

Imperial College London
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**INVESTIGATION OF NEW AND NOVEL TECHNIQUES OF
ACCURATELY MEASURING THE RESPONSE OF
STRUCTURES TO SHORT-DURATION LOADS**

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Doctor of Philosophy
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Declaration of originality

I confirm that this thesis is my own work and that any material from published or unpublished work from others is appropriately referenced. The research has been supervised by Dr Luke A Louca.

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Abstract

Although terrorist and accidental industrial explosions are infrequent, engineers still need to understand how structures respond to the short-duration loads generated by such events. Previous research to measure the response of structures, subject to such loads, has often been limited to before and after measurements. Although there are sensors that can measure the transient response, such as linear voltage displacement transducers, accelerometers and laser displacement gauges, these are often expensive and limited to measuring individual points. This research identifies cheaper methods to accurately measure the transient response caused by short-duration loads. A review of sensors used to measure structural response in short-duration and structural health monitoring (SHM) scenarios, identified inertial measurement units (IMUs) as a potential alternative to traditional sensors. Although the individual IMU gyroscopes and accelerometers may be used in isolation, when combined using data fusion techniques improved accuracy is possible. IMUs have been used for vehicle navigation, and to a limited degree SHM, but minimal evidence of their use to measure the response of structures subject to short-duration loads exists. To validate their use in such environments, an MPU9250 IMU was used to monitor the response of several simple structures to various loads, from static, through dynamic to ballistic loading. In a parallel investigation, two full-bridge strain gauge arrangements were used to measure support rotation. The research proved that using data fusion to combine IMU accelerometer and gyroscope results, produced an accurate representation of the response, quantified using existing error and inequality measure criteria. It also showed that, in certain scenarios, a combination of strain gauges could be used to measure an element's support rotation with a reasonable degree of accuracy. It was concluded that cheap sensor systems, such as IMUs and strain gauges, had the potential to accurately measure the response of structures to short-duration loads.

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Abbreviations

2D	-	Two Dimensional
3D	-	Three Dimensional
AC	-	Alternating Current
Acc	-	Accelerometer
AD0	-	I2C Slave Address LSB
ADC	-	Analogue Digital Converter
AI	-	Analogue Input
AUV	-	Automated Underwater Vehicle
BFF	-	Body-Fixed Frame
BMIB	-	Buncefield Major Incident Investigation Board
BS	-	British Standard
BSi	-	British Standards Institute
C	-	Geers' CSA Comprehensive Error Measure
CCD	-	Charge-Coupled Device
cDAQ	-	Compact Data Acquisition
CF	-	Complementary Filter
CPNI	-	Centre for the Protection of National Infrastructure
CSA	-	Comparative Shock Analysis
DAQ	-	Data Acquisition
DC	-	Direct Current
DIAdem	-	Data management and visualization software
DIC	-	Digital Image Correlation
DIF	-	Dynamic Increase Factor
DIO	-	Digital Input/Output
DMA	-	Direct Memory Access
DMP	-	Digital Motion Processor
DNV-GL	-	Det Norske Veritas - Germanischer Lloyd
DOF	-	Degree of Freedom
d.p.	-	Decimal Place
DTP	-	Displacement Timer Pins
ECF	-	Explicit Complementary Filter
EKF	-	Extended Kalman Filter
EN	-	European Norm
FBG	-	Fiber Bragg Grating
FE	-	Finite Element
FIFO	-	First-in-first-out
FPGA	-	Field Programable Gate Array
FSR	-	Full-Scale Range

GC	-	Geers' Comprehensive Error Measure
GF	-	Gauge Factor
GM	-	Geers' Magnitude Error Measure
GP	-	Geers' Phase Error Measure
GPS	-	Global Positioning Systems
Gyro	-	Gyroscope
GUI	-	Graphical User Interface
HSV	-	High Speed Video
I2C	-	Inter-Integrated Circuit
IC	-	Integrated Circuit
IMU	-	Inertial Measurement Unit
ISO	-	International Standards Organisation
KF	-	Kalman Filter
LabVIEW	-	Laboratory Virtual Instrument Engineering Workbench
LED	-	Light Emitting Diode
LETf	-	Large Explosion Test Facility
LSB	-	Least Significant Bit
LVDT	-	Linear Variable Differential Transformer
LZ	-	Landing Zone
M	-	Geers' CSA Magnitude Error Measure
MEMS	-	Micro Electro-mechanical systems
MF	-	Median Filter
MGB	-	Medium Girder Bridge
MSB	-	Most Significant Bit
NI	-	National Instruments
P	-	Geers' CSA Phase Error Measure
PCB	-	Printed Circuit Board
PI	-	Pressure – Impulse
PWM	-	Pulse Width Modulation
RaDAR	-	Radio Detection and Ranging
RC	-	Russell's Comprehensive Error Measure
RIO	-	Reconfigurable Input Output
RM	-	Russell's Magnitude Error Measure
RP	-	Russell's Phase Error Measure
RSS	-	Root-Sum-Square
SCI	-	Steel Construction Institute
SCL	-	I2C Clock
SD	-	Standard Deviation / Secure Digital
SDA	-	I2C Data
SDO	-	SPI Serial Data Output

SDOF	-	Single Degree of Freedom
SG	-	Strain Gauge
SGC	-	Sprague & Geers' Comprehensive Error Measure
SGM	-	Sprague & Geers' Magnitude Error Measure
SGP	-	Sprague & Geers' Phase Error Measure
SHM	-	Structural Health Monitoring
SME	-	Subject Matter Experts
S/s	-	Samples per Second
SSF	-	Sensitivity Scale Factor
TI	-	Theil's Inequality Coefficient
UAV	-	Unmanned Aerial Vehicle
UHSPC	-	Ultra-High Strength Pre-stressed Concrete
UKF	-	Unscented Kalman Filter
Var	-	Variance
VI	-	Virtual Instrument
WI	-	Whang's Inequality Measure

Variables

A	-	State transition matrix
a	-	Acceleration
<i>a</i>	-	Accelerometer reading
<i>a_{bias}</i>	-	Accelerometer bias
B	-	Control matrix
<i>c</i>	-	Calculated response history
<i>d</i>	-	Displacement of connection
<i>e</i>	-	Error
F	-	Force
<i>g</i>	-	Gravitational acceleration
H	-	Observation matrix
<i>I</i>	-	Identity matrix
K	-	Kalman gain
<i>K_I</i>	-	Integral gain
<i>K_P</i>	-	Proportional gain
<i>L</i>	-	Length of element or original length
ΔL	-	Change in length of element
M	-	Mass
<i>m</i>	-	Measured response history
<i>n</i>	-	Noise
Q	-	Estimated process error covariance
<i>q</i>	-	Quaternion
$ q $	-	Quaternion norm
<i>q[*]</i>	-	Quaternion conjugate
<i>q⁻¹</i>	-	Quaternion inverse
$\dot{q}$	-	Rate of change of quaternion
P	-	Covariance matrix
R	-	Estimated measurement error covariance
<i>R</i>	-	Rotation matrix
S	-	Innovation covariance matrix
<i>T</i>	-	Natural period
<i>t_d</i>	-	Phase duration
<i>u</i>	-	Control input vector
<i>v</i>	-	Measurement noise
$\dot{v}$	-	Linear acceleration
$\hat{v}$	-	Gravitational direction calculated using gyroscope data
$\bar{v}$	-	Gravitational direction calculated using accelerometer data
<i>w</i>	-	Process noise

X_m	-	Total deflection
X_E	-	Deflection at elastic limit
x	-	State update
y	-	Innovation
z	-	Measurement input vector
α	-	Complementary filter coefficient
Δt	-	Sample period
μ	-	Ductility ratio
θ	-	Member support rotation / chord rotation of element / pitch
Φ	-	Roll
Ψ	-	Yaw
τ	-	Time coefficient
ω	-	Gyroscope reading
ω_{bias}	-	Gyroscope bias

Chapter 1 Introduction

Terrorist incidents around the world have demonstrated that the detonation of large improvised explosive devices near to, or within, structures can cause significant damage and may lead to the collapse of the structure. Fortunately, large terrorist or accidental explosions occur relatively infrequently, however when they do occur, and there is damage to structures, they generally result in multiple fatalities and a large number of casualties. Examples of such events where structures have been subject to high-impact loadings from terrorist bombs include the partial collapse of the Murrah Federal Building, Oklahoma City April 1995 (Corley *et al.*, 1998; Weidlinger Associates, 2011) and Khobar Towers, Saudi Arabia June 1996 (Mahin, 2006) and the extensive damage to parts of Manchester City Centre June 1996 (Williams, Batho & Russell, 2000) and the government district in Oslo July 2011 (Wolff & Larsen, 2014; Meyer, Jore & Johansen, 2015). Similarly, the frequency of industrial accidents such as Flixborough, June 1974 (Venart, 2004; Sadee, Samuels & O'Brien, 1977) and Buncefield, December 2005 (Johnson, 2010; BMIIB, 2006; SCI, 2009) are thankfully very low (see Figure 1-1). The challenge for structural engineers is to design structures that are sufficiently robust that they remain standing long enough for people to be evacuated.



Figure 1-1 – Image of damage to warehouse at Buncefield (Author's photograph)

An understanding of how a building behaves under such short-duration large magnitude loads, similar to those produced by an explosion or a vehicle or ballistic impact, is an important step in determining a building's structural characteristics; hence enabling improvements to design procedures to be made. The infrequent nature of such events, either malicious or accidental, means that monitoring buildings to see how they respond under short-

duration large magnitude loads is not feasible, which has meant limited freely accessible research has been conducted in this area to date.

The field of Structural Health Monitoring (SHM), however, uses sensor systems to enable engineers to monitor the condition and response of a structure to in-service loads throughout its life. SHM is used to monitor a variety of structures for a range of different reasons (Karbhari & Ansari, 2009). It has seen increased use for monitoring major bridges or those bridges that are deemed to be at risk and necessitate monitoring. It has also been used for monitoring structures with a safety critical function such as dams or buildings of historical significance or at risk from earthquakes (Brownjohn, 2007). Brownjohn provides a useful overview of the types of civil infrastructure where SHM is used with a brief description of current and future developments in the instrumentation, data collection and the subsequent use of the data collected. To date the use of such technology has mainly been limited to environments where the loads are applied relatively slowly, for instance vehicular loading for bridges and seismic shock loading for a wide range of structures.

This research investigated the use of the technology in scenarios where the loads are applied for a short duration and are of relatively large magnitude. Relative magnitude in the context of this research refers to the magnitude of the load in relation to the size of structure being loaded. For example, a small cantilever tested in the laboratory might be subject to a load of a couple of Newtons in comparison to a large structure that might experience a load of several Kilo or Mega Newtons. Short duration refers to loads that are applied for a duration of milli-seconds rather than seconds.

1.1 Background

Experimental research to investigate the response of structures to short duration loads, such as blast or ballistic loading, has traditionally been limited to relatively simple techniques to establish what is happening to a structure at a particular point in time. Sometimes this might be limited to recording the residual deflections of the target after each test (Chen, Louca & Elghazouli, 2015; Foglar *et al.*, 2017). Where more detailed information is required, a selection of individual sensors such as accelerometers, strain gauges and linear variable differential transformer (LVDT) linear position sensors may be used (Li, Wu & Hao, 2015). When appropriately logged and analysed, the results from such instrumentation has the potential to provide pseudo-real-time information on structural response.

The advent of high-quality digital imaging has seen increased use of high-speed video (HSV) cameras. More recently the use of digital photogrammetry techniques such as Digital Image Correlation (DIC) (Reu, Cooper & Miller, 2010; Hooper, Arora & Dear, 2010; Pickerd *et al.*, 2016; Hehir, 2013; Hooper *et al.*, 2012) has become more widespread. Although such techniques provide the potential for an increased level of accuracy and precision, this comes with the requirement for significant post-processing of the HSV images meaning that currently their use is limited to post-event analysis rather than real-time data recording.

1.2 What is the gap in the knowledge

The author's personal experience¹ has shown that although many full-scale commercial blast tests utilize a lot of the sensors mentioned, the data is often not then used to produce an accurate dynamic measurement of the response of the structure in question. Frequently the outcome of such tests results in 'before and after' data only; what happens to the material or structure in between has not been further investigated. The latter response information is however important for developing better structural response models and understanding how both materials and structures respond throughout all phases of their loading. In addition, multiple sensors may be fielded, however generally little attempt is made to correlate the different sensor readings to come up with the most accurate measurement possible. It is these gaps in current practice that this research addresses.

1.3 Motivations for the research

The motivation for conducting such research has been based upon the author's personal practical trials experience where multiple sensor readings of a particular point, using a number of different sensors, such as displacement gauges, produced different results for each sensor when, theoretically, the results should have all been the same. Based upon this disparate data an initial simple hypothesis was proposed for combining the results to produce a more accurate answer. Figure 1-2 illustrates the concept.

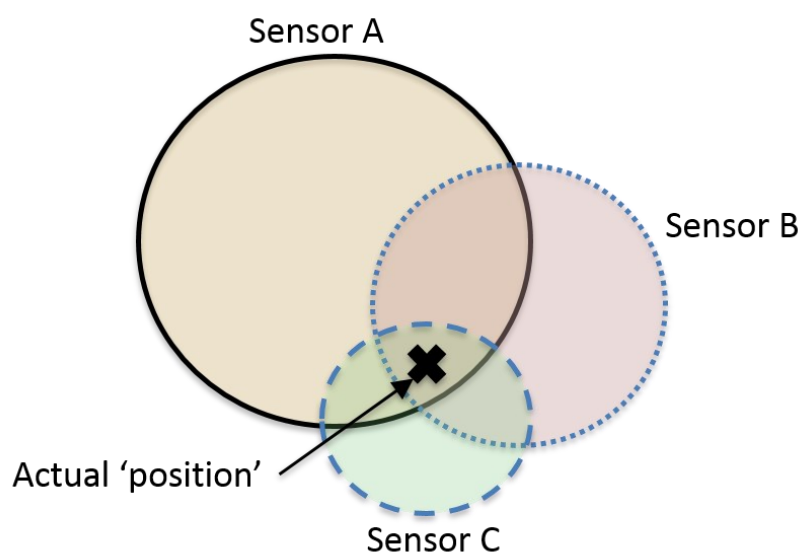


Figure 1-2 – Venn diagram to illustrate concept of data fusion

It shows a Venn diagram representing the results of three measurements. The diameter of each circle represents the level of uncertainty associated with each measurement, the smaller the circle the greater the certainty. The portion of overlap represents a more accurate result, the area of the overlap representing a level of certainty. The

¹ The author is a Chartered Civil Engineer who has been involved in the explosion and ballistic testing of materials and structures for over 15 years. This has included designing and conducting, and subsequent analysis and presentation of data from, large-scale explosion tests that involved the use of a wide range of sensors and monitoring techniques as part of national and international trials programmes. It is through this experience that the author identified the potential gaps in current knowledge that this research addresses.

ideal situation is to have a set of sensors whose readings all fall into a small area, as represented by the overlap of the areas in the centre of the Venn diagram.

As discussed elsewhere in the thesis, the concept of data fusion, with which this hypothesis aligns, is not new. However, evidence of its use in the field of short-duration large magnitude load scenarios is not well documented and therefore warrants further investigation.

High Speed Video (HSV), or other optical sensors, are increasingly being used to measure structural response such as displacement, as mentioned earlier. However, in the arena blast trials environment such methods are severely affected by the intense visual effects of the fireball during the early stages of the blast. This can often mean that visual data is lost or corrupted at the point when the structure starts to respond (Kranzer *et al.*, 2005; Stephens, 2008), therefore limiting the amount of useful data which can be obtained. The challenges of using HSV for monitoring the response of a structure are also highlighted by Pickerd (Pickerd *et al.*, 2016; Pickerd, 2013). In the full-scale fragment and blast tests conducted to observe the response of a steel box to an internal detonation, the challenges of achieving appropriate lighting, ensuring the stability of the HSV equipment and mirrors used and minimising vibration, were all encountered. As will be highlighted in the Literature Review in Chapter 2, the use of HSV imaging techniques, and particularly DIC has seen widespread use in a variety of fields including that of short-duration large magnitude testing such as blast tests. However, based upon the author's own practical experience, the use of such techniques is extremely useful for understanding the dynamic response of a relatively small subject area but, the problem of subject over-illumination is a major challenge, especially when using high explosive detonations to produce a blast wave, which generate the high intensity light produced immediately after detonation.

1.4 Purpose of the research

The aim of this research was to develop measurement and analysis techniques that could accurately measure the response of a structure, and that were not affected by external influences (such as high intensity light or vibration that are often produced by sudden impact loading such as an explosion), by utilising data from a variety of sensors.

1.5 Research questions

As part of the initial investigations for the research a number of questions were identified that could influence the results of the research and therefore needed to be addressed. These are briefly introduced here as a means of setting the context for the research but are covered in more detail in subsequent chapters of the thesis.

1.5.1 What is an acceptable level of accuracy?

Computer models and simulations allow engineers to produce very precise calculations for how a structure might respond however in real-life such precision is unrealistic. For instance, when placing reinforced concrete achieving dimensions of ± 1 mm is difficult to achieve on site, so stipulating it as part of the design may be introducing un-warranted pressures in the construction process. Similarly the use of simplified design methods

may be more efficient in terms of time, but often lead to designs which are considerably different from the ‘exact’ solution (Chancey *et al.*, 2005). Achieving the appropriate level of accuracy, commensurate with the environment that the research is being conducted in, or will be subsequently used, is therefore important.

At this point it is also worth distinguishing between precision and accuracy, which are sometime used interchangeably. For the purpose of this research accuracy (or measurement uncertainty) has been taken to be how close to the true measurement or answer the measurement is, while precision refers to the repeatability or reproducibility of a measurement (Morris & Langari, 2016). The analogy of how close a group of parachutists might land to the landing zone (LZ) each time they jump, as shown in Figure 1-3, illustrates the two concepts well. The figure shows that of the four parachutists, Parachutist A has the best result. All their landings are close together and near, or on, the LZ. By contrast Parachutist D is the worst; their landings are spread apart and not close to the LZ. The other two parachutists are somewhere in between. Parachutist B is close to the LZ but the landings are not close together; Parachutist C on the other hand has a tightly formed group of landings but is not close to the LZ.

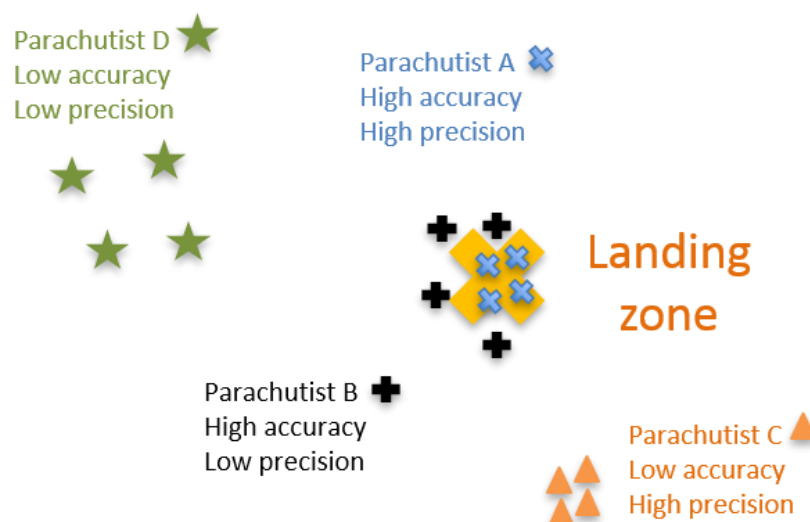


Figure 1-3 – Concept of accuracy and precision

In the context of short-duration high magnitude tests, such as air-blast explosions or ballistic tests, quantifying an accurate applied load is often difficult and therefore there will be some variability in the results obtained. For instance, the ISO standard for testing the explosion resistance of glass subject to air-blast loading in an arena test environment, ISO 16933 (ISO, 2007a), stipulates that for a test to be successful the average percentage variation for either the pressure or the impulse shall not be less than -15%. For the EXV25(X) classification the anticipated mean peak air-blast pressure is 80 kPa which equates to a 12 kPa variation. The acceptable accuracy for the pressure transducers is $\pm 5\%$ so is this an acceptable accuracy for other sensors?

Answering this question early in the research was crucial as it set the rest of the research in context in terms of what was deemed to be acceptable in terms of accuracy and established whether the methods investigated

improved on those that already existed. This question is discussed in more detail in the accuracy and error analysis chapter and included as part of the analysis in each of the experimental results chapters.

1.5.2 How realistic is the model?

In an ideal world it would be hoped that all the experimental results would match those expected, however this is rarely the case. As already illustrated by the author's practical trials experience of three different sensor readings, determining whether a measurement is correct or not is difficult if there is nothing against which to compare it. It is common practice to use a theoretical or computer model as the basis for this comparison. This is fine when the structure or model is relatively simple, but when they are more complex, this may not be easy to achieve. Determining whether the model used as the basis of a particular structural response represents reality is as important as being able to accurately measure that response. The research methodology chapter describes the thought process taken in terms of the author's initial, somewhat optimistic, proposal to use a simple model and rapidly move onto a more complex full-scale structural example. Initial experimental work quickly demonstrated that it was important to keep things simple in order to have confidence that the model was correct, and consequently could be used as the basis against which to compare any measurement data. The existence of a reliable model, or another appropriate technique, against which the chosen methods could be judged was crucial to determining what the level of accuracy was and therefore whether the methods investigated met the chosen success criteria.

1.5.3 At what rate do samples need to be sampled and how many samples are required?

An important aspect of data capture is the rate at which samples need to be recorded. This is particularly important in situations where the state of a structure changes within a short period of time. To accurately record the nature of the response sufficient data samples must be recorded. The sampling rate, which is usually measured in samples per second (S/s), is determined by the data acquisition system and needs to be set high enough to record all the necessary data. In the field of blast testing where an explosion, and the subsequent structural response, may only last a matter of milli-seconds, the data acquisition needs to be fast enough to capture sufficient data. If the sample rate is too slow then important response data might be missed; the correct choice of sample rate and number of samples to be collected is therefore important, and will impact on the methods and sensors that can be used. The challenges of choosing the appropriate sampling criteria are discussed in detail in the methodology chapter, while decisions relating to why a particular sample rate was used are explained within the individual experimental chapters.

1.6 How does this research add to the body of knowledge?

1.6.1 Application of conventional research instruments in new fields of investigation

Conventional sensors such as accelerometers and strain gauges have long been used in the field of blast experimentation (Nassr, 2012; Thiagarajan *et al.*, 2015; Wierschem *et al.*, 2017). This research investigated how the data from such sensors could be 'fused' to fill in any gaps and to provide a more accurate answer. The use of

data fusion algorithms such as the Kalman and complementary filter have been used in areas of Civil Engineering, such as structural health monitoring, but the author has come across only limited evidence of its use in the field of short-duration large magnitude loading, such as blast and ballistic testing. The title of one such paper, ‘Application of a Kalman filter, to blast loaded structures’ by Mediavilla et al (Mediavilla, van Doormaal & Weerheijm, 2007), mentions the Kalman filter however its use in this particular instance was for inverse modelling purposes to determine the material characteristics of blast loaded concrete panels, rather than as a means of fusing sensor data.

1.6.2 Combining disparate concepts in new ways to investigate a conventional issue

Until recently the majority of the sensors used in the field of blast testing were very expensive due to the need for them to withstand the harsh environments of an explosion. This research built on that conducted in the field of inertial navigation and micro-technology by using Micro Electro-mechanical Systems (MEMS) sensors, such as those found in mobile phones and Unmanned Aerial Vehicles (UAV), as sensors to measure impact response. It investigated the use of cheap MEMS Inertial Measurement Units (IMU) and the data fusion algorithms that they use and analysed their ability to accurately measure the response of a variety of structures to both static and short-duration dynamic loads.

1.7 Relating the research to Real World applications

In simple terms structural response is a means of describing how a structure changes when subjected to an external force. The following section describes key aspects of structural response pertinent to structures subject to dynamic loads and how they relate to the research.

1.7.1 Natural frequency and period

The natural frequency of a structure is “that frequency at which a system oscillates when not subjected to a continuous or repeated external force” (Oxford University Press, 2018). The natural period of a structure is the inverse of the natural frequency. The natural period, and hence the natural frequency, are used to determine how a structure will react to a particular type of dynamic load.

In blast design, the natural period of a structure is compared to the positive phase duration of the blast load, which then enables the engineer to classify the structure in terms of the load it receives (Cormie, Mays & Smith, 2009). Where the positive phase duration of the blast load is longer than the natural period of the structure the load is classified as a *quasi-static* load. A short positive phase duration compared to the natural period is classified as an *impulsive* load, while a positive phase duration that is similar to the structure’s natural period is classified as a *dynamic* load. The three regimes are usually summarised in terms of the ratio of phase duration t_d and the structure’s natural period T as follows (Cormie, Mays & Smith, 2009):

Impulsive	$\frac{t_d}{T} < 0.1$
Dynamic	$0.1 < \frac{t_d}{T} < 10$

Quasi-static $10 < \frac{t_d}{T}$

1.7.2 Structural response regimes and their categorisation?

As shown above, the manner in which a structure is loaded dictates the type of response, and depending upon the structure's construction how it will respond; these are usually categorised into *impulsive*, *dynamic* and *quasi-static* response regimes (Cormie, Mays & Smith, 2009). The response of a structure subject to such loads is often shown using pressure-impulse (PI) diagrams (Figure 1-4), while key elements are further categorised by their support rotation and ductility ratio (see Figure 1-5 and Equation 1-1).

$$\text{Ductility ratio} - \mu = \frac{\text{total deflection}}{\text{deflection at elastic limit}} = \frac{x_m}{x_E} \quad (1-1)$$

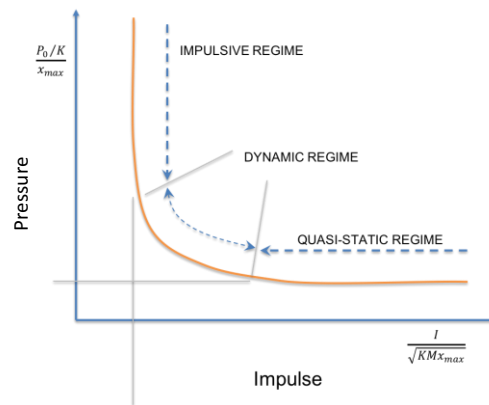


Figure 1-4 – Example of a Pressure-Impulse (PI) diagram (reproduced from (Krauthammer, 2008))

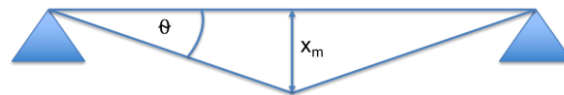


Figure 1-5 – Example of member support rotation

Member support rotations and ductility ratio are further used to determine whether a structure's design will meet pre-determined deformation criteria. For instance the protection categories defined by the United States Department of Defence for accidental loads caused by explosions (DoD, 2008) are widely used and reproduced for completeness in Table 1-1.

For the purposes of most non-military building design, only protection categories one and two are considered as categories three and four refer to the propagation of detonation effects between explosive charges in adjacent areas or structures.

Category	Description (only comments related to structural response included)	Deformation limits Structural Steel (Reinforced Concrete)	
		Support Rotation θ	Ductility Ratio μ
1	“...attenuate blast pressures and structural motion to a level consistent with personnel tolerances; and shield personnel from primary and secondary fragments and falling portions of the structure and/or equipment.”	2° (2°)	10 N/A
2	“Protect equipment, supplies and stored explosives from fragment impact, blast pressures and structural response”	12° (4°)	20 N/A
3	“Prevent communication of detonation by fragments, high-blast pressures, and structural response.”	See Note	See Note
4	“Prevent mass detonation of explosives as a result of subsequent detonations produced by communication of detonation between adjoining areas and/or structures”.	See Note	See Note

Table 1-1 – Protection categories and associated values for support rotation and ductility ratio (DoD, 2008)

Note: There are no deformation limits for Protection Categories 3 and 4 as can be seen in Figure 1-6.

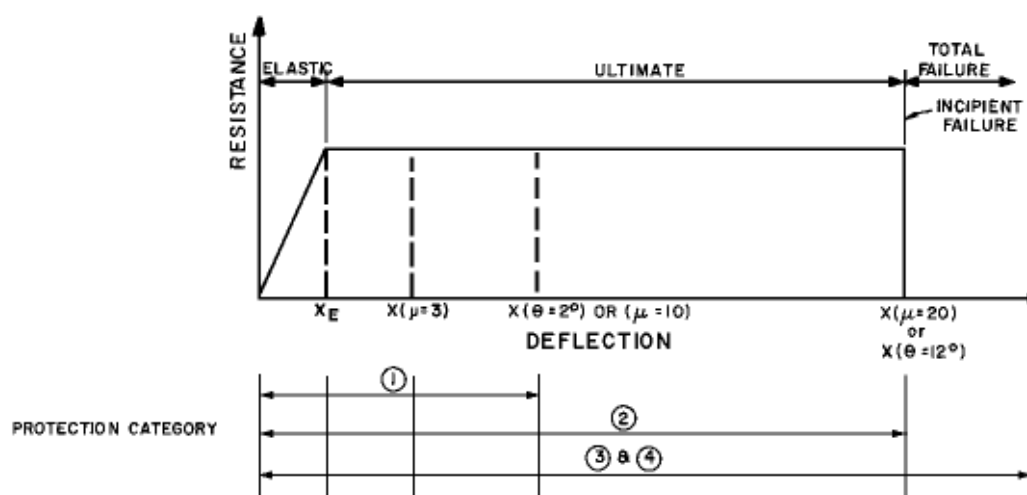


Figure 1-6 – Graphical representation of protection categories and associated values for support rotation and ductility ratio (reproduced from (DoD, 2008))

It will be noted that the descriptions provided in Table 1-1 relate to explosive storage rather than non-military structures; Cormie et al (Cormie, Mays & Smith, 2009) provide a slightly different form which is more appropriate to the latter type of structure as follows:

- Protection Category 1. “Protection of personnel and equipment through the attenuation of blast pressures and to shield them from the effects of primary and secondary fragmentation and falling portions of the structure” (Cormie, Mays & Smith, 2009).

- Protection Category 2. “Protection of structural elements themselves from collapse under the action of blast loading” (Cormie, Mays & Smith, 2009).

The relationship between the protection categories, support rotation and ductility ratio are illustrated graphically in Figure 1-6. The relevance of these criteria, specifically the support rotation, is discussed in more detail in the research methodology chapter where the use of the various sensors to calculate angles of rotation of the various test elements is examined.

1.8 Research objectives

With the previously mentioned challenges and research questions in mind, three research objectives were postulated which are summarised below:

1.8.1 Objective One

The research identified characteristics of structural response that could be used to assess how a structure responds by:

- a. Identifying key structural response characteristics that could be monitored using a variety of sensors,
- b. Establishing the level of accuracy required to correctly determine such a response(s),
- c. Determining the acceptable level of uncertainty that enables the data to be of use,
- d. Developing a suitable remote monitoring system to provide an estimate against which to compare experimental data, and
- e. Where appropriate developing theoretical models of simple structures to confirm the assumptions made and provide an estimate against which to compare experimental data.

1.8.2 Objective Two

The research identified different types of sensor that could be used in short-duration large magnitude loading situations by:

- a. Assessing how each sensor responded to such loading using both simple and more complex experimental arrangements,
- b. Comparing the experimental data with the remote monitoring system or theoretical models developed in Objective One, and
- c. Analysing the results to determine their accuracy and uncertainty and verifying that it met the criteria established in Objective One

1.8.3 Objective three

The research evaluated different methods of utilizing data from a variety of sources to provide an accurate measure of response by:

- a. Identifying existing techniques used for SHM, other response monitoring regimes and data fusion applications,

- b. Assessing the monitoring and data fusion techniques to determine whether they could be applied to short-duration large magnitude loading events, and
- c. Developing a ‘data fusing’ technique that provided an accurate measure of a structure’s response to such loading, while also providing an indication of the uncertainty associated with that measurement.

1.9 Thesis outline

The thesis is broken down into a further eight chapters. Chapter 2 is a literature review that summarizes previous research conducted to investigate the response of structures to short-duration large magnitude loading and existing methods to measure such response. The literature review also considers how data fusion has been utilized in the field of structural monitoring and its use in the field of short-duration large magnitude loading scenarios, such as blast loading, and concludes by looking at research into the accuracy required for structural response analysis.

How the research methodology was applied is discussed in Chapter 3. This includes the basis upon which the research was designed, the methodology behind the research, how the data was collected and a review of sensor technology relevant to the research. Chapter 4 considers data fusion and its relevance to the research, starting with a review of the concept and proceeding with a discussion of the specific techniques considered. Measurement accuracy was a fundamental aspect of the research, so Chapter 5 builds on the brief introduction of section 1.5.1 describing how accuracy and uncertainty was handled and the methods used to quantify errors.

Chapter 6 presents the data, and its analysis, for two simple ‘static’ experimental arrangements. The first was a simple inverted pendulum to measure angular displacement using accelerometers and gyroscopes and the second a vertical cantilever to introduce the use of strain gauges in monitoring structural response. These experiments were used to verify the proposed data fusion techniques and to confirm that the sensors used were producing reliable, accurate results. Chapter 7 moved the analysis up a level of complexity by subjecting the simple vertical cantilever to a short duration ‘dynamic’ load to verify that the proposed techniques worked in a dynamic environment. It also includes the results from two ballistic tests where a number of bullets were fired at two targets: the previously tested vertical cantilever and a mobile monolith. Each of the experimental chapters concludes by discussing the accuracy of the data, any anomalies or unexpected results, and the significance of data fusion for each set of data.

The discussions and observations for all the experimental work are drawn together in Chapter 8 where the efficacy of the proposed techniques is summarised, and outcomes compared against those predicted. This chapter also concludes the thesis with a summary of how the research has contributed to the body of knowledge, how it might be implemented and provides suggestions for future research.

Chapter 2 Literature Review

2.1 Introduction

Research in the field of short-duration high-magnitude loading can broadly be divided into that which investigates the performance of materials, structural elements or full-scale structures. Due to its expense and complexity, research, and hence literature, covering testing of full-scale structures is limited (Yankelevsky, Schwarz & Brosh, 2013). Public access to the research that has been conducted is usually restricted for reasons of security or commercial sensitivity (Morrill *et al.*, 2005). Research into the response of materials and structural components is generally more widely available, and although this research often includes details of the instrumentation used to measure this response, it does not usually include a detailed analysis of the sensors used. This review highlights the methods traditionally used to measure or record the response of these different configurations to the loadings under consideration and analyses their effectiveness in the context of this research.

This thesis also makes use of data fusion as a means of combining the data from different sensors and so this review briefly discusses those applications of this process most relevant to this research. Data fusion has been widely used in several fields usually related to vehicle navigation, such as Unmanned Aerial Vehicles (UAVs), Automated Underwater Vehicles (AUVs) and self-driving vehicles (Zhang *et al.*, 2016; Metni *et al.*, 2005), but also in SHM applications that are more closely aligned to this research. Limited evidence of its use for the short-duration load environment has been found, and that which has, related to material and load quantification examples.

The final important aspect of the research is how the accuracy of the data is approached in terms of what is optimal for the chosen environment. Although the issue of accuracy is frequently covered by researchers, there is little evidence of whether the chosen level of accuracy is acceptable or appropriate, something that this review attempts to address.

2.2 Existing methods of measuring response in elements and structures subject to short-duration high magnitude loading

In most branches of science and engineering standards have been developed to formalise the testing process to enable direct comparisons between similar materials and products to be readily made. The field of dynamic testing is no different, however, for specialities such as explosion and ballistic testing such standards are limited. Ballistic protection standards, for example those for determining the bullet-resistance of structural elements such as glass (BS EN 1063 (BSi, 2000) or ISO 16935 (ISO, 2007b)) or windows and doors (BS EN 1522/1523 (BSi, 1999a, 1999b)), generally rely upon pass or fail criteria. They do not usually require any specialist instrumentation except that to accurately record the velocity of the bullet prior to impact with the target. Similarly, standards for determining the explosion resistance of materials (e.g. glass ISO 16933 (ISO, 2007a)) and products (e.g. windows, doors and shutters BS EN 13124-2 (BSi, 2004)), limit their instrumentation requirements to pressure gauges to determine that the explosive charge has fully detonated. None of the aforementioned standards require any form

of instrumentation to measure the response of the sample under test, solely relying upon the post-test analysis of the condition of the sample. Test designers who want to determine the transient response of a structure are therefore left to their own devices in terms of the instrumentation they might use to achieve this.

Much of the research relating to shock and vibration testing, of which short-duration high-magnitude testing is a subset, often relates to the use of pressure sensors and the determination of the pressures' effect on a structure. Where research makes use of other instrumentation this has traditionally made use of relatively simple sensors such as displacement gauges, linear voltage displacement transducers (LVDT), accelerometers, strain gauges and more recently laser displacement gauges. These sensors all measure individual points on a test sample and as such provide a limited amount of information on the sample's overall response to any load, and therefore need careful positioning to ensure the best response data is recorded. For simple test samples, such as one-way spanning panels with the same fixing condition at each end of the panel, mid-span sensor placement enables the maximum and minimum deflections to be recorded. For example in one such test mechanical displacement devices were used to record the maximum and minimum mid-panel displacements of a selection of horizontal one-way spanning ultra-high strength pre-stressed concrete (UHSPC) panels subject to large air-blast loading reported by Ngo et al (Ngo, Mendis & Krauthammer, 2007) (see Figure 2-1).



Figure 2-1 – Image of one of the UHSPC panels tested by Ngo et al (Author's photograph)

A similar study was conducted by Barnett et al in 2009 (Barnett *et al.*, 2010). In these tests the dynamic displacement of the large fibre-reinforced vertical one-way spanning panels (Figure 2-2 (Left)) was measured using a laser gauge, and the peak deflection was recorded using a 'broom-stick gauge' (see Figure 2-2 (Right)). In the aforementioned tests the panels were all mounted in test structures so that panels were orientated at right-angles to the ground. Tests on similar panels using smaller charges were conducted by Li et al used a single linear voltage displacement transducer (LVDT) to measure the central deflection (Li 2015). In this instance the panels were mounted on supports so that the target face was parallel to the ground. In each of these examples the

displacement measurements were only recorded for the panel mid-point locations and were then generally used to validate computer models. Although the laser gauges and LVDTs were able to measure the response time histories for the chosen points, LVDT sensor and fixing failures for the tests conducted by Li et al meant that in only one test was it possible to record the full displacement time history, highlighting the fallibility of such sensors that rely upon fixity to both the test sample and a robust reference point.



Figure 2-2 – Sensor arrangement for fibre-reinforced concrete panel tests conducted by Barnett et al. (Left) External view of panel in test cubicle. (Right) Pre-test image of ‘broomstick’ and laser gauges. (Author’s photographs)

The ‘broomstick’ gauge used by Barnett et al is a simple, albeit crude, system for determining the maximum deflection of a test sample. It must be carefully adjusted to ensure the appropriate amount of friction is applied to the mount fixing; sufficient to stop the impact of the sample hitting the shaft propelling the broomstick out of its mount, and yet loose enough to allow free movement. It also has the limitation of only being able to measure the deflection at a single point. To address this limitation other researchers have taken the concept further by using multiple, smaller ‘broomsticks’. Such a device, consisting of multiple steel needles inserted into a barrel of sand (see Figure 2-3), was used by Yao et al to measure the deflection of a reinforced concrete beam subject to a small explosive blast load (Yao *et al.*, 2016). The steel needles were inserted into the sand in such a manner that their tips were parallel to the underside of the beam. When the beam deflected it hit the needle tips and pushed them into the sand in such a manner that the maximum angle of the beam rotation could be calculated. At a similar time the concept of multiple pins was being further developed by Fay et al by introducing the additional ability of being able to measure the time at which the pins were impacted (Fay *et al.*, 2015). The displacement timer pins (DTP) that were developed consisted of thin hollow steel tubes to which a single strain gauge was affixed at a set distance from the tip of the pin. A series of these pins was then positioned in a special mount below the target plate. Each pin was set at a specified distance from the rear face of the target plate in such a manner that when the plate deflected under the blast load, the time at which each pin was impacted was recorded by a sudden increase in magnitude of the voltage observed for each of the strain gauges. From this it was possible to determine the

transient deflection of the plate. The relatively robust, inexpensive nature of the system meant that it could be used close into a target without concern of damaging a more expensive sensor.

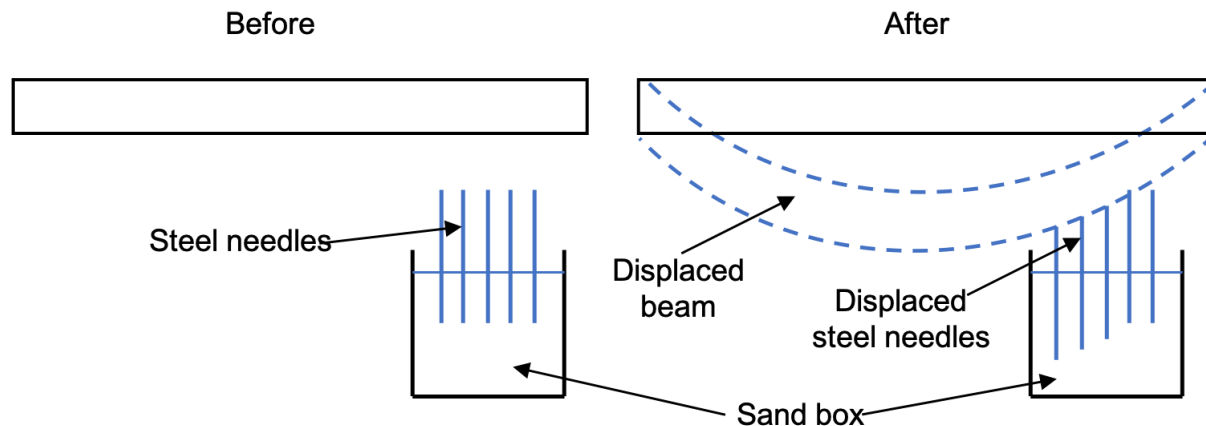


Figure 2-3 – Deflection measurement device used by Yao et al (Author's image).

Although some traditional sensors are able to measure response time histories, their general inability to measure multiple points on a test sample or structure is a significant limitation when quantifying overall response. Optical techniques such as those using HSV can be used to measure response time histories and have sometimes been used to measure the response of structures subject to short-duration dynamic loads. A single Phantom HSV camera was used by Stephens in 2006 to measure the mid-plane dynamic deflection of laminated glass following an air-blast load from an explosion (Stephens, 2008) (Figure 2-4 (Left)). The deflection of the rear face of the glass pane was recorded through an angled mirror at the base of the window and verified using a laser deflection gauge pointed at the centre of the pane (see Figure 2-4 (Right)). The individual HSV images were each calibrated using a pre-recorded grid-scale panel held at mid-pane before each test, which enabled an accurate displacement to be calculated. The resultant deflection time history generated from the individual HSV images compared well with the laser deflection gauge measurement. Although this technique enabled a displacement-time history to be generated it required a considerable amount of post-processing, and only provided the dynamic displacement for individual points. In addition, the effects of the post-detonation intense light had to be addressed by painting the rear face of the glass panes black (Figure 2-4 (Right)) and covering the front face with an aluminium foil sheet (Figure 2-4 (Left)). An example of two of the HSV images demonstrating the effect of the post-detonation light on the image quality are shown in Figure 2-5.



Figure 2-4 – Test arrangement for laminated glass deflection tests conducted by Stephens. (Left) Front of test cubicle showing glass pane with aluminium foil facing. (Right) Rear view of glass pane post-test showing HSV camera and mirror arrangement. (Author’s photographs)

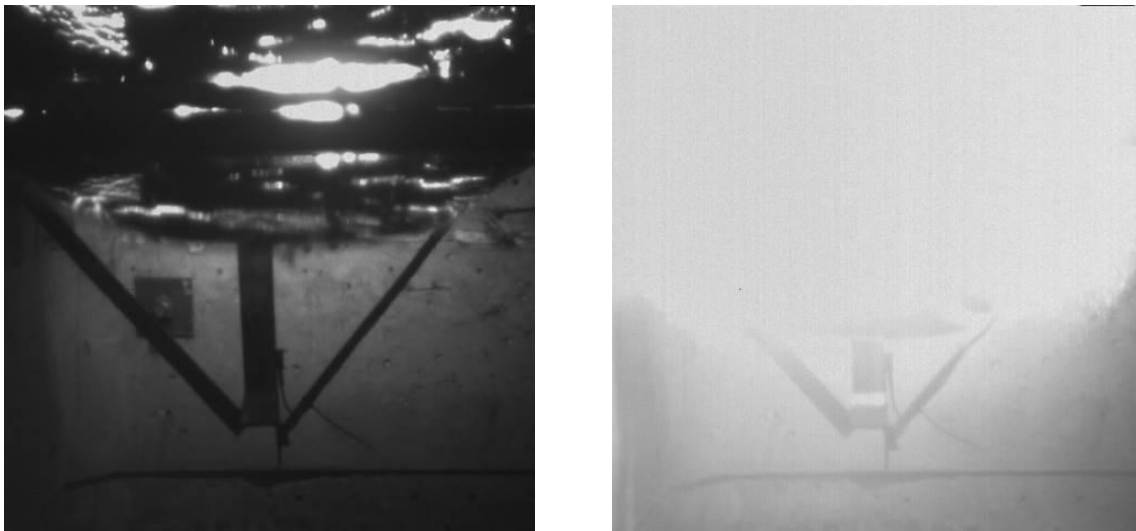


Figure 2-5 – HSV images from tests conducted by Stephens showing effects of post-detonation light on image quality. (Left) Black and white video still image at the point that the laminated glass pane started to tear. (Right) Black and white video still image taken 10 frames after Figure 2-5 (Left) illustrating the effects of post-detonation light. (Author’s photographs)

An almost identical test instrumentation arrangement was used by Idriss and Lowak to measure the mid-point deflection of laminated glass whilst investigating the window frame reaction forces (Idriss & Lowak, 2014). As the tests were conducted in a shock-tube, problems due to post-detonation excess light were avoided. A more recent example of the technique was used by Denney and Clubley to measure the peak deflection of I-section cantilever beams subject to long-duration blast loads in a series of four full-scale tests conducted in a large shock-

tube (Denny & Clubley, 2019). In this study the data from the HSV was used in conjunction with a scale board placed adjacent to the test sample. Because the tests were conducted in a large shock-tube, post-detonation excess light complications were again avoided. Although it was possible to use the HSV to obtain the displacement-time histories for each of these tests, these were still only for individual points on the test samples.

The steady increase in the computational power of computers, and improvements in the resolution of HSV camera optical sensors, has seen the emergence of photogrammetric techniques such as DIC that use twin-synchronised HSV cameras (Figure 2-6 (Left)). DIC enables the three-dimensional response of a surface or structure to be recorded and analysed, thereby avoiding the limitation of only being able to record single data points on a structure as discussed so far. Hooper used DIC to measure the response of laminated glass in order to investigate the effects of blast loading on silicone-bonded laminated glass (Hooper, 2011). He also used strain gauges mounted on the surrounding window frame to measure the axial strain at the edge of the laminated glass. As the air-blast was generated by an explosive charge, the effects of post-detonation light were mitigated (Hooper, 2011; Hooper, Arora & Dear, 2010) by painting the front face of the glass with black acrylic paint while using white paint for the rear face. The white rear face was then covered with black speckles that were analysed by the DIC software to determine changes in appearance (Figure 2-6 (Right)). A drawback of using DIC is the requirement for consistent lighting conditions throughout the test and careful calibration procedures to ensure that the DIC data is accurate. As with the previous mentioned techniques using HSV there is also a significant post-processing requirement.



Figure 2-6 – Test arrangement for DIC laminated glass deflection tests conducted by Hooper. (Author’s photographs)

DIC was also used by Pickerd et al to investigate the strain and deformation of a welded steel container subject to a small internal explosion (Pickerd *et al.*, 2016). The system, which utilised mirrors to ensure that the HSV cameras were not hit by fragmentation, required careful calibration and after the charge detonated, and the air-

blast shockwave passed over the mirrors, the resulting vibration caused the DIC data to become corrupted. Fortunately, these effects occurred after the major container failures had happened and therefore did not impact on the experimental results, and because this test involved an internal detonation, there were minimal issues with post-detonation light effects.

As with all sensor types, optical techniques have their own set of pros and cons. As already mentioned, when used for experiments involving the detonation of explosives the intense light generated by an explosion can corrupt the image, usually at the instance of most importance when the test sample is starting to respond. Careful placement and shielding of the cameras can mitigate this effect. These problems do not generally arise when using gas-driven shock tubes, however it must always be borne in mind that the blast waves generated are usually of longer duration and lower magnitude than those associated with high-explosive detonations. Vibration due to the blast wave impacting surrounding structures, ground or the target can also affect the quality of the HSV images such that they become corrupted.

The use of DIC is not limited to tests investigating the effects of explosions on materials. Yu *et al* used DIC to measure the ballistic impact of a small solid steel ball on two different types of polymer (Yu *et al.*, 2009). Although the level of detail obtained varied between the three- and two-dimensional cases, and as mentioned elsewhere errors were generated due to background vibration, it was possible to differentiate between the strain histories of the two materials, something that had not previously been possible.

So far, the instrumentation examples discussed relate to displacement sensors or methods, all of which rely upon a robust fixing point, whether that be a rigid mounting for the LDVT and mechanical displacement device, or a remote mount for the laser displacement gauge and HSV or DIC. Such mounts need to be isolated from any external movement that might add an error into the displacement reading; if the mount vibrates or moves the resultant measurement could be incorrect. The alternative is to mount the sensor on the structure being monitored. Examples of such sensors include accelerometers and strain gauges. Although these suffer similar problems to the displacement sensors in that they can only monitor an individual point, they do not require additional attachment points and are not affected by extraneous light. Their intimate contact with the test structure does however make them more susceptible to vibration which can add unwanted noise to the sensor reading.

Accelerometers, strain gauges and displacement potentiometers were all used by Nassr to determine the response parameters of explosive blast tests conducted on a selection of vertical, simply supported steel I-Columns (Nassr, 2012). The accelerometer and displacement potentiometer readings were used to compare the experimental results with single degree of freedom (SDOF) models, while the strain rates calculated using the strain gauge data were used as part of an analysis of the validity of the dynamic increase factor (DIF) used. Similar tests were conducted by Schleyer *et al* using accelerometers and strain gauges as part of a study to determine the response of metal blast-walls subject to shock load from a shock-tube (Schleyer *et al.*, 2007). In this instance the accelerometers and strain gauges were used to monitor the transient dynamic effects of the panels that each initially underwent an elastic response without any permanent deformation, followed by a plastic response with permanent deformation.

In addition to using accelerometers and strain gauges for individual elements such sensors have also been used to measure the transient response of large structures; Yankelevsky et al used accelerometers and strain gauges to measure the response of a full-scale reinforced concrete building subject to a number of large air-blasts (Yankelevsky, Schwarz & Brosh, 2013). Following appropriate filtering and integration the accelerometer data was used to calculate velocity and displacement to obtain the respective response time histories. The building's natural frequency was also calculated from the accelerometer data. On a slightly smaller scale Park et al used accelerometers and Fiber Bragg Grating (FBG²) strain gauges to measure the response of a scaled two-storey frame to an impact load (Park, Kim & Oh, 2017). For these tests the accelerometers were used to identify the modal frequencies of the structures, whilst the strain data obtained from the FBG sensors was used to validate the associated computer model.

Structural response characteristics such as member support rotations that were introduced in Chapter 1, have been used by various researchers to quantify the amount of damage done to a structure. Angular rotation was calculated by Puryear et al when evaluating blast-damaged steel connections as part of tests to investigate the performance of various bolted connections (Puryear *et al.*, 2010). Rather than use sensors the rotation was calculated using the arctangent of the chord as shown in Figure 2-7. The angle of rotation of an object, such as a beam, can be calculated by integrating the rate of change of an angle with respect to the time between samples.

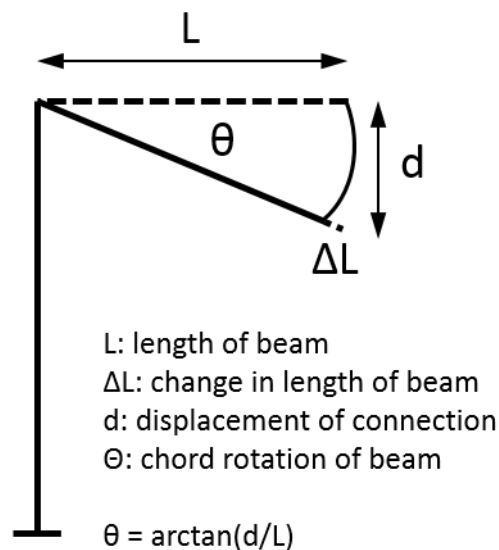


Figure 2-7 – Calculation of angle of rotation of a beam (Puryear *et al.*, 2010) (Author's image)

One type of sensor which is designed to measure the rate of change of an angle is the gyroscope, and although they have not traditionally been used in the shock and vibration field, they are found in the field of navigation where they are frequently used in conjunction with other sensors such as accelerometers using sensor/data fusion. Examples of such uses are discussed later in the next section dealing with data fusion.

² Fiber Bragg Gratings are discussed in more detail in Chapter 3, Methodology.

2.3 Use of data fusion in structural health monitoring

Structural health monitoring (SHM) is the term given to the process of monitoring structures to determine their ‘fitness for purpose’ (Brownjohn, 2007). SHM systems can be found in many walks of life including aircraft (Sbarufatti, Manes & Giglio, 2014; An, Kim & Sohn, 2012) and infrastructure such as bridges (Omenzetter & Brownjohn, 2004; Ai *et al.*, 2014) and buildings (Brownjohn, 2007). Monitoring such structures is usually undertaken by using appropriately placed instrumentation to monitor for changes in the structural response caused by overloading or damage to the structure itself. Many of the instruments mentioned in the previous section have been used to do this. Due to the size of many of the structures, researchers have had to investigate methods that utilise a limited number of sensors (to reduce costs) to ascertain the structural integrity; data fusion is one such method that will be reviewed in the following section.

In full-scale SHM applications, for example large bridges, the use of direct measurement techniques such as fixed displacement sensors (mechanical displacement devices and LDVTs) are impractical due to the distances involved, and the requirement for robust remote fixing points. In such situations, indirect measurement techniques like those that make use of accelerometers and strain gauges, that are secured directly to the structure, are more appropriate. Both sensor types have limitations which mean that it is difficult to use them in isolation to determine structural response; accelerometers suffer from low-frequency drift if double-integrated to obtain displacement, whilst strain gauges are difficult to use where high-frequency modes are present (Park, Sim & Jung, 2013). Fusing the data together to make the most of the different sensors’ strengths is an approach that can enable the sensor reading to be effectively used.

The fusion of data from accelerometers and strain gauges was used by Park *et al* to estimate non-zero mean dynamic displacements (Park, Sim & Jung, 2013). The approach was validated numerically and by practical application to a cable stayed bridge. An improvement on this method was proposed by Cho *et al* that involved mode shapes generated from finite element (FE) models of a structure rather than assuming a sinusoidal mode shape (Cho, Yun & Sim, 2015). Although the various techniques were successfully validated using numerical models of different bridge structures, and in the field using a prestressed concrete bridge, both methods rely upon the generation of appropriate mode shapes of the structural response, which for complex structures such as buildings may be difficult.

It has already been shown that both accelerometers and strain gauges have their limitations in terms of poor low frequency response and susceptibility to noise respectively. To address this, Palanisamy *et al* conducted a study using accelerometers, strain gauges and tilt sensors to determine the optimum combination of the different sensor types (Palanisamy *et al.*, 2015). The data from the accelerometers was combined with both the strain gauge data and then the tilt sensor data using a Kalman filter-based state estimator. The different methods were validated using a numerical model and a simple beam experiment and showed that the best results were obtained when the readings from the accelerometer and tilt sensors were combined, as the strain readings generally had a high noise-level that affected the output.

Instead of using strain gauges and tilt sensors, some researchers have opted to use accelerometer and displacement data. These sensors were fused together using a multi-rate Kalman filter in a proposal by Smyth and Wu (Smyth & Wu, 2007). The multi-rate filter was chosen to allow for data sampled at differing rates and the method was validated using simulated data. In this instance the Kalman filter was used to improve the velocity and displacement data obtained by double-integration of the accelerometer data and adjusted by less frequent samples of more accurate displacement data. A similar approach was used for the fusion of accelerometer and displacement data in an investigation conducted by Kim et al (Kim, Kim & Sohn, 2014). In this instance the Kalman filter was used as part of the error estimation process for calculating errors due to double-integration of the acceleration data to obtain velocity and subsequently displacement. The calculated displacement was then compared with the measured displacement to obtain the displacement error. The proposed method was validated using a vertical cantilever, similar to that used in this research, and subject to three load cases: arbitrary finger-tapping, impact load and non-oscillatory pseudo-static motion (i.e. moving the beam whilst holding the tip). A comparison with the method proposed by Smyth and Wu showed marginal improvement for the finger tap and impact load cases and significant improvement for the pseudo-static motion.

Where it is impractical to use displacement measuring sensors, some researchers have opted to use accelerometers and gyroscopes, such as used as part of a multi-scale structural health sensing scheme proposed by Sung et al to identify damage in bridges (Sung *et al.*, 2013). In their study they used a two-stage process; first the accelerometers were used to identify damage features in the bridge beams, and then the gyroscopes were used to identify damage near to the supports that the accelerometers were unable to identify. Like other sensor fusion methods, the proposed scheme used the strengths of each of the sensor types to determine the optimum solution but did not use traditional fusion algorithms such as the Kalman and complementary filter to optimise the solution. A complementary filter was used to fuse data from a MEMS tri-axial accelerometer and gyroscope by Liu et al when calculating the tilt angle of large civil engineering structures such as wind turbine masts (Liu *et al.*, 2017). A similar approach was taken by Testoni et al who used a MEMS tri-axial accelerometer and gyroscope with a real-time complementary filter to fuse the complementary data from the two sensor types to obtain an accurate tilt angle (Testoni *et al.*, 2019). The chosen algorithm was designed to optimise the individual data signals prior to data fusion using a window filter function. The system was validated in both a static condition by rotating the sensor through a fixed angle, and also in a dynamic condition using the vibration of a simply supported beam. Although similar to aspects proposed by this research, the study did not consider short-duration loading, and as shown in the following section, there is limited evidence in the literature that this type of loading scenario has been actively researched in combination with sensor or data fusion.

2.4 Use of data fusion in short-duration loading large magnitude situations

Although there is evidence of data fusion techniques such as Kalman and complementary filtering being used in other areas of civil engineering such as SHM, there is little evidence that it has been used in investigations involving structures subject to short-duration loading from impact or blast. An Extended Kalman Filter (EKF) was used to determine the material characteristics of a concrete slab subject to blast loading by Mediavilla et al (Mediavilla, van Doormaal & Weerheijm, 2007). An inverse application of the EKF was used with bending displacement data from a finite element model and experimental data for a reinforced concrete slab subject to a blast load. Minimisation of the error between the computational and experimental data enabled the unknown concrete material properties to be back-calculated. A similar approach was taken to determine the blast load experienced by a Medium Girder Bridge (MGB) by Lee and Chen (Lee & Chen, 2008). In a similar manner to the previous example, an inverse formulation of a Kalman filter was used to estimate the blast loads on a simple two degree-of-freedom model of the MGB knowing the response history and structural and material characteristics of the simple model. In this instance the process validation was based upon the simple model and did not include any experimental verification.

2.5 Accuracy requirements for structural response analysis

Having determined how a structure has responded using a combination of sensors, and possibly sensor fusion algorithms, the challenge for the research engineers that have designed those structures is to decide on the criteria against which any structural response will be measured (Phan *et al.*, 2013). Where the aesthetics or continued serviceability of a structure are important, serviceability criteria, such as those to limit damaging deflections, may be specified. Where the magnitude or dynamic characteristics of a load are such that damage will occur, such criteria are unlikely to be suitable and protection categories like those introduced in Chapter 1 may be more appropriate. Table 2-1 provides some examples of typical serviceability deflection criteria along with the associated deflections for the support rotations for Protection Categories 1 and 2 for a steel structure (Cobb, 2014; Davison & Owens, 2012; Cormie, Mays & Smith, 2009; DoD, 2008).

Case	Deflection criteria	Deflection criteria for 20m element (mm)	Support rotation for 20m element (deg (rad))	5% of support rotation (deg (rad))
Beams carrying plaster or other brittle finishes	Span / 360	55.6	0.319 (0.0056)	0.0160 (0.00028)
Other beams (except purlins and sheeting rails)	Span / 200	100	0.573 (0.0100)	0.0287 (0.00050)
Sway for single-storey building	Height / 300	66.7	0.191 (0.0033)	0.0097 (0.00017)
Beam Protection Category 1	N/A	349	2 (0.035)	0.1 (0.00175)
Beam Protection Category 2	N/A	2126	12 (0.209)	0.6 (0.01045)

Table 2-1 – Examples of deflection criteria and associated support rotations

These examples illustrate well the vast difference in deflection values between those that are deemed acceptable for serviceability conditions and those for ensuring a structure does not fail catastrophically. In such instances applying the same level of accuracy and precision may not be appropriate. For instance, applying the same 5%

support rotation tolerance for a beam using serviceability criteria (0.0287°) might be deemed rather too stringent for a Protection Category where there is an order of magnitude difference when compared to the respective 5% tolerance for the Protection Categories (0.1 or 0.6). The concept of the 5% tolerance applied to the serviceability state for Other beams, and Protection Category 1 is shown in Figure 2-8.

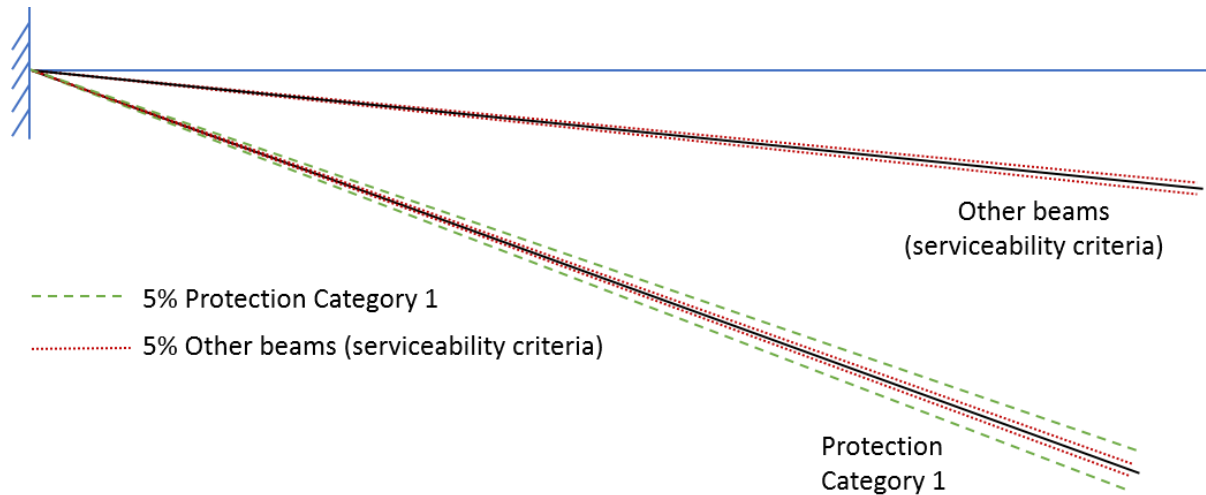


Figure 2-8 – Concept of different tolerances applied to different acceptance criteria

To enable the relative differences to be shown more clearly, the support rotation angles (in degrees) and the 5% of support rotation values (also in degrees) have been multiplied by a factor of ten. With reference to the Protection Category 1 line this shows the quite large difference between the 5% applied to the Protection Category value (green dashed line), compared with the 5% value applied to the serviceability criteria value (red dotted line). From this it can be concluded that applying the same level of tolerance for the serviceability criteria to the Protection Category would be an excessive constraint, however applying the Protection Category level of tolerance to the serviceability criteria would lead to the serviceability criteria being breached.

This example illustrates the importance of choosing the appropriate level of accuracy and precision. There is limited evidence of research being conducted into this area; the question of justifiable precision and accuracy was addressed by Chancey et al who were concerned that because of engineers' increased reliance on computers, they had been drawn into the trap of accepting the answers given as being gospel (Chancey *et al.*, 2005). They advocated that engineers "must take responsibility to understand the limitations of precision to which structures should be designed as a function of the limitations of design codes, specifications, and methodology". Allied to the use of computer models, the problem of variability in data used in structural response models, particularly in relation to blast loads, was identified by Campidelli et al who suggested that there were various sources of discrepancy that could lead to an incorrect model being applied including "parameter uncertainty, model error and inherent variability" (Campidelli *et al.*, 2015). Establishing what is appropriate or acceptable in terms of accuracy and precision for both the models and design, but also the systems being used to measure any response is therefore important.

2.6 Conclusions

There are a selection of sensors and fusion techniques that enable the response of structural elements or complete structures to be monitored whilst subject to loading. Whilst some of these sensors are capable of accurately measuring the response to short-duration loads, there has been limited research conducted to investigate methods that fuse these sensor outputs into an improved response history; this is the focus of Chapter 4 discussing Data Fusion and Chapter 6 and Chapter 7 describing and analysing the subsequent test data. Limited research has also been undertaken to address the question of what level of precision and accuracy is appropriate or acceptable for research in this field; this is explored in more detail in Chapter 5 (Accuracy and Error Estimation) and then discussed in each of the experimental chapters.

Chapter 3 Research Methodology

3.1 Introduction

In order to monitor the response of a structure it was first necessary to identify particular structural response characteristics that could then be monitored using a combination of sensors and sensor/data fusion techniques. As there was limited evidence of such techniques being used in the shock and vibration field it was therefore necessary to investigate and develop one.

3.2 Identification of structural response characteristics

3.2.1 Background

To understand how the chosen structural response characteristics were identified, it is important to appreciate the history behind the initial research proposal. This was based upon the author's involvement in the development and design of a Large Explosion Test Facility (LETF) at the DNV-GL test facility at RAF Spadeadam, Cumbria (see Figure 3-1).



Figure 3-1 – Photograph of Large Explosion Test Facility

The purpose of the LETF was to provide a full-scale building structure that could be used to investigate a variety of aspects of structural response with the ultimate aim of conducting a full-scale progressive collapse of the

structure. The initial research proposal was to develop the instrumentation plan for the structure and then analyse the results. As the main structure had to be able to withstand a number of full-scale blast tests first, many of the key structural elements such as columns and beams were ‘over-designed’, and although this resulted in a robust structure, it meant that developing a realistic model was not a simple proposition. Simple modelling of the LETF conducted by two MSc students (Hehir, 2013; Avramidou, 2013) at Imperial College London confirmed that despite access to reasonable structural drawings of the structure, accurately replicating its response was not simple. In addition, data recorded by a small number of sensors during commissioning blast-tests, such as the accelerometer reading shown in Figure 3-2, highlighted that not only would this data require careful analysis, but the limited number of sensors would need to be carefully positioned to ensure the optimum amount of response data was collected.

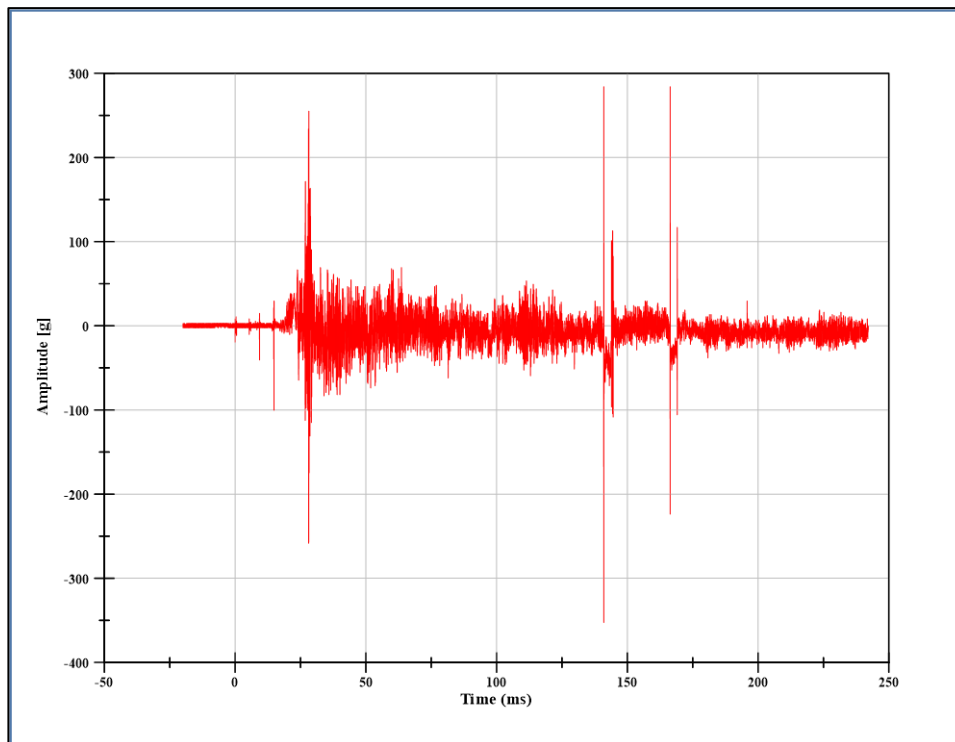


Figure 3-2 – Accelerometer reading from LETF

To address the issue of structural complexity a phased approach was proposed for the experimental research starting with a simple 'structure', in the first instance a simply supported beam, and progressively making the 'structure' more complex by introducing further dimensions. A variety of sensors were to be installed on these structures to enable the structural response to be monitored and to ascertain the sensor suitability.

With the LETF structural frame in mind, a 1.5m long mild-steel simply supported I-section beam was chosen as it was assumed that it best represented a component from the LETF, was relatively easy to model and the chosen section could be manhandled and investigated using the laboratory facilities at Imperial College. The section of beam chosen was 127x76x13 which was the smallest readily available; the experimental setup is shown in Figure 3-3 and a closeup of the beam in Figure 3-4.

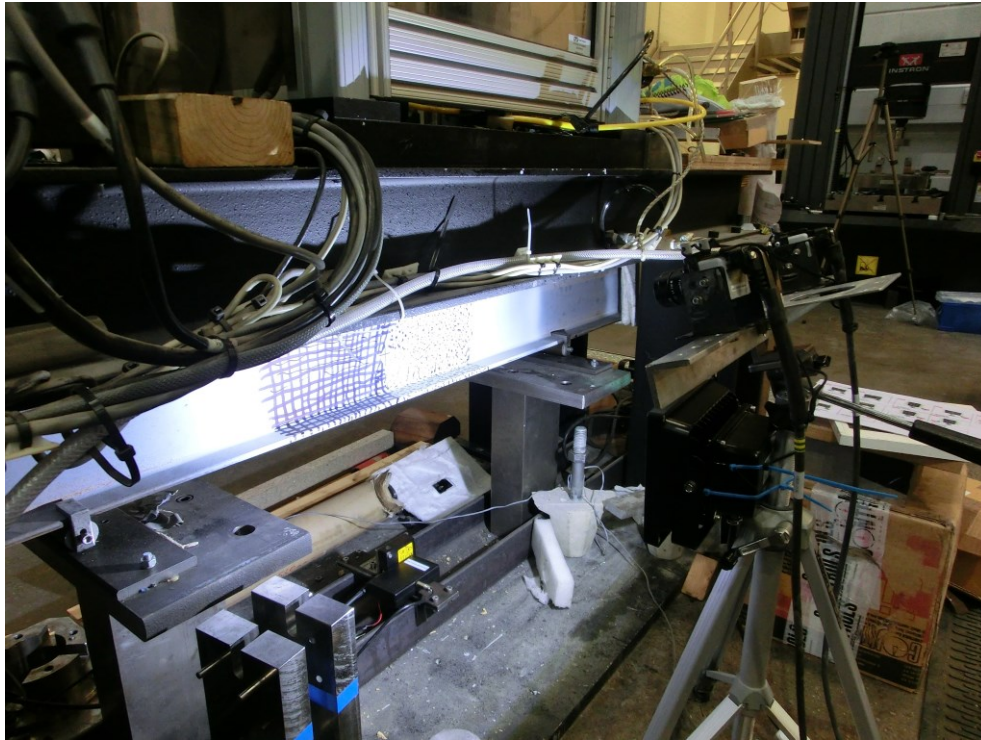


Figure 3-3 – Photograph of beam under impact test rig



Figure 3-4 – Close-up photograph of test beam

An impact load was applied to the mid-point of the beam using an Instron dynamic test rig however due to the relatively stiff nature of the beam compared to the magnitude of the impact load imparted, the recorded deflections were only a few millimetres. Ordinarily this would not have been an issue, however as being able to easily monitor

a response was important, in this case a mid-span deflection, it was decided that an even simpler experimental setup was required. In addition, at this point the author left full-time employment and started to self-fund the research which necessitated keeping the cost of any experimental setup down and designing it such that it could be operated in a home laboratory. To achieve these aims it was identified that any experimental arrangement should:

- Produce a relatively large response in terms of deflection or rotation that could be easily monitored by the chosen sensors.
- Be easily monitored using inexpensive HSV cameras or modelled using simple theoretical or numerical models.
- Produce a response with a similar order of timescale, i.e. milliseconds, when subject to a simple dynamic load but not necessarily the same order of magnitude, as might be caused by a high velocity impact or explosive blast-wave.
- Be easily manufactured and be relatively cheap (sensors should cost under £20).

The requirement for an easily manufactured and relatively cheap system was seen as an advantage. This provided the potential to use more sensors without necessarily having to worry about whether they were destroyed due to their relative cost and availability compared to more expensive traditional sensors.

3.3 Chosen experimental arrangement

The experimental arrangements that were ultimately chosen were based upon an inverted pendulum and vertical cantilever because they enabled large consistent responses to be easily generated. Both could be easily modelled and analysed within the environment of a home laboratory. In addition, by initially using simple structures it was possible to reduce any potential external influences that might affect the sensor readings. Once it was established that the sensors were reading correctly, and the results were consistent with those produced by the monitoring equipment, more ‘complex’ scenarios including dynamic loading were introduced.

3.3.1 Experiment One – Static rotation of inverted pendulum

The premise behind Experiment One was to have a simple method of producing a structural response in the form of a deflection or rotation of a vertical column. To achieve this a rigid bar was mounted as an inverted pendulum with the sensors being tested placed at the top of the bar (see Figure 3-5). As the pendulum was a rigid bar, and moved freely around a pivot at its base, minimal longitudinal stresses were developed so it was not practical to measure strain. A structural response was created by moving the inverted pendulum a set angle, where it was left for a few second before being moved on to the next position. This was achieved using a linear actuator programmed to move a set distance and pause before moving to the next position. The data recorded was then analysed and compared to that recorded by an HSV camera that was used to independently monitor the response for this, and all the subsequent experiments. This simple experiment was used to verify that the sensors being used were performing correctly and to validate the data fusion algorithms and Virtual Instruments (VI)³.

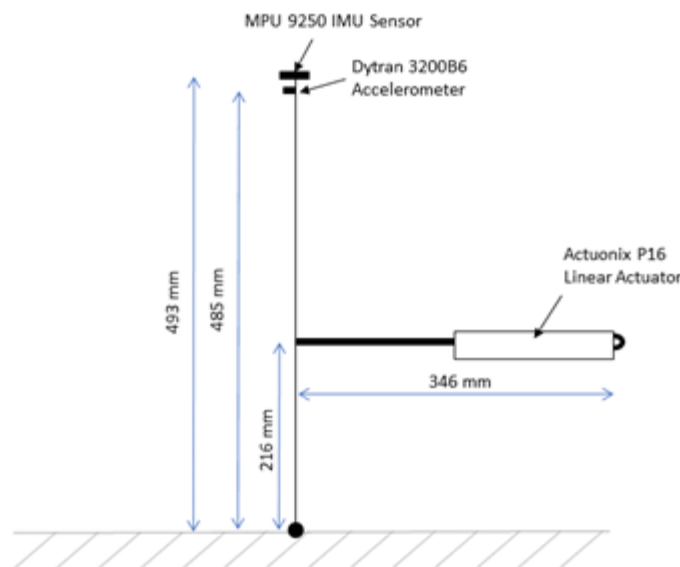


Figure 3-5 – Experiment One key components

A detailed description of the experimental setup for Experiment One can be found in Chapter Six.

³ Virtual Instruments are the building blocks of the National Instruments (NI) LabVIEW (Laboratory Virtual Instrument Engineering Workbench) software that was used to collect and analyse data from the experimental tests.

3.3.2 Experiment Two – Static deflection of cantilever beam

As it had not been possible to measure the longitudinal stresses using the inverted pendulum, a vertical cantilever was used for Experiment Two. This was made from a thin steel strip with a set of strain gauges fixed towards the base (as shown in Figure 3-6). A static deflection, and hence rotation of the tip of the cantilever, was produced by moving the tip of the cantilever using the same actuator and movement programme used in Experiment One.

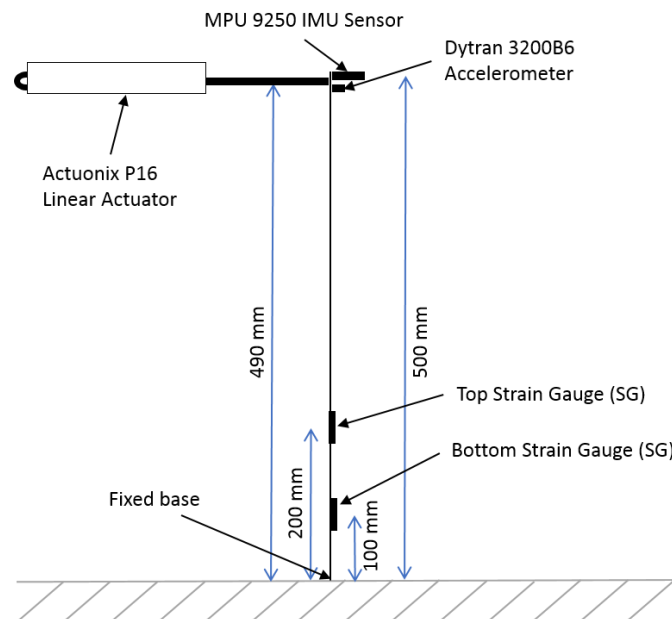


Figure 3-6 – Experiment Two key components

A detailed description of the experimental setup for Experiment Two can be found in Chapter Six.

3.3.3 Experiment Three – Dynamic deflection of cantilever beam

Experiment One and Two provided ‘static’ responses in that the movement was generated by an actuator moving the pendulum or cantilever to elicit a response. To enable a ‘dynamic’ response to be generated Experiment Three used the same vertical cantilever arrangement used for Experiment Two but introduced a novel ‘shear pin’ arrangement to instigate the ‘dynamic’ motion. The tip of the cantilever beam was loaded horizontally using the linear actuator used in the previous experiments, however for this experiment the actuator was attached to the tip of the cantilever using a ‘shear pin’ (see Figure 3-7) instead of the usual bolt, in the arrangement shown in Figure 3-8. The actuator was retracted until the ‘shear pin’ failed. The sudden removal of the load caused the cantilever to oscillate freely until damping effects caused the oscillation to stop.

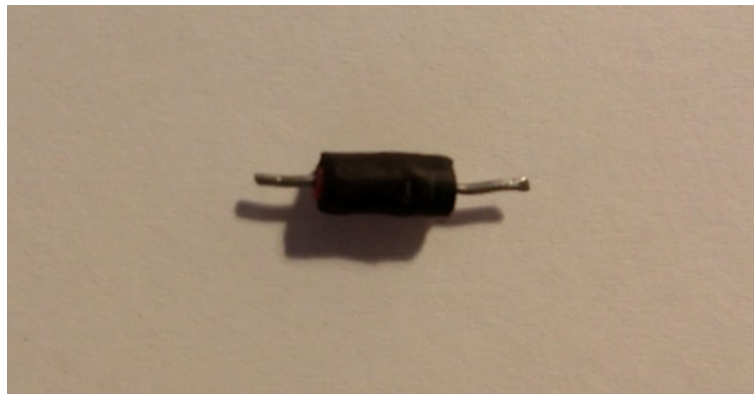


Figure 3-7 – Photograph of ‘shear pin’

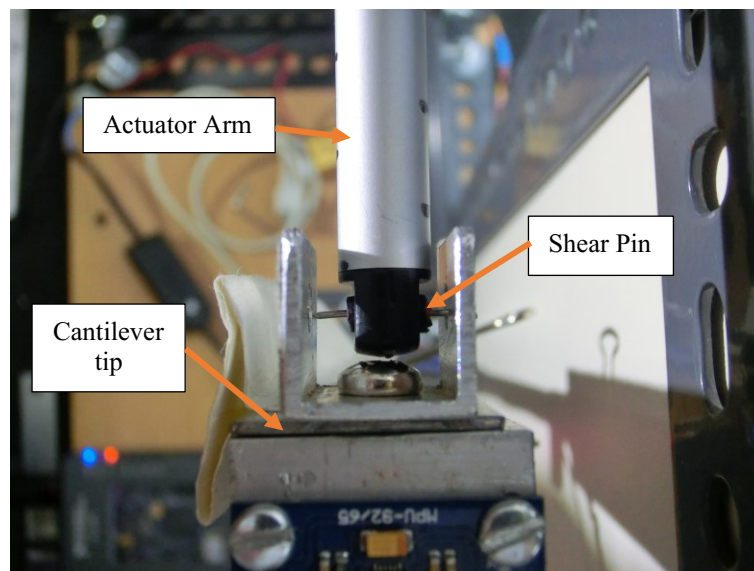


Figure 3-8 – Photograph of cantilever beam ‘shear pin’ in position at the tip of the cantilever

A detailed description of the experimental setup for Experiment Three can be found in Chapter Seven.

3.3.4 Experiment Four – Ballistic loading of cantilever beam

The response observed in the first three experiments was produced by a linear actuator, and although it was possible to produce a pseudo-dynamic response by suddenly releasing the actuator load, it was important to confirm that the proposed technique worked when using a dynamic response created by a short-duration load. The fourth experiment was similar to Experiment Three in that a dynamic response was generated by subjecting the tip of the cantilever to a horizontal force, but instead of using the linear actuator, a series of bullets were fired at a target that had been secured to the top of the cantilever (see Figure 3-9). The sudden load applied by the bullet caused the cantilever to oscillate in a similar manner to that observed in Experiment Three.

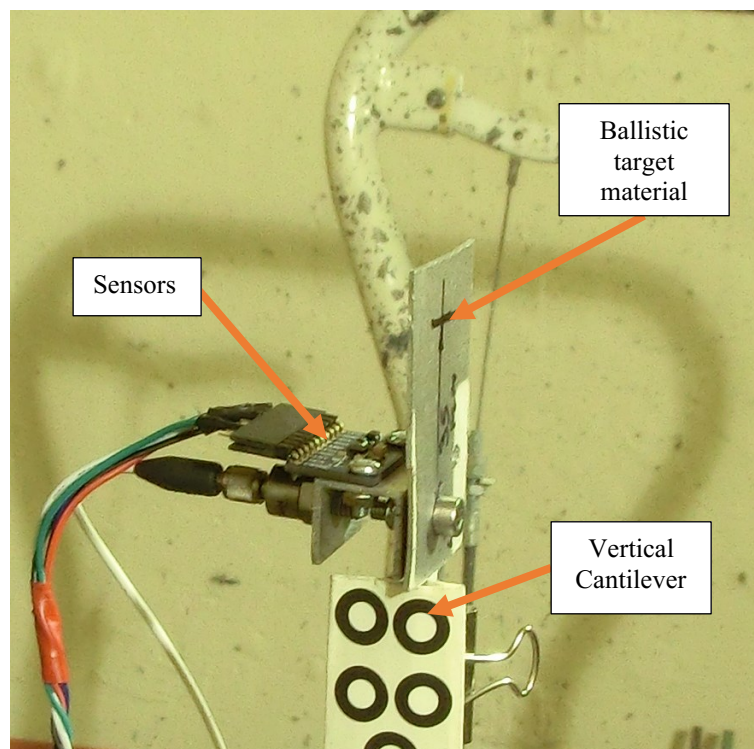


Figure 3-9 – Photograph of ballistic target at the tip of the cantilever

A detailed description of the experimental setup for Experiment Four can be found in Chapter Seven.

3.3.5 Experiment Five – Ballistic loading of free-standing monolith sign

The first four experiments made use of simple experimental arrangements that had been designed to produce a response that could be measured in a laboratory environment. Through the author's work on a commercial project the opportunity arose to test the proposed sensor arrangement on a practical test sample arrangement. This fifth, and final experiment was conducted to investigate the response, and ultimately the stability, of a vertical monolith sign when it was subject to a ballistic attack (Figure 3-10).

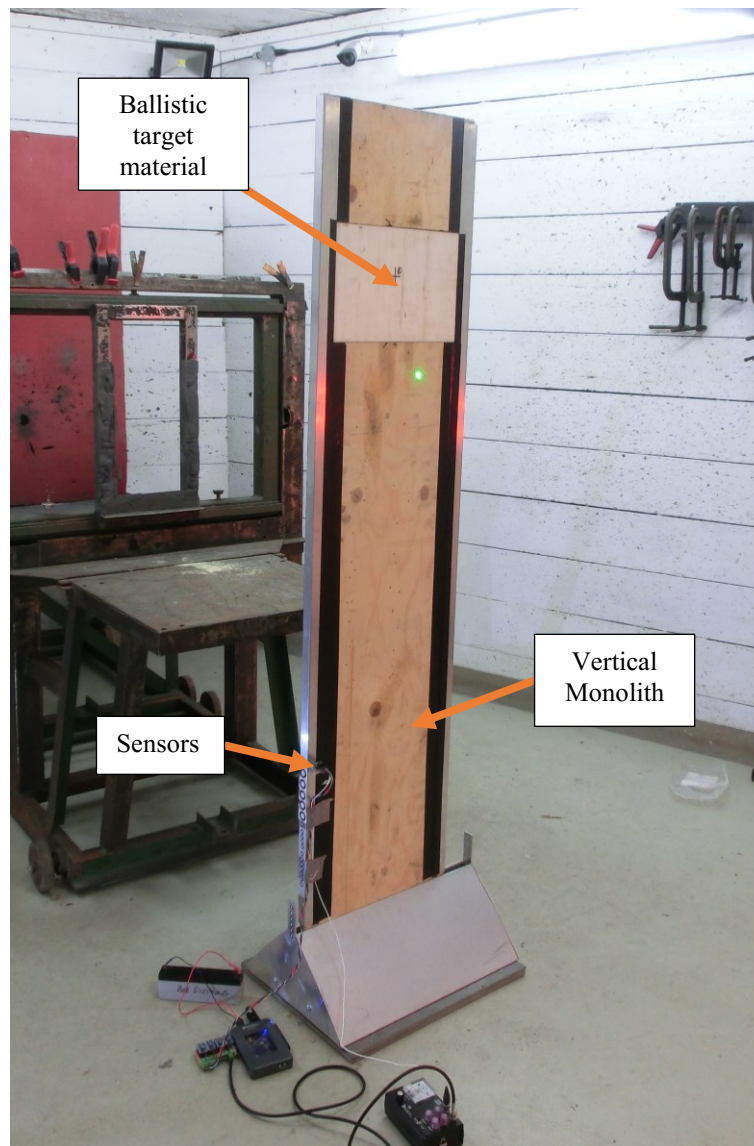


Figure 3-10 – Photograph of vertical monolith sign used in Experiment Five

Similar to Experiment Four, the vertical monolith panel was subject to an impact load from a high-velocity bullet fired at a section of ballistic resistant material towards the top of the panel. The dynamic response of the panel following the ballistic loading was measured using the same sensors used to measure the deflection of the inverted

pendulum in Experiment One and monitored using a remote HSV camera. A detailed description of the experimental setup for Experiment Five can be found in Chapter Seven.

3.4 Measuring the response

The response characteristic displayed by each of the experimental arrangements was that of displacement or rotation and therefore sensors that could measure these characteristics were investigated. Where the structure under test is located within a fixed structure, measurements are usually taken in relation to a robust fixed point that is isolated from the test sample. In the case of individual components such as walls or windows this may be a purpose-built reaction structure or cubicle that supports or holds the component. For larger structures, such as complete buildings, any fixed point is likely to be external to the test structure and depending upon how the load is being applied, will need to be isolated from the effects of the load. Where the sample being tested moves, or the response being observed is relative to the sample, sensors may be placed in contact with the sample and any subsequent response is therefore recorded in relation to the initial response. For the purpose of this research those sensors that are either situated at a distance and not fixed to the structure under test, or that are fixed to the structure and a fixed reference point, are defined as *remote sensors*, while those sensors that are only fixed to the structure under test are defined as *contact sensors*.

3.4.1 Remote sensor – fixed both ends

Examples of sensors that measure response by determining the difference between a fixed reference point and one on the test sample include Linear Variable Differential Transformers (LVDT) and string potentiometers. The theory is shown in Figure 3-11.

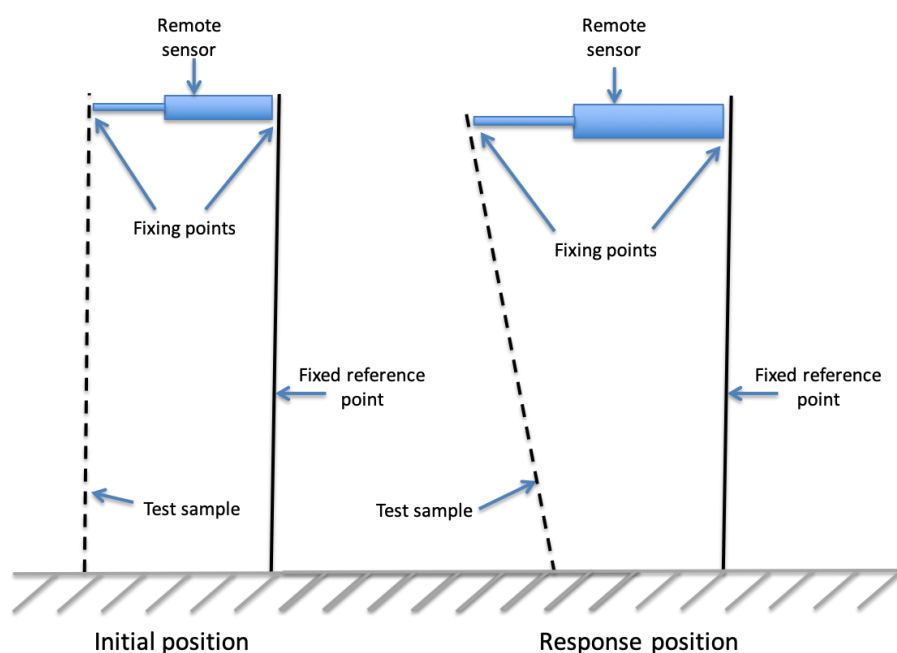


Figure 3-11 – Remote sensing when sensor is fixed at both ends

The challenge is to ensure that the rigid mounting point does not move and that the fixings are sufficiently robust. Many of these type of sensors are also limited in terms of the maximum response that they can measure which may limit their use for large structures or test samples. For this reason, and the fact that many of the sensors were quite expensive (greater than £100), it was decided that they would not form part of the investigation.

3.4.2 Remote sensor – fixed to reference point only

Examples of sensors that are positioned on a fixed reference point and then use some means to ‘sense’ a response include laser, ultrasonic distance sensors and photogrammetric sensors such as video cameras and DIC. The theory is shown in Figure 3-12.

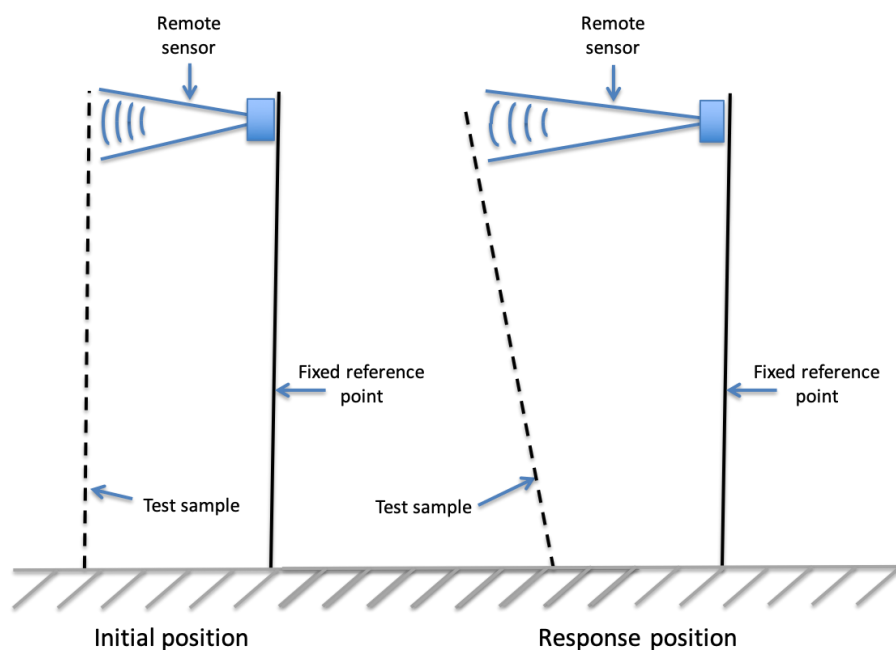


Figure 3-12 – Remote sensing when sensor is only located on fixed reference point

Unlike remote sensors that are fixed at both ends, remote sensors that are only fixed to a fixed reference point are not as constrained in terms of the maximum distance they can measure; there are range limitations, but these are usually due to sensor power and resolution rather than any physical aspect of the sensor.

3.4.2.1 Laser distance measuring sensors

Laser distance measuring sensors use the principle of laser beam triangulation to measure the distance between the sensor and object being measured as shown in Figure 3-13 (Figure 3-14 is a photograph of the sensor itself). The distance, or response, is determined by calculating the angle between the emitted laser beam and the line projected back from the spot created by the beam. Precision optics project the image of the ‘spot’ back onto an array of charge-coupled device (CCD) sensors, the data from which is analysed to determine any change in angle, and hence position. Objects closer to the sensor will create a large angle whilst those further away will produce a smaller angle.

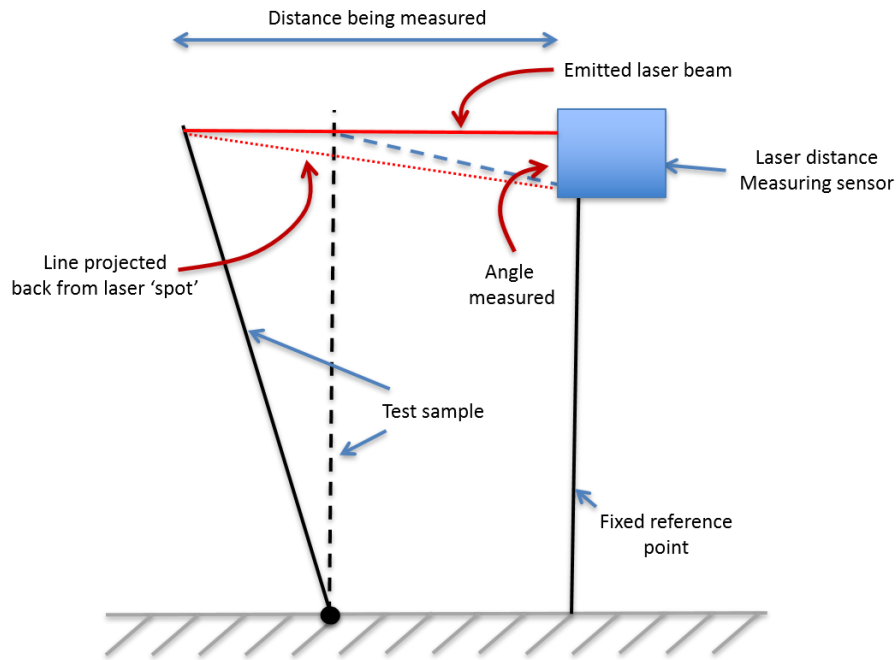


Figure 3-13 – Laser distance measuring operation

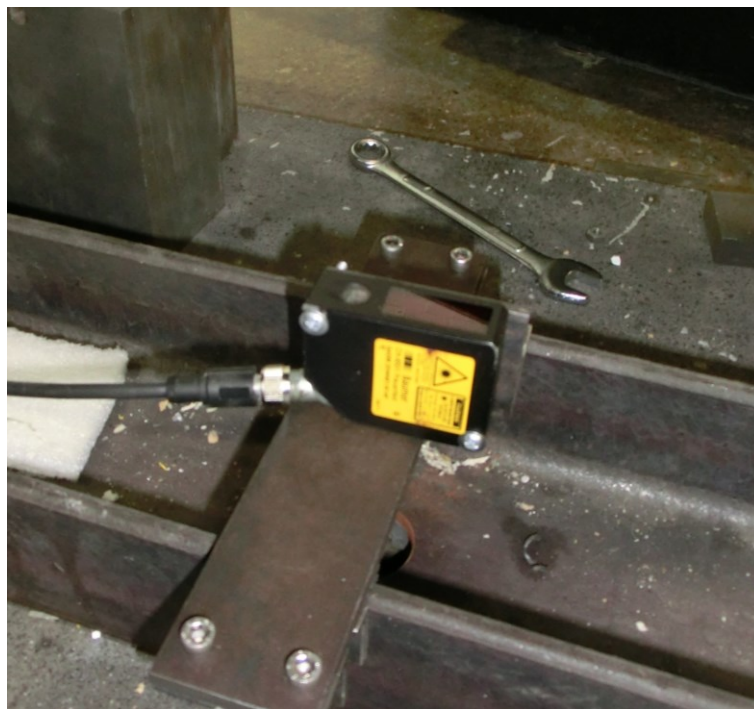


Figure 3-14 – Laser distance measuring sensor

A laser distance measuring sensor was borrowed for the initial laboratory tests conducted at Imperial College, however due to operational requirements it was not possible to extend the loan for the home tests. As the cost of purchasing a replacement was decided to be too expensive, their use was not investigated further. Additionally, the previous use of such sensors in short-duration shock and vibration tests meant that unless they were to be used in a different manner, their use in the research would not have added any novel element.

3.4.2.2 Ultrasonic distance measuring sensors

Ultrasonic sensors use ultrasound waves to measure a variety of properties including fluid flow rates and translational displacement (Morris & Langari, 2016). Ultrasound waves are found in frequency bands greater than 20 kHz and as such cannot usually be detected by humans. Ultrasonic sensors usually consist of a transmitter and a receiver. The time taken for the wave to travel between the two units can be used to determine any change in the property being measured. Alternatively the change in frequency or phase of the wave may be used (Morris & Langari, 2016). An example of a simple ultrasonic distance measuring sensor is shown in Figure 3-15. The transmitter and receiver are mounted side-by-side and the distance is calculated based upon the time it takes for the ultrasound wave to be reflected off the object being measured, and the reflected signal being received by the receiver as shown in Figure 3-16.



Figure 3-15 – Simple ultrasonic distance measuring sensor

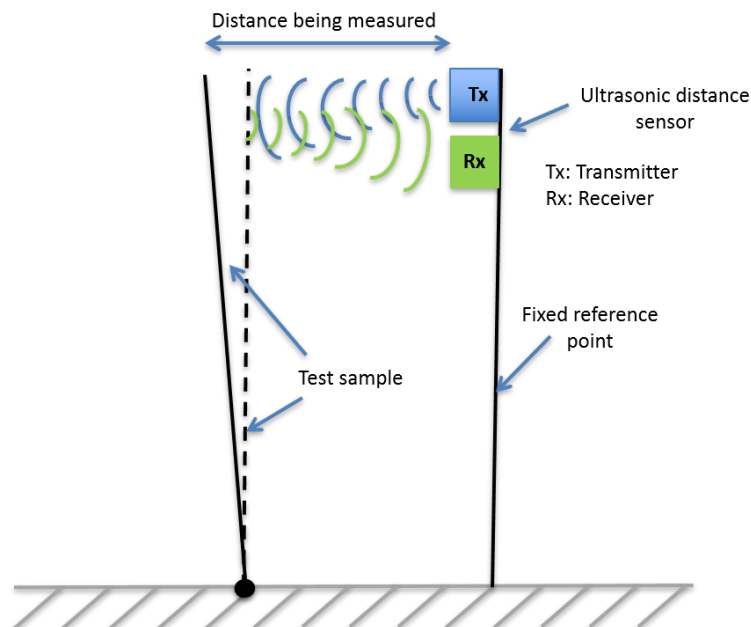


Figure 3-16 – Ultrasonic distance measurement operation

Some initial experiments were conducted using the sensor shown in Figure 3-15 however due to limitations on the range over which the sensor could be used (20 mm to 4000 mm), and variability in the results that were recorded, it was decided not to investigate the use of such sensors any further.

3.4.2.3 Photogrammetric and digital imaging systems

Photogrammetric and digital imaging systems use video or still images and special algorithms to analyse the difference between neighbouring video frames or still images. In its simplest form a fixed camera is used to capture the movement of an object and individual frames are manually analysed to determine the extent of any change. Such a system was used by Stephens ((Stephens, 2008)) to measure the deflection of laminated glass when subject to a blast load using a HSV camera. An angled mirror was used to view the rear face of the glass so that the camera could be in a position where it would not be damaged should the glass fail (Figure 3-17).

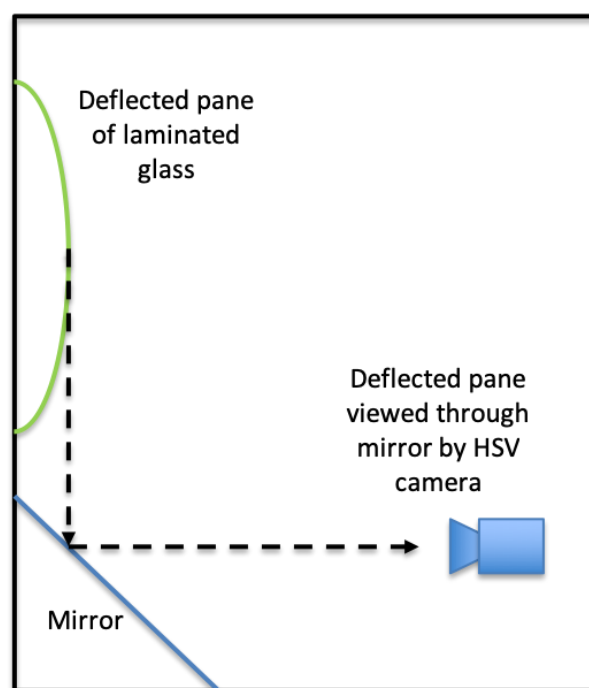


Figure 3-17 – Experimental setup for laminated glass deflection measurement

To enable the extent of the deformation to be calculated, a clear screen with a grid pattern marked upon it (Figure 3-18), was positioned at right angles to the mid-point of the rear face of the glass (Figure 3-19). This calibration image was superimposed on the individual frames of the HSV file and an associated displacement calculated.

Although this technique provided a new way of measuring the displacement of a sample subject to short-duration blast loads, the HSV images were easily corrupted by extraneous light from the detonating explosive and a significant amount of post-processing was required to obtain a displacement time-history of an individual point. Advances in analysis software, such as the GOM Correlate (GOM mbH, 2017) software used to analyse the HSV data for the experimental monitoring, meant that hand-processing was no longer required.

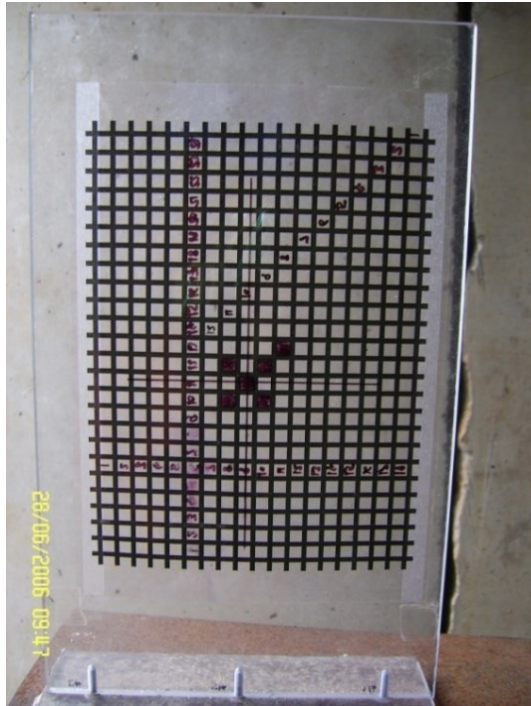


Figure 3-18 – Calibration screen used to calibrate HSV images



Figure 3-19 – Positioning calibration screen to calibrate HSV images

Although this form of photogrammetric technique was briefly considered, based upon the author's practical experience of its limitations in applications where there was the potential for intense light, it was decided that it would not be used as part of the sensor system under investigation but instead only as an independent means of monitoring each of the experiments. An explanation of its use in this context has been included later in this chapter in the section on the sensors and instrumentation used.

3.4.2.4 Digital image correlation (DIC)

DIC is a specialised photogrammetric technique that makes use of synchronised HSV cameras to measure the response of areas of test samples rather than individual points. Similar to other photogrammetric techniques, the use of HSV means that DIC can also be influenced by external factors such as extraneous light and vibration. It requires careful setup and configuration, and although it is being more widely used in short-duration experimental research, its expense meant that it was not a viable option for the proposed research scenario. Therefore, it was decided not to investigate DIC any further.

3.4.3 Contact sensors

Unlike remote sensors, contact sensors have the advantage of not requiring an external reference or fixing point as they are physically attached to the structure under test. However, in order that the structural response can be determined, the location of such sensors must be known, either in terms of their exact position or in terms of their initial orientation. Sensors that fall into the category of contact sensors include magnetometers, accelerometers, gyroscopes and strain gauges.

3.4.3.1 Magnetometers

Magnetometers are used to measure changes in the strength of the earth's magnetic field and consequently a change in response. Although they can be used by themselves, they are more often used in combination with other sensors such as accelerometers and gyroscopes to improve the attitude or orientation result provided. As the magnetic field varies depending upon where you are in the World, magnetometers need to be carefully calibrated prior to use. If they are to be attached to an object that is either itself largely made of steel or will be located in the proximity of a large steel structure, additional calibration measures are required. As it was proposed that one of the main applications of the sensors developed by this research was to monitor large civil engineering structures, it was decided that because of the problems related to using these sensors in such environments, they would not be investigated any further.

3.4.3.2 Accelerometers

As the name suggests, accelerometers are sensors that are designed to measure the acceleration of a body or structure. Most accelerometers consist of a mass suspended within a housing by a spring and some form of damping arrangement (Morris & Langari, 2016). The accelerometer is rigidly attached to the item to be measured so that any accelerations experienced are transmitted to the mass. Any accelerations cause the mass to generate a force that is then measured by some form of internal sensor. Some typical forms of accelerometer are shown in Figure 3-20 (Usher & Keating, 1996).

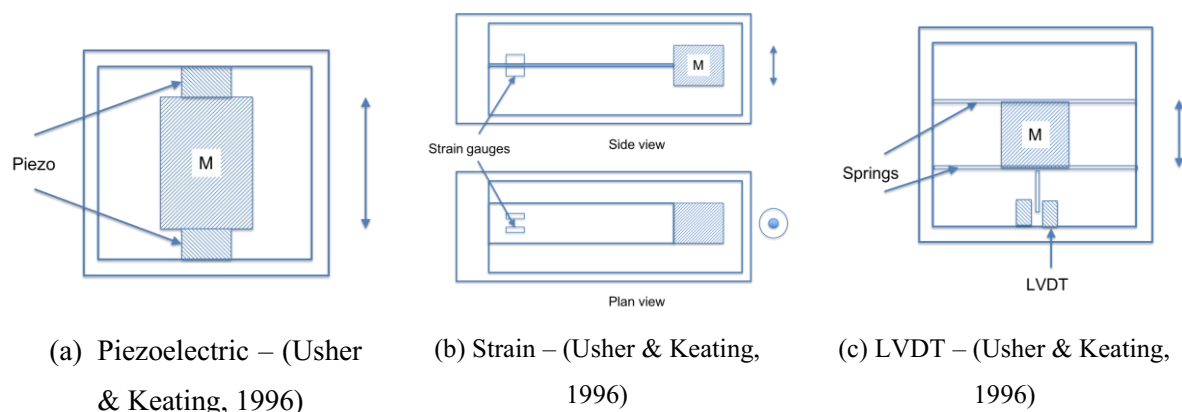


Figure 3-20 – Common forms of accelerometer

Each of the accelerometer types shown in Figure 3-20 measures a response using the principles of Newton's second law (where F is force in Newtons (N), M is mass in kg and a is acceleration in m/s^2):

$$F = M \times a \quad (3-1)$$

Accelerometers are usually placed so that they can measure the acceleration experienced by each 'degree of freedom' (DOF) of the system being investigated. Improvements in technology, such as those seen in the development of microelectromechanical systems (MEMS) sensors, now means that a single integrated chip (IC) can be used to house multi-degree of freedom sensors. Therefore, where it would have been traditional to mount three accelerometers at the point of interest, it is now possible to mount a single sensor to measure the acceleration in the x, y and z axes.

3.4.3.3 Gyroscopes

Gyroscopes are used to measure the rate of rotation of the subject, either in deg/s or rad/s and as with accelerometers, gyroscopes are usually orientated so that they measure the rate of rotation about the principal axes. When integrated with respect to small time intervals it is possible to determine the angle of rotation of the object, however due to errors that accumulate the resultant angle is subject to drift from the true value. When used in conjunction with other sensors, such as accelerometers, it is possible to compensate for this; how this is achieved is explained in more detail in Chapter 4 where the specifics of data and sensor fusion are discussed.

3.4.3.4 Inertial Measurement Units (IMUs)

The advent of MEMS has enabled manufacturers to miniaturise sensors to such an extent that they are now used in many everyday objects such as smart phones and computer tablets. Many of the MEMS sensors on the market combine a number of different sensors into an integrated package further increasing their utility. The inertial measurement unit (IMU) is one such package that integrates a selection of common sensors to enable the output to be used for navigational purposes. The base units commonly combine a 3-axis accelerometer with a 3-axis gyroscope to produce a 6-DOF IMU. The more sophisticated versions add a 3-axis magnetometer and a barometer to the package to produce a 10-DOF IMU. Further enhancements include data processing, fusion and filtering within the MEMS chip itself in addition to the ability to read signals from other sensors such as Global Positioning Systems (GPS). Although a number of such units were considered as part of the initial research, the 9-DOF MPU9250 IMU chosen for the experiments was relatively inexpensive (£4) and simple to interface with the control software selected and is discussed in more detail in the instrumentation used section later in this chapter.

3.4.3.5 Strain gauges

Strain is the measure of the ‘*elongation per unit length*’ as represented by the following equation:

$$\epsilon = \frac{\delta}{L} \quad (3-2)$$

Where:

ϵ = strain

δ = elongation

L = original length

Traditionally strain was measured using strain gauges formed by mounted a thin wire in a zig-zag pattern on a thin backing film. More recently improved manufacturing techniques have enabled metal-foil or semiconductor to be developed (Morris & Langari, 2016). An example of a foil backed strain gauge similar to that used on the vertical cantilever is shown in Figure 3-21.

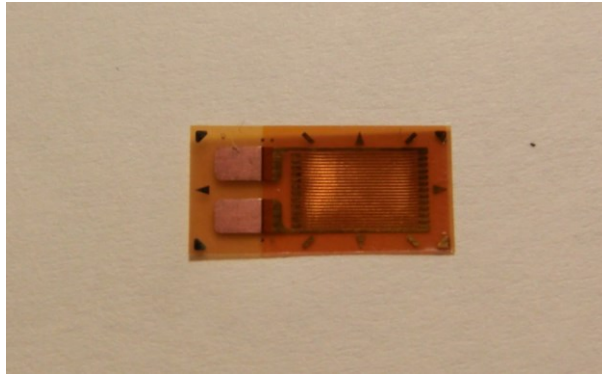


Figure 3-21 – Example of a foil backed strain gauges

Traditionally strain has been measured using small individual wire or foil strain gauges that must be carefully bonded to the surface of the item to be measured to ensure that there is a full transfer of the induced strains. As such strain gauges only cover a relatively small area, they can only measure very localised strain. Measuring the strain of a large object or surface therefore requires a large number of these strain gauges and the associated data logging facilities. Recent improvements in materials and technology has seen the introduction of fibre-optic strain gauges, Fiber Bragg Grating (FBG), which are able to measure the strain at a number of points along a structure such as a beam using a single sensor/reader. Due to the costs involved it was decided that it was not practical to consider the use of FBG strain gauges for this research.

3.5 Experimental arrangement

Schematics showing how the instrumentation was used for the different research experiments are presented in Figure 3-22; details of the instrumentation are described in detail in the following section. The block diagram in Figure 3-23 highlights the relationship between the different aspects of the experimental setup including the different software packages that were used. Experiment Five used the same sensors as Experiment One, but the load was produced by a bullet rather than a linear actuator. The load in Experiment Four was applied by a bullet rather than a linear actuator.

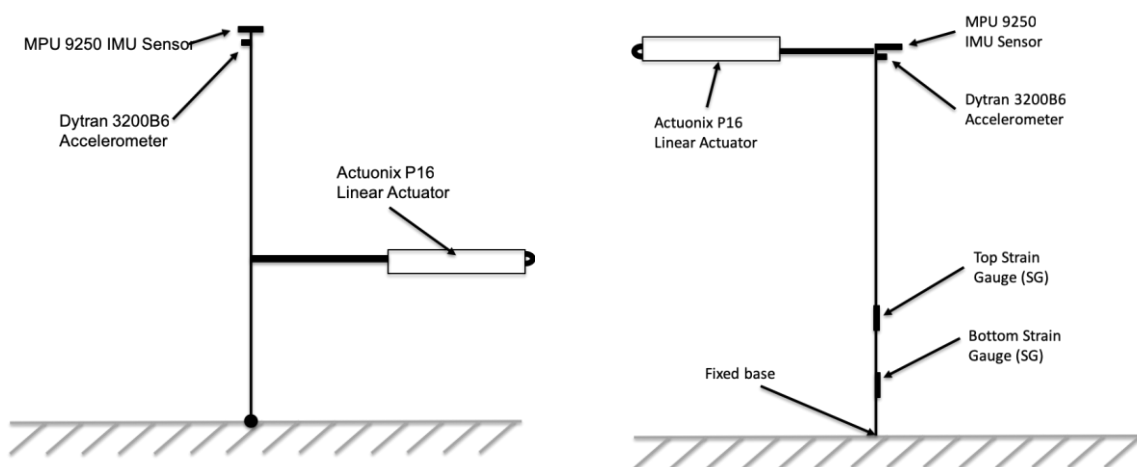


Figure 3-22 – Schematics of how the instrumentation was used in the experiments (Experiment One (left), Experiment Two, Three & Four (right))

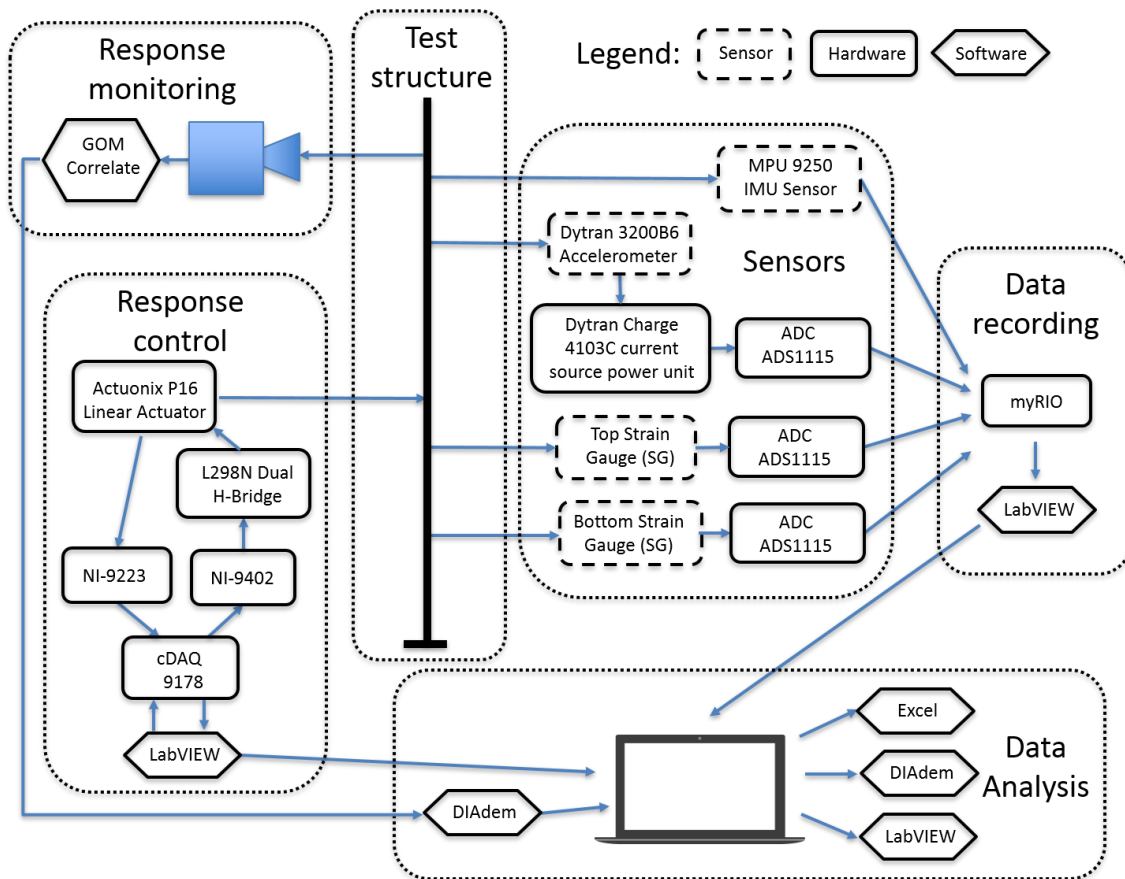


Figure 3-23 – Block diagram showing relationship between instrumentation

3.5.1 Analogue versus digital sensors

Analogue sensors produce a continuous signal whilst the signal from digital sensors is made up of a series of discrete readings as illustrated by the diagram in Figure 3-24, which shows the analogue signal as solid straight lines whilst the digital signal is represented by the collection of bars.

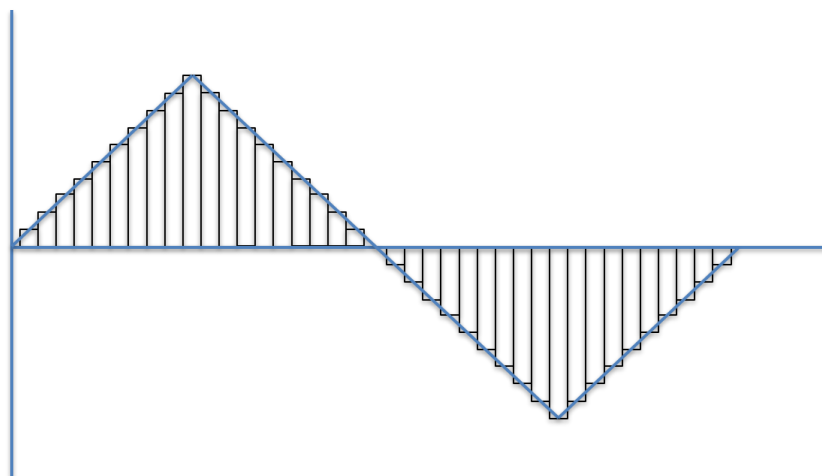


Figure 3-24 – Analogue and digital signals

Examples of a simple analogue and digital sensor are the mechanical dial gauge and digital protractor shown in Figure 3-25.



Figure 3-25 – Analogue dial gauge sensor (left) and digital protractor (right)

The dial gauge provides a continuous reading, albeit the accuracy is dependent upon how the dial is read, whilst the digital protractor provides readings in stepped intervals in this case to one decimal place of precision. Although analogue sensors have the advantage of providing a continuous sensor signal, the use of computers to record and analyse such signals, means that the analogue signals have to be converted to digital signals at some stage. The advent of more precise digital sensors means that the advantage of using an analogue sensor is reduced. In addition, analogue sensors usually require their own separate data logging channels whereas digital communication protocols, such as the inter-integrated circuit (I2C) protocol, enable multiple sensors to utilize a single data logging channel thereby increasing the number of sensors that can be handled by a single data logging device.

3.5.2 Inter-Integrated Circuit (I2C) communications

In addition to enabling more sensors to be logged by a data recording device, the I2C communications protocol enables each of the sensors to be more easily synchronized. The general arrangement for the I2C protocol is illustrated in the diagram in Figure 3-26.

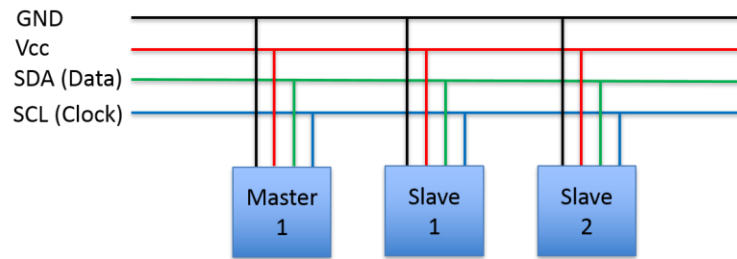


Figure 3-26 – General I2C arrangement

This shows a *master* device and two *slave* devices linked together by four wires; two for power (GND and Vcc) and two for data synchronisation and control (SDA and SCL). To enable multiple devices to be used each must have its own unique address. As many of the IMU devices considered for this research made use of the I2C protocol it was decided that this protocol would be adopted as it offered simple synchronisation and control of multiple sensors, and the flexibility to add additional sensors without the need for more data logging channels.

3.5.3 Sensors

3.5.3.1 Accelerometer

A single Dytran 3200B6 single axis accelerometer (Figure 3-27) was used as a means of comparing a more traditional sensor used in short duration loading tests, with the accelerometer readings obtained from the IMU. This piezoelectric accelerometer required a separate current source power unit in the form of a Dytran 4103C (Figure 3-28). The output signal was an analogue voltage which was input into an analogue to digital convertor (ADC) prior to being read by the datalogger. The sensitivity for the particular sensor was 1.86 mV/g which was used by the datalogging software to convert the raw signal into an acceleration in units of gravitational force g.



Figure 3-27 – Dytran single axis accelerometer



Figure 3-28 – Dytran Charge 4103C current source power unit

3.5.3.2 Inertial Measurement Unit

The 9-DOF InvenSense MPU 9250 IMU (InvenSense, 2016) was chosen as the main sensor for the investigation because of its low-cost, ease of configuration and flexibility (Figure 3-29).

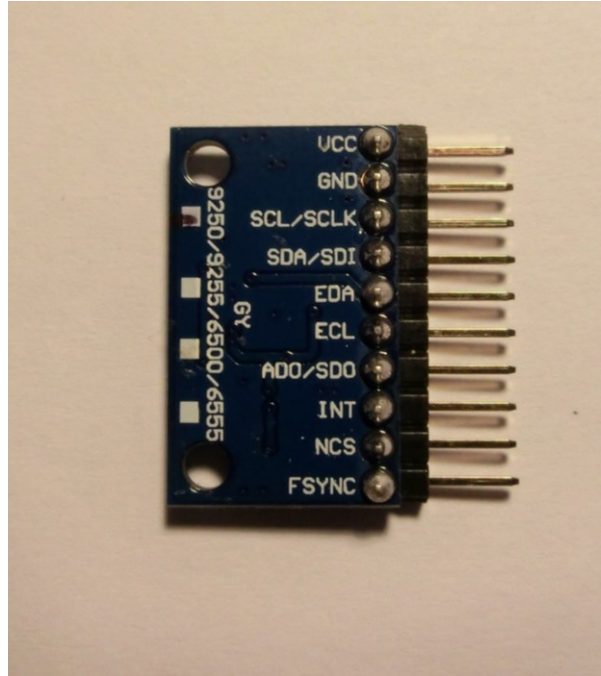


Figure 3-29 – Example of MPU9250 IMU used for experiments

The scale settings for each of the IMU's sensors are summarised in Table 3-1 and illustrate the flexibility available in terms of matching the sensor to the response environment that it might expect to be subject to.

Sensor	Full-scale range options
Gyroscope	± 250 °/s, ± 500 °/s, ± 1000 °/s, ± 2000 °/s,
Accelerometer	± 2 g, ± 4 g, ± 8 g, ± 16 g,
Magnetometer	± 4800 μ T

Table 3-1 – MPU 9250 Sensor scale options

The IMU was controlled using the I2C protocol and had the advantage of being able to be configured so that two versions of the same sensor could be used simultaneously by simply changing the input to the AD0/SDO pin and the address for the module. Although this feature was not used during the research, it provided the flexibility to utilise multiple sensors if required in the future. The magnetometer was not used during the research.

3.5.3.3 Analogue to digital convertors

Four 16-bit I2C ADC (ADS1115) (Texas Instruments, 2018) modules (Figure 3-30) were mounted on a specially designed board that plugged into the expansion ports of the myRIO datalogger. These ADCs were used to convert, and amplify, the analogue signals from the strain gauges and Dytran accelerometer into digital signals. An I2C ADC was used as this could be easily integrated with the IMU using a single I2C port. This had the advantage of meaning the different sensors all used the same timing clock ensuring that the signals were closely synchronised.

Each of the ADC modules had its own unique address and was initialised by setting the appropriate pins on the breakout board.

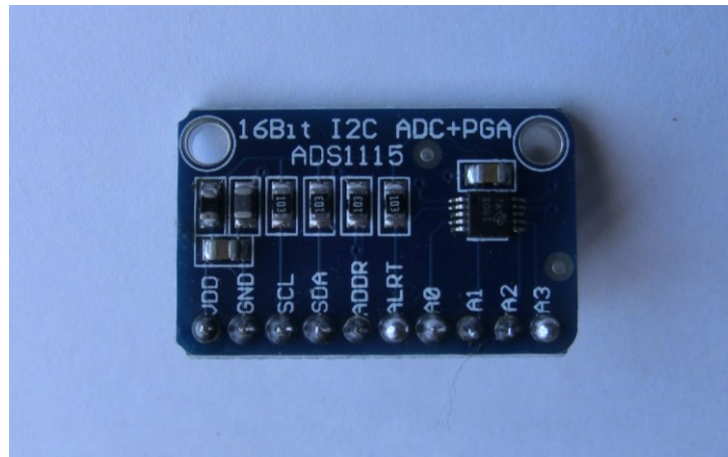


Figure 3-30 – Example of ADS1115 I2C ADC used for experiments

3.5.3.4 Strain gauges

Experiment Two, Three and Four measured the response of a vertical cantilever and made use of a series of strain gauges mounted in full bridge (Configuration 1) towards the base of the cantilever (see Figure 3-31) (National Instruments Corporation., 2016a).

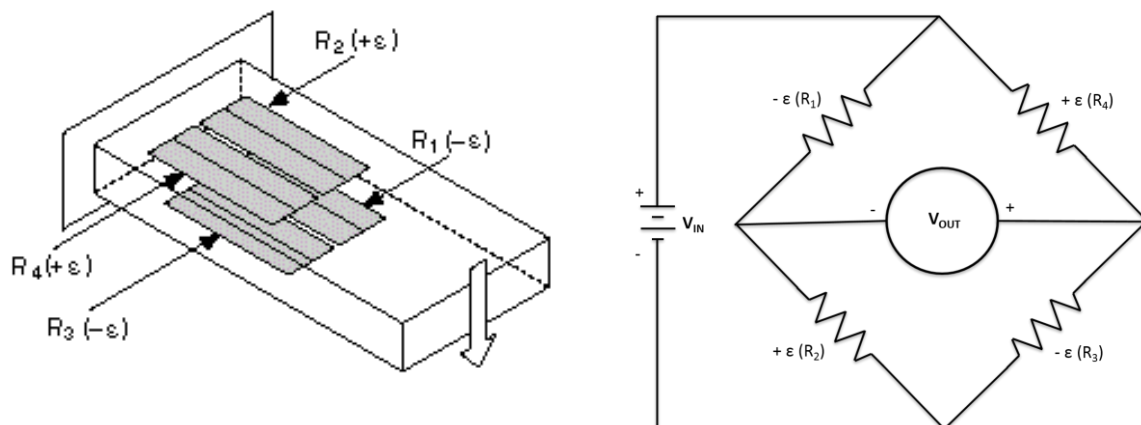


Figure 3-31 – Full bridge strain gauge - Configuration 1

The direct current (DC) excitation voltage V_{IN} was supplied by a battery powered DC power converter to minimise the potential for extraneous noise being introduced into the system by using an alternating current (AC) supply Figure 3-32.

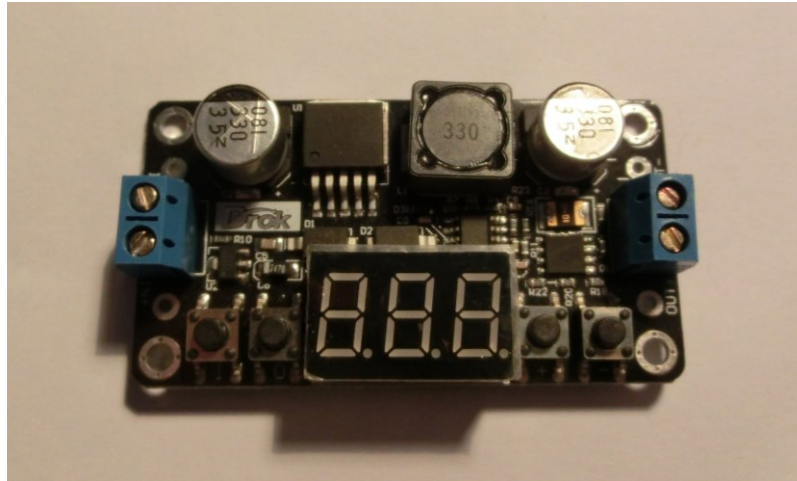


Figure 3-32 – DC Power Converter

Two full-bridge strain gauge arrangements, as shown in Figure 3-33, were used as this enabled the deflection/angle of rotation of the member to be calculated.

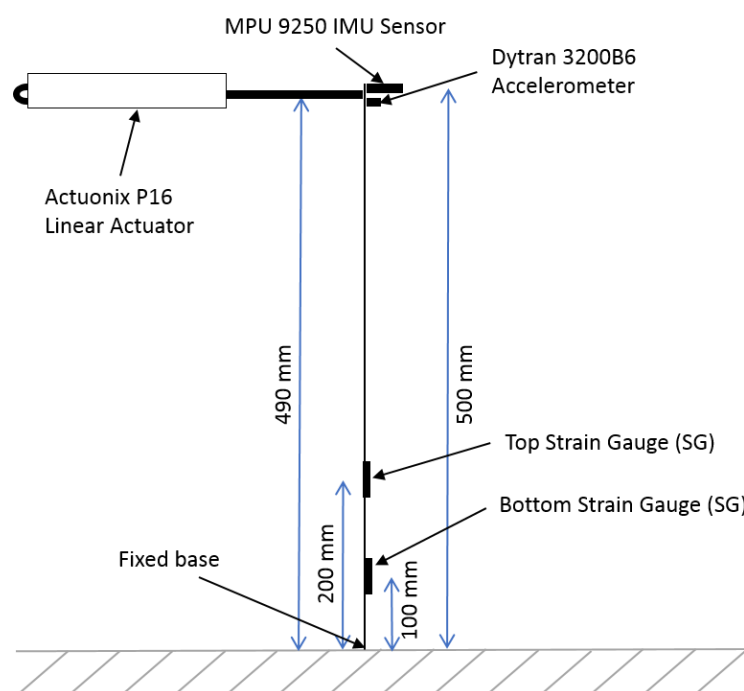


Figure 3-33 – Strain gauge positioning on vertical cantilever

The following section describes the theoretical model used to calculate the rotation at the tip of the cantilever based upon the strain measured in the two sets of full-bridge strain gauges at the bottom of the vertical cantilever. The basis of the model is the basic beam theory (Gere & Timoshenko, 1999) in relation to the deflection of a cantilever beam as shown in Figure 3-34 and Figure 3-35 which is applicable when deflections/rotations are small.

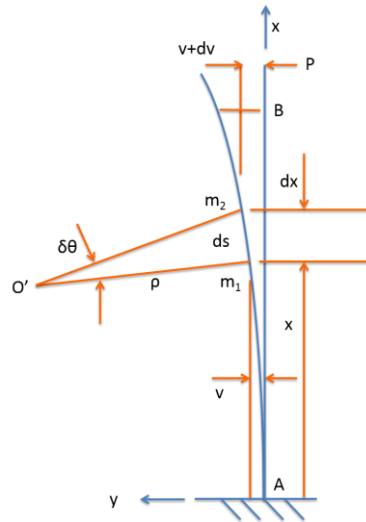


Figure 3-34 – Curvature of cantilever beam

Consider the curvature of a beam κ due to a point load P applied at the tip of the cantilever:

$$\kappa = \frac{1}{\rho} = \frac{d\theta}{ds} \quad (3-3)$$

Where ρ is the radius of curvature, $d\theta$ a small change in angle and ds is the respective small change in curvature. Consider the difference in deflection dv for the deflection v at a point x from the base of the cantilever as shown in Figure 3-35:

$$\frac{dv}{dx} = \tan\theta \quad (3-4)$$

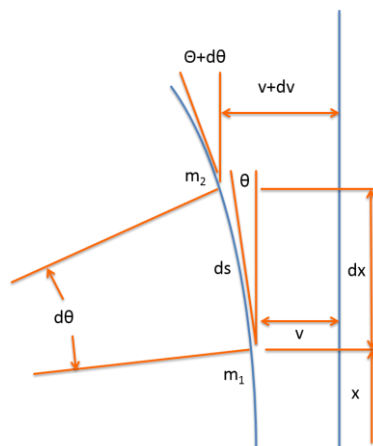


Figure 3-35 – Close-up of beam curvature shown in Figure 3-34

Rearranging in terms of the rotation at the tip of the cantilever θ :

$$\theta = \arctan \frac{dv}{dx} \quad (3-5)$$

for small rotation angles $\theta \Rightarrow \frac{dv}{dx}$. Taking the derivative of θ with respect to x gives:

$$\frac{d\theta}{dx} = \frac{d^2v}{dx^2} \quad (3-6)$$

Consider the moment-curvature equation (Gere & Timoshenko, 1999)

$$\kappa = \frac{1}{\rho} = \frac{M}{EI} \quad (3-7)$$

Equating equations 3-3, 3-6 and 3-7 gives:

$$\kappa = \frac{1}{\rho} = \frac{M}{EI} = \frac{d\theta}{ds} = \frac{d^2v}{dx^2} \quad (3-8)$$

Consider the beam bending for a free-body element of a small slice of beam carrying a uniform load $p(x)$ as shown in Figure 3-36 (based upon (Benham & Warnock, 1976; Kelly, n.d.)):

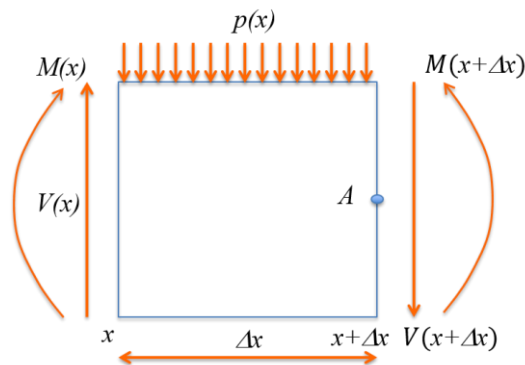


Figure 3-36 – Deflection of beam element

Taking the vertical equilibrium (where positive is downwards):

$$-V(x) + p(x)\Delta x + V(x + \Delta x) = 0$$

$$p(x)\Delta x = V(x) - V(x + \Delta x)$$

$$p(x) = \frac{V(x) - V(x + \Delta x)}{\Delta x} \quad (3-9)$$

Now consider moment equilibrium about the point A (where positive is anti-clockwise):

$$-M(x) - V(x)\Delta x + p(x)\Delta x \frac{\Delta x}{2} + M(x + \Delta x) = 0 \quad (3-10)$$

$$M(x + \Delta x) - M(x) = V(x)\Delta x - p(x) \frac{\Delta x^2}{2} \quad (3-11)$$

If the Δx^2 terms are ignored as they will tend to zero as the element width $\Delta x \Rightarrow 0$ thus Equation 3-11 becomes:

$$M(x + \Delta x) - M(x) = V(x)\Delta x \quad (3-12)$$

$$V(x) = \frac{M(x+\Delta x) - M(x)}{\Delta x} \quad (3-13)$$

And as the element width $\Delta x \Rightarrow 0$ then Equation 3-13 becomes:

$$V(x) = \frac{dM}{dx} \quad (3-14)$$

Consider the vertical cantilever shown in Figure 3-37. If the following assumptions are made:

$$P = V(x), dM = M_2 - M_1, dx = b - a$$

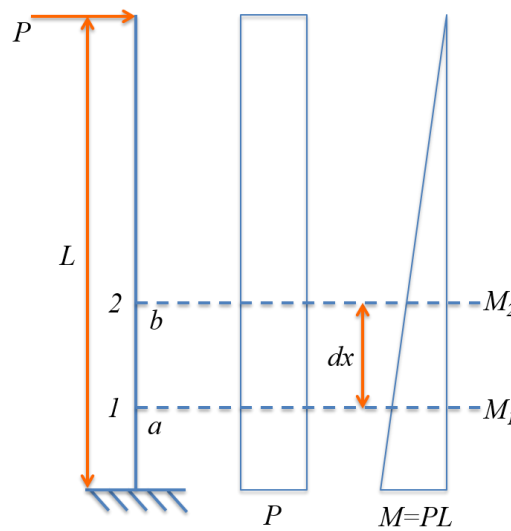


Figure 3-37 – Bending moment and shear force diagrams for vertical cantilever loaded with concentrated load

Then Equation 3-14 can be rewritten as:

$$P = \frac{M_2 - M_1}{b - a} \quad (3-15)$$

The bending stress in the beam can be calculated using:

$$\sigma = \frac{-My}{I} \quad (3-16)$$

Rearranging Equation 3-16 gives:

$$M = -\frac{\sigma I}{y} \quad (3-17)$$

Remembering Hooke's law:

$$E = \frac{\sigma}{\epsilon} \quad (3-18)$$

Rearranging Equation 3-18 gives:

$$\sigma = E\varepsilon \quad (3-19)$$

Inserting Equation 3-19 into Equation 3-17 gives:

$$M = \frac{-EI}{y} \varepsilon \quad (3-20)$$

Inserting Equation 3-20 into Equation 3-15:

$$P = \frac{EI (\varepsilon_1 - \varepsilon_2)}{y (b-a)} \quad (3-21)$$

The rotation of the tip of the cantilever beam is given by (Gere & Timoshenko, 1999):

$$\theta = \frac{Pl^2}{2EI} \quad (3-22)$$

Inserting Equation 3-21 into Equation 3-22 gives:

$$\theta = \frac{EI (\varepsilon_1 - \varepsilon_2)}{y (b-a)} \frac{l^2}{2EI} = \frac{(\varepsilon_1 - \varepsilon_2) l^2}{(b-a) 2y} \quad (3-23)$$

The neutral axis of the cantilever $y = t/2$ where t is the thickness of the cantilever. Inputting into Equation 3-23:

$$\theta = \frac{(\varepsilon_1 - \varepsilon_2) l^2}{(b-a) t} \quad (3-24)$$

The voltage recorded by the strain gauge, adjusted to correct for any offset, is calculated as follows:

$$V_r = \left(\frac{V_{SG}}{V_{ex}} \right)_{strained} - \left(\frac{V_{SG}}{V_{ex}} \right)_{unstrained} = \frac{V_{zero-shifted}}{V_{ex}} \quad (3-25)$$

Where V_{ex} is the wheat-stone bridge excitation voltage and V_{SG} is the voltage recorded at the output of the wheat-stone bridge. To enable the corrected voltage to be calculated, both unstrained and strained readings were taken. The corrected voltage readings were then used to calculate the strain using the following equation:

$$\varepsilon = \frac{-V_r}{GF} \quad (3-26)$$

Where GF is the ‘gauge factor’ for the strain gauge. Using Equation (3-26) the strain for the full-bridge strain gauge arrangements at the base of the vertical cantilever were given by the following equations:

$$\varepsilon_{Bot} = \frac{-V_{zero-shifted-Bot}}{GF \cdot V_{ex}} = \varepsilon_1 \quad (3-27)$$

$$\varepsilon_{Top} = \frac{-V_{zero-shifted-Top}}{GF \cdot V_{ex}} = \varepsilon_2 \quad (3-28)$$

Inputting Equation 3-27 and Equation 3-28 into Equation 3-24 gives:

$$\theta_{SG} = \frac{l^2 (\varepsilon_{Bot} - \varepsilon_{Top})}{t (x_{Top} - x_{Bot})} = \frac{(V_{zero-shifted-Bot} - V_{zero-shifted-Top}) l^2}{(b-a) \cdot t \cdot GF \cdot V_{ex}} \quad (3-29)$$

3.6 Response monitoring

A selection of HSV cameras was used to monitor the response of the experimental structures and to provide a ‘true representation’ that could be compared against the experimental data. Table 3-2 details the cameras used and summarises the relevant parameters, and the experiments for which they were utilised.

