

A STUDY OF BUBBLE DYNAMICS IN NUCLEATE BOILING

by

Sarath Ananda Kumara Abayawardana

B.Sc. (Eng.), Dip. Chem. Eng. (UCL)

May 1974

A thesis submitted for the degree of
Doctor of Philosophy of the University
of London and for the Diploma of Imperial
College of Science and Technology

Department of Chemical Engineering and Chemical Technology
Imperial College of Science and Technology
London S.W.7.

ABSTRACT

A model suitable for obtaining computer solutions for the growth and departure of fast growing bubbles on a heated surface under pool boiling conditions is developed. The restriction of a shape assumption for the bubble profile, normally employed in bubble growth analyses, is removed by the consideration of growth at the base perimeter and the top of the curved interface of the bubble separately. By this approach the effects due to the hydrostatic head corresponding to bubble height and the viscous drag force near the surface restraining the growth of the base perimeter are also included. An approximate expression is developed for this viscous drag force.

A numerical solution of the governing equations is obtained taking into account both the microlayer evaporation and the heat transfer across the curved interface. The pressure and temperature variation within the bubble is also taken into account. A method is proposed for the evaluation of the wall temperature variation underneath the microlayer, which has a considerable influence on the microlayer evaporation in some cases. The proposed model for bubble growth also predicts the time of departure of the bubble and hence its size at departure without the need to consider a force balance on the bubble as a whole.

The model is successfully tested against experimental data quoted in the literature covering a wide range of boiling situations. Computer time required in a CDC 6400 machine to follow the growth of a typical bubble from initiation to departure is about 200 s..

ACKNOWLEDGEMENTS

I wish to express my sincere thanks and deep appreciation to Mr. G.H. Anderson for his helpful advice, guidance and concern throughout the course of the present research. His help in many aspects during my stay here has been invaluable.

I also wish to thank Professor A.R.J.P. Ubbelohde, C.B.E., F.R.S. and Professor R.W.H. Sargent of the Department of Chemical Engineering and Chemical Technology, in whose department the research was carried out.

The financial support provided by the Education Department, Sri Lanka in the form of a Sri Lanka Government University Scholarship and by the Overseas Development Ministry of the British Government, administered through the British Council, are gratefully acknowledged.

I wish to extend a special thanks to the Ceylon Institute of Scientific and Industrial Research for making available the required period of study leave to complete this study.

I am deeply indebted to my parents for many a sacrifice they have made in order to make this venture possible.

Considerable aid given by Miss Helen Regester in typing the thesis in the very limited time available is acknowledged with appreciation.

Finally, thanks are also extended to many of my friends, colleagues and the staff of the Department of Chemical Engineering and Chemical Technology for the help, generosity and encouragement offered during the course of this research.

CONTENTS

	Page
ABSTRACT	2
ACKNOWLEDGEMENTS	3
CONTENTS	4
NOMENCLATURE	9
<u>CHAPTER 1</u>	<u>INTRODUCTION</u>
	14
1.1	General
	14
1.2	Bubble Dynamics and Boiling
	14
1.3	Scope of the Present Work
	16
<u>CHAPTER 2</u>	<u>BUBBLE DYNAMICS - LITERATURE SURVEY</u>
	18
2.1	Introduction
	18
2.2	Mechanisms of Heat Transfer in Boiling
	18
2.2.1	Bubble Agitation Model
	18
2.2.2	Vapour Liquid Exchange Mechanism
	19
2.2.3	Evaporative Mechanism
	20
2.3	Development of Bubble Growth Theories
	21
2.3.1	Bubble Growth in Uniformly Superheated Liquids
	21
2.3.2	Bubble Growth in Non-uniform Temperature Fields
	23
2.3.3	Bubble Growth with Microlayer Evaporation
	30
2.4	Existing Theories of Bubble Departure
	33
2.5	Bubble Shapes in Nucleate Boiling
	39
2.6	Existing Experimental Work on Bubble Growth and Departure used in the Present Work
	41

		<u>Page</u>
<u>CHAPTER 3</u>	<u>MODEL FOR ANALYSIS AND GOVERNING HYDRODYNAMIC</u>	47
	<u>EQUATIONS</u>	
3.1	Introduction	47
3.2	The Model	49
3.2.1	Description of the Model	49
3.2.2	Outline of the Method of Solution	51
3.3	Criterion of Bubble Inception	53
3.4	Initial Conditions for Bubble Growth	55
	Computation	
3.5	Governing Hydrodynamic Equations	57
3.5.1	Development of the Extended Rayleigh Equation	58
3.5.2	Adaptation of the Extended Rayleigh Equation	60
	for Different Growths at Top and the Base of	
	the Curved Interface	
3.5.2.1	Growth of the Top of the Curved Interface	60
3.5.2.2	Growth at the Base of the Curved Interface	61
3.6	Derivation of the Viscous Drag Force at	62
	Bubble Base	
3.7	Evaluation of Pressure and Temperature	67
	Variation inside the Bubble	
3.8	Conclusion	69
 <u>CHAPTER 4</u>	 <u>HEAT TRANSFER INTO THE BUBBLE</u>	 76
4.1	Introduction	76
4.2	Heat Transfer across the Curved Interface of	76
	the bubble	
4.2.1	Initial Temperature Profile in the Liquid	77
	at Bubble Initiation	
4.2.2	Waiting Time Evaluation	78

4.2.3.	Heat Transfer across the curved interface	79
4.2.3.1	Derivation of the Temperature Gradient at the Curved Interface	80
4.2.3.2	Evaluation of the Rate of Heat Transfer	85
4.2.4	Final Set of Equations to be solved to obtain the Rate of Heat Transfer across the Curved Interface	86
4.3	Heat Transfer across the Microlayer	87
4.3.1	Microlayer Thickness	87
4.3.2	Microlayer Evaporation in a Poorly Conducting Liquid on a Highly Conducting Wall	89
4.3.3	Microlayer Evaporation in a Highly Conducting Liquid on a Poorly Conducting Wall	94
4.3.4	Microlayer Evaporation in the case of Intermediate Wall Conductivity	97
<u>CHAPTER 5</u>	<u>RESULTS AND DISCUSSION</u>	108
5.1	Introduction	108
5.2	Comparison of Bubble Growth with Experimental Observations	108
5.2.1	Han and Griffith (25) Data - Bubbles A, B and C	109
5.2.2	Hospeti and Mesler (52) Data - Bubbles D and E	111
5.2.3	Cole and Shulman (22) Data - Bubbles F to L	112
5.2.4	Stewart and Cole (32) Data - Bubbles M to O	115
5.2.5	Merry (53) Data - Bubbles P to Q	116
5.3	Comparison of Bubble Growth with some of the Existing Theories	117

	<u>Page</u>
5.3.1 Comparison with Cole and Shulman (22) Expression	119
5.3.2 Comparison with Cooper and Vijuk (33) Expression	120
5.4 Bubble Departure Estimation	121
5.4.1 Prediction from Present Theory	121
5.4.2 Comparison with Conventional Departure Value Estimations	122
5.5 Special Features of the Computation and Related Aspects of Bubble Growth	124
5.5.1 Variation of Growth Rate with Cavity Size	124
5.5.2 Effect of Waiting Time on Bubble Growth	126
5.5.3 Effect of the Time Step Size on Overall Growth	128
5.5.4 Effect of the Viscous Drag Force Term at the Bubble Base	129
5.5.5 Microlayer and the Dry Patch	130
5.5.6 Wall Temperature Variation	133
 <u>CHAPTER 6</u>	
<u>GENERAL DISCUSSION AND CONCLUSIONS</u>	175
6.1 General Discussion	175
6.2 Applicability of the Method	178
6.3 Suggestions for Future Work	179
6.4 Conclusions	180
 <u>REFERENCES</u>	182
 <u>APPENDIX I</u>	
APPLICATION OF THE RUNGE-KUTTA FOURTH ORDER METHOD TO THE PRESENT PROBLEM	189

	<u>Page</u>
<u>APPENDIX II</u> EXPERIMENTAL DATA VALUES AND CORRESPONDING PHYSICAL PROPERTY VALUES	200
<u>APPENDIX III</u> DESCRIPTION AND USE OF THE COMPUTER PROGRAMMES	207

NOMENCLATURE

(SI units are used throughout unless otherwise mentioned in the text)

A	- Growth constant in $R = At^{\frac{1}{2}}$
	- Constant in Classius-Clapeyron equation
a	- Constant in microlayer profile expression
B	- Constant in Classius-Clapeyron equation
b	- Constant in microlayer profile expression
C_2	- Constant in $\delta_o = \frac{1}{2} \sqrt{t_g}$
C_D	- Drag Coefficient
C_{pl}	- Specific heat of liquid
C_{pv}	- Specific heat of vapour
C_{pw}	- Specific heat of wall material
c	- Constant in microlayer profile expression
D	- Diameter of bubble
D_B	- Diameter of bubble base
D_d	- Departure diameter
D_o	- Departure diameter at zero departure velocity
F_{Bu}	- Buoyancy Force
F_D	- Drag Force
F_I	- Inertia Force
F_N	- Net Force
F_S	- Surface tension Force
G	- Ideal gas constant
g	- Acceleration due to gravity

g_c, g_n	- Gravitation Conversion Factor
h	- Heat transfer coefficient
k	- Thermal Conductivity
k_l	- Thermal Conductivity of Liquid
k_w	- Thermal Conductivity of solid
L	- Latent heat of vaporization
ℓ	- Scaling Length, boundary layer thickness
m	- Mass of Vapour in bubble
δm_{cs}	- Mass of Vapour evaporated across curved interface
δm_m	- Mass of Vapour evaporated from the microlayer
δm_{m1}	- Mass of Vapour evaporated from R_1 to R'
δm_{m2}	- Mass of Vapour evaporated from R' to R_2
N_{ja}	- Jakob number $\frac{C_{pl} \cdot \rho_l \Delta T}{L \rho_v}$
$N_{ja(w)}$	- Jakob number w. r. t. wall temperature
$N_{ja(\infty)}$	- Jakob number w. r. t. bulk temperature
$N_{pr(l)}$	- Prandtl Number of liquid
N_{Re}	- Bubble Reynolds Number
p	- Pressure
p_i	- Pressure of vapour within the bubble
p_d	- Pressure corresponding to drag force
Δp_d	- Difference between p_d at R and p_d at ∞
q_{av}	- Average heat flux
q_{cs}	- Heat flux across curved surface
R	- Radius of spherical segment passing through known points of bubble profile

- R_1 - Radius of the dry patch
- R_2 - Same as Base Radius
- R_B - Base Radius
- $\dot{R}_B$ - Interfacial velocity at base perimeter $(\frac{dR_B}{dt})$
- $\ddot{R}_B$ - Interfacial accelerations at base perimeter $(\frac{d^2 R_B}{dt^2})$
- R_c - Cavity radius
- R_{es} - Radius of hemisphere with equal surface area
- R_T - Radius at top of curved interface-height of bubble
- $\dot{R}_T$ - Interfacial velocity at top $(\frac{dR_T}{dt})$
- $\ddot{R}_T$ - Interfacial acceleration at top $(\frac{d^2 R_T}{dt^2})$
- R' - The radius up to which the microlayer dries up due to evaporation
- R^* - Dimensionless bubble radius R/ℓ
- r - radial coordinate, radius of curvature
- r^* - Dimensionless radial coordinate
- r_o - Orifice radius
- r', r'' - Integrating variables
- T - Temperature
- T_i - Temperature of vapour inside the bubble
- T_s - Saturation temperature at bulk pressure
- T_w - Wall temperature. Used instead of T_{wo} in cases where wall temperature variation is not taken into account
- T'_w - Uncorrected wall temperature
- T_{wo} - Initial wall temperature

T_{∞}	- Bulk temperature
T^*	- Dimensionless Temperature
ΔT	- Superheat temperature, bulk or wall
t	- Time
t_g	- Time taken by base perimeter to grow to these point under consideration
t_w	- Waiting time
t^*	- Dimensionless time
U_b	- Bubble rise velocity
U_r, u_r	- Radial velocity
u_{ϕ}	- Velocity in the ϕ -direction
V	- Volume
V_b	- Bubble Volume
V_d	- Departure Volume
v_l	- Specific volume of liquid
v_v	- Specific volume of vapour

GREEK SYMBOLS

α	- Thermal diffusivity
α_l	- Thermal diffusivity of liquid
α_w	- Thermal diffusivity of solid
β	- Constant (defined where used)
δ	- Microlayer thickness
δ_o	- Initial microlayer thickness
θ	- Coordinate
λ	- Penetration thickness

μ	- Dynamic Viscosity
ν	- Kinematic Viscosity
ρ_l	- Density of liquid
ρ_v	- Density of vapour
ρ_w	- Density of wall
σ	- Surface Tension
ϕ	- Coordinate, Contact angle
ϕ_c	- Curvature factor

SUBSCRIPTS (others defined where used)

av.	- average
B	- Base perimeter of bubble
b	- Bubble
cs	- Curved surface
d	- Departure
g	- Growth time in 't'
i	- Within the bubble
l, l	- Liquid
m	- Microlayer
o	- Initial, orifice
r	- r-direction
s	- Saturation
T	- Top of the curved interface
v	- Vapour
w	- Wall, surface, solid; waiting time in 't'
ϕ	- ϕ -direction
∞	- Bulk

CHAPTER 1

INTRODUCTION

1.1 GENERAL

Nucleate boiling, which is the formation, growth and detachment of vapour bubbles at a heated surface, is a commonly used phenomenon in modern technology. The two main fields where boiling is employed are in vapour generation, as in steam boilers and in distillation equipment, and in instances where large amounts of heat have to be extracted across relatively small areas or where the allowable or available temperature differences across the surfaces are limited.

Although boiling is advantageous as a means of attaining high heat transfer rates, it gives rise to a number of problems frequently occurring both in instances where boiling is intentionally employed and in unwanted boiling situations. A study of nucleate boiling, therefore, has a twofold purpose; one in the investigation of boiling from the point of view of obtaining improved heat transfer and the other in obtaining a better understanding of the problems associated with boiling.

1.2 BUBBLE DYNAMICS AND BOILING

There are various theories existing as to the exact nature of the mechanism which causes the enhancement of heat transfer in boiling situations. Earlier schools tended to think that this is mainly due to agitation of the thermal layer by the bubbles while the more recent theory is that a major proportion of this increase is due to direct evaporation into the bubbles. According to this theory a thin microlayer of liquid exists underneath a fast growing bubble which

rapidly evaporates, contributing a large part to the total evaporation. Consequently, it is possible for a dry patch to form and grow under a bubble where the microlayer has totally evaporated.

Evaporation of the microlayer in particular, and bubble growth in general, leads to concentration of non-volatile or less volatile matter near the surface. This leads to increased scaling and corrosion of the surface thus deteriorating its heat transfer characteristics. As the concentration of this non-volatile or less volatile matter takes place near the active nucleation sites distributed over the heating surface, concentration cells are formed, introducing another corrosion phenomenon. Concentrated solutions immediately adjacent to the heating surface may rise in temperature above the average of the bulk and as a result suffer decomposition and possibly cause further deposition of solids. In many chemical and food processing industries such degenerative chemical changes also reduce the quality of the product, in addition to the problem of deterioration of the surface. The degree of degeneration is normally a function of time and temperature. Even if the equipment is so designed to avoid this degeneration in the bulk liquid complete evaporation to dryness at a dry spot may be harmful to the product over and above the temperature and time effect.

Another important consideration in situations where heat transfer is heat flux determined, is the maximum heat flux that can be attained before the equipment fails due to an excessive rise of surface temperature. This limit arises from the insulating effect of a vapour film formed over the surface. A prelude to this condition is sometimes the formation of a dry area underneath individual bubbles. The formation of these dry areas does not necessarily lead to 'burn out' but if the surface temperature can rise above the Lidenfrost point before the fluid returns to rewet the surface a permanent dry

patch is formed which will spread. In any case, the dry patch formation and expansion is undesirable as the temperature variations at these points lead to thermal stressing of the heat transfer surface which again encourages corrosion.

Accurate predictions of boiling heat transfer coefficients that satisfactorily account for all the parameters involved are still not available, but fortunately in many applications the boiling coefficients are so high that their inaccuracies do not affect the design criteria. There are, however, other processes such as those where heat is supplied by the condensation of a vapour, where the achievement of higher coefficients or greater accuracy in their prediction is an important factor. Most of the existing correlations of heat flux-temperature difference are empirical in nature and are reliable mostly only for the particular case for which they were evolved. Extrapolation of these to different kinds of liquids, surfaces and surface geometries give unreliable results.

An investigation of bubble dynamics is useful in giving an idea of the amount of vapour carried away by a bubble. In an ideal situation, a knowledge of bubble growth rates and departure sizes, used together with the number of bubbles per unit area and the bubble frequency at a particular site, should yield a correlation for heat transfer. However, bubble growth and departure phenomena are not very steady even for the same site. If this variation is of a statistical nature, a reliable model of bubble behaviour would help in formulating a correlation of wider applicability.

1.3 SCOPE OF THE PRESENT WORK

The above discussion stresses the need for bubble growth models which not only successfully predict growth and departure but also

facilitate investigation of local features in bubble growth such as microlayer evaporation and dry patch expansion. But most of the growth theories that exist today tend to take account of various influential factors separately for particular situations, in a manner which limits their application. Further attempts to develop comprehensive theories by the extension of analytical solutions can be extremely difficult and the results so obtained could be very limiting. Accurate solutions require the simultaneous solution of the Navier-Stokes equation and the energy equations governing heat transfer into the bubble. Complete numerical solutions of these require excessive computing resources and lead to impractical solutions. A realistic simplification of this problem giving rise to a comprehensive growth theory with increased applicability should prove very valuable. Investigation and development of such a model, which produces reasonably accurate solutions without involving too much computing is presented in this thesis. A number of restrictions normally used in bubble growth theories have been removed.

A criterion for the prediction of departure values which enables the growth calculation to be continued to the point of departure, without a need for considering an overall force balance on the bubble is also presented.

CHAPTER 2

BUBBLE DYNAMICS - LITERATURE SURVEY

2.1 INTRODUCTION

A large amount of research has been carried out in order to determine the mechanisms that enhance heat transfer rates in boiling. As a result three main lines of thought have developed where the enhancement is attributed to the formation of the bubbles on the surface. These are discussed briefly in conjunction with theories on bubble growth in this chapter. An attempt is made to trace the development of bubble growth relationships. Existing methods of obtaining departure sizes of bubbles and a brief survey of the literature on the shape of bubbles is also included. The final section contains a description of some of the existing experimental work on growth rates and departure sizes, the data from which are used at a later stage for comparison with the present theory.

2.2 MECHANISMS OF HEAT TRANSFER IN BOILING

2.2.1 Bubble Agitation Model

One of the earliest mechanisms, proposed by Rohsenow and Clark (6) with contributions by some others, assume that the enhancement of heat transfer is due to increased agitation near the surface caused by the departing bubbles. According to this line of thought these growing and detaching bubbles create random micro-convection currents along the surface, destroying the thermal layer under the bubble and over some unknown distance around the base which is known as the area of influence of the bubble (Fig 2.1 (a)). Hsu and Graham (7) showed by means of Schlieren motion pictures that the agitation during bubble growth and departure greatly affected the thermal layer around the base of the bubble as far away as one

bubble diameter. It has been conclusively established that there is enhancement of heat transfer due to this agitation. But the question is, to what level would this enhancement take place. Forster and Greif (8) showed that the rate of heat transfer depends to a large extent on the temperature difference $T_w - T_s$ and not $T_w - T_\infty$ as would be expected from the agitation model. Nixon et al. (9) investigated the enhancement of heat transfer at the heating surface by electrolytic generation of gas bubbles, and found that this was dependent on $T_w - T_\infty$ as expected, and that the agitation increased heat transfer up to fourfold, which is not quite as high as one would expect in a nucleate boiling situation.

2.2.2 Vapour-Liquid Exchange Mechanism (Fig. 2.1 (b))

Forster and Greif (8) went on to propose a new model where they suggested that the bubble acts as a piston of a pump which displaces superheated liquid away from the surface to be replaced by cooler liquid from the bulk. This cooler liquid picks up heat from the surface before being displaced by the next growing bubble into the bulk of the liquid. Forster and Greif, using Gunther's (10) data and the assumption that the volume of the departing bubble was a measure of the evaporation taking place, obtained a value of the order of 100 for the ratio of heat capacity of a liquid slug to the latent heat content of a comparable volume of vapour. They did not consider the possibility of simultaneous evaporation and condensation while the bubble was growing despite the fact that Gunther conducted his experiments in subcooled liquid. Another assumption employed in their evaluation was that this departing liquid slug is at a temperature which is the mean of the bulk and wall temperatures. This in turn supposes that thermal layers of the same order of magnitude as bubble radii exist. Experimental evidence clearly shows that bubbles generally grow to much higher radii than thermal layer thicknesses. Again, there is no doubt that there is enhancement of heat transfer to a certain degree by this

mechanism, but due to the reasons discussed it cannot be as high as Forster and Greif's estimation.

2.2.3 Evaporative Mechanism (Fig 2.1 (c))

A third model places emphasis on the evaporative processes associated with the growth of bubbles. A major portion of the evaporative heat transfer to the bubble was thought to occur through the curved surface of the bubble where it is in contact with the superheated thermal layer. It was Snyder and Edwards (11), who first hypothesised the existence of a thin layer of liquid underneath the bubble base and proposed that this may result in very high evaporative heat fluxes across the base of the bubble. Rapid evaporation of a thin layer of liquid in contact with the heating surface would have the effect of drawing large amounts of heat directly from the surface. Moore and Mesler (13) observed the temperature-time profile of a heating surface using rapid response thermocouples, and noted an exceedingly rapid drop in surface temperature and a more gradual reattainment of the initial temperature occurring in a regular manner. At this stage they did not know whether these temperature drops occurred with the same frequency as bubble formation, nor did they have an exact idea of the position of these temperature drops relative to the bubbles formed. But the rapidity of the temperature drop immediately suggested an evaporative mechanism. Moore and Mesler (13) calculated the maximum heat flux obtainable from surface flooding and found this to be insufficient for the magnitude of the temperature drop. They also calculated that 70 to 90% of the total heat transfer could be attributed to this temperature drop. If this temperature drop is indeed caused by evaporation of such a layer, then latent heat transport is many times as important as had been thought previously. This work renewed interest in the Snyder hypothesis and led a number of workers to (Hendricks and Sharp (14), Bonnet et al. (15), Cooper and Lloyd (16)) successfully correlate this temperature drop to the

actual bubble position. They noted that this rapid temperature drop takes place under the base of the bubble and that the drop occurs when the base perimeter passes over the point of measurement. Fig. 2.2 reproduced from the report by Hendricks and Sharp (14) clearly illustrate the above observation. The microlayer has been directly observed by Sharp (17) using an interference method. According to the bubble agitation model and the liquid-vapour exchange model the cooling of the surface should occur after bubble departure when the colder bulk liquid comes into contact with the heating surface.

These considerations show that evaporation into the bubble from the microlayer must be considered in any bubble growth computation. At high bubble growth rates the evaporative heat fluxes across the base should in fact be more prominent than the heat flux across the curved surface of the bubble.

2.3 DEVELOPMENT OF BUBBLE GROWTH THEORIES

2.3.1 Bubble Growth in Uniformly Superheated Liquids

Foundations for most of the bubble growth analyses were laid by Plesset and Zwick (18) and Forster and Zuber (20). They used the assumption that in the later part of bubble life, the inertia of the surrounding liquid and the surface tension forces are negligible. This implies that the pressure difference across the bubble interface required to cause bubble expansion is very small and hence the growth rate is governed entirely by the rate at which heat can diffuse to the bubble interface. This phase of growth is generally known as the heat diffusion controlled growth or the asymptotic region. In this region, the problem of bubble growth was treated by Plesset and Zwick (18) and Forster and Zuber (20) as one of transient heat conduction from the liquid to the bubble and relationships of the form

$$R \propto f(\text{physical properties, } \Delta T) \times t^{\frac{1}{2}}$$

were derived.

Later, Scriven (21) obtained an exact solution to the transient heat conduction equation in the heat diffusion controlled region which, for large Jakob Numbers, reduces to the same as the Plesset and Zwick (18) expression. Cole and Shulman (22) represented these results in a very convenient form by the use of the Jakob Number.

Jakob Number is normally defined as

$$N_{ja} = \frac{\Delta T C_{pl} \cdot \rho_l}{\rho_v L}$$

$\Delta T = T_\infty - T_s$ in the case of uniformly superheated liquids.

$\Delta T = T_w - T_s$ in the case of bubble growth on a surface.

Scriven's (21) complete solution was given by

$$R = 2\beta \sqrt{\alpha_l t}$$

where the constant β , at large Jakob Numbers, reduces to

$$\beta = \frac{\phi_c}{\sqrt{\pi}} \left\{ \frac{T_\infty - T_s}{\frac{\rho_v}{\rho_l} \left[\frac{L}{C_{pl}} + \frac{C_{pl} - C_{pv}}{C_{pl}} (T_\infty - T_s) \right]} \right\}$$

for heat diffusion controlled growth.

But for non-metallic fluids

$$\frac{T_\infty - T_s}{\frac{\rho_v}{\rho_l} \left[\frac{L}{C_{pl}} + \frac{C_{pl} - C_{pv}}{C_{pl}} (T_\infty - T_s) \right]} = N_{ja}$$

With this definition it is possible to summarise the results of the above three papers as follows:

$$R = \phi_c \frac{2}{\pi} N_{ja} \sqrt{\pi \alpha_l t}$$

where ϕ_c , the sphericity factor

$$= \sqrt{3} \text{ for Plesset and Zwick (18) and Scriven (21)}$$

$$= \frac{\pi}{2} \text{ for Forster and Zuber (20)}$$

A common feature of these relationships is that the variation of bubble radius is proportional to $t^{\frac{1}{2}}$ and differences are only in the magnitude of the growth constant. It must be stressed that this simple representation of results is valid only in the case of heat diffusion controlled growth region.

2.3.2 Bubble Growth in Non-uniform Temperature Fields

The initial models on bubble growth in non-uniform temperature fields by Griffith (23), Savic (24), Skinner and Bankoff (27), Han and Griffith (25), Zuber (26), Cole and Shulman (22) and Mikic and Rohsenow (28) all apply for heat diffusion controlled growth. Most of these models assume that as the bubble grows it carries with it the thermal layer, formed over the heating surface, as shown in Fig 2.3 and do not penetrate it.

Griffith (23), Savic (24) and Skinner and Bankoff (27) are some of the authors who attempted to solve the problem of heat transfer into a bubble growing in a non-uniform temperature field using three dimensional models. Griffith assumed a hemispherical bubble shape throughout the entire growth period and also assumed a linear variation of temperature near the surface; $T = T_w$ at $x = 0$ and $T = T_\infty$ at $x = \delta$ where δ is the thermal layer thickness defined by the usual term $\sqrt{\pi \alpha t_w}$. He used a 3-D model with cylindrical symmetry around the vertical axis.

Savic (24) assumed that the temperature field is spherically symmetric and hence wholly radial. The superheated layer thickness was obtained by averaging the actual layer thickness over the hemisphere.

By far the most accurate, three-dimensional analysis of this problem was given by Skinner and Bankoff (27). They first obtained solutions assuming spherically symmetric temperature fields and applied this to

investigate asymptotic bubble growth. Their method is quite general in that a variety of initial temperature distributions could be used to obtain the solution. Some of their results are outlined below

Uniform initial superheat Case:

$$R = 2 \left(R_{t=0}^2 + \frac{4 N_{ja}^2 \alpha_l t}{\pi} \right)^{\frac{1}{2}} \cos \left[\frac{\pi}{6} + \frac{1}{3} \sin^{-1} \left(1 + \frac{4 N_{ja}^2 \alpha_l t}{\pi R_{t=0}^2} \right)^{-3/2} \right]$$

This expression when $R_{t=0} \rightarrow 0$ reduces to

$$R = 2 N_{ja} \left(\frac{3 \alpha_l t}{\pi} \right)^{\frac{1}{2}} \quad \text{which is equivalent to the Plesset}$$

and Zwick (18) result.

Non-uniform initial superheat Case:

In analysing a bubble growing on a surface they assume an initial temperature profile in the thermal layer of a form

$$T(r, 0) = T_{\infty} + (T_w - T_{\infty}) \exp \left(- \frac{r^6}{9 \ell^6} \right)$$

where ' ℓ ' represents the boundary layer thickness and obtained the solution

$$R^3(y) = \left(\frac{6 N_{ja}}{\sqrt{\pi}} \right) \left[y^3 - \omega (y^3 - \frac{1}{2} \tan^{-1} 2y^3) \right]$$

where $\omega = (T_w - T_{\infty}) / (T_w - T_s)$

$$y = \ell^{\frac{1}{6}} \quad ; \quad \ell = \frac{\alpha}{l^6} \int_0^t R^4 dt$$

Inversion to real time can be carried out with

$$\frac{\alpha_l t}{l^2} = \frac{(\pi^2/6)^{1/3}}{N_{ja}^{4/3}} \int_0^y \frac{y \cdot dy}{\left[1 - \omega \left(1 - (1/2y^3) \tan^{-1} 2y^3 \right) \right]^{4/3}}$$

which has to be evaluated numerically.

In another paper Skinner and Bankoff (29) extend the work described above to exclude the restriction that the temperature fields must be spherically symmetric. This analysis applies to situations where the temperature field and hence the temperature gradient at the interface of the bubble varies over the bubble interface, which makes it very suitable for use with bubble growth on surfaces. They achieve the solution of this complicated problem by introducing dimensionless variables and reducing the original equations to forms in which spherical symmetry of temperature fields exist. Then the solution can be obtained by the application of their previous result. In this case it is not necessary to assume the deformation of the thermal layer around the bubble as used in the model shown in Fig. 2.3. They have obtained solutions for two forms of initial temperature profiles near the surface; one a linear decay of the temperature with distance from the surface and the other an exponential decay.

The expressions obtained by authors in the above group are too complicated to be of direct practical use and hence attempts to evaluate approximate heat transfer relationships using one dimensional models have persisted.

Han and Griffith (25) used the one dimensional model given in Fig. 2.3 for the calculation of heat transfer into the bubble. To account for sphericity they introduced a curvature factor, the magnitude of which was

obtained by comparison of their result with Scriven's (21) earlier work at the limiting situation of uniform temperature fields. The bubble surface was assumed to be spherical (ie. a truncated sphere). Han and Griffith were among the first workers to introduce the concept of waiting time into their model. The longer the waiting time before bubble inception, the thicker the thermal layer will be. This affects the initial temperature profile in the thermal layer, which greatly influences the heat transfer calculation, especially during the early stages of bubble growth. It must also be noted that Han and Griffith linearised the initial temperature profile in the thermal layer in the calculation of both the waiting time and the heat input to the bubble. They took account of heat transfer from the heating surface to the bubble across the base by using an expression of the form:

Heat transfer coefficient x Base Area x Wall Superheat.

Being uncertain of the existence of a liquid layer beneath the bubble, they employed heat transfer coefficients far smaller than those now known to be applicable.

Mikic and Rohsenow (28) analysed the problem of bubble growth in non-uniform temperature fields in a somewhat similar manner to Han and Griffith (25) but neglected the heat transfer into the base of the bubble. Their model did not use linearisation of the initial temperature profile in the thermal layer. Breaking down the problem of transient heat conduction to the bubble interface into two equivalent problems and superpositioning of these two solutions enabled them to obtain a relatively simple expression for the temperature profile in the thermal layer. Their expressions again take into account the effect of non-uniformity of the temperature profile by the inclusion of the parameter 'waiting time- t_w '. The rate of evaporation corresponding to the heat transfer rate thus

obtained was equated to the rate of increase of bubble volume to obtain the bubble growth curve. These growth expressions, at infinite waiting time, approach the limiting situation of uniform temperature fields, and a comparison at these conditions with Scriven (21) or Plesset and Zwick (18) expressions yielded the curvature factor, to account for sphericity. Their final expression for bubble growth was of the form:

$$R = \frac{2}{\pi} \sqrt{3} N_{ja} \sqrt{\pi \alpha_l t} \left\{ 1 - \left(\frac{T_w - T_\infty}{T_w - T_s} \right) \left[\left(1 + \frac{t_w}{t} \right)^{\frac{1}{2}} - \left(\frac{t_w}{t} \right)^{\frac{1}{2}} \right] \right\}$$

This result is very similar to the results obtained for growth in uniform temperature fields but with a correction factor introduced to take into account the non-uniformity of the temperature field.

Mikic, Rohsenow and Griffith (30) attempted to obtain a solution for bubble growth rates valid in both the inertia controlled and heat diffusion controlled growth regions; first for uniform temperature field conditions and then an approximate extension to non-uniform temperature field conditions. To include the effects of inertia they employed a simplified form of the energy equation for the liquid surrounding the growing bubble, instead of the more conventional momentum equation. They obtained reasonably convenient relationships for the case of uniformly superheated liquids which agreed well with some experimental data. One assumption they make in this procedure is that the variation of the internal pressure for each individual bubble during its growth is not large (ie. $\frac{dp_i}{dR}$ is small). This is used to simplify the energy equation. In view of this they are right to point out that these relationships are not applicable in the very initial stages of bubble growth.

Most of the models so far discussed yield results too complicated to be of much practical use, and this led a number of workers to develop simpler models. Zuber (26), Cole and Shulman (22), Van Stralen (31) and some others belong in this category. A common feature of all these simplified models

is that they have been derived by a modification of the bubble growth relationships obtained for uniform temperature fields. There is no clear reason to support the applicability of analyses of bubble growth in uniformly superheated liquids of an infinite extent to growth of a bubble on a surface. However, the simplicity of these expressions justified a serious look as to the effectiveness of modified forms of these, in the case of non-uniform temperature fields.

Zuber (26) contended that the rate of evaporation into a bubble growing on a heated surface is equal to the evaporation into a bubble growing in a uniform temperature field less the average heat flux from the boiling surface. With a correction for sphericity his assumption modified the usual bubble growth relationships in the following manner.

$$L\rho_v \frac{dR}{dt} = \phi_c k_\ell \frac{T_w - T_s}{\sqrt{\pi \alpha_\ell t}} - q_{av}.$$

Integration yielded

$$R = \phi_c \frac{2}{\pi} N_{ja} \sqrt{\pi \alpha_\ell t} \left(1 - \frac{q_{av} \sqrt{\pi \alpha_\ell t}}{2 k_\ell (T_w - T_s)} \right)$$

Cole and Shulman (22) argued that since the thermal layer in which the bubble is growing is not uniformly superheated, it might be better to use an average driving force $\frac{1}{2} (T_w - T_s)$, so that they could use the solution for growth in a uniformly superheated liquid

$$R = \phi_c \frac{2}{\pi} \sqrt{\pi \alpha_\ell t} N_{ja}$$

with a modified Jakob Number

$$N_{ja} = \frac{\frac{1}{2} (T_w - T_s) C_{pl} \rho_\ell}{L\rho_v}$$

They confirmed that this approach yielded better results than the previously described Zuber's (26) method for most of their experimental data on bubble growth which were carried out at high Jakob Numbers ranging from 24 to 792. Agreement was better for Jakob Numbers less than 100. They also observed that their experimental growth curves tended to a simple $t^{\frac{1}{2}}$ variation. They plotted the experimental values of constant ' β ' in

$D = 4\beta \sqrt{\alpha_l t}$ against the Jakob Number and developed a relationship $\beta \approx \frac{5}{4} N_{ja}^{\frac{3}{4}}$. Using this, they concluded that a relationship of the form

$D = 5 N_{ja}^{\frac{3}{4}} \sqrt{\alpha_l t}$ might be useful over a wide range of Jakob Numbers.

Van Stralen (31) suggested that a bubble growing on a surface is not wholly covered by the superheated liquid layer and hence the Plesset and Zwick (18) results for bubble growth must be multiplied by a factor less than 1. In addition he assumed that the wall superheat is a function of time so that the bubble growth rate tends to zero at departure.

Stewart and Cole (32) in a paper published in 1972, obtained more data for bubble growth at higher Jakob Numbers (955-1112) which they compared with most of the models described above. They noticed order of magnitude of discrepancies with most of these models. Closest agreement was obtained with the theories of Mikic, Rohsenow and Griffith (30) and Cooper and Vijuk (33), which take into account the effect on bubble growth of liquid inertia. Cooper and Vijuk (33) theory is based on the microlayer evaporation theory, while the other is an extension of the ordinary non uniform superheat theory. Stewart and Cole (32) plotted various force terms (Inertia, buoyancy, surface tension, viscous) for one of their bubbles during its growth period and found that the inertia force term predominated during the entire growth period of the bubble, which confirmed their belief in the importance of inertial effects in large Jakob Number situations. It must also be noted that their experimental data did not confirm the now standard theory, that

higher growth rates result from higher waiting times and vice versa.

Most of the bubble growth estimations discussed above neglect the evaporation of the liquid from the microlayer underneath the bubble base, which according to the works of Moore and Mesler (13) etc. should constitute a major proportion of the total evaporation into the bubble. Some of the above works assume a form of heat transfer across the bubble base, but without the assumption of the existence of a thin liquid layer underneath the bubble these would be far below the true value of the heat input, especially for rapidly growing bubbles. It is surprising that even after neglecting a contributory proportion of the heat input as discussed above, how these relationships manage to agree reasonably well with most of the existing experimental data. It is possible that the neglect of inertia forces increases the growth which compensates for the reduction of growth due to neglect of heat transfer across the base. The assumption of the existence of a spherically symmetric superheated thermal layer, entirely covering the curved interface of the bubble, also may have helped to increase the growth rate.

2.3.3 Bubble Growth with Microlayer Evaporation

Very few attempts have been made to formulate bubble growth relationships which take into account the microlayer evaporation as well as the evaporation across its curved interface.

Cooper (34) made such an analysis based on assumptions derived from observations made in some experiments, carried out mainly by Cooper and Lloyd (35), (16), where the existence of the microlayer was established. These assumptions ensure that the bubble grows to sizes greatly exceeding the thermal boundary layer and as a consequence the microlayer evaporation predominates over the heat transfer across the curved interface. To ensure a fast growing hemispherical bubble with a microlayer he stipulated that

$$\begin{array}{l}
 p_i \gg p_s \\
 p_d \gg p_i \text{ and } p_i \gg p_g \\
 p_i \gg p_v
 \end{array}$$

where p_i , p_s and p_v correspond to the respective pressure terms in the extended Rayleigh Equation $\rho(R\ddot{R} + \frac{3}{2}\dot{R}^2)$, $\frac{2\sigma}{R}$, $4\mu\frac{\dot{R}}{R}$, and

$$p_g = (\rho_l - \rho_v)gR$$

$$p_d = (T_{wo} - T_s) \left(\frac{dp}{dT} \right)_{sat}$$

where $\left(\frac{dp}{dT} \right)_{sat}$ is the slope of the saturation line on the p, T chart at system pressure. The second assumption states that the rate of increase of the bubble volume is very much higher than the rate of evaporation at the curved interface. The third assumption was regarding the behaviour of the wall temperature under the bubble. According to this assumption either $T_{wo} - T_s \gg T_{wo} - T_w$ or $T_{wo} - T_s \gg T_w - T_s$ is satisfied during the growth time of a bubble. They respectively represent situations where the wall temperature remains constant or drops very rapidly to the saturation temperature. As a first step in his evaluation he completely ignored the evaporation across the curved interface. Assuming a growth relation of the type $R = A t^n$ and a hemispherical bubble, he developed the relationship

$$R = \frac{2}{C_2} \frac{N_{ja}}{\sqrt{N_{pr(l)}}} \sqrt{\alpha_l t}$$

for the constant wall temperature case, where C_2 is the constant in the expression for the initial microlayer thickness $\delta_o = C_2 \sqrt{vt_g}$.

He successfully applied this theory to bubbles growing in uniformly superheated liquids and found surprising similarities with the results obtained from Scriven's (21) expression, even though the mechanism and the theory are quite different. Agreement with experimental growth rates, where the assumed conditions approximately hold, was obtained to within ± 25 per cent. An extension of this to take into account the heat transfer across the curved interface, where the initial bulk temperature is uniform, was also made by Cooper (34) with the following results,

$$R = \frac{2}{C_2} \frac{N_{ja(w)}}{\sqrt{N_{pr(l)}}} \sqrt{\alpha_l t} + 2 \sqrt{\frac{3}{\pi}} N_{ja(\infty)} \sqrt{\alpha_l t}$$

- for the constant wall temperature case, and

$$R = \frac{2}{\sqrt{\pi}} \left(\frac{k_w \rho_w C_{pw}}{k_l \rho_l C_{pl}} \right)^{\frac{1}{2}} \cdot N_{ja(w)} \sqrt{\alpha_l t} + 2 \sqrt{\frac{3}{\pi}} N_{ja(\infty)} \sqrt{\alpha_l t}$$

- for low conductivity of the wall

Cooper and Vijuk (33) have attempted to modify Cooper's (34) expressions for a wider range of applications while retaining the restriction of an isolated hemispherical bubble. A main disadvantage is that they have attempted to remove the existing limitations, one at a time, and hence it is difficult to apply the analysis in situations where a number of these restrictions occur together. The following considerations have been included into Cooper's (34) treatment by Cooper and Vijuk (33).

- (i) Non-Uniform Liquid temperature
- (ii) Intermediate conductivity of the wall
- (iii) Variation of temperature at the interface between the microlayer and the vapour inside the bubble
- (iv) Thermal capacity of the microlayer

However, some of the results of this paper tend to go beyond the 'analytic simplicity' that Cooper was aiming at in his previous paper, but this is inevitable as a result of the factors taken into account.

Dzakowic and Frost (38) present an analysis of bubble growth, again taking into account the evaporation from the microlayer. This investigation has been carried out by a consideration of the evaporative mass fluxes at the bubble interface coupled with an estimation of the interface temperature. The temperature at the microlayer interface was obtained by using simplified gas kinetic theory. In obtaining the curved interface temperature, the energy equation has been used with either of two extreme approximations; one where conduction terms are neglected and the other where convection terms are neglected. Rates of evaporation, estimated from the above information, were equated to the rate of increase of bubble volume and the solution for bubble growth was obtained by digital computation. They also assumed a hemispherical bubble shape, and a microlayer of constant thickness under the bubble, in this analysis. Growth rates have been presented for different conditions of boiling where the results using the convection approximation are expected to represent a lower limit for bubble growth. They also present the conclusions that the microlayer evaporation is predominant for Jakob Numbers less than twenty, and for values of Jakob Numbers greater than hundred the microlayer evaporation rate diminishes relative to the evaporation rate across the curved interface.

2.4 EXISTING THEORIES OF BUBBLE DEPARTURE

Cole and Shulman (40) give an excellent review of the literature available on bubble departure mechanisms up to 1966 in their paper on bubble departure diameters at subatmospheric pressures.

One of the very first correlations of departure diameter of bubbles

was given by Fritz (39)

$$\sqrt[3]{V_d} = 0.0119 \phi \sqrt{\frac{2g_c \sigma}{g(\rho_l - \rho_v)}}$$

where the contact angle ' ϕ ' is in degrees, and

g_c is the gravitational conversion factor.

If the bubble can be assumed to be of a spherical shape at departure the above equation becomes

$$D_d = 0.0208 \phi \sqrt{\frac{g_c \sigma}{g(\rho_l - \rho_v)}}$$

Cole (41) went on to show that the above equation results from a force balance equating the buoyancy and surface tension forces on the bubble at departure. Zuber (42) in 1959 obtained an expression for the departure diameter of a vapour bubble from a simplified analysis of gas bubbles formed at an orifice and assuming ' $\sin \phi$ ' equal to unity, an approximation of the departure diameter is given by

$$D_d = \left[\frac{12 r_o g_c \sigma}{g(\rho_l - \rho_v)} \right]^{1/3}$$

where r_o is the orifice radius

This would be identical to the Fritz expression if r_o were the bubble radius and ' $\sin \phi$ ' had been assumed proportional to ϕ . Zuber went on to relate a surface cavity diameter, from which a vapour bubble would nucleate, to other heat transfer data, assuming equivalence between the thermal layer and the cavity diameter. This cavity radius when substituted for above r_o would yield

$$D_d = \left[\frac{6g_c \sigma}{g(\rho_l - \rho_v)} \cdot \frac{k\Delta T}{Q} \right]^{1/3}$$

Staniszewski (43) showed that these relationships do not correlate very well the departure diameters of rapidly growing bubbles and that they attain higher departure diameters than given by the Fritz expression. He tried to account for this by the inclusion of a dynamic growth factor using the velocity of the bubble at departure, by a relationship of the form

$$D_d = D_o + \alpha \cdot \frac{dD}{dt}$$

the final correlation given was

$$D_d = 0.0071 \phi \left[\frac{2g_c \sigma}{g(\rho_l - \rho_v)} \right]^{\frac{1}{2}} \left[1 + 0.435 \frac{dD}{dt} \right] \quad \text{where}$$

$\frac{dD}{dt}$ is in inches per second.

Semeria (45) attempted to correlate the departure diameter data, with pressure, and obtained the relationship

$$D_d = 0.242 P^{-0.5} \quad \text{for } 2 \text{ atm.} < P < 20 \text{ atm.}$$

D_d in inches and P in lb/in^2 absolute.

Han and Griffith (25) successfully correlated their bubble departure data using the Fritz expression but with a dynamic contact angle measured from bubble tracings, rather than a static contact angle. They felt that the use of this contact angle would minimise the departure diameter dependence on the bubble velocity.

A number of workers, later have attempted to analyse bubble departure by taking into account other forces besides buoyancy and surface tension.

Roll and Myers (44) considered the effects of buoyancy, drag and acceleration forces, neglecting the surface tension force. They argued that the acceptance of the microlayer theory for bubble growth introduces a

contradiction to the Fritz model of bubble departure. In the Fritz model a large perimeter of contact for the metal-vapour-liquid triple interface is a requirement whereas the microlayer evaporation model precludes such a large perimeter. The balance of forces was as follows:

$$\frac{\pi D^3}{6} g (\rho_l - \rho_v) - \frac{C_D \pi D^2 \rho_l U_b^2}{8} - \frac{\pi D^3 \rho_l}{12} \left(1 + \frac{3}{8}\right) \frac{dU_b}{dt} = 0$$

where C_D = drag coefficient

U_b = bubble velocity

He assumed the virtual mass of the accelerating bubble as $\frac{1}{2} \left(1 + \frac{3}{8}\right)$ times the displaced liquid following Milne-Thompson (67). Significant bubble velocity was defined as $U_b = 2 \cdot \frac{dR}{dt} = \frac{dD}{dt}$ with $\frac{dR}{dt}$ obtained from Forster and Zuber (20) as

$$\frac{dR}{dt} = \frac{\pi}{D} \left[\frac{\rho_l C_{pl} \Delta T \alpha^{\frac{1}{2}}}{\rho_v L} \right]^2$$

As presented by Cole and Shulman (40), Ruckenstein in a similar analysis has obtained an expression for departure diameter by considering only the buoyancy and drag forces

$$\frac{\pi D^3}{6} g (\rho_l - \rho_v) = C_D \frac{\pi D^2}{4} \rho_l \frac{U_b^2}{2}$$

Using the expression for bubble velocity from Forster and Zuber (20) and assuming the drag coefficient to be unity, one obtains

$$D_d = \left(\frac{3 \pi^2 \rho_l \alpha^2}{g (\rho_l - \rho_v)} \right)^{\frac{1}{3}} \cdot \left(\frac{\rho_l C_{pl} (T_w - T_s)}{L \rho_v} \right)^{\frac{4}{3}}$$

Cole and Shulman (40) attempted to find empirical constants to fit an equation of the form

$$\frac{D_d}{D_F} = 1 + C \cdot \left(\frac{dD}{dt}\right)^{\frac{3}{2}}$$

which would fit their data and agree with the Fritz equation when the departure velocity is zero, and obtained

$$\frac{D_d}{D_F} = 1 + 0.0025 \left(\frac{dD}{dt}\right)^{\frac{3}{2}}$$

where D_F is the diameter as predicted by the Fritz equation and $\left(\frac{dD}{dt}\right)$ is in mm./sec.

Cole and Shulman (40) applied the above expression to their data, as well as some data from Keshock and Siegel (47) and Han and Griffith (25) and obtained satisfactory agreement.

Keshock and Siegel (47) analysed the forces acting on bubble growing under reduced gravity conditions and came to the conclusion that the departure diameter is dependent on the relative magnitude of the buoyancy, inertial and surface tension forces. They used a spherical bubble model, as their work was mainly concerned with slow bubble growth, in low heat flux situations. A brief outline of the force expressions obtained by Keshock and Siegel (47) is given below.

(i) Inertia Force:

$$F_I = \frac{d}{dt} (mu) = \frac{d}{dt} \left[\left(\frac{11}{16} \frac{\rho_l}{g_c} \frac{\pi D^3}{6} \right) \frac{1}{2} \cdot \frac{dD}{dt} \right]$$

Concept of virtual mass was employed here by the use of the same expression as by Roll and Myers (44).

(ii) Buoyancy Force:

$$F_{BU} = \frac{\pi D^3}{6} (\rho_l - \rho_v) \frac{g}{g_n}$$

for an unattached bubble

This was modified for an attached bubble to obtain

$$F_{BU} = \frac{\pi D^3}{6} (\rho_l - \rho_v) \frac{g}{g_n} + \Delta p_{base} \frac{\pi D_B^2}{4}$$

where Δp_{base} = pressure difference across the bubble interface at the base.

Cochran and Aydelott (48) later modified this force term where they considered Δp to be the pressure difference across the top of the curved interface.

(iii) Surface Tension Force:

$$F_S = \pi D_B \sigma \sin \phi$$

where ϕ is the contact angle

(iv) Drag Force:

Drag coefficient used was $C_D = 45/N_{Re}$ where the Reynold's number for the bubble was defined as

$$N_{Re} = \frac{\rho_l D}{2\mu_l} \frac{dD}{dt}$$

The drag force is then obtained from

$$\begin{aligned} F_D &= \frac{1}{2} \rho_l C_D \frac{\pi D^2}{4} \left(\frac{dD}{dt} \right)^2 \\ &= \frac{\pi}{16} \cdot 45 \cdot \mu_l \cdot D \frac{dD}{dt} \end{aligned}$$

They concluded, after comparing these force terms for their data, that the drag force plays a very insignificant part at departure and therefore used only the other three force terms for their force balance to obtain departure conditions.

Similar analyses have been carried out by Cochran and Aydelott (48) and Hatton, James and Liew (49) with various modifications of the force terms. Cochran and Aydelott (48) investigated boiling of subcooled water at normal and reduced gravity conditions. Relative effects of these conditions on the force terms were discussed in their paper. Hatton et al. (49) also obtained reasonable predictions of departure values by a similar balance of forces for different liquids at different pressures. The authors of these two papers also agree that the magnitude of the drag force is insignificant in departure considerations.

In all these theories certain parameters are employed which are not very well defined or where a reasonable choice of them cannot be made due to lack of experimental evidence. As a result, departure estimations depends to a large extent on the individual worker's interpretation of various force terms and his method of approach. Confirmation of the existence of the microlayer also casts some confusion as to the methods of evaluation of some of these force terms, notably in the case of the surface tension force.

All these factors indicate that a re-examination of the existing theories or pursuance of a new approach towards obtaining departure may be more profitable at this stage.

2.5 BUBBLE SHAPES IN NUCLEATE BOILING

The importance of the shapes of bubbles growing on a surface and its relation to bubble growth and departure was first pointed out by Johnson, de La Pena and Mesler (50). In the very early works it had been customary to assume spherical shapes. However, later observations have shown that bubble shapes can differ considerably from spherical, at least in the very

early stages of growth or where rapid bubble growth takes place. Johnson et al. (50) broadly categorised bubbles of varying shapes into three classes; spherical, hemispherical and oblate. They observed and reported the relationship of bubble shapes to other factors such as bubble growth rates, bubble departure values, and waiting times etc.. Rapidly growing bubbles were found to be hemispherical in shape and slow growing bubble were spherical. Bubbles with intermediate growth rates had an oblate shape. These observations agree very well with the microlayer theory of bubble growth. They explained the different shapes of bubbles on the basis of the relative importance of inertia and surface tension forces. Using the terms in the extended Rayleigh Equation corresponding to the inertial and surface tension forces $\left[R\ddot{R} + \frac{3}{2} \dot{R}^2, \frac{1}{\rho_l} \frac{2\sigma}{R} \right]$ they illustrated that inertia forces are always higher than surface tension forces for hemispherical bubbles and vice versa for spherical bubbles.

The next substantial quantitative investigation of bubble shapes was carried out by Madhavan and Mesler (51) where they attempted to obtain a numerical solution of the Navier-Stokes equation describing the flow around a bubble, without considering the energy equation. To start off the calculation an initial bubble profile and a velocity field had to be supplied. They assumed a zero-velocity field but do not give an idea of the initial bubble profile. In order to compensate for this assumption, the bubble was given an initial acceleration by maintaining an excess pressure along the bubble profile. They then used a chosen time dependent function for the pressure term in the Navier-Stokes equation, such that the experimental and theoretical growth rates matched, and carried on the solution to obtain the bubble shape variation. The predictions of bubble shapes they obtained for 8 ms. of growth agreed reasonably well with their experimental observations. An inspection of the only set of results they presented leads one to presume that they used an approximately hemispherical bubble profile of relatively large size ($R \approx 2$ mm.) as their

initial bubble profile in the calculations. If that was the case, the existence of a bubble of such a shape and size at zero-velocity fields is a highly improbable situation and is likely to introduce severe errors into their computation.

They also treated an oblate bubble profile as a static bubble, in order to determine its behaviour at departure and observed a change to an approximately spherical shape with very small contact radius, indicating readiness to detach. They repeated this in a zero gravity field to observe similar behaviour and as a result arrived at the conclusion that the contribution towards bubble detachment by the buoyancy forces are very small compared to the contribution by the surface tension forces.

It is very difficult to see how the use of initial bubble profiles as employed in this work at zero-velocity fields can be justified for a computation of this nature. As expected, they also had to use tremendous amounts of computer resources in order to advance the growth of a bubble for a very short period.

