

A NUMERICAL METHOD FOR THE ANALYSIS
OF UNLINED PRESSURE TUNNELS IN JOINTED ROCK

by

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ABSTRACT

Because the costs of lining hydraulic pressure tunnels are so high, considerable economic advantage may be gained if pressure tunnels can remain unlined, at least in part. The empirical methods currently used to establish the required locations and lengths of lining are unsatisfactory. This thesis presents a numerical method developed specifically for use in design analyses of unlined pressure tunnels in hard, jointed rocks, which may possibly be able to produce better designs than the traditional methods.

The thesis consists of six chapters. Chapter 1 is an introduction, in which a short general description of hydraulic pressure tunnels is given followed by an analysis of failure mechanisms and a review of design criteria and methods for unlined pressure tunnels. The objective of this thesis is also introduced.

Chapter 2 describes a theoretical analysis of conditions for joint opening and hydraulic jacking obtained from analytical solutions based on the elasticity theory. The results of the analysis may be used in the preliminary design of the unlined pressure tunnels.

Chapter 3 presents the joint model developed, in which the effective normal stress - displacement relation is taken to be hyperbolic, the modelling of shear behaviour is based on Barton's empirical joint model with dilation being accounted for using a non-associated flow rule, and non-linear computation is implemented by a secant stiffness iteration method. The computer program developed is tested for a range of problems.

Chapter 4 describes the proposed numerical method including the complete plane strain model, linked finite element and boundary element scheme, determination of the loads due to water pressure acting on the joint walls, and the computational procedure.

In Chapter 5 the proposed numerical method is used to analyze some generic cases of unlined pressure tunnels. Three major failure modes, hydraulic jacking and hydraulic shearing of joints and uplift of the ground surface, are identified.

In Chapter 6 some conclusions are drawn and an approach to design analysis of unlined pressure tunnels is suggested.

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TABLE OF CONTENTS

	Page
ABSTRACT	2
ACKNOWLEDGEMENTS	4
TABLE OF CONTENTS	5
LIST OF FIGURES	10
LIST OF TABLES	18
NOTATION	19
CHAPTER 1 INTRODUCTION	22
1.1 Pressure Tunnels in Hydroelectric Schemes	22
1.2 Failure Mechanisms in Unlined Pressure Tunnels	28
1.2.1 Pressure tunnel failures	29
1.2.2 Analysis of pressure tunnel failure mechanisms	36
1.3 Design Criteria and Methods for Unlined Pressure Tunnels	39
1.3.1 The rule of thumb and its modifications	39
1.3.2 The modern Norwegian method	47
1.3.3 Jaeger's analytical solutions	51
1.3.4 Hoek's method	52
1.3.5 Other methods	57
1.4 Purpose and Components of This Thesis	59
1.4.1 Purpose of the thesis	59
1.4.2 Components of the thesis	61
CHAPTER 2 ANALYTICAL SOLUTIONS	63
2.1 Introduction	63

2.2	Analytical Solutions in the Plane Strain Condition	64
2.2.1	Cases $K_1 = K_2 = K$	64
2.2.2	Cases $K_1 \neq K_2$	69
2.3	Analytical Solutions in the Complete Plane Strain Condition	75
2.3.1	The concept of complete plane strain	75
2.3.2	Basic equations for calculating the stress components	76
2.3.3	Criteria for joint opening	80
2.3.4	Criteria for hydraulic jacking	82
2.3.5	Analyses of conditions for joint opening and hydraulic jacking	83
2.4	Three Dimensional Analytical Solutions in Complete Plane Strain Condition on Microcomputer	94
2.4.1	Conditions for joint opening and hydraulic jacking on the tunnel boundary	94
2.4.2	Conditions for joint opening and hydraulic jacking inside the rock mass surrounding a pressure tunnel	99
2.4.3	Example for implementation of ASJOHJ	101
2.5	Summary	103
CHAPTER 3	NUMERICAL MODEL FOR DISCONTINUITIES	105
3.1	Choice of the Physical Model	105
3.1.1	Review of models for discontinuities	105
3.1.2	Discussion	117
3.2	Review of Joint Elements	120
3.3	Proposed Joint Model	124
3.3.1	Normal stress-displacement relation	125
3.3.2	Shear behaviour	126

3.3.3	Dilation	131
3.3.4	Determination of JRC_m	135
3.3.5	Elasto-plastic model	136
3.3.6	Joint elements	140
3.3.7	Non-linear computation	144
3.3.8	Program test with single joint element	152
3.4	Summary	162
CHAPTER 4	THE PROPOSED NUMERICAL METHOD	164
4.1	Introduction	164
4.2	The Complete Plane Strain Model	168
4.3	The Combined FE and BE Method	173
4.3.1	Coupling of FE and BE	173
4.3.2	Finite element formulation	175
4.3.3	Boundary element formulation	179
4.4	Calculation of Water Pressure Acting on Joint Walls	186
4.4.1	Models for seepage in rock masses	186
4.4.2	FE formulation	187
4.4.3	Estimation of permeability coefficient	189
4.4.4	Evaluation of nodal forces caused by water pressure	190
4.5	Computation Procedure	192
4.5.1	Initial stress field	192
4.5.2	Excavation simulation	196
4.5.3	Application of water pressure	196
4.6	The Program	198
CHAPTER 5	ANALYSIS OF GENERIC CASES OF UNLINED PRESSURE TUNNELS	203
5.1	Introduction	203

5.2	Generic Case 1 - Tunnel and Ground Surface Connected by a Single Vertical Joint	205
5.2.1	General description	205
5.2.2	Cases with varying H	209
5.2.3	Cases with varying α	217
5.2.4	Cases with varying K_1	224
5.3	Generic Case 2 - A Horizontal Joint Parallel to the Ground Surface and the Tunnel Connected by a Single Vertical Joint	227
5.3.1	General description	227
5.3.2	Results for the case of $H_1=1.7\text{m}$	229
5.3.3	Influence of overburden thickness	234
5.4	Generic Case 3 - Vertical Joints Connected to the Tunnel Boundary by a Horizontal Joint	238
5.5	Simulation of a Hypothetical Real Case	246
5.6	Summary	254
CHAPTER 6	SUMMARY AND CONCLUSIONS	257
6.1	Conclusions	257
6.2	Suggested Approach and Guidelines to Unlined Pressure Tunnel Design Analysis	258
6.2.1	Suggested approach to design analysis	258
6.2.2	Guidelines to unlined pressure tunnel design	265
6.3	Suggestions for Further Research Work	267
APPENDIX 1	PRESSURE TUNNEL FAILURES	269
APPENDIX 2	JAEGER'S ANALYTICAL SOLUTIONS FOR FAILURE CONDITIONS OF UNLINED PRESSURE TUNNELS	278
APPENDIX 3	HOEK'S SOLUTIONS FOR CONDITIONS FOR JOINT	

OPENING AND HYDRAULIC JACKING	282
APPENDIX 4 LISTING OF PROGRAM ASJOHJ	289
APPENDIX 5 DERIVATION OF EXPRESSION FOR JRC_m	291
APPENDIX 6 DERIVATION OF ELASTO-PLASTIC STRESS-DISPLACEMENT MATRIX	294
APPENDIX 7 EFFECT OF σ_n ON $\tau - \delta_s$ PLOT	298
APPENDIX 8 VERIFICATION OF COUPLING OF FINITE ELEMENTS AND BOUNDARY ELEMENTS	301
REFERENCES	307

LIST OF FIGURES

- Figure 1.1 General perspective of underground structures of Shintoyone pumped storage scheme, Japan
- Figure 1.2 Longitudinal section and plane view of pressure tunnel system of Hovatn hydropower development, Norway
- Figure 1.3 Longitudinal section of pressure tunnel of Cruachen hydropower project, Great Britain
- Figure 1.4 Hydraulic head along a pressure tunnel
- Figure 1.5 Failure of pressure tunnel in Kandergrund hydropower project
- Figure 1.6 Failure of pressure tunnel in Herlandsfoss project
- Figure 1.7 Failure of pressure shaft in Huangqin project
- Figure 1.8 Failure of pressure shaft in Kotmale hydropower project
- Figure 1.9 Rockfalls in Kemano pressure tunnel
- Figure 1.10 Failure of pressure shaft in Gerlos project
- Figure 1.11 Generic cases of unlined pressure tunnel failure
- Figure 1.12 Original rule of thumb for determining the maximum water pressure for unlined pressure tunnel
- Figure 1.13 Equivalent rock cover adopted in Snowy Mountains Scheme
- Figure 1.14 Rule of thumb adopted in Norway
- Figure 1.15 Reznikov's method for determining the maximum water pressure for omitting steel lining
- Figure 1.16 Sketch and result of finite element analysis for SYSET/STEGGJE pressure tunnel

- Figure 1.17 Example of design chart for unlined pressure tunnel used in Norway
- Figure 1.18 In situ stress measurement results
- Figure 1.19 Hoek's method for determination of length of steel lining
- Figure 1.20 Result of a model test
- Figure 1.21 Arrangement of field pressure test carried out in Ffestiniog hydropower project
- Figure 2.1 Coordinate system and loads for cases in plane strain condition
- Figure 2.2 Conditions for joint opening when $K_1=K_2=K$, $\nu=0.2$
- Figure 2.3 Conditions for hydraulic jacking when $K_1=K_2=K$, $\nu=0.2$
- Figure 2.4 Conditions for joint opening when $K_1=K_2=K$, $\nu=0.3$
- Figure 2.5 Conditions for hydraulic jacking when $K_1=K_2=K$, $\nu=0.3$
- Figure 2.6 Conditions for joint opening when $K_1 \neq K_2$, $\nu=0.2$
- Figure 2.7 Conditions for hydraulic jacking when $K_1 \neq K_2$, $\nu=0.2$
- Figure 2.8 Conditions for joint opening when $K_1 \neq K_2$, $\nu=0.3$
- Figure 2.9 Conditions for hydraulic jacking when $K_1 \neq K_2$, $\nu=0.3$
- Figure 2.10 Cases $K_2 < K_1$
- Figure 2.11 Cases $K_1 < K_2$
- Figure 2.12 Coordinate transformation for complete plane strain case
- Figure 2.13 Relation between in situ stress directions and the tunnel axis
- Figure 2.14 Conditions for joint opening and hydraulic jacking when $\beta=0^\circ$, $K_1=0.5$, $\nu=0.2$
- Figure 2.15 Conditions for joint opening and hydraulic jacking when $\beta=0^\circ$, $K_1=1.0$, $\nu=0.2$
- Figure 2.16 Conditions for joint opening and hydraulic jacking

- when $\beta=0^\circ$, $K_1=2.0$, $\nu=0.2$
- Figure 2.17 Conditions for joint opening and hydraulic jacking
when $\beta=0^\circ$, $K_1=3.0$, $\nu=0.2$
- Figure 2.18 Conditions for joint opening and hydraulic jacking
when $\beta=30^\circ$, $K_1=0.5$, $\nu=0.2$
- Figure 2.19 Conditions for joint opening and hydraulic jacking
when $\beta=30^\circ$, $K_1=1.0$, $\nu=0.2$
- Figure 2.20 Conditions for joint opening and hydraulic jacking
when $\beta=30^\circ$, $K_1=2.0$, $\nu=0.2$
- Figure 2.21 Conditions for joint opening and hydraulic jacking
when $\beta=30^\circ$, $K_1=3.0$, $\nu=0.2$
- Figure 2.22 Conditions for joint opening and hydraulic jacking
when $\beta=45^\circ$, $K_1=0.5$, $\nu=0.2$
- Figure 2.23 Conditions for joint opening and hydraulic jacking
when $\beta=45^\circ$, $K_1=1.0$, $\nu=0.2$
- Figure 2.24 Conditions for joint opening and hydraulic jacking
when $\beta=45^\circ$, $K_1=2.0$, $\nu=0.2$
- Figure 2.25 Conditions for joint opening and hydraulic jacking
when $\beta=45^\circ$, $K_1=3.0$, $\nu=0.2$
- Figure 2.26 Conditions for joint opening and hydraulic jacking
when $\beta=60^\circ$, $K_1=0.5$, $\nu=0.2$
- Figure 2.27 Conditions for joint opening and hydraulic jacking
when $\beta=60^\circ$, $K_1=1.0$, $\nu=0.2$
- Figure 2.28 Conditions for joint opening and hydraulic jacking
when $\beta=60^\circ$, $K_1=2.0$, $\nu=0.2$
- Figure 2.29 Conditions for joint opening and hydraulic jacking
when $\beta=60^\circ$, $K_1=3.0$, $\nu=0.2$
- Figure 2.30 Conditions for joint opening and hydraulic jacking
when $\beta=90^\circ$, $K_1=0.5$, $\nu=0.2$

- Figure 2.31 Conditions for joint opening and hydraulic jacking when $\beta=90^\circ$, $K_1=1.0$, $\nu=0.2$
- Figure 2.32 Conditions for joint opening and hydraulic jacking when $\beta=90^\circ$, $K_1=2.0$, $\nu=0.2$
- Figure 2.33 Conditions for joint opening and hydraulic jacking when $\beta=90^\circ$, $K_1=3.0$, $\nu=0.2$
- Figure 2.34 Coordinate systems for the three dimensional analyses of the critical water pressure for joint opening and hydraulic jacking
- Figure 2.35 Flow chart of program ASJOHJ
- Figure 2.36 Variation of critical water pressure for joint opening and hydraulic jacking with distance from the tunnel boundary - an example of three dimensional analyses
- Figure 3.1 Mohr-Coulomb criterion
- Figure 3.2 Specimen in Patton's tests
- Figure 3.3 Patton's peak shear strength envelope
- Figure 3.4 Jaeger's peak shear strength envelope
- Figure 3.5 Ladanyi and Archambault's joint model
- Figure 3.6 Heuze's joint model
- Figure 3.7 Heuze's peak shear strength criterion
- Figure 3.8 Gerrard's peak shear strength model
- Figure 3.9 Goodman's joint element and its model
- Figure 3.10 One dimensional interface example
- Figure 3.11 Ge's joint element
- Figure 3.12 Relation between normal stress and normal displacement
- Figure 3.13 Influence of JRC, JCS and ϕ_r on peak shear strength in Barton's model
- Figure 3.14 Proposed elasto - plastic model
- Figure 3.15 Dimensionless model of shear behaviour of joints

Figure 3.16	Distribution of peak and initial dilation angles and their relationship with the asperity component of shear strength
Figure 3.17	Dilation angle of Barton's model and its revision
Figure 3.18	Different stress transfer methods
Figure 3.19	Ordinary joint element
Figure 3.20	T shaped joint element
Figure 3.21	Cross joint element
Figure 3.22	Shear stiffness coefficients
Figure 3.23	An abrupt increment in shear strength at very low normal stress
Figure 3.24	Non-linear computation method
Figure 3.25	Transition from unloaded state to plastic state
Figure 3.26	Simple tension and compression
Figure 3.27	Numerical simulation of direct shear test under constant normal stress
Figure 3.28	Effect of test mode on shear deformation curves for dilatant joints
Figure 3.29	Numerical simulation of direct shear tests under varying normal stress
Figure 3.30	Simulation of hydraulic shearing
Figure 3.31	Effect of σ_n on $\tau - \delta_s$ curve
Figure 3.32	Numerical simulation of unloading and reloading
Figure 3.33	Experimental result of unloading and reloading test
Figure 4.1	Computation scheme for numerical analysis
Figure 4.2	Coordinate system notation
Figure 4.3	Limitation of the complete plane strain model
Figure 4.4	Four-node finite element connecting one joint element
Figure 4.5	Four-node finite element connecting two joint elements

- Figure 4.6 Eight-node finite element connecting two joint elements
- Figure 4.7 Eight-node finite element connecting four joint elements
- Figure 4.8 Two-node linear boundary element
- Figure 4.9 Three-node quadratic boundary element
- Figure 4.10 Two-node infinite boundary element
- Figure 4.11 Displacement models for two-node infinite boundary element
- Figure 4.12 Water pressure forces acting on joint walls
- Figure 4.13 General computation scheme
- Figure 4.14 General flow chart of program DPTJR
- Figure 5.1 Generic case 1
- Figure 5.2 Finite element mesh for the case $H=3.5D$ of generic case 1 - Mesh 1
- Figure 5.3 Finite element mesh for the case $H=5.0D$ of generic case 1 - Mesh 2
- Figure 5.4 Finite element mesh for the case $H=7.5D$ of generic case 1 - Mesh 3
- Figure 5.5 Variation of normal stress with excavation and water pressure
- Figure 5.6 Uplift of ground surface for generic case 1
- Figure 5.7 Variation of joint opening with water pressure for generic case 1
- Figure 5.8 Variation of critical water pressure with overburden thickness
- Figure 5.9 Development of shear displacement under reduced effective normal stress and constant shear stress
- Figure 5.10 Variation of shear displacement with water pressure

for generic case 1

- Figure 5.11 Variation of joint opening with water pressure for generic case 1
- Figure 5.12 Influence of K_1 on joint opening for generic case 1
- Figure 5.13 Variation of critical water pressure with K_1 for generic case 1
- Figure 5.14 Generic case 2
- Figure 5.15 Finite element mesh for the case $H_1=1.7\text{m}$ of generic case 2 - Mesh 4
- Figure 5.16 Finite element mesh for the case $H_1=0.9\text{m}$ of generic case 2 - Mesh 5
- Figure 5.17 Joint opening for generic case 2
- Figure 5.18 Uplift of ground surface for generic case 2
- Figure 5.19 Shear displacement on horizontal joint for generic 2
- Figure 5.20 Effective normal stress for generic case 2
- Figure 5.21 Generic case 3
- Figure 5.22 Finite element mesh for generic case 3 - Mesh 6
- Figure 5.23 Two different boundary conditions for joint element
- Figure 5.24 Effective normal stress for generic case 3
- Figure 5.25 Shear stress for generic case 3
- Figure 5.26 Joint opening for generic case 3
- Figure 5.27 Uplift of ground surface for generic case 3
- Figure 5.28 A hypothetical real case
- Figure 5.29 Finite element mesh for the hypothetical real case - Mesh 7
- Figure 5.30 Effective normal stress on horizontal and inclined joints for the hypothetical real case
- Figure 5.31 Opening of upper horizontal joint and uplift of ground surface for the hypothetical real case

- Figure 5.32 Shear displacement on inclined joint for the hypothetical real case
- Figure 5.33 Uplift of ground surface for the hypothetical real case
- Figure 6.1 Suggested method for determining the length of steel lining in preliminary design
- Figure A.2.1 Jaeger's solution for hard, unfissured rock
- Figure A.2.2 Jaeger's solution for radially fissured rock
- Figure A.2.3 Jaeger's solution for plastic rock
- Figure A.2.4 Jaeger's solution for the tunnel adjacent to a rock slope
- Figure A.3.1 Coordinate and loads for Hoek's solutions
- Figure A.3.2 Hoek's solution for conditions for joint opening
- Figure A.3.3 Hoek's solutions for conditions for hydraulic jacking
- Figure A.7.1 Effect of σ_n on $\tau - \delta_s$ plot
- Figure A.8.1 Example 1 for testing coupling of finite element and boundary element
- Figure A.8.2 Finite element and boundary element mesh
- Figure A.8.3 Example 2 for testing coupling of finite element and boundary element

LIST OF TABLES

Table 2.1	Critical water pressure for joint opening and hydraulic jacking for $K=(K_1+K_2)/2=1.0$
Table 2.2	An example of $K_1=0.5$, $\beta=30^\circ$, $\nu=0.2$
Table 5.1	Numbers of elements and nodes of Mesh 1, 2, 3
Table 5.2	Failure mode and critical water pressure for cases of varying α
Table 5.3	The critical water pressures for cases of different K_1
Table 5.4	Numbers of finite elements, joint elements and nodes and critical water pressures for generic case 2
Table A.8.1	Radial displacement of example 1
Table A.8.2	u_x at $z=0$ of example 2
Table A.8.3	u_y at $z=0$ of example 2
Table A.8.4	u_z at $z=0$ of example 2
Table A.8.5	u_x at $x=0$ of example 2
Table A.8.6	u_y at $x=0$ of example 2
Table A.8.7	u_z at $x=0$ of example 2

NOTATION

The following notation is used unless defined otherwise.

c	: cohesion
D	: diameter of tunnel
E_r	: Young's modulus of intact rock
F	: yield function
F_s	: factor of safety
g	: acceleration due to gravity
G	: plastic potential function
h	: depth of tunnel measured from the tunnel crown
H	: depth of tunnel measured from the tunnel center
H_0	: total static head
JCS	: joint wall compressive strength
JRC	: joint roughness coefficient
JRC_m	: mobilized joint roughness coefficient
JRC_p	: peak joint roughness coefficient
K, K_1, K_2	: in situ horizontal and vertical stress ratio
K_n	: normal stiffness coefficient
K_{ni}	: initial normal stiffness
K_{sx}, K_{sy}	: shear stiffness coefficients
K_p	: permeability coefficient
k_p	: specific permeability coefficient
L	: joint length
l, m, n	: direction cosines
N_1, N_2	: shape functions
p	: internal water pressure
p_c	: critical water pressure
p_d	: dynamic water pressure

w_m	: maximum joint closure
w^p	: plastic work
P_x, P_y, P_z	: in situ stresses
u, v, w	: displacement components
α, β, ϕ	: angles
γ	: specific weight of rock
γ_w	: unit weight of water
Δw	: joint closure
δ_A	: shear displacement at the transition point from elastic state to plastic state
δ_{ij}	: Kronecker delta
δ_p	: shear displacement at peak strength
δ_s	: resultant shear displacement
ε	: strain
ν	: Poisson's ratio
ν_m	: kinematic viscosity of water
$\sigma_1, \sigma_2, \sigma_3$	: principal stresses
σ_r	: radial stress
σ_θ	: circumferential stress
σ_y	: normal stress in the tunnel direction
σ_n	: normal stress of joint
σ'_n	: effective normal stress of joint
σ_n^*	: transition low normal stress
τ	: shear stress
τ_m	: mobilized shear strength of joint
τ_p	: peak shear strength of joint
Φ	: seepage potential function
ϕ_{dm}	: mobilized dilational angle
ϕ_{dpk}	: peak dilational angle

ϕ_r : residual frictional angle

CHAPTER 1

INTRODUCTION

1.1 Pressure Tunnels in Hydraulic Schemes

Hydroelectric power projects are those utilizing the falling head of water to generate electricity. A hydroelectric power project consists chiefly of four parts: retaining structure, discharge structure, the power house and the conveyance system. The function of the retaining structure is to form a reservoir and enhance the hydraulic head. In most cases it is a dam. The purpose of the discharge structure is to discharge excessive water, adjust water levels in the reservoir and ensure the safety of the whole project, especially the dam. The discharge structure may be a discharge orifice, a spillway or a discharge tunnel. The power house accommodates the turbine and generator units and the central control room, and may be situated behind the dam, by the river or underground. The conveyance system is a series of structures taking water from the upstream reservoir, supplying water to the turbine and conveying water to the downstream river channel after the water has been used to generate electricity. The conveyance system may include an intake, a headrace tunnel, a pressure shaft, a penstock, a surge tank, a tailrace tunnel/channel and an outlet. Fig.1.1, Fig.1.2, and Fig.1.3 give the general layout of the power system of three hydropower projects: the Shintoyone pumped storage scheme in Japan, the Hovatn hydropower development in Norway and the Cruachan hydropower scheme in Great Britain.

The term pressure tunnel usually refers to a section of upstream conduit between the intake and the steel pipe (penstock) leading to the power house, including a headrace tunnel and/or a pressure shaft for underground or pumped storage hydroelectric schemes. Fig.1.4 shows a sketch of a longitudinal profile

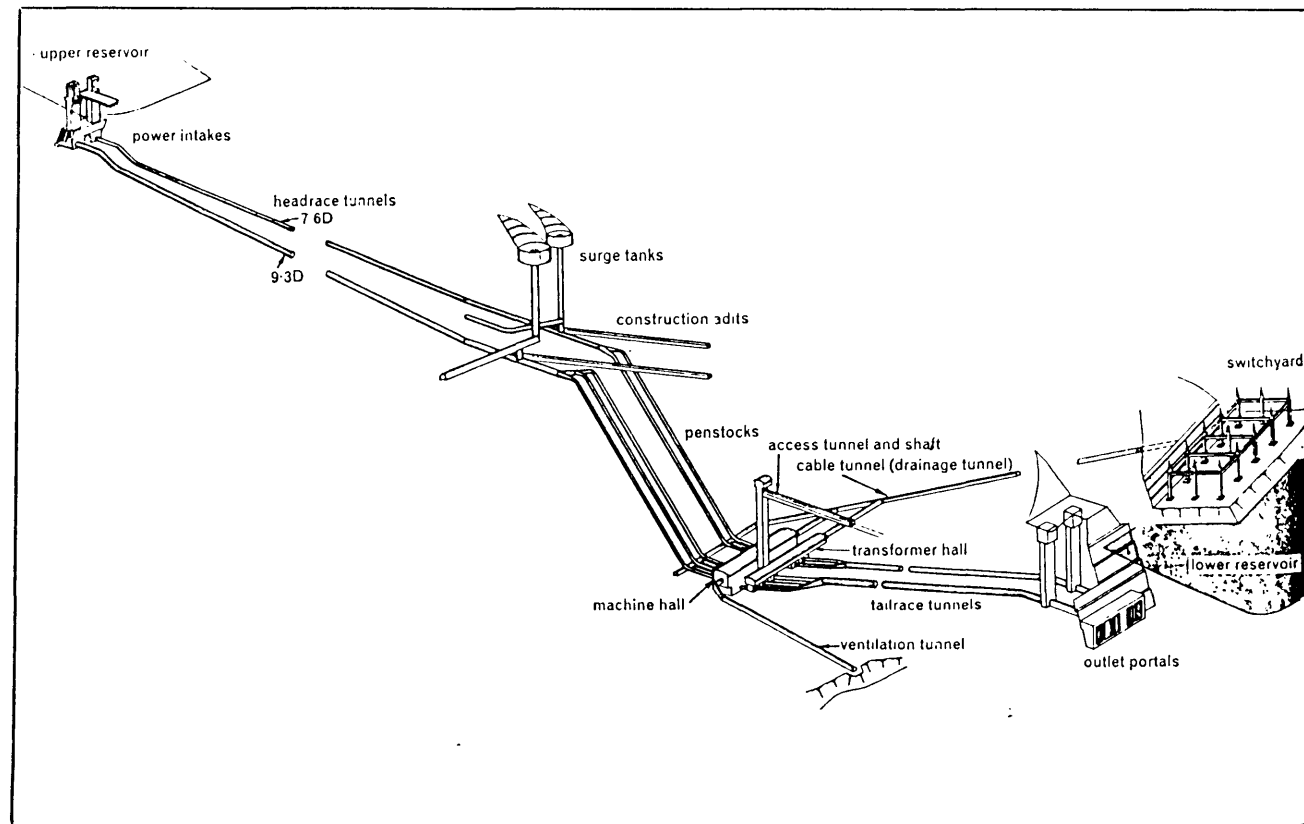


Figure 1.1 General perspective of underground structures of Shintoyone pumped storage scheme, Japan (After Takahashi, 1972)

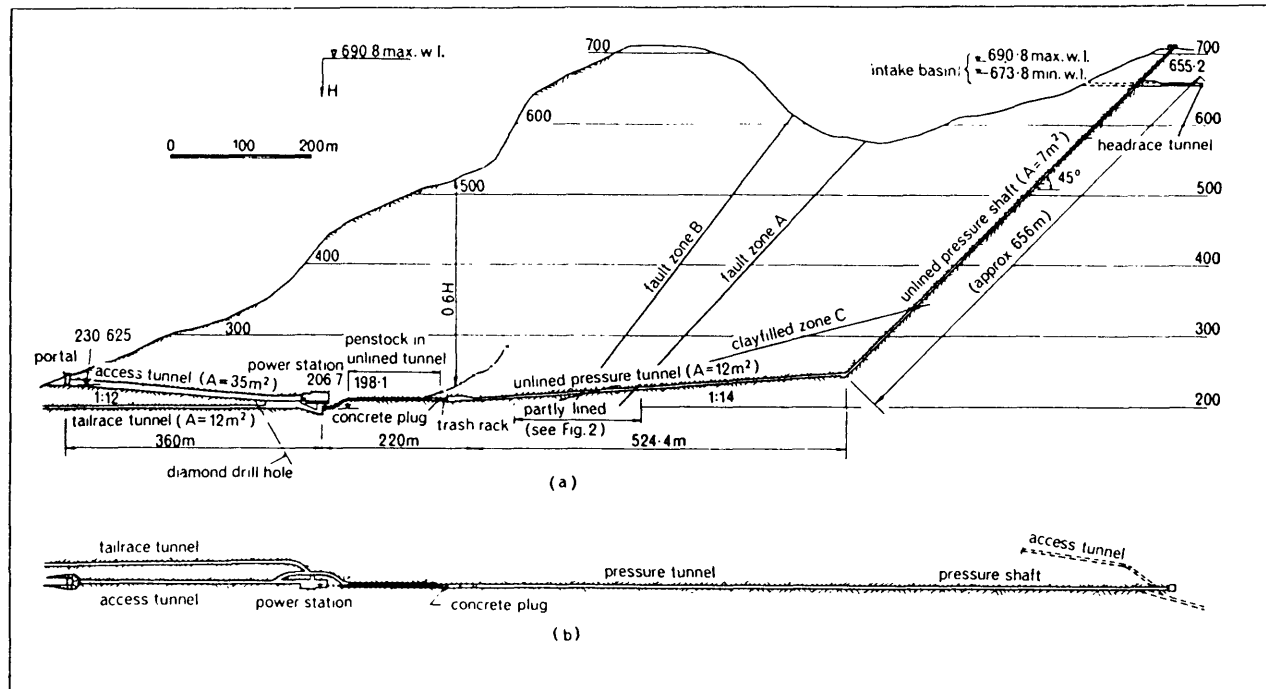


Figure 1.2 Longitudinal section and plane view of pressure tunnel system of Hovtn hydropower development, Norway (After Berdal, 1972)

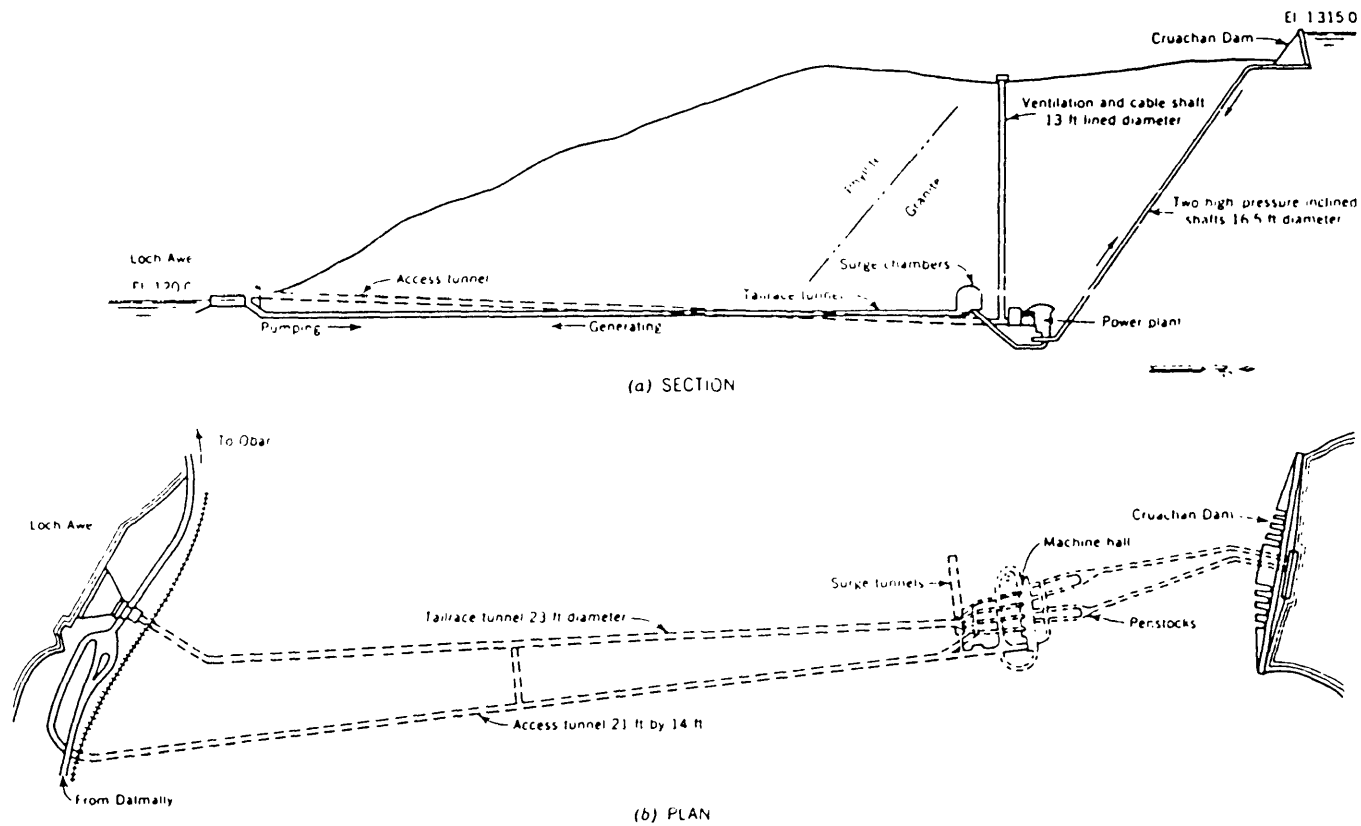


Figure 1.3 Longitudinal section of pressure tunnel of Cruachen hydropower project,
Great Britain

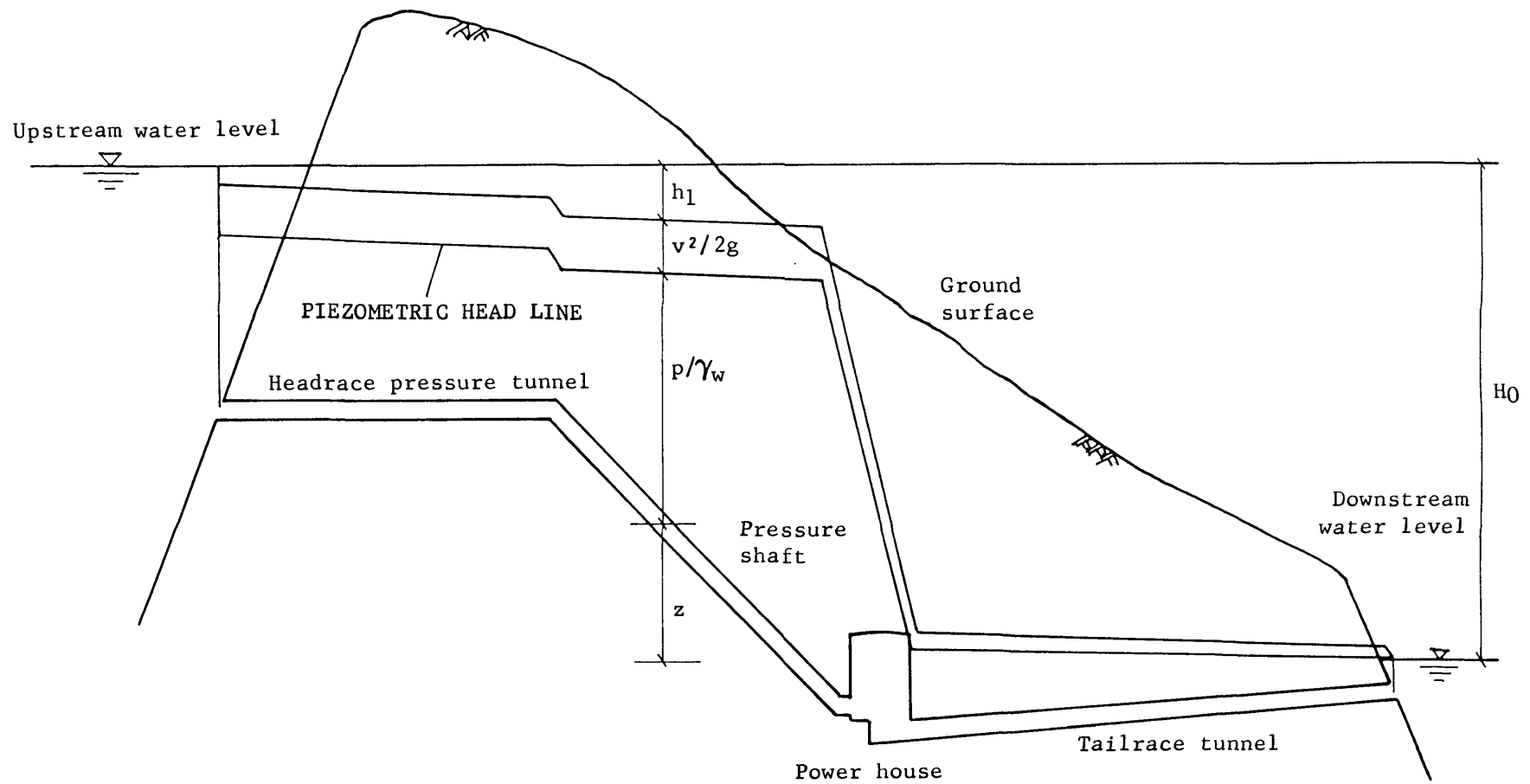


Figure 1.4 Hydraulic head along a pressure tunnel

of the power system of a hydropower station. The water level difference H_0 between the upstream and the downstream is the total static hydraulic head. According to Bernoulli's theorem the summation of the pressure head, $\frac{p}{\gamma_w}$, the location head, z , the velocity head, $\frac{v^2}{2g}$, and the head losses, h_l , is constant at any point along the tunnel. If the downstream water level is taken to be the reference, then:

$$\frac{p}{\gamma_w} + z + \frac{v^2}{2g} + h_l = H_0 \quad (1.1)$$

where p = water pressure,

γ_w = unit weight of water,

z = height of the tunnel centre above the reference level,

v = flow velocity,

g = acceleration due to gravity,

h_l = head loss.

$(\frac{p}{\gamma_w} + z)$ is usually called the piezometric head. From Fig.1.4 it can be seen that $\frac{p}{\gamma_w}$ is simply the vertical difference between the piezometric line and the corresponding tunnel centre. $\frac{p}{\gamma_w}$ will be called the hydraulic head throughout this thesis. It is this hydraulic head that acts on the tunnel boundary and affects the stability of the rock mass surrounding the pressure tunnel. The maximum static head occurs when the power plant is shutdown and the tunnel is filled with water, i.e. $v = 0$, $\frac{p}{\gamma_w} = H_0 - z$. Nevertheless, the dynamic head caused by a water hammer or water fluctuations may be even higher.

The hydraulic head (water pressure) varies along the tunnel. It is usually higher in the pressure shaft than in the headrace pressure tunnel. The biggest hydraulic head occurs at the intersection of the pressure shaft and the turbine

inlet. This is one of reasons why the section of pressure shaft immediately connecting the turbine (penstock) is always lined with steel. In the tailrace tunnel, however, the water pressure is usually very low, and it can sometimes become a non-pressure tunnel.

Two major factors must be taken into account when aligning and designing a pressure tunnel. One factor is concerned with rock mechanics, i.e. the stability of the rock mass surrounding the tunnel and the strength of the liner (if one is present). The other factor is concerned with hydraulics, i.e. to reduce the head losses as much as possible and exploit fully the energy potential. A good design should consider both factors in a comprehensive way. However, the dimensions of the cross section of the tunnel are mainly determined from hydraulic considerations, while decisions on the lining are taken on the basis of rock mechanics analyses.

Originally, most pressure tunnels were lined with steel. With the development of construction technology, reinforced concrete, unreinforced concrete and grout prestressed concrete liners were introduced successively in the pressure tunnels. Unlined pressure tunnels were introduced some 80 years ago. Their use developed rapidly after the 1960's due to the continuing need to minimize costs and progress in rock mechanics and in underground construction techniques.

1.2 Failure Mechanisms in Unlined Pressure Tunnels

In the last few decades a number of failures to varying extent have occurred in pressure tunnels. A comprehensive survey on the failures in 50 projects was conducted (Appendix 1), based on which an attempt was made to analyze failure mechanisms of pressure tunnels.

1.2.1 Pressure tunnel failures

Pressure tunnel failures may be divided into three categories in terms of reasons which caused the failure: hydraulics, rock mechanics and construction problems. These three categories are not mutually independent. It is sometimes the case that hydraulic effects lead to the revelation of geological defects which results in problems of rock mechanics. At the same time no adequate rock reinforcement measures are taken. Therefore, finally, a failure occurs.

The hydraulic reason may be an unexpected increase in water pressure resulting from a water hammer and /or pressure fluctuations due to inadequate operation of the scheme, causing joint opening and leakage. For example, in Kandergrund hydroelectric power project, Switzerland, a rupture occurred causing a rock slide above the pressure tunnel (Fig.1.5, Jaeger 1955, 1963, 1979, Hoek 1982). Three 25mm wide, 30m long fissures along the tunnel crown and the springs of the tunnel soffit caused excessive water leakage. As a result, a forest was destroyed, a farmhouse crushed and two people killed. The reason of the disaster was a water hammer pressure wave which penetrated the tunnel and lifted the concrete lined soffit in a region where rock was weak and rock cover was inadequate.

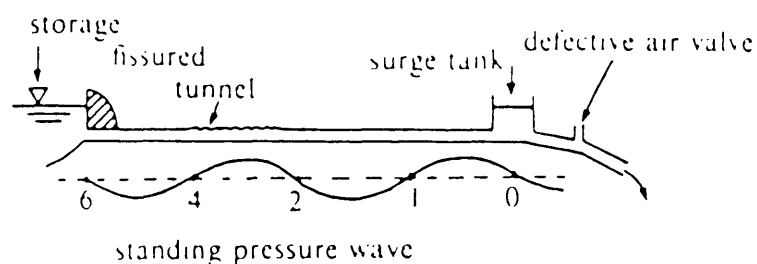


Figure 1.5 Failure of pressure tunnel in Kandergrund hydropower project (After Jaeger 1979)

The failure in the New Calgate pressure tunnel in Yuba hydropower scheme, U.S.A., might be another example of failure due to hydraulic reasons. In the tunnel, discoloured water and a pressure drop were noticed three years after it was put into service. Dewatered inspection revealed some 76000sqyd rock fallout. The accident was attributed to the automatic control which led to continuing pressure fluctuations which consequently resulted in the difference between the tunnel water pressure and pore water pressure. The pressure difference caused an expansion failure and disintegration of sandy gouge and other shear zone material. Consequently, rock blocks loosed and tended to fallout (Lang et al 1976, Hoek 1982).

Hydraulic jacking due to insufficient rock cover above a pressure tunnel, an unfavourable in situ stress field and erosion of filling materials in discontinuities can constitute the rock mechanics reason. Insufficient rock cover has been considered to be one of the major reasons for pressure tunnel failures (Hoek 1982, Broch 1982, Christensen 1982). When the rock cover is inadequate hydraulic jacking can readily occur resulting in leakage and/or rockfalls. The failure of the pressure tunnel of the Herlandsfoss project can be attributed to this reason (Buen 1984, Hoek 1982, Olsen 1970, Patterson et al 1957). The project was completed in 1919. The main rock type in the area is hornblende schist with a seam of mica schist intersecting the tunnel about 100 metres upstream from the power house (Fig.1.6). The original steel and concrete lining was 50 metres long, leaving the rest of the 150 metres unlined. During the first filling when the static hydraulic head reached 120 metres, cracks appeared in the rock along the tunnel springlines and leakage through the mica seam was observed. Then a 50cm thick reinforced concrete lining was placed, but the lining cracked again when the head reached 129 metres. Finally, the entire horizontal section was lined with steel. Since then the power project has operated normally at a head of up to 133 metres. The reason for the accident was considered to be

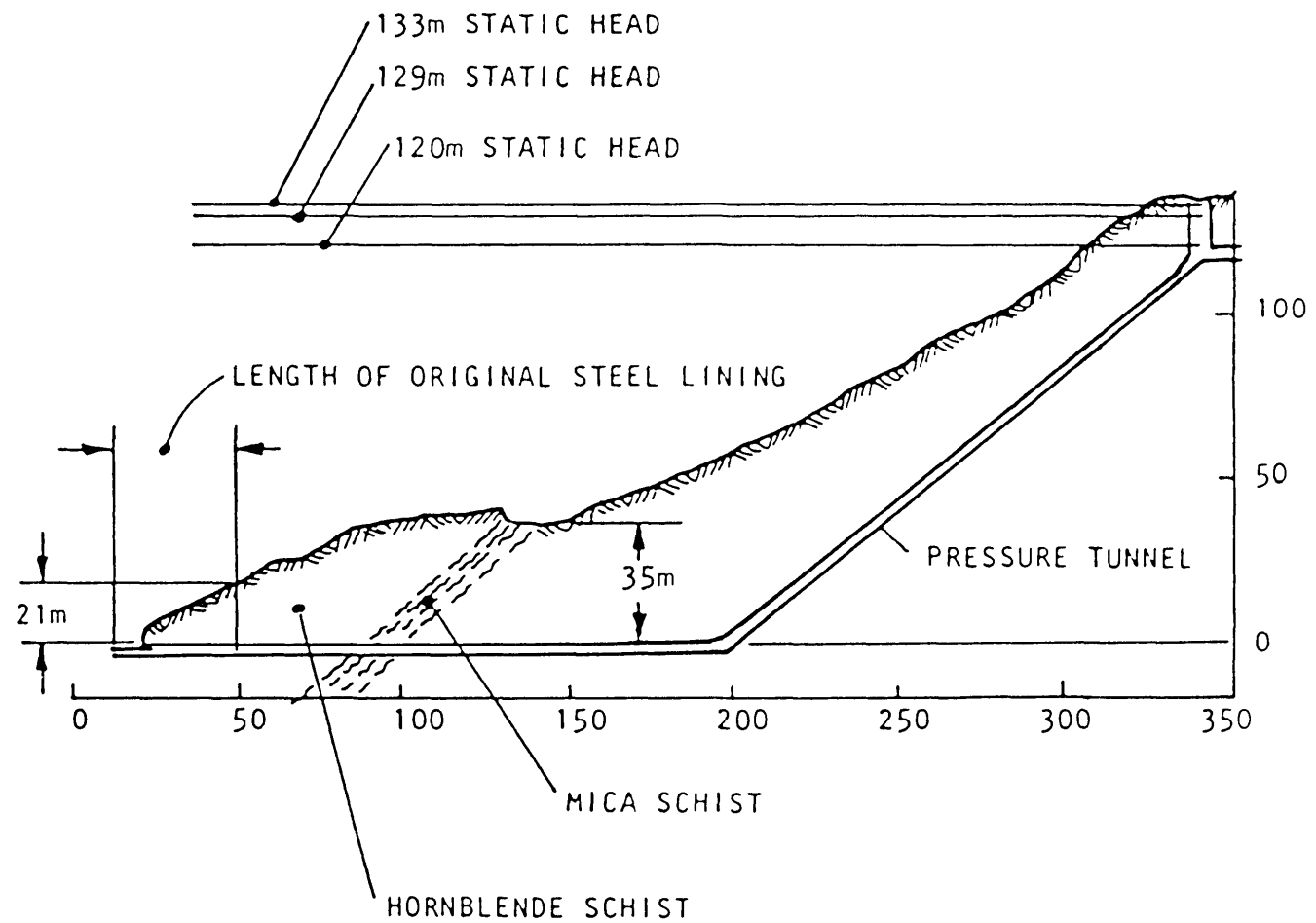


Figure 1.6 Failure of pressure tunnel in Herlandfoss project (After Hoek, 1982)

insufficient rock cover.

The pressure tunnel of Huangqin project, China, was lined with grout prestressed concrete (Fig.1.7). The minimum side cover was 60 metres while the maximum hydraulic head was 300 metres. The tunnel was filled in March 1979 and failure of the lining occurred in February 1981. Longitudinal horizontal cracks of 130 metre long appeared in the pressure shaft and extended to the end of the steel lining (Gu et al 1982, 1983, CHDI 1981). The reason for the failure was likely to be insufficient side cover.

The high pressure shaft of Kotmale hydropower scheme, Sri Lanka, is concrete lined (Fig.1.8). An accident occurred in 1985, when longitudinal cracks appeared in both crown and invert of the concrete lining through which seepage reached 50 litres per second emerging in an old construction adit beneath. It was thought that too low horizontal in situ stresses led to the failure (Anon 1985 a,c).

Kemano pressure tunnel is a part of Kitimat hydropower development located in British Columbia, Canada (Fig.1.9). The first filling was in 1954. There were no problems until 1961 when an excessive head loss was observed. Investigations found that a large amount of rockfalls had occurred in an unlined section. This was thought to be caused by erosion of the mylonite gouge filling in a fault (Hoek 1982, Cooke et al 1962, Lawton 1957).

Various construction problems can arise, such as inadequate excavation methods, inadequate amount and arrangement of drainage holes, incorrect grouting operations and insufficient rock traps. The pressure shaft of Gerlos project, Austria, was steel lined (Fig.1.10). Rupture of the liner occurred in four places along a line parallel to the drainage gallery causing a large amount of rock to be washed away, leaving a large opening in the rock mass (Jaeger

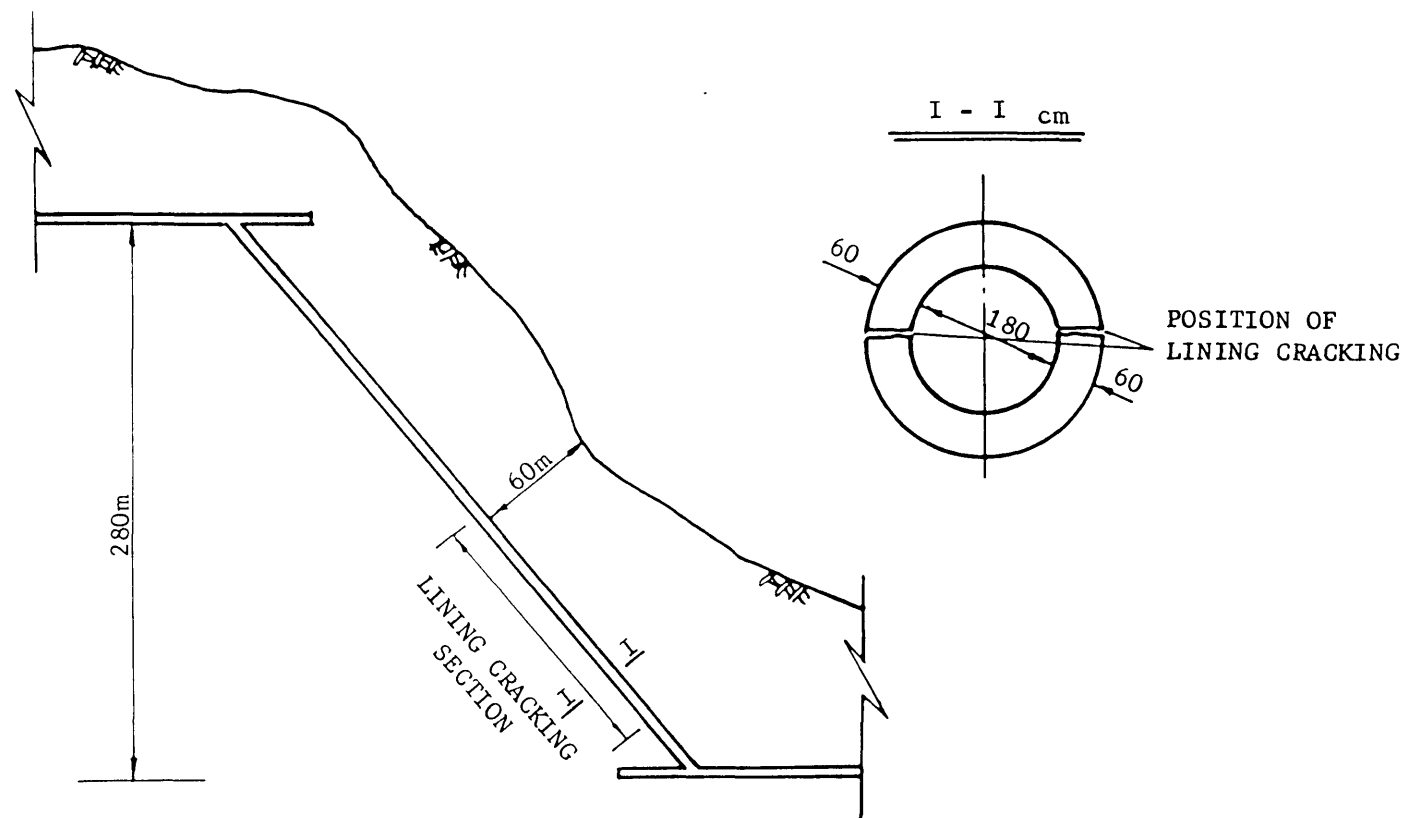


Figure 1.7 Failure of pressure shaft in Huangqin project (After HHDI, 1977)

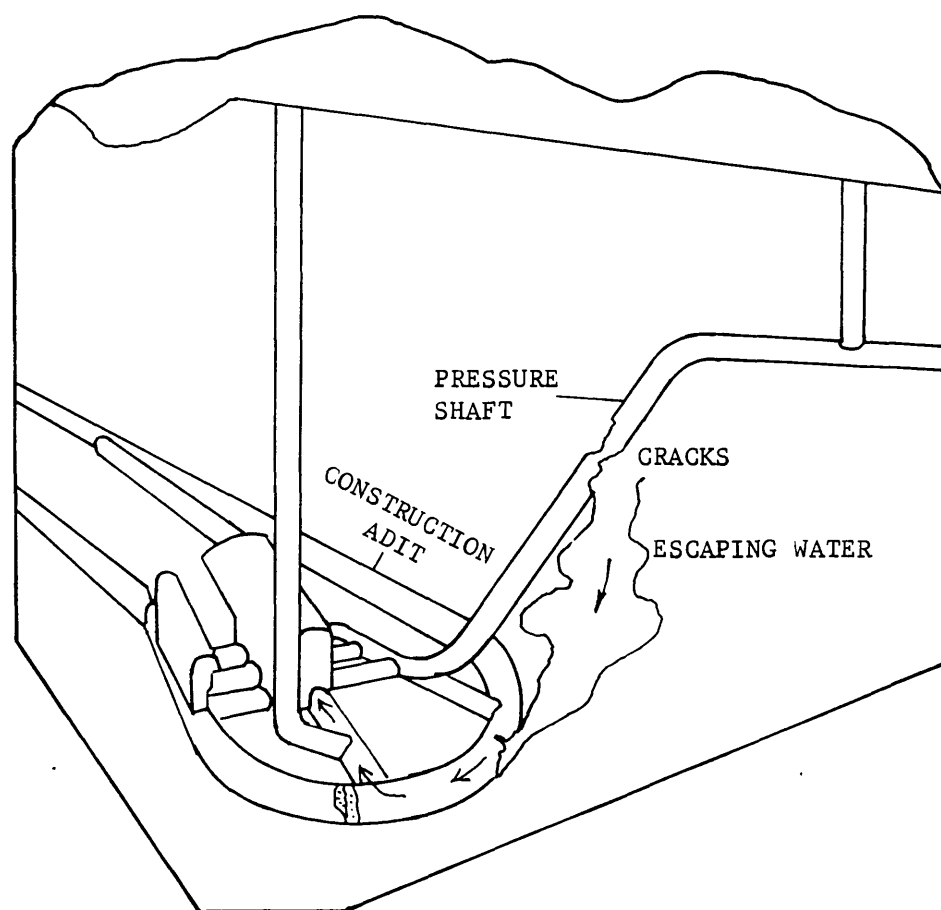


Figure Figure 1.8 Failure of pressure shaft in Kctmale hydropower project (After NCE, 1985)

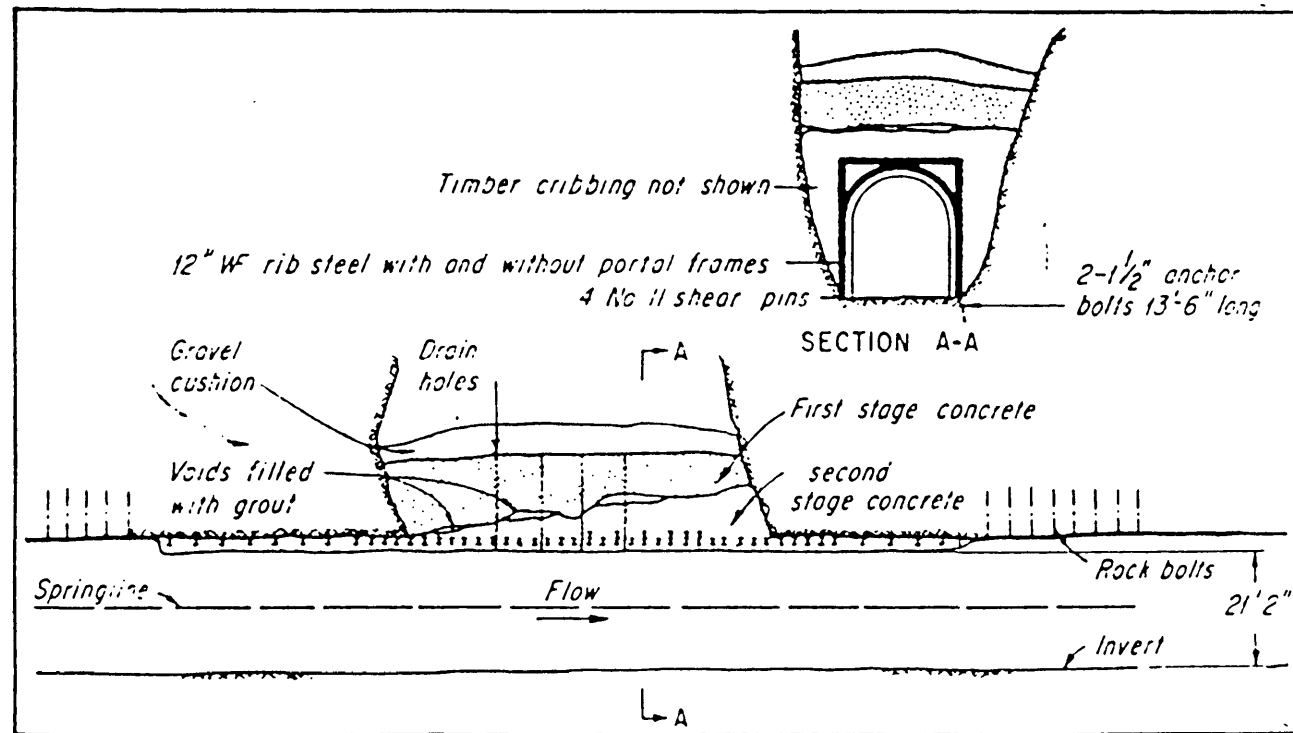


Figure 1.9 Rockfalls in Kemano pressure tunnel (After Cooke, 1962)

1955). This failure was produced by poor quality and uneven subsidence of the concrete which developed excessive bending stresses in the steel liner.

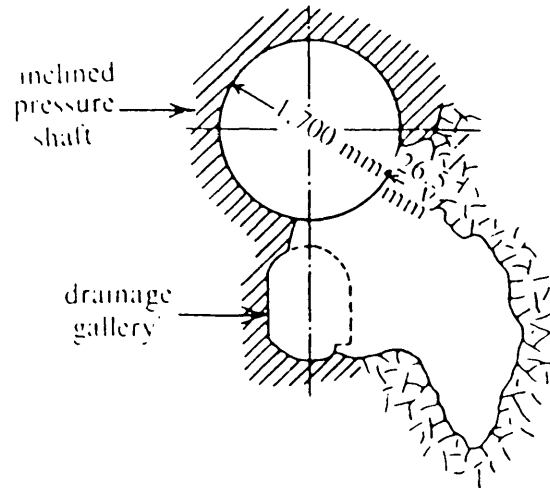


Figure 1.10 Failure of pressure shaft in Gerlos project
(After Jaeger, 1979)

When rock conditions are not good and inadequate reinforcement measures are taken, the results can sometimes be different from those predicted. For example, in the Sydney water supply tunnel, Australia, prestressed grouting was performed to strengthen the surrounding sandstone. Horizontal longitudinal cracks formed during a pressure test causing a high water loss (Jaeger 1979, 1961, Hoek 1982). This was probably due to the low Young's modulus of the sandstone, which could not sustain the grouting pressure.

1.2.2 Analysis of pressure tunnel failure mechanisms

An attempt was made to analyze the failure mechanisms of pressure tunnels in jointed rocks based on the failures described in Section 1.2.1 and Appendix 1. Hydraulic pressure tunnels are often constructed in strong, massive rock which inevitably contains some discontinuities. Some discontinuities

are clean, i.e. without filling, and were called 'extension joints' by Goodman (Goodman 1975). Some are filled with soft materials.

Under application of the internal water pressure, joints, which intersect the tunnel boundary and were originally closed under the initial compressive stress field, will tend to open. In the case where the water pressure is sufficiently high and/or in situ stresses are unfavourable, joint opening will unstably propagate, i.e. hydraulic jacking occurs. For vertical joints (Fig.1.11 a) or joints perpendicular to a valley side (Fig.1.11 b), when the rock cover or side cover is not thick enough, a seepage path from the tunnel boundary to the ground surface will be formed resulting in leakage. If the tunnel is only intersected by horizontal joints (Fig.1.11 c), although the seepage path cannot be formed directly, the horizontal joints might intersect other vertical joints, through which the seepage path may be formed at a high water pressure.

An old theory thought that the water pressure acting on the tunnel boundary might uplift the overburden rock, but such a failure has rarely taken place in practice. This is because the overlying rock cover is usually not that thin. However, when some horizontal joints near the ground surface intersect vertical joints, which intersect the tunnel boundary (Fig.1.11 d) and have opened under the internal water pressure, water under pressure can enter the horizontal joints through the vertical joints. As a result, the ground surface may be lifted and some vertical fissures may appear between the ground surface and the horizontal joints, and leakage can occur (Failure of Bryant tunnel may be an example of this. see Appendix 1). If this situation appears in a valley side (Fig.1.11 e), a landslide may occur on the valley slope.

When the rock surrounding a pressure tunnel is heavily jointed (Fig.1.11 f), application of a water pressure may lead to opening and sliding of joints, resulting in loosening and falling of rock blocks near the tunnel boundary and

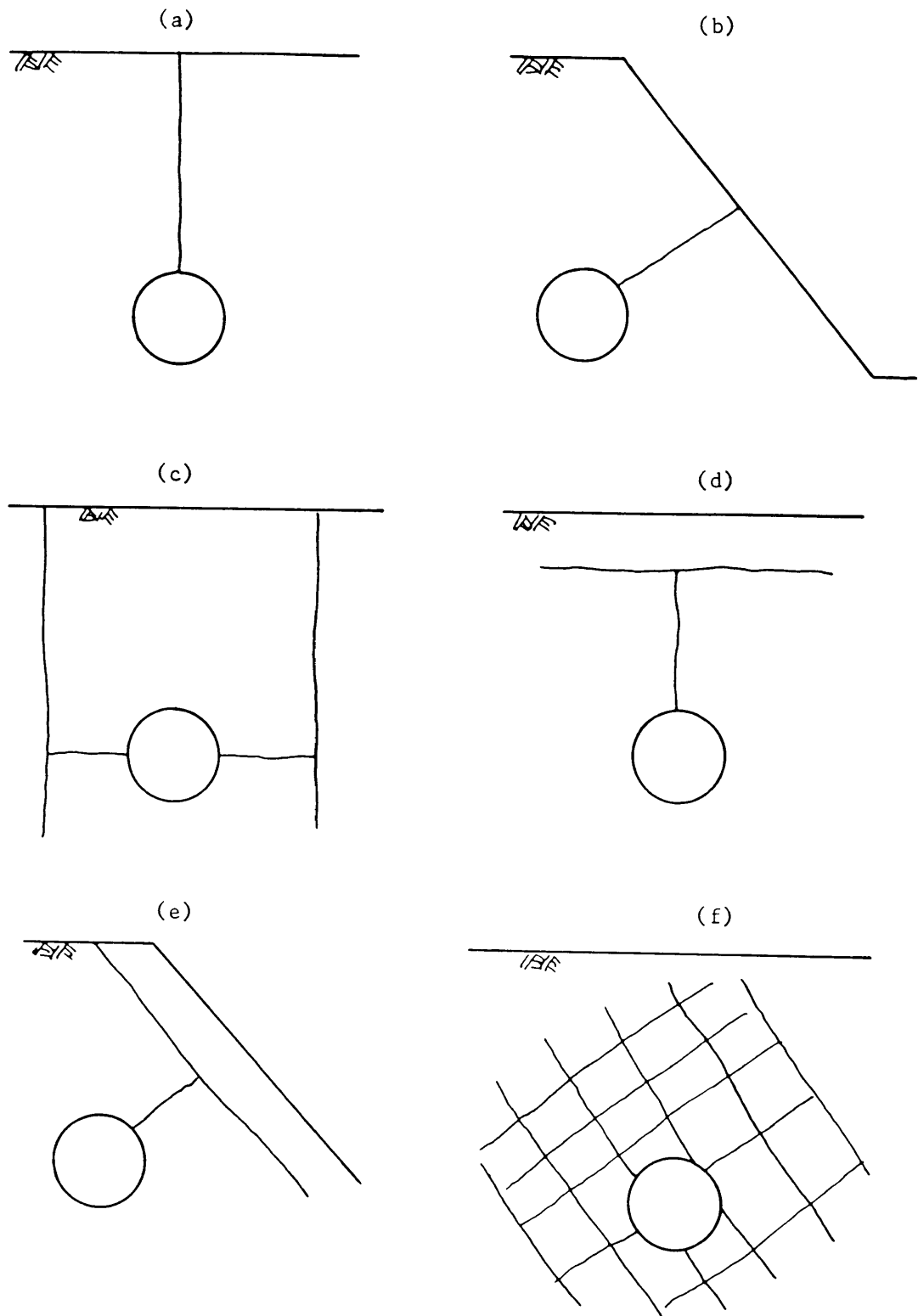


Figure 1.11 Generic cases of unlined pressure tunnel failure

also leakage from the ground surface.

When joints are filled with erosion-susceptible materials such as montmorillonite, mylonite and calcite, the action of seepage water on these materials is likely to result in the joint filling being eroded and washed away. This causes a remarkable decrease in the strength along the joint planes. On the one hand, this leads to a great amount of leakage and even landslide at the ground surface. On the other hand, it loosens the rock blocks near the tunnel boundary causing rockfalls. Modern automatic operation of hydropower stations makes the situation even worse. As the load of a power station increases, the water pressure decreases. Conversely, as the load decreases, the water pressure increases. These frequent pressure fluctuations speed up the erosion process.

It is worth noting that not only are rockfalls and water leakage harmful in terms of the stability of tunnel, but also from an energy point of view they result in head losses, leading to adverse operating conditions for the power station.

1.3 Design Criteria and Methods for Unlined Pressure Tunnels

Several criteria and methods have been adopted in designing more than 200 unlined pressure tunnels in the world over the past 80 years.

1.3.1 The rule of thumb and its modifications

The earliest rule of thumb for designing unlined pressure tunnels was that the rock cover must be such that the water pressure force is balanced by the weight of the superincumbent rock with a width of $B = 2R$, where R is the

radius of the tunnel, i.e. (Jaeger 1979)

$$p \leq \frac{\gamma h}{F_s} \quad (1.2)$$

or

$$h \geq \frac{F_s p}{\gamma} \quad (1.2')$$

where p = internal static water pressure,

γ = specific weight of rock,

F_s = factor of safety,

and h and H are as shown in Fig.1.12. Initially Eqn.1.2 was written as

$$\frac{p_d}{\gamma_w} \leq \lambda H \quad (1.3)$$

where γ_w is the specific weight of water and λ was usually given a value of 0.5 when p_d took the dynamic water pressure (Jaeger 1979). Comparing Eqns 1.2 and 1.3 and assuming $\gamma = 2.5t/m^3$, $H = h$ the factor of safety is 5.

Another fundamental relationship that has been known for many years is (Terzaghi 1962)

$$h \geq \frac{p}{\gamma} \quad (1.4)$$

If $\gamma = 2.6t/m^3$, Eqn.1.4 becomes

$$h \geq 0.385 \frac{p}{\gamma_w} \quad (1.5)$$

Considering a factor of safety of 1.3 the following relationship is obtained,

$$h \geq 0.5 \frac{p}{\gamma_w} \quad (1.6)$$

A number of pressure tunnels were constructed in the Snowy Mountains Scheme, Australia, in the 1950's and 1960's, in which the criterion adopted

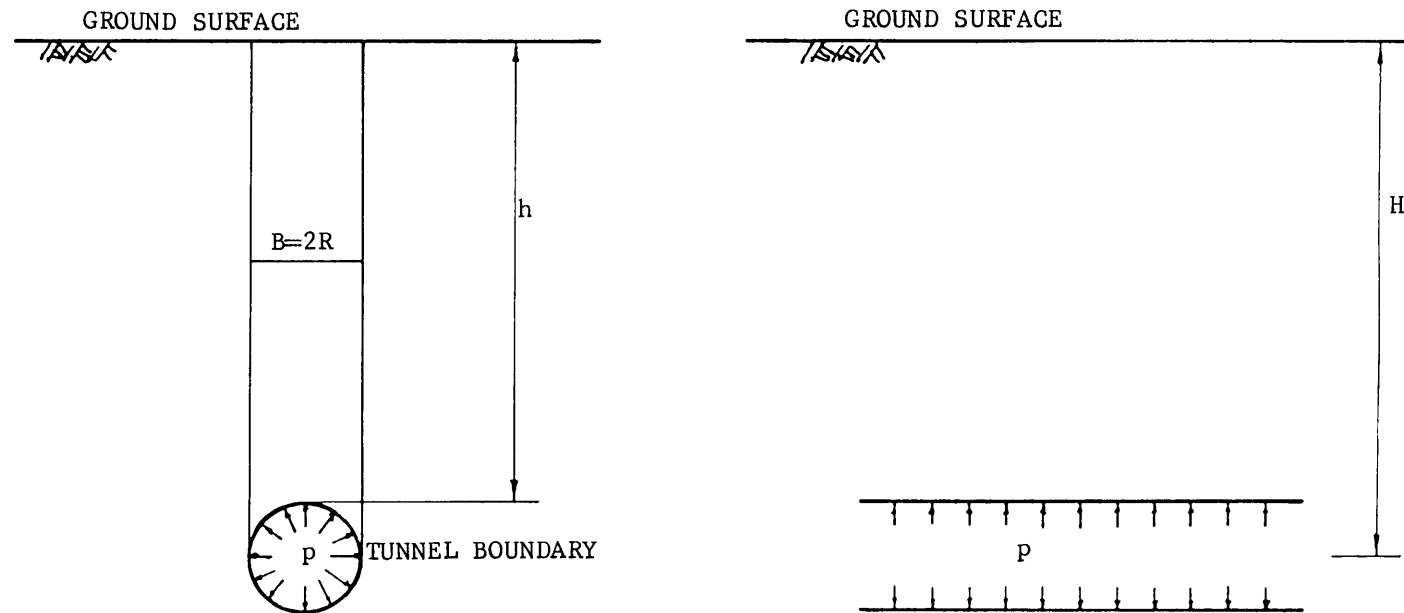


Figure 1.12 Original rule of thumb for determining the maximum water pressure for unlined pressure tunnel

in determining whether a pressure tunnel can remain unlined was based on Eqn.1.4 with the vertical overburden h being replaced by an equivalent rock cover c_v , considering the side cover (Dann et al 1964, Fig.1.13).

Patterson et al (1957) described the design of pressure conduits for three hydroelectric developments built in 1950's in the province of Quebec, Canada, in which the minimum rock cover for omitting steel lining was 50 per cent of the maximum internal water pressure. This was based on an assumption that the horizontal and vertical in situ stress ratio was unlikely to be less than 0.46 and vertical cracking would not occur under the rock cover.

Unlined pressure tunnels have been adopted for over 70 years in California, U.S.A., particularly in the Big Creek development in which the pressure tunnels are predominantly unlined (Spencer et al 1964). The 42 miles of unlined pressure tunnels are mainly located in hard and massive granodiorite. The need for lining was restricted to approximately 7% of the total length. A lot of experience was gained in geological surveys, design and construction. The vertical rock cover for the unlined pressure tunnel was more than 75% of static head and no special requirements were set for the minimum side cover. Some designers required that in sound granite formations side cover should be no less than three times the static head. Observation indicated that there was no leakage from the tunnel with cover equal to or greater than 75% of static head. In cases where the cover was less than 75% of static head, leakage developed in two tunnels; one was because of inadequate side cover and weak rock conditions (tunnel 2), the other due to leakage occurred in pre-existing joints (Mammoth Pool tunnel). However, tunnel 5 experienced no leakage although the cover was only 53% of the static head at two locations.

Eristov et al (1970) recommended a criterion based on the results obtained

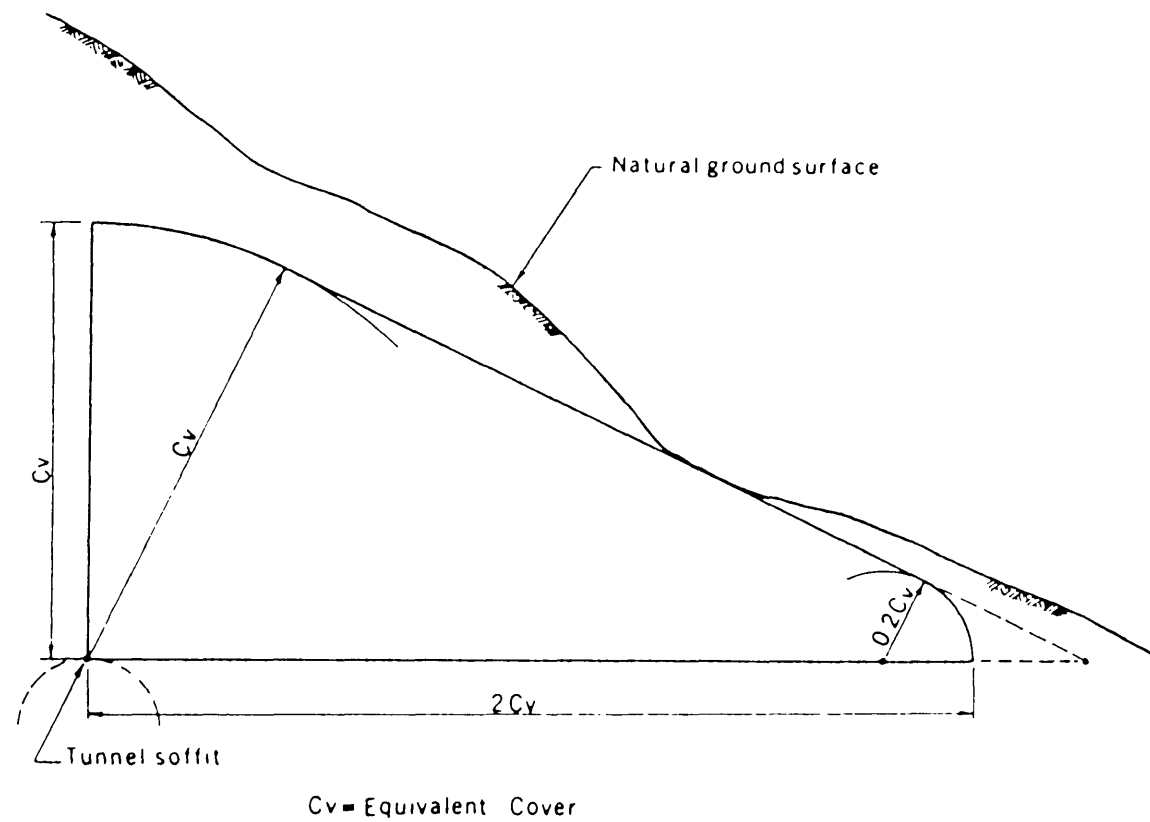


Figure 1.13 Equivalent rock cover adopted in Snowy Mountains scheme (After Dann et al, 1964)

from a series of model tests. That is

$$h \geq 0.25 \frac{p}{\gamma_w} \quad (1.7)$$

In Norway a criterion (Fig.1.14)

$$h \geq \xi \frac{p}{\gamma_w} \quad (1.8)$$

was used before 1968, where the constant ξ was 0.6 for valley sides with inclination up to 35° and increased to 1.3 for those of 60° (Broch 1982). After failure of the unlined pressure tunnel in Byrte hydropower project in 1968, which had an uncommon inclination of 60° , Olsen presented a revised rule of thumb as

$$h \geq \frac{p}{\gamma \cos \alpha} \quad (1.9)$$

(Olsen 1970).

This criterion further considered the effect of the inclination of the pressure tunnel. However, another failure occurred in the unlined pressure tunnel of Askora project (see Appendix 1), that was thought to be connected with very steep valley side. As a result, a new rule of thumb was introduced by Christensen and Dannerig with consideration of the shortest distance between the ground surface and the point studied and the average inclination of the valley side, β , (Olsen 1970, Broch 1982). The criterion was written as

$$L \geq \frac{p}{\gamma \cos \beta} \quad (1.10)$$

Seeber et al (1968) proposed a criterion based on equilibrium analysis assuming that the rock mass bulging above the tunnel is limited by planes emerging from the vault head at a 45° degree angle towards the horizontal ground surface, i.e.

$$h \geq \sqrt{\frac{Dp}{\gamma_w}} \quad (1.11)$$

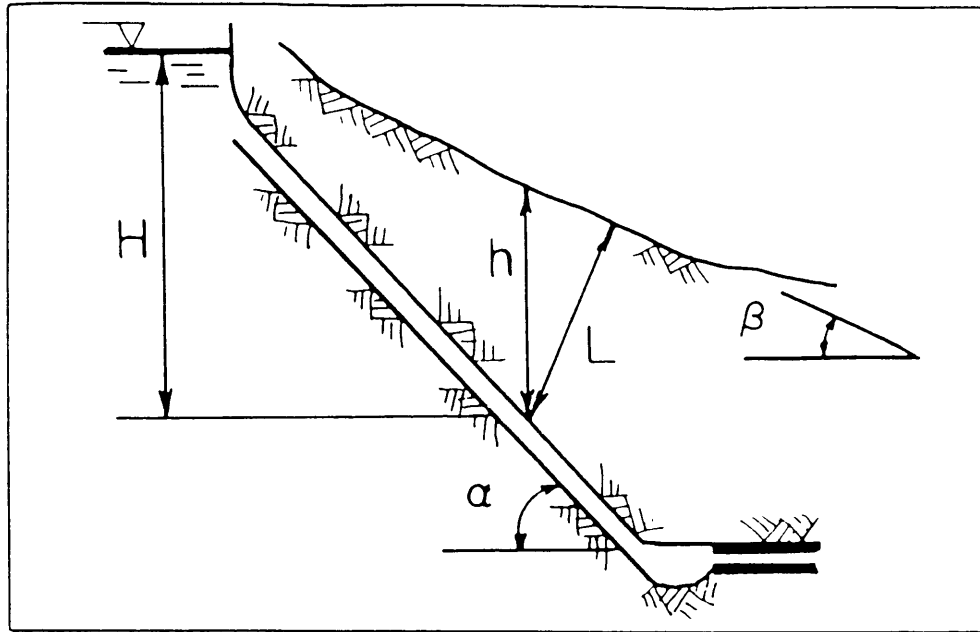


Figure 1.14 Rule of thumb adopted in Norway (After Broch, 1982)

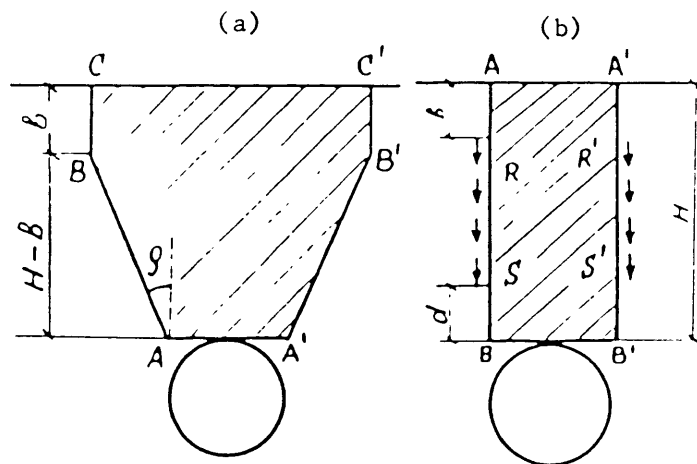


Figure 1.15 Reznikov's method for determining the maximum water pressure for omitting steel lining (After Reznikov et al 1984)

Reznikov et al (1984) assumed that protrusion of the rock above the tunnel could be in two forms as shown in Fig.1.15. For case (a), in which the shear resistance was not taken into account, the weight of the rock block is

$$G_1 = \gamma[HD + (H^2 - B^2) \tan \rho] \quad (1.12)$$

where B is the depth of the surface soil and weathered rock zone. In case (b), the shear resistance was taken into account and the total weight and the shear resistance are

$$G_2 = \gamma\{HD + (H - B - d)\left[\frac{2c}{\gamma} + \lambda \tan \phi(H - d + B)\right]\} \quad (1.13)$$

where c and ϕ are cohesion and frictional angle respectively; d is the depth of the excavation disturbed zone. The criterion was written as

$$\varepsilon \frac{p}{\gamma_w} \geq \frac{G}{F_s D} \quad (1.14)$$

where G can be either G_1 or G_2 ; ε is share of pressure of the rock. The constant ε can be determined on the basis of in situ mechanical tests and geophysical investigations. The angle ρ can sometimes takes a value of $5^\circ - 10^\circ$.

The rule of thumb is basically a crude equilibrium method. It provides a very simple approach to judge if a pressure tunnel can remain unlined. However, this criterion does not take into account the existence of discontinuities, effects of water under pressure in the joints and the in situ stress state. It is more suited to massive intact rocks under a simple and regular topography and a favourable in situ stress field. The method can be used to give a preliminary estimation when there is no geological information available. In general, for hard massive rocks it is likely to produce a conservative design. For rock masses containing adverse joints or where the in situ stresses are unfavourable the rule of thumb may lead to a dangerous design. In fact, some pressure tunnels that do not satisfy these criteria have performed quite well. For example,

in the pressure tunnel of Spray hydropower project, Canada, the equivalent rock cover at the end of the steel lining is only 216 *ft* and the maximum static head is 900 *ft* and the maximum dynamic head is 1220 *ft*, so $\frac{c_u}{p/\gamma_w}$ is only 0.24 for the static head and 0.18 for the dynamic head. However, it has operated successfully to date (Eckenfelder 1952, Hoek 1982).

1.3.2 The modern Norwegian method

Norway is probably one of the most experienced countries in designing and constructing unlined pressure tunnels. Generally massive hard metamorphic rocks, mainly gneiss and granite, have proved to be suitable hosts for unlined pressure tunnels. Up to 1984 a total 79 unlined pressure tunnels and shafts with water heads varying from 150 to 870 metres had been built (Buen 1984, Olsen 1985).

The modern Norwegian method was developed from the rule of thumb. This method pays attention to overburden and side cover, but it is expressed in terms of stresses. It is based on the concept that nowhere along the pressure tunnel or shaft should the internal water pressure exceed the minor in situ principal stress in the surrounding rock in order to avoid the risk of hydraulic fracturing (Olsen 1974, Christensen and Kjølberg 1982, Broch 1982). The minor in situ principal stress is evaluated from two dimensional finite element analysis with the rock being linearly elastic and no discontinuities being introduced under a plane strain condition. Fig.1.16 shows the computational model and the results for the shaft at Nyset/Steggje project. Some design charts for the idealized typical valley sides based on the finite element analysis have been prepared. An example is shown in Fig.1.17, in which the inclination of the valley side $\beta = 40^\circ$, specific weight of the rock mass $\gamma = 2.75 \text{ t/m}^3$, Poisson's ratio $\nu = 0.2$ and the ratio $\sigma_{hor}/\sigma_{ver}$ is 0.5. The curved lines run through the points where the internal water pressure equals the minor principal stress. The

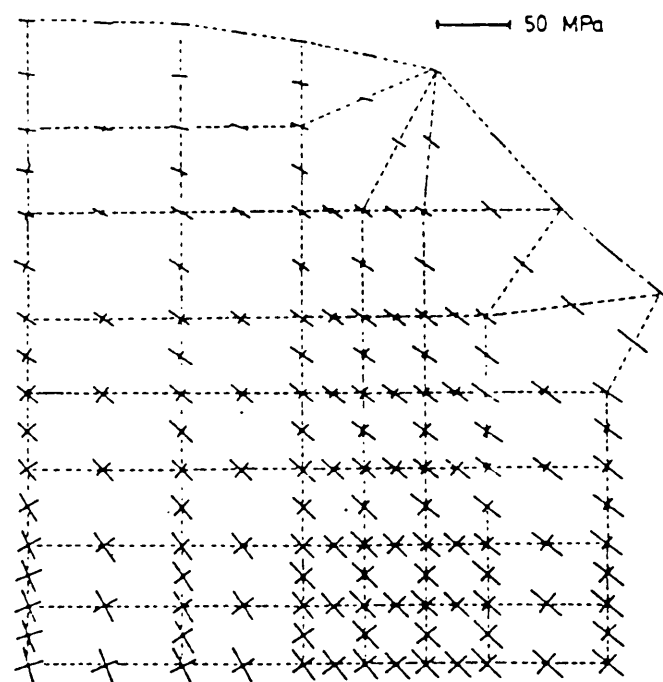
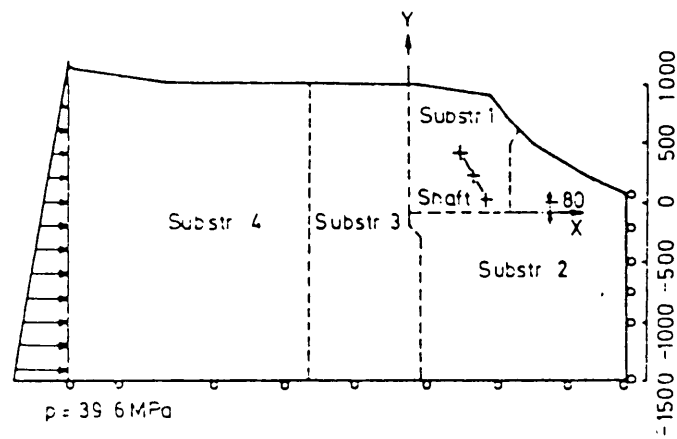


Figure 1.16 Sketch and result of finite element analysis for SYSET/STEGGJE pressure tunnel (After Christensen and Kjolberg, 1982)

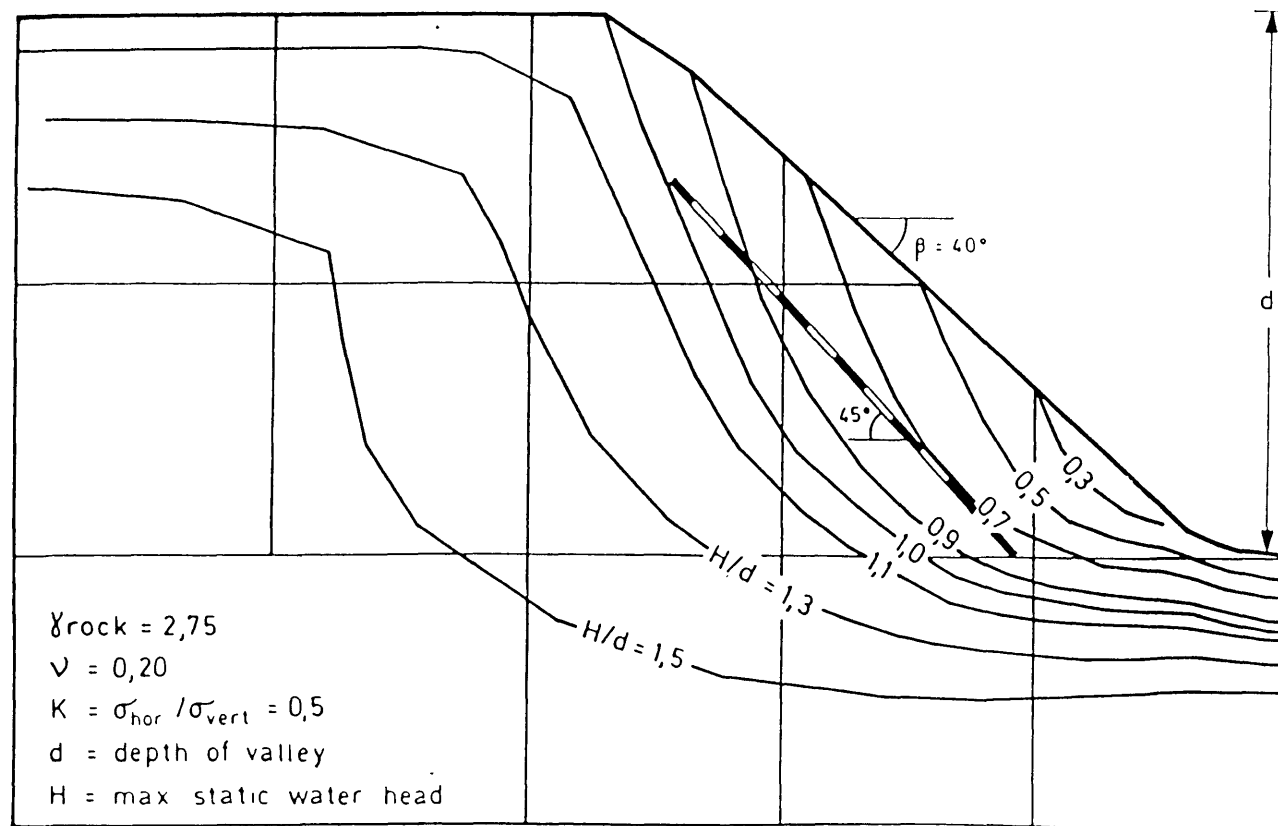


Figure 1.17 Example of design chart for unlined pressure tunnel used in Norway
(After Broch, 1982)

charts for different side inclinations are available and interpolation is possible (Broch 1982).

