

A STUDY OF DIELESS DRAWING AND ISOTHERMAL FORMING

by

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SUMMARY

The first study in the thesis is devoted to dieless drawing, a process for the reduction of long specimens which does not involve any tools actually contacting the specimen. A localised weakness is produced in the specimen by heating and this is moved along the specimen at one speed while simultaneously the end of the specimen is drawn away at another speed to cause the reduction. Earlier work on the process is reviewed and then three different theoretical analyses of it are presented. The first is an analytical technique based on a concept of equalization of work rates at cross-sections through the specimen, the second is a one-dimensional slab method developed for simple numerical solution, and the third is a finite element method. This last solution uses a method which minimises the work rate through the specimen and uses a penalty function to ensure incompressibility. The work is concentrated in this thesis on Ti6Al4V, which is a common alloy of titanium.

The second study of the thesis is concerned with isothermal forming which is a high temperature slow speed forging method. Experiments are described with cylindrical compression, ring tests and profiled specimens mainly composed of Ti6Al4V but also of alloy steel. Very high compressions and dimensional accuracy are achieved in one operation with this process. The finite element method is also applied

to this process and the method has been developed to consider very high compressions, a variety of geometries and, in particular, to include a new method of determining stress distributions through the deforming specimen.

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PREFACE

The two processes studied in this thesis form a part of on-going research at Imperial College into processes which can reduce metal wastage and increase manufacturing efficiency. The two processes are unified because they are most likely to be applied to materials which are difficult to form and which also exhibit strain rate sensitivity. To overcome the forming difficulties, higher temperatures than are usual in metal forming are applied to the material, and in these two processes the method of induction heating is normally adopted. In isothermal forming the strain rate sensitivity is utilised by using low crosshead speeds in the forging. This results in a lowering of the total loads that are required and thus enable very large forgings to be produced (see Fix(1)). In dieless drawing, the stability of the process is entirely dependent on the existence of some strain rate sensitivity in the material (see Turner(2)). Another feature which unifies the two processes is that they have both received most attention in their application to titanium aerospace alloys. These alloys are difficult to form by conventional methods and the expense of any wastage of material makes large scale machining unattractive.

The main emphasis in this thesis is on the theoretical modelling of the processes. In the case of dieless drawing, there is a need to understand the underlying requirements

of the material and the process conditions which enable successful drawing. For isothermal forming, the modelling of the process will have its greatest benefit if it becomes possible to design all the forging arrangements, such as the shape of the starting billet and the dies. One of the main techniques used in the thesis is the finite element method which, while it is routinely applied to elastic stress analyses for example, has yet to be widely applied to the study of metal forming. This is because the length of computing times required for the non-linear types of analysis needed for metal forming can often appear to be prohibitive. The formulation of the finite element method adopted in this thesis is one in which computing times are kept to a minimum.

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CHAPTER 1

INTRODUCTION TO DIELESS DRAWING

- 1.1 A Description of the Process
- 1.2 Review of Previous Experimental Work
- 1.3 Previous Theoretical Analyses
- 1.4 The Theoretical Prediction of the Temperature
Distribution

Chapter 1 - INTRODUCTION TO DIELESS DRAWING

1.1 A Description of the Process

Dieless drawing is an unusual process and the exact mechanisms of its working are difficult to visualise. Essentially it consists in creating a region of weakness in a long prismatic specimen and subjecting it to tensile loading. As the specimen necks in the region of weakness, the region is moved along the specimen so that necking ceases where it was formerly occurring and starts elsewhere. If this is done continuously, a controlled reduction in cross-section can be obtained along the specimen. The region of weakness is created by localised heating and the loading is provided by a moving crosshead. Figure 1.1 is a diagram of such an arrangement. As can be seen, there are many unusual features of this process but, in particular, there are no tools actually touching the deforming material and thus there is no friction involved, and the range of temperatures and strain rates encountered during steady state operation of the process is large.

The number of variables involved in the process is considerable. Turner(2) lists fifteen variables which he thought were of importance in his experiments. These variables and some others can be grouped into the following categories.

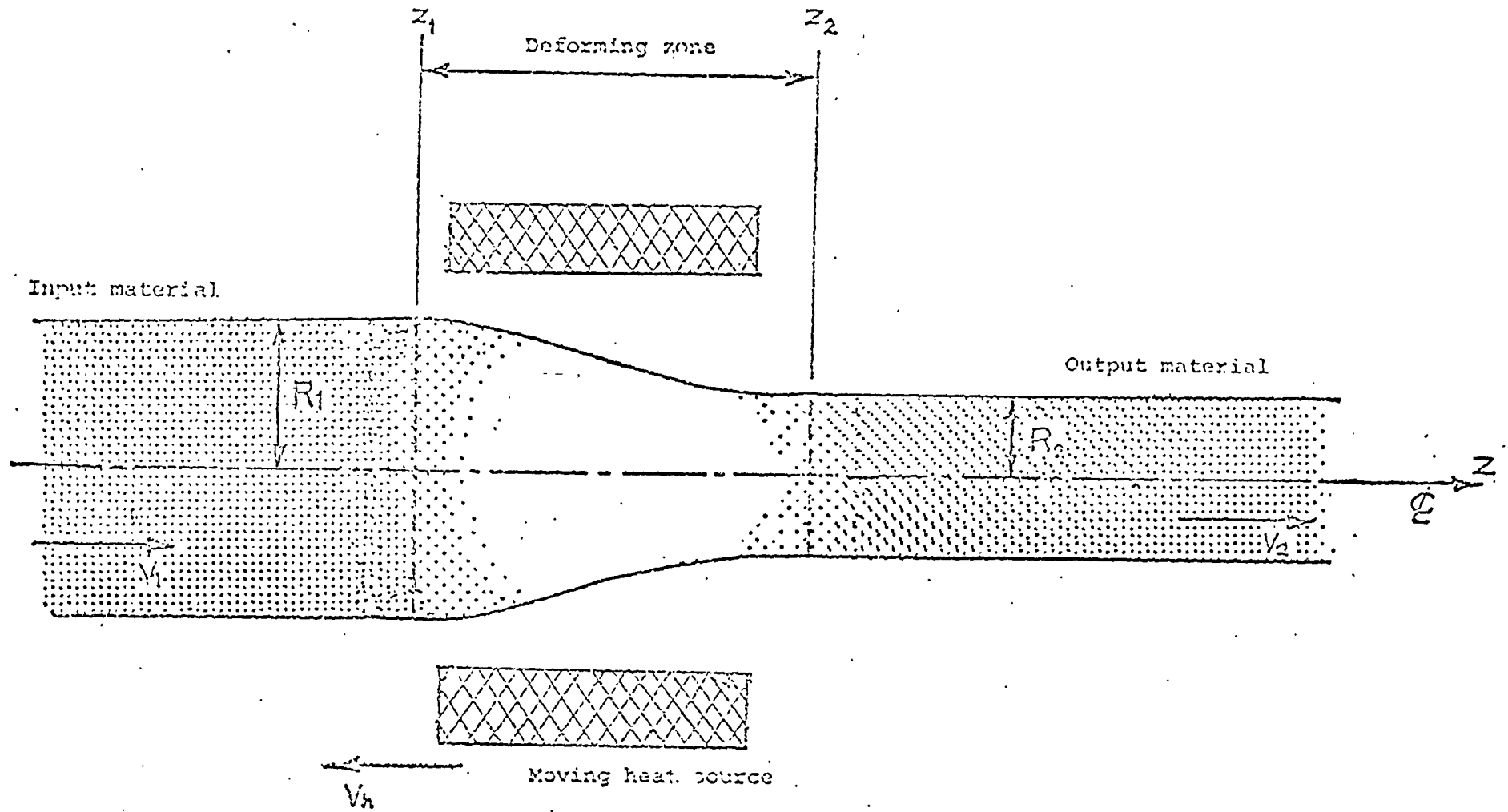


Figure 1.1 Diagram of Dieless Drawing showing notation

- (i) Method of application of tension.

The tensile load can be applied in several ways, for example by a constant speed crosshead or by a machine that applies constant load. Large scale continuous production would demand feed and take up rollers to apply the tension to very long billets.

- (ii) Diameter of input material.

- (iii) Total reduction achieved.

- (iv) Speed of crosshead movement.

The last two factors determine the speed of the heating source because they are linked by the requirement of the material to maintain constant volume. Using the nomenclature of figure 1.1 this connection can be written

$$\frac{R_2}{R_1} = \sqrt{\frac{v_h + v_1}{v_h + v_2}} \quad (1.1)$$

- (v) Material properties of the specimen.

Almost every material property is relevant to the process. The relationships between stress, strain, temperature and strain rate throughout a large range of conditions are involved. Whether the material will develop an oxide scale or otherwise react with the atmosphere must be considered as must the specimen's microstructure, grain size and phase transformations

under the various conditions of temperature and strain. The heat conduction, convection and radiation properties are important, as are the electrical properties if electrical heating is used.

(vi) The temperature distribution throughout the specimen.

This depends not only on all the above-mentioned factors but also on the exact nature of the heating and any cooling used. If induction heating is used (a method which is particularly suitable for dieless drawing) the exact design of the coil, its length and shape and the frequency and strength of the current are all important factors. Some sort of cooling of the exit material may also be required.

With this large array of factors influencing the process, it is necessary first of all to decide that some will have to be ignored in any analysis. From that stage, approaches might be developed which will allow some idea of the significance of the remaining factors. In order to determine this significance, there is no substitute for a simple analytical solution, since such a solution will make the influence of the variables easily discernible. Such a solution would be valuable in selecting the best materials, temperature profiles, machinery and possible reductions, a selection which is critical if the process is to achieve much application in industry. With these problems in mind, a simple analytical solution was sought and is set out in Chapter 2.

A large range of approximations is required to achieve the analytical solution; so this solution may have to be confirmed by a more accurate solution in order to obtain a better picture of the process before attempting its application. Two other solutions were thus attempted, and in Chapter 3 the 'numerical slab method' and in Chapter 4 a finite element solution are proposed.

1.2 Review of Previous Experimental Work

While there is not a large body of work concerning dieless drawing, those papers that there are cover a wide range of interests. Both experimental and theoretical studies have been carried out by Alexander and Turner(4), Weiss and Kott(5), Sekiguchi, Kobatake and Osakada(3) and R.H. Johnson(6). A very extensive description of experiments and theoretical studies are included in the thesis of Turner(2), whose equipment and theoretical work were made available to the present author.

The experiments emphasize that the process can achieve many things which are difficult to perform in more conventional metal forming processes. For example, the cross-section of the output can be made to vary in size and the shape of the output cross-section is dependent on but different from the shape of the input cross-section (except in the case of axisymmetric specimens). It is possible to draw tube stock into different sized tube and to draw tapered products.

High reductions are possible (up to 70-80% depending on the specimen material) and the process may have economic advantages especially for materials that are difficult to work and for which dies present problems. It may be particularly attractive for short runs because no special tools would have to be prepared for every small change in product size.

The process, however, does present several difficulties. The process is difficult to control with metals with a low strain rate sensitivity, but as the strain rate sensitivity increases the process becomes easier to use. With superplastic materials, a very high success rate is achieved. The oxidation of the outside surface can be a problem, and if the oxide scale is thick enough it prevents the cooling of the metal and will cause the specimen to break. Because the process is carried out with a significant axial tensile stress, there is a likelihood of cracking on the surface or the opening up of voids and internal cracking around inclusions.

Some of the difficulties of the process are metallurgical problems and mean that certain materials may not be suitable for use. However problems may be reduced by correct selection of process parameters. It is in this connection that theoretical analyses of the process are important.

1.3 Previous Theoretical Analyses

A very interesting analysis is presented by Cornfield (7) in which the stability of the process can be determined. The analysis is essentially a one-dimensional approach and uses a strain rate/temperature/stress relationship which can be given as

$$\dot{\epsilon}_z = C \exp(-Q/T) \sigma_z^{\frac{1}{n}} \quad (1.2)$$

where C and Q are constants, T is the temperature, and $\dot{\epsilon}_z$ and σ_z are axial strain rate and stress respectively, and n is the strain rate sensitivity index (the symbols are also described in the List of Symbols included at the end of the thesis). The conditions of equilibrium and constant volume and the one-dimensional heat conduction equation are also used. The analysis starts with an assumed deforming zone outside profile and then, in order to make the problem linear, considers small perturbations of temperature and velocity at a series of cross-sections through the deforming zone. The analysis concludes that if these perturbations are inclined to reduce with time and process is stable.

The paper expresses the time derivatives of the perturbations as a linear function of the actual perturbations (ignoring a large number of second order terms). This can be expressed as follows: if the number of elements into which the deforming zone is divided is N, and p_j is one of the perturbations, then

$$\dot{P}_I = Q_{IJ}P_J \quad (1.3)$$

where the summation convention is carried out over all $2N$ perturbations, the dot indicates derivation by time and Q_{IJ} is an element of $2N$ by $2N$ matrix. If the eigen values of the matrix Q are negative, then equation 1.3 has solutions which are asymptotically stable.

Using particular heating conditions and coil speeds, v_h , the analysis was carried out for a wide range of strain rate sensitivity index, n , and reductions. In some combinations instability was indicated while in others the process appeared to be stable. Thus the graphs of n versus reduction can be divided into two areas, one where the process is stable and the other where it is not. These graphs suggest that the larger reductions are most easily obtainable with

- (i) slower forming speeds;
- (ii) shorter heating lengths;
- (iii) high values of n . (As n falls below 0.5, the maximum possible reduction falls very quickly.)

The paper only gives three examples of the calculations, since the technique requires a large number of trial calculations to obtain a single graph of n versus feasible reduction.

Two different solutions of dieless drawing are given by Turner(2) and Alexander and Turner(4). The third solution mentioned in these references is really a method of analysing

the experimental results in order to derive the changes of a single unknown parameter through the deforming zone of a particular experiment. The parameters that were estimated by this one-dimensional or 'plane sections remain plane' technique were temperature and strain rate sensitivity index, n .

The 'simple tension' solution is quick and simple and provides a reasonable description of the profile of the outside surface of the deforming zone. The approach is not purely one-dimensional since the region being analysed is broken into numerous concentric ring elements. The average axial strain rate of these elements is considered to be a function of the temperature of the element and the mean stress acting on the surface of the element on the side towards the input material. The strain rate thus determined is used to find the axial coordinates of so far undetermined nodes, while their radial coordinates are determined by the constant volume requirements. An iterative solution is required because the stress and temperature in an element are dependent on the geometry which is itself the primary unknown. Figure 1.2 compares various strain rate distributions from these theories and from experiments.

Although references (2) and (4) only consider axial stresses and strains with the simple tension method, the method could potentially involve other stress or strain rate components and may even be an alternative to the finite

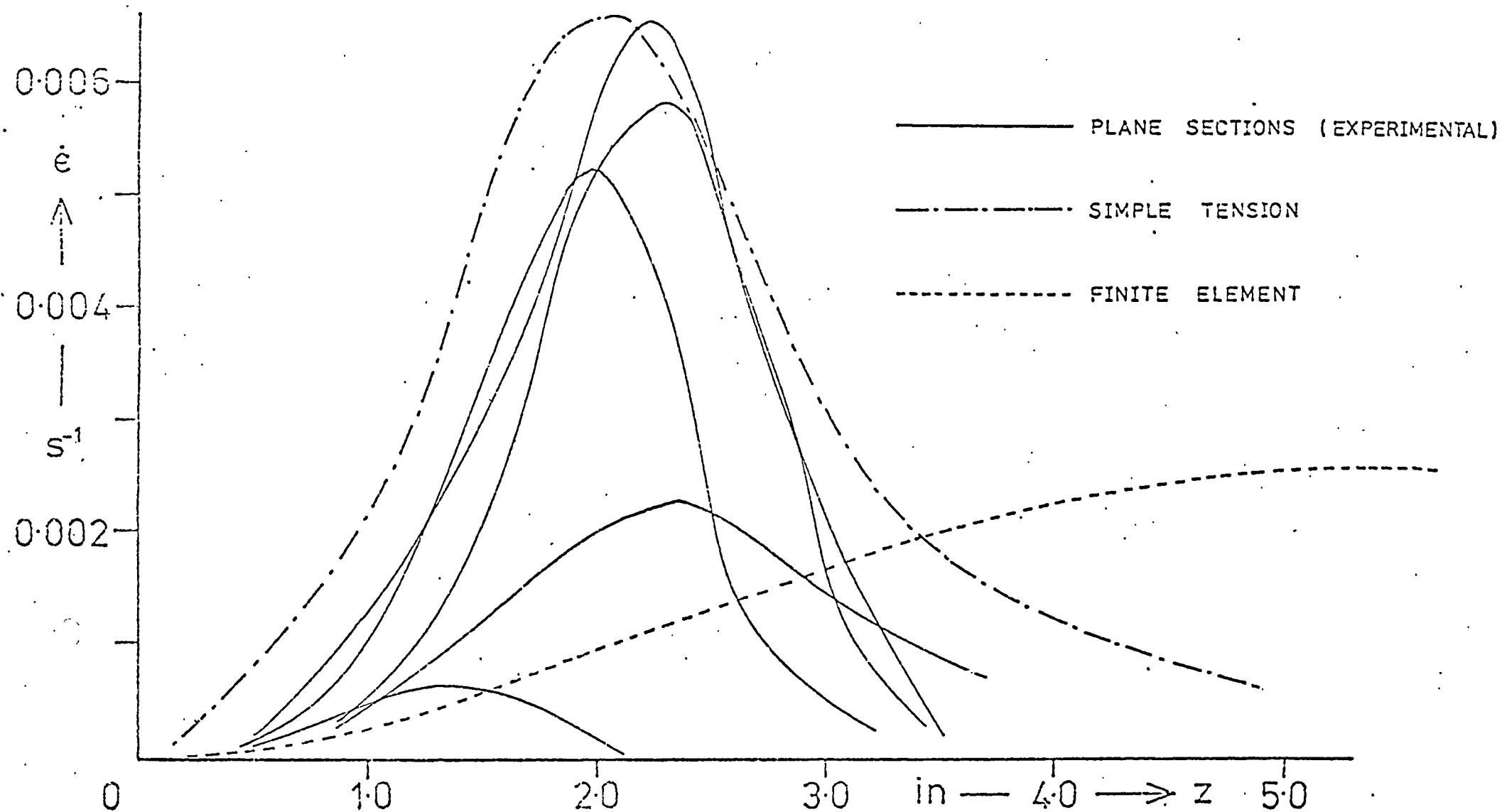


Figure 1.2 Figure 42 from Turner (2). Compares strain rate distributions from various sources for the deforming zone in dieless drawing.

element method for incompressible materials. The chief problem with the method is that the boundary conditions are given by load and not by velocity considerations. In most metal forming processes, including dieless drawing, the process is determined by velocity boundaries and the loading conditions are dependent on these. In this thesis a slightly simpler one-dimensional, plane section remain plane or slab approach is presented for which velocity boundary conditions can be used. Velocity conditions could probably be applied in the same way to the simple tension solution as they are to the numerical slab method of Chapter 3.

The second solution provided by references (2) and (4) is a finite element solution. Discussion of this solution is left until Section 4.5.4.

1.4 The Theoretical Prediction of the Temperature Distribution

The temperature distribution inside the work piece was calculated by both Cornfield(7) and Turner(2). Cornfield's solution is essentially a simple, one-dimensional heat conduction solution which presumably assumes a constant rate of heat generation per unit volume in the region of the heating coil. Turner uses similar assumptions, but applied to an axisymmetric, finite element solution which allows for a particular outside profile of the work piece, Turner considers the movement of the billet but determines that it was of secondary importance.

In the case of induction heating, the range of factors influencing the temperature distribution inside the material can be judged from Lozinskii(8) and Brown et al.(9). The main factors include the position and geometry of the heating coil, frequency and magnitude of the electric current, and the permittivity and resistivity of the metal. These last two factors are strongly dependent on temperature, especially in the case of a material with a Curie point such as steel or nickel. The cooling of the body is also important and the heat is carried away by radiation, conduction, physical movement of the billet and convection. Under the last of these items must be included any additional forced cooling device such as the air blowers used by Turner(2) and Sekiguchi et al.(3). Obviously a highly accurate theoretical solution to the temperature distribution for an axisymmetric work piece in dieless drawing represents a complex non-linear problem, which would, for example, have to be tackled by an extensive finite element solution. Since such a solution would be beyond the scope of this work, temperature distributions were based either on measurements made with an optical pyrometer during actual experiments for simple cases, or on Turner's solution for two-dimensional cases. Turner's solution was accepted as a reasonable approximation for, despite the reservations discussed above, it does take into account the factors of primary importance.

CHAPTER 2

AN ANALYTICAL APPROACH TO DIELESS DRAWING

- 2.1 Basis of an Approach to an Analytical Solution
- 2.2 The Analysis
- 2.3 Work Rate Computations
- 2.4 Direct Application of the Equations

Chapter 2 - AN ANALYTICAL APPROACH TO DIELESS DRAWING

2.1 Basis of an Approach to an Analytical Solution

As will be noted from the discussion presented in Chapter 1, dieless drawing is a difficult process to analyse on a theoretical basis. It appears that the process cannot be easily treated by the methods of Sachs (the slab approach), by slip line field theory or limit analysis. The approach presented in this Chapter introduces another idea which might be described as 'equalization of work rates'. The problem is considered from the outset to be a dynamic problem but is simplified by considering only one dimension. The work rate at each planar cross-section is determined from both geometry/velocity considerations and also from the equilibrium of forces acting on the section. These two work rates must be equal, and this equality is used to eliminate certain unknowns.

As has been mentioned in Section 1.1, there is a large number of variables which have an effect on the dieless drawing process. The analytical solution presented below is an attempt to extract the most significant of these variables and then arrive at some evaluation of their effect. In order to achieve a simple result, a number of assumptions are involved and several unusual calculations must be made. The main assumptions are:

(a) The temperature profile is linear, rising to a maximum temperature, then decreasing as illustrated in figure 2.1.

(b) Plane sections remain plane and thus the only contribution to the work of plastic deformation is the axial stress and strain, that is, the approach is entirely one-dimensional.

(c) The stress required to cause plastic deformation is a function of strain rate and temperature only. Strain hardening should be very small at the high temperatures of dieless drawing (see reference (10) and there is more evidence about this in Chapter 6). If the initial material is annealed, the holding time at elevated temperature should in most cases have little effect(11) as should the heating rates involved (12). The function that will be assumed as the relationship between stress and strain rate will be a simple one used only in this analytical solution:

$$\sigma_z = K(T_c - T)\dot{\epsilon}_z^n \quad (2.1)$$

where K , T_c are constants, n is a strain rate sensitivity index. This kind of equation is suggested by graphs in Fix(1) but they are probably an approximation since the data of Lee and Backofen(13) are probably more accurate and suggests a much more complicated relationship.

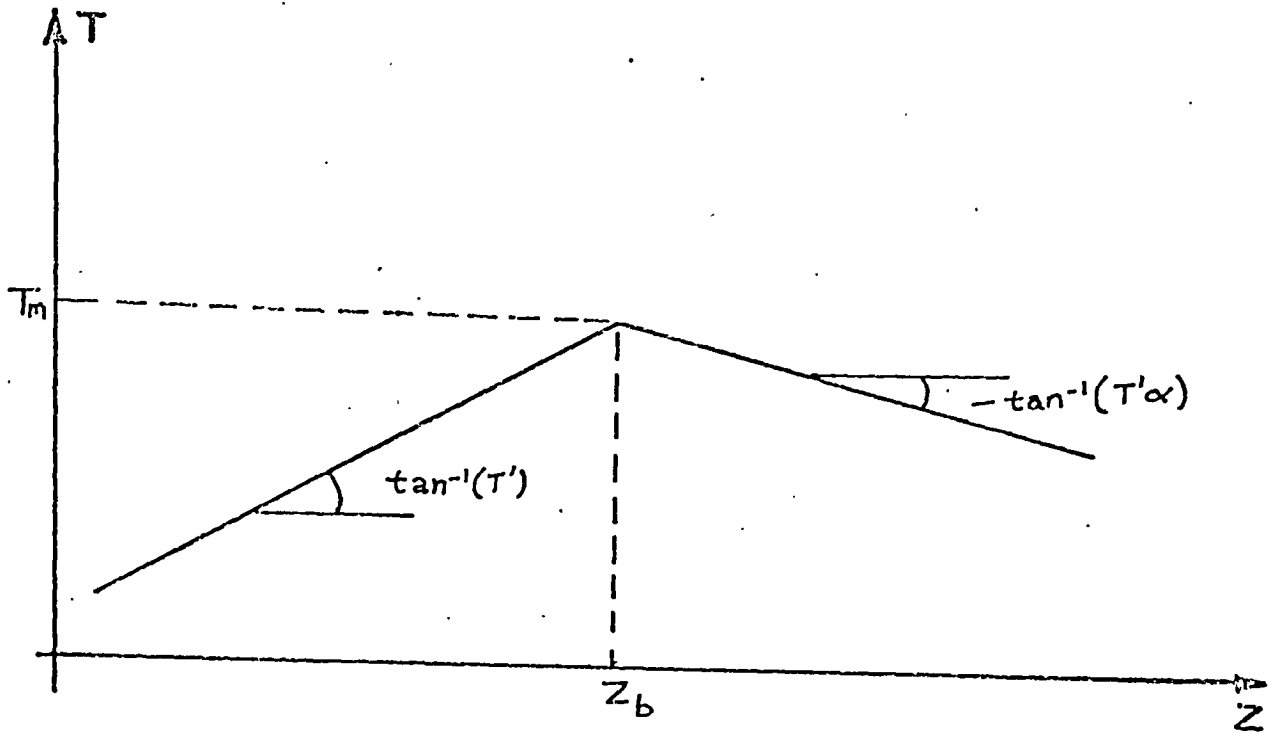


Figure 2.1 Notation for Temperature Distribution for Chapter 2.

(d) The normal assumptions of plasticity theory are used as presented, for example, by Hill(14). In particular, the material maintains a constant volume, there is an equilibrium of forces and the incremental plastic work per unit volume caused by the deformation is given, in tensor notation, by

$$dW = \sigma_{ij} d\epsilon_{ij} \quad (2.2)$$

Because of the low stresses and high temperatures, elastic contributions may be ignored.

(e) The process has achieved steady state.

The first three of these assumptions greatly simplify the real situation, but they are required if a simple analytical solution is to be achieved. These assumptions are steadily reduced in the numerical computer solutions described in later Chapters.

The basic notation of this Chapter conforms to the List of Symbols at the end of the thesis, but a special set of abbreviations is introduced to simplify the equations later.

2.2 The Analysis

Using the assumption that plane sections remain plane an increment of work of deformation per unit volume becomes

$$dW = \sigma_z d\epsilon_z \quad (2.3)$$

Now

$$\epsilon_z = \frac{\partial u_z}{\partial z} \quad (2.4)$$

and using the assumption of constant volume

$$v = v_1 R_1^2 / r^2 \quad (\text{the coil is considered fixed, } v_h = 0)$$

Thus

$$\frac{\partial v}{\partial z} = \frac{\partial^2 u_z}{\partial t \partial z} = - \frac{v_1 R_1^2}{r^3} 2 \frac{\partial r}{\partial z} = \dot{\epsilon}_z \quad (2.5)$$

which can be written

$$\dot{\epsilon}_z = \frac{2v_1 R_1^2}{r^3} (-r'), \text{ where } r' = \frac{\partial r}{\partial z} \quad (2.6)$$

Now from equation 2.1

$$\sigma_z = f(T) \dot{\epsilon}_z^n \quad (2.7)$$

or using 2.6

$$\sigma_z = f(T) \left(\frac{2v_1 R_1^2 (-r')}{r^3} \right)^n \quad (2.8)$$

Now equation 2.3 can be written in terms of work rate per unit volume

$$\dot{W} = \sigma_z \dot{\epsilon}_z \quad (2.9)$$

and equations 2.6, 2.8 and 2.9 give

$$\dot{W} = f(T) \left(\frac{2v_1 R_1^2 (-r')}{r^3} \right)^{n+1} \quad (2.10)$$

This result for the work rate per unit volume at the cross-section of radius r and temperature T , uses mainly the constant

volume assumption. As it stands, equation (2.10) does not guarantee equilibrium of forces. The following equations use the equilibrium equation as their starting point since the load, L , must be constant throughout the specimen, so that at every cross-section

$$\sigma_z = L/\pi r^2 \quad (2.11)$$

$$\text{Using } \dot{\epsilon}_z = (\sigma_z/f(T))^{\frac{1}{n}} \quad (2.12)$$

from equation (2.7) and substituting in equation (2.9)

$$\dot{W} = \sigma_z \left(\frac{\sigma_z}{f(T)} \right)^{\frac{1}{n}}$$

$$\text{thus } \dot{W} = \left(\frac{L}{\pi r^2} \right)^{1+\frac{1}{n}} \left(\frac{1}{f(T)} \right)^{\frac{1}{n}} \quad (2.13)$$

Now at any cross-section of the specimen, the work rate per unit volume given by equation (2.10) must be equal to that given by equation (2.13). The equivalence of these two equations will thus guarantee that all the assumptions so far discussed are simultaneously satisfied.

$$\text{Thus } \left(\frac{L}{\pi r^2} \right)^{1+\frac{1}{n}} \left(\frac{1}{f(T)} \right)^{\frac{1}{n}} = f(T) \left(\frac{2v_1 R_1^2 (-r')}{r^3} \right)^{n+1}$$

$$\text{Thus } \left(\frac{L}{\pi r^2 f(T)} \right)^{\frac{1}{n}} = \frac{2v_1 R_1^2}{r^3} (-r') \quad (2.14)$$

At this point in the solution the linear temperature profile is introduced and $f(T)$ replaced by the linear function of equation (2.1). The profile illustrated on figure 2.1 can be written

$$T = T_m - T' |z_b - z| - \alpha T' |z - z_b| \quad (2.15)$$

where T' and α are positive and the notation $|X|$ indicates that $|X| = X$ when $X \geq 0$ and $|X| = 0$ when $X < 0$. If $f(T)$ is taken to be a linear function of T such that

$$\begin{aligned} f(T) &= K(T_c - T) \\ &= K(T_c - T_m + T' |z_b - z| + T' |z - z_b|) \end{aligned} \quad (2.16)$$

If

$$\tau = \frac{1}{v_l R_1^2} \left(\frac{L}{\pi K} \right)^{1/n}$$

then equation 2.14 can be rewritten

$$\frac{\tau dz}{(T_c - T_m + T' |z_b - z| + T' |z - z_b|)^{1/n}} = -2r^{2/n-3} dr$$

Integration can be carried out in two stages.

For $z \leq z_b$

$$\frac{\tau}{1-\frac{1}{n}} \left[\frac{(T_c - T_m + T'(z_b - z))^{1-1/n}}{T'} \right]_{z_1}^z = \frac{-2}{\frac{2}{n}-2} \left[r^{2/n-2} \right]_{R_1}^r$$

If $m = \frac{1}{n} - 1$ (since $0 < n < 1$, m is greater than unity) the above equation can be written

$$r^{2m} = R_1^{2m} + \frac{\tau}{T'} \{ (T_c - T_m + T'(z_b - z_1))^{-m} - (T_c - T_m + T'(z_b - z))^{-m} \} \quad (2.17)$$

If a further notation is introduced, namely

$$\Delta = (T_c - T_m + T'|z_b - z| + \alpha T'|z - z_b|)^{-m}$$

where $\Delta_1 = \Delta$ at z_1 ; $\Delta_b = \Delta$ at z_b , etc., then equation (2.17) can be reduced to

$$r^{2m} = R_1^{2m} + \frac{\tau}{T'} (\Delta_1 - \Delta) \text{ for } z \leq z_b \quad (2.18)$$

For $z > z_b$ the integration produces

$$r^{2m} = R_1^{2m} + \frac{\tau}{T'} (\Delta_1 - \Delta_b - \frac{1}{\alpha}(\Delta_b - \Delta)) \quad (2.19)$$

R_2 can thus be found from

$$R_2^{2m} = R_1^{2m} + \frac{\tau}{T'} (\Delta_1 - \Delta_b - \frac{1}{\alpha}(\Delta_b - \Delta_2)) \quad (2.20)$$

Now Δ_1 and Δ_2 are unknown because z_1 and z_2 are unknown. At present in the literature there is no generally accepted way of estimating the start and finish of deformation in this situation. To make such an estimate, a yield criterion is required, but the plastic deformation is a hot temperature, dynamic flow analogous to creep but occurring at higher strain rates than are normally associated with creep. Creep equations generally do not give lower limits of temperature and

stress below which creep will not occur, nor do isothermal plastic yield equations such as von Mises(14) cover the situation where yield is caused by rising temperature and not because of rising stress. Experimentation into the kind of yielding that occurs in dieless drawing is rare, an exception being the work of Smith and Robinson(12) who plot yield stress versus temperature for various alloys.

There are two approaches to this problem which will enable calculations to be carried out. The first is to note that at z_1 and z_2 the strain rate, $\dot{\epsilon}_z$, must approach zero or at least is much smaller than the strain rate in the middle of the deforming zone. Now, using equations (2.7) and (2.11),

$$\dot{\epsilon}_z = \left(\frac{1}{K(T_c - T)} \right)^{1/n} \left(\frac{L}{\pi r^2} \right)^{1/n}$$

thus
$$\frac{1}{T_c - T} = \dot{\epsilon}_z^n \frac{\pi r^2 K}{L}$$

thus
$$\Delta = \dot{\epsilon}_z^{1-n} \left(\frac{\pi r^2 K}{L} \right)^{1/n-1}$$

Thus if $\dot{\epsilon}_z$ becomes very small, Δ will also become small. Also it is evident that Δ must become smaller as $(T_c - T)$ increases as it does towards the ends of the deforming zone. Thus it may be possible to consider that Δ_1 and Δ_2 are negligible and may be omitted.

The alternative possibility is for z_1 and z_2 to be fixed arbitrarily. For example, it is obvious when a dieless drawing experiment is carried out that the deforming

zone is unlikely to be longer than 400mm in normal practice. Hence z_1 can be set at -200mm, z_b at zero, and z_2 at +200mm, as is done in some examples in Section 2.4.

Taking the case where Δ_1 and Δ_2 are considered to be negligible, equation (2.20) can be re-arranged to produce an expression for the load, L , in terms of the material and process parameters. Using $R_2 = \beta R_1$, and replacing τ with its equivalent, equation (2.20) becomes

$$(\beta R_1)^{2/n-2} = R_1^{2/n-2} - \frac{1}{v_1 R_1^2} \left(\frac{L}{\pi K} \right)^{1/n} \cdot \frac{1}{T'} \cdot (1+1/\alpha) \left(\frac{1}{T_c - T_m} \right)^{1/n-1}$$

thus

$$L = \pi K v_1^n R_1^2 (1-\beta^{2/n-2})^n T'^n (T_c - T_m)^{1-n} \left(\frac{\alpha}{1+\alpha} \right)^n \quad (2.21a)$$

This can be arranged into dimensionless groups

$$1 = \left(\frac{K}{L} R_1^{2-n} (T_c - T_m) v_1^n \right) \times \left(\frac{T' R_1}{T_c - T_m} \right)^n \times \left(\frac{\alpha}{1+\alpha} \right)^n \times (1-\beta^{2/n-2})^n \times \pi \quad (2.21b)$$

In equation (2.21b), it is clear that π acts as a geometrical factor and could be replaced by another constant if the cross-section of the feed material is not found. If Δ_1 and Δ_2 are not considered to be negligible, then (2.21a) becomes

$$L = \pi K v_1^n R_1^2 (1-\beta^{2/n-2})^n \left(\frac{\Delta_b - \Delta_1}{T'} - \frac{\Delta_2 - \Delta_b}{\alpha T'} \right)^{-n} \quad (2.22)$$

2.3 Work Rate Computations

The total mechanical work rate performed on the deforming zone is

$$I = Lv_2 - Lv$$

i.e.

$$I = Lv_1 \frac{1}{\beta^2} - 1 \quad (2.23)$$

This is the same as the volume integral of the plastic work rate

$$I = \int_{z_2}^{z_1} \dot{W} \pi r^2 dz \quad (2.24)$$

Now from equation 2.13 for all z , the work rate per unit volume

$$\begin{aligned} \dot{W} &= \left(\frac{L}{\pi r^2} \right)^{1+1/n} \left(\frac{1}{f(T)} \right)^{1/n} \\ &= \frac{L}{\pi}^{1+1/n} \frac{1}{r^{2+2/n}} \{ K(T_c - T_m + T' |z_b - z| + \alpha T' |z - z_b|) \}^{-1/n} \end{aligned}$$

This can be integrated in the axial direction (the z direction) between any two cross-sections to obtain the plastic work rate as shown in equation (2.24); r can be replaced by use of equations (2.18) and (2.19). The work rate between the input cross-section and the cross-section of maximum temperature (i.e. between z_1 and z_b) can be shown to be given by

$$I_1 = Lv_1 R_1^2 \left\{ (R_1^{2/n-2} + \frac{\tau}{T'} (\Delta_1 - \Delta_b))^{n/(n-1)} - R_1^{-2} \right\}$$

which can be simplified using equation (2.18)

$$I_1 = Lv_1 R_1^2 \left(\frac{1}{R_b^2} - \frac{1}{R_1^2} \right)$$

Similarly I_2 , the work rate between R_b and R_2 can be calculated:

$$\begin{aligned} I_2 &= Lv_1 R_1^2 \left\{ (R_1^{2/n-2} + \frac{\tau}{T^1} (\Delta_1 - \Delta_b) - \frac{\tau}{\alpha T^1} (\Delta_b - \Delta_2))^{n/(n-1)} \right. \\ &\quad \left. - (R_1^{2/n-2} + \frac{\tau}{T^1} (\Delta_1 - \Delta_b))^{n/(n-1)} \right\} \\ &= Lv_1 R_1^2 \left(\frac{1}{R_2^2} - \frac{1}{R_b^2} \right) \end{aligned}$$

Thus the total work rate in deformation is

$$\begin{aligned} I &= I_1 + I_2 \\ &= Lv_1 R_1^2 \left(\frac{1}{R_2^2} - \frac{1}{R_1^2} \right) \\ &= Lv_1 \left(\frac{1}{\beta^2} - 1 \right) \end{aligned}$$

which corresponds to equation (2.23).

It is to be noted that the drawing machine expends more energy than this and that

$$I' = \frac{Lv_1}{\beta^2} = Lv_2 \quad (2.25)$$

The residual work rate (Lv_1) is consumed not in plastic deformation but in moving the deforming zone into the feed material.

2.4 Direct Application of the Equations

The equations outlined above provide several pieces of information. For a particular material (which fixes n , T_c , K) some investigation of the temperature profile (involving T_m , T' , and α) and the process parameters (R_1 , β and v_1) can be made. Alternatively, a search can be made for the most suitable material for a particular set of process and temperature conditions. Some idea can also be gained as to the likely stability of the process as will be outlined below.

There are two important features that must be considered when applying the equations. Firstly, the material parameters (n, T_c, K) can be difficult to estimate from data because no material really behaves in the manner suggested by equation (2.1). In the case of the Titanium 6Al4V which is used for this example, both Lee and Backofen(13) and Wu and Lowrie(15) give graphs of n versus strain rate and temperature. These graphs show variations of n between 0.85 and 0.15 and thus it is only for specific conditions that an accurate estimate for n is obtainable. Because no equation may be wholly satisfactory, more detailed numerical methods of determining material properties (described in Section 3.2) were developed. However, results can be obtained by choosing some representative value of n , in the case of Ti6Al4V it seems reasonable for the temperatures and strain rates involved in the process to choose a value of 0.6 from the graphs in references (13) and (15). The results, as will be shown, are rather

insensitive to the value of T_c , and K has no effect except that the load, L , is directly proportional to it.

The second feature to be cautious about is that the equations give the impression that the variables are independent of each other. In fact, almost all the variables are strongly related to each other. An example of this might be seen if the input velocity, v_1 , is considered. If, for example, v_1 were doubled but β and R_1 remained the same it would not be possible to keep the temperature profile the same (even if the energy input from the heater were increased). Moreover, the strain rates encountered in the deforming zone would be directly affected (see equation (2.6)) and the representative values of the material parameters might have to be re-evaluated.

Figure 2.2 shows a typical result from the equations for the shape of the deforming zone and can be compared to figure 2.3 which is experimental profiles drawn from Turner's work(2). The material parameters, n , T_c and K , have been estimated from the data of Lee and Backofen(13) and Fix(1); the temperature profile and process parameters were estimated from an experiment carried out by the author but are similar to those encountered in Turner's experiments(2). The values used on figure 2.2 are given on table 2.1 which also gives the load result.

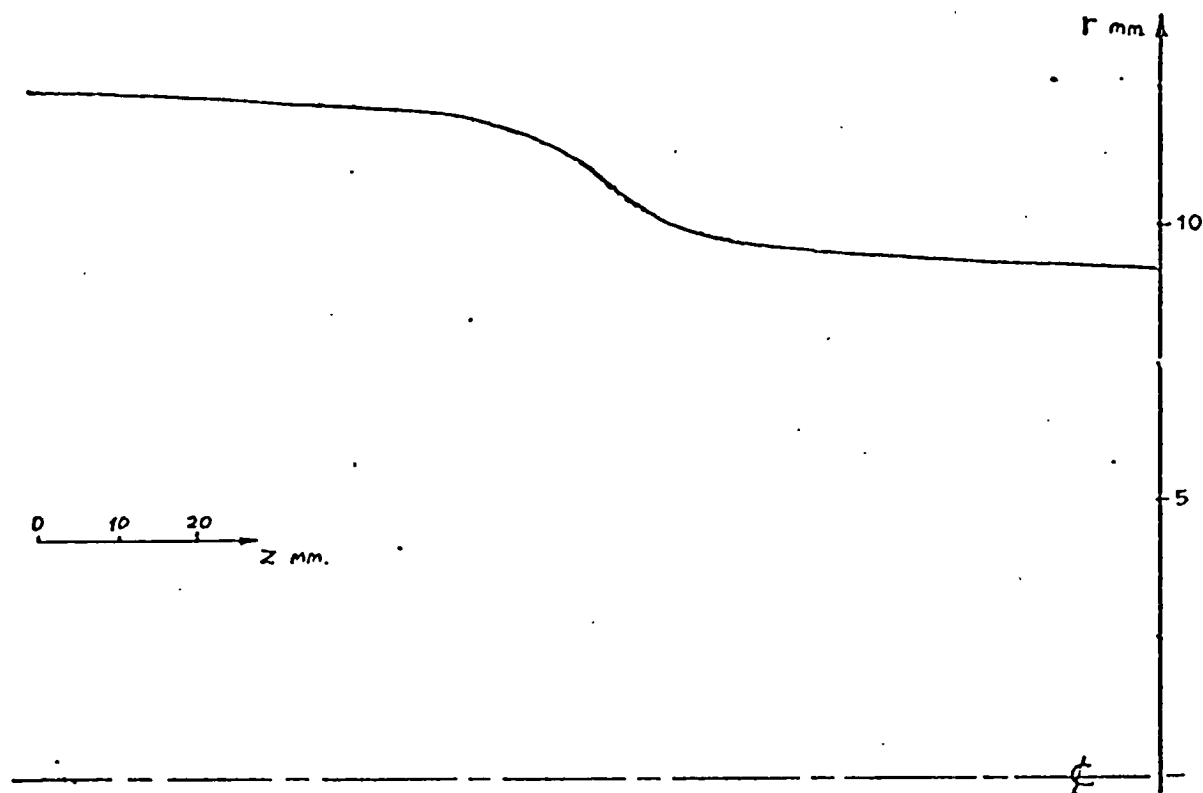


Figure 2.2 Typical Result for the Analytical Prediction of the Outside Profile for Dieless Drawing. Parameters used are listed on Table 2.1
Load Predicted is 15.1 kN.