2.6 EXISTING EXPERIMENTAL WORK ON BUBBLE GROWTH AND DEPARTURE USED IN THE PRESENT WORK

Even though a large amount of bubble growth data exist in the literature today, it is difficult to use most of these in the present model due to lack of sufficient information as to the boiling conditions. However, data obtained from five different works are used, with suitable estimation of unknown parameters to facilitate the application of the present model. Experimental methods used by these five sources are briefly discussed here, in order to give a general idea of the experimental conditions under which they were carried out.

Han and Griffith (25) in their paper published in 1965 presented growth data for three bubbles in subcooled water growing on a gold plated copper surface under atmospheric pressure. The boiling took place on the

top surface of a thin flanged cylindrical copper block at the bottom end of which electrical heaters were embedded. The bubble growth on the finely polished top surface was photographed using a high speed camera. This set of data has the added advantage that the waiting time was recorded in addition to the usual observations of surface temperature, system pressure and bulk liquid temperature etc.. Bubble growth was observed up to departure.

Hospeti and Mesler's (52) experiments were carried out with demineralized water as the boiling liquid and a chromel strip as the heater. The bulk liquid was saturated. Waiting times are also reported in this work. The surface temperature under a bubble was measured using a fast response thermocouple and, as a result, they were able to record the initial surface temperature for each individual bubble, in addition to the initial average surface temperature. Bubble growth was not observed up to departure.

Cole and Shulman (22) presented a large amount of bubble growth and departure data for a number of organic liquids and water, boiling under different subatmospheric pressures at saturation conditions. Even though they recorded all the standard observations defining the boiling situation, an important omission, as regards to the present growth calculation, was the waiting time of bubbles. The heater in this case was a thin zirconium ribbon which was heated electrically and submerged in the saturated test liquid. They also report that the average maximum error in the wall superheat is estimated to be ± 50 per cent.

Stewart and Cole (32) obtained bubble growth data for very high Jakob numbers ranging from 955 to 1112, for water boiling on a polished copper surface. This surface formed one end of a flanged copper rod with electrical heaters embedded at the other end. They recorded all the relevant information to describe the boiling conditions including the waiting times except the cavity radii values. Vapour bubbles in these experiments grew to extreme sizes and it was not possible to observe bubble departure.

Data for bubble growth for the case where the wall has intermediate conductivity is very rare upto the present day. Merry (53) presented one of these rare sets of data where he boiled subcooled water on a glass surface at low pressures. He reports the usual parameters defining the boiling situation. The bubble initiation, in this case, was carried out artificially by the passage of a pulse current through a heater element. A waiting time would be non-existent and meaningless in such a situation. He also observed the temperature variation at four different sites on the surface, under the bubbles, by means of thin film resistance thermometers.

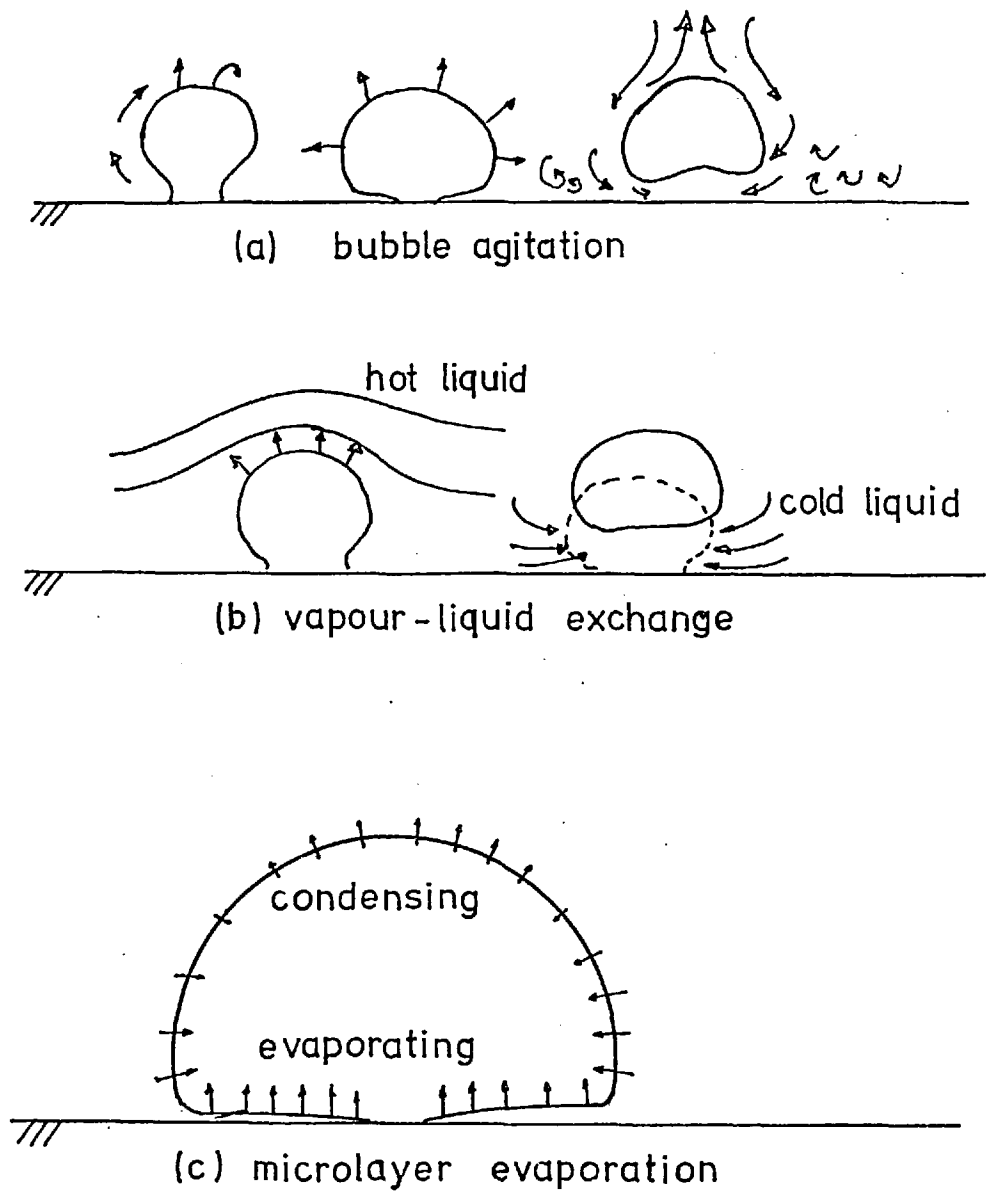
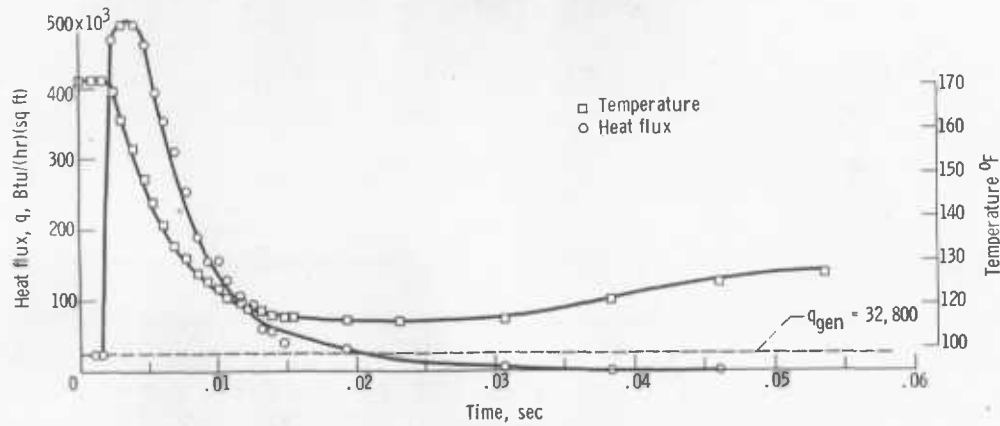
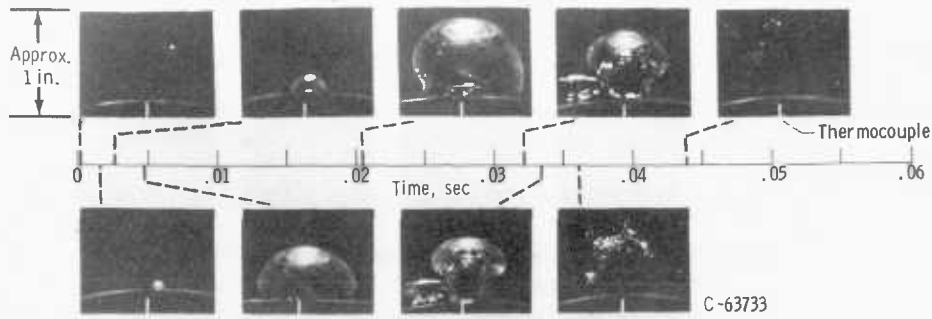
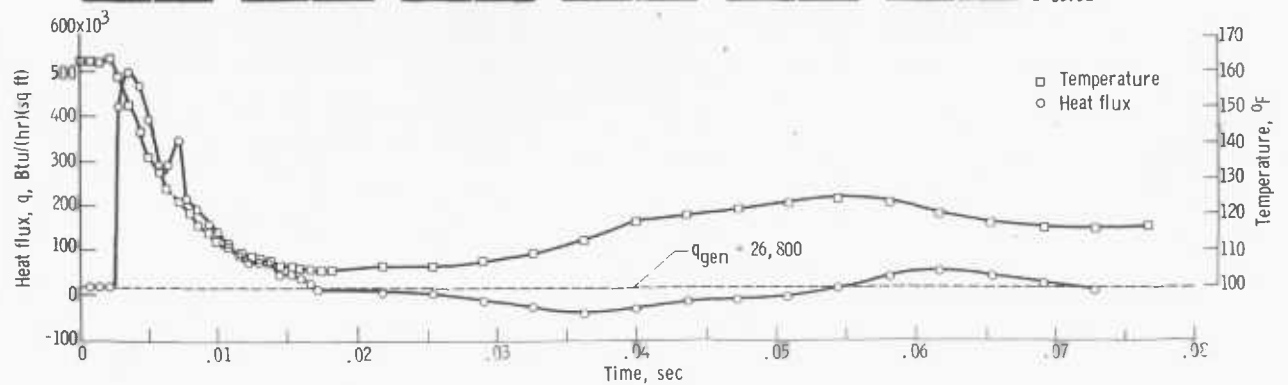
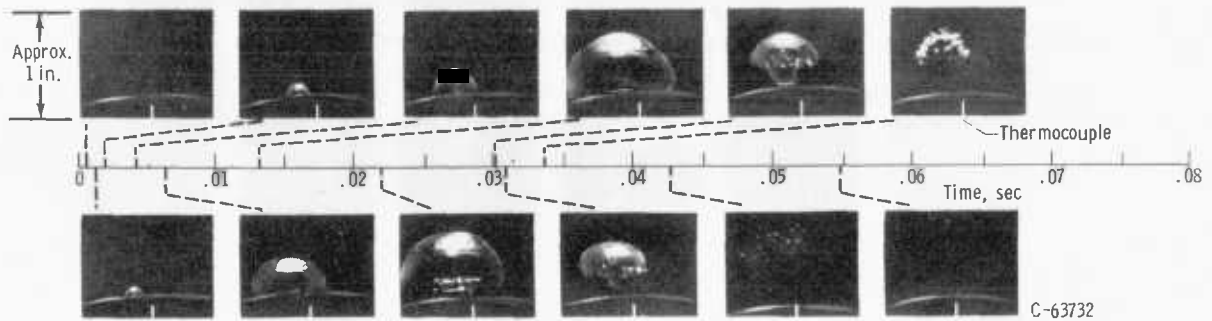


Fig.2.1 Heat Transfer Mechanisms in Boiling



(a) Generated heat flux, 32,800 Btu per hour per square foot; liquid bulk temperature, 84° F.



(b) Generated heat flux, 26,800 Btu per hour per square foot; liquid bulk temperature, 83° F.

Fig.2-2 Temperature and heat flux beneath growing bubble*

*Hendricks & Sharp (14)

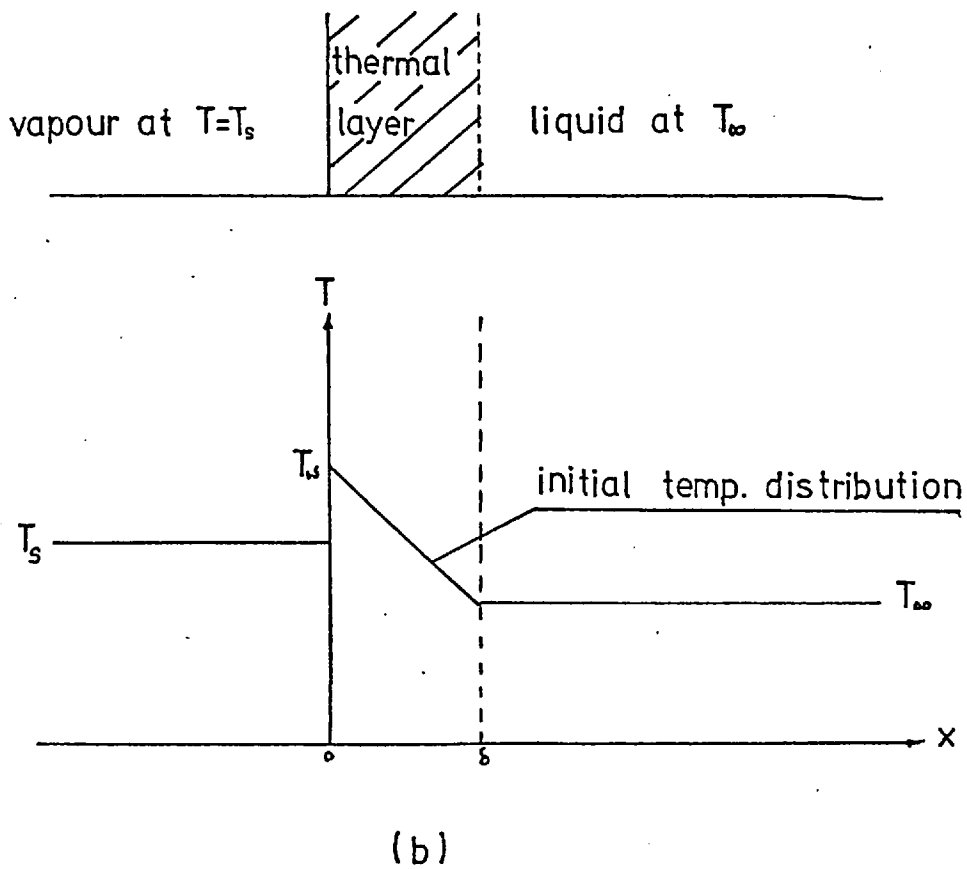
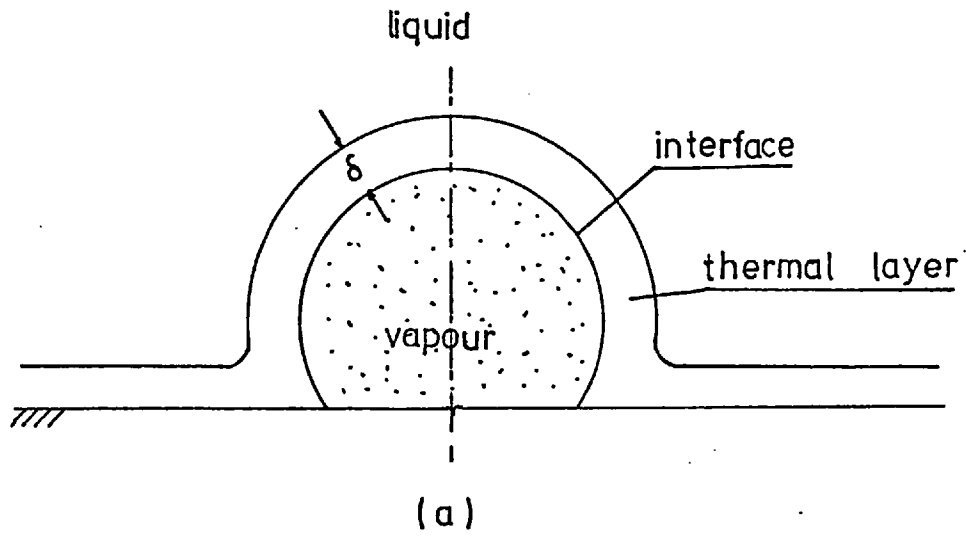


FIG. 2-3 1-D MODEL OF HEAT TRANSFER*

*HAN & GRIFFITH (25)

CHAPTER 3

MODEL FOR ANALYSIS AND GOVERNING

HYDRODYNAMIC EQUATIONS

3.1 INTRODUCTION

Theories of bubble growth well away from surfaces, under uniform superheat conditions are well established. Far less work has been done on bubble growth in non-uniform temperature fields, in the vicinity of heating surfaces. Most of these theories do not take into account the effects of inertial forces in the liquid on the bubble. Another major exclusion from most is the neglect of evaporation from the microlayer underneath the bubble base, which is now proven to exist in the case of rapidly growing bubbles. Add to this, the assumption of a fixed shape of the growing bubble and the validity of a large number of these expressions are open to doubt. Stewart and Cole (32) have clearly shown this when they compared growth data obtained at very high Jakob Numbers (955-1122) with existing bubble growth theories and discovered order of magnitude discrepancies.

Bubble departure criteria are usually developed by balancing various forces acting on the bubble as a whole. A vapour bubble is by no means a rigid body, so that, the force pattern (or pressure) on the interface governs the bubble shape, instead of the force pattern being a function of the shape. Restriction of the shape of the bubble profile, immediately fixes the force pattern. This fact weakens the validity of the existing approach of departure estimation considerably. It has been observed by many experimenters that a hemispherically growing bubble gradually changes shape to spherical near departure. In normal analytical solutions, it is not possible to make clear how this transformation of shape takes place. The present approach investigates this shape

variation, by considering the growth at the bubble base separately from the growth of the top, and makes use of the result to predict departure values.

Most of the previously mentioned restrictions imposed on earlier analyses are due to the complexity of the problem of bubble growth on a surface. Simplifications are invariably necessary in order to obtain analytical results to a problem of this nature, but with computer resources abundant in the present day, it is worthwhile and practical to develop a model which has less simplifications and approximations than in these earlier analyses. A complete solution of the problem of fluid flow and heat transfer around a vapour bubble growing on a heated surface, involves a simultaneous numerical solution of the Navier-Stokes equation and the energy equation governing the heat flow into the bubble. This is no simple task even with the help of computer techniques. Some attempts to obtain computer solutions have been made in the past.

Cooper and Lloyd (35) developed a computer program assuming hemispherical growth and taking into account both evaporation across the microlayer and evaporation or condensation across the curved interface. Bubble growth rate was obtained by equating the total rate of evaporation into the bubble to the rate of increase of bubble volume. Dzakowic and Frost (38) carried out a similar computation for a hemispherical bubble with a different method of evaluation of heat transfer across the microlayer, taking into account the wall temperature variations under the bubble, but assuming a constant microlayer thickness across the bubble base. This paper and the attempt by Madhavan and Mesler (51) to solve the Navier-Stokes equation are discussed in more detail in chapter 2. Cooper and Lloyd's (35) solution is of a much simpler nature than the other two, but cannot be used to describe early bubble growth accurately as they themselves point out. Madhavan and

Mesler (51) method is only an investigation of the shape variation of the bubble, and cannot be considered as a prediction method. Moreover their method uses up a tremendous amount of computer resources. For example, to advance the calculations for 8 msec., using time steps of 0.4 msec. they required computer times of the order of 70 mins. in a GE-635 computer.

The present work attempts to develop a more versatile model of bubble growth suitable for computer applications, without aiming at a complete solution. Instead of a solution for the Navier-Stokes equation, the extended Rayleigh equation is employed to obtain growth together with an approximate idea of the bubble shape variation leading to bubble departure, yet at the same time presenting a fuller picture of the boiling situation by the inclusion of the appropriate energy inputs and drag forces etc.. Models of this nature utilise less computer resources and are easily adaptable to varying conditions.

3.2 THE MODEL

3.2.1 Description of the model

The major difference in the present model to any other is the fact that the growth of the bubble interface near the base and the growth of the bubble interface at the top (peak of the curved interface) are considered independently. It was argued that the assumption of a fixed shape for the bubble results in fixing the pressure distribution around it, and hence would affect the force calculation at departure. If, for example, hemispherical growth is assumed, as most authors have done, it implies that the bubble interface near the base and the bubble interface near the top grow at the same rate. An examination of the situation immediately suggests that the growth rate at the top must be higher than the growth rate at the base. If boiling is on a horizontal plane, as has been the case in most experiments, the bubble top is at

a lower hydrostatic head than the bubble base. In addition the growth of the base perimeter has to push away the liquid which is in the vicinity of the surface. This imposes a high viscous resistance against the growth of the base perimeter. If it is assumed that the pressure of the vapour inside the bubble is uniform, it is clear that the growth of the bubble top is faster than the growth of the bubble base perimeter. Imposing a restriction on the position of the bubble base perimeter in relation to the position of the bubble top, would not yield the true picture of bubble growth. In this work the growth at bubble top and the growth at bubble base perimeter are considered independently. Figs. 3-1 and 3-2 indicate some bubble tracings obtained by Cole and Shulman (22) and Han and Griffith (25). An examination of these tracings illustrate these differences of growth rate at the top and at the base very clearly. It can also be seen that, initially, the base perimeter grows outward from the nucleation centre almost at the same rate as the top of the curved interface, but towards later stages the base perimeter turns back and recedes towards the nucleation centre while the top continues to grow outward. This results, from an excess outward pressure acting near the top of the bubble and a inward pressure acting near the bubble base perimeter.

To facilitate the separate application of the governing hydrodynamic equations (section 3.5.2) to these two positions of the interface under consideration, local spherical curvature is assumed at these two points, with the centre of both the curvatures being at the nucleation site centre as indicated in Fig 3.3 (a). With this assumption the radius of curvature for the bubble top ' R_T ' will be the bubble height and the radius of the bubble base perimeter ' R_B ' will be the base radius. This assumption simplifies the calculations and is not inconsistent with general bubble growth observations (figs. 3.1 and 3.2).

For the purposes of calculation of the bubble volume and the heat transfer across the curved interface, the bubble profile is assumed to be a spherical segment passing through the known position of the bubble top and the known position of the base perimeter.

Using the above model, with the approximations outlined, modified forms of the extended Rayleigh equation are solved separately, for growth at the top and the base, by a step by step method.

3.2.2 Outline of the method of solution

As some of the equations in the next few sections have been derived in the light of the proposed method of solution, a qualitative outline of the step by step method is given here.

The growth of the top of the curved interface and the growth of the base perimeter are considered separately as mentioned earlier. Therefore, every calculation carried out corresponding to one position is repeated for the other position as well. The method of solution can be described very conveniently in stages as given below. For the calculation at each stage, the variation of variables during subsequent stages are taken into account by the mathematical method of solution (Runge-Kutta fourth order) as detailed in Appendix I.

Stage I

A hemispherical bubble nucleus is assumed and initial values are given to the variables, R_T , $\dot{R}_T$, $\ddot{R}_T$, R_B , $\dot{R}_B$, $\ddot{R}_B$ and V_b . The bulk pressure, bulk temperature, the average heat flux and the average initial wall temperature are normally available as experimental data. A value is assumed for the cavity radius, if not available. Using these values the initial equilibrium pressure and temperature at the bubble interface is calculated.

(Section 3.4).

Stage II

Several methods of evaluating heat transfer into the bubble across

the microlayer, depending on the approximations used, are given in Chapter 4, together with the method of evaluation of heat transfer across the curved surface. By these methods the total heat transfer into the bubble is evaluated. The corresponding increase of mass of the vapour in the bubble is then calculated. The new radius of the dry area under the bubble is also estimated.

Stage III

Using the known values of radii and velocities at the top and the base of the curved interface at the beginning of the time step, incremental growths during the time step are calculated separately. Using these the increase in bubble volume is estimated, giving the new volume.

Stage IV

A solution to the Clausius-Clapeyron equation together with the ideal gas law is obtained, using the new vapour mass and volume, by an iterative technique, in order to establish the new equilibrium pressure and temperature at the interface at the end of the time step (Section 3.7).

Stage V

Interfacial velocities (at the top and the base of the curved interface) at the end of the time step are evaluated using the known values of velocities and accelerations at the beginning.

Stage VI

Use of these derived values at the end of the time step, in the governing hydrodynamic equations 3.15 and 3.17 (for top and base) yield new values for the interfacial accelerations.

Stage VII

Now a new set of values are available for $R_T, \dot{R}_T, \ddot{R}_T, R_B, \dot{R}_B, \ddot{R}_B,$

p_i , T_i and V_b corresponding to the time at the end of the time step. For advancement of growth, the calculation is repeated starting from stage II.

3.3 CRITERION OF BUBBLE INCEPTION

In boiling systems nucleation takes place at or certainly very close to the heated solid-liquid interface. This has been conclusively established by a number of workers such as Clark, Streng and Westwater (54) and Griffith and Wallis (55) by experimental observations.

The postulation is that a surface contains many cavities which have some gas or vapour trapped within them. If liquid of sufficiently high superheat surrounds such a vapour nucleus, a bubble grows from it and detaches, leaving some vapour behind in the cavity to act as the nucleus for the next bubble.

As the liquid-vapour interface of the vapour in the cavity grows, it has to pass through a minimum radius of curvature which requires the equilibrium temperature at the interface to attain the highest value. Fig. 3.4 taken from a paper by Rohsenow (56) illustrates the variation of the radius of curvature with the volume of vapour in the nucleus. The relationship between the radius of curvature and the saturation temperature at the interface can be obtained by the application of principles of thermodynamic equilibrium as follows. Consider a curved interface of liquid-vapour at static equilibrium. For static equilibrium

$$p_i - p_b = \sigma \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$$

where r_1 and r_2 are the principal radii of curvature.

For a spherical surface $r_1 = r_2 = r$

$$p_i - p_b = \frac{2 \sigma}{r} \quad 3.1$$

For a liquid-vapour interface in thermodynamic equilibrium the Clausius-Clapeyron relation must hold

$$\frac{dT}{dp} = \frac{T(v_v - v_l)}{L} \quad 3.2$$

From the ideal gas law 'pv = GT'

$$v = \frac{GT}{p}$$

Substituting this in equation 3.2 (neglecting v_l compared with v_v)

$$\frac{dT}{dp} = \frac{GT^2}{Lp}$$

Intergration gives

$$\ln p = -\frac{L}{G} \frac{1}{T} + A (= \text{constant})$$

$$\therefore \ln p = A - \frac{B}{T} \quad 3.3$$

$$\text{where } B = \frac{L}{G} \quad 3.4$$

Equation 3.3 used together with equations, 3.4 and 3.1 give the equilibrium temperature at the interface, and is used throughout the calculations as a relationship between the saturation temperature and pressure of the vapour.

A more approximate form of obtaining this is commonly used, by replacing the differential in equation 3.2 by a finite difference form

$$\frac{T_i - T_s}{p_i - p_s} = \frac{T_s v_v}{L}$$

$$T_i - T_s = \frac{T_s}{L\rho_v} (p_i - p_s)$$

Using the equation 3.1 in the above, we obtain

$$T_i - T_s = \frac{T_s}{L\rho_v} \cdot \frac{2\sigma}{r}$$

$$T_i = T_s + \frac{T_s}{L\rho_v} \cdot \frac{2\sigma}{r} \quad 3.5$$

Equation 3.5 clearly shows that T_i , the saturation temperature at the interface is inversely proportional to the radius of curvature 'r'. Fig. 3.4 illustrates that the nucleus during its growth passes through a minimum radius of curvature which is equal to the cavity radius. Until the temperature of the liquid surrounding this hemispherical nucleus reaches a value above the saturation temperature at the interface indicated by equation 3.5, the nucleus will not start to grow. If the temperature of the liquid in the thermal layer cannot exceed the temperature given by equation 3.5 a bubble will not develop from the cavity under consideration.

3.4 INITIAL CONDITIONS FOR BUBBLE GROWTH COMPUTATION

When the colder bulk liquid comes into contact with the surface as a result of the previous departing bubble, a certain period of time must lapse before the cooler liquid is heated to the required temperature by conduction of heat from the surface. This period of time is known as the 'waiting time (t_w)' and it is reasonable to assume that during this time the nucleus remains static on the cavity and is of a hemispherical shape having a radius equal to the cavity radius. The bubble growth computation is started from this static hemispherical nucleus. The set of initial conditions used are laid

down below. Top of the curved interface is denoted by a subscript 'T' and the base perimeter is denoted by the subscript 'B'.

Conditions at time $t = 0$;

$$R_T = R_c ; \dot{R}_T = 0 ; \ddot{R}_T = 0 \quad 3.6$$

$$R_B = R_c ; \dot{R}_B = 0 ; \ddot{R}_B = 0 \quad 3.7$$

$$\left[\dot{R} = \frac{d}{dt} R, \quad \ddot{R} = \frac{d^2}{dt^2} R, \quad R_c - \text{cavity radius} \right]$$

The initial pressure of the bubble is obtained by equation 3.1

$$p_i = p_\infty + \frac{2\sigma}{R_c} \quad 3.8$$

The initial temperature is obtained by substitution of equation 3.8 in equation 3.3

$$\ln p_i = A - \frac{B}{T_i}$$

$$T_i = \frac{B}{A + \ln p_i} \quad 3.9$$

According to these initial conditions, the cavity radius is a necessary variable to start the calculation. Some of the experimental data available give this value while most do not. In the absence of data a reasonable estimate of the cavity radius must be made.

It is possible to obtain an estimate of the cavity radius if the waiting time of a particular bubble is given, using the analysis given in section 4.2.2 for the waiting time calculation.

In the absence of the waiting time observation, an arbitrary value for the cavity radius must be chosen. Experimental observations of natural nucleation sites are very scarce, but numerous works exist where nucleation has been successful with artificial sites of known dimensions. Heled et al. (57) have investigated pool boiling of organic liquids from large arrays of nucleation sites, where the sites were prepared by drilling. These sites were deep with a diameter of about 0.008 in. (200 μm .) and a depth of about 0.039 in. (1000 μm .). Howell and Siegel (58) carried out tests with prepared nucleation sites in the size range 0.0015 to 0.032 in. radius (75 to 1500 μm .). The cavity depths ranged from 0.010 to 0.020 in. (250 to 500 μm .) R.K. Young (59) has employed very shallow cavities made by a chemical etching process where the diameters ranged from 0.010 in. to 0.035 in. (250 μm . to 875 μm .). Griffith and Wallis (55) have studied the role of surface conditions on nucleate boiling by using conical cavities of approximate mouth diameter 0.0027 in. (70 μm .). Gaddis and Hall (60) have observed nucleation from artificial sites in the flow boiling case. The cavity diameters ranged from 25 to 375 μm . Clark, Stenge and Westwater (54) were amongst some of the few to study natural nucleation sites. Their photographs showed that pits with diameters between 0.0003 and 0.003 in. (7.5 μm . to 75 μm .) were very active sites. It is noticeable that in most of the above works the cavity radii ranged from about 25 μm . to 1000 μm . By virtue of this fact, cavity radii in the same range are assumed in the present calculations and normally, values between 100 μm . and 200 μm . have been employed.

3.5 GOVERNING HYDRODYNAMIC EQUATIONS

In the model described in section 3.2 spherical symmetry for the bubble as a whole is not assumed. However, for both the growth at the top of the interface and the growth at the base of the curved

interface, sphericity is assumed locally. Hence, the Rayleigh Equation developed for the motion of curved interfaces with perfect spherical symmetry is used as the governing equation both for the top and the base of the curved interface.

3.5.1 Development of the extended Rayleigh equation

The treatment used by Scriven (21) is outlined below briefly to indicate the development of the governing equation.

The equation of continuity for the liquid, in the case of radial motion of the curved interface, with perfect spherical symmetry is given by

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u_r) = 0 \quad \text{where } u_r \text{ is the radial velocity}$$

$$\text{i.e. } r^2 u_r = \text{constant}$$

At the bubble interface $r = R$,

$$r^2 u_r = R^2 \dot{R} \quad 3.10$$

The Navier-Poisson equation governing the motion of the liquid for spherical symmetry, incompressible newtonian fluid and no external body forces reduces to

$$\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} = \frac{1}{\rho_l} \frac{\partial p_r}{\partial r} - 2 \frac{\mu}{\rho_l} \frac{\partial^2 u_r}{\partial r^2} \quad 3.11$$

Substitution of equation 3.10 in equation 3.11 yields

$$\frac{1}{r^2} \left[R^2 \ddot{R} + 2 R \dot{R}^2 \right] - 2 (R^2 \dot{R})^2 \frac{1}{r^5} = - \frac{1}{\rho_l} \frac{\partial p_r}{\partial r} - 12 \frac{\mu}{\rho_l} R^2 \dot{R} \frac{1}{r^4} \quad 3.12$$

Integrating the above with respect to 'r' between the limits

$r = R$ to $r = \infty$,

$$-\frac{[p_r]_R^\infty}{\rho_l} = R\ddot{R} + \frac{3}{2} \dot{R}^2 + 4 \frac{\mu}{\rho_l} \frac{\dot{R}}{R}$$

Let p_∞ be the pressure in the bulk liquid (at infinity) and p_R be the pressure in the liquid just outside the bubble interface, then

$$-\frac{(p_\infty - p_R)}{\rho_l} = R\ddot{R} + \frac{3}{2} \dot{R}^2 + 4 \frac{\mu}{\rho_l} \frac{\dot{R}}{R} \quad 3.13$$

The pressure inside the bubble is related to the pressure outside the bubble by an equation similar to equation 3.8 given by

$$p_R = p_i - \frac{2\sigma}{R}$$

Substituting this in equation 3.13 and simplifying, the extended Rayleigh Equation is obtained.

$$\frac{p_i - p_\infty}{\rho_l} = R\ddot{R} + \frac{3}{2} \dot{R}^2 + \frac{1}{\rho_l} \frac{2\sigma}{R} + 4 \frac{\mu}{\rho_l} \frac{\dot{R}}{R} \quad 3.14$$

The above equation can be rearranged to give

$$\ddot{R} = \frac{1}{R} \left[\frac{p_i - p_\infty}{\rho_l} - \frac{3}{2} \dot{R}^2 - \frac{1}{\rho_l} \frac{2\sigma}{R} - 4 \frac{\mu}{\rho_l} \frac{\dot{R}}{R} \right] \quad 3.14(a)$$

Differently modified forms of equation 3.14(a) is used for the calculation of growth at the top and the base separately.

3.5.2 Adaptation of the extended Rayleigh equation for different growths at top and the base of the curved interface

As described in section 3.2 the bubble base and the top grow at different rates, for two reasons:

- (a) Hydrostatic head between the bubble base and the top.
- (b) Higher viscous drag force restraining the growth at the bubble base.

Terms taking into account these two factors must be included in the governing equation.

3.5.2.1 Growth of the Top of the Curved Interface

' p_{∞} ' was defined in the earlier section as the pressure in the bulk liquid at infinity. However, this is not strictly correct for the growth at the top as the hydrostatic head had not been taken into account in the derivation of the extended Rayleigh equation. Therefore p_{∞} can be redefined at this stage as the pressure in the liquid outside the interface in the absence of inertia forces without any loss in accuracy.

Now, in order to take into account the hydrostatic head due to bubble height, a datum must be fixed at which the bulk pressure of the liquid is defined. The level of the heating surface is very convenient for this. If p_{∞} is the bulk pressure at the level of the heating surface, then the pressure in the liquid at the level of the top of the curved interface is lower by an amount equivalent to the hydrostatic head imposed by virtue of bubble height, in the case of boiling on a horizontal surface. Therefore, to take into account this effect, the term p_{∞} in equation 3.14(a) must be replaced by $(p_{\infty} - R_T \rho_l g)$. Therefore the governing equation for growth of the top of the curved interface is given by

$$\ddot{R}_T = \frac{1}{R_T} \left[\frac{p_i - (p_\infty - R_T \rho_\ell g)}{\rho_\ell} - \frac{3}{2} \dot{R}_T^2 - \frac{1}{\rho_\ell} \frac{2\sigma}{R_T} - \frac{4\mu}{\rho_\ell} \frac{\dot{R}_T}{R_T} \right] \quad 3.15$$

3.5.2.2 Growth at the Base of the Curved Interface

Equation 3.14 indicates that the available pressure difference (ie. $p_i - p_\infty$) is used in overcoming an inertia force, a surface tension force and a very small viscous force. The viscous force term $4\mu \frac{\dot{R}}{R}$ does not fully represent the higher viscous drag force that must exist near the surface. If the drop in the available pressure due to this higher viscous force term is represented by Δp_d , equation 3.14 can be rewritten as

$$\frac{p_i - p_\infty}{\rho_\ell} - \frac{\Delta p_d}{\rho_\ell} = R\ddot{R} + \frac{3}{2} \dot{R}^2 + \frac{1}{\rho_\ell} \frac{2\sigma}{R} \quad 3.16$$

This can be rearranged, with R replaced by R_B for the base, to give the governing equation for the growth of the bubble base perimeter

$$\ddot{R}_B = \frac{1}{R_B} \left[\frac{p_i - p_\infty}{\rho_\ell} - \frac{\Delta p_d}{\rho_\ell} - \frac{3}{2} \dot{R}_B^2 - \frac{1}{\rho_\ell} \frac{2\sigma}{R_B} \right] \quad 3.17$$

Equation 3.17, when used with a suitable expression for Δp_d , will describe the flow situation near the bubble base better than the ordinary Rayleigh equation. An expression for Δp_d is developed in the next section.

Δp_d , being the pressure drop equivalent to the viscous drag force cannot be used in the above equation until the viscous boundary layer near the surface has been properly built up. Furthermore, due to the assumption of a static bubble as a initial condition very high interfacial velocities are reached in the initial stages of the calculation, producing very high values for Δp_d . The computed variation of interfacial velocities and accelerations with

time, for one bubble, are illustrated in Fig. 3.5. Han and Griffith (25) termed the short period of time during which high interfacial velocities and accelerations are achieved as the unbinding period. It is reasonable to assume that at the end of this unbinding period, the boundary layer would be properly developed and the velocities would have settled down to more gradual variations. Therefore, Δp_d is applied in the governing equation only after this period. This viscous drag force term is disregarded during the recession of the bubble base perimeter, as a viscous boundary layer would not exist in such a situation.

3.6 DERIVATION OF THE VISCOUS DRAG FORCE AT BUBBLE BASE

To obtain an estimate of the viscous drag force imposed on the liquid by the heater surface, the flow of liquid in Region I of Fig 3.6 (the viscous boundary layer) must be studied. An adaptation of the Olander and Watts (61) approach is used for this study. The thickness of the viscous boundary layer in the liquid is assumed to be very small compared with bubble dimensions. As a result all the effective velocities in the liquid in this region stem from the outward velocity of the liquid interface at the bubble base perimeter.

Under the assumption of axial symmetry the momentum equation (in spherical coordinates) applicable in region I reduces to

$$\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\phi}{r} \frac{\partial u_r}{\partial \phi} = -\frac{1}{\rho_l} \frac{\partial p}{\partial r} + \frac{v}{r^2} \left(\frac{\partial^2 u_r}{\partial \phi^2} \right) \quad 3.18$$

where u_r - boundary layer velocity in radial direction

u_ϕ - boundary layer velocity in ' ϕ ' direction (Fig 3.6)

In region II, the momentum equation is given by

$$\frac{\partial U_r}{\partial t} + U_r \frac{\partial U_r}{\partial r} = - \frac{1}{\rho_l} \frac{\partial p}{\partial r} \quad 3.19$$

where U_r - Radial velocity of liquid outside the boundary layer

The continuity equation in region II is given by

$$\frac{\partial}{\partial r} (r^2 U_r) = 0 \quad 3.20$$

Integration of equation 3.20 using the boundary condition $U_r = \dot{R}$ at $r = R$ (ie, at bubble interface) yields

$$U_r (r, t) = \frac{R^2 \dot{R}}{r^2} \quad 3.21$$

Eliminating the pressure term from the equations 3.18 and 3.19, we obtain

$$\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\phi}{r} \cdot \frac{\partial u_r}{\partial \phi} = \frac{\partial U_r}{\partial t} + U_r \frac{\partial U_r}{\partial r} + \frac{\nu}{r^2} \frac{\partial^2 u_r}{\partial \phi^2} \quad 3.22$$

A first approximate solution for u_r can be obtained by linearising the above equation. In a very thin boundary layer the convective terms will be small compared to the viscous terms. Hence, the term $\frac{u_\phi}{r} \frac{\partial u_r}{\partial \phi}$ and the difference $(u_r \frac{\partial u_r}{\partial r} - U_r \frac{\partial U_r}{\partial r})$ are neglected, to obtain,

$$\frac{\partial u_r}{\partial t} - \frac{\partial U_r}{\partial t} = \frac{\nu}{r^2} \frac{\partial^2 u_r}{\partial \phi^2} \quad 3.23$$

Since U_r is not a function of ' ϕ ' equation 3.23 can be rewritten as

$$\frac{\partial (u_r - U_r)}{\partial t} = \frac{v}{r^2} \frac{\partial^2 (u_r - U_r)}{\partial \phi^2} \quad 3.24$$

with the initial condition

$$u_r - U_r = 0 \quad \text{at} \quad t = 0$$

and boundary conditions

$$\left. \begin{aligned} u_r - U_r &= 0 \quad \text{at} \quad r\phi = \delta(r) \\ u_r - U_r &= -U_r \quad \text{at} \quad r\phi = 0 \end{aligned} \right\} \quad \text{for } t > 0 \quad 3.25$$

In order to obtain a solution for equation 3.24 with the initial and boundary conditions 3.25, it is necessary to know U_r as a function of time. Most experimenters suggest that bubble growth follows a curve of the form $R = At^{\frac{1}{2}}$. An assumption of this nature, at this point, in order to develop an approximate velocity profile in the viscous boundary layer, would not have serious reflections on the final bubble growth. Therefore ' U_r ' can be obtained as a function of time by the use of $R = At^{\frac{1}{2}}$ in equation 3.21. With this assumption equations 3.24 and 3.25 can be solved using methods given in Carslaw and Jaeger (37) (pg. 62, 2nd Ed.) to obtain,

$$u_r - U_r = -U_r \sqrt{\pi} \operatorname{erfc} \left[r\phi/2 \sqrt{vt} \right] \quad 3.26$$

$$\left[\operatorname{erfc} x = \int_x^\infty \operatorname{erfc} x \cdot dx = \frac{1}{\sqrt{\pi}} e^{-x^2} x \operatorname{erfc} x \right]$$

Therefore, the velocity profile in the viscous boundary layer, in terms of bubble interfacial velocity, is given by

$$u_r = U_r \left\{ 1 + \sqrt{\pi} \left[\frac{r\phi}{2\sqrt{vt}} \operatorname{erfc} \left(\frac{r\phi}{2\sqrt{vt}} \right) - \frac{1}{\sqrt{\pi}} e^{-\left(\frac{r\phi}{2\sqrt{vt}} \right)^2} \right] \right\} \quad 3.27$$

This velocity profile is used in equation 3.18 to develop an expression for the pressure drop due to the viscous drag force.

Equation 3.18 can be written in the following form, when the convective term $\frac{u}{r} \frac{\partial u_r}{\partial \phi}$ is neglected.

$$\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} = - \frac{1}{\rho_l} \frac{\partial p}{\partial r} + \frac{\nu}{r^2} \frac{\partial^2 u_r}{\partial \phi^2} \quad 3.28$$

The pressure gradient $\frac{\partial p}{\partial r}$ in the above equation overcomes the restraints due to liquid inertia and viscous forces. In the governing equation for bubble growth represented by equation 3.16 the term $R\ddot{R} + \frac{3}{2}\dot{R}^2$ takes into account the contribution of inertia to the total restraint force. What is required now is to evaluate the contribution of the viscous drag force towards the total restraint force. If 'p_d' denotes the part of the pressure used in overcoming the drag force and p' the part of the pressure that overcomes the inertia force, equation 3.28 can be rewritten as

$$\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} = - \frac{1}{\rho_l} \frac{\partial p'}{\partial r} \left(- \frac{1}{\rho_l} \frac{\partial p_d}{\partial r} + \frac{\nu}{r^2} \frac{\partial^2 u_r}{\partial \phi^2} \right)$$

where $p = p' + p_d$

From the above p_d is defined such that

$$- \frac{1}{\rho_l} \frac{\partial p_d}{\partial r} = - \frac{\nu}{r^2} \frac{\partial^2 u_r}{\partial \phi^2} \quad 3.29$$

From equation 3.29, the pressure difference Δp_d (as discussed in section 3.5.2.2) used to overcome the viscous drag force can be evaluated.

$\frac{\partial^2 u_r}{\partial \phi^2}$ can be evaluated from equations 3.21 and 3.27.

$$\frac{\partial^2 u_r}{\partial \phi^2} = -2 R \dot{R} \left(\frac{1}{2\sqrt{vt}} \right)^2 e^{-\left(\frac{r\phi}{2\sqrt{vt}} \right)^2}$$

Substituting the above in equation 3.29

$$-\frac{1}{\rho_l} \frac{\partial p_d}{\partial r} = -\frac{\nu}{r^2} x - 2R \dot{R} \left(\frac{1}{2\sqrt{vt}} \right)^2 e^{-\left(\frac{r\phi}{2\sqrt{vt}} \right)^2}$$

Integration of the above from $r = R$ to $r = \infty$ using the transformation

$$z = \frac{r\phi}{2\sqrt{vt}} \text{ yields}$$

$$\frac{1}{\rho_l} (p_{dR} - p_{d\infty}) = \frac{R\dot{R}}{2t} \left[e^{-\left(\frac{R\phi}{2\sqrt{vt}} \right)^2} - \sqrt{\pi} \frac{R\phi}{2\sqrt{vt}} \operatorname{erfc} \frac{R\phi}{2\sqrt{vt}} \right] \quad 3.30$$

The viscous boundary layer develops from the outer edge of the base of the bubble and, therefore, is assumed to have a thickness equal to the initial microlayer thickness at that point. At the base perimeter of the bubble $R = R_B$. With the above assumption, $R_B \phi$ represents the initial microlayer thickness at the bubble base perimeter which is represented by δ_0 . Therefore, Δp_d is given by

$$\frac{\Delta p_d}{\rho_l} = \frac{1}{\rho_l} (p_{dR_B} - p_{d\infty}) = \frac{R_B \dot{R}_B}{2t} \left[e^{-\left(\frac{\delta_o}{2\sqrt{vt}}\right)^2} - \sqrt{\pi} \frac{\delta_o}{2\sqrt{vt}} \operatorname{erfc} \frac{\delta_o}{2\sqrt{vt}} \right] \quad 3.31$$

The initial microlayer thickness δ_o , used in this equation is obtained as discussed in section 4.3.1.

3.7 EVALUATION OF PRESSURE AND TEMPERATURE VARIATION INSIDE THE BUBBLE

In order to obtain a solution for the growth of the bubble interface, using equations 3.15 and 3.17, it is necessary to evaluate the variation of pressure and temperature inside the bubble. In asymptotic growth calculations the pressure and temperature inside the bubble are taken to be constant throughout its growth period. In the present method, as the calculation is advanced by each step, the new pressure and the new temperature inside the bubble are evaluated using the following approach.

For this calculation, it is necessary to know the increment of mass of vapour inside the bubble and the increment of volume of the bubble for each step. The evaluation of the mass increment is outlined in chapter 4. For the purposes of calculation of the volume increment, the bubble is assumed to be a segment of a sphere passing through the known points, namely the peak of the bubble and the base perimeter. With the above assumption, the volume of the bubble ' V_b ' is given by

$$V_b = \frac{\pi}{2} \left(R_B^2 R_T + \frac{R_T^3}{3} \right) \quad 3.22$$

The increase in volume due to increments dR_T and dR_B

$$\delta V_b = \frac{\partial V_b}{\partial R_T} \cdot dR_T + \frac{\partial V_b}{\partial R_B} \cdot dR_B$$

$$\delta V_b = \frac{\pi}{2} \left[R_B^2 + R_T^2 \right] \cdot dR_T + \pi R_B R_T \cdot dR_B \quad 3.33$$

Increment of radii, dR_T and dR_B , are evaluated for each step using equations 3.15 and 3.17.

If m_i and V_{bi} were the mass and the volume of the bubble at the i^{th} step m_{i+1} and $V_{b(i+1)}$ can be obtained as discussed above.

$$m_{i+1} = m_i + \delta m$$

$$V_{b(i+1)} = V_{bi} + \delta V_b$$

For thermodynamic equilibrium the pressure and temperature of the vapour must obey the relationship given by equation 3.3.

$$\ln(p) = A - \frac{B}{T} \quad 3.3$$

Assuming that the vapour inside the bubble obeys the ideal gas law

$$pV = mGT \quad 3.34$$

Substitution of the known m_{i+1} and $V_{b(i+1)}$ in equations 3.3 and 3.32 yields

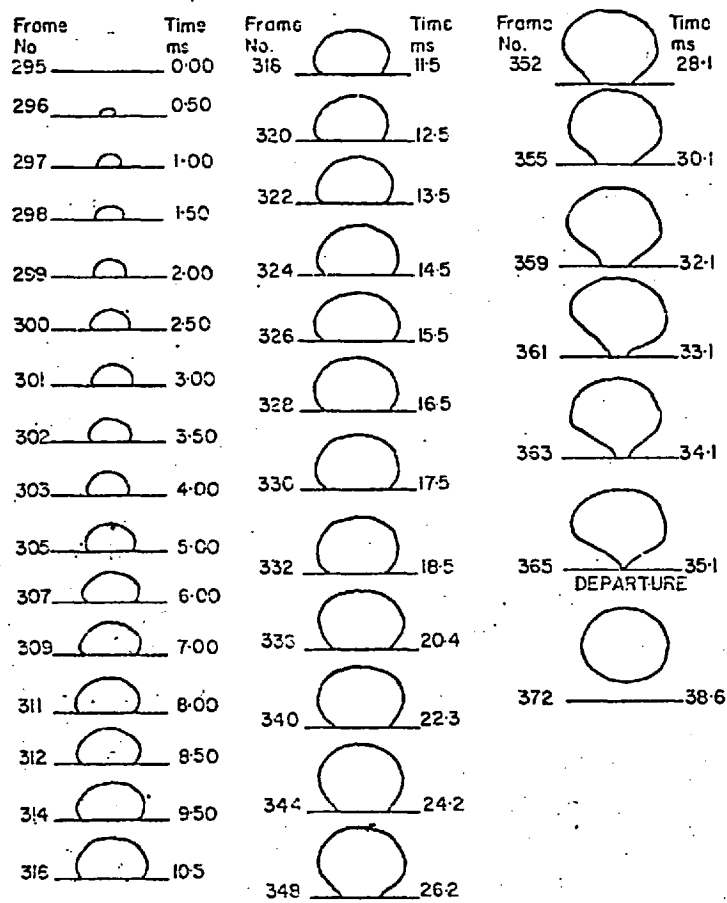
$$\ln p_{i+1} = A - \frac{B}{T_{i+1}}$$

$$\text{and } p_{i+1} V_{b(i+1)} = m_{i+1} GT_{i+1}$$

The above two equations are implicit but can be readily solved by an iterative technique, to obtain p_{i+1} and T_{i+1} .

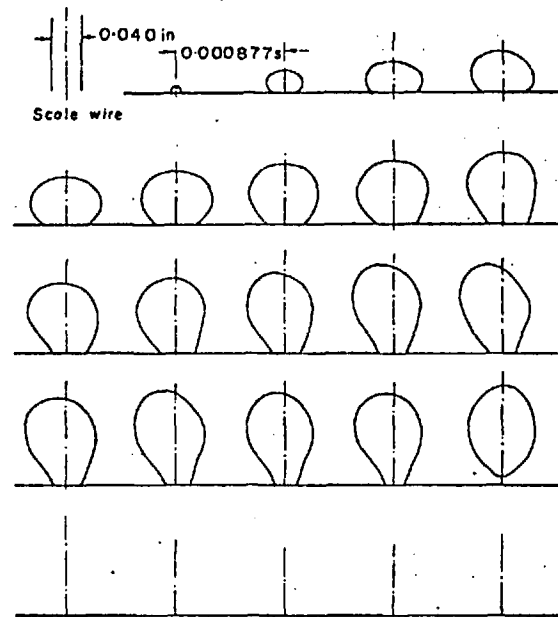
3.8 CONCLUSION

Governing equations for the growth of the bubble interface at the top and the base perimeter are given by equations 3.15 and 3.17 respectively. These two equations, together with the other associated factors outlined in this chapter and chapter 4 are solved using a fourth-order Runge-Kutta method. A brief description of the application of this method to the present problem is given in Appendix I.

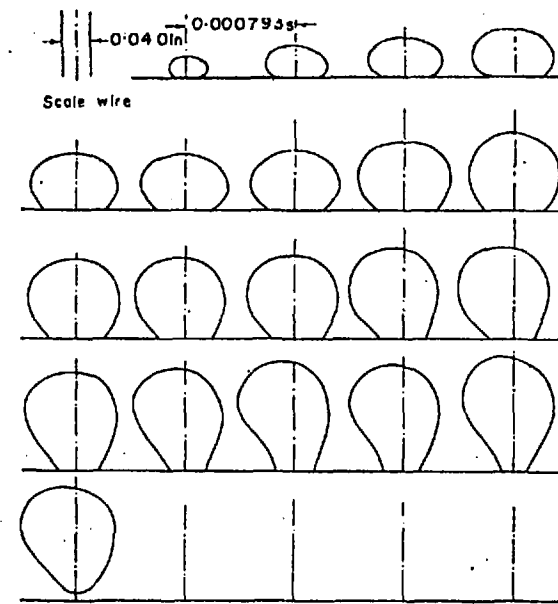


Bubble growth tracings for acetone bubble number 3.