When the tensile strength of the rock is higher than the intended water pressure, a potential composite failure due to hydraulic fracturing would have to develop along pre-existing joints (Christensen 1982). In this case the criterion is that the native normal stress on the joint plane should be less than the intended water pressure. Therefore, it is necessary to perform detailed geological site investigations and logging of joint sets and the finite element analysis.

Field work is an important component of the Norwegian method. This may include careful mapping of all types of discontinuities (their strikes, dips and filling materials) and performance of hydraulic fracturing tests and in situ stress measurements (Broch 1982, Christensen 1982). A two phase field investigation was recommended, i.e. a geological survey prior to excavation and detailed geological logging and hydraulic fracturing and in situ stress measurements during excavation. If the measurements conducted during excavation indicate more adverse conditions than expected, a fallback alternative to place steel lining at the lower level might be implemented (Christensen 1982, 1985, Deere 1983). Hydraulic splitting tests are performed to determine the $\sigma_3 = p/\gamma_w$ lines and estimate directly the risk of hydraulic fracturing. A simple hydraulic splitting test technique was developed by Norwegian Geotechnical Institute in collaboration with the Norwegian Consultants Berdal A/S and utilized in several projects. The in situ stress measurements (usually overcoring) are carried out to obtain a three dimensional stress field only when the tectonic stresses are complicated, because of the high cost (Christensen 1985).

Monitoring during excavation is another feature of the Norwegian method. It is important to observe the change of all leakages into the tunnel or shaft, even small dripping or seeping leakage (Broch 1982), because these are likely

to become water flow paths after the tunnel is filled.

The Norwegian method is much more advanced than the rule of thumb. It provides a comprehensive procedure to estimate the possibility for a pressure tunnel to remain unlined. In particular, the in situ investigations are essential in designing unlined pressure tunnels. This method has been successfully used in a number of projects in Norway. However, the method was worked out based on the specific geological conditions in Norway (Generally massive hard granite and gneiss). When this method is used in complicated geological formations, care must be taken, especially when the rock mass is jointed and permeable. This method tries to restrict joint opening by means of letting the water pressure be less than the minor in situ principal stress at the tunnel boundary. However, this principal stress is calculated without considering the existence of the tunnel and discontinuities. After the tunnel is excavated, the stress distribution will become different. Furthermore, if joint opening only occurs at the tunnel boundary or within a limited distance around the tunnel, i.e. joint opening does not propagate into the interior of the surrounding rock and to the ground surface, the dangerous situation will not arise. Finally, as described in the failure mechanism analysis (Section 1.2) the pre-existing discontinuities play a decisive role in some cases if their locations are unfavourable. Although this method admits the importance of pre-existing discontinuities, no efficient approach was proposed to deal with this problem. Therefore, for an important project in a jointed and permeable rock mass the Norwegian method is not good enough. A more detailed analysis is needed.

1.3.3 Jaeger's analytical solutions

Jaeger (1961, 1979) analyzed the conditions for failure of hard unfissured rock, radially fissured rock and plastic rock surrounding a circular unlined pressure tunnel by means of elastic or plastic analytical solutions and obtained

corresponding critical internal water pressures. Jaeger also analyzed the case in which a pressure tunnel is situated near a rock slope. A detailed description about these solutions is given in Appendix 2. In order to obtain these solutions a number of assumptions had to be made, such as the rock is homogeneous and isotropic; the rock mass is an infinite body; the tensile circumferential stress induced by internal water pressure for the radially fissured rock is proportional to R/x (R is the radius of the tunnel and x is the distance from the point under study to the tunnel centre). However, no matter how idealized these assumptions are, this was the first attempt to use classical elasticity and plasticity theories to analyze the failure of unlined pressure tunnels and the solutions were the first to provide a quantitative estimate of the critical water pressure. What is more, in these solutions the influence of the dimensions of the tunnel was first introduced. This is a very significant feature for tunnels situated at shallow depths and all other known methods ignore it. Therefore, this is an important and useful tool in analyzing the safety of an unlined pressure tunnel.

As in all other analytical solutions, complicated geological structures, non-linear material characteristics and heterogeneity of rock masses are not taken into account. This makes these solutions to some extent unrealistic. As a result, these solutions cannot be taken as the only basis for the design of an unlined pressure tunnel. They can only be used for preliminary design.

1.3.4 Hoek's method

During consulting work for the Cat Arm hydroelectric project, Canada, Hoek (1982) proposed a complete approach for determining the maximum internal water pressure for unlined pressure tunnels and corresponding length of steel lining. An analytical solution for conditions for joint opening and hydraulic jacking in plane strain was carried out which is developed further to

complete plane strain in Chapter 2 of this thesis. A failure mechanism analysis was also presented. The method includes the following features:

(1) The minimum vertical rock cover C_v for an unlined pressure tunnel with a relatively flat ground surface (say less than 30°), can be calculated from the following equation:

$$C_v = \frac{pF_s}{\gamma} \quad (1.15)$$

If p is the maximum dynamic water pressure, $F_s = 1.1$ can be used. When the maximum dynamic pressure is not known, $F_s = 1.3$ should be taken with p being the maximum static pressure.

(2) The validity condition of Eqn.1.15 is that the horizontal in situ stress is greater than the vertical in situ stress (plane strain condition), i.e. horizontal and vertical stress ratio is greater than unity. The condition is not guaranteed if the tunnel is situated in a steep topography because of stress relief. Therefore, it is essential for the local lateral stress to be checked. A finite element stress analysis on a two dimensional section parallel or at right angles to the tunnel is recommended for checking the horizontal stress distribution. The virgin horizontal and vertical stress ratio K to be applied to the finite element model can be approximately estimated from Fig.1.18 if no measured in situ stress values are available.

(3) Having calculated the minimum rock cover from Eqn.1.15 and checked its validity by means of finite element analysis, the length of steel lining can be determined as shown in Fig.1.19. However, before the final decision is made, it is strongly recommended that a simple acceptance test be carried out as follows. A borehole is drilled to tunnel elevation at the location of the end of the steel lining. The last three to five metres of the borehole are isolated by means of a high pressure inflatable packer and water is injected into this packed-off section at a pressure up to the maximum dynamic water pressure anticipated under

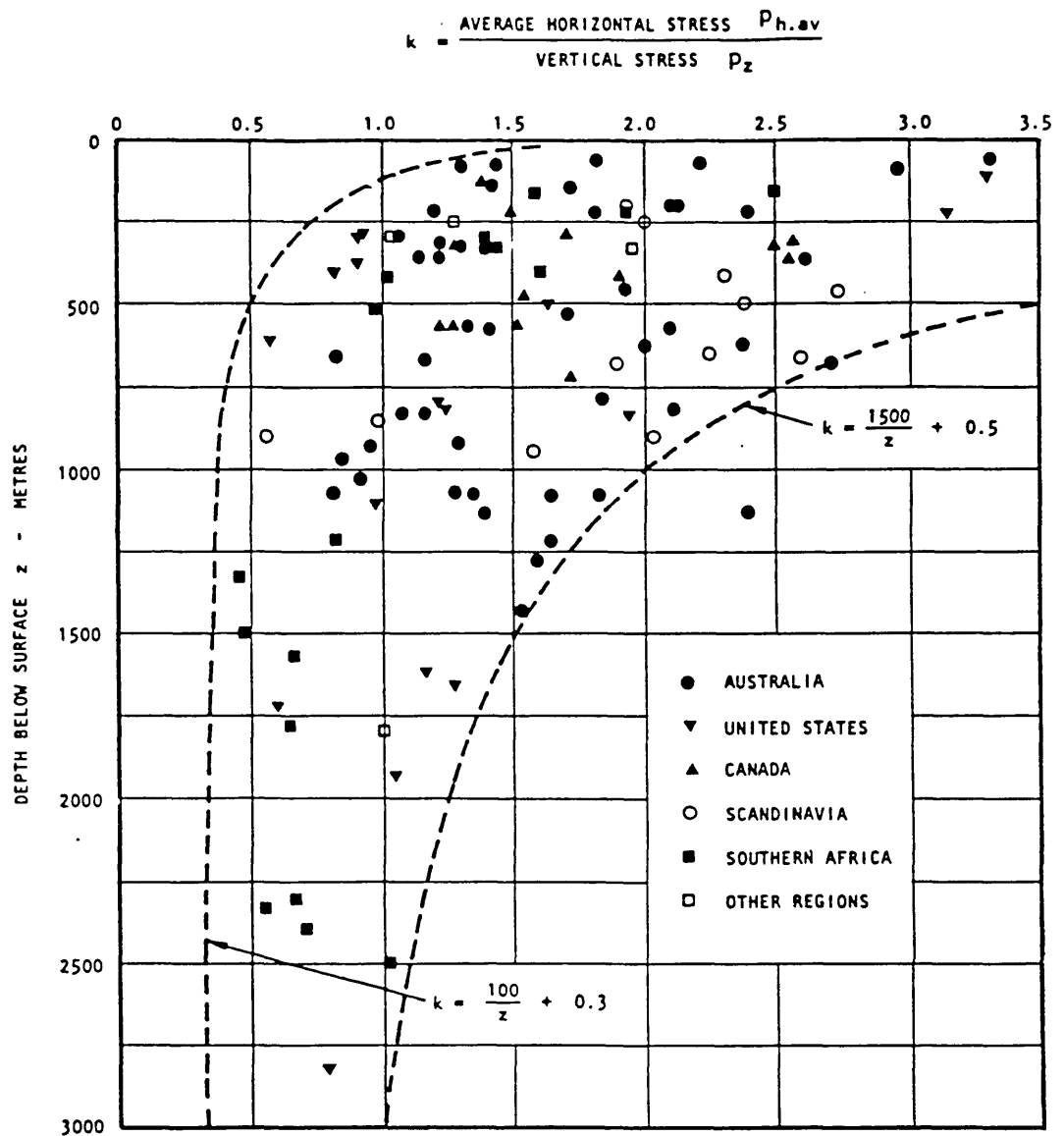


Figure 1.18 In situ stress measurement results (After Hoek and Brown, 1980)

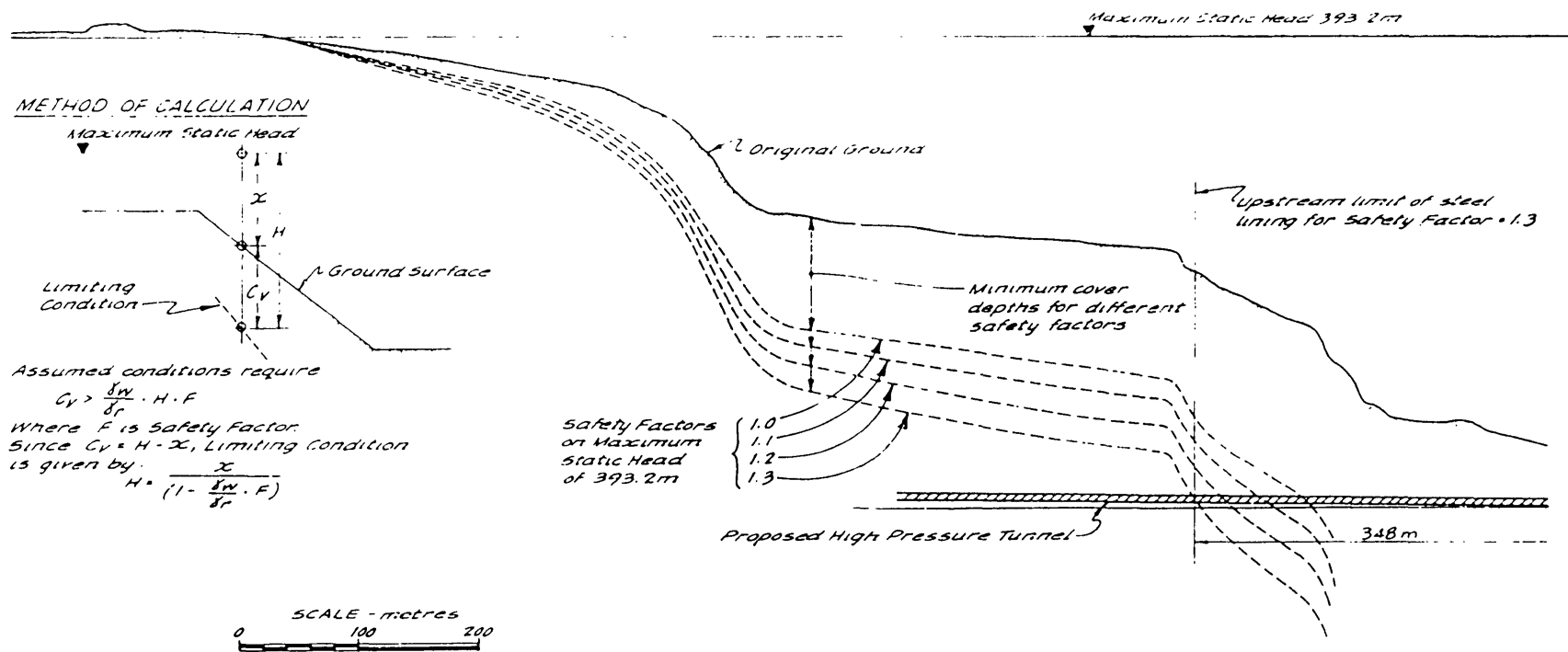


Figure 1.19 Hoek's method for determination of length of steel lining (After Hoek, 1982)

plant operating conditions. This test pressure should be maintained for several hours to check whether any hydraulic fracturing or significant leakage occurs.

(4) During construction of the tunnel, geological mapping is of great importance with attention being paid to weak zones, clay-rich rocks, active clay filled joints or faults or other features which may cause progressive deterioration of the rock mass under operating conditions. In this way some appropriate remedial action may be decided when necessary.

To the author's knowledge, this method is the most comprehensive and advanced to date. It presents a complete procedure for designing unlined pressure tunnels including a simple approach to determine the length of the steel lining, finite element analysis check, an acceptance test prior to excavation and monitoring during construction. One of the outstanding features of this method in comparison with other known methods is the emphasis on the importance of the horizontal and vertical stress ratio. It is correct that the conventional method for determining the maximum internal water pressure for an unlined pressure tunnel is preconditioned by $K > 1$. However, for a real situation of the jointed rock the influence of K is much more complicated. It cannot be simply divided to two cases of greater or less than unity. As will be seen in Chapter 5, in many cases the maximum water pressure increases as K increases. What is more, although in most cases K is greater than unity for hard rocks near the ground surface, sometimes it drops to be less than unity (Broch 1982, Deere 1983). Therefore, a successful method should be capable of analyzing cases with varying values of K . On the other hand, in this method the direction of the tunnel is assumed to be parallel to one of the in situ principal stress directions and only horizontal and vertical joints are taken into account due to the limitation of the plane strain model. In real engineering practice, this is not always the case. The tunnel axis may be orientated at an arbitrary angle

to the in situ principal stresses and the joints may be inclined.

1.3.5 Other methods

Besides the foregoing four criteria and methods some alternatives have also been adopted. Model tests were conducted in Russia in the 1970's (Reznikov et al 1984, Erikov et al 1970). These tests were designed to determine the rock cover that permits the steel lining to be omitted with different tunnel locations ($2D$, $3.5D$ and $5D$) and varying stratification inclinations (horizontal, vertical and inclined by 45 degrees). The dimensions of the models were $80 \times 80 \text{ cm}$ overall, 8 cm thick. Fig.1.20 illustrates the results from one of the tests.

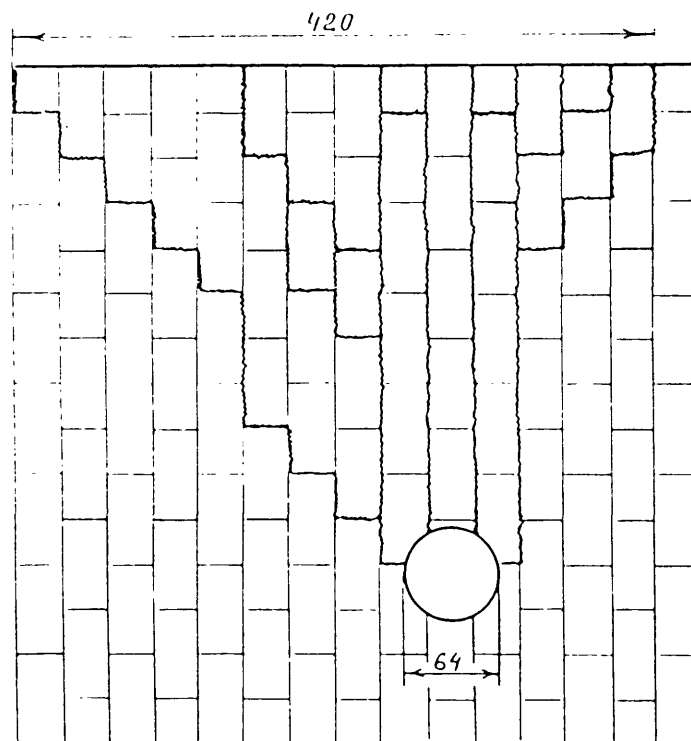


Figure 1.20 Result of a model test (After Reznikov et al, 1984)

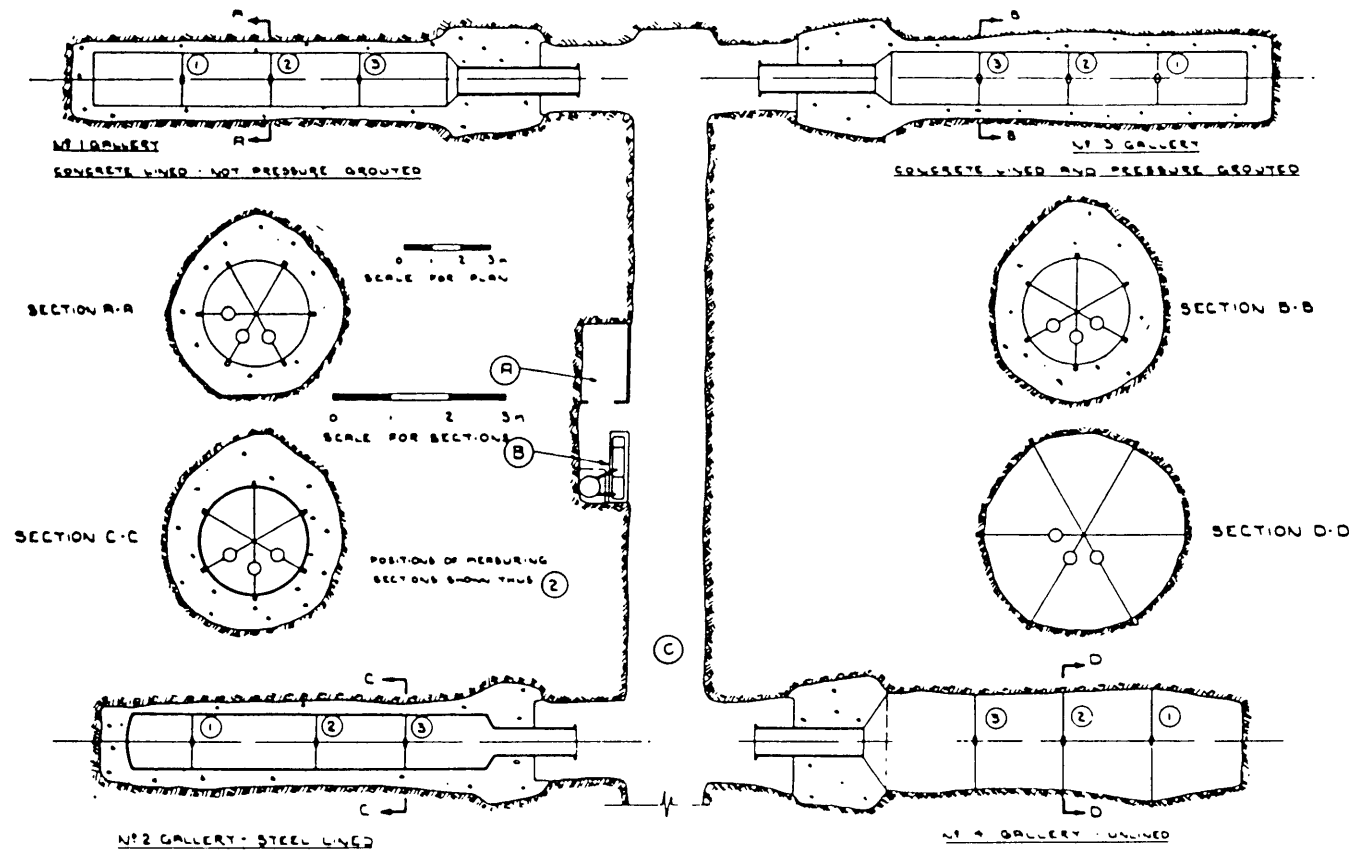


Figure 1.21 Arrangement of field pressure test carried out in Ffestiniog hydropower project (After Chapman, 1961)

The field pressure tests were carried out in some projects such as Sellrian Sily hydroelectric power scheme, Austria (Bonapace 1983), Ffestiniog pumped storage plant, Great Britain (Chapman 1961), Huangqin project, China (HHDI 1977) and Baishan hydropower project, China, (DHDI 1979). Usually the test tunnel is chosen at a site with the same or similar geological formations as the prototype tunnel and parallel to it. In the Ffestiniog project pressure tests were performed in four test tunnels, one of which was unlined (Fig.1.21). Deflections were measured by special extensometers and water losses were measured by flow meters. In the unlined tunnel the rock cover was 65 metres. When the internal water pressure was raised up to 35 kg/cm^2 there were no signs of tensile stress and water losses were surprisingly low. This method can provide direct results, but it is so expensive that it cannot be used in ordinary projects as a conventional design analysis tool.

Madhav and Panesh (1966) presented some design charts obtained from analytical analyses for the design of pressure tunnels.

1.4 Purpose and Components of This Thesis

1.4.1 Purpose of the thesis

Unlined pressure tunnels have great economical advantages over lined pressure tunnels, especially steel lined pressure tunnels. An enormous expenditure can be saved simply by minimizing the length of the steel lined section. Although over 200 unlined pressure tunnels have been built throughout the world, and a lot of experience has been gained, some unreasonable design criteria and methods are still in use. Sometimes they are too conservative; sometimes they lead to dangerous designs and even cause failure. Therefore, it is an essential task to produce a comprehensive and rational design method that will judge

the condition for failure of unlined pressure tunnels more correctly and more accurately, so that the length of the steel lining can be minimized. This study is aimed at developing a new design analysis method which can analyze all the fundamental failure modes introduced in the previous sections and determine the critical internal water pressure for a given pressure tunnel with a certain geological formation.

From the foregoing analyses, it is clear that the pre-existing discontinuities and initial stress state play a decisive role in the failure of unlined pressure tunnels and shafts. The intended design method must be able to model the mechanical and the hydraulic behaviour of discontinuities properly and consider the various initial stress conditions, including the case in which the tunnel is not parallel to one of the in situ principal stresses. Also, in order to obtain realistic results the method must be capable of simulating the whole process from excavation of a tunnel in an initial stress field, application of an internal water pressure, until occurrence of failure. Computerized numerical analysis is the best candidate for this task. It is the most convenient and most feasible approach to deal with such a complex problem. Therefore, the numerical method, basically the finite element method, has been chosen for the new design method to be developed.

Although the computerized numerical method has been proved to be one of the most powerful tools in the field of rock mechanics, it cannot solve any problems independently. It needs collaboration with in situ work to obtain the initial stresses, locations, strikes, dips and dip directions of the discontinuities and with the laboratory tests to get data about the mechanical and hydraulic properties of the intact rock and the discontinuities. The validity of the numerical analysis depends to a large extent upon the accuracy of the input data. Not only does it need input data from in situ and laboratory tests, but also

sometimes the results obtained from the numerical analysis require further confirmation by field tests or/and model tests. Therefore, a complete and proper design method should include all aspects of in situ, laboratory and numerical work. However, due to the time limit this thesis is restricted to numerical analysis.

1.4.2 Components of the thesis

In addition to this introductory chapter there are five chapters in the thesis. In Chapter 2 the analytical solutions for joint opening and hydraulic jacking in complete plane strain condition developed from Hoek's solutions are presented. This gives a general conception about this problem.

In Chapter 3 a new numerical joint model, which is based on Barton's empirical joint model and the elasto-plasticity theory, is put forward including its mechanical and hydraulic features, geometric characteristics, non-linear computational method and its verification.

Chapter 4 deals with the entire proposed numerical method for design analyses of unlined pressure tunnels. It contains the formulation of finite and boundary elements, determination of water pressure acting on joint walls, estimation of the initial stress field, simulation of excavation and computational procedure. A general description of the developed computer program DPTJR is also presented in this chapter.

In Chapter 5 the proposed numerical method is utilized to analyze some generic cases of unlined pressure tunnel failures, from which an analysis of failure modes is carried out and three main failure modes, hydraulic jacking, hydraulic shearing and uplift of ground surface, are identified. The numerical method is also used to analyze a hypothetical real tunnel excavated in a rela-

tively complex geological formation for further verifying the capability of the method.

Finally, in Chapter 6 some conclusions are drawn. A suggested approach to design analysis of unlined pressure tunnels based on the analyses described in preceding chapters and some useful guidelines for design of unlined pressure tunnels are presented. Some suggestions for further research work are also proposed.

CHAPTER 2

ANALYTICAL SOLUTIONS

2.1 Introduction

As mentioned in Chapter 1 Hoek carried out analytical analyses of conditions for joint opening and hydraulic jacking for a circular tunnel (Hoek 1982, Appendix 3). In the analyses the rock surrounding the tunnel was assumed to be homogeneous, continuous, isotropic and elastic and the horizontal in situ stresses were assumed to be equal in all directions. The plane strain condition was adopted in the analyses. The Kirsch equations were used in calculating the stress components. Hoek defined that joint opening occurs when the internal water pressure exceeds the stress acting across the joint plane at the tunnel boundary. The critical stress condition can be established by considering the tangential stress and the stress acting parallel to the tunnel axis. He also defined that once a joint has opened and water can enter the joint, conditions could arise where water pressure could be transmitted into the rock mass surrounding the tunnel. If the opening of the joint is sufficiently large that no pressure drop occurs as the water moves along the joint, hydraulic jacking of the rock mass can occur. This means that, mathematically speaking, the conditions for hydraulic jacking are, firstly, occurrence of joint opening and, secondly, the water pressure being greater than the corresponding in situ stress normal to the joint plane.

The results obtained from the analyses provide simple and direct grounds for determining the location and length of the steel lining. From the results, design charts can be drawn for use in design analyses. Therefore, expanded analyses from the simple plane strain case to the complete plane strain case were carried out. Some modifications to Hoek's solutions were also made.

The following assumptions are made in the analyses.

(1) The rock mass surrounding the pressure tunnel is assumed to be homogeneous, isotropic, continuous and elastic. The joints considered are all conceptual joints. In calculating the stress components the effect of the existence of the joints was not taken into account.

(2) The tunnel is assumed to be situated sufficiently deep, so that the Kirsch equations can be used in calculating the stress components. The influence of the half space associated with the ground surface is ignored.

2.2 Analytical Solutions in the Plane Strain Condition

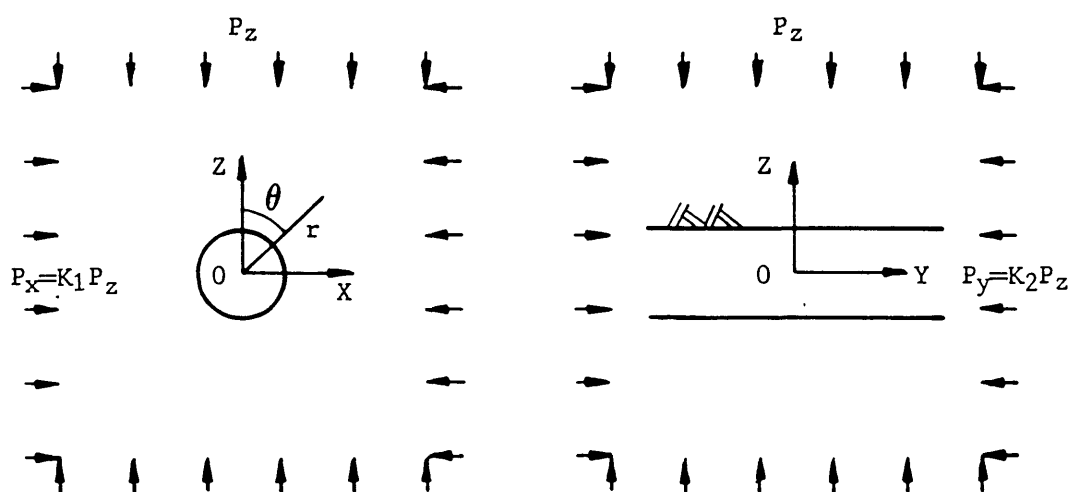


Figure 2.1 Coordinate system and loads for cases in plane strain condition

2.2.1 Cases $K_1 = K_2 = K$

Hoek analyzed the simple case in which the tunnel axis is perpendicular to the vertical in situ stress and the horizontal in situ stresses are equal in all directions, i.e. $K_1 = K_2 = K$ (Fig.2.1, Fig.A.3.1, Appendix 3). In Hoek's solutions the normal stress in the direction parallel to the tunnel axis Y is cal-

culated from Eqn.A.3.7 based on the plane strain condition. A better definition for the plane strain condition would be that excavation does not result in any variations in the strain in Y direction, i.e.

$$\Delta\epsilon_y = 0 \quad (2.1)$$

where $\Delta\epsilon_y$ is the strain induced by excavation in Y direction. Thus,

$$\sigma_y = KP_z + \nu(\Delta\sigma_1 + \Delta\sigma_3) = KP_z + \nu(\Delta\sigma_r + \Delta\sigma_\theta) \quad (2.2)$$

where $\Delta\sigma_r$ and $\Delta\sigma_\theta$ are the radial and tangential stress increments resulting from excavation.

The radial and tangential stresses are still calculated from Eqns (A.3.1) and (A.3.2) without taking the contribution of the water pressure into account as in Hoek's solutions.

Any joint which intersects the tunnel boundary will be forced open when the internal water pressure exceeds the stress acting across the joint plane since the effective normal stress will become tensile. The critical stress conditions can be established by considering the tangential stress and the stress acting parallel to the tunnel axis. On the boundary of the tunnel these stresses reduce to:

When $\theta = 0^\circ$ or 180°

$$\sigma_\theta = P_z(3K - 1) \quad (2.3)$$

$$\sigma_y = P_z[K + 2\nu(K - 1)] \quad (2.4)$$

When $\theta = 90^\circ$ or 270°

$$\sigma_\theta = P_z(3 - K) \quad (2.5)$$

$$\sigma_y = P_z[K + 2\nu(1 - K)] \quad (2.6)$$

Conditions for joint opening are established when the internal water pressure p is greater than the lesser of σ_θ and σ_y providing a joint exists in the required orientation. For the case $\nu = 0.2$, which was analyzed by Hoek, σ_y is

$$\sigma_y = P_z(1.4K - 0.4) \quad (\theta = 0^\circ/180^\circ) \quad (2.7)$$

$$\sigma_y = P_z(0.6K + 0.4) \quad (\theta = 90^\circ/270^\circ) \quad (2.8)$$

The limiting conditions for joint opening are plotted in Fig.2.2. The diagram can be divided into five regions.

$$\underline{K < 0.333}$$

Opening of vertical joints parallel to the tunnel will occur before water pressure is applied.

$$\underline{0.333 < K < 0.375}$$

Opening of vertical joints parallel to the tunnel will occur when $p/P_z > 3K - 1$.

$$\underline{0.375 < K < 1.625}$$

Opening of vertical joints perpendicular to the tunnel will occur when $p/P_z > 1.4K - 0.4$ ($0.375 < K < 1.0$) or $p/P_z > 0.6K + 0.4$ ($1.0 < K < 1.625$).

$$\underline{1.625 < K < 3.0}$$

Opening of horizontal joints will occur when $p/P_z > 3 - K$.

$$\underline{K > 3.0}$$

Opening of horizontal joints will occur before water pressure is applied.

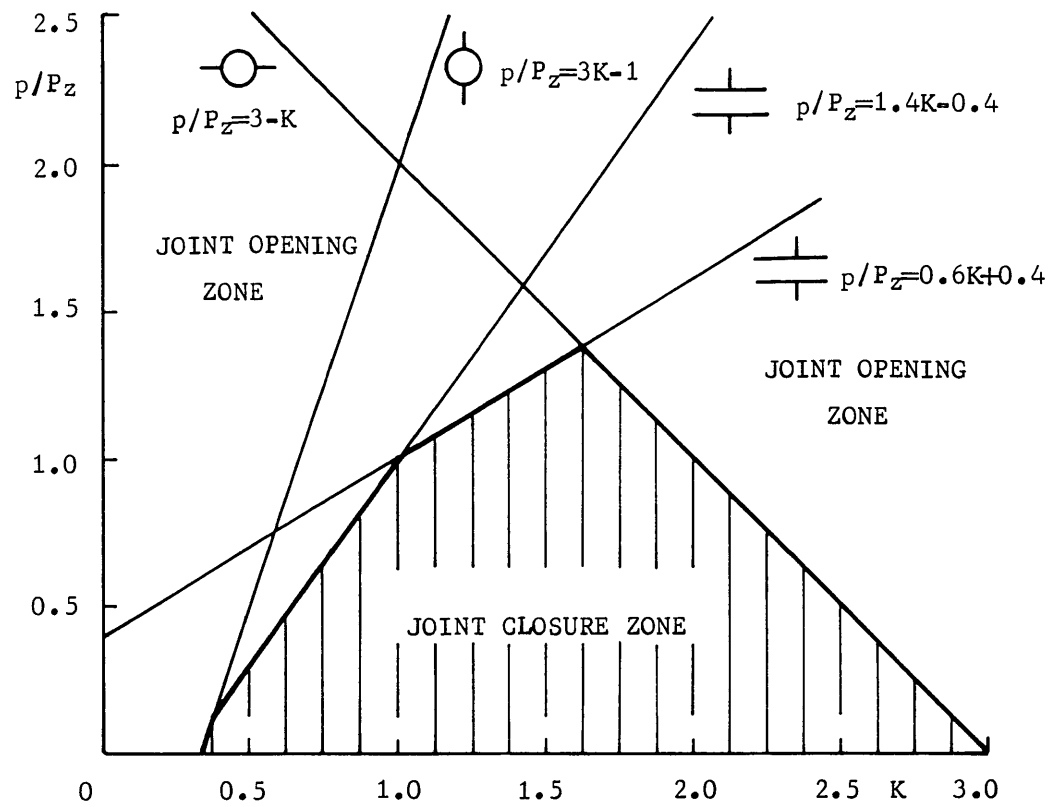


Figure 2.2 Conditions for joint opening when $K_1=K_2=K$, $\nu=0.2$

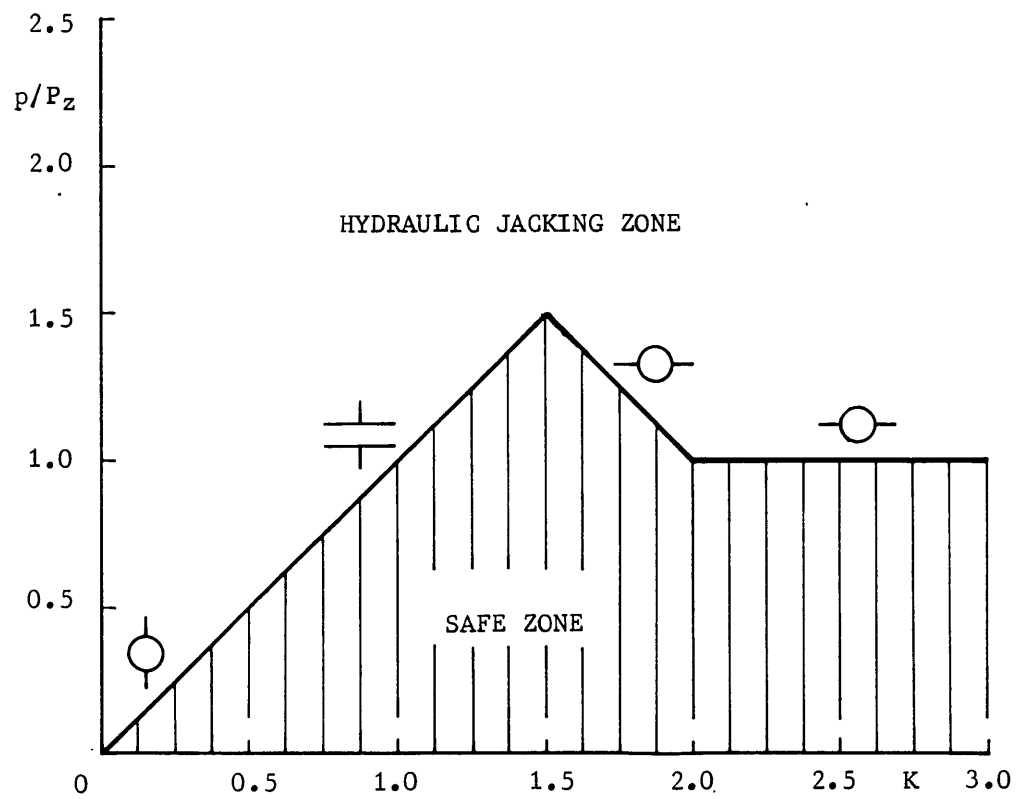


Figure 2.3 Conditions for hydraulic jacking when $K_1=K_2=K$, $\nu=0.2$

Conditions for hydraulic jacking are established when the water pressure is greater than both the water pressure for joint opening and the corresponding in situ stress normal to the joint plane. The limiting conditions for the case $\nu = 0.2$ are plotted in Fig.2.3. The diagram can be divided into four regions.

$$\underline{K < 0.375}$$

When opening occurs on the vertical joints parallel to the tunnel the water pressure is always less than the horizontal in situ stress $K P_z$. Therefore, $p/P_z > K$ is the condition for hydraulic jacking in this region.

$$\underline{0.375 < K < 1.5}$$

When opening occurs on vertical joints perpendicular to the tunnel the water pressure is always less than the horizontal in situ stress normal to the joint plane $K P_z$. Therefore, the condition for hydraulic jacking in this region is also $p/P_z > K$.

$$\underline{1.5 < K < 2.0}$$

When opening occurs on horizontal joint the water pressure is always greater than the corresponding vertical in situ stress P_z . Therefore, the condition for hydraulic jacking in this region is $p/P_z > 3 - K$.

$$\underline{K > 2.0}$$

When opening of horizontal joint occurs the water pressure is always less than the corresponding vertical in situ stress P . Therefore, the condition for hydraulic jacking in this region is $p > P_z$.

If there exist horizontal joints above the tunnel and open vertical joints can transmit the full internal water pressure into the existing horizontal joints

hydraulic jacking may occur on the horizontal joints because $p > P_z$.

The conditions for joint opening and hydraulic jacking for the case $\nu = 0.3$ are shown in Fig.2.4 and 2.5 respectively.

The following conclusions can be drawn from the foregoing analyses.

(1) When the horizontal in situ stresses are low ($K < 1.0$), hydraulic jacking can readily occur on vertical joints under a low water pressure, $p < KP_z$. This is the most dangerous case.

(2) $1.0 < K < 2.0$ is the safest region in terms of hydraulic jacking because it can only occur under a high water pressure.

(3) When $K > 2.0$, although joint opening can occur under a low water pressure hydraulic jacking can only occur when $p > P_z$. This means that even though joint opening occur under $p < P_z$, it cannot propagate unstably. Therefore, a dangerous case will not occur.

2.2.2 Cases $K_1 \neq K_2$

When the in situ horizontal stresses defined by K_1 and K_2 are not equal, σ_θ and σ_y at the tunnel boundary are

When $\theta = 0^\circ$ or 180° ,

$$\sigma_\theta = P_z(3K_1 - 1) \quad (2.9)$$

$$\sigma_y = P_z[K_2 + 2\nu(K_1 - 1)] \quad (2.10)$$

When $\theta = 90^\circ$ or 270° ,

$$\sigma_\theta = P_z(3 - K_1) \quad (2.11)$$

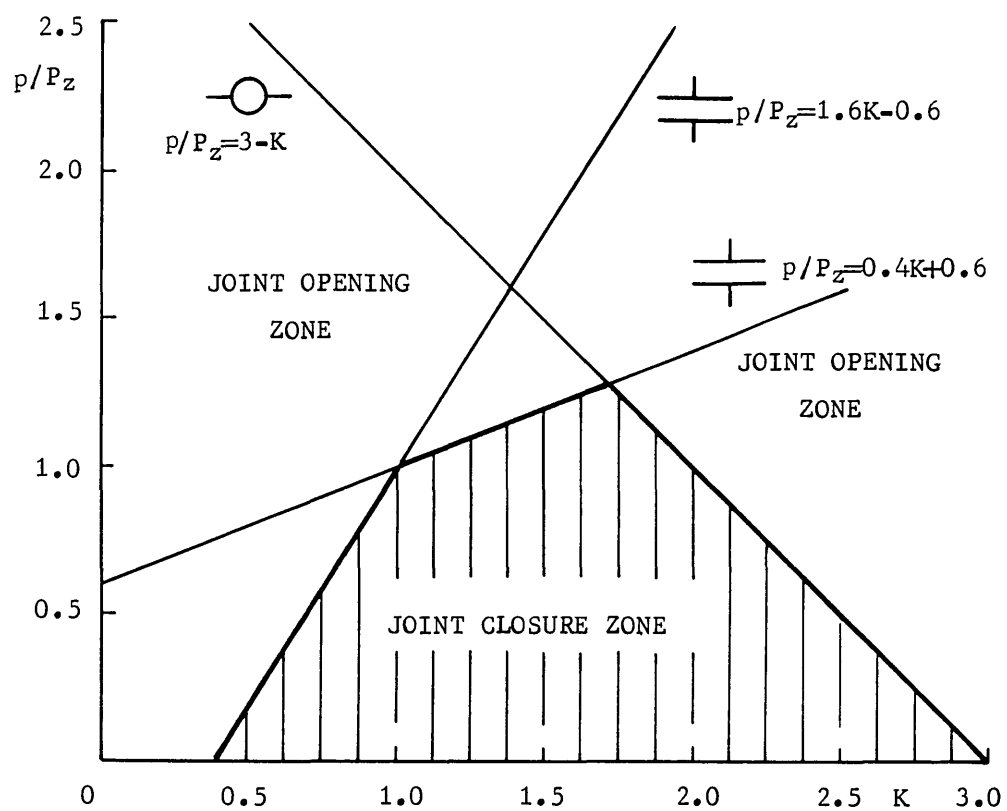


Figure 2.4 Conditions for joint opening when $K_1=K_2=K$, $\nu=0.3$

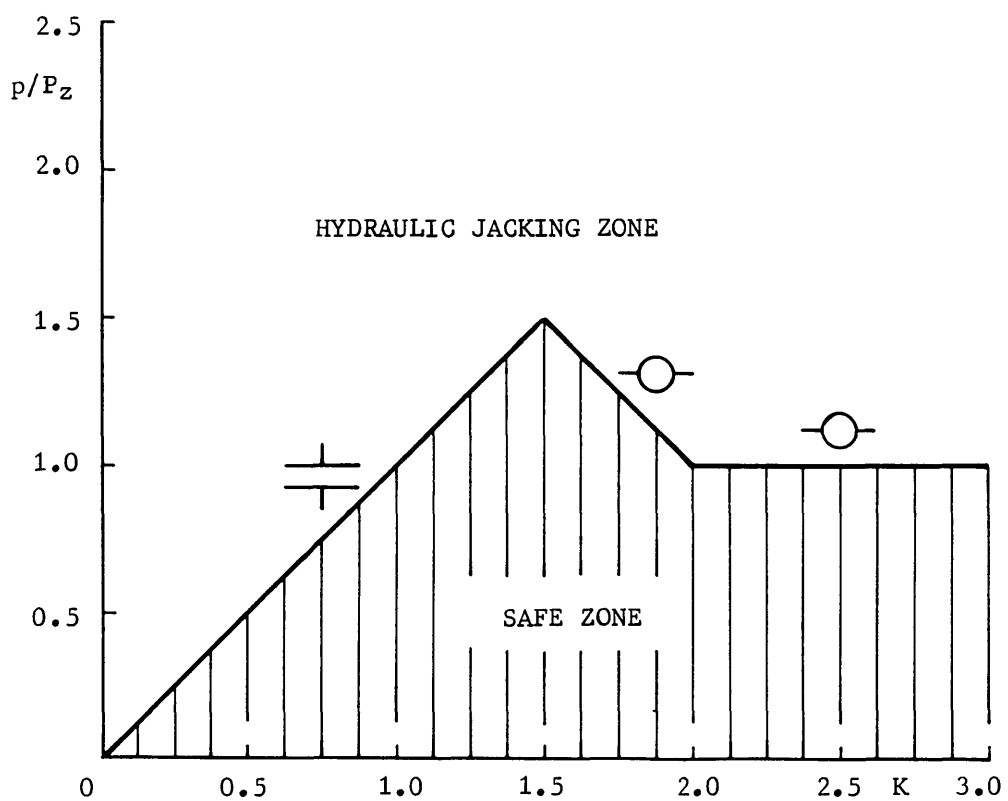


Figure 2.5 Conditions for hydraulic jacking when $K_1=K_2=K$, $\nu=0.3$

$$\sigma_y = P_z[K_2 + 2\nu(1 - K_1)] \quad (2.12)$$

When $\nu = 0.2$, σ_y is

$$\sigma_y = P_z[K_2 + 0.4(K_1 - 1)] \quad (\theta = 0^\circ/180^\circ) \quad (2.13)$$

$$\sigma_y = P_z[K_2 + 0.4(1 - K_1)] \quad (\theta = 90^\circ/270^\circ) \quad (2.14)$$

The cases $K_2 = 0.5K_1$ and $K_2 = 2K_1$ are analyzed. For these two cases Eqns (2.13) and (2.14) reduce to

$$K_2 = 0.5K_1$$

$$\sigma_y = P_z(0.9K_1 - 0.4) \quad (\theta = 0^\circ/180^\circ) \quad (2.15)$$

$$\sigma_y = P_z(0.1K_1 + 0.4) \quad (\theta = 90^\circ/270^\circ) \quad (2.16)$$

$$K_2 = 2K_1$$

$$\sigma_y = P_z(2.4K_1 - 0.4) \quad (\theta = 0^\circ/180^\circ) \quad (2.17)$$

$$\sigma_y = P_z(1.6K + 0.4) \quad (\theta = 90^\circ/270^\circ) \quad (2.18)$$

The conditions for joint opening and hydraulic jacking are plotted in Figs 2.6 and 2.7 respectively. The corresponding results for the cases $\nu = 0.3$ are plotted in Figs 2.8 and 2.9 respectively.

From Figs 2.6 and 2.7 it can be seen that the difference between K_1 and K_2 has a major influence on the conditions for joint opening and hydraulic jacking within commonly encountered values of K_1 and K_2 . Adoption of an average value may lead to a significant error. Table 2.1 shows the critical water pressures for joint opening $p_c/P_z(\text{JO})$ and hydraulic jacking $p_c/P_z(\text{HJ})$ for an example in which the average $K = (K_1 + K_2)/2 = 1.0$.

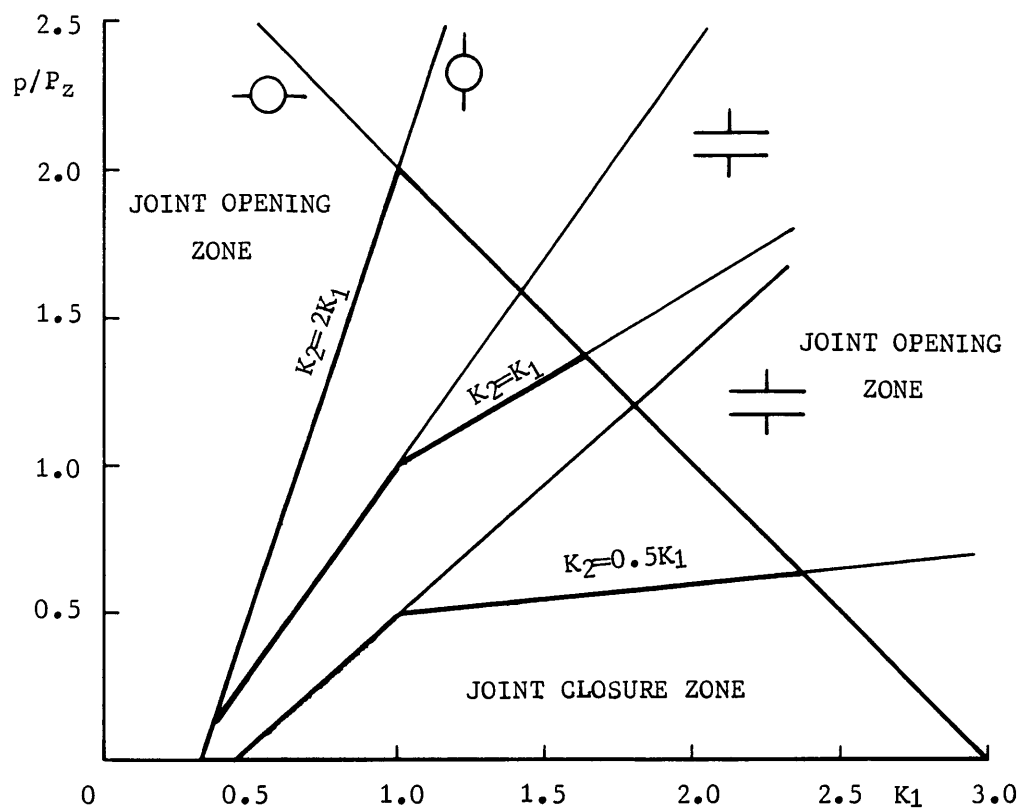


Figure 2.6 Conditions for joint opening when $K_1 \neq K_2$, $\nu = 0.2$

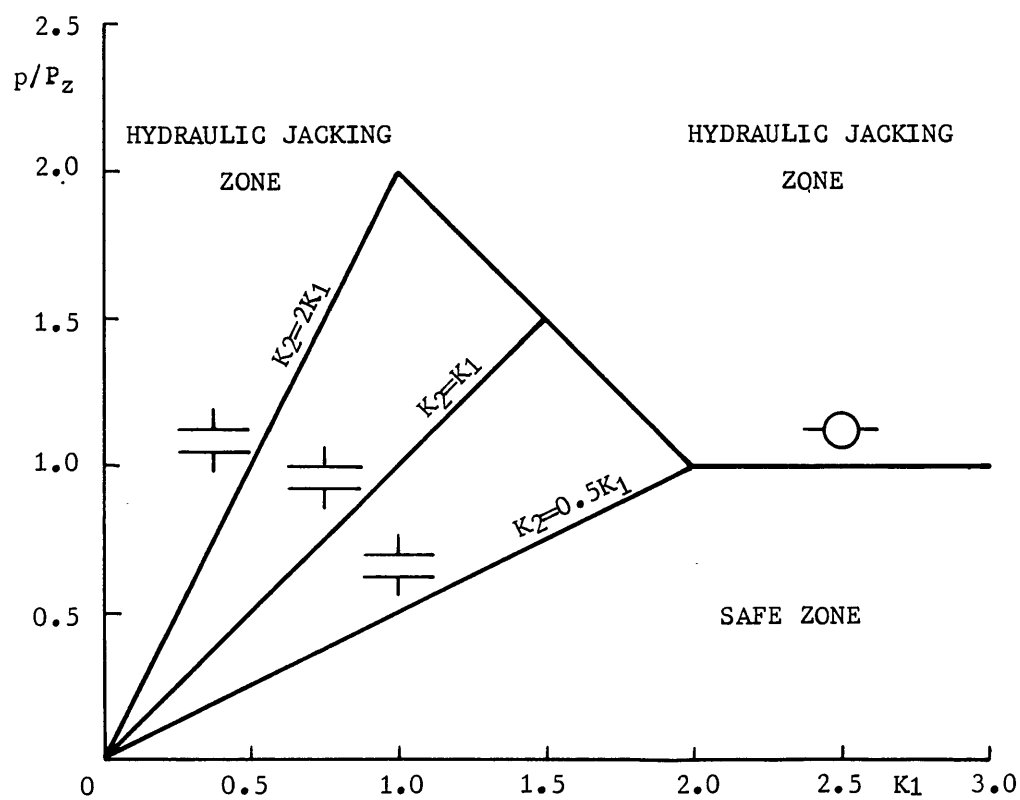


Figure 2.7 Conditions for hydraulic jacking when $K_1 \neq K_2$, $\nu = 0.2$

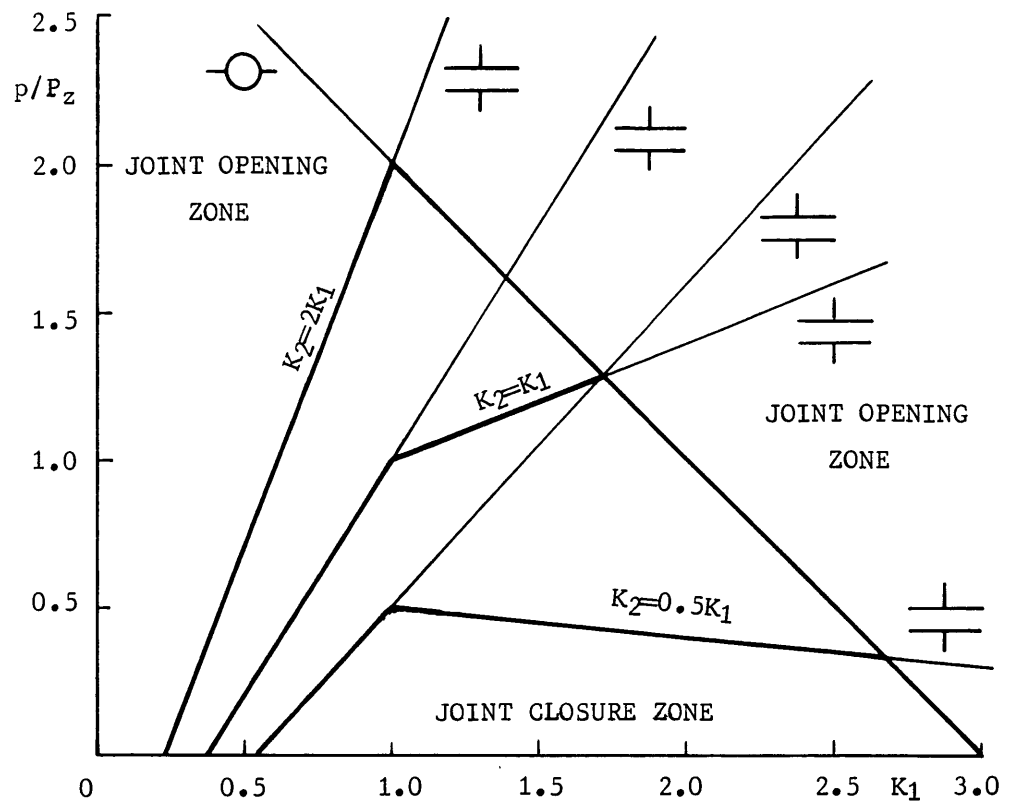


Figure 2.8 Conditions for joint opening when $K_2 \neq K_1$, $\nu = 0.3$

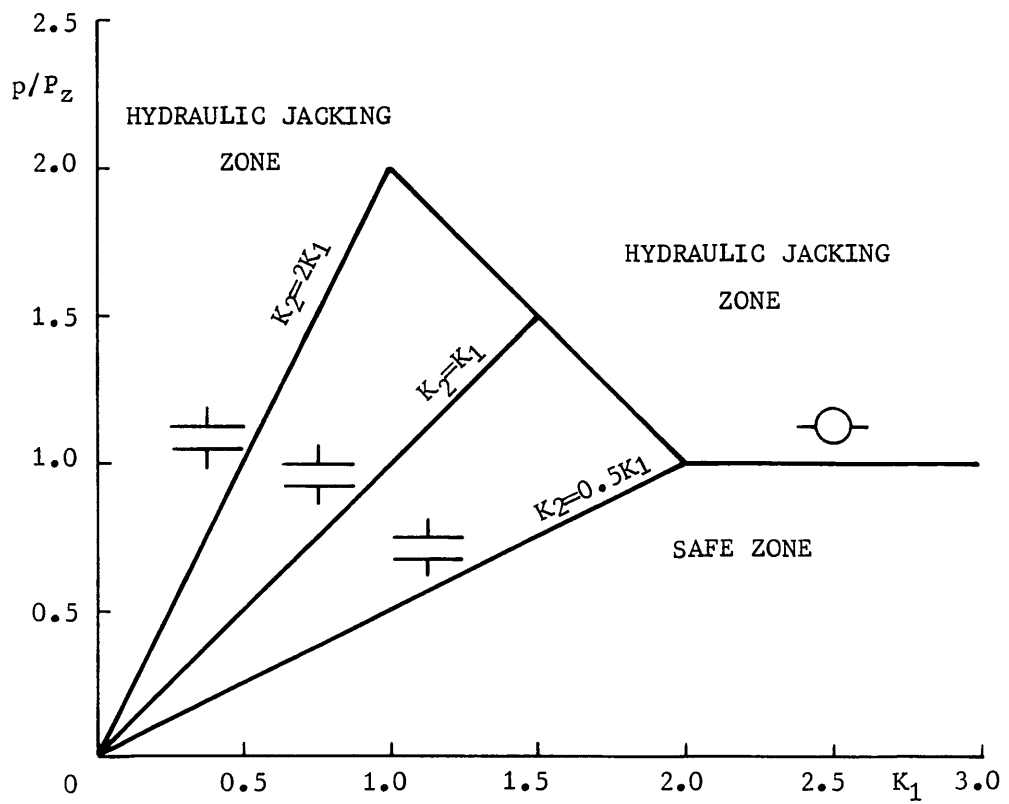


Figure 2.9 Conditions for hydraulic jacking when $K_2 \neq K_1$, $\nu = 0.3$

Table 2.1 Critical water pressure for joint opening and hydraulic jacking for $K=(K_1+K_2)/2=1.0$

K_1	K_2	$p_c/P_z(JO)$	$p_c/P_z(HJ)$	Failure Mode
0.667	1.333	1.000	1.333	
1.000	1.000	1.000	1.000	
1.333	0.667	0.533	0.667	

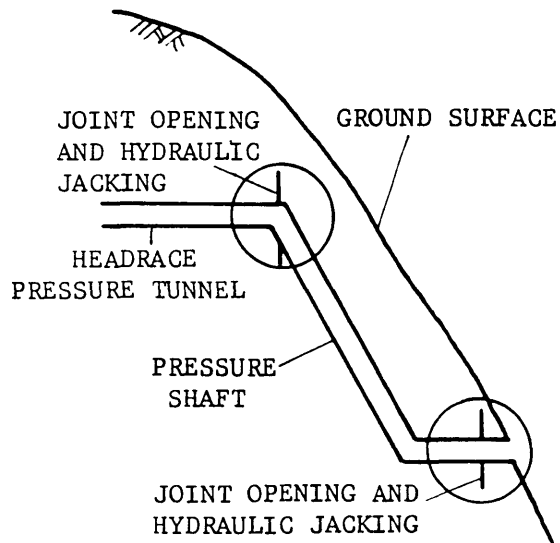


Figure 2.10 Cases $K_2 < K_1$

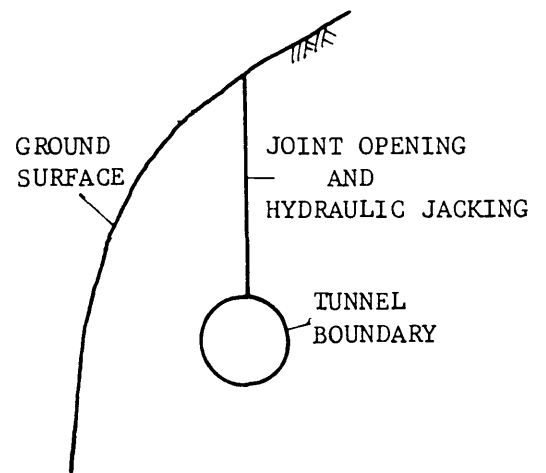


Figure 2.11 Cases $K_1 < K_2$

These results are particularly useful in analysis of a tunnel section near a valley side. At the end of a headrace tunnel and the portal and exit of a pressure tunnel, as shown in Fig.2.10, K_2 is usually less than K_1 and hydraulic jacking is most likely to occur on the transverse joint at a low water pressure. On the contrary, if a tunnel runs parallel to a valley side (Fig.2.11), K_2 is often less than K_1 . In this case, hydraulic jacking may readily occur on the vertical joints parallel to the tunnel. Both these two cases have great potential to cause landslide and can be dangerous.

Comparison of results for $\nu = 0.2$ and $\nu = 0.3$ indicates that the value of ν has some influence on the conditions for joint opening, but almost has no influence on the conditions for hydraulic jacking. For hard rock ν is usually takes a value of 0.15 to 0.25. Therefore, in analyzing the conditions for hydraulic jacking the effect of ν can be reasonably omitted.

2.3 Analytical Solutions in the Complete Plane Strain Condition

2.3.1 The concept of complete plane strain

The analyses described in the last section were for the plane strain condition in which the tunnel axis must be parallel to one of the in situ principal stresses. However, in practice, the tunnel axis is usually inclined to all in situ principal stress directions. If at the same time $K_1 \neq K_2$, the shear stress components parallel to the tunnel axis are not zero, in this case the plane strain model is no longer suitable. The complete plane strain model developed by Brady and Bray (Brady and Bray 1978, Bray 1979) can be used to overcome this difficulty. In the plane strain model, the displacement v in the direction of the tunnel axis Y is zero and the two other displacements u and w in the X

and Z directions are functions of x and z only, i.e.

$$\begin{cases} u = u(x, z) \\ v = 0 \\ w = w(x, z) \end{cases} \quad (2.19)$$

Therefore, the shear strain components γ_{xy} and γ_{yz} must be zero,

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0 \quad (2.20 \ a)$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = 0 \quad (2.20 \ b)$$

This leads to a result that the tunnel axis Y must be parallel to the direction of one of the three in situ principal stresses. In the complete plane strain model, however, the displacement in the Y direction is function of x and z only,

$$v = v(x, z) \quad (2.21)$$

So, in general, it is not necessary for γ_{xy} and γ_{yz} to be zero and accordingly it is not necessary for the tunnel axis to be coincident with one of the in situ principal directions. This means that a complete plane strain analysis can deal with more general cases than a plane strain analysis.

2.3.2 Basic equations for calculating the stress components

The stress components at a point (r, θ) in the rock surrounding a pressure tunnel subjected to a three dimensional stress field, P_x , P_y , P_z , P_{yz} , P_{zx} and P_{xy} , at infinity (Fig.2.12) in the complete plane strain condition can be calculated as follows:

$$\sigma_r = \frac{1}{2}(P_{x'} + P_{z'})(1 - a^2/r^2) + \frac{1}{2}(P_{x'} - P_{z'})(1 - 4a^2/r^2 + 3a^4/r^4) \cos 2(\theta - \alpha) \quad (2.22)$$

$$\sigma_\theta = \frac{1}{2}(P_{x'} + P_{z'})(1 + a^2/r^2) - \frac{1}{2}(P_{x'} - P_{z'})(1 + 3a^4/r^4) \cos 2(\theta - \alpha) \quad (2.23)$$

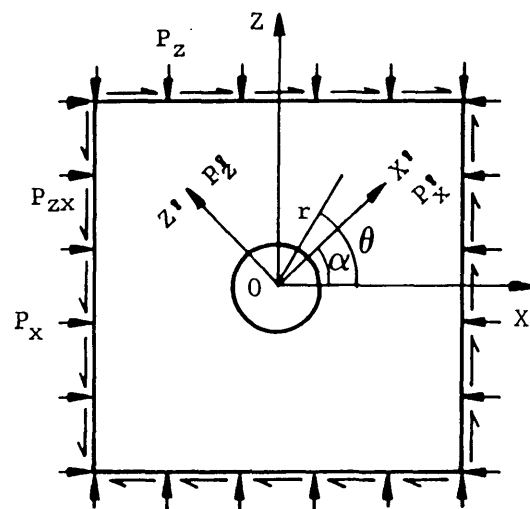


Figure 2.12 Coordinate transformation for complete plane strain cases

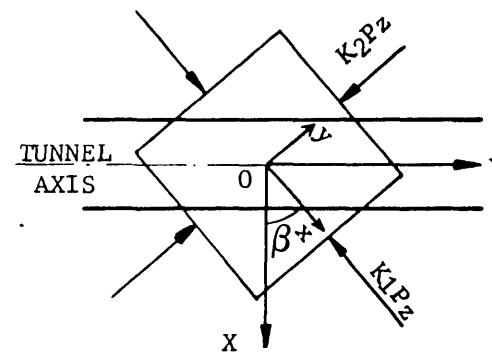


Figure 2.13 Relation between in situ stress directions and the tunnel axis

$$\sigma_y = P_y + \nu(\Delta\sigma_r + \Delta\sigma_\theta) \quad (2.24)$$

$$\tau_{r\theta} = \frac{1}{2}(P_{x'} - P_{z'})(1 + 2a^2/r^2 - 3a^4/r^4) \sin 2(\theta - \alpha) \quad (2.25)$$

$$\tau_{ry} = (P_{xy} \frac{x}{r} + P_{yz} \frac{z}{r})(1 - a^2/r^2) \quad (2.26)$$

$$\tau_{\theta y} = (-P_{xy} \frac{z}{r} + P_{yz} \frac{x}{r})(1 + a^2/r^2) \quad (2.27)$$

where

$$P_{x'} = \frac{P_x + P_z}{2} + \left[P_{xz}^2 + \left(\frac{P_x - P_z}{2} \right)^2 \right]^{\frac{1}{2}} \quad (2.28)$$

$$P_{z'} = \frac{P_x + P_z}{2} - \left[P_{xz}^2 + \left(\frac{P_x - P_z}{2} \right)^2 \right]^{\frac{1}{2}} \quad (2.29)$$

$$\alpha = \frac{1}{2} \tan^{-1} \left| \frac{2P_{xz}}{P_x - P_z} \right| \quad (2.30)$$

At the tunnel boundary Eqns (2.22) to (2.27) reduce to

$$\sigma_r = 0 \quad (2.31)$$

$$\sigma_\theta = (P_{x'} + P_{z'}) - 2(P_{x'} - P_{z'}) \cos 2(\theta - \alpha) \quad (2.32)$$

$$\sigma_y = P_y + \nu(\Delta\sigma_r + \Delta\sigma_\theta) \quad (2.33)$$

$$\tau_{r\theta} = 0 \quad (2.34)$$

$$\tau_{ry} = 0 \quad (2.35)$$

$$\tau_{\theta y} = 2(-P_{xy} \frac{z}{r} + P_{yz} \frac{x}{r}) \quad (2.36)$$

For horizontal tunnels near the ground surface the axis Z is usually co-incident with the vertical in situ principal stress, P_z . In this simplified case, which will be analyzed in this section, $P_{zx} = 0$, $\alpha = 0$, $P_{x'} = P_x$, $P_{z'} = P_z$. Therefore, Eqns (2.31) to (2.36) reduce to

$$\sigma_r = 0 \quad (2.37)$$

$$\sigma_\theta = (P_x + P_z) - 2(P_x - P_z) \cos 2\theta \quad (2.38)$$

$$\sigma_y = P_y + \nu(\Delta\sigma_r + \Delta\sigma_\theta) \quad (2.39)$$

$$\tau_{r\theta} = 0 \quad (2.40)$$

$$\tau_{ry} = 0 \quad (2.41)$$

$$\tau_{\theta y} = 2(-P_{xy} \frac{z}{r} + P_{yz} \frac{x}{r}) \quad (2.42)$$

Assuming that the other two in situ principal directions, x and y intersect the X and Y axes at an angle β (Fig.2.13) and

$$P_x = K_1 P_z \quad (2.43 \text{ a})$$

$$P_y = K_2 P_z \quad (2.43 \text{ b})$$

the stresses at infinity are

$$P_x = P_z(K_1 \cos^2 \beta + K_2 \sin^2 \beta) \quad (2.44 \text{ a})$$

$$P_y = P_z(K_1 \sin^2 \beta + K_2 \cos^2 \beta) \quad (2.44 \text{ b})$$

$$P_z = P_z \quad (2.44 \text{ c})$$

$$P_{xy} = P_z \frac{K_1 - K_2}{2} \sin 2\beta \quad (2.44 \text{ d})$$

$$P_{xz} = P_{yz} = 0 \quad (2.44 \text{ e})$$

The final expressions for stresses at the tunnel boundary are

(i) $\theta = 0^\circ$ or 180° ,

$$\sigma_r = \tau_{r\theta} = \tau_{ry} = \tau_{\theta y} = 0 \quad (2.45 \text{ a})$$

$$\sigma_\theta = P_z(3 - K_1 \cos^2 \beta - K_2 \sin^2 \beta) \quad (2.45 \text{ b})$$

$$\sigma_y = P_z[\nu(2 - 2K_1 \cos^2 \beta - 2K_2 \sin^2 \beta) + K_1 \sin^2 \beta + K_2 \cos^2 \beta] \quad (2.45 \text{ c})$$

(ii) $\theta = 90^\circ$ or 270° ,

$$\sigma_r = \tau_{r\theta} = \tau_{ry} = 0 \quad (2.46 \ a)$$

$$\sigma_\theta = P_z(3K_1 \cos^2 \beta + 3K_2 \sin^2 \beta - 1) \quad (2.46 \ b)$$

$$\sigma_y = P_z[\nu(2K_1 \cos^2 \beta + 2K_2 \sin^2 \beta - 2) + K_1 \sin^2 \beta + K_2 \cos^2 \beta] \quad (2.46 \ c)$$

$$\tau_{\theta y} = \pm P_z(K_2 - K_1) \sin 2\beta \quad (2.46 \ d)$$

2.3.3 Criteria for joint opening

From Eqns (2.45) and (2.46) three possible modes of joint opening can be identified:

(1) Opening of horizontal joint at tunnel side walls, when

$$p/P_z > 3 - K_1 \cos^2 \beta - K_2 \sin^2 \beta \quad (2.47)$$

(2) Opening of vertical joints perpendicular to the tunnel axis at side walls, when

$$p/P_z > \nu(2 - 2K_1 \cos^2 \beta - 2K_2 \sin^2 \beta) + K_1 \sin^2 \beta + K_2 \cos^2 \beta \quad (2.48)$$

(3) Opening of vertical joints intersecting the tunnel axis at an angle at the crown and the invert of the tunnel, when the internal water pressure is greater than the in situ stress acting perpendicular to the joint plane. This can be determined by calculating the minimum principal stress from σ_θ , σ_y and $\tau_{\theta y}$ obtained from Eqn.(2.46)

$$\sigma_3 = \frac{\sigma_\theta + \sigma_y}{2} - \left[\left(\frac{\sigma_\theta - \sigma_y}{2} \right)^2 + \tau_{\theta y}^2 \right]^{\frac{1}{2}} \quad (2.49)$$

$$\alpha = \frac{1}{2} \tan^{-1} \left| \frac{2\tau_{\theta y}}{\sigma_\theta - \sigma_y} \right| \quad (2.50)$$

where σ_3 = the minimum principal stress,

α = the angle between the tunnel axis and the normal of the joint plane.

If the internal water pressure is greater than σ_3 , joint opening occurs, i.e.

$$p > \sigma_3 \quad (2.51)$$

Here the case $K_1 = 0.5$, $\beta = 30^\circ$, $\nu = 0.2$ is taken as an example to demonstrate the calculation procedure. For $\beta = 30^\circ$,

$$\sin^2 \beta = 0.25, \quad \cos^2 \beta = 0.75, \quad \sin 2\beta = 0.866$$

From Eqn.(2.45), when $\theta = 0^\circ$ or 180° ,

$$\sigma_\theta = P_z(2.625 - 0.25K_2)$$

$$\sigma_y = P_z(0.375 + 0.65K_2)$$

From Eqn.(2.46), when $\theta = 90^\circ$ or 270° ,

$$\sigma_\theta = P_z(0.125 + 0.75K_2)$$

$$\sigma_y = P_z(-0.125 + 0.85K_2)$$

$$\tau_{\theta y} = \begin{cases} P_z(0.866K_2 - 0.433) & (\theta = 90^\circ) \\ -P_z(0.866K_2 - 0.433) & (\theta = 270^\circ) \end{cases}$$

The condition for opening of horizontal joints is

$$p/P_z \geq (2.625 - 0.25K_2)$$

The condition for opening of vertical joints perpendicular to the tunnel at walls is

$$p/P_z \geq (0.375 + 0.65K_2)$$

Determination of the condition for opening of vertical joints inclined to the tunnel axis is shown in Table 2.2.

Table 2.2 An example with $K_1=0.5$, $\beta=30^\circ$, $\nu=0.2$

K_2	σ_θ	σ_y	τ_{90}	σ_1	σ_3	α_{90}
0	0.125	-0.125	-0.433	0.451	-0.451	36.9
0.5	0.500	0.300	0	0.500	0.300	0
1.0	0.875	0.725	0.433	1.239	0.361	40.1
1.5	1.250	1.150	0.866	2.067	0.333	41.9
2.0	1.625	1.575	1.299	2.899	0.301	44.4
2.5	2.000	2.000	1.732	3.732	0.268	45.0
3.0	2.375	2.425	2.165	4.565	0.235	45.3

2.3.4 Criteria for hydraulic jacking

Once a joint has opened and the water pressure is greater than the in situ stress normal to the joint plane hydraulic jacking can occur. For the horizontal joint at the tunnel walls this condition is

$$p \geq \sigma_\theta|_{\theta=0^\circ} \quad \text{and} \quad p \geq P_z \quad (2.52)$$

For the vertical joints perpendicular to the tunnel axis the condition is

$$p \geq \sigma_y|_{\theta=0^\circ} \quad \text{and} \quad p \geq P_y \quad (2.53)$$

For the vertical joints inclined to the the tunnel axis the condition is

$$p \geq \sigma_3 \quad \text{and} \quad p \geq P_\alpha \quad (2.54)$$

where σ_3 is obtained from Eqn.(2.49) and P_α is the in situ stress normal to the joint plane which can be evaluated as:

$$P_\alpha = P_z \left[\frac{K_1 + K_2}{2} + \frac{K_1 - K_2}{2} \cos 2\alpha \right] \quad (2.55)$$

where α is determined from Eqn.(2.50).

2.3.5 Analyses of conditions for joint opening and hydraulic jacking

Based on the equations derived above the conditions for joint opening and hydraulic jacking for cases $K_1 = 0.5, 1.0, 2.0, 3.0$; $\beta = 0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$; $\nu = 0.2$ are calculated and the results are plotted in Fig.2.14 to Fig.2.33. These figures provide a series of design charts with which the critical water pressure for any case (ν, β, K_1, K_2) can be obtained by means of interpolation. Some general trends can be identified from these graphs.

(1) When $\beta \neq 0^\circ$ and 90° , that is the tunnel axis is not coincident with one of principal directions, joint opening and hydraulic jacking take place on the vertical joints inclined to the tunnel axis in many cases, especially if K_1 or K_2 is less than unity. This indicates that dealing with such cases with a plane strain model is not adequate and may lead to dangerous designs.

(2) When both K_1 and K_2 have high values (> 1.0), joint opening and hydraulic jacking usually occur on the horizontal joints and the critical internal water pressure against hydraulic jacking is usually equal to the vertical in situ stress, P_z .

(3) The best values of the internal water pressure against hydraulic jacking are obtained when both K_1 and K_2 are between 1.0 and 2.0, while the critical water pressure is usually greater than the vertical in situ stress, P_z .