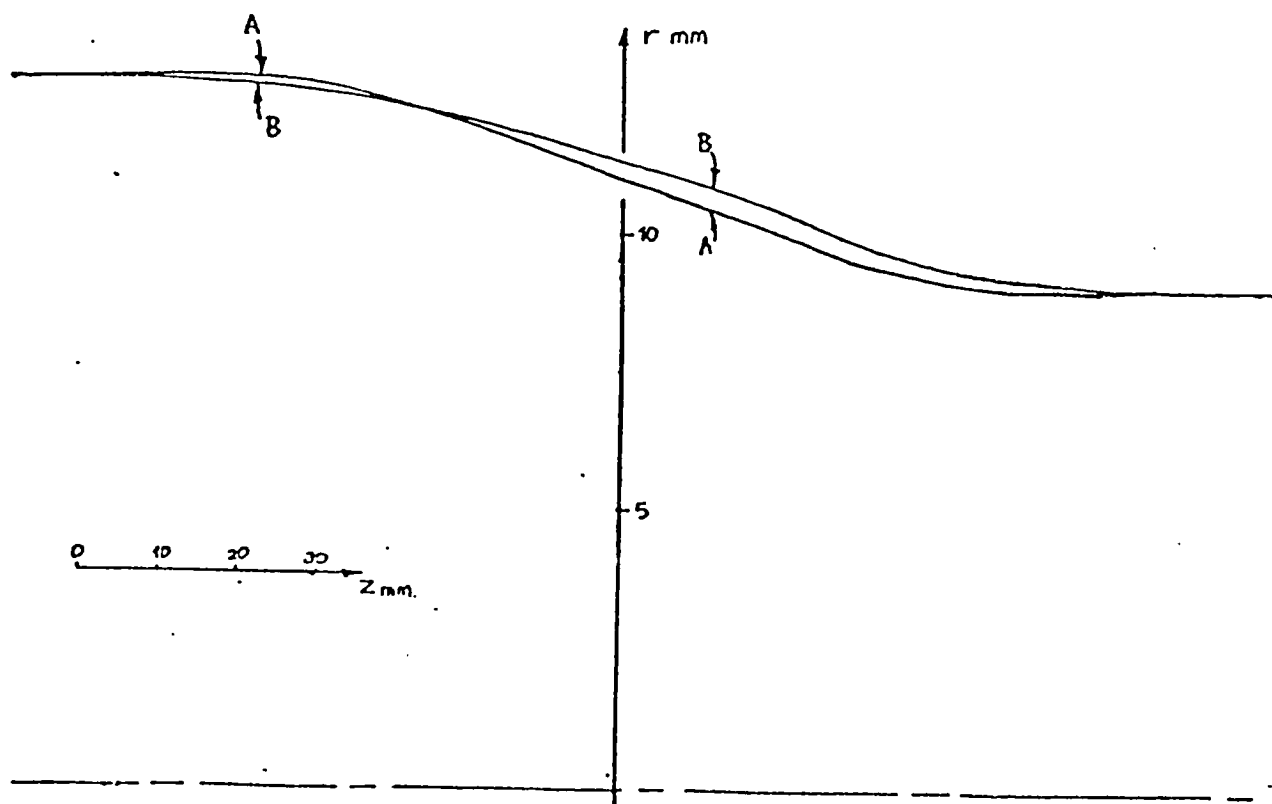


Figure 2.3 Experimental Results for Outside Profile for Dieless Drawing.
From Turner (2).

TABLE 2.1

$n = 0.6$	$T_c = 980^{\circ}\text{C}$	$K = 2.5 \text{ MPa (sec)}^n/^{\circ}\text{C}$
$R_1 = 12.7\text{mm}$	$\beta = .707$ (i.e. 50% reduction in cross-section)	
$v_1 = 1\text{mm/sec}$	$T_m = 950^{\circ}\text{C}$	$T' = 3^{\circ}\text{C/mm}$
$\alpha = -1$ (i.e. input and output temperature gradients are the same)		
$\Delta_1 = \Delta_2 = 0$ i.e. $z_1 = -z_2 = \infty$, $z_b = 0$		
The load calculated for this situation was 15.1 kN (1.52 tons) which is a satisfactory estimate when compared with experimental results.		

Figures 2.4 to 2.7 show results of varying a single one of the parameters while the other parameters are kept at the values given on table 2.1. The parameters not shown are v_1 and K , neither of which have any effect on the profile shape. The load, L , is directly proportional to K and so the results can be calculated from table 2.1. Table 2.2 gives the load for various values of v_1 .

TABLE 2.2

$v_1 = 1.0\text{mm/sec}$	$L = 15.1 \text{ kN}$
10.	60.1
0.1	3.8

With adjustments of several of the parameters from their table 2.1 values, it may be possible to produce a very accurate representation of figure 2.3, the experimental results. However, it must be remembered that the analytical solution is only a first approximation and there is no justification for putting any reliance on the parameters estimated in this way.

Equation (2.25), which gives the total work rate required of the machinery, is analogous to work rate equations in wire drawing or extrusion. However, the energy requirements of the machinery are very low because the loads are very low and the efficiency of the process depends almost entirely on the efficiency of the heating. Some of the difficulties in calculating heating efficiency can be gauged from Section 1.4 and thus theoretical efficiency comparisons with other forming processes are complicated.

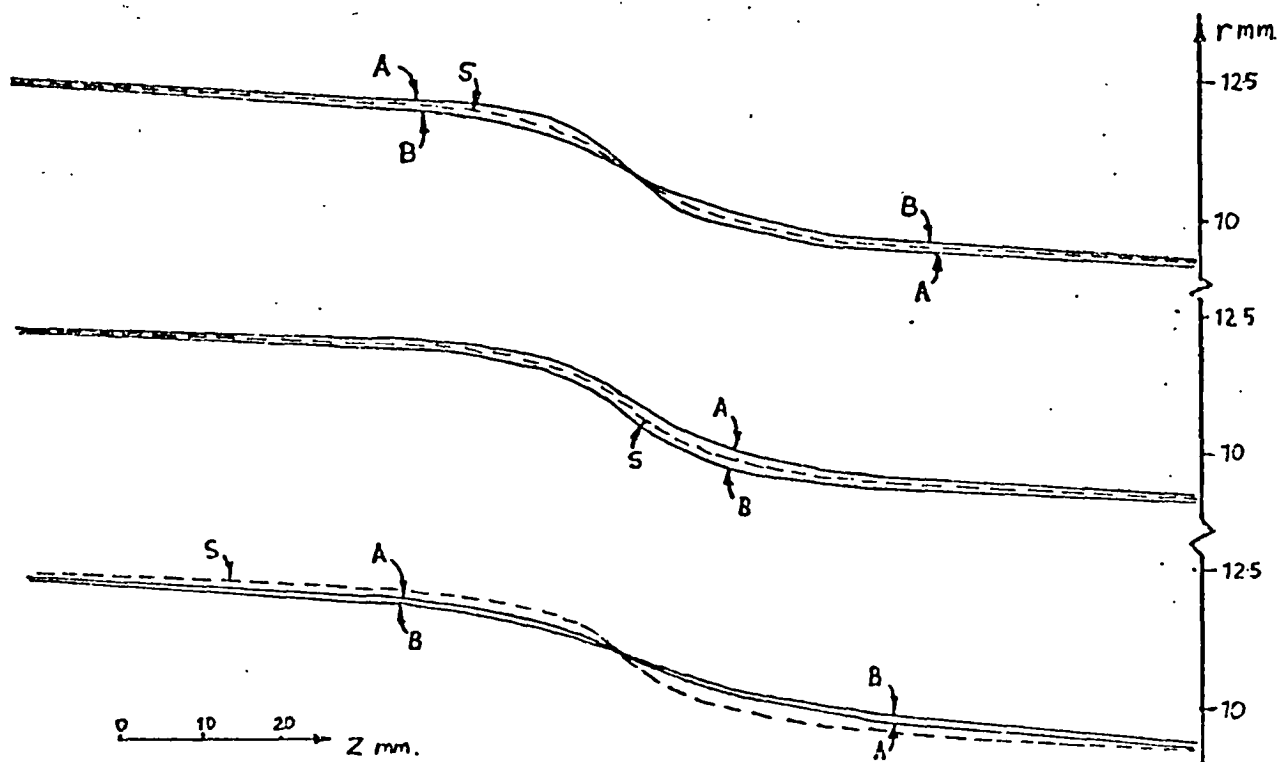


Figure 2.4 Outside Profiles for Different Temperature Distributions.

S is standard curve as for figure 2.2, Table 2.1

($T' = 3.0$, $\alpha = -1.0$, $T_m = 950^\circ\text{C}$, Load = 15.1 kN.)

TOP	Varyiny T'	A:	$T' = 4.0^\circ\text{C/mm}$	Load = 17.6 kN
		B:	$T' = 2.0$	Load = 12.2
CENTRE	Varying α	A:	$\alpha = -0.8$	Load = 14.2 kN
		B:	$\alpha = -1.2$	Load = 15.8
BOTTOM	Varying T_m	A:	$T_m = 925^\circ\text{C}$	Load = 20.1 kN
		B:	$T_m = 900^\circ\text{C}$	Load = 24.2

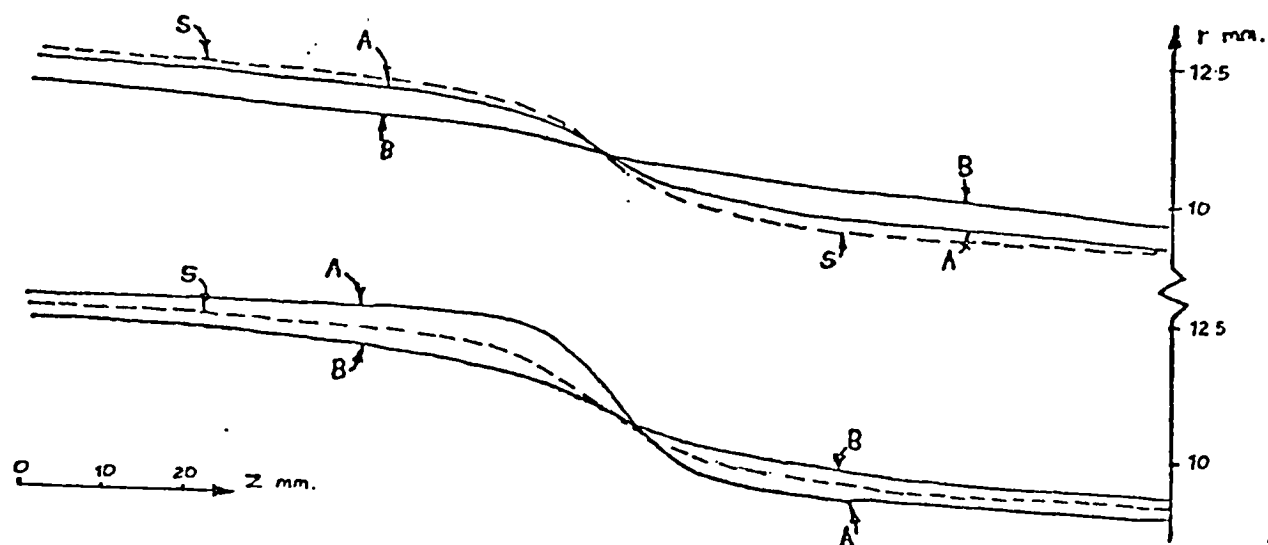


Figure 2.5 Outside Profiles for Different Material Properties.

S is standard curve as for figure 2.2, Table 2.1

($n = 0.6$, $T_c = 980^\circ\text{C}$, Load = 15.1 kN.)

TOP	Varying T_c	A:	$T_c = 1000^\circ\text{C}$	Load = 19.2 kN
		B:	$T_c = 1200$	Load = 46.0
BOTTOM	Varying n	A:	$n = 0.4$	Load = 38.7 kN
		B:	$n = 0.8$	Load = 5.26

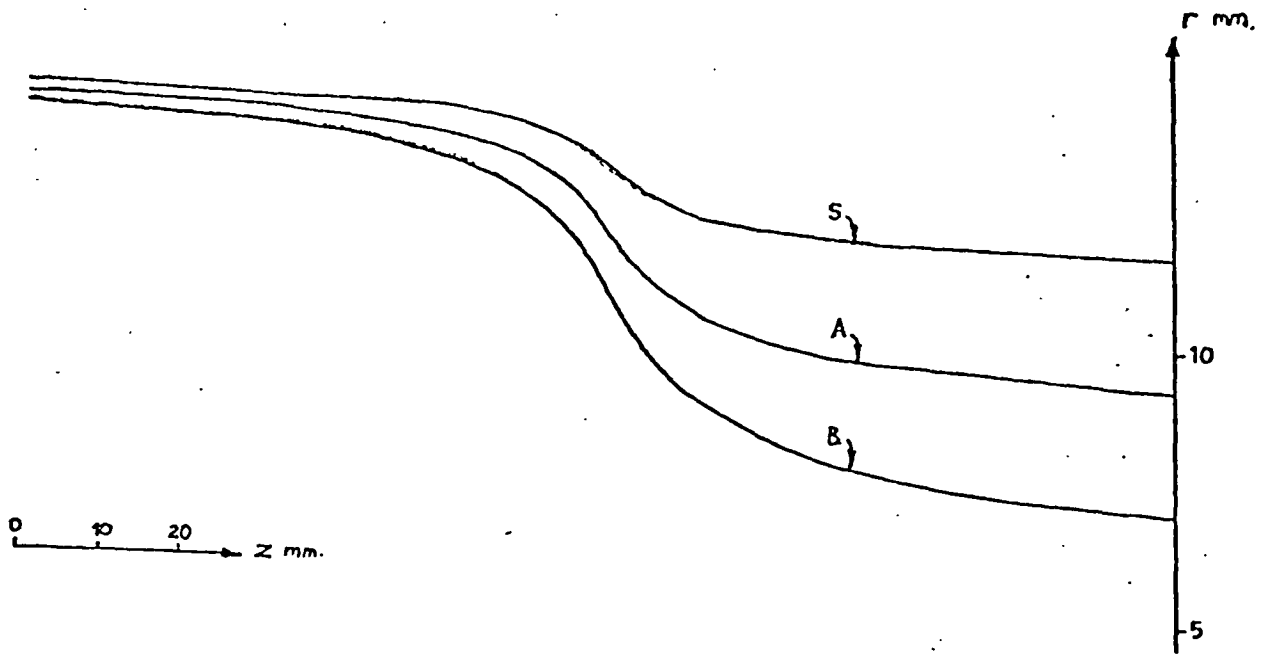


Figure 2.6 Different Reductions (varying β).

S:	$\beta = 0.707$	Load = 15.1 kN	(see table 2.1)
A:	$\beta = 0.5$	Load = 20.2	
B:	$\beta = 0.3$	Load = 24.0	

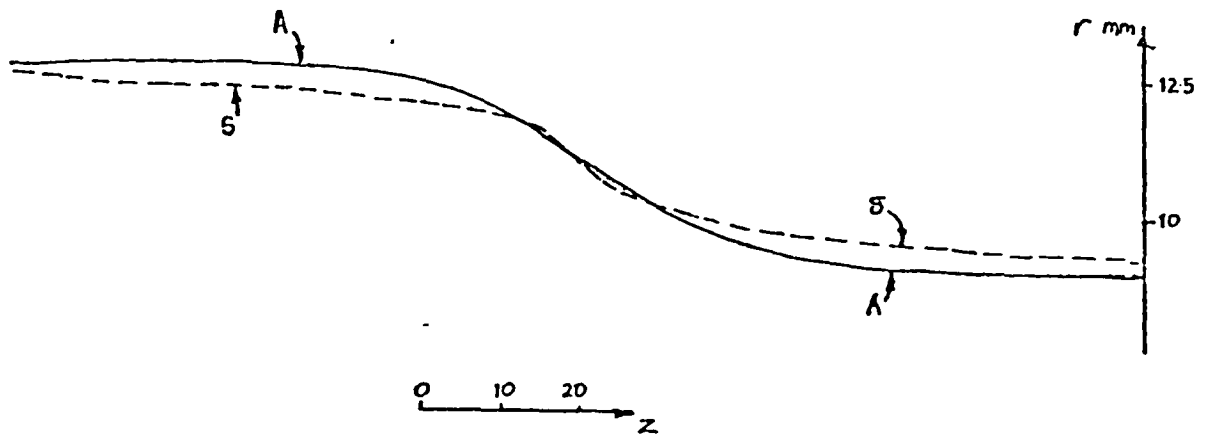


Figure 2.7 Different Deforming Zone Lengths.

S:	$Z_1 = -Z_2 = \infty$	Load = 15.1 kN	(Table 2.1)
A:	$Z_1 = -Z_2 = 200\text{mm}$	Load = 12.0 kN.	

CHAPTER 3

A SIMPLE NUMERICAL APPROACH - THE SLAB METHOD

- 3.1 Introduction
- 3.2 Viscosity
- 3.3 Details of the Program
- 3.4 Application of the Program

Chapter 3 - A SIMPLE NUMERICAL APPROACH - THE SLAB METHOD

3.1 Introduction

The slab method is a numerical technique and was developed to analyse dieless drawing for the following reasons:

(a) A simple, high speed method based on easily confirmed principles was desired.

(b) The method of 'simple tension' developed by Turner (2) has the difficulty that there is no provision for the application of velocity boundaries which means it is difficult to use to solve for specific reductions.

(c) An analytical solution such as the one described in Chapter 2 could never be modified to deal with the complexities of real material properties or temperature distributions.

To achieve these goals, a non-linear computational modification of the slab or Sachs' method of analysing metals in plastic flow was used. Some quite complicated examples of the application of this method are to be found in the work of Altan(16) and Lippmann(17), who apply it to forging and rolling problems requiring computational analysis. The use of simple elements to analyse plasticity problems is illustrated in the texts of Johnson and Mellor(18) and Alexander and Brewer(19).

The slab method used here basically follows the same assumptions as are listed for the 'analytical method' in Section 2.1 except that the material properties are not required to correspond to a particular equation (assumption c) and the temperature profile, while remaining one-dimensional, can take any desired shape (assumption a). Thus two of the worst assumptions of the analytical method can be avoided. The deforming zone of dieless drawing (see figure 1.1) is divided up into a large number of short equal length solid cylinders, the planes dividing it remaining fixed in space. Only axial strain rates and deviatoric stresses are taken into account as controlling the flow within the cylinders; the strain rate that occurs being determined by the equation

$$\dot{\epsilon}_{ij} = \sigma'_{ij}/2\eta \quad (3.1)$$

3.2 Viscosity

η in equation 3.1 is a 'viscosity' which is related to the $d\lambda$ used by Hill(14) in his notation for the Levy-Mises equation

$$d\epsilon_{ij} = \sigma'_{ij}d\lambda \quad (3.2)$$

However, the formulation henceforth used in this thesis will be that of equation 3.1 since

(a) at no time are elastic calculations considered in this thesis so the incremental rendering of equation 3.2 is not necessary;

(b) equation 3.1 is identical to the constitutive equation for viscous incompressible fluids (see Prager(20)) and thus can be interpreted as a viscosity or viscosity coefficient, and

(c) the use of strain rates enables numerical values to be assigned to the viscosity.

The viscosity is a function primarily of temperature and strain rate. At the temperatures concerned in dieless drawing, continuous annealing occurs and thus strain hardening is not a significant consideration (see also Chapter 6 for experimental evidence). Section 2.1 mentions the justification of other assumptions, in particular it is to be noted that the low value of stresses and the very high strains observed allow the elastic strains to be neglected, and as in most work about the creep of metals (for example Schoek(10)), it is assumed that plastic strains develop on the application of even infinitesimal loads.

Given these assumptions, the viscosity can be calculated by numerical interpolation on experimental stress/strain rate/temperature data. The inter-relation between these three variables is complex (as has been shown by Lee and Backofen(13) and Wu and Lowrie(15)) and it cannot be expected that any suitable equation can be found to express the connection over a large range of conditions. To overcome this difficulty, a technique was developed for direct interpolation

(or occasional extrapolation) from the experimental data of Lee and Backofen for Ti6Al4V. All the data from that paper (illustrated on figure 3.1) was stored as data triplets in the computer program and interpolation was carried out between them following an equation suggested by Schoek(10):

$$\dot{\epsilon}_z = C \exp(-Q/T) \sigma_z^{\frac{1}{n}} \quad (3.3 \text{ Cf. 1.2})$$

where C and Q are constants, n is the strain rate sensitivity and T is the temperature. This form of the equation is used since the data from Lee and Backofen is reported from uniaxial tensile tests. In the uniaxial situation

$$\sigma_z = \frac{3}{2} \sigma'_z = \bar{\sigma} \text{ and } \dot{\epsilon}_z = \dot{\epsilon} \quad (3.4)$$

In order to carry out the interpolation equation 3.3 is linearised by taking logarithms

$$\log \dot{\epsilon}_z = \log C - \frac{Q}{T} + \frac{1}{n} \log \sigma_z \quad (3.5)$$

Hence it is possible to interpolate easily along lines of constant T, σ_z , or $\dot{\epsilon}_z$ using the closest data points available as starting points. If a pair of variables ($\dot{\epsilon}_z$ and T or σ_z and T) are provided, the viscosity is uniquely determined and it is calculated by a subroutine. First the subroutine calculates the third variable (σ_z or $\dot{\epsilon}_z$) using equation 3.5 and then the viscosity from the relation

$$\eta = \frac{\sigma'_z}{2\dot{\epsilon}_z} = \frac{\sigma_z}{3\dot{\epsilon}_z} \quad (3.6)$$

(see equations 3.1 and 3.4).

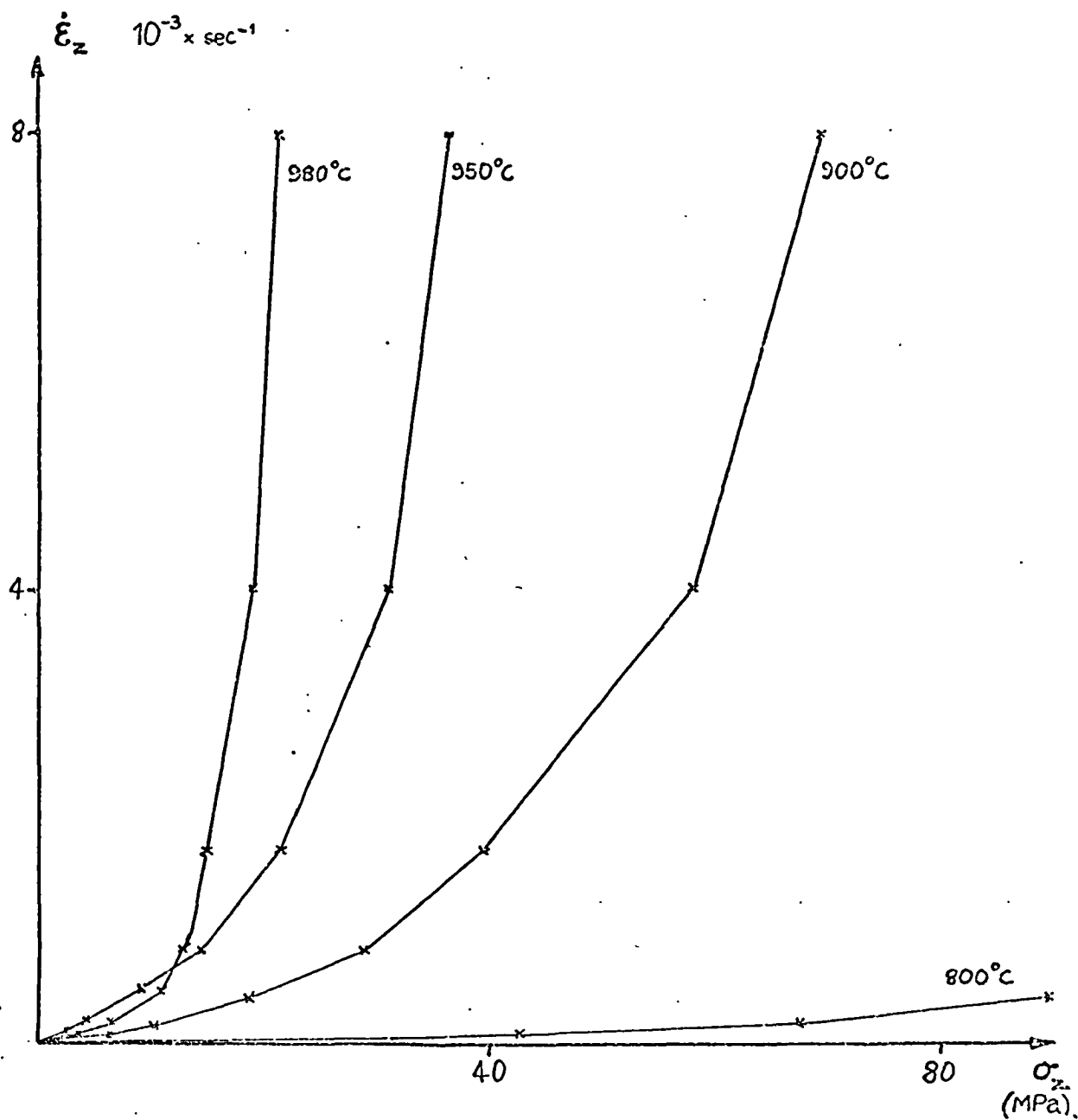


Figure 3.1 Examples of the data for Ti6Al4V from Lee and Backofen (13) covering the range of values encountered in dieless drawing.

3.3 Details of the Program

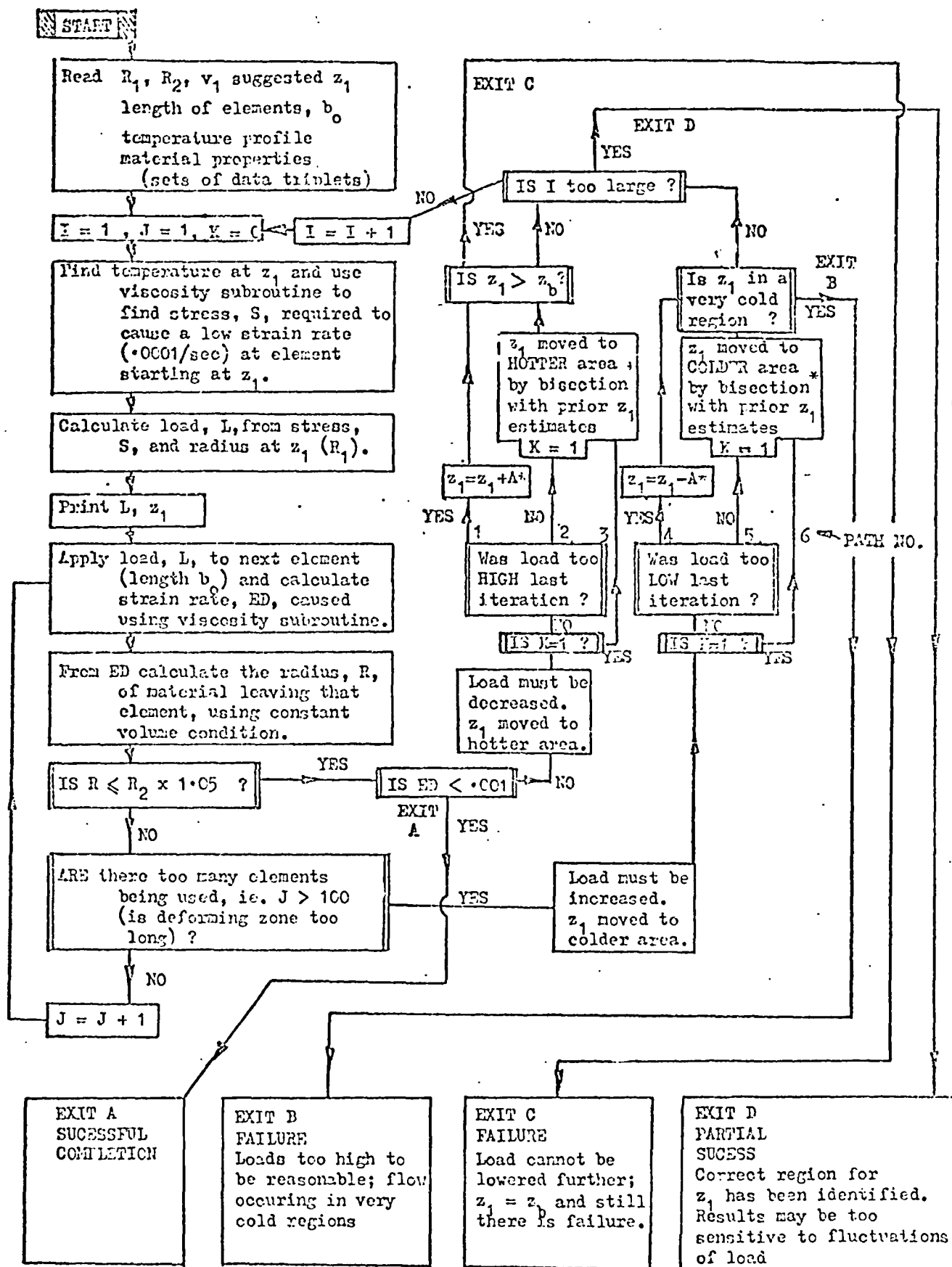
The basis of this program can be summarised as follows. It is assumed that there is a position, z_1 , fixed relative to the coil at which deformation just starts with a low strain rate (for example .0001/sec). Since position z_1 has a temperature defined by a temperature distribution ascertained elsewhere, the stress that is required to cause this strain rate at z_1 can be calculated using the methods of Section 3.2. Thus since the radius of the material at z_1 is the input radius, R_1 , the load which must be applied to the specimen can be calculated.

With this load and the given temperature distribution, the strain rate, $\dot{\epsilon}$, that occurs in the next element (all elements have the length b_0) can be calculated. Using the constant volume requirement, the exit radius from that element can be found and thus the stress on the next element, the next element's strain rate, and so on.

Sometimes it will be found that the total strain occurring throughout the whole deforming zone is calculated to be too high, that is, there is a 'fracture' or the exit radius is too small. In that case the load must be lowered and to do this the position z_1 is moved to a hotter point in the material. On other occasions the total strain estimated will be too low, the deforming zone too long (too many elements required), or the strain rate will fall below the value in the starting element at z_1 before the necessary

reduction has been achieved. In this situation the load will have to be increased by moving z_1 to a colder region. Eventually it is likely that a close approach can be made to estimating a z_1 where the total deformation produces a desired output radius.

The exact nature of the program is complicated to describe and a flowchart is given on figure 3.2. The coding of the program is, however, quite simple and running time for any one solution will only be two or three seconds on a CDC 6400. The flowchart on figure 3.2 does illustrate several other details of the program, including some possible causes for the program to record that the drawing was a failure. Exit A is a successful completion as described above. However, the requirements of exit A are rather stringent as presented on the flowchart and may require a large number of iterations to be achieved. Alternatively, exit D is often encountered but this exit may indicate that the particular situation is sensitive to small changes in load which may mean that the process is unstable. To judge this, the output of loads, L , at each iteration can be helpful for the total strain obtained may oscillate a great deal for small changes in load. Exits B and C are only likely to be encountered if the description of the experimental conditions is unreasonable. For example, exit C would be encountered if a material with zero strain rate sensitivity was postulated and exit B if the temperature gradients were very shallow.



*Notes A is an arbitrary interval used to move z_1 in early iterations ($\approx 1\text{cm.}$)
The bisection technique assumes that the midpoint between one answer that is too high and another that is too low must be better than one of the answers.

Figure 3.2 Flowchart for Plane Sections Remain Plane Program

3.4 Application of the Program

Figure 3.3 is an illustration of the progression of the outside profiles of the deforming zone as the program seeks a solution. Table 3.1 indicates the load, starting position z_1 for each iteration and the path (see figure 3.2) followed after each iteration. The conditions used for this solution are the standard conditions for this Chapter, which are detailed on table 3.2.

The progress of the program can be described as follows:

for curve 1 the starting position z_1 was set at 0mm for which the reduction was very high, hence

for curve 2 z_1 was moved to 10mm, where the loads were so low that very little straining occurred at all, thus

for curve 3 the midpoint between the two former choices of z_1 was chosen for the next estimate of z_1 , since that point would be closer to the correct value, but that had too high a load, so

for curve 4 a z_1 intermediate between those for curves 3 and 2 was chosen.

The program continued in this manner until exit A ('successful completion') was achieved after only eight iterations.

The other figures demonstrate further results. Figure 3.4 illustrates three different temperature profiles and the corresponding results. Figure 3.5 shows variations

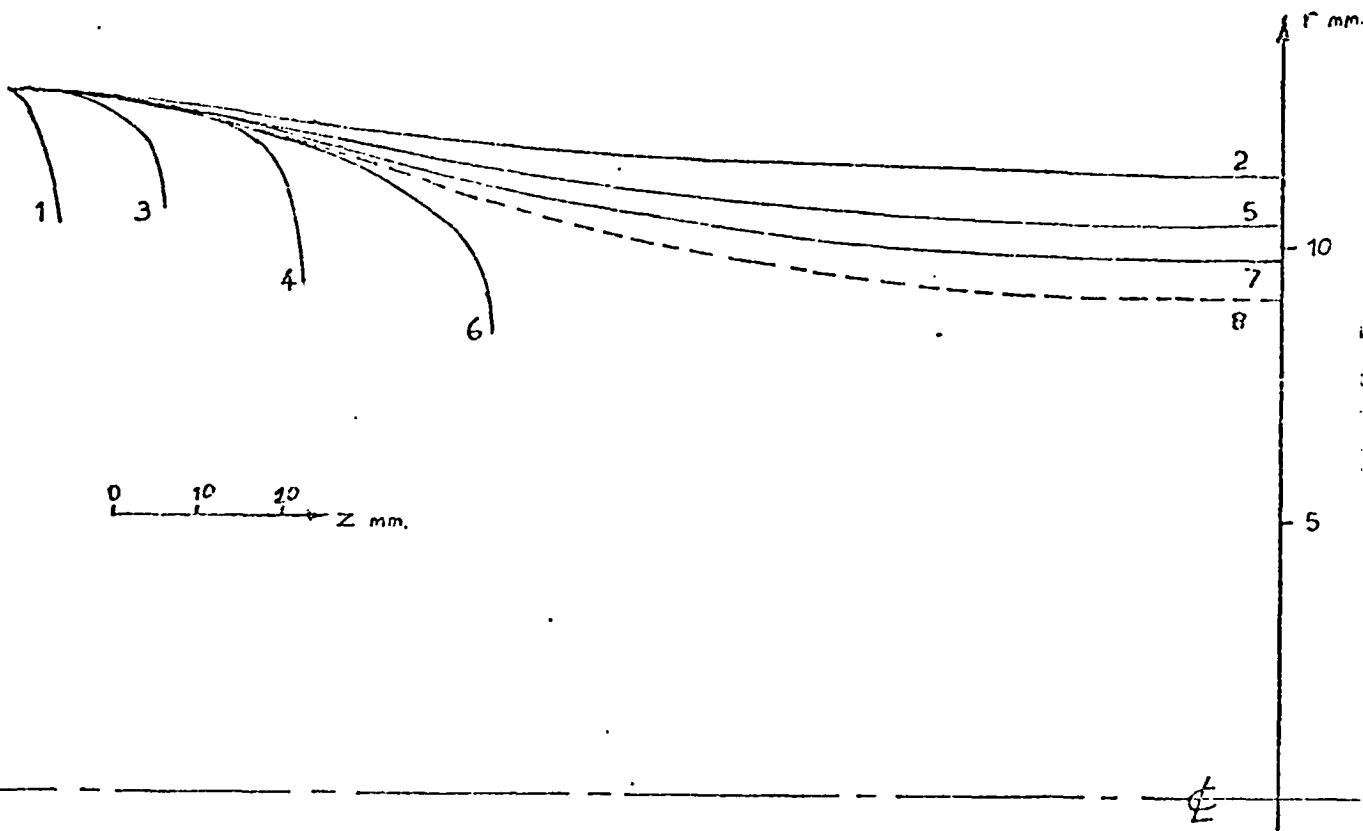


Figure 3.3 The first eight iterations as the program seeks a solution (outside profiles illustrated).

of velocities, and this figure confirms the suggestions of the analytical method in Section 2.4 that the velocity has little effect on the outside profile but the load is strongly affected (the load results on figure 3.5 should be much more accurate than those of table 2.3). Figure 3.6 shows that the method is almost independent of element length, b_o , within the range of lengths investigated. Figure 3.7 gives profile results for varying reductions, but it is to be noted that for the reductions greater than the standard reduction the results were obtained with great difficulty after a large number of iterations. This was possibly due to the temperature profile being unsatisfactory for the reductions imposed.

The result for the standard curve, S, makes a very satisfactory comparison with the experiments results, figure 2.3.

TABLE 3.1

Details of Curve on Figure 3.3

Curve	1	Load 40.6 kN	z_1	Omm	Subsequent path	1
2		13.5		10		5
3		23.5		5		3
4		17.8		7.5		3
5		15.5		8.75		6
6		16.6		8.125		3
7		16.0		8.437		6
8		16.35		8.281		Exit A

TABLE 3.2

Standard conditions for results in this Chapter marked curve S and shown with a dashed line

Input radius, R_1	12.7mm
Input velocity, v_1	1.0mm/sec
Reduction is 50%	($\beta = 0.707$)

Temperature distribution is given on figure 3.4 as curve S.

All distributions are taken from figure 22 in Turner(2)

Material properties are taken from Lee and Backofen(13), see figure 3.1

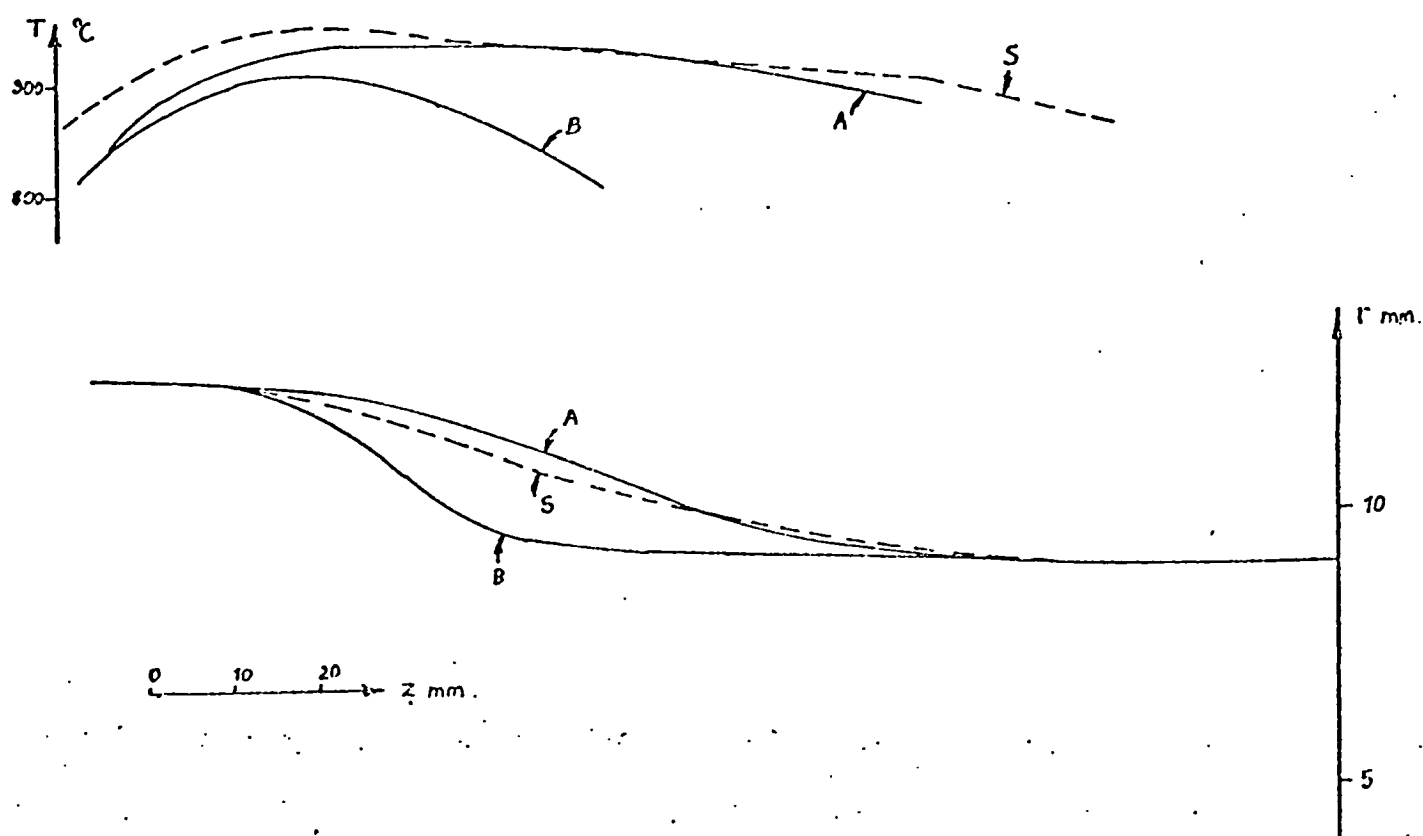


Figure 3.4 TOP Temperature Distributions along forming region.

BOTTOM Corresponding solutions found by the slab method.

S:	Maximum Temperature	950°C	Load = 16.4 kN
A:		935°C	Load = 18.7
B:		909°C	Load = 28.8

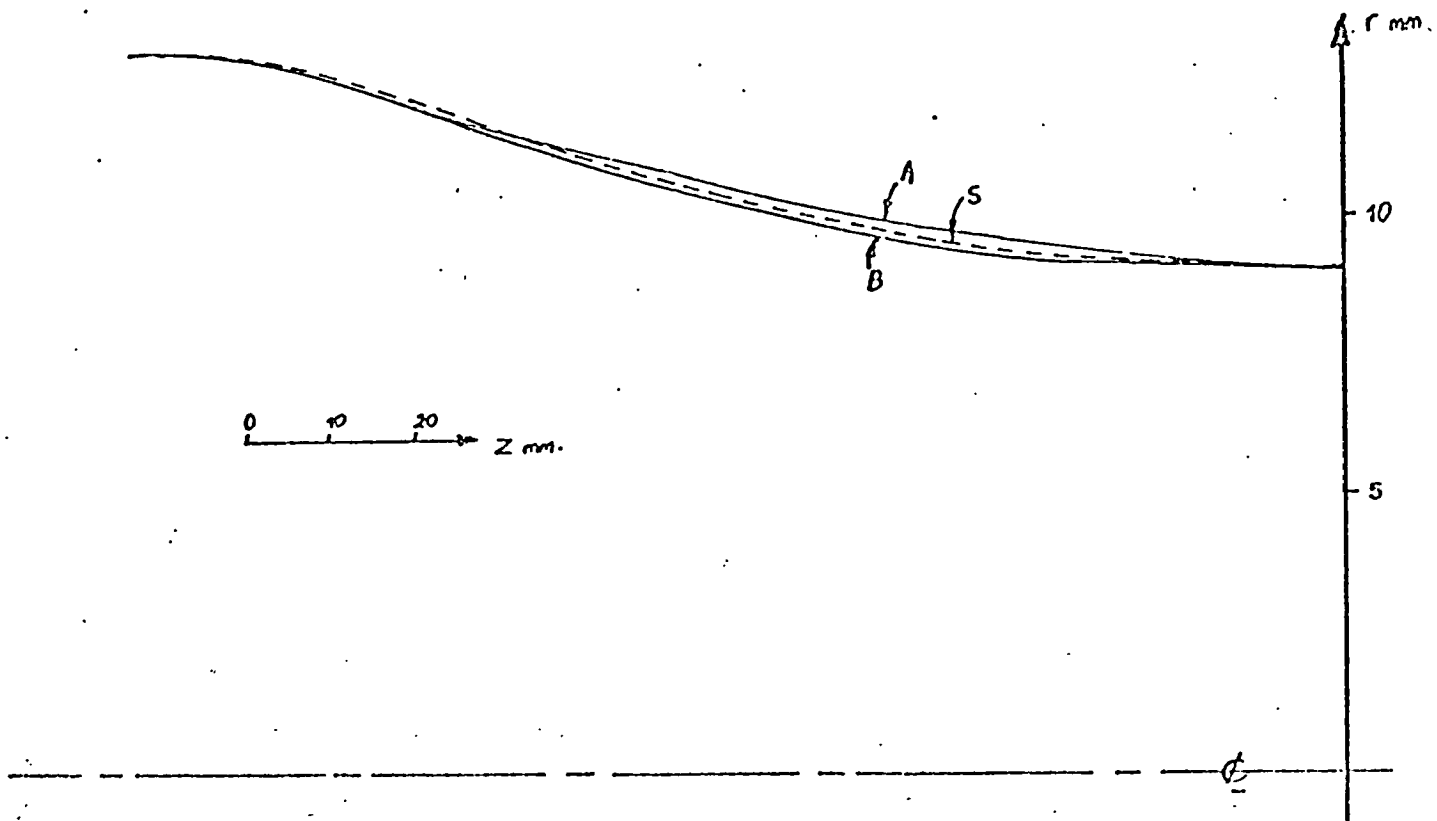


Figure 3.5 Varying the velocities.

