frac{\partial y^j}{\partial \tau}\right)_{\mu\nu} \qquad \text{(II 1.59)}$$

removes the solenoids.

Dirac's action is given by

$$W = W_1 + W_2 + W_3 \qquad \text{(II 1.60)}$$

$$W_1 = -\sum_i m_i \int_{\Gamma_i} ds \qquad \text{(II 1.61)}$$

$$W_2 = -\tfrac{1}{4}\int d^4x\, F^2 \qquad \text{(II 1.62)}$$

$$W_3 = -\int d^4x\, J\cdot A \qquad \text{(II 1.63)}$$

Variation of the trajectories and the vector potential gives

$$\delta W_1 = \sum_i m_i \int_{\Gamma_i} ds \frac{d^2 z^i}{ds^2}\cdot \delta z^i \qquad \text{(II 1.64)}$$

$$\delta W_2 = -\int d^4x\, \partial_\nu F_{\mu\nu}\delta A_\mu - \sum_j g_j \int_{\Gamma_j} dz^j_\nu F^{ext}_{\mu\nu}\delta z^j_\mu \qquad \text{(II 1.65)}$$

$$\delta W_3 = -\sum_i e_i \int_{\Gamma_i} dz^i_\nu \left[(\partial_\mu A_\nu - \partial_\nu A_\mu)\delta z^i_\mu + \delta A_\nu\right] \qquad \text{(II 1.66)}$$

The principle of least action with respect to these variations, together with eqs.(II 1.58) and (II 1.59),lead to all the equations of motion except the generalized Lorentz force law. This can only be obtained if the action is redefined such that it maps into the circle of circumference V.

A direct connection between the Schwinger-Yan form of the action and the Dirac action can be established. Choose for the operator $(n\cdot\partial)^{-1}$ the form (II 1.33) instead of (II 1.11), and for the string the specific form

$$y_j(s,\tau) = z_j(s) + \tau n. \qquad \text{(II 1.67)}$$

With these choices

$$\sum_j G^j = (n\cdot\partial)^{-1} K\wedge n \qquad \text{(II 1.68)}$$

Substitute for F in the Schwinger-Yan action the form (II 1.44) which is exactly the same with (II 1.58) given eq.(II 1.68). We have

$$-\int d^4x\ \bar{B}\cdot K = -\int d^4x\,[(n\partial)^{-1} nF^d]K = \tfrac{1}{2}\int d^4x F\cdot(n\partial)^{-1}(n\wedge K)^d$$

Next observe that

$$-\int d^4x \tfrac{1}{2}F\cdot(\partial\wedge A) - \int d^4x \bar{B}K = -\tfrac{1}{2}\int d^4x F[\partial\wedge A - (n\partial)^{-1}(n\wedge K)^d] =$$

$$= -\tfrac{1}{2}\int d^4x F^2 \qquad \text{(II 1.69)}$$

Therefore

$$I_2 + I_3 \to -\int d^4x \tfrac{1}{2}F^2 - \int d^4x JA = W_2 + W_3 \qquad \text{(II 1.70)}$$

which means that, with the choice (II 1.33) for $(n\cdot\partial)^{-1}$ and (II 1.67) for the Dirac string, the transformation

$$F \to \partial\wedge A - (n\partial)^{-1}(n\wedge K)^d \ , \ A \to A \ , \ \Gamma_i \to \Gamma_i \qquad \text{(II 1.71)}$$

maps the Schwinger-Yan action into the Dirac action [13].

For reasons completely obscure for the moment, we can consider a generalization of these classical particle theories in which there is a contribution from magnetic and electric moments to the classical currents:

$$J_\mu(x) = \sum_i \int_{-\infty}^{+\infty} ds\,[e_i z_{i\mu}'(s) + \sigma_{i\mu\nu}(s)\partial_\nu]\,\delta^4(x - z_i(s)) \qquad \text{(II 1.72)}$$

$$K_\mu(x) = \sum_i \int_{-\infty}^{+\infty} ds\,[g_i z_{i\mu}'(s) + \lambda_{i\mu\nu}(s)\partial_\nu]\,\delta^4(x - z_i(s)) \qquad \text{(II 1.73)}$$

Now let us apply the combined string rotation and singular gauge transformation (II 1.53), (II 1.54) to the modified classical particle action. The additional change in the action, due to this transformation, is

$$\sum_i [\tfrac{1}{2}\int ds\ \sigma_{i\mu\nu}(s)(\partial\wedge\lambda)_{\mu\nu} + \tfrac{1}{2}\int ds\,\lambda_{i\mu\nu}(s)(\partial\wedge\delta\bar{B})_{\mu\nu}] =$$

$$=\sum_i [\tfrac{1}{2}\int ds\, \sigma_{i\,\mu\nu}^{\alpha}(s)[n'(n'\partial)^{-1}-n(n\partial)^{-1}]\Lambda K+\tfrac{1}{2}\int ds\,\lambda_{i\,\mu\nu}^{\alpha}(s)[\partial\Lambda(n'(n'\partial)^{-1}-n(n\partial)^{-1})F^{\alpha}]] \qquad \text{(II 1.74)}$$

which vanishes if particle trajectories do not intersect strings.

2. Monopoles in quantum mechanics

Dirac [1], in 1931, introduced for the first time monopoles in quantum mechanics. His motivation was to understand the reason for the existence of a smallest electric charge e, given approximately, in $\hbar=c=1$ units, by

$$4\pi/e^2=137. \qquad \text{(II 2.1)}$$

His theory did not finally explain the relation (II 2.1) but only established a connection between the smallest electric charge and the smallest magnetic one.

Following Dirac [1], we consider a particle whose motion is described by a wave function $\Psi(x)$. This can be represented as

$$\Psi = Ae^{i\gamma} \qquad \text{(II 2.2)}$$

where A and γ are real functions of the coordinates, denoting the amplitude and the phase of the wave function respectively. For a given state of motion of the particle, Ψ is determined only up to an arbitrary phase, if Ψ is normalized. This means that γ has no physical meaning at a particular point but only differences between its value at two different points are important.

This suggests a generalization of the formalism. We are free to assume that γ has no definite value at a particular point but only a definite difference in values for any two points. We can further assume that this difference is only definite if the two points are neighboring. For two distant points the phase difference is a function of the curve joining them, which has as a consequence a non

zero, in general, change in phase as one goes round a closed curve.

It turns out that this generalized formalism, involving non integrable phases for the wave functions, gives rise to no ambiguity in the physical applications of the theory if the change in phase of a wave function along any curve is the same for all wave functions. This becomes evident if we notice that observables have always the form of the square of the modulus of objects like

$$\int \psi_1^{*} \psi_2 d^3x \qquad \text{(II 2.3)}$$

where ψ_1 , ψ_2 are two different wave functions. In order that (II 2.3) has an unambiguously defined modulus, the integrand must have a well defined phase difference between any two points no matter if they are neighboring or not. This is guaranteed if all wave functions have the same phase difference between two points along any curve joining them.

The result that the change in phase round a closed curve must be the same for all wave functions, means that this change must be determined by the dynamical system itself and must be independent of the state of the system considered. For a single particle, the non-integrability of phase must be connected with the field of force in which the particle moves.

It is convenient to express ψ as a product

$$\psi = \psi' e^{i\beta} \qquad \text{(II 2.4)}$$

where ψ' is any ordinary wave function, with a definite phase at each point, whose modulus is equal to the modulus of ψ . The uncertainty of phase is put in the factor $e^{i\beta}$. This requires that β shall not be a definite function of position and time but its derivatives

$$l_\mu = \frac{\partial \beta}{\partial x^\mu} \qquad \text{(II 2.5)}$$

will be. Moreover these derivatives will not, in general, satisfy the integrability conditions. The change in phase round a closed curve will now be according to Stokes' theorem

$$\int \mathbf{l} \cdot d\mathbf{s} = \int (\partial \Lambda \mathbf{l}) \cdot d\mathbf{S} \qquad \text{(II 2.6)}$$

where ds is an element of arc of the closed curve and dS is an element of a surface whose boundary is the closed curve. This change of phase does not depend on ψ'.

The form (II 2.4) makes it clear that the non-integrability of phase is consistent with the principle of superposition of wave functions, since all wave functions are expressible in this form with the same β. In fact, we are not even obliged to consider a common β and $\mathbf{l}$, since only $\partial \Lambda \mathbf{l}$ is fixed by the requirement that the change of phase around any closed curve shall be common to all wave functions. $\mathbf{l}$'s differing only by the gradient of a scalar function are allowed.

From (II 2.4) we obtain

$$-i \partial_\mu \psi = e^{i\beta} (-i\partial_\mu + l_\mu) \psi' \qquad \text{(II 2.7)}$$

It follows that, if ψ satisfies any wave equation, ψ' satisfies a corresponding wave equation in which $i\partial_\mu$ is replaced by $i\partial_\mu - l_\mu$.

If we assume that ψ satisfies the wave equation for a free particle, then ψ' satisfies the wave equation for a particle with charge $-e$ moving in an electromagnetic field with 4-potential

$$\Lambda_\mu = l_\mu / e \qquad \text{(II 2.8)}$$

We thus see that we can describe the motion of a charged particle in an electromagnetic field, either in terms of a generalized wave function with a non-integrable phase satisfying the free wave equation, or in terms of an ordinary wave function with definite phase but satisfying the wave equation for a particle in an electromagnetic field. In the description in terms of a generalized wave function the

whole effect of the field is to make the phase non-integrable.

With the identification (II 2.8) the components of $\partial\Lambda$ l in Maxwell's theory are, apart from numerical coefficients,equal to the components of the electric and magnetic fields:

$$\text{curl}\vec{l} = e\vec{H}\ ,\ \text{grad}l_0 - \partial_0\vec{l} = e\vec{E} \qquad (II\ 2.9)$$

The connection between non-integrability of phase and the electromagnetic field is another way of expressing the gauge invariance of electrodynamics.

The immediate interpretation of the non-integrable derivatives of the phase of the wave function in terms of the electromagnetic field, has only been possible under the tacit assumption that they are not singular. This is however not always true. We did not take into account the fact that a phase is always undetermined to the extent of an arbitrary integral multiple of 2π. We therefore need to reconsider the connection between the l's and the electromagnetic field.

We saw that the condition for an unambiguous physical interpretation of the theory was that the change in phase round a closed curve should be the same for all wave functions. This change was then interpreted as propotional to the total flux through the closed curve of the 6-vector E,H describing the electromagnetic field. If this change however may be different for differenf wave functions, by arbitrary multiples of 2π, it is not sufficiently well defined to be interpreted in terms of the electromagnetic field.

Let us consider first a very small closed curve. Since the wave equation requires, in general, the wave function to be continuous, the change in phase around a small closed curve must be small, and therefore cannot be different by multiples of 2π for different wave functions. It must have one definite value and may therefore be interpreted without ambiguity in terms of the electric and magnetic flux through the small closed curve.

These considerations do not apply however when the wave function vanishes. As the wave function is complex, its vanishing will require two conditions, so that it will vanish along a line (two-dimensional surface) which we call nodal line. For a wave function having a nodal line passing through our small closed curve, continuity arguments will not enable us to draw the conclusion that the change in phase round the small closed curve must be small. We will only be able to infer that this change is close to $2\pi\hat{n}$ for $\hat{n}$ integer. This integer will be a characteristic of the nodal line, and its sign can be associated with a direction along the nodal line.

The difference between the change in phase round the small closed curve and the nearest $2\pi\hat{n}$ must be the same as the change in phase round the closed curve for a wave function with no nodal line through it. This difference must be interpreted in terms of the flux of $\vec{E}$ and $\vec{H}$ through the closed curve. For a closed curve in three-dimensional space we get for the change in phase

$$2\pi\hat{n} + e\int\vec{H}\cdot dS \qquad \text{(II 2.10)}$$

A large closed curve can be treated by dividing it up into a network of small closed curves lying in a surface bounded by the large closed curve. The total change in phase round the large closed curve will equal the sum of all the changes round the small closed curves and will be equal to

$$2\pi\Sigma\hat{n} + e\int\vec{H}\cdot dS \qquad \text{(II 2.11)}$$

the integration being taken over the surface and the summation over all nodal lines that pass through it. The integral is common to all wave functions but the sum may be different.

The expression (II 2.11) applied to a closed surface gives

$$\Sigma\hat{n} = -\frac{e}{2\pi}\int\vec{H}\cdot dS \qquad \text{(II 2.12)}$$

If $\sum \hat{n}$ does not vanish, some nodal lines must have end points inside the closed surface and its value will equal the sum of the values of $\hat{n}$ for these nodal lines. This sum must be the same for all wave functions. Since this result applies to any closed surface, the end points of nodal lines must be the same for all wave functions and they correspond to points of singularity of the electromagnetic field. The total magnetic flux crossing a small closed surface surrounding one of these points is

$$4\pi g = 2\pi\hat{n}/e \qquad \text{(II 2.13)}$$

where $\hat{n}$ is associated with the nodal line that ends there, or is the sum of the characteristics of the nodal lines ending there. This end point corresponds to a magnetic monopole of strengh

$$g = \hat{n}/2e \qquad \text{(II 2.14)}$$

Magnetic monopoles can therefore be introduced in quantum mechanics, provided the value of their charge is quantized and related to the value of the smallest electric charge.

Another formulation of quantum mechanics, equivalent to the Schroedinger and Heisenberg one, is the path integral formulation, suggested by Dirac [14] and further developed by Feynman [15]. It is instructive to discuss the quantum mechanics of monopoles and electric charges in this language since the path integral formulation is particularly suited for the discussion of the quantum field theory of electric and magnetic charges in interaction with photons.

In the path integral formulation of quantum mechanics the wave function $\psi(x)$ of a particle, can be expressed as an integral over all the classical particle paths ending at x, of the exponential of i times the classical action, i.e.

$$\psi(x) = \int d\Gamma_x \, e^{iS(\Gamma_x)} \qquad \text{(II 2.15)}$$

For an electric charge e in the field of a monopole g, S is any of the classical actions discussed in section 1. We choose for example the Schwinger-Yan action with the choice (II 1.33) for $(n\cdot\partial)^{-1}$.

As an application of this formulation we propose to discuss the problem of the string dependence of the quantum mechanics of an electrically charged particle in the field of a fixed magnetic monopole. Dirac's arguments show that the theory is consistent if the condition (II 2.14) is satisfied. It is interesting to confirm this from another point of view.

Let us exhibit explicitly the n-dependence of ψ and the action.

$$\psi_n(x) = \int d\Gamma_x \, e^{iI_n(\Gamma_x)} \qquad \text{(II 2.16)}$$

We now remember the invariance of the Schwinger-Yan action under the combined string rotation and singular gauge transformation (II 1.53) for closed trajectories. We emphasize that this invariance is only present in the redefined classical Schwinger-Yan action that maps into a circle of circumference V. This definition modulo V, of the classical action cannot be retained in quantum mechanics. In quantum mechanics the action appears in the exponent (see (II 2.15)) and becomes automatically defined only modulo 2Π. Dirac's quantization condition is the imposition of the agreement between those two periods

$$V = 2\eta\hat{n} \qquad \text{(II 2.17)}$$

Quantum mechanics quantizes the radia of the circles into which the action maps.

The statement that the Schwinger-Yan action with the further condition (II 2.17) is invariant under the transformation (II 1.53) for closed trajectories is equivalent to the statement that its change for open trajectories is independent of the path. Therefore [13], the wave function changes only by a multiplicative phase when we move the string. This phase cancels out of observables.

3.Classical field theory of charges and monopoles

The classical field theory of electrically and magnetically charged fields is described by the Maxwell equations

$$\partial \cdot F = J \tag{II 3.1}$$

$$\partial \cdot {}^{d}F = K \tag{II 3.2}$$

together with the equations of motion of the charged fields. Here the point-like currents (II 1.4), (II 1.5), are substituted by the continuous distributions

$$J^{(0)}_{\mu} = \sum_i [ie_i \phi_i^{+} \overleftrightarrow{\partial_\mu} \phi_i - 2e_i(e_i A_\mu + g_i B_\mu)\phi_i^{+}\phi_i] \tag{II 3.3}$$

$$K^{(0)}_{\mu} = \sum_i [ig_i \phi_i^{+} \overleftrightarrow{\partial_\mu} \phi_i - 2g_i(e_i A_\mu + g_i B_\mu)\phi_i^{+}\phi_i] \tag{II 3.4}$$

for the complex spin-0-fields ϕ_i, or by

$$J^{(1)}_{\mu} = \sum_i e_i \bar{\psi}_i \gamma_\mu \psi_i \tag{II 3.5}$$

$$K^{(1)}_{\mu} = \sum_i g_i \bar{\psi}_i \gamma_\mu \psi_i \tag{II 3.6}$$

for the spin-$\frac{1}{2}$-fields ψ_i. The equations of motion are

$$[(i\partial - e_i A - g_i B)^2 - m_i^2]\phi_i = 0 \tag{II 3.7}$$

for the complex spin-0-fields ϕ_i, and

$$(i\not{\partial} - m_i - e_i \not{A} - g_i \not{B})\psi_i = 0 \tag{II 3.8}$$

for the spin-$\frac{1}{2}$-fields ψ_i. In these equations (A,B) is a pair of potentials describing the electromagnetic field.

These equations can be derived variationally from the action

$$S^{(0)}_{M} = S_3 + S^{(0)} \tag{II 3.9}$$

for the spin-0-field case, or from the action

$$S^{(1)}_{M} = S_3 + S^{(1)} \tag{II 3.10}$$

for the spin-½-field case. Here S_3 is the Zwanziger action given in (II 1.21), $S^{(0)}$ is given by

$$S^{(0)} = \sum_i \int dx [(-i\partial - e_i A - g_i B)\phi_i^+ (i\partial - e_i A - g_i B)\phi_i - m_i^2 \phi_i^+ \phi_i] \qquad \text{(II 3.11)}$$

and $S^{(1)}$ is given by

$$S^{(1)} = \sum_i \int dx\, \bar{\psi}_i (i\not{\partial} - m_i - e_i \not{A} - g_i \not{B}) \psi_i . \qquad \text{(II 3.12)}$$

The theory has an exact U(1)xU(1) local gauge symmetry.

This classical field theory is never consistent. With D_μ^i defined by

$$D_\mu^i = \partial_\mu + i e_i A_\mu + i g_i B_\mu \qquad \text{(II 3.13)}$$

we can show that the Jacobi identity

$$\sum [D_\kappa, [D_\lambda, D_\mu]] = 0 \qquad \text{(II 3.14)}$$

is violated. Indeed, using the relation

$$[D_\mu^i, D_\nu^i] = i(e_i F_{\mu\nu} + g_i F^d_{\mu\nu}) \qquad \text{(II 3.15)}$$

we have

$$\sum [D_\kappa^i, [D_\lambda^i, D_\mu^i]] = i \sum_{\substack{\rho \\ \nu \neq \kappa,\lambda,\mu}} [D_\rho^i, e_i F^d_{\rho\nu} - g_i F_{\rho\nu}] = i(e_i K_\nu - g_i J_\nu) \qquad \text{(II 3.16)}$$

with use of Maxwell's equations.

In the following the possibility will be examined, that quantization of this inconsistent classical field theory may lead to a consistent quantum field theory.

4. Quantum field theory of charges and monopoles

We will consider quantum field theories with spin 0 or ½ fields carrying electric and magnetic charge, in general, and interacting with the electromagnetic field.

The quantum field theory action is obtained from the corresponding classical field theory one, by adding a gauge fixing term, plus possible self-interaction terms in the spin 0 matter field case, e.g. terms like

$$-\sum_{i,j}\int\lambda_{ij}(\phi_i^{\dagger}\phi_i)(\phi_j^{\dagger}\phi_j) \qquad \text{(II 4.1)}$$

For example, the manifestly local Zwanziger quantum field theory action with the inclusion of Zwanziger's gauge fixing term

$$\int\tfrac{1}{2}[\partial\cdot(n\cdot A)]^2+\int\tfrac{1}{2}[\partial(n\cdot B)]^2 \qquad \text{(II 4.2)}$$

and matter fields (with the choice $n^2=1$) is

$$S_\eta^{(o)} = S_\eta + S^{(o)} \qquad \text{(II 4.3)}$$

with

$$S_\eta = -\tfrac{1}{2}\int d^4x\left\{[n\cdot(\partial\wedge A)]\cdot[n\cdot(\partial\wedge B)^{d}]+[n\cdot(\partial\wedge A)]^2-[\partial(n\cdot A)]^2+(A\to B, B\to -A)\right\} \qquad \text{(II 4.4)}$$

and

$$S^{(o)}=\sum_i\int d^4x[(-i\partial-e_iA-g_iB)\phi_i^{\dagger}(i\partial-e_iA-g_iB)\phi_i-m_i^2\phi_i^{\dagger}\phi_i-\lambda_i(\phi_i^{\dagger}\phi_i)^2] \qquad \text{(II 4.5)}$$

for the spin 0 matter field case, and

$$S_\eta^{(1)} = S_\eta + S^{(1)} \qquad \text{(II 4.6)}$$

with S_n given by (II 4.4) and $S^{(1)}$ given by

$$S^{(1)} = \sum_i\int d^4x\,\bar{\psi}_i(i\not{\partial}-m_i-e_i\not{A}-g_i\not{B})\psi_i \qquad \text{(II 4.7)}$$

for the spin $\frac{1}{2}$ matter field case.

Variation of A and B gives the correct Maxwell equations (II 3.1) (II 3.2) with the continuous distribution currents of the corresponding classical field theories and also the gauge fixing equations

$$\partial^2 n\cdot A = \partial^2 n\cdot B = 0 \qquad \text{(II 4.8)}$$

Variation of the charged fields gives the correct equations (II 3.7), (II 3.8), if $\lambda_i = 0$.

As already mentioned in section 2, field theory quantization is much more conveniently carried out in the path integral formulation of quantum mechanics. Green functions of local operators are given by functional integrals over the classical fields of the form

$$\langle 0|T\,\theta_1 \ldots \theta_k|0\rangle = N\int \prod d(\text{gauge fields}) \prod d(\text{matter fields}) \times$$

$$\times \theta_1 \ldots \theta_k \exp[i\ \text{action}] \qquad \text{(II 4.9)}$$

where $\prod d(\text{gauge fields})$ and $\prod d(\text{matter fields})$ indicate collectively the measure of the integration over the different fields and the action includes a gauge fixing term, for gauge theories. These Green functions are conveniently obtained from a generating functional

$$Z[J_{gauge}, J_{matter}] = N\int \prod d(\text{gauge}) \prod d(\text{matter}) \times$$

$$\times \exp[i\ \text{action} + i\int J_{gauge}(\text{gauge fields}) +$$

$$+ i\int J_{matter}(\text{matter fields})] \qquad \text{(II 4.10)}$$

of Green functions (a functional of external sources for the different fields) by functionally differentiating with respect to the sources and subsequently setting them to zero.