(4) The most dangerous cases occur when both K_1 and K_2 are less than

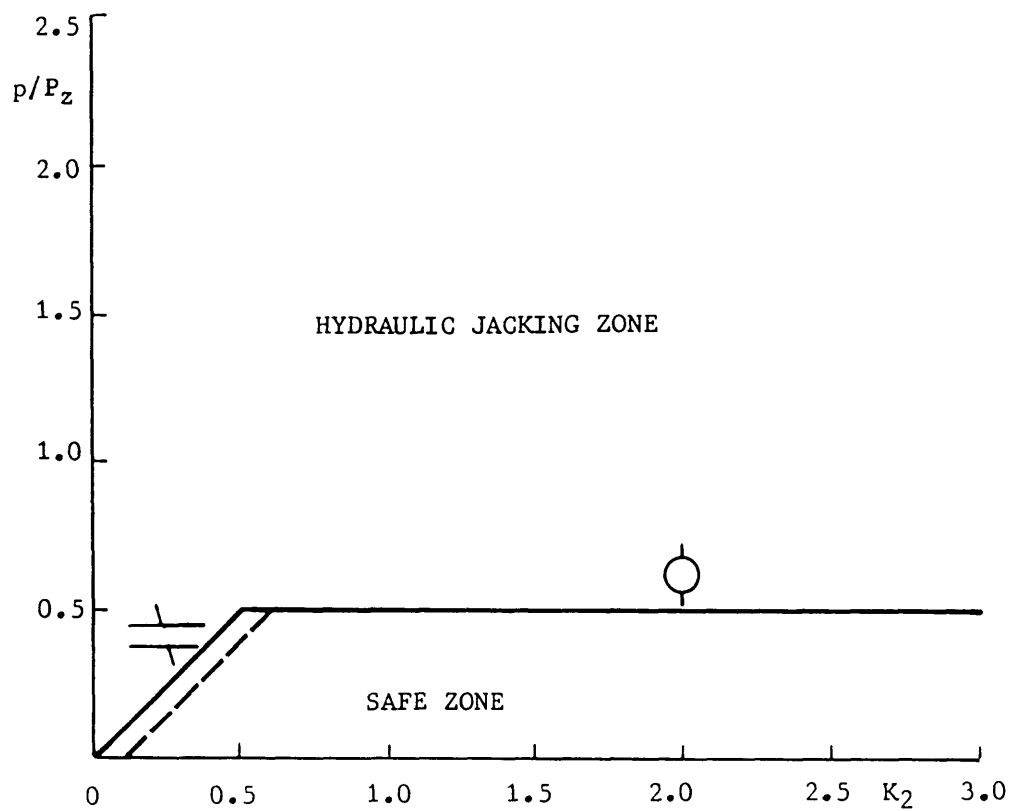


Figure 2.14 Conditions for joint opening and hydraulic jacking
when $\beta=0^\circ$, $K_1=0.5$, $\nu=0.2$

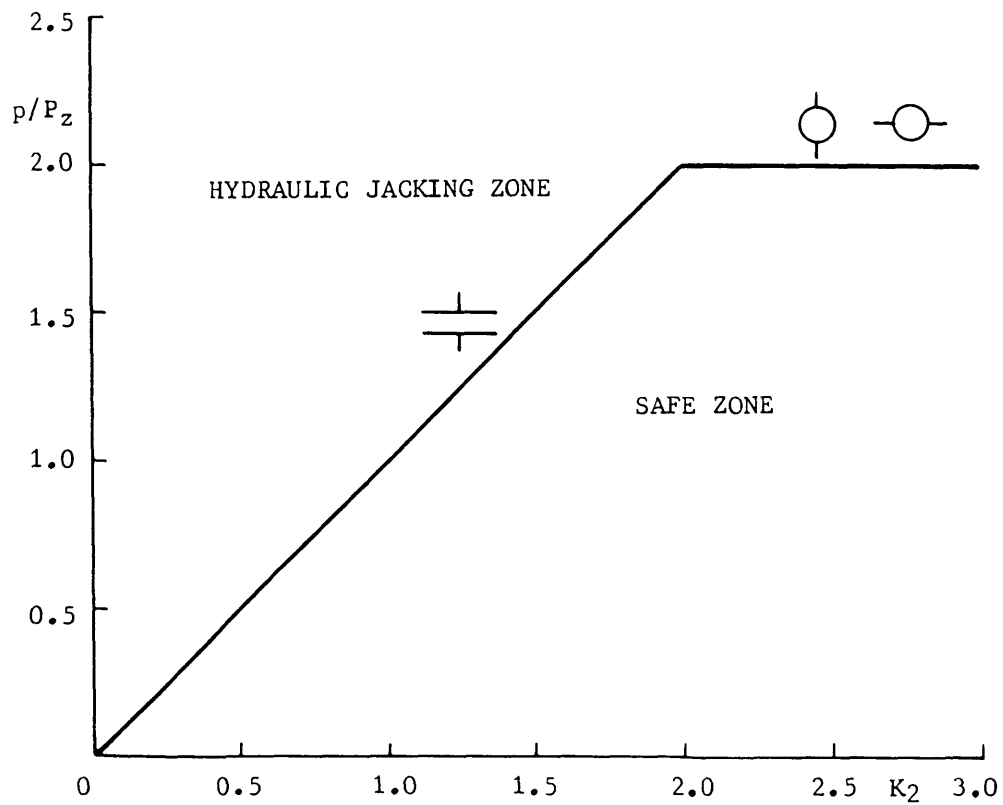


Figure 2.15 Conditions for joint opening and hydraulic jacking
when $\beta=0^\circ$, $K_1=1.0$, $\nu=0.2$

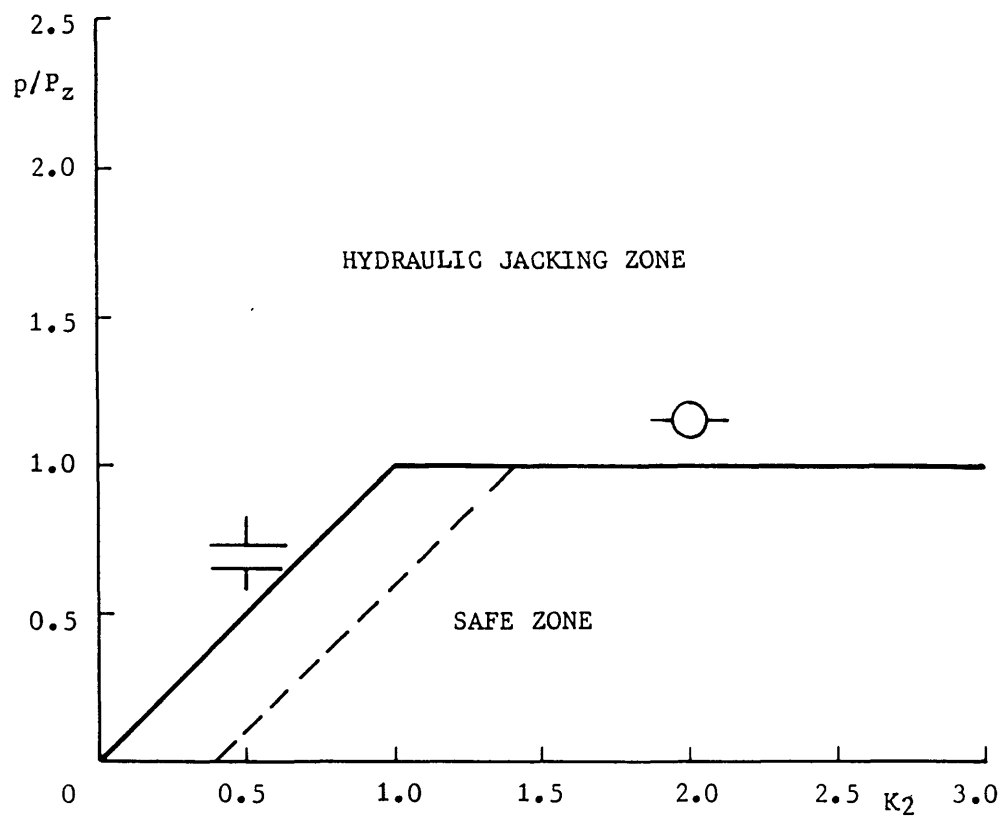


Figure 2.16 Conditions for joint opening and hydraulic jacking
when $\beta=0^\circ$, $K_1=2.0$, $\nu=0.2$

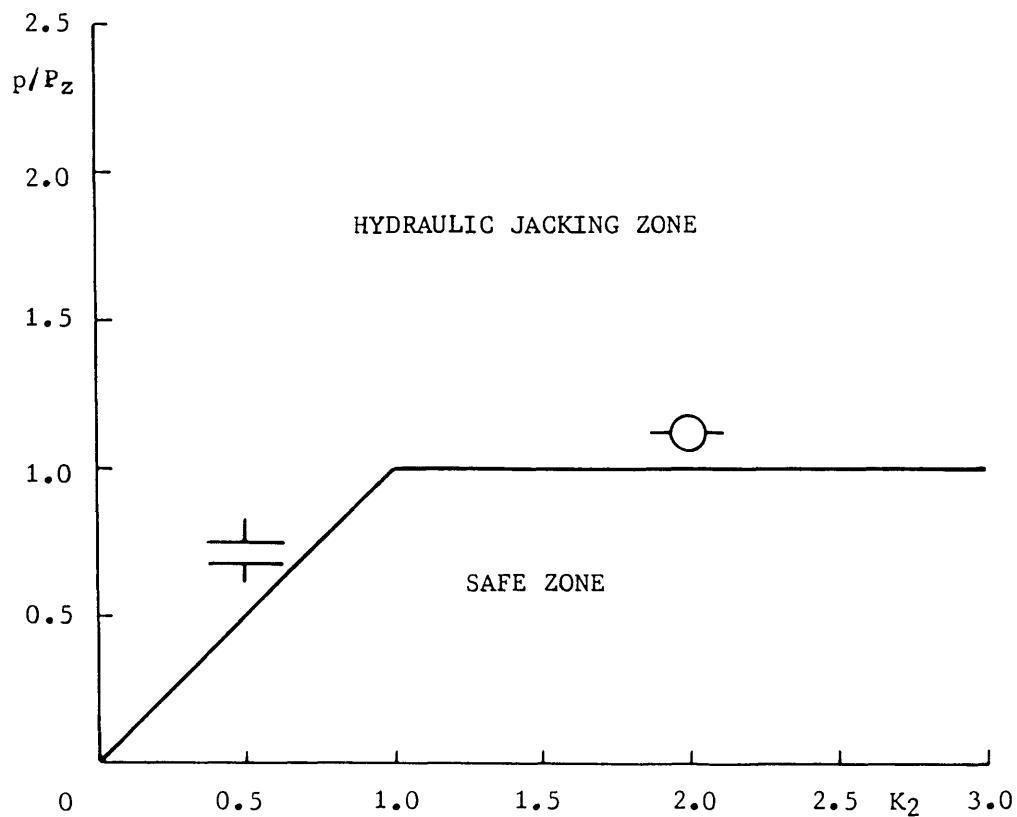


Figure 2.17 Conditions for joint opening and hydraulic jacking
when $\beta=0^\circ$, $K_1=3.0$, $\nu=0.2$

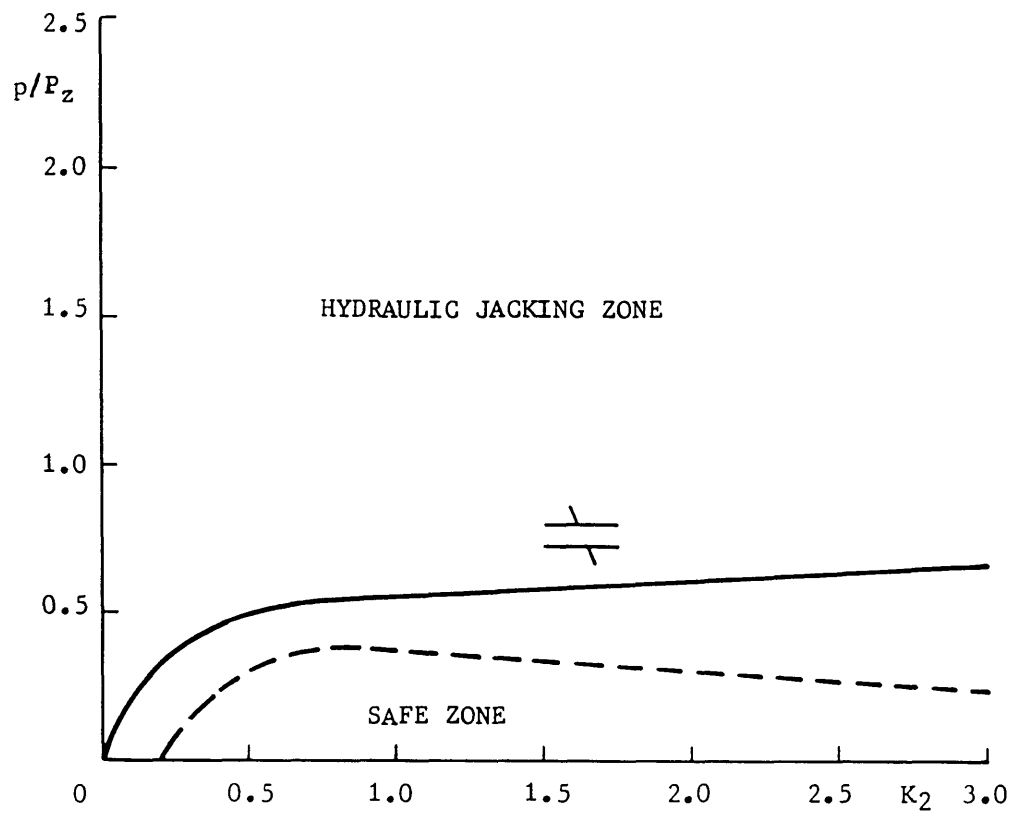


Figure 2.18 Conditions for joint opening and hydraulic jacking when $\beta=30^\circ$, $K_1=0.5$, $\nu=0.2$

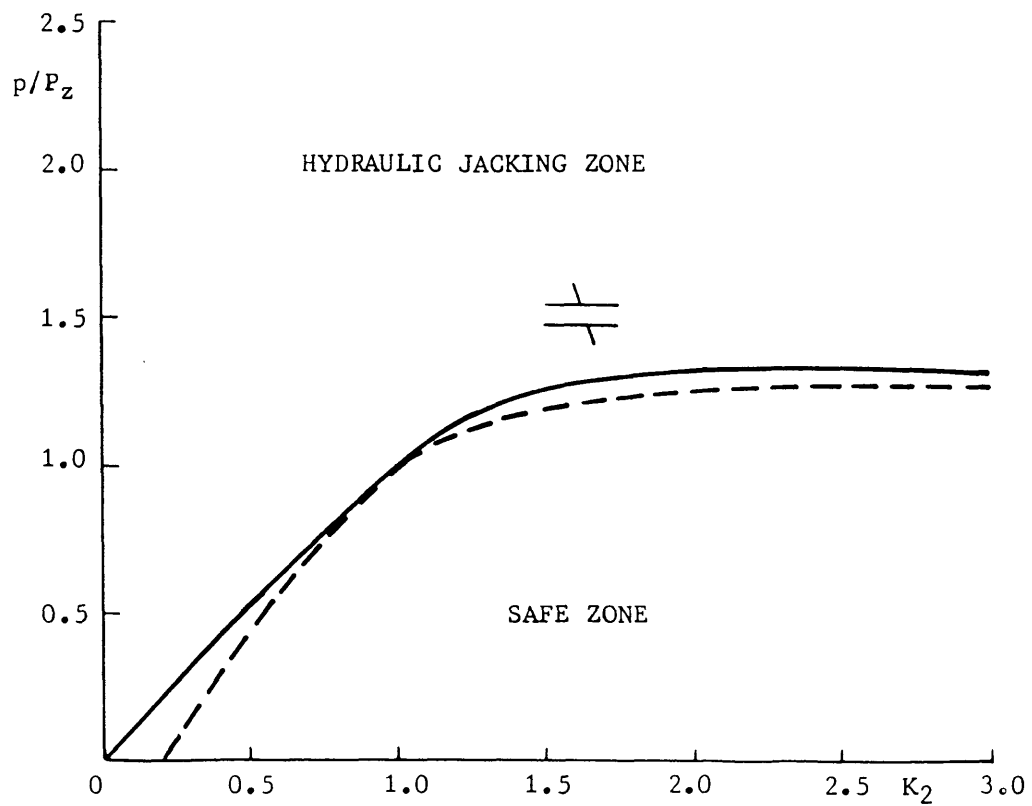


Figure 2.19 Conditions for joint opening and hydraulic jacking when $\beta=30^\circ$, $K_1=1.0$, $\nu=0.2$

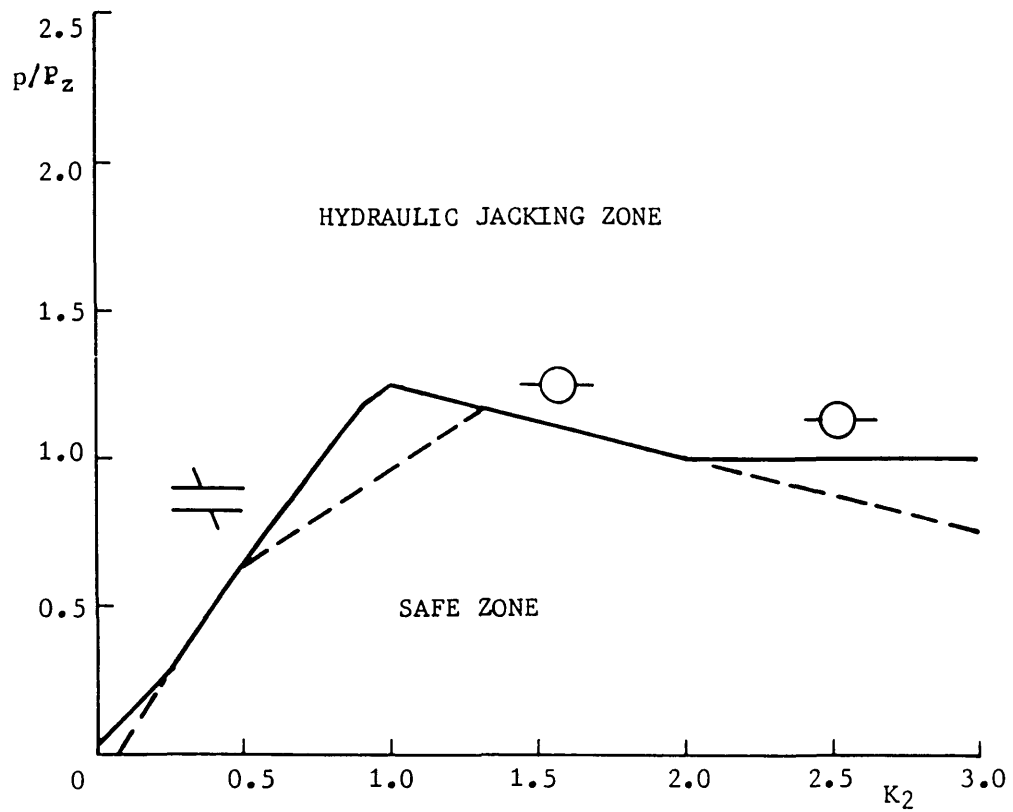


Figure 2.20 Conditions for joint opening and hydraulic jacking when $\beta=30^\circ$, $K_1=2.0$, $\nu=0.2$

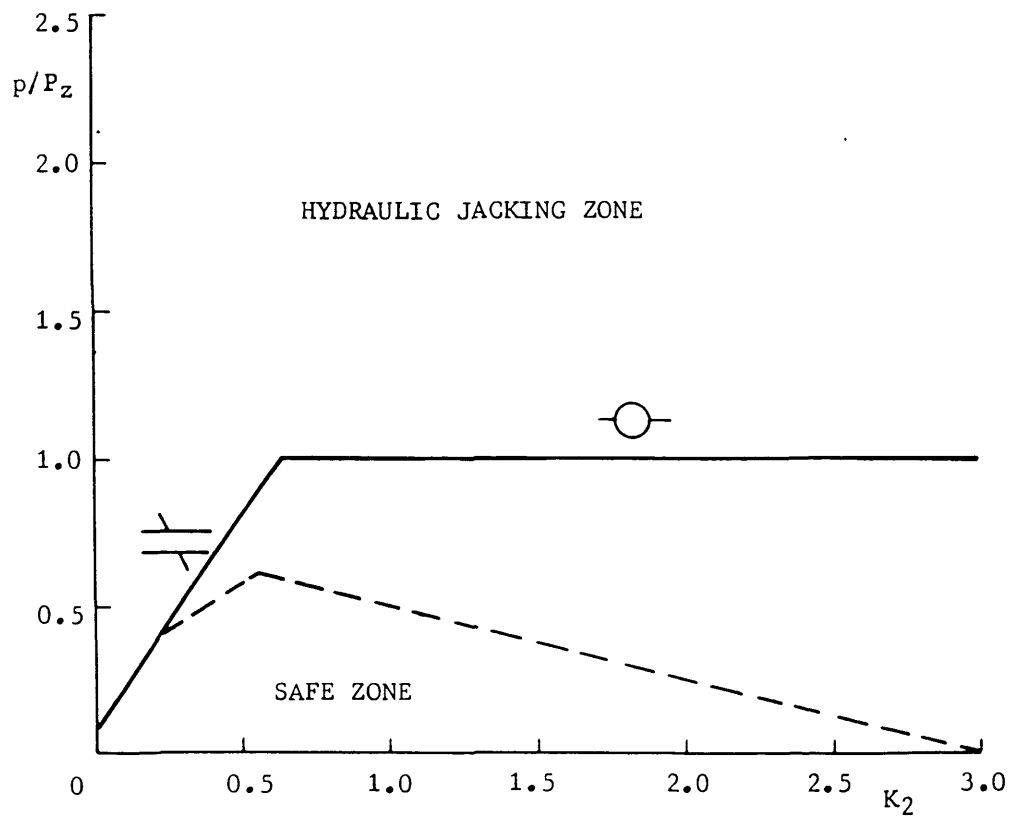


Figure 2.21 Conditions for joint opening and hydraulic jacking when $\beta=30^\circ$, $K_1=3.0$, $\nu=0.2$

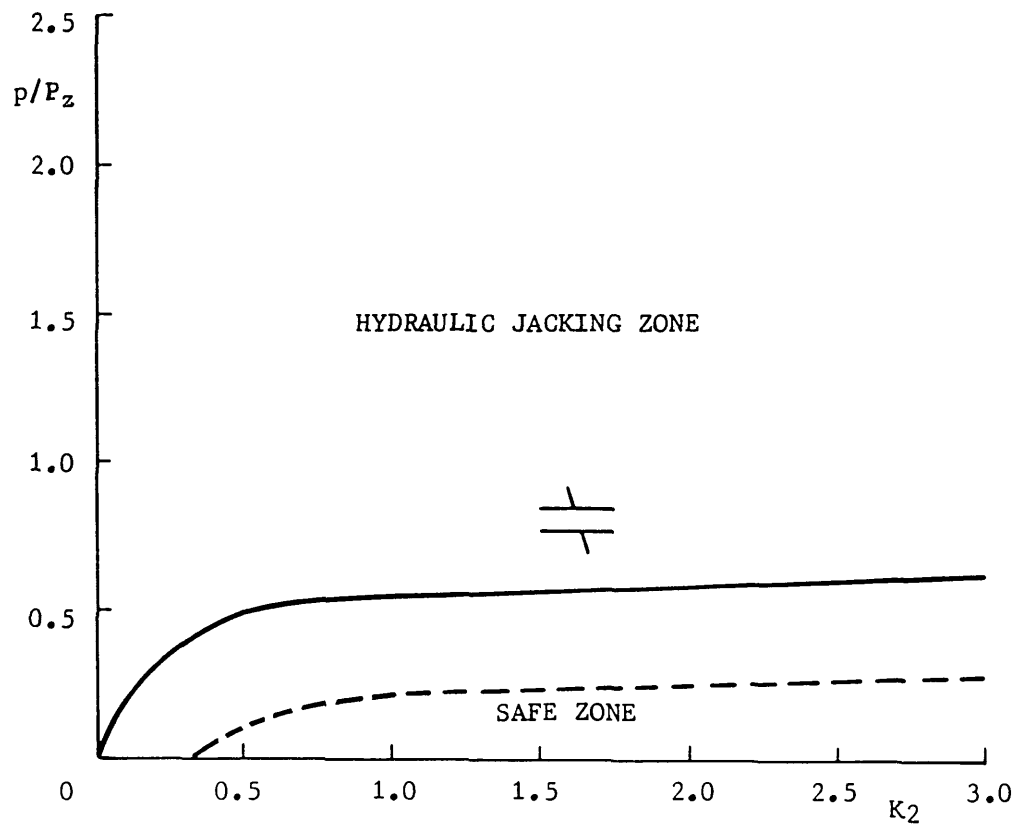


Figure 2.22 Conditions for joint opening and hydraulic jacking
when $\beta=45^\circ$, $K_1=0.5$, $\nu=0.2$

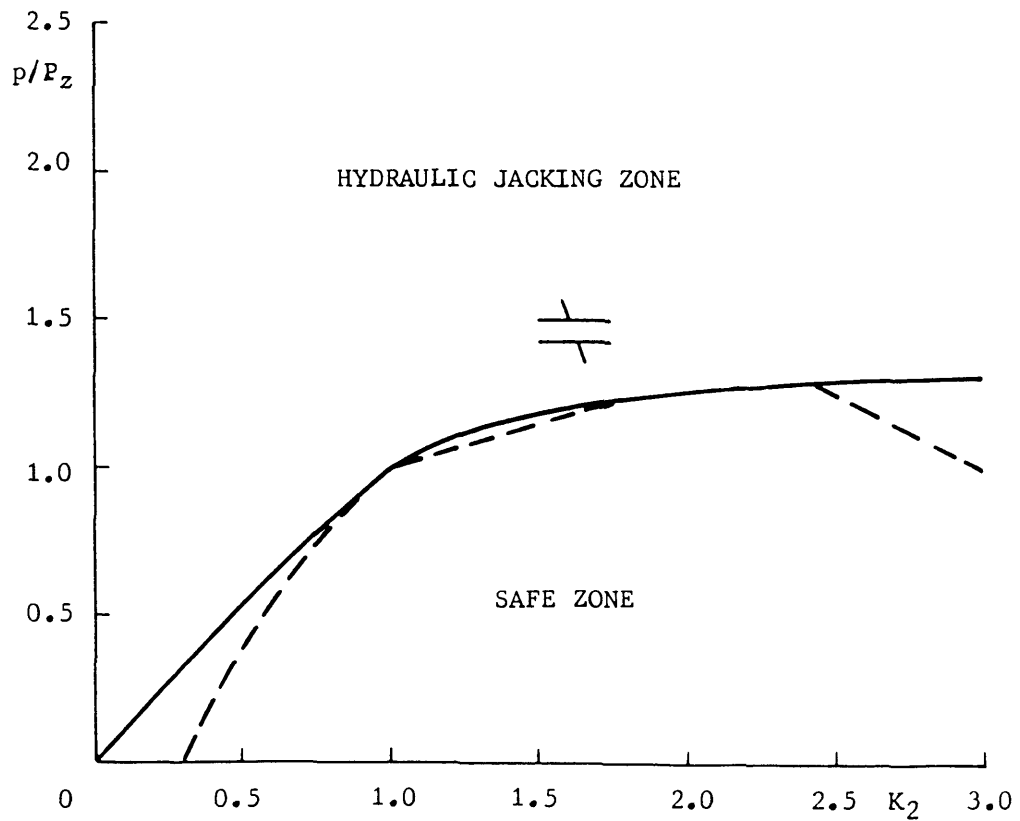


Figure 2.23 Conditions for joint opening and hydraulic jacking
when $\beta=45^\circ$, $K_1=1.0$, $\nu=0.2$

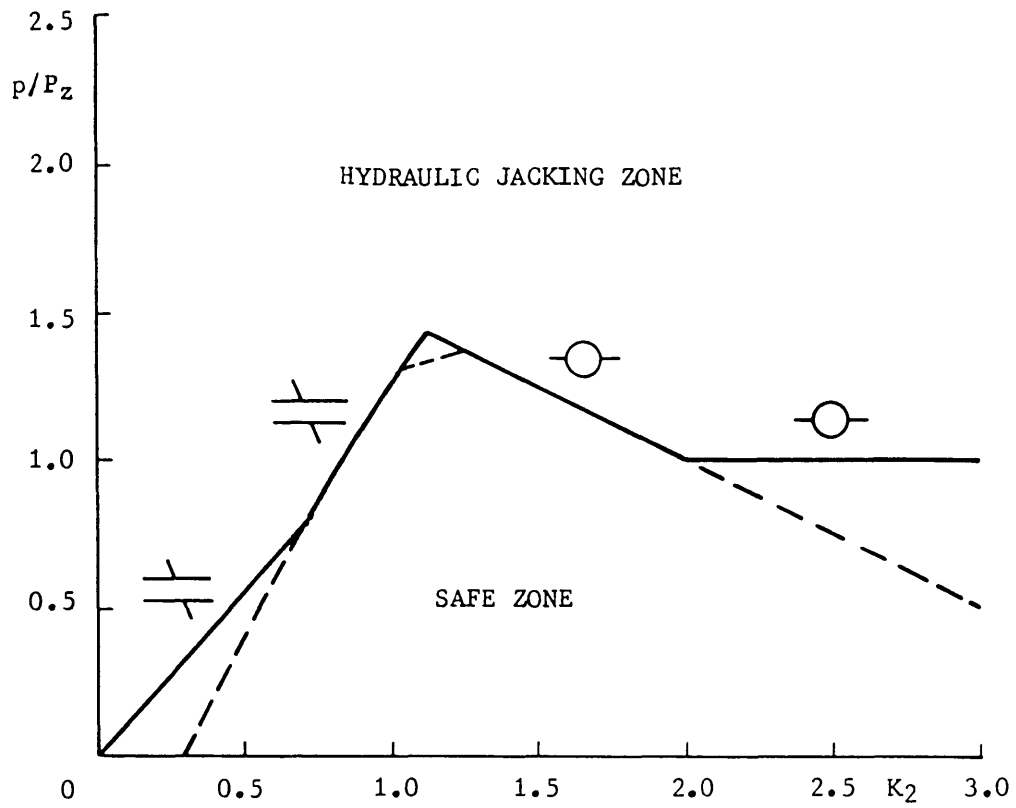


Figure 2.24 Conditions for joint opening and hydraulic jacking
when $\beta=45^\circ$, $K_1=2.0$, $\nu=0.2$

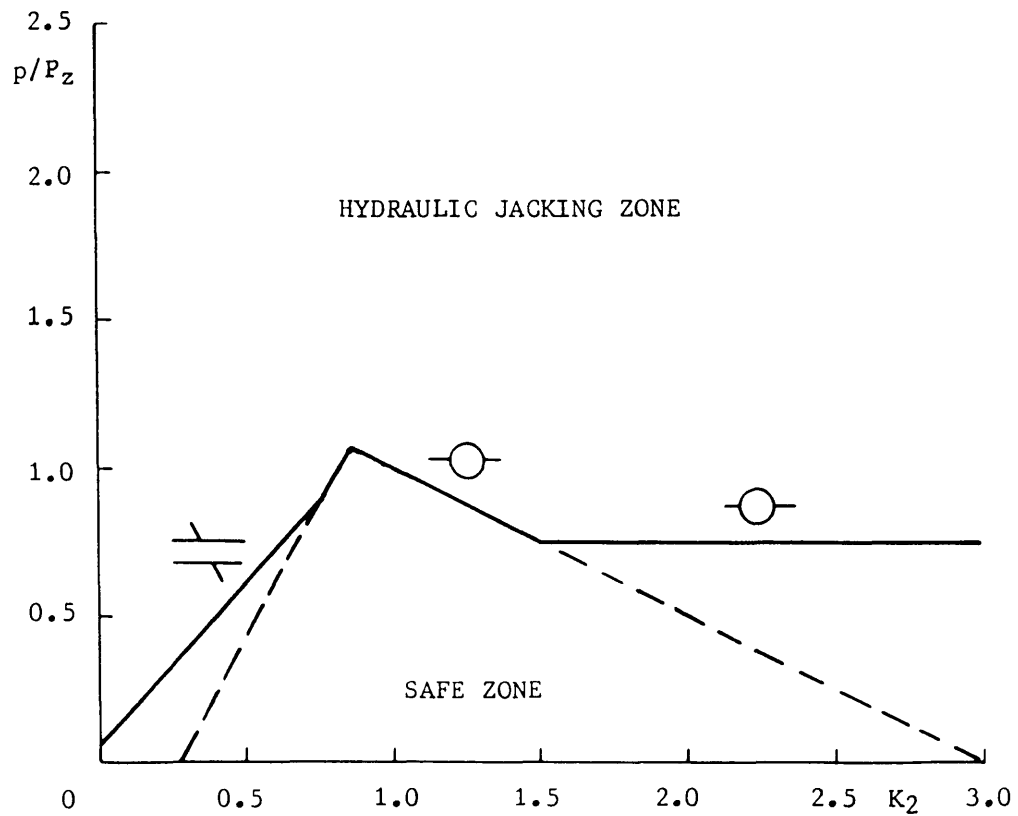


Figure 2.25 Conditions for joint opening and hydraulic jacking
when $\beta=45^\circ$, $K_1=3.0$, $\nu=0.2$

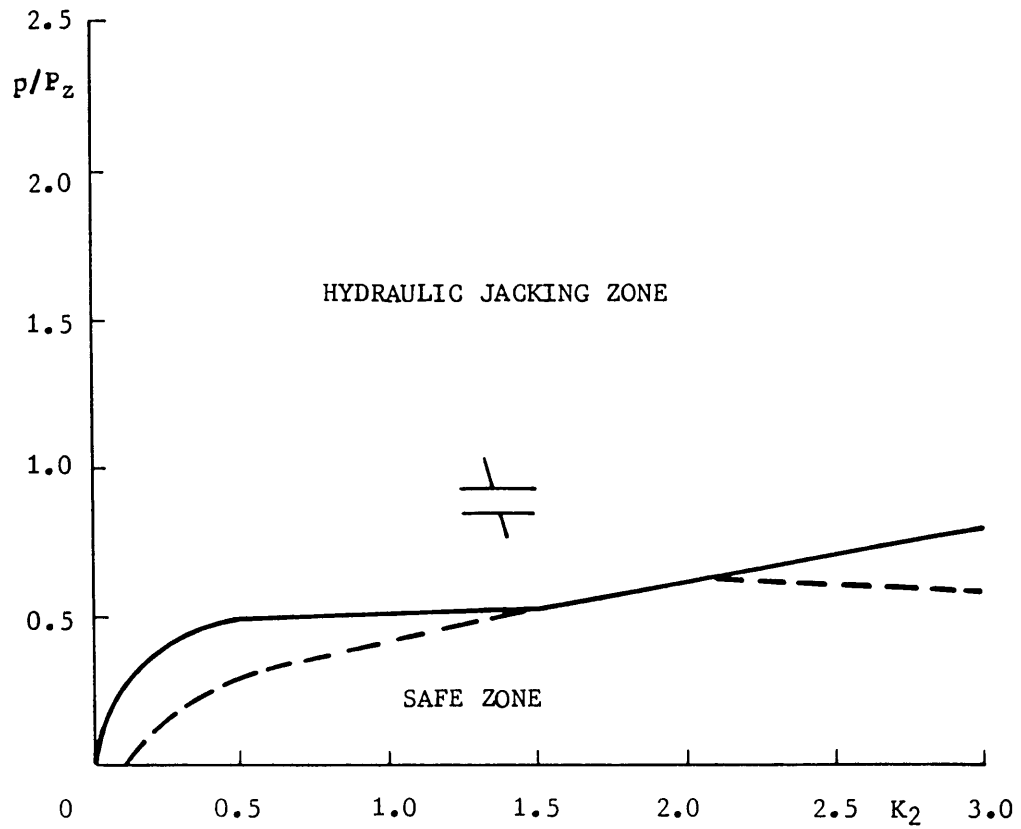


Figure 2.26 Conditions for joint opening and hydraulic jacking when $\beta=60^\circ$, $K_1=0.5$, $\nu=0.2$

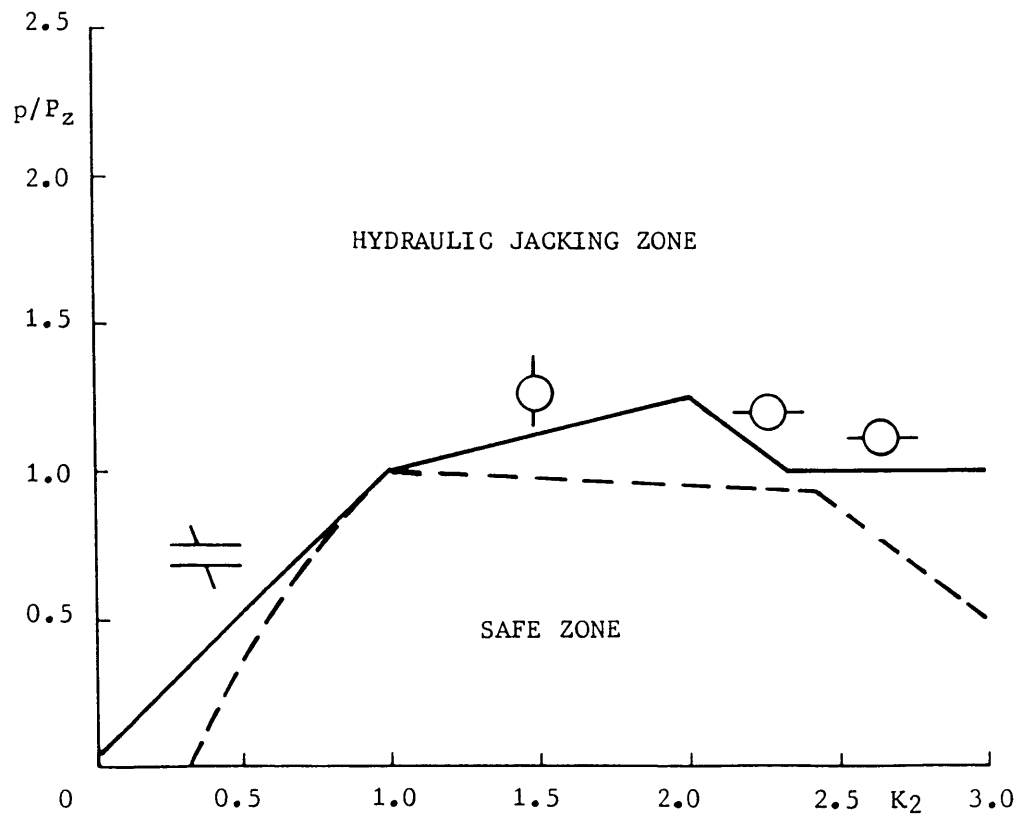


Figure 2.27 Conditions for joint opening and hydraulic jacking when $\beta=60^\circ$, $K_1=1.0$, $\nu=0.2$

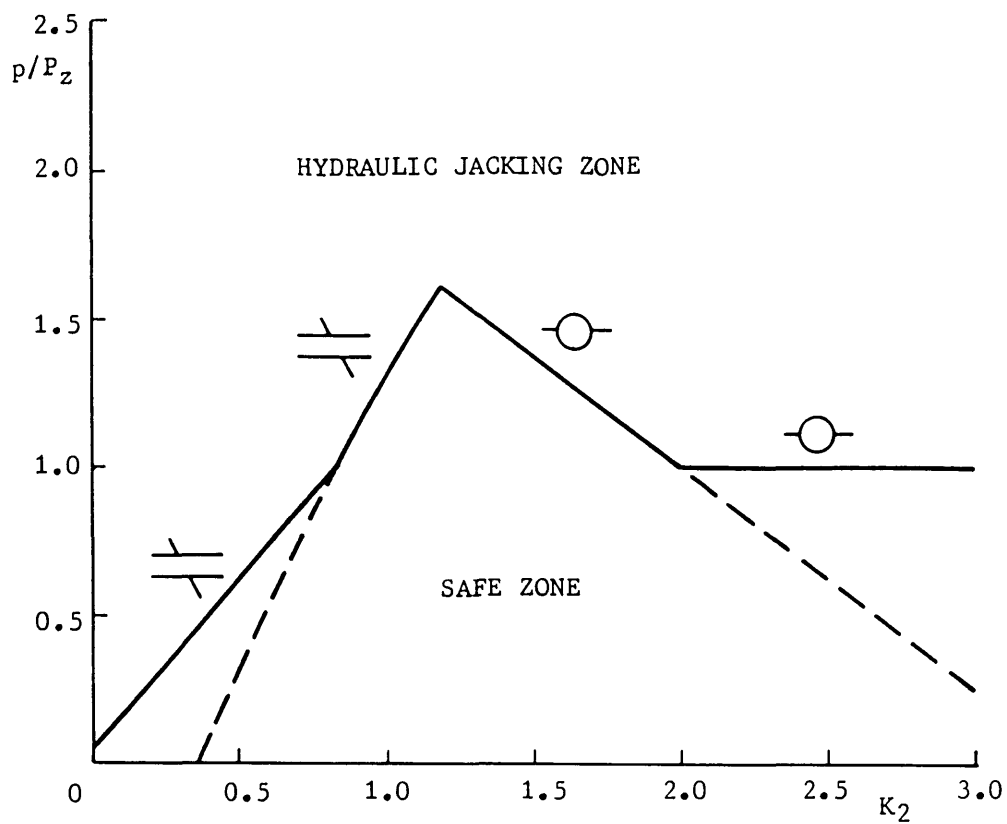


Figure 2.28 Conditions for joint opening and hydraulic jacking
when $\beta=60^\circ$, $K_1=2.0$, $\nu=0.2$

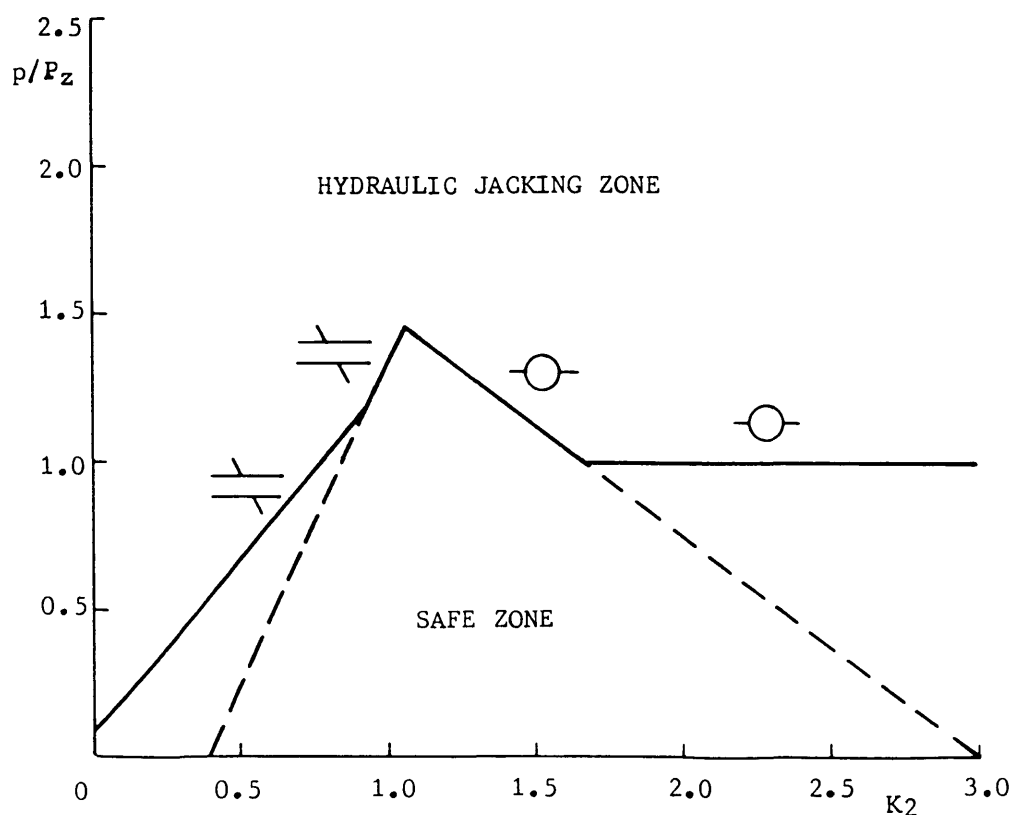


Figure 2.29 Conditions for joint opening and hydraulic jacking
when $\beta=60^\circ$, $K_1=3.0$, $\nu=0.2$

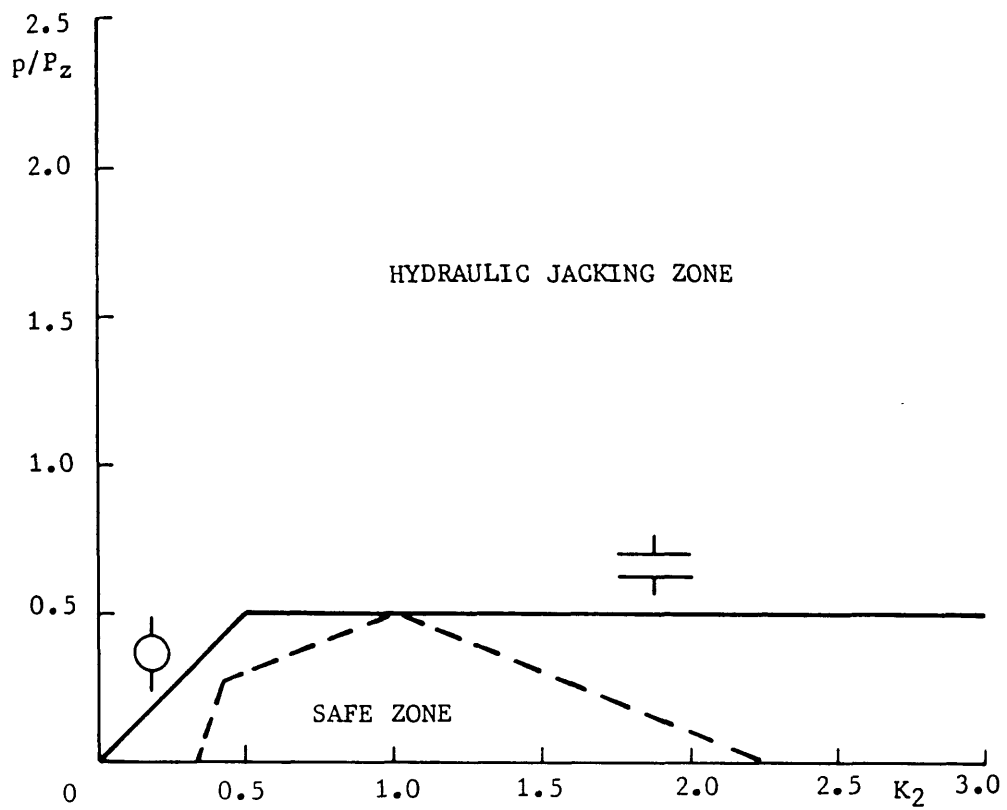


Figure 2.30 Conditions for joint opening and hydraulic jacking when $\beta=90^\circ$, $K_1=0.5$, $\nu=0.2$

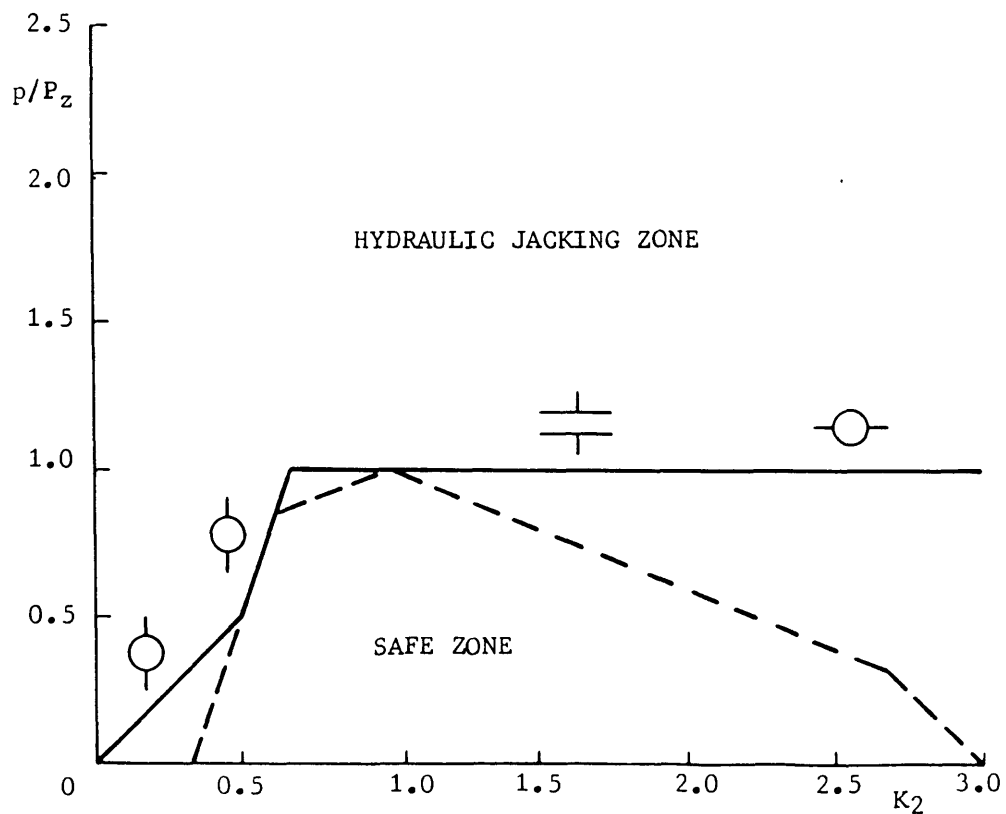


Figure 2.31 Conditions for joint opening and hydraulic jacking when $\beta=90^\circ$, $K_1=1.0$, $\nu=0.2$

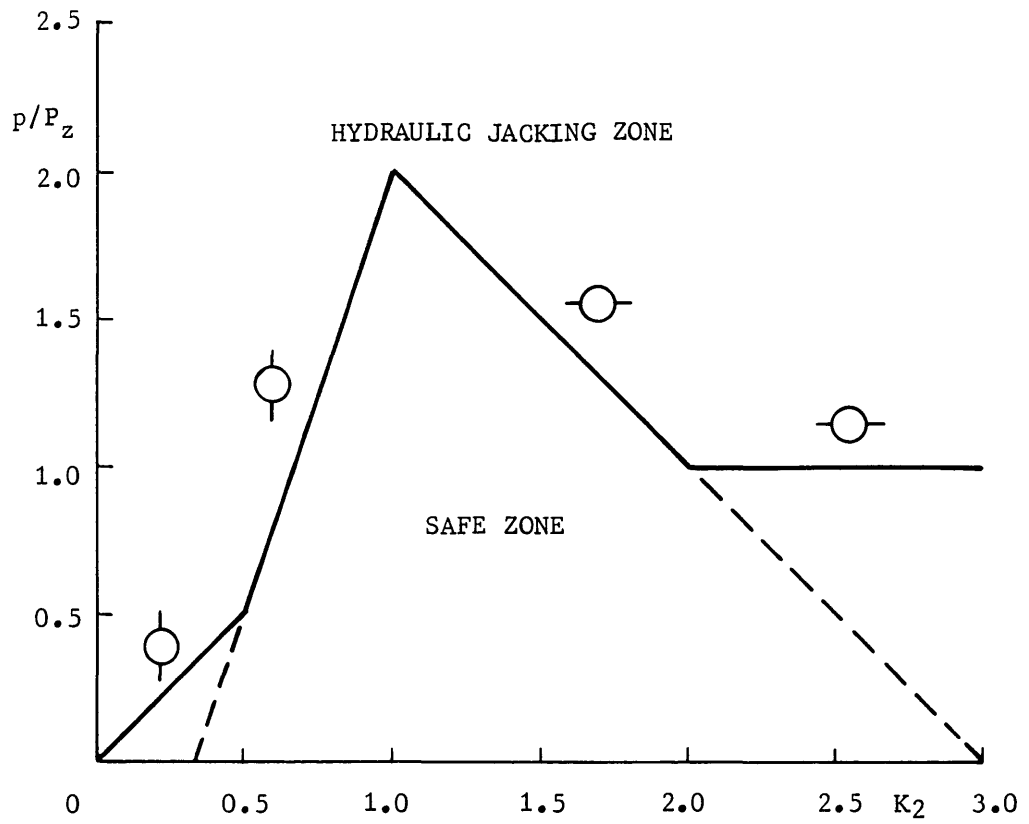


Figure 2.32 Conditions for joint opening and hydraulic jacking when $\beta=90^\circ$, $K_1=2.0$, $\nu=0.2$

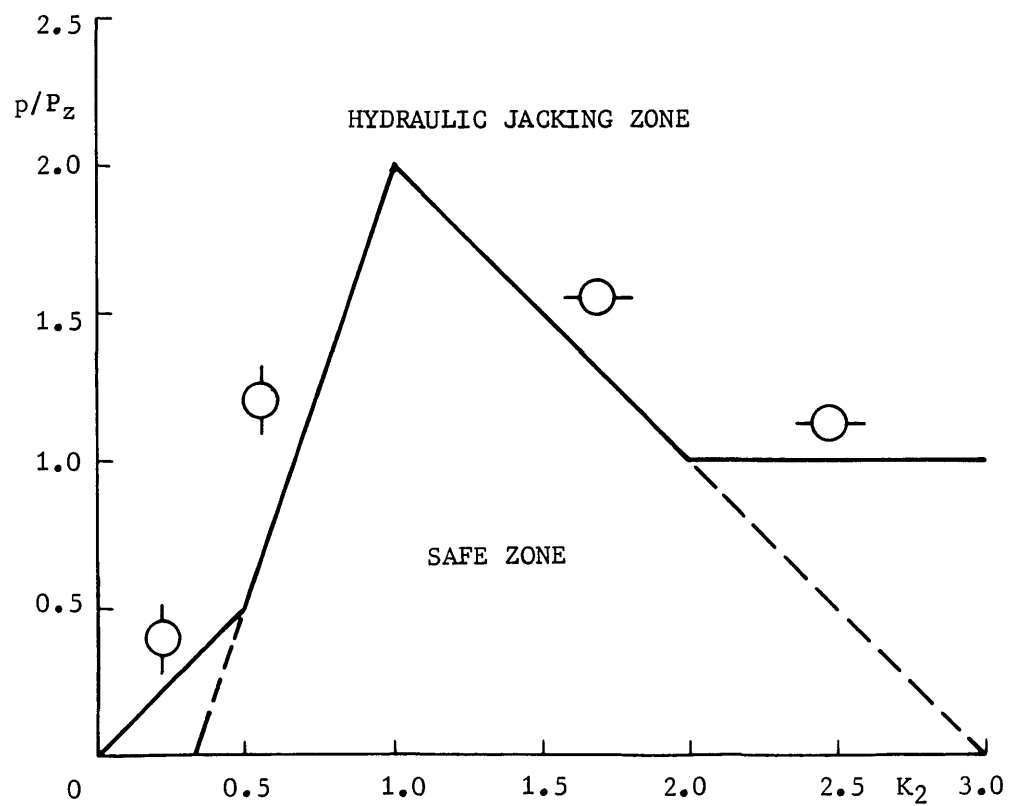


Figure 2.33 Conditions for joint opening and hydraulic jacking when $\beta=90^\circ$, $K_1=3.0$, $\nu=0.2$

unity. In these cases, the critical water pressure for hydraulic jacking is usually less than the vertical in situ stress, P_z . If either K_1 or K_2 is less than 0.5, this gives rise to extremely adverse cases, since the critical water pressure may drop to under half the vertical in situ stress, P_z .

2.4 Three Dimensional Analytical Solutions for Complete Plane Strain Condition on Microcomputer

In Section 2.3 analytical solutions for the complete plane strain condition for the particular case where the tunnel axis is perpendicular to the vertical in situ principal stress were presented. The solutions are suitable for tunnels situated under a flat ground surface. When the surface topography is irregular or the tunnel is situated near a valley side, however, the tunnel axis may be inclined to all three in situ principal directions. In this case the solutions are much more complicated. Therefore, the calculations were carried out on microcomputers.

2.4.1 Conditions for joint opening and hydraulic jacking at the tunnel boundary

The criteria for joint opening and hydraulic jacking are that joint opening occurs when the internal water pressure exceeds the minimum principal stress at the tunnel boundary and hydraulic jacking occurs when the water pressure is greater than both the minimum principal stress and the in situ normal stress in the direction parallel to the minimum principal stress.

The coordinate systems are shown in Fig.2.34, in which X , Y and Z are the three in situ principal directions and Y' is the tunnel axis. Z' is set up in such a way that it is in the plane formed by Z and Y' axes. α , β and γ are

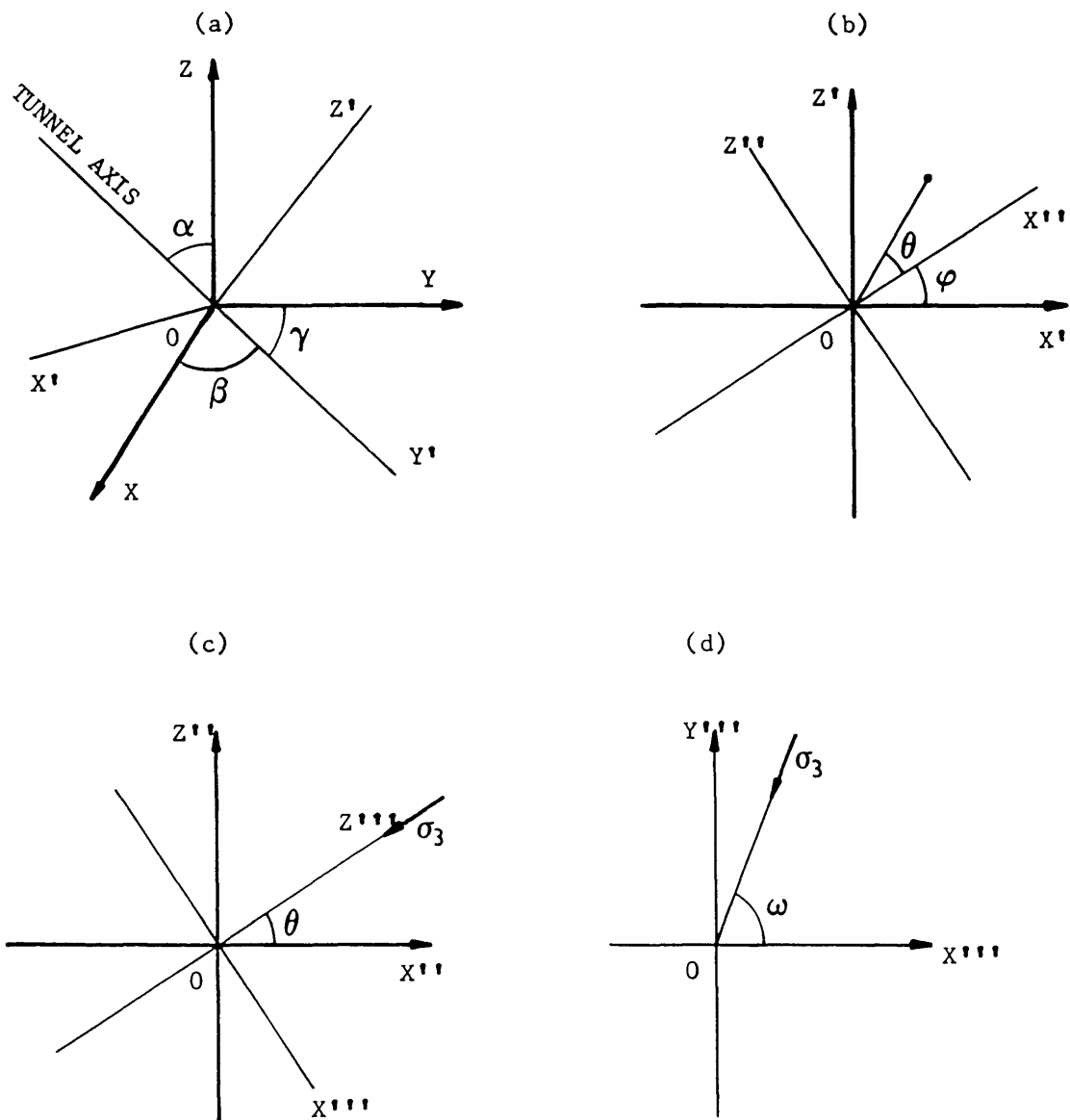


Figure 2.34 Coordinate systems for the three dimensional analyses of the critical water pressure for joint opening and hydraulic jacking

the angles between the X, Y, Z axes and the Y' axis respectively. Of the three angles only two are independent since

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 \quad (2.57)$$

Assuming α and β are independent

$$\cos \gamma = \sqrt{1 - \cos^2 \alpha - \cos^2 \beta}$$

Thus, the direction cosines of the X', Y' and Z' axes in the XYZ system are

$$\begin{aligned} l_1 &= \frac{m_2}{\sqrt{l_2^2 + m_2^2}} & m_1 &= -\frac{l_2}{\sqrt{l_2^2 + m_2^2}} & n_1 &= 0 \\ l_2 &= \cos \alpha & m_2 &= \cos \beta & n_2 &= -\sqrt{1 - l_2^2 - m_2^2} \\ l_3 &= \frac{m_1 n_2}{\sqrt{l_1^2 + m_1^2}} & m_3 &= -\frac{l_1 n_2}{\sqrt{l_1^2 + m_1^2}} & n_3 &= \sqrt{1 - n_2^2} \end{aligned} \quad (2.58)$$

Therefore, the boundary stresses at infinity in the $X'Y'Z'$ system are

$$P_{x'} = P_z(l_1^2 K_1 + m_1^2 K_2 + n_1^2) \quad (2.59 a)$$

$$P_{y'} = P_z(l_2^2 K_1 + m_2^2 K_2 + n_2^2) \quad (2.59 b)$$

$$P_{z'} = P_z(l_3^2 K_1 + m_3^2 K_2 + n_3^2) \quad (2.59 c)$$

$$P_{x'y'} = P_z(l_1 l_2 K_1 + m_1 m_2 K_2 + n_1 n_2) \quad (2.59 d)$$

$$P_{y'z'} = P_z(l_2 l_3 K_1 + m_2 m_3 K_2 + n_2 n_3) \quad (2.59 e)$$

$$P_{z'x'} = P_z(l_3 l_1 K_1 + m_3 m_1 K_2 + n_3 n_1) \quad (2.59 f)$$

Rotating X' and Z' to X'' and Z'' by an angle ϕ so that $P_{z'x'}$ vanishes, the boundary stresses in the $X''Y''Z''$ system (Y'' is coincident with Y') are

$$P_{x''} = \frac{P_{x'} + P_{z'}}{2} + \left[\left(\frac{P_{x'} - P_{z'}}{2} \right)^2 + P_{x'z'}^2 \right]^{\frac{1}{2}} \quad (2.60 a)$$

$$P_{y''} = P_{y'} \quad (2.60 b)$$

$$P_{z''} = \frac{P_{x'} + P_{z'}}{2} - \left[\left(\frac{P_{x'} - P_{z'}}{2} \right)^2 + P_{x'z'}^2 \right]^{\frac{1}{2}} \quad (2.60 \text{ c})$$

$$P_{x''y''} = P_{x'y'} \quad (2.60 \text{ d})$$

$$P_{y''z''} = P_{y'z'} \quad (2.60 \text{ e})$$

$$P_{z''x''} = 0 \quad (2.60 \text{ f})$$

and

$$\phi = \frac{1}{2} \tan^{-1} \left| \frac{2P_{z'x'}}{P_{x'} - P_{z'}} \right| \quad (2.61)$$

The stress components at the tunnel boundary are

$$\sigma_{\theta} = (P_{x''} + P_{z''}) - 2(P_{x''} - P_{z''}) \cos 2\theta \quad (2.62 \text{ a})$$

$$\sigma_y = \nu(\Delta\sigma_r + \Delta\sigma_{\theta}) + P_{y''} = \nu(\sigma_{\theta} - P_{\theta} - P_r) + P_{y''} \quad (2.62 \text{ b})$$

$$\tau_{\theta y} = 2P_{y''z''} \cos(\theta + \phi) - 2P_{x''y''} \sin(\theta + \phi) \quad (2.62 \text{ c})$$

$$\sigma_r = \tau_{r\theta} = \tau_{ry} = 0 \quad (2.62 \text{ d})$$

where θ is shown in Fig.2.34 (b).

The next step is to determine P_r and P_{θ} . The direction cosines of the X'' , Y'' and Z'' axes in the $X'Y'Z'$ system are

$$\begin{aligned} \bar{l}_1 &= \cos \phi & \bar{m}_1 &= 0 & \bar{n}_1 &= \sin \phi \\ \bar{l}_2 &= 0 & \bar{m}_2 &= 1 & \bar{n}_2 &= 0 \\ \bar{l}_3 &= -\sin \phi & \bar{m}_3 &= 0 & \bar{n}_3 &= \cos \phi \end{aligned} \quad (2.63)$$

The direction cosines of vector r and θ in $X''Y''Z''$ system are

$$l_r = \cos \theta \quad m_r = 0 \quad n_r = \sin \theta \quad (2.64)$$

$$l_{\theta} = -\sin \theta \quad m_{\theta} = 0 \quad n_{\theta} = \cos \theta \quad (2.65)$$

Thus, the direction cosines of the vectors r and θ in the XYZ system are

$$\begin{Bmatrix} l'_r \\ m'_r \\ n'_r \end{Bmatrix} = \begin{bmatrix} \bar{l}_1 & \bar{m}_1 & \bar{n}_1 \\ \bar{l}_2 & \bar{m}_2 & \bar{n}_2 \\ \bar{l}_3 & \bar{m}_3 & \bar{n}_3 \end{bmatrix} \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{Bmatrix} l_r \\ m_r \\ n_r \end{Bmatrix} \quad (2.66)$$

$$\begin{Bmatrix} l'_\theta \\ m'_\theta \\ n'_\theta \end{Bmatrix} = \begin{bmatrix} \bar{l}_1 & \bar{m}_1 & \bar{n}_1 \\ \bar{l}_2 & \bar{m}_2 & \bar{n}_2 \\ \bar{l}_3 & \bar{m}_3 & \bar{n}_3 \end{bmatrix} \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{Bmatrix} l_\theta \\ m_\theta \\ n_\theta \end{Bmatrix} \quad (2.67)$$

Therefore,

$$P_r = P_z(l'_r l'_r K_1 + m'_r m'_r K_2 + n'_r n'_r) \quad (2.68)$$

$$P_\theta = P_z(l'_\theta l'_\theta K_1 + m'_\theta m'_\theta K_2 + n'_\theta n'_\theta) \quad (2.69)$$

The minimum principal stress and its direction are calculated as follows:

$$\sigma_3 = \frac{\sigma_\theta + \sigma_y}{2} - \sqrt{\tau_{\theta y}^2 + \left(\frac{\sigma_\theta - \sigma_y}{2}\right)^2} = \sigma_3(\theta) \quad (2.70)$$

$$\omega = \frac{1}{2} \tan^{-1} \left| \frac{2\tau_{\theta y}}{\sigma_\theta - \sigma_y} \right| = \omega(\theta) \quad (2.71)$$

For specified K_1 , K_2 , P_z and β , σ_3 is a function of θ only. Theoretically, the minimum value of σ_3 can be obtained by differentiating $\sigma_3(\theta)$ with respect to θ . However, it is hard to derive an explicit expression for $\frac{d\sigma_3}{d\theta}$. So, the limiting value of σ_3 , $\tilde{\sigma}_3$, and the corresponding angle, $\tilde{\omega}$, are obtained numerically by trial and error.

The condition for joint opening is

$$p \geq \tilde{\sigma}_3 \quad (2.72)$$

In order to obtain the condition for hydraulic jacking, the in situ stress in the direction parallel to $\tilde{\sigma}_3$, $P_{\theta\omega}$, must be determined. To do this, the fourth coordinate system $X'''Y'''Z'''$ has to be set up (Fig.2.34, c). The direction cosines of the X''' , Y''' and Z''' axes in the $X''Y''Z''$ system are

$$\begin{aligned} \tilde{l}_1 &= \sin \theta & \tilde{m}_1 &= 0 & \tilde{n}_1 &= -\cos \theta \\ \tilde{l}_2 &= 0 & \tilde{m}_2 &= 1 & \tilde{n}_2 &= 0 \\ \tilde{l}_3 &= \cos \theta & \tilde{m}_3 &= 0 & \tilde{n}_3 &= \sin \theta \end{aligned} \quad (2.73)$$

The direction cosines of in $X'''Y'''Z'''$ system are

$$l = \cos \omega \quad m = \sin \omega \quad n = 0 \quad (2.74)$$

So, the direction cosines of $\tilde{\sigma}_3$ in the XYZ system are

$$\begin{Bmatrix} l' \\ m' \\ n' \end{Bmatrix} = \begin{bmatrix} \tilde{l}_1 & \tilde{m}_1 & \tilde{n}_1 \\ \tilde{l}_2 & \tilde{m}_2 & \tilde{n}_2 \\ \tilde{l}_3 & \tilde{m}_3 & \tilde{n}_3 \end{bmatrix} \begin{bmatrix} \bar{l}_1 & \bar{m}_1 & \bar{n}_1 \\ \bar{l}_2 & \bar{m}_2 & \bar{n}_2 \\ \bar{l}_3 & \bar{m}_3 & \bar{n}_3 \end{bmatrix} \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{Bmatrix} l \\ m \\ n \end{Bmatrix} \quad (2.75)$$

Finally, $P_{\theta\omega}$ is obtained as

$$P_{\theta\omega} = P_z(l'l'K_1 + m'm'K_2 + n'n') \quad (2.76)$$

The condition for hydraulic jacking is

$$\begin{cases} p \geq \tilde{\sigma}_3 \\ p \geq P_{\theta\omega} \end{cases} \quad (2.77)$$

A program titled ASJOHJ is written in BASIC to implement the calculations. The flow chart of the program is given in Fig.2.35 and a listing in Appendix 4.

2.4.2 Conditions for joint opening and hydraulic jacking inside the rock mass surrounding a pressure tunnel

In some circumstances it is not necessary to prevent joints from opening and jacking at the tunnel boundary. Joint opening and jacking may occur in the vicinity of the tunnel provided they are restricted to within a certain distance from the tunnel boundary. Outside this region opening and/or jacking are not permissible, so that the tunnel is still safe. Thus, a higher water pressure can be admitted than if no joint opening or jacking is permitted. An attempt was made to investigate the limiting conditions for joint opening and hydraulic jacking inside the rock mass surrounding the tunnel.

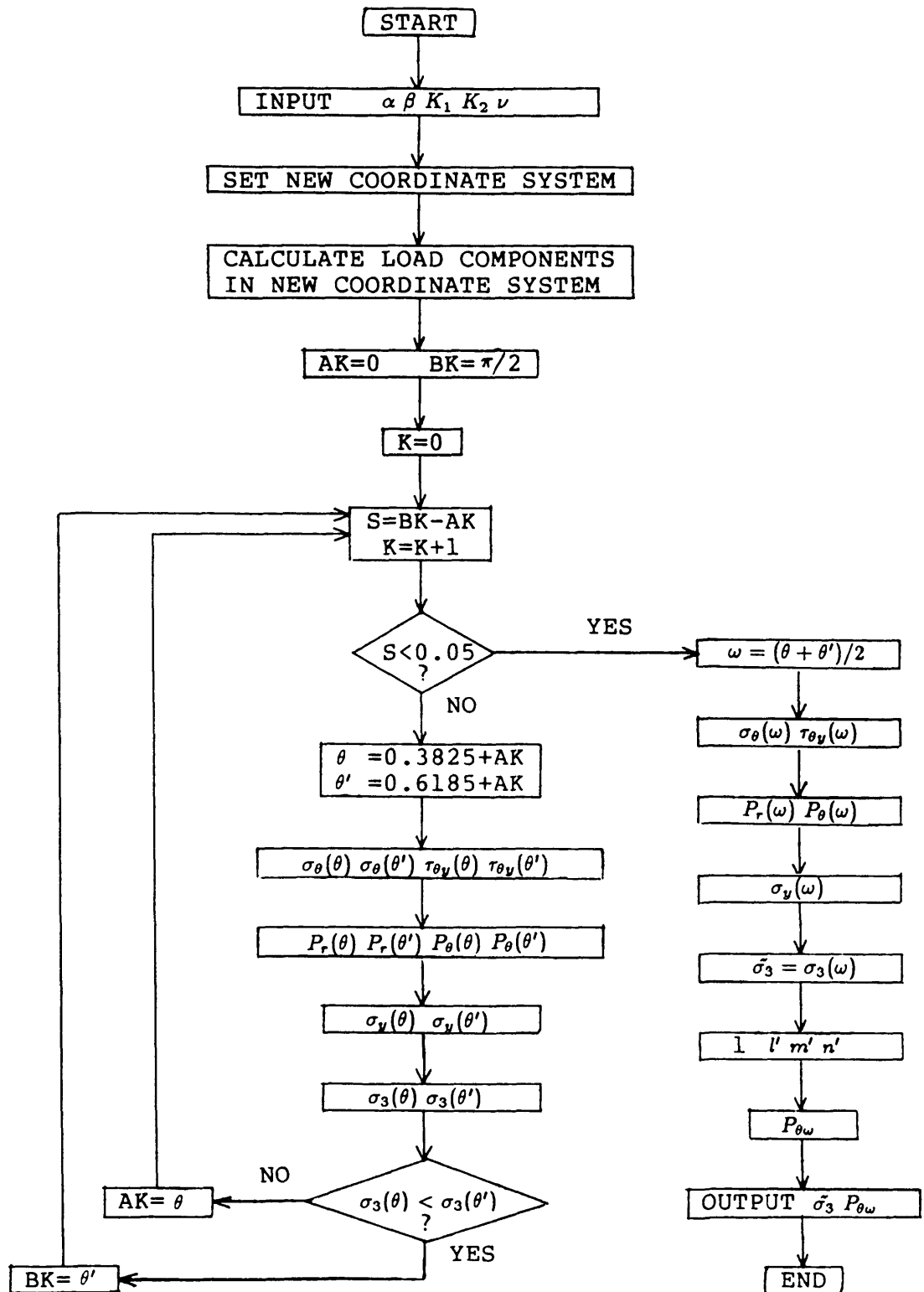


Figure 2.35 Flow chart of program ASJOHJ

The general calculation procedure is the same as that for the boundary case. The major difference is that only three stress components are non-zero for the boundary case while all six components are, in general, non-zero for the interior case. The minimum principal stress should be calculated from the six stress components. But, the radial stress, σ_r , is usually very low in the region near the tunnel boundary. As a result, it is easy for it to become the minimum principal stress. For this analysis, however, joint opening and jacking on the circumferential joints (perpendicular to the direction of σ_r) is not of interest because these joints do not intersect the tunnel boundary. Therefore, for the cases discussed here the minimum principal stress is still calculated from σ_θ , σ_y and $\tau_{\theta y}$, as for the boundary cases.

The program ASJOHJ described in Section 2.4.1 is designed to be capable of covering these cases as well.

2.4.3 Example of application of program ASJOHJ

All six parameters required for the calculation, K_1 , K_2 , α , β , ν and r/a , are input from the keyboard.

Herein the operating procedure on BBC microcomputer and output of the results are illustrated with an example, in which $K_1 = 1.5$, $K_2 = 2.0$, $\alpha = 45^\circ$, $\beta = 75^\circ$, $\nu = 0.25$. What follows is the input and output for this example when $r/a = 1.5$.

```
>*LOAD "ASJOHJ"
>RUN
K1=?1.5
K2=?2.0
ALPHA=?45
BETA=?75
MU=0.25
r/a=?1.5
      K1=1.5      K2=2      ALPHA=45      BETA=75
MU=0.25      r/a=1.5
      S3=0.950564238      P3=1.06658076
      WATER PRESSURE FOR JOINT OPENING:      0.950564238 Pz
      WATER PRESSURE FOR HYDRAULIC JACKING:      1.06658076 Pz
```

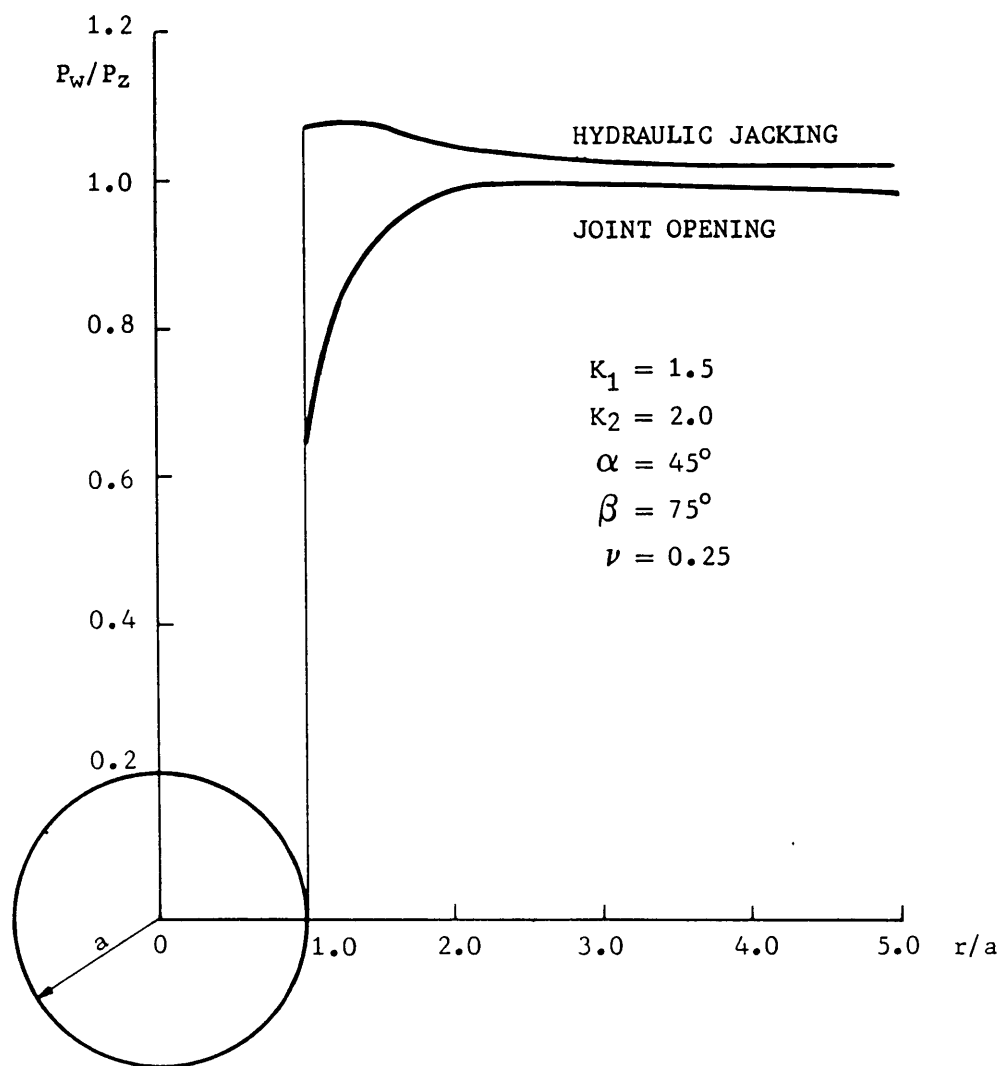


Figure 2.36 Variation of critical water pressure for joint opening and hydraulic jacking with distance from the tunnel boundary - an example of three dimensional analyses

The variation of critical water pressure for joint opening and hydraulic jacking with r/a is shown in Fig.2.36. It illustrates that the critical water pressure for joint opening increases as the point considered moves away from the tunnel boundary.

2.5 Summary

The analytical solutions described in this chapter provide a simple and useful tool for estimating the critical internal water pressure for joint opening and hydraulic jacking provided that the in situ stresses and the orientation of the tunnel are known. Based on these solutions it is easy to determine the required length and position of any steel lining. The design charts presented in Fig.2.14 to Fig.2.33 can be used to find the critical water pressure straightforwardly for the commonly encountered simple cases, i.e. the tunnel axis is perpendicular to the vertical in situ stress. For more general cases, a simple program ASJOHJ, which can be run on ordinary microcomputers, can give the water pressure quickly.

There is no denying the fact that these solutions have a number of limitations which must be noted when using them in the design of pressure tunnels.

(1) The conditions for joint opening and hydraulic jacking are based on the condition that there is a pre-existing joint in the required most adverse orientation. This is not always the case.

(2) The minimum principal stress is calculated without considering the contribution of the water pressure acting on the tunnel boundary. The calculation results indicate that this may change the critical water pressure for joint opening, but will have little influence on the critical water pressure for

hydraulic jacking.

(3) Under some circumstances sliding on the joints may become more critical than joint opening and hydraulic jacking. This may occur when the difference between the in situ stresses is large or the angles between the tunnel axis and the in situ principal stresses are big. This possibility is not taken into account in the analytical solutions.

CHAPTER 3

NUMERICAL MODEL FOR DISCONTINUITIES

3.1 Choice of the Physical Model

3.1.1 Review of models for discontinuities

From the early time of the development of rock mechanics the significance of discontinuities in slope stability problems had been well recognized (Jaeger 1979, Hoek and Bray 1981). Since then a great number of laboratory and in situ tests have been conducted to study the strength, deformability, conductivity and heat transfer behaviour of rock discontinuities. Several models have been suggested, of which some have been adopted in the finite element method, the displacement discontinuity method or the limiting equilibrium method to analyze engineering problems.

The first primitive joint strength model was the Mohr-Coulomb criterion,

$$\tau_p = c + \mu \sigma'_n = c + \sigma'_n \tan \phi \quad (3.1)$$

where τ_p = peak shear strength,

ϕ = frictional angle,

μ = frictional coefficient,

σ'_n = effective normal stress,

c = cohesion.

Parameters c , μ and ϕ are regarded as the material constants and can be evaluated from the laboratory and in situ shear tests (Stagg 1968). Variety of

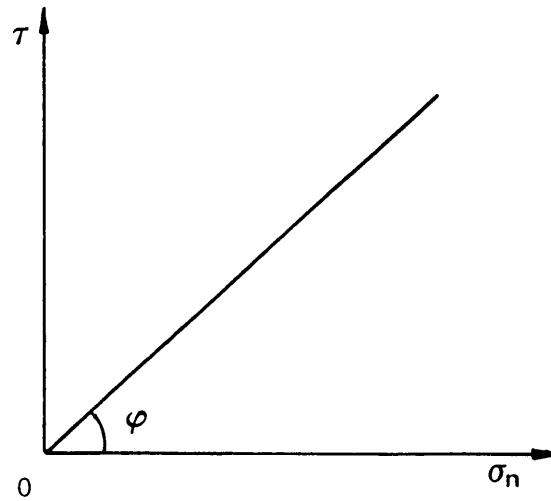


Figure 3.1 Mohr-Coulomb criterion

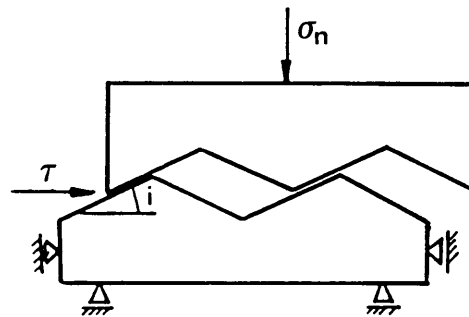


Figure 3.2 Specimen in Patton's tests

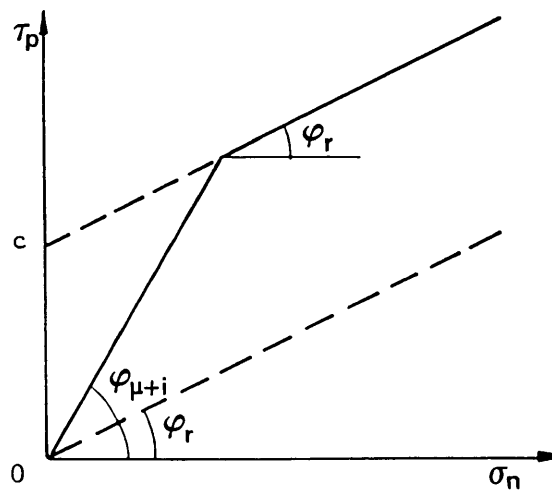


Figure 3.3 Patton's bilinear peak shear strength envelope (after Sofianos, 1984)

values for different types of rock masses have been suggested by Jaeger (1959), Jaeger and Rosengren (1969) and Hoskins et al (1968).

Patton (1966) suggested a bilinear failure envelope from a series of tests conducted on artificial specimens, in which the discontinuities had a set of asperities inclined at a constant angle to the direction of shearing like saw teeth (Fig.3.2). At low normal stress failure occurs by sliding along the inclined surfaces and overcoming frictional resistance. When the normal stress is high enough the failure mode is changed to one of shearing through the bases of the teeth. The bilinear peak shear strength criterion can be expressed as:

$$\tau_p = \sigma'_n \tan (\phi_\mu + i) \quad (\sigma'_n < \sigma_n^*) \quad (3.2 \text{ a})$$

$$\tau_p = c + \sigma'_n \tan \phi_r \quad (\sigma'_n \geq \sigma_n^*) \quad (3.2 \text{ b})$$

in which ϕ_μ = frictional angle of the sliding resistance,

ϕ_r = residual friction angle of the rock comprising the asperities,

i = dilational angle,

c = shear strength of the asperities.

The critical normal stress σ_n^* is obtained from the following equation:

$$\sigma_n^* = c / [\tan (\phi_\mu + i) - \tan \phi_r] \quad (3.3)$$

Jaeger (1971) thought that the abrupt change of failure mode is unrealistic and some form of rounding off is needed considering the increasing use of computers. He suggested an exponentially varying cohesion law for the peak shear strength, which can be written as

$$\tau_p = c(1 - e^{-b\sigma'_n}) + \sigma'_n \tan \phi_r \quad (3.4)$$

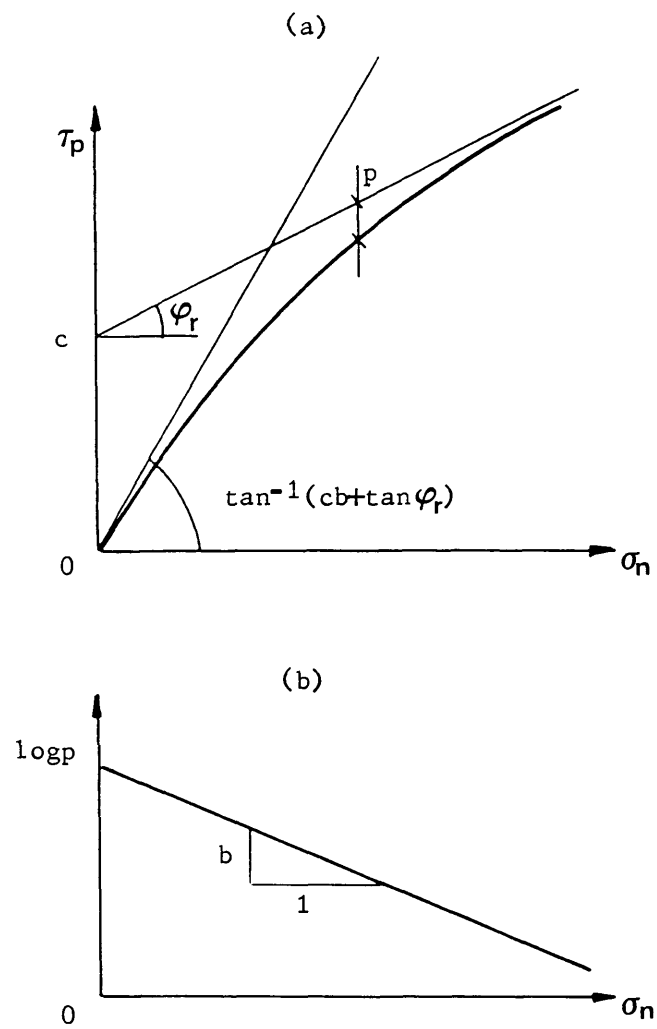


Figure 3.4 Jaeger's peak shear strength envelope
(after Goodman, 1975)

At $\sigma'_n = 0$ the inclination of the $\tau_p - \sigma'_n$ curve is

$$\left(\frac{\partial \tau_p}{\partial \sigma'_n} \right)_{\sigma'_n=0} = \tan \phi_r + cb \quad (3.5)$$

When σ'_n is large the peak shear strength will become asymptotic to the upper line of the Patton's model, i.e.

$$\tau_p = c + \sigma'_n \tan \phi_r$$

The parameter b can be obtained graphically as shown in Fig.3.4.

Ladanyi and Archambault (1970) developed a general joint model for irregular asperities based on energy considerations. The total shear force S acting on the joint plane is assumed to be the resultant of four parts

$$S = (S_1 + S_2 + S_3)(1 - a_s) + S_4 a_s \quad (3.6)$$

where S_1 = component due to external work done in dilating against the external force N ,

S_2 = component due to additional internal work in friction due to dilation,

S_3 = component due to work done in internal friction,

S_4 = component due to work done in shearing off of all asperity teeth at the base,

a_s = ratio of sheared area to the total area of the joint plane.

The peak shear strength can be determined from Eqn.(3.6) as

$$\tau_p = \frac{\sigma'_n(1 - a_s)(\dot{v} + \tan \phi_\mu) + a_s R_s}{1 - (1 - a_s)\dot{v} \tan \phi_f} \quad (3.7)$$

where $\dot{v}$ = ratio of dilation at failure,

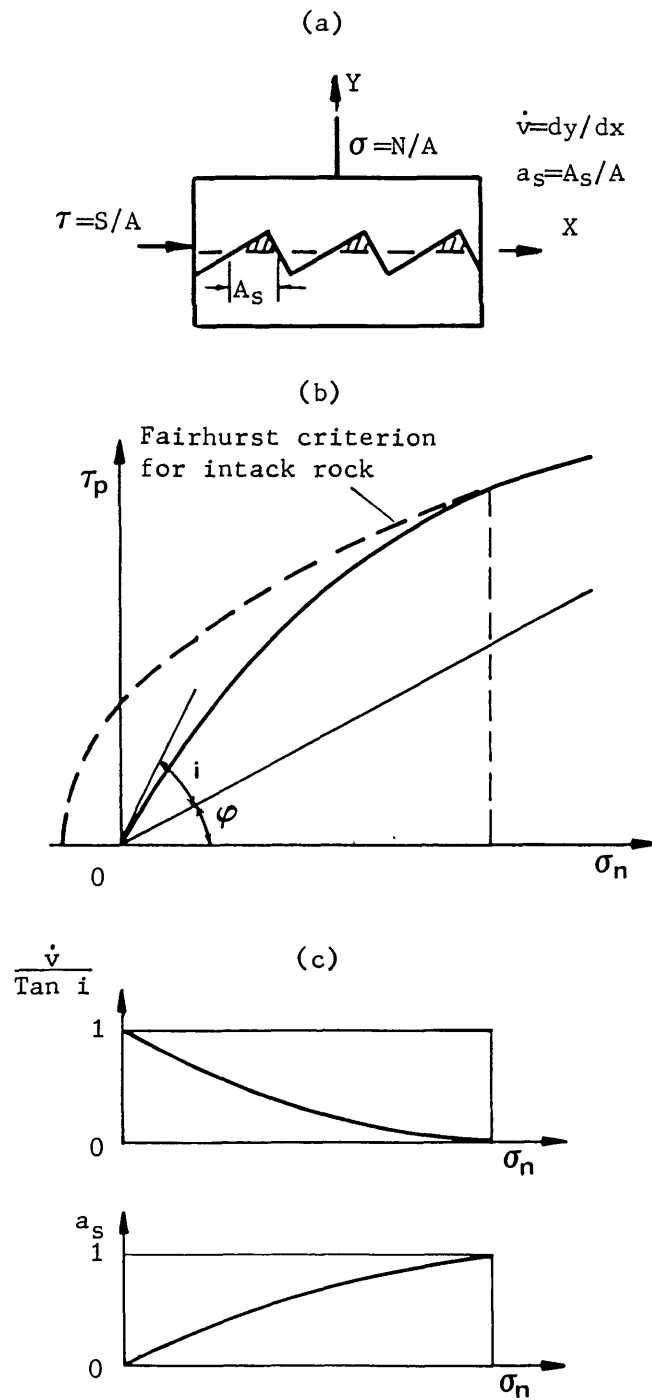


Figure 3.5 Ladanyi and Archambault's joint model (after Ladanyi and Archambault 1970)

R_s = shear strength of the rock comprising the asperities,

ϕ_f = statistical average value of the friction angle when sliding occurs along irregularities of different orientations.

The parameters are evaluated empirically from the following equations,

$$a_s = \begin{cases} 1 - (1 - \sigma'_n/\sigma_T)^{k_1}, & \text{if } \sigma'_n \leq \sigma_T; \\ 1, & \text{if } \sigma'_n > \sigma_T. \end{cases} \quad (3.8)$$

$$\dot{v} = \begin{cases} (1 - \sigma'_n/\sigma_T)^{k_2} \tan i, & \text{if } \sigma'_n \leq \sigma_T; \\ 0, & \text{if } \sigma'_n > \sigma_T. \end{cases} \quad (3.9)$$

The suggested values for k_1 and k_2 are 1.5 and 4.0.

At extremely low normal stresses, $a_s = 0$, $\dot{v} = \tan i$. When only regular teeth are assumed, $\phi_f = \phi_\mu$, Eqn.(3.7) reduces to

$$\tau_p = \sigma'_n \tan (\phi_\mu + i) \quad (3.10)$$

that is the lower part of the Patton's bilinear criterion. At very high normal stresses ($\sigma'_n > \sigma_T$) all teeth are sheared off, $a_s = 1$, $\dot{v} = 0$, and the strength of intact rock becomes dominant,

$$\tau_p = R_s \quad (3.11)$$

R_s can be expressed as the upper part of the Patton's model. However, Fairhurst's parabolic law is preferred by Ladanyi and Archambault, i.e.

$$R_s = c_0 \frac{\sqrt{n+1} - 1}{n} \left(1 + n \frac{\sigma'_n}{c_0}\right)^{\frac{1}{2}} \quad (3.12)$$

where c_0 = unconfined compressive strength of the asperity rock,

n = ratio of compressive strength to tensile strength of the asperity rock.

The transition stress σ_T can be obtained from the point of intersection between the maximum strength envelope of the rock and an envelope representing the frictional strength of the irregular rock surfaces, as suggested by Mogi (1966) and Byerlee (1968).

Heuze (1979, 1981, 1982) proposed a three parameter parabolic model for the peak shear strength of natural joints under relatively low normal stresses ($\sigma'_n < \sigma_c$), i.e.

$$\tau_p = A\sigma'_n + B\sigma'^2_n + C\sigma'^3_n \quad (3.13 \ a)$$

where

$$A = \tan \phi_p \quad (3.13 \ b)$$

$$B = \frac{3c_p^*}{\sigma_c^2} - \frac{2(\tan \phi_p - \tan \phi_R)}{\sigma_c} \quad (3.13c)$$

$$C = -\frac{2c_p^*}{\sigma_c^3} + \frac{\tan \phi_p - \tan \phi_R}{\sigma_c^2} \quad (3.13 \ d)$$

ϕ_R = residual friction angle,

$\phi_p = \phi_R + \delta_0$,

δ_0 = initial dilational angle,

c_p^* = apparent cohesion intercept of the residual envelope,

σ_c = critical normal stress beyond which there is no dilation.

When the normal stress is high, $\sigma'_n > \sigma_c$, the linear envelope is assumed, the same as the upper part of Patton's model,

$$\tau_p = c_p^* + \sigma'_n \tan \phi_R \quad (3.14)$$

The instantaneous dilation angle δ is calculated from $\frac{d\tau}{d\sigma'_n} = \tan(\phi_R + \delta)$.

Therefore,

$$\delta = \tan^{-1} (A + 2B\sigma'_n + 3C\sigma'^2_n) - \phi_R \quad (3.15)$$

The deformation behaviour is defined by considering the stiffnesses of both the joint and the adjacent rock. In the direction normal to the joint plane the deformation relation is

$$\Delta\sigma'_n = \tan \delta \frac{KN * KNEFF}{KN + KNEFF} \Delta u \quad (3.16)$$

The shear stiffness of the joint is

$$KS^* = \frac{1}{\cos^2 \delta} \left(KS + \sin^2 \delta \frac{KN * KNEFF}{KN + KNEFF} \right) \quad (3.17)$$

where KN = normal stiffness coefficient of the joint,

KS = shear stiffness coefficient of a smooth surface of the joint material,

$KNEFF$ = stiffness of the adjacent structure transverse to the mean joint plane. $KNEFF$ can be numerically determined, according to Heuze, by relaxing a small excess normal stress in the joint from a position of equilibrium (Heuze 1982).

Having discussed the shear behaviour of the initially mated joints, Gerrard (1985, 1986) pointed out that the following features should be included as the normal stress increases from 0 to σ_f :

(1) The slope of the peak shear strength envelope ($d\tau_p/d\sigma'_n$) should gradually decrease from μ_0 to μ_f .

(2) The tangent of the dilation angle at failure, g , should gradually decrease from g_0 to 0 with the rate, $dg/d\sigma'_n$ tending to 0 when σ'_n approaches σ_f . The slope of the equipotential lines of the plastic flow function for peak condition ($Q = \text{constant}$) is equal to g .

(3) The coefficient of friction for sliding without dilation, μ , should gradually change from μ_s to μ_f .