$v_2 = 2v_1$ in all cases. Other details are listed on table 3.2

A:	$v_1 = 0.1 \text{ mm/sec}$	Load = 5.7 kN
S:	$v_1 = 1.0$	Load = 16.4
B:	$v_1 = 10.0$	Load = 24.4

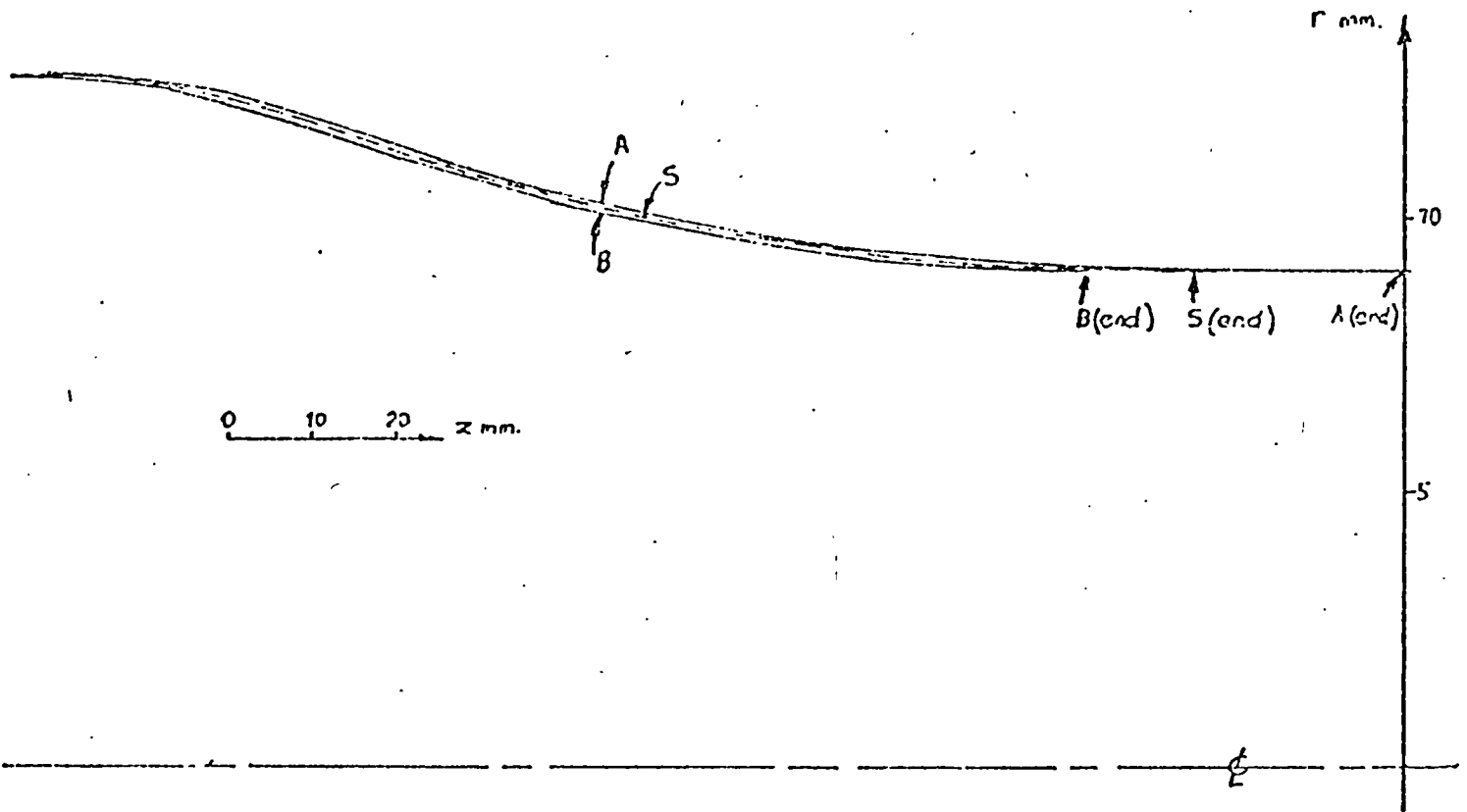


Figure 3.6 Varying slab thicknesses.

All details are listed on Table 3.2 except b_o .

A:	$b_o = 1.75 \text{ mm}$	Load = 16.47 kN
S:	$b_o = 3.5$	Load = 16.35
B:	$b_o = 7.0$	Load = 16.17

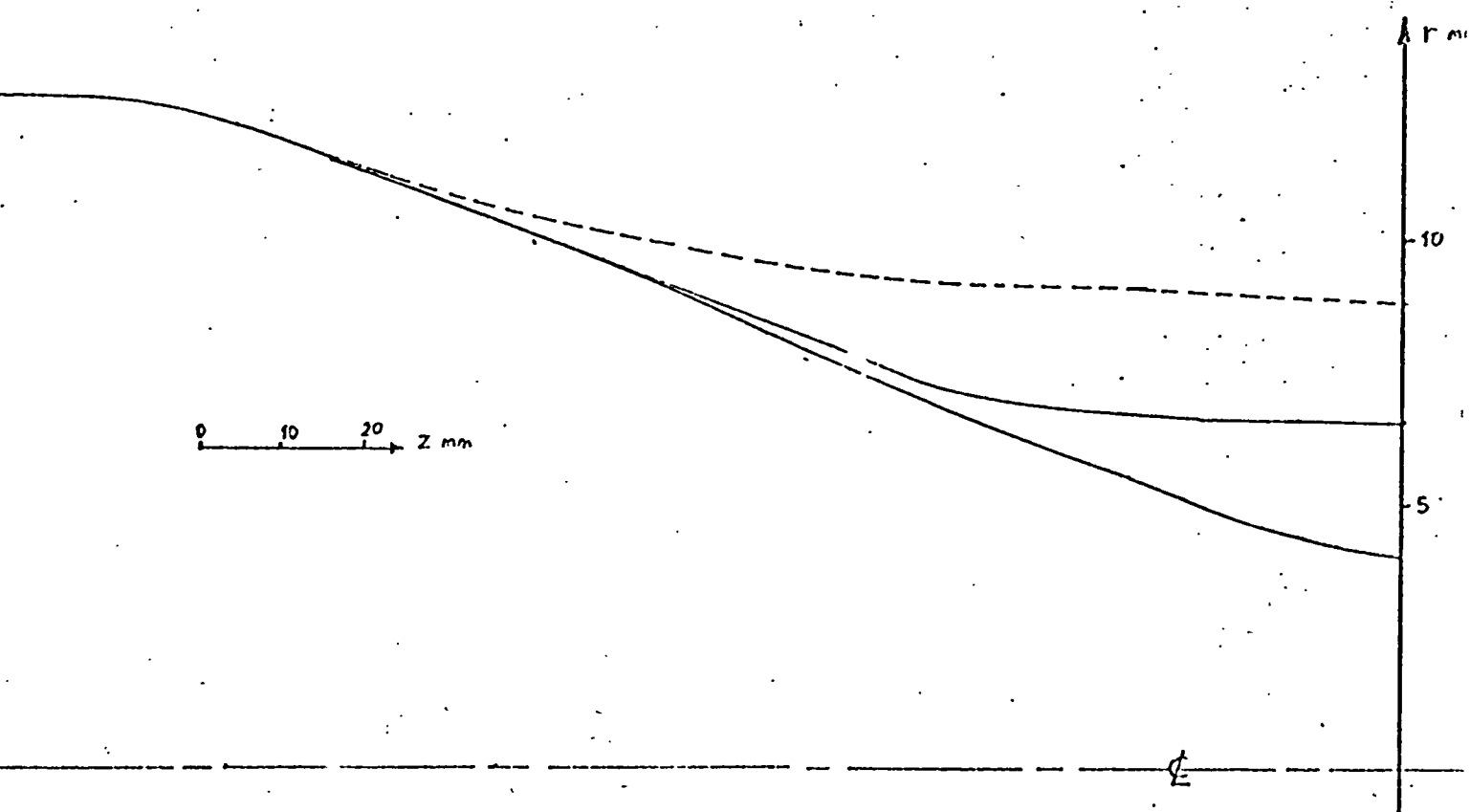


Figure 3.7 Different Reductions.

All details are as listed on Table 3.2 except that reduction is altered.

Top curve 50% reduction in area,

Centre curve 75%,

Bottom curve 90%.

CHAPTER 4

FINITE ELEMENT METHODS FOR HOT METAL FORMING

- 4.1 Introduction
- 4.2 Elastic-Plastic Formulations
- 4.3 Review of Literature of Fully Plastic Analysis
- 4.4 Minimization of Work Rate
- 4.5 Constant Volume Requirement
 - 4.5.1 The Stream Function Method
 - 4.5.2 Lagrange Multiplier/Mean Stress Method
 - 4.5.3 The Penalty Function Approach
 - 4.5.4 Use of Poisson's Ratio Close to 0.5
- 4.6 Specific Application in this Thesis

Chapter 4 - FINITE ELEMENT METHODS FOR HOT METAL FORMING

4.1 Introduction

The finite element method represents one of the most flexible and comprehensive theoretical analysis which can at present be used in continuum mechanics. The large range of applications and the details of the results that the method is capable of achieving can be judged by an examination of the basic text of Zienkiewicz(21). Nowadays the method has a significant application for solving engineering problems for fully elastic structures, but the use of the method for the study of metal forming is comparatively undeveloped. The method presents the possibility that problems which at present can be solved only by experimental and approximate mathematical techniques will eventually be routinely and thoroughly investigated by computational method. The method can also provide solutions for situations where experimental investigations present great difficulties, for example, stress and strain distributions can be calculated inside bodies that are hot and undergoing considerable strain.

The development of the finite element method to include plastic strains at first followed the tendency of mechanical testing literature to show the metal initially behaving elastically, then yielding then developing 'instantaneous irreversible (plastic) strains' without time entering directly into consideration. 'This is at best only a convenient

mathematical fiction.' (Quotations from Zienkiewicz and Cormeau(22).) Two seminal papers which show this attitude are Argyris(23) and Marcal and King(24). In many circumstances this approach is, of course, perfectly acceptable and much literature on mechanical testing does not consider rate effects to be very important.

In the review of the literature that follows, the most important differentiation between various approaches to plasticity are demonstrated by the differences in the constitutive equations that are used. As is suggested above, the majority of the papers that deal with plasticity use a constitutive equation containing stress as a function mainly of total strain and in which strain rate effects are not normally considered. Since the elastic properties of the material can be included, these methods can be described as elastic-plastic analyses. Methods which are unable to involve elastic behaviour can be described as fully plastic or rigid-plastic analyses. In the constitutive equations for the latter formulations, strain rates are normally considered explicitly but for the former approach special provisions have to be made if they are to be considered. (Both approaches are amenable to the addition of temperature and other effects.) As has been discussed in both Sections 2.1 and 3.2, in hot metal forming the stresses are low and elastic strains are negligible, while the stresses must be seen essentially as functions of local strain rate and temperature.. It is for these reasons that the fully plastic formulations

are ideal for hot metal forming. A discussion of elastic-plastic techniques are included in Section 4.2 because of the importance of this work in the literature.

This and subsequent chapters will not include the basic infrastructure of a finite element program and the method which are best described by Zienkiewicz(21), Desai and Abel(25) and Fenner(26). The notation used is based mainly on the texts of Hill(14) and Zienkiewicz(21) and is fully described in the List of Symbols.

4.2 Elastic-Plastic Formulations

The range of the possibilities of applying elastic finite element programs to non-linear materials are neatly summarised by Zienkiewicz(21) who presents a general constitutive equation of the form

$$\sigma_{ij} = D_{ijkl}(\epsilon_{kl} + \epsilon_{kl}^0) - \sigma_{ij}^0 \quad (4.1)$$

where σ_{kl}^0 and ϵ_{kl}^0 are initial stresses and strains respectively. As it is written this equation is linear, but it can be made non-linear if the matrix D or if the initial stress or strain are allowed to vary during the solution. One common approach for elastic-plastic work is to use an equation derived from equation 4.1 (assuming initial strain and stress are constant)

$$d\sigma_{ij} = D_{ijkl}d\epsilon_{kl} \quad (4.2)$$

The value of D is calculated by reference to current local stress state (or strain state) and consideration of the elastic properties, yield criterion and, if the material has yielded, the plastic properties. It is necessary to carry out the application of the boundary conditions incrementally (except if the material is elastic throughout). The exact details of the method differs between authors though the most significant formative papers are Marcal and King(24) and Yamada et al.(27). Examples of the use of this method can also be found in Swedlow(28), Nagamatsu et al.(29), Iwata et al. (30) and Gunasekera and Alexander(31). Equation 4.2 can also be differentiated with respect to time to produce

$$\dot{\sigma}_{ij} = D_{ijkl} \dot{\epsilon}_{kl} \quad (4.3)$$

as has been used by C.H. Lee and Kobayashi(32) and McMeeking and Rice(33).

The technique of varying the initial stress is illustrated by Zienkiewicz, Valiappan and King(34) and an initial strain technique is illustrated by Zienkiewicz and Cormeau (22). The last paper is interesting in that it takes into account creep strains that develop under a constant or changing stress state. Visco-elastic materials were considered by Zienkiewicz, Watson and King(35).

There are a number of approximations inherent in the finite element method and apart from dealing with both elastic and plastic behaviour in the same analysis many

researchers have attempted to reduce other approximations:

(a) More thorough definitions of stress and strain. Most finite element analyses such as described by Zienkiewicz (21) use an infinitesimal definition of strain and the Cauchy stress (defined in Mase(36)). However, more refined definitions have been considered by E.H. Lee(37), Hibbitt, Marcal and Rice(38) and McMeeking and Rice(33) who have been concerned among other things with situations in which there is considerable rotation of the elements. For example, McMeeking and Rice (33) and E.H. Lee(39) show that for plastic materials an error can occur in a time differentiation of stress in which rotational or convective terms are ignored, and that this error is not reduced by reducing the time interval or extent of deformation. However, most other approximations (such as using infinitesimal strain instead of finite strain) are small if the deformed state is very close to the undeformed state.

(b) More developed finite elements. The elements used in the finite element analysis have been considered and certain higher order elements can produce better results as illustrated in Zienkiewicz(21). The constant strain triangle that dominates the early literature has the difficulty that a stepped stress and strain distribution is evaluated.

(c) Improved description of the material properties. No material really behaves in the manner described by the normal elastic and plasticity equations and yield criterion

but few efforts have been made to improve the material description. However, anisotropy has been considered for fully plastic sheet material by S.H. Lee and Kobayashi(40).

(d) Other improvements such as three dimensional computations and temperature calculations being included as part of the analysis (see A. Blass(41)).

Each refinement to the method normally involves further complication, more computing time and computer memory. Thus all improvements to the basic scheme have to be judged from the point of view of necessity or value to the particular problem.

One other approach that should be mentioned in this review is that of Herrman(42) whose technique is designed primarily to investigate nearly incompressible elastic materials. Incompressible materials present the difficulty that if the constitutive equations are expressed in the form of equation 4.1, a division by zero is implied. This can be seen if the relation for isotropic elasticity is examined (see Mase(36)):

$$\sigma_{ij} = \frac{E}{1+\nu} (\epsilon_{ij} + \frac{\nu}{1-2\nu} \delta_{ij} \epsilon_{kk}) \quad (4.4)$$

where E is Young's modulus and ν is Poisson's ratio. If this equation is written to express strain in terms of stress:

$$\epsilon_{ij} = \frac{1}{E} (\sigma_{ij} (1+\nu) - \nu \delta_{ij} \sigma_{kk}) \quad (4.5)$$

it can be seen that incompressibility, $\epsilon_v = \epsilon_{kk} = 0$, is achieved identically if Poisson's ratio is equal to 0.5. Returning to equation 4.4, it is seen that if $\nu = 0.5$, a division by zero occurs, but Herrman proposes a constitutive equation which is a readjustment of equation 4.4, namely

$$\sigma_{ij} = \frac{E}{1+\nu} \epsilon_{ij} + \delta_{ij} \frac{3\nu}{1+\nu} \sigma \quad (4.6)$$

where σ is mean stress ($\frac{1}{3}\sigma_{kk}$). Stated in this way the value of σ has to be evaluated throughout the body as well as the strains but the value of ν is permitted to be 0.5. (In finite element programs this means that extra variable must be evaluated at each node, a requirement that is discussed in paragraphs 4.5.1 and 4.5.2). Putting $\nu = 0.5$ yields

$$\sigma_{ij} = \frac{2E}{3} \epsilon_{ij} + \delta_{ij} \sigma \quad (4.7)$$

If the material being considered is non-linear, the strains should be applied incrementally. This strictly requires that equation 4.7 be differentiated with respect to time and this may introduce problems with some of the terms. The time derivative of E , the Young's modulus, is not normally defined, and moreover when the time derivatives of the stresses are taken it may be necessary to choose a rotationally invariant formulation (see refs. (37)-(39)).

4.3 Review of Literature of Fully Plastic Analysis

Fully plastic or rigid plastic analyses of deformation involve a somewhat different type of constitutive relation to elastic-plastic analyses. If the elastic properties of a material can be ignored, the equations can be simplified and problems connected with the yield criterion and conversion of regions of the material from elastic to plastic properties are removed. The constitutive equation that is relevant to this situation for an incompressible material is the Levy-Mises equation

$$d\epsilon_{ij} = \sigma'_{ij} d\lambda \quad (3.2)$$

Here the strains represent only plastic strains since elastic strains have been ignored. This equation can also be differentiated with respect to time to produce the viscous flow equation (as was discussed in Section 3.2):

$$\sigma'_{ij} = 2\eta\dot{\epsilon}_{ij} \quad (4.8)$$

Equation 4.8 differs from equations 4.1-4.3 in that deviatoric stresses are used and the strain rates are related directly to them. Thus the rate effects that can be significant in plasticity are explicitly considered in this formulation. The value of the viscosity, η , is related to local strain rates and other effects such as temperature and total strain so far achieved.

This approach has been applied to metals by Shah and Kobayashi(43), C.H. Lee and Kobayashi(44) and Zienkiewicz and Godbole(45-47). Similar constitutive equations have been used in finite element analyses for viscous fluids by Atkinson, Card and Irons(48) and Palit and Fenner(49,50).

As it stands, equation 4.8 does not guarantee constancy of volume of the deforming continuum, although it is a necessary condition. The methods for ensuring incompressibility can be applied separately to the constitutive equation and are reviewed in Section 4.5.

Another approach is illustrated in the work of Turner (2) which is discussed also in Alexander and Turner(4). In this technique, equation 4.8 is reformed

$$\dot{\epsilon}_{ij} = \frac{1}{2\eta}(\sigma_{ij} - \delta_{ij}\sigma) \quad (4.9)$$

Thus

$$\dot{\epsilon}_{ij} = \frac{2}{3} \frac{1}{2\eta}(\sigma_{ij}(1+\frac{1}{2}) - \frac{1}{2}\delta_{ij}\sigma_{kk}) \quad (4.10)$$

which is similar to equation 4.5 except that v is replaced by $\frac{1}{2}$, E by 3η and ϵ_{ij} by $\dot{\epsilon}_{ij}$. In the analogous situation, the inversion of equation 4.5 is given in equation 4.4, so the inversion of 4.10 should be

$$\sigma_{ij} = \frac{3\eta}{1+\frac{1}{2}} (\dot{\epsilon}_{ij} + \frac{\frac{1}{2}}{1-2(\frac{1}{2})} \delta_{ij}\dot{\epsilon}_{kk}) \quad (4.11)$$

Since equation 4.11 involves division by zero, the argument continues that if the factor $\frac{1}{2}$ is, in fact, not exactly 0.5 but some close value like 0.49995, then the condition of constancy of volume will be approached. The effect of this is to produce an equation

$$\sigma_{ij} = \beta \dot{\epsilon}_{ij} + \alpha \delta_{ij} \dot{\epsilon}_{kk} \quad (4.12)$$

where β is slightly larger than 2η and α is a large number. The significance of this equation is discussed further in paragraph 4.5.4.

4.4 Minimization of Work Rate

The finite element method can be seen as determining the strain distribution within a body by minimization of the total internal work (or work rate) given particular boundary conditions. An increment of work per unit volume at a point is given by

$$dW = \sigma_{ij} d\epsilon_{ij}$$

Henceforth, however, only fully plastic materials will be considered and for an incompressible medium obeying the Levy-Mises equations, 4.8, additional relations apply (see Mase(36)):

$$dW = \sigma_{ij} d\epsilon_{ij} \quad (4.13a)$$

$$= \sigma_{ij}^i d\epsilon_{ij} \quad (4.13b)$$

$$= \bar{\sigma} d\bar{\epsilon} \quad (4.13c)$$

where $\bar{\sigma}$ is effective stress and $d\bar{\epsilon}$ is an increment of effective (plastic) strain. These equations can be differentiated with respect to time to give a volumetric work rate. For example, differentiating (4.13b) yields

$$\dot{W} = \sigma'_{ij} \dot{\epsilon}_{ij} \quad (4.14)$$

In the fully plastic isotropic case, equations 4.14 and 4.8 can be combined to give

$$\dot{W} = 2\eta \dot{\epsilon}_{ij} \dot{\epsilon}_{ij} \quad (4.15)$$

This equation provides the basis from which the finite element formulation can be derived. The body is divided into elements and within each element the strain rate is defined as a function of the nodal velocities as follows

$$\dot{\epsilon}_{ij} = B_{ijkN} v_{kN} \quad (4.16)$$

where N is summed over all the nodes in the body but $B_{ijkN} = 0$ if the node N is not connected with the element in which $\dot{\epsilon}_{ij}$ is being determined; B_{ijkN} can be a function of position inside the element; v_{kN} is the velocity of node N in direction x_k . The analogy between this flow type formulation and the normal incremental elastic formulation is clearly presented by Zienkiewicz(51). The exact nature of B_{ijkN} differs between programs and two forms used in this thesis will be described in later chapters.

The total plastic work rate in the body is given by the $\dot{W}$ of equation 4.15 being integrated over the volume of the entire deforming body. The external work done on the body by tractions and body forces must be considered and so the energy rate which is to be minimized is given by

$$\dot{X} = \int_V \dot{W} dV - \int_V F_{kN} v_{kN} dV - \int_S T_{kN} v_{kN} dS \quad (4.17)$$

where N is summed over all the nodes in the body, V is the volume and S the surface of the body, F_{kN} are the body forces (per unit volume) and T_{kN} are the surface shear stresses (both of these must be distributed to appropriate nodes in the body). In slow speed plasticity the body forces (for example weight) are considered negligible: $F_{kN} = 0$. The surface shear stresses are zero on free boundaries, but are finite on surfaces with friction. In the case of sticking between the body and rigid tools, the surface velocities parallel to the boundary are zero so the last integral makes no contribution.

The minimum of the work rate with respect to each nodal variable is given by

$$\frac{\partial \dot{X}}{\partial (\text{nodal variable})} = 0 \quad (4.18)$$

In the case given above, the minimum of $\dot{X}$ with respect to each v_{kN} is given by

$$\frac{\partial X}{\partial v_{kN}} = \int_V 2\eta B_{ijlM} B_{ijkN} v_{lM} dv = f_{kN} + t_{kN} \quad (4.19)$$

where f_{kN} and t_{kN} are the resolved forces acting parallel to v_{kN} . This can be rewritten

$$\int_V v_{lM} 2\eta B_{ijkN} B_{ijlM} dv = f_{kN} + t_{kN} \quad (4.20)$$

which is a system of k times N simultaneous linear equations in k times N unknowns. The method for solving these equations is different for different programs and two methods will be introduced in later chapters. Once all v_{kN} have been found then the strain rates can be found using equation 4.16 and from these the stresses can be found using the constitutive equation.

To continue the generalization a further degree, a constitutive equation of the form

$$\sigma_{ij}^* = D_{ijpq} \dot{\epsilon}_{pq} \quad (4.21)$$

can be considered. (σ_{ij}^* can be defined as deviatoric or total stress as desired.) In this case, equation 4.20 becomes

$$\int_V v_{lM} D_{ijpq} B_{ijkN} B_{pqlM} dv = f_{kN} + t_{kN} \quad (4.22)$$

4.5 Constant Volume Requirement

Neither in equation 4.8 or 4.15 is there any explicit provision to ensure that the material obeys the condition of

incompressibility, $\epsilon_v = 0$, even though this is a necessary assumption. Several possibilities for enforcing incompressibility have been presented in the literature and a description of these follows.

4.5.1 The Stream Function Method

This method was used for metals by Godbole (52) and Zienkiewicz and Godbole(46). For plane strain and axisymmetric strain a function, ψ , can be defined which guarantees that the sum of the strains is zero. For the plane strain case

$$v_x = -\frac{\partial \psi}{\partial y}, \quad v_y = \frac{\partial \psi}{\partial x}$$

and thus

$$\dot{\epsilon}_v = \dot{\epsilon}_x + \dot{\epsilon}_y = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0.$$

For the axisymmetric case

$$v_r = \frac{1}{r} \frac{\partial \psi}{\partial z}, \quad v_z = -\frac{1}{r} \frac{\partial \psi}{\partial r}$$

and thus

$$\dot{\epsilon}_v = \dot{\epsilon}_r + \dot{\epsilon}_z + \dot{\epsilon}_t = \frac{\partial v_r}{\partial r} + \frac{\partial v_z}{\partial z} + \frac{v_r}{r} = 0$$

In the finite element program, three nodal variables must be found at each node if ψ is to be found throughout the body. In the axisymmetric case the three variables that must be found are

$$\psi, \quad \frac{\partial \psi}{\partial r}, \quad \frac{\partial \psi}{\partial z}$$

the function and its two gradients (these last are proportional to the velocities). This formulation is suitable for the flow of viscous, incompressible fluids and plastics as illustrated by Palit and Fenner(49,50), Atkinson et al.(48) and Card(53). In such a case the flow obeys the equation

$$\nabla^4 \psi = 0$$

but as will be seen by an examination of Atkinson et al.(48), this produces an equation identical to 4.15 except that the work rate is termed 'viscous dissipation'. There are three main difficulties with this approach.

(a) If the nodal variables are denoted by a vector $\underline{y}$ and the basic equation 4.22 can be rewritten in a matrix form

$$\underline{y} \underline{K} = \underline{f} \quad (4.23)$$

where $\underline{K}$ is described as a 'stiffness matrix' and $\underline{f}$ is the 'force vector' (see Zienkiewicz(21)). If there are N nodes in the body, then for the case where three variables have to be found at each node there are $3N$ elements in both $\underline{y}$ and $\underline{f}$. The equation 4.22 suggests that an axisymmetric problem may be solvable with a minimum of only $2N$ elements in both $\underline{y}$ and $\underline{f}$. If the number of elements in $\underline{y}$ is taken to be M then the number of operations required to solve 4.23 is not directly proportional to M . In fact, even with efficient techniques that take account of every simplifying possibility in the form of $\underline{K}$, the number of operations is more likely to be

proportional to M^2 (see Zienkiewicz(21)). Thus the requirement of having three variables at each node instead of two will more than double computation time for solving equation 4.23.

(b) The knowledge of the value of ψ throughout the body is of no real use in the study of metal forming. Also the method yields its answers in the form of deviatoric stress (through equation 4.8) which must be converted to total stress if useful stress distributions are to be found.

(c) The boundary conditions can be difficult to define if velocities are not prescribed on the boundary (see Godbole (52)).

4.5.2 Lagrange Multiplier/Mean Stress Method

Since equation 4.14 does not make allowance for variation in mean stress (being specifically excluded by use of deviatoric stress), the work rate equations can be written

$$\dot{W} = \sigma'_{ij} \dot{\epsilon}_{ij} + \sigma \dot{\epsilon}_{kk}$$

$$\text{and} \quad \sigma \dot{\epsilon}_{kk} = 0 \quad (4.24)$$

σ , the mean stress at any point may be defined as

$$\sigma = R_p \sigma_p \quad (4.25)$$

where σ_p are the nodal values of mean stress and R_p is zero if σ is being measured at a point not inside an element with

which node P is connected. (Alternatively σ_P could be elemental values of mean stress and $R_P = 1$ inside the element P and zero outside, a formulation used in references (40,43,44).) When the differentiations indicated in 4.18 take place with respect to v_{kN} and σ_P the following two results occur:

$$\int_V 2\eta B_{ijkN} B_{ijlM} v_{lM} dV + \int_V R_P \sigma_P B_{qqkN} dV = f_{kN} + t_{kN}$$

and
$$\int_V R_P B_{qqkN} v_{kN} dV = 0$$

These equations represent $k \times N + P$ equations in $k \times N + P$ unknowns and thus can be solved. The situation is identical to a Lagrangian multiplier technique where the multiplier, λ , is used to ensure that the condition

$$\lambda \int_V \dot{\epsilon}_v dV = 0 \quad (4.26)$$

is met except that λ is also equal to mean stress. The method is similar in respect to the approach of Herrman (equation 4.7) except that technique is for linear materials. The advantage of this technique over the stream function method discussed above is that the values of σ can be used to establish a useful stress distribution within the body. However, once again there are more than two variables per node and the solution time is not at the minimum possible. One difficulty with a Lagrangian formulation is that zeros

are introduced into the leading diagonal of the stiffness matrix, $\underline{K}$, in equation 4.23, which may mean that standard solution techniques cannot be used (see Zienkiewicz and Godbole(45)). The Lagrangian approach is illustrated in papers by S.H. Lee and Kobayashi(40), Shah and Kobayashi(43) and C.H. Lee and Kobayashi(44).

4.5.3 The Penalty Function Approach

In this technique a large positive number, α , is multiplied by the square of the integrated strain rates, for example, using equation 4.16,

$$\alpha \int_V \dot{\epsilon}_V^2 dV = \alpha \int_V B_{ppkN} v_{kN} B_{qqLM} v_{LM} dV$$

and this is added to the functional X in equation 4.17. If $\dot{\epsilon}_V^2$ deviates at any point from being close to zero, then a large positive 'penalty' will be applied to the functional. Thus when the functional is minimized, the volumetric strain rate will be made close to zero. Carrying out the differentiations indicated in equation 4.18, the following is obtained

$$2\eta \int_V v_{LM} B_{ijkN} B_{ijLM} dV + \alpha \int_V v_{LM} B_{qqkN} B_{rrLM} dV = f_{kN} + t_{kN} \quad (4.27)$$

This can be inserted into a program as a constitutive equation

$$\sigma'_{ij} = 2\eta \dot{\epsilon}_{ij} + \alpha \delta_{ij} \dot{\epsilon}_{kk} \quad (4.28)$$

This method is extremely simple to insert into the program requiring the small change of equation 4.28 and only two

variables at each node. It is thus quickest to obtain a solution with this method than with the two others described above. It suffers from the disadvantage of not enabling a direct calculation of mean stress. The use of the method is illustrated in the work of Godbole(52) and Zienkiewicz and Godbole(45,47).

4.5.4 Use of a Poisson's Ratio Close to 0.5

The bases of this approach have been described above in equations 4.9 to 4.12 and there are similarities between this method and the penalty function approach (compare equations 4.12 and 4.28). As has been noted, Turner(2) used this method to analyse dieless drawing, but he found difficulties in obtaining satisfactory agreement at some stages with his experimental results. An example of these difficulties is illustrated on figure 1.2 which is drawn from his work. In this connection, it is interesting to note that Zienkiewicz and Godbole(45) make the following comment:

'For simple finite element representation this approach usually leads to inaccurate results but with isoparametric parabolic elements good accuracy can be found with Poisson's ratio as high as 0.49995.'

Turner used simple triangular elements with constant strain and it is possible that with incompressible elements of this type there are not sufficient independent variables at the nodes (as noted by E.H. Lee(39)).

This procedure offers the greatest simplicity in adaption to existing elastic programs, and according to equation 4.12 it appears to yield total stresses as the result. However, it would seem there is a conflict between equations 4.12 and 4.8 and that if 4.12 is to yield total stresses, then the fact that β does not quite equal 2η and that $\dot{\epsilon}_{kk}$ does not quite equal zero must be taken into account. The multiplication of α , which is a large number, by $\dot{\epsilon}_{kk}$, which is very small, may cause errors as well. Since the constitutive equation is not the same for this method as for the penalty function method, the author does not agree with Zienkiewicz and Godbole(45) that the two methods are identical. Nor is α in either method to be identified with the bulk modulus, for as presented here it is an arbitrary number.

4.6 Specific Application in this Thesis

Throughout this thesis the following techniques were applied. The material is assumed to be fully plastic and incompressible, with the constitutive equation given by the Levy-Mises flow rule (equation 4.8). The viscosity is dependent on local effective strain rate and temperature (determined from a prior increment). The detailed calculation of viscosity has been described in Section 3.2. The penalty function approach is used to ensure constant volume. The disadvantage that only deviatoric stresses are determined in this approach is overcome by a technique described in Chapter 8, a technique which has no direct bearing on

the finite element solution for strain rates. An infinitesimal definition of strain rate is used but the mesh should not be allowed to deform greatly in any one element, which ensures that the approximation is not great.

CHAPTER 5

APPLICATION OF THE FINITE ELEMENT METHOD TO DIELESS DRAWING

- 5.1 The Program Used
- 5.2 The Boundaries of the Problem
- 5.3 Optimum Penalty Function
- 5.4 Strain Rate Distribution and Rigid Regions
- 5.5 Conclusions

Chapter 5 - APPLICATION OF THE FINITE ELEMENT METHOD TO DIELESS DRAWING

5.1 The Program Used

The program used for dieless drawing can be summarized as a program for a continuous process using constant strain rate elements and the solution technique is that of Gauss-Seidel iteration.

Dieless drawing can be considered as a continuous process once the deforming zone is moving at a constant speed. In fact, the situation is simplified by considering the forming zone to be stationary ($v_h = 0$ on figure 1.1) as has been done in Chapters 2 and 3. In order to analyse a continuous process, the deforming zone is at first set up in an arbitrary manner in the computer program and then the conditions of non-equilibrium are removed by repeated iterations. In the case of dieless drawing the deforming zone is given a simple arbitrary shape (for example, it is given a conical outside surface), the nodes are all given velocities conforming to this shape, and the elements start with some guessed viscosity. Then the program recalculates the nodal velocities, from which a new outside profile is calculated and the strain rates through the body are estimated. From the new strain rates and the temperature distribution viscosities can be calculated for each point in the body. Then the program returns to recalculate the nodal velocities and continues repeating the calculations until the results begin to converge. Thus the

two main unknowns at the beginning of the program, the viscosities throughout the deforming zone and the shape of the external surface, are found and the velocity distribution is also calculated. The method of determining viscosities from temperature and strain rate data is described in Section 3.2, while the method for finding the profile of the forming zone is discussed below in Section 5.2.

For constant strain rate triangular elements the following technique can be used to determine the values of B_{ijkN} in the equation

$$\dot{\epsilon}_{ij} = B_{ijkN} v_{kN} \quad (4.16)$$

Suitable local coordinates for such an element can be given as

$$L_i = (a_i + b_{\alpha i} x_{\alpha}) / 2A$$

The Roman subscripts taking the values 1,2,3 for the three nodes, the Greek subscripts have the values 1,2 for the two coordinate directions (this being a two dimensional problem). x with a single subscript is a global, variable coordinate and A is the area of the triangle, and

$$\begin{aligned} a_i &= x_{\alpha j} x_{\beta k} - x_{\alpha k} x_{\beta j} \\ b_{\alpha k} &= x_{\beta j} - x_{\beta k} \end{aligned} \quad (5.1)$$

where $x_{\alpha i}$ is the coordinate x_{α} for node i , and for equations

5.1 only, $i \neq j \neq k$, $\alpha \neq \beta$. The velocity at any point in the element can then be defined as

$$v_{\alpha} = L_i v_{\alpha i}$$

where v_{α} is the velocity in the x_{α} direction and $v_{\alpha i}$ is the velocity in the same direction at node i . Since the dieless drawing program always uses an axisymmetric formation let

$x_1 = z$ and $x_2 = r$. Then

$$\dot{\epsilon}_r = \frac{\partial v_2}{\partial x_2} = \frac{b_{2i}}{2A} v_{2i}$$

$$\dot{\epsilon}_z = \frac{\partial v_1}{\partial x_1} = \frac{b_{1i}}{2A} v_{1i}$$

$$\dot{\epsilon}_{rz} = \frac{1}{2} \left(\frac{\partial v_1}{\partial x_2} + \frac{\partial v_2}{\partial x_1} \right) = \frac{1}{4A} (b_{2i} v_{1i} + b_{1j} v_{2j})$$

$$\dot{\epsilon}_t = \frac{v_2}{x_2} = \frac{L_i v_{2i}}{x_2}$$

$\dot{\epsilon}_t$ still contains variables L_i and x_2 which can be removed as follows.

The area integral of L_i is

$$\int_A L_i dx_1 dx_2 = \frac{A}{3} \quad (\text{see Desai and Abel(25)})$$

Thus the average value of L_i over the element is $1/3$ and the average value for x_2 is the radius of the centroid of the triangle, $\bar{r}$. Thus an approximate expression for the average of $\dot{\epsilon}_t$ is

$$\dot{\epsilon}_t = \frac{1}{3\bar{r}} (v_{21} + v_{22} + v_{23})$$

The Gauss-Seidel iterative technique of solving the equation

$$\underline{K} \underline{v} = \underline{F} \quad (4.23)$$

is fully described (with the same symbols) on page 454 of Zienkiewicz(21).

5.2 The Boundaries of the Problem

The boundary conditions of the outside surface of the deforming zone are those of a free surface, but the position of this surface is unknown. On the input surface the nodes are given an axial velocity of $v_z = v_1$ and on the exit surface the nodes have $v_z = v_2$ where v_1 and v_2 are related by equation 1.1 to the input and output radii:

$$\frac{R_2}{R_1} = \sqrt{\frac{v_h + v_1}{v_h + v_2}} = \sqrt{\frac{v_1}{v_2}} \text{ if } v_h = 0$$

It is not normally necessary to fix the radial velocities of these nodes, for if the input and output surfaces have been chosen at a sufficiently great distance from the deforming zone, the radial velocities should be calculated by the program to be close to zero. In practice, however, the convergence of the program to a solution is aided if the radial velocities at the exit and input planes are set at zero. Figure 5.1 gives the position of the external node adjacent to the output end for the first nine iterations of the programme. The programs differ only in the detail of the boundary condition at the exit and input planes. In the diagram

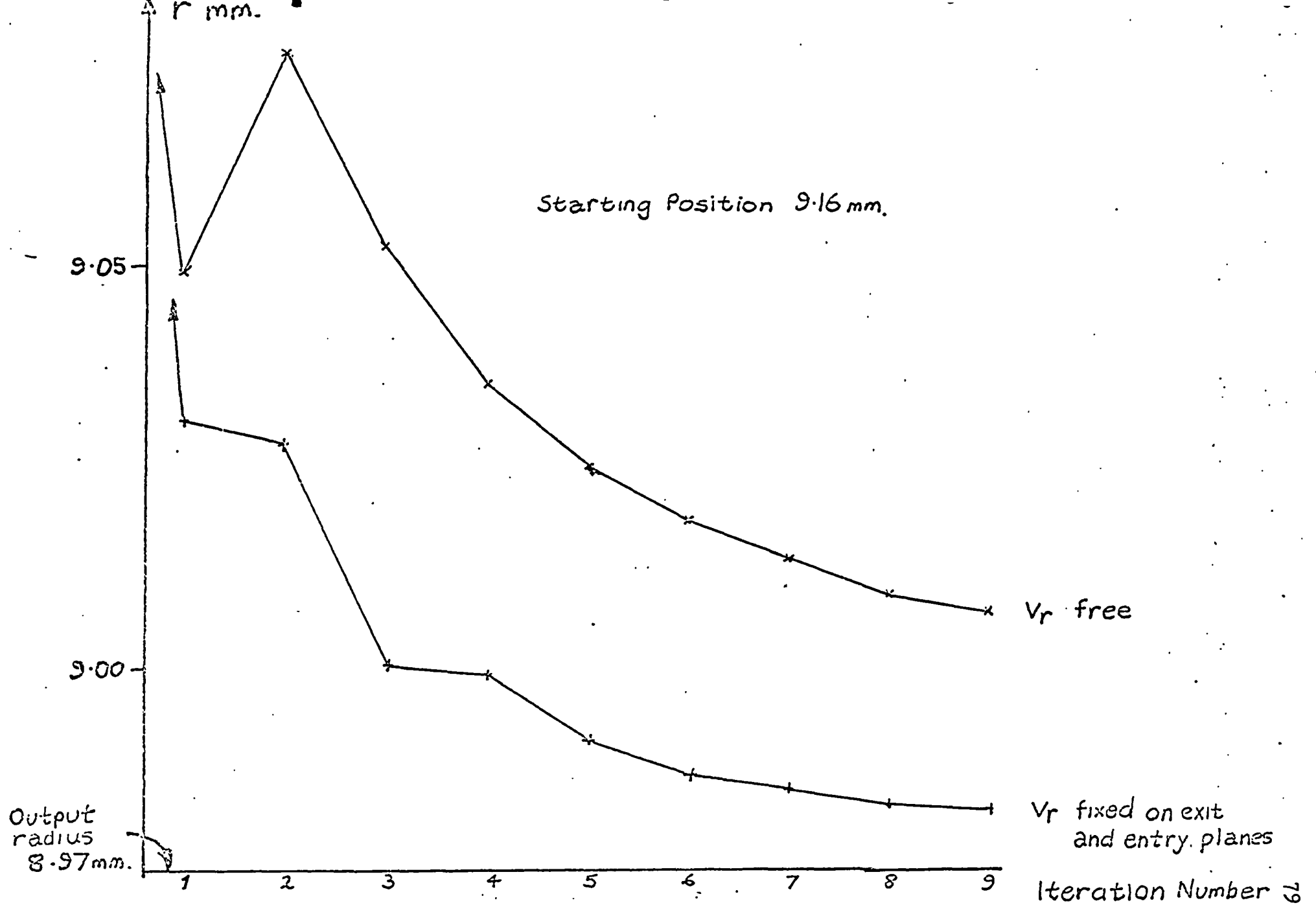


Figure 5.1 Position of the outside node adjacent to the output end for the first nine iterations.

the node starts at an arbitrary position of 9.16mm and as the iterations proceed it should approach but will always be greater than the desired output radius of 8.975mm.