At the classical level the existence of a transformation bringing the Zwanziger action into the Schwinger-Yan action and vice-versa is sufficient to guarantee their equivalence. At the quantum level, in turn, it is the generating functional of Green functions that determines the quantum theory. It is not obvious, only from the existence of this transformation, that the corresponding generating functionals are equal. If Zwanziger's form of the action is used, the functional integration is over all the classical A and B field configurations with a gauge fixing condition for both of them, due to the

huge U(1)xU(1) local gauge symmetry of the action. In contrast, if the Schwinger-Yan action is used, the functional integration is over the A and F variables of which only A needs a gauge fixing. Nevertheless we will show that expressions like (II 4.9) in both formulations are equivalent if θ's are gauge invariant operators.

Indeed [13] let

$$W_Z \sim \int dA dB d(\text{matter}) \exp[iS_n + \text{sources}] \delta(G) \det\left(\frac{\delta G}{\delta \omega}\right) \qquad \text{(II 4.11)}$$

be the generating functional of Green functions in the Zwanziger case and

$$W_S \sim \int dA dF d(\text{matter}) \exp[iI_n + \text{sources}] \delta(g_1) \det\left(\frac{\delta g_1}{\delta \alpha}\right) \qquad \text{(II 4.12)}$$

the generating functional of Green functions in the Schwinger-Yan case. Here the source terms are assumed gauge invariant and equal in the two cases, $G(A,B)=0$ is a (two component) gauge fixing condition for A and B, $\omega=(\gamma,\beta)$ are gauge parameters, and $g_1(A)=0$ is a gauge fixing condition for A. We will show that W_Z and W_S are transformed into one another, up to a constant factor, under the transformation

$$F \to \left\{ n\Lambda[n\cdot(\partial\Lambda A)] \right\} - \left\{ n\Lambda[n\cdot(\partial\Lambda B)] \right\}^{\alpha} \qquad \text{(II 4.13)}$$

Given our discussion in section 1, it suffices to show that the integration measure in W_Z and W_S is the same (up to a constant factor). To see this, we write (II 4.13) as

$$F(A,B) = f_1(\partial\Lambda A) + f_2(\partial\Lambda B) \qquad \text{(II 4.14)}$$

so that

$$dA dF = dA df_2 \quad . \qquad \text{(II 4.15)}$$

The function f_2 determines $\partial\Lambda B$ and therefore B only up to a gauge transformation. If β is a gauge parameter

$$B = b_2(f_2) + b_1(\beta) \qquad \text{(II 4.16)}$$

where b_1 and b_2 are linear and b_1 is local. Now

$$df_2 = df_2 d\beta \delta(\beta - h) \qquad \text{(II 4.17)}$$

for any function h, provided the integrand is gauge invariant. The measure on the right-hand-side of the last equation is proportional to dB, since $dB/df_2 d\beta$ is a constant, and the δ function can be written as

$$\delta(g_2)\det\left(\frac{dg_2}{d\beta}\right) \qquad \text{(II 4.18)}$$

if $\beta = h$ is the only solution to $g_2 = 0$. Therefore

$$df_2 \sim dB\delta(g_2)\det\left(\frac{\delta g_2}{\delta \beta}\right) \qquad \text{(II 4.19)}$$

and substitution of this expression into W_s gives W_z up to a constant factor. This proves the equivalence of the two actions in the quantum field theory as long as we limit ourselves to gauge invariant Green functions.

We should here make a remark concerning the doubling of variables describing the electromagnetic field [4]. In the usual quantum electrodynamics there is only one potential satisfying a second order differential equation. Since Zwanziger's action contains two of them they should satisfy a first order equation. The equations of motion given in (II 1.17),(II 1.18) (before gauge fixing), with the terms $\partial^2 n\cdot A$ and $\partial^2 n\cdot B$ excluded, are second order trivially, since the second order differential operator which appears there factorizes into $(n\cdot\partial)$ and another first order differential operator. Without gauge fixing the Lagrangian has the huge gauge symmetry:

$$A \rightarrow A + \partial\chi_e \qquad \text{(II 4.20)}$$

$$B \rightarrow B + \partial\chi_g \qquad \text{(II 4.21)}$$

$$\phi_i \rightarrow \exp[-ie_i\chi_e - ig_i\chi_g]\,\phi_i \qquad \text{(Ii 4.22)}$$

$$\psi_j \to \exp[-ie_j\chi_e - ig_j\chi_g]\psi_j \qquad \text{(II 4.23)}$$

where χ_e and χ_g are arbitrary scalar functions. This symmetry has to be reduced by a gauge fixing condition. We previously introduced the Zwanziger gauge fixing term

$$\tfrac{1}{2}[\partial(n\cdot A)]^2 + \tfrac{1}{2}[\partial(n\cdot B)]^2 \qquad \text{(II 4.24)}$$

in the Lagrangian. Imposing instead the axial gauge condition

$$n\cdot A = n\cdot B = 0 \qquad \text{(II 4.25)}$$

we get rid of the unwanted terms in the equations of motion and the potentials satisfy a first order differential equation. If n has zero time component and τ is a time-like vector orthogonal to n, contracting eqs. (II 1.17), (II 1.18) with τ, we get

$$\frac{1}{\eta^2}(n\cdot\partial)[n\cdot\partial\,\tau\cdot A - \tau\cdot(\partial\wedge B)\cdot n] - \tau\cdot J = 0 \qquad \text{(II 4.26)}$$

$$\frac{1}{\eta^2}(n\cdot\partial)[n\cdot\partial\,\tau\cdot B + \tau\cdot(\partial\wedge A)\cdot n] - \tau\cdot K = 0 \qquad \text{(II 4.27)}$$

These equations constitute another pair of constraints, since they contain no time derivatives. Thus they reduce the four degrees of freedom of the pair of potentials A and B to the two degrees of freedom of the photon.

Finally, let us turn to the Feynman rules. With the notation $V^1_\mu = A_\mu$, $V^2_\mu = B_\mu$ and $\epsilon^{12}=1$, $\epsilon^{21}=-1$, the free ($e_i=g_i=0$) gauge field propagator can be written compactly in momentum space, with Zwanziger's gauge fixing as [9]

$$D^{ab}_{\mu\nu}(k) = \int d^4x\, e^{ik\cdot x}\langle 0|T V^a_\mu(x) V^b_\nu(0)|0\rangle$$

$$= \left[\left(-g_{\mu\nu} + \frac{k_\mu n_\nu + n_\mu k_\nu}{n\cdot k}\right)\delta^{ab} - 1/(nk)\,\epsilon_{\mu\nu\rho\sigma}\, n_\rho k_\sigma \epsilon^{ab}\right]\frac{1}{k^2+i\varepsilon} \qquad \text{(II 4.28)}$$

Here the prescription for $1/(n\cdot k)$ (in the mixed propagator) is the principal value one. The rest of the Feynman rules are the usual ones. For our abelian theory no Faddeev-Popov ghosts are present.

5. Path integral representations

In the present section we will derive path integral representations of matrix elements of certain unitary operators. These representations will be extremely useful in the next section where the Lorentz invariance of the theory will be discussed.

Consider the operators X_μ and P_μ, satisfying the canonical commutation relations

$$[X_\mu, P_\nu] = -ig_{\mu\nu} \tag{II 5.1}$$

Introduce formal states $|x\rangle$ and $|p\rangle$ such that

$$X|x\rangle = x|x\rangle \tag{II 5.2}$$

$$P|x\rangle = -i\partial|x\rangle \tag{II 5.3}$$

$$X|p\rangle = i\partial_p|p\rangle \tag{II 5.4}$$

$$P|p\rangle = p|p\rangle \tag{II 5.5}$$

with

$$\langle x|p\rangle = e^{-ip\cdot x} \tag{II 5.6}$$

The normalizations are

$$\langle x|x'\rangle = \delta^4(x-x') \qquad \langle p|p'\rangle = (2\pi)^4\delta^4(p-p') \tag{II 5.7}$$

and the completeness relations

$$\int d^4x\,|x\rangle\langle x| = 1 \qquad \int d^4p/(2\pi)^4\,|p\rangle\langle p| = 1 \tag{II 5.8}$$

The matrix element of interest for bosons is

$$W(\tau,x,x') = \langle x|\exp(\tfrac{1}{2}i\tau\{[P-\alpha(X)]^2-m^2\})|x'\rangle \tag{II 5.9}$$

where $\alpha(x)$ is an arbitrary function. Following Feynman [15] we will derive a path integral representation for it.

We divide the interval $(0,\tau)$ into 2N segments of length $\Delta\tau$

$$\tau_0 = 0,\ \tau_1,\ \tau_2,\ \ldots,\ \tau_{2N} = \tau\ ;\ \Delta\tau = \tau/2N.$$

With these τ-values we associate the variables

$$x_0 = x',\ p'_1,\ x_2,\ \ldots,\ x_{2N-2},\ p'_{2N-1},\ x_{2N} = x$$

We write the $\exp(\tfrac{1}{2}i\tau\{[P-\alpha(X)]^2-m^2\})$ as

$$\lim_{N\to\infty}(\exp[i\Delta\tau\{[P-\alpha(X)]^2-m^2\}])^N \qquad \text{(II 5.10)}$$

Now we substitute (II 5.10) in (II 5.9) and insert complete sets of states

$$\langle x|\exp(\tfrac{1}{2}i\tau\{[P-\alpha(X)]^2-m^2\})|x'\rangle =$$

$$=\lim_{N\to\infty}\prod_{j=1}^{N-1}\int dx_{2j}\,\langle x|\exp(i\Delta\tau\{[P-\alpha(X)]^2-m^2\})|x_{2N-2}\rangle\ldots$$

$$\langle x_2|\exp(i\Delta\tau\{[P-\alpha(X)]^2-m^2\})|x'\rangle \qquad \text{(II 5.11)}$$

Let us consider each factor individually. Assume N large enough and insert a complete set of states:

$$\langle x_{2j+2}|\exp(i\Delta\tau\{[P-\alpha(X)]^2-m^2\})|x_{2j}\rangle \simeq \langle x_{2j+2}|\ (1+i\Delta\tau\{[P-\alpha(X)]^2-m^2\}+\ldots)|x_{2j}\rangle$$

$$\simeq\int\frac{d^4p'_{2j+1}}{(2\pi)^4}\exp(-2i\Delta\tau p'_{2j+1}(x_{2j+1}-x_{2j})/2\Delta\tau)\exp(i\Delta\tau\{[p'_{2j+1}-\alpha((x_{2j+2}+x_{2j})/2)]^2-m^2\}) \qquad \text{(II 5.12)}$$

We introduce the notation

$$x_{2j+2}-x_{2j} = \Delta x_{2j+1} \qquad (x_{2j+2}+x_{2j})/2 = x_{2j+1} \qquad \text{(II 5.13)}$$

We then change variables

$$p'_{2j+1} - \alpha(x_{2j+1}) = p_{2j+1} \qquad \text{(II 5.14)}$$

In terms of p_{2j+1} eq. (II 5.12) reads

$$\langle x_{2j+2}|\exp(i\Delta\tau\{[P-\alpha(X)]^2-m^2\})|x_{2j}\rangle = \int\frac{d^4p_{2j+1}}{(2\pi)^4}\exp[-i\Delta\tau m^2-i2\Delta\tau\alpha(x_{2j+1})\cdot(\frac{\Delta x_{2j+1}}{2\Delta\tau})$$

$$\times\exp[2i\Delta\tau\tfrac{1}{2}p^2_{2j+1}-2i\Delta\tau p_{2j+1}\cdot(\frac{\Delta x_{2j+1}}{2\Delta\tau})] \qquad \text{(II 5.15)}$$

We can perform the p_{2j+1} integration to get

$$\langle x_{2j+2}|\exp(i\Delta\tau\{[P-\alpha(X)]^2-m^2\})|x_{2j}\rangle \sim e^{-i\Delta\tau m^2}\exp(-2\Delta\tau[\tfrac{1}{2}(\frac{\Delta x_{2j+1}}{2\Delta\tau})^2+\alpha(x_{2j+1})\cdot(\frac{\Delta x_{2j+1}}{2\Delta\tau})]) \tag{II 5.16}$$

Substituting back into (II 5.11) we obtain

$$W(\tau,x,x')\sim \lim_{N\to\infty}\prod_{j=1}^{N-1}\int dx_{2j}\exp(-i\tfrac{1}{2}\tau m^2\sum_{j=0}^{N-1}\tfrac{\tau}{N}[\tfrac{1}{2}(\frac{\Delta x_{2j+1}}{\tau/N})^2+\alpha(x_{2j+1})\cdot(\frac{\Delta x_{2j+1}}{\tau/N})] \tag{II 5.17}$$

from which

$$W(\tau,x,x')=N\int d\Gamma(\tau;x,x')\exp(-\tfrac{1}{2}im^2\tau-i\int_0^\tau d\tau'[\tfrac{1}{2}\dot{z}^2(\tau')+\alpha(z(\tau'))\cdot\dot{z}(\tau')]) \tag{II 5.18}$$

where the integration is over all x-space paths $\Gamma(\tau;x,x')$ between the points $x'=z(0)$ and $x=z(\tau)$. This is the desired result.

In the case of fermions we are interested in the matrix element

$$V(\tau;x,x')=\langle x|\exp\{i\tau[\not{p}-\not{\alpha}(X)-m]\}|x'\rangle \tag{II 5.19}$$

We will derive also for it a formal path integral representation.

We begin by using the Trotter formula

$$e^{i(A+B)}=\lim_{N\to\infty}(e^{iA/N}e^{iB/N})^N \tag{II 5.20}$$

in order to write

$$e^{i\tau[\not{p}-\not{\alpha}(X)-m]}=\lim_{N\to\infty}(e^{i\tau/N[\not{p}-m]}e^{-i\tau/N\not{\alpha}(X)})^N \tag{II 5.21}$$

We substitute (II 5.21) in (II 5.19) and insert complete sets of states

$$V(\tau;x,x')=\lim_{N\to\infty}\prod_{j=1}^{N-1}\int dx_j\langle x|e^{i\tau/N[\not{p}-m]}e^{-i\tau/N\not{\alpha}(X)}|x_{N-1}\rangle\ldots$$
$$\ldots\langle x_1|e^{i\tau/N[\not{p}-m]}e^{-i\tau/N\not{\alpha}(X)}|x'\rangle \tag{II 5.22}$$

We again deal with each factor individually

$$\langle x_{j+1}|e^{i\tau/N[\not{p}-m]}e^{-i\tau/N\not{\alpha}(X)}|x_j\rangle=$$
$$=\int\frac{d^4p_j'}{(2\pi)^4}e^{-i(\tau/N)p_j'\frac{\Delta x_j}{\tau/N}-i(\tau/N)m}e^{i(\tau/N)[\not{p}_j'-\not{\alpha}(x_j)]} \tag{II 5.23}$$

where

$$\Delta x_j = x_{j+1} - x_j \qquad \text{(II 5.24)}$$

We now perform the change of variables

$$p_j' - \alpha(x_j) = p_j \qquad \text{(II 5.25)}$$

in (II 5.23). The result is

$$\langle x_{j+1}|e^{i\tau/N[\not{p}-m]}e^{-i\tau/N\not{\alpha}(X)}|x_j\rangle = \int\frac{d^4p_j}{(2\pi)^4}e^{-i\tau/N\alpha(x_j)\cdot(\frac{\Delta x_j}{\tau/N})}e^{-i\tau/N\, m}e^{i\tau/N\not{p}_j}e^{-i\tau/N p_j\cdot(\frac{\Delta x_j}{\tau/N})} \qquad \text{(II 5.26)}$$

Using (II 5.26) we can represent $V(\ ;x,x)$ as

$$V(\tau;x,x') = \int d\Gamma(\tau;x,x')\int dQ\, T e^{i\int_0^\tau d\tau'[\not{p}(\tau')-p(\tau')\cdot\dot{z}(\tau')]}e^{[-i\int_0^\tau d\tau'\not{\alpha}(z(\tau'))\cdot\dot{z}(\tau')-i\tau m]} \qquad \text{(II 5.27)}$$

in which the integrations are over all x-space paths $\Gamma(\tau;x,x')$ from $x'=z(0)$ to $x=z(\tau)$ and all p-space paths Q between unrestricted points $p(0)$ and $p(\tau)$, and T denotes τ' ordering.

To emphasize the similarity of the expression for V to the one for W we rewrite (II 5.27) as

$$V(\tau;x,x') = \int d\Gamma(\tau;x,x')F(\Gamma(\tau;x,x'))\exp[-i\int_0^\tau d\tau'\not{\alpha}(\ (\tau'))\cdot\dot{z}(\tau')-i\tau m] \qquad \text{(II 5.28)}$$

with

$$F(\Gamma) \equiv \int dQ\, T\exp(i\int_c^\tau d\tau'[\not{p}(\tau')-p(\tau')\cdot\dot{z}(\tau')]) \qquad \text{(II 5.29)}$$

independent of α and m. This expression seems to exist only formally. This is analogous to the formal existence of $\delta^4(x+\tau\gamma)$ [5]. We therefore will use it only for plausibility arguments. We now turn to a representation of another matrix element which will be used for the discussion of spin $\frac{1}{2}$ fields in the next section.

We start by giving a functional integral representation for

$$U_{ij}(\tau) = [T\exp(-i\int_0^\tau d\tau' h(\tau'))]_{ij} \qquad i,j=1,\ldots,M \qquad \text{(ii 5.30)}$$

where $h(\tau)_{ij}$ is an arbitrary MxM matrix function of τ. This can be

represented as [5]

$$U_{ij}(\tau)=\int\prod_{0\le\tau'\le\tau}\prod_{\kappa=1}^{M}[ds_\kappa(\tau')d\bar{s}_\kappa(\tau')/(2\pi i)]\; s_i(\tau)\bar{s}_j(0)\;x$$

$$x\;\exp(-\bar{s}_i(\tau)s_i(\tau)+i\int_0^\tau d\tau'[-i\dot{\bar{s}}_i(\tau')s_i(\tau')-\bar{s}_i(\tau')h_{ij}(\tau')s_j(\tau')]) \qquad \text{(II 5.31)}$$

Using this formula we turn to a path integral representation for the matrix element

$$W(\tau;x,x')_{ij}=\langle x|[\exp(\tfrac{1}{2}i\tau\{[P-\alpha(X)]^2-m^2-\tfrac{1}{2}i\gamma^\mu\gamma^\nu f_{\mu\nu}(X)\})]_{ij}|x'\rangle \qquad \text{(II 5.32)}$$

where $\alpha_\mu(x)$ and $f_{\mu\nu}(x)$ are arbitrary functions, and i,j are Dirac spinor indices. The procedure used previously gives

$$W(\tau;x,x')_{ij}=\int d\Gamma(\tau;x,x')\,T\exp\{-\tfrac{1}{2}im^2\tau-i\int_0^\tau d\tau'[\tfrac{1}{2}\dot{z}^2(\tau')+\dot{z}(\tau')\cdot\alpha(z(\tau'))+\tfrac{1}{2}i\gamma^\mu\gamma^\nu f_{\mu\nu}(z(\tau'))]\}_{ij} \qquad \text{(II 5.33)}$$

Using (II 5.31) we get finally

$$W(\tau;x,x')=\int d\Gamma(\tau;x,x')\exp\{-\tfrac{1}{2}im^2\tau-i\int_0^\tau d\tau'[\tfrac{1}{2}\dot{z}^2(\tau')+\alpha(z(\tau'))\cdot\dot{z}(\tau')]\}x$$

$$x\int d\Sigma(\tau)s_i(\tau)\bar{s}_j(0)\exp[-i\tfrac{1}{2}\int_0^\tau d\tau' f_{\mu\nu}(z(\tau'))\sigma^{\mu\nu}(\tau')] \qquad \text{(II 5.34)}$$

where

$$d\Sigma(\tau)\equiv\prod_{\tau'}(d\bar{s}ds/(2\pi i))\exp[-\bar{s}_i(\tau)s_i(\tau)-\int_0^\tau d\tau'\dot{\bar{s}}_i(\tau')s_i(\tau')] \qquad \text{(II 5.35)}$$

and

$$\sigma^{\mu\nu}(\tau)=\bar{s}(\tau)i\tfrac{1}{2}[\gamma^\mu,\gamma^\nu]s(\tau) \qquad \text{(II 5.36)}$$

This formula is the desired one and was derived by Brandt, Neri and Zwanziger [5].

6.Lorentz invariance of the quantum field theory

The classical particle theory of charges and monopoles in interaction with the electromagnetic field is Lorentz invariant, in spite of the fact that the actions used involve the "string". This is obvious, given that the equations of motion are Lorentz invariant and

string independent, and the classical theory is completely specified by the equations of motion. We also have seen that the quantized particle theory is Lorentz invariant as well, if the Dirac quantization condition between the electric and magnetic charges is satisfied. Observables turned out to be independent of n. This is equivalent to the statement of Lorentz invariance given that invariance under a Lorentz transformation, with the string kept fixed, is equivalent to invariance under a movement of the string with the inverse of this Lorentz transformation. In the present section we will describe the proof [5] that Green functions of the electric and magnetic currents with a given number of closed particle loops (matter loops) and all photon corrections are n independent.

Let us consider the generating functional of Green functions of the electric and magnetic currents

$$W_n(a,b)=\langle 0|\,T\exp[-i\int d^4x(J\cdot a + K\cdot b)]\,|0\rangle \qquad (II.6.1)$$

where $a_\mu(x)$ and $b_\mu(x)$ are arbitrary functions. The current Green functions are obtained from (II 6.1) by functional differentiation with respect to a and b, followed by setting a and b to zero. In the notation $W_n(a,b)$ the n refers to the n dependence through the rest of the action with exclusion of the gauge fixing term (since another gauge fixing is as good as the one chosen). As we already discussed, the Lorentz invariance of $W_n(a,b)$ is equivalent to its n independence. Hence we must show that $W_n(a,b)=W_{n'}(a,b)$.

We will deal with the spin 0 and spin $\frac{1}{2}$ cases separately.

a) The spin 0 theory: This is specified by the action $S_n^{(o)}$, (II 4.3)-(II 4.5). The functional integral representation of (II 6.1) is

$$W_n(a,b)=N\int dA\,dB\prod_i d\phi_i\,d\phi_i^\dagger \exp\{i[S_n^{(o)}(A,B,\phi_i,\phi_i^\dagger)-\int d^4x(Ja+Kb)] \qquad (II\ 6.2)$$

We can, without loss of generality, omit the self-interaction term of the matter fields since this term can be created by functional differentiation with respect to a position dependent mass, whose inclusion does not affect the arguments at all.