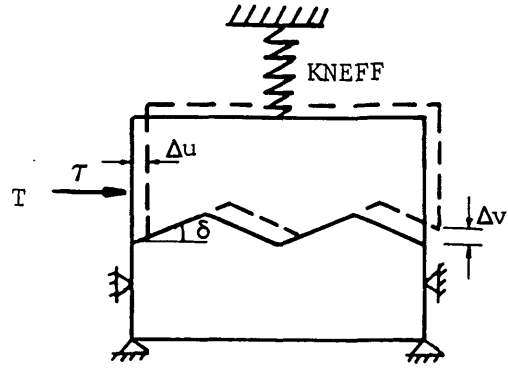


Figure 3.6 Heuze's joint model (after Heuze, 1981)

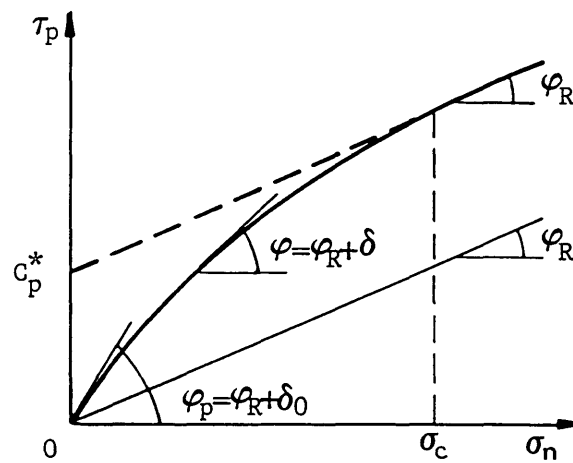


Figure 3.7 Heuze's peak shear strength criterion (after Heuze, 1981)

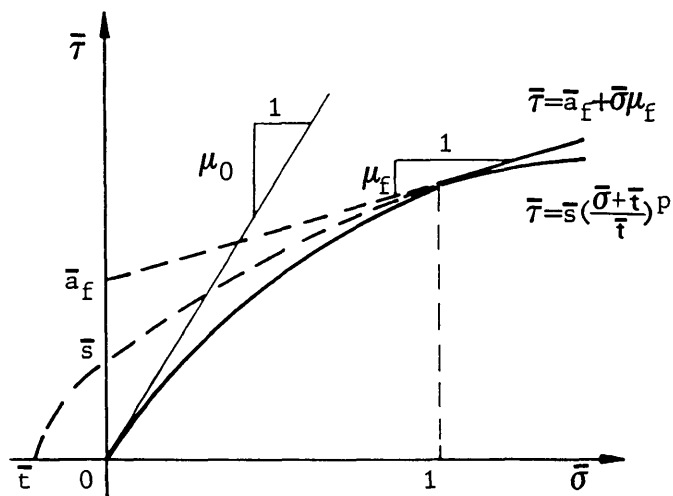


Figure 3.8 Gerrard's peak shear strength model (after Gerrard, 1986)

(4) The asperity strength, a , should increase gradually from 0 to a_f with the rate $da/d\sigma'_n$ tending to 0 for both $\sigma'_n \rightarrow 0$ and $\sigma'_n \rightarrow \sigma_f$.

σ_f is a high normal stress, evaluation of which will be discussed later.

Gerrard proposed a dimensionless total formulation for the peak shear strength to meet the foregoing four features. When $\bar{\sigma}'_n < 1$

$$\bar{\tau}_p = \bar{a}_f [1 - (1 - \bar{\sigma}'_n)^m] - \mu_f \bar{\sigma}'_n \quad (3.18)$$

where

$$m = \frac{\mu_0 - \mu_f}{\bar{a}_f} \quad (3.19)$$

$$\bar{\tau}_p = \tau_p / \sigma_f \quad (3.20 \text{ a})$$

$$\bar{a}_f = a_f / \sigma_f \quad (3.20 \text{ b})$$

$$\bar{\sigma}'_n = \sigma'_n / \sigma_f \quad (3.20 \text{ c})$$

a_f = intercept on the τ_p axis of the tangent to the intersection of the two strength curves, which is equivalent to the maximum asperity strength,

μ_f = effective coefficient of friction appropriate to shearing through the asperities,

μ_0 = slope of the peak shear envelope at $\bar{\sigma}'_n = 0$,

$$\mu_0 = \frac{\mu_s + g_0}{1 - \mu_s g_0} \quad (3.21)$$

μ_s = coefficient of friction without dilation at $\bar{\sigma}'_n = 0$,

g_0 = tangent of the maximum dilation angle.

The stress level of σ_f depends on the surface topography of the joint and the hardness and toughness of the asperities. Ladanyi and Archambault (1970)

suggested that this stress corresponds to the 'transition stress' that marks the change from brittle to ductile behaviour as defined by Mogi (1966) and Byerlee (1968); Goodman (1975) suggested it is the unconfined compressive strength of the asperity material.

When $\bar{\sigma}'_n > 1$ the failure envelope for the material comprising the asperities is adopted, which can be a straight line

$$\bar{\tau} = \bar{a}_f + \bar{\sigma}'_n \mu_f \quad (3.22)$$

or a curve proposed by Hoek and Brown (1980), as suggested by Gerrard,

$$\bar{\tau} = \bar{s} \left(\frac{\bar{\sigma} + \bar{t}}{\bar{t}} \right)^p \quad (3.23)$$

The tangent of the instantaneous dilation angle is

$$g = g_0 (1 - \bar{\sigma}'_n)^n \quad (3.24)$$

The instantaneous coefficient of friction is

$$\mu = (\mu_s - \mu_f)(1 - \bar{\sigma}'_n) + \mu_f \quad (3.25)$$

where constants $g > 1$, $n > 1$.

From a great number of laboratory tests on artificial specimens Barton (1972, 1973, 1976, 1986), Barton and Choubey (1977), Bandis, Lumsden and Barton (1983), Barton, Bandis and Bakhtar (1985) found that the joint behaviour is based to a large extent on joint surface characteristics expressed with three indices: *JRC* - joint roughness coefficient; *JCS* - joint wall compressive strength and ϕ_r - residual friction angle. *JRC* is an overall index denoting the roughness of joint walls varying from 0 to 20 for the flattest to the roughest. It can be estimated from simple tilt tests (Barton and Choubey 1977, Barton et al 1985). *JCS* can be as high as the unconfined compressive

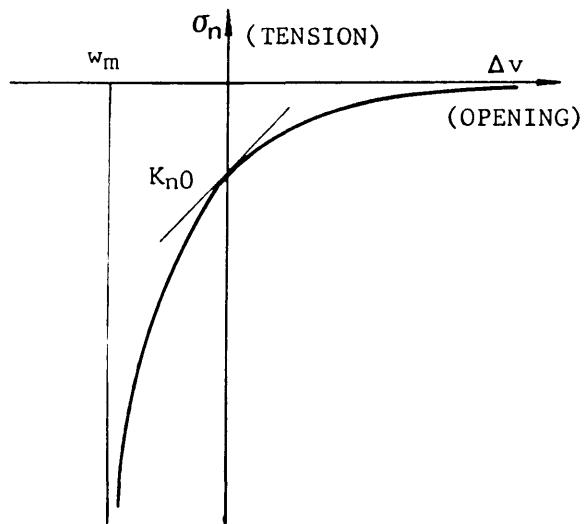
strength σ_c for unweathered joints or may reduce to approximately $1/4\sigma_c$ for weathered joints. The JCS value can be obtained from Schmidt hammer index test (Barton and Choubey 1977). ϕ_r can be experimentally determined in two stages: firstly a Schmidt hammer test on the joint wall and on the unweathered rock and secondly the residual tilt test (Barton and Choubey 1977).

3.1.2 Discussion

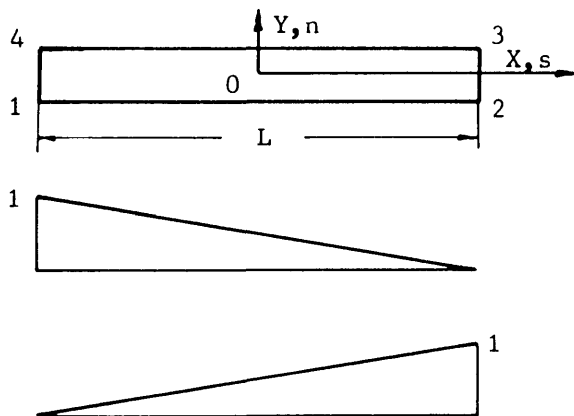
From the foregoing description it is clear that many authors have realized that the joint behaviour including shear strength, deformation and dilation is governed to a large extent by the characteristics of the joint surface asperity. A great effort has been made to take into account the characteristics in formulating the feature of shear strength and deformation. Patton's model simply considers the artificial regular asperities. Compared with Patton's and Jaeger's models, Ladanyi and Archambault's model, Heuze's model and Gerrard's model are relatively advanced. These models are derived theoretically with some parameters determined empirically or experimentally. They attempt to consider the effect by means of adopting a number of parameters, such as coefficients of friction, frictional angles and dilational angles. Each parameter must be evaluated either by an empirical equation or by means of experiments. For example, in Ladanyi and Archambault's model all a_s , ϕ_r , ψ and i are related to the surface asperity, but they must be determined separately. The same thing occurs in the Gerrard's model, i.e. all a_f , μ_f , σ_f and g_0 are governed by surface topography. Evaluation of these parameters are not easy. Barton's model, however, is a purely empirical model. In this model the surface asperity is characterized by a unique parameter JRC . All relations between the joint surface asperity and the mechanical features including shear strength, deformation, dilation and even conductivity are expressed by JRC only. Some empirical relations have been established from a large number of experiments.

JRC is an overall index representing joint surface roughness and can be readily evaluated from a simple index test without much scatter for a given joint specimen. Therefore, Barton's model provides a simple and reliable means to characterize joint roughness and its effect on mechanical and hydraulic features of rock joints.

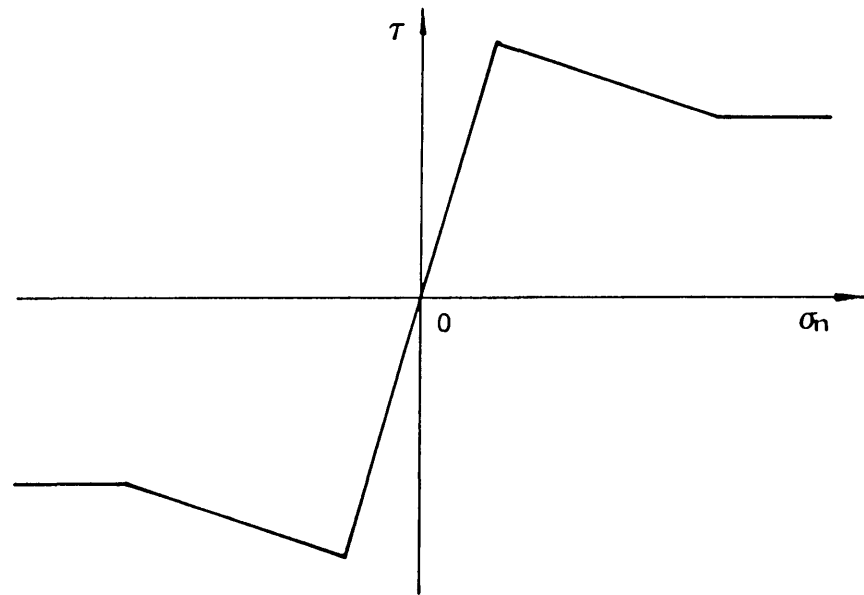
Experimental results show that shearing through asperities starts before the peak shear strength is reached. This is a very important feature for the elasto-plastic analysis of rock joints. At the very onset of shearing through asperities failure starts to occur and shear deformation becomes irrecoverable. So plastic behaviour begins. This indicates that from the beginning of shearing through asperities to the peak shear strength is reached rock joints response elasto-plastically. This pre-peak plastic behaviour is an important feature of rock joints. In the theory of plasticity this can be regarded as the strain hardening plastic behaviour. After the peak point is passed, the strength cannot drop to the residual strength immediately, but declines gradually in a curved way and finally reaches the residual value after a large shear displacement. As long as the residual strength is reached, the shear strength will remain constant. This post-peak plastic behaviour is another important characteristic of rock joints. In the theory of plasticity the region from the peak point, P, to the residual point, R, can be regarded as the strain softening behaviour, while after the residual point perfectly plastic. It is one of the main advantages of Barton's model to account well for the pre- and the post- peak plasticity by introducing the mobilized joint roughness coefficient JRC_m . In Barton's model the pre- and post- peak shear strength is explicitly expressed by a single equation. For any other joint model, however, either there is no explicit expression or no expressions at all. So a linear relation for both pre- and post- peak strength has to be assumed like Goodman's and Ge's joint element (Fig.3.9 b, 3.11 b) if the model is chosen for numerical analysis. Hence the pre- peak plastic behaviour



(a) Relation between normal stress and normal displacement



(c) Geometry of joint element



(b) Shear deformation relation

Figure 3.9 Goodman's joint element and its model
(after Goodman, 1975)

disappears, instead whole pre- peak zone becomes elastic.

In comparison with other advanced joint models, such as Ladanyi and Archambault's model, Heuze's model and Gerrard's model, Barton's model gives the possibly simplest expressions for shear strength, deformation and dilation, that makes the numerical analysis of plasticity for rock joints feasible in practice. From Appendix 6 it can be seen that even for such a simple model the elasto-plastic stress- displacement matrix is very complicated already. If other model is used much more complicated expression will be obtained.

From the point of view of water flow through rock joints, Barton's model is the only model which provides a whole set of equations to evaluate the joint aperture and conductivity. Moreover, Barton's model utilizes the same joint surface parameters to formulate joint conductivity as that describing behaviour of strength and deformation. That makes the whole numerical procedure consistent. To the author's knowledge, there has not been formulation for conductivity in correspondence with other models.

From the discussion it is clear that Barton's model is the simplest, most reliable empirical joint model available so far that is suitable to be used in the numerical plasticity analysis of rock joints. Therefore, it has been chosen as the prototype of the numerical model to be used in this research project. That does not necessarily mean that Barton's model is perfect. Some shortcomings and appropriate modifications will be described in Section 3.3, where a detailed introduction to the numerical model will be given.

3.2 Review of Joint Elements

A number of joint elements using various failure criteria and deformation

relations, fitting different geometry requests for particular purposes, and utilizing various computational methods have been proposed over past 20 years. Goodman et al (1968) developed the first joint element which is a two dimensional four node linear joint element and was further improved afterwards (Goodman 1975, 1977). In this joint element the peak shear strength criterion is chosen from the Ladanyi and Archambault model; the relation between normal stress and displacement is described by a hyperbola; the shear stress- displacement relation is shown in Fig.3.9 (a); the effect of rotation was taken into account; the stress transfer technique is used for non-linear computation. This element was extended to three dimensions by Mahtab and Goodman (1970).

Ghaboussi et al (1973) and Wilson (1977) illustrated numerical ill conditioning associated with high normal stiffness and proposed a new type of interface element which takes the relative displacements of two walls to be the essential and independent unknowns rather than the displacements at each wall nodes. This is illustrated by a one-dimensional example shown in Fig.3.10.

The equilibrium equation is

$$\begin{bmatrix} 1001.5 & -1000.0 \\ -1000.0 & 1001.5 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1.0 \end{Bmatrix}$$

If five significant figures are retained in the stiffness matrix the solution is

$$\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 0.33283 \\ 0.33333 \end{Bmatrix}$$

If only four significant figures are retained the solution is

$$\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 0.4995 \\ 0.5000 \end{Bmatrix}$$

If only three significant figures are taken the equation becomes singular and $u_1 = u_2 = \infty$.

On the other hand, if the relative displacement $\Delta = u_2 - u_1$ is taken to

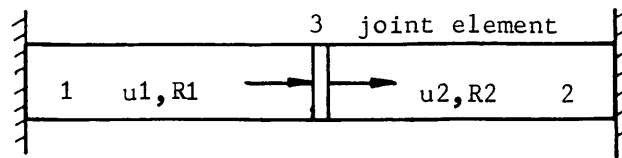
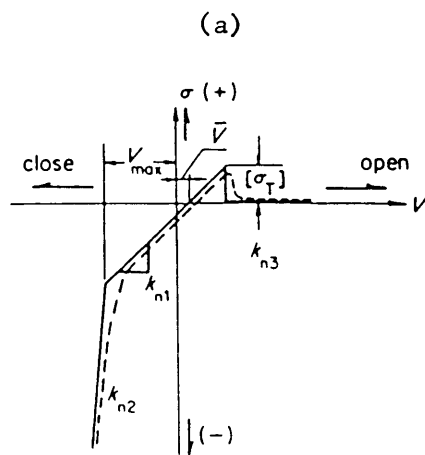
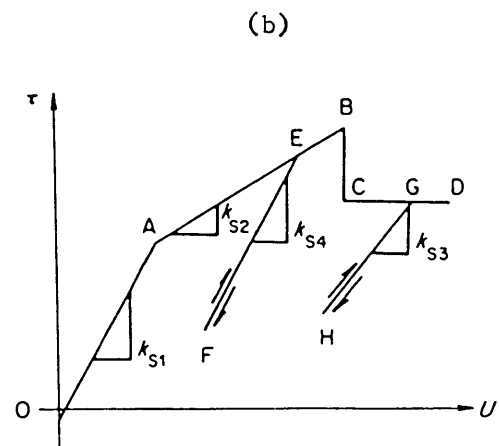


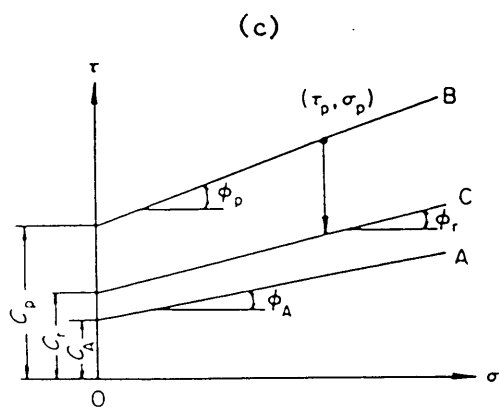
Figure 3.10 One dimensional interface example
(after Wilson, 1977)



The normal deformation model



The shear displacement model



The shear strength model

Figure 3.11 Ge's joint element
model (after Ge, 1981)

be the essential unknown the equilibrium equation is

$$\begin{bmatrix} 3.0 & 1.5 \\ 1.5 & 1001.5 \end{bmatrix} \begin{Bmatrix} u_1 \\ \Delta \end{Bmatrix} = \begin{Bmatrix} 1.0 \\ 1.0 \end{Bmatrix}$$

The solution for four significant figures of the stiffness matrix is

$$\begin{Bmatrix} u_1 \\ \Delta \end{Bmatrix} = \begin{Bmatrix} 0.33308 \\ 0.00050 \end{Bmatrix}$$

and $u_2 = 0.33358$.

If three significant figures are taken only little change occurs in the solution, i.e.

$$\begin{Bmatrix} u_1 \\ \Delta \end{Bmatrix} = \begin{Bmatrix} 0.33000 \\ 0.00050 \end{Bmatrix}$$

This example convincingly illustrates the advantage of taking the relative displacements as independent unknowns.

Ge (1981) employed elasto-plastic theory to simulate joint behaviour. In his joint model the Mohr-Coulomb shear strength envelope is adopted as the failure criterion,

$$\tau_p = \sigma_n \tan \phi + c$$

in which ϕ and c take different values at different stages of deformation as shown in Fig.3.11 (b),(c). After asperities are sheared off joints are treated as perfectly plastic with an associated flow rule. For non-linear computation the stress transfer technique is used.

Having illustrated the great difference between the measured dilation angles and those calculated from the associated flow rule, Pande and Xiong (1982) and Pande (1985) pointed out the associated flow rule overestimates the effect of dilation. They suggested a complex implicit expression for the potential function, that, according to the authors, leads to dilation angles which approximate experimental results. A similar influence of plastic potential on the

dilation effect was detected by Yin and Qu (1981), but no quantitative results were given.

Zienkiewicz and Pande (1977) developed an elasto-viscoplastic multi laminate model for jointed rocks, in which the Mohr-Coulomb shear failure criterion is assumed for the joint system. This model was further improved by Pande and Xiong (1982) and by Olofsson (1985) by introducing a new peak shear strength criterion and new visco-plastic potential function. Pande and Xiong used this model in the analysis of a rock slope, while Olofsson analysed a circular tunnel in a rock mass cut by two sets of joints subjected to a hydrostatic stress field in a two dimensional case.

Carol and Alonso (1983) used a six-node isoparametric element. Desai (1977, 1983, 1984) presented two-dimensional and three-dimensional isoparametric thin layer elements for filled joints or the interface between different materials. Heuze (1981, 1982) developed an axisymmetric joint element and used it in an embedded circular footing. Beer (1985) presented an interface element for shell and solid contact. Beer et al (1985) presented a three dimensional isoparametric joint element with the stress transfer method for non-linear computation.

Roberds and Einstein (1978) presented a comprehensive joint model which can describe the whole process from creation of a discontinuity to its behaviour before and after sliding. Sofianos (1984) made a literature review of joint models up to 1984 in a calendar order. Gerrard (1986) compared the peak shear strength and dilation angle of six different joint models.

3.3 Proposed Joint Model

This numerical joint model uses elasto-plasticity theory to deal with the full range of shearing and compression problems with Barton's numerical model being a prototype. Some modifications and mathematical treatments are made to make it suitable for numerical analysis.

3.3.1 Normal stress displacement relation

In the direction perpendicular to the joint plane no tensile stress is allowed. The normal stiffness increases as joint closure develops. The relation between the effective normal stress σ'_n and relative normal displacement Δw is described by a hyperbola (Fig.3.12),

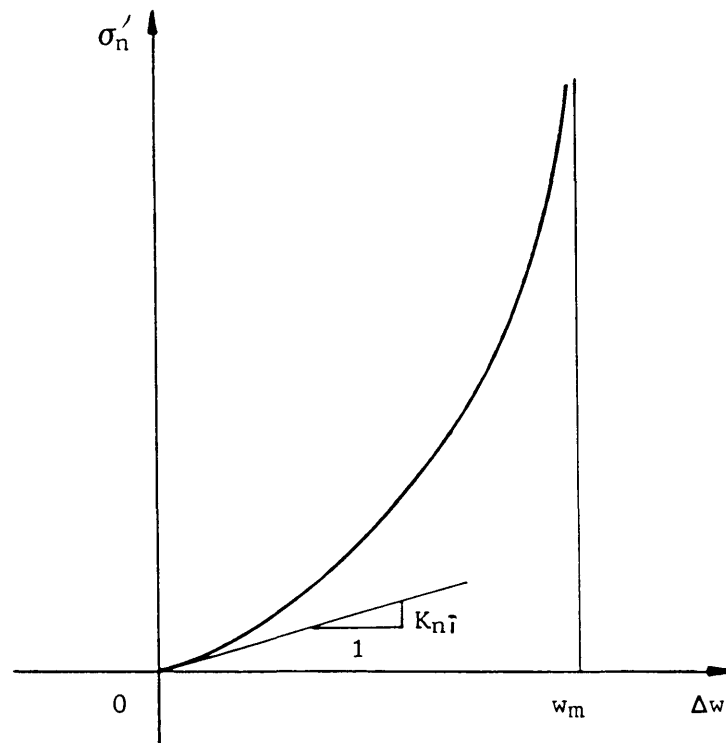


Figure 3.12 Relation between normal stress and normal displacement

$$\sigma'_n = \frac{w_m \Delta w K_{ni}}{w_m - \Delta w} \quad (3.26)$$

where w_m = maximum joint closure,

K_{ni} = initial normal stiffness coefficient.

w_m and K_{ni} are input data.

The normal stiffness coefficient is determined from either σ'_n or Δw as

$$K_n = \frac{K_{ni}}{(1 - \Delta w/w_m)^2} = \frac{K_{ni}}{(1 - \frac{\sigma'_n}{w_m K_{ni} + \sigma'_n})^2} \quad (3.27)$$

The results of the laboratory tests conducted on the artificial samples unveil the hysteresis and permanent deformation in the normal loading and unloading circles (Barton et al 1985). This indicates that the normal stiffness in tension is greater than in compression. However, as the sample is experienced more and more circles of loading and unloading, the hysteresis and the permanent deformation become less and less. It is inferred that for the in situ cases the hysteresis and the permanent deformation are likely very limited because the joints have probably experienced a number of times of geological movements in which loading and unloading may be associated. Therefore, for simplicity, this feature is not taken into account in the proposed joint model.

3.3.2 Shear behaviour

Barton (1973) suggested an empirical peak shear strength criterion. At low effective normal stress σ'_n , the shear strength depends to some extent upon the ratio JCS/σ'_n . At high normal stress ($\sigma'_n > JCS$) the confinement is taken into account due to increased contact area between joint walls. The τ_p/σ'_n relation at extremely low normal stress is assumed to be linear.

$$\tau_p = \sigma'_n \tan \left[JRC_p \log_{10} \left(\frac{JCS}{\sigma'_n} \right) + \phi_r \right] \quad 1 \leq \frac{JCS}{\sigma'_n} \leq 100 \quad (3.28)$$

$$\tau_p = \sigma'_n \tan [JRC_p \log_{10}(\frac{\sigma_1 - \sigma_3}{\sigma'_n}) + \phi_r] \quad \frac{JCS}{\sigma'_n} < 1 \quad (3.29)$$

$$\tau_p = \sigma'_n \tan (2JRC_p + \phi_r) \quad \frac{JCS}{\sigma'_n} > 100 \quad (3.30)$$

Fig.3.13 shows $\tau_p - \sigma'_n$ plots at different JRC_p , JCS and ϕ_r values.

Barton et al (1985) introduced the concepts of mobilized joint roughness coefficient, JRC_m , mobilized shear strength, τ_m , and mobilized dilation angle, ϕ_{dm} , at any stage of shearing to describe the pre- and post- peak shear behaviour. The mobilized shear strength is

$$\tau_m = \sigma'_n \tan [JRC_m \log_{10}(\frac{JCS}{\sigma'_n}) + \phi_r] \quad 1 \leq \frac{JCS}{\sigma'_n} \leq 100 \quad (3.31)$$

$$\tau_m = \sigma'_n \tan [JRC_m \log_{10}(\frac{\sigma_1 - \sigma_3}{\sigma'_n}) + \phi_r] \quad \frac{JCS}{\sigma'_n} < 1 \quad (3.32)$$

$$\tau_m = \sigma'_n \tan (2JRC_m + \phi_r) \quad \frac{JCS}{\sigma'_n} > 100 \quad (3.33)$$

Therefore, the entire shearing process illustrated in Fig.3.14 can be described in the following way.

- (1) Frictional resistance is mobilized when shearing begins (from point O).
- (2) Dilation begins when roughness is mobilized from $JRC_m = 0$ (from point A).
- (3) Peak shear strength is reached when $JRC_m = JRC_p$ and shear displacement $\delta_s = \delta_p$ (at point P).
- (4) Dilation declines as roughness is destroyed past the peak and JRC_m is reduced (from point P).
- (5) Residual strength may be reached after a large shear displacement (at point R).

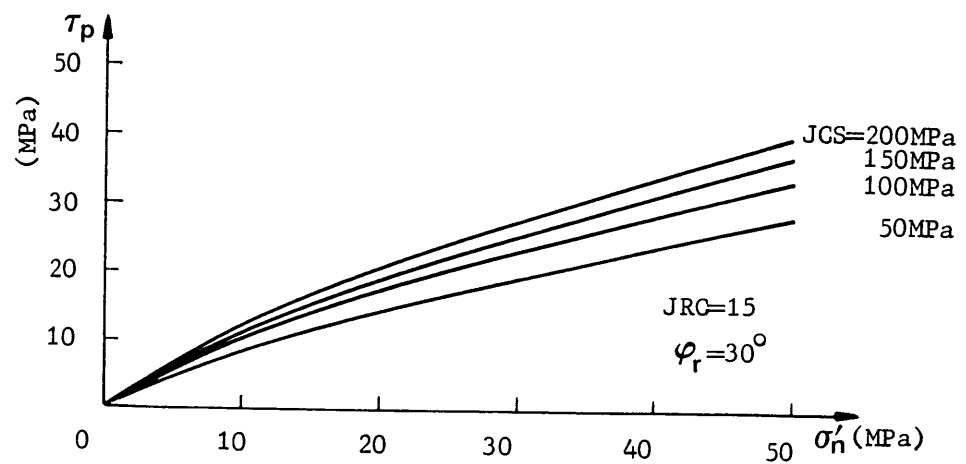
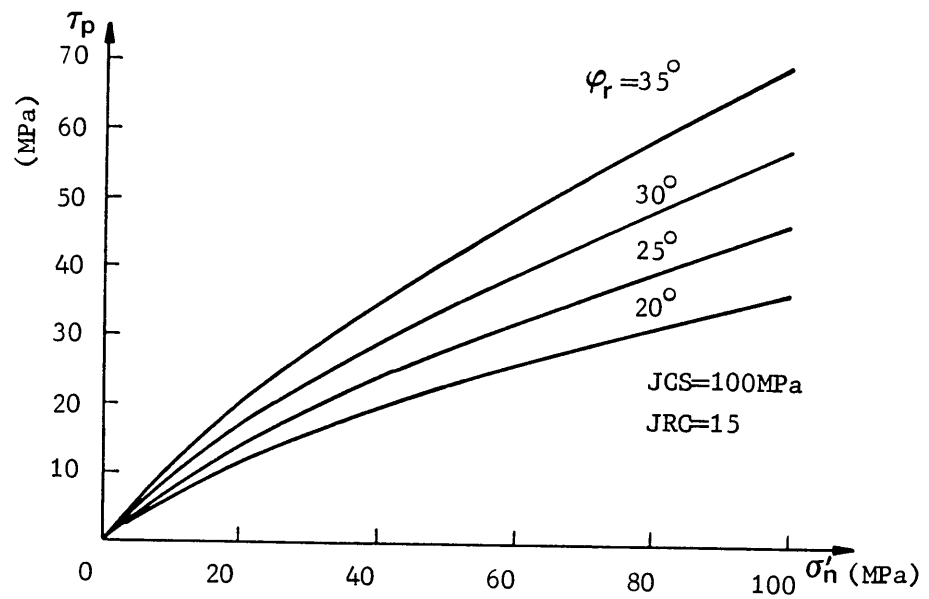
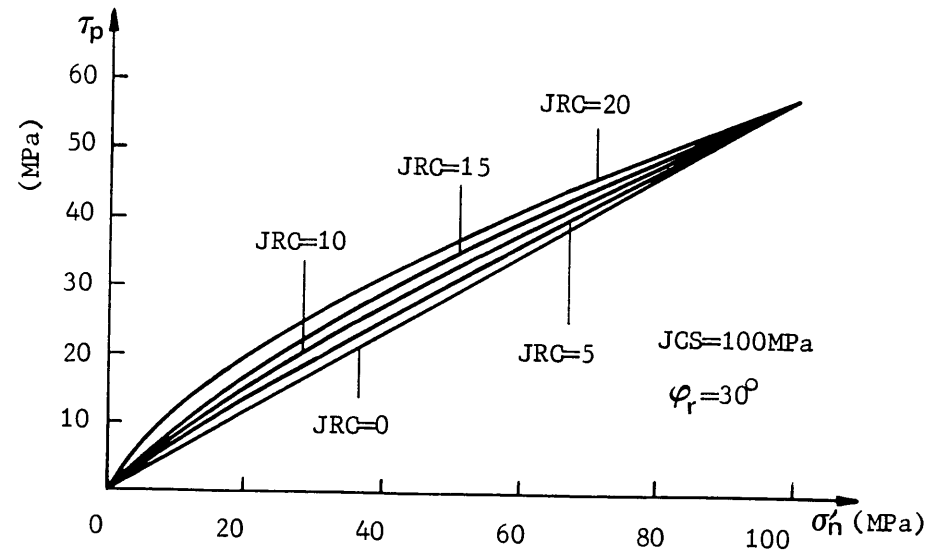


Figure 3.13 Influence of JRC, JCS and φ_r on peak shear strength in Barton's model

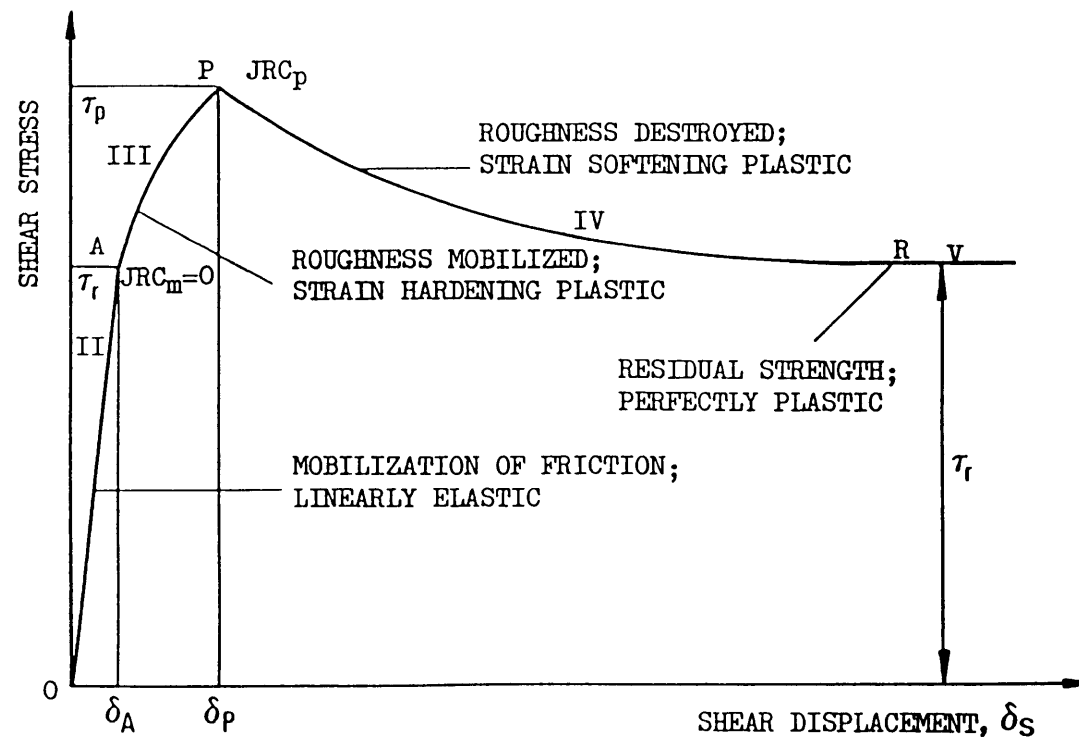


Figure 3.14 Proposed elasto - plastic model

Coincidence between the peak shear strength and the peak shear displacement δ_p is assumed. δ_p is evaluated by an empirical equation (Barton et al 1985)

$$\delta_p = \frac{L}{500} \left[\frac{JRC_p}{L} \right]^{0.33} \quad (3.34)$$

where L is the length of the sample used in the laboratory tests for determination of JRC . For the in situ cases it may be a scaling parameter relevant to the length of the joint.

The relation between shear stress and shear displacement is determined from Fig.3.15 and the shear strength relation (Eqn.3.31 - 33).

3.3.3 Dilation

From the tests on 136 individual jointed specimens Barton and Choubey (1977) found that the majority of peak dilation angles fell between the limits:

$$0.5JRC_p \log_{10}\left(\frac{JCS}{\sigma'_n}\right) < \phi_{dpk} < 2JRC_p \log_{10}\left(\frac{JCS}{\sigma'_n}\right) \quad (3.35)$$

where ϕ_{dpk} is the peak dilation angle.

Barton (1985, 1986) suggested that for general cases the peak dilation angle can be empirically approximated by the lower bound, i.e.

$$\phi_{dpk} = \frac{1}{2}JRC_p \log_{10}\left(\frac{JCS}{\sigma'_n}\right) \quad (3.36)$$

Substitution of JRC_p by JRC_m will yield the mobilized dilation angle,

$$\phi_{dm} = \frac{1}{2}JRC_m \log_{10}\left(\frac{JCS}{\sigma'_n}\right) \quad (3.37)$$

Barton also pointed out that when σ'_n is very low or JCS is very high the following equation may be used,

$$\phi_{dm} = JRC_m \log_{10}\left(\frac{JCS}{\sigma'_n}\right) \quad (3.38)$$

In this joint model the dilation angle is calculated from Eqn.(3.38) based on the following considerations.

(1) From Fig.3.16, Eqn.(3.38) presents the overall mean value of the peak dilation angles.

(2) In some cases too small a dilation angle produces numerical difficulties.

However, problems appear in determining dilation angle at very low normal stresses. The peak dilation angle calculated from Eqn.(3.38) will approach infinity when σ'_n tends to 0 as shown in Fig.3.17. Not only is an infinite dilation angle meaningless in the physical terms, but also it will produce numerical confusion due to oscillation distribution of the tangent function, which may result in negative dilation. In the joint element used herein a revision has been made for this from the following considerations: (1) The dilation angle should never be greater than 50° for any case, i.e. $\phi_{dpk} = 50^\circ$ when $\sigma'_n = 0$; (2) When $JCS/\sigma'_n = 100$, $JRC_p = 20$ (maximum value for JRC_p),

$$JRC_p \log_{10}\left(\frac{JCS}{\sigma'_n}\right) = 40^\circ$$

So there is no problem when $JCS/\sigma'_n < 100$. For extremely low normal stresses a linear relation is assumed. The entire equation for the peak dilation angle is

$$\begin{aligned} \phi_{dpk}^\circ &= JRC_p \log_{10}\left(\frac{JCS}{\sigma'_n}\right) & 1 \leq \frac{JCS}{\sigma'_n} \leq 100 \\ \phi_{dpk}^\circ &= 0 & \frac{JCS}{\sigma'_n} < 1 \\ \phi_{dpk}^\circ &= 100 \frac{\sigma'_n}{JCS} (2JRC_p - 50) + 50 & \frac{JCS}{\sigma'_n} > 100 \end{aligned} \quad (3.39)$$

Similarly the mobilized dilation angle is

$$\begin{aligned} \phi_{dm}^\circ &= JRC_m \log_{10}\left(\frac{JCS}{\sigma'_n}\right) & 1 \leq \frac{JCS}{\sigma'_n} \leq 100 \\ \phi_{dm}^\circ &= 0 & \frac{JCS}{\sigma'_n} < 1 \end{aligned} \quad (3.40)$$

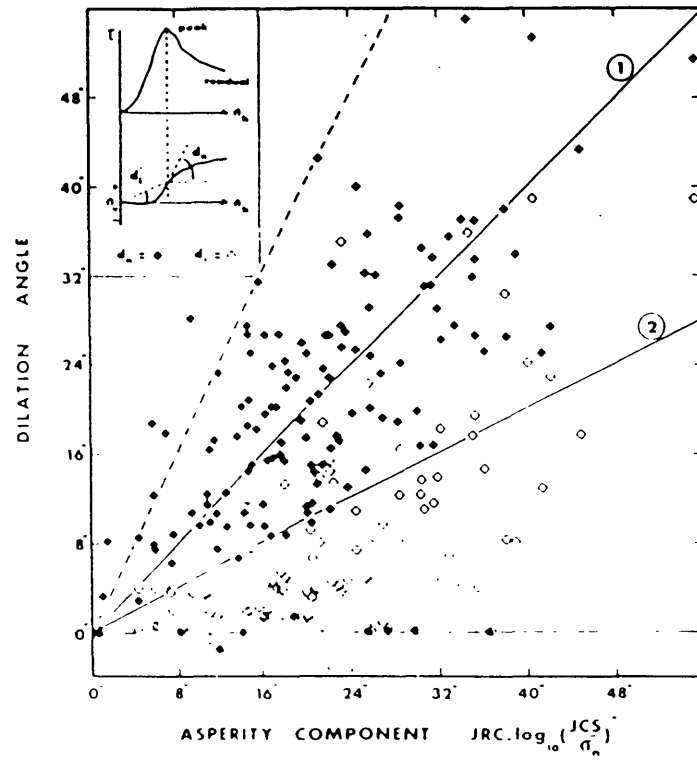


Figure 3.16 Distribution of peak and initial dilation angle and their relationship with the asperity component of shear strength (after Barton and Choubey 1977)

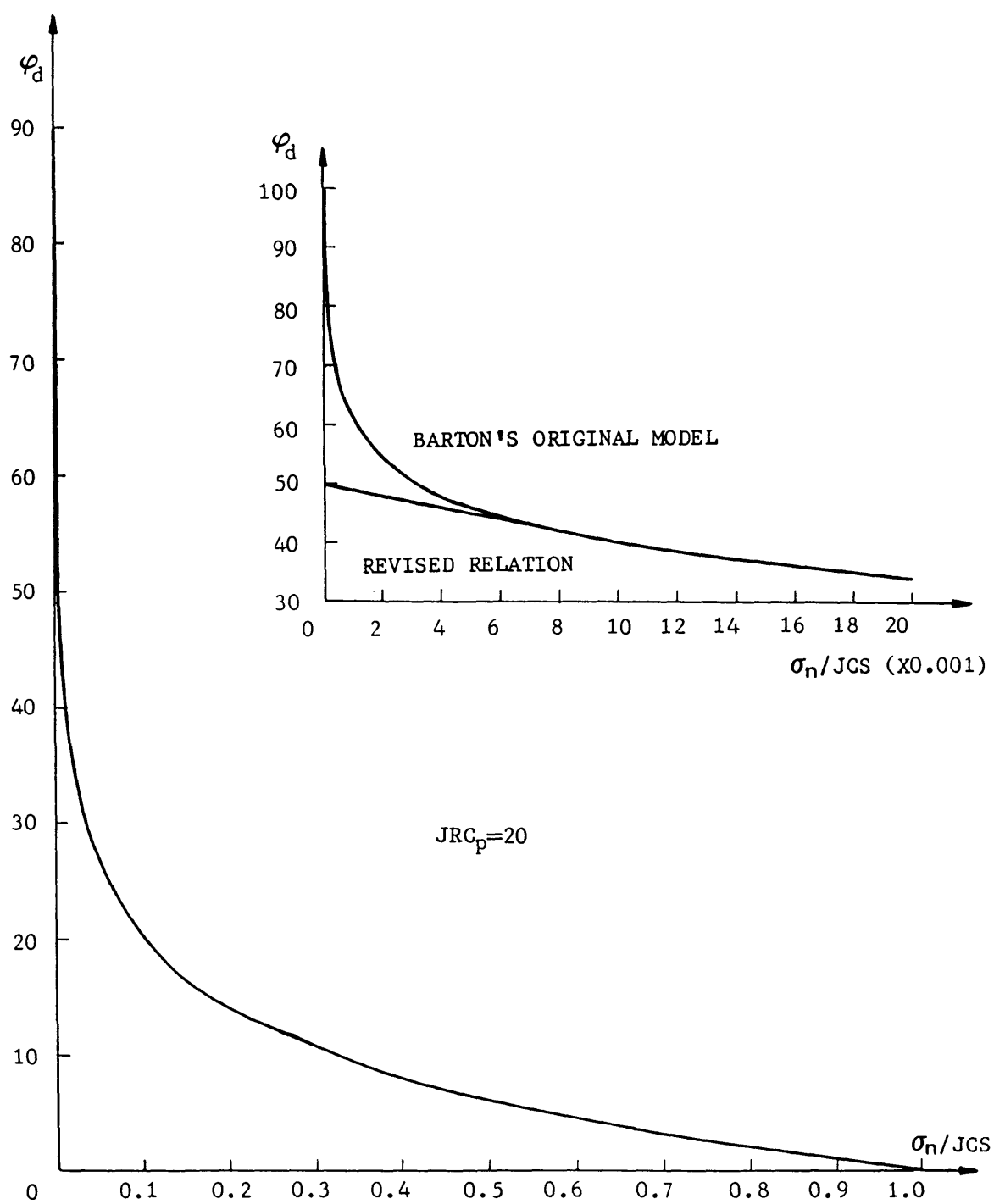


Figure 3.17 Dilation angle of Barton's joint model and its revision

$$\phi_{dm}^o = 100 \frac{\sigma'_n}{JCS} (2JRC_m - 50) + 50 \quad \frac{JCS}{\sigma'_n} > 100$$

3.3.4 Determination of JRC_m

The mobilized joint roughness coefficient JRC_m is a function of the peak joint roughness coefficient JRC_p and the resultant shear displacement δ_s . Fig.3.15 presents some particular points on the $JRC_m/JRC_p - \delta_s/\delta_p$ curve. However, for numerical analysis a continuous and smooth (at least piecewise smooth) relation between JRC_m/JRC_p and δ_s/δ_p is required. So the task is to find out a smooth or piecewise smooth curve that passes the particular points and reasonably represents the relation. Results obtained follow (a detailed derivation is given in Appendix 5).

In region III, i.e. $\delta_A < \delta_s < \delta_p$, a third order parabola is assumed,

$$\begin{aligned} \frac{JRC_m}{JRC_p} = & -1.51913265 + 6.63265306 \frac{\delta_s}{\delta_p} - 5.70790816 \left(\frac{\delta_s}{\delta_p} \right)^2 \\ & + 1.59438776 \left(\frac{\delta_s}{\delta_p} \right)^3 \end{aligned} \quad (3.41)$$

In the first part of the region IV, i.e. $1 < \frac{\delta_s}{\delta_p} < 10$, a fourth order parabola is adopted,

$$\begin{aligned} \frac{JRC_m}{JRC_p} = & 1.23546811 - 0.2888567386 \frac{\delta_s}{\delta_p} + 0.0591628083 \left(\frac{\delta_s}{\delta_p} \right)^2 \\ & - 0.00599762081 \left(\frac{\delta_s}{\delta_p} \right)^3 + 0.000223443926 \left(\frac{\delta_s}{\delta_p} \right)^4 \end{aligned} \quad (3.42)$$

For the second part of the region IV, i.e. $10\delta_p < \delta_s < \delta_R$, a second order parabola is used,

$$\frac{JRC_m}{JRC_p} = 0.617283951 - 0.012345679 \frac{\delta_s}{\delta_p} + 0.0000617283951 \left(\frac{\delta_s}{\delta_p} \right)^2 \quad (3.43)$$

δ_A and δ_R are the shear displacements at point A and R in Fig.3.14 respectively and can be evaluated as

$$\delta_A = 0.3\delta_p \quad \delta_R = 100\delta_p \quad (3.44)$$

according to Barton and Bandis (1983).

3.3.5 Elasto-plastic model

When the strength condition and deformation relation have been specified using the 'no-tension' and 'laminar' models (Zienkiewicz 1977) and the stress transfer technique (Wittke 1977), as done by many authors, the response of joints to external loads can be estimated. As pointed out by Yin et al (1979) and Qu (1981), however, in some cases this method may produce negative strain energy, which is in contradiction with a fundamental theorem of plasticity. Their argument can be simply demonstrated as follows (Fig.3.18).

Assume that the stress state of a joint element is at the point A in the previous load step. In the current load step, the first trial stress calculated from the global stiffness matrix and the elastic stress strain matrix is at the point B_0 , which does not exist actually because it is beyond the yield surface. So, according to Zienkiewicz and Wittke, it should be withdrawn to the yield surface. This is done by retaining σ'_n unchanged and adjusting the shear stress to fit the yield surface, i.e. point B_1 . Yin et al argued that instead of fixing σ'_n and adjusting τ one can also keep τ unchanged and adjust σ'_n . In this way the stress state can be drawn back to the yield surface as well, but at a different point B_2 . Similarly, one can get $B_3, B_4, \dots, B_n$. This means that the result is not unique. This is because the elasto- plastic stress strain relation is not introduced. This does not necessarily mean that the stress transfer method will always produce a large error. But it does give evidence that the method has a theoretical shortcoming. The only solution is to introduce the elasto-plastic stress strain relation that will make the solution unique and avoid the negative strain energy.

In this model the mechanical responses of joints are accounted for by

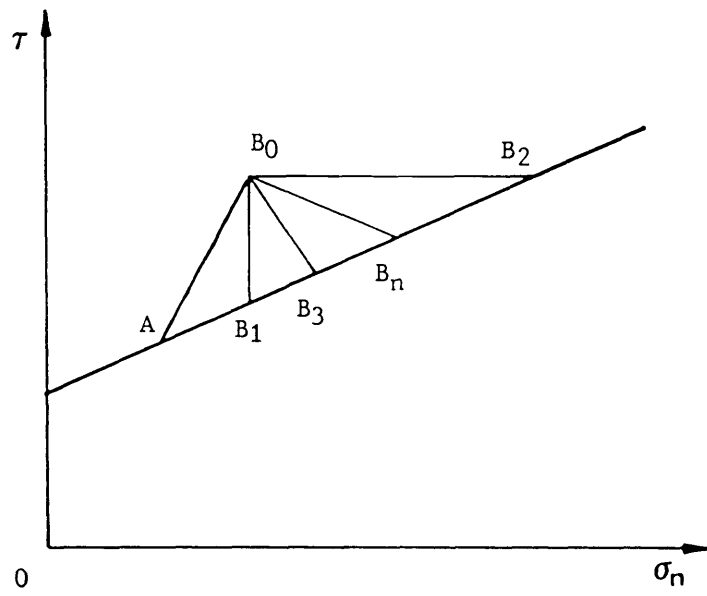


Figure 3.18 Different stress transfer methods

the elasto-plasticity theory. In zone II (Fig.3.14), with just frictional resistance, the joints behave elastically. In zone III, however, roughness, and consequently shear strength, is gradually mobilized, and the joints behave in a strain-hardening elasto-plastic manner. In zone IV, both roughness and shear strength decline, so strain softening elasto-plastic behaviour is assumed. In zone V, no roughness or dilation exist; shear strength is constant and so perfectly plastic behaviour is specified.

The yield function is taken from the shear strength condition,

$$F = \tau + \sigma'_n \tan[JRC_m \log_{10}(\frac{JCS}{\sigma'_n}) + \phi_r] = 0 \quad (3.45)$$

The tangent of the dilation angle can be taken to be the ratio of the normal plastic strain rate to the shear plastic strain rate, which are $\partial G/\partial \sigma'_n$ and $\partial G/\partial \tau$ respectively according to the definition of the plastic potential function,

$$\tan \phi_{dm} = \frac{\dot{\epsilon}_n^p}{\dot{\epsilon}_s^p} = \frac{\partial G/\partial \sigma'_n}{\partial G/\partial \tau} \quad (3.46)$$

Assume

$$G = \tau + f(\sigma'_n) = \text{constant} \quad (3.47)$$

$$\frac{\partial G}{\partial \sigma'_n} = \frac{\partial f}{\partial \sigma'_n} \quad (3.48)$$

$$\frac{\partial G}{\partial \tau} = 1 \quad (3.49)$$

By substituting Eqns (3.38) (3.48) (3.49) into (3.46) we get

$$\tan[JRC_m \log_{10}(\frac{JCS}{\sigma'_n})] = \frac{\partial f}{\partial \sigma'_n} \quad (3.50)$$

By integrating Eqn.(3.50) an explicit expression of f , and consequently for G , can be obtained. However, for the derivation of elasto-plastic stress displacement matrix what is needed is not G itself but $\partial G/\partial \sigma'_n$, $\partial G/\partial \tau$. So far, from

Eqns (3.48) (3.49) (3.50) their explicit expressions have been obtained. Therefore, it is not necessary to derive an explicit expression for G , which can be expressed implicitly as

$$G = \tau + \int_0^{\sigma'_n} \tan[JRC_m \log_{10}(\frac{JCS}{\sigma'_n})] d\sigma'_n \quad (3.51)$$

It can be seen that G is different from F . That means that a non-associated flow rule is adopted. In this way dilation can be simulated properly.

The elasto-plastic stress-displacement matrix $[D^{ep}]$ is derived from a fundamental equation in computational plasticity (Nayak and Zienkiewicz 1972),

$$[D^{ep}] = [D^e] - [D^p] \quad (3.52)$$

$$[D^p] = \frac{[D^e] \left[\frac{\partial G}{\partial \{\sigma\}} \right] \left[\frac{\partial F}{\partial \{\sigma\}} \right]^T [D^e]}{A + \left[\frac{\partial F}{\partial \{\sigma\}} \right]^T [D^e] \left[\frac{\partial G}{\partial \{\sigma\}} \right]} \quad (3.53)$$

where $\{\sigma\}$ = elemental stress vector in the local system. For the complete plane strain model used in this project,

$$\{\sigma\} = [\tau_{zx} \quad \tau_{yz} \quad \sigma'_n]^T \quad (3.54)$$

The corresponding elemental displacement vector is

$$\{\delta\} = [\Delta u \quad \Delta v \quad \Delta w]^T \quad (3.55)$$

in which Δu , Δv , Δw are relative displacements at the element centre in the X' , Y' , Z' directions of the local system.

The coordinate systems are shown in Fig.3.19 in which the disappeared axes, Y and Y' , are parallel to the tunnel axis. For simplicity, the subscripts denoting the directions of the stress components will be expressed by x , y and z instead of x' , y' and z' in this thesis.

$[D^e]$ is the elastic stress displacement matrix. For the complete plane strain model this is

$$[D^e] = \begin{bmatrix} K_{sx} & 0 & 0 \\ 0 & K_{sy} & 0 \\ 0 & 0 & K_n \end{bmatrix} \quad (3.56)$$

A is a parameter associated with plastic hardening or plastic softening, and can be expressed as a function of plastic work as follows (Zienkiewicz 1977):

$$A = -\frac{\partial F}{\partial w^p} \{\sigma\}^T \frac{\partial G}{\partial \{\sigma\}} \quad (3.57)$$

$[D^p]$ is the plastic stress displacement matrix.

The derivation of the elasto-plastic stress displacement matrix is shown in Appendix 6 and the final explicit expression for $[D^p]$ is

$$[D^p] = \frac{1}{A + K_{sx} \frac{\tau_{sx}^2}{\tau^2} + K_{sy} \frac{\tau_{sy}^2}{\tau^2} + K_n \mu \bar{\mu}} \begin{bmatrix} K_{sx}^2 \frac{\tau_{sx}^2}{\tau^2} & K_{sx} K_{sy} \frac{\tau_{sx} \tau_{sy}}{\tau^2} & K_{sx} K_n \mu \frac{\tau_{sx}}{\tau} \\ K_{sx} K_{sy} \frac{\tau_{sx} \tau_{sy}}{\tau^2} & K_{sy}^2 \frac{\tau_{sy}^2}{\tau^2} & K_{sy} K_n \mu \frac{\tau_{sy}}{\tau} \\ K_{sx} K_n \mu \frac{\tau_{sx}}{\tau} & K_{sy} K_n \mu \frac{\tau_{sy}}{\tau} & K_n^2 \mu \bar{\mu} \end{bmatrix} \quad (3.58)$$

where

$$\mu = \tan[JRC_m \log_{10}(\frac{JCS}{\sigma'_n}) + \phi_r] + \frac{\pi JRC_m}{180 \ln 10 \cos^2[JRC_m \log_{10}(JCS/\sigma'_n) + \phi_r]} \quad (3.59)$$

$$\bar{\mu} = \tan[JRC_m \log_{10}(\frac{JCS}{\sigma'_n})] \quad (3.60)$$

$$A = \frac{\tau + \bar{\mu} \sigma'_n}{\tau} \frac{\sigma'_n \log_{10}(JCS/\sigma'_n)}{\cos^2[JRC_m \log_{10}(JCS/\sigma'_n) + \phi_r]} \frac{dJRC_m}{d\delta_s} \quad (3.61)$$

$\frac{dJRC_m}{d\delta_s}$ can be obtained from Eqns (3.41) - (3.43).

3.3.6 Joint elements

The joint element used in the project is a three dimensional joint element for complete plane strain analysis. The coordinates denoting the elemental

geometry are x and z only, but the displacement, strain and stress components are three dimensional.

In order to avoid numerical ill-conditioning caused by large values of the non-diagonal elements in the stiffness matrix associated with high normal stiffness of joint elements, the relative displacements of the two walls of the joint are taken to be the independent and essential unknowns.

The elemental displacement vector is

$$\{\delta\} = [\Delta u_i \quad \Delta v_i \quad \Delta w_i \quad \Delta u_j \quad \Delta v_j \quad \Delta w_j]^T \quad (3.62)$$

The corresponding nodal force vector is

$$\{F\} = [\Delta F_{xi} \quad \Delta F_{yi} \quad \Delta F_{zi} \quad \Delta F_{xj} \quad \Delta F_{yj} \quad \Delta F_{zj}] \quad (3.63)$$

The elemental stiffness matrix for a single joint shown in Fig.3.19 can be derived from the fundamental equation (Zienkiewicz 1977, Zhu 1979)

$$[K^e] = \int_A [B]^T [D] [B] dv \quad (3.64)$$

where $[B]$ is the strain matrix. For this case

$$[B] = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 \end{bmatrix} \quad (3.65)$$

For the linear joint element

$$N_1 = \frac{1}{2}(1 - \eta) \quad (3.66 \ a)$$

$$N_2 = \frac{1}{2}(1 + \eta) \quad (3.66 \ b)$$

$[D]$ is the stress displacement matrix

$$[D] = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} \quad (3.67)$$

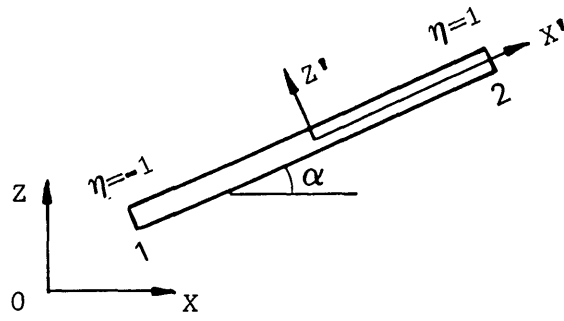


Figure 3.19 Ordinary joint element

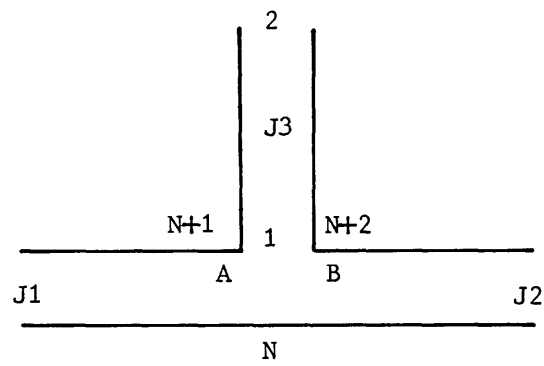


Figure 3.20 T shaped joint element

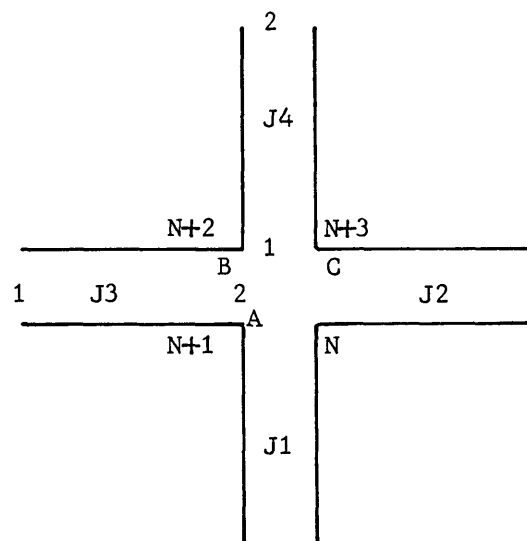


Figure 3.21 Cross joint element

Substituting Eqns (3.65) (3.66) (3.67) into (3.64) the elemental stiffness matrix is obtained,

$$[K^e] = \frac{l}{6} \begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \mathbf{K}_{21} & \mathbf{K}_{22} \end{bmatrix} \quad (3.68)$$

where l = length of the element,

$\mathbf{K}_{ij}$ = submatrices, which are

$$\mathbf{K}_{12} = \mathbf{K}_{21} = [D] \quad (3.69 \ a)$$

$$\mathbf{K}_{11} = \mathbf{K}_{22} = 2[D] \quad (3.69 \ b)$$

To be realistic the joint elements must be able to simulate intersected joints. Therefore the T shaped and the cross joint element are set up.

Fig.3.20 shows the T shaped joint element. It can be viewed as three intersecting joints, J1, J2 and J3. At the intersection the relative displacements between point A and point N and that between point B and point N are taken to be independent unknowns, i.e. $\{\delta_{N+1}\}$ and $\{\delta_{N+2}\}$. Thus J1 and J2 are ordinary joint elements. Only J3 has special features. For J3, the relative displacements between point A and B are $\{\delta_{N+1}\} - \{\delta_{N+2}\}$. We can consider J3 as having three 'nodes': N+1, N+2 and 2. Its elemental stiffness matrix is

$$[K^e] = \begin{bmatrix} \mathbf{K}_{11} & -\mathbf{K}_{11} & \mathbf{K}_{12} \\ -\mathbf{K}_{11} & \mathbf{K}_{11} & -\mathbf{K}_{12} \\ \mathbf{K}_{21} & -\mathbf{K}_{21} & \mathbf{K}_{22} \end{bmatrix} \quad (3.70)$$

Fig.3.21 shows the cross joint element. It can be viewed as four intersecting joints, J1, J2, J3 and J4. The relative displacements between points A and N, B and N, C and N are taken to be the independent unknowns. So J1 and J2 are ordinary joint elements. For J3 the relative displacements between points

C and B are $\{\delta_{N+2}\} - \{\delta_{N+1}\}$. If we define J3 with three nodes, 1, N+2 and N+1, its elemental stiffness matrix is

$$[K^e] = \begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} & -\mathbf{K}_{12} \\ \mathbf{K}_{12} & \mathbf{K}_{22} & -\mathbf{K}_{22} \\ -\mathbf{K}_{12} & -\mathbf{K}_{22} & \mathbf{K}_{22} \end{bmatrix} \quad (3.71)$$

The J4 is the same as J3 for the T shaped joint element.

When a joint is inclined to the global coordinate system, XYZ , the elemental stiffness matrix calculated in the local coordinate system, $X'Y'Z'$, must be transformed to the global system. In order to calculate the stress components the displacements computed in the global system must be transformed to the local system. For the complete plane strain model with the Y -axis being coincident with the Y' -axis, the system transformation matrix is

$$[T] = \begin{bmatrix} \cos \alpha & 0 & -\sin \alpha \\ 0 & 1 & 0 \\ \sin \alpha & 0 & \cos \alpha \end{bmatrix} \quad (3.72)$$

$$\{\delta\} = [T]\{\delta\}' \quad (3.73)$$

$$[K^e] = [L][K^e][L]^T \quad (3.74)$$

where

$$[L] = \begin{bmatrix} [T] & 0 \\ 0 & [T] \end{bmatrix} \quad (3.75)$$

The superscript ' denotes the variables in the local system.

3.3.7 Non-linear computation

As described above the model adopted in this project includes non-linear elasticity, strain hardening plasticity, strain softening plasticity and perfect plasticity, which are all types of non-linear behaviour. In the finite element method, non-linear problems are usually dealt with by three techniques: the incremental method, the iterative method and the mixed method (Desai 1972 a, Desai 1972 b, Zienkiewicz 1977, Zhu 1979 and Owen and Hinton 1980). In the

incremental method the total load is divided into a number of load increments which are applied step by step. The final solution is obtained by accumulating the solutions in each step. In each step a linear relation is assumed. Therefore, the stiffness is constant and can be that at the initial point, the end point or the middle point. In the iterative method the total load is applied simultaneously and the solution is yielded by adjusting the stiffness matrix, the initial stress or the initial strain, to give the variable stiffness method, initial stress method and initial strain method, respectively. The mixed or step iteration method combines the two methods together, i.e. the total load is applied in a step-wise manner. But in each step the iterative procedure is carried out.

In general, the incremental method can handle more general problems, but it is difficult to control the precision and the computing time is longer than that in the iterative method. The iterative method is more efficient but is of less generality due to the convergence problem. It is widely accepted (Desai 1972 a, b, Zienkiewicz 1977 and Zhu 1979) that these methods are valid for nearly all strain hardening and perfectly plastic problems. But difficulties arise in attacking strain softening plastic problems due to a negative stiffness and divergence during iteration.

An incremental secant iteration method is proposed in this project that completely avoids the numerical difficulty associated with negative stiffness coefficients in the strain softening zone and ensures convergence providing that the equilibrium condition is satisfied. In this method the load is applied in a step-wise manner. In each step just the load increment is exerted. A new global stiffness matrix is calculated at the beginning of each load step, which is a linear stiffness matrix and remains constant throughout the step. The coefficients in the stress- displacement matrix for calculating the stiffness matrix are evaluated as follows (see Eqn.3.56) :

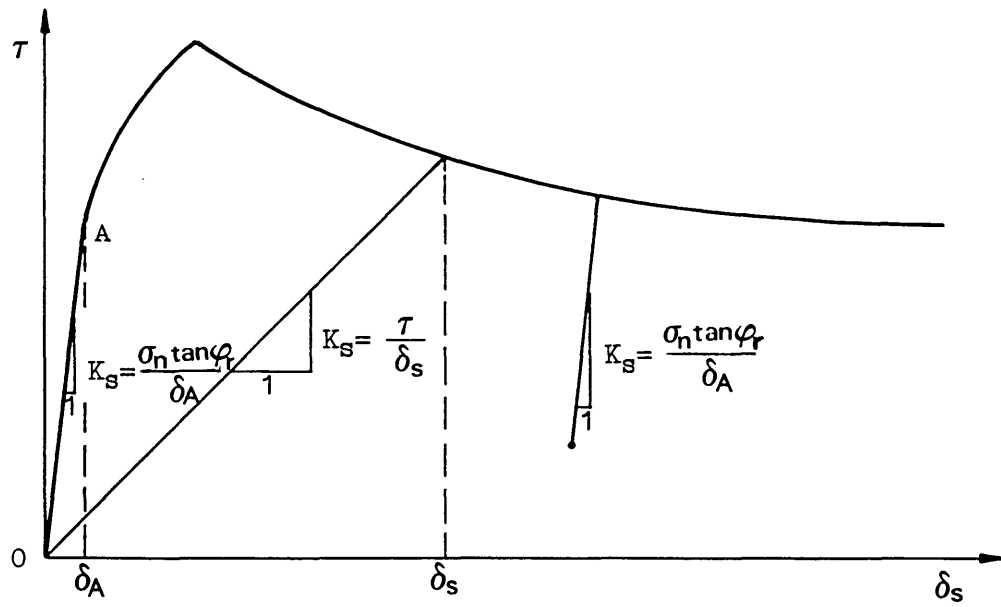


Figure 3.22 Shear stiffness coefficients

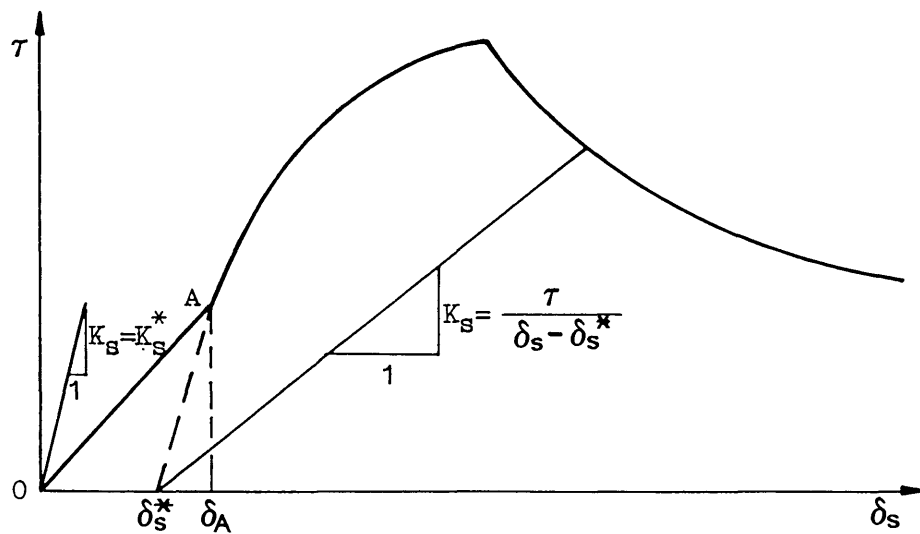


Figure 3.23 An abrupt increase in shear strength at very low normal stress

In the elastic zone, i.e. $\delta_s < \delta_A$,

$$K_{sx} = K_{sy} = \frac{\sigma'_n \tan \phi_r}{\delta_A} \quad (3.76)$$

In the plastic zone, i.e. $\delta_s > \delta_A$,

$$K_{sx} = \frac{\tau_{zx}}{\Delta u} \quad (3.77 a)$$

$$K_{sy} = \frac{\tau_{yz}}{\Delta v} \quad (3.77 b)$$

When a joint element is in the unloading state, the shear stiffness coefficients are calculated from Eqn.(3.76).

When the normal stress is very low an abrupt increment of the tangential stiffness will occur at point A as shown in Fig. 3.23. This type of stress-strain curve will cause numerical difficulties if special care is not taken (Appendix 7) . In the model presented this problem is solved by adopting revised shear stiffness coefficients for both plastic and elastic zones. The tangential stiffness at the critical point A from the plastic zone side is taken to be the shear stiffness coefficient of the elastic zone,

$$K_{sx} = K_{sy} = K_s^* = \frac{\pi \sigma'_n \log_{10} (JCS/\sigma'_n)}{180 \cos^2 \phi_r} \left(\frac{dJRC_m}{d\delta_s} \right)_{\delta_s=\delta_A} \quad (3.78)$$

In the plastic zone

$$K_{sx} = \frac{\tau_{zx}}{\Delta u - \Delta u^*} \quad (3.79 a)$$

$$K_{sy} = \frac{\tau_{yz}}{\Delta v - \Delta v^*} \quad (3.79 b)$$

where

$$\begin{aligned} \Delta u^* &= \frac{\Delta u}{\delta_s} \delta_s^* \\ \Delta v^* &= \frac{\Delta v}{\delta_s} \delta_s^* \\ \delta_s^* &= \delta_A - \frac{180 \tan \phi_r \cos^2 \phi_r}{\pi \log_{10} (JCS/\sigma'_n) \left(\frac{dJRC_m}{d\delta_s} \right)_{\delta_s=\delta_A}} \end{aligned} \quad (3.80)$$

The normal stiffness coefficient is taken to be the tangential stiffness coefficient of the $\sigma'_n - \Delta w$ curve as shown in Eqn.(3.27).

In a load step an iterative method like the revised Newton- Raphson method is used to obtain the solution for the step. The whole computation procedure for the nth load step follows.

(1) Compute the secant global stiffness matrix for the load step as described above.

(2) Apply the load increment for the step and solve the stiffness equation. The first trial displacement increment components are obtained as

$$\{\Delta\delta\}_{n,1} = [K]_n^{-1}\{F\}_n \quad (3.81)$$

(3) Add the displacement increment components to the displacement components obtained in the previous step. The first trial displacement components are obtained.

$$\{\delta\}_{n,1} = \{\delta\}_{n-1} + \{\Delta\delta\}_{n,1} \quad (3.82)$$

(4) Compute the first trial stress components corresponding to the first trial displacement components by adding the first trial stress increments, which are obtained from the elastic stress- displacement matrix and the first trial displacement increments, to the stress components at the end of previous step,

$$\{\sigma^0\}_{n,1} = \{\sigma\}_{n-1} + [D^e]\{\Delta\delta\}_{n,1} \quad (3.83)$$

(5) Compute the real stress components corresponding to the first trial displacement components: if $\delta_s < \delta_A$, $\{\sigma\}_{n,1}$ is obtained from the elastic relation; if $\delta_s > \delta_A$,

$$\{\sigma\}_{n,1} = \{\sigma\}_{n-1} + [D^{ep}]\{\Delta\delta\}_{n,1} \quad (3.84)$$

(6) Compute the first out-of balance force vector

$$\{\Delta F\}_{n,1} = \int_A [B]^T (\{\sigma^0\}_{n,1} - \{\sigma\}_{n,1}) dv \quad (3.85)$$

(7) Compute the second displacement increment components from $\{\Delta F\}_{n,1}$ and $[K]_n$

$$\{\Delta \delta\}_{n,2} = [K]_n^{-1} \{\Delta F\}_{n,1} \quad (3.86)$$

(8) Compute the second trial displacement components

$$\{\delta\}_{n,2} = \{\delta\}_{n,1} + \{\Delta \delta\}_{n,2} \quad (3.87)$$

(9) Compute the second trial stress components

$$\{\sigma^0\}_{n,2} = \{\sigma\}_{n,1} + [D^e] \{\Delta \delta\}_{n,2} \quad (3.88)$$

(10) Compute the second real stress components $\{\sigma\}_{n,2}$ with the same formulation as that in step (5).

(11) Compute the second out-of balance force

$$\{\Delta F\}_{n,2} = \int_A [B]^T (\{\sigma^0\}_{n,2} - \{\sigma\}_{n,2}) dv \quad (3.89)$$

Repeat step (7) to step (11) until the out-of balance force is acceptably small.

Fig.3.24 (a) shows the iteration procedure in the plastic zone, while Fig.3.24 (b) shows that for the normal direction in the elastic zone.

The transition from the elastic state (zone II) to the plastic state (zone III) is judged by the shear displacement, i.e.

$$if \quad \delta_s > \delta_A$$

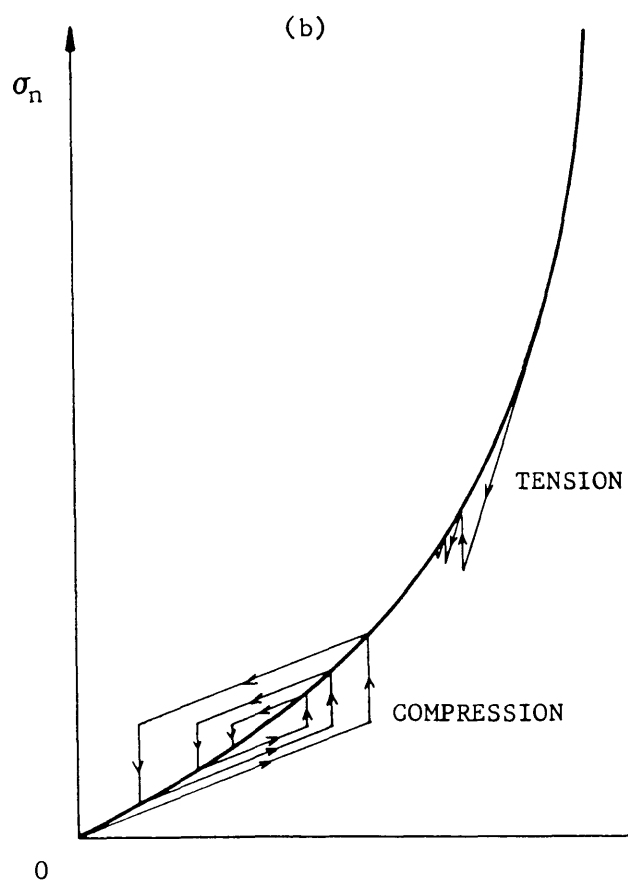
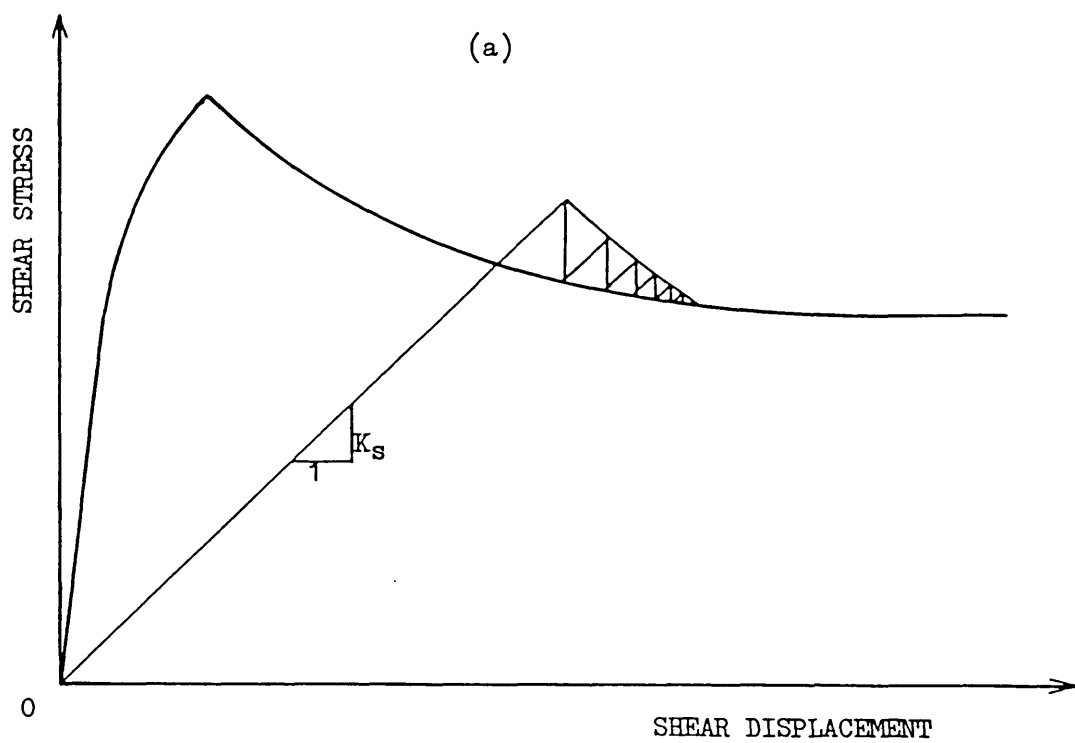


Figure 3.24 Non-linear computation methods

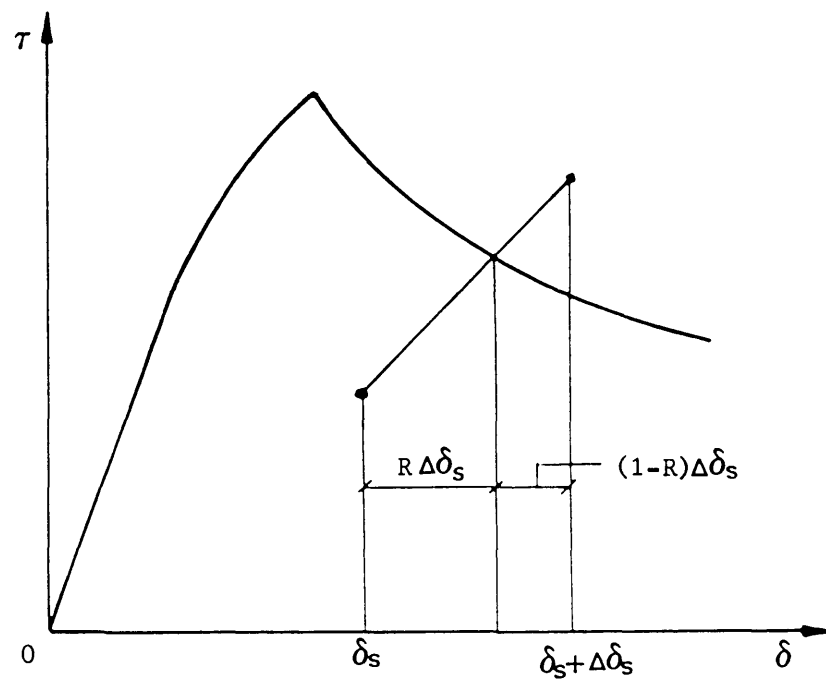


Figure 3.25 Transition from unloaded state to plastic state

In the plastic state the condition of loading and unloading is

$$\frac{\partial F}{\partial \{\sigma\}} d\{\sigma\} \geq 0 \quad \text{plastic loading,}$$

$$\frac{\partial F}{\partial \{\sigma\}} d\{\sigma\} < 0 \quad \text{unloading.}$$

In the unloaded stress state (zone VI), if $\frac{\partial F}{\partial \{\sigma\}} d\{\sigma\} > 0$ and $F > 0$, a transition from the unloaded state to the plastic state occurs. In this case the real stress components must be obtained by dividing $\Delta\delta$ into two parts: $R\Delta\delta$, which produces an elastic stress increment and $(1 - R)\Delta\delta$, which causes a plastic stress increment. The factor R is determined numerically.

3.3.8 Program test with single joint element

The proposed joint model has been tested in a number of examples. The computing results demonstrate the ability of the model to cope with the problems for which it is designed. Some examples involving a single joint follow.

Example 1 *Simple tension and compression*

Simple tension and compression are tested with the example shown in Fig.3.26. Two blocks of rock are separated by a horizontal joint. Vertical forces P are exerted on the top of the upper block increasingly in steps of $0.6MN$ downwards for compression and $1.0MN$ upwards for tension. The figure shows that the numerical result exactly matches the specified model.

Example 2 *Direct shear test under constant normal stress*

A direct shear test under constant normal stress is numerically simulated by the example shown in Fig.3.27. The results indicate very good agreement between the specified model and the numerical results for all elastic, strain hardening plastic, strain softening plastic and perfectly plastic zones.

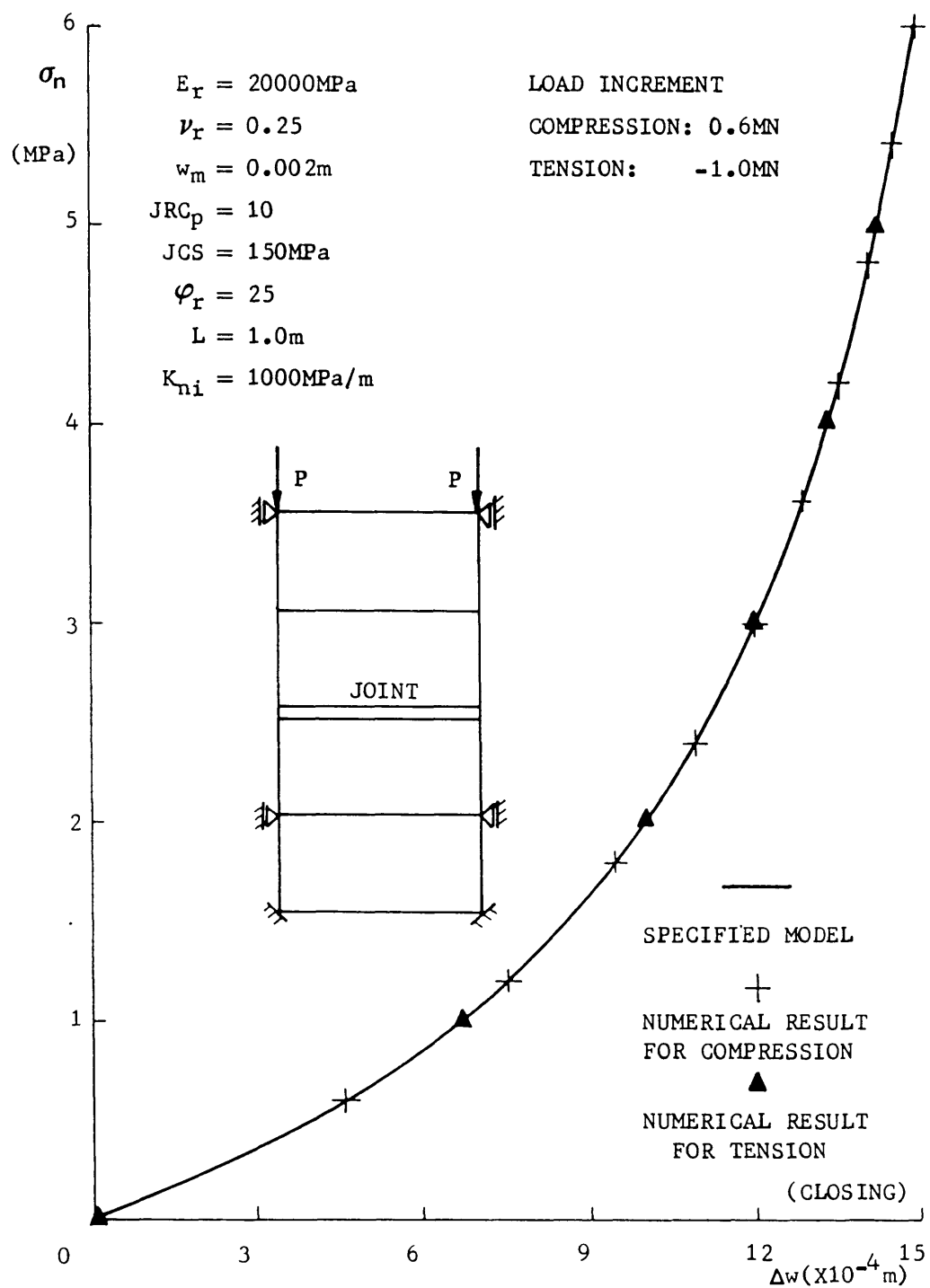


Figure 3.26 Simple compression and tension

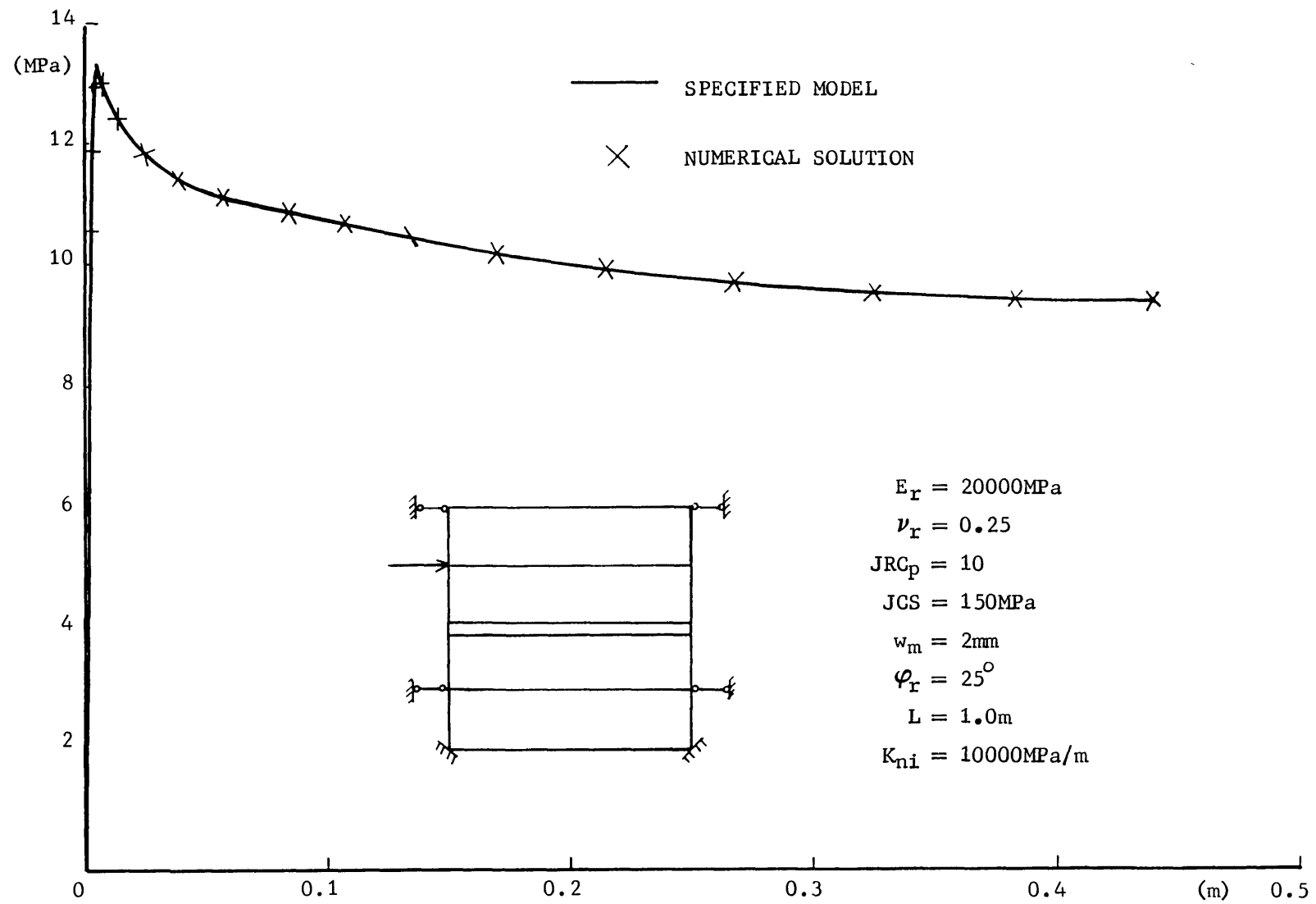


Figure 3.27 Numerical simulation of direct shear test under constant normal stress

Example 3 *Direct shear tests under varying normal stresses*

Example 2 simulates a direct shear test conducted under constant normal stress. If the shear test is carried out under a restricted normal displacement condition a consistently increasing and considerably higher shear strength will result as shown in Fig.3.28.

On the other hand, if tensile forces (e.g. water pressure) are acting on the joint walls as shearing occurs, the normal stress decreases as shear displacement develops. As a result, an even lower shear strength than that obtained from the σ'_n -constant direct shear test will be obtained. This is particularly meaningful when analyzing the effect of pressure water on the joint behaviour.

The proposed model is supposed to be capable of simulating all of these cases. Fig.3.29 (a) and (b) show the computational sketches and the results for the three cases, in which curve 1 is for the constant σ'_n case, curve 2 is for the increasing σ'_n case and curve 3 is for the decreasing σ'_n case. The results show that the model is capable of dealing with these cases.

