

DIGITAL COMPENSATION OF THE DYNAMIC TRANSFER
CHARACTERISTICS OF RF AMPLIFIERS

by

Andrew Stephen Beasley

March 1984

A thesis submitted for the degree of Doctor of Philosophy of the
University of London and for the Diploma of Membership of the
Imperial College

Electrical Engineering Department
Imperial College
London SW7

ABSTRACT

The problem addressed is the correction of memoryless, narrowband, nonlinear effects in RF amplifiers by means of series, digital predistorters. Predistortion, for the removal of both AM/AM and AM/PM effects is considered, and achieved by the use of two orthogonal predistorters.

The nonlinear function implemented by the digital predistorter is easily controlled and modified using an A/D, RAM, D/A form of the predistorter. This enables adaptive systems to be considered, which overcomes the tracking and drift problems associated with analogue predistorters. Possible adaptive systems are enumerated by means of a decision hierarchy, which enables digital predistortion to be viewed as a single subject, rather than as a set of isolated algorithms and techniques.

Systems are considered which use digital predistortion with feedback or feedforward like error adaptation, along with methods of reducing the sampling rates of the A/D and D/A of the nonlinear function generators. Because of this the term "predistortion" no longer seems entirely appropriate and the term "digital compensation" is preferred.

The bulk of the study is concerned with identifying and developing techniques which may be used throughout the hierarchy, particularly the correction of detector nonlinearities and control algorithms.

Software techniques specific to digital compensation are developed to enhance the linearity of the detectors used in the adaptive systems for error pick-up, and to reduce the effective

noise floor of the measurement.

Three distinct types of control algorithm for adaptive behaviour are identified, and mathematical analysis of these algorithms is presented along with experimental results and computer simulations. Two of the three types appear to be highly effective in improving the linearity of the RF amplifiers, with fundamental constraints imposed by anti-alias filter specifications and narrowband with-memory distortion proving to be the limiting factors on ultimate linearity.

ACKNOWLEDGEMENTS

I wish to thank the many people who have made this study possible, whether through devoting their time, effort, advice or support. Particularly generous have been Mr G. Cooper, Dr. M. Cleverley, Professor L. Turner and Mr R. Greatrex. I would also wish to thank the many engineers at McMichael, too numerous to mention here.

Most of all I would thank my wife and family for their forbearance over the four years of this study.

26th March 1984

CONTENTS

	<u>Page</u>
ABSTRACT	1
ACKNOWLEDGEMENTS	3
CONTENTS	4
CHAPTER 1: INTRODUCTION	11
1.1 INTRODUCTION	11
1.2 IMPORTANCE OF NBNL AMPLIFIERS	12
1.3 OUTLINE OF THESIS	13
CHAPTER 2: EXISTING TECHNIQUES OF CORRECTION	16
2.1 INTRODUCTION	16
2.2 MAJOR TECHNIQUES	16
2.2.1 Derating	16
2.2.2 Feedback	17
2.2.3 Feedforward	21
2.2.4 Predistortion	23
2.3 MINOR TECHNIQUES	24
2.3.1 Cascode Networks	25
2.3.2 Selection of Bias Point	25
2.3.3 Helix Modulation	25
2.3.4 Second Harmonic Injection	25
2.3.5 Strong Signal Injection	26
2.3.6 Nonlinear Feedback	26

	<u>Page</u>
2.3.7 Use of Hybrids and Couplers	26
2.3.8 Selection of Frequencies	26
2.3.9 Amplifier Techniques	27
2.4 CONCLUSIONS	27
 CHAPTER 3: DIGITAL COMPENSATION	 38
3.1 INTRODUCTION	38
3.2 RANGE OF APPLICATIONS	39
3.3 HIERARCHY OF DECISIONS	41
3.3.1 Basis of the Representation	41
3.3.2 Vector and Scalar Control	43
3.3.3 IF or Baseband Input to NLFG	44
3.3.4 Choice of Adaptive Algorithm	45
3.4 ANTECEDENTS OF DIGITAL COMPENSATION	52
3.5 AREAS OF STUDY	53
 CHAPTER 4: NONLINEAR FUNCTION GENERATOR CONSIDERATIONS	 73
4.1 INTRODUCTION	73
4.2 POLAR, BASEBAND INPUT, AM MODULATION	73
4.2.1 Bandwidth	73
4.2.2 Quantization Noise and the Envelope NLFG	76
4.2.3 Quantization Noise and the Phase NLFG	77
4.3 POLAR IF INPUT	81
4.3.1 Bandwidth	81
4.3.2 Quantization Noise and the Envelope NLFG	87
4.3.3 Quantization Noise and the Phase NLFG	88
4.4 D/A RESOLUTION	88
4.5 CONCLUSIONS	89

	<u>Page</u>
CHAPTER 5: THE ENVELOPE MODULATOR	105
5.1 INTRODUCTION	105
5.2 CONTROL RANGE	105
5.3 EFFECT OF LIMITED CONTROL RANGE	107
5.3.1 IF Input to NLFG: Effect of Modulator	
Leakthrough	107
5.3.2 AM Baseband System: Effect of Modulator	
Leakthrough	108
5.4 REDUCING LEAKTHROUGH DISTORTION IN AM SYSTEMS	109
5.4.1 Introduction	109
5.4.2 Nulling Methods	109
5.4.3 LO Waveshaping in Diode Modulators	110
5.4.4 Cascading Modulators	114
5.5 CONCLUSIONS	115
 CHAPTER 6: SEPARATION OF AM/AM AND AM/PM	 133
6.1 INTRODUCTION	133
6.2 QUASI-LINEAR METHODS	134
6.2.1 Quasi-linear Envelope Control	135
6.2.2 Quasi-linear Phase Control	139
6.3 ASSUMPTIONS OF QUASI-LINEAR APPROXIMATIONS	142
6.3.1 Envelope Quasi-linear	142
6.3.2 Phase Quasi-linear	144
6.4 ASYMMETRIC EFFECTS	145
6.5 OTHER METHODS	146
6.6 EXPERIMENTAL RESULTS	147
6.7 CONCLUSIONS	148

	<u>Page</u>
CHAPTER 7: THE ENVELOPE DETECTOR	164
7.1 INTRODUCTION	164
7.2 DIODE TYPE	165
7.3 DETECTOR CONFIGURATION	166
7.3.1 Limitations of the Basic Detector	166
7.3.2 Detector Circuitry	168
7.4 FAILURE TO FOLLOW DISTORTION	168
7.4.1 Quantifying Failure to Follow Distortion	170
7.5 THE ENVELOPE DETECTOR AND DIGITAL COMPENSATION	171
7.6 IMPROVEMENTS IN DETECTOR LINEARITY	172
7.6.1 Attenuator Calibrated Detector	173
7.6.2 Quasi-linear Calibrated Detector	177
7.7 CONCLUSIONS	179
 CHAPTER 8: THE PHASE DETECTOR	 204
8.1 INTRODUCTION	204
8.2 EFFECT OF INACCURATE PHASE DETECTION	205
8.3 LIMITER TYPE	208
8.4 PHASE SENSITIVE DEVICE TYPE	210
8.5 EVALUATION OF LIMITER PERFORMANCE	210
8.6 THE PHASE DETECTOR AND DIGITAL COMPENSATION	211
8.7 IMPROVEMENTS IN DETECTOR LINEARITY	211
8.7.1 Attenuator Calibrated Detector	212
8.7.2 Phase Quasi-linear	213
8.8 CONCLUSIONS	214

	<u>Page</u>
CHAPTER 9: TYPE 1 CONTROL ALGORITHMS	227
9.1 INTRODUCTION	227
9.2 ANALYSIS OF TYPE 1	229
9.2.1 Ramp Mode	229
9.2.2 Signal Processing	232
9.2.3 Run Mode	233
9.3 DISCUSSION OF ASSUMPTIONS	237
9.3.1 Imperfect Anti-alias Filter	237
9.3.2 Overlap of Alias Spectra	238
9.3.3 A/D Quantizing Law	239
9.3.4 Detector Linearity	239
9.4 SPECIAL CASE	240
9.5 COMPUTER SIMULATION	240
9.6 EXPERIMENTAL RESULTS	241
9.7 TIME-VARYING CHARACTERISTICS	241
9.8 CONCLUSIONS	243
 CHAPTER 10: TYPE 2 CONTROL ALGORITHMS	 266
10.1 INTRODUCTION	266
10.2 ANALYSIS OF TYPE 2 CONTROL	267
10.3 DISCUSSION OF ASSUMPTIONS	270
10.4 STOCHASTIC SAMPLING OVER A FINITE RANGE	274
10.5 EXPERIMENTAL RESULTS	276
10.6 CONCLUSIONS	276

	<u>Page</u>
CHAPTER 11: TYPE 3 CONTROL ALGORITHMS	281
11.1 INTRODUCTION	281
11.2 SIMPLIFIED ANALYSIS	282
11.2.1 Cascaded Nonlinearities Under Type 3 Control	282
11.2.2 Effect of an Arbitrary Filter on a Nonlinearity	284
11.2.3 Discussion	285
11.3 ANALYSIS	286
11.3.1 Introduction	286
11.3.2 Effect of Small Group Delay Filter and Nonlinearity	287
11.3.3 Algorithm Analysis	288
11.3.4 Spectral Output of the DUT	291
11.3.5 Non-Cosinusoidal Input	294
11.4 DISCUSSION OF ANALYSIS	298
11.5 COMPUTER SIMULATION	299
11.6 PRACTICAL STRUCTURE	304
11.6.1 Description	304
11.6.2 Signal Processing	307
11.6.3 Noise and Quantization Effects	310
11.6.4 Experimental Results	313
11.7 CONCLUSIONS	316
 CHAPTER 12: CONCLUSIONS	 364
12.1 CONCLUSIONS	364
12.2 FUTURE WORK	366

	<u>Page</u>
APPENDIX A BANDWIDTH OF Mth ZONE HARMONIC CAUSED BY Nth ORDER DISTORTION	367
B CHEBYSHEV POLYNOMIALS	368
C SPECTRAL SYMMETRY IN AM/AM AND AM/PM	370
D EXPERIMENTAL DETAILS	372
E LEAKTHROUGH AND QUASI-LINEAR	375
E.1 LOW LEAKTHROUGH ENVELOPE MODULATOR	375
E.2 SETTING RAM 2	375
F EXPERIMENTAL DETAILS	380
G COMMUTATION OF NONLINEAR FUNCTIONS AND CONVOLUTION	382
H STATISTICS OF RANDOM SAMPLING	385
I COMPUTER SIMULATION OF FILTER HYSTERESIS	387
J COMPUTER SIMULATION OF TYPE 3 CONTROL	391
K CASCADE MATRICES	403
K.1 CASCADE MATRICES	403
K.2 EXISTENCE OF [P]	403
REFERENCES	405

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

The problem addressed in this thesis is the problem of compensating narrowband nonlinear distortions in RF and microwave amplifiers. The distortions of narrowband nonlinear (NBNL) amplifiers are often referred to as memoryless distortions, since the output distortion at any instant can be related to an undistorted input envelope occurring at the same instant, without reference to the past.

Thus we are concerned with AM/AM and AM/PM conversion, caused by the nonlinear characteristics of RF amplifiers, embedded in a linear network.

AM/AM and AM/PM effects can arise in networks described by linear differential equations, due to inconstant group delay and frequency dependent attenuation [ref 175]. However, such effects are caused by the highest Q elements in the system, and this is usually not the RF amplifier but the narrowpass filters in the system. Hence AM/AM and AM/PM conversion due to linear effects is not the proper concern of this thesis. Nor do we concern ourselves overmuch with distortions which arise from the interaction of linear and nonlinear networks, since these can be dealt with by manipulation of the linear elements (e.g. Stette [ref 163] describes AM/FM conversion, Sechi [ref 151] discusses AM/PM with memory). Effects caused by the interaction of nonlinear AM/AM and AM/PM (as reported by Berman [ref 18]) are not considered, since they are second order small.

1.2 IMPORTANCE OF NBNL AMPLIFIERS

Narrowband RF amplifiers may be found in nearly all RF/microwave communication links. Because of the widespread use of RF amplifiers, the problem of NBNL distortion in RF amplifiers has been looked at, with emphasis on different specifications on the nonlinear distortions.

In multichannel links (terrestrial and satellite) the resultant intermodulation distortion is of interest as it can act to limit the link's S/N capability or bit error rate [ref 46-48, 69, 72, 78, 79, 83, 89, 158, 165, 172, 180, 181]. For example Bremenson [ref 27] has produced figures showing that the capacity in the FM/FDMA system of Intelsat IV is limited to 685 channels by NBNL S/N effects, as opposed to 852 channels without NBNL effects. Hicks [ref 78] suggests that NBNL effects are one of the major impediments to the use of spectrally efficient modulation schemes on terrestrial microwave links. Even in single channel links the bit error rate can be degraded significantly by RF power amplifier (PA) distortion [ref 39, 66, 93]. This is particularly so for M-QAM systems with high M. For CATV and broadcast VSB-TV the problem is viewed primarily as being one of cross-modulation and inband distortion [ref 15, 37, 45, 69, 119, 139, 175]. In other work the spectral spreading caused by NBNL action is deemed to be of more consequence [ref 47, 72, 85, 106, 183]. This is particularly important in congested terrestrial bands, which look to SSB as a means of providing spectral efficiency.

All of these problems are fundamentally the same since they have the same root cause in the NBNL distortions of the RF amplifier. (The RF power stages are especially likely to give

rise to NBNL distortion near saturation, and to dominate all other NBNL effects). The primary application of this thesis is in relation to these problems. However, there may also be applications to other broadband systems which can be described by a memoryless nonlinearity, for example fibre optic laser modulation systems [ref 112, 166, 184]. There may also be applications to non-amplifier induced nonlinearities, for example in the production of very high linearity I-Q mixer pairs for radar signal processing [ref 40].

1.3 OUTLINE OF THESIS

The development of this thesis with regard to the NBNL problem is as follows:

Chapter 2 briefly outlines existing techniques of correction of RF amplifiers and points out the limitations of these techniques. The chapter concludes that for an important class of amplifiers NBNL problems still constitute a major impediment to system performance.

Chapter 3 suggests a solution to the problems identified. By the use of digitally controlled predistortion (digital compensation) it may be possible to achieve improved performance. The chapter identifies the options available in designing a viable compensation scheme and relates them to the current literature on predistortion. The chapter concludes by identifying AM modulation schemes as having some advantage for the initial testing of digital compensation schemes.

Chapter 4 considers the implications of using a digital nonlinear function generator (digital NLFG) as the predistorting

element. Without being specific as to the type of compensation used, noise, bandwidth and sample rate effects are calculated.

Chapter 5 considers the effects of imperfect envelope modulators. In particular an estimate of the degree of local oscillator breakthrough which is tolerable is given. Methods by which this can be achieved are discussed.

Chapter 6 considers how AM/AM and AM/PM effects may be separated, and quantified individually. Approaches used in the current literature are discussed, but considered unwieldy. Techniques based on digital compensation are put forward and are shown by experimentation to achieve excellent results.

Chapter 7 considers how the effects of imperfect envelope detectors may be reduced by methods specific to digital compensation, and results obtained.

Chapter 8 considers the phase detector, and how improvements comparable with the work of chapter 7, may be obtained.

Chapter 9 lays the theoretical basis for a type 1 algorithm (an algorithm capable of adaptive predistortion). Excellent experimental results are obtained. The limitations of the method are discussed and it is concluded that the method may have application in some areas.

Chapter 10 considers type 2 algorithms. The theoretical basis is presented, along with experimental results. The experimental results are not encouraging, and it is concluded that type 1 and type 3 algorithms hold greater potential for development than type 2 algorithms.

Chapter 11 considers type 3 algorithms. Type 3 algorithms are more mathematically intractable than the other types and convergence of these algorithms can only be shown for certain input waveforms. The theory behind this is verified by computer simulation. Experimental work indicates that convergence is in fact general, and excellent results are obtained. The chapter concludes that the potential future for these algorithms is bright, but that initially they may have to be confined to a number of limited problems.

Chapter 12 concludes the thesis by pointing out that a mathematical and experimental framework has been constructed for dealing with the NBNL problem, and suggesting some areas of future work.

CHAPTER 2

EXISTING TECHNIQUES OF CORRECTION

2.1 INTRODUCTION

In an effort to correct for the NBNL deficiencies of RF amplifiers a number of techniques have been developed. We shall briefly examine the major, and some of the minor, techniques of correction. In dividing between major and minor techniques we follow much the same classification as Abuelma'atti [ref 2]. Green [ref 67] has also carried out a brief review of existing techniques. There appears to be a general consensus as to the advantages and disadvantages of each technique.

2.2 MAJOR TECHNIQUES

We can identify four major techniques in common use: derating, feedback, feedforward, and predistortion.

2.2.1 Derating

Derating an amplifier simply means operating the amplifier away from saturation, where the nonlinear effects are largest, by the amount by which the amplifier is backed off. When the amplifier consists of the summed outputs of individual amplifiers the term "power sharing" is sometimes applied. References 39 and 41 show cases where derating is applied. Figure 2.2.1 shows a simple example where, by paralleling two amplifiers (and hence obtaining a minimum backoff of 3 dB for each), a reduction in intermodulation of 6 dB is obtained. This calculation assumes a third order model of the nonlinearity, as in reference 41.

The principal problem with derating is that to achieve a given intermodulation specification the power handling capability can quickly become ridiculously large. To take a very simple example, given N carriers each of unity envelope voltage, the peak envelope voltage is given by N volts and the peak envelope power (PEP) is given by N^2 watts. Thus if given 10 asynchronous carriers, each of 1 watt average power, the RF PA would need to have a minimum saturated power output of 100W to avoid clipping, and a power of 200W to give 3 dB backoff. Cost, dc-RF efficiency and weight (inclusive of cooling and power supplies), may well rule out such an approach for a 10W average power system.

Derating only applies when the RF amplifier is linear away from the saturated region. In practice, of course, small nonlinearities do exist away from this region even in class A amplifiers. These small nonlinearities can become dominant in class B and class C amplifiers, breaking down the basis of derating [ref 83].

Derating can give a useful reduction of intermodulation distortion (IMD), but does so primarily at the expense of increased cost and dc-RF efficiency.

2.2.2 Feedback

It is useful to recognise a number of different types of feedback: RF feedback, envelope feedback, phase feedback, polar and cartesian feedback.

RF Feedback

Figure 2.2.2 shows the classic feedback arrangement about

an amplifier. Several authors have analysed this arrangement when the amplifier is nonlinear [ref 5, 64, 111, 114, 125], and all conclude that the overall linearity is improved, although higher order distortion effects are produced by the arrangement. Meyer [ref 111], using a Volterra series approach, shows that RF feedback may be optimised to control the division of cross-modulation between phase and amplitude cross-modulation. The effectiveness of feedback is dependent on the open loop gain A and the feedback ratio β . At low frequencies large quantities of feedback can be used (i.e. large $A\beta$), but this is not the case at RF frequencies, especially for power amplifiers. The primary problem is the difficulty of obtaining stability at high gain for feedback amplifiers at these frequencies. (This arises from the time taken for the signal to transit the amplifier being comparable with the time taken for the input signal to change. Hence simultaneous comparison between input and output signals is not possible). Consequently, the effectiveness of the technique diminishes rapidly as frequencies increase, and the total phase shift through the amplifier and feedback loop increases. A secondary problem is that of reduced dc-RF efficiency due to power being coupled through the feedback network.

Despite these problems RF (and IF) feedback is reported to give some reduction [ref 41] in intermodulation distortion (IMD).

Envelope Feedback

Envelope feedback is illustrated in Figure 2.2.3. The linear feedback network of Figure 2.2.2 has been replaced with an envelope detector, and an envelope modulator. By feeding back

just the envelope in this way we can insert gain in the feedback loop without affecting stability (since the RF amplifier and the envelope amplifier operate at different frequencies). Thus the more practical arrangement of Figure 2.2.4 results, and the effectiveness of the technique can be clearly seen as the loop gain can be enormously increased over RF feedback, without incurring stability problems. The technique is widely used in many fields [ref 8, 23, 128, 189].

The disadvantage of the technique is that it relies on obtaining the difference between the input envelope and the output envelope (not the input signal and the output envelope). Consequently the input envelope needs to be derived if it is not already known (as, for example, in AM). Blanke [ref 23] shows how the technique can be extended to systems other than AM. Because of tolerancing effects using Blanke's scheme envelope feedback is not as effective as with AM. Petrovic and Gosling [ref 127, 129] have used envelope feedback with SSB by deriving the SSB envelope prior to amplification.

As the bandwidth of the RF signal increases, so too must the bandwidth of the envelope. References 43 and 135 show that the bandwidth of the envelope of a bandlimited RF signal is in general infinite (AM signals being a special case). When using envelope feedback we refer to the bandwidth, which to all practical intents and purposes, is sufficient to be regarded as the envelope bandwidth. Reference 129 suggests that this should be approximately 15 times the RF bandwidth. However, the RF bandwidth will itself be large due to the high order spurious IMD. Thus, for example, reference 130 uses a total bandwidth of 300 kHz

to handle 3 kHz bandwidth RF.

Leaving aside the special case of single channel AM it is plain that as the RF bandwidth increases the phase shifts through the envelope feedback loop become more significant, leading to stability problems, and the closed loop gain must be diminished to maintain stability [ref 189], with a decrease in effectiveness. Thus envelope feedback, although highly effective in some situations, is not well suited to multichannel and VSB-TV NBNL problems, because of the bandwidths and the lack of a suitable input envelope signal.

Another point that must be made is that envelope feedback can only correct for AM/AM and not for AM/PM distortion. Thus, even using large loop gains, it is possible for IMD to remain high.

Phase Feedback

Phase feedback is a logical extension of envelope feedback, but applied to the phase of the RF amplifier instead of the envelope. The basic method is shown in Figure 2.2.5. References to the technique being used are scarce (e.g. [ref 23, 137, 161]). Probably the main reason for the technique not being widespread is that it does not correct for AM/AM conversion, which is usually the dominant distortion effect near saturation. As with envelope feedback the technique has drawbacks when the phase angle of the signal is not known prior to amplification. Also, as with envelope feedback, the bandwidth of the feedback signal may be large in relation to the RF bandwidth, so that stability problems are met even when the RF bandwidth is low.

Polar and Cartesian Feedback

We take polar feedback to mean that envelope and phase feedback are operated simultaneously. Thus the method put forward by Blanke [ref 23] qualifies as polar feedback, although Blanke did not call it such. The term polar feedback is predominately used in conjunction with the work of Petrovic et al [ref 127, 129, 130], who have been primarily concerned with SSB transmitters. (The need for linear RF amplifiers arises from the desire to stop spectral spreading). Blanke reports no experimental results. The experimental work of Petrovic et al is well documented and shows excellent results. However, the work is somewhat specific to SSB, and although it could be extended to other schemes, the bandwidth/stability problems look formidable and the technique has yet to be reported for use for anything other than 2-3 kHz RF bandwidths.

A polar loop transmitter is shown in Figure 2.2.6 (taken from reference 130).

Cartesian feedback seems the more applicable term to the work of Weaver [ref 181], who has used quadrature demodulators in the feedback loop to correct for a M-QAM system. Weaver reports only computer simulations and no experimental results. Furthermore, Weaver requires that spectral shaping of the M-QAM spectrum takes place post-amplifier, so that only digital signals propagate around the feedback loop.

2.2.3 Feedforward

Feedforward is an old technique, as a low frequency technique it predates feedback [ref 21], which can be adapted for

use at RF [ref 17, 57, 67, 110, 152, 168]. The basic method is shown in Figure 2.2.7. In essence the output from the main RF amplifier is compared with the input to produce an estimate of the RF distortion. This estimate is then summed in anti-phase with the output of the RF amplifier so as to cancel the distortion. The technique involves no feedback, and as such is unconditionally stable and may be applied to signals of any bandwidth.

Results using feedforward systems can be good, but there are a number of drawbacks. The primary drawback is the addition of the summing device at the output of the RF amplifier. This implies that significant RF power may be lost if the summing ratio is low (e.g. a 3 dB coupler will result in a loss of 3 dB of power, a 10 dB coupler will result in a 0.45 dB power loss). Thus to avoid large power loss, with the consequent need to operate the RF amplifier closer to saturation, the tendency is to have high summing ratio's. This results in the error amplifier having to handle significant quantities of power, and its linearity becomes of importance. Thus, aside from the problem of poor dc-RF efficiency, for power amplifiers there comes a point where there is little to be gained, even in terms of linearity, by increasing the summing ratio.

Feedforward also has tolerancing problems imposed by the use of at least two differencing junctions and two time delays. At RF frequencies these can be difficult to fabricate to high accuracy due to problems of signal leakthrough. Thus the practical feedforward system does not have unlimited performance capability. Bennett [ref 17] gives an excellent discussion of the tolerancing accuracy required for a specified performance.

2.2.4 Predistortion

Predistortion is a technique whereby a nonlinear device is placed prior to the RF amplifier, the device being chosen such that the combined system of predistorter and series amplifier is linear (see Figure 2.2.8). Like feedforward the technique is unconditionally stable, and potentially broadband. References 2, 13, 27, 31, 36, 83, 92 and 93 give examples of the technique. Many individual variations exist, usually in the derivation of the predistorter. Reference 13 discusses a commonly used diode network. References 27, 36, 83 and 93 use RF amplifiers as the predistorter so that a better match to the RF amplifier used may be obtained (see Figure 2.2.8). The predistorter must be in series with the RF amplifier, but it can be at IF frequencies [ref 2, 167], or even at baseband [ref 60].

Distortion correction can also be implemented as a postdistortion system [ref 81, 169], although there seems to be no advantage in such a system and considerable disadvantage in terms of dc-RF efficiency.

Predistortion can give good results. Figure 2.2.9 shows the reduction in intermodulation distortion that can be achieved (taken from the work of reference 13). Walker [ref 176] quotes an improvement in bit error rate for QAM systems with predistortion of the order of seven orders of magnitude. To achieve this degree of suppression of intermodulation products (IMP's) requires that the predistorter and RF amplifier be very closely matched in characteristics. However, control over both the amplifier and predistorter characteristics may at best be minimal. [Reference 3 points out that even using diode function generators as

predistorters not all functions may be approximated due to the diode turn on voltage. References 3 and 16 discuss the design of such predistorters]. It follows from this that thermal and long-term drift in either the predistorter or the RF amplifier causes a degradation in IMP suppression. Chakraborty [ref 31] points out that the simple theory of a predistorter places great reliance on the cube of the linear term of the predistorter, see Figure 2.2.8, which suggests extreme temperature stability is required.

Figure 2.2.10 shows a graph from reference 165 which compares predistortion, and feedforward, against a theoretical hard limiter and typical TWT performance. Because of the problem of synthesizing the predistorter to approximate an arbitrary nonlinear function, large suppression of IMD is only possible over part of the dynamic range.

It is illuminating to discuss how VSB-TV transmitters are specified using predistorters in the UK [ref 58]. It is accepted that the predistorter can give large IMP suppression, but only -6 dB of suppression is acceptable since it is not believed that higher suppression can be maintained for any length of time. (Specifications are for IMD to be -52 dB on PEP, at least -46 dB suppression being achieved without use of predistorter).

Predistortion is popular with power amplifiers since no loss of RF power is involved.

2.3 MINOR TECHNIQUES

Aside from the major techniques which can be applied in a wide variety of circumstances there exist a large number of minor

techniques which can be applied according to opportunity, as well as a number of techniques designed to achieve a specific balance between dc-RF efficiency and linearity.

2.3.1 Cascode Networks

The use of cascode amplifiers can improve the linearity of RF amplifiers [ref 113]. Although as a technique it is not suitable for power amplifiers, it is widely used in CATV distribution networks because of the high third order intercept points that can be achieved [ref 171].

2.3.2 Selection of Bias Point

High linearity transistors are designed with bias point very much in mind [ref 15], so that over a narrow range of operation, exceptional linearity can result from third order effects being cancelled by higher order distortions. The idea can be carried over from transistors to amplifiers [ref 126, 151].

2.3.3 Helix Modulation

For TWT's the phase of the output signal depends on the helix voltage. This can be exploited when trying to remove AM/PM effects [ref 10, 121].

2.3.4 Second Harmonic Injection

It has been reported that injection of second harmonic into the input of the TWT's has reduced intermodulation distortion [ref 145, 169].

2.3.5 Strong Signal Injection

Shaft [ref 153] has shown that addition of an out-of-band strong signal can reduce IMD in the passband. The strong signal may be removed by filtering at the output of the amplifier.

2.3.6 Nonlinear Feedback

Essentially a nonlinear device is put into the feedback loop of the amplifier. If the nonlinear device is properly adjusted it will increase the magnitude of the IMP's, whilst keeping the linear term fixed in magnitude. Thus, the system has the appearance of having large loop gain to distortion products, and low loop gain to the linear terms [ref 24, 70].

2.3.7 Use of Hybrids and Couplers

When IMD is caused by another transmitter interacting with the output of the RF amplifier, reduction of the IMD so caused can be achieved by attempting to isolate the output of the RF amplifier and the transmitter. This can be variously achieved by circulators, hybrids, and narrowband filters [ref 55, 57, 80, 101, 120, 131].

2.3.8 Selection of Frequencies

In multichannel operation it may be possible to assign the channel frequencies such that the IMP's lie outside the passband of the receiver. References 42, 88 and 97 discuss how the channel frequencies can be determined.

2.3.9 Amplifier Techniques

There exist a large number of RF amplifier designs which seek to achieve a compromise between dc-RF efficiency and linearity. We are not over-concerned with these techniques since linearity is our prime objective. Clarke [ref 34] gives a good discussion of the more widely used systems and Abuelma'atti [ref 2] provides a fairly complete list. References 56 and 160, taken in conjunction with references 2 and 34, appear to provide an exhaustive list.

2.4 CONCLUSIONS

Existing techniques of correction of NBNL RF amplifiers do, in specific instances, greatly reduce the IMD (e.g. envelope feedback applied to AM, polar feedback for SSB). However, there still exist a wide class of problems where existing techniques are not entirely satisfactory, particularly in relation to those problems associated with relatively large RF bandwidth power amplifiers (e.g. multichannel links and VSB-TV). For such problems feedback cannot be applied due to stability, and feedforward is unattractive due to the large power loss in the output coupler, which leaves only predistortion. Predistortion does not in practice provide large IMP suppression for the reasons discussed above.

FIGURE 2.2.1

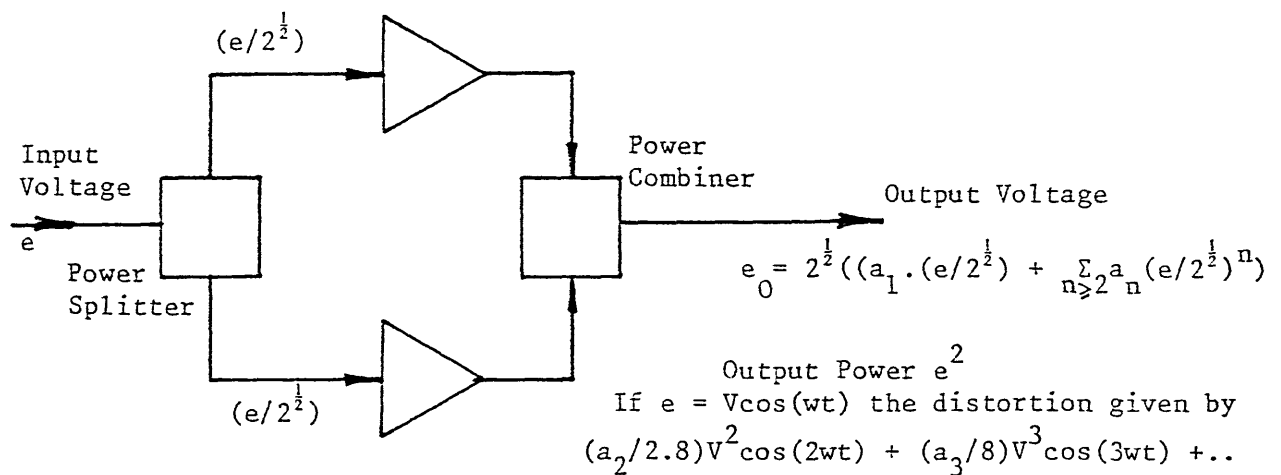
All input, output voltages and transfer functions refer to the envelope voltage.



Output Power e^2

If $e = V \cos(\omega t)$ the output distortion given by

$$(a_2/2)V^2 \cos(2\omega t) + (a_3/4)V^3 \cos(3\omega t) + \text{higher terms}$$



Output Power e^2

If $e = V \cos(\omega t)$ the distortion given by
 $(a_2/2.8)V^2 \cos(2\omega t) + (a_3/8)V^3 \cos(3\omega t) + \dots$

For the same output power the distortion is reduced by the factor $(1/1.41)^{n-1}$

This result is quoted directly from Denny (ref 41)

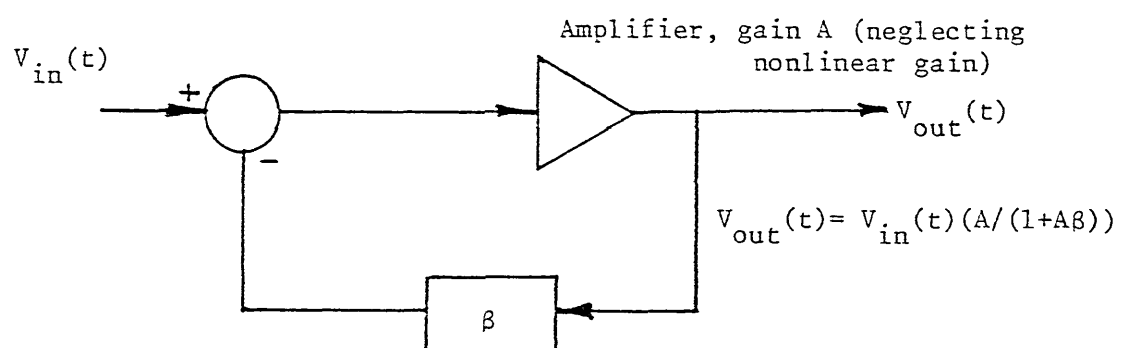
FIGURE 2.2.2

FIGURE 2.2.3

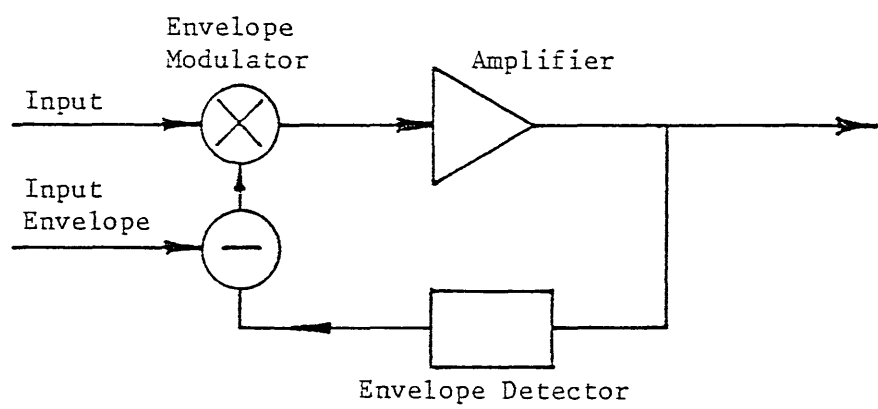


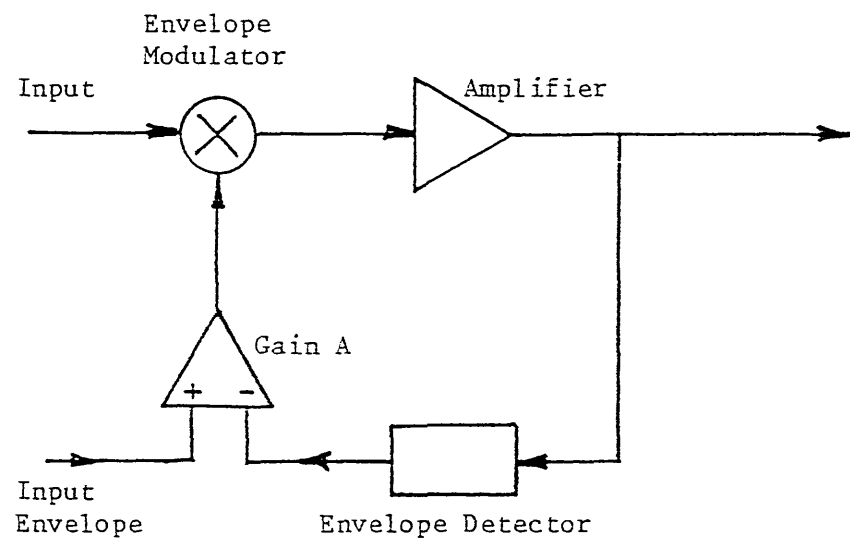
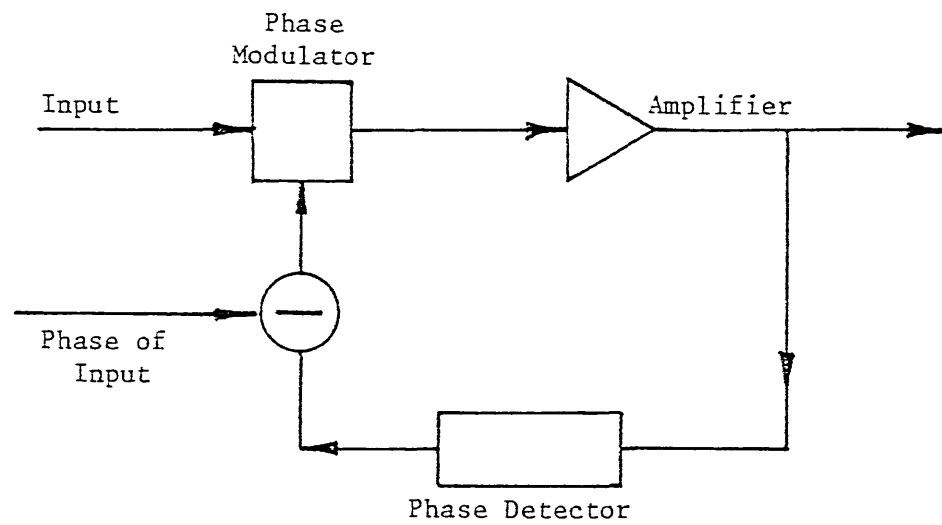
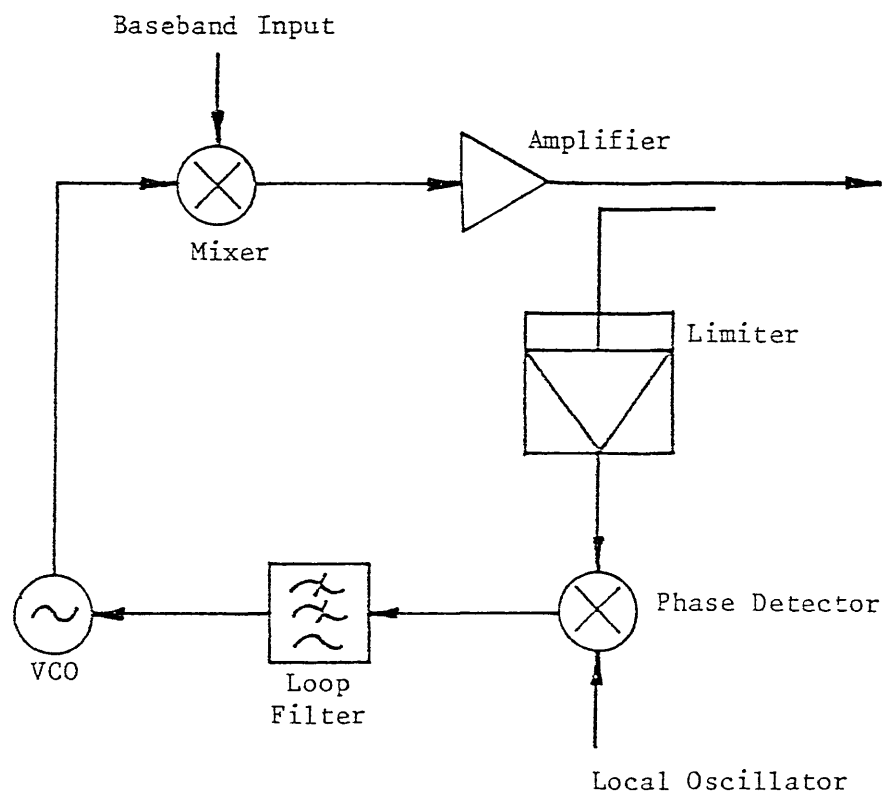
FIGURE 2.2.4

FIGURE 2.2.5



The practical implementation of the above for AM signals is shown below, using phase locked loop (PLL) techniques.



Polar loop transmitter configuration
Taken from ref 130.

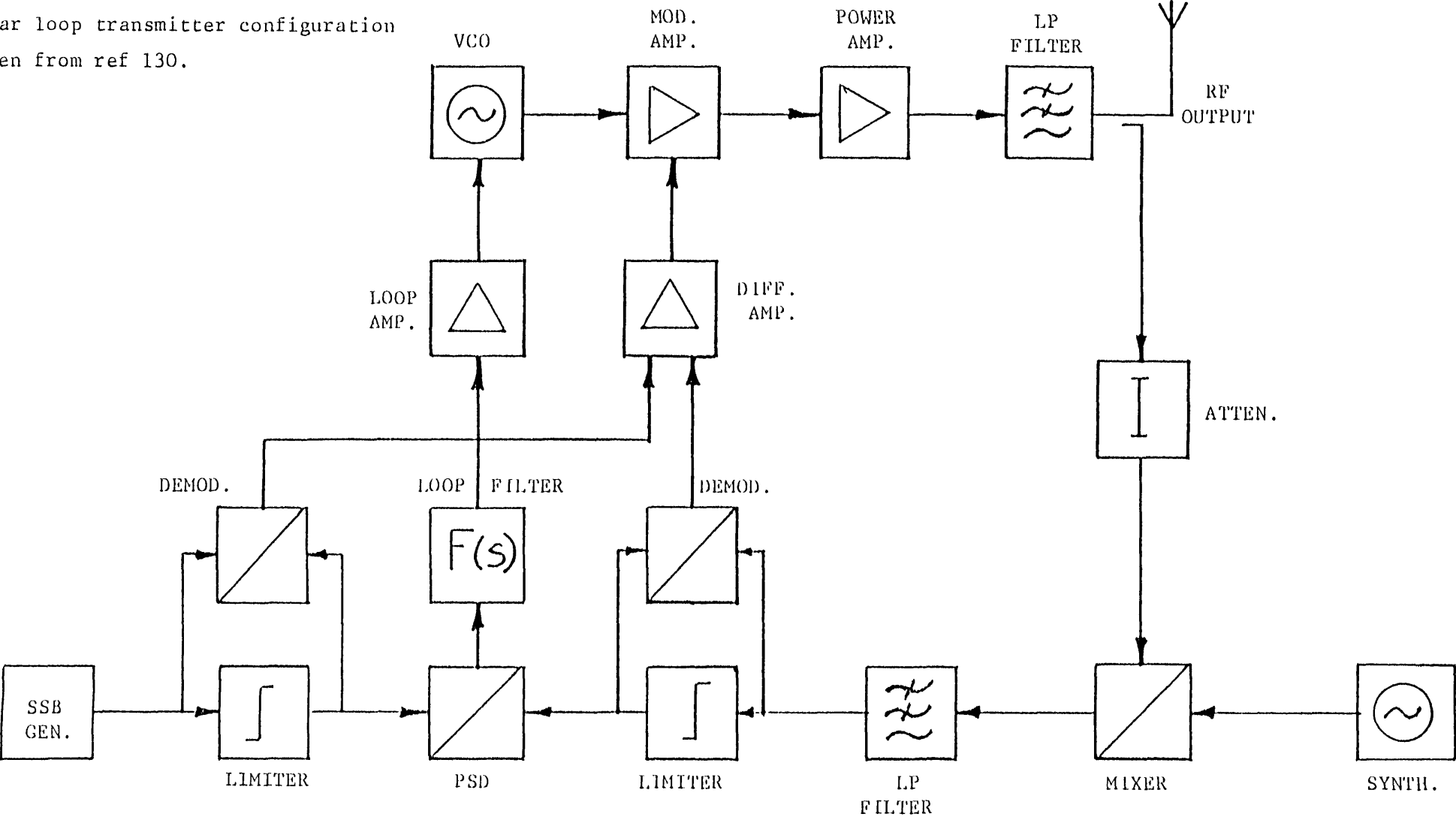


FIGURE 2.2.6

FIGURE 2.2.7

All input, output voltages and transfer functions refer to the envelope voltage.

Basic Feedforward Configuration

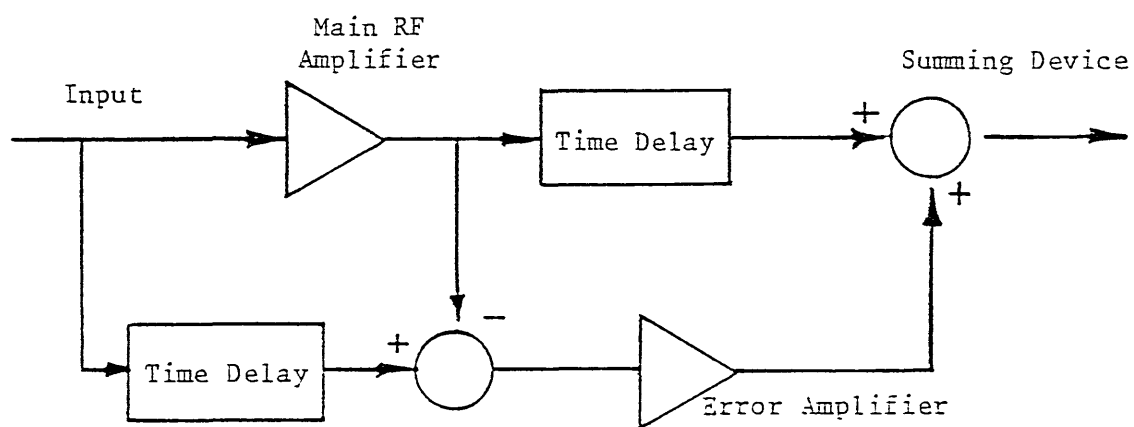
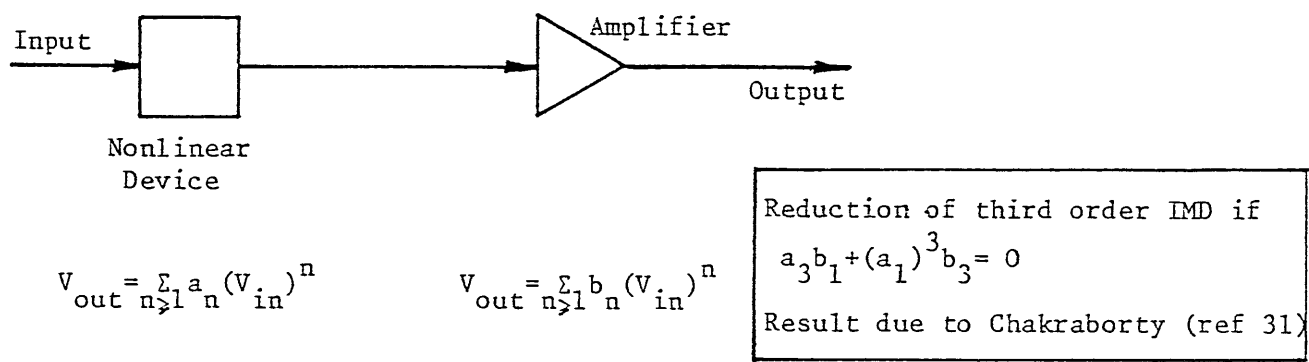
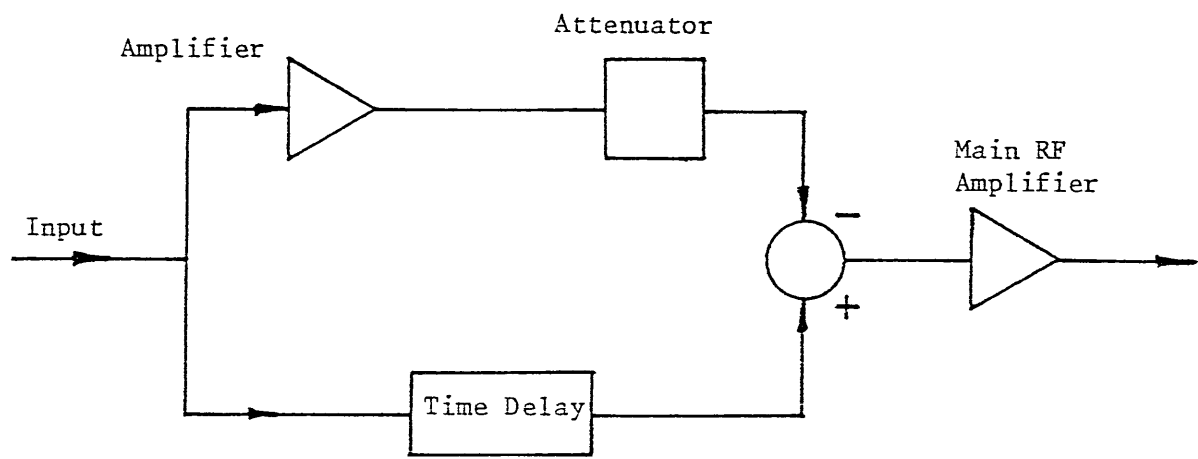


FIGURE 2.2.8

Basic Predistortion



Use of Second Amplifier as the Nonlinear Device



The spectral effects of intermodulation distortion for a five tone test, with and without diode predistortion, are illustrated.
(taken from the work of ref 13)

CENTRE FREQUENCY 154 MHz
10 dB/ vertical
0.5 MHz/ horizontal

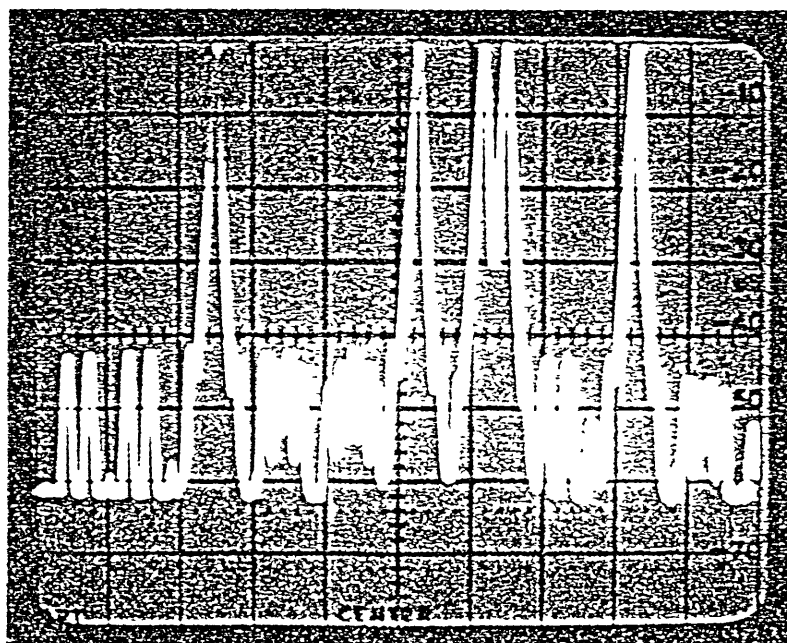
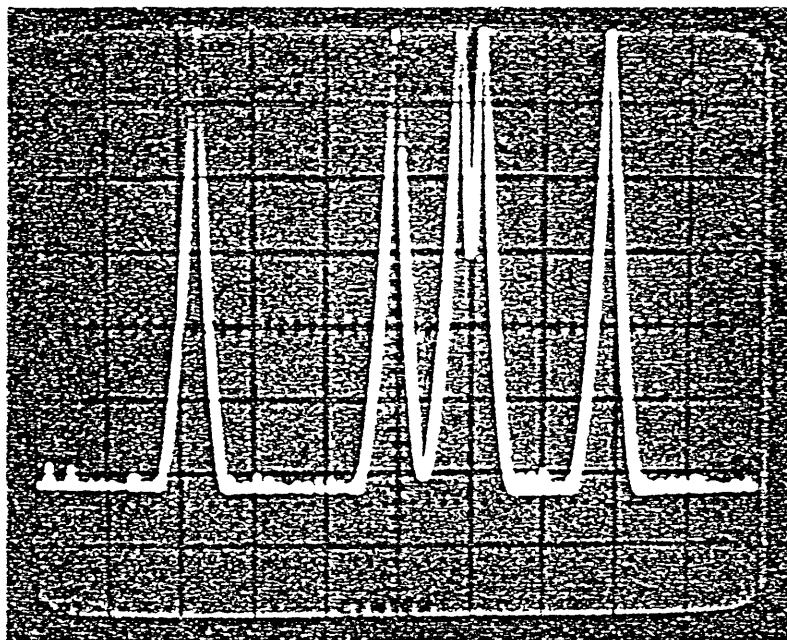
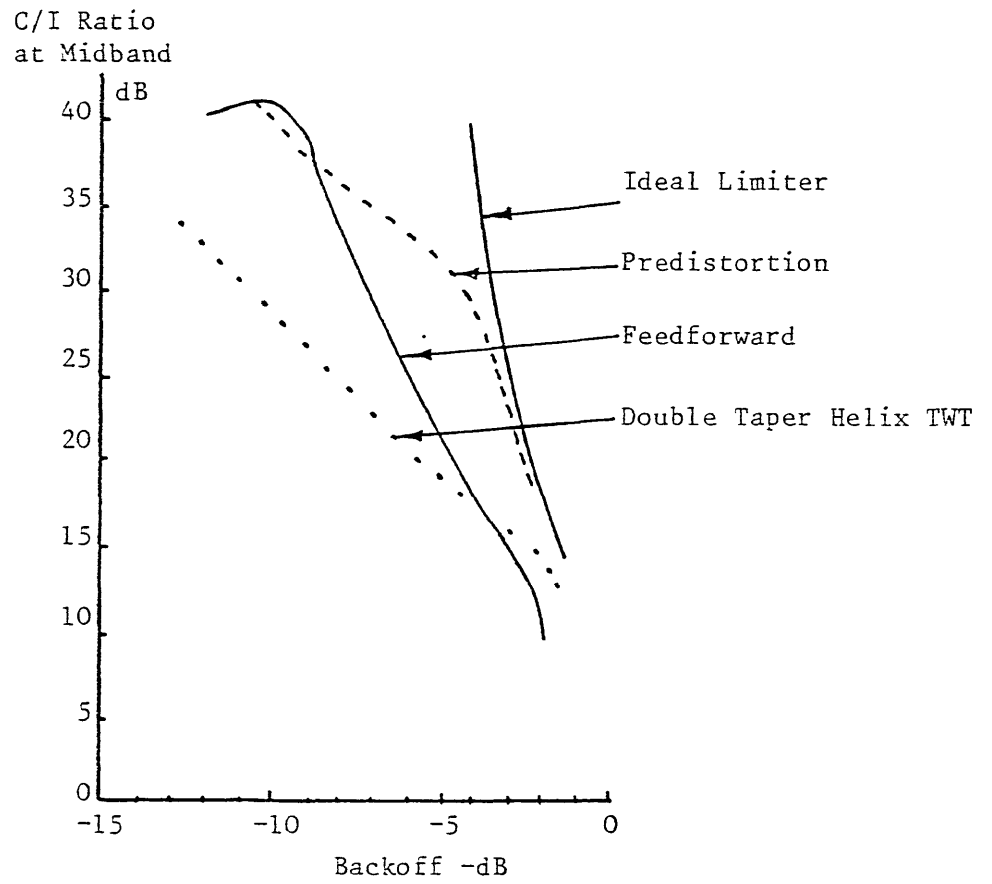


FIGURE 2.2.10

Comparative Effects of Feedforward and Predistortion (from ref 165)



CHAPTER 3

DIGITAL COMPENSATION

3.1 INTRODUCTION

We define digital compensation to mean predistortion using a ~~series~~^{cascaded}, digitally controlled, nonlinear function generator. It is to be hoped that a number of benefits will derive from digital compensation, in that the shortcomings of conventional predistortion may be overcome whilst retaining its more attractive features.

As has been discussed conventional predistortion suffers from an imprecise control over the nonlinear function it is meant to approximate. Worst still, the predistorter and its amplifier chain are prone to drift out of alignment with each other. This results in relatively poor suppression of IMD spectra when compared to other techniques (e.g. polar loop and envelope feedback). However, conventional predistortion is still widely used since it has few bandwidth problems and no stability problems, which is not true of the other techniques.

With digital compensation, and a well designed nonlinear function generator (NLFG), we would aim for much more precise control than can be achieved using analogue circuitry. Furthermore, using a digital NLFG we would expect to be able to control the nonlinear characteristic with considerable ease and flexibility, thus opening up the possibilities of adaptive behaviour to circumvent the problem of drift. Thus we would hope to obtain spectral suppression of IMD approaching that obtainable by feedback techniques (i.e. polar loop and envelope feedback).

It will, of course, remain of prime importance to keep such A/D's and D/A's as we require operating at sampling rates which can be achieved. As one of the great advantages of predistortion is the ability to work with wide bandwidth signals we do not wish to unduly limit digital compensation to narrowband operation. Figure 3.1.1 shows the current state of A/D conversion time versus bit accuracy. The information in Figure 3.1.1 is taken from a variety of sources, [ref 6, 61, 62], and is indicative of the increasing utility of purely digital, as opposed to analogue, approaches to problems.

3.2 RANGE OF APPLICATIONS

Before going on to consider the classification of digital compensation systems it is advantageous to consider a few digital compensation schemes to make the idea more concrete.

Figure 3.2.1 shows a possible arrangement to compensate for NBNL distortion in a single access transmitter link. The transmitting ground station may be passing TV or multichannel signals, but it is the only transmitter using the satellite transponder. In essence the input IF signal envelope is predistorted by a digital NLFG. The NLFG produces an instantaneous voltage transfer characteristic which distorts the IF signal. By bandpass filtering of the first zonal harmonic the effect of the instantaneous characteristic is to produce a digitally controlled envelope transfer characteristic. The resulting signal is then converted up to final frequency and transmitted through the amplifier/antenna chain to the satellite. The satellite receives the signal, frequency translates it to the

downlink frequency and retransmits the signal to be received by all earth receiving stations. The transmit station also receives this signal and measures the envelope of the received signal. In a linear transmit-receive system the receive envelope would be a time-delayed and scaled version of the transmitted envelope, which has also been measured. The two envelopes are compared and the NLFG adjusted to compensate for any nonlinear effects that have arisen. This will make the transmit-receive path linear for the envelope transfer characteristics (i.e. remove AM/AM effects from IF input to IF output).

Provided the receive chain is linear (this is easier to achieve than making the higher power transmit side linear), we are assured that the envelope of the downlink signal, measured at the antenna, is a linear copy of the uplink IF envelope. This will be true when the envelope is measured at other earth stations, thus AM/AM distortion has been removed from the link.

Digital compensation is not confined to satellite links and AM/AM conversion. In Figure 3.2.2 we show an AM transmitter which suffers only from AM/PM. This effect is to be removed by digital compensation. The output phase angle of the transmitter is measured and recorded by the CPU, via the limiter, phase detector and A/D. The CPU also records the input signal (which for AM is equivalent to the output envelope). Over a number of measurements the relationship between the envelope and the phase can be deduced, and the NLFG adjusted to remove the phase modulation, by adding a phase modulation to the RF carrier which is in anti-phase with the phase modulation induced by AM/PM conversion.

3.3 HIERARCHY OF DECISIONS

In attempting to identify potential digital compensation systems it is useful to consider the series of decisions which a hypothetical designer would need to make before producing a final design. By considering the hierarchy of decisions we hope to classify and enumerate systems, whilst at the same time providing a design-aid for designers. The complete hierarchy is shown in Figure 3.3.1 and we shall discuss each set of decisions in turn.

3.3.1 Basis of the Representation

The output of a nonlinear device with input

$$A(t)\cos(\omega t + \phi(t)) \quad (3.1)$$

(where $A(t)$ is the time-varying envelope, ω the centre frequency of operation and $\phi(t)$ the time-varying phase) is conventionally represented in the first harmonic zone by

$$f(A)\cos(\omega t + \phi(t) + g(A)) \quad (3.2)$$

where $f(A)$ is an odd function of $A(t)$ and $g(A)$ is an even function of $A(t)$. [Although the instantaneous voltage transfer function can be described by a mixed polynomial (i.e. even and odd terms), only the odd terms group themselves with the first zonal harmonic for the resultant distortion [ref 20]. Similar reasoning shows that the phase function is an even function of the envelope].

This representation of the output may be termed a polar representation because of the obvious analogy with polar co-ordinates [ref 50, 140]. We identify $f(A)$ as the radial function and $g(A)$ as the angular function, and can associate AM/AM and AM/PM conversion with the functions $f(A)$ and $g(A)$ respectively. However we can rearrange equation 3.2 into a

cartesian form since

$$\begin{aligned}
 & f(A)\cos(wt + \phi(t) + g(A)) \\
 = & f(A)\cos(g(A))\cos(wt + \phi(t)) - f(A)\sin(g(A))\sin(wt + \phi(t)) \\
 = & X(A)\cos(wt + \phi(t)) - Y(A)\sin(wt + \phi(t)) \tag{3.3}
 \end{aligned}$$

where $X(A)$ and $Y(A)$ are odd functions of $A(t)$.

Which basis we choose for the representation of the distortion is determined purely by convenience. Broadly speaking those modulation schemes which rely on the addition of two orthogonal carriers (e.g. filtered QPSK, QAM [ref 47]), are best thought of as having separate distortion functions acting on the in-phase and quadrature signals, and we construct our digital compensation scheme accordingly (i.e. to force $X(A)$ and $Y(A)$ to be linear functions of $A(t)$). Some general purpose modem schemes use orthogonal carrier modulation because of its flexibility [ref 139]. However many of the most important areas where predistortion is currently applied (e.g. VSB-TV, multichannel links), do not use orthogonal modulation methods. Consequently, they are best thought of using the polar representation, and the digital compensation scheme constructed to make $f(A)$ a linear function of $A(t)$ and $g(A)$ a constant.

Note here that by filtered QPSK we mean all the QPSK-like schemes which aim to limit the modulation sidebands by filtering (e.g. bandlimited QPSK, TSI-OQPSK [ref 48], SQORC, MSK [ref 158]). The result of this is to produce a QPSK signal which does not have a constant envelope and hence suffers from AM/AM, AM/PM distortion. The same filtered QPSK can be achieved by orthogonally modulating analogue data streams, rather than discrete digital data streams [ref 158].

Single sideband (SSB) modulation may be produced by orthogonally modulated carriers [ref 138, p.6.65-6.68], or by a single carrier modulation (AM modulation followed by filtering of the unwanted sideband). The majority of SSB transmitters appear to use the latter method, as it gives higher spectral suppression of the unwanted sideband, and so in general we would use the polar representation for SSB.

Plainly it is possible to envisage representations other than cartesian and polar. In practice however it is difficult to see that any other basis will be commonly used.

3.3.2 Vector and Scalar Control

The purpose of digital compensation is to control the distortion by forcing one of $f(A)$, $X(A)$, $Y(A)$ to be a linear function of $A(t)$, or by forcing $g(A)$ to be a constant. Control over any one of these functions will involve at least one NLFG in series with the nonlinear device, giving scalar control over the output signal. If we choose to control the pair $f(A)$ and $g(A)$, or $X(A)$ and $Y(A)$, we still have a digital compensation scheme, but we require two NLFG's and have vector control over the output (i.e. we will control both the envelope and the phase of the output distortion).

To choose between scalar and vector control (i.e. one or two independent NLFG's), we need knowledge of the nonlinear devices and the specifications of the communications link.

For example, in a satellite transmitter link the linearity is often specified by an average intermodulation product (IMP) power level in a given bandwidth [ref 44, p.73]. Provided the

envelope transfer characteristics and the AM/PM characteristic of the transmitter are known, the contribution made to the IMP power density by the removal of distortion associated with one or two of $f(A)$, $g(A)$, $X(A)$, $Y(A)$, may be calculated [ref 20, 155, 164]. From this the need for scalar or vector control may be decided. [Note that, in practice, the form of the input signal must be known, e.g. the number of carriers and their frequencies and power levels. Often the complete calculation is best performed on a computer using a dedicated analysis program [ref 19, 49]].

In other links the specifications may be on the fidelity of the link. In such a case the nature of the receiver is of great importance since it may be more susceptible to one of $f(A)$, $g(A)$, $X(A)$, $Y(A)$. For example in an AM link the distortion at the output of the receiver may depend on the AM/AM conversion and not on the AM/PM conversion, thus scalar control of $f(A)$ would prove adequate.

3.3.3 IF or Baseband Input to NLFG

The input to the NLFG may be at RF, IF, or baseband. However, as there is little functional difference between IF and RF frequencies we take the definition of IF frequencies to include the final RF frequency. For digital compensation at baseband to work we must be able to control one of $f(A)$, $g(A)$, $X(A)$, $Y(A)$, by distorting the baseband signal using an NLFG.

For orthogonal modulation schemes there are usually two baseband signals accessible, $I(t)$ and $Q(t)$ (see Figure 3.3.2). As they undergo distortion by $X(I)$ and $Y(Q)$ respectively, for digital compensation schemes in which the orthogonal representation is

natural, baseband predistortion will generally be preferred in that it requires comparatively low sample rates and little complexity (see Figure 3.3.3).

When the polar representation is preferred only AM modulated links are easily dealt with at baseband, since only with AM modulation is the envelope a baseband signal and hence $f(A)$, $g(A)$ may be deemed to act on baseband signals.

3.3.4 Choice of Adaptive Algorithm

It is important to clarify the difference between a feedback system and an adaptive system.

Feedback requires simultaneous comparison of the input and output signal. Due to finite time delays through the system we cannot achieve an ideal feedback arrangement. As a consequence of this there exists the possibility of oscillation. However, feedback systems do have a rapid response to changes in operating parameters (e.g. response to ambient temperature variations).

In an adaptive system the finite time delay between cause and effect can be taken into account (e.g. by simply recording the input and waiting for the corresponding output to occur). Having observed the cause and effect relationship, the adaptive system then decides how to modify the system and implements a change. The result is that the response to changes in operating parameters is slow (compared to feedback), but that the possibility of oscillation does not exist in the sense used with feedback. What does exist is the possibility of instability in the CPU algorithm, such that a Bounded Input does not produce a Bounded Output (the BIBO criterion of reference 157), or oscillation in the sense of a

closed limit cycle [ref 174]. In the case of digital compensation we may take the view that for the adaptive algorithms the signal per se is of no interest, but the AM/AM and AM/PM transfer characteristics are. It is a consequence of this that allows us to abrogate the need for "real-time" signal processing. In discussions of the input and output signals it should be taken as read that any time delays that exist between the two signals have been accounted for.

The general form of adaptive digital compensation is shown in Figure 3.3.4. The CPU will measure the output parameter of the device under test (DUT) that is of interest (i.e. the envelope, phase, in-phase modulation, or quadrature modulation), and compare it against a reference. On the basis of the difference, decisions can be made as to how the NLFG should be adjusted. The reference may be derived from a number of different sources. We can identify three types of CPU algorithm for making the decision as to how the NLFG should be adjusted, according to the source of the reference signal (see Figure 3.3.5).

1. The reference is known a priori.
2. The reference is derived from a detector placed before the DUT and after the NLFG.
3. The reference is derived from a detector placed before the DUT and prior to the NLFG.

Each of these will result in a completely different CPU algorithm and they together constitute the range of options open. We will briefly discuss each type in turn.

Type 1

For digital compensation the most useful form of this type of CPU algorithm relies on a test waveform being made to pass through the transmitter. The reference associated with this test waveform is known in advance. Figure 3.3.6 shows a possible AM transmitter based on this type of algorithm. The transmitter is based on the polar representation, has scalar control over the AM/AM conversion, and is baseband input to the NLFG. With the input signal disabled the CPU causes the AM modulator to be linearly ramped by the D/A, and the output envelope to be recorded, so that a graph of $f(A)$ may be constructed as in Figure 3.3.6. Using that information the decision as to the functional form of the NLFG may be deduced. We shall attempt to show later that in this type of scheme the signal processing is usually very straightforward and the results of the compensation excellent. However, it is necessary to interrupt the normal transmitter signal to be able to update the NLFG as may be required. Provided that the characteristics of the DUT are only slowly varying (due to ageing, thermal drift, etc), the updating of the NLFG need only be infrequent and thus in many cases unobtrusive. An algorithm of this type might be applied to TV transmitters on the Insertion Test Signal (ITS), and has been suggested before for such a purpose [ref 2]. [For TV signals there exists a period of time in which the video output is in effect gated off (the blanking period [ref 133]). In this period the TV transmitter may be usefully used to transmit test signals of the type proposed].

As another example of this type of processing consider Figure 3.3.7 which is due to Abuelma'atti [ref 2]. This scheme is

not strictly within the confines of digital compensation (the NLFG is analogue). In Figure 3.3.7 the output distortion spectra is measured by a heterodyne voltmeter (the detector), and the dc output of the voltmeter used to control the bias on a diode network. (The method relies on the diode bias/nonlinear characteristic being a close match to the DUT). The processing falls into a type 1 category, since it relies on the separation of the IMP's from signal, by the heterodyne voltmeter, by a priori knowledge of what is, and is not, an IMP.

With a type 1 system we will require that the detector located at the output of the DUT is linear, since any distortion introduced by the detector cannot be distinguished from distortion due to the DUT. Another requirement is that the effect of any change in the NLFG on the output signal is known a priori (i.e. although the form of the compensating function is known it must be implemented with the correct scale factors and these must be known a priori - see Figure 3.3.6). This generally requires knowledge of the A/D and D/A conversion gains in bits/volt and volts/bit respectively and of the conversion gains of the detectors. We also require that adjustment of the NLFG only occurs when a test signal is passing through the DUT, and normal operation of the transmitter may have to be suspended. The ability of a type 1 algorithm to follow thermal drift and ageing depends upon how often this suspension of operation can be allowed.

Type 2

This type of CPU algorithm is illustrated by an AM modulation scheme using a polar representation to remove AM/AM

conversion (see Figure 3.3.8). The scheme might be classified as polar representation, scalar AM/AM control, baseband input. The essence of the scheme is to derive an estimate of the function $f(A)$ for the DUT and to adjust the NLFG accordingly. Unlike the type 1 category there is no need to interrupt the normal operation of the transmitter with a test waveform, as adjustment is continually made in situ.

Operation of Figure 3.3.8 might be as follows: with the switch in position K the CPU records the output envelope ($A_k(t)$) at the instant the modulation signal is $A_m(t)$. Thus over a number of different measurements, with different values of A_m , the functional relationship between A_m and A_k is deduced. The CPU then changes the switch position to J and likewise determines the functional relationship between A_m and A_j . From this data the CPU can determine the relationship between A_j and A_k . But by definition this is the relationship between the input and output envelope of the DUT $f(A)$. Hence $f(A)$ is determined and the NLFG may be adjusted accordingly (see Figure 3.3.9). The process of measurement/adjustment is continually made so that any slow changes in the AM/AM characteristics (due to ageing or thermal drift), may be corrected as they occur.

Figure 3.3.10 shows a system designed to compensate for AM/PM conversion in an AM transmitter (polar representation, scalar control, baseband input). Since it is a type 2 algorithm the processing is as before to derive our estimate of $g(A)$ (i.e. we deduce A_m/ϕ_k , A_m/ϕ_j and hence $g(A)$). Having deduced $g(A)$ the NLFG may be adjusted as in Figure 3.3.11.

Figure 3.3.12 shows the vector system that results

from combining both scalar systems. The algorithms can run independently and suppression of AM/AM and AM/PM results.

A type 2 algorithm requires the detector located at the output of the DUT to be linear, since any distortion it introduces cannot be distinguished from the distortion of the DUT. The effect of changes in the NLFG on the output signal must be known a priori. Unlike a type 1 algorithm, however, there is no need to interrupt the normal transmitter operation.

Type 3

A cartesian, vector, baseband system for removing distortion effects in a QAM system is shown in Figure 3.3.13. The system is due to Grabowski [ref 66]. The output signal is synchronously demodulated to give the in-phase and quadrature data as measured after the DUT. This is compared with the in-phase and quadrature data from prior to the NLFG. By comparing the two sets of data streams the NLFG can be adjusted to remove the distortion effects (i.e. force the two sets of data streams to become identical). It is important to notice that in this scheme the higher order sidebands are removed by a RF shaping filter placed after the output detector. Hence the data streams are digital bit streams. In some systems of QAM the RF shaping is done by filtering the digital data stream at baseband, removing the necessity for the high Q RF filter. Grabowski's work has been limited to the consideration of digital data streams only. This limits his work to such systems where the shaping filter can be made with finite Q elements.

A polar, scalar, AM/AM, IF system is shown in Figure

3.3.14. The CPU makes a series of measurements of the input envelope and the output envelope. From this the AM/AM characteristic for the whole series combination of DUT and NLFG can be deduced. The NLFG is adjusted, and the measurements repeated. Unlike type 1 and type 2 algorithms, if the DUT characteristic remains unchanged we shall not record the same set of values as before since the NLFG has changed, i.e. the NLFG and DUT combined characteristics are always measured, and it is from this that the distinguishing features of type 3 processing arise.

Figure 3.3.15 shows the vector implementation of Figure 3.3.14. The operation is as before with the proviso that the combined AM/PM as well as AM/AM characteristics of the DUT and NLFG are always measured.

Figure 3.3.14 and Figure 3.3.15 can be usefully implemented with the aid of switches, to select between input and output measurements of the detector, as in Figure 3.3.8.

With a type 3 algorithm there may not be a requirement for the detectors to be linear detectors, if the detector characteristics in the input and output positions are identical. (Provided the input and output signals are both at the same frequency this is trivially assured by switching, but for baseband systems as in Figure 3.3.12 this is not the case). With a type 3 algorithm there is no need to have a priori knowledge of the effects of changing the NLFG on the output, since the overall linearity of the series combination is continually measured. Clearly there is no requirement to interrupt the normal transmitter operation. With a type 3 system there does exist the possibility of BIBO instability. (This arises from the fact that

any changes made in the NLFG will cause a change in the values measured by the detector. Since this in turn is used to modify the NLFG we have the possibility of errors in the NLFG growing and BIBO instability resulting). When considering the question of instability the effect of filters in the signal path is of prime importance.

3.4 ANTECEDENTS OF DIGITAL COMPENSATION

Digital compensation has not been considered before within the framework of a general classification. However, there do exist isolated instances of digital compensation schemes being proposed. Smith [ref 159] suggested a cartesian, vector, baseband, type 3 system for M-QAM in 1976, but never built the system. Grabowski and Davis [ref 66], working independently from Smith, built a similar system for M-QAM in 1982 and reported good results. Their system relied on the $I(t)$ and $Q(t)$ data streams being digital and they themselves state that this limits the use to systems where modulation sideband shaping can be done at RF. Liskov and Wood [ref 105] are reported by Weaver [ref 181] to have produced an adaptive predistorter, but no details were given.

Adaptive predistortion schemes (analogue NLFG) are rare. Fehr reports computer studies of an analogue scheme for filtered QPSK [ref 60]. Abuelma'atti [ref 2] reports experimentation with an analogue scheme. Both schemes appear somewhat limited in application, Fehr's due to the specialisation on filtered QPSK, Abuelma'atti's by the need to produce the analogue NLFG close to the required characteristics, and by the need to match dc bias/NLFG characteristics as well.

Outside of the narrowband nonlinear (NBNL) problem digital compensation is more common, especially type 1 algorithms for calibration of test equipment and sensors [ref 182]. However, because these problems are detached from the NBNL problem they have little to contribute to this thesis.

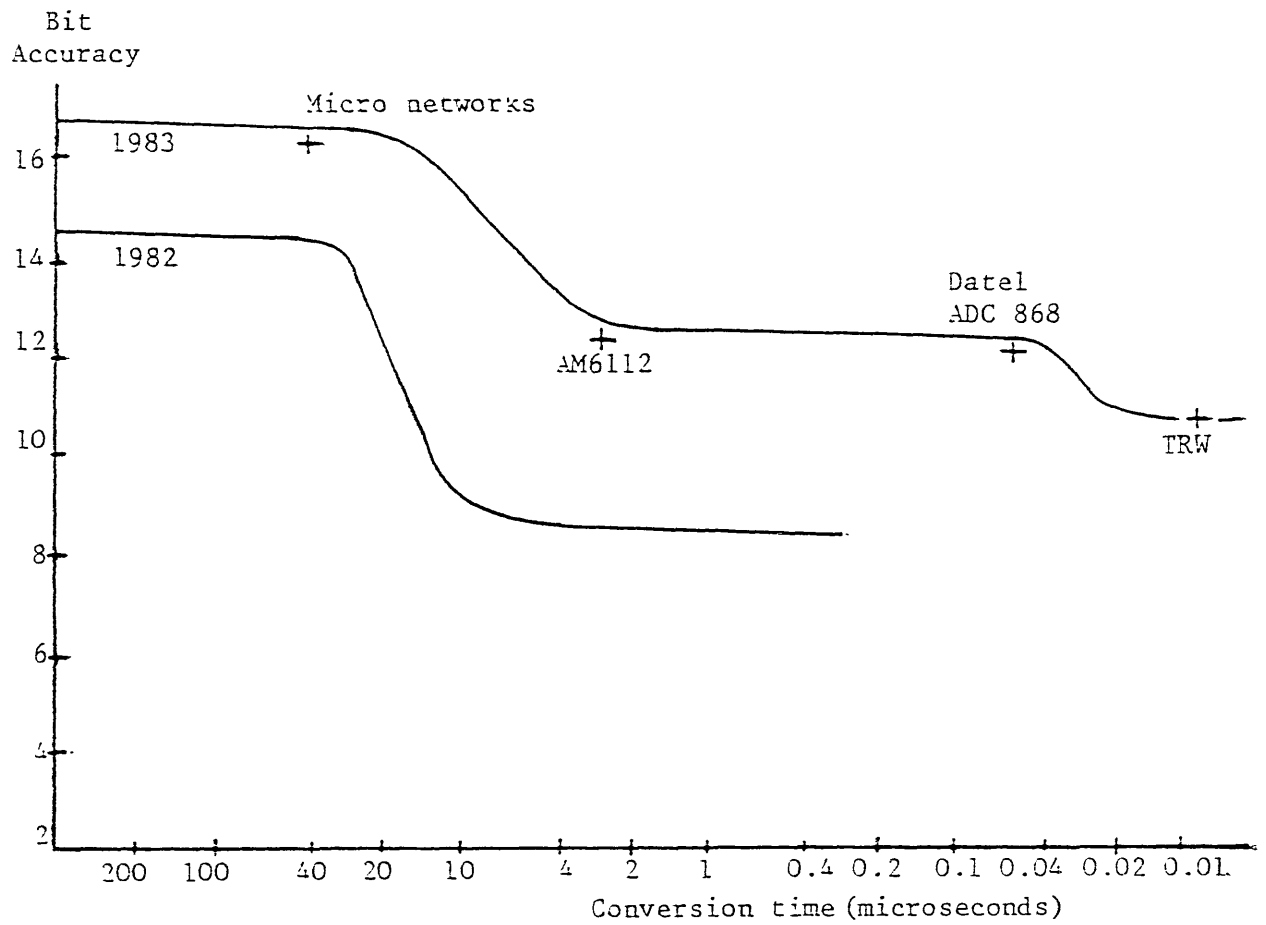
3.5 AREAS OF STUDY

As there are so many diverse systems covered by the decision hierarchy it is not possible to thoroughly analyse each system. Instead this thesis concentrates on the study of polar, vector, baseband AM, systems of types 1, 2 and 3, with a cursory look at polar, vector, IF, systems.

The reasons for choosing the polar representative is its far wider applicability than cartesian systems. Investigation of full vector control is obviously more generally applicable than scalar control. The investigation of AM systems allows the use of baseband inputs to the NLFG, and hence keeps down the sampling rates of the component A/D's and D/A's. This allows the use of monolithic devices with high resolution (e.g. 12-16 bit), and enables the very extremes of spectral control (i.e. IMP suppression) that digital compensation can achieve to be reached, and fundamental limitations examined. Since M-QAM and filtered QPSK may be regarded as a subset of analogue AM in terms of distortion (i.e. the addition of a phase modulator will allow us to implement M-QAM and filtered QPSK, but this will not affect the distortion which is envelope dependent), we can also regard an investigation of AM as a natural expansion of the current interest in M-QAM and filtered QPSK to analogue data streams.

Investigating all three algorithm types will complete the generality of the study.

FIGURE 3.1.1



Approximate status of A/D converters, 1982, 1983

Sources, References 6, 61, 62.

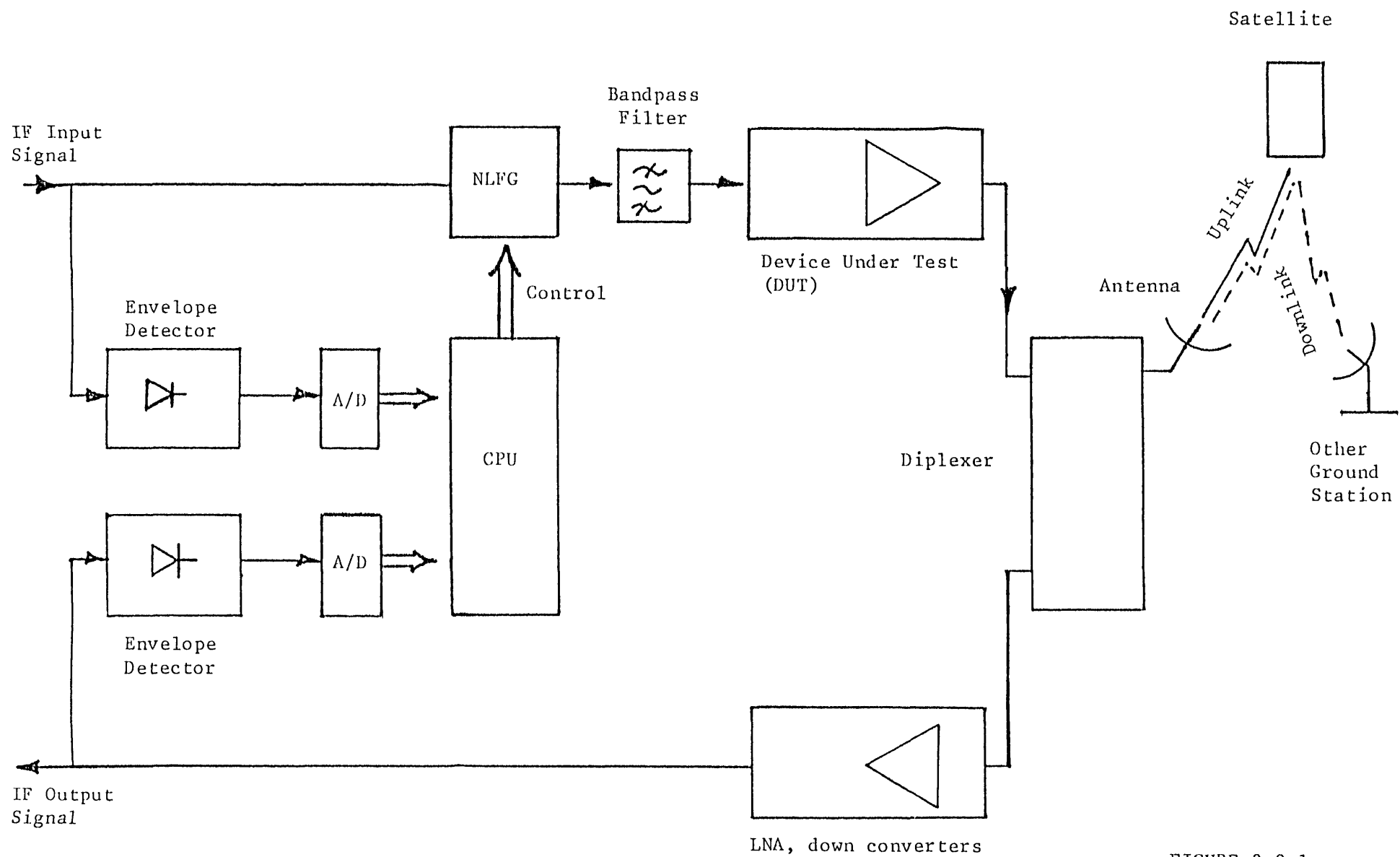


FIGURE 3.2.1

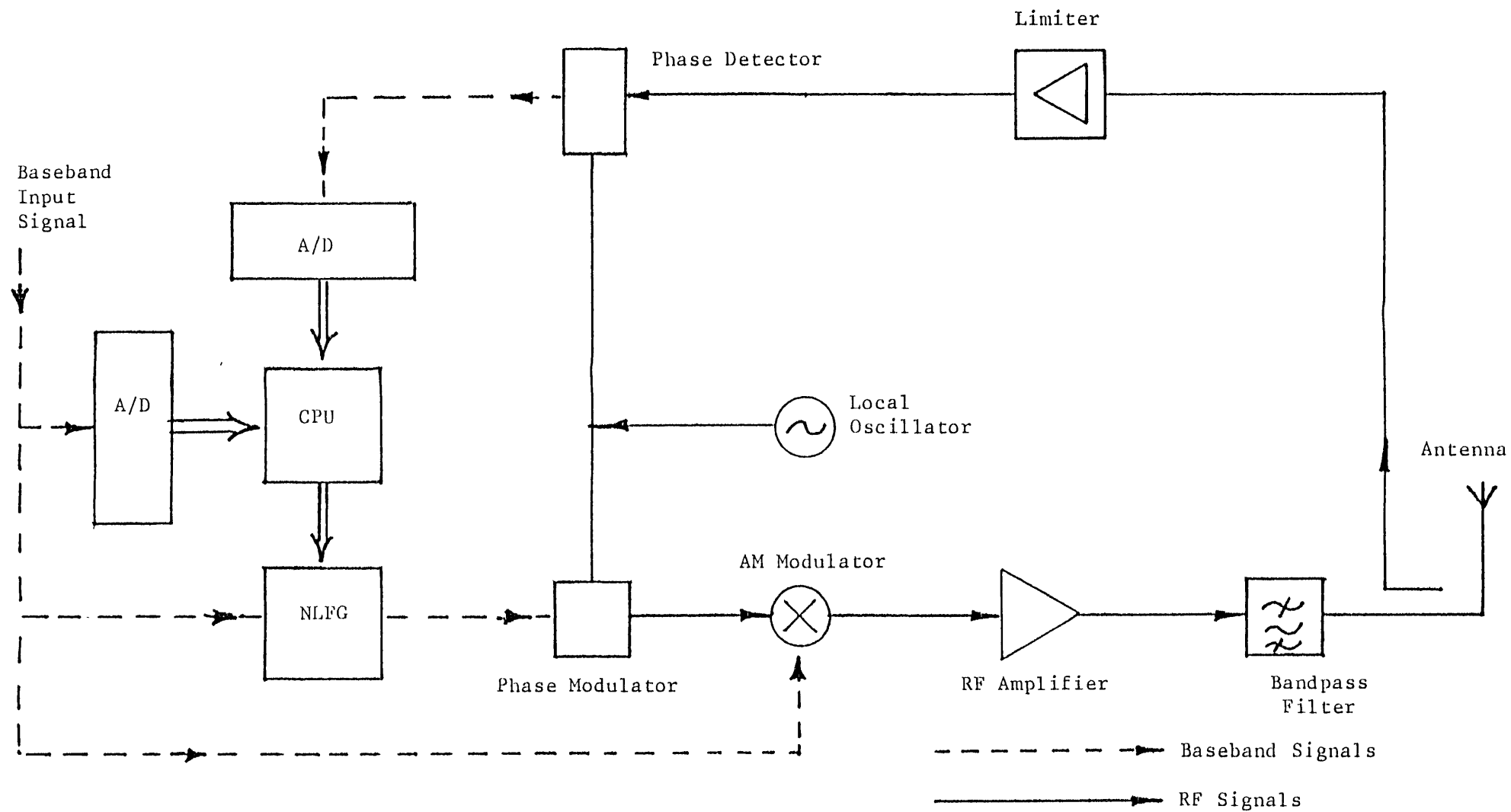


FIGURE 3.2.2

FIGURE 3.3.1

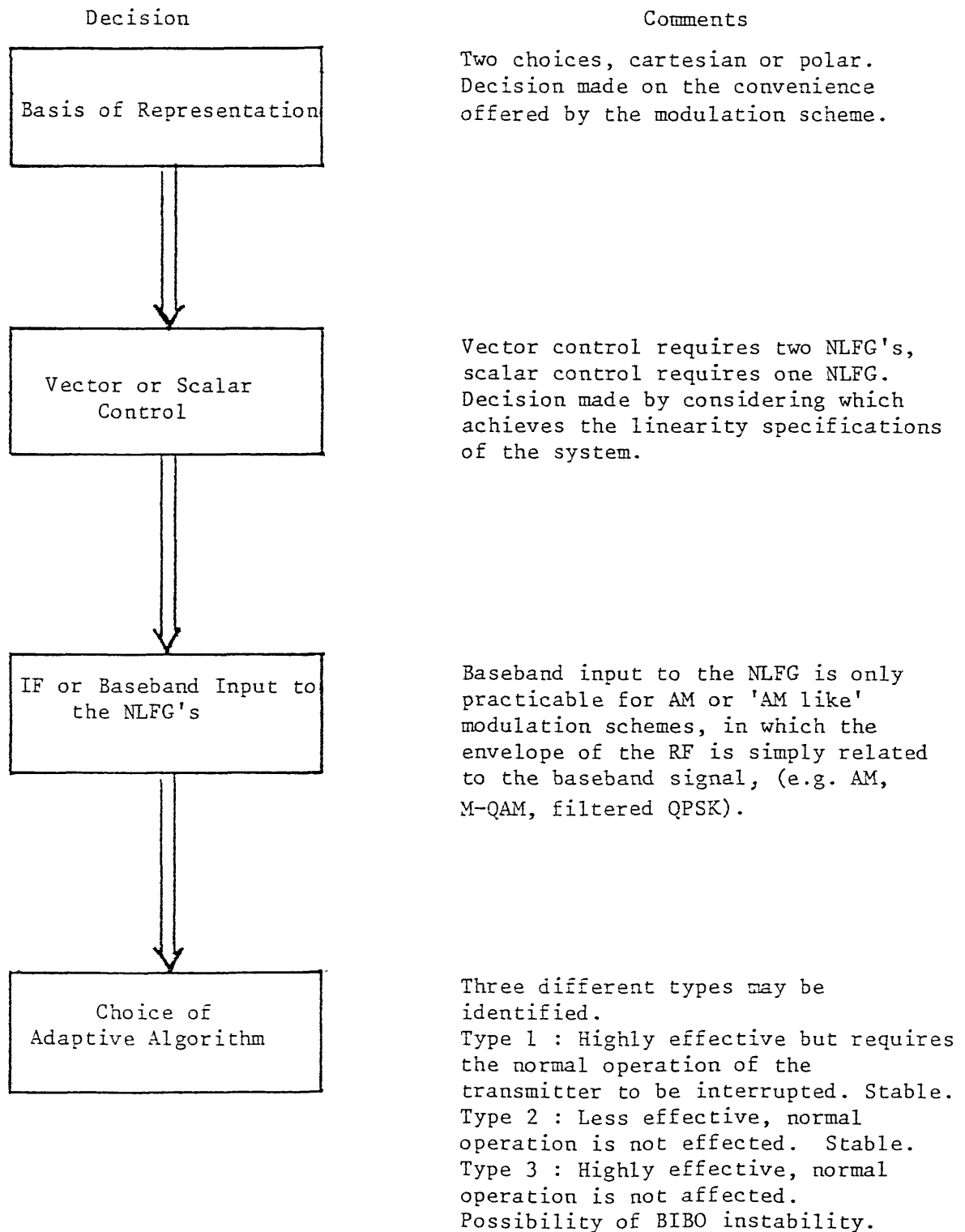
The Decision Hierarchy

FIGURE 3.3.2

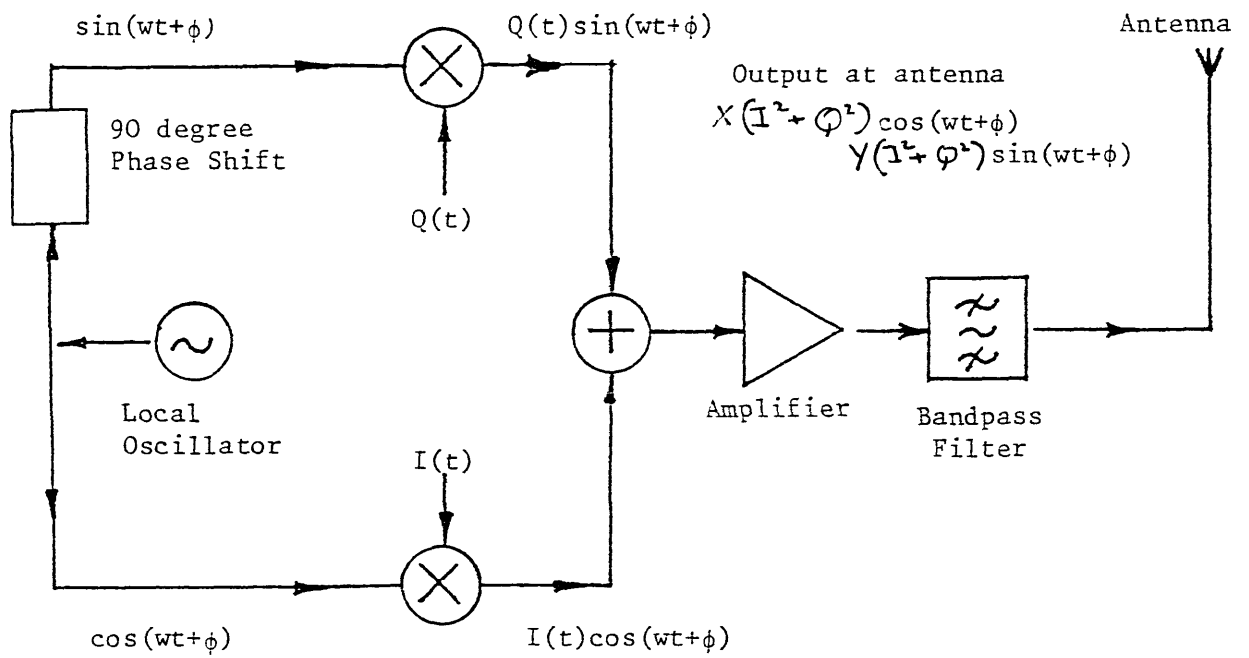
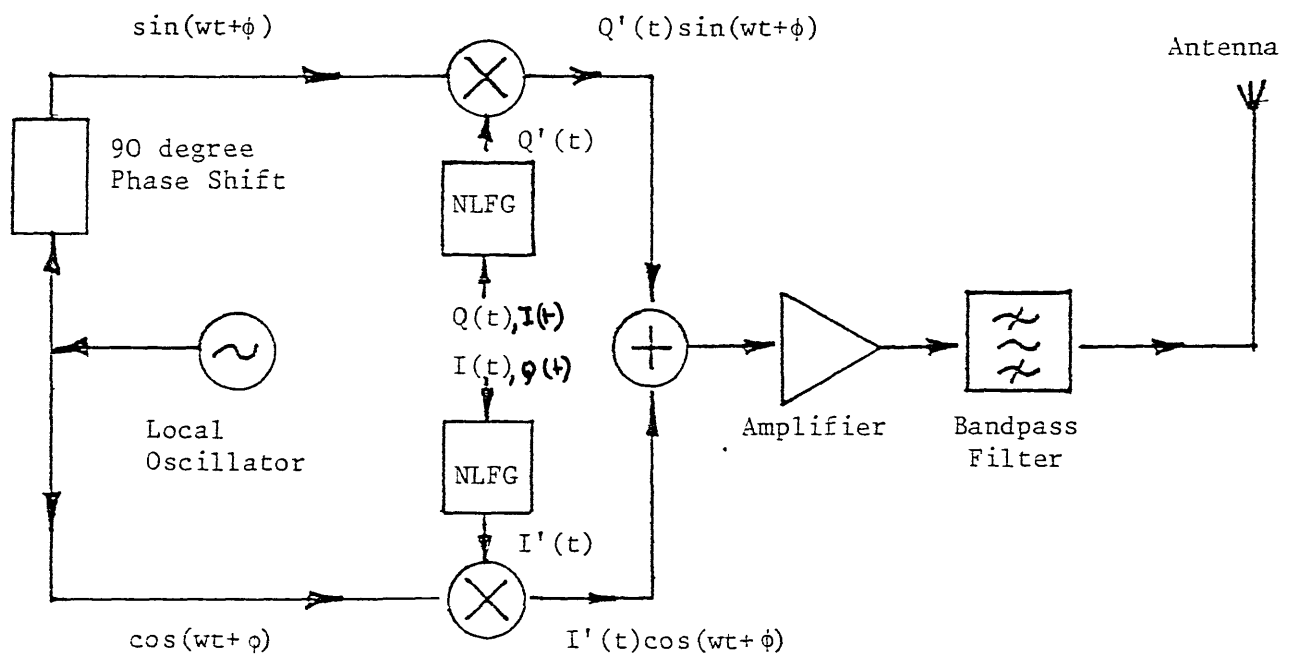


FIGURE 3.3.3



Output at antenna $X(I'(t))\cos(\omega t + \phi) + Y(Q'(t))\sin(\omega t + \phi) =$
 $I(t)\cos(\omega t + \phi) + Q(t)\sin(\omega t + \phi)$ when the NLFG's are correctly set.

FIGURE 3.3.4

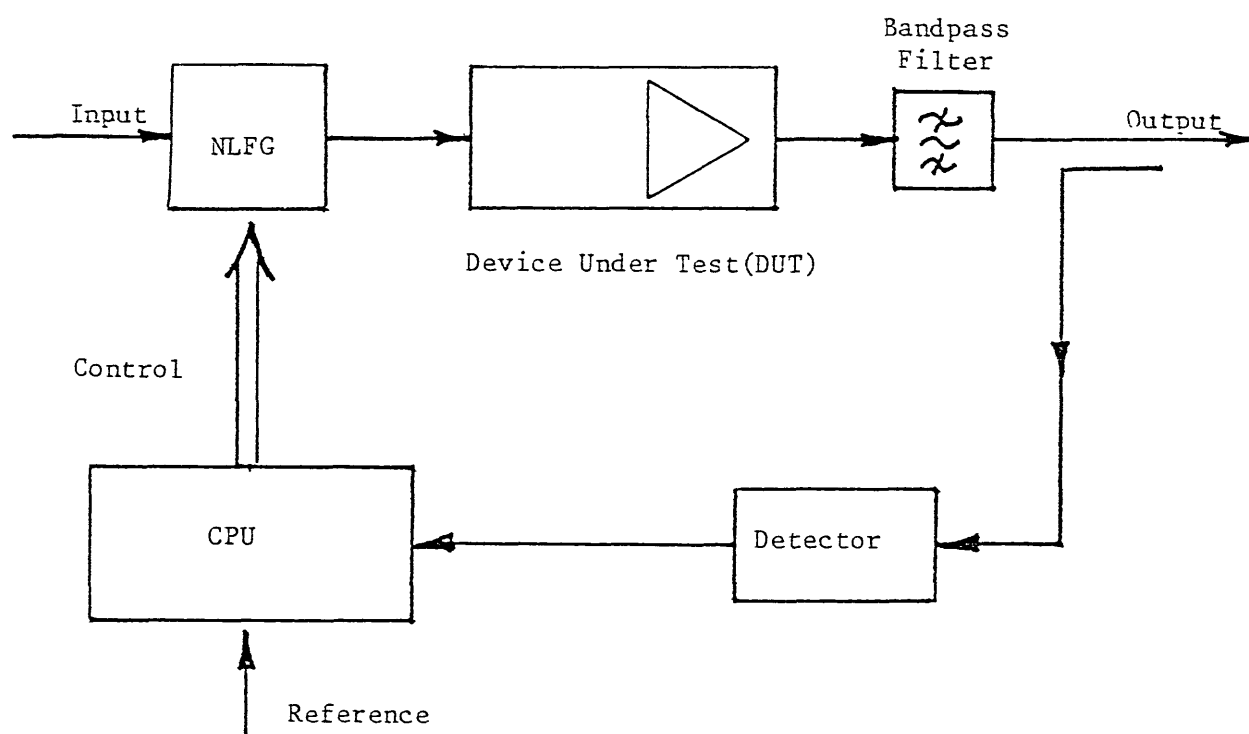


FIGURE 3.3.5

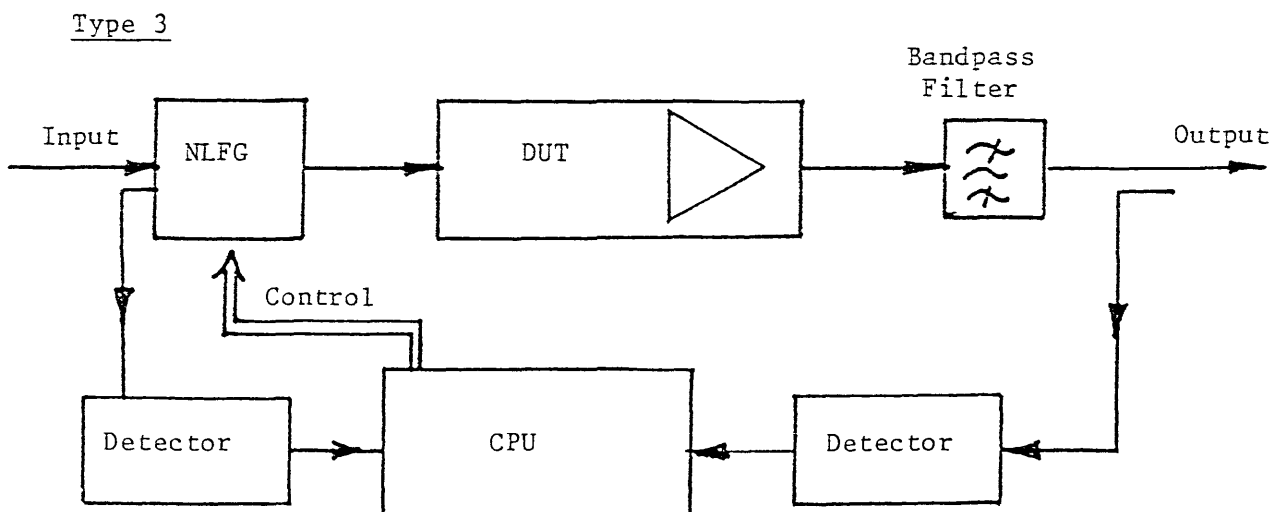
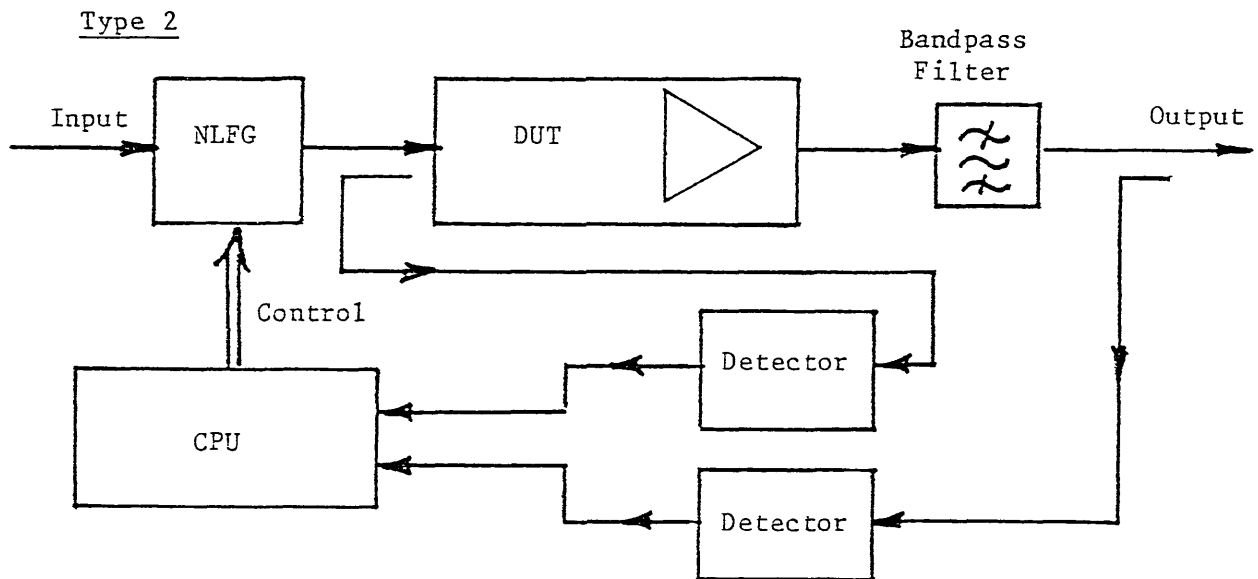
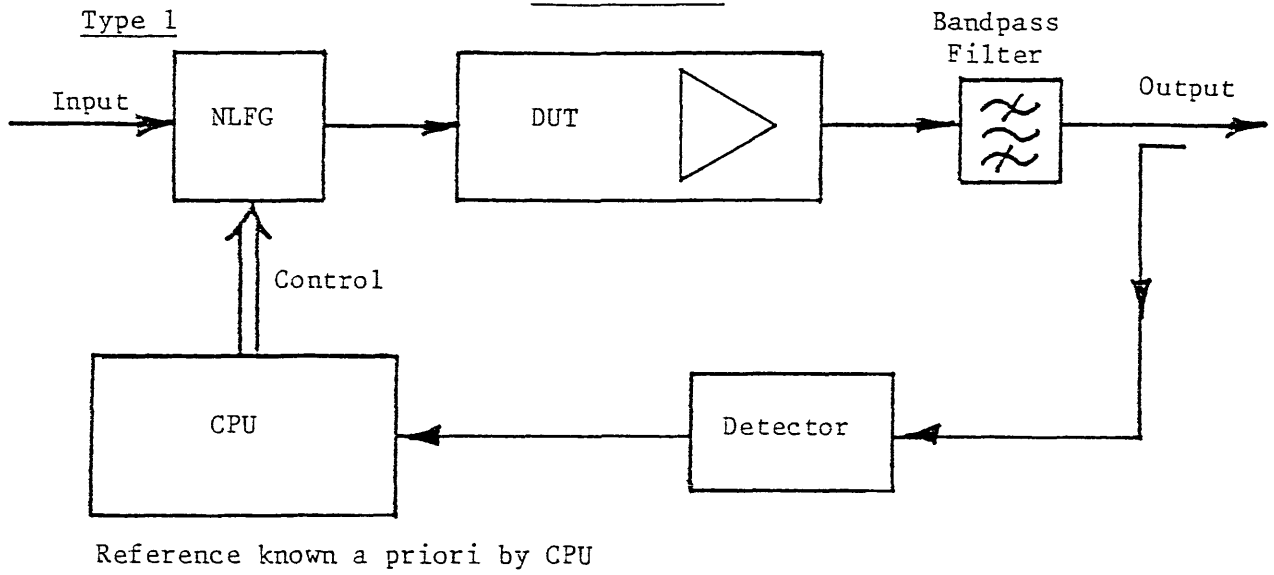
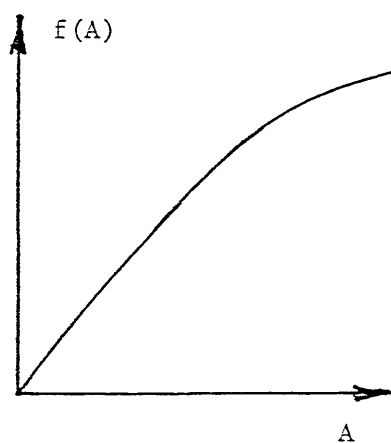
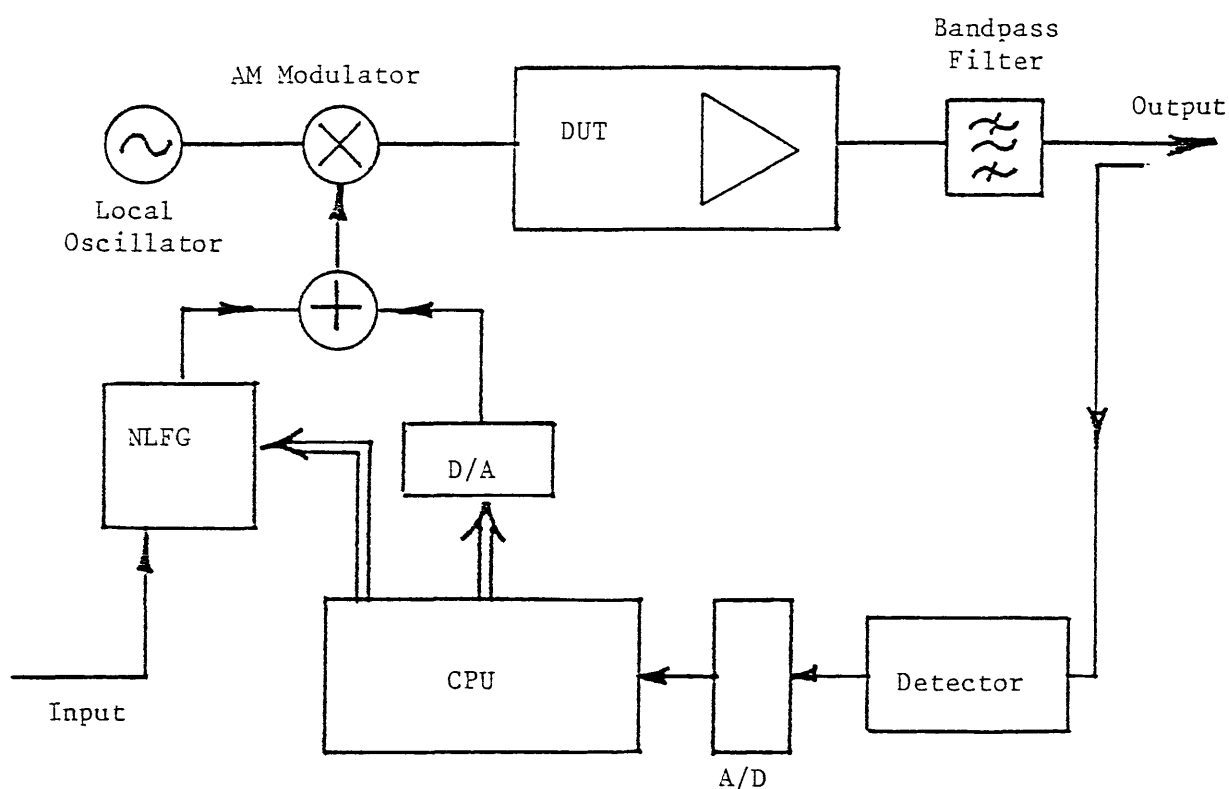
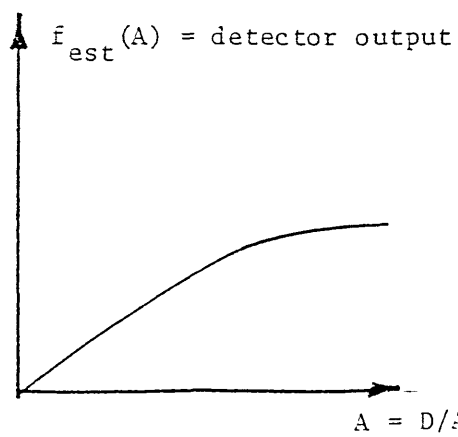


FIGURE 3.3.6



The AM/AM conversion characteristic of the Device under Test (DUT)



Estimate of $f(A)$ is obtained by ramping the D/A through all states, when the input signal is disabled, and recording the detector output. Our estimate is the correct functional form of $f(A)$, but the scale factors of the detector, A/D and D/A must be known to deduce $f(A)$.