We first carry out the integrations over the charged scalar fields in (II 6.2). The result is well-known to be

$$W_n(a,b)=N\int dA dB e^{iS_n}\exp(-1\sum_i \mathrm{Tr}\ln\{[(P-C_i)^2-m^2]/(P^2-m^2)\}) \quad , \qquad \text{(II 6.3)}$$

where

$$C_{i\mu}=e_i(A_\mu+a_\mu)+g_i(B_\mu+b_\mu) \qquad \text{(II 6.4)}$$

Using the formula

$$\ln G = -\int_0^\infty d\tau/\tau \exp[i\tfrac{1}{2}\tau G] \qquad \text{(II 6.5)}$$

the expression in the second exponent in (II 6.3) can be represented as

$$\mathrm{Tr}\ln\{[(P-C)^2-m^2]/(P^2-m^2)\} =-\int_0^\infty \frac{d\tau}{\tau} dx \langle x|\left\{e^{i\frac{1}{2}\tau[(P-C)^2-m^2]} - e^{i\frac{1}{2}\tau[P^2-m^2]}\right\}|x\rangle \qquad \text{(II 6.6)}$$

We now observe that we have at our disposal a path integral representation for the expression on the right-hand side of (II 6.6). It is the formula (II 5.18) derived in the previous section. We use it to get

$$\mathrm{Tr}\ln\{[(P-C)^2-m^2]/(P^2-m^2)\}=-\int_0^\infty \frac{d\tau}{\tau} e^{-\frac{1}{2}i\tau m^2}\int d\Gamma(\tau) f(\Gamma(\tau))[\exp(-i\int_0^\tau d\tau' \dot{z}\cdot C)-1] \qquad \text{(II 6.7)}$$

where the integration $d\Gamma(\tau)$ is over all closed paths $\Gamma(\tau)$, and

$$f(\Gamma)=\exp[-\tfrac{1}{2}i\int_0^\tau d\tau' \dot{z}^2(\tau')] \qquad \text{(II 6.8)}$$

Using (II 6.7) we can write $W_n(a,b)$ as

$$W_n(a,b)=N\int dA dB e^{iS_n}\exp\left\{\sum_i \int_0^\infty d\tau/\tau\, e^{-\frac{1}{2}i\tau m_i^2}\int d\Gamma(\tau) f(\tau)[-1+\exp((-i)\int_0^\tau d\tau' \dot{z}\cdot C_i)]\right\} \qquad \text{(II 6.9)}$$

Finally expanding $\exp(\sum_i)$ in a power series, we get

$$W_n(a,b)=\sum_{K=0}^\infty W_n^{(K)}(a,b) \quad , \qquad \text{(II 6.10)}$$

with

$$W_n^{(K)}=N/K!\int dAdB\ e^{iS_n}\int\prod_{r=1}^{K}\left[\frac{d\zeta_r}{\zeta_r}d\Gamma_r(\zeta_r)f(\Gamma_r)\right]x$$

$$x\sum_{\rho}C_\rho\exp(-i\tfrac{1}{2}\sum m_{i_r}^2\tau_r)\exp\left\{-i\int dx[J_\rho^\mu(A+a)+K_\rho^\mu(B+b)]\right\} \qquad \text{(II 6.11)}$$

where ρ represents a particular assignment of the indices $i=1,\cdots,K$, for $r=1,\ldots,K$, C_ρ is a combinatorial factor, and J_ρ and K_ρ are the corresponding classical currents associated with the charges e_i and g_i and (closed) trajectories $\Gamma_r(\zeta)$:

$$J_\rho^\mu(x)=\sum e_{i_r}\oint_{\Gamma_r}dz^\mu\delta^4(x-z) \qquad \text{(II 6.12)}$$

$$K_\rho^\mu(x)=\sum g_{i_r}\oint_{\Gamma_r}dz^\mu\delta^4(x-z) \qquad \text{(II 6.13)}$$

We see that apart from integrations over some real variables and some other factors depending on classical paths Γ, each $W_n^{(K)}$ is just a sum of generating functionals for classical currents, i.e. of functional integrals over classical fields A and B and (closed) trajectories Γ weighted with a classical action $S_n(A,B)-\int[J\cdot(A+a)+K\cdot(B+b)]$:

$$W_n^{(K)}(a,b)\sim\sum\int d\Gamma\int dAdB\ \exp[iS_n(A,B)]\ \exp[-i\int[J\cdot(A+a)+K\cdot(B+b)]] \qquad \text{(II 6.14)}$$

One way of verifying the n independence of $W_n^{(K)}$, given the Dirac quantization condition, is to exploit the invariance, modulo V (given by (II 1.34)), of the classical action under a combined string rotation and singular gauge transformation, and the gauge invariance of the functional measure. We make, to be specific, a change of variables from the set (A,B,Γ) to the set (A,F,Γ). This means that we get an action of the Schwinger-Yan type. We emphasize that this change of variables at the quantum level leads to a Schwinger-Yan type of action if we consider gauge invariant quantities only. Now, instead of $[J\cdot(A+a)+K\cdot(B+b)]$ we have $[J\cdot(A+a)+K\cdot(\bar{B}+b)]$ in (II 6.14), with $\bar{B}$ given in terms of F by (II 1.41). The only n dependence in each term of the

sum (II 6.10) in the transformed version of (II 6.14) lies in the K·B term. This transformed classical action has, if the Dirac condition is satisfied, the invariance property under the string rotation and singular gauge transformation given in (II 1.53), (II 1.54). Since the combined transformation is a symmetry that leaves W_n invariant, showing that $W_n=W_{n'}$ under $n\to n'$ only, is equivalent to showing that the gauge transformation alone leaves W_n invariant. But this is true since the invariance of the measure dA under $A\to A+\partial\lambda=A'$ makes possible the absorption of this change in the form of a change of variables. This completes the proof of the relation $W_n=W_{n'}$. We shall refer to this version of the proof as the "indirect" one.

A more "direct" proof involves an explicit integration of the A and B variables in (II 6.14). Each term then reduces to something propotional to

$$\int d\Gamma e^{-i\int(J\cdot a+K\cdot b)}\exp[-\tfrac{1}{2}i\iint(J\cdot D_{AA}\cdot J+J\cdot D_{AB}\cdot K+K\cdot D_{BA}\cdot J+K\cdot D_{BB}\cdot K] \quad \text{(II 6.15)}$$

where $D_{AA}=D^{11}, D_{BB}=D^{22}, D_{BA}=D^{21}$ are the propagators of the A and B fields. By current conservation, only the mixed propagators in (II 6.15) contribute to the possible n dependence. Using the form of the mixed propagator, the n dependent part of the exponent in (II 6.15) is readily seen to have the form of sum of terms like

$$i\tfrac{1}{2}(e_ig_j-g_ie_j)\oint_{\Gamma_i}dx_i^{\mu}\oint_{\Gamma_j}dx_j^{\nu}\epsilon_{\mu\nu\alpha\beta}n^{\alpha}\partial^{\beta}(n\cdot\partial)^{-1}\frac{1}{\partial^2}(x_i-x_j) \quad \text{(II 6.16)}$$

This last expression, with use of Stokes' theorem, becomes (up to signs which are irrelevant)

$$i\tfrac{1}{2}(e_ig_j-g_ie_j)\int_{\Sigma_i}dS_i^{\mu\rho}\oint_{\Gamma_j}dx_j^{\nu}(\partial_{\rho}\epsilon_{\mu\nu\alpha\beta}-\partial_{\mu}\epsilon_{\rho\nu\alpha\beta})n^{\alpha}\partial^{\beta}(n\cdot\partial)^{-1}\frac{1}{\partial^2}(x_i-x_j) \quad \text{(II 6.17)}$$

This expression is equal (up to a sign) to

$$i\tfrac{1}{2}(e_ig_j-g_ie_j)\int_{\Sigma_i}dS_i^{d\kappa\lambda}\epsilon_{\kappa\lambda}{}^{\mu\rho}\oint_{\Gamma_j}dx_j^{\nu}\partial_{\rho}\epsilon_{\mu\nu\alpha\beta}n^{\alpha}\partial^{\beta}(n\cdot\partial)^{-1}\frac{1}{\partial^2}(x_i-x_j)=$$

$$=-i(e_ig_j-g_ie_j)\int_{\Sigma_i}dS_i^{d\kappa\lambda}\oint_{\Gamma_j}dx_j^{\kappa}n^{\lambda}\partial^2(n\cdot\partial)^{-1}\frac{1}{\partial^2}(x_i-x_j)$$

$$+ i(e_i g_j - g_i e_j) \int_{\Sigma_i} dS_{i\kappa} \oint_{\Gamma_j} dx_j^{\kappa} \partial^{\lambda} \frac{1}{\partial^2}(x_i - x_j) \qquad \text{(II 6.18)}$$

In this last expression only the first term is n dependent. Using the form of $(n\cdot\partial)^{-1}$ given in (II 1.11), the n dependent term becomes (up to a sign)

$$\tfrac{1}{2} i(e_i g_j - g_i e_j) \int_{\Sigma_i} dS_i^{\mu\nu} \oint_{\Gamma_j} dx_{j\mu} n_\nu \int_0^\infty ds [\delta^4(x_i - x_j - ns) - \delta^4(x_i - x_j + ns)] \qquad \text{(II 6.19)}$$

Because of the δ functions, the integrations in (II 6.19) simply count the number of times the loop Γ_j intersects the oriented three-surface Σ_i $x \pm ns$ $(0 \leq s \leq \infty)$. This is an integer N_{ij} for almost all paths. It is only ill defined on a set of measure zero in the space of paths. These paths cannot change the answer. Thus all the n dependence in (II 6.11) lies in factors of the form

$$\exp[i\tfrac{1}{2}(e_i g_j - g_i e_j) N_{ij}] \qquad \text{(II 6.20)}$$

which are equal to 1, if the stronger

$$e_i g_j - g_i e_j = 4\pi\hat{n} \qquad \hat{n} \text{ integer} \qquad \text{(II 6.21)}$$

Schwinger-type quantization condition is assumed. If the form of $(n\cdot\partial)^{-1}$ given in (II 1.33) is used the Dirac type of quantization

$$e\cdot g = 2\pi\hat{n} \qquad \text{(II 6.22)}$$

is sufficient.

b) The spin $\frac{1}{2}$ theoty: This is specified by the action $S_n^{(1)}$ (II 4.6). The functional integral representation of (II 6.1) is

$$W_n(a,b) = N \int dA dB \left(\prod_i d\psi_i d\bar{\psi}_i\right) \exp\left\{ i[S_n^{(1)}(A,B,\psi_i,\bar{\psi}_i) - \int dx (J\cdot a + K\cdot b)] \right\} \qquad \text{(II 6.23)}$$

Performing explicitly the spinor integrations we get

$$W_n(a,b) = N \int dA dB e^{iS_n} \exp\left\{ \sum_i \mathrm{Tr} \ln[(i\partial\!\!\!/ - m_i - C_i)/(i\partial\!\!\!/ - m_i)] \right\} \qquad \text{(II 6.24)}$$

where again

$$C_{i\mu} = e_i(A_\mu + a_\mu) + g_i(B_\mu + b_\mu) \qquad \text{(II 6.25)}$$

For a formal argument we can proceed as in the scalar case. We represent the second exponent in (II 6.24) as

$$\mathrm{Tr}\ln[(\not p-\not\zeta-m)/(\not p-m)] = -\int_0^\infty \frac{d\tau}{\tau}\int dx \langle x| \mathrm{tr}\left(e^{i\tau(\not p-\not\zeta-m)} - e^{i\tau(\not p-m)}\right)|x\rangle \quad \text{(II 6.26)}$$

where the trace in the right-hand-side is only over the spinor indices. Substitution of the path integral representation (II 5.28), (II 5.29) gives

$$\mathrm{Tr}\ln[(\not p-\not\zeta-m)/(\not p-m)] = -\int_0^\infty \frac{d\tau}{\tau} e^{-im\tau}\int d\Gamma(\tau)\, g(\Gamma(\tau))\left[1-e^{-i\int_0^\tau d\tau' \dot z\cdot C}\right] , \quad \text{(II 6.27)}$$

where

$$g(\Gamma) = \mathrm{tr}F(\Gamma) , \quad \text{(II 6.28)}$$

with $F(\Gamma)$ given in (II 5.29). This expression is completely analogous to the one in the scalar case, and the whole proof goes through without any change. However due to the only formal existence of the path integral representation (II 6.27) we will follow a different line for the spin $\frac{1}{2}$ case.

Note that

$$\begin{aligned}
&\mathrm{Tr}\ln(\not p-\not\zeta-m+i\varepsilon) - \mathrm{Tr}\ln(\not p-m+i\varepsilon) = \\
&= \mathrm{Tr}\int_m^\infty d\mu\left[1/(\not p-\not\zeta-\mu+i\varepsilon) - 1/(\not p-\mu+i\varepsilon)\right] \\
&= \mathrm{Tr}\int_m^\infty d\mu\left[(\not p-\not\zeta+\mu)\frac{1}{(\not p-\not\zeta)^2-\mu^2+i\varepsilon} - (\not p+\mu)\frac{1}{p^2-\mu^2+i\varepsilon}\right] \\
&= \mathrm{Tr}\int_m^\infty d\mu\left[1/((\not p-\not\zeta)^2-\mu^2+i\varepsilon) - 1/(p^2-\mu^2+i\varepsilon)\right] \\
&= \tfrac{1}{2}\mathrm{Tr}\int_m^\infty d\mu^2\left[1/((\not p-\not\zeta)^2-\mu^2+i\varepsilon) - 1/(p^2-\mu^2+i\varepsilon)\right] \\
&= \tfrac{1}{2}\mathrm{Tr}\ln[(\not p-\not\zeta)^2-m^2+i\varepsilon] - \tfrac{1}{2}\mathrm{Tr}\ln[p^2-m^2+i\varepsilon] \qquad \text{(II 6.29)}
\end{aligned}$$

We now proceed as in the scalar case

$$-\mathrm{Tr}\ln(\not p-\not\zeta-m+i\varepsilon) + \mathrm{Tr}\ln(\not p-m+i\varepsilon) =$$

$$=\tfrac{1}{2}\mathrm{Tr}\int_0^\infty \frac{d\tau}{\tau}\left[\exp\left(\tfrac{1}{2}i\tau\left[(\not{P}-\not{C})^2-m^2\right]\right)-\exp\left(\tfrac{1}{2}i\tau\left[\not{P}^2-m^2\right]\right)\right]$$

$$=\tfrac{1}{2}\mathrm{Tr}\int_0^\infty \frac{d\tau}{\tau}\left[\exp\left(\tfrac{1}{2}i\tau\left[(P-C)^2-\tfrac{1}{2}i\gamma^\mu\gamma^\nu(\partial\wedge C)_{\mu\nu}-m^2\right]\right)-\exp\left(\tfrac{1}{2}i\tau(P^2-m^2)\right)\right] \qquad \text{(II 6.30)}$$

The exponential in the last expression is of the form (II 5.32) and therefore we can use the representation (II 5.34) for it to obtain

$$-\mathrm{Tr}\ln(\not{P}-\not{C}-m)+\mathrm{Tr}\ln(\not{P}-m)=$$

$$=\tfrac{1}{2}\int_0^\infty \frac{d\tau}{\tau}\exp\left(-\tfrac{1}{2}im^2\tau\right)\int d\Gamma(\tau)f(\Gamma(\tau))\int d\xi(\tau)s_i(\tau)\bar{s}_i(0)$$

$$x\left[\exp\left(-i\int_0^\tau d\tau'\left[\dot{z}\cdot C+\tfrac{1}{2}(\partial\wedge C)_{\mu\nu}\sigma^{\mu\nu}\right]\right)-1\right] \qquad \text{(II 6.31)}$$

where $f(\Gamma)$ and $d\xi(\tau)$ are given by (II 6.8), (II 5.35) and

$$\sigma^{\mu\nu}(\tau)=\bar{s}(\tau)\tfrac{1}{2}i[\gamma_\mu,\gamma_\nu]s(\tau) \qquad \text{(II 6.32)}$$

The generating functional now gets the form

$$W_n(a,b)=N\int dAdBe^{iS_n}\exp\left\{\sum_i\tfrac{1}{2}\int_0^\infty\frac{d\tau}{\tau}\exp\left(-\tfrac{1}{2}im_i^2\tau\right)\int d\Gamma(\tau)f(\Gamma)\int d\xi(\tau)\bar{s}_i(\tau)s_i(0)\right.$$

$$\left.x\left[-\exp\left(-i\int_0^\tau d\tau'\left[\dot{z}\cdot C_i+\tfrac{1}{2}(\partial\wedge C_i)_{\mu\nu}\sigma^{\mu\nu}\right]\right)+1\right]\right\} \qquad \text{(II 6.33)}$$

which differs from the spin 0 expression only by the $\sigma^{\mu\nu}$ contribution to the exponential. Now we expand in a power series

$$W_n(a,b)=\sum_{\kappa=0}^\infty W_n^{(\kappa)}(a,b) \qquad \text{(II 6.34)}$$

with

$$W_n^{(\kappa)}(a,b)=\frac{N}{k!}\int dAdBe^{iS_n}\int\prod_{r=1}^{\kappa}\left[\frac{d\tau_r}{\tau_r}d\Gamma_r(\tau_r)f(\Gamma_r)\right]$$

$$x\sum_\rho C_\rho\exp\left\{-\tfrac{1}{2}i\sum m_{i_r}^2\tau_r-i\int d^4x\left[J_\rho\cdot(A+a)+K_\rho\cdot(B+b)\right]\right\} \qquad \text{(II 6.35)}$$

where the currents are

$$J_\rho^\mu(x)=\sum e_{i_r}\int_0^{\tau_r}d\tau_r'\left[\dot{z}_r^\mu(\tau_r')+\tfrac{1}{2}\sigma_r^{\mu\nu}(\tau_r')\partial_\nu\right]\delta^4(x-z_r) \qquad \text{(II 6.36)}$$

$$K_\rho^\mu(x)=\sum g_{i_r}\int_0^{\tau_r}d\tau_r'\left[\dot{z}_r^\mu(\tau_r')+\tfrac{1}{2}\sigma_r^{\mu\nu}(\tau_r')\partial_\nu\right]\delta^4(x-z_r) \qquad \text{(II 6.37)}$$

The only additional feature, relative to the scalar case, is the existence of first-moment contributions to the classical currents. Remembering the discussion at the end of section 1, we immediately realize that the "indirect" version of the proof goes through, except from a problem that arises if a particle trajectory hits a string. If we believe that such a collision corresponds to a set of measure zero in the space of paths, string independence follows.

The "direct" version of the proof also goes through, using again the "set of measure zero" argument.

In conclusion we should emphasize the assumptions that go into the proof:

1) It is assumed that the generating functional may be expressed as a sum over the number of charged particle loops. This assumption is weaker than the assumption of convergence of the perturbation series, since each term contains an infinite number of perturbation terms.

2) It is assumed that, although the Feynman measure is not really a measure, some exceptional configurations, like the ones corresponding to intersection of particle trajectories with the string, do not contribute to the final answer.

3) It is assumed that renormalization will not affect the result. This question is going to be the main topic in the following chapters.

III REGULARIZATION

1. Introduction

We saw in the previous chapter that the quantum theory of electric charges and magnetic monopoles is not automatically Lorentz invariant, in contrast to the classical particle theory whose Lorentz invariance is apparent in the Lorentz invariant form of the equations of motion. At the quantum level the particle theory was shown to be consistent and Lorentz invariant provided the Dirac quantization condition between the electric and magnetic charges is satisfied. At the quantum field theory level Brandt, Neri and Zwanziger [5] gave a proof of the statement that Green functions of the gauge invariant electric and magnetic currents with a given number of charged particle loops and all photon corrections are Lorentz invariant, provided, again, the Dirac quantization condition is satisfied. We remind the reader that the statement of Lorentz invariance, in the Zwanziger local formulation, is equivalent to the statement of independence from the 4-vector n which is the direction of the Dirac "string".

Several assumptions went into the proof of this rather unbelievable result. These assumptions were listed at the end of the previous chapter. In the present and later chapters we will be mainly concerned with the last assumption, namely the assumption that renormalization will not affect this result.

It is, by now, well known that all quantum field theories in four dimensions, in perturbation theory at least, are full of ultraviolet divergences. As a result, it is necessary to find first a suitable regularization procedure that gives an unambiguous meaning to the divergent quantities by making them temporarily finite and then to remove, in a well defined way the regulator. The procedure of removal of the regulator in a way that the resulting theory is finite and has all

the properties of a well defined physical theory, is called renormalization. In order to show that the resulting theory satisfies all the requirements of a physical theory it is convenient that the regularization satisfies certain conditions. For example it should keep as many of the symmetries of the original (bare) theory as possible. Finding a gauge invariant regulator is an important step towards a proof of the unitarity of a gauge field theory.

The proof of n independence of the Green functions of the gauge invariant electric and magnetic currents, given in the previous chapter, obviously refers to the bare theory. The proof is using the path integral formulation of quantum field theory, without any explicit reference to Feynman diagrams.However the path integral can be expanded in terms, corresponding to Feynman graphs, most of which are ultraviolet divergent. From this point of view this proof can be considered as only formal. One could, of course, argue that simply the fact that each term in the expansion of the path integral, corresponding to the perturbation series, is ultraviolet divergent does not necessarily imply the divergence of the path integral, if the path integral represents their sum in any sense. Although a cancellation of the infinities is very attractive and not in principle impossible, it is al least highly improbable. Perturbation theory has taught us that infinities in higher orders become only worse. So, unless a regularization is provided which will give a meaning to these divergent quantities, statements like their n independence are not unambiguous. The danger of working formally with these infinite quantities was first realized by the authors of the original paper who stated as an open question the examination of the possibility that renormalization invalidates their conclusion. We believe that a regularization of the bare theory in a way that their proof can be repeated, is a good proof that the renormalized theory, in the sense of any reasonable subtraction on individual graphs or on their sum, is n independent, although it would not give a definite ne-

gative answer if this program were unsuccessful. Here by a reasonable subtraction is meant any subtraction that subtracts something that is present (i.e. an infinite term) and does not introduce something not present in the first place (in the unsubtracted theory in the presence of the regulators). If the full, bare, regularized theory does not have n anywhere, any subtraction should not introduce it. Of course it has to be found a subtraction satisfying these requirements.