Camera	Frame rate	Resolution	Sensor type/size	Experiments
Casio EX-F1 (CASIO Computer Co Ltd, 2009)	30 – 1200 fps	6 Million Pixels	1/1.8-inch high-speed CMOS-Sensor	One, Two, Three, Four & Five
Casio EX-ZR100 (CASIO Computer Co Ltd, 2012)	30 – 1000 fps	12.10 Million Pixels	1/2.3-inch High-sensitivity CMOS sensor	Two
Photron Fastcam MC2 (Photron USA, 2014)	250 – 2000 fps	512 x 512 Pixels (262144 Pixels)	CMOS (Bayer system colour, single sensor) with 10 μ m pixel	Three

Table 3-2 – HSV Cameras used during research

Despite the reliability of using the digital imagery, there were a number of limitations related to the use of digital cameras, including HSV.

3.6.1 *Frame rate.*

The frame rate is usually quoted in terms of frames per second (fps). This governs the number of individual frames taken by the camera of the subject. A camera with a high frame rate usually means that a better image can be obtained for moving subjects. The disadvantage of a higher frame rate generally results in the resolution of the image being reduced.

3.6.2 *Lighting.*

Allied to the resolution of an image was the intensity and manner in which the subject was lit. This was particularly important when the video image was to be processed using image analysis and manipulation software. Such software packages rely upon sharp images with a balance of contrast between the different portions of the image. If the image was lit too brightly, or insufficiently, it was difficult for the software to distinguish key features.

3.6.3 *Glare on subject.*

If the object being filmed had a smooth or reflective surface there was the potential for glare to be created by any harsh or direct lighting which compounded the previously mentioned contrast problems. Angling or reducing the intensity of the illumination source reduced this.

3.6.4 *Subject image sharpness and contrast.*

The image sharpness and contrast were usually dictated by the various factors already discussed. It can also be affected by the optical quality, setup of the camera used and the focus accuracy. It was therefore important to

ensure that the HSV cameras were focused correctly prior to starting any experiments and checked periodically during the test programme.

3.6.5 Illumination source.

To minimise any effects that might be caused by the frequency of the light source, an LED lights source was used to illuminate Experiments One, Two and Three (Figure 3-38). In Experiment Four where halogen lights were used to illuminate the target a noticeable flickering was evident.



Figure 3-38 – LED light source

3.6.6 Image analysis software

GOM Correlate software was used to analyse the actual deflection of the different experimental response structures. The software can be used to calculate 2D or 3D movement of a structure depending upon the number of cameras and their orientation. As a single camera image only was recorded for all the experiments, only the 2D feature was utilised. A flow diagram summarising the process that was used to prepare each video file, and subsequently produce a data file that could be compared with the measured data is shown in Figure 3-39.

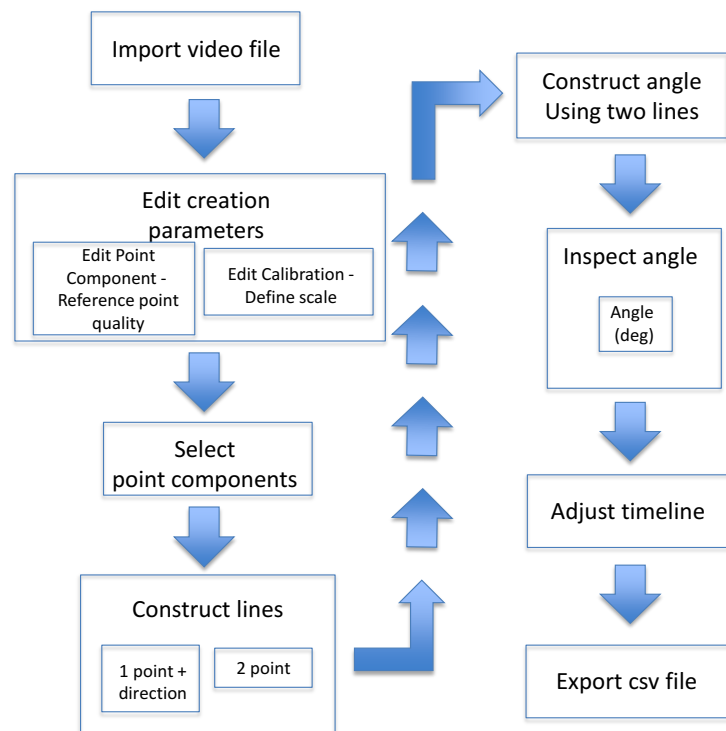


Figure 3-39 – GOM Correlate software data preparation and analysis process

A detailed description of the process is included at Annex A for reference.

3.7 Data recording

The data from the sensors monitoring the structural response of each of the experimental structures was recorded using a National Instruments (NI) myRIO data logger (National Instruments Corporation., 2013) (Figure 3-40). This multi-purpose datalogger had a selection of both analogue and digital input terminals enabling the user to use a wide variety of sensors. The myRIO also had a field-programable gate array (FPGA) that enabled rapid data logging and analysis of data using the onboard microprocessor; a feature that was utilised by the VIs developed for the research.



Figure 3-40 – National Instruments myRIO

To enable the analogue inputs from the Dytran accelerometer and strain gauges to be correlated with the inputs for the I2C IMU two special modules were made incorporating four ADCs (ADS1115) and terminals to accept the analogue and digital sensor inputs. These modules were designed to fit into the expansion ports on the myRIO. Power for the I2C IMUs was also supplied from these modules. An image of the modules and schematic of the circuit used are shown in Figure 3-41 and Figure 3-42 respectively.

The NI myRIO data logger and control module was programmed using NI's proprietary design and analysis LabVIEW⁴ software. This versatile software was programmed using a graphical user interface (GUI) and a library of VIs. Each VI was interconnected to form any process that the user wished. An example of the sub-VI created to read the data from the ADS1115 ADC is shown in Figure 3-43.

The sensor data was processed by the FPGA in batches which were stored in a first-in-first-out (FIFO) buffer and transferred by direct memory access (DMA) from the FPGA to the Host; in this case the myRIO onboard computer. This data was then saved to a USB memory drive that was plugged into the myRIO USB socket.

⁴ LabVIEW – Laboratory Virtual Instrument Engineering Workbench

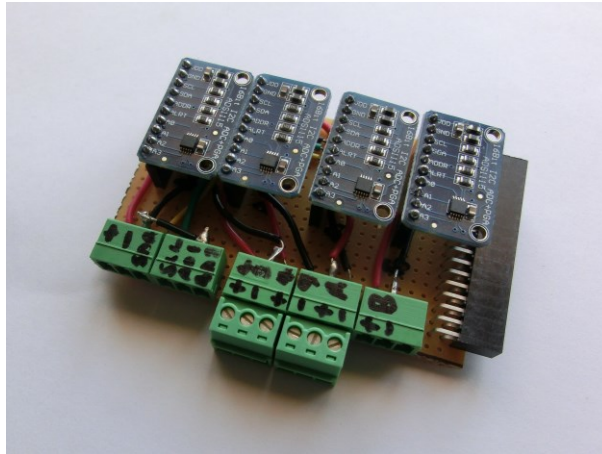


Figure 3-41 – Photograph of myRIO expansion module

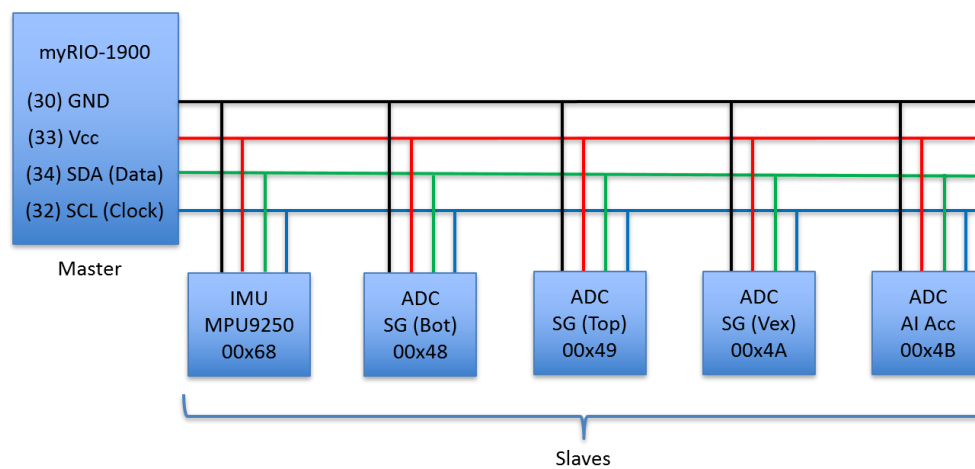


Figure 3-42 – Schematic of myRIO expansion module

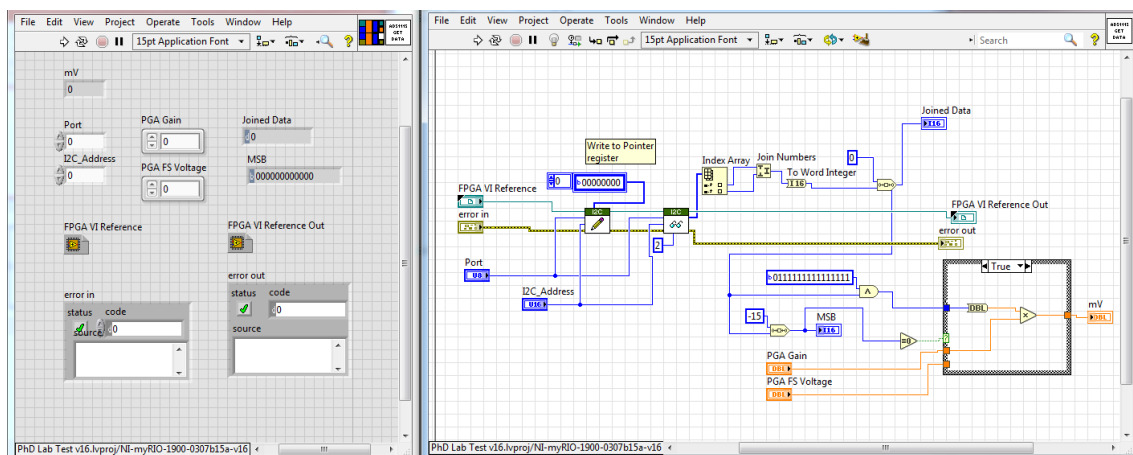


Figure 3-43 – LabVIEW sub-VI created to read data from the ADS1115 ADC

To ensure no data was lost, the amount of data to be transferred during each cycle had to be carefully controlled by adjusting the amount of data transferred and how quickly this occurred. If too many elements were transferred, and the buffer was not of the appropriate size, then data could be lost. Equally, if the transfer rate was too fast,

and the buffer was not completely filled with data, or too many elements had been requested to be transferred, there could be gaps in the data. A schematic of the process is shown in Figure 3-44.

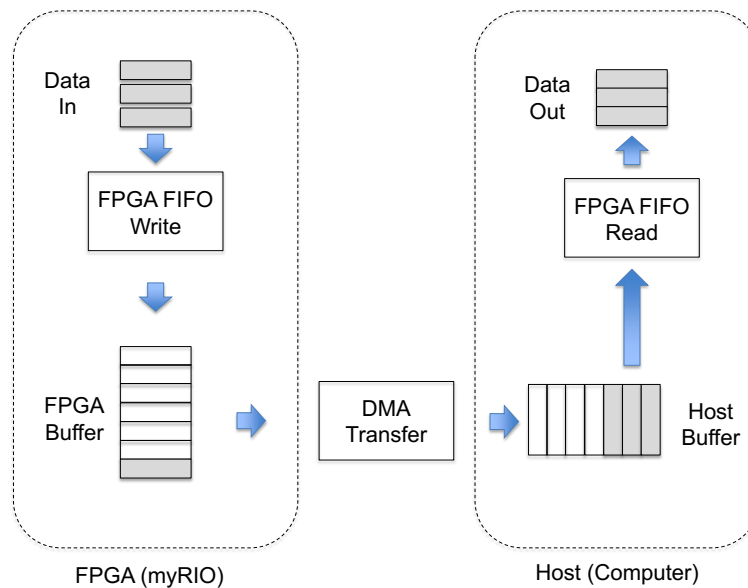


Figure 3-44 – Schematic of FPGA data transfer process

3.8 Response control

The response for Experiments One, Two and Three was produced using a linear actuator that was controlled, and its position recorded using a separate data acquisition (DAQ) device. The different elements of the system are described in the following section and shown schematically in Figure 3-45.

3.8.1 Data acquisition and actuator control

The linear actuator was controlled using a 4-channel NI-9402 Digital Input/Output (DI/O) module (Figure 3-46 (Left))(National Instruments Corporation, 2016b) mounted in an 8-slot NI Compact Data Acquisition (cDAQ) 9178 chassis (Figure 3-46 (Centre))(National Instruments Corporation, 2016c). One channel of the NI-9402 module produced a pulse-width modulated (PWM) signal while two of the other channels controlled the actuator direction of travel.

The position of the actuator arm was monitored using a 4-channel NI-9223 Analogue Input module (Figure 3-46 (Right))(National Instruments Corporation, 2016a). One channel of the NI-9223 module recorded the difference between the positive and negative actuator reference rails, while a second recorded the voltage from the wiper. LabVIEW was also used to create a VI to control and record the actuator movement using the cDAQ 9178 chassis.

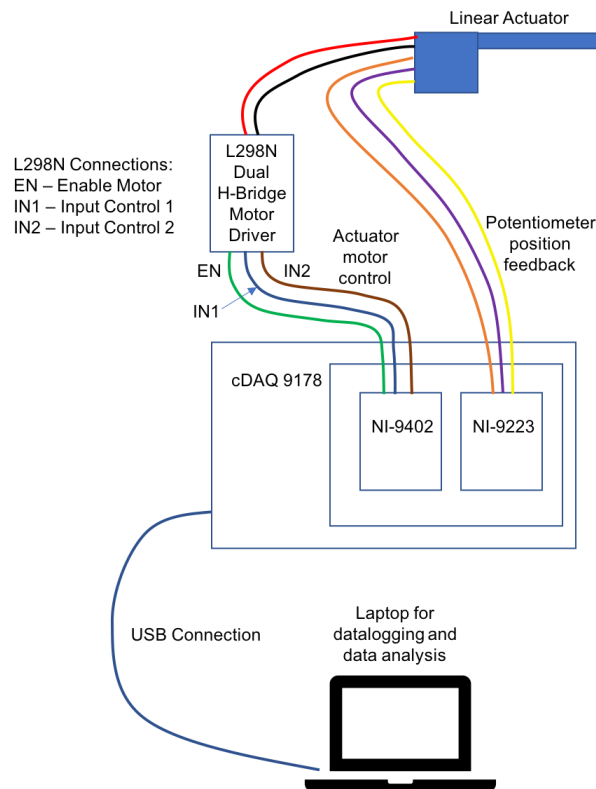


Figure 3-45 – Schematic of actuator control and logging system



Figure 3-46 – NI-9402 (Left), CDAQ 9178 (Centre), NI-9223 (Right)

3.8.1.1 Linear actuator

An Actuonix P-16-200-22-12-P miniature linear actuator was used to provide linear movement for the experiments (Figure 3-47). This actuator had a 200mm stroke length, maximum ‘no load’ speed of 46mm/s and an in-built potentiometer providing an analogue feedback signal of the actuator’s position (Actuonix, 2016).

3.8.1.2 L298N Dual H-Bridge Motor Drive

The movement of the actuator was controlled using an L298N Dual H-Bridge Motor Drive module (Figure 3-48) (STMicroelectronics, 2000) which enabled the direction of the actuator motor to be controlled using a combination of pulses sent from the NI-9402 digital module.

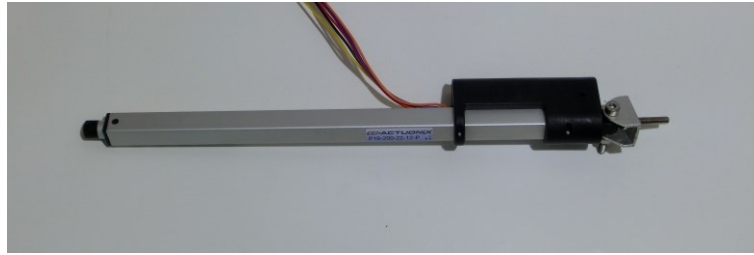


Figure 3-47 – Actuator P-16-200-22-12-P Linear Actuator

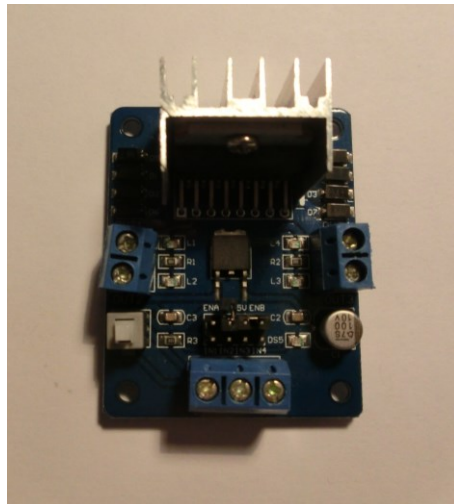


Figure 3-48 – L298N Dual H-Bridge Motor Drive

3.9 Data analysis

Three software packages were used to analyse the data from the sensors and the HSV camera.

3.9.1 *DIAdem*

DIAdem was the primary software package for analysing the data produced by the research (National Instruments Corporation., 2015). This software package was developed by the National Instruments Corporation to enable the management and analysis of large amounts of instrumentation data. It was used to analyse and compare the experimental test data and to produce graphs of appropriate results.

3.9.2 *LabVIEW software*

Although the LabVIEW software was primarily used for data logging it was also used to create VIs to analyse the IMU sensor signals and perform filter functions. Examples of the VIs created for filter and signals analysis are described in subsequent chapters.

3.9.3 Excel spreadsheet

Microsoft Excel (Microsoft Corporation, 2018) was used in scenarios where it was necessary to undertake simple calculations or data analysis that did not warrant the use of DIAdem or LabVIEW.

3.10 Sampling rate

The sampling rate was set by adjusting the timing loop period in the data logging VI (see Figure 3-49).

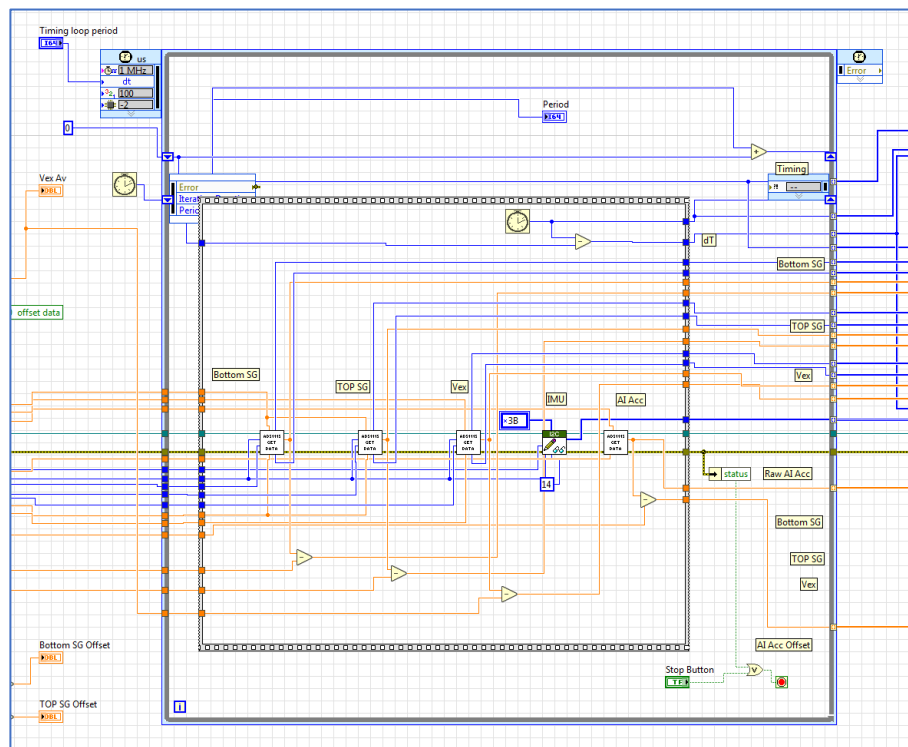


Figure 3-49 – Timing Loop Control used in data logging VI

Following initial tests, it was determined that using a timing loop period of 1000μs and clock speed of 1MHz provided a reasonable balance between accuracy without producing overly large data files. The number of samples transferred from the FPGA depended upon the number of processes taking place within the timing loop. This meant that more samples were transferred for the Experiments One and Five tests, where data was only being recorded for the IMU and analogue accelerometer sensors, and the other three experiments where data was recorded for these sensors and the strain gauges. This translated into approximately 100 samples per second (S/s) for Experiments One and Five and 35 S/s for Experiments Two, Three and Four.

3.11 Chapter conclusions

Following the thought process outlined in this chapter it has been shown how a simple experimental procedure was established to enable a multi-sensor system to be developed to measure the response of simple structures subject to short duration loads. How the data from this system was fused together to provide an improved result,

and the methods used to quantify the errors and accuracy of the data are discussed in Chapters Four and Five respectively. Detailed discussions of the conduct of the individual experiments, presentation and subsequent analysis and discussion of the results are contained in Chapters Six and Seven.

Chapter 4 Data Fusion

4.1 Introduction

Accurately measuring the response of a structure is a significant aspect of the research, and although it has been shown that this can be achieved using a variety of sensors by themselves, it is possible to enhance the result by combining the data from different sensors using data or sensor fusion. The concept of data fusion is not new and has been applied to many fields but there is limited evidence of its use for applications involving short-duration loads. This chapter provides a brief background of the concept's use before evaluating the most common data fusion algorithms in the context of the research.

4.1.1 What is data fusion?

There are a variety of definitions used to define data fusion, and that developed for the Data Fusion community of the United States military by Franklin White in 1991 (White, 1991) is widely recognized as the accepted definition; *"A process dealing with the association, correlation, and combination of data and information from single and multiple sources to achieve refined position and identity estimates, and complete and timely assessments of situations and threats as well as their significance"*. A similar definition is provided by Hall and Llinas (Hall & Llinas, 1997) *"Data fusion techniques combine data from multiple sensors, and related information from associated databases, to achieve improved accuracies and more specific inference than could be achieved by the use of a single sensor alone"*. These definitions have their basis in the military domain where they were originally associated with technologies such as radar and missile guidance and tracking, but also battlefield analytics such as automated threat recognition systems (Hall & Llinas, 1997). They are equally applicable to civilian applications such as robotics, inertial navigation and biometrics where data fusion has become widespread.

The term 'data fusion' is quite broad often referring to the techniques of 'fusing' different types of information or data together. In the context of the research covered by this thesis it has been taken to refer to the fusion of data, specifically sensor and structural data, and therefore a more accurate description is 'multi-sensor data fusion' and it is this description that will generally be used unless the reference is to a more general application in which case the 'data fusion' descriptor will be used.

4.1.2 Concept of multi-sensor data fusion

Multi-sensor data fusion is not a new concept; as humans we employ multi-sensory data fusion on a daily basis, continuously using our senses to study the world around us then analysing these observations to decide what to do. Take the act of crossing a busy road: we start by observing our surroundings, we estimate the speed and distance of approaching vehicles, we use our knowledge of how quickly we can walk and we gauge the distance across the road. Based upon our prior experience and knowledge we fuse the data together to determine the optimum position and time to cross the road safely. The concept of multi-sensor data fusion is no different, except

that it usually involves a computer rather than human intervention, and in the context of response measurement generally involves the use of sensors whose data can be readily related.

An example of how multi-sensor data fusion might be applied using two sensors, that measure the position of an object, is shown in Figure 4-1. The relative accuracy of each of the sensors is represented by the length of the error bars; in this instance Sensor A is more accurate than Sensor B. In addition, this relative accuracy may also be used to provide an indication of the reliability of the sensor, and therefore how much value is placed in the sensors' reading. This is represented by the relative position of each of the sensor lines; as Sensor A is more accurate than Sensor B, it is therefore deemed to be more reliable and therefore any estimate that is calculated will be closer to Sensor A and also the actual position.

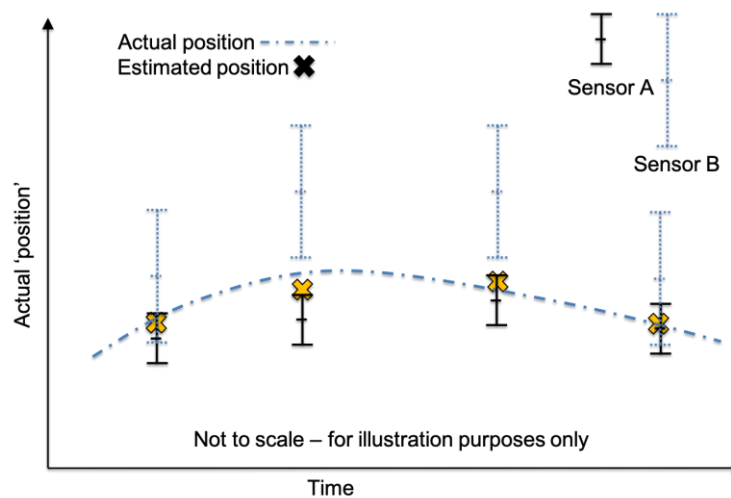


Figure 4-1 – Multi-sensor data fusion timeline

Conversely Sensor B is less accurate and therefore does not influence the estimated position as much, and when these readings are further away they also have less of an influence. Data fusion algorithms, such as the Kalman and Complementary filter that are discussed later in this chapter, all utilise some form of weighting function to proportion the degree of significance given to a particular sensor or system feature.

Another common feature of data fusion algorithms is the utilisation of previous data to influence future estimates. This is illustrated in the continuous measurement and estimation cycle shown in Figure 4-2. In this example after a measurement is taken it is compared to the update from the previous cycle before an estimate is made. How this process is handled is what generally defines the different data fusion algorithms and is discussed in more detail in the subsequent sections describing the different data fusion processes.

To enable the response of an object or structure to be measured it is necessary to estimate its attitude in relation to its environment; therefore, before discussing the data fusion processes investigated as part of this research it is important to understand how the attitude is estimated and described.

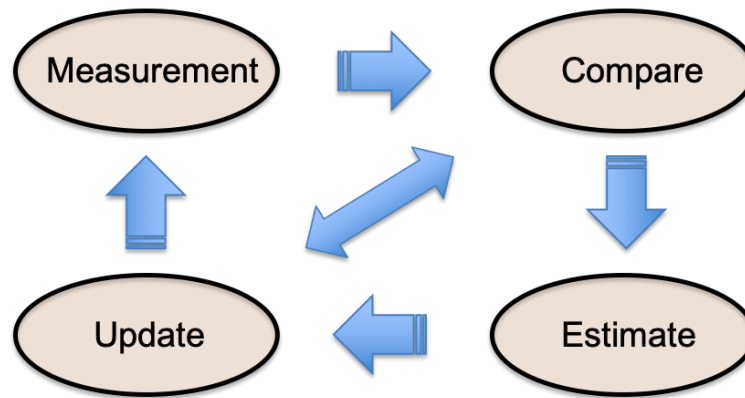


Figure 4-2 – General data fusion process

4.2 Attitude estimation

4.2.1 Euler angles

The attitude of an object or structure can be described using a variety of methods. A common approach used in navigational applications is based on the Euler Angles describing the Roll (Φ), Pitch (θ) or Yaw (ψ) of the vehicle or object being observed (see Figure 4-3 (Left)). In this example, commonly referred to as the Aerospace frame, the standard right-hand rule (Kuipers, 2002) is applied such that a positive Roll is shown as a clockwise rotation about the x-axis, a positive Pitch is shown as a clockwise motion about the y-axis and a positive Yaw is shown as a clockwise motion about the z-axis. The axes are orientated such that the x-axis points forwards, the y-axis points right and the z-axis points down.

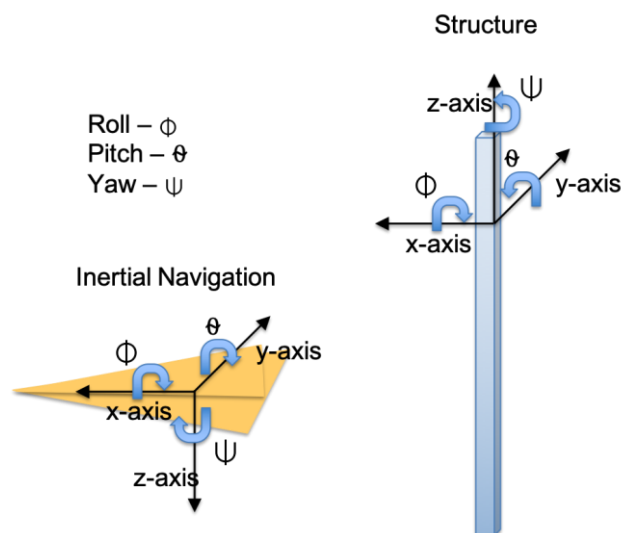


Figure 4-3 – Attitude angles and axes used

As the response measured in the various tests was a form of Pitch, a similar approach was proposed for describing the attitude and therefore the response of the structures tested (see Figure 4-3 (Right)). For the experimental tests the axes used changed slightly to match those of the IMU used; whilst the x and y axes remained unchanged, the z-axis was rotated through 180 degrees to point upwards. Similarly, the Roll and Yaw remained positive for a

clockwise rotation about the axes, however a positive rotation for the Pitch was now anti-clockwise about the y-axis. As the primary response measured in each of the experimental tests was Pitch, calculations for Roll and Yaw were not conducted during the analysis.

4.2.2 Calculation of pitch using acceleration

A variety of methods have been proposed in the literature to calculate the Pitch angle, however the one adopted for this research used accelerometer sensor readings. The simplest is based upon basic trigonometry as shown in Figure 4-4 (STMicroelectronics, 2010; Tuck, 2007):

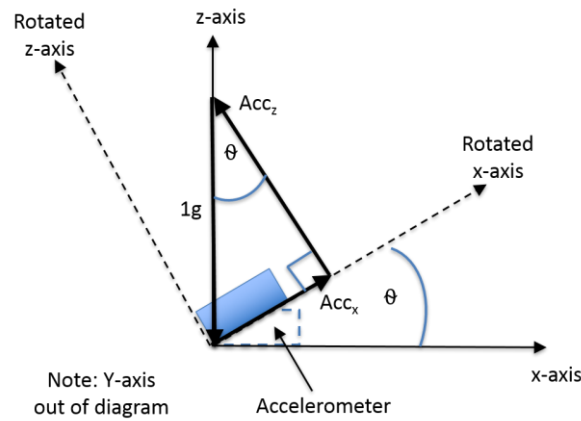


Figure 4-4 – Calculation of tilt angle using accelerometer readings

$$\sin \theta = \frac{Acc_x}{1g} \rightarrow Pitch = \theta = \sin^{-1} \left(\frac{Acc_x}{1g} \right) \quad (4-1)$$

Or:

$$\tan \theta = \frac{Acc_x}{Acc_z} \rightarrow Pitch = \theta = \tan^{-1} \left(\frac{Acc_x}{Acc_z} \right) \quad (4-2)$$

A more advanced trigonometric form involving all three accelerometer directions is given by (STMicroelectronics, 2010; Pedley, 2013; Tuck, 2007):

$$Pitch = \theta = \tan^{-1} \left(\frac{Acc_x}{\sqrt{Acc_y^2 + Acc_z^2}} \right) \quad (4-3)$$

A problem that can occur when using the Euler angles equations involve scenarios where the angles involved are extreme i.e. $\pm 90^\circ$ which result in singularities that cause ‘Gimbal Lock’ (Gebre-Egziabher, Hayward & Powell, 2004; Fresk & Nikolakopoulos, 2013; Pedley, 2013). This phenomenon occurs when two of the rotational axes align causing the ‘system’ to lock resulting in a loss of a degree of freedom for a 3D-system (Figure 4-5). Traditionally associated with mechanical gyroscopes (as illustrated in Figure 4-5), the phenomenon also applies to trigonometric rotation operations.

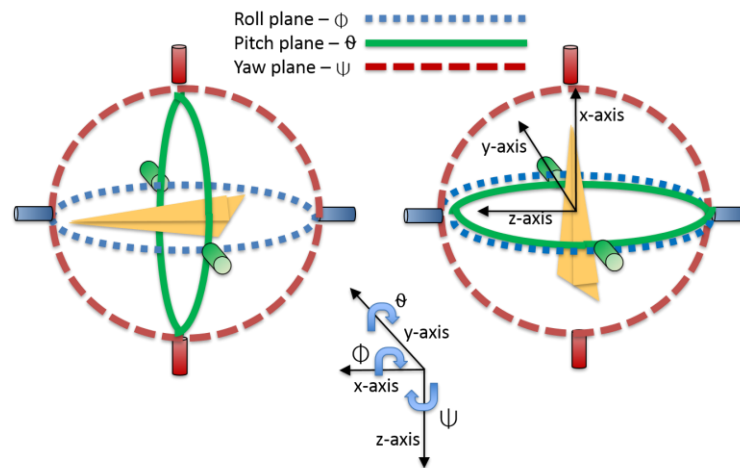


Figure 4-5 – Gimbal Lock

The scenario displayed in Figure 4-5 shows an aircraft that has pitched up 90° to such an attitude that it is no longer able to roll about its forward, x-axis because the roll and pitch planes have coalesced. To avoid this problem when using IMUs care must be taken to ensure that the orientation of the sensors is such that they are unable to operate near the regions where gimbal lock might occur, special algorithms are used to cover this eventuality. An alternative is to use quaternions.

4.3 Kalman filters

The Kalman Filter (KF) was originally developed by Rudolf Kalman in the 1960s (Kalman, 1960) and is a form of predictive filter that is used for linear filtering problems (Welch & Bishop, 2006; Goldenstein, 2004). A great deal of research has been conducted since its introduction with various forms being developed, however they are all based upon the general cycle shown in Figure 4-6.

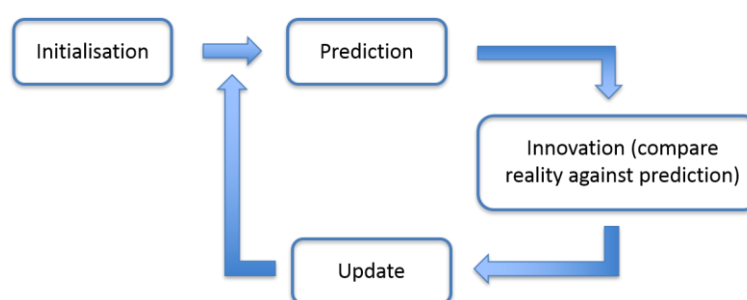


Figure 4-6 – General Kalman Filter cycle

Whilst the KF is appropriate for linear problems it is less reliable for non-linear scenarios. In these instances, the Extended Kalman Filter (EKF) is widely used although another variation of KF, the Unscented Kalman Filter (UKF), has also become equally prominent. The latter two forms of the KF are briefly described here due to their increased use in applications involving the rotation of rigid bodies, however due to their relative complexity (and hence increased computational overhead), the main discussions relate to the discrete KF which was the one used in the research.

4.3.1 Extended Kalman Filter

The EKF is used in situations where the linearity requirements of the KF make it difficult to model dynamics of the system being analysed. In these situations the non-linear problems are linearized to enable the normal KF to be used by using first order methods to approximate the non-linear elements (van der Merwe, Wan & Julier, 2012; Julier & Uhlmann, 1997). As noted by Julier and Uhlmann, the linearization process can cause instabilities in the filter, and the derivation of the Jacobian matrices is complex leading to significant application issues. As it was proposed that the algorithms could be deployed on microprocessors with limited computational power, it was with these limitations in mind that it was decided not to investigate the EKF any further.

4.3.2 Unscented Kalman Filter

The UKF is a variation of a Sigma-point Kalman Filter that uses a selection of sigma-points (sampling points), that are generated based upon the current estimate of the model's state and its covariance, and then translates these to the new state using non-linear transformation equations as shown in Figure 4-7.

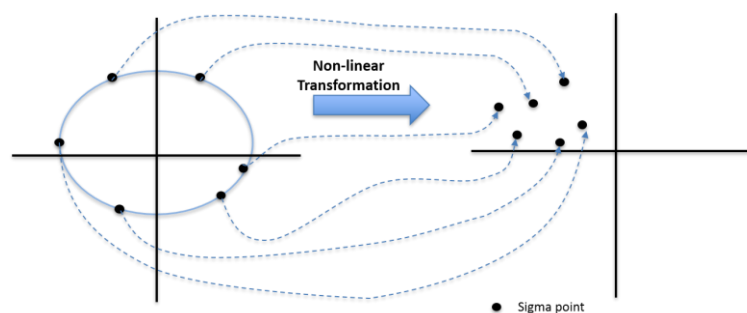


Figure 4-7 – The concept of the unscented transform

Goldenstein postulated that because the UKF used unscented transforms to translate the samples between different states instead of linearizing the non-linear problem, it was more accurate than the EKF (Goldenstein, 2004). Despite this perceived advantage, the additional requirements of choosing suitable sampling points and the associated non-linear transformation algorithms introduce an additional computational overhead, and therefore it was decided that the UKF would not be investigated any further.

4.3.3 Discrete Kalman Filter

Although KFs can be used to model both continuous and discrete time systems, it is the latter type of system that was considered for this research since the data collected had discrete time-steps rather than being continuous. The process cycle for a discrete KF is shown in Figure 4-8 and follows the general KF filter process presented in Figure 4-6. How the KF was implemented in the research, illustrated by its application to some test data, is discussed in Section 4.5.

KFs are generally applied to linear dynamic systems that have both measurement and process noise, or error, and that has a Gaussian (normal) probability distribution (Welch & Bishop, 2006). Although the literature contains

numerous examples of notation to describe the process, a combination of those proposed by Faragher, Czerniak and Salmony were adopted here for their relative simplicity and intuitive descriptions (Czerniak, 2015; Faragher, 2012; Philip Salmony, 2018), and follow the process illustrated in Figure 4-8 and described in the subsequent sections. The notation used is summarized, and where appropriate its purpose explained, in Table 4-1. To illustrate the process followed, a hypothetical example based upon measurement data obtained from a gyroscope (denoted as $(\dot{\theta})$) and an angle calculated from accelerometer data (denoted as (θ)) was chosen, and where appropriate relevant variables and constants shown (Gui, Tang & Mukhopadhyay, 2004).

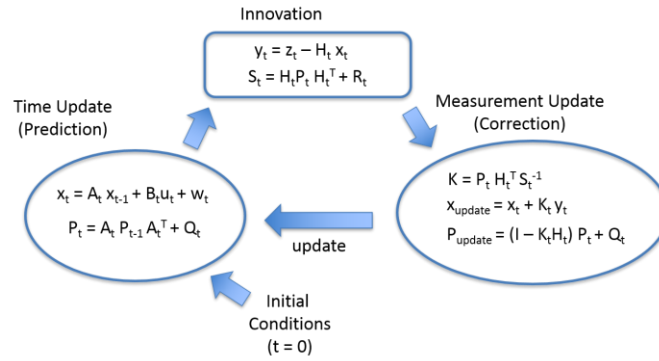


Figure 4-8 – Discrete Kalman Filter cycle

4.3.3.1 Initialization

At the start of the process the filter is initialized with the following variables and constants:

Variables: Control Vector (U_0), Measurement vector (Z_0), State update (x_0), Covariance update (P_0)

Constants: State transition matrix (A), Control matrix (B), Observation matrix (H), Estimated process error covariance (Q), Estimated measurement error covariance (R)

4.3.3.2 Kalman Filter Standard expressions

Equations 4-4 and 4-5 are the standard KF expressions used to describe the system state and measurement inputs:

$$x_t = A_t x_{t-1} + B_t u_t + w_t \quad (4-4)$$

$$z_t = H_t x_t + v_t \quad (4-5)$$

where the variables and constants are described in Table 4-1, and the process to implement them is described in the following sections. It should be noted that w_t and v_t are the process and measurement noise respectively which are included in the standard expressions but are incorporated into the covariance matrices Q and R respectively.

4.3.3.3 Prediction (Time update)

The first step is to update the state prediction using the previous (or initial state prediction) and the most recent measurement update as shown in Equation 4-6:

$$x_t = A_t x_{t-1} + B_t u_t \quad (4-6)$$

Type	Notation	Description	Remarks	Matrix Dimensions
Inputs	u_t	Control input vector	Indicates the magnitude of any control system or user's control on the situation	$(M \times 1)$
	z_t	Measurement input vector	Contains the real-world measurement received in the time step	$(M \times 1)$
Outputs	x_t	State update	Newest estimate of the current "true" state	$(N \times 1)$
	P_t	Covariance update	Newest estimate of the average error for each part of the state and how close the state is to the actual value	$(N \times N)$
Constants	A_t	State transition matrix	Used to get a prediction of the state for the next time step, by multiply with the state and control factors	$(N \times N)$
	B_t	Control matrix	Used to define the linear equations for any control factors	$(N \times M)$
	H_t	Observation matrix	Used to translate a state vector to a measurement vector	$(M \times N)$
	Q_t	Estimated process error covariance	Relates to process noise	$(N \times N)$
	R_t	Estimated measurement error covariance	Relates to measurement noise	$(M \times M)$
Intermediary variables	x_p	Current prediction of state		$(N \times 1)$
	x_{t-1}	Previous value of state		$(N \times 1)$
	$P_{predicted}$	Current prediction of covariance		$(N \times N)$
	P_{t-1}	Previous value of covariance		$(N \times N)$
	y_t	Innovation	Compare reality (the measured state) with the predicted state	$(N \times 1)$
	S_t	Innovation covariance	Compare the real error with the predicted error	$(M \times M)$
	S_t^{-1}	Innovation covariance inverse		$(M \times M)$
	K_t	Kalman gain	Used to moderate the prediction	$(N \times M)$
	I	Identity matrix		$(N \times N)$
Noise/Error	w_t	Process noise	May be Gaussian white noise or other random process	$(N \times 1)$
	v_t	Measurement noise	Usually unknown initially	$(M \times 1)$

Table 4-1 – Summary of Kalman Filter notation used

Where the different forms of the state update are $x_t = \begin{bmatrix} \theta \\ \dot{\theta}_b \end{bmatrix}_t$, the control vectors are $u_t = \dot{\theta}_t$, u_t is the gyro measurement in radians per second at time t , and $\dot{\theta}_b$ is the gyro bias, or the amount that the gyro has drifted. The state transition matrix is $A_t = \begin{bmatrix} 1 & -\Delta t \\ 0 & 1 \end{bmatrix}$ and the control matrix is $B_t = \begin{bmatrix} \Delta t \\ 0 \end{bmatrix}$ where Δt is the time between samples. This leads to:

$$x_t = \begin{bmatrix} 1 & -\Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta}_b \end{bmatrix}_{t-1} + \begin{bmatrix} \Delta t \\ 0 \end{bmatrix} \dot{\theta}_t = \begin{bmatrix} \theta_{t-1} + (\dot{\theta}_t - \dot{\theta}_{b,t-1})\Delta t \\ \dot{\theta}_{b,t-1} \end{bmatrix} \quad (4-7)$$

The prediction of the covariance P_t (or the amount of error in the prediction) is calculated using the previous covariance prediction and the latest update of process error Q_t as shown in Equation 4-8:

$$\mathbf{P}_t = \mathbf{A}_t \mathbf{P}_{t-1} \mathbf{A}_t^T + \mathbf{Q}_t \quad (4-8)$$

Where the error covariance matrix $\mathbf{P}_t = \begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix}$, the process error covariance $\mathbf{Q}_t = \begin{bmatrix} Q_\theta & 0 \\ 0 & Q_{\dot{\theta}_b} \end{bmatrix} \Delta t$, and Q_θ and $Q_{\dot{\theta}_b}$ are the accelerometer and gyro bias variances respectively. This gives:

$$\begin{aligned} \mathbf{P}_t &= \begin{bmatrix} 1 & -\Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix}_{n-1} \begin{bmatrix} 1 & 0 \\ -\Delta t & 1 \end{bmatrix} + \begin{bmatrix} Q_\theta & 0 \\ 0 & Q_{\dot{\theta}_b} \end{bmatrix} \Delta t \\ &= \begin{bmatrix} P_{00} + \Delta t(\Delta t P_{11} - P_{01} - P_{10} + Q_\theta) & P_{01} - \Delta t P_{11} \\ P_{10} - \Delta t P_{11} & P_{11} + Q_{\dot{\theta}_b} \Delta t \end{bmatrix} \end{aligned} \quad (4-9)$$

4.3.3.4 Innovation

The innovation step of the KF process is often combined with the measurement update step however it has been separated here to allow the individual components to be discussed. The innovation y_t compares reality in the form of the measurements with the current prediction:

$$y_t = z_t - \mathbf{H}_t x_t \quad (4-10)$$

Where the observation matrix $\mathbf{H}_t = [1 \ 0]$ which gives:

$$\begin{aligned} y_t &= z_t - [1 \ 0] \begin{bmatrix} \theta_{t-1} + (\theta_n - \dot{\theta}_{b_{t-1}}) \Delta t \\ \dot{\theta}_{b_{t-1}} \end{bmatrix} \\ &= z_t - (\theta_{t-1} + (\theta_n - \dot{\theta}_{b_{t-1}}) \Delta t) \end{aligned} \quad (4-11)$$

The innovation covariance $\mathbf{S}_t$ compares the real errors (associated with the measurements by $\mathbf{R}_t$), with the current state error described by the covariance matrix $\mathbf{P}_t$ as shown in Equation 4-12:

$$\begin{aligned} \mathbf{S}_t &= \mathbf{H}_t \mathbf{P}_t \mathbf{H}_t^T + \mathbf{R}_t \\ &= [1 \ 0] \begin{bmatrix} P_{00} + \Delta t(\Delta t P_{11} - P_{01} - P_{10} + Q_\theta) & P_{01} - \Delta t P_{11} \\ P_{10} - \Delta t P_{11} & P_{11} + Q_{\dot{\theta}_b} \Delta t \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \mathbf{R}_t \\ &= [P_{00} + \Delta t(\Delta t P_{11} - P_{01} - P_{10} + Q_\theta) \quad P_{01} - \Delta t P_{11}] \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \mathbf{R}_t \\ &= (P_{00} + \Delta t(\Delta t P_{11} - P_{01} - P_{10} + Q_\theta)) + \mathbf{R}_t \end{aligned} \quad (4-12)$$

4.3.3.5 Correction (Measurement update)

The final step before starting a new iteration is to update the Kalman Gain $\mathbf{K}_t$ and produce an update to the state x_{update} and its covariance $\mathbf{P}_{update}$. The Kalman Gain, which is used to moderate the errors, is calculated as follows:

$$\mathbf{K}_t = \mathbf{P}_t \mathbf{H}_t^T \mathbf{S}_t^{-1} \quad (4-13)$$

The new estimate in the form of the state update is:

$$x_{update} = x_t + \mathbf{K}_t y_t \quad (4-14)$$

The new error in the form of the covariance update is:

$$\mathbf{P}_{update} = (\mathbf{I} - \mathbf{K}_t \mathbf{H}_t) \mathbf{P}_t + \mathbf{Q}_t \quad (4-15)$$

For the next iteration of the filter $x_t = x_{update}$ and $\mathbf{P}_t = \mathbf{P}_{update}$

4.3.3.6 Adjustment of error and measurement covariance matrices $\mathbf{Q}$ and $\mathbf{R}$

Determining values for the measurement and process noise covariance is frequently discussed and analysed in the literature. The measurement noise covariance is generally the easier of the two to determine, as it is often possible to conduct off-line sampling of the sensors to determine the variance of the measurement noise. It is more difficult to determine the process noise as it is generally not possible to observe the process being modelled. In both situations it is possible to use a constant value for these variables and allow the filter to stabilise over time (Welch & Bishop, 2006). As the filter process was conducted using a LabVIEW VI as a separate exercise after the data had been collected, it was possible to adjust these values to see which produced the optimum result.

4.4 Complementary filter

Complementary filters (CF) combine data from different sensors by adjusting the proportions of the data used depending upon the reliability of the sensors. For attitude and tilt estimation using accelerometers and gyroscopes the basic process used is shown in Figure 4-9.

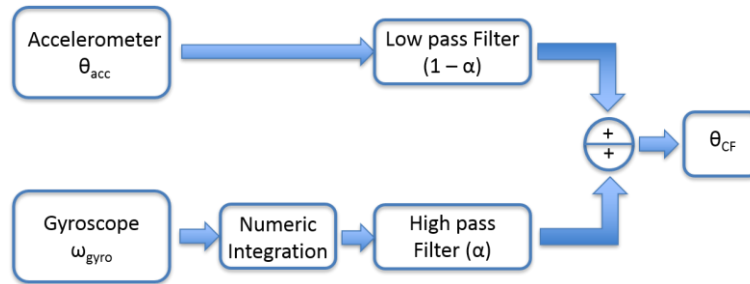


Figure 4-9 – Complementary filter process

Over a short time interval gyroscopic data is reasonably precise and not affected by external influences, however over longer periods errors accumulate which can cause the gyroscope results to drift. Conversely, accelerometers are affected by noise over a short time interval, but over longer periods are more reliable and can therefore be used to counter the effects of drift on the gyroscope.

4.4.1 General algorithm for Complementary Filters

The general algorithm that describes this is illustrated in Figure 4-9 and presented arithmetically in Equation 4-16.

$$\theta_{CF} = \alpha \times (\theta_{CF} + \theta_{gyro}) + (1 - \alpha) \times \theta_{acc} \quad (4-16)$$

Where θ_{CF} is the angle of tilt (in this research the pitch) being monitored, ω_{gyro} and θ_{acc} are the angles calculated using the data from the gyroscope and accelerometer respectively, and α is the coefficient used to adjust the sensitivity of the filter, calculated using the time coefficient τ and the sample period Δt as shown in Equation 4-17.

$$\alpha = \frac{\tau}{\tau - \Delta t} \quad (4-17)$$

As accelerometers cannot provide a direct reading of angle, the previously discussed Euler Angle equation, Equation 4-3 can be used to calculate θ_{acc} whilst for small time durations integrating the gyroscope readings ω_{gyro} with the sample period Δt , enable the angle due to the gyroscope θ_{gyro} to be calculated ($\theta_{gyro} = \omega_{gyro} \times \Delta t$). As with the KF, how the CF was implemented in the research, illustrated by its application to a set of test data, is discussed in Section 4.5.

4.4.2 Representing the Complementary Filter using quaternions

The general CF algorithm described in equation 4-16 is simple to implement, however it is susceptible to the previously discussed problems associated with singularities and Gimbal Lock. To avoid these issues a number of researchers have utilised quaternions to produce quaternion based CFs, notably Mahony, Euston and Madgwick. (Mahony, Hamel & Pflimlin, 2008; Euston *et al.*, 2008; Madgwick, Harrison & Vaidyanathan, 2011; Madgwick, 2010). A number of these utilised magnetometers as part of the filter process but as these were not investigated as part of this research these methods were briefly reviewed but not investigated further. The Explicit Complementary Filter (ECF) proposed by Mahony and Euston was initially investigated, however inconsistencies in the data generated using this method meant that only the data using the KF and CF data was subsequently analysed as part of the research. Information on quaternions and the basic concept of the ECF using quaternions is contained in Annex B for information.

4.5 Kalman and Complementary filter example

To illustrate how this and the other filter algorithms were applied to the research each of the algorithms described in the previous section were converted to LabVIEW VIs so that they could be used to analyse the test data, an example of which is shown at the end of the section. The diagrams in the following sections show the ‘block diagrams’ that mimicked the algorithms.

4.5.1 Kalman Filter

The KF described in Section 4.3.3 was converted to the LabVIEW VI shown in Figure 4-10.

4.5.2 Complementary Filter

The CF filter described in Section 4.4.1 was converted to the LabVIEW VI shown in Figure 4-11.

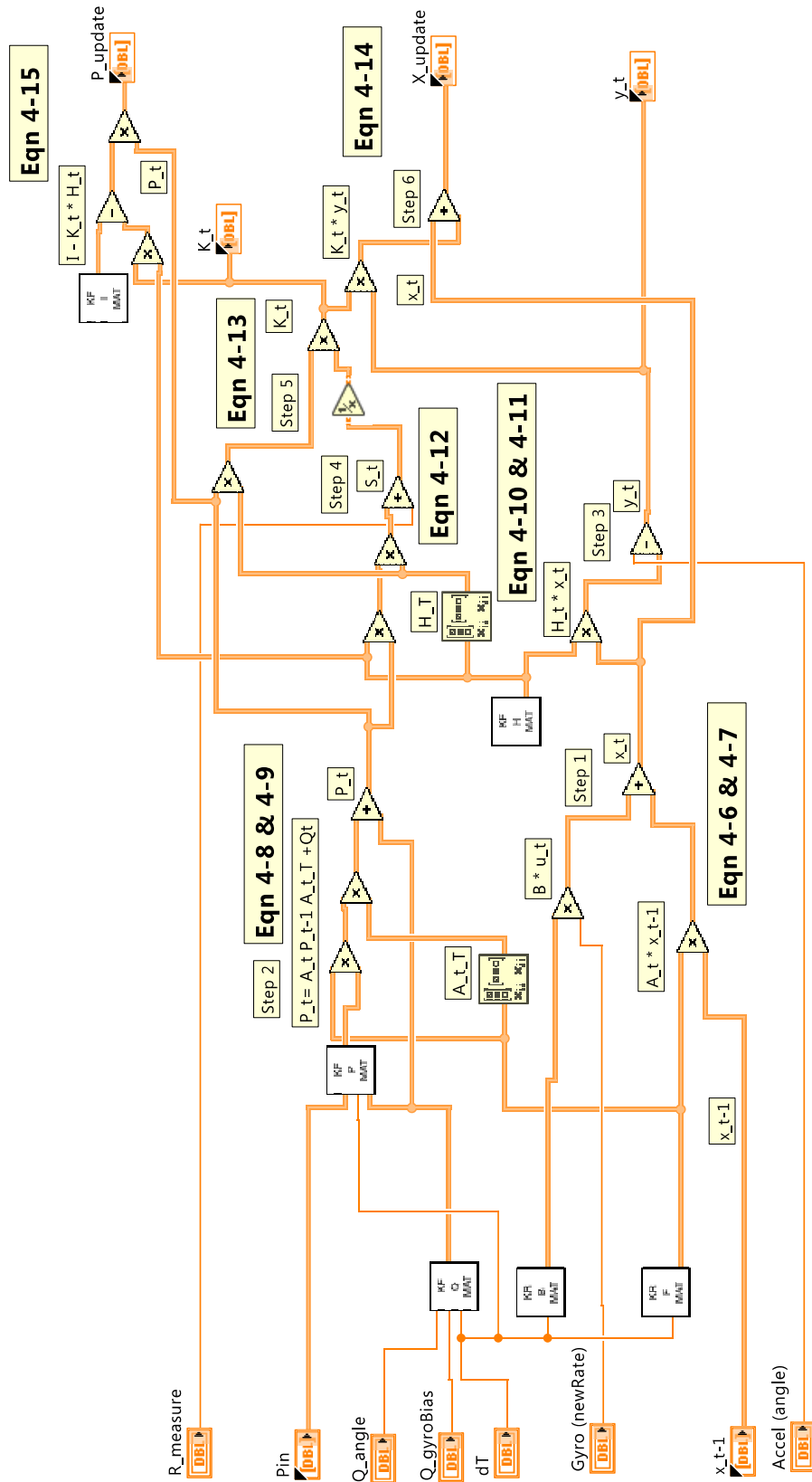


Figure 4-10 – Discrete Kalman Filter VI Block Diagram

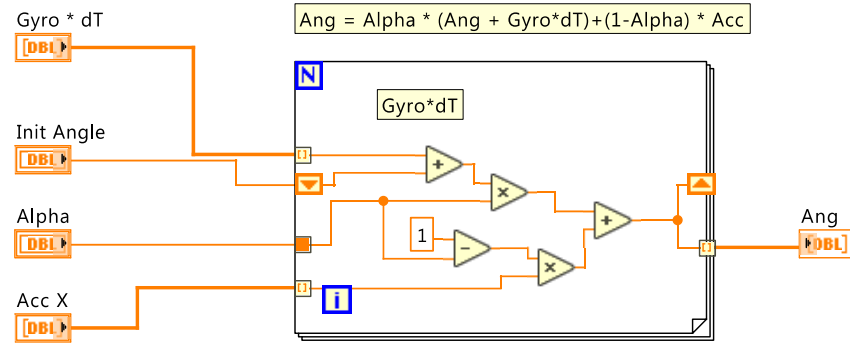


Figure 4-11 – Complementary Filter VI Block Diagram

4.5.3 Comparison of methods using test data

All of the data was analysed using the two algorithms; the results from one of the tests conducted as part of Experiment Five is shown in Figure 4-12. This diagram shows the filtered data computed using the KF and CF algorithms, compared to the base data recorded by the HSV and is typical of the test results obtained from each of the experiments.

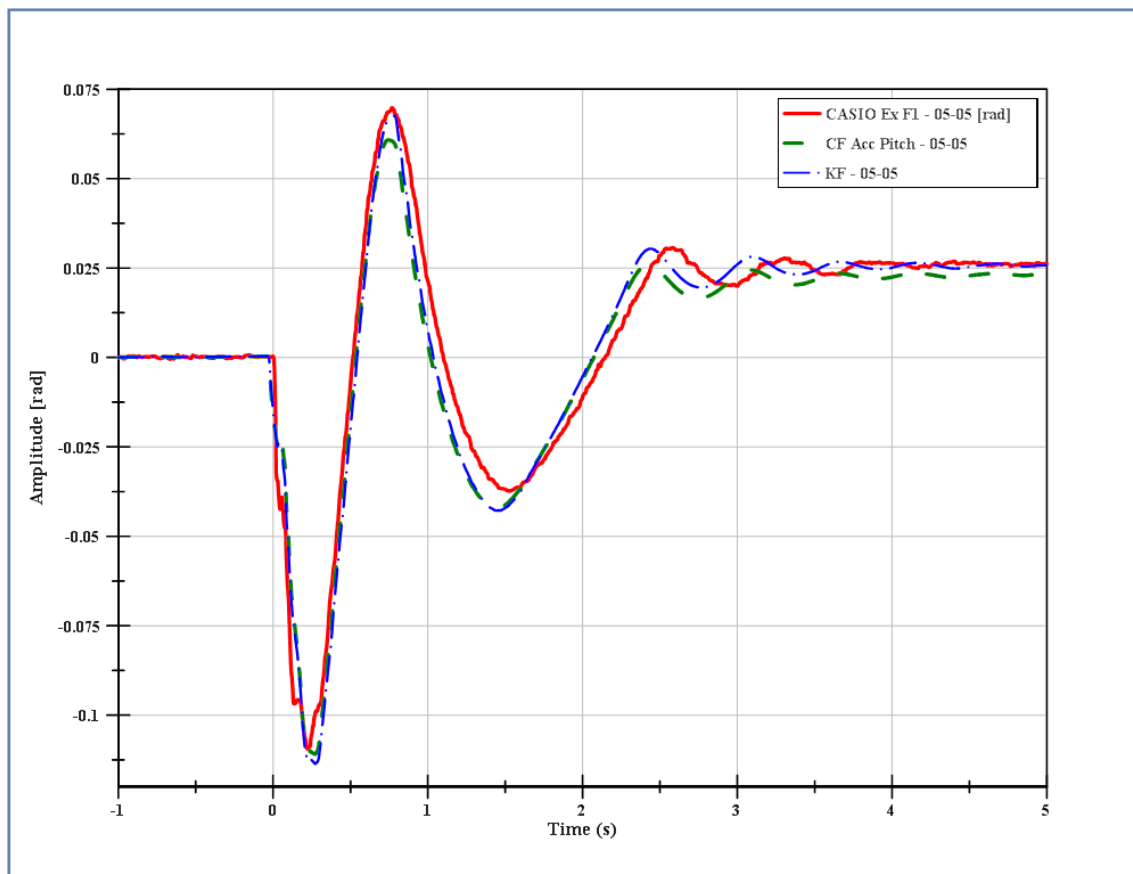


Figure 4-12 – Example of test data analysed using filter algorithms

4.6 Chapter conclusions

Various data fusion methodologies are available however this Chapter has highlighted some of the limitations associated with the more common ones, specifically in relation to their adaption for use in situations where ‘computing power’ may be limited. The chosen KF and CF algorithms are programmatically simple to implement and detailed analysis of their effectiveness for short-duration load scenarios is discussed in detail in Chapters Six and Seven.

Chapter 5 Accuracy and Error Analysis

5.1 Introduction

Understanding the level of accuracy and precision of the test data to correctly determine any structural response and quantifying the acceptable level of uncertainty to enable it to be of use, were essential parts of the research. To provide consistency in relation to terminology, this chapter begins by defining the terms that have been used in this and subsequent chapters in relation to accuracy and precision. A discussion of the different sources of error that were encountered during the research follows, including explanations of how these were handled, before finally describing the methods used to analyse and quantify these errors.

5.2 Definitions

As mentioned in earlier chapters the definitions used in relation to the accuracy and precision of a measurement can be varied. For the purpose of this research the terms and definitions as found in the ISO Standard 5725 for determining the ‘Accuracy (trueness and precision) of measurement methods and results’ were adopted (referred to as the Standard) (BSi, 1994). The main ones applicable to this research have been reproduced below. Figure 5-1 provides a graphical interpretation of the relationship between some of the terms and concepts described in Table 5-1.

Definition	Description	Remarks
Accepted reference value	The Standard defines what the agreed-upon reference might be based upon, including; “scientifically derived, established or theoretical values; a certified value based upon experimental work; or a value based upon the collaborative work of a scientific or engineering group”.	For this research the ‘accepted reference value’ was taken to be the independent record of measurement recorded by the remote HSV. As will be seen later in this Chapter when discussing the error measures, the accepted reference value is also referred to as the calculated value.
Observed value	The value of a characteristic obtained as the result of a single observation.	For this research the observed value was assumed to relate to the raw sensor data prior to any data analysis or manipulation.
Test result	The test result may consist of several observations or relate to the function of an observation.	It was assumed for this research that the test result was the response-time history after it had been analysed, and if appropriate adjusted to convert it into the same format as the reference value. As the reference value related to the angle of rotation of the test item, inverted pendulum, vertical cantilever or monolith, these values were in radians.
Accuracy	The closeness of agreement between a test result and the accepted reference value.	Figure 5-1 illustrates the concept of accuracy well; the group of test results towards the bottom left of the diagram are least accurate due to their dispersed nature, whilst those towards the top right have the greatest accuracy as depicted by their close proximity to each other and the accepted reference value.
Trueness	The closeness of agreement between the average value obtained from a large series of test results and an accepted reference value.	Again, the concept of trueness is well illustrated by the top two clusters of test values in Figure 5-1. Their relatively even distribution about the accepted reference value is a measure of their trueness compared to the distributions of the lower groups, which although similarly spaced, are offset to one side of the accepted reference value.

Definition	Description	Remarks
Bias	The difference between the expectation of the test results and the accepted reference value.	The concept of bias is demonstrated by the distance between the 'centre' of the lower right group of test results in Figure 5-1 in relation to the accepted reference value. For the group of test results in the bottom left the centre of the group is further away from the accepted reference value and therefore the bias is greater than that for the lower right group. Conversely the relatively even dispersion of the test results about the accepted reference value for the top two groups is an indication that the bias is close to, or equal to zero.
Precision	The closeness of agreement between independent test results obtained under stipulated conditions. The Standard divides precision into two categories, repeatable and reproducible depending upon the conditions under which the tests are conducted.	The concept of precision is illustrated in Figure 5-1 by the relative closeness of the test results. The cluster of test results towards the right are more precise because they are closer together than those towards the left that are more dispersed.
Precision - Repeatability conditions	Conditions where independent test results are obtained with the same method on identical test items in the same laboratory by the same operator using the same equipment within short intervals of time.	The concepts of repeatability and reproducibility defined by the Standard have been produced to relate to those that might be employed by test laboratories conducting tests in accordance with a particular standard. For instance, the ballistic tests that were conducted as part of this research, Experiment Four and Experiment Five (discussed in more detailed in Chapter 7), were undertaken at two different test laboratories. In terms of repeatability, each of the experiments produced a set of 'repeatable' results for the test items under investigation, however as a different test item was tested at each test laboratory, the results from the two experiments could not be classified as having been 'reproduced'. For this to have been the case, an identical test item would have had to have been tested as part of each experiment, and the tests conducted by different operators. This means that in accordance with the Standard, the tests conducted as part of this research can be classified as repeatable but not reproducible. An instance where it might be argued that a test be deemed reproducible however, is where the same test is conducted a number of times, but for each test, an identical but different sensor is used.
Precision - Repeatability	Precision under repeatability conditions.	
Precision Reproducibility conditions	Conditions where test results are obtained with same method on identical test items in different laboratories with different operators using different equipment.	
Precision Reproducibility	Precision under reproducibility conditions.	
Outlier	A member of a set of values which is inconsistent with the other members of that set.	An example of an outlier is shown near the bottom left hand cluster of test results shown in Figure 5-1. Its remoteness to the rest of the cluster is an indication that it is potentially an outlier. Types of outlier encountered during the research, and how they were dealt with are discussed in subsequent sections.

Table 5-1 – ISO 5725 Standard Definitions applicable to the research

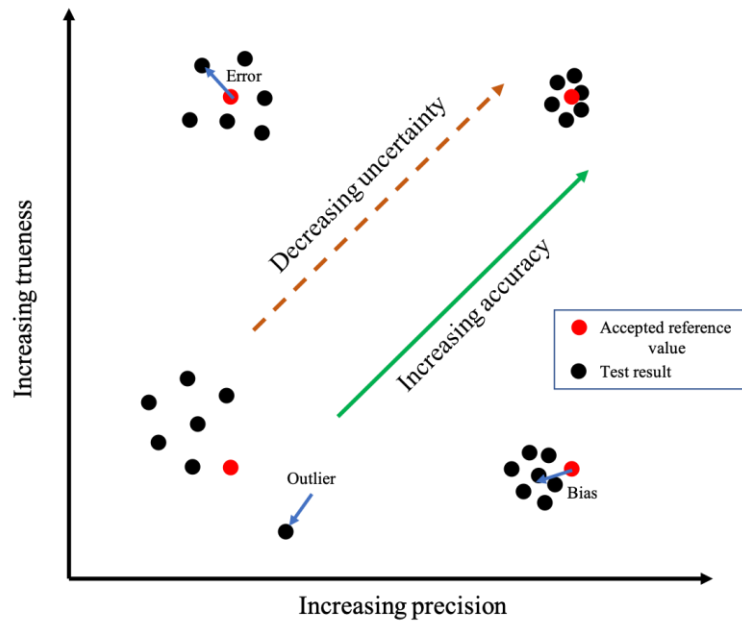


Figure 5-1 – Relationship between accuracy, trueness and precision

5.3 Sensor precision

To obtain the maximum precision for a test, the full-scale range of the chosen sensor had to be carefully matched to the maximum anticipated event. For example, if the maximum anticipated acceleration event was 3.25g then a sensor with a full-scale range greater than 3.25g had to be chosen. Where the anticipated maximum event was easily predicted this was not a problem, otherwise a greater full-scale range had to be chosen to ensure data was not lost, the downside of which was a reduction in precision.

For the tests conducted as part of this research the IMU sensor precision, in terms of the sensor's sensitivity scale factor, was directly related to the sensor full-scale range set at the beginning of each test. The sensitivity scale factors for the MPU-9250 IMU used are summarised in Table 5-2.

Gyroscope specifications		Accelerometer specifications	
Full-scale range	Sensitivity Scale Factor	Full-scale range	Sensitivity Scale Factor
±250 °/s	131 LSB/(°/s)	±2 g	16,384 LSB/g
±500 °/s	65.5 LSB/(°/s)	±4 g	8,192 LSB/g
±1000 °/s	32.8 LSB/(°/s)	±8 g	4,096 LSB/g
±2000 °/s	16.4 LSB/(°/s)	±16 g	2,048 LSB/g

Table 5-2 – MPU-9250 IMU sensor range and sensitivity values (InvenSense, 2016)

This shows that the sensitivity was dependent upon the number of 'least significant bits' (LSB); the greater the LSB/unit the more sensitive the sensor. Both the IMU gyroscope and accelerometers used 16-bit ADCs to convert the analogue motion into digital signals which represented a total of $2^{16} = 65,536$ bits. The sensitivity scale factor was therefore calculated by dividing the total number of bits by the total full-scale range. For example, as shown

in Table 5-2, the full-scale range for ± 4 g was 8 g and therefore the associated sensitivity scale factor was $65,536 \div 8 = 8,192$ LSB/g.

The purpose of Figure 5-2 is to graphically represent that data summarised in Table 5-2 to help illustrate the relationship between an IMU sensors' full-scale range and its associated sensitivity scale factor and how this relates to precision.

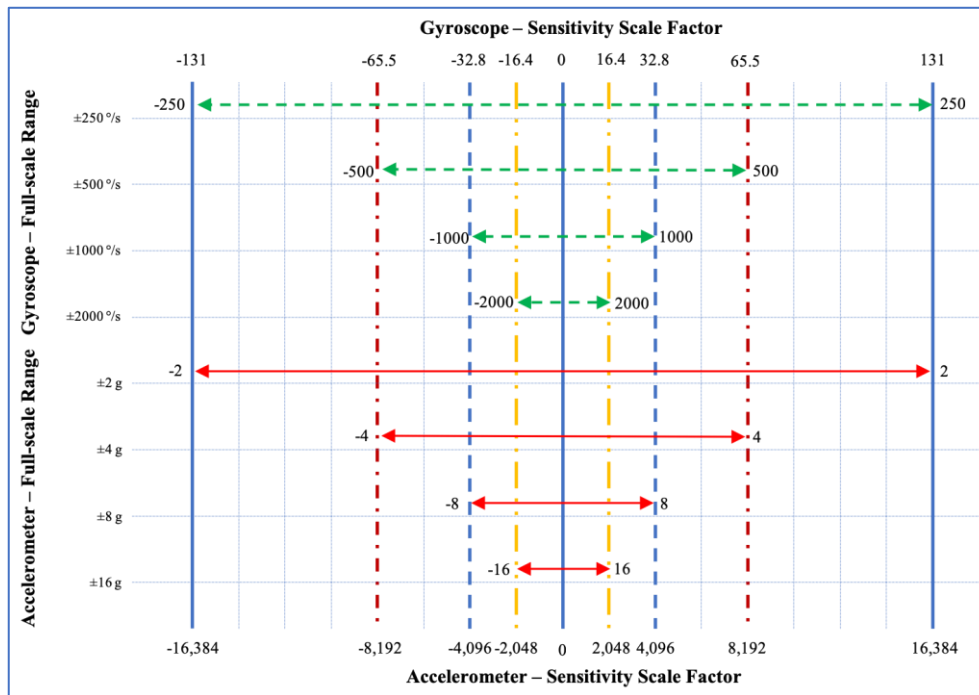


Figure 5-2 – Diagram showing relationship between IMU sensor Full-scale Range and Sensitivity Scale Factor

The relative precision of the sensor settings is shown by the length of the horizontal lines; for the gyroscope the ± 250 °/s setting is more precise than the ± 2000 °/s one, and the ± 2 g accelerometer setting is more precise than the ± 16 g one. This reinforces the importance of correct sensor sensitivity selection to ensure the highest precision and therefore greatest accuracy. How both the IMU gyroscope and IMU accelerometer sensitivities affected the test results was investigated for Experiments One, Two and Three by conducting a series of tests where different combinations of the two sensor sensitivities were selected. The results of these tests are discussed in detail in Chapter 6 and Chapter 7.

5.4 Types of error

Menditto et al (Menditto, Patriarca & Magnusson, 2007) provided a useful discussion of the relationship between accuracy, trueness, precision and errors, and highlighted that despite the existence of a number of national and international standards the concepts are often misinterpreted. To illustrate the relationship between the different concepts and the related errors, they produced the diagram shown in Figure 5-3. As highlighted in Figure 5-1 and shown in Figure 5-3, the accuracy of a test result can be determined by its precision and trueness in relation to the

accepted reference value. The extent of the trueness and precision depends upon the magnitude of the associated errors which are then quantified by the bias and standard deviation respectively.

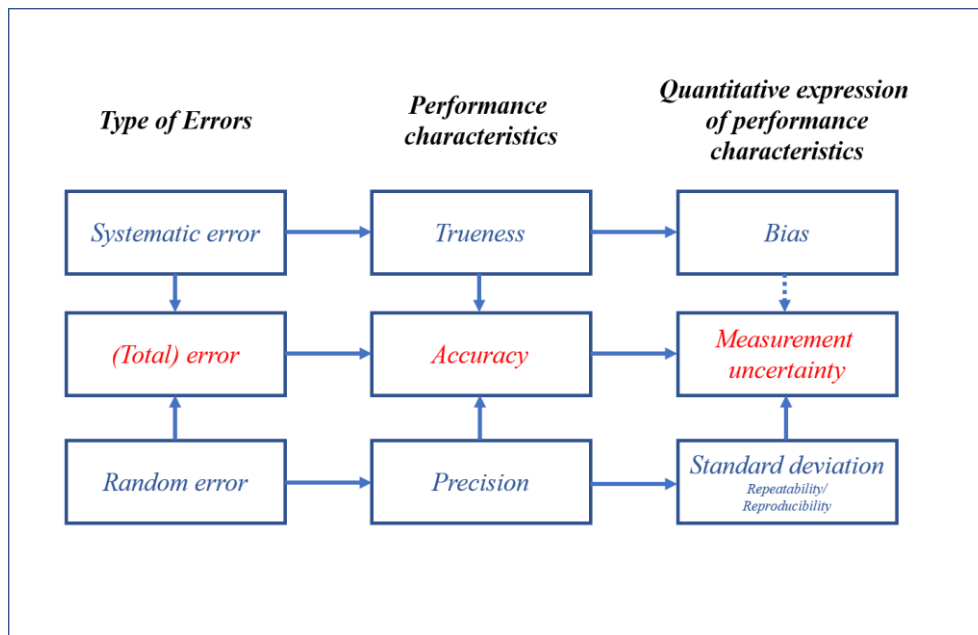


Figure 5-3 – Relationship between accuracy, trueness, precision, error and performance characteristics (based upon a diagram developed by Menditto et al (Menditto, Patriarca & Magnusson, 2007))

As shown in Figure 5-3 errors can be categorized into two types: systematic and random. An explanation and examples of these errors that were encountered as part of the research can be found below.

5.4.1 Systematic errors

Systematic errors are those that occur systematically and will affect all the results produced by a sensor (Murty, 2010), measurement or calculation and are typically represented as bias in a test result (Figure 5-1 and Figure 5-3).

5.4.1.1 Calculation errors

An example of a calculation error is when an incorrect value is used as part of a calculation. If the incorrect value is not identified and continues to be used, it will result in a consistent error in all the test results. Such errors can be avoided by rigorous checking of any calculations, and where possible conducting a simple, easily defined, initial test to ensure that the accepted reference value and the test results are as close as possible. This was the purpose of Experiments One and Two discussed in Chapter 6. An example of such an error was the value of Young's Modulus chosen to calculate the deflection of the cantilever beam which could have a significant influence on the resultant response calculation. Fortunately, the proposed method of determining the rotation/deflection using two sets of strain gauges, resulted in it not being necessary to know either the Young's Modulus or the Second Moment of Area of the beam, mitigating the potential effect of using the wrong material property values.

5.4.1.2 Sensor orientation

Determining the orientation of a sensor was important particularly if any observed sensor data was then used to calculate a final test result. This was especially important with the IMU sensors used. The MPU-9250 sensor used for the research had combined 3-DOF accelerometers and 3-DOF gyroscopes mounted on a printed circuit board (PCB). Figure 5-4 shows two possible orientations for the same sensor which could lead to different gyroscope readings.

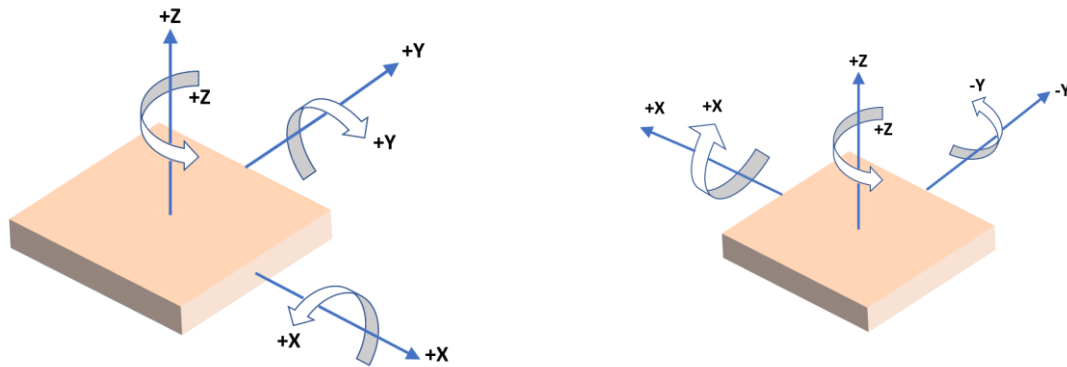


Figure 5-4 – IMU orientations – Standard orientation (Left) – Orientation used for research (Right)

The diagram on the left shows the standard orientation of the IMU, while that on the right shows the orientation used for the research discussed in this thesis. It will be noted that although the x-axis and z-axis orientations remain positive, the 180-degree rotation of the IMU means that for the pitch, roll and yaw arrangement described in Chapter 4, the orientation of the y-axis results is a negative value.

An example of the accelerometer and gyroscope data recorded for one of the tests is shown in Figure 5-5. This shows offset corrected observed values for the x-axis accelerometer (top) and the y-axis gyroscope (middle and bottom). The middle plot shows the data recorded from the IMU while the bottom plot is for that multiplied by a factor of negative one to adjust the data for the different gyroscope orientation.

Figure 5-6 is a diagram showing both the negative and positive y-axis gyroscope data after it had been analysed, along with the related accelerometer data, using the Complementary and Kalman filters which was then compared to the accepted reference value provided by an HSV (depicted by the black solid curve, Angle [rad] in Figure 5-6). The red (dashed and dot) and the blue (dashed) curves represent the analysis for the Complementary and Kalman filters computed using the positive gyroscope data, while the orange (with cross) and green (with diamond) curves are for the analysis for the Complementary and Kalman filters computed using the negative gyroscope data. It is quite clear that the Complementary and Kalman filter curves calculated using the negative gyroscope data do not match the accepted reference value curve. By comparison the Complementary and Kalman filter curves generated using the positive gyroscope data closely match the accepted reference curve. This demonstrates the importance of correct sensor alignment, and if this is not possible making sure the data used for calculations is appropriately adjusted.

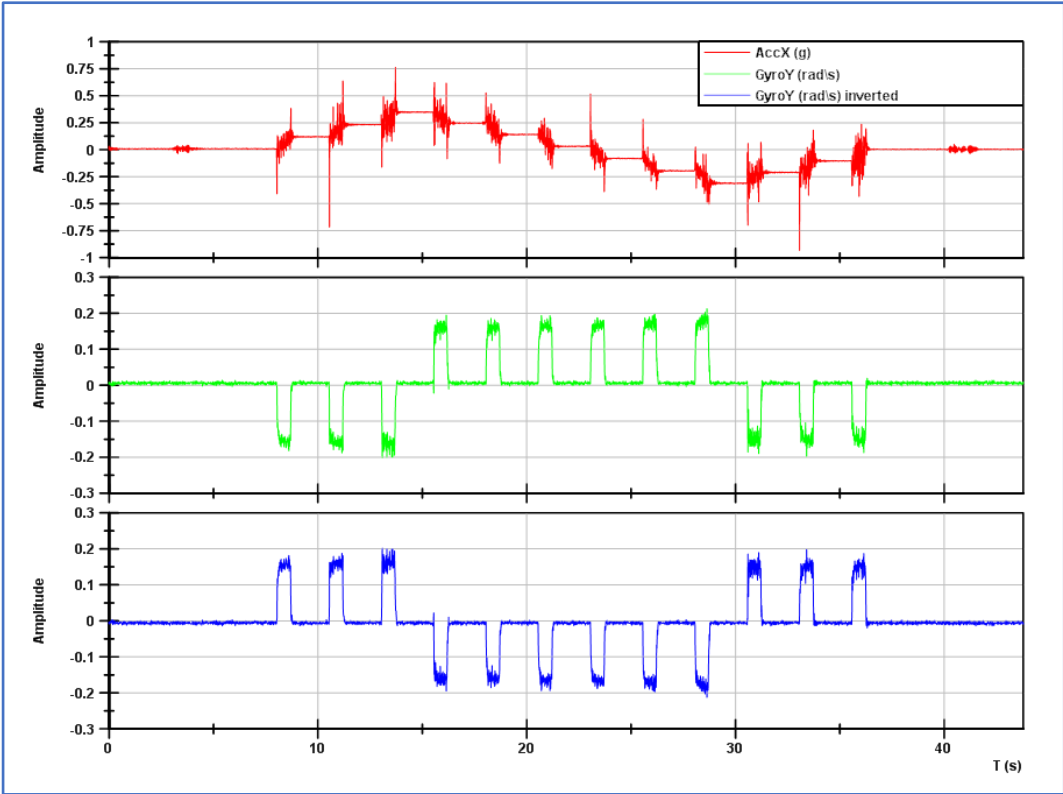


Figure 5-5 – Sensor orientation errors

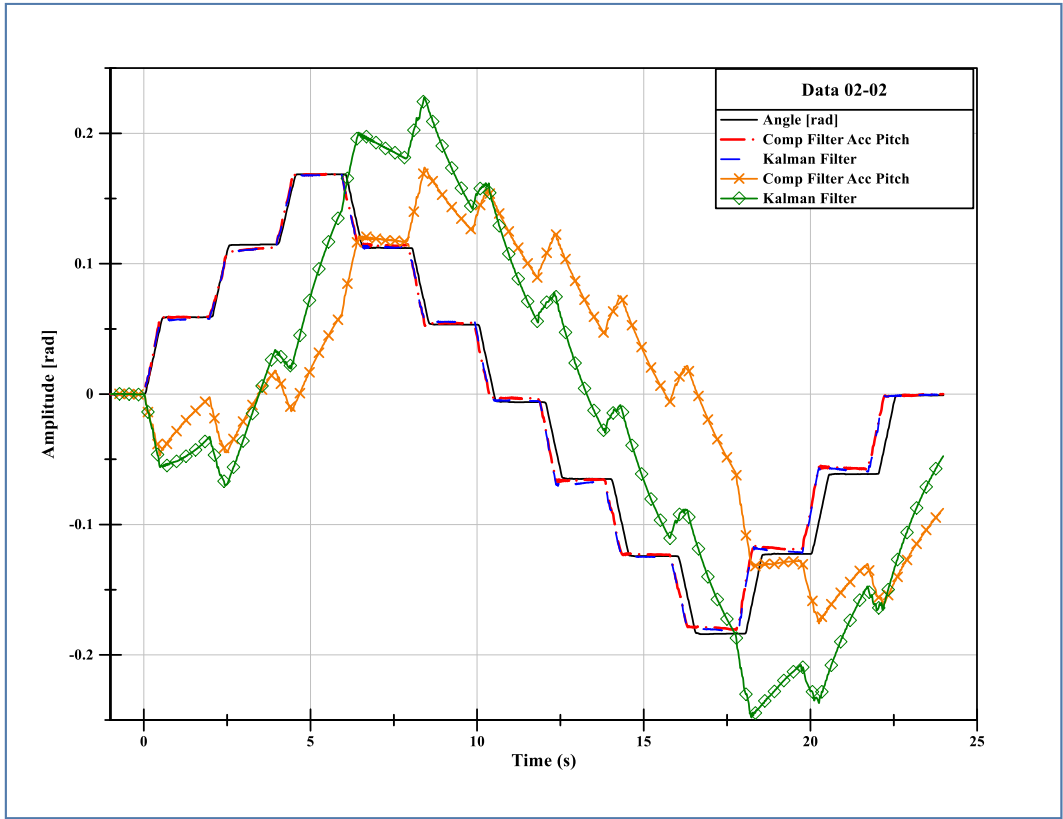


Figure 5-6 – Comparison of analysed test results using gyroscope data with different orientations

5.4.1.3 Sensor calibration

As part of the sensor calibration process, the sensitivity for both the IMU and ADCs was set at the beginning of each test using the LabVIEW VI, to ensure that the captured data was of the optimum quality. The IMU sensor range and sensitivity scale factors that were shown in Table 5-2, were programmed into the LabVIEW VI to enable the chosen settings to be quickly changed at the start of each test. As discussed in Section 5.3 the appropriate selection of these parameters was important to ensure that the data subsequently recorded was as precise as possible. This was particularly important for the IMU accelerometer during the dynamic tests where, during some of the initial tests, the initial peak acceleration was seen to be ‘clipped’ at the full-scale range of the sensor. An example of this is shown in Figure 5-7 and Figure 5-8 which show the x-axis IMU accelerometer data for four of the Experiment Three tests.

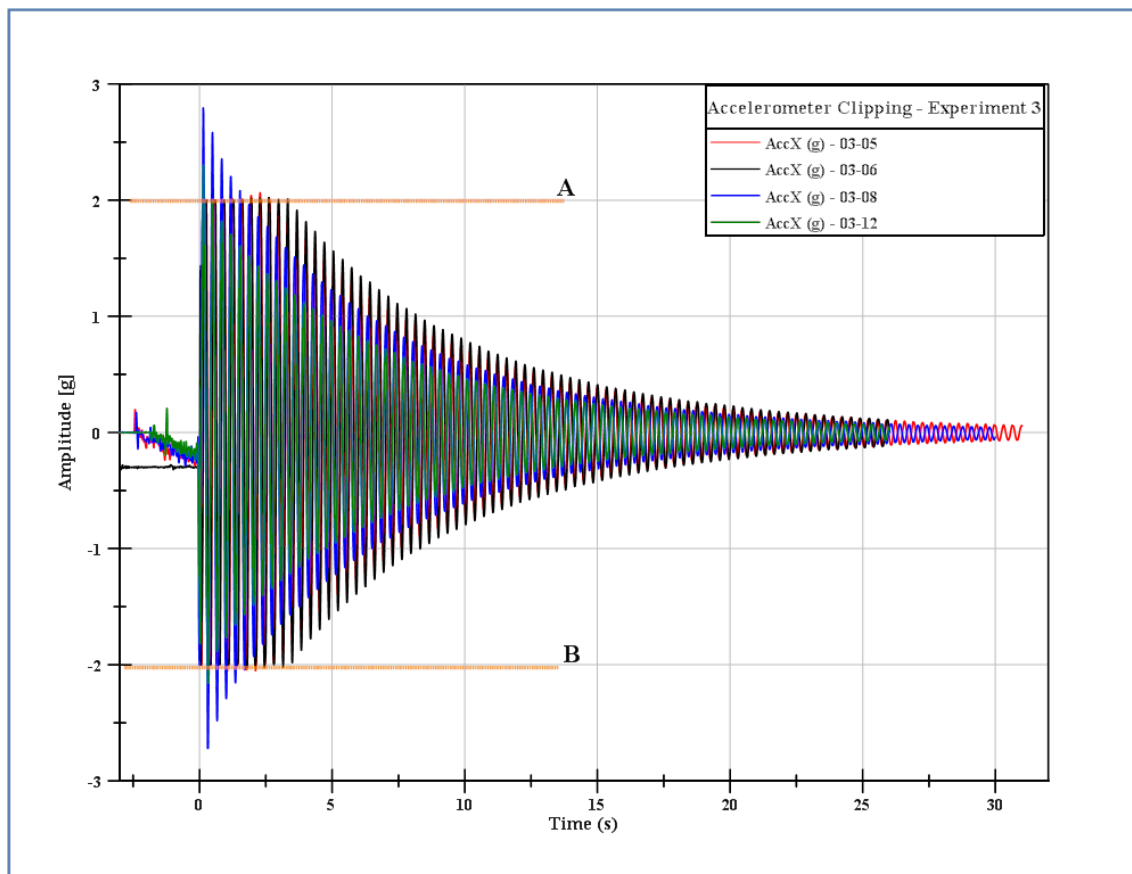


Figure 5-7 – Example of ‘data clipping’ for IMU accelerometer data

The horizontal lines, A and B, highlight the points at which the data for tests 03-05 and 03-06 (red and black dashed lines) have been ‘clipped’ at ± 2 g; although this can be seen in Figure 5-7, it is more evident in Figure 5-8 between the 0 and 2 second interval. The full-scale range for the IMU accelerometer for both tests 03-05 and 03-06 had been set at ± 2 g. By contrast, the full-scale range setting used for tests 03-08 and 03-12 was set for ± 4 g and therefore the initial peak data did not become ‘clipped’.

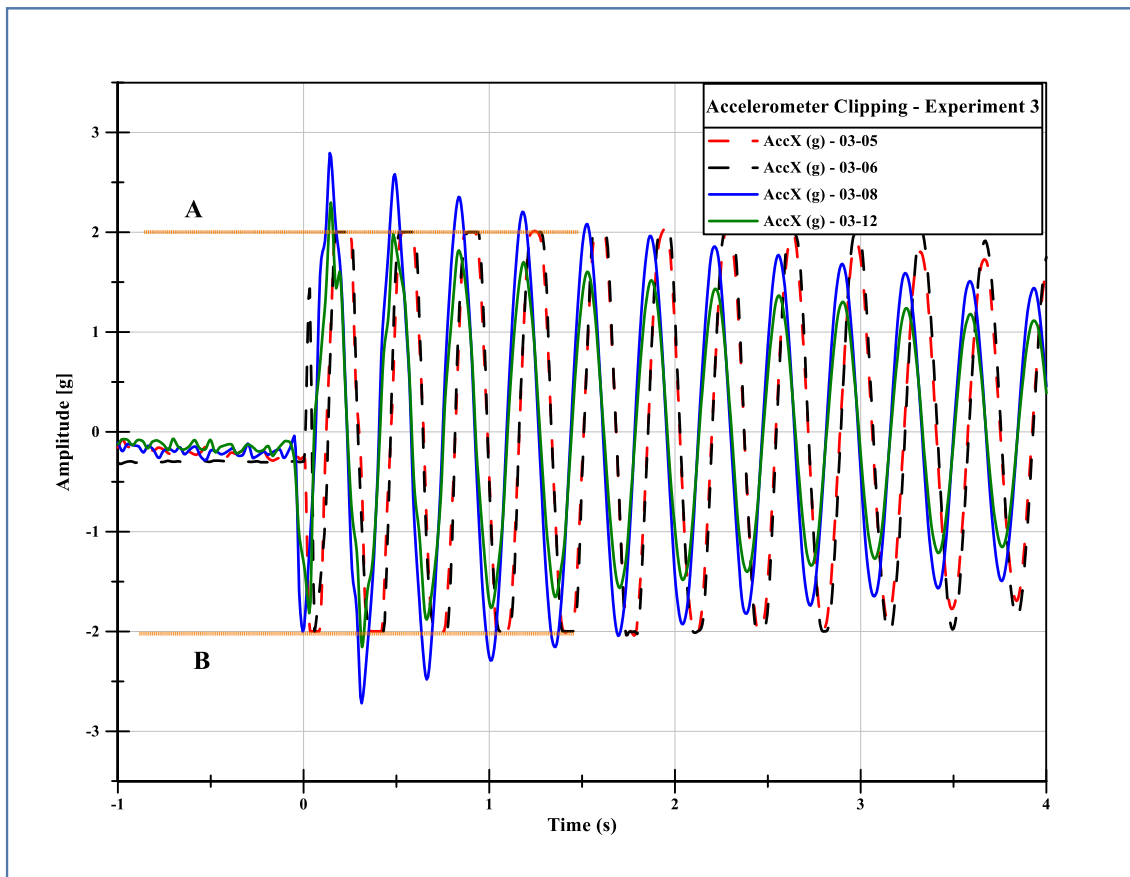


Figure 5-8 – Close-up of ‘data clipping’ for IMU accelerometer data

In addition to the appropriate full-scale range settings being used for the IMU accelerometer data, the IMU gyroscope readings had to be adjusted to convert the data from degrees per second ($^{\circ}/s$) output from the sensor, to radians per second (rad/s) for the subsequent data analysis and data fusion.

5.4.1.4 Sensor offset

Figure 5-9 shows the individual sensor signals for the three axes of the MPU-9250 accelerometer. The areas A and B refer to close-up views that can be seen in Figure 5-10, which shows a more detailed view of the offset error for each of the axes. If the Z-axis of the accelerometer points straight-up, and at right-angles to the Earth’s surface it will be parallel to the Earth’s gravitational field and will therefore have a value of one. Similarly, if the X and Y axes are at right-angles to the Z-axis, and therefore parallel to the earth’s surface, but at right-angles to the gravitational field, they should both read zero.

The left-hand graph in Figure 5-10 is a close-up view of the offset error that can be identified in areas A and B of Figure 5-9. The graph shows that the data for all three axes contained a small offset error. This error may have been due to manufacturing flaws within the sensor or potentially minor sensor mis-alignments. To compensate for manufacturing flaws correction factors are provided by manufacturers in their specification sheets. Sensor mis-alignments can be minimized by careful installation and calibration of the sensor prior to testing. Point 1 and Point 2 in the left-hand diagram in Figure 5-10 refer to the offset error for the x-axis and y-axis accelerometers respectively; Point 3 relates to the offset error for the z-axis.

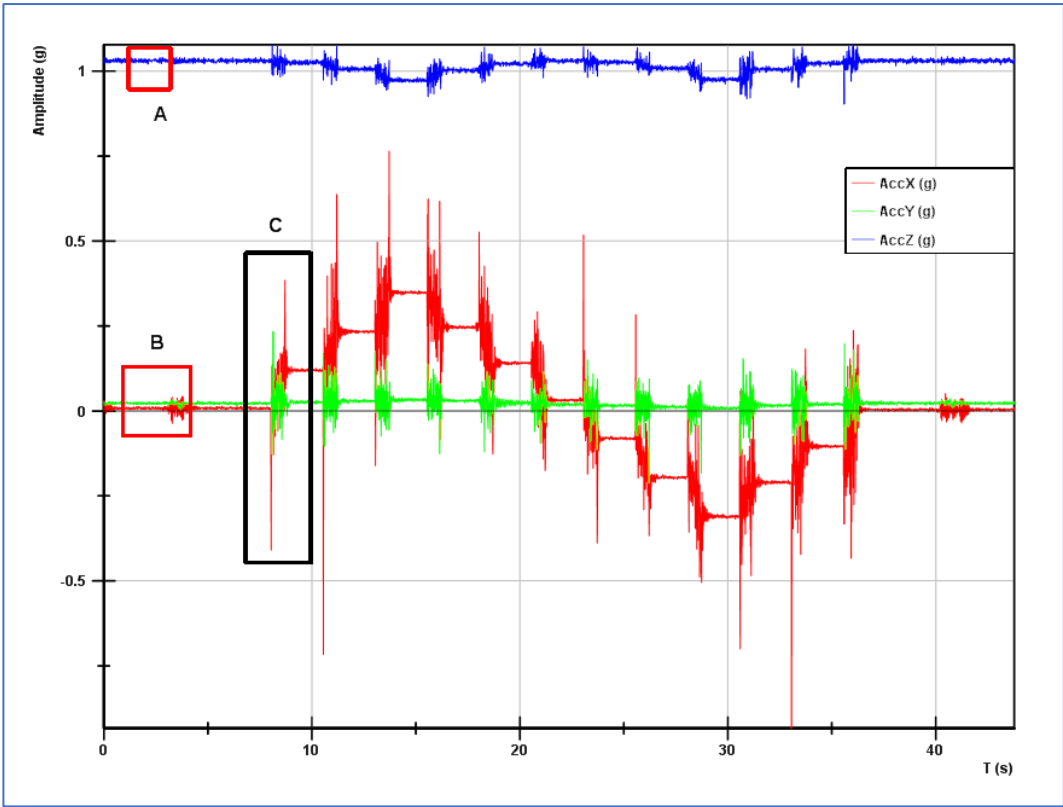


Figure 5-9 – Sensor offset and outlier errors

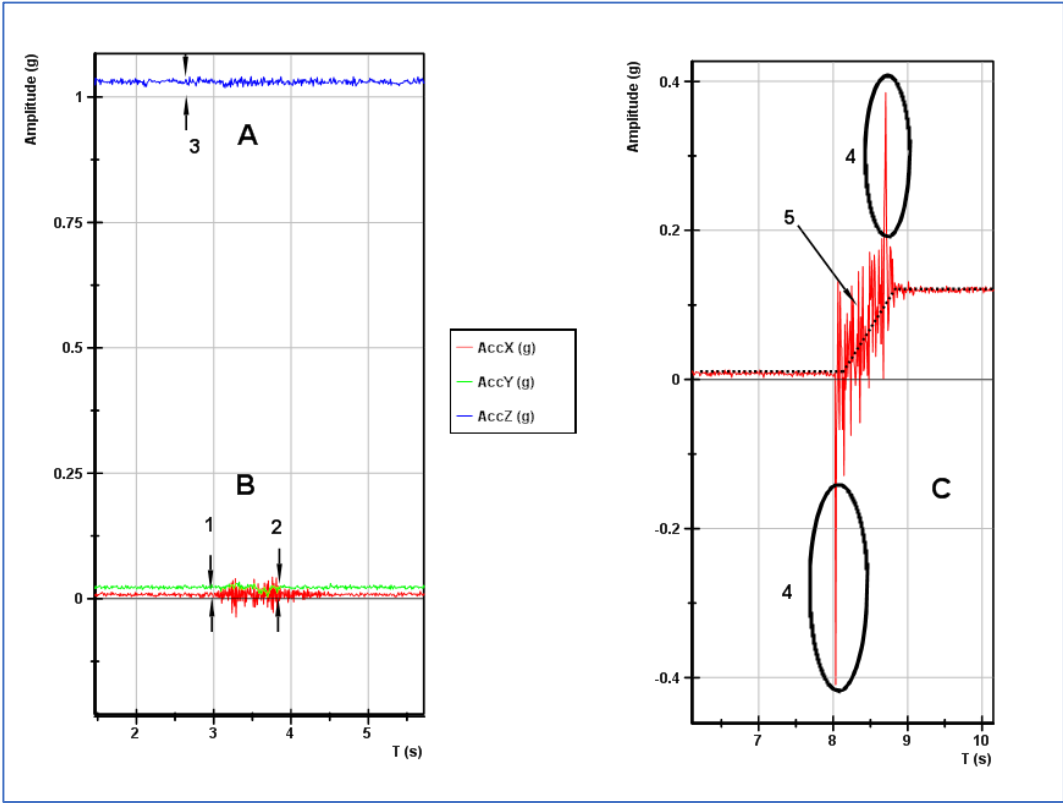


Figure 5-10 – Sensor offset errors (left) and outlier and noise (right)

5.4.2 *Random errors*

Random errors are those that do not follow a pattern and are often caused by external factors such as induced noise and environmental disturbances. Examples of those encountered during the research are discussed in the following section.

5.4.2.1 *Measurement errors*

Where measurements are taken by a human, such as the reading of an analogue meter or scale, there is scope for random errors to occur as how an individual reads a particular instrument may result in an incorrect reading if not done correctly (Morris & Langari, 2016). If more than one individual is involved in the process, the more chance that erroneous readings are taken unless clear procedures are in place to ensure consistency. If one individual is responsible for taking the readings, and these are taken in the same manner each time, then the errors become more systematic. As discussed in Chapter 3, Methodology, to avoid such errors this research made use of the LabVIEW software that output digital readings, while the HSV image data was analysed using the GOM Correlate software that enabled accurate interpretation of the calibration points and thereby reduced such errors.

5.4.2.2 *Noise*

The right-hand image in Figure 5-10 shows two types of noise that were frequently encountered in the raw data collected during the research. The two Point 4s relate to random ‘spikes’ in the data or outliers, while Point 5 is typical of random noise, that in this instance was probably caused by vibrations transmitted to the test item by the linear actuator, that was used to produce a consistent input force. The two spikes were most likely due to the initial sudden application of the actuator load with the test item (inverted pendulum or vertical cantilever), which caused a sudden increase in acceleration. As there was a small amount of play between the bolt securing the test item to the actuator, this could cause a slight ‘jerk’ in the motion at the beginning and end of the actuator motion, as indicated by the sudden increase/decrease in acceleration. It may have been possible to reduce this affect by ensuring a tighter fit between the actuator and fixing bolt, but this would have been at the expense of introducing unwanted friction into the system. Had the test items been larger it might have been possible to use a frictionless bearing, however due to the relatively small dimensions of the test items this was not a practical option for this research.

Noise can also be introduced by poorly grounded equipment or external sources such as electrical power cables placed near to instrumentation and sensor cables; Figure 5-11 shows an example of a signal with such noise clearly evident. The data shown in Figure 5-11 was for a strain gauge reading taken early in the research whilst learning how to use the data logging equipment and the appropriate use of basic instrumentation. On close inspection it can be seen that there are 5 cycles between 0 sec and 0.1 sec which is equal to 5 cycles every 0.1 sec or 50 cycles per second or 50 Hz. The 50 Hz uniform nature of the signal is an indicator that it is most likely caused by interference generated by an AC mains power supply in the vicinity of the instrumentation setup. To reduce the chances of this type of noise occurring in subsequent tests, where possible, shielded or twisted pair instrumentation cables were used, and were routed as far away from mains power supplies as practical, or battery power supplies were used to remove the potential for this type of noise.

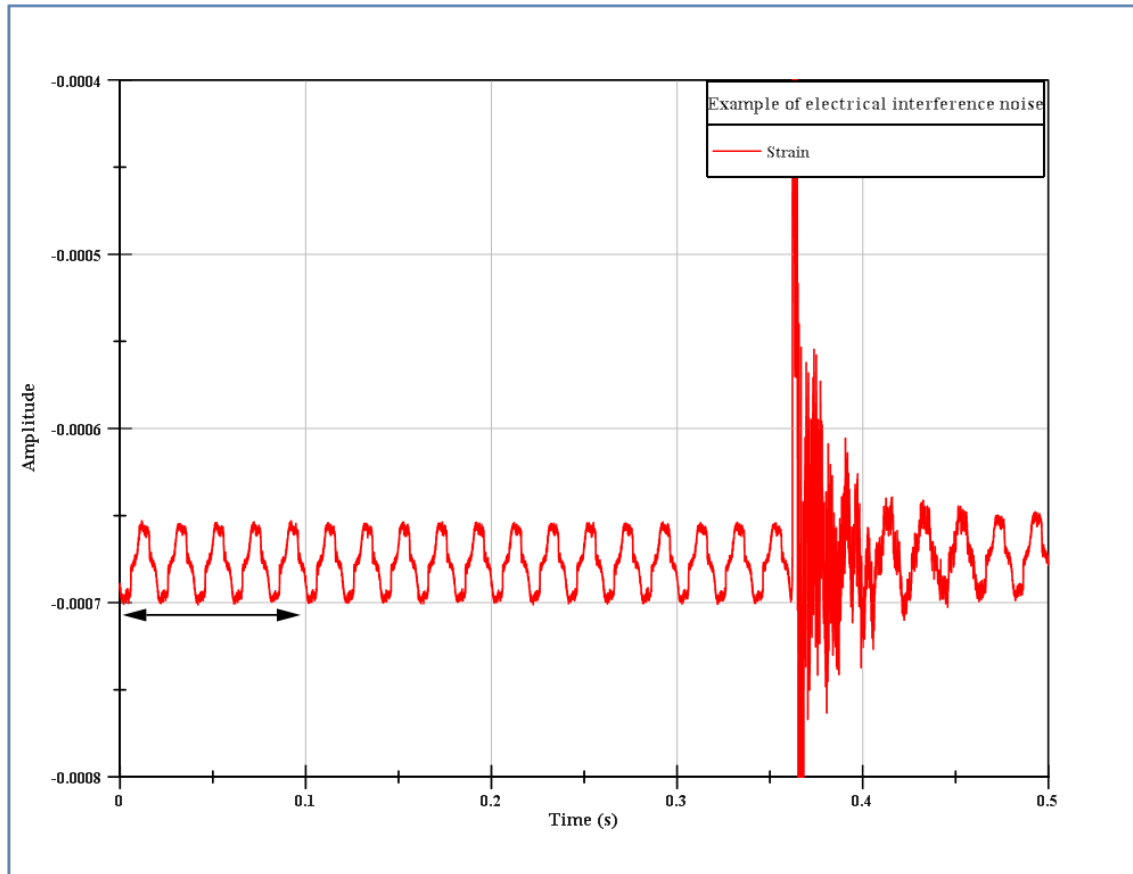


Figure 5-11 – Example of external electrical noise in sensor signal

5.4.3 Gross Errors

In addition to random and systematic errors a third group of errors is sometimes discussed; Gross errors (Murty, 2010). These are errors that come about due to the person conducting the experiment producing a poor experimental design and not taking care whilst conducting the experiment. An example of this might be the failure to ensure the instruments are ‘zeroed’, or not making an allowance for such requirements to be conducted prior to analysing the results. For the research discussed in this thesis, the IMU sensors and strain gauges used for the experiments had offset errors that were adjusted as part of the data manipulation to produce the test results that were used during the subsequent data analysis and data fusion.

A lack of knowledge and understanding of the aims of the experiment and the types of instrumentation that might be suitable may also be a factor. To avoid gross errors, it is important that the person conducting the experiment understands any limitations that the proposed instrumentation might have and plans the experimental procedure. This was one of the main reasons for conducting simple ‘static’ experiments before moving onto more complex ‘dynamic’ experiments and was to ensure that any limitations of the instrumentation and any inherent errors within the experimental setup, were fully understood at an early stage and could therefore be accounted for in subsequent analysis.

5.4.4 Smoothing and outlier removal

The data received from the sensors often had a range of errors and noise associated with it. Where possible the experimental setup was adjusted to reduce or remove any errors or noise, however where this was not possible post-processing techniques such as smoothing, and outlier removal were employed.

5.4.4.1 Smoothing

There were a variety of data smoothing techniques available however, where required, the unweighted moving average smoothing algorithm supplied as a smoothing function within the DIAdem data post-processing software was used (National Instruments Corporation., 2015). This function calculates an unweighted mean for the value in question based upon it, and a specified number of neighbouring values. The smoothing algorithm used by the software is shown in the following equation:

$$f(y_i) = \frac{1}{2N + 1} \sum_{k=1-N}^{k=i+N} y_k, m - N > i > N$$

An example of a data signal that was smoothed using this algorithm is shown in Figure 5-12 and close-ups of the area surrounded by the green circle have been reproduced in Figure 5-13. The diagram consists of four curves; the original curve (AccX (g)) and three curves that represent this curve after it had been smoothed using different numbers of neighbouring values, 8, 12 and 16. The data signal was taken from Experiment Two where the vertical cantilever had been deflected and paused in equal steps using a linear actuator. The portions of the data signal that were horizontal are where the actuator was paused, and the diagonal increasing/decreasing areas were where the actuator was moving outwards or inwards.

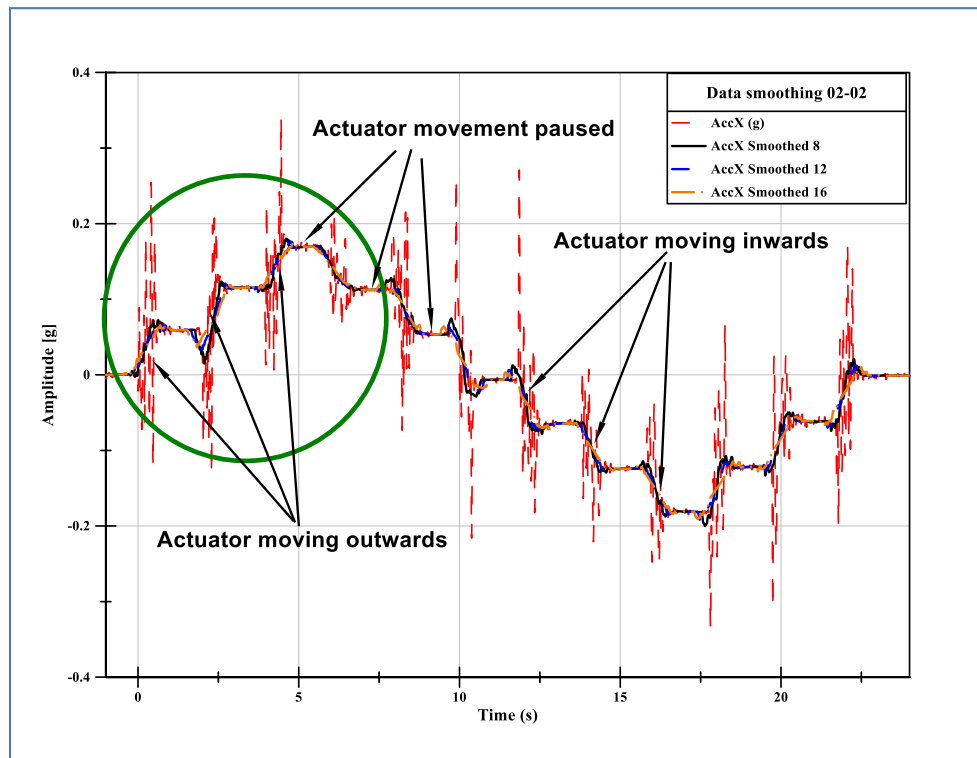


Figure 5-12 – Example of a data signal where smoothing was used to remove noise from the signal

A close examination of the curves at Points A, B and C in Figure 5-13 follows, showing how varying the number of values used to calculate the unweighted moving average changed how the original curve was smoothed.

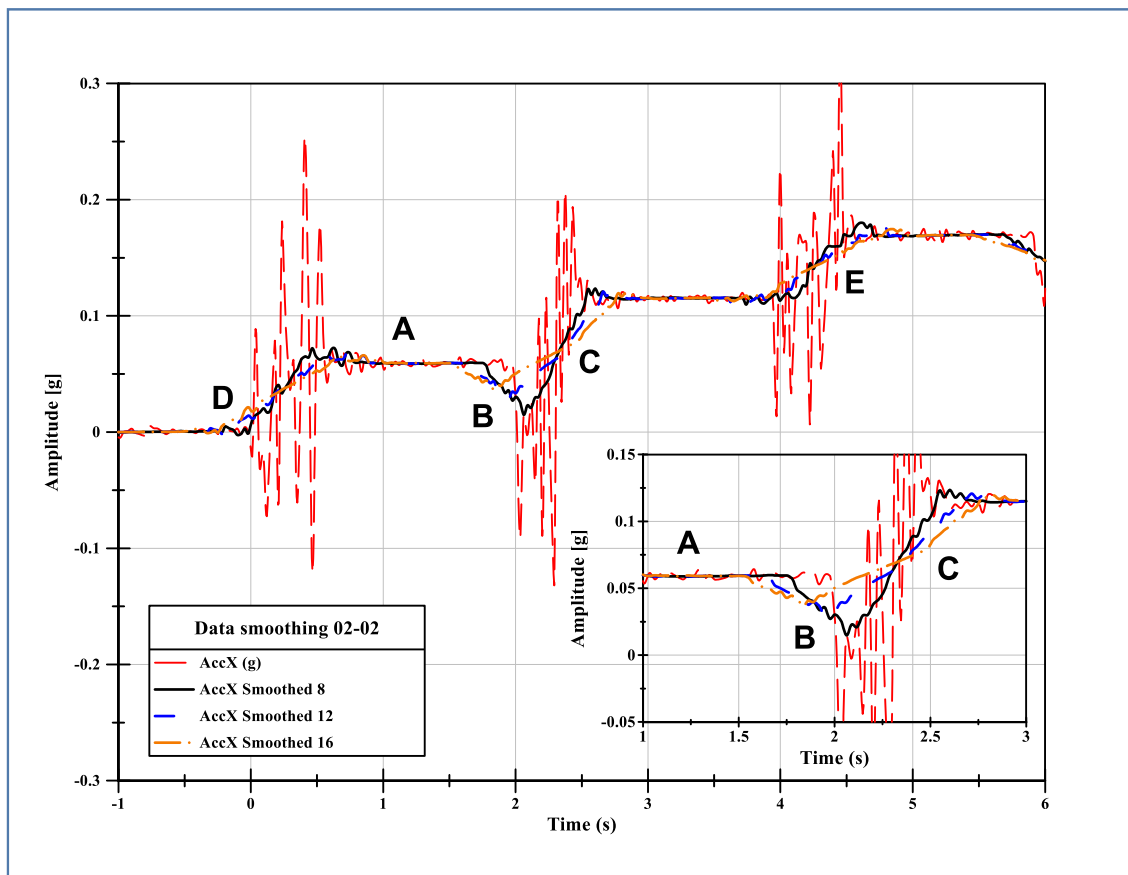


Figure 5-13 – Close-up of smoothed data shown in Figure 5-12

Point A shows how the smoothing process handled those areas of the data where the actuator movement was paused, and as might be expected because there was little signal variation or noise, the number of values had little affect except at the extremities where there was a change in movement and hence increased noise. At the extremities smoothing using a greater number of values, as seen in the orange (dash and dot) AccX Smoothed 16 curve, produced a shallower transition that started earlier and finished later. The opposite was true if less values were used, as seen in the solid black AccX Smoothed 8 curve.

Point B shows how the smoothing process handled sudden changes in the signal, in this instance the start of actuator movement. As discussed for Point A, the more values used to average the data the shallower the smoothed signal, and the more closely it matched the anticipated transition. Again, as might be expected, using fewer values to average the signal produced a smoothed signal closer to the original unsmoothed signal with a pronounced dip in the smoothed signal.

Point C depicted the area where the actuator was moving and hence the original unsmoothed signal was quite noisy. In this instance the smoothing process that used the least number of values produced a closer approximation to the original unsmoothed signal, as can be seen in the solid black AccX Smoothed 8 curve. In this instance the

shallower angle of the smoothed curve that used a greater number of values for averaging led to a curve that did not closely match the original unsmoothed curve in this area of interest.

The discussion in this section has highlighted the dilemma that arises when an unweighted moving average process is used to smooth a curve where there are sudden transitions in data attitude coupled with an increase in signal noise. It should be noted that the effects discussed are not as pronounced in the similar areas, D and E, on either side of the point in question, leading to the reservations as to whether the odd spike in the data is of major concern. A review of the overall data shown in Figure 5-12 would suggest that all of the smoothed curves remove the majority of noise, and it is therefore a compromise in terms of removing uncharacteristic spikes in the data and maintaining a smoothed signal that most closely matches the general trend of the original unsmoothed data.

5.4.4.2 Outlier removal

Where the data signal contained uncharacteristic spikes or outliers such as those seen in Figure 5-10, a median filter (MF) was used in preference to the unweighted moving average smoothing algorithm described above. As the DIAdem software was not supplied with a median filter, a LabVIEW VI (National Instruments Corporation., 2016b) was produced instead.

Figure 5-14 shows the same data that was smoothed using the DIAdem smoothing algorithm in the previous section, but with the addition of curves that represent data where the outliers were removed using the median filter algorithm. As with the smoothed data, close-ups of the area highlighted within the green circle in Figure 5-14 have been reproduced in Figure 5-15.

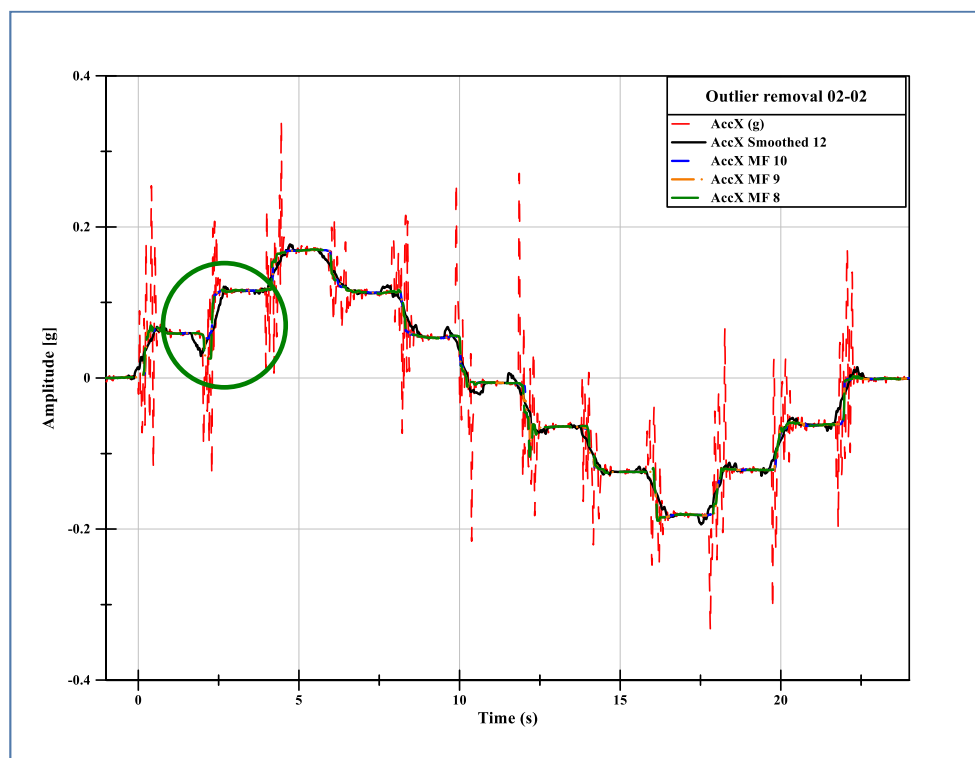


Figure 5-14 – Example of a data signal where a median filter was used to remove outliers from a signal

The diagram consists of five curves; the original curve (AccX (g)), one curve that represents this curve after it had been smoothed using 12 neighbouring values, and three curves showing the curve after a median filter had been applied.

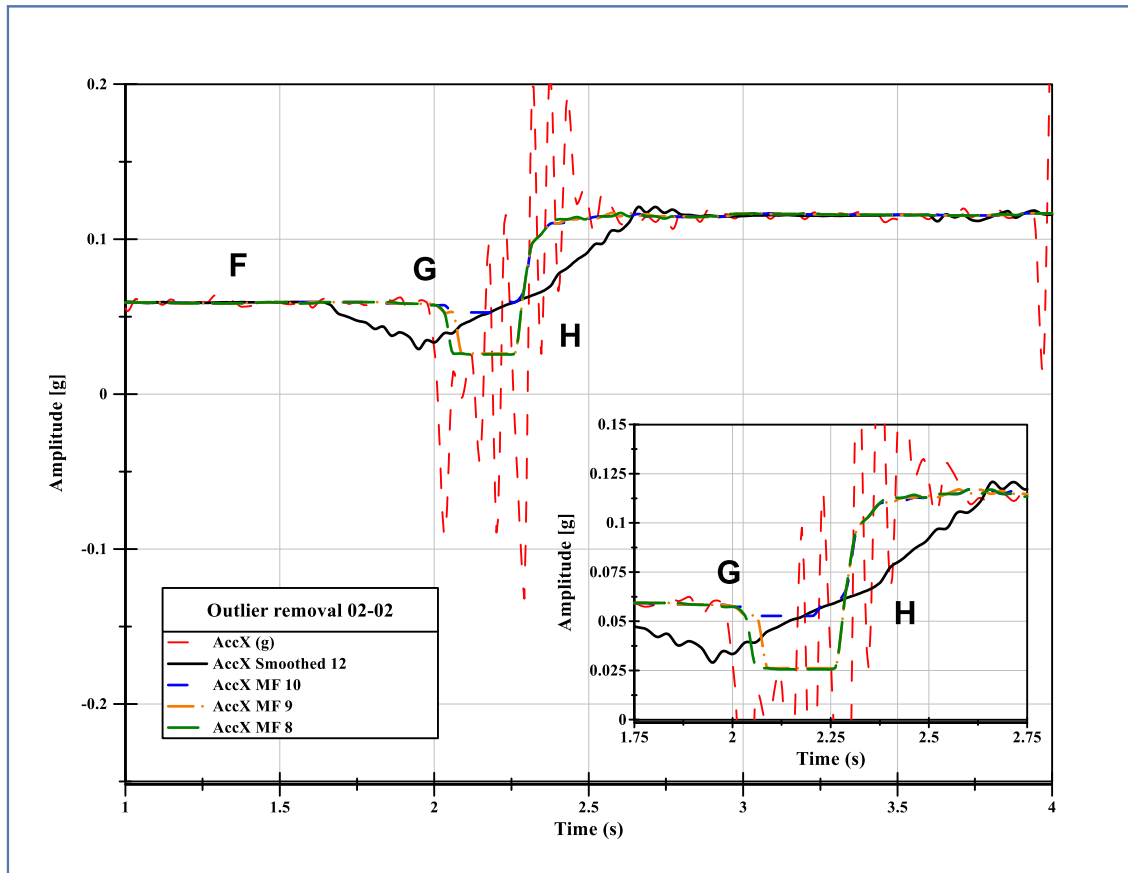


Figure 5-15 – Close-up of outlier removal shown in Figure 5-14

A close examination of the curves at Points F, G and H in Figure 5-15 follows, showing how varying the number of values used to calculate the moving median changed how effectively outliers were removed from the original data signal.

Point F is similar to Point A in the smoothing process in that it represents the portion of the data signal where there was no actuator movement, and little noise, and therefore the effect of the moving median was similar regardless of the number of values used.

Point G is similar to Point B in the smoothing process in that it represents a portion of the original data where the actuator had started to move producing a sudden spike in the data signal. A comparison between the solid black smoothed data curve, AccX Smoothed 12, and the other curves representing the median filter, shows a marked difference in the impact that the median filter had on removing the outliers. It shows that a lower value of numbers used for the median filter can lead to a pronounced dip in the filtered curve, as shown by the green (dashed) and orange (dash and dot) curves representing AccX MF 8 and AccX MF 9 respectively. However, when the optimum

number is chosen, in this instance 10 values, as represented by the blue (dashed) curve AccX MF 10, a close approximation to the original data signal can be achieved.

Point H is similar to Point C in the smoothing process in that it represents a portion of the original data where the actuator was moving that resulted in an increase in noise due to vibrations generated by the actuator movement. In this instance the number of values used for the median filter had no impact on the attitude of the resultant curve, and therefore all the median filter curves followed a similar path that quite closely matched that of the original data signal.

The above discussion has highlighted the relative benefits of smoothing using an unweighted moving average and the median filter for outlier removal, however, due to the prominence of outliers in much of the raw data it was decided that the median filter would generally be used in preference to smoothing. The impact of this decision is discussed in more detail in the subsequent results chapters where the influence of such post-processing on the final data analysis is investigated.

5.5 Error analysis and quantification

The quantification of errors and differences between sensor signals can be a subjective process as one person's interpretation of a result can differ from someone else's, unless clear interpretation criteria have been defined beforehand. The data shown in Figure 5-16 illustrates this well; it shows a selection of curves, some of which are closely aligned and one which is not.

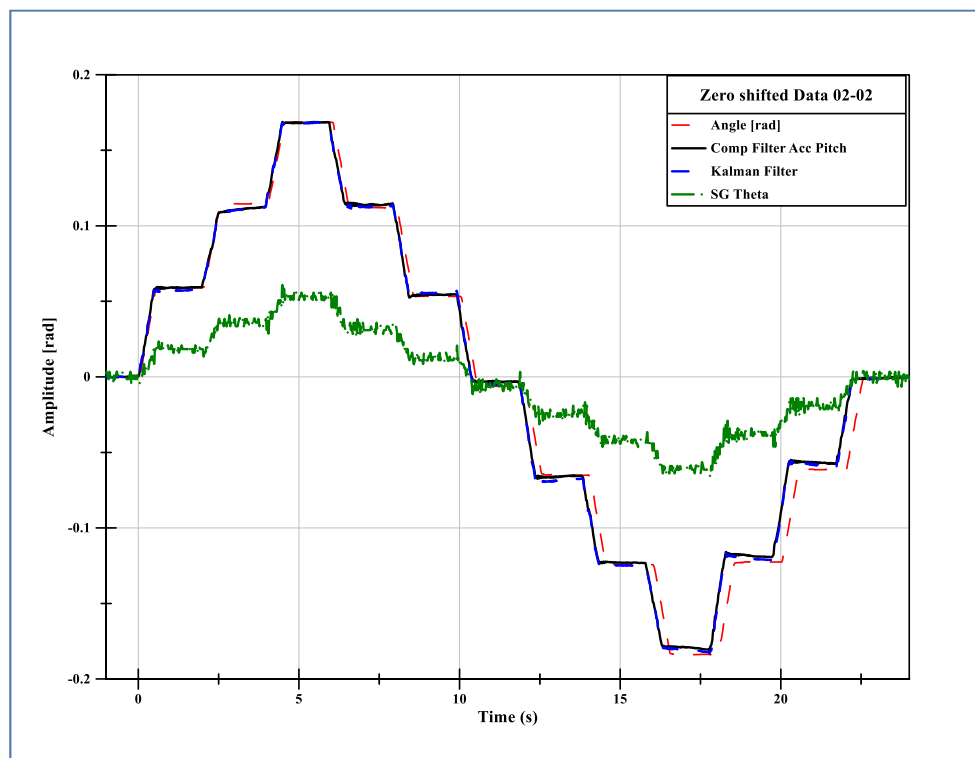


Figure 5-16 – Comparison of Data from Experiment Two

Terms that might be used to describe the similarity or otherwise of the curves include ‘good agreement’ or ‘close correlation’. Unfortunately, both the words ‘good’ and ‘close’ provide no real measure of the ‘agreement’ or ‘correlation’ between the different curves, because as suggested above, they are subjective and rely on an individual’s perception of the concepts. To address this problem various authors have proposed metrics with the aim of quantifying the extent of any agreement or correlation. Generally, the approaches used compare a ‘computed’ (accepted reference value) and a ‘measurement’ (test result) response history and then output metrics to describe the difference in magnitude, phase and a combined or comprehensive metric that relates both the differences in magnitude and phase.

Various examples of the use of error measures and metrics in the shock and vibration field can be found in the literature, and as such are relevant to this research. In his papers Schwer (Schwer, 2007, 2006) discussed the use of a number of the better known error measures such as Sprague and Geers (Sprague & Geers, 2003, 2004, 1999). He considered these metrics against measured velocity data and compared the outcome with the opinion of a panel of Subject Matter Experts (SME) to see whether the error measures could mirror the decisions of the SME panel. When the error measure metric results and the SME opinions were compared, they all fell within one standard deviation of each other suggesting that there was reasonable agreement between the chosen approaches.

Sarin et al (Sarin *et al.*, 2008) produced a thorough review of a selection of error metrics in particular those that included combined or comprehensive error measures. The data used for the comparisons related to the time histories for vehicle safety assessments response models, and although not directly related to the shock and vibration field, their analysis of measures and metrics in relation to time histories was considered relevant. The review included the previously identified Sprague and Geers’ error measure and also Russell’s (Russell, 1997a) error measure, and a separate study that compared the results from the error measures with the data gathered from SMEs. As part of their review, Sarin et al used a simple example to identify a limitation with both the Sprague and Geers’ and Russell’s error measures that limited these measures’ ability to quantify the magnitude when there was a difference in shape. As the data curves produced as part of this research were all of a similar shape to each other, it was decided that this limitation was unlikely to apply, and therefore both the Sprague and Geers’ and Russell’s error measures were included as part of the investigation that is described later in this chapter.

Many of the error measure methods assume that there is a ‘true’ value against which other data is compared. As it may be difficult to ascertain whether the ‘true’ value is actually the correct value, Whang et al (Whang, Gilbert & Zilliacus, 1994) proposed the use of inequality measures that looked at the combined effects and output a single inequality index that was not reliant upon a true value. Two of the inequality measures evaluated by Whang et al, Whang’s inequality index and Theil’s inequality coefficient, were chosen to be used as part of this research and provide a comparison with the other multi-component error measures discussed above.

The remainder of this section considers a selection of the more popular metrics and measures, displaying the formula used to calculate each value alongside the complementary LabVIEW VI that was used to automate this aspect of the analysis process. In each case c_i represents the ‘computed’ or accepted reference value, whilst m_i represents the ‘measurement’ value or test result, and i the specific element number being considered. It will be

seen that many of the measures use combinations of the root-sum-square (RSS) of the history of which $\sqrt{\sum c_i^2}$ is one such example.

5.5.1 Geers' error measures

Since the mid-1980s Thomas Geer has been seen as a pioneer of error measures within the US Navy's shock and vibration community (Whang, Gilbert & Zilliacus, 1994). In 1984 he presented his original error measures to enable an objective method of comparing the difference between transient response histories (Geers, 1984). These included measures to compare the magnitude (GM) and phase (GP) of the response histories that were then combined to produce a comprehensive error measure (GC). The phase measure presented below is a revised version of the original phase measure to remedy the problem of sign reversal when the sign of one of the curves is reversed (Whang, Gilbert & Zilliacus, 1994).

5.5.1.1 Equations

$$GM = \frac{\sqrt{\sum c_i^2}}{\sqrt{\sum m_i^2}} - 1.0 \quad (5-1)$$

$$GP = 1.0 - \frac{\sum c_i m_i}{\sqrt{\sum c_i^2} \sqrt{\sum m_i^2}} \quad (5-2)$$

$$GC = \sqrt{GM^2 + GP^2} \quad (5-3)$$

5.5.1.2 LabVIEW VI

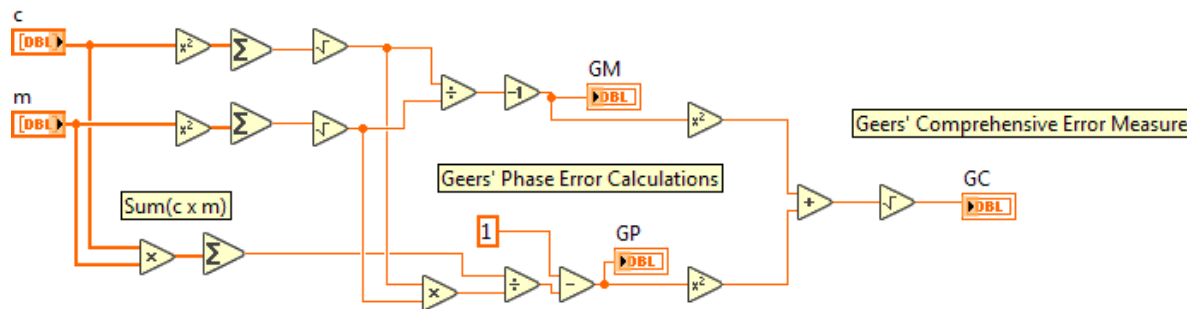


Figure 5-17 – LabVIEW SubVI for Geer's error measures

5.5.1.3 Criteria

	< 0	= 0	> 0
GM	$c_M < m_M$	$c_M = m_M$	$c_M > m_M$
GP	Not applicable (NA)	$c_P = m_P$	$c_P \neq m_P$
GC	Not applicable (NA)	$c_C = m_C$	$c_C \neq m_C$

Table 5-3 – Geers' error measure criteria

Where c_M, c_P, c_C and m_M, m_P, m_C respectively refer to the differences in magnitude, phase and combined factors between the calculated and measured response histories. For the Geers' error measure criteria summarized in Table 5-3 the following apply:

- When the magnitude of the measured response history is greater than that for the calculated response history, GM will be a negative number.
- When the calculated and measured response histories are identical all the criteria, GM, GP and GC will equal zero.
- When the magnitude of the calculated response history is greater than that for the measured response history, GM will be a positive number.
- When the calculated and measured response histories are not identical GP and GC will be positive numbers.

5.5.2 Geers' Comparative Shock Analysis (CSA) Program error measures

The Comparative Shock Analysis (CSA) Programme was part of the US SEAWOLF Main Propulsion Unit project (Geers CSA 1987). As part of this project, the previously mentioned Geers' error measures were updated to offer supplementary information to aid a better understanding of differences between two data signals.

5.5.2.1 Equations

$$M = \frac{\sqrt{\sum c_i^2}}{\sqrt{\sum m_i^2}} - 1.0 \quad (5-4)$$

$$P = 1.0 - \frac{|\sum c_i m_i|}{\sqrt{\sum c_i^2} \sqrt{\sum m_i^2}} \quad (5-5)$$

$$C = \text{sign}(\sum c_i m_i) \sqrt{M^2 + P^2} \quad (5-6)$$

5.5.2.2 LabVIEW VI

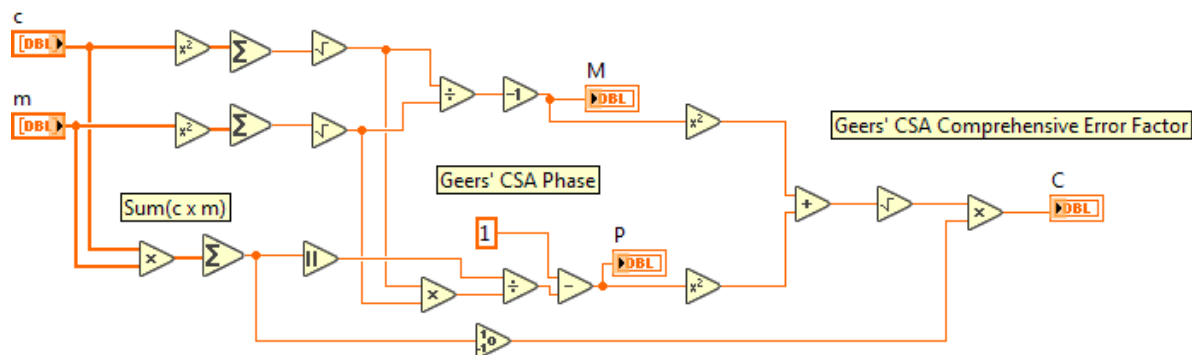


Figure 5-18 – LabVIEW SubVI for Geer's CSA error measures

5.5.2.3 Criteria

	< 0	= 0	> 0
M	$c_M < m_M$	$c_M = m_M$	$c_M > m_M$
P	Not applicable (NA)	$c_P = m_P$	$c_P \neq m_P$
C	$c_C \leq 0.5m_C$	$c_C = m_C$	$c_C > 0.5m_C$

Table 5-4 – Geers’ CSA error measure criteria

Comparing the criteria relating to the Geers’ CSA error measures summarized in Table 5-4, with those for the Geers’ error measure in Table 5-3, shows that the criteria are identical except for $C < 0$. In this instance a negative value for C relates to a relative difference between the phase for the calculated and measured response histories of 0.5 or less.

5.5.3 Russell’s error measures

In 1997 Russell published a new error measure that was designed to address limitations in existing error measures where biasing towards one of the sets of data being compared, was reduced (Russell, 1997a). By doing this it was hoped to be able to make the resultant error measures better suited to subsequent statistical analysis. As with the previously discussed Geers’ error measures, Russell’s error measures included magnitude (RM), phase (RP) and comprehensive (RC) that are shown and discussed below.

5.5.3.1 Equations

$$RM = \text{sign}(m) \log_{10}(1 + |m|) \quad (5-7)$$

$$m = (\sum c_i^2 - \sum m_i^2) / \sqrt{\sum c_i^2 \sum m_i^2} \quad (5-8)$$

$$RP = \frac{1}{\pi} \cos^{-1} \left(\frac{\sum c_i m_i}{\sqrt{\sum c_i^2 \sum m_i^2}} \right) \quad (5-9)$$

$$RC = \sqrt{\frac{\pi}{4} (RM^2 + RP^2)} \quad (5-10)$$

5.5.3.2 LabVIEW VI

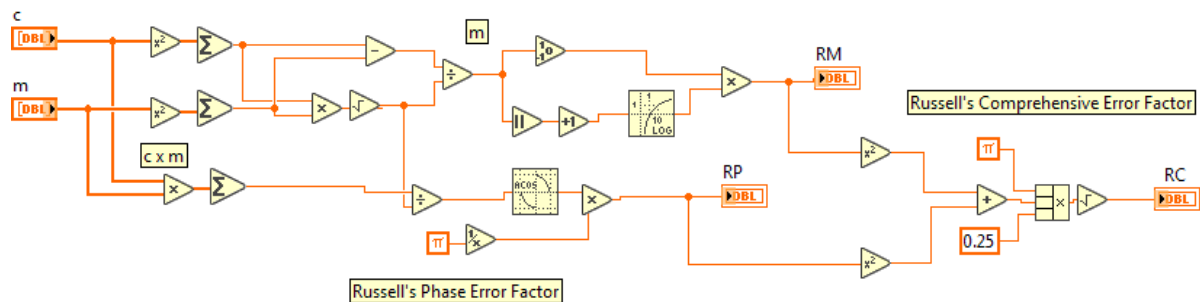


Figure 5-19 – LabVIEW SubVI for Russell’s error measures

5.5.3.3 Criteria

	< 0	= 0	> 0
RM	$c_M < m_M$	$c_M = m_M$	$c_M > m_M$
RP	Not applicable (NA)	$c_P = m_P$	$c_P \neq m_P$
RC	Not applicable (NA)	$c_C = m_C$	$c_C \neq m_C$

Table 5-5 – Russell’s error measure criteria

The Russell error measures have advantages over some of the other error measures in that it is possible to use the magnitude and phase measures to provide a degree of interpretation of the difference between the response histories being compared. For instance:

- A positive magnitude error measure indicates that the calculated response is greater than the measured response, however unlike other error measures, the value is not weighted to one or other of the responses.
- Although the magnitude error measure is not bounded it remains relatively small with an order of magnitude difference being approximately equal to 1.0.
- The phase error measure provides an indication of the proportion of time that the responses being compared are in or out of phase. A value of 0.5 therefore suggests that the responses are by-in-large in phase for 50% of the time and out of phase for the other 50%.