FIGURE 3.3.7

Modified scheme due to Abuelma'atti (ref 2)

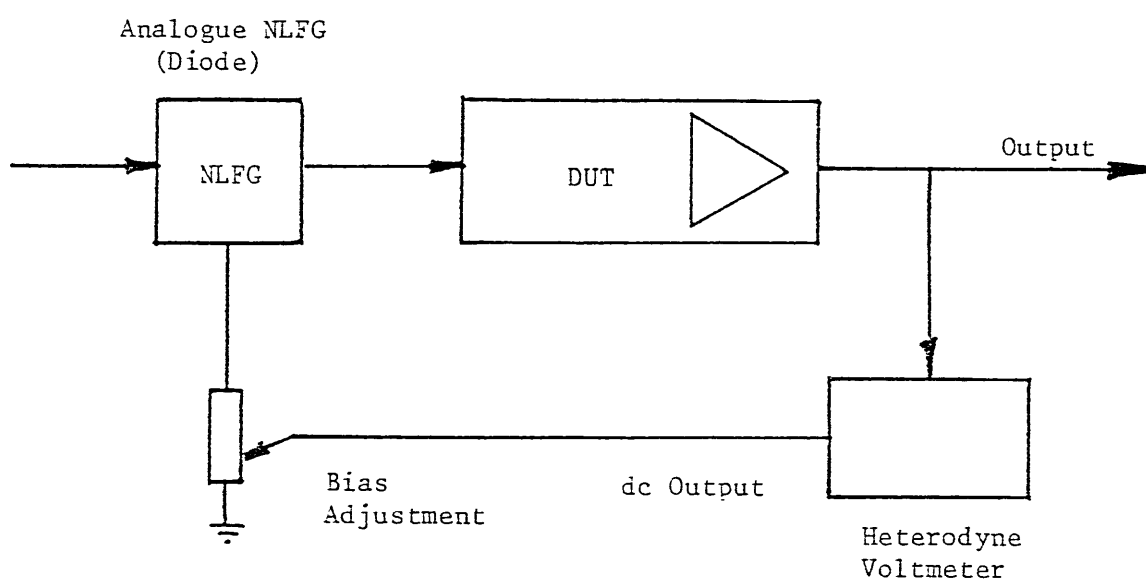


FIGURE 3.3.8

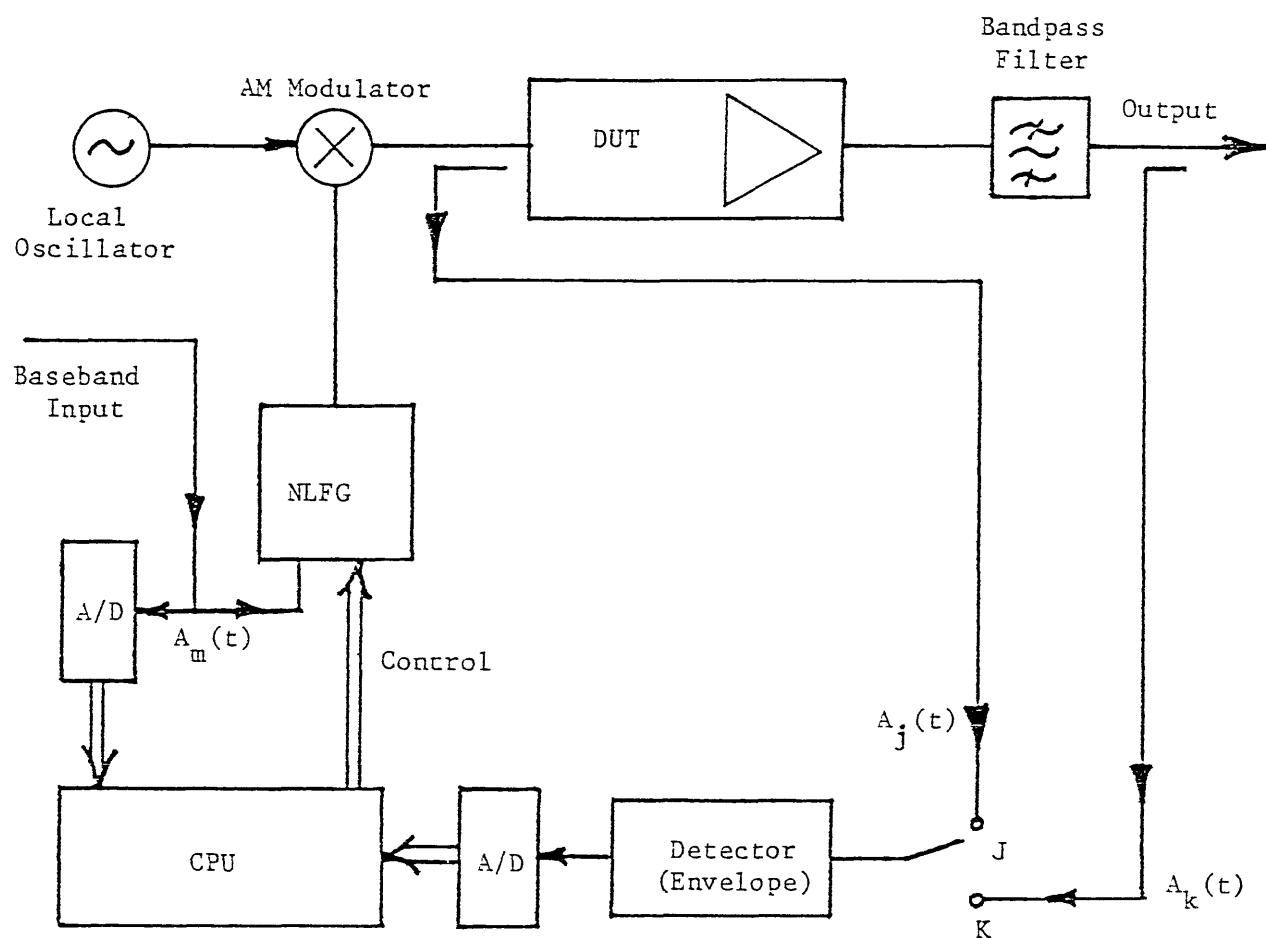
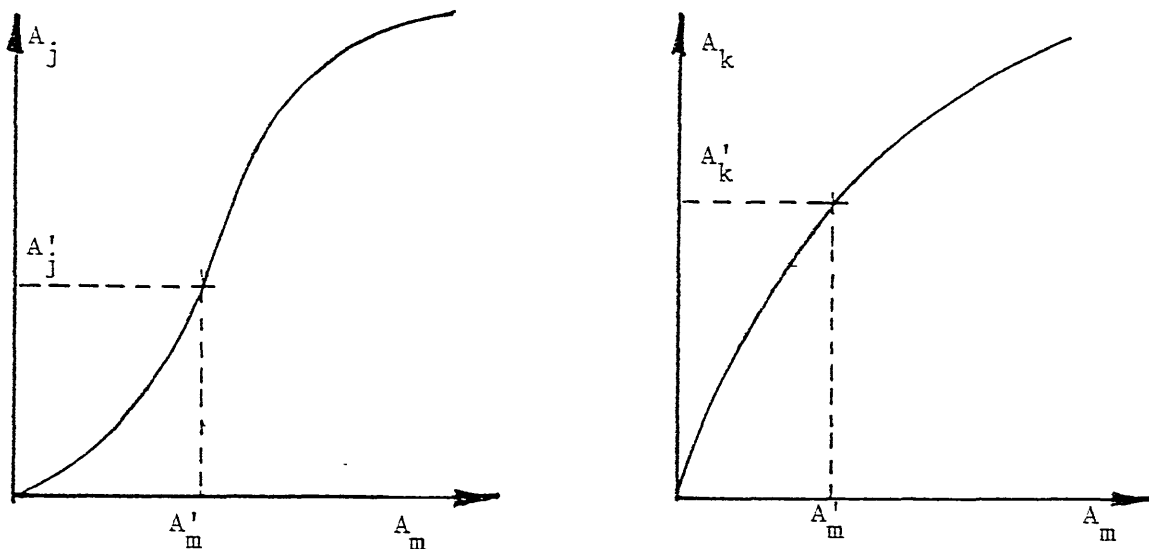
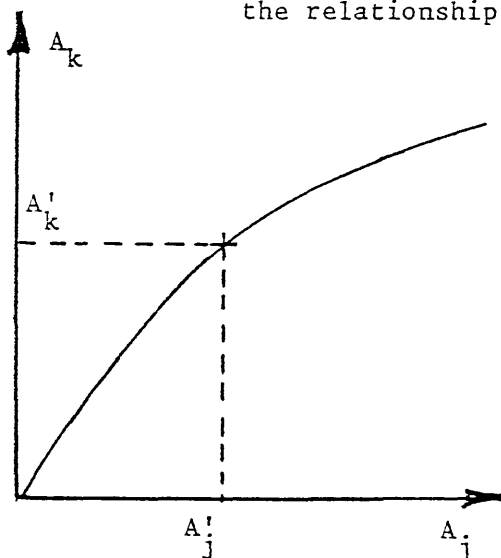


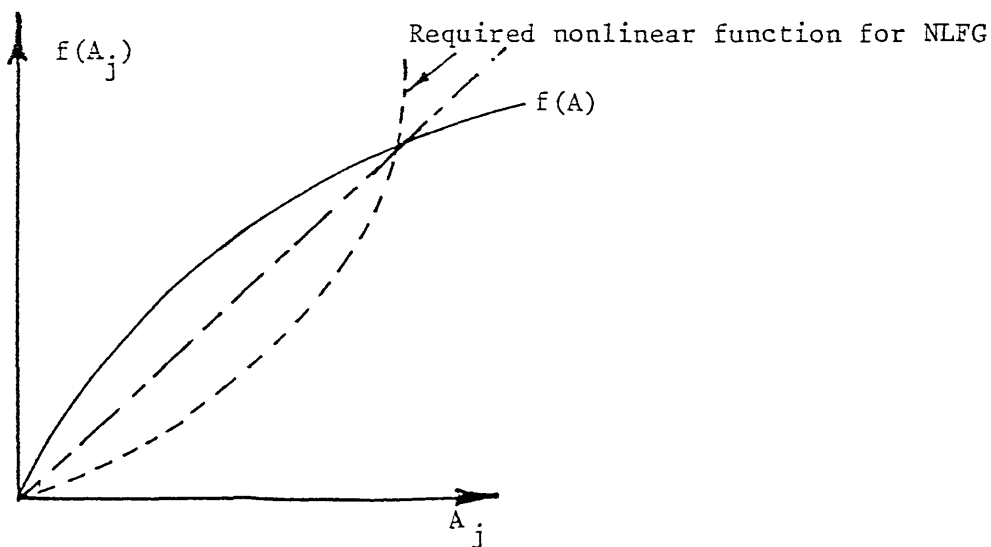
FIGURE 3.3.9



We record the relationships between A_j and A_m , A_k and A_m from which the relationship between A_k and A_j can be deduced



But by definition $f(A_j) = A_k$ and from this we can see the required nonlinear function (NLF)



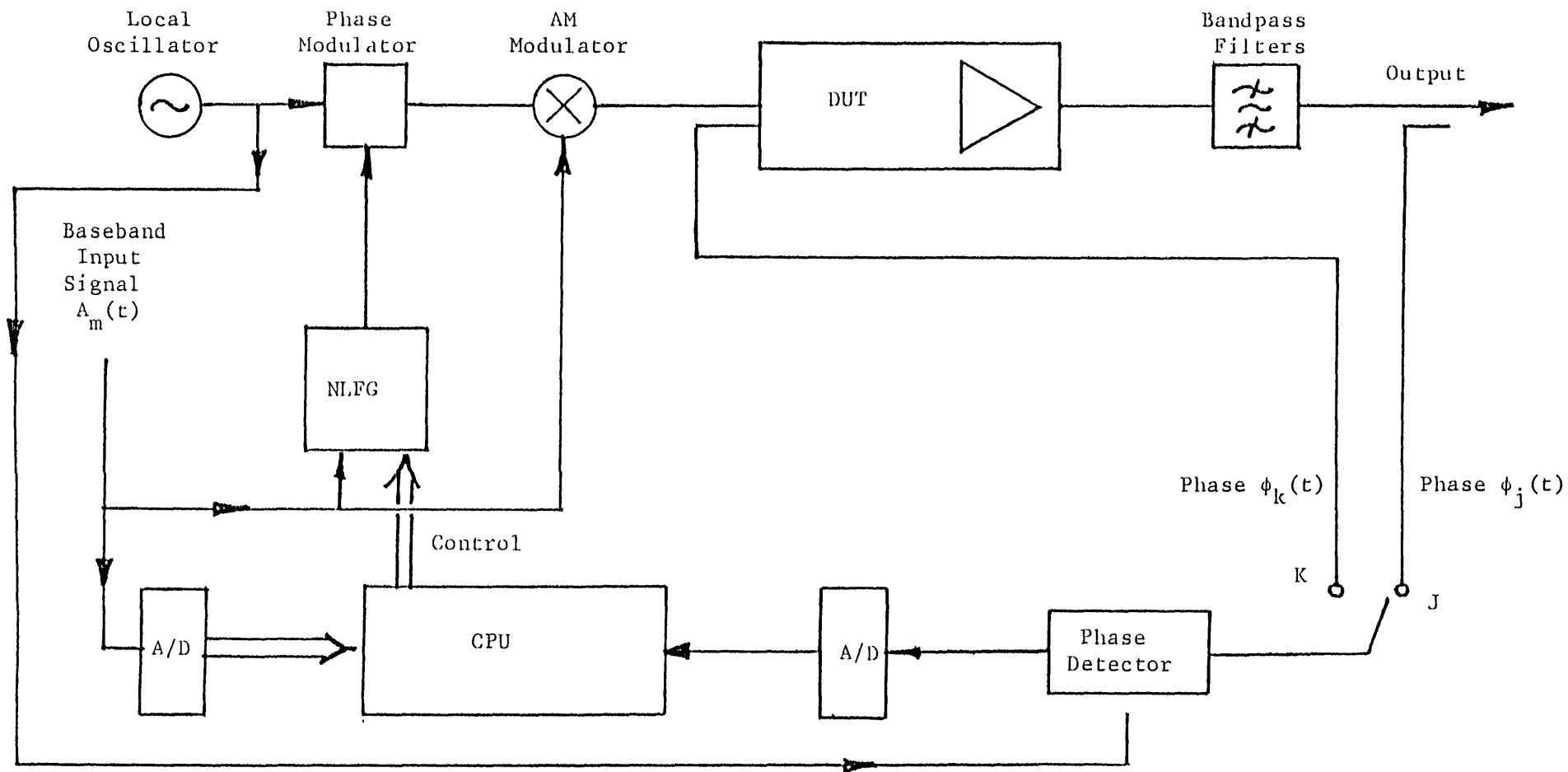
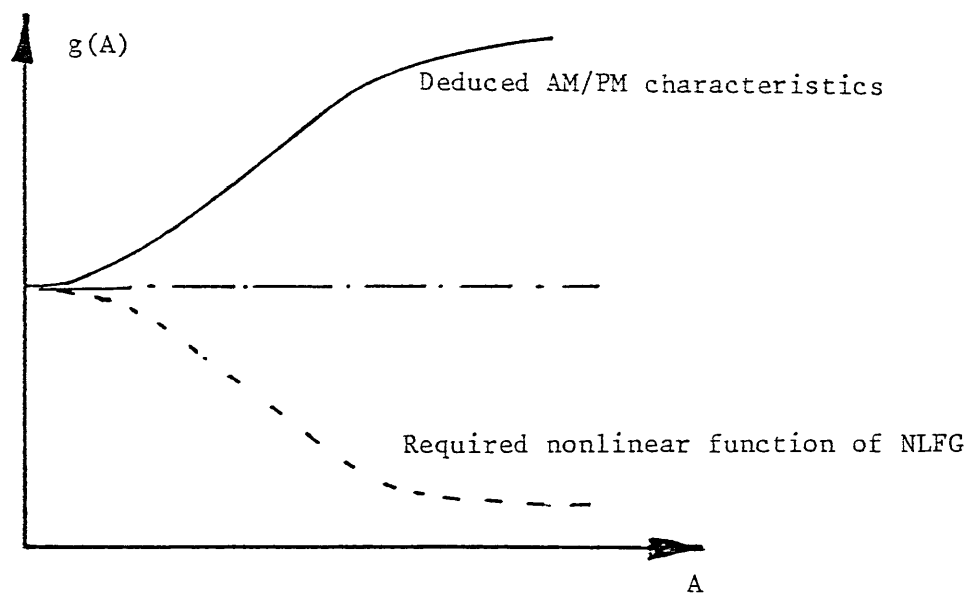


FIGURE 3.3.10

FIGURE 3.3.11

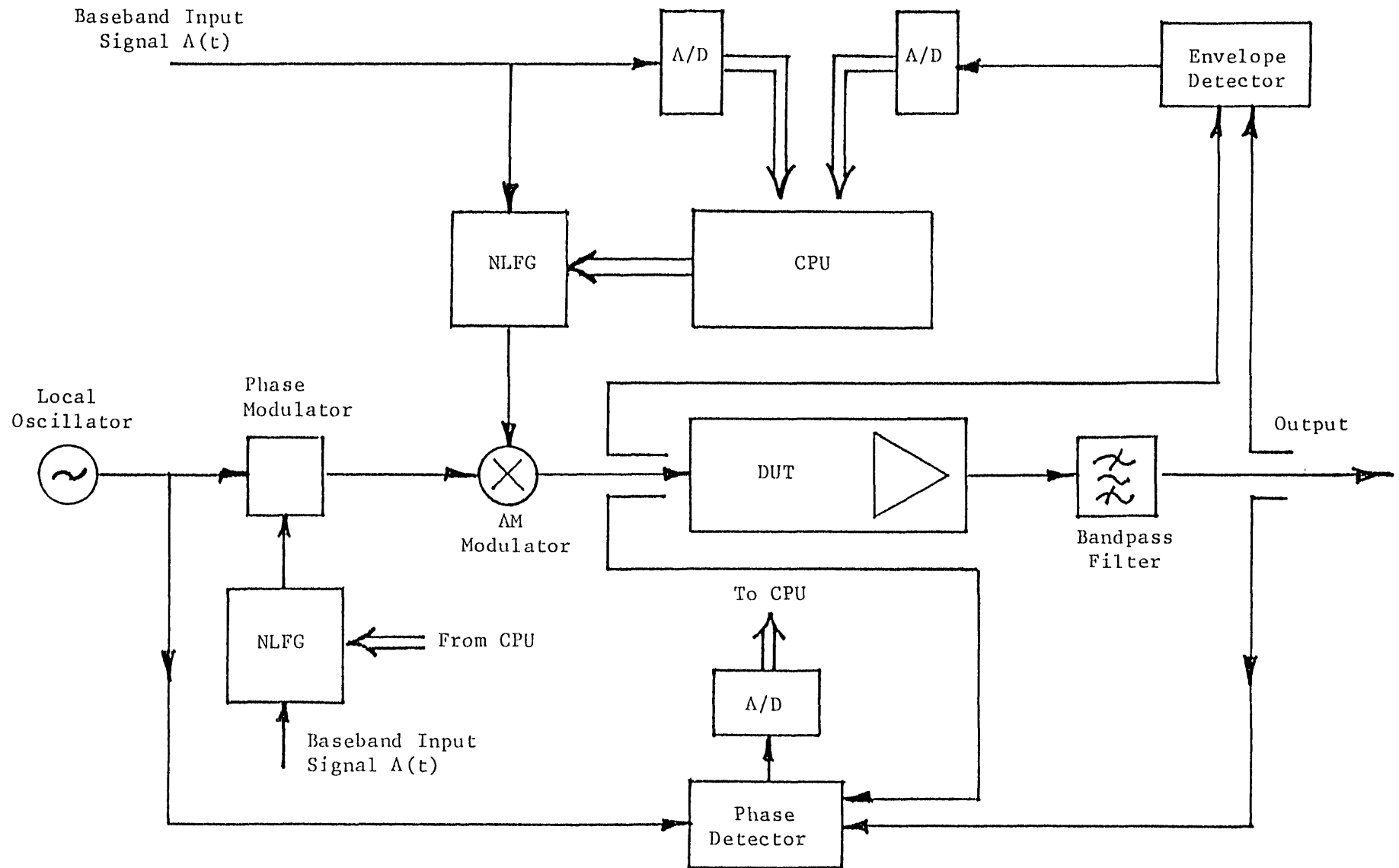
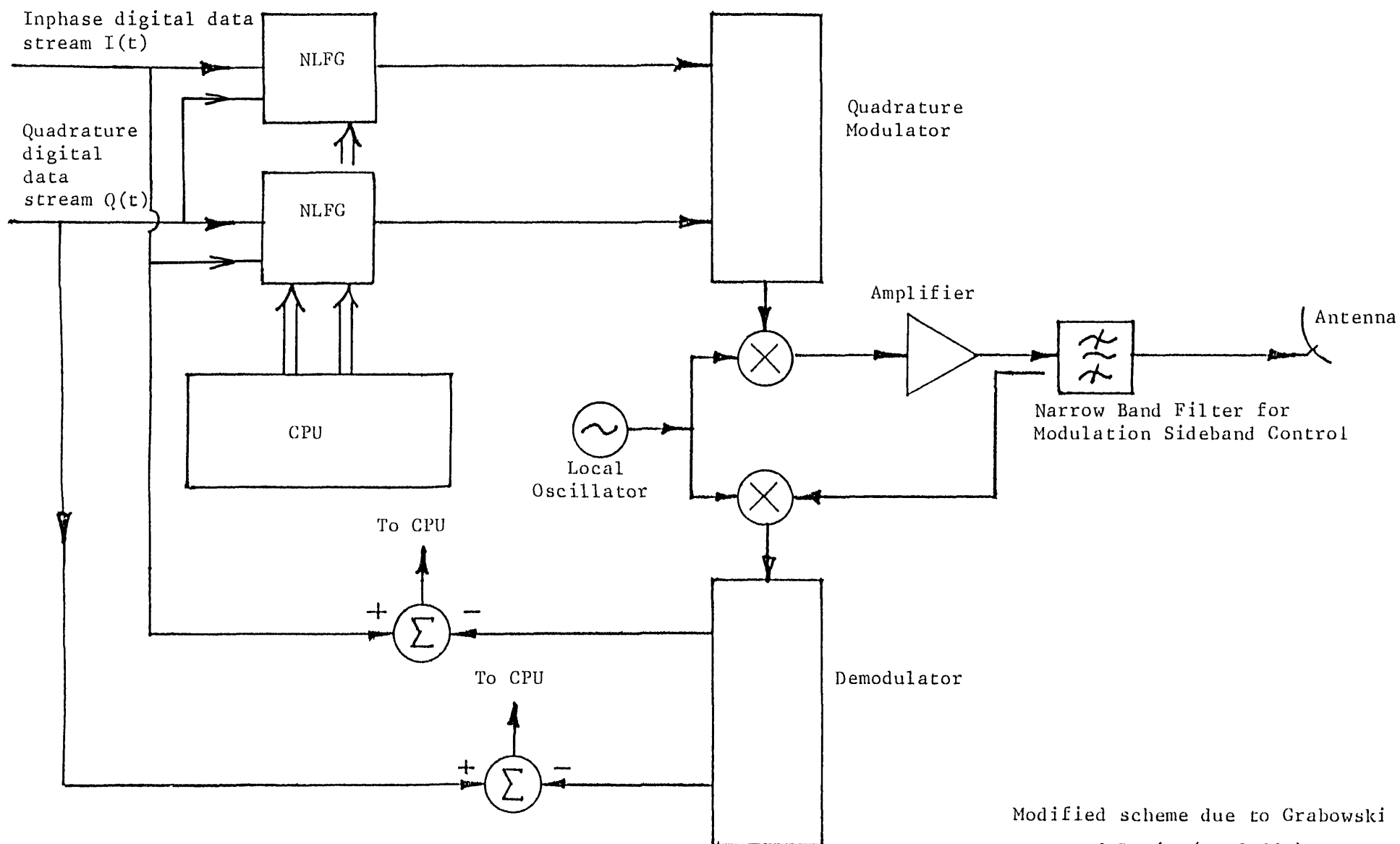


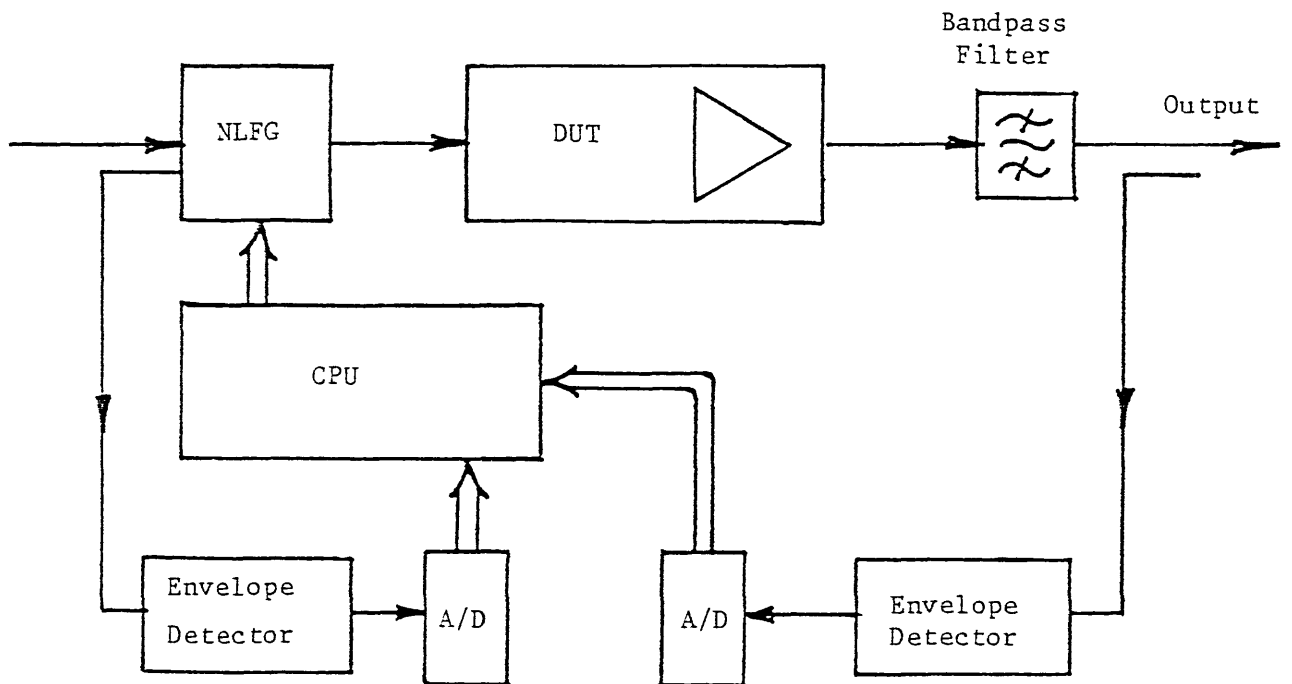
FIGURE 3.3.12



Modified scheme due to Grabowski
and Davis (ref 66)

FIGURE 3.3.13

FIGURE 3.3.14



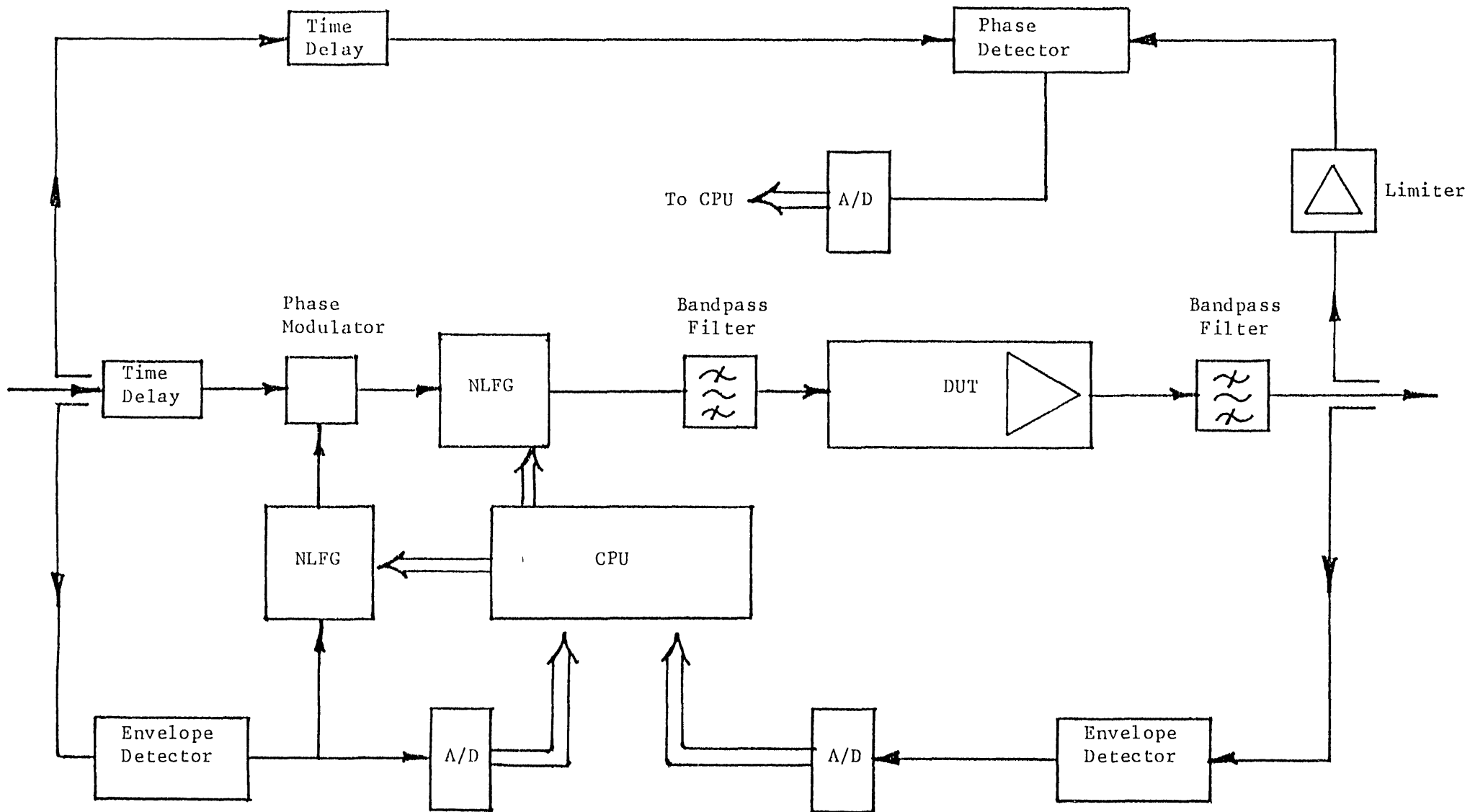


FIGURE 3.3.15

CHAPTER 4

NONLINEAR FUNCTION GENERATOR CONSIDERATIONS

4.1 INTRODUCTION

The use of a digital nonlinear function generator (NLFG) provides the tight control over the nonlinear function that we require. It does inevitably involve constraints on quantization noise and sampling rates, and these constraints must be shown not to restrict the RF signal bandwidth and signal to noise ratio to the extent that a digital NLFG becomes unacceptable. The purpose of this chapter is to set out the boundaries of these constraints.

4.2 POLAR, BASEBAND INPUT, AM MODULATION

4.2.1 Bandwidth

Several different digital NLFG's can be constructed. Reference 103 shows a digital/analogue NLFG which can well approximate bilinear functions. Reference 80, p.216-217,363 shows schemes based on multiplying D/A converters and a general purpose A/D, CPU, RAM, D/A scheme. The latter is shown in Figure 4.2.1. The input signal is sampled by the A/D [in this thesis when referring to an A/D we mean the total circuitry necessary to quantize the signal - bandlimiting filters, sample and holds, etc (a discussion of these elements may be found in reference 191)], and the sample values passed to the CPU which performs interpolation between sample points. The interpolated sample points are used as the addresses for the RAM. The data points held at these addresses are passed to the D/A and turned into analogue voltage levels. The D/A is followed by a conventional

anti-alias filter with $\sin(x)/x$ correction [ref 136]. The nonlinear function to be approximated is determined in functional form by the data/address relationship in the RAM table. The A/D and D/A quantizing levels then scale this function to the observed input voltage/output voltage levels (see Figure 4.2.2). The purpose of the CPU is primarily to match the A/D sampling rate to the D/A sampling rate. If we allow the A/D and D/A to have identical sampling rates then the NLFG of Figure 4.2.3 results. The A/D samples the input signal, the quantized sample is then used as the address for the RAM table as before, the output of the RAM being passed to the D/A. The purpose of the CPU is merely to set up and modify the RAM table as required.

Referring to Figure 4.2.1, assuming the input signal is bandlimited to frequencies below B Hz, we require the A/D sample rate f_A (Hz) be such that

$$f_A > 2B \quad (4.1)$$

to satisfy the sampling theorem [ref 26, 154].

If the RAM table can be well approximated by a polynomial of order n , then since the spectral width of the output of the NLFG is nB Hz [ref 20], to satisfy ^{the Sampling Theorem [ref 26]} ~~Shannon [ref 154]~~ we require that the D/A sampling rate f_D (Hz) satisfies

$$f_D > 2nB \quad (4.2)$$

When the NLFG of Figure 4.2.3 is used we require

$$f_A = f_D > 2nB \quad (4.3)$$

If we have some knowledge of the nonlinear RF amplifier chain we can calculate the spectral effects of different values of n for any given test. The more linear the amplifier chain the more likely it is that the required value of n will be low. However we

might conjecture that $n=7$ will cover most potential applications, on the crude assumption that $n=7$ will drastically reduce up to seventh order effects. (Distortion above seventh order can usually be corrected by conventional linear design).

Figure 4.2.4 shows a possible polar, vector, baseband AM, digital compensation scheme, with the adaptive detectors removed for clarity.

Referring to Figure 4.2.4, if the AM/AM and AM/PM characteristics of the DUT are represented by $f(A)$ and $g(A)$ respectively, then the purpose of the digital compensation is to set the envelope NLFG such that

$$f(A'(t)) = A(t) \quad (4.4)$$

and to set the phase NLFG (which implements the transfer characteristic $p(A)$) such that

$$g(A'(t)) + hp(A(t)) = k \quad (4.5)$$

where k is an arbitrary constant (which we can set equal to zero without loss of generality), and h is the phase modulator gain in radians/volt. Under these circumstances the output of the DUT is given by

$$\begin{aligned} & f(A'(t))\cos(\omega t + g(A'(t)) + hp(A(t))) \\ &= A(t)\cos(\omega t + k) \end{aligned} \quad (4.6)$$

The function required of the phase NLFG is given explicitly by equation 4.5. The function required of the envelope NLFG is defined by equation 4.4. Only in the range in which $f(A)$ is a function (i.e. the range in which there is a ~~one to one~~ ^{single valued} relationship between A and $f(A')$), can we solve equation 4.4. Reference 133 discusses the range over which the inverse of $f(A)$ exists and concludes that the inverse of $f(A)$ can only be defined

between stationary points of $f(A)$ as in Figure 4.2.5. In practice $f(A)$ is primarily determined by the characteristics of the output amplifier, and for this element we will find that $f(A)$ is a monotonically increasing function of A up to saturation. After saturation it is possible for $f(A)$ to decrease [ref 18].

4.2.2 Quantization Noise and the Envelope NLFG

We first consider the quantization noise associated with the envelope NLFG in equation 4.4 (this equation is representative of Figure 4.2.4). Panter [ref 125, p.633-652], gives an excellent discussion of quantization noise and we quote results from his work.

If the input A/D can quantize the input voltage into one of M equally spaced levels, and if the input signal is equally likely to occupy any one of these M levels, then the RMS signal power to RMS noise power ratio $(S/N)_{\text{RMS}}$ associated with the quantization noise is given by

$$\begin{aligned} (S/N)_{\text{RMS}} &= M^2 - 1 \\ &\approx M^2 \quad \text{if } M^2 \gg 1 \end{aligned} \quad (4.7)$$

The more often quoted peak signal power to RMS noise power ratio (S/N) is given by

$$\begin{aligned} (S/N) &= 12(M^2 - 1) \\ &\approx 12M^2 \quad \text{if } M^2 \gg 1 \end{aligned} \quad (4.8)$$

Provided that the resolution of the D/A is greater than the A/D, we can derive the same output from the anti-alias filter by regarding the NLFG as perfect (no quantization noise, and the series combination of NLFG and DUT behaving linearly), and the input to the A/D as having a (S/N) ratio given by equation 4.8.

Thus, since the series combination of NLFG and DUT can be regarded as linear, the output envelope to noise power ratio (E/N) is also given by equation 4.8. Hence, under the conditions outlined, a 12 bit A/D gives rise to $(E/N) = 83$ dB, and a 16 bit A/D gives rise to $(E/N) = 107$ dB.

In the case where not all quantized levels are equally likely, better results can be obtained by companding before the A/D [ref 125], and carrying out the necessary expanding in the RAM. Assuming that the required signal at the output of the anti-alias filter is unchanged no change in the D/A sampling rate is required. Likewise no change in the A/D sample rate is required.

4.2.3 Quantization Noise and the Phase NLFG

Clearly the phase noise we must consider is due to the quantization noise associated with

$$g(A'(t)) + hp(A(t)) = 0$$

$$\text{i.e.} \quad g(A'(t)) = -hp(A(t)) \quad (4.9)$$

(reproducing equation 4.5 with $k=0$). Each of these two terms introduce quantization noise. However, since the quantization noise for both derive from the same A/D (see Figure 4.2.4), and since clearly the two terms are in anti-phase, the noise contributions of each must add synchronously and cancel.

Thus, under the conditions outlined (A/D the prime source of quantization noise, and the digital compensation achieving near-perfect removal of distortion), there is no phase noise due to quantization effects. [Physically we are saying that the A/D introduces a quantized error when it measures the input signal,

and that $g(A)$ and $hp(A)$ both contain a spurious phase angle due to this error. Provided the D/A associated with the phase NLFG has adequate resolution the spurious phase angles due to $g(A)$ and $hp(A)$ are always in anti-phase].

In practice these conditions may not apply; perfect removal of distortion effects may not be achieved, and there will exist small timing errors between the phase modulation and the envelope modulation. (These timing errors may arise due to non-identical anti-alias filters in the phase and envelope NLFG's, unaccounted delays in the phase and envelope modulators, and imperfections in the synchronisation between phase and envelope CPU algorithms). We shall take as a pessimistic upper bound for the phase noise the asynchronous sum of the $g(A'(t))$ and $hp(A(t))$ terms.

Considering first the $p(A)$ term, we treat the phase NLFG as perfect and consider the input envelope ($V(t)$) to contain both noise ($n(t)$) and signal ($A(t)$). The signal to noise ratio for the input signal is given by equation 4.8. Now by definition

$$hp(V(t)) = hp(A(t) + n(t)) \quad (4.10)$$

Taylor expanding the right-hand side of equation 4.10 gives

$$hp(V(t)) = hp(A(t)) + h(dp/dA)n(t) + O(n^2(t)) \quad (4.11)$$

If we assume that AM/PM conversion follows an essentially quadratic phase law then $p(A)$ is approximately equal to cA^2 (where c is a constant). Thus $(dp/dA)=2cA$ and we can identify the phase noise $\phi(t)$ as

$$\phi(t) = 2hcA(t)n(t) \quad (4.12)$$

Now reference 125 allows us to identify the RMS noise of an A/D converter as

$$\langle n(t) \rangle_{\text{RMS}} = A_{\text{max}} / ((12)^{0.5} M) \quad (4.13)$$

where $A_{\max}$ is the maximum input signal level and M is the total number of equally spaced quantized levels. Similarly

$$\langle A(t) \rangle_{\text{RMS}} = A_{\max}/3^{0.5} \quad (4.14)$$

assuming all M levels equally likely. Thus we identify the RMS phase noise $\langle \phi \rangle_{\text{RMS}}$ as

$$\langle \phi \rangle_{\text{RMS}} = 2hc(A_{\max})^2/((36)^{0.5M}) \quad (4.15)$$

But as the maximum phase deviation due to AM/PM conversion $\phi_{\max}$ is given by

$$\phi_{\max} = hc(A_{\max})^2 \quad (4.16)$$

we can write equation 4.15 as

$$\langle \phi \rangle_{\text{RMS}} = 2\phi_{\max}/((36)^{0.5M}) \quad (4.17)$$

However, reference 148 shows that for small phase angles the RMS phase is related to the single sideband phase noise L (dB/Hz), conventionally defined in a 1 Hz bandwidth, by

$$L = 10\log(0.5(\langle \phi \rangle_{\text{RMS}})^2) \quad (4.18)$$

Combining with equation 4.17 we get

$$\begin{aligned} L &= 10\log((\phi_{\max})^2/(18M^2)) \\ &= 20\log(\phi_{\max}/M) - 12.5 \end{aligned} \quad (4.19)$$

Equation 4.19 describes the phase noise associated with a quadratic AM/PM, due to quantization errors in the phase NLFG. We assume that under worst case conditions the phase noise due to quantization in the envelope NLFG is of equal magnitude, but asynchronous. Thus we derive an upper bound for the phase noise due to all quantization errors as

$$\begin{aligned} L_{\max} &\leq 20\log(\phi_{\max}/M) - 12.5 + 3 \\ \text{i.e.} \quad L_{\max} &\leq 20\log(\phi_{\max}/M) - 9.5 \end{aligned} \quad (4.20)$$

Figure 4.2.6 shows an experimental graph of the AM/PM conversion of an amplifier. The maximum phase shift is 40 degrees

(approximately 0.7 radians). Hence to correct for this we would have to arrange such that

$$\phi_{\max} > 0.7 \quad (4.21)$$

Thus equation 4.20 gives upper limits for the single sideband phase noise in a 1 Hz bandwidth as -85 dB/Hz and -109 dB/Hz, for 12 and 16 bit A/D's respectively. If these upper limits do not suffice the alternatives are:

1. Reduce the maximum required phase shift by analogue modification of the RF amplifier chain (e.g. re-bias all class C amplifiers to class AB, introduce RF feedback, add a series, analogue, phase predistorter). Figure 4.2.7 shows the AM/PM of the amplifier of Figure 4.2.6 when modified by a phase locked loop (discussed later). For the modified amplifier $\phi_{\max} = 3$ degrees (0.05 radians), and the SSB phase noise improves by 23 dB.
2. Increase the value of M. In practice this may have consequences on the sample rate, since high resolution A/D's are generally slower than low resolution A/D's.
3. Determine the maximum phase noise permissible, set the digital compensation so as not to exceed that level (i.e. set $\phi_{\max}$), and accept any residual AM/PM distortion. For example, in Figure 4.2.7 the bulk of the phase shift lies within a one degree band (2-3 degrees). If we set $\phi_{\max} = 1$ degree then from Figure 4.2.7 we would not expect much residual AM/PM conversion.

4.3 POLAR IF INPUT

4.3.1 Bandwidth

We consider first simply using the NLFG of Figure 4.2.1, and requiring that the input is at IF rather than at baseband.

Let the input signal have a bandwidth of B Hz, centred on f_C as in Figure 4.3.1. Clearly ^{Sample Theory}~~Shannon~~ requires that

$$f_A > 2f_{\max} \quad (4.22)$$

$$\text{and} \quad f_D > 2nf_{\max} \quad (4.23)$$

$$\text{and} \quad f_{\max} > B \quad (4.24)$$

where f_A , f_D and n have the same meanings as before. It appears then, that provided we choose a low IF frequency, we have incurred little additional penalty in using the system of Figure 4.2.1 as a NLFG for IF signals. However, unlike the AM case, the NLFG does not have the signal envelope as its input. Consequently, we must regard the NLFG as behaving as an instantaneous voltage transfer function rather than as an envelope transfer function. Thus we must modify Figure 4.2.1 to remove all but the first zone harmonics of the IF (as in Figure 4.3.2).

This will result in the NLFG acting purely on the envelope of the input IF. The relationship between the RAM table (which is defining the instantaneous voltage transfer function) and the envelope transfer function is given in reference 20.

As a result of this it is necessary to impose further conditions to ensure that the first zone harmonic does not alias with other zones, although aliasing of higher zones is not of concern to us since we will filter them out anyway. Figure 4.3.3 gives some simple examples.

The following analysis assumes that even order distortion

effects are small and can be neglected.

For n odd, the bandwidths of the odd harmonics are all nB (Hz), up to the n^{th} harmonic, (for proof see Appendix A). So to avoid overlap of the first and third zonal harmonics we require

$$f_C + nB/2 \leq 3f_C - nB/2$$

i.e. $f_C \geq nB/2$ (4.25)

but as $f_C = f_{\max} - B/2$ we require

$$f_{\max} \geq (n + 1)B/2 \quad (4.26)$$

By similar reasoning we can show that this condition prevents folding of the first zone at dc.

Another condition to be imposed is that the D/A sample rate (f_D) is chosen such that there is no aliasing within the first zone (see Figure 4.3.3). This is a condition quite distinct from that of equation 4.26, which was to stop overlap of harmonic zones. Thus we require

$$f_C + nB/2 \leq f_D - nf_{\max} \quad (4.27)$$

but as $f_C = f_{\max} - B/2$ we have

$$f_{\max} - B/2 + nB/2 \leq f_D - nf_{\max}$$

i.e. $f_D \geq (n + 1)f_{\max} + (n - 1)B/2$ (4.28)

(N.B. $n=1$ in equation 4.28 gives Nyquist's result).

Combining equation 4.26 and equation 4.28 we derive the absolute minimum D/A sample rate ($f_{D\min}$), which holds only if we take equation 4.26 as an equality. Hence

$$f_{D\min} = n(n + 3)B/2 \quad (4.29)$$

A typical polar, vector, IF application would be VSB-TV, when the video bandwidth is 6 MHz. [It is common in TV transmission to amplify the sound and video signals by separate RF amplifier chains and combine the two signals only at the antenna. This is a

more linear way of amplifying than sending both signals through the same amplifier chain. The total bandwidth of the video (sideband and vestigial sideband), may be taken as 6 MHz [ref 187]]. Thus the D/A sample rate for $n=7$ would need to be at least 210 MHz (in practice this would not leave enough of a guard band between zones and alias products, so a larger sample rate would be required). If we wished to avoid CPU interpolation the A/D sample rate would need to be the same as the D/A sample rate. Since these sample rates are not currently achievable using high bit accuracy converters (see Figure 3.1.1), we conclude that the NLFG's as described by Figure 4.3.2 are not well-suited for IF applications.

The problem arises from using the A/D, RAM, D/A as a means of controlling the instantaneous voltage transfer function, rather than the envelope transfer function, as was the case with baseband AM. Using the instantaneous voltage approach is a highly inefficient means of utilising the bandwidth.

An obvious course of action is to strip off the envelope from the IF signal and use that as the input to the NLFG. Figure 4.3.4 shows the idea. The system shown in Figure 4.3.4 is a feedforward system, but with certain important differences over conventional feedforward:

1. Only one matched time delay is required.
2. Correction takes place prior to the amplifier, hence there is no loss of dc-RF efficiency.
3. There is no requirement for a secondary amplifier. Hence problems associated with the linearity of the secondary amplifier are of no relevance here.

The method also has some parallels with the polar loop feedback technique, in that we attempt to strip off the envelope for processing. Unfortunately, just as that technique has severe bandwidth constraints, so has this course of action. Consider the IF signal, with a total signal bandwidth $2d$ Hz, described by

$$\begin{aligned} & \cos((w + d)t + \phi) + \cos((w - d)t - \phi) \\ &= 2\cos(dt + \phi)\cos(wt) \\ &= 2 |\cos(dt + \phi)| \cos(wt + n(t)) \end{aligned} \quad (4.30)$$

where $n(t) = 0$ if $\cos(dt + \phi) > 0$

$$n(t) = \pi \text{ otherwise}$$

and thus we identify the envelope as

$$2 |\cos(dt + \phi)| \quad (4.31)$$

The envelope is sketched in Figure 4.3.5. Clearly the discontinuity in the first derivative of the envelope gives rise to a spectral width of the envelope much greater than d Hz.

[Reference 30 shows that the harmonics of a periodic function discontinuous in the first derivative fall at a rate proportional to m^2 , where m is the order of the harmonic. Thus, to include signals that are less than -40 dB on the first harmonic, we may need to consider up to the tenth harmonic].

Powers [ref 135] and Dujundji [ref 43] show that the envelope of a signal of bandwidth B Hz has infinite bandwidth (AM being a special case). Hardly surprising then that the polar loop transmitter has fundamental bandwidth constraints. However, the square of the envelope has a maximum bandwidth of B Hz [ref 43, 135], where B is the bandwidth of the RF signal.

Thus, to go back to our example, if the envelope is given by equation 4.31 then the envelope squared is given by

$$4\cos^2(dt + \phi) \quad (4.32)$$

which can be written as

$$2(1 + \cos(2dt + 2\phi)) \quad (4.33)$$

and thus the result is as expected as the total bandwidth is now $2d$ Hz.

Figure 4.3.6 shows a polar, vector, IF scheme based on this result. By squaring the IF signal and detecting the envelope of the second harmonic the NLFG is fed with the square of the signal envelope, which is bandlimited to the signal bandwidth. Another well-known way of producing the envelope squared is to pass the RF through a square law device and simply extract the low frequency/dc component. The two techniques are equivalent except that the latter must be dc coupled. The scheme is arranged so that no NLFG has to deal with an infinite bandwidth signal. Thus, referring to the envelope NLFG in Figure 4.3.6, we find that to implement a n^{th} odd order distortion of the envelope, the NLFG only has to output a signal with $(n-1)$ even order distortion of the envelope, which has a bandwidth (Hz) given by

$$B(n - 1)/2 \quad (4.34)$$

and thus to satisfy Nyquist

$$f_D \geq B(n - 1) \quad (4.35)$$

and for the A/D (assuming independent sampling from the D/A)

$$f_A \geq B \quad (4.36)$$

For the phase NLFG we require that it can be described by a m^{th} even order polynomial, from which we deduce that the output bandwidth must be $mB/2$ Hz (see Figure 4.3.6). Thus Nyquist requires

$$f_{Dp} > mB \quad (4.37)$$

where f_{Dp} represents the sample rate of the phase D/A. (For the polar, baseband AM scheme considered previously there was no need to differentiate between phase and envelope sample rates, since they both followed the same equations). In practice, the CPU interpolation may be neglected if we make

$$f_A = f_D = f_{Dp} \quad (4.38)$$

whilst satisfying equations 4.34-36.

Hence for a 6 Mhz video application with $n=7$, $m < n$, and assuming that no interpolation is used, we require A/D and D/A sample rates of 36 MHz, which is in the 10-12 bit quantization realm of Figure 3.1.1.

In practice, of course, we cannot fabricate perfect square law devices (although JFET's may give good performance). Despite this, we are still assured that the envelope of the second harmonic is defined by an even function of the envelope of the fundamental [ref 20], i.e. if the IF signal is described by

$$A(t)\cos(\omega t + \phi(t)) \quad (4.39)$$

then the second harmonic envelope ($A_s(t)$) must be described by

$$A_s(t) = a_2 A^2(t) + a_4 A^4(t) + \dots + a_j A^j(t) \quad j \text{ even} \quad (4.40)$$

Hence the input to the A/D will have to quantize a signal not of bandwidth B Hz but of $jB/2$ Hz, to take account of equation 4.40. Since it will usually be the case that the A/D is operating at the D/A sampling rate (so as to avoid fast interpolation), provided that $j < m$ and $j < n$ no increase in sample rate for the A/D is required. Since the output required from the NLFG is unaffected by the square law device imperfections, the D/A's sample rate is unaffected and equations 4.34-36 still hold. Thus the scheme proposed is little affected by imperfections in the square law

characteristic.

4.3.2 Quantization Noise and the Envelope NLFG

Following the same line of argument as with the AM NLFG, we can consider the input to the A/D (in Figure 4.3.6) to be signal and noise, the ratio between the two being

$$(S/N) = 12(M^2 - 1) \quad (4.41)$$

as before (equation 4.8). We must translate this power ratio at the A/D to a ratio of signal to noise prior to the square law device. This is most simply done by noting that if the input to the square law device is given by

$$A(t) + n(t) \quad (4.42)$$

where $A(t)$ represents the signal and $n(t)$ the noise, then the output envelope of the square law device will be proportional to

$$(A(t) + n(t))^2 = A^2(t) + 2n(t)A(t) + n^2(t) \quad (4.43)$$

Neglecting the last term as second order small, we see that the noise voltage is represented by $2n(t)A(t)$, whilst the signal voltage is represented by $A^2(t)$. Thus on passing through the square law device there is a 6 dB degradation in signal to noise.

Hence, provided the D/A resolution is larger than that of the A/D, we can represent the quantization effects of the A/D as being the same as if the NLFG was perfect (no quantization effects, linear when in series with the DUT), and the input envelope to noise power ratio (E/N), measured in dB, given by

$$\begin{aligned} (E/N) &= 20\log(M) + 6 + 10.8 \\ &= 20\log(M) + 16.8 \quad (M^2 \gg 1) \end{aligned} \quad (4.44)$$

Thus for a perfect NLFG the same envelope to noise ratio results at the output of the DUT (e.g. for 12 bit systems $(E/N) = 89$ dB,

whilst for 16 bit systems $(E/N) = 113 \text{ dB}$).

4.3.3 Quantization Noise and the Phase NLFG

Our arguments concerning the phase noise are little affected by the addition of the square law device. Since the noise contributions from the AM/PM conversion and from the direct phase modulation are still both synchronous and in anti-phase, under ideal conditions there is no net phase noise introduced which is associated with the NLFG's. However, as before, we take a worst case pessimistic attitude and calculate the phase noise associated with the two effects adding asynchronously.

As before we argue that we can represent the input to the A/D as containing signal and noise, the ratio of which is defined by equation 4.8, and the NLFG as near-perfect. This produces a single sideband noise power ratio in a 1 Hz bandwidth ($L \text{ (dB/Hz)}$), with an upper limit given by

$$L_{\max} \leq 20\log(\phi_{\max}/M) - 9.5 \quad (4.45)$$

exactly as before (equation 4.20). (Arguments of baseband AM follow through directly from equation 4.11, without need to refer to the signal to noise ratio at the input to the A/D, prior to the square law device).

Naturally, the arguments as to how the upper limit to the phase noise may be improved are the same as for baseband input, AM modulation (see section 4.2.3).

4.4 D/A RESOLUTION

All of the preceding analysis (both IF and baseband), assumed that the D/A resolution was greater than that of the A/D,

and so could be neglected. If we wished we could attempt to analyse for finite resolution D/A's, by attempting to approximate the D/A resolution as being a degradation in the "effective" A/D resolution. In fact we do not attempt this analysis as the D/A can easily have greater resolution than the A/D.

Clearly the last statement can be justified in many cases, (e.g. 12 bit A/D and 16 bit D/A). However, in other cases it may not be so simple as using a D/A IC with higher resolution. In such cases the approach of Figure 4.4.1 may be used. In essence we use two D/A's, one to act as the coarse voltage adjustment and the other as the fine. Provided the nonlinear function (NLF) we wish to approximate lies in Region 1 (see Figure 4.4.1), we effectively achieve higher resolution. In Figure 4.4.1 the coarse D/A is not preceded by a RAM, thus the coarse D/A, in conjunction with the A/D, simply acts as a linear transfer function. With most of the NLF's that are met this is all that is needed of the coarse voltage setting, and is an example of trend removal as found in time series analysis [ref 116].

4.5 CONCLUSIONS

For IF systems we would advise the use of the NLFG's of Figure 4.3.6 rather than of Figure 4.3.4 or Figure 4.3.2 because of superior bandwidth considerations. Since the lower the sample rate the higher the bit accuracy that can be achieved, the NLFG's of Figure 4.3.6 will also have superior noise performance in practice. It is worthy of note that for IF systems the NLFG has antecedents in feedforward, as much as in conventional predistortion. It is this fact which helps to justify identifying

digital compensation as a technique separate from predistortion.

For baseband systems the NLFG is more straightforward, sample rates are lower and there is no need for time delays to be built into the system. However, baseband systems are only easily applied when the RF signal envelope is trivially related to the baseband information (as in AM for example). Thus, although baseband systems are preferred, it must be conceded that IF systems are a more general implementation of digital compensation.

FIGURE 4.2.1

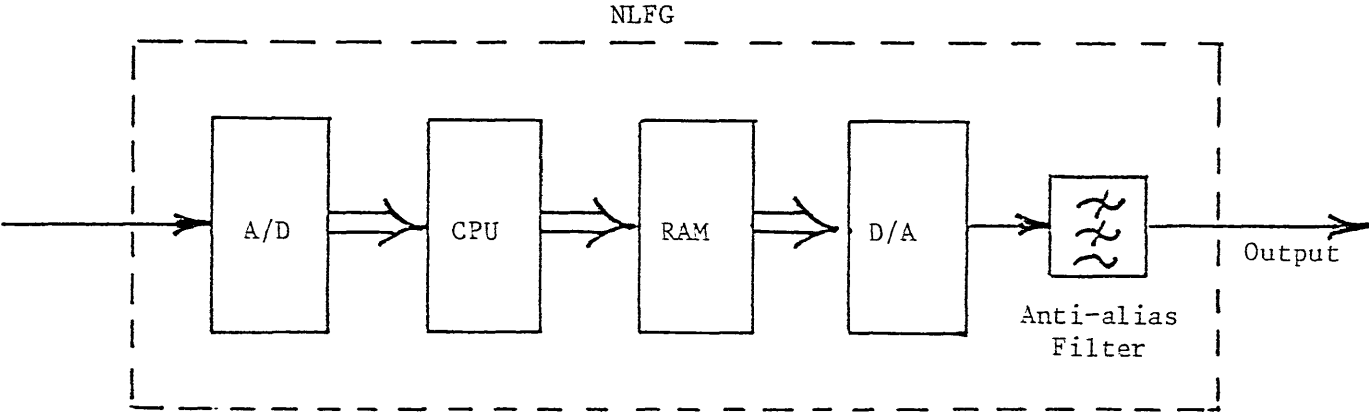


FIGURE 4.2.2

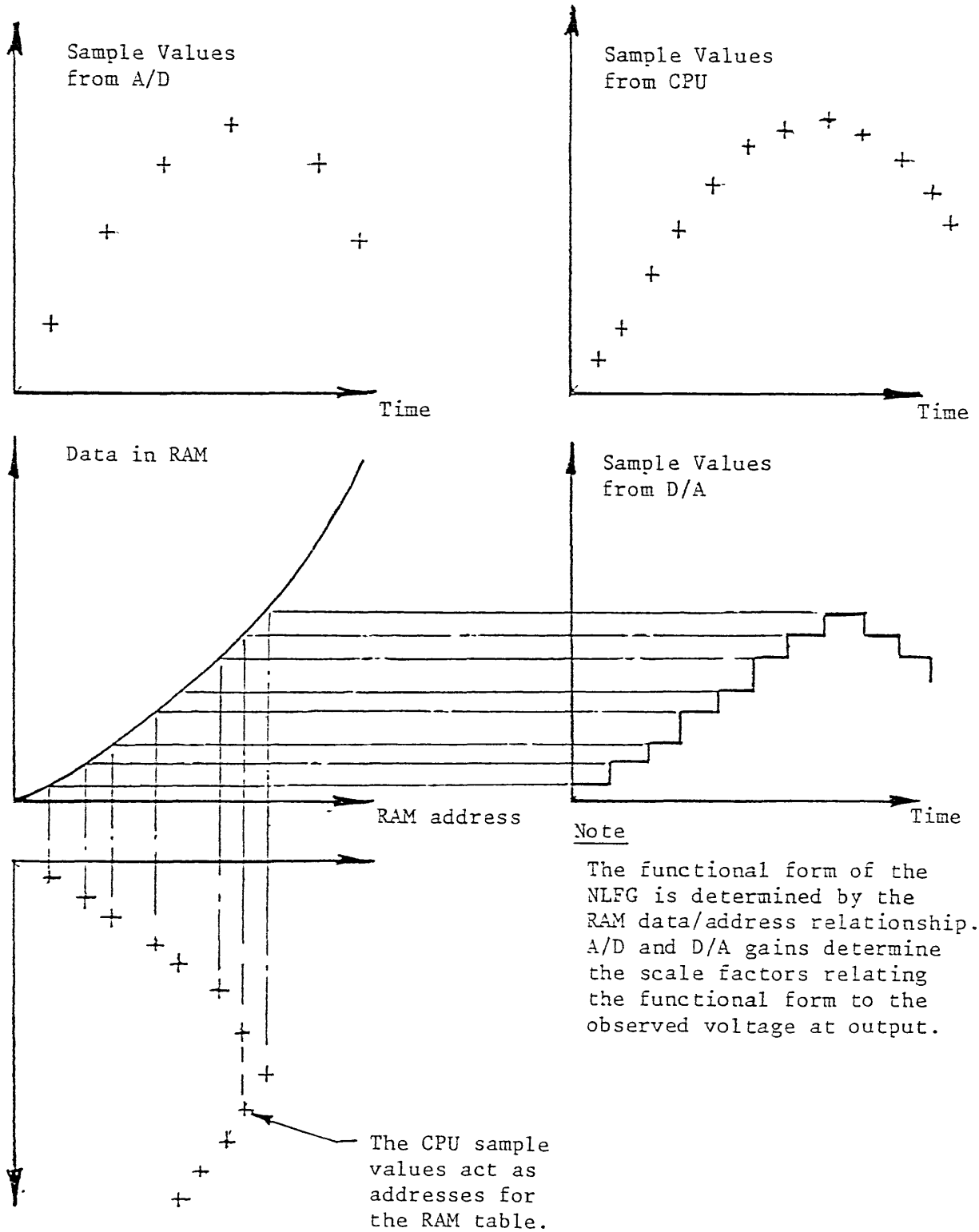


FIGURE 4.2.3

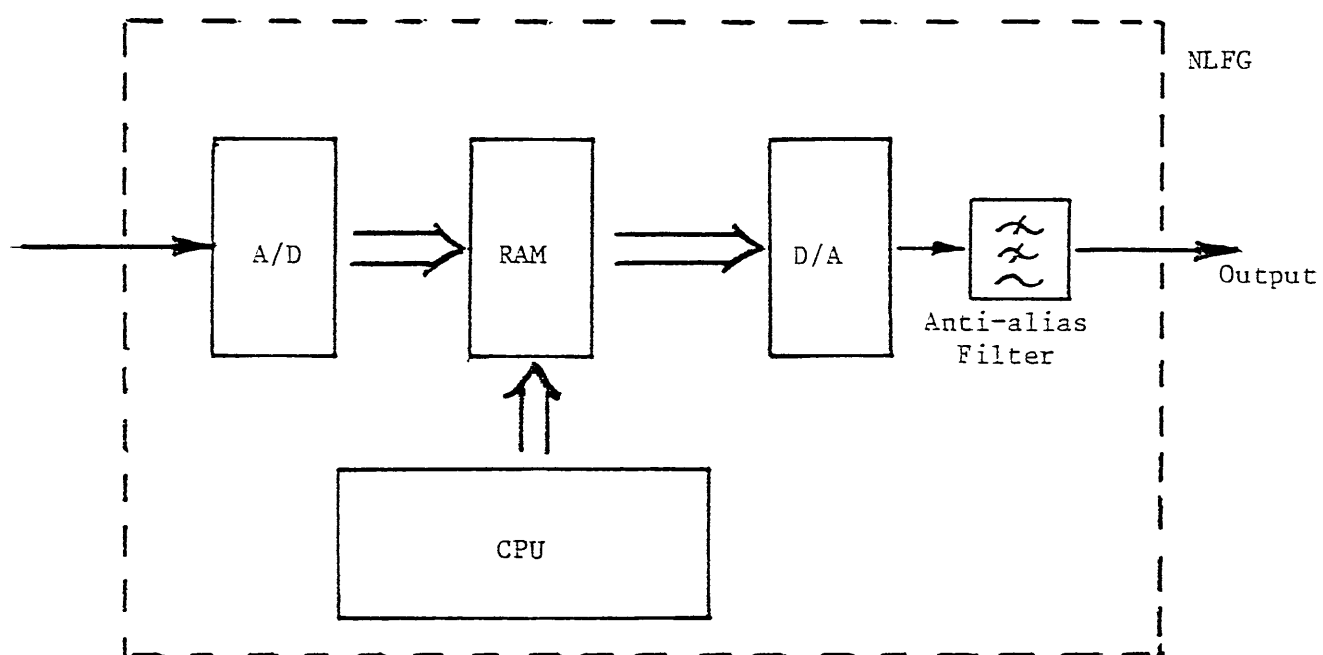


FIGURE 4.2.4

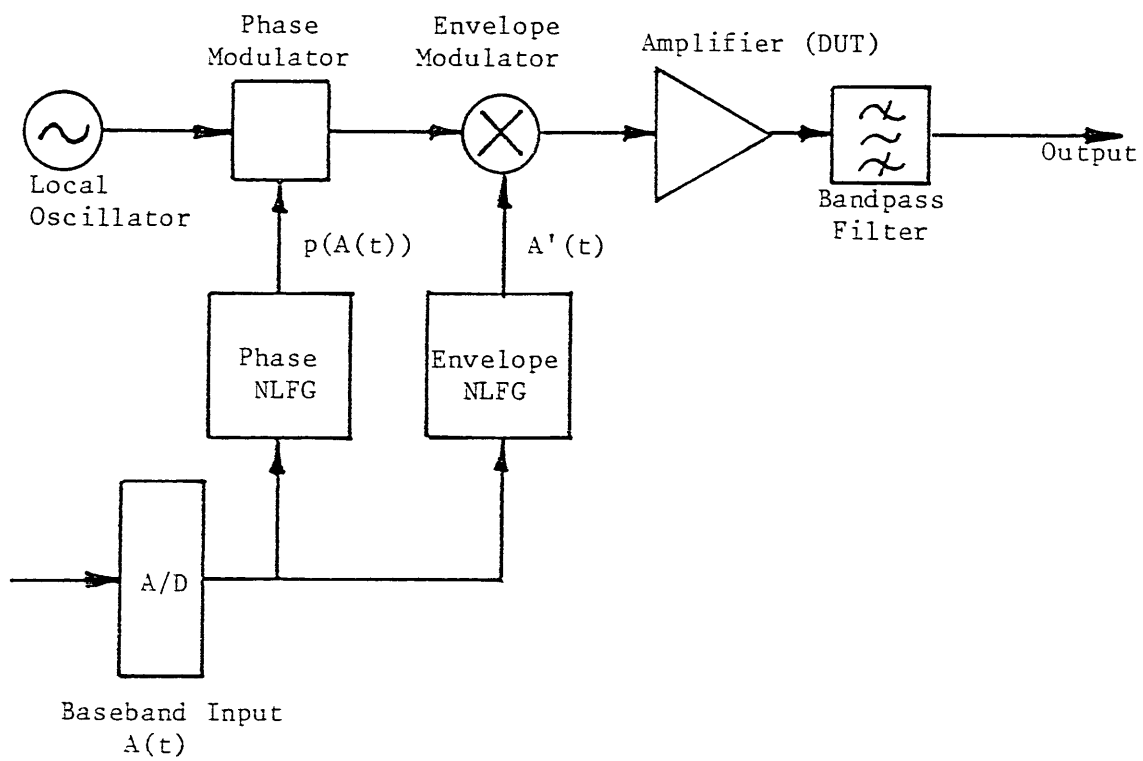
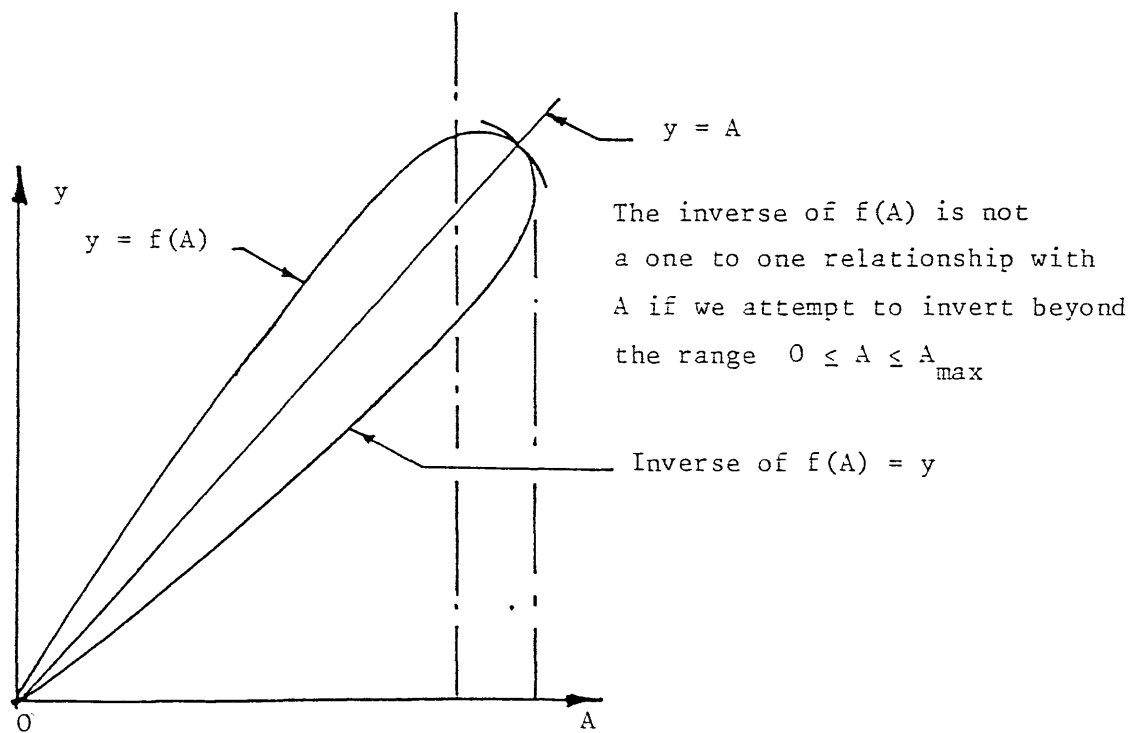
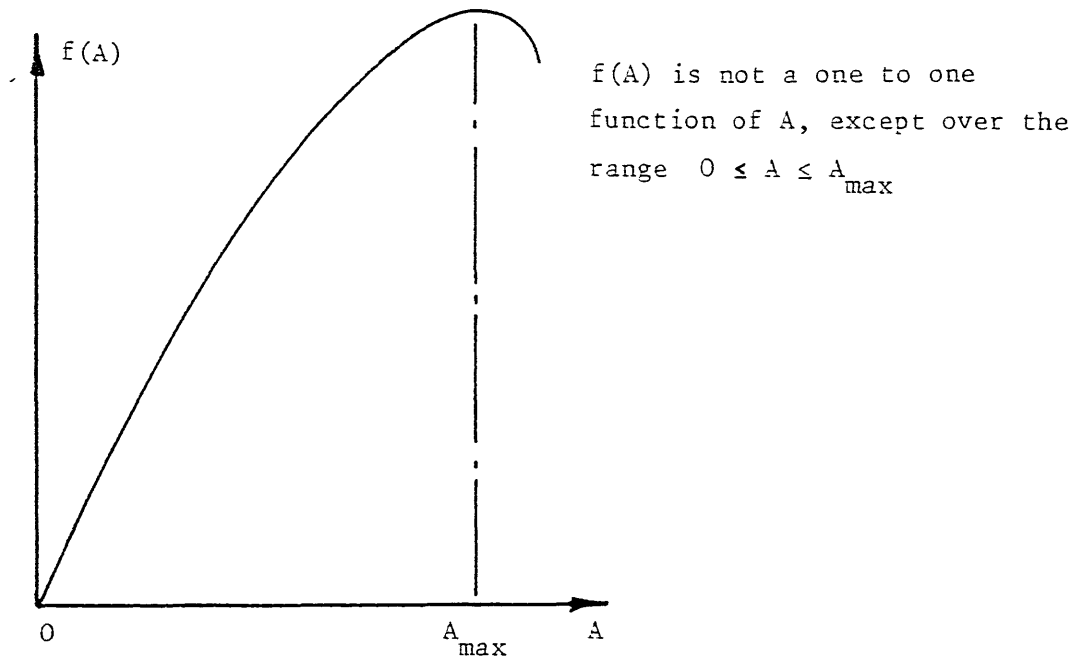


FIGURE 4.2.5



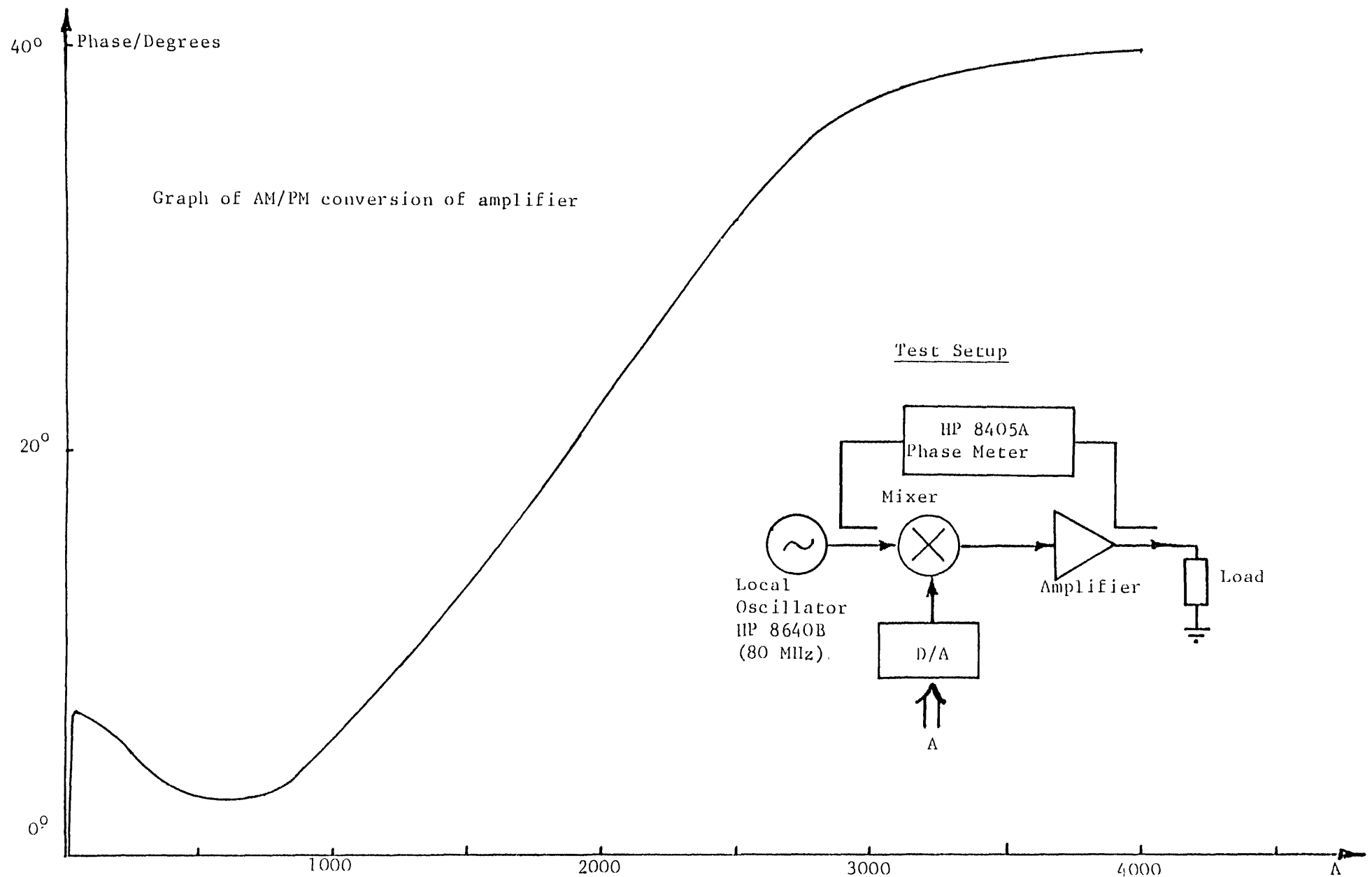
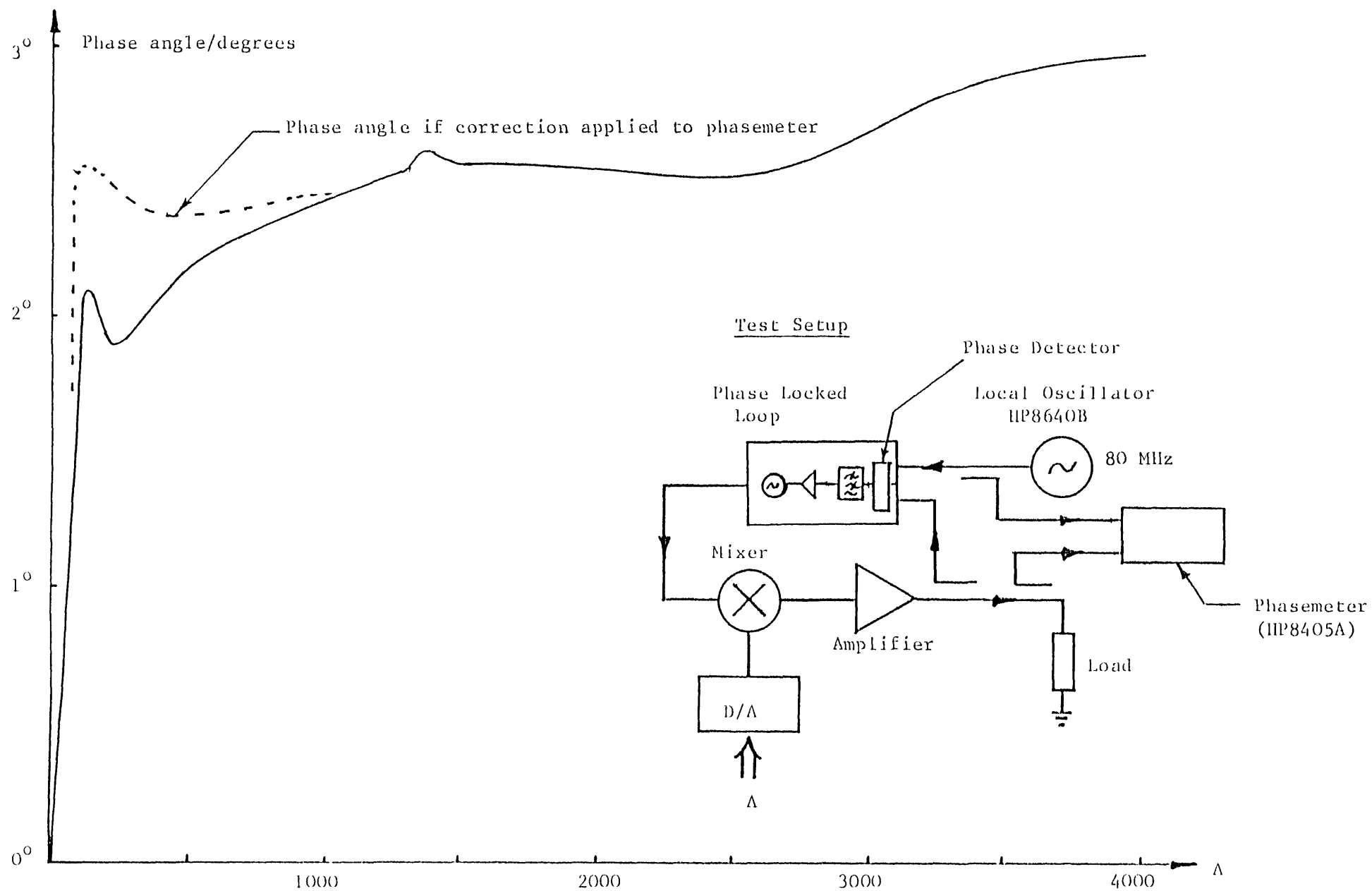


FIGURE 4.2.6



Graph of AM/PM conversion of amplifier with phase locked loop correction as with polar loops

FIGURE 4.2.7

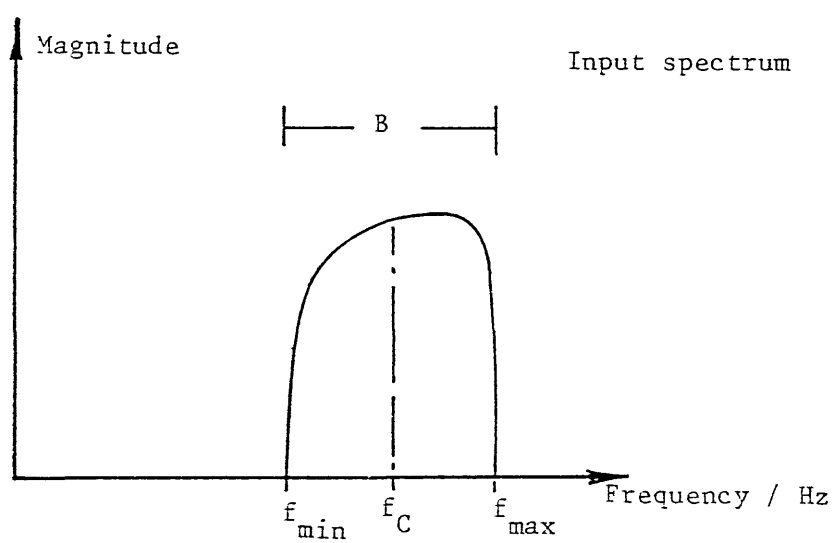
FIGURE 4.3.1

FIGURE 4.3.2

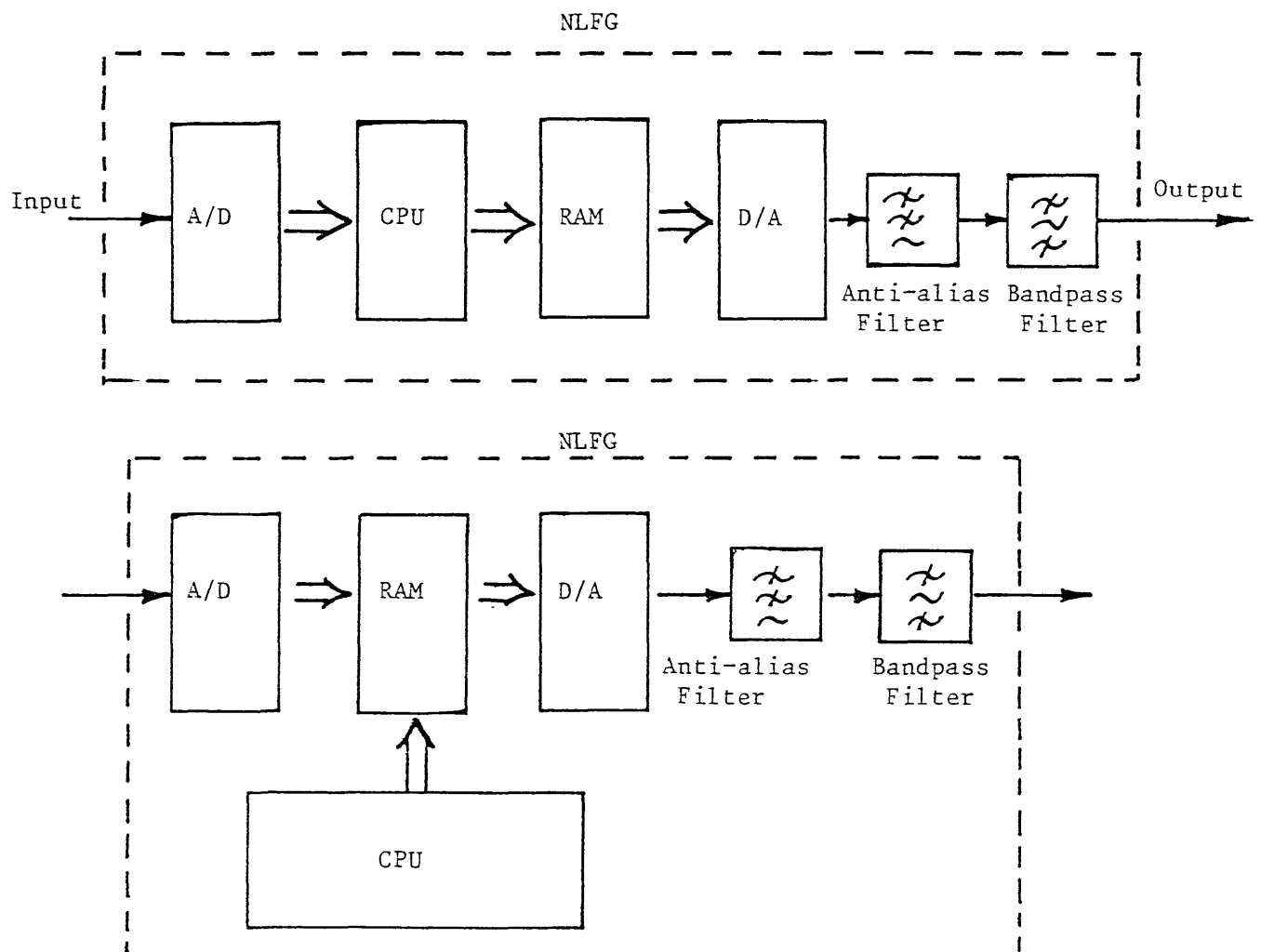


FIGURE 4.3.3

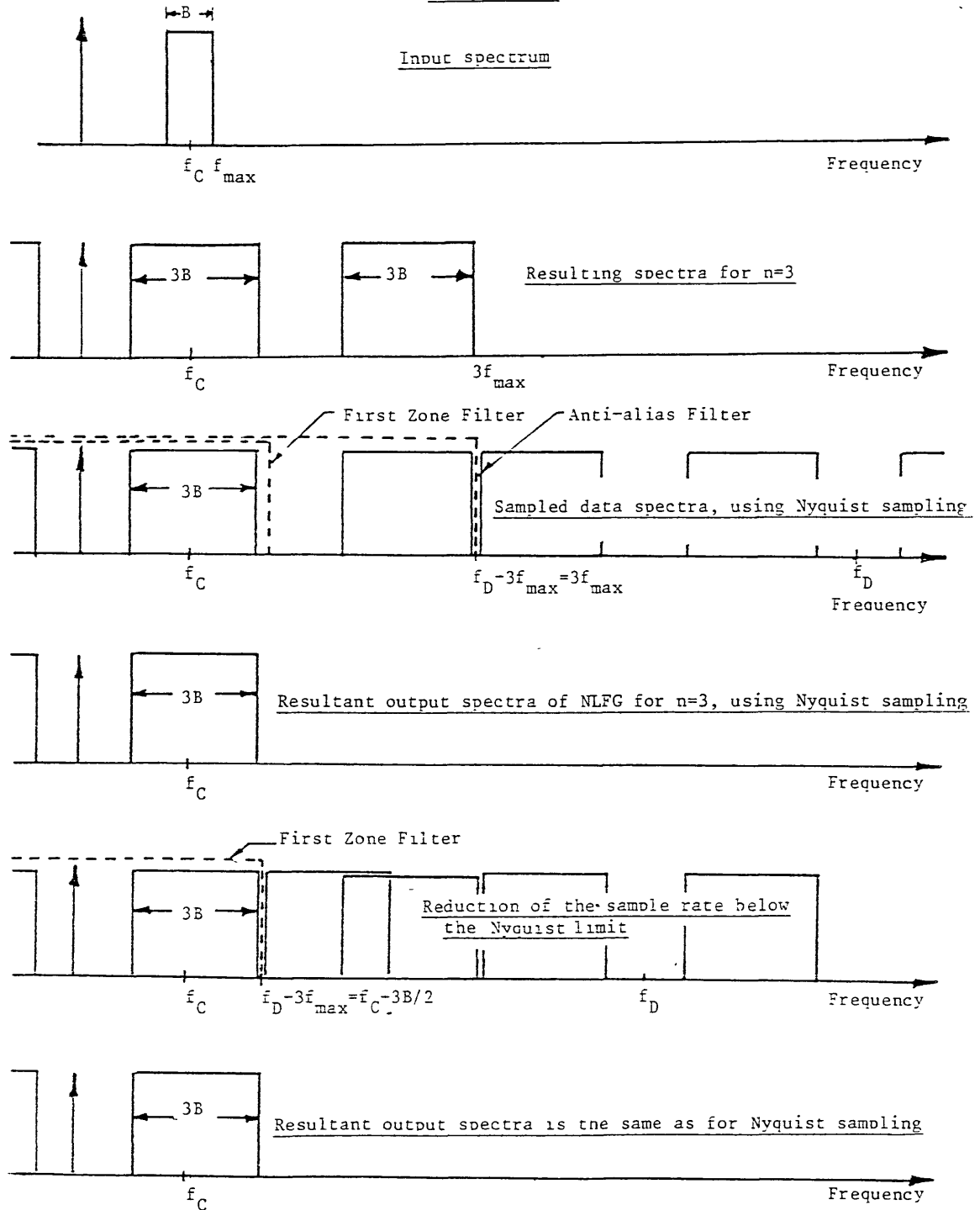


FIGURE 4.3.4

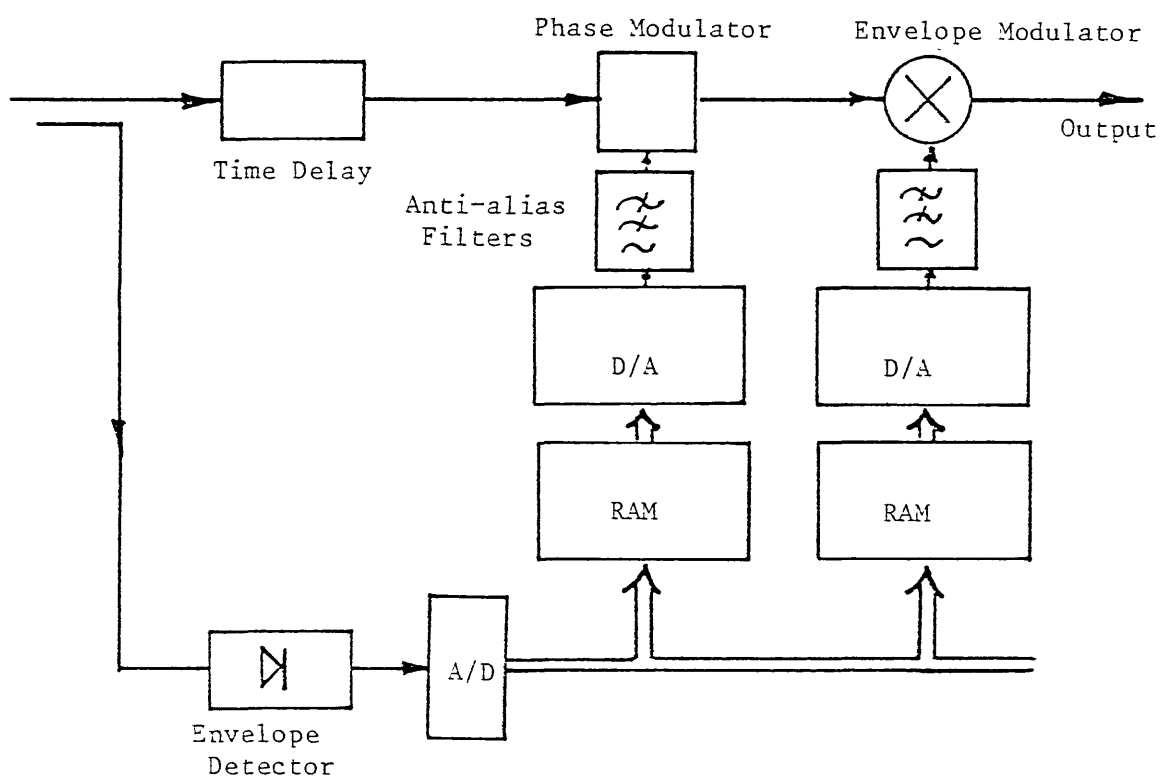


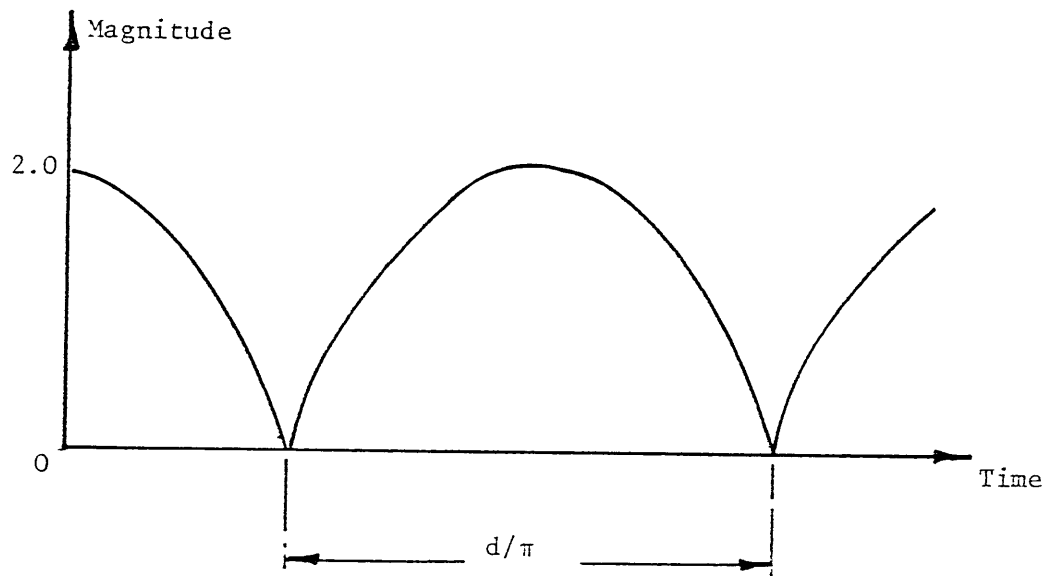
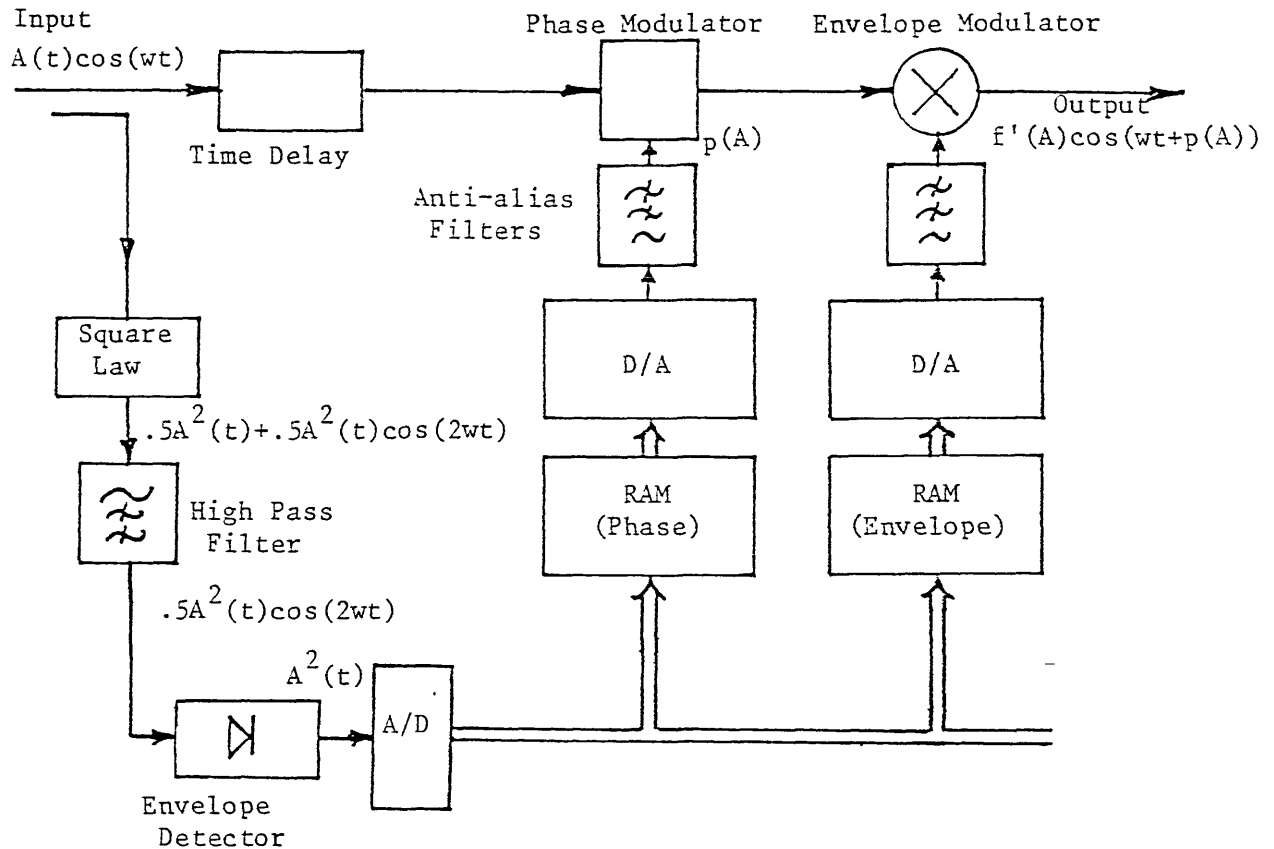
FIGURE 4.3.5

FIGURE 4.3.6



Function implemented by phase RAM

$$p(A) = p_0 + p_2 A^2 + p_4 A^4 + p_6 A^6 + \dots + p_m A^m$$

$$= p_0 + p_2 A^2 + p_4 (A^2)^2 + p_6 (A^2)^3 + p_8 (A^2)^4 + \dots + p_m (A^2)^{m/2}$$

Hence if bandwidth of $A^2(t)$ is B Hz total bandwidth for $p(A)$ is $Bm/2$ Hz.

Function implemented by envelope RAM

$$f'(A) = a_1 A + a_3 A^3 + a_5 A^5 + \dots + a_n A^n$$

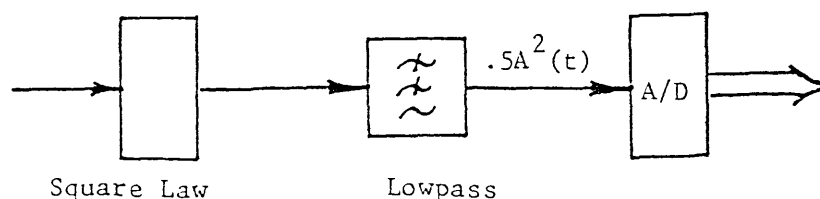
$$= A (a_1 + a_3 A^2 + a_5 (A^2)^2 + \dots + a_n (A^2)^{(n-1)/2})$$

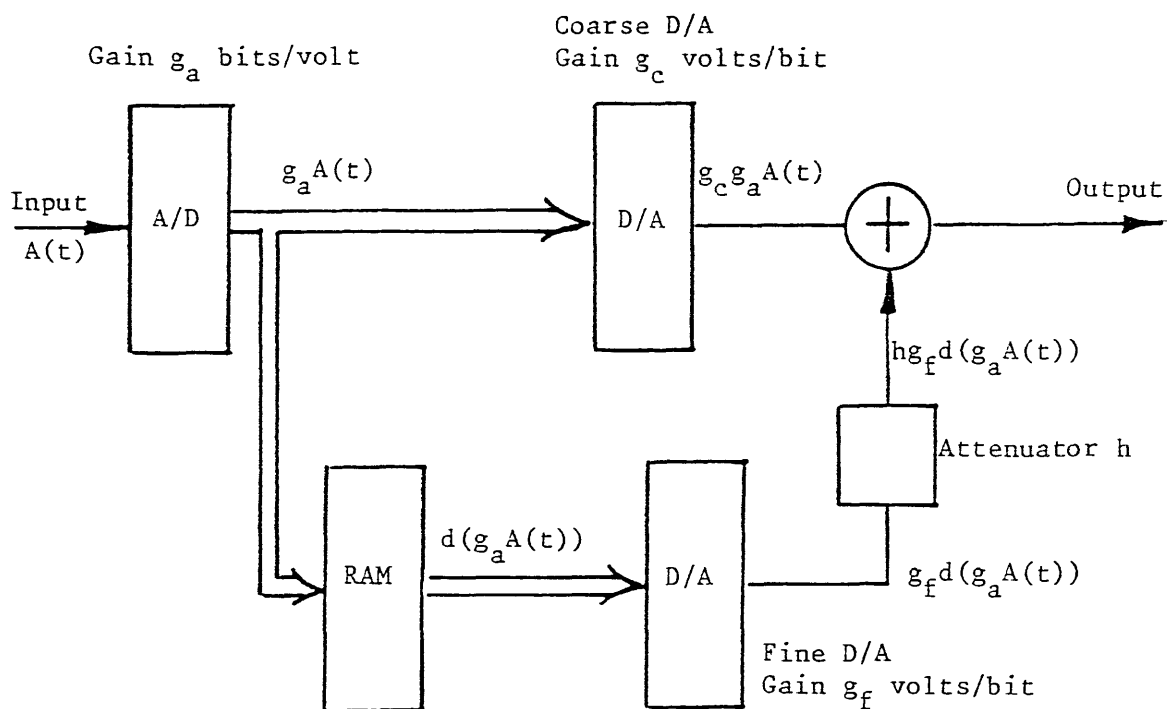
Hence if bandwidth of $A^2(t)$ is B Hz then the bandwidth out of the anti-alias filter is given by the bandwidth of

$$(a_1 + a_3 A^2 + a_5 (A^2)^2 + \dots + a_n (A^2)^{(n-1)/2})$$

which is $(n-1)B/2$ Hz.

The square law, filter, A/D shown below may be used instead of the arrangement above.



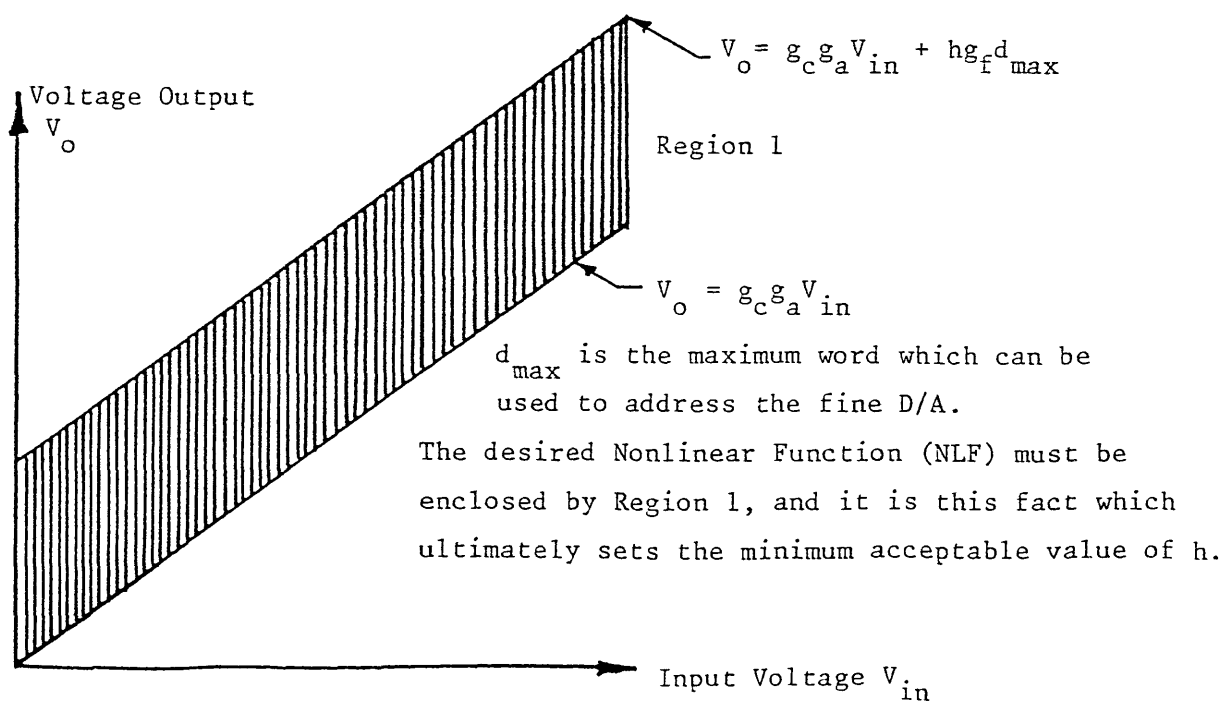


$d(A)$ represents the data value stored at RAM address A

$$\text{Output} = g_c g_a A(t) + h g_f d(g_a A(t))$$

$$\text{Minimum output voltage resolution} = h g_f$$

By suitable choice of h , the required resolution can always be reached.



CHAPTER 5

THE ENVELOPE MODULATOR

5.1 INTRODUCTION

Our objective is to predistort the RF envelope, but in general it is a baseband signal which passes through the A/D, RAM, D/A combination. As the RF signal does not pass through the NLFG the need for an envelope modulator is inevitable. Many devices are available for achieving this. The most notable are the JFET, dual gate FET, schottky diode and pin diode modulators [ref 73]. These, and other, devices and systems capable of achieving envelope modulation are discussed in reference 7. In general there is little advantage in not using FET or diode modulators, so other modulators are not considered here. For digital compensation we do not require a bipolar envelope modulator, but we do require the control range of the modulator to be large, and that the modulator be linear. In this chapter we consider the effects of control range and ways by which it can be improved. We do not consider linearity, as the modulator nonlinearity can be added to, and corrected with, the DUT nonlinearity.

5.2 CONTROL RANGE

Envelope modulators have a finite control range caused by leakthrough of the RF signal. The envelope observed at the output of the modulator results from the vector sum of the leakthrough and the controlled envelope (see Figure 5.2.1). Referring to Figure 5.2.1 (which is a typical component of AM systems) the output of the modulator is given by

$$A(t)\cos(\omega t + \phi(t)) + a\cos(\omega t + \phi(t) + \theta) \\ = [A^2(t) + a^2 + 2A(t)a\cos(\theta)]^{0.5}(\cos(\omega t + \phi(t) + p)) \quad (5.1)$$

where

$$p = \arctan(\sin(\theta)/(A(t) + a\cos(\theta))) \quad (5.2)$$

θ is the phase shift associated with leakthrough, and a is its magnitude. Thus the envelope $E(t)$ of the output of the modulator is given by

$$E(t) = [A^2(t) + a^2 + 2A(t)a\cos(\theta)]^{0.5} \quad (5.3)$$

where $A(t)$ is determined by the linearity of the device and the voltage of the modulating signal. Without loss of generality we can assume that $-1 < A(t) < +1$ and that the PEP is associated with $A(t) = \pm 1$.

Figure 5.2.2 shows the variation of output of the modulator with A , taken for $a = 0.003$ (-50 dB relative to the PEP) and for $\theta = 100$ degrees. From Figure 5.2.2 it is clear that there exists a "dead zone" of range E_w . The modulator cannot produce an output in this range, regardless of the baseband input, modulator linearity or modulator type. Clearly this range is given by consideration of $(dE/dA)=0$. The condition for this to occur from equation 5.3 is

$$(dE/dA) = 0.5[A^2 + a^2 + 2Aa\cos(\theta)]^{-0.5}(2A + 2a\cos(\theta)) = 0 \\ \text{i.e.} \quad A + a\cos(\theta) = 0 \\ \text{i.e.} \quad A = -a\cos(\theta) \quad (5.4)$$

Combining with equation 5.3 to determine the "dead zone" width

$$E_w = [a^2\cos^2(\theta) + a^2 - 2a^2\cos^2(\theta)]^{0.5} \\ = [a^2 - a^2\cos^2(\theta)]^{0.5} \\ = a\sin(\theta) \quad (5.5)$$

Hence to produce a modulator with a complete control range

($E_w=0$), we require either

$$a = 0 \quad (5.6)$$

or $\theta = n\pi \quad n \text{ integer} \quad (5.7)$

Generally the magnitude of the feedthrough increases with increasing frequency, due to capacitive and other effects [ref 73]. Consequently we aim to use the envelope modulator at the lowest possible IF frequency comparable with the bandwidth required for the modulation sidebands. Typically a low IF will reduce local oscillator feedthrough to 65-60 dB depending on the modulator [see reference 117 for the MC1496 active double-balanced modulator at 0.5 MHz].

With AM systems using a low IF frequency increases the complexity, in that multiple IF's and image reject filters will now be required, whereas before they were not. In some cases low IF frequencies are unacceptable anyway. For example, if the baseband signal occupies 8 MHz then for $n=7$ the minimum IF frequency would be 56 MHz. In practice a higher IF frequency would be chosen so that the envelope modulator does not have to be dc-coupled to the DUT. Typical feedthrough at 60 MHz is of the order of 50 to 40 dB [ref 73].

5.3 EFFECT OF LIMITED CONTROL RANGE

The result of a limited control range ($E_w \neq 0$) depends on the classification of the digital compensation.

5.3.1 IF Input to NLFG: Effect of Modulator Leakthrough

For the recommended envelope-squared scheme of Figure 5.3.1, provided the DUT is approximately linear in transfer

characteristic, it is unlikely that a large control range will be demanded of the envelope modulator. This is because for Figure 5.3.1, the output of the envelope modulator (with leakthrough) is given by

$$A(t)B(t)\cos(\omega t + \phi(t)) + aA(t)\cos(\omega t + \phi(t) + \theta) \quad (5.8)$$

from which we identify the envelope $E(t)$ as

$$E(t) = A(t)[B^2(t) + a^2 + 2B(t)a\cos(\theta)]^{0.5} \quad (5.9)$$

Now for the DUT approximately linear, the range required to be spanned by the term raised to the power 0.5 in equation 5.9 will be small, as $E(t)$ is approximately equal to $A(t)$ if the DUT is nearly linear, and so there is little chance that this term will ever be required to take a value in the dead zone.

5.3.2 AM Baseband System: Effect of Modulator Leakthrough

For the baseband AM schemes we have been discussing we will, in general, require the modulator to traverse the complete control range and thus, unlike IF systems, we are affected by modulator leakthrough. Figure 5.3.2 shows the envelope waveshape typical of leakthrough on a 100% AM modulation test. Although the leakthrough on its own is spectrally unimportant, its presence causes the adaptive system to produce a new envelope. It is the spectral distortion of this new envelope that we must consider. This new envelope is also shown in Figure 5.3.2 for sinusoidal AM modulation. The new envelope is a clipped sinusoid, the clipping occurring at E_w . The spectral distortion associated with this new envelope is found with the aid of reference 51, which gives the spectra of a clipped sine wave (see Figure 5.3.3).

In Figure 5.3.3 we have plotted the magnitude of the

distortion products (relative to PEP), against the LO leakthrough suppression (relative to PEP). By equation 4.8 we know the noise floor (relative to PEP) for any A/D bit accuracy. Figure 5.3.3 shows the LO suppression needed, if the resulting distortion is to be in the noise floor of the RF spectra, for various A/D bit accuracies. Although Figure 5.3.3 strictly only applies on the 100% AM sine wave test, we take it to be representative of the required LO suppression.

Once the LO suppression for a modulator is known Figure 5.3.3 defines the level of distortion which will occur. The important point to grasp is that leakthrough induced distortion cannot be corrected by adjusting the NLFG, unlike most other sources of distortion.

5.4 REDUCING LEAKTHROUGH DISTORTION IN AM SYSTEMS

5.4.1 Introduction

Leakthrough induced errors are a problem only with AM baseband systems. Consequently they represent a problem which is not general to digital compensation, and hence not of prime importance to this thesis. However, it is important that the errors associated with leakthrough do not dominate the AM baseband systems constructed, and so we are forced to give some brief consideration to methods of suppressing local oscillator leakthrough.

5.4.2 Nulling Methods

A standard way to reduce leakthrough in envelope modulators is to add a small quantity of the local oscillator voltage in

anti-phase to the output of the modulator [ref 68], see Figure

5.4.1. Referring to Figure 5.4.1 the output is described by

$$A(t)\cos(\omega t) + a\cos(\omega t + \theta) + b\cos(\omega t + \phi) \quad (5.10)$$

hence provided

$$a = b \quad \text{and} \quad \theta = \phi + \pi \quad (5.11)$$

the overall effect is to remove the leakthrough. Unfortunately component drift and tolerancing cause a , θ , b and ϕ to vary, so that only finite suppression can be achieved. Figure 5.4.2 shows the effect of temperature on a double balanced schottky diode modulator (taken from reference 73, p.180). The effect of phase and amplitude errors on the leakthrough are shown in Figure 5.4.3. By examining Figure 5.4.2 we can see that even over a 25°C temperature range the leakthrough of the modulator will vary by typically 1 dB. Using Figure 5.4.3 we see that the nulling circuit cannot achieve better than 19 dB suppression with this variation. This is in agreement with the 20 dB suppression reported in reference 68. This enables us to achieve at least 60 dB suppression in the VHF range, which is comparable with 16 bit accuracy.

5.4.3 LO Waveshaping in Diode Modulators

What we propose here is believed to be a new method of reducing leakthrough, in that although diode modulators have been extensively examined in the literature [ref 28, 32, 71, 75, 87, 118], and although LO waveshape has been considered in relation to conversion loss [ref 38], no consideration has been given to LO waveshape and leakthrough suppression.

As a simple model of a double balanced diode mixer consider

Figure 5.4.4. The LO is sinusoidal. Referring to the simplified model of Figure 5.4.4 points A, B, A' and B' are at a dc level controlled by the IF port. Assume the diodes are identical. A full analysis would proceed to calculate $i_L(t)$ (the current in the load) by using the diode law and Kirchoff's equations. We approximate the diode characteristic into three regions (and not the conventional two regions of reference 75), as in Figure 5.4.5.

In region 1 the diode is turned on and its resistance low, in region 3 the diode is turned off and its resistance high. Region 2 is the intermediate region.

Referring to Figure 5.4.4, when one pair of diodes are in region 1 the other pair are in region 2, and thus there is little current flow of the LO signal through the load. As the LO signal falls both sets of diodes enter region 2 and thus current flows through the load.

Assuming sinusoidal LO waveshape $V(t) = V \sin(\omega t)$ across the diodes (see Figure 5.4.6), we would expect both sets of diodes to enter region 2 when the LO voltage ($V(t)$) is approximately given by

$$V(t) = \pm 2\phi \quad (5.12)$$

where ϕ is the barrier potential for the diode, typically 0.2 volts for schottky diodes. Hence the time (t_s) in region 2 is given by

$$t_s = (1/\omega) \arcsin(2\phi/V) \quad (5.13)$$

$$\text{and hence } t_s = 2\phi/\omega V \quad \text{for } V \gg 2\phi \quad (5.14)$$

Alternatively we can obtain this result by noting that the LO waveshape has slope ωV near the origin, and thus the time taken to transit 2ϕ is as equation 5.14.

We approximate the shape of the current pulses in the load by

$$i_p(t) = -4I_0(t/t_s)\exp(-\pi(t/t_s)^2) \quad (5.15)$$

(see Figure 5.4.6). Bracewell [ref 26] gives the Fourier transform of equation 5.15 as

$$F(f) = 4jfI_0(t_s)^2\exp(-\pi(ft_s)^2) \quad (5.16)$$

Using this we deduce that the amplitude of the fundamental (L0) flowing in the load due to periodic current pulses is given by

$$8jI_0(t_s)^2\exp(-\pi(t_s/T)^2) \quad (5.17)$$

where $T = 2\pi/w$ (5.18)

Equation 5.17 is derived by treating the periodic function of current pulses as being formed by a convolution of $\delta(t/T)$ and a convolution of $-\delta((t/T)-\frac{1}{2})$.

If we now attempt to approximate a square wave L0 waveshape by the first two terms in a harmonic expansion, the L0 voltage across the diodes is given by

$$V(t) = V(\sin(wt) + 0.33\sin(3wt)) \quad (5.19)$$

and the slope of the L0 waveshape near zero is now

$$2Vw \quad (5.20)$$

Hence the time (t_{sq}) taken to transit 2ϕ is given by

$$t_{sq} = 2\phi/2Vw = \phi/Vw \quad (5.21)$$

Thus the analysis of equation 5.17 applies if we replace t_s by t_{sq} . (We note however that $(t_s/t_{sq})=2$). So using equation 5.17 we can find the ratio of leakthrough for sine L0 to sine wave-plus-third harmonic as Q where

$$Q = [8jI_0(t_s)^2\exp(-\pi(t_s/T)^2)]/[8jI_0(t_{sq})^2\exp(-\pi(t_{sq}/T)^2)]$$

i.e. $Q = 4\exp(-\pi(t_s/T)^2(1-1/4))$ (5.22)

Expanding to first order

$$Q = 4(1 - 0.75\pi(t_s/T)^2 \dots) \quad (5.23)$$

but as $(t_s/T) \ll 1$ for $wV \gg 2\phi$

$$Q = 4 \quad (5.24)$$

Thus we would expect an improvement in leakthrough of 12 dB.