In this chapter therefore we are looking for a regularization procedure that renders all graphs finite, that preserves the U(1)xU(1) gauge symmetry of Zwanziger's action and that can be incorporated into the proof of n independence given by Brandt, Neri and Zwanziger. Unfortunately, the best regularization up to now which would leave the proof untouched by inspection, dimensional regularization [16], cannot be used since the theory contains the Levi-Civita tensor which has no dimensional continuation. From the rest of the regulators none is satisfactory by itself. That is why we will employ a combination of different regularization procedures.

In section 2 we describe the regularization scheme. In section 3 we show that the regularization described in section 2 satisfies the condition that it regularizes the theory in a way that the proof of n independence of the Green functions of the gauge invariant electric and magnetic currents goes through. In section 4 we describe another slightly different regularization, which applies to the spin $\frac{1}{2}$ matter field case only. This regularization has the advantage that it provides, as a by-product, regularized propagators of the gauge fields which are easier to work with in all cases. Section 5 contains some conclusions drawn from the discussions of this chapter. Throughout the choice $n^2=1$ for the "string" is made.

2.Regularization

The lagrangian density of the Zwanziger action without matter fields (II 4.4), denoted from now on L_{em}, can be written (after some trivial algebra) as

$$L_{em}=-\tfrac{1}{2}[\ (n\cdot\partial\wedge A)]\cdot[n\cdot(\partial\wedge B)^{d}]+\tfrac{1}{2}[n\cdot(\partial\wedge B)]\cdot[n\cdot(\partial\wedge A)^{d}]$$

$$-\tfrac{1}{2}[(n\cdot\partial A)^{2}-2(n\cdot\partial A)\cdot(\partial n\cdot A)+(n\cdot\partial B)^{2}-2(n\cdot\partial B)\cdot(\partial n\cdot B)] \qquad \text{(III 2.1)}$$

We shall add to this lagrangian density a regularizing part, in order to modify the free propagators ($e_i=g_i=0$) of the gauge fields:

$$L_{R}=-\tfrac{1}{2}\frac{\alpha}{\Lambda^{2}}[\partial^{\sigma}[n\wedge[n\cdot(\partial\wedge A)]-\{n\wedge[n\cdot(\partial\wedge B)]\}^{d}]]$$

$$\cdot[\partial_{\sigma}[n\wedge[n\cdot(\partial\wedge A)]-\{n\wedge[n\cdot(\partial\wedge B)]\}^{d}]]$$

$$-\tfrac{1}{2}\frac{\beta}{\Lambda^{4}}[\partial^{2}[n\wedge[n\cdot(\partial\wedge A)]-\{n\wedge[n\cdot(\partial\wedge B)]\}^{d}]]$$

$$[\partial^{2}[n\wedge[n\cdot(\partial\wedge A)]-\{n\wedge[n\cdot(\partial\wedge B)]\}^{d}]] \qquad \text{(III 2.2)}$$

In L_R, α and β are arbitrary constants and Λ is a cut-off parameter with dimensions of mass introduced to maintain the correct dimensionality of the Lagrangian. Letting Λ go to infinity removes the regulators ($L_R \longrightarrow 0$). Regulators of that form have been introduced for the first time by Slavnov [17] in connection with problems associated with non-linear chiral Lagrangians and used by Slavnov and Lee and Zinn-Justin [18] to provide a regularization for non-abelian gauge theories. This possibility was also known to K.Johnson.

The propagator obtained from eqs. (III 2.1) and (III 2.2) has a rather large expression and its presentation is not very illuminating; therefore, its explicit form will not be given here. Its only interesting property is its leading behavior for large k^2 which, except for terms proportional to k_μ, is $1/k^6$ due to an inverse power of Γ, with

$$\Gamma = \frac{\alpha}{\Lambda^2} k^2 + \frac{\beta}{\Lambda^4} k^4 \qquad \text{(III 2.3)}$$

Let us examine the various cases, according to the spin of the matter fields, one by one.

(i) Spin $\frac{1}{2}$ case: Let us first consider graphs with any number of closed particle loops and all kinds of corrections inside the particle loops and between particle loops. A graph of this form is shown in fig 1. Graphs containing closed particle loops without any corrections inside the closed particle loop will be considered later (fig 2). The modified propagator for the gauge fields would be enough to regularize graphs like those in fig. 1, if the terms containing a k_μ were absent. Fortunately, it is possible to eliminate these terms and no longer consider them, by current conservation. In fact, this is true only if there is a gauge invariant regularization that makes possible the shift of the origin of integration in expressions of the amplitudes of the diagrams to which they contribute [19]. A satisfactory regularization is the analytic continuation [20], in which we modify the propagators essentially by elevating their denominators to some power λ that can be fixed such that the amplitude is finite. This regularization is known not to be gauge invariant in QED [20], when the propagators of the fermion fields around a closed particle loop are modified and not the vertices, e.g. in the computation of the vacuum polarization at the one loop level. Graphs like that fall into the category of fig.2 and are irrelevant for this discussion, since in these graphs there is no gauge field propagator in the loops and it is this propagator that contains the terms that we want to drop. For the relevant graphs analytic continuation is good, provided we modify only the gauge field propagators. This is sufficient. After we eliminate the k_μ terms, analytic continuation is no longer needed. Its existence only is sufficient to decide that k_μ terms really give zero contribution. From now on we completely forget the k_μ terms and the analytic continuation.

Let us now turn to the subgraphs of the form shown in fig. 2 (and graphs containing them). For the spin $\frac{1}{2}$ case that we are considering there is in fact only one divergent graph of that form, the one shown in fig.2. The graphs with a closed particle loop, no photon corrections inside the particle loop and 1 or 3 external legs are zero because of Furry's theorem [21], and the one with no corrections and 4 external gauge boson legs is finite, although by power counting infinite, because of gauge invariance. If the number of external legs is greater than four, the graphs of this form are finite. The divergent graphs of this category can be regularized by means of Gupta regulators [22], i.e. by including in the Lagrangian auxiliary spinor fields coupled minimally to the gauge fields with some of them obeying the wrong statistics. They have arbitrarily large masses which prevent them from appearing as final particles. Gupta has explicitly calculated the graph of fig.2, and shown that it is regularized in a gauge invariant way. Putting the mass equal to infinity decouples the regulators.

(ii) Spin 0 case: The discussion of the previous case concerning graphs with closed matter field loops with corrections (like the ones in fig.1) applies to the spin 0 case too.

The graphs of the second category that diverge (in this case all graphs with up to four external gauge boson legs) (fig.2) can also be regularized using auxiliary massive boson fields minimally coupled to the gauge fields and some of them obeying the wrong statistics. The only difference is that in this case some of them appear with a Lagrangian having an overall minus sign. Calculations with this kind of Gupta regulators have been performed by Lee [23].

As already discussed, the propagator that we get after adding to the Zwanziger Lagrangian the regularizing Lagrangian L_R in (III 2.2), behaves, for large momentum k as k^{-6} except for the gauge dependent

terms proportional to k_μ which remain unregularized. The reason is the gauge invariance of the regularizing part L_R. If we break this gauge invariance, the resulting gauge propagator will behave for large momentum k as k^{-6}, including the gauge dependent terms. This is accomplished by replacing Zwanziger's gauge fixing term, by the following ghost-free [29] cutoff-dependent one:

$$L_{GF}=\tfrac{1}{2}\xi^2\left[(n\cdot A)^2+\frac{1}{\Lambda^2}(n\cdot A)\partial^2(n\cdot A)+\frac{1}{\Lambda^4}(n\cdot A)\partial^4(n\cdot A)\right.$$
$$\left.+(n\cdot B)^2-\frac{1}{\Lambda^2}(n\cdot B)\partial^2(n\cdot B)-\frac{1}{\Lambda^4}(n\cdot B)\partial^4(n\cdot B)\right] \qquad \text{(III 2.4)}$$

where ξ and Λ are parameters with dimensions of mass, Λ being the cutoff. Therefore, if L_3 is the Zwanziger Lagrangian without gauge fixing, the Lagrangian

$$L = L_3 + L_{GF} + L_{Gupta} + L_{matter} \qquad \text{(III 2.5)}$$

gives rise to a regularized field theory of charges and monopoles.

3. Proof of Lorentz invariance in the presence of regulators

With the modified Lagrangian

$$L_{em} + L_R \qquad \text{(III 3.1)}$$

for the gauge field action, the "indirect" version of the proof of n independence given in the previous chapter goes through trivially. It suffices to note that, under the change of variables that takes us from the functional integral weighted with the exponential of the Zwanziger action to the functional integral weighted with the exponential of the Schwinger-Yan action, after integrating out the matter fields, L_R goes to an expression containing only derivatives of F and no n dependence:

$$L_R \to -\tfrac{1}{4}\frac{\alpha}{\Lambda^2}(\partial^\sigma F)\cdot(\partial_\sigma F)-\tfrac{1}{4}\frac{\beta}{\Lambda^4}(\partial^2 F)\cdot(\partial^2 F) \qquad \text{(III 3.2)}$$

The singular gauge transformation

$$A \to A + \partial\lambda \quad , \quad F \to F \quad , \quad \Gamma_i \to \Gamma_i \tag{III 3.2}$$

with the scalar function $\lambda(x)$ satisfying the condition

$$\partial\wedge\partial\lambda = \left\{ [(n'\cdot\partial)^{-1} n' - (n\cdot\partial)^{-1} n] \wedge K \right\}^{d} \tag{III 3.4}$$

which is equivalent to a string rotation $n \to n'$, leaves L_R invariant and the proof remains exactly the same.

The Gupta regulators do not cause any trouble, since they can be treated like the rest of the matter fields. The fact that some of them obey the wrong statistics changes only a sign in front of a trln term that we get when we integrate them out. The minus sign of the overall Lagrangian of the regulator field in the spin 0 case simply gives an n independent factor in front of the path integral over the gauge fields and can be absorbed in the normalization constant.

4. An alternative regularization for the spin ½ case

Let us try instead of the L_R in (III 2.2) the following combination, denoted $L_{R'}$:

$$\begin{aligned}
L_{R'} = &-\tfrac{1}{2}\frac{\alpha'}{\Lambda^2}[n\cdot\partial^{\sigma}(\partial\wedge A)]\cdot[n\cdot\partial_{\sigma}(\partial\wedge B)^{d}] + \tfrac{1}{2}\frac{\alpha'}{\Lambda^2}[n\cdot\partial^{\sigma}(\partial\wedge B)]\cdot[n\cdot\partial_{\sigma}(\partial\wedge A)^{d}] \\
&-\tfrac{1}{2}\frac{\beta'}{\Lambda^4}[n\cdot\partial^{2}(\partial\wedge A)]\cdot[n\cdot\partial^{2}(\partial\wedge B)^{d}] + \tfrac{1}{2}\frac{\beta'}{\Lambda^4}[n\cdot\partial^{2}(\partial\wedge B)]\cdot[n\cdot\partial^{2}(\partial\wedge A)^{d}] \\
&-\tfrac{1}{2}\frac{\gamma'}{\Lambda^2}[n\cdot\partial^{\sigma}(\partial\wedge A)]\cdot[n\cdot\partial_{\sigma}(\partial\wedge A)] - \tfrac{1}{2}\frac{\gamma'}{\Lambda^2}[n\cdot\partial^{\sigma}(\partial\wedge B)]\cdot[n\cdot\partial_{\sigma}(\partial\wedge B)] \\
&-\tfrac{1}{2}\frac{\delta'}{\Lambda^4}[n\cdot\partial^{2}(\partial\wedge A)]\cdot[n\cdot\partial^{2}(\partial\wedge A)] - \tfrac{1}{2}\frac{\delta'}{\Lambda^4}[n\cdot\partial^{2}(\partial\wedge B)]\cdot[n\cdot\partial^{2}(\partial\wedge B)] \\
&-\tfrac{1}{2}\frac{\zeta'}{\Lambda^4}[n\cdot\partial\partial^{\rho}(\partial\wedge A)]\cdot[n\cdot\partial_{\rho}\partial(\partial\wedge B)^{d}] + \tfrac{1}{2}\frac{\zeta'}{\Lambda^4}[n\cdot\partial\partial^{\rho}(\partial\wedge B)]\cdot[n\cdot\partial_{\rho}\partial(\partial\wedge A)^{d}] \\
&-\tfrac{1}{2}\frac{\theta'}{\Lambda^4}[n\cdot\partial\partial^{\rho}(\partial\wedge A)]\cdot[n\cdot\partial_{\rho}\partial(\partial\wedge A)] - \tfrac{1}{2}\frac{\theta'}{\Lambda^4}[n\cdot\partial\partial^{\rho}(\partial\wedge B)]\cdot[n\cdot\partial_{\rho}\partial(\partial\wedge B)]
\end{aligned} \tag{III 4.1}$$

where $\alpha', \beta', \gamma', \delta', \lambda', \theta'$ are arbitrary constants. For arbitrary values of these constants the resulting gauge propagator is very complicated and of no use. The combination $\gamma' = \alpha', \delta' = \beta', \theta' = \lambda'$ yields the much more tractable propagator $(\omega' = \delta' + \theta')$:

$$D^{ab}_{\mu\nu} = (nk)^{-1}\left(1+\left(\frac{\gamma'}{\Lambda^2}\right)k^2+\left(\frac{\omega'}{\Lambda^4}\right)k^4\right)^{-1}\left\{-\frac{n\cdot k}{k^2}g_{\mu\nu}+\frac{\left(\frac{\gamma'}{\Lambda^2}\right)k^2+\left(\frac{\omega'}{\Lambda^4}\right)k^4}{n\cdot k}\frac{k_\mu k_\nu}{k^2}+\frac{k_\mu n_\nu + n_\mu k_\nu}{k^2}\right\}\delta^{ab}$$

$$-1/k^2\,\epsilon_{\mu\nu\rho\sigma}n^\rho k^\sigma \epsilon^{abl}. \qquad \text{(III 4.2)}$$

We see in this case too that for large momenta the modified gauge propagator behaves like k^{-6} except for the gauge dependent terms which can be neglected, as we have already discussed.

What is different now is that the proof of Lorentz invariance does not go through. In fact it is the "direct" version of the proof that is better applicable in this case. The diagonal elements of the gauge propagator do not contribute again to the n dependence because of current conservation. However if we repeat the arguments given in the "direct" proof in the previous chapter for the mixed propagator (off-diagonal elements) we still find two terms, after the use of Stokes' theorem. Out of these terms, the first is again n independent but the second, the n dependent one, is not an integer times $i(e_i g_j - g_i e_j)/2$ for arbitrary values of the regulator parameter. Therefore we see that we can repeat the proof of Lorentz invariance only if we remove the cutoff from the mixed propagator. This means that all graphs containing mixed propagators are not, in general, regularized. Nevertheless we can remove the cutoff from the mixed propagators and still leave the theory regularized in the spin $\frac{1}{2}$ case. The spin $\frac{1}{2}$ limitation is due to the fact that only in theories with spin $\frac{1}{2}$ matter fields are the "box" graphs of fig.5 that serve as bridges between electric charge and monopole loops (fig.6) finite.

We still have to answer a question: does the removal of the cutoff from the mixed propagator only violate the abelian gauge symmetry?

The answer to this question is "no", or better "not the Ward identities!" It is easy to see it in QED. There, we can regularize the photon propagator in the same way by introducing in the Lagragian the term

$$\frac{\alpha}{\Lambda^2}(\partial^\sigma F_{\mu\nu})(\partial_\sigma F^{\mu\nu}) + \frac{\beta}{\Lambda^4}(\partial^2 F_{\mu\nu})(\partial^2 F^{\mu\nu}) \qquad \text{(III 4.3)}$$

Now, if in any graph we vary the cut-off independently in two photon lines, this does not violate any symmetry. The photon lines do not communicate directly by a vertex. The fermion lines in between are not affected at all by the regulator and all vertices belong to a theory with the same gauge symmetry. The Ward identities are based on the identity

$$\partial/\partial p^\mu \frac{1}{(\not{p}-m)} = -\frac{1}{(\not{p}-m)}\gamma_\mu\frac{1}{(\not{p}-m)} \qquad \text{(III 4.4)}$$

saying that differentiation of a free fermion propagator with respect to momentum is equivalent to the insertion of a zero momentum photon. If the regulator parameters in the different photon lines are different, the Ward identity does not care, since we can always label the momenta in the photon lines by independent parameters and leave the dependence on the external momenta only in the fermion lines. The identity (III 4.4) also shows some things that are not allowed in a gauge invariant regularization. We cannot introduce, for instance, regulators that give different masses to particle lines coupled through a common vertex to a gauge boson line. This violates the local gauge symmetry.

Throughout the discussion of this section it is assumed the introduction of Gupta regulators that take care of the graphs of fig.2. The proof of Lorentz invariance is not affected by their presence in any essential way, as discussed in section 3.

Finally we may remark that although the propagator (III 4.2) is not good for the proof of n independence in all cases, once we know that such a proof exists, it can be used for calculations in all cases. In fact, when the terms containing a k_μ are neglected, because of current conservation, the rest is a rather simple expression to work with.

5.Conclusions

In this chapter we gave a gauge invariant regularization for the quantum field theory of electric charges and magnetic monopoles, in the presence of which the proof of Lorentz invariance for the Green functions of the gauge invariant electric and magnetic currents can be repeated. The existence of these regulators renders the proof of n independence given by Brandt, Neri and Zwanziger much less formal,as far as the bare theory is concerned, since the ultraviolet divergences of the theory, present in the unregularized version, are not present any more. However, in order that we are able to conclude that the theory after renormalization will be Lorentz invariant, a scheme of removal of the divergences has to be exhibited which, on top of making the theory finite, will keep its n independence (in the closed particle loop sector). This leads to the next chapter.

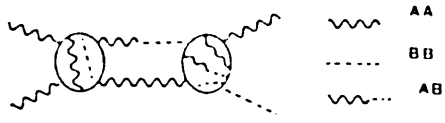


Fig. 1.



Fig. 2.

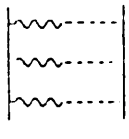


Fig. 3.

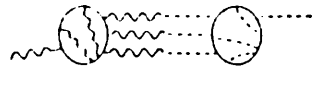


Fig. 4.

IV RENORMALIZATION

1. Introduction

We saw in the previous chapters that there exists a local Lagrangian for the quantum field theory of charges and monopoles, due to Zwanziger, which, although manifestly local, it still retains an element reminiscent of older non local theories, namely the dependence on an arbitrary but fixed four-vector, say n_μ, the remnant of the old Dirac "string" [2]. Brandt, Neri and Zwanziger [5] gave a proof that full Green functions of the gauge invariant electric and magnetic currents are n independent, if Dirac's quantization condition

$$e_i g_j / 4\pi = \hat{n}_{ij}/2 \qquad \hat{n}_{ij} = 0, \pm 1, \pm 2, \ldots \tag{IV 1.1}$$

or the (stronger) Schwinger one

$$(e_i g_j - g_i e_j)/4\pi = \hat{n}_{ij} \qquad \hat{n}_{ij} = 0, \pm 1, \pm 2, \ldots \tag{IV 1.2}$$

depending on the definition of $(n\cdot\partial)^{-1}$, among the electric and magnetic charges e_i, g_i is satisfied. There exists also a gauge invariant regularization of this quantum field theory in the presence of which the proof of n independence can be repeated.

As emphasized in the previous chapter, the existence of these regulators guarantees the n independence of the finite Green functions that might result after any reasonable subtraction, provided that such a subtraction scheme can be found. By reasonable subtraction we mean any subtraction that does not introduce an n dependence in the subtracted Green functions which are n independent in the bare theory. Our purpose in the present chapter is to investigate the existence of such a subtraction scheme.

It should be emphasized that the result of n independence is not extendable to Green functions of the gauge non invariant matter fields.

For the S-matrix, in the context of a coherent state treatment without renormalization, it is known [25] that it depends, in general, on n but this dependence is at most through a phase factor (e.g. in electron-monopole scattering) which cancels out when we multiply the amplitude with its complex conjugate to compute probabilities.

The whole discussion is carried out in the context of the standard LSZ reduction formula which has been used as a guide to the correct renormalization of the parameters of the theory. Also, in this context, we will prove the n independence of the renormalized S-matrix. This result is only valid if the standard reduction is used. However, it should be stressed that the LSZ formulation does not take into account the long range nature of the electromagnetic interaction. This long range nature, if not taken care of properly, manifests itself in infrared divergences. The long range monopole-electron interaction gives also rise to the n dependent phase, as we mentioned before, which reflects the non-scalar nature of the S-matrix in this theory. At this stage we can only hope that our treatment of the ultraviolet problem will not be invalidated by a complete treatment of the infrared one. For a treatment of the infrared problem see reference [26].

In the use of the LSZ formulation the assumption of the existence of asymptotic states is inherent. This means that we assume the existence of an n independent pole (branch point) in the full matter field propagators. [Indeed, if such a pole does not exist, the theory would contain only confined states and therefore the study of scattering amplitudes would not make sense and our discussion would be futile.]

We will confine ourselves to the case of a spin $\frac{1}{2}$ electrically charged particle with charge e and a spin $\frac{1}{2}$ monopole with magnetic charge g. We will see (as mentioned already) that for the non gauge invariant fields it is the on shell S-matrix only that is n independent (Lorentz invariant). After that we will discuss the vacuum pola-

rization and charge, wave function and vertex renormalization in the full theory. Then a subtraction to all orders is described and the finiteness and Lorentz invariance of the resulting theory is discussed. We also comment on the complications that arise when we renormalize the charges, because of the quantization condition. Afterwards the formal expansion in closed charged particle loops is described and its subtraction along the same lines is discussed. Finally, before the conclusions we make some remarks stressing the assumptions made and the cases not covered. Throughout the Dirac (weaker) quantization condition is assumed.

2.Definition of the theory

The theory we are considering is defined by the action

$$S = S_1 + S_2 \tag{IV 2.1}$$

with

$$S_1=-\tfrac{1}{2}\int d^4x \left\{[n\cdot\partial\wedge A)][n\cdot\partial\wedge B)^d]+[n\cdot\partial\wedge A)]^2-[\partial(nA)]^2+(A\to B, B\to -A)\right\} \tag{IV 2.2}$$

and

$$S_2=\sum_i\int d^4x\,\bar{\psi}_i(i\partial\!\!\!/-m_i-e_i A\!\!\!/-g_i B\!\!\!/)\psi_i \quad i=1,2 \tag{IV 2.3}$$

where A_μ and B_μ are independent vector potentials, n_μ an arbitrary but fixed four-vector with $n^2=1$, and $\psi_1(\psi_2)$ is an electrically(magnetically) charged spin $\frac{1}{2}$ matter field with electric(magnetic) charge $e_1=e(g_2=g)$, magnetic(electric) charge $g_1=0(e_2=0)$ and mass $m_1(m_2)$. The notation supresses Lorentz indices like in the previous chapters. Again $F^d_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$ is the dual of $F_{\mu\nu}$.