5.5.4 Sprague and Geers’ error measures

Sprague and Geers (Sprague & Geers, 2004) used a modified version of Geers’ error measure to quantify the error between transient response histories and that of a benchmark solution. As with the previously described error measures, the one they developed consisted of magnitude (SGM), phase (SGP) and comprehensive (SGC) measures of similar form and were based upon the RSS of combinations of the computed and measured response history.

5.5.4.1 Equations

$$SGM = \sqrt{\sum c_i^2 / \sum m_i^2} - 1 \quad (5-11)$$

$$SGP = \frac{1}{\pi} \cos^{-1} \left(\sum c_i m_i / \sqrt{\sum c_i^2 \sum m_i^2} \right) \quad (5-12)$$

$$SGC = \sqrt{SGM^2 + SMP^2} \quad (5-13)$$

5.5.4.2 LabVIEW VI

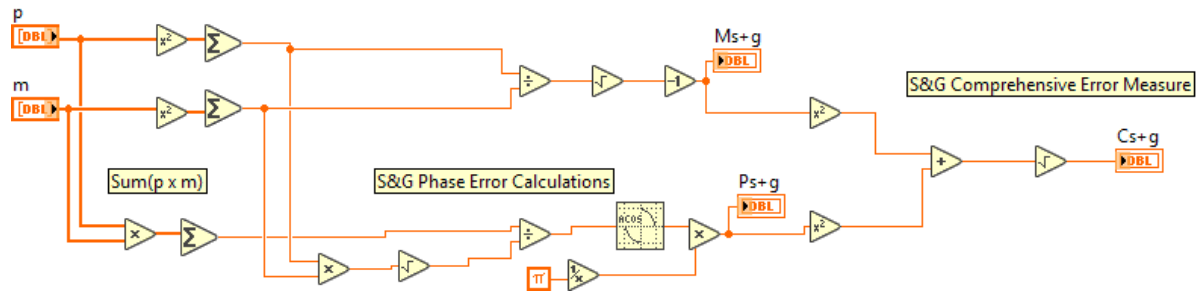


Figure 5-20 – LabVIEW SubVI for Sprague and Geers' error measures

5.5.4.3 Criteria

	$-1 < x < 0$	$= 0$	$0 > x > 1$
SGM	$c_M < m_M$	$c_M = m_M$	$c_M > m_M$
SGP	Not applicable (NA)	$c_P = m_P$	$c_P \neq m_P$
SGC	Not applicable (NA)	$c_C = m_C$	$c_C \neq m_C$

Table 5-6 – Sprague and Geers' error measure criteria

It will be noted that the criteria for the Sprague and Geers' error measures in Table 5-6 are similar to those for the Geer's error measures summarized in Table 5-3 except that they are bounded between 1 and -1. (Note: x relates to the values SGM, SGP or SGC).

5.5.5 Whang's inequality measure

Unlike the previously mentioned error measures, Whang's inequality measure provides a single measure to describe the 'inequality' between two response curves. (Whang, Gilbert & Zilliacus, 1994)

5.5.5.1 Equation

$$WI = \frac{\sum |c_i - m_i|}{\sum |c_i| + \sum |m_i|} \quad (5-14)$$

5.5.5.2 LabVIEW VI

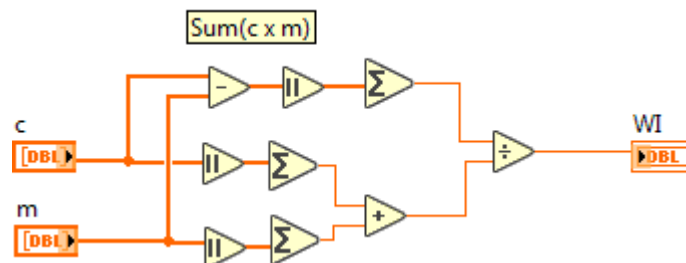


Figure 5-21 – LabVIEW SubVI for Whang's inequality measure

5.5.5.3 Criteria

	= 0	> 0
WI	$c_i = m_i$	$c_i \neq m_i$

Table 5-7 – Whang’s inequality measure criteria

This inequality measure computes the ratio of the difference between the two time histories and the sum of their areas, and is bounded between zero and one, with zero being a perfect correlation and one being poor.

5.5.6 Theil’s inequality error coefficient

Theil’s inequality coefficient is similar to Whang’s inequality measure in that it also provides a single figure to represent the inequality between two curves (Whang, Gilbert & Zilliaccus, 1994).

5.5.6.1 Equation

$$TI = \frac{\sqrt{\sum (c_i - m_i)^2}}{\sqrt{\sum c_i^2} + \sqrt{\sum m_i^2}} \quad (5-15)$$

5.5.6.2 LabVIEW VI

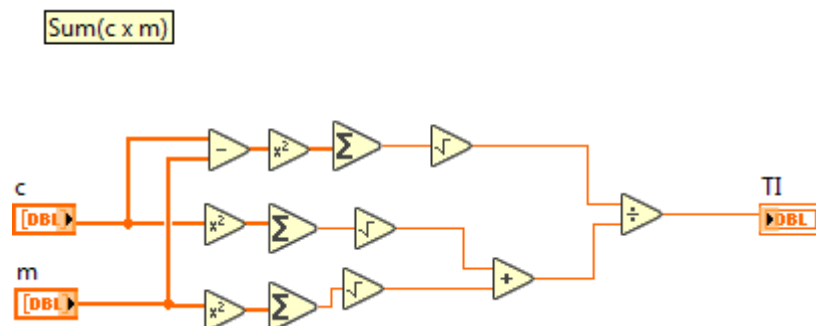


Figure 5-22 – LabVIEW SubVI for Theil’s inequality coefficient

5.5.6.3 Criteria

	= 0	> 0
TI	$c_i = m_i$	$c_i \neq m_i$

Table 5-8 – Theil’s inequality coefficient criteria

Similar to the Whang inequality measure, this measure is bounded between zero and one, with zero being a perfect correlation and one being poor.

5.6 Error analysis case studies

To illustrate the results of using the different error measures, and illustrate their relationship, a set of generated curves and an example of some test data were used as case studies to compare each error measure/metric.

5.6.1 Error measure analysis example

The generated data used for the case study is shown in Figure 5-23. The stepped curve format was chosen because it was simple to generate and similar to the format seen in Experiments One and Two (see Chapter 6). Table 5-9 summarizes all the calculated error metrics and measures generated for this example (the abbreviations in the left-hand column of the table relate to the metric/measure abbreviations discussed in Section 5.5).

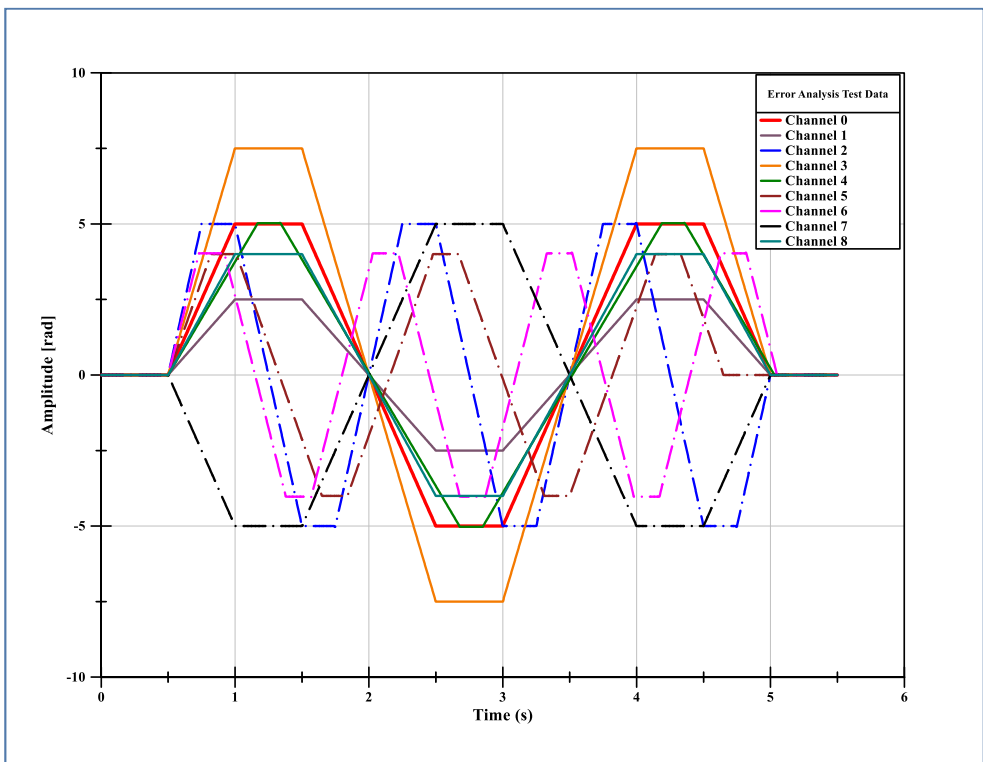


Figure 5-23 – Diagram showing plots for error measure example

For the purpose of this example Channel 0 (Ch0) was chosen as the ‘calculation’ or accepted reference value channel, whilst the remaining channels were taken to be the ‘measurement’ or test result channels that were then compared with Ch0. The data generated for the measurement channels was chosen so that it would represent differences in magnitude, phase and a combination of magnitude and phase, to enable the variation in metrics and measures to be fully investigated: Channels 1, 3, 4, 7 and 8 were used to investigate different magnitudes; Channels 2, 5, 6 and 7 were used to investigate different phases; whilst Channels 5, 6 and 7 were used to investigate different combinations of magnitude and phase.

It will be noted that Channel 7 was used in all three investigations; the reason for this will become evident in the remaining sections that discuss the results of each investigation in more detail.

	Ch0/Ch1	Ch0/Ch2	Ch0/Ch3	Ch0/Ch4	Ch0/Ch5	Ch0/Ch6	Ch0/Ch7	Ch0/Ch8
GM	1.0000	-0.0001	-0.3333	0.1567	0.3916	0.2668	0.0000	0.2500
GP	0.0000	1.0000	0.0000	0.0076	0.8846	1.0125	2.0000	0.0000
GC	1.0000	1.0000	0.3333	0.1569	0.9674	1.0470	2.0000	0.2500
M	1.0000	-0.0001	-0.3333	0.1567	0.3916	0.2668	0.0000	0.2500
P	0.0000	1.0000	0.0000	0.0076	0.8846	0.9875	0.0000	0.0000
C	1.0000	-1.0000	0.3333	0.1569	0.9674	-1.0229	0.0000	0.2500
RM	0.3979	-0.0001	-0.2632	0.1113	0.2235	0.1695	0.0000	0.1614
RP	0.0000	0.5000	0.0000	0.0393	0.4632	0.5040	1.0000	0.0000
RC	0.3527	0.4431	0.2333	0.1046	0.4558	0.4712	0.8862	0.1430
SGM	1.0000	-0.0001	-0.3333	0.1567	0.3916	0.2668	0.0000	0.2500
SGP	0.0000	0.5000	0.0000	0.0393	0.4632	0.5040	1.0000	0.0000
SGC	1.0000	0.5000	0.3333	0.1615	0.6066	0.5702	1.0000	0.2500
WI	0.3333	0.6250	0.2000	0.0870	0.6259	0.6642	1.0000	0.1111
TI	0.3333	0.7071	0.2000	0.0952	0.6762	0.7163	1.0000	0.1111

Table 5-9 – Summary of Error Measure values for error measure example

5.6.2 Error measure analysis example – Magnitude analysis

The data from Channels 1, 3, 4, 7 and 8 was compared against the data from Channel 0 in order to investigate how a change in magnitude of the amplitude of a curve, altered the different calculated metrics and measures (Figure 5-24). Table 5-10 summarizes the differences between the magnitude of the channels shown in Figure 5-24 alongside the relevant metrics and error measures from Table 5-9.

	Difference in Amplitude	Difference in pause period length	GM	M	RM	SGM	WI	TI
Channel 1	0.5	1.0	1.0000	1.0000	0.3979	1.0000	0.3333	0.3333
Channel 3	1.5	1.0	-0.3333	-0.3333	-0.2632	-0.3333	0.2000	0.2000
Channel 4	1.0	0.34	0.1567	0.1567	0.1113	0.1567	0.0870	0.0952
Channel 7	-1.0	1.0	0.0000	0.0000	0.0000	0.0000	1.0000	1.0000
Channel 8	0.8	1.0	0.2500	0.2500	0.1614	0.2500	0.1111	0.1111

Table 5-10 – Summary of channel differences for magnitude investigation

The first thing to note is that magnitude error measure for the Geers', Geers' CSA and Sprague and Geers' methods all produced the same result; this was as expected because they each used the same calculation. The following observations were also made:

- As a measurement channel's magnitude approaches that of the calculation channel, the appropriate error measures approach zero. A comparison of Channels 1, 4 and 8 illustrate this well. The Geers' magnitude error measure for each of these channels is 1.0000, 0.1567 and 0.2500 respectively. Similarly, the Russell's magnitude error measure is 0.3979, 0.1113 and 0.1614 respectively. This means that

Channel 4's magnitude is closer to that of Channel 0 than either Channel 1 or Channel 8, with Channel 1 having the greatest difference and Channel 8 being somewhere in the middle.

- A negative magnitude error measure indicates that the measurement channel's magnitude is greater than the calculation channel's magnitude (as illustrated by the magnitude error measures for Channel 3).

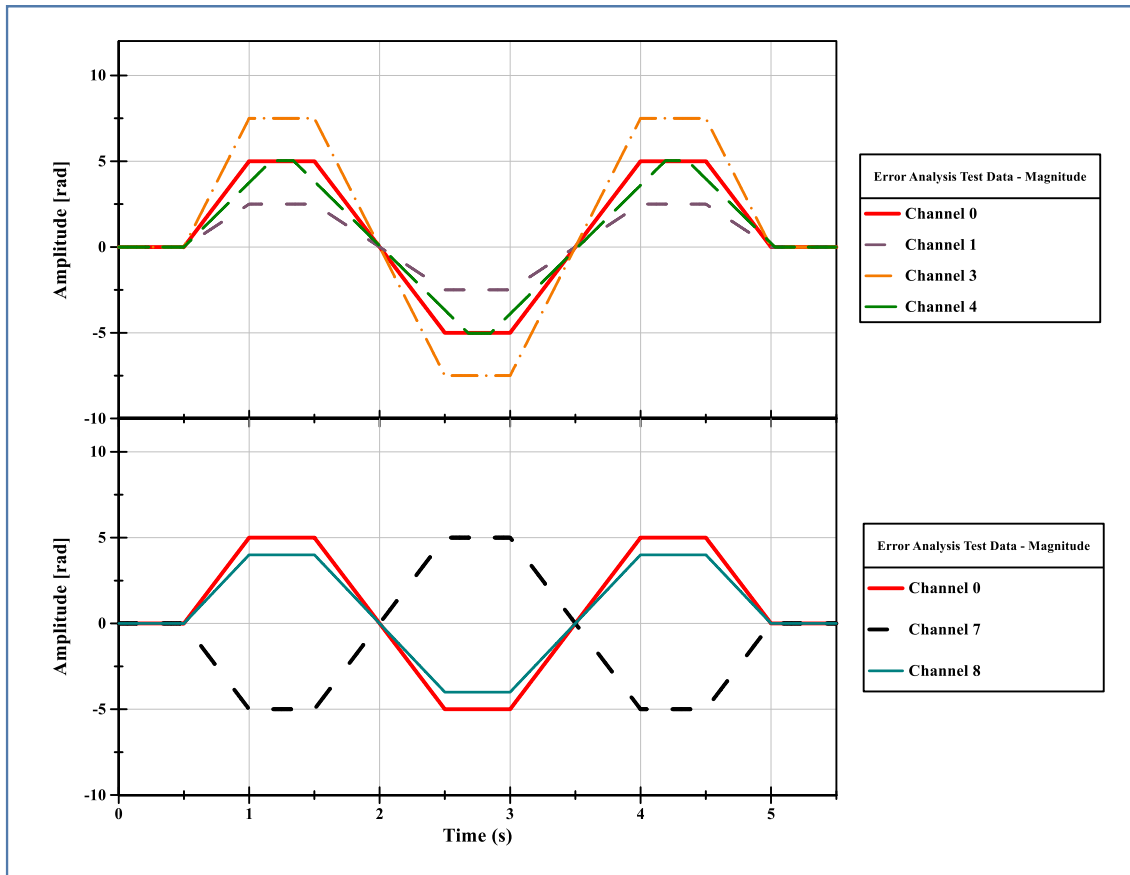


Figure 5-24 – Diagram showing magnitude plots for error measure example

- If the measurement and calculation channel's magnitudes are the same, the magnitude error measures equal zero as illustrated by the error measures for Channel 7. It should be noted that although Channel 7 has the same magnitude as Channel 0, it is of the opposite sign. This illustrates the importance of having a measure of the phase difference between the different data channels. In this particular instance the fact that both the Whang and Theil inequality measures are 1.0000 indicates that although the magnitude is the same, the phase is not. The relevant phase error measure is discussed in the next section.
- When comparing magnitude by itself, both the Whang and Theil inequality measures provide a reasonable method of evaluating the similarity of data channels as long as the measurement channels being compared have the same relation to the calculation channel i.e. they all either have a smaller or larger magnitude than the calculation channel. This is illustrated by the inequality measures for Channels 1, 4 and 8 of 0.3333, 0.0870 and 0.1111 respectively reaffirming that Channel 4 had a magnitude closest to Channel 0 while Channel 1 was the least close and Channel 8 was somewhere in between. It will however be noted that both the inequality measures for Channel 3 are 0.2000 suggesting that this channel

has a magnitude somewhere between that of Channel 8 and Channel 4. However, unlike the other channels, Channel 3's magnitude is greater than that of Channel 0 and also all the other channels. This highlights the fact that although the Whang and Theil inequality measures can differentiate between measurement data of similar magnitude to the calculation channel, they are unable to differentiate between measurement channel magnitudes when one is greater, and one is lesser than, the calculation channel. This is as expected because both of these methods do not differentiate between the 'true' and measured values.

5.6.3 Error measure analysis example – Phase analysis

The data from Channels 2, 5, 6 and 7 was compared against the data from Channel 0 in order to investigate how a change in the phase of a curve altered the different calculated phase metrics and measures (Figure 5-25). Table 5-11 summarizes the differences between the phase of the channels shown in Figure 5-25 alongside the relevant metrics and error measures from Table 5-9.

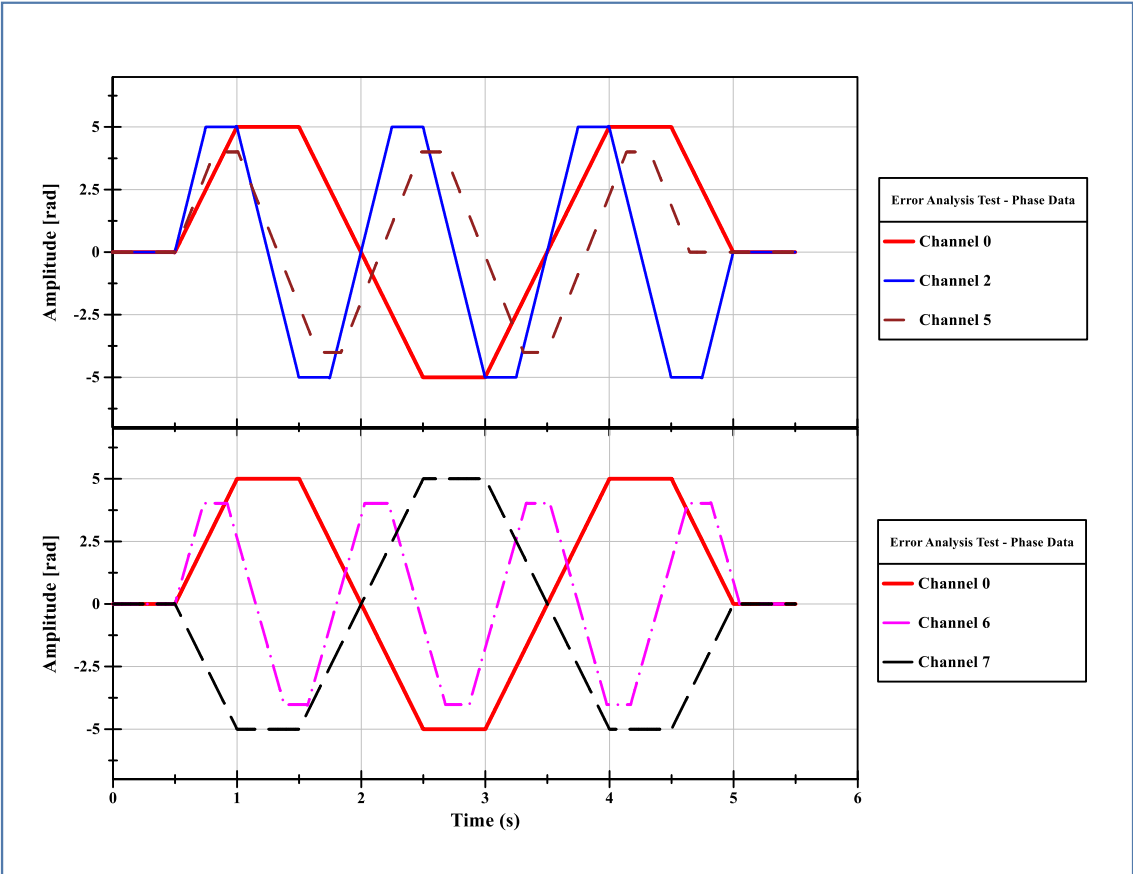


Figure 5-25 – Diagram showing phase plots for error measure example

	Difference in Frequency	GP	P	RP	SGP	WI	TI
Channel 2	2.0	1.0000	1.0000	0.5000	0.5000	0.6250	0.7071
Channel 5	1.83	0.8846	0.8846	0.4632	0.4632	0.6259	0.6762
Channel 6	2.26	1.0125	0.9875	0.5040	0.5040	0.6642	0.7163
Channel 7	1.0	2.0000	0.0000	1.0000	1.0000	1.0000	1.0000

Table 5-11 – Summary of channel differences for phase investigation

As with the magnitude error measures the following observations were made for the related phase error measures:

- Channel 2 had double the frequency of Channel 0. For the Geers' and Geers' CSA the error measure was 1.0000; for the Russell and Sprague and Geers' the error measure was 0.5000; and for the Whang's and Theil's inequality measures were 0.6250 and 0.7071 respectively. The 0.5 value for RP indicates that the two curves are in-and-out of phase for 50% of the time which is consistent with the difference in frequency being double. Similarly, the WI and TI values indicate that the curves are out of phase more often than not.
- Channel 7 had the same frequency as Channel 0 but is out of phase by half a cycle. The associated error measures and metrics were: Geers' error measure was 2.0000; Geers' CSA error measure was 0.000; whilst the remaining error and inequality measures were 1.0000. The values of one for RP, WI and TI suggest that the curves are not correlated which is consistent with them being out-of-phase.

5.6.4 Error measure analysis example – Combined analysis

The data from Channels 5, 6 and 7 was compared against the data from Channel 0 in order to investigate how a change in both the magnitude of the amplitude and the phase of a curve altered the different calculated comprehensive metrics and measures (Figure 5-26). Table 5-12 summarizes the differences between the combined magnitude and phase of the channels shown in Figure 5-26 alongside the relevant metrics and error measures from Table 5-9.

Observations for channels that had both magnitude and phase differences included:

- Channels 5 and 6 were similar in magnitude but with slightly different frequencies; the former being just under twice the frequency of the Channel 0, whilst the latter was just over twice the frequency. The slight increase in all the comprehensive measures (except C and SGC) indicated this.
- For the out of phase example, Channel 7, the comprehensive measures for GC, C, SGC, WI and TI were the same as the associated phase measures suggesting a bias towards the phase for these measures in such situations, however RC was 0.8862 suggesting some independence in terms of how the combined measure was calculated (validating Russell's attempt to reduce bias to either magnitude or phase).

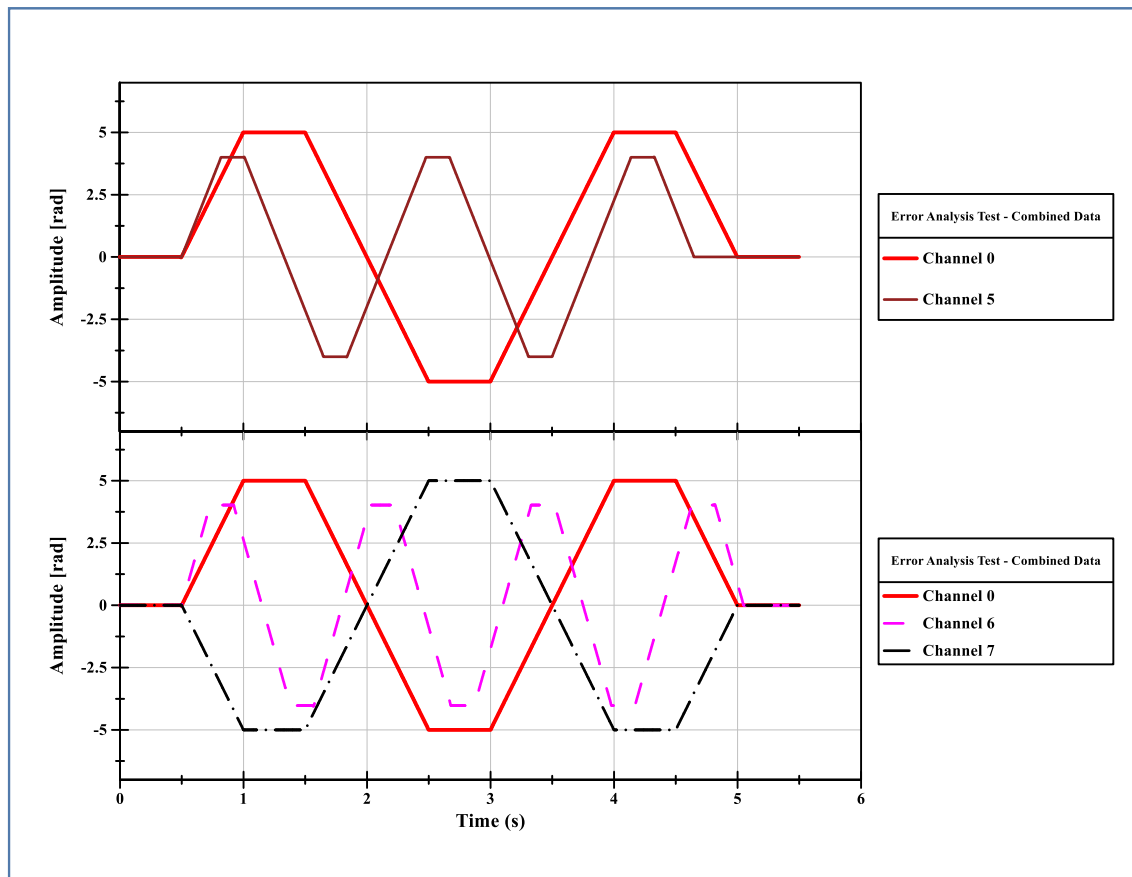


Figure 5-26 – Diagram showing ‘combined’ plots for error measure example

	Difference in Amplitude	Difference in pause period length	Difference in Frequency	GC	C	RC	SGC	WI	TI
Channel 5	0.800	0.38	1.83	0.9674	0.9674	0.4558	0.6066	0.6259	0.6762
Channel 6	0.805	0.38	2.26	1.0470	-1.0229	0.4712	0.5702	0.6642	0.7163
Channel 7	-1.0	1.0	1.0	2.0000	0.0000	0.8862	1.0000	1.0000	1.0000

Table 5-12 – Summary of channel differences for combined magnitude and phase investigation

5.6.5 Error measure analysis - Experiment Two example

The error measures were applied to some actual test results collected during Experiment Two as shown in Figure 5-27, with a close-up of the first part of the data shown in Figure 5-28. Each of the diagrams shows four curves; the accepted reference value (Angle [rad]), two curves showing the results for the Complementary and Kalman filters (Comp Filter Acc Pitch and Kalman Filter respectively), and finally one curve for the angular rotation calculated using data from the strain gauges (SG Theta). To enable the error measure calculations to be correctly processed, the same number of samples were required for both the HSV and measured data. This necessitated resampling the measured data using DIAdem to get matching sample numbers; this was a requirement for all the tests conducted as part of the research. The error measures calculated for the data shown in Figure 5-27 and Figure

5-28 are summarised in Table 5-13. Table 5-13 refers to Ch0, Ch1, Ch2 and Ch3 that respectively relate to Angle [rad], Comp Filter Acc Pitch, Kalman Filter and SG Theta. The following observations could be made:

- Figure 5-27 clearly shows that the curve for SG Theta was approximately one third of the magnitude of the curve for Angle [rad]. The positive magnitude error measure of 2.1470 in Table 5-13 confirms that the measurement channel (SG Theta) has a magnitude less than the calculation channel (Angle [rad]).
- The lower magnitude error measure value for the Kalman Filter (Ch2) compared to the Complementary Filter (Ch1) indicates that the latter has a slightly greater magnitude.
- The Kalman Filter phase error measure is lower than the Complementary Filter, although both are quite close (if considering 3 d.p. (decimal places) they are the same). This indicates very similar phase differences. The Strain Gauge phase difference is slightly greater than that for both the Kalman and Complementary Filters, so is more out of phase with the reference value.
- The Whang and Theil inequality measures place the curves in order Kalman Filter, Complementary Filter and Strain Gauge which agrees with the ranking produced by the other error measures.

	Ch0/Ch1	Ch0/Ch2	Ch0/Ch3
GM	0.0219	0.0164	2.1470
GP	0.0061	0.0059	0.0104
GC	0.0227	0.0175	2.1470
M	0.0219	0.0164	2.1470
P	0.0061	0.0059	0.0104
C	0.0227	0.0175	2.1470
RM	0.0184	0.0139	0.5831
RP	0.0352	0.0347	0.0458
RC	0.0352	0.0332	0.5184
SGM	0.0219	0.0164	2.1470
SGP	0.0352	0.0347	0.0458
SGC	0.0414	0.0384	2.1475
WI	0.0383	0.0369	0.5234
TI	0.0563	0.0551	0.5214

Table 5-13 – Summary of Error Measure values for error measure example for Experiment Two data

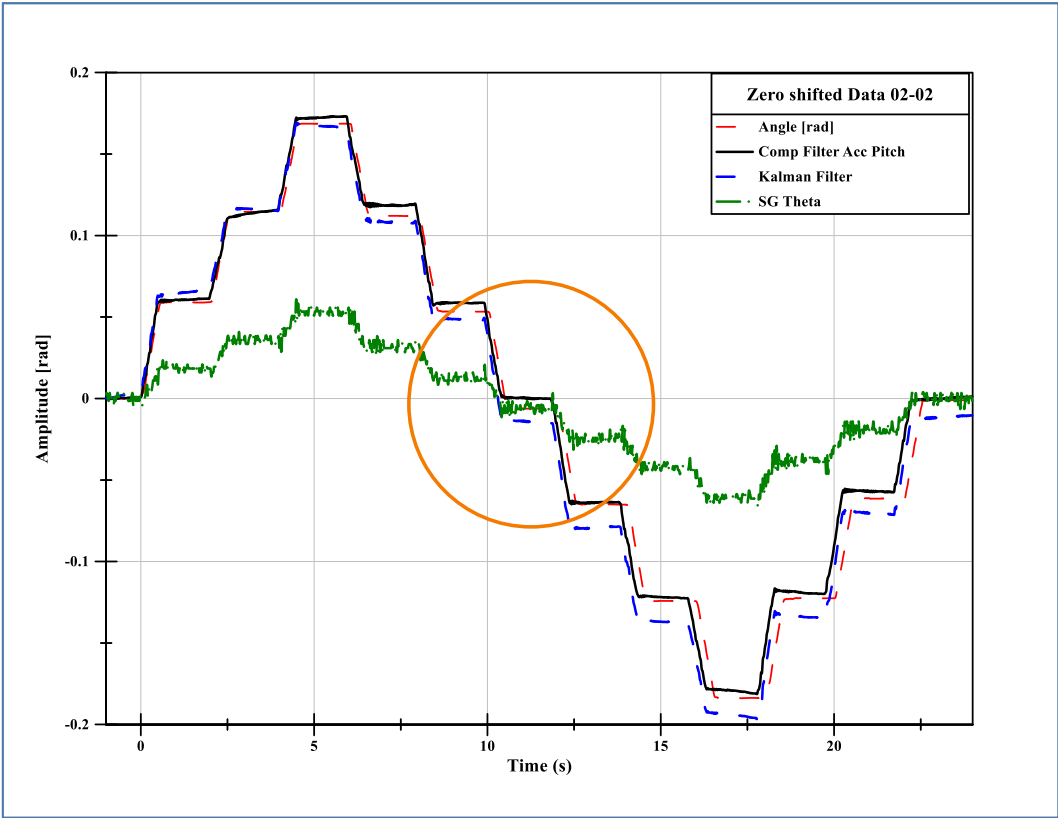


Figure 5-27 – Diagram showing plots for error measure example for Experiment Two data

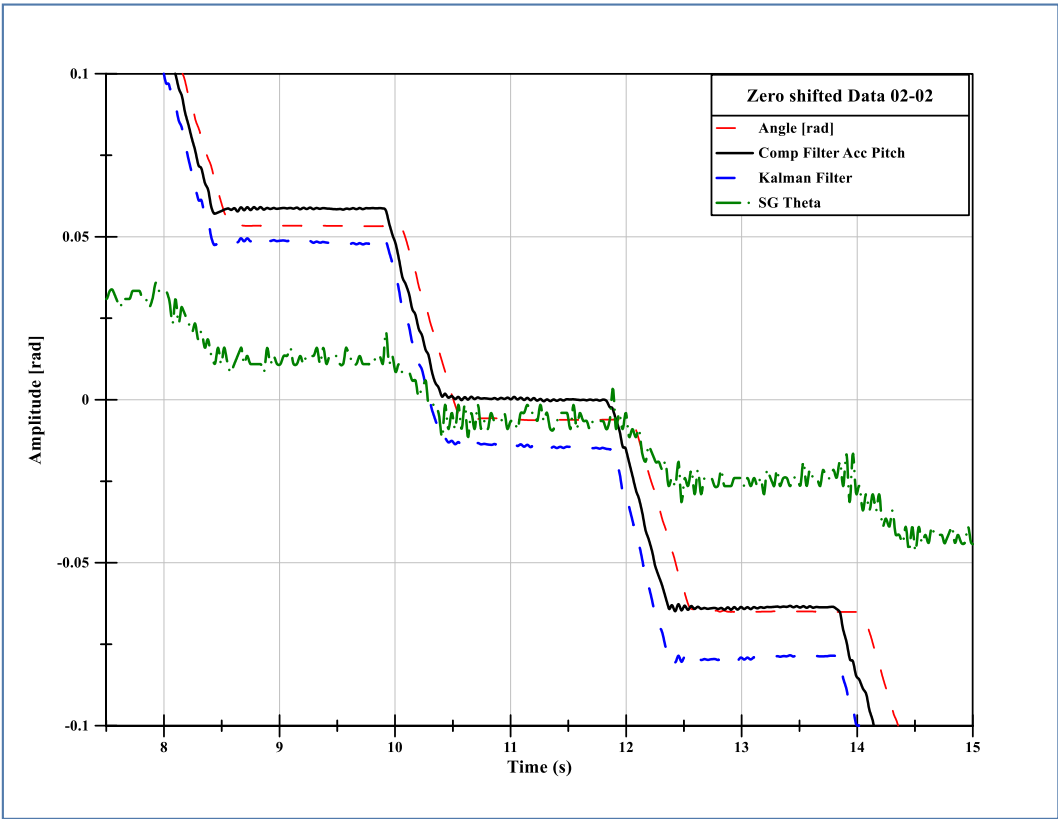


Figure 5-28 – Diagram showing close-up plots of Experiment Two data

5.6.6 *Acceptance criteria*

It is frequently difficult to determine what is acceptable in terms of accuracy and errors often leading to subjective decisions being made. Error measures, such as those considered for this research, enable what is acceptable to be quantified with a degree of certainty. Russell proposed that for his error measures that an acceptable limit might be 0.2, or 0.25 in instances where there was known variability or imprecision such as in shock and vibration measurements and tests (Russell, 1997a, 1997b). Based upon such limits the Kalman and Complementary Filter curves presented in the experimental case study were both extremely good representations of the accepted reference value.

5.7 Chapter conclusions

The nature of experimental work means that although every effort may be made to obtain precise, accurate measurements through careful preparation and planning, this is not always possible, and errors will sometimes be encountered. Where possible it is preferable to remove the cause of such anomalies, however when this is not achievable, using techniques such as those discussed in this chapter enable them to be objectively quantified. Once quantified it is simply a matter of deciding whether they meet predetermined acceptance criteria. Although several error and inequality measures were investigated, a number produced identical or similar results. To avoid duplication, it was therefore decided that only the Russell and Sprague and Geer's error measures, and the Theil inequality coefficient would be used to quantify the errors in the experimental work discussed in Chapters Six and Seven.

Chapter 6 Simple Static Analysis

6.1 Introduction

The primary purpose of the experiments described in this chapter was to provide a means of producing a simple structural response that could be measured by the chosen sensors to confirm their operation, and to verify the selected data fusion algorithms. Experiment One was an inverted pendulum and Experiment Two a vertical cantilever that were each subject to a ‘static’ load which was monitored using an HSV camera. There are two sections, one for each experiment, covering the following areas in detail:

- Experimental arrangement
- Experimental procedure
- Presentation of results
- Analysis and discussion of results (where applicable)

6.2 Experiment One – Statically loaded inverted pendulum

The main objective of this experiment was to provide initial verification that each of the sensors was operating as anticipated, and that the data analysis and fusion algorithms were functioning correctly.

6.2.1 *Experimental arrangement*

The inverted pendulum was constructed from a length of aluminium channel section, the lower end of which was attached to the test rig using a bolt as a pivot, whilst a linear actuator was attached approximately one-third the way up the pendulum. The sensors, consisting of one MPU9250 IMU unit and a single axis Dytran 3200B6 accelerometer⁵, were secured towards the top of the pendulum. The main components with the key dimensions are displayed in Figure 1-1 and a photograph showing a general view of the experimental setup is at Figure 6-2.

6.2.2 *Experimental procedure*

At the start of each test the inverted pendulum was placed in an approximately vertical/‘Twelve o’clock’ position that represented zero radians. The main LabVIEW sensor logging VI⁶ was started which began with a routine to initialise and calibrate the IMU sensors by setting the appropriate sensitivity settings, and then collecting a set number of readings whilst the pendulum and sensors were at rest. These readings were used to calculate an average ‘at rest’ sensor value that provided the offset to compensate for any IMU installation errors. Once the IMU had been calibrated the data logging sequence began; at this point the HSV camera and actuator control VI were started which caused the linear actuator to start moving in a set pattern.

⁵ The data recorded from the Dytran accelerometer was inconsistent during Experiments One and Two so was not included in the results or analysis.

⁶ Images of the VIs developed for this research have been included at Annex C.

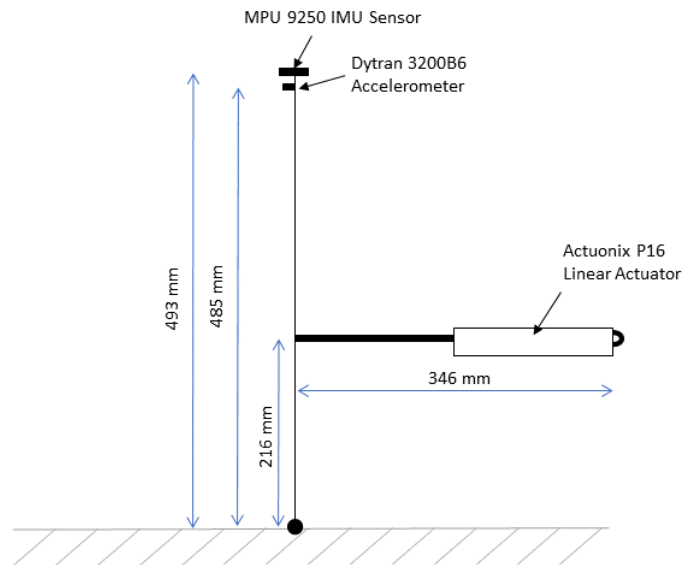


Figure 6-1 – Experiment One main components and key dimensions

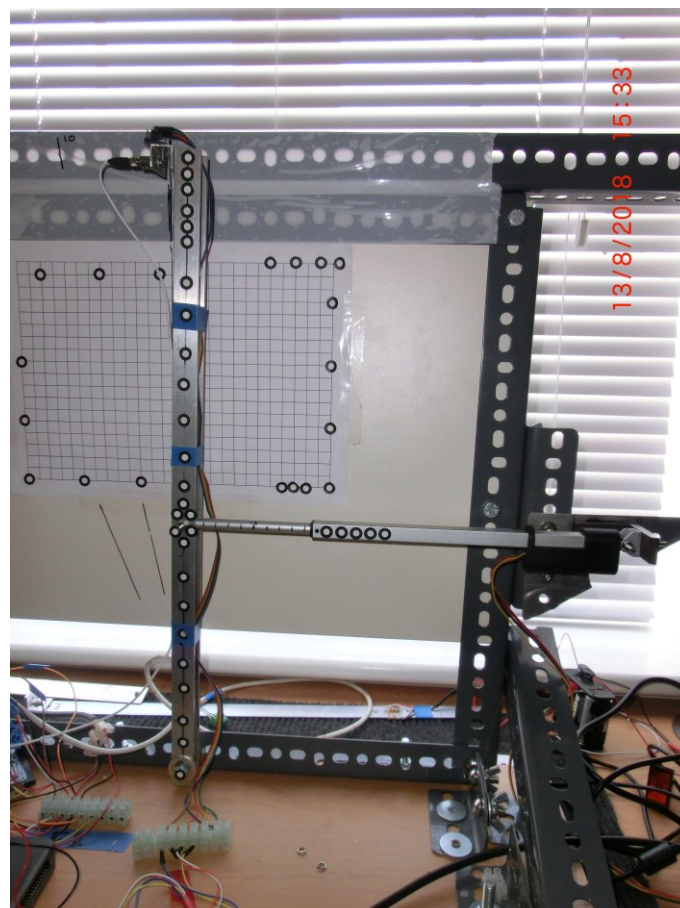


Figure 6-2 – Photograph showing the general arrangement for Experiment One

This pattern consisted of a series of timed movements starting with 3 steps anti-clockwise. Between each movement the actuator was paused for between 1.0 and 1.5 seconds before being moved again. Each movement

lasted between 0.5 and 0.75 seconds⁷. This was followed by a similar series of six clockwise steps with the same pause and movement timings. Finally, the pendulum was returned to the starting position using a series of three anti-clockwise movements. The full motion is summarised visually in the diagram at the top of Figure 6-3 and also indicates the different sensor signal regions that are used later to relate the motion to the data from the IMU accelerometers and gyroscopes.

Once the pendulum was back in the vertical position, the HSV camera and main LabVIEW sensor logging VI were stopped. Due to the slow motion of the actuator, and thus the pendulum, it was possible to use the CASIO Ex F1 and CASIO Ex ZR100 in high definition (HD) mode which therefore enabled high resolution HSV images to be obtained.

6.2.2.1 *Typical IMU Accelerometer data*

An example of the scaled observed IMU accelerometer data for three axes is shown in the top chart in Figure 6-3, and clearly illustrates how the observed acceleration values differed depending upon the orientation of the individual accelerometers (the IMU axes applied to this experimental setup are shown in Figure 6-4). For instance, Acc Z (dark green curve) which was the accelerometer orientated parallel to the Z-axis, had a value of approximately 1g when stationary. As the pendulum was slowly rotated anti-clockwise this value decreased slightly as the influence of gravity had less of an effect on this accelerometer.

By comparison, Acc X (bright red curve) which was the accelerometer orientated parallel to the X-axis, steadily increased as the pendulum was rotated anti-clockwise until the maximum rotation was reached. The acceleration then decreased as the pendulum was rotated clockwise and until the maximum clockwise rotation was achieved when the acceleration reached a maximum negative value. As the pendulum was finally rotated anti-clockwise, to return to its starting position, the acceleration became zero.

The value for Acc Y (brown curve), which was the accelerometer orientated parallel to the Y-axis, did not change significantly compared to the other two accelerometer results. The slight variations in sign that can be seen in Figure 6-3 are probably due to minor sensor misalignments. All of the accelerometer results exhibited noise particularly when the actuator started to move and when it came to a stop, which as previously mentioned, resulted in spikes in the data. Both Acc X and Acc Y were meant to be zero when they were perpendicular to the gravitational force.

6.2.2.2 *Typical IMU Gyroscope data*

The IMU gyroscopes measured the angular velocity about each of the principal axes; Gyro X (orange curve) measured the angular velocity about the X-axis (roll); Gyro Y about the Y-axis (pitch) (blue curve); and Gyro Z about the Z-axis (yaw) (grey curve). The bottom diagram in Figure 6-3 is a plot of the typical data recorded for the gyroscope.

⁷ Longer pauses and movements were used for the initial tests but these were shortened for later tests to reduce the overall length of each test.

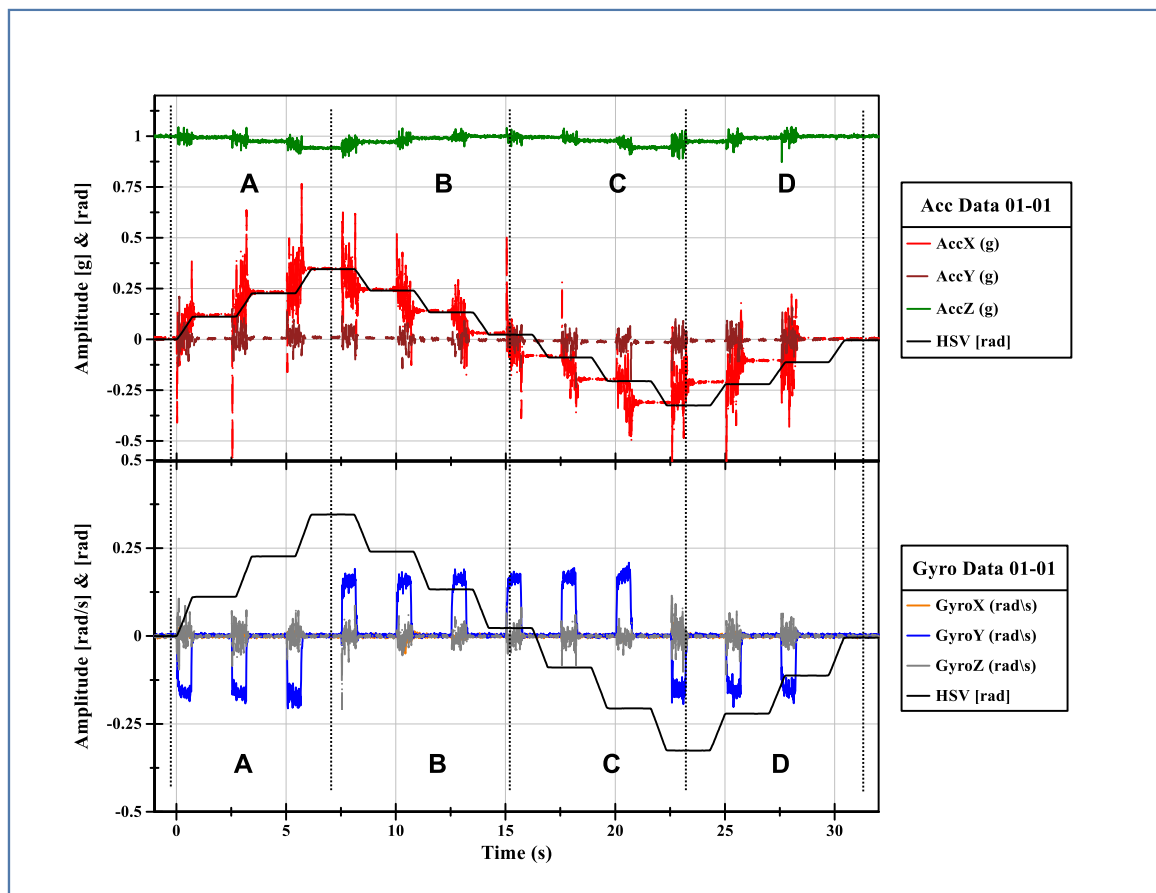


Figure 6-3 – Experiment One – Typical IMU accelerometer sensor data

As the main pendulum movement was in pitch about the Y-axis the Gyro-Y reading showed the greatest differences. Because the pendulum was moving in the plane of the X-axis the Gyro-X reading in Figure 6-3 is steady. The Gyro-Z readings show some slight difference which was due to the changes in orientation of Z-axis as it moved away from the true-vertical.

The relationship between the accelerometer and gyroscope readings and the rotation recorded by the HSV is shown in Figure 6-5. As previously discussed, the orientation of the IMU meant that the Gyro Y (blue curve) was negative, however for the subsequent data analysis the sign had to be reversed (see Section 5.4.1.2). The Gyro Y

curve shown in Figure 6-5 provides an indication of pendulum motion and when there is a change of direction; a non-zero value such as that shown at points 1 and 2 meant that the pendulum was undergoing a rotation, and the different sign meant that the pendulum was rotating in the opposite direction. Similarly, the increase and decrease in the Acc X (red curve) reading provide a measure of the magnitude of the pendulum rotation where a constant reading indicates that the pendulum was at rest.

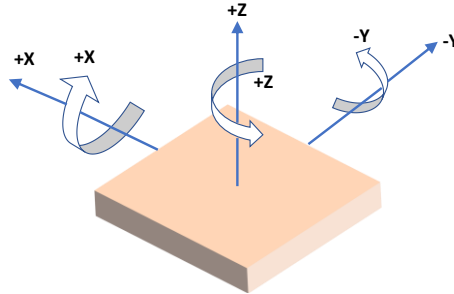


Figure 6-4 – Experiment One – IMU axes alignment

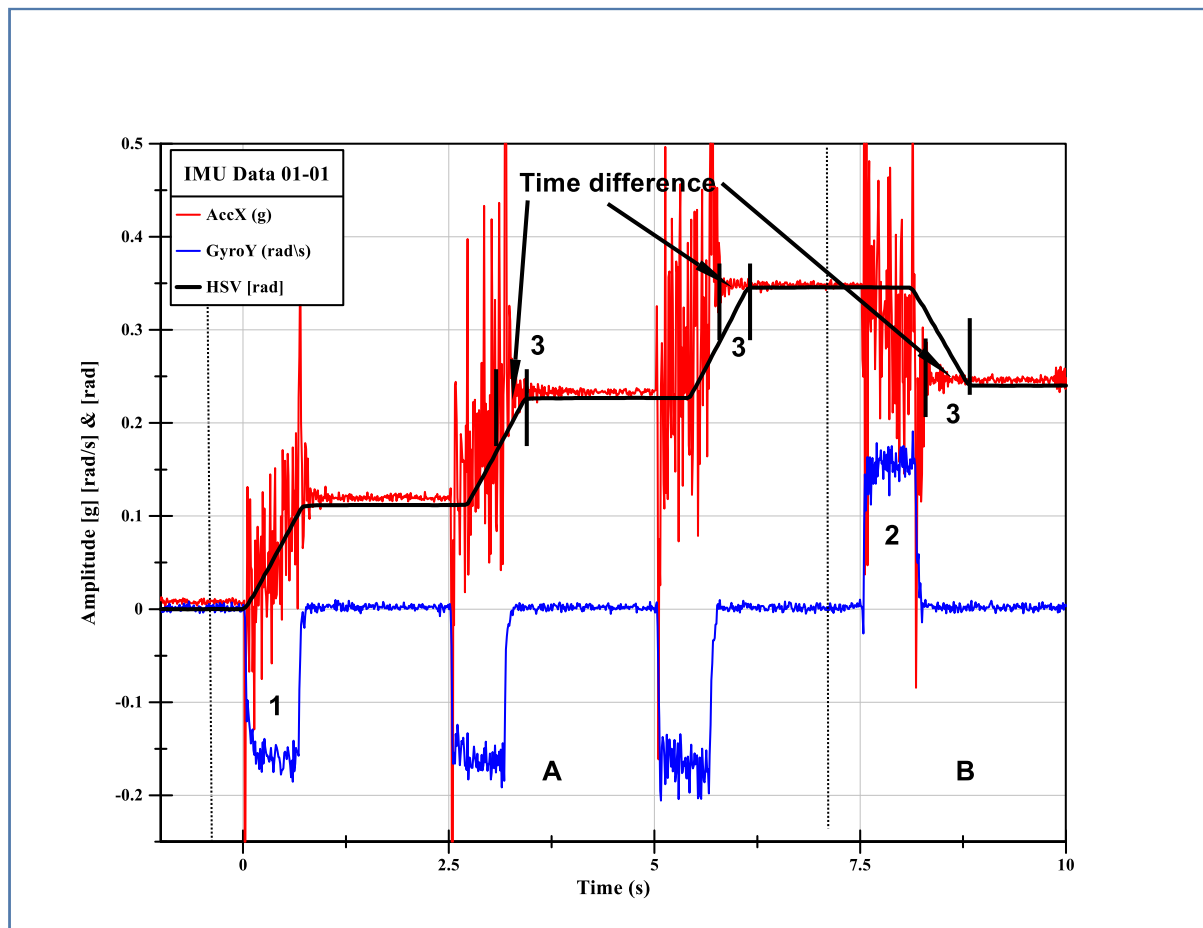


Figure 6-5 – Experiment One – General relationship between different sensor data

During the initial tests it became apparent that there was a proportional discrepancy between the timings for the measured sensor data and the pendulum rotation calculated using the HSV footage. This is clearly visible at the three point 3s in Figure 6-5. As this was a common feature for Experiment One it is discussed in more detail in Section 6.2.3.7.

6.2.3 Presentation of results

Table 6-1 summarises the tests carried out as part of Experiment One. The table details the different combinations of the IMU gyroscope and accelerometer sensitivities investigated during the experiment to ascertain the impact that the combinations might have on the accuracy of the final results, and therefore try to determine the optimum combination.

Test No	Gyro Setting (deg/s)	Acc Setting (g)	Test No	Gyro Setting (deg/s)	Acc Setting (g)	Test No	Gyro Setting (deg/s)	Acc Setting (g)	Test No	Gyro Setting (deg/s)	Acc Setting (g)
01-01	500	2	01-15	1000	2	01-29	250	2	01-43	2000	2
01-02	500	2	01-16	1000	2	01-30	250	2	01-44	2000	4
01-03	500	2	01-17	1000	2	01-31	250	2	01-45	2000	8
01-04	500	2	01-18	1000	2	01-32	250	4	01-46	2000	16
01-05	500	2	01-19	1000	2	01-33	250	8	01-47	250	2
01-06	500	2	01-20	500	2	01-34	250	16	01-48	250	2
01-07	500	2	01-21	500	2	01-35	500	2	01-49	250	2
01-08	500	2	01-22	500	2	01-36	500	4	01-50	250	2
01-09	500	2	01-23	500	2	01-37	500	8	01-51	250	2
01-10	500	2	01-24	500	2	01-38	500	16	01-52	250	2
01-11	250	2	01-25	500	2	01-39	1000	2	01-53	250	2
01-12	250	2	01-26	500	2	01-40	1000	4	01-54	250	2
01-13	250	2	01-27	500	2	01-41	1000	8	01-55	250	2
01-14	1000	2	01-28	500	2	01-42	1000	16			

Table 6-1 – Summary of Gyroscope and Accelerometer sensitivity settings for Experiment One

The different shaded areas of the table refer to specific parts of the Experiment that were used to investigate different aspects of the experimental procedure.

6.2.3.1 Experiment One - Tests 01 to 07 – Varying actuator speed and phase

The extension speed of the linear actuator used to produce a consistent load for Experiments One and Two was controlled using a pulse width modulated (PWM) signal. This was achieved by adjusting the frequency and ‘duty-cycle’ of the signal. Producing a consistent, repeatable actuator extension was important to enable comparisons to be made between different tests, so the first series of tests for this Experiment investigated how different combinations of the PWM parameters affected the actuator performance. Three extension speeds were investigated, the parameters for which are summarised in Table 6-2, and illustrated in Figure 6-6 and Figure 6-7. The individual extension speeds were determined by calculating the slope of the individual rise/falls of each curve on the extension-time chart shown in Figure 6-8, and taking the average for the twelve values for each of the extension speeds.

Figure 6-6 and Figure 6-7 show that there was a discernible difference between the different speeds in terms of the repeatability of the results, which is borne out in the error measure calculations summarised in Table 6-3. These indicate that there was an order of magnitude difference between the results for the slow actuator extension compared to the other two extension rates, and that the results for the medium actuator extension were between approximately two to five times the fast extension rate results. From this it could be concluded that using the fast actuator extension would produce the most consistent results, however as continued use of the actuator at high duty cycles could reduce the actuator’s life, it was decided to use the medium extension rate as this still produced consistent results.

Test No	Speed	Average Speed (mm/s)	Duty Cycle	Frequency
01-01	Medium	30.73	50%	50Hz
01-02	Medium	30.72	50%	50Hz
01-03	Slow	19.25	20%	50Hz
01-04	Fast	34.63	90%	50Hz
01-05	Fast	34.47	90%	100Hz
01-06	Medium	30.40	50%	100Hz
01-07	Slow	18.20	20%	100Hz

Table 6-2 – Summary of parameters used for actuator extension speed investigation

Test No	Speed	RM	RP	RC	SGM	SGP	SGC	TI
01-03 / 01-07	Slow	0.0098	0.0348	0.0321	0.0114	0.0349	0.0367	0.0550
01-01 / 01-02	Medium	-0.0009	0.0030	0.0028	-0.0010	0.0030	0.0031	0.0047
01-01 / 01-06	Medium	0.0044	0.0062	0.0067	0.0051	0.0062	0.0080	0.0100
01-04 / 01-05	Fast	0.0011	0.0010	0.0014	0.0013	0.0010	0.0017	0.0018

Table 6-3 – Error measures for actuator speed comparison

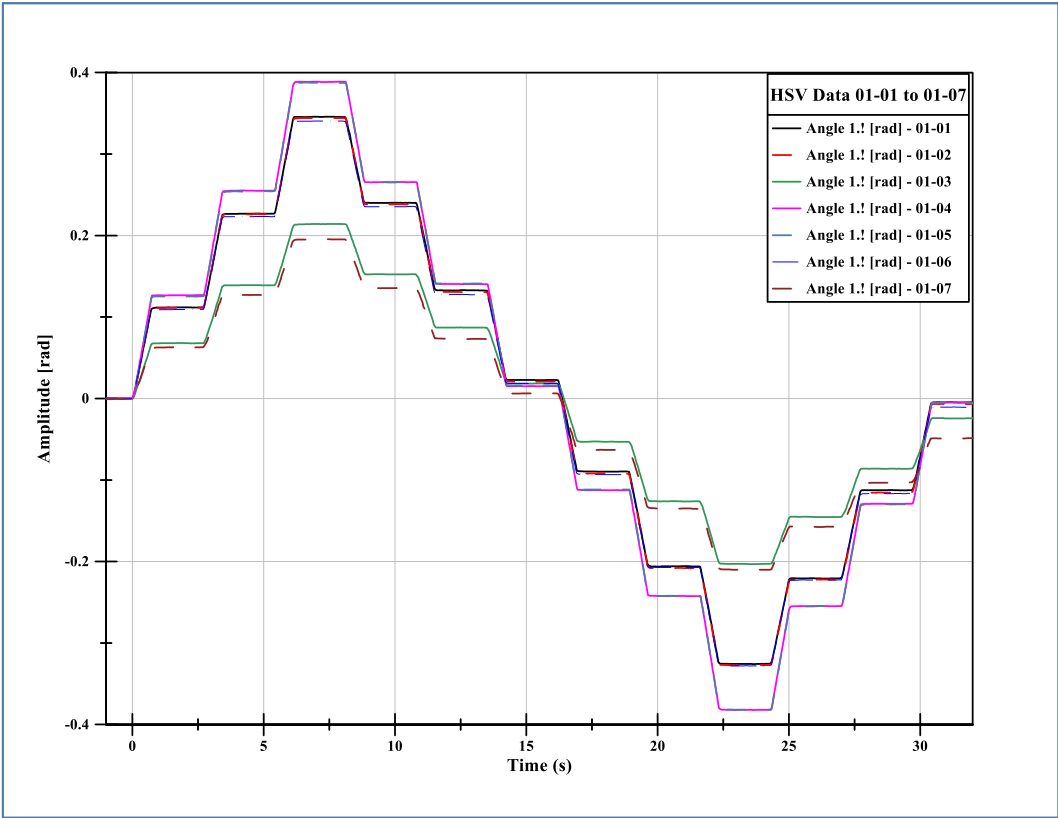


Figure 6-6 – Comparison of HSV data curves for different actuator speeds

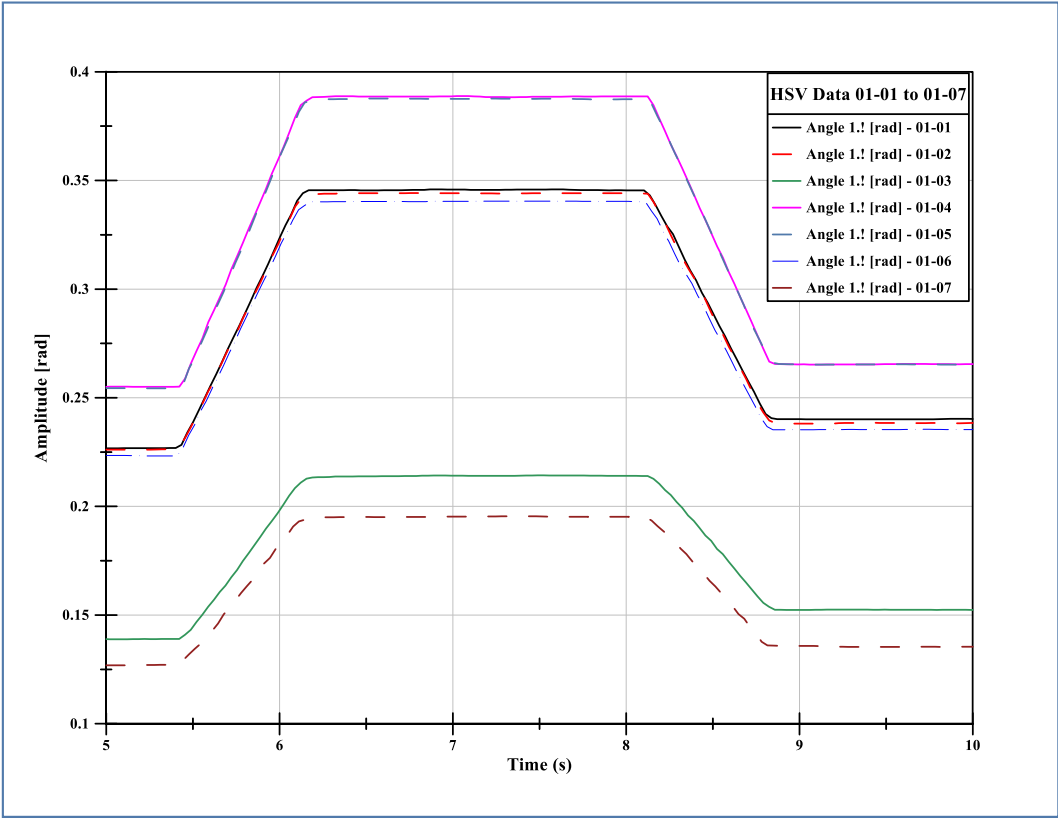


Figure 6-7 – Close-up of HSV data curves for different actuator speeds

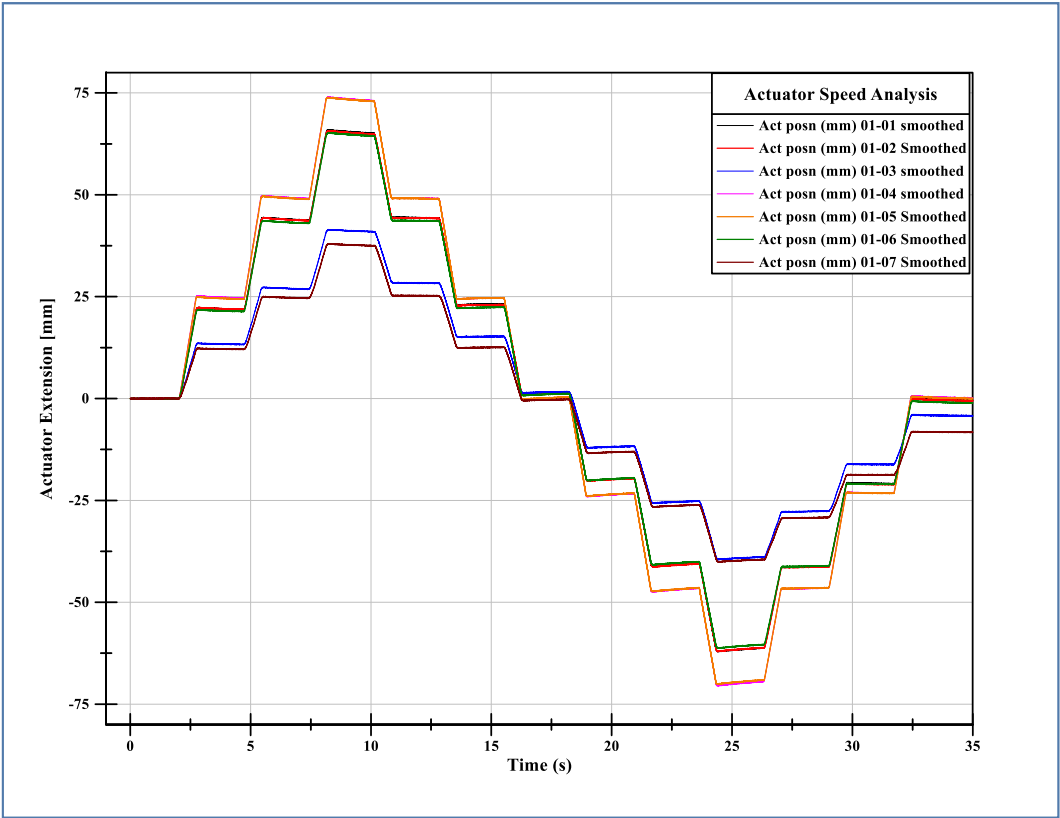


Figure 6-8 – Extension time chart for different actuator speeds

6.2.3.2 Experiment One – Tests 01, 02 and 06

The actuator speed analysis discussed in the previous section was only based upon data obtained from the HSV monitoring system. To evaluate the efficacy of the data fusion algorithms, this data was then compared to the results calculated using the data fusion algorithms. The HSV data for the medium speed tests, Tests 01-01, 01-02 and 01-06, compared to the corresponding CF and KF calculated data is shown in Figure 6-9 and Figure 6-10.

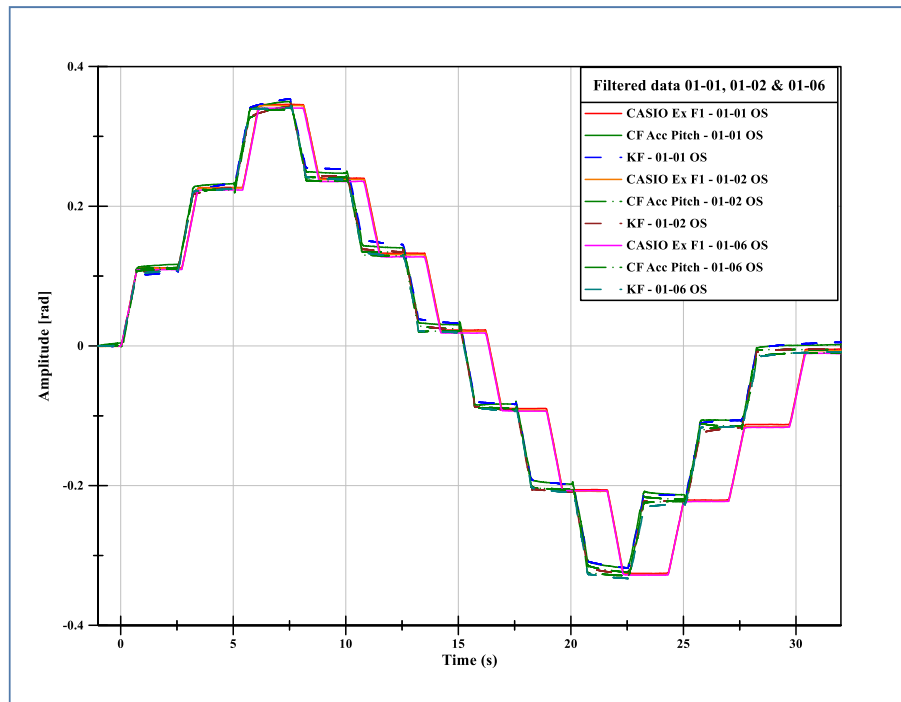


Figure 6-9 – Filtered data 01-01, 01-02 & 01-06

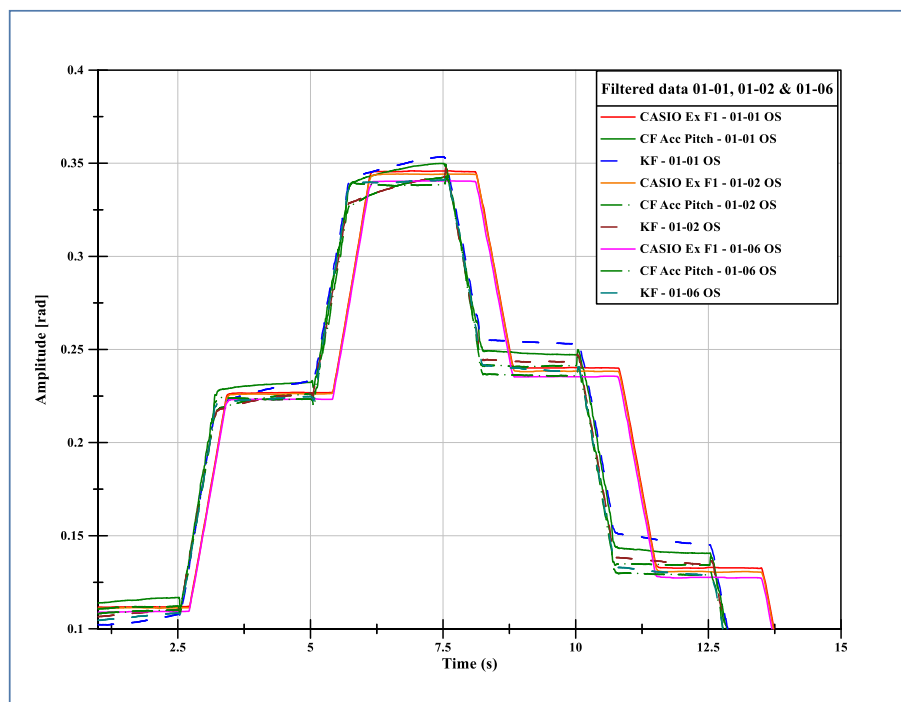


Figure 6-10 – Filtered data close-up 01-01, 01-02 & 01-06

Using the combined plots, the close alignment of all the data curves made visual interpretation challenging; to make analysis easier, the data for each individual test was separated. The results of this exercise for Tests 01-01, 01-02 and 01-06, can be seen in Figure 6-11, Figure 6-12 and Figure 6-13 respectively. To augment the analysis process, the data fusion curves were compared to the HSV data curves using the error measures, the results of which are shown in Table 6-4.

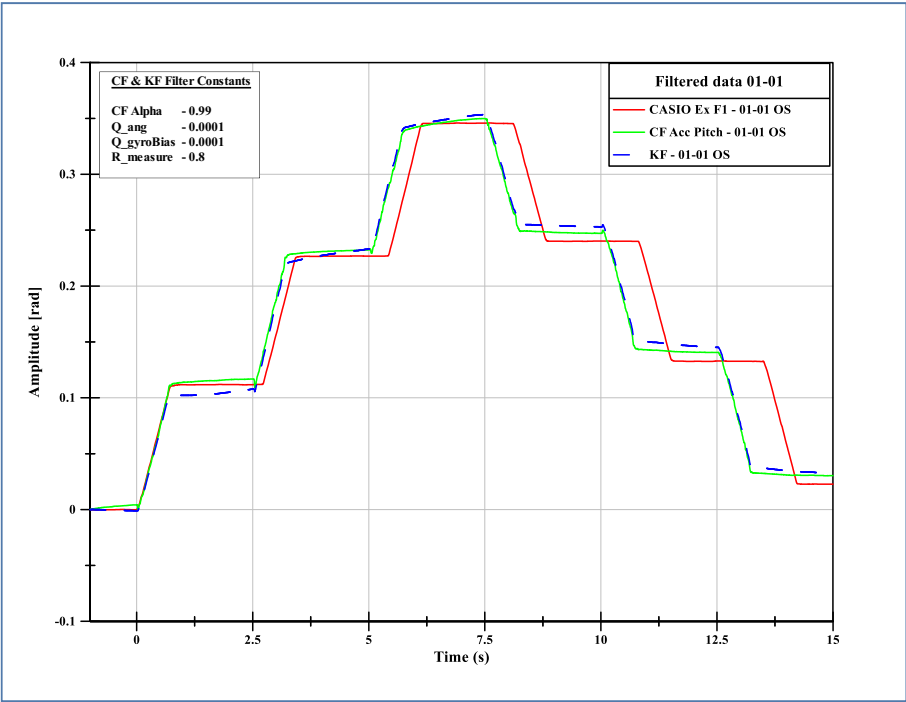


Figure 6-11 – Filtered data close-up 01-01

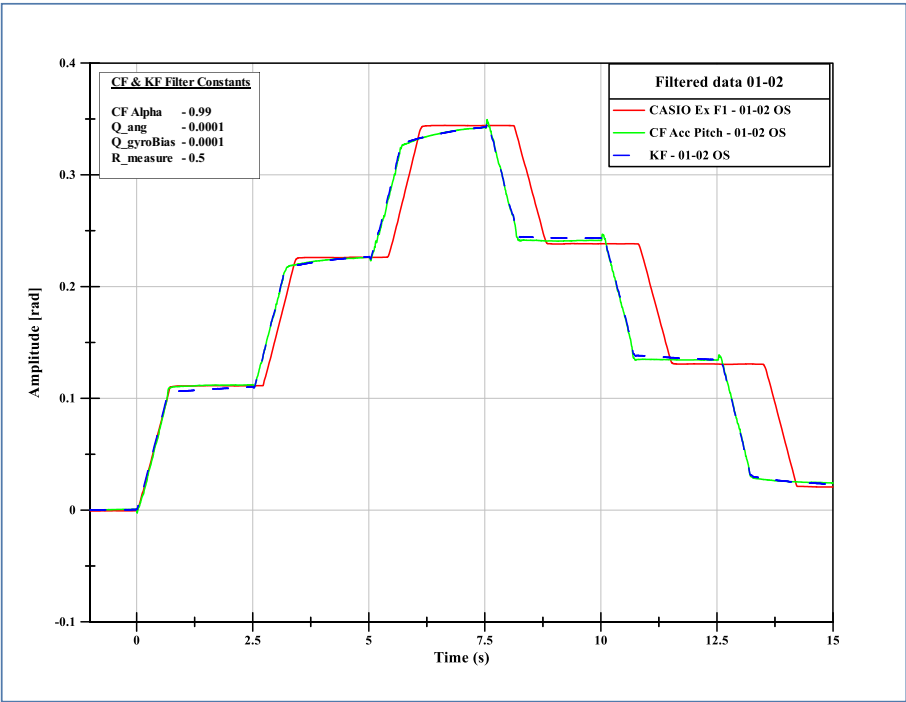


Figure 6-12 – Filtered data close-up 01-02

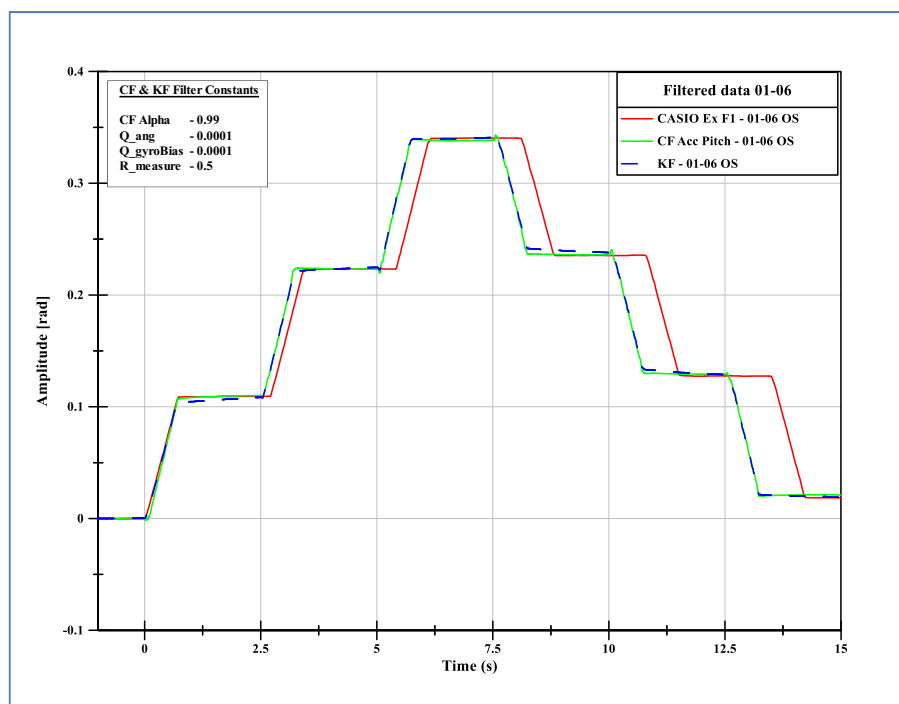


Figure 6-13 – Filtered data close-up 01-06

Test No	Filter	RM	RP	RC	SGM	SGP	SGC	TI
01-01	CF	0.0389	0.1003	<u>0.0954</u>	0.0480	0.1003	0.1112	0.1586
01-01	KF	0.0350	0.0989	0.0930	0.0429	0.0989	0.1078	0.1562
01-02	CF	0.0403	0.0991	0.0948	0.0497	0.0991	0.1109	0.1568
01-02	KF	0.0343	0.0985	0.0925	0.0420	0.0985	0.1071	0.1555
01-06	CF	0.0326	0.0997	0.0930	0.0398	0.0997	0.1073	0.1571
01-06	KF	0.0255	0.0989	<u>0.0905</u>	0.0307	0.0989	0.1036	0.1554

Table 6-4 – Error measures for filtered data 01-01, 01-02 & 01-06

The following observations were made:

- The previously mentioned difference in timings between the sensor and HSV data was clearly evident for each of the tests.
- For each of the tests there was visually very little difference between the CF and KF, however the slightly lower KF error measure values indicated that, for the chosen filter parameters, the latter was very slightly better at predicting the pendulum movement.
- All the tests produced very similar results; this was indicated from a cursory visual inspection but confirmed by the error measure results that were nearly all the same if only two decimal places were considered.
- Of the three tests, 01-06 was marginally better than the other two tests in terms of the magnitude error.
- Based upon the acceptance criteria of 0.2 suggested by Russell (Russell, 1997a, 1997b) all three tests provided a very good agreement with the HSV monitoring data. The comprehensive error measure RC was between 0.0954 and 0.0905, approximately half that suggested by Russell; a review of the acceptance criteria appropriate for this research was delayed until data for Experiment Three had been analysed.

6.2.3.3 Experiment One – Tests 08 to 19 – Investigation of data repeatability

Having determined the appropriate actuator speed, a series of tests was conducted to investigate the consistency of the data when the same actuator configuration and sensor sensitivities were used. The actuator step timings were decreased for Test 01-08 to 01-19 to reduce the overall length of the test, and to investigate whether this influenced the difference in timings between the HSV and sensor readings.

The increased number of data curves to be evaluated also necessitated a different approach to that adopted for the first series of tests. Rather than consider individual tests in isolation it was decided that each data set, HSV, CF and KF would be evaluated together using a single HSV channel as a comparator. In this way it was possible to calculate error measures relating to the comparator for each of the curves in a data set. As the same comparator was used for each of the data sets it was also possible to directly compare each of the data sets. This was done by calculating the statistical parameters (standard deviation (SD), variance (Var), median and mean) for each of the error measures, which could then be related to each other to determine the level of correlation between the data sets. As Experiments One and Two consisted of several series of tests, which each included multiple tests, this approach was implemented where there were more than three or four tests to be evaluated together.

An overall image of the HSV and filtered data for Test 01-08 to 01-19 is shown in Figure 6-14, and a close-up of the area surrounded by the red rectangle in Figure 6-15. The purpose of these two figures was to provide a general impression of the correlation of the filter data in relation to the monitoring data provided by the HSV.

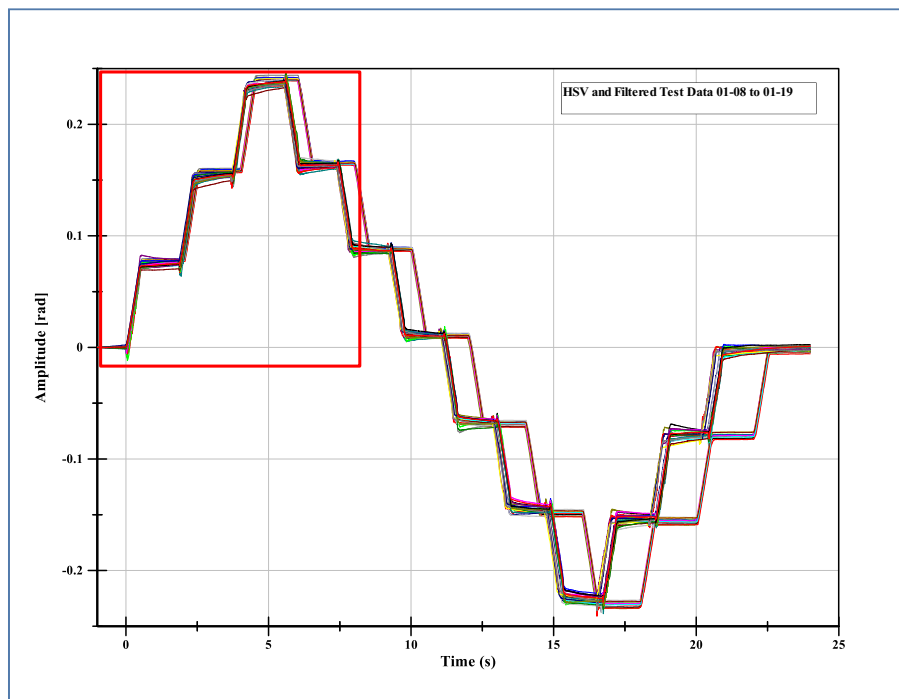


Figure 6-14 – HSV, CF and KF data for tests 01-08 to 01-19

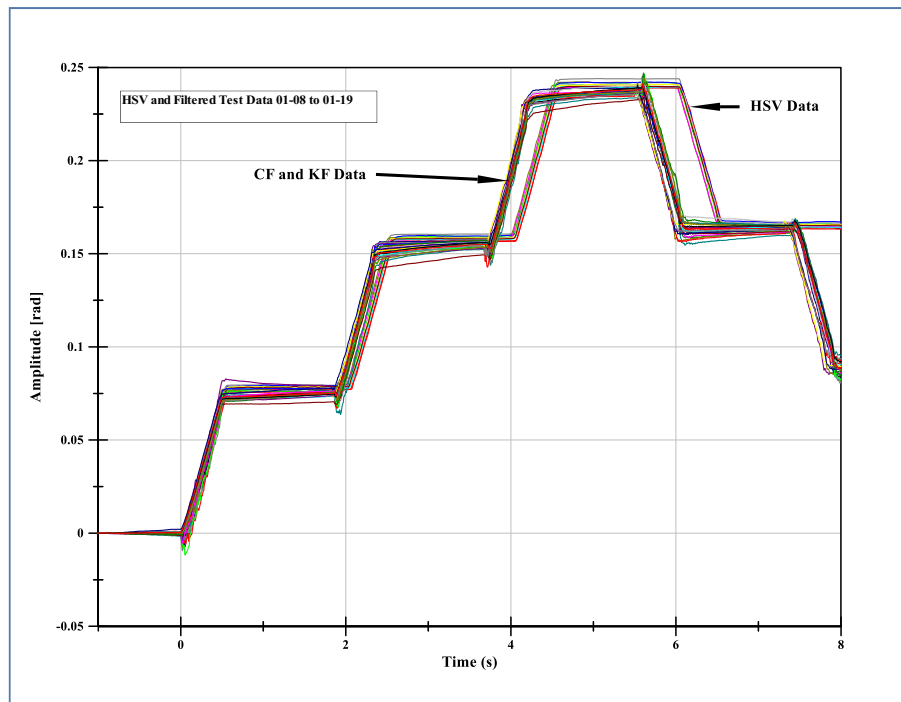


Figure 6-15 – Close-up of HSV, CF and KF data for tests 01-08 to 01-19

A summary of the filter coefficients used for each test is shown in Table 6-5. This shows that the same constants were used for each of the tests except for tests 01-11, 01-12, 01-16 and 01-17. In these tests the CF Alpha constant, and the KF co-variance Q values, had to be adjusted very slightly to ensure that the associated filter curves best matched the HSV data. Making such adjustments was usually required when the sensor data was slightly noisier than normal.

	01-08	01-09	01-10	01-11	01-12	01-13	01-14	01-15	01-16	01-17	01-18	01-19
CF Alpha	0.99	0.99	0.99	0.985	0.985	0.99	0.99	0.99	0.985	0.985	0.99	0.99
Q Angle	0.0001	0.0001	0.0001	0.0001	0.0005	0.0001	0.0001	0.0001	0.0003	0.0003	0.0001	0.0001
Q Gyro Bias	0.0001	0.0001	0.0001	0.0001	0.0005	0.0001	0.0001	0.0001	0.0003	0.0003	0.0001	0.0001
R Measure	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Table 6-5 – Filter constants for tests 01-08 to 01-19

To enable a better comparison of the data to be made, each of the components was analysed separately starting with the HSV data, a close-up of which is shown in Figure 6-16, and the corresponding error measures summarised in Table 6-6. For the error measure calculations each of the HSV data curves was compared against the HSV data for Test 01-08. A cursory review of the HSV curves in Figure 6-16 suggested that the curves were all reasonably consistent, which was borne out by the error measures in Table 6-6 which were all less than 0.02.

A similar process was adopted for analysing the consistency of the CF and KF data. To enable comparisons to be made between all the error measure values, each of the CF and KF data curves was compared to the HSV data for Test 01-08. The results for the CF and KF data are visually summarised in Figure 6-17 and Figure 6-18 respectively, whilst the error measure data is shown in Table 6-7 and Table 6-8 respectively.

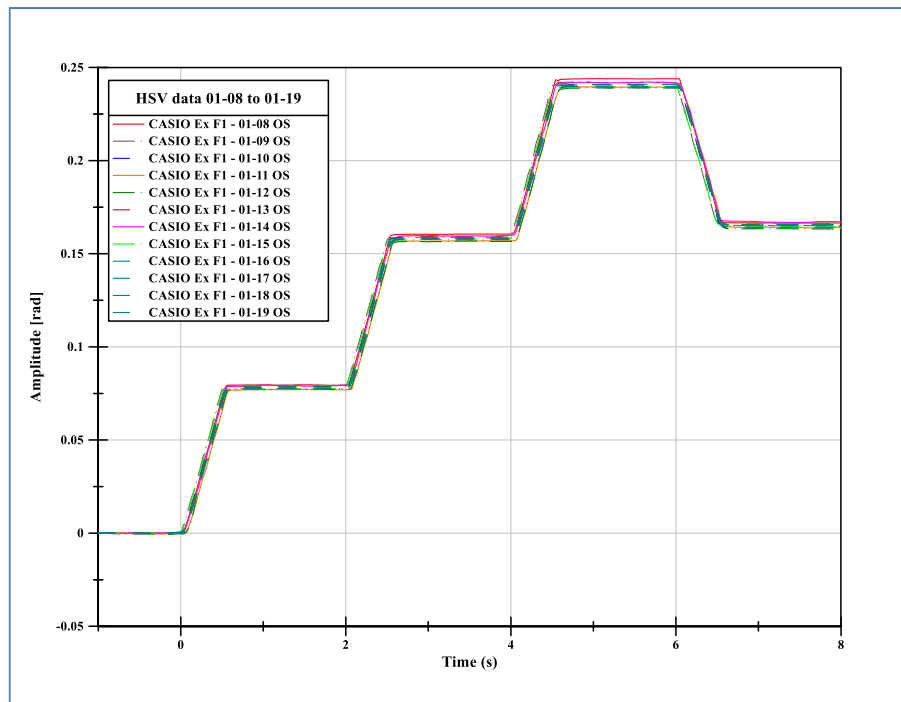


Figure 6-16 – Close-up of HSV data for tests 01-08 to 01-19

A visual inspection of Figure 6-17 and Figure 6-18 indicated that the CF and KF data curves were not as closely matched as the HSV data and, as in previous tests, exhibited a noticeable difference in timings between the HSV and filtered data. These results were confirmed by the error measure data which was an order of magnitude greater than the HSV error measure data for all the error measures, and in particular the phase and comprehensive measures RP, SGP, RC and SGC which were all greater than 0.1 (when considering values to one decimal place (dp)).

The difference between the various error measures was further verified by calculating the SD and Var for each measure along with their median and average values which are summarised in Table 6-9. The wider distribution of the CF and KF curves, compared to the HSV curves, is emphasised by the greater SD and variance values for the magnitude and comprehensive error measures. Similarly, the greater SD and variance values for the phase error measures confirm the difference in timings between the HSV and filtered data.

The following observations were made for this series of tests:

- The previously mentioned difference in timings between the sensor and HSV data was clearly evident for each of the tests.
- The combined CF and KF data curves shown in Figure 6-17 and Figure 6-18 were indistinguishable, however the slightly lower KF error measure values in Table 6-8 indicated that, for the chosen filter parameters, the latter was very slightly better at predicting the pendulum movement.
- Even though each of these tests was well below Russell's acceptance criteria of 0.2, and would therefore be deemed to have passed, the obvious difference in timings reinforced the need to review what was an appropriate acceptance criterion following the completion of Experiment Three.

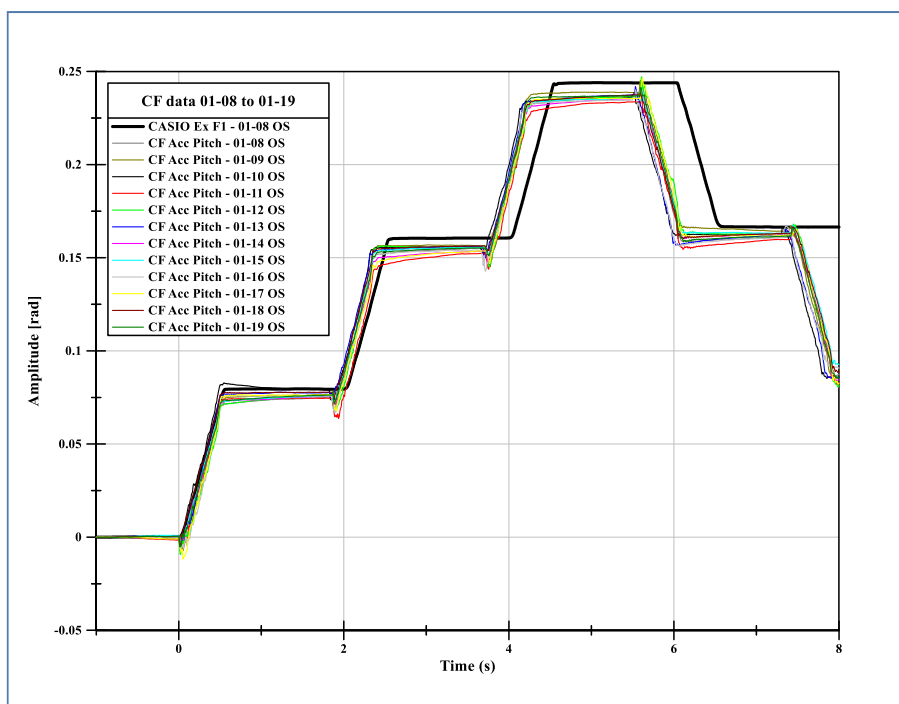


Figure 6-17 – Close-up of CF data for tests 01-08 to 01-19

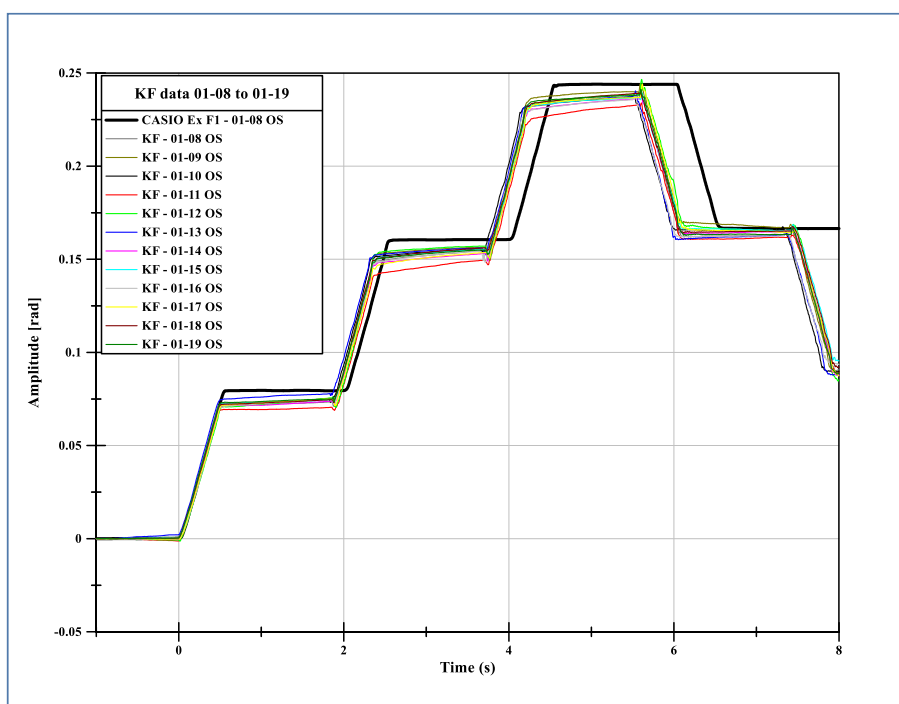


Figure 6-18 – Close-up of KF data for tests 01-08 to 01-19

HSV/ HSV	01-08/ 01-09	01-08/ 01-10	01-08/ 01-11	01-08/ 01-12	01-08/ 01-13	01-08/ 01-14	01-08/ 01-15	01-08/ 01-16	01-08/ 01-17	01-08/ 01-18	01-08/ 01-19
RM	<u>0.0008</u>	0.0069	0.0114	0.0059	0.0121	0.0090	0.0132	0.0114	<u>0.0170</u>	0.0147	0.0165
RP	0.0078	0.0043	0.0067	0.0079	0.0029	0.0043	0.0051	0.0037	0.0059	0.0034	0.0032
RC	0.0070	0.0072	0.0117	0.0087	0.0110	0.0088	0.0125	0.0107	0.0159	0.0134	0.0149
SGM	<u>0.0009</u>	0.0081	0.0134	0.0069	0.0142	0.0105	0.0155	0.0134	<u>0.0202</u>	0.0174	0.0196
SGP	0.0078	0.0043	0.0067	0.0079	0.0029	0.0043	0.0051	0.0037	0.0059	0.0034	0.0032
SGC	0.0079	0.0092	0.0150	0.0105	0.0145	0.0113	0.0163	0.0139	0.0210	0.0177	0.0199
TI	0.0123	0.0079	0.0124	0.0128	0.0084	0.0086	0.0111	0.0089	0.0136	0.0101	0.0109

Table 6-6 – HSV Error measures for tests 01-08 to 01-19

HSV/ CF	01-08/ 01-08	01-08/ 01-09	01-08/ 01-10	01-08/ 01-11	01-08/ 01-12	01-08/ 01-13	01-08/ 01-14	01-08/ 01-15	01-08/ 01-16	01-08/ 01-17	01-08/ 01-18	01-08/ 01-19
RM	0.0550	0.0465	0.0635	0.0625	0.0513	0.0701	0.0718	0.0584	0.0679	0.0590	0.0634	0.0682
RP	0.1005	0.1062	0.1118	0.1013	0.1023	0.1119	0.1020	0.1009	0.1111	0.1034	0.1016	0.1023
RC	0.1016	0.1028	0.1139	0.1055	0.1014	0.1170	0.1106	0.1033	0.1154	0.1055	0.1062	0.1090
SGM	0.0698	0.0581	0.0818	0.0803	0.0646	0.0915	0.0940	0.0745	0.0881	0.0754	0.0817	0.0887
SGP	0.1005	0.1062	0.1118	0.1013	0.1023	0.1119	0.1020	0.1009	0.1111	0.1034	0.1016	0.1023
SGC	0.1224	0.1211	0.1385	0.1293	0.1210	0.1445	0.1387	0.1254	0.1418	0.1280	0.1304	0.1354
TI	0.1607	0.1684	0.1789	0.1630	0.1630	0.1801	0.1656	0.1617	0.1786	0.1657	0.1636	0.1654

Table 6-7 – CF Error measures for tests 01-08 to 01-19

HSV/ KF	01-08/ 01-08	01-08/ 01-09	01-08/ 01-10	01-08/ 01-11	01-08/ 01-12	01-08/ 01-13	01-08/ 01-14	01-08/ 01-15	01-08/ 01-16	01-08/ 01-17	01-08/ 01-18	01-08/ 01-19
RM	0.0519	0.0386	0.0573	0.0619	0.0458	0.0617	0.0675	0.0511	0.0633	0.0539	0.0575	0.0616
RP	0.0988	0.1045	0.1093	0.0976	0.1019	0.1096	0.1007	0.0990	0.1085	0.1015	0.0997	0.1002
RC	0.0989	0.0988	0.1093	0.1024	0.0990	0.1115	0.1075	0.0988	0.1113	0.1018	0.1020	0.1043
SGM	0.0655	0.0475	0.0731	0.0795	0.0571	0.0792	0.0876	0.0644	0.0815	0.0682	0.0733	0.0792
SGP	0.0988	0.1045	0.1093	0.0976	0.1019	0.1096	0.1007	0.0990	0.1085	0.1015	0.0997	0.1002
SGC	0.1186	0.1148	0.1314	0.1258	0.1168	0.1353	0.1335	0.1181	0.1357	0.1223	0.1238	0.1277
TI	0.1578	0.1651	0.1743	0.1572	0.1617	0.1754	0.1629	0.1580	0.1739	0.1621	0.1599	0.1612

Table 6-8 – KF Error measures for tests 01-08 to 01-19

	HSV				CF				KF			
	SD	Var	Median	Average	SD	Var	Median	Average	SD	Var	Median	Average
RM	0.0046	0.00002	0.0114	0.0108	0.0074	0.00006	0.0630	0.0615	0.0079	0.00006	0.0574	0.0560
RP	0.0017	0.00000	0.0044	0.0050	0.0043	0.00002	0.1023	0.1046	0.0041	0.00002	0.1011	0.1026
RC	<u>0.0028</u>	<u>0.00001</u>	0.0110	0.0111	<u>0.0052</u>	<u>0.00003</u>	0.1058	0.1077	<u>0.0047</u>	<u>0.00002</u>	0.1022	0.1038
SGM	0.0055	0.00003	0.0134	0.0127	0.0106	0.00011	0.0810	0.0790	0.0109	0.00012	0.0732	0.0714
SGP	0.0017	0.00000	0.0044	0.0050	0.0043	0.00002	0.1023	0.1046	0.0041	0.00002	0.1011	0.1026
SGC	<u>0.0041</u>	<u>0.00002</u>	0.0145	0.0143	<u>0.0079</u>	<u>0.00006</u>	0.1299	0.1314	<u>0.0071</u>	<u>0.00005</u>	0.1248	0.1253
TI	<u>0.0019</u>	<u>0.00000</u>	0.0110	0.0106	<u>0.0068</u>	<u>0.00005</u>	0.1655	0.1679	<u>0.0064</u>	<u>0.00004</u>	0.1619	0.1641

Table 6-9 – Summary of statistical parameters for tests 01-08 to 01-19 Error measures

6.2.3.4 Experiment One – Tests 20 to 28 – Comparison with different cameras

To ascertain whether the difference in data timings was caused by the camera frame rate, a simple series of tests was conducted using a CASIO Ex ZR100 digital camera instead of a CASIO Ex F1. An overview of the data for these tests is shown in Figure 6-19 and a close-up of the data in the red rectangle is shown in Figure 6-20. The same filter constants were used for each test as summarised in Table 6-10.

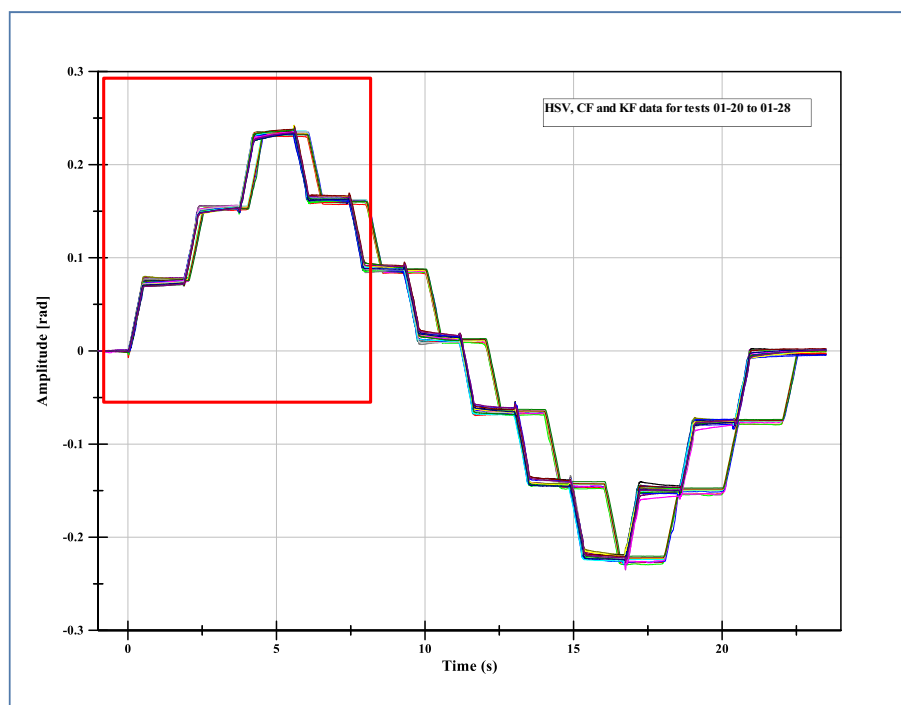


Figure 6-19 – HSV, CF and KF data for tests 01-20 to 01-28

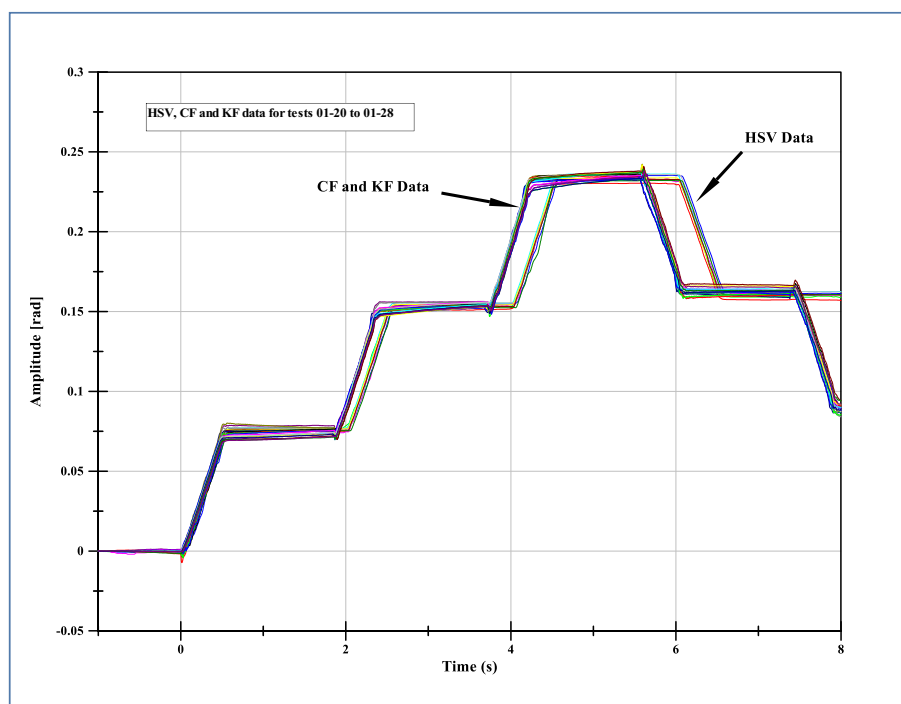


Figure 6-20 – Close-up of HSV, CF and KF data for tests 01-20 to 01-28

	01-20	01-21	01-22	01-23	01-24	01-25	01-26	01-27	01-28
CF Alpha	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
Q Angle	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Q Gyro Bias	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
R Measure	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Table 6-10 – Filter constants for tests 01-20 to 01-28

To enable a comparison to be made between the HSV data and the filtered data, the first channel of HSV data (CASIO EX ZR100 – 01-20 OS) was chosen as the comparator. The data curves for the HSV are shown in Figure 6-21 and the respective error measures using the 01-20 data curve comparator are summarised in Table 6-11. As in the previous series of tests, both a visual inspection of the data curves in Figure 6-21 and a review of the error measures, indicated that all the HSV curves were quite closely matched with none of the values being greater than 0.02. The negative magnitude values RM and SGM for HSV curves 01-21 to 01-25 signify that these curves had a greater magnitude than the comparator, 01-20.

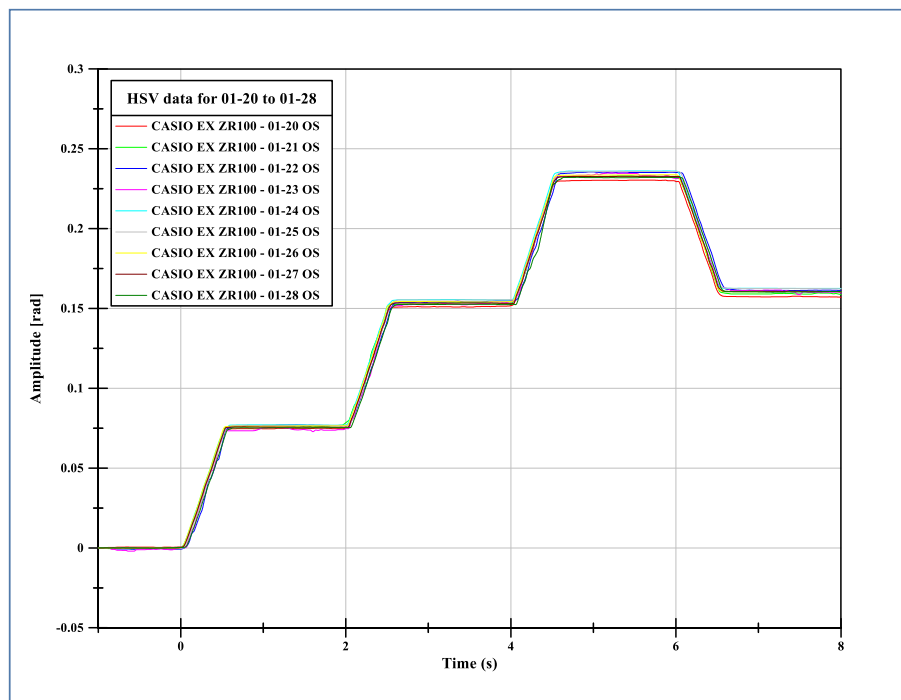


Figure 6-21 – Close-up of HSV data for tests 01-20 to 01-28

The same process was applied to the filtered data; the CF data curves are shown in Figure 6-22 whilst the KF data curves are shown in Figure 6-23. The error measures for the CF and KF are summarised in Table 6-12 and Table 6-13 respectively. The HSV comparator curve is shown as a solid red line in both Figure 6-22 and Figure 6-23 and clearly showed the previously identified timing differences were present for this series of tests, implying that the camera frame rates were not the cause of this anomaly. A visual inspection of these two charts suggested that they were very similar which was also confirmed by the similarity in the error measures. This was also verified by comparing the statistical parameters in Table 6-14 that also displayed a close resemblance.

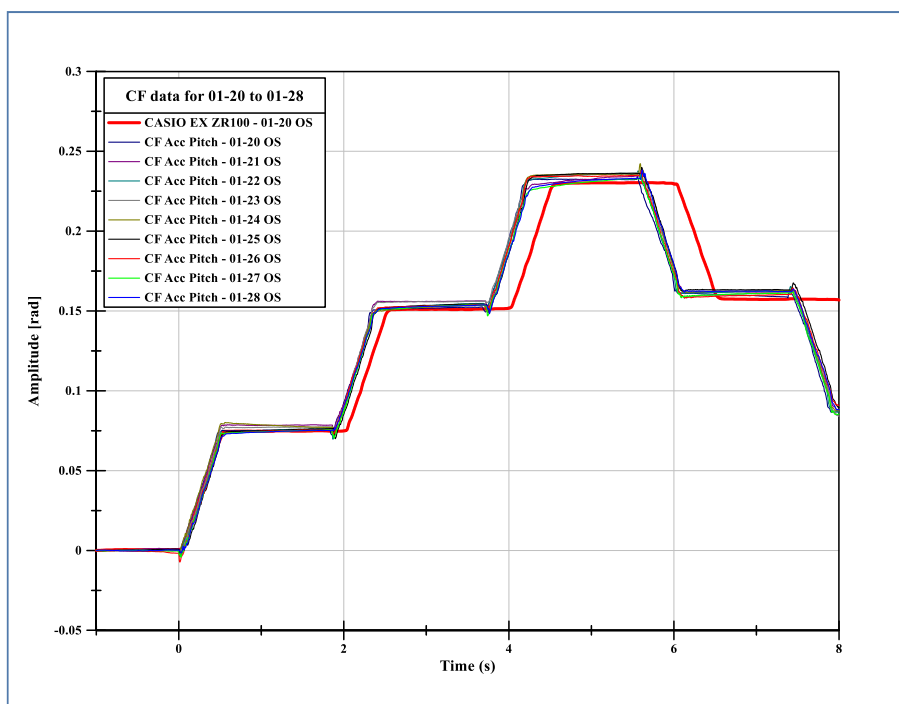


Figure 6-22 – Close-up of CF data for tests 01-20 to 01-28

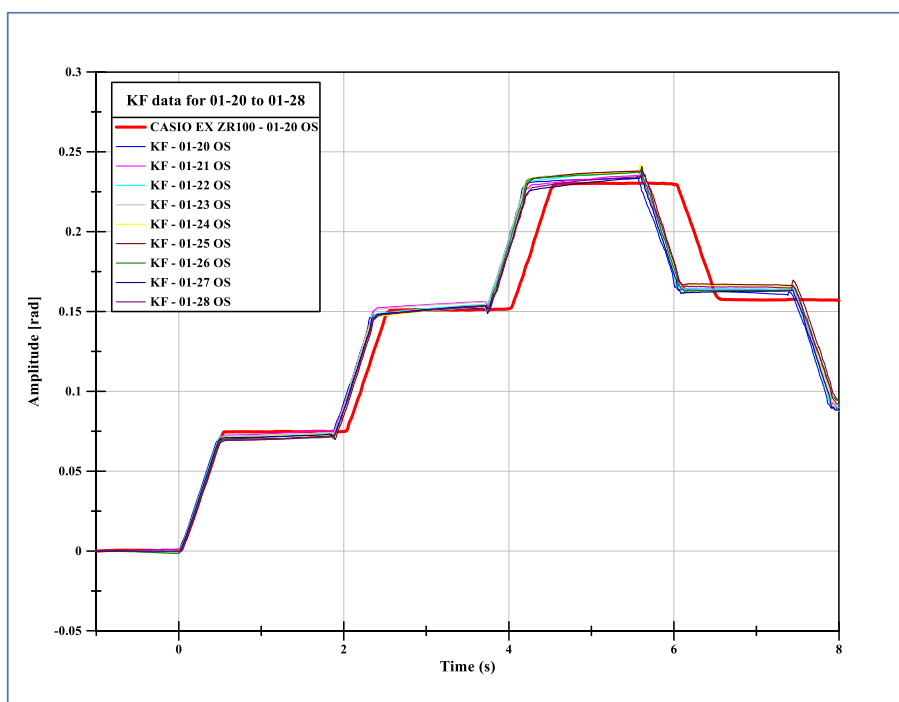


Figure 6-23 – Close-up of KF data for tests 01-20 to 01-28

The statistical parameters for the HSV data comparison shown in Table 6-14 indicated that the spread of the HSV data curves for these tests was slightly more than in previous tests, indicated by the greater variance values. A close inspection of the difference between the upper and lower bound magnitude error measures confirmed this (see bold, underlined figures in Table 6-6 and Table 6-11 – RM 0.0162 compared to 0.0186 and SGM 0.0193

compared to 0.0217). It was concluded that similar observations could be made for this series of tests as for Tests 01-08 to 01-19.

HSV/ HSV	01-20/ 01-21	01-20/ 01-22	01-20/ 01-23	01-20/ 01-24	01-20/ 01-25	01-20/ 01-26	01-20/ 01-27	01-20/ 01-28
RM	<u>-0.0087</u>	-0.0069	-0.0039	-0.0046	-0.0012	0.0021	0.0046	<u>0.0099</u>
RP	0.0025	0.0103	0.0055	0.0088	0.0112	0.0077	0.0086	0.0128
RC	0.0080	0.0110	0.0060	0.0088	0.0100	0.0070	0.0087	0.0144
SGM	<u>-0.0101</u>	-0.0080	-0.0045	-0.0053	-0.0014	0.0024	0.0054	<u>0.0116</u>
SGP	0.0025	0.0103	0.0055	0.0088	0.0112	0.0077	0.0086	0.0128
SGC	0.0104	0.0130	0.0071	0.0102	0.0113	0.0080	0.0101	0.0173
TI	0.0064	0.0166	0.0089	0.0140	0.0176	0.0121	0.0138	0.0210

Table 6-11 – HSV Error measures for tests 01-20 to 01-28

HSV/ CF	01-20/ 01-20	01-20/ 01-21	01-20/ 01-22	01-20/ 01-23	01-20/ 01-24	01-20/ 01-25	01-20/ 01-26	01-20/ 01-27	01-20/ 01-28
RM	0.0388	0.0310	0.0352	0.0355	0.0378	0.0398	0.0316	0.0400	0.0421
RP	0.1055	0.0999	0.1073	0.1036	0.1024	0.1021	0.1018	0.1039	0.1001
RC	0.0996	0.0927	0.1001	0.0971	0.0967	0.0971	0.0945	0.0987	0.0962
SGM	0.0479	0.0376	0.0431	0.0435	0.0465	0.0491	0.0385	0.0494	0.0522
SGP	0.1055	0.0999	0.1073	0.1036	0.1024	0.1021	0.1018	0.1039	0.1001
SGC	0.1159	0.1067	0.1156	0.1124	0.1124	0.1133	0.1089	0.1150	0.1129
TI	0.1666	0.1573	0.1690	0.1634	0.1617	0.1614	0.1604	0.1642	0.1586

Table 6-12 – CF Error measures for tests 01-20 to 01-28

HSV/ KF	01-20/ 01-20	01-20/ 01-21	01-20/ 01-22	01-20/ 01-23	01-20/ 01-24	01-20/ 01-25	01-20/ 01-26	01-20/ 01-27	01-20/ 01-28
RM	0.0314	0.0243	0.0278	0.0308	0.0328	0.0354	0.0282	0.0349	0.0370
RP	0.1034	0.0976	0.1053	0.1018	0.1003	0.1002	0.1006	0.1022	0.0979
RC	0.0958	0.0891	0.0965	0.0943	0.0935	0.0941	0.0926	0.0957	0.0927
SGM	0.0382	0.0292	0.0336	0.0374	0.0399	0.0434	0.0341	0.0427	0.0454
SGP	0.1034	0.0976	0.1053	0.1018	0.1003	0.1002	0.1006	0.1022	0.0979
SGC	0.1103	0.1019	0.1106	0.1085	0.1080	0.1092	0.1062	0.1108	0.1079
TI	0.1628	0.1533	0.1655	0.1602	0.1582	0.1581	0.1582	0.1612	0.1547

Table 6-13 – KF Error measures for tests 01-20 to 01-28

	HSV				CF				KF			
	SD	Var	Median	Average	SD	Var	Median	Average	SD	Var	Median	Average
RM	0.0059	0.000034	-0.0025	-0.0011	0.0036	0.000013	0.0378	0.0369	0.0039	0.000015	0.0314	0.0314
RP	0.0031	0.000009	0.0087	0.0084	0.0023	0.000005	0.1024	0.1029	0.0023	0.000005	0.1006	0.1010
RC	0.0024	0.000006	0.0087	0.0092	0.0022	0.000005	0.0971	0.0970	0.0021	0.000004	0.0941	0.0938
SGM	0.0068	0.000047	-0.0029	-0.0012	0.0047	0.000022	0.0465	0.0453	0.0050	0.000025	0.0382	0.0382
SGP	0.0031	0.000009	0.0087	0.0084	0.0023	0.000005	0.1024	0.1029	0.0023	0.000005	0.1006	0.1010
SGC	0.0029	0.000009	0.0103	0.0109	0.0029	0.000008	0.1129	0.1126	0.0026	0.000007	0.1085	0.1081
TI	0.0044	0.000020	0.0139	0.0138	0.0035	0.000012	0.1617	0.1625	0.0036	0.000013	0.1582	0.1591

Table 6-14 – Summary of statistical parameters for tests 01-20 to 01-28 Error measures

6.2.3.5 Experiment One – Tests 29 to 46 – Comparison of different IMU sensitivities

In order to investigate whether changing the sensitivity of the IMU sensors had an effect on the results, a series of tests were conducted where the gyroscope sensitivity was fixed, and the accelerometer sensitivity was changed after each individual test. This process was repeated for each gyroscope sensitivity setting as indicated in Table 6-1. An overview of the data for these tests is shown in Figure 6-24 and a close-up of the data in the red rectangle is shown in Figure 6-25.

These figures show that the timing difference between the HSV and IMU data was still evident. Except for Test 01-40, each of the other tests used the same filter constants as shown in Table 6-15.

	01-29	01-30	01-31	01-32	01-33	01-34	01-35	01-36	01-37
CF Alpha	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
Q Angle	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Q Gyro Bias	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
R Measure	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
	01-38	01-39	01-40	01-41	01-42	01-43	01-44	01-45	01-46
CF Alpha	0.99	0.99	0.985	0.99	0.99	0.99	0.99	0.99	0.99
Q Angle	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Q Gyro Bias	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
R Measure	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Table 6-15 – Filter constants for tests 01-29 to 01-46

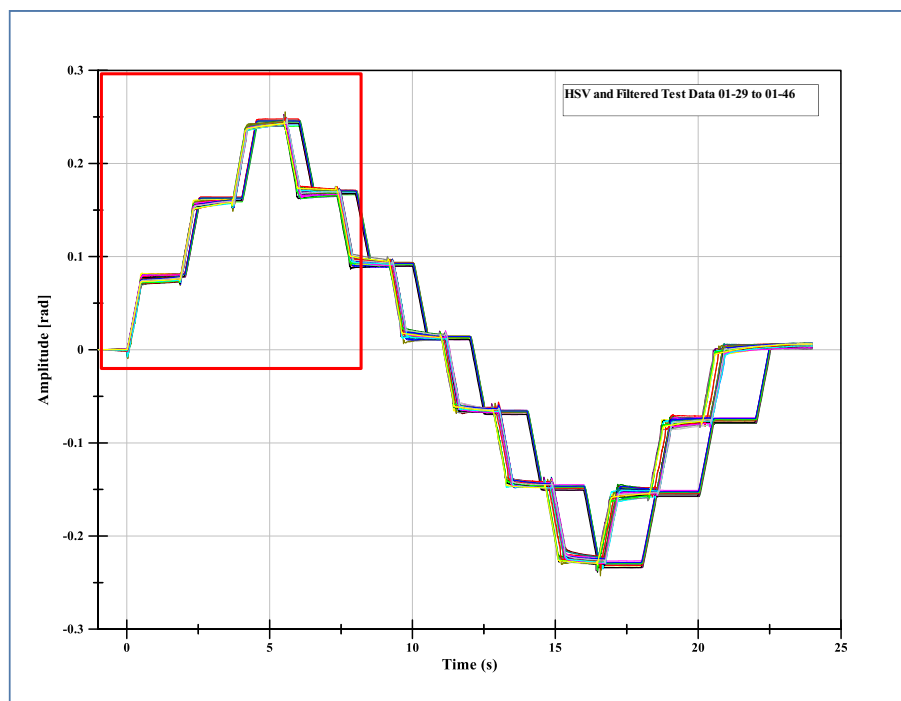


Figure 6-24 – HSV, CF and KF data for tests 01-29 to 01-46

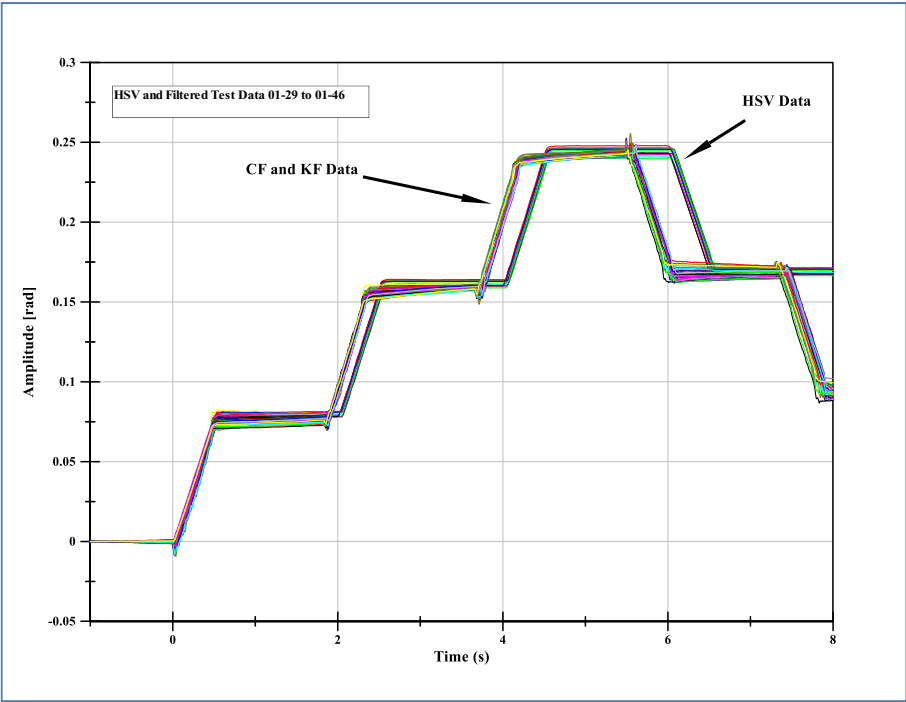


Figure 6-25 – Close-up of HSV, CF and KF data for tests 01-29 to 01-46

The HSV data is shown in Figure 6-26 with the respective error measures summarised in Table 6-16. For these tests the HSV data curve for Test 01-29 was used as the comparator. A review of the error measures showed that all of the data curves except for those for Tests 01-30, 01-32 and 01-33 had magnitudes that were greater than the comparator.

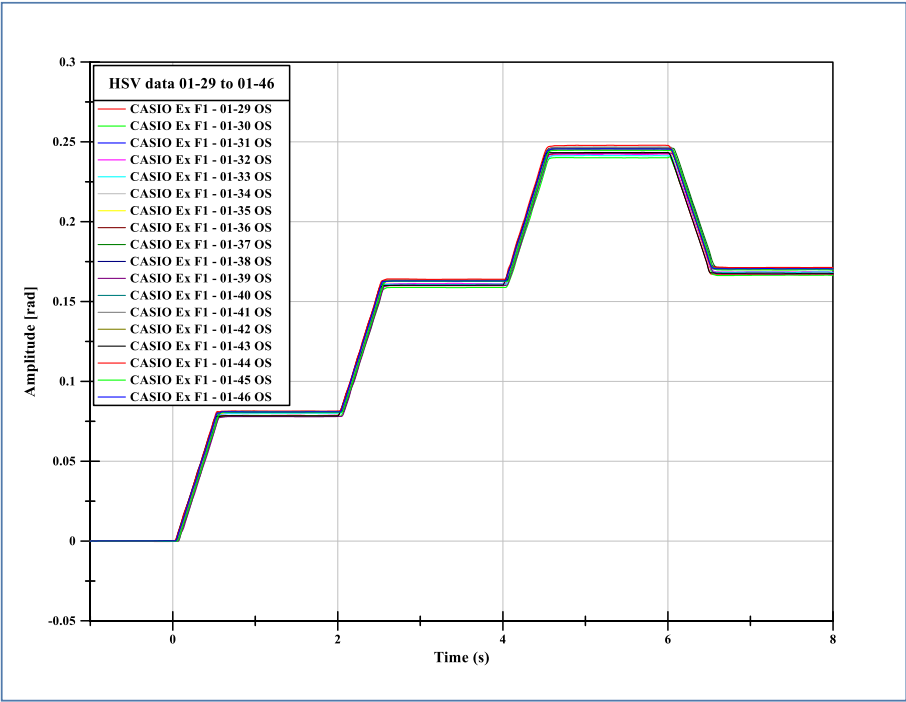


Figure 6-26 – Close-up of HSV data for tests 01-29 to 01-46

The same process was applied to the filtered data; the CF data curves are shown in Figure 6-27 whilst the KF data curves are shown in Figure 6-28. The error measures for the CF and KF are summarised in Table 6-17 and Table 6-18 respectively, while the statistical parameters for all the data sets are summarised in Table 6-19.

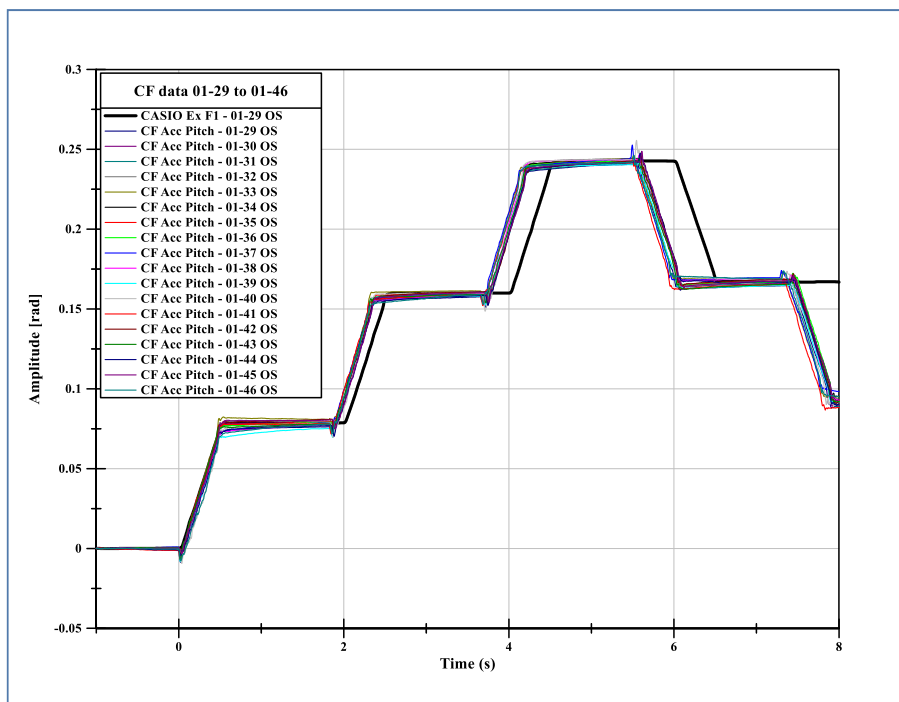


Figure 6-27 – Close-up of CF data for tests 01-29 to 01-46

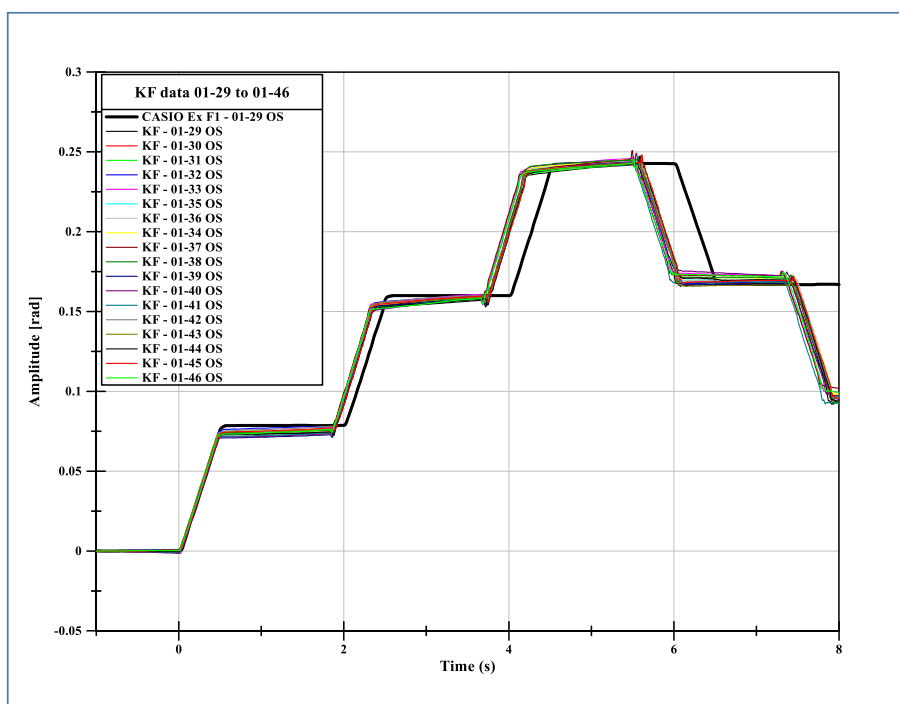


Figure 6-28 – Close-up of KF data for tests 01-29 to 01-46

HSV/ HSV	01-29/ 01-30	01-29/ 01-31	01-29/ 01-32	01-29/ 01-33	01-29/ 01-34	01-29/ 01-35	01-29/ 01-36	01-29/ 01-37	01-29/ 01-38
RM	0.0068	-0.0091	0.0066	0.0040	-0.0082	-0.0086	-0.0090	-0.0131	-0.0064
RP	0.0047	0.0044	0.0058	0.0063	0.0069	0.0030	0.0054	0.0090	0.0070
RC	0.0073	0.0090	0.0078	0.0066	0.0095	0.0081	0.0093	0.0141	0.0084
SGM	0.0080	-0.0105	0.0077	0.0047	-0.0095	-0.0100	-0.0104	-0.0152	-0.0074
SGP	0.0047	0.0044	0.0058	0.0063	0.0069	0.0030	0.0054	0.0090	0.0070
SGC	0.0092	0.0114	0.0096	0.0078	0.0118	0.0104	0.0117	0.0177	0.0102
TI	0.0084	0.0087	0.0099	0.0102	0.0119	0.0069	0.0100	0.0161	0.0116
HSV/ HSV	01-29/ 01-39	01-29/ 01-40	01-29/ 01-41	01-29/ 01-42	01-29/ 01-43	01-29/ 01-44	01-29/ 01-45	01-29/ 01-46	
RM	-0.0068	-0.0087	-0.0089	-0.0036	-0.0074	-0.0118	-0.0040	-0.0032	
RP	0.0060	0.0047	0.0073	0.0070	0.0038	0.0056	0.0069	0.0064	
RC	0.0081	0.0087	0.0102	0.0069	0.0073	0.0116	0.0070	0.0064	
SGM	-0.0079	-0.0101	-0.0103	-0.0041	-0.0085	-0.0137	-0.0046	-0.0037	
SGP	0.0060	0.0047	0.0073	0.0070	0.0038	0.0056	0.0069	0.0064	
SGC	0.0099	0.0111	0.0126	0.0081	0.0093	0.0148	0.0083	0.0074	
TI	0.0103	0.0089	0.0126	0.0111	0.0073	0.0112	0.0110	0.0103	

Table 6-16 – HSV Error measures for tests 01-29 to 01-46

The cells of the CF and KF error measure tables have been highlighted with different colours to distinguish between the different IMU sensor sensitivity settings as follows:

Headings (Gyro sensitivity) (deg/sec)	250		500		1000		2000	
Columns (Acc sensitivity) (g)	2		4		8		16	

HSV/ CF	01-29/ 01-29	01-29/ 01-30	01-29/ 01-31	01-29/ 01-32	01-29/ 01-33	01-29/ 01-34	01-29/ 01-35	01-29/ 01-36	01-29/ 01-37
RM	0.0421	0.0422	0.0486	0.0418	0.0427	0.0463	0.0463	0.0419	0.0437
RP	0.1046	0.1008	0.1009	0.1034	0.1127	0.1027	0.1054	0.1020	0.1121
RC	0.1000	0.0968	0.0993	0.0988	0.1068	0.0998	0.1020	0.0977	0.1066
SGM	0.0522	0.0523	0.0609	0.0518	0.0529	0.0578	0.0578	0.0519	0.0543
SGP	0.1046	0.1008	0.1009	0.1034	0.1127	0.1027	0.1054	0.1020	0.1121
SGC	0.1170	0.1136	0.1179	0.1156	0.1245	0.1178	0.1202	0.1144	0.1246
TI	0.1656	0.1597	0.1605	0.1636	0.1779	0.1629	0.1671	0.1615	0.1771
HSV/ CF	01-29/ 01-38	01-29/ 01-39	01-29/ 01-40	01-29/ 01-41	01-29/ 01-42	01-29/ 01-43	01-29/ 01-44	01-29/ 01-45	01-29/ 01-46
RM	0.0436	0.0433	0.0402	0.0461	0.0486	0.0445	0.0449	0.0457	0.0466
RP	0.1052	0.1033	0.1100	0.1136	0.1025	0.1029	0.1015	0.1010	0.1141
RC	0.1009	0.0993	0.1038	0.1086	0.1005	0.0994	0.0984	0.0982	0.1092
SGM	0.0542	0.0538	0.0496	0.0576	0.0609	0.0554	0.0559	0.0570	0.0582
SGP	0.1052	0.1033	0.1100	0.1136	0.1025	0.1029	0.1015	0.1010	0.1141
SGC	0.1184	0.1165	0.1207	0.1273	0.1192	0.1169	0.1159	0.1159	0.1281
TI	0.1666	0.1637	0.1736	0.1796	0.1629	0.1632	0.1611	0.1603	0.1804

Table 6-17 – CF Error measures for tests 01-29 to 01-46

HSV/ KF	01-29/ 01-29	01-29/ 01-30	01-29/ 01-31	01-29/ 01-32	01-29/ 01-33	01-29/ 01-34	01-29/ 01-35	01-29/ 01-36	01-29/ 01-37
RM	0.0379	0.0366	0.0440	0.0350	0.0373	0.0413	0.0368	0.0418	0.0369
RP	0.1026	0.0984	0.0994	0.1017	0.1100	0.1034	0.1003	0.1013	0.1091
RC	0.0969	0.0930	0.0963	0.0954	0.1030	0.0987	0.0947	0.0971	0.1021
SGM	0.0466	0.0450	0.0547	0.0429	0.0459	0.0511	0.0452	0.0518	0.0454
SGP	0.1026	0.0984	0.0994	0.1017	0.1100	0.1034	0.1003	0.1013	0.1091
SGC	0.1127	0.1082	0.1135	0.1104	0.1192	0.1154	0.1100	0.1138	0.1182
TI	0.1621	0.1554	0.1577	0.1605	0.1734	0.1636	0.1584	0.1603	0.1720
HSV/ KF	01-29/ 01-38	01-29/ 01-39	01-29/ 01-40	01-29/ 01-41	01-29/ 01-42	01-29/ 01-43	01-29/ 01-44	01-29/ 01-45	01-29/ 01-46
RM	0.0372	0.0374	0.0387	0.0423	0.0425	0.0370	0.0388	0.0394	0.0381
RP	0.1030	0.1013	0.1051	0.1107	0.1005	0.1012	0.1000	0.0988	0.1113
RC	0.0971	0.0957	0.0993	0.1050	0.0967	0.0955	0.0950	0.0943	0.1042
SGM	0.0458	0.0460	0.0477	0.0525	0.0528	0.0454	0.0478	0.0486	0.0468
SGP	0.1030	0.1013	0.1051	0.1107	0.1005	0.1012	0.1000	0.0988	0.1113
SGC	0.1127	0.1112	0.1154	0.1225	0.1135	0.1109	0.1108	0.1101	0.1207
TI	0.1627	0.1600	0.1660	0.1748	0.1592	0.1598	0.1581	0.1563	0.1754

Table 6-18 – KF Error measures for tests 01-29 to 01-46

	HSV				CF				KF			
	SD	Var	Median	Average	SD	Var	Median	Average	SD	Var	Median	Average
RM	0.0058	0.000033	-0.0074	-0.0054	0.0023	0.000005	0.0441	0.0444	0.0024	0.000006	0.0380	0.0388
RP	0.0014	0.000002	0.0060	0.0059	0.0046	0.000021	0.1034	0.1055	0.0041	0.000017	0.1015	0.1032
RC	0.0019	0.000004	0.0081	0.0086	0.0038	0.000014	0.0999	0.1015	0.0035	0.000012	0.0968	0.0978
SGM	0.0067	0.000045	-0.0085	-0.0062	0.0031	0.000010	0.0549	0.0553	0.0032	0.000010	0.0467	0.0479
SGP	0.0014	0.000002	0.0060	0.0059	0.0046	0.000021	0.1034	0.1055	0.0041	0.000017	0.1015	0.1032
SGC	0.0025	0.000006	0.0102	0.0107	0.0042	0.000018	0.1178	0.1191	0.0039	0.000015	0.1131	0.1138
TI	0.0021	0.000004	0.0103	0.0104	0.0070	0.000049	0.1636	0.1671	0.0063	0.000040	0.1604	0.1631

Table 6-19 – Summary of statistical parameters for tests 01-29 to 01-46 Error measures

The different setting combinations have been combined into individual charts to aid visual interpretation of the results along with a set of respective statistical parameters for each of these new data sets. The relevant charts and tables for parameters are summarised in Table 6-20.