Example 4 *Hydraulic shearing*

In general, the joints in the rock masses surrounding pressure tunnels are pre-stressed in shear before the water under pressure penetrates into them. Under the water pressure, although there is no further shear load applied the shear displacement will continuously develop due to the decrease in the normal stress (this is called hydraulic shearing and will be discussed in detail in section 5.2.3).

An example similar to example 2 is tested with the load being applied in this way: the horizontal force H is exerted first in steps of $5MN$ until the shear displacement exceeds $0.002m$. Then H is stopped and the tensile forces

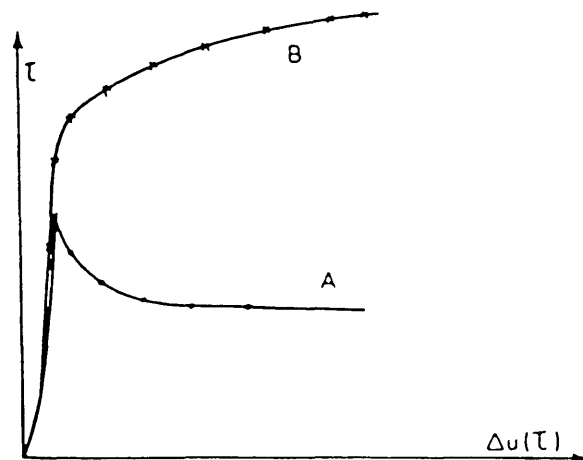


Figure 3.28 Effect of test mode on shear deformation curves
for dilatant joints (after Goodman, 1975)

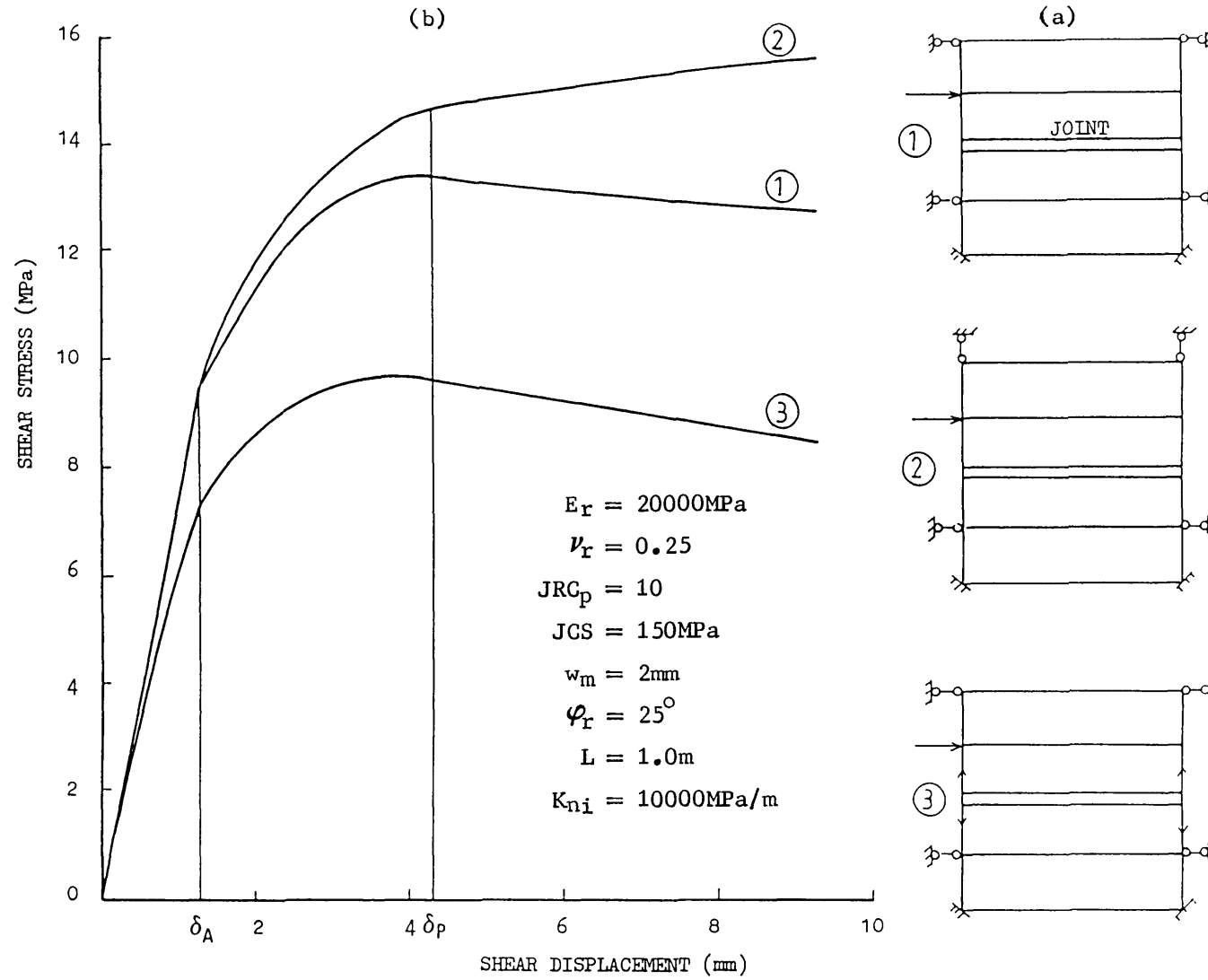


Figure 3.29 Numerical simulation of direct shear tests under varying normal stresses

P are applied in steps of $1MN$, which is the simulation of water pressure. The numerical result shown by curve 1 of Fig.3.30 indicates that the shear displacement develops as the water pressure decreases and in the meantime the shear strength reduces rapidly.

If there are no constraints on the upper block, when the water pressure is applied, the shear displacement develops. But the shear stress remains constant due to the equilibrium requirement. However, after the peak shear displacement is reached, convergence cannot be obtained. The reason for this is that from the strength condition the shear stress should decrease; but in order to maintain equilibrium the shear stress must remain constant. This is a contradiction. From the physical point of view, the lack of convergence indicates considerable shearing.

Example 5 Low normal stress case

As described in section 3.3.7 a special numerical treatment is included in the proposed model to deal with low normal stress cases. The example shown in Fig.3.31 is computed to test the validity of the model in this case. From Eqn.(A.7.6) the critical normal stress for this example is $\sigma_n^* = 18.986MPa$. Cases with three different σ_n' : $60MPa (> \sigma_n^*)$, $18.986MPa (= \sigma_n^*)$ and $3.0MPa (< \sigma_n^*)$ have been considered. The results show that the proposed method solved this problem well.

Example 6 Unloading case

Under the effect of excavation some joints with initial shear stresses may be unloaded due to the increase of the normal stress or the decrease of shear forces. Modelling unloading and reloading is one of the significant abilities for a successful joint model. The example used to test this is like example 2, but

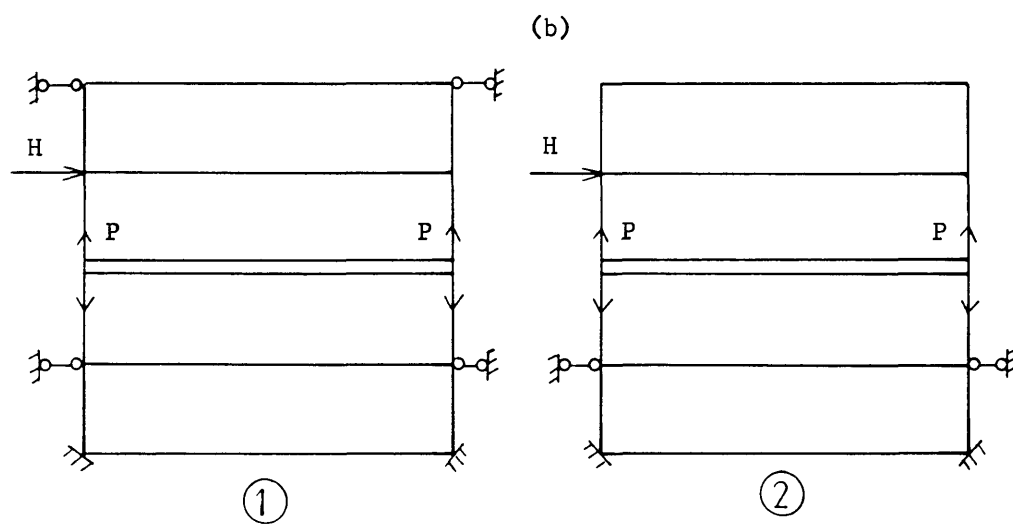
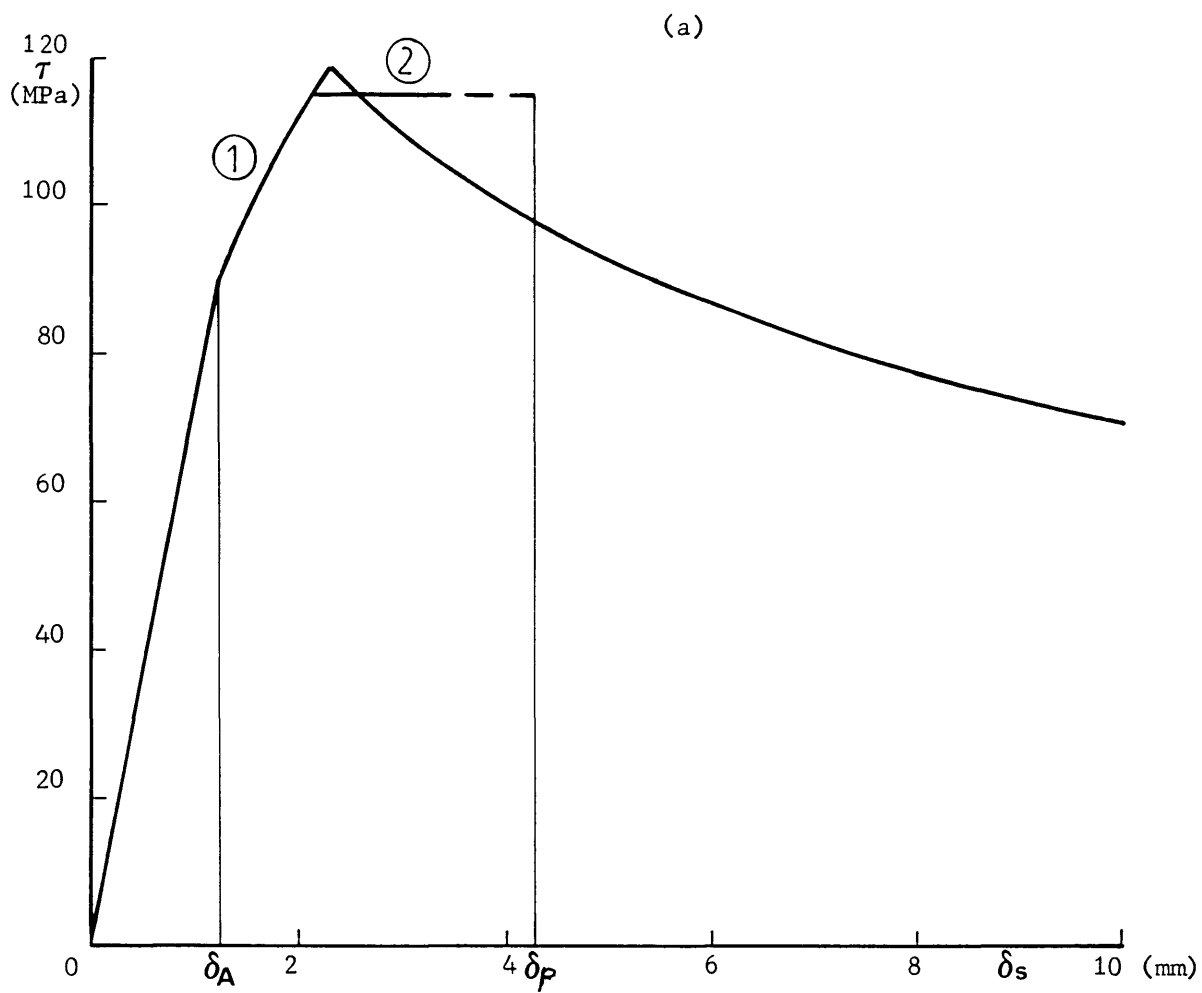


Figure 3.30 Simulation of hydraulic shearing

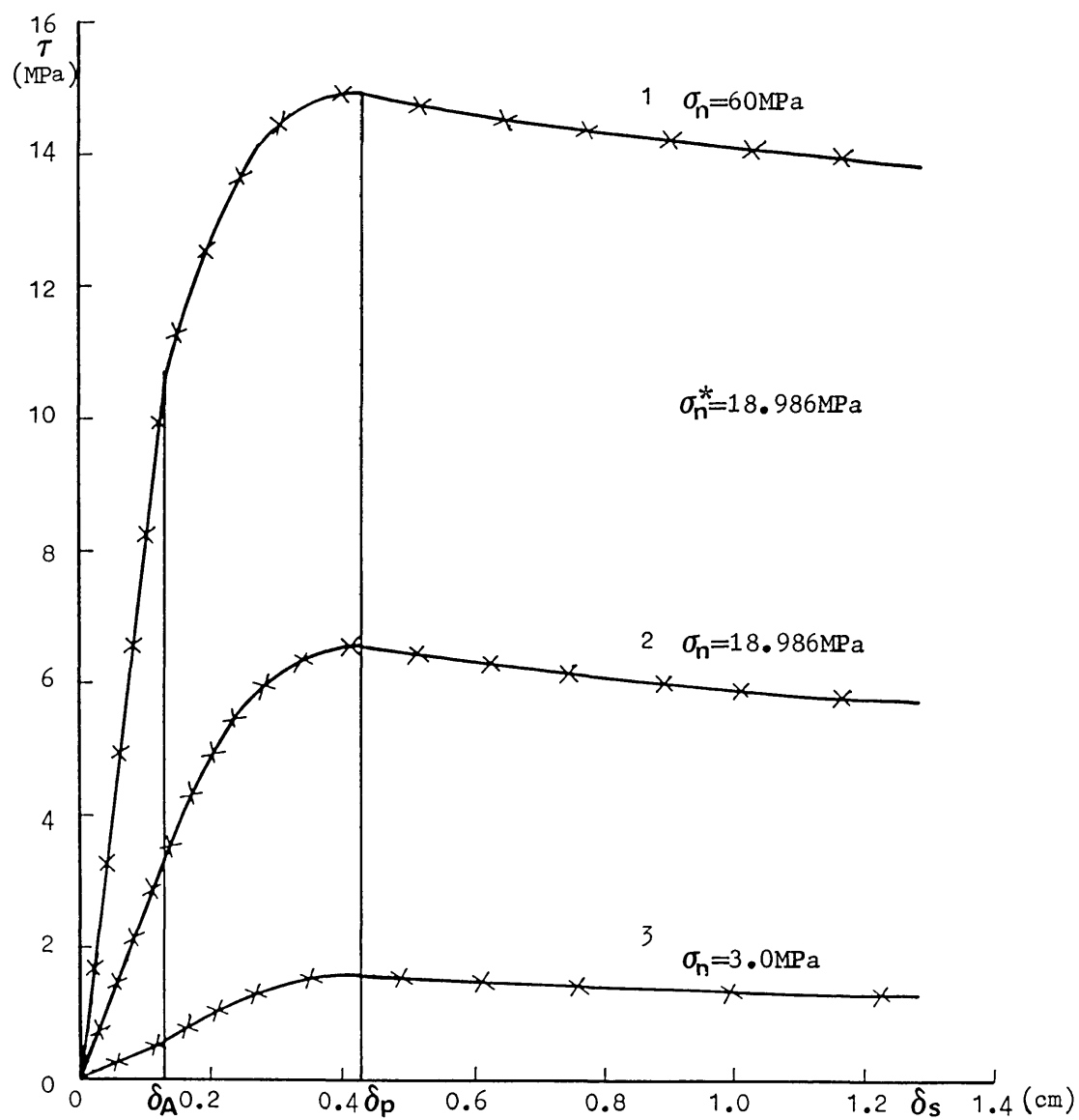


Figure 3.31 Effect of σ_h on $\tau - \delta_s$ curve

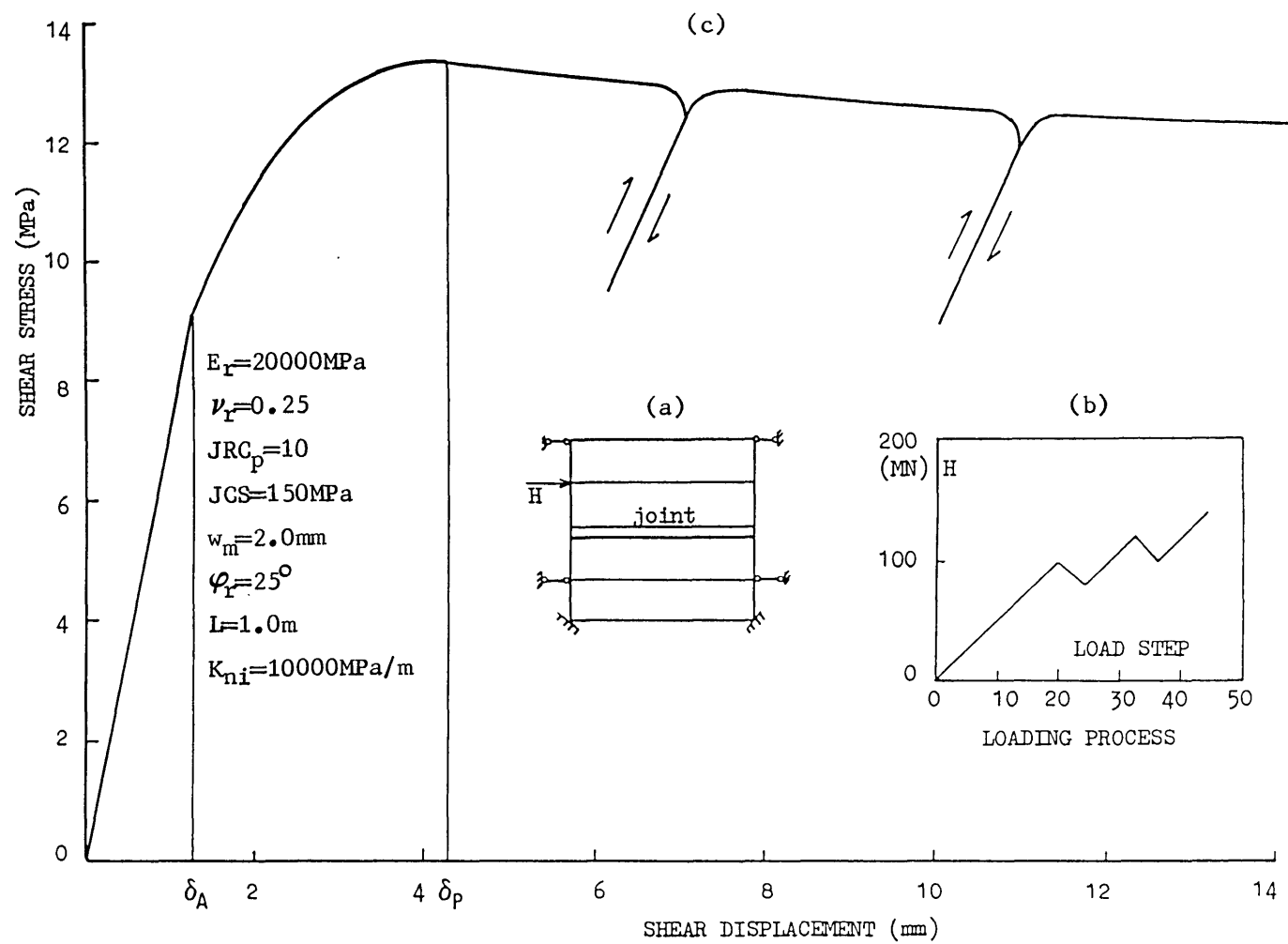


Figure 3.32 Numerical simulation of unloading and reloading

with the horizontal force being applied in the following way: it is first applied increasingly in a step-wise manner from zero to $100MN$, then it is decreased to $80MN$, then increased again to $120MN$, then decreased once more to $100MN$ and finally increased again, as shown in Fig.3.32 (b). Fig.3.32 (c) shows the numerical result, which is in good agreement with that obtained from direct shear tests (Yoshinaka and Yamabe 1986, Fig.3.33).

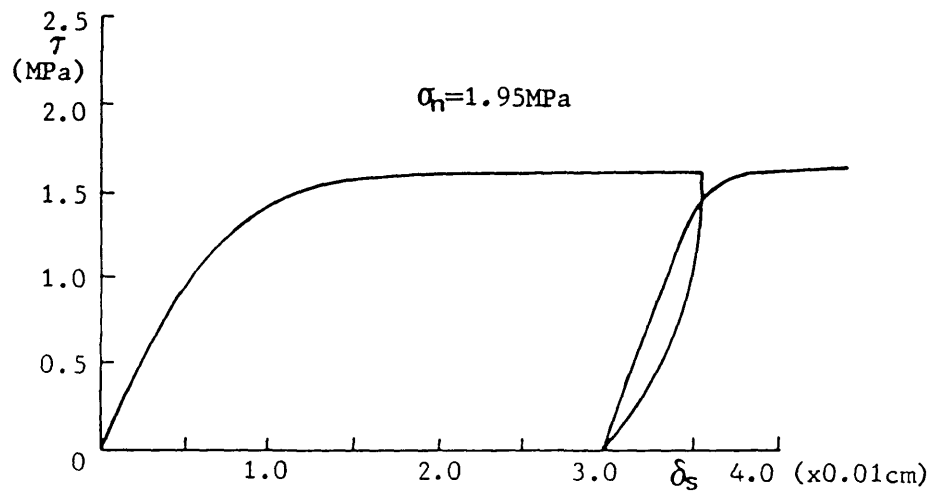


Figure 3.33 Experimental result of unloading and reloading test (after Yoshinaka, 1986)

3.4 Summary

A numerical joint model based on Barton's empirical model, in which the shear strength and deformation behaviour is expressed by some joint surface characteristics, has been developed. It uses the theory of elasto-plasticity with non-associated flow rule to reproduce the experimentally observed pre- and post- peak shear behaviour and associated dilation characteristics of rock joints. The pre- and post- peak shear behaviour is simulated by strain hardening and strain softening plastic models respectively, while dilation is quantitatively modelled by adopting a non-associated flow rule.

A secant stiffness iterative numerical procedure has been proposed that ensures numerical convergence in all non-linear cases such as opening, closing and shearing. Its advantage is clearly demonstrated in strain softening case for which the commonly used incremental procedure is invalid.

Types of joint elements with the relative displacements being independent unknowns including T shaped and cross joint elements have been developed. Adopting these joint elements avoids numerical ill-conditioning, makes computations associated with water pressure easier and makes it possible to model more realistic cases.

Test runs on single joint specimen indicate validity of this model in cases of opening and closing, pre- and post- shearing, loading and unloading. The mechanical response of rock joints under water pressure acting on joint walls has also been tested, although a further verification will be given in Chapter 5. On the whole, the results have shown that the model is capable of dealing with the cases intended.

CHAPTER 4

THE PROPOSED NUMERICAL METHOD

4.1 Introduction

In the analytical solution described in Chapter 2 no explicit discontinuities were introduced. All joints are conceptual ones, i.e. their existence was considered to have no influence on the stress distribution. In fact, however, the presence of discontinuities may render the initial stress field heterogeneous (Price 1966, Brady and Brown 1985). Following excavation, and especially under the water pressure, slip on discontinuities may completely change the stress distribution (Hudson 1987). The analytical solution given in Chapter 2 provides an approximate estimation of the critical water pressure required for opening and hydraulic jacking of a simple single joint. However, real situations are much more complex. Frequently, rock mass is cut by several sets of joints. The analytical solution is intractable in such cases. Generally speaking, the analytical solution may be used to identify general trends and to analyze some problems of less importance and those with relatively simple geological structures or may be used in the preliminary design. If the problem is important or the geological condition is complicated some other means must be used.

As mentioned in section 1.4, the task is to analyze the response of the jointed rock surrounding an unlined tunnel under internal water pressure in an initial ground stress field. In addition to the analytical solution, there are two general ways of studying the problem - in situ testing and numerical analysis. This project is concerned with development of a suitable numerical method.

Unlined hydraulic pressure tunnels are often constructed in hard, massive rocks containing a few discontinuities. Usually compressive and shear failure

will not readily occur in the intact rock due to relatively high ratio of compressive and shear strengths to the resulting stresses. In some cases, however, tensile failure may occur under the application of internal water pressure. In these cases, the rock is usually considered to be linearly elastic with a simple tensile cut-off strength criterion, i.e.

$$\sigma_3 - R_t \geq 0 \quad (4.1)$$

where R_t = tensile strength of the intact rock.

When a tensile failure occurs in a rock block fissures appear. If the fissures intersect the tunnel boundary water under pressure can penetrate into the fissures. As the water pressure increases the fissures may propagate and hydraulic fracturing may occur. The problem of initiation and propagation of a crack under water pressure is quite complicated. Solving this problem may requires the knowledge of energy theorem and fracture mechanics. For simplicity, a simple stress transfer technique is used to estimate the tensile failure zone rather than to simulate the process of crack initiation and propagation explicitly.

Rock joints are usually formed in sets. A set of joints is a group of joints with an approximately equal strike and dip directions and for which a mean spacing may be identified. In numerical analyses rock masses containing sets of joints may be treated as anisotropic materials; different mechanical constants are assigned in the directions parallel and perpendicular to the joint planes (Amadei and Goodman 1981, Gerrard 1982, Gao 1983 and Lu 1987). In this way the whole rock mass can be regarded as homogeneous continuum, so that computation can be simplified.

By definition, the rock mass surrounding an unlined pressure tunnel usually will not contain several sets of closely spaced joints. Generally, only rel-

atively few widely spaced joints will exist within the rock mass. Therefore the rock mass need not be treated as an anisotropic homogeneous continuum. Instead, the rock mass may be regarded as being comprised of homogeneous and continuous rock blocks connected by joints, which need special analytical treatment.

There are two types of joints in terms of filling. Some joints are clean and free of filling, while others are filled with soft materials. Those joints commonly encountered in rock masses surrounding unlined pressure tunnels are usually clean joints or, as noted by Goodman (1975), extension joints. If an unlined pressure tunnel is intersected by a joint which is filled with soft and erosion-susceptible materials, it is usually treated by locally shotcreting or lining the tunnel surface or/and grouting the joints before water is supplied (Hoek 1982, Jaeger 1979). In this project only the extension joints are taken into account.

Strictly speaking, the stability of the rock mass surrounding pressure tunnels is a three dimensional problem since the topographical and geological conditions may vary all the way along the tunnel axis. But for simplicity a complete plane strain model is used rather than a fully three dimensional one. This does not produce much difference when the geological condition does not vary severely and the section under consideration is not close to intersections and changes of cross section.

In general, two methods of numerical analysis are used for the analysis of underground excavations in rock - the finite element method (FEM) and the boundary element method (BEM). The FEM has advantages in dealing with non-linear problems, which will be encountered in analyzing joint behaviour. But it is not very good at simulating the boundary condition at infinity unless infinite finite elements are used or the computation domain is sufficiently large. On the contrary, the BEM is readily able to model an infinite body. In the

proposed numerical method the FEM is mainly used, but subroutines for linear and quadratic complete plane strain boundary elements are included so that the BEM may be used to deal with boundary conditions. In this case the computational scheme is as shown in Fig.4.1. The rock mass adjacent to the tunnel boundary is modelled by finite elements, which are surrounded by boundary elements. If the ground surface is included infinite boundary elements must be used to deal with the half space problem.

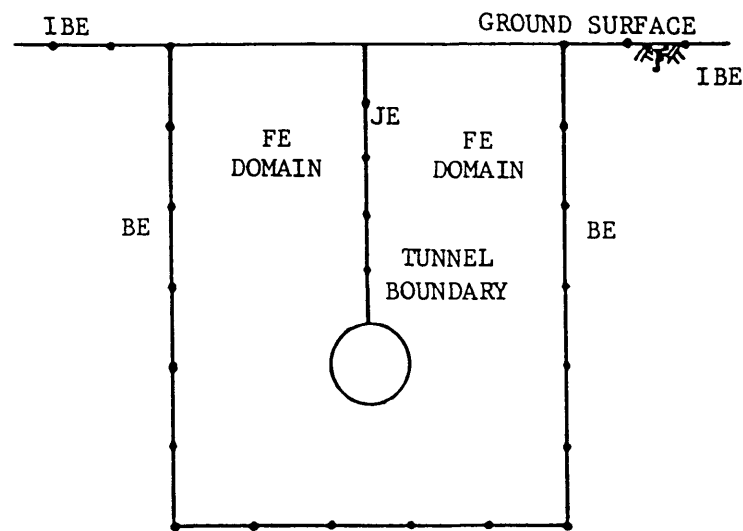


Figure 4.1 Computation scheme for numerical analysis

When the tunnel is filled with water under pressure seepage will occur along the pre-existing joints from the tunnel boundary towards a free surface, which is either the ground surface or the boundary of another opening. The water pressure acting on the joint walls must be taken into account because of its significance in initiating failure of unlined pressure tunnels. The water pressure at the tunnel boundary varies in accordance with the operating characteristics of the power station. In some cases the water pressure may vary from the peak value to zero three times a day; in other cases it may be kept

constant over months. For simplicity, no dynamic effects associated with the variation of water pressure are considered. In other words, the water flow is taken to be steady.

Rock blocks are permeable materials. However, compared with joints their permeabilities are considerably low (Kranz et al 1979, Jaeger 1979). In the cases under consideration, the major water flow will occur along joints, so it will be assumed that the intact rock is impermeable and that water flow can only occur along joints.

Ground water will be present in some cases. For concrete or reinforced concrete lined pressure tunnels the effect of ground water is to offset the effect of the internal water pressure producing tensile strength in the lining. For steel lined pressure tunnels, in addition to the preceding effect, ground water may result in buckling of the steel liner. For unlined pressure tunnels, however, the presence of ground water does not make much difference for failure of unlined pressure tunnels to take place in terms of hydraulic jacking and hydraulic shearing. For simplicity, ground water will not be taken into account in this analysis.

4.2 The Complete Plane Strain Model

The rock mass surrounding a pressure tunnel is a three dimensional body subjected to a three dimensional stress field. However, fully three dimensional analyses are complicated and expensive to carry out. Therefore, in order to simplify the computations the complete plane strain model developed by Brady and Bray (Brady and Bray 1978, Brady 1979) will be adopted. In the complete plane strain model, only two coordinates (x and z) are used to describe the geometry of the elements, but the displacement and stress components are three

dimensional. The difference between this and the fully three dimensional model is that, in the complete plane strain model, all displacement components are functions of x and z only, and independent of the coordinate in the direction of the tunnel axis Y . The coordinate system notation shown in Fig.4.2 is adopted throughout the thesis. As described in Section 2.3.1 this model has a great advantage over the plane strain model that it can deal with the cases in which the tunnel axis is not coincident with one of the in situ principal directions and some aspects of the three dimensional nature can be allowed for.

However, the complete plane strain model is not fully three dimensional. The limitation imposed by the adoption of the complete plane strain model in this project is that all joint planes must be parallel to the tunnel axis. Joints like those shown in Fig.4.3 (a) can be simulated without any difficulty while joints like those in Fig.4.3 (b) cannot be modelled.

In the complete plane strain model the nodal displacement vector is

$$\{\delta\} = [u_1 \quad v_1 \quad w_1 \quad u_2 \quad v_2 \quad w_2 \quad \dots \quad u_n \quad v_n \quad w_n]^T \quad (4.2)$$

where u_i , v_i , w_i are the displacements at the i th node in the X , Y , Z directions, respectively.

The strain vector for the rock blocks is

$$\{\varepsilon\} = [\varepsilon_x \quad \varepsilon_y \quad \varepsilon_z \quad \gamma_{yz} \quad \gamma_{zx} \quad \gamma_{xy}]^T \quad (4.3)$$

The stress vector for the rock blocks is

$$\{\sigma^r\} = [\sigma_x \quad \sigma_y \quad \sigma_z \quad \tau_{yz} \quad \tau_{zx} \quad \tau_{xy}]^T \quad (4.4)$$

The stress vector for the joints is

$$\{\sigma\} = [\tau_{zx} \quad \tau_{yz} \quad \sigma'_n]^T \quad (4.5)$$

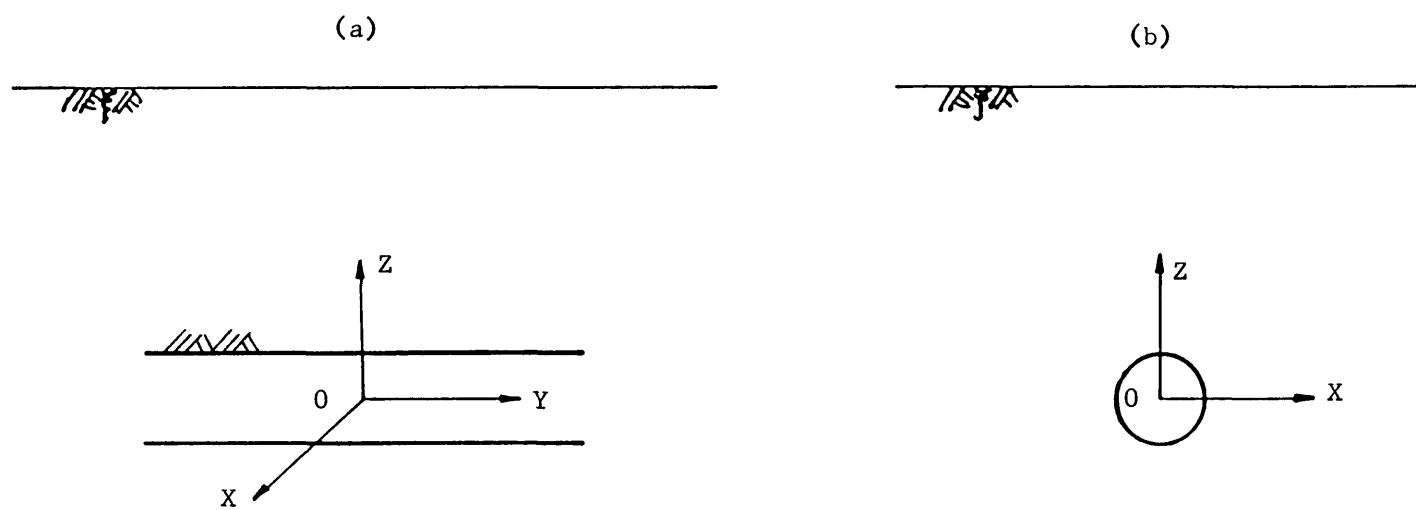


Figure 4.2 Coordinate system notation

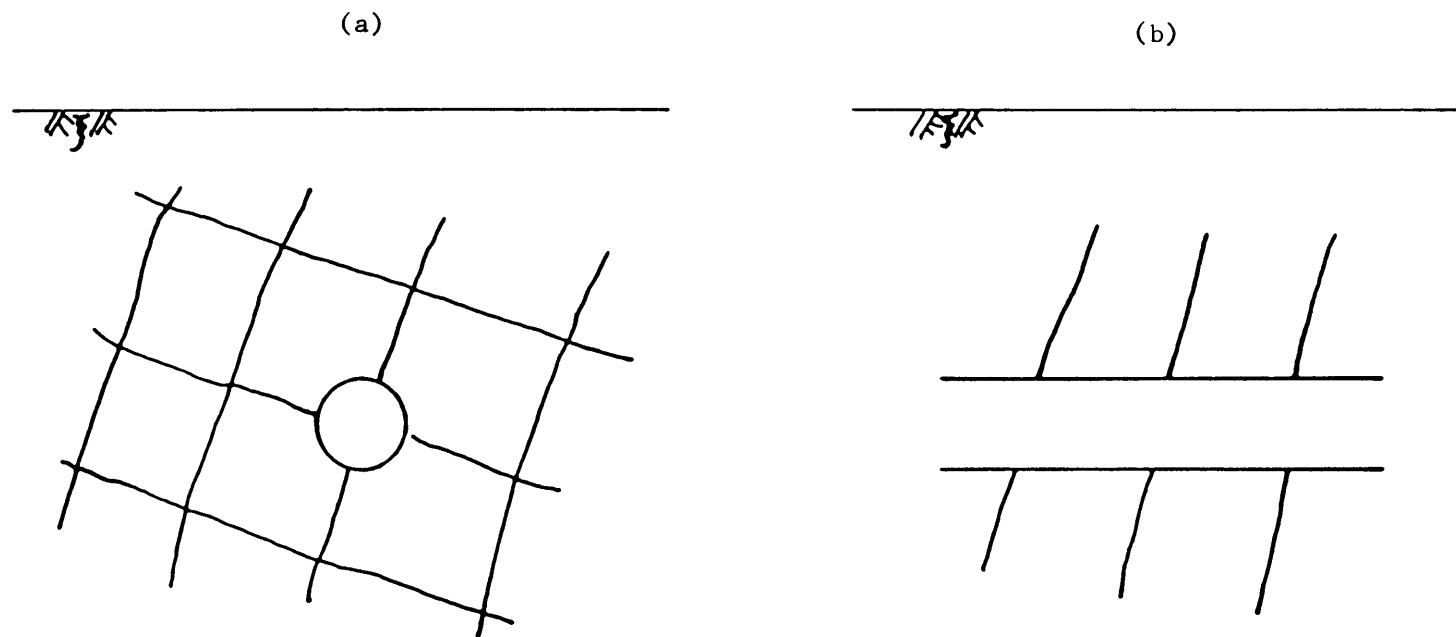


Figure 4.3 Limitation of complete plane strain model

The strain components of the rock blocks in the complete plane strain model are calculated as follows,

$$\varepsilon_x = \frac{\partial u}{\partial x} \quad (4.6 \ a)$$

$$\varepsilon_y = 0 \quad (4.6 \ b)$$

$$\varepsilon_z = \frac{\partial w}{\partial z} \quad (4.6 \ c)$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} \quad (4.6 \ d)$$

$$\gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \quad (4.6 \ e)$$

$$\gamma_{xy} = \frac{\partial v}{\partial x} \quad (4.6 \ f)$$

In the FEM

$$\{\varepsilon\} = [B]\{\delta\} \quad (4.7)$$

$$[B] = [\mathbf{B}_1 \ \mathbf{B}_2 \ \dots \ \mathbf{B}_n]^T \quad (4.8)$$

For the complete plane strain case

$$\mathbf{B}_i = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial z} \\ 0 & \frac{\partial N_i}{\partial z} & 0 \\ \frac{\partial N_i}{\partial z} & 0 & \frac{\partial N_i}{\partial x} \\ 0 & \frac{\partial N_i}{\partial x} & 0 \end{bmatrix} \quad (4.9)$$

where N_i is the shape function of the i th node.

The stress-strain matrix used to calculate the stress vector from the strain

vector is the same as that used in the fully three dimensional analysis, i.e.

$$[D^r] = \begin{bmatrix} K + 4G/3 & K - 2G/3 & K - 2G/3 & 0 & 0 & 0 \\ K - 2G/3 & K + 4G/3 & K - 2G/3 & 0 & 0 & 0 \\ K - 2G/3 & K - 2G/3 & K + 4G/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{bmatrix} \quad (4.10)$$

where

$$K = \frac{E}{3(1 - 2\nu)} \quad (4.11)$$

$$G = \frac{E}{2(1 + \nu)} \quad (4.12)$$

E = Young's modulus,

ν = Poisson's ratio,

K = volumetric modulus,

G = shear modulus.

Eqns (4.9) and (4.10) are used to calculate the elemental stiffness matrix as well.

4.3 The Combined FE and BE Method

4.3.1 Coupling of FE and BE

The basic equation of the FEM is

$$[K]^F \{u\} = \{F\} \quad (4.13)$$

where $[K]^F$ = global stiffness matrix,

$\{u\}$ = unknown nodal displacement vector,

$\{F\}$ = applied nodal load vector.

$[K]^F$ can be calculated following the standard procedure (Zienkiewicz 1977, Zhu 1979).

The basic equation relating nodal tractions and displacements of the direct formulation of the BEM is (Beer 1983, Brady 1979, Brady and Wassyng 1981, Brebbia 1980),

$$[T]\{u\} = [U]\{t\} \quad (4.14)$$

where $[T]$ = traction matrix caused by unit line load acting on each boundary node,

$\{u\}$ = unknown nodal displacement vector,

$[U]$ = nodal displacement matrix caused by unit line load acting on each boundary node,

$\{t\}$ = nodal traction vector.

$\{t\}$ can be expressed in terms of the nodal load vector,

$$\{t\} = [b]^{-1}\{F\} \quad (4.15)$$

Substituting Eqn.(4.15) into (4.14) gives

$$[T]\{u\} = [U][b]^{-1}\{F\}$$

or

$$[K]^B\{u\} = \{F\} \quad (4.16)$$

where

$$[K]^B = [b][U]^{-1}[T] \quad (4.17)$$

The calculation of $[U]$ and $[T]$ will be described later in this chapter.

Combining Eqns (4.13) and (4.16) gives

$$[K]\{u\} = \{F\} \quad (4.18)$$

where

$$[K] = [K]^F + [K]^B \quad (4.19)$$

In this way the finite elements and the boundary elements are coupled. However, to ensure adequate coupling the compatibility condition must be satisfied. This is fulfilled by utilizing identical shape functions in both the FE and the BE domains along the interface. The equilibrium condition is satisfied in an average sense when the nodal forces are equal and opposite at a common node (Beer 1983, Brady and Wassying 1981). Solving Eqn.(4.18) gives the nodal displacements. Then following the FEM procedure, the stress components in the FE domain can be obtained.

For a linearly elastic material, in the complete plane strain analysis the whole problem can be divided into two parts: one in the plane, the other in the anti-plane, thus:

$$[K] = [K]_{xz} + [K]_y \quad (4.20)$$

4.3.2 Finite element formulation

To suit the needs of a variety of geometries 4-node linear and 8-node quadratic isoparametric finite elements have been included in the program. For the ordinary 4-node finite elements, the elemental nodal displacement vector, the elemental nodal force vector and the elemental stiffness matrix are

$$\{\delta\} = [\delta_1 \quad \delta_2 \quad \delta_3 \quad \delta_4]^T \quad (4.21)$$

$$\{F\} = [F_1 \quad F_2 \quad F_3 \quad F_4]^T \quad (4.22)$$

$$[K^e] = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{32} & K_{33} & K_{34} \\ K_{41} & K_{42} & K_{43} & K_{44} \end{bmatrix} \quad (4.23)$$

where

$$\delta_i = [\delta_{ix} \quad \delta_{iy} \quad \delta_{iz}] \quad (4.24)$$

$$F_i = [F_{ix} \quad F_{iy} \quad F_{iz}] \quad (4.25)$$

K_{ij} is a 3x3 submatrix and can be determined by the standard FEM procedure.

Because the joint elements in this model take the relative displacements to be the essential and independent unknowns the finite elements 'above' a joint element must be treated specially. For the 4-node finite elements connecting one joint element (Fig.4.4),

$$\{\delta_{1*}\} = \{\delta_1\} + \{\Delta_1\} \quad (4.26)$$

$$\{\delta_{2*}\} = \{\delta_2\} + \{\Delta_2\} \quad (4.27)$$

where

$$\{\delta_{i*}\} = [\delta_{i*x} \quad \delta_{i*y} \quad \delta_{i*z}]^T \quad (4.28)$$

$$\{\Delta_i\} = [\Delta_{ix} \quad \Delta_{iy} \quad \Delta_{iz}]^T \quad (4.29)$$

The elemental nodal displacement vector and the elemental force vector are

$$\{\delta\} = [\delta_1 \quad \Delta_1 \quad \delta_2 \quad \Delta_2 \quad \delta_3 \quad \delta_4]^T \quad (4.30)$$

$$\{F\} = [F_1 \quad \Delta F_1 \quad F_2 \quad \Delta F_2 \quad F_3 \quad F_4]^T \quad (4.31)$$

So the elemental stiffness matrix is

$$[K^e] = \begin{bmatrix} K_{11} & K_{11} & K_{12} & K_{12} & K_{13} & K_{14} \\ K_{11} & K_{11} & K_{12} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{21} & K_{22} & K_{22} & K_{23} & K_{24} \\ K_{21} & K_{21} & K_{22} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{31} & K_{32} & K_{32} & K_{33} & K_{34} \\ K_{41} & K_{41} & K_{42} & K_{42} & K_{43} & K_{44} \end{bmatrix} \quad (4.32)$$

If a 4-node finite element connects two joint elements (at a T shaped or a cross joint as in Fig.4.5) the corresponding elemental nodal displacement vector, the elemental nodal force vector and the elemental stiffness matrix are

$$\{\delta\} = [\delta_1 \quad \Delta_1 \quad \delta_2 \quad \Delta_2 \quad \delta_3 \quad \delta_4 \quad \Delta_4]^T \quad (4.33)$$

$$\{F\} = [F_1 \quad \Delta F_1 \quad F_2 \quad \Delta F_2 \quad F_3 \quad F_4 \quad \Delta F_4]^T \quad (4.34)$$

$$[K^e] = \begin{bmatrix} K_{11} & K_{11} & K_{12} & K_{12} & K_{13} & K_{14} & K_{14} \\ K_{11} & K_{11} & K_{12} & K_{12} & K_{13} & K_{14} & K_{14} \\ K_{21} & K_{21} & K_{22} & K_{22} & K_{23} & K_{24} & K_{24} \\ K_{21} & K_{21} & K_{22} & K_{22} & K_{23} & K_{24} & K_{24} \\ K_{31} & K_{31} & K_{32} & K_{32} & K_{33} & K_{34} & K_{34} \\ K_{41} & K_{41} & K_{42} & K_{42} & K_{43} & K_{44} & K_{44} \\ K_{41} & K_{41} & K_{42} & K_{42} & K_{43} & K_{44} & K_{44} \end{bmatrix} \quad (4.35)$$

For the 8-node finite elements connecting two joint elements the corresponding expressions are (Fig.4.6)

$$\{\delta\} = [\delta_1 \quad \Delta_1 \quad \delta_2 \quad \Delta_2 \quad \delta_3 \quad \delta_4 \quad \delta_5 \quad \Delta_5 \quad \delta_6 \quad \delta_7 \quad \delta_8]^T \quad (4.36)$$

$$\{F\} = [F_1 \quad \Delta F_1 \quad F_2 \quad \Delta F_2 \quad F_3 \quad F_4 \quad F_5 \quad \Delta F_5 \quad F_6 \quad F_7 \quad F_8]^T \quad (4.37)$$

$$[K^e] = \begin{bmatrix} K_{11} & K_{11} & K_{12} & K_{12} & K_{13} & K_{14} & K_{15} & K_{15} & K_{16} & K_{17} & K_{18} \\ K_{11} & K_{11} & K_{12} & K_{12} & K_{13} & K_{14} & K_{15} & K_{15} & K_{16} & K_{17} & K_{18} \\ K_{21} & K_{21} & K_{22} & K_{22} & K_{23} & K_{24} & K_{25} & K_{25} & K_{26} & K_{27} & K_{28} \\ K_{21} & K_{21} & K_{22} & K_{22} & K_{23} & K_{24} & K_{25} & K_{25} & K_{26} & K_{27} & K_{28} \\ K_{31} & K_{31} & K_{32} & K_{32} & K_{33} & K_{34} & K_{35} & K_{35} & K_{36} & K_{37} & K_{38} \\ K_{41} & K_{41} & K_{42} & K_{42} & K_{43} & K_{44} & K_{45} & K_{45} & K_{46} & K_{47} & K_{48} \\ K_{51} & K_{51} & K_{52} & K_{52} & K_{53} & K_{54} & K_{55} & K_{55} & K_{56} & K_{57} & K_{58} \\ K_{51} & K_{51} & K_{52} & K_{52} & K_{53} & K_{54} & K_{55} & K_{55} & K_{56} & K_{57} & K_{58} \\ K_{61} & K_{61} & K_{62} & K_{62} & K_{63} & K_{64} & K_{65} & K_{65} & K_{66} & K_{67} & K_{68} \\ K_{71} & K_{71} & K_{72} & K_{72} & K_{73} & K_{74} & K_{75} & K_{75} & K_{76} & K_{77} & K_{78} \\ K_{81} & K_{81} & K_{82} & K_{82} & K_{83} & K_{84} & K_{85} & K_{85} & K_{86} & K_{87} & K_{88} \end{bmatrix} \quad (4.38)$$

For the 8-node finite elements connecting four joint elements the corresponding items are (Fig.4.7)

$$\{\delta\} = [\delta_1 \quad \Delta_1 \quad \delta_2 \quad \Delta_2 \quad \delta_3 \quad \delta_4 \quad \Delta_4 \quad \delta_5 \quad \Delta_5 \quad \delta_6 \quad \delta_7 \quad \delta_8 \quad \Delta_8]^T \quad (4.39)$$

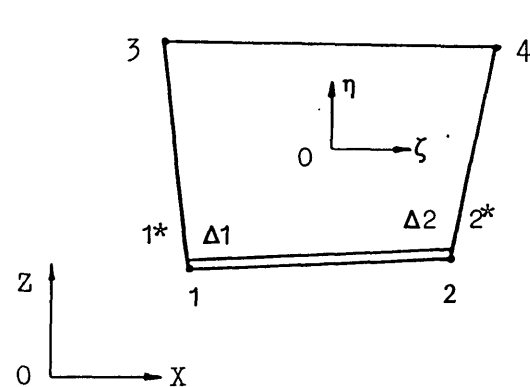


Figure 4.4 Four-node finite element connecting one joint element

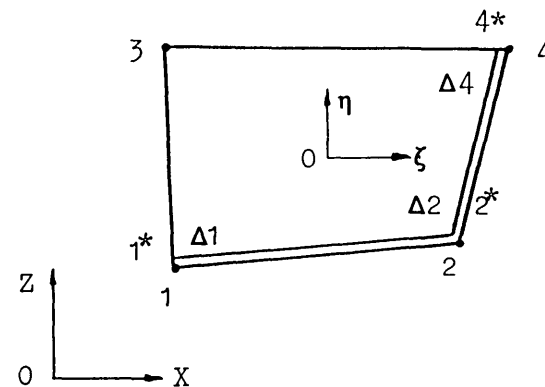


Figure 4.5 Four-node finite element connecting two joint elements

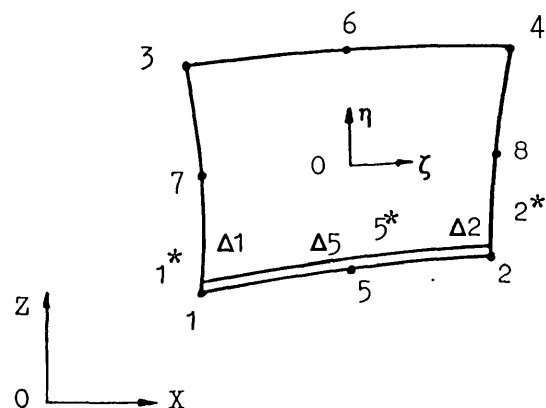


Figure 4.6 Eight-node finite element connecting two joint elements

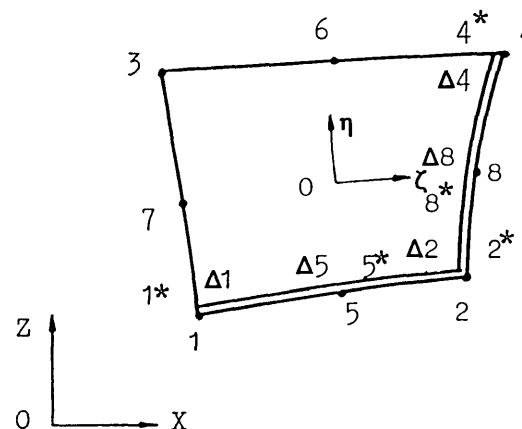


Figure 4.7 Eight-node finite element connecting four joint elements

$$[F] = [F_1 \quad \Delta F_1 \quad F_2 \quad \Delta F_2 \quad F_3 \quad F_4 \quad \Delta F_4 \quad F_5 \quad \Delta F_5 \quad F_6 \quad F_7 \quad F_8 \quad \Delta F_8]^T \quad (4.40)$$

$$[K^e] = \begin{bmatrix} K_{11} & K_{11} & K_{12} & K_{12} & K_{13} & K_{14} & K_{14} & K_{15} & K_{15} & K_{16} & K_{17} & K_{18} & K_{18} \\ K_{11} & K_{11} & K_{12} & K_{12} & K_{13} & K_{14} & K_{14} & K_{15} & K_{15} & K_{16} & K_{17} & K_{18} & K_{18} \\ K_{21} & K_{21} & K_{22} & K_{22} & K_{23} & K_{24} & K_{24} & K_{25} & K_{25} & K_{26} & K_{27} & K_{28} & K_{28} \\ K_{21} & K_{21} & K_{22} & K_{22} & K_{23} & K_{24} & K_{24} & K_{25} & K_{25} & K_{26} & K_{27} & K_{28} & K_{28} \\ K_{31} & K_{31} & K_{32} & K_{32} & K_{33} & K_{34} & K_{34} & K_{35} & K_{35} & K_{36} & K_{37} & K_{38} & K_{38} \\ K_{41} & K_{41} & K_{42} & K_{42} & K_{43} & K_{44} & K_{44} & K_{45} & K_{45} & K_{46} & K_{47} & K_{48} & K_{48} \\ K_{41} & K_{41} & K_{42} & K_{42} & K_{43} & K_{44} & K_{44} & K_{45} & K_{45} & K_{46} & K_{47} & K_{48} & K_{48} \\ K_{51} & K_{51} & K_{52} & K_{52} & K_{53} & K_{54} & K_{54} & K_{55} & K_{55} & K_{56} & K_{57} & K_{58} & K_{58} \\ K_{51} & K_{51} & K_{52} & K_{52} & K_{53} & K_{54} & K_{54} & K_{55} & K_{55} & K_{56} & K_{57} & K_{58} & K_{58} \\ K_{61} & K_{61} & K_{62} & K_{62} & K_{63} & K_{64} & K_{64} & K_{65} & K_{65} & K_{66} & K_{67} & K_{68} & K_{68} \\ K_{71} & K_{71} & K_{72} & K_{72} & K_{73} & K_{74} & K_{74} & K_{75} & K_{75} & K_{76} & K_{77} & K_{78} & K_{78} \\ K_{81} & K_{81} & K_{82} & K_{82} & K_{83} & K_{84} & K_{84} & K_{85} & K_{85} & K_{86} & K_{87} & K_{88} & K_{88} \\ K_{81} & K_{81} & K_{82} & K_{82} & K_{83} & K_{84} & K_{84} & K_{85} & K_{85} & K_{86} & K_{87} & K_{88} & K_{88} \end{bmatrix} \quad (4.41)$$

4.3.3 Boundary element formulation

Three types of boundary elements are included in the program: 2-node linear boundary elements to connect the 4-node finite elements, 3-node boundary elements to connect the 8-node finite elements, and infinite boundary elements to deal with the ground surface.

Because the boundary elements are introduced simply to handle the boundary conditions in this project, all that needs to be done with the boundary elements is to compute the elemental stiffness matrix and to couple them into the global stiffness matrix. The coupling procedure has been described in section 4.3.1 and the elemental stiffness matrix is calculated from Eqn.(4.17), in which $[b]$, $[U]$ and $[T]$ are estimated in the following way.

The basic equation for the traction caused by unit line load in the $X - Z$

plane is (Brebbia 1980, Watson 1979)

$$T_{ij} = -\frac{1}{4\pi(1-\nu)r} \left\{ \frac{\partial r}{\partial n} [(1-2\nu)\delta_{ij} + 2\frac{\partial r}{\partial x_i} \frac{\partial r}{\partial x_j}] - (1-2\nu) \left(\frac{\partial r}{\partial x_j} n_i - \frac{\partial r}{\partial x_i} n_j \right) \right\} \quad (4.42)$$

where x_i , x_j can be x and z coordinates,

x_i = the direction of traction,

x_j = the direction in which the unit line load is applied,

r = distance from the node at which the unit line load is applied to the node under consideration,

n = external normal at the point under consideration,

δ_{ij} = Kronecker delta,

n_i = direction cosine of n to the X_i axis,

n_j = direction cosine of n to the X_j axis.

When the node at which the unit line load is applied is coincident with none of the nodes of the element under consideration, the coefficients of $[T]$ for this element are evaluated by integrating Eqn.(4.42) using Gaussian quadrature. When the unit line load is applied at one of the nodes of the element the node becomes a singular point. So the corresponding leading diagonal coefficients cannot be determined by the foregoing integration method. Fortunately, these coefficients can be evaluated by rigid body movement considerations after the other coefficients in the row have been calculated.

$$T(N_{ix}, N_{ix}) = 1 - \sum_{j=1}^n T(N_{ix}, N_{jx}) \quad (j \neq i) \quad (4.43 a)$$

$$T(N_{iz}, N_{iz}) = 1 - \sum_{j=1}^n T(N_{iz}, N_{jz}) \quad (j \neq i) \quad (4.43 \text{ b})$$

$$T(N_{ix}, N_{iz}) = - \sum_{j=1}^n T(N_{ix}, N_{jz}) \quad (j \neq i) \quad (4.43 \text{ c})$$

$$T(N_{iz}, N_{ix}) = - \sum_{j=1}^n T(N_{iz}, N_{jx}) \quad (j \neq i) \quad (4.43 \text{ d})$$

The shear stresses in the Y direction caused by unit line load are

$$\tau_{xy} = -\frac{1}{2\pi} \frac{\Delta x}{r^2} \quad (4.44 \text{ a})$$

$$\tau_{yz} = -\frac{1}{2\pi} \frac{\Delta z}{r^2} \quad (4.44 \text{ b})$$

where $\Delta x, \Delta z$ = coordinate differences in the X and Z directions between the node at which the unit line load is applied and the node under consideration.

The coefficient T_{yy} is obtained by integrating the shear stresses

$$T_{yy} = \int (\tau_{yz} dx - \tau_{xy} dz) \quad (4.45)$$

When unit line load is applied at one of nodes of the element the leading diagonal coefficient is evaluated in the same way as in the $X - Z$ plane, i.e. by rigid body movement considerations,

$$T(N_{iy}, N_{iy}) = 1 - \sum_{j=1}^n T(N_{iy}, N_{jy}) \quad (j \neq i) \quad (4.46)$$

The coefficients of the matrix $[U]$ in the $X - Z$ plane are obtained by integrating the fundamental solutions u_{xx} , u_{xz} , u_{zx} and u_{zz} over the element under consideration using Gaussian quadrature. The fundamental solutions are

$$u_{xx} = \frac{1}{8\pi G(1-\nu)} \left[\frac{\Delta x^2}{r^2} - (3-4\nu) \ln r \right] \quad (4.47 \text{ a})$$

$$u_{zz} = \frac{1}{8\pi G(1-\nu)} \left[\frac{\Delta z^2}{r^2} - (3-4\nu) \ln r \right] \quad (4.47 \text{ b})$$

$$u_{xz} = u_{zx} = \frac{1}{8\pi G(1-\nu)} \frac{\Delta x \Delta z}{r^2} \quad (4.47 \text{ c})$$

The corresponding fundamental solution in the Y direction is

$$u_{yy} = -\frac{1}{2\pi G} \ln r \quad (4.48)$$

When unit line load is applied at one of the nodes of the element, due to singularity, a special Gaussian quadrature in the form $\int_0^1 f(\eta) \ln \eta d\eta$ is used to perform the integration (Anderson 1965).

The transformation matrix $[b]$ for the 3-node boundary elements is derived as follows:

$$\{F\} = [\mathbf{F}_1 \quad \mathbf{F}_2 \quad \mathbf{F}_3]^T \quad (4.49 \text{ a})$$

$$\mathbf{F}_i = [F_{ix} \quad F_{iy} \quad F_{iz}] \quad (4.49 \text{ b})$$

$$\{t\} = [\mathbf{t}_1 \quad \mathbf{t}_2 \quad \mathbf{t}_3]^T \quad (4.49 \text{ c})$$

$$\mathbf{t}_i = [t_{ix} \quad t_{iy} \quad t_{iz}] \quad (4.49 \text{ d})$$

From Eqn.(4.15)

$$\{F\} = [b]\{t\}$$

where

$$[b] = \begin{bmatrix} \mathbf{b}_{11} & \mathbf{b}_{12} & \mathbf{b}_{13} \\ \mathbf{b}_{21} & \mathbf{b}_{22} & \mathbf{b}_{23} \\ \mathbf{b}_{31} & \mathbf{b}_{32} & \mathbf{b}_{33} \end{bmatrix} \quad (4.50)$$

$$\mathbf{b}_{ij} = \int \begin{bmatrix} N_i N_j & 0 & 0 \\ 0 & N_i N_j & 0 \\ 0 & 0 & N_i N_j \end{bmatrix} ds \quad (4.51)$$

For the 2-node boundary elements

$$[b] = \begin{bmatrix} \mathbf{b}_{11} & \mathbf{b}_{12} \\ \mathbf{b}_{21} & \mathbf{b}_{22} \end{bmatrix} \quad (4.52)$$

where b_{ij} is still calculated from Eqn.(4.51).

The boundary element method provides an easy way to deal with both the entire infinite body and the finite body. However, for problems concerned with a semi-infinite body, e.g. a half space, which are encountered when a tunnel is located near the ground surface, some special treatment must be used. There are two possible approaches. The first is to use Melan's solution (Melan 1932) or Mindlin's solution (Mindlin 1936) instead of Kelvin's solution for the displacement u resulting from an unit line load. This introduces the need to derive the traction t from u ; this is a rather difficult task. The other approach is to form a special boundary element at the surface of the semi-infinite body with special shape functions and displacement models, so that the Kelvin's fundamental solutions can still be used. This method has been chosen because of its simplicity.

The idea is that as $\eta \rightarrow 1$ the coordinates approach infinity and the displacements tend to zero (see Fig.4.10). The shape functions are

$$N_1 = -\frac{\eta}{1-\eta} \quad (4.53 \ a)$$

$$N_2 = \frac{1}{1-\eta} \quad (4.53 \ b)$$

Note that

$$N_2|_{\eta=0} = 1$$

$$N_1|_{\eta=0} = 0$$

$$N_1 + N_2 = 1$$

The displacement and traction models are

$$u(\eta) = u_2 \left(\frac{r_0}{\tilde{r}} \right)^\alpha \quad (4.54 \ a)$$

$$t(\eta) = t_2 \left(\frac{r_0}{\tilde{r}} \right)^2 \quad (4.54 \ b)$$

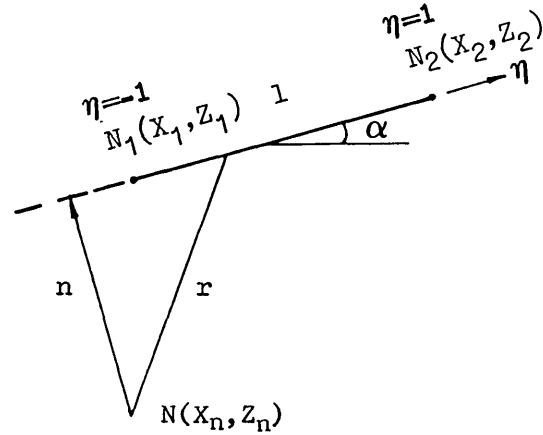


Figure 4.8 Two-node linear boundary element

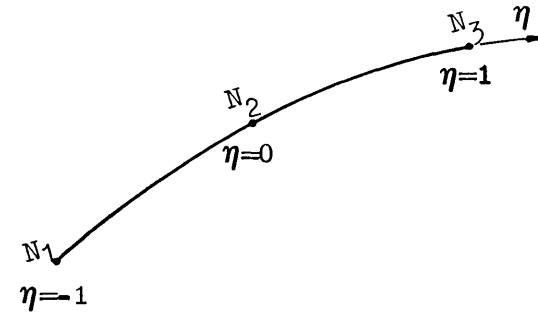


Figure 4.9 Three-node quadratic boundary element

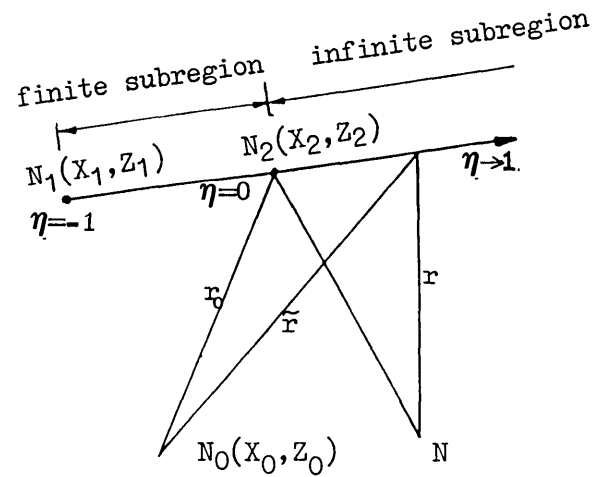


Figure 4.10 2-node infinite boundary element

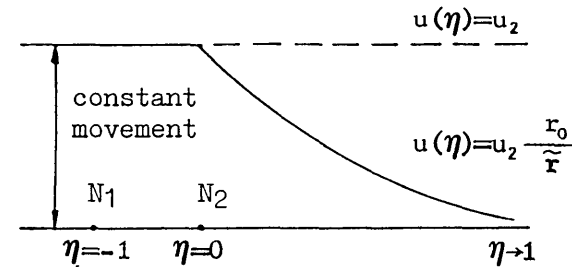


Figure 4.11 Displacement models for 2-node infinite boundary element

where u_2, t_2 = displacement and traction at node 2,

r_0 = distance from node 2 to the reference point,

$\tilde{r}$ = distance from the point under consideration to the reference point,

a = coefficient greater than unity.

Integrating the fundamental solution over the whole element with consideration of Eqns (4.53) and (4.54) gives the coefficients of $[T]$ and $[U]$. There is no difficulty for the finite subregion, but care must be taken for the infinite subregion, so that convergence of the integration is ensured. The leading diagonal coefficients of $[T]$ are obtained by rigid body movement considerations after the other elements of the same row have been evaluated. Instead of using unity in the ordinary boundary elements (see Eqn.4.46) 0.5 has been adopted in the infinite boundary elements i.e.

$$T(N_{ix}, N_{ix}) = 0.5 - \sum_{j=1}^n T(N_{ix}, N_{jx}) \quad (j \neq i) \quad (4.55 \text{ a})$$

$$T(N_{iz}, N_{iz}) = 0.5 - \sum_{j=1}^n T(N_{iz}, N_{jz}) \quad (j \neq i) \quad (4.55 \text{ b})$$

$$T(N_{iy}, N_{iy}) = 0.5 - \sum_{j=1}^n T(N_{iy}, N_{jy}) \quad (j \neq i) \quad (4.55 \text{ c})$$

It is worth noticing that the rigid body movement technique used to evaluate the leading diagonal elements of $[T]$ is based on the concept that when a constant movement is given at any point in a body no traction is produced. If with $a = 1$ in Eqn.(4.54 a), a constant movement is given at each node, the movement at line of the infinite elements from $\eta = 0$ to $\eta = 1$ is not constant as shown in Fig.4.11. The displacement varies from the constant value to zero as η approaches unity. As a result, Eqn.(4.55) is no longer valid, i.e., the coefficient 0.5 does not work in this case. Putting $a = 0$ in Eqn.(4.54 a), i.e. $u(\eta) = u_2$,

can overcome this difficulty, since in this case constant body movement will occur in the whole half space. However, this in turn results in two problems. The first is that because $u(\eta) = u_2$ the integrand in the integral for calculating $[T]$ is proportional to $\frac{1}{r}$, i.e.

$$-\frac{1}{4\pi(1-\nu)}\left\{\frac{\partial r}{\partial n}[(1-2\nu)\delta_{ij} + 2\frac{\partial r}{\partial x_i}\frac{\partial r}{\partial x_j}] - (1-2\nu)\left(\frac{\partial r}{\partial x_j}n_i - \frac{\partial r}{\partial x_i}n_j\right)\right\}\frac{1}{r} \propto \frac{1}{r} \quad (4.56)$$

This makes the integral non-existent because an integral like

$$\int_a^\infty \frac{1}{x^p} dx$$

does not exist when $p \leq 1$. As long as $p > 1$, however, the integral does exist. So it is necessary to choose a value of a slightly greater than 0, say $a = 10E-10$, in order to ensure that the integral exists. As a result, the numerical integration can be performed.

Secondly, assuming $u(\eta) = u_2$ will produce errors for some cases since in fact $u(\eta)$ should decay to zero when $\eta \rightarrow 1$.

4.4 Calculation of the Water Pressure Acting on Joint Walls

4.4.1 Models for seepage in rock masses

There are three mathematical models to deal with the problems of seepage in rock masses. The first is the homogeneous isotropic porous seepage model which is suited to massive rock masses with closed joints, i.e. the seepage takes place through the entire rock mass. The second is the homogeneous anisotropic permeable medium model which is used for a stratified rock mass with water flow along the direction of layers through the whole rock mass. The third is the discontinuous medium model, in which water flow occurs only along particular discontinuities. For this project is concerned, the discontinuous

medium model is the most appropriate since, according to our assumptions, the rock is impermeable and water flow can take place only along joints.

4.4.2 FE formulations

The basic equation for joint flow is (Zienkiewicz 1977, Zhu 1979)

$$\frac{\partial}{\partial s}(K_p \frac{\partial \Phi}{\partial s}) - Q = 0 \quad (4.57)$$

where K_p = permeability coefficient along the joint,

Φ = seepage potential function

$$\Phi = \frac{p_w}{\gamma_w} + z \quad (4.58)$$

p_w = water pressure acting on the joint walls,

γ_w = unit weight of water,

Q = inward or outward flow rate,

s = direction of passage of water flow.

There are two types of boundary conditions.

(1) The water pressure, hence the seepage potential is given, i.e.

$$\Phi = \Phi_B \quad (4.59)$$

or

(2) The flow rate is given, i.e.

$$K_p \frac{\partial \Phi}{\partial s} = -q_B \quad (4.60)$$

For our case only the first type of boundary condition is relevant. The water pressure anywhere along the tunnel boundary is known, p_w , and the water pressure anywhere along the ground surface is atmospheric pressure, i.e. zero.

The FE discretization of Eqn.(4.57) can be written as

$$[H]\{\Phi\} = \{F_p\} \quad (4.61)$$

where $[H]$ = global seepage 'stiffness matrix',

$\{\Phi\}$ = seepage potential vector, i.e. the vector of the piezometric head at each node,

$\{F_p\}$ = seepage vector in terms of the flow rate at each node.

The elements of $[H]$ are calculated from each joint flow element as

$$H_{ij} = \sum h_{ij}^e \quad (4.62)$$

where

$$h_{ij}^e = \int_s K_p \frac{\partial N_i}{\partial s} \frac{\partial N_j}{\partial s} ds$$

For a two node joint flow element (see Fig.3.19),

$$[H^e] = \frac{K_p}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad (4.63)$$

where l is the length of the joint element.

The water pressure at each node along the joint network is obtained by solving Eqn.(4.61). Special care must be taken when solving the system of simultaneous equations since at some nodes the water pressure is known with the flow rate unknown, while at other nodes the flow rate is known with the water pressure unknown. This means that all unknowns are distributed on both sides of the equal sign.

It is worth mentioning that taking the relative displacements of joint walls as the essential and independent unknowns (see section 3.3.6) makes the FE procedure for determining the water pressure acting on the joint walls much easier and saves computer time since only the relative displacements are related to the water pressure.

4.4.3 Estimation of permeability coefficient

The permeability coefficient is evaluated from the joint aperture based on Barton's modified cubic flow law (Barton et al 1985). According to Barton et al the initial mechanical aperture E_0 of a joint depends upon its surface characteristics and is determined from the following equation:

$$E_0 = \frac{JRC_p}{5} (0.2 \frac{\sigma_c}{JCS} - 0.1) (mm) \quad (4.64)$$

where σ_c = unconfined compressive strength of the rock adjacent to the joint wall.

If the joint walls are assumed to be unweathered then

$$\sigma_c = JCS \quad (4.65)$$

Substituting Eqn.(4.65) into (4.64), E_0 becomes

$$E_0 = \frac{JRC_p}{50} (mm)$$

The real aperture E is

$$E = E_0 - \Delta w_j \quad (4.66)$$

where Δw_j is joint closure, which is obtained from the FE computation.

Barton's modified cubic flow law is adopted to determine the permeability aperture e

$$e = \frac{JRC_p^{2.5}}{(E/e)^{2.0}} (\mu m) \quad (4.67)$$

so

$$e = \frac{E^2}{JRC_p^{2.5}} (\mu m) \quad (4.68)$$

From e the specific permeability coefficient k_p is calculated from the following equation

$$e = (12k_p)^{0.5}$$

$$k_p = \frac{e^2}{12} \quad (4.69)$$

From k_p the permeability coefficient K_p can be obtained

$$K_p = \frac{k_p g}{\nu_m} \quad (4.70)$$

where ν_m is the kinematic viscosity of water.

4.4.4 Evaluation of forces caused by water pressure

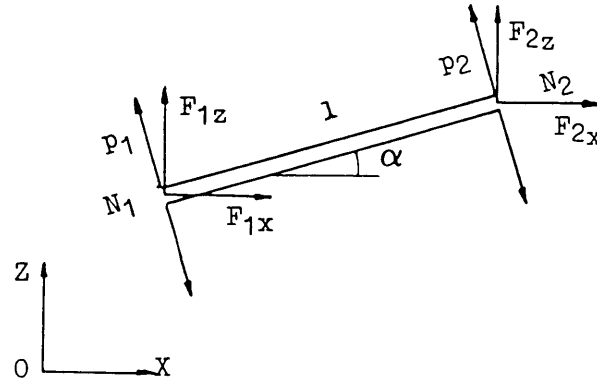


Figure 4.12 Water pressure forces acting on joint walls

After the water pressure at each node has been calculated the water forces are readily obtained as follows (Fig.4.12):

$$F_1 = \frac{p_1 l}{3} + \frac{p_2 l}{6} \quad (4.71 a)$$

$$F_2 = \frac{p_1 l}{6} + \frac{p_2 l}{3} \quad (4.71 \text{ b})$$

$$F_{1x} = -F_1 \sin \alpha \quad (4.71 \text{ c})$$

$$F_{1z} = F_1 \cos \alpha \quad (4.71 \text{ d})$$

$$F_{2x} = -F_2 \sin \alpha \quad (4.71 \text{ e})$$

$$F_{2z} = F_2 \cos \alpha \quad (4.71 \text{ f})$$

For the ordinary joint elements, F_{1x} , F_{1z} , F_{2x} and F_{2z} are directly applied at the corresponding joint node.

For the J3 of the T shaped joint element (see Fig.3.20),

$$F_{N+1} = F_1$$

$$F_{N+2} = -F_1 \quad (4.72)$$

$$F_2 = F_2$$

For the J3 of the cross joint element (see Fig.3.21),

$$F_1 = F_1$$

$$F_{N+1} = -F_2 \quad (4.73)$$

$$F_{N+2} = F_2$$

For the J4 of the cross joint element,

$$F_{N+2} = F_1$$

$$F_{N+3} = -F_1 \quad (4.74)$$

$$F_2 = F_2$$

4.5 Computation Procedure

The proposed numerical method analyzes the stability of rock masses surrounding unlined pressure tunnels in the following steps:

- (1) Estimate the initial stress field.
- (2) Compute the second stress field induced by excavation.
- (3) Apply the water pressure increasingly until failure occurs.

4.5.1 Initial stress field

The problem of the initial stress is rather complicated. The usual way to estimate the initial stress is the in situ measurement, by which (e.g. the over-coring procedure) the three dimensional stress components at the measurement locations are obtained. Due to the large expenditure in the in situ measurement, it is practically impossible to measure the in situ stresses at a large amount of locations. However, for the finite element analysis what is needed is an entire initial stress field within the computation domain. Some authors developed approaches to estimate the initial stress field from the known in situ stress values at a limited number of locations by means of the finite element method. Some programs are available for this job. If sufficient reliable results of the in situ stress measurements are obtained, this approach may be recommended to be used in the determination of the initial stress field for the finite element analysis. In case that there is no measured data available, especially when the tunnel is situated near the ground surface, an initial stress field can be assumed according to some general trends (Hoek and Brown 1980, Brady and Brown 1985). For the homogeneous, isotropic, continuous and elastic rock mass this assumed initial stress field can be directly used for the following computations. If the rock mass is heterogeneous, anisotropic, discontinuous

and non-linear materials, however, it is difficult to assume a compatible initial stress field. Therefore, after assuming an initial stress field it is necessary to adjust the stresses at each element to make the stress field acceptable.

Hoek and Brown (1980) presented results of an investigation of data obtained from in situ stress measurements. They pointed out that the vertical in situ stress P_z can be fairly predicted by the depth z of the point and the unit weight of the rock, as $P_z = \gamma z$. For the horizontal in situ stresses at a shallow depth, however, there is a large amount of scatter. Even though, it is clear that for hard rocks the horizontal and vertical stress ratio is usually greater than unity and it is not zero at the ground surface.

In this program, the initial stress field for tunnels situated near the ground surface is calculated by a series of linear computations followed by a stress adjustment process. The computation scheme is shown in Fig.4.13. A horizontal boundary load is applied at either both or single lateral sides of the computation domain with the selfweight force being applied vertically.

The selfweight and the lateral load are calculated as follows:

$$\begin{aligned} P_x &= P_{x0} + K_1(h)\gamma h \cos^2 \alpha + K_2(h)\gamma h \sin^2 \alpha \\ P_y &= P_{y0} + K_1(h)\gamma h \sin^2 \alpha + K_2(h)\gamma h \cos^2 \alpha \\ P_z &= \gamma h \end{aligned} \tag{4.75}$$

where K_1, K_2 = lateral pressure coefficients of the initial stress field, which vary with the depth h , i.e. $K_1 = K_1(h), K_2 = K_2(h)$,

γ = specific weight of the rock,

α = angle between the direction of the major horizontal principal stress and the X -axis,

P_{x0}, P_{y0} = horizontal stresses at the ground surface.

$K_1, K_2, P_{x0}, P_{y0}, \gamma$ and α are all input data.

In the examples shown in Chapter 5, due to lack of measured data, for simplicity it is assumed that $P_{x0} = P_{y0} = 0$ and K_1, K_2 are constant, i.e. independent of depth. So Eqn.(4.75) becomes

$$\begin{aligned} P_x &= K_1 \gamma h \cos^2 \alpha + K_2 \gamma h \sin^2 \alpha \\ P_y &= K_1 \gamma h \sin^2 \alpha + K_2 \gamma h \cos^2 \alpha \\ P_z &= \gamma h \end{aligned} \quad (4.76)$$

Because in calculating the stiffness matrices of joint elements the normal stress σ_n on each joint element must be known in advance we have to assume a value of σ_n for each joint element. The initially assumed σ_n is calculated from the following assumed stress field considering the assumptions that $P_{x0} = P_{y0} = 0$ and K_1, K_2 are independent of depth h ,

$$\sigma_x = K_1 \gamma h \cos^2 \alpha + K_2 \gamma h \sin^2 \alpha \quad (4.77 \text{ a})$$

$$\sigma_y = K_1 \gamma h \sin^2 \alpha + K_2 \gamma h \cos^2 \alpha \quad (4.77 \text{ b})$$

$$\sigma_z = \gamma h \quad (4.77 \text{ c})$$

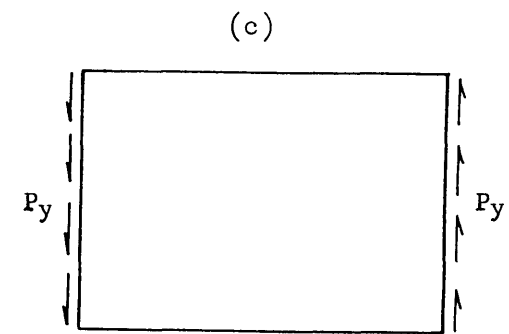
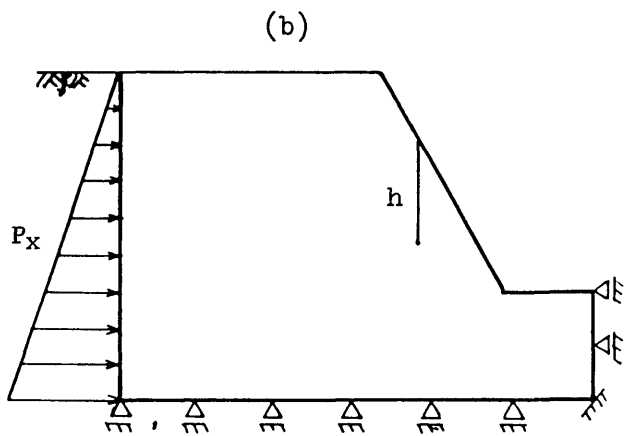
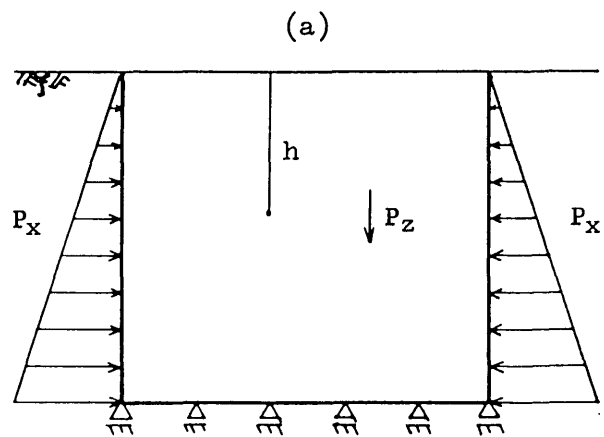
$$\tau_{yz} = 0 \quad (4.77 \text{ d})$$

$$\tau_{zx} = 0 \quad (4.77 \text{ e})$$

$$\tau_{xy} = (K_1 - K_2) \gamma h \sin \alpha \cos \alpha \quad (4.77 \text{ f})$$

From these six stress components, using the coordinate transformation formulation the normal stress of each joint element can be obtained. Due to the coincidence of the Y and the Y' axes, however, σ_n is only a function of σ_x, σ_z and α in our case:

$$\sigma_n = \sigma_x \sin^2 \alpha + \sigma_z \cos^2 \alpha \quad (4.78)$$



4.13 General computation scheme

So far an initial stress field and an initial displacement field have been determined. Usually they are not compatible; that is the stress and displacement components do not match the prescribed stress displacement relationship. Therefore, a stress adjustment must be carried out. This is done by using the stress transfer technique (Zienkiewicz 1977, Wittke 1968), in which an iteration process is inevitable. Thus, a compatible three dimensional initial stress field is obtained.

4.5.2 Excavation simulation

The excavation process can be simulated by removing the material inside the tunnel and applying the equivalent nodal forces on the tunnel boundary or by gradually reducing the stiffness of the rock inside the tunnel. In the former method, after the initial stress field has been obtained, recalculate the global stiffness matrix excluding the elements inside the tunnel boundary. Then apply the nodal forces equivalent to the stresses at the tunnel boundary in a step-wise manner (say in 10 steps). In each step the global stiffness matrix must be recalculated due to the non-linearity. In the latter method, the stiffness of the rock inside the tunnel boundary is diminished step by step until it reaches zero. At each step the global stiffness matrix is recalculated and the boundary load identical to that for calculating the initial stress field is applied. As a result, a new stress and displacement fields are obtained. This method takes into account to some extent the effect of the progressive advance of the excavation face. It is more reasonable than the first method and can produce a better result for the elastic case. For non-linear material problems, however, difficulties arise in determining the stiffnesses for each step due to their non-linear variation with the stress components. Therefore, the first method is adopted to simulate the excavation process in this project.

4.5.3 Application of water pressure

After the second stress field has been obtained the water pressure is applied increasingly in a step-wise manner on both the tunnel boundary and joint walls until convergence cannot be reached. In each water pressure step a new global stiffness matrix is calculated according to the stress and displacement components obtained in the previous step. The water pressure acting on the joint walls must also be recalculated from the joint aperture at the beginning of each water pressure increment and the current tunnel water pressure.

During the iteration for each water pressure increment the convergence occurs if the maximum relative displacement at any of the nodes is less than a prescribed allowable value $|\varepsilon|$,

$$\varepsilon = \text{Max} \left(\frac{\Delta\delta_i}{\delta_i} \right) < |\varepsilon| \quad (4.79)$$

where $\Delta\delta_i$ = displacement increment at node i for this iteration,

δ_i = total displacement at node i for this iteration.

For the computations in this project $|\varepsilon|$ takes a value of 0.0001.

At a certain water pressure the computation is sometimes obviously divergent, i.e. ε becomes larger and larger. This water pressure can be taken to be the critical water pressure and the computation can be terminated. Sometimes, however, the computation has a convergent tendency but it converges rather slowly. This means that failure almost occurs at this water pressure. Continuing the computation takes a long computer time and is meaningless. So in the program it has been specified that if the relation shown in Eqn.(4.79) cannot be satisfied after 100 iterations, no matter whether the tendency is convergent or divergent, the computation is ended and the corresponding water pressure is taken to be the critical water pressure.

4.6 Program

A program entitled DPTJR in FORTRAN 77 with some 4500 statements has been written to implement computation based on the proposed method. Fig.4.14 shows the general flow chart of the program DPTJR. The program consists of a main program and 46 subroutines. The main program controls the computing procedure by calling each subroutine to implement a particular function. The subroutines and their functions are:

ININF: input information and data,

FMNOE: resolve the input elemental information,

IOBD: deal with boundary conditions,

FNP1: form directive arrays for the asymmetric equation solver of the global equilibrium equation,

FNP2: same as FNP1,

FNNP1: form directive arrays for the equation solver for determining the water pressure acting on joint walls,

FNNP2: same as FNNP1,

ANGJ: calculate angles between the X -axis and the direction of each joint element,

INTST: calculate the initial stress field,

STSTB: calculate the nodal stresses at the tunnel boundary for 4-node finite elements,

STSTB1: calculate the nodal stresses at the tunnel boundary for 8-node finite elements,

EQFORW: calculate the equivalent nodal forces of selfweight of 4-node finite elements,

EQFORW1: as above for 8-node finite elements,

FFLS: calculate the lateral load for computing the initial stress field,

EILD: calculate the excavation equivalent forces acting on the tunnel boundary for 4-node finite elements,

EILD1: calculate the excavation equivalent forces acting on the tunnel boundary for 8-node finite elements,

IWLD: calculate the water pressure acting on the tunnel boundary for 4-node finite elements,

IWLD1: calculate the water pressure acting on the tunnel boundary for 8-node finite elements,

JWP: calculate the water pressure acting on joint walls,

REM: calculate the elastic matrix of the intact rock for evaluating its stiffness matrix,

GSTIF: calculate the global stiffness matrix,

SMFE4: stiffness matrix of 4-node finite elements connecting no joint elements,

SMFE41: stiffness matrix of 4-node finite elements connecting one joint element,

SMFE42: stiffness matrix of 4-node finite elements connecting two joint elements,

SMFE8: stiffness matrix of 8-node finite elements connecting no joint elements,

SMFE81: stiffness matrix of 8-node finite elements connecting two joint elements,

SMFE82: stiffness matrix of 8-node finite elements connecting four joint elements,

SMJE: stiffness matrix of ordinary joint elements,

SMJE3: stiffness matrix of the T shaped and the cross joint elements,

SMBE2: stiffness matrix of 2-node boundary elements including the infinite boundary elements,

SMBE3: stiffness matrix of 3-node boundary elements,

SEQ: equation solver for calculating displacements,

SEQ1: equation solver for calculating the water pressure acting on joint walls,

AESTGS: transform the elements of the global stiffness matrix from 2-D to 1-D array,

AESTGS1: transform the elements of the seepage stiffness matrix from 2-D to 1-D array,

FDN4: calculate the partial differentiations of the shape functions to the global coordinates for 4-node finite elements,

FDN8: calculate the partial differentiations of the shape functions to the global coordinates for the 8-node finite elements,

FISF: stress and displacement adjustment for determining the initial stress field,

STRESS: calculate the stress components of joint elements,

STRESS1: calculate the stress components of intact rock elements,

FR: calculate the proportional ratio R at which the critical state between the elastic and plastic is reached by iterations,

FVF: calculate the value of the yield function for joint elements,

TSSM: calculate the plastic stress strain matrices of joints,

JRCM: calculate the mobilized joint roughness coefficients,

OOBF: calculate the out-of balance forces for the ordinary joint elements,

OOBF1: calculate the out-of balance forces for the T shaped and the cross joint elements.