With the notation

$$V^1_\mu = A_\mu, V^2_\mu = B_\mu \tag{IV 2.4}$$

and

$$\epsilon^{12} = -\epsilon^{21} = 1 \qquad \text{(IV 2.5)}$$

the free gauge field propagators are [4]

$$D^{ab}_{\mu\nu} \equiv \int dx e^{ikx} \langle 0 | T V^a_\mu(x) V^b_\nu(0) | 0 \rangle = [(-g_{\mu\nu} + \frac{K_\mu n_\nu + n_\mu K_\nu}{n\cdot K}) \delta^{ab} - \frac{1}{n\cdot K} \epsilon_{\mu\nu\rho\sigma} n^\rho k^\sigma \epsilon^{ab}] \qquad \text{(IV 2.6)}$$

The remaining Feynman rules can be correctly read-off directly from the Lagrangian. They are the usual ones for spinor fields with minimal coupling to a gauge field.

3. The n independence of the S-matrix

It was shown in a previous chapter that full Green functions of the gauge invariant electric and magnetic currents, are n independent. The same is not true for the full Green functions of the gauge non invariant fields ψ_i and $\bar{\psi}_i$. In discussing the renormalization of the theory we need to make statements about all physical processes. Knowledge of the n independence of the bare theory in the closed charged matter loop sector is not sufficient. In this section we will extend the Brandt-Neri-Zwanziger proof of Lorentz invariance to the (on shell) S-matrix. Unfortunately, as already discussed in the introduction, we will use crude means in order to obtain the result. We will slaughter the Green functions of the spinor fields through the LSZ reduction formula, ignoring infrared problems which might be severe. We hope that the infrared problem can be treated independently of the ultraviolet one and a formal proof of renormalizability will not be invalidated by the infrared complications. Details concerning the steps of the proof that are common to the one for the Green functions of the currents can be found in the review of the paper of Brandt, Neri and Zwanziger given before. Here we will only stress differencies.

We introduce into the generating functional of Green functions

source terms for the spinor fields. It is essential for the proof that these source terms be gauge invariant. Such a gauge invariant source is the path dependent source [27], with coupling to the fermions

$$\bar{J}^{\Psi_i}(x)\,[\exp ia_i \int_{-\infty}^{x} dx'^{\mu} V_{\mu}^{i}(x')]\,\Psi_i(x) \qquad (a_1=e, a_2=g) \tag{IV 3.1}$$

where the integration is taken along a line from $-\infty$ to the point x. This source emits single particles in lowest order and additional photons in higher order. These new couplings of the source contribute only to external line renormalizations and not to the S-matrix.

We integrate out the spinor fields in the generating functional $W_n(J^{\Psi})$ and we express the result as a functional integral over classical particle paths. This is weighted with the classical particle Zwanziger action. We perform a change of variables from the Zwanziger classical particle action to the Schwinger-Yan non local classical particle action. It is at this stage that the gauge invariance of the spinor source terms is needed. We should remember from our discussion in section 4 of chapter II, that this change of variables can only be done without complications if the path integral is over gauge invariant quantities. After this change of variables, we perform the change of integration variable

$$A \rightarrow A + \partial\lambda \tag{IV 3.2}$$

where λ is a singular n dependent function satisfying the condition

$$\partial\Lambda\partial\lambda = \left\{[(n'\partial)^{-1} n' - (n\partial)^{-1} n]\Lambda K\right\}^{d} \tag{IV 3.3}$$

and such that the gauge transformation (IV 3.2) in the Schwinger-Yan action is equivalent to the change

$$n \rightarrow n' \tag{IV 3.4}$$

Then we move the "string" from n to n'. Using the invariance of the dA

measure under the change of variables (IV 3.2) we see that $W_n'(J^{\psi_i})$ differs from $W_n(J^{\psi_i})$ only in effective couplings of the source J^{ψ_i} with the gauge and spinor fields, depending on n. To get the (on shell) S-matrix we apply to these string dependent Green functions the LSZ reduction formula. When we cut the external legs according to the standard reduction procedure, from the graphs that these new couplings give rise to, only the contributions of those survive which have a pole in the physical mass of the ψ_i. So, in general, the wave function renormalization constant of ψ_i depends on n but not the S-matrix. This, in particular, has as a consequence that when

$$\not{p} \to m_i \quad \text{(physical)} \qquad \text{(IV 3.5)}$$

the full propagator has the form

$$Z_2(n)_i / (\not{p} - m_i) \quad + \quad \text{regular terms} \quad , \qquad \text{(IV 3.6)}$$

where the wave function renormalization Z_{2i} depends, in general, on n. The situation is parallel to the gauge dependence of the matter field wave function renormalization constant in ordinary gauge theories.

The imposition of an n independent mass is nacessary if we want to have a unitary theory. The thresholds in gauge invariant Green functions, that are also n independent, cannot depend on n. If these thresholds correspond to the creation of real particles, their mass is certainly n independent.

The proof on n independence of the S-matrix goes through without any change in the presence of the regulators of the previous chapter.

4.Vacuum polarization

Let us first use the theorem of n independence of the full Green functions of the electric and magnetic currents (J_μ and K_μ respectively) in order to find the form of the full gauge boson propagators. The

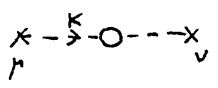
full Green function of two electric currents $\langle T(J_\mu J_\nu)\rangle$ represented by



is of the form

$$i(g_{\mu\nu}k^2 - k_\mu k_\nu)C(k^2) \qquad \text{(IV 4.1)}$$

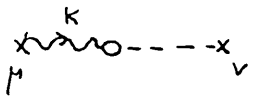
using the gauge invariance and n independence to all orders. The full Green function of two magnetic currents $\langle T(K_\mu K_\nu)\rangle$ represented by



is of the form

$$i(g_{\mu\nu}k^2 - k_\mu k_\nu)D(k^2) \qquad \text{(IV 4.2)}$$

for the same reasons. The same holds for the full Green function of one electric and one magnetic current $\langle T(J_\mu K_\nu)\rangle$ represented by

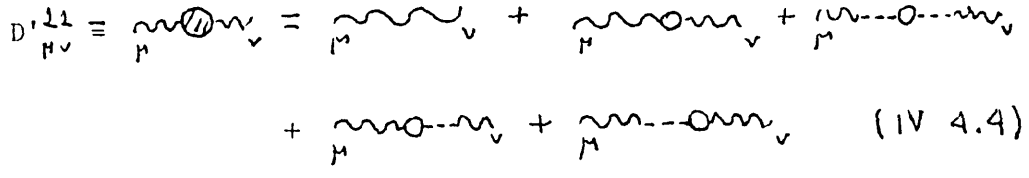


and having the form

$$i(g_{\mu\nu}k^2 - k_\mu k_\nu)F(k^2) \qquad \text{(IV 4.3)}$$

The scalar functions C, D and F do not depend on n. The external legs are amputated everywhere.

The full (AA) propagator, denoted $D'^{11}_{\mu\nu}$, takes the following contributions



(IV 4.4)

where the external legs are not amputated. Because of current conservation, the terms proportional to k_μ in the expressions of the blobs can be dropped. The last two terms cancel each other since the mixed propagator is antisymmetric in its dual space indices and even in mo-

mentum. The three first terms give

$$D'^{11}_{\mu\nu} = -\frac{\delta_{\mu\nu}}{K^2}[1 + C(k^2) + D(k^2)] - (n_\mu n_\nu - g_{\mu\nu})/(nk)^2 D(k^2) =$$

$$= -\frac{\delta_{\mu\nu}}{K^2}[1 + C(k^2) + D(k^2)] + T_{\mu\nu}/(nk)^2 D(k^2) \qquad \text{(IV 4.5)}$$

where

$$T_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu \qquad \text{(IV 4.6)}$$

Terms proportional to k_μ have been dropped.

The same steps give

$$D'^{22}_{\mu\nu} = -g_{\mu\nu}/k^2 \; [1 + C(k^2) + D(k^2)] + T_{\mu\nu}/(nk)^2 \; C(k^2) \qquad \text{(IV 4.7)}$$

We see that the full (AA) and (BB) propagators are string dependent and not multiplicatively renormalizable. This makes obvious the fact, that the theory cannot be renormalizable in the strict sense, namely all Green functions cannot be made finite without introduction of counter terms of different form than the terms originally present in the Lagrangian. The trouble in this case is that, if we introduce even a finite number of counterterms of different form than the original terms, the theory stops being Lorentz invariant (in the closed charged loop sector). So, unless the string dependent terms effectively disappear, the theory cannot be made finite.

A problem related to the string dependence of the full (AA) and (BB) propagators, is the definition of the screening of electric and magnetic charges due to vacuum palarization. This screening is a physical process and should not be string dependent. The best candidate for the object corresponding to the usual Z_3 of QED, is the 1+C+D multiplying the $g_{\mu\nu}$ term of both propagators at $k^2=0$. It is the residue of the pole of the photon propagator on shell and determines the scale of the photon field. This scale, in turn, determines the contribution of the vacuum polarization to charge renormalization. In the following

we will argue that the string dependent terms will effectively be absent in all physical processes and therefore there is no ambiguity in the definition of Z_3. But even if the string dependent terms do not cancel completely, which would be fatal for the finiteness of the renormalized theory, the correct definition of the contribution of vacuum polarization to charge renormalization can be unambiguously obtained by studing Compton-like phenomena, as we will do at the end of this section. After we make these points more clear and justify our choice of the definition of Z_3, we will have proved in a new, probably much easier, way the result obtained by Schwinger [28] in 1966, that the electric and magnetic charges acquire the same renormalization due to vacuum polarization. In fact Schwinger proved this under the assumption of the stronger quantization condition for the charges (which would allow the change of the position of the string) and assuming that the string is not parallel to the line connecting the charges. Here, in contrast, we make use of the string independence of the theory.

Consider the sum of all graphs to all orders contributing to a physical process. for example e-e scattering, with all external legs amputated according to the reduction formula, and on shell. According to the theorem proved in section 3, this sum is n independent. Since we consider the sum of graphs to all orders, we can replace every gauge boson propagator by a full propagator. Then, each (AA) and (BB) propagator has the form given above. We assume that the string dependent bit will cancel according to the theorem and the full propagator will be effectively proportional to $g_{\mu\nu}$. If this is true, then the (AA) and (BB) full propagators look effectively the same, which means that they are multiplicatively renormalizable when embedded in a sum of graphs which is n independent.

We define $Z_3=1+C(0)+D(0)$. We also define renormalized (AA) and (BB) propagators by

$$D'^{11}_{R\mu\nu} = D'^{11}_{\mu\nu}/Z_3 \qquad \text{(IV 4.8)}$$

$$D'^{22}_{R\mu\nu} = D'^{22}_{\mu\nu}/Z_3 \qquad \text{(IV 4.9)}$$

and renormalized charges by

$$\sqrt{Z_3}e = e_R \qquad \text{(IV 4.10)}$$

$$\sqrt{Z_3}g = g_R \qquad \text{(IV 4.11)}$$

This amounts to defining a renormalized photon field by

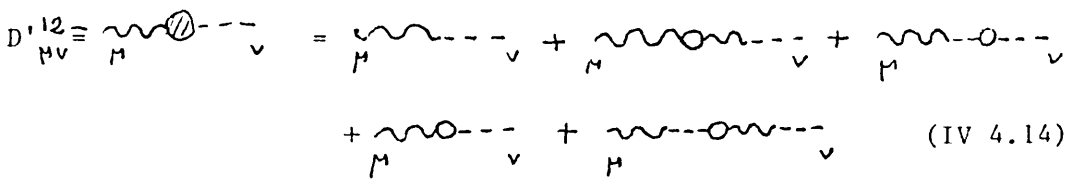
$$\sqrt{Z_3}\, A^{\mu}_R = A^{\mu} \qquad \text{(IV 4.12)}$$

$$\sqrt{Z_3}\, B^{\mu}_R = B^{\mu} \qquad \text{(IV 4.13)}$$

We see that the renormalization of these propagators can be absorbed into a renormalization of the charges at the vertices connected through these propagators. Both charges are renormalized in the same way due to vacuum polarization.

Once the contribution of vacuum polarization to charge renormalization has been defined in such a way as to absorb the photon field renormalization, the renormalization of the mixed propagator is also bound to be the same as that of the (AA) and (BB) propagators if we do not want to introduce counterterms that would spoil Lorentz invariance. This is so since the A and B fields are essentially one field, the photon field. We turn now to the computation of the full mixed propagator in order to see whether this renormalization is sufficient to remove the infinity of the residue of the pole at $k^2 = 0$

The full mixed propagator (AB), denoted $D'^{12}_{\mu\nu}$ includes the following contributions



(IV 4.14)

The three first terms give a contribution proportional to the tree mixed propagator

$$-\frac{\epsilon_{\mu\nu\rho\sigma} n^\rho K^\sigma}{K^2 (n\cdot K)}[1 + C(\vec{k}^2) + D(\vec{k}^2)] \qquad \text{(IV 4.15)}$$

The last two terms are identically zero. The reason for that is that all graphs contributing to $F(\vec{k}^2)$ have, because of Furry's theorem [21], an odd number of mixed propagators and therefore an odd number of $\epsilon_{\mu\nu\rho\sigma}$ terms. An odd number of $\epsilon_{\mu\nu\rho\sigma}$ cannot be contracted completely. One of them, at least, should survive the contractions. But by the n independence theorem, $\epsilon_{\mu\nu\rho\sigma}$ cannot appear in the final answer for $\langle T(J_\mu K_\nu)\rangle$. Hence

$$F(\vec{k}^2)=0 \qquad \text{(IV 4.16)}$$

This argument is essentially due to J.Strathdee. Since $F(\vec{k}^2)$ is zero the mixed propagator is multiplicatively renormalizable and has the exact correction in order to have a finite residue at $\vec{k}^2=0$ after renormalization. We see that (technically) we could have defined the photon field (and charge) renormalization starting from the mixed propagator. We also see that, as it happens with the (AA) and (BB) propagators, the mixed propagator renormalization can be absorbed into equal renormalization of the electric and magnetic charges at the vertices connected by the mixed propagator.

If $F(\vec{k}^2)$ were not zero (for instance in the presence of dyons) a new term

$$-T_{\mu\nu}/(nk)^2 F(\vec{k}^2) \qquad \text{(IV 4.17)}$$

would contribute to the full mixed propagator. To this term would apply the same discussion given previously for the corresponding terms in the (AA) and (BB) propagators.

We can check now that this definition of Z_3 is the correct one

and consistent with the reduction formula for cutting external photon legs of Green functions in order to obtain S-matrix elements. We consider a process in which, among other things happening, an external photon line is attached to a charged particle line. The reduction formula, as far as the photon leg is concerned, takes the form:

$$1/\sqrt{Z_3}\, V^{\alpha\mu}_{ext}\, D^{-1\,ab}_{\mu\nu} \langle\, V^{b\nu}\, \psi\bar{\psi}\ldots \,\rangle \qquad \text{(IV 4.18)}$$

where $V^1_\mu = A_\mu$, $V^2_\mu = B_\mu$, $D^{-1\,ab}_{\mu\nu}$ is the inverse propagator as a 2x2 matrix and V^α_{ext} is the wave function for the V^α field, satisfying the equation

$$D^{-1\,ab}_{\mu\nu} V^{b\nu}_{ext} = 0 \qquad \text{(IV 4.19)}$$

The solution can be taken in the form

$$e^{-ikx}\, f^{a\nu} \qquad \text{(IV 4.20)}$$

with $f^{a\nu}$ orthogonal to n^μ and k^μ and f^1 orthogonal to f^2. f^1 and f^2 are not zero only if $k^2 = 0$ so that A and B really describe photons. Because the action of V^α_{ext} on the inverse propagator creates a zero at $k^2 = 0$, only corrections to (AA) and (BB) propagators having a pole at $k^2 = 0$ can contribute to Z_3. This guarantees that the string dependent terms proportional to $T_{\mu\nu}(nk)^{-2}$ in the full (AA) and (BB) propagators do not contribute to vacuum polarization, consistently with the definition we gave before.

5. Proper vertices and fermion propagators

We consider first the sum of proper vertex parts with an A field external leg and two electrically charged external matter field legs. We regard it as embedded in a sum of larger graphs with all kinds of corrections to all orders and with their external legs cut and on shell. According to the theorem of n independence, all string dependence from the latter sum of graphs drops. Since the sum of proper ver-

tex parts contains all graphs to all orders we can replace all gauge boson propagators with full ones. We know that all n dependence of this full vertex will disappear as a part of the extensive cancellations that take place in the larger graph. For example, some n dependence in the vertex under consideration might cancel some n dependence in another part of the larger graph. Now, after neglecting terms that should cancel, the vertex, when

$$p' \to p \qquad \text{(IV 5.1)}$$

with

$$p^2 = m^2 \qquad \text{(IV 5.2)}$$

(p is the momentum of the fermion line and m the fermion mass) will have the form

$$G \not{p} \qquad \text{(IV 5.3)}$$

where G is a constant.

Analogous things happen in connection with the full fermion propagator. In fact there is a correspondence between graphs contributing to the fermion proper self energy and the proper vertex part. This correspondence is the usual Ward identity. When the fermion line is on shell the full propagator behaves, neglecting n dependence that will cancel, like

$$H/(\not{p}-m) \qquad \text{(IV 5.4)}$$

The Ward identity, if it is respected by the cancellations, guarantees that

$$G=H \qquad \text{(IV 5.5)}$$

So, in a sense, the infinity of the vertex that survives the n cancellation will be cancelled by the infinities in the overall scale of the full nearby propagators that survive the n cancellations. In

any on shell sum of graphs to all orders and amputated according to the reduction formula, the fermion propagators and the A-fermion-fermion vertices "renormalize" each other and the infinities of the full gauge boson propagators are absorbed into charge renormalization. This is exactly the situation in ordinary QED if we forget the lack of complications due to the string.

Of course, we should remember that we have already defined the wave function renormalization of the full fermion propagator. It is an n dependent (in general) Z_2 that we have used in order to cut the n dependent external fermion legs, according to the reduction formula. After these legs are cut, the S-matrix becomes n independent and the rest of the arguments are applicable. The two pictures, in external legs and inside the graph, that we have described, approach each other if we adopt the following formal renormalization definitions. The renormalized full ψ_1 propagator S'_{1R} is defined by

$$S'_{1R} = S'_1 / Z_2 \qquad \text{(IV 5.6)}$$

with S'_1 the unrenormalized full ψ_1 propagator, and the renormalized full proper $A\text{-}\psi_1\text{-}\psi_1$ vertex $\Gamma^1_{\mu R}$ is related to the unrenormalized one Γ^1_μ by the relation

$$\Gamma^1_{\mu R} = Z_2 \Gamma^1_\mu \qquad \text{(IV 5.7)}$$

The charges do not get any new renormalization beyond the one coming from vacuum polarization.

With these definitions we cut the external legs correctly, we do not introduce any additional charge renormalization as we saw happening inside the graphs and we do not destroy the balance of infinities between the vertex and the propagators.

These results could have been obtained formally by applying the Ward identity relating the proper vertex and the fermion proper self-

energy, while ignoring the n dependence. The result would be the old $Z_1=Z_2$ relation of QED (Z_1 is the vertex renormalization) which seems to be effectively correct.

The $B-\psi_2-\psi_2$ vertex and ψ_2 propagator can be treated in exactly the same way.

Fermion mass renormalization presents no problem. A renormalized mass can be defined by

$$m_R=m+\delta m \qquad \text{(IV 5.8)}$$

and δm can be fixed in such a way that the n independent pole of the full fermion propagator is found in the right place.

A new complication that arises in this theory is the $A-\psi_2-\psi_2$ and $B-\psi_1-\psi_1$ vertices. These vertices do not exist at the tree level, but are created in higher orders. If these proper vertices are overall divergent and need an overall subtraction, the theory is in trouble since we have no counterterms for them. Fortunately, these graphs are finite. We see this as follows. Graphs contributing to the $B-\psi_1-\psi_1$ proper vertex parts are like the ones in fig.1. The Ward identity relates the sum of these graphs with a zero momentum photon, to the derivative of the graph of fig.2 which is zero by Furry's theorem [21]. This means that we do not make an overall subtraction of these proper vertices.

6. Subtraction and finiteness

We consider the sum of all graphs to all orders contributing to a given physical process, with all the external legs cut and replaced with wave functions according to the reduction formula. We know that at this stage the strings cancel. We contemplate the renormalizations described in the previous sections at the same time to all orders. The renormalization constants are now multiple infinite power series in the coupling constants. In QED we get something like that to all or-

ders but with the important difference that we can start from lower orders and construct the whole series in steps. Here we cannot say the same since at this stage we only know that the n cancellation occurs only when we sum an infinite series of graphs. Also in QED each graph either is primitively divergent and needs an overall subtraction or is not and is finite given that in both cases all self-energy and proper vertex part insertions are made finite in a previous step. Here the difference is that there is no previous step. All steps have to be taken at the same time.

If we have a look at the Feynman rules of Zwanziger's theory we can see that they give rise to graphs with the same structure as the graphs in QED and with the same kind of infinities. In this theory we also have Ward identities of the same form as in QED. These two facts guarantee that one subtraction, as described, is enough to make the theory finite. The complication of this theory is due to the fact that we have in finite orders infinities that need counter terms with different tensor structure than the terms present in the Lagrangian, because of the n dependence, and not to the presence of worse (let us say quadratic) divergences. After the n cancellation counterterms of this kind are not needed anymore.

7.Lorentz invariance of the renormalized theory

According to the renormalization program discussed in the previous sections, we renormalize each vector boson propagator by dividing with an n independent constant Z_3, and both charges by multiplying the bare ones with $\sqrt{Z_3}$. Each fermion propagator is renormalized by dividing with a (possibly n dependent) constant Z_2. Multiplication with the same constant renormalizes the vertex of two fermion lines and one line of the vector field coupling to the fermion. There is also a mass counter term.

Among all these renormalization constants, only the wave function renormalization of the fermions is (in general) n dependent. This should be so in order to remove the n dependent external leg corrections and leave the S-matrix n independent. By renormalizing the corresponding vertices by multiplying them with the inverse of this n dependent constant, we make sure that the whole subtraction scheme does lead to a Lorentz invariant theory.