Setting	Chart (Combined CF & KF)	CF Statistical Parameters	KF Statistical Parameters
Accelerometer sensitivity 2g	Figure 6-29	Table 6-21	Table 6-22
Accelerometer sensitivity 4g	Figure 6-30		
Accelerometer sensitivity 8g	Figure 6-31		
Accelerometer sensitivity 16g	Figure 6-32		
Gyroscope sensitivity 250 deg/s	Figure 6-33	Table 6-23	Table 6-24
Gyroscope sensitivity 500 deg/s	Figure 6-34		
Gyroscope sensitivity 1000 deg/s	Figure 6-35		
Gyroscope sensitivity 2000 deg/s	Figure 6-36		

Table 6-20 – Summary of chart and parameters for different IMU sensor sensitivities

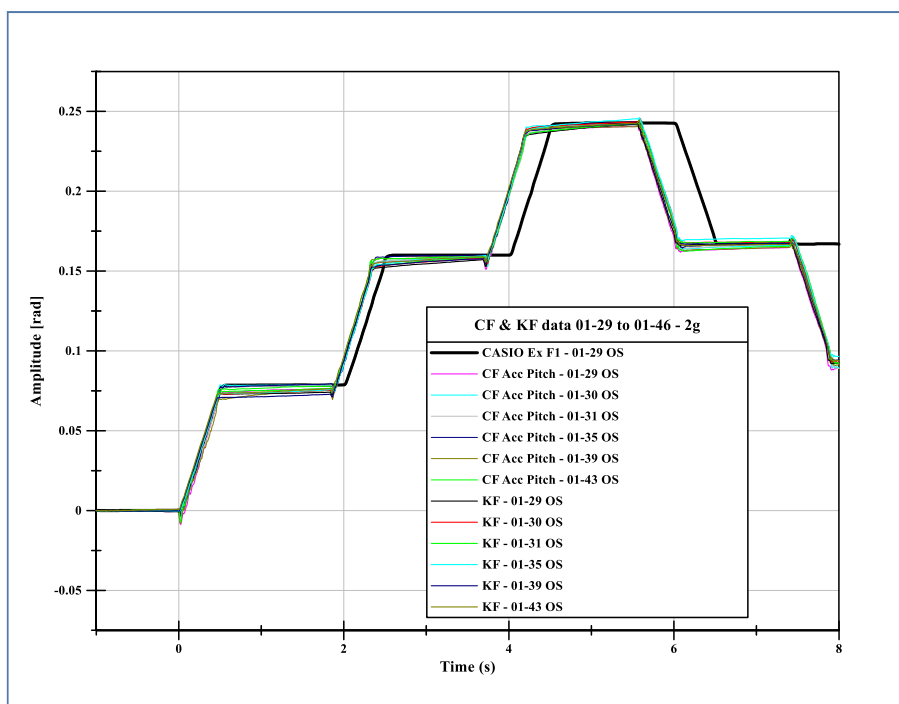


Figure 6-29 – Close-up of CF and KF data for tests 01-29 to 01-46 – Accelerometer sensitivity 2g

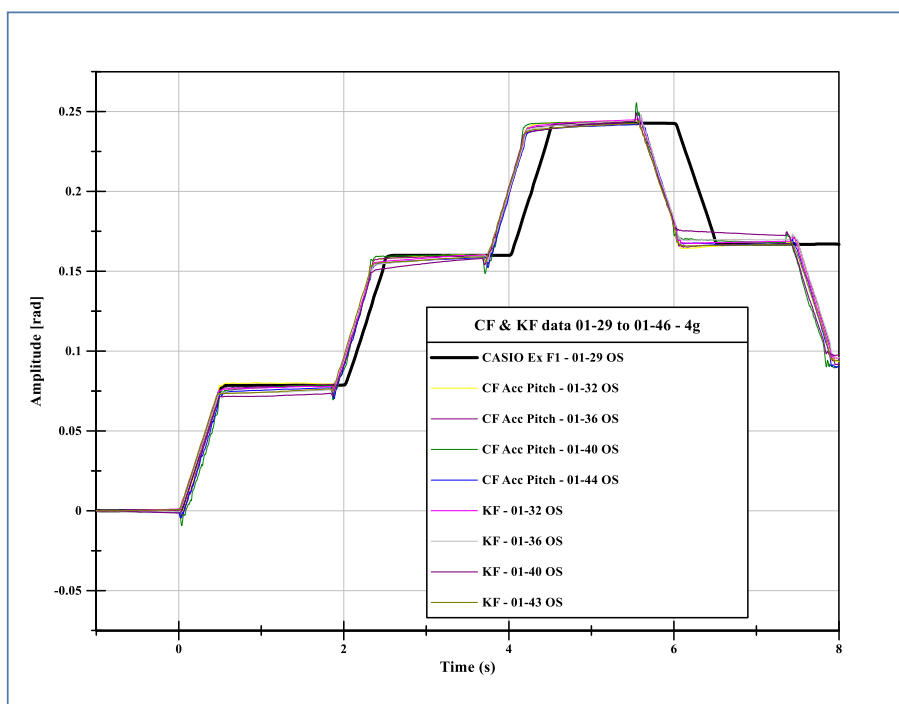


Figure 6-30 – Close-up of CF and KF data for tests 01-29 to 01-46 – Accelerometer sensitivity 4g

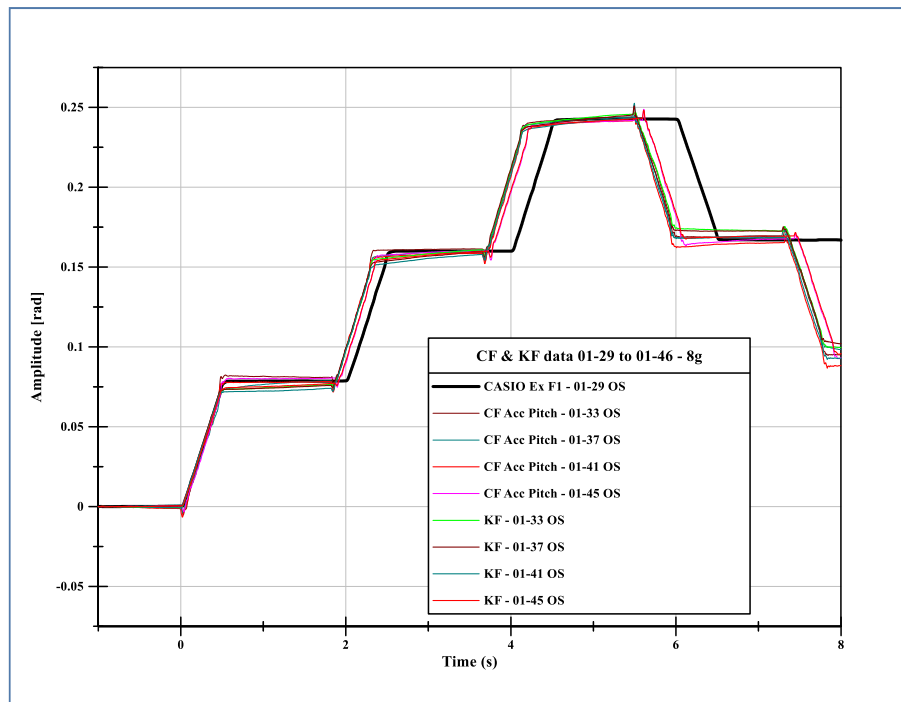


Figure 6-31 – Close-up of CF and KF data for tests 01-29 to 01-46 – Accelerometer sensitivity 8g

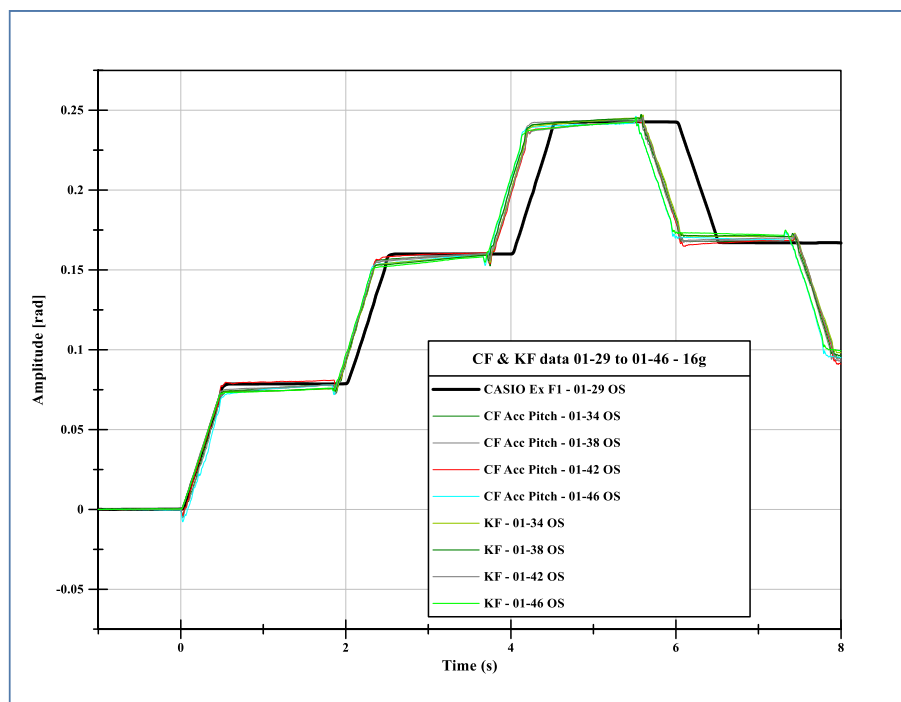


Figure 6-32 – Close-up of CF and KF data for tests 01-29 to 01-46 – Accelerometer sensitivity 16g

A visual inspection of the charts for the data curves collated using accelerometer sensitivities (Figure 6-29 to Figure 6-32) indicated that they were all very similar. This was verified by the statistical parameters in Table 6-21 and Table 6-22. Although these figures suggested that the different data sets were all quite close in terms of their distribution, the comprehensive error measures RC and SGC, and the inequality TI, implied that the 2g data sets

were the most closely matched followed by the 4g data sets (see the bold underlined values in Table 6-21 and Table 6-22).

	2g				4g			
	Std Dev	Var	Median	Average	Std Dev	Var	Median	Average
RM	0.0023	0.000005	0.0439	0.0445	0.0017	0.000003	0.0419	0.0422
RP	0.0017	0.000003	0.1031	0.1030	0.0034	0.000012	0.1027	0.1042
RC	<u>0.0015</u>	<u>0.000002</u>	0.0993	0.0994	<u>0.0024</u>	<u>0.000006</u>	0.0986	0.0997
SGM	0.0031	0.000010	0.0546	0.0554	0.0023	0.000005	0.0519	0.0523
SGP	0.0017	0.000003	0.1031	0.1030	0.0034	0.000012	0.1027	0.1042
SGC	<u>0.0020</u>	<u>0.000004</u>	0.1169	0.1170	<u>0.0024</u>	<u>0.000006</u>	0.1158	0.1167
TI	<u>0.0026</u>	<u>0.000007</u>	0.1634	0.1633	<u>0.0051</u>	<u>0.000026</u>	0.1625	0.1649
	8g				16g			
	Std Dev	Var	Median	Average	Std Dev	Var	Median	Average
RM	0.0014	0.000002	0.0447	0.0445	0.0018	0.000003	0.0464	0.0463
RP	0.0052	0.000027	0.1124	0.1098	0.0047	0.000022	0.1039	0.1061
RC	<u>0.0040</u>	<u>0.000016</u>	0.1067	0.1051	<u>0.0038</u>	<u>0.000015</u>	0.1007	0.1026
SGM	0.0019	0.000004	0.0557	0.0555	0.0024	0.000006	0.0580	0.0578
SGP	0.0052	0.000027	0.1124	0.1098	0.0047	0.000022	0.1039	0.1061
SGC	<u>0.0043</u>	<u>0.000018</u>	0.1245	0.1231	<u>0.0042</u>	<u>0.000018</u>	0.1188	0.1209
TI	<u>0.0078</u>	<u>0.000061</u>	0.1775	0.1737	<u>0.0072</u>	<u>0.000052</u>	0.1648	0.1682

Table 6-21 – Summary of statistical parameters for CF data for tests 01-29 to 01-46 Accelerometers error measures

	2g				4g			
	Std Dev	Var	Median	Average	Std Dev	Var	Median	Average
RM	0.0026	0.000007	0.0372	0.0383	0.0024	0.000006	0.0388	0.0386
RP	0.0014	0.000002	0.1007	0.1005	0.0019	0.000004	0.1015	0.1020
RC	<u>0.0013</u>	<u>0.000002</u>	0.0956	0.0954	<u>0.0017</u>	<u>0.000003</u>	0.0962	0.0967
SGM	0.0034	0.000012	0.0457	0.0472	0.0032	0.000010	0.0478	0.0476
SGP	0.0014	0.000002	0.1007	0.1005	0.0019	0.000004	0.1015	0.1020
SGC	<u>0.0017</u>	<u>0.000003</u>	0.1111	0.1111	<u>0.0021</u>	<u>0.000004</u>	0.1123	0.1126
TI	<u>0.0021</u>	<u>0.000004</u>	0.1591	0.1589	<u>0.0029</u>	<u>0.000008</u>	0.1604	0.1612
	8g				16g			
	Std Dev	Var	Median	Average	Std Dev	Var	Median	Average
RM	0.0021	0.000005	0.0371	0.0335	0.0022	0.000005	0.0397	0.0398
RP	0.0049	0.000024	0.1096	0.1109	0.0041	0.000016	0.1032	0.1046
RC	<u>0.0041</u>	<u>0.000017</u>	0.1025	0.1030	<u>0.0030</u>	<u>0.000009</u>	0.0979	0.0992
SGM	0.0028	0.000008	0.0456	0.0410	0.0029	0.000009	0.0490	0.0491
SGP	0.0049	0.000024	0.1096	0.1109	0.0041	0.000016	0.1032	0.1046
SGC	<u>0.0046</u>	<u>0.000021</u>	0.1187	0.1189	<u>0.0031</u>	<u>0.000010</u>	0.1144	0.1156
TI	<u>0.0075</u>	<u>0.000056</u>	0.1727	0.1746	<u>0.0061</u>	<u>0.000037</u>	0.1631	0.1652

Table 6-22 – Summary of statistical parameters for KF data for tests 01-29 to 01-46 Accelerometers error measures

A similar visual inspection of the charts for the data curves collated using gyroscope sensitivities (Figure 6-33 to Figure 6-36) indicated that these data sets were also all very similar. This was again verified by the statistical parameters in Table 6-23 and Table 6-24. However, in this instance no one data set could be identified as being the most closely matched.

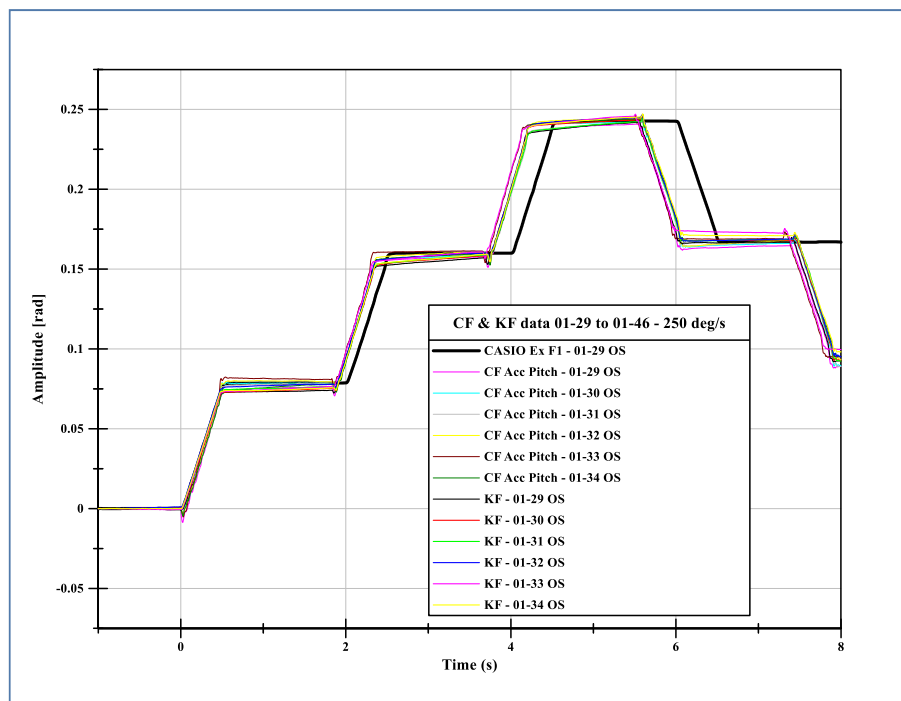


Figure 6-33 – Close-up of CF and KF data for tests 01-29 to 01-46 – Gyroscope sensitivity 250 deg/s

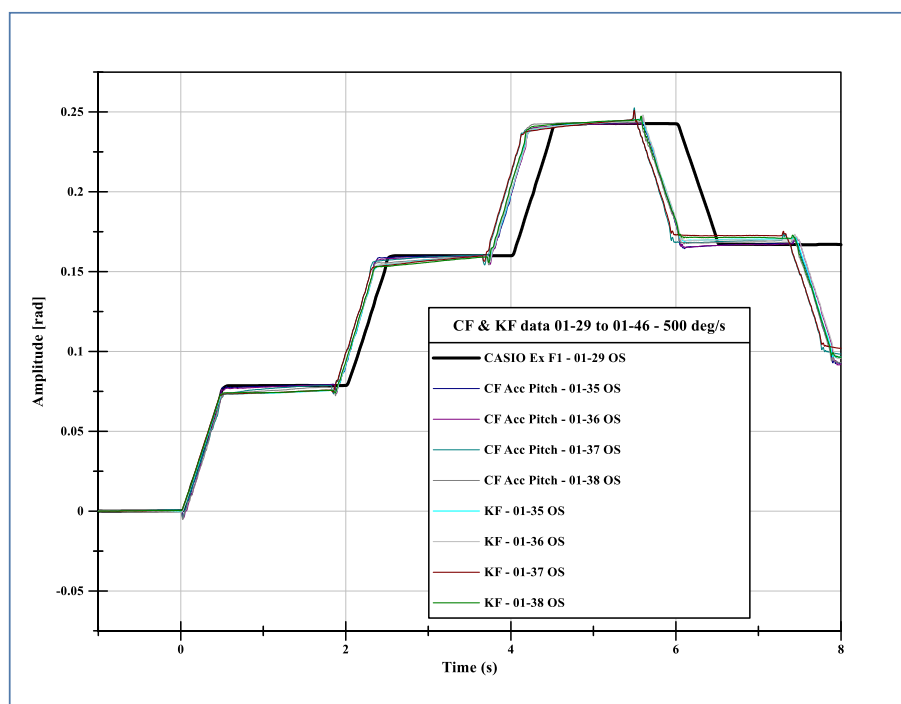


Figure 6-34 – Close-up of CF and KF data for tests 01-29 to 01-46 – Gyroscope sensitivity 500 deg/s

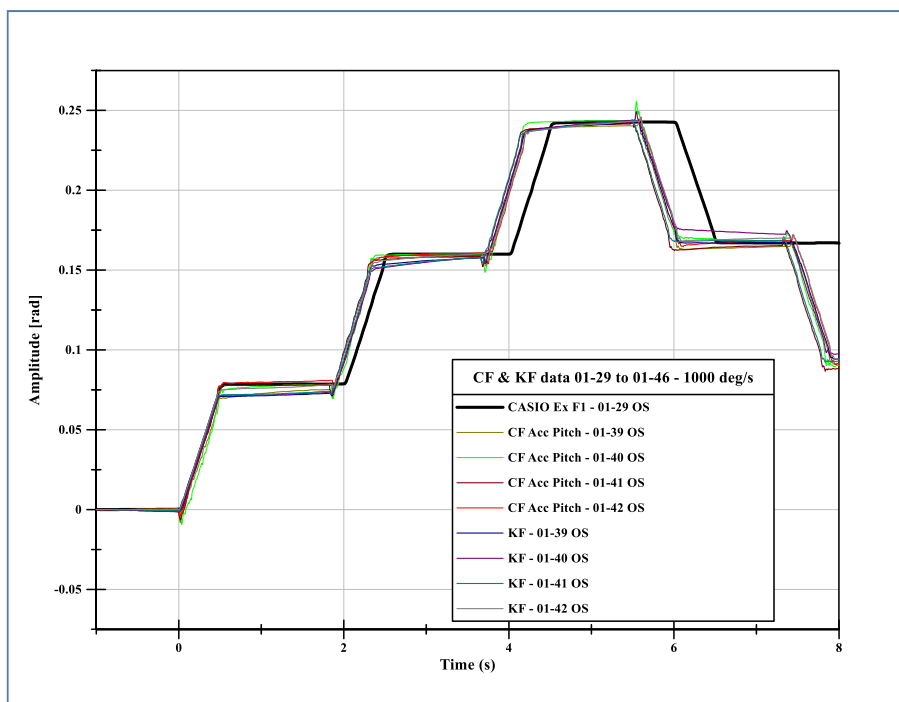


Figure 6-35 – Close-up of CF and KF data for tests 01-29 to 01-46 – Gyroscope sensitivity 1000 deg/s

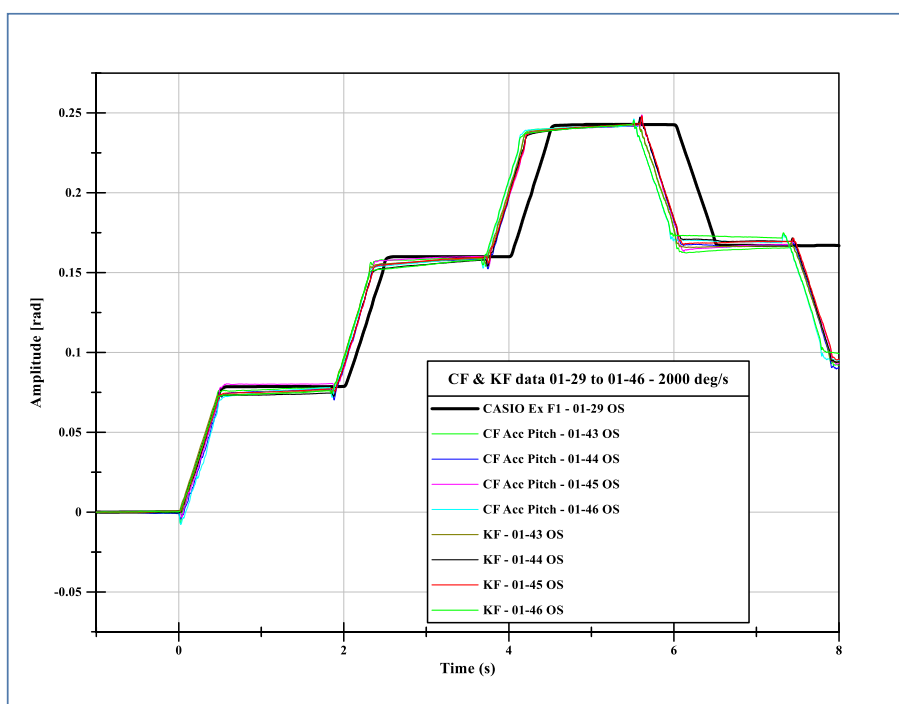


Figure 6-36 – Close-up of CF and KF data for tests 01-29 to 01-46 – Gyroscope sensitivity 2000 deg/s

From this analysis it could be concluded that for the slow rate motion scenarios demonstrated by this series of tests, the sensitivity chosen for the gyroscope had little impact on the results, however using the more sensitive 2g accelerometer setting marginally improved the consistency of the readings.

	250 deg/s				500 deg/s			
	Std Dev	Var	Median	Average	Std Dev	Var	Median	Average
RM	0.0026	0.000007	0.0424	0.0439	0.0016	0.000002	0.0437	0.0439
RP	0.0040	0.000016	0.1030	0.1042	0.0037	0.000014	0.1053	0.1062
RC	<u>0.0031</u>	<u>0.000010</u>	0.0995	0.1002	<u>0.0032</u>	<u>0.000010</u>	0.1015	0.1018
SGM	0.0035	0.000012	0.0526	0.0547	0.0021	0.000004	0.0543	0.0546
SGP	0.0040	0.000016	0.1030	0.1042	0.0037	0.000014	0.1053	0.1062
SGC	<u>0.0034</u>	<u>0.000011</u>	0.1174	0.1177	<u>0.0036</u>	<u>0.000013</u>	0.1193	0.1194
TI	<u>0.0061</u>	<u>0.000037</u>	0.1633	0.1650	<u>0.0057</u>	<u>0.000032</u>	0.1668	0.1681
	1000 degs/s				2000 deg/s			
	Std Dev	Var	Median	Average	Std Dev	Var	Median	Average
RM	0.0031	0.000010	0.0447	0.0445	0.0042	0.000018	0.0417	0.0423
RP	0.0046	0.000021	0.1067	0.1074	0.0044	0.000020	0.1067	0.1072
RC	<u>0.0036</u>	<u>0.000013</u>	<u>0.1022</u>	0.1031	<u>0.0025</u>	<u>0.000006</u>	0.1022	0.1022
SGM	0.0042	0.000018	0.0557	0.0555	0.0056	0.000032	0.0517	0.0525
SGP	0.0046	0.000021	0.1067	0.1074	0.0044	0.000020	0.1067	0.1072
SGC	<u>0.0040</u>	<u>0.000016</u>	0.1200	0.1209	<u>0.0020</u>	<u>0.000004</u>	0.1200	0.1196
TI	<u>0.0070</u>	<u>0.000049</u>	0.1686	0.1699	<u>0.0064</u>	<u>0.000041</u>	0.1686	0.1695

Table 6-23 – Summary of statistical parameters for CF data for tests 01-29 to 01-46 Gyroscope error measures

	250 deg/s				500 deg/s			
	Std Dev	Var	Median	Average	Std Dev	Var	Median	Average
RM	0.0030	0.000009	0.0376	0.0387	0.0021	0.000004	0.0371	0.0382
RP	0.0038	0.000014	0.1022	0.1026	0.0034	0.000012	0.1022	0.1034
RC	<u>0.0031</u>	<u>0.000010</u>	0.0966	0.0972	<u>0.0027</u>	<u>0.000007</u>	0.0971	0.0977
SGM	0.0040	0.000016	0.0462	0.0477	0.0028	0.000008	0.0456	0.0470
SGP	0.0038	0.000014	0.1022	0.1026	0.0034	0.000012	0.1022	0.1034
SGC	<u>0.0035</u>	<u>0.000012</u>	0.1131	0.1132	<u>0.0029</u>	<u>0.000009</u>	0.1132	0.1137
TI	<u>0.0057</u>	<u>0.000033</u>	0.1613	0.1621	<u>0.0052</u>	<u>0.000027</u>	0.1615	0.1633
	1000 degs/s				2000 deg/s			
	Std Dev	Var	Median	Average	Std Dev	Var	Median	Average
RM	0.0022	0.000005	0.0405	0.0402	0.0009	0.000001	0.0384	0.0383
RP	0.0040	0.000016	0.1032	0.1044	0.0050	0.000025	0.1006	0.1028
RC	<u>0.0036</u>	<u>0.000013</u>	0.0980	0.0992	<u>0.0041</u>	<u>0.000016</u>	0.0953	0.0972
SGM	0.0030	0.000009	0.0501	0.0497	0.0012	0.000001	0.0473	0.0472
SGP	0.0040	0.000016	0.1032	0.1044	0.0050	0.000025	0.1006	0.1028
SGC	<u>0.0042</u>	<u>0.000018</u>	0.1145	0.1157	<u>0.0044</u>	<u>0.000019</u>	0.1109	0.1131
TI	<u>0.0063</u>	<u>0.000039</u>	0.1630	0.1650	<u>0.0076</u>	<u>0.000058</u>	0.1589	0.1624

Table 6-24 – Summary of statistical parameters for KF data for tests 01-29 to 01-46 Gyroscope error measures

6.2.3.6 Experiment One – Tests 47 to 55 – Comparison of different sampling rates

The final series of tests for Experiment One were conducted to investigate the influence of changing the sampling rate on the results. An overview of the data for these tests is shown in Figure 6-37 and a close-up of the data in the red rectangle is shown in Figure 6-38. The same filter constants were used for each test as summarised in Table 6-25.

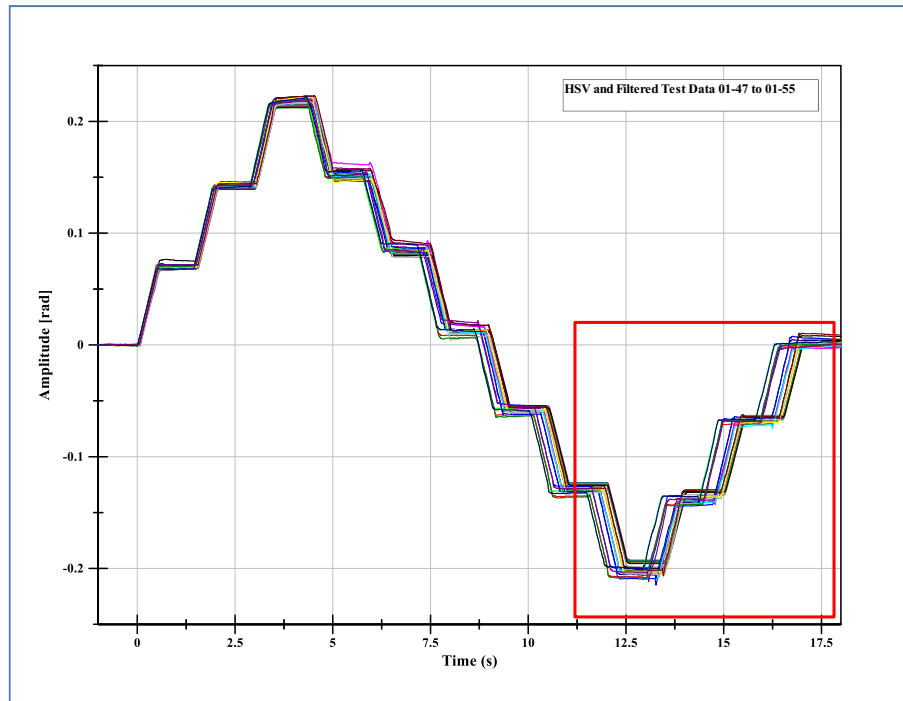


Figure 6-37 – HSV, CF and KF data for tests 01-47 to 01-55

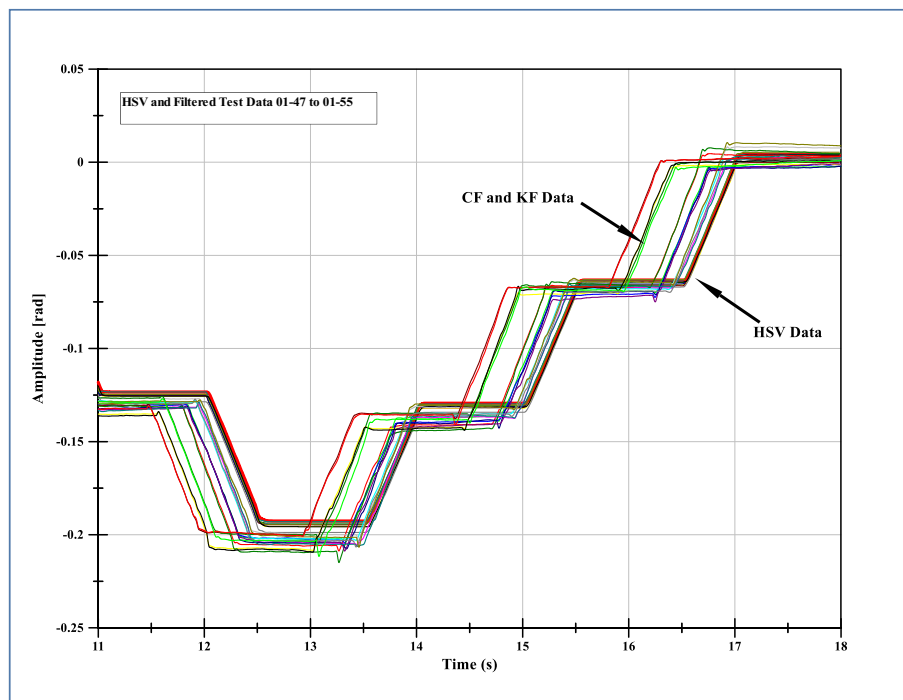


Figure 6-38 – Close-up of HSV, CF and KF data for tests 01-47 to 01-55

	01-47	01-48	01-49	01-50	01-51	01-52	01-53	01-54	01-55
CF Alpha	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
Q Angle	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Q Gyro Bias	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
R Measure	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Table 6-25 – Filter constants for tests 01-47 to 01-55

The sampling rate parameters chosen for these tests are summarised in Table 6-26. The timing loop period was used to set how long the timed loop, within the LabVIEW datalogging VI, waited before starting the next iteration and therefore a longer wait time decreased the number of samples recorded. This meant that tests 01-50 to 01-52 were sampled at twice the rate of tests 01-47 to 01-49 while tests 01-53 to 01-55 were sampled at half the rate. A timing loop period of 1000 μ S was used for the majority of tests conducted for the research and described in this thesis.

	01-47	01-48	01-49	01-50	01-51	01-52	01-53	01-54	01-55
Clock speed (MHz)	1	1	1	1	1	1	1	1	1
Timing loop period (μS)	1000	1000	1000	500	500	500	2000	2000	2000
Gyro sensitivity (deg/s)	250	250	250	250	250	250	250	250	250
Acc sensitivity (g)	2	2	2	2	2	2	2	2	2

Table 6-26 – Sample rate parameters for tests 01-47 to 01-55

The HSV data is shown in Figure 6-39 with the respective error measures summarised in Table 6-27. For these tests the HSV data curve for Test 01-51 was used as the comparator. A review of the error measures showed that all of the data curves, except for Test 01-49, had magnitudes that were greater than the comparator.

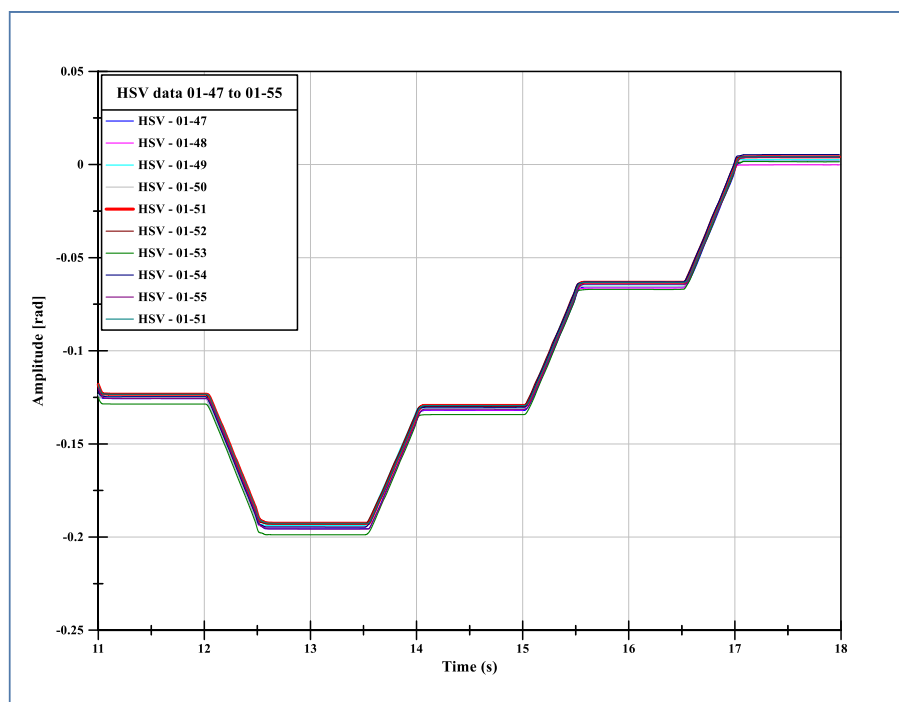


Figure 6-39 – Close-up of HSV data for tests 01-47 to 01-55

The same process was applied to the filtered data; the CF data curves are shown in Figure 6-40 whilst the KF data curves are shown in Figure 6-41. The error measures for the CF and KF are summarised in Table 6-28 and Table 6-29 respectively, while the statistical parameters for all the data sets are summarised in Table 6-30.

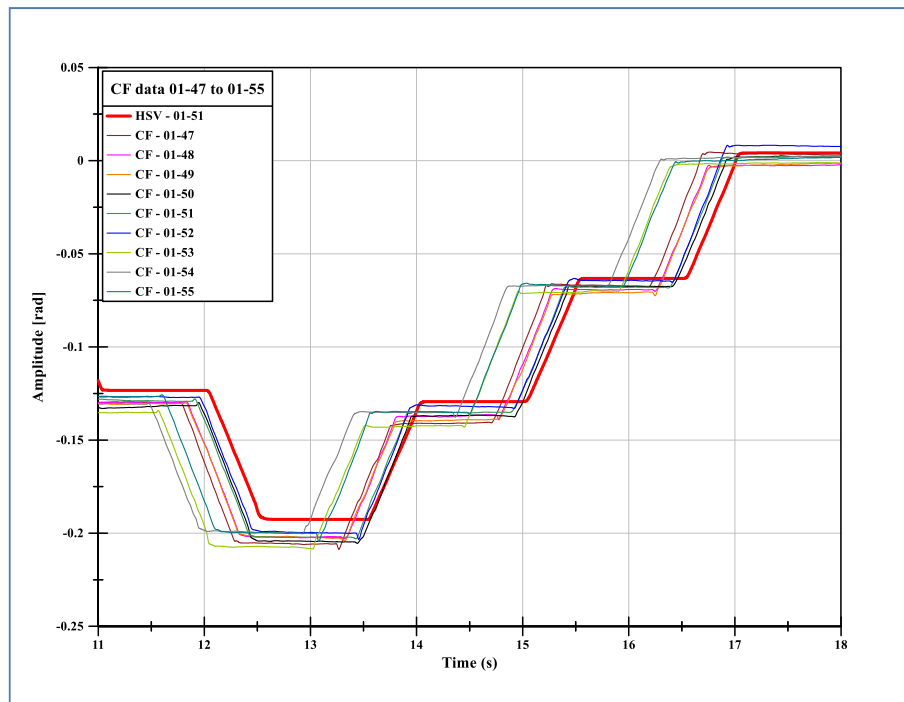


Figure 6-40 – Close-up of CF data for tests 01-47 to 01-55

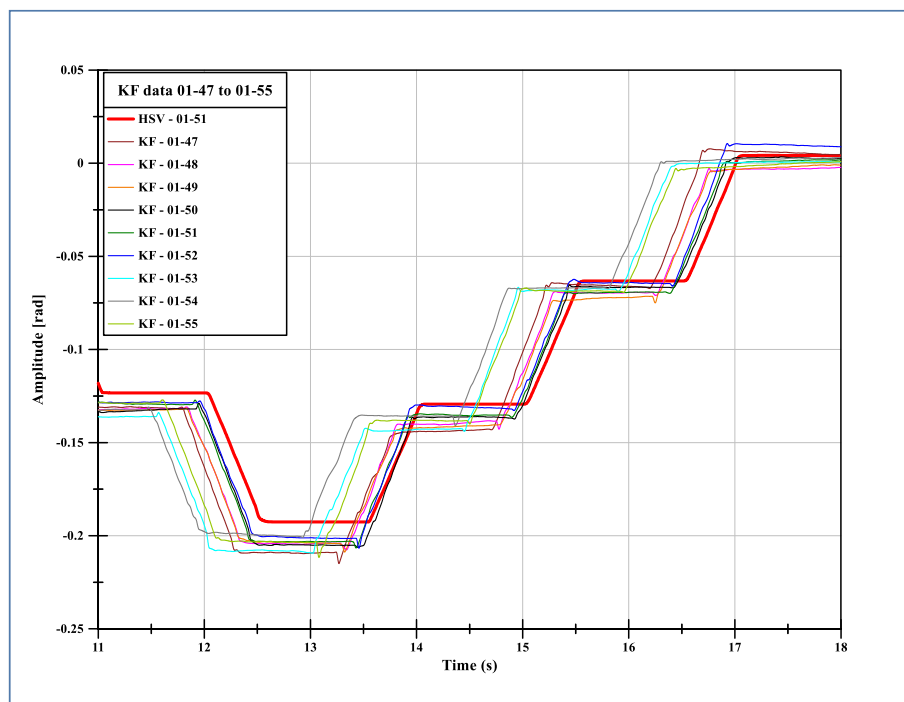


Figure 6-41 – Close-up of KF data for tests 01-47 to 01-55

HSV/ HSV	01-51/ 01-47	01-51/ 01-48	01-51/ 01-49	01-51/ 01-50	01-51/ 01-52	01-51/ 01-53	01-51/ 01-54	01-51/ 01-55
RM	-0.0054	-0.0036	0.0007	-0.0017	-0.0021	-0.0132	-0.0094	-0.0089
RP	0.0045	0.0066	0.0065	0.0066	0.0044	0.0081	0.0034	0.0023
RC	0.0062	0.0067	0.0058	0.0060	0.0043	0.0137	0.0089	0.0082
SGM	-0.0062	-0.0042	0.0008	-0.0019	-0.0024	-0.0153	-0.0109	-0.0103
SGP	0.0045	0.0066	0.0065	0.0066	0.0044	0.0081	0.0034	0.0023
SGC	0.0077	0.0079	0.0065	0.0069	0.0050	0.0173	0.0115	0.0106
TI	0.0077	0.0106	0.0102	0.0104	0.0069	0.0148	0.0077	0.0064

Table 6-27 – HSV Error measures for tests 01-47 to 01-55

HSV/ CF	01-51/ 01-47	01-51/ 01-48	01-51/ 01-49	01-51/ 01-50	01-51/ 01-51	01-51/ 01-52	01-51/ 01-53	01-51/ 01-54	01-51/ 01-55
RM	-0.0289	-0.0284	-0.0225	-0.0217	-0.0227	-0.0305	-0.0255	-0.0128	-0.0073
RP	0.0387	0.0284	0.0295	0.0174	0.0184	0.0150	0.0667	0.0728	0.0568
RC	<u>0.0428</u>	<u>0.0356</u>	<u>0.0329</u>	<u>0.0246</u>	<u>0.0259</u>	<u>0.0301</u>	<u>0.0633</u>	<u>0.0655</u>	<u>0.0508</u>
SGM	-0.0338	-0.0332	-0.0262	-0.0253	-0.0265	-0.0357	-0.0298	-0.0149	-0.0084
SGP	0.0387	0.0284	0.0295	0.0174	0.0184	0.0150	0.0667	0.0728	0.0568
SGC	<u>0.0514</u>	<u>0.0437</u>	<u>0.0394</u>	<u>0.0307</u>	<u>0.0323</u>	<u>0.0387</u>	<u>0.0731</u>	<u>0.0743</u>	<u>0.0574</u>
TI	<u>0.0631</u>	<u>0.0476</u>	<u>0.0482</u>	<u>0.0302</u>	<u>0.0319</u>	<u>0.0298</u>	<u>0.1057</u>	<u>0.1144</u>	<u>0.0892</u>

Table 6-28 – CF Error measures for tests 01-47 to 01-55

HSV/ KF	01-51/ 01-47	01-51/ 01-48	01-51/ 01-49	01-51/ 01-50	01-51/ 01-51	01-51/ 01-52	01-51/ 01-53	01-51/ 01-54	01-51/ 01-55
RM	-0.0400	-0.0391	-0.0281	-0.0256	-0.0269	-0.0315	-0.0248	-0.0168	-0.0195
RP	0.0388	0.0275	0.0285	0.0160	0.0170	0.0159	0.0675	0.0717	0.0546
RC	<u>0.0494</u>	<u>0.0424</u>	<u>0.0355</u>	<u>0.0268</u>	<u>0.0282</u>	<u>0.0313</u>	<u>0.0637</u>	<u>0.0653</u>	<u>0.0514</u>
SGM	-0.0471	-0.0460	-0.0329	-0.0300	-0.0314	-0.0369	-0.0290	-0.0195	-0.0227
SGP	0.0388	0.0275	0.0285	0.0160	0.0170	0.0159	0.0675	0.0717	0.0546
SGC	<u>0.0610</u>	<u>0.0536</u>	<u>0.0435</u>	<u>0.0339</u>	<u>0.0357</u>	<u>0.0402</u>	<u>0.0734</u>	<u>0.0743</u>	<u>0.0592</u>
TI	<u>0.0654</u>	<u>0.0492</u>	<u>0.0478</u>	<u>0.0293</u>	<u>0.0311</u>	<u>0.0312</u>	<u>0.1068</u>	<u>0.1128</u>	<u>0.0865</u>

Table 6-29 – KF Error measures for tests 01-47 to 01-55

	HSV				CF				KF			
	SD	Var	Median	Average	SD	Var	Median	Average	SD	Var	Median	Average
RM	0.0044	0.00002	-0.0045	-0.0055	0.0073	0.00005	-0.0227	-0.0222	0.0074	0.00005	-0.0269	-0.0280
RP	0.0018	0.00000	0.0055	0.0053	0.0208	0.00043	0.0295	0.0382	0.0208	0.00043	0.0285	0.0375
RC	<u>0.0027</u>	<u>0.00001</u>	0.0065	0.0075	<u>0.0146</u>	<u>0.00021</u>	0.0356	0.0413	<u>0.0138</u>	<u>0.00019</u>	0.0424	0.0438
SGM	0.0051	0.00003	-0.0052	-0.0063	0.0085	0.00007	-0.0265	-0.0260	0.0088	0.00008	-0.0314	-0.0328
SGP	0.0018	0.00000	0.0055	0.0053	0.0208	0.00043	0.0295	0.0382	0.0208	0.00043	0.0285	0.0375
SGC	<u>0.0037</u>	<u>0.00001</u>	0.0078	0.0091	<u>0.0154</u>	<u>0.00024</u>	0.0437	0.0490	<u>0.0145</u>	<u>0.00021</u>	0.0536	0.0528
TI	<u>0.0026</u>	<u>0.00001</u>	0.0089	0.0093	<u>0.0312</u>	<u>0.00097</u>	0.0482	0.0622	<u>0.0308</u>	<u>0.00095</u>	0.0492	0.0622

Table 6-30 – Summary of statistical parameters for tests 01-47 to 01-55 Error measures

Unlike previous tests, the CF and KF data curves were more widely dispersed which meant that the statistical parameters in Table 6-30 did not provide any clear distinction, besides the fact that they were similar, indicating that the two filter methods produced comparable results. To aid interpretation of the results, the data was therefore collated in terms of the selected timing loop period. The results for the 1000 μS loop period (01-47 to 01-49) are shown in Figure 6-42, the 500 μS loop period (01-50 to 01-52) in Figure 6-43 and the 2000 μS loop period (01-53 to 01-55) in Figure 6-44.

The key observation that arose from collating the data in this manner was that there appeared to be a direct correlation between the timing loop period and how the filtered data sets matched the HSV comparator; the lower the timing loop period the better the correlation (see Figure 6-43), and conversely a higher timing loop period the worse the correlation (see Figure 6-44). This was also borne out in the numerical values for the comprehensive error measures RC and SGC, and the inequality measure TI in Table 6-28 and Table 6-29 (see bold, underlined numbers). This showed that a lower value denoted a better correlation against the comparator whilst a higher value indicated less correlation. Using these measures, the data set correlation could be ranked as 500 μS being the best followed by 1000 μS and then 2000 μS .

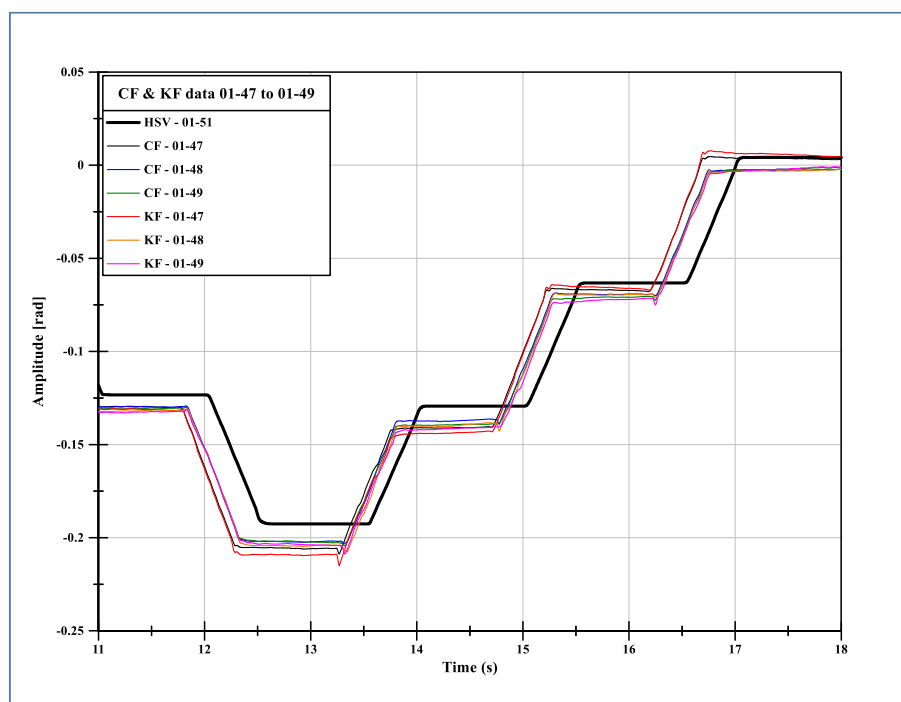


Figure 6-42 – Close-up of CF and KF data for tests 01-47 to 01-49

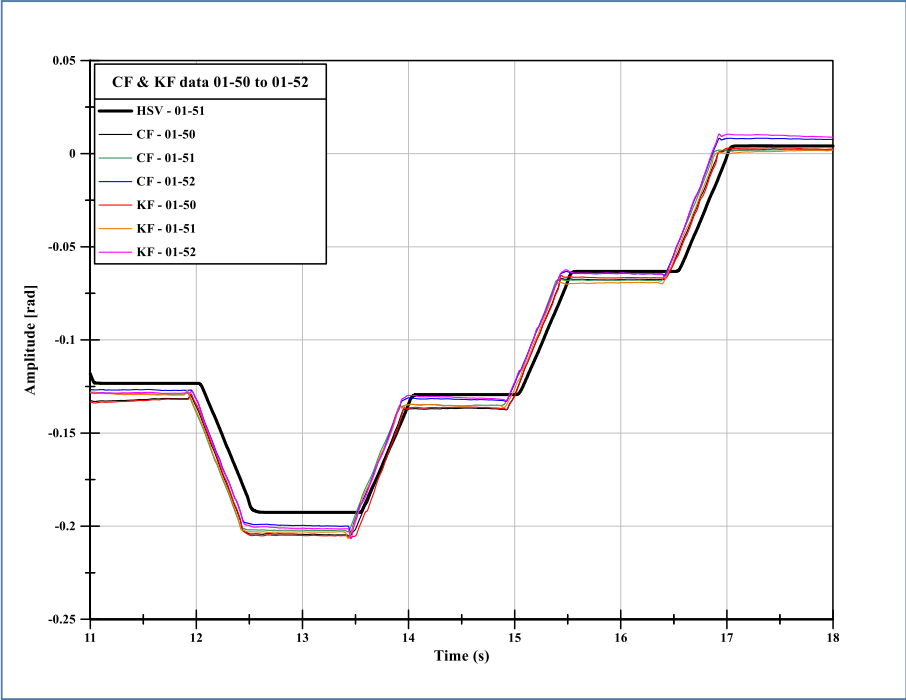


Figure 6-43 – Close-up of CF and KF data for tests 01-50 to 01-52

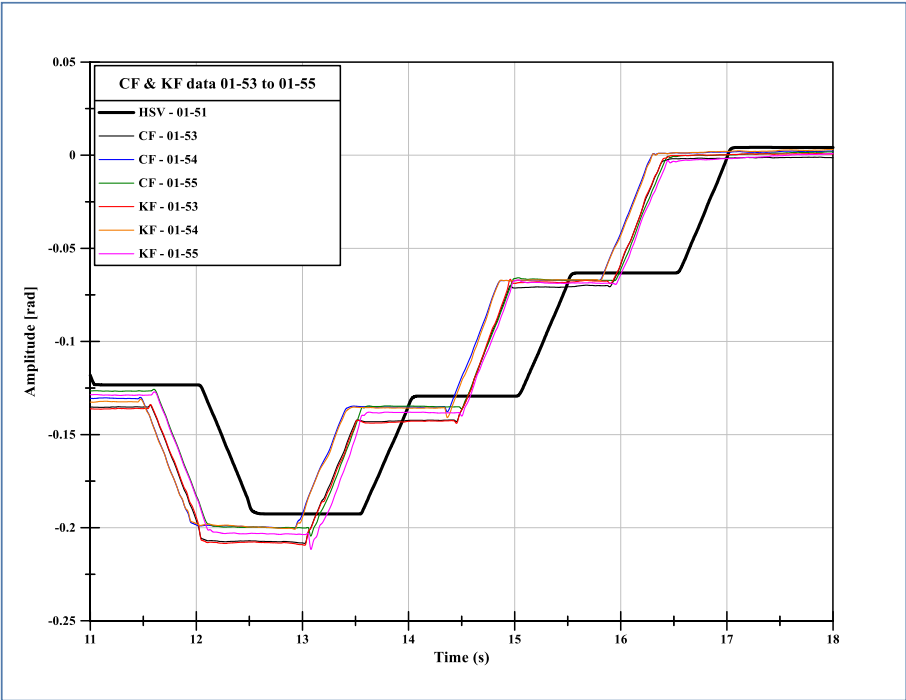


Figure 6-44 – Close-up of CF and KF data for tests 01-53 to 01-55

6.2.3.7 Time channel offset analysis

The difference between the timings for the HSV and the filtered data was shown to be a potential issue, and as it appeared to be a systematic error, possibly related to the data logging, a simple process was developed using DIAdem (National Instruments Corporation., 2015) and Excel to determine how consistent the error was.

The first step involved pinpointing easily identifiable features within the data curves using the Flag function within the DIAdem VIEW tab as shown in Figure 6-45.

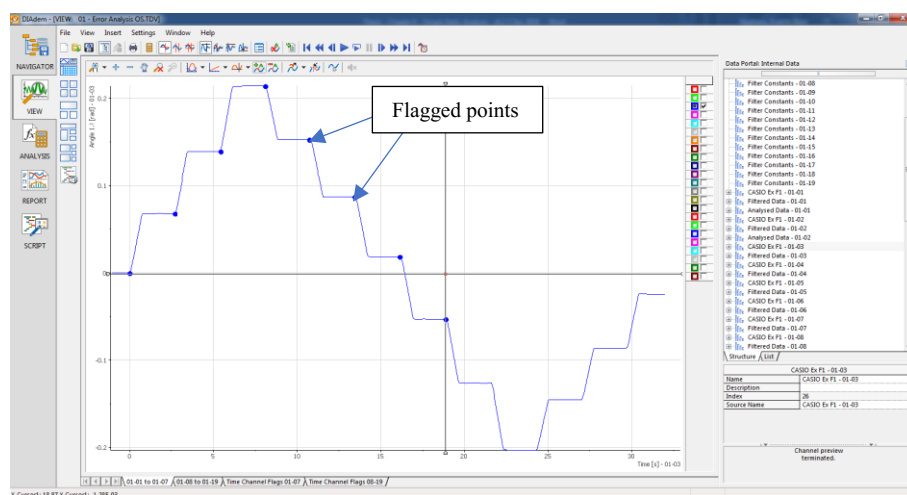


Figure 6-45 – Experiment One – Identify transition points using Flag function

The copy data points function, also in the VIEW tab, was then used to copy the flag points identified in the previous step to individual channels for each data curve. During this process missing points were replaced with NOVALUE (see Figure 6-46) which were then removed using the Remove NOVALUE function found in the ANALYSIS tab. The data with the NOVALUE values removed was then selected in the DIAdem VIEW tab and copy-and-pasted into an Excel spreadsheet (Figure 6-47 and Figure 6-48).

The NOVALUE data that was input into the spreadsheet was used to produce a straight line for the HSV, and one for the filtered data curves, using the timing data for each of these curves and the corresponding ‘step no’ for the respective time steps (as shown in Figure 6-48). The ratio of the slope factors from the best-fit lines for each of these straight lines gave the scale factor between the time channel for the HSV data curve, and that of the filtered data curve. This was verified by multiplying the filtered data curve time channel by the scale factor using the DIAdem scaling function in the ANALYSIS tab (Figure 6-49) and replotting the data curves using the new time channel. The resultant plot, shown in Figure 6-50, revealed that the previous time channel offset had been removed. In order to ascertain the consistency of this procedure it was applied to the CF data for 01-01 to 01-19. The resultant scale factors and mean and median for tests 01-01 to 01-07 and 01-08 to 01-19 are summarised in Table 6-31 and Table 6-32, and showed that the mean and median for each of these data sets were similar, and if only three significant figures were considered, they were in fact the same at 1.08.

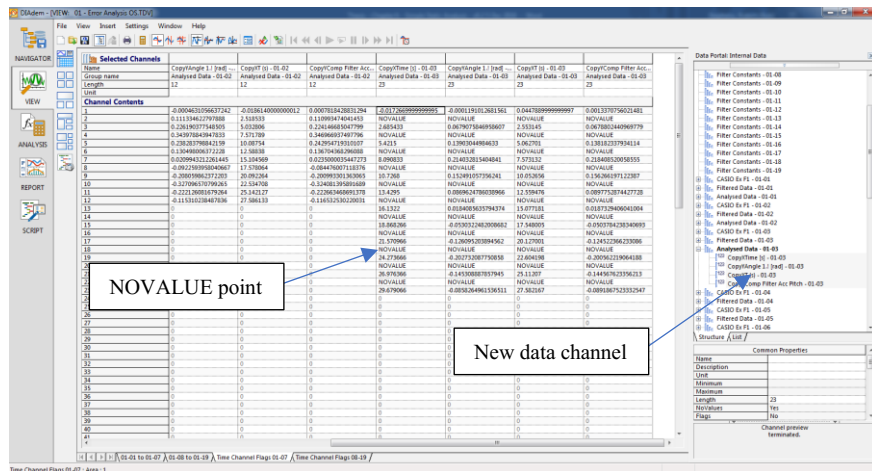


Figure 6-46 – Experiment One – Manipulating flagged data

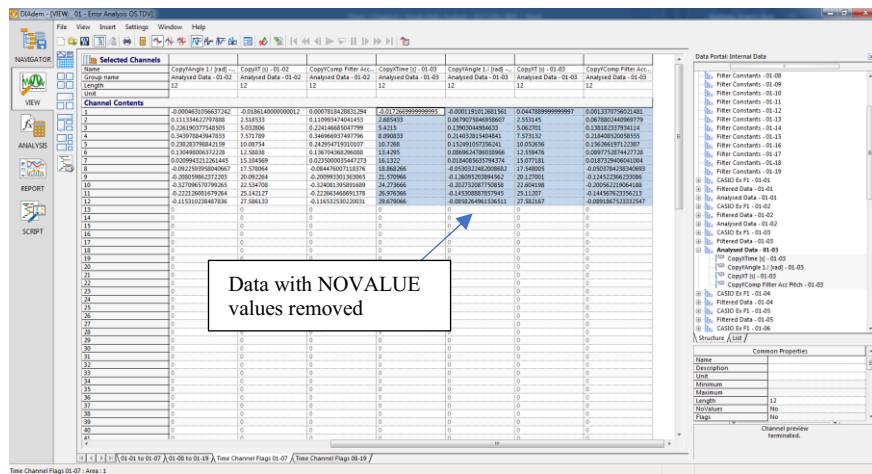


Figure 6-47 – Experiment One – Copy flagged intersection points

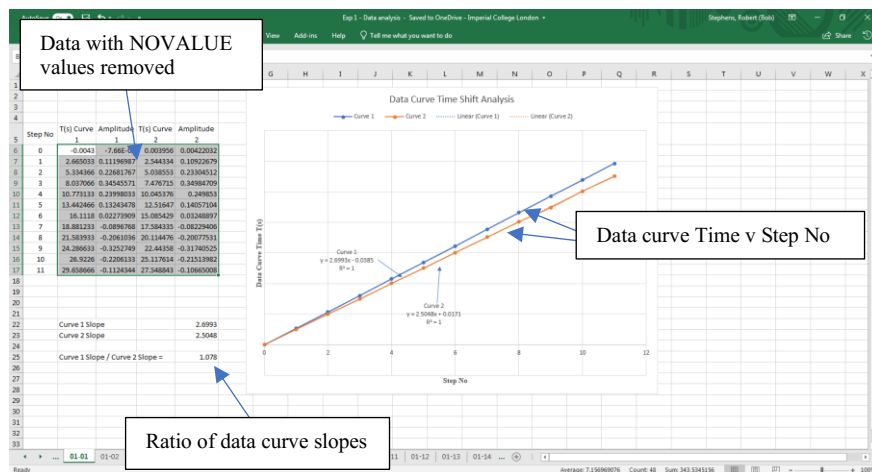


Figure 6-48 – Experiment One – Calculate time channel offset scale-factor between HSV and IMU data using Excel spreadsheet

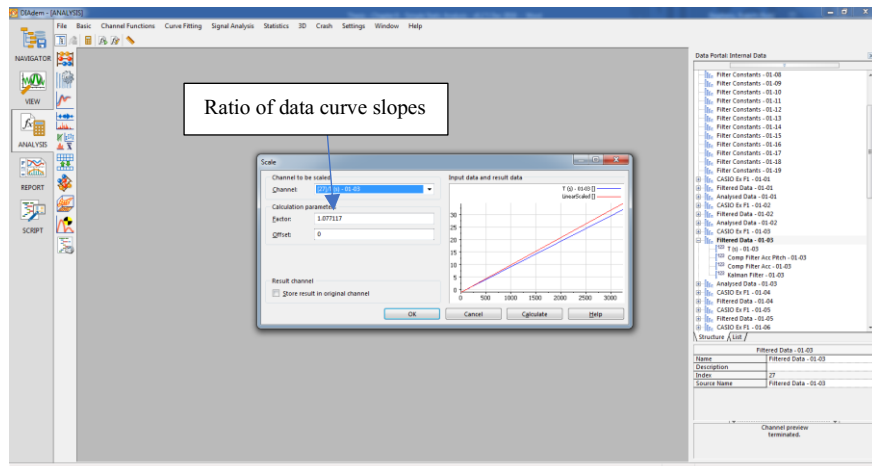


Figure 6-49 – Experiment One – Adjust IMU complementary filter time channel using previously calculated scale-factor

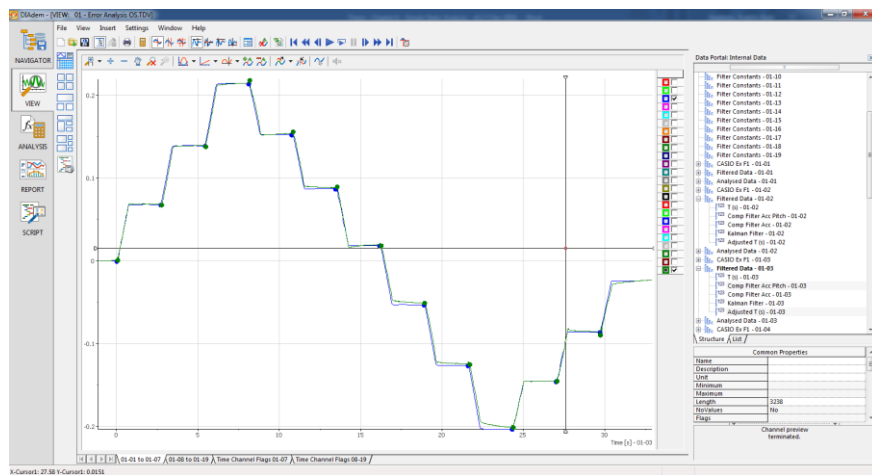


Figure 6-50 – Experiment One – Comparison of time adjusted IMU complementary filter data and HSV data

Test No	01-01	01-02	01-03	01-04	Mean
Scale Factor	1.078	1.076	1.077	1.078	1.077
Test No	01-05	01-06	01-07		Median
Scale Factor	1.077	1.079	1.077		1.077

Table 6-31 – Time channel offset scale factors for tests 01-01 to 01-07

Test No	01-08	01-09	01-10	01-11	01-12	01-13	Mean
Scale Factor	1.077	1.084	1.092	1.076	1.081	1.088	1.081
Test No	01-14	01-15	01-16	01-17	01-18	01-19	Median
Scale Factor	1.079	1.078	1.095	1.079	1.073	1.077	1.079

Table 6-32 – Time channel offset scale factors for tests 01-08 to 01-19

6.3 Experiment Two – Statically loaded vertical cantilever

Similar to Experiment One, the main purpose of Experiment Two was to verify that each of the sensors was operating as anticipated, and that the data analysis and fusion algorithms were again functioning correctly but using a vertical cantilever instead of an inverted pendulum. In addition to the MPU9250 IMU unit and a single axis Dytran accelerometer that were secured towards the top of the cantilever, strain gauges (SG) were attached near the base of the vertical cantilever which were also used as part of the data analysis.

6.3.1 Experimental arrangement

This experiment consisted of a vertical cantilever constructed from a 23.5mm x 1.2mm flexible metal strip. The bottom of the strip was rigidly fixed to a wooden stand using two, back-to-back steel angle sections, effectively creating an encastre connection. Two sets of full-bridge strain gauges were adhered to the base of the cantilever whilst the IMU and accelerometer used in Experiment One were fixed to the tip of the cantilever opposite the linear actuator as shown in Figure 6-51.

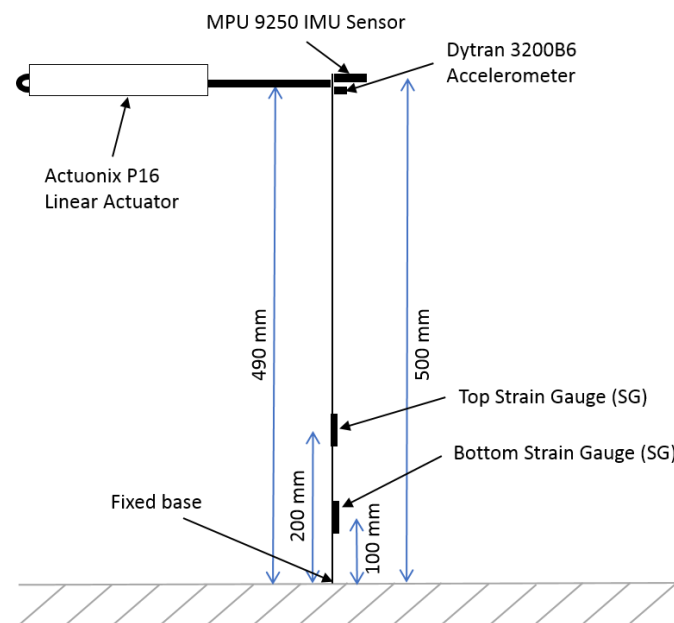


Figure 6-51 – Experiment Two main components and key dimensions

A general view of the experimental setup is shown in Figure 6-52 whilst the linear actuator used to horizontally load the tip of the vertical cantilever is shown in Figure 6-53.

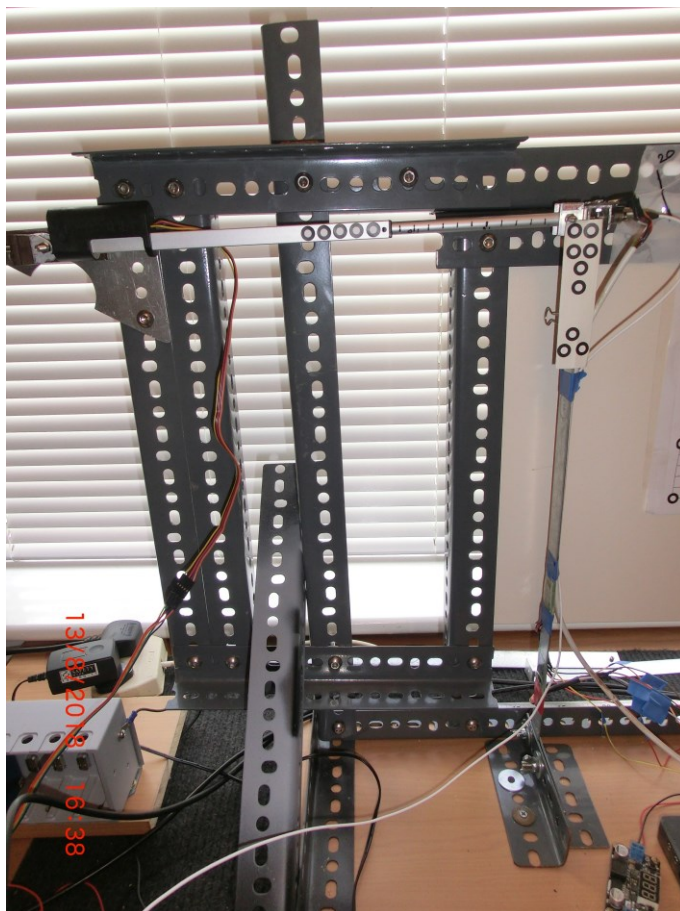


Figure 6-52 – Photograph of Experiment Two

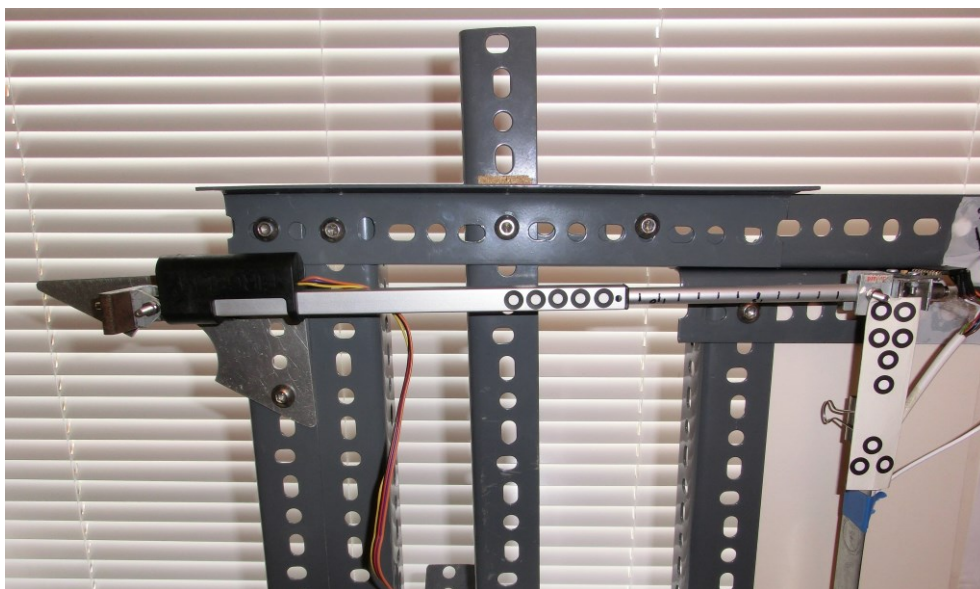


Figure 6-53 – Photograph of loading arrangement for Experiment Two

6.3.2 Experimental procedure

A similar procedure to that adopted in Experiment One was used in terms of loading and unloading the cantilever in fixed steps using the linear actuator controlled by a LabVIEW VI. As in Experiment One the procedure started by running the main LabVIEW sensor logging VI to obtain steady state readings for all the sensors and strain gauges to enable zero-offsets to be calculated if required. Once this calibration procedure had been completed the CASIO HSV was set to record in HD mode and the actuator VI was initiated. At the end of the actuator stepped cycle, which stopped automatically, the HSV camera and main LabVIEW sensor logging VI were also stopped.

6.3.3 Presentation of results

Table 6-33 summarises the tests carried out as part of Experiment Two. The table details the different combinations of the IMU gyroscope and accelerometer sensitivities investigated during the experiment to ascertain the impact that the combinations might have on the accuracy of the final results, and therefore try to determine the optimum combination.

Test No	Gyro Setting (deg/s)	Acc Setting (g)	Test No	Gyro Setting (deg/s)	Acc Setting (g)	Test No	Gyro Setting (deg/s)	Acc Setting (g)
02-01	500	2	02-12	500	4	02-23	250	2
02-02	500	2	02-13	500	8	02-24	250	2
02-03	500	2	02-14	500	16	02-25	250	2
02-04	500	2	02-15	1000	2	02-26	250	2
02-05	500	2	02-16	1000	4	02-27	250	2
02-06	500	2	02-17	1000	8	02-28	250	2
02-07	250	2	02-18	1000	16	02-29	250	2
02-08	250	4	02-19	2000	2	02-30	250	2
02-09	250	8	02-20	2000	4	02-31	250	2
02-10	250	16	02-21	2000	8			
02-11	500	2	02-22	2000	16			

Table 6-33 – Summary of Gyroscope and Accelerometer sensitivity settings for Experiment Two

The different shaded areas of the table refer to specific parts of the Experiment that were used to investigate different aspects of the experimental procedure. The analysis procedures developed during Experiment One were adopted for this experiment which, where appropriate, enabled the analysis to be condensed.

6.3.3.1 Experiment Two – Tests 01 to 06 – Investigation of data repeatability

A short series of tests was conducted to ascertain how using a flexible cantilever influenced the repeatability of the results. An overall image of the HSV and filtered data for Test 02-01 to 02-06 is shown in Figure 6-54 and a close-up of the area surrounded by the red rectangle in Figure 6-55. The purpose of these two figures was to provide a general impression of the correlation of the filter data in relation to the monitoring data provided by the HSV. Also included in these figures is the data curve for the cantilever rotation calculated using the strain gauge data. The same filter constants were used for each test as summarised in Table 6-34.

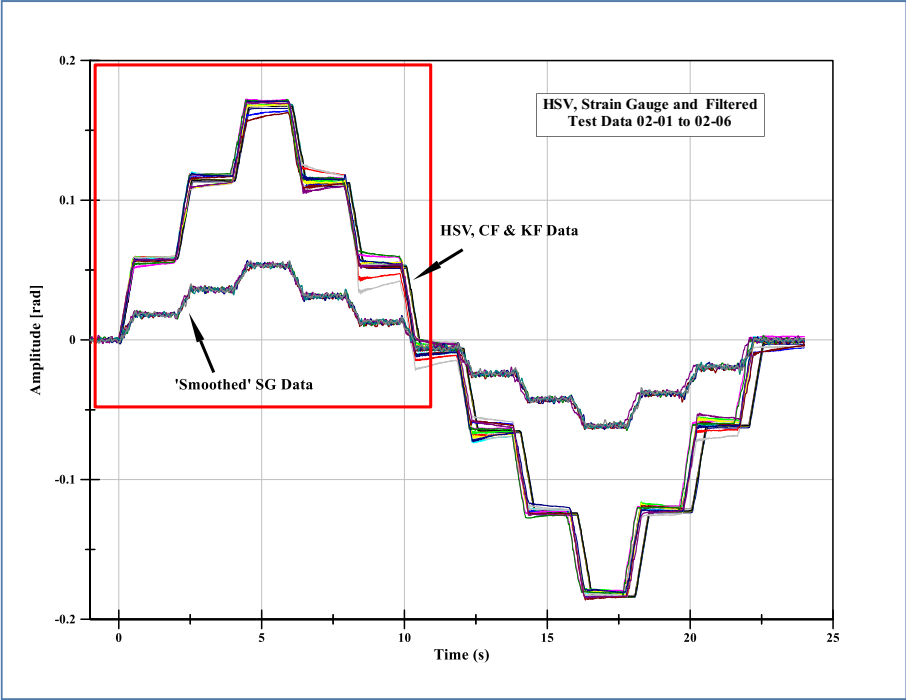


Figure 6-54 – HSV, Strain Gauge, CF and KF data for tests 02-01 to 02-06

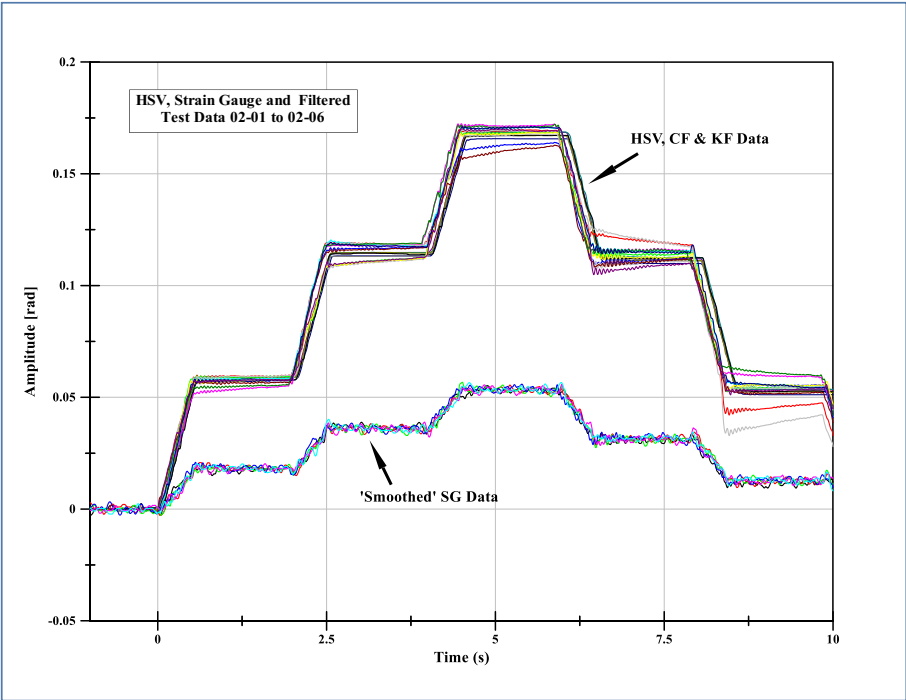


Figure 6-55 – Close-up of HSV, Strain Gauge, CF and KF data for tests 02-01 to 02-06

	02-01	02-02	02-03	02-04	02-05	02-06
CF Alpha	0.99	0.99	0.99	0.99	0.99	0.99
Q Angle	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Q Gyro Bias	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
R Measure	0.5	0.5	0.5	0.5	0.5	0.5

Table 6-34 – Filter constants for tests 02-01 to 02-06

The HSV data curves are shown in Figure 6-56 and the respective error measures using the 02-01 data curve as a comparator are summarised in Table 6-35. As in Experiment One, both a visual inspection of the data curves in Figure 6-56 and a review of the error measures, indicated that all the HSV curves were quite closely matched with most of the values less than 0.008 and only one value greater than 0.01. All the data curves had a magnitude greater than the comparator 02-01, as signified by the negative values of RM and SGM, except for data curve 02-04.

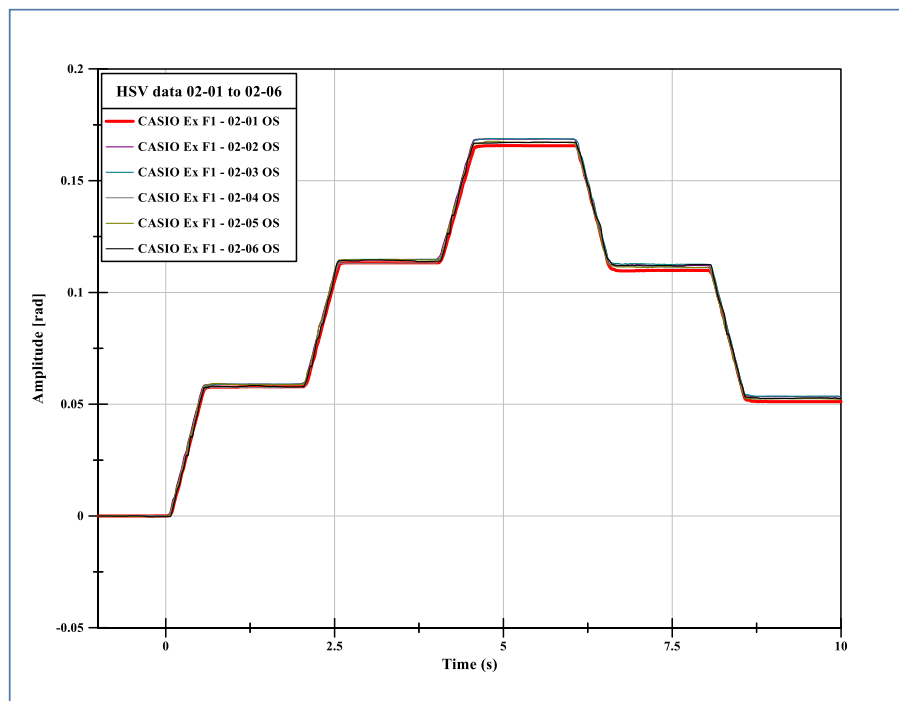


Figure 6-56 – Close-up of HSV data for tests 02-01 to 02-06

The same process was applied to the filtered data and the strain gauge data; close-ups of the CF data curves are shown in Figure 6-57, the KF data curves in Figure 6-58 and the smoothed strain gauge data curves in Figure 6-59. The HSV comparator curve 02-01 is shown as a solid black line in each of the figures and clearly showed the previously identified timing differences were not as evident as in Experiment One. The error measures for the filtered data sets are summarised in Table 6-36 and Table 6-37 respectively.

A visual inspection of the CF and KF charts suggested that they were similar which was also confirmed by the similarity in the error measures, however compared to the tests conducted in Experiment One, the data curves appeared to be more widely dispersed. As in previous tests, the KF appeared, on balance, to be slightly more

accurate as indicated by the lower statistical parameters for RC, SGC and TI in Table 6-38 highlighted with bold and underlining.

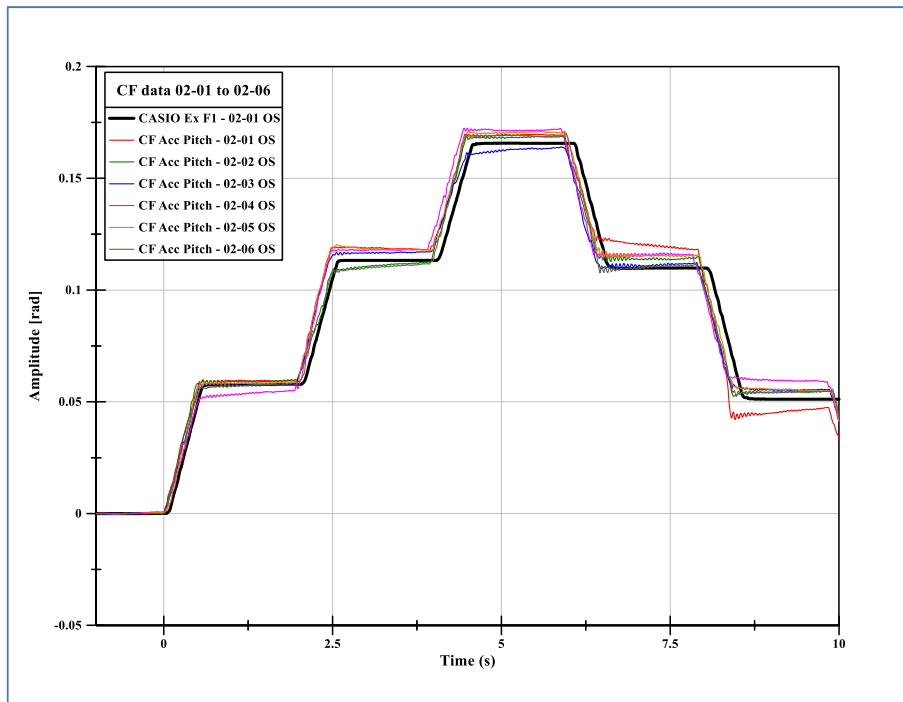


Figure 6-57 – Close-up of CF data for tests 02-01 to 02-06

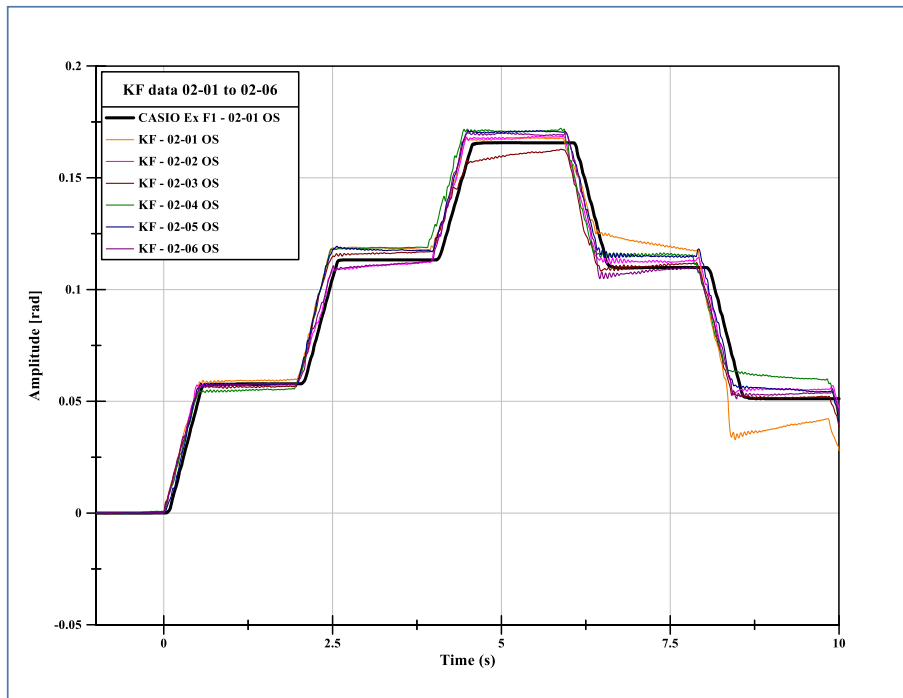


Figure 6-58 – Close-up of KF data for tests 02-01 to 02-06

HSV/ HSV	02-01/ 02-02	02-01/ 02-03	02-01/ 02-04	02-01/ 02-05	02-01/ 02-06
RM	-0.0033	-0.0056	0.0036	-0.0029	-0.0001
RP	0.0067	0.0049	0.0057	0.0048	0.0050
RC	0.0066	0.0066	0.0059	0.0050	0.0044
SGM	-0.0038	-0.0064	0.0041	-0.0034	-0.0001
SGP	0.0067	0.0049	0.0057	0.0048	0.0050
SGC	0.0077	0.0081	0.0070	0.0059	0.0050
TI	0.0107	0.0084	0.0091	0.0077	0.0079

Table 6-35 – HSV Error measures for 02-01 to 02-06

HSV/ HSV	02-01/ 02-01	02-01/ 02-02	02-01/ 02-03	02-01/ 02-04	02-01/ 02-05	02-01/ 02-06
RM	0.0020	0.0152	0.0127	0.0072	-0.0013	0.0107
RP	0.0435	0.0393	0.0373	0.0521	0.0404	0.0358
RC	0.0386	0.0374	0.0349	0.0466	0.0358	0.0331
SGM	0.0023	0.0180	0.0150	0.0084	-0.0015	0.0126
SGP	0.0435	0.0393	0.0373	0.0521	0.0404	0.0358
SGC	0.0435	0.0433	0.0402	0.0528	0.0404	0.0379
TI	0.0683	0.0624	0.0591	0.0818	0.0634	0.0565

Table 6-36 – CF Error measures for tests 02-01 to 02-06

HSV/ HSV	02-01/ 02-01	02-01/ 02-02	02-01/ 02-03	02-01/ 02-04	02-01/ 02-05	02-01/ 02-06
RM	0.0005	0.0107	0.0163	0.0004	0.0041	0.0081
RP	0.0442	0.0386	0.0383	0.0507	0.0389	0.0376
RC	0.0392	0.0355	0.0369	0.0449	0.0346	0.0341
SGM	0.0005	0.0126	0.0193	0.0004	0.0048	0.0095
SGP	0.0442	0.0386	0.0383	0.0507	0.0389	0.0376
SGC	0.0442	0.0405	0.0429	0.0507	0.0392	0.0387
TI	0.0694	0.0608	0.0609	0.0796	0.0611	0.0592

Table 6-37 – KF Error measures for tests 02-01 to 02-06

	HSV				CF				KF			
	SD	Var	Median	Average	SD	Var	Median	Average	SD	Var	Median	Average
RM	0.0031	0.000010	-0.0029	-0.0016	0.0058	0.000034	0.0090	0.0078	0.0057	0.000033	0.0061	0.0067
RP	0.0007	0.000001	0.0050	0.0054	0.0054	0.000029	0.0398	0.0414	0.0047	0.000022	0.0387	0.0414
RC	0.0009	0.000001	0.0059	0.0057	0.0043	0.000019	0.0366	0.0377	0.0037	0.000014	0.0362	0.0375
SGM	0.0036	0.000013	-0.0034	-0.0019	0.0069	0.000047	0.0105	0.0091	0.0067	0.000046	0.0071	0.0078
SGP	0.0007	0.000001	0.0050	0.0054	0.0054	0.000029	0.0398	0.0414	0.0047	0.000022	0.0387	0.0414
SGC	0.0011	0.000001	0.0070	0.0067	0.0048	0.000023	0.0418	0.0430	0.0041	0.000017	0.0417	0.0427
TI	0.0011	0.000001	0.0084	0.0088	0.0083	0.000068	0.0629	0.0652	0.0072	0.000052	0.0610	0.0652

Table 6-38 – Summary of statistical parameters for tests 02-01 to 02-06 HSV, CF and KF error measures

The strain gauge data curves shown in Figure 6-59 clearly did not align well with the HSV comparator curve; they had approximately a third of the magnitude but were reasonably closely matched in terms of their phase. This was confirmed by the large RM and SGM values and the lower RP and SGP values in Table 6-39.

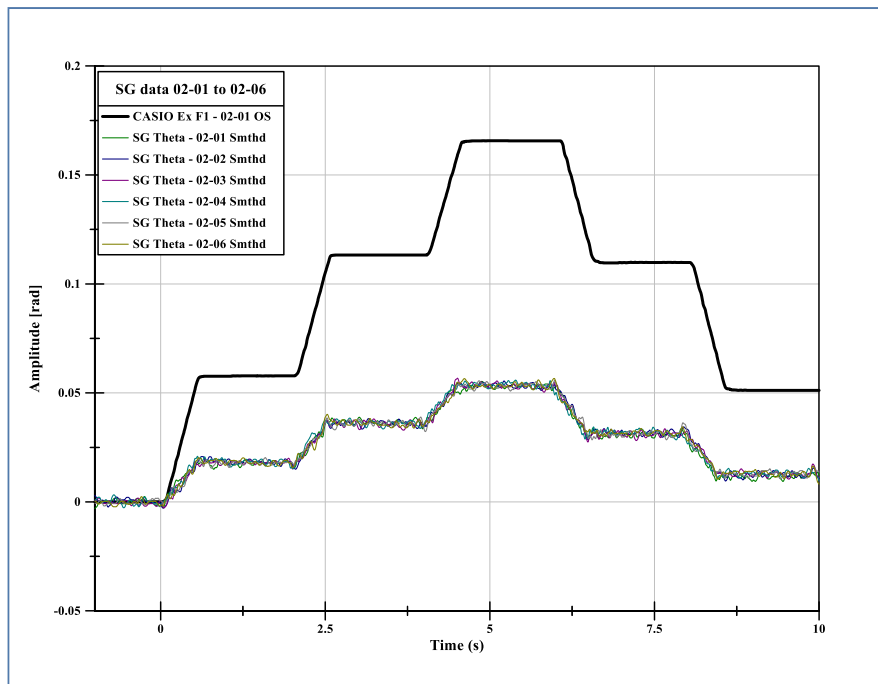


Figure 6-59 – Close-up of Strain Gauge Smoothed data for tests 02-01 to 02-06

HSV/ HSV	02-01/ 02-01	02-01/ 02-02	02-01/ 02-03	02-01/ 02-04	02-01/ 02-05	02-01/ 02-06
RM	<u>0.5768</u>	<u>0.5822</u>	<u>0.5798</u>	<u>0.5827</u>	<u>0.5824</u>	<u>0.5827</u>
RP	<u>0.0504</u>	<u>0.0438</u>	<u>0.0433</u>	<u>0.0573</u>	<u>0.0447</u>	<u>0.0437</u>
RC	0.5131	0.5174	0.5152	0.5189	0.5177	0.5179
SGM	<u>2.0968</u>	<u>2.1399</u>	<u>2.1204</u>	<u>2.1434</u>	<u>2.1413</u>	<u>2.1440</u>
SGP	<u>0.0504</u>	<u>0.0438</u>	<u>0.0433</u>	<u>0.0573</u>	<u>0.0447</u>	<u>0.0437</u>
SGC	2.0974	2.1404	2.1209	2.1442	2.1417	2.1445
TI	0.5163	0.5202	0.5179	0.5230	0.5205	0.5207

Table 6-39 – Strain Gauge Smoothed Error measures for tests 02-01 to 02-06

	SG			
	SD	Var	Median	Average
RM	0.0022	0.000005	0.5823	0.5811
RP	0.0051	0.000026	0.0443	0.0472
RC	<u>0.0019</u>	<u>0.000004</u>	0.5175	0.5167
SGM	0.0173	0.000298	2.1406	2.1310
SGP	0.0051	0.000026	0.0443	0.0472
SGC	<u>0.0173</u>	<u>0.000298</u>	2.1411	2.1315
TI	<u>0.0021</u>	<u>0.000005</u>	0.5204	0.5198

Table 6-40 – Summary of statistical parameters for tests 02-01 to 02-06 Strain Gauge error measures

Whilst the strain gauge data did not correlate well with the other data, the smoothed data curves were reasonably consistent as demonstrated by the statistical parameters in Table 6-40. The RC and TI parameters are comparable to those for the CF and KF data; the SGC value is higher which was due to the major difference in magnitude between the data set and the comparator.

The following observations were made for this series of tests:

- The previously mentioned difference in timings between the sensor and HSV data was less evident for each of the tests but would require further investigation once all tests for the Experiment had been completed.
- The CF and KF data were not as closely correlated as in the tests in Experiment One. This was possibly due to the more flexible nature of the vertical cantilever being less stiff and therefore more susceptible to vibrations.
- The rotations calculated using the strain gauge data were approximately a third of those recorded using the HSV and required monitoring to identify whether this was a consistent feature.

6.3.3.2 Experiment Two – Tests 07 to 22 – Comparison of different IMU sensitivities

As in Experiment One, a series of tests were conducted to investigate whether changing the sensitivity of the IMU sensors influenced the results. First a series of tests were conducted where the gyroscope sensitivity was fixed, and the accelerometer sensitivity was changed after each individual test. This process was repeated for each gyroscope sensitivity setting as indicated in Table 6-33.

An overview of the data for these tests is shown in Figure 6-60 and a close-up of the data in the red rectangle is shown in Figure 6-61. As in Tests 02-01 to 02-06, these figures show that the timing difference between the HSV and IMU data was no longer as evident as in Experiment One. Except for Test 02-18, each of the tests used the same filter constants as shown in Table 6-41.

	02-07	02-08	02-09	02-10	02-11	02-12	02-13	02-14
CF Alpha	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
Q Angle	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Q Gyro Bias	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
R Measure	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
	02-15	02-16	02-17	02-18	02-19	02-20	02-21	02-22
CF Alpha	0.99	0.99	0.99	0.988	0.99	0.99	0.99	0.99
Q Angle	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Q Gyro Bias	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
R Measure	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Table 6-41 – Filter constants for tests 02-07 to 02-22

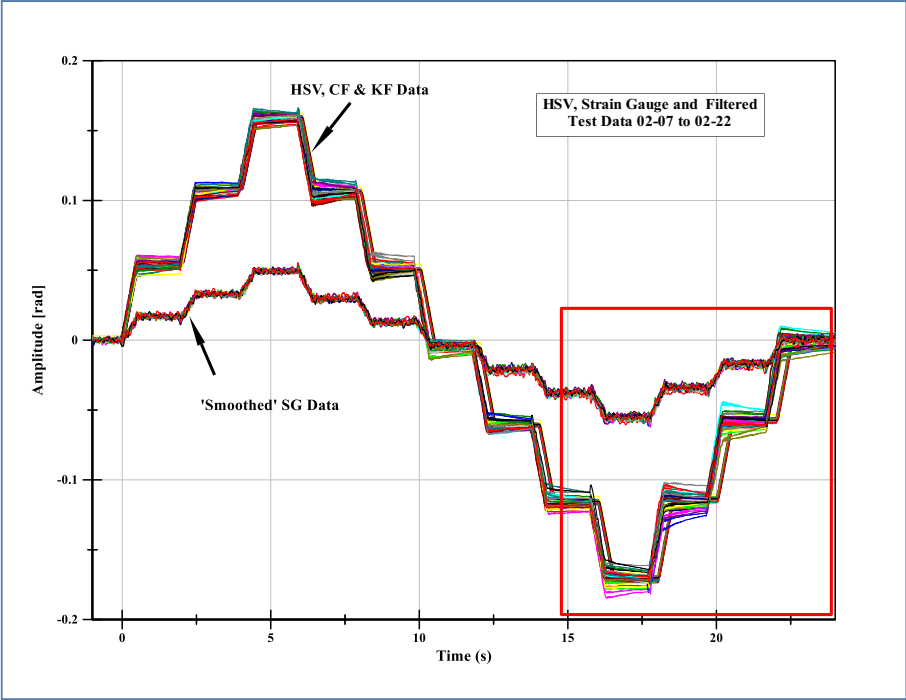


Figure 6-60 – HSV, CF and KF data for tests 02-07 to 02-22

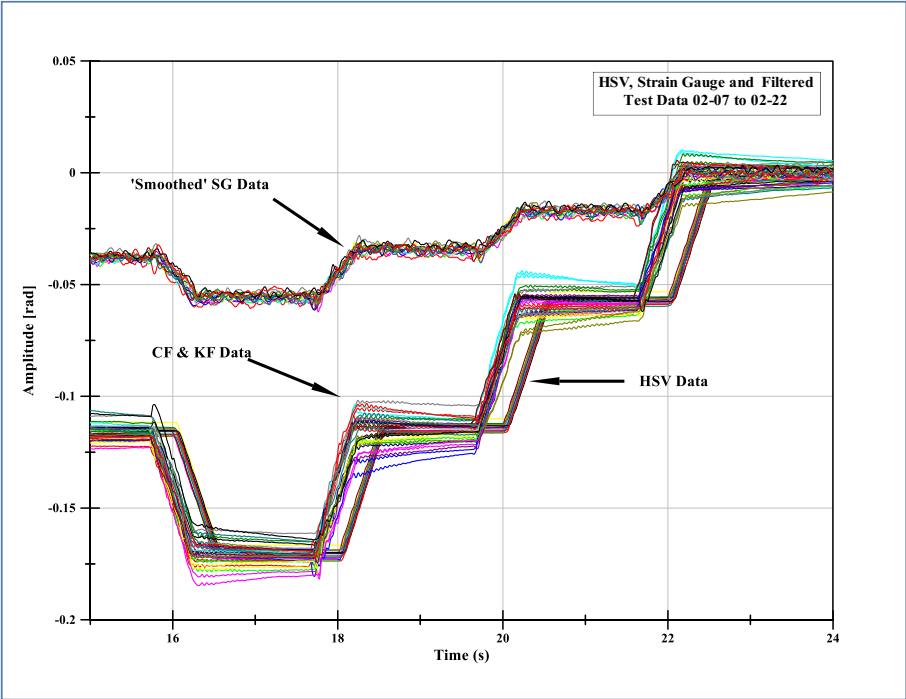


Figure 6-61 – Close-up of HSV, CF and KF data for tests 02-07 to 02-22

The data curves for the HSV are shown in Figure 6-62 and the respective error measures using the 02-14 data curve as a comparator are summarised in Table 6-42. As in previous tests both a visual inspection of the data curves in Figure 6-62, and a review of the error measures, indicated that all the HSV curves were reasonably closely matched. This was further highlighted by the statistical parameters for the RC, SGC and TI values in Table 6-43 (bold, underlined values). All the data curves had a magnitude less than the comparator 02-14, except for data curves 02-07 and 02-08.

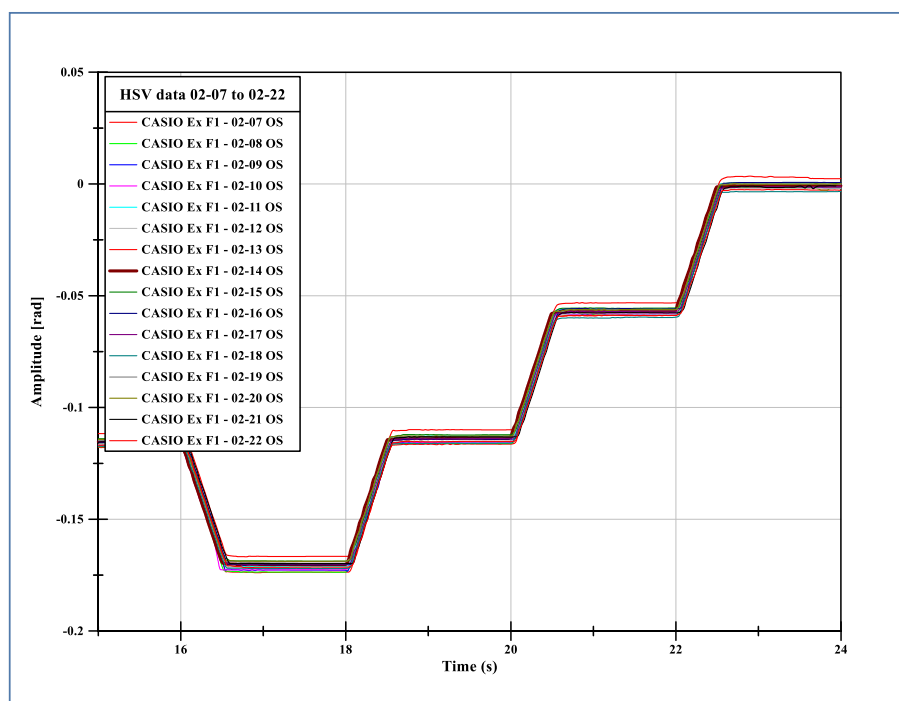


Figure 6-62 – Close-up of HSV data for tests 02-07 to 02-22

HSV/ HSV	02-14/ 02-07	02-14/ 02-08	02-14/ 02-09	02-14/ 02-10	02-14/ 02-11	02-14/ 02-12	02-14/ 02-13	02-14/ 02-15
RM	-0.0026	-0.0005	0.0001	0.0098	0.0064	0.0129	0.0120	0.0153
RP	0.0055	0.0045	0.0078	0.0065	0.0035	0.0147	0.0095	0.0067
RC	0.0054	0.0040	0.0069	0.0105	0.0064	0.0173	0.0135	0.0148
SGM	-0.0030	-0.0006	0.0001	0.0115	0.0074	0.0152	0.0141	0.0181
SGP	0.0055	0.0045	0.0078	0.0065	0.0035	0.0147	0.0095	0.0067
SGC	0.0063	0.0046	0.0078	0.0132	0.0082	0.0211	0.0170	0.0193
TI	0.0088	0.0071	0.0123	0.0117	0.0066	0.0243	0.0164	0.0139
HSV/ HSV	02-14/ 02-16	02-14/ 02-17	02-14/ 02-18	02-14/ 02-19	02-14/ 02-20	02-14/ 02-21	02-14/ 02-22	
RM	0.0127	0.0122	0.0069	0.0145	0.0146	0.0107	0.0062	
RP	0.0036	0.0032	0.0041	0.0060	0.0064	0.0053	0.0031	
RC	0.0117	0.0112	0.0071	0.0139	0.0142	0.0105	0.0061	
SGM	0.0150	0.0143	0.0080	0.0171	0.0173	0.0125	0.0072	
SGP	0.0036	0.0032	0.0041	0.0060	0.0064	0.0053	0.0031	
SGC	0.0154	0.0147	0.0090	0.0181	0.0184	0.0136	0.0078	
TI	0.0094	0.0087	0.0076	0.0127	0.0133	0.0103	0.0061	

Table 6-42 – HSV Error measures for tests 02-07 to 02-22

	HSV			
	SD	Var	Median	Average
RM	0.0056	0.000032	0.0107	0.0087
RP	0.0029	0.000008	0.0055	0.0060
RC	<u>0.0039</u>	<u>0.000015</u>	0.0105	0.0102
SGM	0.0066	0.000044	0.0125	0.0103
SGP	0.0029	0.000008	0.0055	0.0060
SGC	<u>0.0051</u>	<u>0.000026</u>	0.0136	0.0130
TI	<u>0.0045</u>	<u>0.000020</u>	0.0103	0.0113

Table 6-43 – Summary of statistical parameters for tests 02-07 to 02-22 for HSV error measures

The same process was applied to the filtered data and the strain gauge data, however for these tests only close-up images of the data sets for the different IMU sensor sensitivity settings have been included. The error measures for the CF and KF data are summarised in Table 6-44 and Table 6-45 respectively, and the cells of the CF and KF error measure tables have been highlighted with different colours to distinguish between the different IMU sensor sensitivity settings as follows:

Headings (Gyro sensitivity) (deg/sec)	250		500		1000		2000	
Columns (Acc sensitivity) (g)	2		4		8		16	

HSV/ CF	02-14/ 02-07	02-14/ 02-08	02-14/ 02-09	02-14/ 02-10	02-14/ 02-11	02-14/ 02-12	02-14/ 02-13	02-14/ 02-14
RM	0.0020	-0.0090	-0.0186	-0.0253	0.0184	0.0107	-0.0134	0.0112
RP	0.0413	0.0365	0.0333	0.0310	0.0438	0.0351	0.0347	0.0440
RC	0.0366	0.0333	0.0338	0.0354	0.0421	0.0326	0.0330	0.0403
SGM	0.0023	-0.0104	-0.0217	-0.0295	0.0219	0.0126	-0.0156	0.0132
SGP	0.0413	0.0365	0.0333	0.0310	0.0438	0.0351	0.0347	0.0440
SGC	0.0413	0.0380	0.0398	0.0428	0.0489	0.0373	0.0381	0.0460
TI	0.0648	0.0576	0.0534	0.0509	0.0696	0.0555	0.0551	0.0694
HSV/ CF	02-14/ 02-15	02-14/ 02-16	02-14/ 02-17	02-14/ 02-18	02-14/ 02-19	02-14/ 02-20	02-14/ 02-21	02-14/ 02-22
RM	0.0188	-0.0024	0.0217	0.0129	0.0234	-0.0020	0.0170	0.0226
RP	0.0356	0.0327	0.0366	0.0338	0.0391	0.0361	0.0315	0.0377
RC	0.0357	0.0291	0.0377	0.0320	0.0404	0.0320	0.0317	0.0390
SGM	0.0224	-0.0028	0.0259	0.0151	0.0280	-0.0023	0.0202	0.0270
SGP	0.0356	0.0327	0.0366	0.0338	0.0391	0.0361	0.0315	0.0377
SGC	0.0420	0.0329	0.0449	0.0370	0.0481	0.0362	0.0374	0.0464
TI	0.0569	0.0514	0.0588	0.0535	0.0629	0.0567	0.0504	0.0607

Table 6-44 – CF Error measures for tests 02-07 to 02-22

HSV/ KF	02-14/ 02-07	02-14/ 02-08	02-14/ 02-09	02-14/ 02-10	02-14/ 02-11	02-14/ 02-12	02-14/ 02-13	02-14/ 02-14
RM	0.0074	-0.0022	-0.0256	-0.0330	0.0225	0.0062	-0.0101	0.0099
RP	0.0384	0.0322	0.0323	0.0323	0.0440	0.0330	0.0329	0.0447
RC	0.0346	0.0286	0.0365	0.0409	0.0438	0.0298	0.0305	0.0406
SGM	0.0086	-0.0026	-0.0299	-0.0387	0.0269	0.0072	-0.0117	0.0116
SGP	0.0384	0.0322	0.0323	0.0323	0.0440	0.0330	0.0329	0.0447
SGC	0.0393	0.0323	0.0440	0.0504	0.0515	0.0338	0.0349	0.0462
TI	0.0604	0.0506	0.0529	0.0544	0.0703	0.0520	0.0520	0.0705
HSV/ CF	02-14/ 02-15	02-14/ 02-16	02-14/ 02-17	02-14/ 02-18	02-14/ 02-19	02-14/ 02-20	02-14/ 02-21	02-14/ 02-22
RM	0.0098	0.0207	-0.0219	-0.0130	0.0033	0.0032	0.0260	0.0184
RP	0.0373	0.0345	0.0318	0.0317	0.0328	0.0337	0.0276	0.0370
RC	0.0342	0.0357	0.0342	0.0304	0.0292	0.0300	0.0336	0.0366
SGM	0.0115	0.0247	-0.0256	-0.0151	0.0038	0.0037	0.0314	0.0218
SGP	0.0373	0.0345	0.0318	0.0317	0.0328	0.0337	0.0276	0.0370
SGC	0.0390	0.0424	0.0408	0.0351	0.0330	0.0339	0.0417	0.0429
TI	0.0588	0.0555	0.0515	0.0504	0.0516	0.0530	0.0460	0.0590

Table 6-45 – KF Error measures for tests 02-07 to 02-22

The different setting combinations have been combined into individual charts to aid visual interpretation of the results along with a set of respective statistical parameters for each of these data sets. The relevant charts and tables for the statistical parameters are summarised in Table 6-46.

Setting	Chart (Combined CF & KF)	CF Statistical Parameters	KF Statistical Parameters
Accelerometer sensitivity 2g	Figure 6-63	Table 6-47	Table 6-48
Accelerometer sensitivity 4g	Figure 6-64		
Accelerometer sensitivity 8g	Figure 6-65		
Accelerometer sensitivity 16g	Figure 6-66		
Gyroscope sensitivity 250 deg/s	Figure 6-67	Table 6-49	Table 6-50
Gyroscope sensitivity 500 deg/s	Figure 6-68		
Gyroscope sensitivity 1000 deg/s	Figure 6-69		
Gyroscope sensitivity 2000 deg/s	Figure 6-70		

Table 6-46 – Summary of chart and parameters for different IMU sensor sensitivities

A visual inspection of the charts for the data curves collated using accelerometer sensitivities (Figure 6-63 to Figure 6-66) indicated that they all appeared to have similar data dispersions. This was verified by the statistical parameters in Table 6-47 and Table 6-48, which also implied that there was no discernible trend in terms of which of the sensitivity settings was more accurate.

A similar visual inspection of the charts for the data curves collated using gyroscope sensitivities (Figure 6-67 to Figure 6-70) indicated that these data sets also appeared to have similar data dispersions. This was again verified by the statistical parameters in Table 6-49 and Table 6-50. As for the accelerometer sensitivities no one data set could be identified as being the most closely matched.

From this analysis it could be concluded that for the slow rate motion scenarios demonstrated by this series of tests, the sensitivity chosen for the both the accelerometer and gyroscope had little impact on the results.

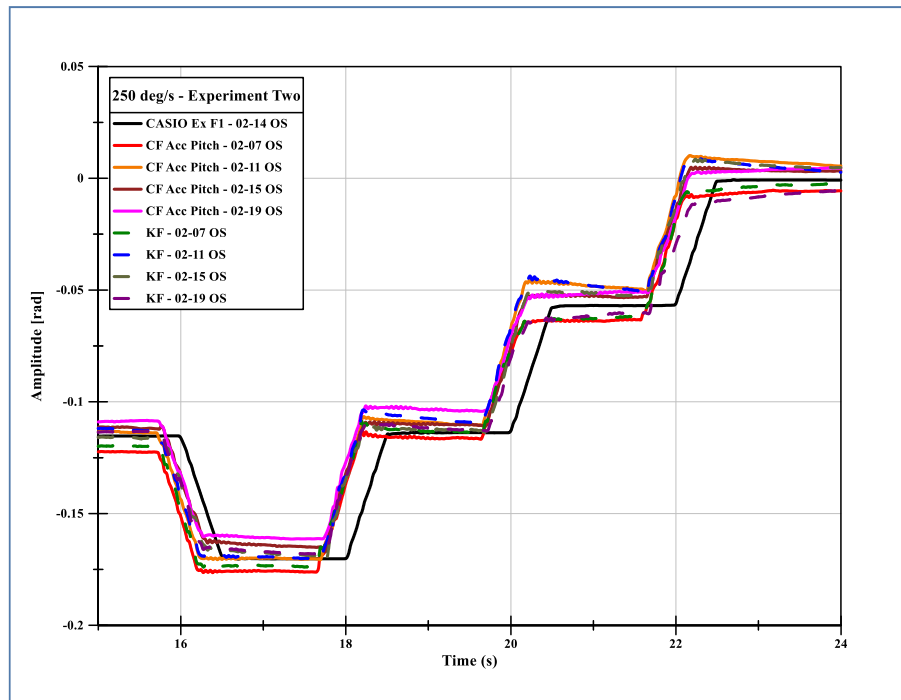


Figure 6-63 – Close-up of CF and KF data for tests 02-07 to 02-22 – Accelerometer sensitivity 2g

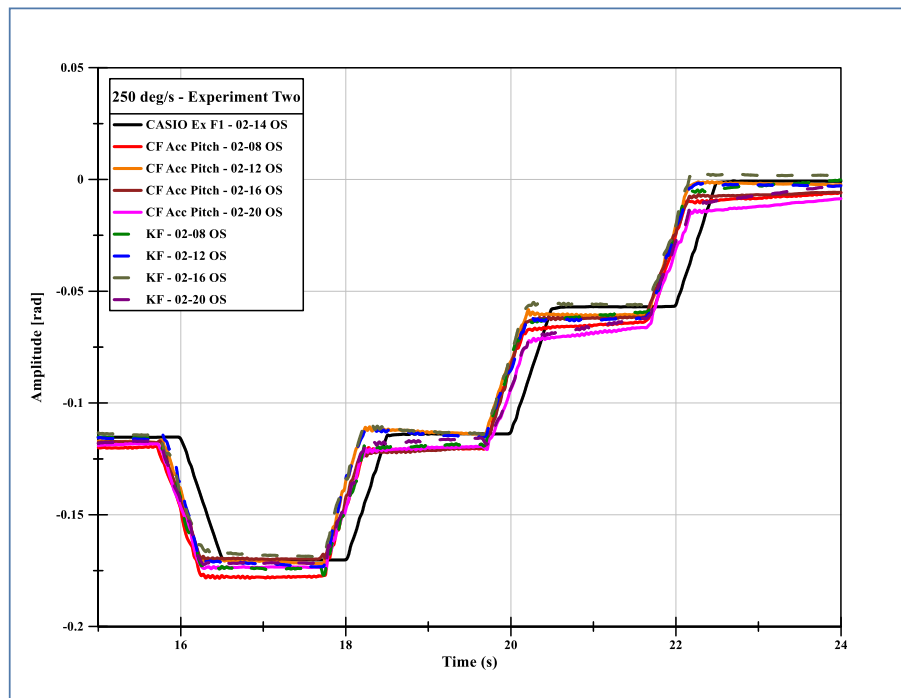


Figure 6-64 – Close-up of CF and KF data for tests 02-07 to 02-22 – Accelerometer sensitivity 4g

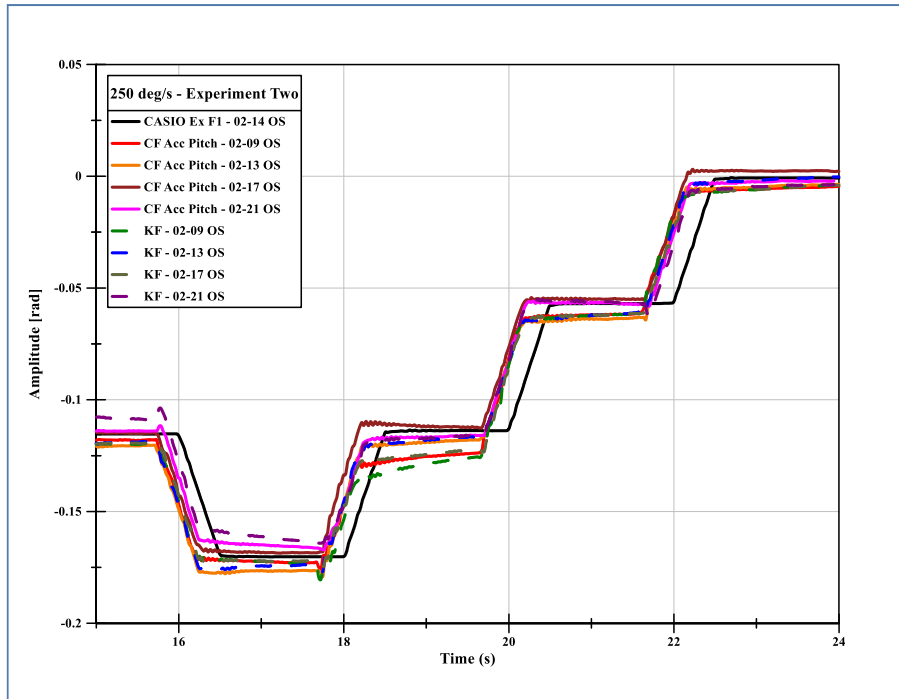


Figure 6-65 – Close-up of CF and KF data for tests 02-07 to 02-22 – Accelerometer sensitivity 8g

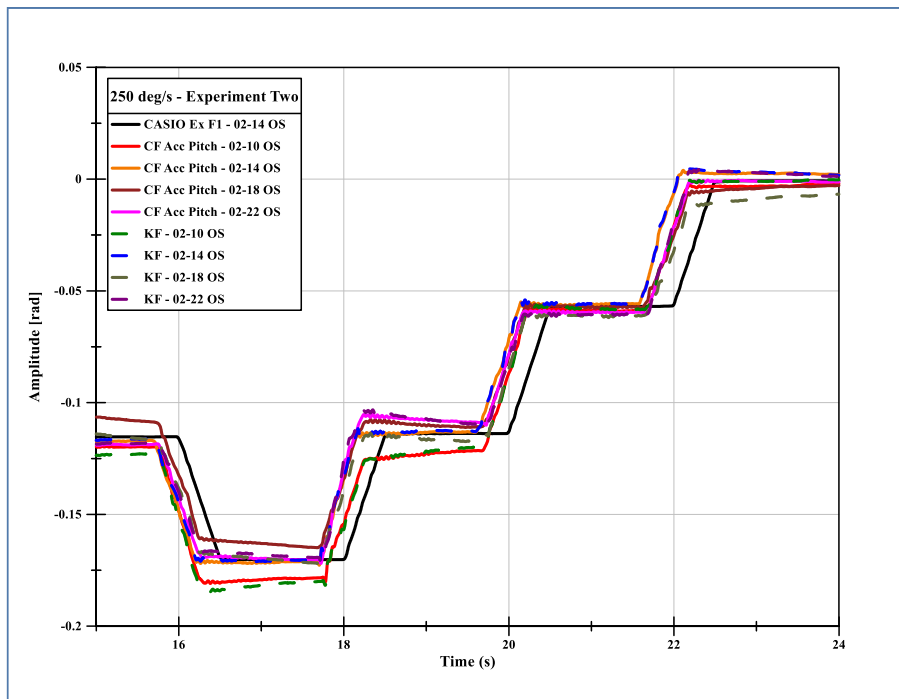


Figure 6-66 – Close-up of CF and KF data for tests 02-07 to 02-22 – Accelerometer sensitivity 16g

	2g				4g			
	Std Dev	Var	Median	Average	Std Dev	Var	Median	Average
RM	0.0081	0.000066	0.0186	0.0156	0.0071	0.000051	-0.0022	-0.0007
RP	0.0030	0.000009	0.0402	0.0399	0.0015	0.000002	0.0356	0.0351
RC	<u>0.0026</u>	<u>0.000007</u>	0.0385	0.0387	<u>0.0016</u>	<u>0.000003</u>	0.0323	0.0318
SGM	0.0097	0.000095	0.0221	0.0186	0.0083	0.000070	-0.0026	-0.0007
SGP	0.0030	0.000009	0.0402	0.0399	0.0015	0.000002	0.0356	0.0351
SGC	<u>0.0034</u>	<u>0.000012</u>	0.0451	0.0451	<u>0.0020</u>	<u>0.000004</u>	0.0367	0.0361
TI	<u>0.0045</u>	<u>0.000020</u>	0.0639	0.0636	<u>0.0024</u>	<u>0.000006</u>	0.0561	0.0553
	8g				16g			
	Std Dev	Var	Median	Average	Std Dev	Var	Median	Average
RM	0.0179	0.000319	0.0018	0.0017	0.0182	0.000331	0.0120	0.0053
RP	0.0019	0.000003	0.0340	0.0340	0.0049	0.000024	0.0358	0.0366
RC	<u>0.0022</u>	<u>0.000005</u>	0.0334	0.0341	<u>0.0032</u>	<u>0.000010</u>	0.0372	0.0367
SGM	0.0211	0.000443	0.0023	0.0022	0.0214	0.000459	0.0142	0.0065
SGP	0.0019	0.000003	0.0340	0.0340	0.0049	0.000024	0.0358	0.0366
SGC	<u>0.0029</u>	<u>0.000009</u>	0.0389	0.0400	<u>0.0038</u>	<u>0.000014</u>	0.0444	0.0430
TI	<u>0.0030</u>	<u>0.000009</u>	0.0543	0.0545	<u>0.0072</u>	<u>0.000052</u>	0.0571	0.0587

Table 6-47 – Summary of statistical parameters for CF data for tests 02-07 to 02-22 Accelerometers error measures

	2g				4g			
	Std Dev	Var	Median	Average	Std Dev	Var	Median	Average
RM	0.0072	0.000051	0.0086	0.0107	0.0085	0.000072	0.0047	0.0069
RP	0.0040	0.000016	0.0378	0.0381	0.0008	0.000001	0.0334	0.0334
RC	<u>0.0052</u>	<u>0.000028</u>	0.0344	0.0354	<u>0.0027</u>	<u>0.000007</u>	0.0299	0.0310
SGM	0.0087	0.000075	0.0101	0.0127	0.0101	0.000102	0.0054	0.0082
SGP	0.0040	0.000016	0.0378	0.0381	0.0008	0.000001	0.0334	0.0334
SGC	<u>0.0067</u>	<u>0.000045</u>	0.0392	0.0407	<u>0.0040</u>	<u>0.000016</u>	0.0339	0.0356
TI	<u>0.0067</u>	<u>0.000044</u>	0.0596	0.0603	<u>0.0018</u>	<u>0.000003</u>	0.0525	0.0528
	8g				16g			
	Std Dev	Var	Median	Average	Std Dev	Var	Median	Average
RM	0.0204	0.000417	-0.0160	-0.0079	0.0201	0.000403	-0.0015	-0.0044
RP	0.0021	0.000004	0.0320	0.0311	0.0052	0.000027	0.0346	0.0364
RC	<u>0.0021</u>	<u>0.000005</u>	0.0339	0.0337	<u>0.0042</u>	<u>0.000018</u>	0.0386	0.0371
SGM	0.0242	0.000587	-0.0186	-0.0090	0.0236	0.000557	-0.0017	-0.0051
SGP	0.0021	0.000004	0.0320	0.0311	0.0052	0.000027	0.0346	0.0364
SGC	<u>0.0033</u>	<u>0.000011</u>	0.0413	0.0404	<u>0.0056</u>	<u>0.000031</u>	0.0446	0.0437
TI	<u>0.0027</u>	<u>0.000007</u>	0.0518	0.0506	<u>0.0075</u>	<u>0.000056</u>	0.0567	0.0586

Table 6-48 – Summary of statistical parameters for KF data for tests 02-07 to 02-22 Accelerometers error measures

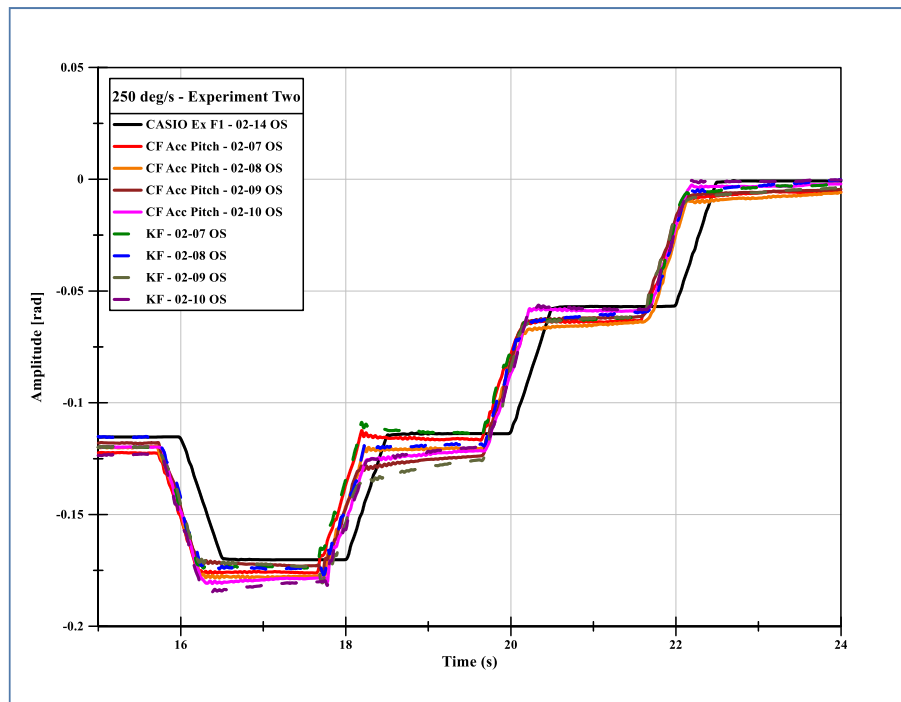


Figure 6-67 – Close-up of CF and KF data for tests 02-07 to 02-22 – Gyroscope sensitivity 250 deg/s

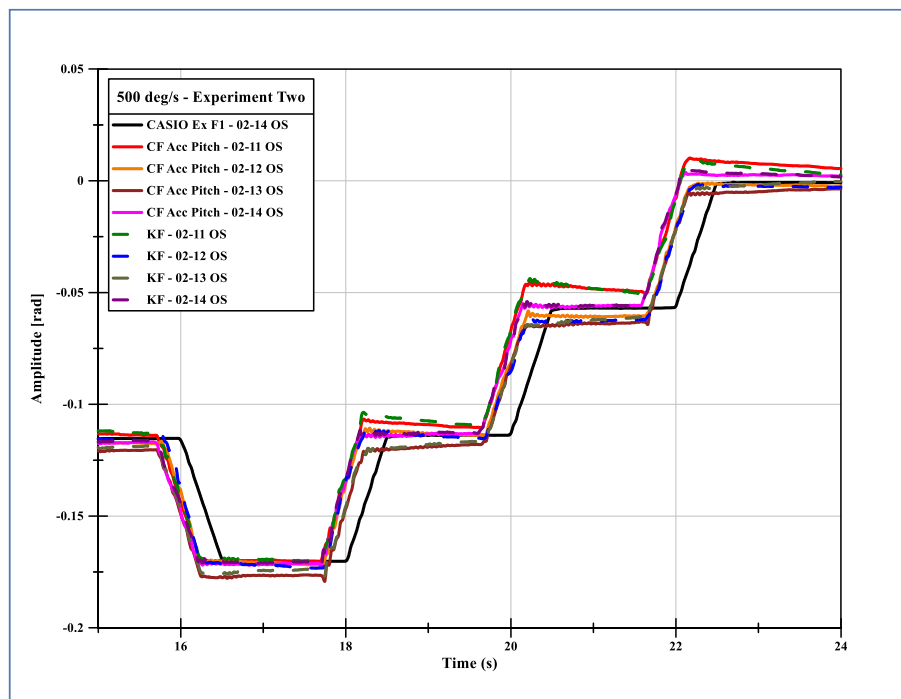


Figure 6-68 – Close-up of CF and KF data for tests 02-07 to 02-22 – Gyroscope sensitivity 500 deg/s

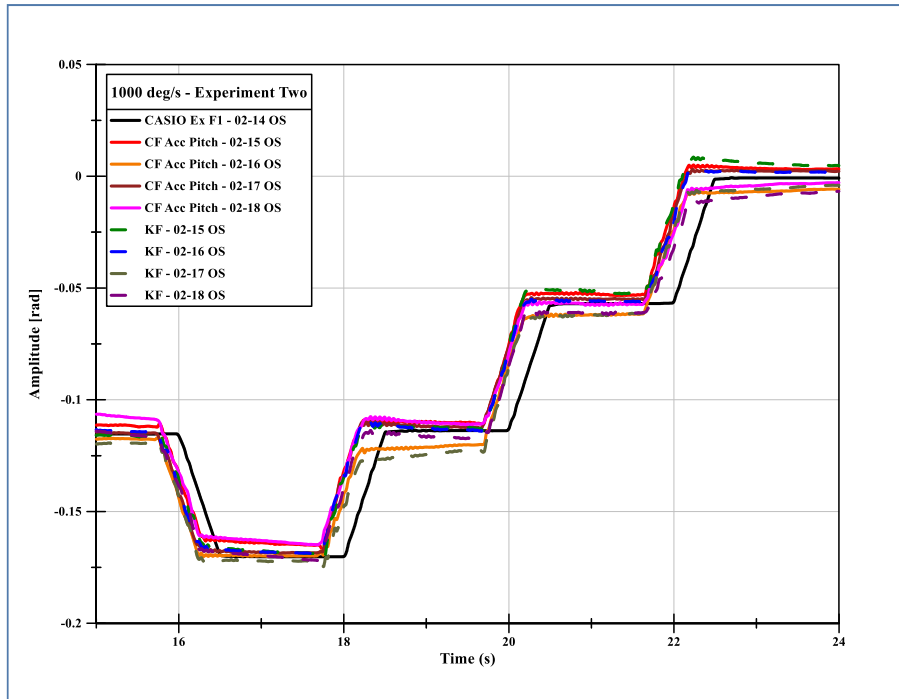


Figure 6-69 – Close-up of CF and KF data for tests 02-07 to 02-22 – Gyroscope sensitivity 1000 deg/s

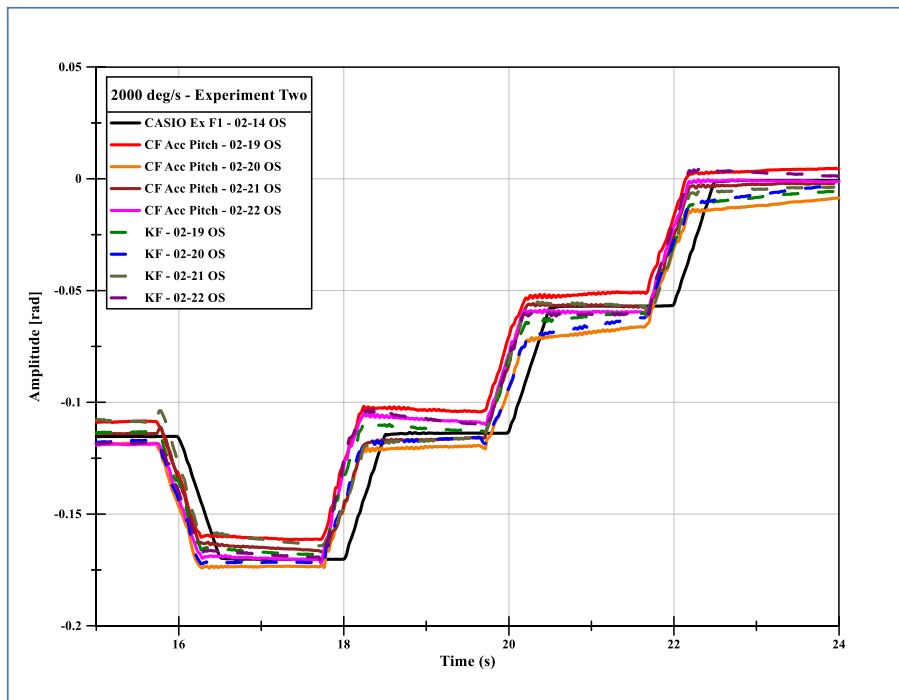


Figure 6-70 – Close-up of CF and KF data for tests 02-07 to 02-22 – Gyroscope sensitivity 2000 deg/s

	250 deg/s				500 deg/s			
	Std Dev	Var	Median	Average	Std Dev	Var	Median	Average
RM	0.0103	0.000106	-0.0138	-0.0127	0.0120	0.000145	0.0110	0.0067
RP	0.0039	0.000015	0.0349	0.0355	0.0045	0.000020	0.0395	0.0394
RC	<u>0.0013</u>	<u>0.000002</u>	0.0346	0.0348	<u>0.0043</u>	<u>0.000018</u>	0.0366	0.0370
SGM	0.0120	0.000144	-0.0161	-0.0148	0.0141	0.000199	0.0129	0.0080
SGP	0.0039	0.000015	0.0349	0.0355	0.0045	0.000020	0.0395	0.0394
SGC	<u>0.0018</u>	<u>0.000003</u>	0.0405	0.0405	<u>0.0050</u>	<u>0.000025</u>	0.0420	0.0426
TI	<u>0.0053</u>	<u>0.000028</u>	0.0555	0.0567	<u>0.0071</u>	<u>0.000050</u>	0.0625	0.0624
	1000 degs/s				2000 deg/s			
	Std Dev	Var	Median	Average	Std Dev	Var	Median	Average
RM	0.0093	0.000087	0.0158	0.0127	0.0103	0.000105	0.0198	0.0152
RP	0.0015	0.000002	0.0347	0.0347	0.0029	0.000008	0.0369	0.0361
RC	<u>0.0033</u>	<u>0.000011</u>	0.0338	0.0336	<u>0.0039</u>	<u>0.000015</u>	0.0355	0.0358
SGM	0.0111	0.000123	0.0188	0.0152	0.0122	0.000150	0.0236	0.0182
SGP	0.0015	0.000002	0.0347	0.0347	0.0029	0.000008	0.0369	0.0361
SGC	<u>0.0046</u>	<u>0.000021</u>	0.0395	0.0392	<u>0.0053</u>	<u>0.000028</u>	0.0419	0.0420
TI	<u>0.0029</u>	<u>0.000008</u>	0.0552	0.0552	<u>0.0047</u>	<u>0.000022</u>	0.0587	0.0577

Table 6-49 – Summary of statistical parameters for CF data for tests 02-07 to 02-22 Gyroscope error measures

	250 deg/s				500 deg/s			
	Std Dev	Var	Median	Average	Std Dev	Var	Median	Average
RM	0.0165	0.000273	-0.0139	-0.0134	0.0116	0.000135	0.0080	0.0071
RP	0.0026	0.000007	0.0323	0.0338	0.0057	0.000032	0.0385	0.0387
RC	<u>0.0044</u>	<u>0.000019</u>	0.0356	0.0352	<u>0.0061</u>	<u>0.000038</u>	0.0356	0.0362
SGM	0.0193	0.000374	-0.0163	-0.0156	0.0138	0.000190	0.0094	0.0085
SGP	0.0026	0.000007	0.0323	0.0338	0.0057	0.000032	0.0385	0.0387
SGC	<u>0.0066</u>	<u>0.000043</u>	0.0417	0.0415	<u>0.0075</u>	<u>0.000056</u>	0.0406	0.0416
TI	<u>0.0036</u>	<u>0.000013</u>	0.0536	0.0546	<u>0.0092</u>	<u>0.000084</u>	0.0611	0.0612
	1000 degs/s				2000 deg/s			
	Std Dev	Var	Median	Average	Std Dev	Var	Median	Average
RM	0.0171	0.000292	-0.0016	-0.0011	0.0099	0.000097	0.0108	0.0127
RP	0.0023	0.000005	0.0331	0.0338	0.0034	0.000011	0.0333	0.0328
RC	<u>0.0020</u>	<u>0.000004</u>	0.0342	0.0336	<u>0.0029</u>	<u>0.000009</u>	0.0318	0.0324
SGM	0.0201	0.000404	-0.0018	-0.0011	0.0119	0.000142	0.0128	0.0151
SGP	0.0023	0.000005	0.0331	0.0338	0.0034	0.000011	0.0333	0.0328
SGC	<u>0.0027</u>	<u>0.000007</u>	0.0399	0.0393	<u>0.0045</u>	<u>0.000020</u>	0.0378	0.0379
TI	<u>0.0034</u>	<u>0.000011</u>	0.0535	0.0541	<u>0.0046</u>	<u>0.000022</u>	0.0523	0.0524

Table 6-50 – Summary of statistical parameters for KF data for tests 02-07 to 02-22 Gyroscope error measures

As in Tests 02-01 to 02-06, the strain gauge data curves shown in Figure 6-71 clearly did not align well with the HSV comparator curve; they were again approximately a third of the magnitude and it was evident that there was a slight time difference. This was confirmed by the large RM and SGM values and the lower RP and SGP values in Table 6-51.

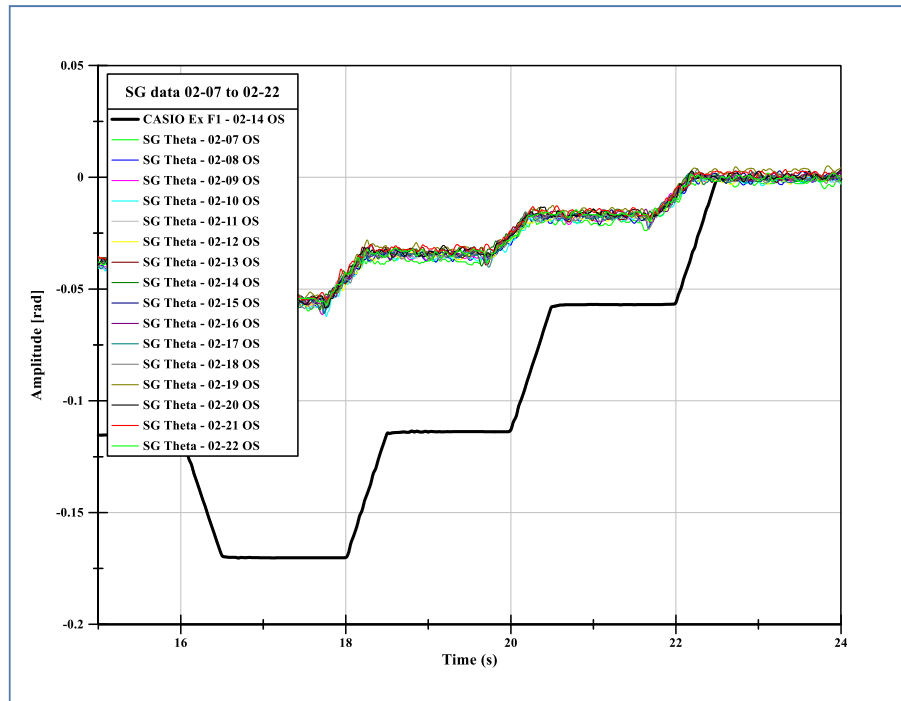


Figure 6-71 – Close-up of Strain Gauge Smoothed data for tests 02-07 to 02-22

HSV/ SG	02-14/ 02-07	02-14/ 02-08	02-14/ 02-09	02-14/ 02-10	02-14/ 02-11	02-14/ 02-12	02-14/ 02-13	02-14/ 02-14
RM	<u>0.5700</u>	<u>0.5846</u>	<u>0.5859</u>	<u>0.5870</u>	<u>0.5927</u>	<u>0.5915</u>	<u>0.5925</u>	<u>0.5928</u>
RP	<u>0.0462</u>	<u>0.0436</u>	<u>0.0477</u>	<u>0.0456</u>	<u>0.0492</u>	<u>0.0448</u>	<u>0.0461</u>	<u>0.0505</u>
RC	0.5068	0.5195	0.5210	0.5218	0.5271	0.5257	0.5267	0.5273
SGM	<u>2.0439</u>	<u>2.1589</u>	<u>2.1697</u>	<u>2.1787</u>	<u>2.2252</u>	<u>2.2150</u>	<u>2.2230</u>	<u>2.2257</u>
SGP	<u>0.0462</u>	<u>0.0436</u>	<u>0.0477</u>	<u>0.0456</u>	<u>0.0492</u>	<u>0.0448</u>	<u>0.0461</u>	<u>0.0505</u>
SGC	2.0444	2.1593	2.1703	2.1792	2.2257	2.2155	2.2234	2.2263
TI	0.5093	0.5224	0.5243	0.5249	0.5307	0.5289	0.5300	0.5310
HSV/ SG	02-14/ 02-15	02-14/ 02-16	02-14/ 02-17	02-14/ 02-18	02-14/ 02-19	02-14/ 02-20	02-14/ 02-21	02-14/ 02-22
RM	<u>0.5940</u>	<u>0.5942</u>	<u>0.5917</u>	<u>0.5892</u>	<u>0.5979</u>	<u>0.6002</u>	<u>0.6035</u>	<u>0.6005</u>
RP	<u>0.0436</u>	<u>0.0467</u>	<u>0.0479</u>	<u>0.0474</u>	<u>0.0441</u>	<u>0.0459</u>	<u>0.0425</u>	<u>0.0434</u>
RC	0.5279	0.5283	0.5261	0.5238	0.5313	0.5335	0.5361	0.5336
SGM	<u>2.2359</u>	<u>2.2375</u>	<u>2.2163</u>	<u>2.1960</u>	<u>2.2682</u>	<u>2.2872</u>	<u>2.3147</u>	<u>2.2897</u>
SGP	<u>0.0436</u>	<u>0.0467</u>	<u>0.0479</u>	<u>0.0474</u>	<u>0.0441</u>	<u>0.0459</u>	<u>0.0425</u>	<u>0.0434</u>
SGC	2.2363	2.2380	2.2169	2.1965	2.2686	2.2877	2.3151	2.2901
TI	0.5310	0.5317	0.5295	0.5272	0.5346	0.5370	0.5394	0.5369

Table 6-51 – Strain Gauge Smoothed Error measures for tests 02-07 to 02-22

Whilst the strain gauge data did not correlate well with the other data, the smoothed data curves were reasonably consistent as demonstrated by the statistical parameters in Table 6-52. The RC and TI parameters are comparable to those for the CF and KF data; the SGC value is higher which was again due to the major difference in magnitude between the data set and the comparator.