Comments

The theory is clearly rather crude in that it ignores capacitive coupling. As a result equation 5.17 would lead us to believe that leakthrough will decrease with decreasing L_0 period, whereas the observed effect is an increase. Despite this, the theory seems useful in approximating the effect of increasing diode switching speed, in a highly nonlinear environment, even if its ability to predict the absolute level is poor.

We might conjecture that the effect of capacitive coupling is simply to unbalance the diodes and to produce a higher leakthrough effect than would otherwise be obtained, as both sets of diodes enter region 2 when the applied voltage across the load is greater than 2ϕ . We might further conjecture that the dominant leakthrough/frequency relationship is determined by the capacitive balance shifting with frequency. Regarding capacitive effects as simply unbalancing the diodes is a common assumption [ref 75]. In equation 5.17 this would make t_s a function of T .

The approximation that the current pulses are described by an exponential is not significant, see Figure 5.4.7.

Figure 5.4.8 shows the experimental evidence, and Figure 5.4.9 and Figure 5.4.10 the experimental details. A 10-15 dB improvement in suppression results, over and above that which is otherwise achieved.

The significance of the method is that it gives suppression

which is additive to the nulling method and thus can be seen as a way of improving the suppression provided by nulling. Using this method LO leakthrough at 80 MHz was routinely kept to 75-80 dB. That degree of suppression could not be maintained for any length of time using nulling alone. The method may also be used alone, as an alternative to nulling.

5.4.4 Cascading Modulators

Figure 5.4.11 shows the method. We cascade two modulators, so that the output is given by

$$\begin{aligned} & A(t)(A(t)\cos(\omega t) + a\cos(\omega t + \theta)) \\ & + a(A(t)\cos(\omega t + \theta) + a\cos(\omega t + 2\theta)) \end{aligned} \quad (5.25)$$

$$= A^2(t)\cos(\omega t) + 2aA(t)\cos(\omega t + \theta) + a^2\cos(\omega t + 2\theta) \quad (5.26)$$

With the exception of the a^2 term all other terms have an amplitude controllable by the IF input. Thus, if the leakthrough is -40 dB for a single modulator, for the cascade the leakthrough is -80 dB.

The technique is broadband, and can obviously be extended to more than two modulators. The prime drawback is that the modulator is no longer linear but a square law device (output envelope related to the square of the IF input voltage). Thus, to obtain the same envelope as before, the NLFG output which controls the IF voltage must be the square root of the single modulator voltage. This extends the bandwidth required for the IF signal and thus we must either increase the D/A sample rate or decrease the power of n which describes the NLFG.

Provided the modulator characteristics are carefully chosen, this problem can at least be minimised (see Figure 5.4.11). Reference 82 discusses how pin diode modulators may be

made to give linear control.

In essence, for pin diode modulators, the logarithm of the RF attenuation is linear with the logarithm of the IF current [ref 73, 82]. By means of a compensating diode (which has an exponential voltage/current relationship), it is possible to force the RF attenuation to be linear with IF voltage. The technique requires temperature compensation.

5.5 CONCLUSIONS

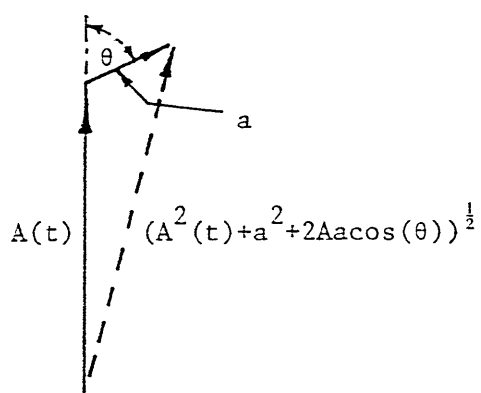
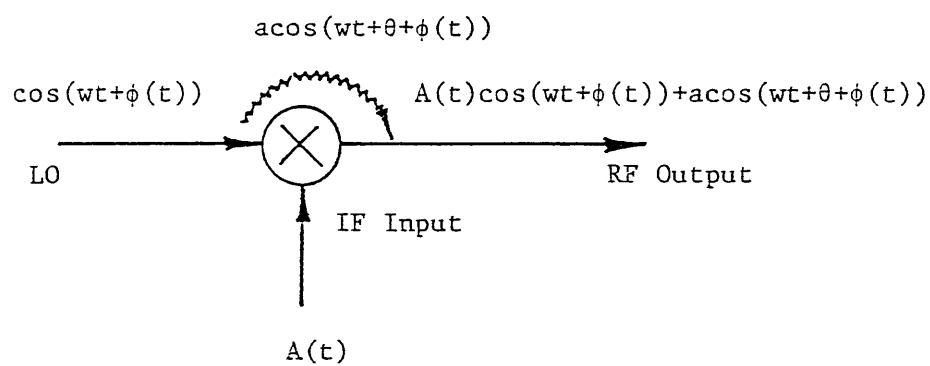
For polar AM systems the performance of digital compensation may be limited by the local oscillator leakthrough of the envelope modulator. Although diode modulators have been primarily examined (schottky and pin), the results should hold true for other modulator types.

As a first step in suppressing leakthrough the choice of a low IF frequency is important. If this is not possible, is undesirable, or does not provide the required suppression, then there are three possibilities:

1. Nulling may be used to obtain a further 20 dB suppression. The frequency and temperature range over which nulling may work are limited.
2. Control over LO waveshape appears to give 10-15 dB suppression, and does not appear to be sensitive to environment. If this 10-15 dB improvement is not enough it is possible to add a nulling network. The effect is additive with nulling, so that a 30-35 dB suppression which is frequency and temperature sensitive is produced.
3. Cascading modulators offers broadband, temperature

insensitive, suppression of high degree (e.g. 80 dB at VHF). However, it produces a nonlinear modulator which forces a compromise on the degree of the polynomial the NLFG can approximate, or demands an increase in the D/A sample rate. Nevertheless, by careful modulator design these problems can be minimised.

FIGURE 5.2.1



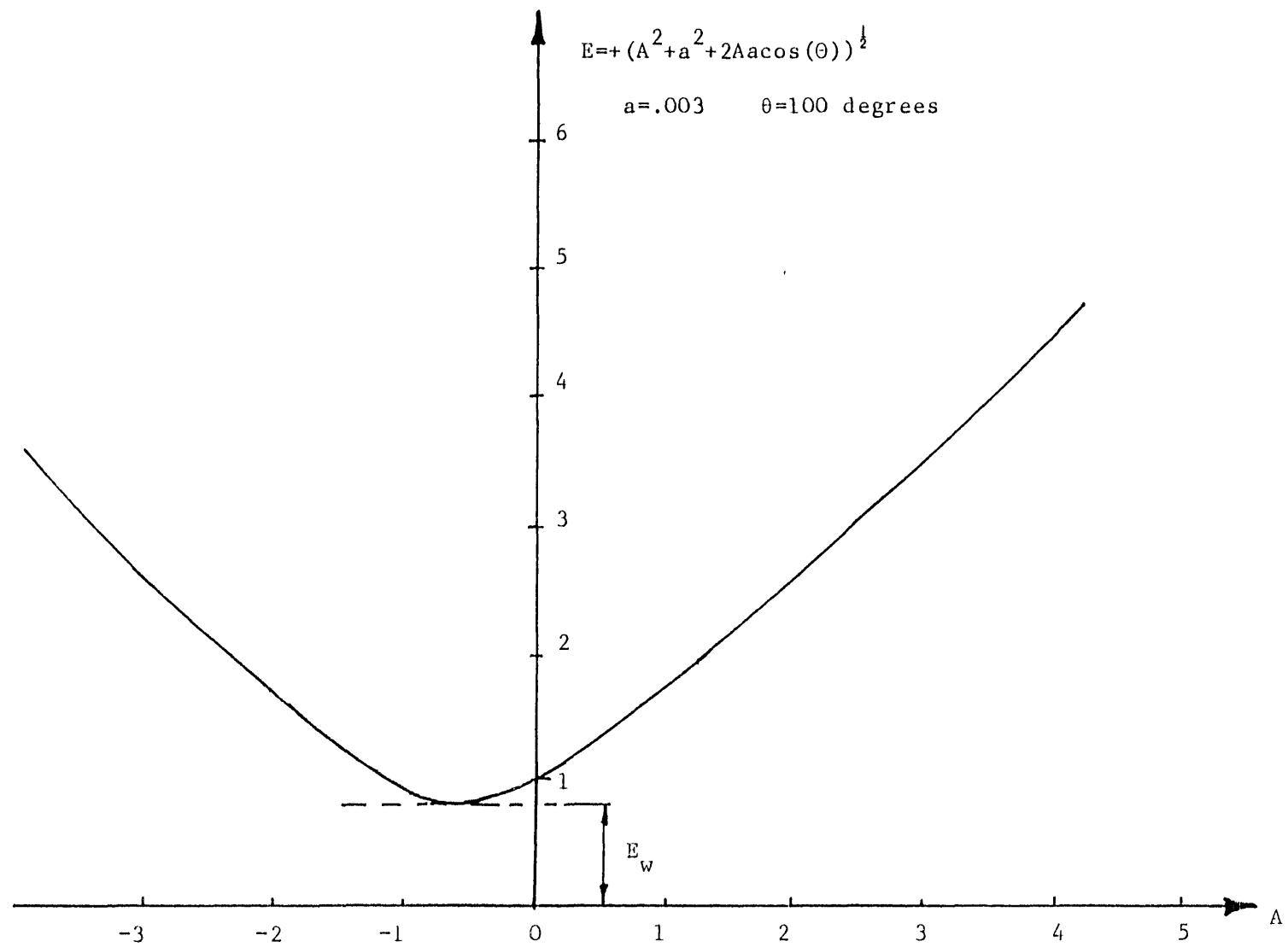


FIGURE 5.2.2

FIGURE 5.3.1

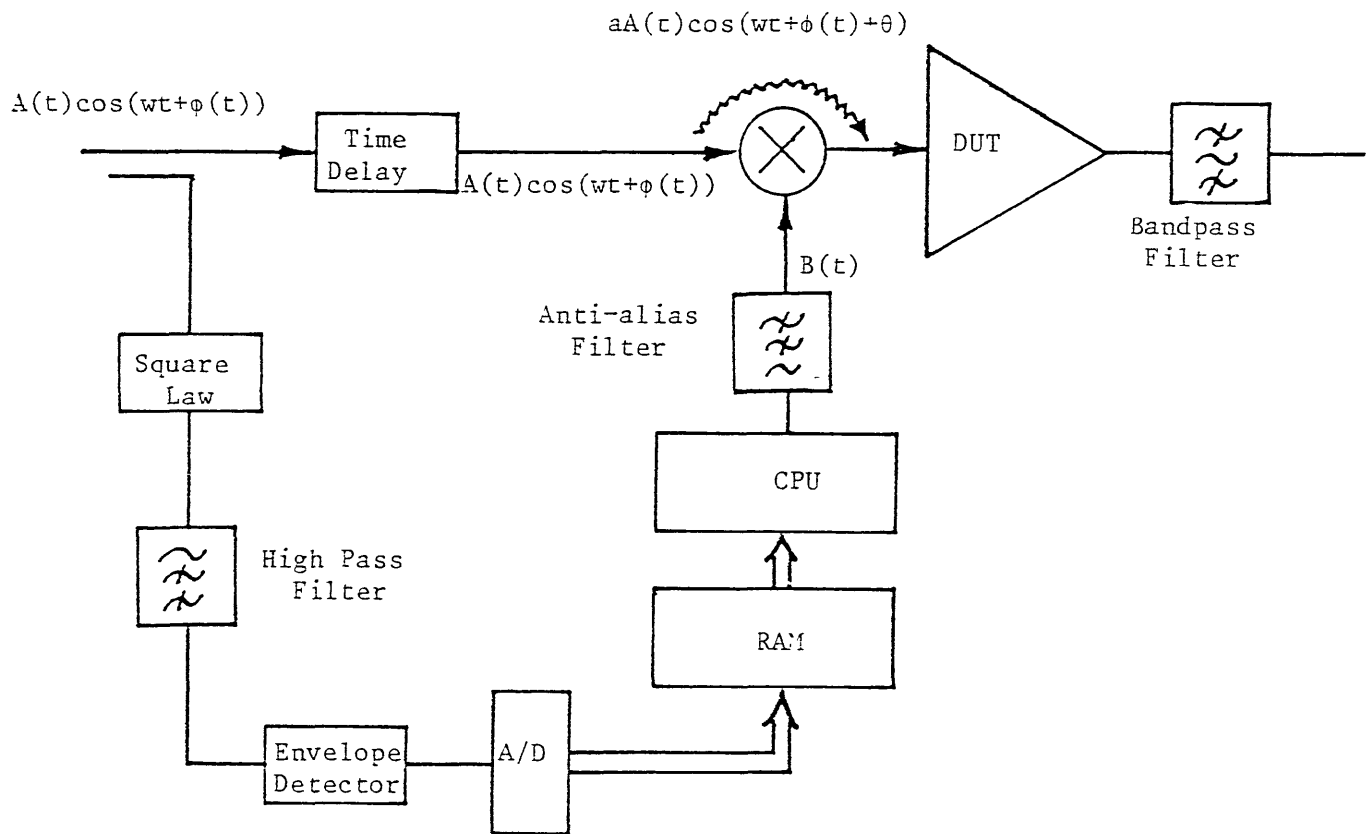
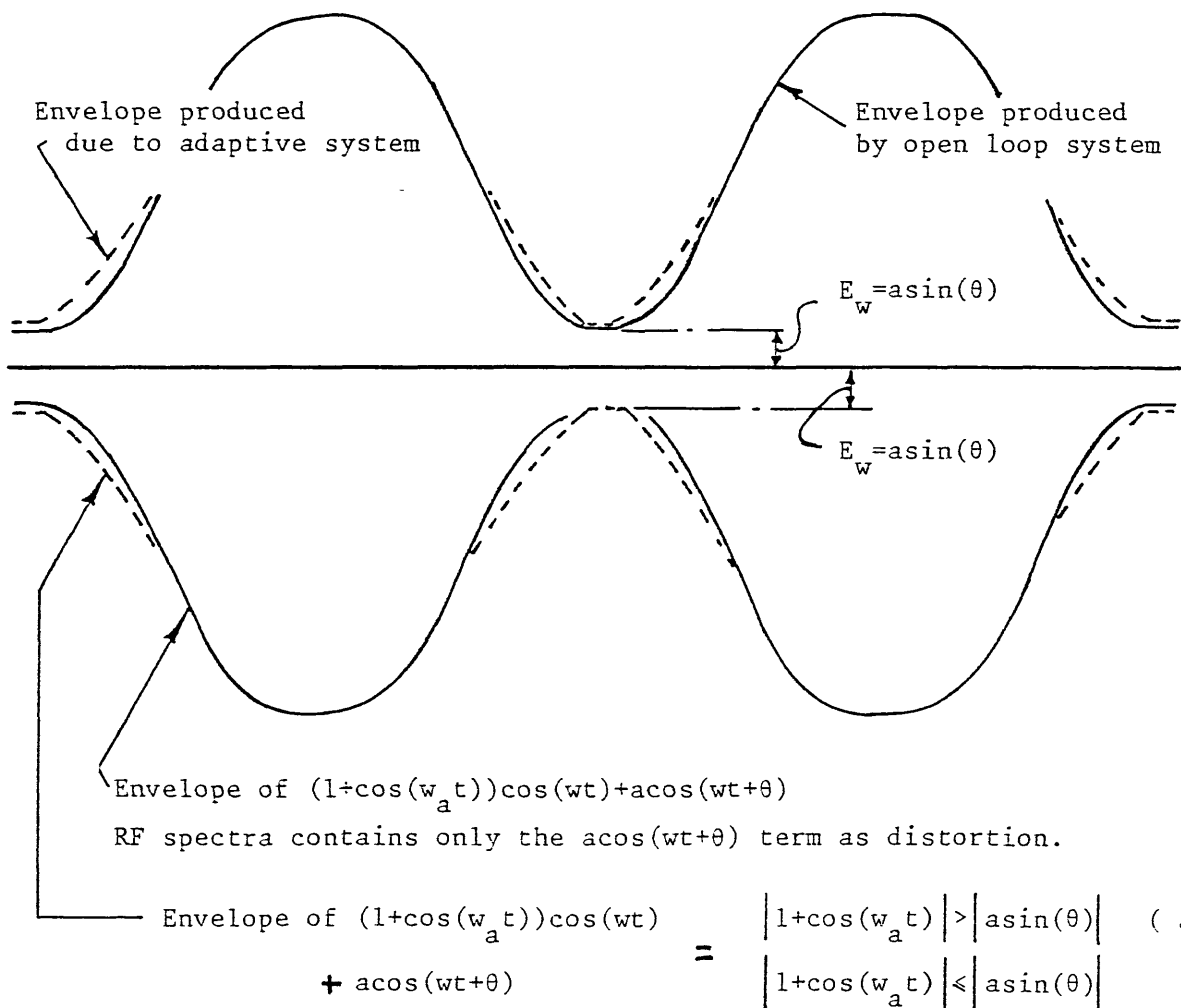


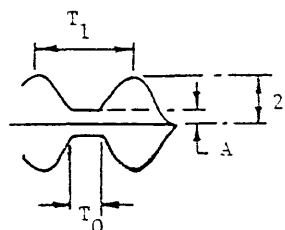
FIGURE 5.3.2

RF envelope for 100% AM with LO leakthrough



RF spectra contains high order distortion.

FIGURE 5.3.3



Reference 51 gives the fourier series for a clipped sinusoid as $\sum a_m \cos(m\omega t)$

$$a_m = \frac{2A}{m} T \left((\text{sinc}((m-1)T) + (\text{sinc}(m+1)T)) - 2\cos\pi T \text{sinc} mT \right)$$

$$T = T_0 / T_1$$

This immediately gives the magnitude of the m^{th} harmonic term due to the distortion relative to the modulating sideband. For 100% AM the peak envelope power is 12 dB above the sideband level.

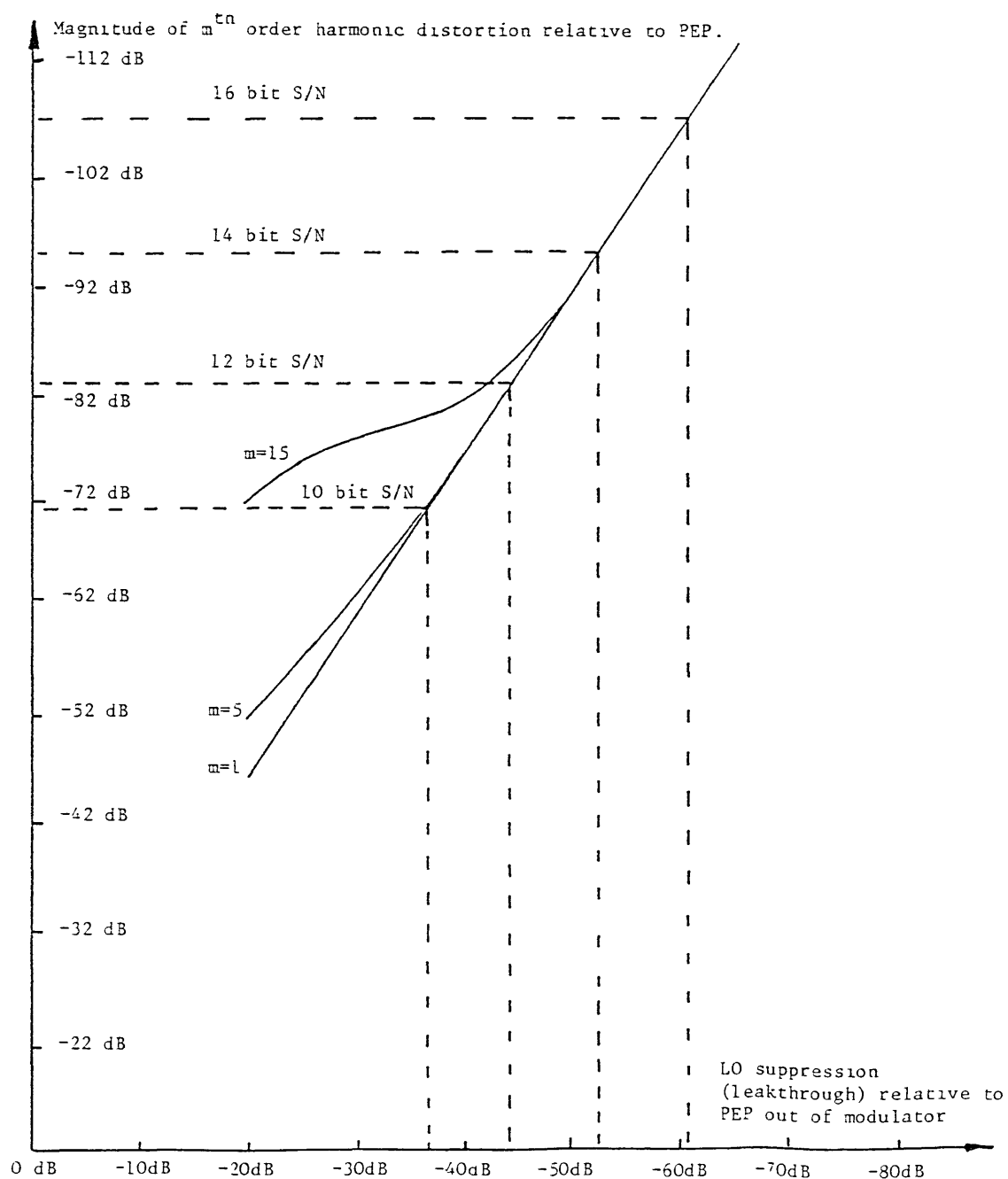


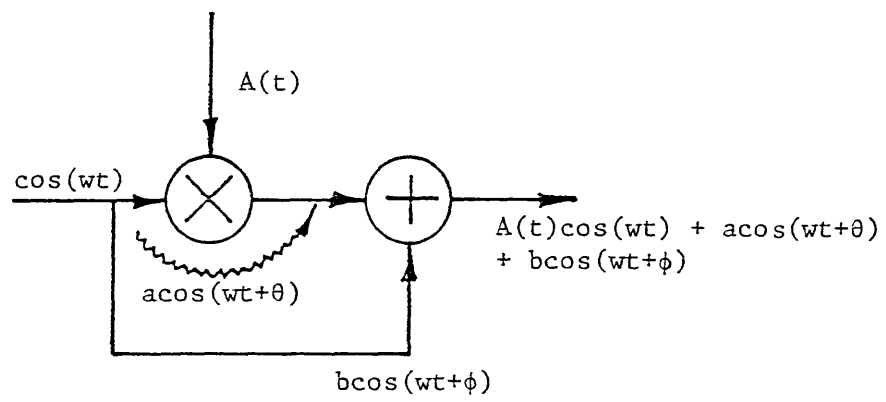
FIGURE 5.4.1

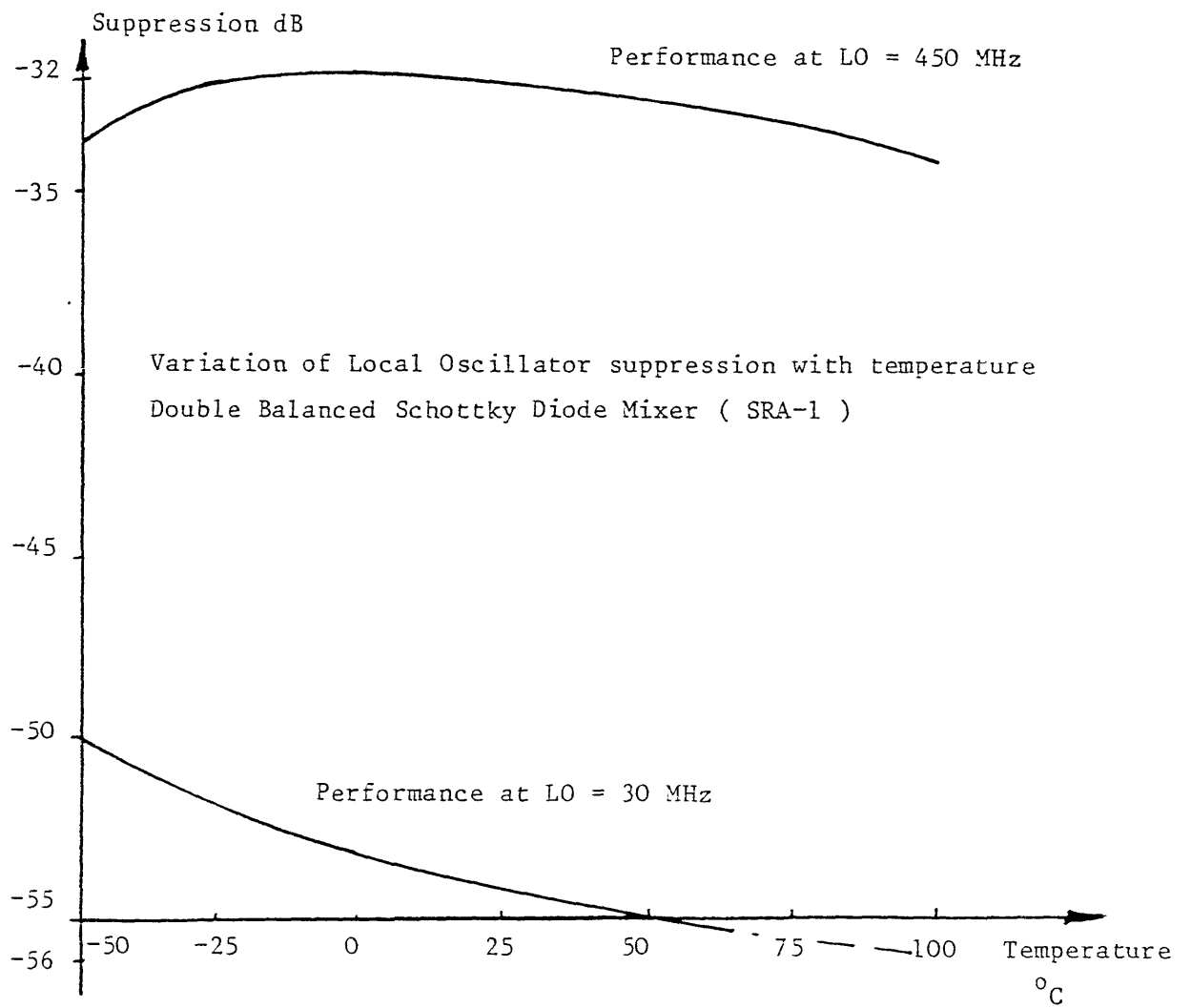
FIGURE 5.4.2

FIGURE 5.4.3

Addition of two anti-phase sine waves: effect of amplitude and phase error

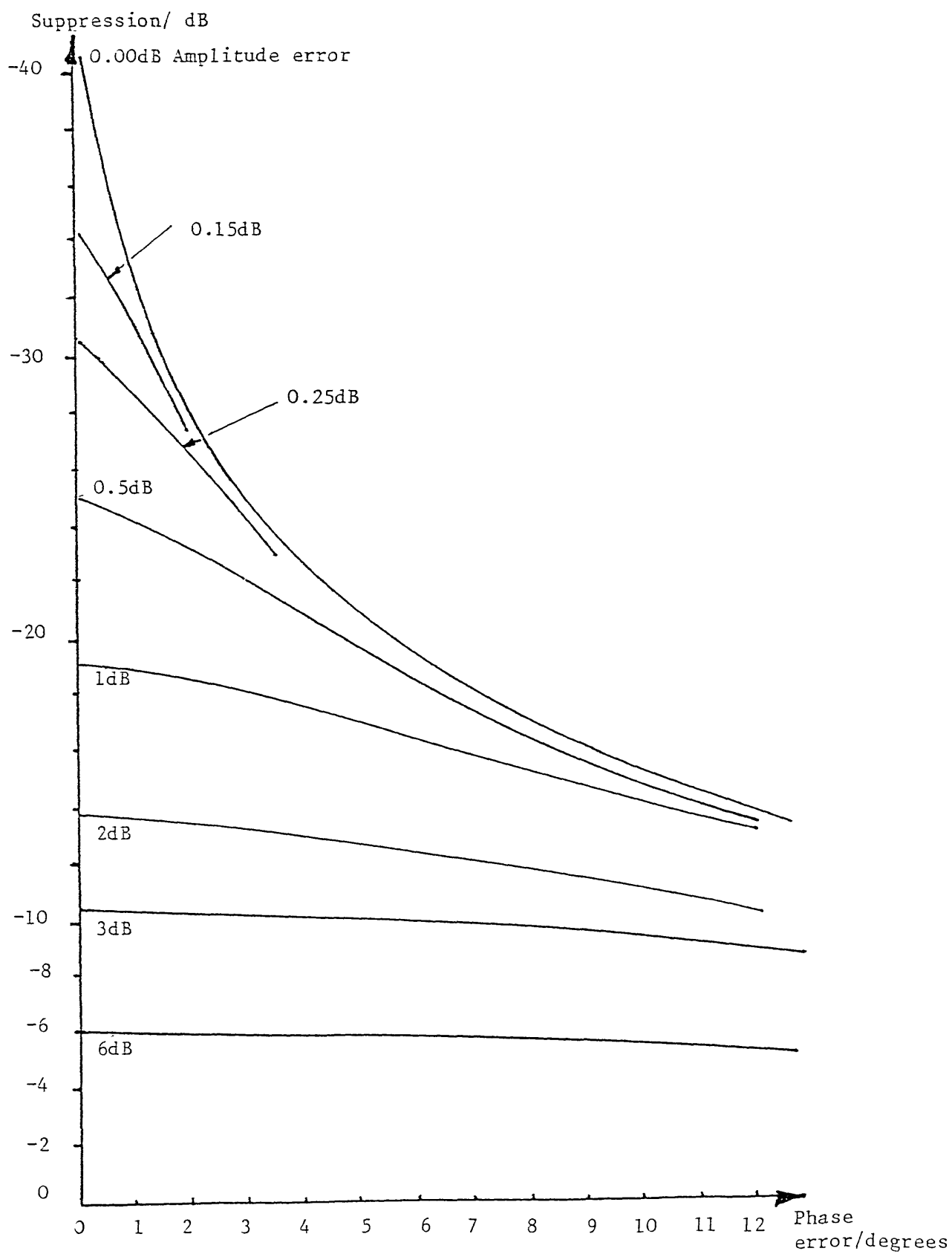
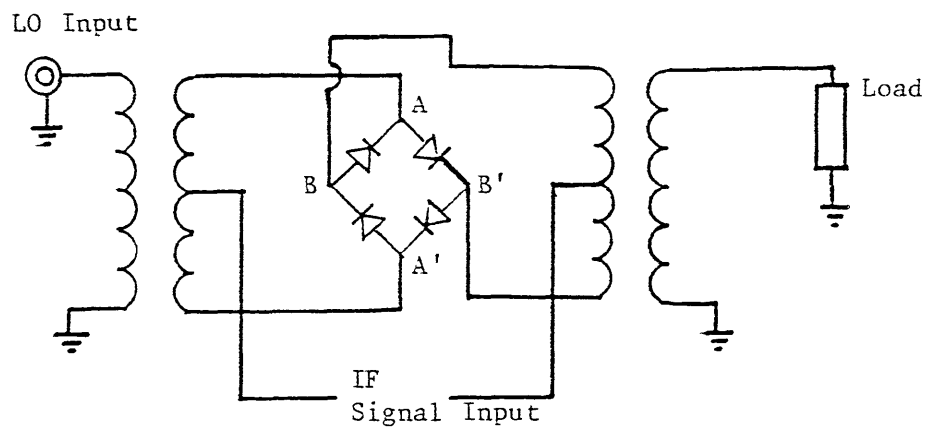
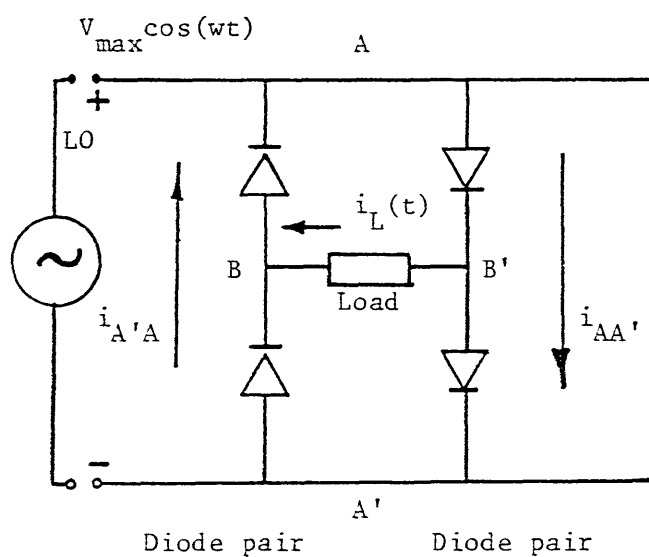


FIGURE 5.4.4

Standard Double Balanced Diode Mixer



Simplified Double Balanced Mixer



dc levels are set by the IF input signal.

Maximum current flows in the load when one diode pair is leaving Region 1 to Region 2, and the other pair are leaving Region 3 to Region 2. When both diode pairs are in Region 2 no current flows in the load, by reason of balance.

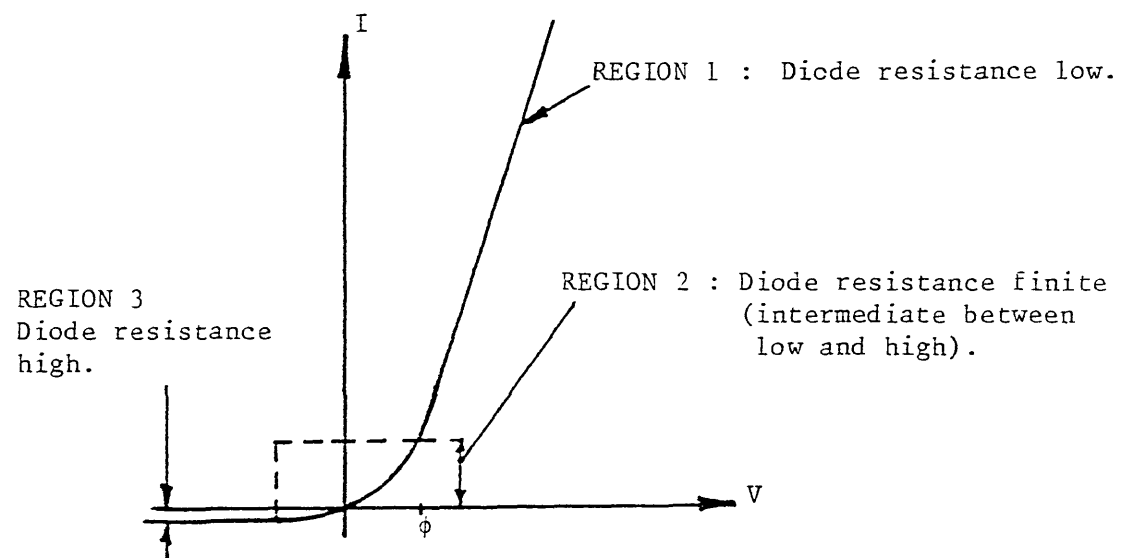
FIGURE 5.4.5

FIGURE 5.4.6

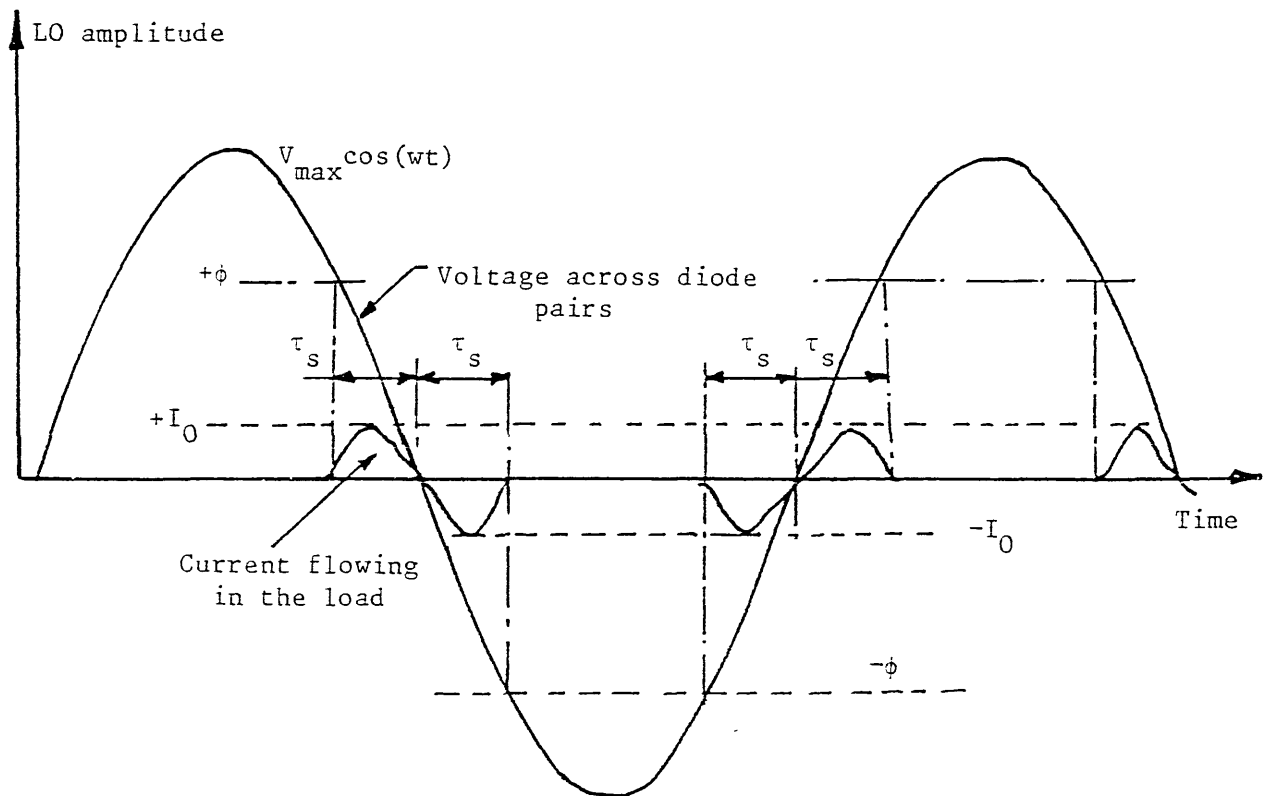
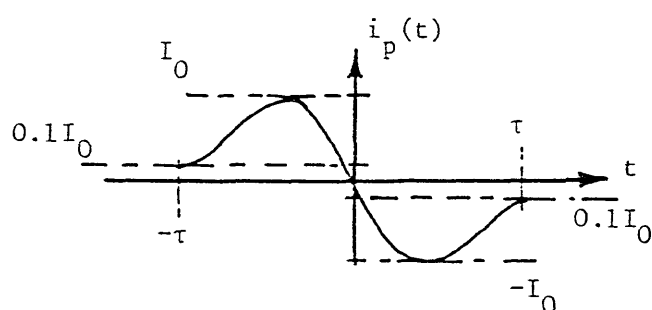


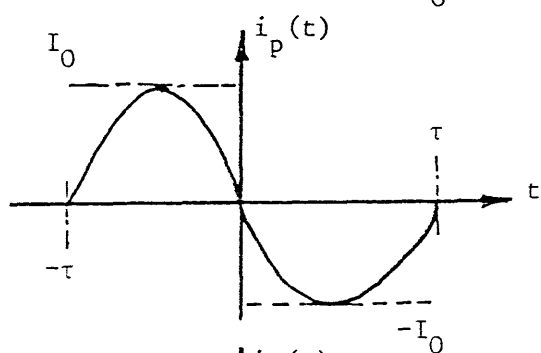
FIGURE 5.4.7

All of the current/pulse waveforms below give rise to the same conclusions over LO leakthrough. The only assumption necessary is that the current waveform about the origin is linear.



Function

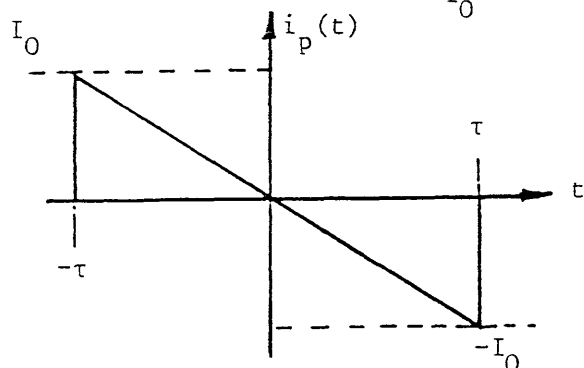
$$i_p(t) = -2I_0(t/\tau) \exp(-\pi(t/\tau)^2)$$



Function

$$i_p(t) = -I_0 \sin(\pi t/\tau) \quad |t| < \tau$$

$$= 0 \quad \text{otherwise}$$



Function

$$i_p(t) = -I_0(t/\tau) \quad |t| < \tau$$

$$= 0 \quad \text{otherwise}$$

FIGURE 5.4.8

Experimental results on LO feedthrough on a SRA-1 diode mixer at 80 MHz

LO waveshaping gives a 10 dB improvement when used on its own
and a 14 dB improvement when used in conjunction with nulling.

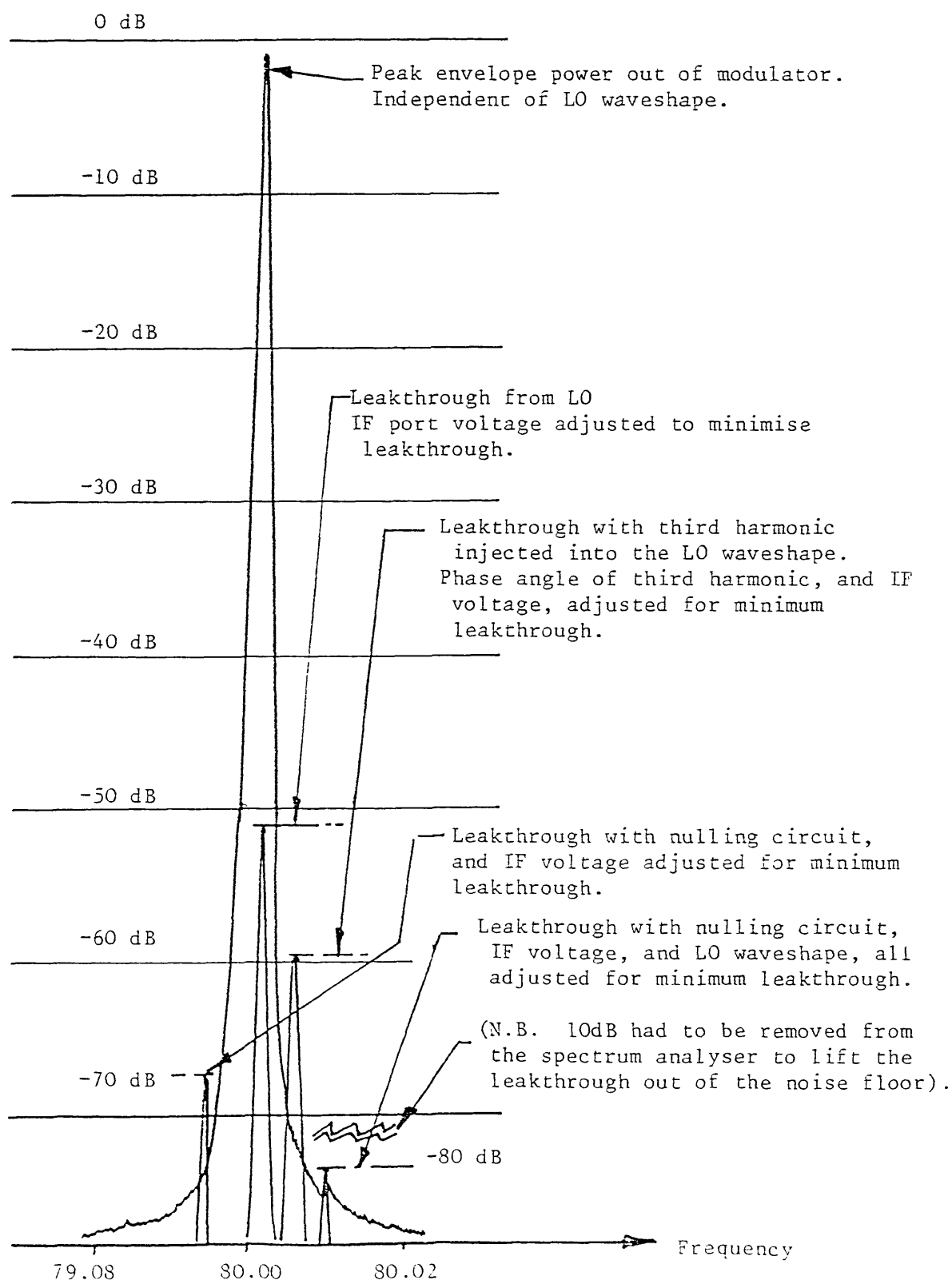
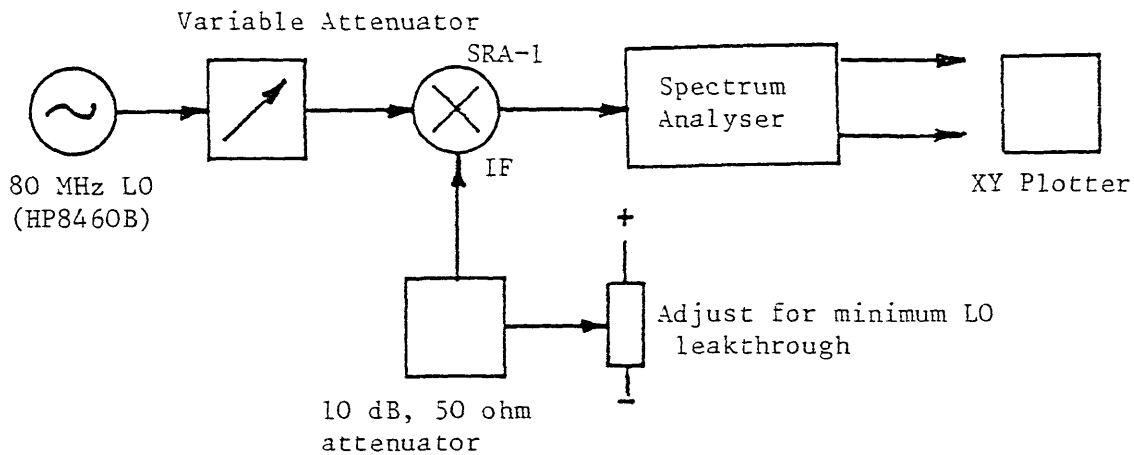


FIGURE 5.4.9

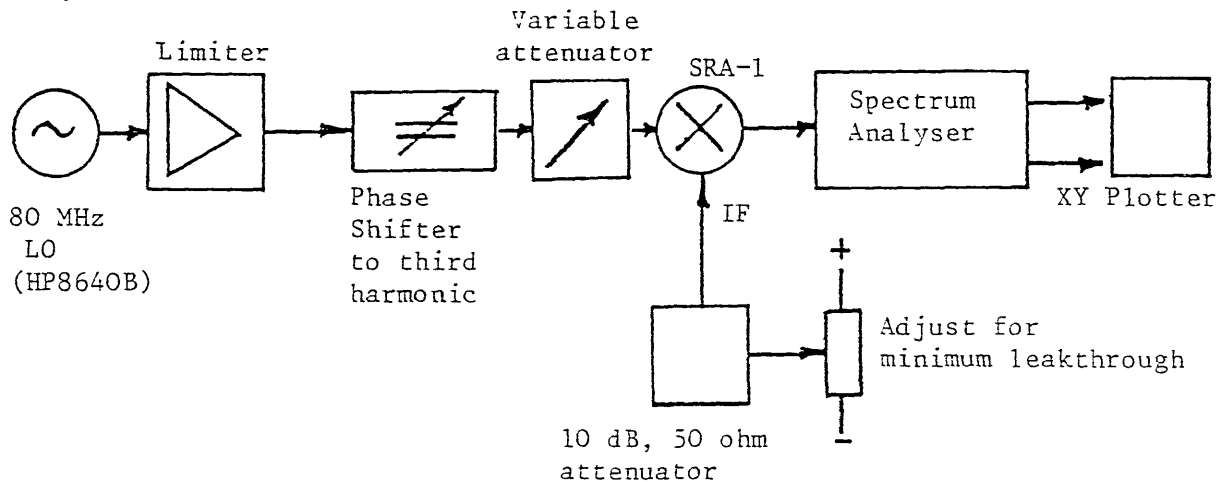
Experiment 1



The spectrum analyser is operated with 10dB input attenuation to provide a broadband 50 ohm match to the mixer output.

Peak signal power out of mixer is recorded, and the IF voltage adjusted for minimum leakthrough, and the mixer output recorded.

Experiment 2



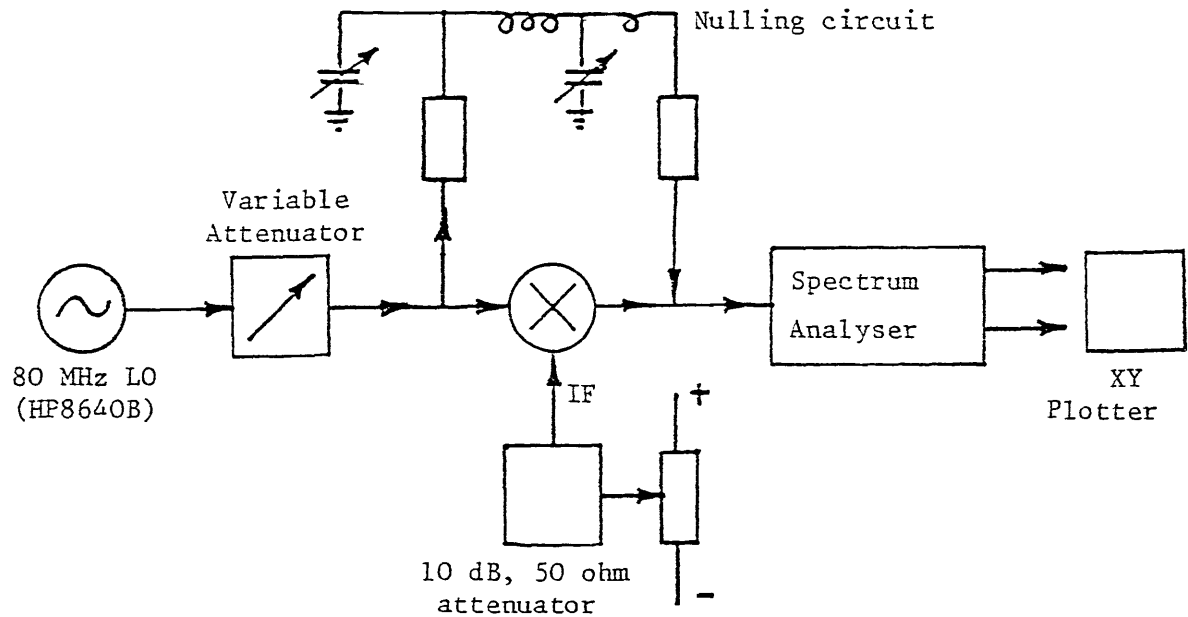
The variable attenuator is adjusted to give the same maximum PEP for the same maximum IF voltage as in Experiment 1.

The phase shifter (which acts primarily to filter out higher harmonics than third, and to phase shift the third harmonic), and IF voltage are both adjusted for minimum leakthrough and the results recorded.

The limiter consisted of a dual gate fet, operated in saturation to change the sinusoidal drive to square wave output. The limiter had significant power gain at 300 MHz. The spectrum analyser was used to confirm that the third harmonic out of the attenuator was approximately 10 dB down on fundamental, as would be expected.

FIGURE 5.4.10

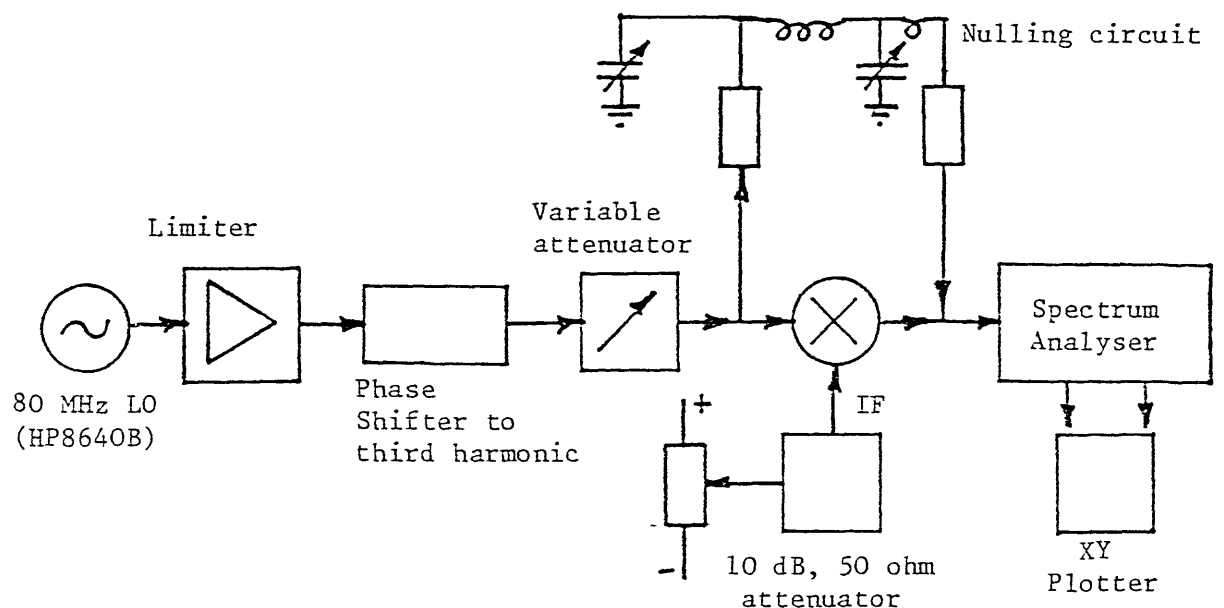
Experiment 3



Variable attenuator adjusted to give same PEP for same IF voltage as Experiment 1.

Nulling circuit and IF voltage adjusted to minimise the leakthrough, and the results recorded.

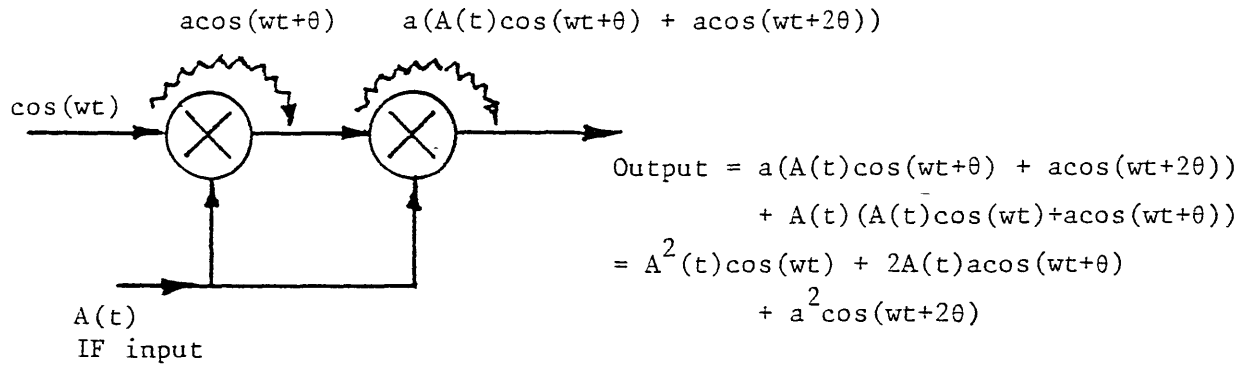
Experiment 4



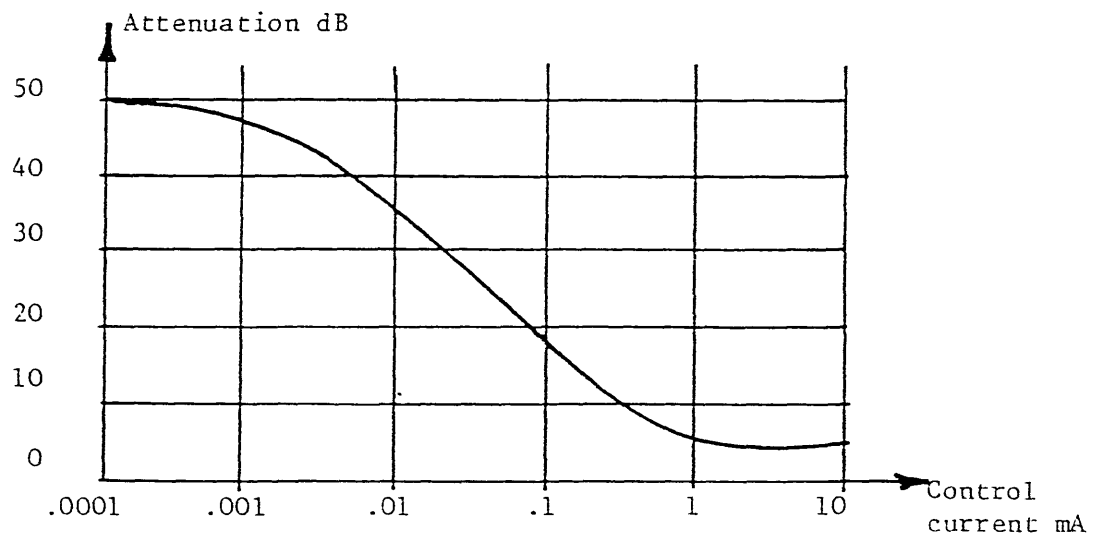
Variable attenuator adjusted to give same PEP as Experiment 1, for the same IF voltage.

All variables, (phase shifter, IF voltage, nulling circuit) adjusted for minimum leakthrough and the results recorded.

FIGURE 5.4.11



Characteristics of PAS 3 PIN diode modulator (see Ref 73)



The disadvantage of the cascaded modulator is the nonlinear modulation characteristic. Reference 82 shows how the problem might be minimised for PIN diode modulators (typical characteristics shown above).

Over the bulk of the control range, the attenuation on a logarithmic scale, is linear with the current on a logarithmic scale. By means of a compensation diode in the IF path (a diode has exponential current to voltage relationship) the log of the attenuation can be made linear with the control voltage. Trivially, by scaling the input voltage, the attenuation can be made proportional to the square of the input voltage as required. The method needs temperature compensation.

CHAPTER 6

SEPARATION OF AM/AM AND AM/PM

6.1 INTRODUCTION

Before considering the envelope and phase detectors necessary for digital compensation, we consider how AM/AM and AM/PM effects may be distinguished from each other.⁺ It is often essential to be able to quantify and separate AM/AM and the AM/PM content of the RF spectrum. This is because in the polar representation different circuitry (NLFG's, detectors, etc) are associated with the different distortion mechanisms. Thus, if any experimental understanding of the limiting factors in the system is to be obtained, the effects must be separated. As the RF spectrum is generally given as a power spectrum (via a spectrum analyser), it is clear that special techniques will need to be applied.

The requirement to separate the AM/AM and AM/PM effects does not normally arise in predistortion, feedforward or feedback systems, since the circuitry does not distinguish between the distortion mechanisms. Polar loop feedback could clearly benefit from separating the effects, but nothing has been published on experimental polar loop techniques.

A survey of the literature establishes that a reasonable approach is as follows:

1. Evaluate the characteristics of the amplifier (whether it be by direct measurement of AM/AM, AM/PM characteristics or of parameters in a nonlinear model).
2. Calculate the separate spectral effects of AM/AM and AM/PM using the model.

⁺ In order to make empirical measurements using a spectrum analyser meaningful, distortion effects have to be separated by some means, so that their contribution to the power spectrum can be assessed.

Step 1 can vary depending on the model used: Volterra theory [ref 86, 102, 111, 143], Wiener theory [ref 149], piecewise harmonic balance [ref 63], state variable methods [ref 124], Fourier methods [ref 144], and multi-tone tests [ref 4, 91, 109]. References 99, 115 and 175 concentrate on direct measurement of the AM/AM or AM/PM characteristic.

Step 2 can vary from model to model, but it is worth noting Shimbo's work [ref 155] relating the spectrum directly to the AM/AM and AM/PM characteristics, and Stette's method of calculation [ref 164] along with purpose-written computer programs capable of the calculation [ref 49].

Unfortunately, the approach of inferring the separate AM/AM and AM/PM content of a spectrum lacks experimental utility, since the need is for an instant measure of the worth of a particular circuit variation. Furthermore, for the accuracy that will be required, some of the measurements are non-trivial in metrology.

Consequently, a set of techniques have been developed (collectively referred to as quasi-linear techniques), which are specifically designed for practical application with digital compensation.

6.2 QUASI-LINEAR METHODS

For the purposes of our analysis the test apparatus is assumed to be that of Figure 6.2.1. (In point of fact the actual test rig used the coarse/fine D/A technique of chapter 4, but this is a somewhat minor modification of quasi-linear which we ignore for the purposes of analysis).

Quasi-linear methods allow us to control the power

spectrum, by iterative modifications to the RAM tables. We shall show that, provided certain conditions are met, if we force the power spectrum to resemble that of an AM RF signal, we can remove either AM/AM or AM/PM effects at will, to show the AM/PM or AM/AM distortion of the DUT. As a by-product of this methodology, the envelope RAM table contains the inverse of the DUT AM/AM characteristics, and the phase angles of individual IMP's are determined.

6.2.1 Quasi-linear Envelope Control

Consider the setup as in Figure 6.2.2. The audio input to the A/D is described by

$$V_{in}(t) = a + b\cos(w_at) \quad (6.1)$$

and the envelope ($E(t)$) of the RF signal is proportional to

$$V_{in}(t) + x\cos(mw_at) + y\sin(mw_at) + \text{other terms} \quad (6.2)$$

Substituting for $V_{in}(t)$ from equation 6.1 we get that $E(t)$ is proportional to

$$a + b\cos(w_at) + x\cos(mw_at) + y\sin(mw_at) + \text{other terms} \quad (6.3)$$

We have little interest in the other terms, in that they do not involve the m^{th} harmonic of the audio input w_a . (Note here that we have not imposed any conditions on the RAM table, the distortion terms may be due either to the RAM table or to the DUT. Nor do we take much interest in the A/D, D/A, or modulator gains, which set the proportionality constant in equation 6.3). We denote the current state of the RAM table by $d_E(A)$ where A represents the RAM address and $d_E(A)$ the data found at that address.

We now reason that the m^{th} harmonic distortion would be

reduced, for a system that is approximately linear, if the input signal to the A/D, $V_{in}(t)$, were transformed by

$$\begin{aligned} V_{in}(t) &\rightarrow V_{in}(t) - x\cos(mw_at) \\ &= a + b\cos(w_at) - x\cos(mw_at) \end{aligned} \quad (6.4)$$

and all other parameters (RAM, A/D, D/A, modulator gains) remained constant. This is the quasi-linear approximation for the envelope.

We now describe two different situations which result in an identical set of numbers from the envelope RAM table of Figure 6.2.1.

Situation 1

The input signal is given by equation 6.4 while the RAM may be described by $d_E(A)$. The output of the RAM would be described by

$$d_E(g_A(a + b\cos(w_at) - x\cos(mw_at))) \quad (6.5)$$

where g_A is the A/D gain in bits/volt.

Situation 2

The input signal is given by equation 6.1 and the RAM table modified to d'_E where

$$d'_E(g_A V_{in}(t)) = d_E(g_A(a + b\cos(w_at) - x\cos(mw_at))) \quad (6.6)$$

We can express equation 6.6 more compactly by putting

$$A = g_A(a + b\cos(w_at)) \quad (6.7)$$

$$\text{i.e.} \quad \cos(w_at) = (A - ag_A)/bg_A \quad (6.8)$$

$$\text{and} \quad \cos(mw_at) = T_m((A - ag_A)/bg_A) \quad (6.9)$$

where $T_m(x)$ is a Chebyshev polynomial of the first kind. The properties of Chebyshev polynomials are given in references 1, 14

and 54. They can be defined with different constants of proportionality. This thesis uses the same constants as Abramowitz and Stegun [ref 1]. A list of the relevant properties of Chebyshev polynomials may be found in Appendix B. Hence equation 6.6 may be written

$$d'_E(A) = d_E(A - xg_A T_m((A - ag_A)/bg_A)) \quad (6.10)$$

Comment

In either of the two situations outlined above the output of the RAM table is the same. We chose to implement changes using equation 6.10 to "reshuffle" the RAM table as follows:

1. The CPU records the maximum and the minimum values given by the A/D. For an analogue input $V_{in}(t)=a+b\cos(w_at)$ the maximum value is given by $(a+b)g_A$ and the minimum value by $(a-b)g_A$. Clearly then ag_A and bg_A are found for equation 6.10. We never, in fact, need to know g_A .
2. From the spectrum analyser we measure the ratio of the m^{th} distortion term to the fundamental term, i.e. we determine an approximate value for (x/b) , but since bg_A is known from above we obtain an approximate value for xg_A in equation 6.10.
3. Provided the CPU knows a value for m , it can reshuffle the RAM table for all values of A in equation 6.10, applying linear interpolation between points in the envelope RAM table where necessary.

We can only find an approximate value for (x/b) since we cannot separate the in-phase from the quadrature term, i.e. we can only measure the ratio

$$(x^2 + y^2)^{0.5}/b \quad (6.11)$$

Worse still, we do not know from the analyser the sign of x .

Neither of these two problems prove insurmountable in practice. We rely on the following:

1. The wrong choice of sign will increase the distortion levels, whereas the correct choice of sign will decrease the distortion levels. See Figure 6.2.3.
2. Where there is no difference between the distortion levels obtained with one choice of sign, and with the other, the distortion is a pure quadrature signal ($x=0$, $y \neq 0$). See Figure 6.2.3.
3. We can iteratively use equation 6.10 as many times as we like. Consequently, over a number of iterations, we can reduce the in-phase component of the m^{th} harmonic to zero, even though we never know the exact value of the ratio x/b at any iteration.

Our action then is to guess the sign, and to approximate (x/b) by

$$(x^2 + y^2)^{0.5}/b \quad (6.12)$$

By observing the change in spectra brought about by this value of (x/b) in equation 6.10 we can deduce the required sign, and iteratively force the in-phase component to zero.

We repeat this course of events for all values of $m > 1$ which cause distortion of sufficient magnitude to be of concern, until the remaining distortion spectra is all quadrature.

The quadrature terms cannot be produced by AM modulation and are clearly due to AM/PM effects.

6.2.2 Quasi-linear Phase Control

Although we keep within the definition of quasi-linear (control of the power spectrum by iterative modifications to the RAM), we have to derive a new set of equations. The setup is as in Figure 6.2.4.

The general form of a distorted output for a NBNL device in Figure 6.2.4 is

$$V_{out}(t) = f(A')\cos(\omega t + g(A') + hp(A)) \\ = f(A')[\cos(g(A') + hp(A))\cos(\omega t) - \sin(g(A') + hp(A))\sin(\omega t)] \quad (6.13)$$

We now assume that $(g(A') + hp(A))$ is small so that we may approximate $V_{out}(t)$ by

$$f(A')\cos(\omega t) - f(A')(g(A') + hp(A))\sin(\omega t) \quad (6.14)$$

This is the quasi-linear assumption for phase. Provided the assumption holds true, we determine that the quadrature terms represent the totality of effects of AM/PM (i.e. AM/PM conversion only causes quadrature effects, and is the only cause of quadrature effects).

With this assumption we can consider the RF spectrum which contains only AM/PM distortion (i.e. envelope quasi-linear has been applied to remove all in-phase distortion). Hence the RF output from Figure 6.2.4 is

$$V_{out}(t) = f(A')\cos(\omega t) - f(A')(hp(A) + g(A'))\sin(\omega t) \quad (6.15)$$

but $f(A')=A(t)$ since we are free from AM/AM conversion. Hence

$$f(A') = A(t) = a + b\cos(\omega_a t) \quad (6.16)$$

$$\text{and } V_{out}(t) = A(t)\cos(\omega t) - A(t)(hp(A) + g(A'))\sin(\omega t) \quad (6.17)$$

Consider the case where the RF power spectra is described by Figure 6.2.5 which only has second harmonic distortion of the audio, caused by the modulating term of the quadrature RF in

equation 6.17. Consider what would happen if $hp(A)$ is transformed to $hp'(A)$ where

$$hp'(A) = hp(A) + hk\cos(w_a t) \quad (6.18)$$

in the modulation of the quadrature term of equation 6.17 (k is a constant to be determined). Combining equation 6.18 with equation 6.17 we see that $-A(t)(hp(A) + g(A'))$ becomes

$$-A(t)(hp(A) + hk\cos(w_a t) + g(A')) \quad (6.19)$$

Clearly we introduce a new distortion term given by

$$\begin{aligned} & -A(t)hk\cos(w_a t) \\ & = -(a + b\cos(w_a t))hk\cos(w_a t) \\ & = -ahk\cos(w_a t) - (bhk/2)(1 + \cos(2w_a t)) \end{aligned} \quad (6.20)$$

We would like this new distortion term to be in anti-phase with the second harmonic distortion. Thus we would like to choose k such that

$$y\cos(2w_a t) = ahk\cos(w_a t) + (bhk/2)(1 + \cos(2w_a t)) \quad (6.21)$$

There is no solution to equation 6.21 in general, so we choose instead to equate the powers of w_a , and hence choose k such that

$$y\cos(2w_a t) = (bhk/2)\cos(2w_a t)$$

$$\text{i.e.} \quad k = 2y/hb \quad (6.22)$$

The ratio (y/b) is given directly by the power spectrum (see Figure 6.2.5). Provided the approximate value of h is known a priori, we can deduce k . The change in the RAM indicated by the transformation of $hp(A)$ in equation 6.18 is implemented by reshuffling the phase RAM ($d_p(A)$). Hence we transform $d_p(A)$ to $d_p'(A)$ where

$$d_p'(A) = d_p(A + g_A k T_1 [(A - ag_A)/bg_A]) \quad (6.23)$$

Hence the procedure would be:

1. Find ag_A and bg_A as in quasi-linear for envelope.

2. Calculate k for the second harmonic by equation 6.22 knowing the approximate value of h .
3. Knowing the approximate value of g_A , apply equation 6.23 to obtain a new RAM table. (An approximate value for g_A can always be found using common laboratory test equipment).

As before we do not know the sign of the value of k , but as before this is not difficult in practice, since both signs can be tried and the correct one quickly determined.

We can extend the method to take account of any high order (m^{th}) harmonic, by modifying equation 6.18 to include terms up to $(m-1)$. Thus for the third harmonic we consider the transformation of $hp(A)$ to $hp'(A)$ where

$$hp'(A) = hp(A) + hk_1 \cos(w_a t) + hk_2 \cos(2w_a t) \quad (6.24)$$

causing an extra modulation of the quadrature term of

$$- h(a + b \cos(w_a t))(k_1 \cos(w_a t) + k_2 \cos(2w_a t)) \quad (6.25)$$

Equating equation 6.25 with the distortion $y \cos(3w_a t)$ and equating second and third harmonics to zero gives

$$k_1 = -4ya/hb^2 \quad (6.26)$$

$$k_2 = 2y/hb \quad (6.27)$$

leaving us to reshuffle the phase RAM by transforming $d_p(A)$ to

$$d_p(A + g_A k_1 T_1 [(A - g_A a)/g_A b] + g_A k_2 T_2 [(A - g_A a)/g_A b]) \quad (6.28)$$

The derivation of the equations for higher order distortion than third is straightforward though tedious. In practice there has been little need to consider higher than third harmonics for phase.

Quasi-linear for the phase always leaves a residual quadrature term which acts on the zeroth and first harmonics of the audio sidebands. However, phase quasi-linear does remove the

AM/PM effects which produce higher sidebands, and so it fulfils its task in allowing us to remove and isolate AM/PM effects in the higher sidebands, ($m > 1$).

Since phase quasi-linear is iterative there is no need to have precise knowledge of h and g_A , as any errors introduced by an imprecise knowledge may be reduced at the next iteration.

6.3 ASSUMPTIONS OF QUASI-LINEAR APPROXIMATIONS

We look briefly at the assumptions of quasi-linear, to determine under what circumstances they hold.

6.3.1 Envelope Quasi-linear

For envelope quasi-linear the input to the A/D was described by

$$V_{in}(t) = a + b \cos(\omega_a t) \quad (6.29)$$

It is convenient to redefine the volt such that $b=1$, thus

$$V_{in}(t) = A + \cos(\omega_a t) \quad (6.30)$$

where $A=(a/b)$. Now the envelope transfer characteristics of any DUT can be described by

$$E(t) = \sum_n c_n (\langle V_{in}(t) \rangle) T_n(V_{in}(t) - \langle V_{in}(t) \rangle) \quad (6.31)$$

where $\langle \rangle$ denotes finding the average (or dc) value.

Equation 6.31 simply says that the envelope transfer function can be written for the ac term as a Chebyshev series, with coefficients determined by the dc term. Thus from equation 6.30 we have

$$E(t) = \sum_n c_n(A) T_n(\cos(\omega_a t)) \quad (6.32)$$

Now from the properties of Chebyshev polynomials we know that the magnitude of the m^{th} harmonic of $\cos(\omega_a t)$ is simply given by

$c_m(A)$. Hence when we replace $V_{in}(t)=a+b\cos(w_at)$ with

$$V_{in}(t) = a + b\cos(w_at) - x\cos(mw_at) \quad (6.33)$$

this produces a new envelope $E'(t)$ where

$$E'(t) = \sum_n c_n(A) T_n(\cos(w_at) - (x/b)T_m(\cos(w_at))) \quad (6.34)$$

We shall now expand equation 6.34 and compare coefficients with equation 6.32 to determine if the m^{th} harmonic is reduced, and under what circumstances the changes in the other harmonic terms can be neglected. Putting $X=\cos(w_at)$ for convenience we examine

$$E'(t) = \sum_n c_n(A) T_n(X - (x/b)T_m(X)) \quad (6.35)$$

Now by direct substitution into $T_0(x)$ and $T_1(x)$

$$T_0(X - (x/b)T_m(X)) = T_0(X) \quad (6.36)$$

$$\text{and} \quad T_1(X - (x/b)T_m(X)) = T_1(X) - (x/b)T_m(X) \quad (6.37)$$

For higher order we use a Taylor expansion

$$T_n(X - (x/b)T_m(X)) = T_n(X) - (x/b)T_m(X)(dT_n/dX) + O((x/b)^2) \quad (6.38)$$

We now note that $|T_m(X)| < 1$ and that $|dT_n/dX| < n^2$ (see Appendix B). Thus

$$T_m(X)(dT_n/dX) < n^2 \quad (6.39)$$

$$\text{so} \quad T_n(X - (x/b)T_m(X)) \sim T_n(X) + O((x/b)n^2) \quad (6.40)$$

Substituting these results into equation 6.35 we obtain

$$E'(t) = \sum_n [c_n(A)T_n(X)] - (x/b)T_m(X) + O(\sum_n c_n(A)(x/b)n^2) \quad (6.41)$$

From equation 6.41 we see that proper choice of (x/b) will enable us to reduce the coefficient of $c_m(A)$ and hence directly to reduce the m^{th} distortion term.

From equation 6.41 we infer the weak condition that to operate quasi-linear requires that

$$\sum_n c_n(A)(x/b)n^2 \quad (6.42)$$

converges to a finite limit. If we apply the ratio test for convergence we require

$$\lim_{n \rightarrow \infty} (c_{n+1}(A)(x/b)(n+1)^2)/(c_n(A)(x/b)n^2) < 1$$

i.e. $\lim_{n \rightarrow \infty} (c_{n+1}(A)/c_n(A)) < 1$ (6.43)

From this we conjecture the following: provided that the harmonic levels fall at a rate which satisfies equation 6.43 quasi-linear may safely be applied. It is of interest to note that reference 30 shows that the envelope harmonics will in general fall like $(1/n)^m$, provided that the envelope waveform is continuous in the first $(m-1)$ derivatives. Thus our conjecture implies that provided the envelope transfer function is continuous in at least the first derivative quasi-linear may safely be applied. Furthermore, quicker convergence may be anticipated if the transfer function is continuous in higher derivatives.

Consider for example the spectra of Figure 6.3.1. In this figure the harmonics fall faster than $(1/n)^2$, thus we would expect quasi-linear to hold, and the envelope to be controllable by quasi-linear (i.e. if we apply quasi-linear techniques we should be able to force the envelope to what we wish; if equation 6.43 did not hold this is not certain).

In practice all of the DUT's dealt with in this study satisfy equation 6.43 so that envelope quasi-linear can always be applied.

6.3.2 Phase Quasi-linear

The assumption was that we could regard the spectral effects of phase modulation as simply AM modulation of the quadrature RF signal

$$\begin{aligned} \text{i.e.} \quad & f(A')\cos(\omega t + g(A') + hp(A)) \\ = & f(A')(\cos(g(A') + hp(A))\cos(\omega t) - \sin(g(A') + hp(A))\sin(\omega t)) \end{aligned} \quad (6.44)$$

which is approximately equal to

$$f(A')(\cos(wt) - (g(A') + h(p(A))\sin(wt))) \quad (6.45)$$

Here we have assumed that

$$g(A') + hp(A) \ll 1 \quad (6.46)$$

$$\text{so that } \cos(g(A') + hp(A)) \sim 1 \quad (6.47)$$

$$\text{and } \sin(g(A') + hp(A)) \sim g(A') + hp(A) \quad (6.48)$$

If initially the phase RAM is given by $p(A)=0$ then by equation 6.46 we require the maximum phase shift due to AM/PM to be less than 57 degrees. In practice this may well be the case (see Figure 4.2.6 for example). Thereafter, as we obtain better and better estimates of $p(A)$, the sum of the two terms in equation 6.46 will diminish making the quasi-linear assumption true.

6.4 ASYMMETRIC EFFECTS

Using quasi-linear methods it is usually the case that there will remain an asymmetric spectra (as in Figure 6.6.8). Such asymmetric spectra are occasionally shown in the literature [e.g. reference 41, Figure 13]. That the asymmetry is not due to AM/AM or AM/PM effects for an input envelope such as

$$V_{in}(t) = a + b\cos(w_a t) \quad (6.49)$$

is shown in Appendix C. This asymmetry has been investigated [ref 94, 96, 151] and shown to be due to finite time delays between the AM/AM and AM/PM functions $f(A(t))$ and $g(A(t))$. For example Khadra [ref 94] shows that the difference in magnitude between the upper sideband of the n^{th} harmonic (A_n) and the lower sideband (A_{-n}) for a signal described by

$$V(t) = (1 + a\cos(w_a t + \theta))\cos(wt + b\cos(w_a t)) \quad (6.50)$$

$$\text{is } |(A_n)^2 - (A_{-n})^2| = |(4an/b)J_n(b)\sin(\theta)| \quad (6.51)$$

where $J_n(x)$ is a Bessel function [ref 1].

These asymmetric effects cannot be compensated for by digital compensation, since AM/AM and AM/PM correction can only introduce symmetrical spectra (on the AM tests used). Possible causes of asymmetry are:

1. Bias networks not presenting a low enough impedance to the envelope waveform [ref 151].
2. Any harmonics of the RF carrier (which have AM or PM modulation), undergoing a large group delay, and then being remixed back to fundamental. (A common case in interstage amplifier networks, and in reactively terminated mixers). Should the group delay be small, AM/PM conversion only results.
3. Delay of the PM modulation in any phase locked loop filters which may be present.
4. Poor synchronisation of the envelope and phase correcting NLFG's.

When asymmetric spectra are met, the cause has to be investigated and eliminated by good circuit design practice (e.g. improved bias networks, harmonic filters in interstage networks, proper matching of mixers, etc).

6.5 OTHER METHODS

One other method for separating AM/AM and AM/PM effects has been investigated, and is illustrated in Figure 6.5.1. The AM modulator, including the envelope NLFG, is enclosed by a phase locked loop arrangement, which acts to keep the output phase angle of the RF constant. By this means AM/PM conversion is removed, and whatever distortion remains is due to AM/AM, and is removed by

envelope quasi-linear methods. The phase locked loop arrangement is typical of polar loop transmitters [ref 127, 129, 130]. The method is useful provided the envelope does not swing over too large a dynamic range, when AM/PM in the limiter will start to cause problems [ref 52, 53, 130]. The limiters used were Plessey SL532C's, which are specified for low AM/PM distortion.

6.6 EXPERIMENTAL RESULTS

Figure 6.6.1 and Figure 6.6.2 show the "before and after" spectra which result from applying envelope quasi-linear to the AM sidebands. In Figure 6.6.2 the AM/AM distortion has been removed to show that asymmetrical effects are rife, and in fact of more importance in this instance than AM/PM.

Figure 6.6.3 and Figure 6.6.4 show the "before and after" effects of applying phase quasi-linear to the third audio sideband. Slight asymmetry in Figure 6.6.3 becomes much more pronounced when phase quasi-linear is applied.

Figure 6.6.5 shows the phase modulator characteristics used for phase quasi-linear.

Figure 6.6.6 and Figure 6.6.7 show the "before and after" effects of adding a phase locked loop arrangement to remove AM/PM. The remaining distortion sidebands are due to AM/AM distortion. In Figure 6.6.8 we removed the AM/AM distortion by means of a type 1 control algorithm, to leave a small residual asymmetry. Thus Figure 6.6.8 shows how the RF spectra can be improved (over Figure 6.6.6), using digital compensation and phase feedback. The type 1 control algorithm uses an envelope detector calibrated using envelope quasi-linear (see chapter 7).

6.7 CONCLUSIONS

Using the techniques described, AM/AM and AM/PM effects may be separated, and this is of great benefit in the design of digital compensation systems, since it allows us to see where effort must be applied to improve linearity. The techniques used seem ultimately limited by asymmetric effects, which in this work appeared at -60 to -70 dB on PEP, as compared to distortion levels of -30 to -40 dB on PEP.

Envelope quasi-linear is quick and effective, with few problems associated with its implementation (the need for interpolation between RAM data points has not been discussed but is straightforward).

Phase locked loop techniques are preferable over phase quasi-linear techniques, since they do not rely on the recognition of the phase effects as being quadrature effects. However, when the dynamic range of the envelope is large, the phase locked loop method inevitably breaks down due to AM/PM conversion in the limiters. Phase quasi-linear is potentially free from dynamic range problems.

It would make sense to combine phase locked loop and phase quasi-linear methods to get the best of both techniques (i.e. to use the phase locked loop to force the phase distortion to very low levels, so that it is effectively in quadrature, and to use phase quasi-linear to remove this small residual phase distortion, and to correct for AM/PM conversion in the limiters). This was not attempted as it was felt that the asymmetric effects were dominant over phase locked and quasi-linear errors.

FIGURE 6.2.1

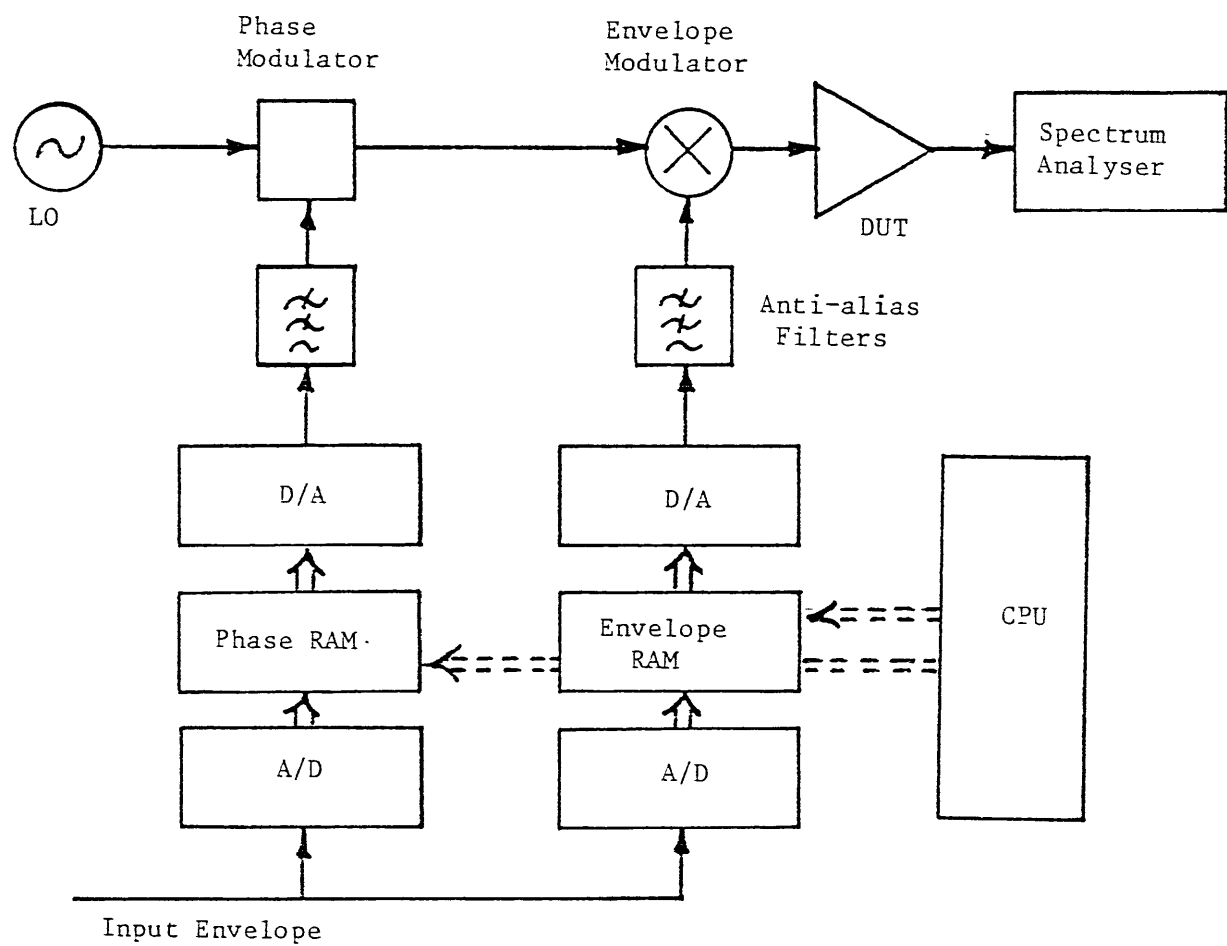


FIGURE 6.2.2

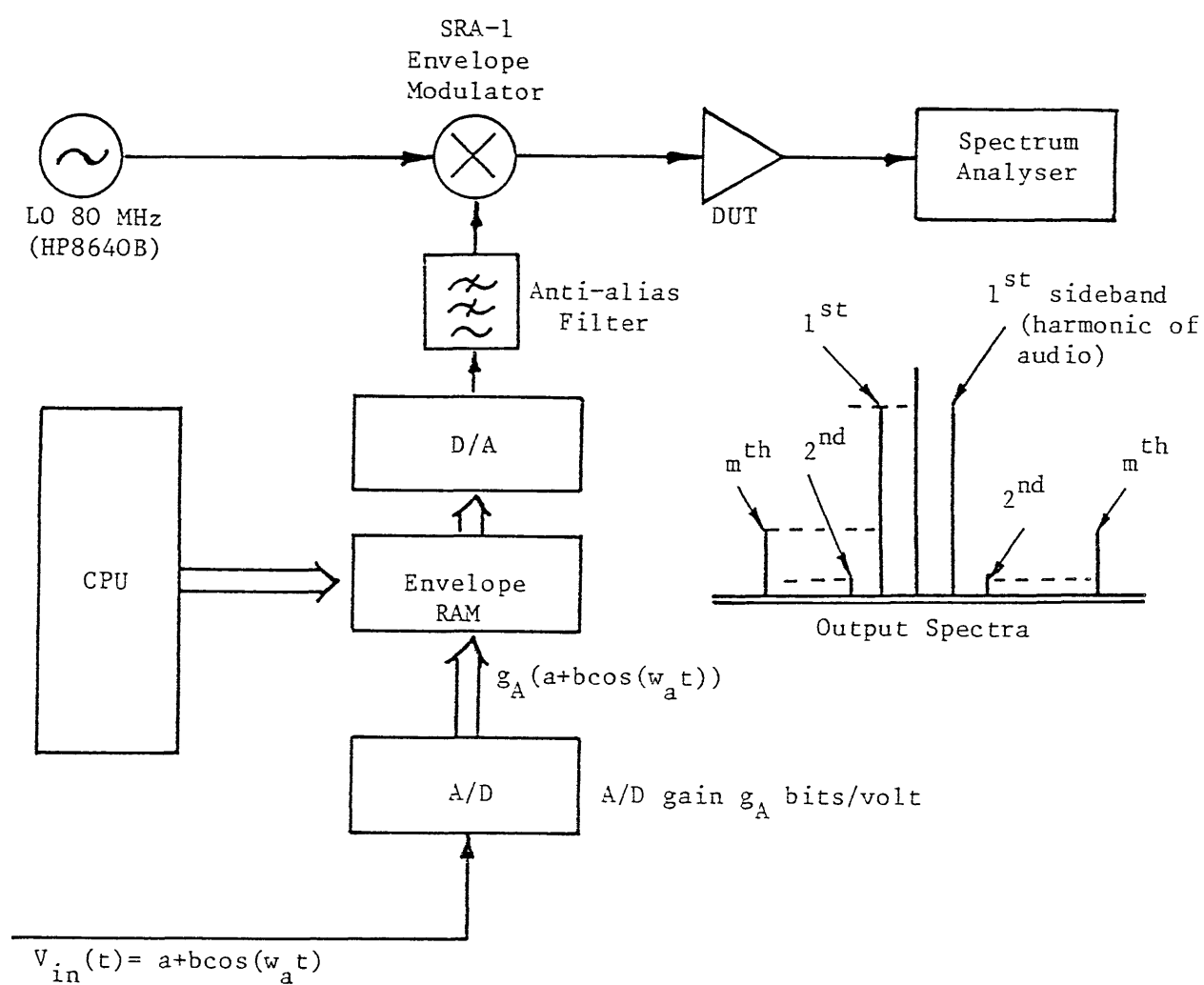
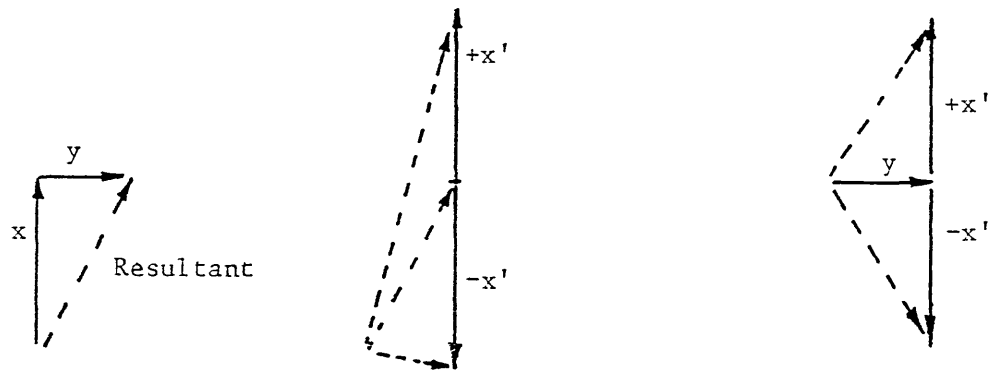


FIGURE 6.2.3



The magnitude of the sideband is given by the vector addition of the inphase and quadrature component.

We chose the sign of our estimate of x . Wrong choice of sign results in a larger sideband level than correct choice of sign.

If the sideband contains no inphase term, then whatever the choice of sign, the magnitude of the resultant sideband is the same.

FIGURE 6.2.4

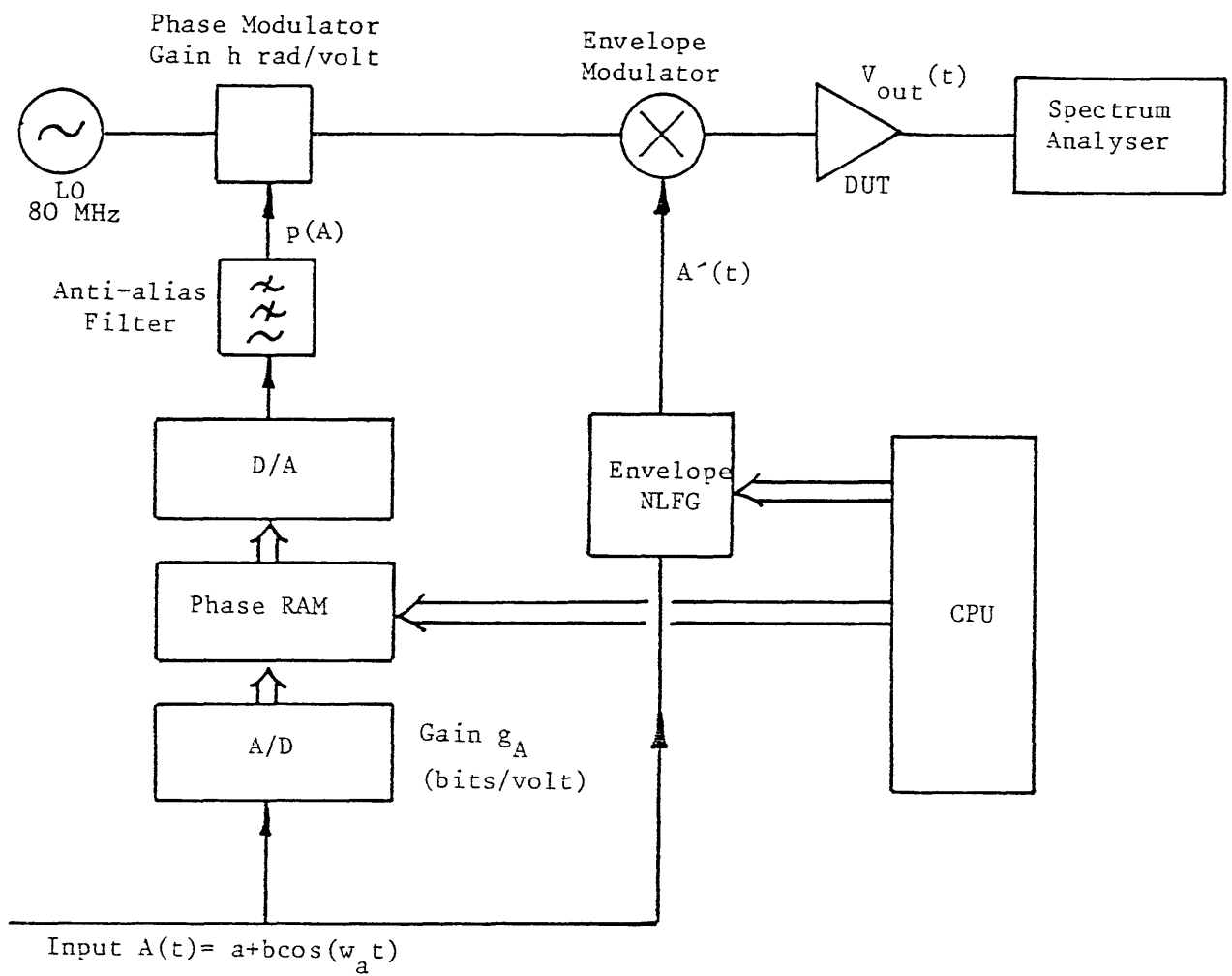


FIGURE 6.2.5

RF Spectrum

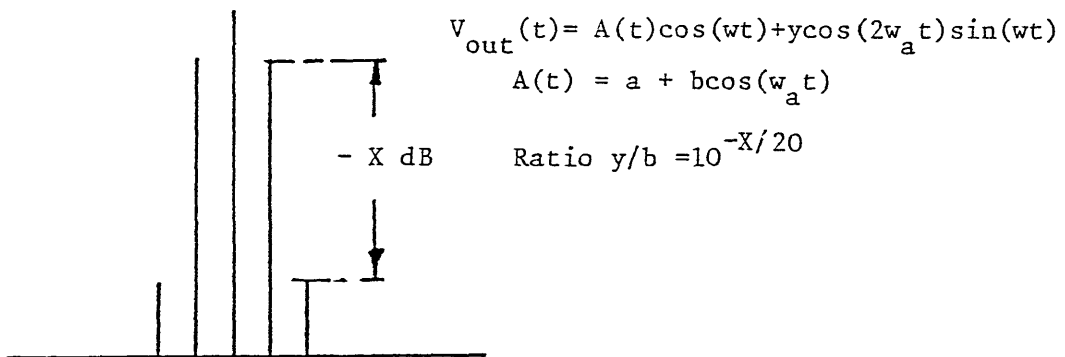


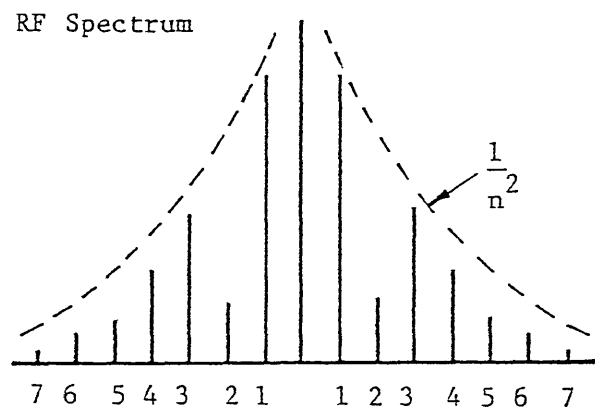
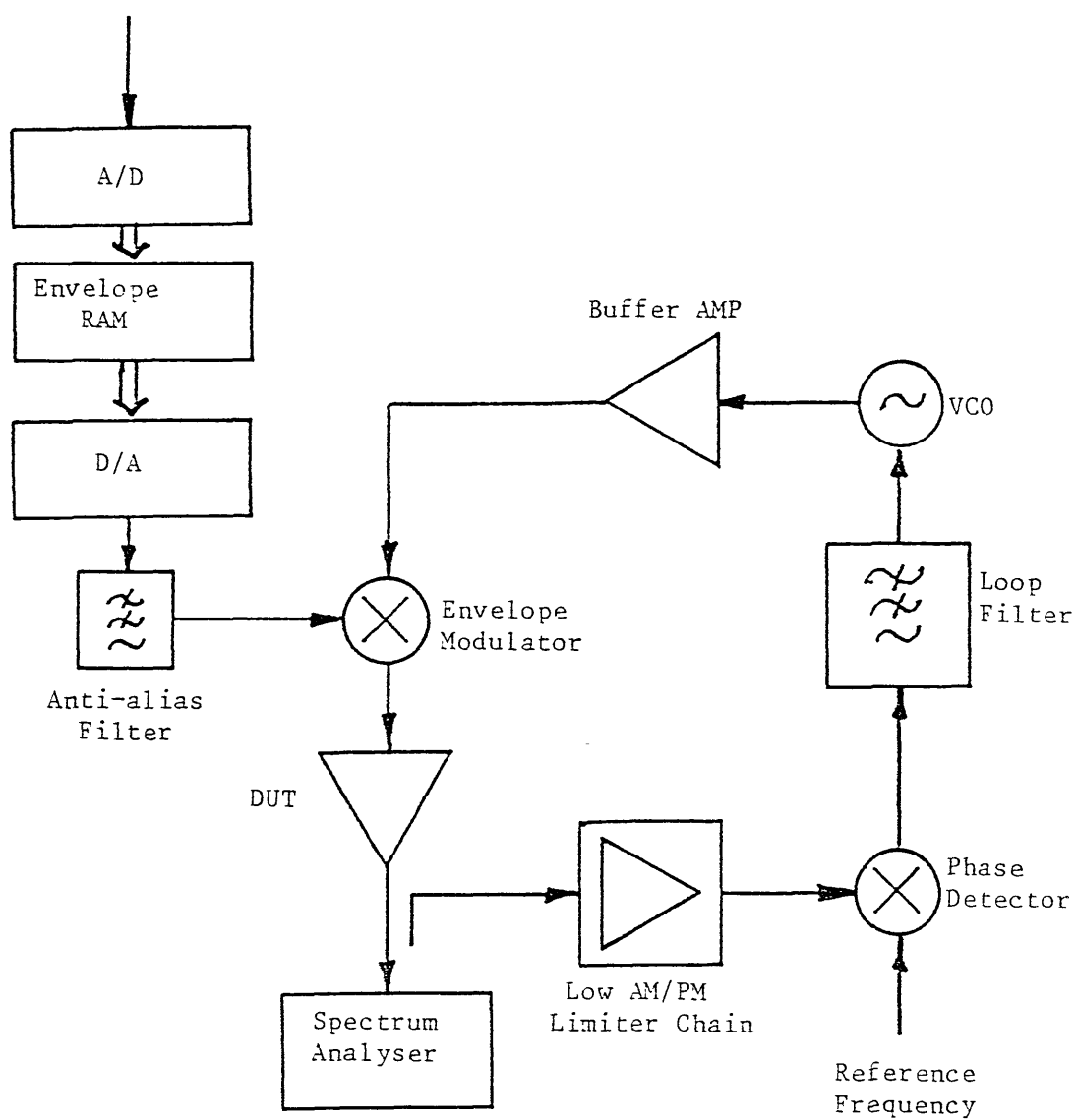
FIGURE 6.3.1

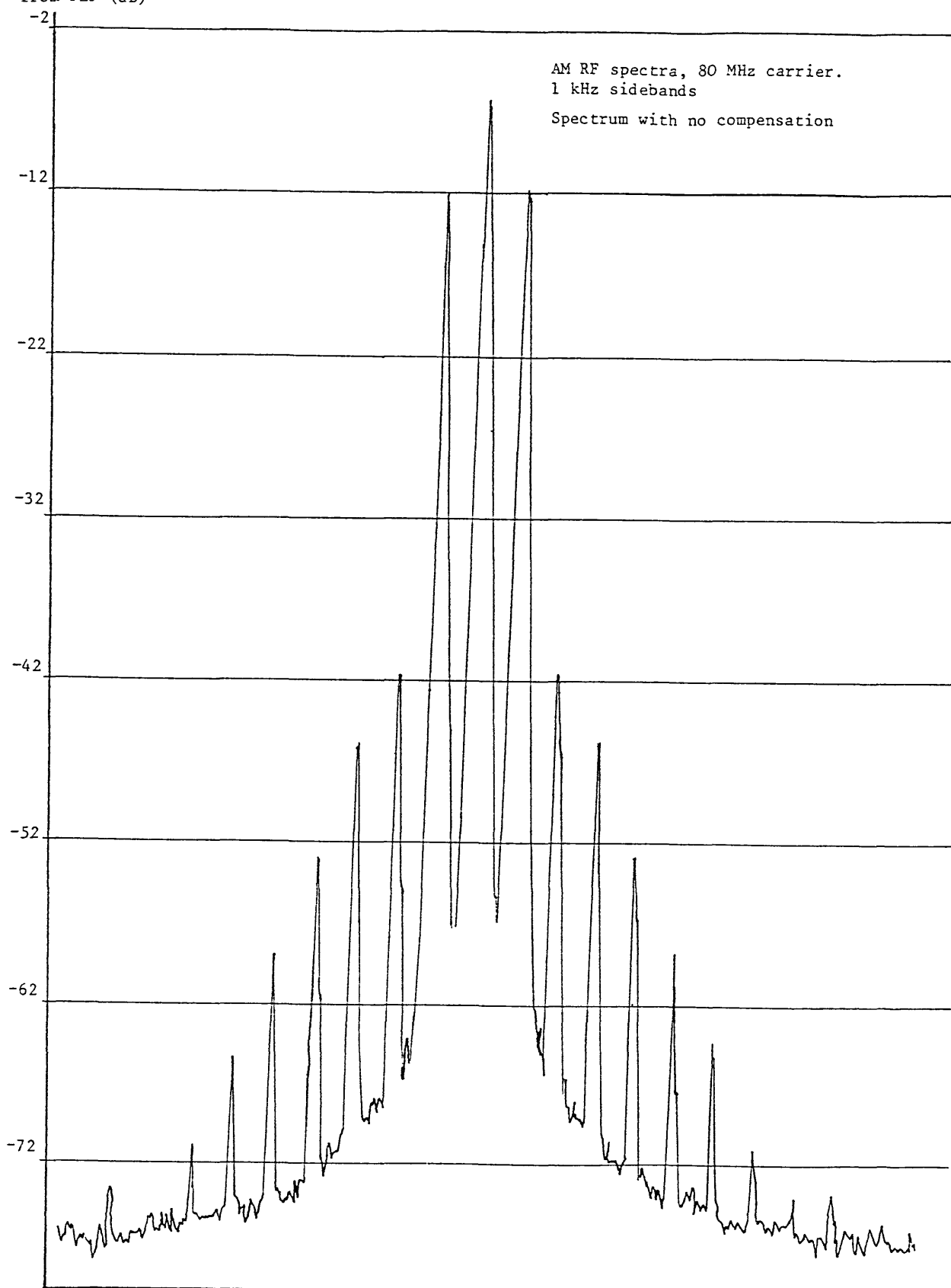
FIGURE 6.5.1



Addition of a phase locked loop around the DUT results in the phase angle of the output being constant, provided no AM/PM conversion occurs in the limiters.

RF level referenced
from PEP (dB)

FIGURE 6.6.1



RF level referenced
from PEP (dB)

FIGURE 6.6.2

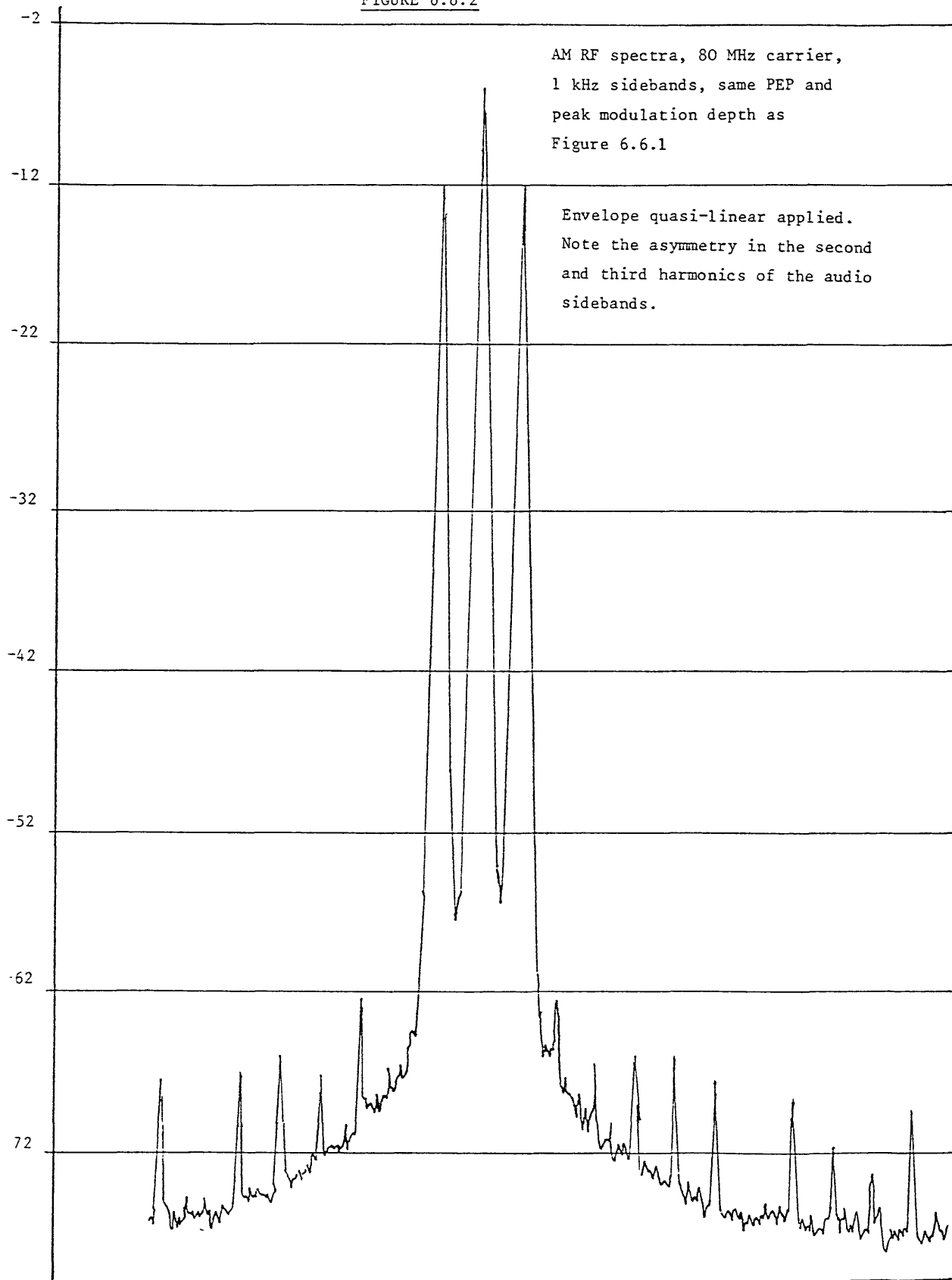


FIGURE 6.6.3

AM RF spectra, 80 MHz carrier,
1kHz sidebands.
RF level referenced
from PEP (dB)

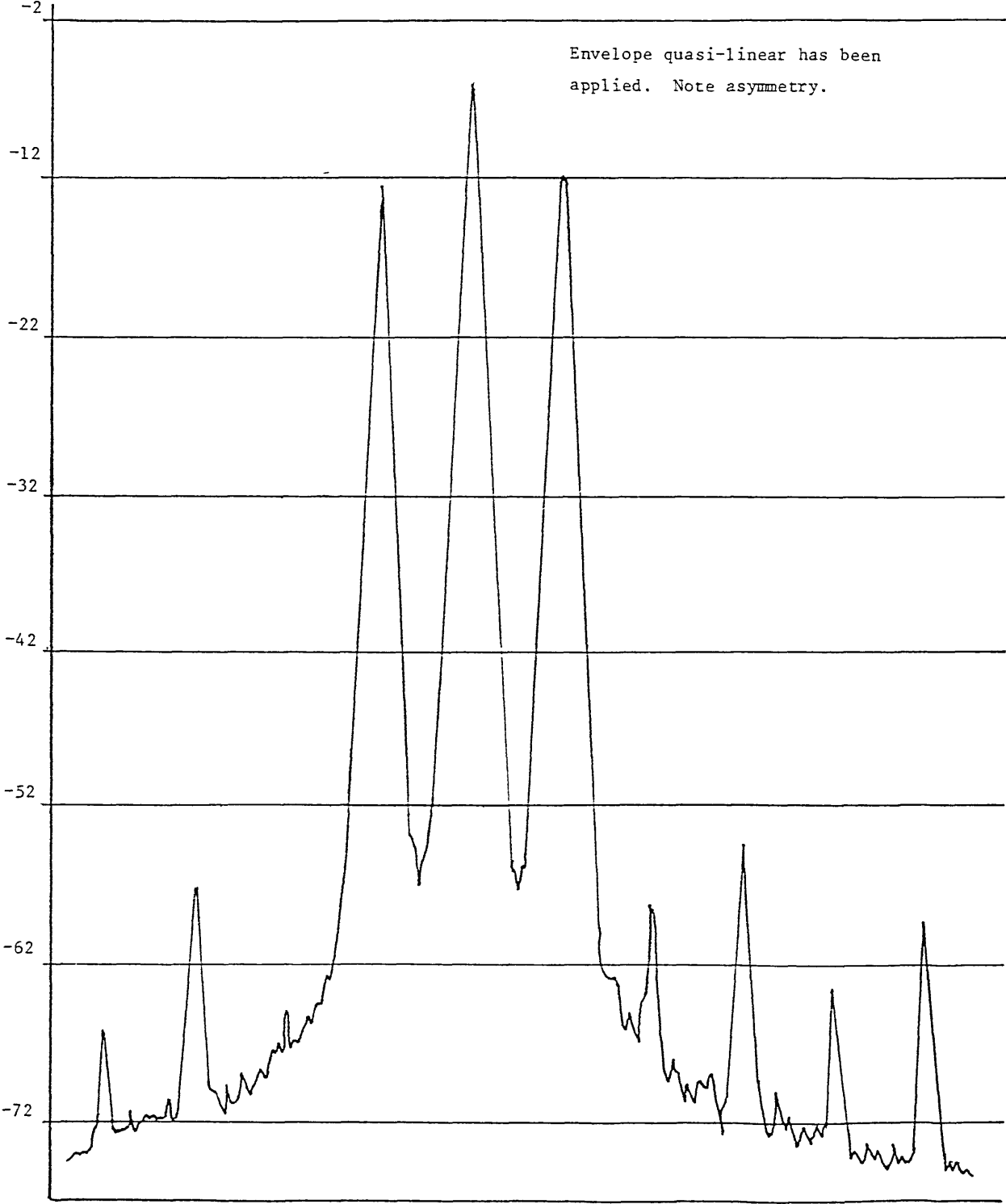
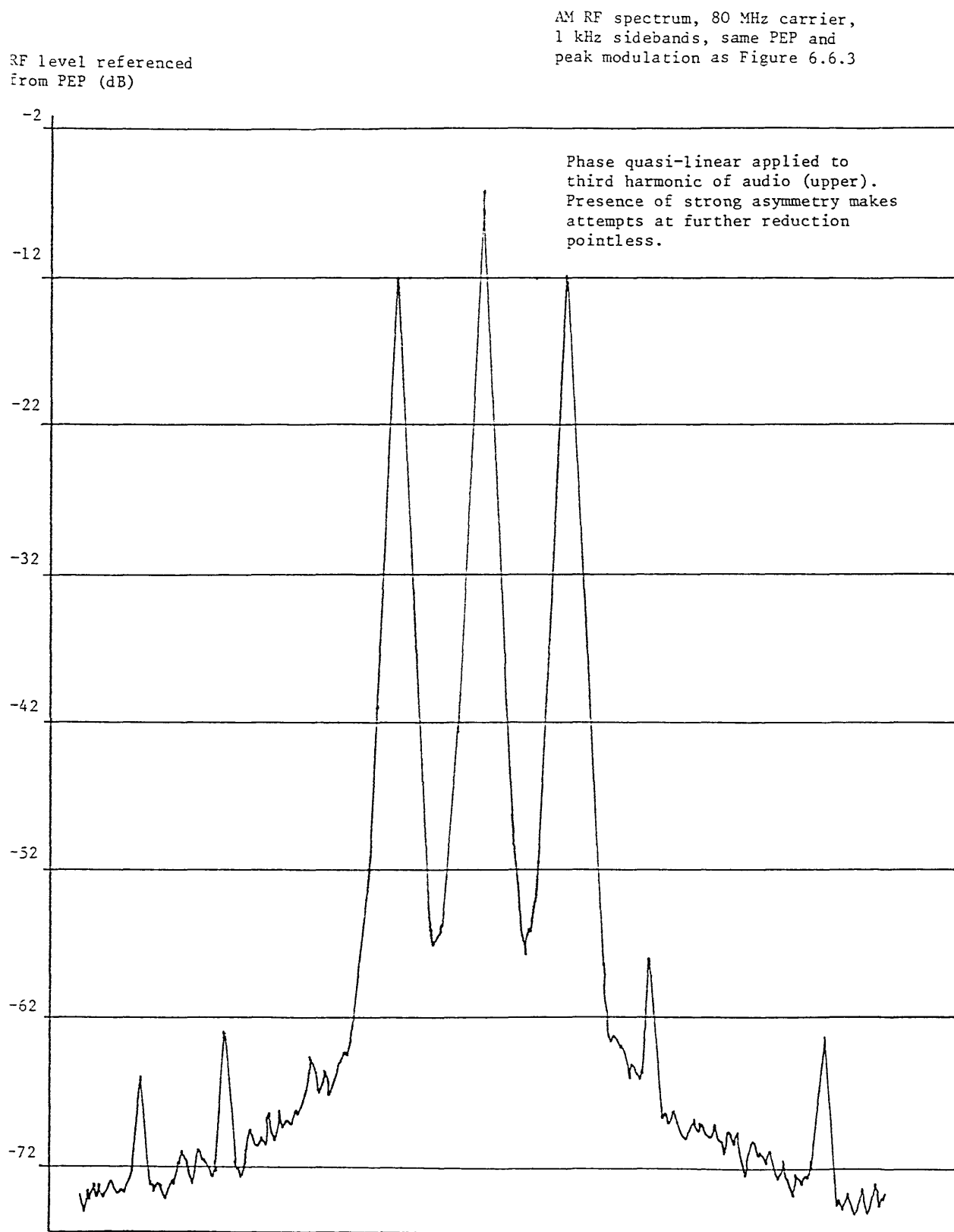


FIGURE 6.6.4



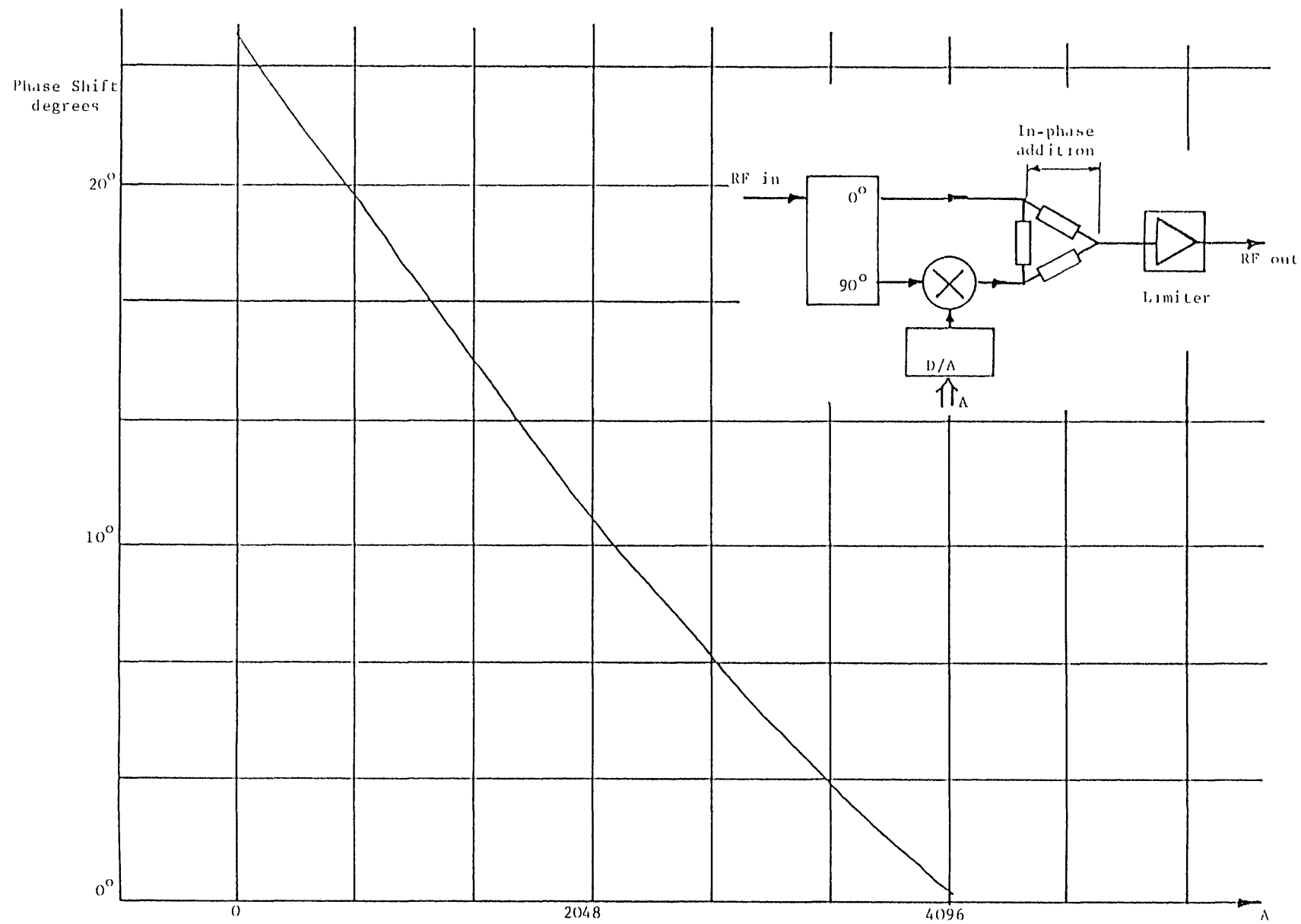


FIGURE 6.6.5

FIGURE 6.6.6

RF level referenced
from the PEP level
(dB)

AM RF spectrum, 30 MHz carrier,
1 kHz audio sidebands.