Formally the Lorentz invariance of the renormalized theory can be checked directly. Expressing the Lagrangian in terms of the renormalized quantities, we see that Z_2 does not appear in the coupling of the fermions to the gauge fields. The appearence of Z_2 in the fermionic Lagrangian does not affect the closed particle loop sector. With rescaling of the photon field the proof of n independence of Green functions of the currents can be trivially repeated. For the S-matrix we need again, of course, the LSZ reduction.

8.Charge renormalization and Dirac's quantization condition

With the definitions

$$\sqrt{Z_3}\, e = e_R \qquad \text{(IV 8.1)}$$

and

$$\sqrt{Z_3}\, g = g_R \qquad \text{(IV 8.2)}$$

if we keep e_R and g_R finite and fixed, the whole cuttof dependence goes to the bare charges e and g. But the bare charges satisfy Dirac's quantization condition

$$eg = 2\pi\hat{n} \qquad \hat{n}=0,\pm1,\pm2,\ldots \qquad \text{(IV 8.3)}$$

For our program the Dirac quantization condition is needed for the bare charges, given that the proof of n independence of the S-matrix uses

this condition in an unavoidable way, and the whole subtraction scheme is based on this n independence. However, as argued by Schwinger [28], the theory should be consistent at the level of fields as well as at the level of particles. The ordinary quantum mechanics argument of Dirac applies to the renormalized charges and necessitates the quantization condition for them in order that the quantum particle theory be consistent. Both requirements can be satisfied if the bare charges diverge as the cutoff goes to infinity. We believe that this is not absurd given our experience with QED. In this case we require that the renormalized charges satisfy the condition and we also leave the bare ones to go to infinity in such a way that they also satisfy the condition. This can be done if we introduce a discrete cutoff, which is allowed to take only those values for which $eg(2\eta)^{-1}$ is an integer. In a sense we renormalize the integer present in Dirac's condition and let the bare $\hat{n}$ go to infinity through integer values. If it should happen, for some reason, that the bare charges remain finite the theory would still make sense if Z_3 is a ratio of appropriate integers.

9.The expansion in closed fermion loops

Up to this point we have been considering sums of graphs to all orders, because our renormalization scheme is based on the n cancellation. It is true nevertheless, as we already have seen many times, that the n cancellation theorem asserts something even stronger, namely that this cancellation takes place even if we do not sum all graphs contributing to a given process, but only the graphs with a given number of closed charged particle loops and all radiative corrections.

Everything we have said using the sum of all graphs carries through if we consider the sum of graphs with a fixed number of closed fermion loops and all radiative corrections. The only reason we did not do it from the beginning is that, in our opinion, the argument to all

orders can be presented with less effort. Of course now the contributions to the renormalization constants will be restricted.

Perhaps the use of the closed fermion loop expansion can make the discussion of section 6 a little bit less academic. Using this expansion the necessary renormalization constants can be computed, in principle. This expansion defines a sequence of steps. Each step still requires a summation of an infinite number of graphs, but, at least, this way a "previous step" exists. Taking the next step involves an infinite amount of labor which now is well defined. An infinite number of such steps should reproduce the renormalization constants of the full theory as a multiple power series in the coupling constants.

It should be noted that the expansion in closed fermion loops is not a reasonable one, since the number of closed fermion loops does not have much to do with the size of the contribution. However it is technically satisfying that each sector of the theory which is Lorentz invariant can be made finite (modulo our assumptions).

10. Remarks

a) In order to avoid misunderstandings we would like again to stress the assumptions we have made, which we believe are very reasonable : 1) The infinite summation and the removal of the regulators are interchangeable. 2) The string dependent terms proportional to $T_{\mu\nu}(nk)^{-2}$ in the full (AA) and (BB) gauge boson propagators will cancel completely. 3) The Ward identities are respected by the string cancellations.

b) Whether or not the string dependent terms in the full diagonal gauge boson propagators cancel, the equal renormalization of the two charges, due to vacuum polarization, remains true. Only the finiteness of the theory depends on this cancellation.

c) It does not look obvious that the renormalized charges will be the experimentally measurable ones at large distances. This will be on-

ly true if a low energy theorem holds stating that at zero momentum transfer the full S-matrix element in charge-charge and monopole scattering gets contribution only from the single gauge boson graph with all gauge boson propagator corrections and the string terms absent. We believe that such a theorem should be true if the assumption of exact cancellation of the string dependent terms is correct.

d) The infinite constant H in (IV 5.4) is not equal to Z_2. Since it gets cancelled everywhere we do not need to know how much it is. The only renormalization constants whose knowledge is necessary for the closed fermion loop sector of the theory are Z_3 and δm. Z_2 is only needed in external fermion lines. These constants are, in principle, determinable unambiguously. No assumptions about the way in which the strings cancel enter their determination.

e) The fact that the Zwanziger Lagrangian (without matter fields) gets a counterterm proportional to itself (the proportionality constant being Z_3-1) guarantees that in the renormalized theory there is just one photon, like in the bare theory.

f) If the Ward identities happen not to be respected by the string cancellation, an extra renormalization of the two charges will be needed. This does not violate Lorentz invariance if the new renormalization constant is string independent and a consistent way of computing it is found.

g) We have covered explicitly only the case of spin $\frac{1}{2}$ matter fields with only one type of charge. The only difference, as far as we can see, that the inclusion of dyons might give rise to is the F term in the full mixed gauge propagator. This F would still be zero, if independent electric and magnetic charge conjugation is a symmetry. We have not dealt with the spin 0 matter field case.

11. Conclusions

In this chapter we exhibited a scheme of renormalization of the local quantum field theory of spin $\frac{1}{2}$ charges and spin $\frac{1}{2}$ monopoles in interaction with the electromagnetic field. The two types of charge are equally renormalized. The resulting theory is finite if our assumptions are correct. The result of Brandt, Neri and Zwanziger concerning the Lorentz invariance of the closed matter loop sector of the theory remains intact after renormalization. The Lorentz non invariant S-matrix of the bare theory is rendered finite and Lorentz invariant after renormalization only if the LSZ reduction is used in spite of infrared divergences. Although the discussion is rather academic it shows that the independent constants of the Lagrangian are sufficient to absorb, in principle, the ultraviolet divergences. One naturally wonders whether there is an example that can be worked out explicitly, following these lines. The answer to this question is to be found in the next chapter.

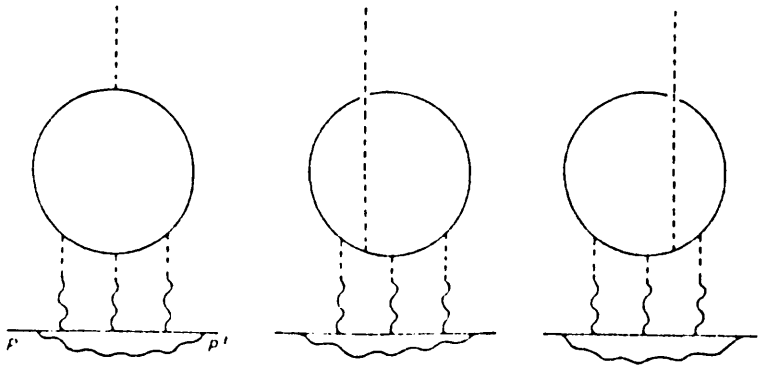


Figure 1

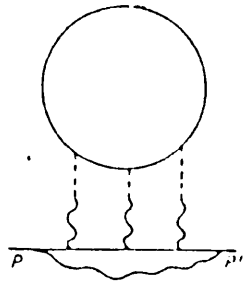


Figure 2

V RENORMALIZATION OF THE QEMD OF A DYON FIELD

1. Introduction

The local quantum field theory of electric charges and magnetic monopoles, proposed by Zwanziger, depends on a fixed direction n , the direction of the old Dirac string. This last dependence, a remnant of the string, has been the source of many difficulties. It violates explicitly Lorentz invariance, at the Lagrangian level at least, and spoils the renormalizability of Zwanziger's theory, in the conventional sense of the word.

Brandt, Neri and Zwanziger [5] using the language of path integrals, presented a formal proof of the restoration of Lorentz invariance in the full Green functions of the gauge invariant electric and magnetic currents. Based on this work, we presented in the last two chapters a program of regularization and subsequently subtraction of the ultraviolet infinities of Zwanziger's theory. This subtraction scheme assumes the cancellation of the string dependence (n dependence) which is expected to occur only after a summation of an infinite number of Feynman diagrams, provided the Dirac quantization condition between the electric and magnetic charges of the matter fields is satisfied. Consequently, this subtraction scheme cannot be implemented order by order in perturbation theory, before the Brandt-Neri-Zwanziger cancellation occurs. Besides that, perturbation theory does not make much sense in the strong coupling charge-monopole quantum electromagnetodynamics (QEMD).

There is however a special case in which this cancellation does happen in finite orders of perturbation theory, and perturbation theory can be made applicable by an appropriate choice of the values of the coupling constants. This is the case of a dyon field carrying electric charge e and magnetic charge g and interacting with the electro-

magnetic field described by the pair of electric and magnetic vector potentials A_μ and B_μ. In this case the "dual invariant" form of Dirac's condition

$$e_i g_j - g_i e_j = 4\pi \hat{n}_{ij} \ , \ \hat{n}_{ij} \text{ integer} \qquad (V\ 1.1)$$

is trivially satisfied and the electric and magnetic charges are unrestricted. The same is true for all cases where $\hat{n}_{ij}=0$.

Our purpose in this chapter is to examine the renormalizability of the QEMD of a dyon field. The discussion will be limited to the one loop level, but we cannot forsee any problems in extending it to higher orders. We will try to be as explicit as possible in order to illustrate the program of renormalization of monopole theories as discussed in abstract terms in chapter IV. The present discussion by no means substitutes the one given in the previous chapter since the case of a single dyon field is much simpler than the general one. Nevertheless, in this context a very clear description can be given of the problems involved in the general case.

We will see that for the specific case at hand, finite answers can be obtained (when the program of the previous chapter is followed) only when a given process is considered. Even in the exceptional case of a single dyon field in which the n cancellation occurs in each order of the loop expansion, Zwanziger's theory is not conventionally renormalizable in the sense that all infinities of the primitively divergent graphs cannot be absorbed into a redefinition of the coupling constants and field renormalizations in the original Lagrangian. Only when the graphs contributing to a given process are summed and some string dependent terms cancel between different graphs, a finite expression is obtained. In the particular case considered, the on shell condition on Green functions is not strictly necessary for finiteness.

We believe that QEMD of a dyon is a unique example of a theory

which is not conventionally renormalizable, is not only on shell finite (as conjectured for some supergravity theories) but can be made finite at the Green function level (except of the two point function of the vector potentials) in each order of the loop expansion. In chapter IV it has been argued that the same is true of the general charge-monopole theory but only after a summation of an infinite number of graphs corresponding to the Brandt-Neri-Zwanziger Lorentz invariance restoration and at the level of the renormalized S-matrix (assuming that such an object can be given a meaning, in spite of infrared divergences). The present example demonstrates that the infinite graph summation and the on shell condition are not strictly necessary, but only non-minimal requirements due to the complexity of the problem in its generality.

2. Definition of the theory

The action that we consider here is (without gauge fixing)

$$S=S_1+S_2 \tag{V 2.1}$$

with

$$S_1=-\tfrac{1}{2}\int d^4x\left\{[n\cdot(\partial\wedge A)]\cdot[n\cdot(\partial\wedge B)^d]+[n\cdot(\partial\wedge A)]^2+(A\to B, B\to -A)\right\} \tag{V 2.2}$$

and

$$S_2=\int d^4x\,\bar{\psi}(i\not{\partial}-m+e\not{A}+g\not{B})\psi \tag{V 2.3}$$

where A_μ and B_μ are independent vector potentials, n_μ an arbitrary but fixed 4-vector with $n^2=1$, and ψ a dyon spin $\frac{1}{2}$ matter field with electric charge e, magnetic charge g and mass m. Our notation is the one used in the previous chapters which suppresses Lorentz indices as much as possible.

We choose to work in Landau gauge ($\partial\cdot A=\partial\cdot B=0$) which shows better

the similarity of the theory with ordinary QED. In this gauge with the notation

$$V^1_\mu = A_\mu, V^2_\mu = B_\mu \quad \text{and} \quad \epsilon^{12} = -\epsilon^{21} = 1 \tag{V 2.4}$$

the propagators of the gauge fields can be written compactly in momentum space as

$$D^{ab}_{\mu\nu} = \int d^4x e^{ik\cdot x} \langle 0| T V^a_\mu(x) V^b_\nu(0) |0\rangle = [(-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2})\delta^{ab} - \frac{1}{\eta\cdot k}\epsilon_{\mu\nu\rho\sigma}\eta^\rho k^\sigma \epsilon^{ab}] \frac{1}{k^2 + i\epsilon} \tag{V 2.5}$$

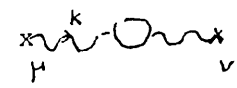
Due to the antisymmetry in its a,b indices the mixed $(V^a_\mu V^b_\nu)(a\neq b)$ propagator, for the case of a dyon field (or whenever $\hat{n}_{ij}=0$) does not contribute between two matter field lines and this accounts for the special characteristics of the example under study. For example for each graph like the one in fig.1 there is one which is exactly opposite given in fig.2. This means that, with the exclusion of the processes involving external photon lines, the mixed propagators, in which the whole n dependence resides, can be omitted from the Feynman rules. Omitting them however, instead of being a simplification, leads to enormous difficulties in renormalizing the theory. Higher order corrections induce a coupling between the A_μ and B_μ vector fields. The infinities of the thus created $(A_\mu B_\nu)$ propagator cannot be removed in an obvious way. This means that although for the processes in question the string is absent, the construction of appropriate counterterms is not trivial. We will return to this point later.

Wavy lines correspond to the A field, broken lines to the B field, continuous lines to the dyon.

3. Vacuum polarization

We start our discussion of the renormalization of the QEMD of a dyon field with the vacuum polarization, following, as closely as possible, the program set up in the previous chapter.

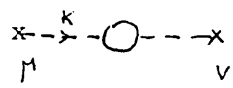
The one loop Green function of two electric currents, represented by



is of the form

$$i(g_{\mu\nu}k^2-k_\mu k_\nu)C(k^2). \qquad \text{(V 3.1)}$$

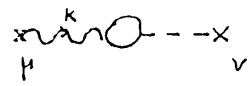
The one loop Green function of two magnetic currents, represented by



is of the form

$$i(g_{\mu\nu}k^2-k_\mu k_\nu)D(k^2) \qquad \text{(V 3.2)}$$

The one loop Green function of one electric and one magnetic current, represented by



is of the form

$$i(g_{\mu\nu}k^2-k_\mu k_\nu)F(k^2). \qquad \text{(V 3.3)}$$

The crosses in these graphs mean that the external legs are amputated. The scalar functions C,D and F do not depend on n and are given by

$$C(k^2)=e^2I(k^2) \qquad \text{(V 3.4)}$$

$$D(k^2)=g^2I(k^2) \qquad \text{(V 3.5)}$$

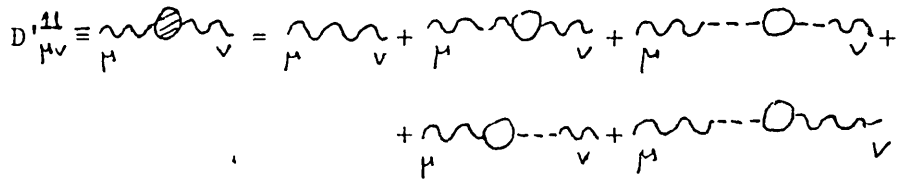
$$F(k^2)=egI(k^2) \qquad \text{(V 3.6)}$$

where

$$I(k^2)=-(12\pi^2)^{-1}\left\{\ln(\Lambda^2/m^2-6\int_0^1 dz\,z(1-z)\ln[1-\frac{k^2}{m^2}z(1-z)]\right\} \qquad \text{(V 3.7)}$$

In (V 3.7) Λ is a regulator mass. In this simple case we can use Pauli-Villars regulators. The more sophisticated regularization of chapter III does not seem to be necessary.

The (AA) propagator, up to one loop, denoted $D'^{11}_{\mu\nu}$ has the following contributions:



(V 3.8)

The result is

$$D'^{11}_{\mu\nu} = -\frac{g_{\mu\nu}}{k^2}\left[1+C(k^2)+D(k^2)\right]+\frac{T_{\mu\nu}}{(n\cdot k)^2}D(k^2) \qquad \text{(V 3.9)}$$

where

$$T_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu \qquad \text{(V 3.10)}$$

Terms proportional to k_μ have been dropped because they do not contribute by current conservation to gauge invariant quantities.

Similarly

$$D'^{22}_{\mu\nu} = -\frac{g_{\mu\nu}}{k^2}\left[1+C(k^2)+D(k^2)\right]+\frac{T_{\mu\nu}}{(n\cdot k)^2}C(k^2) \qquad \text{(V 3.11)}$$

$$D'^{12}_{\mu\nu} = -\frac{\epsilon_{\mu\nu\rho\sigma}n^\rho k^\sigma}{k^2(n\cdot k)}\left[1+C(k)+D(k)\right]-\frac{T_{\mu\nu}}{(n\cdot k)^2}F(k^2) \qquad \text{(V 3.12)}$$

$$D' = \frac{\epsilon_{\mu\nu\rho\sigma}n^\rho k^\sigma}{k^2(n\cdot k)}\left[1+C(k)+D(k)\right]-\frac{T_{\mu\nu}}{(n\cdot k)^2}F(k^2) \qquad \text{(V 3.13)}$$

We see that the one loop corrected gauge propagators are string dependent and not multiplicatively renormalizable. They contain a piece proportional to the tree gauge boson propagator plus string dependent pieces with different infinite coefficients. This makes obvious the fact that the theory cannot be perturbatively renormalizable in the strict sense, i.e. without the introduction of counterterms of a different form than the terms originally present in the Lagrangian, unless the string dependent terms cancel exactly in all physical proc-

esses. The introduction of new types of counterterms is dangerous for the Lorentz invariance of the theory and following the program of chapter IV we will not take this approach.

It will be shown below that the string dependent pieces in the corrected propagators of eqs. (V.3.11) to (V.3.13) will actually cancel in the computation of Green functions corresponding to different processes. Given this, it is clear that the photon wave function renormalization should be defined in terms of the string independent common coefficient of the pieces of the gauge propagators having the same structure as the tree ones.

We define

$$Z_3 = 1+C(0)+D(0) = 1-(e^2+g^2)(12\pi^2)^{-1}\ln(\Lambda^2/m^2) \qquad \text{(V 3.14)}$$

In terms of Z_3 we define renormalized gauge boson propagators by

$$D'^{ab}_{R\,\mu\nu} = D'^{ab}_{\mu\nu} Z_3^{-1} \qquad \text{(V 3.15)}$$

and renormalized charges by

$$\sqrt{Z_3}\, e = e_R \ , \quad \sqrt{Z_3}\, g = g_R \qquad \text{(V 3.16)}$$

We see that both charges are renormalized in the same way due to vacuum polarization. Definition (V 3.15) is equivalent to the following definition of a renormalized photon field

$$\sqrt{Z_3}\, A^\mu_R = A^\mu \ , \quad \sqrt{Z_3}\, B^\mu_R = B^\mu \qquad \text{(V 3.17)}$$

These definitions amount to rewriting the Lagrangian in the form

$$L = -\tfrac{1}{2}\{[n\cdot(\partial\wedge A_R)]\cdot[n\cdot(\partial\wedge B_R)^d] + [n\cdot(\partial\wedge A_R)]^2 + (Z_3-1)[n\cdot(\partial\wedge A_R)]\cdot[n\cdot(\partial\wedge B_R)^d]$$
$$+ (Z_3-1)[n\cdot(\partial\wedge A_R)]^2 + (A_R\to B_R\,, B_R\to -A_R)\} + \bar\psi(i\partial\!\!\!/ - m + e_R A\!\!\!/_R + g_R B\!\!\!/_R)\psi \qquad \text{(V 3.18)}$$

We now turn to a discussion of specific processes in order to demonstrate the cancellation of the extra string dependent pieces in the

corrected gauge propagators whose infinities we were unable to subtract.

First we consider a process in which, among other things, an external photon is attached to a dyon line. The one loop graphs contributing to the process in question, as far as the external photon line is concerned (vertex corrections are irrelevant for this discussion) are shown in fig.3 in terms of the corrected bare propagators. Summing the contributions of graphs a and c we get

$$\frac{\gamma_\mu}{K^2}[1+C(k^2)+D(k^2)]e+\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\nu n^\rho K^\sigma}{K^2(n\cdot K)}[1+C(k^2)+D(k^2)]g-\frac{T_{\mu\nu}\gamma^\nu}{(n\cdot K)^2}eD(k^2)+\frac{T_{\mu\nu}\gamma^\nu}{(n\cdot K)^2}gF(k^2) \quad \text{(V 3.19)}$$

Now eqs.(V 3.5) and (V 3.6) give

$$eD(k^2)-gF(k^2)=0 \quad \text{(V 3.20)}$$

This means that the last two terms in eq (V 3.19) cancel. Similarly, the sum of graphs b and d is

$$\frac{\gamma_\mu}{K^2}[1+C(k^2)+D(k^2)]g-\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\nu n^\rho K^\sigma}{K^2(n\cdot K)}[1+C(k^2)+D(k^2)]e \quad \text{(V 3.21)}$$

using the identity

$$gC(k^2)-eF(k^2)=0 \quad \text{(V 3.22)}$$

We see, as promised, that the string dependent pieces which have not been taken care of by our renormalization definitions actually exactly cancel without need for extra counterterms and without causing any trouble. Now eq. (V 3.19) in terms of the renormalized charges takes the form

$$\text{Sum of [a and c]} \equiv G_{a+c}=\sqrt{Z_3}\,\frac{1+C(K^2)+D(K^2)}{Z_3}\left[\frac{\gamma_\mu}{K^2}e_R+\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\nu n^\rho K^\sigma}{K^2(n\cdot K)}g_R\right]$$

$$=\sqrt{Z_3}\left[1+\frac{e_R^2+g_R^2}{2\pi^2}\int_0^1 dz\,z(1-z)\ln\left(1-\frac{K^2}{m^2}z(1-z)\right)\right]\left[\frac{\gamma_\mu}{K^2}e_R+\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\nu n^\rho K^\sigma}{K^2(n\cdot K)}g_R\right] \quad \text{(V 3.23)}$$

Similarly eq.(V 3.21) takes the form

$$\text{Sum of [b and d]} \equiv G_{b+d}=\sqrt{Z_3}\,\frac{1+C(K^2)+D(K^2)}{Z_3}\left[\frac{\gamma_\mu}{K^2}g_R-\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\nu n^\rho K^\sigma}{K^2(n\cdot K)}e_R\right]$$

$$=\frac{1}{3}\left[1+\frac{e_R^2+g_R^2}{2\pi^2}\int_0^1 dz\, z(1-z)\ln\left(1-\frac{K^2}{m^2}z(1-z)\right)\right]\left[\frac{\gamma_\mu}{K^2}g_R\frac{\epsilon_{\mu\nu\rho\sigma}\gamma_\nu n_\rho K_\sigma}{K^2(n\cdot K)}e_R\right] \qquad \text{(V 3.24)}$$

We can now define renormalized Green functions by

$$G^R_{a+c}=G_{a+c}Z_3^{-1/2}, \quad G^R_{b+d}=G_{b+d}Z_3^{-1/2} \qquad \text{(V 3.25)}$$

which are perfectly finite. This last renormalization corresponds to a renormalization of the photon wave function that will be attached to the photon line when the propagators of external photon legs will be amputated according to the standard reduction formula.

