Equivalent Circuit for Double Annular Aperture Frequency Selective Surfaces

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***Abstract*—In this work a double annular aperture frequency selective surface is analyzed from an equivalent circuit perspective. A comparison between full wave numerical solution and the proposed equivalent circuit results is provided for different examples. A very good agreement is obtained endowing the equivalent circuit with great potential as a very powerful and efficient design tool for advanced space filters.**

I. Introduction

Frequency selective surfaces (FSSs) are classical periodic structures widely used in the microwave regime as spatial filters and polarizers in the framework of radar applications [1]. More recently, they have also been found useful for sensing application given their strong dependency with their surroundings [2]. Therefore, a complete knowledge of the transmission/reflection properties is crucial for design and optimization of such FSS based devices. Equivalent circuit modeling techniques provide a full characterization of these features with very low computational efforts in comparison with full wave simulation schemes [3]–[5].

In this work, an equivalent circuit to model a double annular aperture FSS is presented. The case of a single annular (coaxial) aperture has been studied in depth in [6]. Therein, the multimodal network topology proposed in [5] was used. Here, this approach is combined with the inclusion of two basis functions to characterize the aperture field, as presented in [7], [8].

The obtained equivalent circuit results are compared with full wave numerical simulations with a very good agreement for a selected number of cases. The agreement between the proposed equivalent circuit may be very useful as an efficient design tool for advanced FSS structures working either as filters or sensing devices in the microwave and THz ranges.

II. Geometry and Equivalent Circuit

A portion of the infinite two dimensional structure under study is shown in Fig. 1(a). A periodic array of two concentric annular apertures is drilled in a metallic screen. These apertures are determined by the internal and external radii a_i and b_i (with $i = 1, 2$), respectively. The unit cell periods are given by d_x and d_y. The equivalent circuit topology

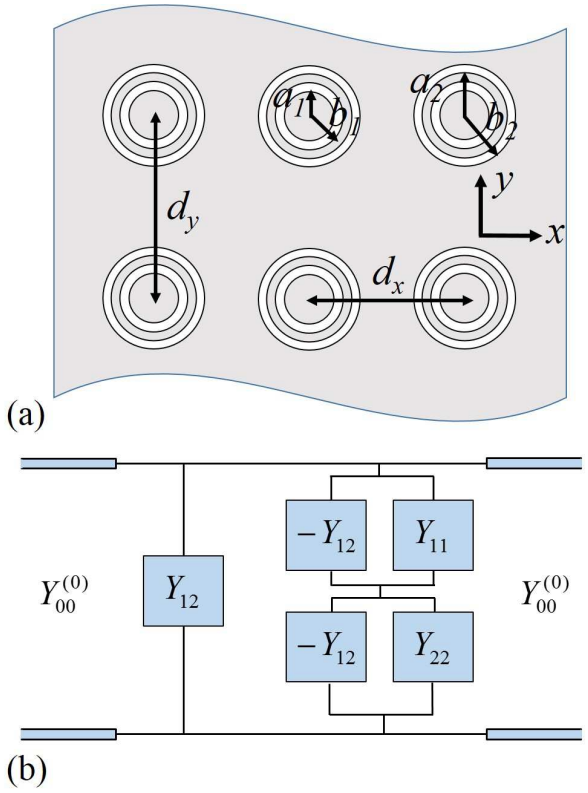


Fig. 1. (a) Schematic of a the infinite structure along with the geometrical parameters. (b) Equivalent circuit model.

used in this work is depicted in Fig. 1(b). This approach has been recently employed for enlarging the operation frequency regime of the model in metallic patch arrays and for predicting the transmission/reflection features of unit cells containing more than one scatterer [8]. In the present case, each annular aperture (1 and 2) is characterized by a field profile with sine/cosine dependence of the radial component as proposed in [3]. This field profile is linked to the TE_{11} mode of each coaxial waveguide accounting for each annular aperture. Then, the basis function is identical but with different boundary

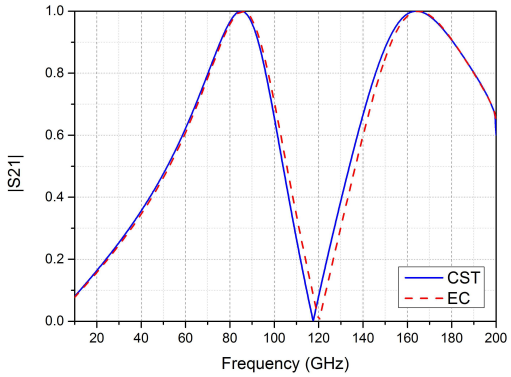


Fig. 2. Transmission coefficient versus frequency under normal incidence for a free-standing FSS with the following geometrical parameters: x-period, d_x = 1.5 mm; y-period, $d_y = d_x$; inner radius of the first aperture, a_1 = 0.3 mm; outer radius of the first aperture, b_1 = 0.4 mm; inner radius of the second aperture, a_2 = 0.55 mm; outer radius of the second aperture, b_2 = 0.65 mm.