	SG			
	SD	Var	Median	Average
RM	0.0076	0.000058	0.5926	0.5918
RP	0.0022	0.000005	0.0460	0.0459
RC	<u>0.0067</u>	<u>0.000045</u>	0.5269	0.5260
SGM	0.0617	0.003801	2.2241	2.2178
SGP	0.0022	0.000005	0.0460	0.0459
SGC	<u>0.0616</u>	<u>0.003798</u>	2.2246	2.2183
TI	<u>0.0069</u>	<u>0.000047</u>	0.5303	0.5293

Table 6-52 – Summary of statistical parameters for tests 02-07 to 02-22 for Strain Gauge error measures

It was concluded that similar observations could be made for this series of tests as for Tests 02-01 to 02-06.

6.3.3.3 Experiment Two - Tests 23 to 31 – Comparison of different sampling rates

The final series of tests for Experiment Two were conducted to investigate the influence of changing the sampling rate on the results using the same process applied to conduct a similar investigation during Experiment One. An overview of the data for these tests is shown in Figure 6-72 and a close-up of the data in the red rectangle is shown in Figure 6-73. The same filter constants were used for each test as summarised in Table 6-53.

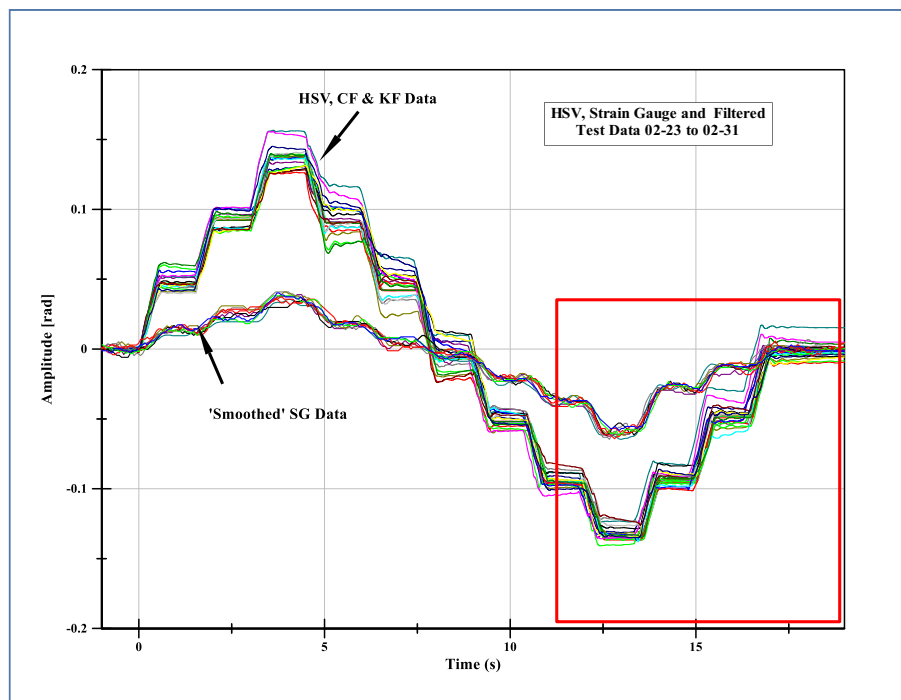


Figure 6-72 – HSV, CF and KF data for tests 02-23 to 02-31

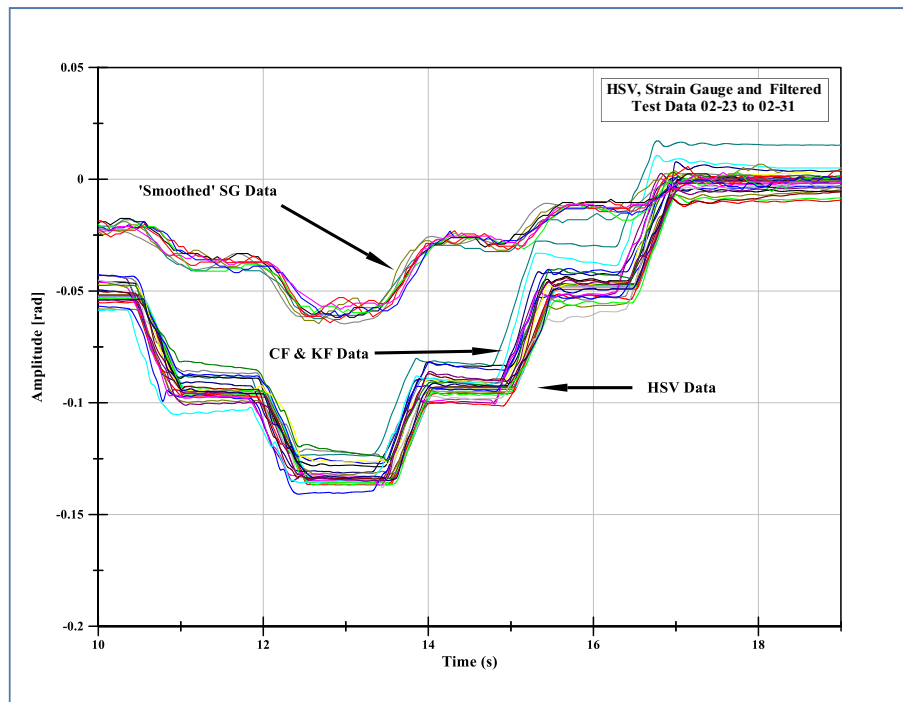


Figure 6-73 – Close-up of HSV, CF and KF data for tests 02-23 to 02-31

	02-23	02-24	02-25	02-26	02-27	02-28	02-29	02-30	02-31
CF Alpha	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
Q Angle	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Q Gyro Bias	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
R Measure	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Table 6-53 – Filter constants for tests 02-23 to 02-31

The sampling rate parameters chosen for these tests, which controlled the LabVIEW data logging VI timed loop, are summarised in Table 6-54.

	02-23	02-24	02-25	02-26	02-27	02-28	02-29	02-30	02-31
Clock speed (MHz)	1	1	1	1	1	1	1	1	1
Timing loop period (μ S)	2000	2000	2000	1000	1000	1000	500	500	500
Gyro sensitivity (deg/s)	250	250	250	250	250	250	250	250	250
Acc sensitivity (g)	2	2	2	2	2	2	2	2	2

Table 6-54 – Sample rate parameters for tests 02-23 to 02-31

The HSV data is shown in Figure 6-74 with the respective error measures summarised in Table 6-55. For these tests the HSV data curve for Test 02-27 was used as the comparator. A review of the error measures showed that all of the data curves, except for Tests 02-29, 02-30 and 02-31 had magnitudes that were greater than the comparator.

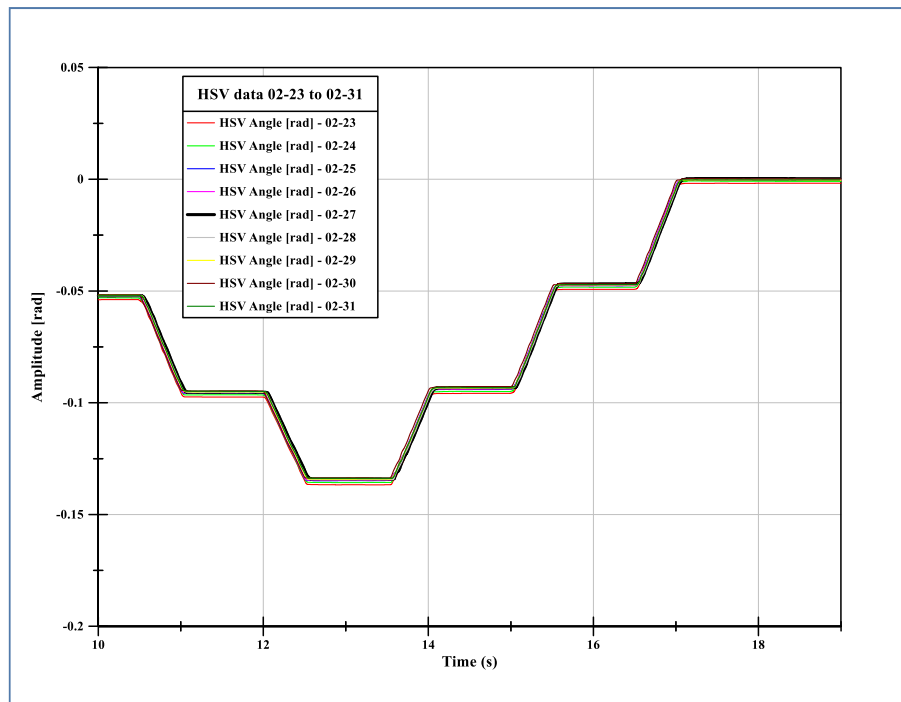


Figure 6-74 – Close-up of HSV data for tests 02-23 to 02-31

HSV/ HSV	02-27/ 02-23	02-27/ 02-24	02-27/ 02-25	02-27/ 02-26	02-27/ 02-28	02-27/ 02-29	02-27/ 02-30	02-27/ 02-31
RM	-0.0121	-0.0094	-0.0041	-0.0040	-0.0013	0.0004	0.0036	0.0031
RP	0.0076	0.0067	0.0074	0.0069	0.0022	0.0047	0.0097	0.0042
RC	0.0126	0.0103	0.0075	0.0071	0.0022	0.0041	0.0092	0.0046
SGM	-0.0140	-0.0109	-0.0047	-0.0046	-0.0015	0.0004	0.0042	0.0036
SGP	0.0076	0.0067	0.0074	0.0069	0.0022	0.0047	0.0097	0.0042
SGC	0.0159	0.0128	0.0088	0.0083	0.0026	0.0047	0.0106	0.0055
TI	0.0138	0.0119	0.0119	0.0111	0.0035	0.0073	0.0153	0.0069

Table 6-55 – HSV Error measures for tests 02-23 to 02-31

The same process was applied to the filtered data; the error measures for the CF and KF are summarised in Table 6-56 and Table 6-57 respectively, while the statistical parameters for all the data sets are summarised in Table 6-58. As in the earlier Experiment Two tests, the CF and KF data curves were more widely dispersed which meant that the statistical parameters in Table 6-58 did not provide any clear distinction. The RC and SGC values suggested that the CF data was more closely aligned (indicated by the lower SD and Var values, underlined and in bold), whilst the higher TI values suggested that the KF values were more closely aligned.

To aid interpretation of the results, the data was collated in terms of the selected timing loop period. The tests for the 2000 μs loop period (02-23 to 02-25) are shown in Figure 6-75, the 1000 μs loop period (02-26 to 02-28) in Figure 6-76 and the 500 μs loop period (02-29 to 02-31) in Figure 6-77.

HSV/ CF	02-27/ 02-23	02-27/ 02-24	02-27/ 02-25	02-27/ 02-26	02-27/ 02-27	02-27/ 02-28	02-27/ 02-29	02-27/ 02-30	02-27/ 02-31
RM	0.0210	-0.0013	-0.0334	0.0483	0.0056	0.0353	0.0119	-0.0091	0.0007
RP	0.0358	0.0316	0.0679	0.0269	0.0361	0.0317	0.0369	0.0292	0.0390
RC	<u>0.0368</u>	<u>0.0281</u>	<u>0.0670</u>	<u>0.0490</u>	<u>0.0324</u>	<u>0.0420</u>	<u>0.0344</u>	<u>0.0271</u>	<u>0.0346</u>
SGM	0.0251	-0.0015	-0.0391	0.0605	0.0065	0.0432	0.0140	-0.0105	0.0008
SGP	0.0358	0.0316	0.0679	0.0269	0.0361	0.0317	0.0369	0.0292	0.0390
SGC	<u>0.0437</u>	<u>0.0317</u>	<u>0.0784</u>	<u>0.0662</u>	<u>0.0367</u>	<u>0.0536</u>	<u>0.0395</u>	<u>0.0310</u>	<u>0.0390</u>
TI	<u>0.0575</u>	<u>0.0497</u>	<u>0.1083</u>	<u>0.0514</u>	<u>0.0568</u>	<u>0.0540</u>	<u>0.0583</u>	<u>0.0462</u>	<u>0.0612</u>

Table 6-56 – CF Error measures for tests 02-23 to 02-31

HSV/ KF	02-27/ 02-23	02-27/ 02-24	02-27/ 02-25	02-27/ 02-26	02-27/ 02-27	02-27/ 02-28	02-27/ 02-29	02-27/ 02-30	02-27/ 02-31
RM	-0.0073	-0.0248	-0.0582	0.0078	0.0265	0.0106	0.0547	-0.0017	-0.0345
RP	0.0347	0.0308	0.0497	0.0244	0.0198	0.0312	0.0300	0.0300	0.0297
RC	<u>0.0314</u>	<u>0.0351</u>	<u>0.0678</u>	<u>0.0227</u>	<u>0.0293</u>	<u>0.0292</u>	<u>0.0553</u>	<u>0.0267</u>	<u>0.0403</u>
SGM	-0.0085	-0.0290	-0.0691	0.0091	0.0319	0.0125	0.0694	-0.0020	-0.0405
SGP	0.0347	0.0308	0.0497	0.0244	0.0198	0.0312	0.0300	0.0300	0.0297
SGC	<u>0.0357</u>	<u>0.0423</u>	<u>0.0851</u>	<u>0.0260</u>	<u>0.0375</u>	<u>0.0336</u>	<u>0.0756</u>	<u>0.0301</u>	<u>0.0502</u>
TI	<u>0.0546</u>	<u>0.0506</u>	<u>0.0858</u>	<u>0.0386</u>	<u>0.0348</u>	<u>0.0494</u>	<u>0.0578</u>	<u>0.0472</u>	<u>0.0510</u>

Table 6-57 – KF Error measures for tests 02-23 to 02-31

	HSV				CF				KF			
	SD	Var	Median	Average	SD	Var	Median	Average	SD	Var	Median	Average
RM	0.0053	0.000028	-0.0027	-0.0030	0.0228	0.000522	0.0056	0.0088	0.0317	0.001004	-0.0017	-0.0030
RP	0.0022	0.000005	0.0068	0.0062	0.0115	0.000131	0.0358	0.0372	0.0077	0.000060	0.0300	0.0311
RC	<u>0.0032</u>	<u>0.000010</u>	0.0073	0.0072	<u>0.0118</u>	<u>0.000138</u>	0.0346	0.0390	<u>0.0140</u>	<u>0.000195</u>	0.0314	0.0375
SGM	0.0061	0.000037	-0.0031	-0.0034	0.0278	0.000772	0.0065	0.0110	0.0385	0.001480	-0.0020	-0.0029
SGP	0.0022	0.000005	0.0068	0.0062	0.0115	0.000131	0.0358	0.0372	0.0077	0.000060	0.0300	0.0311
SGC	<u>0.0041</u>	<u>0.000017</u>	0.0086	0.0087	<u>0.0154</u>	<u>0.000236</u>	0.0395	0.0466	<u>0.0195</u>	<u>0.000380</u>	0.0375	0.0462
TI	<u>0.0037</u>	<u>0.000014</u>	0.0115	0.0102	<u>0.0175</u>	<u>0.000306</u>	0.0568	0.0604	<u>0.0137</u>	<u>0.000188</u>	0.0506	0.0522

Table 6-58 – Summary of statistical parameters for tests 02-23 to 02-31 Error measures

Although not as pronounced as in Experiment One, the correlation between the timing loop period and how the filtered data sets matched the HSV comparator appeared to hold, the lower the timing loop period the better the correlation and conversely a higher timing loop period the worse the correlation. This was borne out by a visual inspection of Figure 6-75, Figure 6-76 and Figure 6-77, but was less conclusive when the values for the comprehensive error measures RC and SGC and the inequality measure TI (see bold, underlined numbers), in Table 6-56 and Table 6-57, were analysed.

As the strain gauge data did not closely correlate to the HSV data, it was decided not to conduct a detailed analysis as it would not add anything that had not previously been identified.

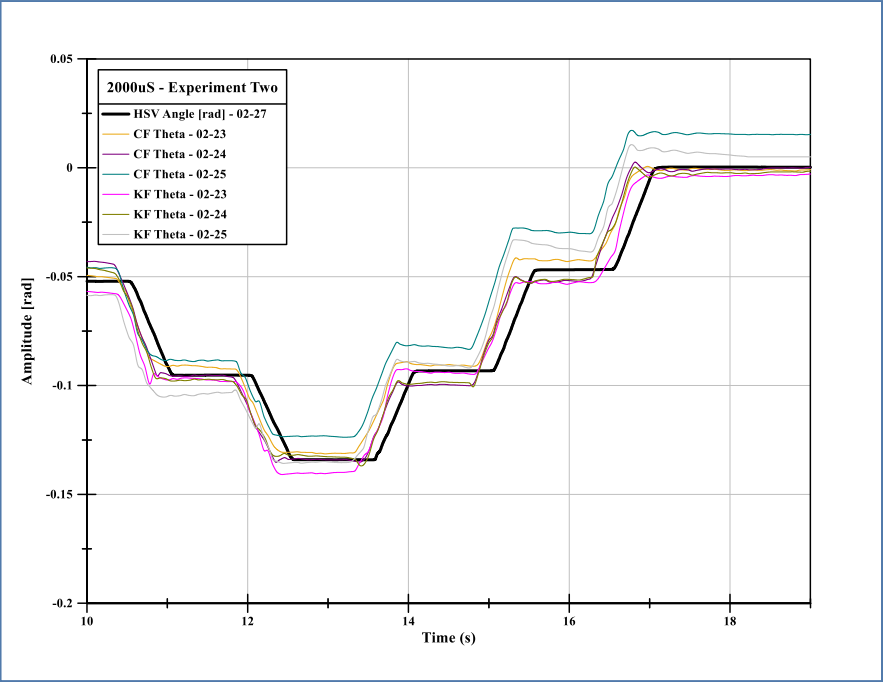


Figure 6-75 – Close-up of CF and KF data for 2000 μ S tests 02-23 to 02-25

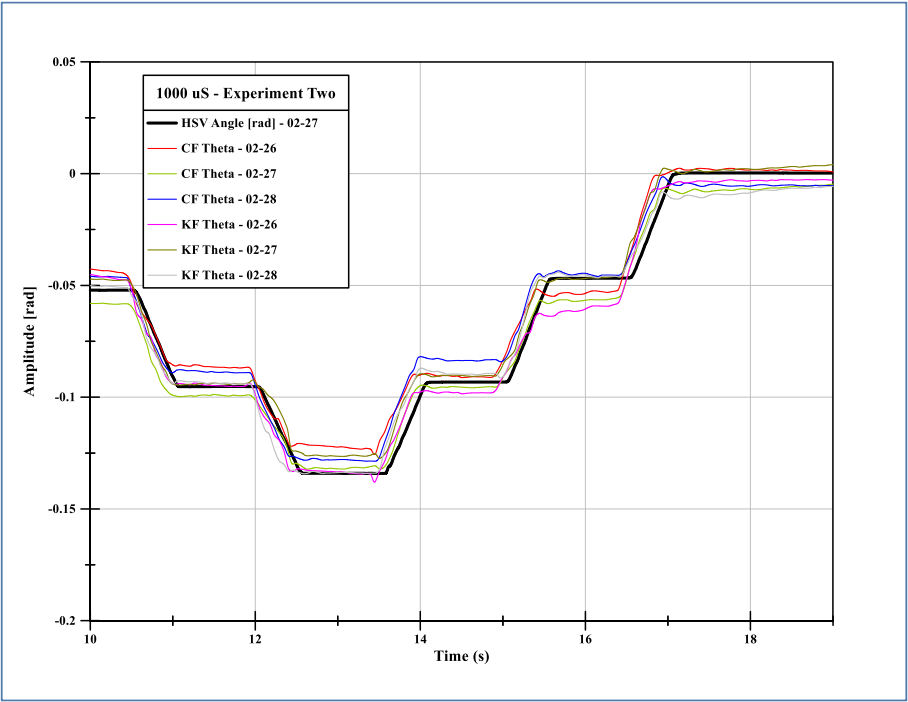


Figure 6-76 – Close-up of CF and KF data for 1000 μ S tests 02-26 to 02-28

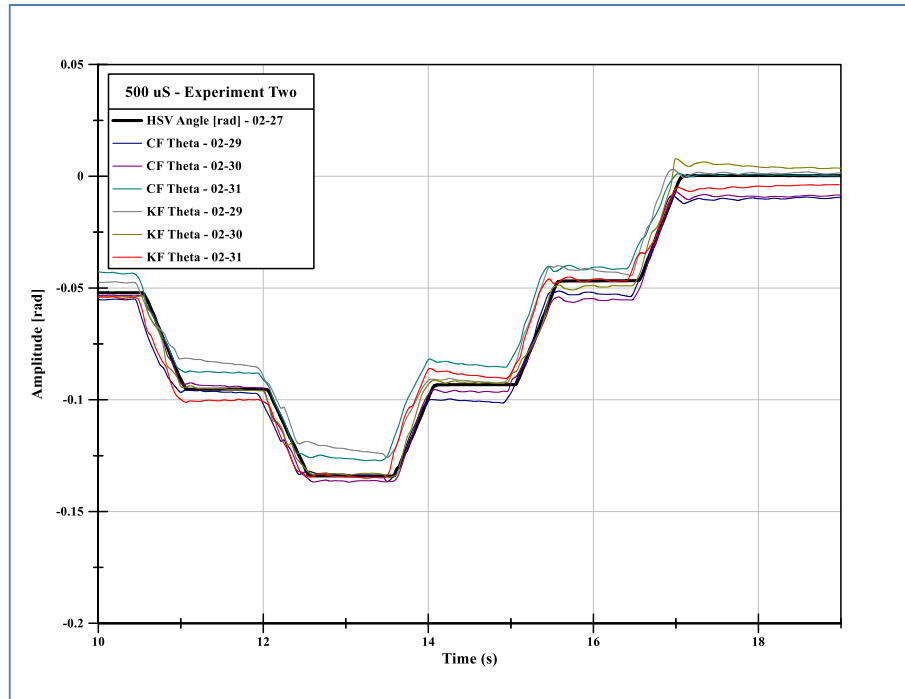


Figure 6-77 – Close-up of CF and KF data for 500 μ S tests 02-29 to 02-31

6.3.4 Analysis and discussion of results

It was assumed that the large difference between the rotation calculated using the strain gauge data and the HSV reference data (as seen in Figure 6-78) was because the calculation of the former was based upon small deflection theory and the rotations observed during Experiment Two were assumed to be large deflections by comparison.

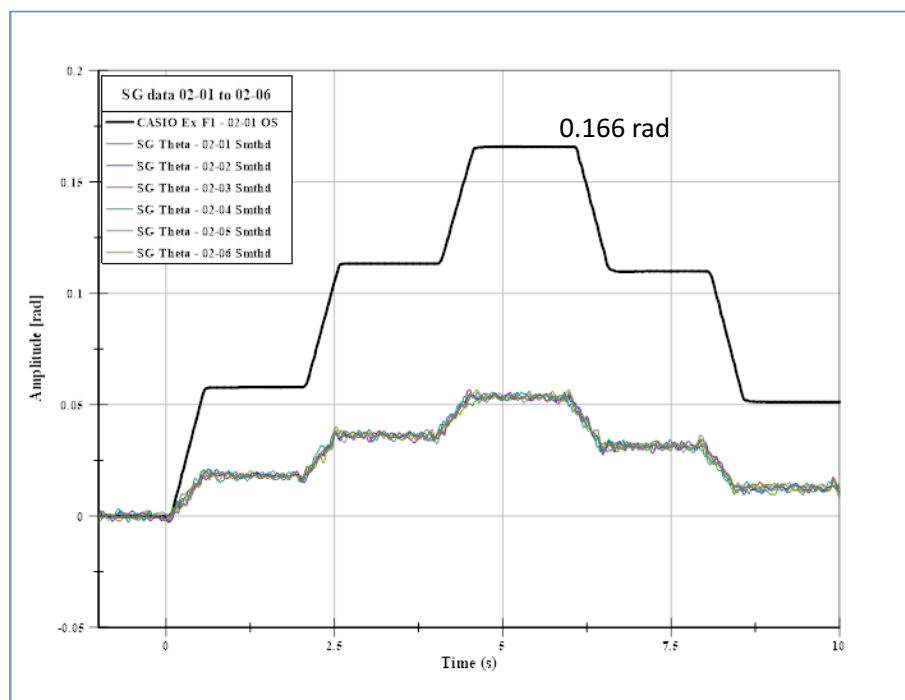


Figure 6-78 – Close-up of Strain Gauge Smoothed data for tests 02-01 to 02-06

Figure 6-78 shows the peak rotation recorded by the HSV as 0.166 radians whilst that calculated using the strain gauge data as being approximately 0.05 radians. A review of the image of the maximum rotation of the cantilever for test 02-01 in Figure 6-79 showed that in the region of the strain gauges at the base of the cantilever the angle of rotation was 0.074 radians compared to 0.166 radians at the tip of the cantilever. This image also further illustrated the difference between the calculated (0.05 radians) and observed rotations (0.074 radians) for the strain gauge data. Whilst this indicated that it might be possible to use strain gauges to calculate member rotation, further investigation would be required to identify the upper bound for which the small rotation theory applied, and also whether it could be used for dynamic load scenarios.

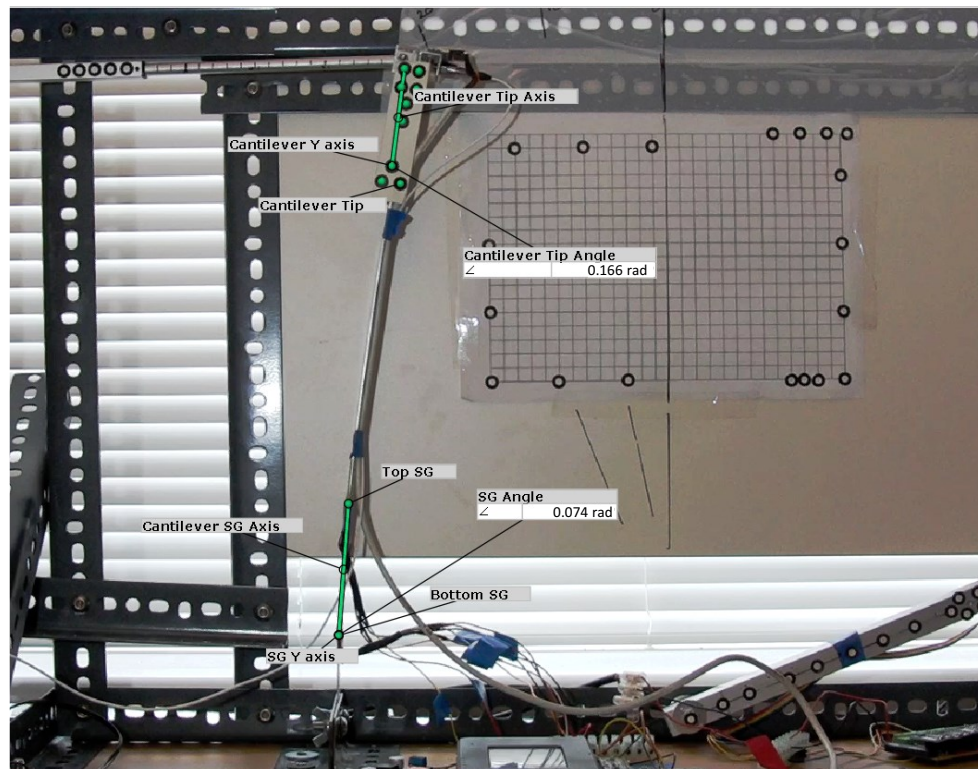


Figure 6-79 – Rotation angles for cantilever deflection

6.4 Chapter conclusions

Experiments One and Two showed, that when coupled with simple data fusion algorithms, IMU sensors could be used to measure the response of simple structural elements subject to static loads with a reasonable degree of accuracy. Although the technique demonstrated that it could generally provide a good representation of the response in terms of amplitude, correlation of the time channel results remained an issue although some of the experimental results indicated that this might be addressed by using a higher sampling rate. The use of error measures to quantify the accuracy of the results enabled an objective analysis to be conducted. However, published acceptance criteria were overly generous and at odds with obvious anomalies in some of the results and therefore require further investigation. Experiment Two demonstrated that, when used on appropriate structural elements, it was possible to use a combination of strain gauges to measure the rotation of the element. However, as the deflections/rotations observed were relatively large, and the strain gauge results had been based upon small

deflection/rotation theory, further investigation was required to confirm whether this was a viable process for use in dynamic load scenarios and to identify the maximum rotation for which this analysis could be applied.

Overall, it was decided that the experiments covered in this Chapter had successfully met their aims, and that the issues of data timing, error measure acceptance criteria and using strain gauges to measure rotations, should be investigated further as part of the dynamic load tests covered in Chapter 7.

Chapter 7 Dynamic Analysis

7.1 Introduction

The primary purpose of the experiments described in this chapter was to provide a means of producing a dynamic structural response, that could be measured by the chosen sensors to confirm their operation, and to verify the selected data fusion algorithms. Experiment Three employed the vertical cantilever used in Experiment Two but subjected to sudden load removal. Experiment Four also employed the vertical cantilever used in Experiment Three but subjected to ballistic load, while Experiment Five was a vertical monolith panel subjected to ballistic load. A general arrangement of each of the experiments is shown in Figure 7-1.

The purpose of each of the experiments was to investigate how the sensors performed under a short duration dynamic load in line with the main objective of the research. A detailed discussion of each of the experiments is contained in subsequent sections that cover the following areas:

- Experimental arrangement
- Experimental procedure
- Presentation of results
- Analysis and discussion of results

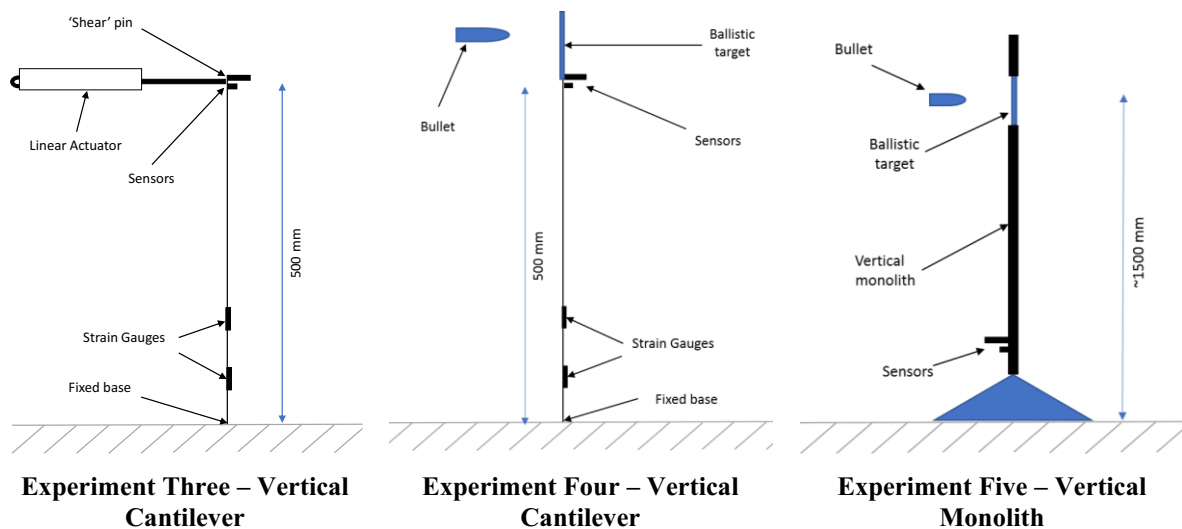


Figure 7-1 – Visual comparison of Experiments Three, Four and Five

7.2 Experiment Three – Vertical cantilever subjected to sudden load removal

7.2.1 Experimental arrangement

This experiment made use of the same vertical cantilever as used in Experiment Two with identical sensor and strain gauge configuration. The same linear actuator was used, however in this experiment it was used to deflect the cantilever tip to one side, before a ‘shear pin’ failed allowing the cantilever to freely oscillate. The main components and dimensions are shown in Figure 1-1, while Figure 7-3 is a photograph of the general experimental arrangement.

The ‘shear pin’ was made from a 15 mm length of solder wire encased in a piece of electric wire plastic sheath, that was further encased in a small piece of rubber heat-shrink tube. The latter was used to ensure that the ‘shear pin’ assembly was just wide enough so that it was securely held in position in the end of the actuator (as shown in Figure 7-4), whilst still enabling it to be removed to replace the solder wire when this broke.

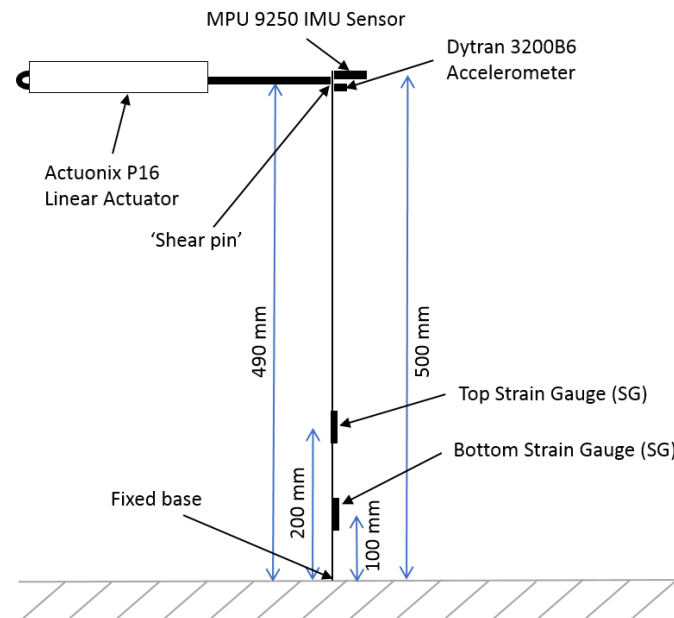


Figure 7-2 – Experiment Three main components and key dimensions

7.2.2 Experimental procedure

As in the previous experiments, the first action was to start the main LabVIEW sensor logging VI, and once the initial configuration and calibration was complete, the HSV camera and the actuator control VI were started. The latter caused the linear actuator to pull the tip of the cantilever to one side to such a point that the ‘shear pin’ failed. At this point the cantilever could freely oscillate, until damping effects caused it to stop. Once the cantilever oscillations were no longer visible, the HSV camera and main LabVIEW sensor logging VI were stopped, and the actuator was repositioned with the ‘shear pin’ readjusted ready for the next test. An image showing a typical response recorded by the HSV is shown in Figure 7-5, and illustrates the decaying periodic motion well.

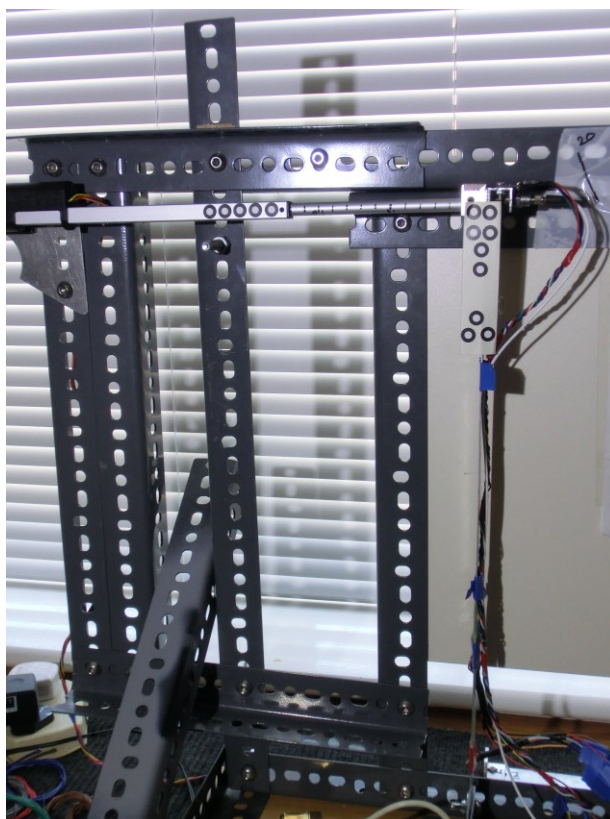


Figure 7-3 – Photograph showing the general arrangement for Experiment Three

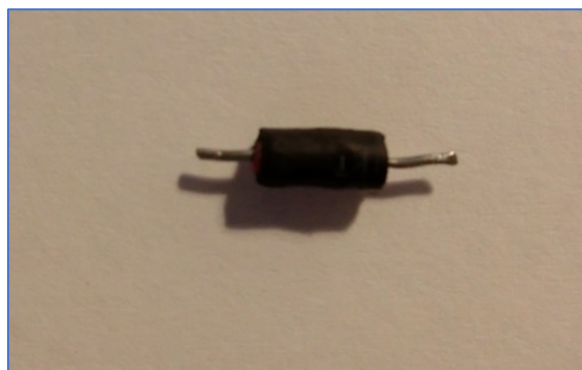


Figure 7-4 – Photograph showing the 'shear pin' in the end of the actuator (left) and a close-up image of the 'shear pin' (right)

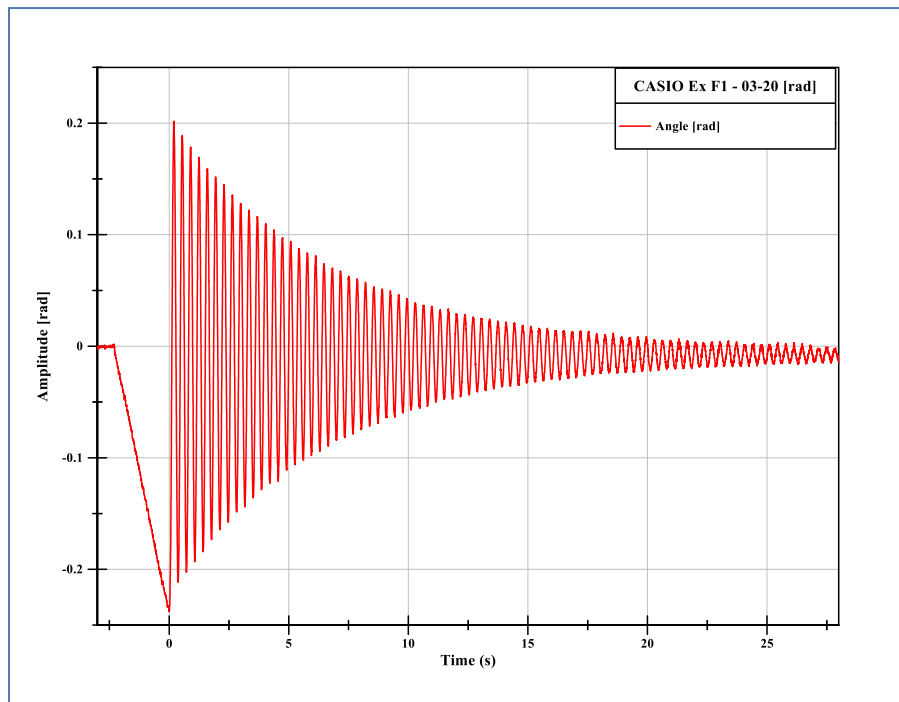


Figure 7-5 – Typical HSV image showing deflection of vertical cantilever subject to sudden load removal

7.2.3 Presentation of results

The tests carried out as part of Experiment Three are summarised in Table 6-1. The table details the different combinations of the IMU gyroscope and accelerometer sensitivities investigated during the experiment, to ascertain the impact that the combinations might have on the accuracy of the final results, and therefore try to determine the optimum combination.

Test No	Gyro Setting (deg/s)	Acc Setting (g)	Test No	Gyro Setting (deg/s)	Acc Setting (g)	Test No	Gyro Setting (deg/s)	Acc Setting (g)
03-01	500	2	03-11	250	16	03-21	1000	16
03-02	500	2	03-12	500	4	03-22	1000	16
03-03	500	2	03-13	500	8	03-23	1000	16
03-04	500	2	03-14	500	16	03-24	1000	16
03-05	500	2	03-15	1000	4	03-25	1000	16
03-06	500	2	03-16	1000	8	03-26	1000	16
03-07	250	4	03-17	1000	16	03-27	1000	16
03-08	250	4	03-18	2000	4	03-28	1000	16
03-09	250	4	03-19	2000	8	03-29	1000	16
03-10	250	8	03-20	2000	16			

Table 7-1 – Summary of Gyroscope and Accelerometer sensitivity settings for Experiment Three

7.2.3.1 Experiment Three – Tests 01 to 08 – Investigation of data repeatability

Similar to the previous two experiments, the first series of tests was used to ascertain the consistency of the results between individual tests. However, unlike the previous experiments, the chosen method of releasing the actuator

load was not as consistent as originally anticipated. Consequently, the load was released at different times resulting in varying peak load, but similar oscillation frequencies, as can be seen in Figure 7-6 and Figure 7-7 which show the results from the HSV for these tests. Figure 7-6 provides a general overview while Figure 7-7 is a close-up of the initial period after load release, where the different amplitudes but same oscillation frequencies, are clearly evident.

The different loading scenarios meant that it was not possible to follow the same analysis procedure as used in Experiments One and Two. Instead, each of the tests had to be analysed separately using the HSV data curve for each test as the comparator. To avoid reproducing large quantities of data in the body of the thesis, charts showing an overall view of the results for each test, along with a corresponding close-up have been included in Annex D, and only where appropriate reproduced in the main text to clarify any issues. Close-up images showing the initial period after release for the CF, KF and strain gauge data for each of the tests have been included in Figure 7-8, Figure 7-9 and Figure 7-10 respectively, to illustrate the consistency of these results in terms of the similar oscillation frequencies.

The same filter parameters were used for each of the tests as summarised in Table 6-5, while the error measures calculated for each test are shown in Table 7-3.

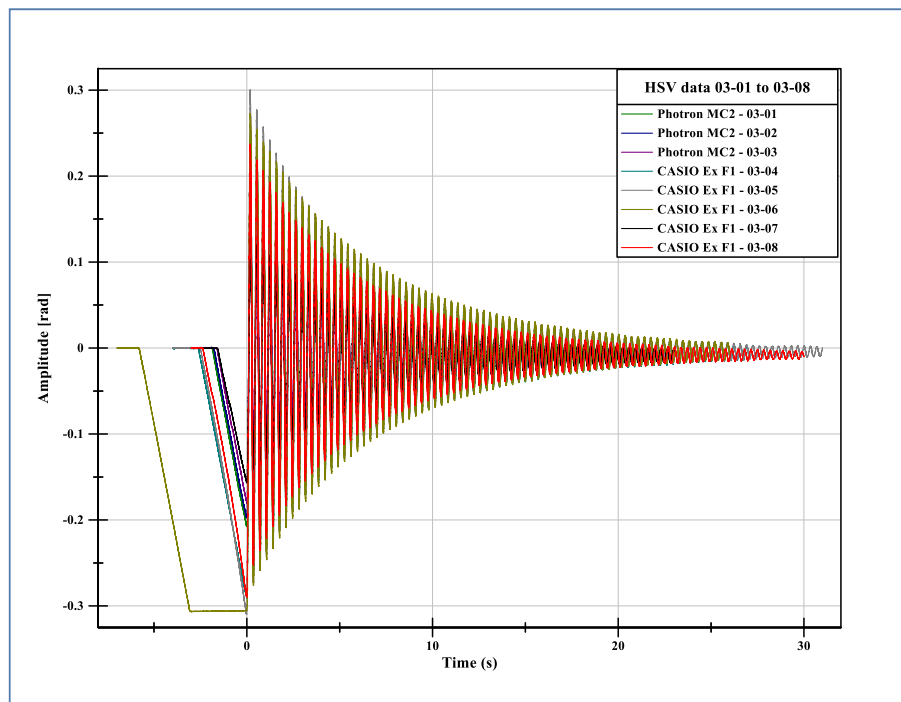


Figure 7-6 – HSV data for tests 03-01 to 03-08

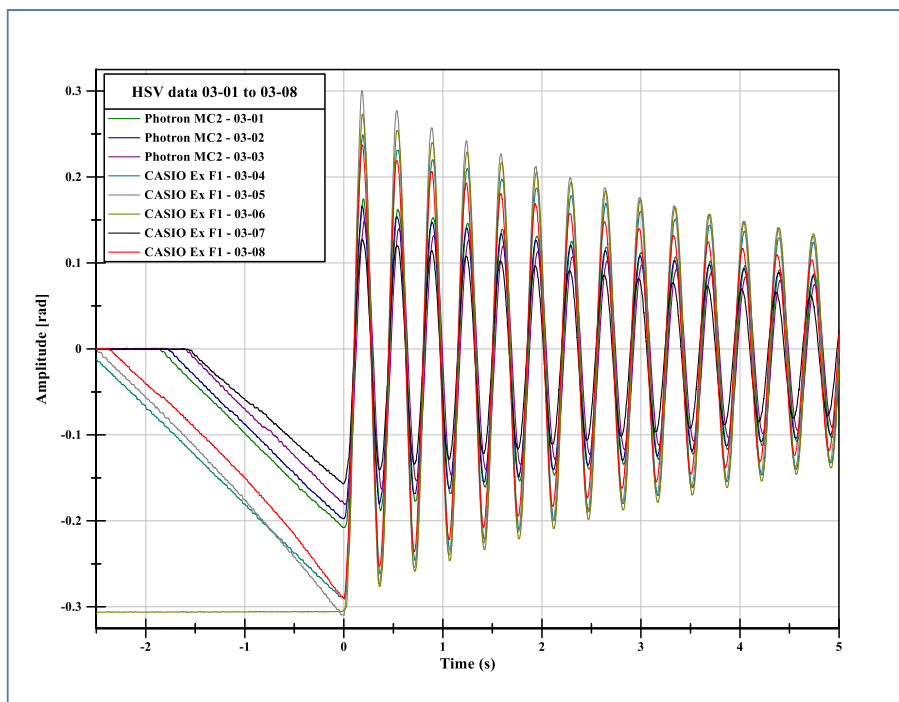


Figure 7-7 – Close-up of HSV data for tests 03-01 to 03-08

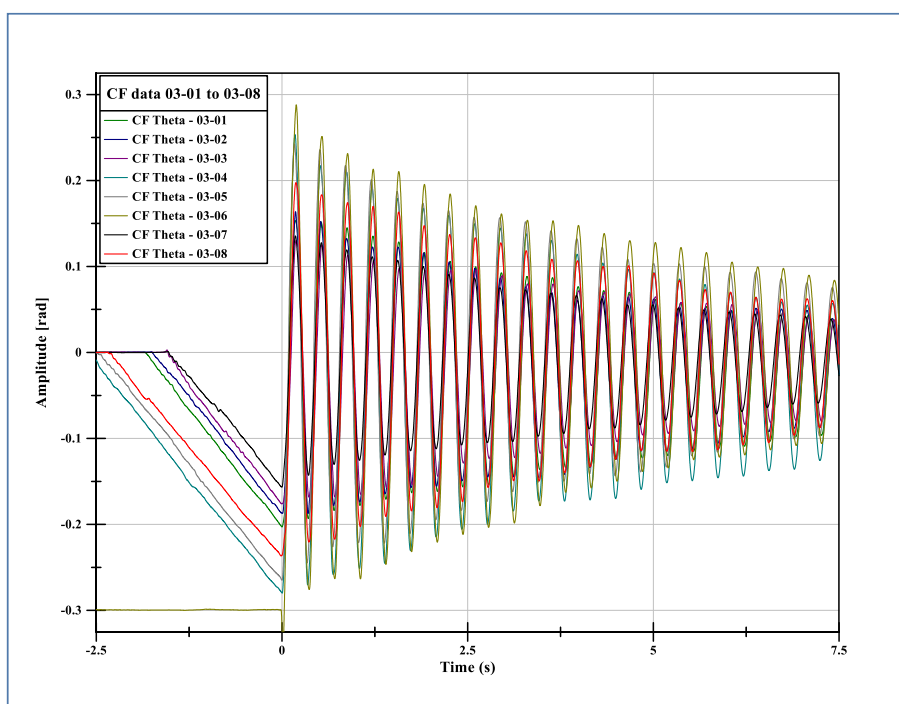


Figure 7-8 – Close-up of CF data for tests 03-01 to 03-08

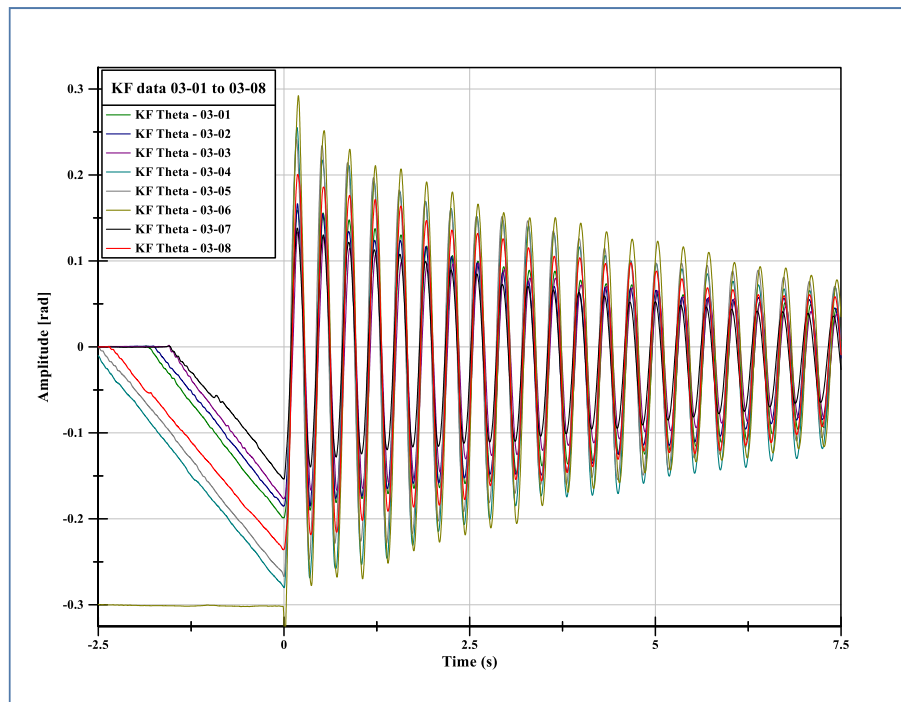


Figure 7-9 – Close-up of KF data for tests 03-01 to 03-08

A visual inspection of Figure 7-8 and Figure 7-9 suggested that the CF and KF data for the tests was similar. This was confirmed by a review of the individual test error measures in Table 7-3, where all the CF and KF error measures were quite close together, with the KF values generally being less. A close inspection of the RC, SGC and TI values (bold and underlined) reinforced the fact. This was further validated by the statistical parameters in Table 7-4, that showed the KF values being more closely correlated, indicated by the lower standard deviation and variance values. This implied that the KF produced better results than the CF for these tests.

When the RC results for the CF and KF values were compared to the 0.2 acceptance criteria suggested by Russell (Russell, 1997a, 1997b), only two of the tests met this criteria (Test 03-06 and Test 03-08). If, as suggested by Russell, a less stringent acceptance criteria of 0.25 was adopted, because the tests involved vibrations, then all of the KF values met the criteria, whilst only Tests 03-02, 03-03 and 03-05 for the CF values failed to meet the criteria.

To better understand what the difference in error measure values meant in relation to the actual tests, close-up portions of the test data for Tests 03-02 and 03-06 have been reproduced in Figure 7-11 and Figure 7-12 respectively. Test 03-02 represented the test with the highest error measures, and therefore the least accurate, whilst Test 03-06 had the lowest error measures and therefore the most accurate. A potential reason why the Test 03-02 values were greater, is shown in the areas identified by A in Figure 7-11. In these areas there is a noticeable difference in the relationship between the HSV curve (black) and the CF and KF curves (red and blue). Although the curves have similar magnitudes, the filter curves dip downwards. The two parallel lines above and below the curves represent the vertical difference, and the fact that the difference between the two lines is similar indicates that the amplitudes at this position are also similar.

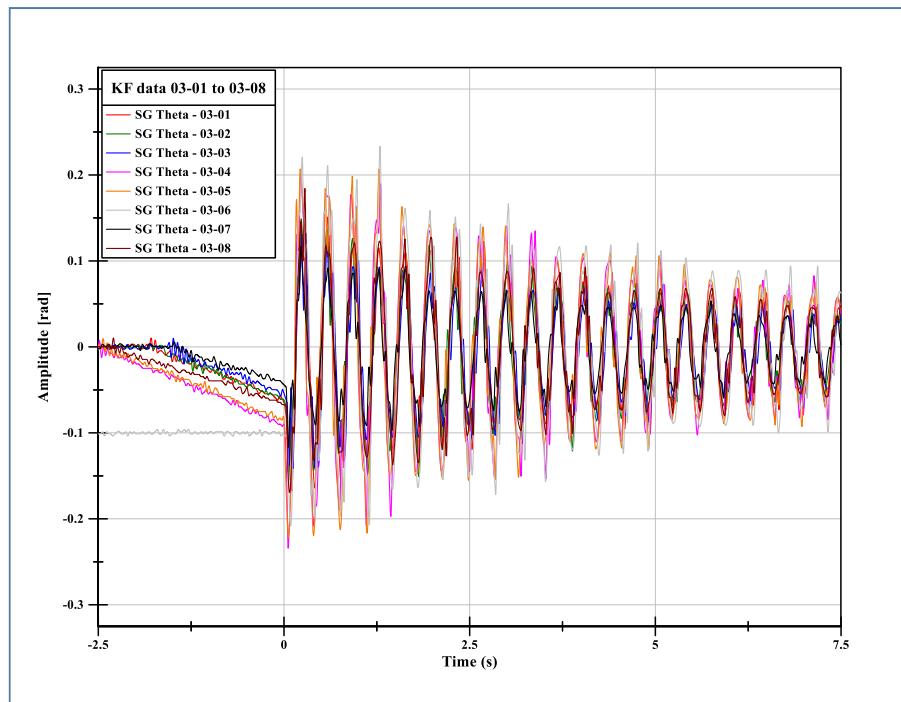


Figure 7-10 – Close-up of strain gauge data for tests 03-01 to 03-08

	03-01	03-02	03-03	03-04	03-05	03-06	03-07	03-08
CF Alpha	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
Q Angle	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Q Gyro Bias	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
R Measure	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Table 7-2 – Filter constants for tests 03-01 to 03-08

By comparison there is only a very slight difference between the curves in the area A in Figure 7-12 for Test 03-06, and therefore the data is better correlated. This is validated by the RC and SGC error measure values in Table 7-3 which are approximately half those for Test 03-02; CF-RC is 0.1331 compared to 0.2562 and CF-SGC is 0.1505 compared to 0.2891. The KF values are similar. In both tests there was a noticeable difference in terms of the data timings, as highlighted at point B in both Figure 7-11 and Figure 7-12, suggesting that the timing difference identified in the static experiments was still present.

The error measures for the strain gauge data were all greater than the Russell's acceptance criteria. This was confirmed by a visual inspection of the data curves, that showed that there was a significant difference in amplitude between the strain gauge data curve (green) and the HSV data curve. However, compared to the significant differences observed for the strain gauge data in Experiment Two, these differences in amplitude were very good. Coupled with the fact that the oscillation frequency of the strain gauge data was similar to that of the HSV, it could be concluded that using the strain gauges in this manner to monitor the rotation merited further investigation.

	03-01			03-02			03-03		
	HSV/ CF	HSV/ KF	HSV/ SG	HSV/ CF	HSV/ KF	HSV/ SG	HSV/ CF	HSV/ KF	HSV/ SG
RM	-0.0080	-0.0061	0.2814	-0.0010	-0.0024	0.2774	-0.0066	-0.0150	0.2658
RP	0.2764	0.2657	0.2501	0.2891	0.2799	0.2734	0.2841	0.2728	0.2558
RC	<u>0.2450</u>	<u>0.2356</u>	<i>0.3336</i>	<u>0.2562</u>	<u>0.2480</u>	<i>0.3452</i>	<u>0.2518</u>	<u>0.2421</u>	<i>0.3269</i>
SGM	-0.0093	-0.0070	0.5548	-0.0012	-0.0028	0.5425	-0.0076	-0.0174	0.5074
SGP	0.2764	0.2657	0.2501	0.2891	0.2799	0.2734	0.2841	0.2728	0.2558
SGC	<u>0.2765</u>	<u>0.2658</u>	<i>0.6085</i>	<u>0.2891</u>	<u>0.2799</u>	<i>0.6075</i>	<u>0.2842</u>	<u>0.2733</u>	<i>0.5682</i>
TI	<u>0.4207</u>	<u>0.4054</u>	<i>0.4322</i>	<u>0.4386</u>	<u>0.4256</u>	<i>0.4594</i>	<u>0.4316</u>	<u>0.4156</u>	<i>0.4332</i>
	03-04			03-05			03-06		
	HSV/ CF	HSV/ KF	HSV/ SG	HSV/ CF	HSV/ KF	HSV/ SG	HSV/ CF	HSV/ KF	HSV/ SG
RM	-0.0023	-0.0021	0.3002	0.0814	0.0689	0.3165	0.0157	0.0084	0.4177
RP	0.2780	0.2678	0.2493	0.2710	0.2611	0.2450	0.1493	0.1432	0.2124
RC	<u>0.2464</u>	<u>0.2373</u>	<i>0.3458</i>	<u>0.2508</u>	<u>0.2393</u>	<i>0.3547</i>	<u>0.1331</u>	<u>0.1271</u>	<i>0.4153</i>
SGM	-0.0027	-0.0024	0.6151	0.1084	0.0897	0.6711	0.0186	0.0098	1.0940
SGP	0.2780	0.2678	0.2493	0.2710	0.2611	0.2450	0.1493	0.1432	0.2124
SGC	<u>0.2780</u>	<u>0.2678</u>	<i>0.6637</i>	<u>0.2919</u>	<u>0.2761</u>	<i>0.7144</i>	<u>0.1505</u>	<u>0.1435</u>	<i>1.1145</i>
TI	<u>0.4230</u>	<u>0.4084</u>	<i>0.4392</i>	<u>0.4157</u>	<u>0.4007</u>	<i>0.4418</i>	<u>0.2326</u>	<u>0.2231</u>	<i>0.4679</i>
	03-07			03-08					
	HSV/ CF	HSV/ KF	HSV/ SG	HSV/ CF	HSV/ KF	HSV/ SG			
RM	0.0012	-0.0124	0.3190	0.0682	0.0572	0.3800			
RP	0.2509	0.2402	0.2473	0.2121	0.2032	0.2640			
RC	<u>0.2224</u>	<u>0.2132</u>	<i>0.3577</i>	<u>0.1975</u>	<u>0.1871</u>	<i>0.4100</i>			
SGM	0.0013	-0.0144	0.6799	0.0887	0.0729	0.9196			
SGP	0.2509	0.2402	0.2473	0.2121	0.2032	0.2640			
SGC	<u>0.2509</u>	<u>0.2406</u>	<i>0.7235</i>	<u>0.2299</u>	<u>0.2159</u>	<i>0.9568</i>			
TI	<u>0.3840</u>	<u>0.3685</u>	<i>0.4456</i>	<u>0.3295</u>	<u>0.3155</u>	<i>0.4954</i>			

Table 7-3 – Error measures for tests 03-01 to 03-08

	CF				KF			
	SD	Var	Median	Average	SD	Var	Median	Average
RM	0.0333	0.0011	0.0001	0.0186	0.0303	0.0009	-0.0023	0.0121
RP	0.0450	0.0020	0.2737	0.2514	0.0437	0.0019	0.2634	0.2417
RC	<u>0.0394</u>	<u>0.0016</u>	0.2457	0.2254	<u>0.0385</u>	<u>0.0015</u>	0.2365	0.2162
SGM	0.0437	0.0019	0.0000	0.0245	0.0387	0.0015	-0.0026	0.0161
SGP	0.0450	0.0020	0.2737	0.2514	0.0437	0.0019	0.2634	0.2417
SGC	<u>0.0446</u>	<u>0.0020</u>	0.2773	0.2564	<u>0.0435</u>	<u>0.0019</u>	0.2668	0.2454
TI	<u>0.0662</u>	<u>0.0044</u>	0.4182	0.3845	<u>0.0646</u>	<u>0.0042</u>	0.4031	0.3704
	SG							
	SD	Var	Median	Average				
RM	0.0497	0.0025	0.3084	0.3198				
RP	0.0167	0.0003	0.2497	0.2497				
RC	<u>0.0312</u>	<u>0.0010</u>	0.3503	0.3612				
SGM	0.1920	0.0369	0.6431	0.6981				
SGP	0.0167	0.0003	0.2497	0.2497				
SGC	<u>0.1797</u>	<u>0.0323</u>	0.6891	0.7446				
TI	<u>0.0201</u>	<u>0.0004</u>	0.4437	0.4518				

Table 7-4 – Statistical parameters for error measures for tests 03-01 to 03-08

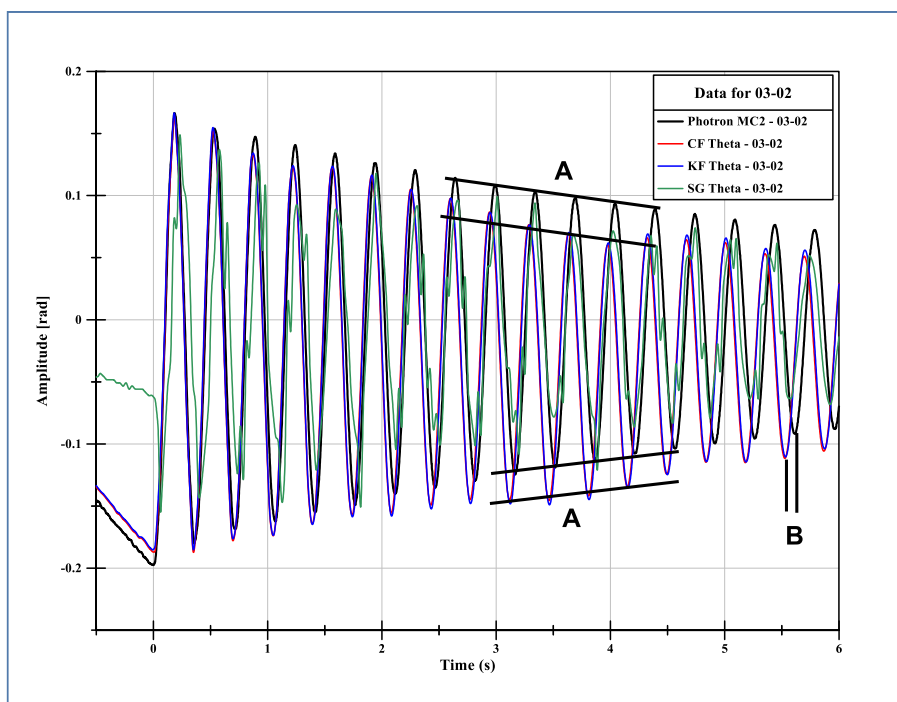


Figure 7-11 – Close-up of data for test 03-02

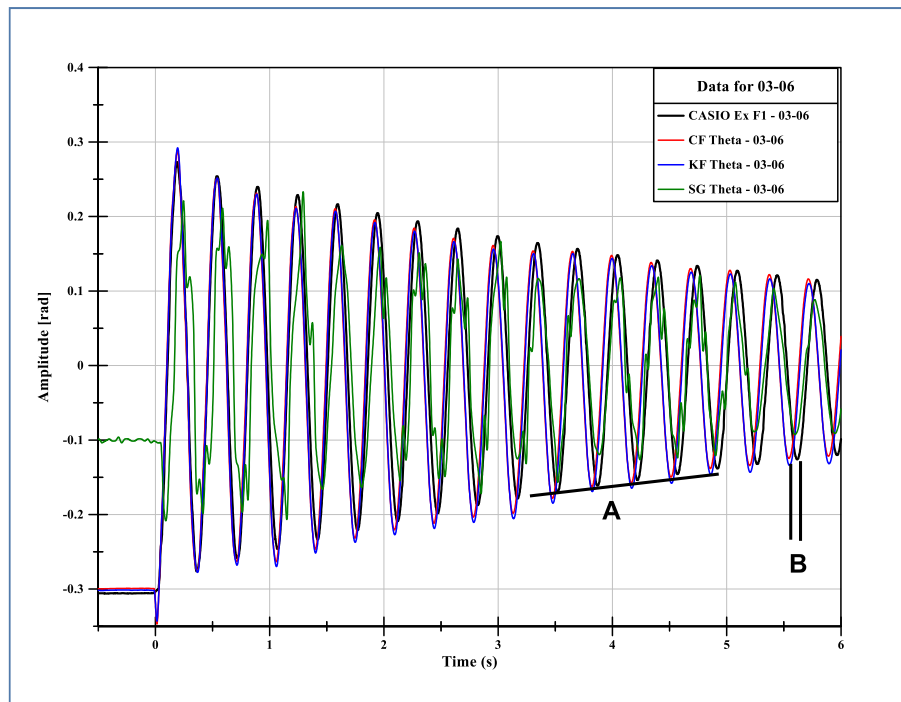


Figure 7-12 – Close-up of data for test 03-06

From these tests the following observations could be made:

- The timing difference observed in Experiments One and Two was still evident but greatly reduced.
- If a higher acceptance criterion of 0.25 was chosen for the filtered data RC error measure, the majority of the tests could be deemed to have passed.
- The strain gauge data matched the HSV data much better than in Experiment Two and therefore deserved further investigation.
- As in previous experiments both the CF and KF produced similar results, however the fact that all the KF results passed the 0.25 acceptance criteria, while three of the CF results did not, indicated that the KF was the more consistent and accurate of the two.

7.2.3.2 Experiment Three – Tests 09 to 20 – Comparison of different IMU sensitivities

The second series of tests was conducted to investigate whether changing the sensitivity of the accelerometer and gyroscope had an impact on the IMU sensors' performance. During initial tests it was identified that the sudden load removal resulted in an acceleration of over 2g, which resulted in some of the acceleration data being 'clipped' (as previously discussed in Section 5.4.1.3). It was therefore decided that the 2g sensitivity setting would not be investigated.

The filter constants used for these tests are summarised in Table 7-5, while the error measures for the filtered and strain gauge data are summarised in Table 6-18. As in the previous experiments, when investigating the impact of changing the sensitivity settings, the cells of this table have been highlighted with different colours to distinguish between the different IMU sensor sensitivity settings.

	03-09	03-10	03-11	03-12	03-13	03-14	03-15	03-16	03-17	03-18	03-19	03-20
CF Alpha	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
Q Angle	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Q Gyro Bias	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
R Measure	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Table 7-5 – Filter constants for tests 03-09 to 03-20

The HSV data for each of the tests is visually summarised in Figure 7-13 and Figure 7-14, while close-up charts for the CF, KF and strain gauge data curves are shown in Figure 7-15, Figure 7-16 and Figure 7-17 respectively.

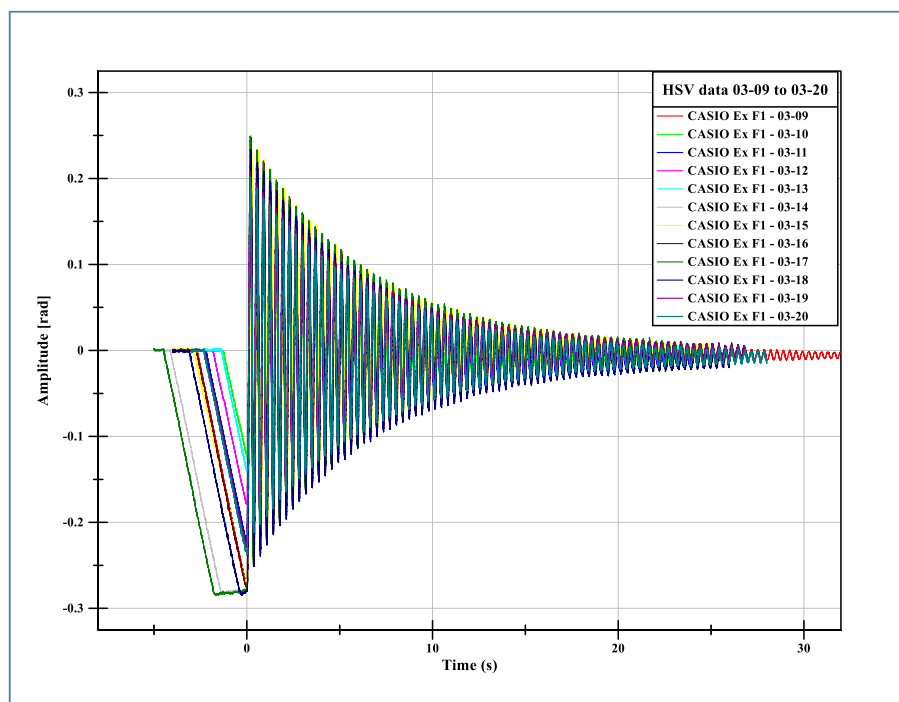


Figure 7-13 – HSV data for tests 03-09 to 03-20

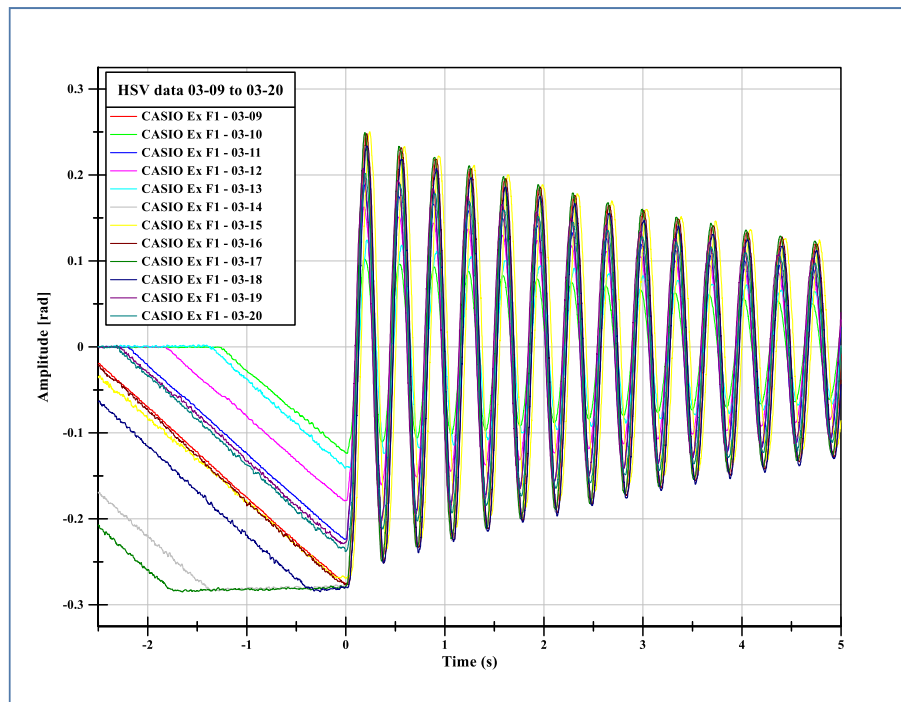


Figure 7-14 – Close-up of HSV data for tests 03-09 to 03-20

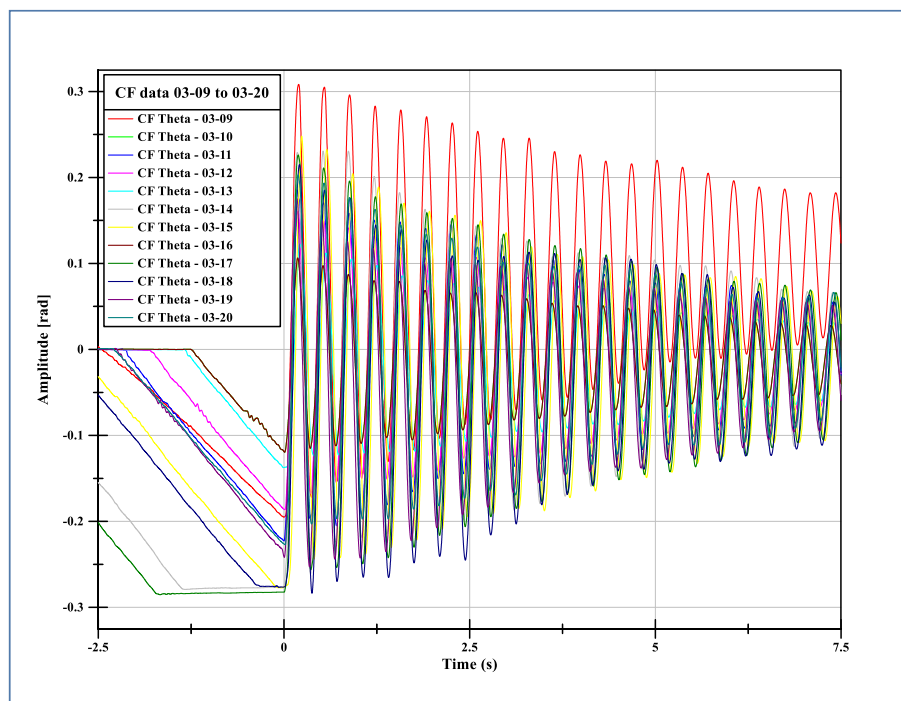


Figure 7-15 – Close-up of CF data for tests 03-09 to 03-20

A close inspection of the CF and KF data showed that the results for Test 03-09 (solid red lines) were noticeably different from those for the rest of the tests. While the CF results for this test in Figure 7-15 were vertically displaced from the remainder of the results, the KF results for the same test in Figure 7-16 gradually converged towards the other results. This behaviour is consistent with the different types of filter. The CF uses a constant value to apportion the amount of the gyroscope or accelerometer data to be used to calculate the filtered value.

This contrasts with the KF which continuously adjusts the results, depending upon previous iterations, and for this research has consistently shown better accuracy than the CF. This is confirmed by the error measures in Table 6-18 where all of the KF values for the comprehensive error measures RC and SGC, and inequality measure TI, are less than the comparable CF values.

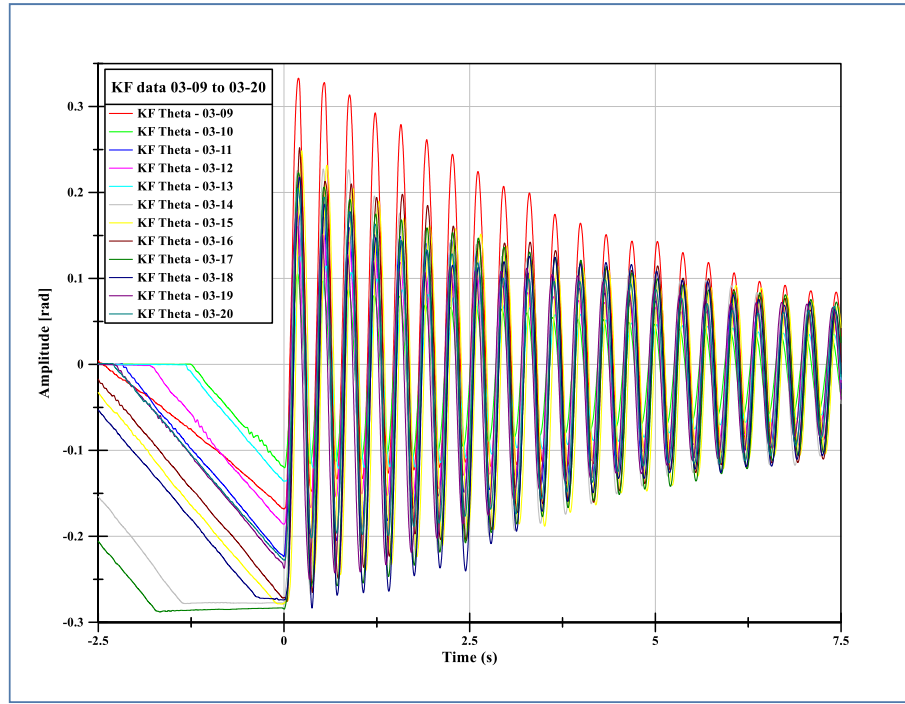


Figure 7-16 – Close-up of KF data for tests 03-09 to 03-20

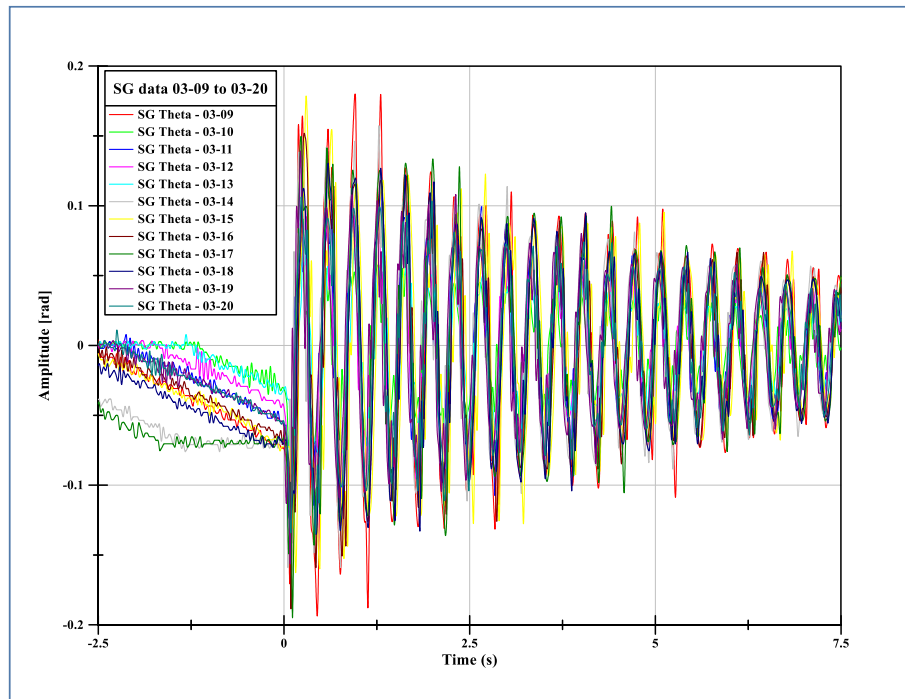


Figure 7-17 – Close-up of strain gauge data for tests 03-09 to 03-20

Headings (Gyro sensitivity) (deg/sec)	250		500		1000		2000	
Columns (Acc sensitivity) (g)	4		8		16			

	03-09			03-10			03-11		
	HSV/ CF	HSV/ KF	HSV/ SG	HSV/ CF	HSV/ KF	HSV/ SG	HSV/ CF	HSV/ KF	HSV/ SG
RM	-0.3287	0.0685	0.3645	-0.0098	-0.0206	0.3519	-0.0041	-0.0174	0.4045
RP	0.4036	0.2717	0.2441	0.2928	0.2813	0.2596	0.2318	0.2225	0.2566
RC	<u>0.4613</u>	<u>0.2484</u>	<u>0.3888</u>	<u>0.2597</u>	<u>0.2499</u>	<u>0.3876</u>	<u>0.2055</u>	<u>0.1978</u>	<u>0.4245</u>
SGM	-0.4168	0.0891	0.8540	-0.0113	-0.0240	0.8033	-0.0047	-0.0202	1.0306
SGP	0.4036	0.2717	0.2441	0.2928	0.2813	0.2596	0.2318	0.2225	0.2566
SGC	<u>0.5802</u>	<u>0.2860</u>	<u>0.8882</u>	<u>0.2930</u>	<u>0.2823</u>	<u>0.8442</u>	<u>0.2318</u>	<u>0.2234</u>	<u>1.0620</u>
TI	<u>0.6291</u>	<u>0.4158</u>	<u>0.4658</u>	<u>0.4439</u>	<u>0.4277</u>	<u>0.4759</u>	<u>0.3561</u>	<u>0.3426</u>	<u>0.5017</u>
	03-12			03-13			03-14		
	HSV/ CF	HSV/ KF	HSV/ SG	HSV/ CF	HSV/ KF	HSV/ SG	HSV/ CF	HSV/ KF	HSV/ SG
RM	-0.0287	-0.0345	0.3754	-0.0226	-0.0222	0.3657	0.0127	0.0086	0.4984
RP	0.2533	0.2419	0.2592	0.2747	0.2640	0.2892	0.2136	0.2073	0.2323
RC	<u>0.2259</u>	<u>0.2165</u>	<u>0.4043</u>	<u>0.2442</u>	<u>0.2348</u>	<u>0.4132</u>	<u>0.1897</u>	<u>0.1839</u>	<u>0.4873</u>
SGM	-0.0335	-0.0405	0.9000	-0.0263	-0.0259	0.8590	0.0150	0.0100	1.5436
SGP	0.2533	0.2419	0.2592	0.2747	0.2640	0.2892	0.2136	0.2073	0.2323
SGC	<u>0.2555</u>	<u>0.2452</u>	<u>0.9366</u>	<u>0.2759</u>	<u>0.2653</u>	<u>0.9064</u>	<u>0.2141</u>	<u>0.2076</u>	<u>1.5610</u>
TI	<u>0.3878</u>	<u>0.3713</u>	<u>0.4879</u>	<u>0.4184</u>	<u>0.4031</u>	<u>0.5153</u>	<u>0.3294</u>	<u>0.3200</u>	<u>0.5412</u>
	03-15			03-16			03-17		
	HSV/ CF	HSV/ KF	HSV/ SG	HSV/ CF	HSV/ KF	HSV/ SG	HSV/ CF	HSV/ KF	HSV/ SG
RM	-0.0343	-0.0388	0.4119	0.0156	0.0066	0.4411	-0.0020	-0.0095	0.5009
RP	0.2427	0.2325	0.2570	0.2388	0.2313	0.2651	0.1557	0.1470	0.2341
RC	<u>0.2173</u>	<u>0.2089</u>	<u>0.4303</u>	<u>0.2121</u>	<u>0.2051</u>	<u>0.4561</u>	<u>0.1380</u>	<u>0.1305</u>	<u>0.4900</u>
SGM	-0.0403	-0.0457	1.0659	0.0185	0.0077	1.2130	-0.0023	-0.0110	1.5597
SGP	0.2427	0.2325	0.2570	0.2388	0.2313	0.2651	0.1557	0.1470	0.2341
SGC	<u>0.2461</u>	<u>0.2370</u>	<u>1.0965</u>	<u>0.2395</u>	<u>0.2314</u>	<u>1.2416</u>	<u>0.1558</u>	<u>0.1474</u>	<u>1.5772</u>
TI	<u>0.3726</u>	<u>0.3579</u>	<u>0.5065</u>	<u>0.3665</u>	<u>0.3554</u>	<u>0.5318</u>	<u>0.2422</u>	<u>0.2289</u>	<u>0.5444</u>
	03-18			03-19			03-20		
	HSV/ CF	HSV/ KF	HSV/ SG	HSV/ CF	HSV/ KF	HSV/ SG	HSV/ CF	HSV/ KF	HSV/ SG
RM	-0.0014	0.0078	0.4644	-0.0382	-0.0273	0.4283	0.0131	0.0013	0.4351
RP	0.2407	0.2333	0.2456	0.2825	0.2700	0.2498	0.2496	0.2413	0.2583
RC	<u>0.2133</u>	<u>0.2069</u>	<u>0.4656</u>	<u>0.2527</u>	<u>0.2405</u>	<u>0.4394</u>	<u>0.2215</u>	<u>0.2138</u>	<u>0.4484</u>
SGM	-0.0017	0.0091	1.3406	-0.0449	-0.0319	1.1470	0.0154	0.0015	1.1816
SGP	0.2407	0.2333	0.2456	0.2825	0.2700	0.2498	0.2496	0.2413	0.2583
SGC	<u>0.2407</u>	<u>0.2335</u>	<u>1.3629</u>	<u>0.2861</u>	<u>0.2719</u>	<u>1.1739</u>	<u>0.2501</u>	<u>0.2413</u>	<u>1.2095</u>
TI	<u>0.3692</u>	<u>0.3584</u>	<u>0.5290</u>	<u>0.4299</u>	<u>0.4118</u>	<u>0.5095</u>	<u>0.3822</u>	<u>0.3700</u>	<u>0.5217</u>

Table 7-6 – Error measures for tests 03-09 to 03-20

To enable comparisons between the different error measures to be made more easily, so that the optimum combination could be identified, the comprehensive error measures RC and SGC and the inequality measure TI, were extracted from Table 6-18. These were collated in terms of the different filtering methods (CF and KF) and the accelerometer and gyroscope settings and are summarised in Table 7-7. This showed that the lowest error measures, and therefore the greatest accuracy, were consistently achieved when the accelerometer was set to 16g and the gyroscope to 1000 deg/s, as shown by the bold double-underlined values in Table 7-7. This was true for both the CF and KF data curves. Similarly, the maximum error measures were also identified, signifying the lowest accuracy, and showed that for the CF the settings were 4g and 250 deg/s while for the KF they were 8g and 250 deg/s (the italic underlined values).

CF - RC	4g	8g	16g	KF - RC	4g	8g	16g
250 deg/s	<u><i>0.4613</i></u>	0.2597	0.2055	250 deg/s	0.2484	<u><i>0.2499</i></u>	0.1978
500 deg/s	0.2259	0.2442	0.1897	500 deg/s	0.2165	0.2348	0.1839
1000 deg/s	0.2173	0.2121	<u>0.138</u>	1000 deg/s	0.2089	0.2051	<u>0.1305</u>
2000 deg/s	0.2133	0.2527	0.2215	2000 deg/s	0.2069	0.2405	0.2138
CF - SGC	4g	8g	16g	KF - SGC	4g	8g	16g
250 deg/s	<u><i>0.5802</i></u>	0.293	0.2318	250 deg/s	<u><i>0.2860</i></u>	0.2823	0.2234
500 deg/s	0.2555	0.2759	0.2141	500 deg/s	0.2452	0.2653	0.2076
1000 deg/s	0.2461	0.2395	<u>0.1558</u>	1000 deg/s	0.2370	0.2314	<u>0.1474</u>
2000 deg/s	0.2407	0.2861	0.2501	2000 deg/s	0.2407	0.2719	0.2413
CF - TI	4g	8g	16g	KF - TI	4g	8g	16g
250 deg/s	<u><i>0.6291</i></u>	0.4439	0.3561	250 deg/s	0.4158	<u><i>0.4277</i></u>	0.3426
500 deg/s	0.3878	0.4184	0.3294	500 deg/s	0.3713	0.4031	0.3200
1000 deg/s	0.3726	0.3665	<u>0.2422</u>	1000 deg/s	0.3579	0.3554	<u>0.2289</u>
2000 deg/s	0.3692	0.4299	0.3822	2000 deg/s	0.3584	0.4118	0.3700

Table 7-7 – Comparison of error measures for tests 03-09 to 03-20 related to sensor sensitivities

It should be borne in mind that these conclusions are only based upon a limited number of values; three values for each of the gyroscope sensitivity comparisons, and four values of each of the accelerometer sensitivity comparisons. However, for the purposes of the research it was decided that the 16g accelerometer and 1000 deg/s gyroscope sensitivity settings should be used for the remainder of the tests for this experiment.

With the exception of the CF data for Tests 03-09, 03-10 and 03-19, all of the other data curves' RC error measure values met the 0.25 Russell's acceptance criteria and could therefore be deemed to have passed.

7.2.3.3 Experiment Three – Tests 21 to 29 – Comparison of different sampling rates

The final series of tests that were undertaken as part of Experiment Three, were conducted to investigate how changing the data logging timing loop period affected the sensor performance. As in the similar tests conducted during Experiments One and Two, three tests were conducted at three different timing loop periods using the same gyroscope and accelerometer sensitivity settings, as summarised in Table 6-26. In this instance sensitivity settings of 1000 deg/s for the gyroscope, and 16g for the accelerometer, were used based upon the results of the previous series of tests.

The filter constants used for these tests are summarised in Table 7-9, while the error measures for the filtered and strain gauge data are summarised in Table 7-10. To aid interpretation of the data, the values in Table 7-10 have been highlighted with the same colours used in Table 6-26 corresponding to the different timing loop periods. There was a problem with the data logging for Test 03-27 which meant that the strain gauge data was not recorded from the point of load release (hence the empty spaces in Table 7-10). As the primary purpose of the tests was to investigate the performance of the IMU sensors, it was decided not to repeat the test as the IMU data was not affected, and there was strain gauge data available from the two other tests at that timing loop setting (Test 03-28 and Test 03-29).

The HSV data for each of the tests is visually summarised in Figure 7-18 and Figure 7-19, while close-up charts for the CF, KF and strain gauge data curves are shown in Figure 7-20, Figure 7-21 and Figure 7-22 respectively.

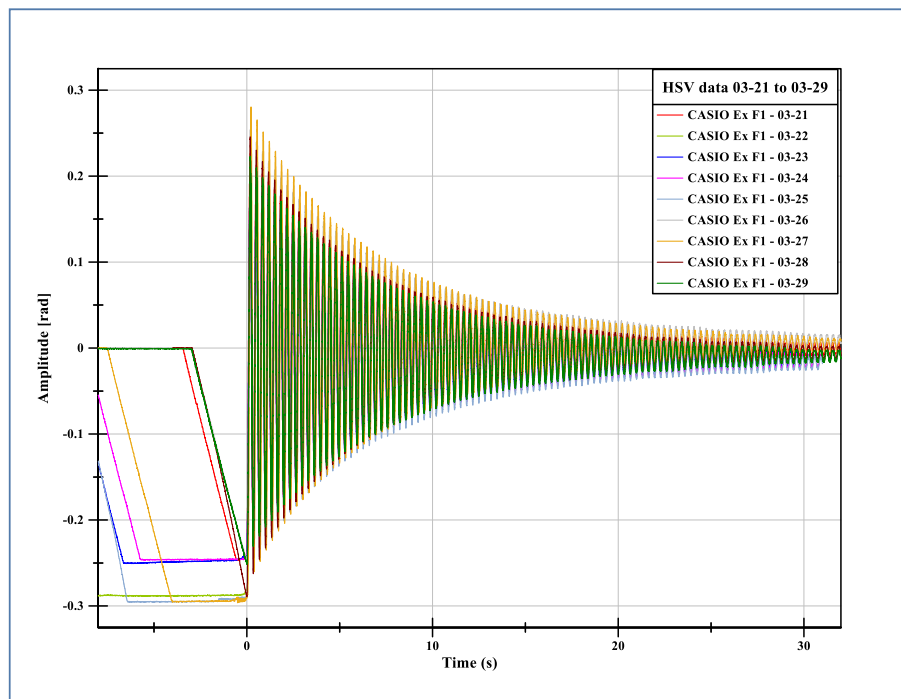


Figure 7-18 – HSV data for tests 03-21 to 03-29

	03-21	03-22	03-23	03-24	03-25	03-26	03-27	03-28	03-29
Clock speed (MHz)	1	1	1	1	1	1	1	1	1
Timing loop period (μS)	2000	2000	2000	1000	1000	1000	500	500	500
Gyro sensitivity (deg/s)	1000	1000	1000	1000	1000	1000	1000	1000	1000
Acc sensitivity (g)	16	16	16	16	16	16	16	16	16

Table 7-8 – Sample rate parameters for tests 03-21 to 03-29

	03-21	03-22	03-23	03-24	03-25	03-26	03-27	03-28	03-29
CF Alpha	0.95	0.92	0.99	0.99	0.99	0.99	0.95	0.99	0.95
Q Angle	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Q Gyro Bias	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
R Measure	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Table 7-9 – Filter constants for tests 03-21 to 03-29

	03-21			03-22			03-23		
	HSV/ CF	HSV/ KF	HSV/ SG	HSV/ CF	HSV/ KF	HSV/ SG	HSV/ CF	HSV/ KF	HSV/ SG
RM	-0.1089	-0.0473	0.2423	0.0233	0.2418	0.5016	0.0751	0.0667	0.4407
RP	0.2878	0.2613	0.3514	0.3558	0.1690	0.2556	0.1792	0.1682	0.2847
RC	<u>0.2727</u>	<u>0.2353</u>	<u>0.3783</u>	<u>0.3160</u>	<u>0.2614</u>	<u>0.4990</u>	<u>0.1722</u>	<u>0.1604</u>	<u>0.4649</u>
SGM	-0.1324	-0.0559	0.4411	0.0279	0.4396	1.5643	0.0988	0.0864	1.2109
SGP	0.2878	0.2613	0.3514	0.3558	0.1690	0.2556	0.1792	0.1682	0.2847
SGC	<u>0.3168</u>	<u>0.2672</u>	<u>0.5639</u>	<u>0.3569</u>	<u>0.4709</u>	<u>1.5850</u>	<u>0.2047</u>	<u>0.1891</u>	<u>1.2439</u>
TI	<u>0.4415</u>	<u>0.3999</u>	<u>0.5464</u>	<u>0.5304</u>	<u>0.3147</u>	<u>0.5621</u>	<u>0.2815</u>	<u>0.2643</u>	<u>0.5501</u>
	03-24			03-25			03-26		
	HSV/ CF	HSV/ KF	HSV/ SG	HSV/ CF	HSV/ KF	HSV/ SG	HSV/ CF	HSV/ KF	HSV/ SG
RM	0.0105	-0.0076	0.4322	0.0607	0.0897	0.4842	-0.0342	-0.0310	0.2110
RP	0.0953	0.0852	0.2980	0.1658	0.1581	0.2304	0.2234	0.2004	0.3701
RC	<u>0.0850</u>	<u>0.0758</u>	<u>0.4653</u>	<u>0.1564</u>	<u>0.1611</u>	<u>0.4752</u>	<u>0.2003</u>	<u>0.1797</u>	<u>0.3776</u>
SGM	0.0123	-0.0088	1.1668	0.0778	0.1212	1.4566	-0.0401	-0.0363	0.3604
SGP	0.0953	0.0852	0.2980	0.1658	0.1581	0.2304	0.2234	0.2004	0.3701
SGC	<u>0.0961</u>	<u>0.0857</u>	<u>1.2043</u>	<u>0.1831</u>	<u>0.1992</u>	<u>1.4747</u>	<u>0.2270</u>	<u>0.2037</u>	<u>0.5166</u>
TI	<u>0.1493</u>	<u>0.1335</u>	<u>0.5583</u>	<u>0.2600</u>	<u>0.2519</u>	<u>0.5298</u>	<u>0.3443</u>	<u>0.3102</u>	<u>0.5638</u>
	03-27			03-28			03-29		
	HSV/ CF	HSV/ KF	HSV/ SG	HSV/ CF	HSV/ KF	HSV/ SG	HSV/ CF	HSV/ KF	HSV/ SG
RM	0.0329	0.0079		-0.0035	0.0088	0.2374	0.0018	-0.0149	0.2114
RP	0.2040	0.1851		0.0720	0.0880	0.5462	0.1663	0.1297	0.3864
RC	<u>0.1831</u>	<u>0.1642</u>		<u>0.0639</u>	<u>0.0784</u>	<u>0.5278</u>	<u>0.1474</u>	<u>0.1157</u>	<u>0.3903</u>
SGM	0.0402	0.0093		-0.0041	0.0102	0.4278	0.0021	-0.0173	0.3615
SGP	0.2040	0.1851		0.0720	0.0880	0.5462	0.1663	0.1297	0.3864
SGC	<u>0.2079</u>	<u>0.1854</u>		<u>0.0721</u>	<u>0.0886</u>	<u>0.6938</u>	<u>0.1663</u>	<u>0.1309</u>	<u>0.5291</u>
TI	<u>0.3155</u>	<u>0.2868</u>		<u>0.1129</u>	<u>0.1379</u>	<u>0.7653</u>	<u>0.2582</u>	<u>0.2026</u>	<u>0.5840</u>

Table 7-10 – Error measures for tests 03-21 to 03-29

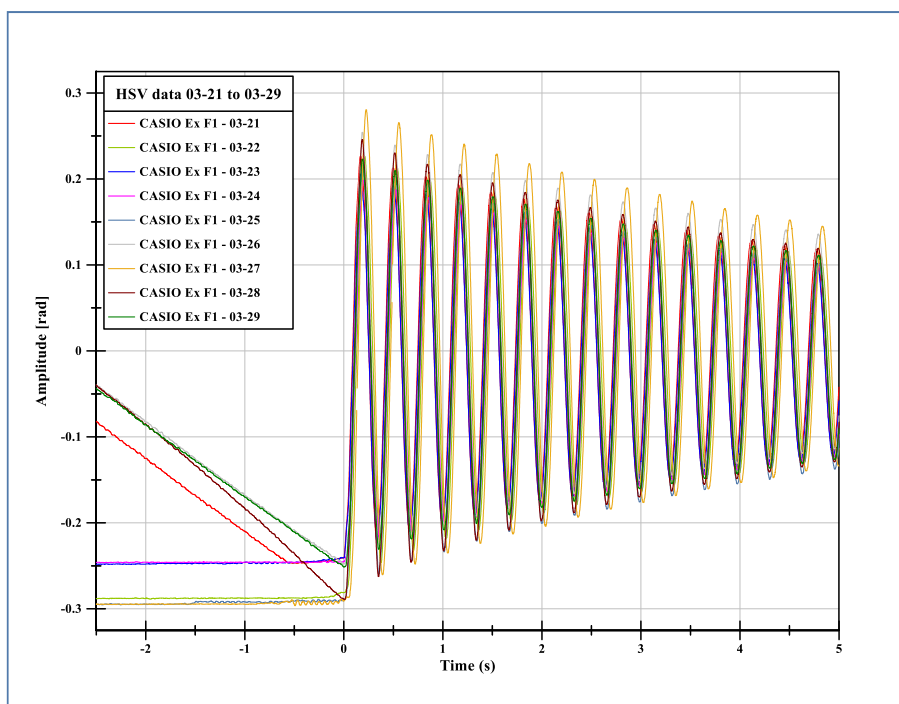


Figure 7-19 – Close-up of HSV data for tests 03-21 to 03-29

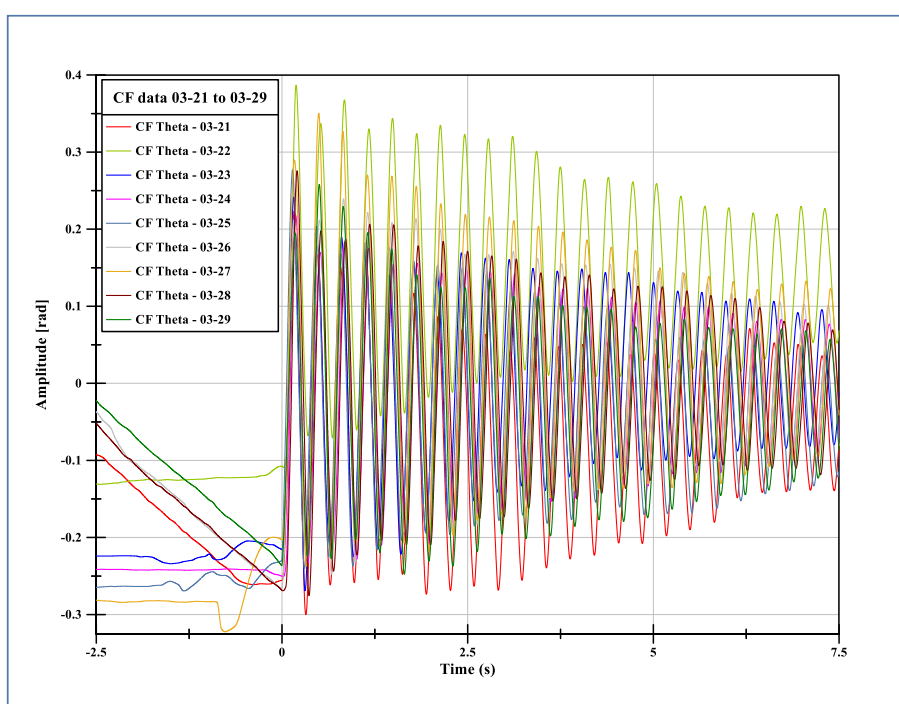


Figure 7-20 – Close-up of CF data for tests 03-21 to 03-29

A visual inspection of the CF data in Figure 7-20 indicates that the data curve for Test 03-22 (solid light green line) was vertically displaced above the other data curves as was seen for Test 03-09. The higher RC value of 0.3160 in Table 7-10 verifies this divergence. It should also be noted that the CF Alpha value for this particular test was 0.92, indicating that adjustments had already been undertaken using the filter constant, however it had not been possible to reduce the discrepancy.

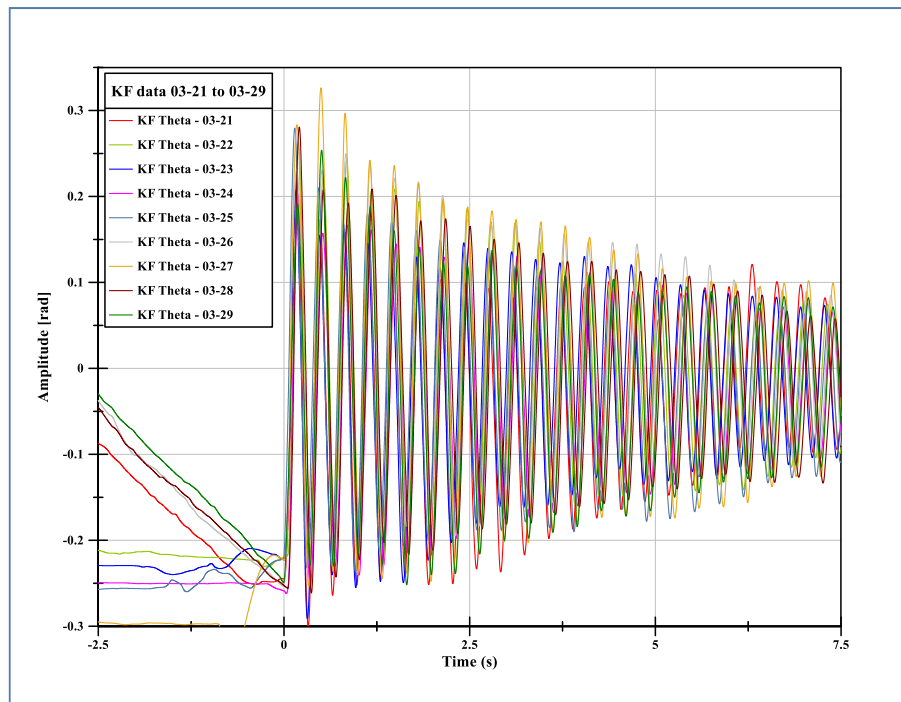


Figure 7-21 – Close-up of KF data for tests 03-21 to 03-29

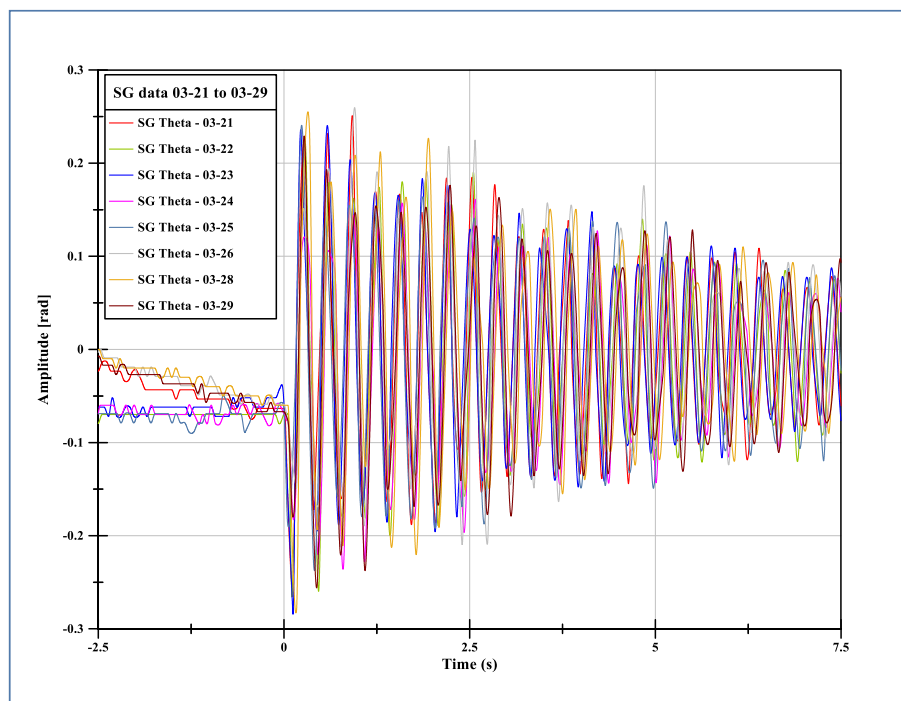


Figure 7-22 – Close-up of strain gauge data for tests 03-21 to 03-29

A review of the error measure data in Table 7-10 showed that all of the tests except for Test 03-21 and Test 03-22 had RC error measures on or below the 0.2 acceptance criteria. When compared with the acceptance criteria results for the earlier tests conducted for this experiment, which were generally between 0.25 and 0.2, this was a better result suggesting that the chosen IMU sensor sensitivity settings resulted in improved accuracy.

In addition to improved results due to selecting the optimum IMU sensitivity settings, altering the timing loop period also had an impact on the accuracy. A review of the error measures for each of the timing loop periods highlights this; to aid this review, the comprehensive error and inequality measures have been copied from Table 7-10 and summarised in Table 7-11 along with their statistical parameters. This table shows the CF data in the top-left quarter, the KF data in the top-right quarter and the SG data in the bottom-left quarter. As in Table 7-10, the same shading has been used to denote the different timing loop periods given in Table 6-26.

CF	03-21	03-22	03-23	SD	Var	KF	03-21	03-22	03-23	SD	Var
RC	<u>0.2727</u>	<u>0.3160</u>	0.1722	0.060	0.004	RC	<u>0.2353</u>	<u>0.2614</u>	0.1604	0.043	0.002
SGC	0.3168	0.3569	0.2047	0.064	0.004	SGC	0.2672	0.4709	0.1891	0.119	0.014
TI	0.4415	0.5304	0.2815	0.103	0.011	TI	0.3999	0.3147	0.2643	0.056	0.003
CF	03-24	03-25	03-26	SD	Var	KF	03-24	03-25	03-26	SD	Var
RC	0.0850	<u>0.1564</u>	<u>0.2003</u>	0.048	0.002	RC	0.0758	<u>0.1611</u>	<u>0.1797</u>	0.045	0.002
SGC	0.0961	0.1831	0.227	0.054	0.003	SGC	0.0857	0.1992	0.2037	0.055	0.003
TI	0.1493	0.2600	0.3443	0.080	0.006	TI	0.1335	0.2519	0.3102	0.074	0.005
CF	03-27	03-28	03-29	SD	Var	KF	03-27	03-28	03-29	SD	Var
RC	0.1831	<u>0.0639</u>	<u>0.1474</u>	0.050	0.002	RC	0.1642	<u>0.0784</u>	<u>0.1157</u>	0.035	0.001
SGC	0.2079	0.0721	0.1663	0.057	0.003	SGC	0.1854	0.0886	0.1309	0.040	0.002
TI	0.3155	0.1129	0.2582	0.085	0.007	TI	0.2868	0.1379	0.2026	0.061	0.004
SG	03-21	03-22	03-23	SD	Var						
RC	0.3783	0.4990	0.4649	0.051	0.003						
SGC	0.5639	1.5850	1.2439	0.424	0.180						
TI	0.5464	0.5621	0.5501	0.007	0.000						
SG	03-24	03-25	03-26	SD	Var						
RC	0.4653	0.4752	0.3776	0.044	0.002						
SGC	1.2043	1.4747	0.5166	0.403	0.163						
TI	0.5583	0.5298	0.5638	0.015	0.000						
SG	03-27	03-28	03-29	SD	Var						
RC		0.5278	0.3903	0.069	0.005						
SGC		0.6938	0.5291	0.082	0.007						
TI		0.7653	0.5840	0.091	0.008						

Table 7-11 – Statistical parameters for tests 03-21 to 03-29

A comparison of the RC values for both the CF and KF data for Tests 03-28 and 03-29, Tests 03-25 and 03-26 and Tests 03-21 and 03-22, which represent the 500 μ S, 1000 μ S and 2000 μ S timing periods respectively, ranks the data in the order 500 μ S, 1000 μ S and 2000 μ S. This would suggest that as in previous experiments, using a lower timing loop period reduces the error measure values thereby indicating an improved accuracy. As these results were based upon only two out of three results for each of the data sets, further similar tests would need to be conducted to confirm this was in fact true.

7.2.4 Analysis and discussion of results

7.2.4.1 Difference in timing between HSV and sensor data

The difference between the timings for the HSV and sensor data was initially identified early in Experiment One, but it was shown that decreasing the timing loop period reduced this effect significantly. Similar tests in Experiment Two also had a positive effect reinforcing this hypothesis. To confirm that this was true for the dynamic tests, similar tests were conducted, and it was shown that the hypothesis also appeared to be true. This is visually shown in Figure 7-23, Figure 7-24 and Figure 7-25 which compare HSV data against the corresponding filtered data for three tests, one for each of the timing loop periods.

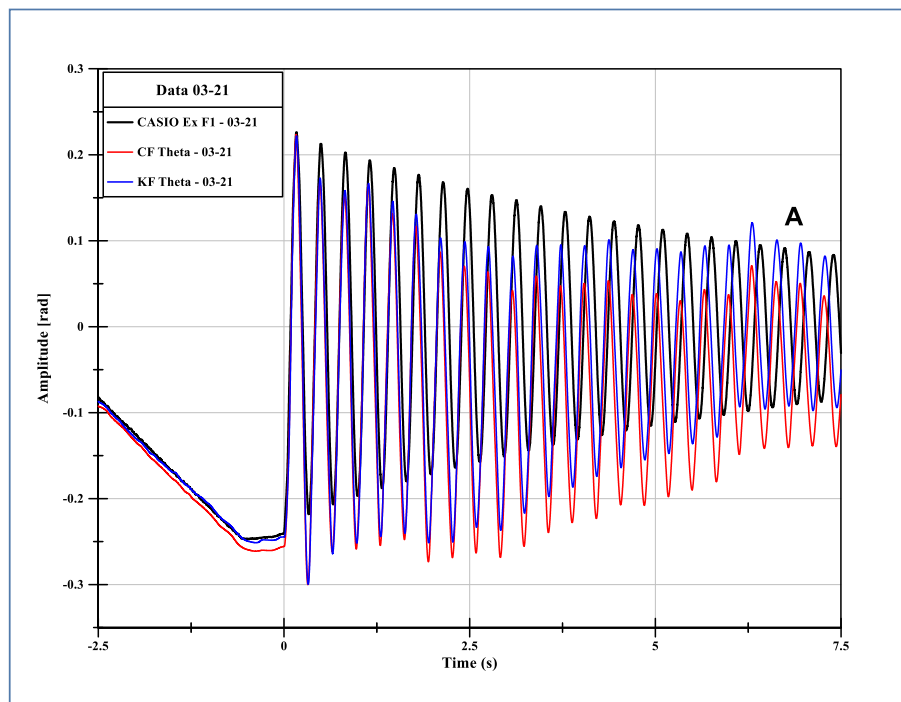


Figure 7-23 – Comparison of HSV, CF and KF data for Test 03-21 (2000 μ S timing loop period)

There is an evident difference between the HSV and filtered data timings at point A in Figure 7-23 which was a test conducted using a 2000 μ S timing loop period. Similarly, at point A in Figure 7-24 there is still a small timing difference; this time the timing loop period was 1000 μ S. Finally, at point A in Figure 7-25 the timing difference is insignificant for the test conducted using a 500 μ S timing loop period.

Although these results indicate that decreasing the timing loop period reduces any timing differences, they only represent a small sample of data and therefore further dynamic tests, using the lower timing loop period would need to be undertaken to confirm that this is a valid theory.

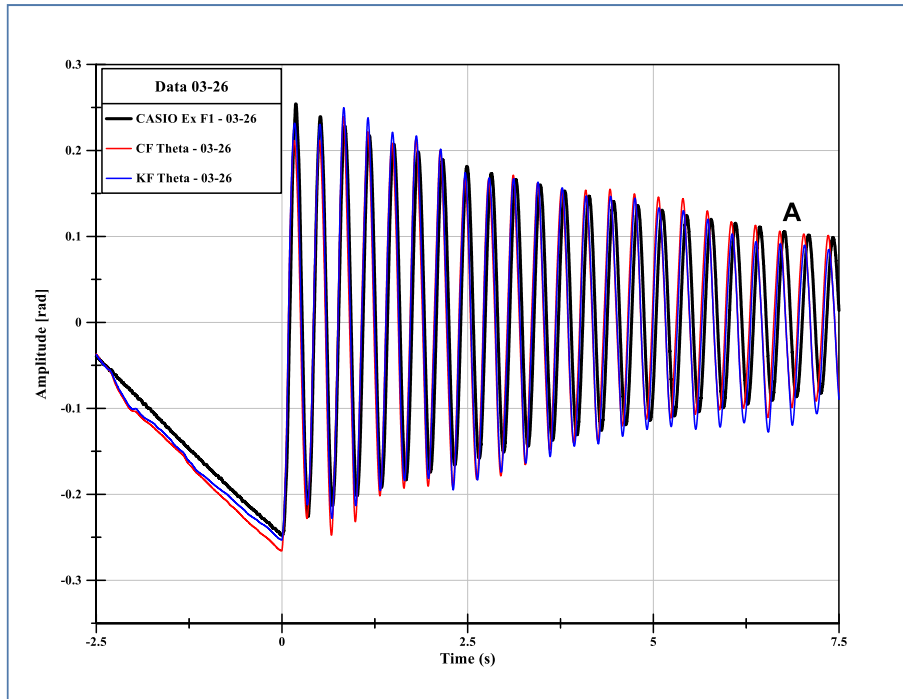


Figure 7-24 – Comparison of HSV, CF and KF data for Test 03-26 (1000 μ S timing loop period)

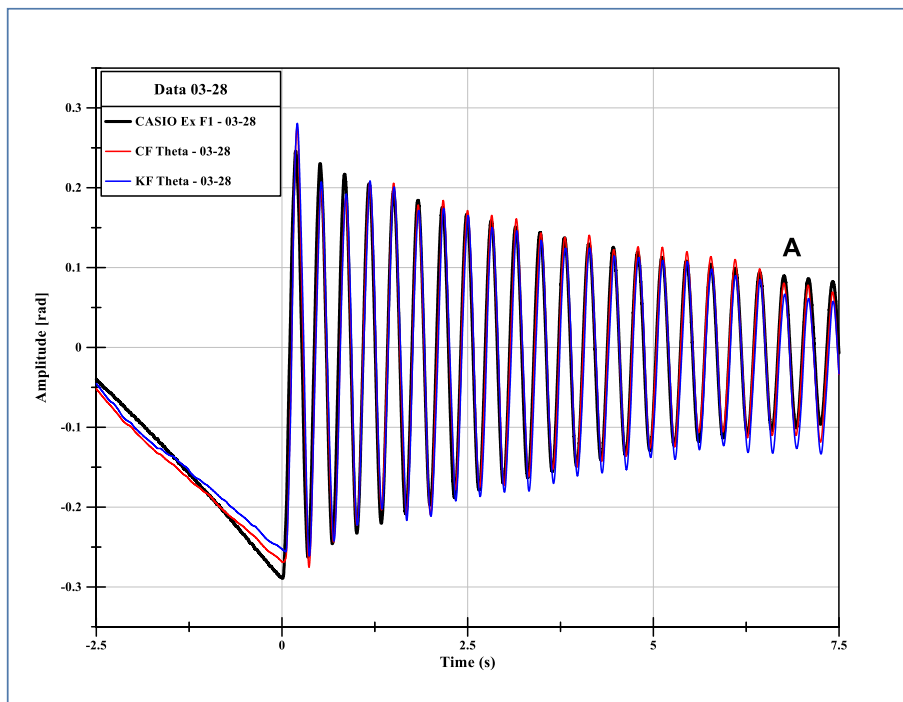


Figure 7-25 – Comparison of HSV, CF and KF data for Test 03-28 (500 μ S timing loop period)

7.2.4.2 Analogue and IMU X-axis Accelerometer comparison

A single axis Dytran 3200B6 accelerometer was used during Experiment Three Tests 03-01 to 03-20 to evaluate the performance of the IMU accelerometers against a more expensive sensor. As the acceleration could be measured along only one axis using this accelerometer, it was decided to orientate it parallel to the IMU x-axis accelerometer, as this was the axis that would be experiencing the most movement. The data for the two accelerometers, for two of the Experiment Three tests (Test 03-04 and Test 03-19) is shown in Figure 7-26 to Figure 7-28.

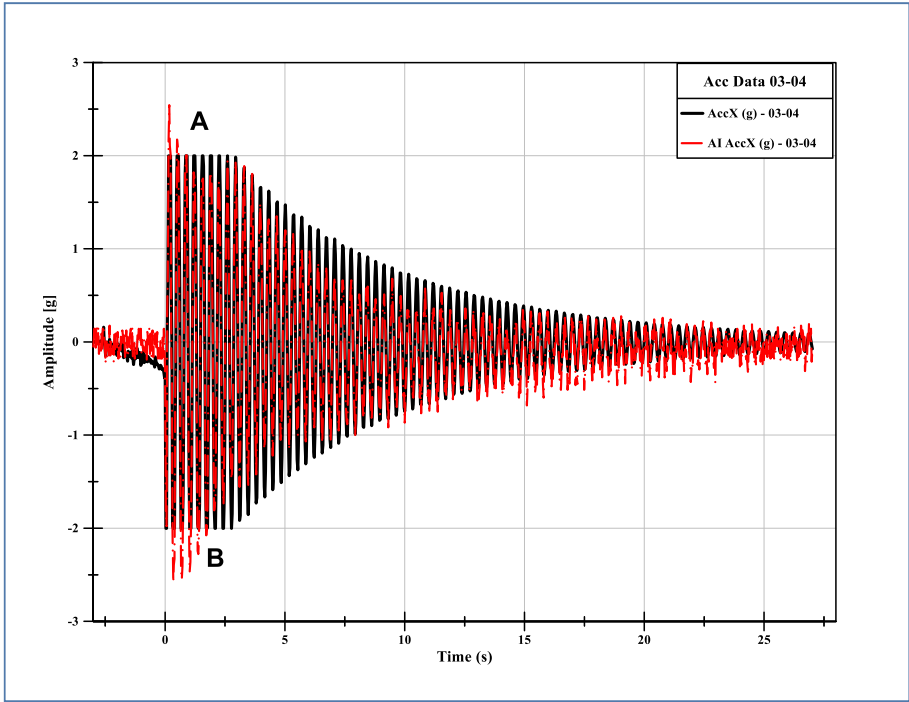


Figure 7-26 – Comparison of Analogue and IMU x-axis accelerometer for Test 03-04

The error measures for these tests, and the other eighteen tests where the Dytran accelerometer was used, are summarised in Table 7-12. The grey-shaded area within this table, covering tests 03-04 to 03-06, are those tests where the IMU accelerometer sensitivity had been set to 2g, however the response measured was greater than this causing the data to be ‘clipped’ (as shown at points A and B in Figure 7-26). For these three tests the comprehensive error measures (RC and SGC) were all below the comparable error measures for the other seventeen tests. This was most likely due to the fact that the reduced amplitude caused by the ‘clipping’ of the IMU data, meant that the curves being compared were more closely aligned.

The charts for Test 03-19 were chosen because the comprehensive and inequality measures for this test were the lowest suggesting the closest alignment. Careful visual inspection of Figure 7-27, but particularly Figure 7-28, confirm this. The latter figure shows that the Dytran accelerometer closely matched the IMU accelerometer in terms of frequency, however there were occasions when the former accelerometer’s amplitude was not comparable. Despite this, it was concluded that the cheaper IMU accelerometer consistently compared well to the more expensive Dytran accelerometer, a fact that was reinforced by the RC error measures in Table 7-12, that were all well below the 0.2 acceptance criteria (the highest RC value was 0.1556 for Test 03-13).

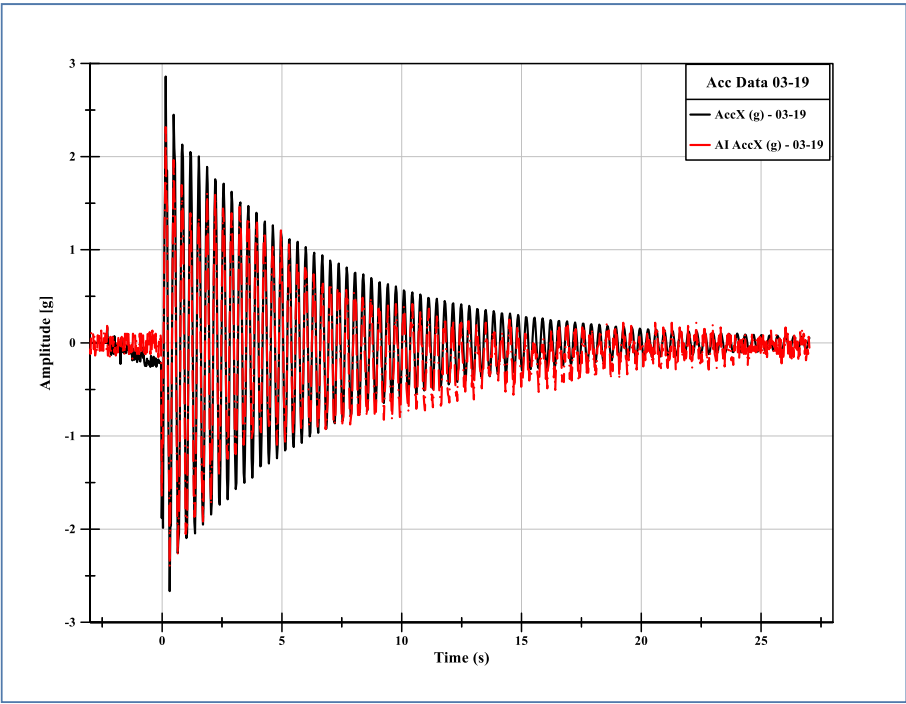


Figure 7-27 – Comparison of Analogue and IMU x-axis accelerometer for Test 03-19

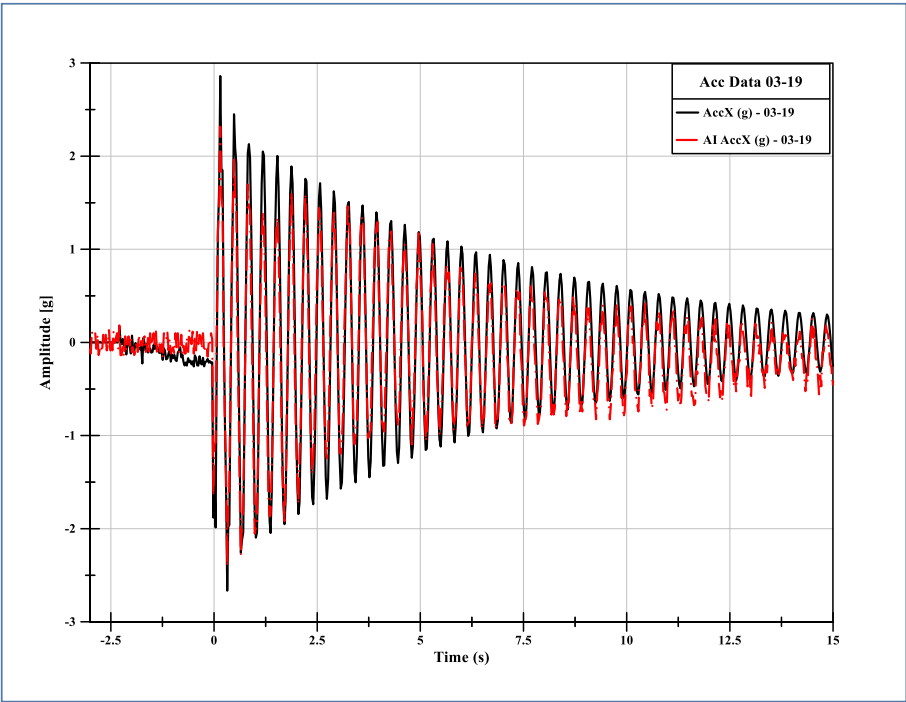


Figure 7-28 – Close-up comparison of Analogue and IMU x-axis accelerometer for Test 03-19

	03-01	03-02	03-03	03-04	03-05	03-06	03-07	03-08	03-09	03-10
RM	0.1195	0.1214	0.1187	0.0891	0.0942	0.0765	0.0963	0.1082	0.1107	0.1023
RP	0.1127	0.0980	0.1084	0.1066	0.1112	0.1163	0.1261	0.1068	0.1055	0.1272
RC	0.1456	0.1383	0.1425	0.1231	0.1292	0.1234	0.1406	0.1348	0.1356	0.1447
SGM	0.1708	0.1742	0.1695	0.1203	0.1285	0.1010	0.1318	0.1515	0.1557	0.1416
SGP	0.1127	0.0980	0.1084	0.1066	0.1112	0.1163	0.1261	0.1068	0.1055	0.1272
SGC	0.2047	0.1999	0.2012	0.1607	0.1699	0.1540	0.1824	0.1853	0.1881	0.1904
TI	0.1925	0.1725	0.1861	0.1757	0.1837	0.1877	0.2060	0.1808	0.1797	0.2088
	03-11	03-12	03-13	03-14	03-15	03-16	03-17	03-18	03-19	03-20
RM	0.0740	0.0989	0.0595	0.1169	0.1095	0.0996	0.1071	0.1026	0.1041	0.0501
RP	0.1354	0.1305	0.1651	0.1006	0.1036	0.1196	0.1206	0.1094	0.1070	0.1636
RC	0.1368	0.1451	<u>0.1556</u>	0.1366	0.1336	0.1379	0.1429	0.1329	<u>0.1323</u>	0.1517
SGM	0.0972	0.1360	0.0762	0.1662	0.1537	0.1371	0.1496	0.1420	0.1445	0.0631
SGP	0.1354	0.1305	0.1651	0.1006	0.1036	0.1196	0.1206	0.1094	0.1070	0.1636
SGC	0.1667	0.1885	<u>0.1818</u>	0.1943	0.1853	0.1819	0.1922	0.1793	<u>0.1798</u>	0.1754
TI	0.2159	0.2129	<u>0.2589</u>	0.1746	0.1767	0.1971	0.2003	0.1831	<u>0.1799</u>	0.2559

Table 7-12 – Error measures for Analogue and IMU x-axis accelerometer comparisons for Tests 03-01 to 03-20

The consistency of the data was further verified by the statistical parameters for the different error measures, collated in Table 7-13, which showed low standard deviations and variances for the RC, SGC and TI values. The low values were determined by comparing them against the comparable error measure statistical parameters for KF data in Table 7-11; RC = 0.008 v 0.035, SGC = 0.013 v 0.040 and TI = 0.024 v 0.061.

	SD	Var	Median	Avg
RM	0.019	0.00036	0.1024	0.0980
RP	0.018	0.00033	0.1120	0.1187
RC	<u>0.008</u>	<u>0.00007</u>	0.1373	0.1381
SGM	0.030	0.00090	0.1418	0.1355
SGP	0.018	0.00033	0.1120	0.1187
SGC	<u>0.013</u>	<u>0.00016</u>	0.1839	0.1831
TI	<u>0.024</u>	<u>0.00058</u>	0.1869	0.1964

Table 7-13 – Statistical parameters for Analogue and IMU x-axis accelerometer comparisons for Tests 03-01 to 03-20

From this it was concluded that for the tests conducted during this research, the cheaper IMU accelerometers performed as well as the more expensive Dytran accelerometer, producing consistent results and not requiring additional hardware to operate them.

7.2.4.3 Review of error measure acceptance criterion.

In Chapter 5 acceptance criterion for Russell's comprehensive error measure (RC) were discussed. Russell suggested acceptance criteria of 0.2, or 0.25 where the data involved shock and vibration data (Russell, 1997a, 1997b). No similar error measures were available for the Sprague and Geer's comprehensive error measure (SGC) (Sprague & Geers, 2004) or the Theil inequality error coefficient (TI) (Whang, Gilbert & Zilliacus, 1994).

To address this, a comparison between the RC, SGC and TI values for Experiment Three was undertaken using an Excel spreadsheet. It was assumed that the relationship between the different error/inequality criteria was linear and would therefore result in multiplication coefficients that could be used convert the RC acceptance criteria to provide similar acceptance criteria for the SGC and TI measures/coefficient. The resultant values for the complementary filter (CF) and Kalman filter (KF) are summarised in Table 7-14.

RC	CF 03-01 to 03-08		CF 03-09 to 03-20		CF 03-21 to 03-29		KF 03-01 to 03-08		KF 03-09 to 03-20		KF 03-21 to 03-29	
	SGC	TI	SGC	TI	SGC	TI	SGC	TI	SGC	TI	SGC	TI
0.2	0.23	0.34	0.23	0.34	0.23	0.34	0.22	0.34	0.23	0.34	0.24	0.33
0.25	0.28	0.43	0.28	0.43	0.29	0.43	0.28	0.43	0.28	0.43	0.31	0.41
RC	Average of CF values		Average of KF values		Acceptance Criteria							
	SGC	TI	SGC	TI	SGC	TI						
0.2	<u>0.23</u>	<u>0.34</u>	<u>0.23</u>	<u>0.34</u>	<u>0.23</u>	<u>0.34</u>						
0.25	<u>0.28</u>	<u>0.43</u>	0.29	<u>0.43</u>	<u>0.28</u>	<u>0.43</u>						

Table 7-14 – Comparative error measure acceptance criteria for Experiment Three

This table shows the average values for each of the measures/coefficients for the three sets of tests conducted as part of Experiment Three; the calculated values for the CF are in the top left quarter and values for the KF in the top right quarter. The values in the bottom left quarter are the averages of the data for the CF and KF in the other areas of the table, and the average of these values that result in comparable acceptance criteria of SGC 0.23 and 0.28 and TI 0.34 and 0.43.

7.2.4.4 Correlation between HSV and strain gauge data

Figure 7-29 and Figure 7-30 show comparisons of the strain gauge data with the corresponding HSV data for Test 03-11. Figure 7-29 provides an overview of the data while Figure 7-30 shows a close-up of the first five seconds. As observed in Experiment Two and discussed in section 7.2.3.1, the strain gauge data did not match the HSV data indicating that the rotations observed potentially related to large rather than small deflections. The relative magnitude between the HSV and strain gauge data was much less prominent than that observed in Experiment Two, between 2.0 and 1.5 (0.22 radians versus 0.12 radians – based upon data for Test 03-11) instead of approximately 3.0 for Experiment Two (0.166 radians versus 0.05 radians based upon data for Test 02-01). Whilst

this might be taken to indicate that an upper rotation limit for the small rotations/deflections was close, the similar maximum rotations (0.166 radians for Experiment Two and 0.22 radians for Experiment Three) suggested that further research was required to determine this upper limit.

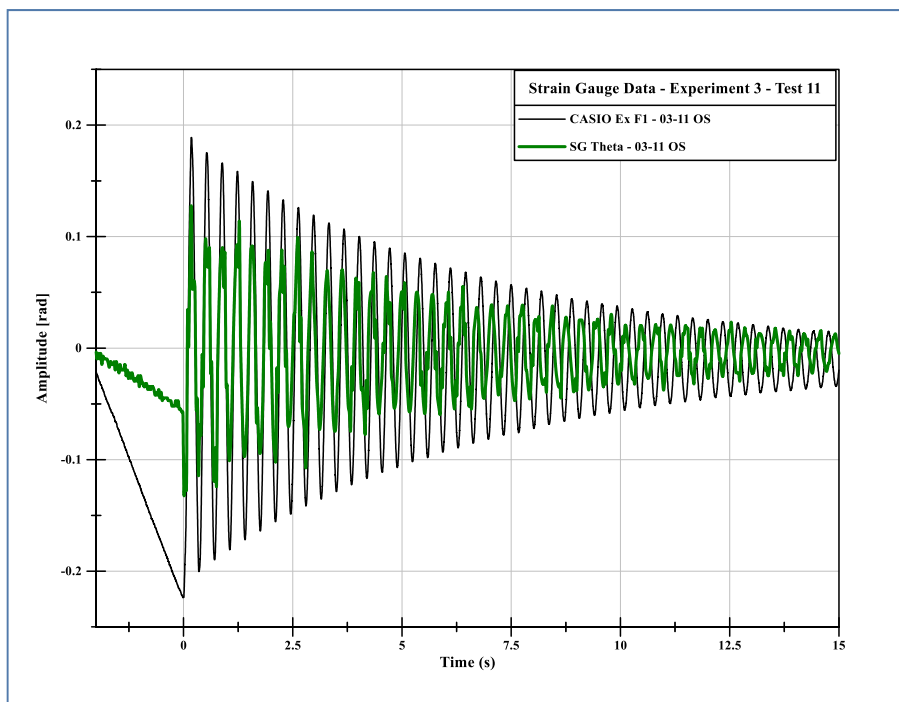


Figure 7-29 – Comparison of HSV and strain gauge data for test 03-11

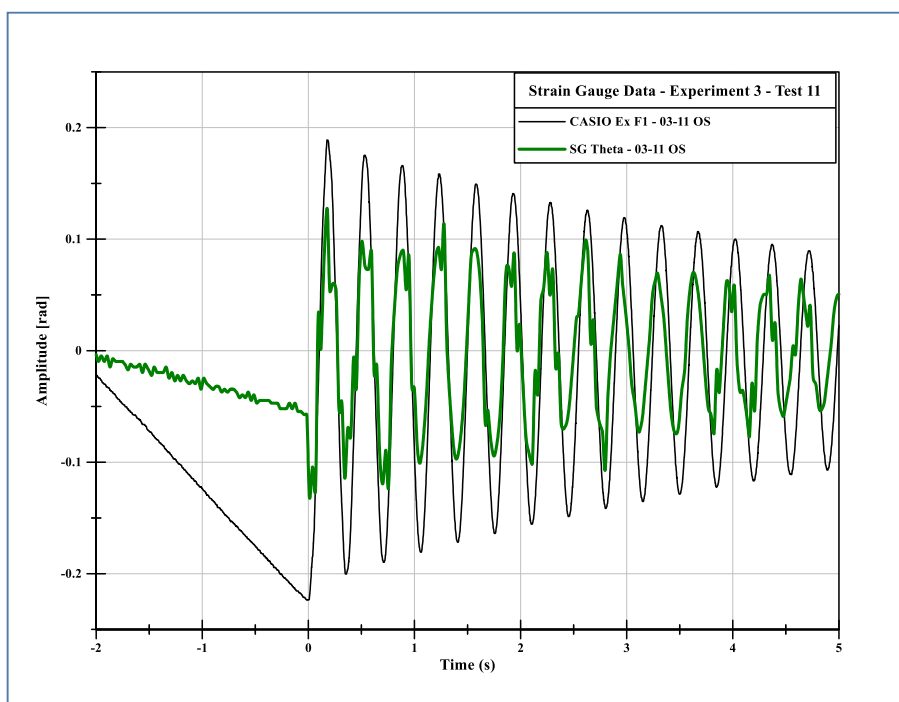


Figure 7-30 – Close-up comparison of HSV and strain gauge data for test 03-11

7.3 Experiment Four – Vertical cantilever subjected to ballistic loading

To investigate the performance of the sensors when subject to real high magnitude, short-duration loads the vertical cantilever experimental assembly used in Experiments Two and Three, was subject to a ballistic load. The main difference was the inclusion of an aluminium ‘target plate’ which was attached to the top of the cantilever above the sensors.

7.3.1 Experimental arrangement

The Experiment used the identical vertical cantilever, sensors and strain gauges as those used in Experiments Two and Three. The key dimensions and components of the experimental setup are shown in Figure 7-31, whilst a photograph showing the key elements can be found in Figure 7-32. A 60mm x 30mm x 1.5mm aluminium target plate was securely fixed to the tip of the cantilever on the opposite side of the cantilever to the sensors. Unlike the previous vertical cantilever experiments, the dynamic load was applied to the tip of the cantilever by firing a 0.357” calibre round (see Figure 7-33) at the aluminium target using a purpose-built proof barrel (see Figure 7-34). As in all the previous experiments the motion of the vertical cantilever was monitored and recorded for subsequent analysis using an HSV camera mounted to the left side of the vertical cantilever (Figure 7-35).