Here we should make two remarks. First we need not renormalize at $k^2=0$ and consequently the off-shell Green functions can be made finite. Second the Green functions in question are not gauge invariant. It is therefore natural that they are not string independent. Nevertheless the renormalized S-matrix is, due to the removal of the string dependent external legs by the LSZ reduction formula.

Now let us examine the validity of these two remarks for the same process in the more general case in which electric and magnetic charges may not be carried by the same field. The computation of $D^{ab}_{\mu\nu}$ and the definition of Z_3 (in terms of C, D and F but not in terms of their specific values of the present example) hold for this case too at the one loop level (where no gauge propagators contribute to the electric and magnetic current Green functions) and to all orders (where C, D and F are string independent) as discussed in chapter IV. However the cancellations of the string dependent dangerous terms, as discussed in the context of the studied process, do not take place. The reason is that in general (V 3.20) and (V 3.22) are not satisfied. For example in the absence of dyons $F(k^2)=0$ at the one loop level (since the corresponding graphs are absent) and in fact to all orders (as a consequence of Furry's theorem and the results of [5] as shown in chapter IV). This means that in this case the off-shell Green functions cannot be made finite.

It is also true that they are not string independent, as expected. At this point the LSZ reduction formula is needed for two purposes. First it removes the dangerous string dependent terms whose infinities have not been absorbed by renormalization, given that these terms have no pole at $k^2=0$. Second it renders the theory string independent by amputating the external photon legs. This irrelevance of the terms proportional to $T_{\mu\nu}$ in the full $D'^{ab}_{\mu\nu}$ propagators to the wave function renormalization of the gauge fields proves that, even in the most general case and in the exact theory, the two charges are renormalized in the same way due to vacuum polarization. This was first proved by Schwinger in the context of his first order formulation, and in chapter IV was shown to be true in the context of Zwanziger's local Lagrangian formulation.

We now go on to discuss processes in which one photon is exchanged between two fermion lines. Vertex corrections will be discussed in a later section. As explained in section 2 for processes like this, in which the photon line has both ends landing on a fermion line, mixed string dependent propagators do not contribute. Nevertheless we believe that if we do not draw the corresponding graphs the discussion of renormalization becomes very difficult. This, of course, is true on the condition that we have not evaluated the necessary counterterms from another process. For example in our case by simply rescaling quantities in the same way as before, on the condition that the introduced counterterms are sufficient to remove all infinities in the theory (up to one loop), we should have no problem. Let us pretend for a moment that we do not know the necessary counterterms. We omit the mixed propagators and we draw all the graphs that involve only $D^{11}_{\mu\nu}$ and $D^{22}_{\mu\nu}$. The relevant graphs are shown in fig.4. These graphs rather suggest a different wave function renormalization for A_μ and B_μ but this is still not sufficient given that there is nothing left to subtract the infinities of the last

two graphs. On the other hand, without omitting the mixed propagators the graphs contributing to the process in question can be classified using the corrected gauge boson propagators as in fig.5. The calculation of these graphs gives

$$i(e^2+g^2)/k^2\,[1+C(k^2)+D(k^2)]\bar{u}(p_4)\gamma_\mu u(p_3)\bar{u}(p_2)\gamma^\mu u(p_1) \qquad (V\ 3.26)$$

In terms of the renormalized charges this result reads

$$i\frac{e_R^2+g_R^2}{k^2}\frac{1+C(k^2)+D(k^2)}{Z_3}\bar{u}(p_4)\gamma_\mu u(p_3)\bar{u}(p_2)\gamma^\mu u(p_1)=$$

$$=i\frac{e_R^2+g_R^2}{k^2}\{1+\frac{e_R^2+g_R^2}{2\pi^2}\int_0^1 dz\,z(1-z)\ln(1-\frac{k^2}{m^2}z(1-z))\}\bar{u}(p_4)\gamma_\mu u(p_3)\bar{u}(p_2)\gamma^\mu u(p_1) \qquad (V\ 3.27)$$

We see once more that although we did not take care of the string dependent terms proportional to $T_{\mu\nu}$(containing potentially dangerous infinities) we were able to get a finite answer. We therefore conclude that in discussing vacuum polarization the mixed propagators should be kept before the evaluation of the counterterms. This is rather relevant for higher orders. Also note that the answer is n independent.

The moral drawn from this last exercise is that we should not worry about the string dependent terms of the corrected gauge propagators provided we consider the contribution of these propagators to a set of n independent graphs (a set whose sum is n independent). We also learn that once we consider a set of n independent graphs in the bare theory (like the set of figs 4 and 5) and we renormalize, rescaling quantities by string independent constants (e.g. $\sqrt{Z_3}\,e=e_R$), the renormalized contribution of these graphs will also be n independent. This is a "reasonable subtraction that does not introduce an n dependence" in the language of chapter III. These two remarks are at the heart of the abstract renormalization scheme of the previous chapter.

It is also worth noting that string dependent counterterms, the ones exhibited in (V 3.18) are employed in order to guarantee that the completely n independent set of graphs of fig.4 is subtracted in a way

that maintains its n independence.

We can ask again what happens in the more general case in which the electric and magnetic charges may be carried by different fields. In this case the blobs of fig.5 can represent one loop contributions or contributions to all orders (which are n independent) as discussed before. In the absence of dyons the last two sets of graphs in fig.5 are absent and the answer comes out to be neither finite nor n independent. The dangerous string dependent pieces of the corrected gauge boson propagators do not cancel. This happens simply because we did not consider an n independent set of graphs. For theories more general than the one of a single dyon field, the set of graphs of fig.5 is not n independent. The smallest known set of n independent graphs in the general case contains an infinite number of them; for example the Green function of four currents with a given number of fermion loops and all photon corrections. It has been argued in chapter IV that once the full gauge boson propagators are considered embedded in a set of n independent graphs, the string dependent pieces for which no counterterms are introduced will cancel without creating any problems. However, due to the fact that in the general case the known sets of such n independent graphs contain an infinite number of them, the assumption was made that the cancellation of these pieces is exact and no infinite remainder of them is left. This is the main assumption of the program of chapter IV. It should also be noted that although the set of graphs of fig.3 in the general case is not n independent, and although the cancellation of infinities does not happen, we still managed to get finite answers by going to the S-matrix and noting that the dangerous terms have no pole at $\vec{k}=0$ and therefore could not survive. This indicates that in computing S-matrix elements we have an extra weapon to "kill" unwanted terms.

The conclusion of this section is that the counterterms of eq. (V 3.18) are sufficient for the subtraction of all infinities due to

vacuum polarization effects in the specific case discussed in this chapter, but analogous counterterms for the general case are only sufficient when an n independent process is discussed.

4.Fermion self energy and proper vertex parts

The graphs contributing to the fermion self energy and proper vertex (up to one loop) are shown in fig.6 and 7, respectively. They form n independent sets of graphs due to the antisymmetry of the mixed gauge propagator in its a,b indices (the last two graphs in each of the two sets cancel each other). Therefore their discussion is analogous to the one of ordinary QED. Due to the usual Ward identity the charges do not get any additional renormalization on top of the one they already received from vacuum polarization. So in the interior of each graph each vertex "renormalizes" the adjacent propagators and no counterterms are needed except for a mass counterterm and a renormalization of the external fermion lines which are taken care of by the definition of a renormalized Green function. Alternatively, this renormalization can be viewed as a renormalization of the spinor wave function that substitutes the external propagators according to the LSZ reduction formula. This wave function renormalization (usually denoted Z_2) is gauge dependent and string independent in the particular case examined. It should be noted that in the case of QEMD of one dyon we do not need to go to the S-matrix (as far as the fermion external lines are concerned) in order to get a finite and string independent answer.

In the general case self energy and proper vertex parts are not known to be n independent. They are therefore expected to need counterterms of a different form than the terms existing in the Lagrangian. The situation is analogous to the discussion of the corrected gauge boson propagators of section 3. There we saw that string dependent terms were created, having different tensor structure from the tree

propagators and infinite coefficients whose infinities we were unable to absorb into the parameters of the Lagrangian. When considering an n independent set of graphs, following our discussion of chapter IV, we need not worry about these extra complications as we saw explicitly in section 3. The Ward identity will guarantee that effectively vertices "renormalize" the adjacent propagators except for external fermion lines. In the general case there is no reason for Z_2 to be n independent and nor for it to be sufficient to remove the infinities of Green functions involving external fermion lines. The extra n dependent pieces with string dependent singularities (like $(nk)^{-2}$ in section 3) will generally have different coefficients. In this case only after going to the S-matrix and therefore killing the extra terms (as we did in section 3 for the process of fig.3 in the general case) we can get finite answers. Green functions with external fermion lines are not gauge invariant, unlike the ones with external photon lines, and therefore not generally n independent. In chapter IV we argued, using the gauge invariant path dependent source, that the full renormalized S-matrix is n independent. The argument was analogous to the one given for the gauge invariance of ordinary gauge theories. Also the fermion wave function renormalization was predicted to depend on n like it depends on the gauge in ordinary gauge theories. Classifying the S-matrix among the n independent objects, together with our discussion of the external legs, allows us to conclude that the renormalized S-matrix is finite and n independent. However the proof of n independence of the S-matrix relies on the LSZ reduction and therefore neglects infrared problems which might be very serious. The result of chapter IV therefore should perhaps be regarded as rather formal.

5.Prescription for higher orders

1) Draw all graphs contributing to a given process without omitting any mixed propagators.

2) Treat vacuum polarization insertions according to the method followed for the process of fig.5. The blobs of fig.5 will of course include higher order graphs but will be n independent due to the antisymmetry of the mixed gauge propagator in its a,b indices. The cancellations that occurred for the process of fig.5 will occur again.

3) Treat external photon lines according to the method followed for the process of fig,3. The blobs will contain again higher order terms but no n dependence due to the cancellations between mixed gauge boson propagators. The cancellation of the dangerous terms will take place again. String independent answers will be obtained after amputating the external photon lines according to the LSZ reduction formula.

4) Omit mixed gauge propagators in the graphs not falling in the cases covered in rules 2 and 3.

5) Define renormalized quantities according to the definitions adopted in the previous sections.

6.Concluding remarks

1) The renormalization procedure of the QEMD of the single dyon field, demonstrated explicitly here up to one loop, can be obviously continued to higher orders. The method should be extendable to all cases in which the dual form of Dirac's condition is trivially satisfied.

2) The Lorentz invariance of the renormalized theory follows from the fact that n cancellations in the bare theory are not affected once the rescaling by n independent constants is made. In the more general case in which we might need an n dependent Z_2 the program does not violate this rule given that Z_2 cancels inside the graphs (it is compensated for by the vertices) and in external legs it is needed in

order to remove an existing n dependence. Even if the Dirac condition is satisfied, the unrenormalized Green functions are not n independent. They become so, once we divide by Z_2 (through the LSZ).

3) The dual invariance of the classical theory is preserved by renormalization due to the fact that the same constant, $\sqrt{Z_3}$, renormalizes both gauge potentials and its inverse the two charges.

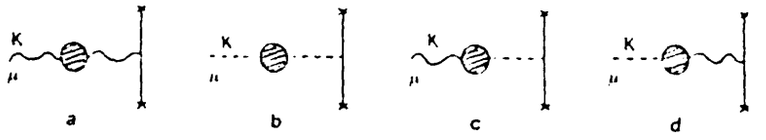


Fig 3

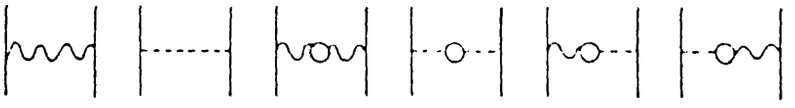


Fig 4

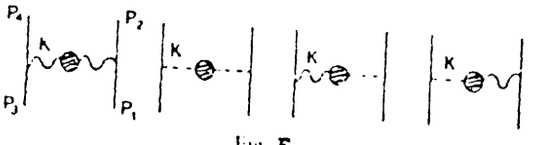


Fig 5

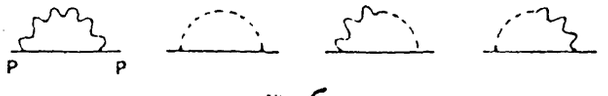


Fig. 6

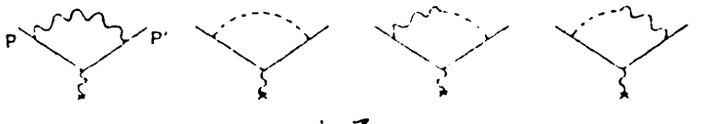


Fig 7.

VI AN ELECTROWEAK MODEL WITH POINT MONOPOLES

1. Introduction

We saw in the previous chapters that Zwanziger's local quantum field theory consists of an appropriate generalization of QED capable of describing the interaction of photons with point electric and magnetic charges. It involves two vector potentials A_μ and A'_μ (notice a change in notation) which couple minimally the one to the electrically and the other to the magnetically charged matter fields.

Zwanziger's Lagrangian, without matter, is given by

$$L = -\tfrac{1}{2} n_\mu F_{\mu\rho} n_\nu F_{\nu\rho} - \tfrac{1}{2} n_\mu F'_{\mu\rho} n_\nu F'_{\nu\rho} - \tfrac{1}{2} n_\mu F_{\mu\rho} n_\nu \tilde{F}'_{\nu\rho} + \tfrac{1}{2} n_\mu F'_{\mu\rho} n_\nu \tilde{F}_{\nu\rho} \qquad \text{(VI 1.1)}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad \text{(VI 1.2)}$$

$$F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu \qquad \text{(VI 1.3)}$$

and

$$\tilde{F}_{\mu\nu} = \tfrac{1}{2} \epsilon_{\mu\nu\rho\sigma} F_{\rho\sigma} \qquad \text{(VI 1.4)}$$

with n_ν a fixed time-like unit 4-vector which violates Lorentz invariance at the Lagrangian level. Lorentz invariance is restored in the full quantum field theory. Although it has a $U(1)_{electric} \times U(1)_{magnetic}$ local gauge symmetry it describes one photon.

Since the time that Zwanziger presented this local Lagrangian our understanding of elementary particle physics has changed considerably. The success of the electroweak unification and of QCD indicates that electromagnetism is not independent of the rest of elementary particle physics but the electromagnetic U(1) comes from the spontaneous breaking of a larger non-abelian gauge group. Therefore it seems natural to

ask what is the generalization of Zwanziger's theory that will take into account the progress made in the rest of physics during the Past decade and discuss the problems that this generalization would give rise to. This is the purpose of the present chapter.

In section 2 we present an extension of the SO(3) model of Georgi and Glashow [29] with point monopoles. The extension of the proof of n independence is given and renormalizability is discussed briefly. In section 3 we present an extension of the standard electroweak SU(2)XU(1) theory. Section 4 contains some conclusions.

2.An SO(3) extended model

The most natural way to extend Zwanziger's theory would be to replace the $U(1)_{cm}$ that comes from the spontaneous breaking of a non-abelian gauge theory with Zwanziger's $U(1)_{electric}XU(1)_{magnetic}$ To be specific let us choose to talk in terms of the SO(3) electroweak model of Georgi and Glashow. The model consists of an SO(3) gauge group with two charged (electrically) gauge bosons W^+,W^- and a neutral one A. It is spontaneously broken down to $U(1)_{em}$ with A as a photon. The breaking is done via a triplet of scalars s^+,s^-,ϕ°, with ϕ° acquiring a vacuum expectation value. The gauge coupling constant is denoted e. The gauge field Lagrangian is

$$L_{gauge}=-\tfrac{1}{2}F^i_{\mu\nu}F^{i\mu\nu}=-\tfrac{1}{2}(\partial_\mu W^+_\nu-\partial_\nu W^+_\mu)\partial^\mu W^{-\nu}-\tfrac{1}{2}(\partial_\mu W^-_\nu-\partial_\nu W^-_\mu)\partial^\mu W^{+\nu}-\tfrac{1}{2}(\partial_\mu A_\nu-\partial_\nu A_\mu)\partial^\mu A^\nu$$
$$+A_\mu[ie(W^-_\nu\partial^\nu W^{+\mu}-W^+_\nu\partial^\nu W^{-\mu}+W^+_\nu\partial^\mu W^{-\nu}-W^-_\nu\partial^\mu W^{+\nu})-e^2(W^+_\nu W^{-\nu}A^\mu-W^{+\mu}W^-_\nu A^\nu)]$$
$$-\tfrac{1}{2}e^2[W^+_\mu W^{-\mu}W^+_\nu W^{-\nu}-W^+_\mu W^{+\mu}W^-_\nu W^{-\nu}] \qquad \text{(VI 2.1)}$$

in terms of the electric charge eigenstates. The scalar Lagrangian is

$$L_{scalar}=\tfrac{1}{2}[\partial_\mu\phi^\circ+ie(W^-_\mu s^+-W^+_\mu s^-)][\partial^\mu\phi^\circ-ie(W^{+\mu}s^--W^{-\mu}s^+)]+$$
$$+(\partial_\mu s^+-ieA_\mu s^++ie\phi^\circ W^+_\mu)(\partial^\mu s^-+ieA^\mu s^--ie\phi^\circ W^{-\mu}) \qquad \text{(VI 2.2)}$$

Consider now another $U(1)_{magnetic}$ commuting with SO(3) with coupling constant g. We connect the two U(1)'s (the kinetic terms) via the "Zwanziger mechanism", namely we replace the

$$F_{\mu\nu}^2 = -\tfrac{1}{2}(\partial_\mu A_\nu - \partial_\nu A_\mu)\partial_\mu A_\nu \qquad \text{(VI 2.3)}$$

term in (VI 2.1) by Zwanziger's Lagrangian (VI 1.1). Monopoles can be introduced via a minimal coupling to A'_μ. This way the model has all the characteristics of a theory of electric and magnetic charge in which the photon comes from a breaking of a non-abelian symmetry. In contrast to the usual case however, the non abelian gauge symmetry $SO(3)XU(1)_{magnetic}$ seems to be explicitly broken down to $U(1)_{electric}XU(1)_{magnetic}$. We propose to investigate the Lorentz invariance (n independence) and the renormalizability of this theory.

For the discussion of n independence, no matter fields except the Higgs will be introduced. We think that no new problems will show up if extra fermions are coupled to the gauge system. Because of the complicated self coupling of the different fields (quartic interactions of W's and scalars) and interactions between them, it is not possible to integrate out all the fields. It seems to be sufficient to keep only the quadratic terms in the neutral fields, to introduce the source terms $J^{\mu\nu}W^+_\mu W^-_\nu$, $J^\mu W^+_\mu s^-$, Js^+s^- and leave unintegrated the neutral fields. The full generating functional is then obtained by using an exponential of appropriately combined functional derivatives with respect to the source terms. So if we show that the simplified action leads to an n independent generating functional, this will have as a consequence that full Green functions of the U(1)XU(1) invariant electric and magnetic currents will be n independent. The functional derivatives with respect to the sources $J^{\mu\nu}, J^\mu, J$ do not introduce an n dependence unless an explicit n is introduced in the interactions that we will create using these derivatives.