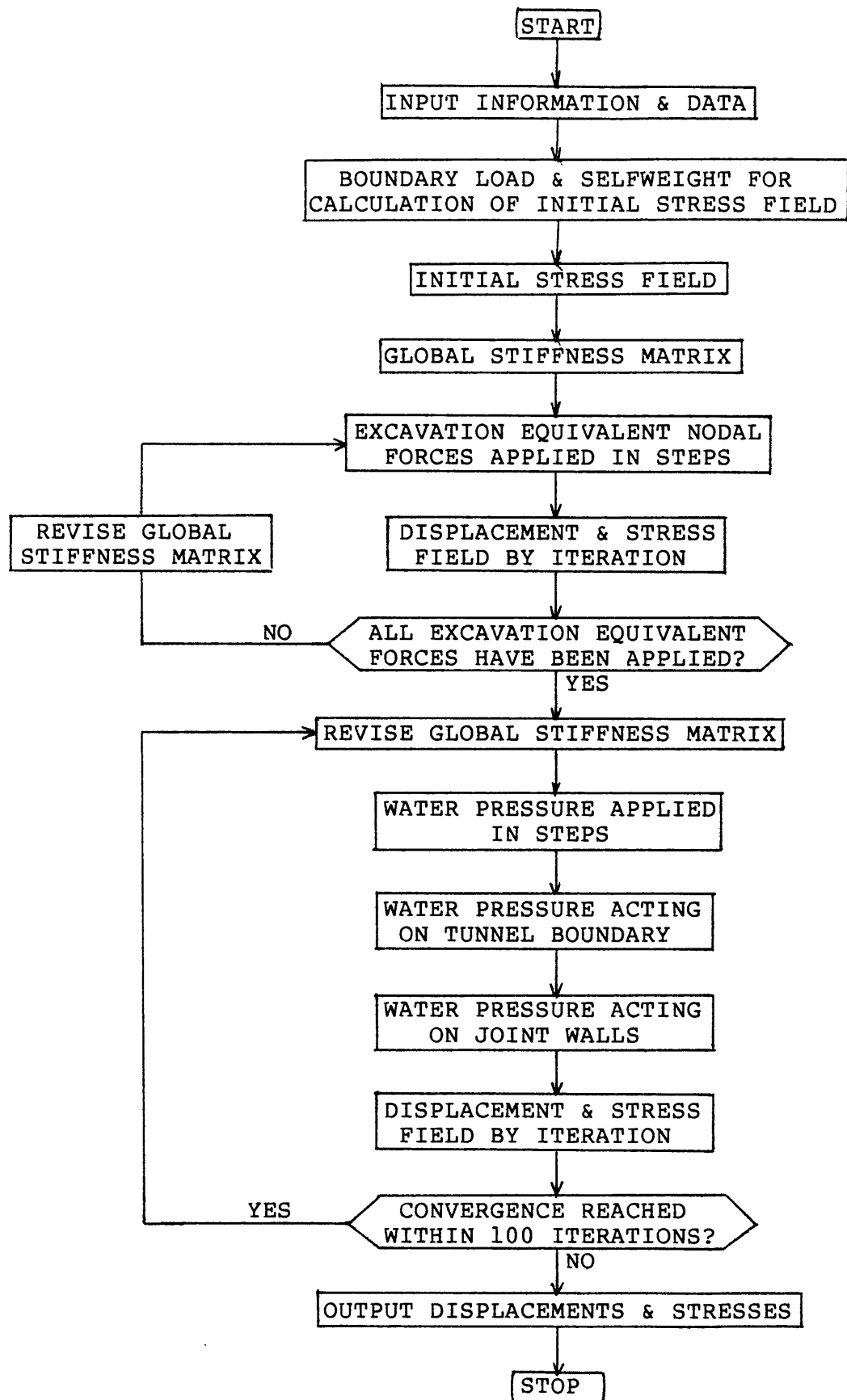


Fig.4.14 General flow chart of program DPTJR

CHAPTER 5

ANALYSIS OF GENERIC CASES OF UNLINED PRESSURE TUNNELS

5.1 Introduction

In Section 1.2.2 the failure mechanisms of unlined pressure tunnels were analyzed and some generic failure cases were summarized (Fig.1.11). Both the currently used traditional design methods and the analytical solutions described in Chapter 2 are unsuitable for dealing with these cases. The numerical method proposed in Chapter 4 including the joint model introduced in Chapter 3, however, was designed to be able to calculate the stress distribution in the rock mass surrounding a pressure tunnel and to evaluate the potential for failure. Thus it can be decided whether it is possible for a given tunnel in particular geological and topographical conditions and under a certain operating water pressure can remain unlined.

In this chapter some results of analyses of generic cases of unlined pressure tunnel failures are presented. On the one hand, the reliability of this numerical method can be further verified (In Chapter 3 only the joint model was tested). On the other hand, some representative failure modes are identified including hydraulic jacking, hydraulic shearing on discontinuities and uplift of the ground surface. This enables us to understand better the coupled mechanical and hydraulic behaviour of discontinuous rock masses.

Finally, results of computations for a hypothetical real unlined pressure tunnel in relatively complicated geological conditions are presented to demonstrate the capability of the proposed method in handling the practical problems.

As mentioned in Chapter 4 the program DPTJR includes both finite el-

elements and boundary elements and can perform coupled FE and BE analyses. This program has been tested with some simple examples and comparison between the numerical results and closed form solutions has been made demonstrating this coupled capability (Appendix 8). However, because of the asymmetry and the full distribution (non-sparseness) of the stiffness matrix of the boundary elements, the coupled FE and BE analysis requires much more computer central memory and more computer time. In addition, according to the results shown in Example 2 of Appendix 8, for the non-uniform problems the finite element domain must be larger than that for the uniform problems. A finite element region of $4D$ is required for a deeply situated uniform tunnel of diameter D (Varadarajan et al 1985). This means that an even bigger central memory is needed for non-uniform problems. Therefore, although the coupled finite element and boundary element method has advantages as mentioned in Section 4.3, some further work has to be done to overcome the shortcomings and to exploit fully the advantages of the coupled method. So, at this stage, the finite element method is still recommended for analyzing real or hypothetical real problems. In the analyses of the generic cases of unlined pressure tunnel failure presented here only finite elements are used.

In order to simulate properly the boundary conditions in finite element analyses, in principle, the larger the computational domain the better, so that the influence of the boundaries can be largely diminished. But an excessively large computational domain is not feasible in practice nor is it necessary. The optimum domain size depends upon the characteristics of the rock mass, the type of problem, the geological conditions etc. Except for the hypothetical real case the numerical analyses presented in this chapter are all qualitative ones. In these cases the analyses are expected to show the general trend in the development of displacements and stresses and an indication of the failure modes instead of an exact value of the critical water pressure for a certain case.

Thus, for these analyses it is not necessary to use a very large computational domain. A minimum horizontal dimension of $7D$ (where D is the diameter of the tunnel) and a vertical distance under the tunnel centre of $3D$ are adopted. For the hypothetical real case a much larger computational domain is chosen so that a realistic result can be obtained.

Two types of boundary conditions are commonly adopted in numerical analyses of rock mechanics: the stress controlled boundary condition and the displacement controlled boundary condition. After the initial stress field is determined by means of that introduced in section 4.5.1, the stress redistributions induced by excavation and water pressure can be calculated with these two boundary conditions. If computational domain is sufficiently large they should produce almost the same results. However, with a limited computational domain one boundary condition may be better than the other based on features of the problem. In order to decide which boundary condition will produce more reasonable results in the problems to be studied, the example $H = 3.5D$ of generic case 1, which will be discussed in detail in the following section, is calculated with these two boundary conditions. For the stress controlled boundary condition, failure occurs at water pressure step 9 and for the displacement controlled boundary it does not occur until water pressure step 40. The correct result should be in between. However, the result obtained from the stress controlled boundary condition is definitely better in this case. Nevertheless, it produces a safe design. Therefore, it has been decided that the stress controlled boundary condition will be adopted in the following analyses.

5.2 Generic Case 1 - The Tunnel and the Ground Surface Connected by a Single Vertical Joint

5.2.1 General description

In Section 1.2.2 it was noted that in the case of a pressure tunnel situated near the ground surface and intersected by a joint, which extends to the ground surface and was originally closed under the initial compressive stress field, as the pressure increases the water under high pressure may penetrate the discontinuity causing it to open and result in hydraulic jacking, hydraulic shearing and water leakage. This is an important and typical failure mode of unlined pressure tunnels. Although from the geological point of view this is a very simple case, but it is very representative as well. By investigating in detail the mechanical and hydraulic responses of the rock mass for this case some fundamental and general features of the failure phenomena of unlined pressure tunnels can be identified. This is very helpful when analyzing commonly encountered real cases, which may have more complicated geological and topographical conditions, but in which general development of failure is more or less similar. On the other hand, the relative simplicity of the problem makes a parametric analysis practically feasible considering the costs of computer time. In this case a parametric analysis considers the influence of the magnitude and the directions of the initial horizontal principal stresses (α, K_1, K_2) and the thickness of the superincumbent rock H , since these factors play a very important role in the failure of unlined pressure tunnels (Chapter 1 and 2). Obviously, the mechanical and hydraulic properties of the rock mass, especially those of the discontinuities, also play an important role in the failure of the tunnel. But this thesis cannot provide a complete systematic parametric study. Therefore, for simplicity variations in rock properties are not taken into account and only one set of material parameters is assumed for the all following problems.

The material parameters for both the intact rock and the discontinuities adopted are:

Young's modulus of the intact rock $E_r = 20000MPa$

Poisson's ratio of the intact rock $\nu = 0.25$

unit weight of the intact rock $\gamma = 0.0265KN/m^3$

maximum closure of the joints $w_m = 0.002m$

peak joint roughness coefficient $JRC_p = 10$

joint wall compressive strength $JCS = 150MPa$

residual frictional angle of the joints $\phi_r = 30^\circ$

initial normal stiffness coefficient $K_{ni} = 500MPa/m$

joint length $L = 1.0m$

These parameters represent relatively hard rocks and ordinary joints. The joint length of $1.0m$ may infer that the joint is intersected by other discontinuities at an approximate spacing of $1.0m$. It seems that these parameters are quite representative of the rock masses surrounding unlined pressure tunnels.

In this case the computational domain is of dimensions $7D$ in the horizontal direction and $3.5D$ from the centre of the tunnel to the bottom of the mesh.

As mentioned in Section 4.5.1 the initial stress field is estimated by applying the triangularly distributed lateral loads to the computational domain with the bottom constrained vertically. The excavation equivalent forces acting on the tunnel boundary are exerted in 10 steps and the water pressures acting on both the tunnel boundary and the joint walls are applied in steps of 2 meter hydraulic head, i.e. $0.0196MPa$.

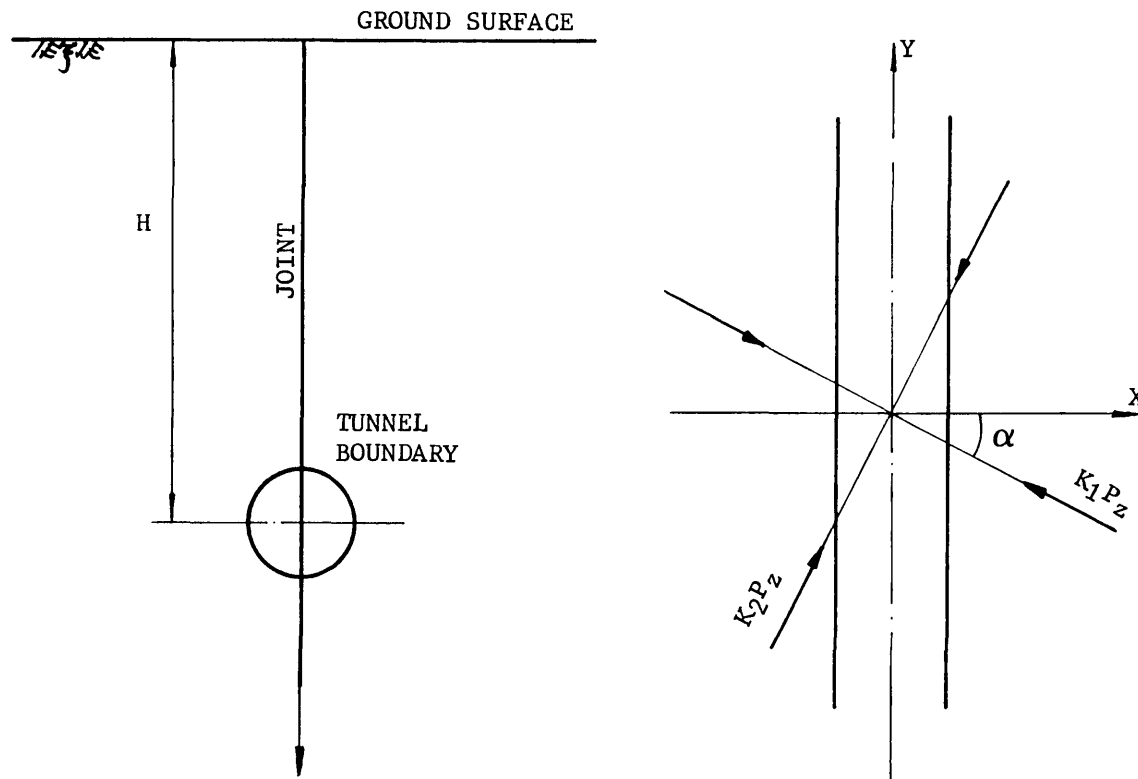


Figure 5.1 Generic case 1

$$\begin{aligned} \gamma_r &= 0.0265 \text{ MN/m}^3 \\ E_r &= 20000 \text{ MPa} \\ \nu &= 0.25 \\ w_m &= 0.002 \text{ m} \\ JRC_p &= 10 \\ JCS &= 150 \text{ MPa} \\ \varphi_r &= 30^\circ \\ L &= 1.0 \text{ m} \\ K_{ni} &= 500 \text{ MPa/m} \end{aligned}$$

5.2.2 Cases with varying H

Fig.5.1 shows a sketch of the problem. One of the horizontal initial principal stresses is parallel to the tunnel axis with a magnitude of $1.0P_z$ and the other is perpendicular to the tunnel axis with a magnitude of $1.5P_z$, i.e. $\alpha = 0^\circ$.

Three different thicknesses of the superincumbent rock are chosen to analyze its influence, i.e. $H = 3.5D$, $5.0D$ and $7.5D$. The corresponding finite element meshes, Mesh 1, Mesh 2 and Mesh 3, are shown in Fig.5.2, 5.3 and 5.4 respectively. The numbers of the elements and nodes of the three meshes are given in Table 5.1.

Table 5.1 Numbers of elements and nodes of Mesh 1, 2, 3

case	H	FE	JE	node
1	3.5D	80	12	102
2	5.0D	84	13	108
3	7.5D	88	14	114

The common tendency of the development of the stresses and displacements is described below for the case of $H = 5.0D$ as an example.

In this case the shear stress component on the joints τ_{xy} is zero because one principal stress is parallel to the tunnel axis. The other shear stress component τ_{yz} is also zero due to the symmetry of both the structure and the load. So the unique non-zero stress component on the joints is the effective normal stress σ'_n . In the initial stress field σ'_n is compressive. As the excavation equivalent forces are exerted increasingly on the tunnel boundary, σ'_n increases as well. This is because that extracting the part of the rock inside the tunnel makes the tunnel

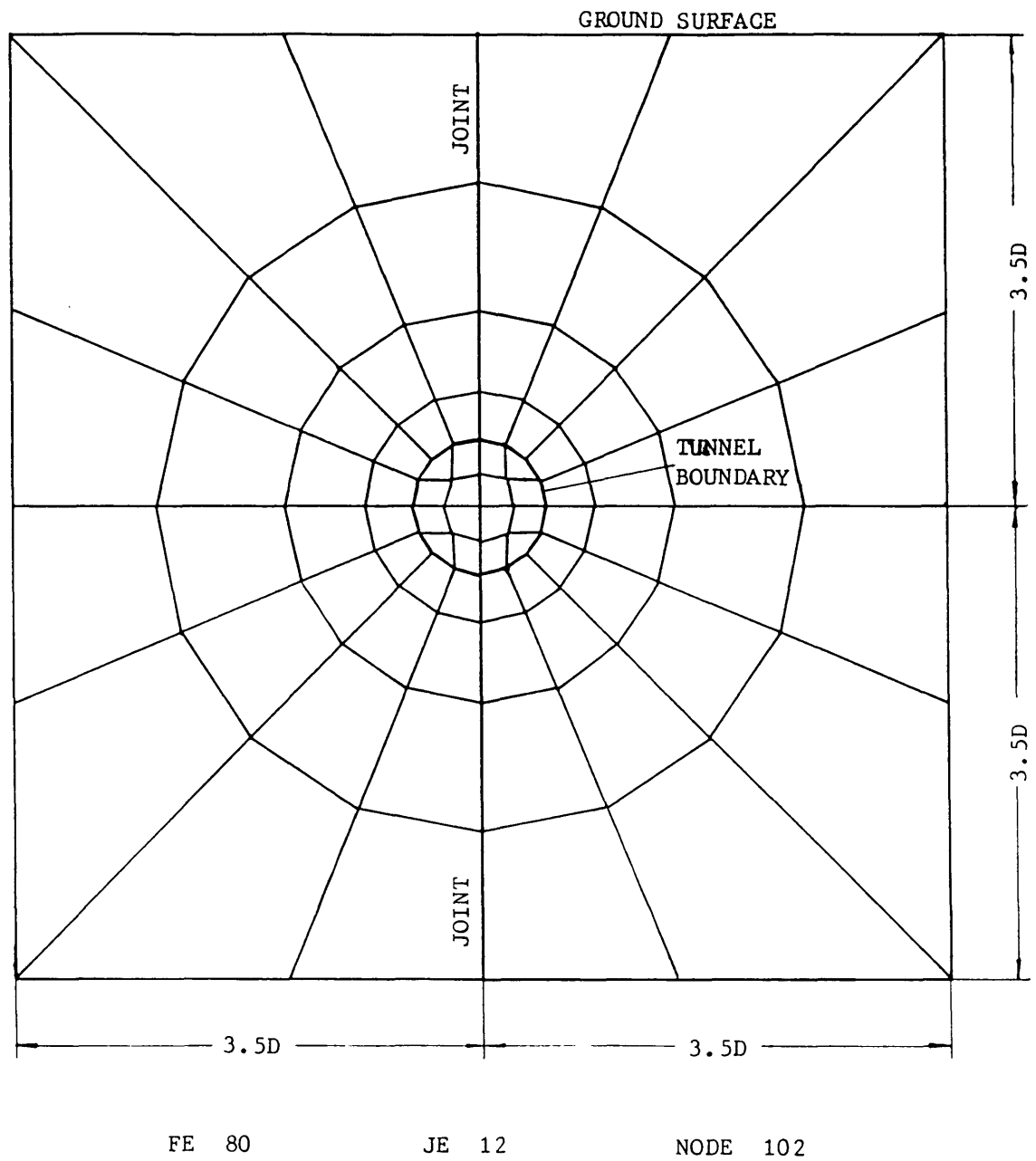


Figure 5.2 Finite element mesh for the case $H=3.5D$ of generic case 1 - Mesh 1

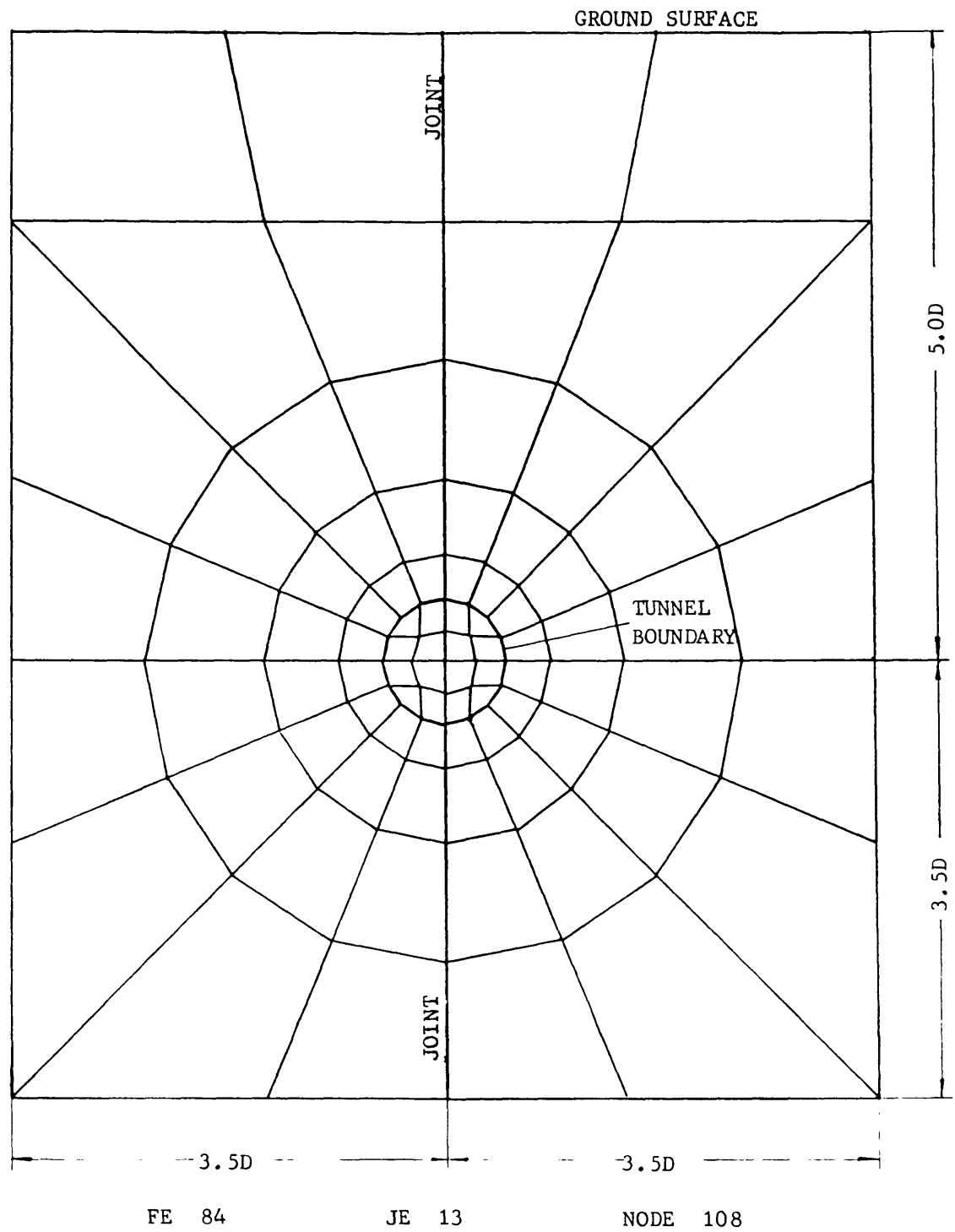


Figure 5.3 Finite element mesh for the case $H=5.0D$ of generic case 1 - Mesh 2

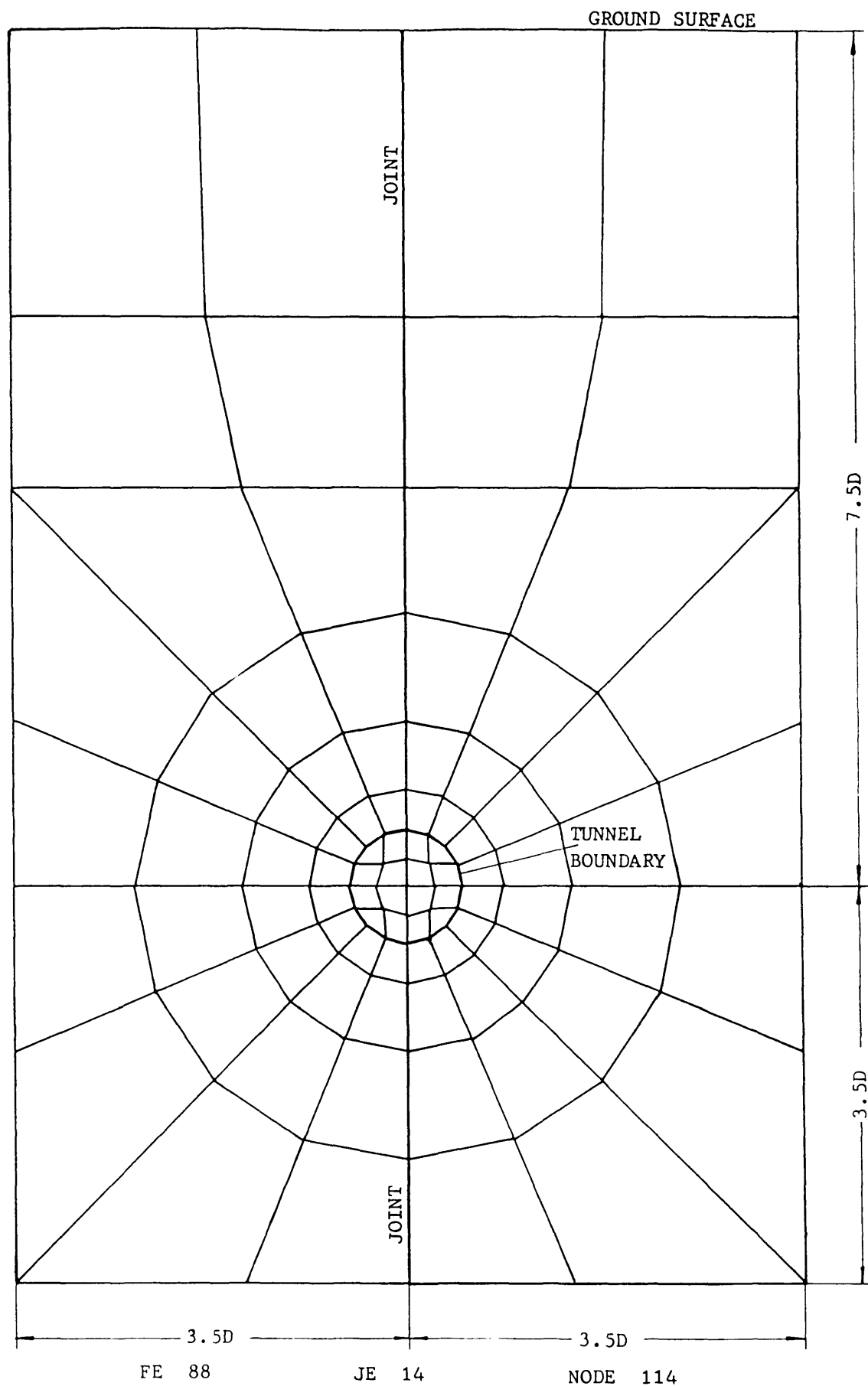


Figure 5.4 Finite element mesh for the case $H=7.5D$ of generic case 1 - Mesh 3

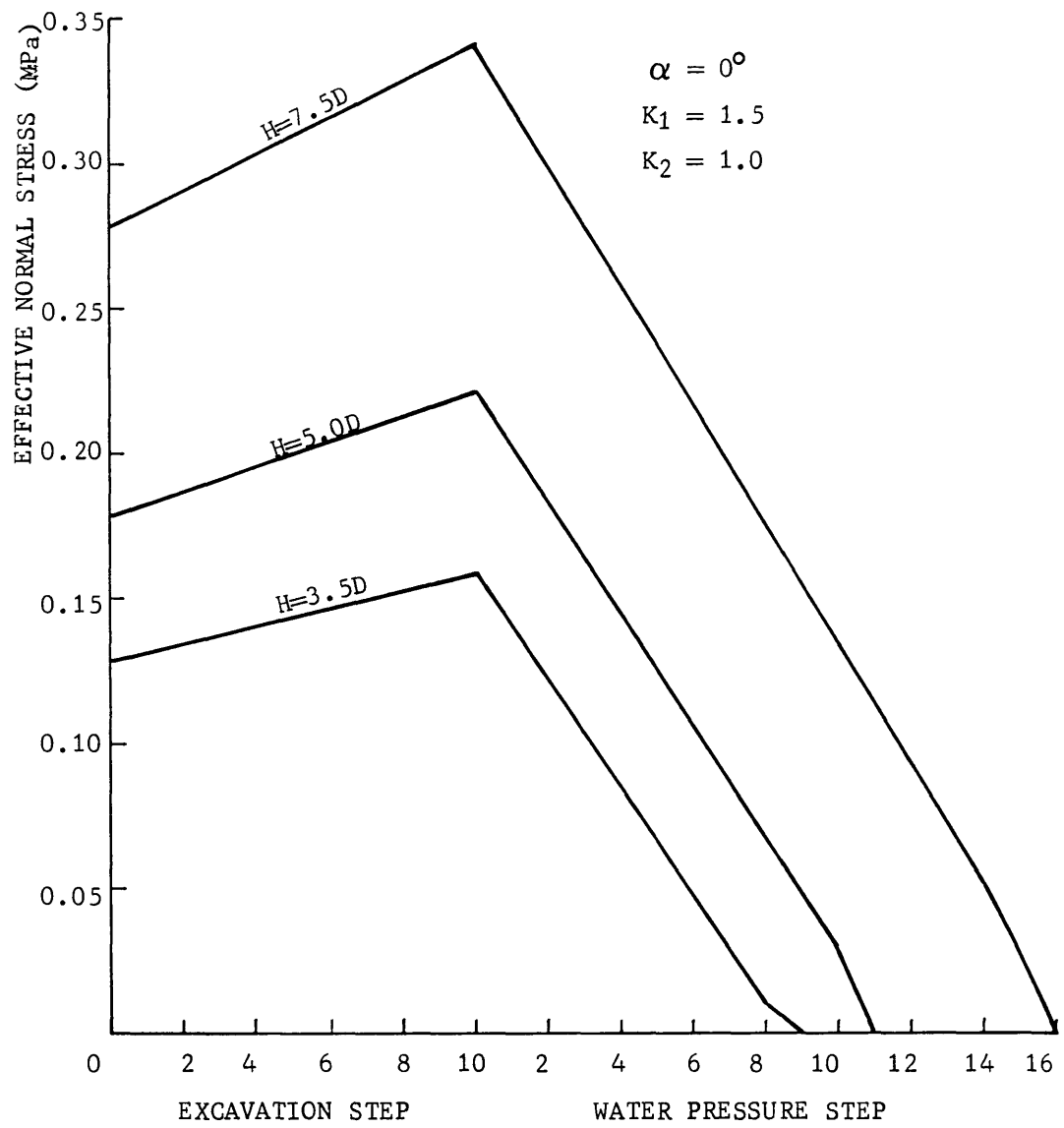


Figure 5.5 Variation of normal stress with excavation and water pressure

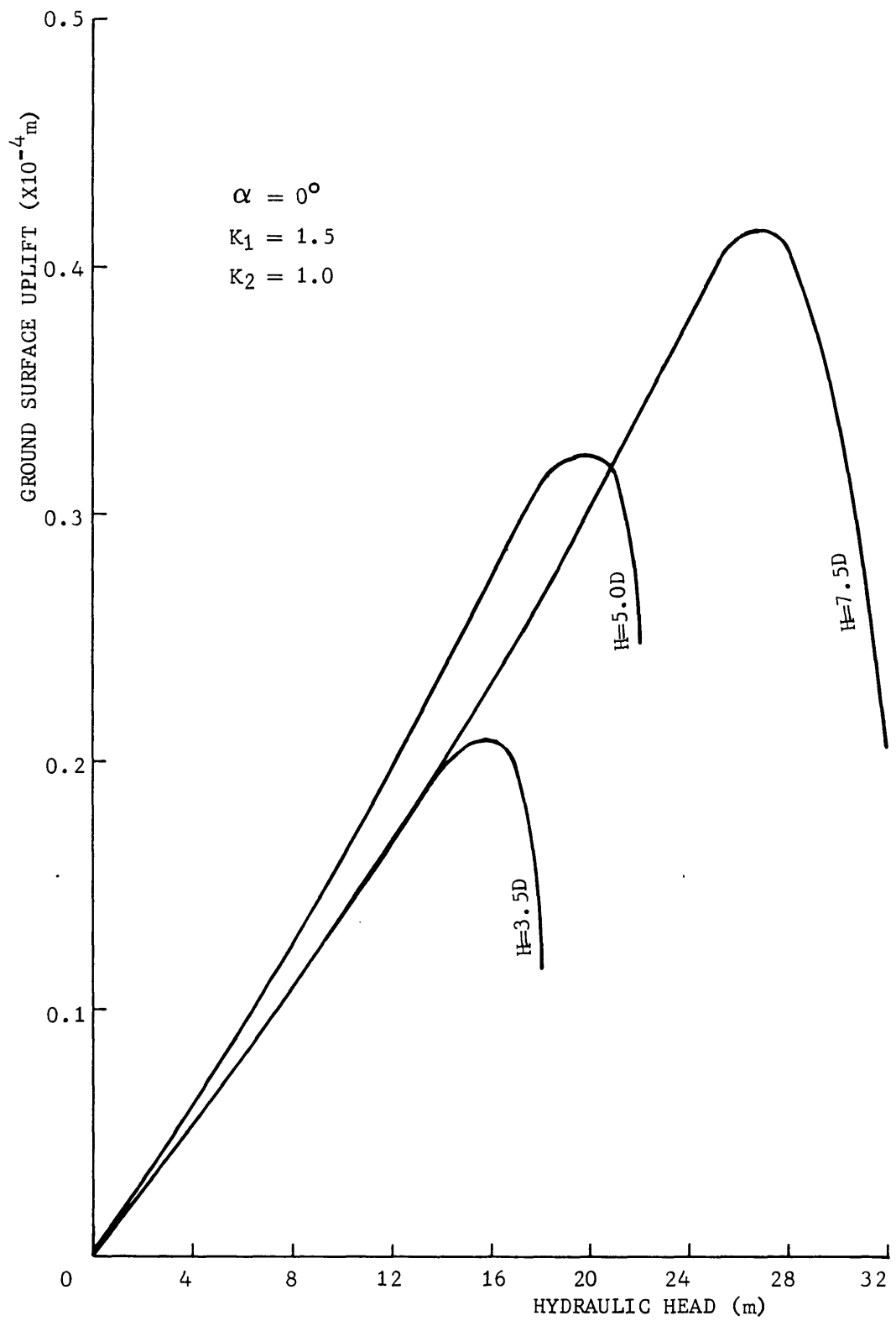


Figure 5.6 Uplift of ground surface for generic case 1

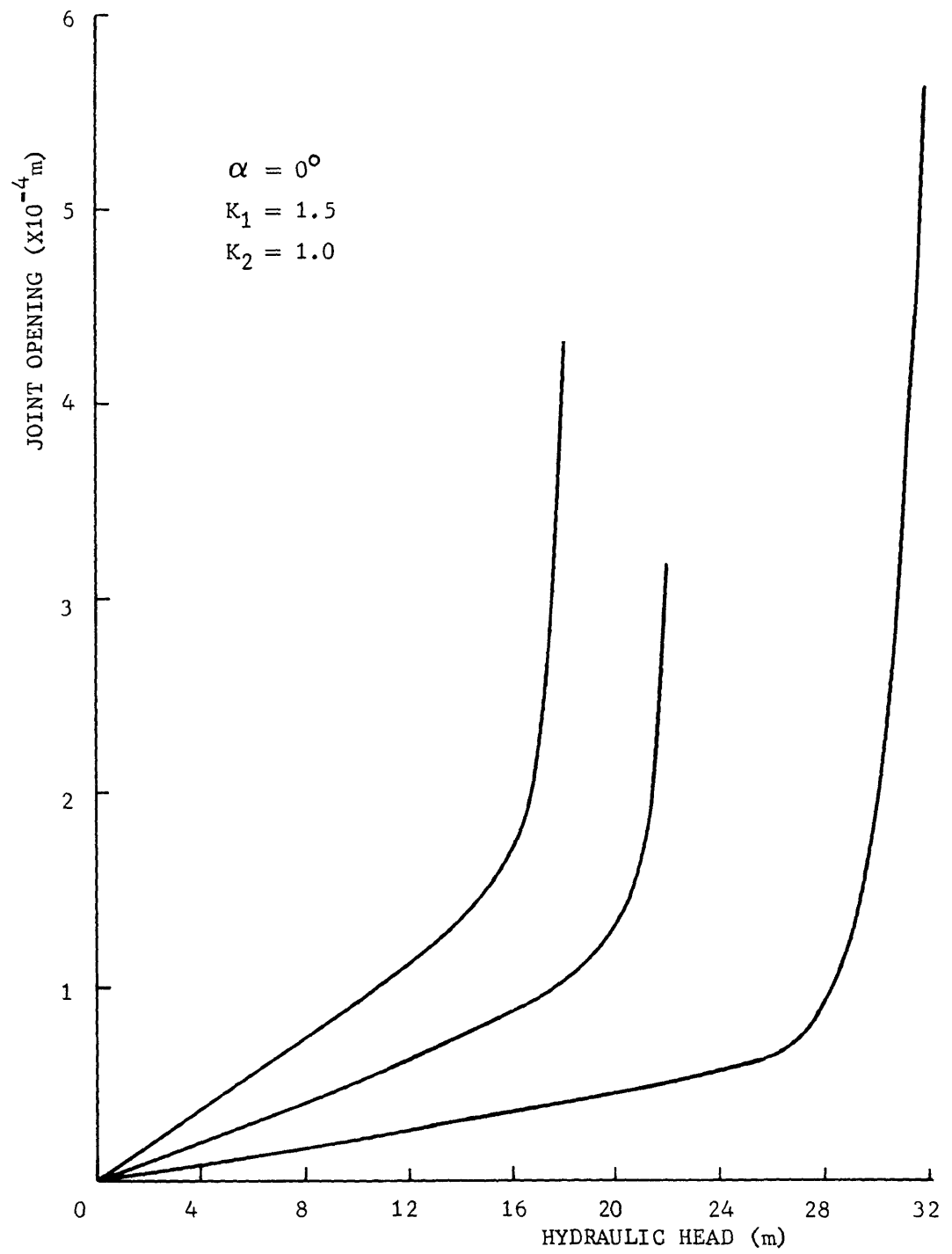


Figure 5.7 Variation of joint opening with water pressure for generic case 1

boundary contract in this particular case. Therefore, the joint intersecting the tunnel boundary must close. When the water pressure is applied, however, the joints may open and the effective normal stress decreases. As the water pressure increases the normal stress on the joints becomes less and less and joint opening becomes larger and larger. Finally, when the water pressure reaches $0.216MPa$ (11 steps), the effective normal stress on all joint elements between the tunnel boundary and the ground surface becomes zero and joint opening reaches a big value. Although no leakage rate is given from computation, it can be inferred that the leakage through the joints must be very considerable. It is defined that hydraulic jacking occurs at this stage and the corresponding water pressure is taken to be the critical water pressure.

If the water pressure continues to increase convergence will not be reached and no stable solution will be obtained, so computation is terminated here.

The uplift of the ground surface also increases as the water pressure increases in the first steps. Then it reaches a peak value followed by a decrease. However, it has considerably lower values than the joint opening. The biggest surface uplift is just $0.0325mm$. Therefore, it can be concluded that the main failure mode for this generic case is hydraulic jacking.

The cases of $H = 3.5D$ and $7.5D$ show the same tendency in the variation in the development of stresses and displacements, but the critical water pressures are different due to the different thicknesses of the superincumbent rock. Fig.5.5 shows the variation of the effective normal stress in the course of excavation and applying the water pressure for the cases of $H = 3.5D$, $5.0D$ and $7.5D$. Fig.5.6 and 5.7 show the variation of the uplift of the ground surface and joint opening with increasing water pressure for the three cases.

Fig.5.8 is the plot of the critical water pressure p_c against the thickness

of the superincumbent rock H . From the curve it can be seen that the critical water pressure rises as the thickness of the superincumbent rock increases. That means that when all other conditions are the same, the deeper the tunnel is situated, the more difficult it is for hydraulic jacking to occur and the tunnel can remain unlined under a higher water pressure.

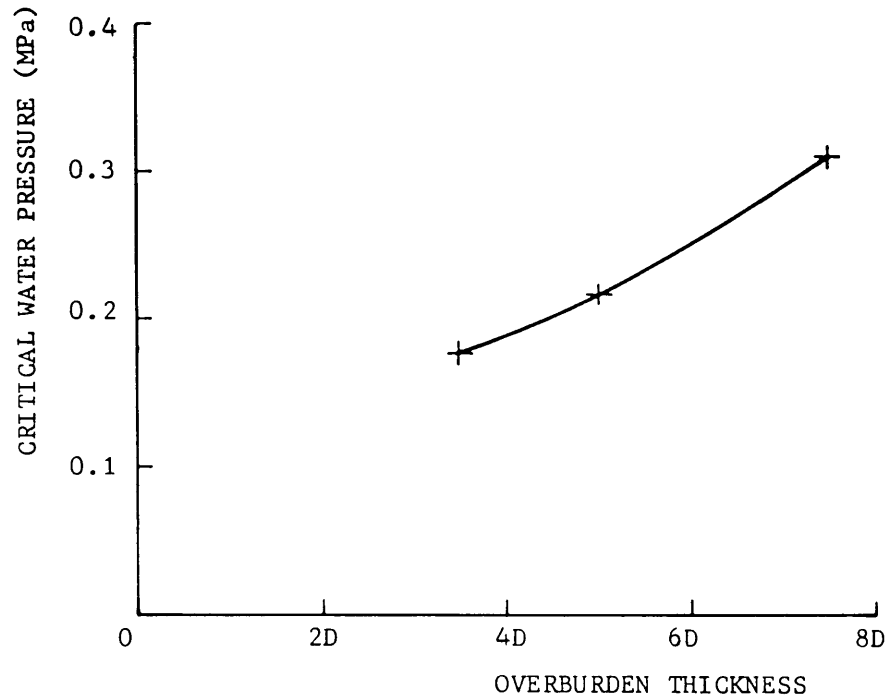


Figure 5.8 Variation of critical water pressure with overburden thickness

5.2.3 Cases with varying α

There are many factors to be considered in aligning a pressure tunnel, such as economy, energy loss, operating conditions, the geological conditions and so on. Generally, it is not the case that the tunnel axis is coincident with one of the initial principal stresses. Therefore, investigation of the behaviour of the unlined pressure tunnels in the cases in which the tunnel axis is not coincident with one of the initial principal stresses is of importance in practice. The complete plane strain model provides a possibility to carry out the

investigation.

An analysis is conducted for the examples $H = 5.0D$, $K_1 = 1.5$, $K_2 = 1.0$. The corresponding finite element mesh is shown in Fig.5.3. Four cases, i.e. $\alpha = 0^\circ, 15^\circ, 30^\circ, 45^\circ$ are analyzed. The result for the case of $\alpha = 0^\circ$ has been described in Section 5.2.2, i.e. hydraulic jacking occurs at the water pressure $p_c = 0.216MPa$. The results for the cases of $\alpha > 0$ are described as follows with the case of $\alpha = 30^\circ$ used as an example.

In the initial stress field the shear stress component τ_{yz} is zero due to the symmetry of the structure and the load. But the other shear stress component τ_{xy} is not zero because of the force components acting in the Y direction. The normal stress on the joint is compressive. The excavation equivalent forces are three dimensional and they produce three dimensional displacement increments. But the three dimensional displacement increment can only cause stress changes in τ_{xy} and σ'_n , while τ_{yz} remains zero due to the symmetry. During the excavation both τ_{xy} and σ'_n increase by small amounts. For example, for the joint element immediately adjacent to the tunnel boundary, $\tau_{xy} = 0.02593MPa$, $\sigma'_n = 0.1645MPa$ in the initial stress field and $\tau_{xy} = 0.03200MPa$, $\sigma'_n = 0.2020MPa$ for the second stress field after the excavation.

Water pressures acting on both the tunnel boundary and the joint walls are two dimensional. The force in the Y direction F_y is zero because the water pressure is always perpendicular to the solid surface on which it acts. Obviously, the normal stress decreases as the water pressure increases. But it seems that there will be no increments in the shear displacement and in the shear stress because there are no forces in the Y direction, $F_y = 0$. As a matter of fact, however, the shear displacement develops as the water pressure increases. This is explained in Fig.5.9. Curve 1 is the original stress displacement plot (under

the original normal stress σ'_{n1}). When it is decreased to σ'_{n2} the plot should be curve 2. But if the shear stress τ_{xy} remains unchanged (because of no force in Y direction) the shear displacement must increase, otherwise the correct relation cannot be obtained. In fact, the shear stresses on all joint elements are redistributed. In some joint elements it increases a little, while in other joint elements it decreases a little but the changes are small.

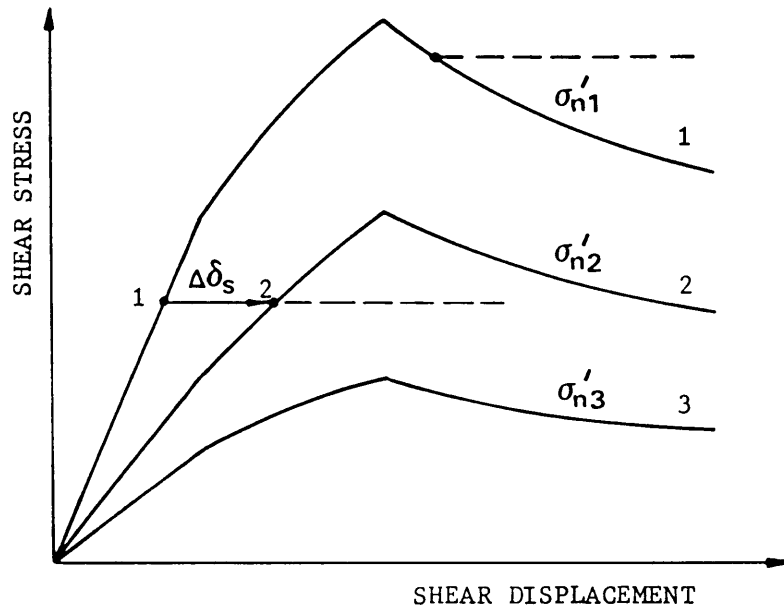


Figure 5.9 Development of shear displacement under reduced effective normal stress and constant shear stress

This phenomenon of the development of shear displacement when water pressure increase reduces the normal stress is called hydraulic shearing. In order to gain a comprehensive understanding of this phenomenon it is worthwhile making a detailed investigation.

In the single joint element case (e.g. example 4 in Section 3.3.8 and Fig.3.30), when the shear force remains constant, as the water pressure increases, the effective normal stress decreases and the shear displacement develops as just mentioned (from point 1 to point 2 in Fig.5.9). When the effective

normal stress is reduced to such a level that the corresponding peak shear strength is lower than the given shear stress the shear displacement will develop infinitely (in Fig.5.9 the lower horizontal dotted line cannot intersect curve 3) and shear failure occurs. In terms of displacement this occurs as long as the shear displacement exceeds the peak value. We say that hydraulic shearing occurs at this stage. The higher the original shear stress (before the water pressure is applied) the easier it is for hydraulic shearing to occur. If the shear stress is originally very low, hydraulic shearing may will not occur until the effective normal stress drops almost to zero.

In the multiple joint element case, when the peak shear strength is lower than the original shear stress on a certain joint element, the excessive shear stress may be transferred to adjacent joint elements for which peak shear strength has not yet been reached. This is shear stress redistribution. Only when in all joint elements the peak shear strength is lower than the corresponding shear stress does hydraulic shearing occur.

Now it is time to give a clear definition of hydraulic jacking and hydraulic shearing.

Hydraulic jacking

A joint intersecting a borehole or a tunnel which is closed by the initial compressive stress field originally may be opened by induced tensile stress when the internal water pressure is applied.

Hydraulic shearing

A joint intersecting a borehole or a tunnel which is in a shear stress state originally may develop its shear displacement to fail by decrease of the normal compressive stress when the internal water pressure is applied.

Hydraulic jacking and hydraulic shearing are two fundamental failure modes for the discontinuities subjected to internal water pressure. When a joint is in a purely normal stress state (shear stress free) the unique potential failure mode is hydraulic jacking. When the joint is in a post-peak shear stress state already before the water pressure is applied, hydraulic shearing will definitely occur as soon as water pressure is applied. When the joint is in a near peak shear stress state hydraulic shearing may be the most likely failure mode. When the joint is in a low shear stress state, however, joint opening may develop as rapidly as shearing does, so that hydraulic jacking and hydraulic shearing may occur at the same water pressure step. But strictly speaking, in physical terms, hydraulic shearing is always the first failure mode to occur although hydraulic jacking may follow immediately.

In the case analyzed, when the water pressure reaches 0.196MPa the shear displacement on all joint elements between the tunnel boundary and the ground surface almost reaches the peak shear displacement and computation cannot converge after 100 iterations. The joint opening also reaches a very high value. Fig.5.10 and 5.11 show the variations of the shear displacement and joint opening at the intersection of the tunnel boundary and the joint with increase of the water pressure for the cases of $\alpha = 15^\circ$, 30° and 45° . From the figures it can be seen that when δ_p is approached the shear displacement develops rapidly for all the cases and no converged results can be obtained. The bigger α the bigger the original shear stress, so that hydraulic shearing occurs at lower water pressures.

When $\alpha = 15^\circ$, hydraulic shearing occurs at the water pressure step 11, for which the corresponding water pressure is 0.216MPa , and hydraulic jacking also occurs at the same water pressure because the original shear stress is not very large. When $\alpha = 30^\circ$ only hydraulic shearing occurs at step 10 and water

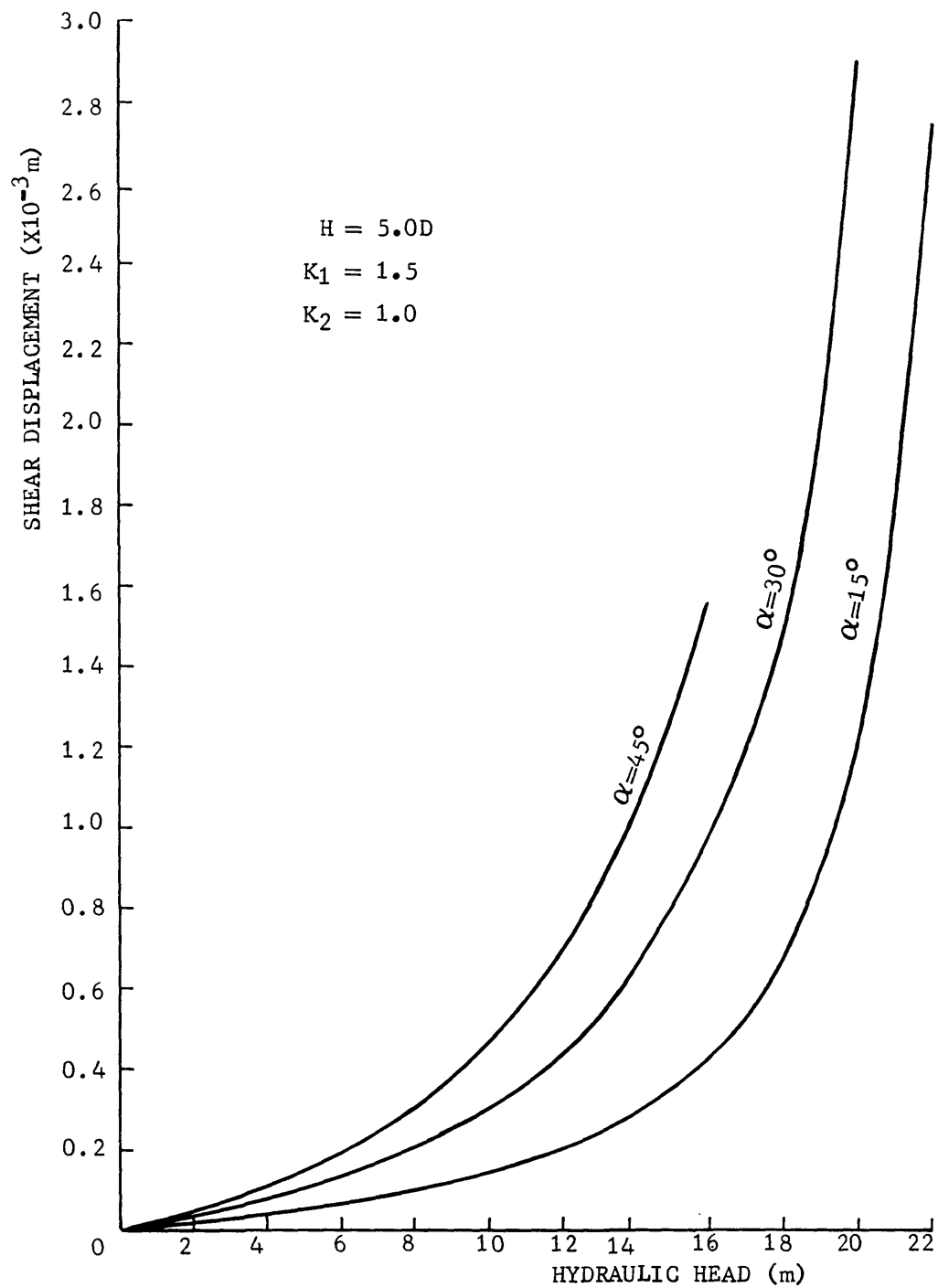


Figure 5.10 Variation of shear displacement with water pressure for generic case 1

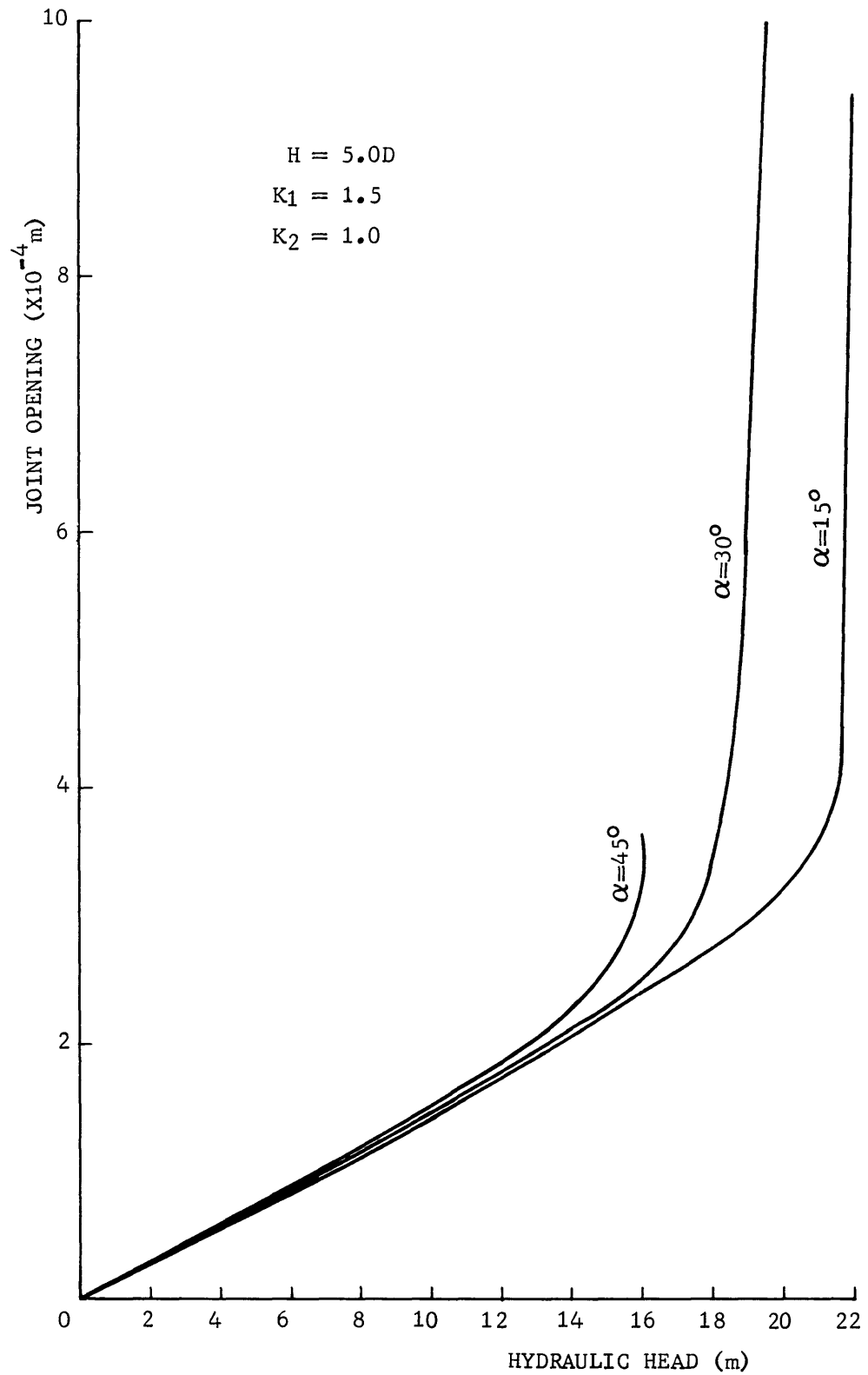


Figure 5.11 Variation of joint opening with water pressure for generic case 1

pressure 0.196MPa . When $\alpha = 45^\circ$, it is step 8 and 0.157MPa . The failure mode and the critical water pressure for each case are summarised in Table 5.2.

Table 5.2 Failure mode and critical water pressure for cases of varying α

α	Failure Mode	Critical Water Pressure
0°	hydraulic jacking	0.216MPa
15°	hydraulic jacking/shearing	0.216MPa
30°	hydraulic shearing	0.196MPa
45°	hydraulic shearing	0.157MPa

The analysis of this case indicates that for this configuration of the tunnel and the joint when the tunnel axis is not parallel to one of the initial principal stresses and $K_1 \neq K_2$ hydraulic shearing may become a more critical failure mode than hydraulic jacking. For the general case attention must be paid to both failure modes, hydraulic jacking and hydraulic shearing.

5.2.4 Cases with varying K_1

In Chapter 2 it was concluded from the analytical solutions that when K_1 is large hydraulic jacking will not easily occur for the plane strain case. Here a numerical analysis is carried out to investigate the influence of K_1 on the critical water pressure under which hydraulic jacking occurs for the rock mass containing a real joint. Case 2 in Section 5.2.2, i.e. $H = 5.0D$, $\alpha = 0$, is used for the analysis. The finite element mesh is shown in Fig.5.3. Four cases, $K_1 = 0.5, 1.0, 1.5$ and 2.0 with K_2 being constant 1.0 are computed (plane strain model).

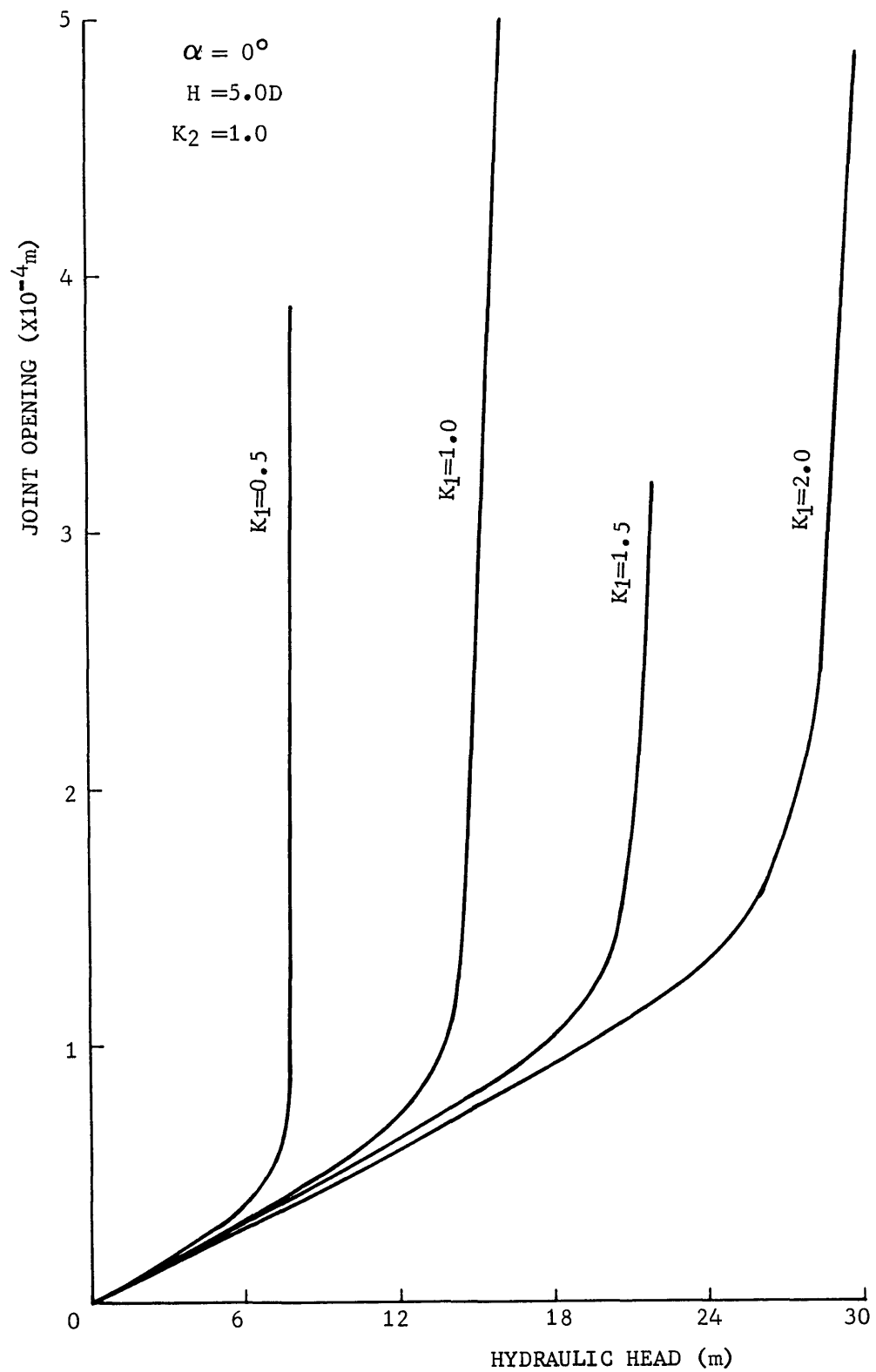


Figure 5.12 Influence of K_1 on joint opening for generic case 1

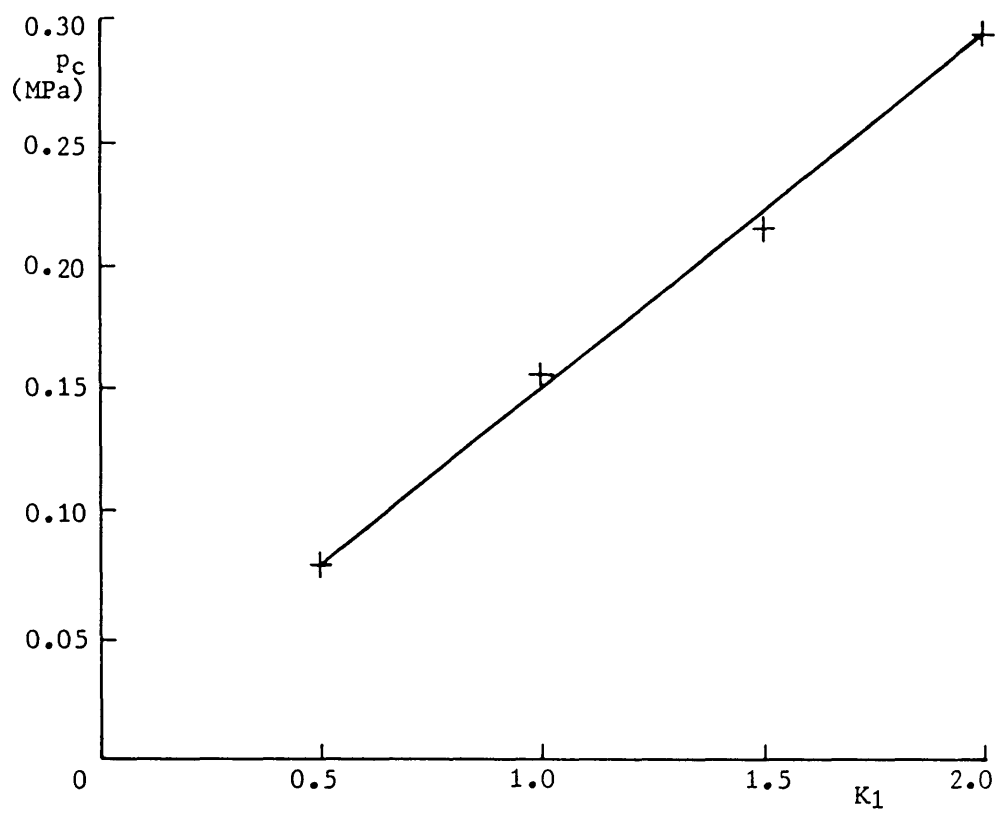


Figure 5.13 Variation of critical water pressure with K_1 for generic case 1

Table 5.3 The critical water pressures for cases of different K_1

K_1	Failure Mode	Critical Water Pressure
0.5	hydraulic jacking	0.078
1.0	hydraulic jacking	0.157
1.5	hydraulic jacking	0.216
2.0	hydraulic jacking	0.294

The case of $K_1 = 1.5$ has been discussed in Section 5.2.2. The variations of the stresses and displacements and the failure mode are the same for the four cases, i.e., hydraulic jacking occurs when the water pressure increases to a certain level. But the critical water pressures are different due to the differences in the initial stress field. The critical water pressures are presented in Table 5.3 and the development of joint opening is shown in Fig.5.12.

Fig. 5.13 shows the variation of the critical water pressure with K_1 . Table 5.3 and Fig.5.13 show the initial horizontal stress has a major influence on the critical water pressure in this case. The higher the initial lateral stress, the more difficult it is for hydraulic jacking to occur. This result indicates that the value of the initial lateral stress is a very influential factor to be considered when locating a pressure tunnel.

5.3 Generic Case 2 - A Horizontal Joint Parallel to the Ground Surface and the Tunnel Connected by a Single Vertical Joint

5.3.1 General description

In the failure mechanism analysis (Section 1.2) it has been pointed out that when horizontal joints near the ground surface intersect a vertical joint, which meets the tunnel boundary and has opened, the pressure water can penetrate the horizontal joints. As a result, the ground surface may be uplifted. This is one of the fundamental failure mechanisms of unlined pressure tunnels. This section presents some computational results for this generic case. Several factors may influence the failure of the tunnel in this case. But there is no doubt that the distance between the ground surface and the horizontal joint is one of the dominant factors to be considered. The problem is shown in Fig.5.14 and two cases of $H_1 = 1.7m$ and $0.9m$ are analyzed with the other parameters

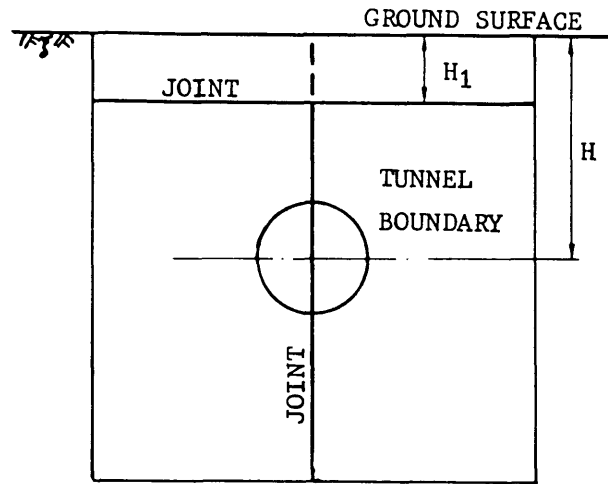


Figure 5.14 Generic case 2

Table 5.4 Numbers of finite elements, joint elements and nodes
and critical water pressures for generic case 2

H_1	FE	JE	Node	Critical water pressure
1.7m	112	22	157	0.049MPa
0.9m	112	23	158	0.029MPa

remaining constant. The finite element meshes for these two cases are shown in Fig.5.15 and 5.16, respectively, and the corresponding numbers of the finite elements, joint elements and nodes are given in Table 5.4. The material constants are the same as those for generic case 1. The initial stress field is defined by $K_1 = 1.5$, $K_2 = 1.0$, and $\alpha = 0^\circ$. The excavation equivalent forces are applied in the same way as in generic case 1, i.e. in 10 steps, but the water pressure increment adopted is 1 meter of hydraulic head, i.e. $0.00981MPa$.

5.3.2 Results for the case $H_1 = 1.7m$

This problem is a plane strain problem ($\alpha = 0^\circ$), so the shear stress component τ_{xy} vanishes. The other shear stress component is also zero for the vertical joint elements due to symmetry, but it is not zero for the horizontal joint elements. In the initial stress field, all joint elements are compressive in the normal direction, but σ'_n is higher on the vertical joint elements than on the horizontal joint elements. After excavation the normal stress on the vertical joint is increased and the shear stress on the horizontal joint is increased with the normal stress altered little.

Under the application of the water pressure, joint opening occurs in both the horizontal and the vertical joints, but it has higher values and it increases at a faster rate in horizontal joints than in vertical joints (Fig.5.17). This is because the overburden is too small. An abrupt increase in joint opening occurs on horizontal joint at water pressure step 5 which indicates occurrence of hydraulic jacking. In the meantime, the rock block above the horizontal joint displaces significantly upwards, which can be viewed explicitly as uplift of the ground surface (Fig.5.18). By comparing Fig.5.17 and Fig.5.18 it can be seen that the uplift of the ground surface and opening of the horizontal joint have almost exactly equal values. The small difference is from the deformation of the rock block above the horizontal joint. The shear displacement of the

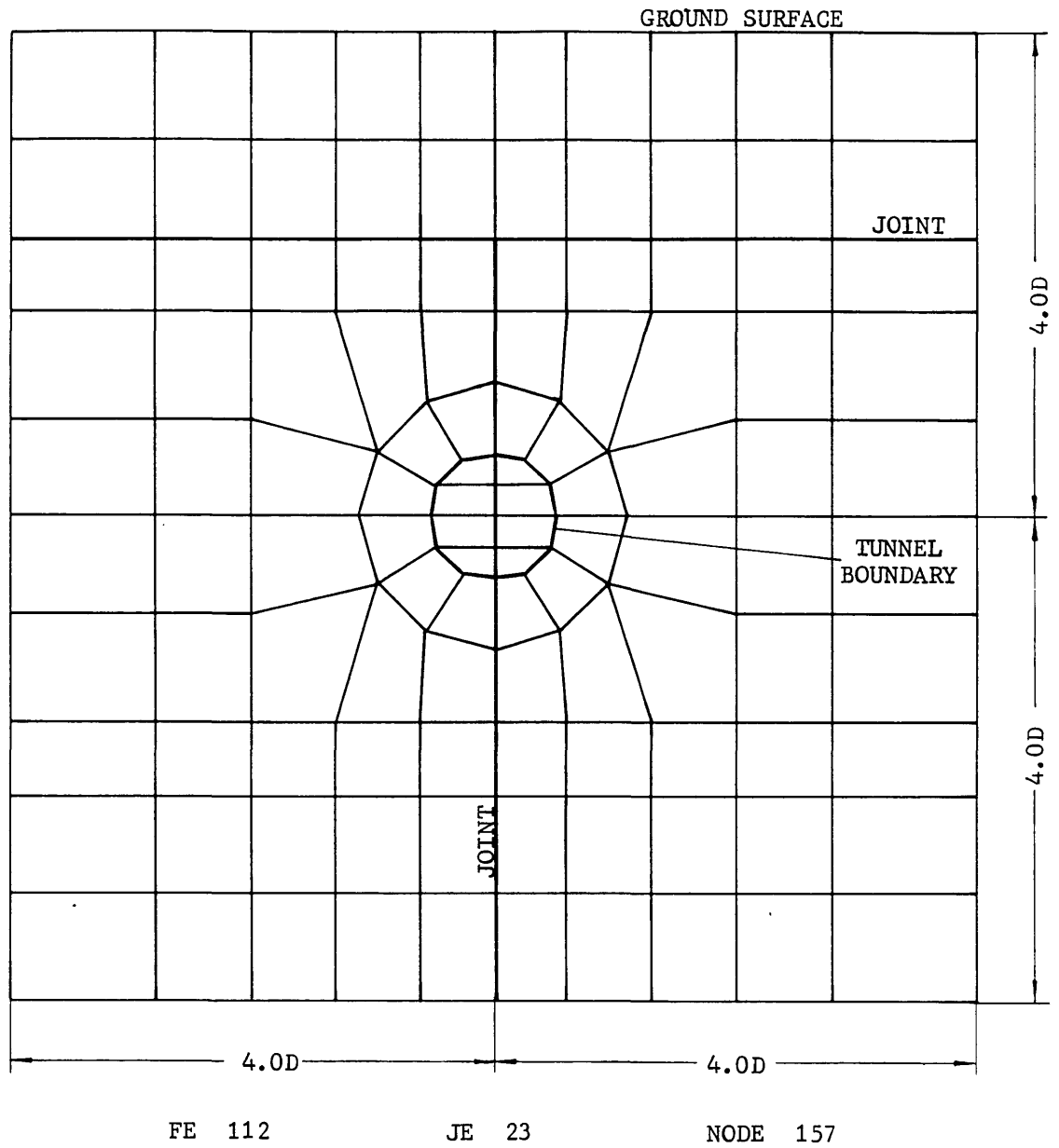


Figure 5.15 Finite element mesh for the case $H_1=1.7\text{m}$ of generic case 2 - Mesh 4

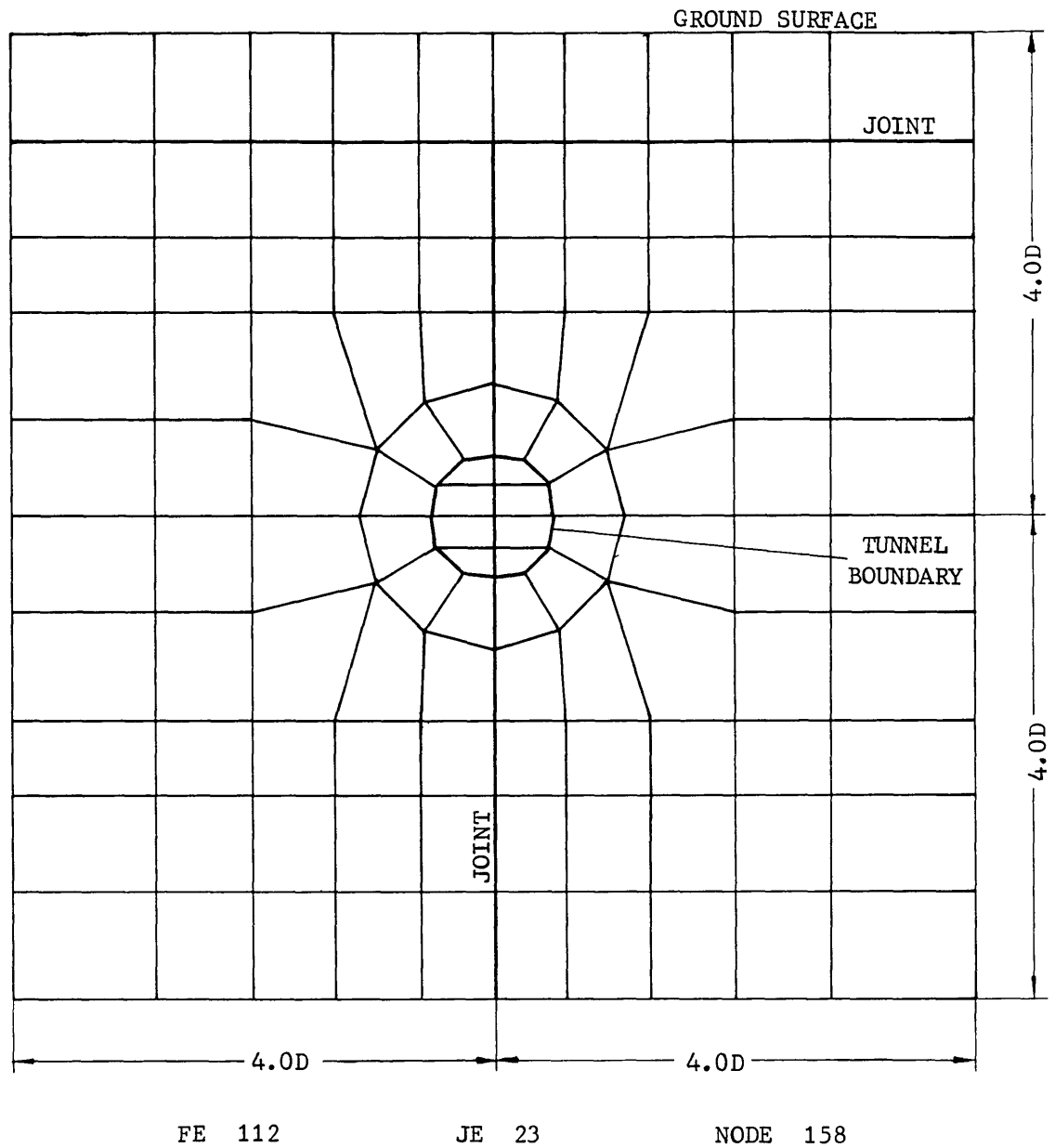


Figure 5.16 Finite element mesh for the case $H_1=0.9m$ of generic case 2 - Mesh 5

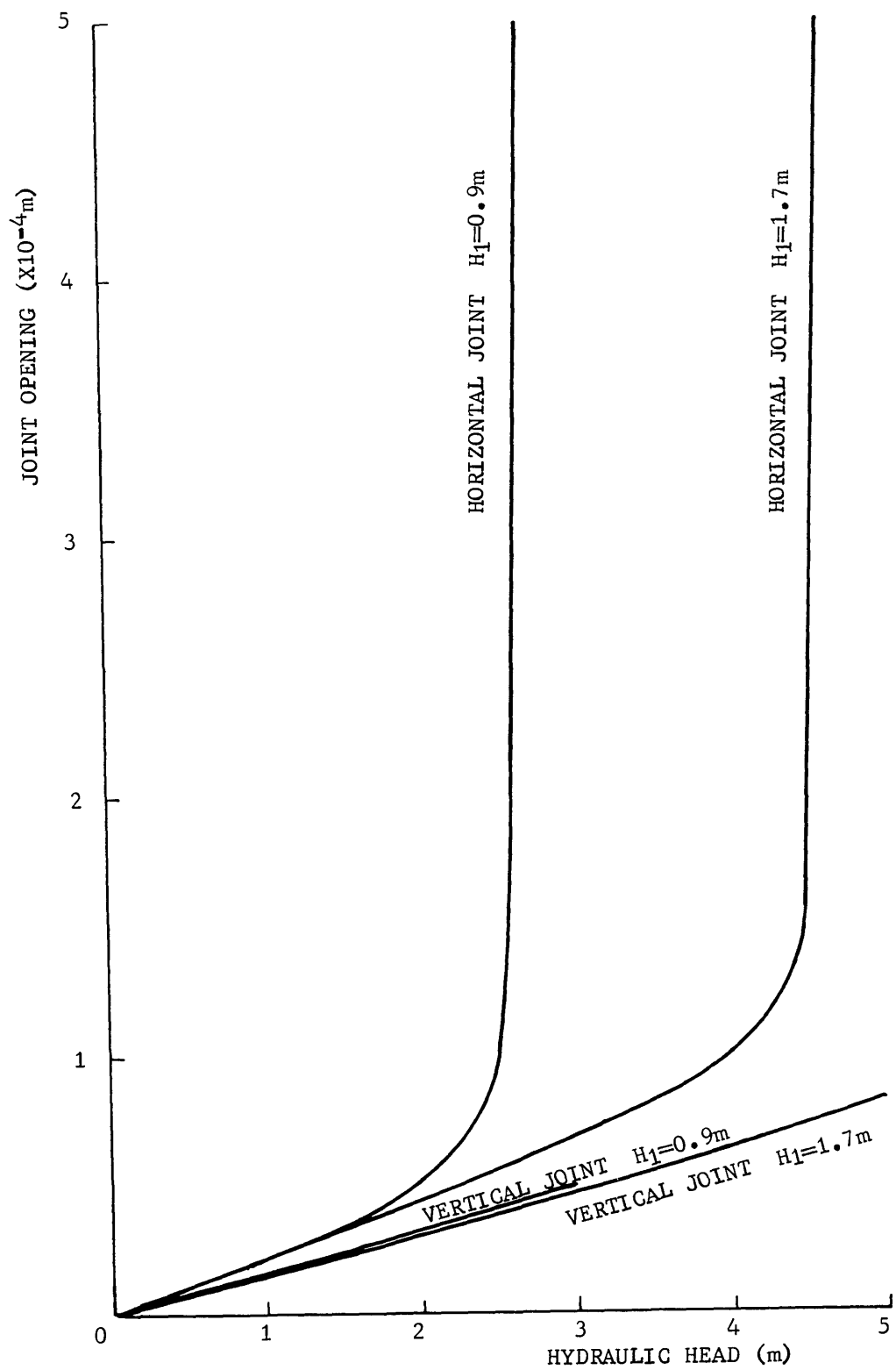


Figure 5.17 Joint opening for generic case 2

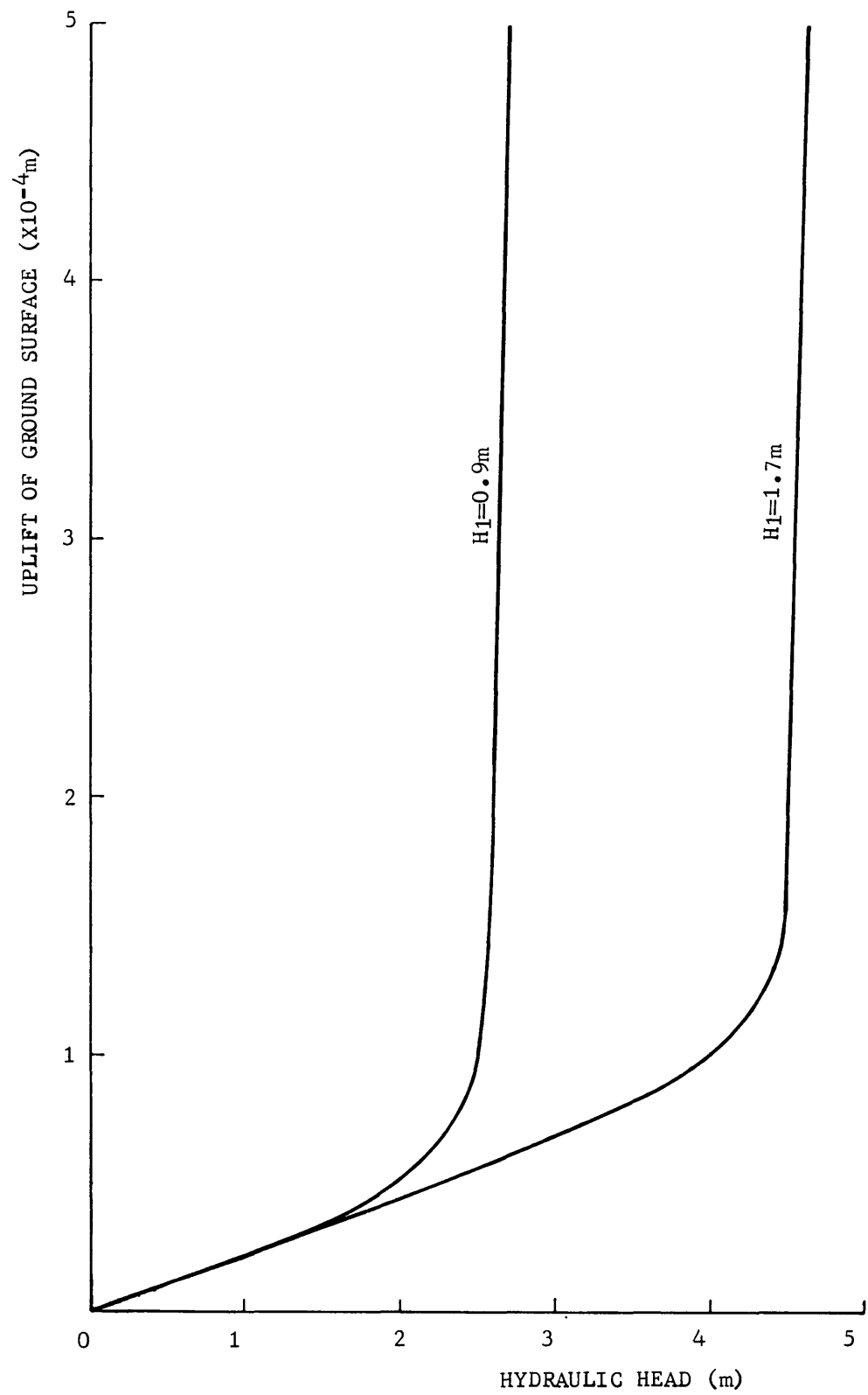


Figure 5.18 Uplift of ground surface for generic case 2

horizontal joint increases with the water pressure, but is much lower than the corresponding joint opening (Fig.5.19).

Fig.5.20 shows the variation of the effective normal stress on both horizontal and the vertical joints. It can be seen that the compressive normal stress decreases on both joints as the water pressure increases, but it has much lower values on the horizontal joint than on the vertical joint. In fact, when the uplift of the ground surface reaches a big value (the computation diverges) the effective normal stress in the horizontal joint becomes zero and hydraulic jacking occurs.

Therefore, it can be concluded that it is the excessive uplift of the ground surface associated with hydraulic jacking of the horizontal joint that governs the failure of the unlined pressure tunnel in this case.

5.3.3 Influence of overburden thickness

It is inferred that when the vertical distance between the ground surface and the horizontal joint is smaller failure will occur more easily, since the initial normal stress is lower in the horizontal joint. The results for the case $H = 0.9m$ show the same general tendency of the development of the stresses and the displacements. However, failure takes place at a much lower water pressure as shown in Table 5.4. Opening of the horizontal joint and uplift of the ground surface for both $H_1 = 1.7m$ and $H_1 = 0.9m$ are shown in Fig.5.17 and 5.18. It is clear that hydraulic jacking of the horizontal joint occurs under much lower water pressure; consequently, the abrupt increase in uplift of the ground surface occurs at much lower water pressure as well for the case $H_1 = 0.9m$ than for the case $H_1 = 1.7m$. These results indicate that the nearer the ground surface to the horizontal joint the more dangerous is the case.