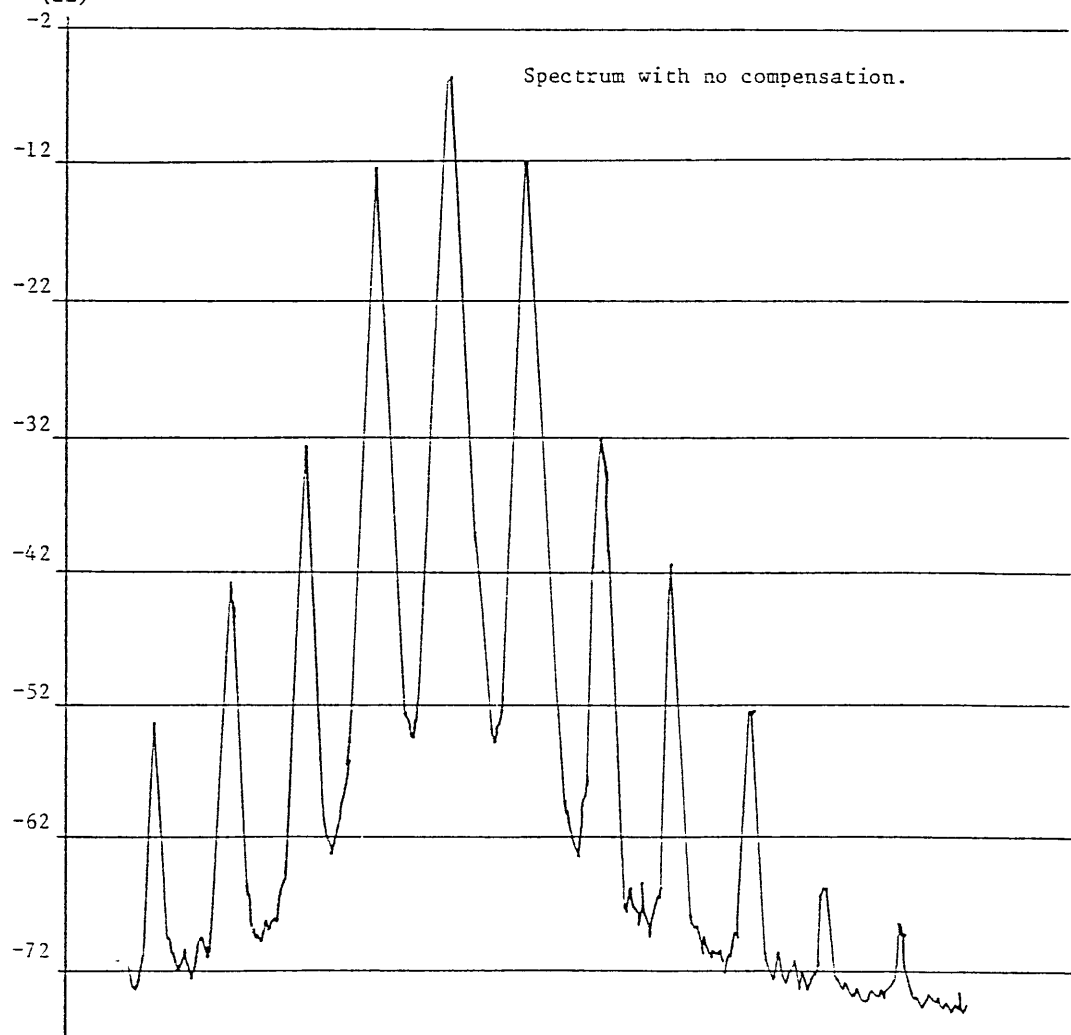


FIGURE 6.6.7

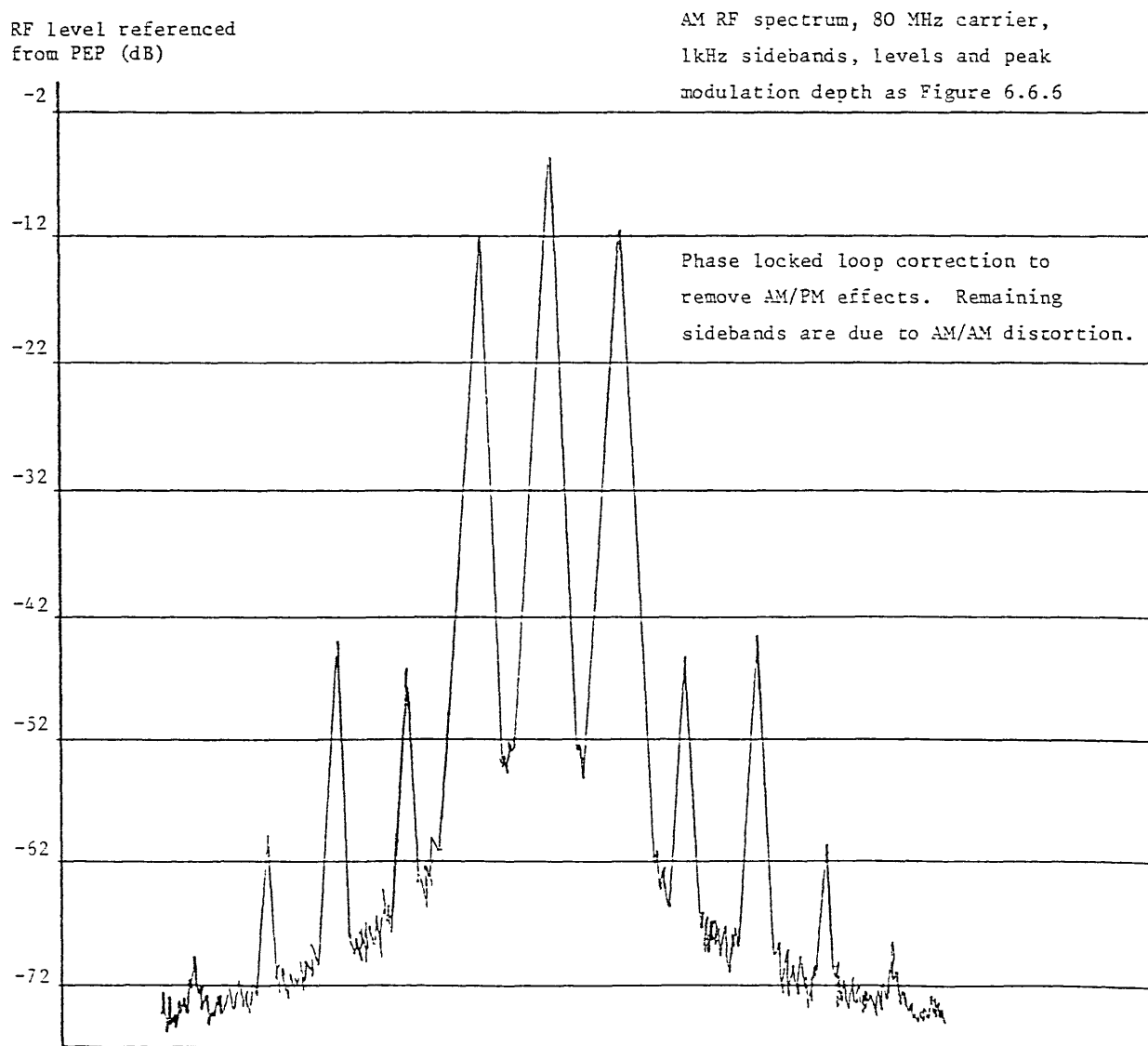
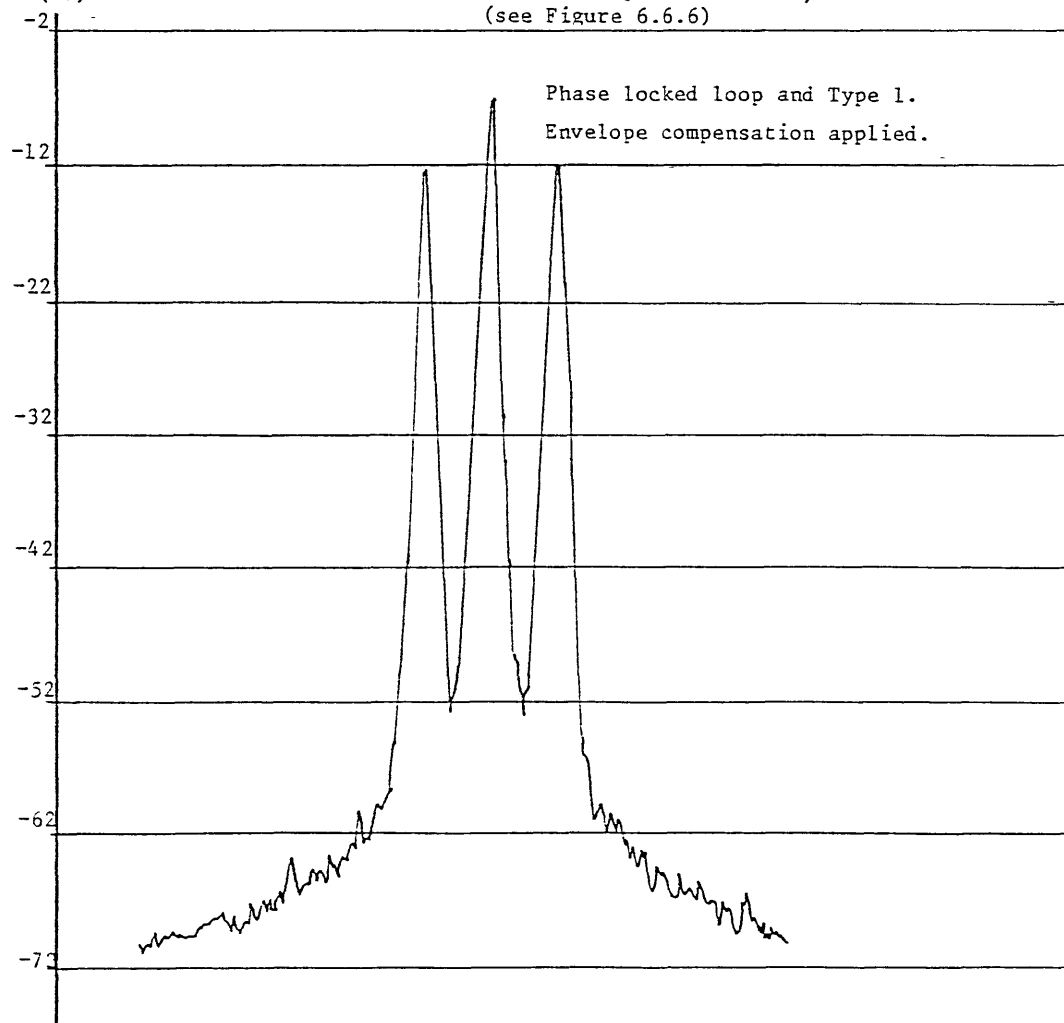


FIGURE 6.6.8

Notice the small asymmetry due to the second and third harmonics of the audio. This small residual asymmetry is commonly met, and difficult to remove entirely.

RF level referenced
from the PEP level
(dB)

AM RF spectrum, 80MHz carrier,
1 kHz audio sidebands, levels and
modulation peak as before,
(see Figure 6.6.6)



CHAPTER 7

THE ENVELOPE DETECTOR

7.1 INTRODUCTION

For any polar adaptive digital compensation scheme the requirement for an envelope detector is inevitable. A wide variety of detectors may be considered: FET, valve, synchronous detectors, diode detectors, may all be used. Of these the diode detector stands out because of its simplicity and reliability. It also has the advantage over synchronous detectors of being totally "phase deaf". The latter is of importance as our ultimate aim is to construct phase and envelope control loops around an amplifier, and interaction between phase and envelope loops could cause instability (in the BIBO sense), or at the very least degradation of performance. For these reasons the diode detector has proved a fairly natural choice for digital compensation, just as it forms a mainstay of envelope feedback. The limiter/mixer detector of Figure 7.1.1 has a higher dynamic range than the envelope detector [ref 98], and does not suffer from failure to follow distortion. However, its use is not considered here, since its complexity is felt to outweigh its advantages.⁺ This type of detector is used with polar loops.

Although the choice of a diode detector is straightforward, the choice of diode type and circuit configuration requires examination. Such limitations that are apparent are examined, and modifications made to overcome these limitations.

⁺ The mixer/limiter envelope detector would best be implemented using IF stages in the limiter chain. This allows gain to be distributed across the spectrum, assisting stability.

7.2 DIODE TYPE

Figure 7.2.1 shows the characteristics for the commonly used diode types (taken from reference 25). Our requirements for a diode detector are:

1. High breakdown voltage, since this parameter sets the upper limit of the RF signal level we can rectify accurately.
2. Sharp change of curvature (preferably about the origin).
This parameter sets the smallest RF voltage which can be rectified with accuracy. This parameter is sometimes, and somewhat mistakenly, confused with low turn on voltage.
(The turn on voltage can always be altered by addition of a dc voltage level).
3. Low capacitance, especially that due to charge storage.
This parameter sets the upper frequency limit that the diode can rectify.

Judged by these criteria no one diode type is ideal. Silicon has the highest reverse breakdown, but has a relatively poor change in curvature. The backward diode has the best change in curvature, and the worst breakdown characteristics. The choice lies primarily between hot carrier schottky diodes and germanium point contact diodes. Both diode types may be purchased with low capacitance (reference 150 suggests values of 0.09pF and 0.16pF respectively) suitable for high frequency work. Schottky diodes may be purchased in matched pairs (which is an advantage in some circuit configurations). Germanium diodes appear to have less repeatable characteristics [ref 150], but (in this study) when carefully chosen appeared to have a superior dynamic range over schottky diodes (typical operating frequencies 80 MHz). Both

schottky and germanium diode types were used in this study.

Figure 7.2.2 shows a table from reference 150 comparing three commonly used microwave detector diodes. Reference 156 gives a further discussion of diode types.

7.3 DETECTOR CONFIGURATION

The basic envelope detector configuration is shown in Figure 7.3.1. It consists of a matching circuit between the RF input impedance and the diode impedance, diode, and RC filter to remove all but the low frequency envelope component. The RF input to the detector is assumed to be free from RF harmonics. If this is not the case the detector should be preceded by a filter.

7.3.1 Limitations of the Basic Detector

The analysis of the basic detector is straightforward and well-known [ref 35, 170] and so we do not repeat it here. From the analysis it is apparent that the detector suffers from a number of limitations:

1. Under some circumstances (most notably with over 100% AM modulation) the diode detector suffers from a form of distortion known variously as failure to follow distortion, or diagonal clipping, [ref 35, 170]. This is illustrated in Figure 7.3.2.
2. The detector does not give a true estimate of the envelope as it behaves in a nonlinear fashion, being a peak detector at high levels and a square law device at low. Lazarus [ref 104] shows the output voltage $V_{out}(t)$ is related to the input envelope $A(t)$ by

$$V_{out}(t) = R i_s [I_0(eA(t)/a) \exp(-eV_{out}(t)/a) - 1] \quad (7.1)$$

where k is Boltzmann's constant, T is the absolute temperature, i_s is the reverse saturation current, I_0 is a modified Bessel function of the first kind [ref 1], e is the charge on the electron, and $a = nkT$ where n is an adjustable parameter (typically $1 < n < 3$). The diode characteristic satisfies

$$i = i_s [\exp(eV/a) - 1] \quad (7.2)$$

and the reverse saturation current satisfies

$$i_s = \exp(-\phi e/a) \quad (7.3)$$

where ϕ is the barrier potential of the diode.

3. For large signal swings the diode law of equation 7.2 does not hold, and the distortion due to nonlinearity increases.
4. The detector characteristics in equation 7.1 are strongly temperature dependent. The reverse saturation current is also strongly temperature dependent.
5. As with any detector there are finite noise figures to contend with. For diodes, noise performance is usually measured by the tangential sensitivity [ref 51, 142, 150, 156].
6. The diode impedance is usually high, and it must be matched to the RF (usually 50 ohms). Consequently, there is a tradeoff between the detector RF bandwidth and the power match that can be obtained. This results in a tradeoff between sensitivity and RF bandwidth.
7. Thermal emf's can bias the diode, and cause additional temperature variations.

7.3.2 Detector Circuitry

A number of detector circuits may be considered. Wetenkamp [ref 185, 186] uses a balanced technique to simultaneously improve noise performance and to reduce thermal and harmonic effects. The method is shown in Figure 7.3.3. Wetenkamp has yet to extend the technique to ac envelopes, and has concentrated primarily on measurement of RF power, and so the failure to follow characteristics of the detector are unproven.

Another common circuit is shown in Figure 7.3.4. No analysis appears to have been presented in the literature, and so we offer one in Figure 7.3.4. The detector has superior linear characteristics over the basic detector, and a much reduced temperature sensitivity. However, the circuit does not have enhanced resistance to failure to follow distortion. A circuit of this type was built, details being given in Appendix D.

A third type of detector in use is shown in Figure 7.3.5 [ref 84]. This detector uses a virtual ground amplifier to keep the charge on the capacitor to a very small quantity (ideally zero). Since failure to follow is associated with the charge on the capacitor, this results in an improved failure to follow distortion. A detector of this type was built, details being given in Appendix D.

7.4 FAILURE TO FOLLOW DISTORTION

Failure to follow distortion occurs at high AM modulation indices, and so can be avoided when the AM modulation index is low (< 95%). However, we wish this investigation of the envelope detector to have wider applicability than low index AM.

Specifically, in the square-and-detect scheme proposed in Figure 4.3.6, which is applicable to such cases as multichannel links or VSB-TV, we are effectively dealing with the detection of 100% modulated AM waveforms. The work of this thesis has been with high modulation indices ($> 97\%$) to keep this study general (high modulation index AM is not in itself an important case).

Clarke and Hess [ref 35, p.492-502] analyse the conditions necessary for failure to follow distortion and we quote results from that analysis. The model of the detector is shown in Figure 7.4.1.

If the RF input to the detector can be represented by the current $i_L(t)$ where

$$i_L(t) = I(1 + m\cos(\omega_a t))\cos(\omega t) \quad (7.4)$$

then the conditions for no failure to follow distortion are

$$(1-n) > m[(\omega_a/\omega_1)^2 + 1]/[(\omega_a/\omega_2)^2 + 1]^{0.5} \quad (7.5)$$

where

$$n = V/IR \quad (7.6)$$

$$\omega_1 = 1/(R_0 C_0) \quad (7.7)$$

$$\omega_2 = (2R + R_0)/(2RR_0(C + C_0)) \quad (7.8)$$

It follows from these equations that provided

$$R_0 C_0 \leq 2RC \quad (7.9)$$

$$\text{and} \quad n = 0 \quad (7.10)$$

modulation depths of 100% ($m=1$) can be achieved without failure to follow distortion. Note here that equation 7.9 is equivalent to

$$\omega_1 \leq \omega_2 \quad (7.11)$$

Increasing the RC time constant to a drastic extent would have catastrophic effects on the bandwidth of the detector, since it is at variance with attempts to power match the RF signal to the

detector. The virtual earth circuit (already discussed) attempts to force $R_0=0$, and thus satisfy equation 7.9. Equation 7.10 also explains why it is usual to apply forward bias to the detectors (i.e. set n approximately equal to zero).

7.4.1 Quantifying Failure to Follow Distortion

The approach taken to quantify failure to follow distortion is shown as a test setup in Figure 7.4.2. From Figure 7.3.2 we note that failure to follow distortion is an asymmetric effect, and thus results in the audio output of the detector having sidebands described by cosine and sine terms. The sine terms are due to failure to follow distortion. By applying quasi-linear to Figure 7.4.2 we can remove the in-phase (cosine) terms in the spurious harmonics, to leave a residual level. This level corresponds to finding the sine terms in the spurious spectra. By taking the magnitude of the residual sine terms as being a measure of failure to follow distortion, we can compare and evaluate failure to follow effects by quantifiable means. Our aim is to have the magnitude of failure to follow distortion (as measured by this method) smaller than, or of the same magnitude as, the noise floor set by the A/D resolution.

Figure 7.4.3 shows the spectrum of the detector output when used to produce a 100% AM RF signal by a type 1 algorithm. The distortion at -72 dB on PEP can be shown by quasi-linear to be due to failure to follow distortion. The RF spectra into the diode detector has already been discussed (see Figure 6.6.8). The RF spectra is constrained by RF asymmetry. [Note here that failure to follow distortion is very sensitive to the AM modulation depth.

This parameter is very difficult to quantify to better than 1%. The method chosen in this experiment was to adjust the RF envelope as viewed on a fast oscilloscope (Tektronic 7904) through a harmonic filter, until the minima of the envelope were separated by 2-4mV and the maxima by 1.4-1.5V. This corresponds to 99.7-99.8% modulation].

7.5 THE ENVELOPE DETECTOR AND DIGITAL COMPENSATION

Due to the nature of digital compensation we are able to contribute to the improvement of the diode detector in a number of respects.

We can improve the effective detector linearity. In essence it should be possible to determine the detector characteristics, and to take account of this when it comes to producing a quantized value for the detector output. A possible scheme is shown in Figure 7.5.1. The A/D, RAM combination in Figure 7.5.1 simply acts as a discrete NLFG to correct for memoryless distortion by the detector. (Such a method would have little application in polar and envelope feedback schemes, where a continuous analogue voltage out of the detector is required). In effect we shall precalibrate the diode detector. Provided failure to follow distortion does not occur, the precalibration scheme will result in improved detector linearity.

The other improvement we can make is in noise performance. In digital compensation the RF signal per se is of no importance, our interests lie with the AM/AM characteristic of the DUT. In essence we shall use the detector to make a series of measurements of the DUT characteristic. Provided the DUT characteristic does

not vary over the course of the measurements, we can average our results and so reduce the effective noise associated with the output of the detector. (We must bear in mind that the noise is Rayleigh and not zero mean Gaussian distributed [ref 150]). However, it is more convenient to postpone discussion of data processing of the detector output until considering algorithm type.

It is worth noting that both of these improvements could be obtained irrespective of the detector type, so much of the following would be directly applicable to Figure 7.1.1 for example.

7.6 IMPROVEMENTS IN DETECTOR LINEARITY

As briefly noted in Chapter 3 (section 3.3.4), a polar, IF, type 3 control algorithm does not necessarily need linear detectors. However, type 3 control has its disadvantages (discussed later). Consequently the linearity of the detector is, in general, of great importance to digital compensation and will directly reflect on the suppression of the spurious AM/AM sideband we can hope to achieve.

To precalibrate the envelope detector we need to compare it against a detector of higher (and preferably traceable) standard, or know in advance the precise nature of the envelope that it is being used to detect.

Two methods have been developed which are especially suitable for digital compensation. The first relies on the known linearity of a passive, resistive attenuator; the second relies on having a known RF envelope.

7.6.1 Attenuator Calibrated Detector

The test setup is shown in Figure 7.6.1. The envelope detector is used to measure a fixed level RF signal (no modulation present) which passes through a switched attenuator. In one position the nominal attenuation is 5 dB, in the other position 8 dB. The switched attenuator is preceded by a continuously variable attenuator. The detector output is passed to an A/D, and displayed via the CPU on a VDU. The test procedure is as follows:

1. The switch is placed in the 5 dB position, and the variable attenuator adjusted until the VDU shows the output of the A/D is at full scale (X_{max}).
2. The switch is thrown so that 3 dB more attenuation occurs (i.e. a total of 8 dB of attenuation). The output of the A/D is noted.
3. The switch is put to the 5 dB position and the variable attenuator adjusted so that the A/D output is as it was at the end of step 2.
4. Steps 2 and 3 are repeated until the A/D reading drops below minimum resolution.

The idea behind this test is best explained with the aid of Figure 7.6.2. This shows a graph of input envelope level to the detector and the output voltage of the A/D output. We define the linear attenuator we wish to calibrate against as being formed by the straight line between the origin and the data point determined by step 1. When we increased the attenuator by 3 dB, the input envelope to the diode detector fell by a factor of k (clearly k is approximately equal to 0.7079). The A/D output fell by a factor

which will be k only if the detector is operating linearly. From the difference in factors a correction for the detector may be deduced (see Figure 7.6.2). The 8 dB attenuator is switched back, and the variable attenuator is used to bring the RF level to the same point as before. The procedure then repeats as indicated in Figure 7.6.2. Notice that:

1. The reduction of the RF by a factor of k is always done by the same attenuator. Thus, although we may have some tolerancing on our knowledge of the value of k , we are certain that k is consistent from one set of readings to the next.
2. There is always a minimum of 5 dB between detector and signal generator. This is a precaution to ensure that the output power of the generator is not affected by poor detector VSWR (which will vary as a function of power level).
3. We end up with a finite number of calibration points relating the envelope transfer characteristic of the detector to an ideal linear detector.

We now consider the effect of tolerancing. Table 7.6.1 shows the experimentally recorded points and Figure 7.6.3 shows the effect of tolerancing due to the finite resolution A/D. In Figure 7.6.3 we show that if a set of measurements are made, the maximum error which may occur in measuring the envelope (V_{error}) satisfies

$$V_{\text{error}} < 0.5/(1 - k) \quad (7.12)$$

Figure 7.6.3 shows how the maximum error varies with choice of k .

Also to be considered are the number of calibration points in a finite dynamic range. If the dynamic range of the detector

is D dB, then the number of calibration points will be given by

$$D/(20\log(k)) \quad (7.13)$$

Of course the dynamic range of the detector must be matched by a comparable dynamic range (bit resolution) for the A/D.

Clearly there is a compromise to be had over the value of k we choose. Choice of k approximately equal to 1 will give us a large number of calibration points, but a comparatively poor maximum error. Choice of $k \ll 1$ results in small errors, but a very limited number of calibration points with which to work. Between calibration points we use linear interpolation. Working on the assumption that our worst error due to quantization will inevitably be approximately equal to 1 bit, we conclude that a reasonable choice of k is 0.707 for the 12 bit A/D used.

The method so far described relies on knowing accurately the value of k . At VHF however, it is difficult to make attenuators with attenuation known to better than 0.1 dB [ref 77, pp.458-460]. Alternatively we can view this tolerancing as a natural consequence of being unable to produce a perfect impedance match for the attenuator (i.e. attenuation is due to the resistive loss and VSWR effects, the latter varying from signal source to signal source and load to load).

Thus, determination of the value of k can present some difficulty, since to high accuracy it varies from circuit to circuit. We now put forward a method of determining k which is accurate and straightforward for digital compensation.

To calculate the value of k we proceed with a second experiment, shown in Figure 7.6.4.

The input to the detector is a 100% AM signal, which passes

through a switched 10 dB attenuator. (The AM/AM, AM/PM distortion of the RF signal is of no interest to us, all we require is that the RF envelope traverse as large a dynamic range as the failure to follow distortion of the detector permits). The detector output is distorted by the NLFG of Figure 7.6.4, the output of which is monitored by a high impedance spectrum analyser, through a second switched attenuator. The operation of the experiment is as follows:

1. We assume a value for k . We then set the NLFG so that if this value of k is correct, the series combination of detector and NLFG is linear.
2. With the switched RF attenuator at minimum attenuation, the second attenuator at maximum, we record the spectrum shown on the analyser.
3. The RF attenuator is then switched to maximum, the second attenuator to minimum, and the spectrum recorded.
4. We repeat this sequence of operations for different values of k in step 1.

If we choose the true value of k , then the series combination of detector and NLFG will be linear. Thus there will be no change in the observed spectrum when the RF is attenuated 10 dB and 10 dB is removed from just prior to the analyser. If we have an incorrect value of k then we would expect the spectrum to be dependent on the RF attenuation and to change between steps 2 and 3. Figures 7.6.5 to 7.6.7 show the experimental results. In those figures we show the level of the harmonics of the audio with, and without, attenuation. We look for minimum change between steps 2 and 3, down to approximately 83 dB on peak voltage (corresponding to the

noise floor for the 12 bit A/D used). For interest, Figure 7.6.8 shows the results that we would obtain if we assume the detector is linear (i.e. we make no attempt to compensate for the detector distortion with a NLFG).

From these results we deduced that the value of k is 0.7128 within an error range of -0.008 , $+0.0016$. The error bounds were defined by the experimental value of k at which degradation from $k=0.7128$ could just be discerned. Thus $k = 2.940 \pm 0.01$ (dB). Hence, given k , the detector may be calibrated. This method appears to compare with the 0.02 dB accuracy of the power meter of reference 77 (p.448), whilst circumventing the problem of attenuator loss when terminated by the detector (i.e. a poor VSWR).

The method, in fact, corrects for any nonlinearities in the series combination of A/D and detector (the same A/D being used in all the experiments). Thus it is important to be sure that the linearity of the A/D is greater than that required of the detector.

7.6.2 Quasi-linear Calibrated Detector

A conceptual test setup is shown in Figure 7.6.9. We require an AM modulated RF signal source which acts as the input to the detector, the output of the detector goes to a NLFG which is connected to a spectrum analyser. The AM source may contain AM/PM distortion, but must not contain AM/AM distortion.

We now apply quasi-linear adjustment to the RAM table so that the output spectrum contains just dc and first audio sideband. The methodology and details of quasi-linear control of

a power spectrum have been set out previously (see chapter 6). Although in that chapter we were dealing with the control of a RF power spectrum the details (and theory) carry over to apply to Figure 7.6.9.

By use of quasi-linear we can force the NLFG to act as the inverse of the detector distortion, and thus determine the detector distortion (see chapter 6, section 6.3.1).

The only assumption we need to make is that the AM source contains no AM/AM distortion. We now examine this assumption.

Some commercial signal generators with AM modulation capability use an envelope feedback circuit to control the AM/AM distortion of the generator (e.g. HP8640B). This means that AM/AM distortion may be good, especially for low modulation index AM (< 50%), but still may not provide the required linearity. (The HP8640B is specified for 3% AM/AM distortion above 50% modulation [ref 77, p.406]).

Kibbert [ref 95] describes an AM modulated RF signal of minimal quantifiable AM/AM conversion (in reference 95 the power spectrum has spurious sidebands -70 dB on PEP. We take this as a crude indication of the AM/AM spurious levels). Kibbert's modulator is now available in commercial form [ref 108], but was not available for this thesis at the time of the study.

As Kibbert's modulator was not available at the time of the study a technique was developed specifically for the requirements of digital compensation. Figure 7.6.10 shows the test setup. The arrangement uses a type 3 control algorithm.

In essence the AM signal is detected, and compared with the audio input, distorted by RAM 2. The difference is passed to the

CPU, which then modifies RAM 1 (and hence the RF), so that the detector output and RAM 2 are always in agreement. If RAM 2 were a perfect representation of the detector distortion, this would mean that the RF spectra was AM/AM free.

We use quasi-linear to control the RF power spectrum and to remove spurious sidebands, the quasi-linear adjustments being made on RAM 2. Thus we can calibrate the detector. [Note here that to calibrate the detector the envelope modulator must be free from LO leakthrough. The necessity of this and the way that this is ensured in the experiment is discussed in Appendix E].

A Fortran program used to generate a PROM table to replace RAM 2 is shown in Appendix E. The program can take either quasi-linear or attenuator experimental results, and produces a formatted hexadecimal listing of the values required in the PROM.

This method of calibrating the detector involves ac envelopes, and thus the possibility of failure to follow distortion. Care must be taken to check that the detector output is free from failure to follow distortion if spurious calibration tables are not to be obtained.

7.7 CONCLUSIONS

Digital compensation offers advantages which can be used to improve the envelope detector noise and linearity performance. Only linearity has been looked at here, but it is plain that a significant improvement is offered (compare Figure 7.6.8 with Figure 7.6.6).

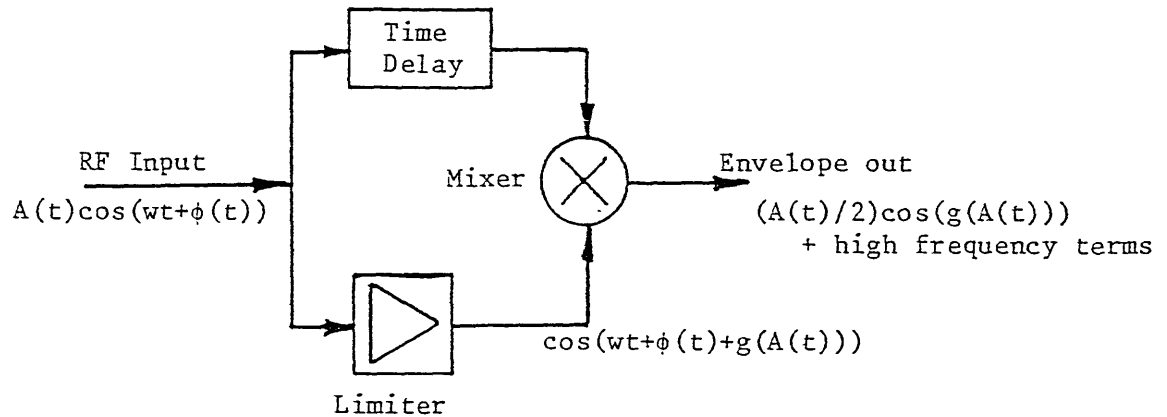
Of the two methods of precalibrating the diode detector discussed, quasi-linear appears to give slightly better results,

although it requires a more complex test setup. Both methods are in broad agreement.

Failure to follow distortion does not appear to be a problem for the modulating frequencies used. The effect can be minimised by circuit design (see Figure 7.1.1 and Figure 7.3.5), although the spectral effects of the circuit of Figure 7.3.4 are not obtrusive (see Figure 7.4.3).

Temperature variable effects may be dealt with by using a constant temperature enclosure for the detectors (see Appendix D for details).

FIGURE 7.1.1



The purpose of the time delay is simply to compensate for the finite time taken by the signal in passing through the limiter. By definition, $A(t)$, the envelope is unipolar, so for the cos term small, the output is unipolar.

In practice all limiters suffer from AM/PM conversion. This gives rise to distortion of the detected envelope.

The detector is capable of wideband operation, with good dynamic range, and operates without failure to follow distortion. However, it is far more complex and costly than the diode detector. Because of the high gain of the limiter, oscillation can occur. Often the limiter will use multiple IF's to circumvent this, leading to more complexity.

FIGURE 7.2.1

Si silicon junction
 CuO copper oxide
 SB silicon backward
 HC hot carrier (schottky)
 GPC germanium point contact

Insert shows the silicon backward diode characteristics about the origin.

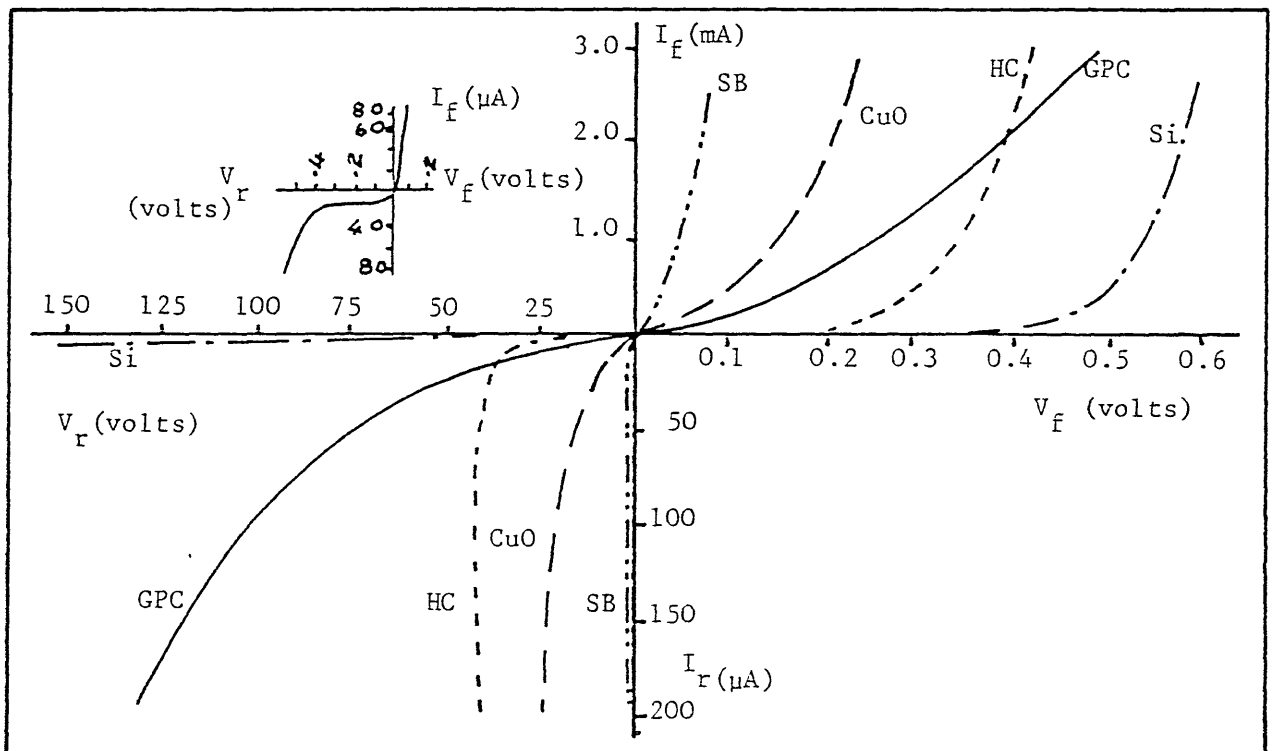


FIGURE 7.2.2

CHARACTERISTIC	POINT CONTACT DIODE	SCHOTTKY DIODE	BACK DIODE
TSS compared at 12GHz in a 10MHz video bandwidth (matched)	-53 dBm	-55 dBm	-48 dBm
Bias considerations	10-50 μ A range for best performance	5-20 μ A range must be employed	100 μ A will typically yield a 1.5 dBm improvement in TSS
Inherent RF impedance (VSWR at -20dBm $Z_0 = 50$ ohms)	20 @ 2 GHz 6 @ 12 GHz with 10 μ A bias	20 @ 2 GHz 5 @ 12 GHz with 10 μ A bias	Low impedance, typically 3 to 5
Typical detected voltage, 12GHz, -20dBm, with a 1000 ohm load	1.5mV	0.2mV	8mV
Flicker noise (1/f) characteristic	High below 100 kHz	Moderately low below 10kHz	Very low
Burn out capability	Good, 200-300mW	Very good, 300-400 mW	Poor to fair, 100-200mW
Reproducibility of diode for matched pair applications	Poor to fair	Good	Good
Video impedance	1500-10,000 ohms @ 10-50 μ A bias	4,000-30,000 ohms @ 10-50 μ A bias	100-300 ohms @ zero bias

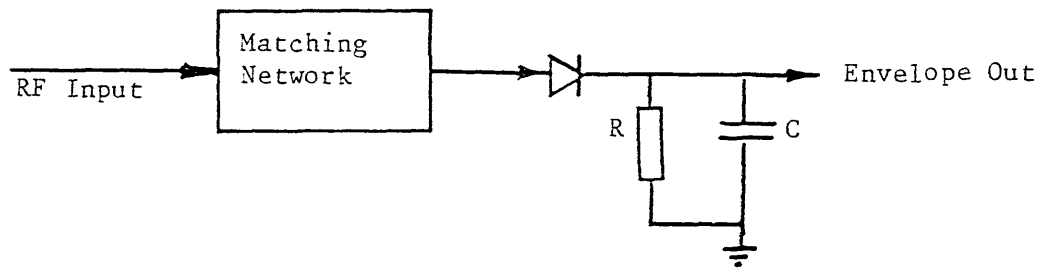
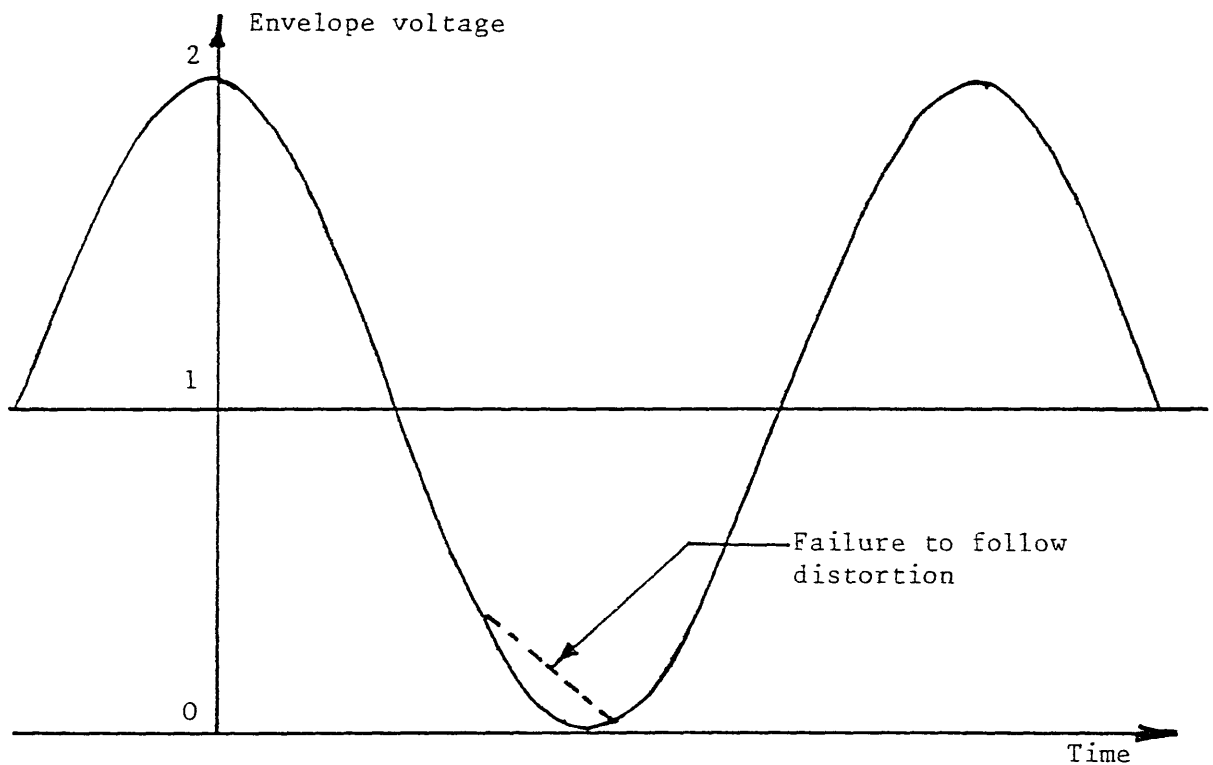
FIGURE 7.3.1

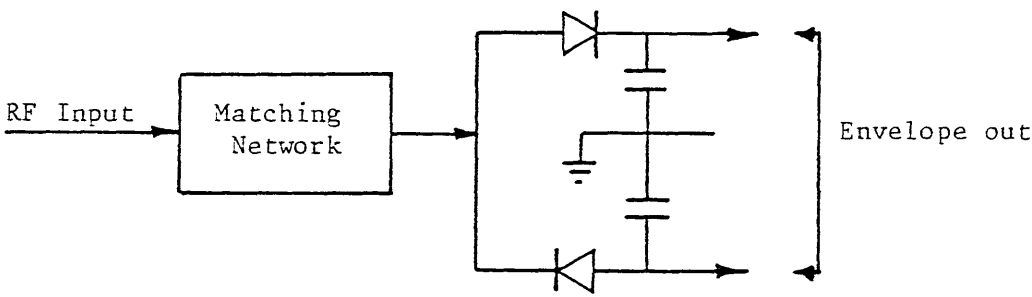
FIGURE 7.3.2

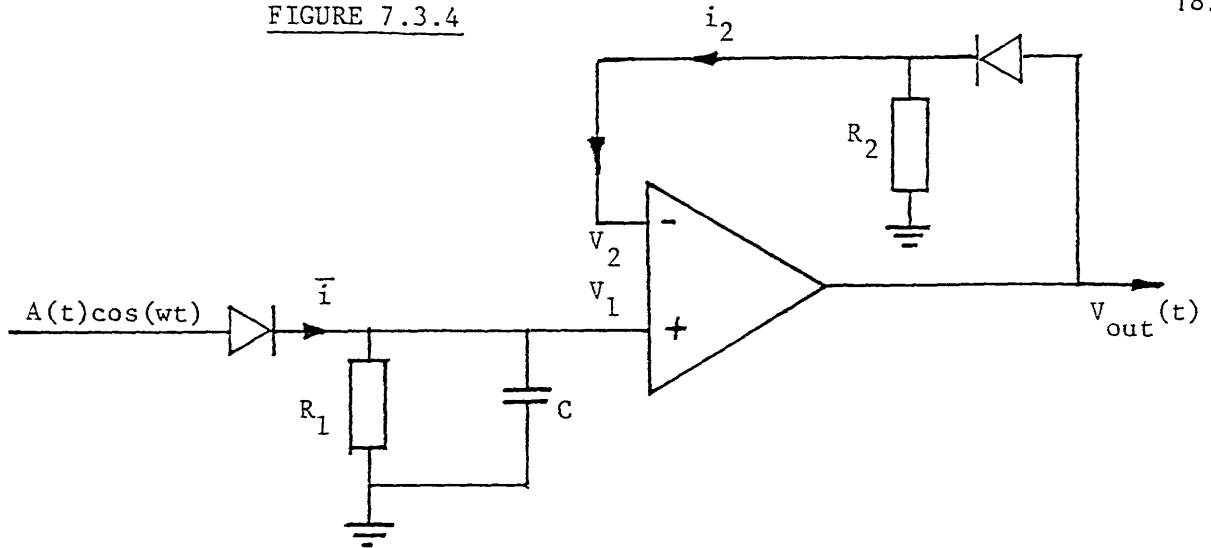


Failure to follow distortion (sometimes called diagonal clipping) is illustrated. It occurs on the falling edge of the envelope voltage. To identify the existence of failure to follow distortion we observe the following:

For AM test waveforms with cosine waves, the true envelope is described by $1+m\cos(\omega t)$ which is even with respect to time. Any AM/AM distortion will preserve this evenness. Failure to follow destroys the evenness, and so introduces quadrature terms into the envelope spectrum. By a quasi-linear approach this can be detected.

FIGURE 7.3.3





Analysis

Diode characteristic equation $i = i_s (\exp(eV/nkT) - 1)$

All symbols have their usual meanings (see section 7.3.1)

Thus the dc current in the resistor R_1 is given by

$$i_{dc} = \bar{i} = i_s \overline{(\exp(e(A(t)\cos(wt) - R_1 \bar{i})/nkT) - 1)} \quad \text{where bars represent averages.}$$

$$= i_s \left[\overline{(\exp(e(A(t)\cos(wt))/nkT)} \cdot (\exp(-eR_1 \bar{i}/nkT) - 1) \right]$$

$$= i_s (I_0(eA(t)/nkT) \cdot \exp(-eR_1 \bar{i}/nkT) - 1)$$

Where $I_0(x)$ is a modified Bessel function (reference 1, 135).

Thus the dc voltage at the noninverting terminal of the op-amp is given by

$$V_1(t) = R_1 \bar{i} = i_s R_1 (I_0(eA(t)/nkT) \cdot \exp(-eV_1(t)/nkT) - 1) \quad \text{equ 1.}$$

Equation 1 is essentially Lazarus's result from reference 104.

Now if we apply the diode equation to the diode in the inverting feedback path

$$V_2(t) = R_1 i_2 = R_1 i_s (\exp(eV_{out}(t)/nkT) \cdot \exp(-eV_2(t)/nkT) - 1) \quad \text{equ 2.}$$

Now assuming a perfect op-amp, we must have $V_1(t) = V_2(t)$ thus comparing equ 1 and equ 2 we find (provided $R_1 = R_2$)

$$I_0(eA(t)/nkT) = \exp(eV_{out}(t)/nkT) \quad \text{equ 3.}$$

$$\text{i.e. } V_{out}(t) = (nkT/e) \cdot \ln(I_0(eA(t)/nkT)) \quad \text{equ 4.}$$

Equation 4 describes the detector characteristics up to large signal RF inputs, when the diode equation no longer applies due to breakdown.

Equation 4 shows improved temperature performance over the basic detector, (due to the removal of the i_s term, which has a strong temperature dependence). The linearity of equation 4 is also considerably improved over the basic detector.

FIGURE 7.3.5

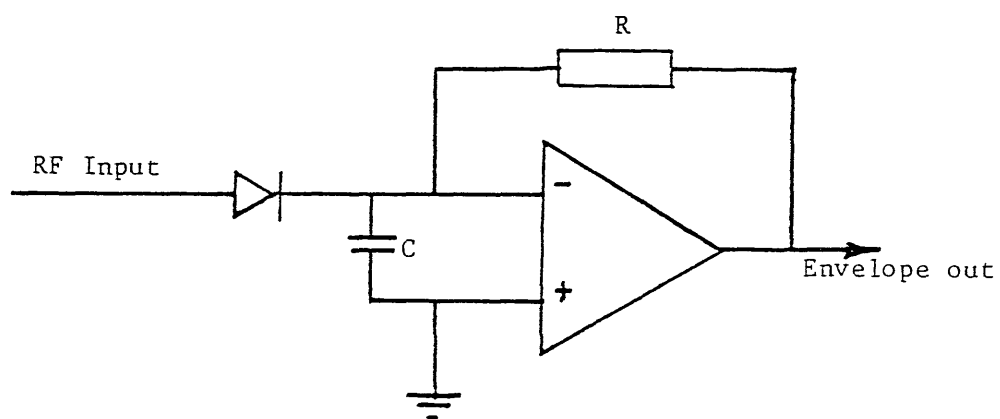
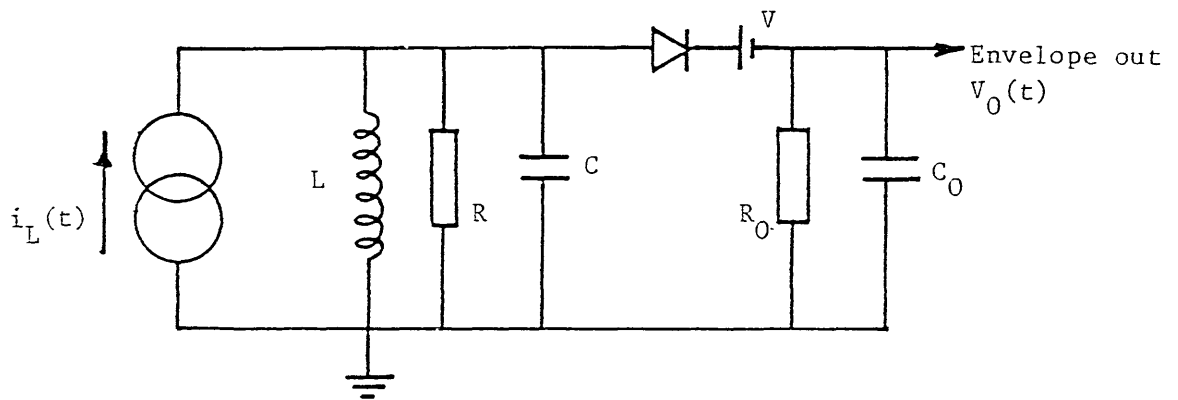


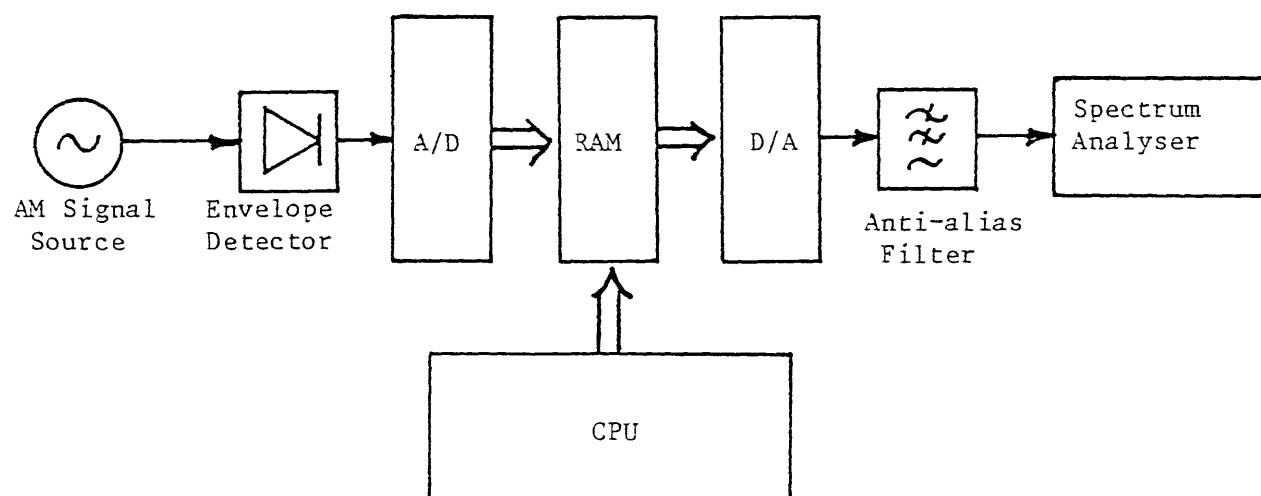
FIGURE 7.4.1



The diode is assumed perfect, any turn on voltage being accounted for by V

$$i_L(t) = I(1 + m\cos(\omega_a t))\cos(\omega t)$$

FIGURE 7.4.2



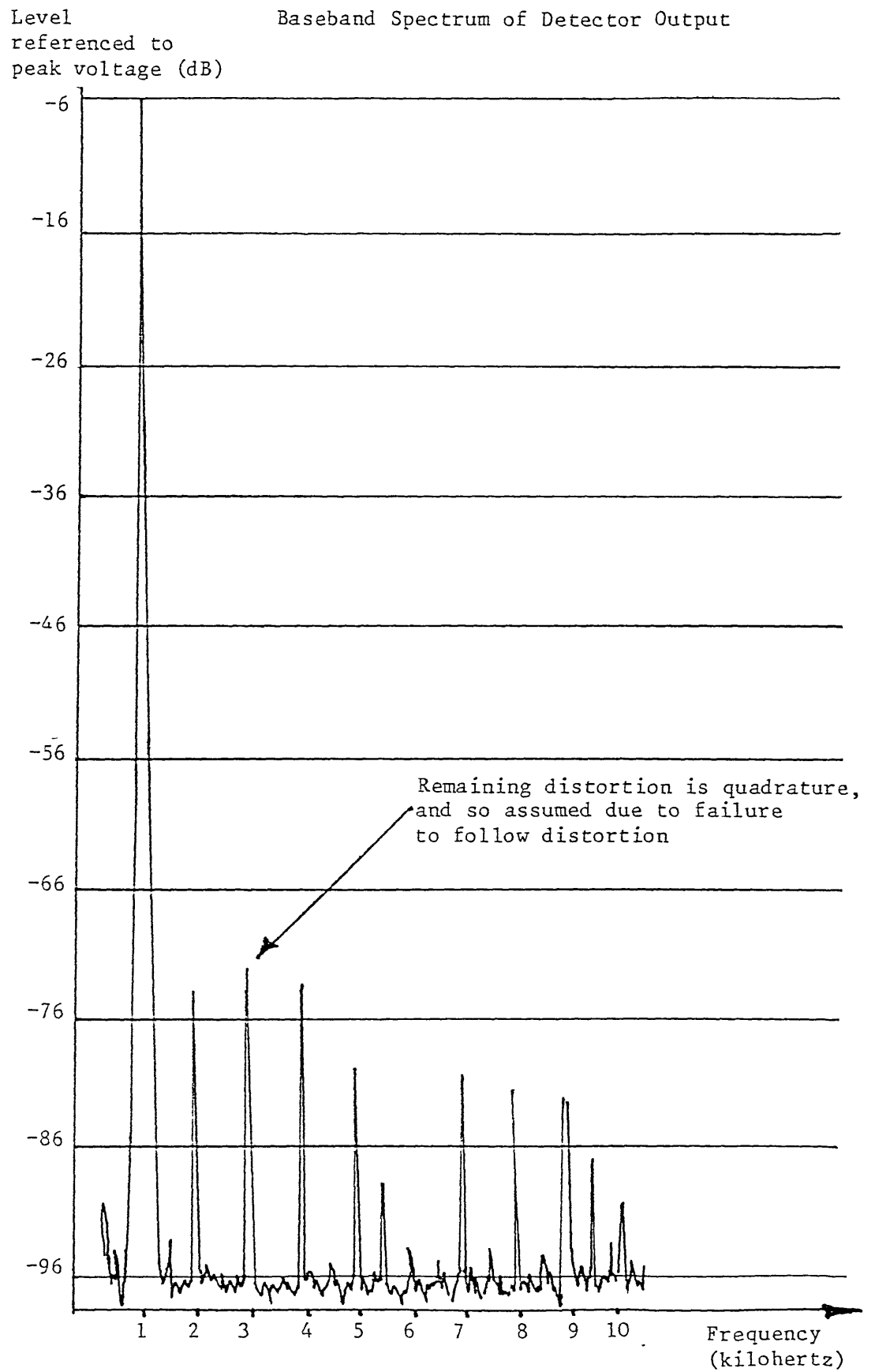


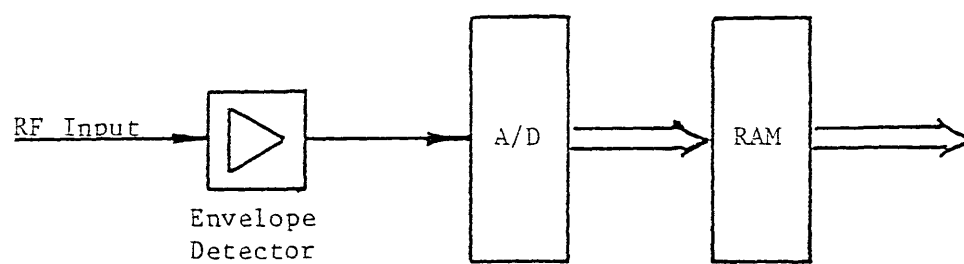
FIGURE 7.5.1

TABLE 7.6.1

A/D Output	Envelope	A/D Output	Envelope	A/D Output	Envelope
4035	$V_{\max} \cdot 1$	281	$V_{\max} \cdot k^9$	27	$V_{\max} \cdot k^{18}$
2866.5	$V_{\max} \cdot k$	208	$V_{\max} \cdot k^{10}$	20	$V_{\max} \cdot k^{19}$
2038.5	$V_{\max} \cdot k^2$	155	$V_{\max} \cdot k^{11}$	14	$V_{\max} \cdot k^{20}$
2039	$V_{\max} \cdot k^3$	118	$V_{\max} \cdot k^{12}$	9	$V_{\max} \cdot k^{21}$
1452.5	$V_{\max} \cdot k^4$	91	$V_{\max} \cdot k^{13}$	5	$V_{\max} \cdot k^{22}$
1035	$V_{\max} \cdot k^5$	71	$V_{\max} \cdot k^{14}$		
740	$V_{\max} \cdot k^6$	57	$V_{\max} \cdot k^{15}$		
532	$V_{\max} \cdot k^7$	45	$V_{\max} \cdot k^{16}$		
385	$V_{\max} \cdot k^8$	35	$V_{\max} \cdot k^{17}$		

Where the A/D output is given as a fraction, this is to account for the A/D's inability to choose between the two nearest integers.

We arbitrarily set $V_{\max} = 4035$ for convenience.

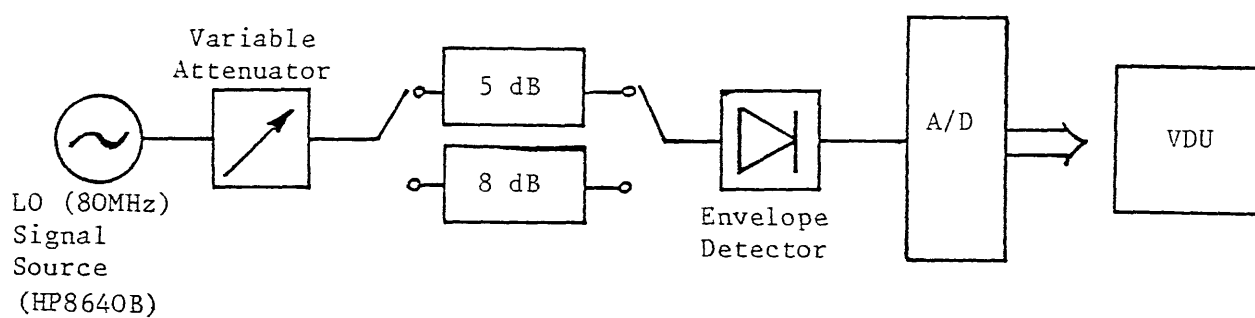
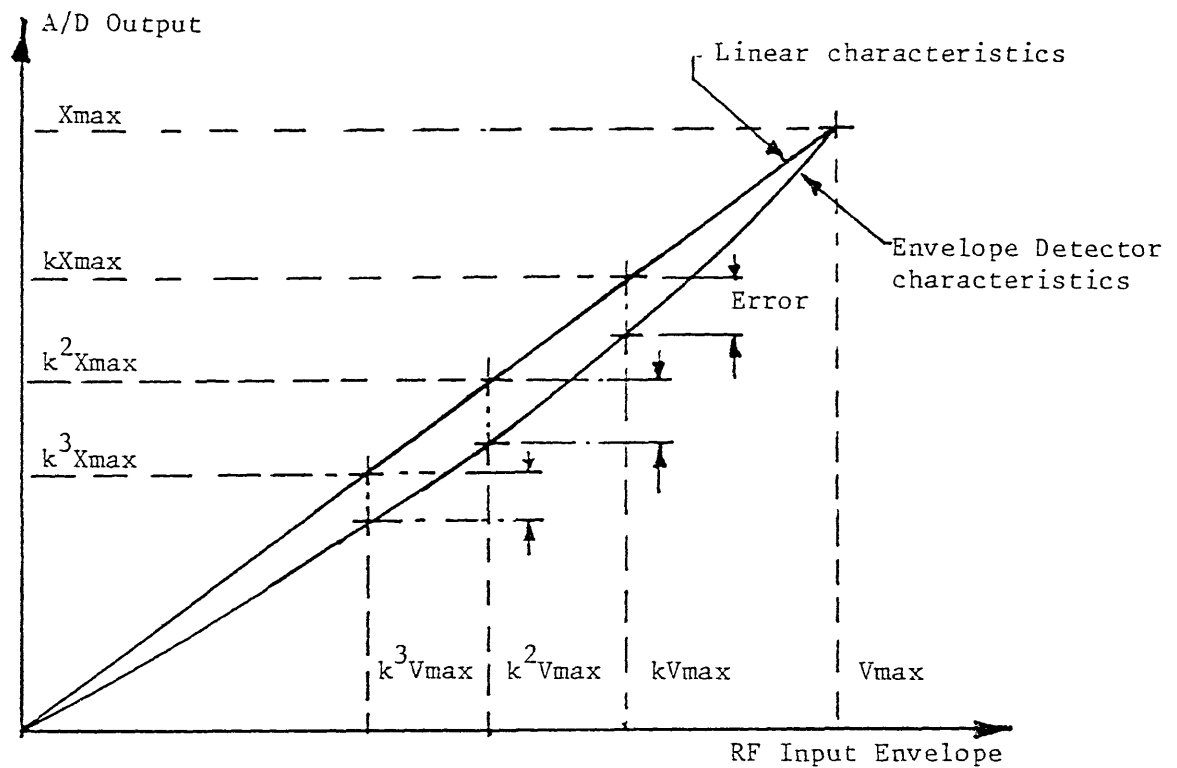
FIGURE 7.6.1

FIGURE 7.6.2



We reduce the RF input envelope by a factor of k . For a linear detector, the detector output should fall by a factor of k .

The difference between the actual fall and the expected fall (k) indicates the error.

Once we know the value of k and the error values, the envelope detector characteristics can be deduced.

FIGURE 7.6.3

We consider the effect of tolerancing.

The A/D output is measured to an accuracy of ± 0.5

After each measurement the envelope is kept constant, when the attenuation is switched out, by means of a variable attenuator.

The accuracy to which it is reset is ± 0.5

Thus if we complete the table with tolerancing it reads:

A/D Output	Envelope
4035 ± 0.5	4035 ± 0.5
2866.5 ± 0.5	$(4095 \pm 0.5) k$
2038.5 ± 0.5	$((4095 \pm 0.5) k \pm 0.5) k$
etc.	etc.

The error in the envelope term ultimately dominates. Plainly this error grows like $.5(1+k+k^2-k^3+k^4\dots\dots)$ assuming worst case choice of sign. If we take the summation to infinity, then by the binomial expansion we find the error tends to $0.5/(1-k)$

This is shown:

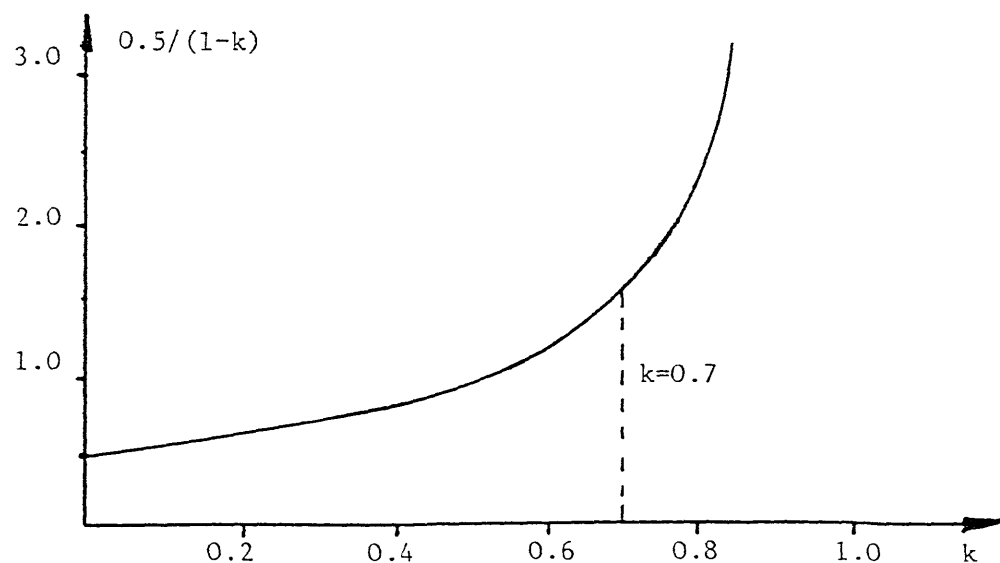
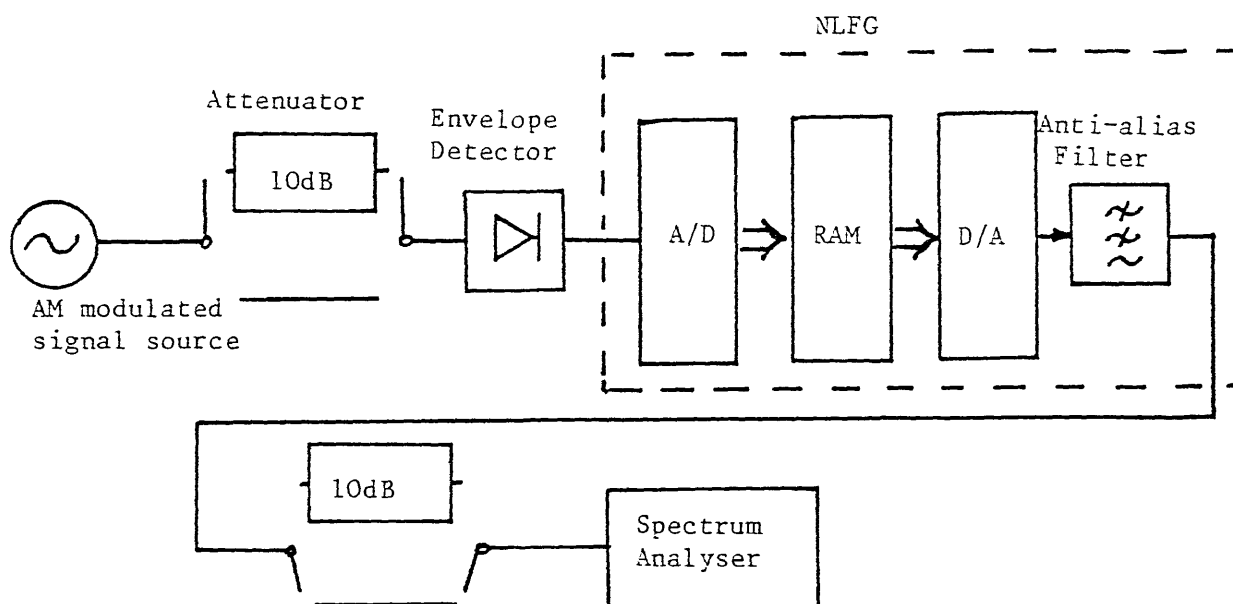


FIGURE 7.6.4



Typical Spectrum

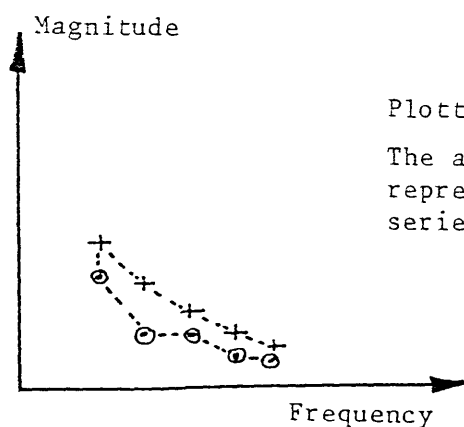
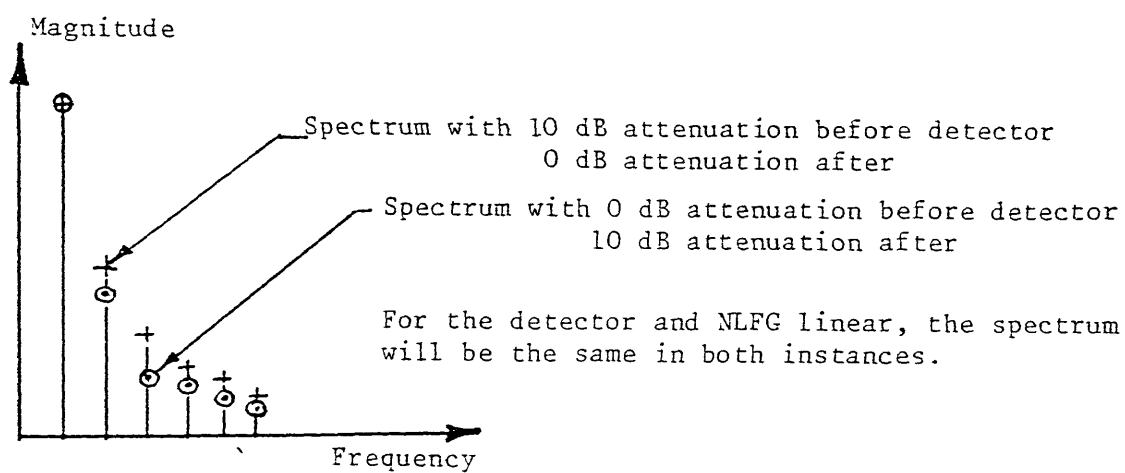


FIGURE 7.6.5

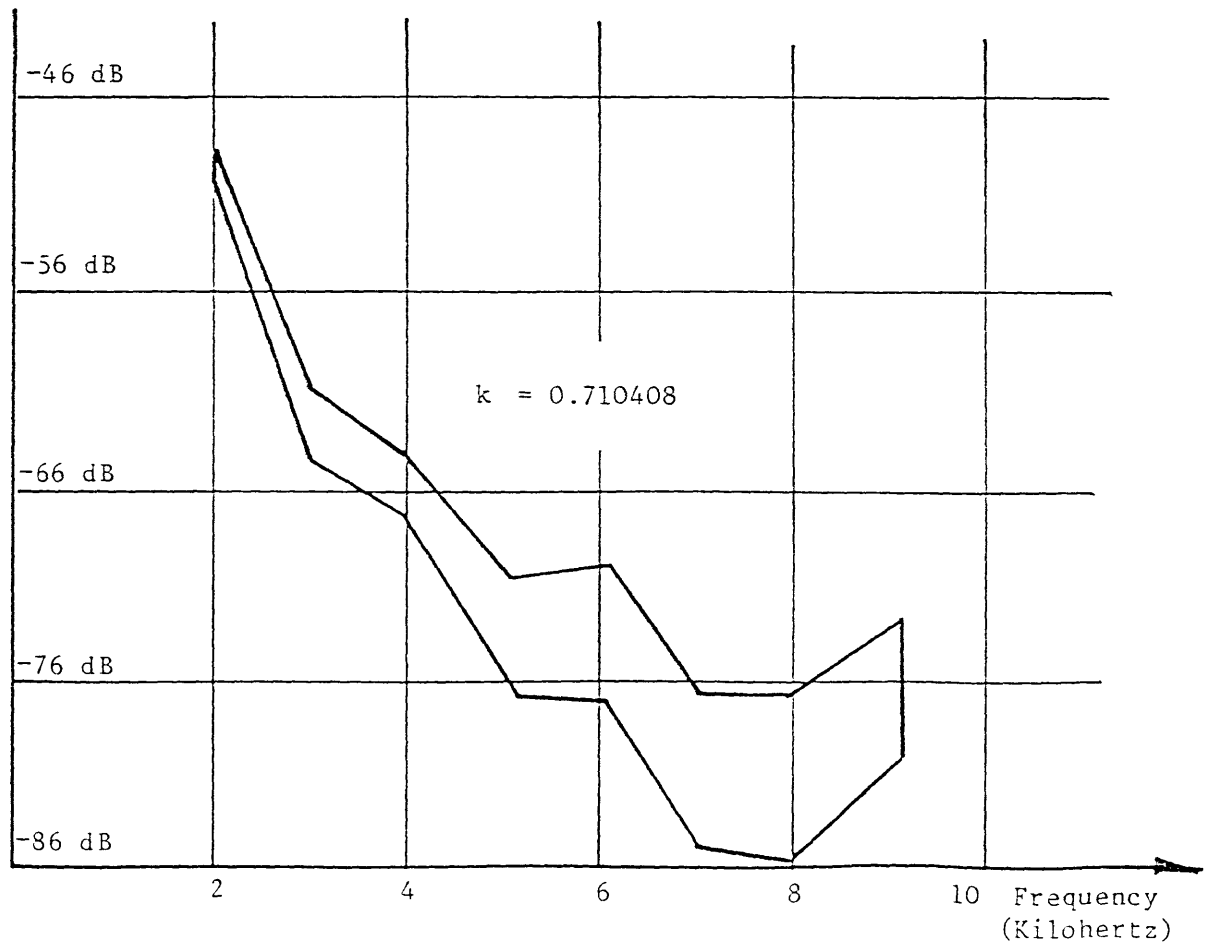
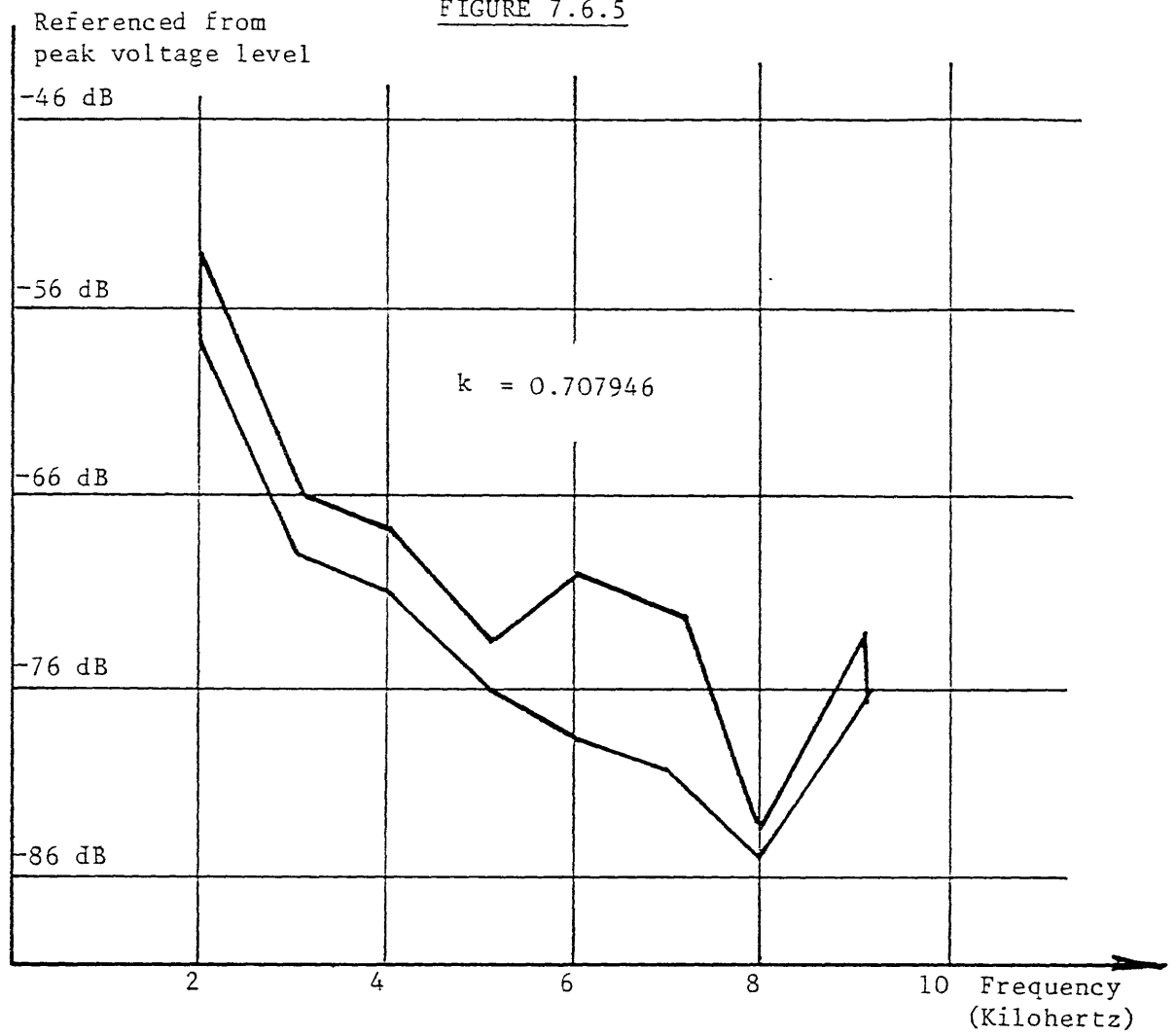


FIGURE 7.6.6

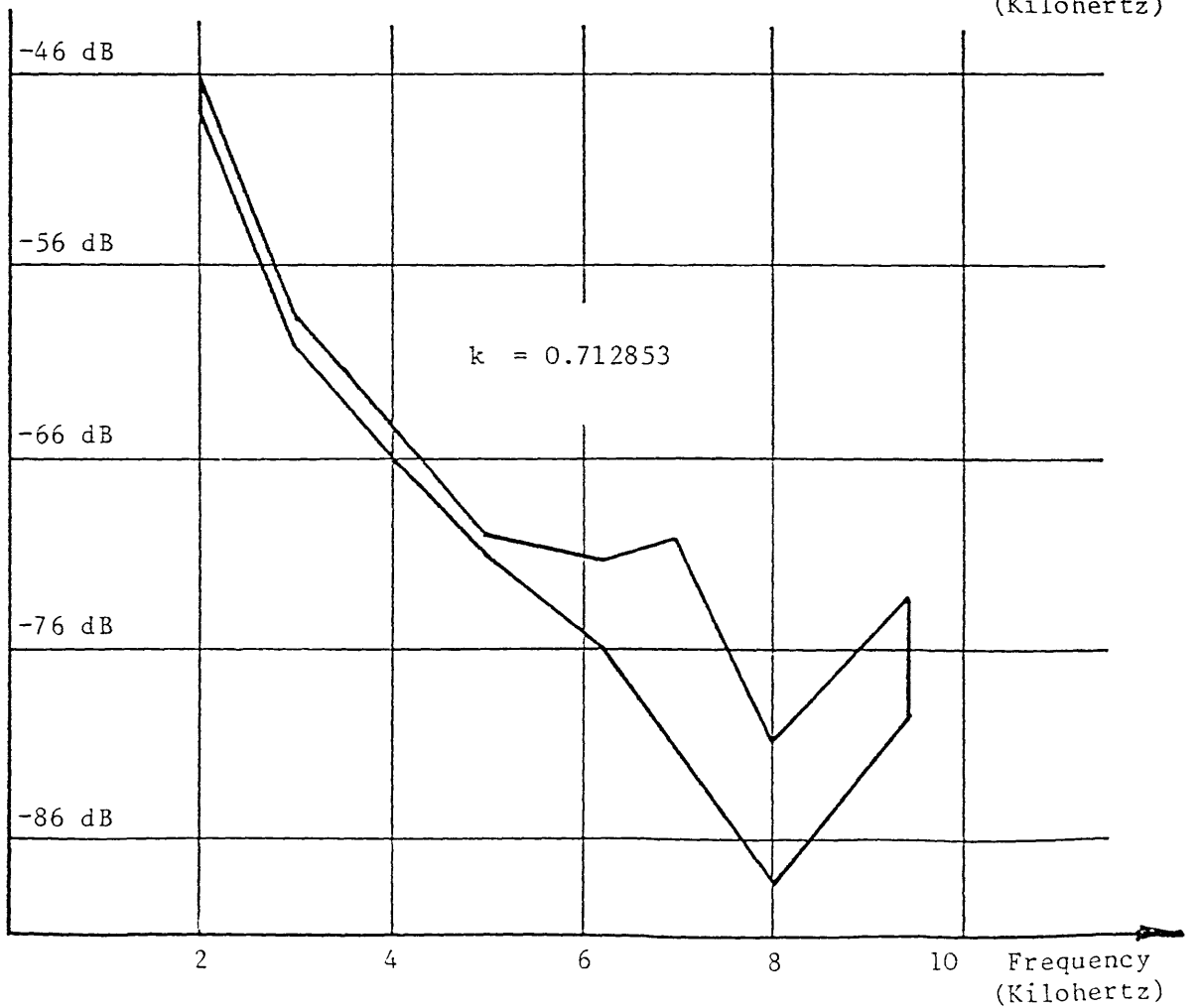
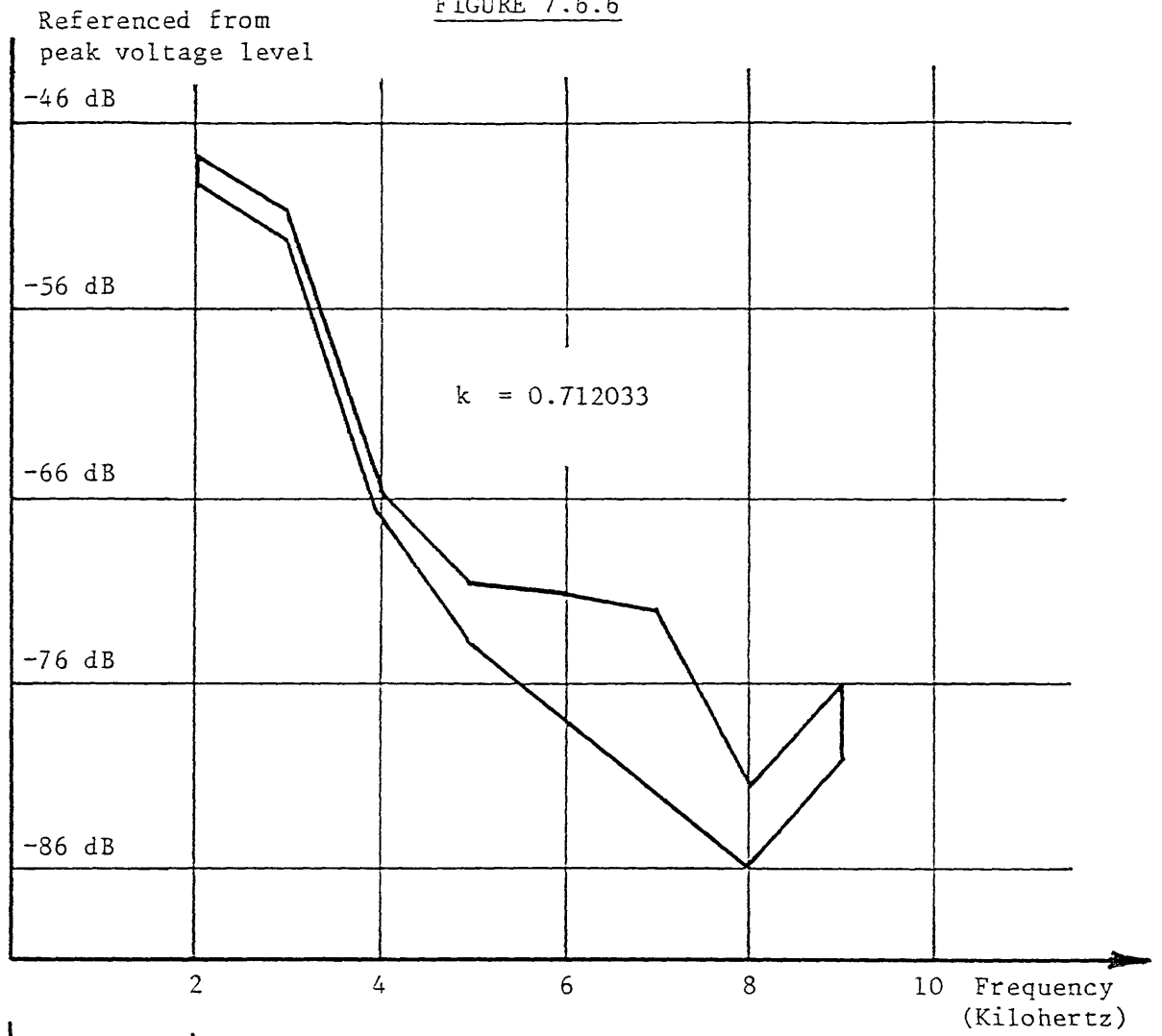


FIGURE 7.6.7

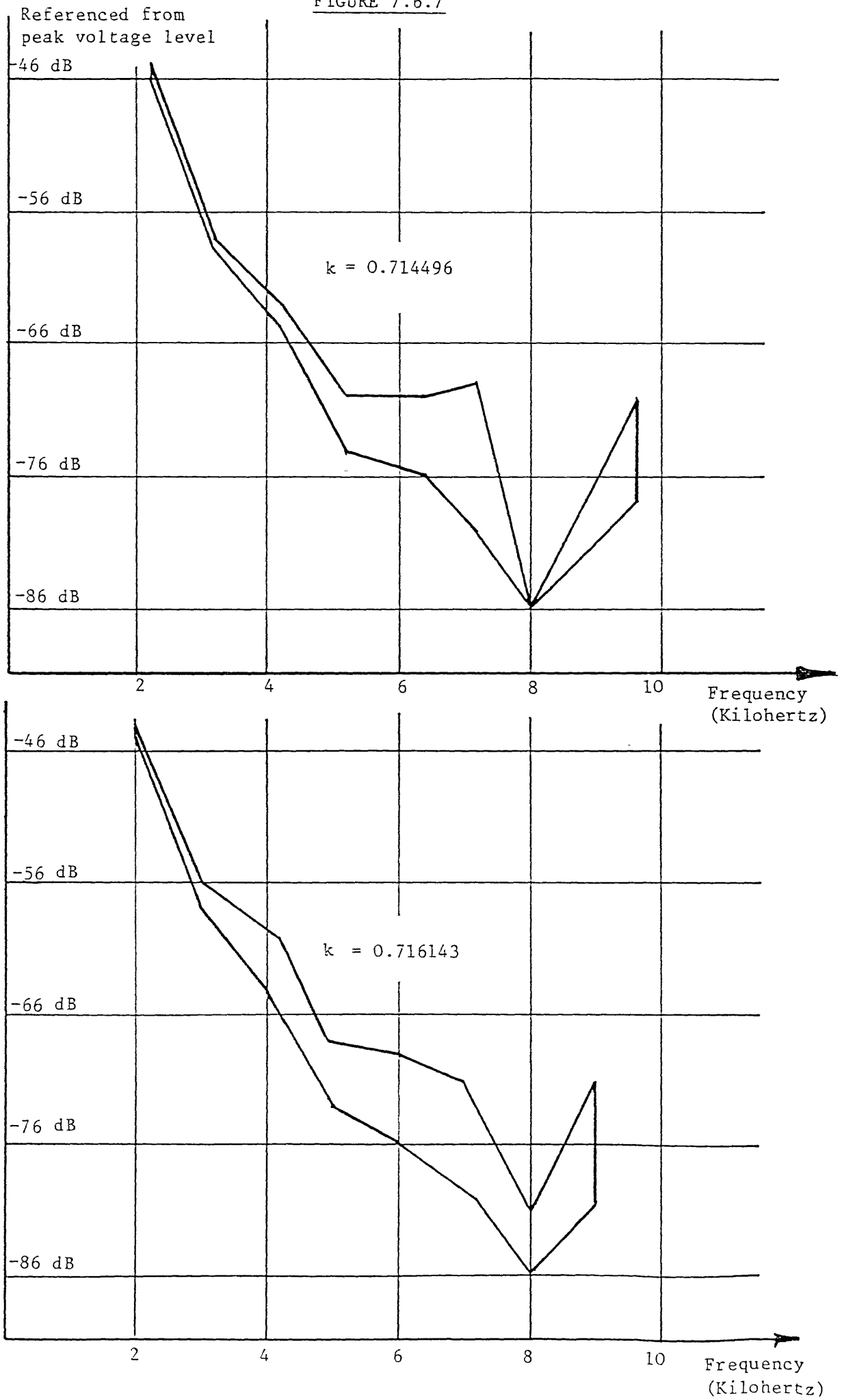


FIGURE 7.6.8

Results of assuming the detector is linear

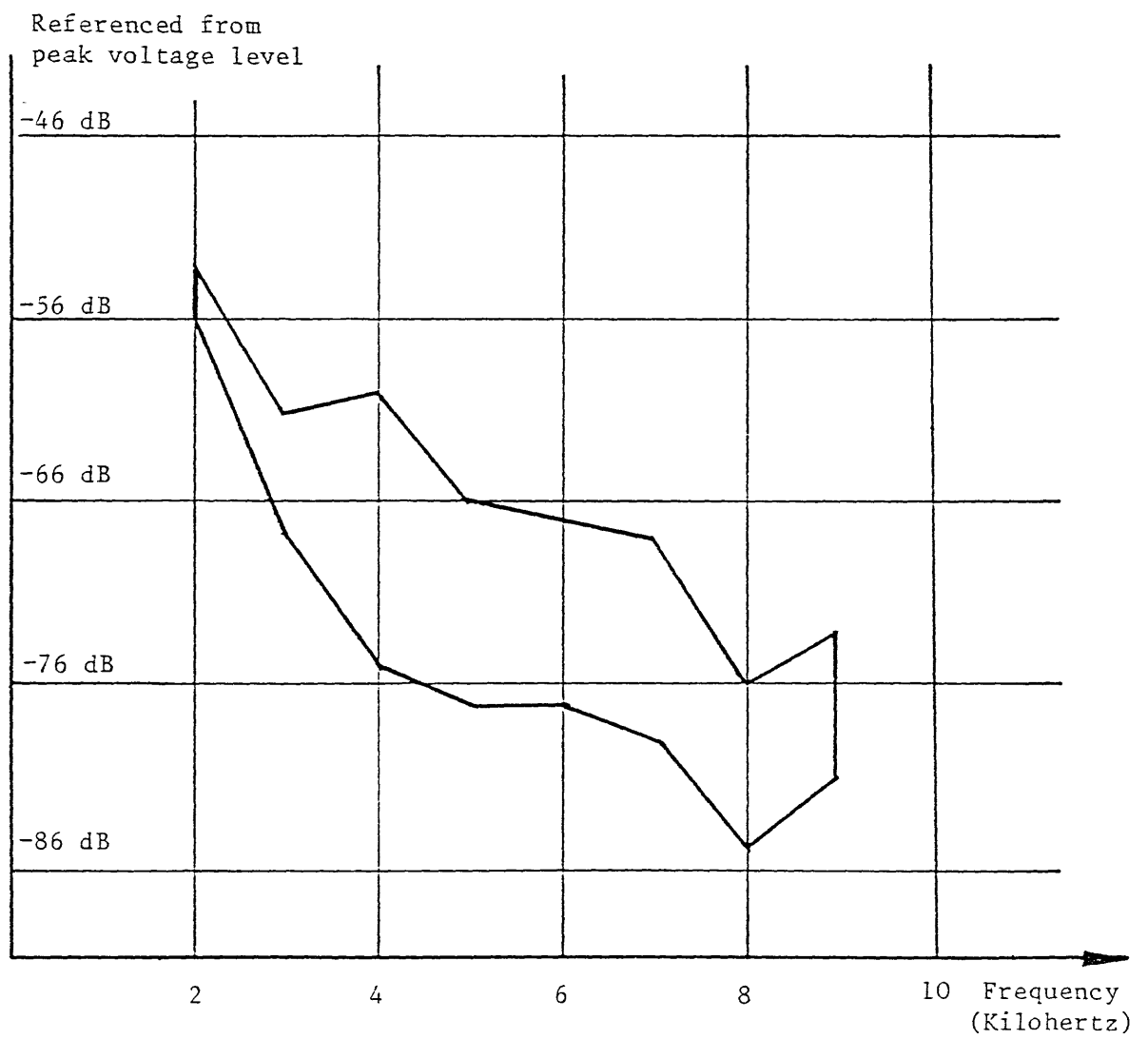


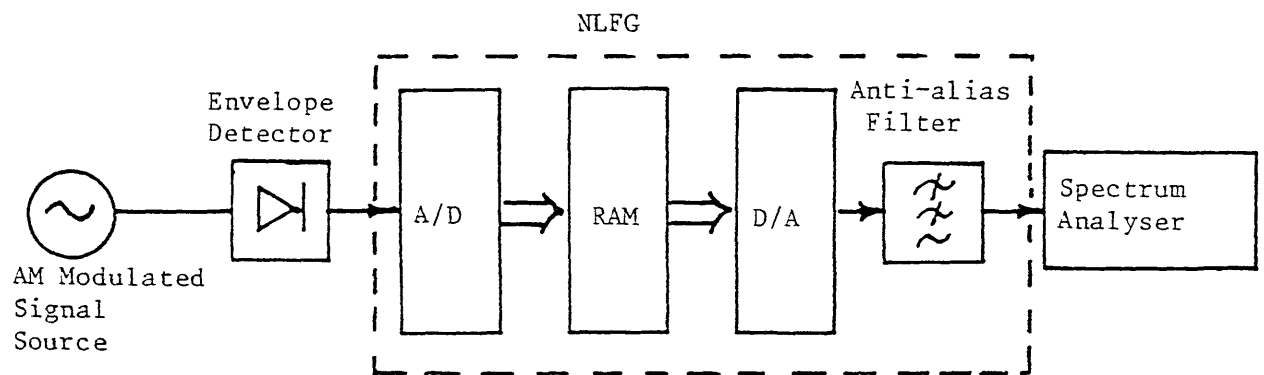
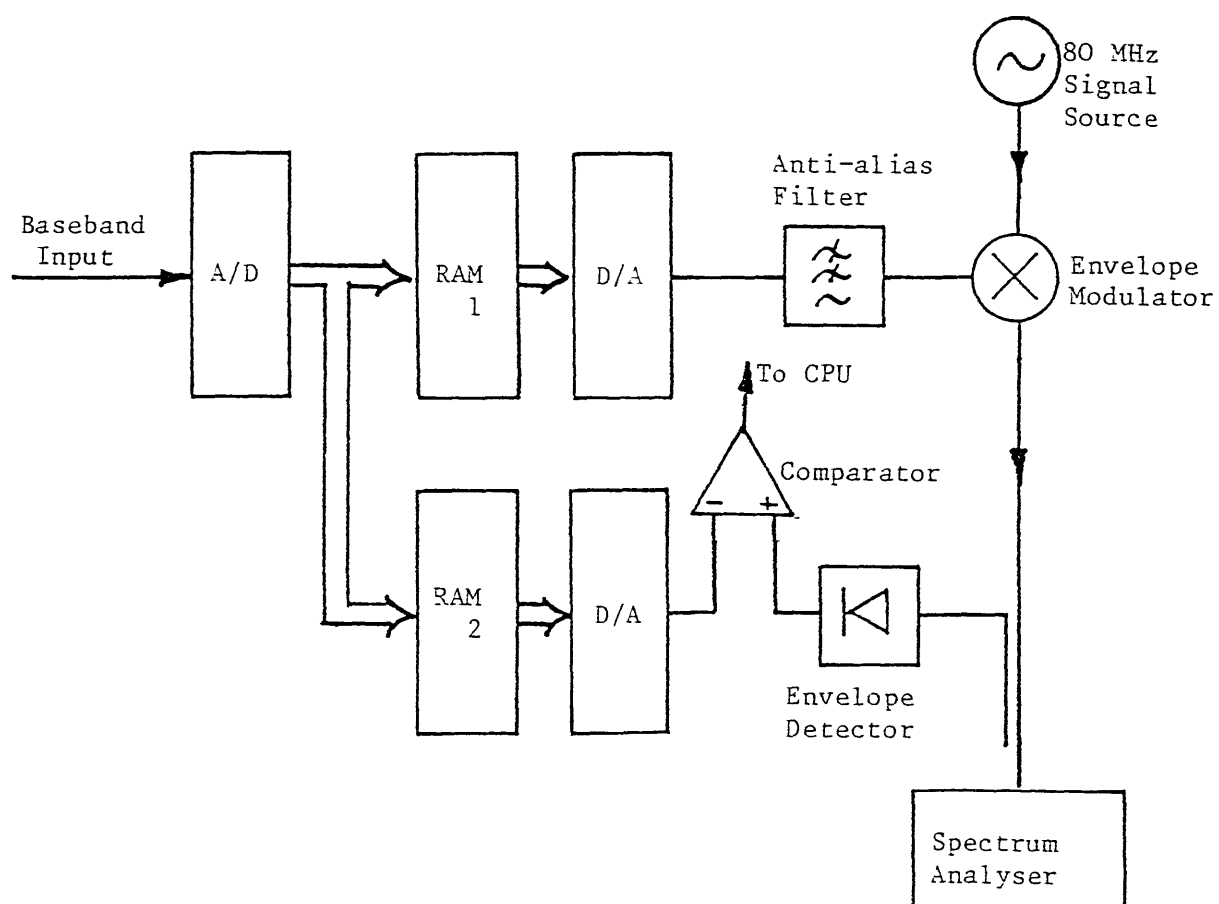
FIGURE 7.6.9

FIGURE 7.6.10



CHAPTER 8

THE PHASE DETECTOR

8.1 INTRODUCTION

For any polar, adaptive, digital compensation scheme the need for a phase detector is inevitable. A variety of phase detectors are in common usage [see ref 100, 146]. This study concentrated on square law (mixer) detectors, preceded by limiters (see Figure 8.1.1). This represents one of the simplest ways of measuring phase for RF signals, and is commonly used at RF frequencies. It is possible to heterodyne the input signal to the detector to a lower frequency and use different phase detection principles, but this represents a degree of complexity that we would rather avoid. Most of the techniques used in this chapter carry over to other phase detector types.

Thus the principal area of study is that of the limiter and the following mixer.

Unlike envelope detectors a phase detector requires two inputs, the test signal and a reference signal. The phase of the test signal is compared against the reference. The way this reference is provided determines the type of control algorithm that is applicable (types 1, 2 and 3 as discussed in chapter 3), but does not affect the design of the phase detector.

In this study the reference was always readily available as a constant envelope signal (since this study dealt primarily with AM systems). Under other circumstances the reference may need to be derived from a time-varying envelope signal. In such cases the limiter work presented here is applicable in minimising errors in

deriving the reference.

8.2 EFFECT OF INACCURATE PHASE DETECTION

For the envelope detector the result of inaccuracies in the envelope detection was to give rise to AM/AM distortion at RF. (Thus, if the envelope detector introduced a spurious sideband at -50 dB on fundamental, the adaptive action would be to remove this sideband by producing an anti-phase sideband at -50 dB at RF). Hence, as the inaccuracies in the envelope transferred over almost directly to the RF spectrum, the envelope detector needed to be linear to the same level as the noise floor of the A/D. For the phase detector no such obvious relationship holds.

We therefore develop a crude model relating the inaccuracies of the limiter to the spectral effects on our standard AM RF test signal. We assume the model of Figure 8.2.1, where the RF signal at the output of the system is given by

$$V_{\max}(1 + m\cos(w_a t))\cos(wt + \phi + 20K\log(V_{\max}(1 + m\cos(w_a t)))) \quad (8.1)$$

where K is the AM/PM conversion coefficient in radians/dB. Now we concentrate on the phase term

$$\begin{aligned} & 20K\log(V_{\max}(1 + m\cos(w_a t))) \\ &= 20K\log(V_{\max}) + 20K\log(1 + m\cos(w_a t)) \end{aligned} \quad (8.2)$$

Neglecting the first term as being a constant we consider

$$20K\log(1 + m\cos(w_a t)) \quad (8.3)$$

Now Carr [ref 29, p.428] gives

$$\begin{aligned} & 0.5\log_e(1 + 2a\cos(x) + a^2) \\ &= \cos(x) - (a^2/2)\cos(2x) + (a^3/3)\cos(3x) \dots \quad \text{if } |a| < 1 \end{aligned} \quad (8.4)$$

Manipulating the left-hand side of equation 8.4 we get

$$0.5\log_e(1 + 2a\cos(x) + a^2)$$

$$\begin{aligned}
&= 1.151 \log[(1 + a^2)(1 + (2a/(1 + a^2))\cos(x))] \\
&= 1.151(\log(1 + a^2) + \log(1 + (2a/(1 + a^2))\cos(x))) \quad (8.5)
\end{aligned}$$

Combining with equation 8.4 we get

$$\begin{aligned}
&\log(1 + (2a/(1 + a^2))\cos(x)) \\
&= (1/1.151)(\cos(x) - (a^2/2)\cos(2x) \dots) - \log(1 + a^2) \quad (8.6)
\end{aligned}$$

Now let

$$m = (2a/(1 + a^2)) \quad (8.7)$$

$$\text{so that} \quad a = [1 \pm (1 - m^2)^{1/2}]/m \quad (8.8)$$

where $|a| < 1$ and $|m| < 1$. Hence from equation 8.6 and equation 8.3 we get

$$\begin{aligned}
&20K \log(1 + m \cos(w_a t)) \\
&= (20K/1.151)(\cos(w_a t) - (a^2/2)\cos(2w_a t) \dots) - 20K \log(1 + a^2) \quad (8.9)
\end{aligned}$$

Now we approximate the right-hand side of equation 8.9 for $|a| < 1$ to get

$$(20K/1.151)\cos(w_a t) + \text{constant} \quad (8.10)$$

Since the constant term in equation 8.10 does not produce any phase modulation sidebands we take little interest in it. We now proceed to substitute from equation 8.10 into equation 8.1. We then expand in terms of Bessel functions, using the well-known result that

$$\begin{aligned}
&\cos(wt + \phi + (20Ka/1.151)\cos(w_a t)) \\
&= \sum_n J_n(20Ka/1.151)\cos(wt + \phi + nw_a t) \quad (8.11)
\end{aligned}$$

and noting that for small x [ref 1]

$$J_n(x) \sim (0.5x)^n/n! \quad \text{if } n > 0 \quad (8.12)$$

$$J_{-n}(x) = J_n(x) \quad \text{if } n \text{ even} \quad (8.13)$$

$$J_n(x) = -J_{-n}(x) \quad \text{if } n \text{ odd} \quad (8.14)$$

$$\text{thus} \quad J_0(x) \approx 1 \quad (8.15)$$

$$\text{and} \quad J_{-1}(x) \approx -x/2 \quad (8.16)$$

$$\text{and} \quad J_1(x) \approx x/2 \quad (8.17)$$

Hence for small arguments of $J_n(x)$ equation 8.11 can be approximated by

$$\begin{aligned} & \cos(\omega t + \phi) + (20Ka/2(1.151))\cos(\omega t + \phi + \omega_a t) \\ & - (20Ka/2(1.151))\cos(\omega t + \phi - \omega_a t) \end{aligned} \quad (8.18)$$

If we now substitute this approximation into equation 8.1, we can obtain the magnitude of the phase sidebands (S) referenced to PEP. Note here that we take as the phase sideband magnitude the magnitude of the first distinguishable phase sideband. This is the upper sideband formed by

$$m\cos(\omega_a t)(20Ka/2.302)\cos(\omega t + \phi + \omega_a t) \quad (8.19)$$

Hence

$$S = 20\log((20Ka/2.302)(m/2)/(1 + m)) \quad (8.20)$$

$$\text{where} \quad m = 2a/(1 + a^2) \quad (8.21)$$

Equation 8.20 holds provided

$$a \gg (a^2/2) \quad \text{and} \quad (20Ka/1.151) \ll 1 \quad (8.22)$$

Using equation 8.20 for a simple example, let $m=0.98$ and $K=2.0E-3\text{rad/dB}$ (approximately 0.11deg/dB), (taken from reference 132 p.78, see Figure 8.2.2), then we obtain $a=0.817$ and $(20Ka/1.151) = 0.0284$. Thus equation 8.22 appears satisfied, and $S=-49\text{ dB}$ from equation 8.20.

Clearly the limiter of reference 132 (p.75-78) is not comparable with the degree of IMD suppression that we require. Using equation 8.20 and letting $m=0.98$ we find that for $S=-72\text{ dB}$ we require $K=1.43E-4$ (approximately 0.008 degrees/dB).

The model used tacitly assumes that the limiter output is always a constant envelope. In those cases where the AM modulation depth does reach 100%, the RF signal actually

disappears entirely, and so it is impossible for the limiter to maintain a constant output envelope. However, if the limiter gain is large, the time for which limiting does not occur is small (see Figure 8.2.3). Thus for large limiter gain, the error caused by assuming that the limiter output envelope is constant, is confined to a very small portion of the perceived AM/PM characteristic (see Figure 8.2.3). In practice the smoothing applied to the perceived characteristic removes the error introduced by the failure of the limiting action.

The distinction between smoothing of the characteristic in the RAM table and, as occurs in phase feedback, at the output of the limiter is important. The latter can cause instability so that some loss of performance at limiter failure is inevitable (e.g. as with polar loops). Filtering the RAM table does not cause instability, and so no loss of performance is involved.

Even if the limiter does not completely fail to maintain a constant envelope output, it may carry some residual AM modulation. For the limiter/mixer type of phase detector discussed here this causes the output of the phase detector to vary i.e. it causes an effect indistinguishable from AM/PM conversion. Thus, provided we define K as the effective AM/PM conversion coefficient of the detector, our model still holds.

8.3 LIMITER TYPE

There are two basic limiter types, the amplifier limiter and the AGC limiter. They are illustrated in Figure 8.3.1 and Figure 8.3.2 respectively. The requirements on the limiter are:

1. High gain, to give as large a limiting range as possible.

2. Low AM/PM conversion, to maintain inherent accuracy of the phase detector.

The first criterion is easily met by cascading limiters to achieve high gain [ref 132 p.75-80], or by heterodyning and using multiple IF's, each with high gain. (The latter spreads the high gain over different frequency bands, and thus is a more stable approach).

The second criterion is less easy to meet. The phase shift of a limiter chain from reference 132 p.75-78 is shown in Figure 8.2.2. Fishstein [ref 52, 53] discusses this problem and points out that AM/PM conversion often determines the fundamental metrological parameter in measuring phase.

Reference 53 puts forward some simple formulae relating the transition frequency (f_t) of an amplifier limiter to its phase shift. The precise relationship is temperature dependent. The higher the transition frequency the lower the AM/PM conversion. Reference 52 discusses some compensation circuits for amplifier limiters.

Another approach to the problem is the AGC limiter, where only one component needs good AM/PM performance (the gain control device), and so by judicious choice of gain controller lower AM/PM conversion can be achieved, along with lower temperature dependence. Reference 53 is a comparative study of an AGC limiter and an amplifier limiter. It concludes that, in general, the AGC limiter provides superior characteristic over the amplifier limiter, although compensation techniques are not so well developed for AGC limiters, reducing the performance difference between the two.

In this study two high performance RF limiters were built,

one an AGC limiter using a pin diode attenuator as the control element, the other used the Plessey SL532C low phase shift limiter (a high f_t amplifier limiter). Details are given in Appendix F.