The result of the path integration over the electrically charged fields is

$$\exp[-\tfrac{1}{2}\mathrm{tr}\ln D^{\alpha}_{\ b}] \qquad \text{(VI 2.4)}$$

where

$$D^{\alpha}_{\ b} = \begin{pmatrix} \nabla^2 g^{\mu}_{\ \nu} - \nabla^{\mu}\nabla_{\nu} + J^{\mu}_{\ \nu} & ie\phi^{\circ}\nabla_{\nu} + ie(\partial_{\nu}\phi^{\circ}) \\ ie\phi^{\circ}\nabla^{\mu} + J^{\mu} & -\nabla^2 + J \end{pmatrix} \qquad \text{(VI 2.5)}$$

with

$$a,b=0,1,2,3,4 \qquad \mu,\nu=0,1,2,3 \qquad \text{(VI 2.6)}$$

and

$$\nabla_{\mu} = \partial_{\mu} + ieA_{\mu} \qquad \text{(VI 2.7)}$$

We will now derive for $D^{\alpha}_{\ b}$ in (VI 2.5) a formal path integral representation. Define $G^{\alpha}_{\ b}$ through

$$D^{\alpha}_{\ b} G^{b}_{\ c}(x,x') = g^{\alpha}_{\ c}\delta^4(x-x') \qquad \text{(VI 2.8)}$$

where

$$\begin{aligned} g^{\alpha}_{\ b} &= g^{\mu}_{\ \nu} \quad \text{for } a=\mu, b=\nu \\ g^{a}_{\ b} &= -1 = g_{ab} \quad \text{for } a,b=4 \end{aligned} \qquad \text{(VI 2.9)}$$

Introduce formal states $|x\ a\rangle$ and $|p\ a\rangle$ such that

$$\langle x\ a \mid x'a'\rangle = g^{a}_{\alpha'}\delta^4(x-x') \qquad \text{(VI 2.10)}$$

$$\langle p\ a \mid p'a'\rangle = g^{\alpha}_{\alpha'}(2\pi)^4\delta^4(x-x') \qquad \text{(VI 2.11)}$$

$$\langle x\ a \mid p\ a'\rangle = g^{\alpha}_{\alpha'}\exp(-ipx) \qquad \text{(VI 2.12)}$$

corresponding to operators X_{μ} and P_{μ} defined through

$$\langle x|X_\mu = x_\mu \langle x| \tag{VI 2.13}$$

$$\langle x|P_\mu = i\partial_\mu \langle x| \tag{VI 2.14}$$

$$\langle p|X_\mu = -i\partial_\mu \langle p| \tag{VI 2.15}$$

$$\langle p|P_\mu = p_\mu \langle p| \tag{VI 2.16}$$

and

$$[X_\mu, P_\nu] = -ig_{\mu\nu} \tag{VI 2.17}$$

Now define G such that

$$\langle x\ a|G|x'\ a'\rangle = G^{\alpha}_{a}(x,x') \tag{VI 2.18}$$

Using (VI 2.18), eq (VI 2.8) becomes

$$D^{\alpha}_{b}\langle x\ b|G|x'c\rangle = \langle x\ b|x'c\rangle \tag{VI 2.19}$$

Introduce now a spin matrix $\Lambda_{\lambda\rho}$ such that

$$\langle x\ \mu|\Lambda^{\rho}_{\lambda}|x'\nu\rangle = \tfrac{1}{2}(g^{\mu}_{\lambda}\, g^{\rho}_{\nu} + g^{\mu\rho} g_{\lambda\nu})\, \delta^{4}(x-x') \tag{VI 2.20}$$

and

$$\Lambda_{44} = 0 \tag{VI 2.21}$$

Also introduce spin matrices M and $\bar{M}$ such that

$$\langle x\ 4|M^{\rho}_{h}|x'\nu\rangle = \delta_{h4}\, g^{\rho}_{\nu} \quad \text{and all other elements zero} \tag{VI 2.22}$$

$$\langle x\mu|\bar{M}^{4}_{\lambda}|x'\nu\rangle = \delta_{f4}\, g^{\mu}_{\lambda} \quad \text{and all othar elements zero} \tag{VI 2.23}$$

Then with

$$\hat{k} = (0,0,0,0,1)$$

we have

$$\Big\{[P^{\lambda}-eA^{\lambda}(X)](g^{\rho}_{\lambda}-\Lambda^{\rho}_{\lambda})[P_{\rho}-eA_{\rho}(X)]-\Lambda^{\rho}_{\lambda}J^{\lambda}_{\rho}(X)+J(X)+e\phi^{\circ}(X)\hat{k}^{h}\hat{M}^{\rho}_{h}[P_{\rho}-eA_{\rho}(X)]$$

$$+ie\hat{k}^{h}\hat{M}^{\rho}_{h}(\partial_{\rho}\phi^{\circ}(X)+e\phi^{\circ}(X)[P^{\lambda}-eA^{\lambda}(X)]\bar{M}^{f_1}_{\lambda}\hat{k}_{f}+J^{\lambda}(X)\bar{M}^{f_1}_{\lambda}\hat{k}_{f}\Big\}G=-1 \qquad \text{(VI 2.25)}$$

or

$$\tilde{D}(P_{\mu}-eA_{\mu},X_{\mu})G=-1$$

from which

$$-G=\tilde{D}^{-1} \qquad \text{(VI 2.27)}$$

Formally

$$\ln(-G)=\int_{0}^{\infty}\frac{d\tau}{\tau}\exp[i\frac{\tau}{2}\tilde{D}] \qquad \text{(VI 2.28)}$$

We have now

$$\ln\det(-G)=\mathrm{Tr}\ln(-G)=\sum_{\alpha}\int d^{4}x\,\langle x\ a|\ln G|a\ x\rangle \qquad \text{(VI 2.29)}$$

which, with use of (VI 2.28) becomes

$$\ln\det(-G)=\int_{0}^{\infty}\frac{d\tau}{\tau}\sum_{\alpha}\int d^{4}x\langle x\ a|\exp(i\frac{\tau}{2}\tilde{D})|a\ x\rangle \qquad \text{(VI 2.30)}$$

Divide the interval $(0,\tau)$ into 2N segments of length $\Delta\tau$

$$\tau_{0}=0\ ,\ \tau_{1},\ \tau_{2},\ \ldots,\tau_{2N}=\tau;\ \Delta\tau=\tau/2N$$

With these τ-values associate the variables

$$x_{0}=x,p'_{1}\ ,x_{2},p'_{3},\ldots,x_{2N-2},p'_{2N-1}\ ,x_{2N}=x$$

We know that

$$\exp(i\frac{\tau}{2}\tilde{D})=\lim_{N\to\infty}[\exp(i\Delta\tau\tilde{D})]^{N} \qquad \text{(VI 2.31)}$$

Inserting complete sets of states we get

$$\langle x\ a|e^{i\frac{\tau}{2}\tilde{D}}|x\ a\rangle=\lim_{N\to\infty}\prod_{j=1}^{N-1}\sum_{\alpha_{2j}}\int dx_{2j}\cdot\langle x\ a|e^{i\Delta\tau\tilde{D}}|x_{2N-2}\ a_{2N-2}\rangle\ldots\langle x_{2}\ a_{2}e^{i\Delta\tau\tilde{D}}|x\ a\rangle \qquad \text{(VI 2.32)}$$

Consider each factor in (VI 2.32) separately:

$$\langle x_{2j+2}\, a_{2j+2} | e^{i\Delta\tau\tilde{D}} | a_{2j}\, x_{2j}\rangle \tilde{=} \langle x_{2j+2}\, a_{2j+2} | (1+i\Delta\tau\tilde{D}+\ldots) | a_{2j}\, x_{2j}\rangle =$$
$$= \sum_{\alpha_{2j+1}} \int dp'_{2j+1} (2\pi)^{-4} e^{-2i\Delta\tau\, p'_{2j+1}\cdot\frac{(x_{2j+2}-x_{2j})}{2\Delta\tau}} g^{\alpha_{2j+2}}_{\alpha_{2j+1}} \langle a_{2j+1} | e^{i\Delta\tau\tilde{D}(p'_{2j+1}-eA, x_{2j})} | a_{2j}\rangle \qquad \text{(VI 2.33)}$$

Changing variables

$$p_{2j+1} = p'_{2j+1} - eA \qquad \text{(VI 2.34)}$$

we get

$$\langle x_{2j+2}\, a_{2j+2} | e^{i\Delta\tau\tilde{D}} | a_{2j}\, x_{2j}\rangle = \sum_{\alpha_{2j+1}} \int \frac{d^4p_{2j+1}}{(2\pi)^4} e^{-2i\Delta\tau\, p_{2j+1}\cdot(x_{2j+2}-x_{2j})/2\Delta\tau}$$
$$\times\, g^{\alpha_{2j+2}}_{\alpha_{2j+1}} \langle a_{2j+1} | e^{i\Delta\tau\tilde{D}(p_{2j+1}, x_{2j})} | a_{2j}\rangle e^{2i\Delta\tau e A\cdot(x_{2j+2}-x_{2j})/2\Delta\tau} \qquad \text{(VI 2.35)}$$

Define

$$\lim_{N\to\infty} \prod_{j=1}^{N-1} \sum_{\alpha_{2j+1}} \frac{d^4p_{2j+1}}{(2\pi)^4} g^{\alpha_{2j+2}}_{\alpha_{2j+1}} \langle a_{2j+1} | e^{i\Delta\tau\tilde{D}(p_{2j+1}, x_{2j})} | a_{2j}\rangle = [dp(\tau)] \qquad \text{(VI 2.36)}$$

$$\lim_{N\to\infty} \prod_{j=1}^{N-1} \sum_{a_{2j}} dx_{2j} = [dx(\tau)] \qquad \text{with } x_\mu(0) = x_\mu(\tau) = x_\mu \qquad \text{(VI 2.37)}$$

Then (VI 2.30) becomes

$$\ln\det(-G) = \int_0^\infty \frac{d\tau}{\tau} \sum_a \int dx^4 \int [dx(\tau)] \int [dp(\tau)] \exp\left\{-i\int_0^\tau d\tau' [p_\mu(\tau') + eA_\mu(x(\tau'))]\dot{x}^\mu(\tau')\right\} \qquad \text{(VI 2.38)}$$

Consider the exponential

$$\exp\left\{-i\int_0^\tau d\tau' eA_\mu(x(\tau'))\dot{x}^\mu(\tau')\right\} = \exp\left\{-ie\oint_{\Gamma(\tau)} dz^\mu A_\mu(z)\right\} = \exp\left\{-i\int dx^4 A_\mu(x)\tilde{J}^\mu(x)\right\} \qquad \text{(VI 2.39)}$$

where Γ is a closed path satisfying $z_\mu(0) = z_\mu(\tau) = z_\mu$ and

$$\tilde{J}_\mu(x) = e\oint_\Gamma dz_\mu \delta^4(x-z) \qquad \text{(VI 2.41)}$$

is a classical current. We choose for $-\mathrm{tr}\ln D^a_b$ the notation

$$-\mathrm{tr}\ln D^a_b = \int_0^\infty \frac{d\tau}{\tau} \int d\Gamma(\tau) f(\Gamma(\tau)) e^{-i\int dx A_\mu(x)\tilde{J}^\mu(x)} \qquad \text{(VI 2.41)}$$

The result of the path integral over the monopole field is

$$\exp(\omega \mathrm{tr}\ln D')=\exp\{\omega\int_0^\infty ds/s\int d\Gamma'(s)f'(\Gamma'(s))e^{-i\int d^4x B_\mu(x)\tilde{K}^\mu(x)}\} \qquad \text{(VI 2.42)}$$

where ω is a numerical factor depending on the spin of the monopole and $\tilde{K}^\mu(x)$ a classical magnetic current. So the generating functional becomes

$$Z[J]=N\int[dA][dB][d\phi]e^{iS_{\phi^0}+iS_z}\exp\{\tfrac{1}{2}\int_0^\infty\frac{d\tau}{\tau}\int d\Gamma(\tau)f(\Gamma(\tau))e^{-i\int d^4x A_\mu(x)J^\mu(x)}$$
$$+\omega\int_0^\infty\frac{ds}{s}\int d\Gamma'(s)f'(\Gamma'(s))e^{-i\int d^4x B_\mu(x)\tilde{K}^\mu(x)} \qquad \text{(VI 2.43)}$$

From now on the proof is the same as in chapter II. It seems that the complicated self interactions of vector and scalar charged fields and the couplings between them do not affect the proof. The only thing needed is the occurence of A_μ and A'_μ, in the result of the path integration over the charged fields, in abelian covariant derivatives which stems from the U(1)XU(1) gauge symmetry being exact.

The main problem that arises is the one of renormalizability. Coupling monopoles to A' without non-abelian quantum numbers breaks the non-abelian symmetry explicitly. This can be seen by replacing the monopole by an electric charge, in which case L_z is equivalent to $F^2_{\mu\nu}$. On top of that, L_z explicitly breaks the non-abelian symmetry as well. Therefore the way of magnetically extending electroweak models followed in this section, although it leads to no new Lorentz invariance violations it should be abandoned due to renormalizability difficulties.

3.A magnetically extended $SU(2)_LXU(1)_Y$ electroweak model

In this section we will use the criterion that the replacement of the monopoles by electric charges should lead to a renormalizable theory as a guide to the choice of the correct extension of Zwanziger's theory. Therefore we are after a theory in which the $U(1)_{em}$ comes from the spontaneous breaking of a non abelian gauge symmetry and at the same time charged particles can couple to the photon without having any non-

abelian quantum numbers.

A model like this is the standard $SU(2)_L XU(1)_Y$ model of electroweak interactions where right-handed electrons can couple to the photon if given weak hypercharge. This leads to the following solution to the posed problem.

We start with the $SU(2)_L XU(1)_Y$ model and we replace $U(1)_Y$ by the Zwanziger $U(1)_Y XU(1)_{magnetic}$, i.e.

$$F^2_{Y\mu\nu} = -\tfrac{1}{2}(\partial_\mu B_\nu - \partial_\nu B_\mu)\partial_\mu B_\nu \qquad \text{(VI 3.1)}$$

is replaced by

$$L_Z = -\tfrac{1}{2} {}^nF_{Y\mu\rho}\, {}^nF_{Y\nu\rho} - \tfrac{1}{2} {}^nF'_{\mu\rho}\, {}^nF'_{\nu\rho} - \tfrac{1}{2} {}^nF_{Y\mu\rho}\, {}^n\tilde{F}'_{\nu\rho} + \tfrac{1}{2} {}^nF'_{\mu\rho}\, {}^n\tilde{F}_{Y\nu\rho} \qquad \text{(VI 3.2)}$$

where

$$F'_{\mu\nu} = \partial_\mu B'_\nu - \partial_\nu B'_\mu \qquad \text{(VI 3.3)}$$

Monopoles are coupled minimally to B'_μ. The usual Higgs doublet gives masses to the $W^{\pm}$ and Z bosons, where $W^{\pm}$ are the (electrically) charged $SU(2)_L$ vector bosons and

$$Z = \cos\theta_w W^3 - \sin\theta_w B \qquad \text{(VI 3.4)}$$

with W^3 the neutral $SU(2)_L$ vector boson. The orthogonal combination, the photon, remains massless.

In this model the SU(2)XU(1) symmetry instead of being explicitly broken, due to the coupling of monopoles, (like in the SO(3) case) is enlarged but the number of gauge degrees of freedom remains the same. After sponteneous symmetry breaking the exact symmetry is U(1)XU(1) but these U(1)'s are not the ones connected through the Zwanziger mechanism. Instead the broken $U(1)_Y$ is connected to the $U(1)_{magnetic}$. The monopole charges are related to the hypercharges of the charged particles through Dirac's quantization condition in order to ensure Lorentz invariance.

As far as renormalizability is concerned the discussion of chapter IV applies.

The replacement of monopoles by electric charges leads to a conventionally renormalizable theory and therefore our criterion is satisfied.

In the limit of $\sin\vartheta_w=0$ the $W^{\pm}$ and Z vector boson masses and $SU(2)_L$ coupling constant tend to infinity with G_F (the Fermi coupling constant) kept fixed, so that the weak interactions decouple from the electromagnetic ones and reduce to the Fermi theory. Also $U(1)_{em}$ coincides with $U(1)_Y$ and Zwanziger's electromagnetodynamics is recovered.

The $\sin\vartheta_w=0$ limit which makes the gauge bosons infinitely heavy can be compared with Scherk's zero slope limit of the dual theory. Thus we can say that as supersymmetry is obtained when we take the zero slope limit of the dual theory, in much the same way the duality between electricity and magnetism is recovered when we take the zero mixing limit of the proposed extended SU(2)XU(1)XU(1) model.

Due to its strong magnetic coupling to the $U(1)_Y$ which is partially in a Higgs phase, the monopole introduced here is presumably confined.

4.Conclusions

In conclusion our model, which does not suffer from worse Lorentz invariance and renormalizability problems than Zwanziger's original theory, gives an explanation for the need for the U(1) factor in the standard electroweak model, explains electric charge quantization (because of hypercharge quantization) without grand unification and predicts strong coupling of monopoles to the Z gauge boson. Due to this coupling, these monopoles, if they exist at all, should be produced as heavy monopole-antimonopole pairs in the new accelerators.

VII A SUPERSYMMETRIC FIELD THEORY OF MONOPOLES

1. Introduction

In this chapter we are going to briefly describe a supersymmetric field theory of charges and monopoles. This theory can be regarded as the supersymmetrization of Zwanziger's theory in the sense that, in the absence of matter, the pure gauge Lagrangian can be shown to be identical to Zwanziger's Lagrangian.

The notation follows the one of reference [30], where the reader can find an introduction to supersymmetry.

2. The supersymmetric Lagrangian

We consider free fields first. The construction of a supersymmetric extension of Zwanziger's Lagrangian proceeds by introducing the following chiral spinor superfield

$$\Psi_\alpha = -(i/2\sqrt{2})\bar{D}_+ D_- (e^{-2V} D_{\alpha+} e^{2V}) \qquad \text{(VII 2.1)}$$

where V is a general real superfield containing the gauge field A_μ, while $D=(\partial/\partial\bar{\theta} - \frac{1}{2} i\not{\partial}\theta)$. In the Wess-Zumino gauge in which V takes the form

$$V=\tfrac{1}{2}\bar{\theta} i\gamma_\nu\gamma_5\theta A_\nu + (i/2\sqrt{2})\bar{\theta}\theta\bar{\theta}\gamma_5\lambda + \frac{1}{16}(\bar{\theta}\theta)^2\partial^2\phi \qquad \text{(VII 2.2)}$$

Ψ becomes

$$\Psi = \exp(-\tfrac{1}{2}\bar{\theta}\not{\partial}\gamma_5\theta)\,[\lambda_+ + (i/\sqrt{2})(\partial^2\phi + \tfrac{1}{2}\sigma_{\mu\nu}F_{\mu\nu})\theta_+ + \tfrac{1}{2}\bar{\theta}_-\theta_+(-i\not{\partial}\lambda_-)] \qquad \text{(VII 2.3)}$$

(the conventional notation for $\partial^2\phi$ is D). We also introduce a primed chiral spinor superfield Ψ' with corresponding components.

The supersymmetric extension of Zwanziger's Lagrangian is given by

$$-\frac{1}{8}\{\bar{\Psi}\not{\partial}\not{n}\not{\partial}[i(n\cdot\partial)/\partial^4]\Psi + \bar{\Psi}'\not{\partial}\not{n}\not{\partial}[i(n\cdot\partial)/\partial^4]\Psi' + \bar{\Psi}\not{\partial}\not{n}\not{\partial}[(n\cdot\partial)/\partial^4]\Psi' - \bar{\Psi}'\not{\partial}\not{n}\not{\partial}[(n\cdot\partial)/\partial^4]\Psi\}_D \qquad \text{(VII 2.4)}$$

where D signifies the coefficient of $[1/32\ (\bar{\theta}\theta)^2]$.

First examine its bosonic sector. Given that the gauge fields enter the Lagrangian through gauge invariant combinations, the resulting theory is gauge invariant. In order to fix the gauge we choose the condition $\partial\cdot A=\partial\cdot A'=0$ or its supersymmetric version $\bar{D}_- D_+ V=\bar{D}_- D_+ V'=0$. Using the transversality of A_μ and A'_μ the bosonic sector reduces to the local form

$$\tfrac{1}{2}[A_\mu(n\cdot\partial)^2 A_\mu+A'_\mu(n\cdot\partial)^2 A'_\mu-\epsilon_{\sigma\rho\mu\tau}An^\rho(n\cdot\partial)\partial^\sigma A'^\tau+\epsilon_{\sigma\rho\mu\tau}A'n^\rho(n\cdot\partial)\partial^\sigma A^\tau$$

$$+\phi(n\cdot\partial)^2\partial^2\phi+\phi'(n\cdot\partial)^2\partial^2\phi'-2\phi(n\cdot\partial)\partial^2(nA')+2\phi'(n\cdot\partial)\partial^2(nA)] \qquad \text{(VII 2.5)}$$

For the free case, the ϕ and ϕ' are easily eliminated using the equations of motion. We obtain the Zwanziger Lagrangian (up to surface terms) in the equivalent form

$$L_z=-\tfrac{1}{2}n_\mu F_{\mu\nu}n_\nu F_{\nu\rho}-\tfrac{1}{2}n_\mu F'_{\mu\rho}n_\nu F'_{\nu\rho}-\tfrac{1}{2}n_\mu F_{\mu\rho}n_\nu \tilde{F}'_{\nu\rho}+\tfrac{1}{2}n_\mu F'_{\mu\rho}n_\nu \tilde{F}_{\nu\rho} \qquad \text{(VII 2.6)}$$

In momentum space the various propagators are

$$(AA)_{\mu\nu}=(A'A')_{\mu\nu}=-g_{\mu\nu}/k^2+k_\mu k_\nu/k^4$$

$$(AA')_{\mu\nu}=-(A'A)_{\mu\nu}=-\epsilon_{\mu\nu\rho\sigma}n^\rho k^\sigma/k^2(nk) \qquad \text{(VII 2.8)}$$

$$(\phi\phi)=(\phi'\phi')=1/k^4 \qquad \text{(VII 2.9)}$$

$$(\phi A')_\nu=-(\phi' A)_\nu=-i[(nk)/k^4]\{n_\nu-[(nk)/k^2]k_\nu\} \qquad \text{(VII 2.10)}$$

The $(\phi\phi)$, $(\phi'\phi')$, $(\phi A')$ and $(\phi' A)$ propagators are a new feature, arising on account of supersymmetry.

The Lagrangian for the "photinos" reads

$$\bar{\lambda}_-[(n\cdot\partial)^2/\partial^2]i\not{\partial}\lambda_-+\bar{\lambda}'_-[(n\cdot\partial)^2/\partial^2]i\not{\partial}\lambda'_--\bar{\lambda}'_-[(n\cdot\partial)/\partial^2][(n\cdot\partial)\not{n}-\not{\partial}]\lambda_-$$

$$-\bar{\lambda}_-[(n\cdot\partial)/\partial^2][(n\cdot\partial)\not{n}-\not{\partial}]\lambda'_- \qquad \text{(VII 2.11)}$$

The propagator matrix it leads to is given by

$$\begin{bmatrix} \lambda_- \bar{\lambda}_- & \lambda_- \bar{\lambda}'_- \\ \lambda'_- \bar{\lambda}_- & \lambda'_- \bar{\lambda}'_- \end{bmatrix} = \begin{bmatrix} 1 & i-i\not{p}/(np) \\ -i+i\not{p}/(np) & 1 \end{bmatrix} \frac{1}{\not{p}} \qquad \text{(VII 2.12)}$$

There is just one pole at $p^2=0$, which has as residue an 8X8 matrix in Dirac and λ, λ' space. This has rank two, which garantees that the (λ,λ') pair of fermions describes only two states, as it should from supersymmetry.

We now turn to the interacting fields and couple matter to the "electromagnetic" field. This coupling is conventional through the superfields V and V', and presents no new problems. Thus if Φ_+, Φ_- are "electrically" charged chiral superfields and Φ'_+, Φ'_- "magnetically" charged chiral superfields, the matter field coupling takes the form

$$\tfrac{1}{4}(\Phi^*_+ e^{2V}\Phi_+ + \Phi^*_- e^{-2V}\Phi_- + \Phi'^*_+ e^{2V'}\Phi'_+ + \Phi'^*_- e^{-2V'}\Phi'_-)_D \qquad \text{(VII 2.13)}$$

[For dyons, $\Phi_+=\Phi'_+$, $\Phi_-=\Phi'_-$ and the coupling is $\tfrac{1}{4}(|\Phi_+|^2 e^{2V+2V'} + |\Phi_-|^2 e^{-2V-2V'})_D$]

We expect the field theory to be Lorentz invariant when the Dirac condition is satisfied in the same sense as Zwanziger's, since it represents a supersymmetrization of Zwanziger's theory. This however has not been proved.

It should be emphasized that supersymmetrization has dynamical consequences. The ultraviolet divergences become less severe and in particular mass counterterms are finite. From this point of view our theory resembles the solitonic non-abelian monopole [31] and may be considered as its field-theoretic representation, in the energy region below its mass, where the non abelian monopole can be considered as an elementary entity.

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