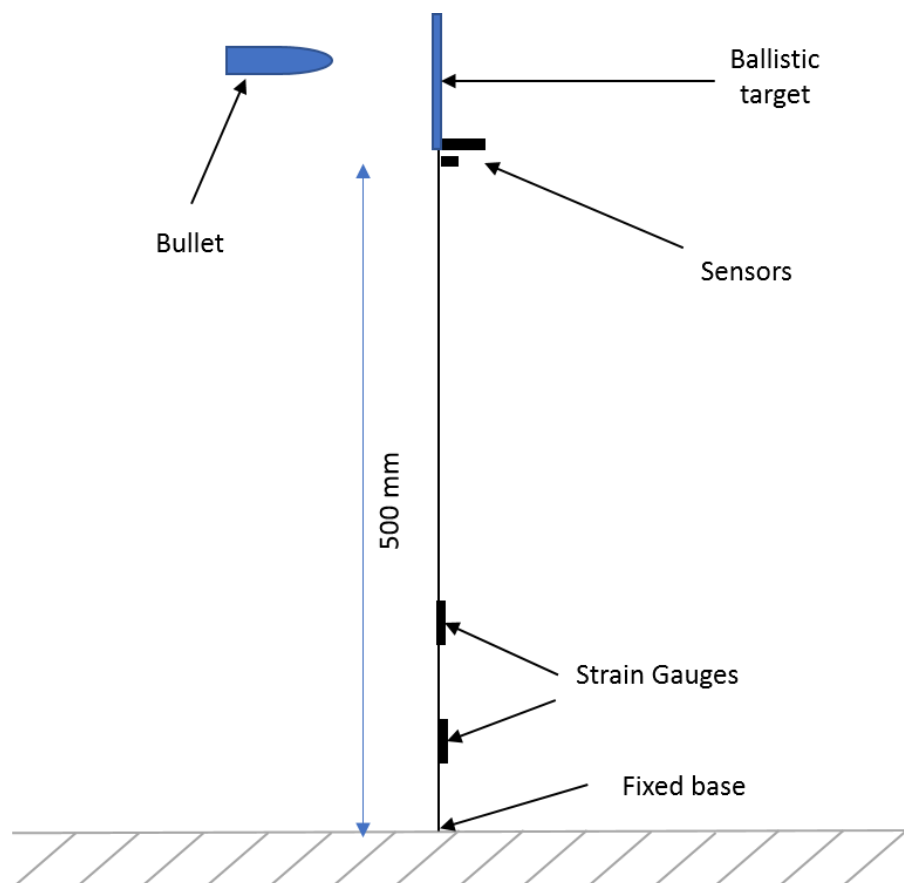


Figure 7-31 – Experiment Four main components and key dimensions

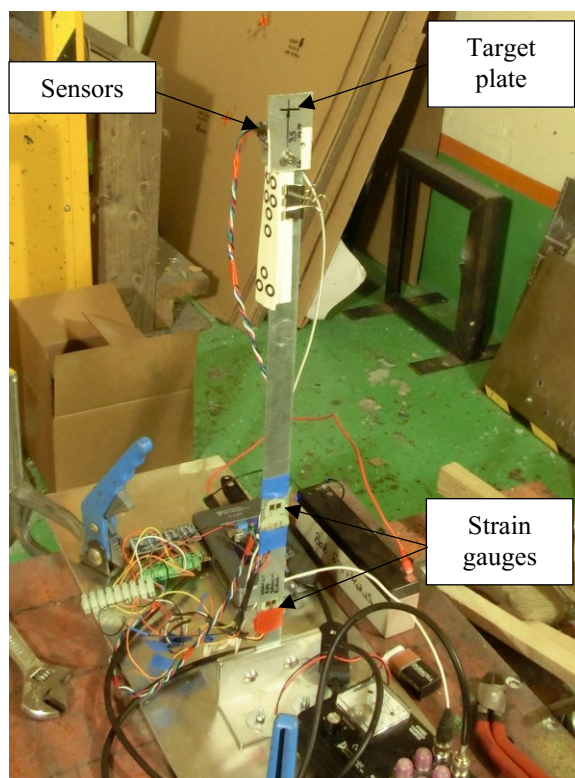


Figure 7-32 – Photograph showing the general arrangement for Experiment Four



Figure 7-33 – Photograph of 0.357" calibre round

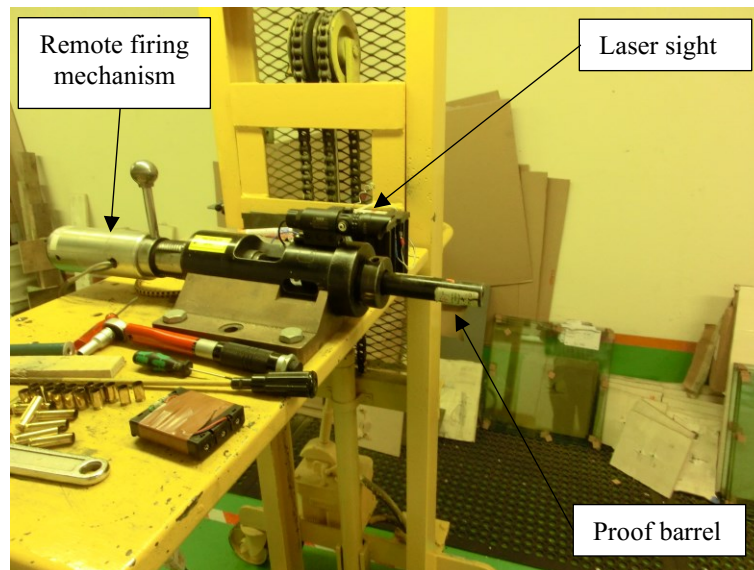


Figure 7-34 – Photograph showing the firing arrangement and proof barrel used to fire the rounds for Experiment Four

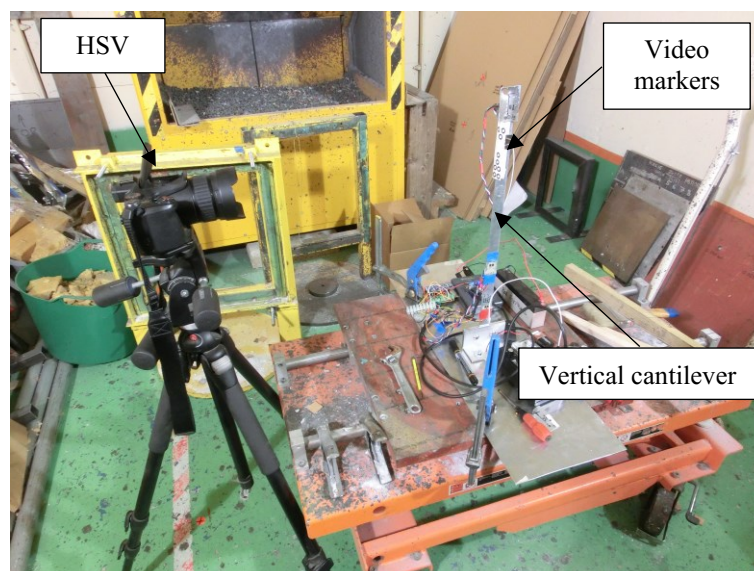


Figure 7-35 – Photograph showing the HSV camera mounted to left of the vertical cantilever

7.3.2 Experimental procedure

The experimental procedure adopted was similar to that used in Experiment Three, however due to the safety procedures required when firing ballistic rounds, the data logging and the HSV camera had to be started in advance of the rounds being loaded into the proof barrel. This meant that both the LabVIEW data and video footage files were substantially longer than in previous experiments, and therefore required editing prior to their analysis. The 0.357" round was fired from the proof barrel remotely having previously been aligned with the target using a laser sight, and the bullet velocity was measured using a velocity screen placed between the barrel and the target.

Figure 7-36 is a photograph of the target plate after the first shot showing the bullet-hole where the round penetrated the target at the top edge. The round impacting the target caused the vertical cantilever to deflect and

then oscillate in a similar manner to that observed in Experiment Three. Figure 7-37 is the HSV data curve for Test 04-02.

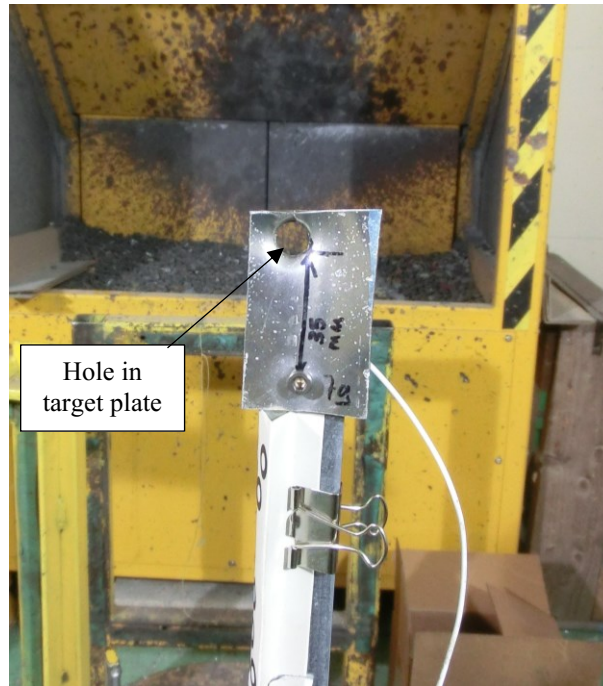


Figure 7-36 – Photograph showing one of the target plates used in Experiment Four after being shot

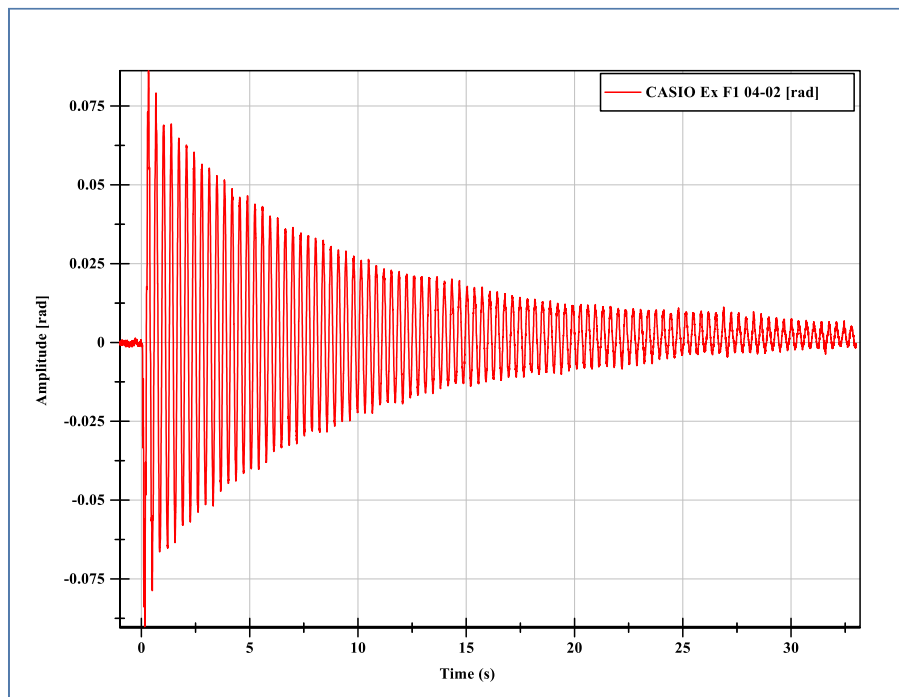


Figure 7-37 – HSV data for Test 04-02

To establish how the ballistic load compared to a typical load used in Experiment Three, the HSV data from Test 04-02 was compared to that from Test 03-20 as shown in Figure 7-38. The green curve is the response of the vertical cantilever subject to the ballistic load while the red curve is the response for the vertical cantilever

following sudden load removal. This shows that the frequency of oscillation for both experiments was similar. However, the amplitude of the oscillations for the vertical cantilever subject to the sudden load removal, was approximately 2.5 times that of the oscillations for the vertical cantilever subject to the ballistic load. This demonstrated that the ballistic load for this experiment imparted less force than the sudden load removal.

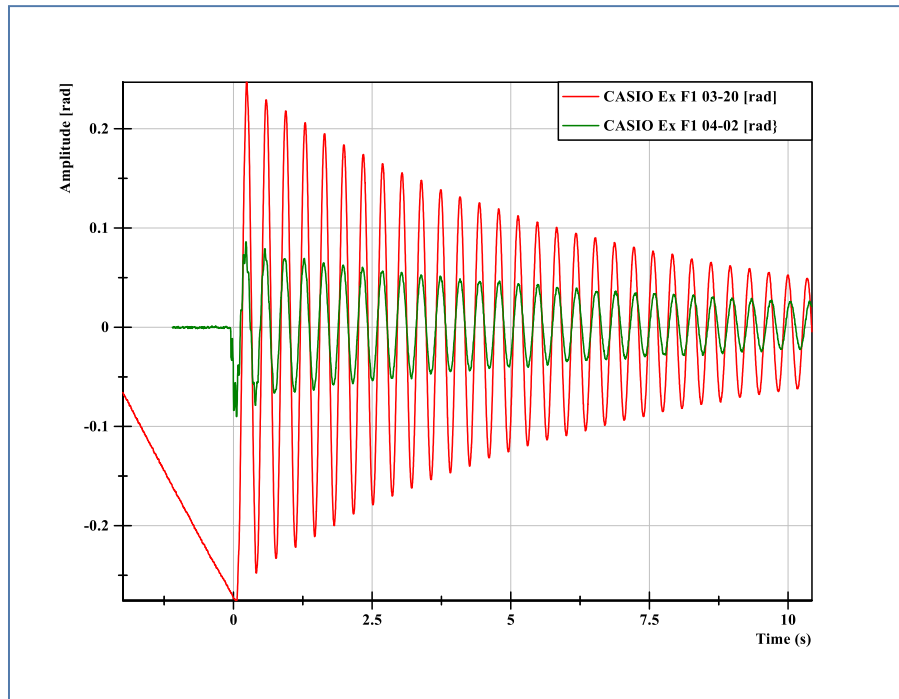


Figure 7-38 – Comparison between HSV data from Experiments Three and Four

7.3.3 Presentation of results

The sensitivity and timing loop settings used for the tests are summarised in Table 7-15 along with the velocity of the bullet just before it hit the target, while charts showing the HSV, filtered and strain gauge data for each test are shown in Figure 7-39 to Figure 7-46. The filter settings applied to the original data are summarised in Table 7-16, and the error measures calculated from the HSV, filtered and strain gauge data, are shown in Table 7-17.

	04-02	04-03	04-04	04-05
Clock speed (MHz)	1	1	1	1
Timing loop period (μS)	1000	1000	1000	1000
Gyro sensitivity (deg/s)	500	500	500	500
Acc sensitivity (g)	8	4	4	4
Bullet Velocity (m/s)	429	425	430	432

Table 7-15 – IMU sensitivity and timing loop settings for tests 04-02 to 04-05

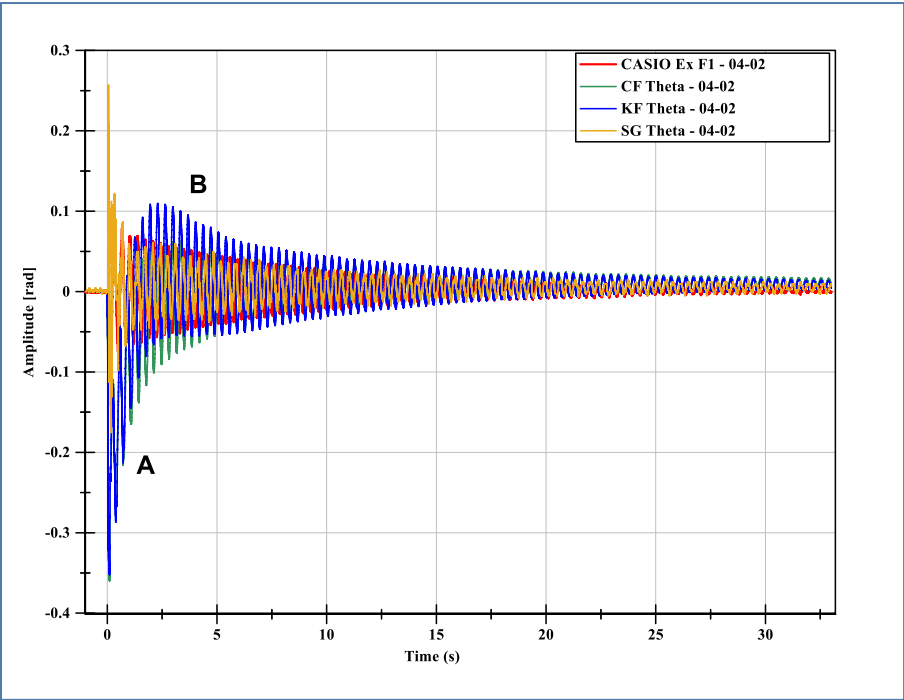


Figure 7-39 – Comparison between HSV, CF, KF and strain gauge data for test 04-02

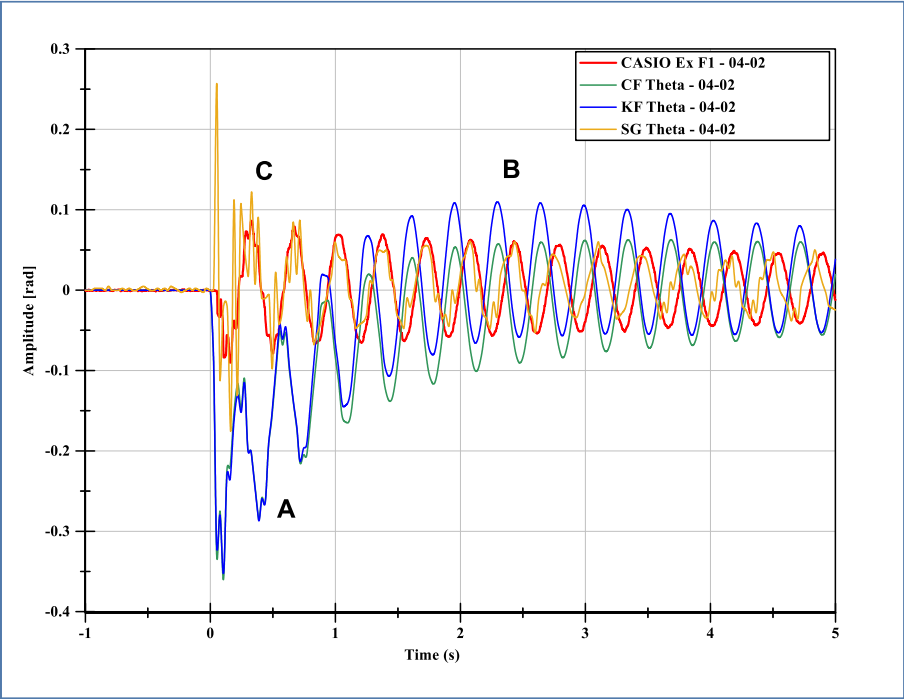


Figure 7-40 – Close-up comparison between HSV, CF, KF and strain gauge data for test 04-02

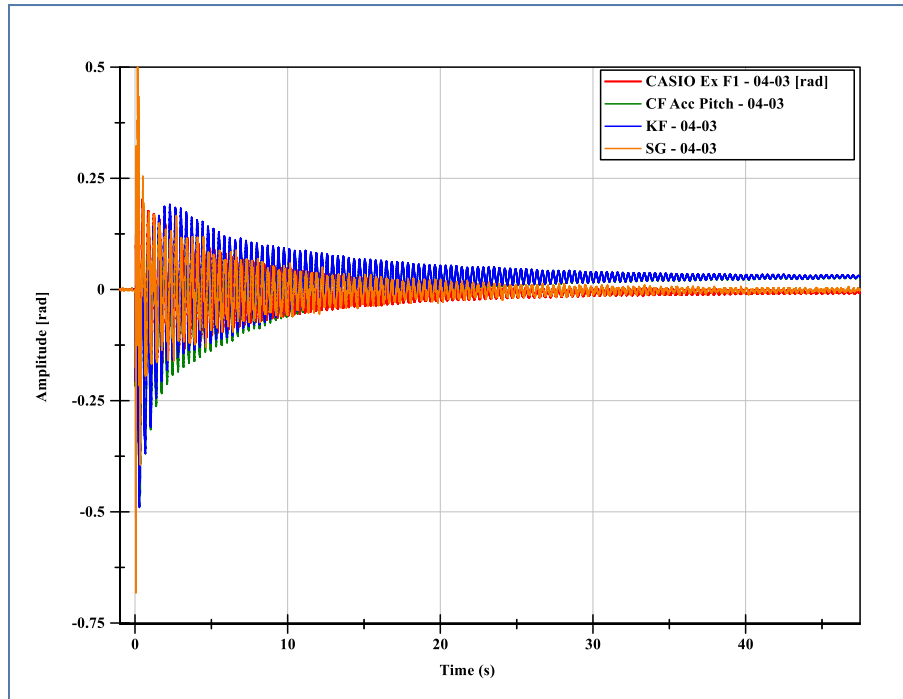


Figure 7-41 – Comparison between HSV, CF, KF and strain gauge data for test 04-03

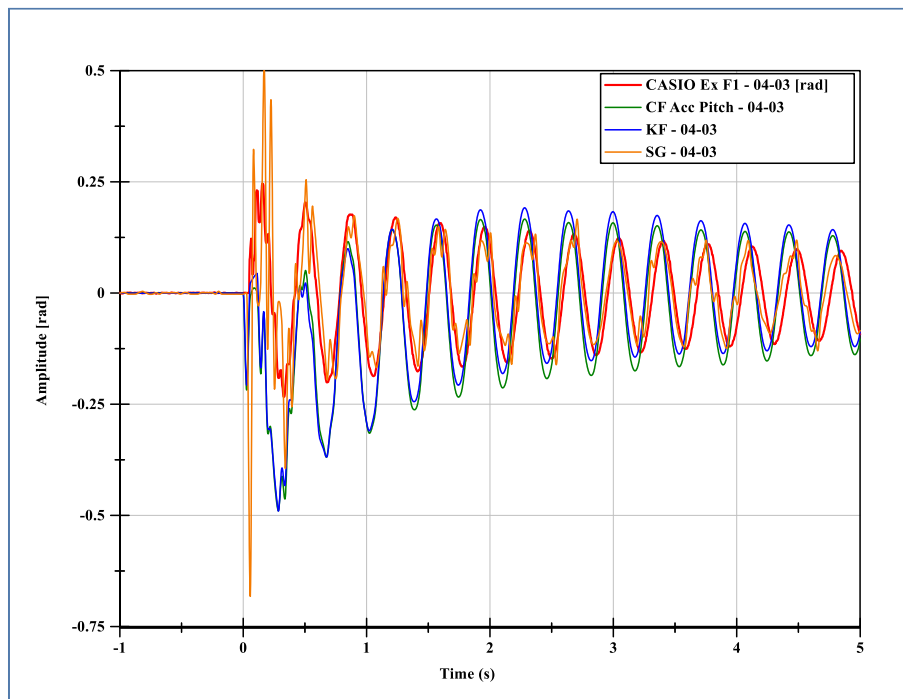


Figure 7-42 – Close-up comparison between HSV, CF, KF and strain gauge data for test 04-03

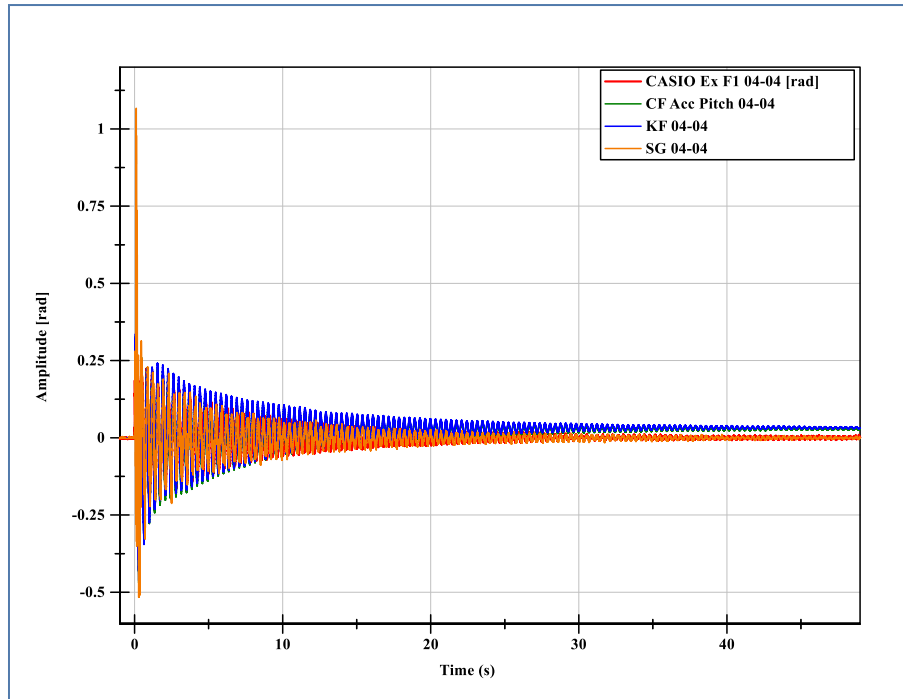


Figure 7-43 – Comparison between HSV, CF, KF and strain gauge data for test 04-04

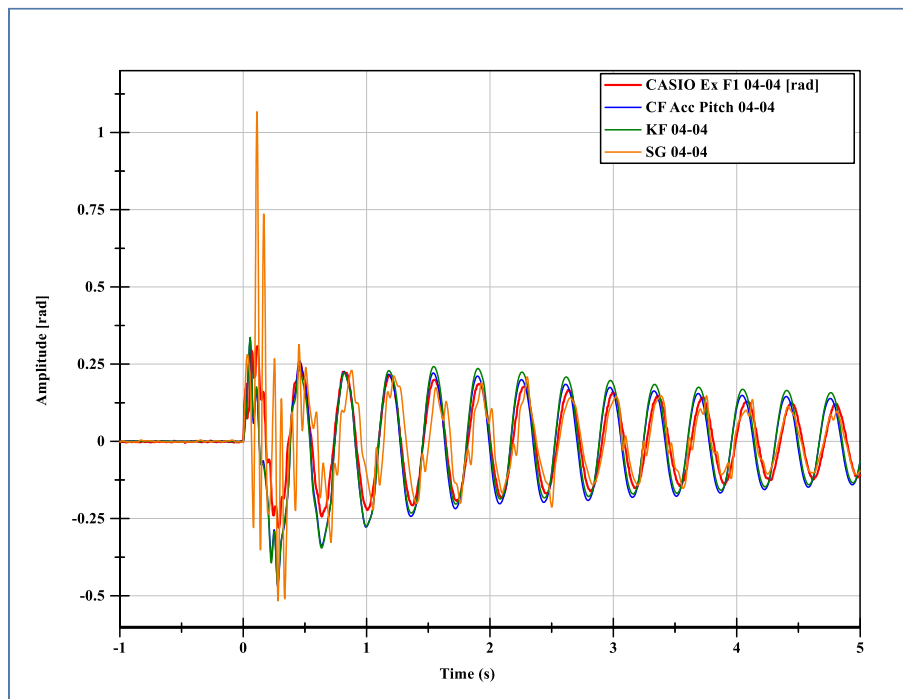


Figure 7-44 – Close-up comparison between HSV, CF, KF and strain gauge data for test 04-04

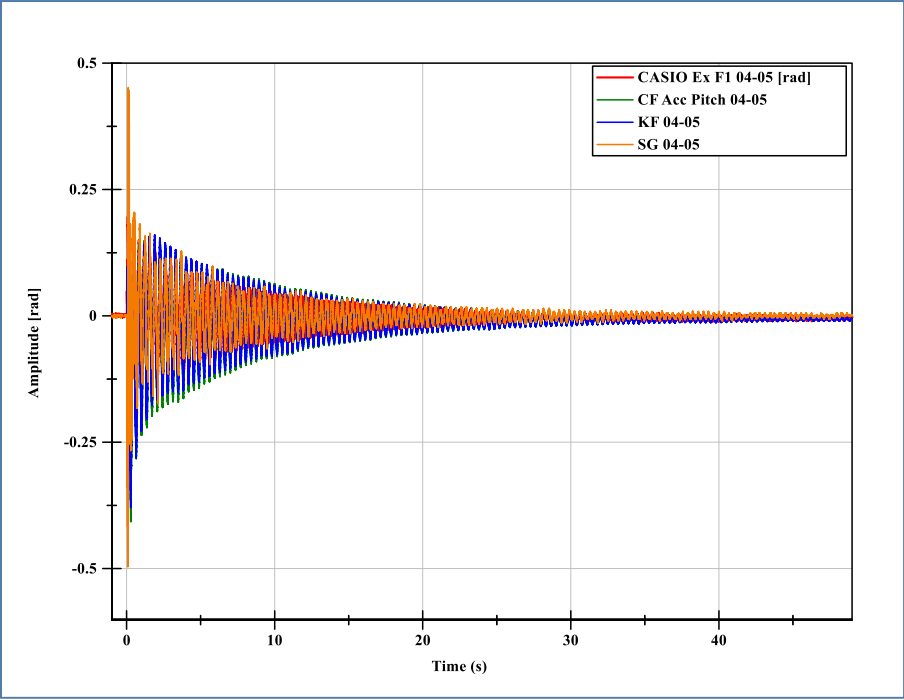


Figure 7-45 – Comparison between HSV, CF, KF and strain gauge data for test 04-05

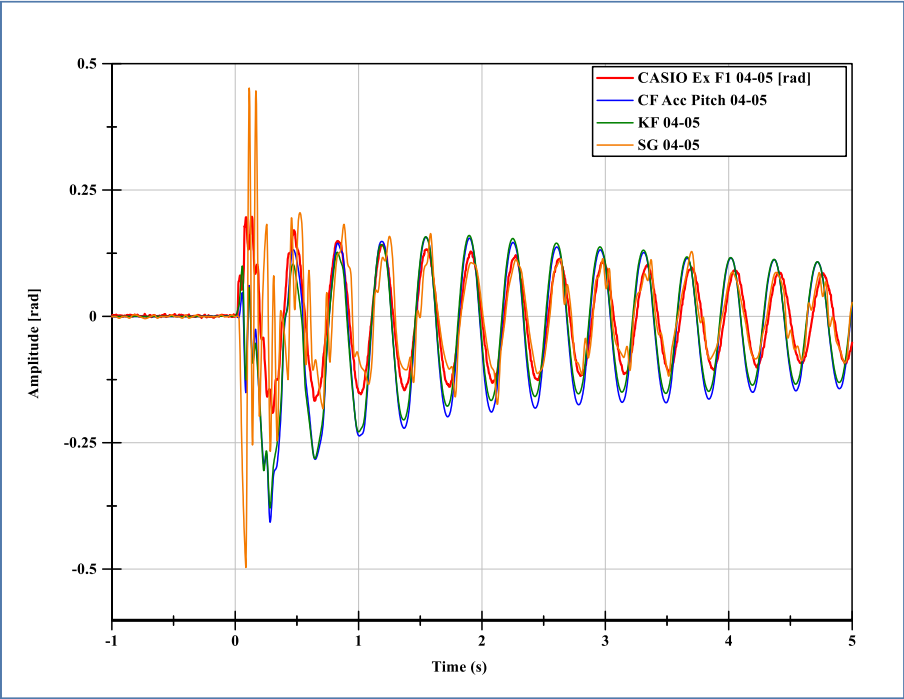


Figure 7-46 – Close-up comparison between HSV, CF, KF and strain gauge data for test 04-05

	04-02	04-03	04-04	04-05
CF Alpha	0.96	0.95	0.96	0.945
Q Angle	0.00001	0.0001	0.0001	0.0001
Q Gyro Bias	0.001	0.0001	0.0001	0.0001
R Measure	0.05	0.008	0.01	0.007

Table 7-16 – Filtered constants for tests 04-02 to 04-05

	04-02			04-03		
	HSV/ CF	HSV/ KF	HSV/ SG	HSV/ CF	HSV/ KF	HSV/ SG
RM	-0.4193	-0.4243	0.0444	-0.2818	-0.2758	-0.0144
RP	0.5820	0.5950	0.3750	0.3967	0.4032	0.3150
RC	<u>0.6357</u>	<u>0.6476</u>	<u>0.3347</u>	<u>0.4313</u>	<u>0.4329</u>	<u>0.2795</u>
SGM	-0.5242	-0.5298	0.0553	-0.3574	-0.3497	-0.0167
SGP	0.5820	0.5950	0.3750	0.3967	0.4032	0.3150
SGC	<u>0.7833</u>	<u>0.7966</u>	<u>0.3791</u>	<u>0.5339</u>	<u>0.5337</u>	<u>0.3154</u>
TI	<u>0.8212</u>	<u>0.8323</u>	<u>0.5561</u>	<u>0.6097</u>	<u>0.6160</u>	<u>0.4749</u>
	04-04			04-05		
	HSV/ CF	HSV/ KF	HSV/ SG	HSV/ CF	HSV/ KF	HSV/ SG
RM	-0.1925	-0.2228	-0.0527	-0.2573	-0.2277	-0.0483
RP	0.2819	0.2845	0.2901	0.3060	0.3108	0.3361
RC	<u>0.3026</u>	<u>0.3203</u>	<u>0.2613</u>	<u>0.3543</u>	<u>0.3414</u>	<u>0.3009</u>
SGM	-0.2408	-0.2805	-0.0625	-0.3256	-0.2869	-0.0571
SGP	0.2819	0.2845	0.2901	0.3060	0.3108	0.3361
SGC	<u>0.3707</u>	<u>0.3996</u>	<u>0.2968</u>	<u>0.4468</u>	<u>0.4230</u>	<u>0.3409</u>
TI	<u>0.4460</u>	<u>0.4566</u>	<u>0.4411</u>	<u>0.4934</u>	<u>0.4918</u>	<u>0.5043</u>

Table 7-17 – Error measures for tests 04-02 to 04-05

7.3.4 Analysis and discussion of results

7.3.4.1 Correlation between HSV and filtered data

A visual inspection of the data charts, and the error measure values, indicates that the results from these tests were not as good as for the other three experiments. The results for Test 04-02 are particularly poor as illustrated well by the divergence of both the CF and KF data curves from the HSV curve; in Figure 7-39 both curves start below the HSV curve (Point A) before rising towards the HSV curve after about 5 seconds (Point B). The KF curve then rises above the HSV curve, whilst the CF curve starts to tend towards the HSV curve. The initial 5 seconds are shown in Figure 7-40 and highlight this divergence even better. The higher comprehensive error measures (RC and SGC), than those seen in the previous experiments, also confirms this lack of correlation.

The reason for this divergence was due to initial high peaks in both the IMU accelerometer and gyroscope data which are shown in Figure 7-47 and Figure 7-48 respectively. The peaks have been marked with an A and compared against the same data from Test 03-20. The significant difference in the initial amplitude of these peaks compared to a ‘normal’ test (Test 03-20) is clearly evident. Both filters needed to be adjusted to cater for these

discrepancies, as can be seen by the adjusted filter constants in Table 7-16, and even then it took over 5 seconds before the data started to converge towards the HSV data.

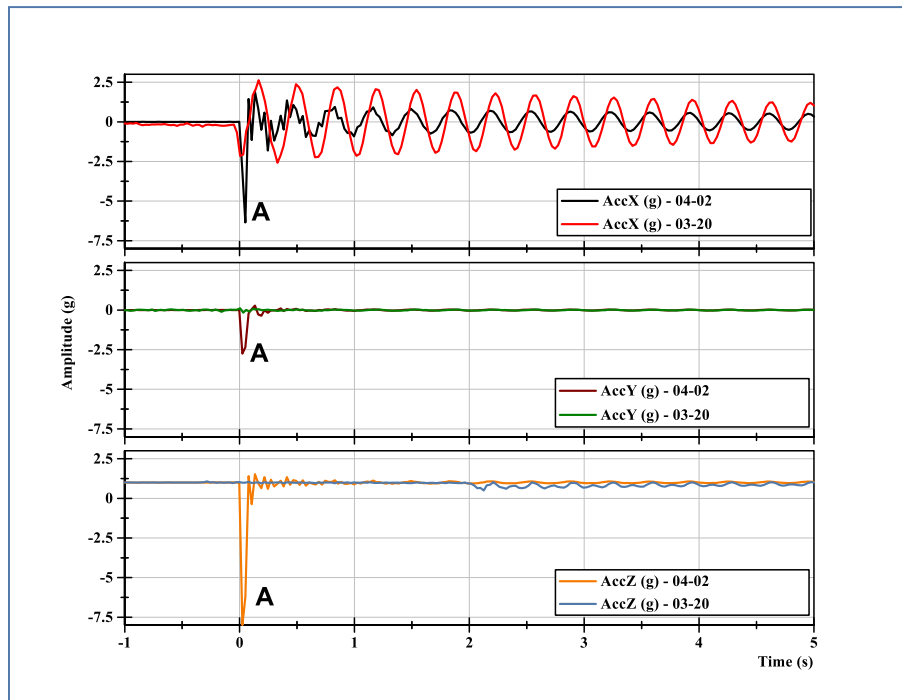


Figure 7-47 – Comparison of Accelerometer Data between Test 04-02 and Test 03-20

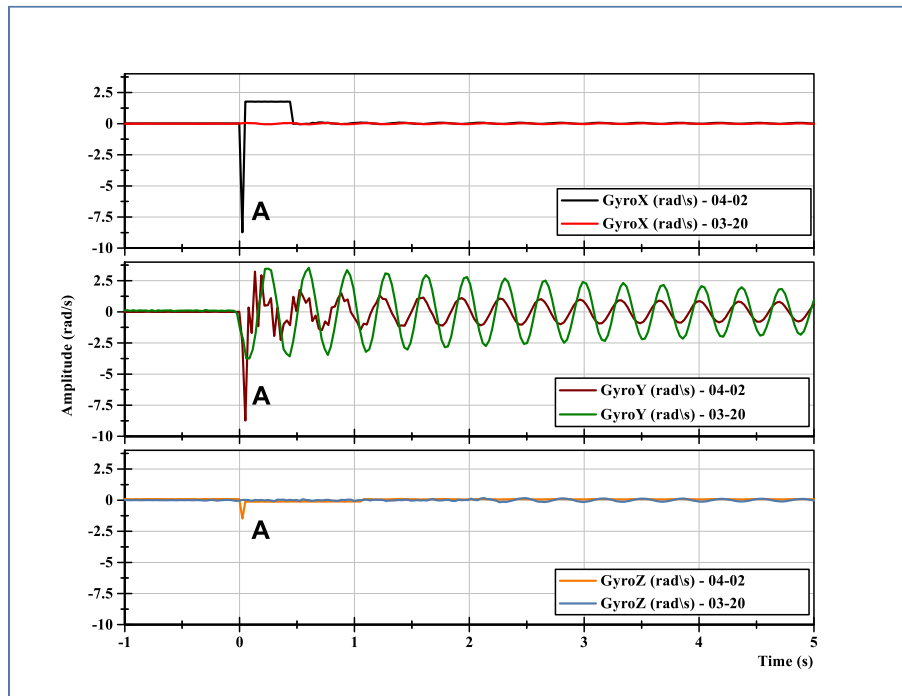


Figure 7-48 – Comparison of Gyroscope Data between Test 04-02 and Test 03-20

The spikes in the data were caused by the initial impact of the bullet with the target plate, and due to the rigid connection between the sensors and the cantilever, this was transmitted to the sensor. Including an absorbent

material between the IMU and the cantilever would probably reduce these peaks, however due to time constraints when conducting the tests, and the limited access to the ballistic facilities it was not possible to investigate this further.

Although the results for the other three tests in this experiment were significantly better than Test 04-02, none of them met Russell's 0.25 acceptance criteria. In addition, all the tests required adjustments to be made to the filter constants to ensure the best correlation was achieved. Despite this, the results indicate that the proposed filter methods can be used in ballistic scenarios however, a means of isolating the IMU from the effects of the initial ballistic impact need to be investigated further.

7.3.4.2 Correlation between HSV and strain gauge data

By comparison, the strain gauge data was much better correlated for all four tests when compared to similar comparisons made for Experiment Two and Experiment Three. This was highlighted by the generally lower comprehensive error measure values related to the comparable filtered data values. Except for the initial one or two seconds after impact, where the strain gauge data contained quite a lot of noise, the remainder of the data matched the HSV data reasonably well. This can be seen in the close-up charts in Figure 7-40, Figure 7-42, Figure 7-44 and Figure 7-46. Both Test 04-03 and Test 04-04 had RC error measures very close to the 0.25 acceptance criteria (0.2795 and 0.2613 respectively), indicating that for these tests the correlation was reasonably close.

The improved results for the strain gauge data in this experiment, suggested that the proposed method did provide the potential for determining the rotation of a structure if the rotations were small. Figure 7-39 to Figure 7-46 show that except for Test 04-02 the peak rotation values for the other three tests (Tests 04-03, 04-04 and 04-05) were -0.25 radians and 0.25 radians; peak rotations for Test 04-02 were -0.10 radians and 0.10 radians. The former peak values are greater than those for Experiments Two and Three as can be seen in Figure 7-51 and Figure 7-52 respectively. The peak value for Experiment Two was 0.166 radians while for Experiment Three was between -0.2 and 0.2. As the same algorithm was employed to calculate the rotation using the strain gauge data for all three experiments it was not possible to confirm that the difference between this and the observed rotations was due to small rotations. Therefore, further research would be required to ascertain the significance of the amplitude differences between the HSV and strain gauge data seen in Experiment Two, and the improved results for Experiment Three and Four. In addition, research to investigate methods to reduce the noise, inherent when using bonded strain gauges, needs to be conducted.

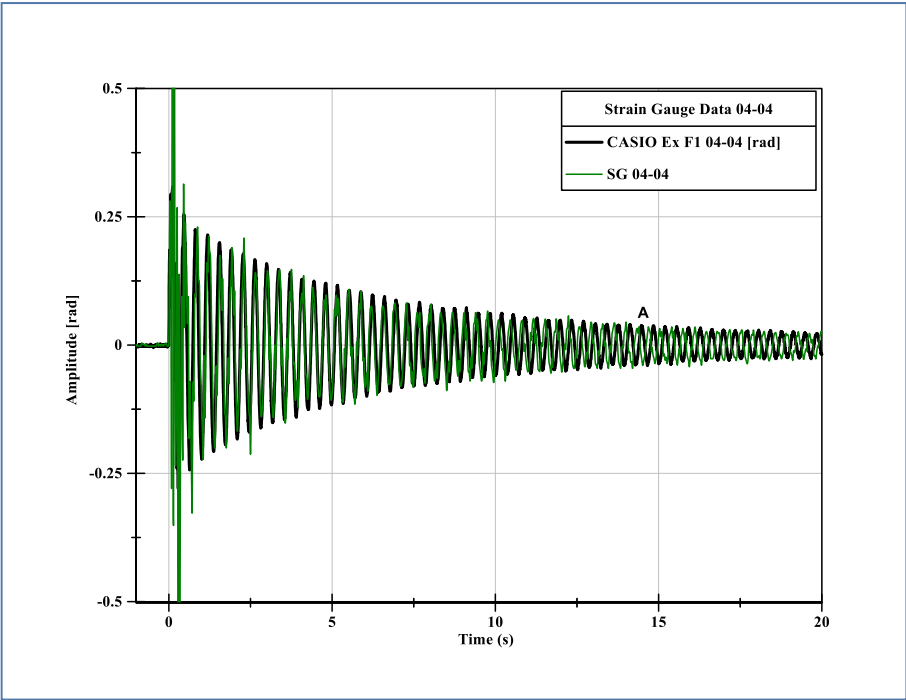


Figure 7-49 – Overall comparison of HSV and strain gauge data for Test 04-04

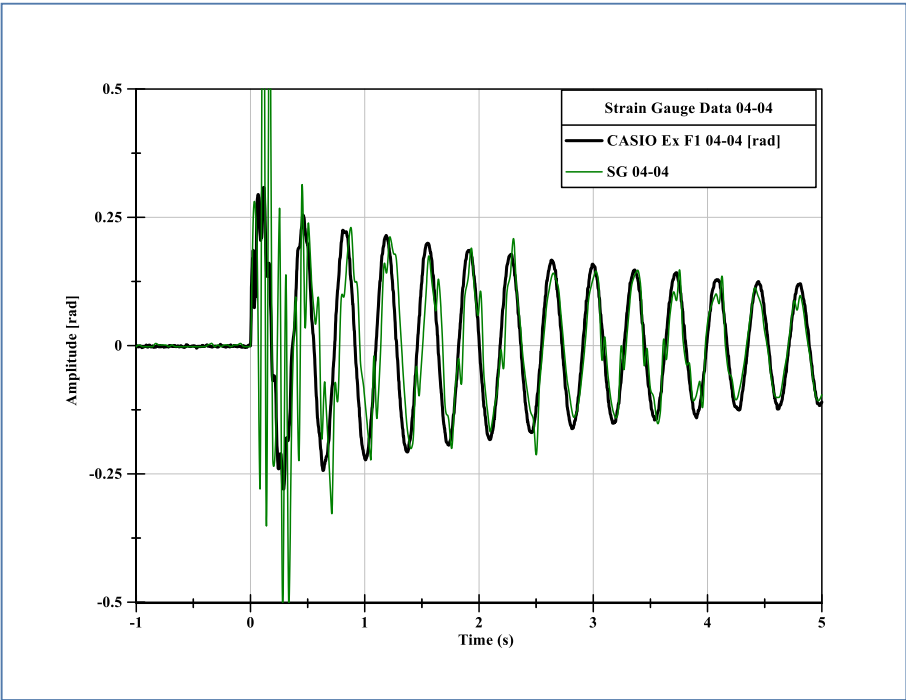


Figure 7-50 – Close-up comparison of HSV and strain gauge data for Test 04-04

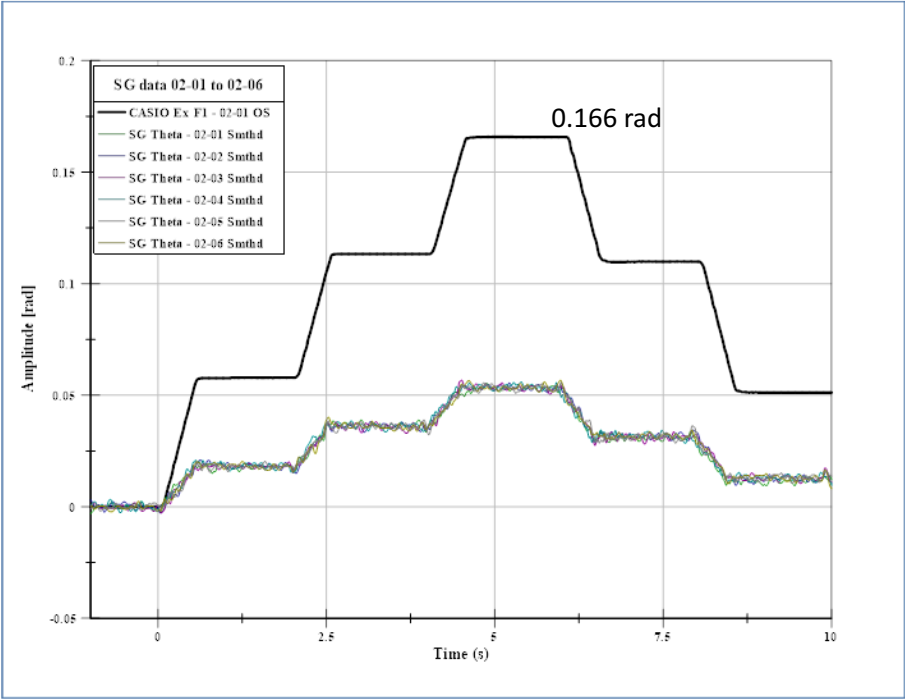


Figure 7-51 – Close-up of Strain Gauge Smoothed data for tests 02-01 to 02-06

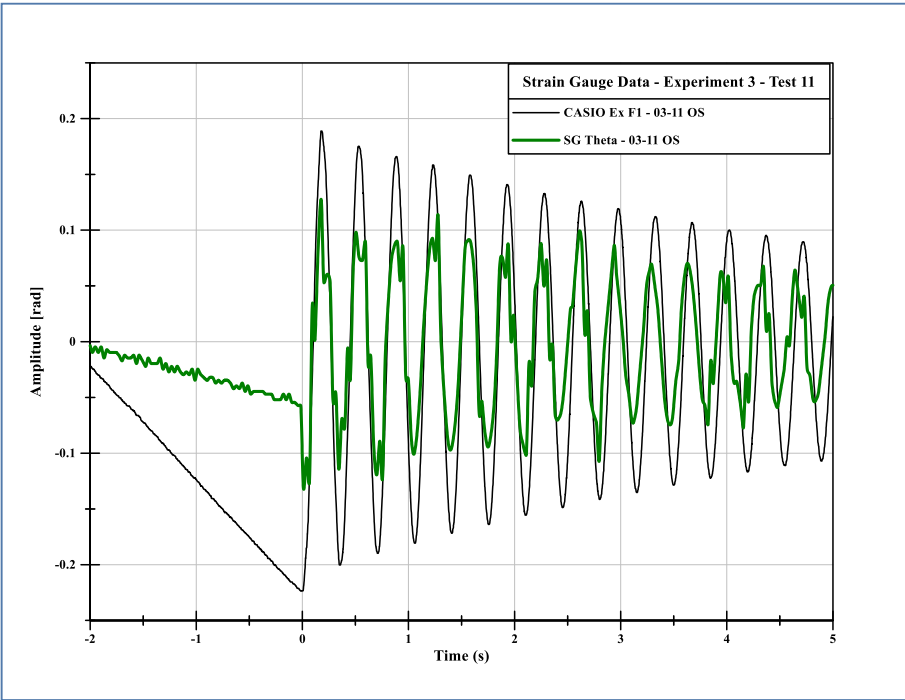


Figure 7-52 – Close-up comparison of HSV and strain gauge data for test 03-11

7.4 Experiment Five – Vertical monolith panel subjected to ballistic loading

For Experiment Five, a vertical monolith panel was subjected to a ballistic load from a 7.62 x 39 calibre round (see Figure 7-53), typically fired from an AK47 assault rifle. The research experiment was conducted as part of the main tests, the purpose of which was to investigate whether the vertical monolith would be overturned when subject to a ballistic attack. These tests were part of a wider research programme to develop a lightweight protective panel, that could be used by firearms officers to seek refuge in the event of being fired upon by an attacker. The sensors from the thesis research were used to monitor the overturning rotation of the panel.



Figure 7-53 – Photograph of 7.62 x 39 round

7.4.1 Experimental arrangement

The main panel was manufactured from 18mm thick plywood, except for a 250mm section centred 1495mm from the ground. This section was manufactured from a 20mm thick panel of bullet-resistant Dynema that formed the ballistic target. This panel was set at this height to maximise the lever-arm created when the panel was subject to the ballistic load, thereby maximising the overturning moment. The key dimensions, and components of the experimental arrangement, are summarised in Figure 7-54, while Figure 7-55 is a general photograph of the panel and sensor arrangement. Two aluminium channel sections were slid down the edge of the plywood/dynema panel to hold the sections together and to provide the panel with some rigidity. As the purpose of the tests was to measure the overturning rotation of the panel, only the IMU sensors were used in this experiment (gyroscope and accelerometer). The IMU was mounted on the aluminium channel section approximately 500mm above floor level.

The tests were conducted at the Wiltshire Ballistic Services (WBS) range in Devizes, Wiltshire. WBS use a custom-made firing arrangement which is shown in Figure 7-56. The rigid mounting framework can be adjusted both vertically and horizontally, thus enabling a number of target points to be selected without having to move the target. It is also possible to adjust the vertical trajectory of the barrel via a screw at the rear of the assembly, to enable additional elevation to be achieved if required. A laser pointer aligned to the axis of the barrel was used

to ensure that each shot was accurately aimed. Figure 7-57 shows the movable velocity screens that were placed in front of the target to measure the velocity of the bullet at a set distance before the target.

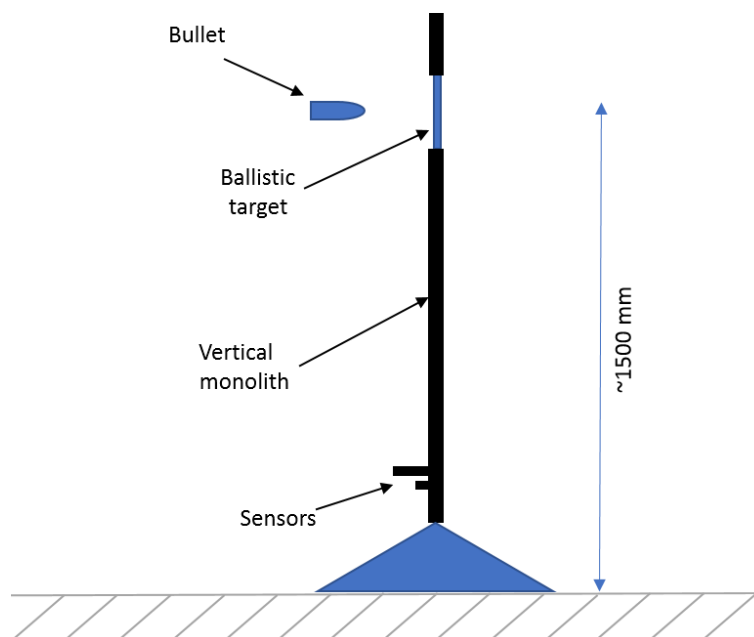


Figure 7-54 – Experiment Five main components and key dimensions

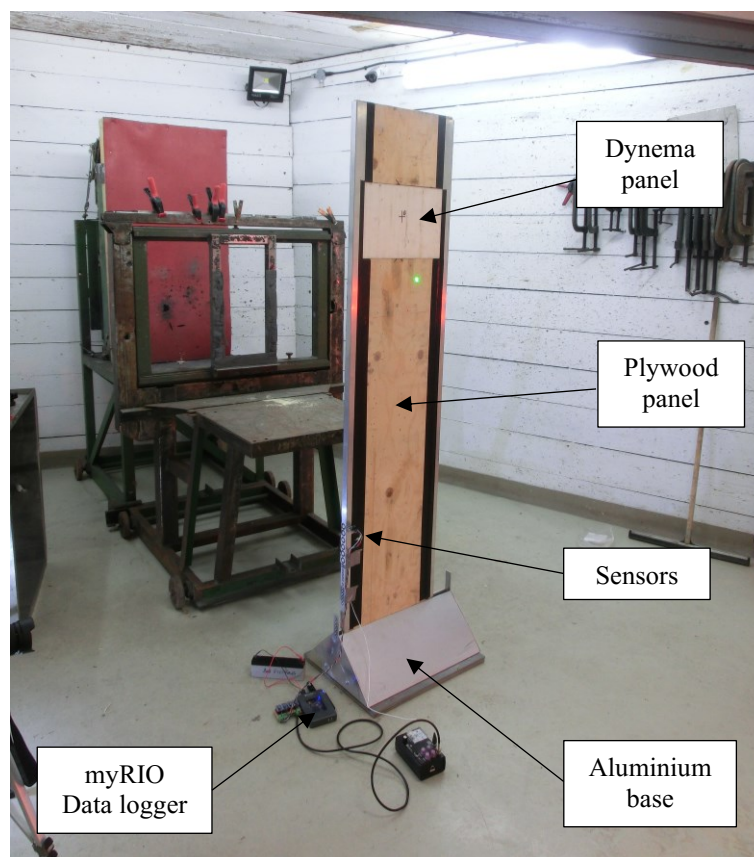


Figure 7-55 – Photograph showing the general arrangement for Experiment Five vertical monolith

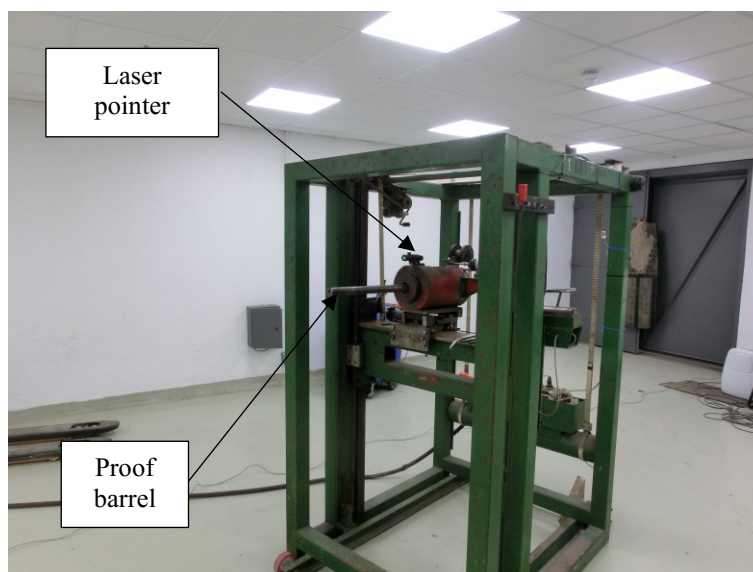


Figure 7-56 – Photograph showing the firing arrangement and proof barrel used to fire the rounds for Experiment Five

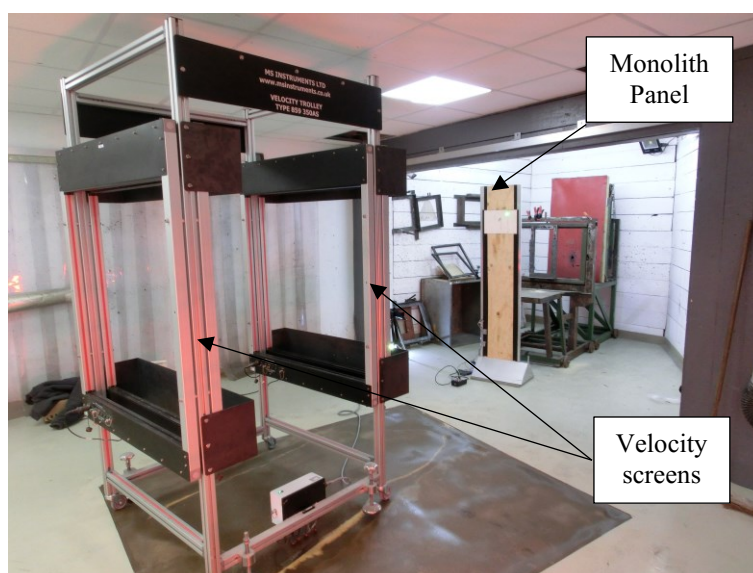


Figure 7-57 – Photograph showing the velocity screens used to measure the round velocities for Experiment Five

Although the results from the test were not being used for any form of certification or approval process, the procedures and criteria contained in BS EN 1522 (BSi, 1999a) and BS EN 1523 (BSi, 1999b) were used to provide a standard reference against which the performance could be measured. The key criteria applicable to these tests are summarised in Table 7-18.

Criteria		
Range from barrel to target	10 m	±0.5 m
Range from velocity screen to target	2.5 m	
Optimum velocity	700 m/s	±0.5 m/s

Table 7-18 – Experiment Five Range test criteria

The purpose of the main test was to investigate the effect of a ballistic attack on the monolith panel arrangement's overall stability, and in order to investigate its ability to resist over-turning, a number of different base unit configurations were tested. The configuration, and hence the overall weight of the base unit, was primarily altered by the addition or removal of the base-plate support tube or base-plates; the individual base elements are shown in the photographs in Figure 7-58.



Figure 7-58 – Photograph showing monolith base unit elements: aluminium base-unit (left); base-plate support tube (centre); base-plate (right).

Details of the different configurations used for the individual tests, along with the recorded bullet velocity prior to impact, are summarised in Table 7-19 and the dimensions and weights for the component parts are shown in Table 7-20.

Test No	Base-plate	Support Tubes	Bullet Velocity (m/s)
05-01	3	2	719.6
05-02	0	2	733.66
05-03	1	2	733.08
05-04	1	2	699.78
05-05	1	2	738.78

Table 7-19 – Experiment Five Range base-plate configurations for each test

Component	Width (mm)	Height (mm)	Thickness (mm)	Mass (kg)
Plywood panel	390	1495	18	8.40
Dynema armour panel	390	218	20	1.55
Base-plate (bottom)	400	495	6	8.80
Base-plate (intermediate)	400	495	6	8.85
Base-plate support bar (50x50x3mm)	390			2.55
Aluminium base				4.40

Table 7-20 – Experiment Five Range base-plate component parameters

7.4.2 Experimental procedure

The experimental procedure adopted was similar to that used in Experiment Four; due to the safety procedures required when firing ballistic rounds all the data logging equipment and HSV cameras had to be started in advance

of each round being fired. This meant that both the LabVIEW data and video footage files were substantially longer than in previous experiments, and therefore required editing prior to their analysis. The 7.62mm round was fired from the proof barrel remotely having previously been aligned with the target using a laser sight. Once each shot had been fired, and the ‘all-clear’ had been given the HSV and LabVIEW VIs were stopped.

7.4.3 Presentation of results

The sensitivity and timing loop settings used for the tests are summarised in Table 7-21, while charts showing the HSV, filtered data for each test are shown in Figure 7-59 to Figure 7-63. The filter settings applied to the original data are summarised in Table 7-22 and the error measures, calculated from the HSV and filtered data, in Table 7-23.

	05-01	05-02	05-03	05-04	05-05
Clock speed (MHz)	1	1	1	1	1
Timing loop period (µS)	1000	1000	1000	1000	1000
Gyro sensitivity (deg/s)	500	500	500	500	500
Acc sensitivity (g)	4	4	4	4	4

Table 7-21 – IMU sensitivity and data logging timing loop settings for tests 05-01 to 05-05

	05-01	05-02	05-03	05-04	05-05
CF Alpha	0.994	0.98	0.9955	0.988	0.989
Q Angle	0.0001	0.001	0.0001	0.0001	0.0001
Q Gyro Bias	0.0001	0.001	0.0001	0.0001	0.0001
R Measure	0.04	0.1	0.1	0.018	0.021

Table 7-22 – Filter parameters for tests 05-01 to 05-05

	05-01		05-02		05-03		05-04		05-05	
	HSV/ CF	HSV/ KF	HSV/ CF	HSV/ KF	HSV/ CF	HSV/ KF	HSV/ CF	HSV/ KF	HSV/ CF	HSV/ KF
RM	0.0957	0.1284	0.0167	0.0159	0.0917	0.0854	0.0783	0.0160	0.0437	-0.0052
RP	0.0539	0.0837	0.1070	0.1062	0.0505	0.0528	0.0701	0.1224	0.0682	0.0608
RC	<u>0.0973</u>	<u>0.1358</u>	<u>0.0960</u>	<u>0.0952</u>	<u>0.0928</u>	<u>0.0890</u>	<u>0.0932</u>	<u>0.1094</u>	<u>0.0718</u>	<u>0.0540</u>
SGM	0.1308	0.1867	0.0198	0.0188	0.1244	0.1145	0.1037	0.0189	0.0544	-0.0060
SGP	0.0539	0.0837	0.1070	0.1062	0.0505	0.0528	0.0701	0.1224	0.0682	0.0608
SGC	<u>0.1414</u>	<u>0.2046</u>	<u>0.1088</u>	<u>0.1079</u>	<u>0.1342</u>	<u>0.1261</u>	<u>0.1252</u>	<u>0.1239</u>	<u>0.0872</u>	<u>0.0611</u>
TI	<u>0.1044</u>	<u>0.1560</u>	<u>0.1675</u>	<u>0.1663</u>	<u>0.0985</u>	<u>0.0989</u>	<u>0.1203</u>	<u>0.1913</u>	<u>0.1101</u>	<u>0.0953</u>

Table 7-23 – Error measures for tests 05-01 to 05-05

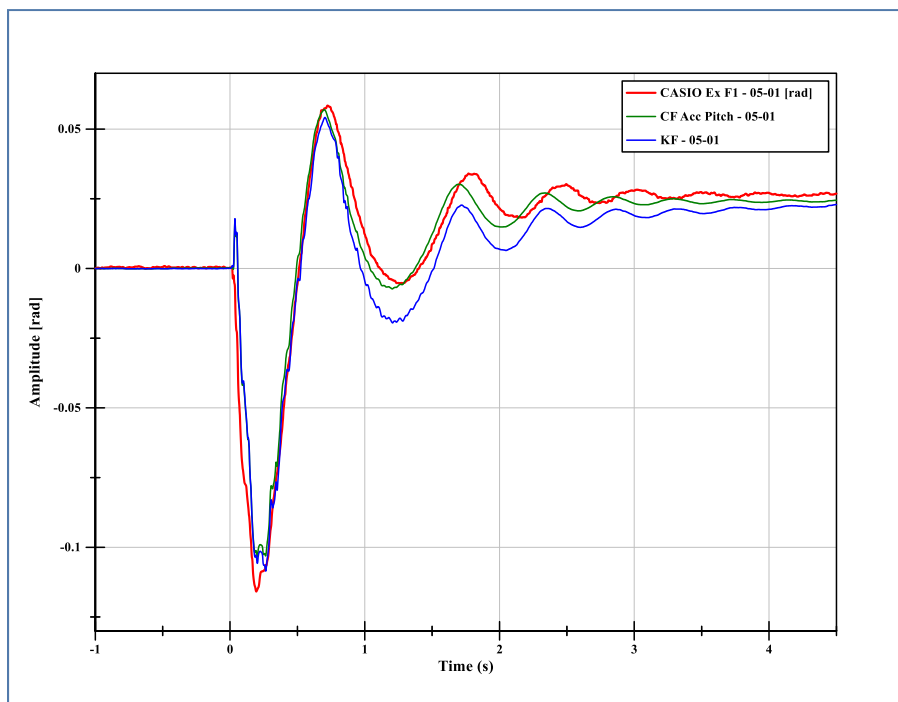


Figure 7-59 – Comparison between HSV, CF and KF data for test 05-01

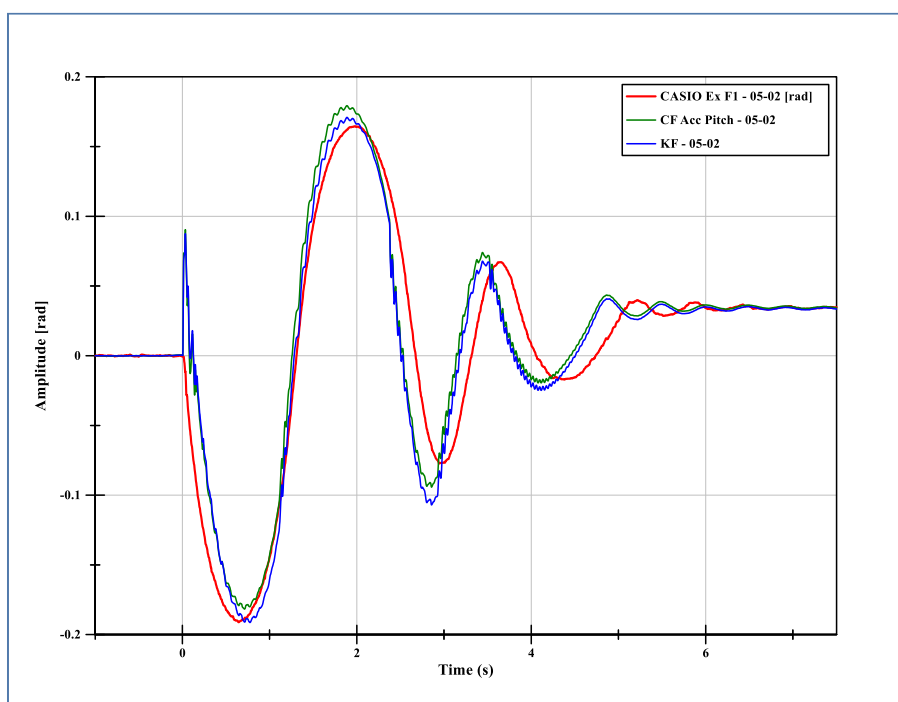


Figure 7-60 – Comparison between HSV, CF and KF data for test 05-02

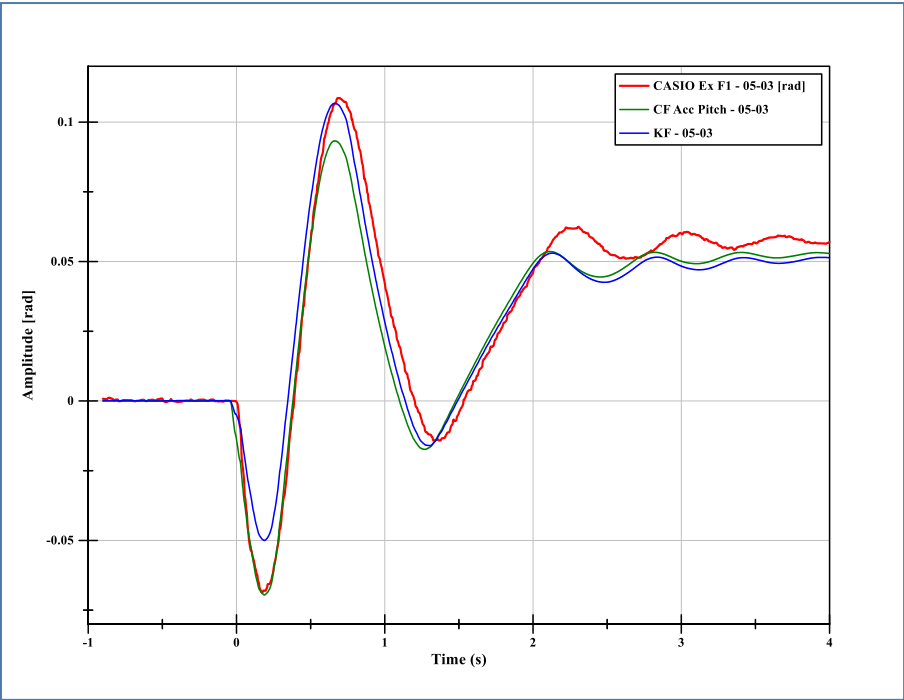


Figure 7-61 – Comparison between HSV, CF and KF data for test 05-03

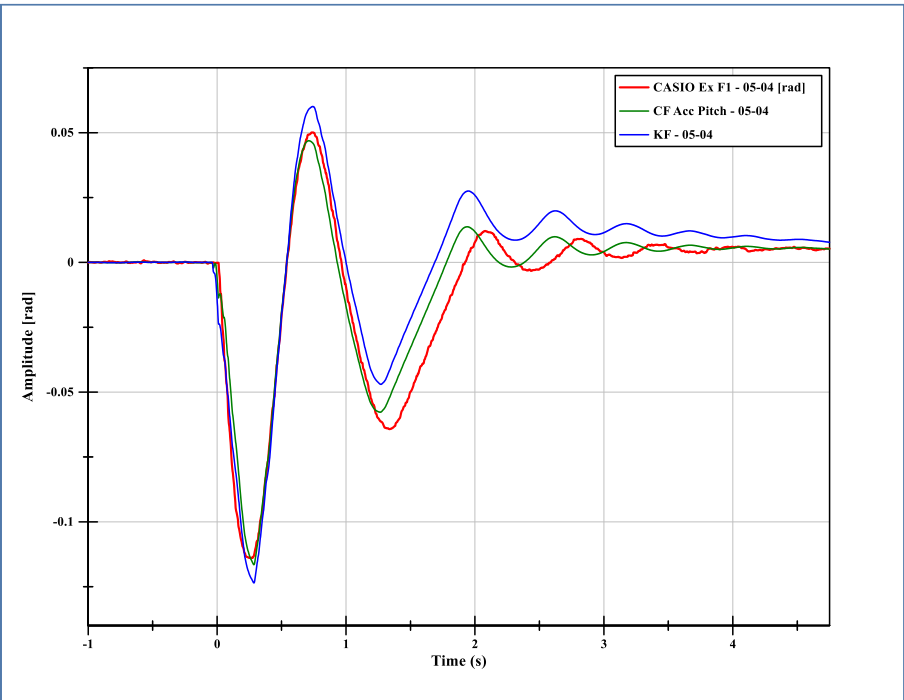


Figure 7-62 – Comparison between HSV, CF and KF data for test 05-04

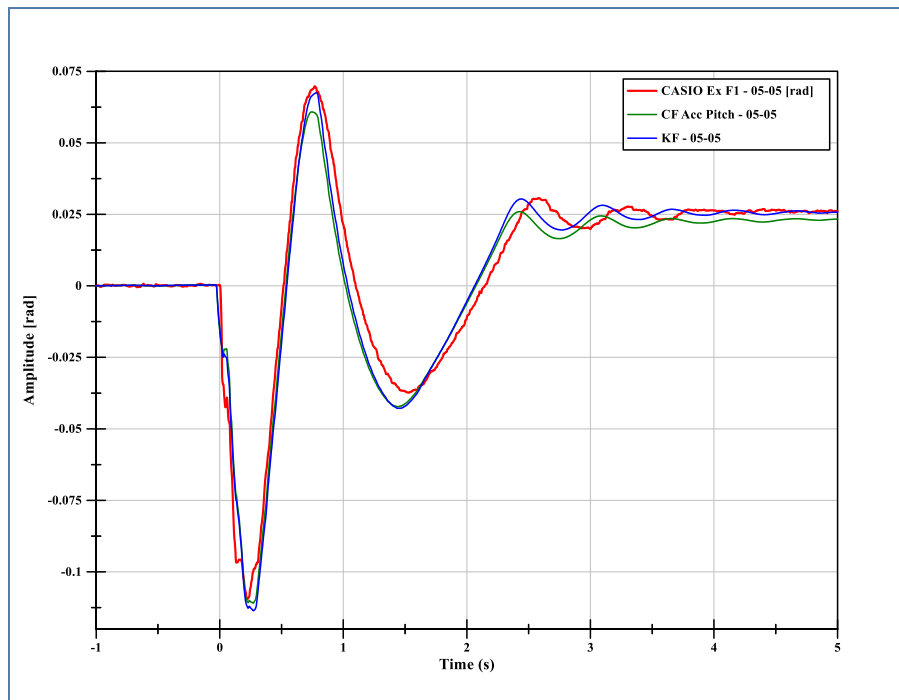


Figure 7-63 – Comparison between HSV, CF and KF data for test 05-05

7.4.4 Analysis and discussion of results

The most noticeable difference between the results for this experiment and those for Experiment Four, was the lower oscillation frequency and shorter overall test duration. This was as expected due to the significant difference in mass between the bullet and the vertical monolith panel, 8g versus 28.25kg. By comparison the difference in mass between the bullet and the vertical cantilever for Experiment Four, was 10g versus 150g. The reduced number of oscillations also meant that there was more potential for better correlation between the HSV data and the filtered data, as shown in the data charts and the error measure values. The filter constants for all tests had to be adjusted, but not as much as in Experiment Four.

All of the tests easily met Russell's 0.2 acceptance criteria, with the highest RC value being 0.1358 for the KF value for Test 05-01, and the lowest 0.0540 for the KF value for Test 05-05. From the results of these tests, it could be concluded that the proposed method worked well in scenarios where the structure was relatively slow to respond to the short-duration load. This was as expected because such scenarios are similar in nature to the sudden movements that might be experienced in navigation situations experienced by robots, aircraft or unmanned aerial vehicles (Pérez-D'arpino *et al.*, 2011).

7.5 Chapter Conclusions

Experiment Three demonstrated that the simple static analysis methods developed in Chapter Six, could be used to measure the response of simple structural elements subject to dynamic loads with a reasonable degree of accuracy. The acceptance criteria that had been considered overly generous in the static loading environment, proved to be appropriate when used in the dynamic scenarios that they had originally been developed for. In addition, the use of a lower timing loop period for data logging appeared to resolve the issue of timing difference between the HSV and filtered data. However, the limited number of tests meant that further similar tests were recommended to validate this.

The ballistic tests conducted as part of Experiments Four and Five provided two very different practical scenarios where a structure was subject to a large magnitude, short-duration load. Experiment Four involved such a load being applied to a light-weight vertical cantilever, or unrestrained column, that responded very quickly, comparable to a structure subject to a dynamic load. By the same token, the structural response investigated in Experiment Five was very different from that seen in Experiment Four, because the load duration was much shorter than the response of the heavier vertical monolith, which could be compared to the impulsive load that a large structure might experience.

Although the RC error measures for Experiment Four did not meet the chosen acceptance criteria, they were close enough to warrant further investigation. This was particularly true as Experiment Three had shown that tests where larger magnitude loads had been applied, when using a similar experimental arrangement, resulted in satisfactory results. By comparison, all the RC error measures for Experiment Five met the chosen acceptance criteria, suggesting that the proposed procedure could be applied to structures subject to impulsive loading.

Overall it was concluded that the experiments discussed in this chapter met the aims of the research, and it was proven that a cheap IMU could be used to monitor the response of structural elements subject to a large magnitude, short-duration load. Additionally, it was shown that careful placement of strain gauges could also be used to provide a reasonably accurate measure of an element's rotation. However further research was required in other experimental, and real-life scenarios to verify this. Several issues arose in terms of how the research had been undertaken that could be used to improve how similar research might be conducted in the future; these are discussed in detail in the concluding chapter.

Chapter 8 Conclusions

8.1 Introduction

This research has proven that the proposed technique of using a combination of IMU sensors and data fusion, to measure the response of structures subject to short-duration loads, can be applied to small structural elements such as flexible vertical cantilevers or columns. A review of the literature has shown that, although IMUs and data fusion have been widely used in other areas of engineering and technology, there was little evidence of their use in the shock and vibration field, and specifically in relation to short-duration load scenarios. This research provides some initial steps to addressing this gap in the knowledge and literature and demonstrates the potential of a cheaper alternative to more expensive traditional sensors. In addition, a method to quantify the accuracy of the technique against the chosen accepted reference value was developed based upon existing error and inequality measure algorithms. This enabled a quantitative assessment of the accuracy of the methods under investigation to be made, thereby permitting the best to be identified.

This chapter begins by reviewing the original research questions to establish whether they were addressed and if not, what questions remained unanswered. It continues by summarising how the research has added to the body of knowledge and provides suggestions for future researchers to use to conduct similar research. The chapter concludes by discussing how the research might be conducted differently, based upon what is now known, and suggesting short and immediate research opportunities.

8.2 Original research questions

8.2.1 *What is an acceptable level of accuracy?*

The notion of acceptable and appropriate accuracy was introduced in the literature review and developed in Chapter Five, particularly in terms what could be regarded as an appropriate acceptance criterion for the chosen error measures. For the Russell's comprehensive error measure (RC), Russell suggested acceptance criteria of 0.2, or 0.25 where the data involved shock and vibration data (Russell, 1997a, 1997b). Similar acceptance criteria did not appear to exist in any of the literature for the Sprague and Geer's combined error measure (SGC) (Sprague & Geers, 2004), or the Theil's inequality coefficient (TI) (Whang, Gilbert & Zilliacus, 1994). Therefore, comparable values were developed as part of the investigation of acceptable accuracy; the resultant figures were 0.23 and 0.28 for SGC and of 0.34 and 0.43 for TI.

In Experiment One and Two it was suggested that these recommended acceptance criterion values were overly conservative as the calculated error measures were all much lower. Subsequently it was shown in Experiments Three, Four and Five that this was not the case, and that they provided a good baseline against which to ascertain the accuracy of the proposed process.

8.2.2 *How realistic is the model?*

Early in the research it was decided to use an HSV camera to monitor the response of the various structures to provide an accepted reference value. As the research was predominantly experiment based it was concluded that this technique was the most appropriate in terms of accurately representing the response of the chosen structures. Although it would have been possible to use numerical or computer models to simulate the response, the accuracy of such models would have been dependent on having the appropriate materials and damping criteria. These models would have still required verification, thereby introducing an additional process.

The use of HSV cameras to monitor the different structures' behaviour, and subsequently provide the accepted reference value, was therefore deemed to provide the most realistic 'model'. This was particularly the case in Experiment Five, where it would have been very difficult to produce a reliable numerical or computer model.

8.2.3 *At what rate do samples need to be sampled and how many samples are required?*

The sample rate for the data logging was controlled by adjusting the timing loop period within the LabVIEW data logging VI. The number of samples collected was therefore dependent upon how long the data logging process took. For the initial tests a timing loop period of 1000 μ S, with a clock rate of 1MHz was chosen as this provided consistent data without producing overly large data files. Towards the end of the experimental process a series of nine tests were conducted for Experiments One, Two and Three which each investigated the impact that changing the timing loop period had on the results. It was shown that reducing the period to 500 μ S appeared to remove a previously identified timing discrepancy between the HSV and measured data.

It was concluded that a timing loop period of 500 μ S or less, ensured that the accepted reference value provided by the HSV data and the measured data were correctly correlated in terms of timing. However, as only a limited number of test results were used to draw this conclusion, it is recommended that further tests should be conducted to investigate the use of lower timing loop periods to fully validate this hypothesis.

8.3 Knowledge added

8.3.1 *Cheap alternative to monitor structural response*

A significant benefit of the research was the identification of the potential of sensors, such as those found in IMU units, to monitor structures subject to short-duration loads. A review of the literature determined that although similar sensors in conjunction with data fusion algorithms have been proposed for the structural health monitoring of civil engineering structures (Liu *et al.*, 2017; Testoni *et al.*, 2019), there was no evidence that such techniques had been used for tests involving short-duration ballistic or blast loads. Tests conducted during Experiment Three proved that the IMU accelerometers offered similar accuracy to the more expensive accelerometers, such as the Dytran 3200B6, but at significantly reduced cost. The comparative cost was approximately £5 for the IMU and approximately £1500 for the Dytran accelerometer, a cost differential of approximately 300. As the Dytran 3200B6 was only a single-axis accelerometer the total cost differential to replicate the capabilities of the

IMU would be far greater. This meant that using IMUs had the potential as an alternative for monitoring the response in harsh environments as multiple modules could be deployed, without significant additional expense, to ensure the appropriate response was captured if any of the IMUs was damaged.

Although the proposed system had been successfully tested using a ballistic load, further work was required to investigate methods of mounting the IMU to isolate it from the initial sudden impacts of a ballistic load, as seen in Experiment Four, which resulted in anomalies in the filtered data. Such impacts were not evident in the Experiment Five data, because they were absorbed by the test sample, however the sensor data relating to the actual movement was not affected. It is recommended that once a suitable mounting arrangement has been developed, further ballistic tests should be conducted followed by similar blast tests.

8.3.2 Factors to consider when conducting research using IMUs and data fusion algorithms in shock and vibration environments

This research has enabled a better understanding to be gained in relation to conducting tests on IMUs when they are subject to short duration loads, specifically when used in conjunction with data fusion techniques such as the complementary and Kalman filter. The following is list of things that should be considered by researchers undertaking similar investigations in the future:

- The combined use of a rigid inverted pendulum and a linear actuator for the initial tests, provided a simple means of ensuring a consistent response that could be used to verify the repeatability of the sensors under test. The rigid nature of the pendulum minimised undesirable vibration that might have introduced excessive noise into the sensor readings.
- The use of a flexible vertical cantilever offered a simple method of producing a structural response to different loads, that could be easily monitored to enable the investigation of sensors such as IMUs and strain gauges. The adaptability of the vertical cantilever enabled the same element to be used in a variety of experiments resulting in similar responses that could be readily compared despite being generated by different loads.
- Large element rotations were produced using the flexible cantilever which were easier to monitor. The use of similar flexible elements, for more complex structural arrangements, would enable the investigation of the response near rigid connections to be more easily investigated (see comments in Section 8.5.3.6).
- The use of relatively cheap HSV cameras, in conjunction with the free GOM correlate software, to accurately monitor the structures being used to investigate sensor performance. This eliminates the need to use computer or numerical models as a means of verification.
- Isolation of sensors to avoid transmission of initial shock, possibly by mounting them on an absorbent layer such as rubber or cork.
- The use of error measure algorithms as a means of quantifying the accuracy of the sensors under investigation, rather than reliance on qualitative terms.

- The use of LabVIEW software to enable quick prototyping of bespoke data logging, analysis and presentation tools.

8.4 What would be done differently, or lessons learned

Having conducted several different types of experiment as part of the research, the following is a summary of some of the things that would be done differently based upon the knowledge that is now known:

- Synchronise the cameras monitoring the response with the data logging to avoid problems with synchronising the timing data. This could be achieved by including a triggering arrangement to activate the HSV cameras and data logging upon load initiation. Although the Photron MC2 HSV cameras had the option to be started using a trigger signal, it was not possible to do this with the CASIO HSV.
- Develop a more consistent method of producing a dynamic load for Experiment Three, possibly using a magnet to ensure consistent load release after a certain force was reached, instead of the ‘shear-pin’ arrangement used.
- Consider a larger experimental arrangement for Experiments One and Two, to enable bearings to be used between the actuator and structural element, to eliminate as much friction and vibration as possible to ensure noise free data.
- Review the sensor mounting arrangement to allow flush rather than perpendicular mounting. As both the IMU gyroscope and accelerometer axes had the same sensitivity settings, the sensor could be mounted in any orientation. For the purpose of the research the IMU was mounted in a horizontal manner to make assimilation with conventional orientation terminology, such as Euler angles, easier. If mounted in any other manner, care would be required to ensure that appropriate axes were used during subsequent data analysis.

8.5 Future research opportunities

Although the research demonstrated the potential that the combination of IMUs and data fusion might be an alternative to more expensive shock and vibration sensors, there are several questions that remain unanswered. Following on there are also some immediate and longer-term research opportunities.

8.5.1 *Questions still to be answered*

8.5.1.1 *Further investigation into the use of quaternions*

Although the use of quaternions was described in Chapter 4, the technique was not utilised in the research due to inconsistencies in the data produced by the algorithms chosen, and subsequently programmed in LabVIEW. As this technique is often used in conjunction with IMUs, in particular to avoid issues such as ‘gimbal-lock’, it is recommended that this technique requires further investigation particular in relation to short-duration load scenarios.

8.5.1.2 Confirm shorter timing loop reduces time difference between HSV and measured data

It was established that reducing the timing loop period to 500 μ S appeared to guarantee that the timing between the accepted reference value and the measured data was correctly synchronised. However, as this was established using a limited number of test results, it is recommended that further tests should be conducted to verify that this was true.

8.5.1.3 Confirm IMU sensitivity setting for dynamic tests

A series of tests was conducted as part of Experiment Three to investigate how changing the sensitivity settings for the IMU sensors impacted on the accuracy of the filtered data. These tests identified that for this particular IMU, the MPU9250, an accelerometer sensitivity setting of 16g and a gyroscope sensitivity setting of 1000 deg/sec, produced the most accurate data. As this was based upon a limited number of test samples, it is recommended that further dynamic tests using similar combinations of sensitivity settings, should be conducted to verify that these settings are the optimum ones for this type of test.

8.5.1.4 Investigate means of isolating the IMU to avoid data spikes

As part of the ballistics tests conducted during Experiment Four, it was identified that large data spikes in the raw IMU data, caused by the initial bullet impact, were amplified during the data fusion process. This resulted in significant differences between the accepted reference value and the filtered data. Reducing or eliminating these data spikes, by introducing a material to isolate any significant shocks, is a potential way of doing this and therefore requires further investigation.

8.5.1.5 Investigate using strain gauges to measure rotation and how to reduce noise in strain gauge data

The use of strain gauges to measure the rotation of the vertical cantilevers in Experiment Two and Three resulted in mixed results. Whilst there were large discrepancies between the accepted reference value produced by the HSV and the calculated data from the strain gauges for Experiment Two, the results for Experiment Three almost met the acceptance criteria. A significant amount of noise was present in the data for both experiments. This noise needs to be eliminated, or at the least reduced, for this technique to be considered a viable option. Further investigations are also required to identify the reasons for the discrepancies in the Experiment Two data.

8.5.2 Immediate opportunities

8.5.2.1 Use of impact pendulum to simulate ballistic test

Conducting ballistic tests is reliant upon having access to the requisite ballistic facilities. To address the problem of data spikes caused by the initial sudden impact, a simpler easily accessible alternative should be considered, such as a rigid pendulum. Such an arrangement is shown in Figure 8-1. When released, the pendulum would hit the pendulum target at the tip of the cantilever very briefly causing an initial sudden impact similar to that exerted by a bullet. Using such a setup would enable different sensor mounting arrangements to be quickly assessed without the requirement for comparatively expensive ballistic tests until absolutely necessary.

8.5.2.2 Using other IMUs

The MPU9250 was chosen because it was relatively inexpensive and offered sensors with a good level of sensitivity. A number of other IMU units are available and listed in Table 8-1 along with the MPU9250. It will be noted that although these IMUs have similar features to the MPU9250, they are both more expensive. However, it is suggested that these, and any other similar IMU modules, should be investigated to confirm whether their performance matches or is better than the MPU9250. As only two MPU9250 IMUs can be used on the same I2C branch, having access to other IMUs with similar performance, means that multiple sensors could be used to provide redundancy, or cater for different sensitivities.

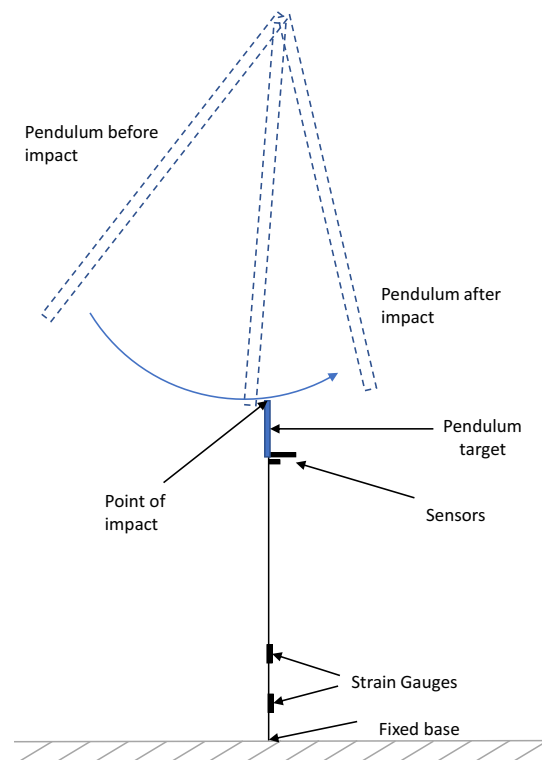


Figure 8-1 – Possible pendulum arrangement for investigating elimination of data spikes caused by sudden initial impacts.

Unit	Approximate cost (£)	Features
MPU9250	5	9 Degrees Of Freedom (9DOF), 16-bit Acc, 16-bit Gyro, 16-bit Mag, on-chip Digital Motion Processor (DMP)
BNO055	35	9DOF, 14-bit Acc, 16-bit Gyro, 16-bit Mag, on-chip DMP
MPU6050	10	6DOF, 16-bit Acc, 16-bit Gyro, on-chip DMP

Table 8-1 – IMU cost and features

8.5.2.3 Using cheaper data logging equipment such as Arduino or Raspberry Pi

The myRIO data logger was used as it offered a flexible system that could be expanded and adapted to enable the investigation of different sensor packages. However, at a cost of approximately £900, it represented a significant investment for a data logging system for everyday use. Therefore, a cheaper alternative that can be used to control the IMU and record the data is recommended. Simple micro-processors such as the Arduino and Raspberry Pi

may be suitable, examples of which are listed in Table 8-2 and shown in Figure 8-2 and Figure 8-3 respectively. These units come with a variety of features including digital input/output (DIO), analogue input (AI) and general-purpose input/output (GPIO) channels, Bluetooth and WiFi for remote operation, and may include a secure digital (SD) card for data logging or have the option to add such a feature.

In addition, because they have their own microprocessors, once programmed they can then often be operated without the need to have access to an external computer. Any research involving such units would need to investigate whether data from the sensors could be recorded at an appropriate sample rate to guarantee there was no data loss.

Unit	Approximate cost (£)	Features
Arduino Uno	20	14 DIO, 6 AI
Arduino 101	35	14 DIO, 6 AI, Bluetooth, 6DOF Acc/Gyro
Arduino Mega	35	54 DIO, 16 AI
Arduino Yun	50	20 DIO, 12 AI, WiFi, SD Card
Raspberry Pi	30	40 GPIO, Bluetooth, WiFi, microSD Card

Table 8-2 – Development board/micro-processor cost and features

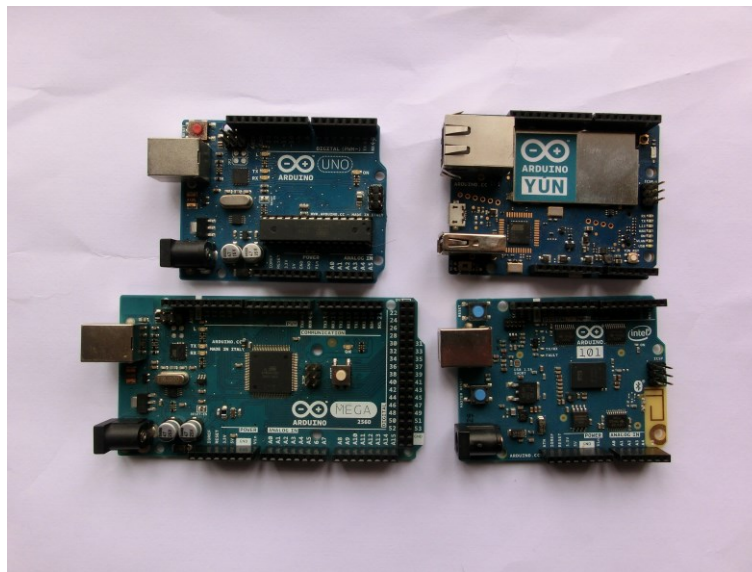


Figure 8-2 – Various Arduino development boards (Uno (Top Left), Yun (Top Right), Mega (Bottom Left) and Arduino 101 (Bottom Right))

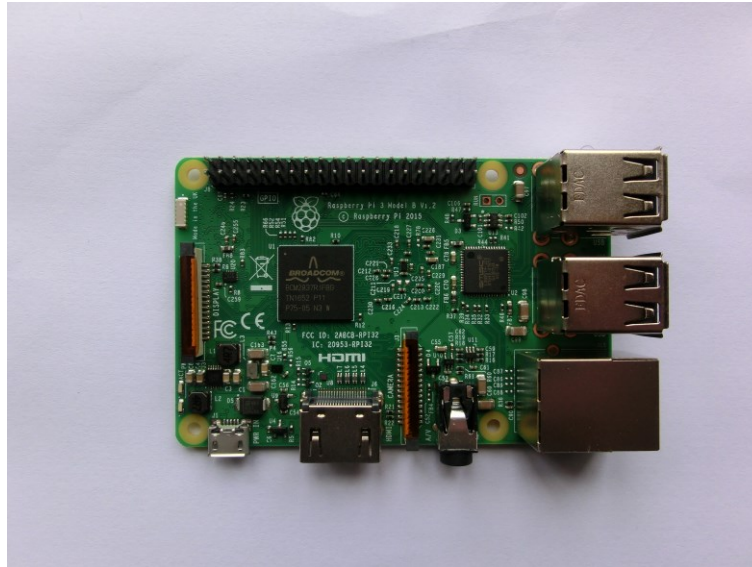


Figure 8-3 – Raspberry Pi 3 micro-computer

8.5.3 Longer-term opportunities

8.5.3.1 Use of multiple IMUs

The research focused on the use of a single IMU. The low cost of the IMU module meant that it would be possible to use multiple IMUs to provide redundancy and to cater for different sensitivity requirements. Depending upon the IMU module chosen, multiple modules could be operated using the same I2C channel by altering the address of each module. However, there are usually limits on the number of similar modules that can be linked together in this manner, and therefore it might be necessary to use a combination of different modules. In addition to investigating the feasibility of such an arrangement it would also be important to investigate how all the sensors are synchronised to ensure close correlation of the results.

8.5.3.2 Using error measures to provide an estimation of the correlation between different sensors

It is proposed that the error measure procedures utilised to monitor the accuracy of the proposed method could be used to monitor the correlation between multiple sensor modules. The procedures investigated during this research were used as part of the post-analysis of the results. Although the same process could be used to verify the correlation of different sensor modules monitoring the same response, investigating whether the error measure procedures could be adapted to provide real-time analysis of the correlation would enhance the capability of the sensor system.

8.5.3.3 Miniaturisation and combining data logging with IMU

Another avenue of investigation is the miniaturisation of the sensor system to enable it to be self-sufficient. As with the use of multiple IMUs, if multiple sensor systems are used to monitor the response, it would be important to ensure that they can be accurately synchronised to guarantee that the data is properly correlated. An example of such a unit, utilising the MPU9250 investigated during this research, is the Sparkfun 9DOF Razor IMU M0 as

shown in Figure 8-4 (Left). This module combines a 9DOF MPU9250 IMU (Figure 8-4 (Right)) with an onboard processing unit based upon an Arduino, and a micro-SD slot to enable data to be logged to a micro-SD card.

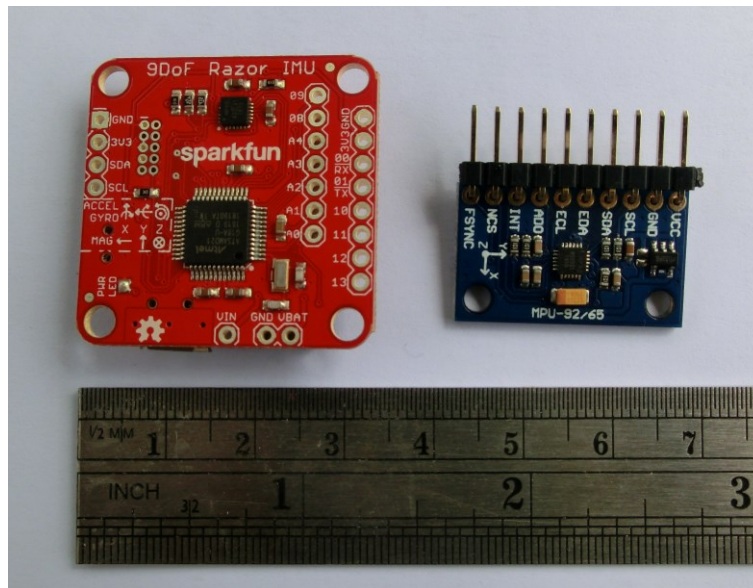


Figure 8-4 – Potential combined IMU and data logger (Left) and MPU 9250 (Right)

8.5.3.4 Further ballistic tests

Once investigations to establish a means of eliminating or reducing the initial data spikes in the IMU data caused by sudden impacts have established a successful mounting arrangement, it is recommended that the revised arrangement be ballistically tested. As the tests in Experiment Four did not include an investigation of sensor sensitivity or timing loop period settings, it is recommended that these should be included as part of any ballistic tests.

8.5.3.5 Use in a blast or large-scale impact environment

Although ballistic tests have been conducted using the proposed sensors it is also recommended that similar tests be conducted using a blast load. As such tests usually involve loads with much larger magnitudes, a different experimental arrangement may be required to ensure the element/structure being monitored was not significantly damaged. Although it might be possible to use the vertical cantilever experimental arrangement used in Experiments Three and Four, the explosive charge or substance used to generate the blast would need to either be quite small, or placed at such a distance from the cantilever, to avoid overloading it. In addition, as highlighted during the literature review, an appropriate method of monitoring the response would be required to avoid the complications associated with excessive light generated by a blast load generated by an explosion.

8.5.3.6 Use of sensors on a large scale

The tests conducted as part of the research were limited to investigating the response of small single structural elements. To understand the viability of the sensor systems in real life situations, it is recommended that multiple sensors be installed on a more complex structure. As highlighted in the methodology chapter, initially this would involve a simple two-dimensional structure that could be easily monitored or modelled, such as the simple frame

shown in Figure 8-5. This structure utilises two of the flexible vertical cantilevers used during the research, and combines them with a flexible horizontal beam element, joined together using rigid angle sections to create fixed connections. Such an arrangement could be made to deflect, as shown by the solid blue line in Figure 8-5, enabling the rotation of each of the elements to be more easily observed. Careful placement of the IMU sensors would enable the deflected shape of the individual members to be determined by the rotation at the sensor locations. The more sensors used, the more accurate the deflected shape produced. As highlighted earlier, if multiple sensors are used they would need to be synchronised to ensure all the data was accurately correlated.

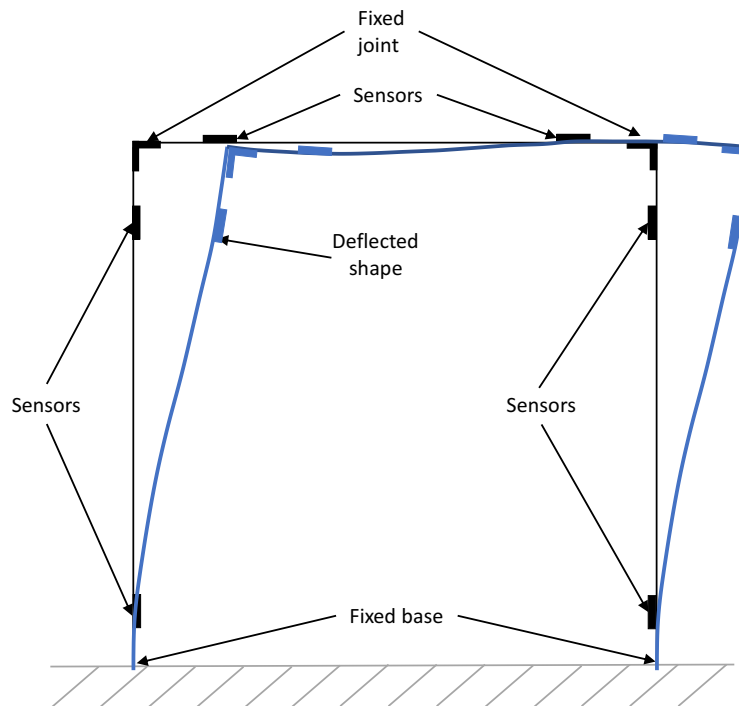


Figure 8-5 – Potential simple frame for use in initial full-scale tests

Successful investigation of a simple structure such as this, could be followed by a similar arrangement with more beams, and fixed or pinned joints. Once such experimental setups had been successfully investigated, research involving small three-dimensional (3D) frames, such as that shown in Figure 8-6 could be conducted. Such structures would be more rigid in nature than the previously tested flexible structures, however they would be a better representation of real structures, such as the Large Explosion Test Facility mentioned in Chapter Three. Tests conducted on these 3D frames could be used to ascertain support rotation conditions and would include blast tests.

8.6 Concluding remarks

This research has proven the potential of IMUs, in conjunction with data fusion algorithms, can provide a significantly cheaper alternative to existing sensor options for accurately monitoring the response of structures subject to short-duration loads. It has also provided a method of quantifying the accuracy and shown how a combination of strain gauges might be used to monitor the rotation of a structural element.

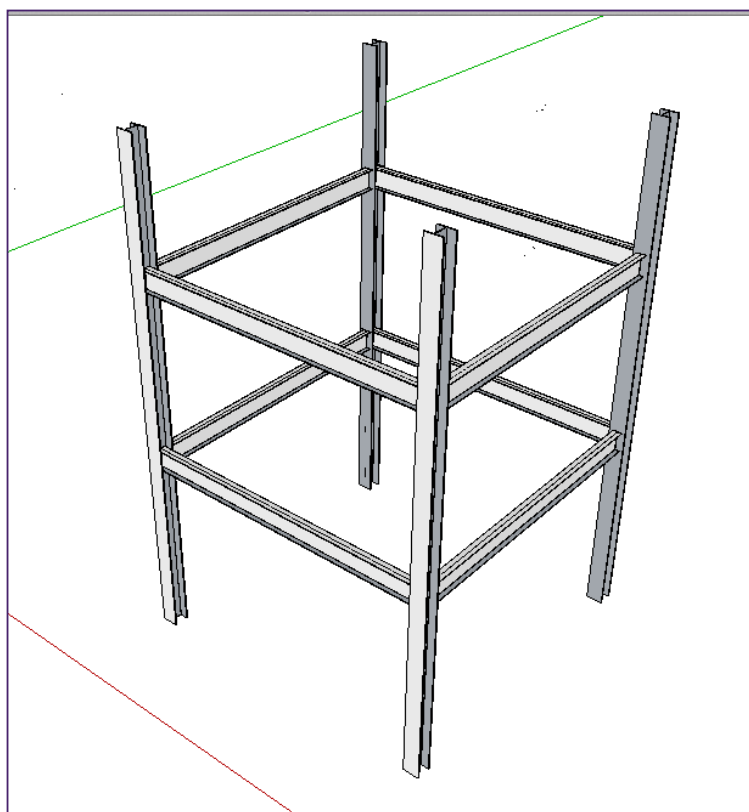


Figure 8-6 – Potential simple 3-D frame for use in full-scale tests

Bibliography

Actuonix (2016) *Minature Linear Motion Series - P16*.

Ai, D., Zhu, H., Luo, H. & Yang, J. (2014) An effective electromechanical impedance technique for steel structural health monitoring. *Construction and Building Materials*. [Online] 73, 97–104. Available from: doi:10.1016/j.conbuildmat.2014.09.029.

An, Y.-K., Kim, M.K. & Sohn, H. (2012) Airplane hot spot monitoring using integrated impedance and guided wave measurements. *Structural Control and Health Monitoring*. [Online] 19 (7), 592–604. Available from: doi:10.1002/stc.1493.

Avramidou, S.-A. (2013) Investigating the accuracy of finite element modelling predictions of a bolted beam to column steel connection under blast loading. *Department of Civil and Environmental Engineering*.

Barnett, S.J., Millard, S.G., Schleyer, G.K. & Tyas, A. (2010) *Briefing: Blast tests of fibre-reinforced concrete panels*. [Online] 163 (3), 127–129. Available from: doi:10.1680/coma.900017.

Benham, P.P. & Warnock, F. V (1976) *Mechanics of Solids and Structures: SI Units*. [Online]. Pitman. Available from:
http://books.google.co.uk/books?id=QzxFswEACAAJ&dq=mechanics+of+solids+and+structures&hl=&cd=8&source=gbs_api.

BMIIB (2006) *The Buncefield Investigation, Progress report*.

Brownjohn, J.M.W. (2007) Structural health monitoring of civil infrastructure. *Philos Trans A Math Phys Eng Sci*. [Online] 365 (1851), 589–622. Available from: doi:10.1098/rsta.2006.1925.

BSi (2000) *BS EN 1063:2000 Glass in building - Security glazing - Testing and classification of resistance against bullet attack*.

BSi (2004) *BS EN 13124-2:2004 Windows, doors and shutters - Explosion resistance - Test method*.

BSi (1999a) *BS EN 1522:1999 Windows, doors, shutters and blinds - Bullet resistance - Requirements and classification*.

BSi (1999b) *BS EN 1523:199 Windows, doors, shutters and blinds - Bullet resistance - Test method*.

BSi (1994) *BS ISO 5725-3:1994 Accuracy (trueness and precision) of measurement methods and results.5*.

Campidelli, M., El-Dakhkhni, W.W., Tait, M.J. & Mekky, W. (2015) Blast Design-Basis Threat Uncertainty and Its Effects on Probabilistic Risk Assessment. *ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering*. [Online] 1 (4), 4015012. Available from: doi:10.1061/AJRUA6.0000823.

CASIO Computer Co Ltd (2009) *CASIO EX-F1 Digital Camera - User's Guide*. [Online]. pp.1–185. Available from: <http://www.exilim.com/> [Accessed: 4 March 2019].

CASIO Computer Co Ltd (2012) *CASIO EX-ZR100 Digital Camera - User's Guide*. [Online]. pp.1–205. Available from: <http://www.exilim.com/> [Accessed: 4 March 2019].

Chancey, R., Sputo, T., Minchin, E. & Turner, J. (2005) Justifiable Precision and Accuracy in Structural Engineering Calculations: In Search of a Little Less Precision and Supposed Accuracy. *Practice Periodical on Structural Design and Construction*. [Online] 10 (3), 154–160. Available from: doi:10.1061/(ASCE)1084-0680(2005)10:3(154).

- Chen, A., Louca, L.A. & Elghazouli, A.Y. (2015) Blast assessment of steel switch boxes under detonation loading scenarios. *International Journal of Impact Engineering*. [Online] 78 (0), 51–63. Available from: doi:10.1016/j.ijimpeng.2014.11.019.
- Cho, S., Yun, C.-B. & Sim, S.-H. (2015) *Displacement estimation of bridge structures using data fusion of acceleration and strain measurement incorporating finite element model*. 15, 645–663.
- Cobb, F. (2014) *Structural Engineer's Pocket Book: Eurocodes*. Eurocodes, Third Edition. [Online]. CRC Press. Available from: doi:10.1201/b17549.
- Corley, W., Sr, P., Sozen, M. & Thornton, C. (1998) The Oklahoma City Bombing: Summary and Recommendations for Multihazard Mitigation. *Journal of Performance of Constructed Facilities*. [Online] 12 (3), 100–112. Available from: doi:10.1061/(ASCE)0887-3828(1998)12:3(100).
- Cormie, D., Mays, G. & Smith, P. (2009) *Blast effects on buildings*. Thomas Telford Services Ltd.
- Czerniak, G. (2015) *Kalman Filters for Undergrads 1*. [Online]. 2015. Greg Czerniak's Website. Available from: <http://greg.czerniak.info/guides/kalman1/> [Accessed: 13 March 2018].
- Davison, B. & Owens, G.W. (2012) *Steel Designers' Manual*. 7th edition. Chichester, Wiley Blackwell.
- Denny, J.W. & Clubley, S.K. (2019) Long-duration blast loading & response of steel column sections at different angles of incidence. *Engineering Structures*. 178, 331–342.
- DoD, U.S. (2008) *UFC 3-340-02 Unified Facilities Criteria (UFC) Structures to resist the effects of accidental explosions*. U S DoD (ed.).
- Euston, M., Coote, P., Mahony, R., Kim, J., et al. (2008) A complementary filter for attitude estimation of a fixed-wing UAV. In: *2008 IEEE/RSJ International Conference on Intelligent Robots and Systems, IROS*. [Online]. 2008 p. Available from: doi:10.1109/IROS.2008.4650766.
- Faragher, R. (2012) Understanding the basis of the Kalman filter via a simple and intuitive derivation. *IEEE Signal Processing*. 128–132.
- Fay, S.D., Rigby, S.E., Tyas, A., Clarke, S.D., et al. (2015) Displacement timer pins: An experimental method for measuring the dynamic deformation of explosively loaded plates. *International Journal of Impact Engineering*. [Online] 86, 124–130. Available from: doi:10.1016/j.ijimpeng.2015.08.002.
- Foglar, M., Hajek, R., Fladr, J., Pachman, J., et al. (2017) Full-scale experimental testing of the blast resistance of HPFRC and UHPFRC bridge decks. *Construction and Building Materials*. [Online] 145, 588–601. Available from: doi:10.1016/j.conbuildmat.2017.04.054.
- Fresk, E. & Nikolakopoulos, G. (2013) Full quaternion based attitude control for a quadrotor. In: *European Control Conference*. [Online]. July 2013 p. Available from: doi:10.23919/ECC.2013.6669617", "publicationTitle": "2013.
- Gebre-Egziabher, D., Hayward, R.C. & Powell, J.D. (2004) Design Of Multi-sensor Attitude Determination Systems. *IEEE Transactions on Aerospace and Electronic Systems*. [Online] 40 (2), 627–649. Available from: doi:10.1109/TAES.2004.1310010.
- Geers, T.L. (1984) An objective error measure for the comparison of calculated and measured transient response histories. *Shock and Vibration*.
- Gere, J.M. & Timoshenko, S.P. (1999) *Mechanics of Materials*. SI Edition. Fourth. Cheltenham, Stanley Thornes.
- Goldenstein, S.K. (2004) A gentle introduction to predictive filters. *RITA*. [Online] 11 (1), 63–90. Available from: doi:https://doi.org/10.22456/2175-2745.5962.

-
- GOM mbH (2017) *GOM Correlate*. [Online]. Available from: <https://www.gom-correlate.com/> [Accessed: 28 February 2019].
- Gui, P., Tang, L. & Mukhopadhyay, S. (2004) MEMS based IMU for tilting measurement: Comparison of complementary and kalman filter based data fusion. In: *2015 IEEE 10th Conference on Industrial Electronics and Applications (ICIEA) IS - SN - VO - VL -*. 2004 pp. 2004–2009.
- Hall, D.L. & Llinas, J. (1997) An introduction to multisensor data fusion. *Proceedings of the IEEE*. [Online] 85 (1), 6–23. Available from: doi:10.1109/5.554205.
- Hehir, J. (2013) *Investigating the Accuracy of Finite Element Model Predictions of Large Steel Structures Subjected to Blast Loading*. London, Imperial College London.
- Hooper, P. (2011) *Blast performance of silicone-bonded laminated glass*. London, Imperial College London.
- Hooper, P., Arora, H. & Dear, J.P. (2010) Blast and Impact Resistance of Laminated Glass Structures. In: *IMPLAST 2010*. 2010 Providence, Rhode Island USA, Society for Experimental Mechanics, Inc. p.
- Hooper, P.A., Sukhram, R.A.M., Blackman, B.R.K. & Dear, J.P. (2012) On the blast resistance of laminated glass. *International Journal of Solids and Structures*. [Online] 49 (6), 899–918. Available from: doi:10.1016/j.ijsolstr.2011.12.008.
- Idriss, J.S. & Lowak, M.J. (2014) Empirical Evaluation of Glazing Systems in Response to Blast Loads. In: *Structures Congress 2009*. [Online]. 2014 Reston, VA, American Society of Civil Engineers. pp. 258–269. Available from: doi:10.1061/9780784413357.024.
- InvenSense (2016) *MPU-2950 Product Specification*.
- ISO (2007a) *ISO 16933:2007 Glass in building - Explosion-resistant security glazing - Test and classification for arena air-blast loading*.
- ISO (2007b) *ISO 16935:2007 Glass in building - Bullet-resistant security glazing - Test and classification*.
- Johnson, D.M. (2010) The potential for vapour cloud explosions - Lessons from the Buncefield accident. *Journal of Loss Prevention in the Process Industries*. [Online] 23 (6), 921–927. Available from: doi:10.1016/j.jlp.2010.06.011.
- Julier, S.J. & Uhlmann, J.K. (1997) New extension of the Kalman filter to nonlinear systems. In: *Signal Processing, Sensor Fusion, and Target Recognition VI*. [Online]. 1997 Orlando, Florida USA, SPIE. p. Available from: doi:10.1117/12.280797.
- Kalman, R.E. (1960) A New Approach to Linear Filtering and Prediction Problems. *Journal of Basic Engineering*. [Online] 82 (1), 35–45. Available from: doi:10.1115/1.3662552.
- Karbhari, V.M. & Ansari, F. (2009) *Structural health monitoring of civil infrastructure systems*. Cambridge, UK; Boca Raton, FL, Woodhead Publishing ; CRC Press.
- Kelly, P. (n.d.) *Solid Mechanics Part I*. An Introduction to Solid Mechanics.
- Kim, J., Kim, K. & Sohn, H. (2014) Autonomous dynamic displacement estimation from data fusion of acceleration and intermittent displacement measurements. *Mechanical Systems and Signal Processing*. [Online] 42 (1 {textendash} 2), 194–205. Available from: doi:10.1016/j.ymssp.2013.09.014.
- Kranzer, C., Gürke, G., Mayrhofer, C., Gürke, G., et al. (2005) Testing of Bomb Resistant Glazing Systems Experimental Investigation of the Time Dependent Deflection of Blast Loaded 7.5 mm Laminated Glass. In: *Glass Processing Days*. 2005 Tampere, Finland. p. 7.
- Krauthammer, T. (2008) *Modern Protective Structures*. CRC Press.
-

-
- Kuipers, J.B. (2002) *Quaternions and Rotation Sequences*. A Primer with Applications to Orbits, Aerospace, and Virtual Reality. Princeton University Press.
- Lee, M.H. & Chen, T.C.C. (2008) Blast Load Input Estimation of the Medium Girder Bridge using Inverse Method. *Defence Science Journal*.
- Li, J., Wu, C. & Hao, H. (2015) An experimental and numerical study of reinforced ultra-high performance concrete slabs under blast loads. *Materials & Design*. [Online] 82, 64–76. Available from: doi:10.1016/j.matdes.2015.05.045.
- Liu, C., Park, J.-W., Spencer Jr., B.F., Moon, D.-S., et al. (2017) Sensor fusion for structural tilt estimation using an acceleration-based tilt sensor and a gyroscope. *Smart Materials and Structures*. [Online] 26 (10), 105005. Available from: doi:10.1088/1361-665X/aa84a0.
- Madgwick, S.O.H. (2010) An efficient orientation filter for inertial and inertial/magnetic sensor arrays. *Report x-io and University of s*. [Online] Available from: doi:10.1109/ICORR.2011.5975346.
- Madgwick, S.O.H., Harrison, A.J.L. & Vaidyanathan, R. (2011) Estimation of IMU and MARG orientation using a gradient descent algorithm. In: *IEEE International Conference on Rehabilitation Robotics*. [Online]. 2011 p. Available from: doi:10.1109/ICORR.2011.5975346.
- Mahin, D.A. (2006) Addressing the Threat: An Examination of Science and Technology Policy Addressing Weapons of Mass Destruction. In: *2006 IEEE International Symposium on Technology and Society*. [Online]. 2006 IEEE. pp. 1–12. Available from: doi:10.1109/ISTAS.2006.4375888.
- Mahony, R., Hamel, T. & Pflimlin, J.M. (2008) Nonlinear Complementary Filters on the Special Orthogonal Group. *IEEE Transactions on Automatic Control*. 53 (5), 1203–1218.
- Mediavilla, J., van Doornaal, A. & Weerheijm, J. (2007) Application of a Kalman filter to blast loaded structures. In: *PROTECT performance, protection and strengthening of structures under extreme loading*. 2007 Vancouver. p.
- Menditto, A., Patriarca, M. & Magnusson, B. (2007) Understanding the meaning of accuracy, trueness and precision. *Springer*.
- van der Merwe, R., Wan, E. & Julier, S. (2012) *Sigma-Point Kalman Filters for Nonlinear Estimation and Sensor-Fusion: Applications to Integrated Navigation*. In: [Online]. 2012 p. Available from: doi:10.2514/6.2004-5120.
- Metni, N., Pflimlin, J., Hamel, T. & Soueres, P. (2005) Attitude and Gyro Bias Estimation for a Flying UAV. In: *2005 IEEE/RSJ International Conference on Intelligent Robots and Systems IS -*. June 2005 pp. 1114–1120.
- Meyer, S.F., Jore, S.H. & Johansen, K.W. (2015) Troublesome trade-offs: balancing urban activities and values when securing a city-centre governmental quarter. *City, Territory and Architecture*. [Online] 2 (1), 308. Available from: doi:10.1186/s40410-015-0025-6.
- Microsoft Corporation (2018) *Microsoft Excel*.
- Morrill, K.B., Crawford, J.E., Brewer, T.R., Weaver, M.K., et al. (2005) System Response of Full-Scale Reinforced Concrete Framing Systems to Column Removal and Blast Loading. *Structures Congress 2009*. [Online] 1173–1182. Available from: doi:10.1061/9780784479117.100.
- Morris, A.S. & Langari, R. (2016) *Measurement and instrumentation : theory and application*. Theory and application. Second. Amsterdam, Elsevier Academic Press.
- Murty, D.V.S. (2010) *Transducers and Instrumentation*. PHI Learning Pvt. Ltd.
-

- Nassr, A.A. (2012) *Experimental and Analytical Study of the Dynamic Response of Steel Beams and Columns to Blast Loading*. McMaster University.
- National Instruments Corporation. (2016a) *Measuring strain with strain gauges*.
- National Instruments Corporation. (2015) *National Instruments DIAdem*.
- National Instruments Corporation. (2016b) *National Instruments LabView*.
- National Instruments Corporation. (2013) *NI myRIO-1900*.
- National Instruments Corporation (2016a) *NI 9223 - Datasheet*. [Online]. pp.1–12. Available from: http://www.ni.com/pdf/manuals/374223a_02.pdf [Accessed: 4 March 2019].
- National Instruments Corporation (2016b) *NI 9402 - Datasheet*. [Online]. pp.1–9. Available from: http://www.ni.com/pdf/manuals/374614a_02.pdf [Accessed: 4 March 2019].
- National Instruments Corporation (2016c) *NI cDAQ TM-9171/9174/9178 User Manual*. [Online]. pp.1–105. Available from: <http://www.ni.com/pdf/manuals/372838e.pdf> [Accessed: 4 March 2019].
- Ngo, T., Mendis, P. & Krauthammer, T. (2007) *Behavior of Ultrahigh-Strength Prestressed Concrete Panels Subjected to Blast Loading*. [Online] 133 (11), 1582–1590. Available from: doi:10.1061/(ASCE)0733-9445(2007)133:11(1582).
- Omenzetter, P. & Brownjohn, J.M.W. (2004) Application of time series and Kalman filtering for structural health monitoring of a bridge. *2 nd European workshop on*
- Oxford University Press (2018) *English Dictionary, Thesaurus, & grammar help | Oxford Dictionaries*. [Online] Available from: <https://en.oxforddictionaries.com>.
- Palanisamy, R., Cho, S., Kim, H. & Sim, S.-H. (2015) Experimental validation of Kalman filter-based strain estimation in structures subjected to non-zero mean input. *Smart Materials and Structures*. 15 (2), 489–503.
- Park, H.S., Kim, Y. & Oh, B.K. (2017) A model updating method with strain measurement from impact test for the safety of steel frame structures. *Measurement*. [Online] 102, 220–229. Available from: doi:10.1016/j.measurement.2017.02.013.
- Park, J., Sim, S. & Jung, H. (2013) Displacement Estimation Using Multimetric Data Fusion. *IEEE/ASME Transactions on Mechatronics*. 18 (6), 1675–1682.
- Pedley, M. (2013) Tilt Sensing Using a Three-Axis Accelerometer. *Freescale semiconductor application notes*. [Online] Available from: doi:10.3390/s17010187.
- Pérez-D'arpino, C., Vigouroux, D., Medina-Meléndez, W., Fermín, L., et al. (2011) Development of a low cost inertial measurement unit for UAV applications with Kalman Filter based attitude determination. In: *2011 IEEE Conference on Technologies for Practical Robot Applications, TePRA 2011*. [Online]. 2011 p. Available from: doi:10.1109/TEPRA.2011.5753503.
- Phan, D.T., Lim, J.B.P., Tanyimboh, T.T., Lawson, R.M., et al. (2013) Effect of serviceability limits on optimal design of steel portal frames. *Journal of Constructional Steel Research*. 86, 74–84.
- Philip Salmony (2018) *IMU Attitude Estimation*. [Online]. 2018. Available from: <http://philsal.co.uk/projects/imu-attitude-estimation> [Accessed: 16 October 2018].
- Photron USA, I. (2014) FASTCAM MC2. *photron.com*. [Online]. Available from: http://www.photron.com/?cmd=product_general&product_id=8&product_name=FASTCAM+MC2.
- Pickerd, V. (2013) *Optimisation and Validation of the ARAMIS Digital Image Correlation System for use in Large-scale High-strain-rate Events*. (DSTO-TN-1203).

-
- Pickerd, V., Bornstein, H., McCarthy, P. & Buckland, M. (2016) Analysis of the structural response and failure of containers subjected to internal blast loading. *International Journal of Impact Engineering*. [Online] 95, 40–53. Available from: doi:10.1016/j.ijimpeng.2016.04.010.
- Puryear, J.M.H., Marchand, K.A., Stevens, D. & Sunshine, D.A. (2010) Performance of blast-damaged steel connections in progressive collapse. In: *MABS 21*. October 2010 Protection Engineering Consultants. p.
- Reu, P.L., Cooper, M.A. & Miller, T.J. (2010) Ultra-high speed imaging and DIC for explosive system observation. In: *IMPLAST 2010 Conference, Society for Experimental Mechanics*. 2010 Providence, Rhode Island USA, Society for Experimental Mechanics. p.
- Russell, D.M. (1997a) Error measures for comparing transient data: Part I: development of a comprehensive error measure. In: *68th Shock and Vibration Symposium*. 1997 Hunt Valley, MD. p.
- Russell, D.M. (1997b) Error measures for comparing transient data: Part II: Error measures case study. In: *68th Shock and Vibration Symposium*. 1997 Hunt Valley, MD. p.
- Sadee, C., Samuels, D.E. & O'Brien, T.P. (1977) The characteristics of the explosion of cyclohexane at the Nypco (UK) Flixborough plant on 1st June 1974. *Journal of Occupational Accidents*. [Online] 1 (3), 203–235. Available from: doi:10.1016/0376-6349(77)90001-3.
- Sarin, H., Kokkolaras, M., Hulbert, G., Papalambros, P., et al. (2008) *A Comprehensive Metric for Comparing Time Histories in Validation of Simulation Models With Emphasis on Vehicle Safety Applications*. In: [Online]. 2008 ASME. pp. 1275–1286. Available from: doi:10.1115/DETC2008-49669.
- Sbarufatti, C., Manes, A. & Giglio, M. (2014) Application of sensor technologies for local and distributed structural health monitoring. *Structural Control and Health Monitoring*. [Online] 21 (7), 1057–1083. Available from: doi:10.1002/stc.1632.
- Schleyer, G.K., Lowak, M.J., Polcyn, M.A. & Langdon, G.S. (2007) Experimental investigation of blast wall panels under shock pressure loading. *International Journal of Impact Engineering*. [Online] 34 (6), 1095–1118. Available from: doi:10.1016/j.ijimpeng.2006.05.006.
- Schwer, L.E. (2006) *Validation Metrics for Response Histories: A Review with Case Studies*.
- Schwer, L.E. (2007) Validation metrics for response histories: perspectives and case studies. *Engineering with Computers*. [Online] 23 (4), 295–309. Available from: doi:10.1007/s00366-007-0070-1.
- SCI (2009) *Buncefield Explosion Mechanism Phase 1* SCI (ed.).1 and 2 (RR718).
- Smyth, A. & Wu, M. (2007) Multi-rate Kalman filtering for the data fusion of displacement and acceleration response measurements in dynamic system monitoring. *Mechanical Systems and Signal Processing*. [Online] 21 (2), 706–723. Available from: doi:10.1016/j.ymssp.2006.03.005.
- Sprague, M.A. & Geers, T.L. (2004) A spectral-element method for modelling cavitation in transient fluid–structure interaction. *International Journal for Numerical Methods in Engineering*. [Online] 60 (15), 2467–2499. Available from: doi:10.1002/nme.1054.
- Sprague, M.A. & Geers, T.L. (1999) Response of Empty and Fluid-Filled, Submerged Spherical Shells to Plane and Spherical, Step-Exponential Acoustic Waves. *Shock and Vibration*. [Online] 6 (3). Available from: doi:10.1155/1999/647090.
- Sprague, M.A. & Geers, T.L. (2003) Spectral elements and field separation for an acoustic fluid subject to cavitation. *Journal of Computational Physics*. 184 (1), 149–162.
- Stephens, R.A.C. (2008) *Determination of the resistance of laminated glass subjected to blast loading using high*
-

- speed video*. Shrivenham, Cranfield University.
- STMicroelectronics (2000) *DUAL FULL-BRIDGE DRIVER - Datasheet*. [Online]. pp.1–13. Available from: <https://www.st.com/resource/en/datasheet/l298.pdf> [Accessed: 4 March 2019].
- STMicroelectronics (2010) Tilt Measurement Using a Low-g 3-axis accelerometer. *AN3182*.
- Sung, S.H., Park, J.W., Park, J.W., Nagayama, T., et al. (2013) A multi-scale sensing and diagnosis system combining accelerometers and gyroscopes for bridge health monitoring. *Smart Materials and Structures*. [Online] 23 (1), 1–14. Available from: doi:10.1088/0964-1726/23/1/015005.
- Testoni, N., Zonzini, F., Marzani, A., Scarponi, V., et al. (2019) A Tilt Sensor Node Embedding a Data-Fusion Algorithm for Vibration-Based SHM. *Electronics*. 8 (1).
- Texas Instruments (2018) *ADS1115 Datasheet*. [Online]. pp.1–54. Available from: www.ti.com [Accessed: 4 March 2019].
- Thiagarajan, G., Kadambi, A. V, Robert, S. & Johnson, C.F. (2015) Experimental and finite element analysis of doubly reinforced concrete slabs subjected to blast loads. *International Journal of Impact Engineering*. [Online] 75, 162–173. Available from: doi:10.1016/j.ijimpeng.2014.07.018.
- Tuck, K. (2007) *Tilt sensing using linear accelerometers - AN3461 Rev 2*.
- Usher, M.J. & Keating, D.A. (1996) *Sensors and Transducers : Characteristics, Applications, Instrumentation, Interfacing*. Second. London, MacMillan Press Ltd.
- Venart, J.E.S. (2004) Flixborough: the Explosion and its Aftermath. *Process Safety and Environmental Protection*. [Online] 82 (2), 105–127. Available from: doi:10.1205/095758204322972753.
- Weidlinger Associates, I. (2011) Preventing Structures from Collapsing. *Homeland Security - Buildings and Infrastructure Protection Series*. (BIPS 05/June 2011).
- Welch, G. & Bishop, G. (2006) *An Introduction to the Kalman Filter*. (TR 95-041).
- Whang, B., Gilbert, W.E. & Ziliacus, S. (1994) Two Visually Meaningful Correlation Measures for Comparing Calculated and Measured Response Histories. *Shock and Vibration*. [Online] 1 (4), 303–316. Available from: doi:10.3233/SAV-1994-1401.
- White, F.E. (1991) Data Fusion Lexicon. *Data Fusion Subpanel of the Joint Directors of Laboratories 15*. 1–13.
- Wierschem, N.E., Hubbard, S.A., Luo, J., Fahnestock, L.A., et al. (2017) Response attenuation in a large-scale structure subjected to blast excitation utilizing a system of essentially nonlinear vibration absorbers. *Journal of Sound and Vibration*. [Online] 389, 52–72. Available from: doi:10.1016/j.jsv.2016.11.003.
- Williams, G., Batho, S. & Russell, L. (2000) Responding to urban crisis: The emergency planning response to the bombing of Manchester city centre. *Cities*. [Online] 17 (4), 293–304. Available from: doi:10.1016/S0264-2751(00)00025-1.
- Wolff, K. & Larsen, S. (2014) Can terrorism make us feel safer? Risk perceptions and worries before and after the July 22nd attacks. *Annals of Tourism Research*. [Online] 44, 200–209. Available from: doi:10.1016/j.annals.2013.10.003.
- Yankelevsky, D., Schwarz, S. & Brosh, B. (2013) *Full Scale Field Blast Tests on Reinforced Concrete Residential Buildings - From Theory to Practice*. [Online] 4 (4), 565–590. Available from: doi:10.1260/2041-4196.4.4.565.
- Yao, S., Zhang, D., Chen, X., Lu, F., et al. (2016) Experimental and numerical study on the dynamic response of RC slabs under blast loading. *Engineering Failure Analysis*. [Online] 66, 120–129. Available from:

doi:10.1016/j.engfailanal.2016.04.027.

- Yu, J.H., Hsieh, A.J., Dehmer, P.G. & Sands, J.M. (2009) Real-Time Full-field Deformation Analysis on the Ballistic Impact of Polymeric Materials Using High-speed Photogrammetry. In: *American Society for Composites - 24th Technical Conference*. 2009 University of Delaware, Newark, DE. p.
- Zhang, S., Yu, S., Liu, C., Yuan, X., et al. (2016) A Dual-Linear Kalman Filter for Real-Time Orientation Determination System Using Low-Cost MEMS Sensors. *Sensors (Basel)*. 16 (2), 264.

Annex A GOM Correlate Process

A.1. Introduction

All of the high-speed video (HSV) data files, that were used to monitor the response of the structural elements used during the research, were analysed using the free GOM Correlate software (GOM mbH, 2017). The process adopted is summarised in Figure A-1 and described visually in the subsequent photographs.