Fig.3.1 Bubble growth tracings - Cole & Shulman (22)

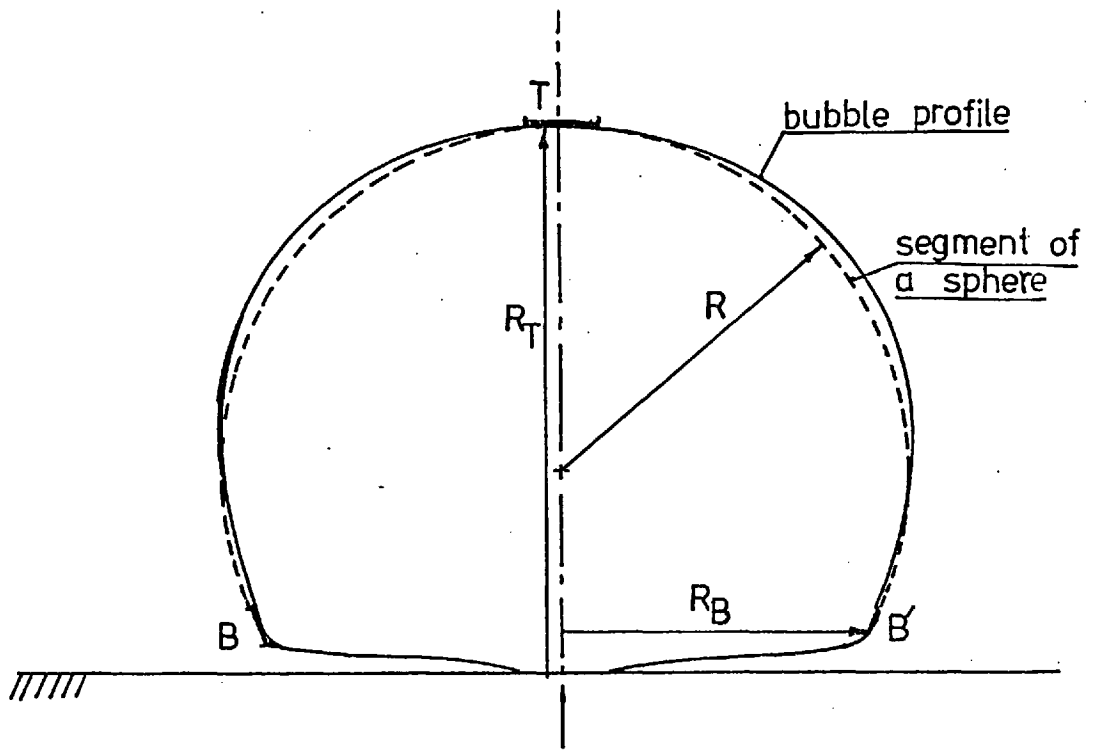


Bubble number 1
 Camera speed = 1140 frames/s
 Waiting period = 29 frames = 0.0254 s
 Departure period = 19 frames = 0.0166 s
 Departure radius = 0.00397 ft
 History of growth of bubble Number 1.

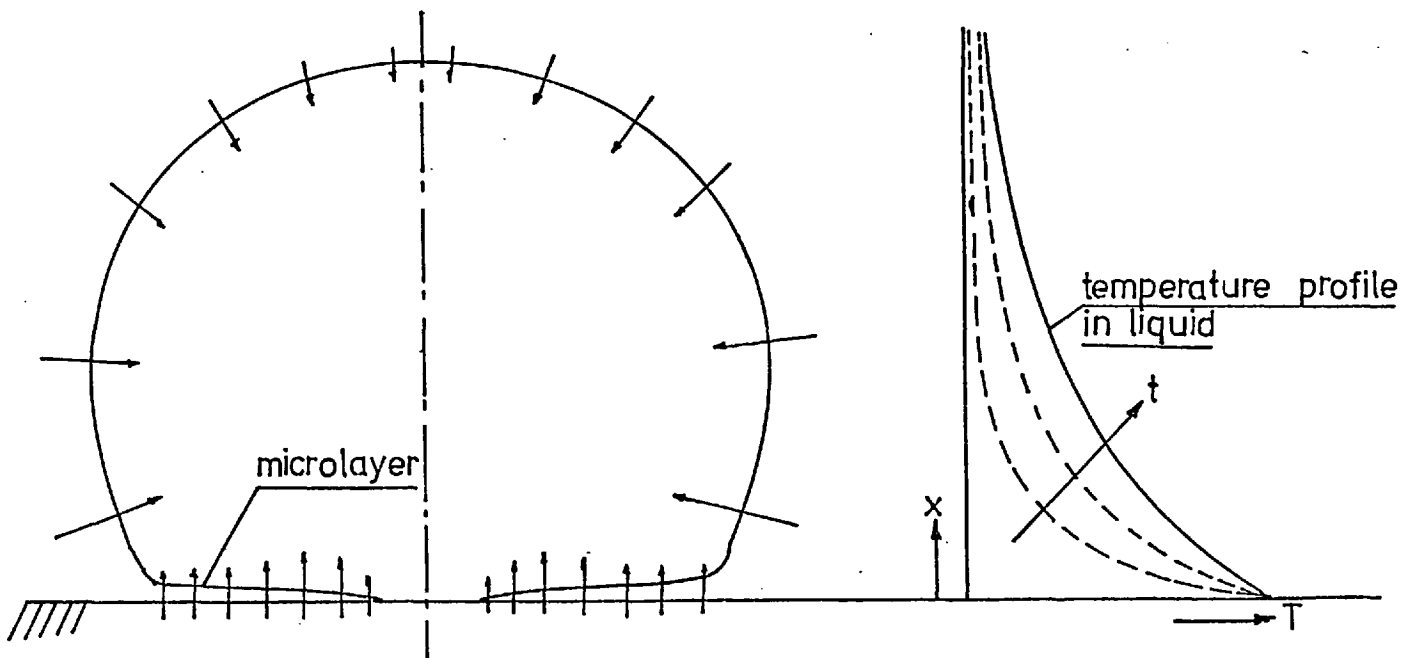


Bubble number 2
 Camera speed = 1260 frames/s
 Waiting period = 55 frames = 0.0437 s
 Departure period = 21 frames = 0.0167 s
 Departure radius = 0.00533 ft
 History of growth of bubble Number 2.

Fig.3-2 Bubble growth tracings - Han & Griffith (25)



(a) Geometry of the Model



(b) Heat Transfer into the Bubble

Fig.3.3 THE MODEL FOR ANALYSIS

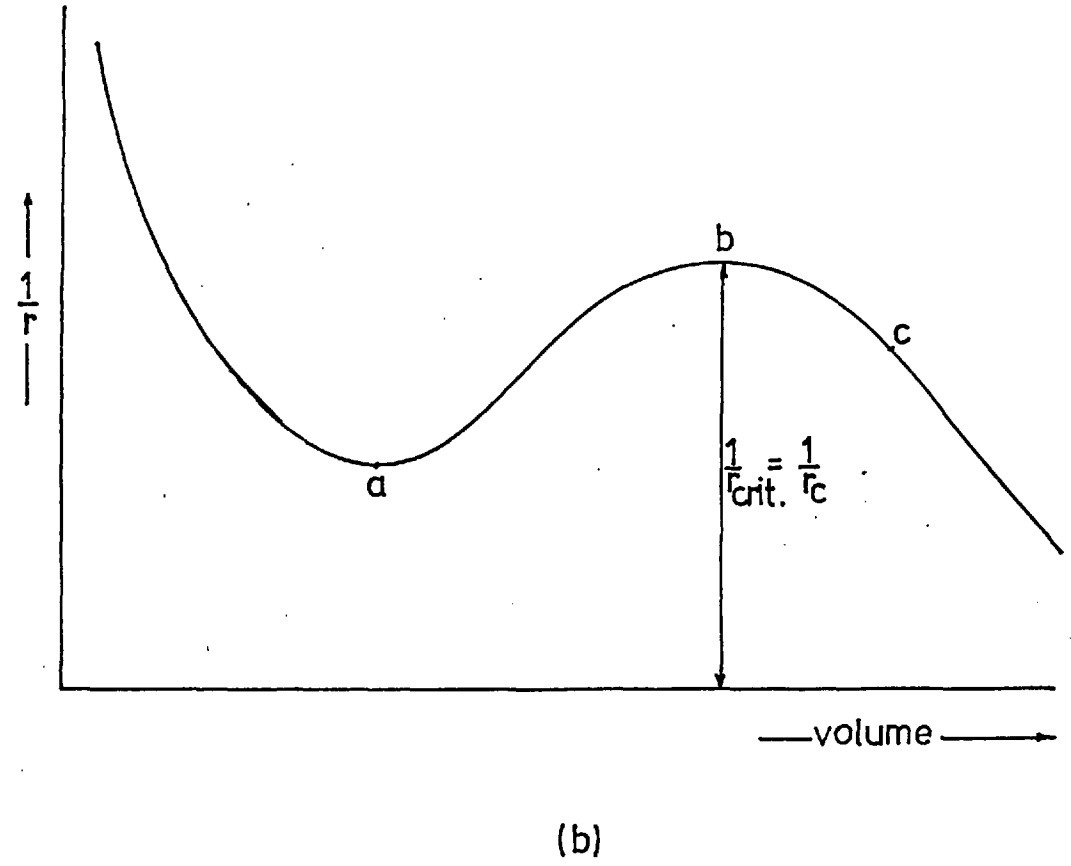
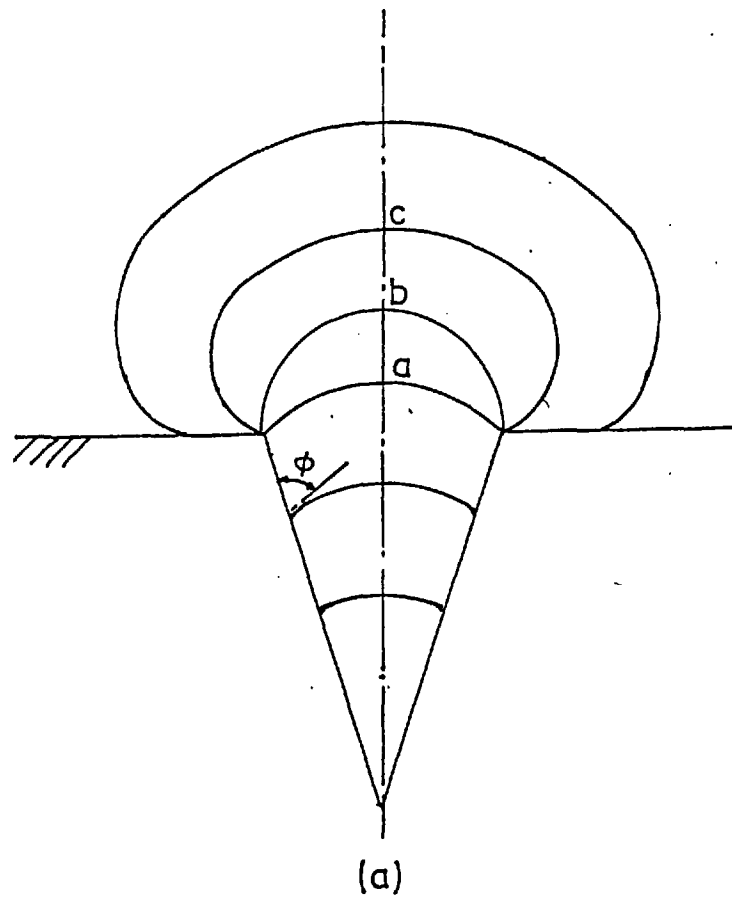


Fig. 3.4 Radius of curvature vs. vapour volume in cavity

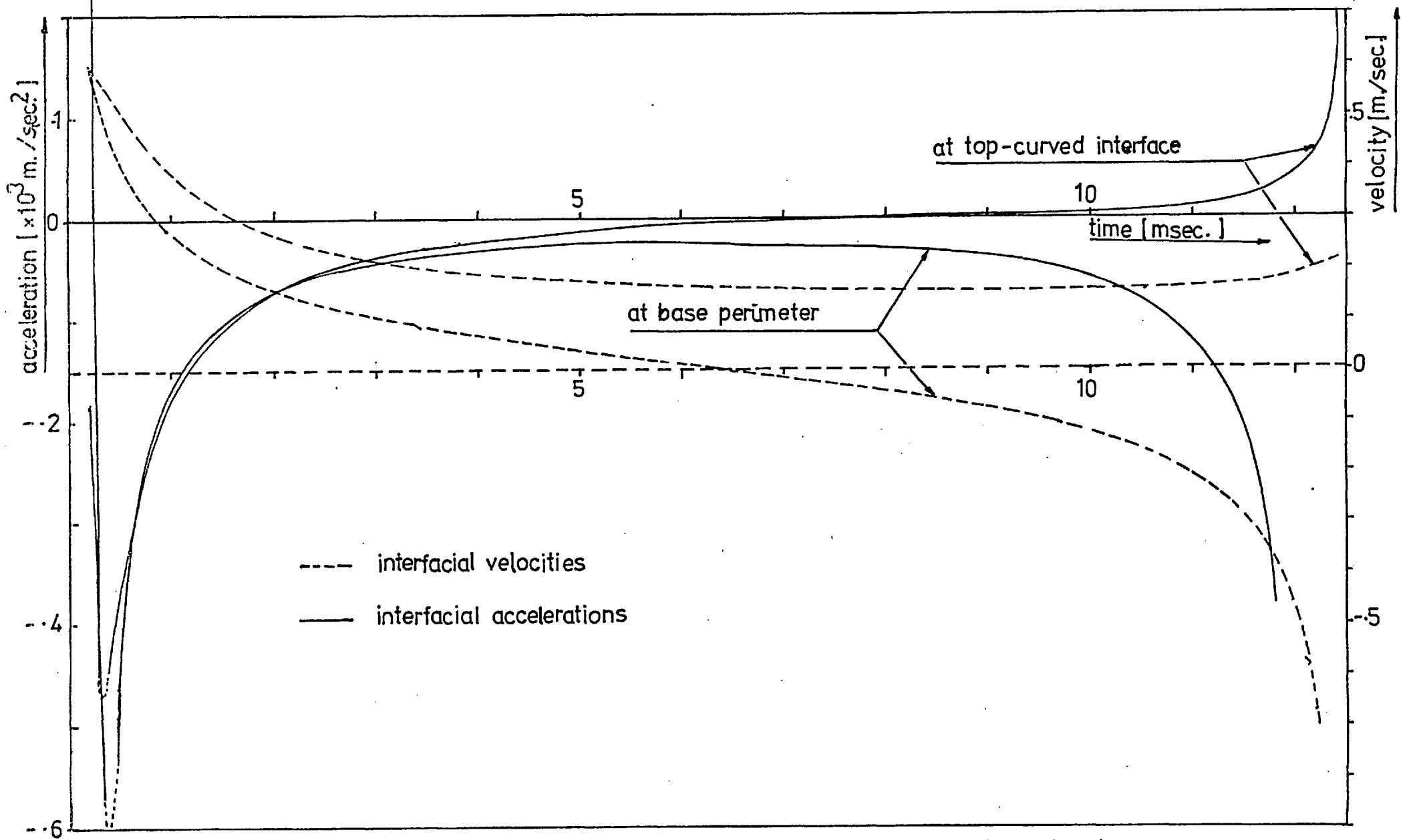


Fig.3-5 Velocities and Accelerations at the interface (bubble D)

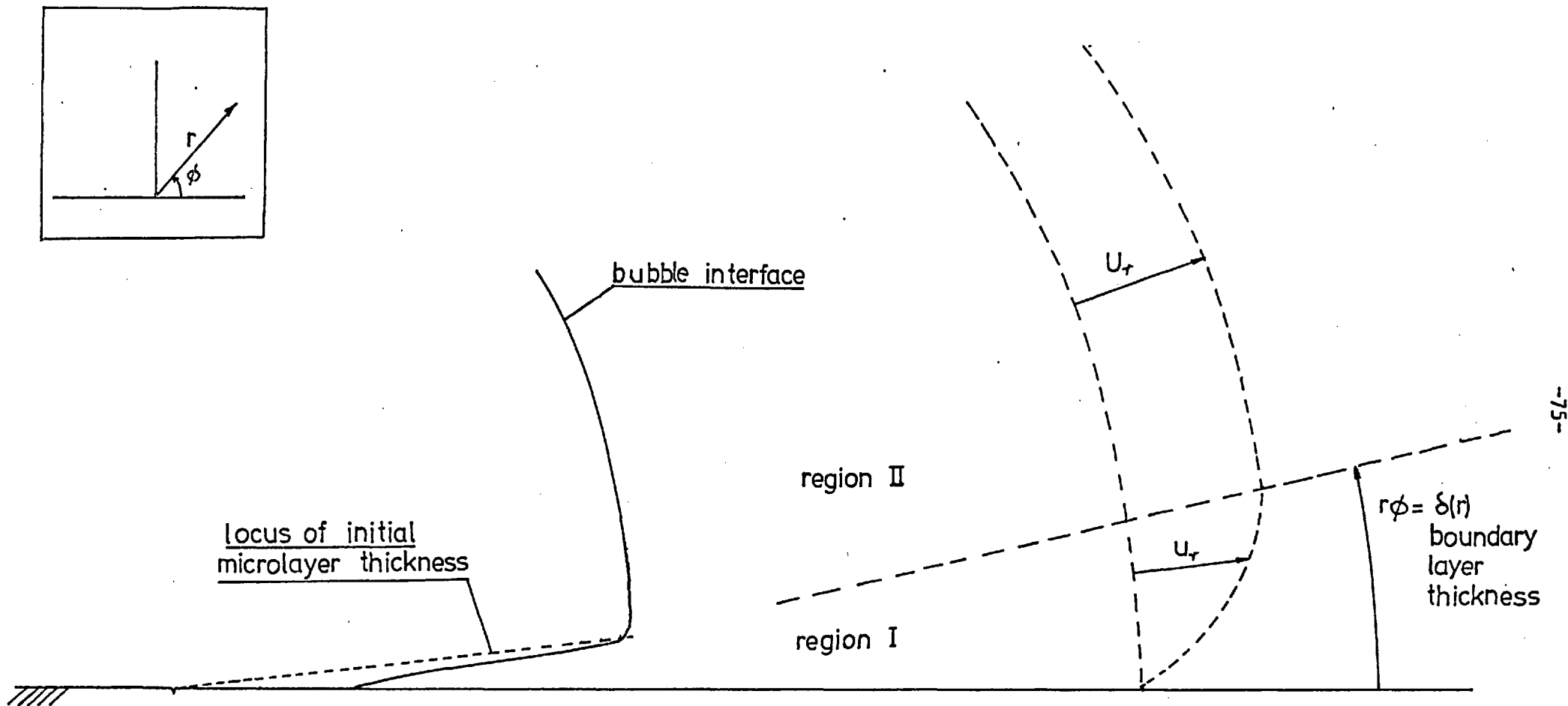


FIG.3-6 FLOW OF LIQUID IMMEDIATELY OUTSIDE A GROWING BUBBLE

CHAPTER 4

HEAT TRANSFER INTO THE BUBBLE

4.1 INTRODUCTION

The numerical solution of the hydrodynamic governing equations is effected in stages as given in section 3.2.2. To carry out stage II of the calculation it is necessary to evaluate the heat transfer into the bubble during each time step. This can be evaluated as two different fractions, namely the heat transfer across the microlayer and across the curved interface.

For heat transfer across the curved interface analytical expressions are developed in this chapter based on the Skinner and Bankoff (29) approach. Methods of evaluation of heat transfer across the microlayer are developed for three different cases, two of which assume expressions defining the microlayer profile, enabling the development of simple analytical expressions for heat transfer. For the third case, where the wall temperature variation is taken into account, a numerical evaluation of the heat transfer across the microlayer is necessary. When either of these three methods are used in stage II, it is also necessary to keep track of the expansion of the dry area underneath the bubble.

4.2 HEAT TRANSFER ACROSS THE CURVED INTERFACE OF THE BUBBLE

In order to estimate the heat transfer across the curved interface of the bubble, a knowledge of the temperature gradient and the area of the curved interface is required. The assumption that a growing bubble carries with it the thermal layer formed over the heating surface has been commonly used. This implies a spherically symmetric temperature field around the bubble, enabling the problem of heat transfer to be treated one dimensionally. A more general case assumes that the bubble grows through the thermal layer.

Fig. 4.1 illustrates the difference between these two models of heat transfer. Irrespective of the model, it is necessary to know the initial temperature profile in the liquid at bubble initiation and also the waiting time.

4.2.1. Initial temperature profile in the liquid at bubble initiation

When a bubble detaches itself from the heating surface and departs, colder liquid from the bulk comes in to replace the volume occupied by the departing bubble. The nucleus of the next bubble in the nucleation site cannot grow until this liquid reaches a sufficiently high superheat. The variation with time of the temperature profile adjacent to the heating surface is obtained by the solution of the transient heat conduction equation, following Han and Griffith (25). It is assumed that very near the surface, where the thermal boundary layer is beginning to develop, convection effects are negligible. Development of this temperature profile according to these assumptions is obtained by a solution of

$$\frac{\partial T}{\partial t} = \alpha_l \cdot \frac{\partial^2 T}{\partial x^2} \quad 4.1$$

$$\text{with the Initial Conditions} \quad \left. \begin{array}{l} T = T_w \quad \text{at } x = 0 \\ T = T_\infty \quad \text{at } x > 0 \end{array} \right\} t = 0$$

$$\text{and the boundary Conditions} \quad \left. \begin{array}{l} T = T_w \quad \text{at } x = 0 \\ T = T_\infty \quad \text{at } x = \infty \end{array} \right\} t > 0$$

The solution is given by

$$T - T_\infty = (T_w - T_\infty) \operatorname{erfc} \frac{x}{2\sqrt{\alpha_l t}} \quad 4.2$$

At the end of the waiting time (when $t = t_w$), when the bubble nucleus starts to grow, the temperature profile in the thermal layer is given by

$$T - T_\infty = (T_w - T_\infty) \operatorname{erfc} \frac{x}{2\sqrt{\alpha_l t_w}} \quad 4.3$$

In spherical coordinates with axial symmetry this becomes

$$T(r, \phi, t_w) - T_\infty = (T_w - T_\infty) \operatorname{erfc} \left(\frac{r \cos \phi}{2 \sqrt{\alpha_l t_w}} \right) \quad 4.4$$

4.2.2. Waiting time evaluation

It was shown in section 3.4, that a hemispherical vapour nucleus must pass through a minimum radius equal to the cavity radius, before the start of bubble growth. The vapour pressure in the bubble will be at a highest when it passes through this minimum radius and the corresponding saturation temperature is given by equations 3.8 and 3.9. Han and Griffith (25) argue that an isothermal line, passing through the top point of a waiting bubble is $\frac{3}{2} R_c$ distant from the heating surface (see Fig 4.3). From equation 4.3, the temperature of the liquid at a point $\frac{3}{2} R_c$ from the surface (ie. $x = \frac{3}{2} R_c$) is obtained

$$T - T_\infty = (T_w - T_\infty) \operatorname{erfc} \left(\frac{\frac{3}{2} R_c}{2 \sqrt{\alpha_l t_w}} \right) \quad 4.5$$

When this temperature is equal to the saturation temperature at bubble interface (T_i) growth can start.

$$\therefore T_i - T_\infty = (T_w - T_\infty) \operatorname{erfc} \left(\frac{\frac{3}{2} R_c}{2 \sqrt{\alpha_l t_w}} \right)$$

$$\operatorname{erfc} \left(\frac{\frac{3}{2} R_c}{2 \sqrt{\alpha_l t_w}} \right) = \frac{T_i - T_\infty}{T_w - T_\infty} \quad 4.6$$

Equation 4.6 can be readily solved for ' t_w ', by a numerical procedure.

Han and Griffith (25) linearised the temperature profile given by equation 4.3, in order to obtain an analytical expression giving the waiting time for particular cavity sizes and heating conditions

and obtained

$$t_w = \frac{9}{4\pi\alpha_l} \left\{ \frac{(T_w - T_\infty) R_c}{T_w - T_s [1 + (2\sigma/R_c \rho_v L)]} \right\}^2 \quad 4.7$$

For the experiments reported in their paper where ' t_w ' was given instead of the cavity radius, the cavity radius has been evaluated using equation 4.7 and compared with the numerical solution of equation 4.6. There is an appreciable difference between the two results, as the following table indicates. Two values of R_c would satisfy equation 4.7 for a given t_w . The higher values were compared.

WAITING TIME (Sec.)	CAVITY RADIUS ($\mu\text{m.}$)	
	FROM EQN. 4.6	FROM EQN. 4.7
0.0245	62.37	52.31
0.0275	66.39	55.64
0.0437	85.63	77.10

TABLE 4.1 CAVITY RADIUS FROM WAITING TIME USING
HAN AND GRIFFITH (25) DATA

In the present computation, equation 4.6 is used throughout to find the waiting time from the cavity radius and vice versa.

4.2.3. Heat Transfer across the curved interface

Skinner and Bankoff (29) evaluated the rate of heat transfer into a spherical bubble across its curved interface and Cooper and Vijuk (33) have applied this analysis with slight variation to hemispherical bubble growth. The present work is concerned with growth while the bubble surface follows an approximately hemispherical shape and hence this later approach is adopted. If the temperature field around the bubble is not assumed to be spherically symmetric

(see Fig.4.1 (b)), derivation of an analytical expression for the rate of heat transfer is extremely difficult without the assumption of a definite shape for the bubble profile.

In Fig.4.4 , the hemispherical profile 'C' is defined such that its surface area is equal to the bubble surface area. The heat transfer rate evaluated for the growth of this hemispherical bubble 'C' using the Skinner and Bankoff approach is then used as a good approximation to calculate actual bubble growth. The approximate surface area of the bubble is evaluated from the spherical segment (B) passing through the three known points of the actual bubble profile as outlined in section 3.2.

$$\text{Approximate surface area of bubble} = 2\pi R. R_T$$

Hence the radius of the equivalent hemispherical bubble is given by

$$2\pi R_{es}^2 = 2\pi R. R_T$$

$$R_{es} = \sqrt{R R_T} \quad 4.8$$

4.2.3.1 Derivation of the Temperature gradient at the curved interface

The variation of the temperature field around the bubble is governed by the energy equation. For an inviscid liquid of constant thermal and physical properties the energy equation in spherical coordinates (for axial symmetry) reduces to:

$$\frac{DT}{Dt} = \alpha_L \nabla^2 T \quad 4.9$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u_r \frac{\partial}{\partial r} \quad ; \quad u_r \text{ is the radial velocity}$$

and

$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \cdot \frac{\partial}{\partial r}) + \frac{1}{r^2 \sin \phi} \frac{\partial}{\partial \phi} (\sin \phi \cdot \frac{\partial}{\partial \phi})$$

The solution of the above equation with the corresponding initial and boundary conditions will give the temperature of the liquid as a function of r , ϕ and t .

$$\text{ie. } T = T(r, \phi, t)$$

The initial condition for the present case is obtained in section 4.2.1 and is given by equation 4.4.

$$T(r, \phi, 0) = T_{\infty} + (T_w - T_{\infty}) \operatorname{erfc} \frac{r \cos \phi}{2 \sqrt{\alpha_l t_w}}$$

The boundary condition, ie. the temperature at the bubble interface, is varying in an unknown pattern. Evaluation of heat transfer into the bubble across the curved interface, taking this into account, is extremely complex. In addition, it has been shown that for rapidly growing bubbles the contribution of this heat transfer is small compared to the heat transfer across the microlayer. Therefore, in the present evaluation, it has been assumed that the bubble interface remains at the saturation temperature corresponding to the bulk pressure. Hence the boundary condition at $r = R$ is

$$T(R, \phi, t) = T_s \quad 4.10$$

Equation 4.9 with the corresponding initial and boundary conditions cannot be solved easily in its original form. Hence the dimensionless temperature ' T^* ' is defined in such a way as to reduce the three dimensional problem to two dimensions.

$$T^* (r, t) = \frac{1}{(T_w - T_s)} \int_0^{\pi/2} (T(r, \phi, t) - T_s) \sin \phi \cdot d\phi \quad 4.11$$

With this definition the set of equations 4.9, 4.4 and 4.10 reduce to the following:

$$\frac{\partial T^*}{\partial t} + \frac{R^2}{r^2} \cdot \frac{\partial T^*}{\partial r} = \alpha_l \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \cdot \frac{\partial T^*}{\partial r} \right) \quad 4.12$$

Initial Condition

$$T^* (r, 0) = \frac{1}{T_w - T_s} \int_0^{\pi/2} [T(r, \phi, 0) - T_s] \sin \phi \cdot d\phi$$

After substitution from equation 4.4 for $T(r, \phi, 0)$ this can be written as

$$T^* (r, 0) = \frac{1}{(T_w - T_s)} \left\{ (T_\infty - T_s) + (T_w - T_\infty) \left[\operatorname{erfc} \frac{r}{2\sqrt{\alpha_l t_w}} \right] + \frac{1}{\sqrt{\pi}} \frac{2\sqrt{\alpha_l t_w}}{r} \left(1 - \exp \left(- \left(\frac{r}{2\sqrt{\alpha_l t_w}} \right)^2 \right) \right) \right\} \quad 4.13$$

Boundary Condition

$$T^* (R, t) = 0 \quad 4.14$$

Now a scaling length 'l' is introduced, and the following dimensionless set of variables defined to reduce the above equation further.

[Following Plesset and Zwick (19) and Skinner and Bankoff (29)]

$$\left. \begin{aligned} r^* &= (r^3 - R^3(t))/3l^3 \\ R^* &= R/l \\ t^* &= \frac{\alpha_l}{l^6} \int_0^t R^4 \cdot dt \end{aligned} \right\} \quad 4.15$$

The energy equation 4.12, when transformed into these new coordinates will take a form

$$\frac{\partial T^*}{\partial t^*} = \frac{\partial}{\partial r^*} \left(\left(1 + \frac{3r^*}{R^3} \right)^{4/3} \cdot \frac{\partial T^*}{\partial r^*} \right) \quad 4.16$$

For the purposes of determining the heat transfer rate into the bubble, what is of interest is the temperature gradient in the 'r' direction near the surface and here the term $\frac{3r^*}{R^3} \left(= \frac{r^3 - R^3}{R^3} \right)$ is very small compared to 1.

Thus, equation 4.16 can be reduced to

$$\frac{\partial T^*}{\partial t^*} = \frac{\partial^2 T^*}{\partial r^{*2}} \quad 4.17$$

The initial condition given by equation 4.13, when transformed using the set of equation 4.15 is given by

$$T^* (r^*, 0) = \frac{1}{T_w - T_s} \left\{ (T_\infty - T_s) + (T_w - T_\infty) \left[\operatorname{erfc} \frac{(3r^{*3} + R_c^3)^{1/3}}{2 \sqrt{\alpha_l t_w}} \right. \right. \\ \left. \left. + \frac{1}{\sqrt{\pi}} \frac{2 \sqrt{\alpha_l t_w}}{(3r^{*3} + R_c^3)^{1/3}} \left(1 - \exp \left(- \frac{(3r^{*3} + R_c^3)^{1/3}}{2 \sqrt{\alpha_l t_w}} \right) \right) \right] \right\} \quad 4.18$$

Denoting this expression by $g(r^*)$

$$T^* (r^*, 0) = g(r^*) \quad 4.19$$

Boundary condition given by equation 4.14 transforms to

$$T^* (0, t^*) = 0 \quad 4.20$$

Thus, the complete problem is now expressed in the new coordinates by the equations 4.17, 4.19 and 4.20.

The solution to this differential equation for any $g(r^*)$ is given by Carslaw and Jaeger (37) [ref. pg 58] .

$$T^* (r^*, t^*) = \frac{1}{2\sqrt{\pi t^*}} \int_0^\infty g(r') \left[e^{-\frac{(r^*-r')^2}{4t^*}} - e^{-\frac{(r^*+r')^2}{4t^*}} \right] . dr' \quad 4.21$$

What is of further interest is the temperature gradient at the bubble interface. In an endeavour to obtain this, the dimensionless temperature gradient is first evaluated from equation 4.21

$$\begin{aligned} \frac{\partial T^*}{\partial r^*} (r^*, t^*) = \frac{1}{2\sqrt{\pi t^*}} \int_0^\infty g(r') \left[-\frac{2(r^*-r')}{4t^*} e^{-\frac{(r^*-r')^2}{4t^*}} \right. \\ \left. + \frac{2(r^*+r')}{4t^*} e^{-\frac{(r^*+r')^2}{4t^*}} \right] . dr' \end{aligned}$$

The dimensionless temperature gradient at the bubble interface is obtained by putting $r^* = 0$ in the above equation

$$\frac{\partial T^*}{\partial r^*} (0, t^*) = \frac{1}{2\sqrt{\pi t^*}} \int_0^\infty g(r') \frac{r'}{t^*} e^{-\frac{r'^2}{4t^*}} . dr' \quad 4.22$$

By conversion of the integrating variable r' to r'' using $r'' = \frac{r'^2}{4t^*}$

$$\frac{\partial T^*}{\partial r^*} (0, t^*) = \frac{1}{\sqrt{\pi t^*}} \int_0^\infty g(2\sqrt{r'' t^*}) e^{-r''} . dr'' \quad 4.23$$

The above expression can be readily integrated by numerical methods. Although transformation of equation 4.23 back to the original coordinate system is difficult, this result can still be used by transforming the required temperature gradient, namely $\frac{\partial T}{\partial r} (R, \phi, t)$ to the new system of coordinates.

4.2.3.2 Evaluation of the Rate of Heat Transfer

The rate of heat input to the bubble across the curved interface is given by

$$q_{cs} = k_{\ell} \int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi/2} R d\phi \times R \sin\phi d\theta \times \frac{\partial T}{\partial r} (R, \phi, t)$$

$$q_{cs} = k_{\ell} \cdot 2\pi R^2 \int_0^{\pi/2} \frac{\partial T}{\partial r} (R, \phi, t) \sin\phi .d\phi \quad 4.24$$

Differentiating equation 4.11 with respect to r

$$\frac{\partial T^*}{\partial r} = \frac{1}{T_w - T_s} \int_0^{\pi/2} \frac{\partial T}{\partial r} (r, \phi, t) \sin\phi .d\phi \quad 4.25$$

Using the set of equations 4.15

$$\frac{\partial T^*}{\partial r} = \frac{\partial T^*}{\partial r^*} \times \frac{r^2}{\ell^3} \quad 4.26$$

when $r = R$ (i.e. $r^* = 0$) equation 4.25 and 4.26 yield

$$\int_0^{\pi/2} \frac{\partial T}{\partial r} (R, \phi, t) \sin\phi .d\phi = (T_w - T_s) \frac{R^2}{\ell^3} \frac{\partial T^*}{\partial r^*} (0, t^*) \quad 4.27$$

Substitution of equation 4.27 in 4.24 yields

$$q_{cs} = k_{\ell} \times 2\pi R^2 \left[(T_w - T_s) \frac{R^2}{\ell^3} \frac{\partial T^*}{\partial r^*} (0, t^*) \right] \quad 4.28$$

Equations 4.18, 4.19, 4.23 and 4.28, together with the set of coordinate transformation equations 4.15, constitute a solution to the heat transfer

problem across the curved interface.

4.2.4. Final set of equations to be solved to obtain the Rate of Heat Transfer across the curved interface

Summarising, the final set of equations used to obtain the rate of heat transfer across the curved interface are,

$$\left. \begin{aligned} r^* &= (r^3 - R^3(t))/3\ell^3 \\ R^* &= R/\ell \\ t^* &= \frac{\alpha_\ell}{\ell^6} \int_0^t R^4 \cdot dt \end{aligned} \right\} \quad 4.15$$

$$T^*(r^*, 0) = g(r^*) \quad 4.19$$

where

$$g(r^*) = \frac{1}{(T_w - T_s)} \left\{ (T_\infty - T_s) + (T_w - T_\infty) \left[\operatorname{erfc} \frac{(3r^*\ell^3 + R_c^3)^{1/3}}{2\sqrt{\alpha_\ell t_w}} \right] \right. \\ \left. + \frac{1}{\sqrt{\pi}} \frac{2\sqrt{\alpha_\ell t_w}}{(3r^*\ell^3 + R_c^3)^{1/3}} \left(1 - \exp \left(- \frac{(3r^*\ell^3 + R_c^3)^{1/3}}{2\sqrt{\alpha_\ell t_w}} \right) \right) \right\} \quad 4.18$$

$$\frac{\partial T^*}{\partial r^*}(0, t^*) = \frac{1}{\sqrt{\pi t^*}} \int_0^\infty g(2\sqrt{r'^* t^*}) e^{-r'^*} \cdot dr'^* \quad 4.23$$

$$q_{cs} = k_\ell \times 2\pi R^2 \left[(T_w - T_s) \frac{R^2}{\ell^3} \frac{\partial T^*}{\partial r^*}(0, t^*) \right] \quad 4.28$$

In striving to obtain the rate of heat transfer across the curved

interface (q_{cs}) from this set of equations, the only obviously lengthy process is the integration represented by equation 4.23. Most of the procedure only involves systematic substitution. It should be noted that the integrand in equation 4.23 tends to zero very rapidly due to the term $e^{-r'}$, with the result that the numerical solution of this equation does not require excessive computer time. Furthermore, it can be shown that the final result is independent of ' l ' and hence any arbitrary value (eg. $l = 1$) can be used.

4.3 HEAT TRANSFER ACROSS THE MICROLAYER

As discussed in chapter 2, a number of workers have shown that a larger part of the evaporation into a rapidly growing bubble takes place from the base of the bubble across a thin microlayer. In order to evaluate this, a knowledge of the thickness of the microlayer and its variation with time and distance from the nucleation centre is necessary. Another point of importance is the variation of surface temperature under the bubble with time which depends on the relative thermal conductivity of the liquid and the wall. Methods are outlined in the following sections for three different cases of relative conductivity of the liquid and the wall. The first analysis is for the case where the thermal conductivity of the wall is very much higher than that of the liquid, giving rise to approximately constant wall temperatures. Low and intermediate relative conductivities of the wall are the other two cases discussed.

4.3.1 Microlayer Thickness

Cooper and Lloyd (35) have considered the flow of liquid around the bubble in the vicinity of the heating surface and using a simplified model have developed a relationship for δ_0 , the initial microlayer thickness at a certain point, of the form $\delta_0 \propto \sqrt{vt_g}$

where ν is the kinematic viscosity of the liquid and t_g is the time taken for the bubble base perimeter to grow upto that point. Their theory predicts a value of 0.8 for the constant of the above proportionality while values from experimental observations vary between 0.5 and 1.0. At about the same time, Olander and Watts (61) using a very similar model but using spherical coordinates with axial symmetry developed a result of the form

$$\delta_o(R) = \frac{\sqrt{\pi\nu}}{4} \frac{R}{A}$$

where A is the growth constant in $R = At^{\frac{1}{2}}$.

But $\frac{R}{A} = \sqrt{t_g}$, Hence

$$\delta_o(R) = \frac{\sqrt{\pi}}{4} \sqrt{\nu t_g}$$

The equivalent expression obtained by Cooper and Lloyd (35) was

$$\delta_o(R) = \frac{\sqrt{\pi}}{2} \sqrt{\nu t_g}$$

Cooper and Lloyd corrected their expression for the effects of the motion of the bubble base perimeter before the newly formed microlayer comes to rest and gave

$$\delta_o(R) = \frac{\pi^2}{\pi^2+1} \frac{\sqrt{\pi}}{2} \sqrt{\nu t_g}$$

$$\text{ie. } \delta_o(R) = 0.8 \sqrt{\nu t_g}$$

Both the above analyses were based on parabolic growth of the

bubble, $R = At^n$ with $n = \frac{1}{2}$. It was stated by Cooper and Lloyd (35) that the argument was generalised for other values of 'n' by Lloyd (64). As presented by Cooper and Vijuk (33) these generalised values are given by $C_2 = \sqrt{(3n)/ (3n+1)}$ for use in $\delta_o = C_2 \sqrt{vt_g}$. The variation of C_2 with 'n' is indicated in the following table.

n	C_2
0.333	1.128
0.500	0.885
0.667	0.705
0.833	0.665
1.000	0.601

Table 4.2

The above table shows that substantial changes in 'n' lead to comparatively small changes in C_2 , and so $C_2 = 0.8$ is accepted in this work.

$$\therefore \delta_o(R) = 0.8 \sqrt{vt_g} \quad 4.29$$

4.3.2 Microlayer Evaporation in a Poorly Conducting Liquid on a Highly Conducting Wall

In most boiling situations the wall is of a highly conducting material, with an organic or inorganic liquid of relatively low conductivity as the boiling liquid. In such cases the drop in wall temperature, expected due to the vaporization of the microlayer, will be very small particularly if the heater is relatively thick as the surface is being replenished quickly with heat from the depths of the wall. Thus the variation of wall temperature is small in relation

to the liquid superheat. If T_{wo} represents the initial wall temperature and T_w the varying wall temperature at any other time, then the above assumption can be represented by

$$T_{wo} - T_s \gg T_{wo} - T_w$$

The heat transfer into the bubble across the microlayer is dependent on the radius up to which the microlayer has dried out due to evaporation. It is further assumed in this treatment, that thermal contact between the wall and vapour is very poor and as a result any heat transfer directly taking place from the wall to the vapour is neglected in comparison with the evaporative heat flux. The thermal capacity of the microlayer is neglected and heat transfer is assumed to take place by pure conduction.

Then, if ' δ ' denotes the thickness of the microlayer at a certain point at a certain time, the heat flux at that point is given by

$$k_l \cdot \frac{(T_w - T_s)}{\delta}$$

and evaporation per unit area is given by

$$-\delta(\delta) = \frac{k_l (T_w - T_s) \cdot \delta t}{\rho_l L \delta} \quad 4.30$$

Integration of the above gives the variation of ' δ ' at a point with time

$$\frac{\delta^2}{2} = - \frac{k_l (T_w - T_s)}{\rho_l L} t + \text{const.}$$

$$\text{At } t = t_g, \quad \delta = \delta_o (= 0.8 \sqrt{v t_g})$$

$$\therefore \frac{\delta^2}{2} = \frac{\delta_o^2}{2} - \frac{k_l (T_w - T_s)}{\rho_l L} (t - t_g) \quad 4.31$$

To obtain an accurate evaluation of heat transfer across the microlayer during each time step, it is necessary to effect a numerical summation of $\delta(\delta)$'s given by equation 4.30 over the bubble base. This procedure, if carried out for each step of advance of bubble growth, is very time consuming. However, it is possible to develop an approximate equation to represent the microlayer profile with radius (measured along the surface) following Cooper (34). Equation 4.31 can be rewritten as

$$\frac{\delta^2}{2} = \frac{(0.8 \sqrt{v})^2}{2} t_g - \frac{k_l (T_w - T_s)}{\rho_l L} (t - t_g) \quad 4.31(a)$$

If a certain point is considered ($t_g = \text{constant}$), then the above equation gives the variation of ' δ ' at that point with time ' t '. However, by considering the values of ' δ ' at all point for a fixed time ' t ' (t_g varying), we see that ' δ ' is a function of t_g given by an equation of the form

$$\delta^2 = c t_g - d \quad 4.32$$

If the above equation can be transformed into a relationship between ' δ ' and ' r ', giving the microlayer profile, it will be very useful as a means of evaluating the total mass of liquid evaporated from the base during each time step. This is possible with the assumption $R \propto t^{\frac{1}{2}}$. In such a situation $t_g \propto r^2$ (t_g is the time taken for the base perimeter to grow to r). Using this in equation 4.32, we obtain

$$\delta^2 = ar^2 - b \quad 4.33$$

The values for ' a ' and ' b ' can be evaluated for each time step, with the knowledge of two points through which the microlayer

profile passes. It must be stressed here that the proportionality $R \propto t^{\frac{1}{2}}$ is used only in order to obtain an approximate idea of the microlayer profile. The constants in equation 4.33 are evaluated without knowledge of the growth constant in $R = At^{\frac{1}{2}}$. Bubble growth was computed for the same set of data, once by using the profile given by equation 4.33, and again by a numerical summation of $\delta(\delta)$'s from equation 4.30 at each time step. The results, discussed in section 5.5.5, justify the use of the simple computation as a first approximation.

The two known values of microlayer thicknesses employed in order to evaluate 'a' and 'b' in equation 4.33 are at $r = R_1$ upto where complete evaporation of the microlayer has taken place, and at $r = R_2$ where the microlayer has just formed and hence is known.

$$\text{At } r = R_1 \quad \delta = 0$$

$$\text{At } r = R_2 \quad \delta = \delta_2 = 0.8 \sqrt{vt_g}$$

Substitution of these conditions in equation 4.33 gives

$$\left. \begin{aligned} a &= \frac{\delta_2^2}{R_2^2 - R_1^2} \\ b &= \frac{\delta_2^2 \cdot R_1^2}{R_2^2 - R_1^2} \end{aligned} \right\} \quad 4.34$$

The amount of liquid corresponding to the area sectioned in Fig. 4.5(b) would have evaporated into the bubble from the base during a small time interval ' δt '. The microlayer would dry up to a radius R' which will be used as R_1 in the next interval of time. The evaluation of the total mass evaporated from the base is carried out in three steps.

(i) Evaluation of R' :

R' is given by the point where the evaporation ' $-\delta(\delta)$ ' is equal to the thickness of the microlayer ' δ '. Thus from equations 4.30 and 4.33, when $-\delta(\delta) = \delta$, we have

$$aR'^2 - b = \frac{k_l (T_w - T_s)}{\rho_l L} \cdot \delta t$$

$$R' = \sqrt{\left[\frac{k_l (T_w - T_s)}{\rho_l L} \delta t + b \right] / a} \quad 4.35$$

(ii) Mass of vapour evaporated from R_1 to R' (δm_{m1}):

$$\delta m_{m1} = \rho_l \int_{R_1}^{R'} \delta \cdot 2\pi r \cdot dr$$

Substituting for δ from equation 4.33

$$\delta m_{m1} = \frac{2\pi\rho_l}{3a} \left\{ (aR'^2 - b)^{3/2} - (aR_1^2 - b)^{3/2} \right\}$$

But $aR_1^2 - b = 0$, hence

$$\delta m_{m1} = \frac{2\pi\rho_l}{3a} (aR'^2 - b)^{3/2} \quad 4.36$$

(iii) Mass of vapour evaporated from R' to R_2 (δm_{m2}):

$$\delta m_{m2} = -2\pi\rho_l \int_{R'}^{R_2} r \cdot \delta(\delta) \cdot dr$$

Substituting from equation 4.30 for $\delta(\delta)$ and from 4.33

for δ

$$\delta m_{m2} = \frac{2\pi\rho_l}{a} \frac{k_l (T_w - T_s)}{\rho_l L} \cdot \delta t \left[\sqrt{aR_2^2 - b} - \sqrt{aR'^2 - b} \right] \quad 4.37$$

The total mass of vapour evaporated from the base during the time interval ' δt ' is given by the sum of the equations 4.36 and 4.37.

4.3.3 Microlayer Evaporation in a Highly Conducting Liquid on a Poorly Conducting Wall

Practical situations where a liquid of relatively high conductivity is boiled on a poorly conducting wall are not very common. In such situations when a liquid microlayer is evaporated the wall temperature at the surface drops very rapidly to a value very near that of the liquid. During the very short initial stage when this is occurring the heat transfer rate is governed by the transient heat conduction equation in the wall. After this stage and for most part of the bubble growth period, the temperature at a point on the surface will be slightly above that of the liquid but, due to the relatively high conductivity of the liquid, can maintain a heat flux approximately equal to the average heat flux. Similar behaviour of wall temperature can be expected even when boiling on a thin wall of high relative conductivity where the heat capacity of the heater material is small. As a result of the penetration depth exceeding the wall thickness in such situations, the wall temperature can rapidly drop to values very near the liquid temperature.

In this section the evaporation from the microlayer is evaluated with the assumption that the heat flux is constant. Hence the rate of evaporation is independent of the microlayer thickness. However, it is necessary to follow a procedure very similar to that described

in the previous section, because of the need to keep track of the dried out area of the microlayer.

Assuming the heat flux to be a constant equal to ' q_{av} ', the evaporation of the microlayer per unit area during each time step is given by

$$-\delta(\delta) = \frac{q_{av}}{\rho_l L} \cdot \delta t \quad 4.38$$

Integration gives

$$\delta = - \frac{q_{av}}{\rho_l L} \cdot t + C$$

$$\text{At } t = t_g, \quad \delta = \delta_o$$

$$\therefore C = \delta_o + \frac{q_{av}}{\rho_l L} t_g$$

$$\text{Using } \delta_o = 0.8 \sqrt{v t_g}$$

$$\delta = 0.8 \sqrt{v t_g} - \frac{q_{av}}{\rho_l L} (t - t_g)$$

With the assumption that $t_g \propto r^2$

$$\delta = ar^2 + br - c \quad 4.39$$

$$\text{where } c = \frac{q_{av}}{\rho_l L} t$$

Equation 4.39 gives the approximate profile of the microlayer for this case. 'a' and 'b' can be evaluated using the known values of microlayer thicknesses; $\delta = 0$ at $r = R_1$ and $\delta = \delta_2 [= 0.8 \sqrt{v t_g}]$ at $r = R_2$. Substitution of these boundary values in equation 4.39 yields

$$\left. \begin{aligned} a &= \frac{\delta_2}{R_2(R_2 - R_1)} - \frac{c}{R_1 R_2} \\ b &= (c - a \cdot R_1^2) / R_1 \\ c &= \frac{q_{av}}{\rho_l L} t \end{aligned} \right\} \quad 4.40$$

Evaluation of mass evaporated from the microlayer during each time step is carried out in the same three steps as in section 4.3.2.

(i) Evaluation of R' :

At R' the evaporation $-\delta(\delta)'$ is equal to the microlayer thickness δ .

From equations 4.38 and 4.39

$$\frac{q_{av}}{\rho_\ell L} \cdot \delta t = aR'^2 + bR' - c$$

$$R' = \frac{-b \pm \sqrt{b^2 + 4a(c + q_{av} \cdot \delta t / \rho_\ell L)}}{2a} \quad 4.41$$

The value of R' which lies between R_1 and R_2 must be used.

(ii) Mass of vapour evaporated from $r = R_1$ to R' (δm_{m1}):

$$\delta m_{m1} = \rho_\ell \int_{R_1}^{R'} 2\pi r \cdot \delta \cdot dr$$

Substituting for δ from equation 4.39 and simplifying,

$$\delta m_{m1} = 2\pi\rho_\ell \left[\frac{a}{4} (R'^4 - R_1^4) + \frac{b}{3} (R'^3 - R_1^3) - \frac{c}{2} (R'^2 - R_1^2) \right] \quad 4.42$$

(iii) Mass of vapour evaporated from R' to R_2 (δm_{m2}):

$$\delta m_{m2} = -\rho_\ell \int_{R'}^{R_2} 2\pi r \cdot \delta(\delta) \cdot dr$$

Substituting for $\delta(\delta)$ from equation 4.38

$$\delta m_{m2} = \pi \rho_{\ell} \left(\frac{q_{av} \cdot \delta t}{\rho_{\ell} L} \right) (R_2^2 - R'^2) \quad 4.4.3$$

The total mass evaporated from the microlayer, during the time interval ' δt ' is given by the sum of equation 4.42 and 4.43

4.3.4 Microlayer Evaporation in the case of Intermediate Wall Conductivity

This is a more general case as compared to the two extreme cases discussed in the preceeding two sections 4.3.2. and 4.3.3. The computation becomes more complex due to the complicated nature of the calculation of the rate of microlayer evaporation. It is pointless in this case to attempt to develop expressions for the microlayer profile without developing similar relationships for the variation of the wall temperature along the surface, under the bubble. Attempts to obtain simple analytical relationships for the microlayer evaporation by employing two such profiles may prove to be difficult. As a result the total amount of microlayer evaporation must be evaluated by a numerical summation of the reduction of the microlayer thicknesses at different points on the bubble base, for each step of time. This represents a fairly large increase in the computational time, but it is still well within the practical limits of most computers. The method described for the evaluation of the temperature variation at a point on the surface is an approximate one, but the trend of the predicted variation agrees reasonably well with the observed variations.

Carslaw and Jaeger (37) (ref. pg 70, 2nd ed.) give a solution to the temperature variation at the surface of a semi-infinite solid of initial constant temperature with the radiation boundary condition

at the surface. It is a modification of this result that is outlined below.

Transient heat conduction equation in the solid, with 'x' measured from the surface under consideration is

$$\frac{\partial T}{\partial x} = \alpha_w \frac{\partial^2 T}{\partial x^2} \quad 4.43$$

Carslaw and Jaeger gave the solution of this equation for the following initial and boundary conditions

$$T = T_o \text{ at } t = 0 \text{ for all } x$$

$$-\frac{\partial T}{\partial x} + \beta T = 0 \text{ at } x = 0 \text{ for all } t.$$

The second condition represents a heat loss at the surface proportional to its temperature. 'β' is a constant which represents the ratio of heat transfer coefficient to the thermal conductivity of the solid (ie. $\beta = h/k_w$). The solution of equation 4.43 for the temperature variation in the block is

$$\frac{T}{T_o} = \text{erf} \left[\frac{x}{2\sqrt{\alpha_w t}} \right] + e^{\beta x + \beta^2 \alpha_w t} \text{erfc} \left[\frac{x}{2\sqrt{\alpha_w t}} + \beta \sqrt{\alpha_w t} \right] \quad 4.44$$

From the above expression the temperature at the surface (ie. $x = 0$) can be obtained as

$$\frac{T_w}{T_o} = e^{\beta^2 \alpha_w t} \text{erfc} \left[\beta \sqrt{\alpha_w t} \right] \quad 4.45$$

Cooper and Vijuk adopted this solution to the case of microlayer evaporation into a vapour at a temperature T_s , on a surface of

initial temperature T_{wo} , and used

$$(T_w' - T_s) = (T_{wo} - T_s) \exp(X^2) \operatorname{erfc}(X) \quad 4.46$$

$$\text{where } X = \frac{k_l}{k_w} \frac{\sqrt{\alpha_w(t-t_g)}}{\delta_o}$$

$$\left[\text{from } \beta \sqrt{\alpha_w t}, \text{ where } \beta = h/k_w \right]$$

to obtain the temperature variation at a point on the surface with time. It should be noted that the initial temperature gradient in the solid has not been taken into account in the above derivation and as a result it is to be expected that the drop in surface temperature occurs somewhat faster than in the actual situation.