8.4 PHASE SENSITIVE DEVICE TYPE

Having obtained a constant envelope signal the final component of the phase detector is the phase sensitive device. A number of different devices may be used (reference 141 gives a discussion of possible devices).

However, the most commonly used RF device is the double balanced mixer, which acts as a square law device. Its operation is described in Figure 8.1.1. References 28 and 179 discuss how imperfections in a mixer and environmental changes affect the performance as a phase detector.

Provided the mixer operates in a constant temperature environment, and is operated near its quadrature point, the detector provides a near linear estimate of the phase. The action of any adaptive control algorithm is to force the mixer towards its quadrature operating point (i.e. the reference and the test signal are approximately 90 degrees out of phase).

8.5 EVALUATION OF LIMITER PERFORMANCE

The test setup is shown in Figure 8.5.1. In essence we remove the AM/AM conversion of a 100% AM modulated RF signal by envelope quasi-linear. The remaining AM/PM is to be removed by the phase locked loop arrangement shown in Figure 8.5.1. By seeing just how suppressed the IMP's are it is possible to competitively evaluate the two limiter types. The results are

shown in Figure 8.5.2 and Figure 8.5.3. Also shown are the levels predicted by equation 8.20 for 100% AM. (We use the manufacturers data sheet values to determine K for the amplifier limiter, and measure the AGC limiter phase shift using a Hewlett Packard vector voltmeter (HP8405A)).

Our conclusions are much as reference 53 would suggest. Although superficially the AGC limiter may be expected to be superior, there is in fact little comparative difference between the two limiters. Whichever limiter is used, the AM/PM conversion is still obtrusive for a 12 bit A/D digital compensation scheme, although for the AGC limiter the asymmetric effects are more dominant.

The test setup of Figure 8.5.1 (set up to compare limiter types), could also have been used to compare phase sensitive devices, although it was not used as such.

8.6 THE PHASE DETECTOR AND DIGITAL COMPENSATION

Just as the digital nature of the compensation technique allowed for improvement in the envelope detector linearity and noise performance, so too for the phase detector.

The linearity of the phase detector may be improved by a precalibration scheme, and the noise performance improved by simple averaging (as the phase noise is assumed uniformly distributed from -180 degrees to +180 degrees with zero mean).

8.7 IMPROVEMENTS IN DETECTOR LINEARITY

We note that a type 3 control algorithm does not require linear detectors, but this type of control algorithm is not always

applicable (discussed later), and so in general digital compensation does require linear phase detectors.

As with the envelope detector we develop two methods, one based on the absence of AM/PM conversion in attenuators, and the other based on a known RF envelope and phase.

A common method for calibrating phase modulators is to modulate with phase deviations that cause nulls of the carrier or other spectral effects. Reference 188 discusses the tolerancing effect of this approach. Although reference 188 does not consider using frequency dividers it is clear from the reference that there will be considerable difficulties in obtaining the accuracies required.

8.7.1 Attenuator Calibrated Detector

The test setup is shown in Figure 8.7.1. The objective is to calibrate a phase detector. For convenience the phase detector chosen was a Hewlett Packard vector voltmeter (HP8405A). This phasemeter is specified to a phase accuracy of 1.5 degrees when both channels have equal voltages (i.e. it uses identical limiters in the A and B channels, the differential phase shift is 1.5 degrees). Resolution of phase is 0.1 degrees. The basic method is illustrated in Figure 8.7.1. The measured phase shift is assumed to be a function of the four parameters:

1. Phase shift due to level x at channel A ($P_a(x)$).
2. Phase shift due to level x at channel B ($P_b(x)$).
3. Phase shift due to n dB attenuator (P_n) assumed independent of RF level.
4. Meter ($Offset$).

The implementation of the method is explained in Figure 8.7.1. By making a series of measurements we can deduce a correction factor for the phase detector.

Proceeding then to the test setup in Figure 8.7.2 the phase shift across the DUT is determined. (In Figure 8.7.2 the phase shift as observed by the HP8405A with, and without, precalibration is shown). From the experiment shown in Figure 8.7.2 the correction required for the phase detector used in the phase locked loop is apparent.

The method appears to be limited by the 0.1 degree resolution of the phasemeter.

8.7.2 Phase Quasi-linear

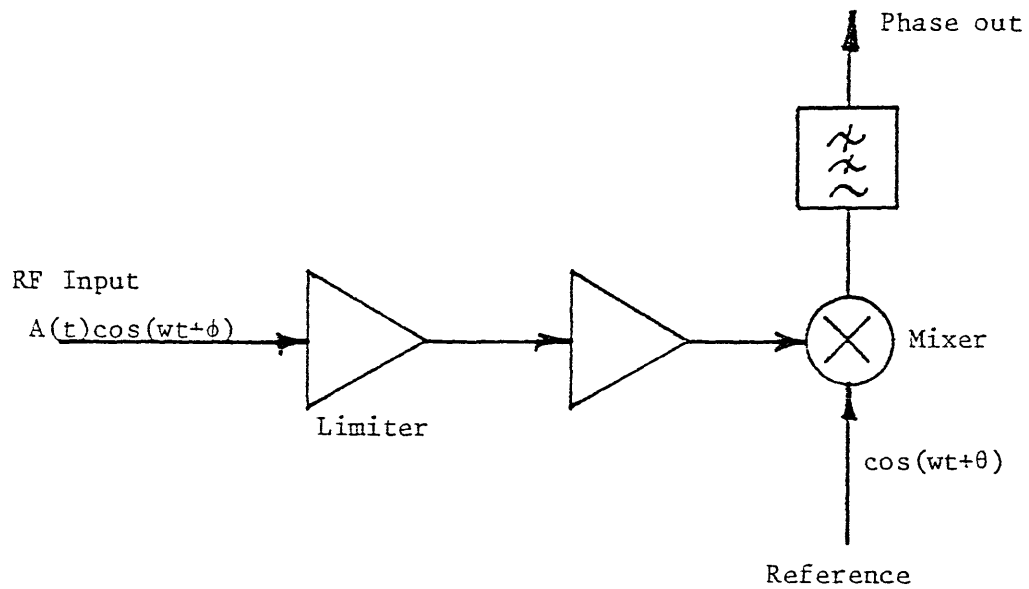
The basic method is shown in Figure 8.7.3. We require an AM modulated RF signal source, which is AM/PM free. This acts as the input for the phase detector. The output of the phase detector goes to an A/D, RAM, D/A, the output of which goes to a spectrum analyser. The output of the phase detector should be constant (i.e. constant phase angle), and we assume that any phase deviations that do exist are due to AM/PM conversion in the detector. Using quasi-linear we remove all ac components of the series combination of phase detector and NLFG. Thus the necessary correction data is formed in the RAM table.

The implementation of this method requires a "perfect" AM modulated RF source. This is provided by the envelope modulator of chapter 7, section 7.6.2, (i.e. cascaded pin diode attenuator). This modulator can be checked for AM/PM conversion by the quasi-linear techniques of chapter 6.

8.8 CONCLUSIONS

Digital compensation can offer advantages for the correction of AM/PM conversion in phase detectors. This has been examined here. It appears from this study that AM/PM spurious sidebands can be reduced to the same level as asymmetric effects, and the calibration techniques can be designed to take the spurious levels below these, although this study has not done so.

FIGURE 8.1.1



Output of limiter

$$f(A)\cos(wt+\phi+g(A))$$

Assuming a square law mixer, output of mixer

$$\begin{aligned} & f(A)\cos(wt+\phi+g(A))\cos(wt+\theta) \\ &= (f(A)/2) (\cos(\phi+g(A)-\theta) + \cos(2wt+\phi+\theta+g(A))) \end{aligned}$$

Thus the phase output is

$$(f(A)/2)\cos(\phi+g(A)-\theta)$$

Typically the circuit is operated with $\theta = \pi/2$

Thus the phase output is

$$(f(A)/2)\sin(\phi+g(A))$$

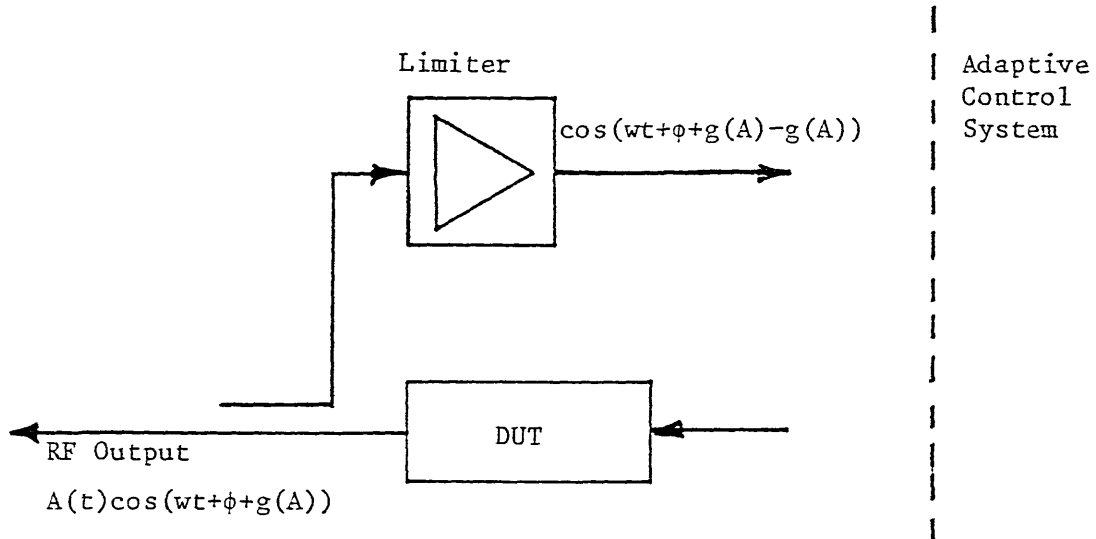
Which for small phase deviations is approximately

$$(f(A)/2) (\phi+g(A))$$

For a perfect limiter $f(A)=\text{constant}$ (arbitrarily set $f(A)=1$) and the AM/PM conversion is zero, thus $g(A)=0$. Hence the output (for an ideal limiter) is proportional to $\phi/2$

FIGURE 8.2.1

We assume the system is running an AM signal



The limiter introduces AM/PM conversion $-g(A)$. The action of the adaptive control system is to phase modulate the RF output by $g(A)$ so that the limiter output is at constant phase.

For our model we take $A(t) = V_{\max}(1+m\cos(w_a t))$

$$g(A) = 20K \log V_{\max}(1+m\cos w_a t)$$

where K is the AM/PM conversion coefficient in radians/dB.

FIGURE 8.2.2

Example of Limiter AM/PM Conversion (from reference 132).

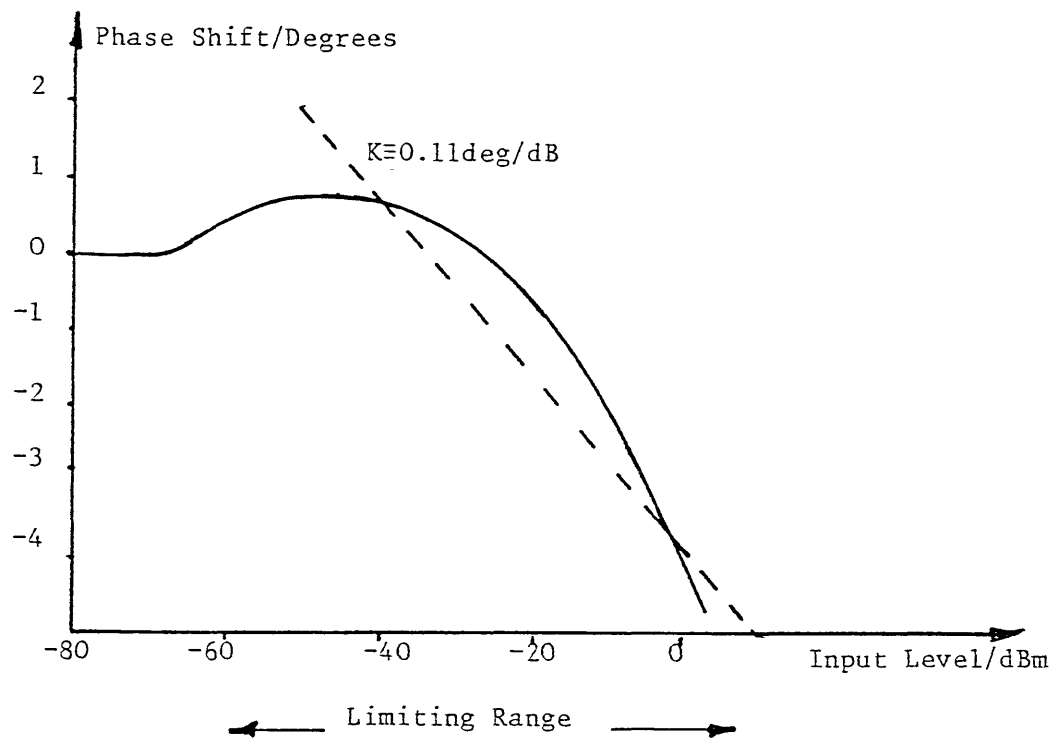


FIGURE 8.2.3

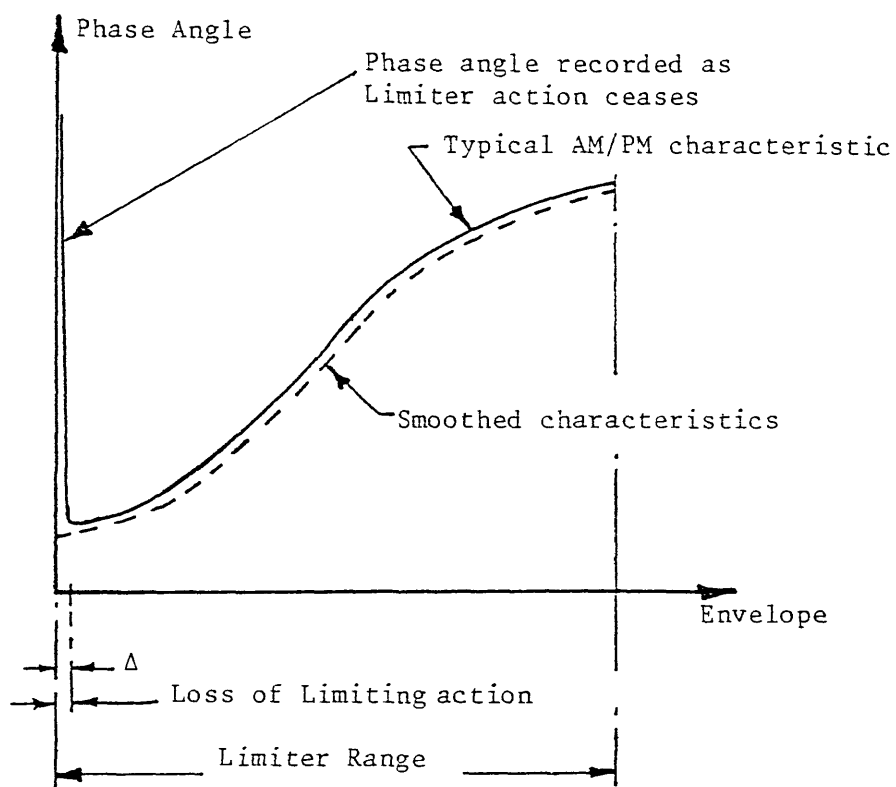
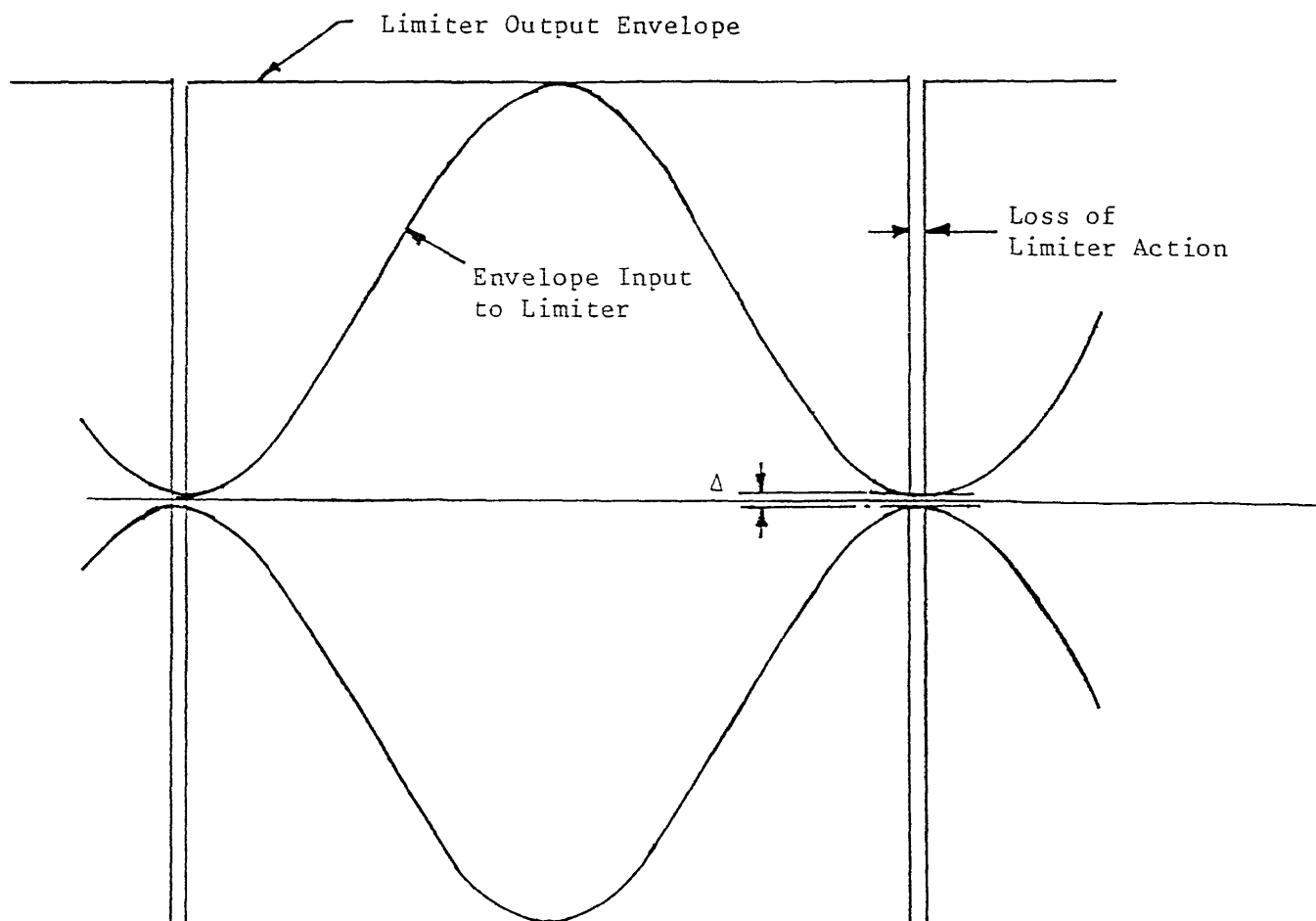
Envelope Waveform

FIGURE 8.3.1

Amplifier Limiter, Limiter action occurs from driving the individual amplifiers into saturation.

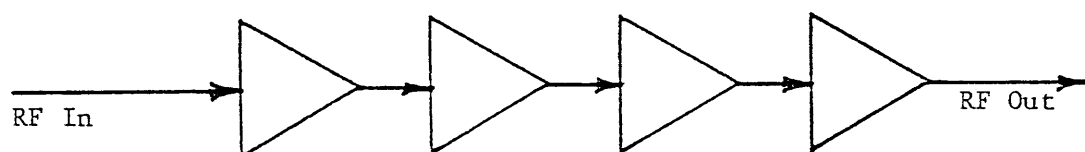
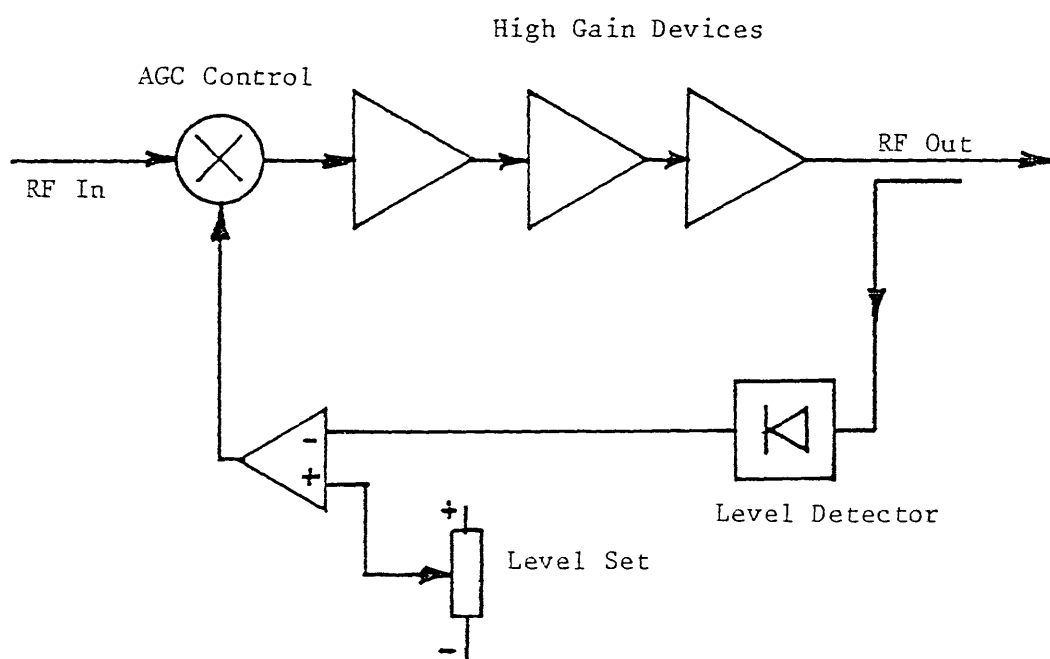
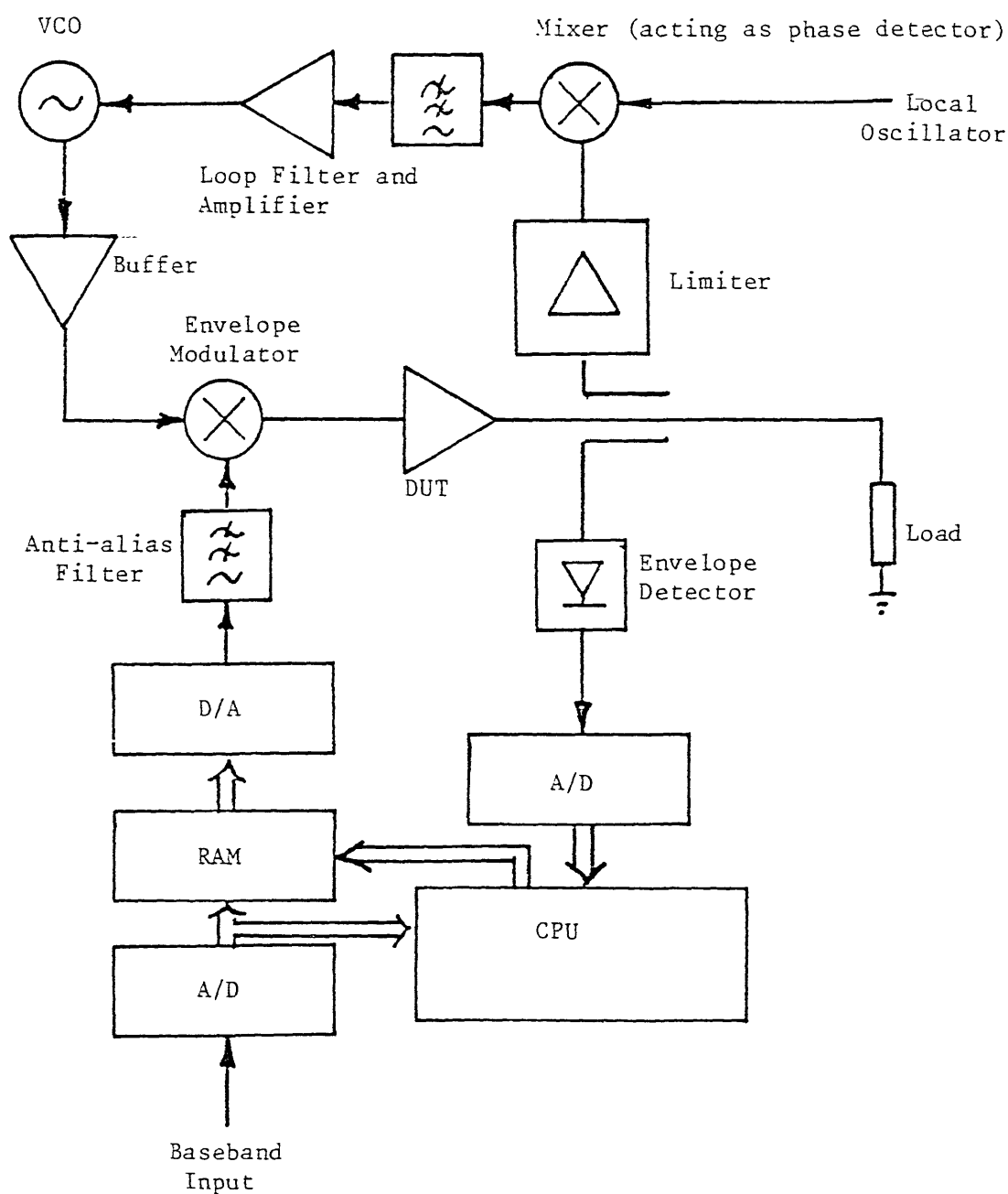


FIGURE 8.3.2



The AGC Limiter operates at constant envelope, except prior to the AGC Control Device.

FIGURE 8.5.1



AM/AM effects are removed by the digital compensation scheme (using either a Type 1 algorithm or quasi-linear).

The remaining spectral distortion is AM/PM. This is reduced by the phase locked loop arrangement. By observing the RF spectrum at the load, comparative tests of the limiters used in the PLL may be made.

FIGURE 8.5.2

RF Spectrum showing the AM/PM sidebands produced using the test setup of Figure 8.5.1 using the AGC limiter.

Levels referenced
from PEP (dB)

Centre frequency 80 MHz, Sideband
frequency 1 KHz.

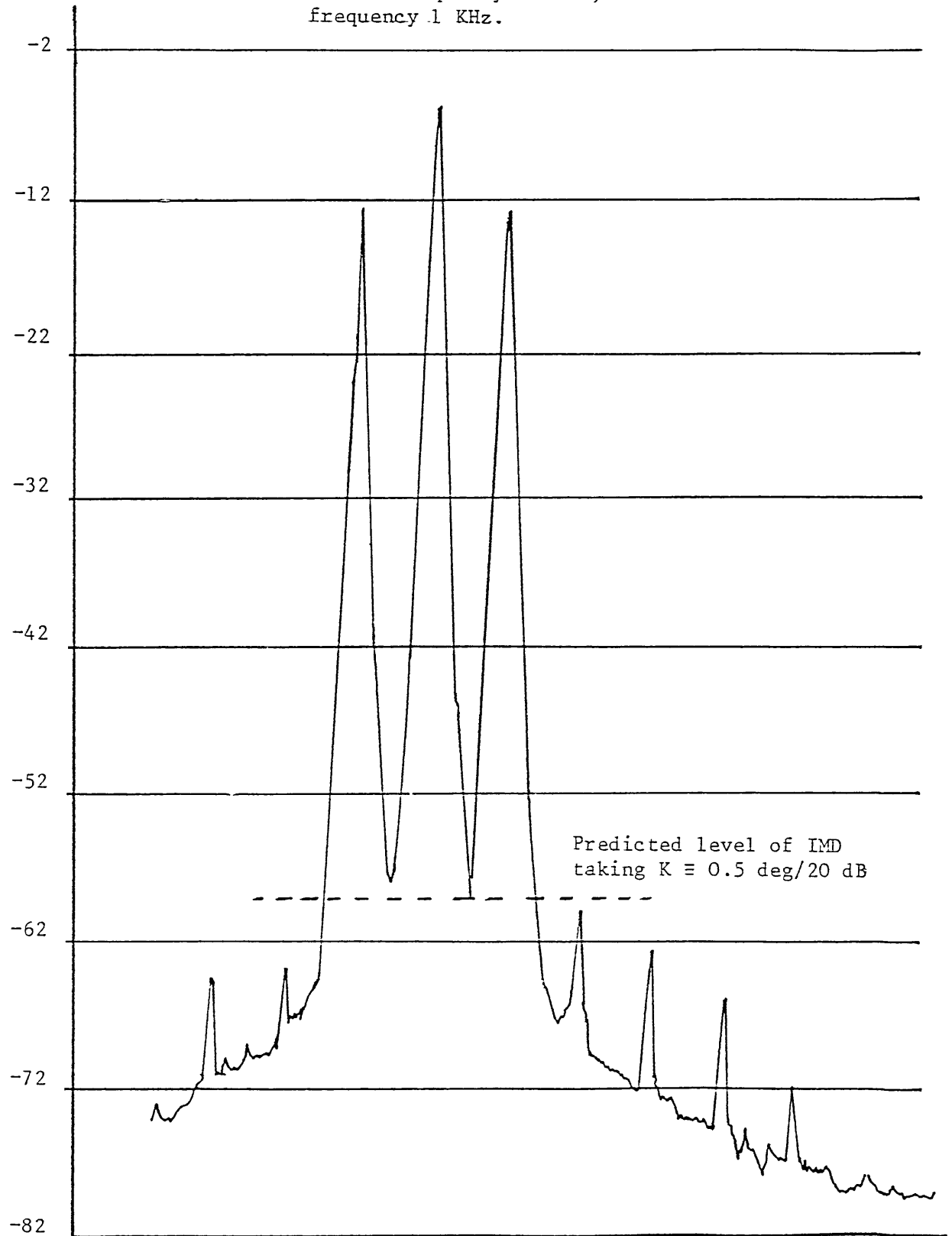


FIGURE 8.5.3

RF Spectrum showing the AM/PM sidebands using the test setup
of Figure 8.5.1 using the Plessey amplifier limiter

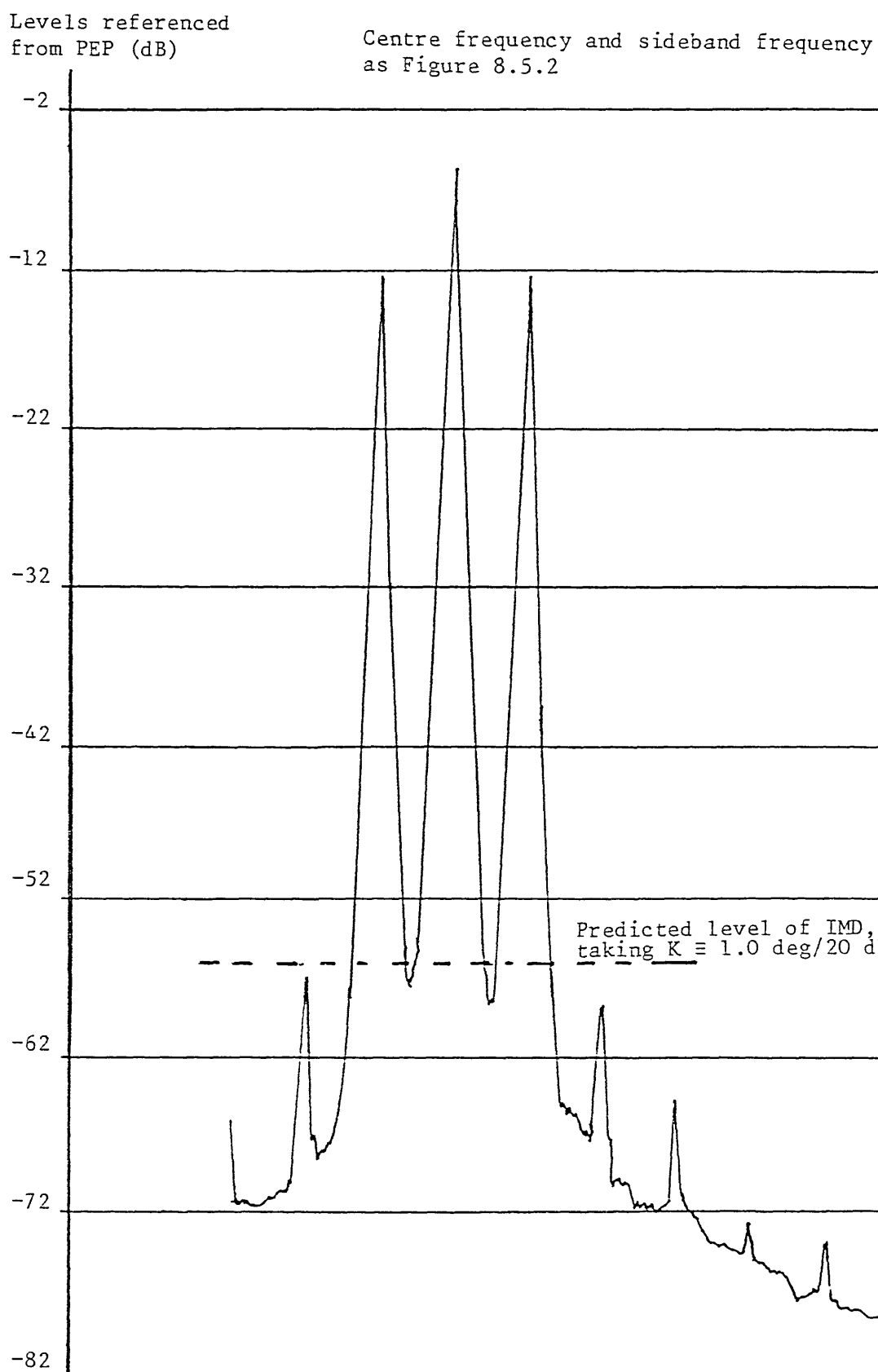
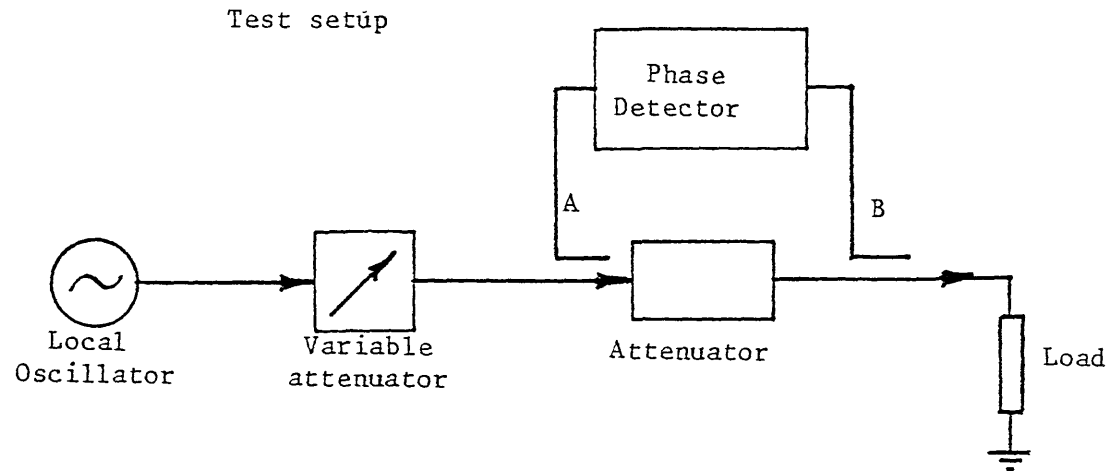
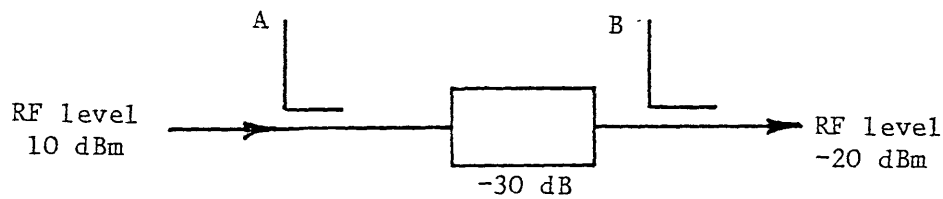


FIGURE 8.7.1

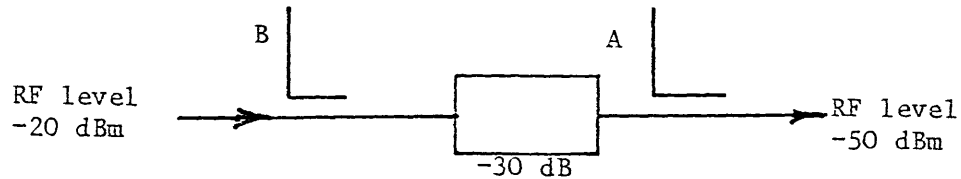


Example

Case 1



Case 2



Phase recorded in Case 1

$$\phi_1 = P_a(10) + P_b(-20) + P_{-30} + O_{\text{ffset}}$$

In Case 2

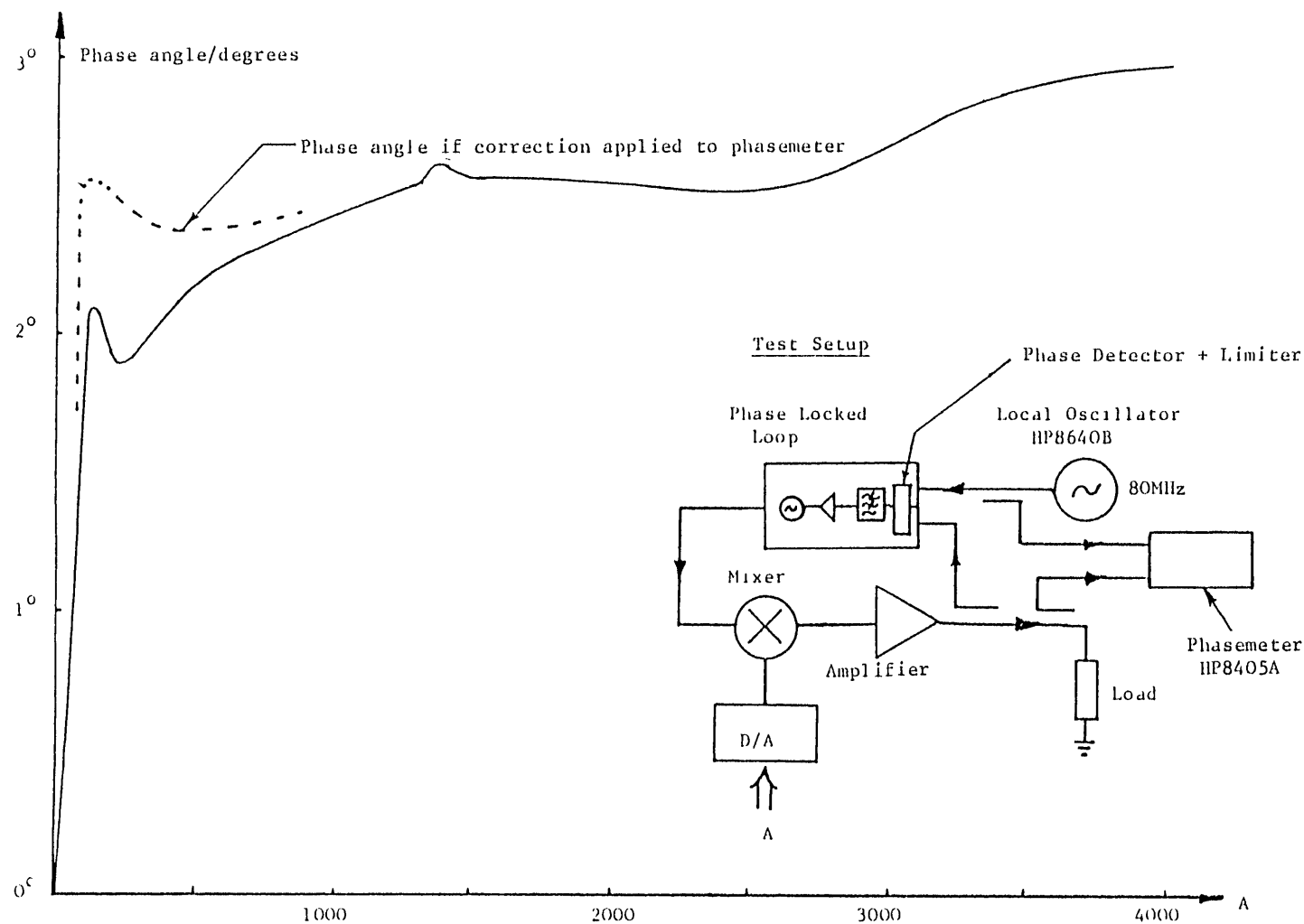
$$\phi_2 = P_a(-50) + P_b(-20) + P_{-30} + O_{\text{ffset}}$$

Hence

$$\phi_1 - \phi_2 = P_a(10) - P_a(-50)$$

By making a series of measurements the AM/PM conversion of channel A is deduced. By a similar set of experiments the conversion due to channel B is deduced.

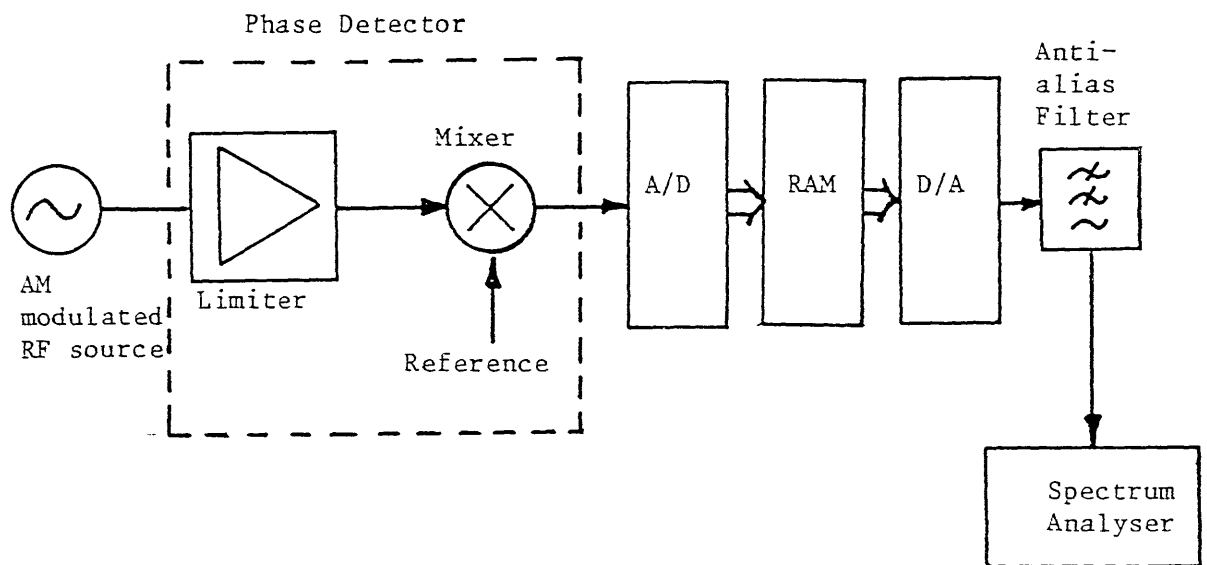
Thus knowing the functions P_a and P_b to within a constant enables a correction factor for the phase to be determined to within a constant. The method is accurate providing the assumption that there are no cross terms (e.g. $P_a \cdot P_b$) in the phase function is true.



Phase shift recorded by HP8405A with and without a correction factor. From this experiment the AM/PM which results from the PLL is apparent, and may be quantified.

FIGURE 8.7.2

FIGURE 8.7.3



CHAPTER 9

TYPE 1 CONTROL ALGORITHMS

9.1 INTRODUCTION

If the waveform passing through the DUT is known a priori, and if this knowledge is used to set up the series NLFG, the control algorithm which sets up the digital NLFG is defined to be a type 1 control algorithm. The a priori waveform is said to be the test waveform.

In any given situation it is possible to envisage a large class of possible test waveforms, and an equally large class of detectors deriving information from the DUT via the test waveform. Nevertheless, it is useful to recognise type 1 algorithms as a single class, since they have much in common. Features common to members of the class include:

1. The requirement for linear detectors.
2. By careful choice of the test waveform some of the deficiencies of the detector may be overcome (e.g. failure to follow distortion).
3. The algorithm can easily be made stable (in the BIBO sense).
4. The interpolation requirements are relatively trivial.
5. There is a requirement that normal operation of the DUT be interrupted for the test waveform.
6. A relatively high sensitivity to thermal changes and drift.
7. Relatively high IMP suppression.

Two type 1 algorithms have already been discussed.

Quasi-linear (envelope and phase), of chapter 6 section

6.2, is a type 1 algorithm. The test waveform used was a 100% AM RF signal, and the detector was a RF spectrum analyser. Linearity of the spectrum analyser was assumed. The conditions for stability and convergence were discussed in chapter 6 section 6.3. Interpolation was required when reshuffling the RAM table, since non-integer address states can occur. High IMP suppression results, but the normal operation of the DUT is terminated whilst using quasi-linear. Because quasi-linear requires a significant time to perform (e.g. several seconds to reduce the spurious spectra on just one iteration), any thermal changes or drift in the characteristics which occur over a comparable time period greatly reduce the effectiveness of the technique.

The other type 1 algorithm briefly touched on is illustrated in Figure 3.3.7 and is due to Abuelma'atti [ref 2]. It consists of running a test waveform, of known spectral character, through the DUT. The IMD can be separated in a heterodyne voltmeter, since the frequency at which IMD occurs can be calculated beforehand. Interpolation as such was not used, as the NLFG was an analogue diode waveshaping circuit. As reported by Abuelma'atti, the scheme is unconditionally stable.

For a deeper investigation of more practical type 1 control algorithms, the scheme illustrated in Figure 9.1.1 is to be examined in detail. This scheme uses a type 1 control algorithm for correction of AM/AM, and analogue (polar) feedback for the correction of AM/PM conversion. Although the algorithm discussed is specific to this setup, the algorithm can trivially be extended to other systems.

9.2 ANALYSIS OF TYPE 1

Referring to Figure 9.1.1 the equipment can be operated in a number of modes, depending upon the switch positions. The first mode to be considered is shown in Figure 9.2.1. This is referred to as ramp mode.

9.2.1 Ramp Mode

Under CPU control the D/A is forced to output a digital ramp, whilst the detector records the DUT output envelope for each ramp value. For each new value of the ramp output from the D/A a settling time must be allowed before recording the envelope (this settling time being determined by the transient response of the anti-alias filter, DUT and detector). The CPU now has a record of the DUT envelope characteristics, relating the data word at the D/A (d) to the A/D output (A). The transfer function of the envelope detector is denoted by $k(V_E)$, where V_E is the output envelope of the DUT. The A/D quantizes according to a quantizing law defined by $Q_A(x)$ (which relates the numerical output of the A/D to the analogue input voltage x).

$$\text{Let} \quad V = k(V_E) \quad (9.1)$$

$$\text{then} \quad A = Q_A(V) + e' \quad (9.2)$$

where e' represents the error due to quantizing.

$$\text{But} \quad V_E = f(Q_D(d)) \quad (9.3)$$

where $Q_D(d)$ is the D/A quantizing law which relates the D/A output voltage to the numerical input d . The DUT is assumed to be described by an AM/AM conversion function $f(x)$ (where x is the input envelope). Thus combining equations 9.1, 9.2 and 9.3 we obtain

$$A = Q_A(k(f(Q_D(d)))) + e' \quad (9.4)$$

Notice that in equation 9.4 the detector and DUT transfer functions are not separated in any way (i.e. from equation 9.4 any nonlinearities in the detector are indistinguishable from any DUT nonlinearity). We define the observed transfer function $f'(x)$ where

$$f'(x) = k(f(x)) \quad (9.5)$$

Thus in equation 9.4 we obtain

$$A = Q_A(f'(Q_D(d))) + e' \quad (9.6)$$

The relationship indicated by equation 9.6 is stored in the CPU, for consecutive integer values of d , at the end of ramp mode.

We now consider the total time taken for this operation. We assume that the dominant transient effect is introduced by the anti-alias filter (i.e. that the DUT and detector are broadband devices in comparison with the anti-alias filter). The settling time is dependent on the filter type chosen (reference 192 gives the step response for all the commonly used filter types). The filter type chosen was a sixth order, 0.1 dB ripple Chebyshev filter. The theoretical step response, and the measured step response, are shown in Figure 9.2.2.

Using such a filter and sampling the detector output at 230 microsecond intervals minimises intersymbol interference, whilst minimising the time required to wait between measurements. (Note that the filter is not designed for zero intersymbol interference measurements as such, but assuming the ramp consists of 1 bit changes in the D/A, and this produces a nominal 1 bit change at the A/D, then we only require settling to 50% of final value anyway).

Thus, using the filter described, for a 12 bit system consisting of 4096 ramp values, the minimum time in which the ramp could be performed is 230×4096 microseconds = 0.94 seconds.

As an engineering approximation for a general lowpass filter with a passband width B Hz, the settling time required may be taken as approximately equal to $(5/B)$ seconds. (This is based on the tables of reference 51, and only applies to filters of reasonably high order). Thus, if the A/D can recognise n levels, the total ramp time is of the order of $(5n/B)$ seconds.

It should be noted that the step size of the ramp at the D/A is chosen to cause approximately a 1 bit change at the A/D, i.e. the number of ramp steps corresponds to the number of levels that the A/D can recognise, not to what the D/A can distinguish.

Thus the minimum time taken to produce the ramp is dependent on the number of discrete levels the A/D can recognise and the anti-alias filter used. Although a faster step response from the anti-alias filter may be obtained by a different filter type, or different choice of bandwidth, this can only be at the expense of amplitude response. The spectral effects of a filter with a reduced amplitude response are discussed later.

In this study, no discrepancy was observed between the characteristic as measured by a slow ramp, and as observed under more dynamic conditions. Such discrepancies have been reported in manually operated systems [ref 67], and thermal changes have been proposed as the cause. For the ramp used in this study thermal time constants appear long compared to total ramp time.

9.2.2 Signal Processing

Our objective in signal processing is to fill the RAM table such that equation 9.6 holds for all address and data locations.

The ramp procedure can be repeated n times, and the average taken from these results in deducing equation 9.6 for any given d . For zero mean Gaussian distributed noise the error term (e') can then be reduced in variance by a factor of n . [Note here that this assumption proves true if the input to the A/D carries random noise of peak deviation ($1/2\text{LSB}$), and zero mean. In practice this either is the case, or can be made to be so. Chapter 11 discusses adding noise to the input of an A/D to increase resolution, and the bulk of that discussion applies here. Reference 136, p.747, also mentions the need for noise to resolve below quantization levels].

However, the noise output of the envelope detector is not Gaussian but Rayleigh distributed, and the signal plus noise Rician distributed [ref 150 p.381]. For large signal levels the noise can be modelled as a zero mean Gaussian distribution, and the assumption of decreasing variance of e' holds. However, for small signal levels the assumption fails, and only knowledge of the noise variance will enable us to reduce e' by averaging. In this work there was no attempt to correct for non-zero mean, non-Gaussian, statistics. However, it would certainly be possible to do so, although the use of a synchronous envelope detector would preserve the zero mean Gaussian nature of the noise, and enable the reduction of the variance of e' to be performed independent of signal level. Thus, where averaging of ramps is possible, the synchronous detector is superior to the diode

detector.

However, this error reduction can only be achieved by increasing the total time devoted to ramp mode by a factor of n . If n is made very large the thermal changes and drift of the DUT may start to cause problems. Thus, in practice, we must choose some reasonably small value of n , whether or not a synchronous detector is used.

In using equation 9.6, where synchronous averaging has been applied, it is best to keep the definition of e' , and introduce a factor $(1/n^{1/2})$ to take account of the change in statistics of the error term, i.e. we obtain in lieu of equation 9.6

$$A = Q_A(f'(Q_D(d))) + (e'/n^{1/2}) \quad (9.7)$$

We can measure the value of A for consecutive values of d in equation 9.7. However, we require the value of d for consecutive values of A (see Figure 9.2.3). This is known in texts on numerical analysis as the inverse interpolation problem [ref 147]. If only a few data points were known then schemes such as Bessel interpolation would be advised. When the density of data points is very high simple linear interpolation is adequate, and this has been used in this study.

It should be noted that the ramp waveform is a monotonically increasing waveform, hence since failure to follow distortion is associated with a falling envelope, with an increasing ramp failure to follow distortion is not a problem.

9.2.3 Run Mode

Having derived the value of d in equation 9.7 for consecutive values of A the CPU may be put into run mode (switch

positions as Figure 9.2.4). We assume that the sample period is 1 Hz without loss of generality. Referring to Figure 9.2.4, if the input voltage to the A/D is $V_{in}(t)$ then the address formed on the output to the A/D at the i^{th} sample instant is

$$A_i = Q_A(V_i(t)) + e \quad (9.8)$$

where e is the quantization error and

$$V_i(t) = V_{in}(t) \left(\sum_i \delta(t - i) \right) \quad (9.9)$$

$\delta(t)$ is the Dirac delta function. $V_i(t)$ is illustrated in Figure 9.2.5 and corresponds to a "perfectly" sampled $V_{in}(t)$.

We have replaced the dependence of A on t in equation 9.8 by the sample label i for convenience of notation. We further assume that the sampling rate is unity, again for convenience of notation.

Assuming that the RAM is described by $d(A)$ the output of the RAM at the i^{th} sample instant (d_i) is related to A_i by equation 9.7. Thus the output envelope of the DUT $V_E(t)$ as viewed by the envelope detector is given by

$$\begin{aligned} V_E(t) &= k(f[Q_D(d_i) * r(t) * h(t)]) \\ &= f'[Q_D(d_i) * r(t) * h(t)] \end{aligned} \quad (9.10)$$

where $r(t) = 1$ if $|t| < 1$

$= 0$ otherwise

$h(t)$ is the anti-alias filter impulse response

$*$ denotes the operation of convolution.

We shall now assume that the convolution may be taken outside of $f'(x)$. This assumption is discussed in much greater detail later. We show in Appendix G that provided:

1. the anti-alias filter is perfect; and
2. the aliased frequencies out of the D/A do not overlap; and

3. the RAM implements the inverse function to $f'(x)$ in the point-wise sense;

there is no approximation in this assumption. We further need to assume that the envelope transfer function is continuous in all derivatives. Thus under the conditions stated

$$\begin{aligned} V_E(t) &= f'[Q_D(d_i)] * r(t) * h(t) \\ &= f'[Q_D(d_i)] * r(t) * h(t) \end{aligned} \quad (9.11)$$

We set up the RAM table according to the relationship of equation 9.7. Restating equation 9.7

$$A = Q_A(f'(Q_D(d))) + (e^{-1/n^{1/2}}) \quad (9.12)$$

$$\text{i.e.} \quad f'(Q_D(d)) = Q_A^{-1}(A - (e^{-1/n^{1/2}})) \quad (9.13)$$

where the function $Q_A^{-1}(A)$ is the inverse of $Q_A(x)$. Thus Q_A^{-1} relates the numerical output of the A/D to the analogue input x . Substituting into equation 9.11 gives

$$V_E(t) = (Q_A^{-1}(A_i - (e^{-1/n^{1/2}}))) * r(t) * h(t) \quad (9.14)$$

Combining with equation 9.8 we get

$$V_E(t) = (Q_A^{-1}(Q_A(V_i) + e - (e^{-1/n^{1/2}}))) * r(t) * h(t) \quad (9.15)$$

Noting that the noise and offset term are small, we can Taylor expand with very little approximation to get that $V_E(t)$ is approximately equal to

$$\begin{aligned} &(Q_A^{-1}(Q_A(V_i)) + [(e - (e^{-1/n^{1/2}}))(dQ_A^{-1}(x)/dx)]) * r(t) * h(t) \\ &= V_i * r(t) * h(t) + [(e - (e^{-1/n^{1/2}}))/g_A] * r(t) * h(t) \\ &= V_{in}(t) + (1/g_A)[(e(t) - (e^{-1/n^{1/2}}))] * r(t) * h(t) \end{aligned} \quad (9.16)$$

where g_A is the approximate gain of the A/D in bits/volt.

In equation 9.16 we have assumed that the anti-alias filter reconstructs the perfectly sampled input waveform

$$\text{i.e.} \quad V_i(t) * r(t) * h(t) = V_{in}(t) \quad (9.17)$$

The following points should be noted in equation 9.16.

1. The output envelope (viewed via the detector) is linearly related to the input waveform.
2. The linearity, or lack of it, of the A/D and D/A does not affect the linearity of the digital compensation, i.e. $Q_A(x)$ and $Q_D(x)$ do not figure in equation 9.16.
3. There exists a small noise contribution in the output envelope due to the finite A/D resolution. For n large, this contribution has the same peak signal to rms noise ratio as we would have expected from the A/D (see chapter 4, equation 4.8).
4. The noise contribution is bandlimited in the frequency domain to the anti-alias frequency cutoff frequency.
5. Equation 9.16 plainly represents a stable system in a static equilibrium (i.e. the RAM table is fixed after ramp, and does not vary thereafter).

It is clear from equation 9.16 that, provided the assumptions we have made are valid, AM/AM distortion is totally removed if

$$f'(x) = k[f(x)] = f(x) \quad (9.18)$$

Thus, if the detector is linear, we may associate the distortion-free output of the envelope detector with a lack of AM/AM conversion on the RF signal. It is also clear that this procedure and analysis could be used verbatim for removing AM/PM nonlinearities via a similar setup.

To obtain a better estimate of the practical performance of this type 1 algorithm it is necessary to investigate the assumptions that had to be made in deriving equation 9.16.

9.3 DISCUSSION OF ASSUMPTIONS

The following assumptions have been made in deriving equation 9.16:

1. The anti-alias filter is perfect.
2. The sampled data output of the D/A does not have overlapping aliased frequencies.
3. The quantizing law of the A/D has an inverse.

Aside from these points it is also useful to look at the effect of nonlinearities in the detector.

The breakdown of any one of these assumptions causes IMD in the output spectrum of the DUT. Below we present an analysis of the breakdown of these assumptions.

9.3.1 Imperfect Anti-alias Filter

In practice any anti-alias filter will deviate slightly from a perfect anti-alias filter. Figure 9.3.1 shows a fairly typical case of 0.1 dB amplitude ripple, and 2 degrees phase deviation from linear in the passband, along with deviations from the ideal in the transition region. Figure 9.3.1 is, in fact, the measured response of the anti-alias filter used in this work.

Consider the output spectrum of the anti-alias filter to be as in Figure 9.3.2. Due to imperfections in the filter this can be regarded as consisting of two components, the "perfect" component and the error component which needs to be added to produce the observed spectrum. The magnitude of the error term can be deduced from the amplitude and phase information of the filter. For example, consider the filter to have a deviation from perfect of 0.1 dB and 2 degrees at the third harmonic. Using

Figure 9.3.3 we can see that the error spectra at this frequency is of magnitude -28 dB on the "perfect" third harmonic spectra. To relate this to the output spectra of the DUT we approximate the DUT as being linear.

If the third harmonic spectra were (for example) -41 dB down on the PEP of the signal, at the output of the anti-alias filter, then we would anticipate a third harmonic term in the envelope, at the output of the DUT. The magnitude of this distortion would be approximately $(-41-28-6) = -75$ dB on PEP. (The factor of -6 dB arises from the RF envelope sidebands being split into two equal upper and lower spectral sidebands, thus lowering their individual magnitude relative to PEP by 6 dB). Following this simple interpretation, we see that the error introduced at RF is dependent on the relative spectral structure of the output of the anti-alias filter, weighted by its deviation from an ideal.

9.3.2 Overlap of Alias Spectra

We treat the problem of overlapping aliased spectra (as shown in Figure 9.3.4) slightly differently from the effect of an imperfect anti-alias filter. The reason for this is that an imperfect anti-alias filter modified all spectral lines, but introduced no new spectral lines. In general overlapping anti-alias spectra introduce new spectral lines, but do not modify any existing lines.

Our first order analysis is to assume that the DUT is linear and apply the superposition theorem. Thus if the spectral components out of the anti-alias filter which overlap are of order

-a dB on PEP, then we expect (to first order) that the RF envelope will contain distortion terms at the same frequencies of order -a dB on PEP (see Figure 9.3.4).

9.3.3 A/D Quantizing Law

For the analysis of run mode to proceed, we had to assume the existence of the inverse quantization law for the A/D, i.e. we assumed that given the envelope voltage, the corresponding address (A) to the RAM is known in equation 9.13, see Figure 9.3.6. The effect of random quantization errors has been accounted for in deriving the noise term of equation 9.16. By the inverse theorem of reference 133 we are in fact insisting that the A/D be monotonic (see Figure 9.3.6). Since this is not an unusual requirement we assume that it is met when designing type 1 systems and take the analysis of non-monotonic A/D's no further. Note no such restriction has been applied to the D/A.

9.3.4 Detector Linearity

As noted previously, the detector transfer function and the DUT envelope transfer function are indistinguishable from one another. Thus, only if $k(x)$ is linear can we obtain the desired result. Our first order interpretation of a nonlinear detector is as follows. Consider the setup of Figure 9.3.5. A first order analysis is achieved by considering the DUT and envelope detector to be approximately linear, and hence by applying the superposition theorem we see that nonlinearities of order -a dB in the detector give rise to distortions of -a dB in the RF spectrum.

Thus, the action of nonlinearities in the detector is to force distortion on the DUT output. For example, to achieve linearity 60 dB down on PEP, the detector must give a linear output to 60 dB down.

For the envelope detectors developed in this work, at very high AM modulation depths, the detector can give rise to errors due to failure to follow distortion. In this study, such errors were typically at a level of -70 dB on PEP and so were not obtrusive.

9.4 SPECIAL CASE

We note with considerable interest that no assumptions need be made in equation 9.11 if the test setup of Figure 9.4.1 is used. The anti-alias filter has been placed after the RF DUT, and acts directly on the envelope. There are, of course, still difficulties in implementing a "perfect" filter, and in implementing the filter at RF frequencies, because of the high Q elements required.

9.5 COMPUTER SIMULATION

The problem we wish to address (via computer simulation) is this. Given the ramp data, (which corresponds to basic data concerning the description of the nonlinearity), what will be the spectral results of imperfections in the anti-alias filter, and in the overlap of aliased components.

The first question will reflect directly on how many stages the anti-alias filter must be designed with in order to meet the system specifications. The second question will determine the

maximum input frequency that can be tolerated whilst still meeting the IMP specification.

9.6 EXPERIMENTAL RESULTS

Figure 9.6.1 shows graphically the contents of the RAM table after a ramp operation. Using this information, along with the theoretical filter response, computer predictions of the levels of the spectral sidebands of the output of the DUT and the anti-alias filter, for a given input, can be made. The results experimentally obtained, and the computer predictions, are shown in Figure 9.6.2 and Figure 9.6.3. Also shown in Figure 9.6.3 are the predictions we would make for the spurious level using the interpretation of the effect of imperfect anti-alias filter given.

Notice that, before the RF levels of the envelope sidebands are recorded, the AM/PM conversion effects have to be removed. The DUT is the same as used for Figures 6.6.6 to 6.6.8.

Figure 9.6.4 shows the output of the envelope detector which was used to measure the ramp data. Clearly, to within the limits set by failure to follow distortion, the detector output is linearly related to the baseband input signal (pure 1 kHz sine wave), as would be predicted by equation 9.16.

9.7 TIME-VARYING CHARACTERISTICS

The primary problem with a type 1 algorithm is the need to interrupt normal operation of the DUT, to set up the RAM table. This may have to be a relatively infrequent occurrence in order to minimise disruption of normal operation. Thus should the

characteristics of the DUT vary after the RAM table has been set up, IMD must be expected. Hence, in any real DUT, we must trade off IMD performance against the frequency with which we can interrupt normal operation.

For a sizeable class of transmitters there may be few problems. For instance, for those transmitters operated in a burst mode (e.g. on a fast data or voice link), there is considerable scope for performing a fast ramp operation between transmissions. [Note that in such circumstances the transmitter output should be switched into a dummy load during the ramp operation, to avoid transmitting the ramp envelope whilst others in the communications net have control].

As has been pointed out in chapter 3, TV transmitters come into the category of burst mode transmitters, because of the blanking period. [Note that in chapter 4 the sample rate for a TV signal was estimated at 36 MHz. Thus, the anti-alias filter bandwidth of 18 MHz will typically require 0.28 microseconds for each level of the ramp. Hence it is conceivable that approximately 4000 ramp steps may be fitted into the field blanking period of 1.1 milliseconds [ref 51, p.20.17], on an "occasional" basis].

The prime short-term cause of changes in transmitter characteristics appears to be thermal changes. If these can be dealt with, then it is reasonable to assume that the long-term changes [ref 134] can also be dealt with.

Figures 9.7.1 to 9.7.4 show the thermal changes present in some class A, B and C amplifiers respectively.

Figure 9.7.1 shows the result of thermal changes on the

envelope characteristic for a small signal amplifier (CA 2818) running in class A. Although the changes in the envelope appear slight, on a 100% AM test the dominant IMP changed from -61 dB on PEP to -56 dB on PEP two hours later, due to the change in thermal equilibrium.

Figure 9.7.2 shows the envelope transfer characteristic of a 80W class B amplifier at 80 MHz. At such high power levels self-heating effects occur, such that the change in characteristic shown in Figure 9.7.2 occurs over a five minute period. However, such a drastic change appears to be a transient effect, as is illustrated by Figure 9.7.3, which shows that a thermal equilibrium is rapidly established.

Figure 9.7.4 shows the same amplifier as Figure 9.7.1, biased into class C. In Figure 9.7.4 one of the two envelope transfer characteristics shown was measured at switch-on (80W PEP, 100% AM signal). After 40 seconds the case temperature of the power transistors had risen from 25°C to 38°C and the corresponding envelope characteristic is shown.

9.8 CONCLUSIONS

Type 1 control algorithms are an identifiable class of algorithms with similar traits. This work has concentrated on a particular type 1 system, although little generality has been lost by so doing.

Type 1 systems can give excellent suppression of IMD, but require the normal operation of the DUT to be interrupted. The frequency of this interruption depends upon the rate of change of the envelope transfer characteristic, and on the degree of

intermodulation suppression required.

The performance appears ultimately to be limited by the imperfections in the anti-alias filter, in the manner discussed. For the tests performed, this gave spurious levels comparable with the asymmetric effects, and so was not obtrusive (see Figure 9.6.3).

The rate of change of the envelope characteristic of amplifiers, due to thermal changes, is a function of the class of operation, power level, and duty cycle, as well as device physics. Low power, low duty cycle, class A amplifiers are the most suitable for a type 1 algorithm; high power, variable duty cycle, class C amplifiers the least suitable.

Due to the interruption required by a type 1 control algorithm it has been suggested [ref 2] that the insertion test signal of VSB-TV be used for ramp mode. It is also possible to suggest that type 1 control could be used with M-QAM systems, with the synchronisation bits replacing the blanking period of VSB-TV.

Another point to be made is that the type 1 algorithm described forces the series combination of NLFG and DUT to approximate a hard limiter. If limiting must inevitably occur (for instance on multicarrier links), this is a sub-optimal way of reducing intermodulation effects [ref 40].

FIGURE 9.1.1

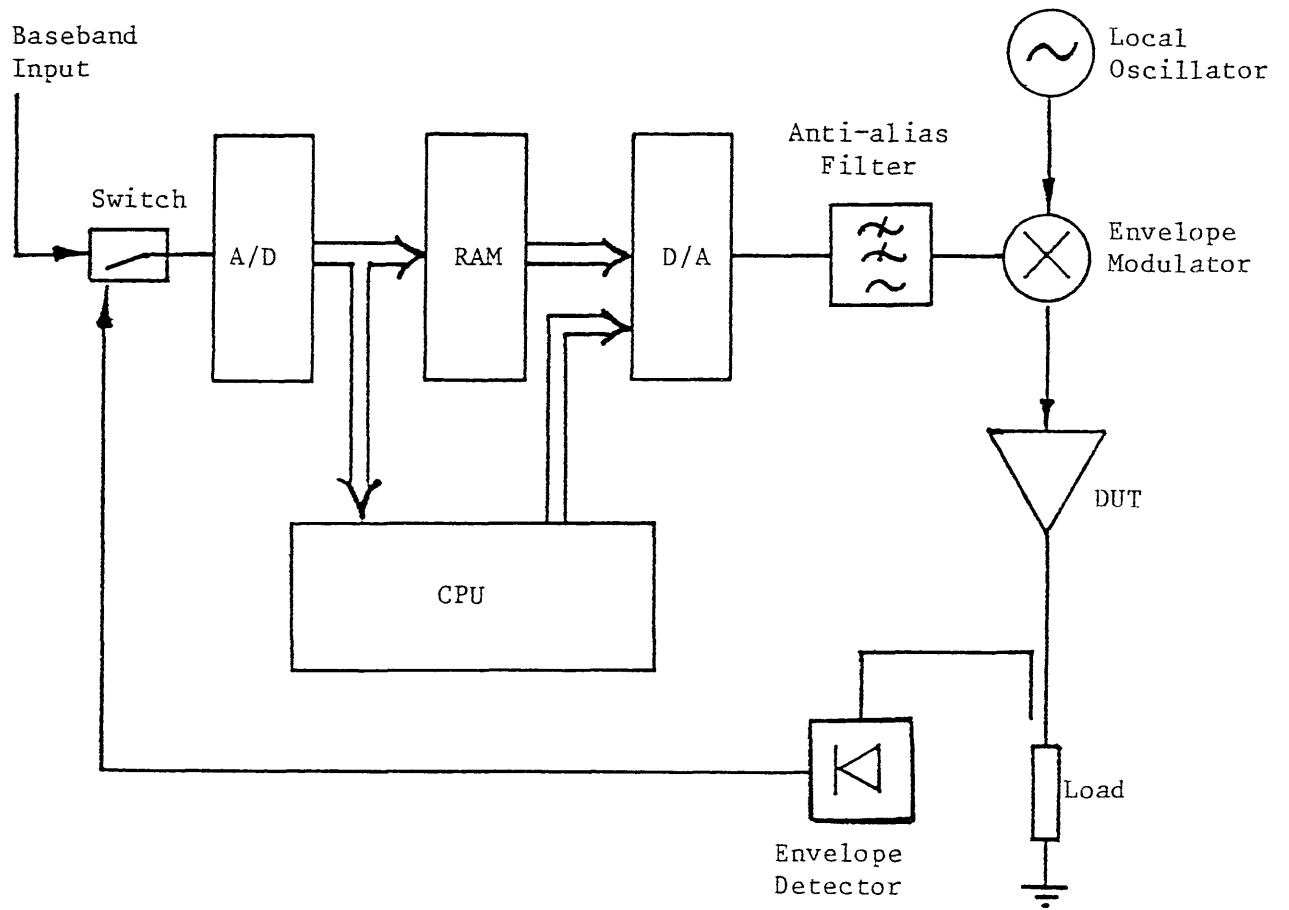
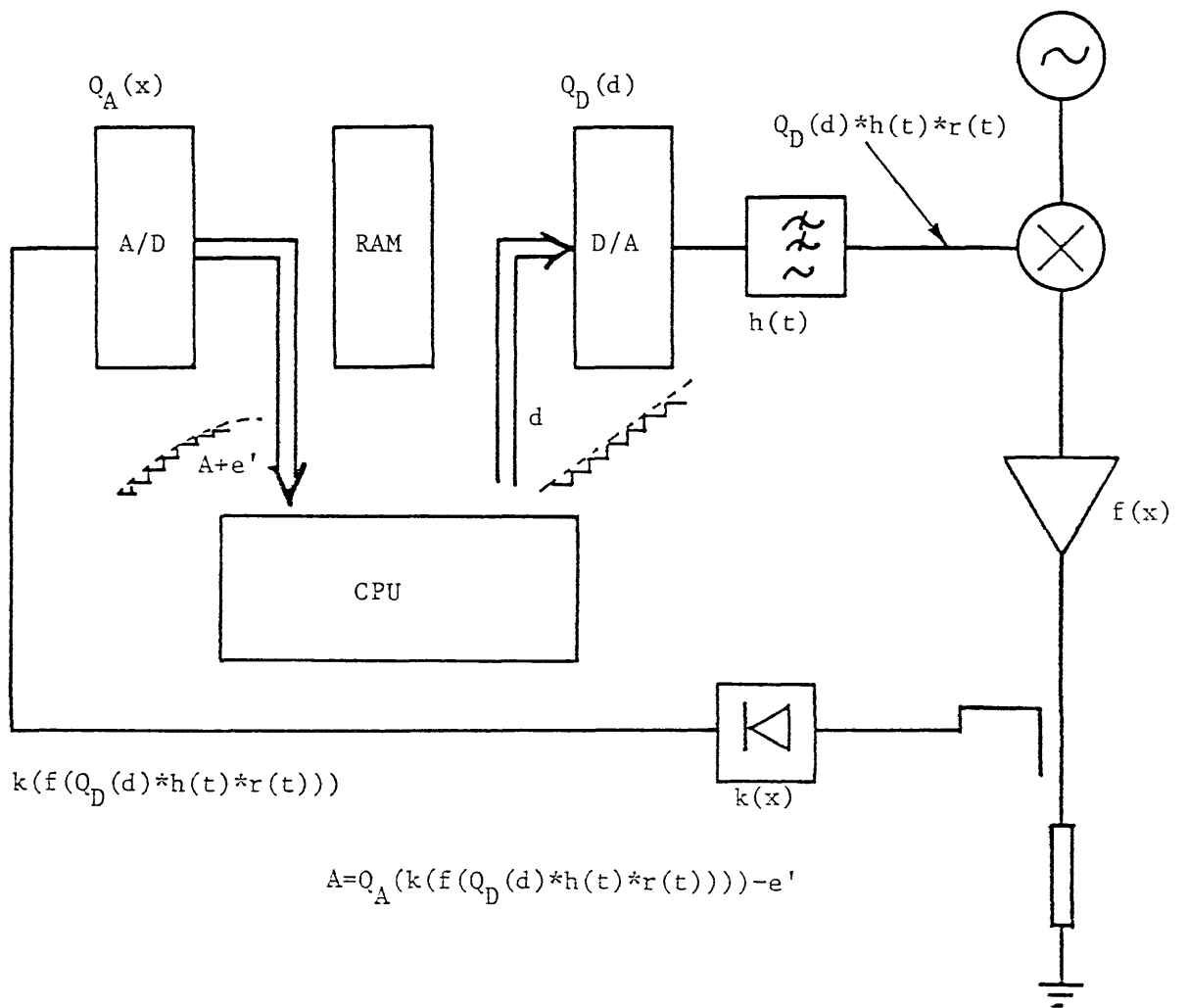
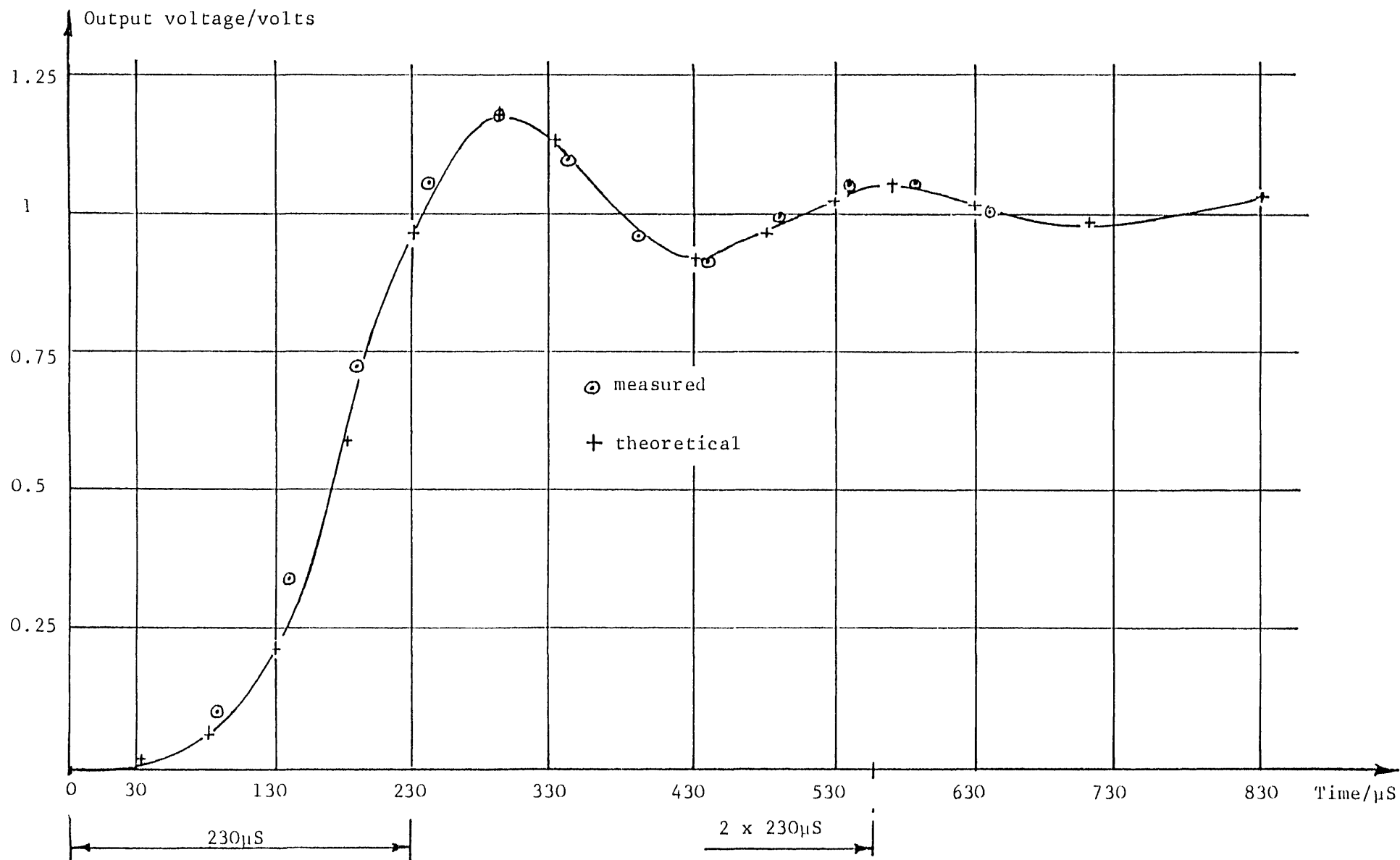


FIGURE 9.2.1

Ramp mode



Step response of 0.1 dB 6th order Chebyshev filter (as used in this study)

FIGURE 9.2.2

FIGURE 9.2.3

Typical results from ramp mode, showing how interpolation is required.

MEASURED		PROCESSED	
d	A	A	d
4000	4000	4000	3999
3999	4000	3999	3997.5
3998	4000	3998	3996.5
3997	3998	3997	3994.5
3996	3998	3996	3993
3995	3997	3995	3991.5
3994	3997	3994	
3993	3996	3993	
3992	3995	3992	
3991	3995	3991	
3990	3994	3990	
.	.	.	
.	.	.	
1000	989	989	1000
999	986	988	999.66
998	983	987	999.33
997	980	986	999

FIGURE 9.2.4

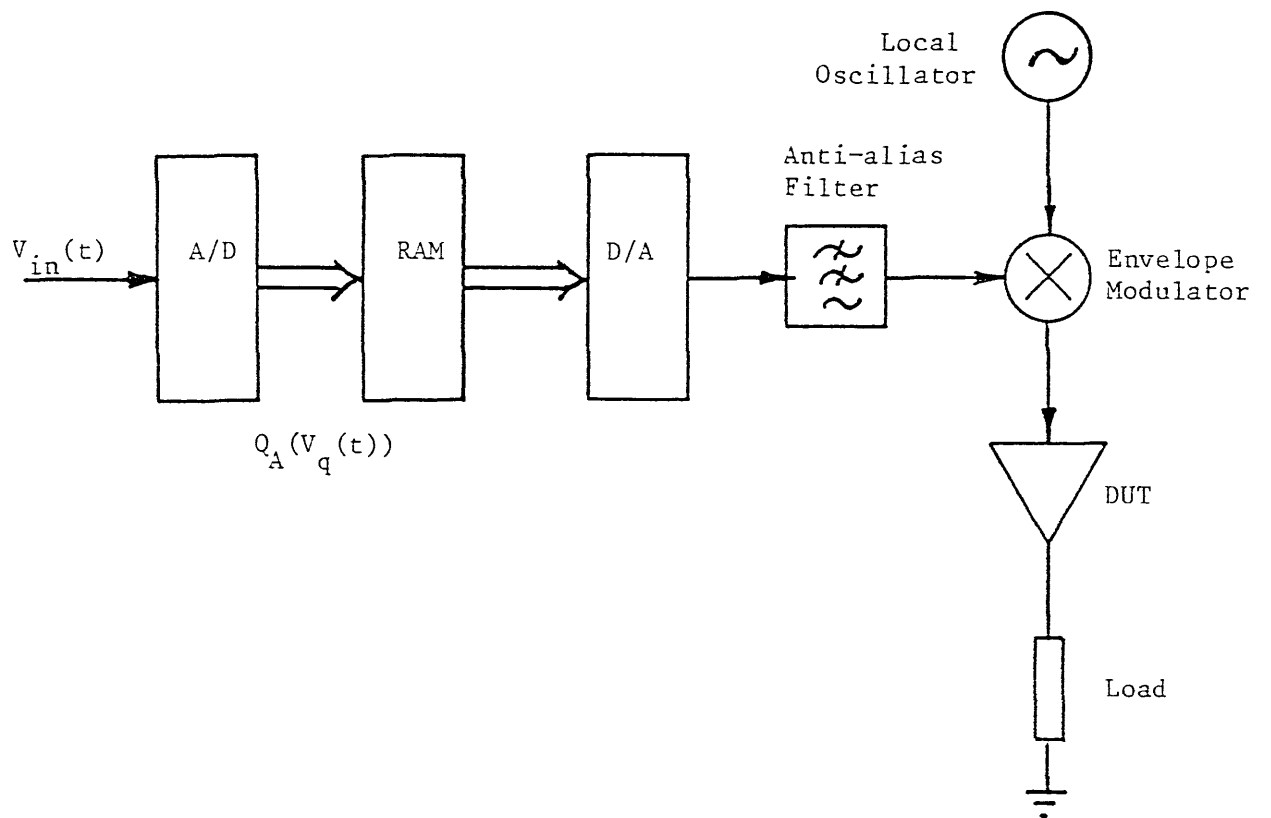
Run mode

FIGURE 9.2.5

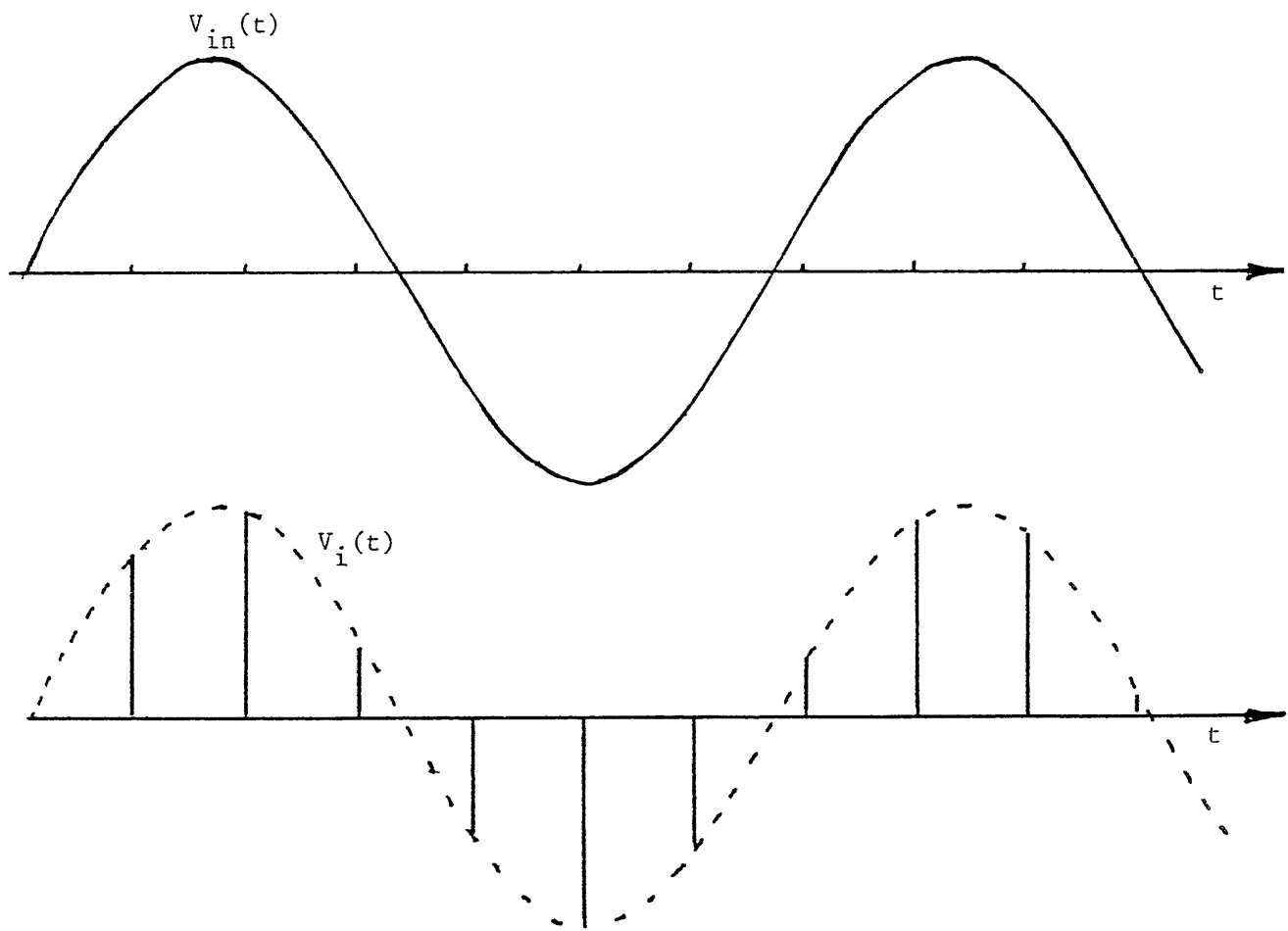
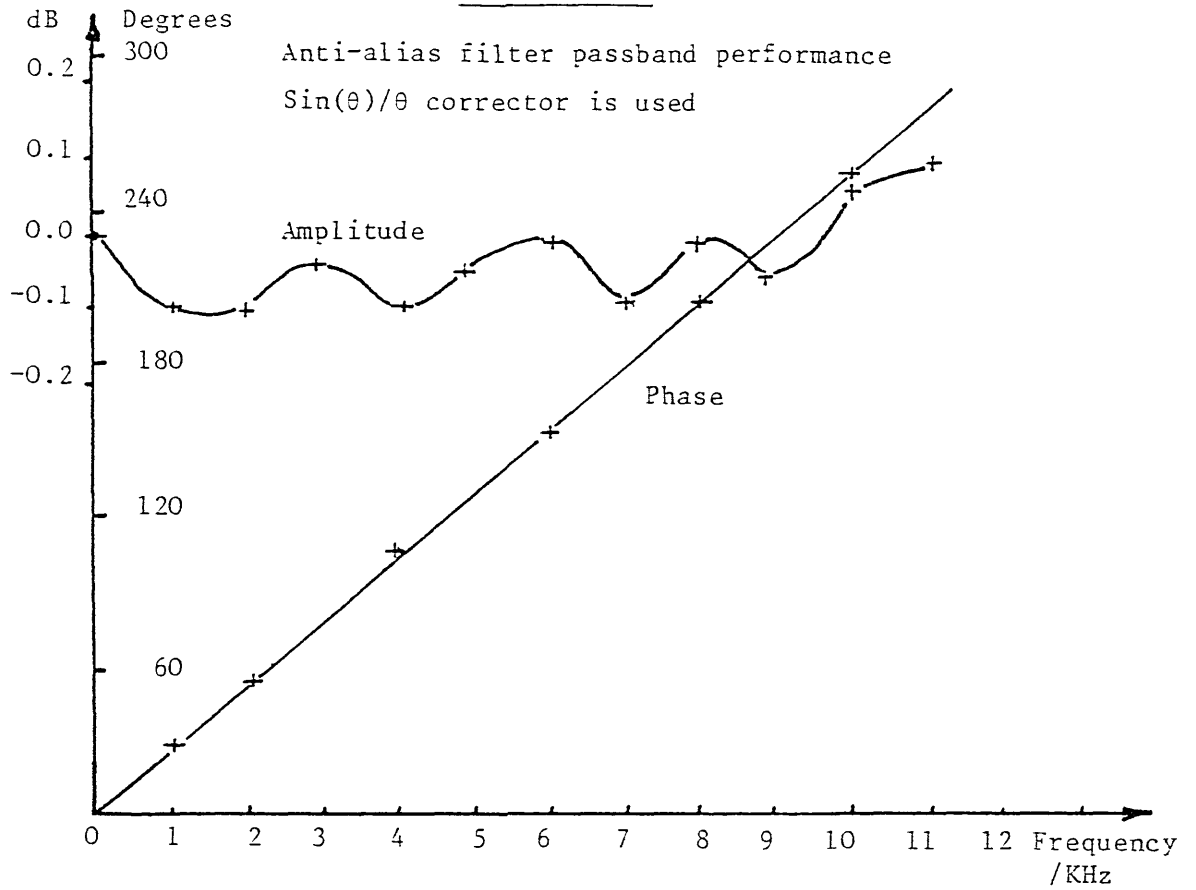
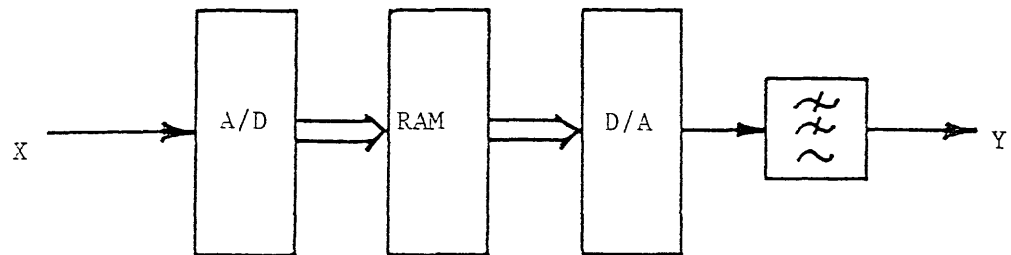


FIGURE 9.3.1



Test setup All measurements of phase and amplitude at the output (Y) are referenced to X



RAM table set
for linear operation

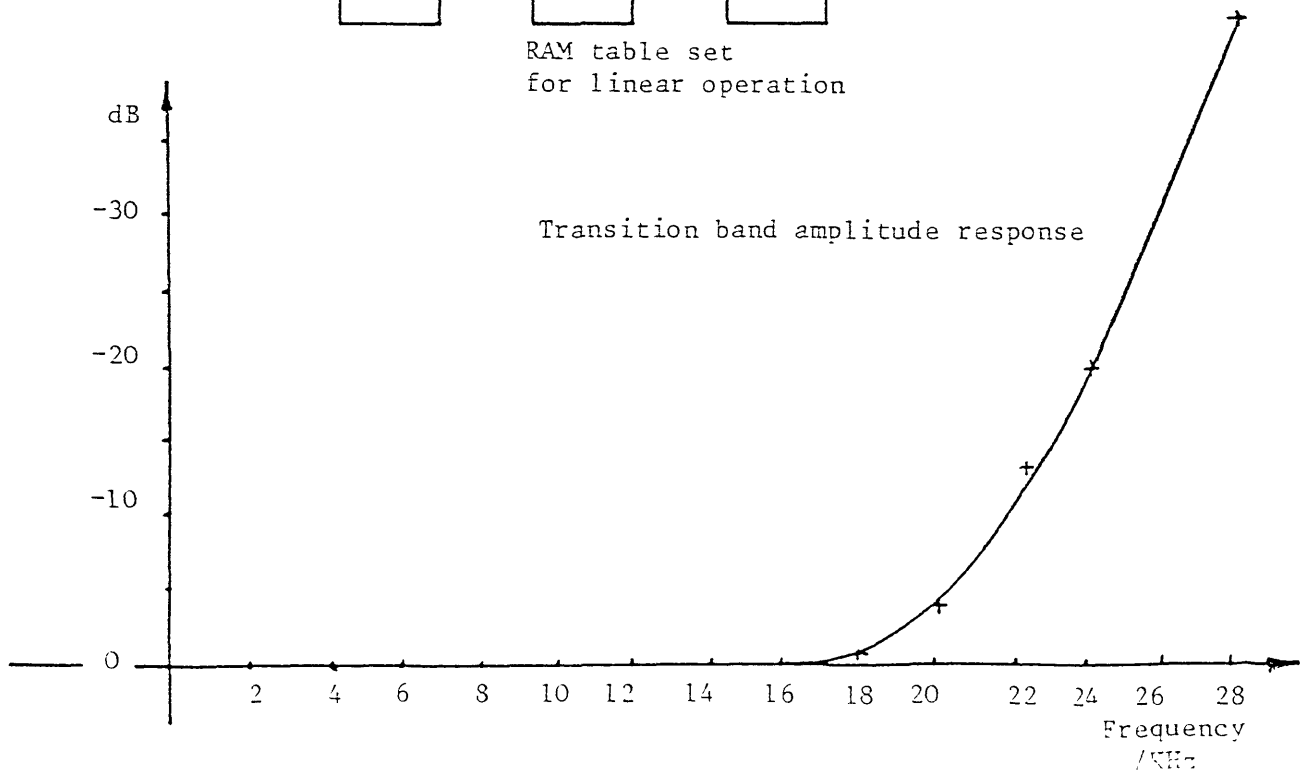
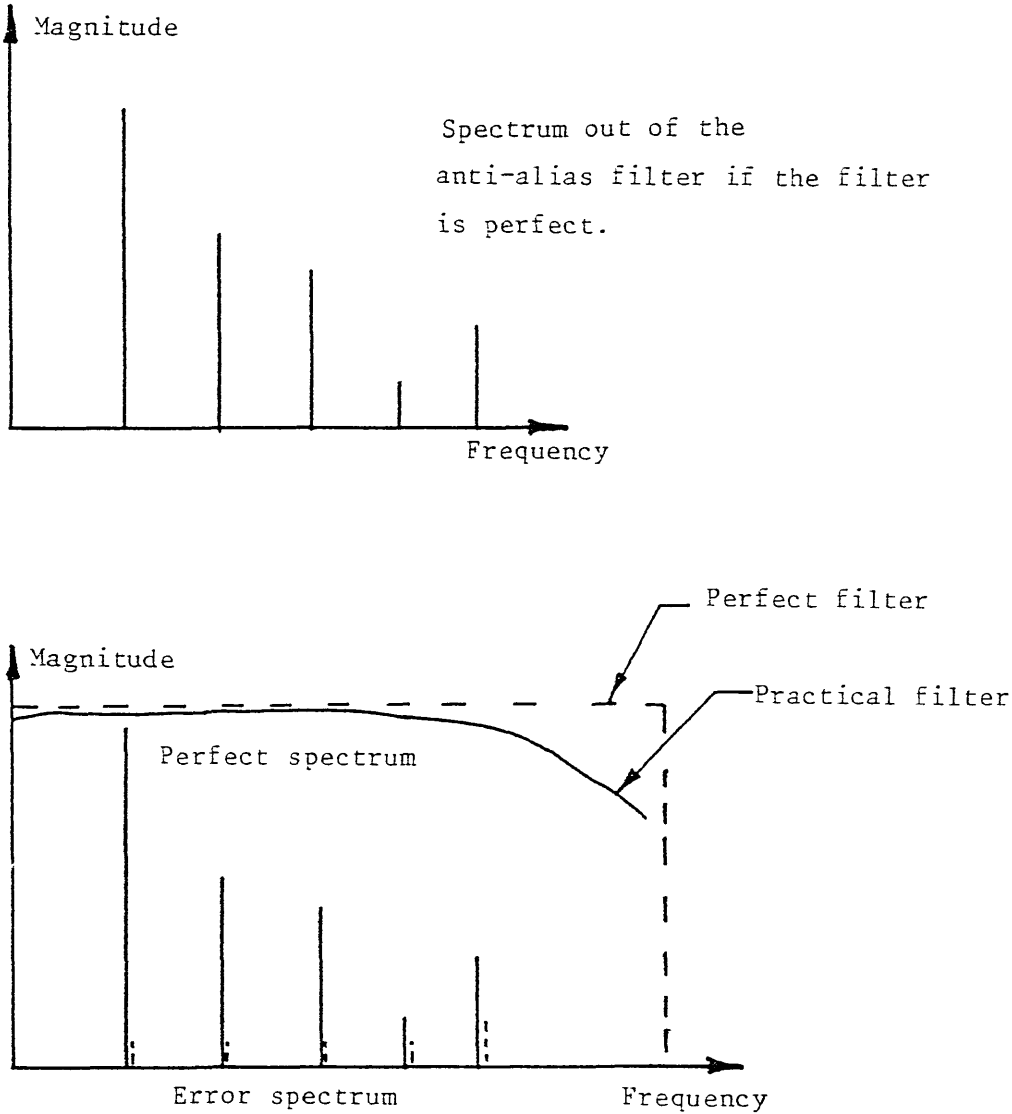


FIGURE 9.3.2



The spectrum out of the practical filter can be considered the vector sum of the "perfect" and error terms.

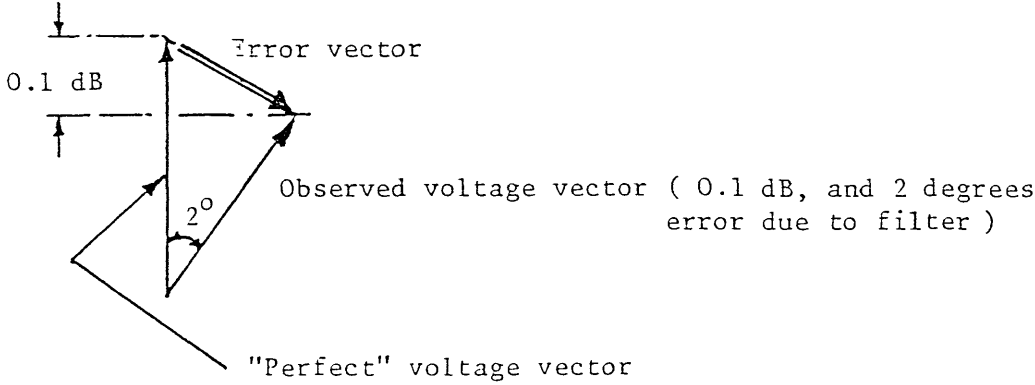


FIGURE 9.3.3

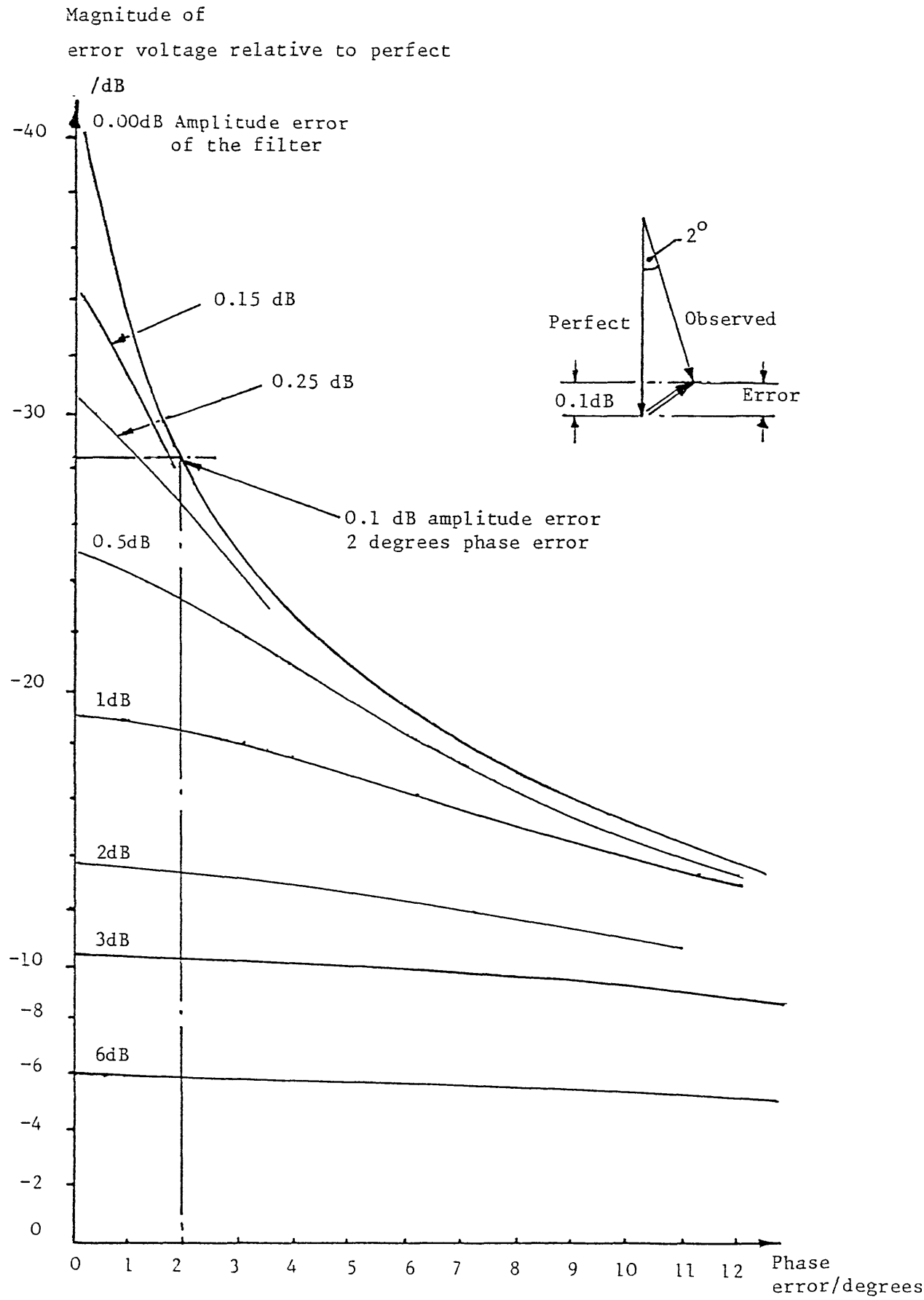
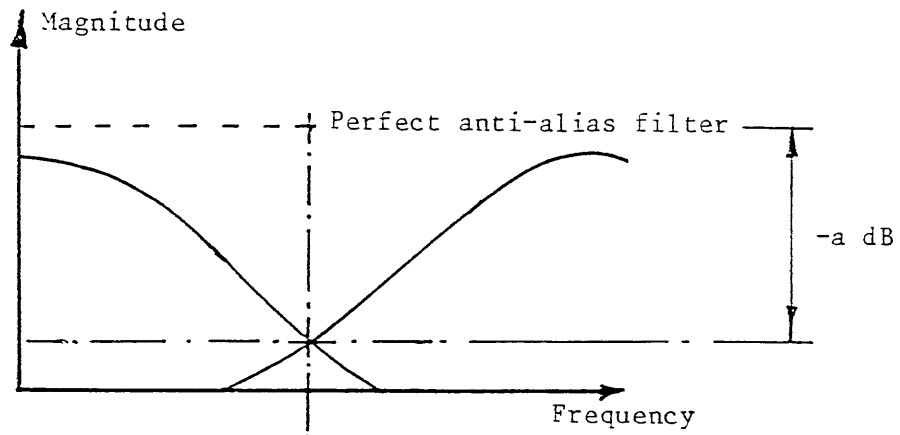


FIGURE 9.3.4



The resultant spectrum can be treated as the vector sum of a perfect and error spectrum, the magnitudes of which are shown below.

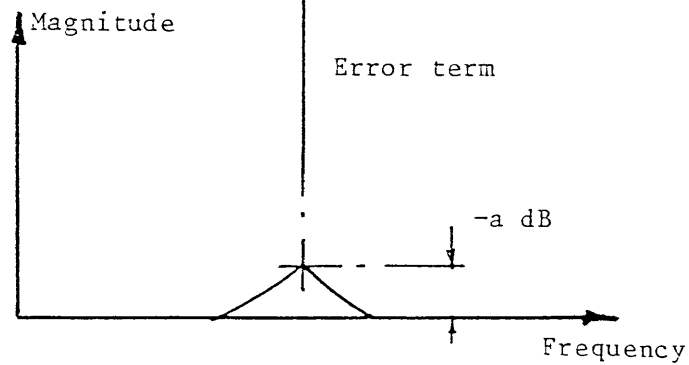
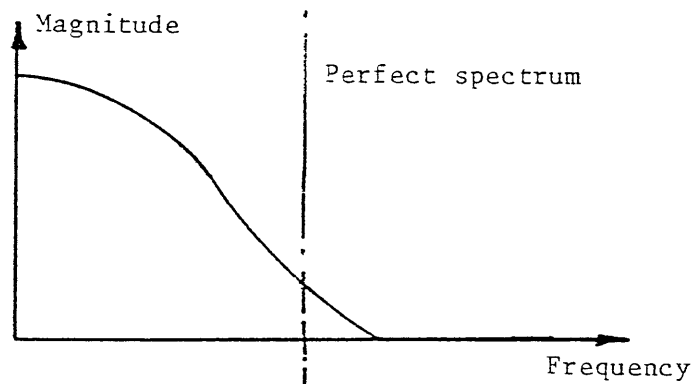
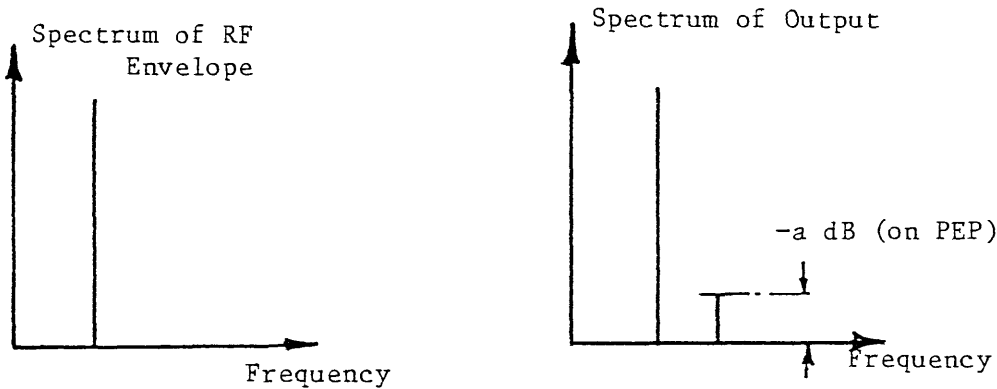
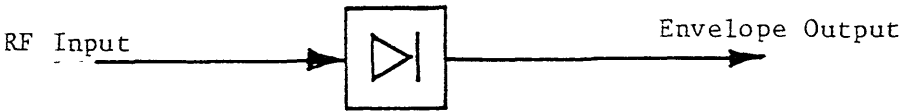


FIGURE 9.3.5



Hence if the detector is approximately linear we can apply the superposition theorem to deduce the RF envelope spectrum when the detector output is linear.

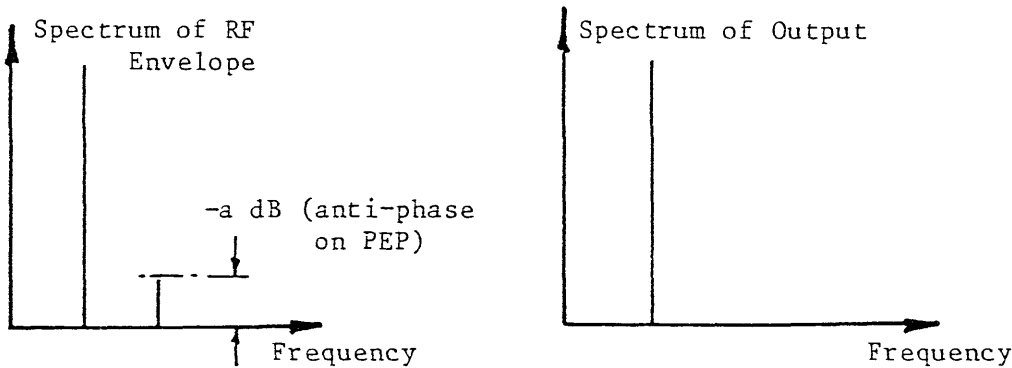


FIGURE 9.3.6

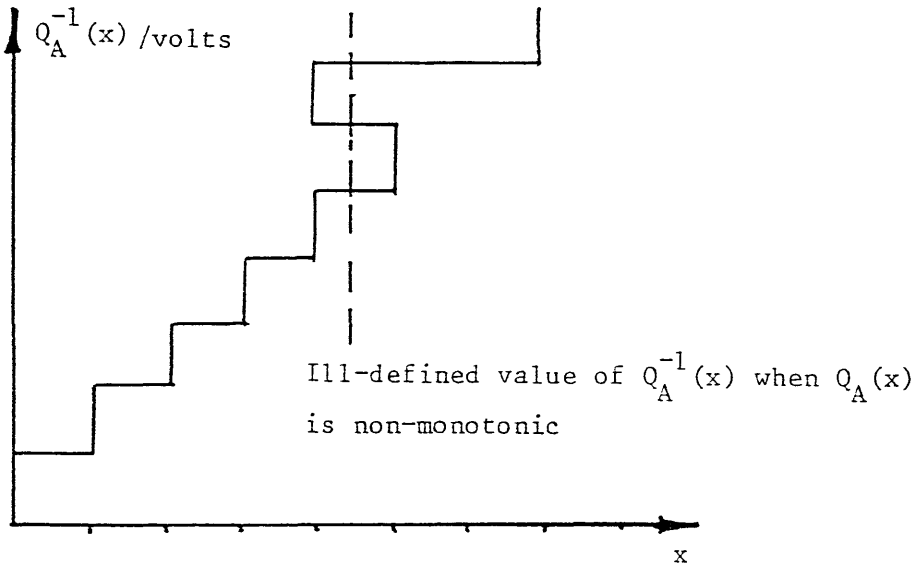
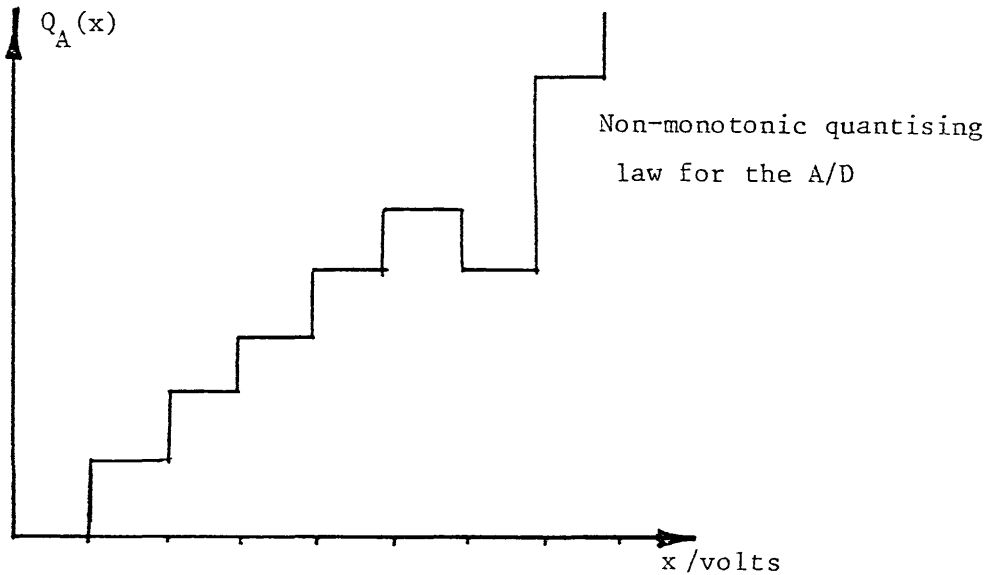
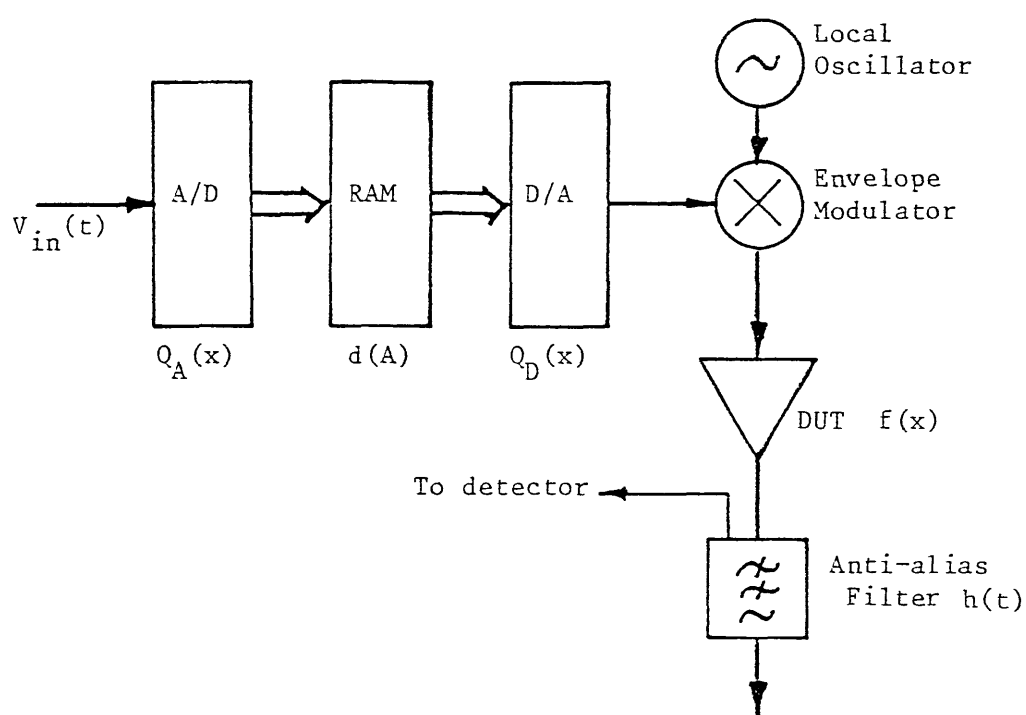


FIGURE 9.4.1



Output envelope

$$V_E(t) = f(Q_D(d_i)) * r(t) * h(t)$$

without any approximation or proviso.