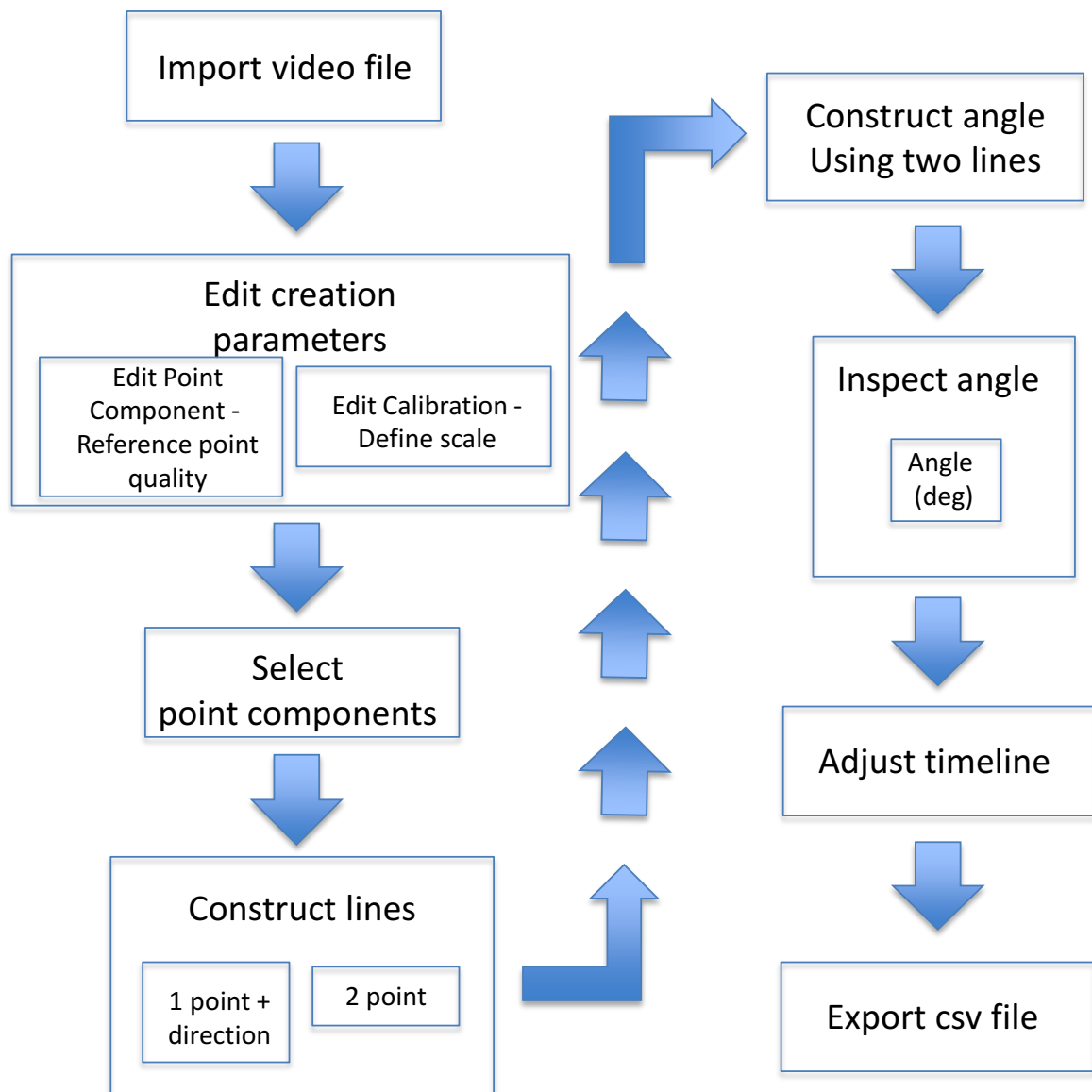


Figure A-1 – Flow chart for GOM Correlate HSV data analysis

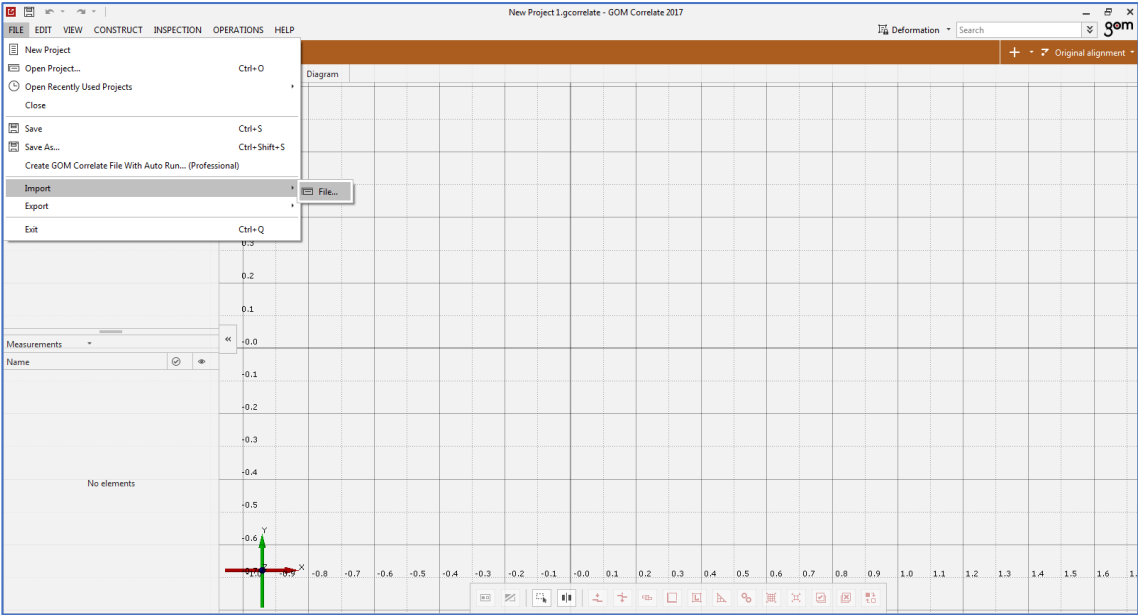


Figure A-2 – Import HSV file into GOM Correlate

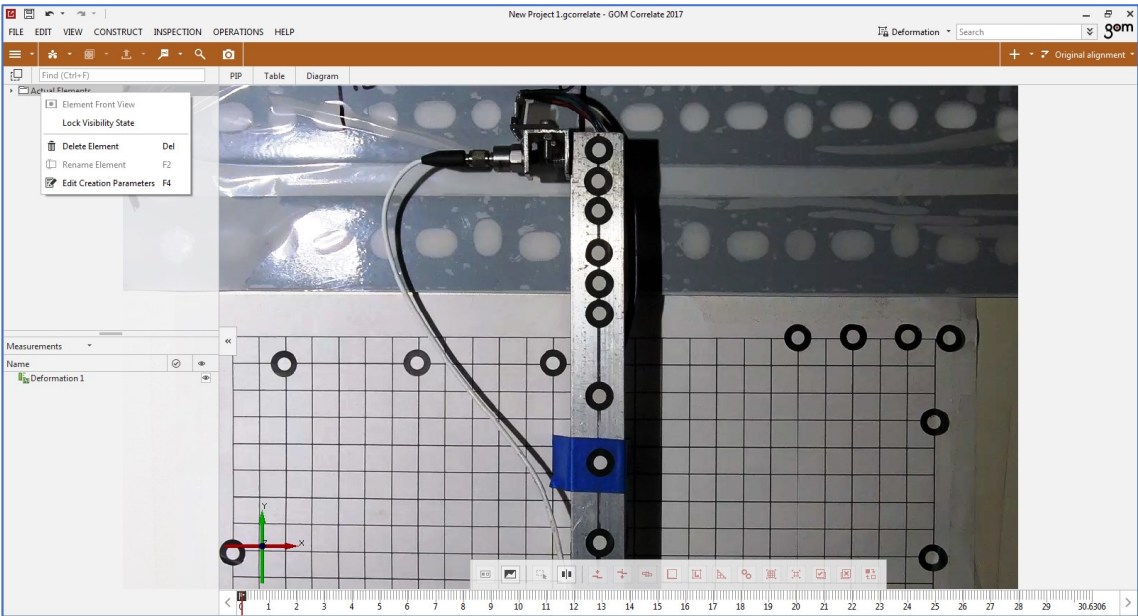


Figure A-3 – Edit creation parameter menu

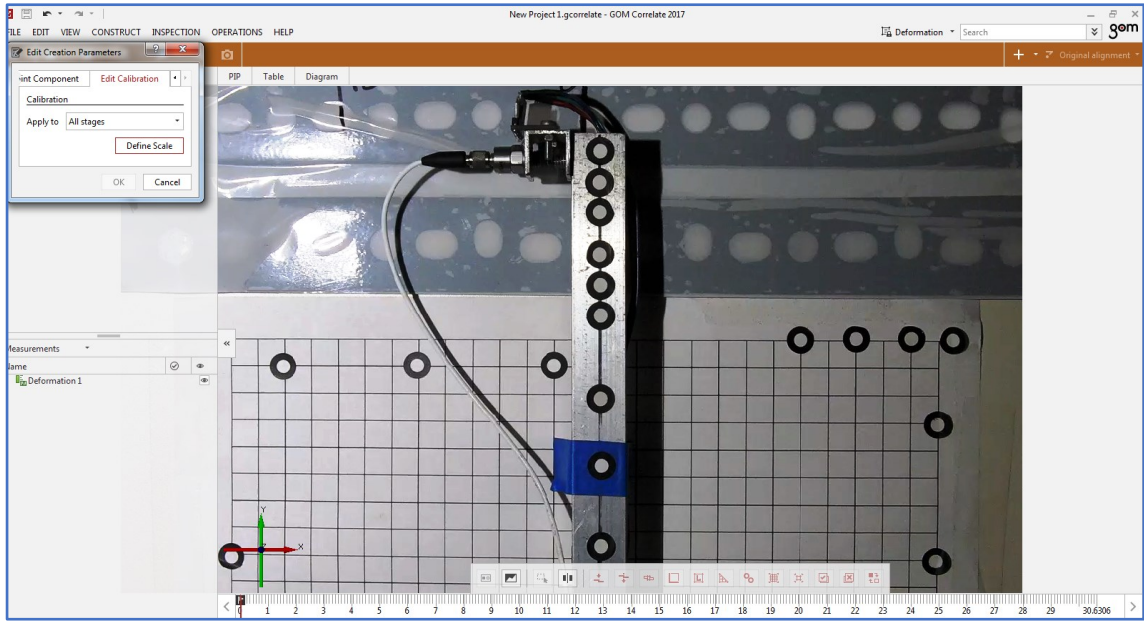


Figure A-4 – Edit Calibration Menu

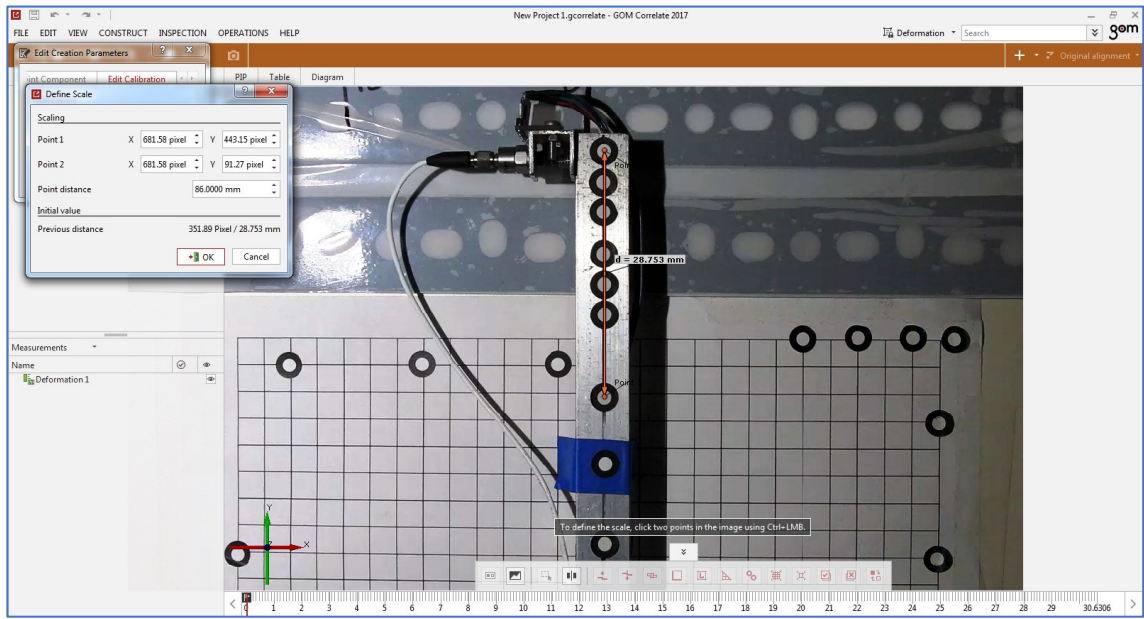


Figure A-5 – Edit Calibration – Define Scale Menu

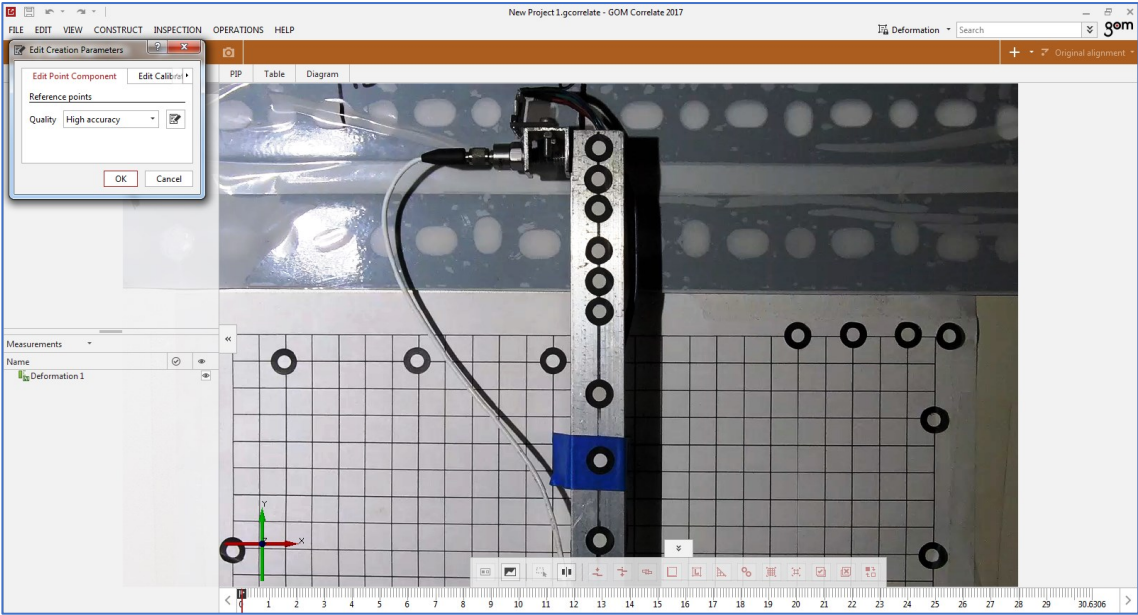


Figure A-6 – Edit Calibration – Edit Point Component

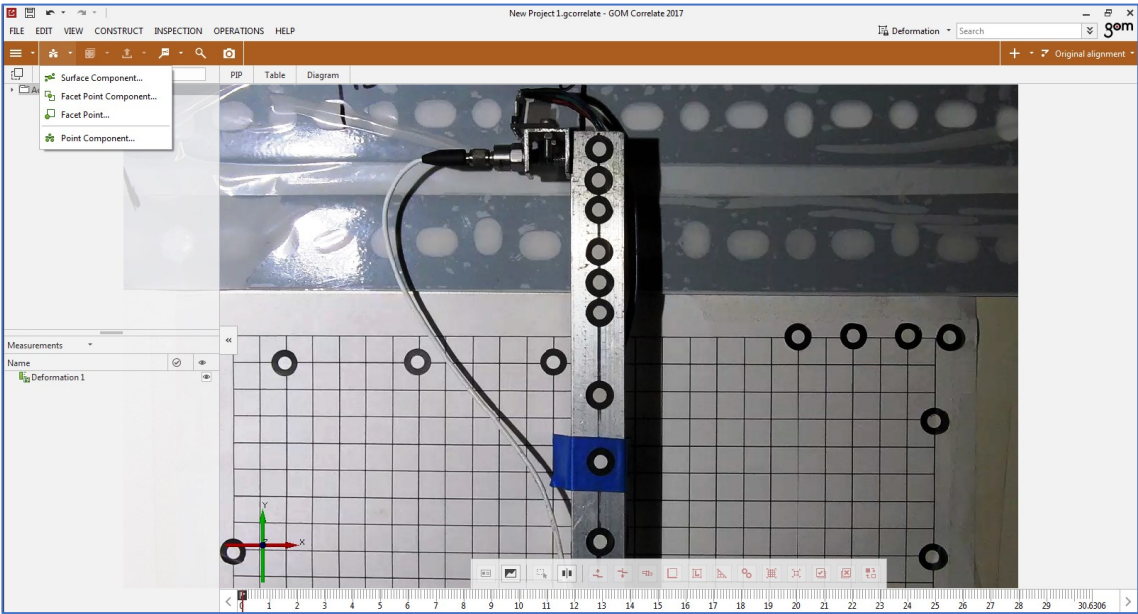


Figure A-7 – Component Menu

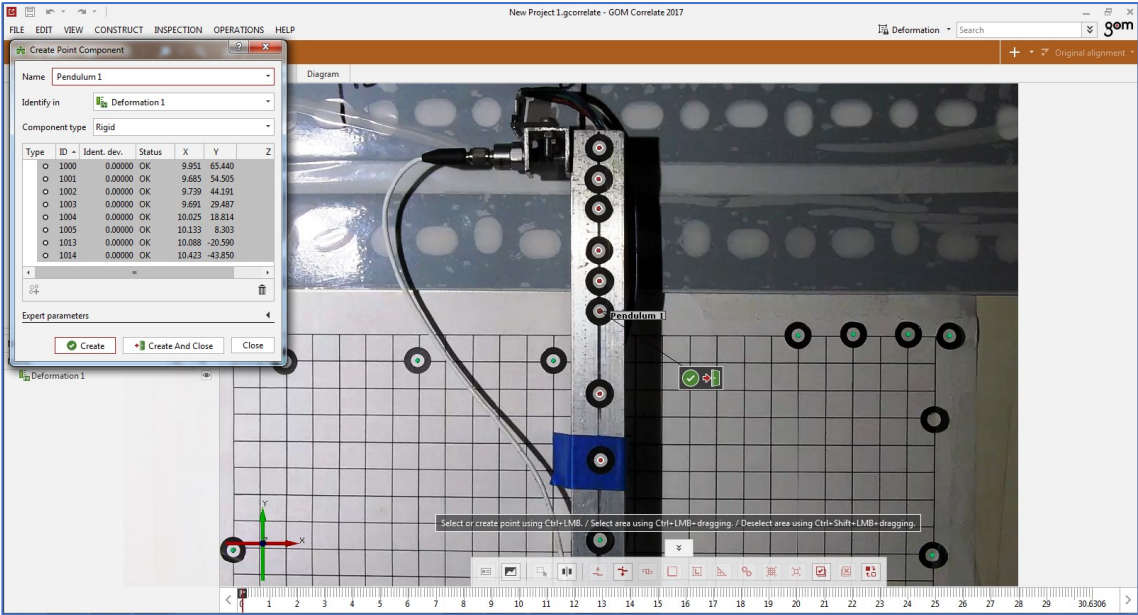


Figure A-8 – Create Point Component Menu

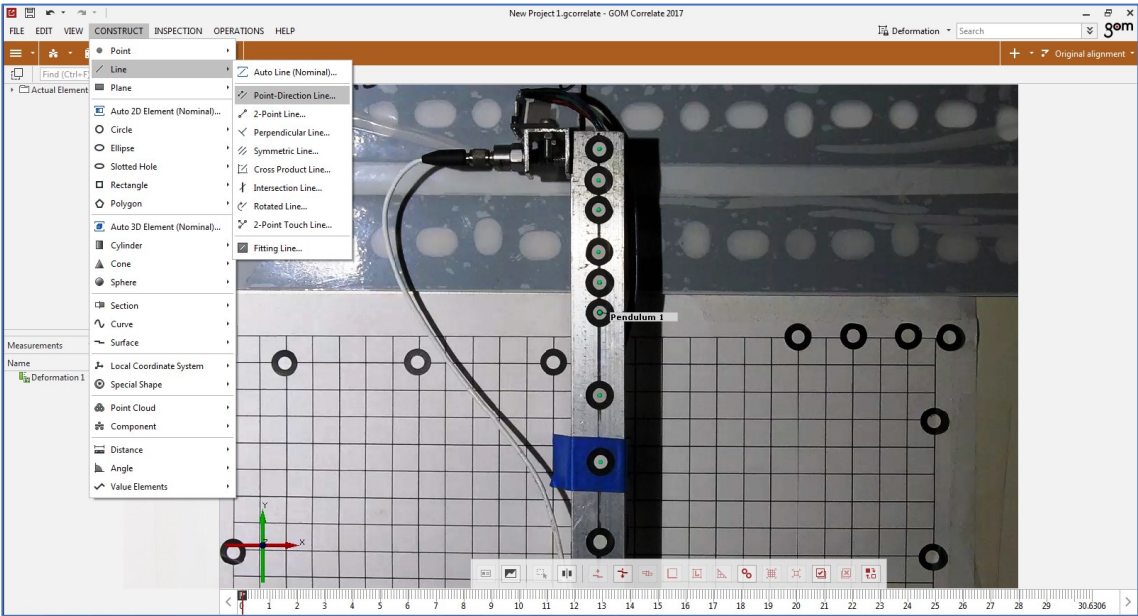


Figure A-9 – Construct Line Menu

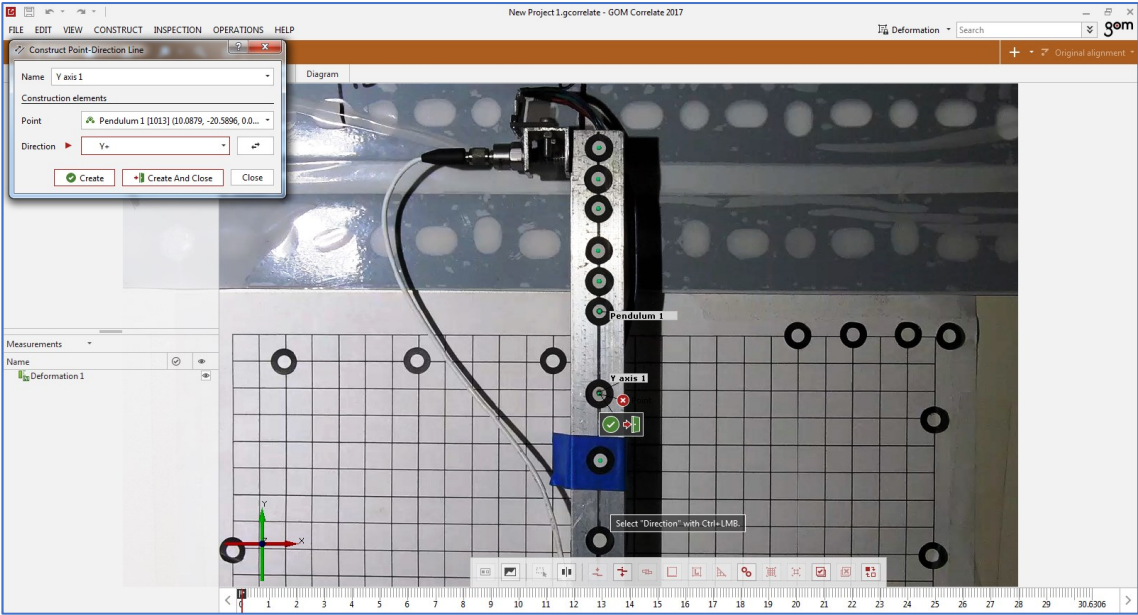


Figure A-10 – Construction Point – Direction Line

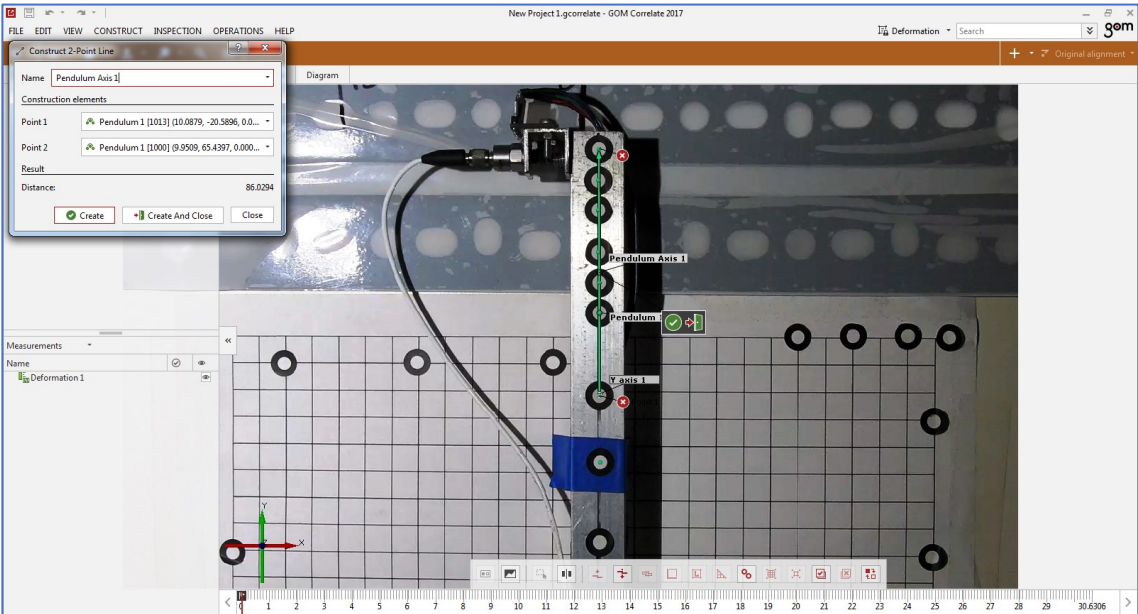


Figure A-11 – Construct Two Point Line

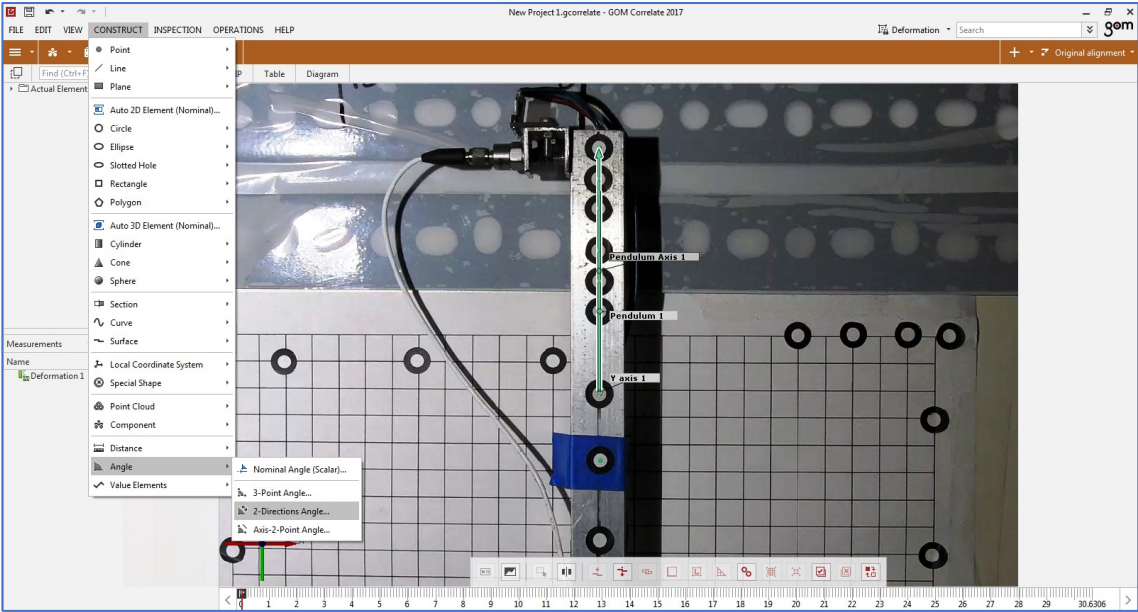


Figure A-12 – Construct Angle Menu

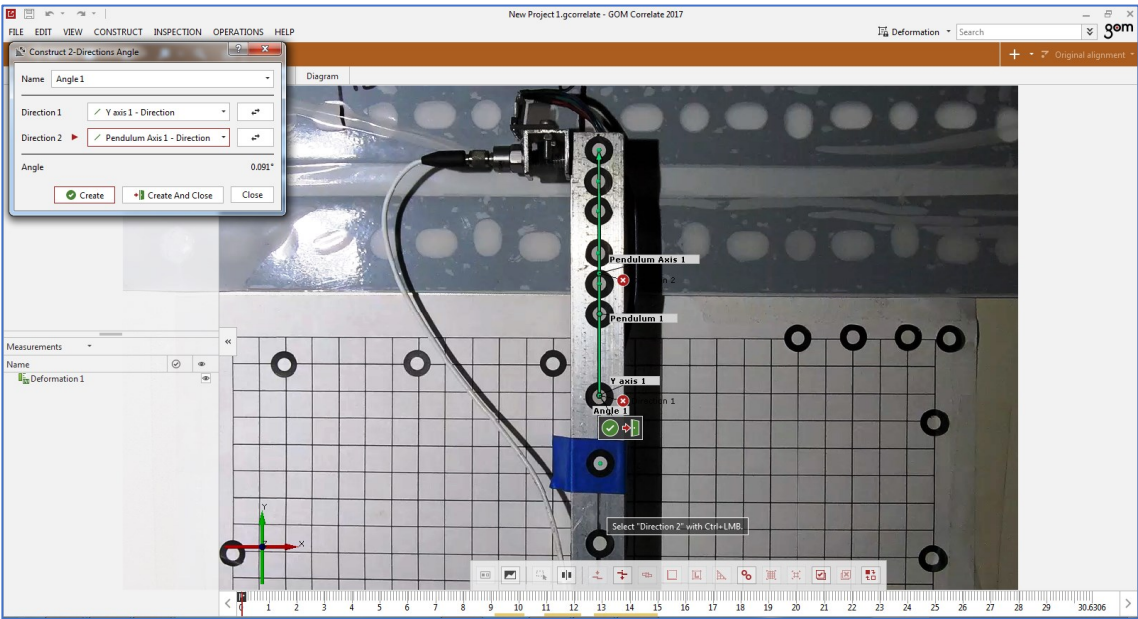


Figure A-13 – Construct 2 Directions Angle

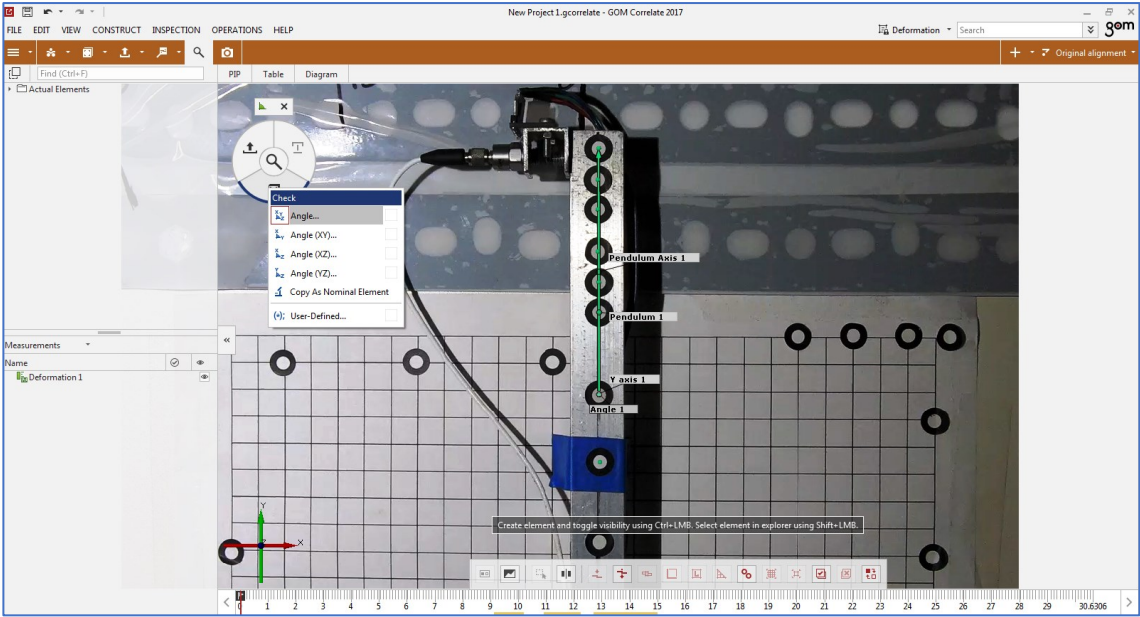


Figure A-14 – Inspect Angle Menu

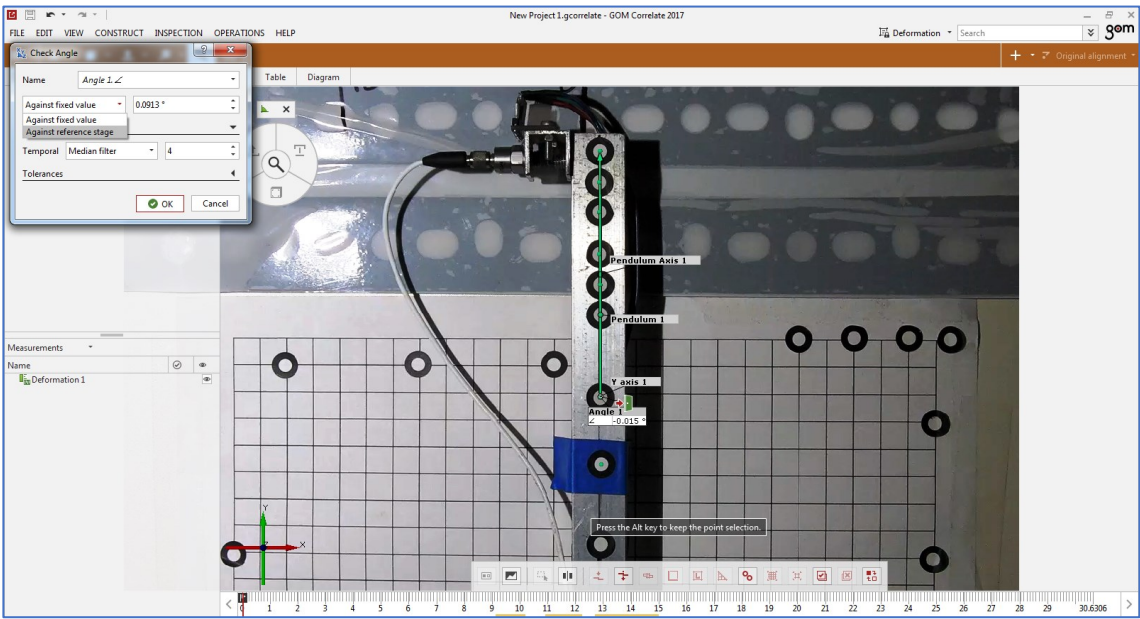


Figure A-15 – Check Angle Menu

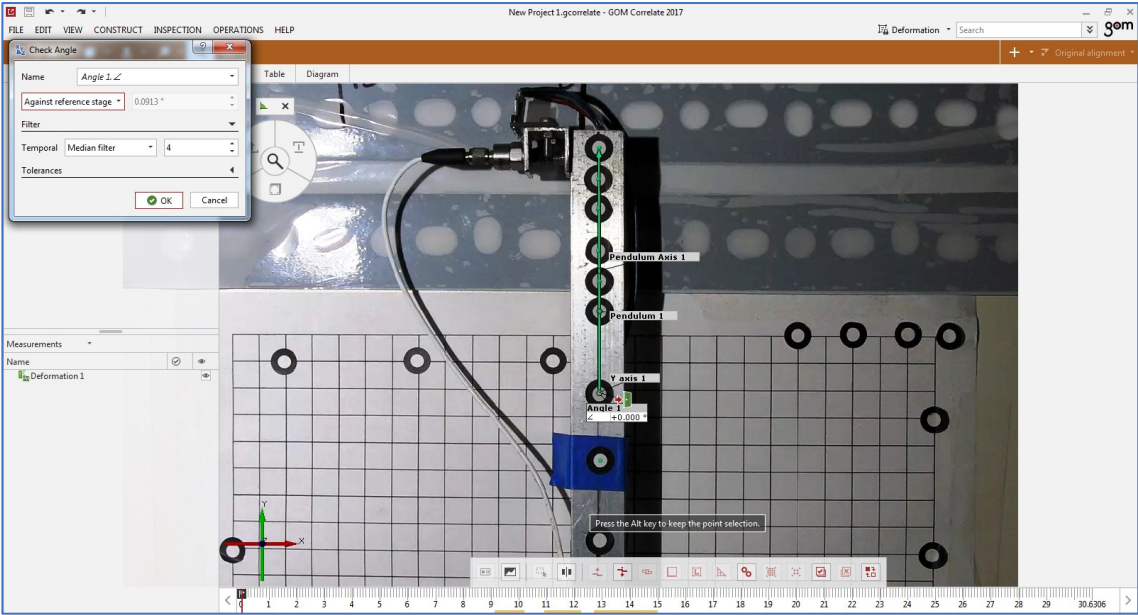


Figure A-16 – Check Angle Menu – Against reference stage

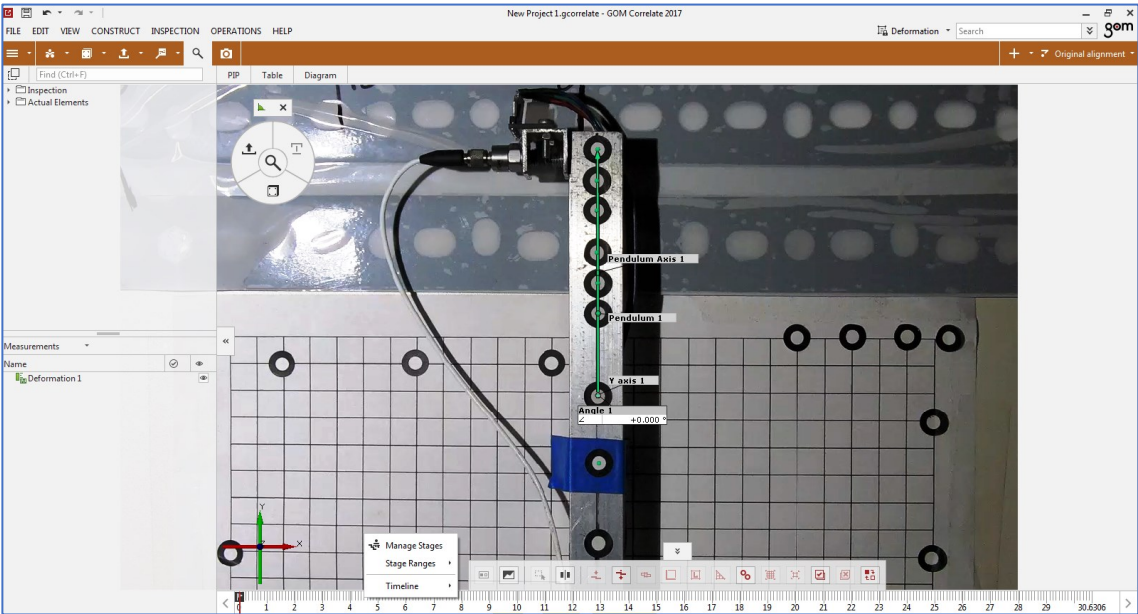


Figure A-17 – Figure A-14 – Adjust timeline

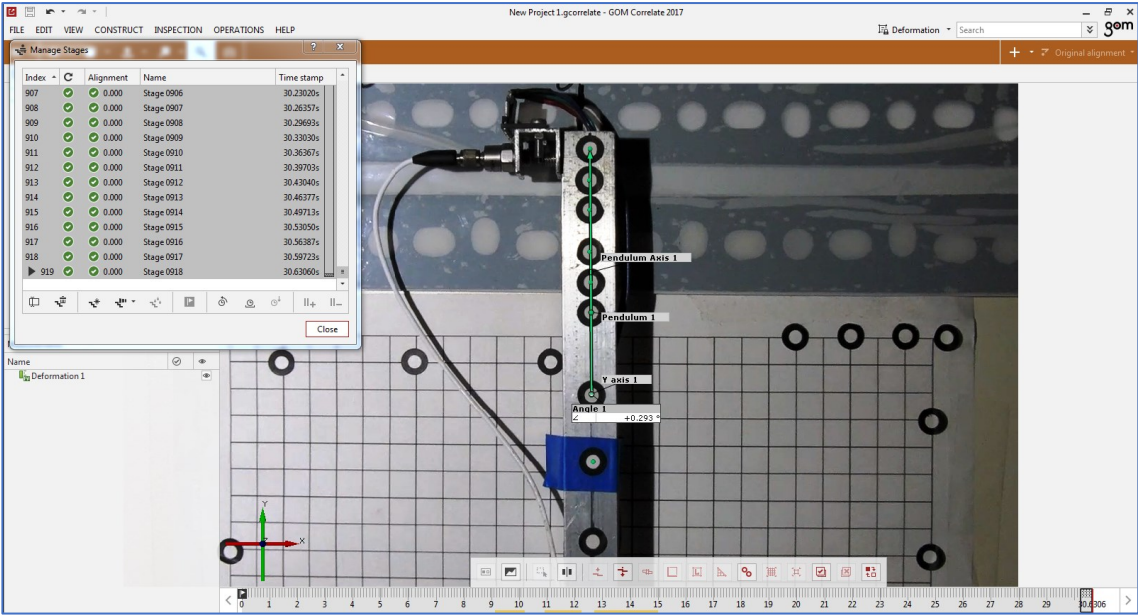


Figure A-18 – Figure A-14 – Manage Stages Menu

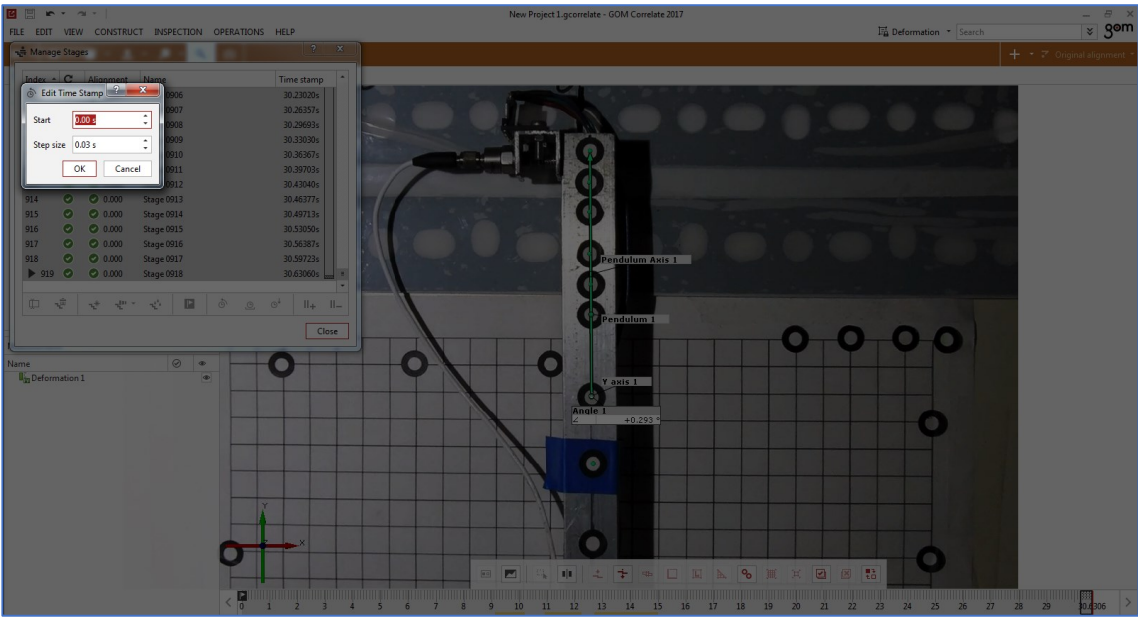


Figure A-19 – Figure A-14 – Edit Time Step Menu

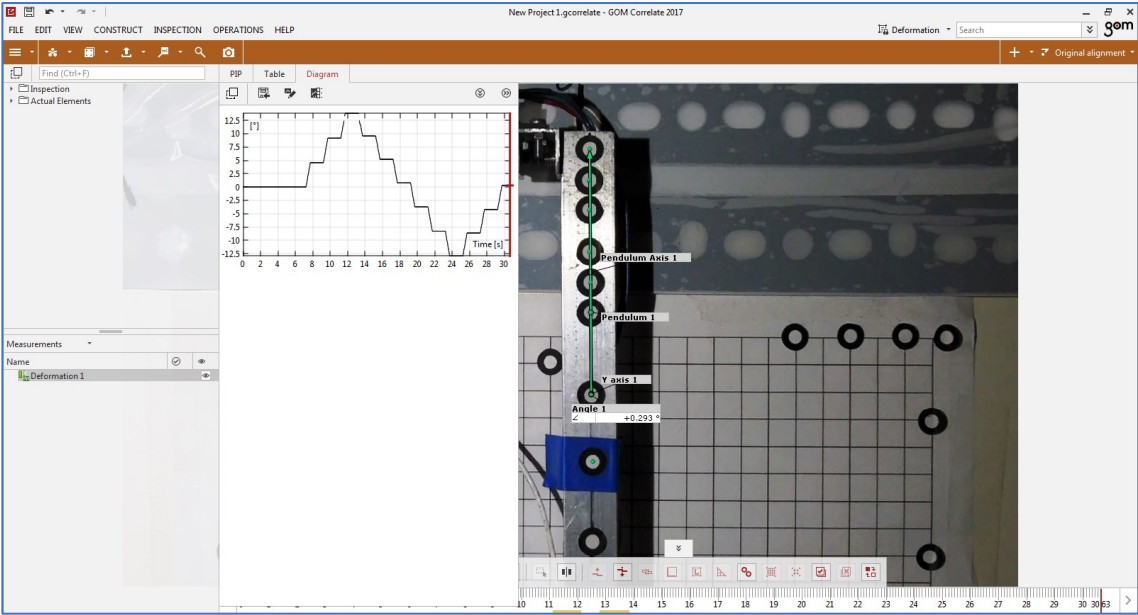


Figure A-20 – Diagram Tab

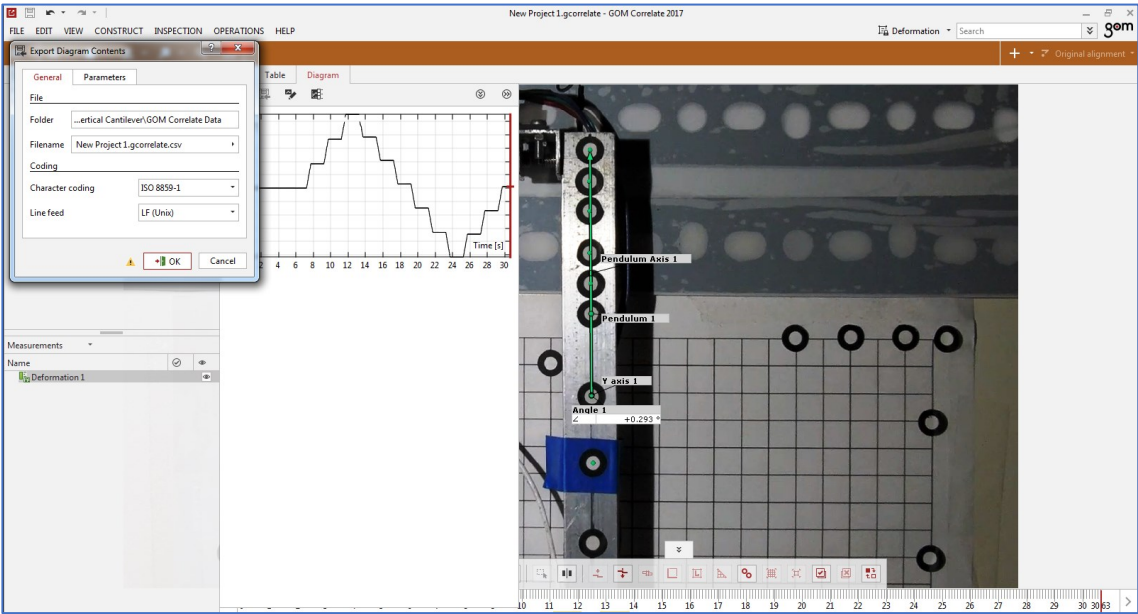


Figure A-21 – Export Diagram Contents Menu

Annex B Quaternions and their derivations

B.1. Background

The use of quaternions was initially investigated in relation to their use with the Explicit Complementary Filter (ECF) proposed by Mahony and Euston. However inconsistent data during testing resulted in the method being excluded from the final analysis. Details have been retained in this annex for information and as a prompt for further investigation in future research into their efficacy in relation to short duration load scenarios.

B.2. Introduction

Quaternions are “hyper-complex numbers” of rank 4 (Kuipers, 2002). Originally invented in 1843 by the Irishman William Rowan Hamilton, they have seen increased use in a variety of fields where the attitude of rigid bodies is being investigated. A 3D representation of a quaternion q is shown in Figure B-1.

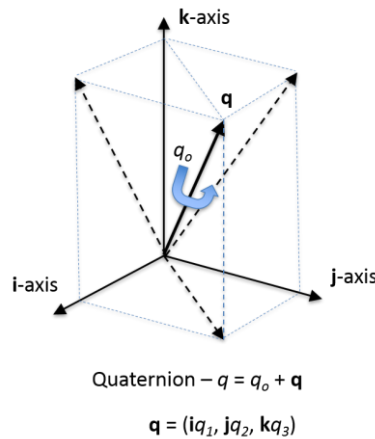


Figure B-1 – Quaternion representation

In this example q comprises a *scalar* part q_0 and a *vector* part $\mathbf{q}$ that has components $\mathbf{i}q_1$, $\mathbf{j}q_2$ and $\mathbf{k}q_3$ where $\mathbf{i} = (1, 0, 0)$, $\mathbf{j} = (0, 1, 0)$, $\mathbf{k} = (0, 0, 1)$ and can be written in the form $q = (q_0, \mathbf{q})$. Although various forms of notation are used in the literature, that proposed by Kuipers was adopted for this research (Kuipers, 2002). Depending upon the situation, quaternions have been represented as follows:

$$q = q_0 + \mathbf{i}q_1 + \mathbf{j}q_2 + \mathbf{k}q_3 \quad \text{or} \quad \mathbf{q} = \begin{bmatrix} q_0 \\ \mathbf{q} \end{bmatrix} = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

where vector elements have been denoted by lower-case bold letters (e.g. $\mathbf{q}, \mathbf{p}$) and scalar elements and quaternions by italic lower-case letters (e.g. q, p or q_0). Examples of basic operations based upon two quaternions, q and p

$$q = q_0 + \mathbf{i}q_1 + \mathbf{j}q_2 + \mathbf{k}q_3, p = p_0 + \mathbf{i}p_1 + \mathbf{j}p_2 + \mathbf{k}p_3$$

are described in the following sections.

B.3. Basic operations

B.3.1. Addition and scalar multiplication

The sum of quaternions q and p is:

$$q + p = (q_0 + p_0) + \mathbf{i}(q_1 + p_1) + \mathbf{j}(q_2 + p_2) + \mathbf{k}(q_3 + p_3) \text{ or } q + p = \begin{bmatrix} q_0 + p_0 \\ q_1 + p_1 \\ q_2 + p_2 \\ q_3 + p_3 \end{bmatrix}$$

A quaternion multiplied by a scalar is:

$$aq = aq_0 + \mathbf{i}aq_1 + \mathbf{j}aq_2 + \mathbf{k}aq_3$$

B.3.2. Quaternion product

The following rules apply to the multiplication of the quaternion elements $\mathbf{i}, \mathbf{j}$ and $\mathbf{k}$:

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -1$$

$$\mathbf{ij} = \mathbf{k} = -\mathbf{ji}$$

$$\mathbf{jk} = \mathbf{i} = -\mathbf{kj}$$

$$\mathbf{ki} = \mathbf{j} = -\mathbf{ik}$$

Therefore, when applied to the product of q and p the result is:

$$q \otimes p = (q_0 + \mathbf{i}q_1 + \mathbf{j}q_2 + \mathbf{k}q_3)(p_0 + \mathbf{i}p_1 + \mathbf{j}p_2 + \mathbf{k}p_3) = q_0p_0 - \mathbf{q} \cdot \mathbf{p} + p_0\mathbf{q} + q_0\mathbf{p} + \mathbf{q} \times \mathbf{p}$$

Where $\otimes$ is the operator for quaternion multiplication, $\mathbf{q} \cdot \mathbf{p}$ is the scalar product and $\mathbf{q} \times \mathbf{p}$ is the cross product:

$$\mathbf{q} \cdot \mathbf{p} = q_1p_1 + q_2p_2 + q_3p_3$$

$$\mathbf{q} \times \mathbf{p} = \mathbf{i}(q_2p_3 - q_3p_2) + \mathbf{j}(q_3p_1 - q_1p_3) + \mathbf{k}(q_1p_2 - q_2p_1)$$

Note also that:

$$\mathbf{q} \times \mathbf{q} = \mathbf{i}(q_2q_3 - q_3q_2) + \mathbf{j}(q_3q_1 - q_1q_3) + \mathbf{k}(q_1q_2 - q_2q_1) = \mathbf{0}$$

In instances where there is limited scope for confusion the quaternion multiplication operator $\otimes$ has been omitted in subsequent sections. Derivation of the product can be found at 0.

B.3.3. Complex conjugate

The complex conjugate of a quaternion is defined as follows:

$$q^* = q_0 - \mathbf{q} = q_0 - \mathbf{i}q_1 - \mathbf{j}q_2 - \mathbf{k}q_3$$

B.3.4. Quaternion norm

The norm of a quaternion is denoted by $N(q)$ or $|q|$ and represents the length of the quaternion and is defined as follows:

$$\begin{aligned} N(q) &= \sqrt{q^* \otimes q} \\ N^2(q) &= q^* \otimes q = (q_0 - \mathbf{q}) \otimes (q_0 + \mathbf{q}) \\ &= (q_0 q_0 - (-\mathbf{q}) \cdot \mathbf{q} + q_0 \mathbf{q} + q_0 (-\mathbf{q}) + (-\mathbf{q}) \times \mathbf{q}) \\ &= (q_0^2 + \mathbf{q} \cdot \mathbf{q} + 0) = q_0^2 + q_1^2 + q_2^2 + q_3^2 \\ N^2(q) &= |q|^2 \end{aligned}$$

B.3.5. Quaternion inverse

The inverse of quaternion q is q^{-1} so:

$$q^{-1}q = qq^{-1} = 1$$

therefore:

$$q^{-1}qq^* = q^*qq^{-1} = q^*$$

it can also be shown that $q^*q = N^2(q)$ and therefore:

$$\begin{aligned} N^2(q)q^{-1} &= q^* \\ q^{-1} &= \frac{q^*}{N^2(q)} = \frac{q^*}{|q|^2} \end{aligned}$$

If q is a unit or normalised quaternion then $N(q) = 1$ and therefore:

$$q^{-1} = q^*$$

B.3.6. Quaternion rotation

Often a primary reason for using quaternions is to enable the rotation of a vector from one frame to another to be accomplished more easily than when using a number of individual rotations as required when using other methods such as Euler angles. When using quaternions this is achieved by creating a rotation quaternion q_R whose norm $|q|$ must equal 1. To rotate a vector from frame S (sensor) to E (Earth), such as that shown in Figure B-2, a conjugate quaternion operation is necessary:

$$q_E = q_R \otimes q_S \otimes q_R^*$$

Where q_E and q_S are pure quaternions and q_R is a rotation quaternion with norm equal to one. A 3D vector can be represented as a pure quaternion by setting the scalar part $q_0 = 0$ as follows:

$$q = 0 + \mathbf{i}q_1 + \mathbf{j}q_2 + \mathbf{k}q_3 = 0 + \mathbf{i}x + \mathbf{j}y + \mathbf{k}z$$

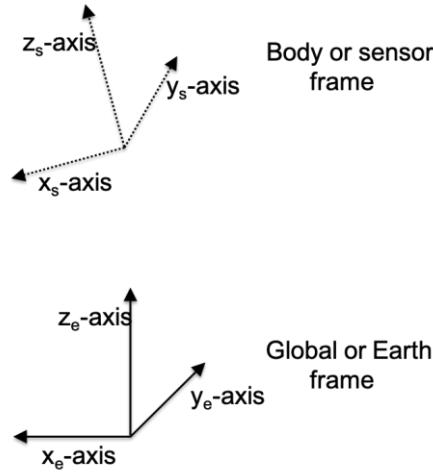


Figure B-2 – Sensor and Earth co-ordinate reference frames

The multiplication of the pure quaternion q_E by the rotation quaternion q_R and its conjugate results in the following:

$$\begin{aligned} q_E &= q_R \otimes q_S \otimes q_R^* = (q_0 + \mathbf{i}q_1 + \mathbf{j}q_2 + \mathbf{k}q_3) (\mathbf{i}x + \mathbf{j}y + \mathbf{k}z) (q_0 - \mathbf{i}q_1 - \mathbf{j}q_2 - \mathbf{k}q_3) \\ &= (x(q_0^2 + q_1^2 - q_2^2 - q_3^2) + 2y(q_1q_2 - q_0q_3) + 2z(q_0q_2 + q_1q_3))\mathbf{i} + \end{aligned}$$

$$(2x(q_0q_3 + q_1q_2) + y(q_0^2 - q_1^2 + q_2^2 - q_3^2) + 2z(q_2q_3 - q_0q_1))\mathbf{j} +$$

$$(2x(q_1q_3 - q_0q_2) + 2y(q_0q_1 + q_2q_3) + z(q_0^2 - q_1^2 - q_2^2 + q_3^2))\mathbf{k}$$

This can be expressed as a rotation matrix R as follows:

$$R = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 - q_0q_3) & 2(q_0q_2 + q_1q_3) \\ 2(q_0q_3 - q_1q_2) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 - q_0q_1) \\ 2(q_1q_3 - q_0q_2) & 2(q_0q_1 + q_2q_3) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}$$

Although it is common for the literature to state these basic operations without any proof, as done here in the interests of brevity, the full derivations have however been verified and included below in sections 0 to 0.

B.3.7. Calculation of rotation quaternion

There are various methods that can be used to calculate the rotation quaternion. If the attitude of the sensor is aligned to the Earth frame and is at rest, then $q_R = (1 + \mathbf{i}0 + \mathbf{j}0 + \mathbf{k}0)$ and the rotation matrix R becomes the identity matrix I . If this is not the case, then it is possible to define the quaternion using the unit vector $\mathbf{v}$ defining the axis of rotation and the rotation angle θ as follows:

$$\mathbf{v} = \begin{bmatrix} \frac{v_1}{|\mathbf{v}|} \\ \frac{v_2}{|\mathbf{v}|} \\ \frac{v_3}{|\mathbf{v}|} \end{bmatrix} \text{ where } |\mathbf{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

$$\text{and } q = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} \cos \frac{\theta}{2} \\ \frac{v_1}{|\mathbf{v}|} \sin \frac{\theta}{2} \\ \frac{v_2}{|\mathbf{v}|} \sin \frac{\theta}{2} \\ \frac{v_3}{|\mathbf{v}|} \sin \frac{\theta}{2} \end{bmatrix}$$

The basic operations described above form the basis for the majority of quaternion operations. Due to their computational efficiency, and the avoidance of issues such as ‘gimbal lock’, quaternions are frequently used in conjunction with data fusion algorithms such as the Kalman and Complementary Filter that are discussed in the more detail in Chapter 4 of the thesis.

B.3.8. Quaternion multiplication

$$\begin{aligned}
q \otimes p &= (q_0 + \mathbf{i}q_1 + \mathbf{j}q_2 + \mathbf{k}q_3)(p_0 + \mathbf{i}p_1 + \mathbf{j}p_2 + \mathbf{k}p_3) = q_0p_0 - \mathbf{q} \cdot \mathbf{p} + p_0\mathbf{q} + q_0\mathbf{p} + \mathbf{q} \times \mathbf{p} \\
&= (q_0p_0 + q_0p_1\mathbf{i} + q_0p_2\mathbf{j} + q_0p_3\mathbf{k}) + (q_1p_0 + q_1p_1\mathbf{i} + q_1p_2\mathbf{j} + q_1p_3\mathbf{k}) + (q_2p_0 + q_2p_1\mathbf{i} + q_2p_2\mathbf{j} + q_2p_3\mathbf{k}) + \\
&\quad (q_3p_0 + q_3p_1\mathbf{i} + q_3p_2\mathbf{j} + q_3p_3\mathbf{k}) \\
&= (q_0p_0 + q_0p_1\mathbf{i} + q_0p_2\mathbf{j} + q_0p_3\mathbf{k}) + (q_1p_0\mathbf{i} + q_1p_1\mathbf{i}^2 + q_1p_2\mathbf{ij} + q_1p_3\mathbf{ik}) + (q_2p_0\mathbf{j} + q_2p_1\mathbf{ji} + q_2p_2\mathbf{j}^2 + q_2p_3\mathbf{jk}) + \\
&\quad (q_3p_0\mathbf{k} + q_3p_1\mathbf{ki} + q_3p_2\mathbf{kj} + q_3p_3\mathbf{k}^2) \\
&= (q_0p_0 + q_1p_1\mathbf{i}^2 + q_2p_2\mathbf{j}^2 + q_3p_3\mathbf{k}^2) + (q_0p_1\mathbf{i} + q_1p_0\mathbf{i} + q_3p_2\mathbf{kj} + q_2p_3\mathbf{jk}) + (q_0p_2\mathbf{j} + q_2p_0\mathbf{j} + q_3p_1\mathbf{ki} + q_1p_3\mathbf{ik}) + \\
&\quad (q_0p_3\mathbf{k} + q_3p_0\mathbf{k} + q_1p_2\mathbf{ij} + q_2p_1\mathbf{ji}) \\
&= (q_0p_0 - q_1p_1 - q_2p_2 - q_3p_3) + (q_0p_1 + q_1p_0 + q_2p_3 - q_3p_2)\mathbf{i} + (q_0p_2 + q_2p_0 - q_1p_3 + q_3p_1)\mathbf{j} + \\
&\quad (q_0p_3 + q_3p_0 + q_1p_2 - q_2p_1)\mathbf{k} \\
&= q_0p_0 - (q_1p_1 + q_2p_2 + q_3p_3) + p_0(q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k}) + q_0(p_1\mathbf{i} + p_2\mathbf{j} + p_3\mathbf{k}) + (q_2p_3 - q_3p_2)\mathbf{i} + (q_3p_1 - q_1p_3)\mathbf{j} + \\
&\quad (q_1p_2 - q_2p_1)\mathbf{k} \\
\mathbf{q} \times \mathbf{p} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ q_1 & q_2 & q_3 \\ p_1 & p_2 & p_3 \end{vmatrix} = (q_2p_3 - q_3p_2)\mathbf{i} + (q_3p_1 - q_1p_3)\mathbf{j} + (q_1p_2 - q_2p_1)\mathbf{k} \\
&= q_0p_0 - \mathbf{q} \cdot \mathbf{p} + p_0\mathbf{q} + q_0\mathbf{p} + \mathbf{q} \times \mathbf{p}
\end{aligned}$$

B.3.9. Quaternion rotation

$$q_B = q_R \otimes q_A \otimes q_R^*$$

where the rotation quaternion and its conjugate are represented as follows:

$$q_R = q_0 + q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k}$$

$$q_R^* = q_0 - q_1\mathbf{i} - q_2\mathbf{j} - q_3\mathbf{k}$$

and the quaternion to be rotated is represented as:

$$q_A = 0 + x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$q_B = q_R \otimes q_A \otimes q_R^* = (q_0 + q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k}) \otimes (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \otimes (q_0 - q_1\mathbf{i} - q_2\mathbf{j} - q_3\mathbf{k})$$

Using the properties of the quaternion imaginary part:

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -1$$

$$\mathbf{ij} = \mathbf{k}, \mathbf{ji} = -\mathbf{k}$$

$$\mathbf{jk} = \mathbf{i}, \mathbf{kj} = -\mathbf{i}$$

$$\mathbf{ki} = \mathbf{j}, \mathbf{ik} = -\mathbf{j}$$

Consider:

$$q_R \otimes q_A = (q_0 + q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k}) \otimes (x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$$

Multiplying out the individual terms:

$$= (q_0\mathbf{x}\mathbf{i} + q_0\mathbf{y}\mathbf{j} + q_0\mathbf{z}\mathbf{k}) + (q_1\mathbf{x}\mathbf{i} + q_1\mathbf{y}\mathbf{j} + q_1\mathbf{z}\mathbf{k}) + (q_2\mathbf{x}\mathbf{j} + q_2\mathbf{y}\mathbf{j} + q_2\mathbf{z}\mathbf{k}) + (q_3\mathbf{x}\mathbf{k} + q_3\mathbf{y}\mathbf{j} + q_3\mathbf{z}\mathbf{k})$$

Collecting the imaginary parts:

$$= (q_0\mathbf{x}\mathbf{i} + q_0\mathbf{y}\mathbf{j} + q_0\mathbf{z}\mathbf{k}) + (q_1\mathbf{x}\mathbf{i}^2 + q_1\mathbf{y}\mathbf{i}\mathbf{j} + q_1\mathbf{z}\mathbf{i}\mathbf{k}) + (q_2\mathbf{x}\mathbf{j}\mathbf{i} + q_2\mathbf{y}\mathbf{j}^2 + q_2\mathbf{z}\mathbf{j}\mathbf{k}) + (q_3\mathbf{x}\mathbf{k}\mathbf{i} + q_3\mathbf{y}\mathbf{k}\mathbf{j} + q_3\mathbf{z}\mathbf{k}^2)$$

Simplifying using quaternion properties:

$$\begin{aligned} &= (q_0\mathbf{x}\mathbf{i} + q_0\mathbf{y}\mathbf{j} + q_0\mathbf{z}\mathbf{k}) + (q_1\mathbf{x}(-1) + q_1\mathbf{y}(\mathbf{k}) + q_1\mathbf{z}(-\mathbf{j})) + (q_2\mathbf{x}(-\mathbf{k}) + q_2\mathbf{y}(-1) + q_2\mathbf{z}(\mathbf{i})) + (q_3\mathbf{x}(\mathbf{j}) + q_3\mathbf{y}(-\mathbf{i}) + q_3\mathbf{z}(-1)) \\ &= (q_0\mathbf{x}\mathbf{i} + q_0\mathbf{y}\mathbf{j} + q_0\mathbf{z}\mathbf{k}) + (-q_1\mathbf{x} + q_1\mathbf{y}\mathbf{k} - q_1\mathbf{z}\mathbf{j}) + (-q_2\mathbf{x}\mathbf{k} - q_2\mathbf{y} + q_2\mathbf{z}\mathbf{i}) + (q_3\mathbf{x}\mathbf{j} - q_3\mathbf{y}\mathbf{i} - q_3\mathbf{z}) \end{aligned}$$

Arrange in terms of scalar and imaginary parts:

$$= (-q_1\mathbf{x} - q_2\mathbf{y} - q_3\mathbf{z}) + (q_0\mathbf{x}\mathbf{i} + q_2\mathbf{z}\mathbf{i} - q_3\mathbf{y}\mathbf{i}) + (q_0\mathbf{y}\mathbf{j} - q_1\mathbf{z}\mathbf{j} + q_3\mathbf{x}\mathbf{j}) + (q_0\mathbf{z}\mathbf{k} + q_1\mathbf{y}\mathbf{k} - q_2\mathbf{x}\mathbf{k})$$

Multiply by the conjugate:

$$\begin{aligned} &= [(-q_1\mathbf{x} - q_2\mathbf{y} - q_3\mathbf{z}) + (q_0\mathbf{x}\mathbf{i} + q_2\mathbf{z}\mathbf{i} - q_3\mathbf{y}\mathbf{i}) + (q_0\mathbf{y}\mathbf{j} - q_1\mathbf{z}\mathbf{j} + q_3\mathbf{x}\mathbf{j}) \\ &\quad + (q_0\mathbf{z}\mathbf{k} + q_1\mathbf{y}\mathbf{k} - q_2\mathbf{x}\mathbf{k})] (q_0 - q_1\mathbf{i} - q_2\mathbf{j} - q_3\mathbf{k}) \\ &= (-q_0q_1\mathbf{x} - q_0q_2\mathbf{y} - q_0q_3\mathbf{z}) + (q_0^2\mathbf{x}\mathbf{i} + q_0q_2\mathbf{z}\mathbf{i} - q_0q_3\mathbf{y}\mathbf{i}) + (q_0^2\mathbf{y}\mathbf{j} - q_0q_1\mathbf{z}\mathbf{j} + q_0q_3\mathbf{x}\mathbf{j}) + (q_0^2\mathbf{z}\mathbf{k} + q_0q_1\mathbf{y}\mathbf{k} - q_0q_2\mathbf{x}\mathbf{k}) \\ &\quad + (q_1^2\mathbf{x}\mathbf{i} + q_1q_2\mathbf{y}\mathbf{i} + q_1q_3\mathbf{z}\mathbf{i}) + (-q_0q_1\mathbf{x}\mathbf{i}^2 - q_1q_2\mathbf{z}\mathbf{i}^2 + q_1q_3\mathbf{y}\mathbf{i}^2) + (-q_0q_1\mathbf{y}\mathbf{j}\mathbf{i} + q_1^2\mathbf{z}\mathbf{j}\mathbf{i} - q_1q_3\mathbf{x}\mathbf{j}\mathbf{i}) \\ &\quad + (-q_0q_1\mathbf{z}\mathbf{k}\mathbf{i} - q_1^2\mathbf{y}\mathbf{k}\mathbf{i} + q_1q_2\mathbf{x}\mathbf{k}\mathbf{i}) \\ &\quad + (q_1q_2\mathbf{x}\mathbf{j} + q_2^2\mathbf{y}\mathbf{j} + q_2q_3\mathbf{z}\mathbf{j}) + (-q_0q_2\mathbf{x}\mathbf{i}\mathbf{j} - q_2^2\mathbf{z}\mathbf{i}\mathbf{j} + q_2q_3\mathbf{y}\mathbf{i}\mathbf{j}) + (-q_0q_2\mathbf{y}\mathbf{j}^2 + q_1q_2\mathbf{z}\mathbf{j}^2 - q_2q_3\mathbf{x}\mathbf{j}^2) \\ &\quad + (-q_0q_2\mathbf{z}\mathbf{k}\mathbf{j} - q_1q_2\mathbf{y}\mathbf{k}\mathbf{j} + q_2^2\mathbf{x}\mathbf{k}\mathbf{j}) \\ &\quad + (q_1q_3\mathbf{x}\mathbf{k} + q_2q_3\mathbf{y}\mathbf{k} - q_3^2\mathbf{z}\mathbf{k}) + (-q_0q_3\mathbf{x}\mathbf{i}\mathbf{k} - q_2q_3\mathbf{z}\mathbf{i}\mathbf{k} + q_3^2\mathbf{y}\mathbf{i}\mathbf{k}) + (-q_0q_3\mathbf{y}\mathbf{j}\mathbf{k} + q_1q_3\mathbf{z}\mathbf{j}\mathbf{k} - q_3^2\mathbf{x}\mathbf{j}\mathbf{k}) \\ &\quad + (-q_0q_3\mathbf{z}\mathbf{k}^2 - q_1q_3\mathbf{y}\mathbf{k}^2 + q_2q_3\mathbf{x}\mathbf{k}^2) \end{aligned}$$

Collecting into groups of similar terms:

$$\begin{aligned} &= (-q_0q_1\mathbf{x} - q_0q_2\mathbf{y} - q_0q_3\mathbf{z}) + (q_0^2\mathbf{x}\mathbf{i} + q_0q_2\mathbf{z}\mathbf{i} - q_0q_3\mathbf{y}\mathbf{i}) + (q_0^2\mathbf{y}\mathbf{j} - q_0q_1\mathbf{z}\mathbf{j} + q_0q_3\mathbf{x}\mathbf{j}) + (q_0^2\mathbf{z}\mathbf{k} + q_0q_1\mathbf{y}\mathbf{k} - q_0q_2\mathbf{x}\mathbf{k}) \\ &\quad + (q_1^2\mathbf{x}\mathbf{i} + q_1q_2\mathbf{y}\mathbf{i} + q_1q_3\mathbf{z}\mathbf{i}) + (q_0q_1\mathbf{x} + q_1q_2\mathbf{z} - q_1q_3\mathbf{y}) + (q_0q_1\mathbf{y}\mathbf{k} - q_1^2\mathbf{z}\mathbf{k} + q_1q_3\mathbf{x}\mathbf{k}) \\ &\quad + (-q_0q_1\mathbf{z}\mathbf{j} - q_1^2\mathbf{y}\mathbf{j} + q_1q_2\mathbf{x}\mathbf{j}) + (q_1q_2\mathbf{x}\mathbf{j} + q_2^2\mathbf{y}\mathbf{j} + q_2q_3\mathbf{z}\mathbf{j}) + (-q_0q_2\mathbf{x}\mathbf{k} - q_2^2\mathbf{z}\mathbf{k} + q_2q_3\mathbf{y}\mathbf{k}) \\ &\quad + (q_0q_2\mathbf{y} - q_1q_2\mathbf{z} + q_2q_3\mathbf{x}) + (q_0q_2\mathbf{z}\mathbf{i} + q_1q_2\mathbf{y}\mathbf{i} - q_2^2\mathbf{x}\mathbf{i}) + (q_1q_3\mathbf{x}\mathbf{k} + q_2q_3\mathbf{y}\mathbf{k} - q_3^2\mathbf{z}\mathbf{k}) + (q_0q_3\mathbf{x}\mathbf{j} + q_2q_3\mathbf{z}\mathbf{j} - q_3^2\mathbf{y}\mathbf{j}) \\ &\quad + (-q_0q_3\mathbf{y}\mathbf{i} + q_1q_3\mathbf{z}\mathbf{i} - q_3^2\mathbf{x}\mathbf{i}) + (q_0q_3\mathbf{z} + q_1q_3\mathbf{y} - q_2q_3\mathbf{x}) \end{aligned}$$

$$\begin{aligned} &= ((q_0^2\mathbf{x} + q_0q_2\mathbf{z} - q_0q_3\mathbf{y}) + (q_1^2\mathbf{x} + q_1q_2\mathbf{y} + q_1q_3\mathbf{z}) + (q_0q_2\mathbf{z} + q_1q_2\mathbf{y} - q_2^2\mathbf{x}) + (-q_0q_3\mathbf{y} + q_1q_3\mathbf{z} - q_3^2\mathbf{x}))\mathbf{i} \\ &\quad + ((q_0^2\mathbf{y} - q_0q_1\mathbf{z} + q_0q_3\mathbf{x}) + (-q_0q_1\mathbf{z} - q_1^2\mathbf{y} + q_1q_2\mathbf{x}) + (q_1q_2\mathbf{x} + q_2^2\mathbf{y} + q_2q_3\mathbf{z}) + (q_0q_3\mathbf{x} + q_2q_3\mathbf{z} - q_3^2\mathbf{y}))\mathbf{j} \\ &\quad + ((q_0^2\mathbf{z} + q_0q_1\mathbf{y} - q_0q_2\mathbf{x}) + (q_0q_1\mathbf{y} - q_1^2\mathbf{z} + q_1q_3\mathbf{x}) + (-q_0q_2\mathbf{x} - q_2^2\mathbf{z} + q_2q_3\mathbf{y}) + (q_1q_3\mathbf{x} + q_2q_3\mathbf{y} - q_3^2\mathbf{z}))\mathbf{k} \end{aligned}$$

$$\begin{aligned} &= (x(q_0^2 + q_1^2 - q_2^2 - q_3^2) + y(-q_0q_3 + q_1q_2 + q_1q_2 - q_0q_3) + z(q_0q_2 + q_1q_3 + q_0q_2 + q_1q_3))\mathbf{i} \\ &\quad + (x(q_0q_3 + q_1q_2 + q_1q_2 + q_0q_3) + y(q_0^2 - q_1^2 + q_2^2 - q_3^2) + z(-q_0q_1 - q_0q_1 + q_2q_3 + q_2q_3))\mathbf{j} + \end{aligned}$$

$$\begin{aligned}
& + (x(-q_0q_2 + q_1q_3 - q_0q_2 + q_1q_3) + y(q_0q_1 + q_0q_1 + q_2q_3 + q_2q_3) + z(q_0^2 - q_1^2 - q_2^2 + q_3^2))\mathbf{k} \\
& = (x(q_0^2 + q_1^2 - q_2^2 - q_3^2) + 2y(q_1q_2 - q_0q_3) + 2z(q_0q_2 + q_1q_3))\mathbf{i} \\
& + (2x(q_0q_3 + q_1q_2) + y(q_0^2 - q_1^2 + q_2^2 - q_3^2) + 2z(q_2q_3 - q_0q_1))\mathbf{j} + \\
& + (2x(q_1q_3 - q_0q_2) + 2y(q_0q_1 + q_2q_3) + z(q_0^2 - q_1^2 - q_2^2 + q_3^2))\mathbf{k}
\end{aligned}$$

B.3.10. Triple products

(Kuipers, 2002) (p124) states that the triple vector products can be written in the following forms:

1. $\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a} \cdot \bar{c})\bar{b} - (\bar{a} \cdot \bar{b})\bar{c}$
2. $(\bar{a} \times \bar{b}) \times \bar{c} = (\bar{a} \cdot \bar{c})\bar{b} - (\bar{b} \cdot \bar{c})\bar{a}$

The derivation of equation 1:

$$\begin{aligned}
\bar{a} \times (\bar{b} \times \bar{c}) &= (\bar{a} \cdot \bar{c})\bar{b} - (\bar{a} \cdot \bar{b})\bar{c} \\
\bar{a} &= a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}, \quad \bar{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k} \quad \text{and} \quad \bar{c} = c_1\mathbf{i} + c_2\mathbf{j} + c_3\mathbf{k} \\
\bar{a} \times (\bar{b} \times \bar{c}) &= \bar{a} \times \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \\
&= (b_2c_3 - b_3c_2)\mathbf{i}, \quad (b_3c_1 - b_1c_3)\mathbf{j}, \quad (b_1c_2 - b_2c_1)\mathbf{k} \\
\bar{a} \times (\bar{b} \times \bar{c}) &= \bar{a} \times \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ (b_2c_3 - b_3c_2) & (b_3c_1 - b_1c_3) & (b_1c_2 - b_2c_1) \end{vmatrix} \\
&= a_2(b_1c_2 - b_2c_1) - a_3(b_3c_1 - b_1c_3) = a_2b_1c_2 - a_2b_2c_1 - a_3b_3c_1 + a_3b_1c_3 \\
&= a_2b_1c_2 + a_3b_1c_3 - a_2b_2c_1 - a_3b_3c_1 = b_1(a_2c_2 + a_3c_3) - c_1(a_2b_2 + a_3b_3) + a_1b_1c_1 - a_1b_1c_1 \\
&= b_1(a_1c_1 + a_2c_2 + a_3c_3) - c_1(a_1b_1 + a_2b_2 + a_3b_3) \\
&= a_3(b_2c_3 - b_3c_2) - a_1(b_1c_2 - b_2c_1) = a_3b_2c_3 - a_3b_3c_2 - a_1b_1c_2 + a_1b_2c_1 \\
&= a_3b_2c_3 + a_1b_2c_1 - a_3b_3c_2 - a_1b_1c_2 = b_2(a_3c_3 + a_1c_1) - c_2(a_3b_3 + a_1b_1) + a_2b_2c_2 - a_2b_2c_2 \\
&= b_2(a_1c_1 + a_2c_2 + a_3c_3) - c_2(a_1b_1 + a_2b_2 + a_3b_3) \\
&= a_1(b_3c_1 - b_1c_3) - a_2(b_2c_3 - b_3c_2) = a_1b_3c_1 - a_1b_1c_3 - a_2b_2c_3 + a_2b_3c_2 \\
&= a_1b_3c_1 + a_2b_3c_2 - a_1b_1c_3 - a_2b_2c_3 = b_3(a_1c_1 + a_2c_2) - c_3(a_1b_1 + a_2b_2) + a_3b_3c_3 - a_3b_3c_3 \\
&= b_3(a_1c_1 + a_2c_2 + a_3c_3) - c_3(a_1b_1 + a_2b_2 + a_3b_3) \\
&= b_1(a_1c_1 + a_2c_2 + a_3c_3) - c_1(a_1b_1 + a_2b_2 + a_3b_3) \\
&= b_2(a_1c_1 + a_2c_2 + a_3c_3) - c_2(a_1b_1 + a_2b_2 + a_3b_3) \\
&= b_3(a_1c_1 + a_2c_2 + a_3c_3) - c_3(a_1b_1 + a_2b_2 + a_3b_3)
\end{aligned}$$

$$= (\bar{a} \cdot \bar{c})\bar{b} - (\bar{a} \cdot \bar{b})\bar{c}$$

The derivation of equation 2:

$$(\bar{a} \times \bar{b}) \times \bar{c} = (\bar{a} \cdot \bar{c})\bar{b} - (\bar{b} \cdot \bar{c})\bar{a}$$

$$\bar{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}, \quad \bar{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k} \quad \text{and} \quad \bar{c} = c_1\mathbf{i} + c_2\mathbf{j} + c_3\mathbf{k}$$

$$(\bar{a} \times \bar{b}) \times \bar{c} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \times \bar{c}$$

$$(a_2b_3 - a_3b_2), (a_3b_1 - a_1b_3), (a_1b_2 - a_2b_1)$$

$$(\bar{a} \times \bar{b}) \times \bar{c} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ (a_2b_3 - a_3b_2) & (a_3b_1 - a_1b_3) & (a_1b_2 - a_2b_1) \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\begin{aligned} (a_3b_1 - a_1b_3)c_3 - (a_1b_2 - a_2b_1)c_2 &= a_3b_1c_3 - a_1b_3c_3 - a_1b_2c_2 + a_2b_1c_2 \\ &= a_3b_1c_3 + a_2b_1c_2 - a_1b_3c_3 - a_1b_2c_2 = b_1(a_2c_2 + a_3c_3) - a_1(b_2c_2 + b_3c_3) + a_1b_1c_1 - a_1b_1c_1 \\ &= b_1(a_1c_1 + a_2c_2 + a_3c_3) - a_1(b_1c_1 + b_2c_2 + b_3c_3) \end{aligned}$$

$$\begin{aligned} (a_1b_2 - a_2b_1)c_1 - (a_2b_3 - a_3b_2)c_3 &= a_1b_2c_1 - a_2b_1c_1 - a_2b_3c_3 + a_3b_2c_3 \\ &= a_1b_2c_1 + a_3b_2c_3 - a_2b_1c_1 - a_2b_3c_3 = b_2(a_1c_1 + a_3c_3) - a_2(b_1c_1 + b_3c_3) + a_2b_2c_2 - a_2b_2c_2 \\ &= b_2(a_1c_1 + a_2c_2 + a_3c_3) - a_2(b_1c_1 + b_2c_2 + b_3c_3) \end{aligned}$$

$$\begin{aligned} (a_2b_3 - a_3b_2)c_2 - (a_3b_1 - a_1b_3)c_1 &= a_2b_3c_2 - a_3b_2c_2 - a_3b_1c_1 + a_1b_3c_1 \\ &= a_2b_3c_2 + a_1b_3c_1 - a_3b_2c_2 - a_3b_1c_1 = b_3(a_1c_1 + a_2c_2) - a_3(b_1c_1 + b_2c_2) + a_3b_3c_3 - a_3b_3c_3 \\ &= b_3(a_1c_1 + a_2c_2 + a_3c_3) - a_3(b_1c_1 + b_2c_2 + b_3c_3) \end{aligned}$$

$$= (\bar{a} \cdot \bar{c})\bar{b} - (\bar{b} \cdot \bar{c})\bar{a}$$

B.4. Representing the Complementary Filter using quaternions

The basic concept of the ECF proposed by Mahony and Euston is shown in Figure B-3 and described in detail in the following section.

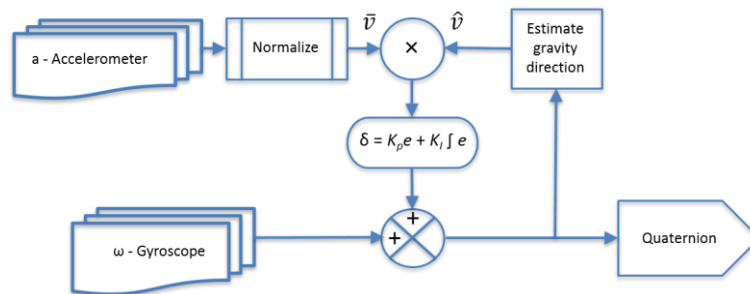


Figure B-3 – General schematic for Explicit Complementary Filter using Quaternions

B.4.1. Initialization

At the beginning of the ECF process initial values of the quaternion $\hat{q}$, sampling frequency Δt and proportional and integral gain (K_P and K_I) are set. It is common to assume that the sensors are at rest and therefore $\hat{q}$ is taken to be a unit quaternion $[1, 0, 0, 0]$. The proportional gain K_P is used to control the relationship between the accelerometer and gyroscope estimates, while the integral gain K_I is used to correct for any gyro bias ω_{bias} .

B.4.2. Read sensor data

The data from the gyroscopes and accelerometers is modelled using the expressions shown in Equations B-1 and B-2 respectively.

$$\omega^b = \omega + \omega_{bias} + n_{gyro} \quad (\text{B-1})$$

$$a = \dot{v} - g + a_{bias} + n_{acc} \quad (\text{B-2})$$

Where ω^b is the gyroscope reading in the Body-Fixed Frame (BFF) (or sensor frame)(see **Figure B-**) which is composed of the gyroscope reading ω , the gyro bias ω_{bias} and the gyro noise n_{gyro} . The accelerometer reading a is composed of the linear acceleration $\dot{v}$, gravitational acceleration g , accelerometer bias a_{bias} and the accelerometer noise n_{acc} . To ensure that the sensor data is in the correct format for subsequent evaluation, it is converted into either a pure quaternion in the case of the gyroscope data, or normalized vector in the case of the accelerometer data as shown in Equations B-3 and B-4 respectively.

$$p(\bar{\omega}^b) = (0, \bar{\omega}^b) = \begin{bmatrix} 0 \\ \bar{\omega}_x^b \\ \bar{\omega}_y^b \\ \bar{\omega}_z^b \end{bmatrix} \quad (\text{B-3})$$

$$\bar{v} = \frac{\hat{g}^b}{|\hat{g}^b|} \quad (\text{B-4})$$

Where $p(\bar{\omega}^b)$ is the pure quaternion of the gyroscope sensor data in the BFF, $\bar{\omega}^b$ the normalised gyroscope components, $\bar{v}$ the measured inertial direction from the accelerometer sensor data, $\hat{g}^b$ the estimate of the gravitational acceleration in the BFF and $|\hat{g}^b|$ is its norm.

B.4.3. Gravity direction estimation

The predicted gravitational direction $\hat{v}$ is calculated using Equation B-5, with the initial quaternion estimate $\hat{q}$ being set as a unit quaternion and subsequently as the algorithm output quaternion $\hat{q}$.

$$\hat{v} = \begin{bmatrix} 2(\hat{q}_0\hat{q}_2 + \hat{q}_1\hat{q}_3) \\ 2(\hat{q}_0\hat{q}_1 + \hat{q}_2\hat{q}_3) \\ \hat{q}_0^2 - \hat{q}_1^2 - \hat{q}_2^2 + \hat{q}_3^2 \end{bmatrix} \quad (\text{B-5})$$

B.4.4. Calculate error

The error between the estimate of the direction of gravity calculated using the gyroscope data $\hat{v}$, and that calculated using the accelerometer data $\bar{v}$, is obtained by computing the ‘cross-product’ of the two as shown in Equation B-6.

$$e = \bar{v} \times \hat{v} \quad (\text{B-6})$$

B.4.5. Calculate rate of change of quaternion

The penultimate step is to differentiate the quaternion using Equation B-7 to compute the rate of change of the quaternion:

$$\dot{\hat{q}}_t = \frac{1}{2} \hat{q}_{t-1} \otimes p(\bar{\omega}^b + \delta) \quad (\text{B-7})$$

Where the proportional and integral gains are used to produce a new error estimate to adjust the gyroscope sensor data as shown in Equation B-8.

$$\delta = K_p e + K_i \int e \Delta t \quad (\text{B-8})$$

B.4.6. Attitude estimation

The final step is to calculate the estimated quaternion for this iteration by integrating the rate of change of the quaternion calculated in the previous step as shown in Equation B-9.

$$\hat{q}_t = \hat{q}_{t-1} + \dot{\hat{q}}_t \Delta t \quad (\text{B-9})$$

The quaternion is normalised prior to being used to compute further iterations, or to compute the attitude in Euler form using Equations B-10, B-11 or B-12 which represent Roll, Pitch and Yaw respectively:

$$\phi = \text{atan2}(2(q_2q_3 - q_0q_1), 2q_0^2 - 1 + 2q_3^2) \quad (\text{B-10})$$

$$\theta = -\arctan\left(\frac{2(q_1q_3 + q_0q_2)}{\sqrt{1 - (2q_1q_3 + 2q_0q_2)^2}}\right) \quad (\text{B-11})$$

$$\psi = \text{atan2}(2(q_1q_2 - q_0q_3), 2q_0^2 - 1 + 2q_1^2) \quad (\text{B-12})$$

B.5. Complementary Filter using Quaternions LabVIEW VI

The CF filter using quaternions described above was converted to the LabVIEW VI shown in Figure B-4.

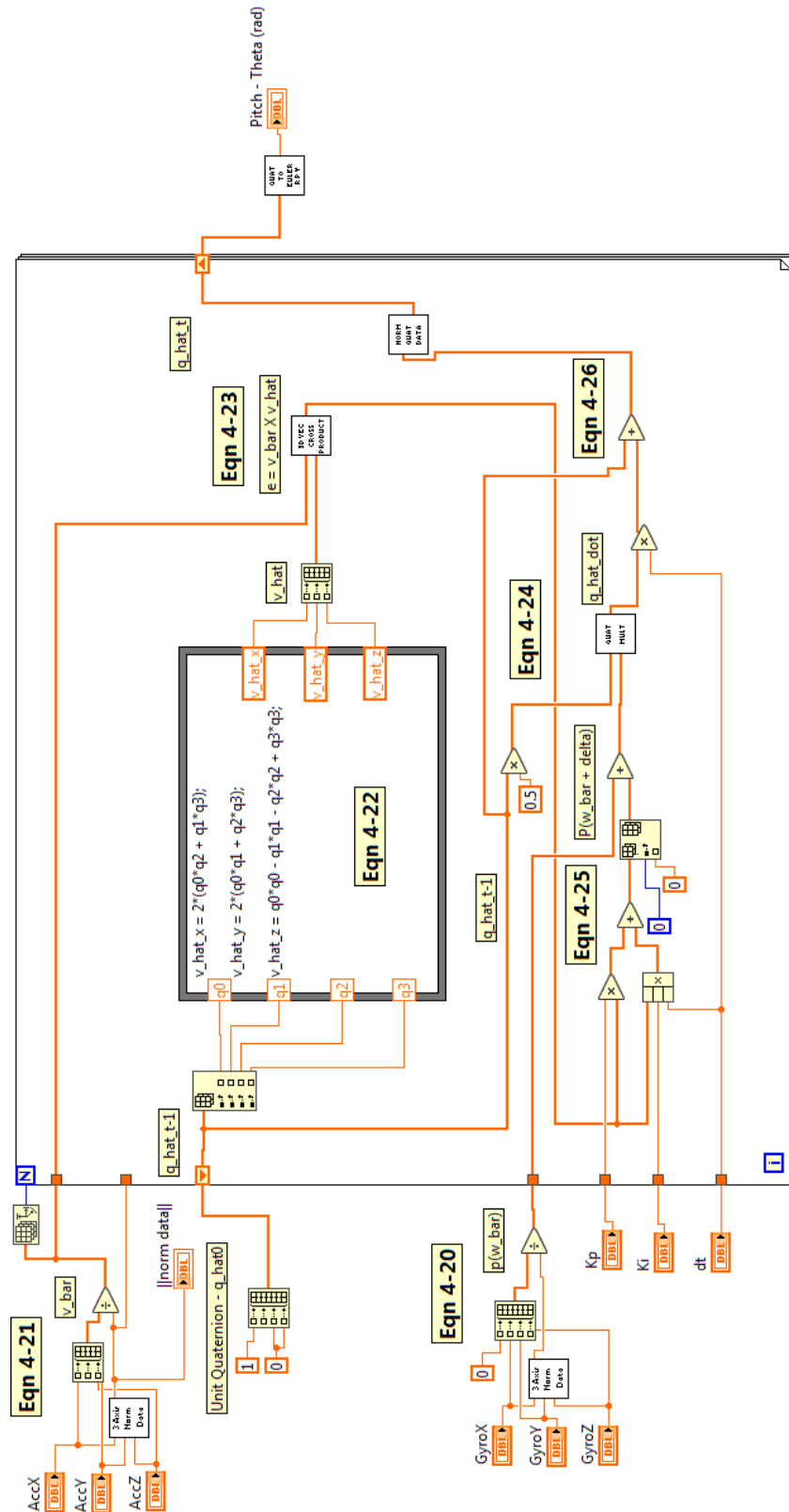


Figure B-4 – Complementary Filter using Quaternions VI Block Diagram

Annex C LabVIEW Data Logging and Control VIs

This Annex provides images of the LabVIEW VIs used for data logging and actuator control.

C.1. Data Logging VIs

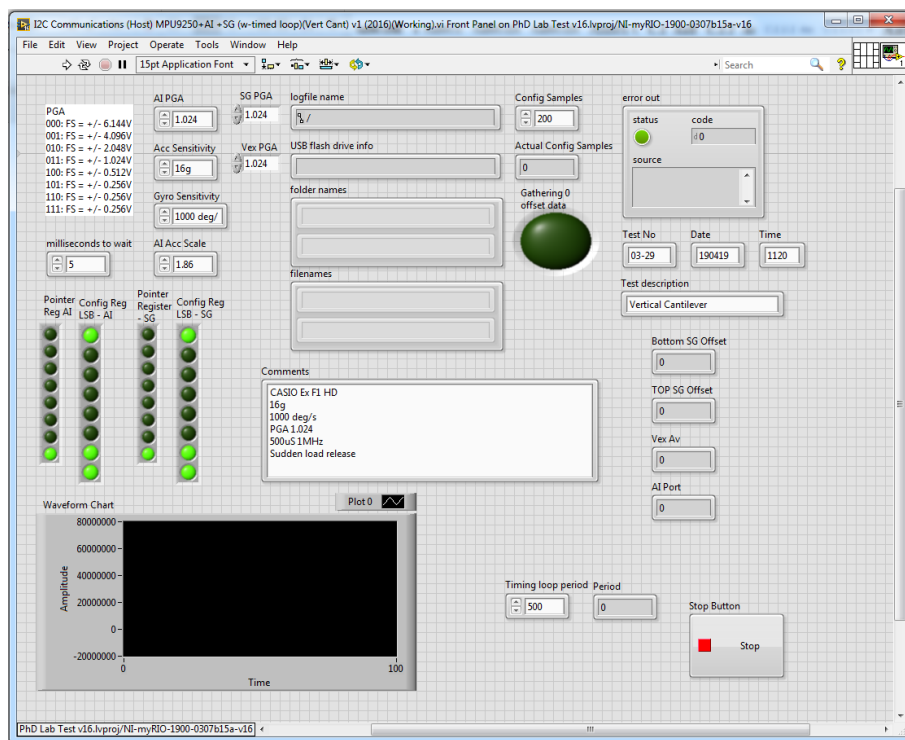


Figure C-1 – Front panel for data logging VI

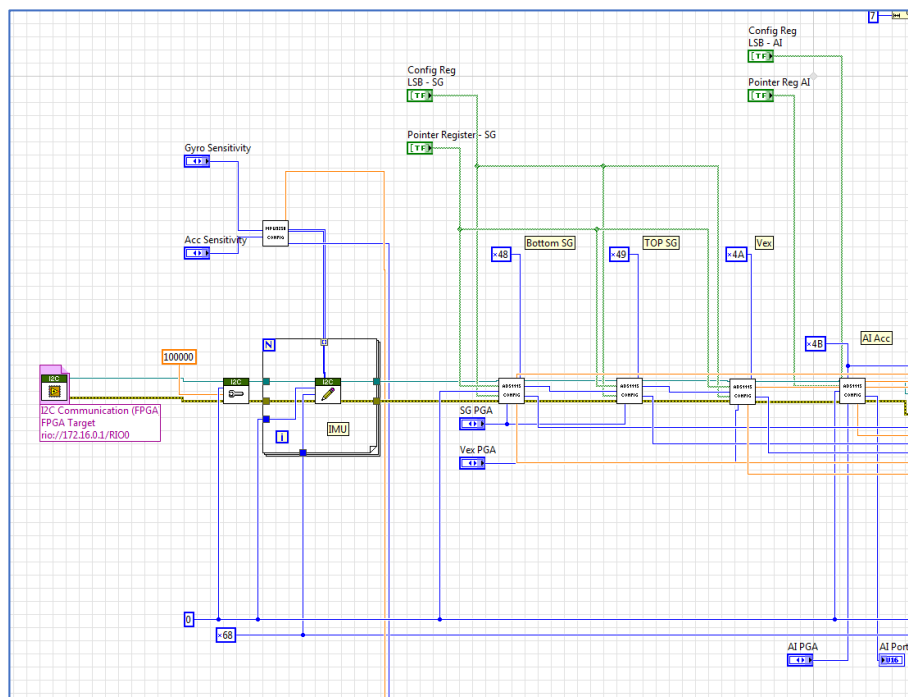


Figure C-2 – Data logging back panel – Sensor configuration

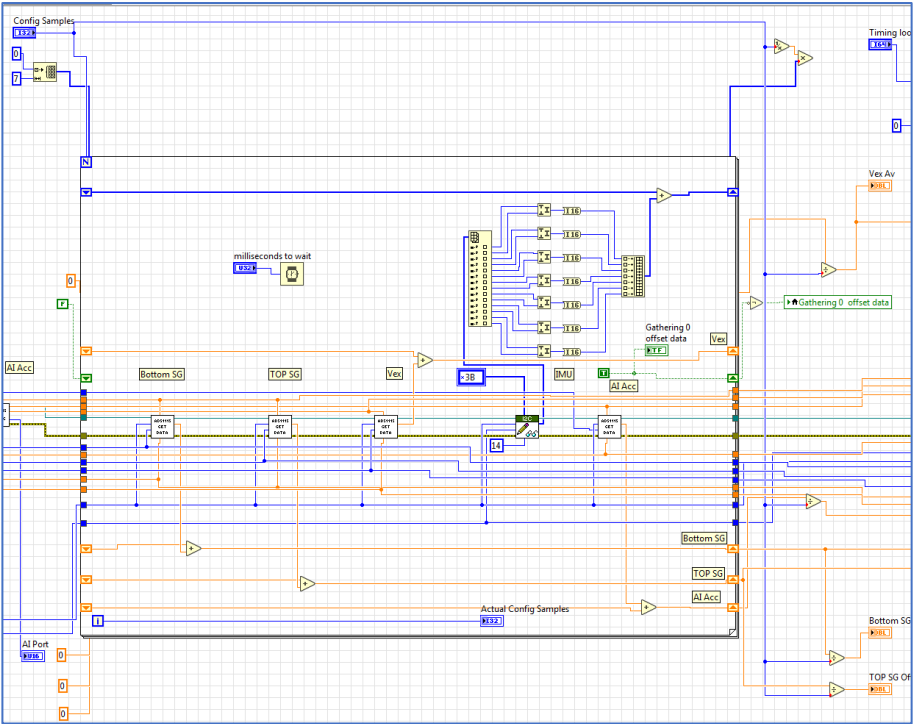


Figure C-3 – Data logging back panel – Averaging initial data to be used to calculate zero-offset

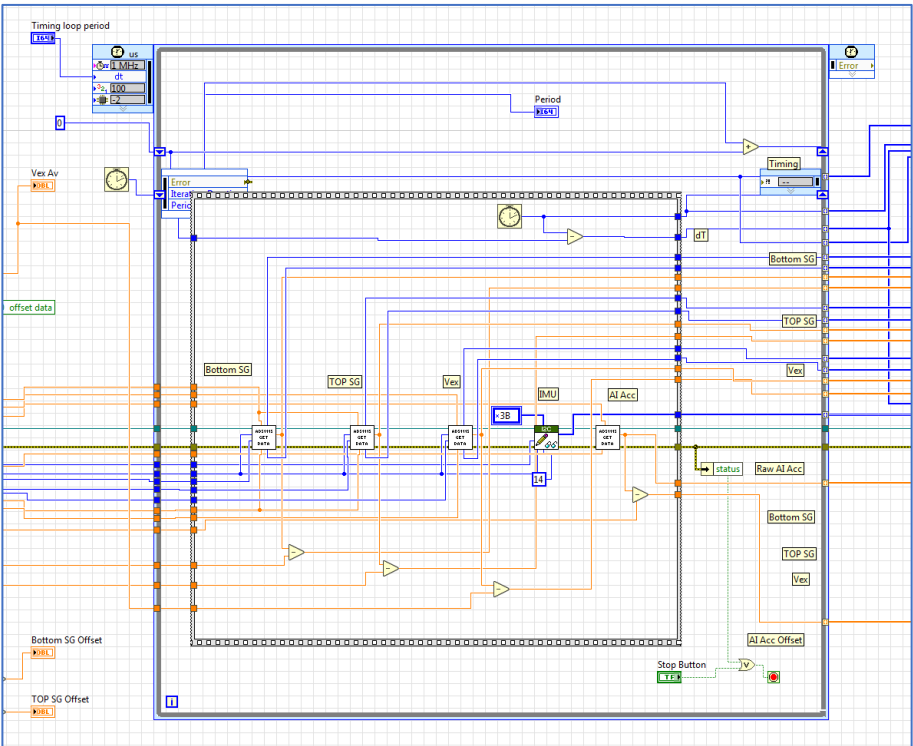


Figure C-4 – Data logging back panel – Data logging timing loop

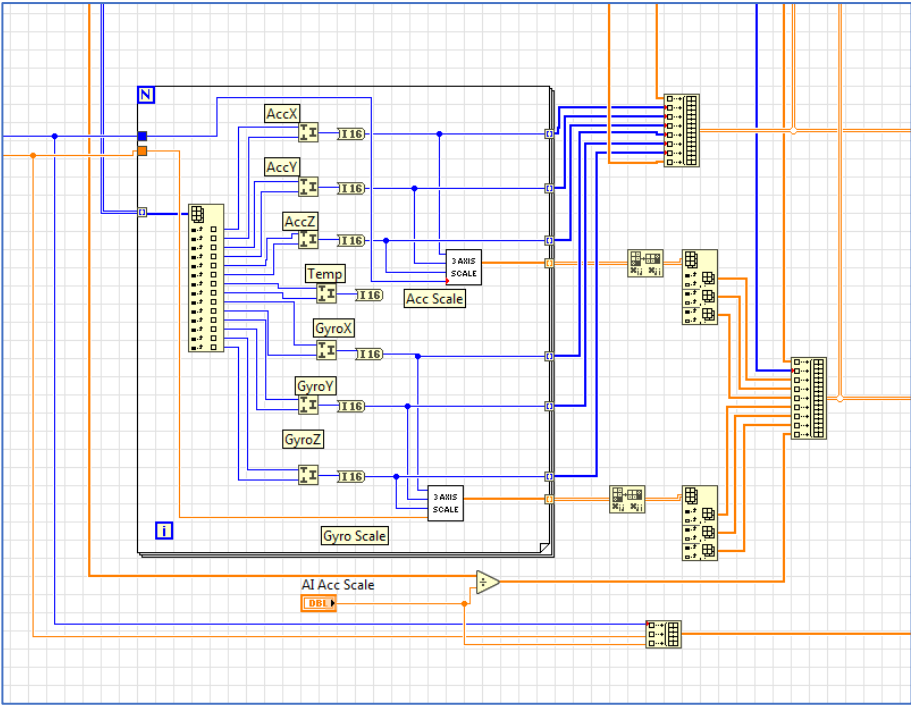


Figure C-5 – Data logging back panel – Scaling raw sensor data

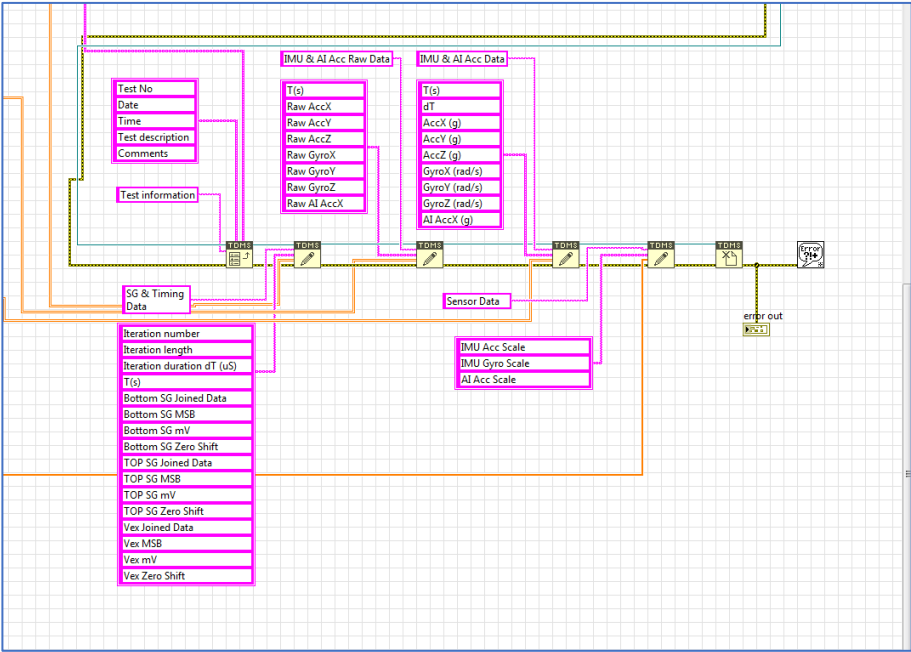


Figure C-6 – Data logging back panel – Create TDMS results file

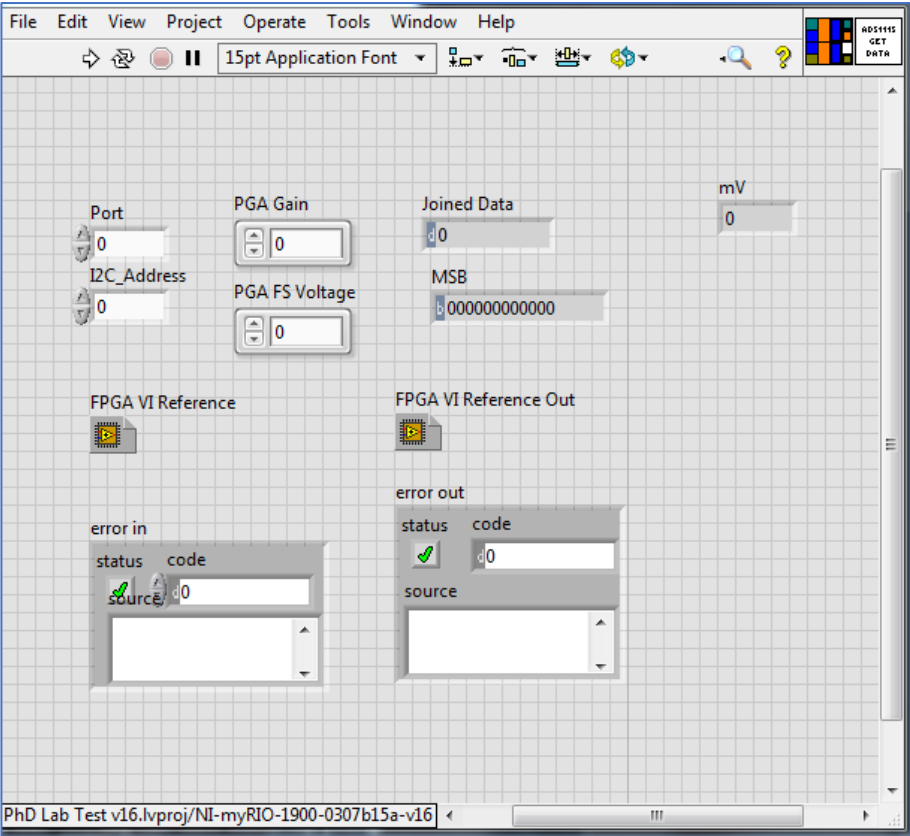


Figure C-7 – Data logging VI – ADS1115 ADC data collection SubVI - Front panel

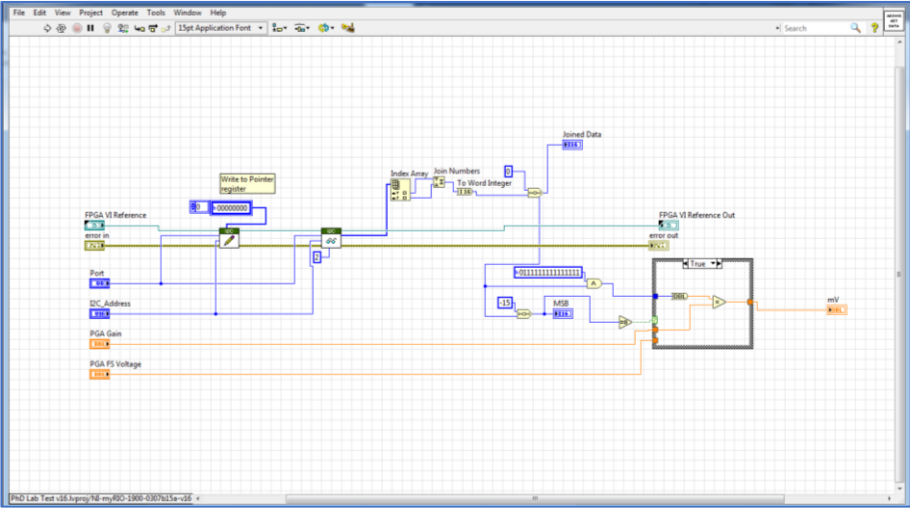


Figure C-8 – Data logging VI – ADS1115 ADC data collection SubVI - Back panel

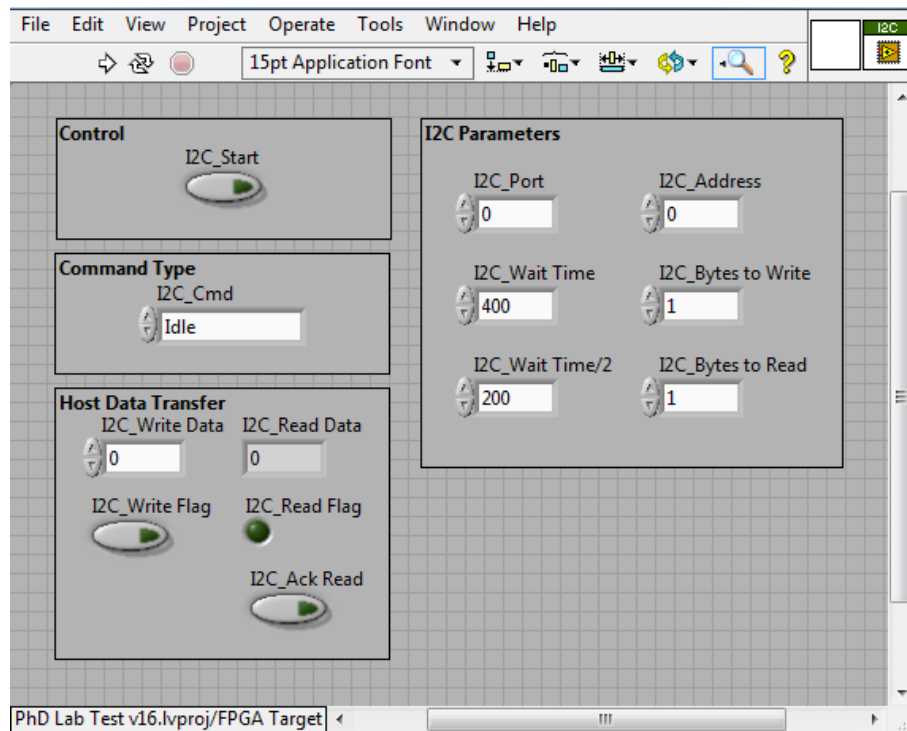


Figure C-9 – Data logging VI – FPGA control - Front panel

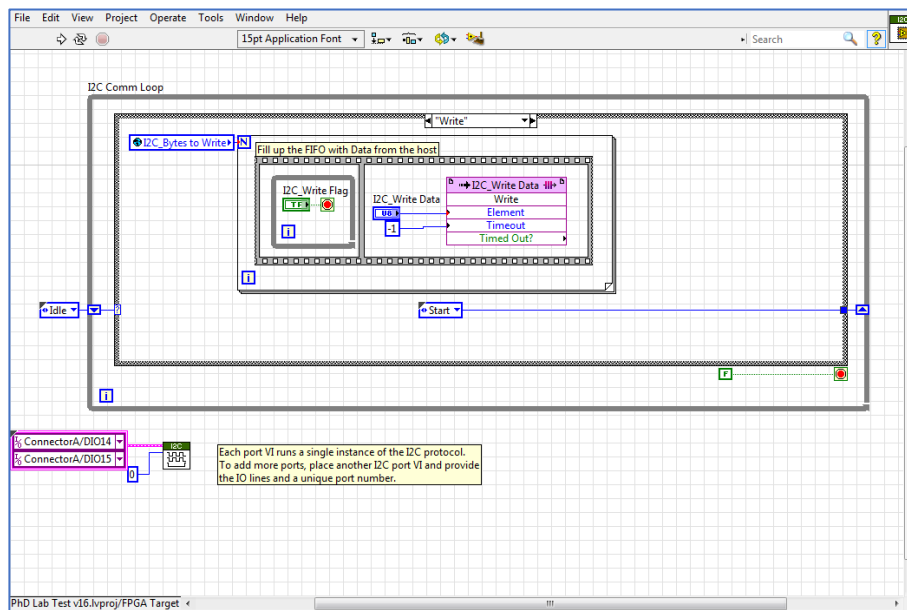


Figure C-10 – Data logging VI – FPGA control - Back panel

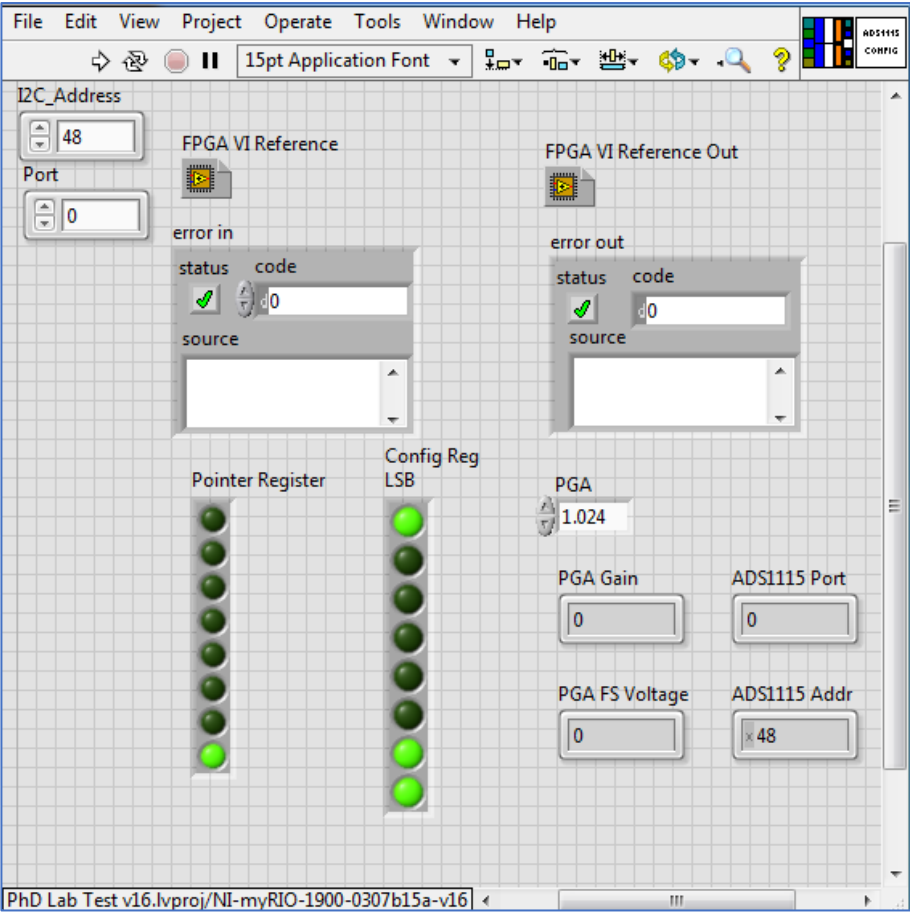


Figure C-11 – Data logging VI – ADS1115 ADC data configuration SubVI - Front panel

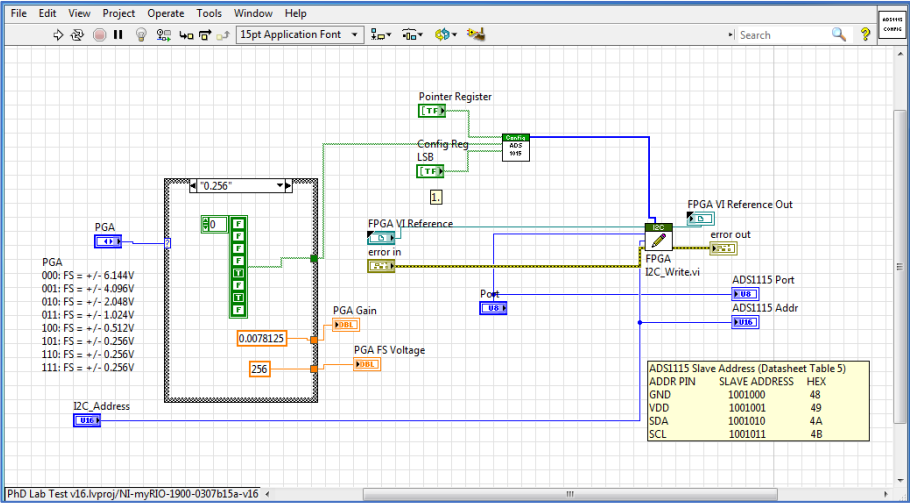


Figure C-12 – Data logging VI – ADS1115 ADC data configuration SubVI - Back panel

C.2. Actuator Control VI

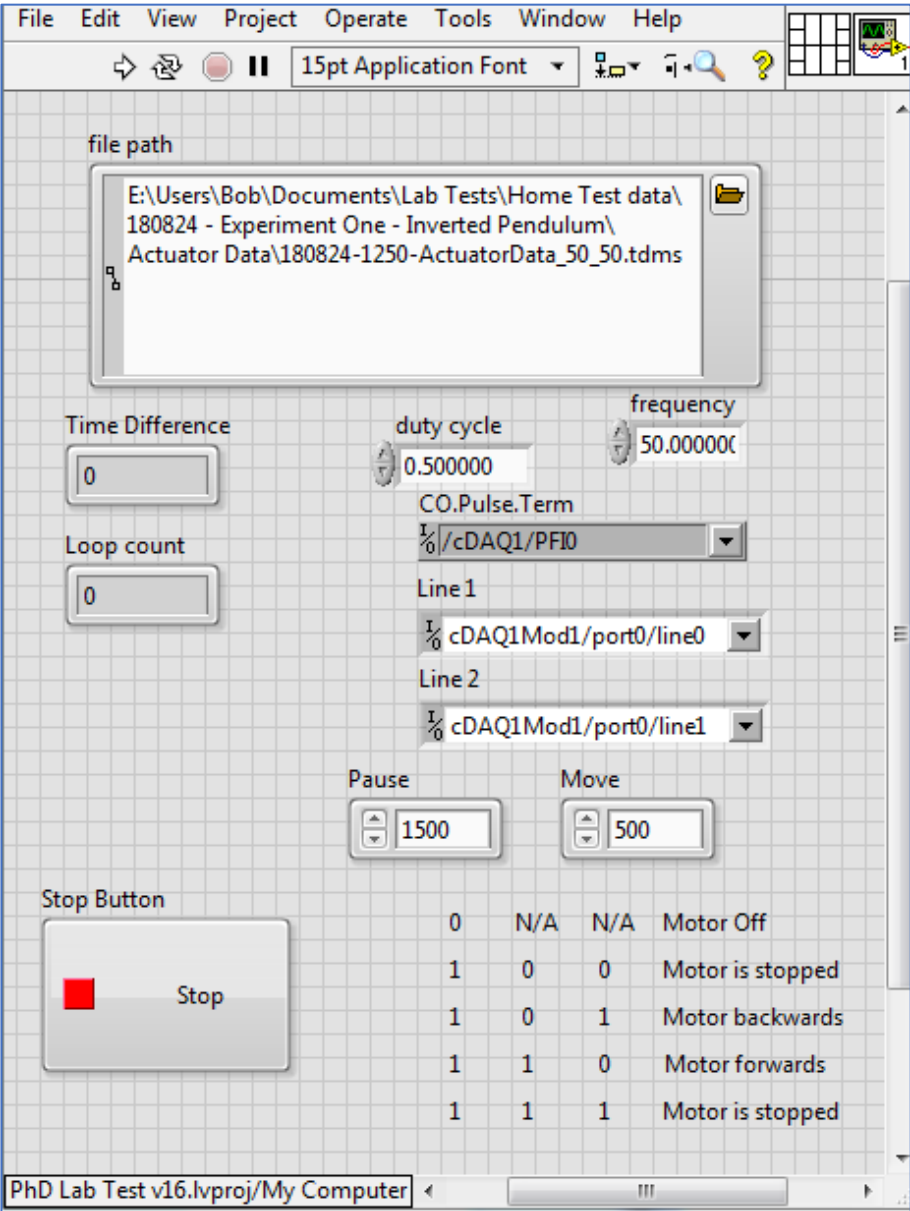


Figure C-13 – Actuator control VI – Front panel

Annex D Experiment Three Data

This Annex contains images of the test data for Experiment Three including close-up images.