To account for this, the surface temperature obtained from equation 4.46 is incremented by an amount based on the initial temperature profile in the solid, assumed to be a straight line with a gradient corresponding to the average heat flux. Using an estimated temperature gradient at the surface a penetration depth ' λ ' is first calculated, assuming a straight line variation of temperature from the surface. The penetration depth represents the depth in the solid to which the variation of temperature is felt to an appreciable degree (see Fig 4.6). Carslaw and Jaeger's solution implies that the unaffected temperature at the depth ' λ ' is equal to the initial surface temperature ' T_{wo} '. A better estimate of this temperature is given by the initial temperature profile at a depth ' λ '. The difference in these two temperatures is applied as a correction to the surface temperature given by equation 4.46.

The uncorrected surface temperature is obtained from the Cooper and Vijuk (33) adaptation of the Carslaw and Jaeger solution.

$$T_w' - T_s = (T_{wo} - T_s) \exp(X^2) \operatorname{erfc}(X) \quad 4.46$$

$$\text{where } X = \frac{k_l}{k_w} \frac{\sqrt{\alpha_w(t - t_g)}}{\delta}$$

An estimate of the temperature gradient at the surface can be obtained using the known corrected surface temperature at the previous time step at that point.

$$\left. \frac{\partial T}{\partial x} \right|_{x=0} = \frac{k_l}{k_w} \frac{(T_w - T_s)}{\delta} \quad 4.47$$

The initial temperature profile in the solid is given by

$$T = T_{wo} + \frac{q_{av}}{k_w} \cdot x \quad 4.48$$

and hence the temperature correction to be applied is

$$\frac{q_{av}}{k_w} \cdot \lambda.$$

The penetration depth ' λ ' shown in Fig.4.6 is now obtained from

$$T_w' = T_{wo} - \left(\left. \frac{\partial T}{\partial x} \right|_{x=0} \right) \cdot \lambda \quad 4.49$$

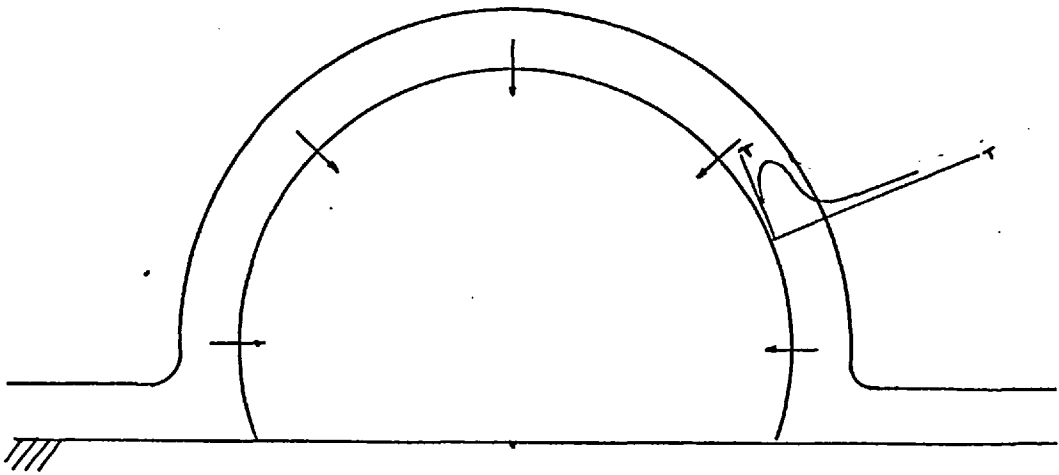
Substitution from equations 4.46 and 4.47 yields

$$\lambda = \frac{(T_{wo} - T_s) [1 - \exp(X^2) \operatorname{erfc}(X)]}{\left. \frac{\partial T}{\partial x} \right|_{x=0}} \quad 4.50$$

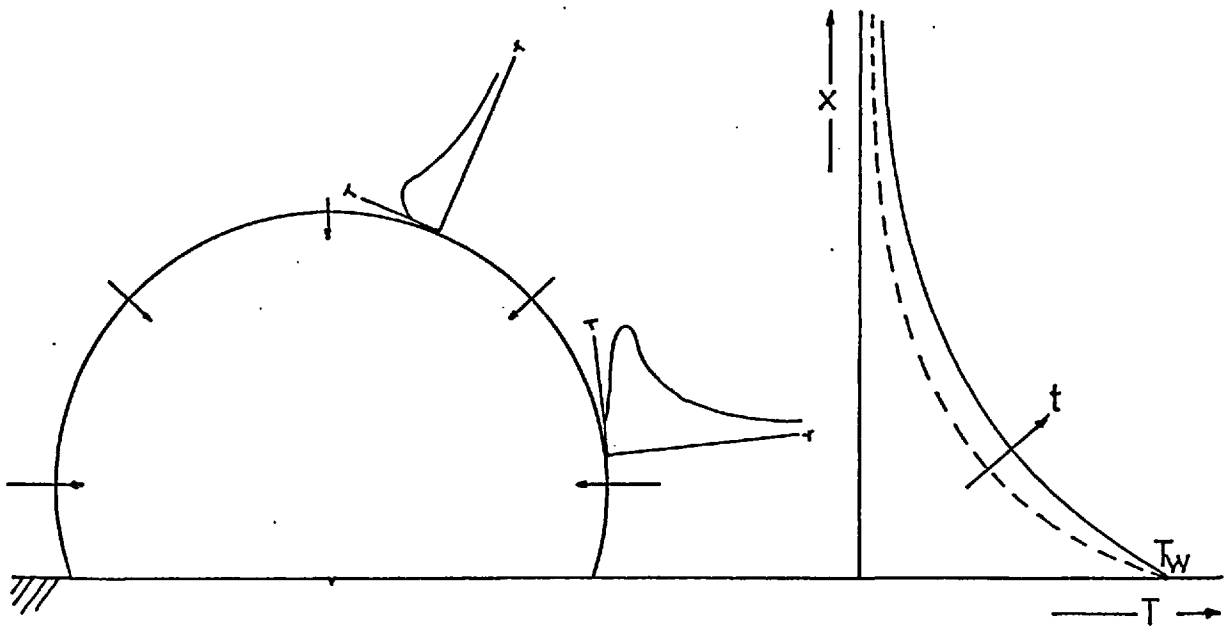
Use of this expression for ' λ ' in $\frac{q_{av}}{k_w} \cdot \lambda$ now yields the

necessary temperature correction to be applied to T_w' .

The corrected value of T_w is then used in equation 4.30 to obtain the evaporation of the microlayer ' $\delta(\delta)$ ' for a small time interval ' δt ', for a series of points under the base of the bubble, from which the total amount of evaporation from the microlayer during that time interval is calculated. It is obvious that a procedure of this nature, to evaluate the mass evaporated from the base for each time step by which the computation is being advanced, would constitute a major increase in the computational time, as compared to the two simpler cases discussed in the preceeding two sections.



(a) spherically symmetric temperature field



(b) spherically non-symmetric temperature field

Fig. 4.1 Different models of heat transfer

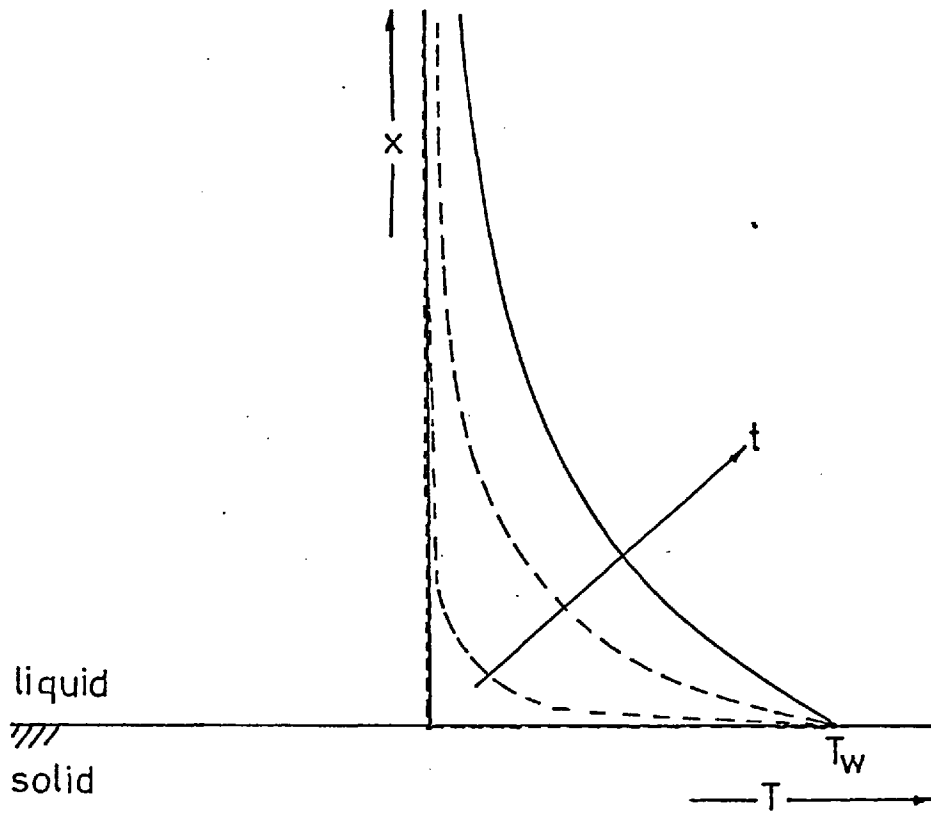


Fig.4·2 Development of the temperature profile in the liquid during the waiting time

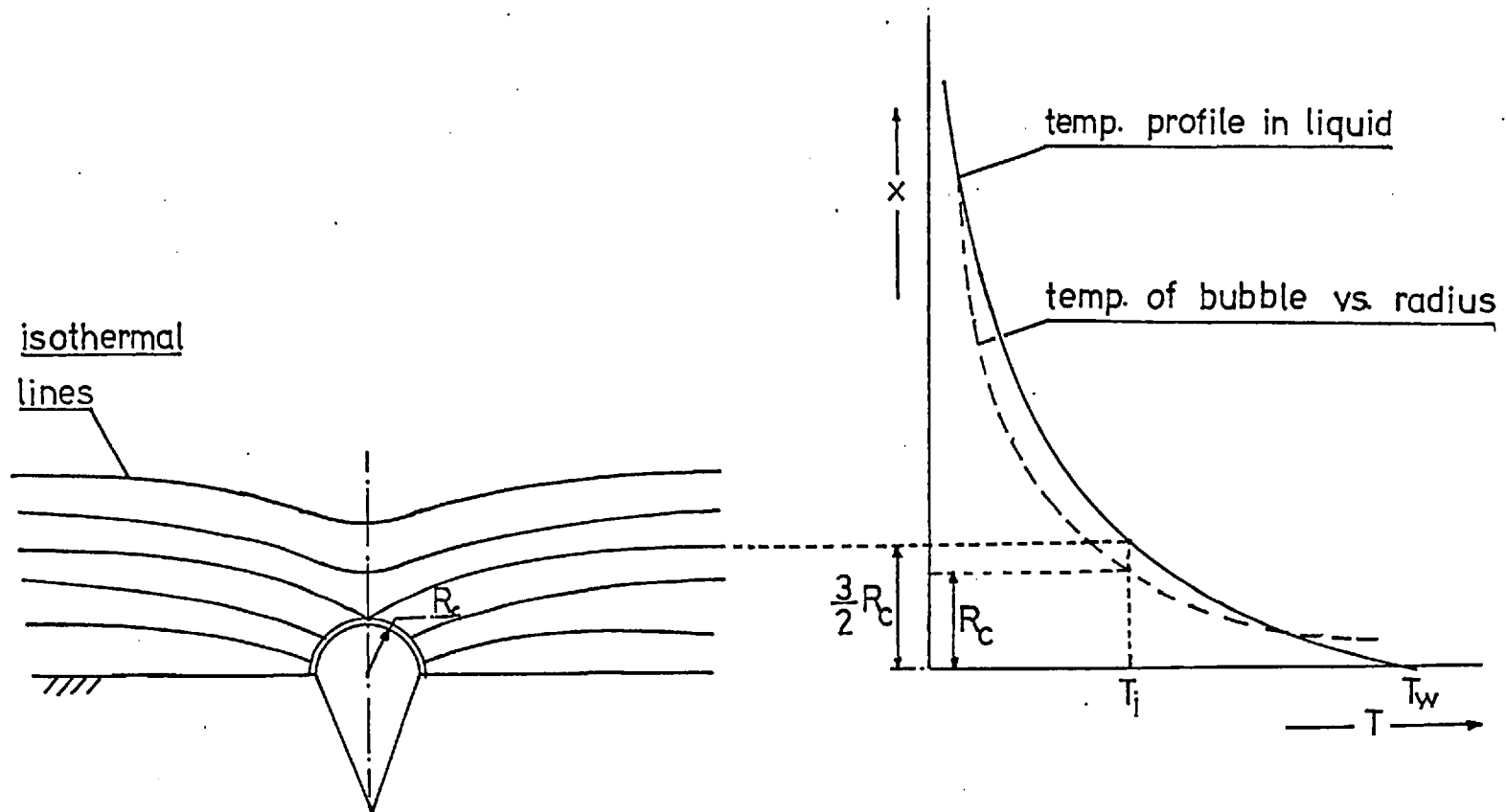
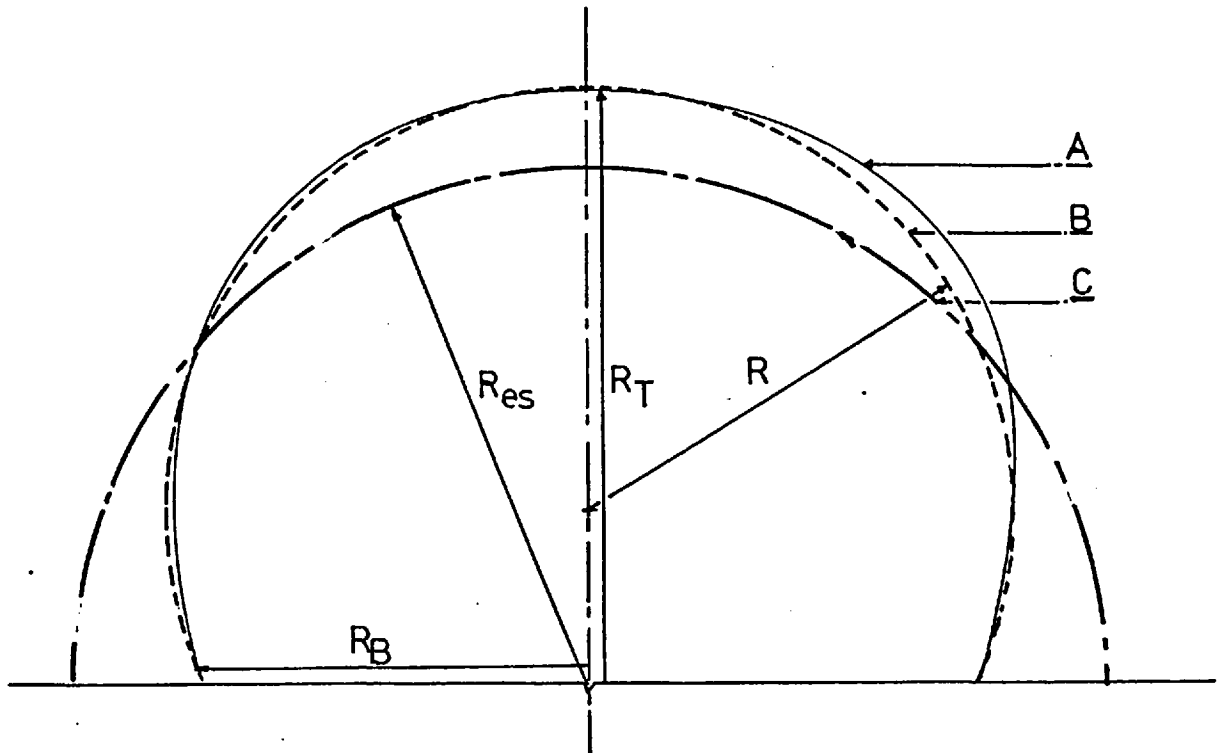
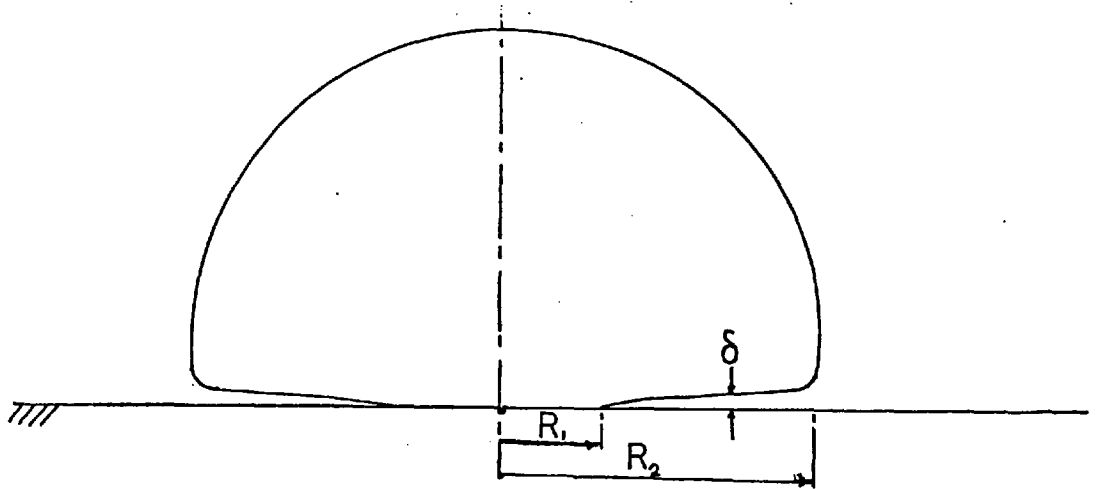


Fig. 4.3 Temperatures of fluid and bubble near a heating surface

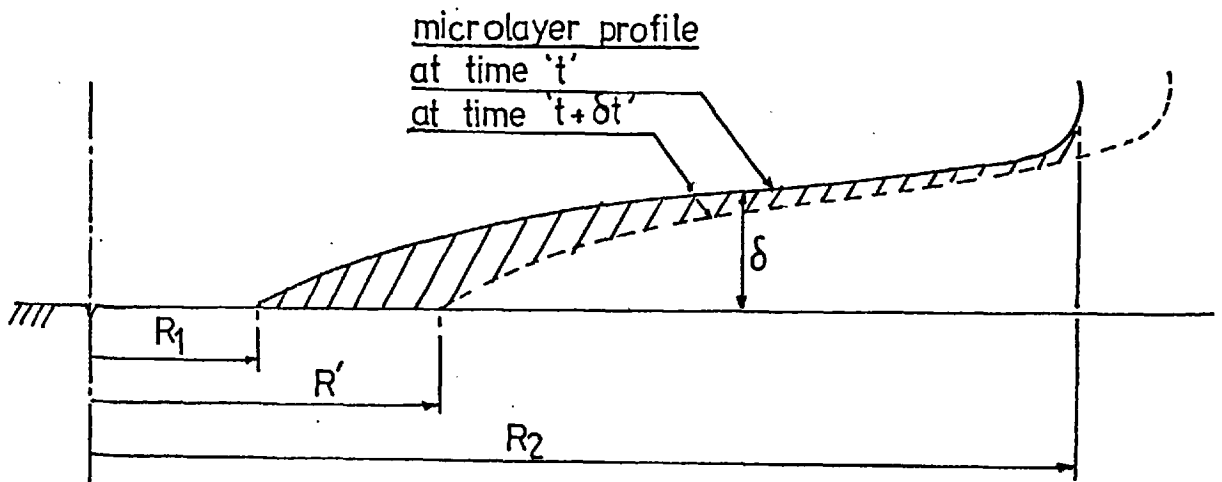


- A - actual bubble profile
- B - spherical segment
- C - hemispherical profile of
equal surface area

Fig. 4·4 Equivalent Hemispherical Profile



(a)



(b)

Fig. 4.5 Evaporation from the microlayer

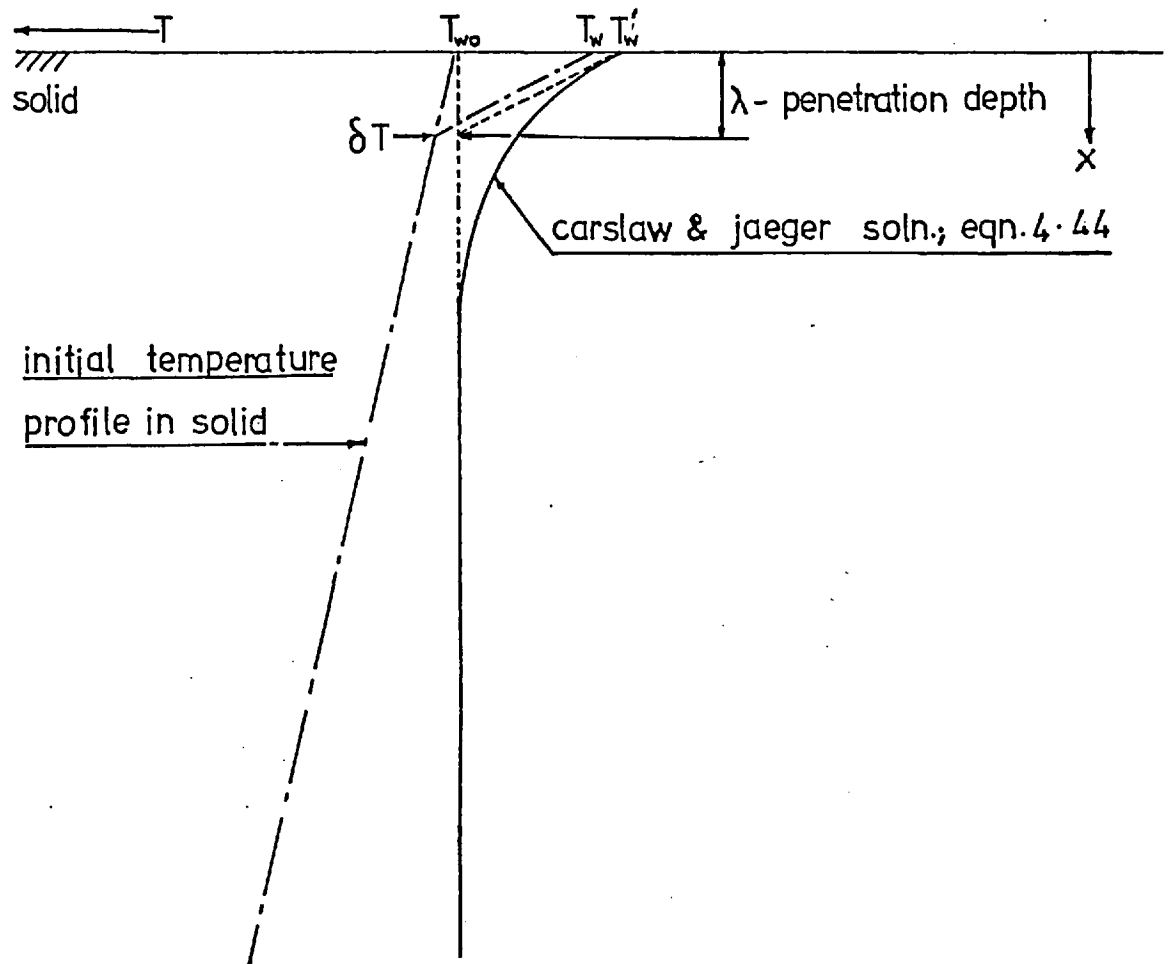


Fig. 4.6 Surface temperature variation under the microlayer

CHAPTER 5

RESULTS AND DISCUSSION

5.1 INTRODUCTION

A simplified model for rapid bubble growth on a heated surface under pool boiling conditions has been developed in the preceeding chapters. The model is suitable for obtaining computer solutions. This model takes into account both the evaporation from the microlayer and heat transfer across the curved interface. Usually neglected features like the liquid inertia force, pressure change in bulk due to gravity and the viscous drag force near the surface have also been taken into account. The restriction of a shape assumption for the bubble profile has been relaxed by the individual consideration of growth at the top of the curved interface and at the base perimeter. An approximate method for evaluating the temperature variation under the bubble has been developed and used in boiling situations where the heater material is of a relatively low conductivity. Departure times and radii can be predicted from the same growth computation without recourse to an overall force balance on the bubble.

The results obtained using this method are presented and discussed in this chapter.

5.2. COMPARISON OF BUBBLE GROWTH WITH EXPERIMENTAL OBSERVATIONS

The computations developed in the present work change in complexity depending on the method of evaluation of the evaporation

from the microlayer. The simplest computation is obtained in the case where a constant wall temperature and equation 4.33 for the microlayer profile are assumed. More computer time and storage are required when evaporation from the microlayer is found by a numerical summation of equation 4.30 over the bubble base, and still more when the wall temperature variation under the bubble is taken into account. Computer programmes for these three cases have been developed and are described in Appendix III.

Figs. 5.1 to 5.17 compare the predicted bubble growth rates with experimental data reported in the literature. In each of these figures the size of the bubble is represented by two radii values; one (curve 1) represents the radius of a spherical segment passing through the known positions of the bubble base perimeter and the uppermost point of the bubble, while curve 2, which is referred to as the equivalent radius, represents the radius of a sphere with the same volume as the bubble. Curve 3 is the base radius and curve 4, where given, represents the velocity of the bubble interface at the top of the curved interface. Table 5.1 describes the conditions under which the experimental data presented in Figs. 5.1 to 5.17 were obtained. Corresponding experimental data points are given in tabulated form in Appendix II, together with the corresponding physical and thermal properties.

The comparisons made here depend on the method of application of the present model and these vary from worker to worker. Hence they are discussed in separate sections.

5.2.1 Han and Griffith (25) Data - Bubbles A, B and C

The predicted values in Figs. 5.1 to 5.3 are obtained using

the simplest computation assuming that the wall temperature is constant and that the microlayer profile is given by equation 4.33. The constant wall temperature assumption is used here as the heater and liquid combination belongs to the category of a poorly conducting liquid on a highly conducting surface. This assumption is further justified as their boiling surface formed one end of a relatively long copper cylinder, giving rise to high heat capacities in the heater material.

The present computation needs the cavity size as an initial condition but this is not reported. However, given the waiting time and other data, it is possible to calculate a cavity radius on the basis of Han and Griffith's theory, as discussed in section 4.2.2. In order to obtain the initial vapour volume of the nucleus, the cavity is assumed to have a volume equal to that of a sphere with radius equal to the cavity radius.

Han and Griffith (25) have not defined the method of evaluation of their reported bubble radii values, and hence it is not known whether comparison should be made with curve 1 or curve 2. In any case, reasonable agreement is obtained for the first two bubbles A and B but for bubble C the predicted growth rates are considerably higher. The waiting times corresponding to Figs. 5.1 to 5.3 gradually decrease from 43.7 ms. to 24.5 ms. A common feature of the predicted growth rates for these bubbles is that the initial growth is higher than experimental growth, but during the latter half of bubble life the predicted values tend to drop below the observed values. This is in direct contrast to the predicted growth rates from the theory developed by Han and Griffith (25), where initial theoretical growth was slower than experimental and vice versa during the latter half. This may be as a direct result of the method of

evaluation of heat transfer across the curved interface. The present work assumes that the bubble profile penetrates the thermal layer into the bulk liquid, whereas Han and Griffith used a 1-D model with the assumption that the bubble profile carries the thermal layer with it.

5.2.2. Hospeti and Mesler (52) data - Bubbles D and E

Hospeti and Mesler (52) in addition to the standard observations of boiling conditions, recorded the waiting time and the initial surface temperature for each bubble. This initial surface temperature was different from the overall average surface temperature, which was also presented. They measured the surface temperature at a point over which the bubble perimeter passed, by a fast response thermocouple. In using the data presented by these authors, there is a problem in deciding the wall temperature to be used. They obtained a value for the initial surface temperature by the addition of the measured maximum surface temperature drop under the bubble to the saturation temperature, thus assuming that the minimum surface temperature was equal to the saturation temperature in the bulk of the liquid. It is noticeable that the initial surface temperature obtained in this way by these authors for certain bubbles was above the temperature averaged over the whole surface. This situation can occur only if complete dryout of the microlayer took place at the point of temperature measurement. In all the temperature-time data presented by these workers the recovery of temperature after microlayer evaporation never reached values higher than the average surface temperature. Consequently, it is difficult to see how they arrived at higher initial wall temperatures. In any case, the dry patch expansion under these conditions is very small, thus making it more correct to use the average wall temperature as the effective wall temperature.

The cavity radii are deduced from the waiting times as for Han and Griffith's data. Hospeti and Mesler (52) have presented their data of bubble size in the form of an equivalent radius equal to the radius of a sphere of the same volume. Therefore the experimental values must be compared with curve 2 of Figs. 5.4 and 5.5. Since bubble growth data were available only for small time periods smaller time steps (5 and 7 μ s.) were used in the analysis of these two bubbles. Bubble D was defined as an oblate bubble while bubble E was a hemispherical bubble. Very good agreement is obtained for both these cases. However, all attempts to apply the present method to a much slower growing spherical bubble, presented by the same authors, failed. The waiting times in the case of bubbles D and E were of an order of 70-80 ms. while for the spherical bubble it was 0.4 ms. Clearly a bubble which begins to grow so shortly after the departure of a previous bubble is growing into the very turbulent wake region of the departing bubble and initial conditions assumed in the theory are unlikely to apply.

Hospeti and Mesler (52) estimated that the maximum errors involved in their observations of bubble radii and bubble volumes were $\pm 5\%$ and $\pm 15\%$ respectively.

5.2.3 Cole and Shulman (22) Data - Bubbles F to L

Cole and Shulman (22) presented a large collection of growth and departure data, at relatively high Jakob numbers. Lack of availability of the corresponding waiting times (or cavity radii) is a serious drawback in the application of the present method to these data. In their paper it can be clearly seen that for each boiling condition, several bubbles have been recorded with varying growth

rates. These could have been bubbles originating from different cavities or from the same cavity with different waiting times. One bubble which was thought to be representative of the several was chosen for comparison.

Data sets for the growth of seven bubbles in different liquids at different pressures from the Cole and Shulman (22) paper are compared with the predictions from the present model in Figs. 5.6 to 5.12. The major problem in using these data was in the choice of the cavity radius. In this situation, it was necessary to make an arbitrary choice of the values for the cavity radii, so that the predicted growth curves are of a reasonably correct shape. This choice of the value of the cavity radius is based on the discussion outlined in section 3.4. The direct dependence of the predicted growth curves on the assumed cavity radius is discussed in section 5.5.1 where it is shown that the growth curves are not particularly sensitive to the cavity size assumed. The waiting times are calculated using the assumed cavity radii and a hemispherical vapour nucleus, by the method given in section 4.2.2.

The growth curves shown in Figs 5.6, 5.7, 5.8, 5.9 and 5.12 for bubbles F, G, H, I and L are obtained assuming constant wall temperature but without assuming a particular microlayer profile. This programme gives better agreement with experiment than the simpler programme using the expression given by equation 4.33 for the microlayer profile. The cavity radii for all the above bubbles are assumed to be 200 μm . The experimental data points should be compared with curve 2, as Cole and Shulman (22) presented their data in the form of equivalent radii. These authors also noted that the maximum error involved in the superheat measurement of the

experiments reported may be as high as $\pm 50\%$.

On the whole, the present method shows favourable agreement with these sets of data, demonstrating that the model is capable of making predictions for systems having different physical properties. Agreement with experimental observations was very good at relatively low Jakob numbers.

Figs. 5.10 and 5.11 show two water bubbles at low pressures, where the predicted growth rates are much higher than those experimentally obtained. Cole and Shulman (22) observed the same effect for these two bubbles with the relationship they proposed for bubble growth as given by the equation

$$R = \frac{N_{ja}}{2} \sqrt{\pi \alpha_l t}$$

$$\text{where } N_{ja} = \frac{\frac{1}{2} (T_w - T_s) C_{pl} \cdot \rho_l}{L \rho_v}$$

The Cooper and Vijuk (33) expression, which takes into account the inertia effects of the liquid to a certain extent, also predicts higher values for these two bubbles as shown in table 5.2. Assuming a smaller cavity size, in order to reduce the waiting time for these two bubbles, made little improvement. Variation of waiting time in this case did not seem to have an effect as marked as in the other cases. It must be noted that the Jakob numbers for these two cases were 192.7 and 298.6; two of the highest out of Cole and Shulman (22) data sets. The growth curves presented in Figs. 5.10 and 5.11 correspond to a cavity radius of 50 μm ., but to a cavity volume much larger than the volume of a sphere of equal radius as the use of a smaller volume results in the failure of the computation to start

growth. This is possibly due to the development of extremely high interfacial velocities in the first few steps of the computation as discussed in section 5.5.1.

5.2.4 Stewart and Cole (32) Data - Bubbles M to O

Stewart and Cole (32) have given growth data for a large number of bubbles at subatmospheric pressures with Jakob numbers of the order of 1000. Their data include equivalent diameters and the waiting time for each of the bubbles reported. The boiling liquid was slightly superheated in these experiments. The data presented by these workers are not all in line with the expected variation of growth rates with waiting times. A higher waiting time is normally expected to produce a bubble with a higher growth rate at the same boiling conditions. The present model also predicts this trend and it is clear that discrepancies will arise with experimental results which are contrary to this trend. Three sets of bubble growth data which appear to follow this expected variation are selected for comparison with the present model. The cavity radii are calculated from the given waiting times. The results obtained are presented in Figs. 5.13 to 5.15. Overall agreement is not particularly good although very good agreement is obtained in Fig. 5.14. A marked peculiarity observed in these comparisons is the lack of dependence of the growth curves on the waiting time as shown in Figs. 5.13 and 5.14 where the boiling conditions are similar except for the waiting times. Under more normal boiling situations any change in waiting time is clearly reflected in the final growth curve. This could be due to two reasons; one being that at the very high Jakob numbers at which these experiments were carried out, the liquid inertia effects may have a more prominent effect on growth than the variation of heat transfer

due to changed waiting time, the other being that the high waiting times observed may not conform to the waiting time theory of Han and Griffith (25) used in the present work.

Stewart and Cole (32) pointed out that existing growth equations which do not take into account the effects of liquid inertia, predict extremely high growth rates under these boiling conditions. They found that, two existing models which take account of liquid inertia were in much closer agreement with their data; one is the microlayer evaporation model of Cooper and Vijuk (33) and the other by Mikic, Rohsenow and Griffith (30) which takes into account the heat transfer across only the curved interface. An examination of Fig. 5.14 reveals that, the interfacial velocity at the top of the curved interface remains at an appreciable value even after 40 ms. of growth under these conditions, unlike the case of boiling under atmospheric conditions at low Jakob numbers. This clarifies why the inertia effects are more important in similar situations and why the growth equations not taking into account this factor show order of magnitude discrepancies with the experimental values.

5.2.5 Merry (53) Data - Bubbles P and Q

The only data analysed for bubble growth situations with intermediate wall conductivity are from Merry's (53) thesis where he boiled subcooled water on glass at subatmospheric pressures. A major difficulty arises in the application of the present model to this data due to the fact that the bubbles were artificially initiated by the passage of a pulse current in Merry's experiments. This posed a problem as the present model needs an initial cavity size and a value for the waiting time to initiate the computation. This

initiation by pulse currents imparts an initial velocity to the bubble interface and to compensate this effect, it was obvious that comparatively high waiting times have to be assumed.

To get an idea of the order of magnitude of growth under these boiling conditions, the simplest programme, where constant wall temperature is assumed, is employed for a number of different cavity radii and corresponding waiting times. For bubble 'P' it is found that a cavity radius assumption of the order of 400 μm . would yield a waiting time of 662 ms. and produce initial growth rates approximately of the same order as the observed growth rates. Bubble 'Q' required waiting times around 200 ms. to get initial growth of the same order as the observed rates. Predicted growth at later stages, using this programme is much higher than the experimental growth rates, possibly due to a drop in the surface temperature due to the relatively low conductivity of the heater material. The same initial cavity radii and waiting times are then used in the programme where the variation of surface temperature is taken into account in the manner discussed in section 4.3.4. Figs. 5.16 and 5.17 show the growth curves obtained by this method compared with the experimental values. Agreement is better than when a constant wall temperature is assumed. Very good agreement would be fortuitous in this case as the initial conditions are arbitrarily chosen to represent non-existent values.

5.3 COMPARISON OF BUBBLE GROWTH WITH SOME OF THE EXISTING THEORIES

Most of the early theoretical attempts at predicting bubble growth have resulted in equations of the type

$$R = f(\text{physical properties, } \Delta T) \times t^{\frac{1}{2}}$$

Even though this kind of relationship had been developed for the uniform superheat case, most of the later models for boiling situations near a surface have used this with variations in the constant of proportionality. Amongst the large number of bubble growth relationships in this class, Cole and Shulman's (22) simple modification stands out because of its simplicity. Cooper and Vijuk's (33) analysis and Mikic, Rohsenow and Griffith's (30) analysis belong to another class of bubble growth theory taking into account the liquid inertia effects. Mikic et al. (30) considered only the heat transfer across the curved interface whereas Cooper and Vijuk (33) considered only the heat transfer across the microlayer. The final solution for bubble growth in both the above analysis must be obtained by an numerical method. But Cooper and Vijuk (33) were successful in producing an algebraic expression which approximated their solution. This seems to be the only simple growth expression available in the literature which takes into account the inertia of the liquid, even in an approximate manner. It must be noted that the expression obtained from Cooper and Vijuk (33) for comparison here does not take into account the heat transfer across the curved interface; they developed a different analysis to take this factor into account but this neglects the liquid inertia effects.

Cole and Shulman's (22) analysis and the Cooper and Vijuk (33) analysis are taken as representative of these two different classes of approach. The validity of the above expressions are limited to saturated boiling conditions. Predictions from the present theory are compared with these expressions for the bubbles D to L where boiling is under saturated conditions.

5.3.1 Comparison with Cole and Shulman (22) Expression

Modification by Cole and Shulman (22) of the standard growth equations for uniform temperature fields, to apply in non-uniform temperature field situations can be represented by

$$R = \frac{N_{ja}}{2} \sqrt{\pi \alpha_L t} \quad 5.1$$

The predicted values of radius against time from the present work, the Cole and Shulman (22) expression and the Cooper and Vijuk (33) expression are tabulated with the experimental values in table 5.2. An inspection of this table shows that the present method in general produces a better and a more consistent prediction than the Cole and Shulman (22) method. Cole and Shulman's expression when applied to Hospeti and Mesler (52) data (bubble E) produces very small bubble growth rates as shown in Fig. 5.18. Fig. 5.19 is for bubble G which was forwarded by Cole and Shulman (22) themselves. For the two bubbles J and K, where water was boiled at very low pressures, the present method and the Cole and Shulman (22) method both overestimate the growth considerably, as indeed do most conventional methods tested by Cole and Shulman. A comparison of the results presented here with the presentations by Cole and Shulman (22) shows that the present method gives better agreement over a wide range of variables than other conventional growth equations.

The present method has several advantages over those similar to Cole and Shulman's (22) method. The capability of the present method to take into account the subcooling of the liquid, inertia forces, and the evaporation from the microlayer are some of these. Relaxation of the assumption of a fixed shape for the bubble profile

is another major advantage in the present method. It is surprising that the class of methods similar to Cole and Shulman's manage to show agreement at least to a certain degree as this approach neglects the microlayer evaporation completely, which is now known to contribute a major part of the total heat transfer into rapidly growing bubbles.

5.3.2 Comparison with Cooper - Vijuk (33) Expression:

Cooper and Vijuk (33) gave the expression

$$R = \frac{1}{\frac{1}{t} \sqrt{\frac{3}{2} \frac{\rho_l}{\rho_v} \frac{T_s}{L(T_w - T_s)} + \frac{1}{M N_{ja} \sqrt{\alpha_l t}}} \quad 5.2$$

$$\text{where } M = 2/(0.8 \sqrt{v/\alpha_l})$$

as an approximation to their derivation of bubble growth, taking into account the liquid inertia effects. The radius predicted by this expression is the radius of a hemispherical bubble. Therefore values obtained by this method are multiplied by $(\frac{1}{2})^{1/3}$, before comparison with the equivalent radii obtained from the present method. Table 5.2 compares the present prediction with their prediction in relation to the experimental sets of data used.

An inspection of this table shows that for most sets of data the present method gives better predictions than the Cooper and Vijuk (33) expression given by equation 5.2. Figs. 5.20, 5.21, and 5.22 show some typical results in graphical form. The present method consistently predicts higher growth rates than the Cooper and Vijuk expression, not surprisingly as in deriving the expression given by equation 5.2, Cooper and Vijuk (33) did not take into

account the heat transfer across the curved interface. This method by Cooper and Vijuk (33) also assumes hemispherical bubble growth, a restriction not imposed in the present case. In addition the present method can also show the variation in growth due to waiting time changes. As a result, the present method represents a more comprehensive approach to bubble growth. It must also be noted that the present method produces departure values as a result of the growth computation.

5.4 BUBBLE DEPARTURE ESTIMATION

5.4.1 Prediction from Present Theory

Development of most of the existing theories for bubble departure have been based on the assumption that an overall force balance can be applied to a vapour bubble and that departure occurs when there is a net force acting on the bubble directed away from the surface. However, it has also been observed that towards departure, a rapidly growing bubble loses its hemispherical shape. With the approach used in the present model it is possible to investigate this shape variation, without considering an overall force balance on the bubble. The application of an overall force balance to a deformable liquid-vapour interface is open to question.

In the present computation the growth at the bubble base is considered independently from the growth at the top of the curved interface. It is possible with this model to observe the outward growth and contraction of the base perimeter. By supposing that departure takes place when the base perimeter recedes to a radius less than the cavity radius, it is possible to evaluate the characteristics of the bubble at departure.

Table 5.3 presents the departure characteristics evaluated in this way for a number of bubbles together with the experimental departure characteristics where available. Cole and Shulman (22) presented the departure diameters for their experiments in a separate paper (40) without the departure times. However, it is possible to extract representative values of departure times and diameters by a comparison of their two papers ((22) and (40)).

Departure sizes predicted by the present approach, without the use of any empirical constants, agree with the bubbles analysed here to within $\pm 16\%$. The agreement of predicted departure times show somewhat wider scatter. However, taking into account the experimental fact that there is a random variation of bubble growth at a particular site under the same boiling conditions, agreement of this nature can be considered to be very satisfactory.

5.4.2 Comparison with Conventional Departure Value Estimations

Cole and Shulman (40) have carried out an extensive analysis of prediction of bubble departure diameters at subatmospheric pressures where they have discussed the importance of departure velocities and accelerations in predicting departure values. They found that none of the existing correlations were satisfactory in departure value estimations, especially under subatmospheric conditions, and they developed an empirical correlation which modified the Fritz departure equation (balance of surface tension and buoyancy forces) by the inclusion of a term containing the departure velocity. Their expression was of a form

$$\frac{D_d}{D_F} = 1 + 0.0025 \left(\frac{dD}{dt} \right)^{3/2} \quad 5.3$$

where $D_F = 0.0208 \phi \sqrt{\frac{g_c \sigma}{g(\rho_l - \rho_v)}}$

$$\left[\begin{array}{l} \phi - \text{contact angle in degrees} \\ \frac{dD}{dt} - \text{interfacial velocity in mm./sec.} \end{array} \right]$$

Later attempts at predicting departure values have balanced a number of forces acting on the bubble as a whole at departure including inertia forces, thus taking into account the effect of departure velocity and acceleration. These were mostly limited to atmospheric conditions and have the disadvantage of being very difficult to apply as they require the evaluation of a number of complicated force terms, some involving empirical constants of a very arbitrary nature. The results from these depend to a large degree on the reliability of the growth equations used in order to obtain various velocity and acceleration terms necessary.

Cole and Shulman's (40) estimation of departure diameters were also not satisfactory at low pressures. They did not attempt to compare values for departure times. Departure times, in these situations, would have to be obtained subsequent to the departure diameter estimation by the use of a suitable growth equation. The errors shown in departure diameters will be magnified in the departure times by such a procedure, due to the decline in bubble growth rate as departure is approached. In view of this the departure time predictions made in the present work, shown in table 5.3, can be considered to be very favourable as compared to an estimation using

equation 5.3. The use of this equation also requires a knowledge of the velocity at departure; Cole and Shulman (40), in their work, obtained these from their experimental growth curves. Under normal circumstances, use of an approximate growth expression is necessary to find values for $\frac{dD}{dt}$ in terms of D. A complicated growth expression complicates equation 5.3 tremendously and reduces the applicability of the above equation.

5.5 SPECIAL FEATURES OF THE COMPUTATION AND RELATED ASPECTS OF BUBBLE GROWTH

5.5.1 Variation of Growth Rate with Cavity Size

Characteristics of the cavity from which a bubble nucleates is one of the necessary pieces of data in the present computation, but one which is seldom a measured variable in experimental work. In situations where the cavity size is not defined a cavity radius is assumed. The corresponding value for the cavity volume is taken as equal to that of a sphere of the same radius. The waiting time is obtained using the value of the cavity radius together with other data defining the boiling condition. Where the waiting times are known as experimental data the cavity radius is found from the waiting time..

An investigation of bubble growth and departure has been carried out for a number of bubbles to find the dependence of the final result on the assumed cavity radius. In all cases the experimental waiting time is used to evaluate the temperature field around the nucleus. In general, smaller cavity radii give rise to higher growth rates. The results of one analysis (bubble D - Hospeti and

Mesler (52) bubble OB1) are presented in Fig. 5.23. The effect on departure time is not as marked as in the case of growth. Table 5.4 gives the departure values for this bubble for five different values of cavity radii.

Even though the final result does not depend to too great a degree on the cavity radius used, in some cases a difficulty is encountered in starting the computation where the cavity radius used is obtained from a knowledge of the waiting time. A good example of this kind of situation is the Stewart and Cole (32) data at subatmospheric pressures, where the waiting times were high.

During the first few steps of the computation for these bubbles, the pressure within the bubble rises very rapidly giving rise to high interfacial velocities and accelerations. But the expansion so caused can rapidly relieve the pressure with the result that the high inertia forces can reverse the direction of movement of the interface. Unless the time steps in the calculation are very small indeed, the computation can become unstable in such circumstances. A convenient way of overcoming this problem is to assume a high cavity volume without increasing the cavity radius, which arrests the extremely large initial pressure rise.

Once the conditions are arrived at, which promotes the growth of the bubble without the above mentioned computational failure occurring, any further variation of cavity size has insignificant effect on the final growth result and the departure estimation. The computation has been repeated for the same bubble as above for a number of different cavity volumes, keeping the cavity radius constant, and the growth results are presented in Fig. 5.24. The results show that

higher cavity volumes reduce overall growth but the variation is so small as to be insignificant. Similarly, the effect of cavity volume variation on departure is also very small.

As a conclusion, it can be said that the estimation of cavity radii from known waiting times is a viable method for use in the present computation for cases with waiting times in the medium value range (20 ~ 150 ms.). For very high waiting times higher cavity volumes may have to be employed, especially at very low pressures.

5.5.2 Effect of Waiting Time on Bubble Growth

The waiting time is a more readily available parameter than the cavity size. If not experimentally available, a reasonable estimation of waiting time is necessary to obtain meaningful bubble growth and departure results. The effect of waiting time is reflected both in the magnitude of growth and the shape of the bubble. As normally accepted, high waiting times yield hemispherical bubbles and high growth rates, low waiting times yield spherical bubbles and low growth rates. The initial wall temperature at bubble inception and the extent to which the thermal layer develops around the bubble are two factors which depend on the waiting time. The model developed by Han and Griffith (25) for the evaluation of waiting time which is used in the present work is based on an ideal situation where the wall temperature remains at a constant value and a quasi-static hemispherical vapour nucleus exists, surrounded by quiescent liquid.

An inspection of the literature reveals that the waiting times for spherical bubbles, in general differ from the waiting times for hemispherical bubbles at least by an order of magnitude. Hospeti

and Mesler (52) give clear examples of this where they have quoted waiting times of the order of 70-80 ms. for hemispherical bubbles while the waiting times for spherical bubbles are around 1 ms.. In attempting to predict bubble growth for these bubbles it is found that the computation of cavity size based on Han and Griffith's model fails. Furthermore, if a cavity size is assumed, the predicted growth rate is initially far higher than the experimental values. It is possible that for these very small waiting times, where the bubble is growing into the highly turbulent wake region of the previous bubble, initiation takes place due to pressure fluctuations in the wake region without the proper development of the thermal boundary layer. If this is the case, it is not surprising that the present method fails to predict growth satisfactorily under these conditions.

Another situation where the present method of evaluation of waiting time may not be valid is for exceedingly high waiting times. Stewart and Cole (32) observed in their boiling experiments under subatmospheric pressures, waiting times upto 1640 ms.. Most of their bubble growth data did not follow the expected variation with waiting time. An investigation of the waiting time variation with cavity radius for their boiling conditions, based on the Han and Griffith (25) theory of waiting time evaluation, reveals that there is a maximum possible waiting time which, for their boiling conditions, is around 200 ms.. Most of their observed waiting times are above this value. This suggests that either the Han and Griffith (25) theory of waiting time evaluation is not applicable to boiling at very low subatmospheric pressures (Stewart and Cole (32) data were obtained at a pressure of about $(1/20)$ th the atmospheric pressure) or some experimental error in the data at these pressures.

In general, it is safe to say that the present computation with the present method of evaluation of waiting time is valid in the medium range of waiting times where the waiting times are larger than the departure times.

5.5.3 Effect of the Time Step Size on Overall Growth

In solving the present equations by a numerical method (fourth-order Runge-Kutta method) the size of the time step chosen invariably has an effect on the final solution obtained. Generally, the smaller the time step, the more stable the solution is but the error is increased due to higher rounding off errors (a function of the number of arithmetic operations carried out). A large time step reduces the rounding off errors but increases the truncation error, which may be propagated and accumulated with advance of time and lead to instability. A fuller discussion of the error estimation is given in Appendix I, together with a description of the Runge-Kutta method.

Time steps of the order of 10 μ s. are very suitable for the present computations. The time step chosen must be such that it is possible to advance the solution for a considerable length of time without increasing the usage of the computer resources too much. With a time step of 10 μ s., it is possible to advance the bubble growth calculation using the simplest programme up to 40 msec. of growth in one of the lowest job categories available in the CDC 6400 computer at the Imperial College. Where the microlayer evaporation is evaluated without using an approximate expression for the microlayer profile, but the wall temperature variation is taken into account, higher job categories have to be employed. A typical job of this kind for 40 ms. of growth time requires about 75 sec. in a CDC 7600

computer. Fig 5.25 shows the computed growth rates for bubble E (Hospeti and Mesler (52) bubble H1) with different time steps. The departure characteristics of the bubble also vary with the time step used and is given in table 5.5.

5.5.4 Effect of the Viscous Drag Force Term at the Bubble Base

An expression has been developed in the present work for the viscous drag force near the surface which restricts the growth of the bubble base perimeter. This is given by equation 3.31. The importance of this drag force on the growth and departure of bubbles is considered here. This force term largely accounts for the differences in growth rates at the top of the curved interface and at the base perimeter. Fig.5.26 illustrates the effect of this force term on the growth of bubble D (Hospeti and Mesler (52) - Bubble O-B1) and its departure.

When this force term is neglected, the growth rate and particularly the departure time are very much higher than when this force term is used. This is not surprising as the introduction of this force reduces the growth at the bubble base and this in turn reduces the evaporation into the bubble across the microlayer, which constitutes a major portion of the total evaporation into rapidly growing bubbles. In the present model, the departure time is also related to growth at the bubble base. For the particular bubble examined here the departure time, if this force term is omitted, is 30 ms.. Even without this force term the base of the bubble has a greater deceleration than the top of the curved interface, as the liquid pressure at the base is higher than that at the top by an amount equivalent to the hydrostatic head corresponding to bubble height.

When the drag force is introduced the departure time is reduced from 30 ms. to 12 ms.

This drag force term has been developed using the equations of motion of the liquid in the viscous boundary layer. It is reasonable to suppose that the bubble interface must move from its initial position before this viscous boundary layer is established. Growth of several bubbles has been investigated varying the time from which this drag force term is used in the computation. If it is used from zero time, the bubble invariably collapses prematurely. Growth and departure increases rapidly with increase of the time at which this force term is introduced. Beyond a time of about 0.4 ms. this variation is much reduced. This point is chosen for convenience to represent the time at which the boundary layer is properly developed, and the viscous drag force term is used from that time onwards, until the base perimeter starts to recede. During the contraction of the base, when the bulk liquid from all directions is moving towards the base, a boundary layer of this form will not exist.

5.5.5 Microlayer and the Dry Patch

As outlined at the beginning of this chapter, three basic computer programmes were developed in the present work. The first two assume constant wall temperature and the simpler of these two assumes that the microlayer profile can be approximated by an algebraic expression given by

$$\delta^2 = ar^2 - b \quad 4.33$$

Fig. 5.27 compares the growth curves obtained using these two

programmes for bubble 'G' (Cole and Shulman (22) - Methanol). The curve of higher growth rate is obtained by the use of the second programme, in which the evaporation from the microlayer is evaluated by a numerical summation of equation 4.30
$$\left[\delta(\delta) = - \frac{k_l (T_w - T_s) \cdot \delta t}{\rho_l L \delta} \right],$$
 over the bubble base for each time step.

It can be seen that simplifying the computation by the assumption of the microlayer profile reduces the growth at later stages. At initial stages the growth is quite similar. This trend is observed in most of the growth computations made, but in a fairly large number the variation is not very marked. However, the departure point does seem to be affected quite considerably, as shown in Fig. 5.27.