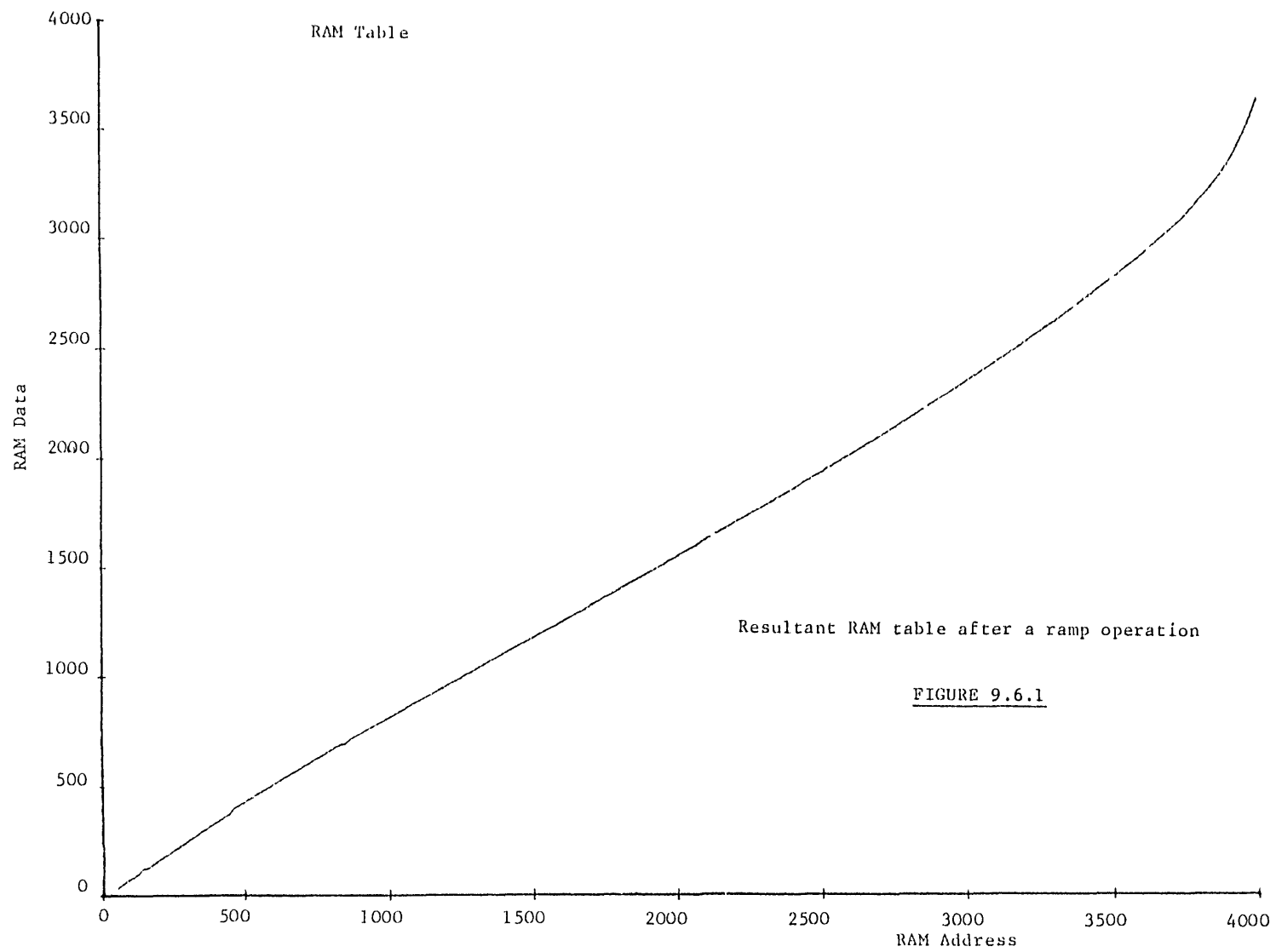


FIGURE 9.6.2

Referenced from
peak voltage

Spectrum out of the anti-alias
filter using RAM table as
Figure 9.6.1. Also shown are
the computer predicted levels.

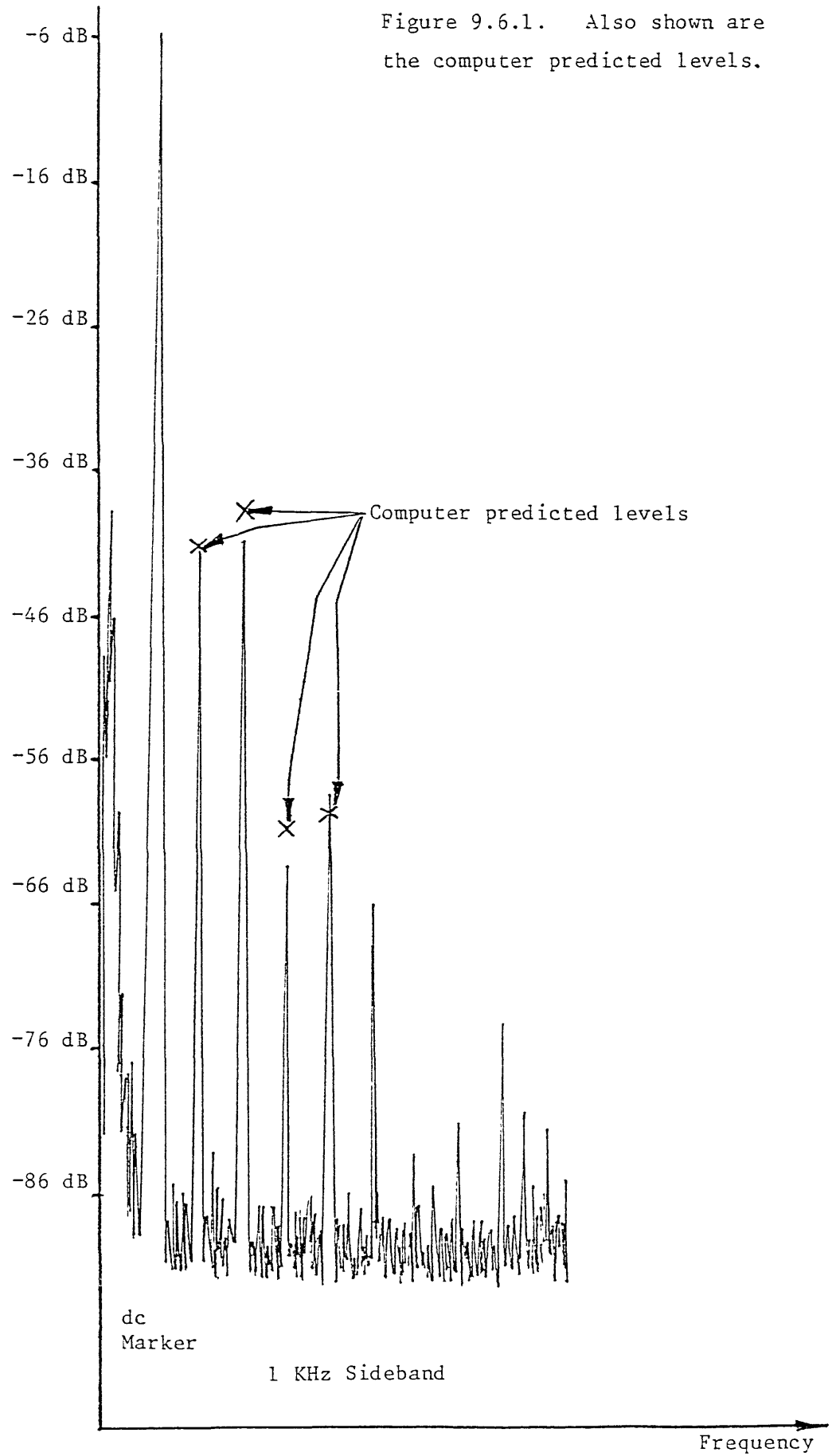


FIGURE 9.6.3

RF spectrum, corresponding to Figures 9.6.1,2,4

Also shown are the computer predicted IMD, and that predicted using the interpretation of Chapter 9 Section 9.3.1

Referenced from
PEP

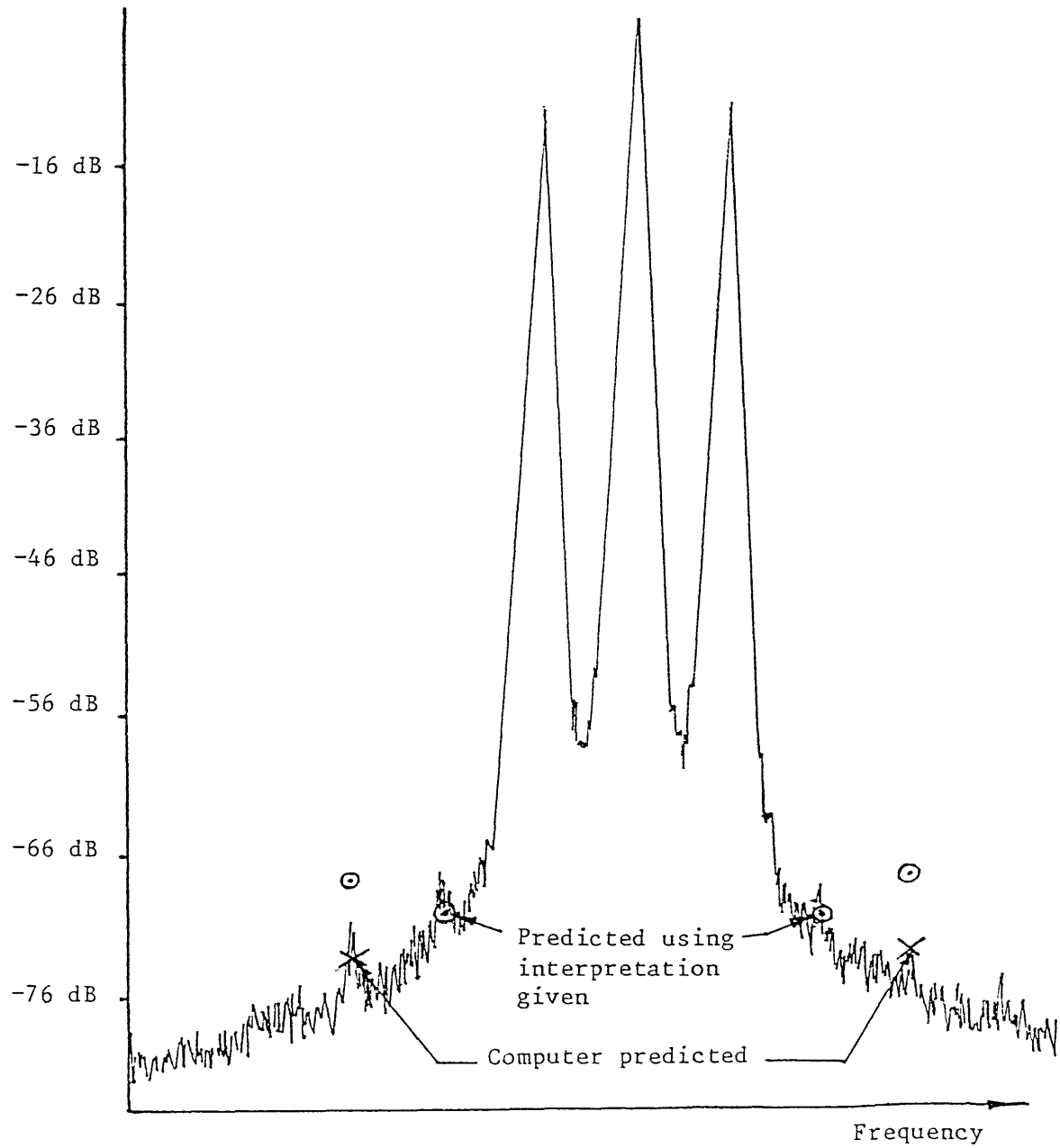
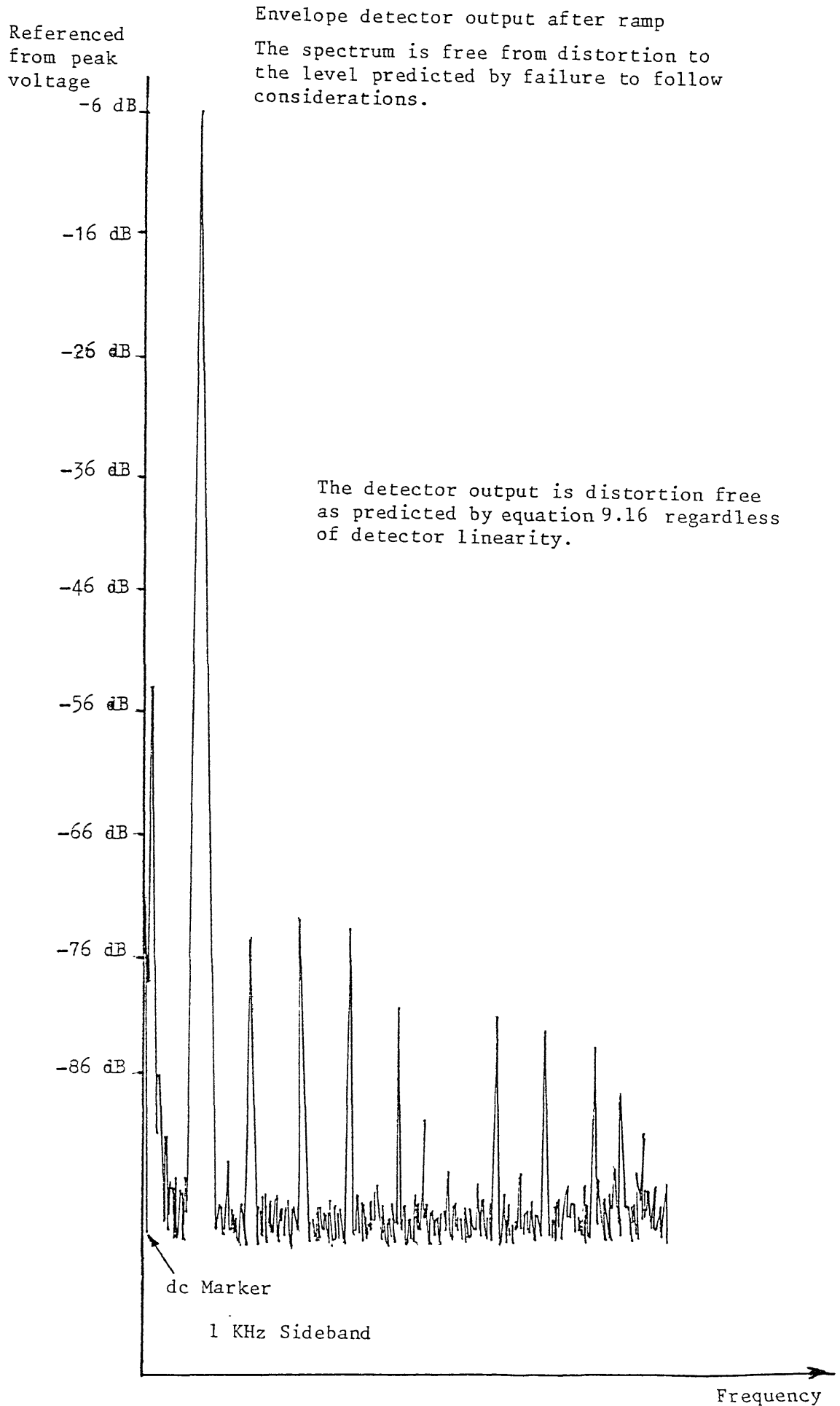


FIGURE 9.6.4



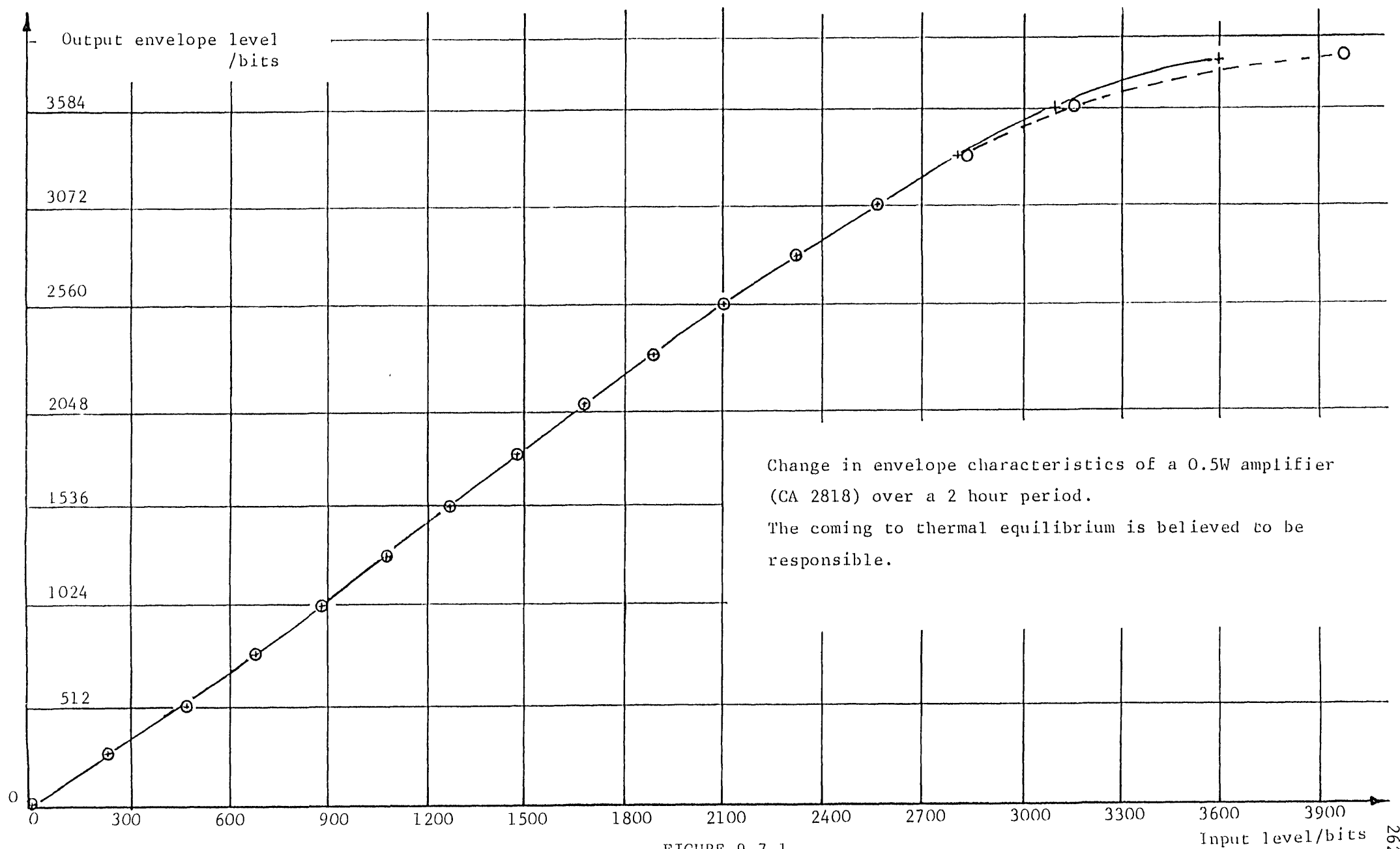


FIGURE 9.7.1

FIGURE 9.7.2

Change in envelope characteristic of a 80W class B amplifier.

Change occurred over a 5 minute period after switch-on.

The change is believed to represent the thermal "switch-on" transient.

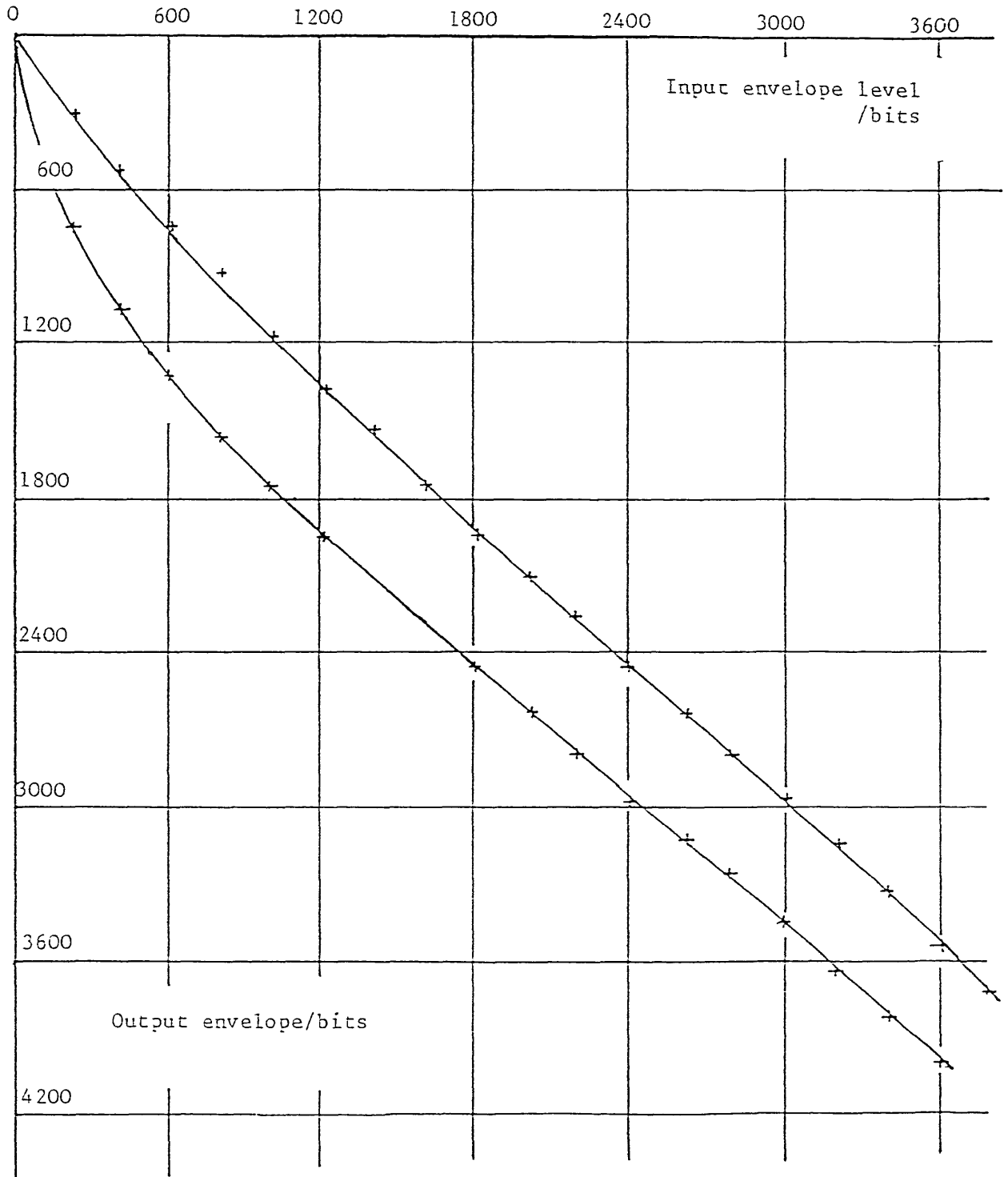
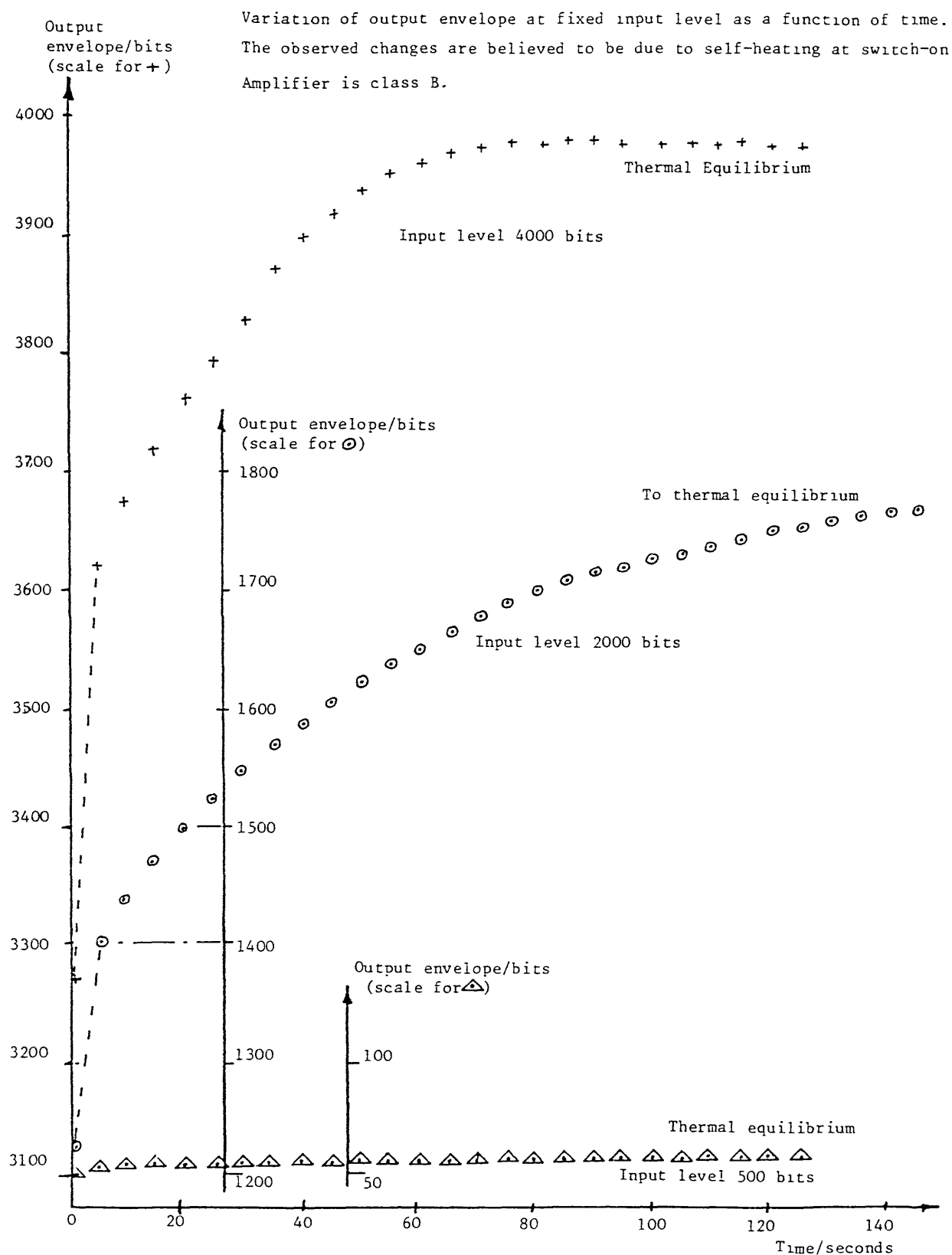
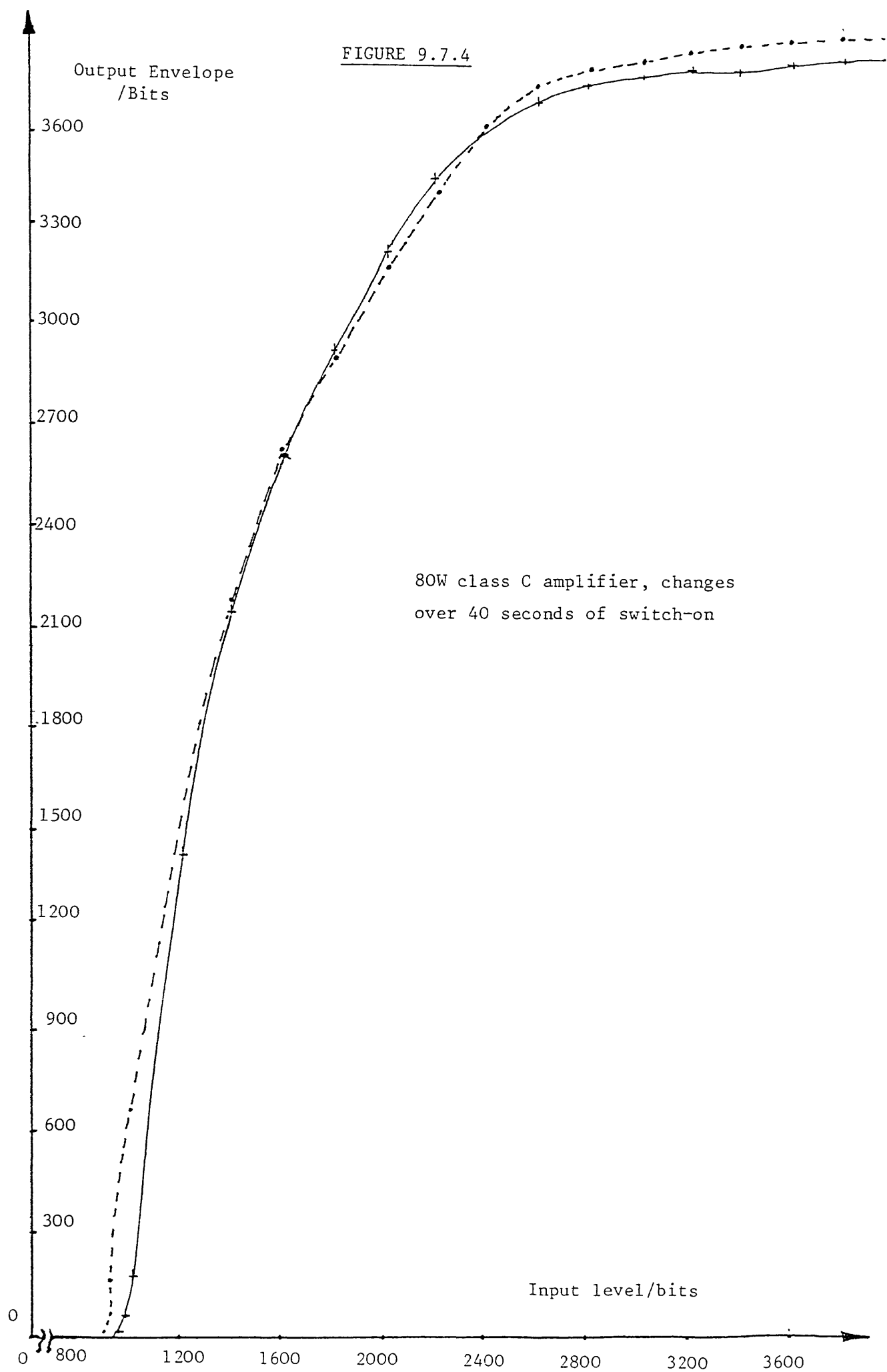


FIGURE 9.7.3





CHAPTER 10

TYPE 2 CONTROL ALGORITHMS

10.1 INTRODUCTION

If the input/output characteristics of the DUT are continuously monitored, and the series NLFG adjusted on the basis of that monitoring, then the control algorithm responsible for the NLFG is said to be a type 2 control algorithm.

Dependent upon the decision hierarchy, detectors used, etc., a large number of type 2 algorithms can be envisaged. Nevertheless, certain common features can be identified for type 2 algorithms which make it worthwhile to distinguish type 2 control. These features include:

1. The detectors need to be linear
2. A priori knowledge of the relationship of the NLFG to the DUT is required (i.e. scaling relationships between the RAM table and the DUT output detectors must be known).
3. There is a need for interpolation to extend the range over which compensation is applied.
4. The algorithm can be made unconditionally stable.
5. Sensitivity to thermal changes and drift is low, as the DUT is continuously monitored.
6. Suppression of intermodulation is not as high as type 1 and type 3 algorithms.

There appear to be no reports of type 2 algorithms being suggested in the literature. This undoubtedly stems from the "open loop" nature of the control. Type 1 control (as discussed in chapter 9), can be regarded as a feedback system in which A/D, D/A, and

modulator gains need not be known to determine the NLFG. It will be shown that the same applies to type 3 control. However, for type 2 control, this is not the case. Any error (or drift) in our specification of A/D, D/A or modulator gains, will directly affect the performance of the IMP suppression. Thus, from the outset, it is clear that type 2 control may prove the least useful of the three control algorithm types.

In consequent of this, our examination of type 2 control is more cursory than our examination of type 1 or type 3 control.

The purpose of this chapter is to investigate type 2 control in a theoretical and experimental context to quantify its limitations. To do this a polar baseband AM/AM compensation scheme was constructed and investigated. The system used is illustrated in Figure 10.1.1. Correction of AM/PM is by analogue phase locked loop. Although the analysis we must perform is specific to the system shown, both the system and analysis can usefully be applied to a much wider variety of circumstances than discussed here.

10.2 ANALYSIS OF TYPE 2 CONTROL

Operation of Figure 10.1.1 is as follows. The baseband input passes through the A/D, RAM, D/A, filter combination, (which acts as a NLFG), to the envelope modulator. The narrowband modulator output is amplified by the DUT. The other circuitry is used to set up and adjust the RAM table. Its operation is as follows. The output of the A/D (A) is passed to D/A2. Likewise, the output of the RAM table (d(A)) is passed to D/A1. Both data words are latched. The outputs of D/A1 and D/A2 correspond to the

current estimates of AM/AM distortion in the DUT. [Note that the value A_i corresponds to the expected envelope output of the DUT when the envelope input to the DUT is $d(A_i)$].

The outputs of D/A1 and D/A2 go to comparators, with latched outputs. At the instant the voltage at the output of the anti-alias filter is $Q_{D1}(d(A_i))$ volts (where $Q_{D1}(x)$ is the quantizing law for D/A1), comparator 1 changes state. Via a time delay network (to take account of finite time delays in the demodulation of the envelope, and in the transit time across the DUT), comparator 1 latches the output of comparator 2, which is comparing $Q_{D2}(A_i)$ with the output of the detector. According to the value latched, the RAM table is decreed to have under/over-estimated the value A_i corresponding to $d(A_i)$. The RAM table is modified accordingly.

The analysis (referring to the variables of Figure 10.1.1, and to those defined in chapter 9) proceeds as follows. The input to the envelope modulator at the firing of comparator 1 is x volts where

$$x = Q_{D1}(d(A_i)) \quad (10.1)$$

$$\text{and} \quad A_i = Q_A(V_i) \quad (10.2)$$

The output of the envelope detector is a function of the input to the envelope modulator

$$k(f(x)) \quad (10.3)$$

where $k(x)$ is the envelope detector characteristic. Thus the algorithm arranges that

$$Q_{D2}(A_i) = k(f(x)) = k(f(Q_{D1}(d(A_i)))) \quad (10.4)$$

Notice that in equation 10.4 $k(x)$ and $f(x)$ are not separable.

Thus any nonlinearities in the detector cannot be distinguished

from those of the DUT. Hence, we need to assume that the detector is linear. Thus if

$$k(x) = px + c \quad (10.5)$$

then from equation 10.4 we have that

$$Q_{D2}(A_i) = pf(Q_{D1}(d(A_i))) + c \quad (10.6)$$

The action of the type 2 algorithm is to force equation 10.6 to be true for all values of A_i . The algorithm is stable, in that $d(A_i)$ will simply be modified until equation 10.6 is true. Because of the absence of time dependence in equation 10.6 instabilities do not arise.

Given equation 10.6 we now wish to determine the output envelope of the DUT. This is given by $V_E(t)$ where

$$V_E(t) = f[Q_D(d(A_i))*r(t)*h(t)] \quad (10.7)$$

$$\text{where } A_i = Q_A(V_i) \quad (10.8)$$

and where $r(t) = 1$ if $|t| < 1$

$$= 0 \text{ otherwise}$$

$h(t)$ is the anti-alias filter impulse response

$*$ denotes the operation of convolution.

Unity sample rate is assumed for convenience of notation.

Now, as discussed in Appendix G, if the anti-alias filter is perfect, the impulse response can be taken outside the function $f(x)$ giving

$$V_E(t) = f[Q_D(d(A_i))]*r(t)*h(t) \quad (10.9)$$

$$\text{Now if } Q_{D1}(x) = Q_D(x) \quad (10.10)$$

then by equation 10.6

$$f(Q_D(d(A_i))) = (Q_{D2}(A_i) - c)/p \quad (10.11)$$

Thus in equation 10.9

$$V_E(t) = [(Q_{D2}(A_i) - c)/p]*r(t)*h(t) \quad (10.12)$$

and hence from equation 10.8

$$V_E(t) = [(Q_{D2}(Q_A(V_i)) - c)/p] * r(t) * h(t) \quad (10.13)$$

If we now assume that

$$Q_{D2}(Q_A(x)) = p'x + c' \quad (10.14)$$

we obtain

$$\begin{aligned} V_E(t) &= [(p'V_i + c' - c)/p] * r(t) * h(t) \\ &= (p'/p)(V_i * r(t) * h(t)) + ((c' - c)/p) * r(t) * h(t) \\ &= (p'/p)V_{in}(t) + \text{constant} \end{aligned} \quad (10.15)$$

Thus, given the assumptions, the output envelope is a linear copy of the baseband input, to within a constant.

10.3 DISCUSSION OF ASSUMPTIONS

The following assumptions have been made:

1. Comparators are perfect.
2. Time delay associated with the transit time of a signal from envelope modulator to demodulator has been taken into account.
3. The detector is linear.
4. That the two D/A's are identical (i.e. equation 10.10 holds).
5. The anti-alias filter is perfect.
6. The back to back combination of the A/D and D/A2 is linear (i.e. equation 10.14 holds).

There is little approximation associated with the first assumption. Any offset errors associated with the comparators are indistinguishable from a modification of c and c' and so can be ignored. Hysteresis can be kept small compared to the quantization errors. Provided switching speeds are in excess of

the time delay from modulator input to demodulator output, little error arises due to switching speed.

There appears to be little error associated with the second assumption. Superficially an error in the time delay compensation could lead to instabilities. In practice the time delay for which we have to compensate is very small in relation to the slew rate of the envelope, and so the voltage error associated with the timing error is comparable with quantization error, and thus of little consequence.

The third assumption is a necessary one, but one which we can force to be true (e.g. quasi-linear and precalibration).

The fourth assumption of identical D/A's arises from the requirement that D/A gains be known for type 2 systems. (In this system the requirement is for identical D/A's. The system was constructed with this in mind, since it was regarded as a less arduous constraint than knowledge of D/A gains). It is a difficult constraint to meet. Dual monolithic D/A's on the same chip are currently only available as 8 bit devices, and not as 12 bit. Use of D/A's not on the same chip gives rise to problems with temperature and drift. Worst still, it is difficult to ensure that the D/A's have the same gain (volts/bit), and offset.

The fifth assumption (perfect anti-alias filter) has been discussed at some length in chapter 9, and will not be repeated here since the same analysis applies. There is one further constraint however. Unlike type 1 algorithms, we are sensitive to any frequency independent scaling caused by the filter. (Attenuation by the filter would be indistinguishable from a decrease in the gain of the main D/A).

The last assumption arises from the need to know A/D and D/A gains. In fact it is a weak condition, since provided the A/D and D/A2 are linear the assumption holds to an accuracy associated with the quantization error. Thus, provided care is taken in the specification of the A/D and D/A2, this condition can hold.

To a good approximation then, it is assumed that the errors which do arise are the same as the errors of chapter 9, plus vector addition of the error associated with the assumption of equation 10.10. (We assume a near linear model, so that the component errors which arise can be summed vectorially to find the total error). Since the prime cause of the failure of equation 10.10 to hold is due to linear effects (i.e. gain and offset differences), we investigate the effect of replacing equation 10.10 with

$$Q_D(x) = (1 + a)Q_{D1}(x) + e \quad (10.16)$$

In equation 10.9 this gives

$$V_E(t) = f[(1 + a)Q_{D1}(d(A_i)) + e]*r(t)*h(t) \quad (10.17)$$

Taylor expanding for a, e small we get that $V_E(t)$ is approximately equal to

$$f(Q_{D1}(d(A_i)))*r(t)*h(t) + [aQ_{D1}(d(A_i)) + e](df/dx)*r(t)*h(t) \quad (10.18)$$

where (df/dx) is evaluated at $x=Q_{D1}(d(A_i))$. Thus assuming a linear detector ($p'=1$, $c'=0$ in equation 10.14), an error arises in equation 10.15 of magnitude

$$(1/p)[aQ_{D1}(d(A_i)) + e](df/dx)*r(t)*h(t) \quad (10.19)$$

If we approximate

$$(df/dx) = b \quad (10.20)$$

where b is a constant and note that typically

$$b \approx p \quad (10.21)$$

we obtain in equation 10.19

$$[aQ_{D1}(d(A_i)) + e]*r(t)*h(t) \quad (10.22)$$

Replacing $Q_{D1}(x)$ with $Q_D(x)$ via equation 10.16 and neglecting the ae term as second order small we obtain

$$[(a/(1 + a))Q_D(d(A_i)) + e]*r(t)*h(t) \quad (10.23)$$

Binomial expanding $1/(1+a)$ and neglecting second order small terms we obtain

$$[aQ_D(d(A_i)) + e]*r(t)*h(t) \quad (10.24)$$

We take equation 10.24 no further. Our interpretation is as follows. The gain error (a) scales the spectrum out of the anti-alias filter, such that harmonic levels are reduced in magnitude, relative to peak voltage. For example, if $a=0.01$ (corresponding to a 1% gain error), then if the output of the anti-alias filter has a spectrum with components $-X$ dB on peak voltage, the errors in the RF/AM spectrum due to equation 10.24 will contain errors at a level $-(X+20+6)$ dB on peak voltage. [The term $e*r(t)*h(t)$ gives rise only to a dc offset in the carrier in this model].

In practice the gain error may be better than 1%. However, a frequency independent 0.09 dB attenuation in the anti-alias filter would have the effect of causing an apparent gain error of this magnitude. Thus, type 2 control algorithms are sensitive to frequency independent attenuation of the filter, by a mechanism which does not exist for type 1, (or type 3), algorithms. This sensitivity is in addition to the problems caused by imperfect anti-alias filters. For type 1 (and type 3) control any frequency independent attenuation was indistinguishable from a redefinition of $Q_D(x)$. Thus, since type 1 (and type 3) control has a

performance independent of $Q_D(x)$, no problems were caused.

10.4 STOCHASTIC SAMPLING OVER A FINITE RANGE

It is important to recognise that, for type 2 control, we are not free to choose the element in the RAM table that corresponds to the measurement, i.e. we simply process the current A/D address and RAM data ($A_i, d(A_i)$), without being free to choose A_i or $d(A_i)$. Thus, how many measurements will we need to make before we have reasonable certainty that we have acquired information on all the elements in the RAM table? This question is discussed in Appendix H, the result being that the required number of measurements is given by

$$M \log_e(M) \quad (10.25)$$

where M is the number of possible values of A_i . Thus, with a 12 bit A/D, $M=4096$ and approximately 34000 measurements need to be made. With the results from these measurements the RAM table can be moved closer to optimal. Taking into account anti-alias filter delays, for the test setup used any machine coded program would have to wait approximately 150 microseconds per measurement (to set up the D/A's, wait for the comparator to fire, store the resultant data). Thus, in the example given ($M=4096$), a total of 5.1 seconds are required to gather information on the DUT. From this information the RAM table can be adjusted by only one bit. (We gather information only as to the DUT output being bigger or smaller than expected). Replacement of the comparators by A/D's would alleviate this problem. For the comparator system used, we can only move the RAM table data up or down one bit on the basis of the information gathered. Hence, if we have a 12 bit D/A, we

may need to make 4096 adjustments of the RAM table before equilibrium is achieved. Thus, for the example given, it may take $4096(5.1)$ seconds = 5.8 hours to achieve convergence.

In general if t is the time taken per measurement and M' is the total number of D/A levels then the maximum possible time to come to equilibrium is given by

$$tM'M\log_e(M) \quad (10.26)$$

In practice, of course, trend removal will ensure that the RAM will start much closer to the final equilibrium solution, by a factor of 4 to 6. However, the time taken to come to equilibrium may still be unacceptable.

The process can be speeded up if, instead of insisting on one measurement per A/D level, we insist on one measurement per A/D bin. A bin consists of a number of adjacent A/D levels (typically 16 adjacent levels). According to the measurements, the centre value of the bin is adjusted up/down, and linear interpolation used between bin centre values. Using bins of width 16 means that for a 12 bit system M becomes 256. Thus, by equation 10.26, the time to come to equilibrium is approximately 14.5 minutes. Given that typically we will require less time by a factor of 4 to 6, the time taken to come to equilibrium appears more acceptable.

Another factor which must be considered for a practical algorithm concerns the range over which information is gathered. In the type 1 algorithm the DUT was evaluated over its full range, (i.e. the ramp took values from zero to full scale). With a type 2 algorithm there is no control over the set of values over which measurements are made (see Figure 10.4.1). Thus, when data on the

RAM table beyond a certain range is not available, we have no choice but to attempt some form of interpolation. Reference 116 gives a good discussion of interpolation, and points out that the variance of the error outside of the range increases with the order of the polynomial used for interpolation. However, since the underlying curve which we are trying to approximate will contain high order polynomial terms, we must inevitably accept imperfections in the RAM table in the interpolated range.

10.5 EXPERIMENTAL RESULTS

The RF spectrum "before and after" type 2 control is shown in Figure 10.5.1. The results are disappointing compared to type 1 (and type 3) control. This is thought to be primarily due to the effects already outlined (imperfect anti-alias filter, non-identical D/A's and non-zero filter attenuation).

The implementation of the control algorithm was not quite as described, because of speed restraints imposed by the CPU we had to allow 500 microseconds per measurement. This is not believed to have affected overall IMP suppression. A simple moving average filter was used on the RAM table to smooth and extend the interpolation range.

10.6 CONCLUSIONS

Type 2 control is not a particularly attractive way of compensating for RF nonlinearities. However, as a system it lies part way between the more important type 1 and type 3 control systems, and so occupies an important conceptual position in digital compensation.

As a method of implementing a reduction of intermodulation distortion, the method used here can be improved. Specifically the following changes could be expected to improve the performance of the system:

1. Use dual, monolithic, matched D/A's, preferably on the same chip. Such devices have just started to become available in 8 bit form (e.g. the Analogue Devices AD7528).
2. Implement a type 1 control algorithm to relate the quantization laws of the D/A's, and correct for the differences in software (effectively precalibrate the differences in the D/A's).

These possibilities were not investigated here since it was felt that type 1 and type 3 algorithms had greater potential, even allowing for such improvements.

FIGURE 10.1.1

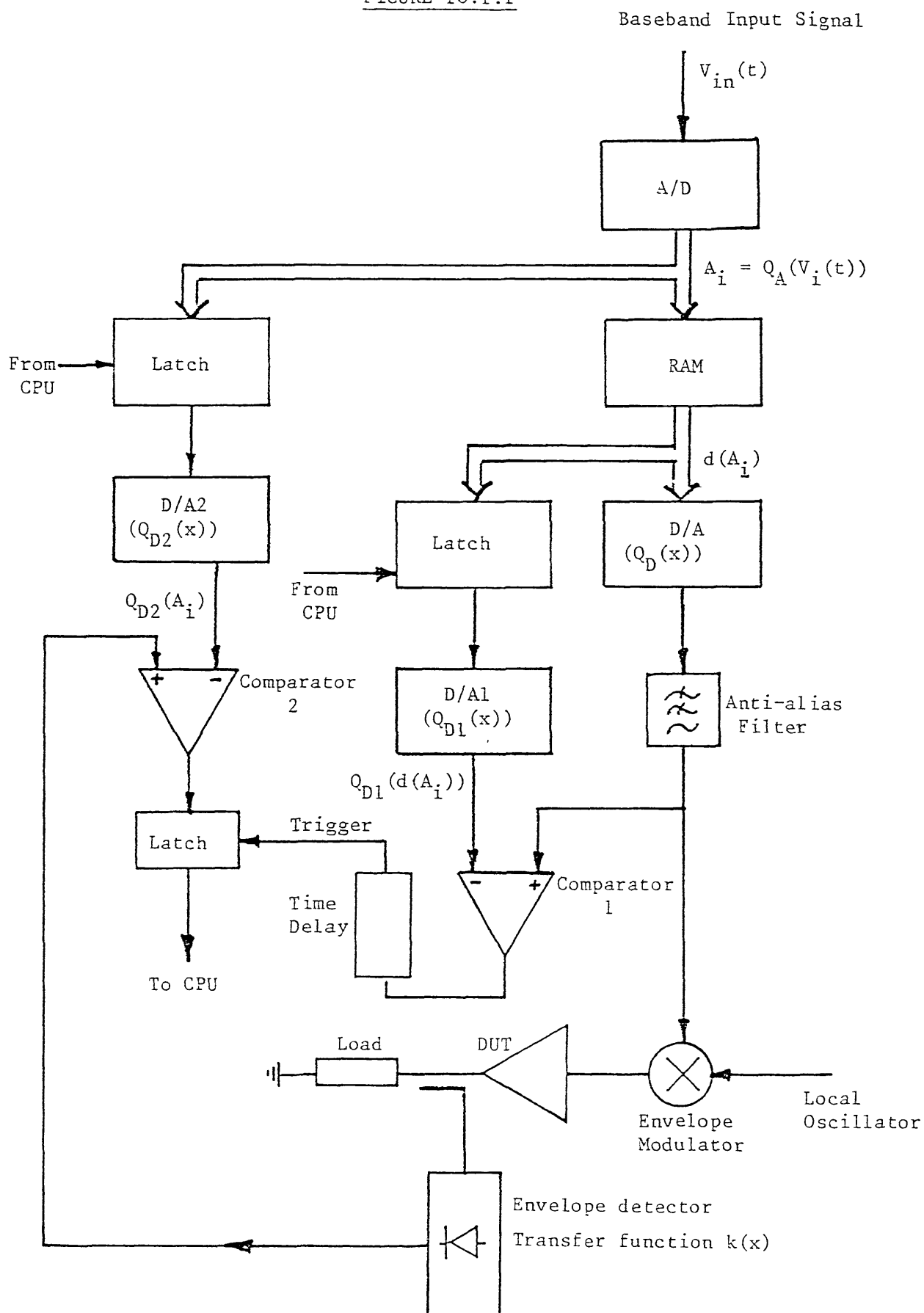
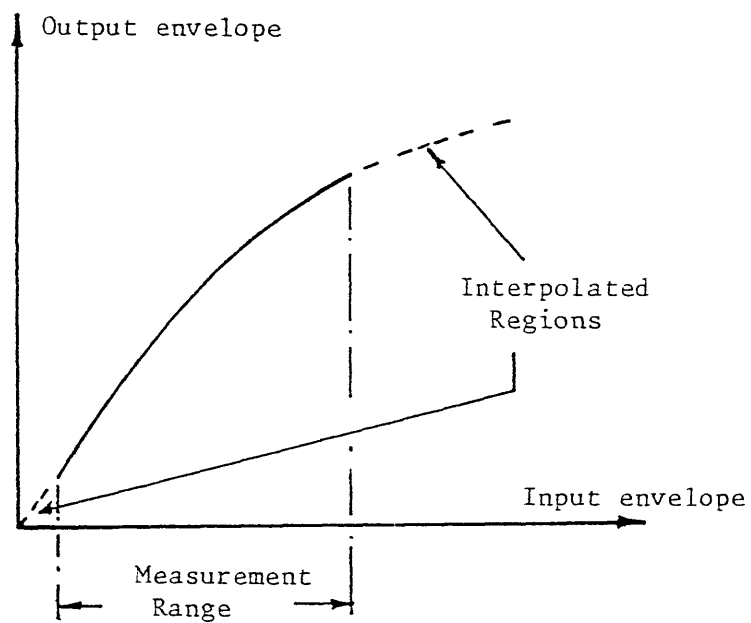
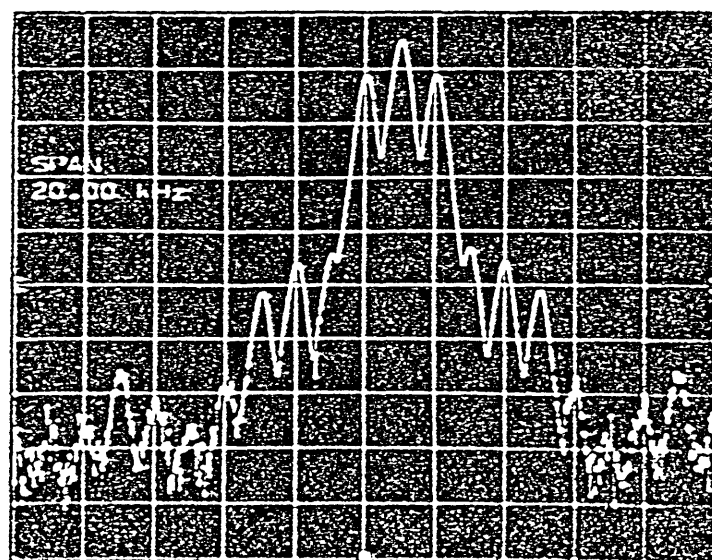
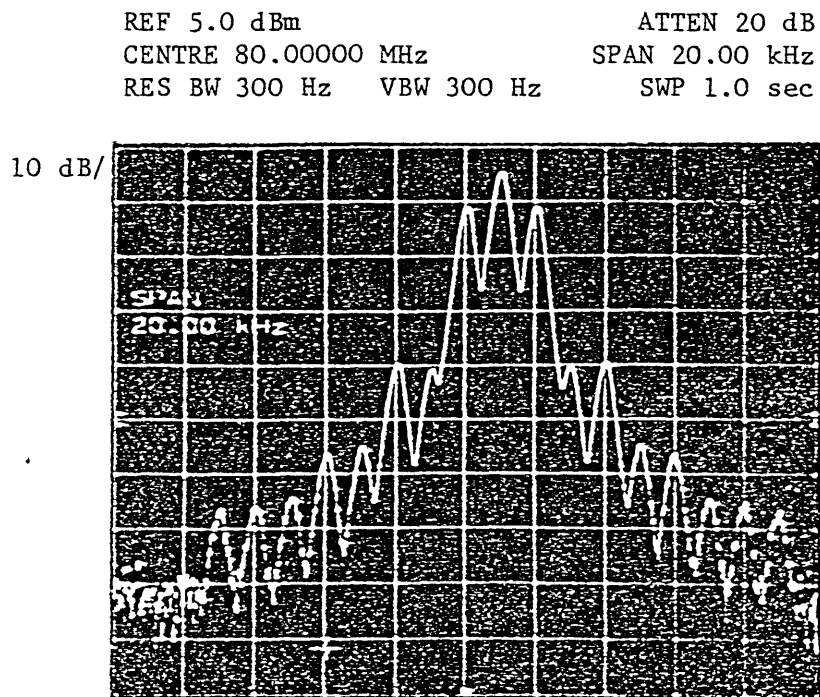


FIGURE 10.4.1

Data as to the form of the envelope transfer characteristics is only received over a limited range corresponding to the statistics of the input signal. Beyond this range we have no choice but to interpolate.

FIGURE 10.5.1



RF spectrum using Type 2 control algorithm and phase locked loop feedback.

The reduction in IMD is almost entirely due to the Type 2 envelope control.

However, it is clear that the remaining distortion is due to AM/AM distortion and that the Type 2 control is not as effective as Type 1 control.

CHAPTER 11

TYPE 3 CONTROL ALGORITHMS

11.1 INTRODUCTION

If the AM/AM or AM/PM characteristics of the series combination of the DUT and NLFG are continuously monitored, and the NLFG continuously adjusted on the basis of that monitoring, the control algorithm responsible for the NLFG is said to be a type 3 control algorithm.

Dependent on the decision hierarchy and on the particular circuitry involved, it is possible to envisage a large class of type 3 control algorithms. Nevertheless, certain features are common to members of the class which make it worthwhile to distinguish type 3 algorithms. These features include:

1. The limitations imposed on the detectors can be made very lax (e.g. the requirement for linear behaviour can be abrogated in IF systems).
2. There is a need for interpolation of the transfer characteristics, outside the range over which measurements are made.
3. The algorithm may be potentially unstable in the BIBO sense.
4. The algorithm will keep the NLFG in a dynamic equilibrium, rather than a static equilibrium.
5. Suppression of IMD can be high.
6. The sensitivity to thermal changes and drift is very low.

Only one type 3 algorithm has been discussed, and that is due to Grabowski and Davis [ref 66]. As outlined in chapter 3, it is

clear that their scheme represents a somewhat special case because of the positioning of the anti-alias filter at RF.

For this study the setup of Figure 11.1.1 was built.

Conceptually Figure 11.1.1 can be considered as Figure 11.1.2.

In Figure 11.1.2 we attempt the envelope correction of an AM modulated RF carrier. The carrier is modulated and then passes through the DUT to the load. The output envelope is detected, and compared with a suitably time-delayed version of the baseband input signal. For AM, this baseband input signal corresponds to the envelope to be expected from a linear DUT. According to the results of the comparison, the CPU modifies the RAM table to obtain a more linear AM envelope.

Although the test setup represents a specialised case, much of the control algorithm analysis transfers over to other cases of type 3 control. The primary drawback of Figure 11.1.2 is that, being a baseband system rather than an IF system, the requirement for the detector to be linear still remains. Although this affects the performance of the system, it does not in fact reflect greatly on the control analysis, which is our principal concern here.

11.2 SIMPLIFIED ANALYSIS

Before going on to consider the problem posed by Figure 11.1.2 we consider some simpler problems, in an effort to gain some insight into the problem.

11.2.1 Cascaded Nonlinearities Under Type 3 Control

In Figure 11.2.1 we have two nonlinearities in cascade,

$d(x)$ and $f(x)$. By monitoring the linearity of the series combination, when the input is X , we use an iterative procedure to adjust the value of $d(X)$, to obtain linearity of the cascade.

The output of the series combination on the n^{th} iteration is described by

$$f(d^n(X)) \quad (11.1)$$

and we require that the series combination as n approaches infinity is described by mX . Consider the iterative algorithm

$$d^{n+1}(X) = G(mX - f(d^n(X))) + d^n(X) \quad (11.2)$$

where G is a constant yet to be determined. If we now define M such that $f(M)=mX$ then we define the error of the n^{th} iteration (e^n) by $e^n=M-d^n(X)$. Thus

$$\begin{aligned} f(d^n(X)) &= f(M - e^n) \\ &\approx f(M) - e^n(df/dx) \\ &= mX - e^n(df/dx) \end{aligned} \quad (11.3)$$

where (df/dx) is evaluated at $x=X$. Substituting into equation 11.2 we get

$$\begin{aligned} M - e^{n+1} &= G(mX - mX + e^n(df/dx)) + M - e^n \\ \text{i.e.} \quad e^{n+1} &= (1 - G(df/dx))e^n \end{aligned} \quad (11.4)$$

Plainly then for the error term in equation 11.4 to vanish as n approaches infinity we require that

$$\begin{aligned} |1 - G(df/dx)| &< 1 \\ \text{i.e.} \quad 0 &< G(df/dx) < 2 \end{aligned} \quad (11.5)$$

Given an initial error e^0 we note that the error e^n after n iterations is given by

$$e^0(1 - G(df/dx))^n \quad (11.6)$$

We can meet the condition imposed by equation 11.5 provided G is small, if the nonlinearity $f(x)$ has a constant sign for its slope

over the range of interest. For a continuous function this corresponds to demanding that $(df/dx) \neq 0$ over the range, i.e. that there are no stationary points in the range (see Figure 11.2.2).

Thus, given the monotonic type of nonlinearity met in the NBNL problem, and given that we shall not attempt to operate at saturation (where $(df/dx) = 0$), it is always possible to choose G sufficiently small to guarantee the convergence of equation 11.2. We note, however, that the algorithm does not converge at the origin for nonlinearities such as class C amplifiers. Away from the origin there is no problem with such an amplifier.

11.2.2 Effect of an Arbitrary Filter on a Nonlinearity

Consider now the situation described by Figure 11.2.3, which consists of a nonlinear function $d(x)$ in series with an arbitrary filter (with impulse response $h(t)$). The input to the system is a single cosine wave. We wish to approximate the system as a single nonlinear function $d'(x)$, and our problem is how this can best be attempted. Trivially we connect oscilloscope probes across the system and observe the X-Y relationship between input and output (we take account of the delay between X and Y channels). We in fact generate Lissajous figures. Computer generated figures are shown in Figures 11.2.4 to 11.2.9. The program used for the generation of these figures is shown in Appendix I. From these figures we note the following:

1. For a filter with constant group delay we can model the series combination as a nonlinear function $d'(x)$ in series with a constant time delay. However, this function is dependent on the amplitude response of the filter, and

hence also on the frequency of the input cosinusoid.

(Compare Figure 11.2.4 with Figure 11.2.5 where the only parameter that has been changed is increased attenuation of the third harmonic. Similarly for Figure 11.2.7 and Figure 11.2.8).

2. Considering just the cosinusoidal input, when the group delay is not constant hysteresis is introduced, making it impossible to define $d'(x)$. (Compare Figure 11.2.4 with Figure 11.2.6 where attenuation and phase shift of just the third harmonic is introduced, and with Figure 11.2.5 where just attenuation is introduced). However, there is a well-defined average to the hysteresis function, which allows us to define $d'(x)$ which is at least minimal in its error in approximating the hysteresis curve (see Figure 11.2.6).
3. For non-cosinusoidal inputs it may not be possible to define $d'(x)$ as a continuous function, in either the pure attenuation case or in the hysteresis case (see Figures 11.2.7 to 11.2.9).

11.2.3 Discussion

The type of system we shall consider is shown in Figure 11.2.10. The filter shown in Figure 11.2.10 is to take account of imperfections in the anti-alias filters of practical systems. If the anti-alias filter could be constructed to give ideal response, then the filter of Figure 11.2.10 could be removed, and the system can be considered as in Figure 11.2.1. Grabowski and Davis [ref 66] used a system as in Figure 11.2.11 in which the anti-alias

filter is placed after the DUT nonlinearity. In such circumstances the analysis of the cascaded nonlinearities applies directly, and the type 3 algorithm such as equation 11.2 can be used without problems. However, in general, it is not always possible to construct the anti-alias filter at RF due to the Q's involved.

This study looks at systems where the anti-alias filter lies between the NLFG and the DUT, and attempts to show that useful systems result.

11.3 ANALYSIS

11.3.1 Introduction

We define the problem with reference to Figure 11.3.1. Consider the series combination of NLFG, filter (the precise nature of the filter being related to the imperfections in the anti-alias filter), and DUT. Consider the observable variables of the DUT output, the NLFG input, and the NLFG output, (trivially known if the NLFG characteristics are known). Using these considerations how best should the iterative control function be constructed to minimise the nonlinear error of the DUT output? Thus referring to the variables defined in Figure 11.3.1 we have that

$$d^{n+1}(V_{in}(t)) = C(V_{in}(t), d^n(V_{in}(t)), f[d^n(V_{in}(t))*h(t)]) \quad (11.7)$$

such that

$$\lim_{n \rightarrow \infty} E = \int_{-\infty}^{+\infty} [f(d^n(V_{in}(t))*h(t)) - V_{in}(t)] dt \quad (11.8)$$

is minimal.

It is clear from the Lissajous figures considered that equation 11.2 represents an optimal control algorithm in systems

where the combination of NLFG and filter can be considered as a nonlinear function in series with a constant time delay. In such circumstances the error defined in equation 11.8 is zero.

For non-constant group delay, or frequency dependent attenuation, the nonlinear function is dependent on the input waveform, and so will be constantly varying, unless the input waveform is fixed. If we now consider the hysteresis effect of non-constant group delay then it is clear that the error in equation 11.8 can never be zero in such circumstances.

Thus, although we cannot solve equation 11.8 in any satisfactory analytic way for the general case, we do have a solution for the case of constant group delay with no frequency dependent attenuation. We attempt to apply this solution to the general case, in the hope that for small group delay/amplitude variations, we are still near-optimal. We shall then investigate the effects of non-cosinusoidal input waveforms.

11.3.2 Effect of Small Group Delay Filter and Nonlinearity

We refer to the situation shown in Figure 11.3.1. Let the input to the NLFG be a cosinusoid of unity magnitude, and let the NLFG $d^n(x)$ be well approximated by a Chebyshev series of coefficients d^n_i . Thus the input to the NLFG is

$$V_{in}(t) = \cos(wt) \quad (11.9)$$

which produces an output from the NLFG of

$$\begin{aligned} V_{out} &= \sum_i d^n_i T_i(V_{in}) \\ &= \sum_i d^n_i T_i(\cos(wt)) \\ &= \sum_i d^n_i \cos(iwt) \end{aligned} \quad (11.10)$$

Now if the filter is described by coefficients h_i and ϕ_i giving

the amplitude and phase shifts at frequency ω , then the output of the filter $V_f(t)$ is given by

$$\begin{aligned} V_f(t) &= \sum_i d_i^n h_i \cos(\omega t + \phi_i) \\ &= \sum_i d_i^n h_i (\cos(\phi_i) \cos(\omega t) - \sin(\phi_i) \sin(\omega t)) \\ &= \sum_i d_i^n h_i \cos(\phi_i) T_i(V_{in}) \pm (1 - (V_{in})^2)^{1/2} (\sum_i d_i^n h_i \sin(\phi_i) (U_{i-1}(V_{in}))) \quad (11.11) \end{aligned}$$

where $U_i(x)$ is a Chebyshev polynomial of the second kind. The properties of Chebyshev polynomials are listed in Appendix B.

Thus, if we drop the t dependence entirely, and think only in terms of a transfer function of an input variable x and the filter output $D^n(x)$ we get

$$D^n(x) = \sum_i d_i^n h_i \cos(\phi_i) T_i(x) \pm (1 - x^2)^{1/2} (\sum_i d_i^n h_i \sin(\phi_i) (U_{i-1}(x))) \quad (11.12)$$

Defining

$$\sum_i d_i^n h_i \cos(\phi_i) T_i(x) = A \quad (11.13)$$

$$\text{and} \quad (1 - x^2)^{1/2} (\sum_i d_i^n h_i \sin(\phi_i) (U_{i-1}(x))) = B \quad (11.14)$$

$$\text{we get} \quad D^n(x) = A \pm B \quad (11.15)$$

The ambiguity of sign in equation 11.15 arises from the square root function being undefined in sign. Equation 11.15 represents the relationship between the input and the output voltage through the NLFG and filter (neglecting constant time delays). Thus the hysteresis which arises from the non-constant group delay ($\phi_i \neq 0$) is represented by the second term.

11.3.3 Algorithm Analysis

If we now examine equation 11.2 using equation 11.15 we obtain

$$d^{n+1}(x) = G(mx - f(D^n(x))) + d^n(x) \quad (11.16)$$

Plainly the function $f(D^n(x))$ contains hysteresis, and so a solution to equation 11.8 cannot be found with $E=0$. We conjecture

that a near-optimal solution may be found by dealing with the function defined by the average of $f(D^n(x))$. Hence we get

$$\langle f(D^n(x)) \rangle = \langle f(A \pm B) \rangle \quad (11.17)$$

Taylor expanding and noting that $B \ll A$ for $\phi_i \ll 1$

$$\begin{aligned} \langle f(D^n(x)) \rangle &= \langle f(A) \rangle + \langle \pm B(df/dx) \rangle + \langle (B^2/2)(d^2f/dx^2) \rangle + \dots \\ &= f(A) + 0 + (B^2/2)(d^2f/dx^2) + \dots \\ &= f(A) + O(\sin^2(\phi_i)) \end{aligned} \quad (11.18)$$

Thus in equation 11.16 we obtain

$$\begin{aligned} d^{n+1}(x) &= G(mx - \langle f(D^n(x)) \rangle) + d^n(x) \\ &\sim G(mx - f(A)) + d^n(x) \end{aligned} \quad (11.19)$$

We now define the Chebyshev coefficients of d to be the sum of a perfect term and an error term, i.e.

$$d^n_i = a_i + e^n_i \quad (11.20)$$

$$\text{and} \quad d^{n+1}_i = a_i + e^{n+1}_i \quad (11.21)$$

$$\text{where} \quad f(\sum_i a_i h_i \cos(\phi_i) T_i(x)) = mx \quad (11.22)$$

Substituting these equations into equation 11.19 and Taylor expanding $f(x)$, assuming the error term is small, gives

$$\begin{aligned} &\sum_i a_i T_i(x) + \sum_i e^{n+1}_i T_i(x) \\ &= \sum_i [a_i T_i(x) + e^n_i T_i(x) - G e^n_i h_i \cos(\phi_i) (T_i(x)) (df/dx)] \end{aligned} \quad (11.23)$$

Hence

$$\sum_i e^{n+1}_i T_i(x) = \sum_i [e^n_i T_i(x) - G e^n_i h_i \cos(\phi_i) (T_i(x)) (df/dx)] \quad (11.24)$$

which becomes

$$\sum_i e^{n+1}_i T_i(x) = \sum_i [1 - G h_i \cos(\phi_i) (df/dx)] e^n_i T_i(x) \quad (11.25)$$

To proceed we make the approximation

$$(df/dx) \approx b \quad (11.26)$$

where b is a constant. Thus we can equate coefficients of $T_i(x)$

in equation 11.25 to get

$$e^{n+1}_i = (1 - G h_i b \cos(\phi_i)) e^n_i \quad (11.27)$$

Plainly for convergence of e^{n+1}_i to zero we require b to be small and the $\cos(\phi_i)$ term to be of fixed sign. Note that the amplitude term h_i is (by definition) always positive.

We note that for $|\phi_i| < \pi/2$ the sign of $\cos(\phi_i)$ is fixed. Now for high order terms (i large), corresponding to frequencies close to the band edge of the filter, we conjecture that ϕ_i may be large, and the sign of $\cos(\phi_i)$ subject to change. Thus, we see the desirability of filtering the RAM table in Chebyshev space, so that the i^{th} Chebyshev coefficient is weighted by F_i . If we do this we must return to equation 11.23 and obtain

$$a_i + e^{n+1}_i = F_i[a_i + e^n_i - Gbe^n_i h_i \cos(\phi_i)] \quad (11.28)$$

$$\text{i.e.} \quad e^{n+1}_i = a_i(F_i - 1) + F_i(1 - Gbh_i \cos(\phi_i))e^n_i \quad (11.29)$$

$$\text{Define} \quad C_i = a_i(F_i - 1) \quad (11.30)$$

$$P_i = F_i(1 - Gbh_i \cos(\phi_i)) \quad (11.31)$$

$$\text{then} \quad e^{n+1}_i = C_i + P_i e^n_i \quad (11.32)$$

Given this relationship, and given an initial error e^0_i in the i^{th} coefficient, the error in the i^{th} coefficient after n iterations is given by

$$\begin{aligned} e^n_i &= (P_i)^n e^0_i + C_i(1 + P_i + (P_i)^2 + (P_i)^3 + \dots + (P_i)^{n-1}) \\ &= (P_i)^n e^0_i + C_i(1 - (P_i)^n)/(1 - P_i) \end{aligned} \quad (11.33)$$

Hence from equation 11.33 we see that the criterion for stability (i.e. to avoid BIBO instability) is that $|P_i| < 1$, i.e.

$$|F_i(1 - Gbh_i \cos(\phi_i))| < 1 \quad (11.34)$$

and that the resultant final error (e^f_i) as n approaches infinity is

$$\begin{aligned} e^f_i &= C_i/(1 - P_i) \\ &= a_i(F_i - 1)/(1 - F_i(1 - Gbh_i \cos(\phi_i))) \end{aligned} \quad (11.35)$$

It is important to note that for the above equation

$$\begin{aligned}
&\text{if } F_i=0 \quad \text{then} \quad e_i^f = -a_i \\
&\text{if } F_i=1 \quad \text{then} \quad e_i^f = 0
\end{aligned}
\tag{11.36}$$

We note the following:

1. We must filter ($0 < F_i < 1$) before the phase error reaches 90 degrees and changes the sign of $\cos(\phi_i)$ in equation 11.34. This will ensure stability.
2. Convergence may be slow if the $Gbh_i\cos(\phi_i)$ term in equation 11.31 is small when $F_i=1$, especially if $h_i \ll 1$. In the passband however, there should be little problem (h_i approximately equal to 1). Since the bulk of the signal will lie in the passband of the anti-alias filter there is little problem with the rate of convergence.
3. Given stability ($|P_i| < 1$), if $F_i=1$ then $C_i=0$ and the final error is zero, irrespective of the values of h_i and ϕ_i .
4. We might suspect that, for a well-designed system, the need to Chebyshev filter will be at such a large value of i that the error term will produce little effect as a_i will be small.
5. The relationship between input frequency and equation 11.34 and equation 11.35 is as follows - increasing the input frequency will change the value of $h_i\cos(\phi_i)$, forcing the RAM table to change (i.e. the value of a_i is changed). However the RAM table will converge to a stable state, provided the frequency change does not impose a change of sign of $\cos(\phi_i)$, while $F_i=1$.

11.3.4 Spectral Output of the DUT

Equation 11.35 deals with the error in the Chebyshev

coefficient in the RAM table, not with the error in the DUT output spectrum, which is of course our real interest. Nor have the spectral effects of the hysteresis associated with the non-constant group delay of the filter been investigated.

By making the assumption that the DUT is approximately linear, and that aliasing overlap does not occur at the NLFG output, we may analyse these effects separately and use superposition to find the combined effects. This, although not exact, gives a useful engineering approximation.

Effect of errors in the RAM table

We can write the final state of the RAM as being described by

$$d^f(x) = \sum_i a_i T_i(x) + \sum_i e_i^f T_i(x) \quad (11.37)$$

where the e_i^f are defined by equation 11.35. Thus the average transfer function observed at the output of the filter is given from equation 11.18 as

$$\langle f(D^f(x)) \rangle \sim f[\sum_i a_i h_i \cos(\phi_i)(T_i(x)) + \sum_i e_i^f h_i \cos(\phi_i)(T_i(x))] \quad (11.38)$$

Taylor expanding and neglecting the error term as small we get

$$\begin{aligned} \langle f(D^f(x)) \rangle &\sim f[\sum_i a_i h_i \cos(\phi_i)(T_i(x))] + \sum_i e_i^f h_i \cos(\phi_i)(T_i(x)) (df/dx) \\ &= mx + \sum_i e_i^f h_i \cos(\phi_i)(T_i(x))b \end{aligned} \quad (11.39)$$

thus as $x = \cos(wt)$ we have

$$\langle f(D^f(x)) \rangle \sim m \cos(wt) + \sum_i e_i^f h_i \cos(\phi_i)(\cos(iwt))b \quad (11.40)$$

Hence the ratio of fundamental to i^{th} harmonic error (K_i^L (dB)) is given by

$$K_i^L = 20 \log[e_i^f h_i \cos(\phi_i)(b/m)] \quad (11.41)$$

Effect of hysteresis

For the analysis we assume that the final RAM state is described by equation 11.37, and that the output of the filter is as equation 11.12, whilst the error term is zero. With these assumptions the DUT output is

$$f(\sum_i a_i h_i \cos(\phi_i)(T_i(x)) \pm (1 - x^2)^{1/2} (\sum_i a_i h_i \sin(\phi_i)(U_{i-1}(x))) \quad (11.42)$$

where $x = \cos(wt)$ as before. Taylor expanding for the second term small we get

$$\begin{aligned} & mx \pm (1 - x^2)^{1/2} (\sum_i a_i h_i \sin(\phi_i)(U_{i-1}(x))b) \\ & = m \cos(wt) + \sum_i a_i h_i \sin(\phi_i)(\sin(iwt))b \end{aligned} \quad (11.43)$$

Thus the ratio of fundamental to i^{th} harmonic error (K_i^H (dB)) is given by

$$K_i^H = 20 \log[a_i h_i \sin(\phi_i)(b/m)] \quad (11.44)$$

Combined error effects

Combining equation 11.41 with equation 11.44 and noting that the effect of the two errors (errors in the RAM table and hysteresis) are in quadrature and must be added vectorially to produce the total error (K_i (dB)) we get

$$\begin{aligned} K_i &= 20 \log[((a_i h_i \sin(\phi_i)(b/m))^2 + (e_i^f h_i \cos(\phi_i)(b/m))^2)^{1/2}] \\ &= 20 \log[(a_i^2 \sin^2(\phi_i) + (e_i^f)^2 \cos^2(\phi_i))^{1/2} (b h_i / m)] \end{aligned} \quad (11.45)$$

In fact the error term due to hysteresis is very similar to the error term due to an imperfect anti-alias filter in chapter 9, except that only the quadrature effect of an imperfect filter contributes. Thus, the term due to hysteresis is in general better than can be achieved using a type 1 algorithm. Within the region $F_i=0$ we have that $e_i^f = -a_i$ from equation 11.35, and so equation 11.45 becomes

$$K_i = 20 \log[(a_i b_i)/m] \quad (11.46)$$

Note that this is measured from fundamental, not from peak voltage. Typically b is approximately equal to m in equation 11.46.

11.3.5 Non-Cosinusoidal Input

The present analysis is somewhat limited, in that it applies to unity magnitude cosine waves, and not to other waveforms. Unlike situations where linear differential equations apply, it is not possible to apply superposition of sums of sine and cosine waves to a nonlinear problem.

The analysis can be extended to a larger class of problems using the matrix formulation developed below. The matrix formulation, when coupled to computer analysis, enables us to avoid the simplifying linear assumptions of the analysis presented above.

Of particular importance is the Chebyshev cascade matrix which has been developed specifically for this study. The cascade matrix bears some relation to a Gram-Schmidt matrix. We have the following theorem:

$$\text{if} \quad f(x) = \sum_i a_i T_i(x) \quad (11.47)$$

$$\text{and} \quad g(x) = \sum_k b_k T_k(x) \quad (11.48)$$

$$\text{then} \quad g(f(x)) = \sum_j d_j T_j(x) \quad (11.49)$$

$$\text{where} \quad \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ d_3 \\ \vdots \end{bmatrix} = \begin{bmatrix} c_{00} & c_{01} & c_{02} & c_{03} & \dots \\ c_{10} & c_{11} & c_{12} & c_{13} & \dots \\ c_{20} & c_{21} & c_{22} & c_{23} & \dots \\ c_{30} & c_{31} & c_{32} & c_{33} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \end{bmatrix} \quad (11.50)$$

$$\text{or} \quad \underline{d} = [\underline{c}]\underline{b} \quad (11.51)$$

$$\text{where} \quad c_{jk} = (1/B) \int_{-1}^1 T_k \left[\sum_i a_i T_i(x) \right] (T_j(x)) / ((1-x^2)^{1/2}) dx \quad (11.52)$$

$$\begin{aligned} \text{and} \quad B &= \pi/2 \quad \text{if } j \neq 0 \\ &= \pi \quad \text{otherwise} \end{aligned} \quad (11.53)$$

The proof of this theorem is given in Appendix K.

The computation of the cascade matrix $[c]$ is dependent only on the definition of $f(x)$, not on $g(x)$. For simple cases, and using the properties of the Chebyshev polynomials given in Appendix B, the cascade matrix can be written down by inspection. Figure 11.3.2 lists cascade matrices for simple cases.

Given the cascade matrix, our approach is as outlined in Figure 11.3.3. The input to the system is a unity magnitude cosine wave, which is modified in waveform by a nonlinearity $M(x)$. The output of $M(x)$ must be an even waveform with respect to time. We shall show that provided the inverse of $M(x)$ exists, there are no further constraints on the form of $M(x)$. The output of $M(x)$ then goes to the same system as before ($d(x)$, filter, DUT). Thus for this system we must consider the convergence of the algorithm

$$d^{n+1}(M(x)) = F * [G(M(x) - \langle f(d^n(M(x))) * h(t) \rangle) + d^n(M(x))] \quad (11.54)$$

where $x = \cos(\omega t)$ as before and the operation $*$ denotes filtering in Chebyshev vector space, and is given a precise meaning in x space in reference 11. From that reference we see that a "convolution-like" interpretation can be put on $*$. Most texts however [ref 14, 54], use the approach of determining the Chebyshev series by orthogonality and truncate the resultant series (i.e. transform to Chebyshev space, filter, and inverse transform). In fact, since we shall soon transform to Chebyshev space, we have little concern with the precise meaning in x space

of $\underline{*}$.

Considering the $d^n(M(x))$ term in equation 11.54, in vector form we can write

$$d^n(M(x)) \equiv [M]\underline{d}^n \quad (11.55)$$

If we now proceed through the same arguments as before for the filter modifying the amplitude and phase of the harmonics, we obtain in vector form

$$\langle d^n(M(x))*h(t) \rangle \equiv [Hcos][M]\underline{d}^n \quad (11.56)$$

where $[Hcos]$ is a matrix with all off-diagonal entries zero and the i^{th} diagonal entry given by

$$h_i \cos(\phi_i) \quad i=0,1,2,3,\dots \quad (11.57)$$

We now approximate $\langle f(d^n(M(x))*h(t)) \rangle$ by

$$f(\langle d^n(M(x))*h(t) \rangle) \quad (11.58)$$

But $\langle d^n(M(x))*h(t) \rangle$ in matrix form is

$$[Hcos][M]\underline{d}^n \quad (11.59)$$

Thus writing

$$\underline{d}^n = \underline{a} + \underline{e}^n \quad (11.60)$$

(as before), and Taylor expanding for small error in equation 11.28, in matrix form we obtain

$$[M](\underline{a} + \underline{e}^{n+1}) = [F]((-Gb[Hcos][M]\underline{e}^n) + [M](\underline{a} + \underline{e}^n)) \quad (11.61)$$

$$\text{i.e. } [M]\underline{e}^{n+1} = ([F] - [I])[M]\underline{a} + [F]([I] - Gb[Hcos])[M]\underline{e}^n$$

$$= \underline{C} + [P][M]\underline{e}^n \quad (11.62)$$

$$\text{where } \underline{C} = ([F] - [I])[M]\underline{a} \quad (11.63)$$

$$\text{and } [P] = [F]([I] - Gb[Hcos]) \quad (11.64)$$

Proceeding as before we can trivially derive [see ref 65, p.200-201]

$$[M]\underline{e}^n = [P]^n[M]\underline{e}^0 + ([I] - [P])^{-1}([I] - [P]^n)([F] - [I])[M]\underline{a} \quad (11.65)$$

Provided $[P]^n$ approaches zero as n approaches infinity we have a stable system for which

$$[M]\underline{e}^f = ([I] - [P])^{-1}([F] - [I])[M]\underline{a} \quad (11.66)$$

Appendix K shows that a necessary and sufficient condition for a real, symmetric, matrix $[P]^n$ to vanish is that the eigenvalues of $[P]$ all have modulus less than one. When $[P]$ is diagonal, this corresponds to saying that all diagonal elements have modulus less than one.

Comments

1. For $[M]=[I]$ equation 11.66 can be written in the scalar form given in equation 11.35.
2. If non-Chebyshev filtering of the RAM table is employed (e.g. the more common Fourier, or least squares schemes), then we can still define an equivalent Chebyshev filter matrix $[F]$, but it will no longer be diagonal. The use of a non-diagonal matrix will plainly increase the error in equation 11.66, since in general zero error is only obtained when $[F]$ is approximately equal to $[I]$.
3. For diagonal $[P]$ and $[F]$ (as is generally the case), the remarks on choice of $[F]$ for stability and minimum error transfer across from the scalar case (i.e. we should choose $F_{ii}=1$ up to $\cos(\phi_i)$ approximately equal to 0).
4. It appears, initially at least, that any periodic, even, waveform is described by equation 11.66 when all the eigenvalues of $[P]$ have modulus less than 1.
5. The assumption that (df/dx) is approximately constant is an unwelcome one in that it assumes the DUT is linear. The matrix approach is far more amenable to analysis for (df/dx) being approximately equal to a Chebyshev

polynomial. The computer program of Appendix J uses this approach. The analytic extension of the matrix approach to (df/dx) being approximately equal to a polynomial is also given in Appendix J.

In deriving equation 11.66 we have made a major assumption on the existence of the inverse of the matrix $[M]$. This is best seen by writing equation 11.66 directly in terms of the RAM error, i.e.

$$\underline{e}^f = [M]^{-1}([I] - [P])^{-1}([F] - [I])[M]\underline{a} \quad (11.67)$$

Unless $[M]^{-1}$ exists it is not possible to define the Chebyshev coefficients of the RAM table. Since $[M]^{-1}$ relates to the function $M(x)$ the non-existence of the inverse of the cascade matrix relates to the non-existence of the inverse function to $M(x)$, i.e. to $(dM/dx)=0$ at some point (see Figure 11.3.4).

Thus, although we can prove the convergence of the algorithm for cosinusoidal inputs, we are restricted in extending this proof to even periodic functions which have the same number of maxima and minima in one period as does a cosine wave.

11.4 DISCUSSION OF ANALYSIS

The convergence and stability conditions for a type 3 control algorithm have been discussed. A particular algorithm was chosen as being near-optimal, whilst at the same time practical from an implementation viewpoint.

Stability and convergence can only be proved for even periodic waveforms containing the same number of maxima and minima in one period as a cosine wave. This is disappointing, in that a more general theorem would prove useful. However, it is clear from the cosine input analysis that stability conditions will

often be met in practice with little or no effort, provided G is small. This inference is obtained as follows. For a well designed anti-alias filter the value of i for which $\cos(\phi_i)$ is approximately equal to zero will in general be large. But (in general) this implies high harmonic frequencies which are naturally smoothed by the rise and fall times of devices.

Practical experimentation indicates that the cosine stability criterion, if met, ensures stability for other input signals, although we have been unable to derive this result analytically.

The matrix representation of the theory is far more easily extended to take into account noise and quantization effects than is the scalar, and is the natural vehicle for the analysis of practical type 3 structures.

Before going on to examine practical structures and their analysis it is useful to check the foregoing theory by computer simulation.

11.5 COMPUTER SIMULATION

The program listed in Appendix J was used to simulate the theory outlined. The program also enables the results of a wider range of filters to be investigated than could be done in practice, as well as enabling us to avoid the simplifying assumptions of the first order theory.

It is important to grasp the strengths and weaknesses of the program in Appendix J, since it is based on the matrix theory touched on here. The program performs a series of matrix manipulations of Chebyshev vectors, to deduce the Chebyshev

series which approximates the series combination of NLFG, anti-alias filter imperfections and DUT. To see the advantage of this approach it is useful to consider the time domain approach to computer simulation of the type 3 control algorithm. This is outlined in Figure 11.5.1.

In Figure 11.5.1 the computer takes the input waveform, passes it through the NLFG, convolves the output with the filter impulse response, passes this through the DUT, and uses the output of the DUT to help deduce how the NLFG should be adjusted. As will be seen in section 11.6.2 of this chapter, approximately 65,000 such measurements must be made before attempting any updating of the NLFG. Approximately 200 to 300 such updates are needed before the NLFG comes to its final equilibrium state. Thus we are forced to conclude that each computer simulation will require approximately 18 million convolutions to complete. (Note that fast fourier transform techniques are of very limited use here because of the nonlinear nature of the problem). Thus simulation in the time domain, while perfectly straightforward, is an inefficient approach.

The matrix approach used in Appendix J is outlined in Figure 11.5.2. The NLFG is described by a known set of Chebyshev coefficients. When the NLFG is in series with the filter, it results in the transfer function being described by a modified set of Chebyshev coefficients as in equation 11.12. These are then cascaded (via a cascade matrix) with the DUT, resulting in the Chebyshev coefficients which describe the transfer function from input to the NLFG to output. These overall coefficients are output from the program. In computing these coefficients, the

Taylor expansion equivalent to equation 11.18 is calculated to include second order effects. By using cascade matrices there is no requirement to approximate (df/dx) by a constant, and any nonlinear DUT characteristic is easily handled, i.e. in lieu of equation 11.18 we compute

$$\langle f(D^n(x)) \rangle = f(A) + (B^2/2)(d^2f/dx^2) \quad (11.68)$$

where $D^n(x) = A + B$ (11.69)

$f(A)$ is calculated by cascade matrices, so there is no requirement for approximation of $f(x)$ as linear, i.e. (df/dx) need not be constant. The $(B^2/2)(d^2f/dx^2)$ term is calculated as a Chebyshev vector.

Using the overall Chebyshev coefficients, the Chebyshev coefficients for the updated NLFG are computed. Approximately 200 to 300 such iterations drive the NLFG to its final equilibrium value.

With this approach (as distinct from the time domain approach), approximately 5 to 6 matrix multiplications are required each update, and thus the program runs in approximately 1000 matrix multiplications. The program uses 15x15 matrices, so it is reasonable to consider each multiplication as the same order of complexity as a convolution. Thus, the matrix approach is approximately 18,000 times more efficient than the time domain approach. However, the Chebyshev approach can only model input waveforms which can be made to satisfy equation 11.67, whereas the time domain approach can model any input waveform. It is this fact which is the main weakness of the program in Appendix J. The other disadvantages are that the spectral effects caused by the hysteresis are not output, nor are effects caused by aliasing

(should any occur).

When working with the output from the program in Appendix J it is convenient to regard the Chebyshev coefficients as the values of the harmonic levels of the demodulated envelope, given a unity magnitude cosinusoidal input envelope. Thus, if the eighth Chebyshev coefficient of the overall transfer function were 0.0001, this corresponds to saying that the output spectrum (exclusive of hysteresis effects), has an eighth order harmonic level in the envelope function -80 dB on fundamental.

Figures 11.5.3 to 11.5.5 show the predicted overall transfer function for the series combination of NLFG, perfect filter, and DUT. The DUT values are obtained from experimental results. In Figure 11.5.3 we show the main program, in Figure 11.5.4 we show the overall Chebyshev coefficients, up to tenth order, for different values of G , for 40 iterations. The system appeared stable in the BIBO sense only up to G approximately equal to 1.6. Figure 11.5.5 shows the same experiment for 300 iterations, illustrating that with a large number of iterations, there is convergence to spurious levels < -160 dB, for values of G which are stable.

Figure 11.5.6 places an imperfect filter between the NLFG and DUT, using the same DUT as Figures 11.5.3 to 11.5.5. The imperfection is represented as a linear phase delay with increasing frequency. (Such an imperfection would be associated with poor timing in determining cause and effect through the anti-alias filter). Figure 11.5.6 demonstrates that the system is unstable for $G=0.2$ in the BIBO sense. This was not the case before the addition of the filter imperfection. The system

appears unstable for any $G \neq 0$. For Figure 11.5.6 the phase delay is such that $\cos(\phi_i) < 0$ for the eighth harmonic and above.

Figure 11.5.7 and Figure 11.5.8 demonstrate that stability is retained if the RAM table is Chebyshev filtered, with severe attenuation for coefficients of seventh order and above.

Stability is only retained up to G approximately equal to 1.6.

Interestingly, optimal results were not obtained by complete removal of Chebyshev coefficients above seventh, although this did result in near-optimal results. The filtering illustrated in Figure 11.5.7 gives near-optimal results.

Figure 11.5.9 illustrates the effect of reducing the RAM table filter cutoff to third order Chebyshev coefficients, for $G=0.2$. For harmonics up to third the suppression of spurious is excellent, above third it is very poor.

Figure 11.5.10 and Figure 11.5.11 illustrate the use of the program in the analysis of non-cosinusoidal inputs. The input is a sawtooth waveform. The main program is set up as in Figure 11.5.10, and the resultant output and the cascade matrix for a sawtooth are shown in Figure 11.5.11.

It is important to note that in these simulations scaling is particularly difficult to apply. This is because of the nonlinear nature of the problem. Before applying computer simulation to a specific system, A/D, D/A gains etc, must be known for setting the scale factors used in the simulation. Thus the value of G used in the program must be carefully related to the practical situation.

11.6 PRACTICAL STRUCTURE

11.6.1 Description

The structure designed to test the type 3 control algorithm is shown in Figure 11.1.1. The baseband input is quantized by the A/D, and the resulting output used to act as the address for the RAM which controls the fine adjust D/A, and acts directly as an input for the coarse adjust D/A. The two outputs from the D/A's are summed, and pass through the anti-alias filter, to the envelope modulator (in fact two cascaded pin diode attenuators). The output envelope of the DUT is detected by the diode detector. The output of the diode detector acts as one input to a fast comparator, the other input to the comparator comes from the A/D RAM2 combination, which acts to correct for the nonlinearities of the diode detector. [It is important to note that the procedure that set up RAM2 also removed the effects of a nonlinear quantizing law from the A/D, i.e. the series combination of A/D and RAM2 has the same transfer function as the diode detector. The setting up of RAM2 by quasi-linear techniques has been discussed in chapter 7]. The comparator acts as a 1 bit quantizer (A/D), and its output is latched and passed direct to the CPU, as is the A/D output. From the information derived the CPU adjusts the RAM table. Phase locked loop techniques are used to remove AM/PM sidebands from the output spectrum. The circuitry to do this is not indicated in Figure 11.1.1 since it is not strictly relevant to consideration of type 3 algorithms.

The 1 bit A/D compares the detector output with a time-delayed version of the A/D output (to compensate for the finite time delay through the RAM, D/A, filter, envelope modulator

and DUT). Although this delay can be adjusted to great accuracy, non-constant group delay in the filter and thermal drift mean that there is a small but finite error.

The detector output is summed with a small signal noise source before going to the 1 bit quantizer. It will be shown that the combination of the small signal noise source and the comparator effectively produce an infinite resolution differential A/D. This comparator/noise source structure is much the preferred arrangement for type 3 processing, since (as may be seen from the analysis of section 11.6.2):

1. The effective resolution is unlimited.
2. The speeds at which the comparator can work are much larger than those which any multi-level A/D can handle.
3. The signal processing is much simplified.
4. Comparator hysteresis is unimportant.

Referring to Figure 11.1.1, if the baseband input to the A/D is

$V_{in}(t)$ the output of the comparator is given by $p(t)$ where

$$p = \text{sign}(mV_{in}(t - t') - y[f(d^n(V_i)*r(t)*h(t))] + n(t)) \quad (11.70)$$

In equation 11.70 we have assumed a linear detector, where $h(t)$ represents the anti-alias filter impulse response, $r(t)$ and $V_i(t)$ represent the same parameters as in chapter 9, $n(t)$ represents the noise source (which has the probability density function as shown in Figure 11.6.1), m and y are scaling parameters determined by A/D, D/A and modulator gains, t' is a time delay set by the CPU, and $\text{sign}(x)$ is the sign function ($\text{sign}(x)=+1$ if $x \geq 0$, $\text{sign}(x)=-1$ if $x < 0$).

Our first concern is to relate the discrete NLFG $d(V_i)$ to a continuous description of the form $d(V_{in})$. We do this with the

aid of Figure 11.6.2. Referring to this figure we see that the output of the discrete NLFG consists of two parts - a set of harmonically related signals (harmonically related to $V_{in}(t)$), and aliased terms (which are not harmonically related to $V_{in}(t)$). To avoid aliasing the terms should not overlap. In practice there may be some small amount of overlap, and we proceed with this assumption. Specifically it is clear that we can write

$$\begin{aligned} d^n(V_i) &= d^n(V_{in}) + N(\sum_{n \neq 0} \cos(nw_D t + \theta_n)(d^n(V_{in}))) \\ &= d^n(V_{in}) + N(t) \end{aligned} \quad (11.71)$$

where w_D is the D/A sample rate, which is not synchronous with the input signal V_{in} , and $N(t)$ is a function of the input signal and harmonics of the D/A sample rate. Clearly $N(t)$ contains no component which is harmonically related to V_{in} . All the harmonically related components of V_{in} are described by the continuous definition of $d(x)$, as in the first term. We substitute into equation 11.70 and obtain

$$p = \text{sign}(mV_{in}(t - t') - y[f([d^n(V_{in}(t)) + N(t)]*r(t)*h(t))] + n(t)) \quad (11.72)$$

We now note that $N(t)*r(t)*h(t)$ is small as the purpose of $h(t)$ is to attenuate the high frequency components which constitute the bulk of the spectra of $N(t)$. Thus we Taylor expand

$f([d^n(V_{in}(t))+N(t)]*r(t)*h(t))$ and obtain an approximation equal

to

$$\begin{aligned} f(d^n(V_{in}(t))*r(t)*h(t)) &+ (N(t)*r(t)*h(t))(df/dx) \\ &+ (N(t)*r(t)*h(t))^2(d^2f/dx^2)/2 \end{aligned} \quad (11.73)$$

We note that only the first term has a harmonic relationship with $V_{in}(t)$ (i.e. is synchronous). We now define

$$\begin{aligned} n'(t) &= n(t) - y[(N(t)*r(t)*h(t))(df/dx) + \\ &\quad (N(t)*r(t)*h(t))^2(d^2f/dx^2)/2] \end{aligned} \quad (11.74)$$

and thus in equation 11.72 we obtain without approximation

$$p = \text{sign}(mV_{in}(t - t') - y[f(d^n(V_{in}(t))*r(t)*h(t))] + n'(t)) \quad (11.75)$$

$$\text{But } d^n(V_{in}(t))*r(t)*h(t) = d^n(V_{in}(t - t'))*h_e(t) \quad (11.76)$$

using the definitions of t' and $h_e(t)$ outlined in Figure 11.6.3.

Thus we obtain

$$p = \text{sign}(mV_{in}(t - t') - y[f(d^n(V_{in}(t - t'))*h_e(t))] + n'(t)) \quad (11.77)$$

Defining

$$x = V_{in}(t - t') \quad (11.78)$$

we obtain

$$p = \text{sign}(mx - y[f(d^n(x)*h_e(t))] + n'(t)) \quad (11.79)$$

Thus the following parameters are passed to the CPU for processing in Figure 11.1.1:

1. The value of x .
2. The value of p defined by equation 11.79. Note that p can only take the values $+1$ or -1 .
3. The value of $d^n(x)$. This last parameter is known since the current state of the RAM table is always known by the CPU.

11.6.2 Signal Processing

We concern ourselves first with the average value of p , observed over a large number of measurements at constant x , i.e.

$$A(x) = \langle \text{sign}(mx - yf[d^n(x)*h_e(t)] + n'(t)) \rangle \quad (11.80)$$

Plainly $A(x)$ need not be an integer. The parameter x is kept to remind us that A is only defined at a given value of x .

We note first that at constant x the term mx is always a constant, but that due to hysteresis the term $yf[d^n(x)*h_e(t)]$ may take a number of discrete values at constant x , each with differing probability. The $n'(t)$ term is, of course, random from

one measurement to the next, since it consists of a pure noise term and terms asynchronous to constant x . The situation is set out in Figure 11.6.4. It is now a comparatively simple problem to calculate the probability of recording $p=+1$ and $p=-1$, and thus of calculating $A(x)$. This is done in Figure 11.6.4 for a three level hysteresis case. The extension to more hysteresis levels is straightforward.

The result found (provided the maximum noise voltage in Figure 11.6.1 is large enough to overcome the hysteresis and aliasing effects), is that

$$A(x) = 2k(mx - y\langle f[d^n(x)*h_e(t)] \rangle) \quad (11.81)$$

where k is a constant of proportionality. It is important to note that in equation 11.81 there is no quantization of the value of $A(x)$, the resolution of the value of A is set purely by the internal bit accuracy of the CPU, and the number of measurements used in the averaging procedure (note that the variance in A would typically decrease inversely with n , where n is the number of trials over which we average). Given this result our procedure is as follows:

1. The total number of recognisable A/D levels (n') is divided up into N bins each of size (n'/N) . Typically $n'=4096$ and $N=256$ giving 16 levels per bin. Initially each bin is empty.
2. A total of gN measurements are made at random in x space. On average each bin contains the results of g measurements. Typically g ranges from 64 to 256. As a minimum, the reasoning of chapter 10 forces us to insist that

$$gN > N \log_e(N)$$

$$\text{i.e. } g > \log_e(N) \quad (11.82)$$

3. Given that each measurement results in a value of p of $+1$ or -1 , the total value of p for each bin is calculated. The result of these additions amount to calculating $A(x)$ where x is the bin centre value. Thus, as each bin now contains an estimate in proportion to $A(x)$, we claim to have calculated

$$G(mx - \langle f[d^n(x)*h_e(t)] \rangle) \quad (11.83)$$

as $A(x)$ is proportional to $(mx - \langle f([d^n(x)*h_e(t)] \rangle)$. G is determined by exact calculation of A/D , D/A , modulator, demodulator gains, and noise considerations.

Notice that to obtain the value of equation 11.83 at fixed x no multiplication or division has been required, due to the use of the comparator/noise source as an A/D .

4. The CPU adds the value determined for equation 11.83 to the current value of the RAM table equivalent to the centre of the bin. This addition is carried out in the CPU's copy of the RAM table, i.e. as yet the RAM table itself has not been modified.

5. The CPU has now computed

$$G(mx - \langle f[d^n(x)*h_e(t)] \rangle) + d^n(x) \quad (11.84)$$

6. The CPU must now filter the copy of the RAM table. Two different algorithms have been used experimentally, a moving average filter (which requires only additions and divide-by-2 operations), and a Chebyshev filter. The former filter is machine coded and the latter runs in high level Basic. Both filters give good results. The Chebyshev filter gives surety of bandlimiting the D/A

output, as well as providing interpolation of the RAM table outside of the immediate range over which data is collected. [Note that outside of this range the likely error caused by interpolation increases with the order of the interpolation polynomial [ref 116], see Figure 11.6.5]. The moving average filter provides no such surety, and can only be used when the D/A sampling rate is greatly in excess of the minimum required, so the signal sees a "near-perfect" anti-alias filter.

Thus we perform

$$F_{\underline{\underline{*}}}(G(mx - \langle f[d^n(x)*h_e(t)] \rangle) + d^n(x)) \quad (11.85)$$

7. The filtered copy of the RAM table is transferred to the working RAM table, performing

$$d^{n+1}(x) = F_{\underline{\underline{*}}}(G(mx - \langle f[d^n(x)*h_e(t)] \rangle) + d^n(x)) \quad (11.86)$$

8. The procedure repeats, with the bins being cleared, and gN measurements being made. (Note that typically $g=256$ and $N=256$ which implies 65,000 measurements before updating the RAM).

11.6.3 Noise and Quantization Effects

As outlined above, it is clear that our first order analysis of the type 3 control structure applies. However, the practical implementation has introduced two further effects, noise (primarily due to the added noise source at the comparator), and quantization (primarily due to the use of a finite number of bins).

We discuss these effects here in order to obtain engineering estimates of their importance. The effects of a

nonlinear detector are as discussed in chapter 9.

Noise effects

The noise added at the comparator needs to be of sufficient magnitude to overcome the hysteresis effects (see Figure 11.6.4). If the detector output is of magnitude X volts, and the noise is of magnitude $(1/2k)$ volts, then the ratio of signal to added noise is

$$20\log(1/(2kX)) \text{ dB} \quad (11.87)$$

The process of averaging, which effectively takes place in the bins, then further reduces the noise variance by a factor of g . For g finite some residual noise will remain, and it will not all be removed by filtering (whether Chebyshev, Fourier, least squares, etc), although it will plainly be reduced by any filtering that occurs. Ultimately some residual noise in the x domain will be transferred over to the RAM table, in the form of increased error in its Chebyshev coefficients, and will result in reduced IMD suppression.

Stability will not be impaired by this process (since RAM convergence is independent of any initial errors that exist in the RAM table). Measured at the NLFG output the noise effect would not be expected to exceed

$$20\log(1/(2kXg^{0.5})) \text{ dB} \quad (11.88)$$

Provided equation 11.88 gives rise to a figure comparable with A/D resolution, little degradation is expected. In the experiments $(1/2k)=15\text{mV}$, $X=10$ volts and $g=64$ giving rise to an anticipated noise level approximately -75 dB on peak voltage.

Should the noise effect become significant, several options

are open:

1. Increase g , as far as other considerations permit.
2. Relate every new iteration of the RAM table to a weighted sum of previous RAM tables. Assuming the DUT characteristics are static this will result in all noise effects being reduced.

The computer simulations in Figure 11.6.6 illustrate the effect of adding noise at the comparator, by introducing a random error in the Chebyshev coefficients, on the final DUT output. As might be expected, the noise produces an error in the DUT output of order approximately equal to (GR) , where R is the rms magnitude of the noise in Chebyshev space.

Quantization effects

Plainly, the action of using bins of finite width introduces a quantization effect into the algorithm. We are forced into introducing finite width bins, since the measurements are made at random, and following the results of chapter 10, the number of measurements we need to make to achieve high probability that all the bins contain a result is given by

$$N \log_e(N) \quad (11.89)$$

Thus were we to choose $N=4096$ (the number of A/D levels), then approximately 34000 measurements would need to be made. If g is kept at 64 then there will be 2.2 million measurements before the RAM table can be modified. Using bins allows us to put $N=256$ whilst keeping the A/D size at 12 bit and $g=64$. This gives us a 24 times decrease in the number of measurements to be made, with a corresponding 24 times increase in speed, and hence in ability to

track thermal changes.

Experimentation and computer simulation both show that there is little practical degradation resulting from using bins, provided $N \gg i_{\max}$ where $i_{\max}$ is the highest order harmonic that is to be generated. Typically then there is no degradation in choosing $N=256$.

There is one other quantization effect that needs to be considered and this is due to the comparator/noise source. Given that the noise has a probability distribution as in Figure 11.6.1, the A/D only has infinite resolution when comparing voltages which are within $(1/k)$ volts of each other. For our experiments we used $(1/k)$ approximately equal to 30mV. Outside of this range, the A/D acts simply as a 1 bit device. Thus the approach to convergence is not strictly as described by equation 11.86. In practice the effects of nonlinear initial approach to convergence are relatively unimportant, provided convergence takes place within a reasonably rapid period. This in turn depends on the values of G , g and N .

The computer simulation results shown in Figures 11.6.7 to 11.6.9 were used to examine the comparator/noise source approach, and to confirm the theory put forward in Figure 11.6.4, as applied in section 11.6.2 of this chapter.

11.6.4 Experimental Results

Figure 11.6.10 shows the "before and after" spectral effects of using type 3 envelope compensation (to correct for AM/AM), with phase locked loop phase correction (to correct for AM/PM). The test waveform is an approximately 100% AM waveform.

Considerable care has to be taken to ensure that the "before and after" compensation comparisons are fair. In this study the criterion has been that the fundamental AM sideband levels be the same in the before and after tests, and that the carrier level be adjusted so that as near as 100% modulation as possible is obtained, whilst ensuring that the modulation depth is never greater than 100% (since this would cause gross distortion with a unipolar envelope modulator).

Considering Figure 11.6.10 suppression of IMD is good, with asymmetric effects clearly dominating the suppression obtained. For the audio frequencies worked with (1 kHz audio modulation), a simple moving average filter was used to ensure stability. Use of Chebyshev filtering would be more applicable to high frequency work, when the A/D's, D/A's do not have such ample sampling rates. Figure 11.6.11 shows the spectral effects of using Chebyshev filtering of the RAM table on the DUT of Figure 11.6.10. The results appear to confirm the theory, in that to first order, removal of Chebyshev polynomials above n^{th} order gives good suppression of IMP up to n^{th} order, but not beyond.

Figure 11.6.12 shows the RAM tables which correspond to Figure 11.6.10 and Figure 11.6.11. The RAM tables shown are for the fine adjust D/A only (i.e. trend removal as described in chapter 4 has been applied). It is plain that the smoother tables generated by Chebyshev filtering produce a more frequency limited output from the D/A than the moving average table.

Figure 11.6.13 shows the performance of the system with non-cosinusoidal inputs. The baseband input is the result of summing the outputs of a number of audio signal generators and

waveform generators (bandlimited to 2.5 kHz). Consequently, the baseband input cannot be described by the theory discussed. Figure 11.6.13 shows "before and after" along with a photograph of the RF envelope showing the non-cosinusoidal nature of the baseband signal. A reduction in the spectral spreading of the RF is apparent in Figure 11.6.13 when compensation is applied. No stability problems were encountered (using moving average or Chebyshev filtering). Figure 11.6.14 shows the effect of Chebyshev filtering. Again, the most noticeable effect is the reduction in spectral spreading. Figure 11.6.15 shows the RAM table for the different filter types, and compares the RAM table with a cosine input. Notice that the RAM table is slightly different from that obtained using the ^{non-}cosinusoidal signal. Theory would predict that they would be different, although the degree of difference is perhaps surprising. Figure 11.6.16 shows the RF spectrum using the moving average RAM table obtained by the cosinusoidal input. Spectrally it is little different from that shown in Figure 11.6.13.

However, despite the fact that there is little spectral difference between Figure 11.6.16 and Figure 11.6.13, the difference in RAM tables looks too large to explain on the basis of the theory put forward. A possible explanation lies with the probability density function of the input signal, and its effect on the signal processing algorithm. The algorithm sums the number of up/down counts of the comparator for a particular bin. Some bins are more likely than others, resulting in variations in the effective value of G from bin to bin. It would be expected that the most likely bins would dominate the least likely bins via the

filtering/interpolation operations. Thus a radical change in the probability density function of the input signal may well have a large effect on the RAM table.

Figure 11.6.17 shows the effect of injecting noise at the comparator, prior to processing the bin data, on the values in the bins. The reduction in variance from bin to bin is readily apparent. Figure 11.6.18 shows the RAM tables which result with, and without, the noise at the comparator.

11.7 CONCLUSIONS

Potentially type 3 control algorithms offer near-optimal suppression of IMD, whilst imposing no requirements for linear detectors (although linear detectors were required for this study, as a baseband system was operated). Furthermore, such algorithms can follow thermal changes and drift in the characteristics of the DUT, whilst not interrupting normal operation.

A near-optimal type 3 algorithm has been investigated here and has proved stable and effective. A novel form of A/D has been developed which is specific to the algorithm (using a noise source and comparator).

The A/D has infinite resolution within a certain voltage range, whilst being able to operate with very high speed signals. Hysteresis and false firing effects in the comparator used are indistinguishable from hysteresis and noise effects in the input signal, and so do not affect the ability to resolve voltage levels. Using the A/D, signal processing of the measurements is much simplified.

For the type 3 algorithm investigated it must be admitted that the stability conditions for an arbitrary input signal have not been derived for the case when significant amplitude and group delay distortions are met. This is not altogether surprising, since the dependence of system stability on input is a well-known phenomena for nonlinear systems [ref 90, 190], which causes problem in proving stability. The standard nonlinear control system techniques of reference 174 do not appear applicable to the problem.

Aside from stability problems, the natural attempt of the algorithm to adapt the RAM table for the particular input signal may give rise to problems, in that "hunting" between two RAM states may be possible, with convergence never being achieved. Furthermore the problem of interpolating beyond the range of the characteristic over which information is gathered remains difficult. Much the same arguments apply as were used in chapter 10 for type 2 algorithms.

Currently type 3 control requires more investigation of the problems outlined. If necessary the technique can usefully be applied to restricted time segments of the signal, for which stability can be ensured. Thus type 3 control may be applied to problems in which the statistics of the input signal are known a priori (e.g. multi-tone FM), or in which a small portion of the DUT transmitted signal carries a known waveform (e.g. VSB-TV in the blanking period, M-QAM during periods of zero intersymbol interference or in the synchronisation data).