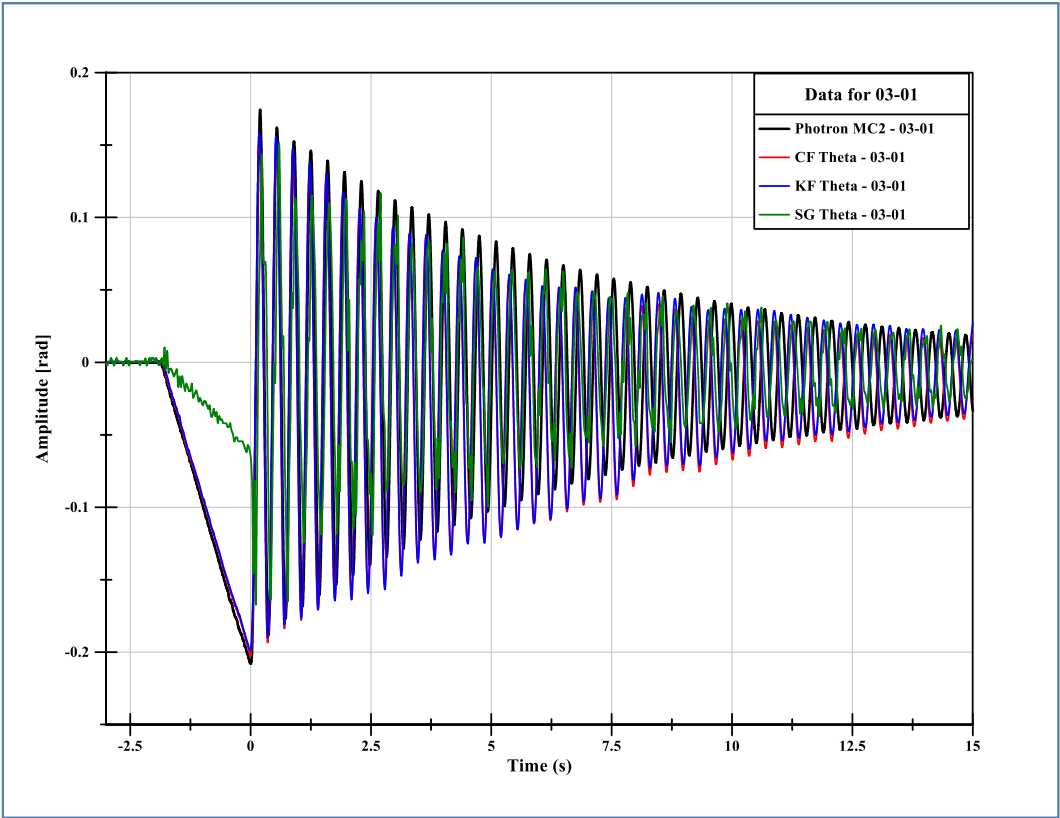


Figure D-1 – Data for 03-01

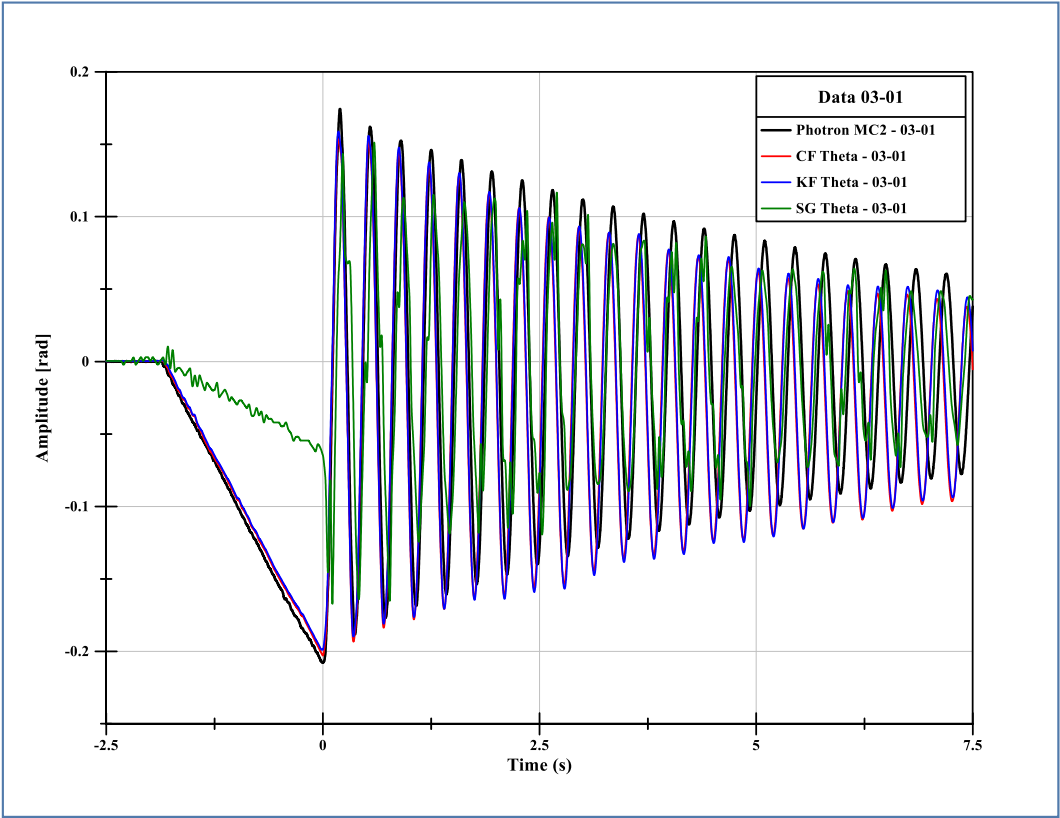


Figure D-2 – Close-up of data for 03-01

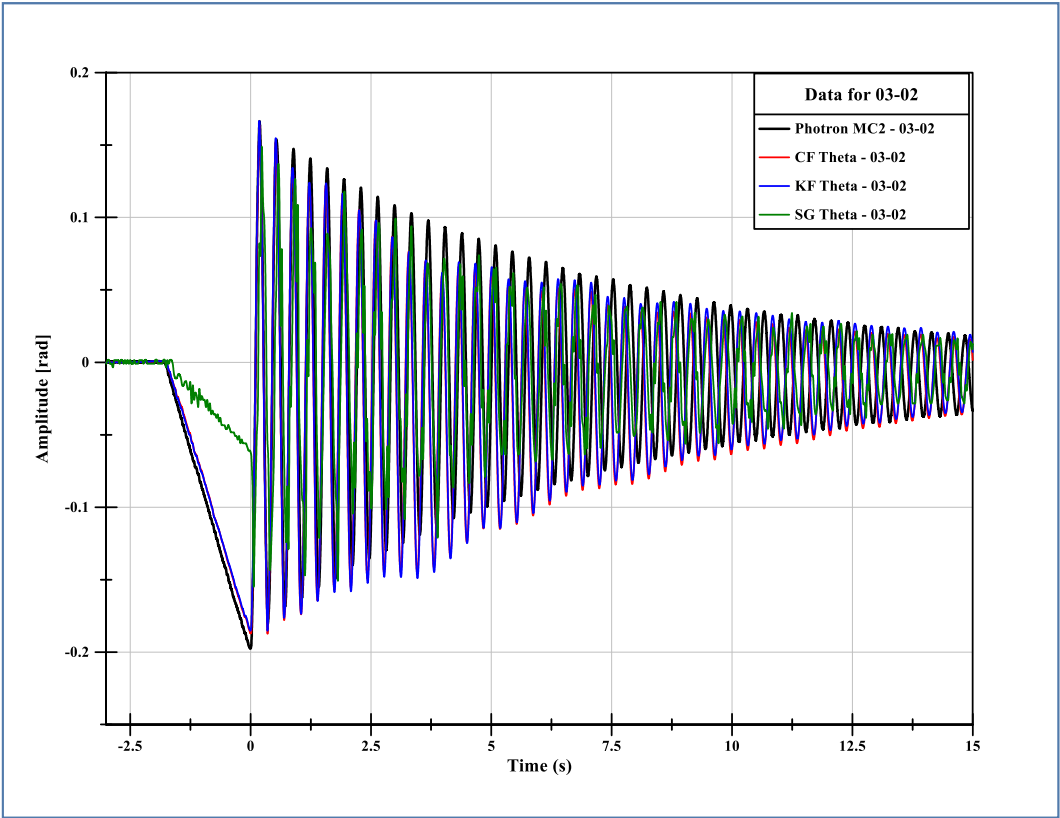


Figure D-3 – Data for 03-02

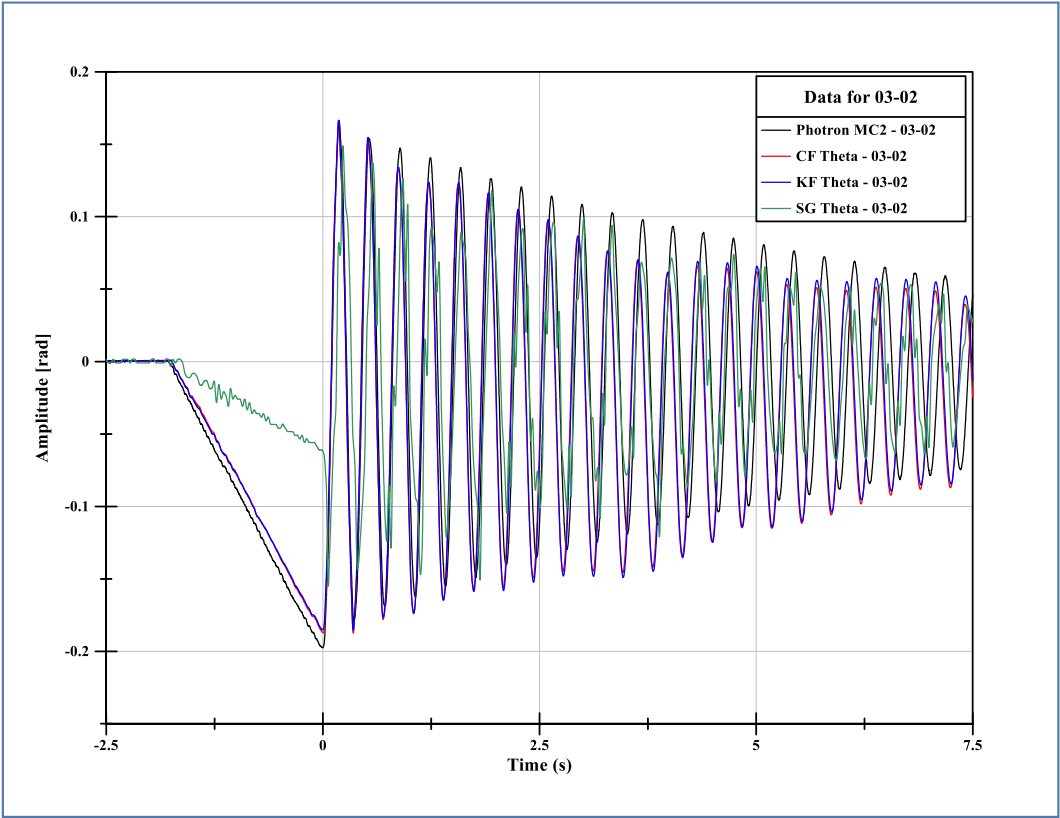


Figure D-4 – Close-up of data for 03-02

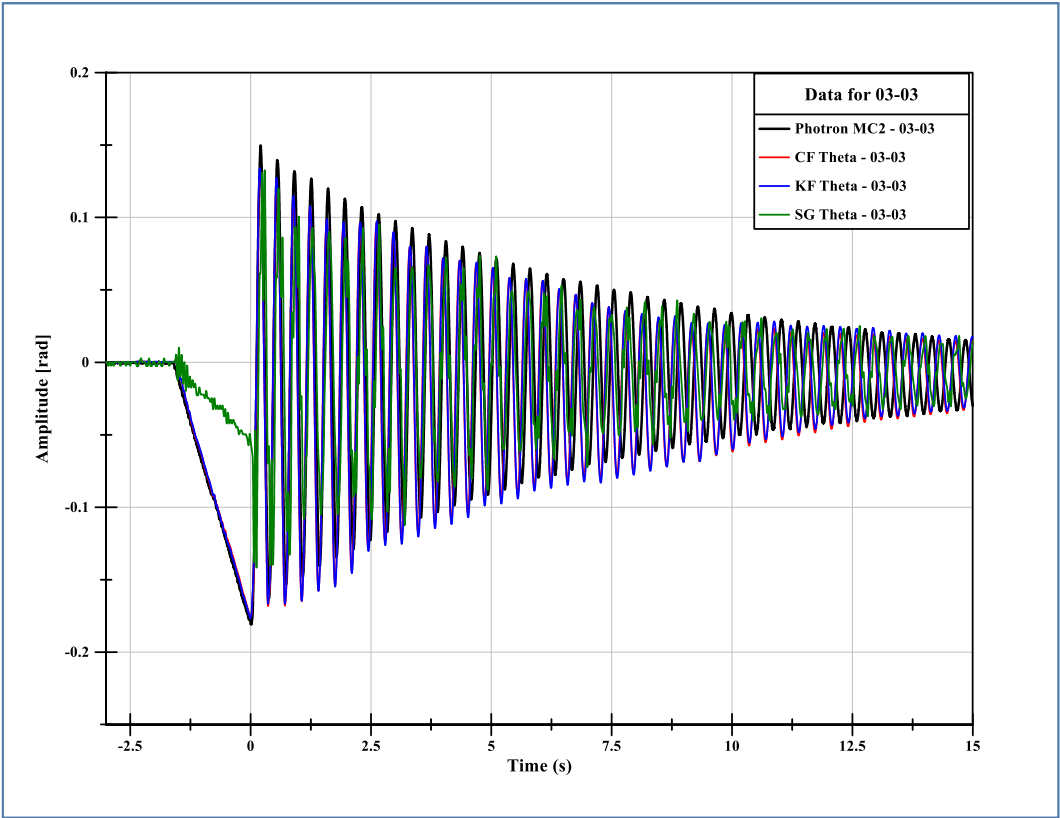


Figure D-5 – Data for 03-03

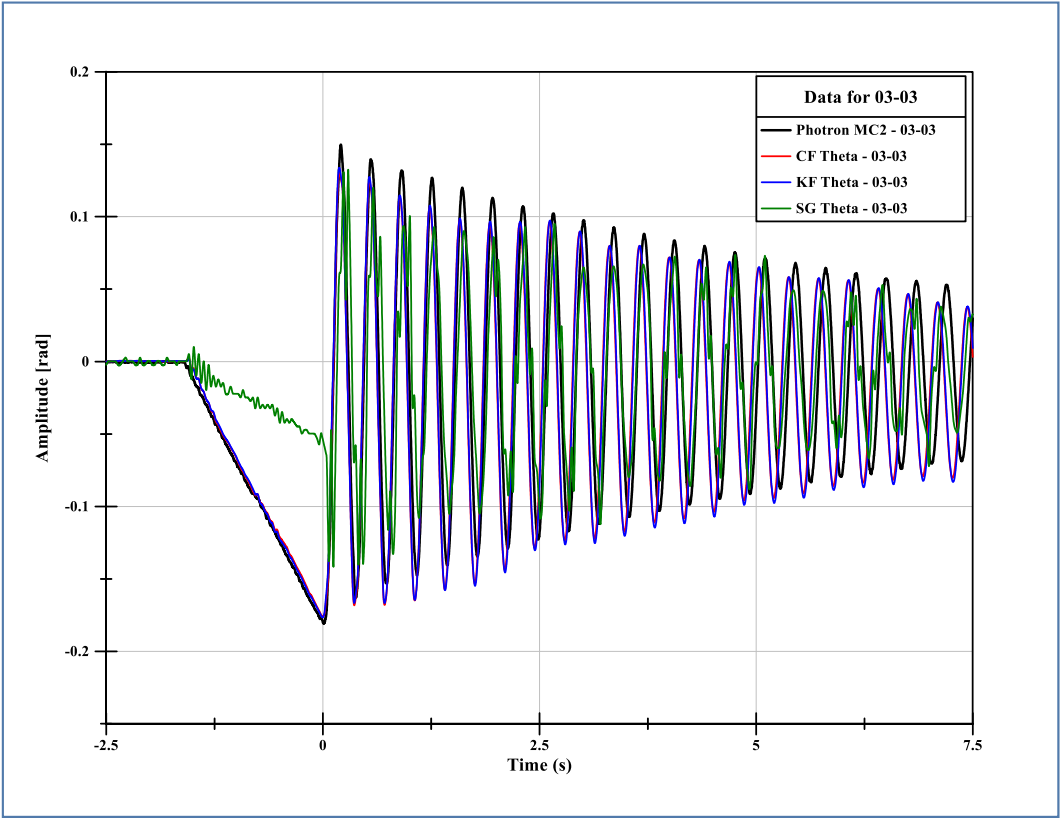


Figure D-6 – Close-up of data for 03-03

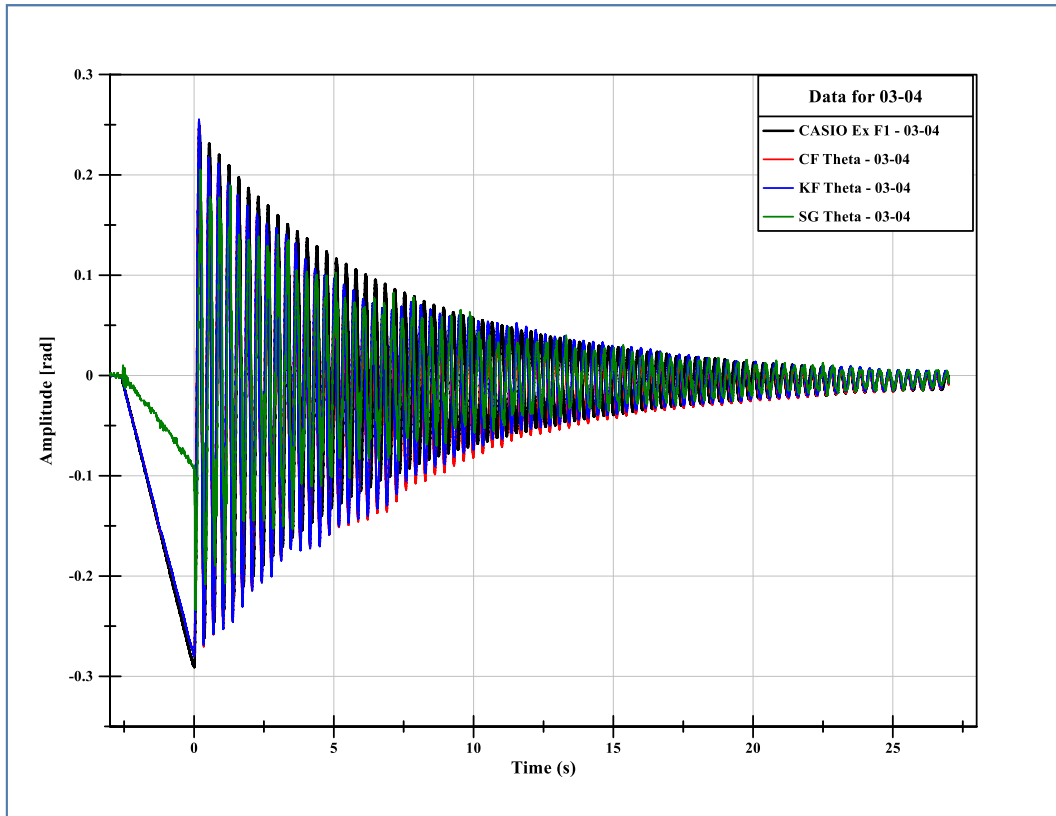


Figure D-7 – Data for 03-04

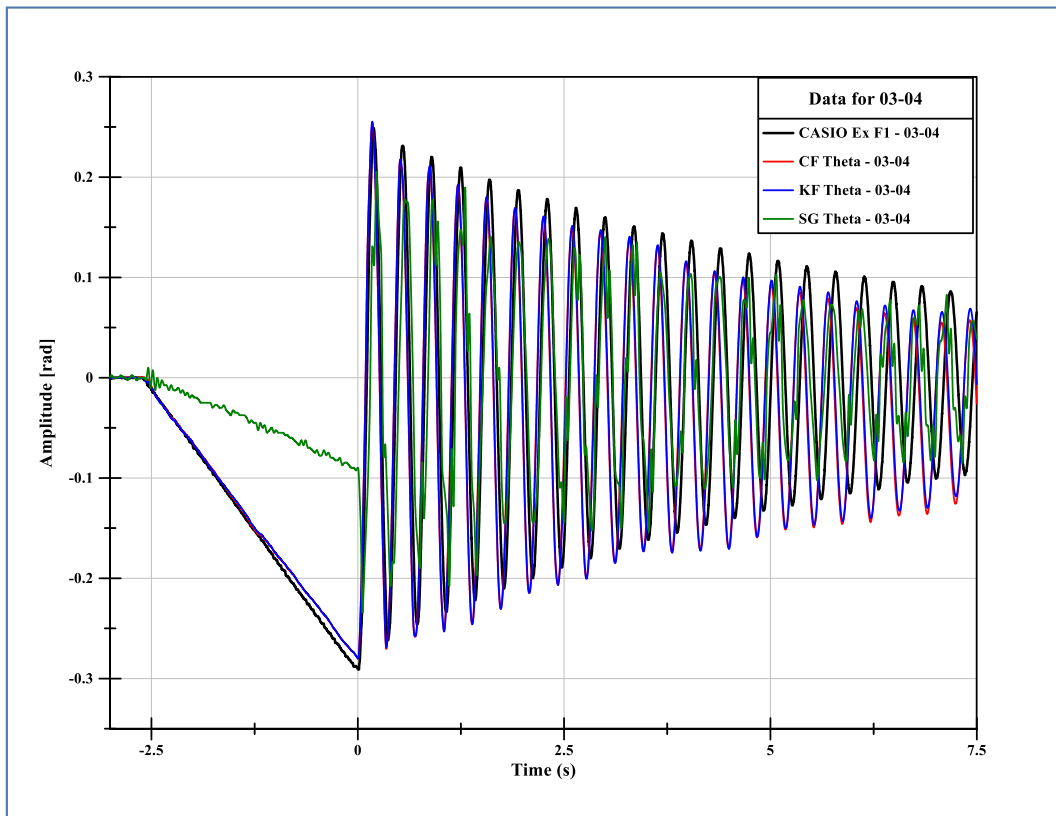


Figure D-8 – Close-up of data for 03-04

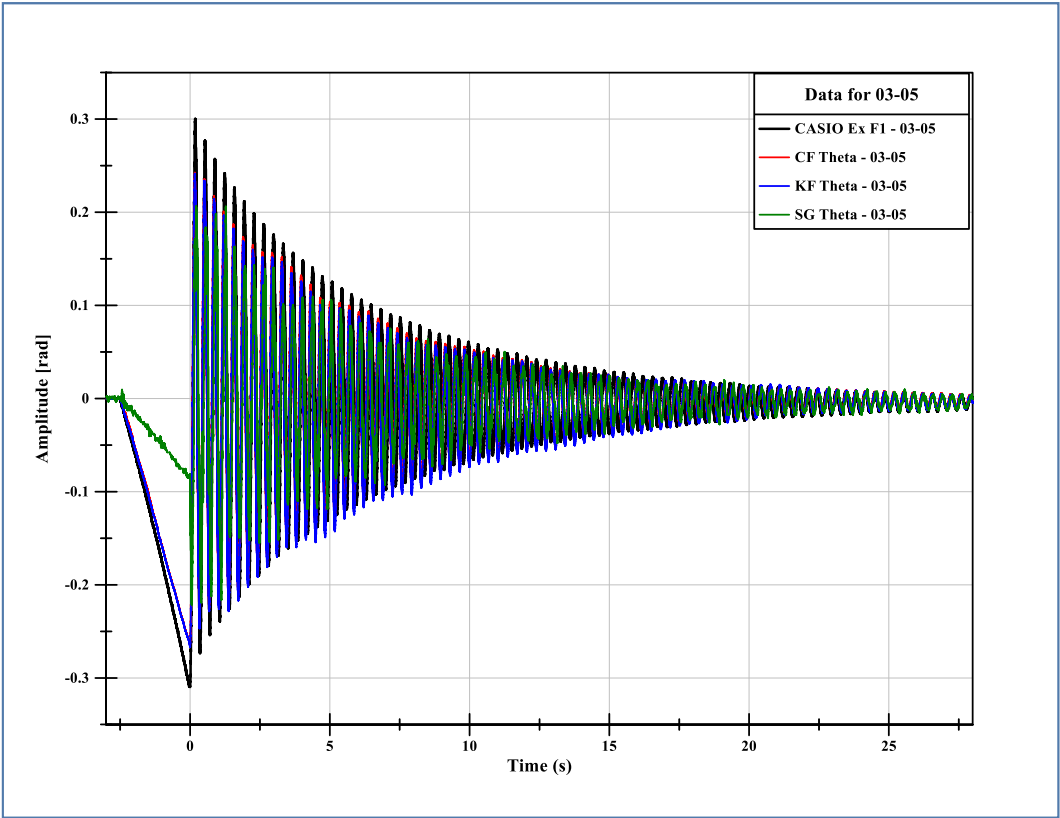


Figure D-9 – Data for 03-05

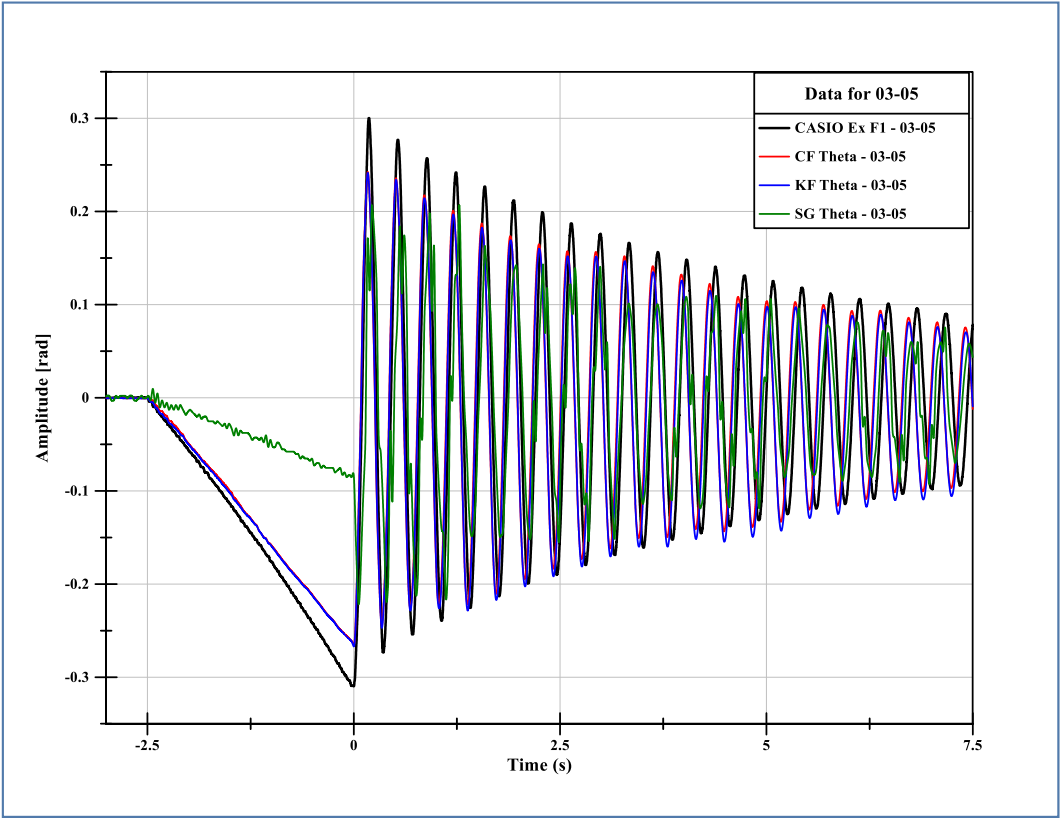


Figure D-10 – Close-up of data for 03-05

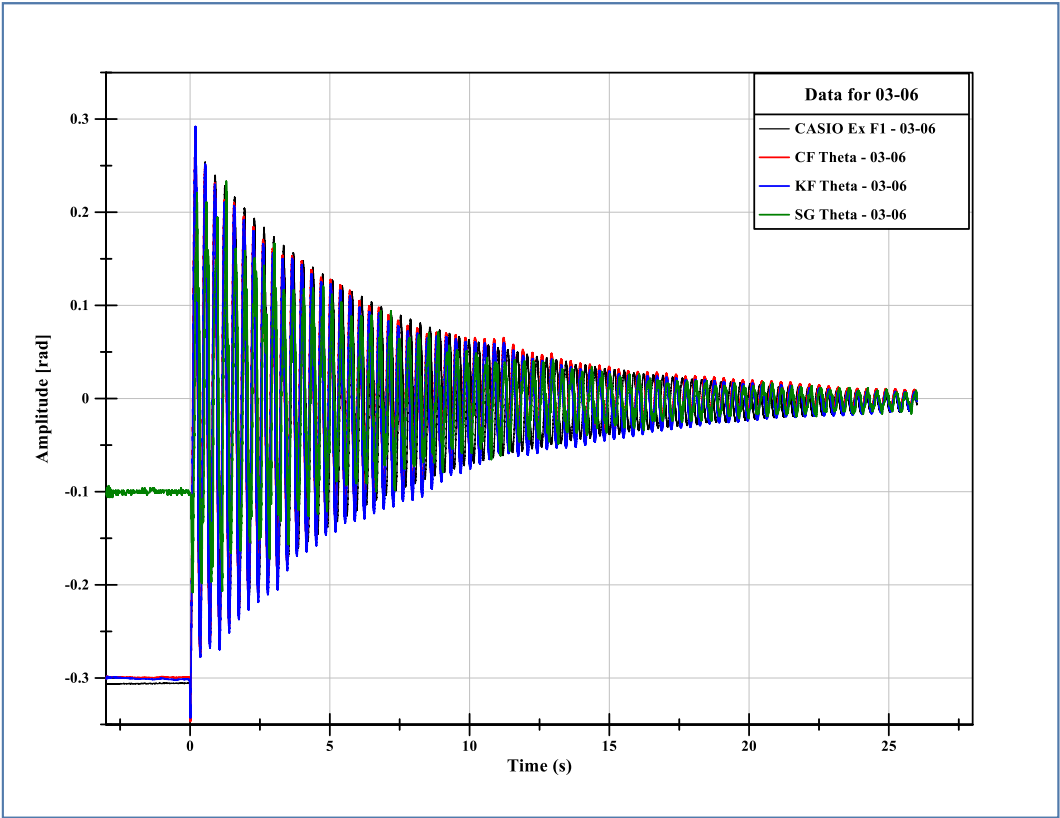


Figure D-11 – Data for 03-06

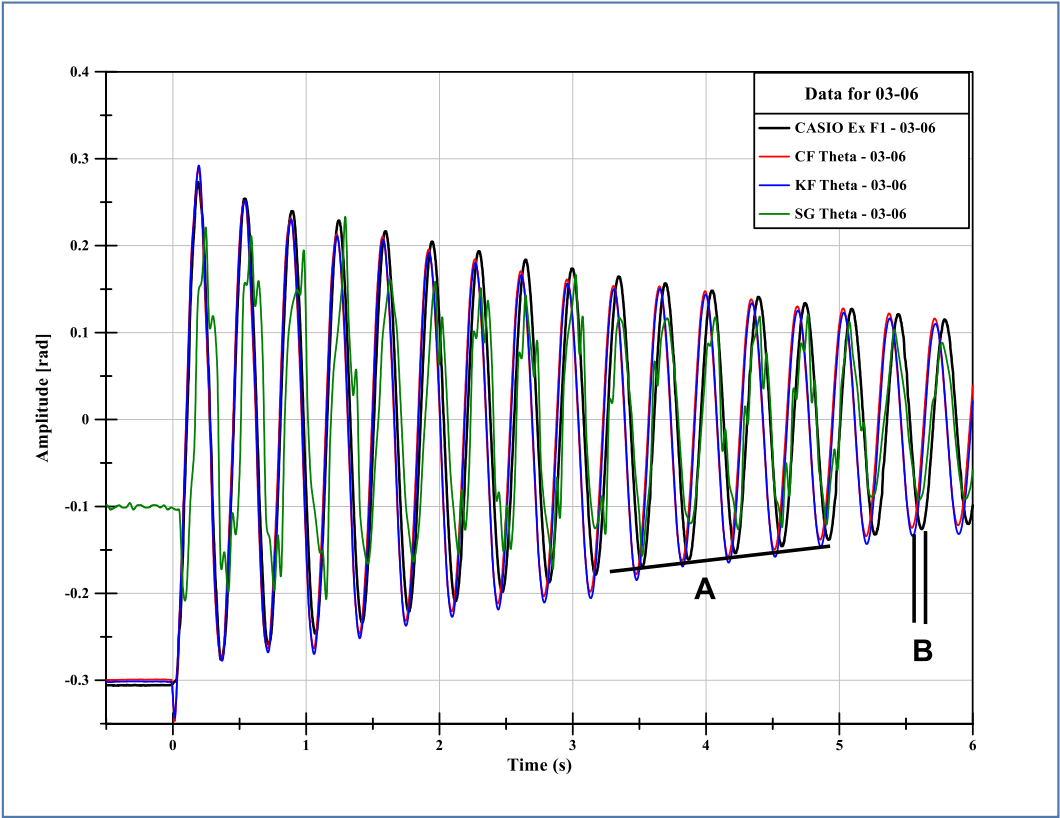


Figure D-12 – Close-up of data for 03-06

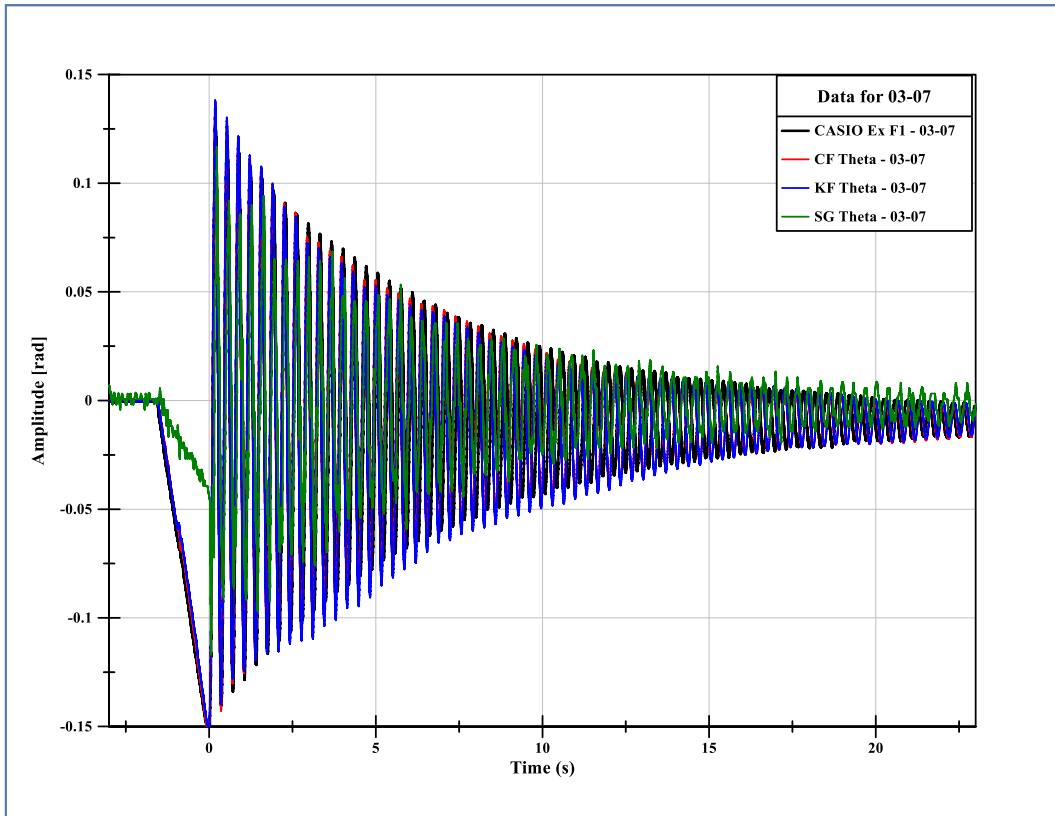


Figure D-13 – Data for 03-07

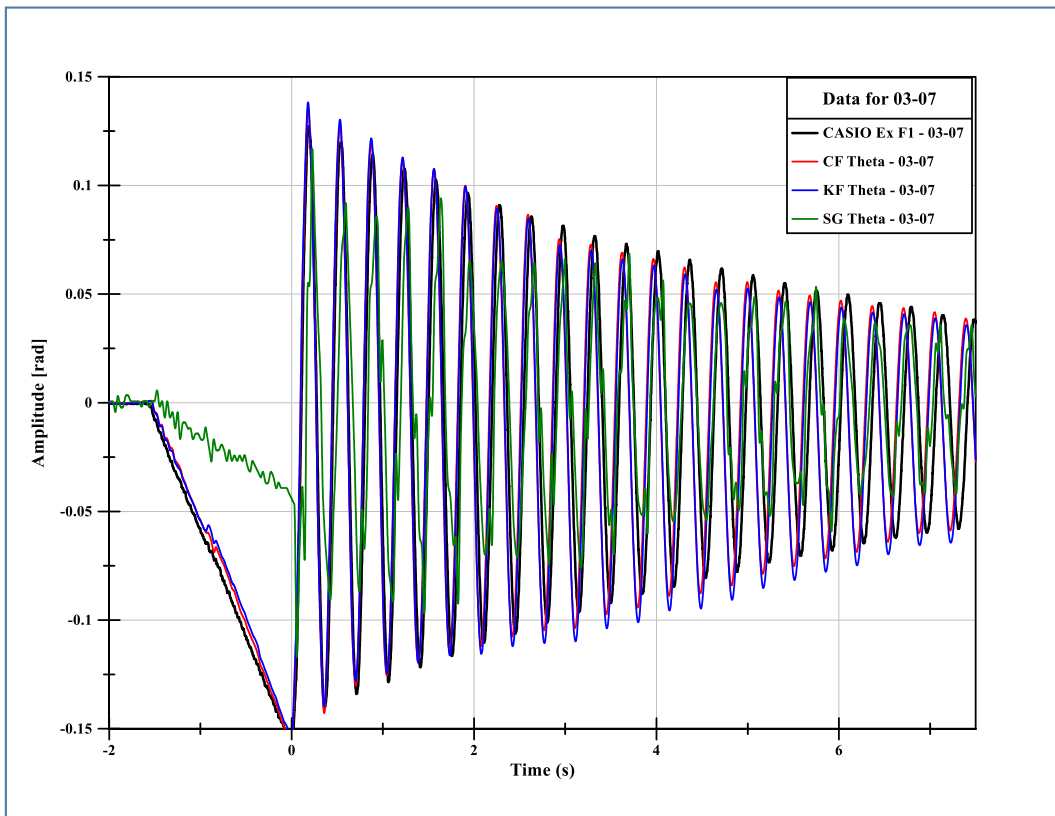


Figure D-14 – Close-up of data for 03-07

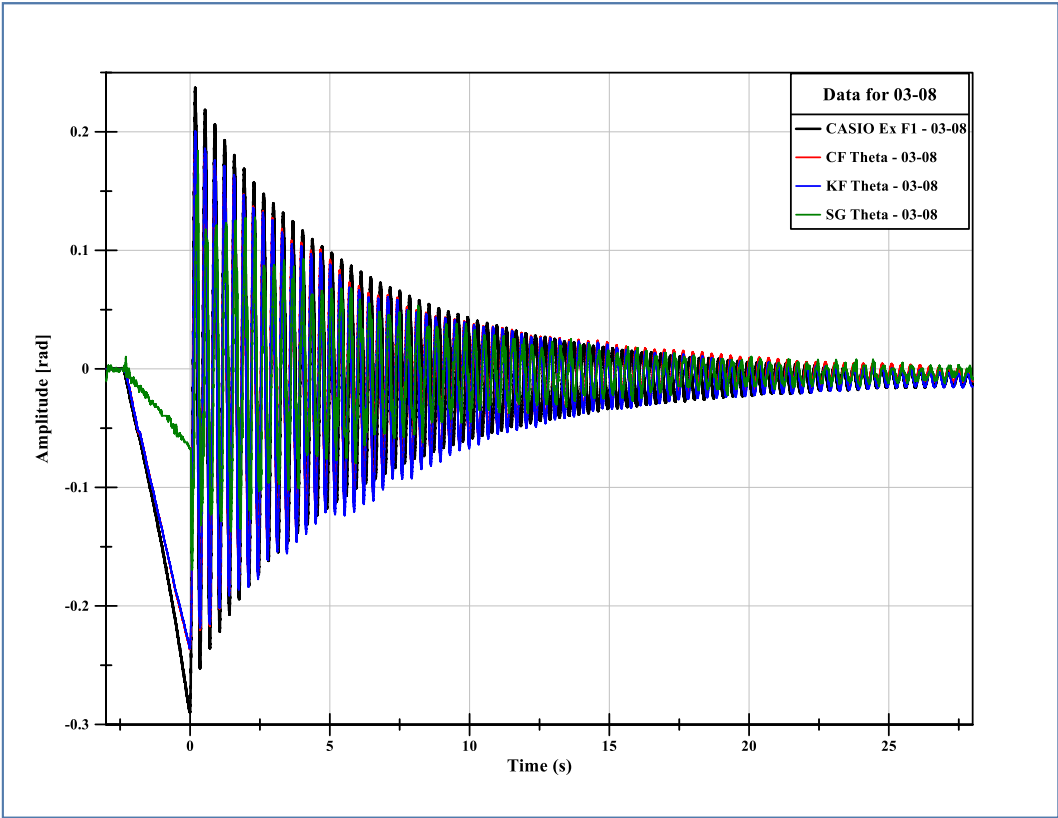


Figure D-15 – Data for 03-08

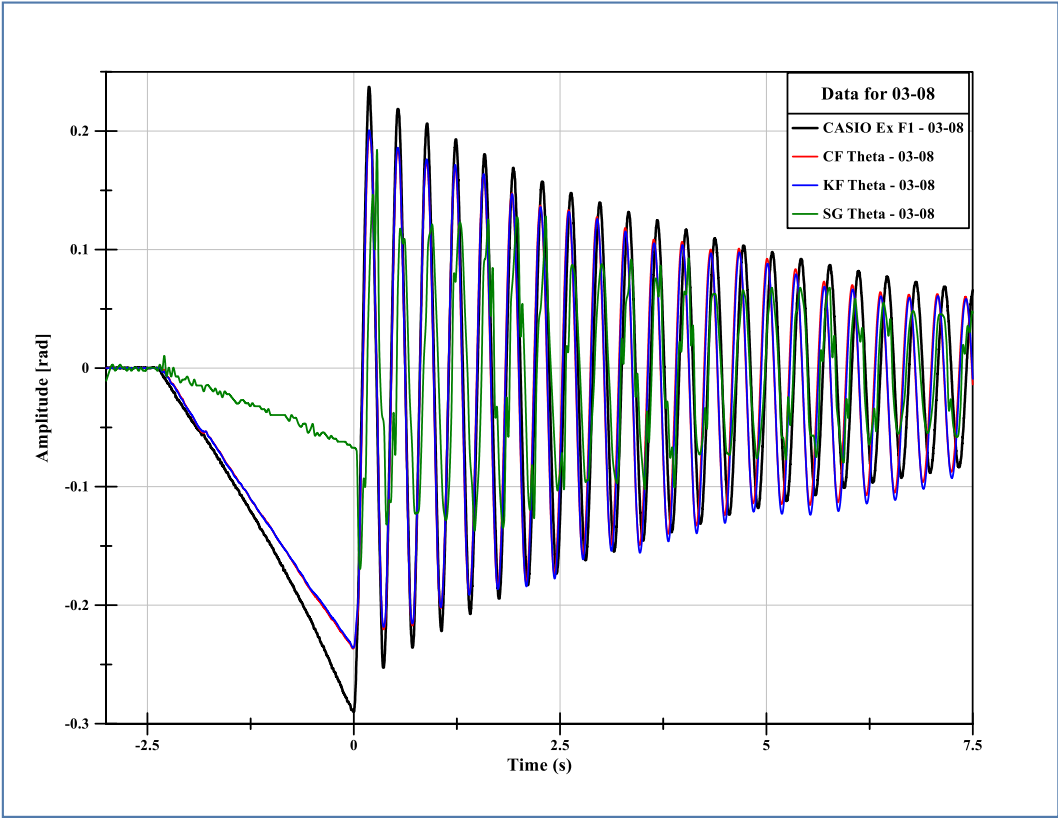


Figure D-16 – Close-up of data for 03-08

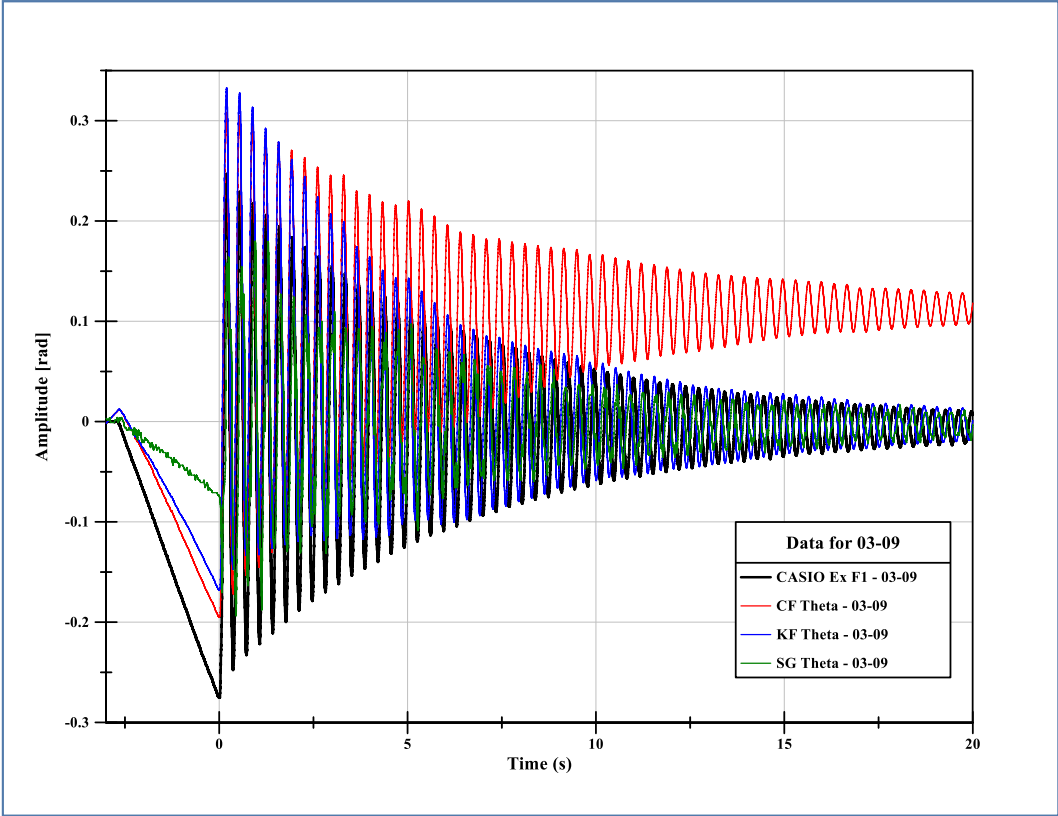


Figure D-17 – Data for 03-09

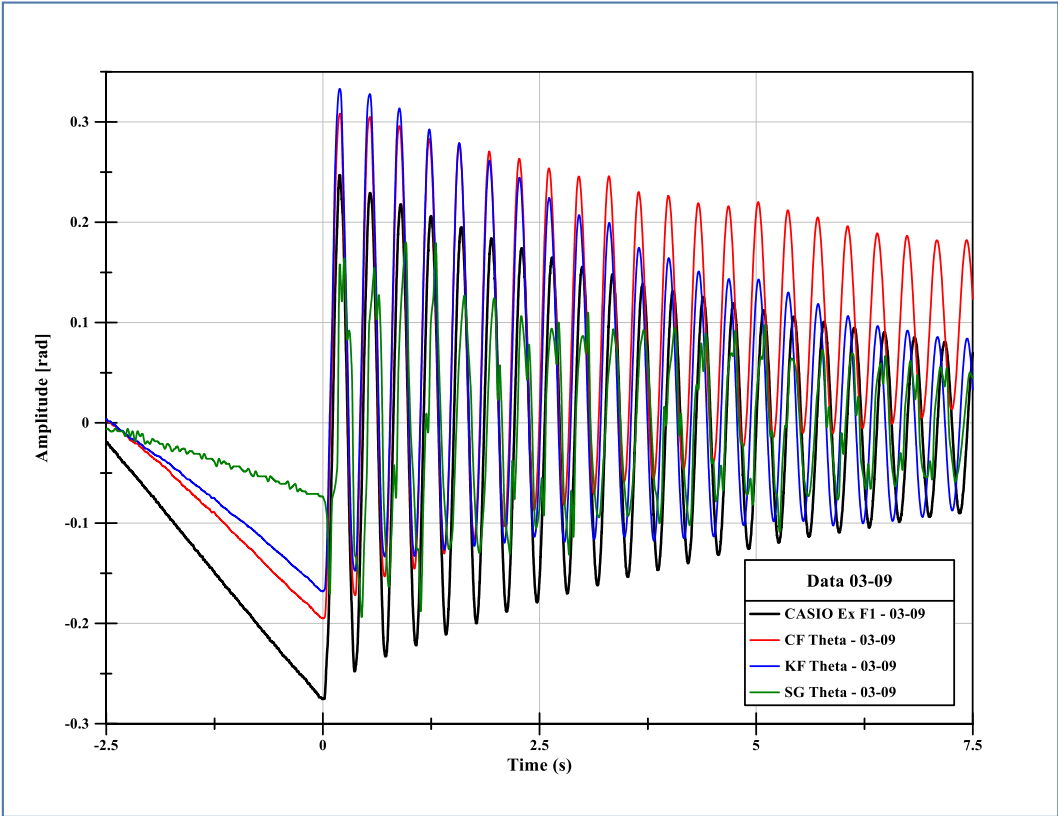


Figure D-18 – Close-up of data for 03-09

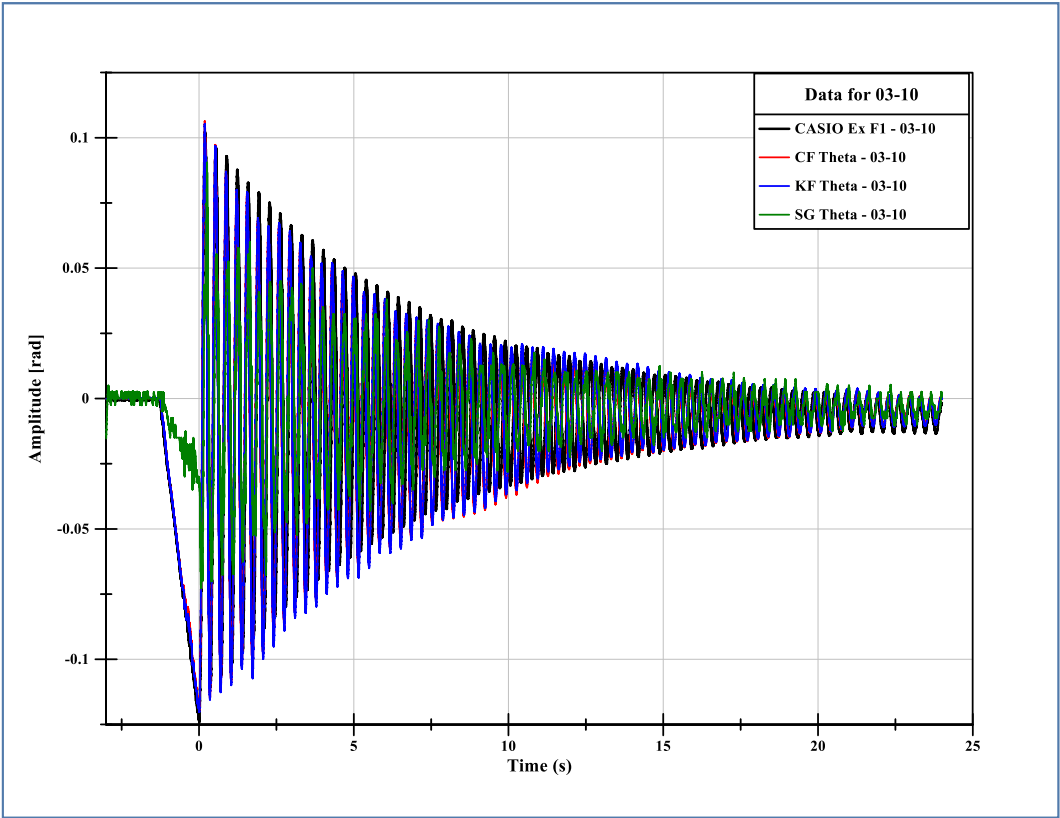


Figure D-19 – Data for 03-10

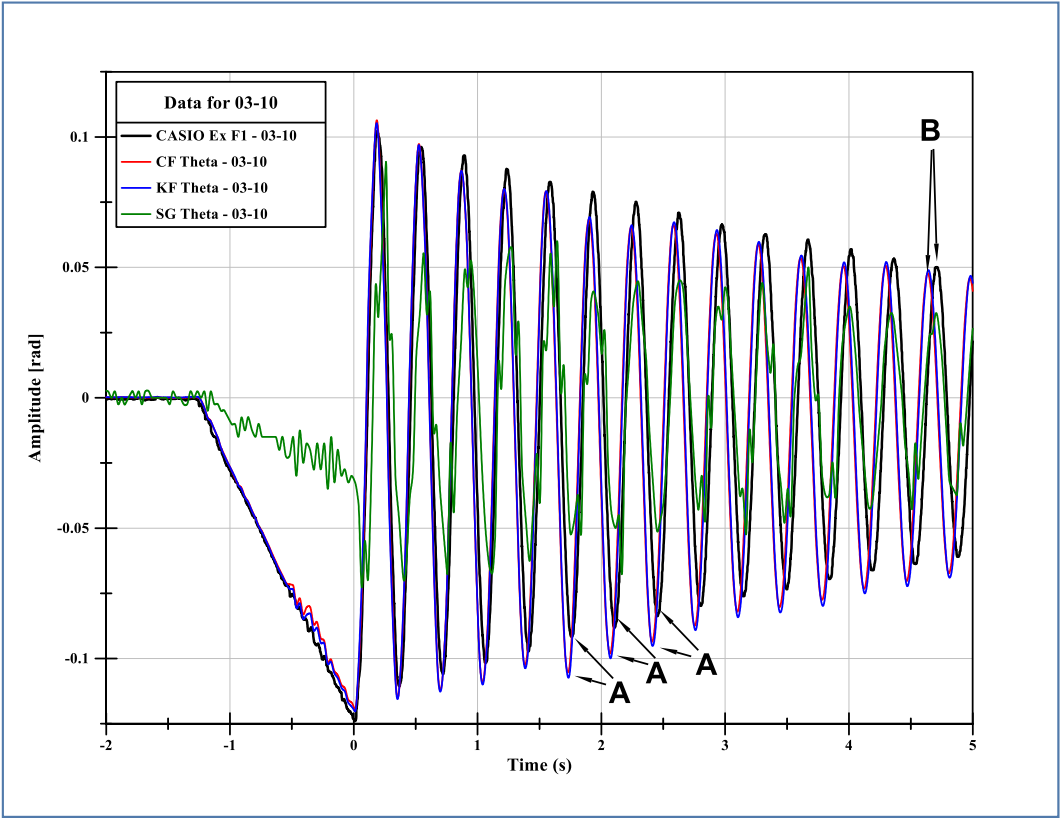


Figure D-20 – Close-up of data for 03-10

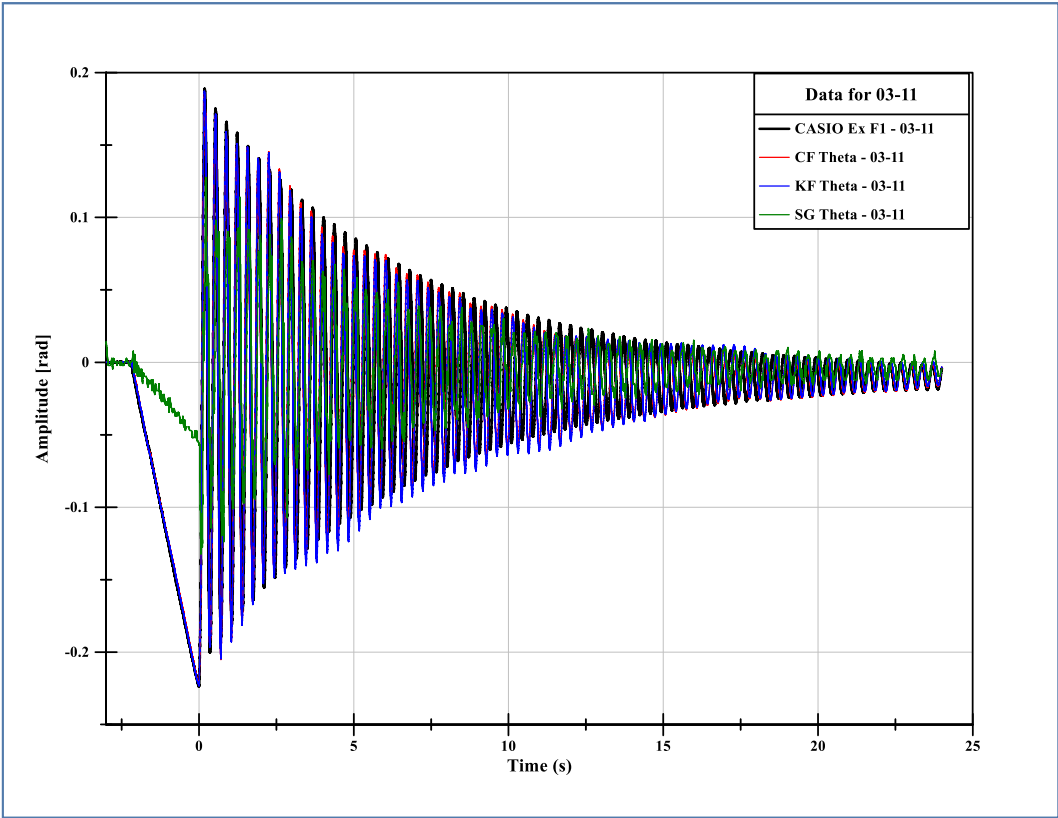


Figure D-21 – Data for 03-11

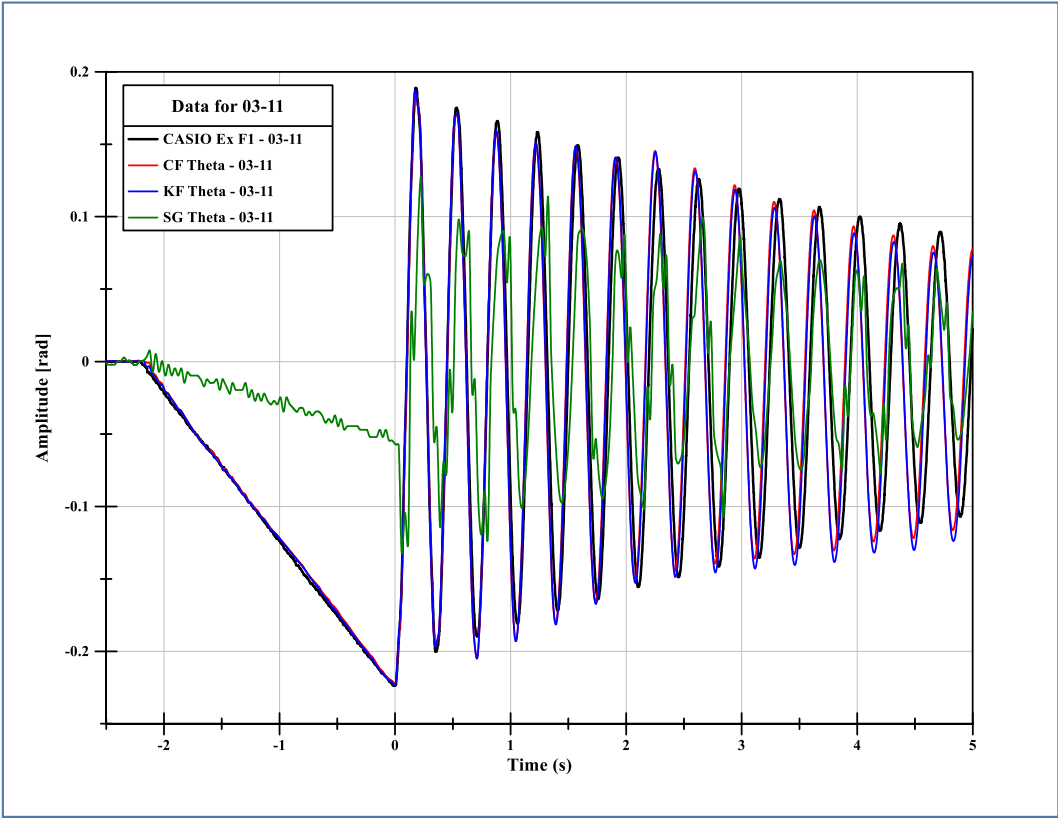


Figure D-22 – Close-up of data for 03-11

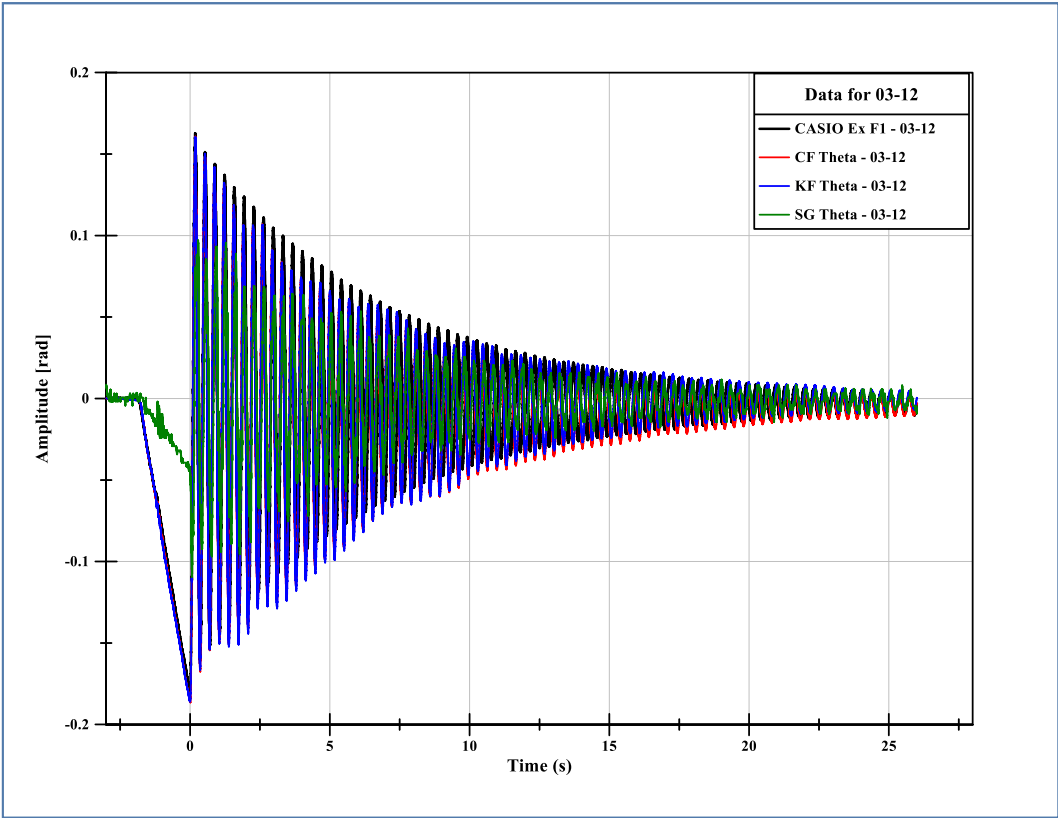


Figure D-23 – Data for 03-12

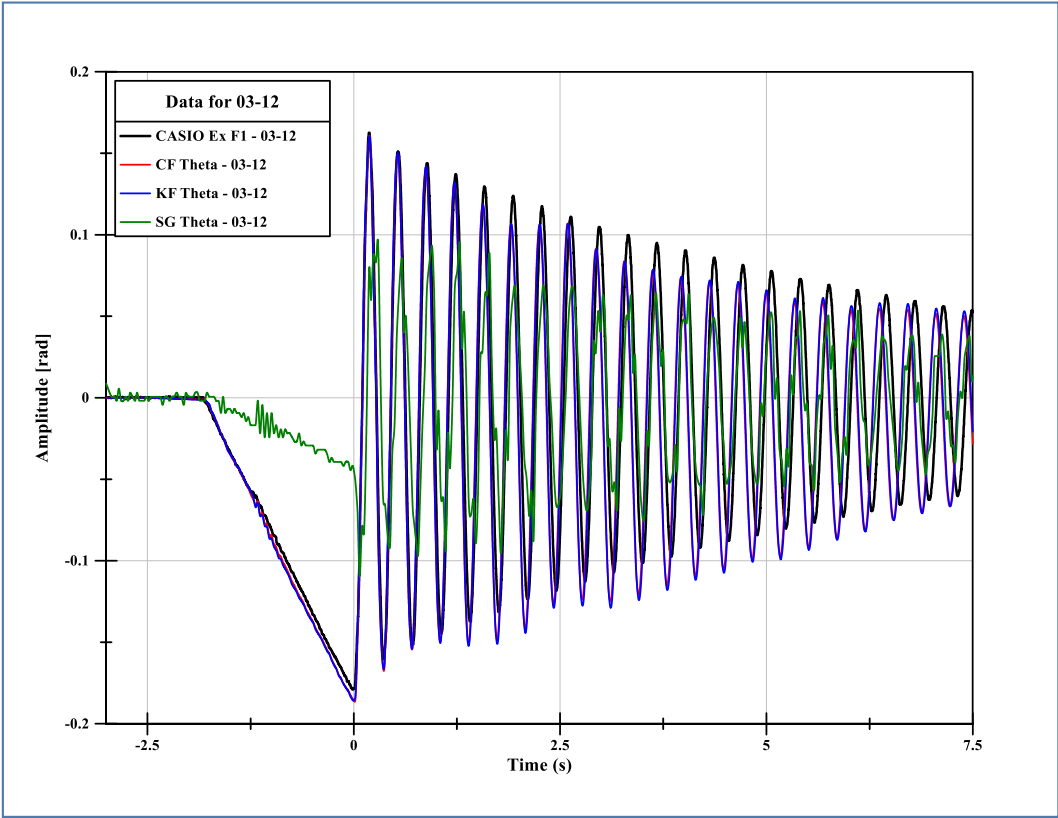


Figure D-24 – Close-up of data for 03-12

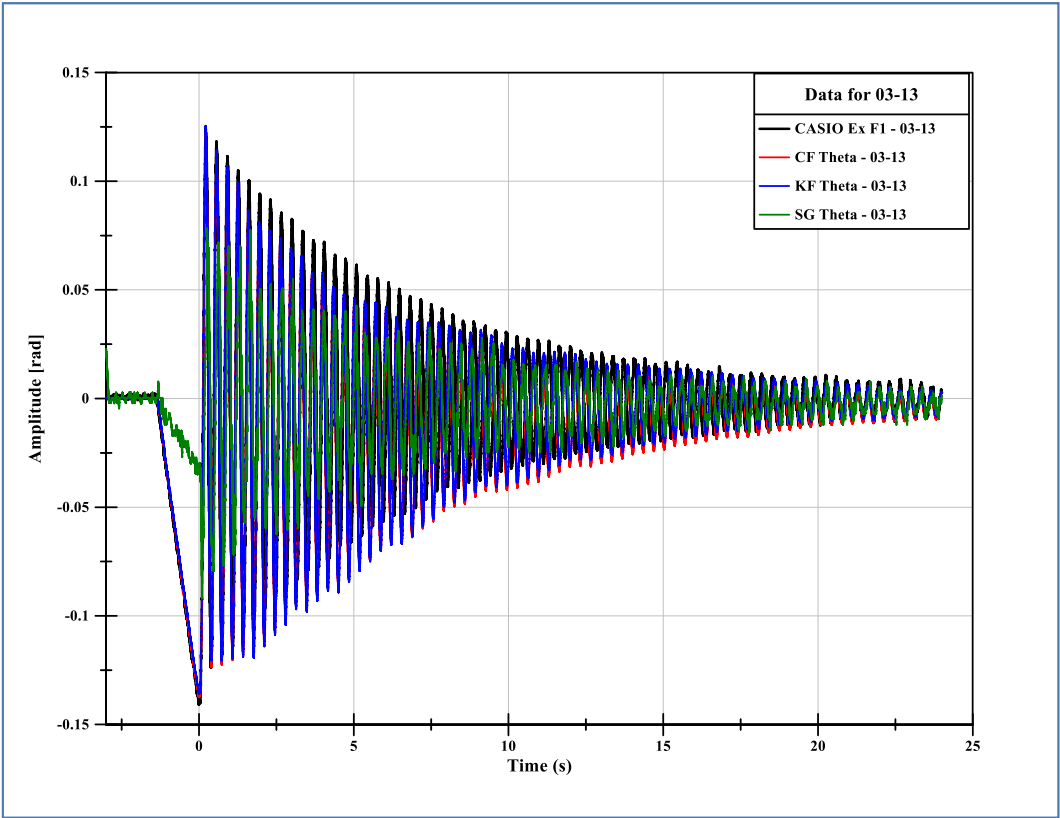


Figure D-25 – Data for 03-13

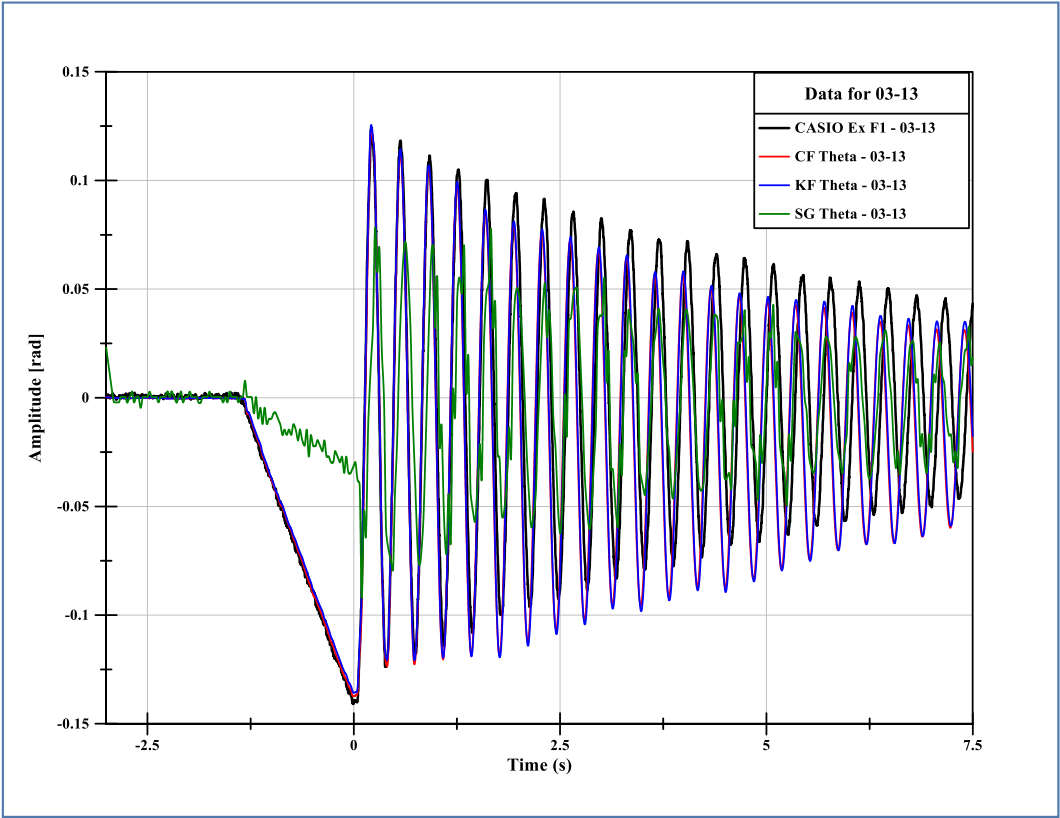


Figure D-26 – Close-up of data for 03-13

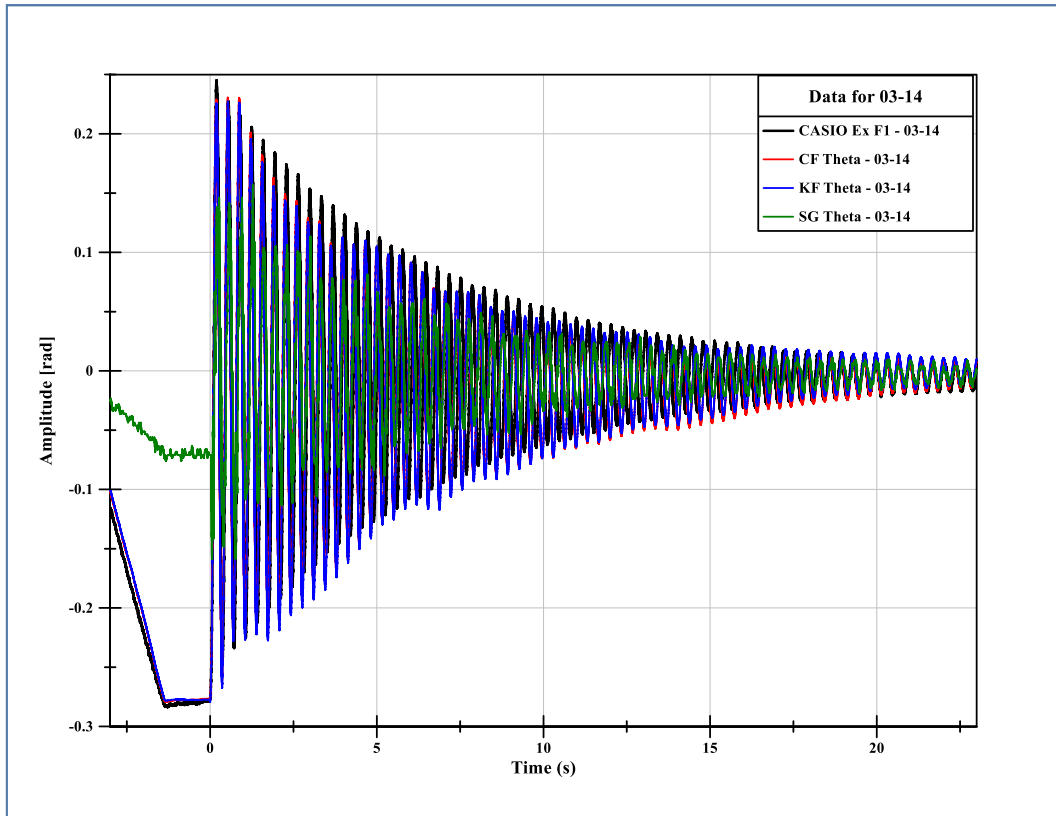


Figure D-27 – Data for 03-14

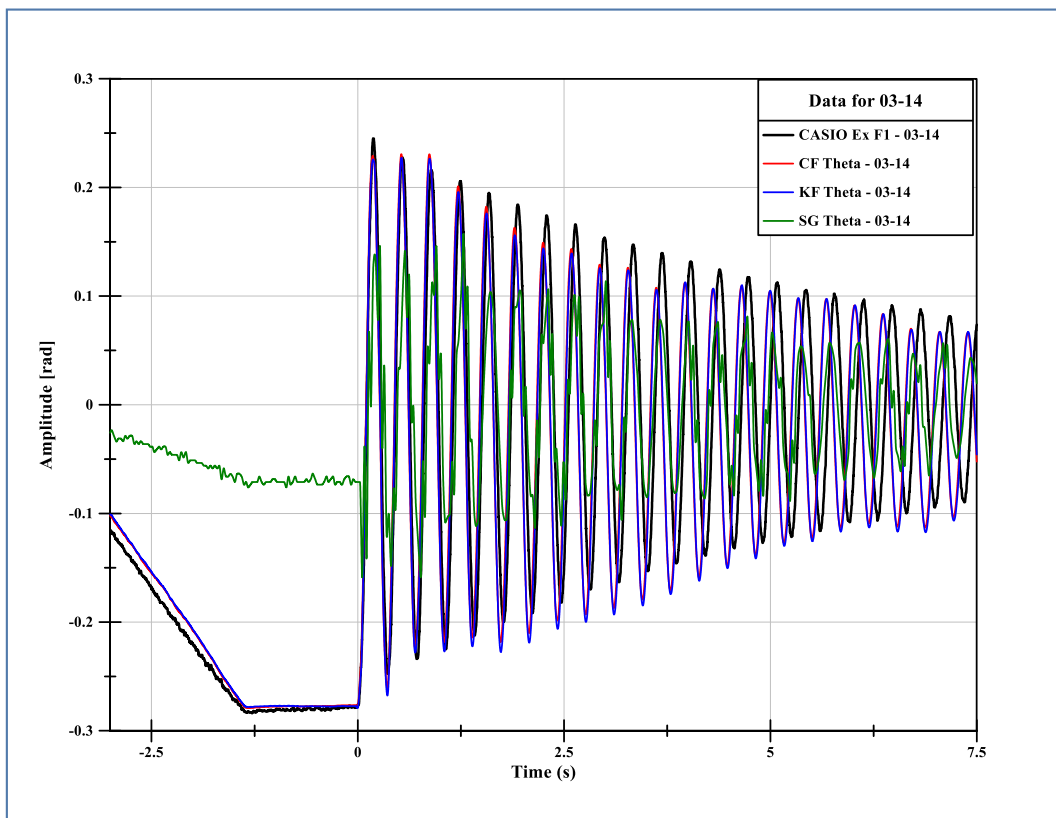


Figure D-28 – Close-up of data for 03-14

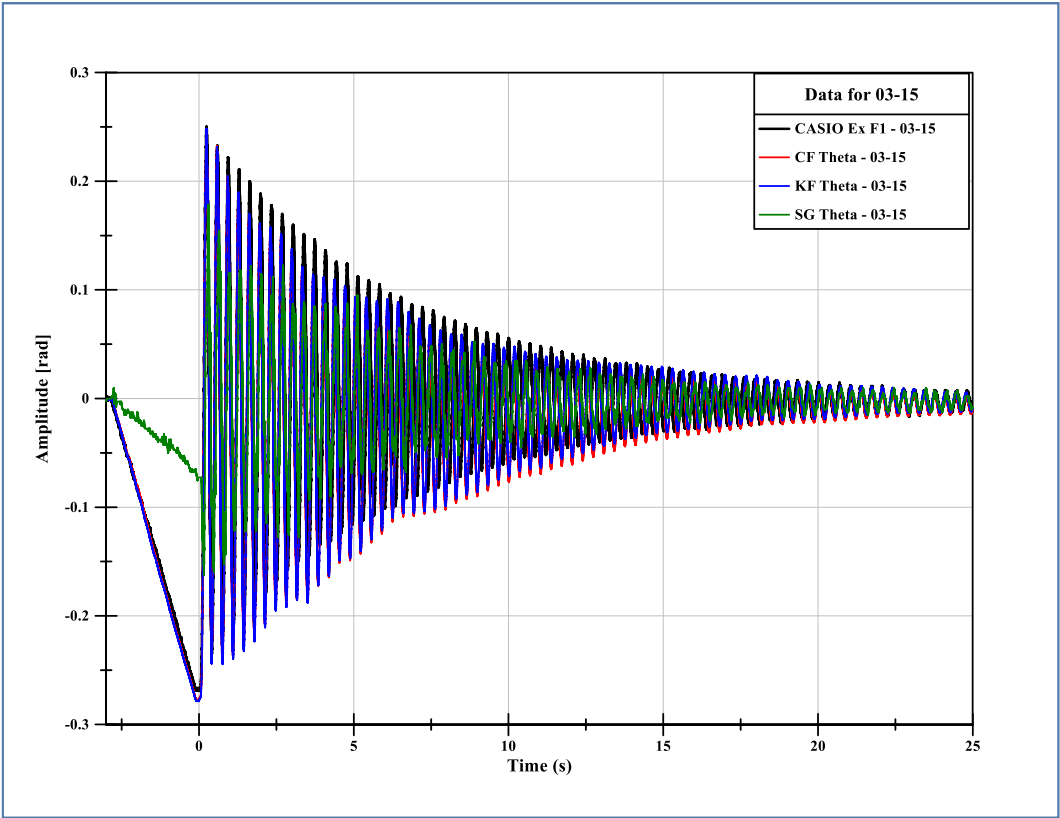


Figure D-29 – Data for 03-15

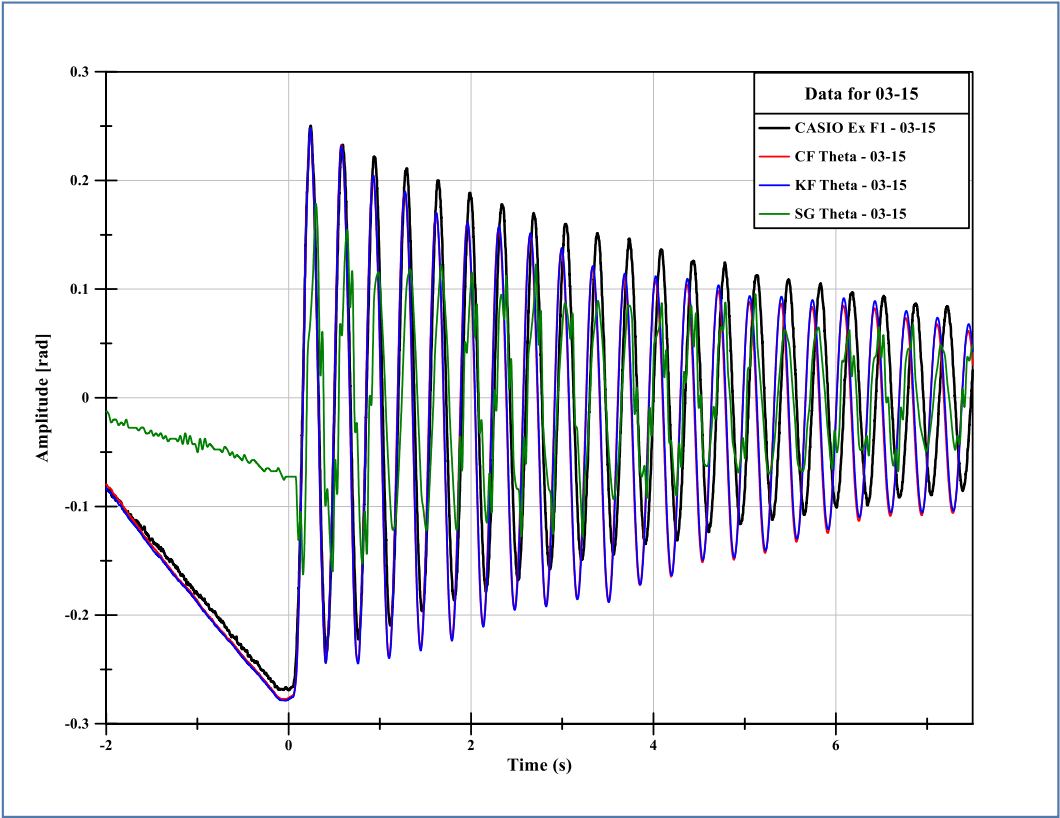


Figure D-30 – Close-up of data for 03-15

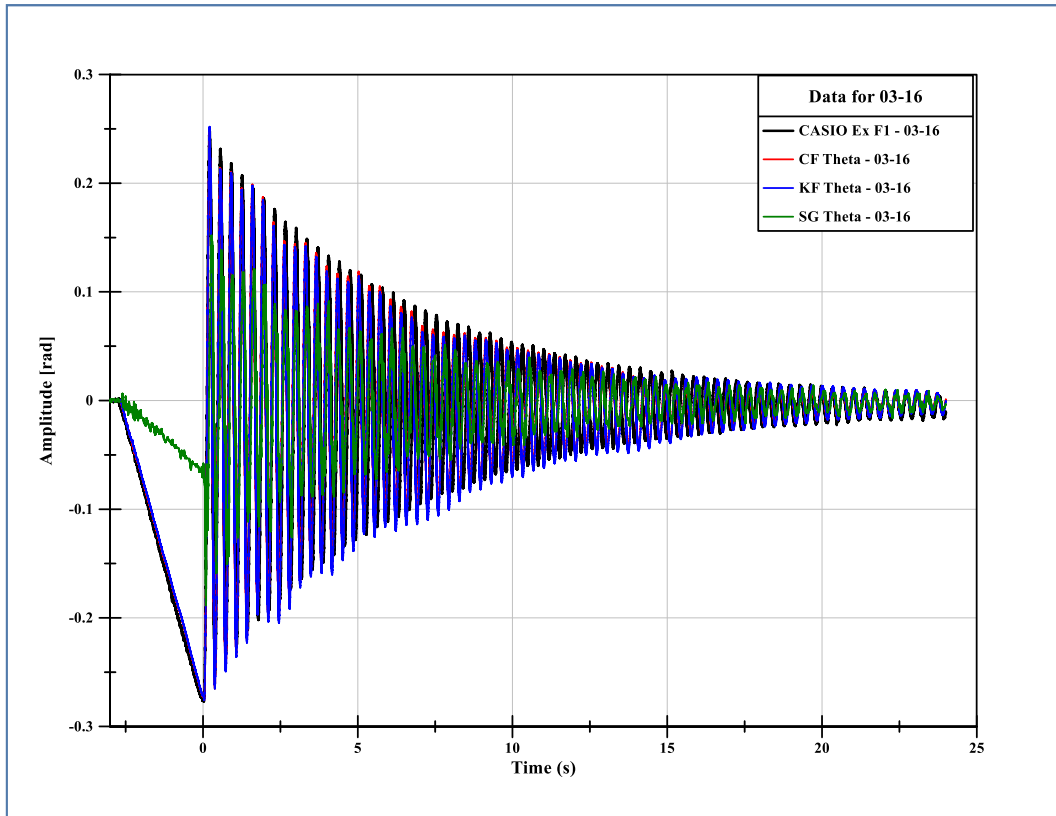


Figure D-31 – Data for 03-16

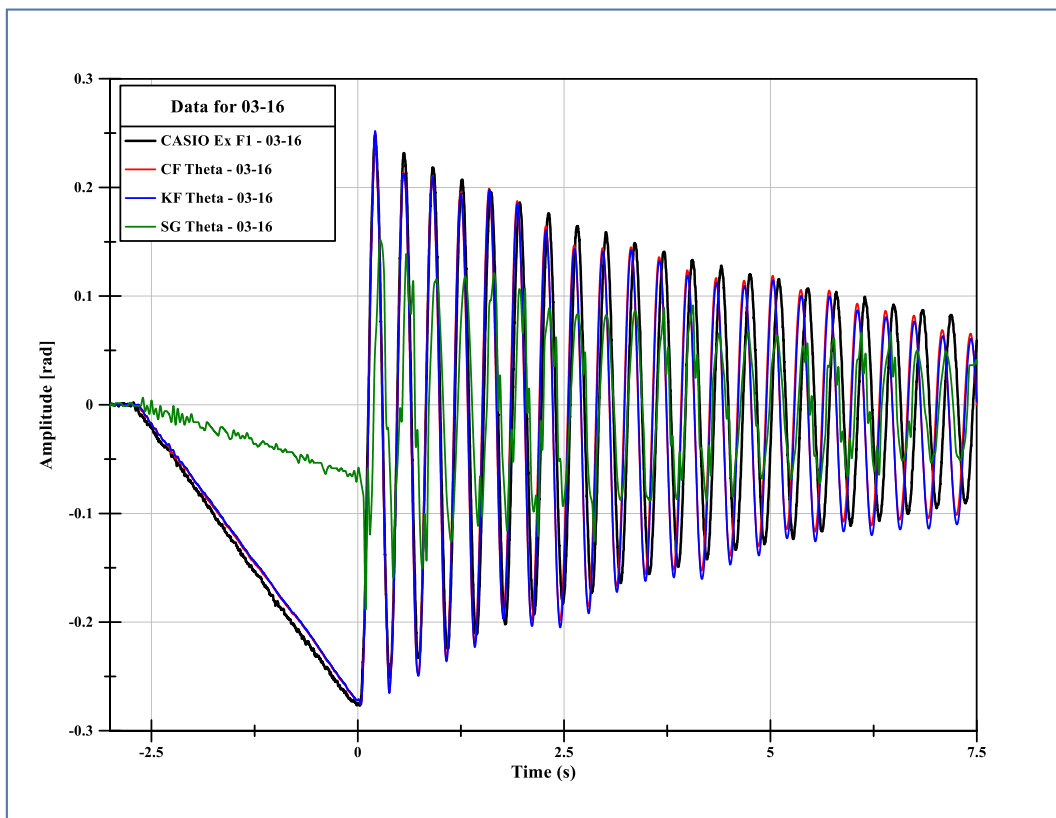


Figure D-32 – Close-up of data for 03-16

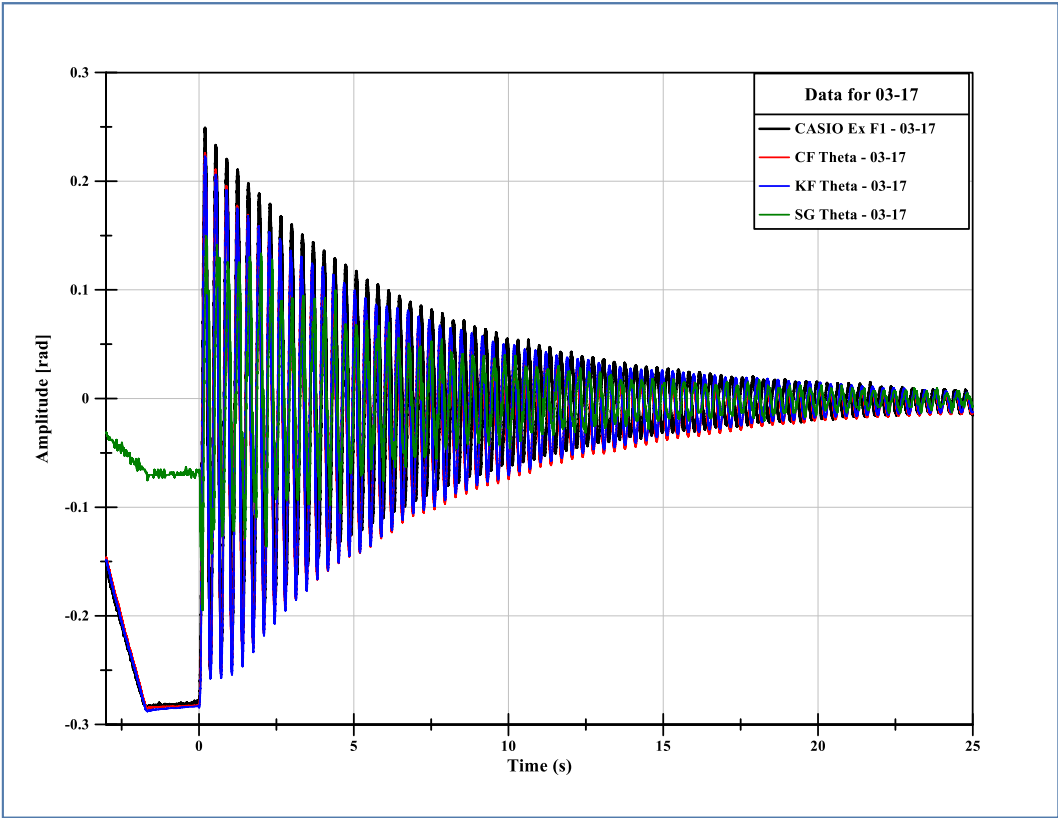


Figure D-33 – Data for 03-17

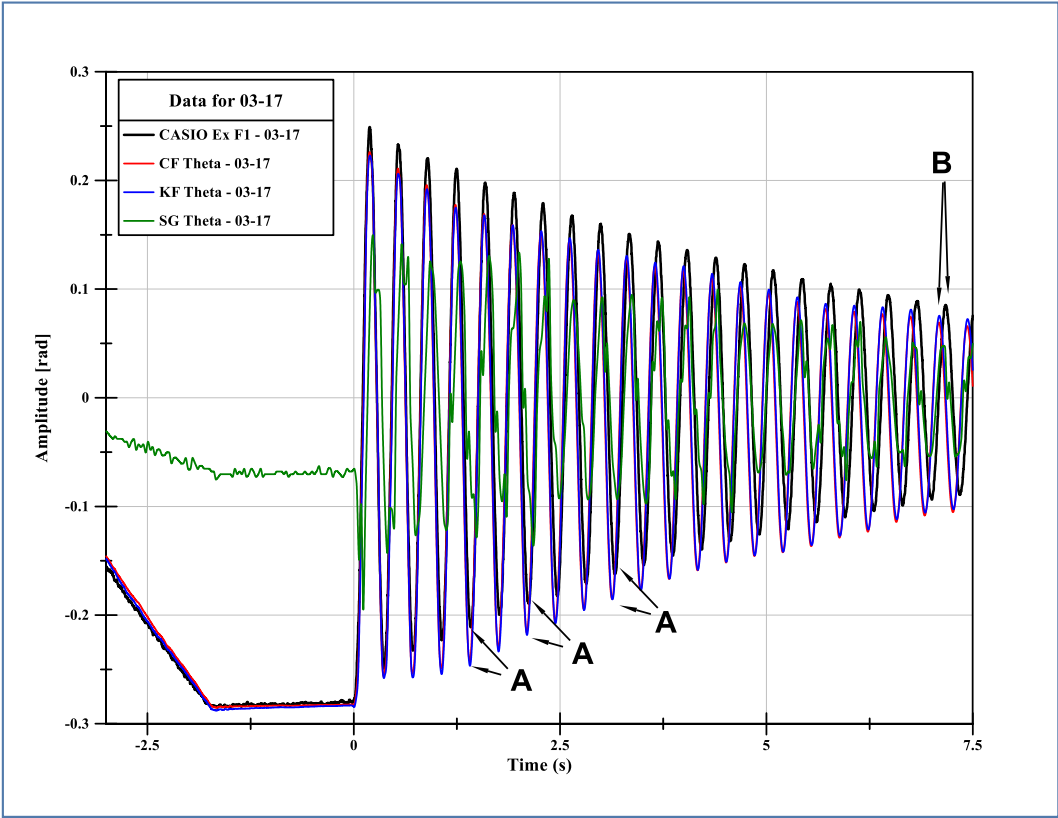


Figure D-34 – Close-up of data for 03-17

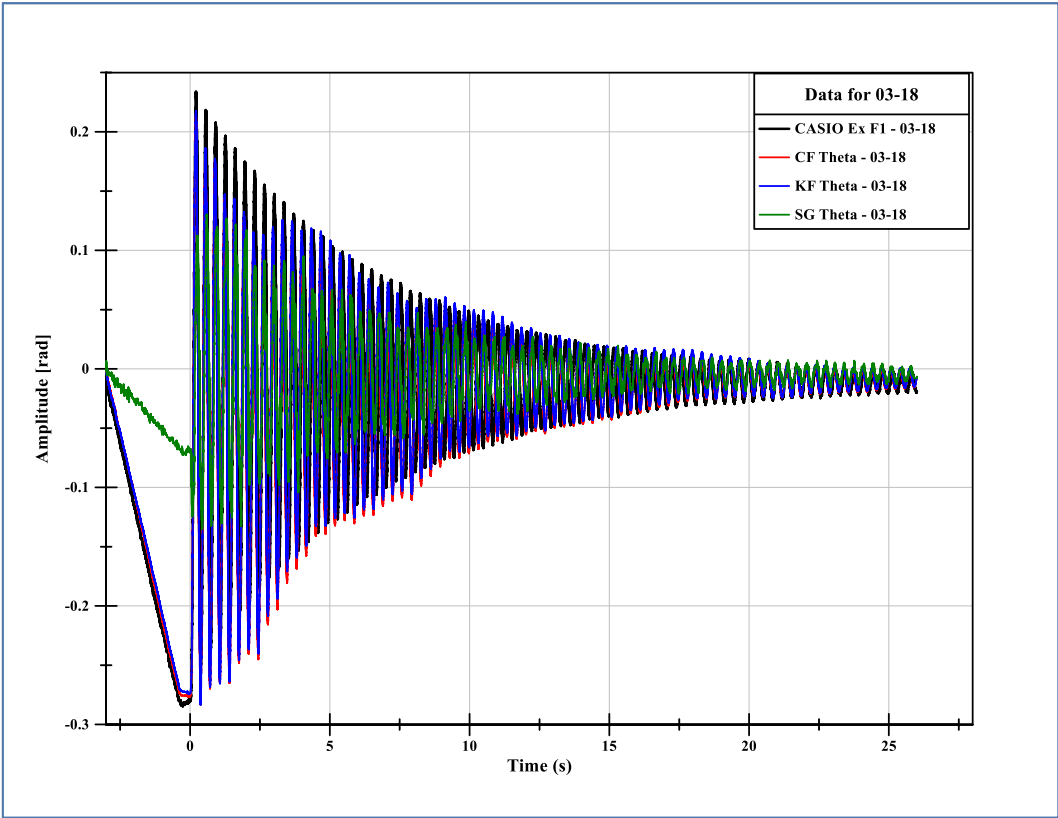


Figure D-35 – Data for 03-18

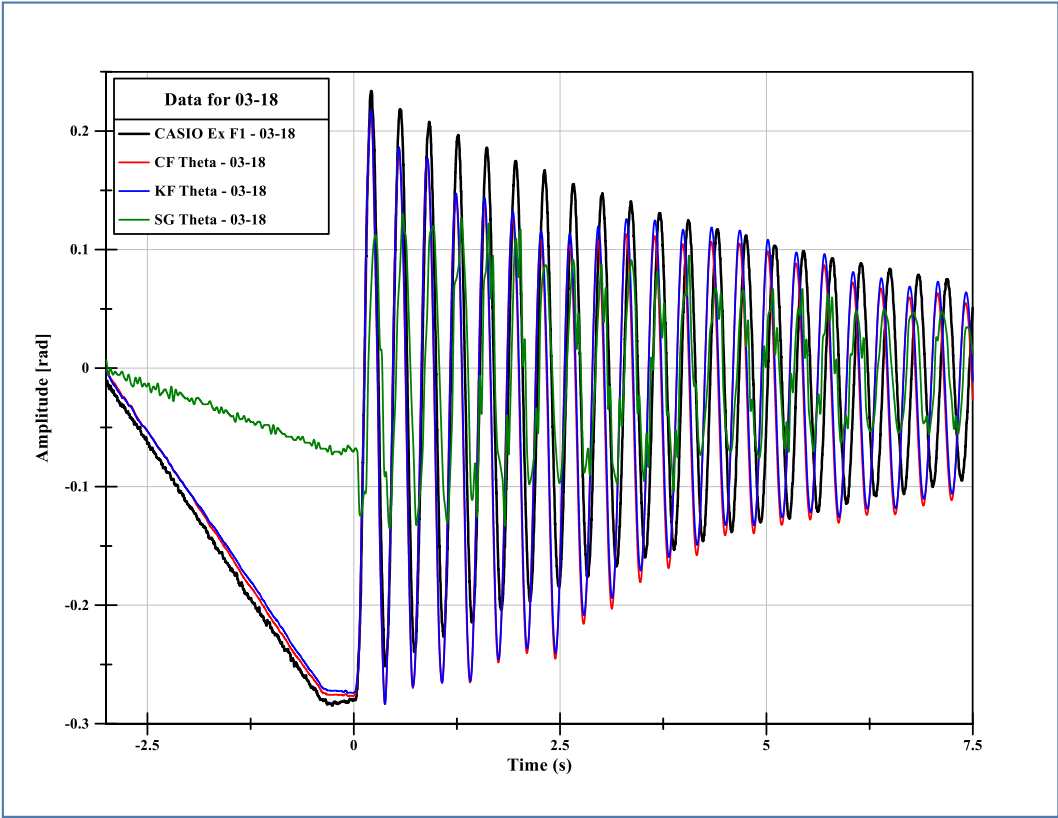


Figure D-36 – Close-up of data for 03-18

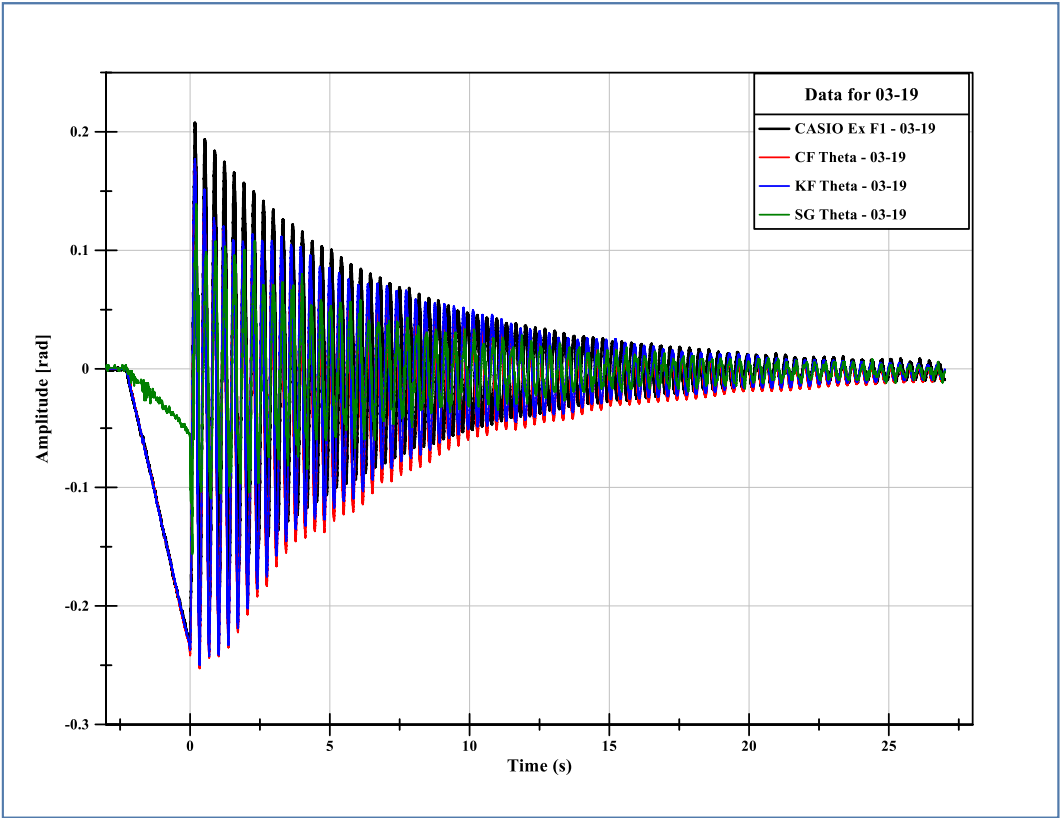


Figure D-37 – Data for 03-19

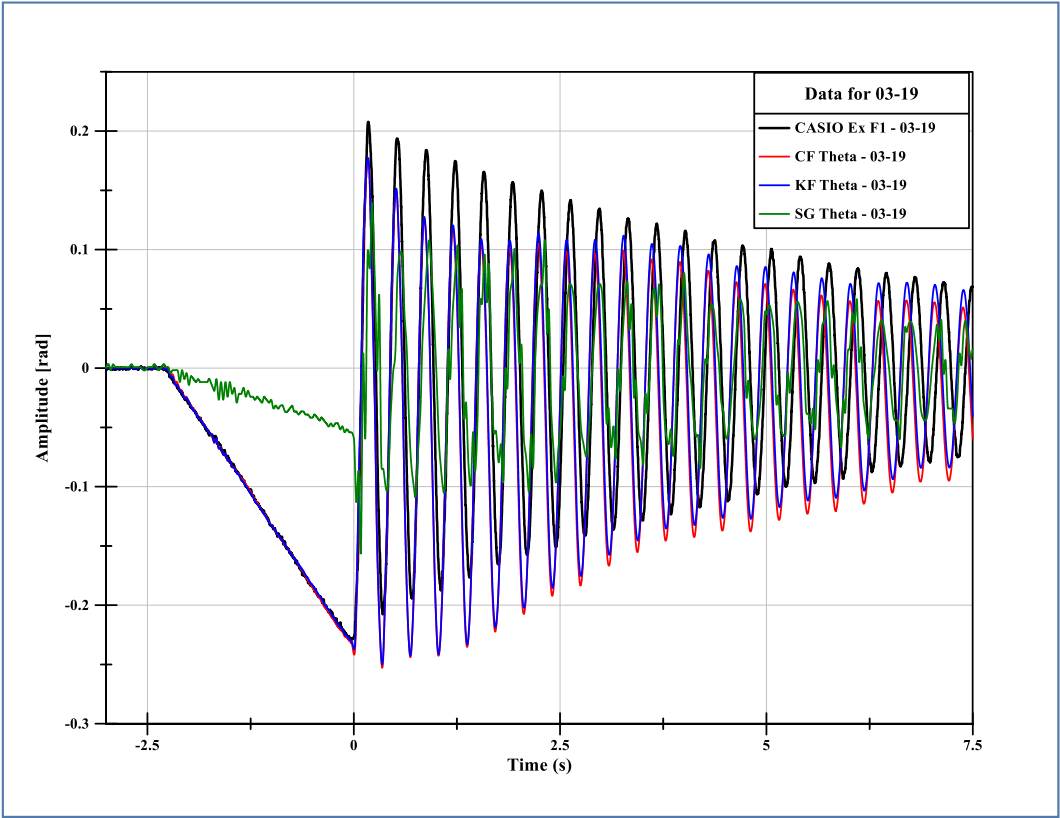


Figure D-38 – Close-up of data for 03-19

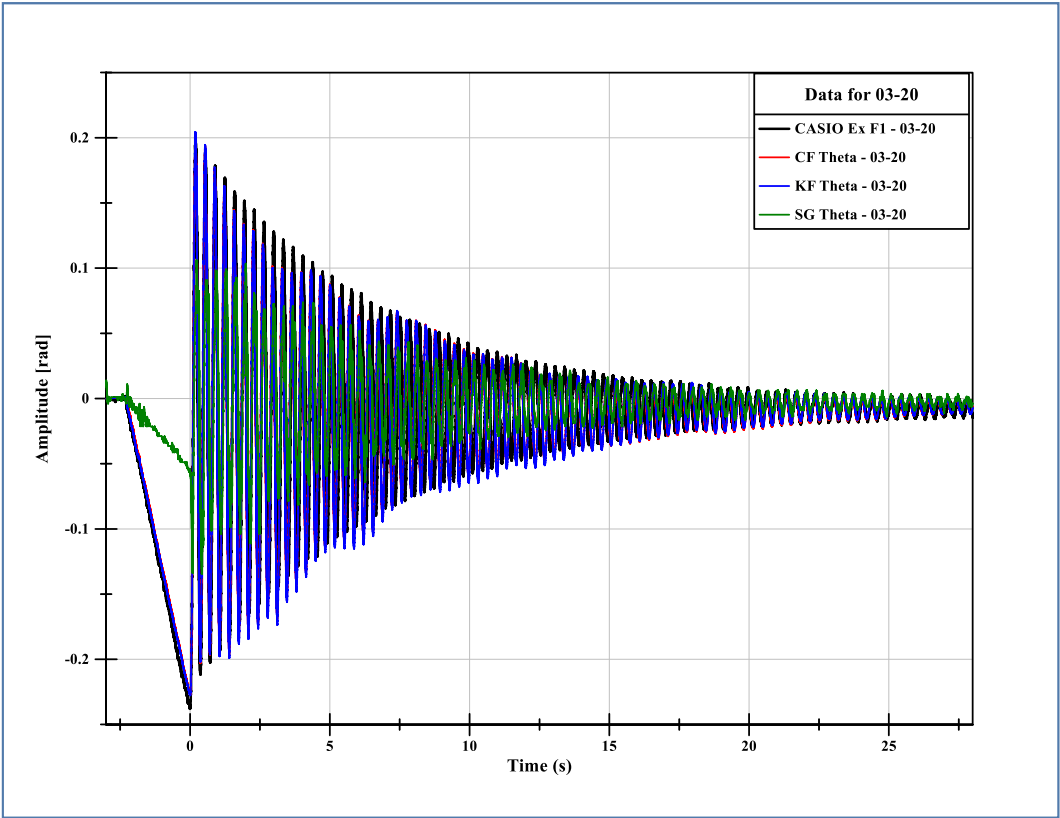


Figure D-39 – Data for 03-20

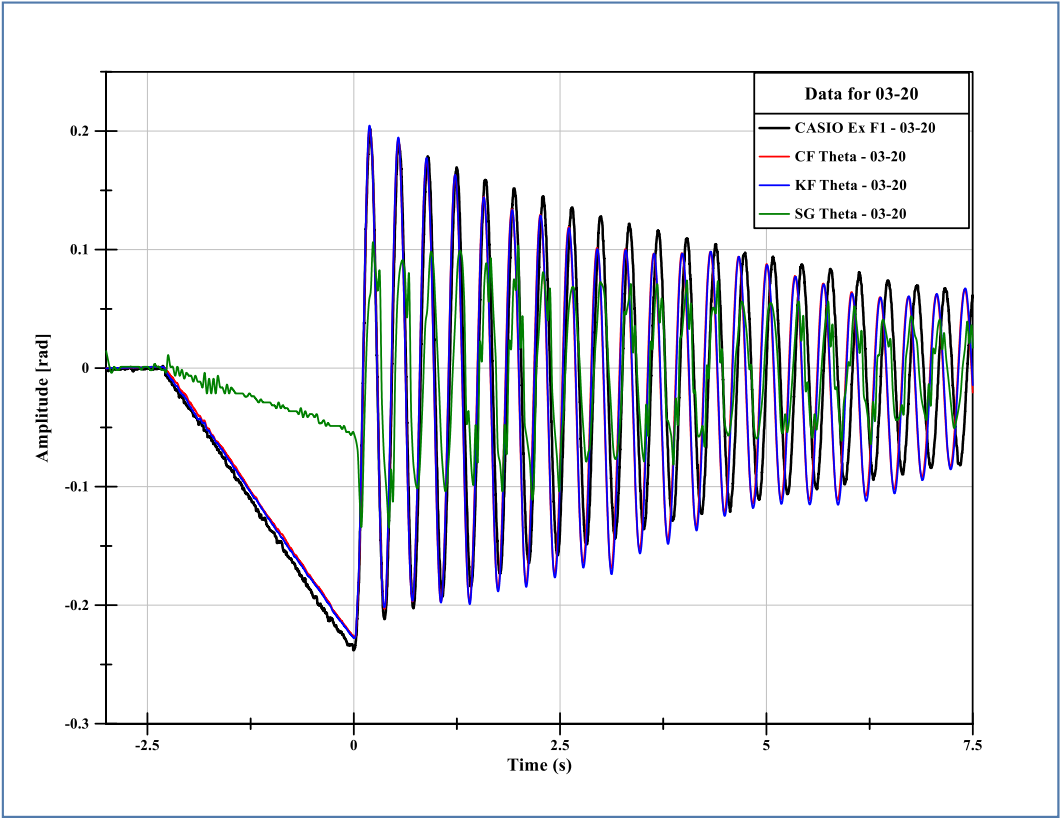


Figure D-40 – Close-up of data for 03-20

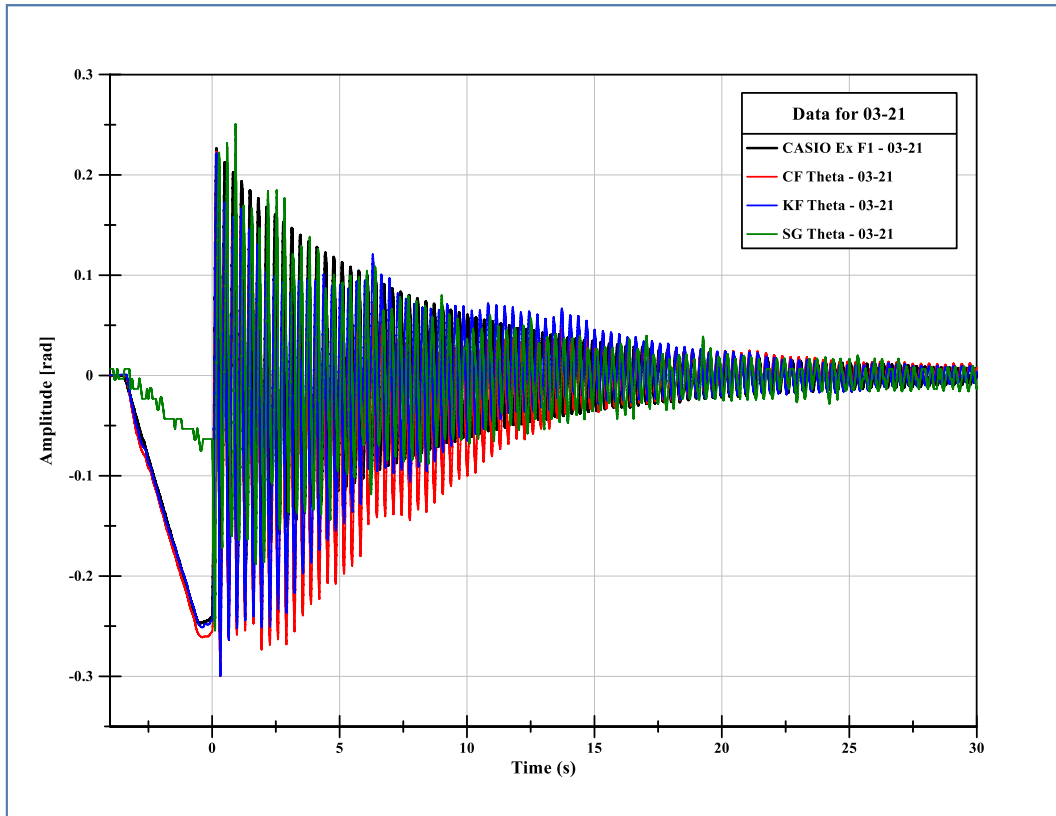


Figure D-41 – Data for 03-21

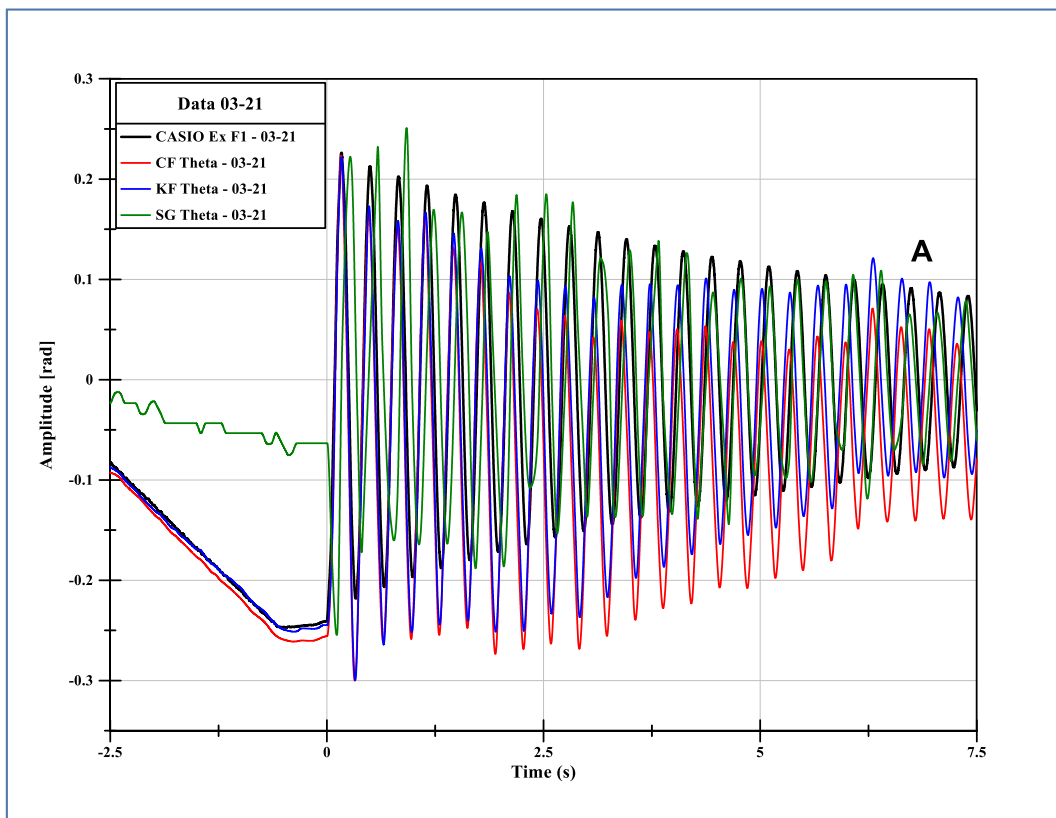


Figure D-42 – Close-up of data for 03-22

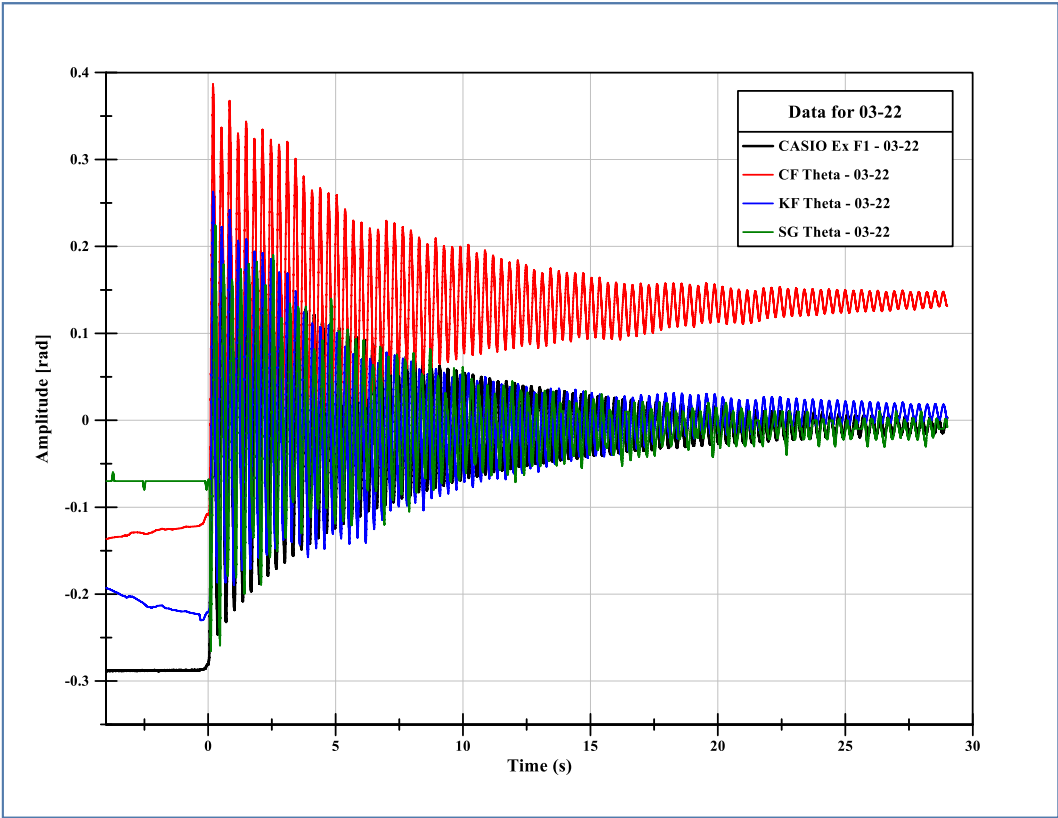


Figure D-43 – Data for 03-22

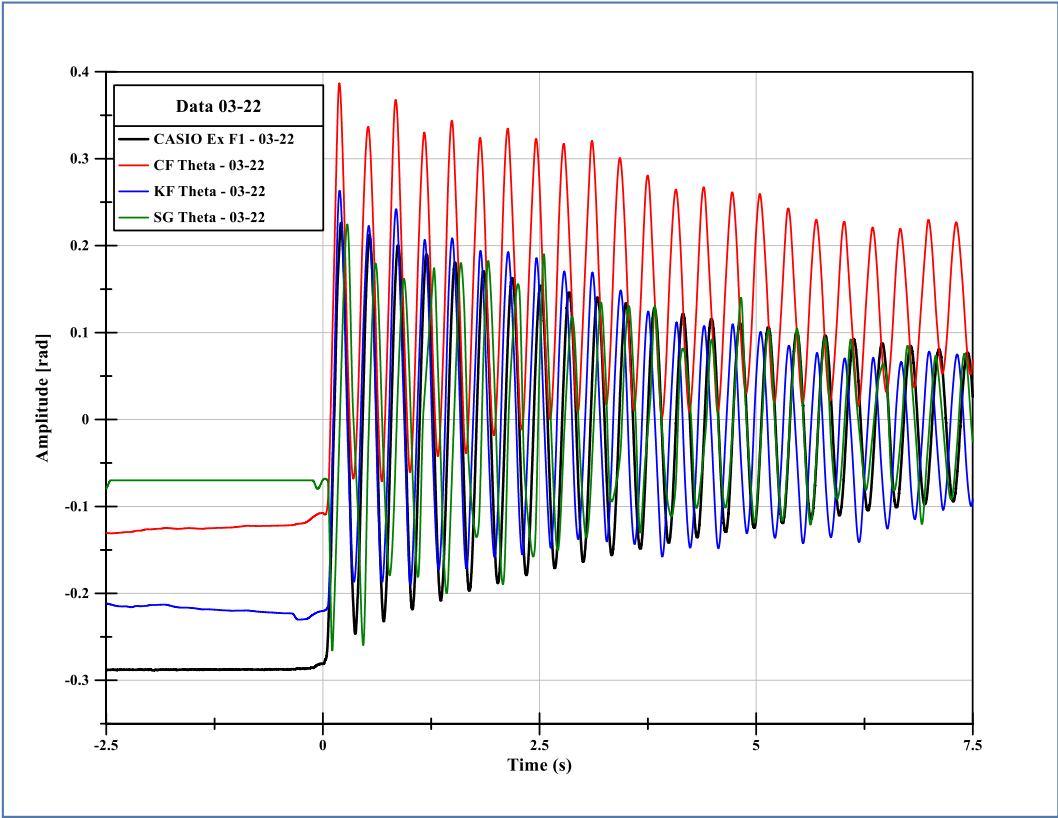


Figure D-44 – Close-up of data for 03-22

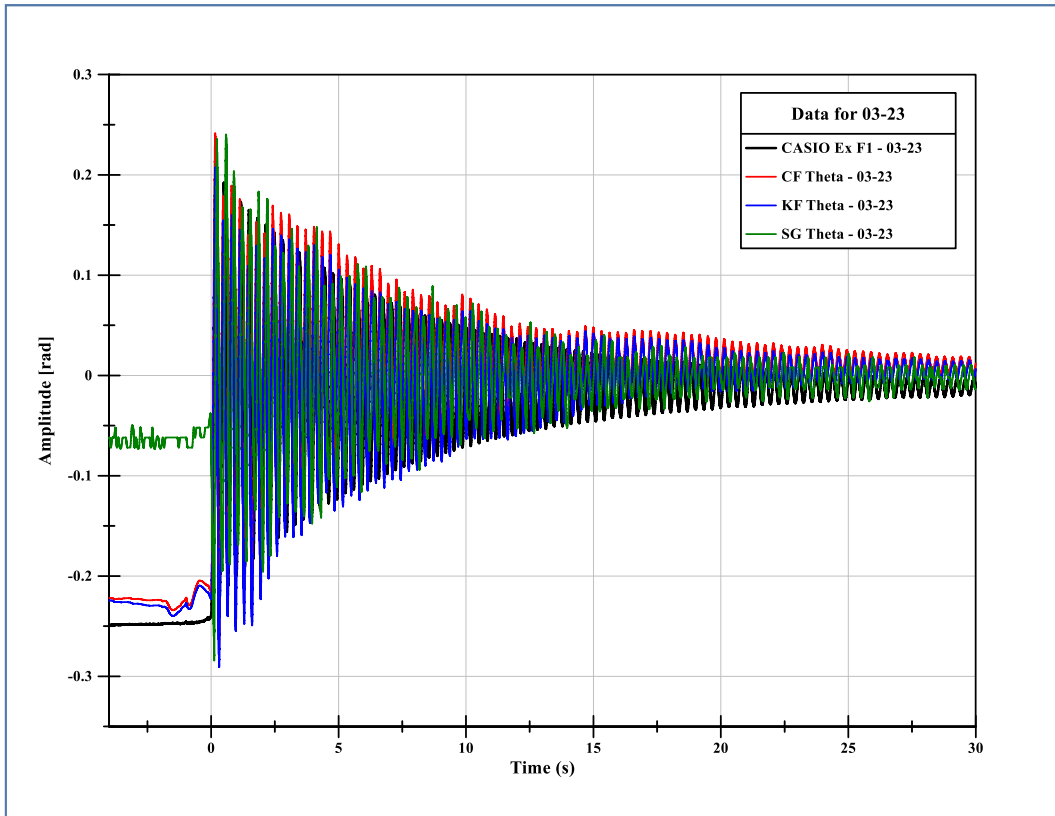


Figure D-45 – Data for 03-23

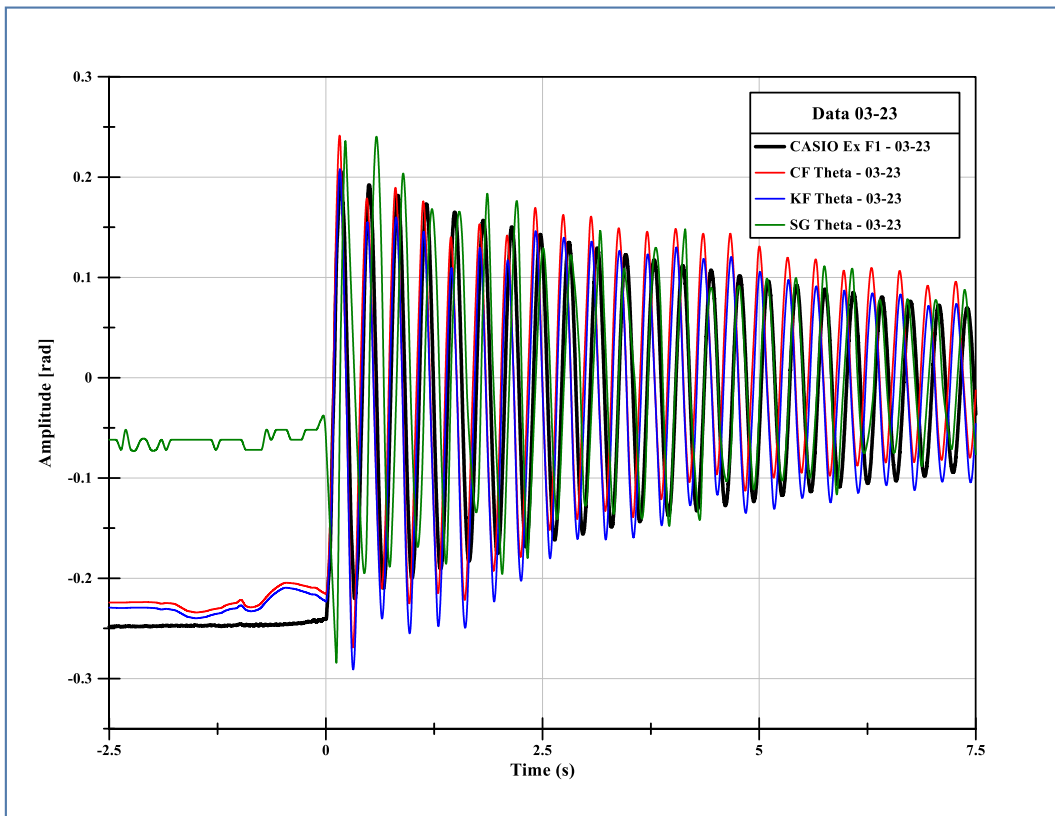


Figure D-46 – Close-up of data for 03-23

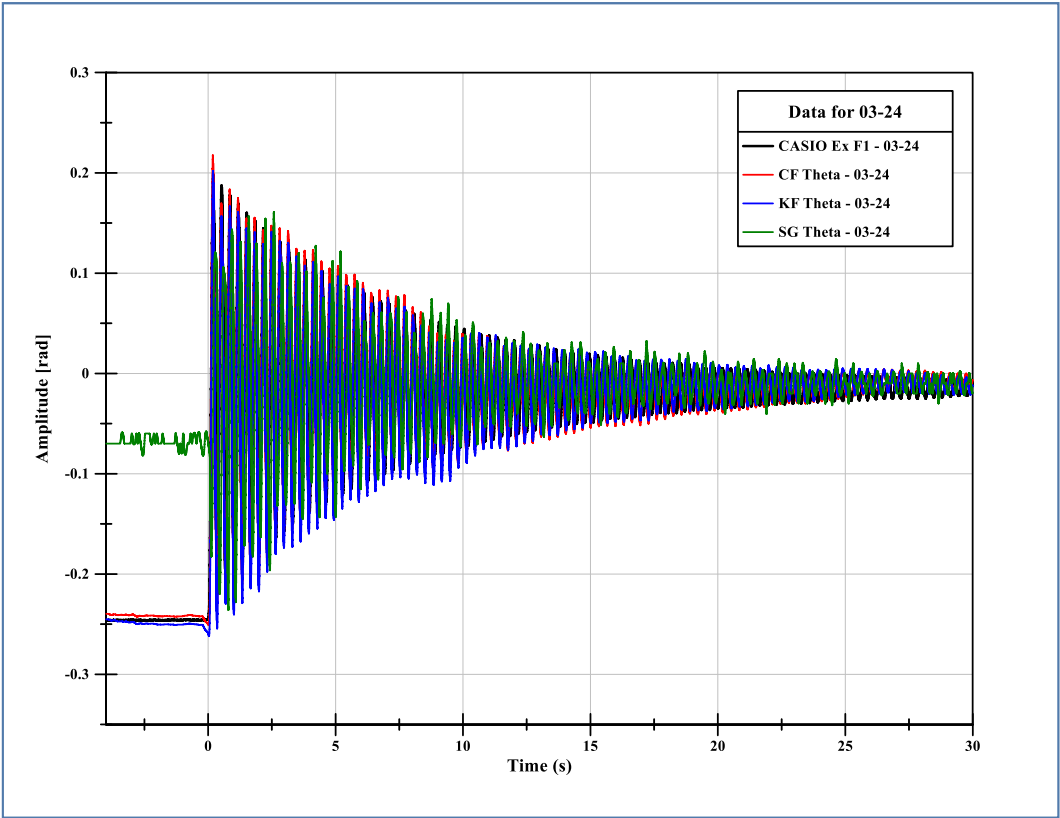


Figure D-47 – Data for 03-24

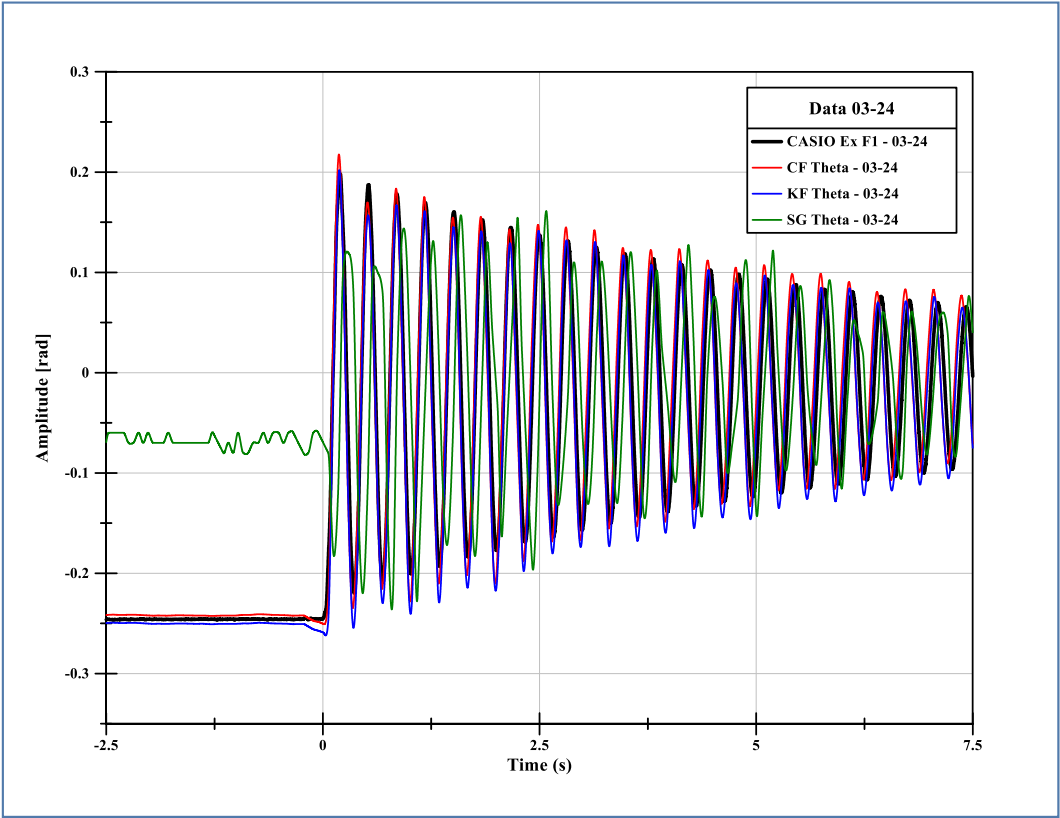


Figure D-48 – Close-up of data for 03-24

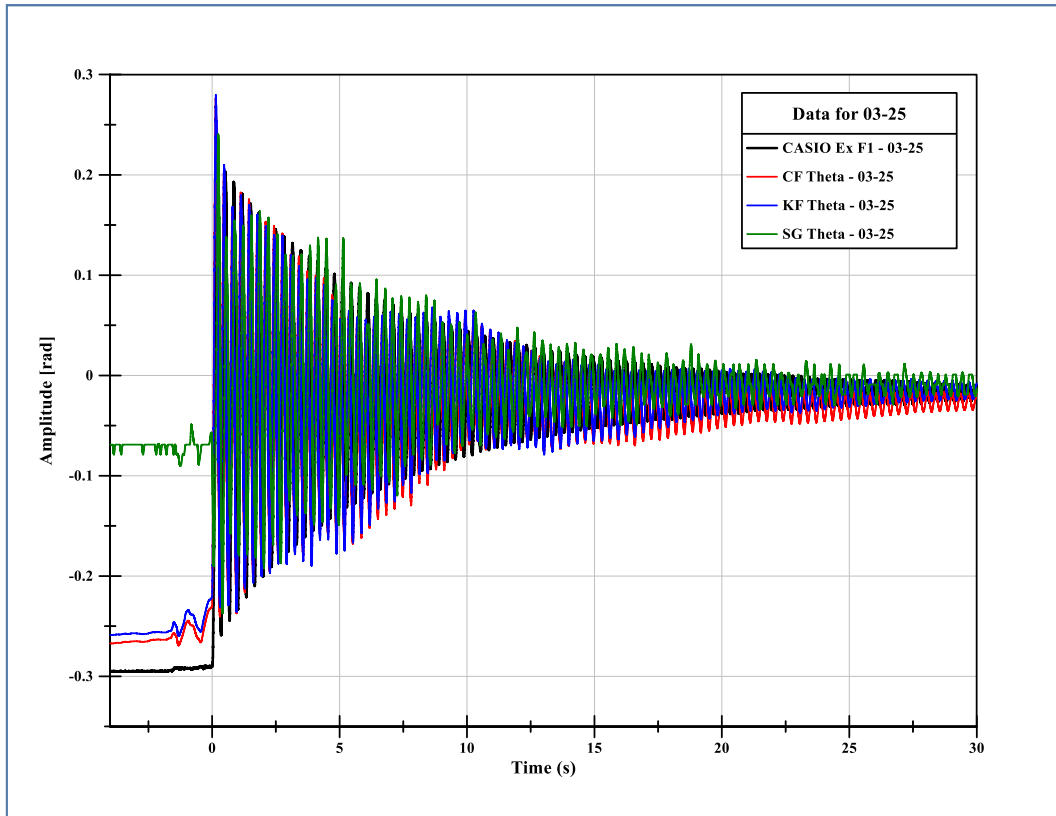


Figure D-49 – Data for 03-25

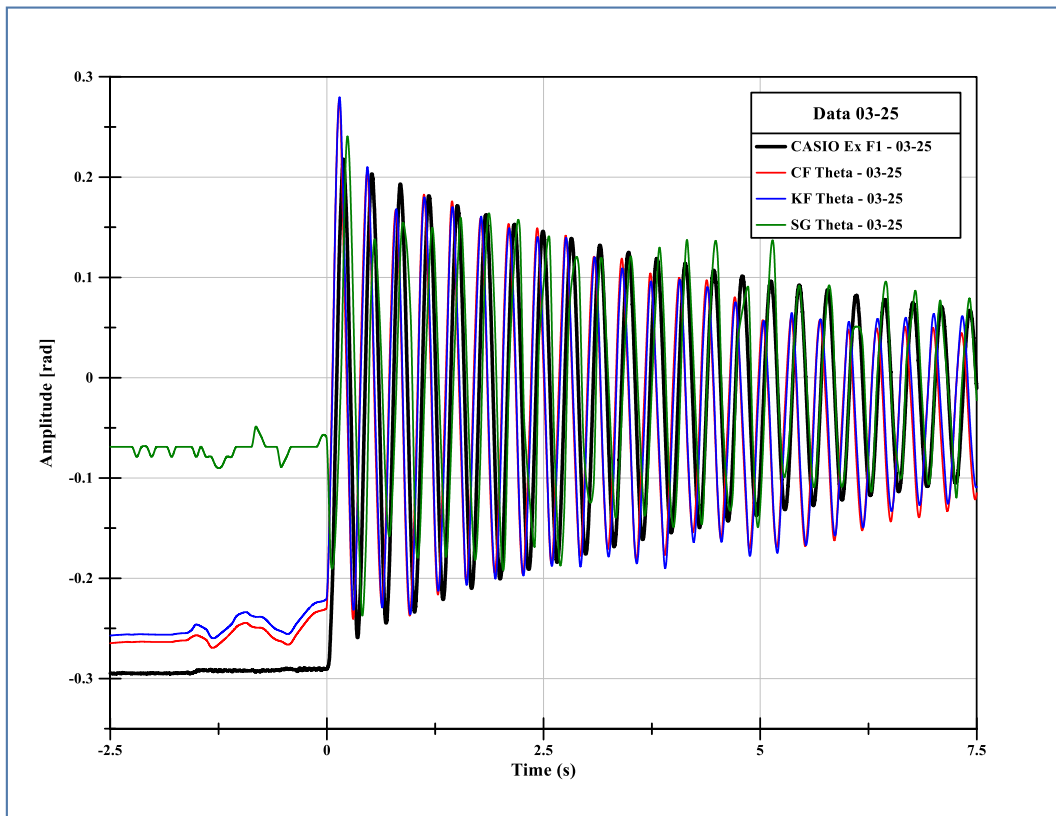


Figure D-50 – Close-up of data for 03-25

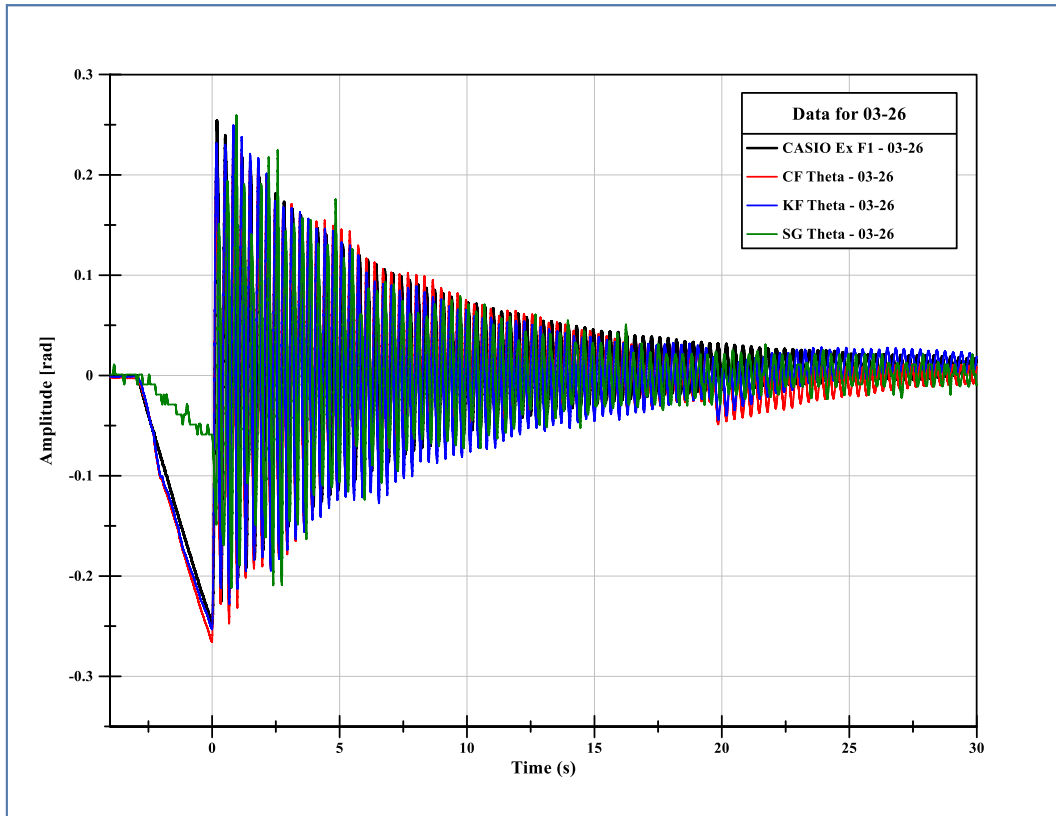


Figure D-51 – Data for 03-26

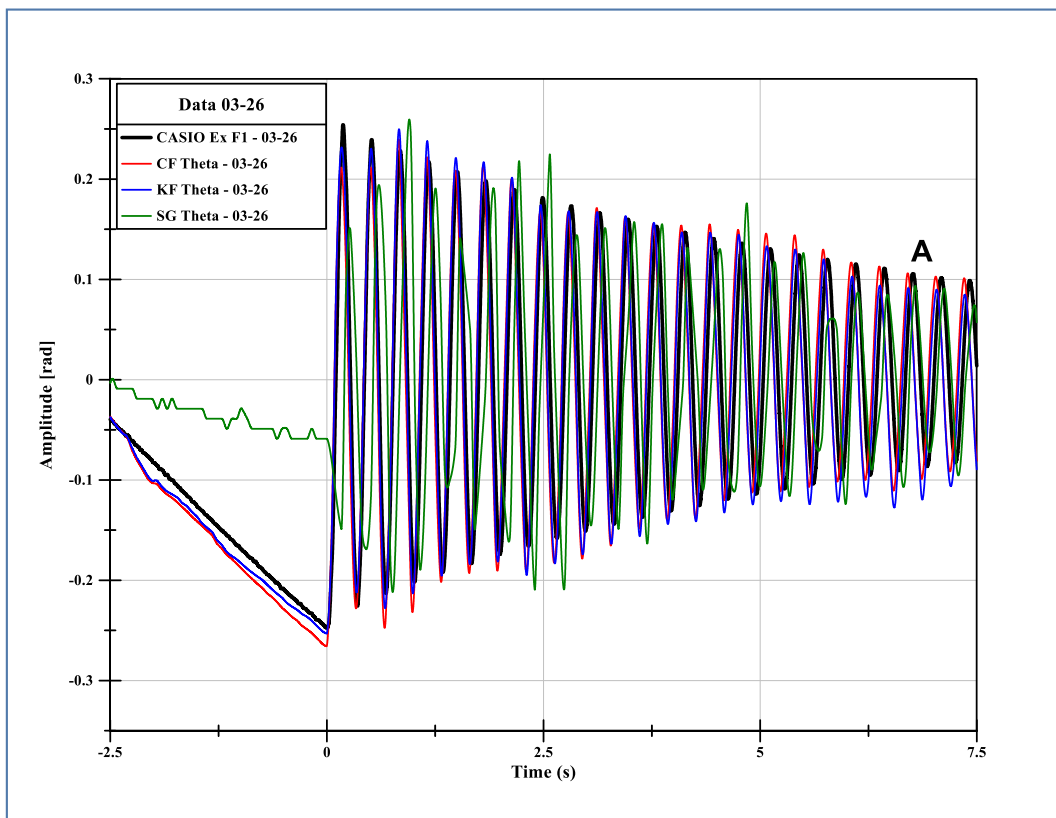


Figure D-52 – Close-up of data for 03-26

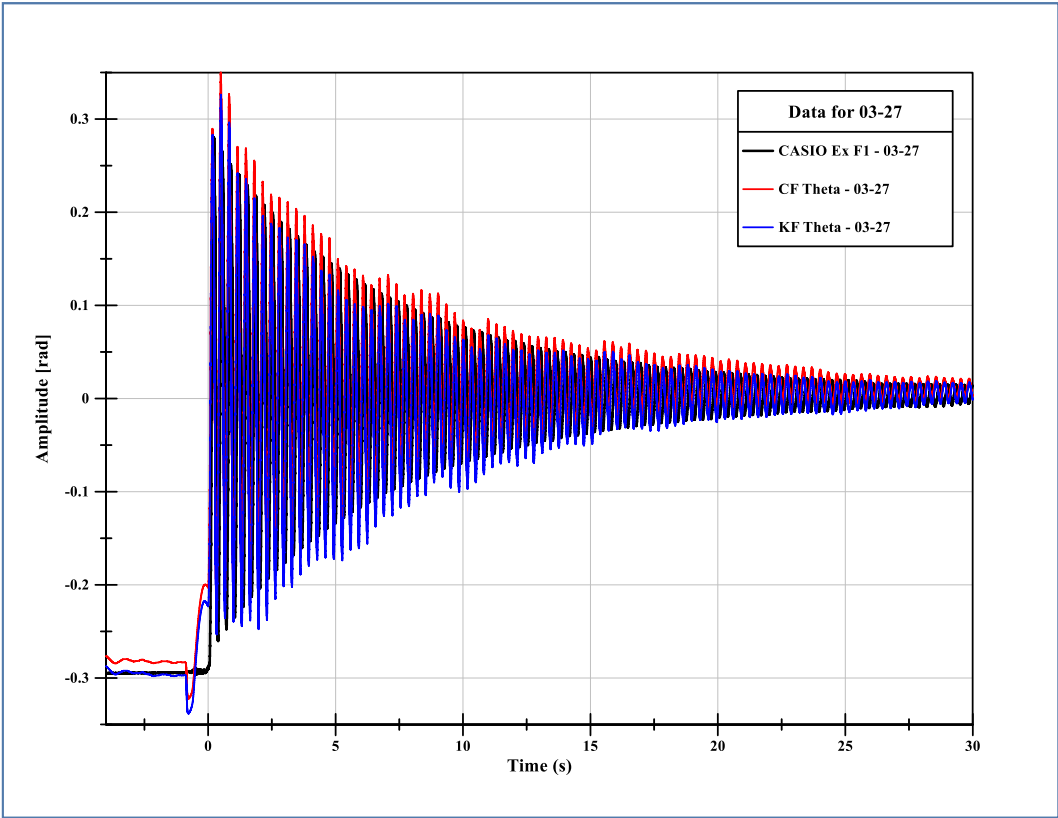


Figure D-53 – Data for 03-27

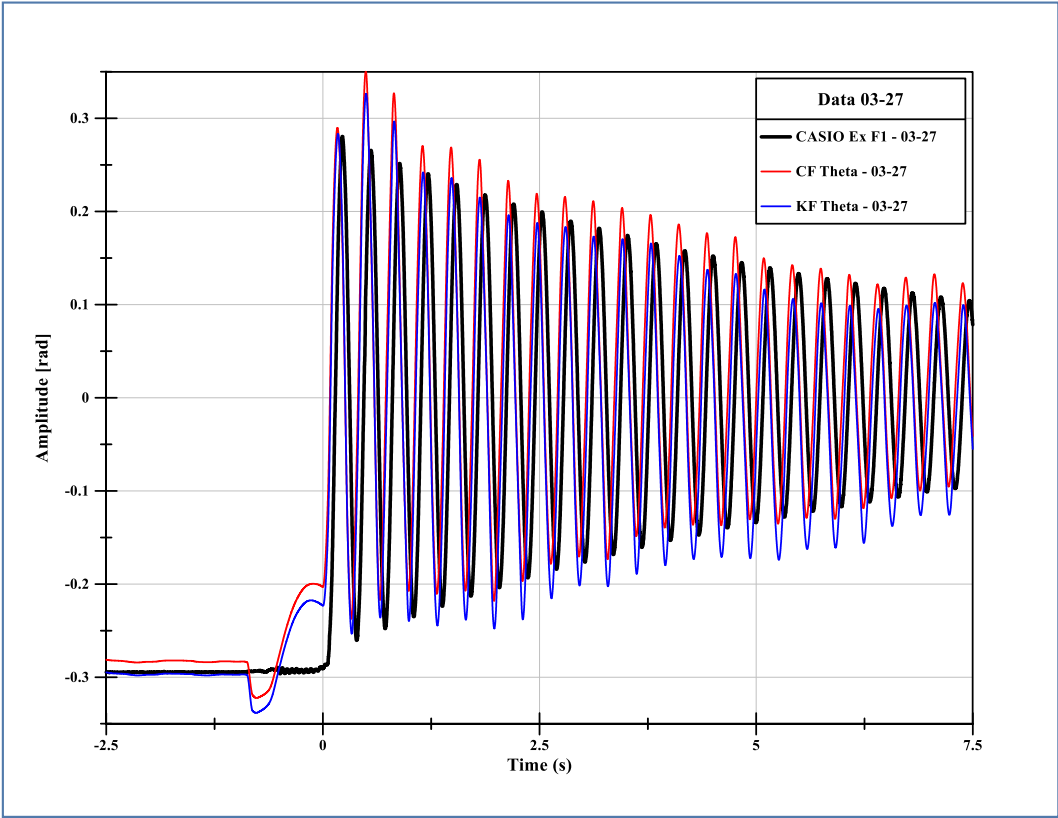


Figure D-54 – Close-up of data for 03-27

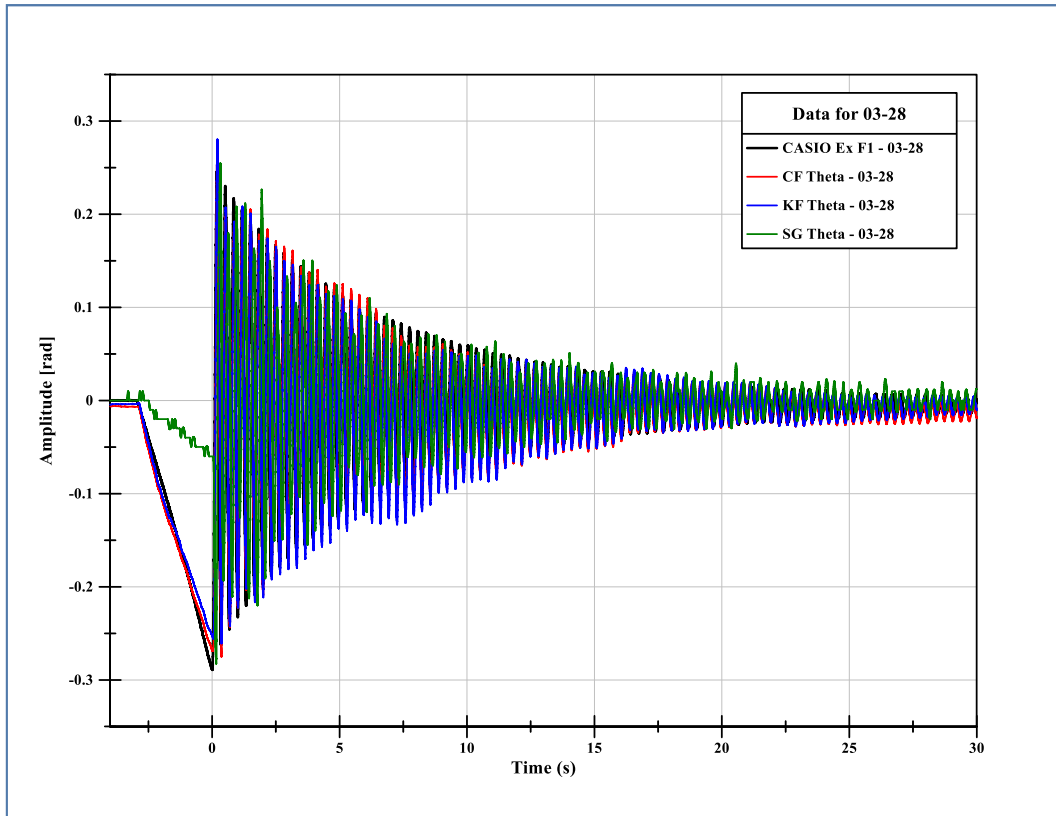


Figure D-55 – Data for 03-28

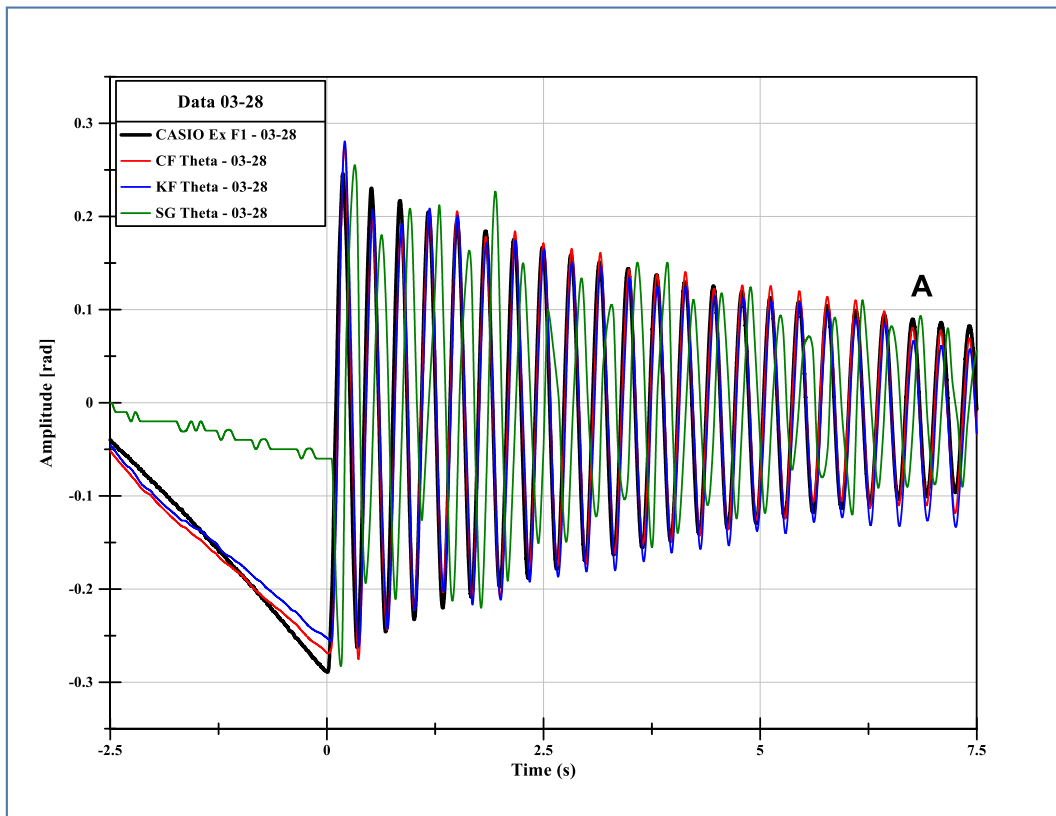


Figure D-56 – Close-up of data for 03-28

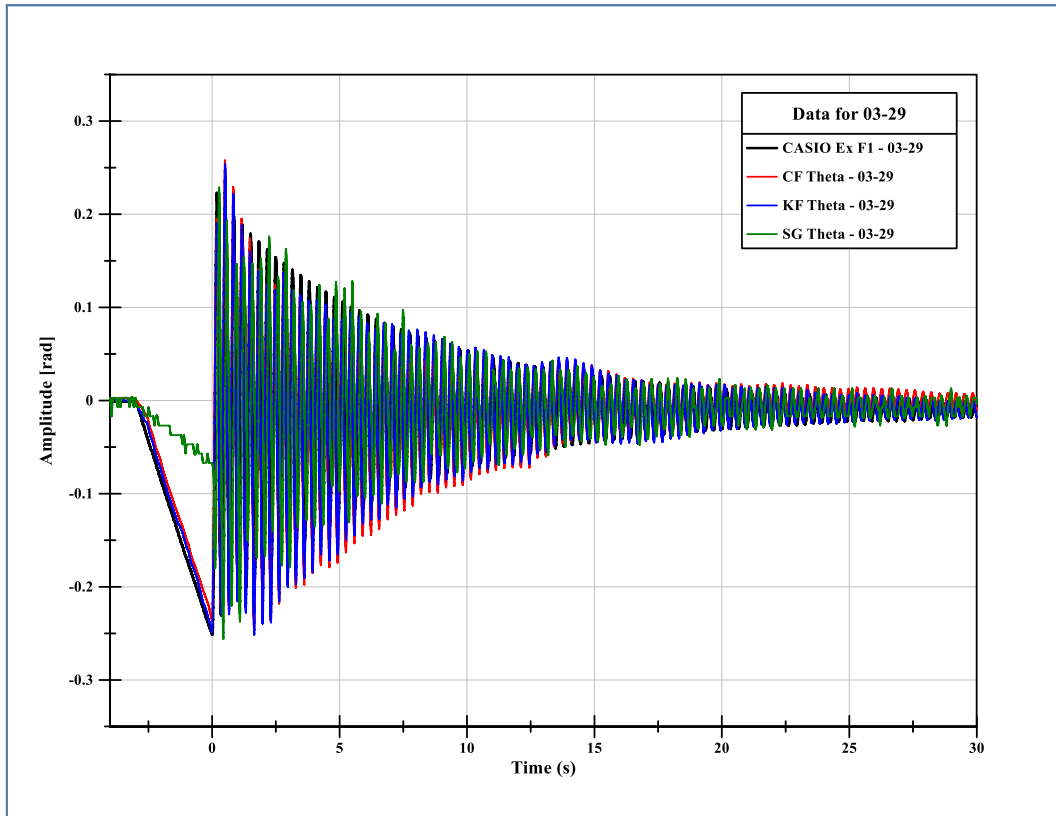


Figure D-57 – Data for 03-29

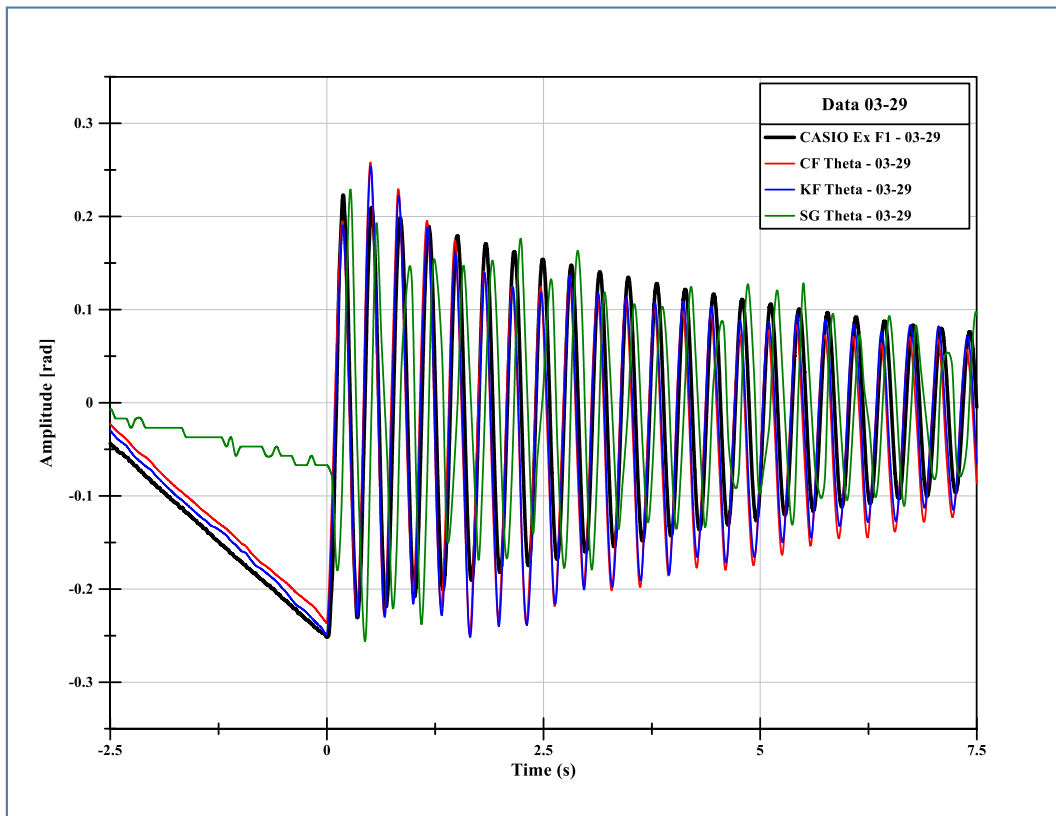


Figure D-58 – Close-up of data for 03-29

Annex E Permission to reproduce images

The Annex contains permissions to use the following images:

Modern Protective Structures - Figure 1-4

Protective Design Center – Figure 1-6

Accreditation and Quality Assurance - Figure 5-3

	
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Order detail ID: 72014355 ISBN: 9781420015423 Publication Type: e-Book Volume: Issue: Start page: Publisher: Taylor and Francis Author/Editor: Krauthammer, Theodor ; Krauthammer, Theodor	Permission Status:  Granted Permission type: Republish or display content Type of use: Thesis/Dissertation Order License Id: 4673091309811
Requestor type Format Portion Number of images/photos requested The requesting person/organization Title or numeric reference of the portion(s) Title of the article or chapter the portion is from Editor of portion(s) Author of portion(s) Volume of serial or monograph Page range of portion Publication date of portion Rights for Duration of use Creation of copies for the disabled With minor editing privileges For distribution to In the following language(s) With incidental promotional use Lifetime unit quantity of new product	Academic Institution Print, Electronic Image/photo 1 Robert Stephens, Imperial College London Fig 8.2 (b) Typical P-I diagram Chapter 8 - Pressure-Impulse Diagrams and Their Applications N/A N/A N/A 328 2008 Main product Life of current edition no no Worldwide Original language of publication no Up to 499 Investigation of new and novel techniques of accurately measuring the response of structures to short duration loads Imperial College London Sep 2019
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Mr. Stephens you request to include the graphic in your PhD thesis, has been approved by DDES, see email traffic below

Bill Seipel

William F. Seipel, PE
 U.S. Army Corps of Engineers
 Protective Design Center CENWO-ED-5
 1616 Capitol Ave, Suite 9000
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 Phone (402)995-2375
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 SIPR Email: william.f.seipel.civ@mail.usil.mil
 SIPR PDC Email: usarmy.cen-omaha.cenwo.mbx.odo@mail.usil.mil

-----Original Message-----

From: Zehrt, William H (Bill) Jr CIV USARMY HQDA DOD ESB (USA)
[\[mailto:william.h.zehrt2.civ@mail.mil\]](mailto:william.h.zehrt2.civ@mail.mil)
 Sent: Friday, September 27, 2019 8:55 AM
 To: Seipel, William F II CIV USARMY CENWO (USA)
William.F.Seipel@usace.army.mil
 Cc: Kreitinger, Timothy J CIV USARMY CENWO (USA)
Timothy.J.Kreitinger@usace.army.mil; Cotton, Lea A CIV USARMY HQDA DOD ESB (USA)
lea.a.cotton.civ@mail.mil
 Subject: RE: Permission to reproduce image in PhD thesis (UNCLASSIFIED)

Hi, Bill - I discussed with Lea Ann this morning, and we're fine with Mr. Stephens using this graphic.

Thanks,

Bill

-----Original Message-----

From: Seipel, William F II CIV USARMY CENWO (USA)
William.F.Seipel@usace.army.mil
 Sent: Monday, September 23, 2019 11:20 AM
 To: Zehrt, William H (Bill) Jr CIV USARMY HQDA DOD ESB (USA)
william.h.zehrt2.civ@mail.mil
 Cc: Kreitinger, Timothy J CIV USARMY CENWO (USA)
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Bill, could you look this over Robert Stephens from the UK is asking to include a graphic (attached) in to his PhD Dissertation

Best Regards

William F. Seipel, PE
 U.S. Army Corps of Engineers
 Protective Design Center CENWO-ED-5
 1616 Capitol Ave, Suite 9000
 Omaha, Nebraska, 68102-4901
 Phone (402)995-2375
 FAX (402)995-2388
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william.f.seipel.civ@mail.usil.mil
 SIPR PDC Email: usarmy.cen-omaha.cenwo.mbx.odo@mail.usil.mil

-----Original Message-----

From: Kreitinger, Timothy J CIV USARMY CENWO (USA)
 Sent: Monday, September 23, 2019 10:15 AM
 To: Seipel, William F II CIV USARMY CENWO (USA)
William.F.Seipel@usace.army.mil

Cc: Revoy, Cassandra L CIV USARMY CENWO (USA)
Cassandra.L.Revoy@usace.army.mil
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Bill,

I believe Huntsville is responsible for this UFC. Who over there does Bob Stephens need to ask.

Thank you,
 Tim

-----Original Message-----

From: Revoy, Cassandra L CIV USARMY CENWO (USA)
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 To: Kreitinger, Timothy J CIV USARMY CENWO (USA)
Timothy.J.Kreitinger@usace.army.mil
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Tim,

Please see email below that was sent to the POC email.

Cassandra Revoy
Civil Engineering Intern
Protective Design Center
U.S. Army Corps of Engineers

-----Original Message-----

From: Stephens, Robert (Bob) [mailto:r.stephens128@imperial.ac.uk]
Sent: Friday, September 20, 2019 7:08 AM
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