From the figure it is clear that if radial velocities are fixed on the exit and input planes the solution converges faster and that the final solution will be closely approached at about nine iterations. There are some irregularities shown on the figure for the first few iterations, probably because the worst errors in the initialization of the deforming zone in the program are being removed.

The position of the external surface of the deforming zone must be calculated in such a way that it converges with continued iterations to a satisfactory result. The position must be such that the velocity at the nodes on the surface is parallel to the surface. Many schemes can be envisaged which will estimate the position of these nodes for a particular iteration from values calculated during the previous iteration. To describe some of the methods, the following notation will be used. r_n is the radius of the external node n , the axial distance between two adjacent nodes is h (assumed to be the same for all the external nodes), the ratio of the velocities at node n will be $t_n = (v_r/v_z)_n$ and there are K nodes along the boundary in question.

One method tried was to determine r_n from

$$r_n = r_{n-1} - \frac{h t_n}{K h t_n} = r_{n-1} - \frac{t_n}{K t_n} \quad (5.2)$$

$n=1 \qquad \qquad \qquad n=1$

and $r_1 = R_1$ (t_n is normally less than zero).

Figure 5.2 illustrates the effect this formation had on the profile. It can be seen that the profile may not be approaching an acceptable shape and that its progress towards a solution is slow. On the ninth iteration the second node from the entry side is shown as having a radius larger than the input radius R_1 , but this is a result that is never observed experimentally. This node has been caused to have a positive radial velocity and it indicates the complex interaction between the formulation for finding the external profile and the results of the finite element solution. Equation 5.2 probably provides an unsatisfactory answer because it does not explicitly conform to the condition that the velocity of each node should be parallel to the surface.

Another approach tried was to suggest that each set of three nodes should be connected by a parabola, for which the following formulation was used

$$r_n = (r_{n+1} + r_{n-1} + (t_{n-1} - t_{n+1})h)/2 \quad (5.3)$$

and $r_1 = R_1$, $r_K = R_2$.

Since r_{n+1} in equation 5.3 has not been determined, an iterative technique was used where the starting approximations of r_{n+1} were replaced by 'better' approximations by repeating the calculations until convergence was achieved. The results, displayed on figure 5.3, were completely

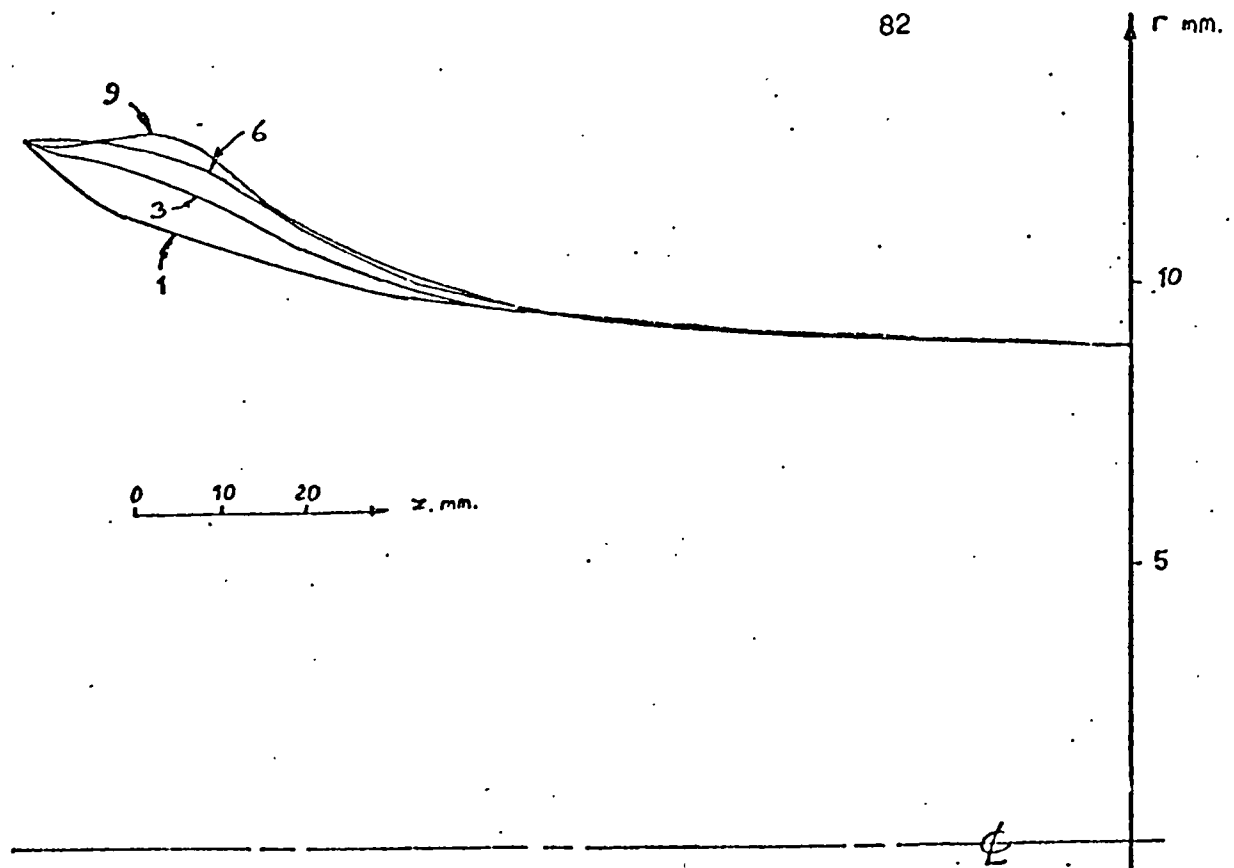


Figure 5.2 Iterations of the outside profile corresponding to equation 5.2

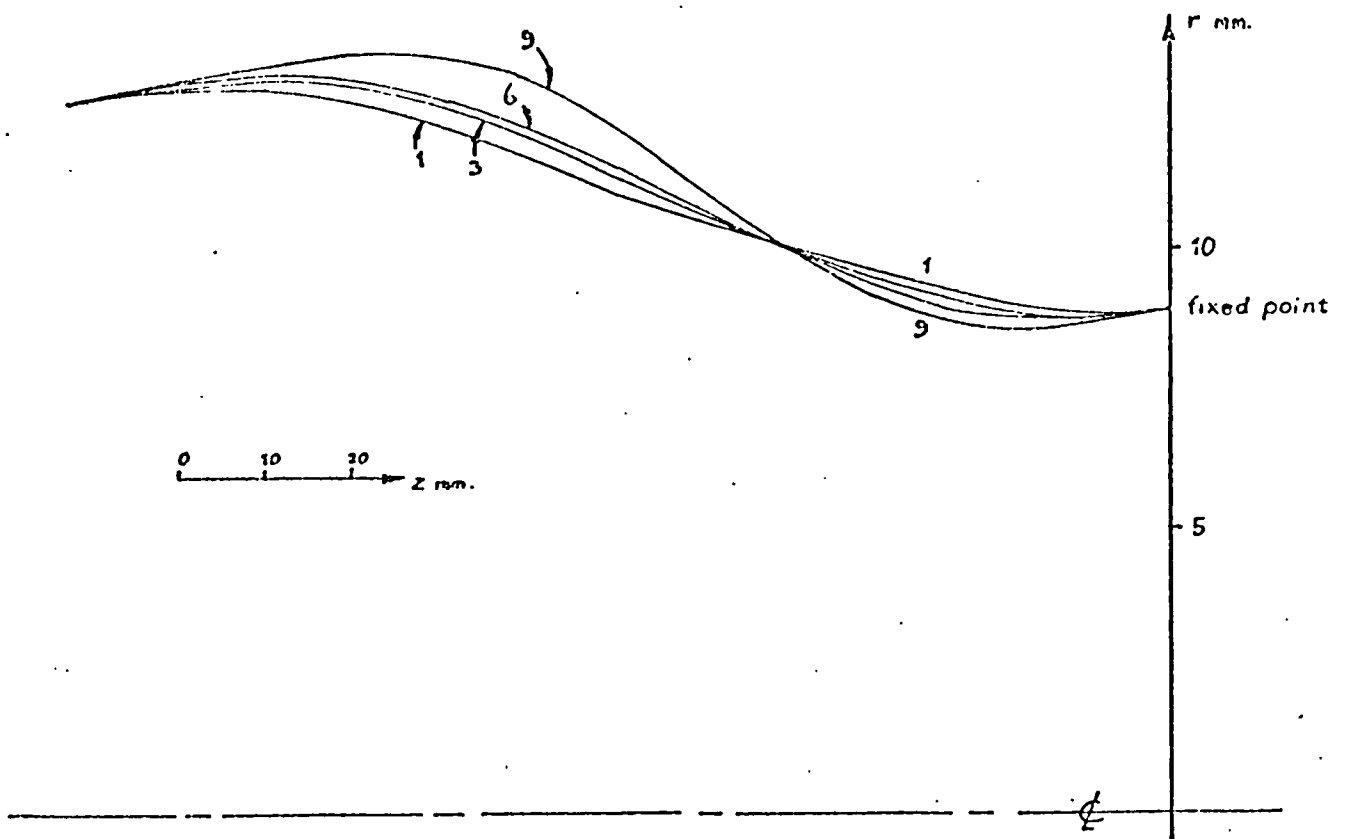


Figure 5.3 Iterations of the outside profile corresponding to equation 5.3

unsatisfactory, the method apparently forcing the profile to adopt an incorrect shape which appears to be a polynomial.

The most satisfactory and quickest method that was tried in this work is the method proposed by Alexander and Turner(4) which follows the formula

$$r_{n+1} = r_n + (t_n + t_{n+1})h/2 \quad (5.4)$$

and $r_1 = R_1$, (r_n is the most recent estimate).

This formulation is a specific attempt to satisfy the criterion that the velocity at the nodes should always be parallel to the surface. By making use only of the adjacent velocities it makes no distinct assertions to the kind of curve that connects the nodes. The method has no tendency to cause there to be positive radial velocities anywhere in the deforming region and using evidence such as presented on figure 5.1 it can be shown that the method is convergent at all nodes. Figure 5.4 presents results for this formulation using the standard conditions for the experiment which are listed on table 5.1. (The same conditions were used for figures 5.2 and 5.3 also.) The result presented in figure 5.4 can be compared with the experimental curves shown in figure 2.3, but exact agreement is not entirely to be expected. This is not only because of approximations inherent in the finite element method but also because the temperature distribution in the experiments may not exactly correspond with the temperature distribution

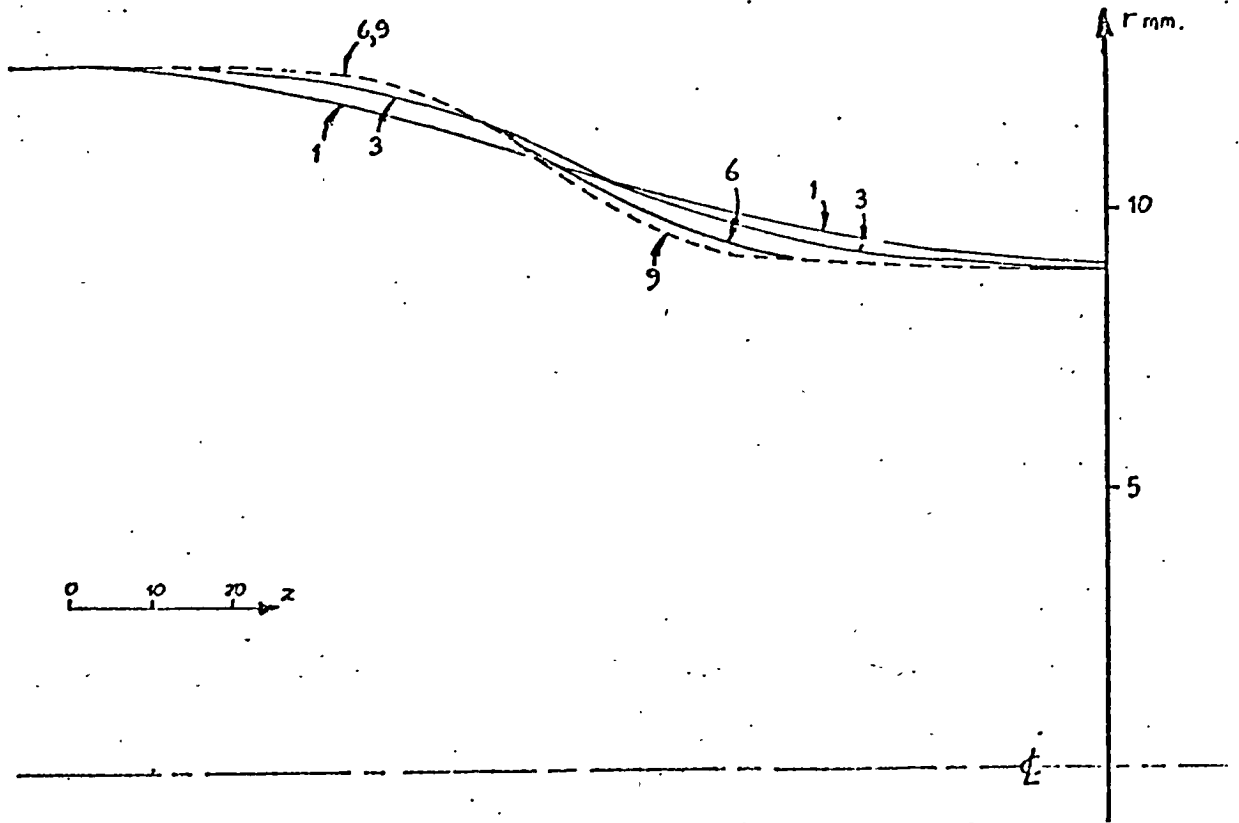


Figure 5.4 Iterations of the outside profile corresponding to equation 5.4

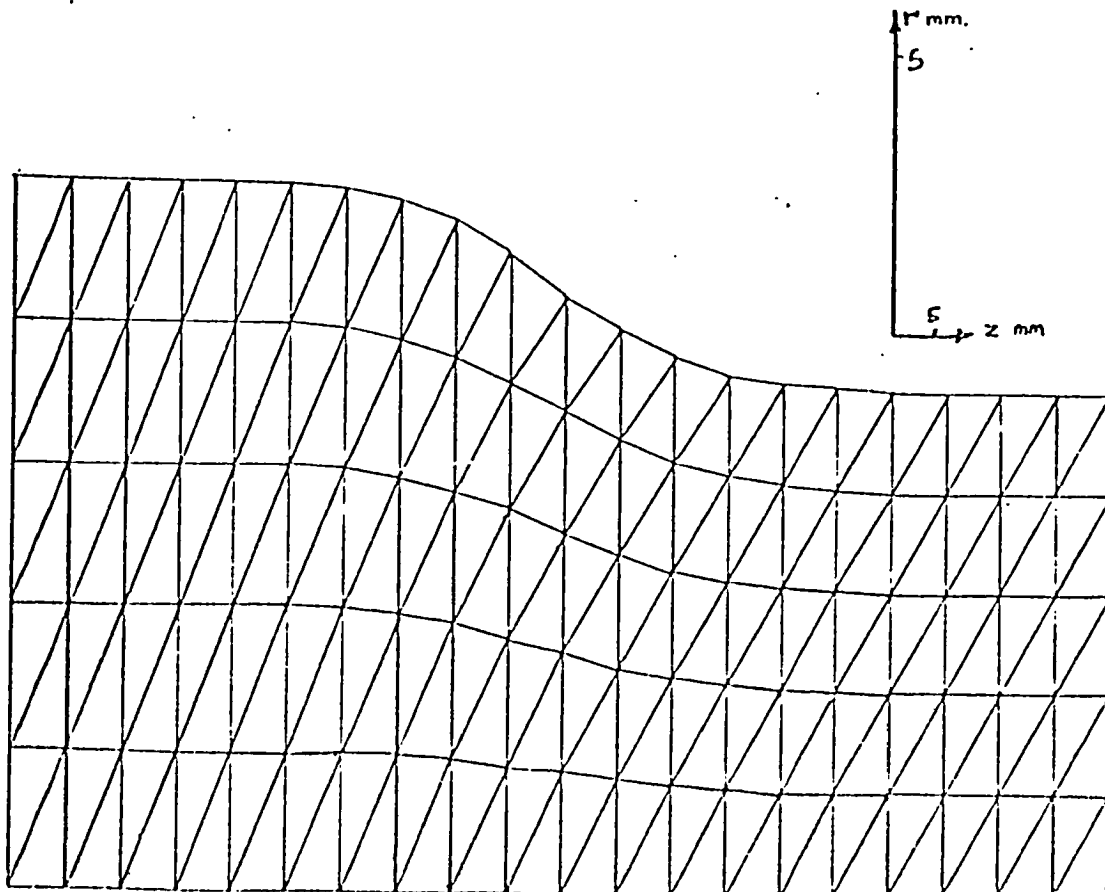


Figure 5.5 The appearance of the elements at the end of the iterations.

used for the calculations. (The difficulties in prediction of temperature distributions for dieless drawing has been discussed in Section 1.4).

The radial position of all the nodes in the deforming zone is always adjusted so that they remain always in the same proportion to the outer radius. The positions of all the nodes and elements on the ninth iteration of figure 5.4 are shown on figure 5.5. This operation is carried out in order to avoid grossly distorted elements being created. The axial positions of the elements never change and their radial positions only a little. Thus it is assumed that the temperature of each element (which is constant within each element) remains the same throughout the whole solution.

TABLE 5.1

Material properties	Lee and Backofen(13) data for Ti 6Al4V
Temperature profile	Following Turner(2) shown on figure 5.5
Input radius	12.7mm
Output radius	8.98mm
Input velocity	1mm/sec
Deformation zone	140mm long, 200 elements in 20 sets of 10 triangular rings (see figure 5.5)

5.3 Optimum Penalty Function

As presented in paragraph 4.5.3 the penalty factor, α , is not given any specific value except that to ensure that the constant volume requirement is closely obeyed it should be as large as possible. However, as the penalty function becomes larger the terms related to it in the stiffness matrix ($\underline{K}$ in equation 4.23) will be much greater than those terms related to the energy rate items. This can be seen in equation 4.27, where the integral multiplied by α will be much larger than the other terms. The maximum allowable value of the penalty function will be determined by the precision obtained with a particular program, for the magnitude of the energy rate terms must be much greater than the rounding off errors on the large penalty function term.

However, the technique by which equation 4.23 is solved for the nodal variables also has an influence on the choice of penalty function. As noted in Section 5.1, the program used for dieless drawing has a Gauss-Seidel iterative solution technique. The progress of this technique towards convergence can be considerably hampered if terms vary considerably in magnitude, and thus as the penalty function is increased the number of iterations required to achieve a satisfactory convergence is increased. This effect is illustrated in figure 5.6 which shows that a much faster solution is obtained with $\alpha = 10^5$ than with $\alpha = 10^6$ (α is expressed in units of MPa sec).

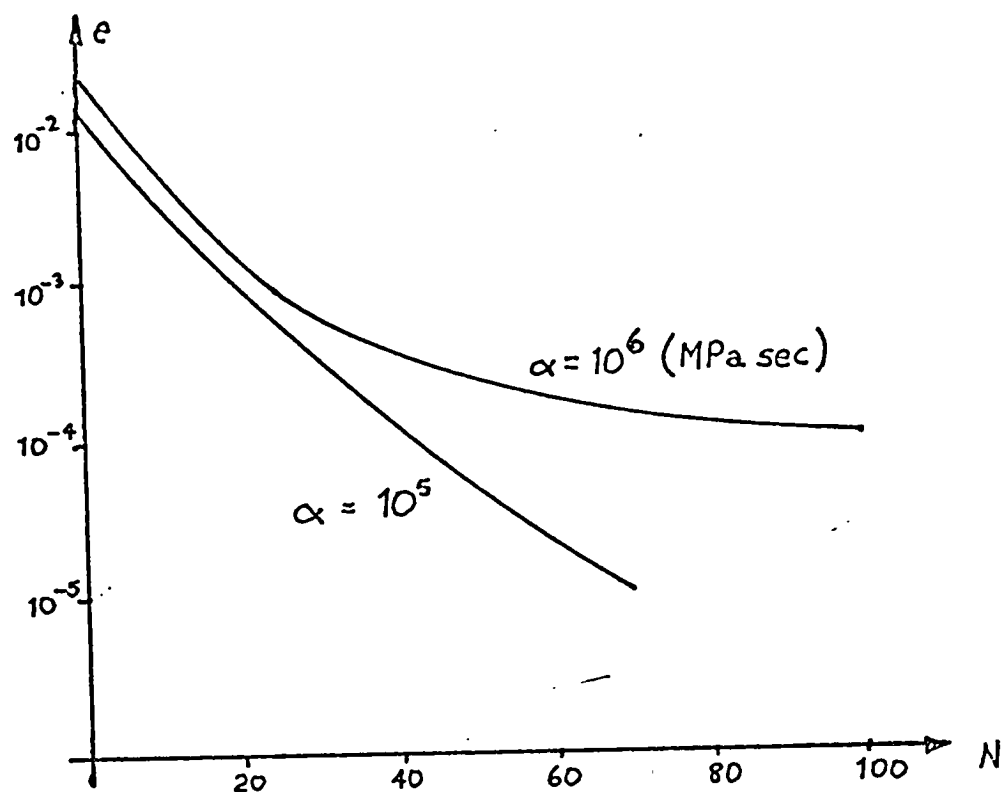


Figure 5.6 Plot of the fractional change in the unknowns, e , versus the number of iterations, N , in the Gauss-Siedel solution for two values of the penalty function, α .

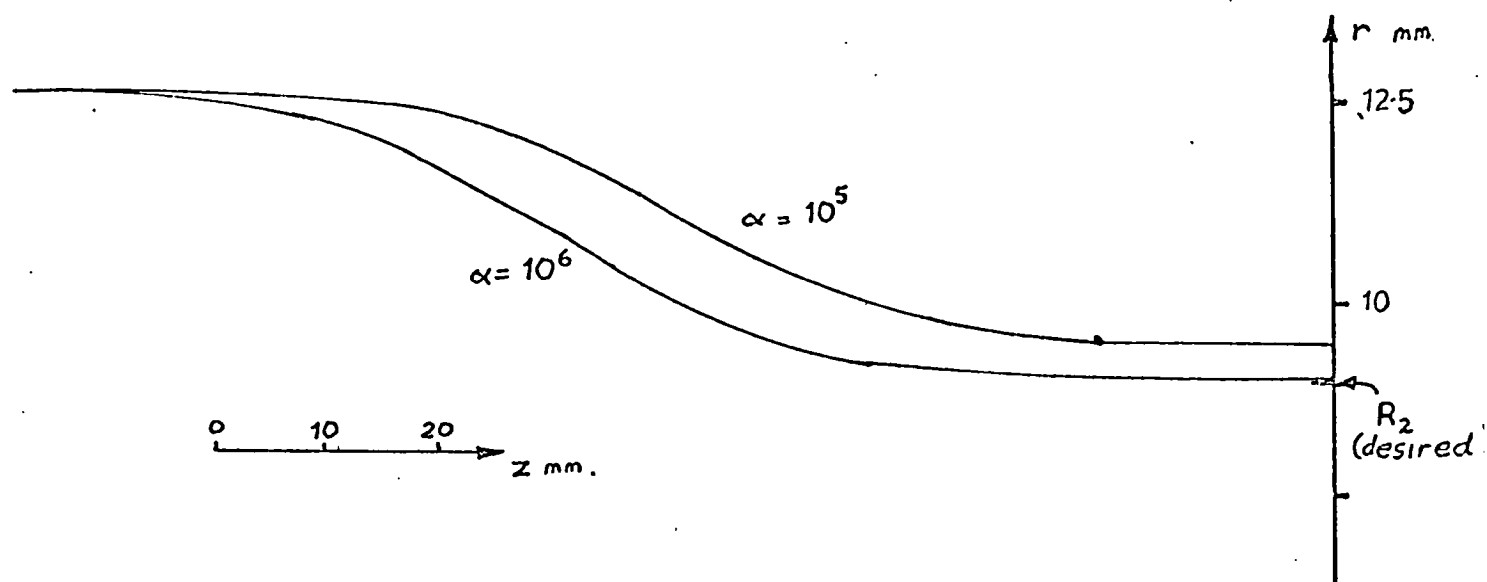


Figure 5.7 The outside profiles determined for two different values of the penalty function, α . (Corresponds to figure 5.6)

A compromise must be sought between the desire for a high value of α for constant volume requirements and a low value for fast Gauss-Seidel solution. In the case of dieless drawing α was lowered to a point where the constant volume condition as defined in equation 1.1 was still satisfied acceptably.

An unusual feature of the method of estimating the position of the external surface devised by Alexander and Turner(4) is that the only outer radius that need be specified throughout the calculations is the input radius, R_1 . Thus if the calculations determine a value of the exit radius R_2 that conforms to equation 1.1, then the condition of constant volume is being met. This is a sensitive method for confirming both that the Alexander and Turner method and the penalty function method are behaving satisfactorily. Figure 5.7 illustrates the outside profile obtained with $\alpha = 10^5$ and $\alpha = 10^6$. As can be seen, the solution is much better for $\alpha = 10^6$, the exit radius being within 1% of the expected value. With higher values of α the exit radius can be made to agree more exactly, but the improvement is barely detectable and there is a very significant increase in computing time required. For these reasons α was chosen to be 10^6 for the dieless drawing work. As will be seen later, in the solution for isothermal forming with the Gaussian elimination technique of solution, larger values of α could be used without increasing computing time or affecting accuracy.

5.4 Strain Rate Distribution and Rigid Regions

Outside the heated deforming region, the material is relatively cold. In these regions the yield stress of the material is high and no plastic deformation occurs. Because the stress encountered is very low it may also be safely assumed that there is very little elastic strain in this material and the regions can be considered to be rigid. Strictly the rigid regions should be removed from the analysis, but in the case of dieless drawing this would involve considerable complications because their exact positions are unknown. If their positions were to be determined, a trial boundary would have to be set and then tested against the solution solution within the plastic region, as illustrated by Shah and Kobayashi(43), and this process repeated until a satisfactory convergence was determined. In the circumstances of dieless drawing, yield data is not readily available in any case because yield is caused in the material by rising temperature and not by rising stress. Hence the program was developed so that results for elements which are found to be located in rigid or nearly rigid regions would not affect the general validity of the analysis.

The basis of this is that the viscosity becomes very large at the strain rate or temperature fall. The equation used for extrapolation of the data (see Section 3.2) yields a relation for the viscosity

$$\eta = \frac{1}{3} C' \exp\left(\frac{Q'}{T}\right) \dot{\epsilon}_z^{n-1} \quad (5.5)$$

the dashes indicating where the constants have been altered from equation 3.3, and n , the strain rate sensitivity index, is less than unity. In the cold regions where T (and also $\dot{\epsilon}_z$) is low the viscosity becomes very large according to equation 5.5 and from

$$\dot{\epsilon}_{ij} = \sigma'_{ij}/2\eta \quad (3.1)$$

it is seen that the strain rates will become low as well. Elements which have a strain rate falling below an arbitrary value suitable to the problem can be identified as being rigid (as done by Shah and Kobayashi(43)), but since the strain rate in these elements is very low there is no real need actually to delete them from the analysis. However, any stress results within such elements must be treated with caution.

Figure 5.8 illustrates the effective strain rate distribution in the standard case as described on table 5.1. Those areas where nodes have a strain rate below (or rather above) $-.0001 \text{ sec}^{-1}$ have been shown as being rigid. This strain rate distribution is quite compatible with the results of Turner's experiments(2) and is a distinct improvement on the strain rate distribution reported by Turner for his finite element solution. Figure 1.2 shows a graph from Turner's thesis for stainless steel, which shows experimental results, simple tension results (a method described in Section 1.3

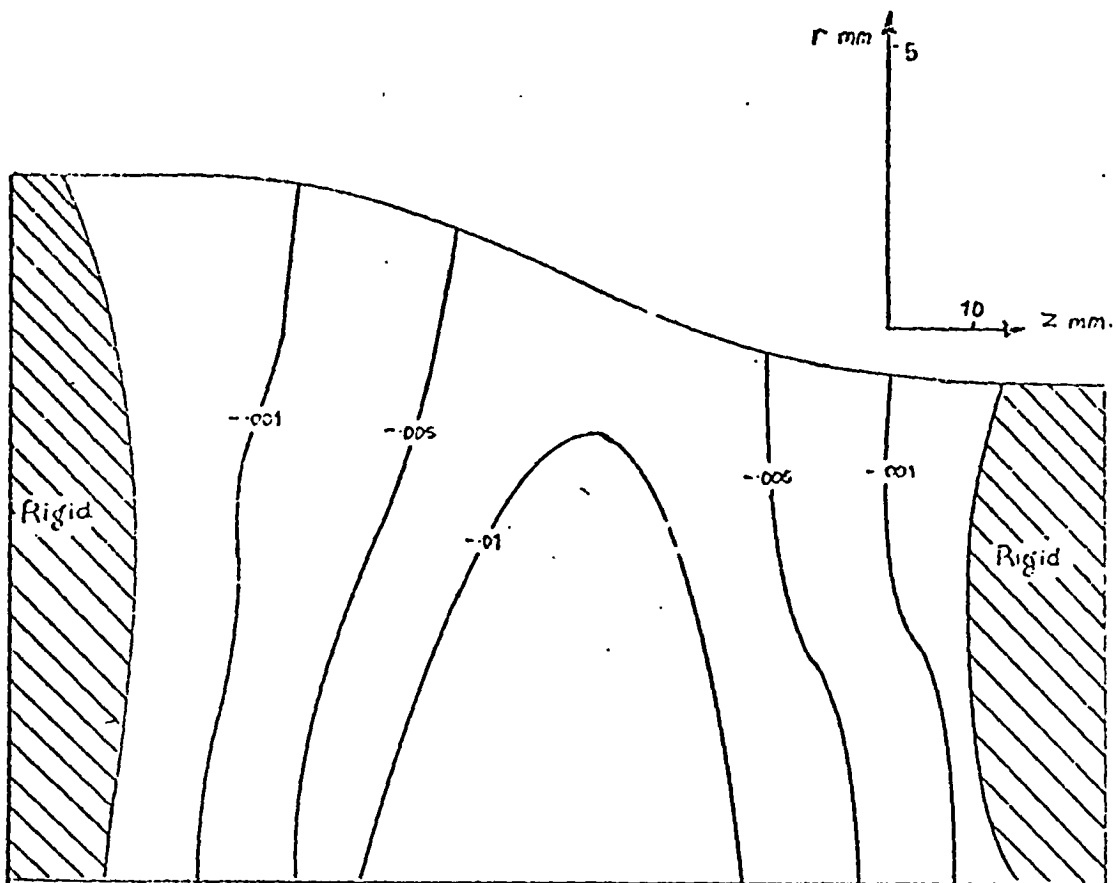


Figure 5.8 Effective strain rate distribution in the forming zone using the values given on table 5.1.

and comparable to the slab approach of Chapter 3) and the rather unsatisfactory finite element results.

5.5 Conclusions

Figures 5.9-5.11 indicate some variations from the conditions on table 5.1 that were tried. Figure 5.9 shows that there was little to be gained by increasing the number of elements. Figure 5.10 shows the results for a higher reduction and figure 5.11 shows the small effect obtained by varying the overall speed of the test. The same temperature distribution was used for each example.

The finite element work on dieless drawing was mainly undertaken to attempt to overcome problems discovered by Turner(2) in his analysis of the process. The results determined during the course of this work indicate that the finite element method used did yield satisfactory results. However, it is to be pointed out that the solution is not significantly better than the slab approach of Chapter 3 but consumes a great deal more computer resources. The slab approach is probably quite well suited to this problem because there are no sharp changes in flow, friction is absent and plane sections do not deform greatly from being plane.

Experience of the finite element method in solving the dieless drawing problem provided the basis for applying the method to the problem of isothermal forming which is considered later in the thesis. In the smooth flowing

situation of dieless drawing the use of constant strain rate triangular elements seems to be satisfactory but in forging problems sharp discontinuities are experienced and may provoke difficulties with the simple elements. Also, whilst the Gauss-Seidel iterative solution seems to have some advantages for non-linear problems (because the solution need not entirely converge before the stiffness matrix is corrected as discussed in Zienkiewicz(21)) the effect the penalty function has on the solution technique suggested that an elimination scheme ought to be investigated (see Section 5.3). For these reasons a new program was sought for the later investigations.

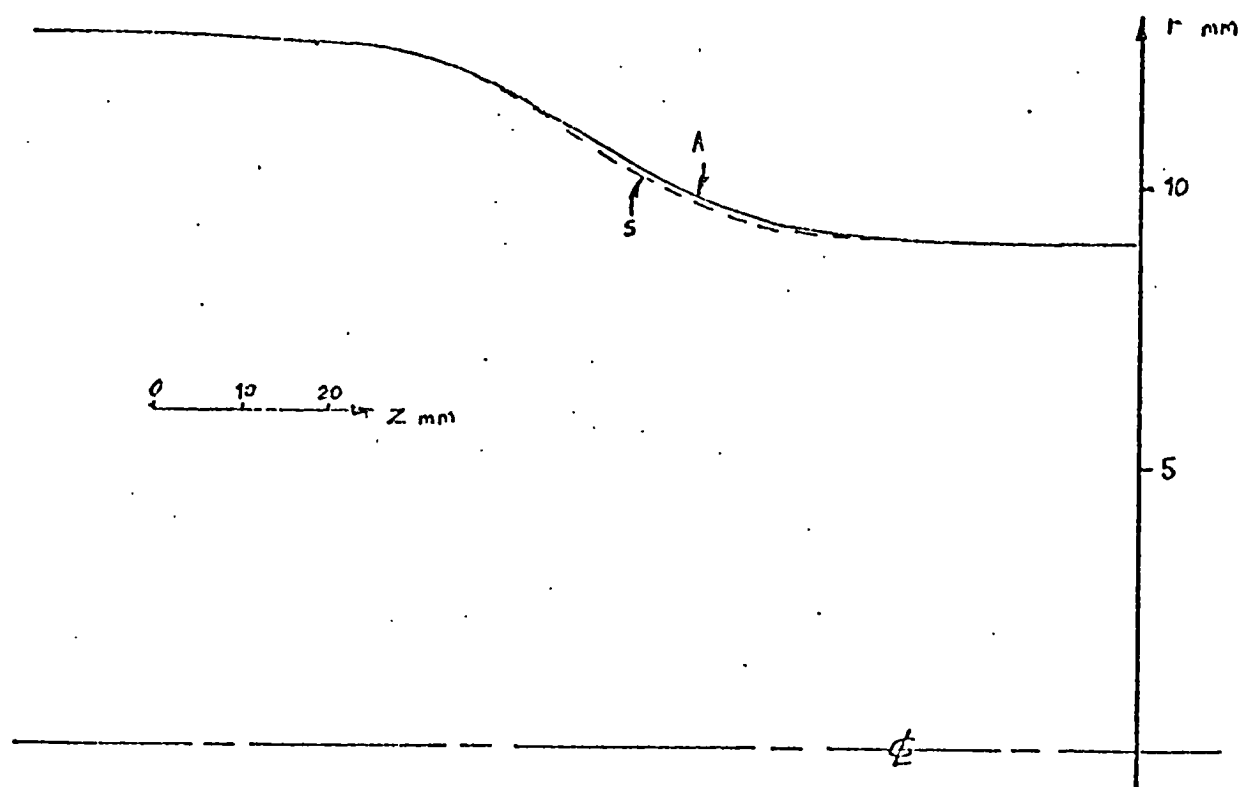


Figure 5.9 The effect of changing the number of elements (data is given on table 5.1).

S: 200 elements

A: 320 elements

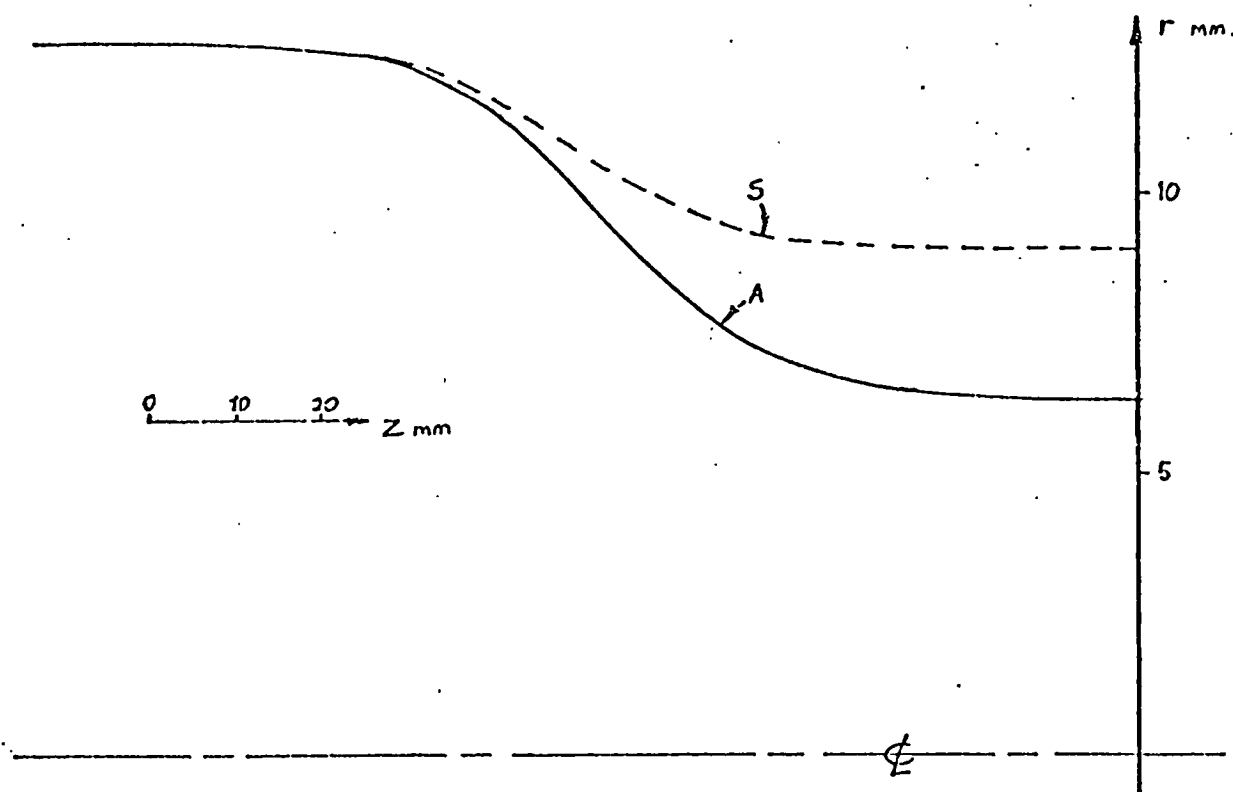


Figure 5.10 The effect of changing the reduction (data for all other parameters is given on table 5.1).

S: 50% reduction in area

A: 75% reduction in area.

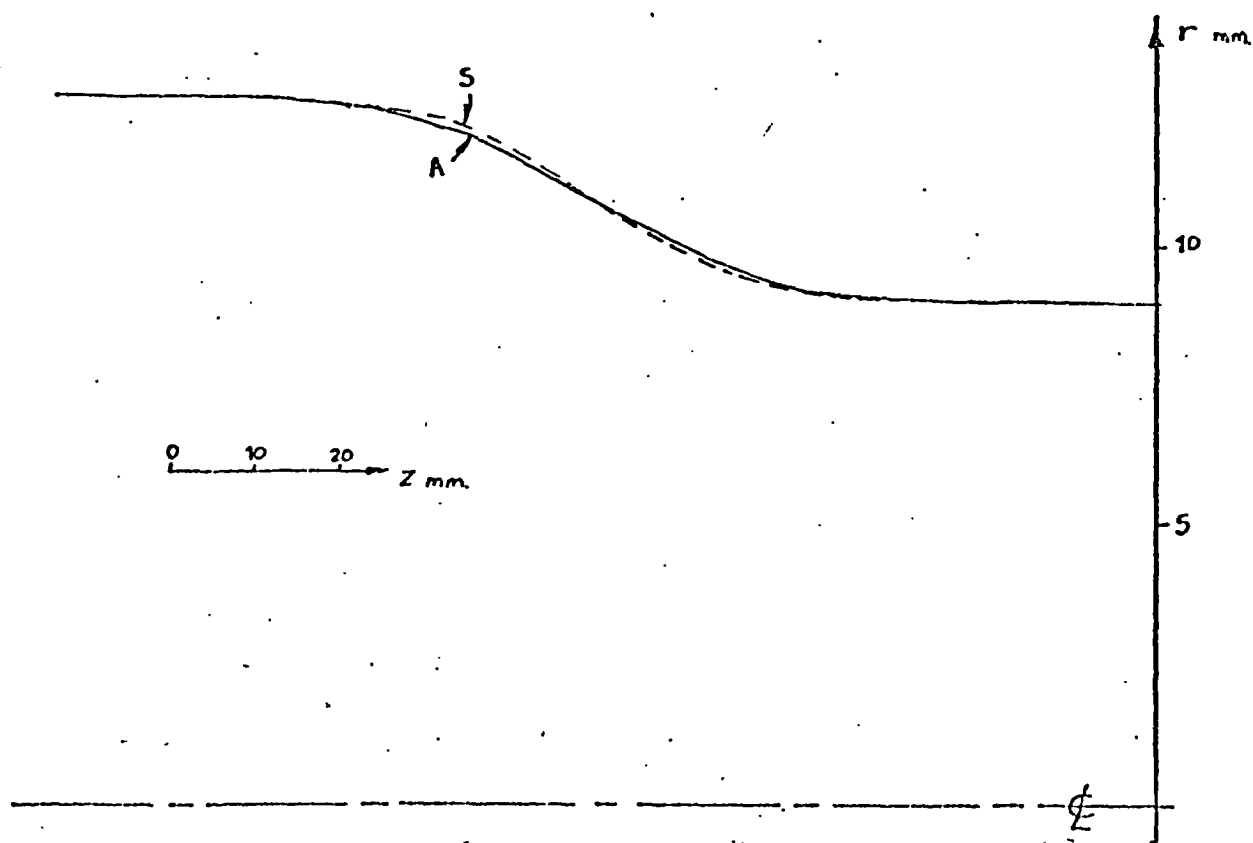


Figure 5.11 The effect of changing the input velocity, v_1 , without changing the reduction (data is given on table 5.1).