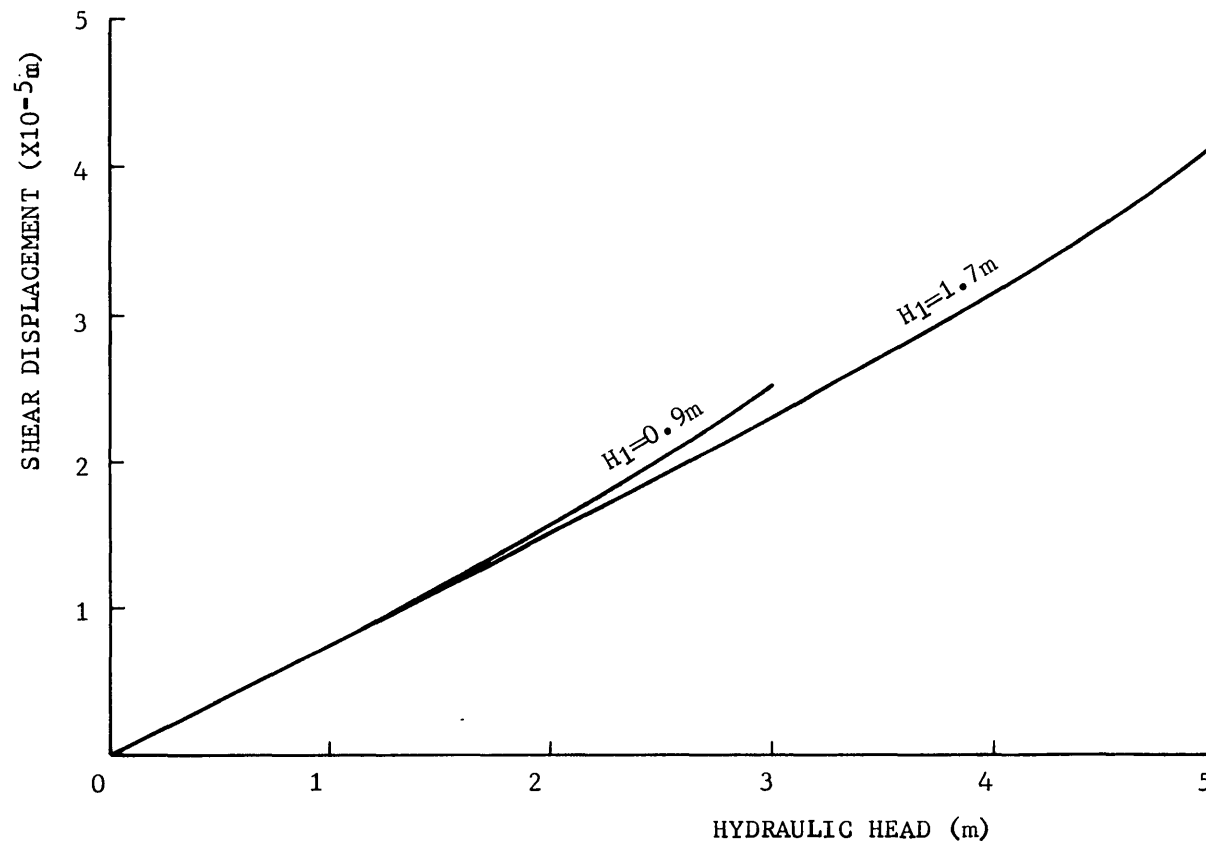


Figure 5.19 Shear displacement on horizontal joint for generic case 2

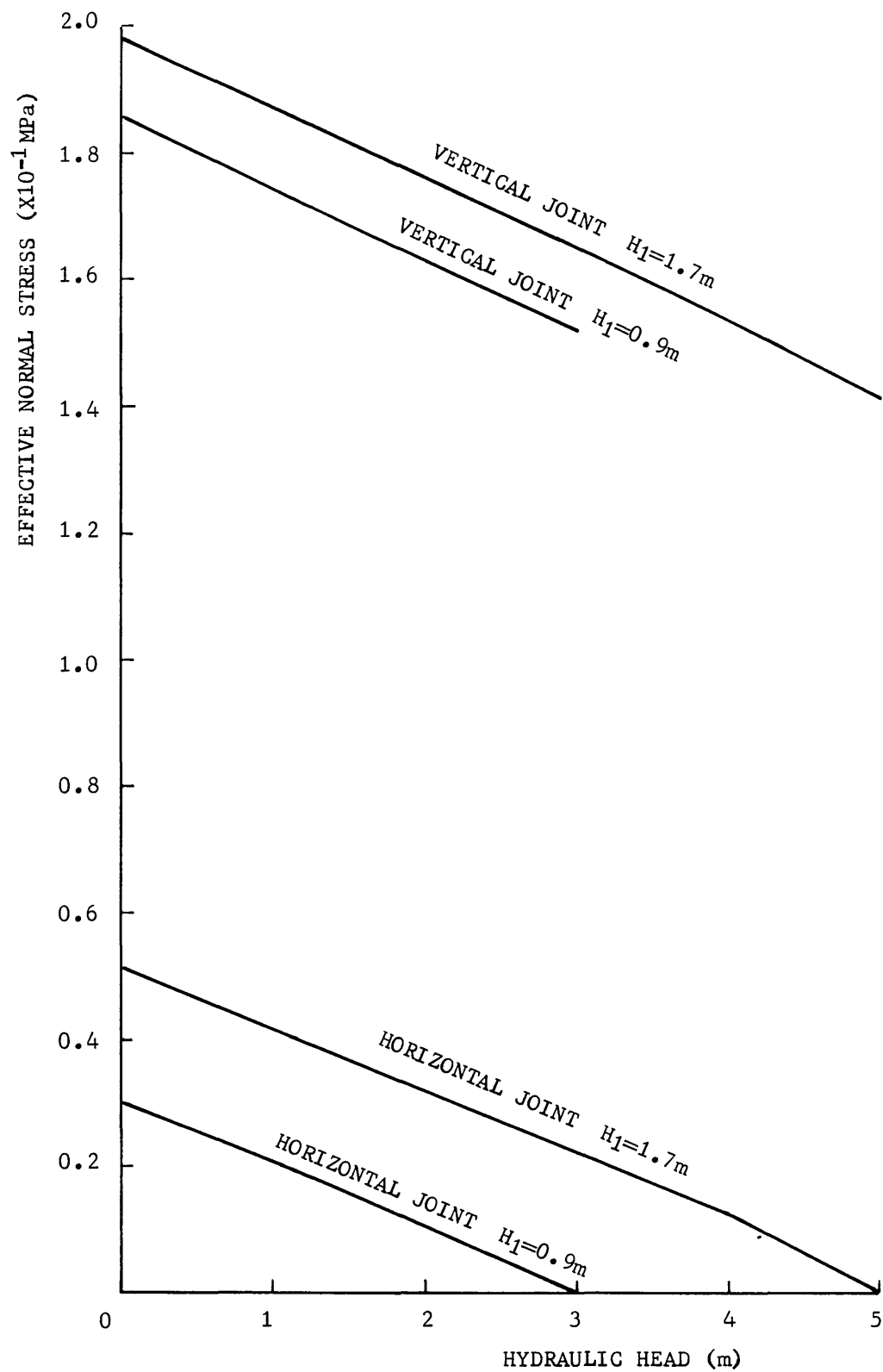


Figure 5.20 Effective normal stress for generic case 2

There is another possible case in which the vertical joint crosses the horizontal joint and extends to the ground surface as shown by the dotted line in the Fig.5.14. In this case water flow occurs in the vertical joint. Accordingly, the water pressure at the intersection of the vertical joint and the horizontal joint, which is equal to the water pressure in the horizontal joint (water pressure is the same anywhere along the horizontal joint), is lower than the corresponding water pressure in the previous case. Therefore, in this case the uplift of the ground surface will become more difficult to induce. However, hydraulic jacking in the vertical joint may occur as described in Section 5.2. The factors determining which mode is the more critical may be the distance between the ground surface and the horizontal joint, the values of the initial stresses etc.

It is worth noting that the critical water pressure for this case is much lower than that in the case of a single vertical joint (Section 5.2.2). This provides a warning that the existence of a near surface horizontal joint connected to the tunnel by a vertical joint is a great danger. Uplift of the ground surface can readily occur under a very low internal water pressure. Great attention must be paid to the engineering treatments required to seal the vertical joint and to prevent it from opening, if no better location can be found for the tunnel.

Sometimes no vertical joint exists between the horizontal joint and the tunnel boundary. There will be no danger in this case since no water can enter the horizontal joint. But if the internal water pressure is so high that tensile cracking occurs in the intact rock between the horizontal joint and the tunnel boundary and fissures penetrate to the horizontal joint, hydraulic jacking and uplift of the ground surface may occur immediately. However, the critical water pressure in this case is that for the tensile opening of the intact rock, that is usually much higher than the critical water pressure for the case of an existing

vertical joint.

5.4 Generic Case 3 - Vertical Joint Connected to the Tunnel Boundary by a Horizontal Joint

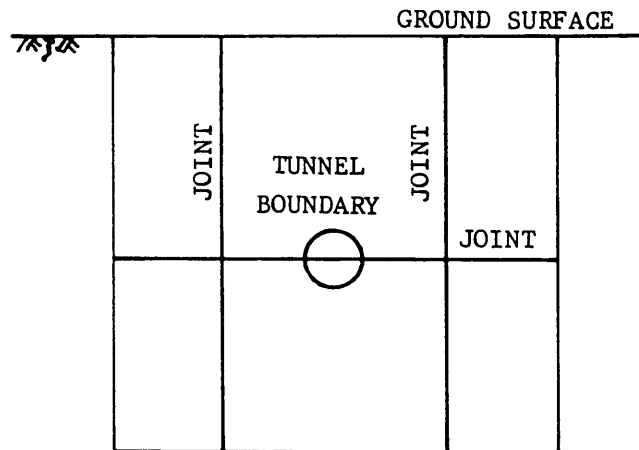


Figure 5.21 Generic case 3

Sometimes there is a set of vertical joints in the rock mass surrounding a pressure tunnel, none of which intersects the tunnel boundary directly. At the same time, there may be a horizontal joint which connects the vertical joints and the tunnel boundary. When the tunnel is put into operation, water under high pressure may penetrate the vertical joints through the horizontal joint and water flow occurs in the joint network. This may cause hydraulic jacking or hydraulic shearing in the vertical joints or uplift of the rock mass above the horizontal joint. This case is analyzed by means of the example shown in Fig.5.21. The corresponding finite element mesh shown in Fig.5.22 includes 102 finite elements, 28 joint elements and 155 nodes. The vertical distance between the ground surface and the tunnel centre is $4D$ and that between the tunnel centre and the bottom of computational domain is $3.5D$. The horizontal

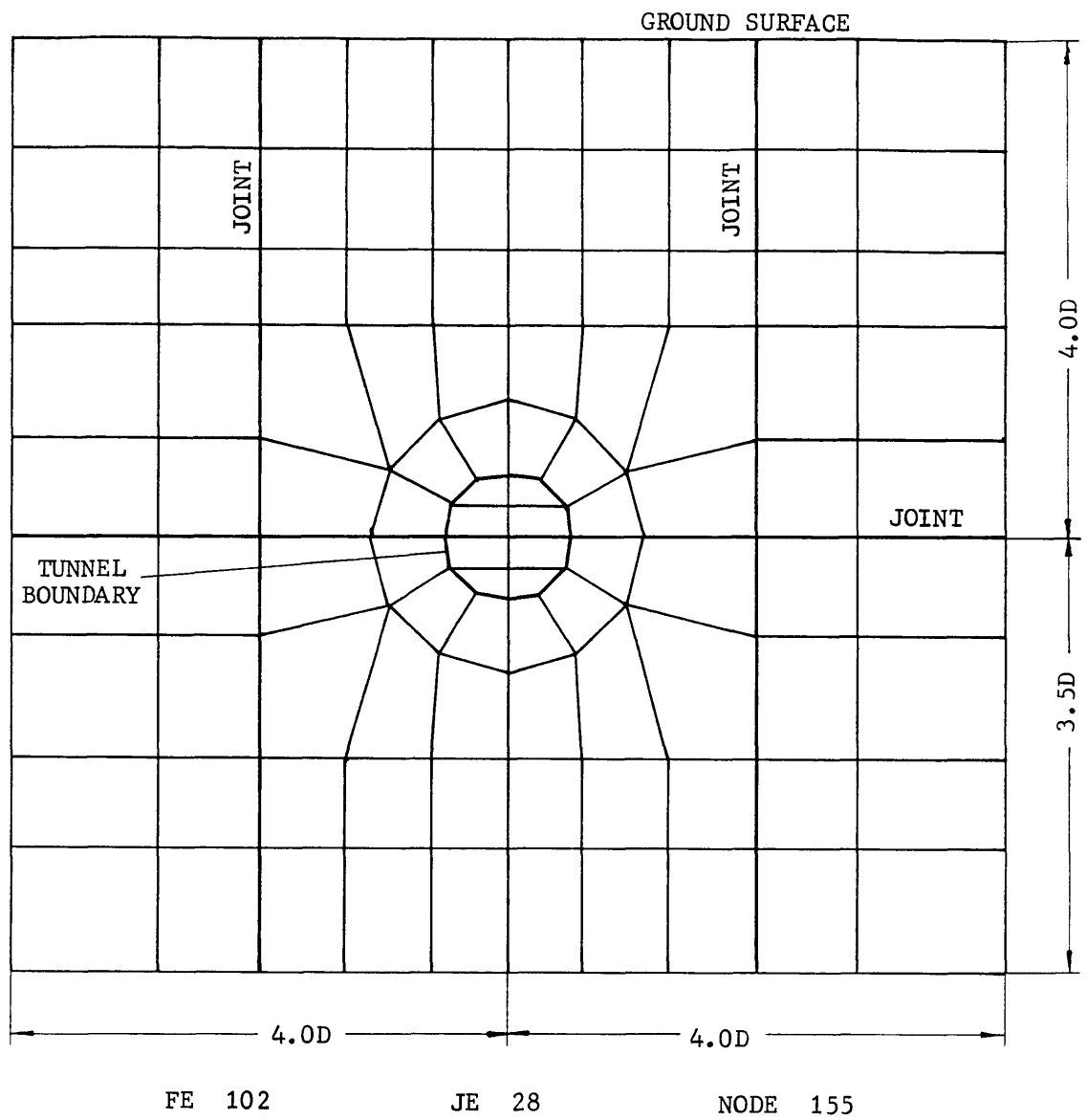


Figure 5.22 Finite element mesh for generic case 3 - Mesh 6

dimension of the domain is $8D$. The horizontal joint is coincident with the mid-height of the tunnel and two vertical joints are located on the left and right hand sides of the tunnel at equal distances of $2D$ from the tunnel centre line. The far left and the far right ends of the horizontal joint are not closed, but remain open (Fig.5.23). This means that opening is allowed along the whole horizontal joint including the two ends. The initial stress field is defined by $K_1 = 1.5$, $K_2 = 1.0$ and $\alpha = 0^\circ$. That means that this is a plane strain condition.

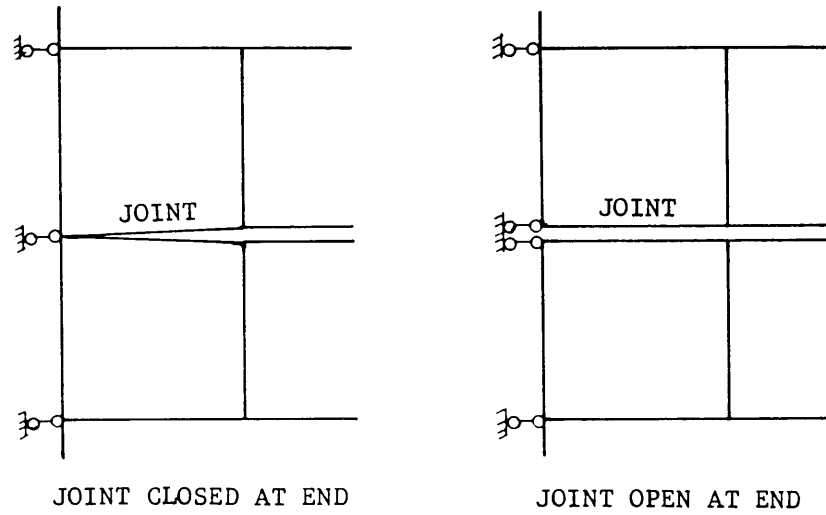


Figure 5.23 Two different boundary conditions for joint element

The shear stress component τ_{xy} disappears due to the plane strain condition. The other shear stress component τ_{yz} and the effective normal stress σ'_n are symmetric on the left and the right vertical joints and the left and the right hand parts of the horizontal joint.

In the initial stress field σ'_n is compressive and τ_{yz} has low values on all joints. During excavation the shear stress τ_{yz} on both vertical and horizontal joints increases; σ'_n also increases slightly (for the vertical joints it is almost

unchanged). As the water pressure is applied, the effective normal stress on both the vertical and the horizontal joints decreases (Fig.5.24) while the shear stress decreases on the horizontal joint and develops in an opposite direction on the vertical joints (Fig.5.25). The water pressure induced shear displacements on both the vertical and the horizontal joints are very small. Joint opening develops in both the vertical and the horizontal joints, but at a much higher rate in the horizontal joint than in the vertical joint (Fig.5.26). An abrupt enhancement occurs at water pressure 0.118MPa (12m hydraulic head) and convergence is not reached in the computation. At the same time, the uplift of the rock block above the horizontal joint increases rapidly as well. Fig.5.27 illustrates the variation of the uplift at the ground surface point directly above the tunnel crown with increasing water pressure. Therefore, the decisive failure mode for this case is uplift of ground surface associated with hydraulic jacking of horizontal joints. However, this is only for this particular initial stress condition. It is imaginable that if the horizontal initial stress ($K_1 P_z$) is lower, hydraulic jacking in the vertical joint may take place first and will become the decisive failure mode.

In this example, due to the boundary conditions adopted the uplift of the whole rock block above the horizontal joint has almost the same value (like a rigid body movement to some extent). If the two ends of the horizontal joint were closed (Fig.5.23), it could be expected that the uplift leading to failure may occur just within the rock block between the two vertical joints and above the horizontal joint. This may be more realistic.

On the whole, for this generic failure case there are two possible failure modes: hydraulic jacking in the vertical joints and hydraulic jacking in the horizontal joint leading to the uplift of the ground surface. Which in fact will be the decisive failure mode depends upon the specific conditions such as initial

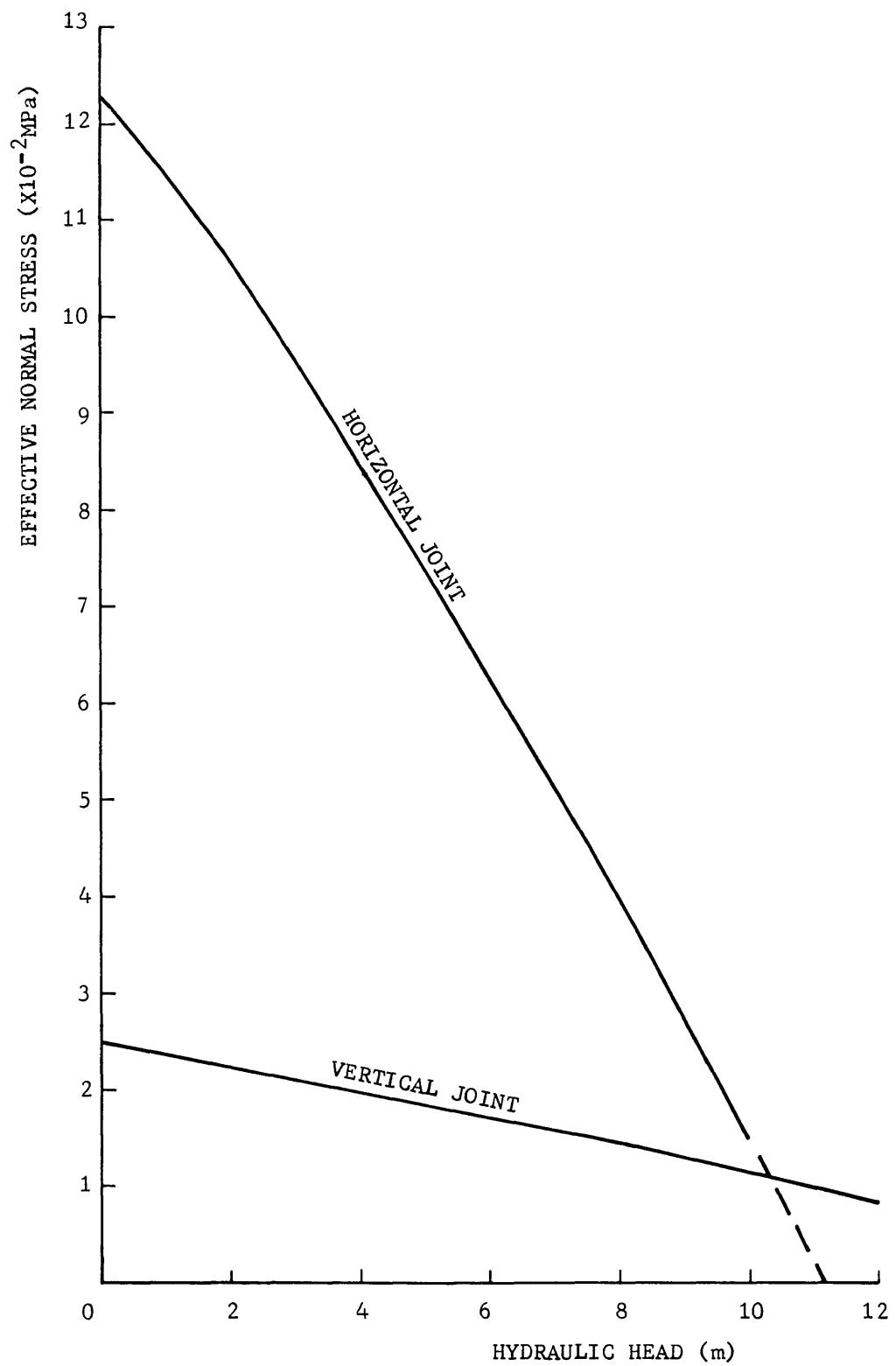


Figure 5.24 Effective normal stress for generic case 3

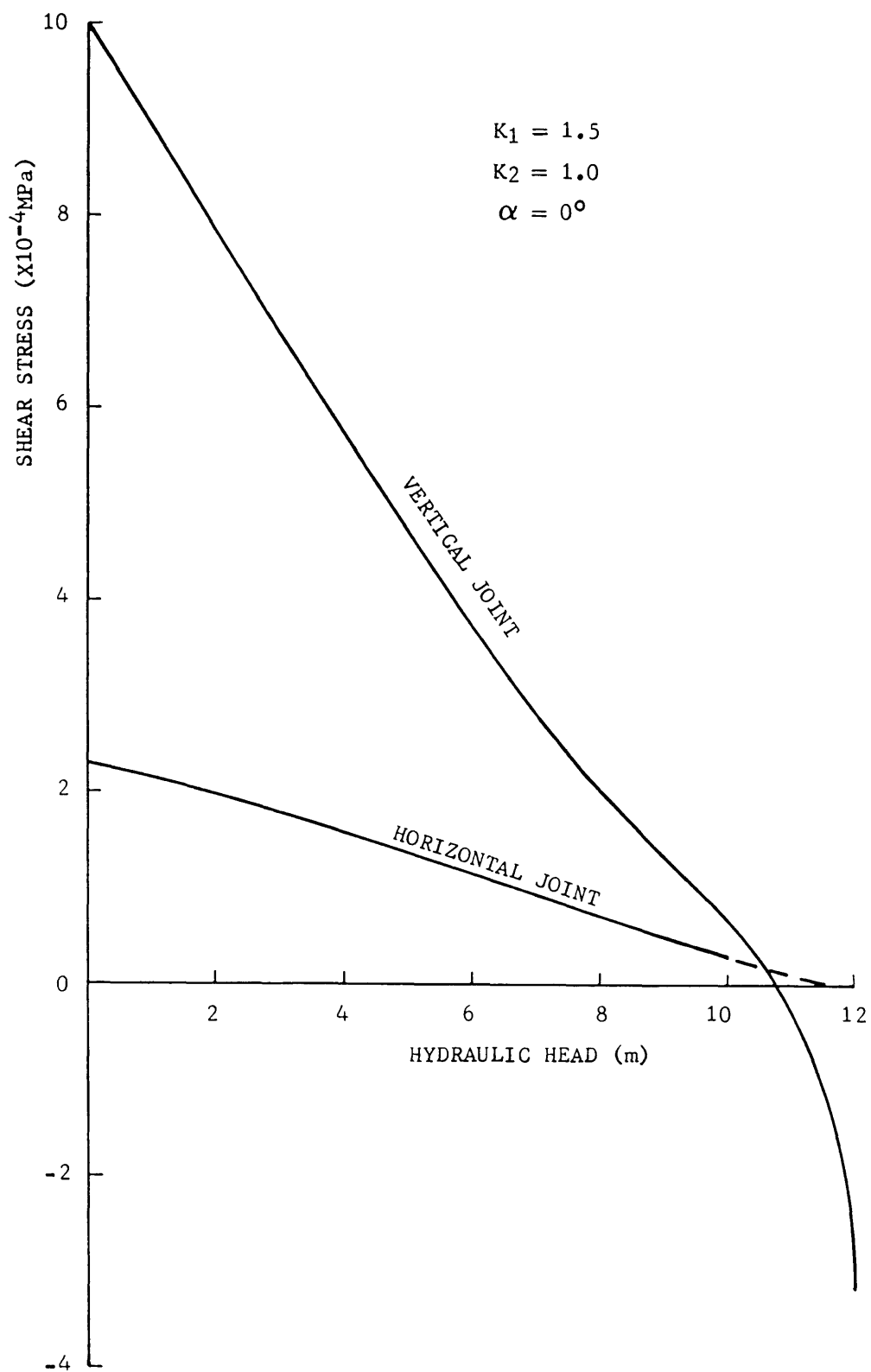


Figure 5.25 Shear stress for generic case 3

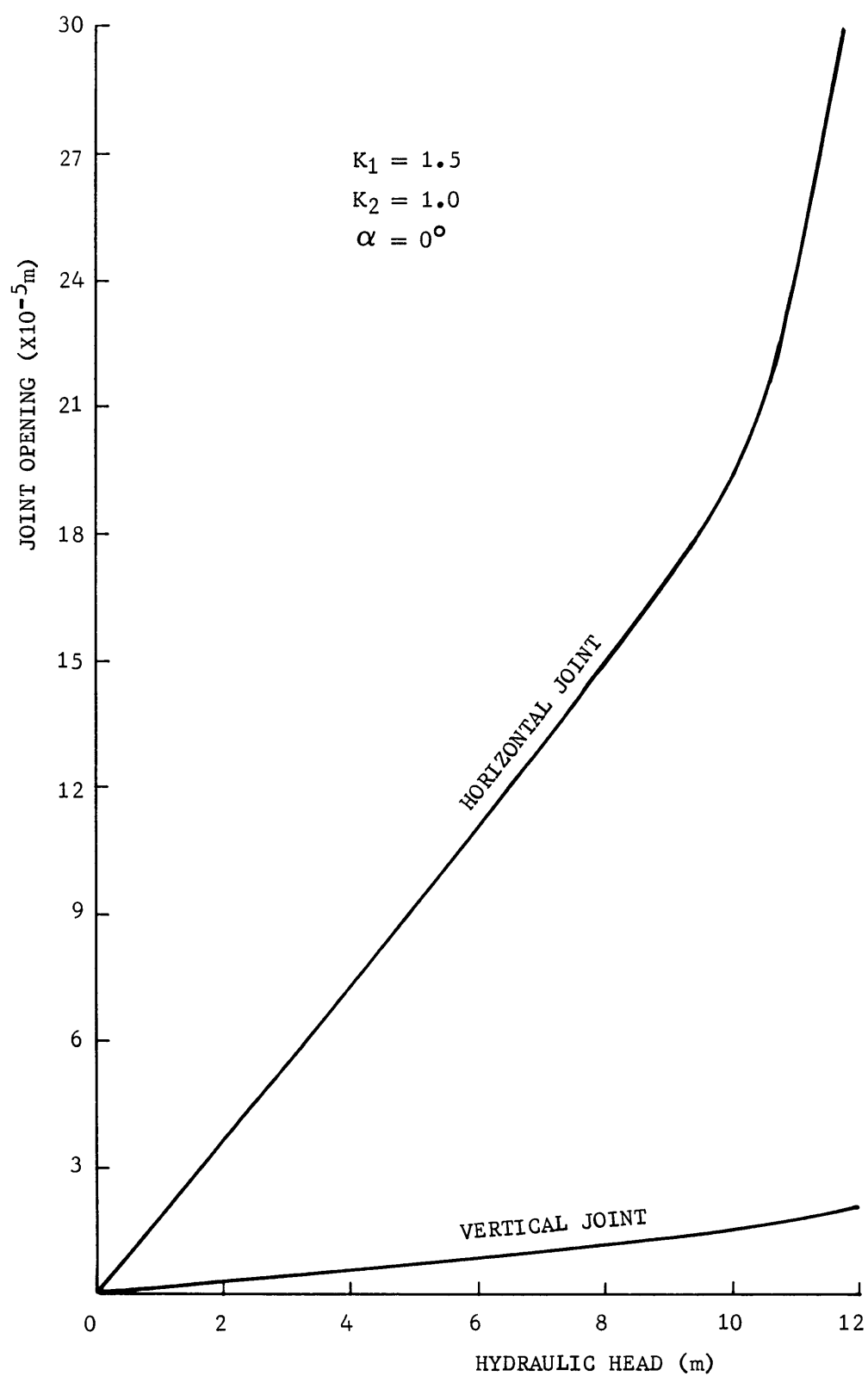


Figure 5.26 Joint opening for generic case 3

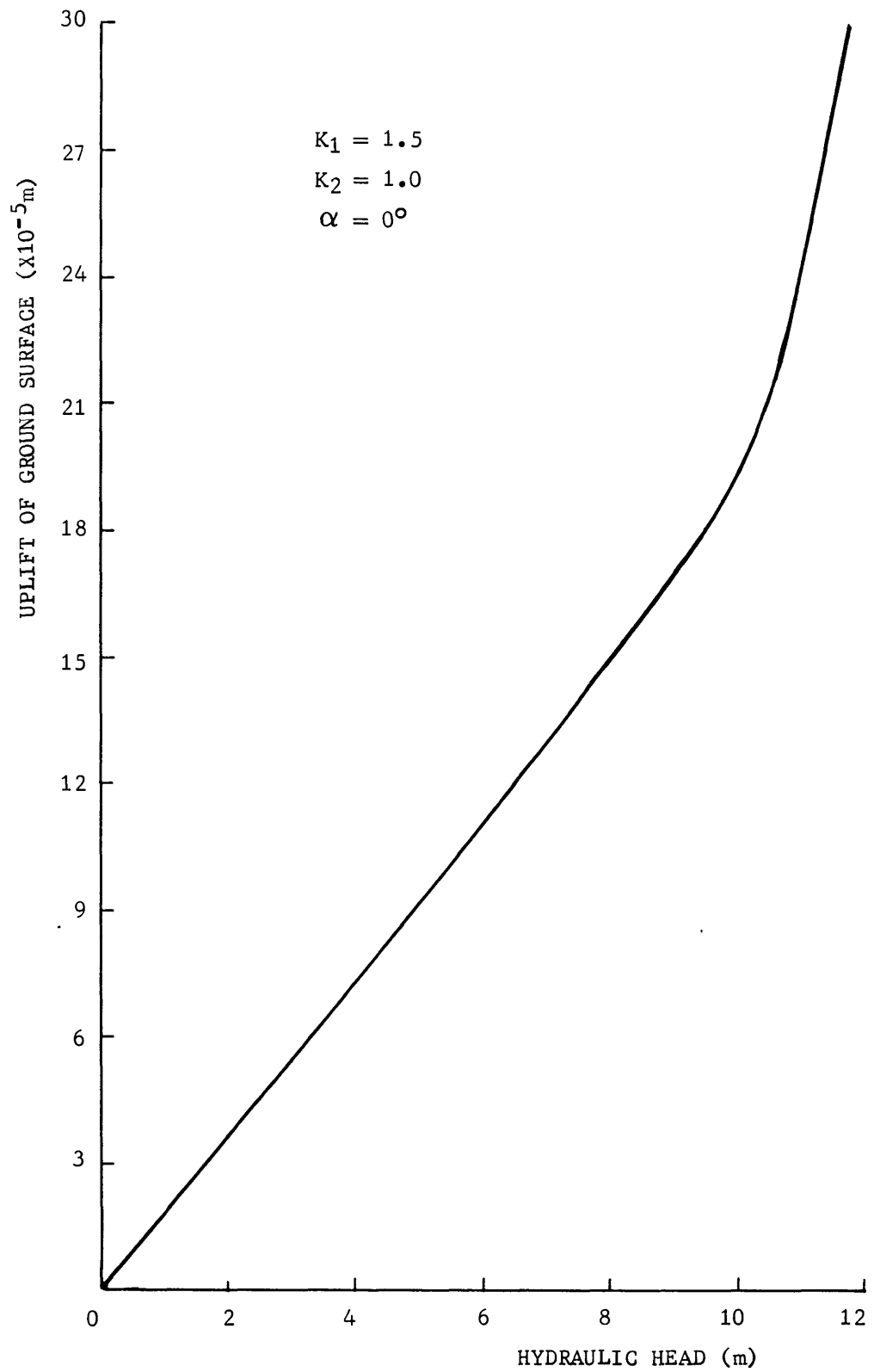


Figure 5.27 Uplift of ground surface for generic case 3

stress field and the thickness of the rock mass above the horizontal joint.

5.5 Simulation of a hypothetical real case

In the previous sections some generic cases of failure of unlined pressure tunnels were analyzed. This shows to some extent the applicability of the proposed numerical method (including the program DPTJR) in design analyses. The examples adopted are much more complex than those with which the analytical method can deal. To further verify the capacity of the proposed method to handle practical problems it would be helpful to analyze a real unlined pressure tunnel. However, there are difficulties in collecting geological information and mechanical properties of the rock mass for a given project. In this section an hypothetical real case is analyzed.

In this case a horseshoe shaped unlined pressure tunnel is excavated $33m$ below the ground surface in a rock mass containing two persistent horizontal and two persistent inclined discontinuities as shown in Fig.5.28. The width of the tunnel invert of is $4.8m$ and the height of the tunnel is $6m$. The material constants of the intact rock and joints are as follows:

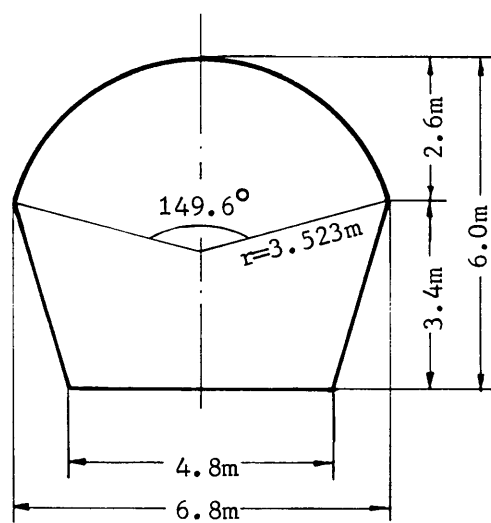
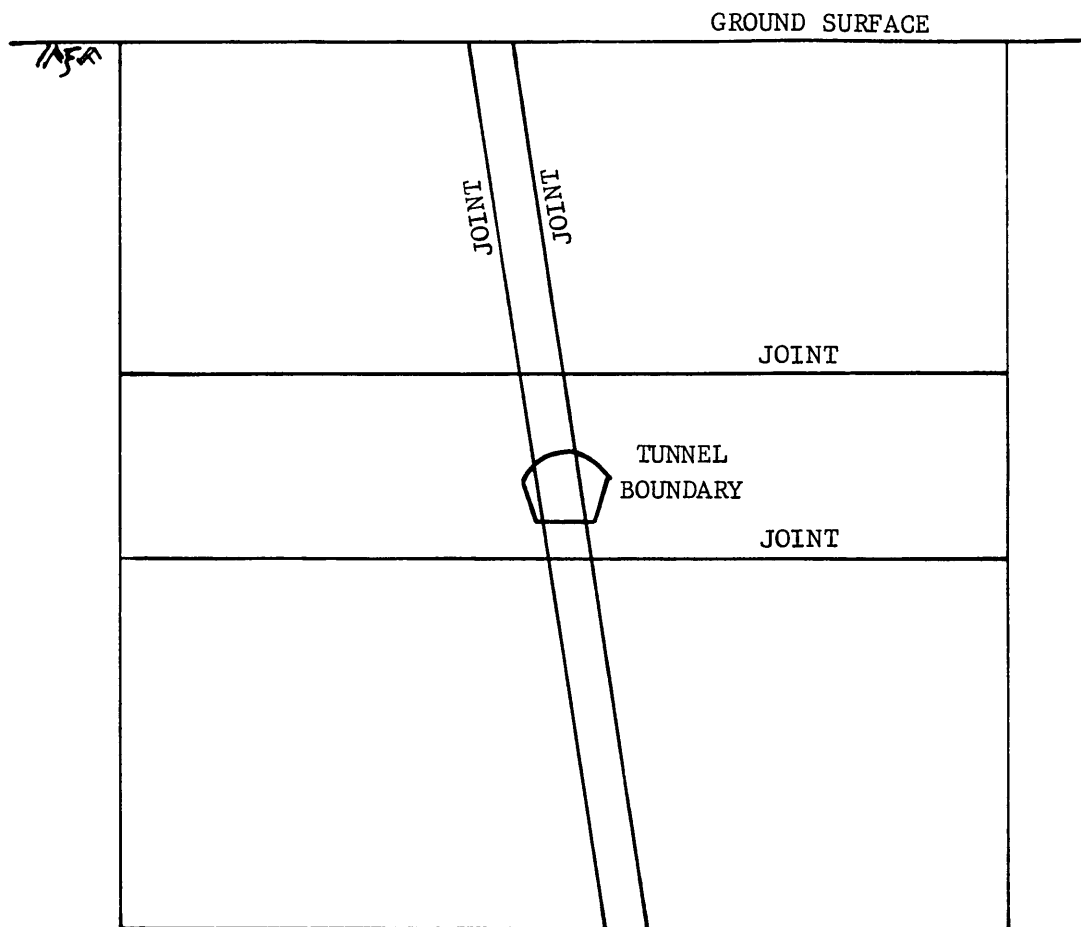
Young's modulus of intact rock $E_r = 10000MPa$

Poisson's ratio of intact rock $\nu = 0.25$

unit weight of intact rock $\gamma = 0.0265KN/m^3$

maximum closure of joint $w_m = 0.002m$

joint roughness coefficient at peak $JRC_p = 10$



$$\begin{aligned}
 E_r &= 10000 \text{ MPa} \\
 \nu &= 0.25 \\
 \gamma_r &= 0.0265 \text{ MN/m} \\
 w_m &= 0.002 \text{ m} \\
 JRC_p &= 10 \\
 JCS &= 100 \text{ MPa} \\
 \varphi_r &= 25^\circ \\
 K_{ni} &= 2000 \text{ MPa/m} \\
 L &= 1.0 \text{ m}
 \end{aligned}$$

Figure 5.28 A hypothetical real case

joint wall compressive strength $JCS = 100MPa$

residual frictional angle of joint $\phi_r = 25^\circ$

initial normal stiffness coefficient $K_{ni} = 2000MPa/m$

joint length $L = 1.0m$

The finite element mesh adopted is shown in Fig.5.29. The horizontal dimension of the computational domain is $72m$, i.e. 12 times of the maximum width of the tunnel. The vertical distance from the tunnel centre to the bottom of the domain is $33m$, i.e. 5.5 times of the height of the tunnel. The mesh contains 106 finite elements, 42 joint elements and 177 nodes.

The parameters defining the initial stress field are $K_1 = 1.5$, $K_2 = 1.0$ and $\alpha = 30^\circ$, i.e. this is the complete plane strain condition. The excavation equivalent forces are applied in ten steps. The internal water pressure is exerted in a step-wise manner in increments of 5 meters hydraulic head, i.e. $0.046MPa$.

In the initial stress field both horizontal and inclined joints have a compressive normal stress and relatively low shear stress. After excavation, the normal stress diminishes on horizontal joint elements and increases on inclined joint elements slightly. As the internal water pressure is applied increasingly the normal stress on all horizontal and inclined joints decreases (Fig.5.30). But, it varies the fastest on upper horizontal joint elements. Opening of horizontal joint elements and uplift of the ground surface also develop as the water pressure increases (Fig.5.31). Fig.5.32 illustrates the variation with water pressure of shear displacements on inclined joints in the direction parallel to the tunnel axis and parallel to the joint dip direction. The final uplift of the ground surface is presented in Fig.5.33. From these two figures it can be seen that the shear displacement along the inclined joints also increases as water pressure is

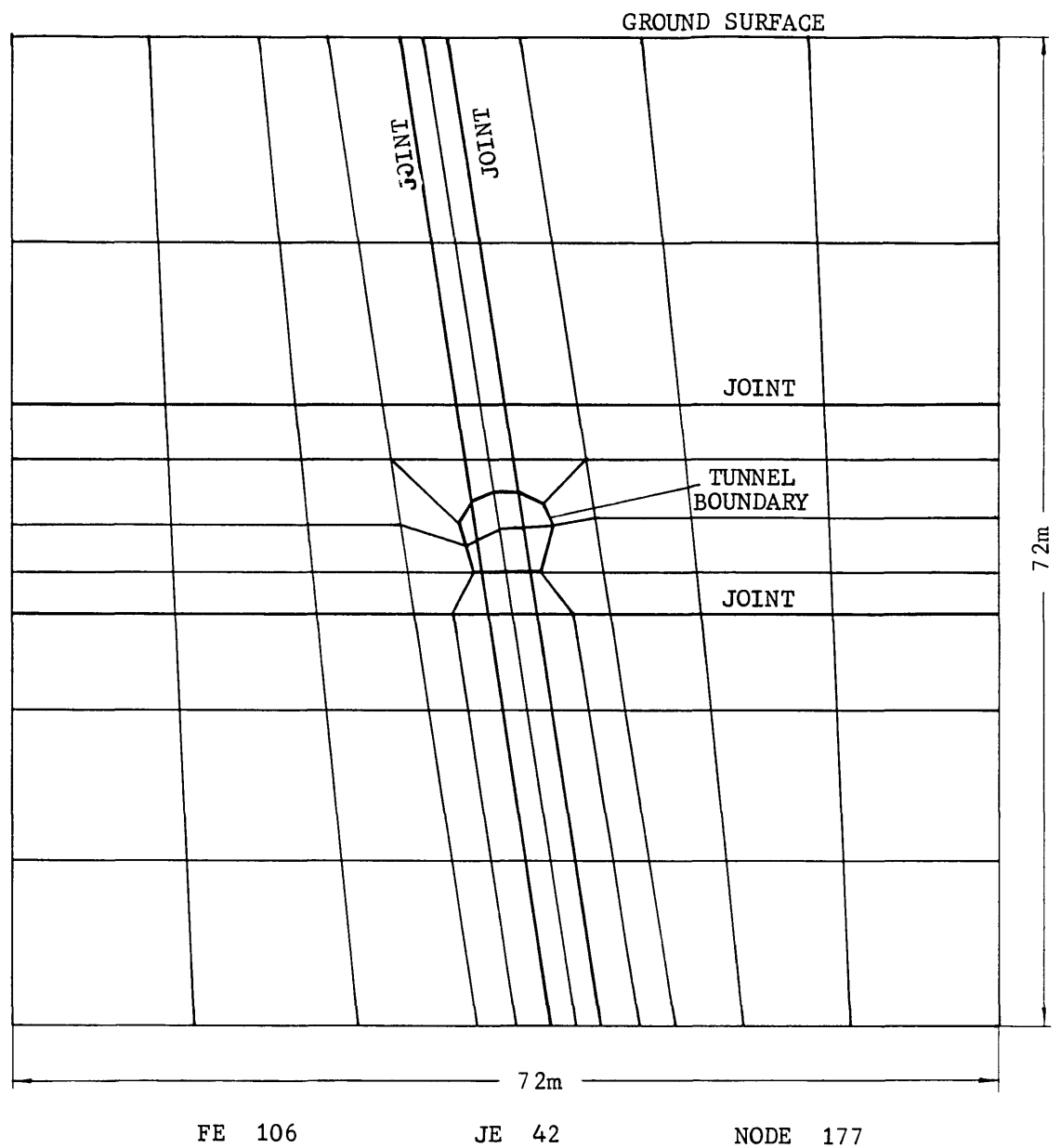


Figure 5.29 Finite element mesh for the hypothetical real case

- Mesh 7

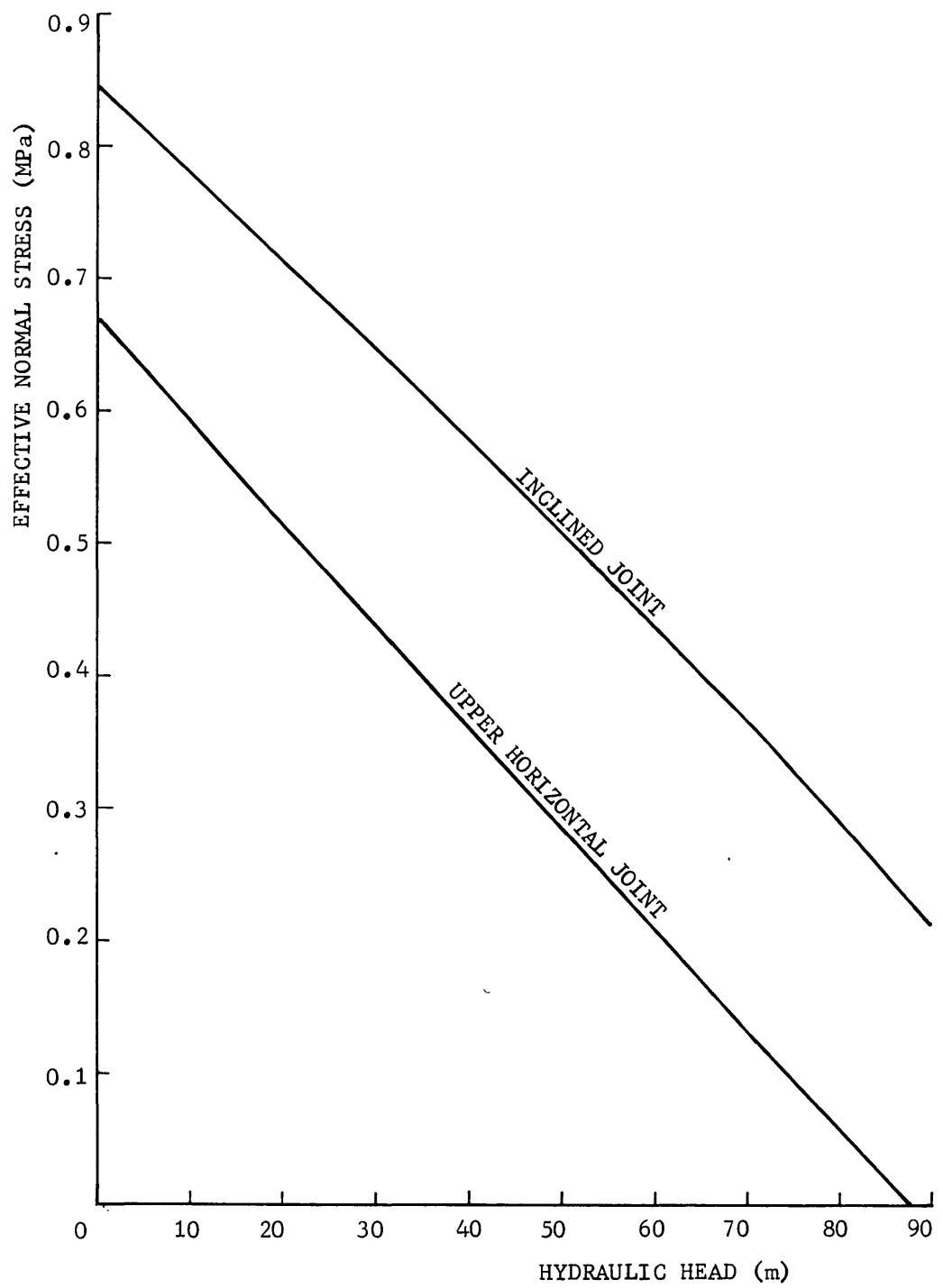


Figure 5.30 Effective normal stress on horizontal and inclined joints for the hypothetical real case

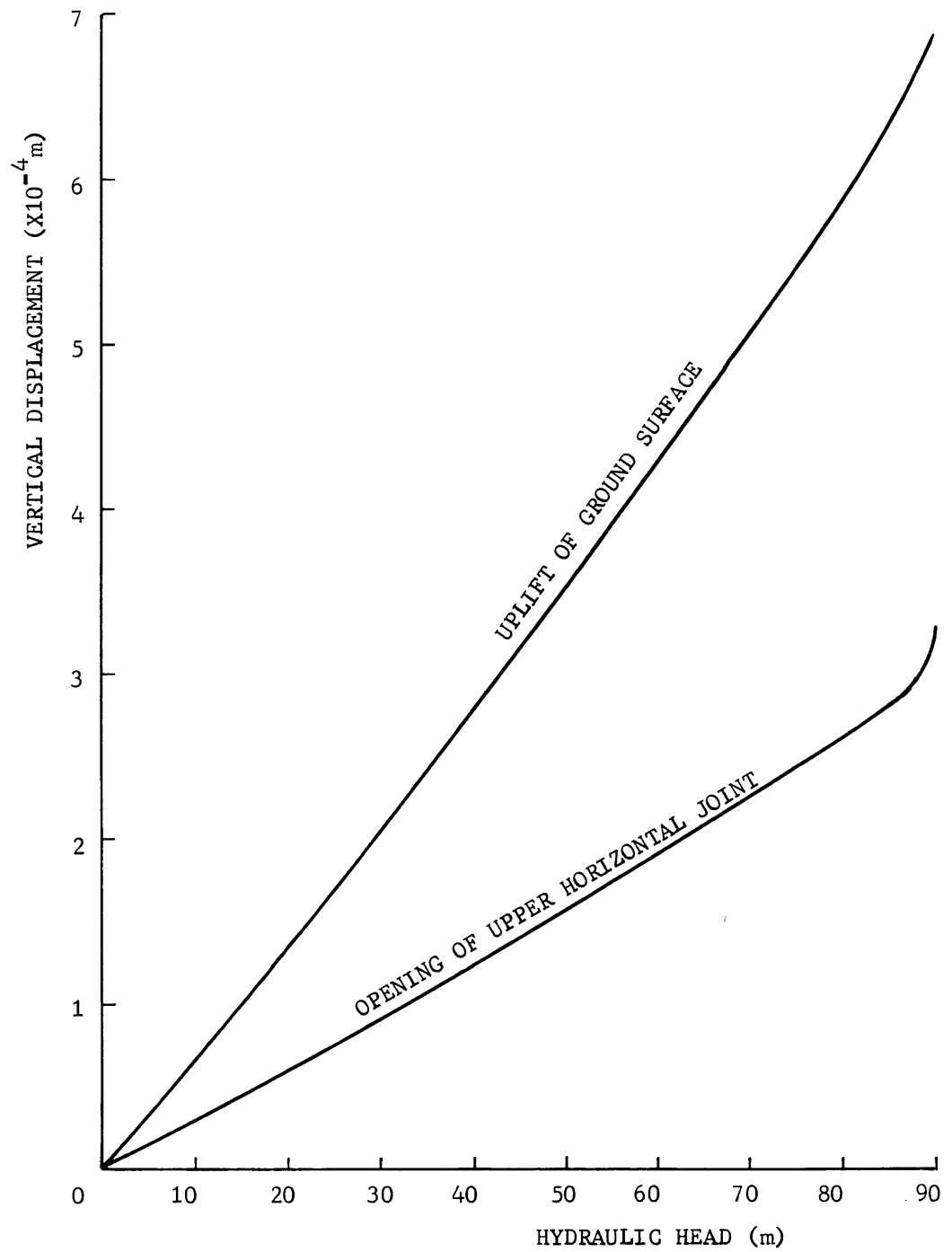


Figure 5.31 Opening of upper horizontal joint and uplift of ground surface for the hypothetical real case

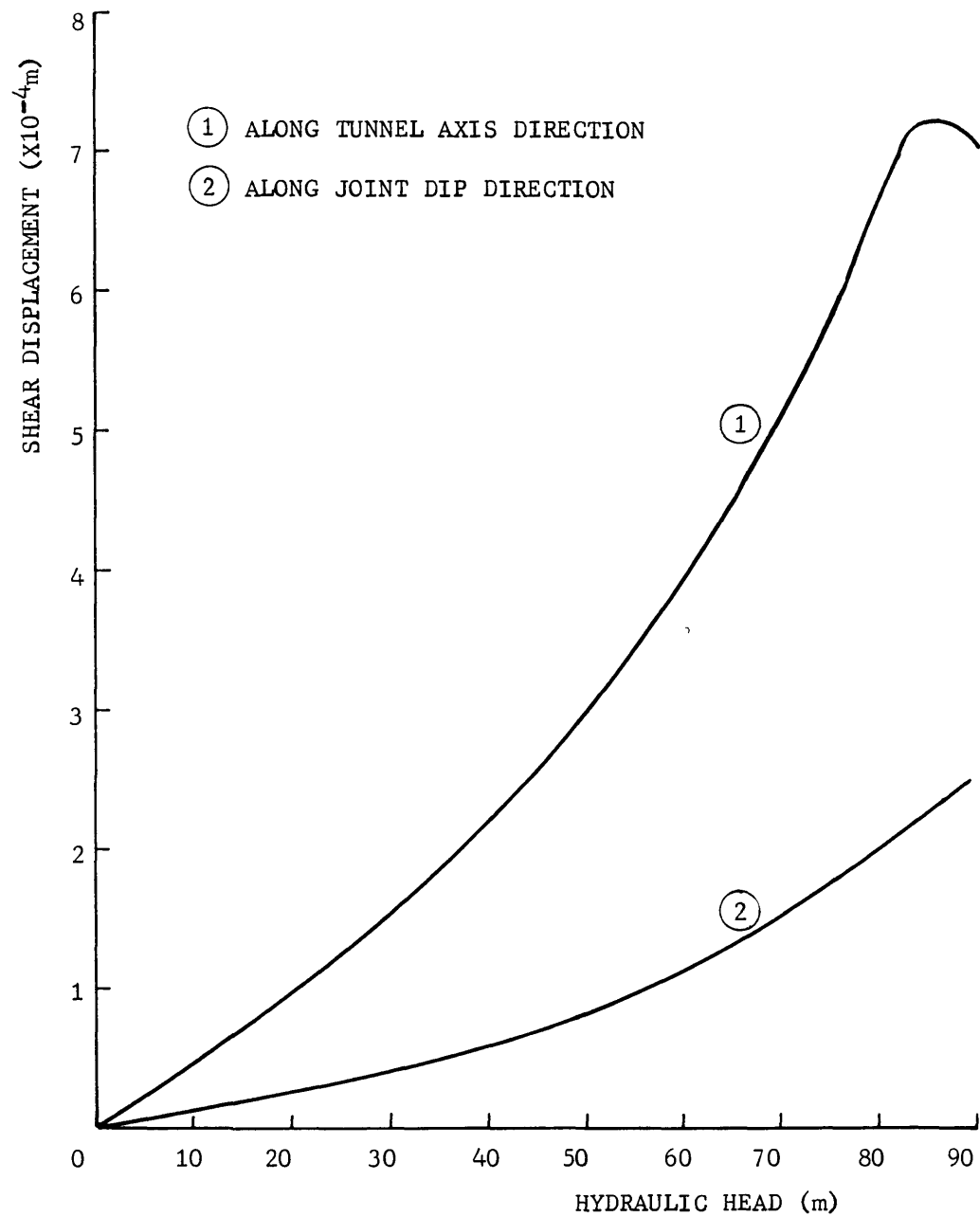


Figure 5.32 Shear displacement on inclined joint for the hypothetical real case

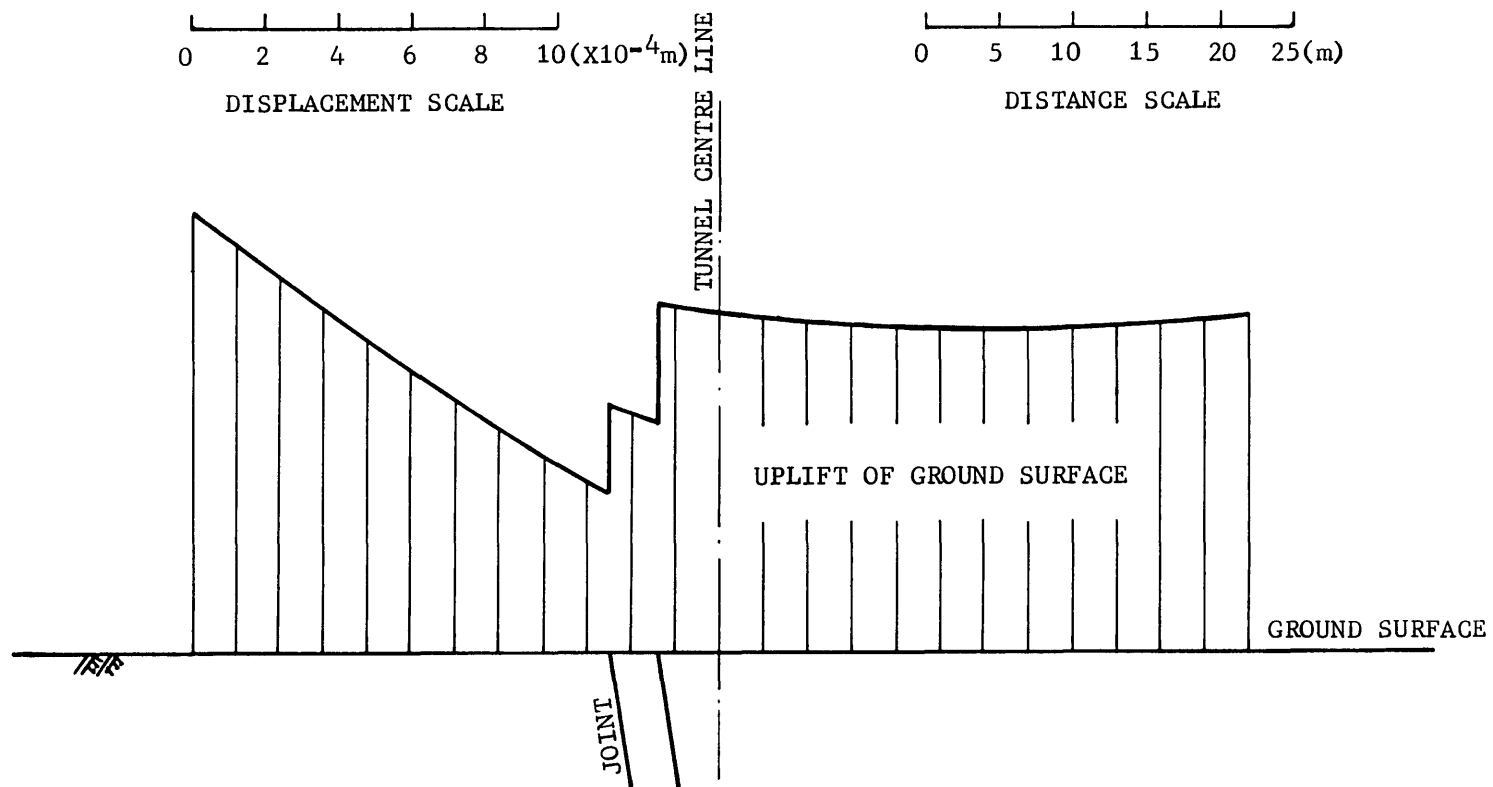


Figure 5.33 Uplift of ground surface for the hypothetical real case

applied. Therefore, for this example there are two possible failure modes, uplift of the ground surface associated with hydraulic jacking of the upper horizontal joint and hydraulic shearing on the inclined joints. For these specific values given for the initial stress field, the normal stress on the upper horizontal joint becomes zero at hydraulic head 90 meters, i.e. water pressure 0.883MPa . At the same time, opening of the upper horizontal joint speeds up abruptly and convergence is not reached in the computation.

The conclusion to be drawn is that for this particular case uplift of the rock blocks above the upper horizontal joint associated with hydraulic jacking of the upper horizontal joint is the decisive mode of failure occurring at an internal water pressure of 0.883MPa .

5.6 Summary

This chapter presents the results of numerical analyses of generic cases of unlined pressure tunnel failure carried out using the proposed numerical method. The essential failure modes of unlined pressure tunnels, hydraulic jacking, hydraulic shearing and the uplift of the ground surface, have been revealed. In the case of absence of horizontal joints near the ground surface hydraulic jacking or hydraulic shearing in the vertical joints is the most likely failure mode. Usually hydraulic jacking accompanied by water leakage is the main failure mode. However, if the tunnel axis is not parallel to the direction of one initial principal stress and $K_1 \neq K_2$ hydraulic shearing may become more critical, especially when the initial shear stresses are at a high level.

The existence of horizontal joints between the tunnel and the ground surface will probably lead to a very dangerous situation if the joints are connected to the tunnel boundary by other joints. In this case the uplift of the ground

surface associated with hydraulic jacking of the horizontal joints is the main failure mode occurring at a relatively low internal water pressure.

In terms of numerical analysis there is no difference between the cases of the ground surface being horizontal or inclined. The three main failure modes are still applicable to the case in which the tunnel is near a valley side. However, it should be mentioned that slope slide may occur if a rock wedge near the valley side is formed by joints and hydraulic jacking occurs in the joints.

The critical water pressure for failure of unlined pressure tunnels may be influenced by the magnitudes and the directions of the initial stress field, the distribution of pre-existing discontinuities and the mechanical and hydraulic properties of the rock mass. A very low level of the initial principal stress perpendicular to the joint plane causes a disadvantageous condition.

In order to leave a pressure tunnel unlined it is necessary to do a relatively detailed geological survey to obtain data about the initial stress field and the distribution of the discontinuities. If the initial horizontal principal stresses are at a reasonably high level and horizontal joints near the ground surface do not exist the tunnel may remain unlined without significant engineering treatments provided that the operating water pressure is lower than the critical water pressure with a sufficient margin. However, in the case of the existence of near surface horizontal joints adequate engineering measures must be taken to ensure that water under pressure cannot enter the horizontal joints. These measures are local treatments and may be shotcrete, grouting or even lining.

The results presented in this chapter further verify the validity of the proposed numerical method and the program DPTJR. From these results it is possible to conclude that the method and the program are capable of dealing

with the design analyses of unlined pressure tunnels in jointed rocks.

CHAPTER 6

SUMMARY AND CONCLUSIONS

6.1 Conclusions

A comprehensive analysis of failure mechanisms of unlined pressure tunnels is carried out based on the investigations into 50 pressure tunnel failures. A review and assessment of currently used design methods and criteria indicates the necessity to develop a new design analysis approach.

A program and design charts for analytical solutions for opening and hydraulic jacking of joints which intersect deep, unlined pressure tunnels under a range of in situ stress magnitudes and orientations are developed based on the Kirsch equations. These solutions are used to determine the length of the steel lining for the preliminary design in the proposed design analysis approach.

A numerical method is developed which uses Barton's empirical joint model and the theory of elasto-plasticity with a non-associated flow rule to reproduce the experimentally observed pre- and post- peak plasticity behaviour and associated dilational characteristics of rock joints. In the joint elements adopted the relative displacements between two joint walls are taken to be the independent and essential unknowns so that the numerical ill-conditioning associated with high normal stiffnesses is avoided. T shaped and cross joint elements are developed to model joint intersections. The incremental secant stiffness iteration procedure is developed that ensures numerical convergence in cases of non-linear behaviour including strain softening. A finite element procedure is developed to calculate the seepage water pressure acting on the joint walls with the permeability coefficient estimated by a coupled mechanical and hydraulic analysis. The complete plane strain model is adopted so that

some aspects of the three dimensional nature of the problem can be allowed for.

The numerical joint model is verified by a number of examples with a single joint element. The proposed numerical method is used in the numerical analyses of some generic cases of unlined pressure tunnel failure. From these analyses three major failure modes, hydraulic jacking, hydraulic shearing and uplift of the ground surface, are identified for unlined pressure tunnels. The effects of in situ stresses, overburden thickness and the locations of joints parallel to the ground surface are demonstrated. Low lateral in situ stresses, thin overburden cover and the existence of a joint parallel and near to the ground surface are adverse conditions for unlined pressure tunnels. The analysis of a hypothetical practical case further verifies the capability of the proposed method in dealing with real engineering problems.

6.2 Suggested Approach and Guidelines to Unlined Pressure Tunnel Design Analysis

The research described indicates the necessity, and provides a sound basis for developing a rational and comprehensive design analysis approach and for establishing some guidelines for use in the design of unlined pressure tunnels.

6.2.1 Suggested approach to design analysis

The proposed approach includes field work, laboratory tests, analytical solutions and numerical analyses. It comprises of the following steps.

- (1) It is essential when designing an unlined pressure tunnel to carry out a detailed geological survey. This survey may include mapping and plotting of all rock exposures along the route of the tunnel, drilling boreholes along the

tunnel axis especially in the area of intersection between the intended steel lined and unlined section, careful mapping of discontinuities and recording leakage into the tunnel. Identification of faults and joints through which the tunnel will pass is the major task of the survey. The following information about the joints should be gained from the investigations.

(i) Location, strike, dip and dip direction of the discontinuities in the rock mass surrounding the tunnel. Attention should be paid not only to the joints intersecting the tunnel but also to those not intersecting the tunnel but near the ground surface or a valley side. After the tunnel is filled a path between the tunnel boundary and the joints may be formed, and as soon as this situation arises hydraulic jacking on the joints can occur readily.

(ii) Infilling in the joints intersecting the tunnel boundary. Rock joints filled with erosion-susceptible materials, such as montmorillonite, mylonite and calcite, are disadvantageous in unlined pressure tunnels. Proper treatment is required for such joints and infilling. Therefore, it is important to identify all such joints and their infilling. However, it is not necessary, and not practical either, to conduct such investigations before the tunnel is excavated. The investigations should be carried out during the excavation of the tunnel.

Some necessary in situ or laboratory tests should be carried out to estimate the mechanical and hydraulic properties of the rock and rock joints, including Young's modulus, Poisson's ratio, joint roughness coefficient, joint wall compressive strength, maximum joint closure and residual frictional angle.

(2) From the analyses described in the foregoing chapters, it is clear that the initial stress field is one of the dominant factors that determine whether or not a pressure tunnel can remain unlined. Both analytical solutions and numerical analyses take the initial stresses as basic input data. Therefore, it

is necessary to estimate the magnitudes and orientations of the initial stresses prior to the excavation. Although a number of in situ measurement results in various geological and geographical conditions are available and some regular patterns have been presented, carrying out in situ measurements at each individual site is still preferable due to the variation of in situ stresses from one location to another.

In the area of uneven ground surface, such as protruding ridge 'noses' between gorges, steep or irregular topography, the lateral in situ stresses are usually reduced by stress relief. If an unlined pressure tunnel is situated in such an area the potential for hydraulic jacking will be increased. What is more, the initial stress field in such an area is often complex and non-uniform and difficult to predict. Therefore, it is essential to determine the stress field by in situ measurements.

Many methods are available for measuring the in situ stresses, such as overcoring, flatjack and hydraulic fracturing. However, there has been no one method to date that has proved to be totally satisfactory because of the difficulties involved (Fairhurst 1986), especially at low depths or near a valley side where the in situ stresses are low. It is suggested that in situ stress measurements be performed and the field stresses calculated, both the magnitudes and orientations, at several carefully chosen locations along the tunnel, where the results of the in situ measurements are reliable. The far field stresses can be estimated from the measured field stresses. Then the field stresses at other locations, where it is difficult to perform reliable in situ stress measurements, can be obtained by surveying the topography and carrying out finite element computations with the far field stresses as boundary loading conditions.

When the construction schedule does not allow the in situ measurements to be carried out prior to the excavation, the in situ stress measurements should be

carried out as soon as the tunnel excavation has started, so that if unexpected adverse conditions emerge some sort of remedial measures can be taken.

(3) Once the pre-excavation initial stress field has been estimated the analytical solutions presented in Chapter 2 can be used to evaluate the critical tunnel water pressure for hydraulic jacking. When the tunnel axis is perpendicular to the vertical in situ stress the design charts shown in Fig.2.14 to 2.33 can be used. If the tunnel axis is inclined to all three in situ principal stresses the program ASJOHJ given in Appendix 4 can be used. This critical water pressure can be taken to be the limiting tunnel water pressure, p_c , for the unlined pressure tunnel. The allowable tunnel water pressure, p , may be obtained by dividing the limiting water pressure, p_c by a factor of safety, F_s ,

$$p \leq \frac{p_c}{F_s} \quad (6.1)$$

The factor of safety is a comprehensive parameter which may be affected by the accuracy of the in situ stresses, complexity of the geological formation, accuracy of the operating water pressure and significance of the project. Considering the margin in the operating water pressure and the double check in the area of complex geological conditions, it is not necessary for the factor of safety to be given too high a value. However, the in situ stresses are difficult to estimate precisely. Taking all these factors into account in a comprehensive way and referring to other design methods and some case histories, it is suggested that a value of 1.1 be taken for the factor of safety, if p is the maximum dynamic pressure, and a factor of safety of 1.3 if p is the maximum static pressure.

Having calculated the allowable water pressure at a series of locations along the tunnel axis an allowable water pressure line can be drawn. Drawing the maximum operating water pressure line on the same diagram, the length

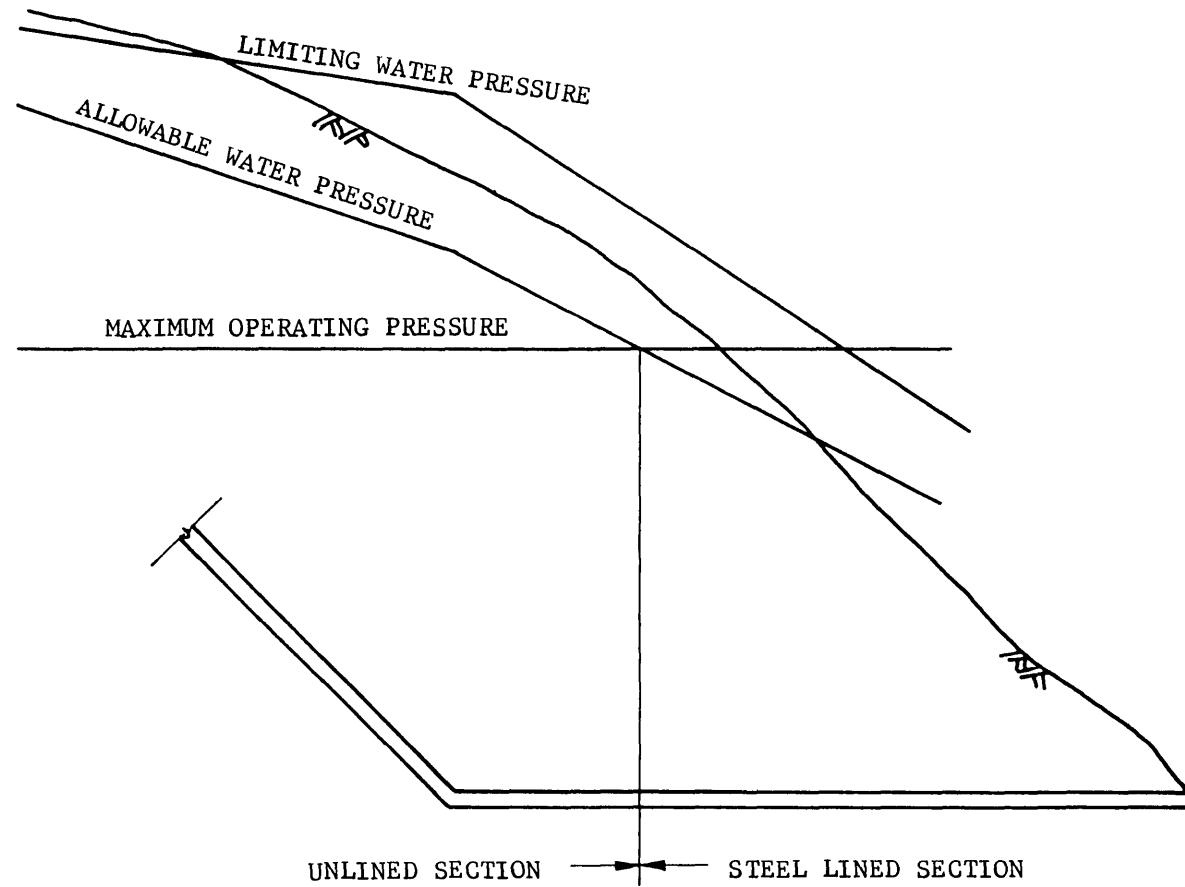


Figure 6.1 Suggested method for determining the length of steel lining in preliminary design

of the steel lining can be determined, such that the intersection of the two lines is the end of the steel lining, as illustrated in Fig.6.1.

(4) The length of the steel lining determined from step (3) can be used in the preliminary design. In the design stage, a numerical analysis following the proposed numerical procedure and using the program DPTJR should be carried out for the cross sections where the tunnel is intersected by rock joints or the rock is weak. The critical water pressure predicted from the numerical analyses, i.e. the water pressure for joint opening, hydraulic jacking, hydraulic shearing, uplift of the ground surface or the tensile failure of the intact rock, can also be substituted into Eqn.6.1 for p_c to determine the allowable tunnel water pressure p . If more adverse conditions are predicted, either some remedial measures must be taken or modifications to the preliminary design must be done, or both. If the location where the adverse conditions occur is near the end of the steel lining, as determined from step (3), it is suggested that the steel lining be extended. Otherwise, local treatment is recommended, such as grouting, shotcreting, concrete or reinforced concrete lining or even steel lining. The major purpose of the treatment is to reduce the permeability along the rock joints to an acceptable level.

(5) Usually the design from the procedure described above is a rational one. However, for further confirmation a conventional hydraulic splitting test or an acceptance test proposed by Hoek (1982) may be carried out. The hydraulic splitting test should be conducted on the major joints. The test site for the Hoek's acceptance test should be chosen in the region of the end of the steel lining and at the tunnel elevation. Considering the large difference in scale between the borehole and the pressure tunnel, this test can be done by using the 'fracture re-opening pressure/flow' observations, rather than the initial 'fracture breakdown pressure/flow' observations (Fairhurst 1986). If the

test does not unveil any problems a final decision on the design can be made. Otherwise, some modifications must be made.

(6) Careful monitoring and supplementary geological investigation and measurement during construction are a vital component of a successful design. No matter how detailed the geological investigations have been, the complete and genuine geological features of the rock mass, surrounding a pressure tunnel, cannot be found until the tunnel has been excavated. Once the tunnel has been excavated, the geology can be better visualized. This affords a good opportunity to check all information that has been obtained from the geological survey. This also gives a final chance to modify the design, if significantly adverse conditions, which violate the condition based on which the design has been made, are identified.

Observing all leakages into the tunnel even small dripping or seeping, during construction, is significant since the paths of the leakages are likely to become water flow paths to the ground surface after the tunnel is filled. All geological features such as joints, faults, dykes and fractured zones which could invalidate the design should be mapped during construction. In situ stress measurements and hydraulic fracturing or hydraulic splitting tests should be carried out during construction, if not enough of such measurements and tests have been performed prior to the excavation and/or if full confidence has not been developed.

If any unanticipated adverse conditions are revealed during construction some modifications to the original design must be made. This may consist of lowering of the level of the steel lining, extending the steel lining or local remedial treatment.

The preceding six steps constitute a comprehensive design analysis ap-

proach to unlined pressure tunnels. It is expected that following these steps will lead to an economic and safe design. However, it is not absolutely necessary for each design to go through the entire procedure, depending on the significance of the project, availability of the facilities for the measurements, tests or computations, and the construction schedule. But if a certain step is not passed, some alternative analysis must be carried out.

6.2.2 Guidelines to unlined pressure tunnel design

For a pressure tunnel to remain unlined in a jointed rock mass the following requirements should generally be met.

(1) The rock surrounding the tunnel is strong, massive rock. High Young's modulus, high tensile strength and low permeability are preferred. If the tensile strength of the rock is too low, tensile failure may occur in the intact rock, starting from the boundary of the tunnel, then propagating into the rock, finally connecting with some pre-existing joints or reaching the ground surface and forming a leakage path. Hard metamorphic rock, such as gneiss and granite, have been proved to be a suitable host for unlined pressure tunnels.

(2) Pre-existing joints play an important role in failure of unlined pressure tunnels. All discontinuities, including those intersecting the tunnel boundary and those not intersecting the tunnel boundary but near the ground surface or a valley side, must be carefully mapped. Special attention should be paid to the joints near the ground surface or a valley side because once water under pressure enters these joints, uplift of the ground surface or landslide associated with hydraulic jacking of the joints may occur. The difficulty is that it is not easy to explore these joints since they do not intersect the tunnel boundary. Boreholes drilled from the ground surface may be able to explore such joints.

All major joints intersecting the tunnel boundary and those joints linking the tunnel boundary and the ground surface must be properly treated to seal the potential water flow path, at least to minimize the permeability.

(3) All joints which intersect the tunnel boundary and are filled with erosion-prone materials must be carefully and properly treated and any potential water flow path to such joints must be completely sealed, so that erosion will not occur. Erosion of the filling in a joint can substantially decrease the strength of the joint and provide a water flow path. This may be a long term process. The tunnel may experience no problems at the onset of operation, but leakage or rockfalls may occur several years later. It is almost impossible to estimate the effects of erosion by analytical or numerical analyses.

(4) A relatively high value of lateral in situ stresses is favoured. Care must be taken when the lateral in situ stresses are low, since opening and hydraulic jacking are likely to occur on the vertical and inclined joints at a low tunnel water pressure. This condition is likely to occur when the tunnel is situated in an irregular topography or near a valley side. Attention must be paid to estimate the field stress in such areas, due to the difficulties in measuring the in situ stresses when the stress components are low.

(5) An unlined pressure tunnel should be so aligned that the overburden rock is sufficiently thick. With too little overburden hydraulic jacking may readily occur on joints intersecting the tunnel. Taking the thickness of the overburden rock as the sole factor for determining the length of the steel lining, as the rule of thumb, is unreasonable. The discontinuities and in situ stresses may play a more important role.

(6) Hydraulic shearing may become the critical failure mode when the difference between the in situ principal stresses is large and the tunnel axis is

inclined to all three in situ stresses. To avoid hydraulic shearing, it had better to prevent water under pressure from entering the joints, if suitable treatment can be performed in practice.

6.3 Suggestions for Further Research Work

(1) Extension of the program to three dimensions

Although the complete plane strain model adopted has great advantage over the standard plane strain model, it is obviously not as good as the fully three dimensional model. The limitations caused by this have been described in Section 4.2. These limitations will be eliminated if the fully three dimensional model is adopted, i.e. joints with any strike, dip and dip direction can be analyzed.

(2) Determination of initial stress field

The accuracy of the results obtained from the numerical analysis depends to a large extent upon the accuracy of the initial stress field. The initial stress field is obtained by a finite element calculation with lateral far field stresses applied on the lateral boundaries of the problem domain. The far field stresses are estimated from in situ stress measurements. It is obvious that the initial stress field estimated in this way cannot properly match the real field situation. The problem of estimating the initial stress field from the results of a limited number of in situ stress measurements has not been resolved to date. Solving this problem not only helps with the design of unlined pressure tunnels, but is also important to all underground excavation problems.

(3) Crack initiation and propagation

Under the application of internal tunnel water pressure, cracks may be initiated in the intact rock on the boundary of the tunnel. As the water pressure increases, the cracks will propagate into the interior of the rock from the tunnel boundary and new joints will be formed. During the next filling, the newly formed joints will behave simply like pre-existing joints. This means that the failure will occur under a tunnel water pressure much lower than that for initiating and propagating a crack. For simplicity this phenomenon is not taken into account in the proposed numerical method. However, to simulate behaviour of unlined pressure tunnels more realistically and comprehensively this feature should be considered.

APPENDIX 1 PRESSURE TUNNEL FAILURES

Project name Location Commissioning year Reference	Description of pressure tunnel	Description of failure
Kandergrund Switzerland 1911 Jaeger 1963 Jaeger 1979	Concrete lined pressure tunnel l=4214m H=312m Weak rock	Three 2.5cm thick fissures about 30m long along the tunnel crown and springs of the tunnel soffit caused water to leak. The reason was that a water hammer pressure wave penetrated the tunnel and lifted the lined soffit in a region of weak rock.
Big Creek tunnel 2 U.S.A. 1913 Spencer 1964	D=3.6m H=22m Fractured and weathered granitic rock	Leakage due to heavily jointed rock and inadequate side cover caused landslide.
Herlandsfoss Norway 1919 Buen 1984 Hoek 1982 Olsen 1970 Broch 1982	Unlined and reinforced concrete lined pressure tunnel H=136m Hornblende schist interbedded with mica schist	Hydraulic fracturing occurred immediately after the first filling and again after repairs with reinforced concrete lining and grouting due to inadequate rock cover. The whole horizontal pressure tunnel was eventually lined with steel.
Ritom Switzerland 1920 Jaeger 1979	Concrete lined pressure tunnel. Geological strata were inclined towards the valley.	Leakage started shortly after being commissioned because horseshoe shape was unfavourable for a tunnel excavated in indifferent rock and that the concrete lining was too stiff for a tunnel adjacent to a rock slope.
Skar Norway	H=142m Granitic gneiss with	Hydraulic fracturing occurred during testing at head of 81m and again at head of 116m due to insufficient

1920 Olsen 1970 Broch 1982	th lenses of amphibolite, partly weathered.	rock cover. The lower tunnel was replaced by a surface penstock.
Ward U.S.A. 1925 Spencer 1964	H=64m Rectangular and horseshoe section, partly lined. Fractured granite	Rockfall caused by removal of gouge in parallel vertical seams by flowing water occurred near the end of concrete lined section.
Sila Italy 1928 Hoek 1982	Reinforced concrete lined pressure tunnel.	Severe leakage occurred in 1971 associated with cracking of lining due to dynamic pressure fluctuations resulting from operating mode of the plant. Possibly inadequate side cover.
Gerlos Austria Before 1943 Jaeger 1955,1979	D=1.6-2.2m, H=620m Steel lined pressure shaft. Schist of doubtful quality.	Rupture occurred in 4 places along lines parallel to the drainage gallery causing a considerable quantity of rock washed away leaving a large opening in the rock masses. That was produced by uneven subsidence of the concrete which developed bending stresses in the steel liner.
Coscile Italy 1950 Hoek 1982	Concrete lined horseshoe shaped tunnel	Erosion of clay-rich rock over 20 year service resulted in cracking of concrete under swelling pressure in the rock.
Tungatinah Australia 1950 Thomas et al 1964	H=306m Completely unlined pressure tunnel without paved invert. Horseshoe shaped. Massive dolerite.	Inspection due to hearing of passing stone found rockfall of more than 50 cu ft with which rock trap were full of. Some faults were treated by concrete and a new larger rock trap was excavated.
Whatshan	D=3.6m	Cracks in the concrete walls let out water which

Canada 1951 Anon 1953	Pressure tunnel partly lined with concrete of poor quality.	seeped through rock faults and caused landslide.
Tasan Sweden 1953 Hoek 1982	H=10-15m Unlined pressure tunnel with rectan- gular section. Gneiss	Rockfalls in stratified compacted gneiss occurred during dewatering. Failure was attributed to erosion and swelling of montmorillonite fissure filling.
Kemano Canada 1954 Cooke et al 1963 Lawton 1957	H=20-40m. Concrete and sprayed-concre- te lined and unlin- ed. Quartzite, sch- ist etc.	Excessive sand passing through turbines resulted in closedown for inspection in 1961. Several collapse occurred where erosion of mylonite-rich fissures had occurred in the area treated with shotcrete only.
Reisach East Bavaria 1954 Naber 1960	D=4.9m, H=184/237m Concrete lined. Firm sound granite.	High leakage was observed during the first trial filling at head of 145m because of insufficient rock cover. Then the steel lining was extended and systematic prestressed grouting was performed.
El Frayle Dam diversion tunnel Peru 1958 Jaeger 1979	Concrete lined tunnel.	Tunnel ruptured when adjacent rock abutment collapsed since too close to rock slope.
Tafjord K3 Norway 1958 Buen 1984	H=286m Unlined pressure shaft. Banded gneiss.	Cracks appeared in concrete lined section with faulted rock probably due to expanding clay. Small leaks into shaft were observed when the shaft was emptied.
Mammoth Pool U.S.A.	D=6m, H=280m Unlined pressure	Leakage to ground surface developed through joints in fissured rock causing a modest guniting and grouting

1960 Spencer 1964	tunnel in partly fractured granite.	repairs.
Tooma-Tumut Australia 1961 Dann et al 1964	H=140m Unlined pressure tunnel. Closely jointed granitic rock	A rockfall of 200 cu yd occurred due to washing out of clay or soft material from seams and loosening of rock along the clay-coated joints which were not detected during construction.
Xiamaling China 1961 Yang 1982 Gu et al 1982	H=80-90m. D=5.6m Concrete and reinforced concrete lined. Limestone, schist with faults.	Rock is so poor that rockfalls occurred at 10 places during construction. Heavy leakage occurred at intersection with a railway tunnel during operation. Finally the reinforced concrete lining collapsed due to insufficient rock cover. Steel lining was placed inside the reinforced concrete lining.
Piastra-Andonno Italy 1962 Matt et al 1978	H=80m Concrete lined	Tunnel lining was cracked to such extent that water losses were unacceptable for proper performance of the plant probably due to erosion through the cracked lining.
Zakucac Yugoslavia 1962 Simic 1979	H=50m Concrete lined. Flysch sandstone, marl, limestone and lime breccia.	Collapse in weathered marl and falls in fractured and weathered limestone occurred after one year of operation.
Agri Italy 1963 Hoek 1982	H=90m Reinforced concrete lined chamber.	Leakage occurred in 1967 and increased until the major leakage and landslide occurred in 1970, that forced closure for repair. Erosion of clay-rich rock was blamed for the failure and the chamber was steel lined in 1973.
Eucumbene-Snowy Australia 1965	Horseshoe shaped unlined pressure tunnel	Significant rockfalls occurred in 3 places that completely blocked the tunnel probably due to erosion of seams and efficient rock support.

Anon 1970	Granite	
Maojianshan China 1966 Yang 1982	D=4.6m Unlined pressure tunnel	Heavy pouring occurred on side slope after the first filling due to insufficient rock cover. The tunnel was steel lined afterwards.
Woh Malasia 1966 Newbery 1970	Unlined pressure tunnel with paved invert. Granite co- taining iron-stain- ed joints and clay- gouge filled fault zone.	Failure occurred twice in weathered zone. Collapse reached the ground level where a depression formed. The tunnel had to be diverted.
Orterren Norway 1966 Buen 1984	H=160. Unlined pre- ssure shaft. Gneiss schistose.	Joint opening and leakage during the first filling. Leak was sealed by grouting and no further problems were reported.
Manlang Malaysia 1967 Newbery 1970	Pressure tunnel Granite	Rockfalls occurred during dewatering as a result of collapse of the rock surrounding the tunnel when subjected to unbalanced water pressure and weight of the saturated altered rock.
Byrte Norway 1968 Olsen 1970 Brekke 1970	H=303m Unlined pressure shaft Gneiss and Granitic Gneiss	Small leak encountered during construction. Hydraulic fracturing occurred due to insufficient rock cover and the whole tunnel had to be steel lined.
Hongrin-Leman Switzerland 1968 Hoek 1982	Prestressed concre- te lined headrace tunnel. Limestone, marls and schists.	Inadequate overburden near surge tank allowed excessive deformation and opening of vertical fissure.

Lemanthyme Austria 1968 Hoek 1982	Mainly unlined pressure tunnel. Sound schist	Collapse occurred between two parallel faults containing crushed fragmented phyllite and slickensided micaceous schist. The zone had been shotcreted only and erosion had released blocks.
Lete-Save Italy 1969 Hoek 1982	H=50m Reinforced concrete lined low pressure tunnel.	Severe leakage and slope movement occurred in area of minimal cover (5-6m) were attributed to erosion of clay-rich rock through leaky lining.
Askara Norway 1970 Broch 1982	H=210m Unlined pressure tunnel. Sandstone	Hydraulic fracturing occurred following sandfilled steeply dipping joints with a strike parallel to the very steep valley side and normal to the tunnel due to insufficient side cover.
Nanshui China 1970 Yang 1982	D=5.5m, H=33.8m Reinforced concrete lined pressure tunnel. Sandstone.	More than 300 cracks parallel to the tunnel axis in the lining occurred and caused heavy leakage.
Yuba (New Calgate tunnel) U.S.A. 1970 Lang et al 1976	D=26.5ft. Full-face driven pressure tunnel.	Discoloured water and pressure drop were noticed 3 years after it was placed in service. Some 76000 cu yd fallout was found in dewatered inspection. The reason was that automatic control led continuing fluctuation in pressure which consequently resulted in the difference between the tunnel water pressure and the pore pressure. The difference caused an expansion failure and disintegration of sandy gouge and other sheared zone materials. On consequent, rock blocks would be loosen and tend to fallout.
Bjerka Norway 1971	H=72m. Unlined pressure tunnel. Foliated gneiss inter-	Deformation and flushing out of clay filled seams due to insufficient rock cover. Steel lining was then extended.