FIGURE 11.1.1

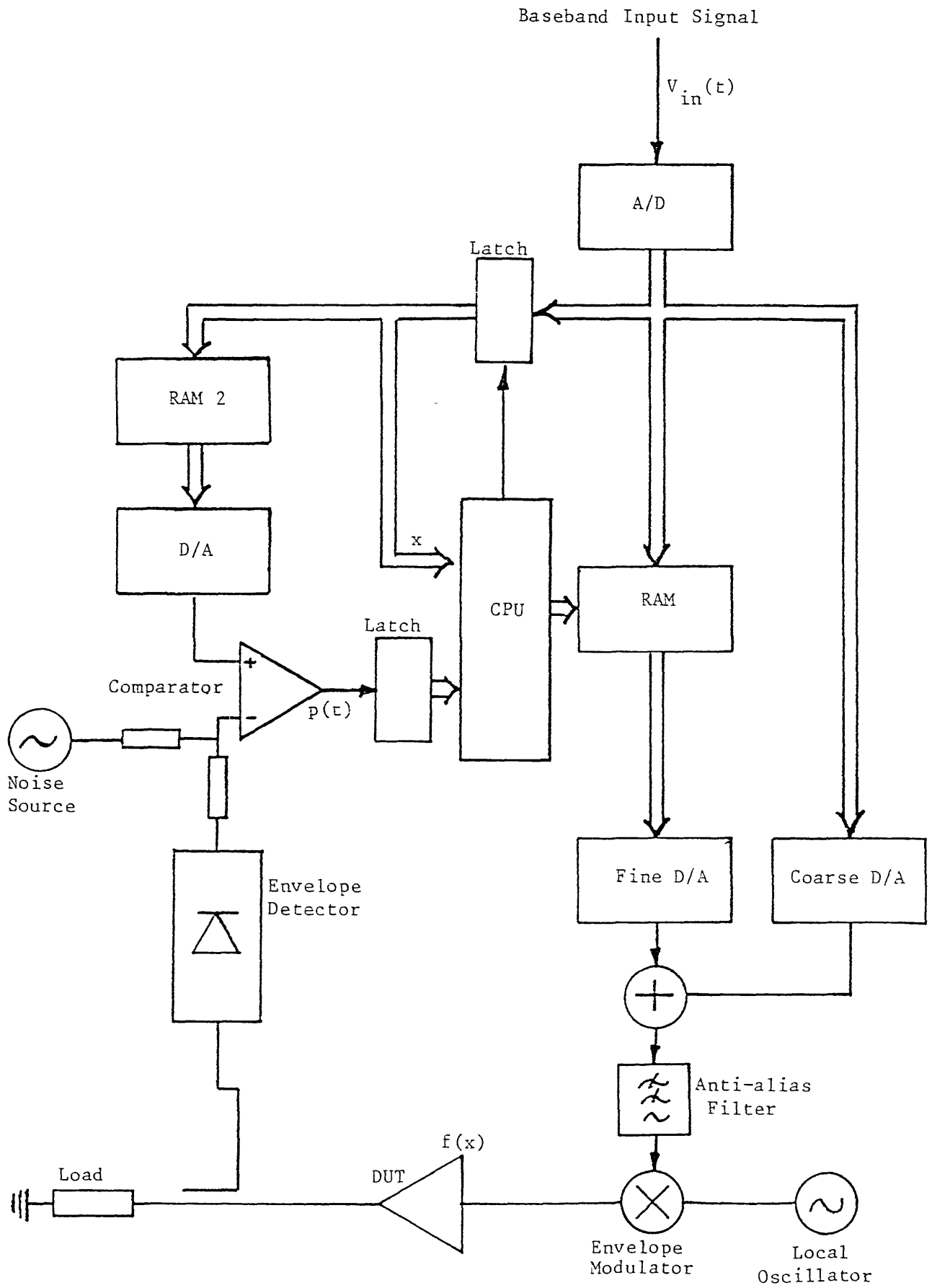


FIGURE 11.1.2

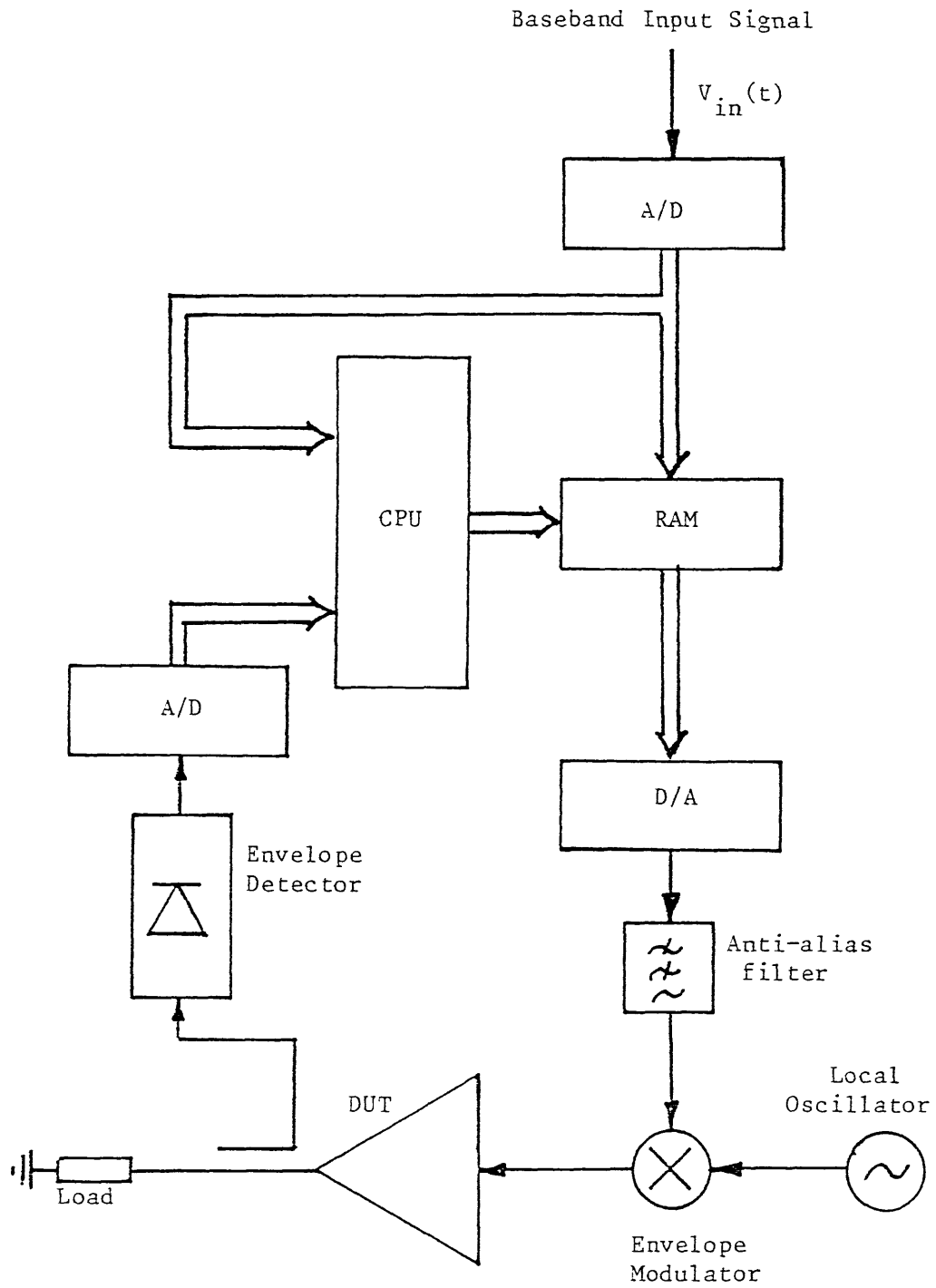


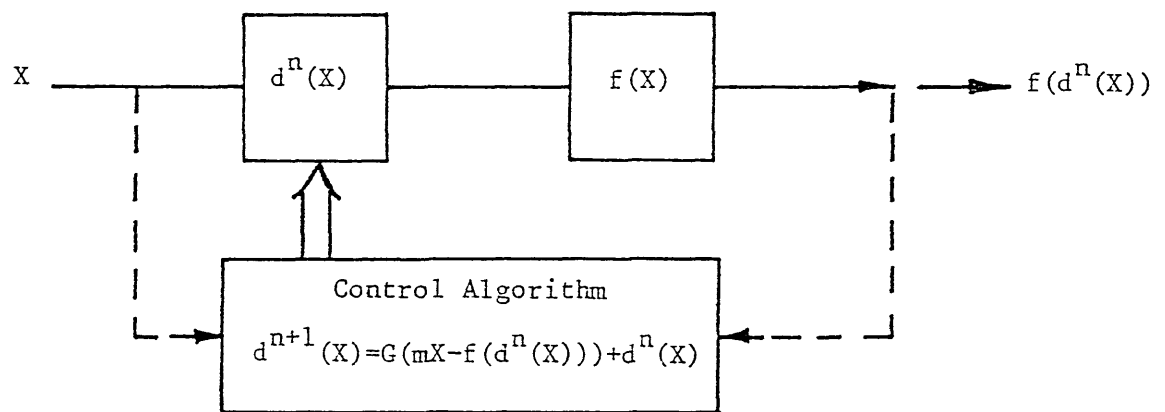
FIGURE 11.2.1

FIGURE 11.2.2

Typical class A/B nonlinearity

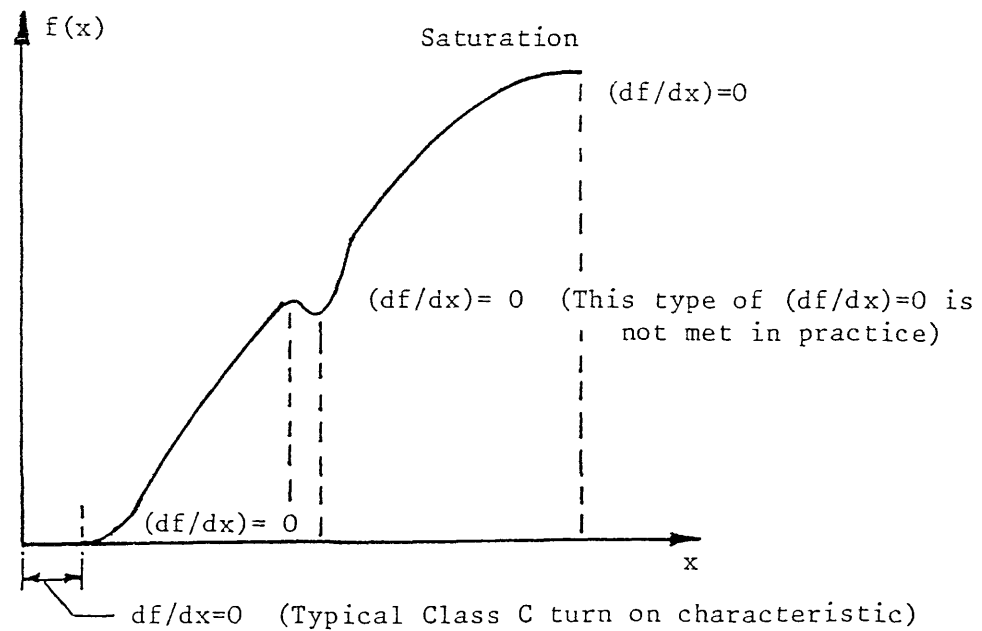
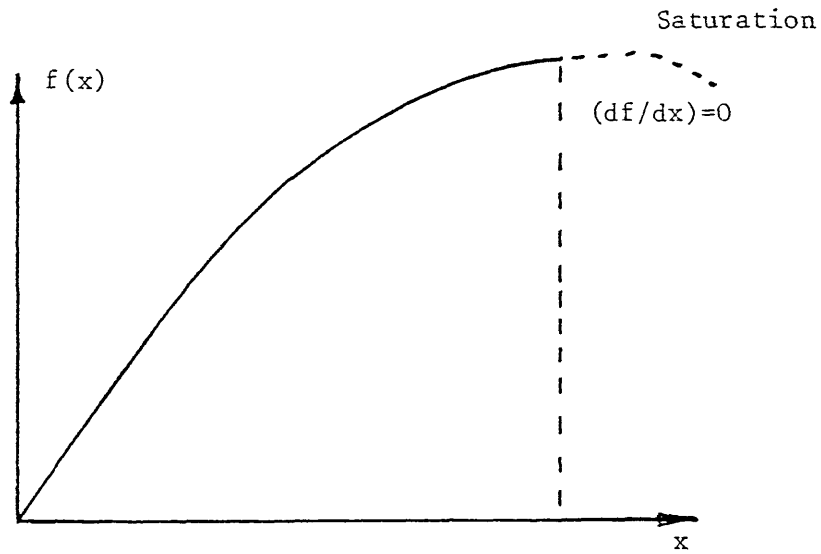


FIGURE 11.2.3

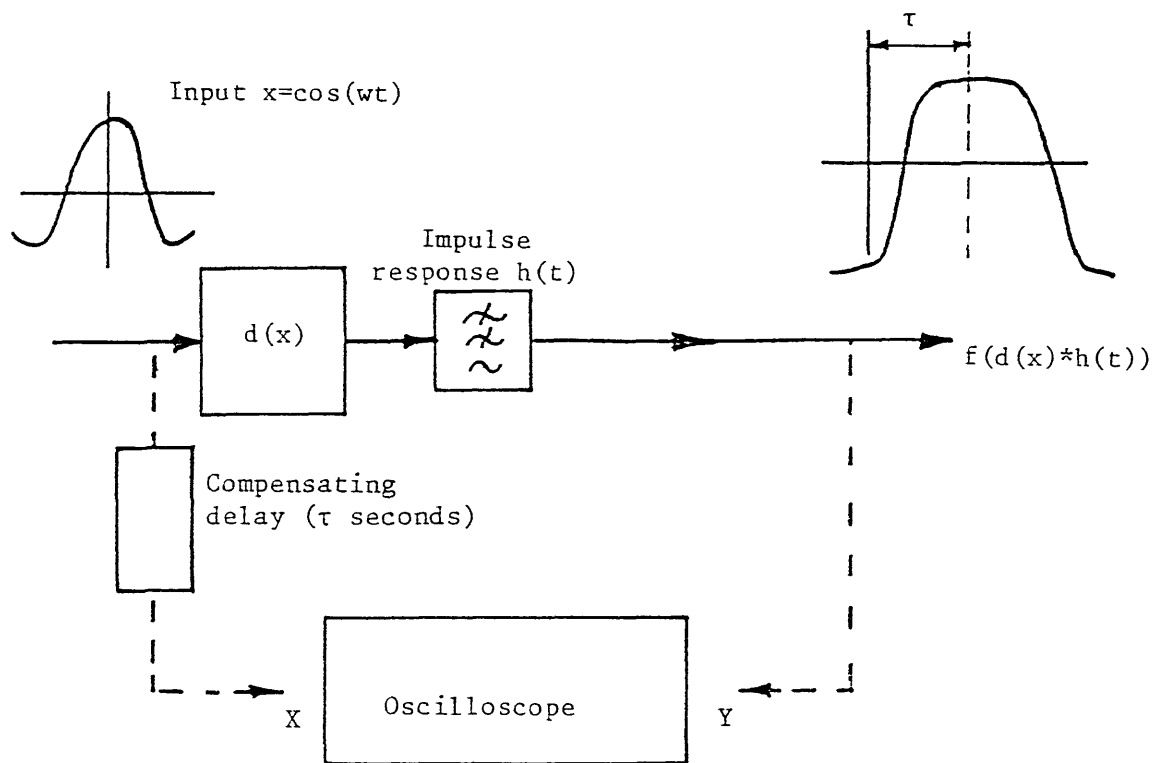
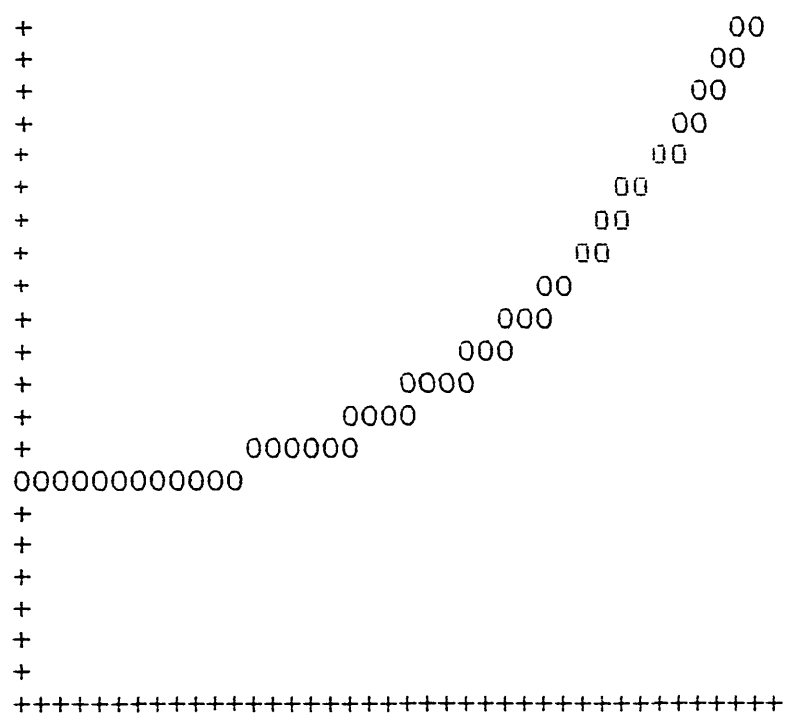


FIGURE 11.2.4

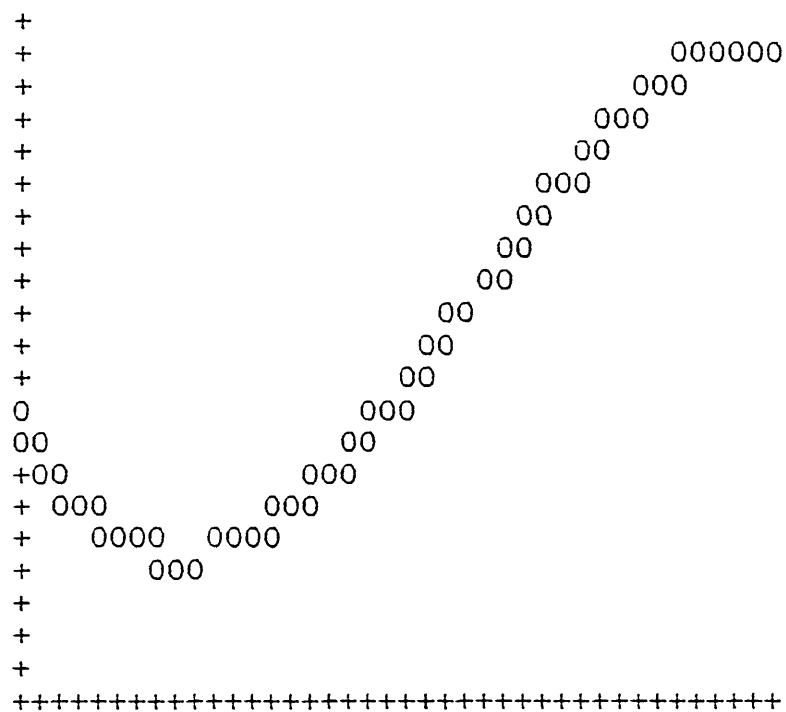


$$X=20+20\cos(T)$$

$$Y=8+0.7(23(X/40)**3)$$

Lissajous figure for $h(t) = \delta(t)$

FIGURE 11.2.5



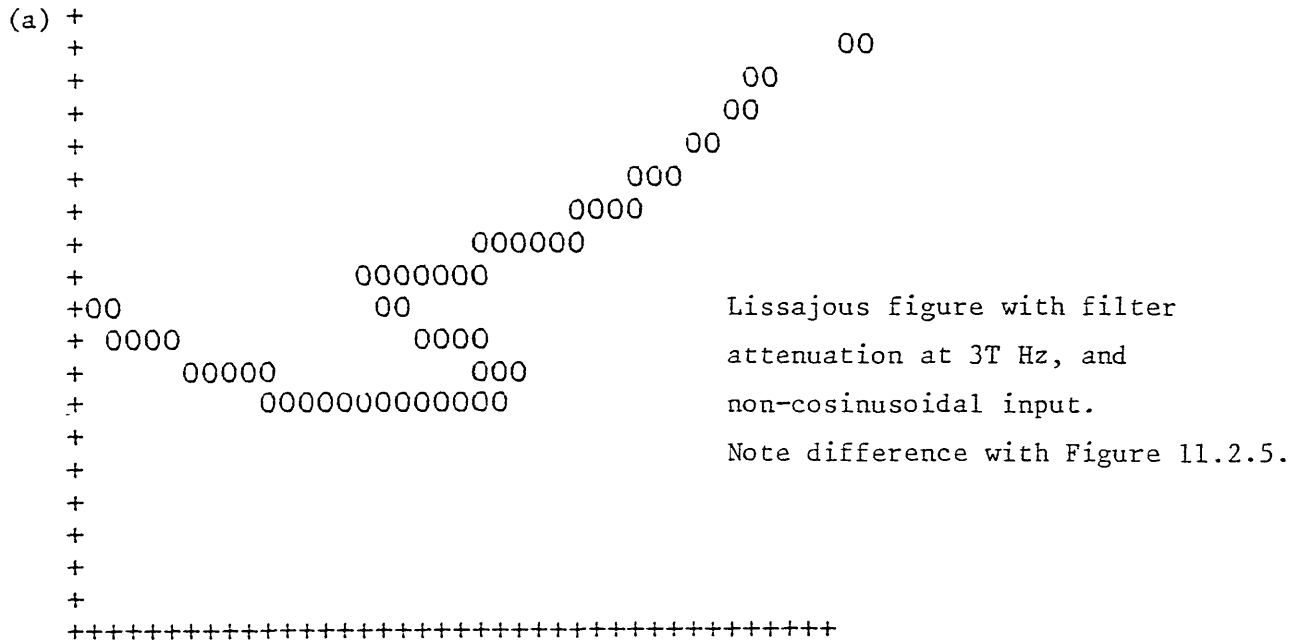
$$X=20+20\cos(T)$$

$$Y=8+0.7(23(X/40)**3-4\cos(3T))$$

Lissajous figure for filter introducing extra attenuation at a frequency of 3T Hz.

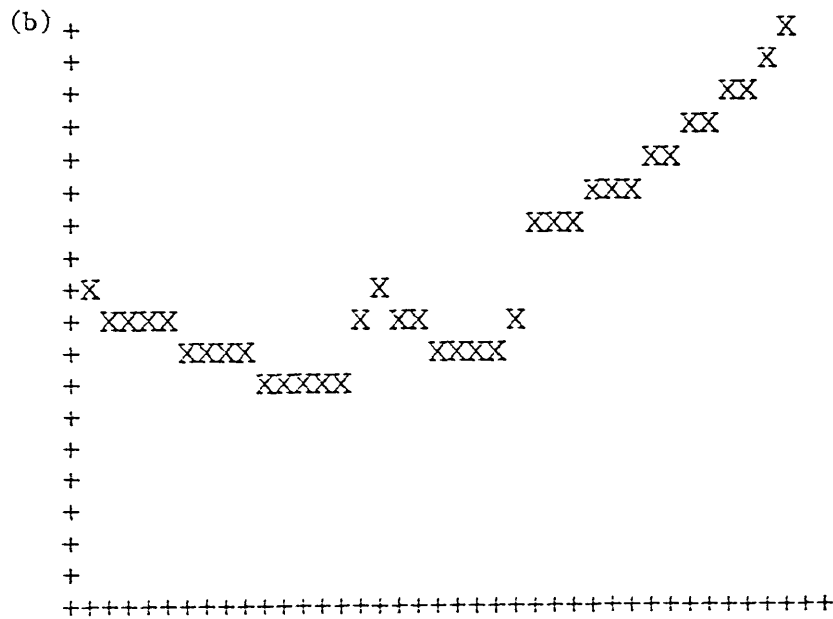
Note change from Figure 11.2.4.

FIGURE 11.2.8



$$X=20+10\cos(T)+8\cos(3T)$$

$$Y=8+0.7(23(X/40)**3-4\cos(4T))$$



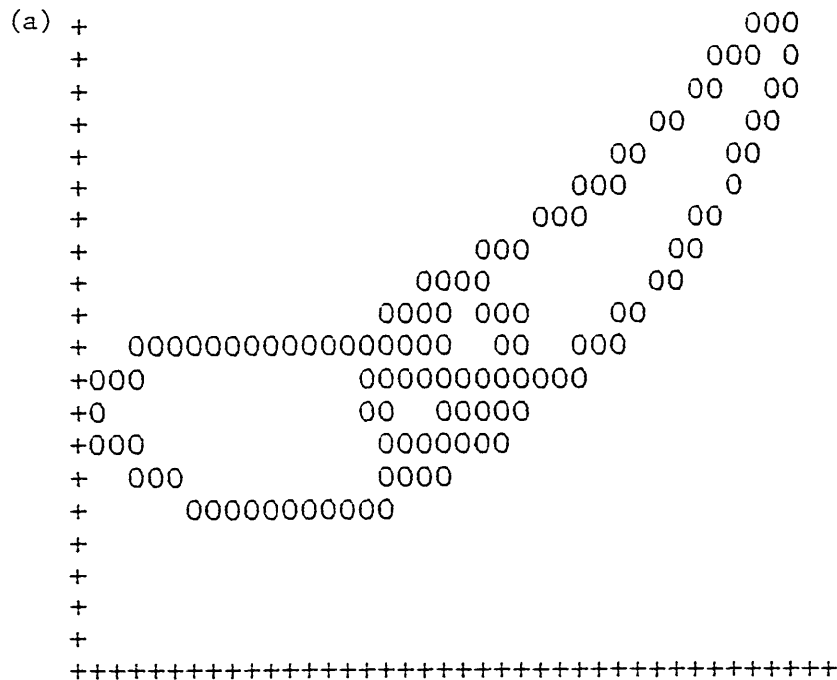
$$X=20+10\cos(T)+8\cos(3T)$$

$$Y=<8+0.7(23(X/40)**3-4\cos(3T))>$$

where < > denotes averaging Y at constant X

Lissajous figure resulting from averaging Y value of Figure 11.2.8(a)

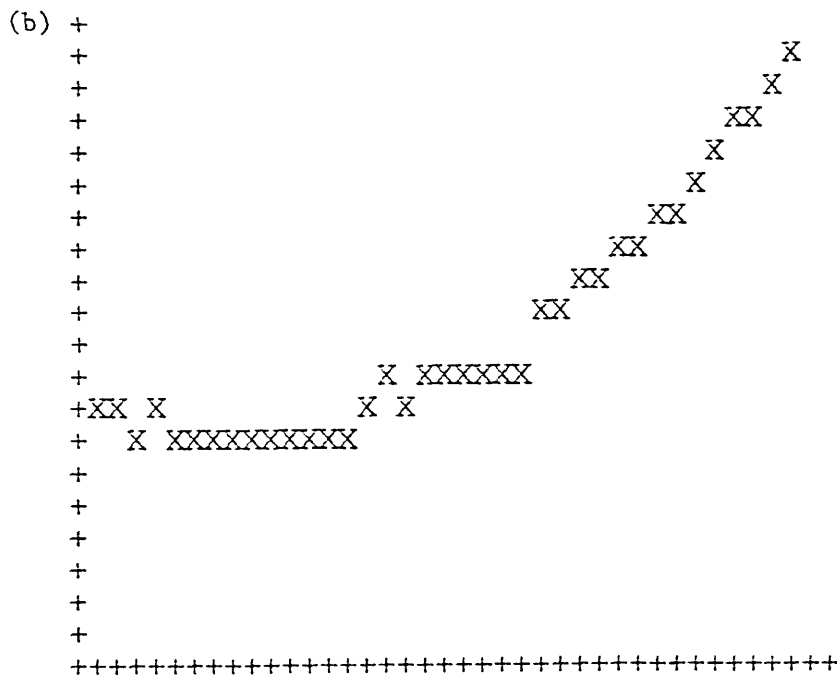
FIGURE 11.2.9



Lissajous figure with
filter attenuation at 3T Hz,
and phase shift.
Non-cosinusoidal input.
Compare with Figure 11.2.6(a)

$$X=20+10\cos(T)+8\cos(3T)$$

$$Y=8+0.7(23(X/40)**3-4\cos(3T+20))$$



$$X=20+10\cos(T)+8\cos(3T)$$

$$Y=<8.0+0.7(23(X/40)**3-4\cos(3T-20))>$$

where < > denotes averaging Y at constant X
Lissajous figure of Figure 11.2.9(a) averaged.
Compare with Figure 11.2.6(b)

FIGURE 11.2.10

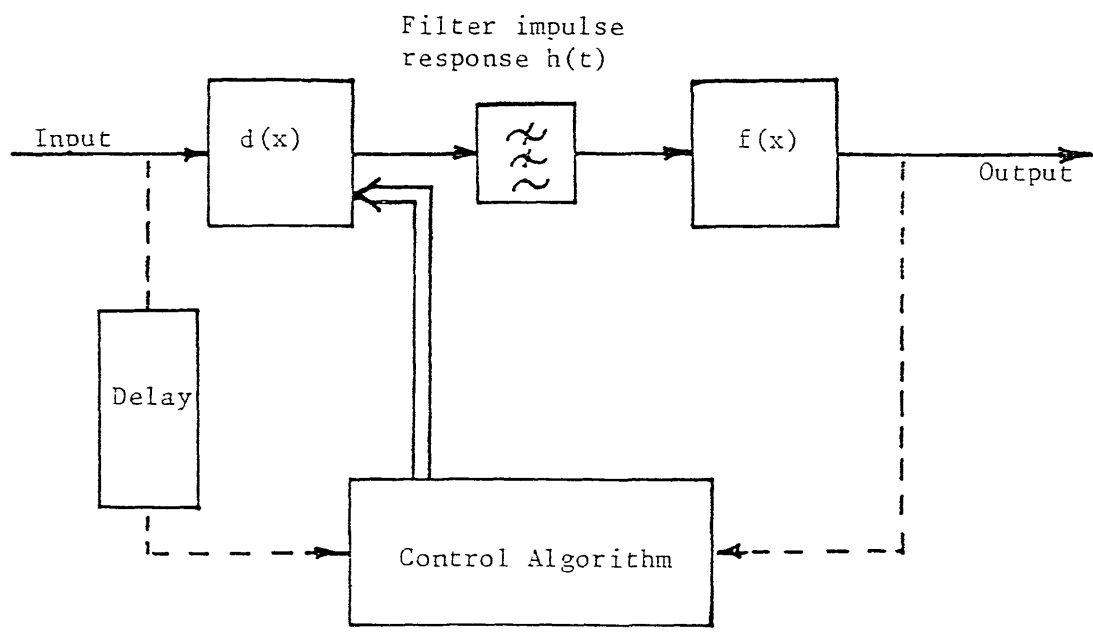


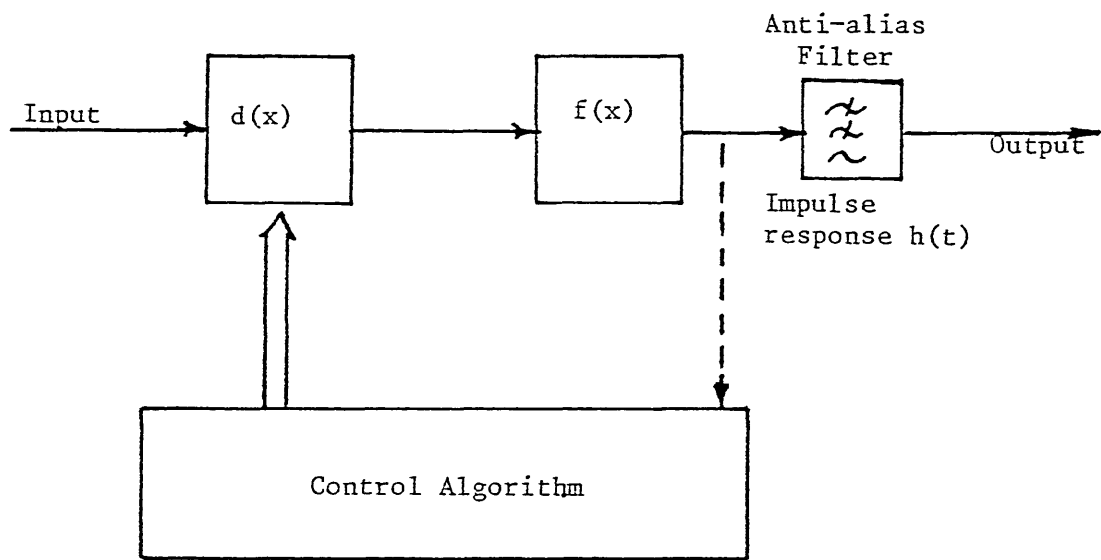
FIGURE 11.2.11

FIGURE 11.3.1

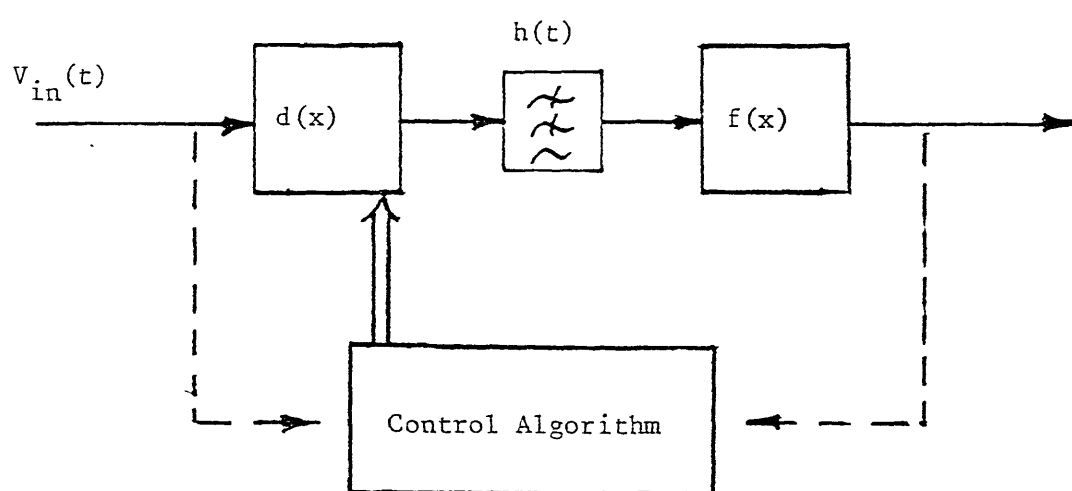


FIGURE 11.3.2

$$\text{If } f(x) = \sum_n a_n T_n(x) = x$$

$$[C] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & 0 & \dots \\ 0 & 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & 0 & 1 & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix} = [I]$$

$$\text{If } f(x) = \sum_n a_n T_n(x) = ax$$

$$[C] = \begin{bmatrix} 1 & 0 & (a-1) & 0 & \dots \\ 0 & a & 0 & 3a(a-1) & \dots \\ 0 & 0 & a & 0 & \dots \\ 0 & 0 & 0 & a & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

$$\text{If } f(x) = \sum_n a_n T_n(x) = b$$

$$[C] = \begin{bmatrix} 1 & b & (2b-1) & \dots \\ 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix}$$

$$\text{If } f(x) = \sum_n a_n T_n(x) = ax+b$$

$$[C] = \begin{bmatrix} 1 & b & (2b+a-1) & \dots \\ 0 & a & 4ab & \dots \\ 0 & 0 & a & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix}$$

$$\text{If } f(x) = \sum_n a_n T_n(x) = T_2(x)$$

$$[C] = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 1 & 0 & \dots \end{bmatrix}$$

FIGURE 11.3.3

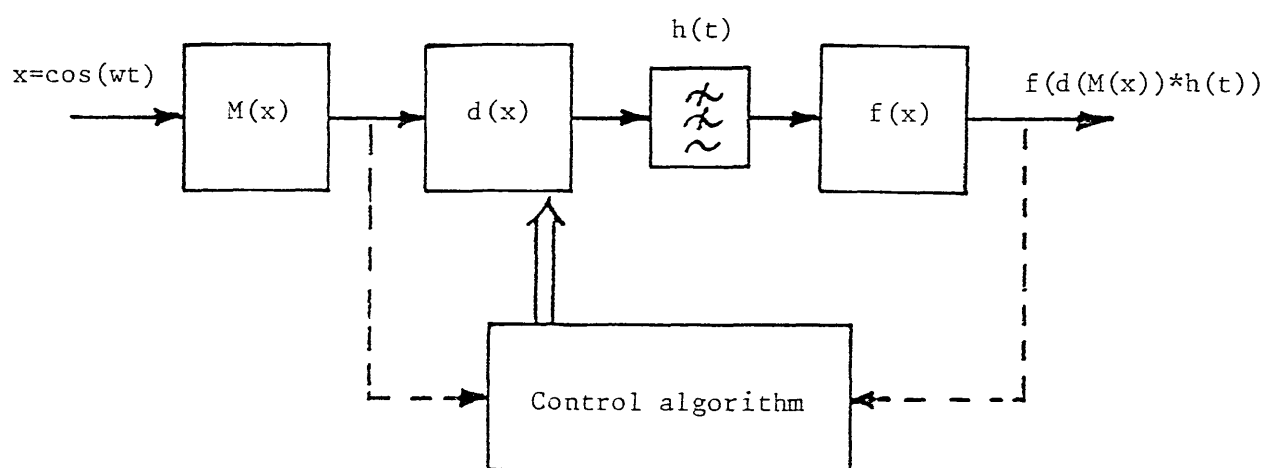


FIGURE 11.3.4

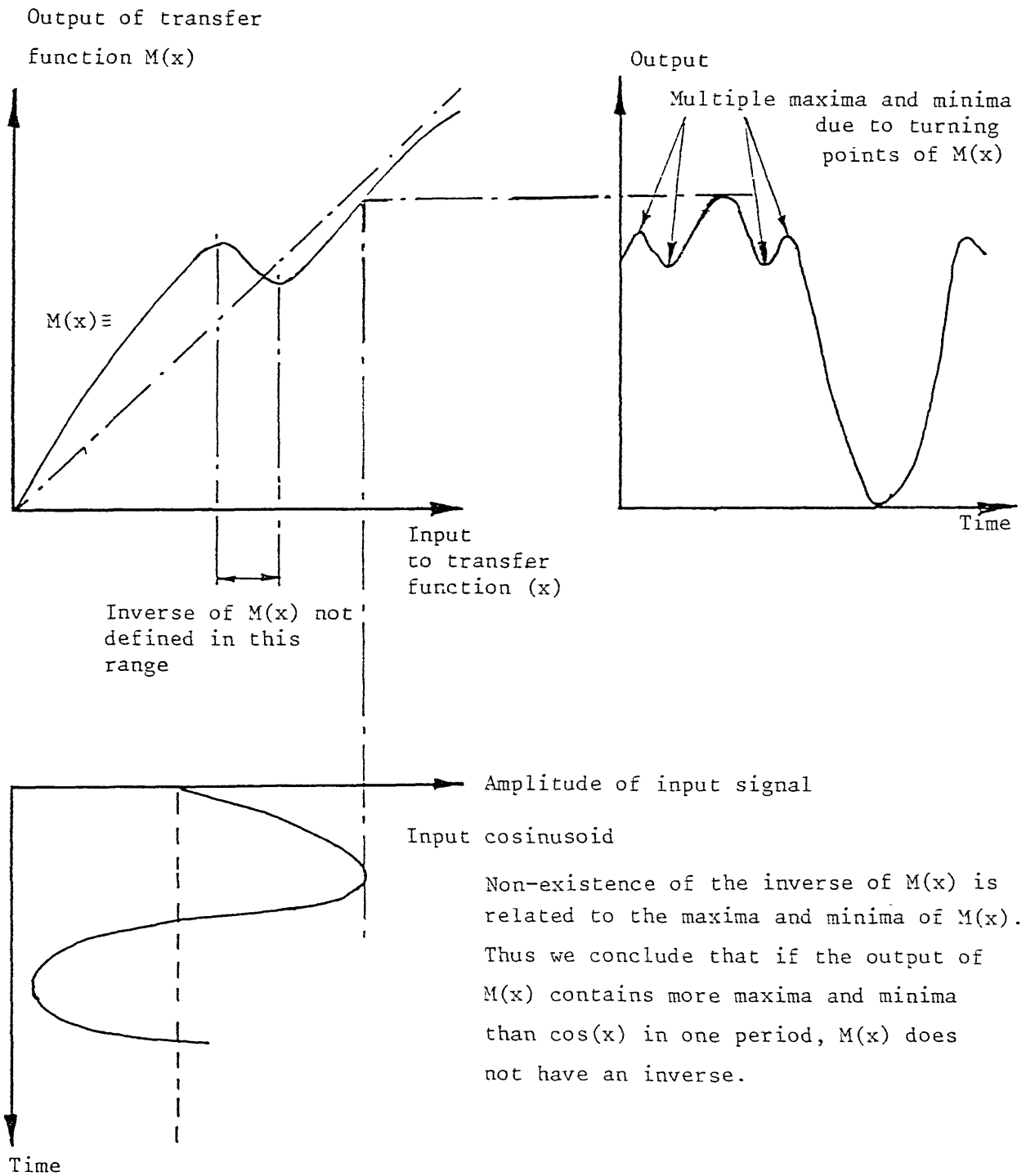


FIGURE 11.5.1

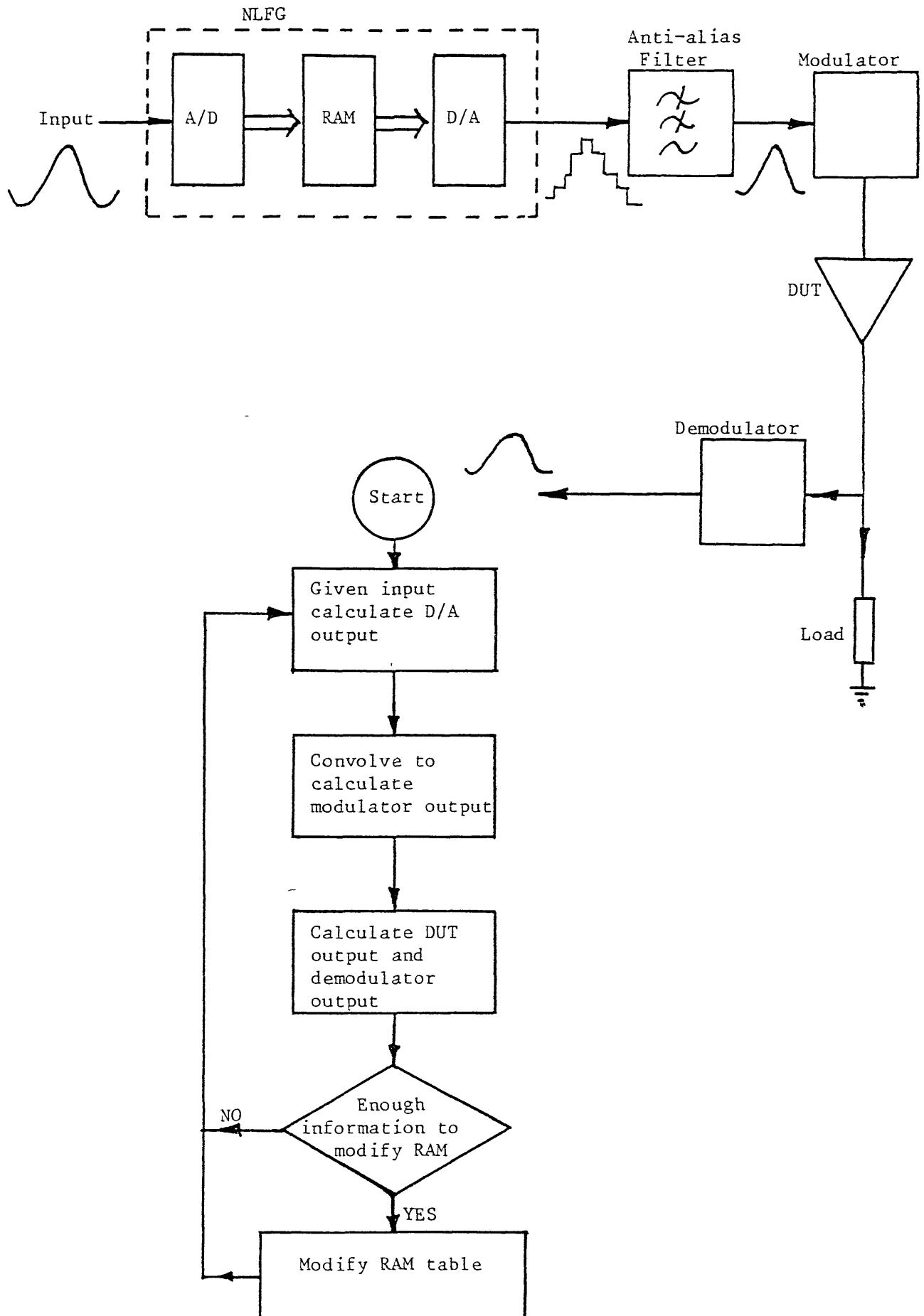


FIGURE 11.5.2

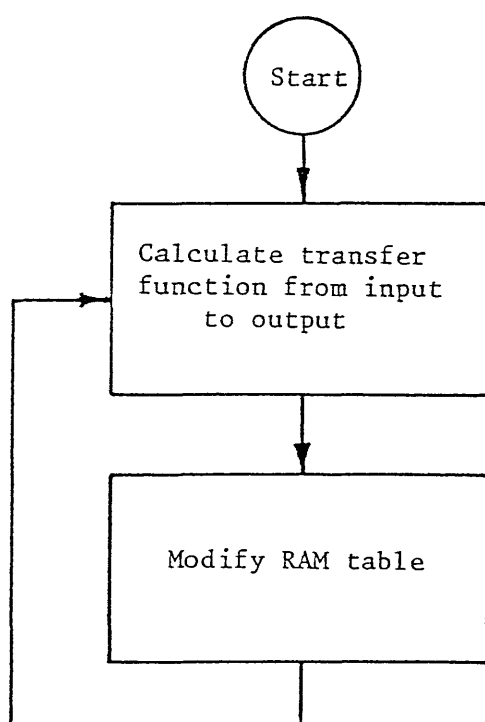
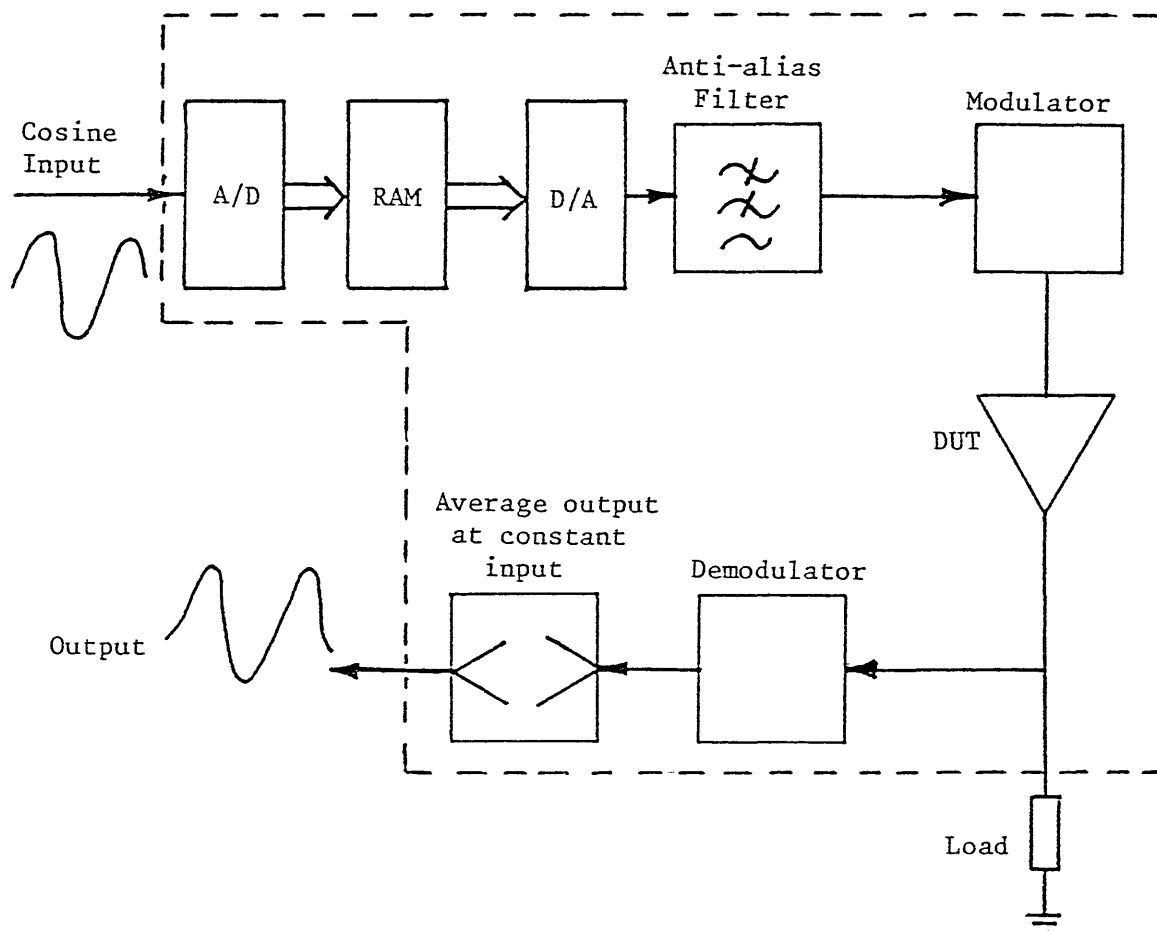


FIGURE 11.5.3

Shown below are the main program statements to run the program in Appendix J for a series NLFG, perfect filter and DUT.

DUT nonlinearity is taken from a measured amplifier.

```

CALL IT
DO B=1,20
PG=.1*B
CALL VH(DES,SIZE)
DO I=0,15
RAT(I)=0
DUT(I)=0
TOT(I)=0
END DO
RAT(1)=1
DUT(1)=.99
DUT(2)=-.0316
DUT(3)=.0177
DUT(4)=-.0056
DUT(5)=.0031
DO N=1,40
CALL CAS(DUT,RAT,TOT)
IF(N.EQ.1.OR.N.EQ.40)CALL DISV(TOT,DES,SIZE,N)
DO I=0,15
IF(I.EQ.1)RAT(I)=RAT(I)-PG*(TOT(I)-1)
IF(I.NE.1)RAT(I)=RAT(I)-PG*TOT(I)
END DO
END DO
END DO
STOP
END

```

FIGURE 11.5.4

We show the overall Chebyshev description of the NLFG, perfect filter and DUT, for 40 iterations, for $G=0.1$ to 1.5. For G greater than 1.5 instability results.

No. of iterations	Coefficient of $T_n(x)$											G
	$n=0$	1	2	3	4	5	6	7	8	9	10	
1	0.00E+00	0.99E+00	-0.32E-01	0.18E-01	-0.56E-02	0.31E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.1
40	0.53E-04	0.10E+01	-0.43E-03	0.17E-03	0.87E-04	-0.21E-04	0.35E-04	0.40E-06	-0.11E-04	0.16E-05	-0.10E-06	
1	0.00E+00	0.99E+00	-0.32E-01	0.18E-01	-0.56E-02	0.31E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.2
40	0.62E-06	0.10E+01	-0.46E-05	0.15E-05	0.21E-05	-0.16E-06	0.18E-06	0.19E-06	-0.28E-06	-0.66E-07	0.52E-07	
1	0.00E+00	0.99E+00	-0.32E-01	0.18E-01	-0.56E-02	0.31E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.3
40	0.63E-08	0.10E+01	-0.28E-07	0.40E-08	0.21E-07	0.47E-09	-0.15E-08	0.14E-08	-0.23E-08	-0.17E-08	0.72E-09	
1	0.00E+00	0.99E+00	-0.32E-01	0.18E-01	-0.56E-02	0.31E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.4
40	0.58E-10	0.10E+01	-0.16E-08	-0.36E-09	0.17E-09	0.00E+00	-0.29E-10	-0.15E-10	-0.18E-11	-0.10E-10	0.36E-11	
1	0.00E+00	0.99E+00	-0.32E-01	0.18E-01	-0.56E-02	0.31E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.5
40	0.58E-10	0.10E+01	-0.16E-08	-0.36E-09	0.17E-09	0.00E+00	0.29E-10	0.00E+00	0.91E-12	-0.18E-11	0.00E+00	
1	0.00E+00	0.99E+00	-0.32E-01	0.18E-01	-0.56E-02	0.31E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.6
40	0.00E+00	0.10E+01	0.70E-09	-0.48E-09	0.23E-09	0.00E+00	0.29E-10	-0.15E-10	0.45E-11	-0.91E-12	0.18E-11	
1	0.00E+00	0.99E+00	-0.32E-01	0.18E-01	-0.56E-02	0.31E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.7
40	0.00E+00	0.10E+01	0.64E-09	-0.42E-09	-0.29E-09	0.00E+00	-0.29E-10	0.15E-10	-0.45E-11	0.00E+00	0.00E+00	
1	0.00E+00	0.99E+00	-0.32E-01	0.18E-01	-0.56E-02	0.31E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.8
40	0.00E+00	0.10E+01	0.70E-09	-0.48E-09	0.23E-09	0.00E+00	0.29E-10	-0.29E-10	0.45E-11	-0.91E-12	0.00E+00	
1	0.00E+00	0.99E+00	-0.32E-01	0.18E-01	-0.56E-02	0.31E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.9
40	0.00E+00	0.10E+01	0.64E-09	-0.42E-09	-0.29E-09	0.00E+00	-0.29E-10	0.29E-10	-0.64E-11	0.18E-11	0.00E+00	
1	0.00E+00	0.99E+00	-0.32E-01	0.18E-01	-0.56E-02	0.31E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.0
40	0.12E-07	0.10E+01	0.15E-07	-0.13E-07	0.45E-08	-0.33E-08	0.55E-09	-0.39E-09	0.13E-09	-0.65E-10	0.22E-10	
1	0.00E+00	0.99E+00	-0.32E-01	0.18E-01	-0.56E-02	0.31E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.1
40	0.14E-07	0.10E+01	0.23E-07	-0.17E-07	0.54E-08	-0.37E-08	0.11E-08	-0.60E-09	0.24E-09	-0.12E-09	0.55E-10	
1	0.00E+00	0.99E+00	-0.32E-01	0.18E-01	-0.56E-02	0.31E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.2
40	0.18E-07	0.10E+01	0.29E-07	-0.19E-07	0.90E-08	-0.56E-08	0.24E-08	-0.14E-08	0.87E-09	-0.59E-09	0.42E-09	
1	0.00E+00	0.99E+00	-0.32E-01	0.18E-01	-0.56E-02	0.31E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.3
40	0.56E-06	0.10E+01	0.11E-05	-0.10E-05	0.95E-06	-0.89E-06	0.81E-06	-0.74E-06	0.66E-06	-0.58E-06	0.50E-06	
1	0.00E+00	0.99E+00	-0.32E-01	0.18E-01	-0.56E-02	0.31E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.4
40	0.26E-03	0.10E+01	0.51E-03	-0.49E-03	0.47E-03	-0.44E-03	0.41E-03	-0.37E-03	0.33E-03	-0.29E-03	0.25E-03	
1	0.00E+00	0.99E+00	-0.32E-01	0.18E-01	-0.56E-02	0.31E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.5
40	0.50E-01	0.90E+00	0.97E-01	-0.93E-01	0.88E-01	-0.82E-01	0.75E-01	-0.67E-01	0.59E-01	-0.51E-01	0.43E-01	

FIGURE 11.5.5

The results shown correspond to the experiment of Figure 11.5.3 and Figure 11.5.4 carried out for 300 iterations for $G=0.1$ to 0.5 . Irrespective of the value of G the coefficients converge to describe a system linear to approximately -160 dB.

No. of iterations	Coefficient of $T_n(x)$												G
	n=0	1	2	3	4	5	6	7	8	9	10		
1	0.00E+00	0.99E+00	-0.32E-01	0.18E-01	-0.56E-02	0.31E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.1
300	0.41E-09	0.10E+01	-0.86E-08	0.30E-08	0.84E-09	0.47E-09	-0.29E-09	0.73E-10	-0.16E-10	-0.82E-11	0.36E-11		
1	0.00E+00	0.99E+00	-0.32E-01	0.18E-01	-0.56E-02	0.31E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.2
300	0.58E-10	0.10E+01	-0.36E-08	0.22E-09	0.15E-09	0.23E-09	-0.15E-09	0.15E-10	-0.55E-11	-0.45E-11	0.18E-11		
1	0.00E+00	0.99E+00	-0.32E-01	0.18E-01	-0.56E-02	0.31E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.3
300	0.17E-09	0.10E+01	-0.23E-08	0.77E-09	0.17E-09	0.00E+00	-0.87E-10	-0.15E-10	-0.27E-11	-0.18E-11	0.00E+00		
1	0.00E+00	0.99E+00	-0.32E-01	0.18E-01	-0.56E-02	0.31E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.4
300	0.58E-10	0.10E+01	-0.16E-08	-0.36E-09	0.17E-09	0.00E+00	-0.29E-10	-0.15E-10	-0.18E-11	-0.91E-12	0.00E+00		
1	0.00E+00	0.99E+00	-0.32E-01	0.18E-01	-0.56E-02	0.31E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.5
300	0.58E-10	0.10E+01	-0.16E-08	-0.36E-09	0.17E-09	0.00E+00	0.29E-10	0.00E+00	0.91E-12	-0.18E-11	0.00E+00		

FIGURE 11.5.6

We show below the effect of adding an imperfect filter to the NLFG, and DUT of Figures 11.5.3-5. Taking $G=0.2$, the program shows the system is unstable, and attempts to produce infinite output beyond the 166th iteration.

No. of iterations	Coefficient of $T_n(x)$										
	$n=0$	1	2	3	4	5	6	7	8	9	10
126	-0.15E-03	0.10E+01	-0.21E-03	0.13E-03	-0.71E-04	0.30E-04	-0.68E-05	0.50E-04	-0.44E-03	0.14E-02	-0.47E-02
127	-0.19E-03	0.10E+01	-0.26E-03	0.16E-03	-0.87E-04	0.36E-04	-0.94E-05	0.55E-04	-0.47E-03	0.15E-02	-0.50E-02
128	-0.23E-03	0.10E+01	-0.31E-03	0.20E-03	-0.11E-03	0.44E-04	-0.12E-04	0.60E-04	-0.50E-03	0.16E-02	-0.54E-02
129	-0.29E-03	0.10E+01	-0.38E-03	0.24E-03	-0.13E-03	0.53E-04	-0.16E-04	0.67E-04	-0.53E-03	0.18E-02	-0.59E-02
130	-0.35E-03	0.10E+01	-0.47E-03	0.30E-03	-0.16E-03	0.64E-04	-0.20E-04	0.73E-04	-0.57E-03	0.19E-02	-0.63E-02
131	-0.43E-03	0.10E+01	-0.58E-03	0.36E-03	-0.19E-03	0.78E-04	-0.25E-04	0.80E-04	-0.61E-03	0.21E-02	-0.68E-02
132	-0.53E-03	0.10E+01	-0.71E-03	0.44E-03	-0.23E-03	0.95E-04	-0.30E-04	0.88E-04	-0.65E-03	0.22E-02	-0.74E-02
133	-0.66E-03	0.10E+01	-0.87E-03	0.54E-03	-0.28E-03	0.12E-03	-0.37E-04	0.96E-04	-0.69E-03	0.24E-02	-0.80E-02
134	-0.81E-03	0.10E+01	-0.11E-02	0.67E-03	-0.34E-03	0.14E-03	-0.45E-04	0.11E-03	-0.74E-03	0.26E-02	-0.86E-02
135	-0.10E-02	0.10E+01	-0.13E-02	0.82E-03	-0.42E-03	0.17E-03	-0.54E-04	0.12E-03	-0.80E-03	0.28E-02	-0.94E-02
136	-0.12E-02	0.10E+01	-0.16E-02	0.10E-02	-0.51E-03	0.21E-03	-0.65E-04	0.13E-03	-0.86E-03	0.31E-02	-0.10E-01
137	-0.15E-02	0.10E+01	-0.20E-02	0.12E-02	-0.62E-03	0.25E-03	-0.79E-04	0.14E-03	-0.93E-03	0.33E-02	-0.11E-01
138	-0.19E-02	0.10E+01	-0.25E-02	0.15E-02	-0.76E-03	0.30E-03	-0.95E-04	0.15E-03	-0.10E-02	0.36E-02	-0.12E-01
139	-0.23E-02	0.10E+01	-0.30E-02	0.18E-02	-0.93E-03	0.37E-03	-0.11E-03	0.17E-03	-0.11E-02	0.40E-02	-0.13E-01
140	-0.28E-02	0.10E+01	-0.37E-02	0.23E-02	-0.11E-02	0.45E-03	-0.14E-03	0.19E-03	-0.12E-02	0.43E-02	-0.14E-01
141	-0.35E-02	0.10E+01	-0.46E-02	0.28E-02	-0.14E-02	0.55E-03	-0.17E-03	0.21E-03	-0.13E-02	0.47E-02	-0.16E-01
142	-0.43E-02	0.10E+01	-0.56E-02	0.34E-02	-0.17E-02	0.66E-03	-0.20E-03	0.23E-03	-0.14E-02	0.52E-02	-0.17E-01
143	-0.54E-02	0.10E+01	-0.69E-02	0.42E-02	-0.21E-02	0.81E-03	-0.24E-03	0.26E-03	-0.15E-02	0.57E-02	-0.19E-01
144	-0.66E-02	0.10E+01	-0.86E-02	0.51E-02	-0.25E-02	0.98E-03	-0.29E-03	0.29E-03	-0.17E-02	0.63E-02	-0.21E-01
145	-0.82E-02	0.10E+01	-0.11E-01	0.63E-02	-0.31E-02	0.12E-02	-0.34E-03	0.33E-03	-0.19E-02	0.70E-02	-0.23E-01
146	-0.10E-01	0.10E+01	-0.13E-01	0.78E-02	-0.38E-02	0.15E-02	-0.41E-03	0.37E-03	-0.21E-02	0.78E-02	-0.25E-01
147	-0.13E-01	0.10E+01	-0.16E-01	0.96E-02	-0.47E-02	0.18E-02	-0.49E-03	0.42E-03	-0.23E-02	0.87E-02	-0.28E-01
148	-0.16E-01	0.10E+01	-0.20E-01	0.12E-01	-0.58E-02	0.22E-02	-0.59E-03	0.48E-03	-0.26E-02	0.98E-02	-0.31E-01
149	-0.20E-01	0.10E+01	-0.25E-01	0.15E-01	-0.71E-02	0.26E-02	-0.70E-03	0.55E-03	-0.29E-02	0.11E-01	-0.35E-01
150	-0.25E-01	0.10E+01	-0.31E-01	0.18E-01	-0.87E-02	0.32E-02	-0.83E-03	0.63E-03	-0.34E-02	0.13E-01	-0.39E-01
151	-0.31E-01	0.11E+01	-0.39E-01	0.23E-01	-0.11E-01	0.39E-02	-0.97E-03	0.73E-03	-0.39E-02	0.14E-01	-0.44E-01
152	-0.39E-01	0.11E+01	-0.49E-01	0.28E-01	-0.13E-01	0.47E-02	-0.11E-02	0.83E-03	-0.45E-02	0.17E-01	-0.51E-01
153	-0.50E-01	0.11E+01	-0.62E-01	0.36E-01	-0.17E-01	0.58E-02	-0.13E-02	0.94E-03	-0.53E-02	0.20E-01	-0.58E-01
154	-0.63E-01	0.11E+01	-0.78E-01	0.45E-01	-0.21E-01	0.70E-02	-0.15E-02	0.11E-02	-0.63E-02	0.23E-01	-0.68E-01
155	-0.82E-01	0.11E+01	-0.10E+00	0.57E-01	-0.26E-01	0.86E-02	-0.16E-02	0.11E-02	-0.75E-02	0.28E-01	-0.80E-01
156	-0.11E+00	0.12E+01	-0.13E+00	0.73E-01	-0.33E-01	0.11E-01	-0.17E-02	0.12E-02	-0.91E-02	0.34E-01	-0.96E-01
157	-0.14E+00	0.12E+01	-0.17E+00	0.95E-01	-0.42E-01	0.13E-01	-0.17E-02	0.11E-02	-0.11E-01	0.41E-01	-0.12E+00
158	-0.19E+00	0.13E+01	-0.22E+00	0.12E+00	-0.54E-01	0.16E-01	-0.14E-02	0.76E-03	-0.13E-01	0.51E-01	-0.14E+00
159	-0.26E+00	0.14E+01	-0.30E+00	0.17E+00	-0.70E-01	0.19E-01	-0.56E-02	-0.68E-04	-0.16E-01	0.64E-01	-0.18E+00
160	-0.35E+00	0.16E+01	-0.41E+00	0.22E+00	-0.89E-01	0.22E-01	0.18E-02	-0.16E-02	-0.20E-01	0.84E-01	-0.23E+00
161	-0.48E+00	0.18E+01	-0.55E+00	0.28E+00	-0.11E+00	0.19E-01	0.86E-02	-0.34E-02	-0.31E-01	0.12E+00	-0.32E+00
162	-0.62E+00	0.21E+01	-0.66E+00	0.31E+00	-0.80E-01	-0.17E-01	0.31E-01	0.29E-03	-0.73E-01	0.23E+00	-0.54E+00
163	-0.60E+00	0.20E+01	-0.48E+00	0.56E-01	0.17E+00	-0.20E+00	0.11E+00	0.51E-01	-0.29E+00	0.69E+00	-0.14E+01
164	0.22E+00	0.49E+00	0.12E+01	-0.15E+01	0.15E+01	-0.10E+01	0.29E+00	0.69E+00	-0.21E+01	0.41E+01	-0.70E+01
165	-0.58E+02	0.11E+03	-0.79E+02	0.57E+02	-0.44E+02	0.39E+02	-0.47E+02	0.71E+02	-0.12E+03	0.19E+03	-0.29E+03
166	0.67E+09	-0.12E+10	0.10E+10	-0.76E+09	0.53E+09	-0.31E+09	0.10E+09	0.10E+09	-0.32E+09	0.57E+09	-0.82E+09

FIGURE 11.5.7

The main program statements necessary to Chebyshev filter the RAM table are shown, (acting on the DUT, imperfect filter and NLFG).

```

CALL IT
CALL DISP(DES)
DO B=1,20
PG=.1*B
CALL VH(DES,SIZE)
DO I=0,15
DUT(I)=0
RAT(I)=0
TOT(I)=0
END DO
RAT(1)=1
DUT(1)=.99
DUT(2)=-.0316
DUT(3)=.0177
DUT(4)=-.0056
DUT(5)=.0031
DO N=1,50
CALL FILTER(RAT,VEC1,DUT,VEC2)
C MAGN(I)=1.0
C PHASE=.2*I
DO I=0,15
TOT(I)=VEC2(I)+VEC1(I)
END DO
IF(N.EQ.1.OR.N.EQ.50)CALL DISV(TOT,DES,SIZE,N)
DO I=0,15
IF(I.EQ.1)RAT(I)=RAT(I)-PG*(TOT(I)-1)
IF(I.NE.1)RAT(I)=RAT(I)-PG*TOT(I)
END DO
CALL RAMFILTER(RAT)
C RAT(I)=RAT(I) IF I.LE.7
C RAT(I)=RAT(I)/I IF I.GT.7
END DO
END DO
STOP
END

```

FIGURE 11.5.8

The results show the effect of Chebyshev filtering the RAM table after the seventh Chebyshev coefficient. Stability results up to $G=1.6$.

No. of iterations	Coefficient of $T_n(x)$											
	n=0	1	2	3	4	5	6	7	8	9	10	G
1	0.26E-04	0.97E+00	-0.29E-01	0.15E-01	-0.39E-02	0.17E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.1
50	-0.28E-05	0.10E+01	-0.23E-03	0.16E-03	0.64E-04	0.74E-05	0.80E-04	-0.18E-04	0.28E-06	-0.40E-05	0.81E-05	
1	0.26E-04	0.97E+00	-0.29E-01	0.15E-01	-0.39E-02	0.17E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.2
50	-0.93E-09	0.10E+01	-0.12E-05	0.17E-05	0.13E-05	0.22E-05	0.10E-04	-0.58E-05	0.68E-06	-0.54E-05	0.94E-05	
1	0.26E-04	0.97E+00	-0.29E-01	0.15E-01	-0.39E-02	0.17E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.3
50	0.12E-08	0.10E+01	0.11E-08	0.17E-07	-0.10E-06	0.45E-06	0.11E-05	-0.22E-05	0.74E-06	-0.57E-05	0.97E-05	
1	0.26E-04	0.97E+00	-0.29E-01	0.15E-01	-0.39E-02	0.17E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.4
50	-0.58E-09	0.10E+01	0.60E-08	0.29E-08	-0.17E-07	0.97E-07	0.55E-07	-0.83E-06	0.75E-06	-0.59E-05	0.98E-05	
1	0.26E-04	0.97E+00	-0.29E-01	0.15E-01	-0.39E-02	0.17E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.5
50	-0.12E-09	0.10E+01	0.28E-08	-0.47E-09	-0.10E-08	0.28E-07	-0.24E-07	-0.32E-06	0.76E-06	-0.59E-05	0.99E-05	
1	0.26E-04	0.97E+00	-0.29E-01	0.15E-01	-0.39E-02	0.17E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.6
50	-0.47E-09	0.10E+01	0.34E-08	-0.93E-09	0.58E-10	0.93E-08	-0.14E-07	-0.12E-06	0.76E-06	-0.60E-05	0.10E-04	
1	0.26E-04	0.97E+00	-0.29E-01	0.15E-01	-0.39E-02	0.17E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.7
50	-0.12E-09	0.10E+01	-0.12E-09	0.14E-08	-0.58E-10	0.36E-08	-0.58E-08	-0.46E-07	0.76E-06	-0.60E-05	0.10E-04	
1	0.26E-04	0.97E+00	-0.29E-01	0.15E-01	-0.39E-02	0.17E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.8
50	0.00E+00	0.10E+01	0.70E-09	-0.35E-09	-0.23E-09	0.14E-08	-0.21E-08	-0.17E-07	0.76E-06	-0.60E-05	0.10E-04	
1	0.26E-04	0.97E+00	-0.29E-01	0.15E-01	-0.39E-02	0.17E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.9
50	-0.35E-09	0.10E+01	-0.87E-09	-0.23E-09	0.58E-10	0.47E-09	-0.81E-09	-0.60E-08	0.77E-06	-0.60E-05	0.10E-04	
1	0.26E-04	0.97E+00	-0.29E-01	0.15E-01	-0.39E-02	0.17E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.0
50	0.00E+00	0.10E+01	-0.58E-09	-0.12E-08	0.29E-09	0.00E+00	-0.23E-09	-0.22E-08	0.77E-06	-0.60E-05	0.10E-04	
1	0.26E-04	0.97E+00	-0.29E-01	0.15E-01	-0.39E-02	0.17E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.1
50	-0.12E-09	0.10E+01	-0.76E-09	-0.70E-09	-0.12E-09	-0.12E-09	-0.58E-10	-0.74E-09	0.77E-06	-0.61E-05	0.10E-04	
1	0.26E-04	0.97E+00	-0.29E-01	0.15E-01	-0.39E-02	0.17E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.2
50	0.47E-09	0.10E+01	0.12E-08	-0.47E-09	-0.41E-09	-0.29E-09	-0.58E-10	-0.28E-09	0.77E-06	-0.61E-05	0.10E-04	
1	0.26E-04	0.97E+00	-0.29E-01	0.15E-01	-0.39E-02	0.17E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.3
50	-0.59E-07	0.10E+01	-0.65E-07	0.33E-07	-0.11E-07	0.35E-08	-0.52E-09	-0.44E-10	0.77E-06	-0.61E-05	0.10E-04	
1	0.26E-04	0.97E+00	-0.29E-01	0.15E-01	-0.39E-02	0.17E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.4
50	-0.95E-07	0.10E+01	-0.10E-06	0.56E-07	-0.19E-07	0.63E-08	-0.11E-08	0.13E-09	0.77E-06	-0.61E-05	0.10E-04	
1	0.26E-04	0.97E+00	-0.29E-01	0.15E-01	-0.39E-02	0.17E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.5
50	-0.45E-04	0.10E+01	-0.50E-04	0.25E-04	-0.97E-05	0.29E-05	-0.59E-06	0.61E-07	0.77E-06	-0.62E-05	0.10E-04	
1	0.26E-04	0.97E+00	-0.29E-01	0.15E-01	-0.39E-02	0.17E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.6
50	-0.25E-01	0.10E+01	-0.28E-01	0.14E-01	-0.54E-02	0.16E-02	-0.33E-03	0.37E-04	0.31E-05	-0.85E-05	0.12E-04	

FIGURE 11.5.9

The main program statements, and the effect after 200 iterations, of Chebyshev filtering with coefficients above third cut off. The value of G is 0.2.

Suppression of distortion due to third order or less is excellent, but suppression is very poor above third order.

```

CALL IT
CALL DISP(DES)
DO B=1,2
PG=.1*B
CALL VH(DES,SIZE)
DO I=0,15
DUT(I)=0
RAT(I)=0
TOT(I)=0
END DO
RAT(1)=1
DUT(1)=.99
DUT(2)=-.0316
DUT(3)=.0177
DUT(4)=-.0056
DUT(5)=.0031
DO N=1,200
CALL FILTER(RAT,VEC1,DUT,VEC2)
C MAGN(I)=1.0
C PHASE =.2*I
DO I=0,15
TOT(I)=VEC2(I)+VEC1(I)
END DO
CALL DISV(TOT,DES,SIZE,N)
DO I=0,15
IF(I.EQ.1)RAT(I)=RAT(I)-PG*(TOT(I)-1)
IF(I.NE.1)RAT(I)=RAT(I)-PG*TOT(I)
END DO
CALL RAMFILTER(RAT)
C RAT(I)=RAT(I) IF I.LE.3
C RAT(I)=RAT(I)/I IF I.GT.3
END DO
END DO
STOP
END

```

Coefficient of $T_n(x)$						
No. of iterations	n=0	1	2	3	4	5
199	0.00E+00	0.10E+01	-0.12E-08	0.40E-08	-0.17E-02	0.89E-03
200	0.00E+00	0.10E+01	-0.12E-08	0.40E-08	-0.17E-02	0.89E-03
	n=6	7	8	9	10	
199	0.28E-03	-0.29E-04	0.23E-05	-0.19E-05	-0.27E-06	
200	0.28E-03	-0.29E-04	0.23E-05	-0.19E-05	-0.27E-06	

FIGURE 11.5.10

Shown are the main program statements necessary to consider the system with a sawtooth input instead of a cosine input.

```

CALL DISP(DES)
DO B=1,1
PG=.1*B
CALL VH(DES,SIZE)
DO I=0,15
DUT(I)=0
RAT(I)=0
M(I)=0
TOT(I)=0
END DO
RAT(1)=1
DUT(1)=.99
DUT(2)=-.0316
DUT(3)=.0177
DUT(4)=-.0056
DUT(5)=.0031
M(9)=5E-3
M(7)=8.26E-3
M(5)=.0162
M(3)=.045
M(1)=.405
M(0)=.5
CALL IT
CALL DISM(MC,DES,SIZE)
CALL MULT(RAT,VEC1,MC)
DO I=0,15
RAT(I)=VEC1(I)
END DO
DO N=1,200
CALL FILTER(RAT,VEC1,DUT,VEC2)
C I.LE.12 MAGN(I)=1.0 I.GT.12 MAGN(I) =.02
C PHASE =.2*I
DO I=0,15
TOT(I)=VEC2(I)+VEC1(I)
END DO
CALL DISV(TOT,DES,SIZE,N)
CALL AVERAGE(TOT,VEC1,7.8E-3)
DO I=0,15
TOT(I)=TOT(I)+VEC1(I)+1E-10*(RAN(VAL)-.5)
RAT(I)=RAT(I)-PG*(TOT(I)-M(I))
END DO
CALL RAMFILTER(RAT)
C RAT(I)=RAT(I) IF I.LE.7
C RAT(I)=RAT(I)/I IF I.GT.7
END DO
END DO
STOP
END

```

FIGURE 11.5.11

The cascade matrix and the results of 200 iterations for $G=0.1$ are shown. The experiment is as outlined in Figure 11.5.10.

Cascade matrix for sawtooth waveform											
	0	1	2	3	4	5	6	7	8	9	10
0	1.00	0.50	-0.33	-0.50	-0.07	0.17	-0.03	-0.17	-0.02	0.09	-0.02
1	0.00	0.41	0.81	0.23	-0.70	-0.72	0.03	0.35	-0.05	-0.31	-0.05
2	0.00	0.00	0.20	0.61	0.56	-0.22	-0.88	-0.55	0.25	0.41	-0.10
3	0.00	0.05	0.09	0.12	0.31	0.50	0.19	-0.57	-0.90	-0.29	0.48
4	0.00	0.00	0.05	0.15	0.19	0.17	0.24	0.24	-0.17	-0.76	-0.74
5	0.00	0.02	0.03	0.05	0.12	0.22	0.20	0.10	0.05	-0.06	-0.42

No. of iterations	n=0	1	2	3	4	5	6	7	8	9	10
185	0.50E+00	0.40E+00	0.80E-05	0.45E-01	0.84E-05	0.16E-01	0.97E-05	0.79E-02	-0.12E-04	-0.17E-03	-0.15E-03
186	0.50E+00	0.40E+00	0.79E-05	0.45E-01	0.83E-05	0.16E-01	0.96E-05	0.79E-02	-0.12E-04	-0.17E-03	-0.15E-03
187	0.50E+00	0.40E+00	0.79E-05	0.45E-01	0.83E-05	0.16E-01	0.95E-05	0.79E-02	-0.12E-04	-0.17E-03	-0.15E-03
188	0.50E+00	0.40E+00	0.79E-05	0.45E-01	0.83E-05	0.16E-01	0.95E-05	0.79E-02	-0.12E-04	-0.17E-03	-0.15E-03
189	0.50E+00	0.40E+00	0.79E-05	0.45E-01	0.82E-05	0.16E-01	0.94E-05	0.79E-02	-0.12E-04	-0.17E-03	-0.15E-03
190	0.50E+00	0.40E+00	0.78E-05	0.45E-01	0.82E-05	0.16E-01	0.93E-05	0.80E-02	-0.12E-04	-0.17E-03	-0.15E-03
191	0.50E+00	0.40E+00	0.78E-05	0.45E-01	0.81E-05	0.16E-01	0.92E-05	0.80E-02	-0.12E-04	-0.17E-03	-0.15E-03
192	0.50E+00	0.40E+00	0.78E-05	0.45E-01	0.81E-05	0.16E-01	0.91E-05	0.80E-02	-0.12E-04	-0.17E-03	-0.15E-03
193	0.50E+00	0.40E+00	0.78E-05	0.45E-01	0.81E-05	0.16E-01	0.91E-05	0.80E-02	-0.12E-04	-0.17E-03	-0.15E-03
194	0.50E+00	0.40E+00	0.77E-05	0.45E-01	0.80E-05	0.16E-01	0.90E-05	0.80E-02	-0.12E-04	-0.17E-03	-0.15E-03
195	0.50E+00	0.40E+00	0.77E-05	0.45E-01	0.80E-05	0.16E-01	0.89E-05	0.80E-02	-0.12E-04	-0.17E-03	-0.15E-03
196	0.50E+00	0.40E+00	0.77E-05	0.45E-01	0.79E-05	0.16E-01	0.89E-05	0.80E-02	-0.12E-04	-0.17E-03	-0.15E-03
197	0.50E+00	0.40E+00	0.77E-05	0.45E-01	0.79E-05	0.16E-01	0.88E-05	0.80E-02	-0.12E-04	-0.17E-03	-0.15E-03
198	0.50E+00	0.40E+00	0.76E-05	0.45E-01	0.79E-05	0.16E-01	0.87E-05	0.80E-02	-0.12E-04	-0.17E-03	-0.15E-03
199	0.50E+00	0.40E+00	0.76E-05	0.45E-01	0.78E-05	0.16E-01	0.87E-05	0.80E-02	-0.12E-04	-0.17E-03	-0.15E-03
200	0.50E+00	0.40E+00	0.76E-05	0.45E-01	0.78E-05	0.16E-01	0.86E-05	0.80E-02	-0.12E-04	-0.17E-03	-0.15E-03

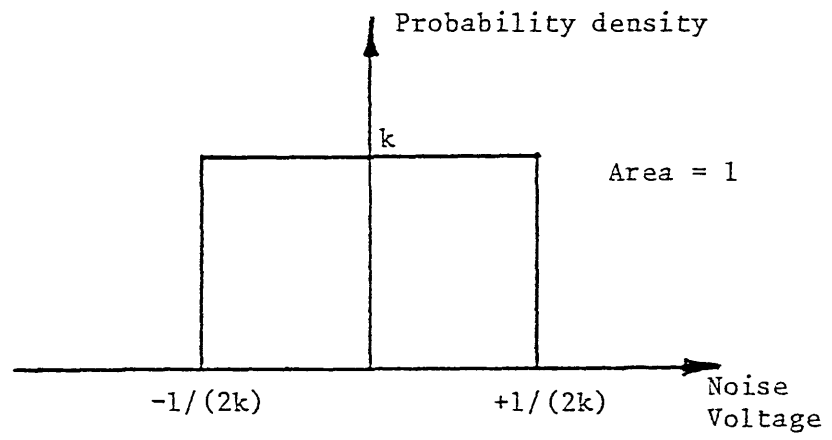
FIGURE 11.6.1

FIGURE 11.6.2

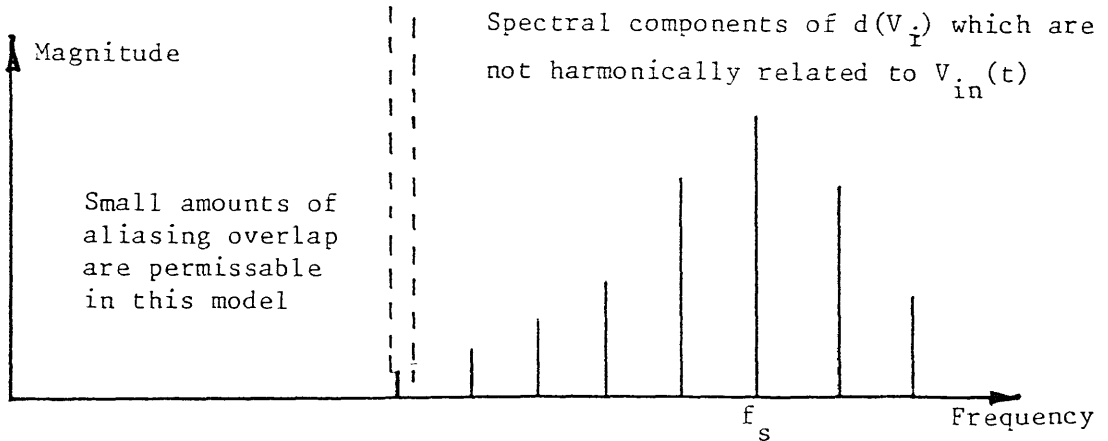
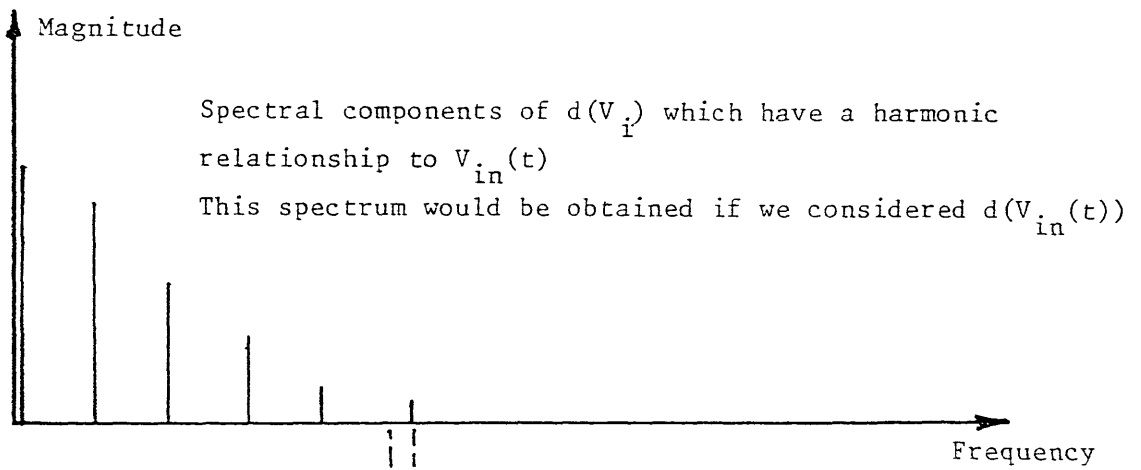
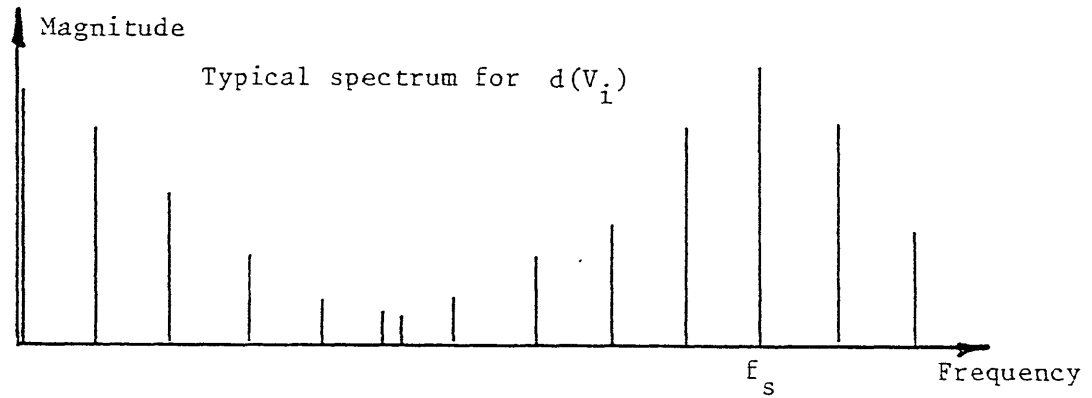
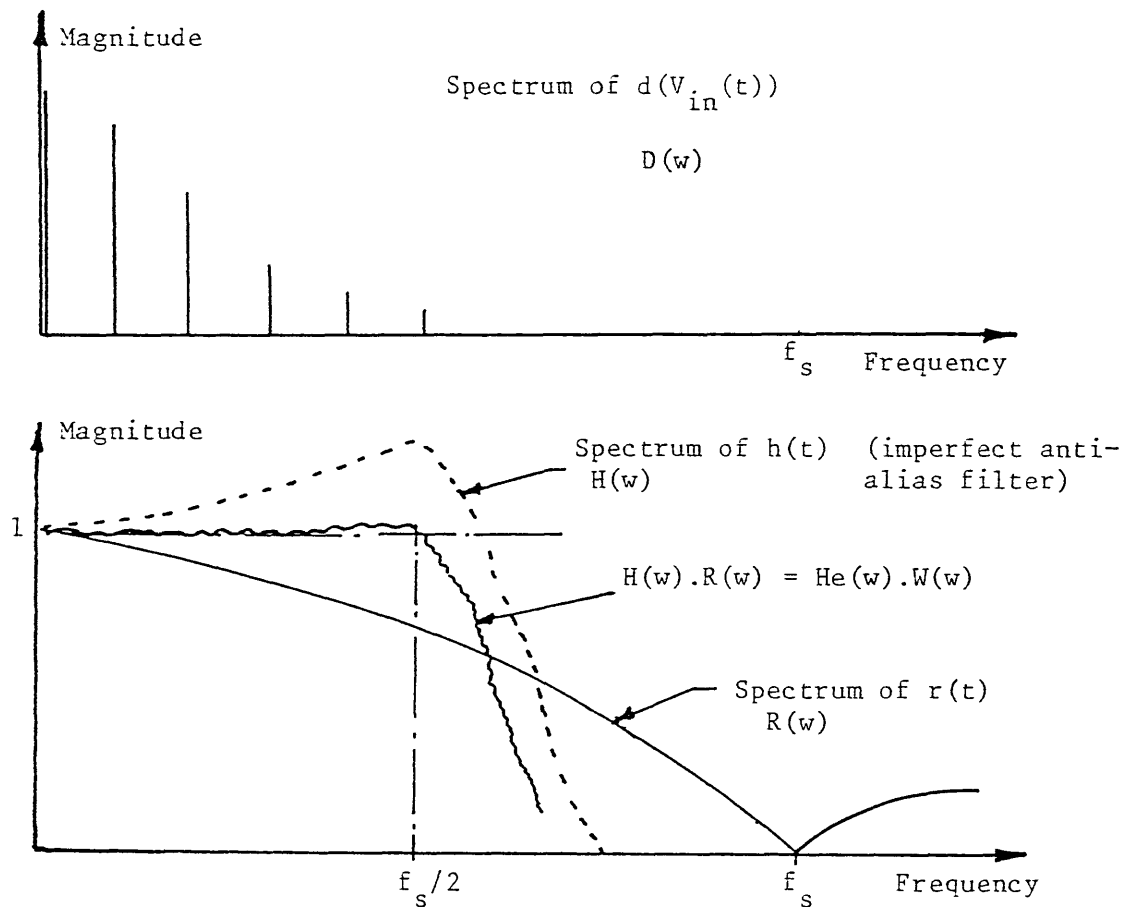


FIGURE 11.6.3



We are interested in $d(V_{in}(t)) * r(t) * h(t)$
 which in the frequency domain is $D(w).R(w).H(w)$

But we define $|He(w)| = |H(w).R(w)|$

Plainly $He(w)$ is near unity in the passband and small in the transition and stop band.

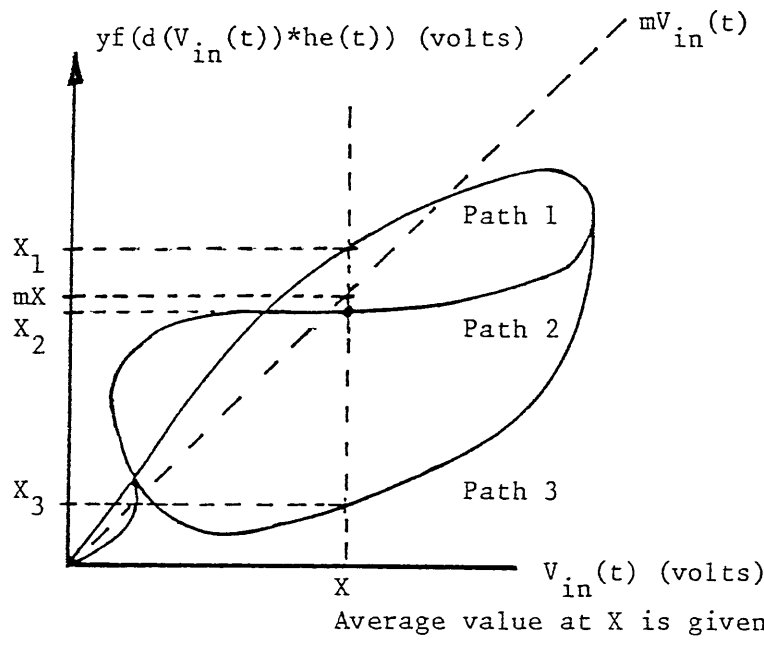
It is convenient to define $W(w)$ as the frequency response equivalent to a pure time delay of t' seconds in the time domain. With $W(w)$ thus defined, and with $H(w).R(w) = He(w).W(w)$ we have defined $He(w)$.

Thus $D(w).H(w).R(w) = D(w).W(w).He(w)$

transferring to the time domain

$$\begin{aligned} d(V_{in}(t)) * h(t) * r(t) &= d(V_{in}(t)) * w(t) * he(t) \\ &= d(V_{in}(t-t')) * he(t) \end{aligned}$$

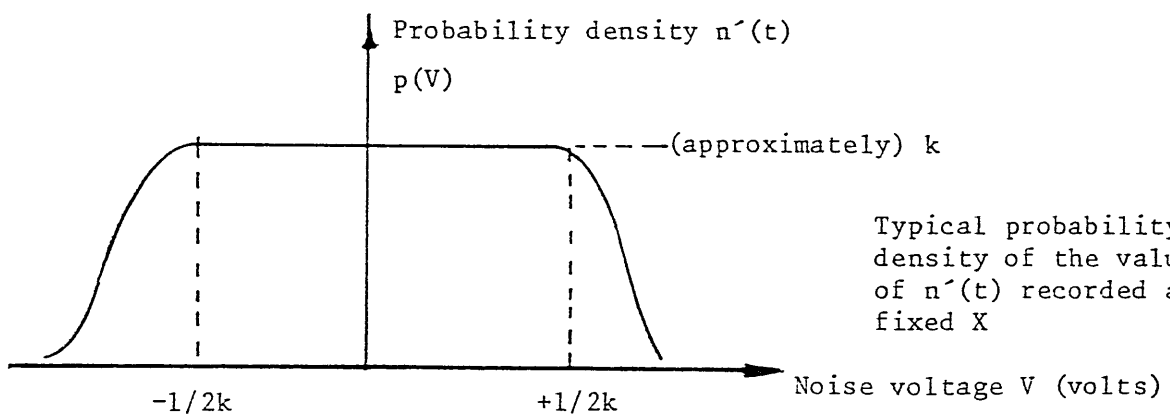
FIGURE 11.6.4



We show a "typical" hysteresis pattern for $V_{in}(t) = \cos(wt)$

Denote the probability of traversing path i by p_i ($i=1,2,3$)

Note that $p_1 + p_2 + p_3 = 1$



The probability that $p = \text{sign}(mX - yf(d(X) * he(t)) + n'(t))$ is $+1$ is given by α where $\alpha = \sum_i p_i \int_{-\infty}^{mX - X_i} p(V) dV$ and thus we have that $A(X) = \langle p \rangle = \alpha - (1 - \alpha) = 2\alpha - 1$

Since we have that $\int_{-\infty}^{mX - X_i} p(V) dV = \int_{-\infty}^{mX} p(V) dV - \int_{mX - X_i}^{mX} p(V) dV$ we can deduce that

$$A(X) = 2(p_1 + p_2 + p_3) \left(\int_{-\infty}^{mX} p(V) dV \right) - 1 - 2 \left(\sum_i p_i \int_{mX - X_i}^{mX} p(V) dV \right)$$

$$= 2 \int_{-\infty}^{mX} p(V) dV - 1 - 2 \left(\sum_i p_i \int_{mX - X_i}^{mX} p(V) dV \right)$$

Now if the values mX and mX_i ($i=1,2,3$) all lie in the range in which $p(V)$ is approximately equal to k , and if we assume that $p(V)$ is zero outside the range $(-1/2k, +1/2k)$ then we have that $A(X)$ is equal to

$$2 \left(\int_{-1/2k}^{mX} k dV \right) - 1 - 2 \left(\sum_i p_i \int_{mX - X_i}^{mX} k dV \right)$$

$$= 2k(mX - \sum_i p_i X_i)$$

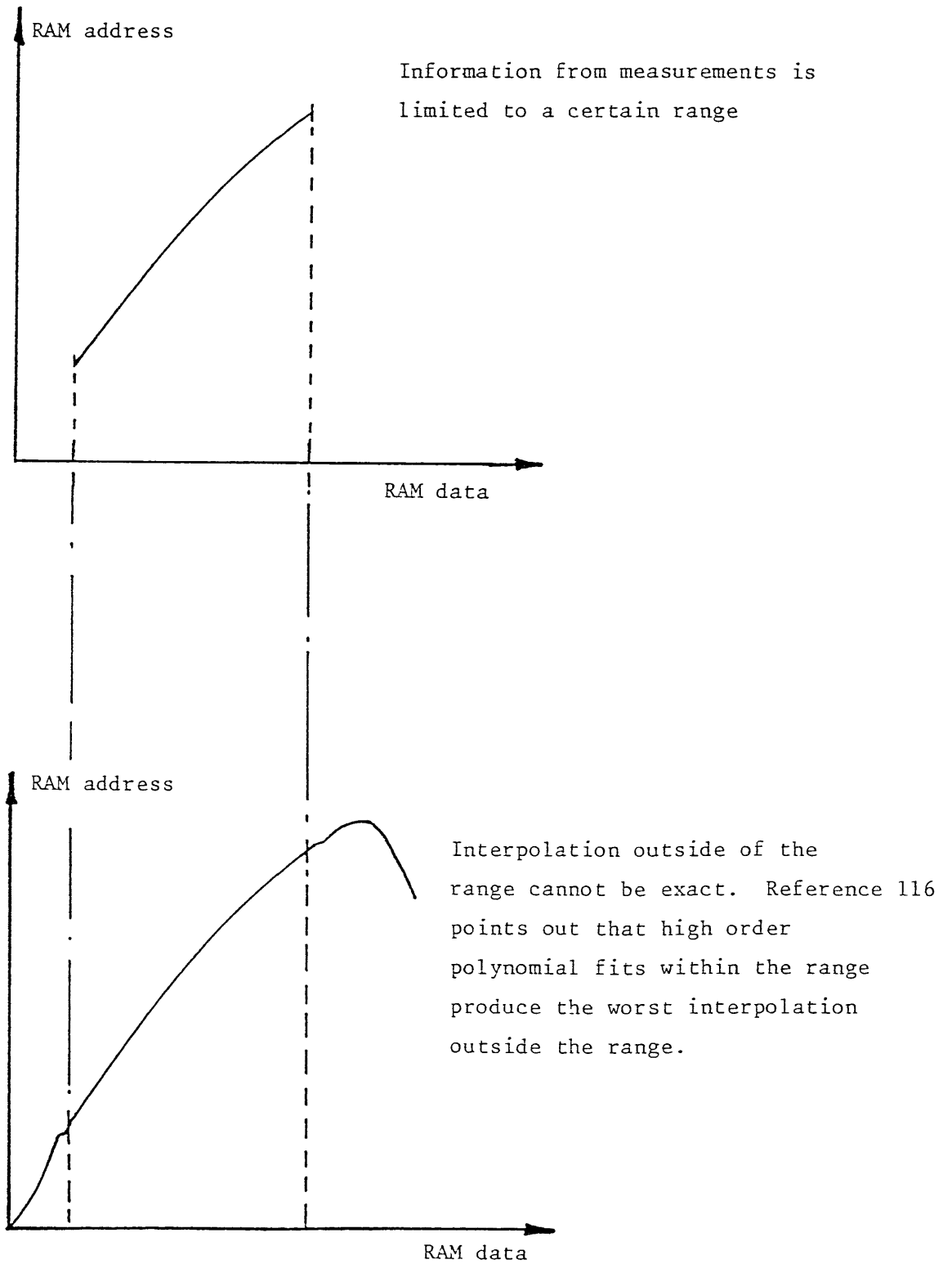
FIGURE 11.6.5

FIGURE 11.6.6

We show below the result of adding noise to the RAM table Chebyshev coefficients. The value of G is 0.1 and the RAM table has a Chebyshev cut-off filter removing coefficients above seventh.

```

CALL IT
CALL DISP(DES)
DO B=1,2
PG=.1*B
CALL VH(DES,SIZE)
DO I=0,15
DUT(I)=0
RAT(I)=0
TOT(I)=0
END DO
RAT(1)=1
DUT(1)=.99
DUT(2)=-.0316
DUT(3)=.0177
DUT(4)=-.0056
DUT(5)=.0031
DO N=1,200
CALL FILTER(RAT,VEC1,DUT,VEC2)
C I.LE.12 MAGN(I)=1.0 I.GT.12 MAGN(I)=.02
C PHASE =.2*I
DO I=0,15
TOT(I)=VEC2(I)+VEC1(I)
END DO
CALL DISV(TOT,DES,SIZE,N)
CALL AVERAGE(TOT,VEC1,7.8E-3)
DO I=0,15
TOT(I)=TOT(I)+VEC1(I)+1E-3*(RAN(VAL)-.5)
IF(I.EQ.1)RAT(I)=RAT(I)-PG*(TOT(I)-1)
IF(I.NE.1)RAT(I)=RAT(I)-PG*TOT(I)
END DO
CALL RAMFILTER(RAT)
C RAT(I)=RAT(I) IF I.LE.7
C RAT(I)=RAT(I)/I IF I.GT.7
END DO
END DO
STOP
END

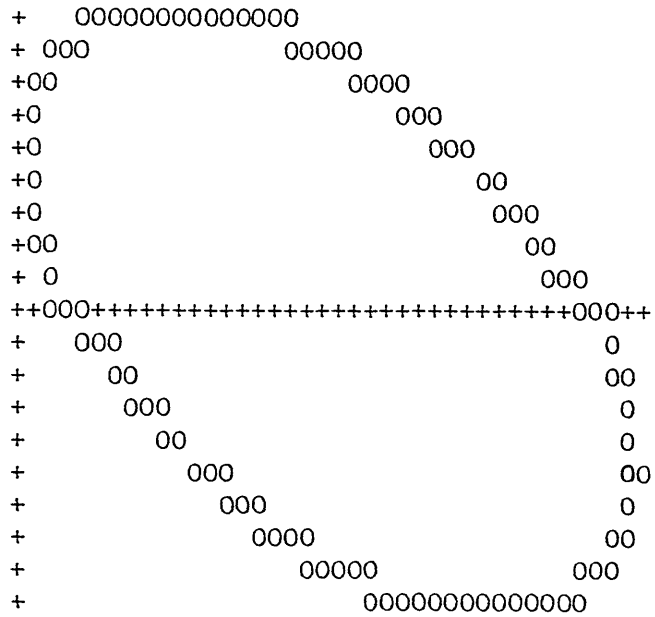
```

Results for iterations 197 to 200

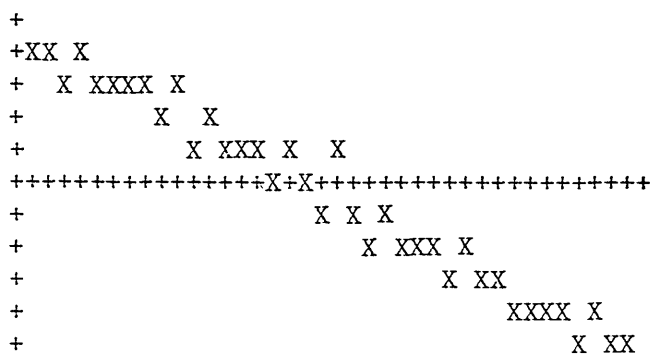
No. of iterations	n=0	1	2	3	4	5
197	-0.19E-03	0.10E+01	0.67E-04	-0.19E-03	0.17E-04	0.20E-05
198	-0.14E-03	0.10E+01	-0.34E-04	-0.18E-03	-0.58E-04	0.45E-04
199	-0.15E-03	0.10E+01	0.54E-04	-0.17E-03	-0.71E-04	-0.11E-04
200	-0.18E-03	0.10E+01	0.11E-03	-0.11E-03	-0.11E-03	-0.21E-04
	n=6	7	8	9	10	
197	0.24E-04	0.98E-05	0.51E-06	-0.55E-05	0.11E-04	
198	0.84E-05	0.24E-04	0.89E-06	-0.70E-05	0.94E-05	
199	0.43E-04	0.28E-04	0.12E-05	-0.80E-05	0.14E-04	
200	0.62E-04	0.22E-04	0.71E-06	-0.84E-05	0.84E-05	

FIGURE 11.6.7

Computer generated
hysteresis curve
representing
 $f(d(x)*he(t))$



Average of hysteresis
curve
 $\langle f(d(x)*he(t)) \rangle$



Result of applying
processing of
section 11.6.2.
No estimate of $\langle \rangle$
obtained except for
bounds on the extent
of the hysteresis.

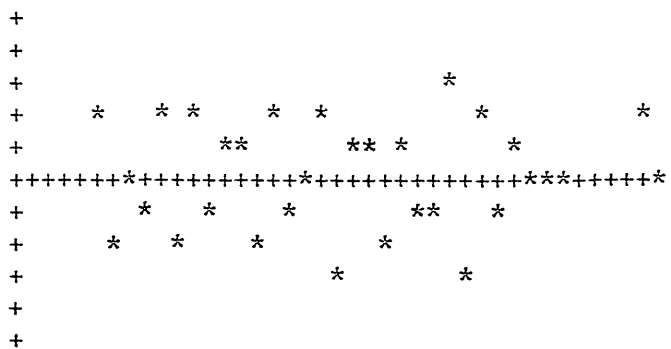
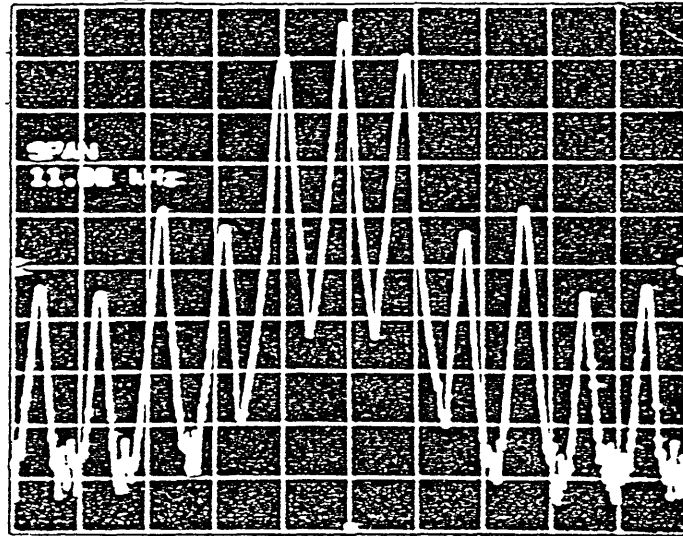


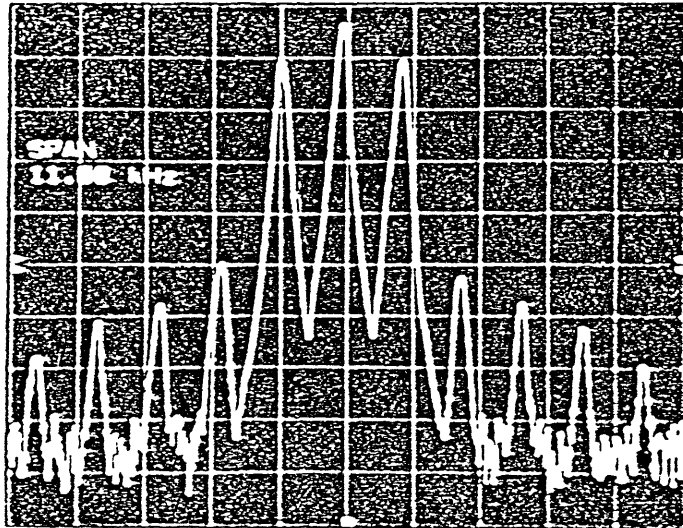
FIGURE 11.6.10

REF 3.5 dBm ATTEN 20 dB
 CENTRE 80.00020 MHz SPAN 11.00 kHz
 RES BW 100 Hz VBW 100 Hz SWP 3.0 sec

10 dB/

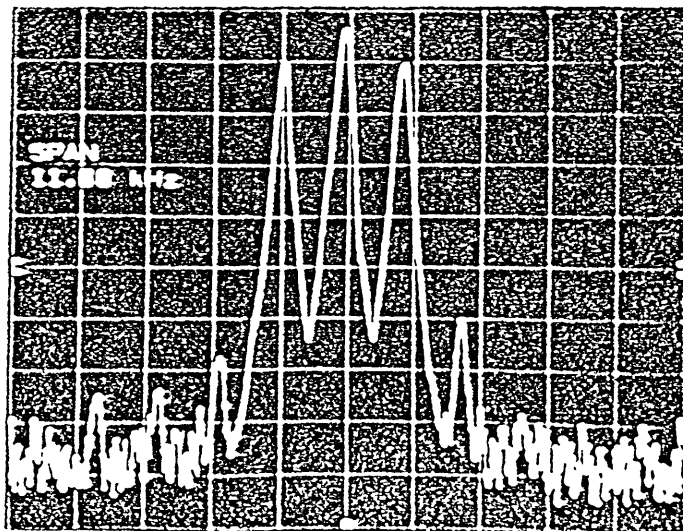


RF spectrum of DUT output. No phase or envelope compensation.



RF spectrum of DUT output with envelope compensation.

Moving average filtering of the RAM table.



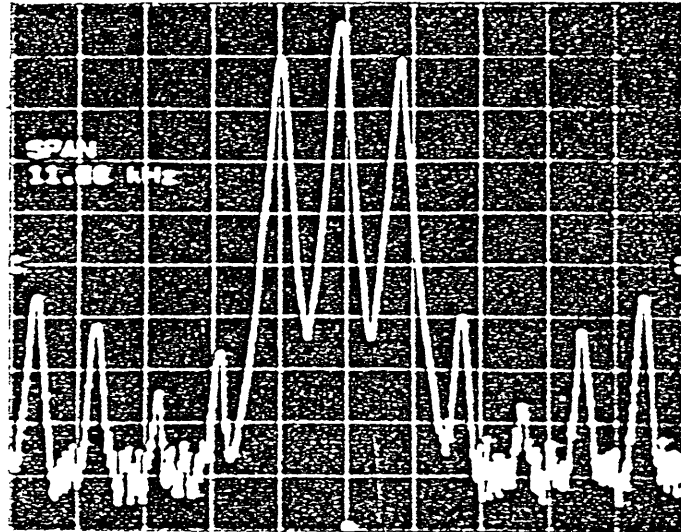
RF spectrum of DUT output with envelope compensation and PLL phase correction.

Asymmetric effects dominate.

Applying Chebyshev filtering to the RAM table appears to suppress effects less than the filter cut-off order, but leaves the higher order effects untouched.

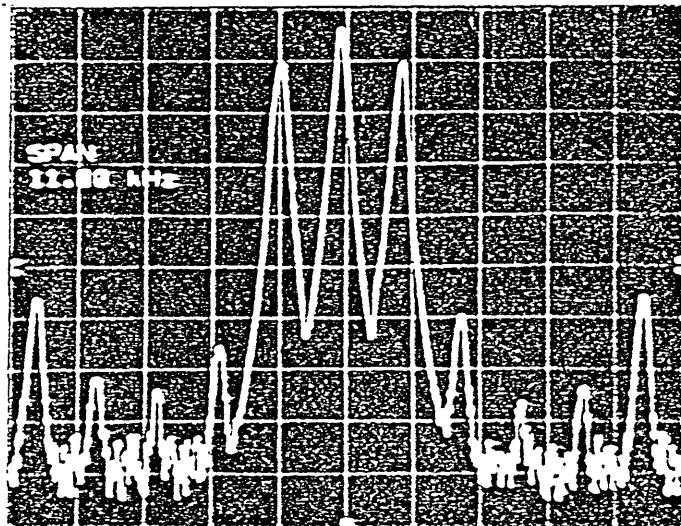
REF 3.5 dBm ATTN 20 dB
CENTRE 80.00020 MHz SPAN 11.00 kHz
RES BW 100 Hz VBW 100 Hz SWP 3.0 sec

10 dB/



RF spectrum out of the DUT, when envelope compensation is applied, and Chebyshev coefficients above 3rd order are removed (from RAM).

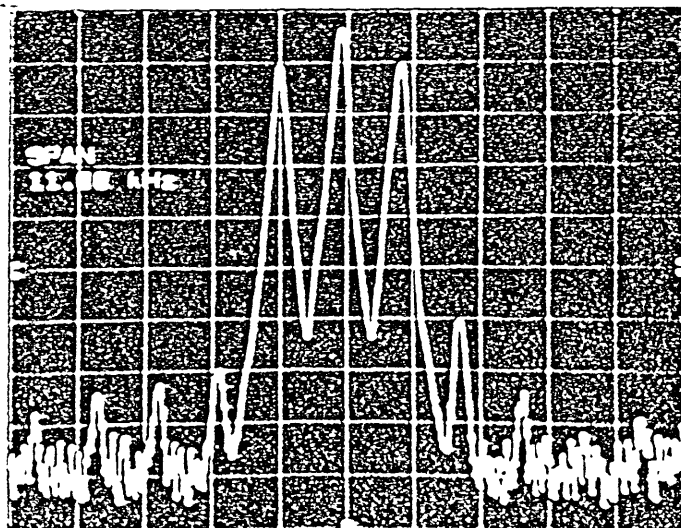
PLL phase correction is applied.



RF spectrum out of the DUT, when envelope compensation is applied.

RAM is filtered to remove Chebyshev coefficients above 4th.

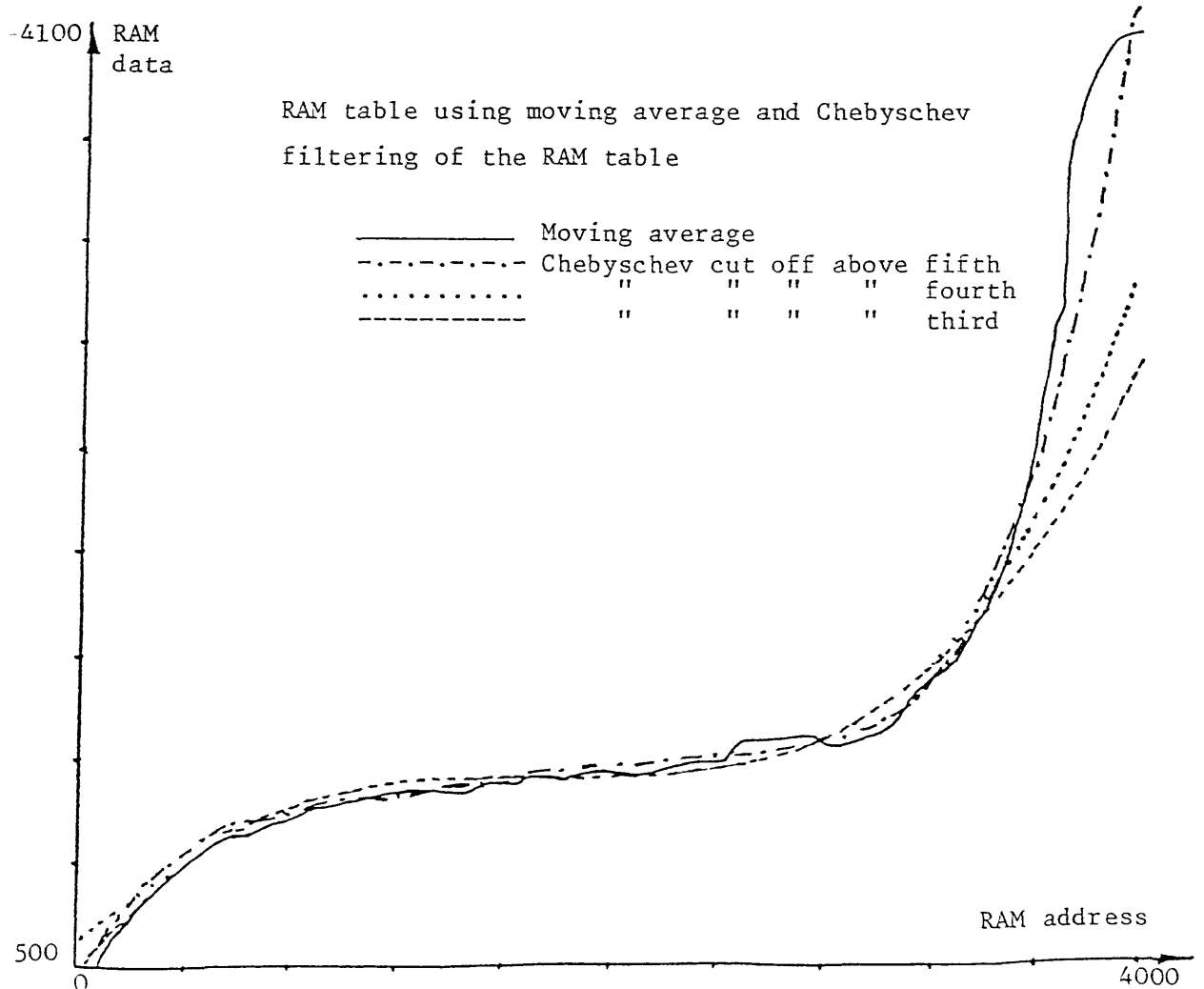
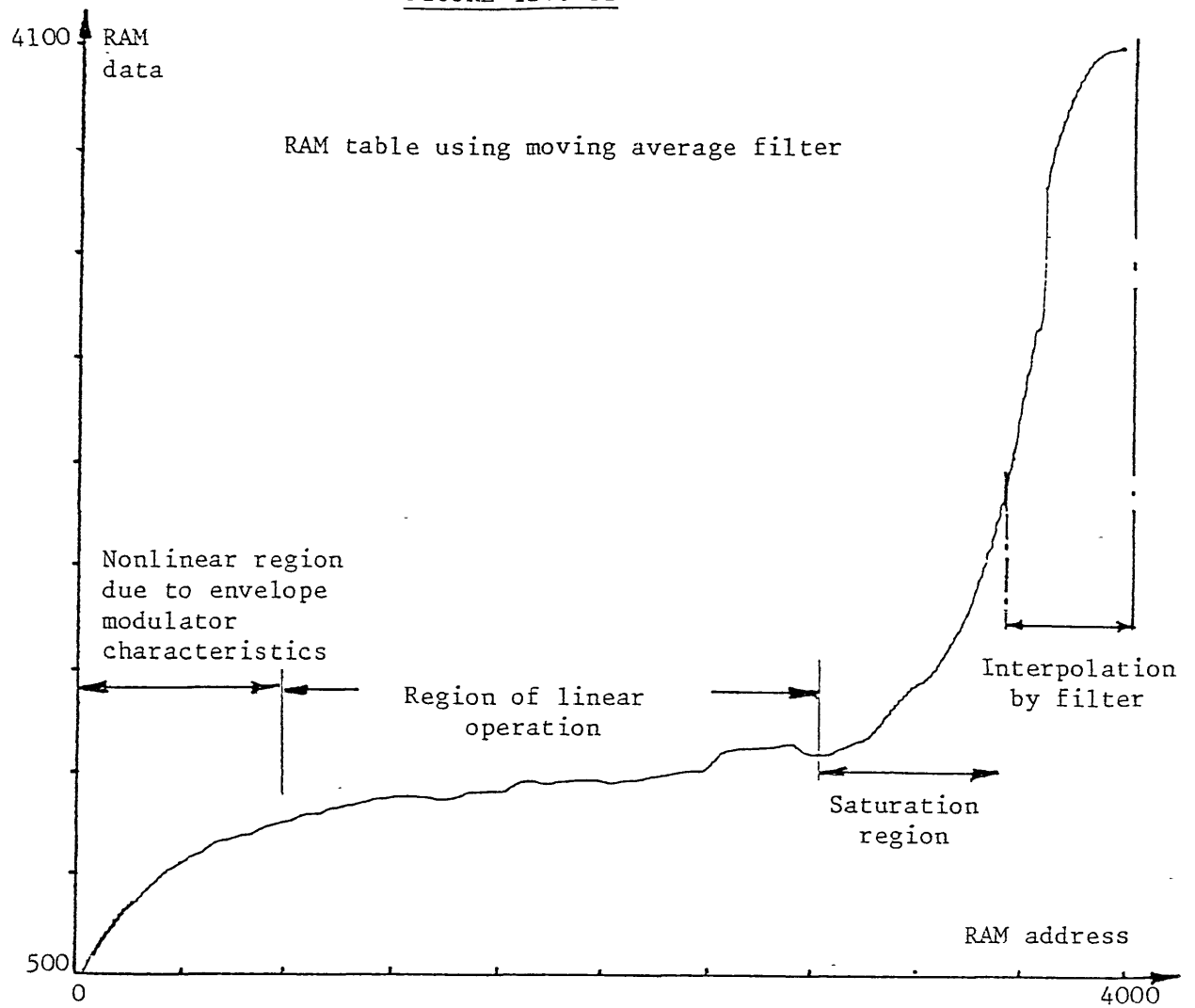
PLL correction is applied to phase.