S: $v_1 = 1.0 \text{ mm/sec}$
 A: $v_1 = 0.1 \text{ mm/sec}$

CHAPTER 6

ISOTHERMAL FORMING - INTRODUCTION AND EXPERIMENTAL

- 6.1 Introduction
- 6.2 Equipment
 - 6.2.1 Specimens
 - 6.2.2 Dies
 - 6.2.3 Heating Techniques
 - 6.2.4 Press and Die Assembly
 - 6.2.5 Instrumentation
- 6.3 Cylindrical Compression Tests
- 6.4 Lubrication
- 6.5 Profile Dies
- 6.6 The Isothermal Forming of Steel

Chapter 6 - ISOTHERMAL FORMING - INTRODUCTION AND
EXPERIMENTAL

6.1 Introduction

The term Isothermal Forming is used here to indicate a process in which a material which is to be forged remains at a uniform high temperature throughout the process. The term Creep Forming is also used since the process is normally carried out at slow strain rates but in fact the strain rates need not necessarily be so slow that they should really be described as creep. The basic idea behind isothermal forming is that at high temperatures and slow strain rates metals tend to deform at low stresses. In contrast to normal hot forging the dies are heated throughout the process to the same temperature as the billet and the forging is completed in one slow loading. As a result, the material has time for extensive deformation, does not cool and harden in regions where it is thin or in contact with the dies and can conform accurately with the dies. As a result, the dimensional accuracy of the product is very good and only a minimum of further working and surface finishing is required to produce a finished product. The dies are subject to lower stresses and thus may not wear at a high rate if they are made of a material that is sufficiently hard and oxidation resistant at the temperature of forging. The process is specially attractive if the production of scrap material from the work-pieces creates significant increases in the cost of the components, as it does, for example, in the case of expensive

titanium alloys (see Alexander(54)). Isothermal forming of titanium alloys has been discussed in patents(55,56) and papers(1,57-63) and the process has also been investigated by Russian workers for the forming of tool steels(64).

The aim of the present experimental work was to examine the process so that a successful practical and theoretical model of the process could be established. No attempt has been made to simulate the complicated forgings that have been produced in various industrially orientated studies(60,61), principally because the dies for such work are very expensive. Instead, tests which are easy to analyse, such as cylindrical and annular compression tests and simple profiled dies, were carried out.

6.2 Equipment

The experimental arrangements were designed to be similar to industrial practice of isothermal forming. Thus specimens of a reasonably large size, up to 75mm diameter, could be tested and an efficient heating and loading cycle were employed. However, this approach meant that some of the detailed care which is required of precise experiments into material properties had to be sacrificed.

6.2.1 Specimens

The specimens were normally cut from round bars of Ti6Al4V which is a titanium alloy with the composition 6%Al, 4%V, 0.08%C, low Nitrogen and Hydrogen, remainder Ti. This

material was chosen for specific study because it is one of the most commonly used aerospace alloys of titanium. This alloy becomes single phase above 950-980°C but below this temperature a two phase alpha and beta structure is stable. At elevated temperatures the material is often identified as being 'superplastic' and the properties are strongly strain rate dependent (see Lee and Backofen(13), Wu and Lowrie(15) and Fix(1)). However, if the material is formed above the transition temperature of 980°C, cooling to room temperature can present problems as there is likely to be precipitation of alpha phase which will embrittle the product. As a result the alloy is best formed at about 950°C where it is both very soft and still below the single phase region.

Steel specimens were also tested as is described in Section 6.6

6.2.2 Dies

To form the titanium specimens it is necessary to use dies that are sufficiently hard and wear resistant at 950°C. The material chosen for the dies was a nickel alloy designated IN-100 which was received in sections of cast bar with the nominal diameter of 76mm. This material is commonly used by other workers in isothermal forming and has the chemical composition 10%Cr, 15%Co, 3%Mo, 7%Ti, 3.5%Al, 1.0%V, 0.06%Zr, 0.18%C, 0.015B and balance Ni. When cast it has an extremely hard acicular grain structure and retains its hardness up to and beyond a temperature of 1000°C.

To produce different shaped dies of the material the main machining was carried out with EDM (spark erosion) with the final finish being obtained by conventional grinding or machining with tungsten carbide tools. The shapes were either flat or with a conical hole as discussed in Section 6.5^{*}.

6.2.3 Heating Techniques

The method used for heating was an induction furnace, which is the usual technique used for isothermal forming in industry. The advantages of this method lie principally in rapid heating and ease of application to the specimen and dies. The machine used was a Raydyne induction set with power variable up to 15kW and a frequency fixed at 3kHz. This frequency produces a depth of heating in the region of 25mm as determined from graphs in Brown et al.(9) and thus enables 'through-heating' of specimen and dies of between 25 and 75mm in diameter. The current is carried to the water cooled and insulated copper heating coil by flexible water cooled leads. These leads are required to be as short as possible and next to each other to reduce energy losses. Total heating time from room temperature to 950°C for the dies and specimens was 10 to 15 minutes. The temperature distribution is not entirely uniform because there is a proximity effect

^{*} All die and specimen materials were kindly provided by Rolls Royce (1971) Ltd.

which occurs with induction heating, where the material closest to the inductor is heated most effectively. However, by control of the heating cycle and the shape of the coil the effect can be reduced so that the outside of the 75mm diameter dies differs by no more than 50°C from the interior of the specimen. The heating method was standardized to minimise variations between different specimens and temperatures were continuously measured by a thermocouple located 2mm from the surface of the die inside the top die (see point X on figure 6.1), but experiments were also attempted with other thermocouples attached to surface of the specimen. A coil shaped as a double cone (illustrated on figure 6.1) was used so that the diameter of the coil in the region of the smaller diameter specimen was smaller, a feature which aids uniformity of heating.

Another heating technique was tested, namely an electrical resistance furnace was placed on the press and the forging occurred inside the furnace. The temperature distribution was not improved significantly when using this method, but the length of the heating time required (6 hours) caused heating of the die holding arrangements and resultant oxidation and deterioration of their mechanical properties. This method was abandoned because of such difficulties.

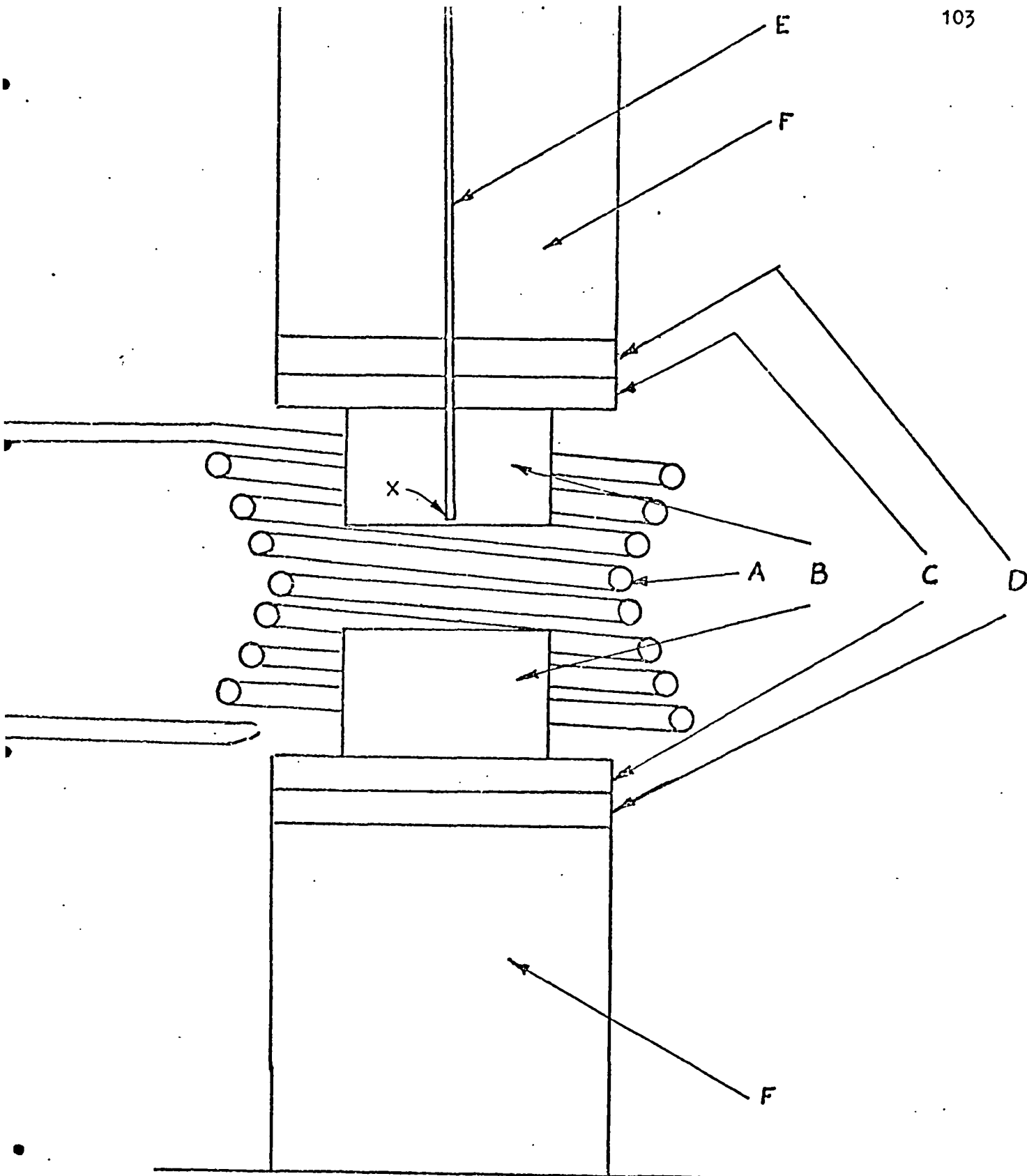


Figure 6.1 Experimental arrangements.

- A: Induction coil, B: Nimonic Dies,
- C: Stainless steel backing plates,
- D: Compressed asbestos insulating disks,
- F: Mild steel extension pieces,
- E: Temperature sensing hole,
- X: Thermocouple junction point.

6.2.4 Press and Die Assembly

The press used was a standard 50kN Denison hydraulic testing machine. This machine enabled a constant crosshead velocity to be obtained or alternatively a constant load to be applied to the specimen. The die assembly was mounted between the compression heads of the press and is composed of two large mild steel extension pieces (marked F on figure 6.1) of diameter 125mm, which served to increase the working area around the specimen and act as a heat sink to save the press from heating up. Between these and the nimonic dies were located two plates of Syndaneo (compressed asbestos) insulating material, and two stainless steel plates to which the dies were attached. All the dies were interchangeable, and could be sited on the top or bottom platen.

6.2.5 Instrumentation

The final location chosen for the displacement transducer was for measuring the movement of the upper crosshead of the press relative to the lower crosshead. A large displacement linear variable displacement transformer (LVDT) was used to measure this displacement. The temperature gradients in the region of the dies led to an unsatisfactory performance of transducers located nearer to the dies, and so this final arrangement was chosen although it meant compensating the readings for the elastic deflection of the die assembly in the digital analysis. An error of about 5% was encountered in these readings. The load was measured

by a displacement transducer connected to the press load measuring mechanism. This mechanism was essentially a dead weight testing lever arm attached to the lower crosshead and no detectable errors occurred in tests against a proving ring. Load cells in the vicinity of the die assembly proved to be too much affected by heat. Temperature measurements were carried out by a thermocouple located in the hole X marked on figure 6.1. The signals from all three transducers were recorded autographically on an ultraviolet light recorder (UV recorder) and converted to punched cards for digital analysis on the computer. Graphs of displacement, time and load readings were produced by computer programs as were uniform stress and strain graphs for each test.

6.3 Cylindrical and Compression Tests

In order to gain some knowledge of the process it was decided to carry out cylindrical compression tests which afford the possibility of simple analysis.

Figures 6.2.1-6.2.3 are typical results from the compression of a cylinder of Titanium 6Al4V to a very high strain, the graphs being part of the output from the computer program mentioned above. The material was formed in this case at 980°C, a graphite lubricant (in water base) was applied lightly to the specimen and the crosshead speed was set at .3mm/sec. The original cylinder was 25.42mm high and 25.45mm in diameter but after compression it was only

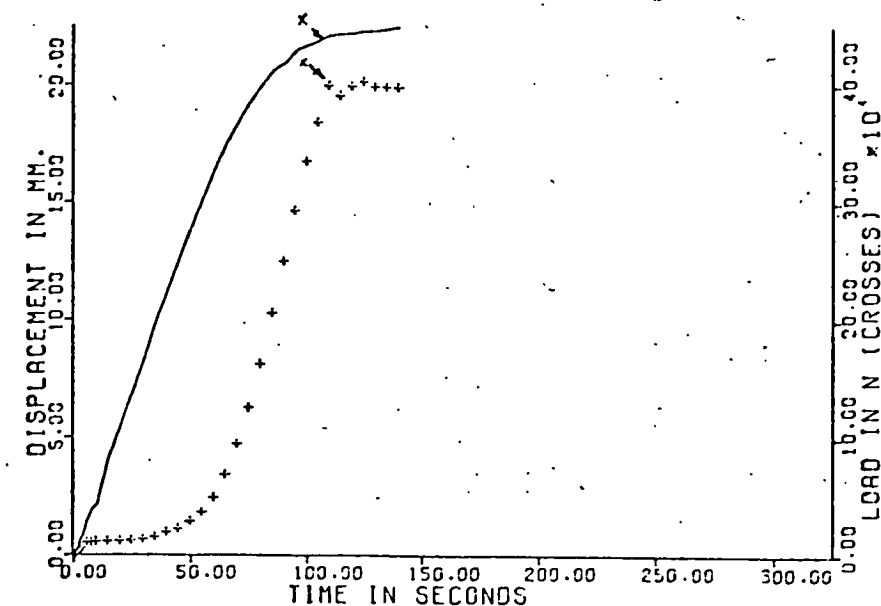


Figure 6.2.1

Load versus time (indicated by crosses, load is shown on right axis).
Displacement versus time.
The specimen is described in section 6.3

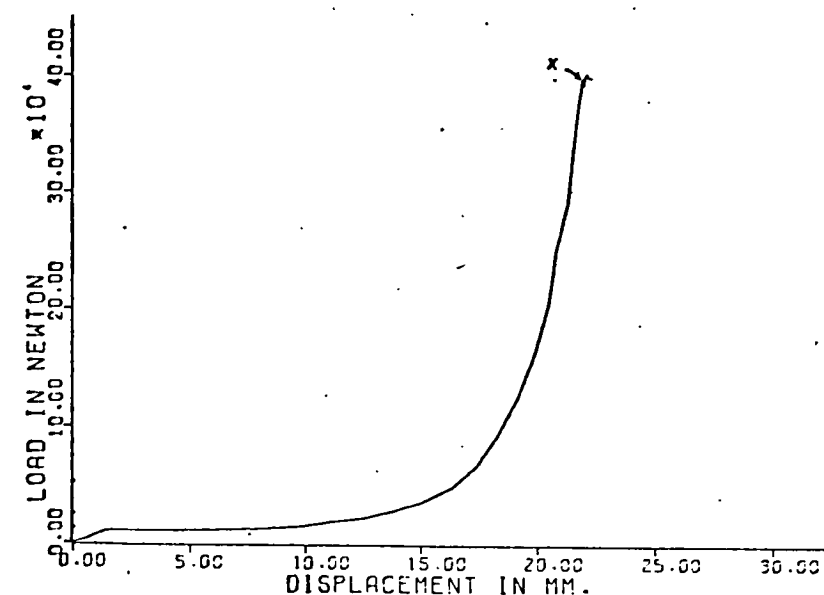


Figure 6.2.2

Load versus displacement.
(A replottting of values from figure 6.2.1)

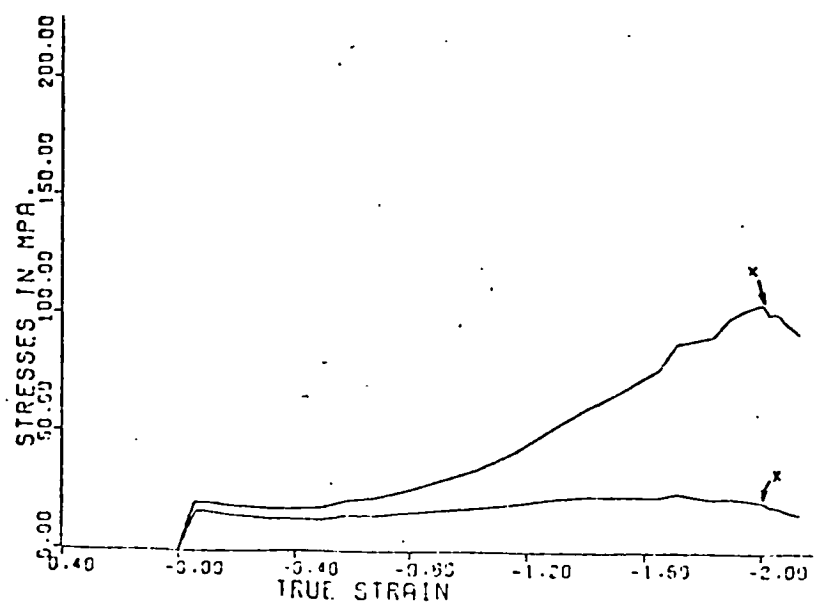


Figure 6.2.3

Mean axial stress versus logarithmic axial strain.
(Calculated from data on figure 6.2.2 and the starting geometry of the specimen.)
Upper curve is directly calculated, lower curve includes the correction for friction effects discussed in section 6.3.

3.1mm high with a diameter of approximately 72mm. This represents a uniform axial true strain of -2.14 , and a reduction of 87.8% in height. On figures 6.2.1 and 6.2.2 two stages of deformation may be discerned separated by the crosses on the figures. The first stage is a smooth curve during which the crosshead moves with an approximately constant speed of .3mm/sec as can be determined from the solid line on figure 6.2.2. Most of the deformation was achieved in the first 10% of the load range (figure 6.2.1). In the region of the cross the machine approached its target load of 400kN. After the cross there is still some further straining at the constant load which may be described as creep forming, but only a small proportion of the displacement occurs in this region (about 2.2%). If figure 6.2.3 is examined it will be seen that quite a large amount of strain occurs after this, since at this stage the specimen is a disc about 3mm thick. The strain during the creeping period goes from -1.96 to -2.14 , or 8.7% of the total.

Figure 6.2.3 is a stress strain diagram for the same specimen, the upper curve on the diagram giving the average axial stress (mean pressure at the interface) versus logarithmic axial strain. Although the effects of specimen geometry are nearly eliminated on such a diagram, the results still include effects due to the friction between the die and the specimen. To evaluate these effects an upper bound analysis (see Avitzur(65)) was carried out and the results of this

are plotted as the lower line on figure 6.2.3. The analysis used assumed a velocity distribution in which the specimen always remains cylindrical, viz

$$u = \frac{rV_o}{2a} \quad , \quad v = \frac{zV_o}{a} \quad (6.1)$$

where the symbols are illustrated on figure 6.3. The friction at the boundary was assumed to be

$$\tau = m \frac{\bar{\sigma}_o}{\sqrt{3}} \quad (6.2)$$

where τ is the shear stress at the boundary, $\bar{\sigma}_o$ is the flow stress of the material (assumed constant), and m is the shear coefficient. Since very high friction effects are experienced, m is given its maximum value of unity. As can be seen from figure 6.2.3, these assumptions seem to analyse the situation for this specimen very well; the flow stress does in fact appear to be constant and the assumption of the velocity distribution appears to be satisfactory. In the region of the cross, the creeping region, the material appears to become softer, which is a reflection of the fact that at the slower strain rates encountered in this region, the flow stresses should be lower, because the material is strain rate sensitive (see paragraph 2.1). It is to be noted, however, that there is no evidence of strain hardening occurring at this temperature.

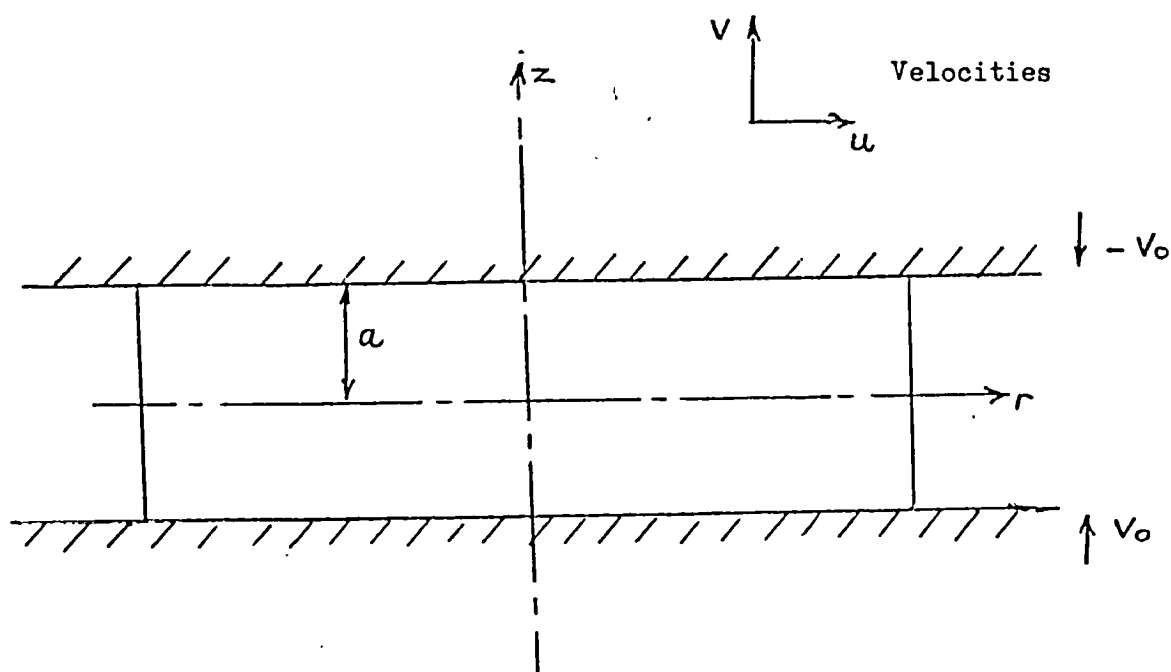


Figure 6.3 Notation for calculations in section 6.3.

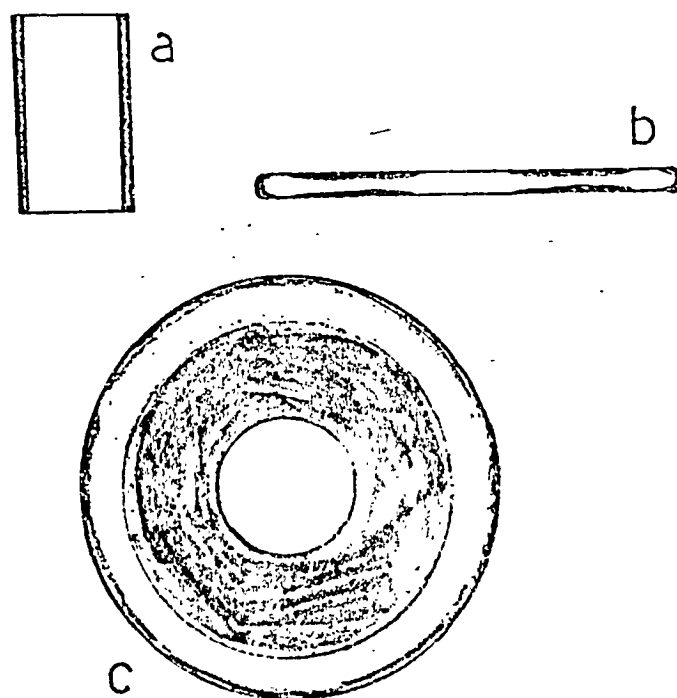


Figure 6.4 Diagram of the results of pressing a plasticine cylindrical specimen composed of two different coloured plasticines.

6.4 Lubrication

In the compression of lubricated graphite specimens, it was found that it could be considered that there was no slip of the specimens relative to the dies. Marks on the ends of the original specimen did not change their radius during the experiment and the large increase in area of the ends that occurs during the experiment is provided by the material from the sides stretching and rolling onto the dies. The graphite thus serves very little purpose in the process and, in fact, isothermal forming can be adequately performed without lubricant. However, in most cases in the experiments a thin film of graphite was applied to the specimen, because it is possible that it may aid the removal of the specimen from the dies and reduce die damage. The contact between the specimen and the dies is probably dominated by the hard oxide coatings on both the specimens and the dies, and as these are likely to come into contact they will impose a high apparent coefficient of friction (see Schey(66)). Plate 6.1 illustrates a series of compression tests with graphite lubricant, and shows the very strong barrelling experienced.

Plate 6.2 illustrates the top of a graphite lubricated specimen, but in this case the specimen has been grossly over-lubricated and there exaggerated effect of the lubricant. The specimen shown was originally 12mm high and 12.8mm in diameter but was deformed to 2.1mm thick and 40mm in diameter. The normal features shown on this plate are the interior

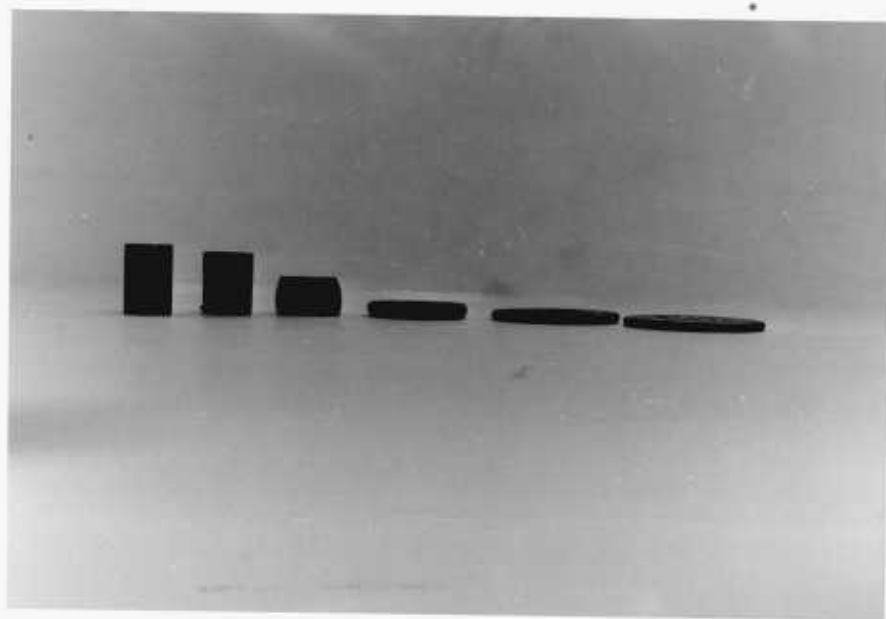


Plate 6.1 A series of isothermal compression tests with graphite lubricant. Original specimens were 12.8 mm in diameter and 19.05 mm high.

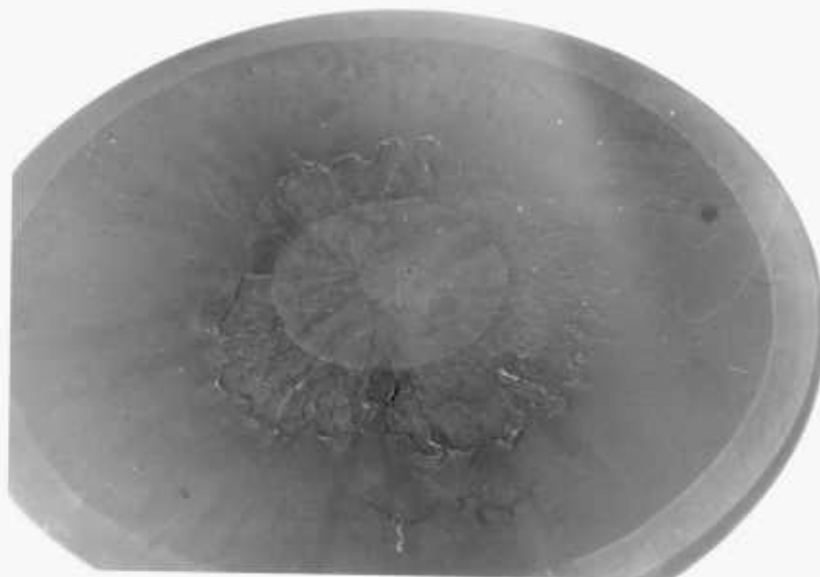


Plate 6.2 The top of a highly compressed cylinder showing the damage caused by an excessive application of graphite lubricant.

circular area which is still 12.5mm in diameter, and out from this a region of darker area and finally a lighter band surrounded by a dark edge. These last features are best understood when figure 6.4 is examined. This figure shows a two colour plasticine cylinder which has been highly compressed. The dark material which was originally on the sides (in a) becomes distended out into a disc (in elevation b and plan c) with some remaining at the very edge of the specimen. The light band (clearly seen in c) is caused by a thinning of the outer material as the interior material approaches very close to the surface of the specimen. The former outer layer is mainly trapped onto the dies at this high strain.

The unusual features of plate 6.2 are connected with the apparent sunburst appearance of the top on the specimen and the damage of the oxide coat. These features are caused by flow or slip of the lubricant as it is extruded radially out between specimen and die. This is caused by a friction hill being developed on the specimens as described by Avitzur (65) and the trapped lubricant is forced from regions of high pressure near the centre of the specimen to the outer regions of lower pressure. The flow of lubricant has been so strong that in some areas it has torn up the oxide coat and forced it outwards. This effect is to be avoided because it not only damages the specimen but also possibly marks the dies. As a result only a thin film of graphite should be applied to a specimen.

Another possible high temperature lubricant is glass, which can be obtained as a powder in alcohol. If a thick layer of low viscosity lubricant is applied to the ends of a cylinder in a compression test it can produce a reverse barrel of hour glass shaped specimen as shown by Douglas and Altan(67) and Hsu(68), and is illustrated by a series of glass lubricated specimens on plate 6.3 from the current experiments. The layer of molten glass has such a low viscosity that its radial extrusion during the compression test causes the specimen to deform outwards more at the die-specimen interface than at the centre of the specimen.

Ring tests of unlubricated, glass and graphite lubricated specimens are shown on plate 6.4. Such tests for friction coefficient were first developed by Male and Cockroft (69) and have been analysed by many other workers including Avitzur(65) and Hawkyard and Johnson(70). The top specimen on plate 6.4 is the starting ring which is 6.45mm high, with an outer diameter of 25.5mm and an inner diameter of 12.9mm. The two specimens on the left are not lubricated and in this case friction is so high that the hole in the middle eventually closes up at high reductions. Using the calibration of Hawkyard and Johnson(70) the shear coefficient in equation 6.2 appears to be equal to one (actually the points fall slightly above the prediction for $m = 1$, but this value is the maximum according to the theory). The centre specimen was lubricated by a thick layer of graphite. Apart from the damage on the surface of the specimen there is no apparent difference in

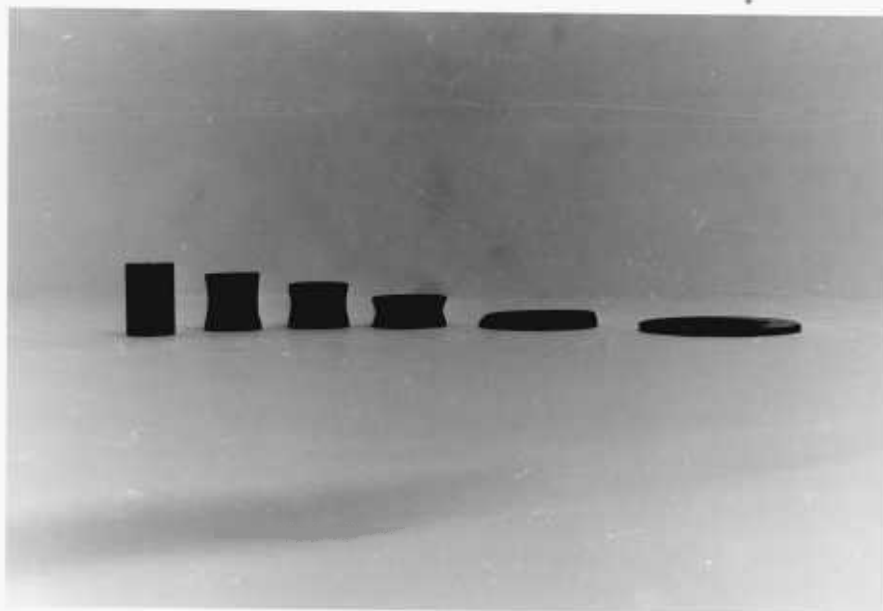


Plate 6.3 A series of isothermal compression tests with a thick glass lubricant. Original specimens were 12.8 mm in diameter and 19.05 mm high.



Plate 6.4 Some ring compression tests shown on an inch scale graph paper.

TOP Original specimen: 6.45 mm thick, outer diameter 25.5 mm, inner diameter 12.9 mm.

LEFT Two stages of compression with no lubricant applied.

CENTRE A graphite lubricated specimen. The excessive application of lubricant has caused some damage on the surface.

RIGHT Two stages of compression of glass lubricated specimens.

performance from the specimens with no lubricant. The two other specimens are lubricated with glass, and they are not so nearly circular as those without lubricant. This effect is caused by the uneven flow of the glass which can flow preferentially in small areas and thus cause these areas to be more distorted. This effect can also be seen to a lesser extent on thickly lubricated graphite specimens. For the glass lubricated rings the calculations of Hawkyard and Johnson indicate a shear coefficient of 0.06.

In general the glass lubricant is probably not satisfactory for isothermal forming when compared to a thin film of graphite. The surface finish on the specimen becomes rougher with glass lubricants and the specimens are less likely to be circular after forming. The glass solidifies as the specimen cools and thus has to be removed physically from the specimen. The specimens are also more difficult to remove from the dies, for at high temperatures the molten glass forms an adhesive mass which can cause an air lock and retard the removal of the specimen, while at low temperatures the solid glass must be fractured.

6.5 Profile Dies

The main profile die experiments have been carried out on two different dies. Both dies have similar dimensions but one die has a more generous fillet radius and taper angle than the other. The basic dimensions of the two dies are

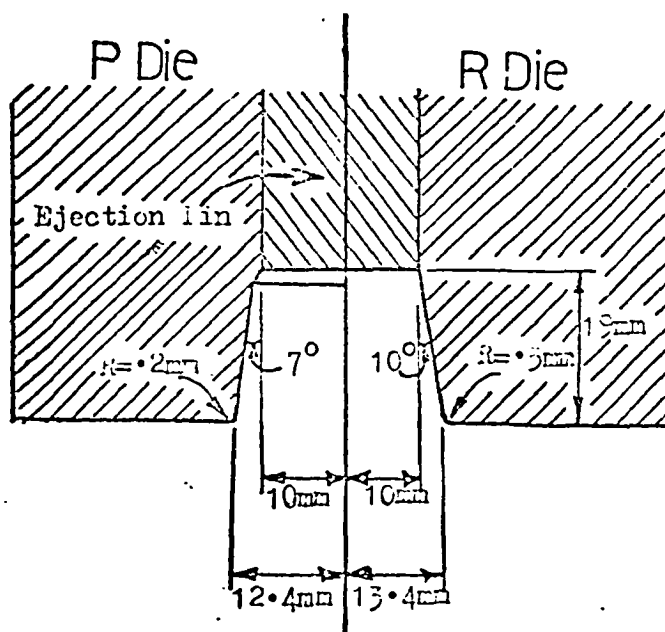


Figure 6.5 Details of the geometry of the P die and the R die.

illustrated on figure 6.5. The final product thus has a truncated conical region, hereafter termed the 'cap', around which there is formed a flange as the dies close. There is no noticeable difference in performance between these two dies (P die and R die). In view of the fact that there is no slipping of the titanium along the horizontal surfaces of the dies the fillet radius makes virtually no difference. There was no problem extracting the finished product from the dies when using a thin film of graphite lubricant, though glass provided considerable problems. A die with a taper angle of 2° was also used but removal of the specimens from this die was more difficult, requiring somewhat more force and even heat expansion of the die. With dies P and R a complete fill of the cap was always possible and a thin flash (0.05-0.2mm) was extruded up where the ejection pin did not fit exactly inside the die.

Plates 6.5-6.7 demonstrate the differences in forming stages when starting with three different billets. Die P was used with graphite lubricant at 950°C , and the three starting shapes all have the same volume ($16,500\text{mm}^3$). The first billet (on plate 6.5) has a diameter that is a little smaller than the ejection pin (19.2mm) and thus it fits into the bottom of the cap of the die. In the first stage the cap is completely filled and the bottom of the specimen is compressed and is thickening. At this stage the load begins to rise and the rest of the test is occupied by what is

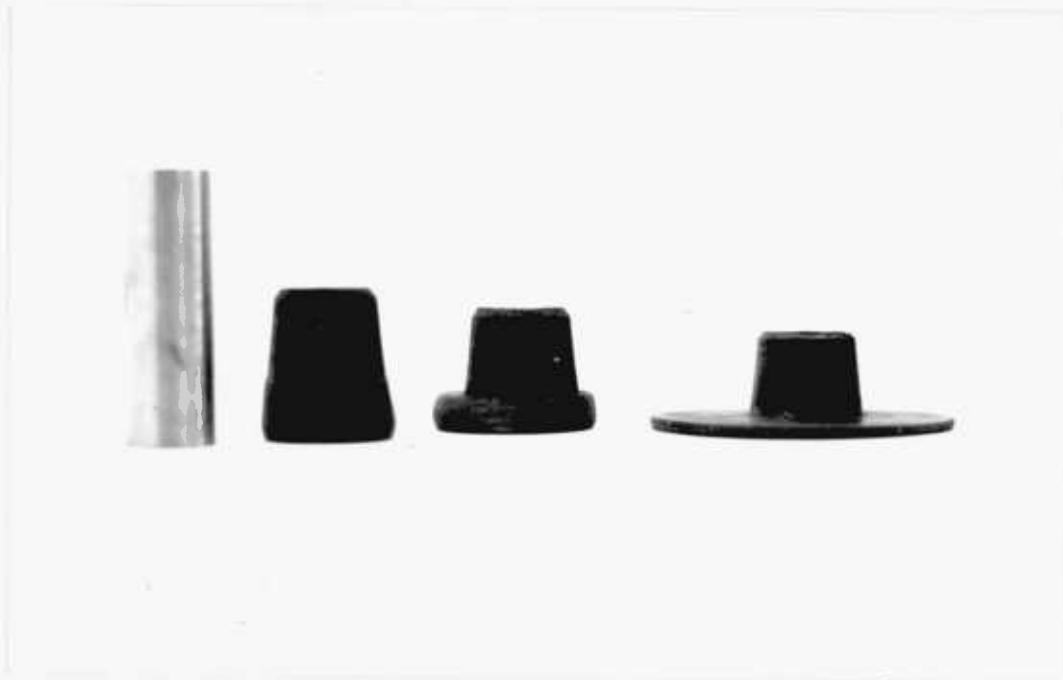


Plate 6.5 Some stages in forming a billet of diameter 19.2 mm with profile dies.



Plate 6.6 Some stages in forming a billet of diameter 25.4 mm with profile dies.
The billets have the same volume as those in Plate 6.5

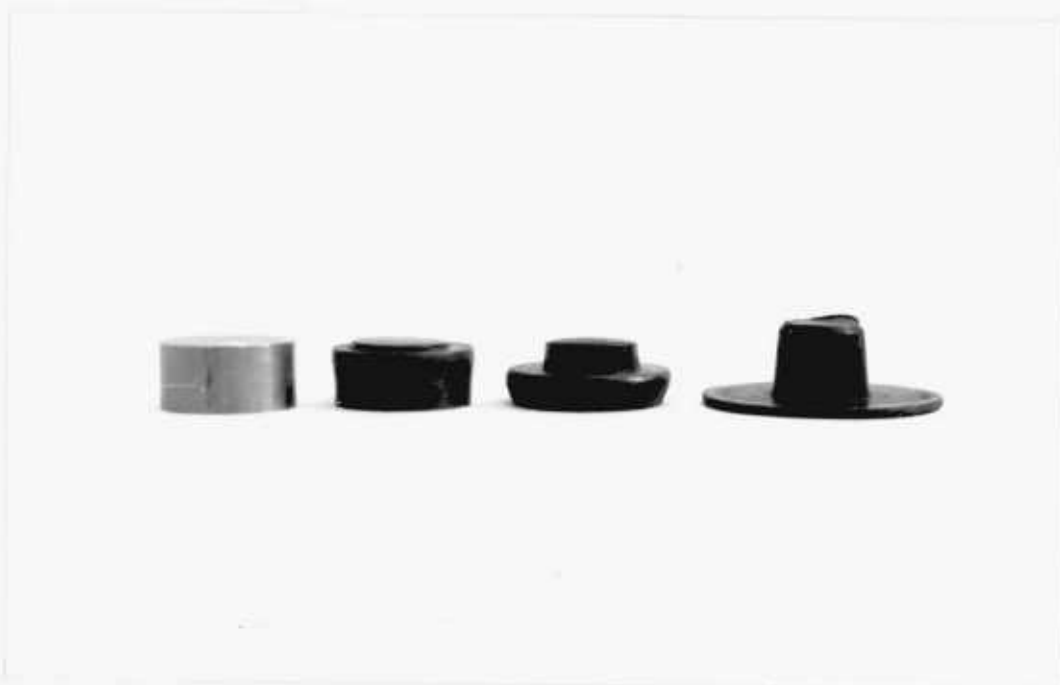


Plate 6.7 Some stages in forming a billet of diameter 35 mm with profile dies. The billets have the same volume as those in Plates 6.5 and 6.6



Plate 6.8 Failure of die and specimen brought about by excessive heating and a long contact time. There appears to have been some alloying of the two materials which caused both of them to become unusually brittle in the cold conditions. The failure occurred when separation was attempted.

virtually the compression of a cylinder to a thin disc, since the material in the cap barely deforms. Plate 6.6 illustrates a specimen of starting diameter 25.4mm which is only slightly smaller than the diameter of the bottom of the cap. As the load is applied this specimen slips at the edges where it contacts the inside of the cap in the die. Such a situation causes a quite severe situation for die wear, and the press shudders a little at this time which possibly indicates that a stick-slip friction situation is occurring. As deformation continues a lip is formed on the specimen which indicates a region of quite severe shearing. The lip disappears in the final specimen which is also the only specimen to show complete filling of the cap. Plate 6.7 illustrates a specimen of 35mm in diameter. In this specimen there is no rising into the cap during the early stages of the experiment, in fact, the total height of the specimen slightly decreases. Finally, however, the flange becomes sufficiently thin for the material to be forced up into the cap, and the height of the specimen increases. At constant crosshead velocity this billet is the quickest of the three to be finished for it requires the least travel of the crosshead.

Plate 6.8 illustrates a die failure that occurred during the experiments. The alloy of which this die was made was a Rolls Royce alloy C2-63 (Co 19-20, Mo 5.6-6.1, Cr 19-20, Ti 1.9-2.4, Balance Ni) which is not normally used now for isothermal forming dies, IN 100 being preferred.

The failure appears to have been occasioned by local heating of the die to such a level that it may have partially melted. The melting point of the alloy is about 1200°C which is only about 250°C above the operating temperature. The melted region appears to have alloyed a little with the specimen, and the resultant mixtures proved to be extremely brittle when the assembly was cooled to room temperature. The die and the specimen failed in the manner illustrated on plate 6.8.

6.6 The Isothermal Forming of Steel

A steel of composition 0.23C, 2V, 4Cr, 5.1Mo, 5.2W was used to test the possibilities of the isothermal forming of steel. The basic purpose of the additions is to provide a durable oxide that could withstand the conditions of isothermal forming. It is probably that a steel with 4% Chromium as its only addition would probably form satisfactorily in an air environment as discussed by Alexander and Turner(4) and, of course, steels with different additions may also prove useful.

Compression tests at 980°C with thin film graphite lubrication using billets 25.4mm height and 25.4mm diameter were performed with the steel alloy and the titanium alloy. The two experiments are compared in table 6.1.