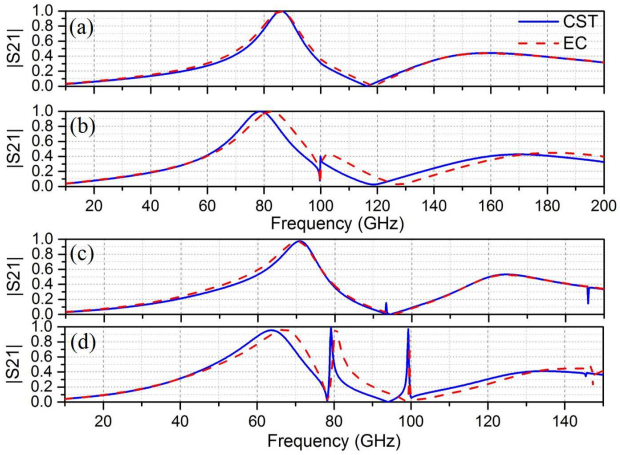


Fig. 3. Transmission coefficient versus frequency under normal incidence for a free-standing FSS with the following geometrical parameters: x-period, d_x = 1.5 mm; y-period, d_y = 3 mm; inner radius of the first aperture, a_1 = 0.3 mm; outer radius of the second aperture, b_1 = 0.4 mm; inner radius of the first aperture, a_2 = 0.55 mm; outer radius of the second aperture, b_2 = 0.65 mm. (a) Free-standing (linear polarization along d_y). (b) Free-standing (linear polarization along d_x). (c) Dielectric backed (linear polarization along d_y). (d) Dielectric backed (linear polarization along d_x).

conditions delimited by the internal and external radii of each aperture. Due to the periodic nature of the structure, the field at the aperture can be expanded in terms of Floquet modes. In fact, under normal incidence, the periodic problem can be reduced to an aperture-discontinuity problem inside a rectangular waveguide with a combination of vertical/horizontal electric/magnetic walls depending on the polarization of the impinging plane wave [9].

The equivalent circuit shown in Fig. 1 (b) consists of an input and output transmission lines whose characteristic admittance, Y_{00}^0, is determined by the medium (superscript 0 denotes air medium for the free-standing case) and the impinging harmonic h (usually TE/TM$_{00}$). The mutual admittance Y_{12} accounts for the interaction between the inner and outer apertures whereas the self-admittances Y_{11} and Y_{22} account for the effect of each individual isolated aperture. These admittances can be evaluated as follows:

$$Y_{ij} = \sum_h \frac{N_{i,h}^* N_{j,h}}{N_{i,00}^* N_{j,00}} \left[Y_{in,h}^{(L)} + Y_{in,h}^{(R)} \right] \qquad (1)$$

where $(i,j) = 1,2$, $N_{i/j,h}$ are the turn ratios accounting for the coupling of the h harmonic and $Y_{in,h}^{(L)}$ and $Y_{in,h}^{(R)}$ are the input admittances seen to the right and left of the discontinuity for each harmonic h. The explicit formulation for these quantities can be found in [5]–[7]. Finally, the scattering parameters can be readily evaluated by applying classical network theory [10].

III. Results

In this section the equivalent circuit model shown in the previous section is applied to different double annular aperture cases. Figure 2 shows the transmission coefficient for a square unit cell ($d_x = d_y$) free-standing structure. As it can be seen the agreement between the full wave simulation (done with the commercial tool CST^{TM} Microwave Studio) [11] and the equivalent circuit (EC) results is very good. The unit cell boundary conditions available for the frequency domain solver are used for emulating an infinite periodic structure with plane wave excitation. The metal sheet used in the simulation was made of perfect electric conductor material and with infinitesimal thickness. In this way, a double pass-band FSS with a deep null between transmission bands can easily be designed with a very low computational cost.

Figure 3 shows results for rectangular unit cells ($d_x \neq d_y$) for free-standing [panels (a) and (b)] and dielectric backed solutions [panels (c) and (d)]. In both cases, the agreement is very good for the case where the impinging wave is linearly polarized parallel to the short period (d_x) and qualitatively good for the opposite case (electric field parallel to the long period, d_y). Although the double band feature vanishes for the rectangular case, this kind of solutions could be useful for applications where polarization selection is important. As it can be seen, the transmission maximum appears at different frequencies depending on the polarization of the impinging wave. In addition, for practical applications, the presence of a dielectric slab as supporting platform should be taken into account [Fig. 3 (c) and (d)]. A dielectric slab of height h_s = 0.5 mm and relative permittivity ε_r = 2.1 was chosen. The obtained equivalent circuit and full wave simulation results are again in good agreement.

IV. Conclusion

In this work an equivalent circuit model has been proposed to characterize double annular apertures on metallic planar screens. Different examples have been tested showing a very good agreement. Therefore, this equivalent circuit can be used as an initial design tool for relatively complex structures such as the one presented. The computational effort required by this approach is negligible in comparison with full wave based simulation tools leading to a great computational time

saving when dealing with this kind of problems. The presented structure can find application as double band space filter for radar applications and if scaled to higher frequencies can be used as a sensing platform for THz applications.

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