Even though the computation is made much simpler by the use of this expression for the microlayer, the validity of this remains to be investigated. Using the more complex program, it is possible to obtain values of the microlayer thickness at different base radii in any one time step. Fig. 5.28, for bubble 'G', shows the variation of the microlayer profile with time. The expression ' $\delta^2 = ar^2 - b$ ', used for the microlayer profile for the simple case of constant wall temperature, has been developed assuming that the base radius grows outward according to the expression $R_b = At^{\frac{1}{2}}$. Obviously this is not the case and the disagreement obtained by the assumed profile with the actual microlayer profile is quite considerable. This agreement is most marked in the middle portion of the curve representing the microlayer profile. Most of the evaporation into the bubble from the microlayer takes place from the innermost third of the microlayer, where it is thinnest, as can be seen in Fig. 5.28. It has been possible to obtain reasonably accurate growth using the expression given by equation 4.33 despite the obvious disagreement, probably

because the evaporation from the portion where the disagreement is most marked, is not considerable compared to the total evaporation. This figure also makes it clear, why consistently lower growth rates are obtained when growth is advanced using equation 4.33 for the microlayer profile. The actual microlayer profile seems to follow a more complex curve. Approximation of a complicated expression, for the microlayer profile, does not serve the purpose unless it is possible to generate simple algebraic expressions for the microlayer evaporation during one time step. It does not seem likely that a simple curve could be fitted to the microlayer which would facilitate the convenient evaluation of microlayer evaporation. However, a reexamination of Figs. 5.27 and 5.28 makes it clear that the present expression used (equation 4.33) does yield reasonable growth curves, and as a first approximation of the present model, this approach could be satisfactorily used.

In the present computation for bubble 'G' and all the other computations, it was observed that the expansion of the dry area takes place relatively slowly. Complete dry out of the area under the bubble base never took place in any of the bubbles considered. For instance, for the present bubble, the dry patch expanded to a maximum value around 0.7 mm., whereas the bubble base grew to a maximum value of about 1.96 mm.. Expansion of the dry patch was of similar proportions for all the other bubbles as well. An analysis of the surface temperature rise in the dried out area, is important in burnout considerations. In the present analysis, where most of the bubbles grew on relatively highly conducting surfaces, there is no likelihood of a high surface temperature rise, due to transverse conduction. However, an analysis of the temperature rise in the dried out area could be very useful in situations where

the surface is poorly conducting.

No flow of liquid in the microlayer or a rolling away phenomenon of the triple interface has been considered in the present analysis.

5.5.6 Wall Temperature Variation

In practice the wall temperature under a bubble decreases due to large amounts of heat being extracted from the wall, for the evaporation from the microlayer, especially in cases where the heater conductivity is relatively low. For the bubbles presented by Merry (53) this was the case as he boiled subcooled water on a glass surface. The third programme developed was for such situations to take into account the variation of the wall temperature.

An approximate modification of a solution given by Carslaw and Jaeger (37) was outlined in section 4.3.4. to obtain the wall temperature variation. In the third programme, during each time step the wall temperature is calculated at certain points under the bubble base and the total microlayer evaporation is obtained by the numerical summation of equation 4.30 as before. It is clear that this constitutes a major increase in the time consumed by the computer programme. However, even in this case the computation to advance the growth calculation to 40 msec., by steps of 10 μ s. was still well within the limits of the normal job categories available at the University of London computers.

Figs. 5.29 and 5.30 show the effect of the wall temperature variation when applied to two of Merry's (53) bubbles. There is a clear difference between this growth rate and the growth rate obtained

with the constant wall temperature assumption. Very good agreement should not be expected in this case because of the artificial nature of the initial conditions assumed. The bubble interfaces would have had initial values for velocities due to the extraneous initiation of these bubbles, and this would increase their growth at the initial stages. The present model assumes a zero initial velocity and has predicted a lower growth than the experimental at the initial stages. Growth curves obtained by using the Cooper and Vijuk (33) adaptation for the wall temperature, given by equation 4.46, are also presented. This expression does not take into account the heat supplied to the surface from the heater and as such produces too sharp a temperature drop giving rise to low growth rates. The approach used here, to obtain the wall temperature variation is not theoretically very sound, but it does give a wall temperature variation of the correct trend. A typical temperature variation at a point under the microlayer is presented in Fig 5.31. The growth prediction using equation 4.46 (ie. Cooper and Vijuk (33) adaptation) for the surface temperature shows very good agreement with the shape of the growth curve, but gives lower growth and early departure. However, due to the arbitrary nature of the initial conditions used, a strict comparison cannot be made here.

The present approach in obtaining the wall temperature has an inherent inaccuracy in it due to the initial assumption of the Carslaw and Jaeger solution (Equation 4.44). This equation gives the correct wall temperature only if the rate of loss of heat from the surface is proportional to its temperature. In the present case the rate of heat transfer at the surface is $\frac{k_l (T_w - T_s)}{\delta}$; δ being the microlayer thickness, which varies with time. As such, the rate

of heat loss at the surface can be assumed to be proportional to the temperature difference only if ' δ ' remains reasonably constant. As observed in most of the present computations, ' δ ' does in fact remain nearly constant over the outer portion of the microlayer, in a similar manner to that given in Fig. 5.28.

Bubble Ref.	Fig No.	Boiling liquid/ Surface	Bulk Pressure $\times 10^{-5} \text{ N/m}^2$	Wall Temperature K	Saturation Temperature K	Bulk Temperature K	Heat Flux W/m^2	Cavity Radius $\times 10^{-6} \text{ m}$	Waiting Time $\times 10^{-3} \text{ s}$	Jakob Number	Source Ref.
A	5.1	Water/Gold Plated Copper	1.013	383.0	373.0	369.4	-	85.1	43.7	30.3	25
B	5.2	" "	1.013	383.0	373.0	369.4	-	66.4	27.5	30.3	25
C	5.3	" "	1.013	383.0	373.0	369.4	-	62.4	24.5	30.3	25
D	5.4	Water/Chromel	1.013	380.9	373.0	373.0	50,000	254.1	75.0	23.8	52
E	5.5	" "	1.013	387.4	373.0	373.0	-	305.7	87.0	43.7	52
F	5.6	Methanol/Zirconium	0.720	350.0	332.2	332.2	29,150	200.0	61.4*	41.5	22
G	5.7	" "	0.529	345.0	325.0	325.0	28,850	200.0	61.0*	59.9	22
H	5.8	" "	0.405	339.0	319.0	319.0	23,250	200.0	63.3*	74.9	22
I	5.9	" "	0.272	336.7	310.0	310.0	28,100	200.0	62.0*	140.4	22
J	5.10	Water/Zirconium	0.260	357.2	338.9	338.9	47,800	50.0	7.2*	192.7	22
K	5.11	" "	0.131	339.2	324.2	324.2	37,400	50.0	14.2*	298.6	22
L	5.12	Acetone/Zirconium	0.296	327.2	300.0	300.0	42,300	200.0	67.7*	126.4	22
M	5.13	Water/Copper	0.049	325.5	305.5	306.4	17,100	233.3	118.0	988.6	32
N	5.14	" "	0.049	325.5	305.5	306.4	17,100	341.7	184.0	988.6	32
O	5.15	" "	0.049	326.8	305.5	306.6	17,100	153.2	65.0	1052.9	32
P	5.16	Water/Glass	0.137	344.8	325.3	321.5	48,300	400.0	662.0*	371.6	53
Q	5.17	" "	0.275	360.6	340.2	339.0	48,300	400.0	200.0*	203.5	53

*calculated using Assumed R_c

TABLE 5.1 BOILING CONDITIONS OF BUBBLES ANALYSED

Time $\times 10^{-3}$ s.	RADIUS $\times 10^{-3}$ m.			
	Experimental	Present Prediction	Cole-Shulman Prediction	Cooper-Vijuk Prediction
BUBBLE D				
1.0	0.66	0.58	0.27	0.41
1.5	0.84	0.70	0.34	0.51
2.0	0.94	0.79	0.39	0.60
3.0	1.10	0.93	0.47	0.74
4.0	1.20	1.05	0.55	0.87
5.0	1.27	1.13	0.61	0.98
6.0	1.31	1.22	0.67	1.08
7.0	1.35	1.28	0.72	1.17
BUBBLE E				
1.0	0.90	0.97	0.50	0.72
1.5	1.14	1.18	0.62	0.91
2.0	1.34	1.35	0.71	1.07
3.0	1.63	1.63	0.87	1.34
4.0	1.87	1.84	1.01	1.56
5.0	2.04	2.02	1.12	1.76
6.0	2.20	2.18	1.23	1.94
7.0	2.33	2.32	1.33	2.11
BUBBLE F				
0.8	0.40	0.47	0.31	0.29
1.8	0.65	0.67	0.47	0.45
3.5	0.94	0.90	0.65	0.64
5.1	1.13	1.05	0.78	0.77
6.1	1.20	1.13	0.86	0.85
9.1	1.40	1.33	1.05	1.04
12.0	1.58	1.47	1.20	1.20
14.8	1.73	1.58	1.34	1.33

TABLE 5.2 BUBBLE GROWTH COMPARISON WITH THEORETICAL PREDICTIONS

Time $\times 10^{-3}$ s.	RADIUS $\times 10^{-3}$ m.			
	Experimental	Present Prediction	Cole-Shulman Prediction	Cooper-Vijuk Prediction
BUBBLE G				
0.8	0.50	0.61	0.46	0.40
1.4	0.73	0.78	0.59	0.53
3.3	1.13	1.18	0.93	0.86
6.0	1.40	1.53	1.25	1.18
7.2	1.60	1.65	1.37	1.29
11.3	1.90	1.99	1.71	1.63
14.5	2.11	2.19	1.94	1.86
17.5	2.30	2.35	2.13	2.04
BUBBLE H				
2.8	1.35	1.31	1.08	0.94
6.5	2.15	1.90	1.64	1.48
10.0	2.60	2.28	2.03	1.85
13.0	2.90	2.53	2.32	2.12
16.9	3.30	2.81	2.64	2.43
20.0	3.55	2.99	2.88	2.65
23.9	3.80	3.19	3.14	2.91
27.8	4.10	3.34	3.39	3.14
BUBBLE I				
1.5	1.25	1.52	1.50	1.12
4.3	2.35	2.56	2.55	2.05
8.3	3.05	3.46	3.54	2.94
12.3	3.65	4.12	4.31	3.63
15.5	4.15	4.54	4.83	4.11
21.3	4.75	5.18	5.67	4.86
26.0	5.30	5.59	6.26	5.40
32.0	5.85	6.03	6.94	6.02

TABLE 5.2 (Continued)

Time $\times 10^{-3}$ s.	RADIUS $\times 10^{-3}$ m.			
	Experimental	Present Prediction	Cole-Shulman Prediction	Cooper-Vijuk Prediction
BUBBLE J				
1.5	1.35	2.06	2.65	2.00
2.7	2.10	3.14	3.56	3.02
6.5	3.75	5.43	5.52	5.41
8.5	4.50	6.32	6.31	6.40
11.0	5.00	7.26	7.18	7.51
13.0	5.65	7.93	7.80	8.32
15.5	6.13	8.67	8.52	9.25
18.5	6.60	9.46	9.31	10.28
BUBBLE K				
1.5	2.25	1.96	4.05	1.87
2.5	3.10	3.00	5.22	2.80
5.0	4.50	5.06	7.39	4.72
7.0	5.50	6.39	8.74	6.02
10.5	6.75	8.35	10.71	7.99
13.0	7.25	9.52	11.92	9.25
14.5	7.85	10.18	12.59	9.95
18.5	8.75	11.75	14.22	11.69
BUBBLE L				
3.6	2.00	2.36	2.02	1.93
8.0	3.15	3.39	3.01	3.00
13.0	4.00	4.17	3.84	3.90
16.0	4.50	4.53	4.26	4.36
24.0	5.50	5.28	5.22	5.40
28.0	6.00	5.57	5.63	5.86
32.0	6.40	5.82	6.02	6.28
35.5	6.60	5.99	6.34	6.63

TABLE 5.2 (Continued)

Bubble Reference	Departure Time ($\times 10^{-3}$ s.)			Departure Radius ($\times 10^{-3}$ m.)		
	Experimental	Present Work	Percentage Agreement	Experimental	Present Work	Percentage Agreement
A	16.7	12.1	-27.5	1.59	1.41	-11.3
B	14.5	11.3	-22.0	1.43	1.29	- 9.8
C	16.6	11.1	-33.1	1.19	1.26	+ 5.9
D	-	12.4	-	-	1.47	-
E	-	26.2	-	-	3.42	-
F	14.8	19.2	+29.7	1.75	1.70	+ 2.9
G	18.0	26.7	+48.3	2.30	2.65	+15.2
H	29.0	31.2	+ 7.6	3.95	3.40	-13.9
I	33.0	45.6	+38.1	6.10	7.03	+15.2
L	35.0	42.4	+21.1	6.50	6.25	- 3.8

TABLE 5.3 COMPARISON OF DEPARTURE CHARACTERISTICS

Cavity Radius $\times 10^{-3}$ m.	Departure Time $\times 10^{-3}$ s.	Departure Radius $\times 10^{-3}$ m.
0.05	12.69	1.59
0.1	12.71	1.58
0.2	12.61	1.52
0.2541	12.40	1.47
0.3	12.12	1.42

TABLE 5.4 EFFECT OF CAVITY RADIUS ON DEPARTURE

Step Size $\times 10^{-5}$ s.	Departure Time $\times 10^{-3}$ s.	Departure Radius $\times 10^{-3}$ m.
0.8	26.6	3.48
1.0	27.6	3.59
1.2	28.8	3.68
1.5	30.0	3.80

TABLE 5.5 EFFECT OF STEP SIZE ON DEPARTURE

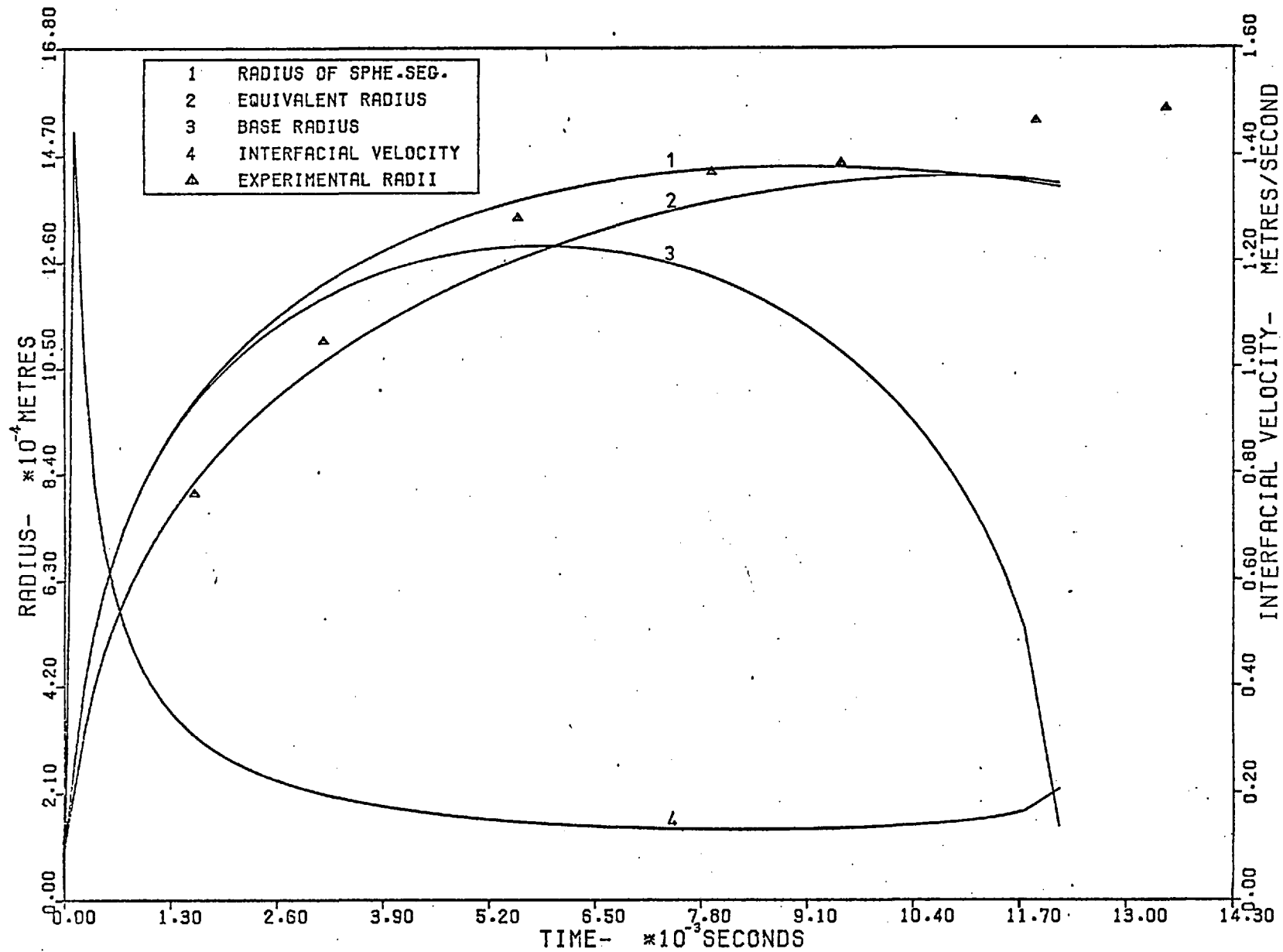


FIG. 5.1 BUBBLE GROWTH COMPARISON -REF/25/BUBBLE 2 (A)

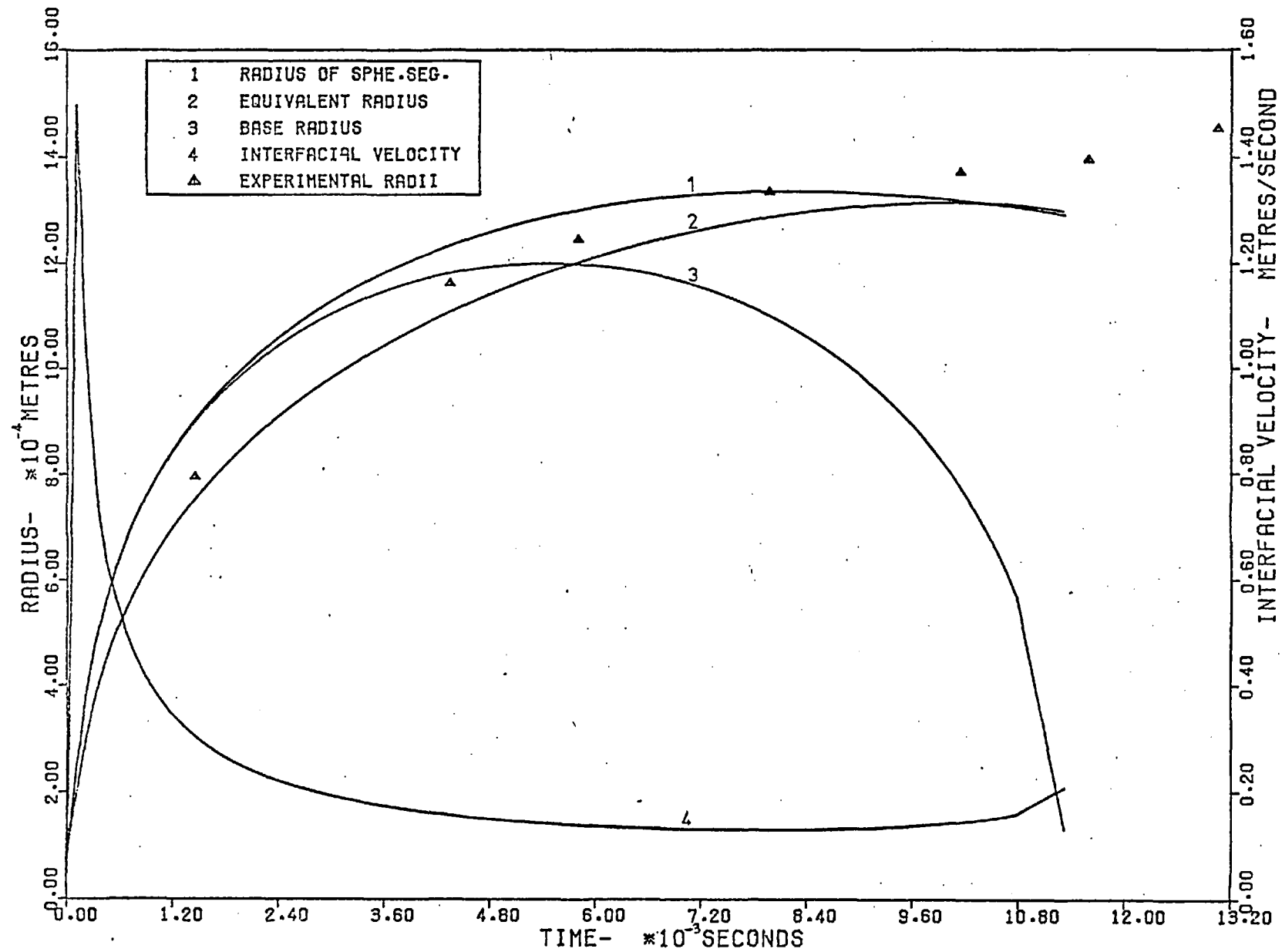


FIG.5.2 BUBBLE GROWTH COMPARISON -REF/25/BUBBLE 3 (B)

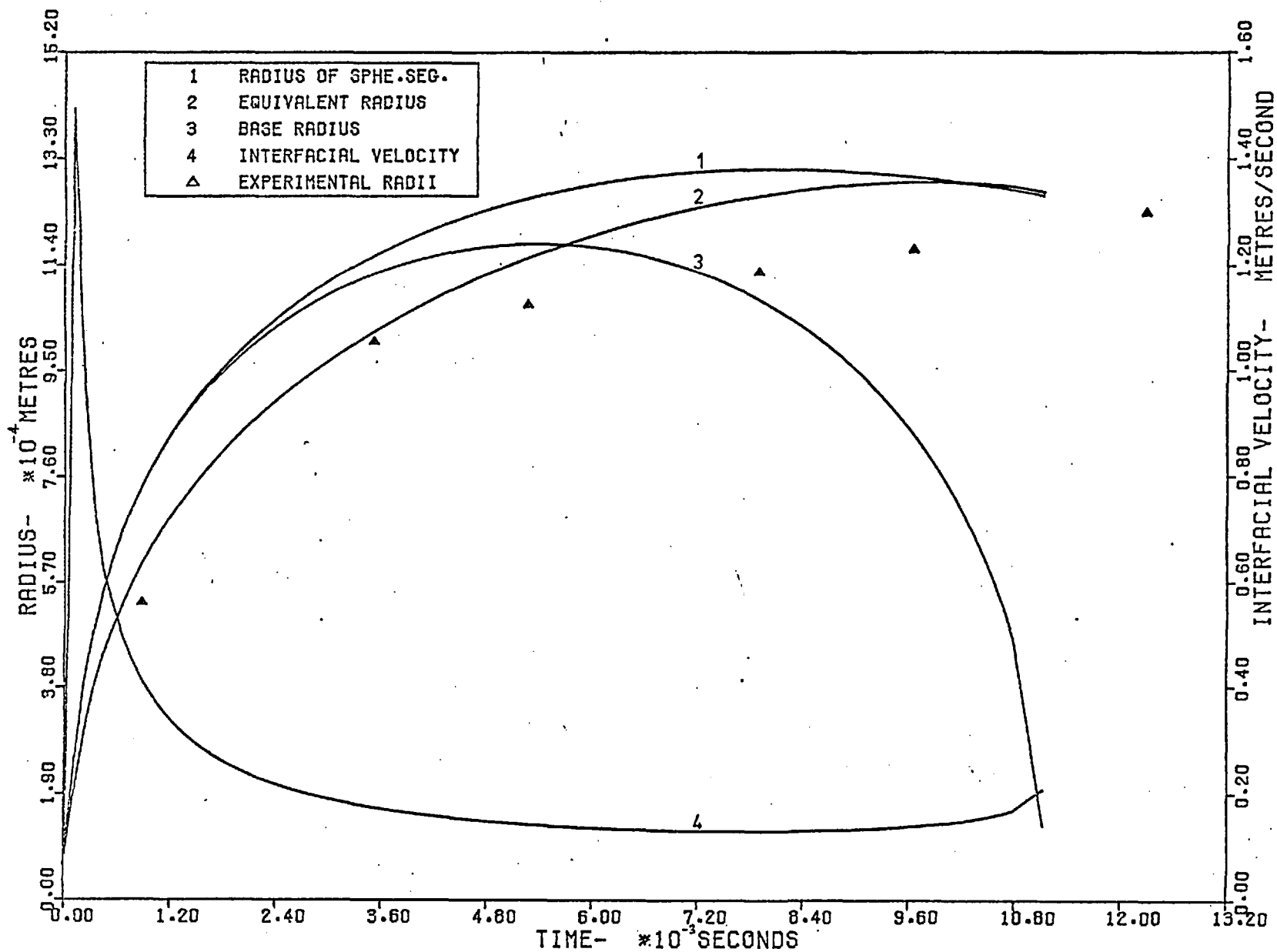


FIG. 5.3 BUBBLE GROWTH COMPARISON -REF/25/BUBBLE 1 (C)

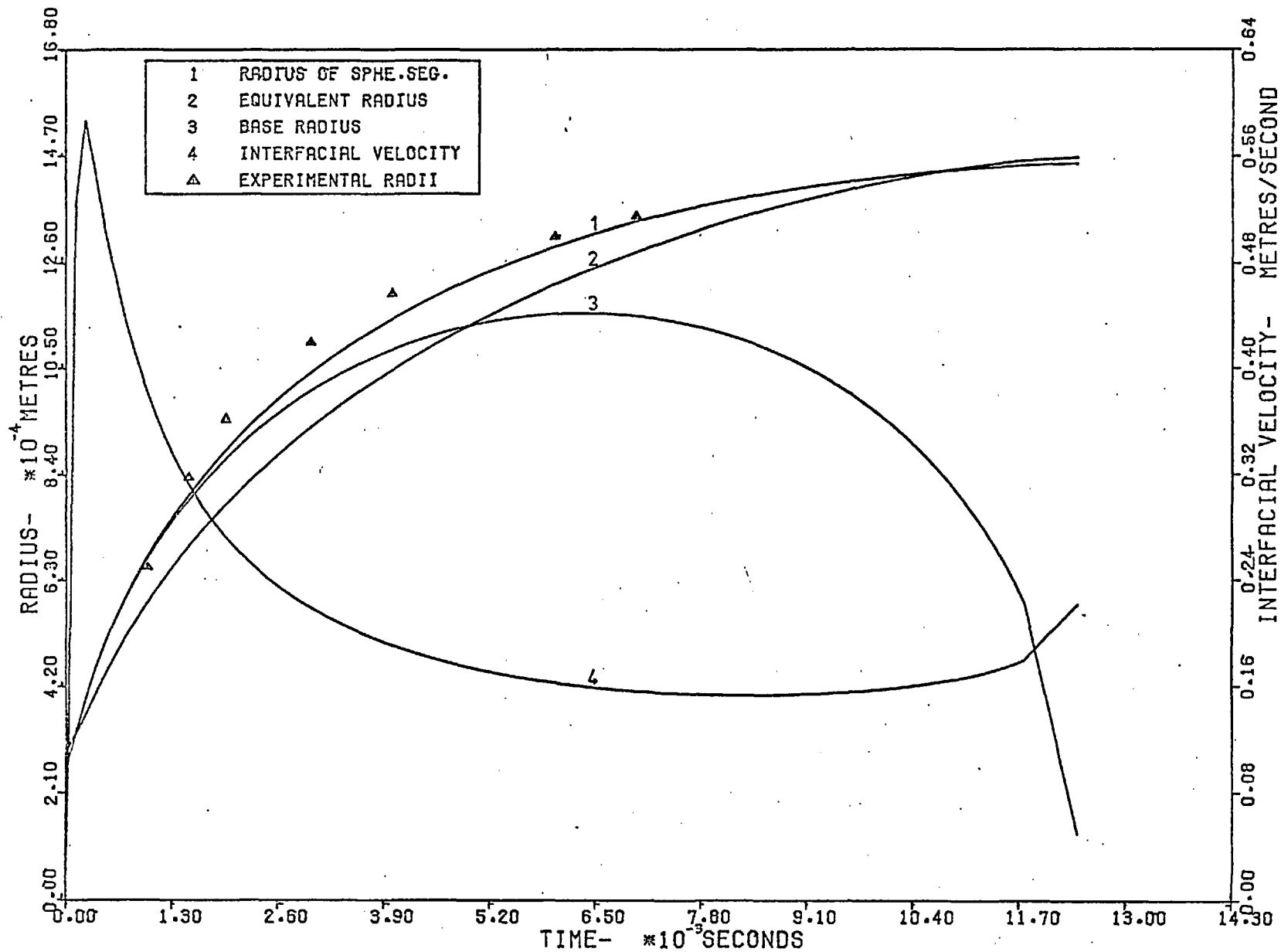


FIG. 5.4 BUBBLE GROWTH COMPARISON -REF/52/FIG.5 BUBBLE 0B1(D)

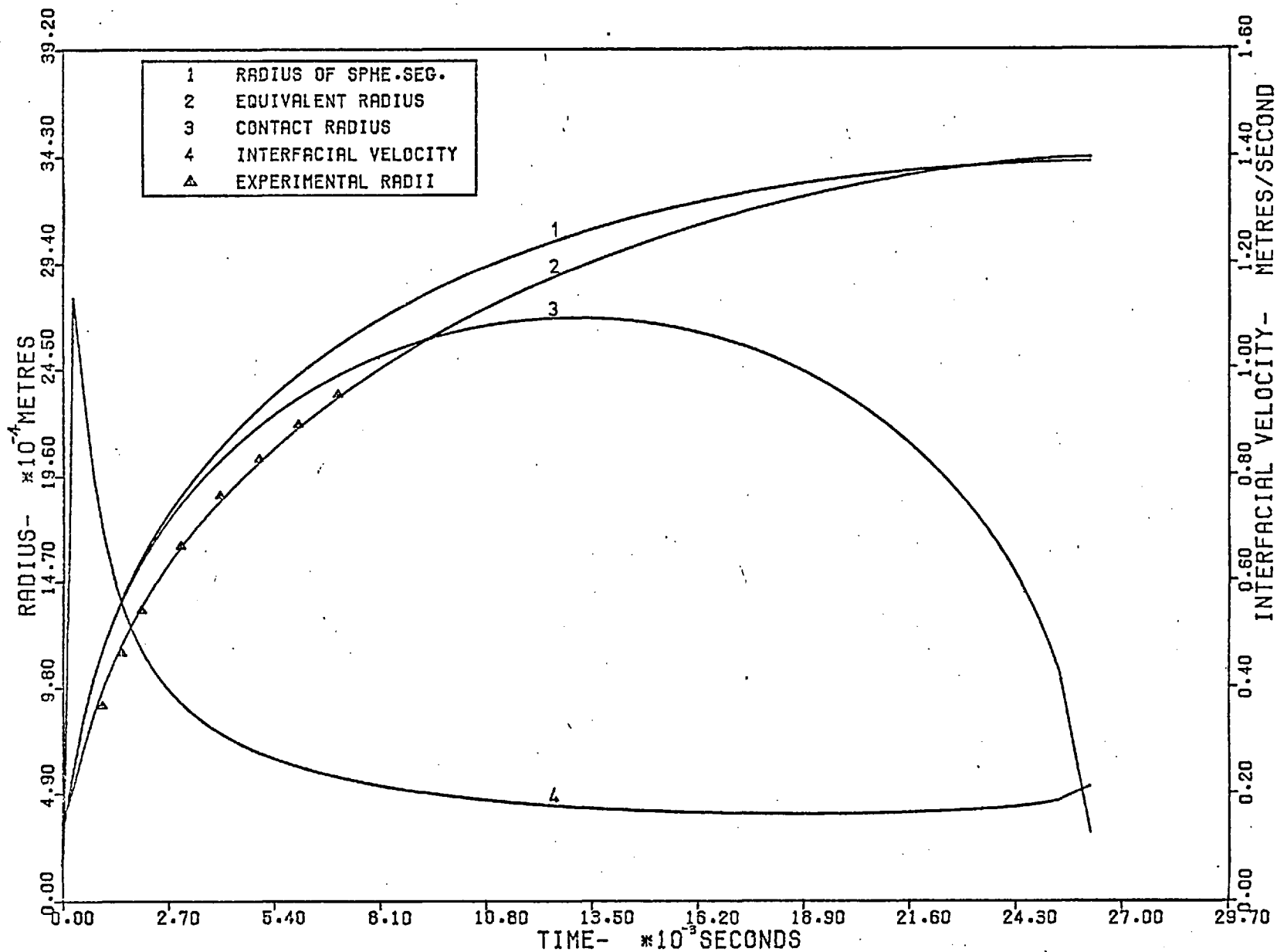


FIG.5.5 BUBBLE GROWTH COMPARISON -REF/52/FIG.6 BUBBLE H1 (E)

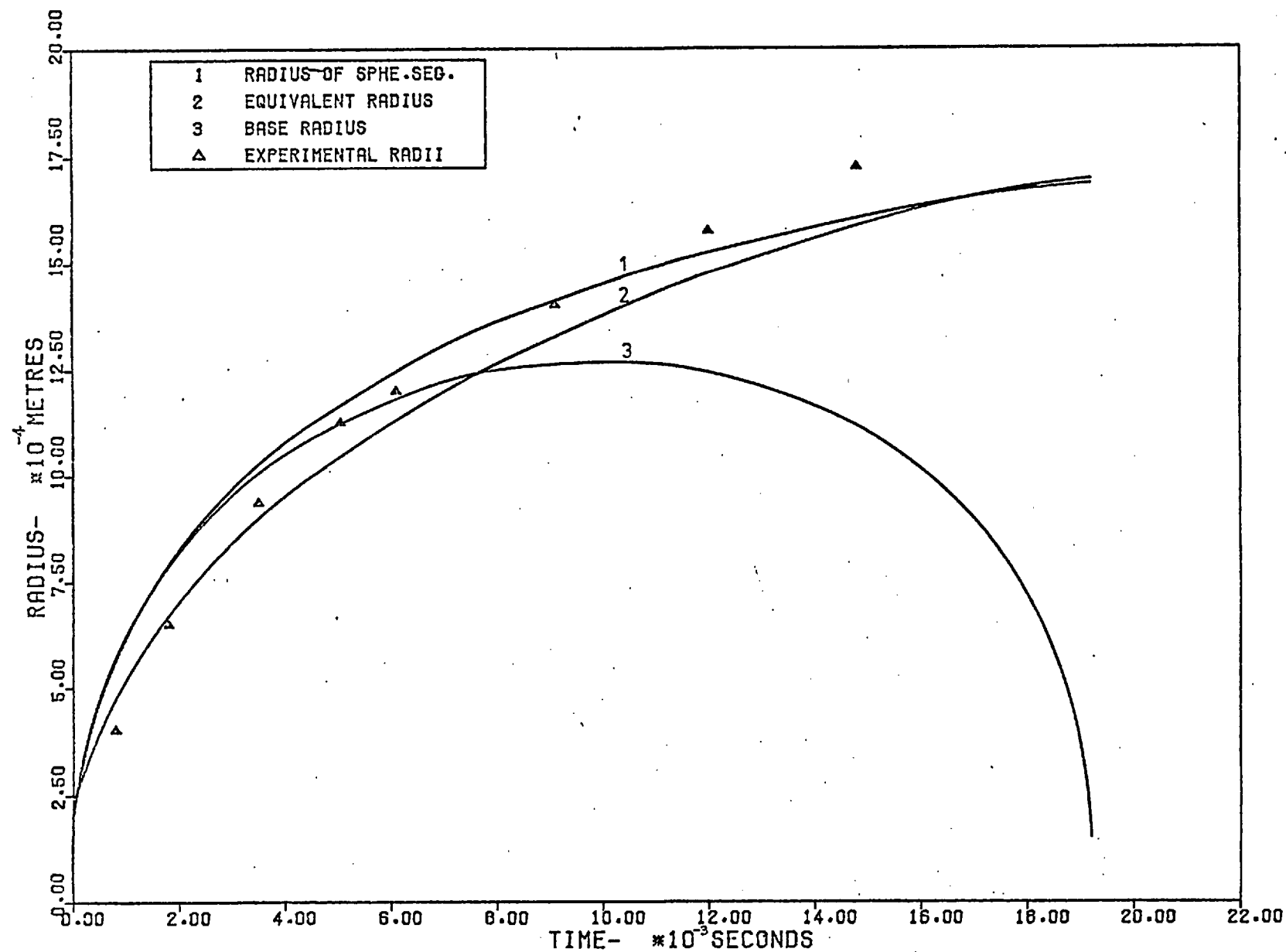


FIG. 5.6 BUBBLE GROWTH COMPARISON -REF/22/FIG.14 BUBBLE 2 (F)

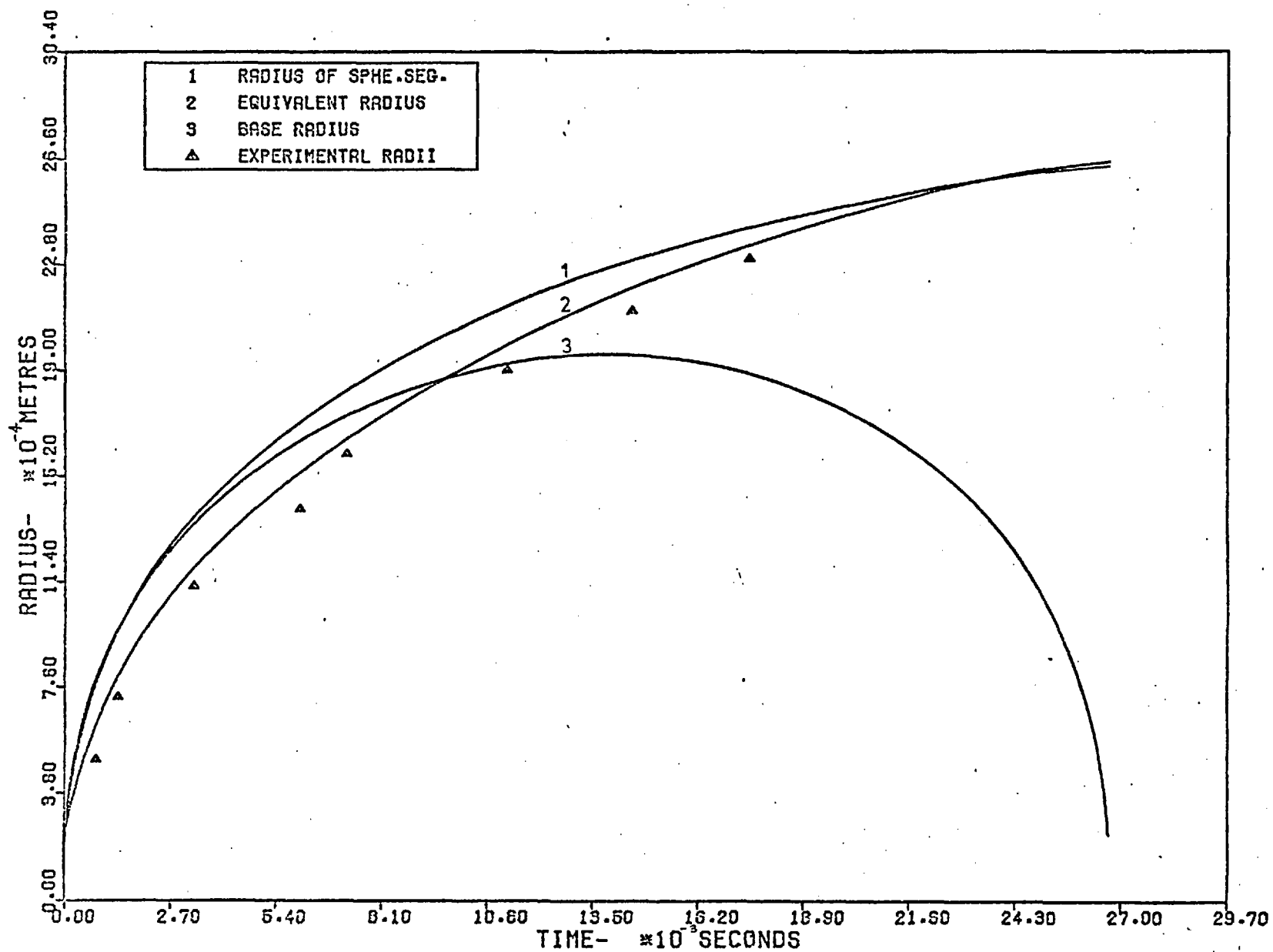


FIG.5.7 BUBBLE GROWTH COMPARISON -REF/22/FIG.17 BUBBLE 8 (G)

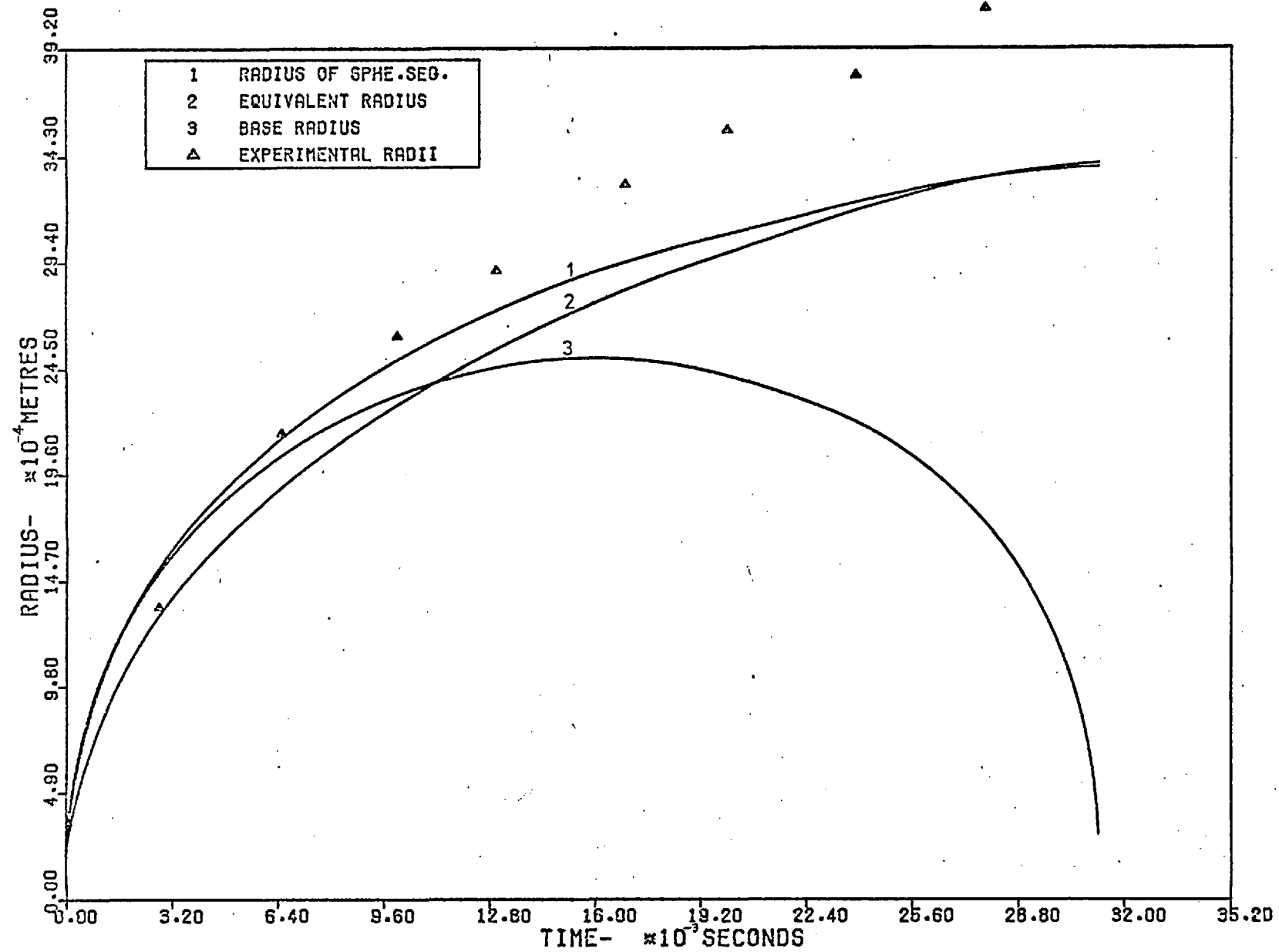


FIG.5.8 BUBBLE GROWTH COMPARISON -REF/22/FIG.18 BUBBLE 5 (H)

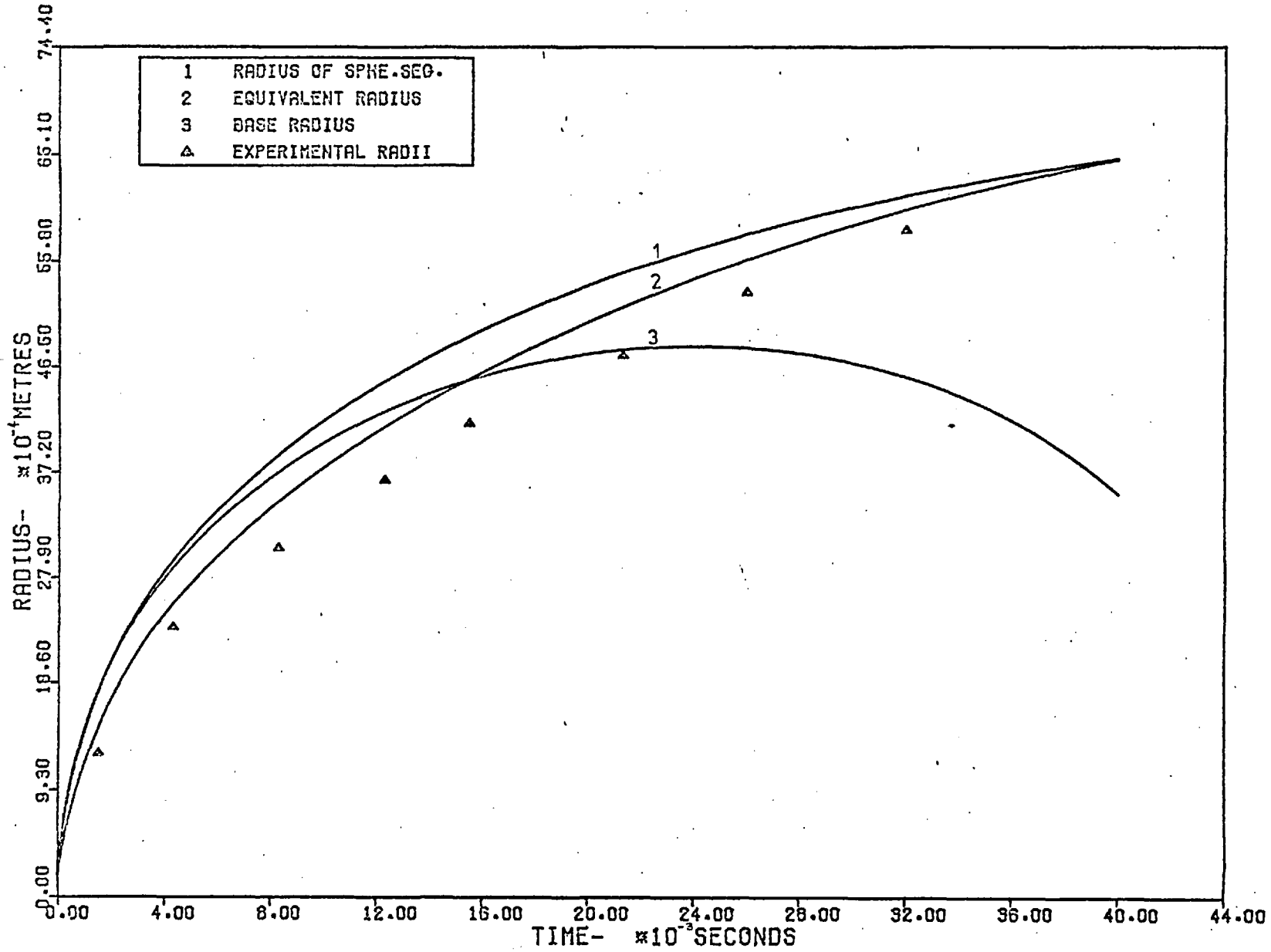


FIG.5.9 BUBBLE GROWTH COMPARISON -REF/22/FIG.20 BUBBLE 1 (I)

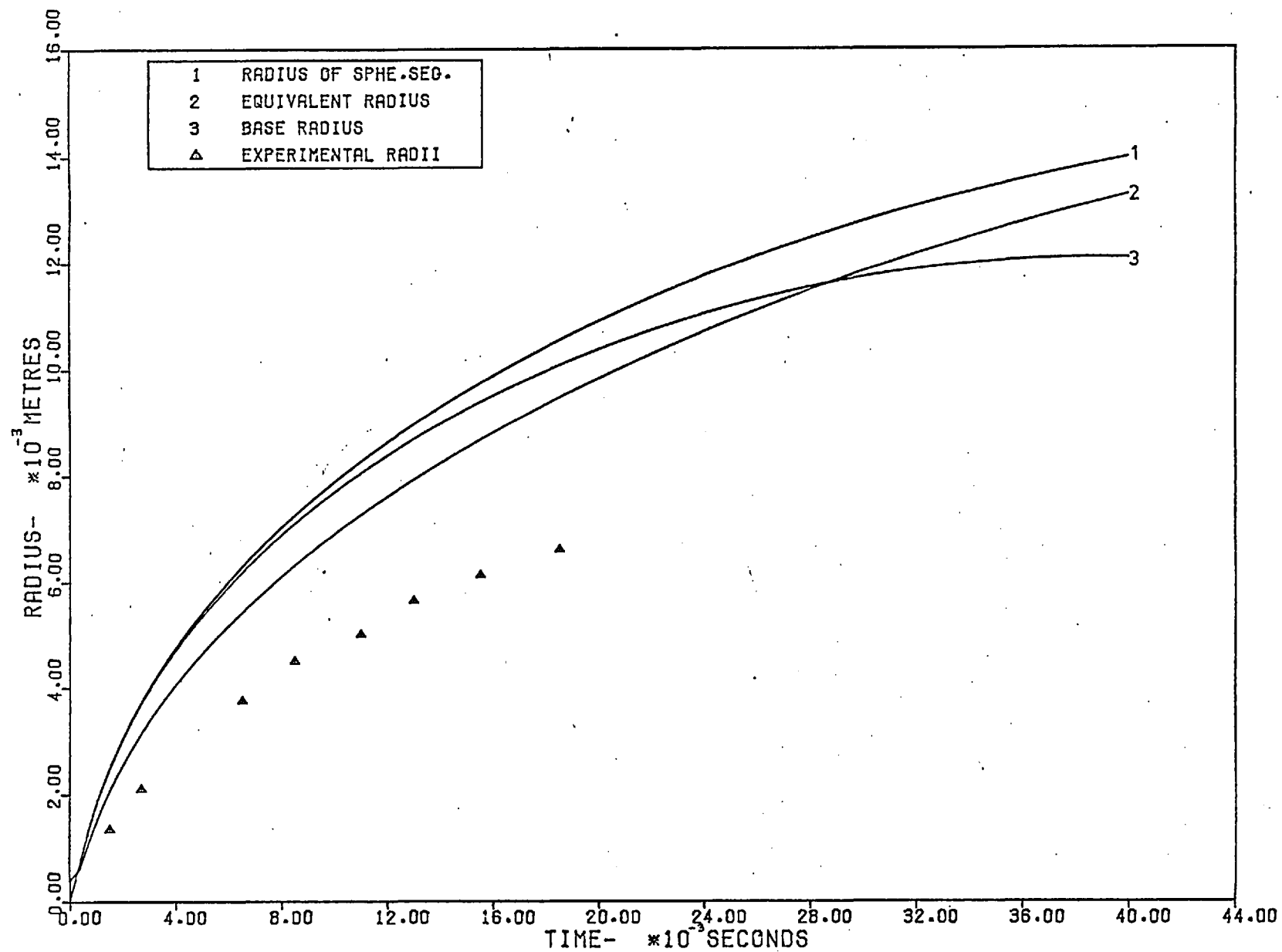


FIG.5.10 BUBBLE GROWTH COMPARISON -REF/22 /FIG.7 BUBBLE 1 (J)

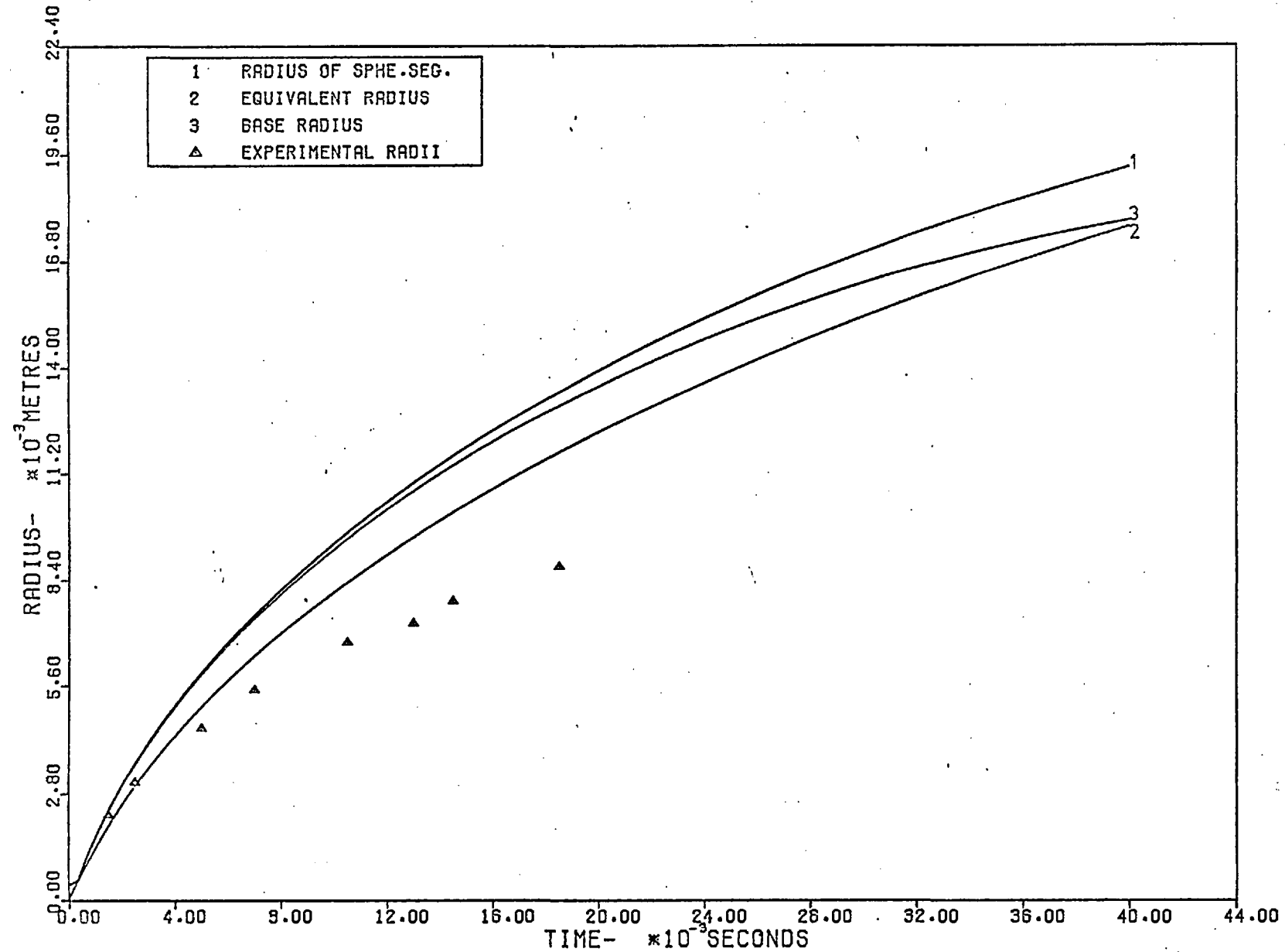


FIG.5.11 BUBBLE GROWTH COMPARISON -REF/22 /FIG.8 BUBBLE 1 (K)