TABLE 6.1

Alloy	Steel	Titanium
Final load	500kN	400kN
Final thickness	4mm	3mm $\pm$.2mm
Final diameter	63mm	72mm
Reduction	84%	88%
Axial strain	-1.9	-2.1
Stress at -.18 strain	51.5 MPa	18.1 MPa
At maximum load	No apparent creeping	Appreciable creeping

CHAPTER 7

APPLICATION OF THE FINITE ELEMENT METHOD TO ISOTHERMAL FORMING

- 7.1 The Program Used
- 7.2 Boundary Conditions
- 7.3 Application of the Program
 - 7.3.1 Cylindrical Compression
 - 7.3.2 Ring Tests
 - 7.3.3 Profile Dies
- 7.4 Discussion

Chapter 7 - APPLICATION OF THE FINITE ELEMENT METHOD TO ISOTHERMAL FORMING

7.1 The Program Used

The program used for isothermal forming can be summarised as a program that applies incremental displacements to the specimen. The elements used were isoparametric quadrilaterals which permit a parabolic displacement function within the elements, and the solution technique is a Gaussian elimination frontal method.

Isothermal forming must be considered by an incremental approach, the specimen starting at its original shape and then the dies being closed at a fixed or variable rate, the solution being found at various times during the process. Initially the elements are all given an arbitrary viscosity since the strain rate distribution through the material is unknown. In isothermal forming the temperature throughout the body is uniform and constant, since the process is continuously heated and slow. An incremental displacement is applied which gives an approximate representation of the strain rate distribution and subsequently the original arbitrary viscosities can be replaced by better approximations, calculated by the methods described in Section 3.2. The increment is again applied, starting with the initial geometry but with improved estimates of viscosity. This process is repeated until a satisfactory convergence is determined, at which point the program can proceed to the next increment of

deformation. A worthwhile economy can be achieved by limiting the quality of the convergence and so the level at which iterations are stopped is when the value of

$$A_n - A_{n-1}$$

falls below 3% of the value of A_n . A_n represents the sum of all the displacements calculated in the nth iteration. This level of convergence can normally be achieved within two or three iterations, and for most increments of deformation the values of the viscosities from the strain rates calculated in the previous increment are sufficient.

The strain rates within an element are given by

$$\dot{\epsilon}_{ij} = B_{ijkN} v_{kN} \quad (4.17)$$

where N is summed over all the nodes, v_{kN} is the value of the velocity in the x_k direction at node N. The local coordinates for a quadrilateral, eight noded isoparametric element can be given as p and q where they have the value indicated on figure 7.1. The velocity at any position in such an element can be defined as

$$v_k = N_N v_{kN} \quad (7.1)$$

For nodes located at corners the shape function can be given as

$$N_N = \frac{1}{4} (1 + pp_N)(1 + qq_N)(pp_N + qq_N - 1) \quad (7.2a)$$

and for the mid-side nodes (taking node 2 on figure 7.1 as an example)

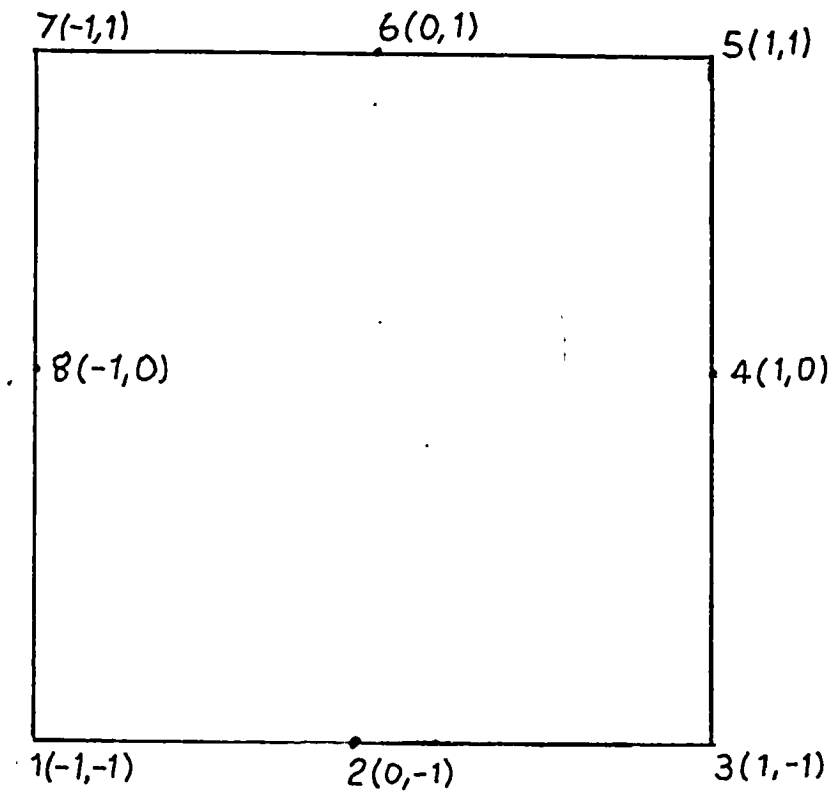


Figure 7.1 Nodal numbering in an element (8 noded quadrilateral) with the local coordinates indicated (p,q).

$$N_2 = \frac{1}{2} (1 + qq_2)(1 - p^2) \quad (7.2b)$$

These equations can be differentiated with respect to p or q , for example, at a corner node

$$\frac{\partial N_N}{\partial p} = \frac{1}{4} p_N (1 + qq_N)(2pp_N + qq_N).$$

The strain rates are defined by the equation

$$\dot{\epsilon}_{ij} = \frac{1}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \quad (7.3)$$

which means that equation 7.1 gives

$$\frac{\partial v_k}{\partial x_j} = \frac{\partial N_N}{\partial x_j} v_{kN}$$

and the shape functions have to be differentiated in local coordinates (these will be termed p_i from now on such that $p_1 = p$ and $p_2 = q$ of equation 7.2),

$$\frac{\partial N_N}{\partial x_j} = \frac{\partial p_1}{\partial x_j} \frac{\partial N_N}{\partial p_1} v_{iN} + \frac{\partial p_k}{\partial x_i} \frac{\partial N_N}{\partial p_k} v_{jN}$$

Thus

$$B_{ijkN} = \left(\frac{\partial p_1}{\partial x_j} \frac{\partial N_N}{\partial p_1} v_{iN} + \frac{\partial p_k}{\partial x_i} \frac{\partial N_N}{\partial p_k} v_{jN} \right) / 2$$

The nature of isoparametric elements is that in analogue to equation 7.1 the position inside the element is defined

$$x_k = N_N x_{kN}$$

where x_{kN} is the position of node N is the coordinate x_k .
Thus

$$\frac{\partial x_k}{\partial p_i} = \frac{\partial N}{\partial p_i} x_{kN} \quad (7.4)$$

where $\frac{\partial x_k}{\partial p_i}$ are elements of the Jacobian matrix and $\frac{\partial p_i}{\partial x_k}$ are elements of the inverse Jacobian (formed by inverting the Jacobian matrix).

The solution technique is a frontal Gaussian elimination technique. The actual details of this method are given in Appendix 3.

7.2 Boundary Conditions

Isothermal forming with graphite lubricant has been described in Section 6.3. For application of the finite element method, this situation presents quite simple boundary conditions. The free surfaces have no loads applied and are permitted to move as the deformation proceeds, the nodal points are all displaced after each increment by the value of the velocity at that point times the time interval over which the increment occurred. The boundaries in contact with the dies are fixed to the dies, because there is no slip between the specimen and the die with graphite lubricant. Thus these boundaries have prescribed velocities determined by the speed of crosshead movement.

The main difficulty in this situation is that as deforming continues parts of the boundary which were originally free come into contact with the die and thus start to have prescribed velocity conditions. In the program the boundary nodes are considered after each increment and any node that is about to come into contact with the die is fixed onto the die and the status of its boundary conditions are changed. The level at which this occurred was when the point came to be less than 2% of the gap between the dies away from the die. In the profile dies a similar arrangement ensured that nodes could become fixed to the conical surfaces as the die closed.

With test conditions that enable slip between the specimen and die, provision must be made for frictional conditions to be applied at the interface. If shear coefficient friction conditions are applied the appropriate formula is

$$\tau = m \frac{\bar{\sigma}_0}{\sqrt{3}} \quad (6.2)$$

where τ is the shear stress at the interface, $\bar{\sigma}_0$ the equivalent flow stress of the deforming material and m is the shear coefficient. The factor of $\sqrt{3}$ is a consequence of the Von Mises flow criterion. In the situation of isothermal forming the factor $\bar{\sigma}_0$ in equation 6.2 cannot be considered as a constant. The value of it must be determined by knowing the strain rate in the element adjacent to the surface and from that deducing the viscosities and hence the formula

$$\bar{\sigma}_0 = 3\eta\dot{\epsilon} \quad (7.5)$$

can be applied (see Section 3.2). This requires an iterative technique and thus must be added to the iterative methods already being used to determine the flow stresses within the deforming material. The direction in which the shear stress in equation 6.2 operates may also be uncertain in regions of the interface, for example, in ring tests where the position of the neutral surface is unknown. This may require extra iterations to determine the approximate position of the neutral surface before the shear stresses are applied to the interface. Convergence speed is slower than with simple boundary conditions because of the addition of non-linear boundaries but the difficulties can be accommodated in less than 50% increase of iterations, these being more necessary at the initiation of calculations than later.

7.3 Application of the Program

7.3.1 Cylindrical Compression

A test situation of a cylindrical compression test with no friction at the interface is a useful check on whether constant volume is obtained in the program. On carrying out such tests it was found that there was no detectable departure from constant volume, the mesh deforming uniformly and remaining always completely cylindrical. Such a test is shown on figure 7.2.

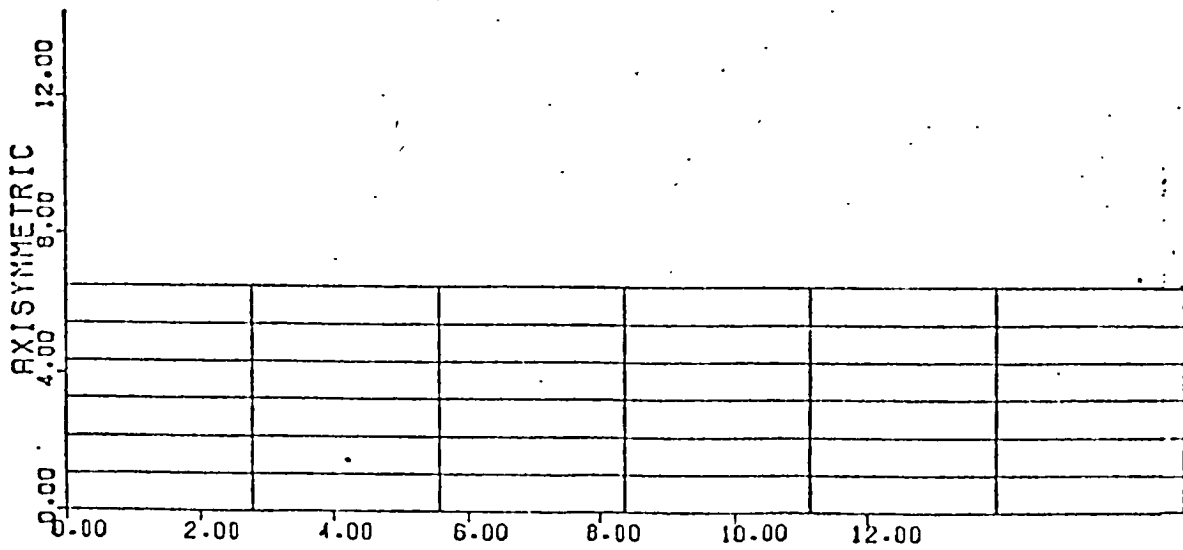
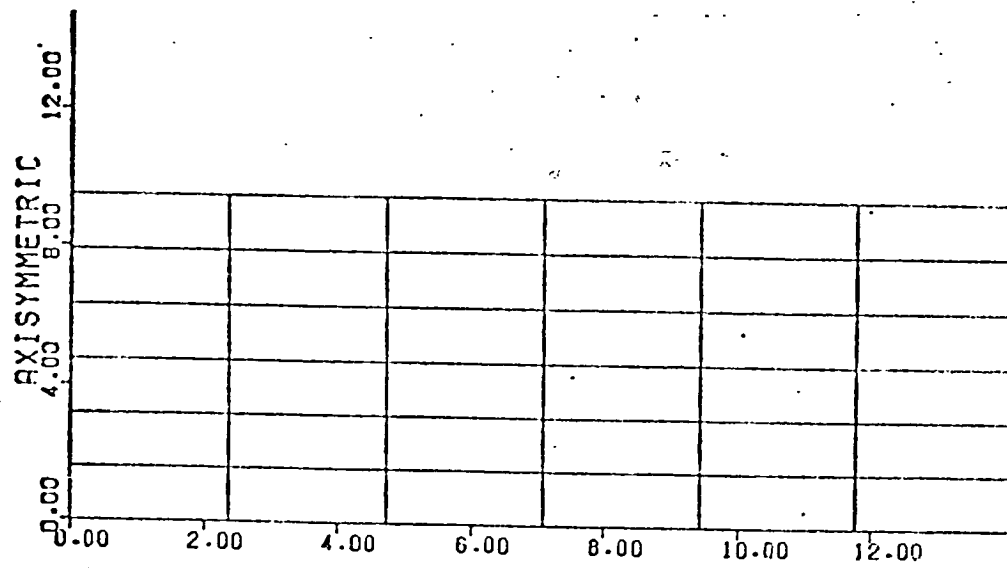
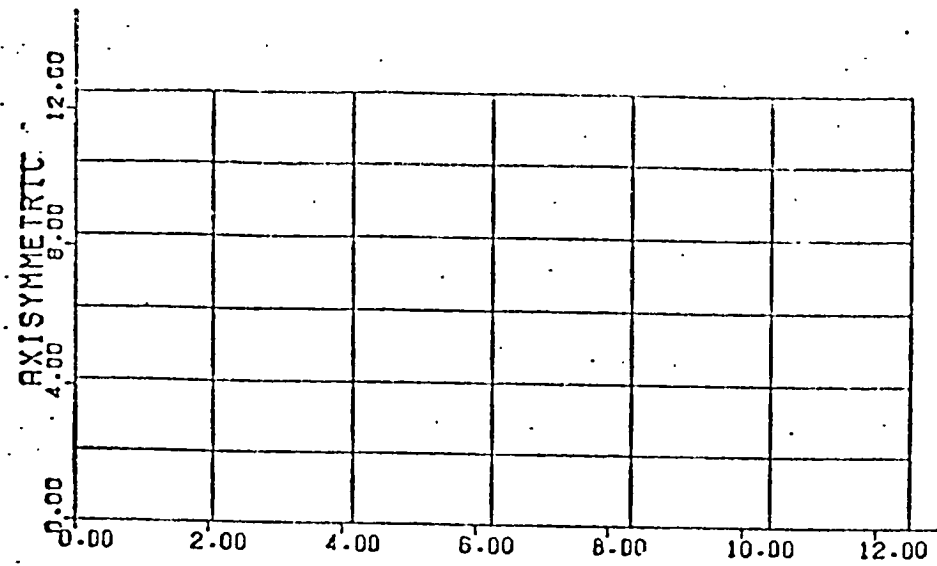


Figure 7.2 Compression of a cylinder without friction, one quadrant shown. Initial dimensions 25mm high, 25mm diameter.

A more difficult situation occurs with sticking friction. Examples are shown on figure 7.3 of the deformation for cylinders of Ti6Al4V at 950°C, using material data from Lee and Backofen(13). The height to diameter ratios were originally $\frac{1}{4}$, 1 and 4 respectively and as yet in none of these examples has any material rolled onto the dies. The most interesting example shown is the tall thin cylinder which exhibits a double bulge effect, a phenomenon seen in experiments on specimens of this ratio. The deformation in the central region of the tall specimen is quite uniform because the effects as the friction at the ends becomes reduced.

Figure 7.3 illustrates a specimen originally of height to diameter ratio $\frac{1}{4}$ which has been taken to a higher strain of 50% reduction. The free surface of this specimen has proceeded to roll onto the dies and also the mesh has been straightened out or reformed on one iteration because the movement of the nodes can cause the elements to cross one another in highly deformed regions. The original volume of this specimen was 12566mm³ but now it is approximately 12145mm³ measured from the diagram which is a loss of about 3.3%.

The loss of volume is due to the fact that constant volume or zero volumetric strain rate is only met exactly at certain points within each of the eight noded quadrilateral elements. This is because these elements can only satisfy C^0 continuity across their boundaries, that is, only the

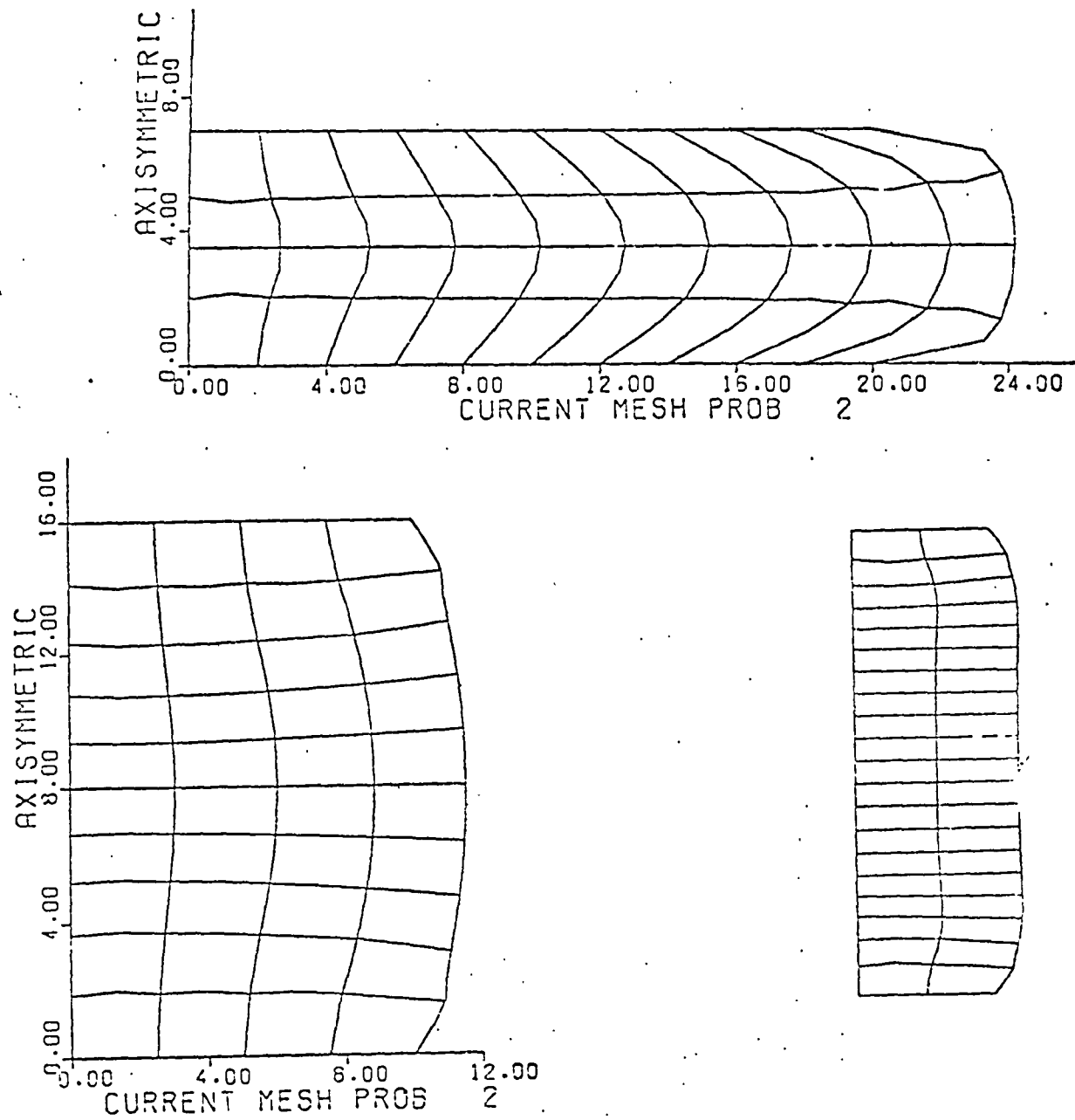


Figure 7.3 Compression of cylinders of ratio $\frac{1}{4}$, 1, and 4 with sticking friction.
Half of each specimen is shown.

velocities have been calculated with a continuous value throughout the body. The derivatives of the velocities are not continuous through the element boundaries and thus the strain rates, which are a function of the derivatives of the velocities (see equation 7.3), must be discontinuous at the element edges. To achieve continuous derivatives, elements which enforce a C^0 continuity are required, but such elements involve considerably more complication and many extra nodal variables. The trade-off in computer effort to achieve C^1 continuity is not justified in most problems (see Strang and Fix(71)). However, it is found that the derivatives of the nodal variables are accurately calculated at certain points within the elements and that the 2 x 2 Gauss points are normally very close to such points (see Naylor(72)). In the cases being investigated it is found that the volumetric strain rate is indeed very close to zero when calculated at the 2 x 2 Gauss points but finite values normally occur at the nodes. The values at the nodes can be either negative or positive and tend to cancel each other out. Further considerations of these problems are made in Appendix 4.

The final set of three figures in this section, 7.4-7.6, illustrate the very high deformation of specimens of ratios 2/3, 1 and 3/2. Here the operation of the subroutines which enable nodes that are on the free surface at the beginning of the deformation to stick on to the dies is fully illustrated. The computing times for these high deformations

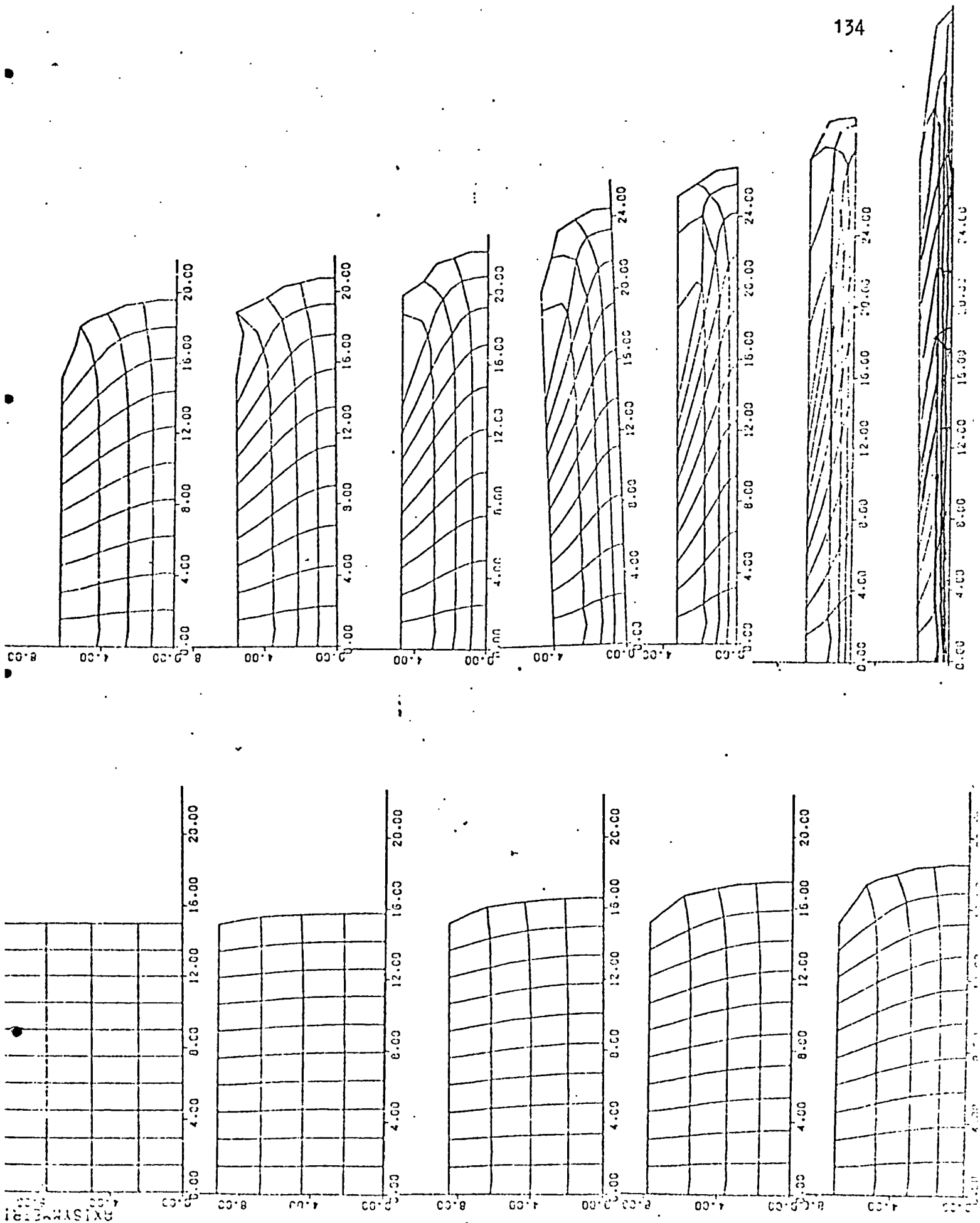


Figure 7.4 Compression of a cylinder of starting height to diameter ratio $2/3$ with sticking friction. Final compression shown is 82.5%.

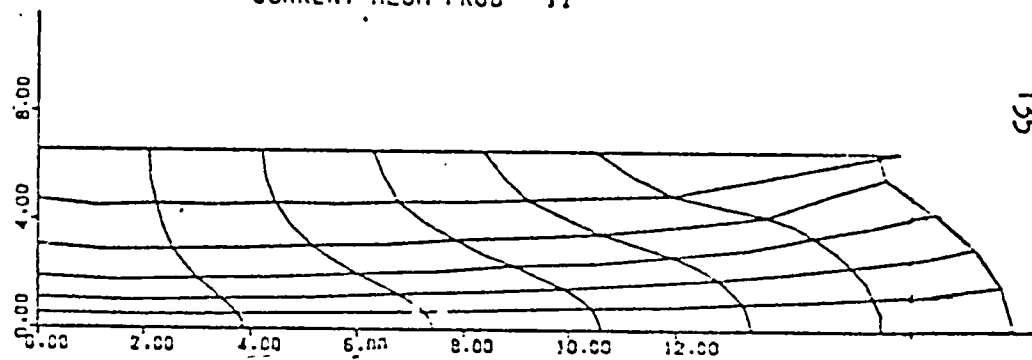
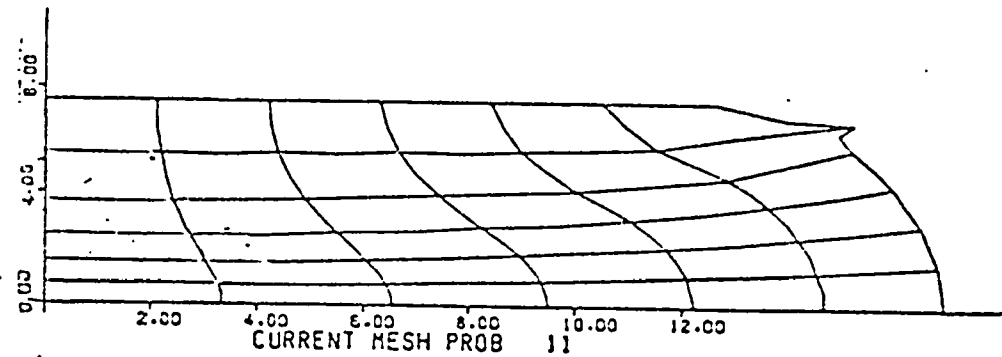
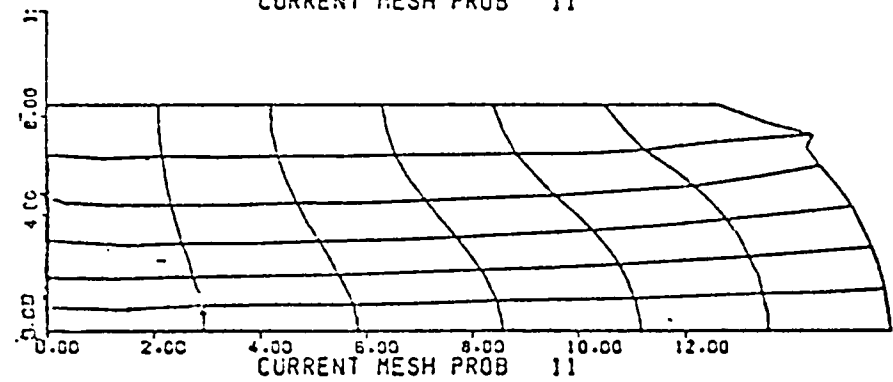
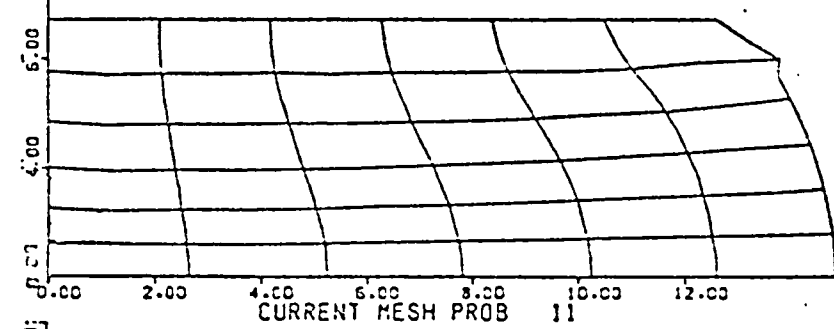
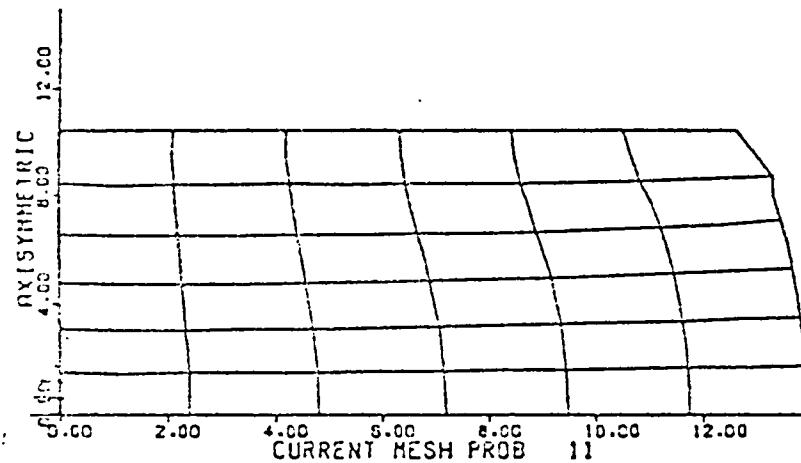
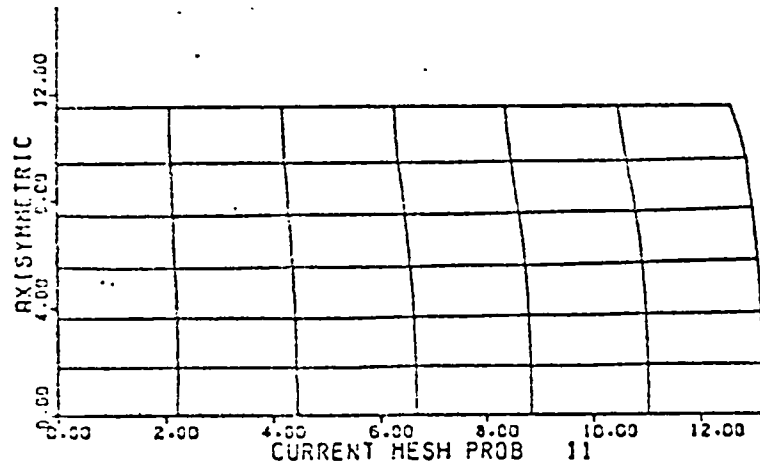
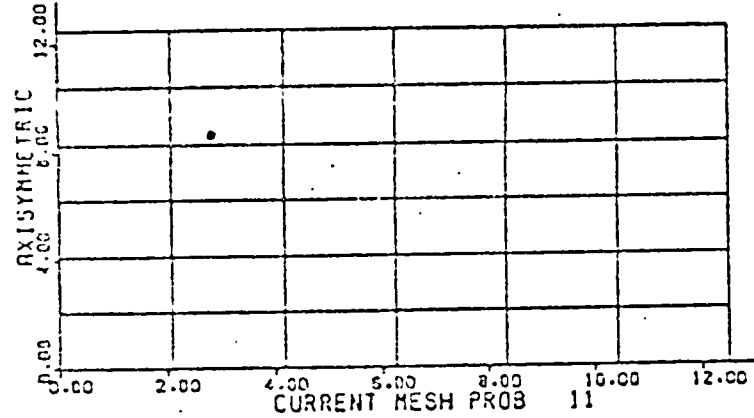


Figure 7.5 Starting height to diameter ratio of 1; sticking friction. 54% compression shown in last figure.

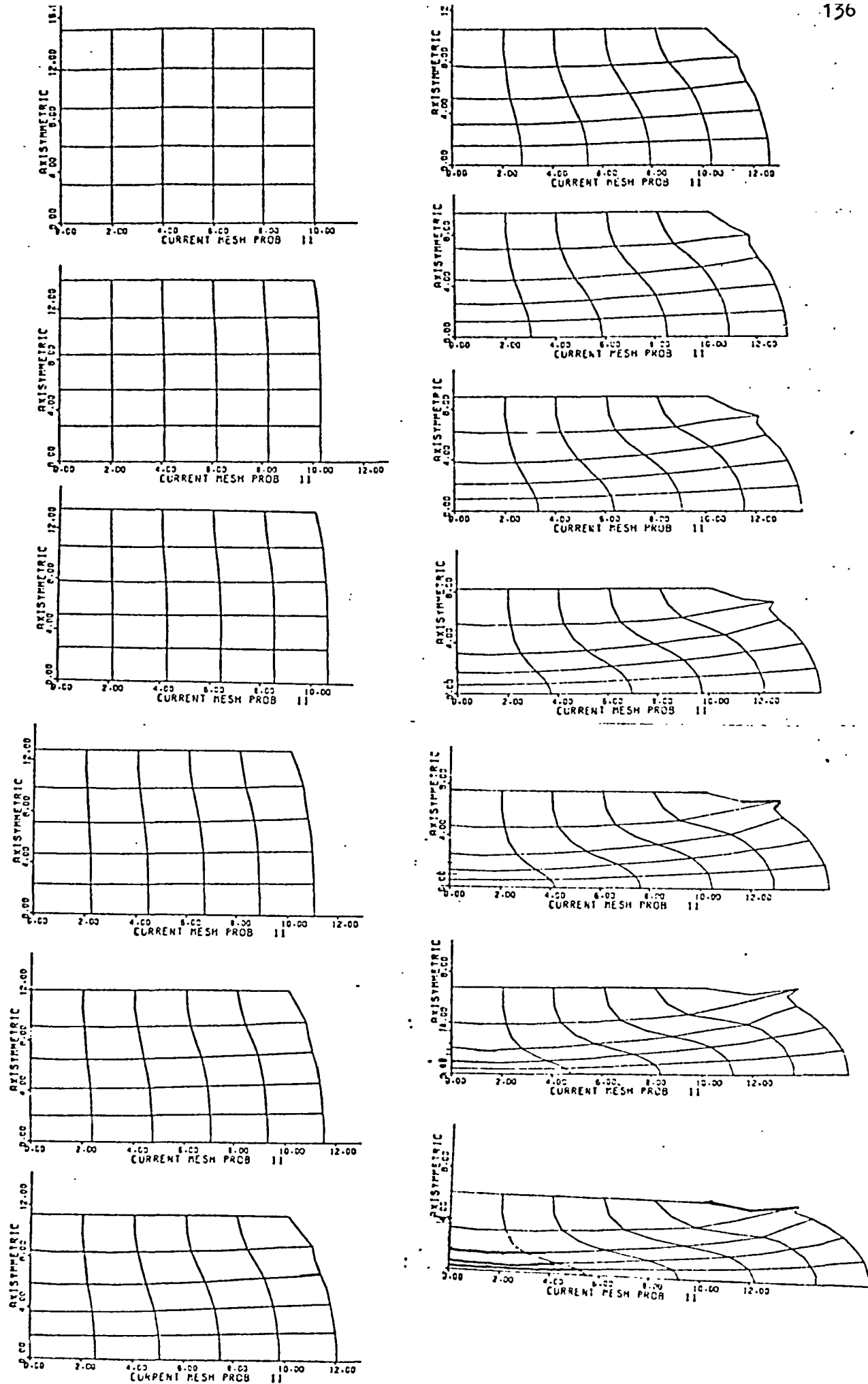


Figure 7.6 Starting height to diameter ratio of 3/2; sticking friction.

Final compression shown is 60%. One quadrant only is shown.

is quite economical with the penalty function approach and these results (without stress calculations) were all obtained in less than two minutes of central processor time on the CDC 6400 at Imperial College.

7.3.2 Ring Tests

Figure 7.7 illustrates a ring test; the compression of an annulus between flat dies. The starting geometry has the ratio 4:2:1 (outer diameter: inner diameter: thickness) and the thickness of the specimen was 6.45mm. The frictional conditions are those of sticking friction and could be represented by a value of shear coefficient of unity, that is, m equals one in equation 6.2. Figure 7.8 illustrates a similarly shaped ring, but in this case the friction subroutine was introduced into the program and a value of 0.06 was used for the shear coefficient. Figure 7.9 illustrates a plot of reduction in inner bore diameter versus reduction in height and compares the results of some experiments, the current finite element results and the predictions of Hawkyard and Johnson(70). The lubricant used for the low friction case in the experiments was molten glass, the high friction case used no lubricant. The mid-section radii have been used for all the comparisons (Hawkyard and Johnson assume plane sections remain plane and thus the mid-section radius for their analysis is the same as the end radii). Complete agreement with experiments cannot be expected because the real friction effects are not likely to be precisely the same

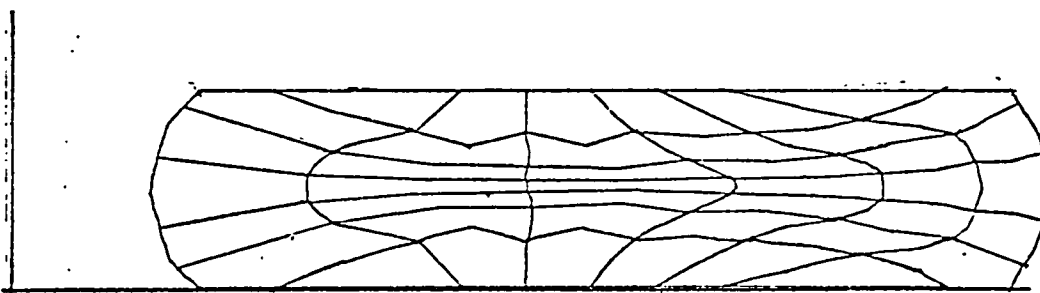
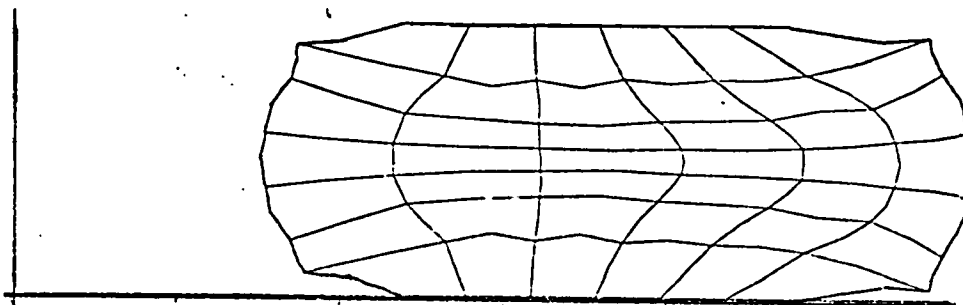
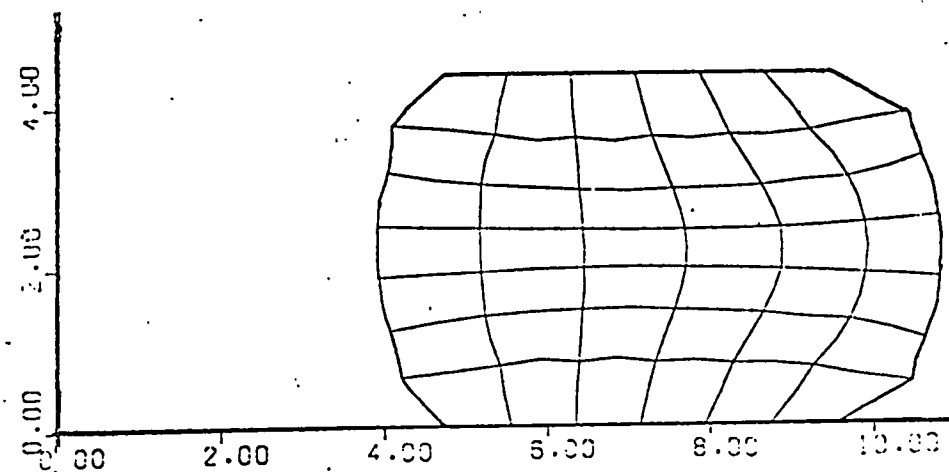
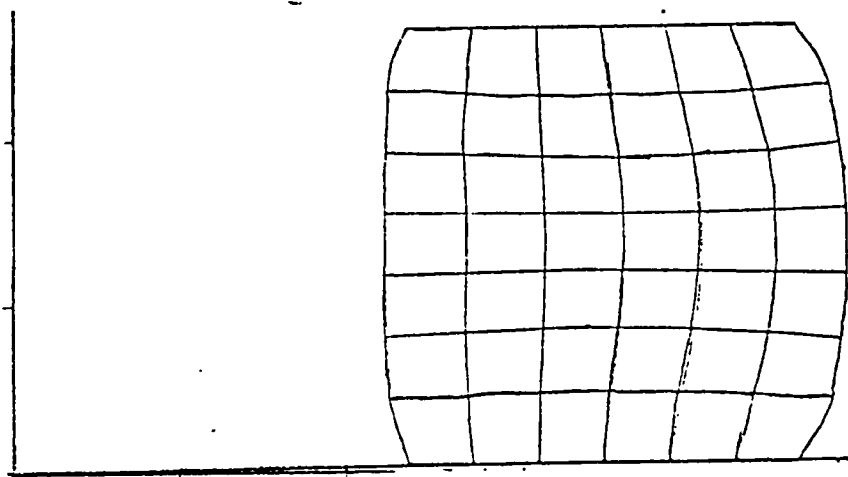
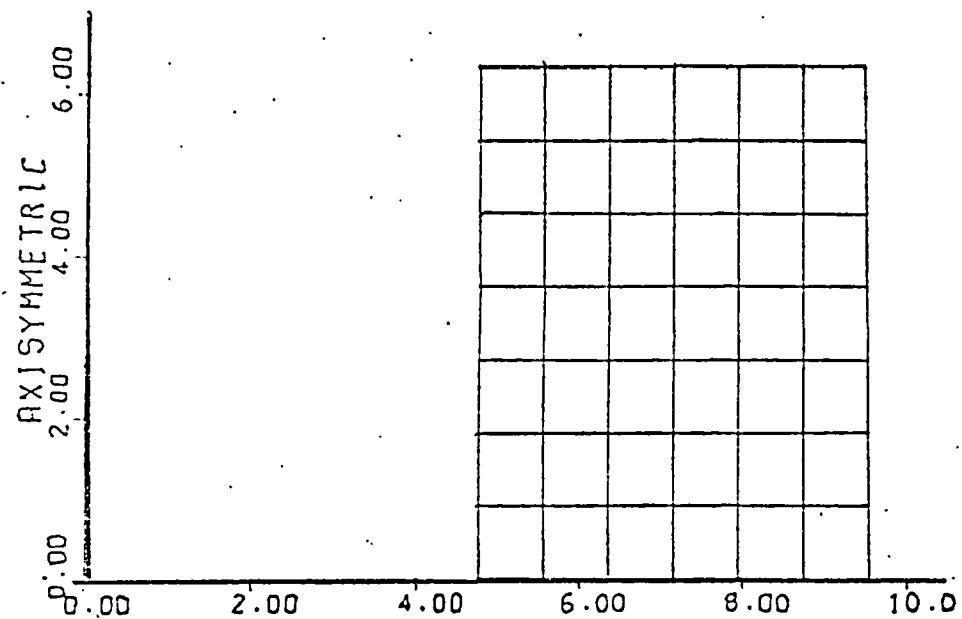


Figure 7.7 Ring test with sticking friction, half a specimen is shown. Final compression is 38.8%.