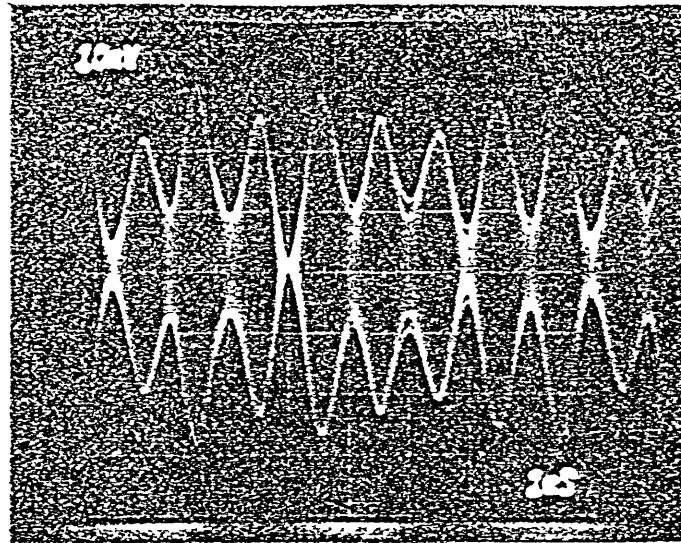


RF spectrum out of the DUT, when envelope compensation is applied.

RAM is filtered to remove Chebyshev coefficients above 5th.

PLL correction is applied to phase.

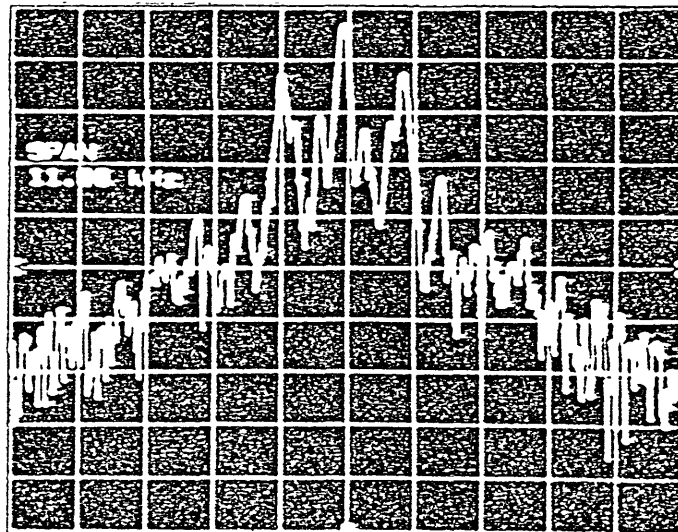




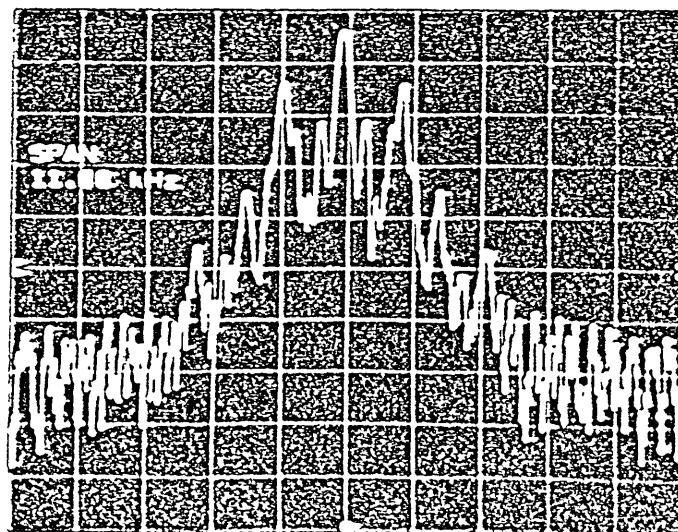
RF envelope for
non-cosinusoidal
baseband input.

REF 3.5 dBm ATTEN 20 dB
CENTRE 80.00020 MHz SPAN 11.00 kHz
RES BW 100 Hz VBW 100 Hz SWP 3.0 sec

10 dB/



RF spectrum of DUT
output with no
correction.



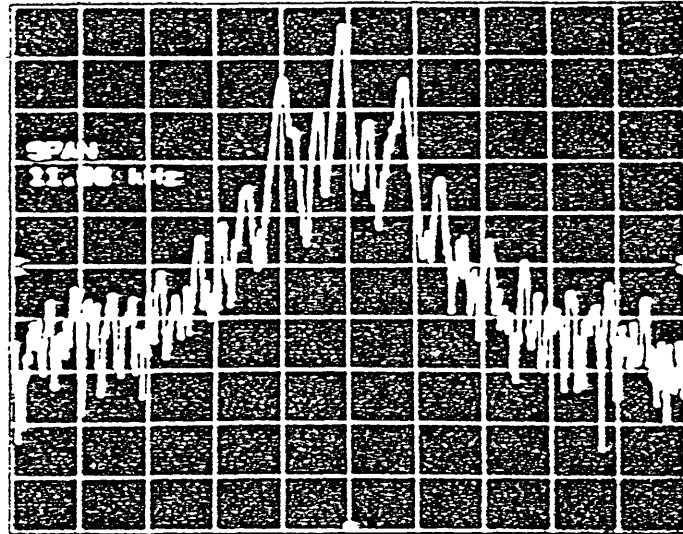
RF spectrum of DUT
output, with envelope
compensation, and PLL
phase correction.

RAM table is moving
average filtered.

FIGURE 11.6.14

REF 3.5 dBm ATTEN 20 dB
CENTRE 80.00020 MHz SPAN 11.00 kHz
RES BW 100 Hz VBW 100 Hz SWP 3.0 sec

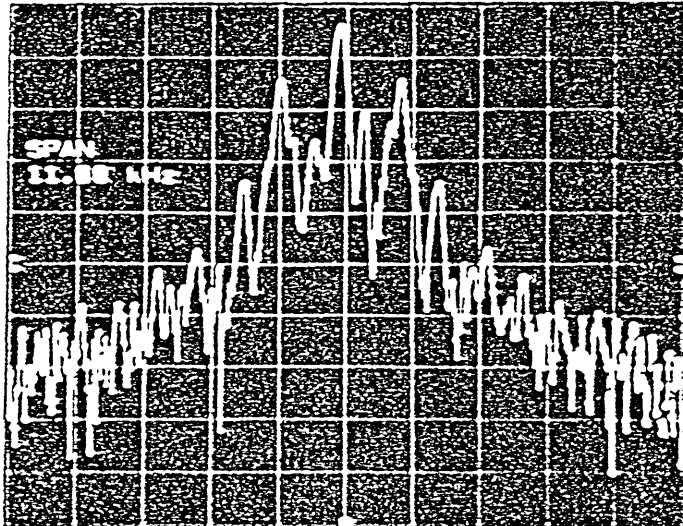
10 dB/



RF spectrum out of DUT.

RAM table Chebyshev
filtered with
coefficients above
3rd removed.

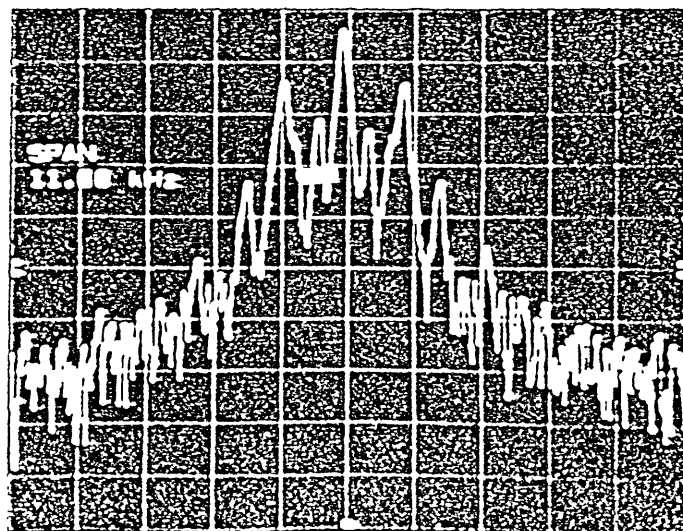
PLL phase correction
applied.



RF spectrum out of DUT.

RAM table Chebyshev
filtered to remove
coefficients above 4th.

PLL phase correction
applied.



RF spectrum out of DUT.

RAM table Chebyshev
filtered to remove
coefficients above 5th.

PLL correction applied
to phase.

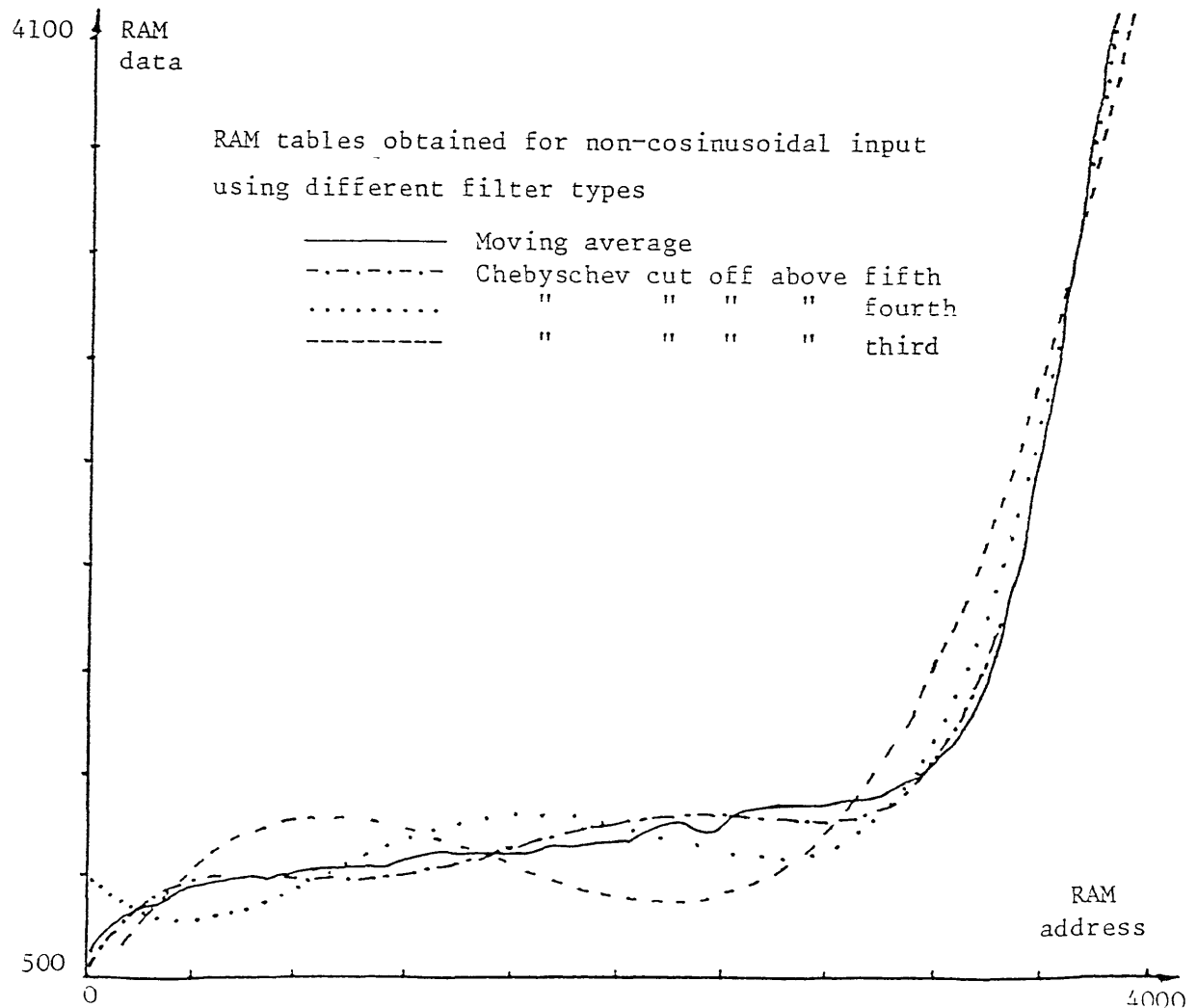
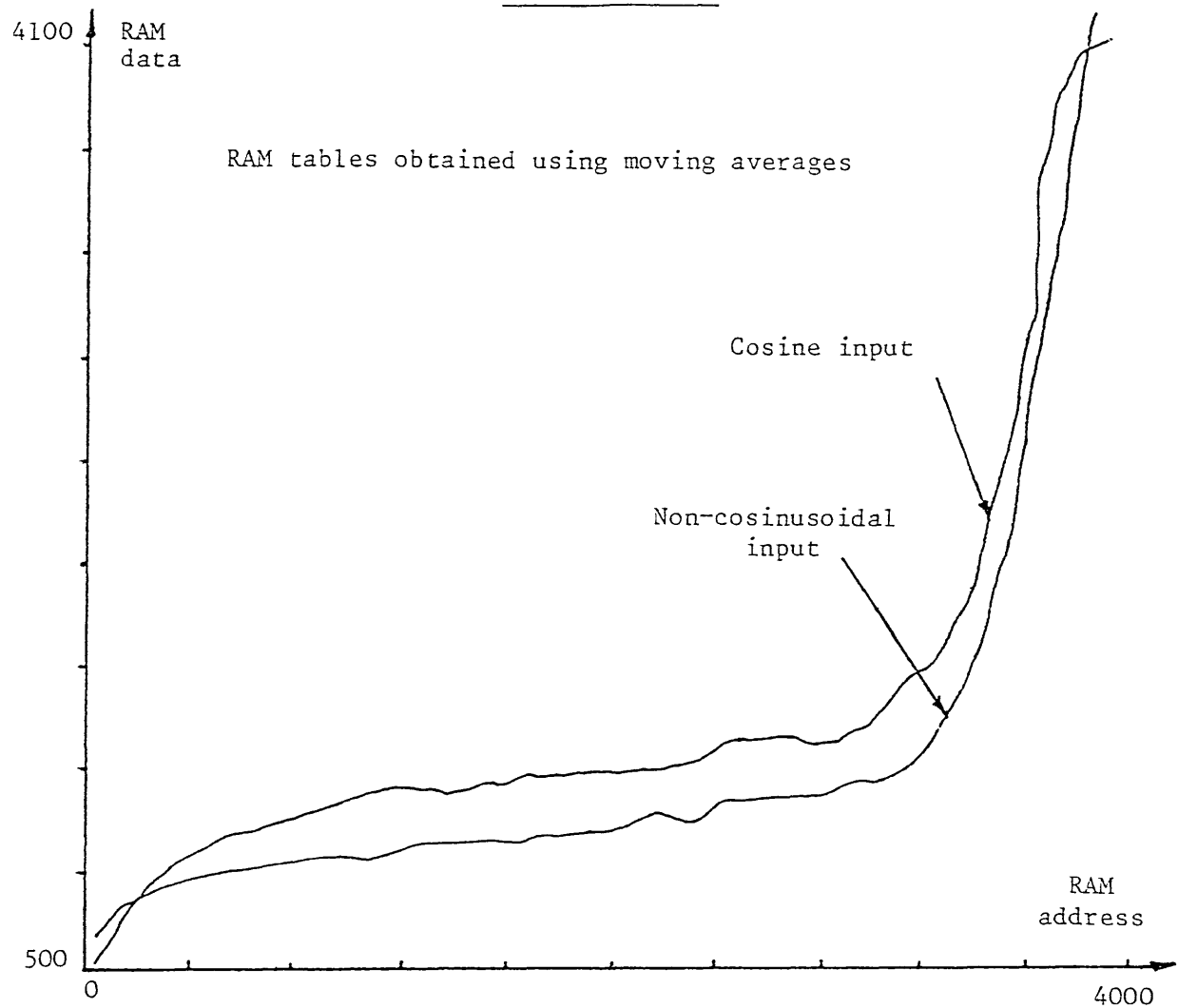


FIGURE 11.6.16

RF spectrum of DUT output using the moving average RAM table produced by a cosine input. The spectral difference between this and Figure 11.6.13 (lower figure) is small.

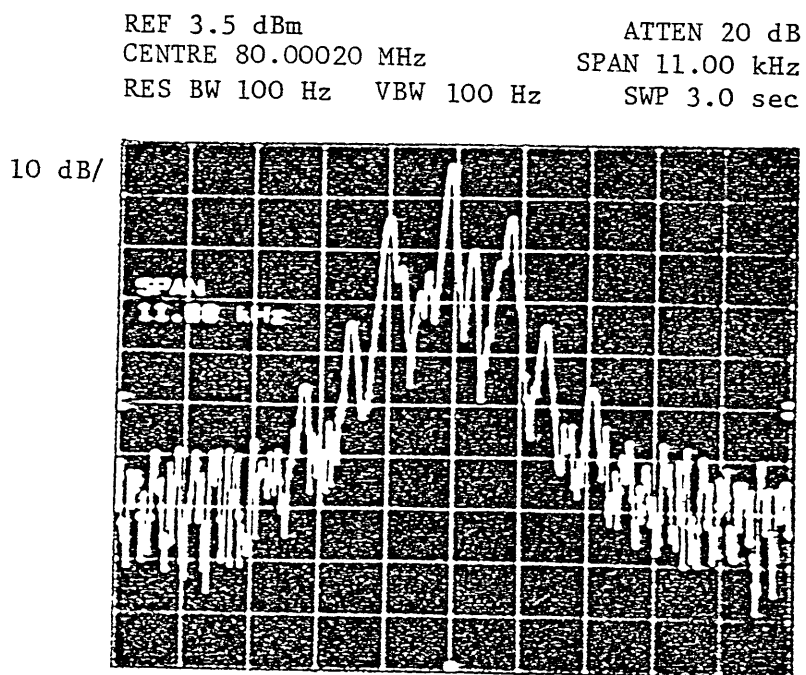


FIGURE 11.6.17

Bin values with and without noise added at the comparator

The variation from bin to bin is less when noise is added, especially at the high and low bin numbers.

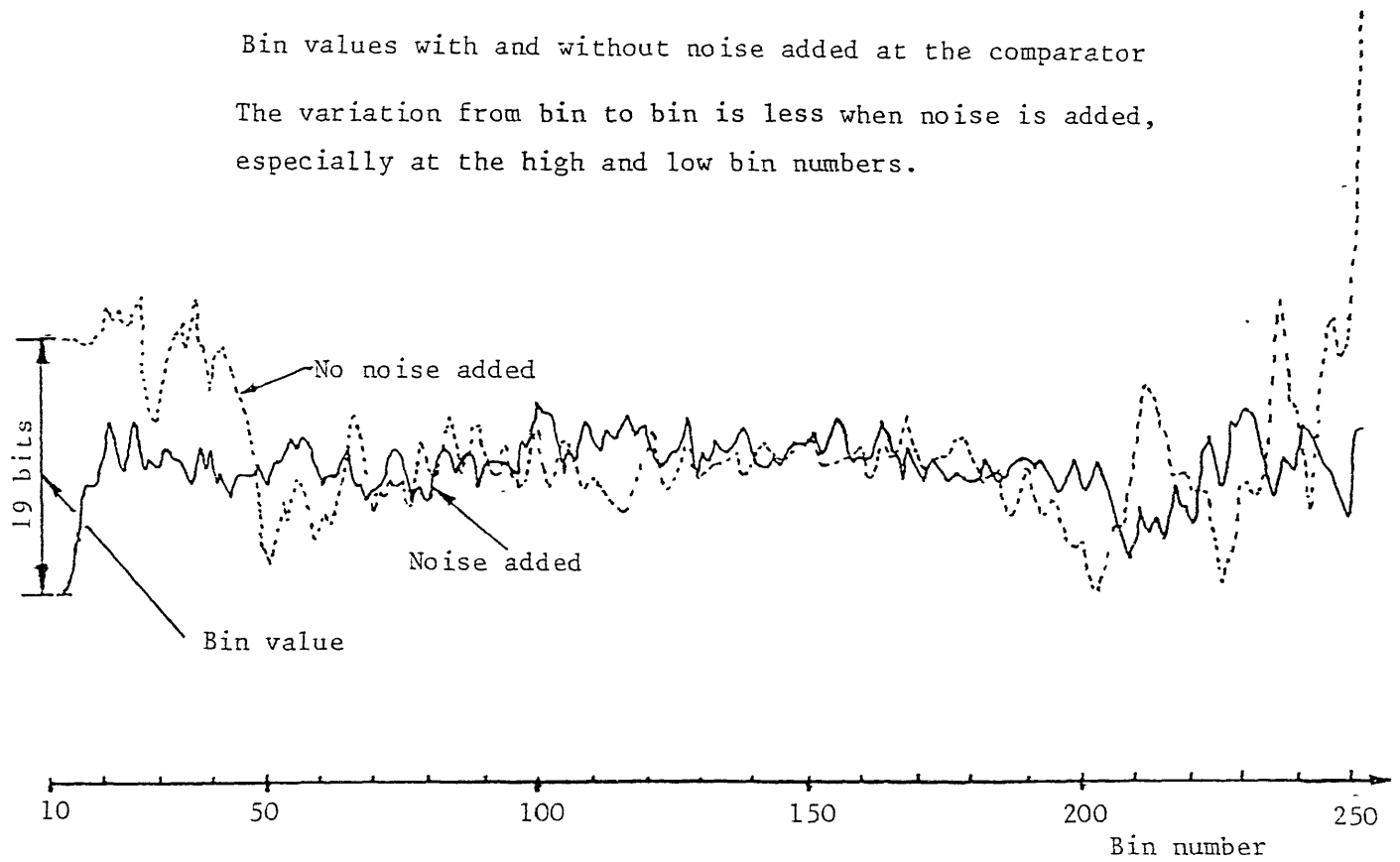
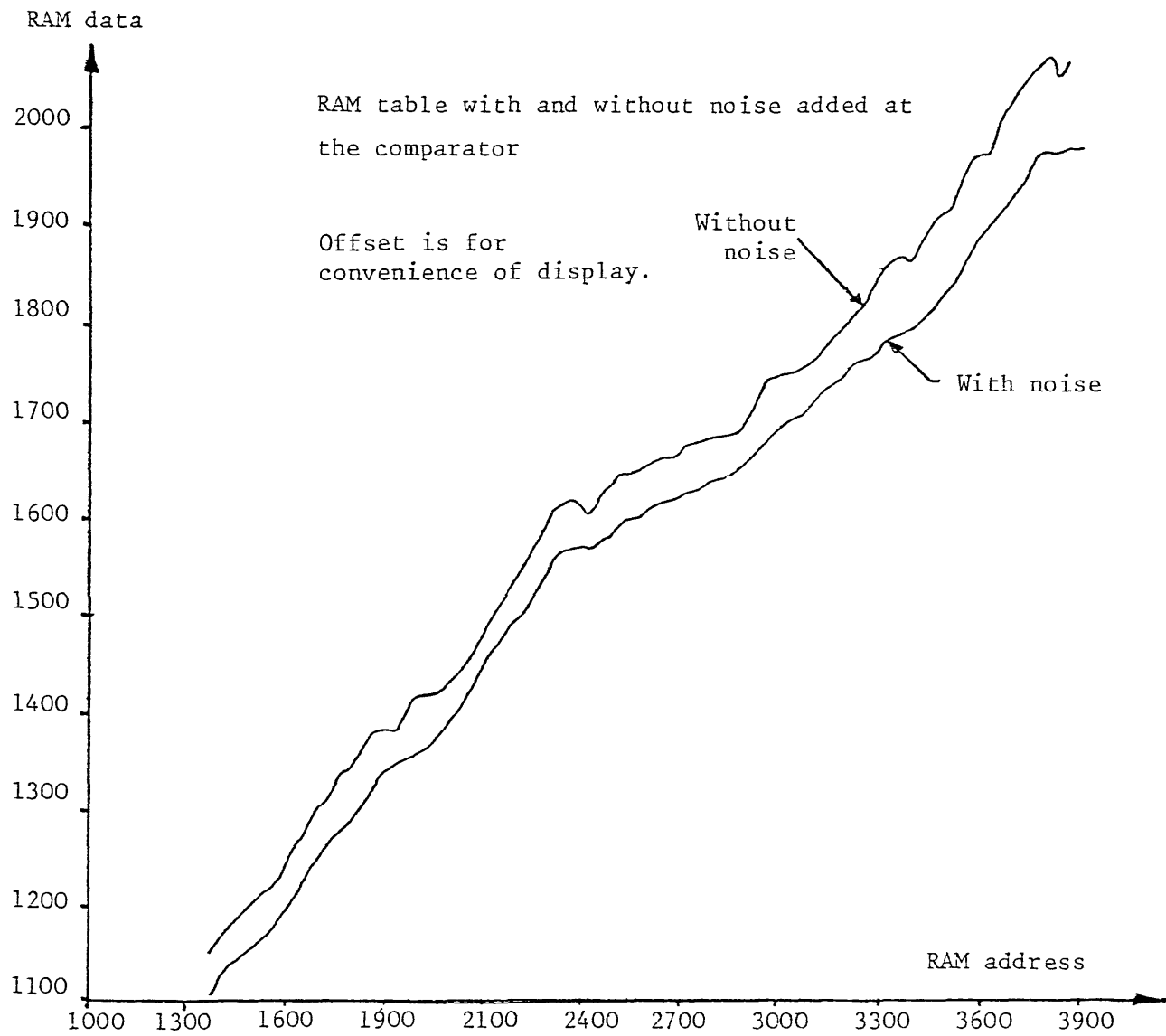


FIGURE 11.6.18



CHAPTER 12

CONCLUSIONS

12.1 CONCLUSIONS

This thesis has attempted to put digital compensation on a more rigorous basis than that used for conventional predistortion. The need for this is natural, given the far more powerful implementation of the NLFG that digital compensation brings. Conventional predistortion has to be on a "one-off" basis, because of the difficulty in constructing a NLFG which matches the DUT. Digital compensation however is far more flexible, and hence the appeal of a wider theoretical framework.

To these ends, the idea of the decision hierarchy stands as a central theme, relating what would otherwise be disparate implementations of digital compensation. Having constructed the hierarchy it is plain that not all implementations can be investigated, and so this thesis concentrated on polar, baseband AM systems, since these can be easily implemented using widely available monolithic A/D's and D/A's. It is important to bear in mind though, that the techniques developed here have wider application than AM systems.

The AM systems dealt with had problems in two categories, those common to all members of the decision hierarchy (effects of digital NLFG, type 1, 2, 3 implementation, stability, detector linearity), and those specific to AM systems (e.g. local oscillator leakthrough in the AM modulator). It is the first category which has been of more concern.

This thesis has tried to show that digital NLFG's do not

impose unacceptable noise, phase noise and bandwidth constraints. At the same time the control algorithms have been examined in some depth, and the mathematical basis of analysis of these algorithms put forward. This basis has included the development of two powerful theorems relating the commutivity of nonlinear functions (see Appendices G and K). This mathematical basis is only partially successful for type 3 algorithms, since general case stability has not been proved. It has been shown experimentally that type 1 and type 3 control algorithms give considerable suppression of intermodulation distortion, far beyond that reliably achievable by conventional predistortion. Type 2 control has proved less effective, although the results obtained in this study may be improved upon by further modifications to the implementation of the system. Digital compensation offers novel ways of overcoming noise and linearity problems in detectors, and the latter have been investigated in some depth, with excellent results.

Thus, it appears that digital compensation can provide effective, reliable, adaptive, predistortion with performance in excess of that reported for conventional predistortion. Digital compensation may most usefully be applied in those circumstances where conventional predistortion currently prevails. However, as technology reduces anti-alias filter imperfections to increasingly lower levels (e.g. by switched capacitor filters, or by far higher sampling rates), it is conceivable that digital compensation will offer a performance approaching that of feedback systems.

12.2 FUTURE WORK

Type 1 performance and theory are well understood, but there remain problems for type 3 algorithms. The lack of a general stability theorem, and an analysis of algorithm performance with non-periodic inputs, requires more work and may be more profitable than improving the type 2 system described.

Aside from the possibility of replacing existing predistortion systems, digital compensation would seem to have particular application to M-QAM. Both type 1 and type 3 control can find application to M-QAM systems with comparative ease. The techniques used in this study would not require the spectral shaping to be done at RF (since digital compensation handles analogue data), in contrast to the proposed cartesian feedback of reference 181, or the adaptive predistortion of reference 66. Thus, digital compensation offers significant advantages for M-QAM over other techniques. M-QAM also offers the possibility of intersymbol interference (ISI) free measurements, allowing useful variations of the methods developed here (e.g. making measurements at ISI free instants and using inverse interpolation to construct the full RAM table).

Thus it is hoped that this study will form the basis for further investigation and expansion of digital compensation techniques.

APPENDIX A

BANDWIDTH OF Mth ZONE HARMONIC CAUSED BY Nth ORDER DISTORTION

We consider the case where the input to the NLFG has the spectral width of B (Hz), a maximum frequency of $f_{\max}$ (Hz), and a minimum frequency of $f_{\min}$ (Hz).

$$\text{Thus} \quad f_{\max} - f_{\min} = B \quad (\text{A.1})$$

If the NLFG can be described by an odd polynomial of maximum order n , then the highest frequency content of the m^{th} zone harmonic is given by the largest value of

$$af_{\max} - bf_{\min} \quad (\text{A.2})$$

$$\text{where} \quad a - b = m \quad (\text{A.3})$$

$$\text{and} \quad a + b = n \quad (\text{A.4})$$

Equation A.3 is the condition to be in the m^{th} harmonic zone and equation A.4 the condition to be a product of n^{th} order distortion. Note that we require a and b to be positive integers. Thus it follows that if n is odd then m must also be odd and less than or equal to n . Solving equation A.3 and equation A.4 we get

$$a = (n + m)/2 \quad (\text{A.5})$$

$$b = (n - m)/2 \quad (\text{A.6})$$

The highest frequency component in the m^{th} zone is hence given by

$$(n + m)f_{\max}/2 - (n - m)f_{\min}/2 \quad (\text{A.7})$$

Similarly the lowest frequency is given by

$$(n + m)f_{\min}/2 - (n - m)f_{\max}/2 \quad (\text{A.8})$$

Hence the bandwidth (Hz) of the m^{th} zone due to the n^{th} order distortion is given by the difference between equation A.7 and equation A.8, i.e.

$$nf_{\max} - nf_{\min} = nB \quad (\text{A.9})$$

APPENDIX B

CHEBYSCHEV POLYNOMIALS

The following identities, concerning Chebyshev polynomials of the first kind, can be found in references 1, 14 and 54. Where these references conflict over definition of arbitrary constants of normalisation, the definition in reference 1 (Abramowitz and Stegun) always prevails. In computer manipulations of Chebyshev polynomials the work of references 9, 14, and 54 has been used.

$$T_0(x) = 1 \quad (B.1)$$

$$T_1(x) = x \quad (B.2)$$

$$T_2(x) = 2x^2 - 1 \quad (B.3)$$

$$T_3(x) = 4x^3 - 3x \quad (B.4)$$

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x) \quad (B.5)$$

$$T_n(x) = \cos(n(\arccos(x))) \quad \text{if } |x| \leq 1 \quad (B.6)$$

$$T_n(\cos(wt)) = \cos(nwt) \quad (B.7)$$

$$|T_n(x)| < 1 \quad \text{if } |x| < 1 \quad (B.8)$$

$$dT_n(x)/dx = n(T_n(x) - xT_{n-1}(x))/(1 - x^2) \quad (B.9)$$

$$dT_n(x)/dx = 2nT_{n-1}(x) + (n/(n-2))(dT_{n-2}(x)/dx) \quad (B.10)$$

$$|dT_n(x)/dx| < n^2 \quad (B.11)$$

$$T_n(T_m(x)) = T_{nm}(x) \quad (B.12)$$

$$2T_n(x)T_m(x) = T_{n+m}(x) - T_{n-m}(x) \quad (n > m) \quad (B.13)$$

The following identities concern Chebyshev polynomials of the second kind.

$$U_0(x) = 1 \quad (B.14)$$

$$U_1(x) = 2x \quad (B.15)$$

$$U_n(x) = 2xU_{n-1}(x) - U_{n-2}(x) \quad (B.16)$$

$$U_{n-1}(x) = \sin(nx)/\sin(x) \quad \text{if } |x| \leq 1 \quad (B.17)$$

$$U_n(\cos(x)) = \sin((n+1)x)/\sin(x) \quad (\text{B.18})$$

$$\sin(nx) = U_{n-1}(\cos(x))(1 - \cos^2(x))^{0.5} \quad (\text{B.19})$$

APPENDIX C

SPECTRAL SYMMETRY IN AM/AM AND AM/PM

We put forward a simple proof that AM/AM and AM/PM conversion give rise to a symmetric power spectrum, for a certain class of input functions. Consider the input envelope described by

$$A(t) = \sum_n a_n \cos(\omega_n t) \quad (C.1)$$

Hence $A(t)$ is an even function with respect to time. Let the output of the NBNL device ($V(t)$) be described by

$$V(t) = f(A) \cos(\omega t + g(A)) \quad (C.2)$$

$$\begin{aligned} &= f(A) (\cos(g(A)) \cos(\omega t) - \sin(g(A)) \sin(\omega t)) \\ &= X(A) \cos(\omega t) - Y(A) \sin(\omega t) \end{aligned} \quad (C.3)$$

where $X(A)$ and $Y(A)$ are even functions with respect to time if equation C.1 is satisfied. (We impose no conditions on the symmetry of $f(A)$ and $g(A)$ with respect to A). Since $X(A)$ and $Y(A)$ are even functions with respect to time we can write

$$X(A) = \sum_n x_n \cos(\omega_n t) \quad (C.4)$$

$$Y(A) = \sum_n y_n \cos(\omega_n t) \quad (C.5)$$

Substituting into equation C.3 gives

$$\begin{aligned} V(t) &= \sum_n [x_n \cos(\omega_n t) \cos(\omega t) - y_n \cos(\omega_n t) \sin(\omega t)] \\ &= 0.5 (\sum_n [x_n \cos((\omega - \omega_n)t) + x_n \cos((\omega + \omega_n)t) \\ &\quad + y_n \sin((\omega - \omega_n)t) + y_n \sin((\omega + \omega_n)t)]) \end{aligned} \quad (C.6)$$

from which we see that the magnitude of the m^{th} order upper and lower sidebands are given by S_m and S_{-m} respectively, where

$$S_m = 0.5((x_m)^2 + (y_m)^2)^{0.5} \quad (C.7)$$

$$\text{and} \quad S_{-m} = 0.5((x_{-m})^2 + (y_{-m})^2)^{0.5} \quad (C.8)$$

Thus it is apparent from equation C.7 and equation C.8 that the

power spectrum should be symmetric provided equation C.1 and equation C.2 are satisfied. An important case of equation C.1 is when

$$A(t) = a_0 + a_1 \cos(\omega t) \quad (C.9)$$

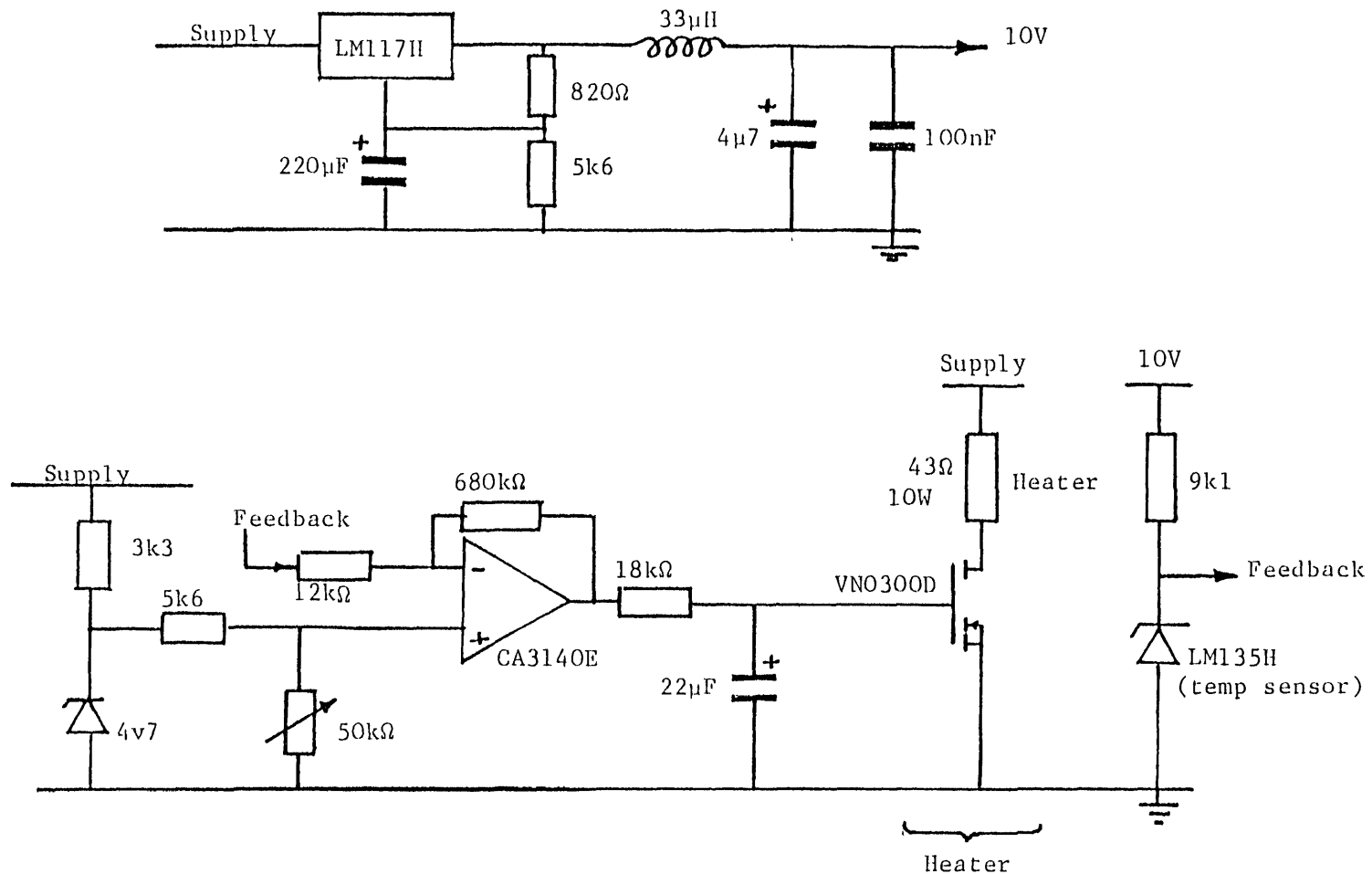
and an AM signal results. This is usually the case with the test signals used in this thesis. The more commonly used two-tone test can also be shown to satisfy equation C.1, and so the power spectrum is symmetric for two tone tests.

APPENDIX D

EXPERIMENTAL DETAILS

To reduce thermal problems the detectors in this study were enclosed in a proportionally controlled oven, the details of which are shown in Figure D.1.

The envelope detectors used in this study are shown in Figure D.2. The operating frequency is narrowband, centred on 80 MHz.



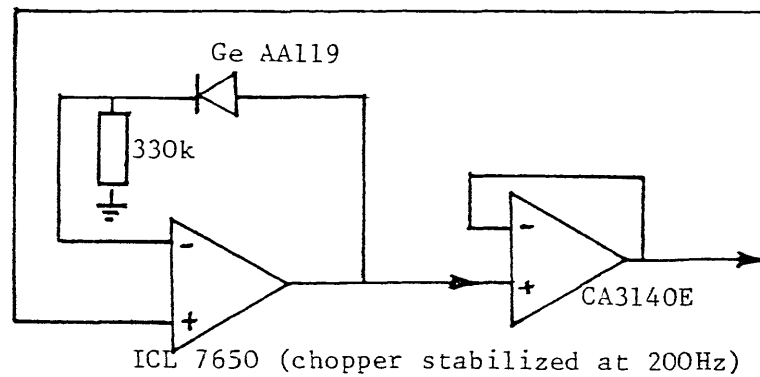
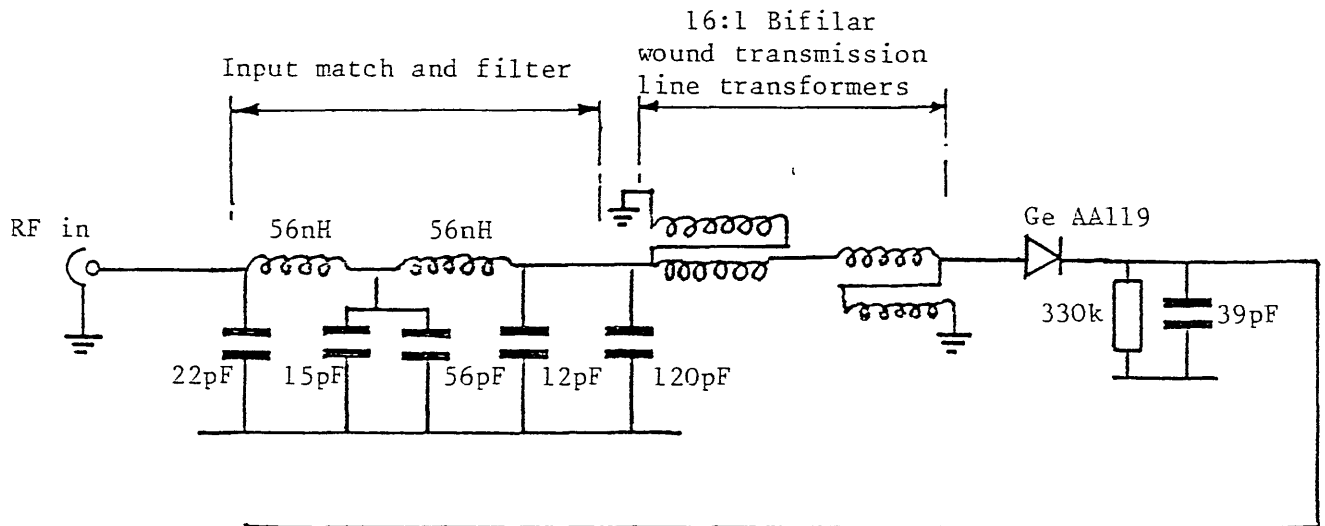
Details of constant temperature heating/control circuit

The circuit, when properly packaged, constitutes a proportionally controlled oven. Both heater and temperature sensor are mounted close to the temperature sensitive elements of the detector. The PCB is then put on spacers in the centre of a polystyrene filled enclosure.

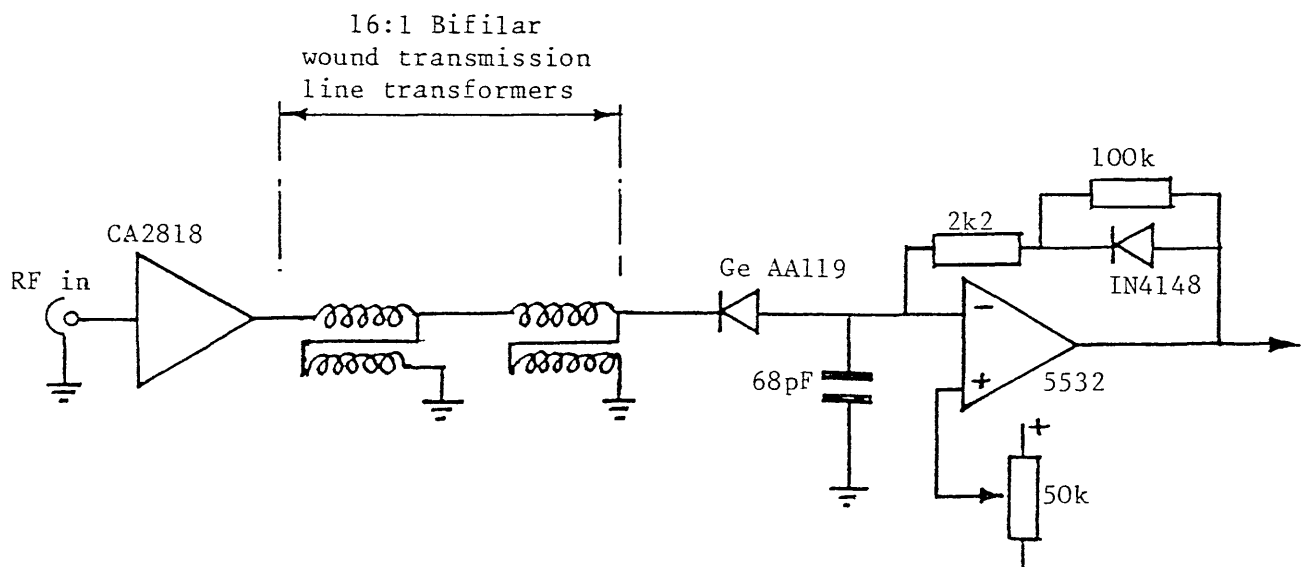
FIGURE D.1

FIGURE D.2

Circuit details of envelope detectors used
 Detectors were operated in the constant temperature enclosure.



Diode type Ge



APPENDIX E

LEAKTHROUGH AND QUASI-LINEAR

E.1 LOW LEAKTHROUGH ENVELOPE MODULATOR

The quasi-linear experiment of Figure 7.6.10 relies on the envelope modulator having low LO leakthrough. As discussed in chapter 5, leakthrough results in the envelope spectra being distorted, and makes it impossible to produce an envelope of the form

$$(1 + m\cos(w_a t)) \quad (E.1)$$

whilst producing a RF spectrum given by

$$(1 + m\cos(w_a t))\cos(w t + \phi) \quad (E.2)$$

Thus, for the experiment shown in Figure 7.6.10, a cascade of two pin diode attenuator modules (MCL PAS-3 double balanced pin attenuators), were used to keep leakthrough at a level compatible with the noise floor of the A/D.

E.2 SETTING RAM 2

The following program can act either on attenuator produced data, or envelope quasi-linear data, to produce the necessary data for RAM 2 in Figure 7.6.10. The program produces a hexadecimal array of numbers suitable for use as data for a 4096*8 bit PROM.

```

INTEGER W(0:4095),A,D,FINI
INTEGER F1(0:4095),F2(0:4095),F3(0:4095)
REAL K,V(23),M

C THE V ARRAY CONTAINS THE CORRECTION POINTS FROM THE
C ATTENUATOR MEASUREMENTS
```

DATA V(1)/4072.5/

DATA V(2)/2752.5/

DATA V(3)/1863.5/

DATA V(4)/1284.5/

DATA V(5)/889.5/

DATA V(6)/621.5/

DATA V(7)/440.5/

DATA V(8)/316.5/

DATA V(9)/230.5/

DATA V(10)/170.5/

DATA V(11)/130.5/

DATA V(12)/98.5/

DATA V(13)/74.5/

DATA V(14)/56.5/

DATA V(15)/40.5/

DATA V(16)/27.0/

DATA V(17)/16.5/

DATA V(18)/8.5/

DATA V(19)/4.5/

DATA V(20)/2.5/

DATA V(21)/2.0/

DATA V(22)/0.5/

DATA V(23)/0.0/

C K IS THE VALUE SET BY THE ATTENUATOR

DATA K/.720328/

DATA A,FINI/1,1/

C SET UP A FILE FORMATED FOR TRANSFER TO PROM OF THE

C DETECTOR CHARACTERISTICS

```

      CHARACTER*50 FILNAME

      WRITE(6,100)

100  FORMAT(' ENTER THE NAME OF THE OUTPUT FILE ')

      READ(5,200)FILNAME

200  FORMAT(A50)

      OPEN(UNIT=2,CARRIAGECONTROL='LIST',FILE=FILENAME,STATUS='NEW')

      WRITE(2,300)

300  FORMAT('!M0000 ',')

      C USING THE V ARRAY WE LINEARLY INTERPOLATE TO DEDUCE THE
      C DIODE CHARACTERISTIC

      DO N=4095,0,-1

      IF(N.GT.V(1))D=INT(V(1)+(N-V(1))*(V(1)-V(2))/(V(1)*(1-K)))

      IF(N.GT.V(1))GOTO 250

      IF(N.LT.V(1)*(K**A))A=A+1

      IF(FINI.EQ.2)GOTO 240

      M=V(A)-V(A+1)

      M=M/(V(1)*(K**(A-1))-V(1)*(K**A))

      D=INT(V(A+1)+M*(N-V(1)*(K**A)))

240  IF(D.LE.0)FINI=2

      IF(D.LT.0)D=0

250  F1(N)=D

      END DO

260  CONTINUE

      C THE ARRAY F1 NOW CONTAINS THE DIODE CHARACTERISTIC
      C I.E. F1(A) IS THE DIODE OUTPUT IF THE INPUT ENVELOPE IS
      C OF VALUE A. WE NOW SHOW SOME EXAMPLES OF APPLYING
      C ENVELOPE QUASI-LINEAR TO THE PROM.

```

```

C ASSUME WE WISH TO INTRODUCE A 4TH AUDIO HARMONIC
C AS A SIDEBAND -47DB ON FUNDAMENTAL SIDEBAND
C ASSUME THE SIGN OF THE HARMONIC IS POSITIVE
C F2(N) IS THE RESULT OF APPLYING QUASI-LINEAR
  DO N=4095,0,-1
    M=ACOS((N-2048)/2048.1)
    D=INT(N+9.148*COS(4*M))
    IF(D.GT.4095)D=4095
    IF(D.LT.0)D=0
    F2(N)=F1(D)
  END DO

C ASSUME WE WISH TO USE ENVELOPE QUASI-LINEAR
C TO PRODUCE A 3RD HARMONIC AUDIO SIDEBAND, -45DB
C ON FUNDAMENTAL, OF POSITIVE SIGN
C F3(N) IS THE RESULT OF APPLYING QUASI-LINEAR
  DO N=4095,0,-1
    M=ACOS((N-2048)/2048.1)
    D=INT(N+11.516*COS(3*M))
    IF(D.GT.4095)D=4095
    IF(D.LT.0)D=0
    F3(N)=F2(D)
  END DO

C WRITE F3 ARRAY TO FILE SUITABLE FOR DOWN-LOADING TO PROM
C W(N)=(W(N)-INT(W(N)/16)*16)*16 TRANSFERS THE LOWER 4 BITS
C TO A PROM
C W(N)=INT(W(N)/16) WOULD TRANSFER THE UPPER 8 BITS TO A PROM
  DO N=4095,0,-1
    W(N)=F3(N)

```

```
      IF(W(N).GT.4095)W(N)=4095  
      IF(W(N).LT.0)W(N)=0  
      W(N)=(W(N)-INT(W(N)/16)*16)*16  
  
      END DO  
  
      WRITE(2,400)(W(I),I=0,4095)  
400  FORMAT(40Z2.2,',')  
  
      END DO  
  
      STOP  
  
      END
```


APPENDIX F

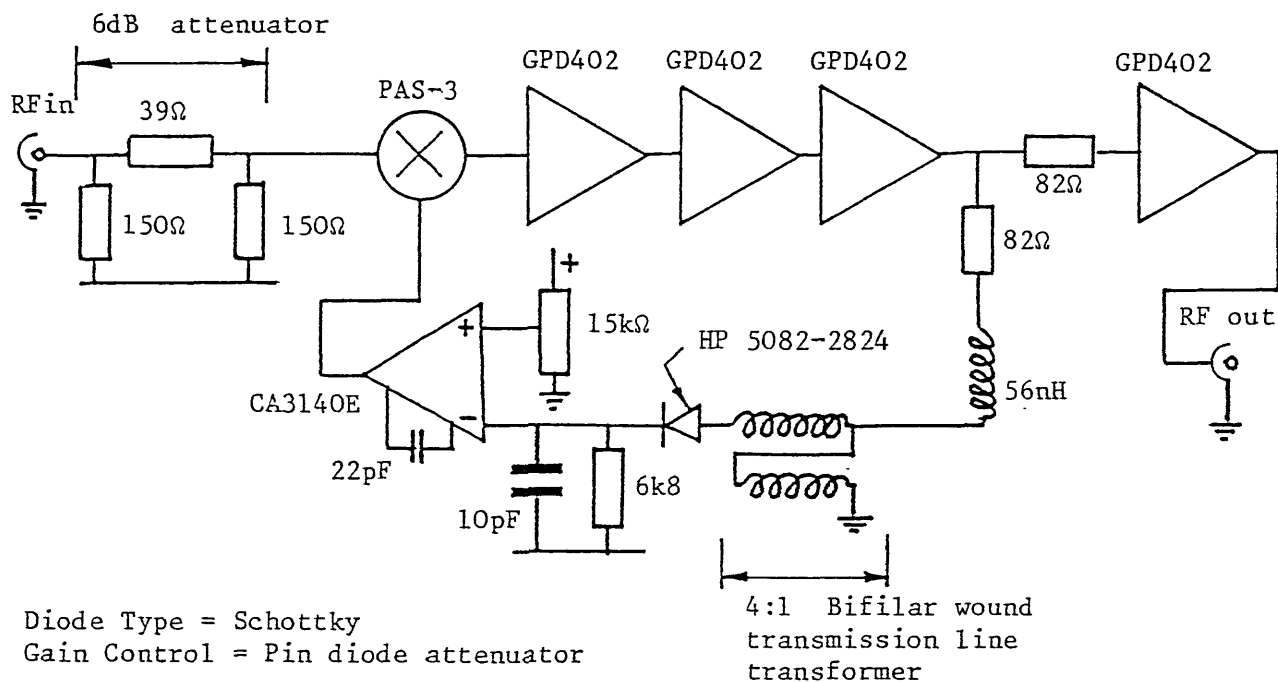
EXPERIMENTAL DETAILS

The details of the two types of limiter built for this study are shown in Figure F.1. Both operated at 80 MHz and were designed for narrowband operation. Temperature induced variations were dealt with by the oven circuit of Figure D.1.

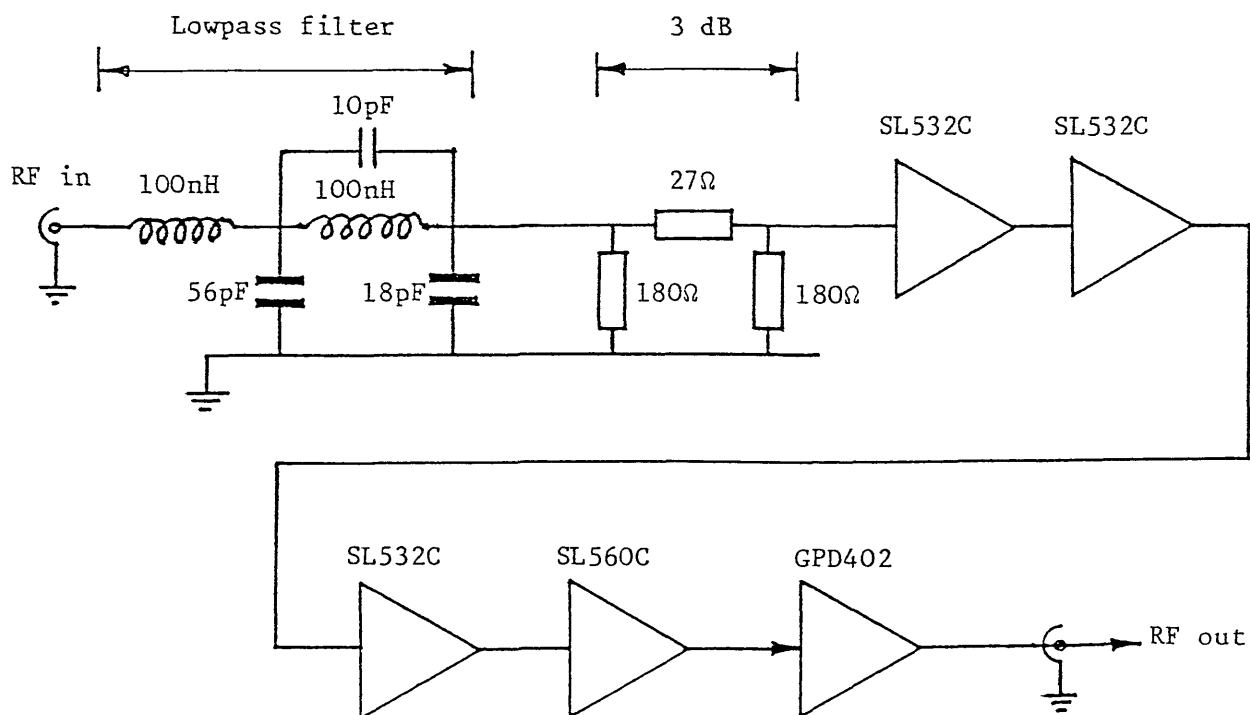
FIGURE F.1

Details of the AGC limiter used

Operating frequency 80 MHz, the AGC control device (PAS3) is a pin diode attenuator.

Details of the amplifier limiter used

The Plessey SL532C is recommended for low AM/PM.



APPENDIX G

COMMUTATION OF NONLINEAR FUNCTIONS AND CONVOLUTION

Theorem

If a nonlinear function $f(x)$ can be described by a polynomial series over the range of interest, and if $x(t)$ is a sampled data waveform whose aliased components do not overlap, then for the perfect reconstruction filter described by $r(t)*h(t)$, we can show that

$$f[x(t)*r(t)*h(t)] = f[x(t)]*r(t)*h(t) \quad (G.1)$$

provided that $f[x(t)*r(t)*h(t)]$ is bandlimited to less than half the sample rate.

Proof

$$\text{Let} \quad f(x) = \sum_n a_n x^n \quad (G.2)$$

$$\text{and} \quad x(t) = w(t) \left(\sum_i \delta(t - i) \right) \quad (G.3)$$

where the spectrum of $w(t)$ is bandlimited to 0.5 Hz to satisfy our criterion for non-overlapping aliased products. We have arbitrarily set the sampling period to unity, but clearly there is no loss of generality in so doing.

$$\begin{aligned} \text{Thus } r(t) &= 1 \quad \text{if } |t| < 1 \\ &= 0 \quad \text{otherwise} \end{aligned} \quad (G.4)$$

$$\text{Now} \quad f[x(t)*r(t)*h(t)] = f(x*r*h) = \sum_n a_n (x*r*h)^n \quad (G.5)$$

We shall denote Fourier transforms by capitals, thus the Fourier transform of $x(t)$ will be denoted by $X(f)$ or more simply by X .

Taking the Fourier transform of equation G.5 and applying the convolution theorem we get

$$F = a_0 \delta(f) + a_1 (XRH) + a_2 (XRH)*(XRH) + a_3 (XRH)*(XRH)*(XRH) + \dots \quad (G.6)$$

But by definition of a perfect reconstruction filter RH represents a brick wall filter of cutoff frequency 0.5 Hz. Thus by the definition of x we have that

$$XRH = W \quad (G.7)$$

$$\text{Hence} \quad F = a_0 \delta(f) + a_1 W + a_2 W*W + a_3 W*W*W + \dots \quad (G.8)$$

But if F is bandlimited to 0.5 Hz we can trivially write

$$F = [F * (\sum_i \delta(f - i))]RH \quad (G.9)$$

Thus equation G.8 becomes

$$\begin{aligned} F = & a_0 \delta(f) * (\sum_i \delta(f - i))RH + a_1 (W * (\sum_i \delta(f - i)))RH \\ & + a_2 (W*W * (\sum_i \delta(f - i)))RH + \dots \end{aligned} \quad (G.10)$$

Taking the inverse Fourier transform and applying the convolution theorem we obtain

$$f(x*r*h) = a_0 *r*h + a_1 (w(\sum_i \delta(t - i)) *r*h + a_2 (w^2(\sum_i \delta(t - i)) *r*h + \dots \quad (G.11)$$

We now note that, for any $n \geq 1$

$$\sum_i \delta(t - i) = (\sum_i \delta(t - i))^n \quad (G.12)$$

and thus we have that equation G.11 becomes

$$\begin{aligned} f(x*r*h) &= a_0 *r*h + a_1 (w(\sum_i \delta(t - i)) *r*h + a_2 (w(\sum_i \delta(t - i))^2 *r*h + \dots \\ &= a_0 *r*h + a_1 x*r*h + a_2 x^2 *r*h + a_3 x^3 *r*h + \dots \\ &= f(x) *r*h \end{aligned} \quad (G.13)$$

Application to type 1 algorithms

In connection with type 1 algorithms the theorem just proved imposes two requirements:

1. That the D/A output does not have overlapping aliased products.
2. That the series NLFG and DUT result in an output of bandwidth of less than half the sample rate.

The latter requirement is invariably justified, since the NLFG is

set up to give a linear (i.e. bandwidth invariant) system when in series with the DUT. Thus, in practice, the theorem simply requires that the D/A output be free from overlapping aliased products, and that the D/A be implementing the inverse function to $f(x)$ in a point-wise sense.

APPENDIX H

STATISTICS OF RANDOM SAMPLING

For type 2 and type 3 control algorithms the information necessary to modify RAM entries is gathered on a random basis. Rather than modify the RAM table as and when information concerning that entry is received, a large number of measurements are made, and then all entries are modified simultaneously. The question then arises as to the total number of measurements that must be made before there is reasonable certainty that all entries have been modified.

The problem may be solved using a Markov chain approach, but a simpler derivation of the necessary result is given here.

Suppose that there are N RAM entries. Assume that there is an equal probability for each entry that a measurement will enable a decision on modification to be made. Initially all entries require modification.

After 1 measurement the probability that an entry cannot be modified is

$$(N - 1)/N \quad (H.1)$$

After A measurements the probability that an entry cannot be modified is

$$((N - 1)/N)^A \quad (H.2)$$

and thus the average number of entries which cannot be modified (Q) is given by

$$Q = N((N - 1)/N)^A \quad (H.3)$$

We shall demand that the total number of measurements that we need to make before updating the entire RAM table is given by requiring

that Q is approximately equal to 1. Thus

$$1 = N((N - 1)/N)^A$$

$$\text{i.e.} \quad 0 = \log_e(N) + A \log_e((N - 1)/N) \quad (\text{H.4})$$

Now $\log_e((N-1)/N) = \log_e(1-1/N)$ and this is approximately equal to $(-1/N)$ for N large by the well-known power series expansion of $\log_e(1+x)$. Thus from equation H.4

$$0 = \log_e(N) - A/N$$

$$\text{i.e.} \quad A = N \log_e(N) \quad (\text{H.5})$$

Our result in the context of type 2 and type 3 control algorithms is as follows. Given N , and given that we shall update the RAM table on a batch basis, we expect to have to make $N \log_e(N)$ measurements before we are in a position to have enough information about the linearity of the system to decide how each RAM entry may be modified. In practice the assumption of "all equally likely" does not hold, but equation H.5 still provides a good estimate of the minimum number of measurements that need to be made. In practice any RAM entries which still cannot be modified once the measurements have been made will rely on interpolation from the rest of the RAM table, but for this to be accurate we need reasonable certainty that the rest of the RAM table is "error free", and so again we are forced to follow equation H.5.

APPENDIX I

COMPUTER SIMULATION OF FILTER HYSTERESIS

The program listed below (written to run on a VAX 750) contains subroutines capable of simulating the input/output transfer characteristics across a filter, and displaying these characteristics, displaying the average value of the characteristic, or of displaying the perceived average using a comparator/noise source approach.

```

INTEGER L,NUM,SUM,RX,RY,MARK,DEST,VAL,I
REAL X,Y

DATA MARK,DEST,I/1HO,12,999/

COMMON L(40,23),SUM(40),NUM(40),VAL(40,2)

C VAL(N,1) CONTAINS THE SUMMATION OF UP/DOWN COUNTS
C VAL(N,2) CONTAINS THE NUMBER OF TIMES N HAS BEEN SELECTED
C DEST IS THE DESTINATION FOR THE OUTPUT (VDU IS 6)
C INITIALISE THE PRINTED GRAPH

CALL BLANK

C INITIALISE NUM,SUM,VAL

CALL ZERO

C CALCULATE WHAT OCCURS AS TIME PASSES

DO T=0,2000,1

X=20+20*COS(T)

Y=8+0.7*(23*((X/40)**3)-4*COS(3*T+20))

RX=NINT(X)

RY=NINT(Y)

IF(RY.GE.1.AND.RY.LE.23)L(RX,RY)=MARK

```



```

      IF(RY.GT.12)VAL(RX,1)=VAL(RX,1)-1
      IF(RY.LE.12)VAL(RX,1)=VAL(RX,1)+1
      VAL(RX,2)=VAL(RX,2)+1
      SUM(RX)=SUM(RX)+RY
      NUM(RX)=NUM(RX)+1
      END DO
      CALL PRINT(DEST)
      CALL BLANK
      CALL AVERAGE
      CALL PRINT(DEST)
      STOP
      END
C SUBROUTINE BLANK SETS THE GRAPH ARRAY TO JUST THE Y=0 AXIS
      SUBROUTINE BLANK
      COMMON L(40,23),SUM(40),NUM(40),VAL(40,2)
      INTEGER EMPTY,PLUS,L,SUM,NUM,VAL
      DATA EMPTY,PLUS/1H ,1H+/
      DO N=1,23,1
      DO M=1,40,1
      L(M,N)=EMPTY
      IF(N.EQ.1)L(M,N)=PLUS
      IF(M.EQ.1)L(M,N)=PLUS
      END DO
      END DO
      RETURN
      END
C SUBROUTINE ZERO SETS NUM,SUM,VAL=0
      SUBROUTINE ZERO

```

```

COMMON L(40,23),SUM(40),NUM(40),VAL(40,2)

INTEGER L,SUM,NUM,VAL

DO N=1,40,1

SUM(N)=0

VAL(N,1)=12

VAL(N,2)=0

NUM(N)=0

END DO

RETURN

END

C SUBROUTINE AVERAGE CALCULATES THE AVERAGE VALUE OF Y AT
C POSITION X AND PUTS THIS DATA INTO THE GRAPHICAL ARRAY

SUBROUTINE AVERAGE

COMMON L(40,23),SUM(40),NUM(40),VAL(40,2)

INTEGER CROSS,L,NUM,SUM,VAL

DATA CROSS/1HX/

DO N=1,40,1

IF(NUM(N).GT.0)L(N,INT(SUM(N)/NUM(N)))=CROSS

END DO

RETURN

END

C SUBROUTINE ESTIMATE TAKES THE INFORMATION IN VAL AND MAKES
C AN ESTIMATE OF THE ERROR. THIS INFORMATION IS PUT INTO
C L(X,Y) AND MAY BE GRAPHICALLY DISPLAYED

SUBROUTINE ESTIMATE

COMMON L(40,23),SUM(40),NUM(40),VAL(40,2)

INTEGER HASH,L,NUM,SUM,VAL,Q

DATA HASH/1HE/

```

```

DO N=1,40,1

Q=100

IF(VAL(N,2).NE.0)Q=NINT(8.0*VAL(N,1)/VAL(N,2))+12

IF(Q.GE.1.AND.Q.LE.23)L(N,Q)=HASH

END DO

RETURN

END

C SUBROUTINE PRINT PRINTS THE GRAPHICAL ARRAY AT DESTINATION
C DEST.  THE ARRAY IS 40*23 (X*Y)

SUBROUTINE PRINT(DEST)

COMMON L(40,23),SUM(40),NUM(40),VAL(40,2)

INTEGER L,SUM,NUM,W(40),DEST,VAL

DO N=23,1,-1

DO M=1,40

W(M)=L(M,N)

END DO

WRITE(DEST,10)W

10 FORMAT(1H ,40A1)

END DO

RETURN

END

```

APPENDIX J

COMPUTER SIMULATION OF TYPE 3 CONTROL

The following program contains the subroutines necessary for the matrix description of a type 3 system, i.e. cascade matrices, matrices which perform differentiation, and matrices which perform multiplication of functions described solely by Chebyshev coefficients. When we need to consider multiplication of functions (e.g. $f(x)(g(x))$) it is useful to use

$$2T_n(x)T_m(x) = T_{n+m}(x) + T_{n-m}(x) \quad (n > m) \quad (J.1)$$

The basis for the fast Chebyshev computation routines can be found in references 9, 14 and 54.

The program trivially handles imperfect anti-alias filters, and specified RAM filters (the latter being specified in the Chebyshev domain). The main program statements illustrating the operation of the subroutines presented below are given in Figures 11.5.3, 11.5.7, 11.5.9, 11.5.10, 11.6.6.

```
COMMON TRAN(0:15,0:15),T(0:15,0:15),TD2(0:15,0:15),M(0:15)
COMMON MC(0:15,0:15),INVMC(0:15,0:15),D(0:15,0:15)
REAL TRAN,T,TD2,VEC1(0:15),VEC2(0:15),VEC3(0:15),RAT(0:15)
REAL DUT(0:15),TOT(0:15),PG,SPA(0:15),NUM,M,MC,INVMC,D
INTEGER SIZE,DES,VAL,B
DATA DES,SIZE,PG,VAL/10,10,1.0,9999/
```

```
+++++
+ Insert main program statements here +
+++++
```

```

C SUBROUTINE DISP DISPLAYS THE MAIN PROGRAM BY WRITING IT TO
C DESTINATION DES.  THIS IS OFTEN USEFUL AS A RECORD OF THE
C DATA WHICH PRODUCED THE RESULTS

SUBROUTINE DISP(DES)

INTEGER DES,FLAG

CHARACTER*80 IN

OPEN(3,STATUS='OLD',FILE='TSC.FOR')

FLAG=0

WRITE(DES,101)
101 FORMAT(1H1)
111 CONTINUE

READ(3,100) IN

IF(INDEX(IN,'STOP').EQ.0) THEN

WRITE(DES,100) IN

ELSE

WRITE(DES,100) IN

READ(3,100) IN

WRITE(DES,100) IN

FLAG=1

END IF

IF(FLAG.EQ.1) GOTO 112

GOTO 111

112 CONTINUE

100 FORMAT(A80)

RETURN

END

```

```

C SUBROUTINE VH DISPLAYS A HEADING ASSOCIATED WITH DISPLAYING
C A VECTOR (SEE DISV).  THE DESTINATION OF THE HEADING IS
C DES, SIZE IS THE HEADING SIZE
      SUBROUTINE VH(DES,SIZE)
      INTEGER DES,SIZE
      WRITE(DES,80)(J,J=0,SIZE)
80  FORMAT(1H ,8X,16(I2,9X))
      RETURN
      END

C SUBROUTINE DISV DISPLAYS THE VECTOR VEC(0:15) AND WRITES IT
C AT DESTINATION DES, SHOWING UP TO VEC(SIZE).  FOR A LINE
C PRINTER SIZE=10, FOR A VDU SIZE=6.  ALSO PRINTED IS THE ID
C (IDENTIFIER) PASSED FROM THE CALLING PROGRAM
      SUBROUTINE DISV(VEC,DES,SIZE,ID)
      REAL VEC(0:15)
      INTEGER DES,SIZE,ID
      WRITE(DES,90)ID,(VEC(I),I=0,SIZE)
90  FORMAT(1H ,I3,1X,16(E9.2,2X))
      RETURN
      END

C SUBROUTINE DISM DISPLAYS THE MATRIX CALLED MAT(0:15,0:15)
C AND WRITES IT AT DESTINATION DES, SHOWING UP TO
C MAT(SIZE,SIZE).  FOR A LINE PRINTER SIZE=10, FOR VDU
C SIZE=6
      SUBROUTINE DISM(MAT,DES,SIZE)
      REAL MAT(0:15,0:15)
      INTEGER DES,SIZE
      WRITE(DES,100)(J,J=0,SIZE)

```

```

      DO I=0,SIZE
      WRITE(DES,110)I,(MAT(I,J),J=0,SIZE)

      END DO

100 FORMAT(1H ,8X,16(I2,9X))
110 FORMAT(1H ,I2,1X,16(F9.2,2X))

      RETURN

      END

C SUBROUTINE IT INTIALISES CERTAIN MATRICES SPECIFICALLY TRAN
C (THE MATRIX TRAN IS AS DESCRIBED IN ASKEY "THEORY AND
C APPLICATION OF SPECIAL FUNCTIONS" [REF 9])

      SUBROUTINE IT

      COMMON TRAN(0:15,0:15),T(0:15,0:15),TD2(0:15,0:15),M(0:15)
      COMMON MC(0:15,0:15),INVMC(0:15,0:15),D(0:15,0:15)
      REAL TRAN,T,TD2,VEC1(0:15),VEC2(0:15),VEC3(0:15),RAT(0:15)
      REAL DUT(0:15),TOT(0:15),PG,SPA(0:15),NUM,M,MC,INVMC,D
      INTEGER SIZE,DES,VAL,B
      REAL DUM(0:15,0:15)

C TRAN IS A MATRIX WHICH TRANSLATES A POLYNOMIAL VECTOR INTO
C A CHEBSYCHEV VECTOR

      DO I=0,15

      DO J=0,15

      TRAN(I,J)=0

      IF(I.EQ.0.AND.ABS(INT(J/2)*2-J).LE.0.1)

1 TRAN(I,J)=(2.0**(-J))*FAC(J,J/2)

      IF(I.NE.0.AND.ABS(INT((I+J)/2)*2-(I+J)).LE.0.1.AND.I.LE.J)

1 TRAN(I,J)=(2.0**(1-J))*FAC(J,(J-I)/2)

      END DO

      END DO

```

```

C THIS SUBROUTINE ALSO CONSTRUCTS THE FIRST 15 CHEBYSCHV
C POLYNOMIALS E.G. T(-,N) CONTAINS THE VECTOR WHICH IS THE
C POLYNOMIAL EQUIVALENT TO  $T_N$  THE MATRIX IS SET UP USING THE
C RECURSION FORMULA  $T_{N+1}=2*X*T_N-T_{N-1}$ 

DO I=0,15

DO N=0,15

T(I,N)=0

DUM(I,N)=0

IF(N+1.EQ.I)DUM(I,N)=2.0

END DO

END DO

T(0,0)=1

T(1,1)=1

DO N=1,14

DO I=0,15

DO J=0,15

T(I,N+1)=DUM(I,J)*T(J,N)+T(I,N+1)

END DO

T(I,N+1)=T(I,N+1)-T(I,N-1)

END DO

END DO

C FORM THE D(15,15) MATRIX WHICH ACTS TO DIFFERENTIATE A
C POLYNOMIAL VECTOR

DO I=0,15

DO J=0,15

D(I,J)=0

IF(I+1.EQ.J)D(I,J)=J*1.0

END DO

```



```

      END DO

      C WE NOW FORM TD2, A MATRIX WHICH WHEN ACTING ON A CHEBYSCHV
      C VECTOR RETURNS ANOTHER CHEBYSCHV VECTOR, WHICH IS
      C EQUIVALENT TO  $d^2( )/dx^2$ 

      DO N=0,15

      DO I=0,15

      VEC1(I)=T(I,N)

      END DO

      CALL MULT(VEC1,VEC2,D)

      CALL MULT(VEC2,VEC1,D)

      CALL MULT(VEC1,VEC2,TRAN)

      DO I=0,15

      TD2(I,N)=VEC2(I)

      END DO

      END DO

      C WE FORM THE CASCADE MATRIX EQUIVALENT TO M(I)

      CALL MCAS(MC,M)

      RETURN

      END

      C SUBROUTINE MULT TAKES THE POLYNOMIAL VECTOR POLY AND FORMS
      C THE RESULT VECTOR RES EQUIVALENT TO MATRIX*POLY. ALL
      C VECTOR SIZES ARE 0:15. MATRIX SIZE MUST BE (0-15)*(0-15)

      SUBROUTINE MULT(POLY,RES,MAT)

      REAL POLY(0:15),RES(0:15),MAT(0:15,0:15)

      DO I=0,15

      RES(I)=0

      DO J=0,15

      RES(I)=MAT(I,J)*POLY(J)+RES(I)

```

```

END DO

END DO

RETURN

END

C SUBROUTINE CAS CASCADES A CHEBYSCHV POLYNOMIAL TSC1 WITH
C TSC2 AND PUTS THE ANSWER IN TSC3 I.E. CASCADE IS
C  $\text{SIGMA}(\text{TSC1}(\text{N}) * \text{T}_\text{N}(\text{SIGMA}(\text{TSC2}(\text{K}) * \text{T}_\text{K}))) = \text{SIGMA}(\text{TSC3}(\text{J}) * \text{T}_\text{J})$ . WE
C MAKE USE OF THE RECURSIVE EXPANSION OF CHEBYSCHV
C POLYNOMIALS

SUBROUTINE CAS(TSC1,TSC2,TSC3)

REAL TSC1(0:15),TSC2(0:15),TSC3(0:15),DUM(0:15,0:15)

CALL MCAS(DUM,TSC2)

CALL MULT(TSC1,TSC3,DUM)

RETURN

END

C SUBROUTINE MCAS FORMS THE CASCADE MATRIX DUM FROM THE
C CHEBYSCHV VECTOR TSC2

SUBROUTINE MCAS(DUM,TSC2)

REAL DUM(0:15,0:15),TSC2(0:15),S

INTEGER Q

DO I=0,15

DO J=0,15

DUM(I,J)=0

IF(I.EQ.0.AND.J.EQ.0)DUM(I,J)=1.0

IF(J.EQ.1)DUM(I,J)=TSC2(I)

END DO

END DO

DO N=2,15

```

```

DO J=0,15
DO K=0,15
S=DUM(K,N-1)*TSC2(J)
DUM(ABS(J-K),N)=S+DUM(ABS(J-K),N)
Q=J+K
IF(Q.GT.15)GOTO 3
DUM(Q,N)=S+DUM(Q,N)
3 CONTINUE
END DO
END DO
DO I=0,15
DUM(I,N)=DUM(I,N)-DUM(I,N-2)
END DO
END DO
RETURN
END

C SUBROUTINE TIM MULTIPLIES TWO CHEBYSCHV VECTORS. WE MAKE
C USE OF  $2T_I T_J = T_{I+J} + T_{I-J}$  ( $I > J$ )
SUBROUTINE TIM(TSC1,TSC2,TSC3)
REAL TSC1(0:15),TSC2(0:15),TSC3(0:15),Q,S
DO I=0,15
TSC3(I)=0
END DO
DO J=0,15
DO K=0,15
S=TSC1(J)*TSC2(K)*.5
TSC3(ABS(J-K))=S+TSC3(ABS(J-K))
Q=J+K

```

```

      IF(Q.GT.15)GOTO 4
      TSC3(Q)=S+TSC3(Q)
4  CONTINUE
      END DO
      END DO
      RETURN
      END
C SUBROUTINE FILTER TAKES A CHEBYSCHEV VECTOR TSC1, MODIFIES
C THE Nth COMPONENT BY MAG(HN)*COS(THETAN), AND CASCADES WITH
C TSC3, THE RESULTING CHEBYSCHEV VECTOR IS TSC4. WE ALSO
C CREATE TSC2 A CHEBYSCHEV VECTOR DEFINED BY SIGMA J SIGMA K
C (TSC1.MAG(HN)*SINE(THETAN)* UN-1*(1-X**2)**.5 )*d2(TSC3)/dX2.
C WE DO NOT HOWEVER ADD TSC4 AND TSC2 TOGETHER.
      SUBROUTINE FILTER(TSC1,TSC2,TSC3,TSC4)
      COMMON TRAN(0:15,0:15),T(0:15,0:15),TD2(0:15,0:15),M(0:15)
      COMMON MC(0:15,0:15),INVMC(0:15,0:15),D(0:15,0:15)
      REAL TRAN,T,TD2,VEC1(0:15),VEC2(0:15),VEC3(0:15),RAT(0:15)
      REAL DUT(0:15),TOT(0:15),PG,SPA(0:15),NUM,M,MC,INVMC,D
      INTEGER SIZE,DES,VAL,B
      REAL TSC1(0:15),TSC2(0:15),TSC3(0:15),MAGN,PHASE,DUM1(0:15)
      REAL DUM2(0:15),DUM3(0:15),DUM4(0:15),TSC4(0:15)
      DO I=0,15
      DUM1(I)=TSC1(I)*MAGN(I)*COS(PHASE(I))
      TSC2(I)=0
      DUM4(I)=0
      END DO
C CONSTRUCT TSC4
      CALL CAS(TSC3,DUM1,TSC4)

```

```

C CONSTRUCT TSC2

CALL MULT(TSC3,DUM2,TD2)

CALL CAS(DUM2,DUM1,DUM3)

C DUM3 IS NOW THE CHEBYSCHV VECTOR CONTAINING  $d^2(TSC3)/dx^2$ 
C EVALUATED AT DUM1

DO I=0,15

DUM2(I)=TSC1(I)*MAGN(I)*SIN(PHASE(I))

END DO

DO I=1,15

DO J=1,15

DUM4(ABS(I-J))=DUM2(I)*DUM2(J)+DUM4(ABS(I-J))

Q=I+J

IF(Q.GT.15)GOTO 7

DUM4(Q)=DUM4(Q)-DUM2(I)*DUM2(J)

7 CONTINUE

END DO

END DO

DO I=0,15

DUM4(I)=DUM4(I)/4

END DO

CALL TIM(DUM4,DUM3,TSC2)

RETURN

END

C FAC IS THE FACTORIAL FUNCTION  $N!/(N-R)!R!$ 

REAL FUNCTION FAC(N,R)

INTEGER N,R

REAL DUM(3)

DUM(1)=0.0

```

```

DUM(3)=0.0

IF(N.NE.0.AND.R.NE.0)THEN

DO I=R+1,N,1

DUM(1)=LOG(I*1.0)+DUM(1)

END DO

DO I=1,(N-R)

DUM(3)=LOG(I*1.0)+DUM(3)

END DO

FAC=DUM(1)-DUM(3)

FAC=EXP(FAC)

END IF

IF(N.EQ.0.OR.R.EQ.0)FAC=1.0

RETURN

END

C MAGN IS THE FUNCTION DEFINING THE MAGNITUDE OF THE FILTER
C RESPONSE AT THE Ith HARMONIC FREQUENCY

REAL FUNCTION MAGN(I)

INTEGER I

IF(I.LE.12)MAGN=1.0

IF(I.GT.12)MAGN=.02

RETURN

END

C PHASE IS THE PHASE RESPONSE IN RADIANS OF THE FILTER AT THE
C Ith HARMONIC FREQUENCY

REAL FUNCTION PHASE(I)

INTEGER I

PHASE=0.2*I

RETURN

```

```

END

C SUBROUTINE RAMFILTER ACTS TO FILTER THE CHEBYSCHV VECTOR
SUBROUTINE RAMFILTER(RAT)
REAL RAT(0:15)
DO I=0,15
IF(I.LE.7)RAT(I)=RAT(I)
IF(I.GT.7)RAT(I)=RAT(I)/I
END DO
RETURN
END

C SUBROUTINE AVERAGE TAKES A CHEBYSCHV VECTOR TSC1 AND
C PRODUCES THE CHEBYSCHV VECTOR TSC2 WHICH IS DEFINED BY
C  $(1/6)*(Q**2)*d^2(TSC1)/dx^2$ . THIS VECTOR WHEN ADDED TO TSC1
C IS EQUIVALENT TO AVERAGING TSC1 OVER +Q TO -Q TO ORDER
C  $Q**4$ . WE DO NOT ADD THE VECTORS TOGETHER
SUBROUTINE AVERAGE(TSC1,TSC2,Q)
COMMON TRAN(0:15,0:15),T(0:15,0:15),TD2(0:15,0:15),M(0:15)
COMMON MC(0:15,0:15),INVMC(0:15,0:15),D(0:15,0:15)
REAL TRAN,T,TD2,VEC1(0:15),VEC2(0:15),VEC3(0:15),RAT(0:15)
REAL DUT(0:15),TOT(0:15),PG,SPA(0:15),NUM,M,MC,INVMC,D
INTEGER SIZE,DES,VAL,B
REAL TSC1(0:15),TSC2(0:15),Q
CALL MULT(TSC1,TSC2,TD2)
DO I=0,15
TSC2(I)=(1/6.0)*(Q*Q)*TSC2(I)
END DO
RETURN
END

```

APPENDIX K

CASCADE MATRICES

K.1 CASCADE MATRICES

We wish to show that if

$$f(x) = \sum_i a_i T_i(x) \quad (K.1)$$

and $g(x) = \sum_k b_k T_k(x) \quad (K.2)$

then $g(f(x)) = \sum_j d_j T_j(x) \quad (K.3)$

where $d_j = \sum_k c_{jk} b_k \quad (K.4)$

and $c_{jk} = (1/B) \int_{-1}^{+1} T_k \left[\sum_i a_i T_i(x) \right] (T_j(x)) / (1 - x^2)^{1/2} dx \quad (K.5)$

where $B = \pi/2 \quad \text{if } j \neq 0$
 $= \pi \quad \text{otherwise} \quad (K.6)$

The proof is as follows

$$\begin{aligned} g(f(x)) &= \sum_k b_k T_k \left[\sum_i a_i T_i(x) \right] \\ &= \sum_j d_j T_j(x) \end{aligned} \quad (K.7)$$

by definition (equation K.3). But using the orthogonal properties of Chebyshev polynomials we have that

$$\begin{aligned} &\int_{-1}^{+1} \left[\sum_s d_s T_s(x) \right] (T_j(x)) / (1 - x^2)^{1/2} dx \\ &= B d_j \\ &= \int_{-1}^{+1} \sum_k b_k T_k \left[\sum_i a_i T_i(x) \right] (T_j(x)) / (1 - x^2)^{1/2} dx \end{aligned} \quad (K.8)$$

Rearranging we get that

$$d_j = \int_{-1}^{+1} \sum_k (1/B) \left[T_k \left(\sum_i a_i T_i(x) \right) (T_j(x)) / (1 - x^2)^{1/2} dx \right] (b_k) \quad (K.9)$$

Hence we can identify c_{jk} as in equation K.5.

K.2 EXISTENCE OF [P]

We wish to show that $[P]^n$ tends to the null matrix, as n tends to infinity, provided that the eigenvalues of $[P]$ all have modulus less than 1. The proof is as follows.

$$\text{Let} \quad [P] = [K]^{-1}[e][K] \quad (\text{K.10})$$

where $[e]$ is a diagonal matrix, the elements of which are the eigenvalues of $[P]$. Provided that $[P]$ is a real symmetric matrix then $[P]$ can always be written in this form [ref 65, p.14]. For the matrices we shall meet this is always the case. Thus

$$\begin{aligned} [P]^2 &= [K]^{-1}[e][K][K]^{-1}[e][K] \\ &= [K]^{-1}[e][I][e][K] \\ &= [K]^{-1}[e]^2[K] \end{aligned} \quad (\text{K.11})$$

Similarly for higher powers of $[P]$ we obtain

$$[P]^n = [K]^{-1}[e]^n[K] \quad (\text{K.12})$$

Thus given equation K.12, and given that $[e]$ is diagonal with entries equal to the eigenvalues of $[P]$, it is clear that if each eigenvalue has modulus less than 1 then $[e]^n$ tends to the null matrix as n tends to infinity, and thus that $[P]$ also tends to the null matrix.

REFERENCES

1. M. Abramowitz and I. Stegun: Handbook of mathematical functions. Dover Publication Inc, 1964.
2. M.T. Abuelma'atti: Reduction of intermodulation distortion in communication amplifiers by pre-correction techniques. PhD thesis, University of Bradford, 1979.
3. M.T. Abuelma'atti: Diode function generators. Wireless World, November 1981.
4. M.T. Abuelma'atti: On the characterisation of a memoryless bandpass nonlinear system from intermodulation measurements. Int. J. Electronics, Vol.54, No.5, p.683-687, 1983.
5. M.T. Abuelma'atti: Intermodulation performance of nonlinear bandpass systems with linear negative feedback. Int. J. of Electronics, Vol.54, No.5, p.673-681, 1983.
6. R. Allan: Military electronics. Electronics Design, p.108, August 4th, 1983.
7. S.W. Amos: AM detectors. Wireless World, April 1980.
8. Arthanayake et al: Linear amplification using envelope feedback. Electronic Letters, Vol.7, No.7, 8th April 1971.
9. R.A. Askey: Theory and application of special functions. Academic Press, 1975.
10. Bava et al: Reduction of phase nonlinearities in TWT's. Alfa Frequenza, Vol.41, No.11, p.836-842, November 1972.
11. H. Bavinck: Jacobi series and approximation. Mathematisch Centrum, Amsterdam, 1972.

12. BBC Research: Pulse code modulation for high quality sound signal distribution - instantaneous companding using diodes as nonlinear elements. BBC Research Dept. Tech. Report No.EL-15, 1968.
13. A.S. Beasley: Vmos transistors score multicarrier power gains. Microwaves and RF, August 1982.
14. P. Beckmann: Orthogonal polynomials for engineers and physicists. The Golem Press 1973.
15. Bell and Clarke: Elimination of cross-modulation in CATV amplifiers. NCTA, Chicago, May 1972.
16. V.G. Bello: Design of a diode function generator using the diode equation and iteration. IEEE Trans on Circuit Theory, Vol.CT-19, p.213-214, March 1972.
17. Bennett and Clements: Feedforward, an alternative approach to amplifier linearisation. The Radio and Electronics Engineer, Vol.44, No.5, p.257-262, May 1974.
18. Berman and Mahle: Nonlinear phase shift in TWT as applied to multiple access communication satellites. IEEE Trans on Comms. Tech., Vol.COM-18, No.1, February 1970.
19. G. Berretta: Predicting multicarrier performance. Microwave Journal, p.35-44, July 1975.
20. N.M. Blachman: Detectors, bandpass nonlinearities and their optimisation, via inversion of the chebyshev transform. IEEE Trans. on Information Theory, Vol.IT-17, No.4, p.398-404, July 1971.
21. H.S. Black, US patent No.1686792, October 1929.
22. Blanchard: Phase locked loops, applications to coherent receiver design. Wiley 1976.

23. Blanke et al: Control theory applied to the problem of reducing intermodulation distortion in nonlinear amplifiers under multicarrier operation. Proc. of the 16th Allerton Conference on Comm. and Control Computing, Monticello IL, USA, p.364-371, 4-6 October 1978.
24. Bogdanovich: A comparison of amplifier linearisation methods. Telecommunications, Vol.24, No.2, 1970.
25. D. Bowers: Precision rectification. Electronic Engineering, June 1980.
26. R. Bracewell: The fourier transform and its application. McGraw-Hill 1978.
27. Bremenson et al: Linearising TWT amplifiers in satellite transponders. AIAA 8th Comm. Satellite Systems Conference, Orlando FL, USA, 20-24 April 1980.
28. Brendel et al: Phase and amplitude modulation effects in a phase detector using an incorrectly balanced mixer. IEEE Tran. On Inst. and Measurement, Vol.IM26, No.2, June 1977.
29. Carr: Formulae and theorems in mathematics. Chelsea Publishing Co. 1970.
30. H.S. Carslaw: Introduction to the theory of fourier series and integrals. 3rd ed., ch.8, p.270-271, MacMillan 1930.
31. Chakraborty et al: Some aspects of wideband HPA lineariser testing. National Telesystems Conference, NTC'82, Galveston TX, IEEE 1982.
32. Cheadle: Selecting mixers for best intermod performance. Microwaves, November 1973.
33. Y.S. Cho: Sensitivity analysis of feedforward amplifiers. IEEE Int. Conf. on Comms., p.3C-1/3C-4, 1974.

34. Clarke: A comparison of current broadcast amplitude modulation techniques. IEEE Trans. on Broadcasting, Vol.BC-21, No.2, June 1975.
35. Clarke and Hess: Communications circuits analysis and design. Addison-Wesley 1971
36. C. Cluniat: Corrections de nonlinearite. Rev. Tech. Thomson-CSF (France), Vol.10, No.2, p.343-390, June 1978.
37. C. Cluniat: Reduced carrier amplifier technique for TV transmitters and transponders. Conf. on Radio Transmitters and Modulation Techniques, London, 24-25 March 1980.
38. J. Cohen: Mixer conversion loss vs. local oscillator waveshape. Microwave Journal, October 1978.
39. J.C. Daly: Fibre optic intermodulation distortion. IEEE Trans. on Comm., Vol.COM-30, No.8, August 1982.
40. C. Davies: Verbal communication with Mr C. Davies, RF group leader, Cambridge Consultants Limited, Cambridge, England.
41. H.W. Denny: Linearisation techniques for broadband transistor amplifiers. IEEE Electromagnetic Compatibility Symposium records, Anaheim CA, USA, p.61-70, 14-16 July 1970.
42. J. Dicks and M. Brown: Intelsat IV-A transmission system design. Comsat technical review, Vol.5, No.1, p.73-103, Spring 1975.
43. J. Dujundji: Envelopes and pre-envelopes of real waveforms. IRE Trans. on Information Theory, March 1958.
44. ECS: ECS Multiservice system specification volume 4 - performance characterisation of Earth stations operating in the ECS satellite multiservice system. p.14, ESA 1983.

45. C.J. Edgcombe et al: Nonlinear behaviour in klystrons for UHF television. Proc. 7th Conf. on Microwave and Optical Generation and Amplification, Hamburg, p.151-156, 1968.
46. EW Communication: The handbook of digital communications. Microwave System News, 1979 (published by EW Communication Inc).
47. Fehr: Digital communications. Prentice-Hall 1981.
48. Fehr et al: TSI-OQPSK for multicarrier satellite systems. National Telesystems Conference NTC'82, Galveston TX, IEEE 1982.
49. Filtronic Components: Spectre signal processing simulator handbook. Filtronic Components Limited, Shipley, West Yorkshire, England.
50. L.M. Fink: Relation between the spectrum and the instantaneous frequency of the signal. Problems in Inform. Transm. (USA), Vol.2, No.4, 1966.
51. L.M. Fink: Electronics engineer handbook, McGraw-Hill 1977.
52. A.M. Fishstein: Phase stable amplifier limiters. Telecommun. and Radio Eng., Part 2, (USA), Vol.35, No.3, p.92-94, March 1980.
53. A.M. Fishstein: A comparative estimate of the amplitude-phase conversion in wideband amplifying limiters and AGC limiters. Telecommun. and Radio Eng., Part 2, (USA), Vol.36, No.2, p.77-78, February 1981.
54. Fox and Parker: Chebyshev polynomials in numerical analysis. Oxford Press 1967.
55. E.A. Franke and A.E. Noorani: Wilkinson hybrid with a quarter-wave offset. RF Design, p.32-38, November/December 1981.

56. S.L. Freeny: TDM/FDM translation as an application of digital signal processing. IEEE Communications Magazine, Vol.18, No.1, January 1980.
57. Gardiner: Amplifiers with feedforward distortion correction in transmitter multi-coupling. Proc. of the Conf. on Land Mobile Radio, Teddington, Middx. (IERE 1975), p.9-14, 18-20 November 1975.
58. T. Gee: Verbal communication with Mr T. Gee, principal engineer, Transmitter Projects, IBA, Crawley Court, Winchester, England.
59. P.R. Geffe: Simplified modern filter design. Iliffe Books 1963.
60. Girard and Fehr: A new baseband lineariser for more efficient utilisation of Earth station amplifiers for QPSK transmission. National Telesystems Conference, NTC'82, Galveston TX, IEEE 1982.
61. F. Goodenough: Analog LSI 1983 technology forecast. Electronics Design, p.194, 6 January 1983.
62. F. Goodenough: Low cost ADC makes 12 bit conversion in 500nS. Electronics Design, p.197, 26 January 1984.
63. K. Gopal et al: Distortion analysis of transistor networks. IEEE Trans. on Circuits and Systems, Vol.CAS-25, No.2, February 1978.
64. W. Gosling: A first course in applied electronics. MacMillan 1975.
65. R.J. Goult et al: Computational methods in linear algebra. Stanley Thornes (Publishers) Limited 1974.

66. Grabowski and Davis: An experimental M-QAM modem using amplifier linearisation and baseband equalisation techniques. National Telesystems Conference, NTC'82, Galveston TX, IEEE 1982.
67. Green: Characterisation and compensation of microwave transmitters. National Telesystems Conference, NTC'82, Galveston TX, IEEE 1982.
68. Gruber: Design strategies aid ELF/VLF receivers. Microwaves and RF, September 1983.
69. Gumm: Phase effects in cross-modulation. IEEE Trans. on Cable Television, Vol.CATV-1, No.1, October 1976.
70. Hall: Nonlinear compensation. IEEE Int. Conf. on Comms., p.208-213, 1968.
71. Hanson et al: A closer look at the ring diode modulator. IEEE Journal of Solid State Circuits, Vol.SC 7, No.3, June 1972.
72. R.A. Harris et al: Factors influencing the choice of shaping filters for the ECS system. ESA SP-120, p.101-120, December 1976.
73. Hayden Publishing: Microwaves product data directory. Hayden Publishing Company Inc. 1980.
74. R.P. Hecken and R.C. Heidt: Predistortion linearisation of the AR 6A transmitter. International Conf. on Comm., Seattle, June 1980.
75. B. Henderson: Reliably predict mixer IM suppression. Microwaves and RF, November 1983, (contains 12 further references).

76. P. Hettrakul and D.P. Taylor: Compensators for bandpass nonlinearities in satellite communications. IEEE Trans. on Aerospace and Electronic Systems, Vol.AES-12, No.4, July 1976.
77. Hewlett Packard: Electronic instrumentation and systems. 1982.
78. R.B. Hicks: Microwave radio for the common carriers. Microwave Journal, p.59-68, January 1983.
79. Y. Hirata et al: Degradation of PSK signals due to intermodulation noise. Electronic and Comms. in Japan, Vol.58-b, No.6, p.91-100, 1975.
80. Hnatek: A user's handbook to D/A and A/D converters. Wiley 1976.
81. Holbrook et al: Reduction of distortion in broadband and narrowband amplifiers by complementary methods. Trans. of the Engineering Institute of Canada, Vol.13, No.C-3, p.1-7, June 1970.
82. G.M. Homer: PIN diode driver allows linear RF attenuation. Microwaves and RF, p.83-84, April 1983.
83. I. Horikawa et al: Design and performance of a 200 Mbit/sec 16-QAM digital radio system. IEEE Trans. on Comm., Vol.COM-27, p.1953-1958, December 1979.
84. R.S. Hughes: Tunnel diodes excel as dc coupled detectors. Microwaves, June 1981.
85. F. Ivanek: Single sideband amplitude modulation in microwave transmission systems. Microwave Journal, April 1972.

86. Jain et al: Volterra transfer function from pulse test for mildly nonlinear channels. National Telesystems Conference, NTC'82, Galveston TX, IEEE 1982.
87. Jaracez et al: Multi-octave double balanced mixer. Microwave Journal, January 1973.
88. K.G. Johannsen and F.L. Paulsen: Limiter intermodulation improvement due to selective carrier spacing. IEEE Trans. on Aerospace and Electronic Systems, Vol.AES-12, No.4, July 1976.
89. P.B. Johns et al: Communications systems analysis, p.98-145, Butterworth 1972.
90. S.B. Karamakar: Stability criteria for a class of nonlinear feedback systems. Journal of Sound and Vibration, Vol.89, No.1, p.1-5, 1983.
91. Kawaguchi: Measurement of transponder nonlinearity using chebyshev transform analysis. Trans. Inst. Elect. Comm. Engineers (Japan), Vol.63, p.255-257, 1980.
92. Kaye et al: Analysis and compensation of bandpass nonlinearities in communications. IEEE Trans. on Comm., Vol.20, p.965-972, October 1972.
93. Kenney: Digital radio for 90 Mb/s 16-QAM transmission at 6 and 11 GHz. Microwave Journal, p.71-80, August 1982.
94. L.M. Khadra and K. Wadi: Multi-tone amplitude and phase modulated signals. IEEE International Symposium on Circuits and Systems, Vol.1, p.94-96, 2-4 May 1983.
95. C.J. Kibbert: AM/FM modulation standard. Proc. IEE, Vol.128, Pt.F, No.6, November 1981.

96. V.C. Kim et al: Wave modulation in nonlinear dispersive media. Phys. Fluids, Vol.23, No.11, p.2250-2257, November 1980.
97. Kotsopoulos: Minimisation of intermodulation interference. IEE Proc., Pt.F, No.4, June 1983.
98. Kotzian and Schmidt: Understanding, specifying and characterising mixers. RF and Microwave Measurement Symposium and Exhibition (HP), September 1981.
99. K. Koyama et al: Measurement of AM/PM conversion coefficients. Telecommunications, Vol.12, No.6, p.25-28, June 1978.
100. S.A. Kravchenko: Tabulated classification of phase measurements. Meas. Tech. (USA), Vol.22, No.2, p.187-192, February 1979.
101. Ku et al: Design techniques and intermodulation analysis of broadband solid state power amplifiers. IEEE Trans. on Electro-magnetic Compatibility, Vol.EMC-19, No.2, p.57-65, May 1977.
102. Y.L. Kuo: Distortion analysis of bipolar transistor circuits. IEEE Trans. on Circuit Theory, Vol.CT-20, p.709-716, November 1973.
103. Kwon: Linearised digital driver for microwave components. Microwave Journal, September 1976.
104. Lazarus: Diode RF rectification, predict it more closely. Microwaves, February 1978.
105. N.A. Liskov and A. Wood: Raytheon Co., verbal communication.

106. D. Lombard and D. Duponteil: PSK transmission over a nonlinear satellite channel. International Conference on Comm., Minneapolis 1974.
107. Manassewitsch: Frequency synthesizers, theory and design. Wiley 1976.
108. Marconi Instruments: 2305 modulation meter handbook. 1983.
109. Maseng: On the characterisation of bandpass nonlinearity by a two tone measurement. IEEE Trans. on Comms., Vol.COM-26, No.6, June 1976.
110. R. Meyer et al: A wideband feedforward amplifier. IEEE Journal of Solid State Circuits, December 1971.
111. R. Meyer: Cross-modulation and intermodulation in amplifiers at high frequencies. IEEE Journal of Solid State Circuits, Vol.SC 7, No.1, February 1972.
112. T.D. Michaelles: Laser diode evaluation for optical analog link. IEEE Trans. Cable TV, Vol.CATV-4, p.30-41, January 1979.
113. B. Miguel: Amplificado-res en clase super A. Revista Espanola de Electronica, September 1982.
114. Millman and Halkias: Integrated electronics. McGraw-Hill 1972.
115. Moffatt: A new method for determining AM/AM and AM/PM conversion in microwave devices. National Telesystems Conference, NTC'82, Galveston TX, IEEE 1982.
116. Morrison: Introduction to sequential smoothing and prediction. McGraw-Hill 1969.
117. Mullard: Mullard technical handbook, book 4, pt.6, signetics analogue IC's, p.14-17, March 1982.

118. Neil: Nonlinear analysis of a balanced diode modulator.
Electronics Letters, Vol.6, 5th March 1970.
119. C.N. O'Loughlin: Intermodulation characteristics of high power klystrons used in frequency transponders for colour television. Proc. of the International Conf. on UHF television, paper no.10, London, December 1965.
120. O'Reilly: Mobile radio transmitters in a hostile environment.
IEE Conf. on Radio Transmitters and Modulation Techniques, March 1980.
121. Ohwaku et al: Common amplification of many signals by a TWT amplifier. Electronics and Comm. in Japan, Vol.51-b, No.5, p.78-86, 1968.
122. E. Oxner: FET's in balanced mixers. Application note AN72-1, FET design catalog, Siliconix 1979, (contains 21 further references).
123. Padovani et al: Linearisation of L band transistor amplifiers to be used with multicarrier signals. European Microwave Conf. Proc., p.410-414, Rome 1976.
124. L.C. Palmer and S. Lebowitz: Computer simulation of solid state amplifiers. Comsat Technical Review, Vol.8, No.2, p.371-408, Fall 1978.
125. Panter: Modulation noise and spectral analysis. McGraw-Hill 1965.
126. Pearce: Intermodulation performance of solid state UHF class C power amplifiers. IEEE Trans. on Comms., Vol.COM-25, No.3, p.304-310, March 1977.
127. V. Petrovic and Gosling: Polar loop transmitters.
Electronics Letters, Vol.15, No.10, 10th May 1979.

128. V. Petrovic and Gosling: VHF/AM transmitter using VMOS technology. Electronic Eng. (GB), Vol.50, No.608, p.65-67.
129. V. Petrovic and Gosling: A radically new approach to SSB transmitter design. IEE Conf. on Radio Transmitters and Modulation Techniques, March 1980.
130. V. Petrovic and Smith: The design of VHF SSB polar loop transmitters. Conf. on Comm. Equipment and Systems, Birmingham, England, 20-22 April 1982.
131. Philips Netherland: Circulators and isolators for reducing transmitter intermodulation. Application note, Philips Netherland Marktgroep, Elanco, Eindhoven, The Netherlands, 1983.
132. Plessey Semiconductors: Linear integrated circuit handbook. Plessey Semiconductors, publication no.P.S.1973, March 1973.
133. T. Poston and I.N. Stewart: Taylor expansions and catastrophies. Pitman 1976.
134. J. Potukuchi and F. Assal: TDMA performance monitoring of communications satellite transmitters. IEEE National Telesystems Conf., (NTC), 1983.
135. Powers: The compatibility problem in single sideband transmission. Proc. of IRE, August 1960.
136. Rabiner and Gold: Theory and application of digital signal processing. Prentice-Hall 1975.
137. Rademacher et al: An automatic phase correction system. IEEE International Conv. Record, Vol.11, Pt.3, p.179-186, 1963.
138. Radio Society: Radio communication handbook. Radio Society of Great Britain, 5th edition, 1982.

139. C.W. Rhodes: Measurement of incidental carrier phase modulation of television transmitters. Conf. on Radio Transmitters and Modulation Techniques, London, 24-25 March 1980.
140. S.O. Rice: Envelopes of narrowband signals. Proc. of the IEEE, Vol.70, No.7, July 1982.
141. U.L. Rohde: Digital PLL frequency synthesizers. Prentice-Hall 1983.
142. Sage Laboratories: Application note, Sage Laboratories Inc., (3 Huron Drive, Natwick, Mass, USA), 1978.
143. Sandberg: On the theory of volterra expansions. National Telesystems Conference, NTC'82, Galveston TX, IEEE 1982.
144. G. Satoh and T. Mizuno: Importance of a new TWTA lineariser upon QPSK/TDMA transmission performance. IEEE Journal on Selected Areas in Comm., Vol.SAC-1, No.1, January 1983.
145. Sauseng et al: Reduction of intermodulation with harmonic injection for wideband travelling wave tubes. Int. Electronic Devices Meeting, Washington D.C., p.411-414, December 1975.
146. F. Schauer: Measurement of very small phase differences. Rev. Sci. Instrum., Vol.52, No.11, November 1981.
147. F. Scheid: Numerical analysis. Schaums's Outline Series, McGraw-Hill 1968.
148. D. Scherer: Generate low phase noise microwave signals. RF and Microwave Measurement Symposium and Exhibition (HP), September 1981.
149. Schetzen: Nonlinear system modelling based on the wiener theory. Proc. of the IEEE, Vol.69, No.12, December 1981.

150. M. Schwartz: Information transmission modulation and noise.
McGraw-Hill, 3rd edition, 1980.
151. F. Sechi: Linearised class B transistor amplifiers. IEEE
Journal of Solid State Circuits, Vol.SC-11, No.2, April 1976.
152. Seidel et al: A feedforward experiment applied to a L-4
carrier system amplifier. IEEE Trans. on Comm. Tech.,
Vol.COM-19, No.3, June 1971.
153. Shaft: Signals through nonlinearities and the suppression of
undesired intermodulation terms. IEEE Trans. on Information
Theory, Vol.IT-18, p.657-659, September 1972.
154. C.E. Shannon: Communications in the presence of noise.
Proc. of IRE, January 1949.
155. O. Shimbo: Effects of intermodulation AM/PM conversion and
additive noise in multicarrier TWT systems. Proc. of IEEE,
Vol.159, No.2, February 1971.
156. B. Siegal and E. Pendleton: Zero bias schottky diodes as
microwave detectors. Microwave Journal, September 1975.
157. D.D. Siljak: Nonlinear systems. Wiley 1969.
158. Simon et al: Performance of staggered quadrature modulations
over nonlinear satellite channels with uplink noise and
intersymbol interference. National Telesystems Conference,
NTC'82, Galveston TX, IEEE 1982.
159. J.G. Smith: Bandwidth compressive modulation system.
AIAA/CASI 6th Comm. Satellite Systems Conf., Montreal,
Canada, April 1976.
160. N.O. Sokal and A.D. Sokal: Class E, a new class of high
efficiency, tuned single ended, switching power amplifiers.
IEEE Journal of Solid State Circuits, p.168-176, June 1975.

161. N.O. Sokal and A.D. Sokal: US Patent No.3900823, August 1975.
162. G.R. Stette and Imboldi: AM/PM conversion and intermodulation in nonlinear devices. Proc. of IEEE, p.796-797, June 1973.
163. G.R. Stette: Intelligible crosstalk in nonlinear amplifiers, calculation of AM/PM transfer. IEEE Trans. on Comms., February 1975.
164. G.R. Stette: Evaluation of linearity by transformation of the amplitude characteristic. IEEE Trans. on Microwave Theory and Techniques, Vol.MTT-23, No.4, April 1975.
165. R. Strauss: Demands on TWT's tighten as carrier density grows. Microwaves, p.84-126, April 1978.
166. K. Stubkjaer et al: Nonlinearities of GaAs lasers, harmonic distortion. IEEE J. Quantum Electronics, Vol.QE-16, p.531-537, May 1980.
167. A.M. Suleh and D.C. Cox: Improving the power added efficiency of FET amplifiers operating with varying envelope signals. IEEE Trans. on Microwave Theory and Techniques, Vol.31, No.1, January 1983.
168. Swanson: A 200 MHz feedforward amplifier. IEE Colloquium on Broadband High Frequency Amplifiers Practice and Theory, November 1979.
169. Tanaka et al: Nonlinear distortion and second harmonics in travelling wave tubes. Electronics and Comm. in Japan, Vol.55-B, No.7, p.62-68, 1972.
170. F.E. Terman: Radio engineers handbook. McGraw-Hill 1943.
171. TRW: RF Semiconductors. 1981 European Edition.

172. B.P. Tunstall: Degradation of satellite links by passively generated intermodulation products. National Telesystems Conf., IEEE 1976.
173. G. van Beijsterveld: The linearisation of transducers with digital techniques. Polytech. Tijdschr. Elektrotech. Elektron. (Netherlands), Vol.36, No.8, p.403-407, August 1981.
174. M. Vidyasagar: Nonlinear system analysis. Prentice-Hall 1978.
175. H. Vifian: Group delay and AM/PM measurement techniques. RF and Microwave Measurement Symposium and Exhibition (HP), London, September 1981.
176. A.C. Walker: Microwave applications in industry. Microwave Journal, p.71-79, January 1983.
177. Warren et al: Application of the polar loop transmitter to HF SSB transmitters. IEE Conference on Radio Transmitters and Modulation, March 1980.
178. C.A. Wass: A table of intermodulation products. Proc. of IEE, pt.3, Vol.95, p.31-39, 1948.
179. Watkins-Johnson Company: Mixers as phase detectors. Watkins-Johnson Technical Notes, Vol.5, No.1, January/February 1978.
180. C.J. Waylan: Intermodulation degradation of PSK signals in a communication satellite. National Telesystems Conf., IEEE 1976.
181. Weaver and Taylor: Feedback linearisation of power amplifiers for digital microwave communications. National Telesystems Conference, NTC'82, Galveston TX, IEEE 1982.

182. M. Weiner: Linearise analogue signals continuously.
Electronics Design, November 23rd 1972.
183. R. Wells: SSB for VHF mobile radio at 5 kHz channel spacing.
IERE Conf. on Radio Receivers and Associated Systems,
p.29-36, July 1978.
184. Western Electric: Transmission systems for communications,
revised 4th edition, chapter 10. Western Electric Company
Inc., Technical Publications, Winston-Salem, North Carolina,
USA, 1971.
185. S. Wetenkamp: Diode power detectors. IEEE-MTTS Symposium on
Microwave Theory and Techniques, Boston MA, 1-3 June 1983.
186. S. Wetenkamp: Comparing single diode and balanced power
detectors. Microwaves and RF, August 1983.
187. R. Wezel and King: Video handbook. Newnes technical books
1981.
188. E.A. Whitman: Phase modulation measurement techniques for
improved accuracy. Microwave Journal, p.113-116, June 1978.
189. Whythe: The nonlinear characteristics of klystron amplifiers.
BBC Engineering monograph, No.74, June 1968.
190. N.Wiener: Response of a nonlinear device to noise. MIT
Radiation Laboratory Report 129. 1942.
191. E.L. Zuch: Data acquisition and conversion handbook.
Datel-intersil Inc., 1979.
192. A.I. Zverev: Handbook of filter synthesis. Wiley 1967.