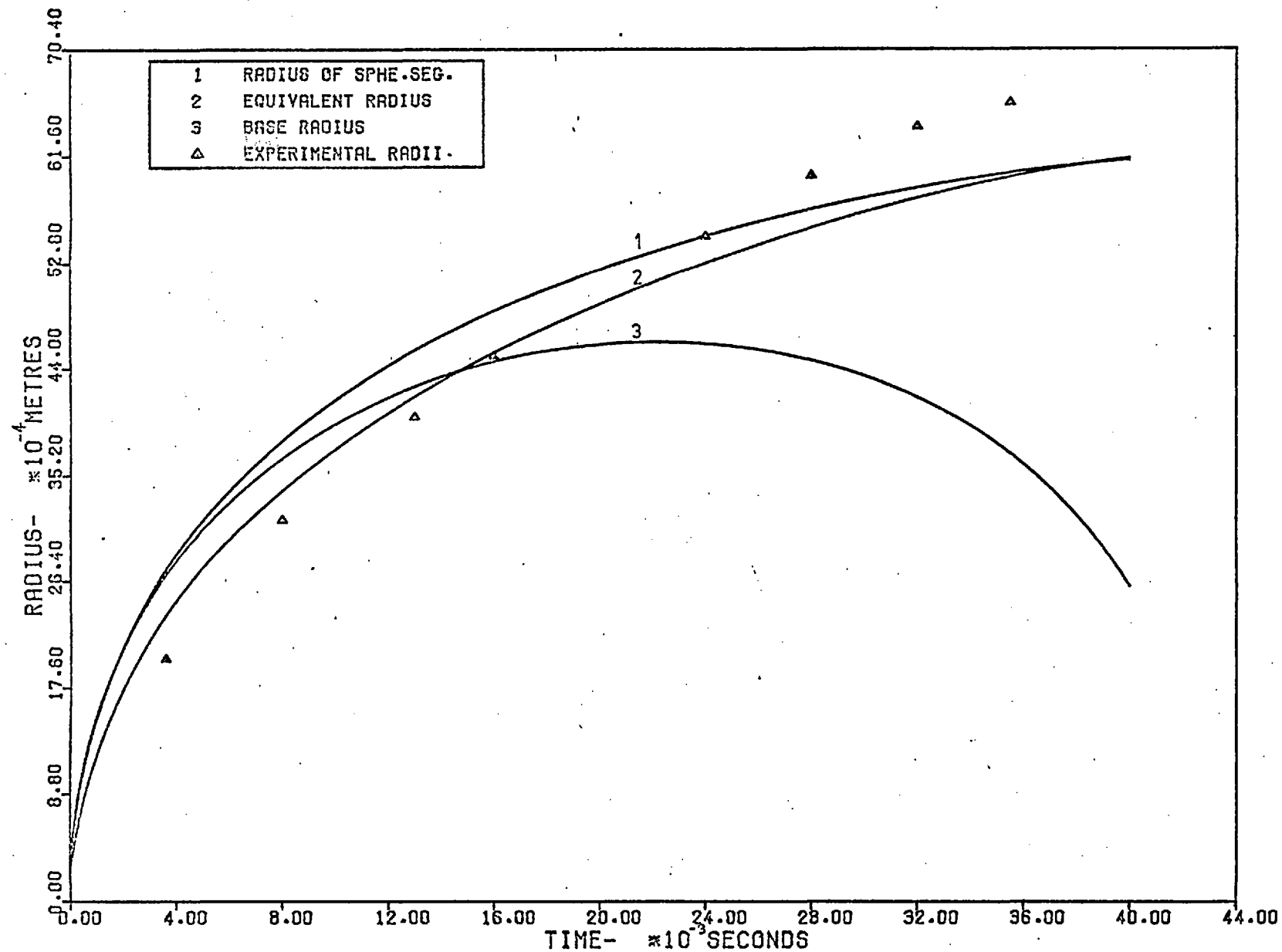


FIG.5.12 BUBBLE GROWTH COMPARISON -REF/22/FIG.24 BUBBLE 3 (L)

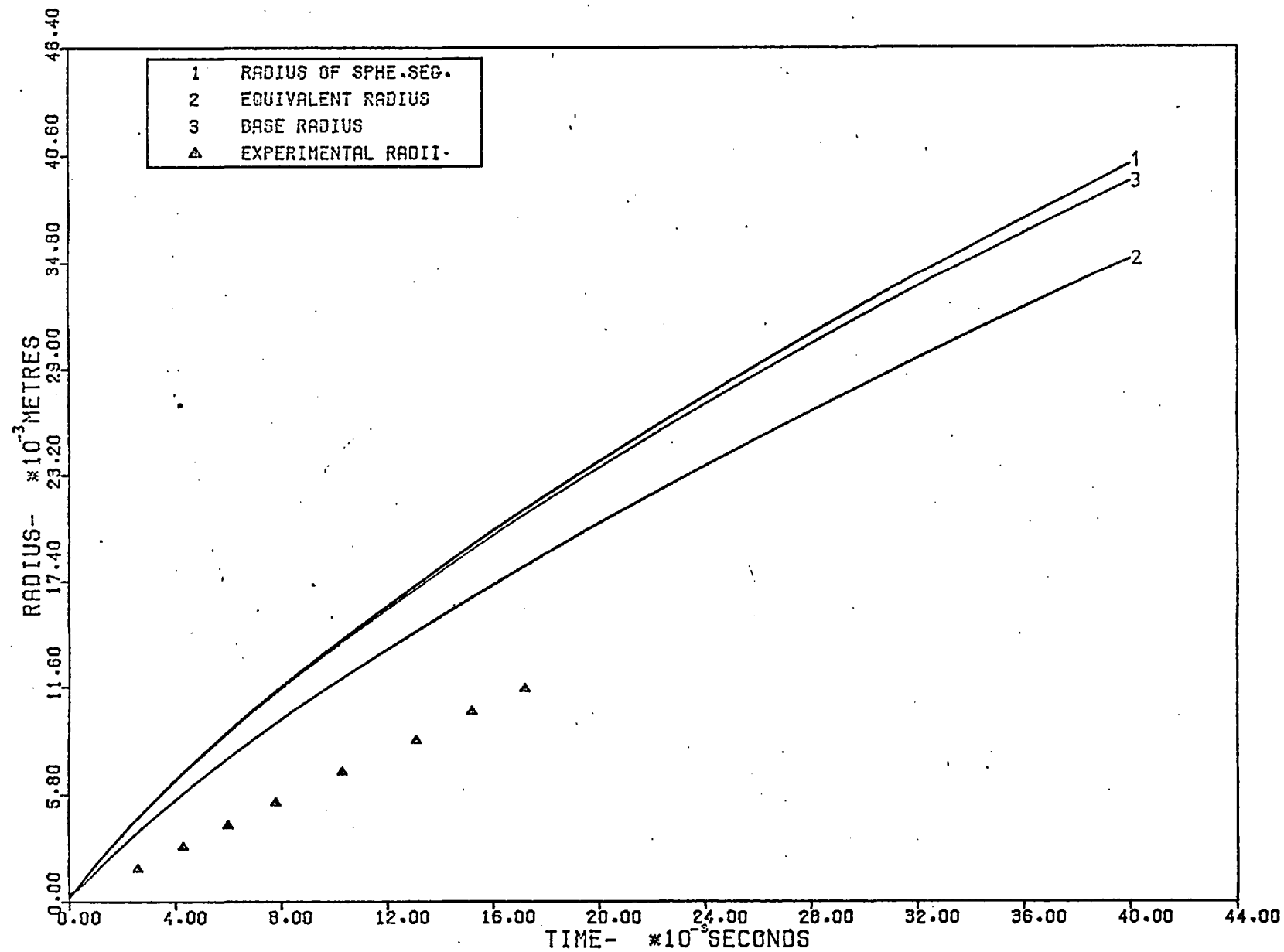


FIG.5.13 BUBBLE GROWTH COMPARISON -REF/32/FIG.7 $t_w=118$ MS.(M)

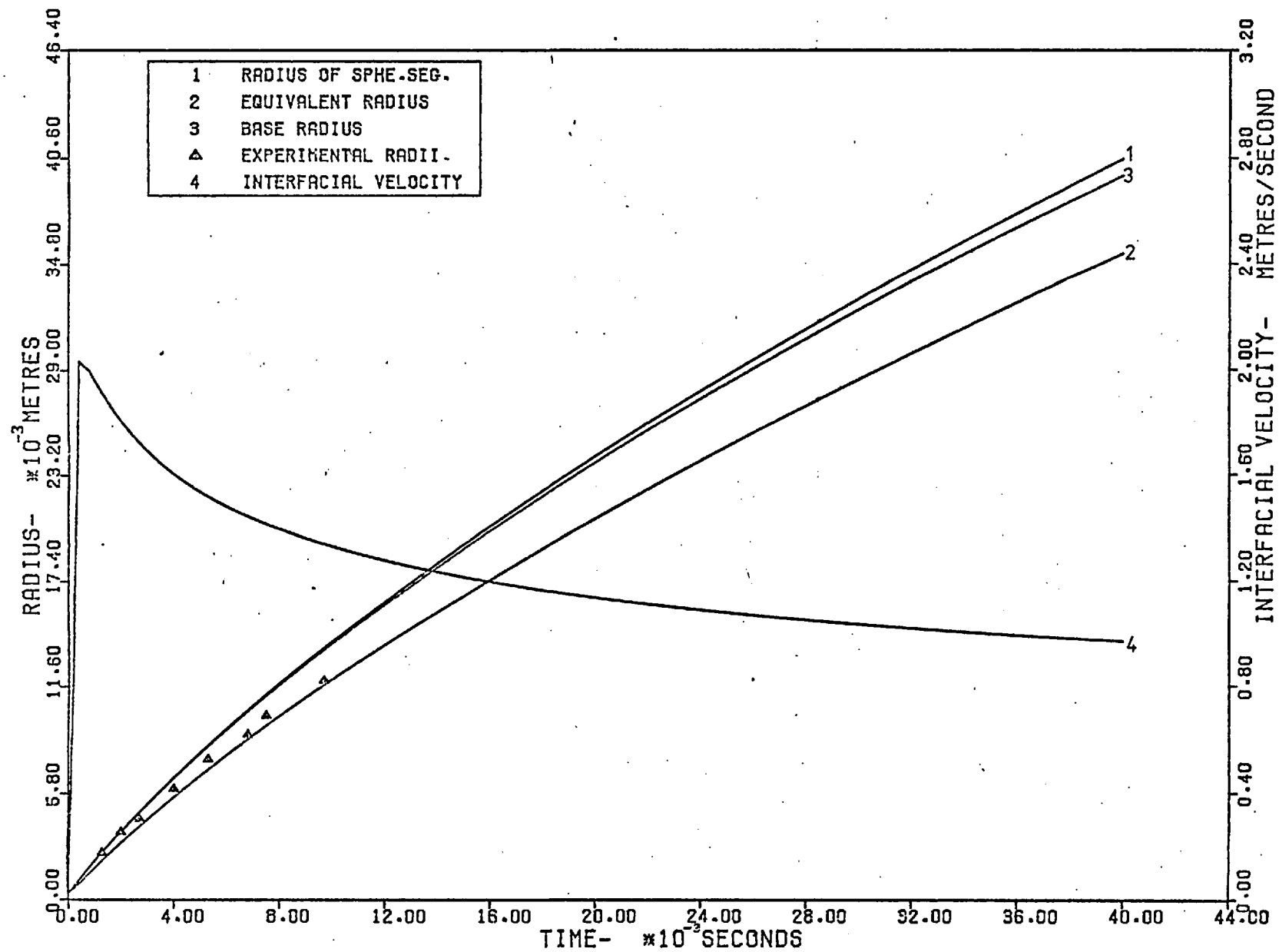


FIG.5.14 BUBBLE GROWTH COMPARISON -REF/32/FIG.7 $t_w=184$ MS.(N)

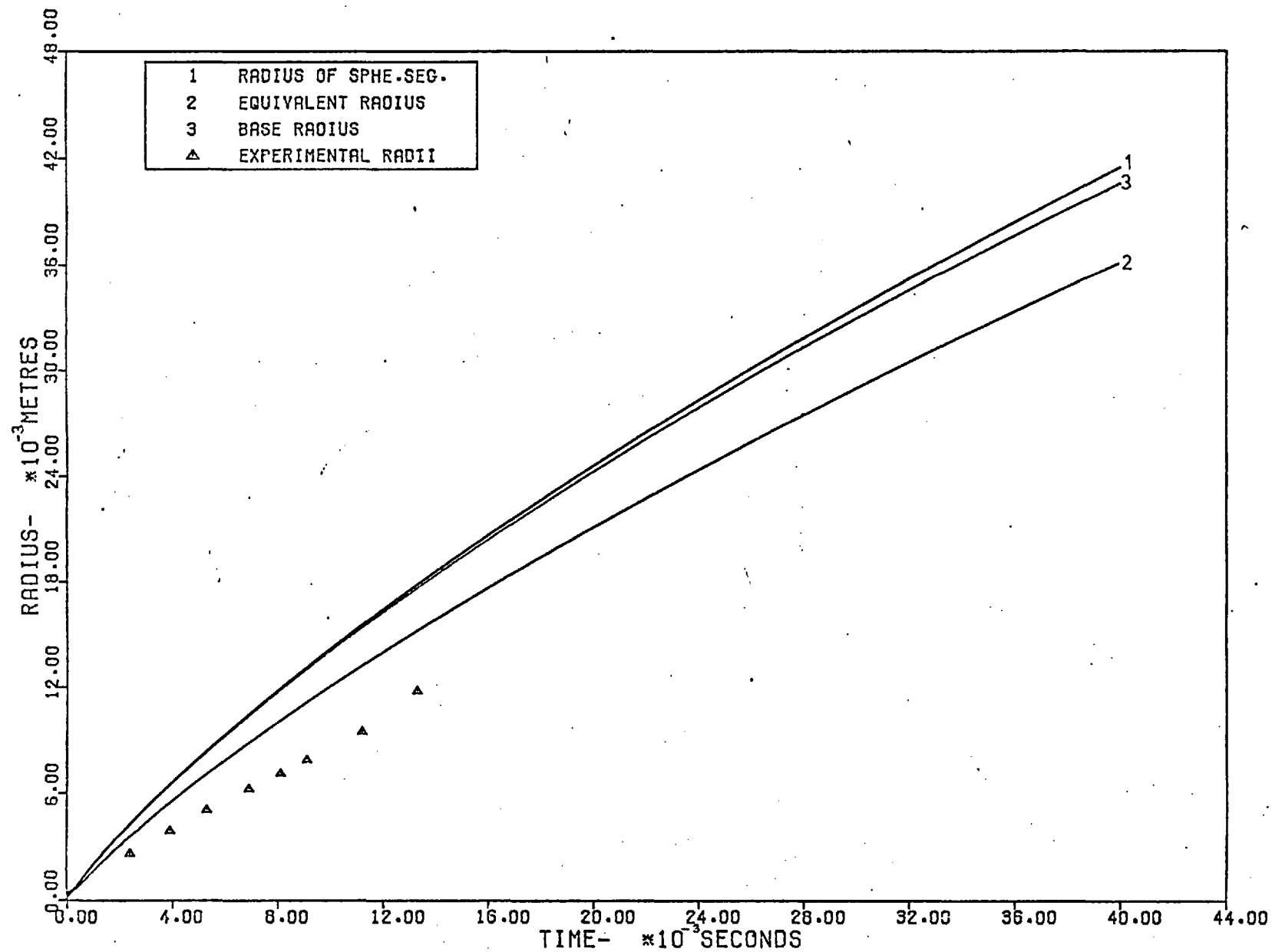


FIG.5-15 BUBBLE GROWTH COMPARISON -REF/32/FIG.12 $t_w=65$ MS.(O)

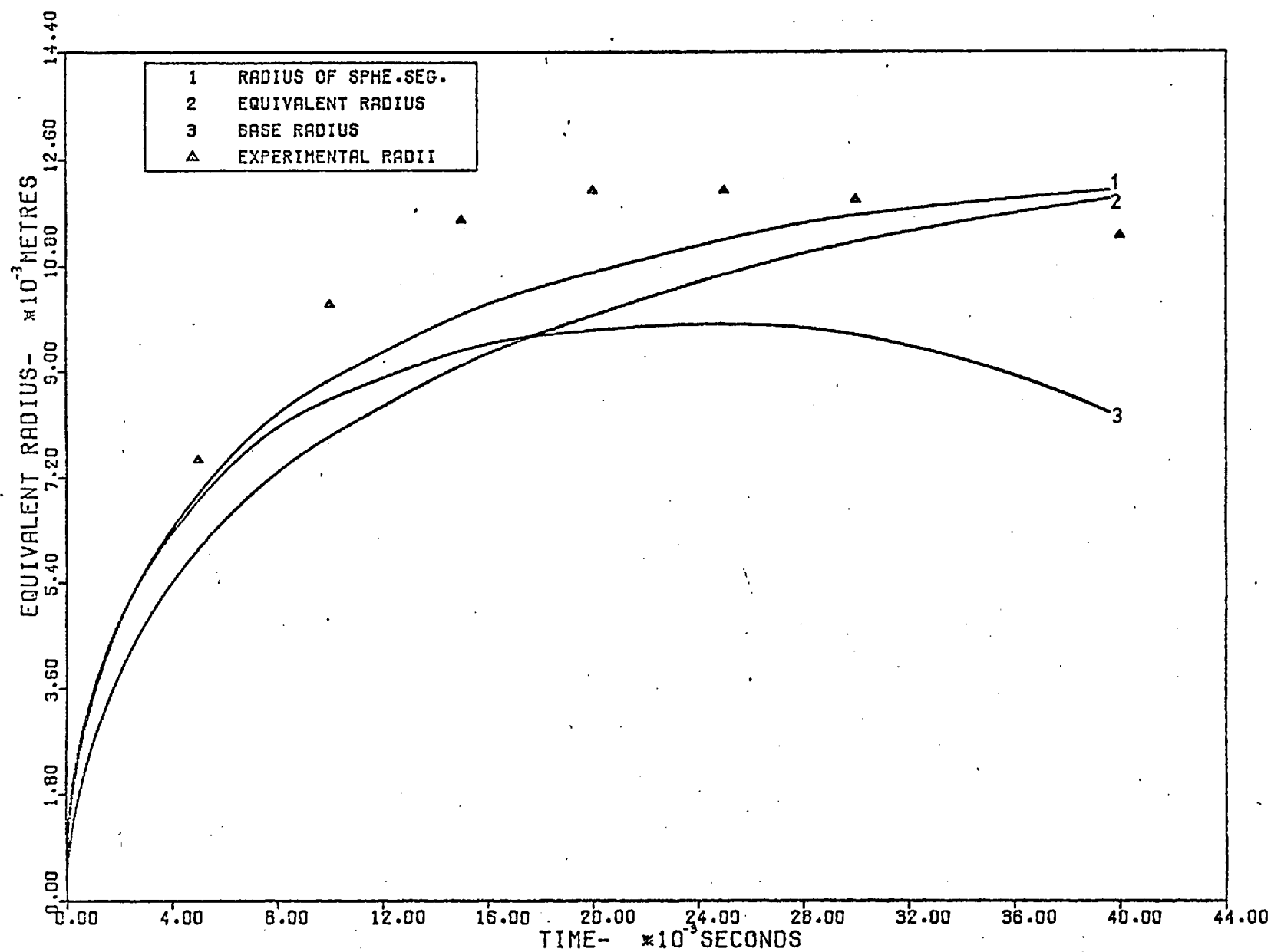


FIG. 5-16 BUBBLE GROWTH COMPARISON -REF/53/ RUN 2 (P)

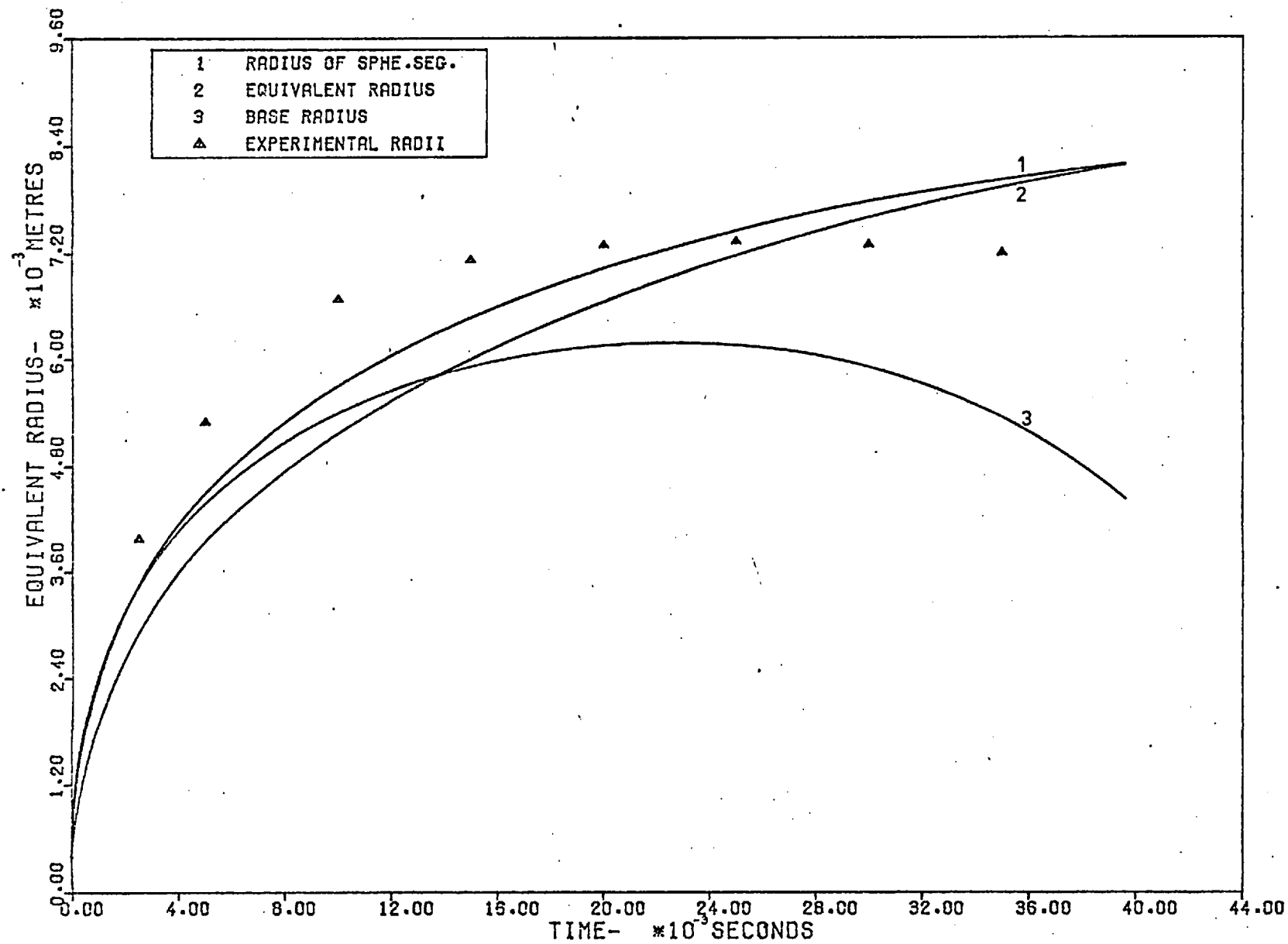


FIG. 5.17 BUBBLE GROWTH COMPARISON -REF/53/ RUN 3 (Q)

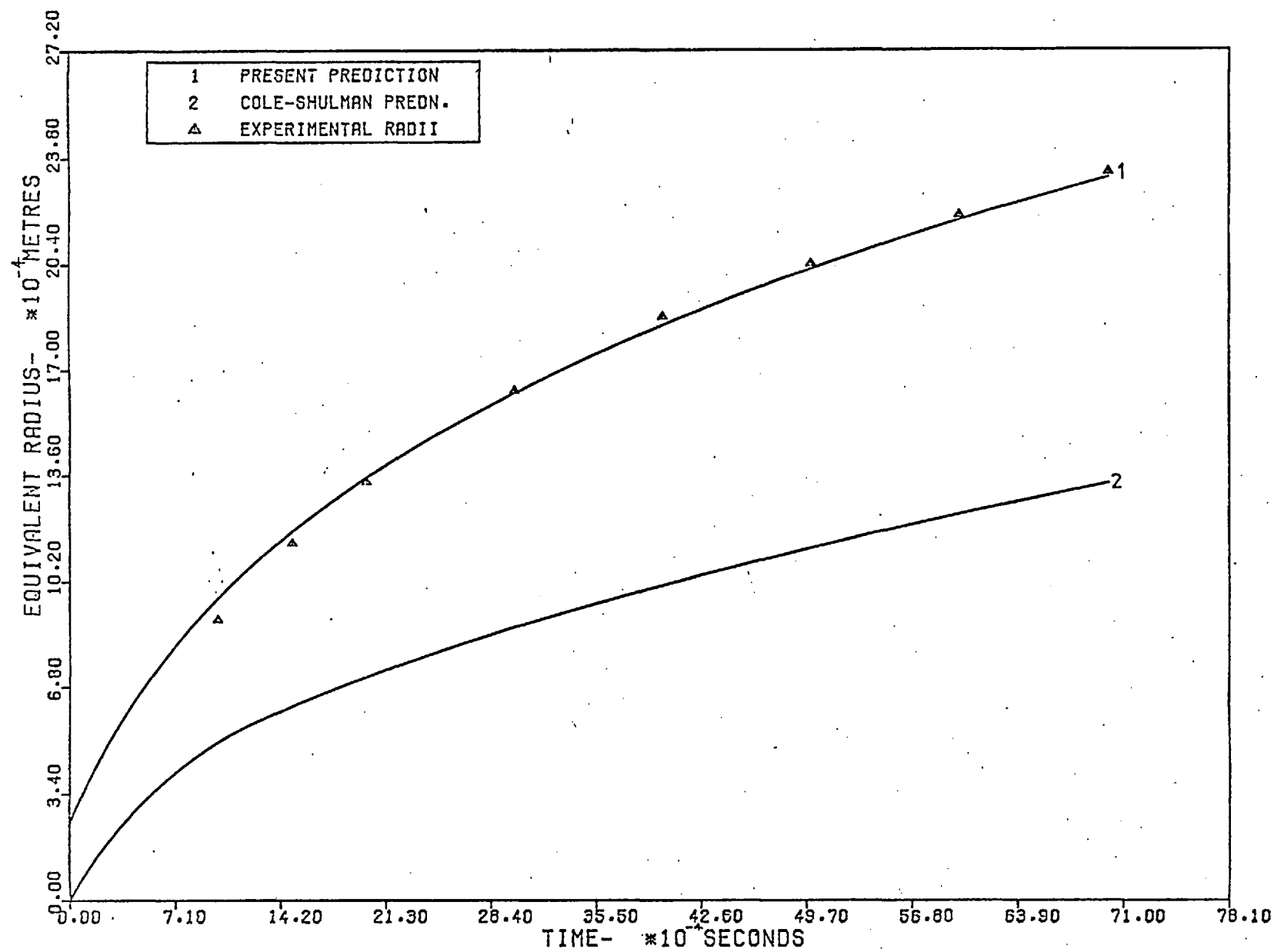


FIG.5-18 COMPARISON OF GROWTH WITH COLE-SHULMAN EQN.(BUBBLE E)

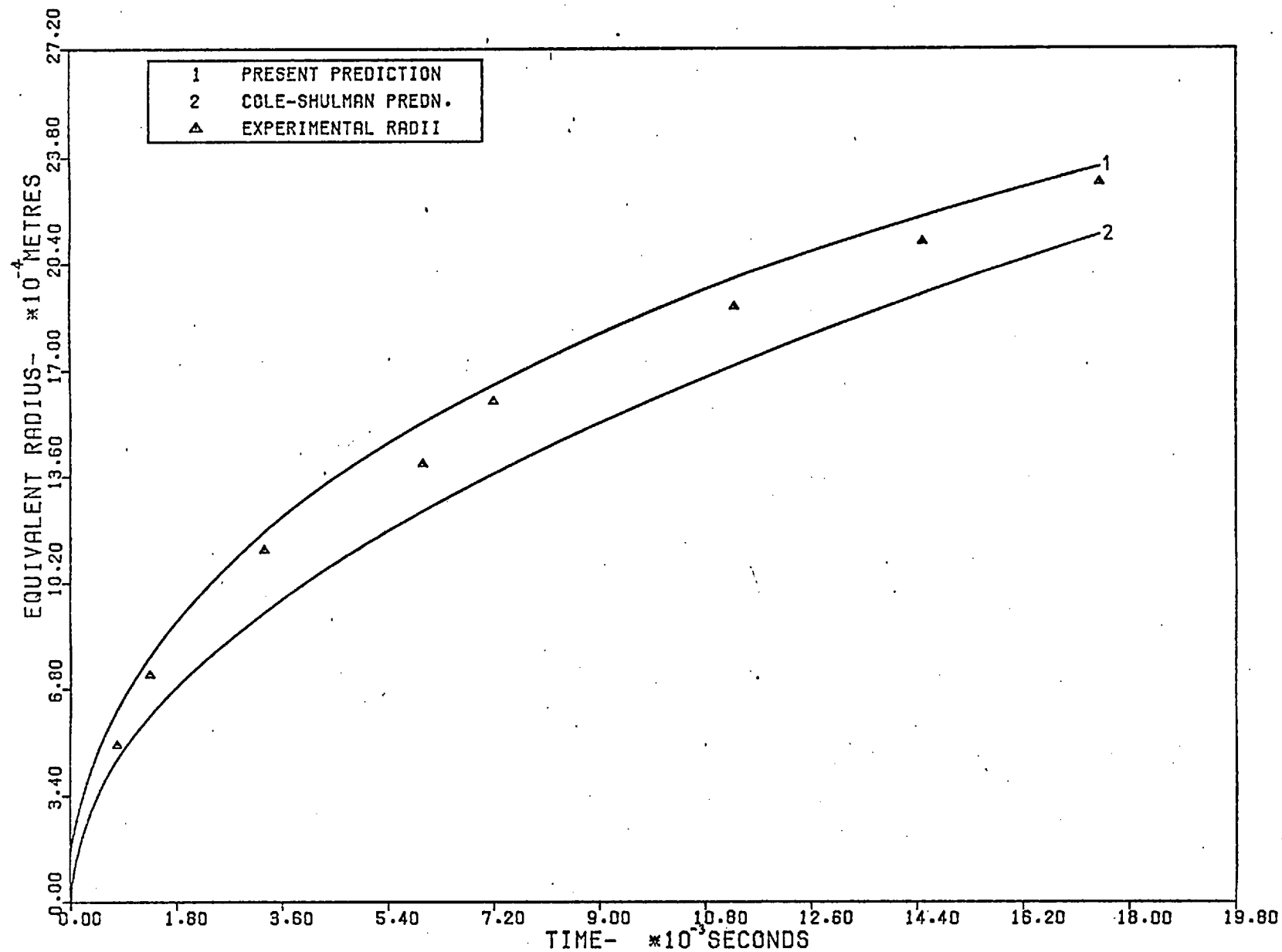


FIG.519 COMPARISON OF GROWTH WITH COLE-SHULMAN EQN.(BUBBLE G)

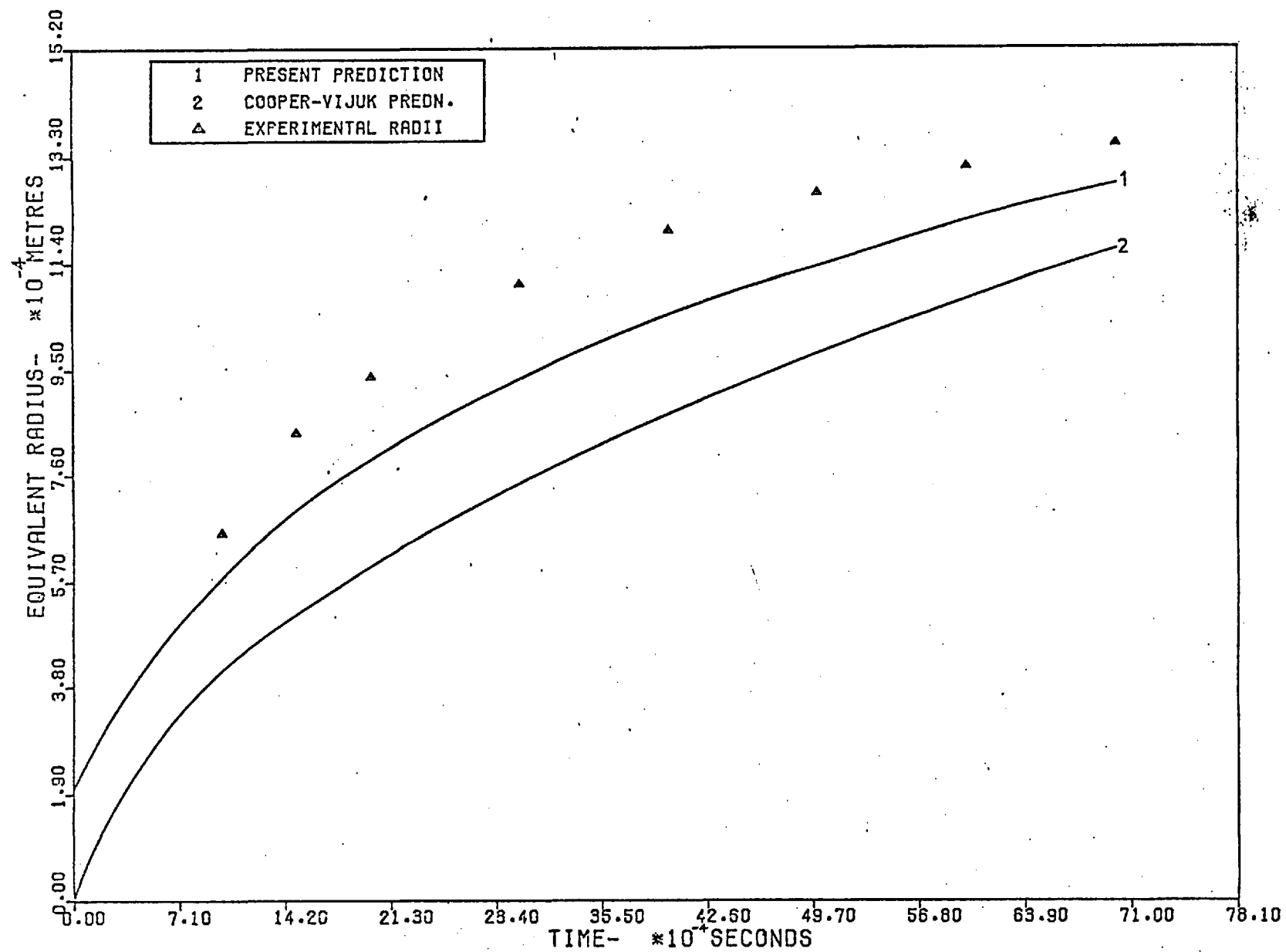


FIG.520 COMPARISON OF GROWTH WITH COOPER-VIJUK EQN.(BUBBLE D)

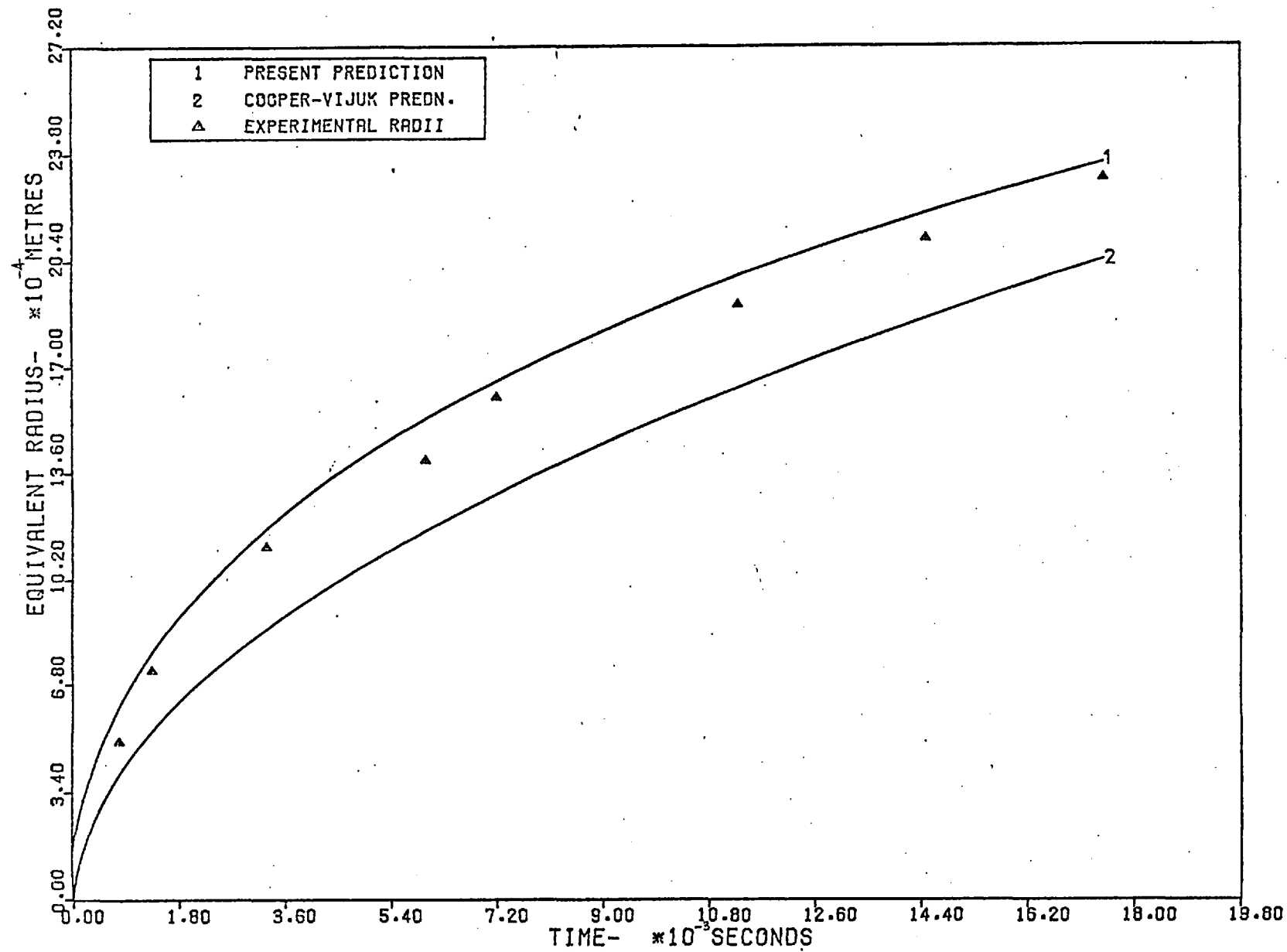


FIG.5.21 COMPARISON OF GROWTH WITH COOPER-VIJUK EQN.(BUBBLE G)

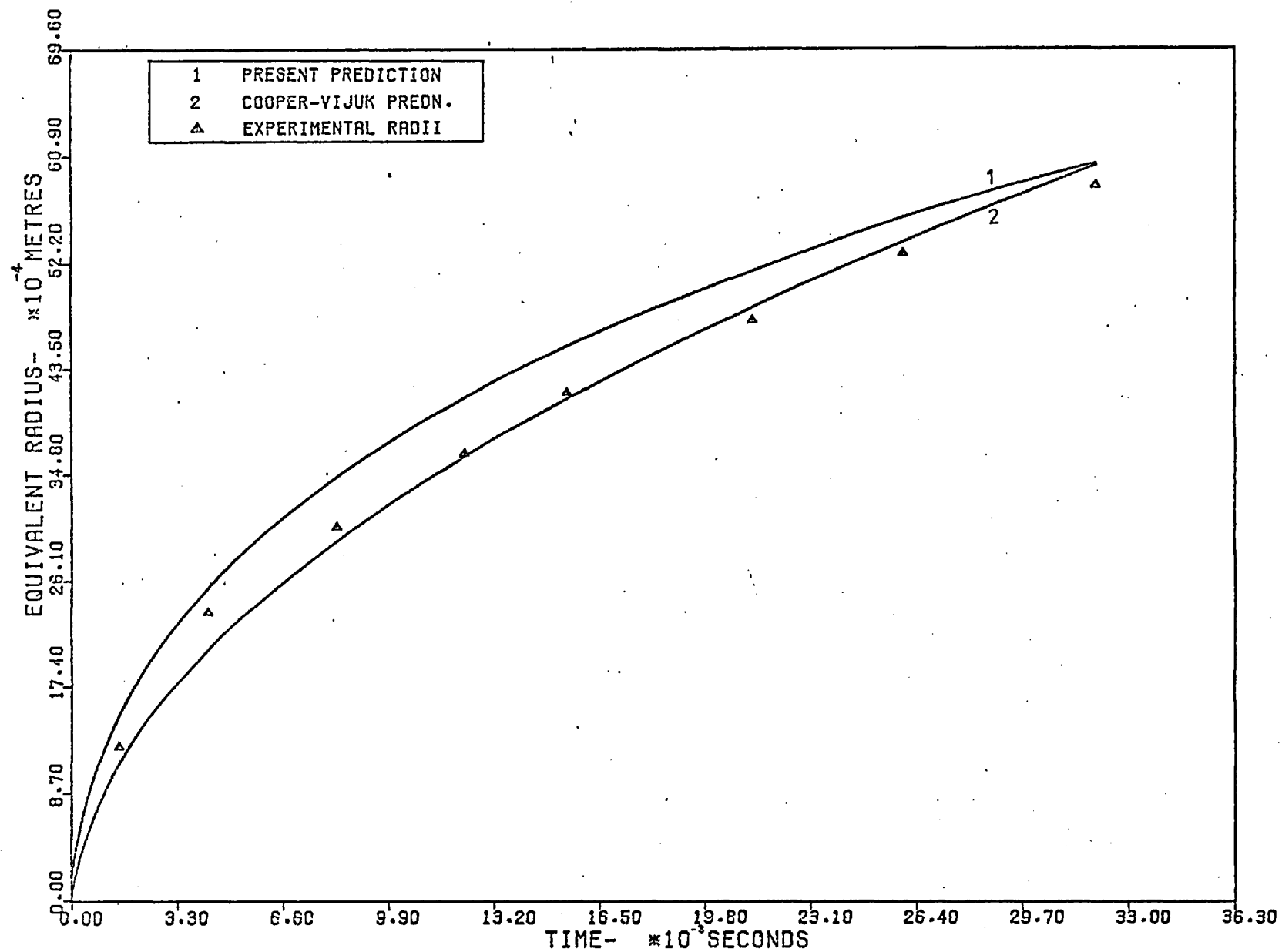


FIG.5.22 COMPARISON OF GROWTH WITH COOPER-VIJUK EQN.(BUBBLE I)

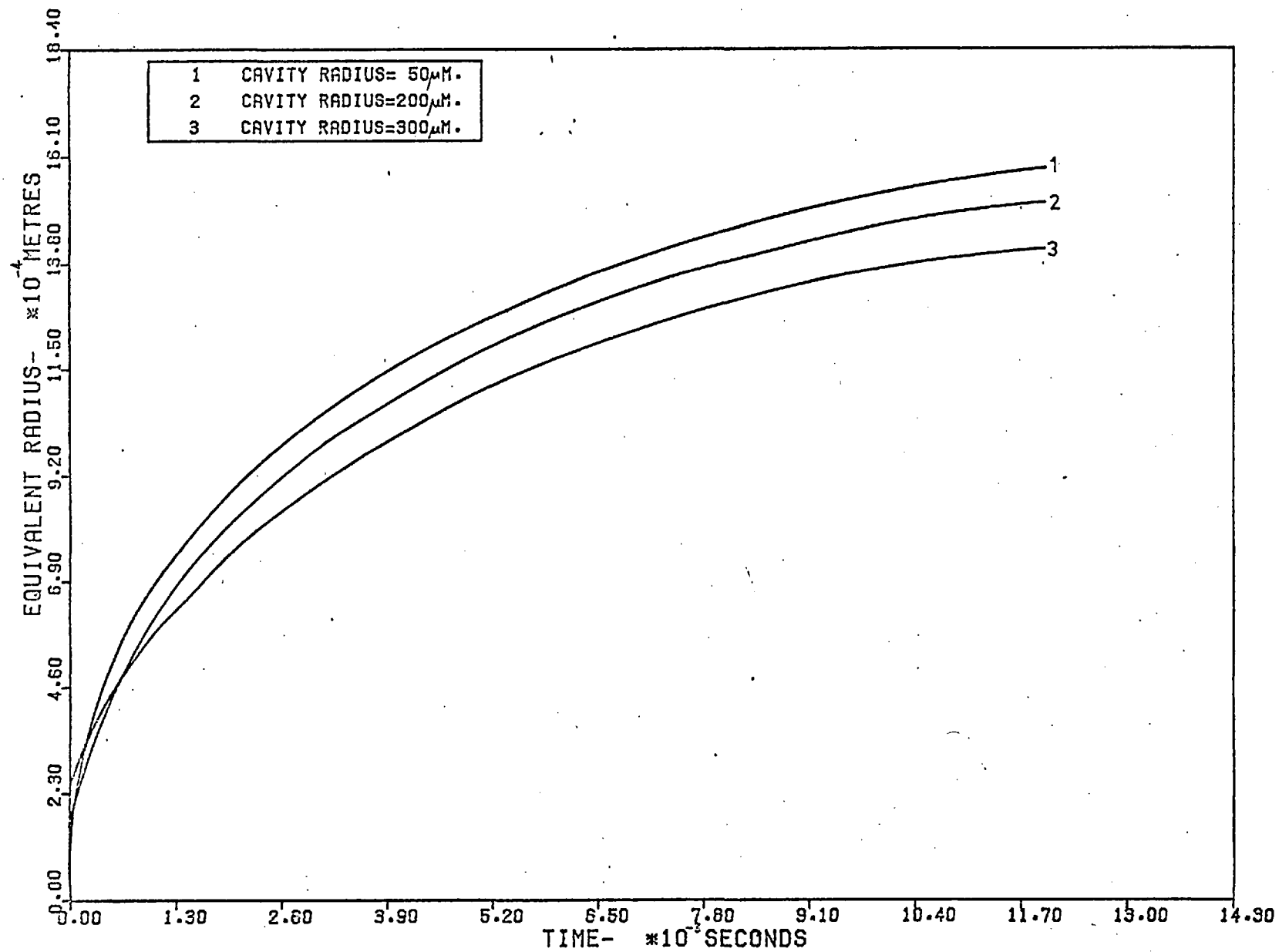


FIG.5-23 VARIATION OF GROWTH WITH CAVITY RADIUS(BUBBLE D)

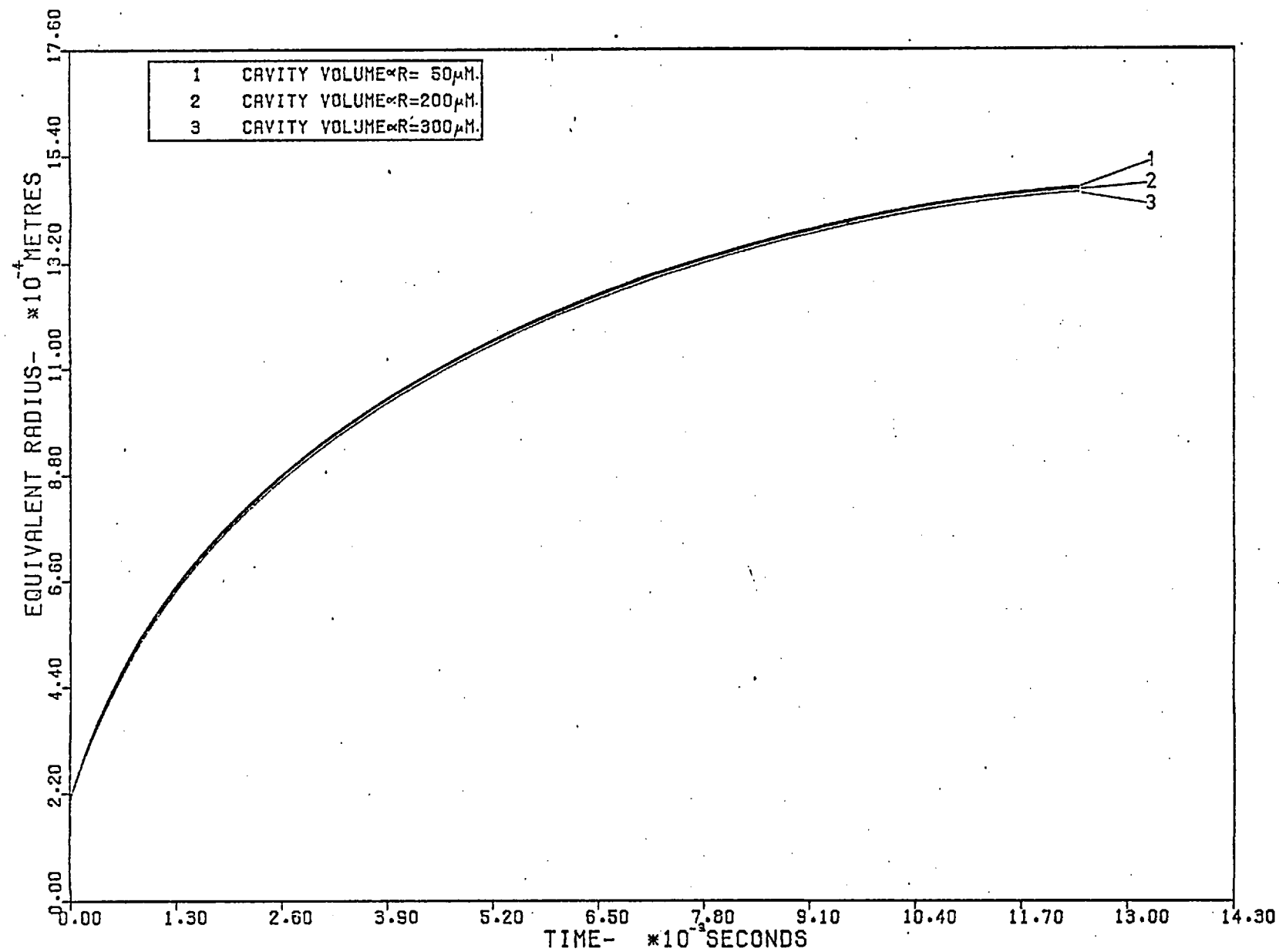


FIG.5-24 EFFECT OF CAVITY VOLUME VARIATION (BUBBLE D)

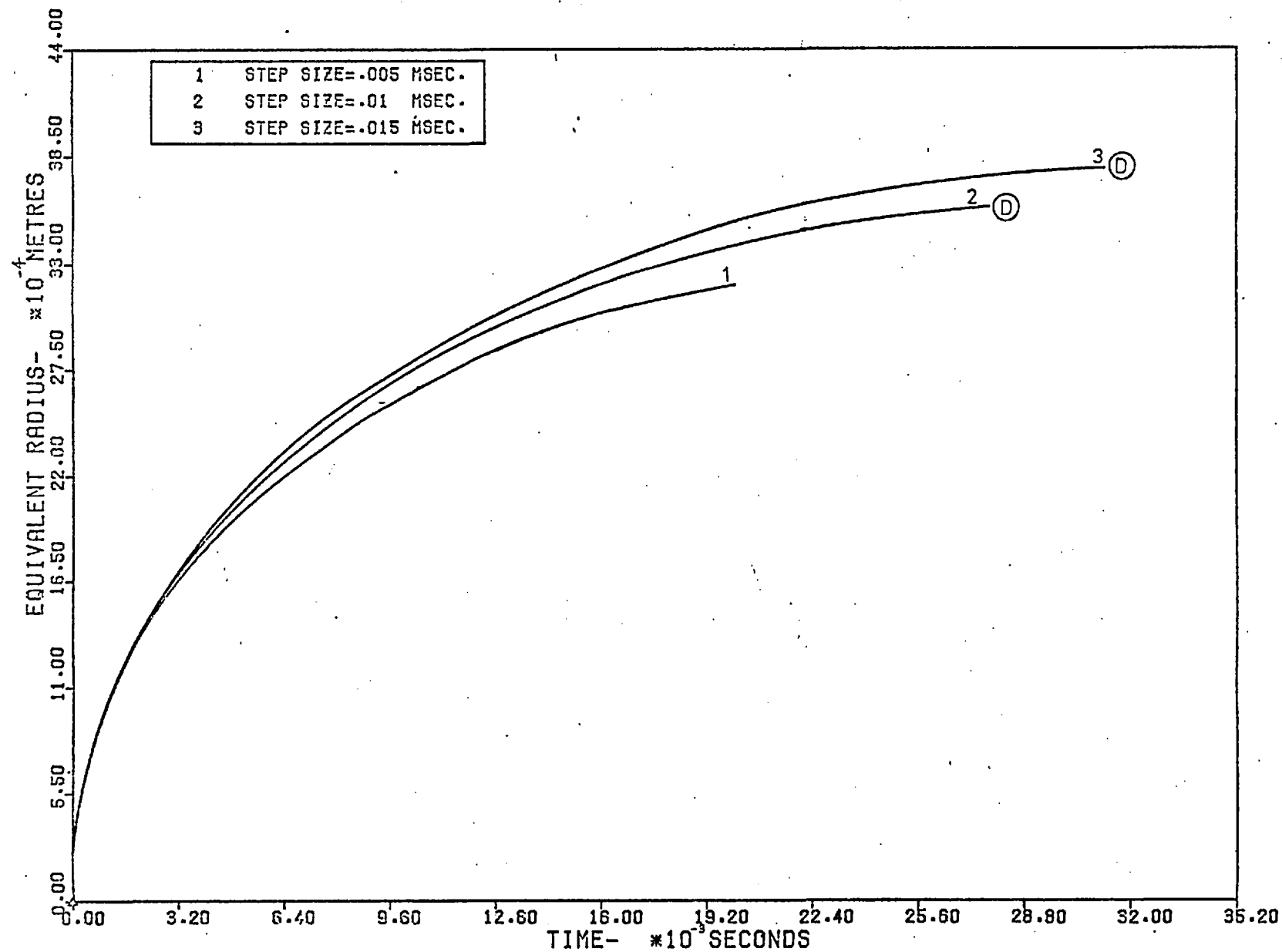


FIG.5.25 EFFECT OF STEP SIZE ON GROWTH RATE (BUBBLE E)