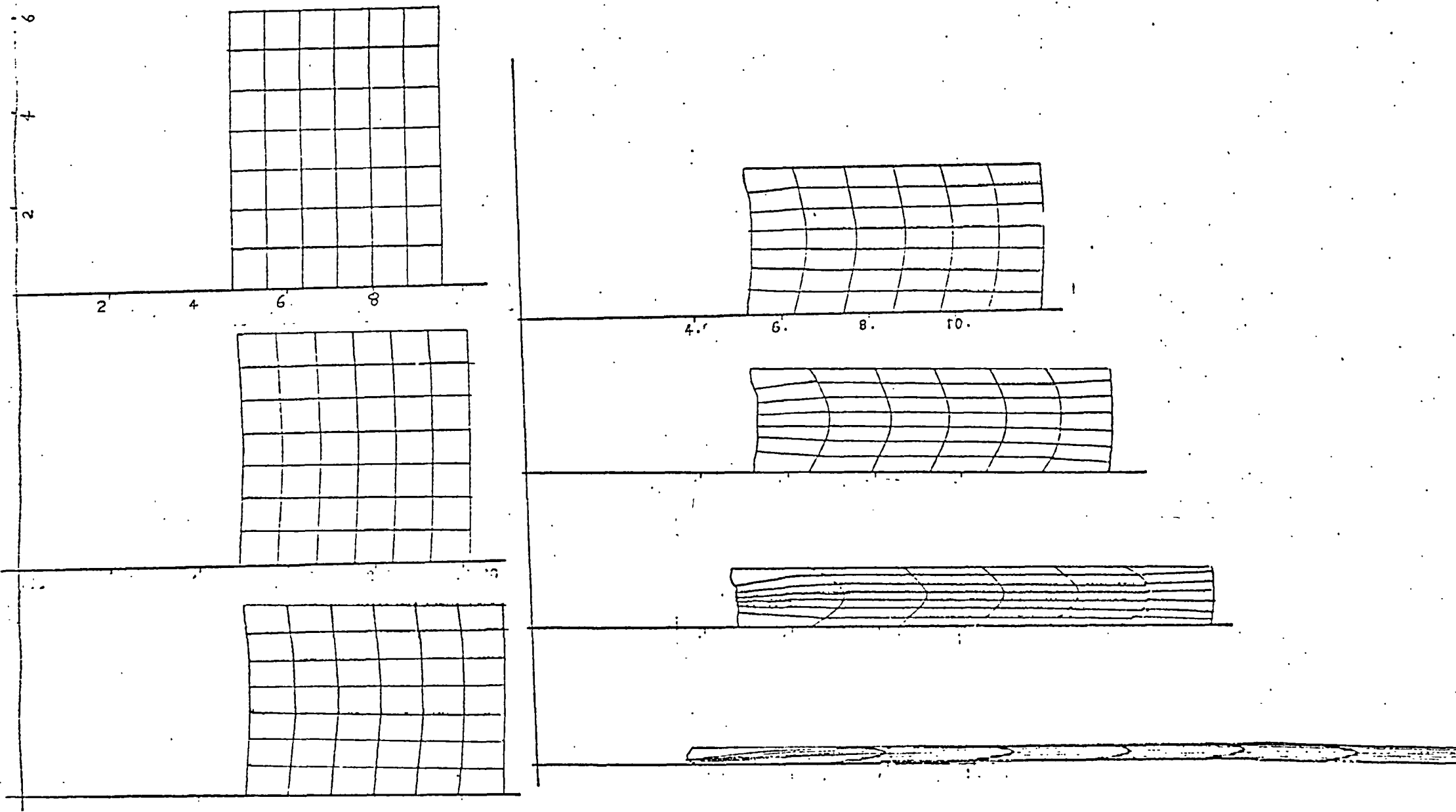


Figure 7.8 Ring test with a shear coefficient of 0.06; half a specimen is shown. Final compression is 92%.

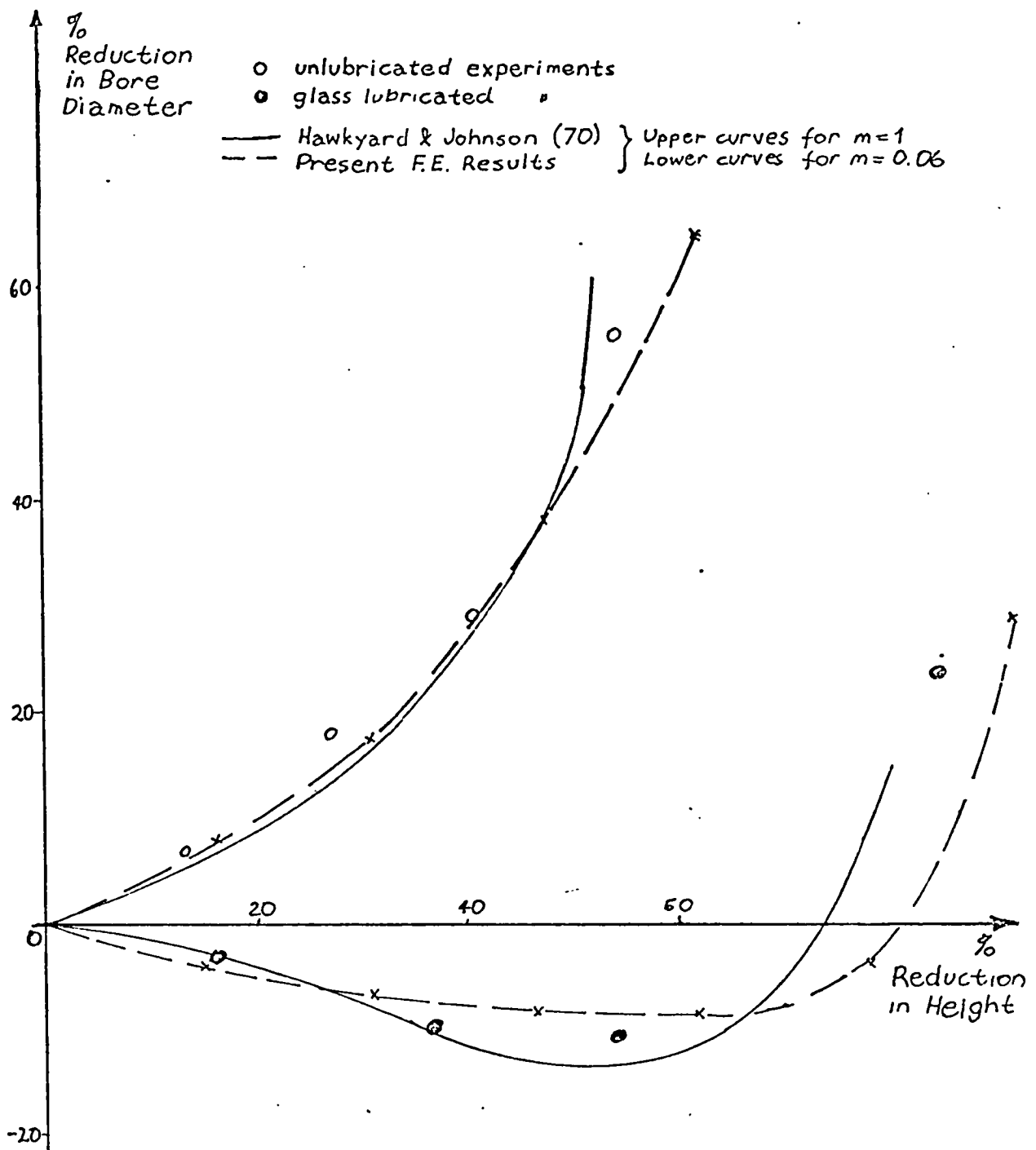


Figure 7.9 Comparison of Hawkyard and Johnson(70) theoretical results, the present finite element predictions and some experiments for the ring tests.

as indicated in equation 6.2 throughout the test, nor are the specimens accurately round after the compression. The net result suggests there is little to choose between the two theoretical method of predicting ring behaviour.

The neutral surface (the surface that does not move) can be determined for the sticking friction case using the hoop strain rate distribution from the finite element analysis. The neutral surface does appear to be very nearly cylindrical as is shown on figure 7.10.

7.3.3 Profile Dies

Figure 7.11 illustrates the results for an extensive compression of a finite element model of the profile die experiments reported in Section 6.5. Subroutines have been included in the program which allow not only the nodes from the outside surface to stick on to the dies, but also allow the nodes on the top surface of the specimen to stick to the conical surface of the top die. As will be noted in the latter stages of this compression, some of the element boundaries tend to cross each other and the die surface in the later stages of compression.

Figure 7.12 illustrates a modification of the program which allows the mesh to be reformed at stages during the compression to remove this problem. Whenever the elements appear to be about to cross, in this modification the whole mesh is reformed in an arbitrary manner, but the exterior

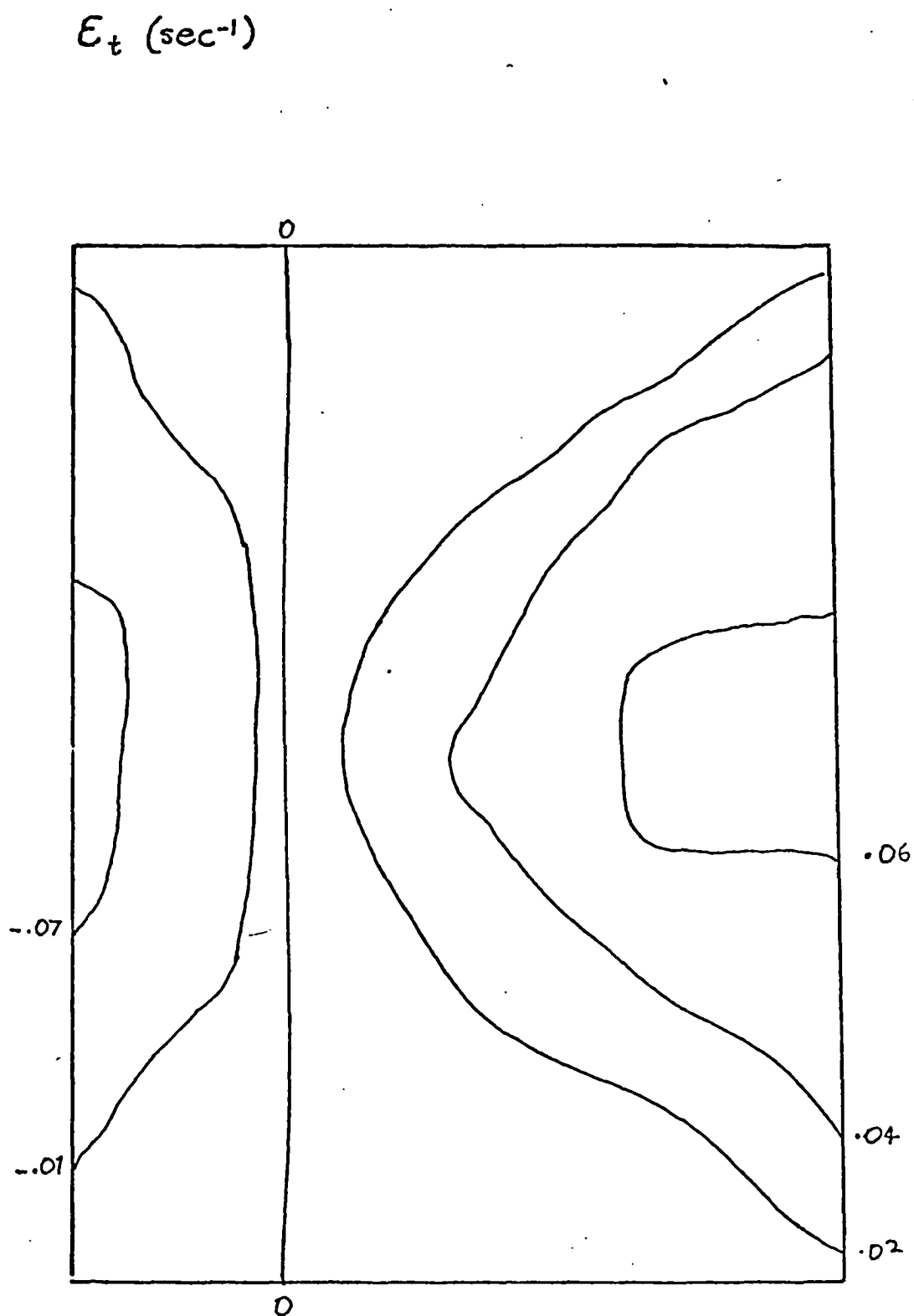


Figure 7.10 Tangential strain rate distribution in the sticking friction ring test of figure 7.7 (first increment). Units are sec^{-1} .

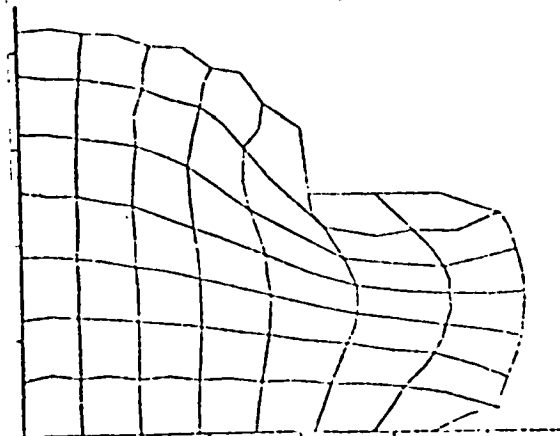
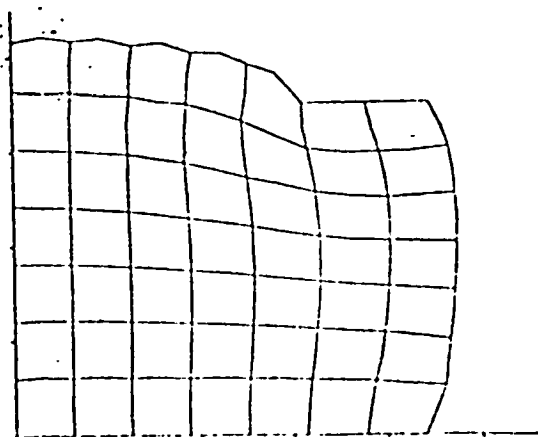
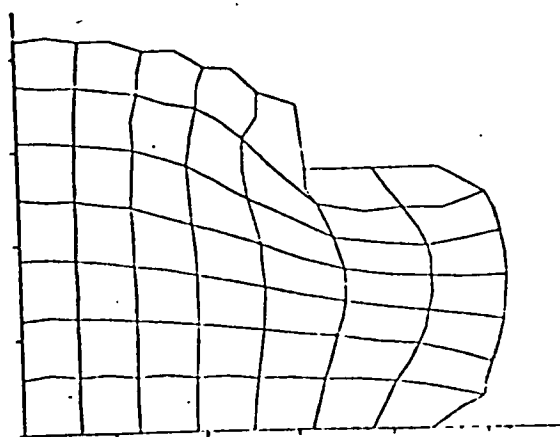
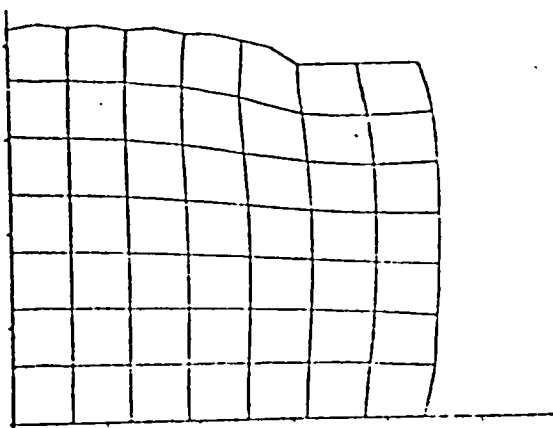
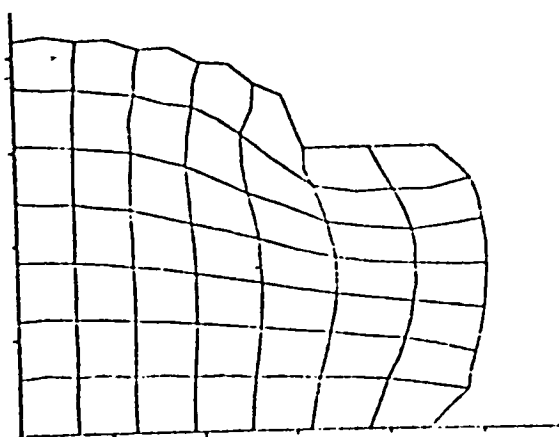
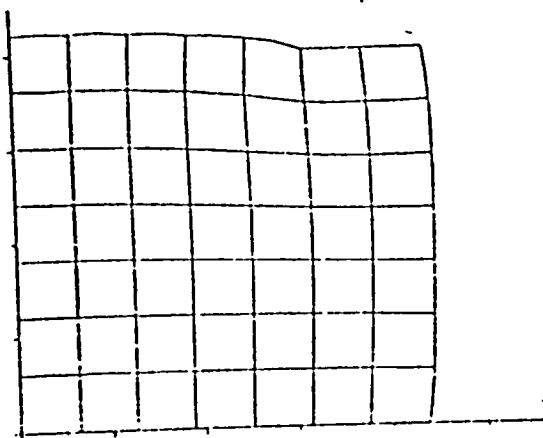
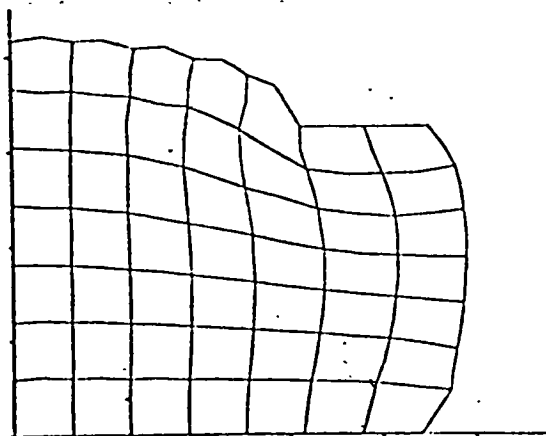
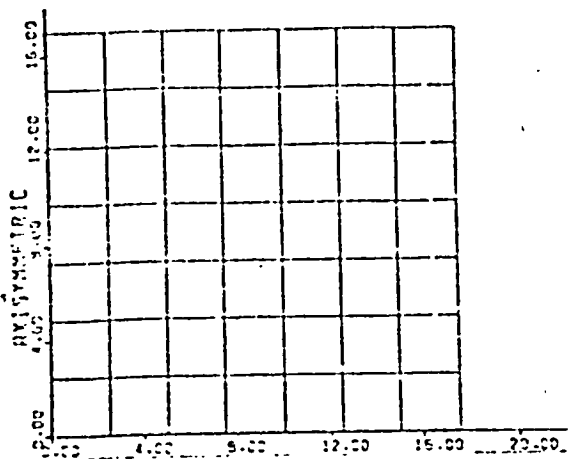


Figure 7.11 (Part one).

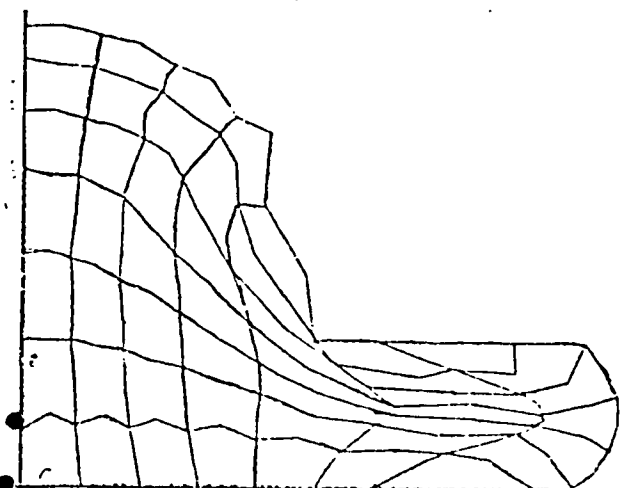
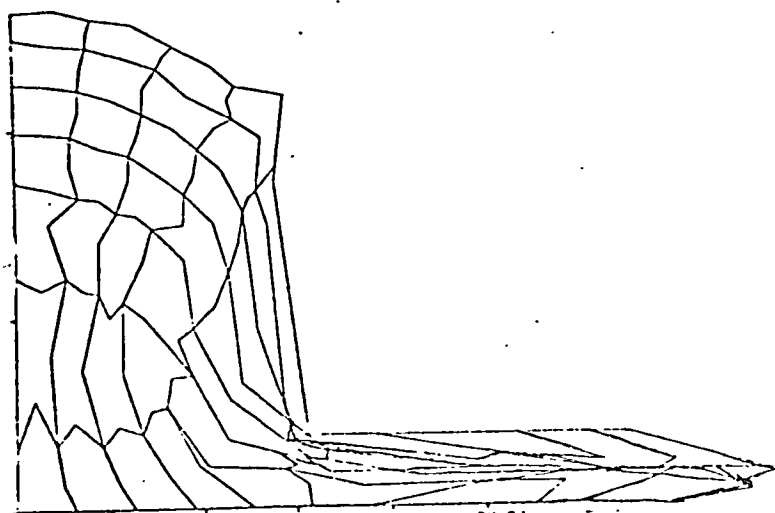
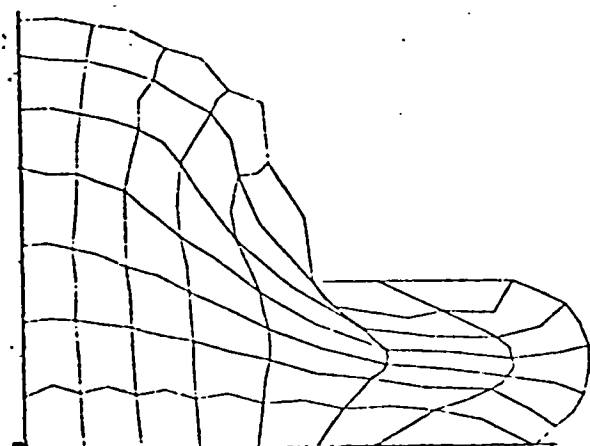
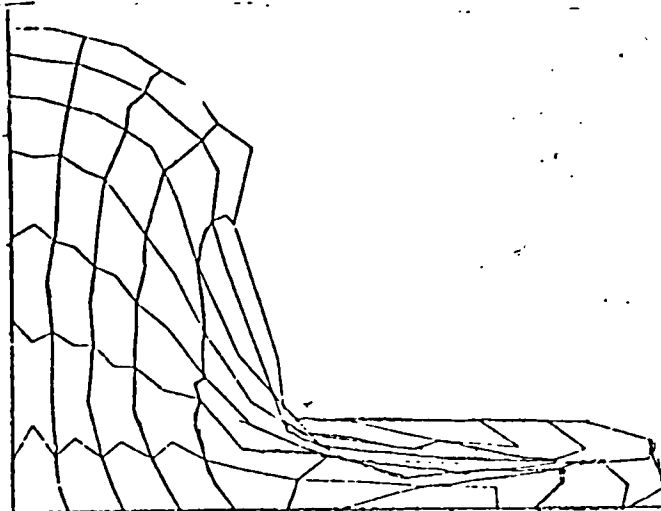
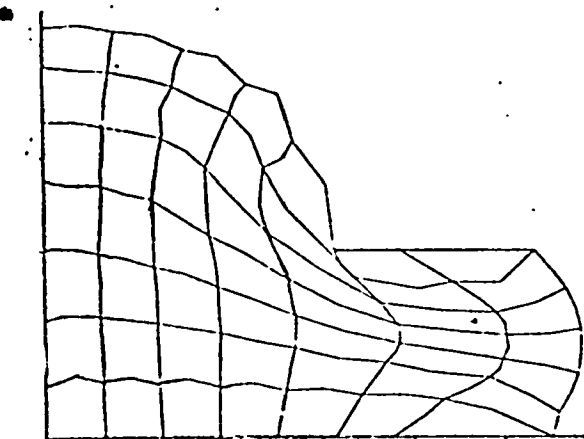
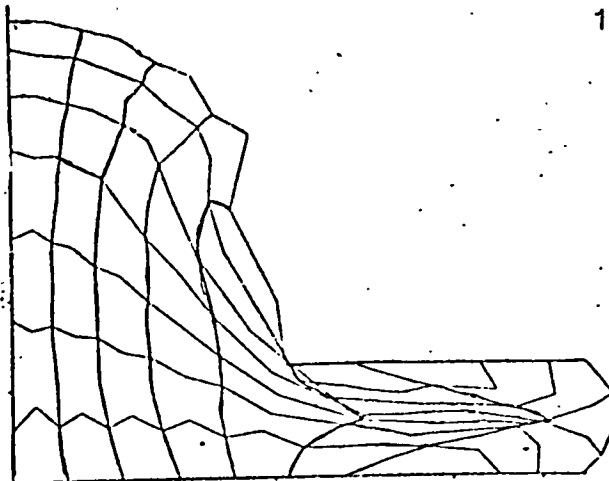
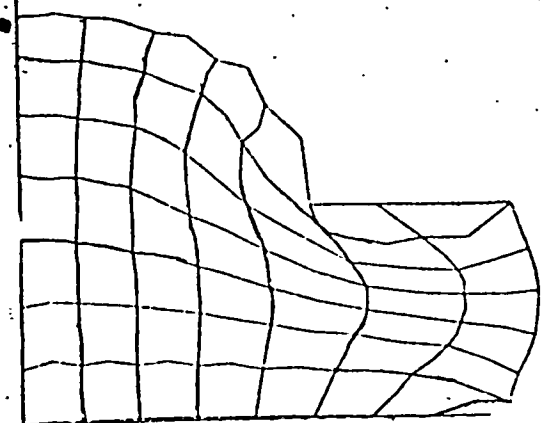


Figure 7.11 (Part two).

Compression of 17.5mm high, 17.5mm
diameter specimen with a profile die.

The mesh is not reformed at any stage.

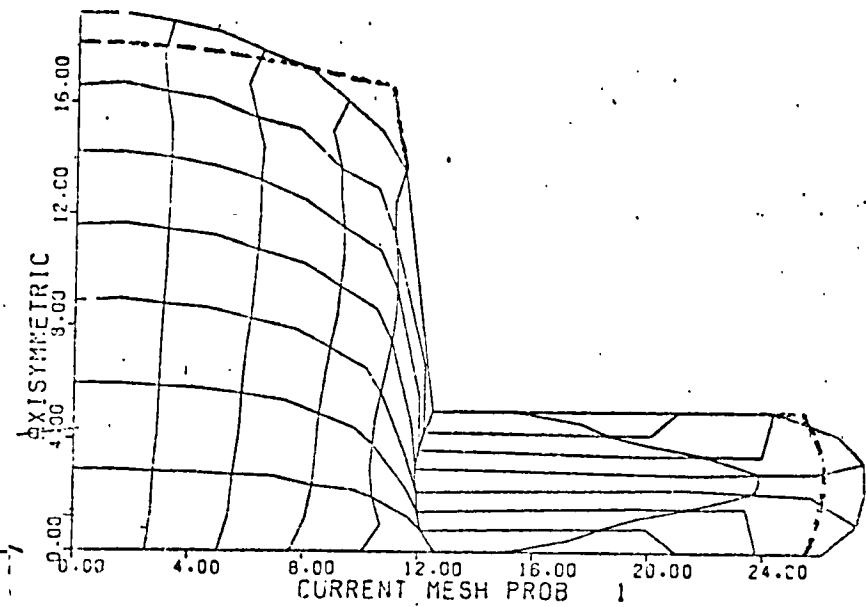
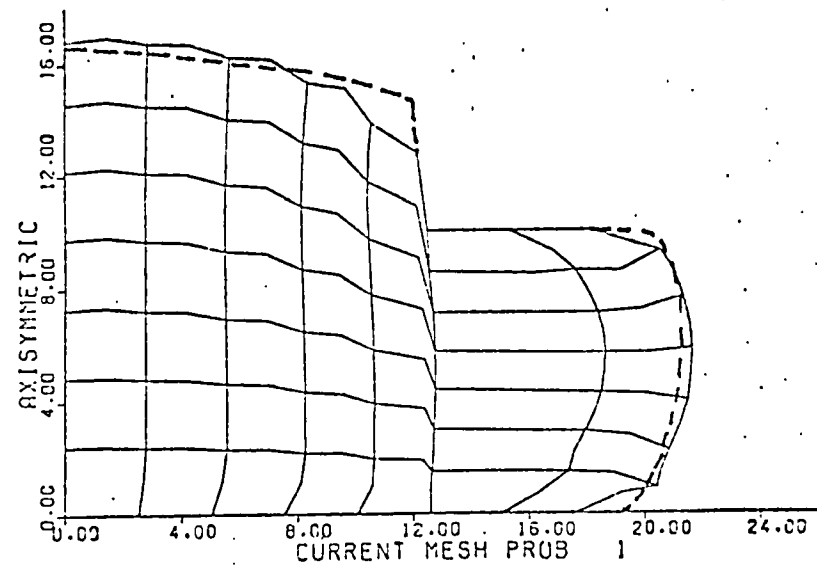
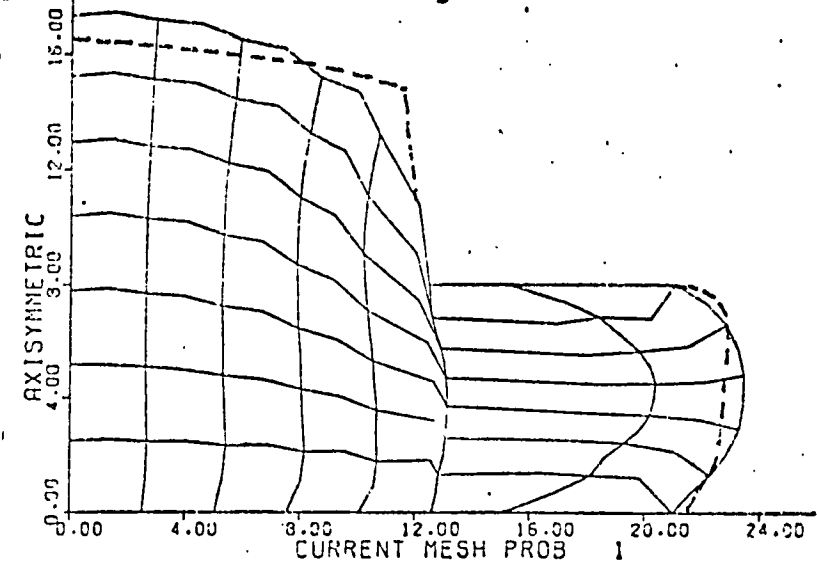
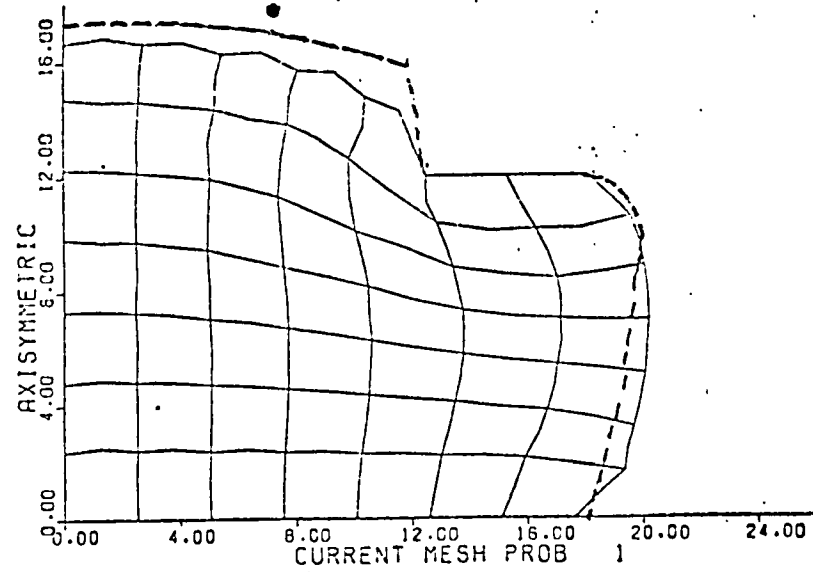


Figure 7.12 Compression with a profile die (CF. figure 7.11). The mesh in this case is reformed at increments 7,12. 34

The dashed line gives some close experimental results for the outside profile of comparable specimens.

surfaces are not altered. The results of the finite element method are compared with experimental profiles that were measured from approximately similar specimens using a projection microscope. As can be seen the comparisons are quite satisfactory.

The most important indicator which occurs during this forming is the reversal of flow when material, instead of flowing into the flanges from the bulk of the specimen, instead flows from the flanges into the conical region in later stages of compression. On the experimental specimens this is detected by the specimen increasing in height, a feature which can be seen to occur (on plate 6.7) in the last stage of the forming. The finite element results on figure 7.12 are presented for a specimen that is as close as possible to the specimen on plate 6.7. As can be seen, if the element boundaries joining the corner of the top die to the surface of the lower die are inspected the flow indeed does reverse in the finite element model and the height does increase in the last stage shown.

Figure 7.13 illustrates the external profiles calculated for the indentation by two opposed punches of a cylinder 25.4mm high by 25.4mm in diameter (only one quadrant of the specimen is shown in the figure). Whilst this experiment was not actually performed, a similar experiment with a single

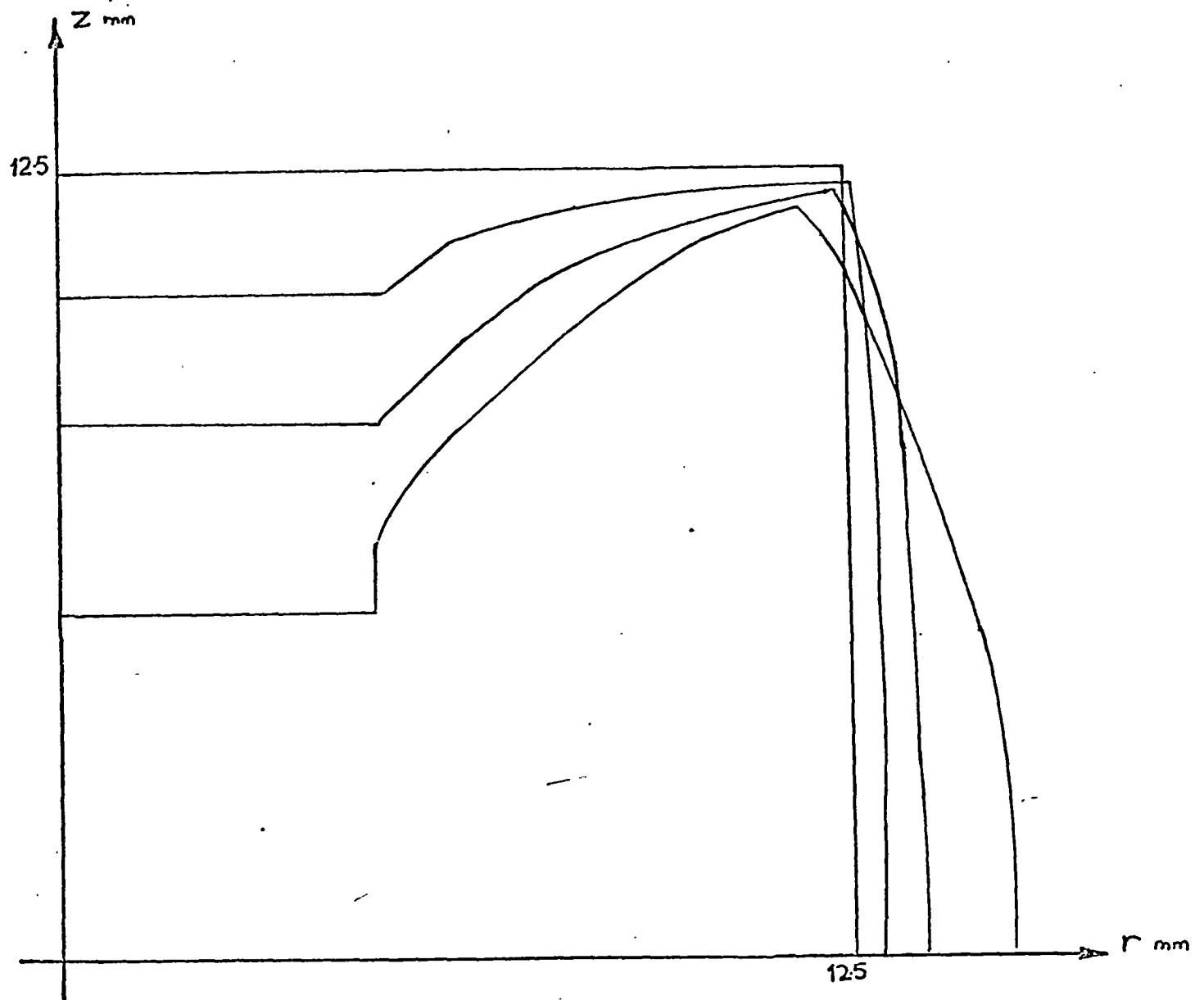


Figure 7.13 Outside profiles for an axisymmetric indentation problem.

The specimen is also symmetrical about the lower axis.

indenter demonstrated similar geometry changes. The chief effects are the formation of a barrel on the outside of the specimen and the tendency of the upper corner to move inwards as the deformation proceeds. The finite element prediction does indicate that material rolls onto the indenter, but this actually occurs earlier and to a greater extent in experiments.

7.4 Discussion

The results of this chapter are extensive and some general comment should be made on them.

The deforming shapes given by the finite element method behave, in general, in the same manner as the specimens behave in experiments. However, in detail there are a number of discrepancies. The most important of these is probably the fact that in the finite element models the material appears to roll onto the dies or indenters at a later stage than is seen in experiments. One approach that can be tried to overcome this problem is to refine the mesh near any region in which the surface will come into contact with the dies, such as the corner in the example of the profile die, or at the edges in simple compression. Such a technique has been tried by Kobayashi(73) in an example of backward extrusion. However, by using smaller elements, the elements near the corners become unacceptable at smaller die movements: the crossing of boundaries illustrated on figure 7.11 tends to occur at very low deformations. As a result, an arbitrary mesh reformation must be introduced at early and frequent

intervals (as is done by Kobayashi). Mesh reforming is an arbitrary procedure and it does, in fact, influence the results of the analysis. This is because regularly shaped elements experience different strain rates when compared to highly deformed elements in the same position. In the theory this is represented by the fact that the values of the elements of the Jacobian (see equation 7.4) are altered inside the elements. An illustration of this is provided on figure 7.14 where the two specimens differ only in that the latter specimen has had its element boundaries straightened at each stage during the compression. The effect on the outside profile is quite profound and is clearly less acceptable for the specimen in which the boundaries have been continuously reformed. The deformed elements in a sense represent the only memory that the finite element model retains of the original shape of the specimen. Mesh reforming is an arbitrary procedure and in the material it might be seen as being analogous to recrystallization. In the case of isothermally formed specimens, microscopic examination reveals that there is not a complete recrystallization in the specimen and that the final deformed specimen retains highly deformed crystals. As a result of these evaluations, it seems that mesh reforming should be left to the absolute minimum if a proper representation of the forming is desired. In addition to these considerations, it should be pointed out that smaller elements require longer computing times and, of course, the mesh reforming is a further complexity in the program.

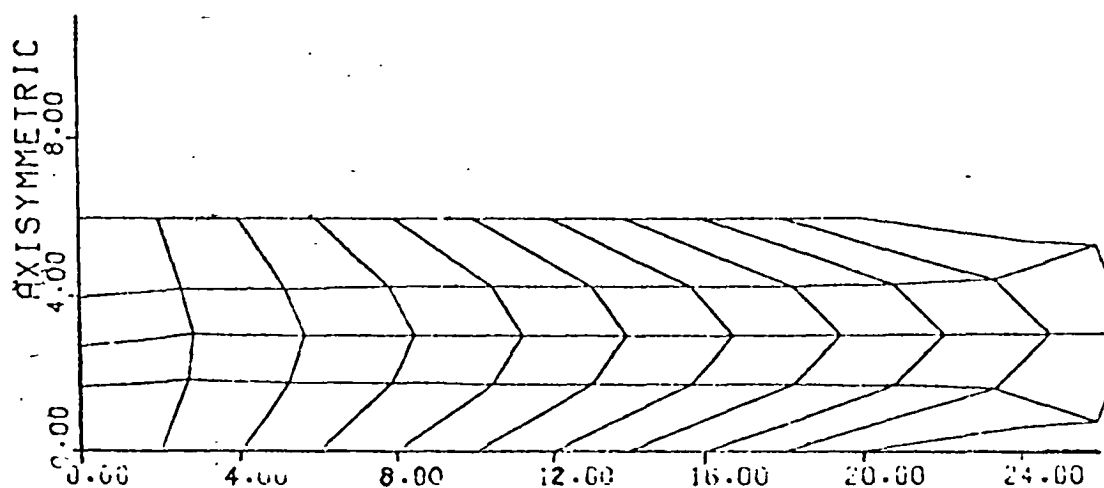
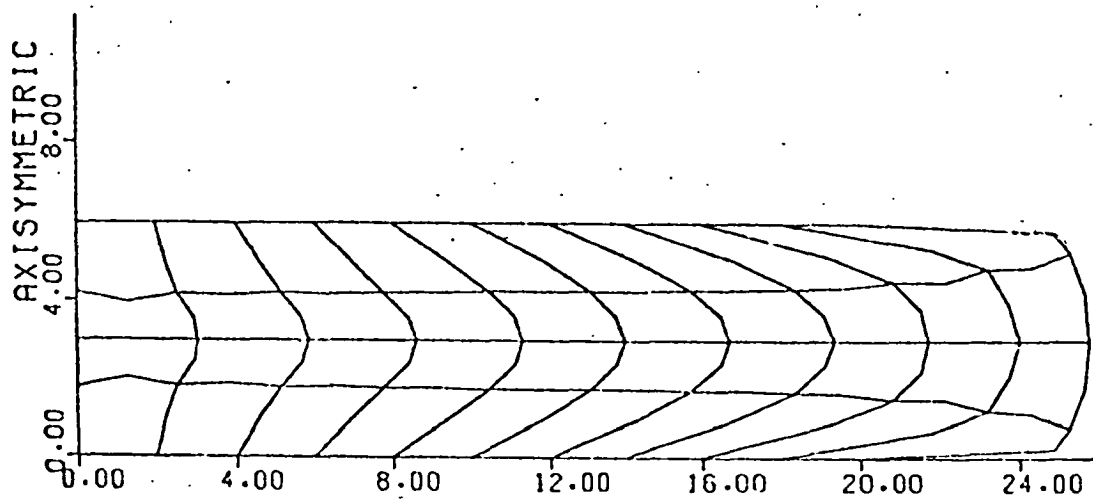


Figure 7.14 Some undesirable effects of mesh reforming. The upper specimen has no mesh reforming applied, the lower specimen has the sides of the elements straightened after each increment.