Buen 1984	beded with mica schist.	
Changhu China 1973 Yang 1982	D=7.0m. Reinforced concrete lined pressure tunnel. Sandstone and phyllite sandstone. H=51m.	Leakage occurred after the first filling resulting in piping probably due to insufficient overburden.
Huamuqiao China 1973 Yang 1982	D=4.5m. Pressure tunnel. Sandstone. Schist.	Rockfall occurred due to erosion of filling materials of a fault.
Yija Norway 1973 Buen 1984	H=210m. Unlined pressure tunnel. Meta-arkose.	Leakage occurred during the first filling probably due to an open joint. Sealing by shotcrete and grouting reduced leakage to approximately 10 l/s.
Malta Austria 1976 Hoek 1982	H=70m. Concrete lined pressure shaft and tunnel.	Extensive cracking occurred in area of poor rock which had been grouted, apparently unsuccessfully.
Al Norway 1951/1978 Buen 1984	H=30m. Unlined pressure tunnel. Gneiss	Considerable but still acceptable leakage occurred due to hydraulic splitting caused by insufficient rock cover. Plug was moved further into the tunnel in order to get sufficient overburden when the power station was renovated in 1978.
Huangqin China 1978 CHDI 1977 Gu et al 1983	H=200m. Grouting prestressed concrete lined pressure tunnel. D=1.8m. Sound rock.	Longitudinal opening occurred along the shaft and extended to the end of steel lining due to insufficient side cover.

Arenal Costa Rica 1979 Anon 1979	H=90m, D=5.0m. Concrete lined pressure tunnel.	Parallel 1km long cracks in both crown and invert and pouring were discovered during test that caused 9 months delay operating due to poor quality of rock.
Bath County U.S.A. 1985 Anon 1985 b Sullenberger 1986	H=1262ft, D=28.5ft Concrete lined pressure tunnel and reinforced concrete lined bifurcation. Sandstone.	Leakage occurred in reinforced concrete lined section of power tunnel 1 during test filling. A 7000gpm of water partially leaked into the other two tunnels and small portion carried out of the side of the mountain causing bulking about 45ft long and narrowing its diameter by about 5ft in the steel liner of the neighbour power tunnel.
Kotmale Sri Lanka 1985 Anon 1985 a Anon 1985 b	D=5.5m Concrete lined pressure shaft. Sound gneiss.	Longitudinal cracks occurred in both crown and invert of concrete lined section through which a seepage reached 50l/s emerging in an old construction adit beneath most likely due to very low horizontal in situ stress. The power plant was shutdown for months.
Hemesdal Norway Hoek 1982	Unlined pressure tunnel with rectangular section. Caledonian rock.	Collapse occurred one year after start of operation between two parallel fissures with montmorillonite filling. The area was dry and stable during construction and need for lining was not anticipated.
Kaunertal Austria Hoek 1982	H=120-170m. Concrete lined headrace pressure tunnel. Variable rock conditions with faults and fissured zones.	Based on full-scale pressure tests, grouting program was developed but proved to be inadequate in very poor rock areas in which erosion of clay filling occurred resulting in cracking.
Bryant Park U.S.A. Jaeger 1979	No details available.	Failure occurred probably because there was a connection from the tunnel to a horizontal fissured zone near the surface and the ground above this was lifted.

Hoek 1982		
Roseland France Hoek 1982	Concrete lined pressure tunnel. Crystalline schist with major near-horizontal faults.	Prestressing the lining by grouting to achieve a water proof conduit was unsuccessful.
Sydney water supply tunnel Australia Jaeger 1955 Jaeger 1979	H=150m. Concrete lined pressure tunnel. Sandstone.	Lining ruptured during test in two places in the form of a horizontal crack and water losses were high probably due to high compressibility of the sandstone. Cement grouting rarely stiffens sandstone rock with a very low modulus of elasticity.
Fisher Australia Hoek 1982	H=250m. Concrete lined pressure tunnel. Permian sediments.	Major landslide occurred in 1973 as a result of leakage from ungrouted section of lining.

APPENDIX 2

JAEGER'S ANALYTICAL SOLUTIONS FOR FAILURE CONDITIONS OF UNLINED PRESSURE TUNNELS

A.2.1 Case of Hard, Unfissured Rock

The vertical and horizontal stresses are assumed to be

$$\sigma_v = \gamma(H - x_1)$$

$$\sigma_h = k\gamma(H - x_1)$$

where γ = unit weight of rock. The circumferential tensile stress caused by water pressure p is

$$\sigma_\theta = p\left(\frac{R}{x_1}\right)^2$$

The condition for the balance of stress at a depth $(H - x_1)$ is

$$-\sigma_\theta \leq \sigma_n$$

Writing that $x_1 = H/n$ and $p = \lambda_1 \gamma_w H$ then

$$\lambda_1 \leq K\left(\frac{\gamma}{\gamma_w}\right)\left(\frac{H}{R}\right)^2(n-1)/n^3 \quad (\text{A.2.1})$$

A.2.2 Case of Radial Fissured Rock

The in situ stresses are the same as those for the unfissured rock. The circumferential tensile stress is assumed to be

$$\sigma_\theta = p\left(\frac{R}{x_2}\right)$$

The critical condition is that at a depth $x_2 = H/n$ above the tunnel centre line no new fissure opening occurs, i.e.

$$p\left(\frac{R}{x_2}\right) = \lambda_2 n \gamma_w R \leq k \gamma_w R(H - x_2) - \sigma_t$$

where σ_t is the tensile strength of the rock. So,

$$\lambda_2 \leq K\left(\frac{\gamma}{\gamma_w}\right)\left(\frac{H}{R}\right)(n-1)/n^2 - \sigma_t/n\gamma_w R \quad (\text{A.2.2})$$

A.2.3 Case of Plastic Rock

From the elasto-plastic solution for a cylindrical tube combined with Tresca's criterion, $\sigma_r - \sigma_\theta = Y$, the water pressure which causes a plastic region of radius d is

$$\frac{p}{Y} = \ln\left(\frac{d}{R}\right) + \frac{1}{2}\left(1 - \frac{d^2}{H^2}\right) \quad (\text{A.2.3})$$

The critical condition is that the circumferential tensile stress σ_θ at a point above the tunnel centre line (in the elastic region) caused by water pressure is equal to the horizontal in situ stress σ_h at the point

$$\sigma_\theta = \sigma_h \quad (\text{A.2.4})$$

where

$$\frac{\sigma_\theta}{Y} = -\frac{1}{2} + \ln\left(\frac{d}{R}\right) - \frac{d^2}{2H^2} \quad (\text{A.2.5})$$

$$\sigma_h = k\gamma(H - d) \quad (\text{A.2.6})$$

Substituting (A.2.3), (A.2.5) (A.2.6) into (A.2.4) and writing $d = H/n$, $p/\gamma_w = \lambda_p H$, the ratio λ_p is obtained (Gu et al 1983),

$$\lambda_p \leq 2k\left(\frac{\gamma}{\gamma_w}\right)\left(\frac{n^2 - n}{n^2 + 1}\right)\left[\ln\left(\frac{H}{nR}\right) + \frac{1}{2}\left(1 - \frac{1}{n^2}\right)\right] \quad (\text{A.2.7})$$

A.2.4 Pressure Tunnels Adjacent to Rock Slope

The in situ stresses are assumed to be

$$\sigma_z = \gamma H \left(\frac{n-1}{n}\right)$$

$$\sigma_h = K_1 \sigma_z$$

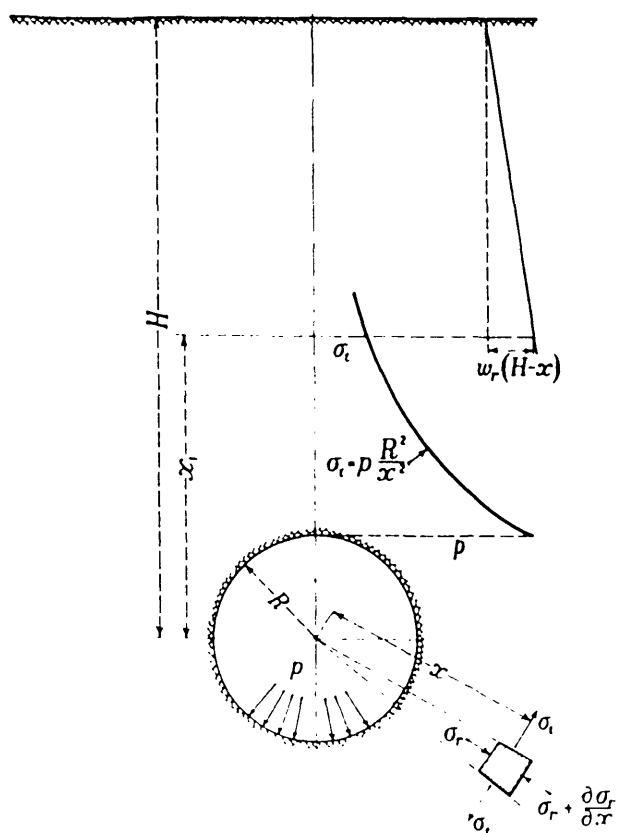


Figure A.2.1 Jaeger's solution for hard, unfissured rock

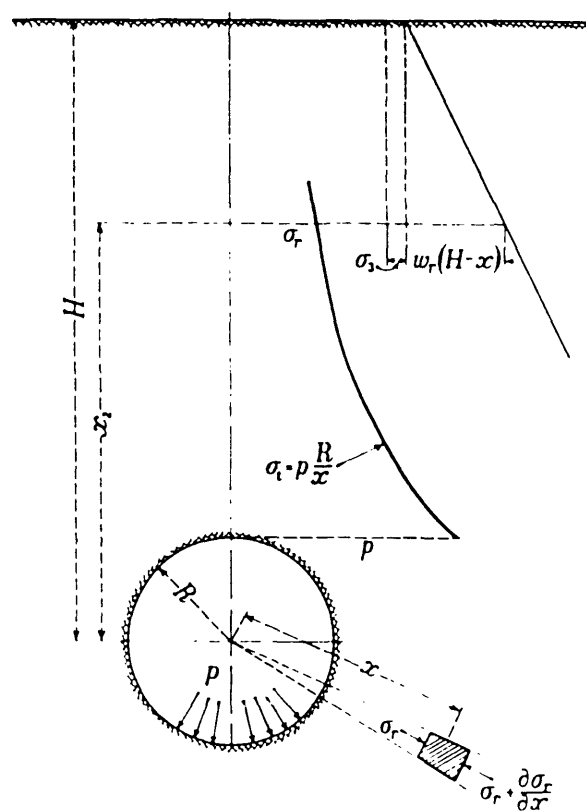


Figure A.2.2 Jaeger's solution for radially fissured rock

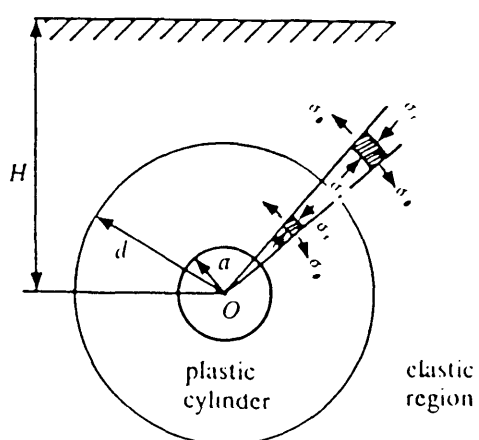


Figure A.2.3 Jaeger's solution for plastic rock

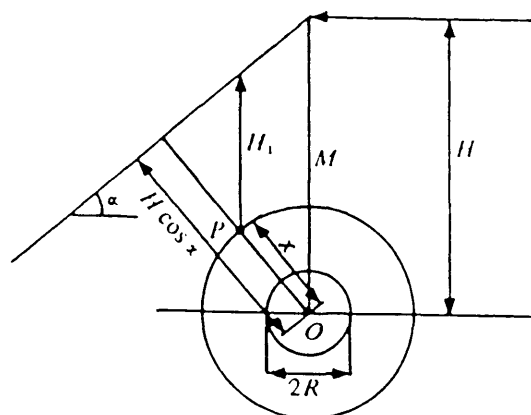


Figure A.2.4 Jaeger's solution for the tunnel adjacent to a rock slope

The horizontal stress component coefficient K_1 decreases progressively from 0 towards point P as $K_1 = 0$ at the surface of the rock slope (stress release). The in situ stress at the point P at angle α is

$$\sigma_\alpha = \frac{\sigma_z + \sigma_h}{2} - \frac{\sigma_z - \sigma_h}{2} \cos 2\alpha$$

So, at the point P

$$\sigma_\alpha = \frac{\gamma}{2} \left[H \frac{(n-1)}{n} (1 + K_1) \right] - \frac{\gamma}{2} \frac{H(n-1)}{n} (1 - K_1) \cos 2\alpha$$

The circumferential tensile stress caused by water pressure $p = \lambda_3 \gamma_w H$ is

$$\sigma_\theta = p \frac{R^2 n^2}{H^2 \cos^2 \alpha} = \lambda_3 \gamma_w \frac{H R^2 n^2}{H^2 \cos^2 \alpha}$$

The balance condition is

$$\sigma_\alpha \geq |\sigma_\theta|$$

Therefore,

$$\lambda_3 \leq \frac{\gamma}{\gamma_w} \frac{(n-1)}{2n^3} \left(\frac{H}{R} \right)^2 [(1 + K_1) - (1 - K_1) \cos 2\alpha] \cos^2 \alpha$$

Similarly for the radially fissured rock

$$\sigma_t = \lambda_4 \gamma_w \left(\frac{Rn}{\cos \alpha} \right)$$

$$\lambda_4 \leq \left(\frac{\gamma}{\gamma_w} \right) \left(\frac{H}{R} \right) \frac{(n-1)}{2n^2} [(1 + K_1) - (1 - K_1) \cos 2\alpha] \cos \alpha - \frac{\sigma_t \cos \alpha}{\gamma_w Rn} \quad (A.2.8)$$

APPENDIX 3 HOEK'S SOLUTIONS FOR CONDITIONS FOR JOINT OPENING AND HYDRAULIC JACKING

A.3.1 Assumptions

Rock is assumed to be hard, strong and elastic and the stresses induced in this rock are high enough to cause opening of joints but not fracture of the intact rock. Plane strain condition is adopted.

A.3.2 Stress Distribution

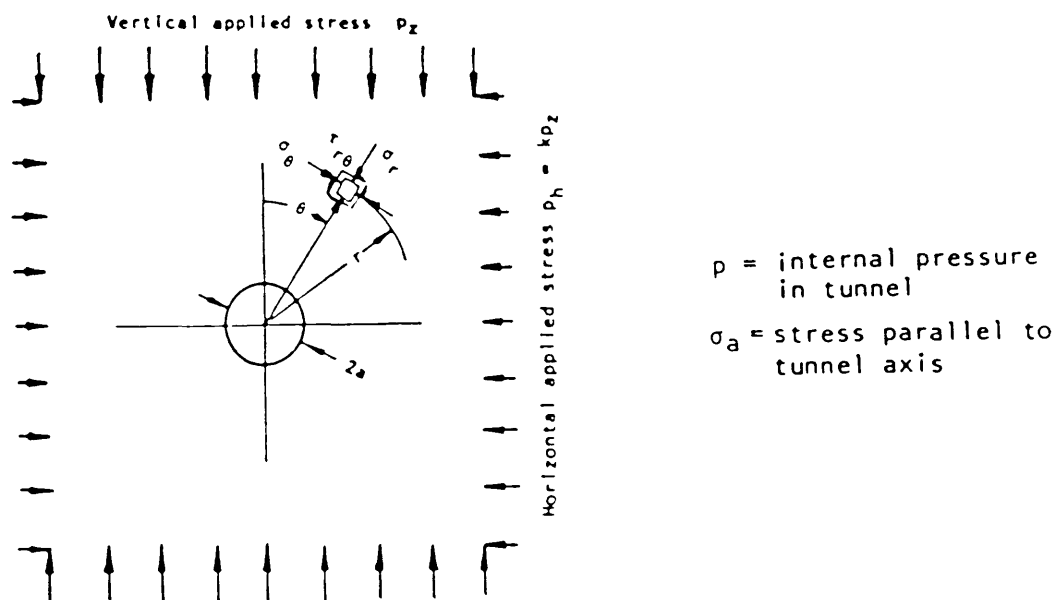


Figure A.3.1 Coordinate and loads for Hoek's solutions
(After Hoek, 1982)

The stresses in the rock surrounding a circular tunnel subjected to internal water pressure p and to triaxial in situ stress condition defined by $K_1 = K_2 = K$

follow.

$$\sigma_r = \frac{1}{2}P_z[(1+K)(1-a^2/r^2) + (1-K)(1-4a^2/r^2 + 3a^4/r^4)\cos 2\theta] + pa^2/r^2 \quad (A.3.1)$$

$$\sigma_\theta = \frac{1}{2}P_z[(1+K)(1+a^2/r^2) - (1-K)(1+3a^4/r^4)\cos 2\theta] - pa^2/r^2 \quad (A.3.2)$$

$$\tau_{r\theta} = \frac{1}{2}P_z[-(1-K)(1+2a^2/r^2 - 3a^4/r^4)\sin 2\theta] \quad (A.3.3)$$

$$\sigma_1 = \frac{1}{2}(\sigma_r + \sigma_\theta) + \left[\frac{1}{4}(\sigma_r - \sigma_\theta)^2 + \tau_{r\theta}^2\right]^{\frac{1}{2}} \quad (A.3.4)$$

$$\sigma_3 = \frac{1}{2}(\sigma_r + \sigma_\theta) - \left[\frac{1}{4}(\sigma_r - \sigma_\theta)^2 + \tau_{r\theta}^2\right]^{\frac{1}{2}} \quad (A.3.5)$$

$$\tan 2\alpha = 2\tau_{r\theta}/(\sigma_\theta - \sigma_r) \quad (A.3.6)$$

$$\sigma_y = KP_z + \nu(\sigma_1 + \sigma_3) \quad (A.3.7)$$

where σ_r = radial stress,

σ_θ = tangential stress,

$\tau_{r\theta}$ = shear stress,

σ_1 = maximum principal stress,

σ_3 = minimum principal stress,

σ_y = principal stress parallel to tunnel axis,

α = inclination of maximum principal stress to radial direction,

ν = Poisson's ratio of rock,

a, r, θ as shown in Fig.A.3.1.

A.3.3 Conditions for Opening of Joints

Any joint which intersects the boundary of the tunnel will be forced open when the internal water pressure exceeds the stress acting across the joint plane. The critical stress conditions can be established by considering the tangential stress and the stress acting parallel to the tunnel axis. At the boundary of the tunnel ($r = a$), these stresses reduce to:

$$\sigma_{\theta} = P_z(3K - 1) \quad (\theta = 0^{\circ} \text{ or } 180^{\circ}) \quad (A.3.8)$$

$$\sigma_{\theta} = P_z(3 - K) \quad (\theta = 90^{\circ} \text{ or } 270^{\circ}) \quad (A.3.9)$$

$$\sigma_y = P_z(K + \nu(3K - 1)) \quad (\theta = 0^{\circ} \text{ or } 180^{\circ}) \quad (A.3.10)$$

$$\sigma_y = P_z(K + \nu(3 - K)) \quad (\theta = 90^{\circ} \text{ or } 270^{\circ}) \quad (A.3.11)$$

The conditions for joint opening are given when the internal water pressure p exceeds either σ_{θ} or σ_y defined by these equations. The limiting conditions are plotted in Fig.A.3.2.

Three zones of particular interest can be identified in this figure:

$$\underline{K < 0.75}$$

Opening of vertical joints in the roof and floor of the tunnel will occur at internal pressure less than P_z , the overburden pressure.

$$\underline{0.75 < K < 2.0}$$

Opening of vertical joints perpendicular to the tunnel axis will occur at internal pressure significantly greater than the overburden pressure.

$$\underline{K > 2.0}$$

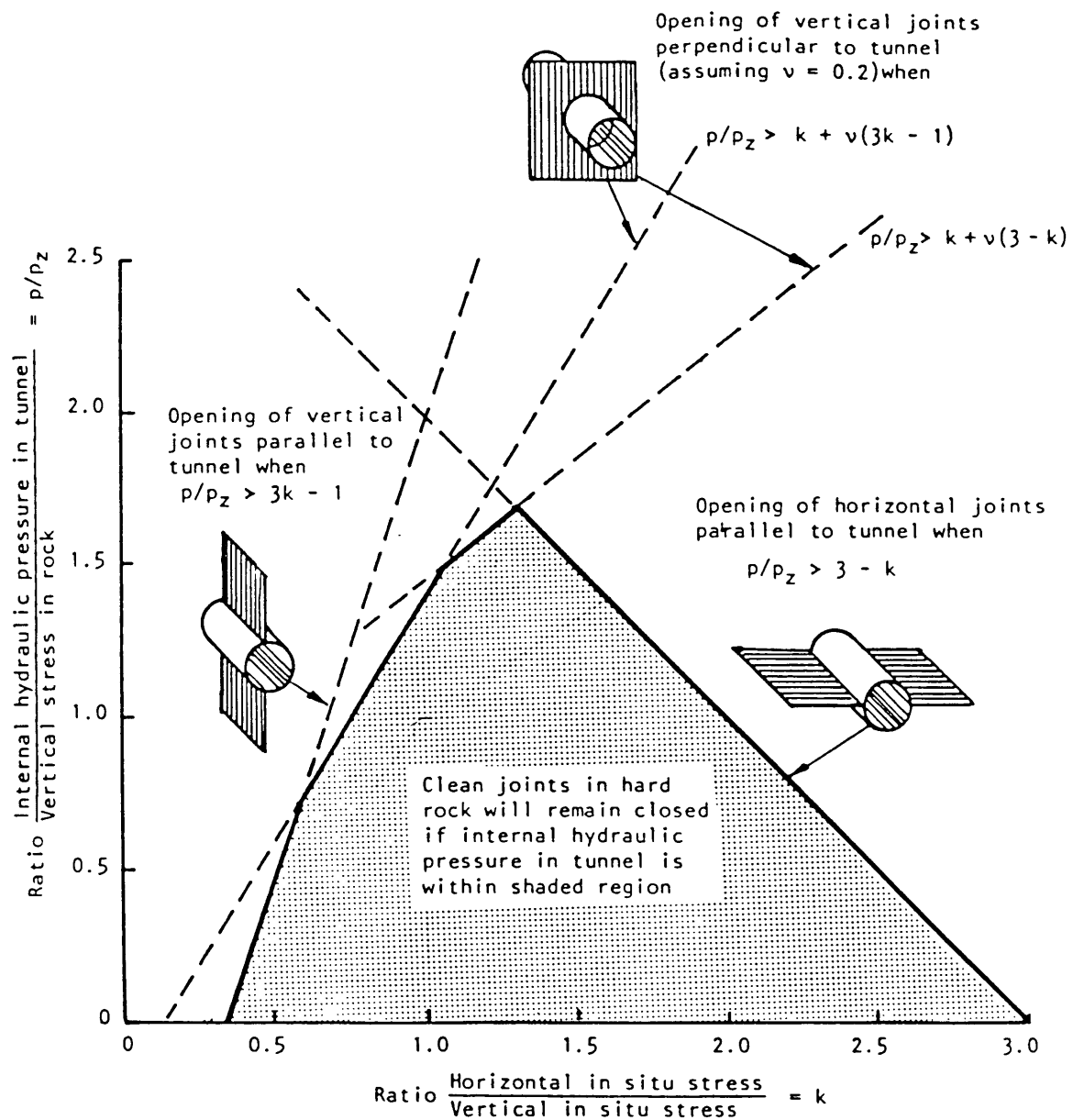


Figure A.3.2 Hoek's solution for conditions for joint opening
(After Hoek, 1982)

Opening of horizontal joints in the sidewalls of the tunnel for internal pressure less than the overburden pressure P_z .

A.3.4 Conditions for Hydraulic Jacking

Once a joint has been opened and water can enter this joint, conditions could arise where water pressure could be transmitted into the rock mass surrounding the tunnel. If the opening of the joint is sufficiently large that no pressure drop occurs as the water moves along the joint, hydraulic jacking of the rock mass can occur. The conditions for hydraulic jacking are illustrated in Fig. A.3.3 and are considered below:

$$\underline{0.5 < K < 1}$$

If a joint opens when the lateral stress acting on the rock mass surrounding the tunnel is less than the vertical stress, unstable propagation of the vertical "crack" can occur because the pressure p in the tunnel (and the vertical crack) will always be greater than the lateral stress KP_z . For joints parallel to the tunnel this condition for hydraulic jacking can occur when

$$p > P_z(3K - 1)$$

For joints perpendicular to the tunnel axis this condition is defined by

$$p > P_z(K + \nu(3K - 1))$$

$$\underline{2 > K > 1}$$

When $K > 1$, the vertical stress P_z is always the minimum principal stress and hydraulic jacking can only occur when the internal pressure exceeds the overburden pressure P_z . When

$$p > P_z(K + \nu(3 - K))$$

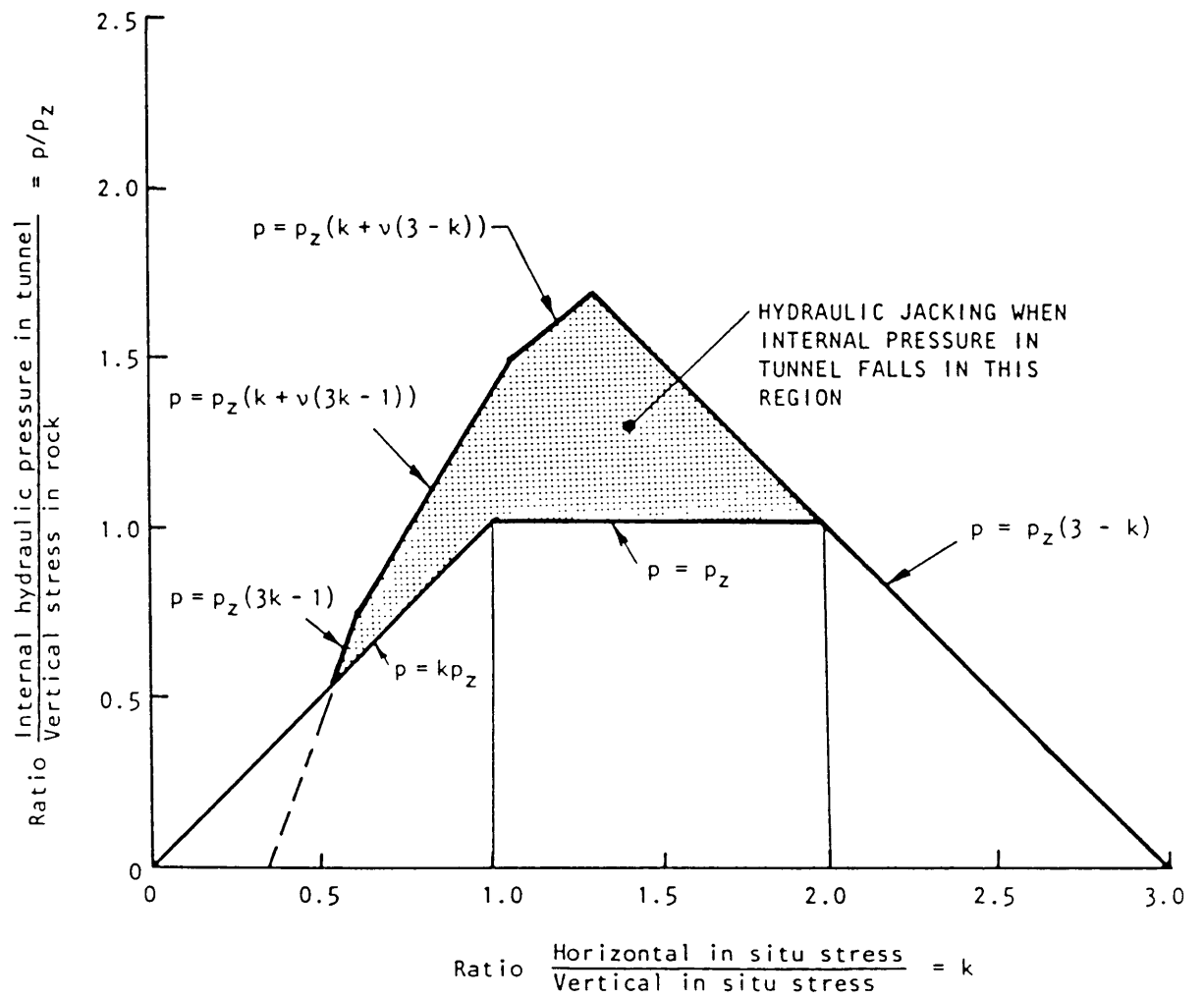


Figure A.3.3 Hoek's solution for condition for hydraulic jacking
(After Hoek, 1982)

vertical joints perpendicular to the tunnel axis will be forced open. If these open joints can transmit the full internal hydraulic pressure to existing horizontal joints in the rock mass, jacking can occur because $p > P_z$. When

$$p > p_z(3 - k)$$

horizontal joints parallel to the tunnel can be forced open and, because $p > P_z$, hydraulic jacking can occur.

$$\underline{K > 2}$$

Under these conditions, opening of horizontal joints parallel to the tunnel can occur when inequality $p > P_z(3 - K)$ is satisfied. However, because this opening will occur when $p < P_z$, hydraulic jacking will not occur. Hydraulic jacking will occur if the internal pressure in the tunnel is increased to a level at which $p > P_z$.

APPENDIX 4

LISTING OF PROGRAM ASJOHJ

```

10 REM This program is to calculate the limiting internal water
    pressure for joint opening and hydraulic jacking inside
    the surrounding rock mass
20 REM Effect of water pressure is not taken into account
30 DIM L(3),L1(3),L2(3),L3(3),M(3),M1(3),M2(3),M3(3)
40 DIM N(3),N1(3),N2(3),N3(3),SGM3(3)
50 PRINT "K1=";:INPUT K1
60 PRINT "K2=";:INPUT K2
70 PRINT "ALPHA=";:INPUT ALPHA
80 PRINT "BETA=";:INPUT BETA
90 PRINT "MU=";:INPUT MU
100 PRINT "r/a=";:INPUT AR
110 PRINT "    K1=";K1;"    K2=";K2;"    ALPHA=";ALPHA;"    BETA=";BETA
120 PRINT "    MU=";MU;"    r/a=";AR
130 AR=1/AR
140 ALPHA=ALPHA*3.14159265/180:BETA=BETA*3.14159265/180
150 L(2)=COS(ALPHA)
160 M(2)=COS(BETA): IF M(2)<.0000001 THEN M(2)=0
170 NN2=1-L(2)*L(2)-M(2)*M(2)
180 IF ABS(NN2)<.00001 THEN NN2=0
190 IF NN2<0 THEN PRINT "THIS CASE CANNOT OCCUR":END
200 N(2)=-SQR(NN2)
210 L(1)=M(2)/SQR(M(2)*M(2)+L(2)*L(2))
220 M(1)=-L(2)/SQR(M(2)*M(2)+L(2)*L(2))
230 N(1)=0
240 M(3)=-L(1)*N(2)/SQR(M(1)*M(1)+L(1)*L(1))
250 L(3)=M(1)*N(2)/SQR(M(1)*M(1)+L(1)*L(1))
260 N(3)=SQR(1-N(2)*N(2))
270 PX=L(1)*L(1)*K1+M(1)*M(1)*K2+N(1)*N(1)
280 PY=L(2)*L(2)*K1+M(2)*M(2)*K2+N(2)*N(2)
290 PZ=L(3)*L(3)*K1+M(3)*M(3)*K2+N(3)*N(3)
300 PXY=L(1)*L(2)*K1+M(1)*M(2)*K2+N(1)*N(2)
310 PYZ=L(2)*L(3)*K1+M(2)*M(3)*K2+N(2)*N(3)
320 PZX=L(3)*L(1)*K1+M(3)*M(1)*K2+N(3)*N(1)
330 P=(PX+PZ)/2:Q=PZX*PZX:R=(PX-PZ)*(PX-PZ)/4:S=SQR(Q+R)
340 IF ABS(PX-PZ)<.001 AND PZX=0 THEN PHI=0:GOTO 420
350 IF ABS(PX-PZ)<.001 AND PZX>0 THEN PHI=-0.785398162:GOTO 420
360 IF ABS(PX-PZ)<.001 AND PZX<0 THEN PHI=0.785398162:GOTO 420
370 PHI=ATN(ABS(2*PZX/(PX-PZ)))/2
380 IF PX>PZ AND PZX>=0 THEN PHI=-PHI
390 IF PX>PZ AND PZX<0 THEN PHI=PHI
400 IF PX<PZ AND PZX>=0 THEN PHI=1.570796325+PHI
410 IF PX<PZ AND PZX<0 THEN PHI=1.570796325-PHI
420 PX=P+S:PZ=P-S
430 AK=0:BK=3.14159265/2
440 S=BK-AK:K=1
450 IF S<.05 THEN T=(TT+TT1)/2:K=0:GOTO 480 ELSE TT=0.382*S+AK
460 TT1=0.618*S+AK:J=1:T=TT
470 SGMR=(PX+PZ)*(1-AR*AR)/2+(PX-PZ)*(1-4*AR*AR+3*AR^4)*COS(2*T)/2
480 SGMT=(PX+PZ)*(1+AR*AR)/2-(PX-PZ)*(1+3*AR^4)*COS(2*T)/2
490 SGMty=(-PXY*SIN(T+PHI)+PYZ*COS(T+PHI))*(1+AR*AR)
500 L1(1)=COS(PHI):M1(1)=0:N1(1)=SIN(PHI)
510 L1(2)=0:M1(2)=1:N1(2)=0
520 L1(3)=-SIN(PHI):M1(3)=0:N1(3)=COS(PHI)
530 LR=COS(T):MR=0:NR=SIN(T):LT=-SIN(T):MT=0:NT=COS(T)

```

```

540 FOR I=1 TO 3
550 L2(I)=L1(I)*L(1)+M1(I)*L(2)+N1(I)*L(3)
560 M2(I)=L1(I)*M(1)+M1(I)*M(2)+N1(I)*M(3)
570 N2(I)=L1(I)*N(1)+M1(I)*N(2)+N1(I)*N(3)
580 NEXT I
590 LLR=L2(1)*LR+L2(2)*MR+L2(3)*NR
600 MMR=M2(1)*LR+M2(2)*MR+M2(3)*NR
610 NNR=N2(1)*LR+N2(2)*MR+N2(3)*NR
620 LLT=L2(1)*LT+L2(2)*MT+L2(3)*NT
630 MMT=M2(1)*LT+M2(2)*MT+M2(3)*NT
640 NNT=N2(1)*LT+N2(2)*MT+N2(3)*NT
650 PR=LLR*LLR*K1+MMR*MMR*K2+NNR*NNR
660 PT=LLT*LLT*K1+MMT*MMT*K2+NNT*NNT
670 SGMY=MU*(SGMR+SGMT-PR-PT)+PY
680 P=(SGMT+SGMY)/2:Q=SGMT-SGMY:Q1=Q*Q/4
690 Q1=SQR(SGMY^2+Q1):SGM3(J)=P-Q1
700 IF K=0 THEN GOTO 740 ELSE J=J+1
710 IF J<2.5 THEN T=TT1:GOTO 480 ELSE
720 IF SGM3(1)>SGM3(2) THEN AK=TT ELSE BK=TT1
730 GOTO 440
740 SGM3B=SGM3(J)
750 IF ABS(SGMT-SGMY)<.001 AND SGMTY=0 THEN OMEGA=0:GOTO 840
760 IF ABS(SGMT-SGMY)<.001 AND SGMTY>0 THEN OMEGA=0.785398162:GOTO 840
770 IF ABS(SGMT-SGMY)<.001 AND SGMTY<0 THEN OMEGA=-0.785398162:GOTO 840
780 OMEGA=ABS(2*SGMTY/(SGMT-SGMY))
790 OMEGA=.5*ATN(OMEGA)
800 IF SGMT<SGMY AND SGMTY>=0 THEN OMEGA=OMEGA
810 IF SGMT<SGMY AND SGMTY<=0 THEN OMEGA=-OMEGA
820 IF SGMT>SGMY AND SGMTY>=0 THEN OMEGA=1.570796325-OMEGA
830 IF SGMT>SGMY AND SGMTY<0 THEN OMEGA=1.570796325+OMEGA
840 L3(1)=SIN(T):L3(2)=0:L3(3)=COS(T)
850 M3(1)=0:M3(2)=1:M3(3)=0
860 N3(1)=-COS(T):N3(2)=0:N3(3)=SIN(T)
870 FOR I=1 TO 3
880 L1(I)=L3(I)*L2(1)+M3(I)*L2(2)+N3(I)*L2(3)
890 M1(I)=L3(I)*M2(1)+M3(I)*M2(2)+N3(I)*M2(3)
900 N1(I)=L3(I)*N2(1)+M3(I)*N2(2)+N3(I)*N2(3)
910 NEXT I
920 LR=COS(OMEGA):MR=SIN(OMEGA):NR=0
930 LT=L1(1)*LR+L1(2)*MR+L1(3)*NR
940 MT=M1(1)*LR+M1(2)*MR+M1(3)*NR
950 NT=N1(1)*LR+N1(2)*MR+N1(3)*NR
960 P3=LT*LT*K1+MT*MT*K2+NT*NT
970 PRINT "      S3=";SGM3B;"          P3=";P3
980 PRINT "      WATER PRESSURE FOR JOINT OPENING:      ";SGM3B;" Pz"
990 IF P3>=SGM3B THEN PHJK=P3 ELSE PHJK=SGM3B
1000 PRINT "      WATER PRESSURE FOR HYDRAULIC JACKING:  ";PHJK;" Pz"
1010 END

```

APPENDIX 5

DERIVATION OF EXPRESSIONS FOR JRC_m

The expressions for JRC_m are derived from coordinates of the points given on Fig.3.15. From the figure it can be seen that point P must be a singular point . Therefore, the whole range can be divided into two parts: region III and region IV. In region III there are three known points available. In order to avoid a minus derivative in the vicinity of the point P an additional condition is imposed that the curve has a horizontal tangent when point P is approached from the left hand side. So there are all together four conditions:

$$\left\{ \begin{array}{ll} x_1 = 0.3 & y_2 = 0 \\ x_2 = 0.6 & y_2 = 0.75 \\ x_3 = 1.0 & y_3 = 1.0 \\ \left(\frac{dy}{dx} \right)_{x=1} = 0 \end{array} \right. \quad (A.5.1)$$

where x denotes $\delta s / \delta p$, y denotes JRC_m / JRC_p .

A third order parabola is assumed for this piece of curve,

$$y = a + bx + cx^2 + dx^3 \quad (A.5.2)$$

where coefficients a, b, c and d are evaluated by substituting Eqn.(A.5.1) into Eqn.(A.5.2) as follows:

$$\left\{ \begin{array}{l} a = -1.51913265 \\ b = 6.63265306 \\ c = -5.70790816 \\ d = 1.59438776 \end{array} \right.$$

So

$$\begin{aligned} \frac{\partial JRC_m}{\partial JRC_p} &= -1.51913265 + 6.63265306 \frac{\delta_s}{\delta_p} \\ &- 5.70790816 \left(\frac{\delta_s}{\delta_p} \right)^2 + 1.59438776 \left(\frac{\delta_s}{\delta_p} \right)^3 \end{aligned} \quad (A.5.3)$$

In region IV, five known points are available and another condition that the curve must have a horizontal tangent at point R should be met as well. So there are all together six conditions, i.e.

$$\left\{ \begin{array}{ll} x_1 = 1 & y_1 = 1 \\ x_2 = 2 & y_2 = 0.85 \\ x_3 = 4 & y_3 = 0.7 \\ x_4 = 10 & y_4 = 0.5 \\ x_5 = 100 & y_5 = 0 \\ \left(\frac{dy}{dx} \right)_{x=100} = 0 \end{array} \right. \quad (A.5.4)$$

If a single function is used to fit the six conditions it is likely that a minus value of y will occur in the domain $x \in (10, 100)$. To avoid this, region IV is divided into two subregions: $x \in (1, 10)$ and $x \in (10, 100)$. For the second subregion it is assumed that

$$y = ax^2 + bx + c \quad (A.5.5)$$

and conditions are

$$\left\{ \begin{array}{ll} x_1 = 10 & y_1 = 0.5 \\ x_2 = 100 & y_2 = 0 \\ \left(\frac{dy}{dx} \right)_{x=100} = 0 \end{array} \right. \quad (A.5.6)$$

Substituting Eqn.(A.5.6) into Eqn.(A.5.5) we get

$$\begin{cases} a = 0.0000617283951 \\ b = -0.012345679 \\ c = 0.617283951 \end{cases}$$

So

$$\frac{JRC_m}{JRC_p} = 0.0000617283951 \left(\frac{\delta_s}{\delta_p} \right)^2 - 0.012345679 \frac{\delta_s}{\delta_p} + 0.617283951 \quad (A.5.7)$$

In the first subregion, in addition to the first four conditions of Eqn.(A.5.4) another condition that the two subcurves must have a coincident derivative at the junction point $x = 10$ must be satisfied to make the curve smooth, i.e.

$$\left(\frac{dy}{dx} \right)_{x=10^-} = \left(\frac{dy}{dx} \right)_{x=10^+}$$

The $\left(\frac{dy}{dx} \right)_{x=10^+}$ is calculated from Eqn.(A.5.7) as

$$\left(\frac{dy}{dx} \right)_{x=10^+} = -0.011111113$$

So all five conditions for the subregion are

$$\begin{cases} x_1 = 1 & y_1 = 1 \\ x_2 = 2 & y_2 = 0.85 \\ x_3 = 4 & y_3 = 0.7 \\ x_4 = 10 & y_4 = 0.5 \\ \left(\frac{dy}{dx} \right)_{x=10} = -0.011111113 \end{cases}$$

The curve is assumed to be

$$y = a + bx + cx^2 + dx^3 + ex^4 \quad (A.5.8)$$

and it is finally obtained

$$\begin{aligned} \frac{JRC_m}{JRC_p} = & 1.23546811 - 0.288856738 \frac{\delta_s}{\delta_p} + 0.0591628083 \left(\frac{\delta_s}{\delta_p} \right)^2 \\ & - 0.00599762081 \left(\frac{\delta_s}{\delta_p} \right)^3 + 0.000223443926 \left(\frac{\delta_s}{\delta_p} \right)^4 \end{aligned} \quad (A.5.9)$$

APPENDIX 6

DERIVATION OF ELASTO-PLASTIC STRESS-DISPLACEMENT MATRIX

The yield function, F , is obtained from the shear strength criterion,

$$F = \tau - \sigma'_n \tan[JRC_m \log_{10}(\frac{JCS}{\sigma'_n}) + \phi_r] = 0 \quad (A.6.1)$$

The plastic potential function, G , is obtained from dilation considerations as follows:

$$\tan \phi_{dm} = \frac{\dot{\epsilon}_n^p}{\dot{\epsilon}_s^p} = \frac{\frac{\partial G}{\partial \sigma'_n}}{\frac{\partial G}{\partial \tau}} \quad (A.6.2)$$

Assume

$$G = \tau + f(\sigma'_n) = \text{constant} \quad (A.6.3)$$

$$\frac{\partial G}{\partial \sigma'_n} = \frac{\partial f}{\partial \sigma'_n} \quad (A.6.4a)$$

$$\frac{\partial G}{\partial \tau} = 1 \quad (A.6.4b)$$

According to the joint model the dilation angle is

$$\phi_{dm}^o = JRC_m \log_{10}(\frac{JCS}{\sigma'_n}) \quad (A.6.5)$$

$$\frac{\partial G}{\partial \sigma'_n} = \tan[JRC_m \log_{10}(\frac{JCS}{\sigma'_n})] \quad (A.6.6)$$

Considering Eqns. (A.6.2) (A.6.4 a) (A.6.4 b) (A.6.6) the plastic potential function becomes

$$G = \tau + \int_0^{\sigma'_n} \tan[JRC_m \log_{10}(\frac{JCS}{\sigma'_n})] d\sigma'_n = \text{constant} \quad (A.6.7)$$

An explicit expression of G can be obtained by integrating Eqn. (A.6.6). However, to derive the elasto-plastic stress-displacement matrix what is needed

is $\partial G/\partial \sigma'_n$ and $\partial G/\partial \tau$ rather than G itself. Therefore, it is not necessary to perform the integration.

$$\tau = (\tau_{zx}^2 + \tau_{yz}^2)^{\frac{1}{2}}$$

$$\frac{\partial G}{\partial \tau_{zx}} = \frac{\tau_{zx}}{\tau} \quad (A.6.8a)$$

$$\frac{\partial G}{\partial \tau_{yz}} = \frac{\tau_{yz}}{\tau} \quad (A.6.8b)$$

Assume

$$\bar{\mu} = \tan[JRC_m \log_{10} (\frac{JCS}{\sigma'_n})] \quad (A.6.9)$$

$$\frac{\partial G}{\partial \{\sigma\}} = \begin{Bmatrix} \frac{\tau_{zx}}{\tau} \\ \frac{\tau_{yz}}{\tau} \\ \bar{\mu} \end{Bmatrix} \quad (A.6.10)$$

Now let's derive $\partial F/\partial \{\sigma\}$

$$\frac{\partial F}{\partial \tau_{zx}} = \frac{\tau_{zx}}{\tau} \quad (A.6.11a)$$

$$\frac{\partial F}{\partial \tau_{yz}} = \frac{\tau_{yz}}{\tau} \quad (A.6.11b)$$

$$\frac{\partial F}{\partial \sigma'_n} = -\tan[JRC_m \log_{10} (\frac{JCS}{\sigma'_n}) + \phi_r] + \frac{\pi JRC_m}{180 \ln 10 \cos^2[JRC_m \log_{10} (\frac{JCS}{\sigma'_n}) + \phi_r]} \quad (A.6.11c)$$

Assume

$$\mu = -\tan[JRC_m \log_{10} (\frac{JCS}{\sigma'_n}) + \phi_r] + \frac{\pi JRC_m}{180 \ln 10 \cos^2[JRC_m \log_{10} (\frac{JCS}{\sigma'_n}) + \phi_r]} \quad (A.6.12)$$

$$\frac{\partial F}{\partial \sigma'_n} = \mu$$

$$\frac{\partial F}{\partial \{\sigma\}} = \begin{bmatrix} \frac{\tau_{zx}}{\tau} & \frac{\tau_{yz}}{\tau} & \mu \end{bmatrix}^T \quad (A.6.13)$$

$$[D^e] \frac{\partial G}{\partial \{\sigma\}} = \begin{bmatrix} k_{sx} & 0 & 0 \\ 0 & K_{sy} & 0 \\ 0 & 0 & K_n \end{bmatrix} \begin{Bmatrix} \frac{\tau_{zx}}{\tau} \\ \frac{\tau_{yz}}{\tau} \\ \bar{\mu} \end{Bmatrix} = \begin{Bmatrix} K_{sx} \frac{\tau_{zx}}{\tau} \\ K_{sy} \frac{\tau_{yz}}{\tau} \\ K_n \bar{\mu} \end{Bmatrix} \quad (A.6.14)$$

$$[D^e] \frac{\partial F}{\partial \{\sigma\}} = \begin{bmatrix} K_{sx} & 0 & 0 \\ 0 & K_{sy} & 0 \\ 0 & 0 & K_n \end{bmatrix} \begin{Bmatrix} \frac{\tau_{zx}}{\tau} \\ \frac{\tau_{yz}}{\tau} \\ \mu \end{Bmatrix} = \begin{Bmatrix} K_{sx} \frac{\tau_{zx}}{\tau} \\ K_{sy} \frac{\tau_{yz}}{\tau} \\ K_n \mu \end{Bmatrix} \quad (A.6.15)$$

$$\begin{aligned} & [D^e] \frac{\partial G}{\partial \{\sigma\}} \left(\frac{\partial F}{\partial \{\sigma\}} \right)^T [D^e] \\ &= \begin{Bmatrix} K_{sx} \frac{\tau_{zx}}{\tau} \\ K_{sy} \frac{\tau_{yz}}{\tau} \\ K_n \mu \end{Bmatrix} [K_{sx} \frac{\tau_{zx}}{\tau} \quad K_{sy} \frac{\tau_{yz}}{\tau} \quad K_n \mu] \\ &= \begin{bmatrix} K_{sx}^2 \frac{\tau_{zx}^2}{\tau^2} & K_{sx} K_{sy} \frac{\tau_{zx} \tau_{yz}}{\tau^2} & K_{sx} K_n \mu \frac{\tau_{zx}}{\tau} \\ K_{sx} K_{sy} \frac{\tau_{zx} \tau_{yz}}{\tau^2} & K_{sy}^2 \frac{\tau_{yz}^2}{\tau^2} & K_{sy} K_n \mu \frac{\tau_{yz}}{\tau} \\ K_{sx} K_n \mu \frac{\tau_{zx}}{\tau} & K_{sy} K_n \mu \frac{\tau_{yz}}{\tau} & K_n^2 \mu \bar{\mu} \end{bmatrix} \end{aligned} \quad (A.6.16)$$

$$\begin{aligned} & \left(\frac{\partial F}{\partial \{\sigma\}} \right)^T [D^e] \frac{\partial G}{\partial \{\sigma\}} \\ &= [K_{sx} \frac{\tau_{zx}}{\tau} \quad K_{sy} \frac{\tau_{yz}}{\tau} \quad K_n \mu] \begin{Bmatrix} \frac{\tau_{zx}}{\tau} \\ \frac{\tau_{yz}}{\tau} \\ \bar{\mu} \end{Bmatrix} \\ &= K_{sx} \frac{\tau_{zx}^2}{\tau^2} + K_{sy} \frac{\tau_{yz}^2}{\tau^2} + K_n \mu \bar{\mu} \end{aligned} \quad (A.6.17)$$

$$A = - \frac{\partial F}{\partial w^p} \{\sigma\}^T \frac{\partial G}{\partial \{\sigma\}} \quad (A.6.18)$$

$$\{\sigma\}^T \frac{\partial G}{\partial \{\sigma\}} = [\tau_{zx} \quad \tau_{yz} \quad \sigma'_n] \begin{Bmatrix} \frac{\tau_{zx}}{\tau} \\ \frac{\tau_{yz}}{\tau} \\ \bar{\mu} \end{Bmatrix} = \tau + \bar{\mu} \sigma'_n \quad (A.6.19)$$

Considering that the normal plastic work done during shearing is considerably smaller than the shear plastic work, the normal plastic work is omitted. Therefore,

$$\frac{\partial F}{\partial w^p} = \frac{1}{\tau} \frac{\partial F}{\partial \delta_s}$$

$$\frac{\partial F}{\partial \delta_s} = \frac{\partial F}{\partial JRC_m} \frac{dJRC_m}{d\delta_s}$$

$$\frac{\partial F}{\partial JRC_m} = - \frac{\sigma'_n \log_{10}(JCS/\sigma'_n)}{\cos^2[JRC_m \log_{10}(JCS/\sigma'_n) + \phi_r]}$$

$$\frac{\partial F}{\partial \delta_s} = - \frac{\sigma'_n \log_{10}(JCS/\sigma'_n)}{\cos^2[JRC_m \log_{10}(JCS/\sigma'_n) + \phi_r]} \frac{dJRC_m}{d\delta_s}$$

$$\frac{\partial F}{\partial w^p} = - \frac{1}{\tau} \frac{\sigma'_n \log_{10}(JCS/\sigma'_n)}{\cos^2[JRC_m \log_{10}(JCS/\sigma'_n) + \phi_r]} \frac{dJRC_m}{d\delta_s}$$

$$A = \frac{\tau + \bar{\mu}\sigma'_n}{\tau} \frac{\sigma'_n \log_{10}(JCS/\sigma'_n)}{\cos^2[JRC_m \log_{10}(JCS/\sigma'_n) + \phi_r]} \frac{dJRC_m}{d\delta_s} \quad (A.6.20)$$

Substituting Eqns (A.6.16) (A.6.17) into Eqn.(3.53) the plastic stress displacement matrix is finally obtained as Eqn.(3.58).

APPENDIX 7

EFFECT OF σ_n ON $\tau - \delta_s$ PLOT

Numerical difficulties arise at the transition area from zone II to zone III when the normal stress, σ_n , has a low value. The difficulties are from the appearance of the $\tau - \delta_s$ plots.

The shear strength model adopted in this project is

$$\tau = \sigma_n \tan[JRC_m \log_{10}(JCS/\sigma_n) + \phi_r] \quad (A.7.1)$$

So,

$$\frac{\partial \tau}{\partial \delta_s} = \frac{\pi \sigma_n \tan[JRC_m \log_{10}(JCS/\sigma_n)]}{180 \cos^2[JRC_m \log_{10}(JCS/\sigma_n) + \phi_r]} \frac{dJRC_m}{d\delta_s} \quad (A.7.2)$$

When $\delta_s > \delta_A$, because the relation between JRC_m and δ_s is prespecified (see Section 3.3.4), $d(JRC_m)/d\delta_s$ is just a function of δ_s . JCS and ϕ_r are input data. JRC_m is a function of δ_s as well. Therefore, if σ_n is constant, $\partial \tau / \partial \delta_s$ is a function of δ_s only. At the point $\delta_s = \delta_A$, $JRC_m = 0$,

$$\frac{\partial \tau}{\partial \delta_s} = \frac{\pi \sigma_n \log_{10}(JCS/\sigma_n)}{180 \cos^2 \phi_r} \left(\frac{dJRC_m}{d\delta_s} \right)_{\delta_s = \delta_A} \quad (A.7.3)$$

This is a function of σ_n only.

On the other hand, in the region $\delta_s \in (0, \delta_A)$ a linear relation between τ and δ_s is assumed

$$\frac{\partial \tau}{\partial \delta_s} = \frac{\sigma_n \tan \phi_r}{\delta_A} \quad (A.7.4)$$

If it is required that at the point $\delta_s = \delta_A$ the $\tau - \delta_s$ curve to be smooth, i.e.

$$\left(\frac{\partial \tau}{\partial \delta_s} \right)_{\delta_s = \delta_A^-} = \left(\frac{\partial \tau}{\partial \delta_s} \right)_{\delta_s = \delta_A^+} \quad (A.7.5)$$

a critical normal stress can be determined

$$\frac{\sigma_n \tan \phi_r}{\delta_A} = \frac{\pi \sigma_n \log_{10}(JCS/\sigma_n)}{180 \cos^2 \phi_r} \left(\frac{dJRC_m}{d\delta_s} \right)_{\delta_s = \delta_A}$$

$$\sigma_n^* = \frac{JCS}{10^{\pi \delta_A \left(\frac{dJRCm}{d\delta_s} \right)_{\delta_s = \delta_A}}} \quad (A.7.6)$$

If $\sigma_n < \sigma_n^*$,

$$\left(\frac{\partial \tau}{\partial \delta_s} \right)_{\delta_s = \delta_A^+} < \left(\frac{\partial \tau}{\partial \delta_s} \right)_{\delta_s = \delta_A^-}$$

If $\sigma_n > \sigma_n^*$,

$$\left(\frac{\partial \tau}{\partial \delta_s} \right)_{\delta_s = \delta_A^+} > \left(\frac{\partial \tau}{\partial \delta_s} \right)_{\delta_s = \delta_A^-}$$

If $\sigma_n = \sigma_n^*$,

$$\left(\frac{\partial \tau}{\partial \delta_s} \right)_{\delta_s = \delta_A^+} = \left(\frac{\partial \tau}{\partial \delta_s} \right)_{\delta_s = \delta_A^-}$$

The $\tau - \delta_s$ plots for these three cases are shown in Fig. A.7.1.

In the case $\sigma_n < \sigma_n^*$ a numerical difficulty arises since when the transition from $\delta_s < \delta_A$ to $\delta_s > \delta_A$ occurs the trial stress is inside the yield surface, i.e. $F < 0$. The numerical treatment for this problem is shown in Fig.3.23, in which substituting stiffness coefficients shown in Eqns (3.78) and (3.79) are adopted.

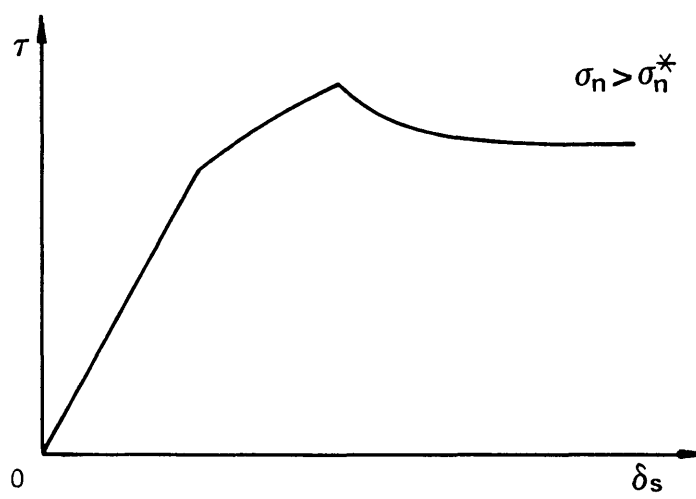
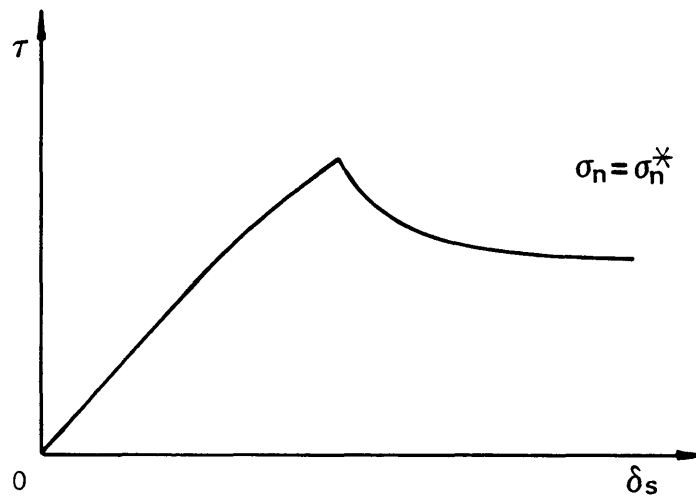
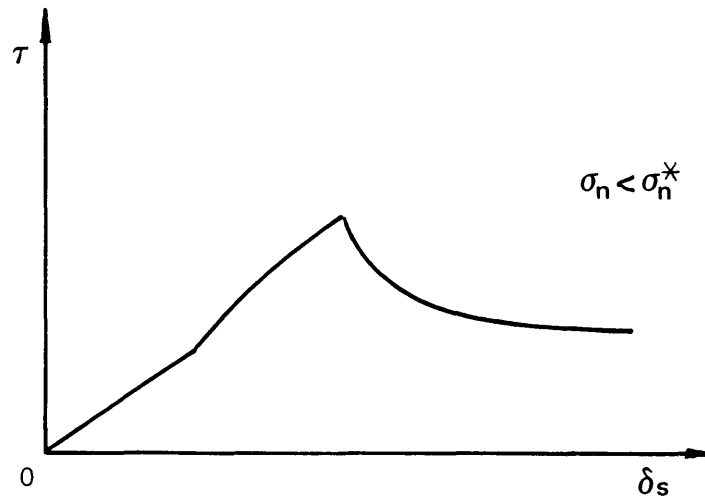


Figure A.7.1 Effect of σ_n on $\tau - \delta_s$ plot

APPENDIX 8
VERIFICATION OF COUPLING
OF FINITE ELEMENTS AND BOUNDARY ELEMENTS

Coupling of finite elements and boundary elements is tested with several examples including the following two.

EXAMPLE 1

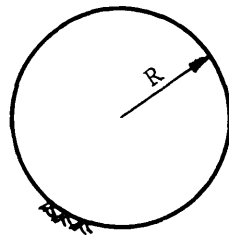
This example is a circular tunnel excavated in a homogeneous infinite rock mass subjected to internal water pressure under plane strain condition, as shown in Fig.A.8.1 (Example 5.1, Brebbia 1980).

The finite-boundary element mesh is given in Fig.A.8.2. It contains 16 eight node finite elements, 8 quadratic boundary elements and 64 nodes. The finite element domain is $3.6D$, where D is the diameter of the tunnel.

The computation results for the radial displacement are shown in Table A.8.1, which indicates good agreement between the closed form solutions and the numerical results. This gives a support to Varadaragan's conclusion that a finite element domain of $4D$ is enough for an appreciate coupling (Varadarajan et al 1985). But this is for the uniform problems.

EXAMPLE 2

This is a tunnel excavated in a non-uniform three dimensional stress field, as shown in Fig.A.8.3.



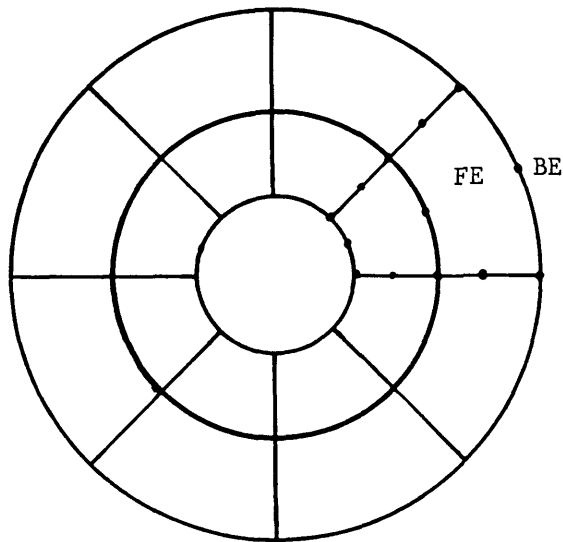
$$E = 207900 \text{ KPa}$$

$$\nu = 0.1$$

$$p = 128.79 \text{ KPa}$$

$$R = 3.0 \text{ m}$$

Figure A.8.1 Example 1 for testing coupling of finite element and boundary element

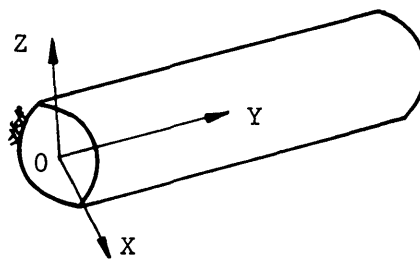
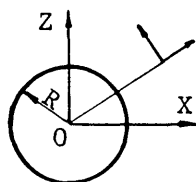


$$\text{FE} \quad 16$$

$$\text{BE} \quad 8$$

$$\text{NODE} \quad 64$$

Figure A.8.2 Finite element and boundary element mesh



$$p_x = 0.397 \text{ MPa}$$

$$p_y = 0.429 \text{ MPa}$$

$$p_z = 0.924 \text{ MPa}$$

$$p_{xy} = 0.116 \text{ MPa}$$

$$p_{yz} = 0.208 \text{ MPa}$$

$$p_{zx} = -0.042 \text{ MPa}$$

$$E = 250 \text{ MPa}$$

$$\nu = 0.25$$

$$R = 1.0 \text{ m}$$

Figure A.8.3 Example 2 for testing coupling of finite element and boundary element

The same finite-boundary element mesh shown in Fig.A.8.2 is chosen to implement the computation. The displacements obtained at different points in the surrounding rock are shown in Table A.8.2 to Table A.8.7. The results indicate that in some directions the numerical results are very close to the closed form solutions; in other directions, however, the errors are considerably large. This means that for non-uniform problems the domain of finite elements $4D$ is not enough for an adequate coupling.

Table A.8.1 Radial displacement for example 1

r/R	CFS	NS	e
	$\times 10^{-2} \text{m}$	$\times 10^{-2} \text{m}$	%
1.0	0.2044	0.2047	0.15
1.5	0.1363	0.1356	0.52
2.2	0.09292	0.09318	0.28
2.8	0.07310	0.07297	-0.056
3.6	0.05679	0.05686	0.125

Table A.8.2 u_x at $z=0$ of example 2, $\times 10^{-3} \text{m}$

r/R	CFS	NS	e
1.0	0.6678	0.8681	30.0
1.5	-0.04272	-0.01879	56.0
2.2	-0.1716	-0.1100	35.9
2.8	-0.1720	-0.1145	33.4
3.6	-0.1522	-0.0925	39.2

Table A.8.3 u_y at $z=0$ of example 2, $\times 10^{-2} \text{m}$

r/R	CFS	NS	e
1.0	0.11600	0.11620	0.18
1.5	0.07733	0.07685	0.62
2.2	0.05273	0.05266	0.13
2.8	0.04143	0.04126	0.41
3.6	0.03222	0.03217	0.16

Table A.8.4 u_z at $z=0$ of example 2, $\times 10^{-3} \text{m}$

r/R	CFS	NS	e
1.0	-0.4200	-0.4041	3.78
1.5	-0.2022	-0.1914	5.30
2.2	-0.1152	-0.1188	3.11
2.8	-0.08456	-0.08315	1.67
3.6	-0.06283	-0.05815	7.44

Table A.8.5 u_x at $x=0$ of example 2, $\times 10^{-3} \text{m}$

r/R	CFS	NS	e
1.0	0.4120	0.4041	3.78
1.5	0.2022	0.1914	5.30
2.2	0.1152	0.1188	3.11
2.8	0.08456	0.08315	1.67
3.6	0.06283	0.05815	7.44

Table A.8.6 u_y at $x=0$ of example 2, $\times 10^{-2} \text{m}$

r/R	CFS	NS	e
1.0	-0.2080	-0.2084	0.18
1.5	-0.1387	-0.1378	0.62
2.2	-0.09454	-0.09442	0.13
2.8	-0.07429	-0.07398	0.42
3.6	-0.05788	-0.05769	0.16

Table A.8.7 u_z at $x=0$ of example 2, $\times 10^{-2}m$

r/R	CFS	NS	e
1.0	-0.5937	-0.5748	3.18
1.5	-0.4446	-0.4401	1.01
2.2	-0.3174	-0.3124	1.58
2.8	-0.2531	-0.2475	2.23
3.6	-0.1987	-0.1931	2.79

REFERENCES

- Amadei, B. and Goodman, R.E. (1981) Formulation of complete plane strain problems for regularly jointed rocks. Rock Mechanics from Research to Application, Proc. 22nd U.S. Symp. Rock Mech., Cambridge Mass., Cambridge Mass.: Mass. Instit. Technol., 265-271
- Anderson, D.G. (1965) Gaussian quadrature formulae for $\int_0^1 -\ln(x)f(x)dx$ Mathematics of Computation 19, 477-481
- Anon (1953) Landslide destruction of power plant blamed on leaks in pressure tunnel. Engng News Record 151, 60, 12 Nov.
- Anon (1970) Cleaning rockfalls in the Eucumbene-Snowy tunnel. Water Power 22, 377-381
- Anon (1979) Cracked tunnel lining delays Arenal Scheme. New Civil Engineer 336, 10, 22 March
- Anon (1985, a) Sri Lankan hydro fizzles. Engng News Record 215, 22-23, 28 Nov.
- Anon (1985, b) Bath County shower: hydro job springs leak during start-up tests. Engng News Record 214, 14-15, 4 April
- Anon (1985, c) Kotmale leaking in high pressure shaft. New Civil Engineer 666, 5, 21 Nov.
- Bandis, S.C., Lumsden, A.C. and Barton, N.R. (1983) Fundamentals of rock joint deformation. Int. J. Rock Mech. Min. Sci. Geomech. Abstr. 20, 249-268
- Barton, N.R. (1972) A model study of rock joint deformation. Int. J. Rock Mech. Min. Sci. Geomech. Abstr. 9, 579-602
- Barton, N.R. (1973) Review of a new shear strength criterion for rock joints. Engng Geology 7, 287-332
- Barton, N.R. (1976) The shear strength of rock and rock joints.

- Int. J. Rock Mech. Min. Sci. Geomech. Abstr. 13, 255-279
- Barton, N.R. (1986) Deformation phenomena in jointed rock. Geotechnique 36, 147-167
- Barton, N.R. and Choubey, V. (1977) The shear strength of rock joints in theory and practice. Rock Mech. 10, 1-54
- Barton, N.R., Bandis, S. and Bakhtar, K. (1985) Strength, deformation and conductivity coupling of rock joints. Int. J. Rock Mech. Min. Sci. Geomech. Abstr. 22, 121-140
- Beer, G. (1983) Finite element, boundary element and coupled analysis of unbounded problems in elastostatics. Int. J. Num. Meth. Engng. 19, 567-580
- Beer, G. (1985) An isoparametric joint/interface element for finite element analysis. Int. J. Num. Meth. Engng. 21, 585- 600
- Beer, G. and Meek, J.L. (1981) Coupled finite element-boundary element analysis of unbounded problems in geomechanics. Proc. Int. Conf. Num. Meth. Coupled Problems, Swansea, E.Hinton, P.Bettess and R.W.Lewis (eds), 605-629. Swansea: Pineridge Press
- Beer, G., Cowling, R. and Bywater, S. (1985) A novel three-dimensional joint element with application to the modelling of mining excavations. Proc. Int. Symp. Fundamentals of Rock Joints, Bjorkliden, O.Stephansson (ed.), 357-365. Lulea: Centek Publishers
- Berdal, A.B. (1972) The Hovatn power plant. Water Power 24, 437-439
- Bonapace, B. (1983) Test and measurements for the pressure tunnels and shafts of the Sellrain Sily hydroelectric power scheme with extremely high head. Proc. 5th Congr. Int. Soc. Rock Mech., Melbourne, 2, D287-292
- Brady, B.H.G. and Bray, J.W. (1978) The boundary element method for

- determining stresses and displacements around long openings in a triaxial stress field. *Int. J. Rock Mech. Min. Sci. Geomech. Abstr.* 15, 21-28
- Brady, B.H.G. (1979) A direct formulation of the boundary element method of stress analysis for complete plane strain. *Int. J. Rock Mech. Min. Sci. Geomech. Abstr.* 16, 235-244
- Brady, B.H.G. and Wassyng, A. (1981) A coupled finite element boundary element method of stress analysis. *Int. J. Rock Mech. Min. Sci. Geomech. Abstr.* 18, 475-485
- Brady, B.H.G. and Brown, E.T. (1985) *Rock Mechanics for Underground Mining* London: George Allen and Unwin
- Brebbia, C.A. (1980) *The Boundary Element Method for Engineers*, 2nd rev. edn. London: Pentech Press
- Brekke, T.L., Bjorlykke, S. and Blindheim, O.T. (1970) Finite element analysis of the Byrte unlined pressure shaft failure. In *Large Permanent Underground Openings*, T.L.Brekke and F.A.Jorstad (eds), 337-342. Oslo: Universitetsforlaget
- Broch, E. (1982) The development of unlined pressure shafts and tunnels in Norway. *Proc. Int. Symp. Rock Mech. Related to Caverns and Pressure Shafts, Aachen, 2*, W.Wittke (ed.), 545-554. Rotterdam: A.A.Balkema
- Buen, B. (1984) *Documentation unlined water conduits in Norway: Part 3* (source unknown)
- Byerlee, J.D. (1968) Brittle-ductile transition in rocks. *J. Geophys. Res.* 73, 4741-4750
- Carol, I. and Alonso, E.E. (1983) A new joint element for the analysis of fractured rock. *Proc. 5th Congr. Int. Soc. Rock Mech., Melbourne, 2*, F147-151
- Chambefort, H. (1977) *The principles and applications of grouting.*

- J. Engng Geol. 10, 57-95
- Chapman, E.J.K. (1961) Pressure tests on rock galleries for the Ffestiniog pumped storage plant. Proc. 7th Int. Conf. Large Dams, Rome, 2, 237-260
- Chengdu Hydropower Design Institute (CHDI, 1981) Collection of tunnels for hydraulic engineering. (in Chinese)
- Christensen, J.B. (1982) Design of unlined pressure shaft at Mauranger power plant, Norway. Proc. Int. Symp. Rock Mech. Related to Caverns and Pressure Shafts 2, W.Wittke (ed.), 531-536. Rotterdam: A.A.Balkema
- Christensen, J.B. (1985) Rock stress measurements for the design of a 965 meter head unlined pressure shaft. Proc. Int. Symp. Rock Stress Rock Stress Measurements, Stockholm, O.Stephansson (ed.), 583-590. Lulea: Centek Publishers
- Christensen, J.B. and Kjolberg, R.S. (1982) Investigations for a 1000 meter head unlined pressure shaft at the Nysey/Steggje project, Norway. Proc. Int. Symp. Rock Mech. Related to Caverns and Pressure Shafts, Aachen, 2, W.Wittke (ed.), 537-543. Rotterdam: A.A.Balkema
- Cooke, J.B., Libby, J.W. and Madill, J.T. (1962) Kemano tunnel repairs: A final report. Engng News Record 169,, 42-44, 4 Oct.
- Dann, H.E., Hartwig, W.P. and Hunter, J.R. (1964) Unlined tunnels of the Snowy Mountains Hydro-electric Authority, Australia. J. Power Div., ASCE, 90, 47-79
- Deere, D.U. (1983) Unique geotechnical problems at some hydroelectric projects. Proc. 7th Pan Amer. Conf. Soil Mech. Found. Engng, Vancouver, 3, 865-888. Montreal: Canadian Geotechnical Society
- Desai, C.S. (1972, a) Introduction to the Finite Element Method New York: Van Nostrand Reinhold Company

- Desai, C.S (1972, b) Overview, trend and projections: Theory and applications of the finite element method in geotechnical engineering, State-of-the-Art. Applications of the Finite Element Method in Geotechnical Engineering, Proc. Symp. held at Vicksburg, Mississippi, C.S.Desai (ed.), 3-90. Vicksburg: U.S. Army Engineer Waterways Experiment Station
- Desai, C.S. (1977) Deep foundations. In Numerical Methods in Geotechnical Engineering, C.S.Desai and J.T.Christian (eds), 235-271. London: McGraw Hill
- Desai, C.S., Eitani, I.M. and Haycocks, C. (1983) An application of finite element procedure for underground structures with non-linear materials and joints. Proc. 5th Congr. Int. Soc. Rock Mech., Melbourne, 2, F209-216
- Desai, C.S. and Zaman, M.M. (1984) Thin-layer element for interfaces and joints. Int. J. Num. Anal. Meth. Geomech. 8, 19-43
- Eristov, U.S., Churakov, A.I. and Kalandarishvili, A.G. (1970) Investigations of pressure tunnels with a concrete lining on models of minimum depth. Hydraulic Structures, No. 3, 45-49 (in Russian)
- Fairhurst, C. (1986) In-situ stress determination - an appraisal of its significance in rock mechanics. Proc. Int. Symp. Rock Stress Rock Stress Measurements, Stockholm, O.Stephansson (ed.), 3-17. Lulea: Centek Publishers
- Gao, A.Z. and Zhang, X.H. (1983) On the stress-strain relationship of anisotropic jointed rock mass. J. Water Power No. 2, 23-44 (in Chinese)
- Ge, X.R. (1981) Non-linear analysis of joint element and its application in rock engineering. Int. J. Num. Anal. Meth. Geomech. 5, 229-245

- Gerrard, C. (1982) Elastic models for rock masses having one, two and three sets of joints. *Int. J. Rock Mech. Min. sci. Geomech. Abstr.* 19, 15-23
- Gerrard, C. (1985) Formulation for the mechanical properties of rock joints. *Proc. Int. Symp. Fundamentals of Rock Joints*, Bjorkliden, O.Stephansson (ed.), 405-422. Lulea: Centek Publishers
- Gerrard, C. (1986) Shear failure of rock joints: Appropriate Constrains for empirical relations. *Int. J. Rock Mech. Min. Sci. Geomech. Abstr.* 23, 421-429
- Ghaboussi, J., Isenberg, V. and Wilson, E.L. (1973) Finite element for rock joints and interface. *J. Soil Mech. Foundns Div., ASCE*, 99, 833-848
- Goodman, R.E., Taylor, R.L. and Brekke, T.C. (1968) A model for the mechanics of jointed rock. *J. Soil Mech. Foundns Div., ASCE*, 94, 637-659
- Goodman, R.E. (1975) *Methods of Geological Engineering in Discontinuous Rock* St. Paul: West Publishing Company
- Goodman, R.E. and St.John, C. (1977) Finite element analysis for discontinuous rock. In *Numerical Methods in Geotechnical Engineering*, C.S.Desai and J.T.Christien (eds), 148-175. London: McGraw Hill
- Gu, Z.Q., Chen, M.Z. and Lu, M. (1982) The capacity undertaking internal water pressure of rock surrounding a pressure tunnel. *Tech. Underground Engng Hydropower Proj.*, 147-155
- Gu, Z.Q., Peng, S.Z. and Chen, M.Z. (1983) *Underground Engineering for Hydropower Developments* Beijing: Tsinghua Univ. Publishing House
- Heuze, F.E. (1979) Dilation effect of rock joints. *Proc 4th Congr.*

- Int. Soc. Rock Mech., Montreux, 1, 169-175
- Heuze, F.E. and Barbour, T.G. (1981) Models for jointed rock structures. Proc. 1st Int. Conf. on Comp. in Civil Engng., 811-824. New York: ASCE
- Heuze, F.E. (1982) New models for rock joints and interfaces. J. Geotech. Engng. Div., ASCE, 108, 757-776
- Hoek, E. (1982) Report on geotechnical aspects of the Cat Arm hydroelectric project. Golder Associates, Newfoundland, Canada
- Hoek, E. and Bray, J. (1981) Rock slope engineering, Rev. 3rd ed. London: The Institution of Mining and Metallurgy
- Hoek, E. and Brown, E.T. (1980, a) Empirical strength criterion for rock masses. J. Geotech. Engng. Div., ASCE, 106, 1013-1035
- Hoek, E. and Brown, E.T. (1980, b) Underground Excavation in Rock London: Inst. Mining and Metallurgy
- Hoskins, E.R., Jaeger, J.C. and Rosengren, K. (1968) A medium scale direct friction experiment. Int. J. Rock Mech. Min. Sci. 5, 143-154
- Hudson, J.A. (1987) The understanding of measured changes in rock structure, in situ stress and water flow caused by underground excavation. Proc. 2nd Int. Symp. Field Measurements Geomech, Kobe, 2, 467-474 (Preprint)
- Hunan Hydropower Design Institute (HHDI, 1977) Test report - high pressure tunnel for Huangqin hydroelectric power station. (in Chinese)
- Jaeger, C. (1955) Present trends in the design of pressure tunnels and shafts for underground hydroelectric power stations. Proc. Instit. Civil Engs 4, 116-200, 545-596
- Jaeger, C. (1961) Rock mechanics and hydro power engineering.

- Water Power 13, 349-360
- Jaeger, C. (1963) Theory of resonance in hydro-power systems.
Water Power 15, 118-125
- Jaeger, C. (1979) Rock Mechanics and Engineering, 2nd edn.
London: Cambridge Univ. Press
- Jaeger, J.C. (1959) The frictional properties of joints in rocks.
Geofis. pura appl. 43, 148-158
- Jaeger, J.C. and Rosengren, K.J. (1969) Friction and sliding of
joints. Proc. Aust. Inst. Min. Metall. 229, 93-104
- Jaeger, J.C. (1971) Friction of rocks and stability of rock
slopes. Geotechnique 21, 93-139
- Jaeger, J.C. and Cook, N.G.W. (1979) Fundamentals of Rock
Mechanics, 3rd edn. London: Chapman and Hall
- Kranz, R.L., Frankel, A.D., Engelder, T. and Scholz, C.H. (1979)
The permeability of whole and jointed Barre granite. Int. J.
Rock Mech. Min. Sci. Geomech. Abstr. 16, 225-234
- Ladanyi, B. and Archambault, G. (1970) Simulation of shear
behaviour of a jointed rock mass. Proc. 11th U.S. Symp. Rock
Mech., Berkeley, W.H. Somerton (ed.), 105-125. New York: AIME
- Lang, T.A., Kendorski, F.S. and Chawla, K.S. (1976) Effect of
rapid water pressure fluctuations on unlined water tunnel
stability. Proc. 1976 Rapid Excav. Tunn. Conf., R.J. Robbins
and R.J. Conlon (eds), 417-429. New York: AIME
- Lawton, F.L. and Sutherland, J.M. (1957) Kemano pressure conduit
engineering investigations. J. Power Div. ASCE, 83, 1396.1-
1396.34
- Lu, M. (1983) Infinite elements and their applications in
geotechnical engineering. Collected Research Papers, Institute
of Water Conservancy and Hydroelectric Power Research 20, 45-

52 (in Chinese)

- Lu, M. (1987) FEM models for jointed rock masses. Collected Research Papers, Institute of Water Conservancy and Hydroelectric Power Research 30, 40-48 (in Chinese)
- Madhav, M.R. and Panesh, M.R. (1966) Nomographs for the design of pressure tunnel to a rock slope. Proc. 1st Congr. Int. Soc. Rock Mech., Lisbon, 2, 243-248
- Mahtab, M.A. and Goodman, R.E. (1970) Three dimensional finite element analysis of jointed rock slopes. Proc. 2nd Congr. Int. Soc. Rock Mech., Belgrade, 3, 7-12
- Matt, P., Thurnhur, F. and Uherkovich, I. (1978) Prestressed concrete pressure tunnels. Water Power Dam Constr. 30, 23-30
- Melan, E. (1932) Point force at internal point in a semi- infinite plate. Math. und Mech. 12, 343-346
- Mindlin, R.D. (1936) Force at a point in the interior of a semi-infinite solid. J. Appl. Phys. 7, 195-202
- Mogi, K. (1966) Pressure dependence of rock strength and transition from brittle fracture to ductile flow. Bulletin, Earthquake Research Institute, Tokyo Univ. 44, 215-232
- Mosonyi, E. (1960) Water power development 2 Budapest: Publishing House of the Hungarian Academy of Science
- Naber, G. (1960) The Jansen pumped storage scheme. Water Power 12, 312-313
- Nayak, G.C. and Zienkiewicz, O.C. (1972) Elasto-plastic stress analysis generalization for various constitutive relations including strain softening. Int. J. Num. Meth. Engng. 5, 113-135
- Newberg, J. (1970) Engineering geology in the investigation and construction of the Batang Padang hydroelectric scheme

- Malaysia. Quart. J. Engng Geol. 13, 151-181
- Olofsson, T. (1985) A non-linear model for the mechanical behaviour of continuous rock joints. Proc. Int. Symp. Fundamentals of rock joints, Bjorkliden, O.Stephansson (ed.), 395-404. Lulea: Centek Publishers
- Olsen, S.R. (1970) Experience with unlined pressure shafts in norway. In Large Permanent Underground Openings, T.L.Brekke and F.A.Jorstad (eds), 327-332. Oslo: Universitetsforlaget
- Olsen, S.R. (1974) Underground openings filled with high- pressure water or air. Bull. Int. Engng Geol. 9, 91-95
- Olsen, S.R. (1985) Experience gained from unlined high pressure tunnels and shafts in hydroelectric power stations in Norway. In Norwegian Hydropower Tunn., 31-40. Trondheim: Tapir Publisher
- Owen, D.J.R. and Hinton, E. (1980) Finite Elements in Plasticity Swansea: Pineridge
- Pande, N.G. and Xiong, W. (1982) An improved multi-laminate model of jointed rock masses. In Numerical models in geomechanics, R.Dunger, G.N.Pande and J.A.Studer (eds), 218- 226. Rotterdam: A.A.Balkema Publisher
- Pande, N.G. (1985) A constitutive model of rock joints. Proc. Int. Symp. Fundamentals of Rock Joints, Bjorkliden, O.Stephansson (ed.), 429-440. Lulea: Centek Publishers
- Paterson, S.J., Hale, G.E. and Ikin, D.B. (1975) Stabilizing a landslide above Fisher penstock, Tasmania. Proc. 2nd Aust. N. Zealand Conf. Geomech., Brisbane,, 314-318. Sydney: The Instit. of Engineers, Australia
- Patterson, F.W., Clinch, R.L. and McCaig, I.W. (1957) Design of large pressure conduits in rock. J. Power Div. ASCE, 83, 1457.1-1457.30

- Patton, F.D. (1966) Multiple modes of shear failure in rock. Proc. 1st Congr. Int. Soc. Rock Mech., Lisbon, 1, 509-513
- Price, N.J. (1966) Fault and Joint Development in Brittle and Semi-brittle Rock Oxford: Pergamon Press
- Qu, S.N. (1981) Personal communication.
- Reznikov, R.A., Khachaturian, N.S., Zolotov, O.N., Kaplan, M.H., and Solochansky, D.I. (1984) Consideration of rock resistance in designing linings for shallow hydropower pressure tunnels. Proc. Int. Symp. Design Perform. Undergr. Excav., E.T.Brown and J.A.Hudson (eds), 165-171. London: British Geotechnical Society
- Roberds, W.J. and Einstein, H.H. (1978) Comprehensive model for rock discontinuities. J. Geotech. Engng. Div., ASCE, 104, 553-569
- Seeber, G., (1968) Diskussion beirage. Zu der Vortragen Hautum, Detzhofer u. Forster u Doring. Felsmechanik u. Ingenieurgeologie Suppl. IV, 279-281
- Simic, R. (1979) Experience in Yugoslavian Pressure Tunnel Construction. Rock Mech. Suppl. 8, 227-248
- Sofianos, A.I. (1984) Numerical simulation of excavations within jointed rock of infinite extent. Ph.D. Thesis, Univ. of London.
- Spencer, R.W., Laverty, B.R. and Barber, D.A. (1964) Unlined tunnels of the Southern California Edison Company. J. Power Div., ASCE, 90, 105-132
- Stagg, K.G. (1968) In situ tests on the rock mass. In Rock Mechanics in Engineering Practice, K.G.Stagg and O.C.Zienkiewicz (eds), 125-156. London: John Wiley & Sons
- Sullenberger, R. (1986) Bath County: the world's largest pumped-storage plant commissioned. Water Power Dam Constr. 38, 12-

- Takahashi, M. (1972) The 1125Mw pumped storage scheme at Shintoyone. *Water Power* 24, 51-56
- Terzaghi, K. (1962) Stability of steep slopes on hard unweathered rock. *Geotechnique* 12, 251-270
- Thomes, H.H. and Whitham, L.S. (1964) Tunnels for hydroelectric power in Tasmania. *J. Power Div. ASCE*, 90, 11-28
- Varadarajan, A., Sharma, K.G. and Singh, R.B. (1985) Some aspects of coupled FEBEM analysis of underground opening. *Int. J. Num. Anal. Meth. Geomech.* 9, 557-571
- Vik, G. and Tunbridge, L. (1985) Hydraulic fracturing - a simple tool for controlling the safety of unlined pressure shaft and headrace tunnels. *Proc. Int. Symp. Rock Stress Rock Stress Measurement, Stockholm*, O.Stephansson (ed.), 591-598. Lulea: Centek Publisher
- Watson, J.O. Advanced implementation of the boundary element method for two and three dimensional elastostatics. In *Developments in boundary element methods-1*, P.K.Banerjee and R.Butterfield (eds), 31-63. London: Applied Science Publisher
- Wilson, E.L. (1977) Finite elements for foundation, joints and fluids. In *Finite Elements in Geomechanics*, G.Gudehus (ed.), 319-350. London: Wiley
- Yang, Q. X. (1982) The layout and alignment of tunnels in hydroelectric projects. *Tech. Undergr. Engng Hydropower Proj.* No. 1, 99-122
- Yin, Y.Q., Qu, S.N. and Liu Jun (1979) The application of non-linear finite element method in engineering geology. *Geological Science* 3, 236-251 (in Chinese).
- Yin, Y.Q. and Qu, S.N. (1981) Constitutive relations used in finite

- element analysis for structures made up of rock-like materials.
J. of Beijing Univ. No. 1, 37-46 (in Chinese)
- Yoshinaka, R. and Yamabe, T. (1986) Joint stiffness and the
deformation behaviour of discontinuous rock. Int. J. Rock Mech.
Min. Sci. Geomech. Abstr. 23, 19-28
- Zhu, B.F. (1979) Theory and Application of Finite Element Method
Beijing: Water Conservancy and Electric Power Press (in
Chinese)
- Zienkiewicz, O.C. (1977) The Finite Element Method in Engineering
Science, 3rd edn. London: McGraw-Hill
- Zienkiewicz, O.C. and Pande, G.N. (1977) Time-dependent
multilaminate model of rocks - A numerical study of deformation
and failure of rock masses. Int. J. Num. Anal. Meth. Geomech.
1, 219-247