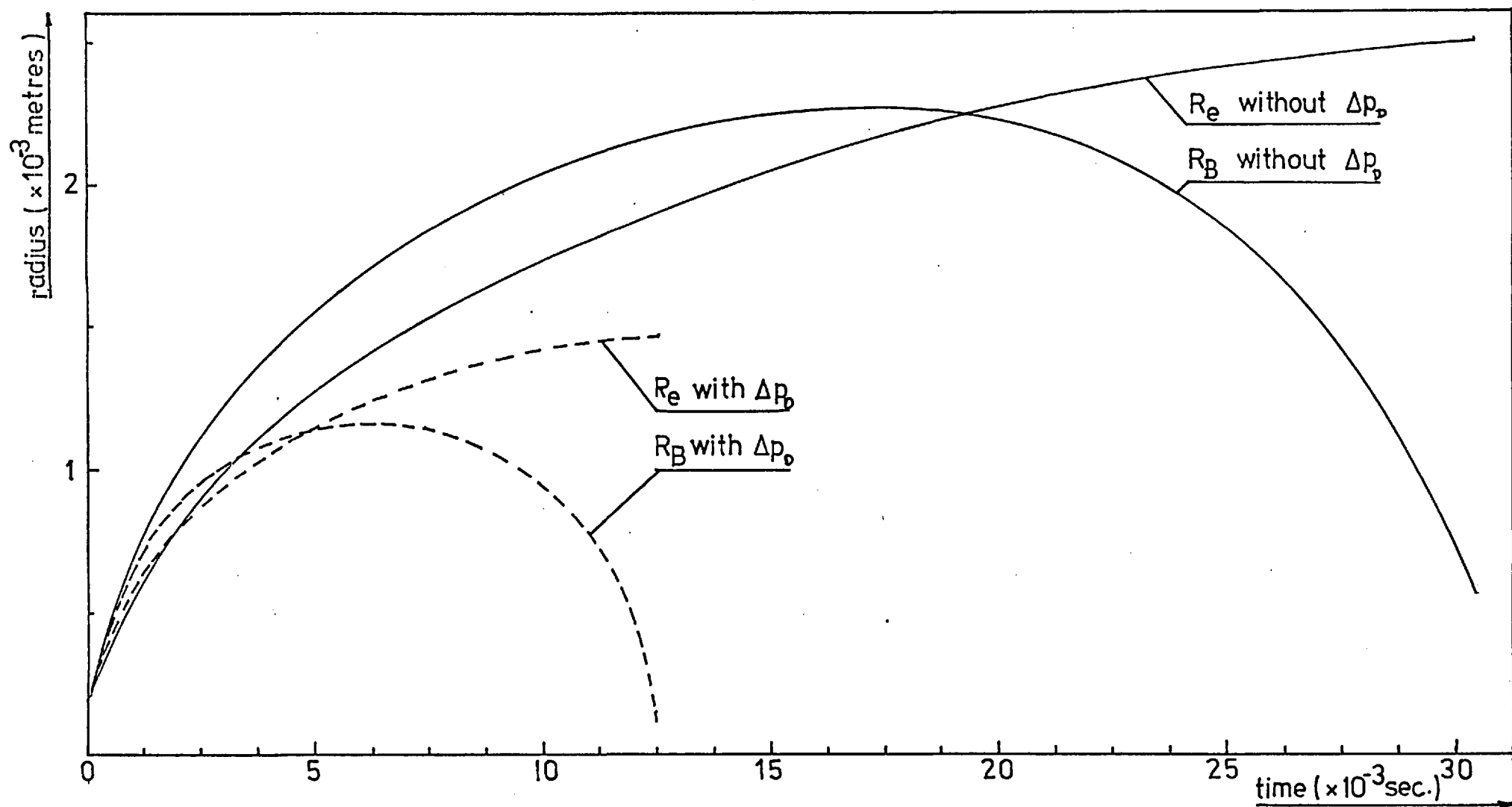


FIG. 5-26 THE EFFECT OF Δp_b ON GROWTH AND DEPARTURE (BUBBLE D)

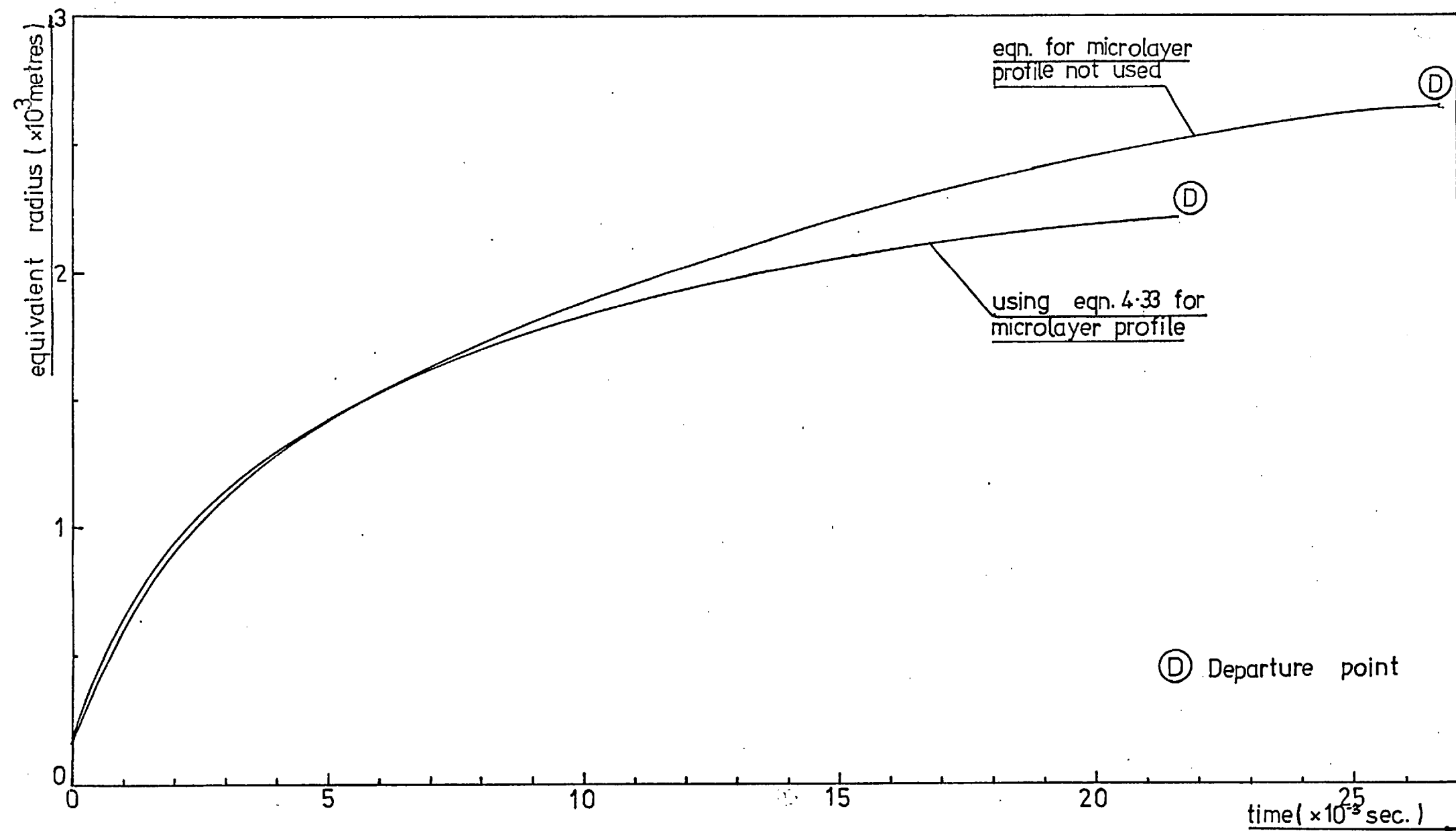


FIG. 5.27 EFFECT OF THE ASSUMPTION OF EQN. 4.33 FOR MICROLAYER PROFILE
(BUBBLE G)

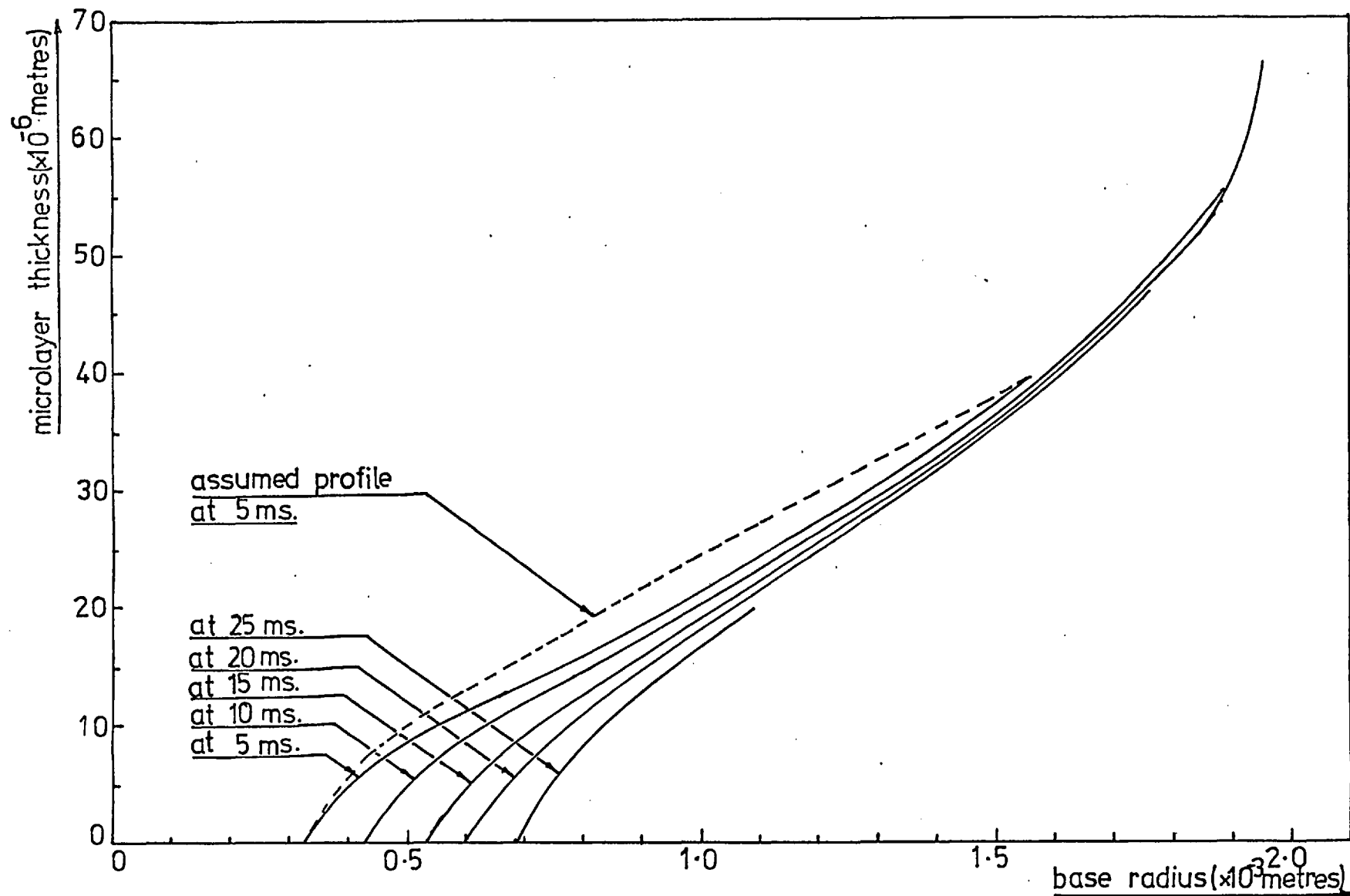


FIG. 5.28 VARIATION OF MICROLAYER PROFILE WITH TIME
(BUBBLE G)

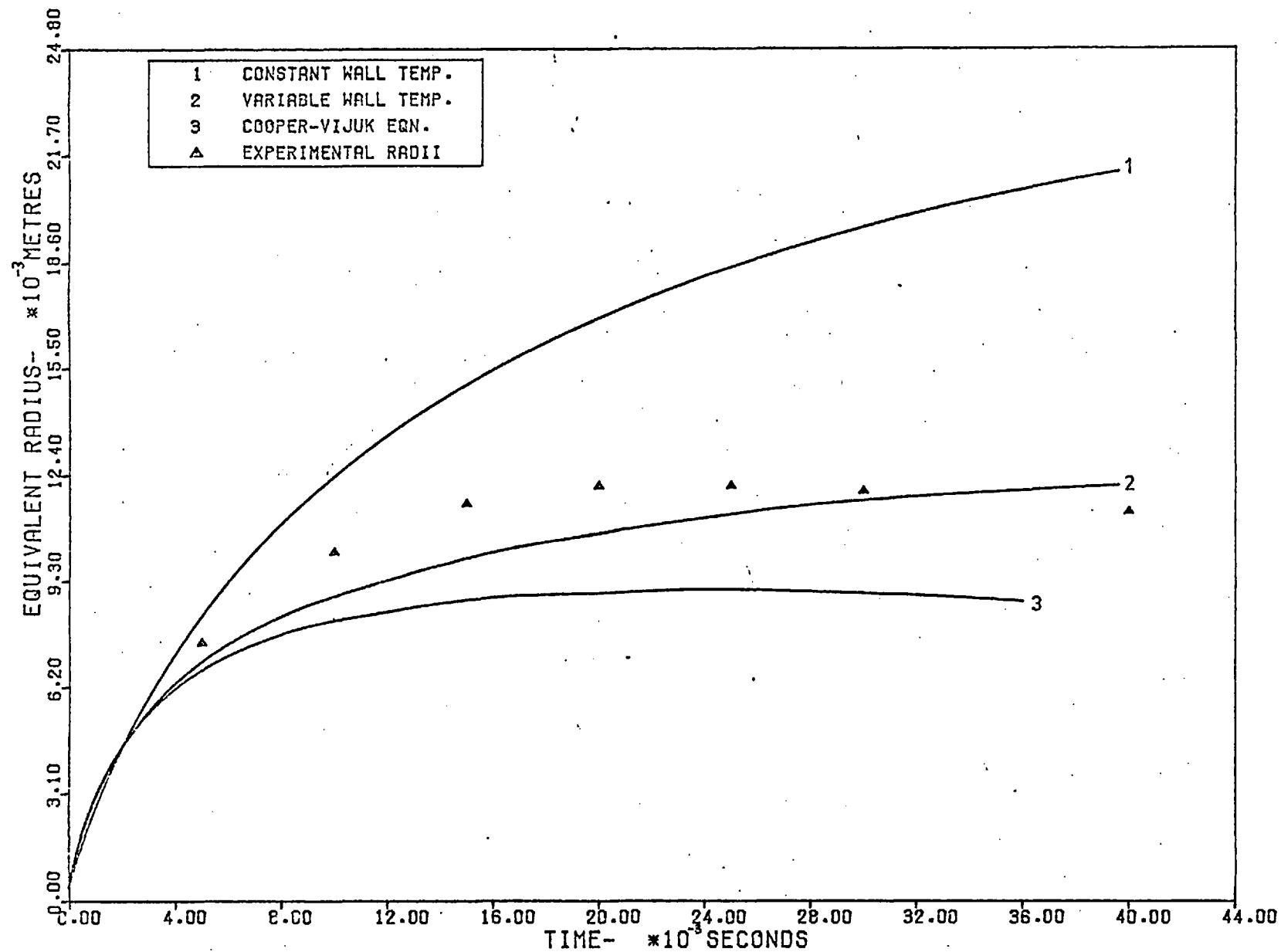


FIG.5-29 GROWTH WITH WALL TEMPERATURE VARIATION (BUBBLE P)

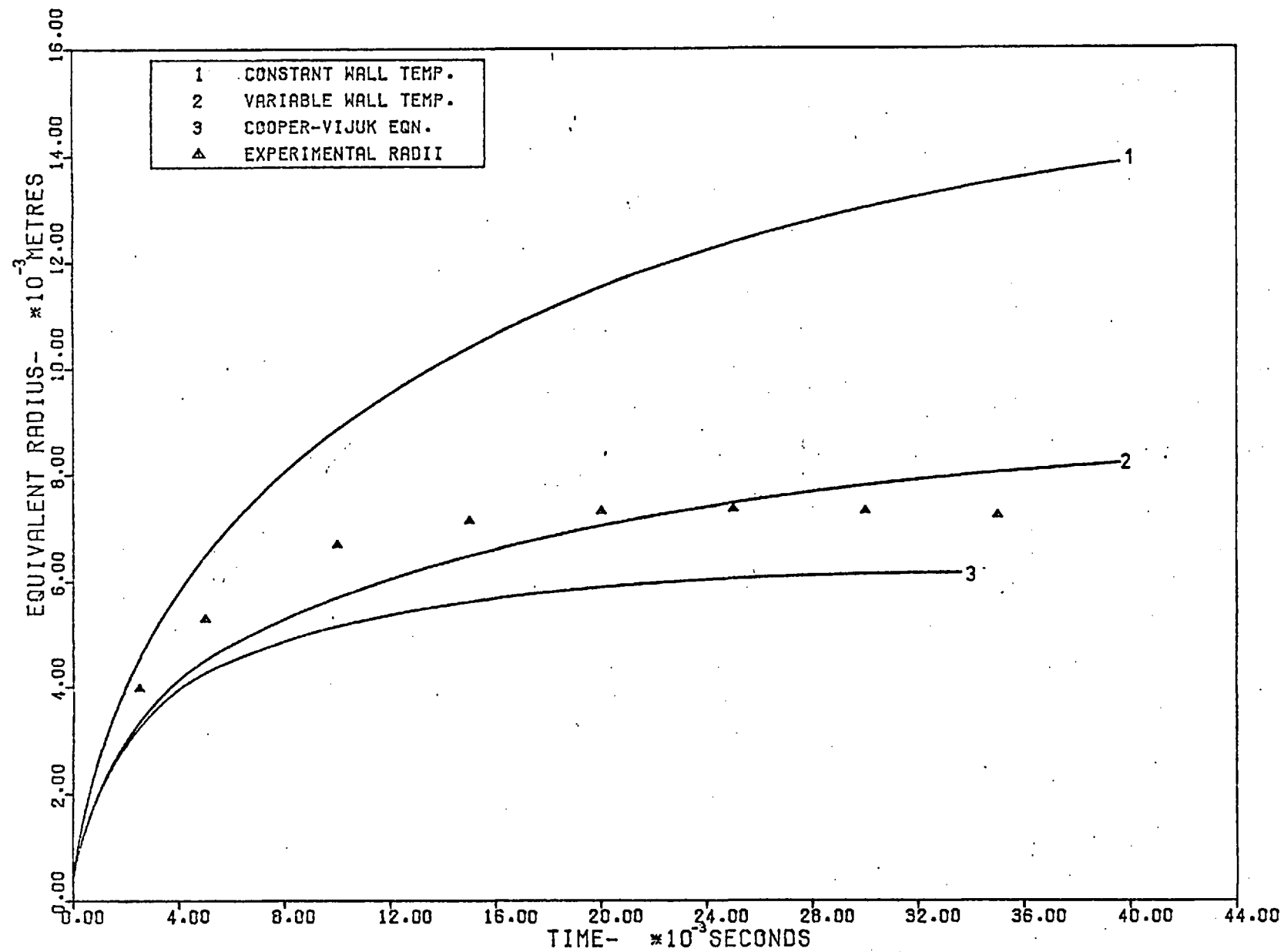


FIG.5-30 GROWTH WITH WALL TEMPERATURE VARIATION (BUBBLE Q)

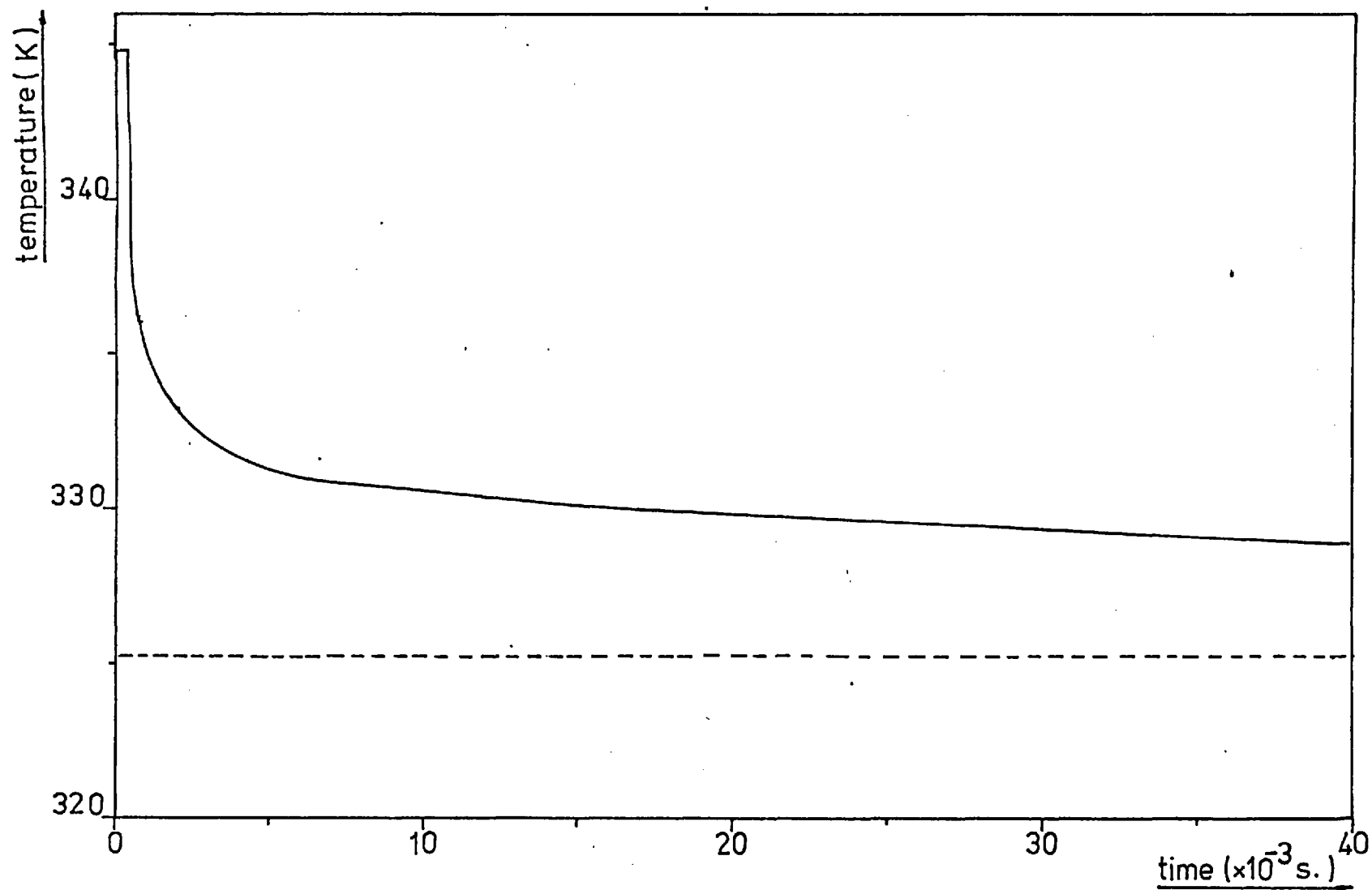


Fig.5.31 Wall Temperature Variation at a Point

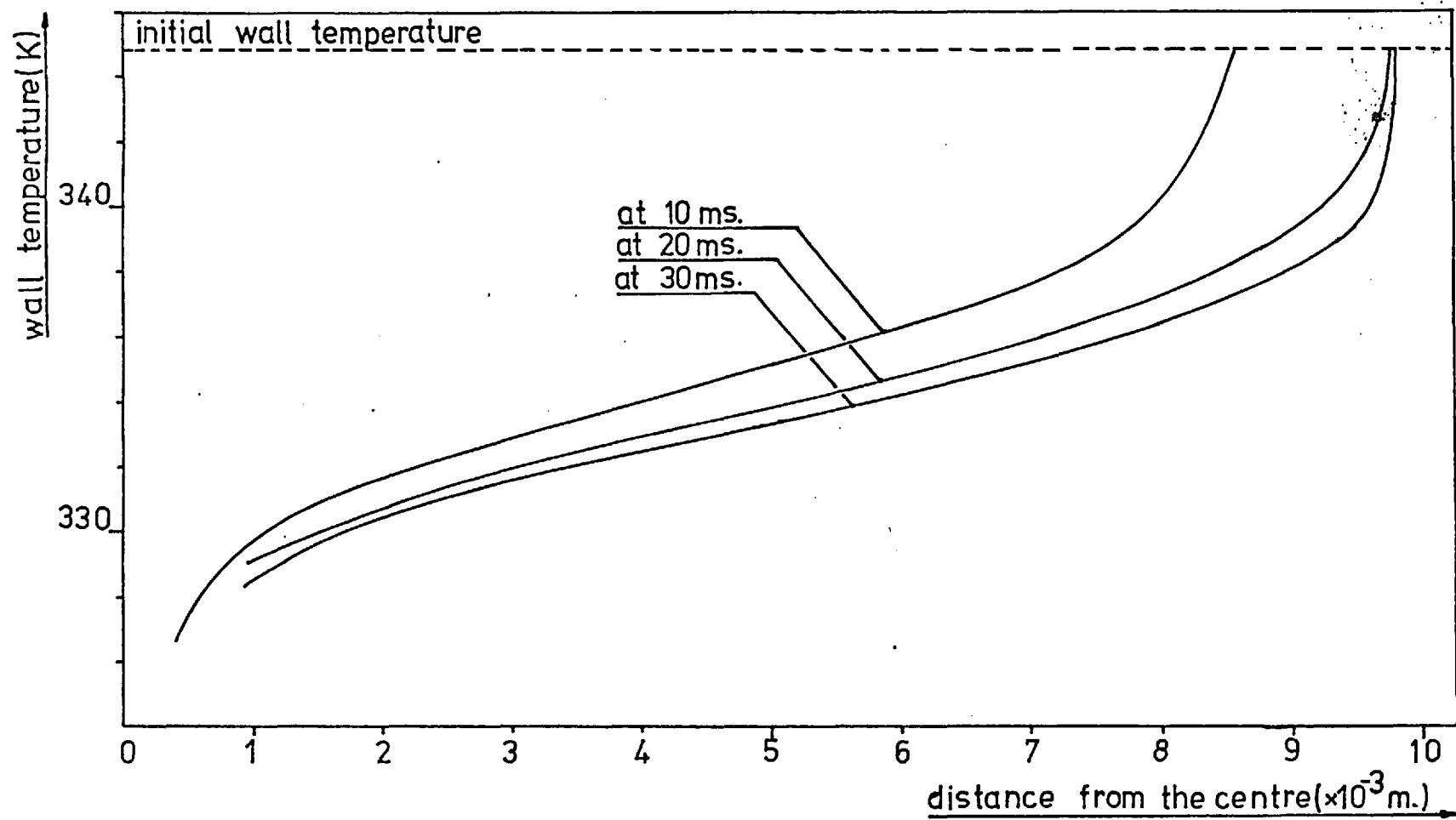


Fig.5-32 Variation of Wall Temperature Profile

CHAPTER 6

GENERAL DISCUSSION AND CONCLUSIONS

6.1 GENERAL DISCUSSION

Extensive work carried out by a number of workers have showed the importance of the microlayer in nucleate boiling. At the same time, there is a dearth of treatments of bubble dynamics taking into account the effects of liquid inertia. This is specially important in boiling at sub-atmospheric pressures as shown by Stewart and Cole (32). A simple model has been proposed which takes into account these two factors which is suitable for obtaining computer solutions for bubble growth in the case of rapidly growing bubbles where a microlayer exists. This model also enables the prediction of departure time and volume without considering an overall force balance on the bubble.

An accurate solution taking into account all these factors would be very complex and involve a simultaneous solution of the Navier-Stokes equation and the energy equation governing heat transfer into the bubble. The only previous attempt to solve the Navier-Stokes equation around a growing bubble on a heated surface was by Madhavan and Mesler (51) and they did not consider the energy equation. Instead they used an arbitrary function for the pressure variation inside the bubble. It is clear from this work that such attempts to predict growth are impractical. The present model proposes a realistic simplification of the problem, which aims to produce a clearer understanding of the mechanisms involved in boiling. The major innovation in this model is the independent consideration of growth at the top of the curved interface and of the base perimeter, thus providing a method of obtaining the approximate shape of the bubble

at all stages and a criterion for departure. Removing the restriction of a rigid shape for the bubble profile is believed to produce a better understanding of the mechanism, both of growth and departure. Suitably modified versions of the extended Rayleigh equation are applied separately to the top and the base to compute growth. Assumption of local spherical curvature is used both at the top and the base for this application. An expression has been developed to account for the higher viscous drag force in the vicinity of the heating surface which has yielded satisfactory results.

The extended Rayleigh equation takes into account the liquid inertia effects, within the limitations of the model, provided correct estimations of the pressure variation inside the bubble can be made. Correct evaluation of the heat transfer rates into the bubble are necessary for this. In the present model heat transfer into the bubble across the microlayer and across the curved interface are calculated separately. The estimate of heat transfer across the curved interface is based on the Cooper and Vijuk (33) adaptation for a hemispherical bubble of the Skinner and Bankoff (29) solution. This assumes a constant temperature at the interface equal to the saturation temperature at bulk pressure and also a hemispherical shape for the bubble. In order to use this approach in the present work an equivalent hemispherical bubble of equal surface area has been defined. Even though the evaluation of expressions for the heat transfer rates across the curved interface by this method initially assume a constant temperature at the interface, equal to the saturation temperature at bulk pressure, the correct temperature difference of $(T_{\infty} - T_i)$ is introduced in the process of inversion of the result to the present coordinates. As the total heat transfer across the curved interface in virtually all the bubble growth computations carried out for fast growing bubbles amounted only to

about 20% of the total heat transfer, errors in the estimation of heat transfer across the curved interface would not greatly affect the final result.

The evaporation from the microlayer has been estimated by three different means. The complexity of the present three programmes depend directly on these three methods. The first uses an approximate expression for the microlayer profile under the bubble. With the aid of this assumption it is possible to derive an expression for the total heat transfer across the microlayer during each time step, making the computer programme very simple. These results have been compared with the results from the second programme where the value of the microlayer thickness at each value of the radius is stored and a numerical summation of evaporation at each of these radii is effected to obtain the total evaporation. The results of these two programmes are in reasonable agreement and show that the first one can be used to obtain a first approximation without too much loss in accuracy. In both these cases constant wall temperatures have been assumed. For highly conducting wall materials this assumption is reasonable, but in the case of boiling on a wall of relatively low conductivity, it results in a considerable overestimate of growth rates and departure values. The third programme takes into account this variation of wall temperature. In this programme, during each time step, the microlayer thickness and the wall temperature are evaluated for each value of the radius and stored for use at the next time step. The complexity of this method increases the computing time by about six fold as compared to the second programme. By adopting the above procedure for the evaluation of microlayer evaporation at every eighth value of the radius instead of each value, computing time is reduced to a more realistic value, yet it produces very similar results. The method employed for the estimation of the

wall temperature is a modification of a solution given by Carslaw and Jaeger (37) to a similar problem. This method gives approximations of temperature variations on the surface of the expected order and yields satisfactory results. In all these three computer programmes drying of the microlayer due to a possible roll up phenomenon and any flow within the microlayer has been neglected. The expression given by Cooper and Lloyd (35) for the initial microlayer thickness has been used quite successfully.

The model gives satisfactory results for the growth of fast growing bubbles initially of hemispherical shape. Very close agreement of any theory with bubble growth data is likely to be fortuitous because of the statistical nature of the variation of growth rates from the same surface under the same boiling conditions, as shown by Streng, Orell and Westwater (36). The criterion for departure as the moment when the base radius recedes to a value less than the cavity radius has proved successful.

6.2 APPLICABILITY OF THE METHOD

The present method is applicable to subcooled boiling, low-gravity boiling and low pressure-high Jakob number situations with a minimum of change. The consideration of the pressure difference between the bubble base and the top, imposed by the hydrostatic head due to bubble height enables the model to take into account any effects due to a change of gravity. Under low gravity conditions the comparative growth of the base is less restricted and hence there is a greater tendency for hemispherical growth. Hydrostatic head due to bubble height is also of particular importance in low pressure boiling situations.

The method does not apply to the growth of spherical bubbles with a very small waiting time. Such bubbles are growing into the

highly turbulent wake region of a previous bubble, whereas the theory used in the present method assumes growth to take place into a comparatively quiescent liquid. It is possible that pressure fluctuations in the wake region give rise to initiation before the proper development of the thermal layer. At the low growth rates of such bubbles, inertia forces are greatly reduced and bubble shape will be influenced to a much greater extent by surface tension. The method of computation presented here assumes initial hemispherical growth which implies that the inertia forces are much greater than the surface tension effects.

6.3 SUGGESTIONS FOR FUTURE WORK

The present research has pointed out a number of inadequacies in the Han and Griffith (25) theory for the relationship between the cavity radius and the waiting time. This is marked in the case of very small and very large waiting times. Therefore, a closer study of the theory of waiting times, aimed at either an improvement of the existing method or achieving a different criterion of initiation will be very useful.

It has been suggested in the literature that burnout may occur under nucleate boiling conditions by the formation of a dry patch under the bubble. It is possible with the present method to keep track of the expansion of the dry patch and its time of contact with the heating surface. With this information, numerical solutions could be developed to evaluate the rise of temperature at the surface and to obtain an idea of the maximum temperature rise. A trace of this rise, throughout a succession of drying and quenching procedures will indicate whether burnout is possible.

In order to understand and alleviate problems of boiling such as scaling, fouling and corrosion, it is necessary to investigate

the behaviour of various micro features in bubble growth. For example, microlayer evaporation dry patch formation etc.. The present method opens up possibilities for a closer examination of these.

6.4 CONCLUSIONS

A comprehensive model has been developed for rapid bubble growth on a heated surface under pool boiling conditions which takes into account a large number of hitherto neglected factors, by the novel approach of considering growth at the top of the curved interface and at the base perimeter separately. By this method a realistic simplification of the application of the complicated governing equations can be achieved, bringing the numerical solution within the capabilities of the present day computers in a practical way. The factors which are included in the present model are

- (a) Heat transfer both across the microlayer and the curved interface, which also takes into account the waiting time.
- (b) Pressure and temperature variation inside the bubble.
- (c) Viscous drag force near the surface restraining the growth of the base perimeter.
- (d) Hydrostatic head due to bubble height, which may be of special importance in low pressure or variable gravity work.
- (e) Wall temperature variation under a bubble, which is of considerable importance in boiling on surfaces of relatively low conductivity.
- (f) Removal of the restriction of a shape assumption for the bubble.

A criterion is proposed which predicts departure times and radii as a result of the same growth computation, without recourse to a conventional overall force balance on the bubble.

The model has been tested against experimental data quoted in the literature, covering a wide range of boiling situations, with satisfactory results. Very consistent results have been obtained showing favourable comparison with existing growth equations. The model does not apply in the case of slow growing spherical bubbles.

The computation confirms that

- (a) Liquid inertia forces are of importance, especially in low pressure-high Jakob number situations.
- (b) The viscous drag force near the surface is an important factor both in growth and departure.
- (c) Assumption of constant wall temperature can lead to considerable errors in cases where the heater wall is of relatively low conductivity or very small heat capacity.

The present method uses cavity size as an initial condition, which is seldom reported in experimental work. However, this parameter has been investigated and found to have little effect on the calculated results.

REFERENCES

1. FIRMAN, E.C.; GARDNER, G.C.; and CLAPP, R.M.: A Review of the Application of Boiling Heat Transfer Developments to Plant Problems. Paper R1, Proc. Inst. Mech. Engrs. 180(3C) 1-15 (1965-66).
2. CLARK, J.A. (ed.): Theory and Fundamental Research in Heat Transfer. Proceedings of the Annual Meeting of the ASME, New York (1960).
3. TONG, L.D.: Boiling Heat Transfer and Two Phase Flow. John Wiley & Sons (1965).
4. TABOREK, J.; AOKI, T.; RITTER, R.B.; and PALEN, J.W.: Fouling: The Major Unresolved Problem in Heat Transfer. Chem. Eng. Progr. 68(2) 59-67 (1972).
5. ROHSENOW, W.M.: Developments in Heat Transfer. M.I.T. Press (1964).
6. ROHSENOW, W.M.; and CLARK, J.A.: A study of the Mechanism of Boiling Heat Transfer. Trans. ASME 73 609-620 (1951).
7. HSU, Y.Y.; and GRAHAM, R.W.: An Analytical and Experimental Study of Thermal Boundary Layer and Ebullition Cycle in Nucleate Boiling. NASA TN D-594 (1961).
8. FORSTER, K.E.; and GREIF, R.: Heat Transfer to a Boiling Liquid - Mechanisms and Correlations. Trans. ASME. J. Heat Transfer 81(1) 43-53 (1959).
9. MIXON, F.O., Jr.; CHON, W.Y.; and BEATTY, K.E., Jr.: The Effect of Electrolytic Gas Evolution on Heat Transfer. A I Ch E Chem. Eng. Progr. Symp. Ser. 56 (30) 75-81 (1960).
10. GUNTHER, F.C.: Photographic Study of Surface-Boiling Heat Transfer to Water with Forced Convection. Trans. ASME 73 115-123 (1951).

11. SNYDER, N.R.; and EDWARDS, D.K.: Post Conference Comments.
Summary of Conference on Bubble Dynamics and Boiling Heat Transfer. Memo No. 20-137, Jet Propulsion Lab., Cal. Inst. Tech. (1956).
12. MADSEN, N.: Temperature Fluctuations at a Heated Surface Supporting Pool Boiling of Water. Paper 14, Proc. Inst. Mech. Engrs. 180(3C) 150-159 (1965-66).
13. MOORE, F.D.; and MESLER, R.B.: The Measurement of Rapid Surface Temperature Fluctuations During Nucleate Boiling of Water. A I Ch E J. 7 (4) 620-624 (1961).
14. HENDRICKS, R.C.; and SHARP, R.R.: Initiation of Cooling Due to Bubble Growth on a Heating Surface. NASA TN D-2290 (1964).
15. BONNET, C.; MACKE, E.; and MORIN, R.: Visualization of Bubble Formation at Atmospheric Pressure and Related Measurement of the Wall Temperature Variations. EUR 1622-e (1965).
16. COOPER, M.G.; and LLOYD, A.J.P.: Transient Local Heat Flux in Nucleate Boiling. Proceedings of the Third International Heat Transfer Conference, Chicago, Illinois, 3 Paper 100, 193-203 (1966).
17. SHARP, R.R.: The Nature of Liquid Film Evaporation During Nucleate Boiling. NASA TN D-1997 (1964).
18. PLESSET, M.S.; and ZWICK, S.A.: The Growth of Vapour Bubbles in Superheated Liquids. J. Appl. Phys. 25 (4) 493-500 (1954).
19. PLESSET, M.S.; and ZWICK, S.A.: A Nonsteady Heat Diffusion Problem with Spherical Symmetry. J. Appl. Phys. 23 (1) 95-98 (1952).

20. FORSTER, H.K.; and ZUBER, N.: Growth of a Vapour Bubble in a Superheated Liquid. J. Appl. Phys. 25 (4) 474-478 (1954).
21. SCRIVEN, L.E.: On the Dynamics of Phase Growth. Chem. Eng. Sci. 10 (1/2) 1-13 (1959).
22. COLE, R.; and SHULMAN, H.L.: Bubble Growth Rates at High Jakob Numbers. Int. J. Heat Mass Transfer 9 (12) 1377-1389 (1966).
23. GRIFFITH, P.: Bubble Growth Rates in Boiling. Trans. ASME, 80 (3) 721-726 (1958).
24. SAVIC, P.: Discussion on 'Bubble Growth Rates in Boiling'. Trans. ASME 80 (3) 726-727 (1958).
25. HAN, C.Y.; and GRIFFITH, P.: The Mechanism of Heat Transfer in Nucleate Pool Boiling - Part 1. Int. J. Heat Mass Transfer 8 (6) 887-904 (1965).
26. ZUBER, N.: Vapour Bubbles in Non-uniform Temperature Field. Int. J. Heat Mass Transfer 2 (1/2) 83-98 (1961).
27. SKINNER, L.A.; and BANKOFF, S.G.: Dynamics of Vapour Bubbles in Spherically Symmetric Temperature Fields of General Variation. The Phys. of Fluids 7 (1) 1-6 (1964).
28. MIKIC, B.B.; and ROHSENOW, W.M.: Bubble Growth Rates in Non-uniform Temperature Field. Progr. in Heat & Mass Transfer 2 283-293 (1969).
29. SKINNER, L.A.; and BANKOFF, S.G.: Dynamics of Vapour Bubbles in General Temperature Fields. The Phys. of Fluids 8 (8) 1417 - 1420 (1965).
30. MIKIC, B.B.; ROHSENOW, W.M.; and GRIFFITH, P.: On Bubble Growth Rates. Int. J. Heat Mass Transfer 13 (4) 657-666 (1970).
31. Van STRALEN, J.D.: The Mechanism of Nucleate Boiling in Pure Liquids and in Binary Mixture, Parts I and II. Int. J. Heat Mass Transfer 9 (10) 995-1046 (1966).

32. STEWART, J.K.; and COLE, R.: Bubble Growth Rates During Nucleate Boiling at High Jakob Numbers. Int. J. Heat Mass Transfer 15 (4) 655-663 (1972).
33. COOPER, M.G.; and VIJUK, R.M.: Bubble Growth in Nucleate Pool Boiling. Proceedings of the 4th International Heat Transfer Conference, Paris-Versailles, V Paper B 2.1 (1970).
34. COOPER, M.G.: The Microlayer and Bubble Growth in Nucleate Pool Boiling. Int. J. Heat Mass Transfer 12 915-933 (1969).
35. COOPER, M.G.; and LLOYD, A.J.P.: The Microlayer in Nucleate Pool Boiling. Int. J. Heat Mass Transfer 12 895-913 (1969).
36. STRENCE, P.H.; ORELL, A.; and WESTWATER, J.W.: Microscopic Study of Bubble Growth During Nucleate Boiling. A I Ch E J. 7 (4) 578-583 (1961).
37. CARSLAW, H.S.; and JAEGER, J.C.: Conduction of Heat in Solids, 2nd ed. Oxford University Press (1959).
38. DZAKOWIC, G.S.; and FROST, W.: Vapour Bubble Growth in Saturated Pool Boiling by Microlayer Evaporation of Liquid at the Heated Surface. Proceedings of the 4th International Heat Transfer Conference, Paris - Versailles V Paper B 2.2 (1970).
39. FRITZ, W.: Phys. Zeitschrift 36 379 (1935).
40. COLE, R.; and SHULMAN, H.L.: Bubble Departure Diameters at Subatmospheric Pressures. A I Ch E Chem. Eng. Progr. Symp. Ser. 62 (64) 6-16 (1966).
41. COLE, R.: A Photographic Study of Pool Boiling in the Region of the Critical Heat Flux. A I Ch E J. 6 (4) 533-538 (1960).
42. ZUBER, N.: AECU - 4439 (June, 1959).
43. STANIZEWSKI, B.E.: Tech. Rept. 16, Div. Sponsored Res., M.I.T. (1959).

44. ROLL, J.B.; and MYERS, J.E.: The Effect of Surface Tension on Factors in Boiling Heat Transfer. A I Ch E J. 10 (4) 530 - 534 (1964).
45. SEMERIA, R.L.: An Experimental Study of the Characteristics of Vapour Bubbles. Symposium on Two Phase Fluid Flow, Inst. Mech. Engrs, London, Paper 7 (1962).
46. MINNS, D.E.: A study of Nucleate Boiling in a Liquid Flowing Through a Vertical Tube. Ph.D. Thesis, Imperial College, University of London (1969).
47. KESHOCK, E.G.; and SIEGEL, R.: Forces Acting on Bubbles in Nucleate Boiling under Normal Gravity Conditions. NASA TN D-2299 (1964).
48. COCHRAN, T.H.; and AYDELOTT, J.C.: Effects of Subcooling and Gravity Level on Boiling in the Discrete Bubble Region. NASA TN D-3449 (1966).
49. HATTON, A.P.; JAMES, D.D.; and LIEW, T.L.: Measurement of Bubble Characteristics for Pool Boiling from Single Cylindrical Cavities. Proceedings of the Fourth International Heat Transfer Conference, Paris - Versailles V Paper B 1.2 (1970).
50. JOHNSON, M.A. Jr.; JAVIER DE LA PENA; and MESLER, R.B.: Bubble Shapes in Nucleate Boiling. A. I. Ch E J. 12 (2) 344-348 (1966).
51. MADHAVAN, S.; and MESLER, R.: A Study of Vapour Bubble Growth on Surfaces. Proceedings of the Fourth International Heat Transfer Conference, Paris - Versailles V Paper B 2.6 (1970).
52. HOSPETI, N.B.; and MESLER, R.B.: Vaporization at the Base of Bubbles of Different Shape During Nucleate Boiling of Water. A I Ch E J. 15 (2) 214-219 (1969).

53. MERRY, J.M.D.: Heat Transfer in Nucleate Boiling - The Microlayer. Ph.D. Thesis, University Engineering Department, Cambridge, (1969).
54. CLARK, H.B.; STRENCE, P.S.; and WESTWATER, J.W.: Active Sites in Nucleate Boiling. A I Ch E Chem. Eng. Progr. Symp. Ser. 55 (29) 103-110 (1959).
55. GRIFFITH, P.; and WALLIS, J.D.: The Role of Surface Conditions in Nucleate Boiling. A I Ch E Chem. Eng. Progr. Symp. Ser. 56 (30) 49-63 (1960).
56. ROHSENOW, W.M.: Nucleation with Boiling Heat Transfer. ASME Paper No. 70-HT-18 (1970).
57. HELED, Y.; RICKLIS, J.; and ORELL, A.: Pool Boiling from large Arrays of Artificial Nucleation Sites. Int. J. Heat Mass Transfer 13 (3) 503-516 (1970).
58. HOWELL, J.R.; and SIEGEL, R.: Activation, Growth and Detachment of Boiling Bubbles in Water from Artificial Nucleation Sites of Known Geometry and Size. NASA TN D-4101 (1967).
59. YOUNG, R.K.: Pool Boiling with Precise Arrays of Active Sites. Ph.D. Thesis, University of Toronto (1968).
60. GADDIS, E.S.; HALL, W.B.: Forced Convection Boiling from Prepared Nucleation Sites. Proceedings of the Fourth International Heat Transfer Conference, Paris - Versailles V Paper B 4.4 (1970).
61. OLANDER, R.R.; and WATTS, R.G.: An Analytical Expression of Microlayer Thickness in Nucleate Boiling. Trans. ASME. J. Heat Transfer 91 (1) 178-180 (1969).
62. HSU, Y.Y.: On the Size Range of Active Nucleation Cavities on a Heating Surface. Trans. ASME, J. Heat Transfer 84 (3) 207-216 (1962).

63. KENNING, D.B.R.; and COOPER, M.G.: Flow Patterns near Nuclei and the Initiation of Boiling during Forced Convection Heat Transfer, Paper 11. Proc. Inst. Mech. Engrs. 180 (3C) 112-123 (1965-66).
64. LLOYD, A.J.P.: Transient Heat Flow in Nucleate Boiling Ph.D. Thesis, University Engineering Department, Cambridge, (1966).
65. COLE, R.: Bubble Frequencies and Departure Volumes at Subatmospheric Pressures. A I Ch E J. 13 (4) 779-783 (1967).
66. KERN, D.Q.: Process Heat Transfer. McGraw-Hill Book Company (1950).
67. MILNE-THOMPSON, L.M.: Theoretical Hydrodynamics, 3rd ed. McMillan, London (1955).
68. CARNAHAN, B.; LUTHER, H.A.; and WILKES, T.O.: Applied Numerical Methods. John Wiley & Co. Limited (1969).
69. KREYSZIG, E.: Advanced Engineering Mathematics, 3rd ed. John Wiley & Sons Inc. (1972).
70. COLLATZ, L.: The Numerical Treatment of Differential Equations, 3rd ed. Springer-Verlag-Berlin (1960).
71. PERRY, R.H.; and CHILTON, C.H. (ed.): Chemical Engineers Handbook, 5th Edition. McGraw-Hill Book Company (1973).
72. WEAST, R.C. (ed): Handbook of Chemistry and Physics, 51st Edition. The Chemical Rubber Company (1970).
73. ENGINEERING SCIENCES DATA UNIT, ROYAL AERONAUTICAL SOCIETY: Physical Data, Chemical Engineering Series, Vol. 1, 2 & 3. (1973).

APPENDIX I

APPLICATION OF THE RUNGE-KUTTA FOURTH ORDER METHOD TO THE PRESENT PROBLEM

I-1 RUNGE-KUTTA METHODS

Runge-Kutta methods are very widely used for numerical integration of Ordinary Differential Equations of higher order. These methods employ a basic forward type integration and are very convenient for obtaining higher order approximations. This is one of the most accurate methods available for initial value problems.

All the Runge-Kutta methods have algorithms of the form

$$y_{n+1} = y_n + h \phi(x_n, y_n, h) \quad \text{I-1}$$

where 'h' is the step size.

A commonly used algorithm for the fourth-order method for the solution of a first order differential equation $y' = f(x, y)$, where $y' = \frac{dy}{dx}$, can be denoted by

$$\left. \begin{aligned} y_{n+1} &= y_n + \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4) \\ k_1 &= hf(x_n, y_n) \\ k_2 &= h.f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1) \\ k_3 &= h.f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2) \\ k_4 &= h.f(x_n + h, y_n + k_3) \end{aligned} \right\} \quad \text{I-2}$$

The points at which k_1, k_2, k_3, k_4 are evaluated are represented graphically in Fig I-1.

A solution using this method to a second order differential equation

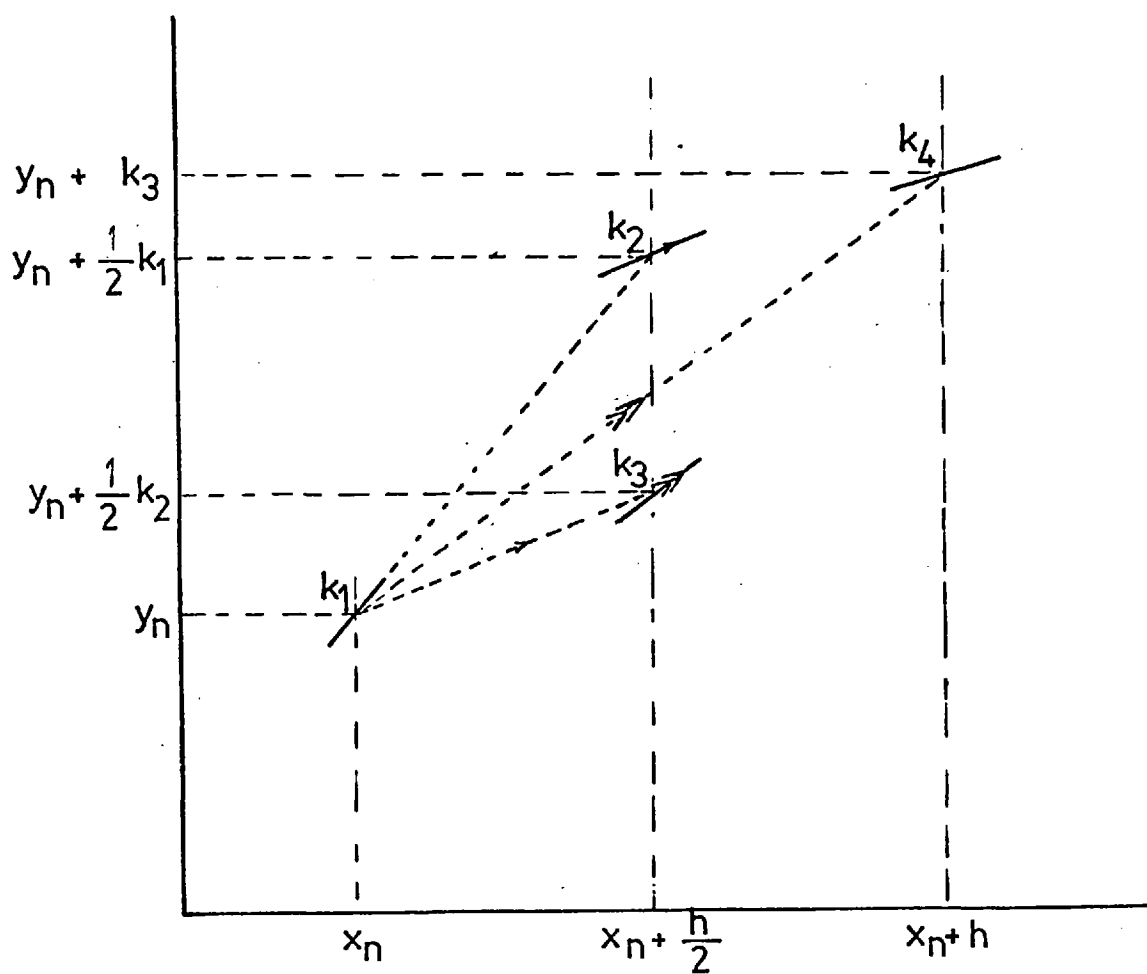


Fig I:1 Runge-Kutta Method

$$y'' = f(x, y, y')$$

can be obtained by reducing the second order equation to two first order equations and the application of the above formula (eqn. I-2). If z is defined such that $z = y'$, the second order equation can be rewritten as

$$\left. \begin{aligned} z &= y' \\ z' &= f(x, y, z) \end{aligned} \right\} \text{I-3}$$

This approach can be effectively represented by the following set of equations.

$$\begin{aligned} y_{n+1} &= y_n + y'_n h + \frac{1}{3} (k_1 + k_2 + k_3) \\ y'_{n+1} &= y'_n + \frac{1}{3h} (k_1 + 2k_2 + 2k_3 + k_4) \\ k_1 &= \frac{h^2}{2} \cdot f(x_n, y_n, y'_n) \\ k_2 &= \frac{h^2}{2} \cdot f(x_n + \frac{1}{2}h, y_n + y'_n \frac{h}{2}, y'_n + \frac{k_1}{h}) \\ k_3 &= \frac{h^2}{2} \cdot f(x_n + \frac{1}{2}h, y_n + y'_n \frac{h}{2} + \frac{k_1}{2}, y'_n + \frac{k_2}{h}) \\ k_4 &= \frac{h^2}{2} \cdot f(x_n + h, y_n + y'_n h + k_2, y'_n + \frac{2k_3}{h}) \end{aligned} \text{I-4}$$

I-2 DISCUSSION OF THE TRUNCATION ERROR, STABILITY AND STEP-SIZE IN THE RUNGE-KUTTA METHODS

Since the fourth order Runge-Kutta methods, generated by requiring that

$$y_{n+1} = y_n + h \phi(x_n, y_n, h),$$

agree with the Taylor's expansion of the solution function $y(x)$

through terms of the order h^4 , the local truncation error e_t is of the form

$$e_t = Kh^5 + O(h^6) \quad \text{I-5}$$

Normally K depends upon $f(x, y)$ and its higher order partial derivatives in a very complicated manner. If ' h ' is sufficiently small $O(h^6)$ terms can be neglected in the error evaluation. One of the serious drawbacks of the Runge-Kutta method is lack of simple means for estimating the error.

In order to choose a reasonable step size, one needs some estimate of the error being committed in integrating across one step. On the one hand step size should be small enough to achieve the required accuracy, on the other it should be as large as possible in order to keep rounding errors (a function of the number of arithmetic operations carried out) under control and to avoid an excessive number of derivative calculations. This latter consideration is important, particularly when the differential equation is complicated and each derivative calculation requires substantial computing time. For the fourth order methods the derivative must be evaluated four times for each integration step.

One approach to the problem of evaluation of truncation errors is to assume that the local truncation errors have the form Kh^5 with K constant, and that the local truncation error dominates the change in total error for one step. Then an indication of the local truncation error can be found in the following manner.

Suppose $y_{n+1,1}$ and $y_{n+1,2}$ are the solutions obtained at x_{n+1} , by integrating between x_n and x_{n+1} , using two different step sizes h_1 and h_2 respectively. As Carnahan et al. (78) have shown, if h_2 is chosen to be $h_1/2$ and $x_{n+1} - x_n = h_1$, the truncation error is given by

$$e_t = Kh_1^{m+1} = \frac{2^m (y_{n+1,2} - y_{n+1,1})}{2^m - 1} \quad \text{for the}$$

m^{th} order method.

For the 4^{th} order method

$$e_t = Kh_1^5 = \frac{16}{15} (y_{n+1,2} - y_{n+1,1}) \quad \text{I-6}$$

This procedure, if used as a monitoring method throughout a computation, unfortunately lengthens the computation threefold. As a compromise the monitoring procedure could be used less frequently. In general, if the local truncation error for a one step method is $O(h^{m+1})$, then the accumulated error will be $O(h^m)$.

All the Runge-Kutta methods can be shown to be convergent, that is, $\lim_{h \rightarrow 0} [y_n - y(x_n)] = 0$. Some equations are inherently unstable, which cannot be solved by any step-by-step method with specified initial conditions. But partial instability is a phenomenon related to the step-size chosen. The error at a point is a combination of the local truncation error and the propagated error. When certain conditions (usually very complicated) which have the step-size as one of their parameters are not satisfied the error at x_n will be amplified in traversing the n^{th} step, and the solution will tend towards instability. Normally, sufficiently low values of 'h' satisfy these conditions and stability can be obtained quite readily in the Runge-Kutta methods. In practice, a suitable step size can be easily chosen by observing the behaviour of the solutions using different step-sizes, with regard to apparent convergence and stability.

A rule-of-thumb method of choosing the step-size, suggested by Collatz (70), stipulates that the difference between k_2 and k_3 should not exceed in magnitude a few percent of the difference between k_1

and k_2 ; otherwise a smaller step should be taken.

Throughout the present computations step sizes of the order of 10^{-5} s. were found to be very satisfactory.

I-3 APPLICATION OF THE METHOD TO THE PRESENT PROBLEM

In the problem considered here two second order ordinary differential equations have to be solved simultaneously to obtain growth at the top and at the base separately. The two equations are given by 3.15 and 3.17

$$\ddot{R}_T = \frac{1}{R_T} \left[\frac{p_i - (p_\infty - R_T \rho_\ell g)}{\rho_\ell} - \frac{3}{2} \dot{R}_T^2 - \frac{1}{\rho_\ell} \cdot \frac{2\sigma}{R_T} - \frac{4\mu}{\rho_\ell} \frac{\dot{R}_T}{R_T} \right] \quad \text{I-7}$$

and

$$\ddot{R}_B = \frac{1}{R_B} \left[\frac{p_i - p_\infty}{\rho_\ell} - \frac{\Delta p_d}{\rho_\ell} - \frac{3}{2} \dot{R}_B^2 - \frac{1}{\rho_\ell} \cdot \frac{2\sigma}{R_B} \right] \quad \text{I-8}$$

where

$$\frac{\Delta p_d}{\rho_\ell} = \frac{R_B \dot{R}_B}{2t} \left[\exp \left(- \left(\frac{\delta_o}{2 \sqrt{vt}} \right)^2 \right) - \sqrt{\pi} \frac{\delta_o}{2 \sqrt{vt}} \operatorname{erfc} \frac{\delta_o}{2 \sqrt{vt}} \right] \quad \text{I-9}$$

When compared with a general equation of the form $y'' = f(x, y, y')$ as used in the preceeding sections,

$$\begin{aligned} x &= t \\ y &= R_T, R_B \\ y' &= \dot{R}_T, \dot{R}_B \\ y'' &= \ddot{R}_T, \ddot{R}_B \end{aligned}$$

The initial conditions for the above equations are given by

$$R_T = R_C, \quad \dot{R}_T = 0, \quad \ddot{R}_T = 0$$

$$R_B = R_C, \quad \dot{R}_B = 0, \quad \ddot{R}_B = 0 \quad \text{I-11}$$

$$p_i = p_\infty + \frac{2\sigma}{R_C} \quad \text{I-12}$$

$$T_i = B/(A + \ln p_i) \quad \text{I-13}$$

In advancing the calculation across one time step by the Runge-Kutta fourth order method, the time step is divided in to two sub-steps, and the derivative calculation is carried out four times during the time step. The procedures involved in these four calculations are very similar and therefore, an outline of the stages in the computation for only one sub-step is given below.

Subscript 'n' denotes the values of the parameters at the beginning of the n^{th} step and the subscript '(n+1)' denotes the values after the first sub-step in the n^{th} step. Let ' δt ' be one sub-step.

$\ddot{R}_{T(n)}$ and $\ddot{R}_{B(n)}$ can be found by substitution of known values at the end of the $(n-1)^{\text{th}}$ step in equation I-7 and I-8. From these known parameters,

$$\left. \begin{aligned} R_{T(n+1)} &= R_{T(n)} + \dot{R}_{T(n)} \cdot \delta t \\ \dot{R}_{T(n+1)} &= \dot{R}_{T(n)} + \ddot{R}_{T(n)} \cdot \delta t \\ R_{B(n+1)} &= R_{B(n)} + \dot{R}_{B(n)} \cdot \delta t \\ \dot{R}_{B(n+1)} &= \dot{R}_{B(n)} + \ddot{R}_{B(n)} \cdot \delta t \end{aligned} \right\} \quad \text{I-14}$$

To find the corresponding interfacial accelerations, it is necessary to evaluate the pressure increase during the sub-step. First the increases in volume and mass have to be evaluated. The volume increase is given by

$$\left. \begin{aligned} \delta V_b &= \frac{\pi}{2} \left[R_{B(n)}^2 + R_{T(n)}^2 \right] \cdot \delta R_T + \pi R_{B(n)} R_{T(n)} \cdot \delta R_B \\ \delta R_T &= \dot{R}_{T(n)} \cdot \delta t \\ \delta R_B &= \dot{R}_{B(n)} \cdot \delta t \end{aligned} \right\} \quad \text{I-15}$$

The Mass Evaporated or condensed across the curved interface is given by the following set of equations (section 4.2.4).

$$\delta m_{cs} = k_l \times 2\pi R_{es}^2 \left[(T_w - T_s) \cdot \frac{R^2}{l^3} \cdot \frac{\partial T^*}{\partial r^*} (0, t^*) \right] \cdot \frac{\delta t}{L} \quad \text{I-16}$$

where

$$\frac{\partial T^*}{\partial r^*} (0, t^*) = \frac{1}{\sqrt{\pi t^*}} \int_0^\infty g(2\sqrt{r'' t^*}) e^{-r''} \cdot dr'' \quad \text{I-17}$$

$$\begin{aligned} g(r^*) &= \frac{1}{(T_w - T_s)} \left\{ (T_\infty - T_s) + (T_w - T_s) \left[\operatorname{erfc} \frac{(3r^{*l^3} + R_c^3)^{1/3}}{2\sqrt{\alpha_l t_w}} \right. \right. \\ &\quad \left. \left. + \frac{1}{\sqrt{\pi}} \frac{2\sqrt{\alpha_l t_w}}{(3r^{*l^3} + R_c^3)^{1/3}} \left(1 - \exp \left(- \frac{(3r^{*l^3} + R_c^3)^{1/3}}{2\sqrt{\alpha_l t_w}} \right) \right) \right] \right\} \quad \text{I-18} \end{aligned}$$

$$t^* = \frac{\alpha_l}{l^6} \int_0^t R_{es}^4 \cdot dt \quad \text{I-19}$$

$$\left[t_{n+1}^* = t_n^* + \frac{\alpha_l}{l^6} \cdot R_{es}^4 \cdot \delta t \right]$$

$$R_{es} = \sqrt{R_{T(n)} \cdot R_{(n)}} \quad \text{I.20}$$

Equation I-17 can be readily evaluated during each time step.

'l' can have any arbitrary value.

The waiting time 't_w' is evaluated at time t = 0 using (section 4.2.2).

$$\text{erfc} \frac{\frac{3}{2} R_c}{2 \sqrt{\alpha_l t_w}} = \frac{T_i - T_\infty}{T_w - T_\infty} \quad \text{I.21}$$

The Mass evaporated from the microlayer during time 'δt' is evaluated using one of three different methods applicable in different situations.

Method 1

For the case of microlayer evaporation in a poorly conducting liquid on a highly conducting wall, and where an expression for the microlayer profile is assumed.

$$\left. \begin{aligned} a &= \frac{\delta_2^2}{R_2^2 - R_1^2} \\ b &= \frac{\delta_2^2 \cdot R_1}{R_2^2 - R_1^2} \end{aligned} \right\} \quad \text{I.22}$$

$$R_1 = R_n'$$

$$R_2 = R_{B(n)}$$

$$R_{(n+1)}' = \sqrt{\left[\frac{k_l (T_w - T_s)}{\rho_l L} \cdot \delta t + b \right] / a} \quad \text{I.23}$$

$$\delta m_{m1} = \frac{2\pi\rho_\ell}{3a} (a R_{n+1}'^2 - b)^{3/2} \quad \text{I-24}$$

$$\delta m_{m2} = \frac{2\pi\rho_\ell}{a} \cdot \frac{k_\ell (T_w - T_s)}{\rho_\ell L} \cdot \delta t \left[\sqrt{a R_2^2 - b} - \sqrt{a R_{n+1}'^2 - b} \right] \quad \text{I-25}$$

$$\delta m_m = \delta m_{m1} + \delta m_{m2} \quad \text{I-26}$$

Method II

Here, no microlayer profile is assumed. Total evaporation is obtained by summing point values.

$$\delta m_m = \sum_{R=R_1}^{R_2} [-\delta(\delta)] \quad \text{I-27}$$

$$-\delta(\delta) = \frac{k_\ell (T_w - T_s) \cdot \delta t}{\rho_\ell L \delta} \quad \text{I-28}$$

Method III

Variation of the wall temperature T_w is taken into account in this case. (section 4.3.4)

$$T_w = T_w' + \delta T \quad \text{I-29}$$

$$\delta T = \frac{q_{av}}{k_w} \frac{(T_{wo} - T_s) [1 - \exp(X^2) \operatorname{erfc}(X)]}{\left. \frac{\partial T}{\partial x} \right|_{x=0}} \quad \text{I-30}$$

$$X = \frac{k_\ell}{k_w} \sqrt{\frac{\alpha_w (t - t_g)}{\delta}} \quad \text{I-31}$$

$$\left. \frac{\partial T}{\partial x} \right|_{x=0} = \frac{k_\ell}{k_w} \frac{(T_{w(n-1)} - T_s)}{\delta} \quad \text{I-32}$$

T_w evaluated in this manner from equation I-29 is used in I-27 to give δm_m .

With the knowledge of the mass and volume increase the new pressure and temperature inside the bubble can be found from the following.

(section 3.7)

$$m_{n+1} = m_n + \delta m_m + \delta m_{cs} \quad \text{I-33}$$

$$V_{b(n+1)} = V_{b(n)} + \delta V_b \quad \text{I-34}$$

$$\ln p_{n+1} = A - B/T_{n+1} \quad \text{I-35}$$

$$p_{n+1} \cdot V_{b(n+1)} = m_{n+1} G \cdot T_{n+1} \quad \text{I-36}$$

Solution of equation I-35 and I-36 yields the new pressure and temperature.

Now sufficient parameters are known at the end of the sub-step for substitution in I-7 and I-8 to obtain $\ddot{R}_{T(n+1)}$ and $\ddot{R}_{B(n+1)}$.

A repetition of this procedure four times in one time step, according to the Runge-Kutta method is necessary to obtain the values for the next time step.

APPENDIX II

EXPERIMENTAL DATA VALUES AND CORRESPONDING PHYSICAL PROPERTY VALUES

Physical property values of fluids used in present computations corresponding to different experimental conditions are given in Table II-1. For some liquids approximate values have been used due to the non-availability of accurate values. All physical properties have been evaluated at the saturation temperature corresponding to the bulk pressure.

Table II-2 gives the values of time and radius for different sets of experimental observations.

Bubble Refs.	Source Ref.	Latent Heat $\times 10^5 \text{ J/kg. } ^\circ\text{K}$	Gas Constant $\text{J/kg. } ^\circ\text{K}$	Surface Tension N/m	Liquid Density kg./m^3	Thermal Conductivity $\text{W/m } ^\circ\text{K}$	Dynamic Viscosity $\times 10^{-3} \text{ Ns/m}^2$	Specific Heat $\text{J/kg } ^\circ\text{K}$
A, B, C	25	22.70	461.9	0.059	958.0	0.681	0.283	4218
D, E	52	22.70	461.9	0.059	958.0	0.681	0.283	4218
F	22	10.89	259.8	0.0201	791.0	0.189	0.345	2680
G	22	11.09	259.8	0.0201	791.0	0.192	0.380	2630
H	22	11.30	259.8	0.0205	791.0	0.194	0.410	2615
I	22	11.40	259.8	0.0210	791.0	0.197	0.469	2560
J	22	23.46	461.9	0.0652	980.0	0.659	0.430	4188
K	22	23.80	461.9	0.0678	987.5	0.644	0.539	4181
L	22	5.47	143.1	0.0230	792.0	0.159	0.300	2215
M, N, O	32	24.22	461.9	0.0708	994.8	0.622	0.762	4179
P*	53	23.85	461.9	0.0680	989.0	0.640	0.540	4190
Q*	53	23.45	461.9	0.0650	977.0	0.664	0.540	4190

*Thermal conductivity of wall (Glass) = $1.173 \text{ W/m } ^\circ\text{K}$

Density of wall material = 2230 kg/m^3

Specific Heat of wall material = $725 \text{ J/kg. } ^\circ\text{K}$

TABLE II-1 PROPERTY VALUES OF FLUIDS

BUBBLE A		BUBBLE B	
TIME (MSEC.)	RADIUS (MM.)	TIME (MSEC.)	RADIUS (MM.)
1.58	.80	1.45	.79
3.17	1.10	4.35	1.16
5.55	1.34	5.80	1.24
7.93	1.43	7.97	1.33
9.51	1.45	10.15	1.37
11.89	1.53	11.60	1.39
13.48	1.56	13.05	1.45
16.65	1.62	14.50	1.46
BUBBLE C		BUBBLE D	
TIME (MSEC.)	RADIUS (MM.)	TIME (MSEC.)	RADIUS (MM.)
.87	.53	1.00	.65
3.50	1.00	1.50	.83
5.26	1.06	2.00	.93
7.89	1.12	3.00	1.10
9.64	1.16	4.00	1.19
12.27	1.23	5.00	1.26
14.03	1.22	6.00	1.31
15.78	1.21	7.00	1.35

TABLE II-2 DATA VALUES FOR TIME AND RADIUS

BUBBLE E		BUBBLE F	
TIME (MSEC.)	RADIUS (MM.)	TIME (MSEC.)	RADIUS (MM.)
1.00	.89	.80	.40
1.50	1.14	1.80	.65
2.00	1.33	3.50	.93
3.00	1.63	5.05	1.12
4.00	1.86	6.10	1.20
5.00	2.04	9.10	1.40
6.00	2.19	12.00	1.57
7.00	2.33	14.80	1.72
BUBBLE G		BUBBLE H	
TIME (MSEC.)	RADIUS (MM.)	TIME (MSEC.)	RADIUS (MM.)
.80	.50	2.80	1.35
1.35	.72	6.50	2.15
3.30	1.12	10.00	2.60
6.00	1.40	13.00	2.90
7.20	1.60	16.90	3.30
11.30	1.90	20.00	3.55
14.50	2.11	23.90	3.80
17.50	2.30	27.80	4.10

TABLE II-2 DATA VALUES FOR TIME AND RADIUS (CTD.)

BUBBLE I		BUBBLE J	
TIME (MSEC.)	RADIUS (MM.)	TIME (MSEC.)	RADIUS (MM.)
1.50	1.25	1.50	1.35
4.30	2.35	2.70	2.10
8.30	3.05	6.50	3.75
12.30	3.65	8.50	4.50
15.50	4.15	11.00	5.00
21.30	4.75	13.00	5.65
26.00	5.30	15.50	6.12
32.00	5.85	18.50	6.60
BUBBLE K		BUBBLE L	
TIME (MSEC.)	RADIUS (MM.)	TIME (MSEC.)	RADIUS (MM.)
1.50	2.25	3.60	2.00
2.50	3.10	8.00	3.15
5.00	4.50	13.00	4.00
7.00	5.50	16.00	4.50
10.50	6.75	24.00	5.50
13.00	7.25	28.00	6.00
14.50	7.85	32.00	6.40
18.50	8.75	35.50	6.60

TABLE II-2 DATA VALUES FOR TIME AND RADIUS (CTD.)

BUBBLE M		BUBBLE N	
TIME (MSEC.)	RADIUS (MM.)	TIME (MSEC.)	RADIUS (MM.)
2.60	1.81	1.30	2.54
4.30	3.00	2.00	3.69
6.00	4.15	2.70	4.42
7.80	5.38	4.00	6.05
10.30	7.00	5.30	7.65
13.10	8.70	6.80	8.97
15.20	10.20	7.50	10.00
17.20	11.48	9.70	11.91
BUBBLE O		BUBBLE P	
TIME (MSEC.)	RADIUS (MM.)	TIME (MSEC.)	RADIUS (MM.)
2.40	2.61	5.00	7.50
3.90	3.88	10.00	10.14
5.30	5.04	15.00	11.57
6.90	6.23	20.00	12.07
8.10	7.11	25.00	12.07
9.10	7.89	30.00	11.92
11.20	9.50	40.00	11.30
13.30	11.78	50.00	10.76

TABLE II-2 DATA VALUES FOR TIME AND RADIUS (CTD.)

BUBBLE Q	
TIME (MSEC.)	RADIUS (MM.)
2.50	3.97
5.00	5.29
10.00	6.68
15.00	7.12
20.00	7.29
25.00	7.33
30.00	7.29
35.00	7.20

TABLE II-2 DATA VALUES FOR TIME AND RADIUS (CTD.)

APPENDIX III

DESCRIPTION AND USE OF THE COMPUTER PROGRAMMES

III-1 THE COMPUTER PROGRAMMES

The following computer programmes have been developed during the course of this project.

- (i) CTWGROW- This is the simplest of all three programmes. Constant wall temperature under the bubble is assumed together with an expression for the microlayer profile.
- (ii) DELGROW - Constant wall temperature under the bubble is assumed, but microlayer evaporation is found by a numerical summation of evaporation at individual points. (Microlayer thickness at each computed radius is stored and modified as the computation proceeds.)
- (iii) VTWGROW - Similar to DELGROW but the wall temperature variation is taken into account.

All the above programmes are similar in most respects. The differences occur in various dimensioning statements and the subroutine QCALC used for calculating the microlayer evaporation. Programme (ii) and (iii) use more storage than program (i), and the computer time utilisation increases in the same order. The steps in the basic algorithm can be outlined as follows.

1. Read in property data and values for experimental radii and times.
2. Print out title and other data.
3. Initialize values; give values for time step etc..
4. Calculate or input waiting time.
5. The calculation is cycled through stages 5 to 10 four times, following the Runge-Kutta fourth order method. For the

first two cycles the time interval (sub-step) is set equal to half the time step and for the other two equal to the defined time step.

6. Calculate interfacial accelerations using known values of pressure, interfacial velocities and radii of the top and the base separately.
7. Calculate new bubble dimensions and the increase in volume at the end of the time interval.
8. Calculate the new interfacial velocities at the end of the time interval.
9. Evaluate heat transfer into the bubble during the time interval and hence the mass increase.
10. Calculate new pressure and temperature of the vapour inside the bubble.
11. Calculate a new set of parameters at the end of the time step using the Runge-Kutta algorithm and print out the values.
12. If the calculation has proceeded upto the required time or departure occurs plot the results.

III-2 NAMELIST

Some of the important fortran symbols used in the programme are given below.

A	- constant in the Clausius-Clapeyron equation
ALPHA	- Thermal diffusivity
ARB	- Interfacial acceleration at base perimeter
ART	- Interfacial acceleration at top.
B	- Constant in Clausius-Clapeyron
CVITY	- Thermal conductivity of liquid
DDEL	- Reduction of microlayer thickness at a point

DELM	- Total increase in mass during time δt
DELMCS	- Mass evaporated from curved interface during δt
DELM	- Mass evaporated from microlayer during δt
DELT	- Time interval (= STEP or STEP/2)
DELTA	- Microlayer thickness
DELV	- Increase in volume during δt
DJN	- Jakob Number
DRAD	- Experimental radius
DTIME	- Experimental time
EL	- Arbitrary constant (set equal to 1)
EMP	- Mass of vapour in bubble
EQRAD	- Equivalent radius
EVAPB	- Total evaporation up to t time 't' from the base
EVAPCS	- Total evaporation upto time 't' from curved interface
GCONST	- Ideal gas constant
GEE	- Acceleration due to gravity
HEATL	- Latent heat of vapourization
IC	- Do loop index
IZ	- Counter
J	- Do loop index
K	- J + 1
LINE	- An array containing characters for plotting results
NAME	- Title of the programme
PBULK	- Bulk pressure
PDRAG	- Pressure drop due to viscous drag force near the base

PDT	- Pressure drop due to viscous forces near the top.
PIP	- Pressure of vapour within the bubble
QFLUX	- Heat flux
QINCS	- Rate of heat transfer across the curved surface
RAD	- Radius of the approximate spherical segment
RBP	- Radius of curvature at base
RDASH	- Radius of dry area
RES	- Radius of the hemisphere with the same surface area
RHO	- Density of the liquid
RHOV	- Density of the vapour at bulk pressure
ROOTPI	- Square root of π
RTP	- Radius of curvature at the top - bubble height
SIGMA	- Surface tension of the fluid
STEP	- Time step
TBULK	- Bulk temperature
TIME	- Instant of time of the calculations
TIMEN	- Instant of time at the end of the time step
TIMEW	- Waiting time
TIP	- Temperature of vapour inside the bubble
TSAT	- Saturation temperature corresponding to bulk pressure
TW	- Wall temperature
VL	- Volume of cavity
VCITY	- Viscosity of the liquid
VEE	- Volume of the bubble
VRB	- Interfacial velocity at the base
VRT	- Interfacial velocity at the top
ZETA	- Dimensionless time for heat transfer calculation.

III-3 DESCRIPTION OF SUBROUTINES

MAIN PROGRAM

This routine calls the subroutines READIN, ERF SOL, RUNGE and PLOT and initialises parameters. The programme assumes the existence of the library functions ERF(X) capable of calculating the value of the error function of X and its compliment.

SUBROUTINE READIN

The data supplied is read in, including the title. A heading is printed.

SUBROUTINE ERF SOL

Using the supplied values of the cavity radius and other data the waiting time is calculated. This involves the evaluation of an inverse error function by a numerical method.

SUBROUTINE RUNGE

The first part of this routine initialises another set of parameters used only in RUNGE which are not included in the common statement. The second part of this routine effects the integration of the differential equations by the Runge-Kutta fourth order method and also calls the subroutines QL7, QCALC and PCALC in the process. The variables are updated at the end of each cycle in this routine and the values are printed after every 40th step. The routine returns control to the main program either when the number of specified steps are completed or when the departure criterion is satisfied.

SUBROUTINE QL7

This has been adopted from the Scientific Subroutine Package supplied by the Control Data Corporation (CDC) to effect a integration of the form $\int_0^{\infty} e^{-x} \cdot f(x) \cdot dx$. The functions $f(x)$ is evaluated in a function subroutine FCT(x)

SUBROUTINE QCALC

Called by RUNGE this calculates the mass of liquid evaporated or condensed at the interface during the time interval δt . The complexity of the developed programmes depend directly on the complexity of this routine. The changes from programme CTWGROW to DELGROW and to VTWGROW can be effected by changing this routine with minor changes in the others.

SUBROUTINE PCALC

This routine solves a set of two implicit algebraic equations, the ideal gas law and the integrated form of the Clausius-Clapeyron equation, to obtain the new pressure and temperature within the bubble.

SUBROUTINE PLOT

This routine produces a graphical plot of the results through the line printer for easier and quicker interpretation of the results.

III-4 USE OF THE COMPUTER PROGRAMMES

These programmes have been run on the CDC 6000 series and CDC 7600 series machines, under different operating systems (Scope 2.0, Scope 3.2, Kronos 2.0.II). Compilation has always been carried out with the University of Minnesota Fortran Compiler PSR 12.

There are a number of non-standard fortran statements, especially the free format input statements, and care should be taken if a different compiler is to be used.

A listing of the programme DELGROW follows.


```

SUBROUTINE READIN
C
C   *   READ IN DATA AND PRINT OUT TITLE
C
C   DIMENSION RAD(1042), VRT(1042), RBP(1042), DELTA(1042)
C   DIMENSION DRAD(10), DTIME(10), EQRAOP(100), LINE(86), NAME(7)
C
C   COMMON A,ALPHA,B,CEE,CONST1,CVITY,DELM,DELM1,DELMCS,DELT,DELTA
C   1,DELV,DRAD,DTIME,EL,EM,EMP,EQRAOP,GCONST,GEE,HEATL,IC,IZ,J,K
C   2,PBULK,PI,PIP,QFLUX,RAD,RBP,RC,RDASH,RHO,ROOTPI,RTP,SIGMA,STEP
C   3,TBULK,THETA,TI,TIME,TINEM,TIMEW,TIP,TSAT,TW,VC,VCITY,VEE,VRS,VRT
C
C   *** WARNING - FREE FORMAT INPUT NON STANDARD
C   *   NAME CAN HAVE A MAXIMUM OF 70 CHARACTERS
C   READ (7,) NAME
C   READ (7,) RC,VC,TW,TBULK,TSAT,PBULK,QFLUX
C   READ (7,) HEATL,GCONST,SIGMA,RHO,CVITY,VCITY,CEE
C   READ (7,) (DRAD(LO),LO=1,8),(DTIME(LO),LO=1,8)
C
C   *   CALCULATE JAKOB NUMBER
C   *   WRITE TITLE OF PROGRAM
C   RHOV=PBULK/(GCONST*TSAT)
C   DJN=CEE*RHO*(TW-TSAT)/(RHOV*HEATL)
C   WRITE (6,1010) NAME
C   *** WARNING - NON STANDARD OUTPUT LIST
C   WRITE (6,1020) RC,TW-TBULK,QFLUX,DJN
C
C   *   WRITE OUT DATA VALUES
C   WRITE (6,1030) RC,VC,TW,TBULK,TSAT,PBULK,QFLUX
C   WRITE (6,1030) HEATL,GCONST,SIGMA,RHO,CVITY,VCITY,CEE
C   WRITE (6,1040) (DTIME(LO),LO=1,8),(CRAD(LO),LO=1,8)
C
C   RETURN
C
C   1010 FORMAT (/1X,7A10/1X,*-----*)
C   1020 FORMAT (1X,*RC=*,E10.3,*TW-TBULK=*,F6.2,*QFLUX=*,F10.3,*NJ=*,E11.4
C   1/1X,*-----*/)
C   1030 FORMAT (1X,7E11.4)
C   1040 FORMAT (/1X,*DATA CONVERTED TO S.I. UNITS*/1X,*TIME *.8E12.3/1X,6H
C   1RADIUS,8E12.4)
C
C   END

```

```

SUBROUTINE QCALC (IT,GRAD)
C
C      *   HEAT INPUT CALCULATION
C
C      DIMENSION RAD(1042), VRT(1042), RBP(1042), DELTA(1042)
C      DIMENSION DRAD(10), DTIME(10), EQRADP(100), LINE(86), NAME(7)
C
C      COMMON A,ALPHA,B,CEE,CONST 1,CVITY,DELM,DELMH,DELMCS,DELT,DELTA
C      1,DELV,DRAD,DTIME,EL,EM,EMP,EQRADP,GCONST,GEE,HEATL,IC,IZ,J,K
C      2,PBULK,PI,PIP,QFLUX,RAD,RBP,RC,RDASH,RHO,ROOTPI,RTP,SIGMA,STEP
C      3,TBULK,THETA,TI,TIME,TIMEN,TIMEW,TIP,TSAT,TW,VC,VCITY,VEE,VRS,VRT
C
C      *   CALCULATE MEAN VALUES OF EQUIVALENT RADIUS AND TIME
C      RADMN=SQRT(RAD(J)*RTP)
C      TIMEMN=(TIME+TIMEN)/2.0
C
C      GO TO (10,90), IT
C
C      *   CALCULATE EVAPORATION FROM MICROLAYER
C      10 DELMM=E.
C
C      *   DELTA(IA) IS CHANGED AFTER EVERY LAST REF. TO 100
C      *   IN EACH STEP. ADVANCE COUNTER AND FIND REMAINDER.
C      IZ=IZ+1
C      IFI=IZ-2*(IZ/2)
C
C      IF (J.LT.3) GO TO 90
C      IF (RBP(K).GT.RBP(J)) GO TO 40
C
C      *   DETERMINE WHERE THE MICROLAYER PROFILE ENDS
C      *   NEAREST 'RBP' TO 'RBP(K)' IS CHOSEN AS END POINT
C      DO 30 IB=2,J
C      IF (RBP(IB).LT.RBP(K)) GO TO 30
C      IF (RBP(IB)-RBP(K).GT.RBP(K)-RBP(IB-1)) GO TO 20
C      IC=IB
C      GO TO 50
C      20 IC=IB-1
C      GO TO 50
C      30 CONTINUE
C
C      40 IC=J
C
C      *   COMPUTE DDEL AND EVALUATE MASS EVAPORATED FROM BASE
C      *   DDEL IS EVALUATED AT EVERY FOURTH STEP STARTING FROM J=3
C      50 DO 80 IA=3,IC,4
C      IF (DELTA(IA).EQ.0.) GO TO 60
C      DDEL=CVITY*(TW-TIP)*DELT/(RHO*HEATL*DELTA(IA))
C      IF (DDEL.GT.DELTA(IA)) DDEL=DELTA(IA)
C      GO TO 70
C      60 DDEL=0.0
C      RDASH=RBP(IA)
C      70 IH=IA+2
C      IL=IA-2
C      IF (IH.GE.(IC-1)) IH=IC
C      ELM=2.*3.1416*RBP(IA)*(RBP(IH)-RBP(IL))*DDEL*RHO
C      DELMM=DELMM+ELM
C      IF (IFI.NE.0) GO TO 80
C      DELTA(IA)=DELTA(IA)-DDEL
C      80 CONTINUE

```

```
C
C
C      *  HEAT TRANSFER ACROSS CURVED INTERFACE
90    IF (RADMM.LT.0.0) GO TO 100
      QINCS=CONST 1*RADMM*RADMM*GRAD
      GO TO 110
100   QINCS=0.0
110   DELMCS=QINCS*DELT/HEATL
C
C      *  TOTAL INCREASE IN MASS DURING TIME DELT
      DELM=DELM1+DELMCS
C
      RETURN
      END
```

SUBROUTINE PCALC

```

C
C      *   CALCULATION OF PRESSURE AND TEMPERATURE INSIDE THE BUBBLE.
C
      DIMENSION RAD(1042), VRT(1042), RBP(1042), DELTA(1042)
      DIMENSION DRAD(10), DTIME(10), EQRADP(100), LINE(86), NAME(7)
C
      COMMON A,ALPHA,B,CEE,CONST1,CVITY,DELM,DELM1,DELMCS,DELT,DELTA
      1,DELV,DRAD,DTIME,EL,EM,EMP,EQRADP,GCONST,GEE,HEATL,IC,IZ,J,K
      2,PGULK,PI,PIP,QFLUX,RAD,RBP,RC,RDASH,RHO,ROOTPI,RT,STEP
      3,TBULK,THETA,TI,TIME,TIMEN,TIMEX,TIP,TSAT,TW,VC,VCITY,VEE,VRE,VRT
C
C      EM=EMP+DELM
      TI=TIP
C
C      *   NEW PI IS FOUND AT THE PREVIOUS TI,BY GAS LAW
      PI=EM*GCONST*TI/(VEE+DELV)
C
C      *   ITERATIVE METHOD IS EMPLOYED IN FINDING TRUE
C      *   NEW PI AT THE CORRESPONDING NEW TI
      DO 10 I=1,50
C      *   NEW TIN FROM THE CLAUSIUS-CLAPEYRON EQN.
      IF (PI.LT.0.0) RETURN
      TIN=B/(A-ALOG(PI))
      PI=PI*TIN/TI
      EF=TIN/TI
      TI=TIN
      IF (EF.GT.0.99999.AND.EF.LT.1.00001) RETURN
10    CONTINUE
C
      RETURN
      END

```

```

SUBROUTINE RUNGE
C
C      *   SOLUTION BY RUNGE-KUTTA FOURTH ORDER METHOD
C      *   FOR MOTION OF TOP RADIUS IS TAKEN AS THE HEIGHT
C      *   FOR MOTION OF BASE PERIMETER RADIUS IS BASE RADIUS
C
C      DIMENSION RAD(1042), VRT(1042), RBP(1042), DELTA(1042)
C      DIMENSION DRAD(10), DTIME(10), EQRADP(100), LINE(86), NAME(7)
C
C      COMMON A,ALPHA,B,CEE,CONST1,CVITY,DELM,DELMH,DELMCS,DELT,DELTA
C      1,DELV,DRAD,DTIME,EL,EN,EMP,EQRADP,GCONST,GEE,HEATL,IC,IZ,J,K
C      2,PBULK,PI,PIP,QFLUX,RAD,RBP,RC,RDASH,RHO,ROOTPI,RTP,SIGMA,STEP
C      3,TBULK,THETA,TI,TIME,TIMEN,TIMEW,TIP,TSAT,TW,VC,VCITY,VEE,VRB,VRT
C
C      COMMON /AA/ TS,TB,TWALL
C      EXTERNAL FCT
C
C      *   WRITE HEADING FOR RESULTS
C      WRITE (6,1070)
C
C      TS=TSAT
C      TB=TBULK
C      TWALL=TW
C
C      *   INITIALISATION - 2
C      EVAPB=0.0
C      EVAPCS=0.0
C      RB=RBP(1)
C      RES=SQRT(RC*RTP)
C      RDASH=RBP(1)
C      PDAG=0.0
C      POT=0.0
C      JX=4000
C      JT=4000
C      PHI=0.4
C      CERPHI=ERFC(PHI)
C      EXPHI=EXP(PHI*PHI)
C
C      *   TRANSFORMATION OF TIME TO DIMENSIONLESS ZETA
C      *   FOR CALCULATION OF HEAT INPUT
C      ELW=2.0*SQRT(ALPHA*TIMEW)
C      RC3=RC*RC*RC
C      EL3=EL*EL*EL
C      ZETA=ALPHA*RC3*RC*STEP/(2.0*EL3*EL3)
C
C
C      *   THE LOOP FOR RUNGE-KUTTA SOLN. IS STARTED HERE
C      DO 60 J=1,1041
C      K=J+1
C
C      *   CALCULATION OF CONSTANTS NECESSARY FOR
C      *   HEAT INPUT CALCULATION
C      CALL QL7 (FCT,Y,ZETA,RC3,EL3,ELW)
C      GRAD=Y*RES*RES/(EL3*SQRT(3.1416*ZETA))
C      CONST1=CVITY*2.0*3.1416*(TW-TIP)
C      DELTA(J)=0.8*SQRT(VCITY*TIME/RHO)

```

```

C      *   CALCULATE PRESSURE TERM INCLUDING SURFACE TENSION
      PI 1=PBULK+2.0*SIGMA/RTP
      IF (RBP(J).LE.RDASH) GO TO 10
      PI 2=PBULK+SIGMA*(1.0/RAD(J)+1.0/RBP(J))
      GO TO 20
10     PI 2=PBULK+SIGMA/RBP(J)
20     CONTINUE

C      DELT=STEP/2.0
      TIMEN=TIME+DELT

C      CYNE=SIN(THETA)
      COCYNE=COS(THETA)
      RB2=RBP(J)*RBP(J)
      GP 1=1.5708*(RB2+RTP*RTP)
      GP 2=3.1416*RBP(J)*RTP
      VRT 1=VRT(J)
      VRB 1=VRB
      ART 1=((PI-PI 1)/RHO+RTP*GEE-POT-1.5*VRT 1*VRT 1)/RTP
      ARB 1=((PI-PI 2)/RHO-PDRAG-1.5*VRB 1*VRB 1)/RB
      ART=ART 1
      ARB=ARB 1

C
C      VRT 2=VRT 1+ART 1*DELT
      VRB 2=VRB 1+ARB 1*DELT
      DELRT=VRT 1*DELT
      DELRB=VRB 1*DELT
      RB=RBP(J)+DELRB
      RBP(K)=RB
      TANG=RB/(RTP+DELRT)
      THETAN=2.0*ATAN(TANG)
      RAD(K)=RB/SIN(THETAN)
      CALL QCALC(1,GRAD)
      DELV=GP 1*DELRT+GP 2*DELRB
      CALL PCALC

C      *   INCLUDE VISCOUS DRAG NEAR THE SURFACE AFTER 40 STEPS
      IF (J.LT.40) GO TO 30
      IF (J.GE.JT) GO TO 30
      PDRAG=RBP(J)*VRE*(EXPHI-ROOTPI*PHI*CERPHI)/(2.0*TIMEN)
      PDT=4.*VCITY*VRT(J)/(RTP*RHO)
      GO TO 40
30     PDRAG=0.0
40     CONTINUE
      ART 2=((PI-PI 1)/RHO+RTP*GEE-PDT-1.5*VRT 2*VRT 2)/(RTP+DELRT)
      ARB 2=((PI-PI 2)/RHO-PDRAG-1.5*VRB 2*VRB 2)/RB

C
C      VRT 3=VRT 1+ART 2*DELT
      VRB 3=VRB 1+ARB 2*DELT
      DELRT=VRT 2*DELT
      DELRB=VRB 2*DELT
      RB=RBP(J)+DELRB
      RBP(K)=RB
      TANG=RB/(RTP+DELRT)
      THETAN=2.0*ATAN(TANG)
      RAD(K)=RB/SIN(THETAN)
      CALL QCALC(2,GRAD)
      DELV=GP 1*DELRT+GP 2*DELRB
      CALL PCALC
      ART 3=((PI-PI 1)/RHO+RTP*GEE-PDT-1.5*VRT 3*VRT 3)/(RTP+DELRT)
      ARB 3=((PI-PI 2)/RHO-PDRAG-1.5*VRB 3*VRB 3)/RB

```

```

DELT=STEP
TIMEN=TIME+DELT
C
VRT 4=VRT 1+ART 3*DELT
VRB 4=VRB 1+ARB 3*DELT
DELAT=VRT 3*DELT
DELRB=VRB 3*DELT
RB=RBP (J)+DELRB
RBP (K)=RB
TANG=RB/(RTP+DELAT)
THETA=2.0*ATAN(TANG)
RAD (K)=RB/SIN(THETA)
CALL QCALC (1,GRAD)
DELV=GP 1*DELAT+GP 2*DELRB
CALL PCALC
ART 4=((PI-PI 1)/RHO+RTP*GEE-PDT-1.5*VRT 4*VRT 4)/(RTP+DELAT)
ARB 4=((PI-PI 2)/RHO-PDRAG-1.5*VRB 4*VRB 4)/RB
EQRAD=(VEE*0.75/3.1416)**0.333
C
C
C
C
* WRITE OUT RESULTS. EVERY 40TH STEP WRITTEN;
* STARTING AT TIME=0. (IE.J=1)
IREM=J-(J/40)*40
IF (IREM.NE.1) GO TO 50
DEGREE=THETA*180./3.1416
WRITE (6,1080) TIME,RAD (J),VRT (J),EVAPB,EVAPCS,EQRAD,TIP,VRB
1,RBP (J),RDASH
L=J/40
IF (L.EQ.0) GO TO 50
EQRADP (L)=EQRAD
C
C
C
C
* FIND VALUES AFTER TIME INTERVAL -STEP-
* AND UPDATE OTHER VARIABLES
50 VRT (K)=VRT (J)+(ART 1+2.*ART 2+2.*ART 3+ART 4)*STEP/6.0
VRB=VRB+(ARB 1+ARB 2*2.0+2.0*ARB 3+ARB 4)*STEP/6.0
DELAT=(VRT 1+2.0*VRT 2+2.0*VRT 3+VRT 4)*STEP/6.0
DELRB=(VRB 1+2.0*VRB 2+2.0*VRB 3+VRB 4)*STEP/6.0
RB=RBP (J)+DELRB
RBP (K)=RB
RTP=RTP+DELAT
TANG=RBP (K)/RTP
THETA=2.0*ATAN(TANG)
RAD (K)=RB/SIN(THETA)
CALL QCALC (2,GRAD)
DELV=GP 1*DELAT+GP 2*DELRB
CALL PCALC
EMP=EMP+DELM
VEE=VEE+DELV
TIP=TI
PIP=PI
EVAPB=EVAPB+DELM
EVAPCS=EVAPCS+DELMCS
TIME=TIMEN
JT=4000
IF (RBP (K).LT.RBP (J)) JT=J
C
RES=SQRT (RAD (K)*RTP)
ZETA=ZETA+ALPHA*RES*RES*RES*RES*STEP/(EL 3*EL 3)

```

```
C      * IF RBP IS LESS THAN RC DEPARTURE OCCURS
      IF (RBP(K).LT.RC) RETURN
C
C
C 60    CONTINUE
C
C
C      RETURN
C
1070  FORMAT (1X,* RUNGE-KUTTA SOLUTION **//3X,* TIME *,6X,* RADIUS *,6X,
      1* VRT *,6X,* EV'PE*,6X,* EVAPCS *,6X,* EQRAD*,6X,* TIP  *,6X,*VR
      2B*,6X,* RBP *)
1080  FORMAT (1X,9(E11.4,2X),E8.2)
C
      END
```

```

SUBROUTINE PLOT
C
C * SUBROUTINE FOR PLOTTING THE RESULTS
C
C DIMENSION RAD(1042), VRT(1042), RBP(1042), DELTA(1042)
C DIMENSION DRAD(10), DTIME(10), EQRADP(100), LINE(86), NAME(7)
C
C COMMON A,ALPHA,B,CEE,CONST1,CVITY,DELM,DELM,DELMCS,DELT,DELTA
C 1,DELV,DRAD,DTIME,EL,EM,EMP,EQRADP,GCONST,GEE,HEATL,IC,IZ,J,K
C 2,PGULK,PI,PIP,QFLUX,RAD,RBP,RC,ROASH,RHO,ROOTPI,RTP,SIGMA,STEP
C 3,TBULK,THETA,II,TIME,TIMEN,TIMEN,TIP,TSAT,TW,VC,VCITY,VEE,VRE,VRT
C
C INTEGER BLANK,DOT,STAR,CROSS,I,MINUS,EX
C DATA BLANK,DOT,STAR,CROSS,I,MINUS,EX/ 1H , 1H., 1H*, 1H+, 1HI, 1H-, 1HX/
C
C * FIND THE MAXIMUM VALUES OF RAD AND VRT
C RADMAX=RAD(1)
C RDOTMX=VRT(1)
C DO 10 IA=1,J
C IF (RAD(IA).GT.RADMAX) RADMAX=RAD(IA)
C IF (VRT(IA).GT.RDOTMX) RDOTMX=VRT(IA)
10 CONTINUE
C RADMAX=1.1*RADMAX
C
C * NORMALISE AND ARRANGE TO PRINT IN THE LAST 80 POSITIONS
C WRITE (6,1110)
C
C * PRINT THE FIRST LINE
C DO 20 IB=1,86
C LINE(IB)=MINUS
20 CONTINUE
C IA=1
C TIME=0.0
C N1=(RAD(IA)/RADMAX)*80.0+1.0
C N2=(VRT(IA)/RDOTMX)*80.0+1.0
C N4=(RBP(IA)/RADMAX)*80.0+1.0
C LINE(N1)=STAR
C LINE(N2)=DOT
C LINE(N4)=EX
C WRITE (6,1120) TIME,RAD(IA),RBP(IA),LINE
C
C
C * EVALUATE PRINTING POSITIONS AND PRINT
C LO=1
C L=1
C DO 100 IA=41,J,40
C TIME=STEP*FLOAT(IA-1)
C N1=(RAD(IA)/RADMAX)*80.0+1.0
C N2=(VRT(IA)/RDOTMX)*80.0+1.0
C N4=(RBP(IA)/RADMAX)*80.0+1.0
C N5=(EQRADP(L)/RADMAX)*80.0+1.0
C DO 30 IB=1,86
C LINE(IB)=BLANK
30 CONTINUE
C LINE(1)=I
C IF (N1.GT.86.OR.N1.LT.1) GO TO 40
C LINE(N1)=STAR
40 IF (N2.GT.86.OR.N2.LT.1) GO TO 50
C LINE(N2)=DOT
50 IF (N4.GT.86.OR.N4.LT.1) GO TO 60
C LINE(N4)=EX

```

```
60 IF (N5.GT.86.OR.N5.LT.1) GO TO 70
   LINE(N5)=MINUS
70 IF (LO.EQ.9) GO TO 90
C  * PRINTING POSITIONS FOR EXPTL. DATA POINTS (8 POINTS)
   IF (ABS(TIME-OTIME(LO)).LE.20.*STEP) GO TO 80
   GO TO 90
80 N3=(ORAD(LO)/RADMAX)*80.0+1.0
   LO=LO+1
   IF (N3.GT.86.OR.N3.LT.1) GO TO 90
   LINE(N3)=CROSS
90 WRITE (6,1130) TIME,RAD(IA),RBP(IA),LINE
   L=L+1
100 CONTINUE
C
C
   RETURN
C
1110 FORMAT (///1X,* SOLUTION USING RUNGE-KUTTA METHOD *//1X,* TIME *,7
            1X,* RADIUS *,5X,* RBP*)
1120 FORMAT (1X,3(E10.4,2X),3X,86A1)
1130 FORMAT (1X,3(E10.4,2X),3X,86A1)
C
   END
```

SUBROUTINE QL7 (FCT,Y,ZETA,RC3,EL3,ELW)

```
C
C  * SUBROUTINE FOR INTEGRATION OF (EXP(-X)*FCT(X)) FROM
C  * 0 TO INFINITY. ROUTINE ADOPTED FROM SSPBIN LIBRARY
C  * SUPPLIED BY THE CONTROL DATA CORPORATION.
C
C  * FCT - NAME OF EXTERNAL SUBPROGRAM FOR EVALUATION OF
C  * FUNCTION
C  * Y - THE RESULTING INTEGRAL VALUE
```

COMMON /AA/ TS, TB, TWALL

```

C
X=19.39573
X=(6.0*SQRT(ZETA*X)*EL3+RC3)**0.333
X=X/ELW
Y=.3170315E-7*FCT(X)
X=12.73418
X=(6.0*SQRT(ZETA*X)*EL3+RC3)**0.333
X=X/ELW
Y=Y+.1586546E-4*FCT(X)
X=8.182153
X=(6.0*SQRT(ZETA*X)*EL3+RC3)**0.333
X=X/ELW
Y=Y+.1074011E-2*FCT(X)
X=4.900353
X=(6.0*SQRT(ZETA*X)*EL3+RC3)**0.333
X=X/ELW
Y=Y+.02063351*FCT(X)
X=2.567877
X=(6.0*SQRT(ZETA*X)*EL3+RC3)**0.333
X=X/ELW
Y=Y+.1471263*FCT(X)
X=1.026665
X=(6.0*SQRT(ZETA*X)*EL3+RC3)**0.333
X=X/ELW
Y=Y+.4218313*FCT(X)
X=.1930437
X=(6.0*SQRT(ZETA*X)*EL3+RC3)**0.333
X=X/ELW
Y=Y+.4093190*FCT(X)

```

CC

RETURN
END

FUNCTION FCT (X)

C
C
C

* SUBPROGRAM FOR EVALUATION OF FCT FOR USE IN QL7

COMMON /AA/ TS,TB,TWALL

CERFX=ERFC(X)

FCT=((TB-TS)+(TWALL-TB)*(CERFX+(1.0-EXP(-X*X))/(X*SQRT(3.1416))))/
1(TWALL-TS)

C

RETURN
END

```

SUBROUTINE ERF SOL
C
C      *   TO FIND THE INVERSE ERROR FUNCTION IN ORDER TO
C      *   ESTIMATE THE VELOCITY OF PROPAGATION OF A TEMPERATURE
C      *   FRONT TI WHICH GIVES THE WAITING TIME
C
C      DIMENSION RAD(1042), VRT(1042), RBP(1042), DELTA(1042)
C      DIMENSION DRAD(10), DTIME(10), EQRADP(100), LINE(86), NAME(7)
C
C      COMMON A,ALPHA,B,CEE,CONST1,CVITY,DELM,DELM1,DELMCS,DELT,DELTA
C      1,DELV,DRAD,DTIME,EL,EN,EMP,EQRADP,GCONST,GEE,HEATL,IC,IZ,J,K
C      2,PBULK,PI,PIP,QFLUX,RAD,RBP,RC,ROASH,RHO,ROOTPI,RTP,SIGMA,STEP
C      3,TBULK,THETA,TI,TIME,TIMEN,TIMEW,TIP,TSAT,TW,VC,VCITY,VEE,VRB,VRT
C
C      *   THE EQUATION TO BE SOLVED IS
C      *    $ERF(X/2.\sqrt{ALPHA*TIME}) = (TW-TI)/(TW-TBULK)$ 
C      *   LET  $PHI = X/(2.\sqrt{ALPHA*TIME})$ 
C
C      *   SOLVE  $PHI = ERF(PHI) - FACT$  WHERE  $FACT = (TW-TI)/(TW-TBULK)$ 
C      *   AS A FIRST APPROXIMATION TAKE  $PHI = 0.5$ 
C      FACT=(TW-TIP)/(TW-TBULK)
C      PHI=0.5
C      DO 10 IA=1,50
C      FPHI=ERF(PHI)-FACT
C      DFPHI=2./((ROOTPI*EXP(PHI*PHI)))
C      PHINEW=PHI-FPHI/DFPHI
C      IF (ABS(PHINEW-PHI)).LT.0.000001) GO TO 20
C      PHI=PHINEW
10  CONTINUE
C
C      *   WAITING TIME USING  $X = C1*TIME^{.5}$ 
C      *   IN THIS CASE  $X = 1.5*RTP$  AND  $TIME = TIMEW$ 
C      *   [  $C1 = PHI*2.\sqrt{ALPHA}$  ]
C      20  C1=PHI*2.0*SQRT(ALPHA)
C      TIMEW=(1.5*RTP/C1)*(1.5*RTP/C1)
C      WRITE (6,1030) TIMEW
C
C      RETURN
C
C      1030 FORMAT (/1X,* WAITING TIME = *,E10.3)
C
C      END

```