Another feature which deserves comment is that in the case of simple geometries, the finite element method does not necessarily give more accurate or useful results than simpler methods of calculation. However, for the more complicated examples such as the profile die and indentation problems presented in Section 7.3.2, two or three-dimensional non-linear techniques are required, and the finite element method is one of the best approaches in such cases. As will be mentioned in Chapter 9, there is scope for the methods described in this thesis to be developed for systematic general application to metal forming; though such development will demand a considerable application of resources.

CHAPTER 8

CALCULATIONS OF MEAN AND TOTAL STRESSES

- 8.1 Introduction
- 8.2 Determination of Mean Stress from Deviatoric Stresses
- 8.3 Stress Distributions
- 8.4 Discussion and Comparison with other Workers
- 8.5 Conclusions

Chapter 8 - CALCULATIONS OF MEAN AND TOTAL STRESSES

8.1 Introduction

As has been discussed in Chapter 4, the finite element method used in this thesis calculates a velocity distribution throughout the deforming body. This can be converted to a strain rate distribution and from that the deviatoric stresses σ'_{ij} , can be calculated using the equation

$$\sigma'_{ij} = 2n\dot{\epsilon}_{ij} \quad (4.8)$$

The chief problem with this approach is that the deviatoric stresses are not particularly useful for the study of metal working, because the total stresses are required for evaluation of the process. Examples of the evaluations for which total stresses are required are the determination of the likelihood and position of failure in the workpiece and the stress distribution on the dies. The latter information can be used to find deformations, wear points and possible failure areas on the dies.

Deviatoric stresses are related to total stresses (Cauchy stresses) by the equation

$$\sigma_{ij} = \sigma'_{ij} + \delta_{ij}\sigma \quad (8.1)$$

where σ is the mean stress defined by

$$\sigma = \frac{1}{3} (\sigma_{ii}) \quad (8.2)$$

(see Mase(36)). From equation 8.1 it is seen that deviatoric shear stress is identical to total shear stress, so no conversion is required. However, in order to calculate the normal stress components (and the principal stresses), additions must be made to the program presented in this thesis and this is detailed in Section 8.2.

One great difficulty with theoretical stress determinations in highly deforming materials is that there are no good methods of experimental verification. Most experimental stress measurements can only be made on elastic materials or materials with only a very small plastic strain. This thesis is concerned with processes where not only is the material being very heavily deformed but is also subjected to very high temperatures. The only way the stress distributions found by the finite element work can be evaluated is by comparing the results with simpler results which give some idea of what can be expected in certain regions.

The best experimental methods for determining stresses at the surface of a deforming specimen are those that rely on measuring the deformation of elastic members in contact with the specimen. Such approaches have, for example, been used by van Rooyen and Backofen(74), Nagamatsu et.al.(29) and for an extrusion situation, Said and Alexander(75). Simple theoretical analyses for cylindrical compression can be found, for example, in the work of Bishop(76) and Avitzur(77). Of particular interest is the existence of a 'friction hill'

where the normal stress at the die/specimen interface is found to be greater at the centre of the specimen than at the circumference because of frictional effects. The simple analyses always predict this, its value being independent of specimen height, and they were in general confirmed by van Rooyen and Backofen(74). However, Nagamatsu et al.(29) found that the friction hill in their experiments was, in fact, changed completely by a change of height to diameter ratio because for tall specimens the effect was reversed (i.e. the normal stress was lower nearer the axis). Van Rooyen and Backofen only report measurements for fairly thin discs (height to diameter ratio of $\frac{1}{4}$ at the thickest), so it is conceivable that these experimental results are not necessarily in conflict. Such findings tend to undermine confidence in the simple theoretical results but arguments as to the reason for these errors, such as the fact that bulging is incorrectly treated or that the friction effects are wrongly considered, have not been developed to a sufficient degree of rigour to be convincing. Given that these difficulties exist with the simple results, it becomes difficult to prove the correctness of complex methods of analysis such as the finite element method. In addition to these difficulties, there is little guidance available as to what should be expected in the type of material that is being considered in this thesis, namely a strain rate dependent material as opposed to a strain hardening material.

Some details, however, can be agreed upon. In particular, the stress normal to the free surface and the shear stress along such a surface should be zero. Moreover, most authors agree that a circumferential tension stress should be detected at the outer surface of a bulging specimen and that this tends to be the cause of the vertical fracture which is often detected in this region in cold formed specimens. (Practical examples of this are given, for example, by Thomason(78).) These features are quite useful checks on the results at least at the 'debugging' stage.

8.2 Determination of Mean Stress from Deviatoric Stresses

The method adopted to find the distribution of the mean stress, σ , is to use the deviatoric stresses already calculated at the nodes by the program. Equation 8.1 gives

$$\sigma_{ij} = \sigma'_{ij} + \delta_{ij}\sigma \quad (8.1)$$

and the equilibrium equations are given by

$$\sigma_{ij,j} = 0 \quad (8.3)$$

where the comma plus subscript represents partial differentiation (by x_j). Equation 8.1 can be differentiated and the total stresses eliminated using equation 8.3. This results in

$$\sigma_{,i} = -\sigma'_{ij,j} \quad (8.4)$$

To find the mean stress at any point, equation 8.4 is integrated which can be done as a line integral within a single element. To do this, equations 8.3 and 8.4 are transcribed to local coordinates written as p_I (the capital subscripts indicating local coordinates). The transformation is written

$$\frac{\partial}{\partial x_i} = \frac{\partial p_J}{\partial x_i} \frac{\partial}{\partial p_J} = L_{iJ} \frac{\partial}{\partial p_J}$$

where L_{iJ} are elements of the inverse Jacobian (see equation 7.4). Equation 8.4 thus becomes

$$\sigma_{,i} = L_{iJ} \sigma_{,J} = L_{iJ} \sigma'_{il,K} \quad (8.5)$$

An increment of mean stress can be given as

$$d\sigma = \sigma_{,K} dp_K \quad (8.6)$$

Thus if the mean stress at a point is known to be equal to σ_a then the mean stress at point b is

$$\sigma_b = \int_a^b d\sigma + \sigma_a = \int_a^b \sigma_{,i} dx_i + \sigma_a \quad (8.7)$$

which is a line integral. Converting this to local coordinates involves

$$dx_i = J_{iK} dp_K$$

where J_{iK} are the elements of the Jacobian and equal $\frac{\partial x_i}{\partial p_K}$

(see equation 7.4). Thus expanding equation 8.7

$$\begin{aligned}\sigma_b &= \int_a^b \sigma_{,i}^J \delta_{iK} dp_K + \sigma_a \\ &= \int_a^b L_{1J} \sigma'_{il,J} \delta_{iK} dp_K + \sigma_a\end{aligned}\quad (8.8)$$

This integral can be easily calculated by Gaussian integration if the term $\sigma'_{il,J}$ can be determined at any point within an element. Since these calculations are to be inserted into a finite element program the terms $\sigma'_{il,J}$ can be most easily found if the values of deviatoric stress already calculated at the nodes and the existing shape functions can be used. Hence if σ'_{ij}^N is a deviatoric stress component at node N, then the deviatoric stresses at a point c can be written

$$\sigma'_{ij} = N(c)_{N,J} \sigma'_{ij}^N$$

The terms $N(c)_{N,J}$ are easily formed, since in the finite element program they already exist to enable evaluations such as in equation 7.4.

To start the calculations of mean stress in an element, it is necessary to know the mean stress at at least one point in the element. Along the free surfaces the deviatoric stress normal to the boundary which has already been calculated in the program must be equal to minus the mean stress, since the total stress normal to such a surface is zero. Thus σ is known at the free surfaces and the integration in equation

8.8 can be carried out to any point within the adjoining elements. Thus σ can be calculated at nodes which do not fall on the free surface, and from these nodes the calculation can be carried to all elements in the specimen.

There is a cumulative error occurring in all the integrations, both because of inaccuracies in the values of deviatoric stresses at the nodes (due to the sources of errors involved in the finite element calculations and the specification of material properties) and in the Gaussian integration used to evaluate the integral of equation 8.8. As a result the number of calculations to go from the free surface to the more distant nodes in the specimen should be kept to a minimum.

8.3 Stress Distributions

Figure 8.1 gives the stress distribution determined at the first iteration for the first increment on a specimen 25mm high by 25mm in diameter, subject to an axial compression at a speed of 0.3mm/sec. The material properties being used are those from Lee and Backofen(13) for Ti6Al4V at 980°C and thus these results compare with the experiments reported on figures 6.2.1-6.2.3. The initial peak on the experimentally measured mean stress curve is at 20.6 MPa, and for the curve corrected for friction effects the stress is 17.0 MPa. The diagram for the theoretically determined normal stress, σ_z , gives the average stress at the surface as 20.6 MPa, while the average centre line axial stress is 13.8 MPa (which should be approximately equivalent to the experimentally

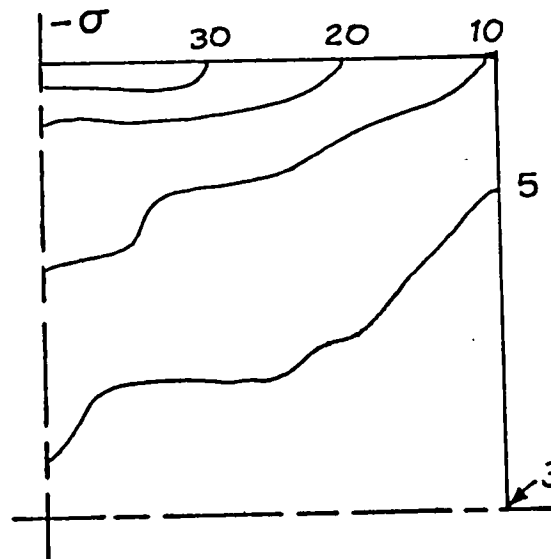
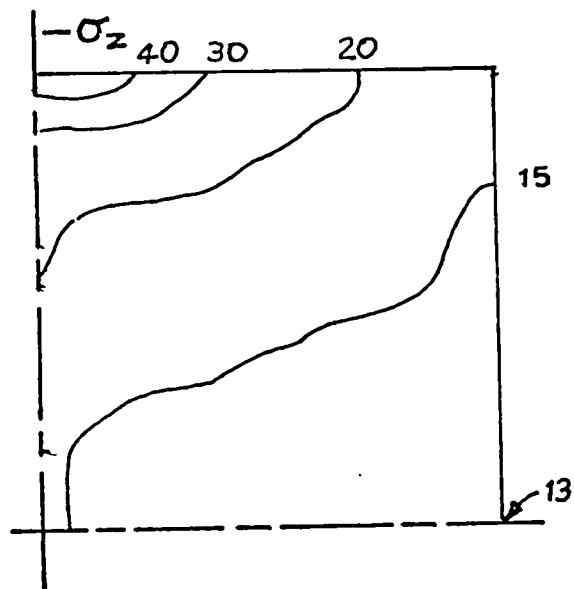
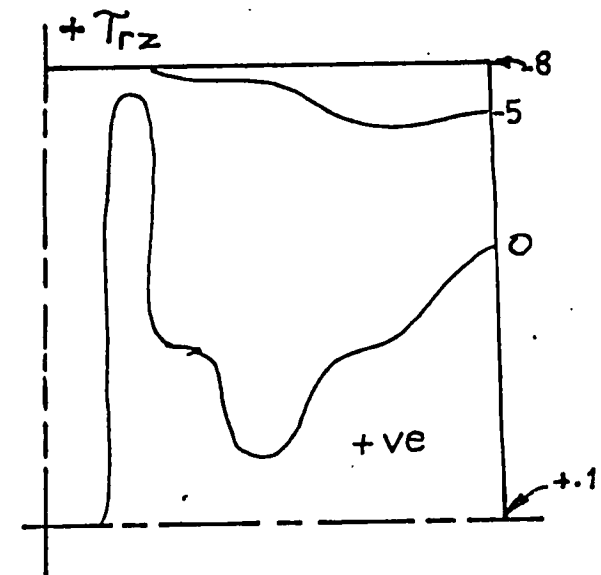
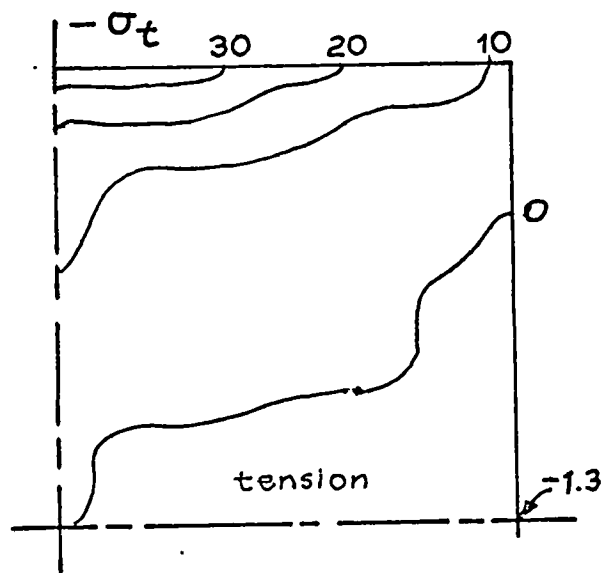
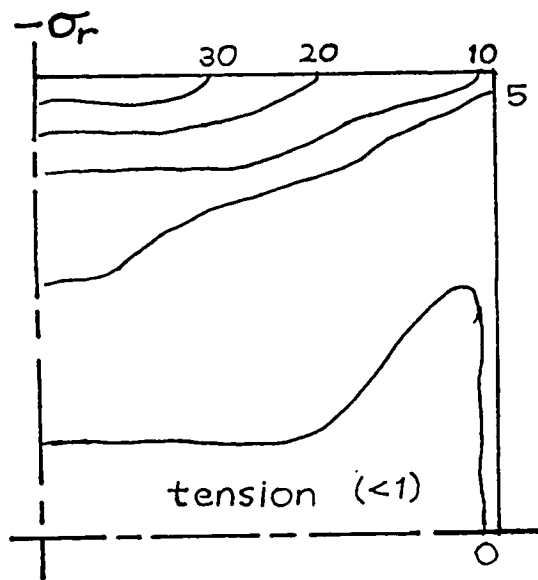


Figure 8.1

Stresses in a cylinder (in MPa).

FIRST increment.

FIRST iteration.

Size: 25mm high by 25mm in diameter.
(one quadrant shown).

Material: Ti6Al4V at 980°C.

Crosshead Speed: 3mm/sec.

determined value of stress corrected for friction). Other features of interest on figure 8.1 are that there is a tension developed in the circumferential stress, σ_t , towards the outside of the bulge; there is quite a marked friction hill detected on the axial stress, σ_z , at the surface; and the radial and shear stresses, σ_r and τ_{rz} , are close to zero at the outside free surface. The error in these last results, which should be exactly zero, is due partially to the fact that a bulge is developing on the specimen and also due to the point of discontinuity occurring at the corner node which lies both on the free surface and is also attached to the die.

There are one or two disturbing features on figure 8.1, for example, the region of slight tension in the σ_r plot and the size of the region of tension on the σ_t plot. The determination of stresses in this non-linear system is dependent on the velocity distribution, which is in turn dependent on the viscosities which are in their turn dependent on velocities. Presumably a more realistic result will be approached if the program is iterated, recalculating velocities, viscosities and stresses. Figure 8.2 indicates the results from the eleventh iteration of the first increment of the same problem as figure 8.1. As can be seen, the results have changed very little in magnitude in most regions, only altering by one or two MPa at most. The mean stress normal to the surface is now calculated to be

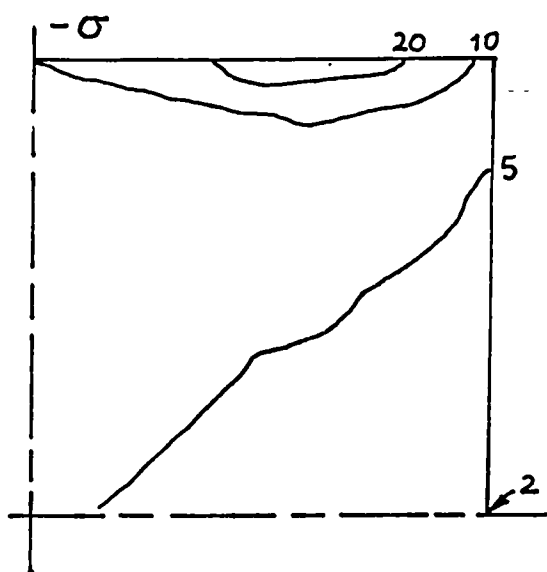
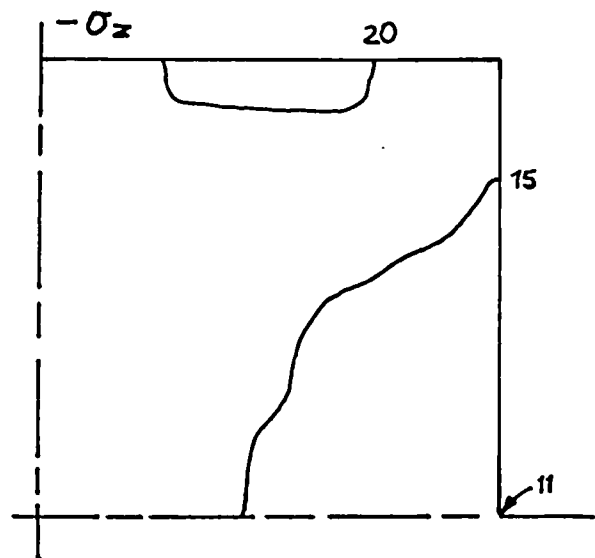
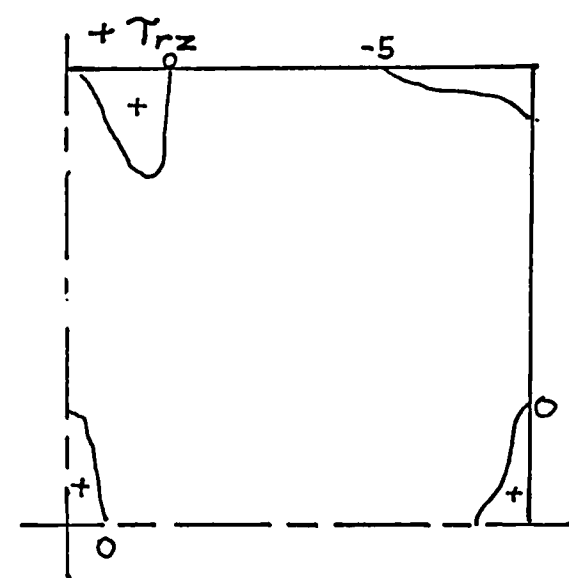
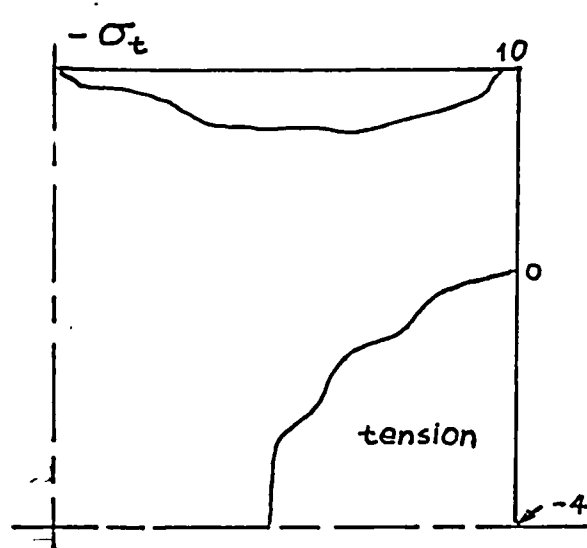
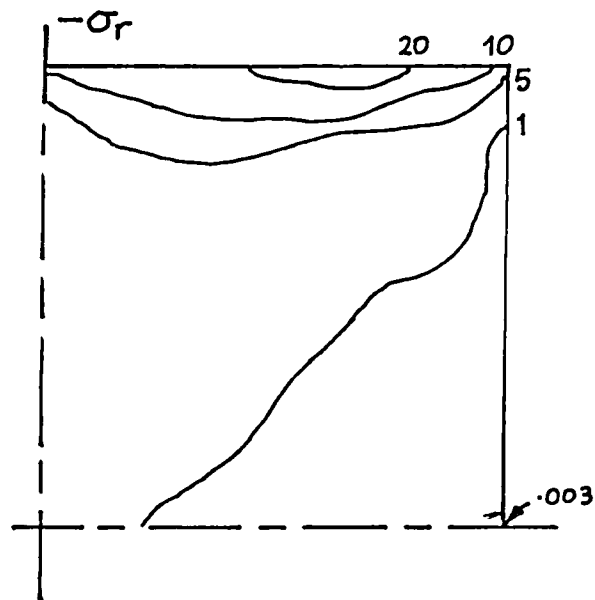


Figure 8.2
Stresses in a cylinder (in MPa).
As for figure 8.1 except
FIRST increment,
ELEVENTH iteration.

19.2 MPa and the average mid-section plane stress is 14.2 MPa. Some improvements to be noted are the removal of the tension region in the σ_r plot, the concentration of the tension region in the σ_t plot and the reduced number of positive readings in the shear stress plot (some positive values may be expected in the region of the bulge).

Another difference can be seen when the σ plots for figure 8.1 and 8.2 are compared. The most significant change is that the friction hill effects have been reduced and the maximum normal stress has moved to an annulus away from the axis. The change is also recorded in the general lowering of all stresses near the die/specimen interface near the axis. The effect of the iterations on the axial stress at the surface and mid-section is shown in more detail in figure 8.3. This figure shows firstly that the changes in stress levels are not great at the mid-section but are more significant at the die/specimen interface near the axis. The most likely explanation for this is that this region is most nearly rigid; the strain rates are very low as can be seen from figure 8.4 (11th iteration calculations). This means that the stress raising features due to friction are being counteracted by stress lowering in the region due to lower stresses being required to maintain the lower strain rates encountered there. The lowering of axial stresses near the axis may also be a reflection of the effects of tall specimens tending to have a 'concave' stress distribution, as reported by Nagamatsu et al.(29) (see Section 8.1).

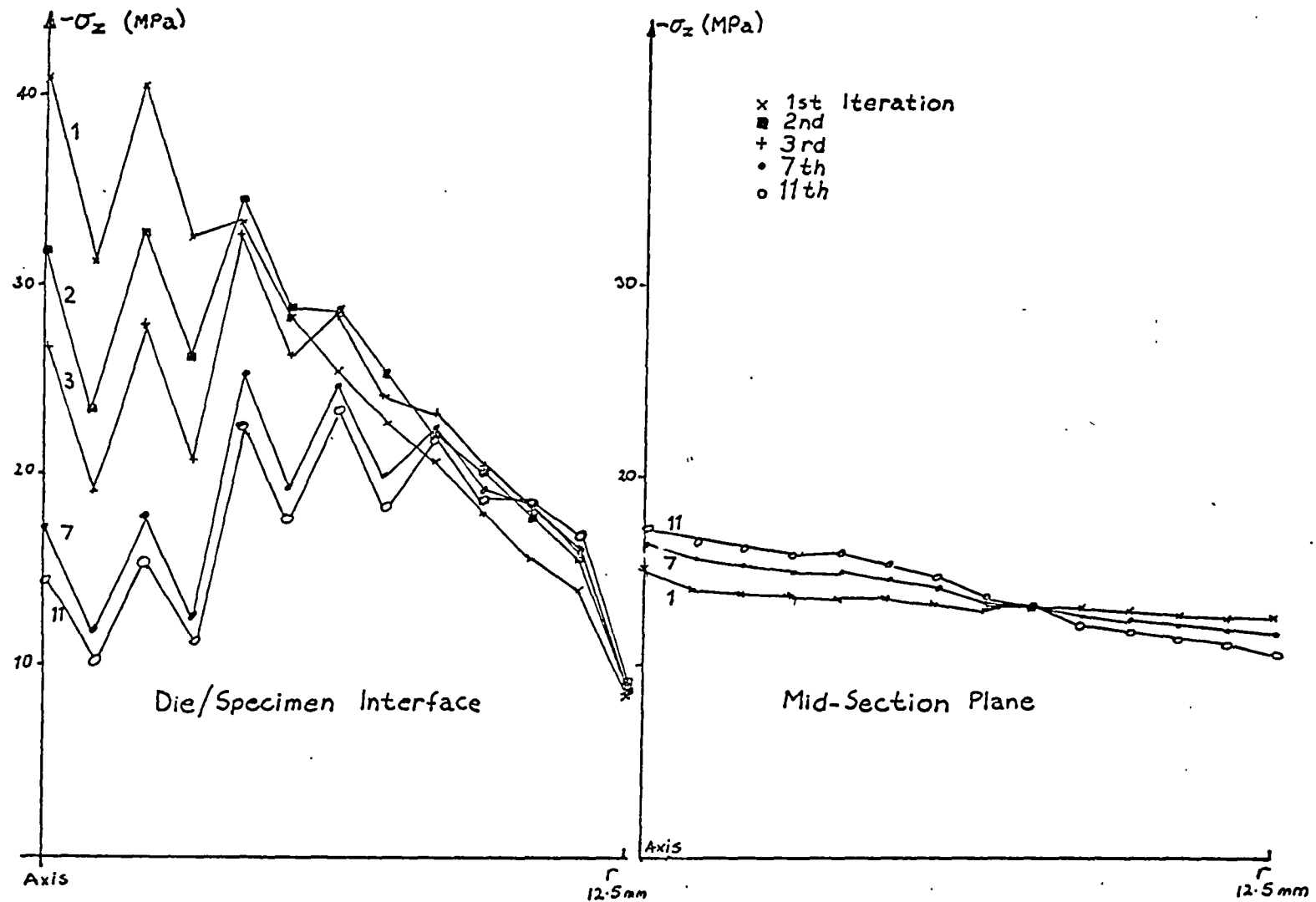


Figure 8.3 The axial stress for the compression test of figures 8.1 and 8.2 reported on two planes in the specimen.

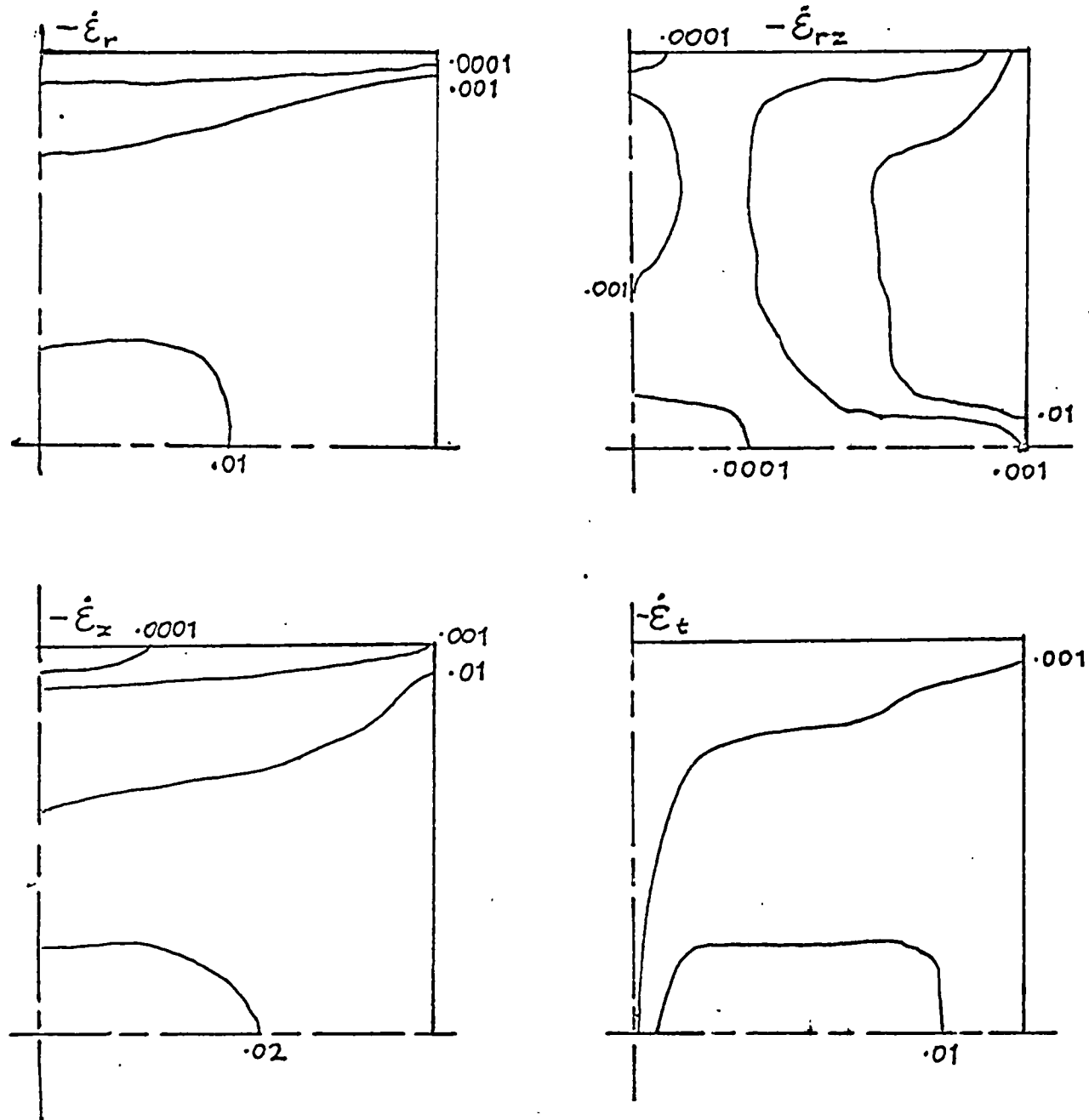


Figure 8.4 Strain rate distributions for the same specimen as illustrated in figure 8.2. Units are sec^{-1} .

Figure 8.3 also reveals an oscillation of stresses which is clearly connected with the nature of the elements being used. The mid-side nodes of elements at the die/specimen interface tend to show lower stress levels than the corner nodes. This is most probably due to an oscillatory error for elements that are highly constrained, a matter which is discussed in Appendix 4.

Figure 8.5 gives some σ_z distributions for a specimen undergoing a ten times greater crosshead speed (3mm/sec). The results given are for the first increment, second iteration and the specimen is identical to the specimen of figures 8.1-8.4. It is noted that the stress levels are in general higher than for the slower speed example and this corresponds to the higher strain rates within the specimen. The same features of the stress distribution are also seen, namely a friction hill effect (which lowers with repeated iterations), quite stable stresses at the mid-section plane (and throughout the bulk of the specimen), and oscillatory stresses at the die/specimen interface near the axis.

For the frictionless specimen that was shown in figure 7.2, the axial stress level in the first and subsequent increments is very nearly the same at all nodes and almost exactly -18.177 MPa (the crosshead speed was 3mm/sec). The values of σ_r , σ_z and τ_{rz} are all less than 10^{-3} at all nodes. The only exception to these generally excellent results are along the axis where the values of σ_t rise to

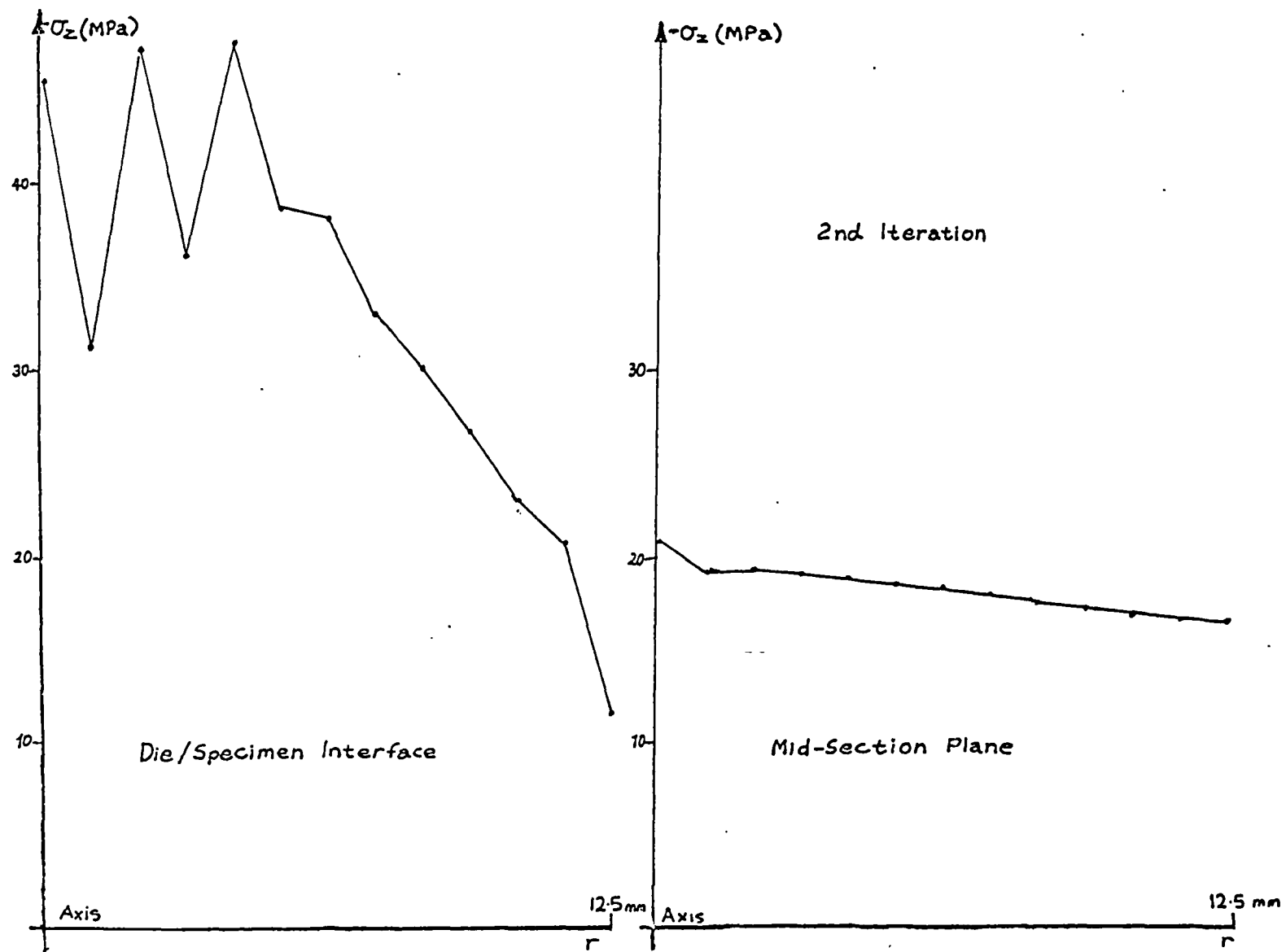


Figure 8.5 Axial stresses at two planes for a high speed specimen. (Cf. figure 8.3.)

-6.5 MPa, and the value of σ_z rises slightly to -19.7 MPa. Thus the axis nodes display an anomalous behaviour which is presumably due to the fact that divisions by the radius r which are involved in some of the calculations of tangential stresses and strain rates, must become difficult as r approaches zero. Figure 8.6 illustrates that at the surface there is no evidence of a friction hill in the values of σ_z , but the nodes in element 1 at the axis are still showing some erroneous behaviour. The restraint on this element must contribute to this as suggested in Appendix 4.

Figure 8.7 displays σ_z along the mid-section plane and the top surface of a ring test at the first increment, second iteration. The ring has a height of 6.45mm, an inner diameter of 12.7mm, an outer diameter of 25.4mm and the crosshead speed is 0.3mm/sec. As can be seen, the axial stress rises to a peak at the interior of the specimen very close to the neutral surface and then decreases to the inner hole. The peak is much more pronounced at the die/specimen interface than on the central plane, and there is some evidence of element dependent oscillations in the area of low strain rates near the neutral axis (see figure 7.10 for a typical distribution of tangential strain rates). Figure 8.8 shows the full stress distribution for the first increment of this ring test and figure 8.9 shows the distribution for the same test at 31% compression (the specimen has been squared off in the latter figure for ease of plotting). As can be seen, the friction hill increases in severity as the compression

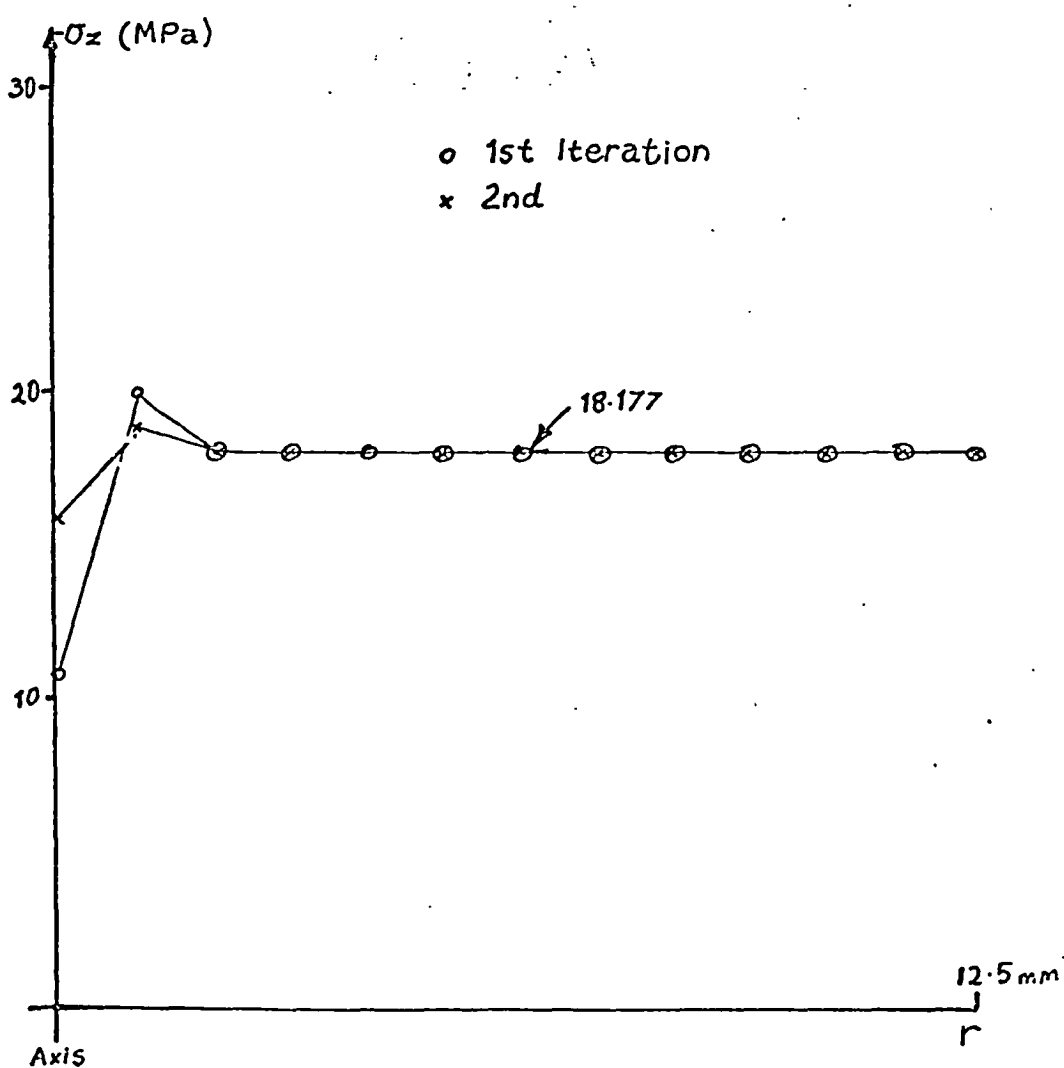


Figure 8.6 Axial stress distribution along the die/surface interface for the frictionless specimen illustrated on figure 7.2.

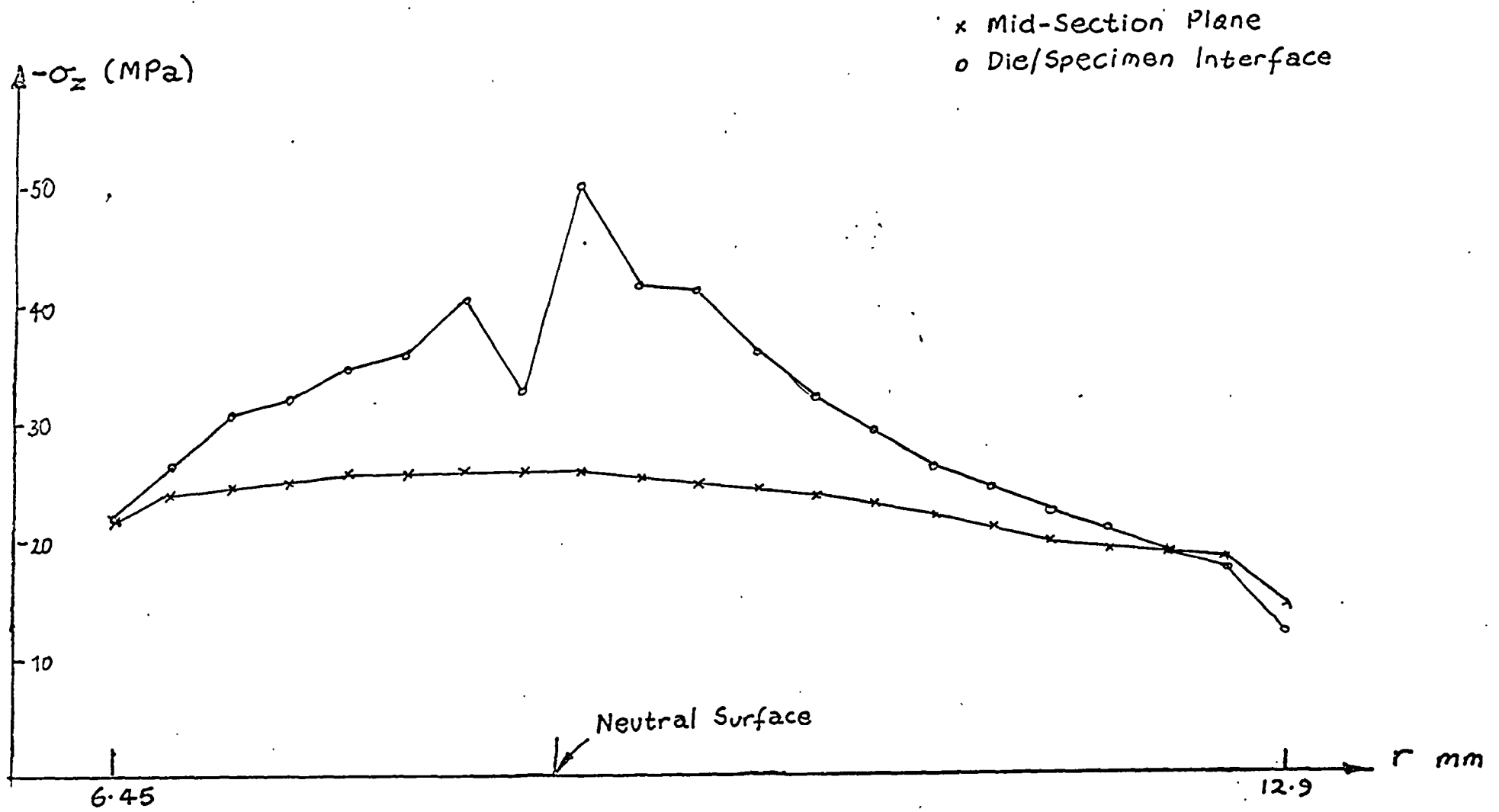


Figure 8.7 Axial stress distributions along two planes in a ring test. First increment, second iteration shown.

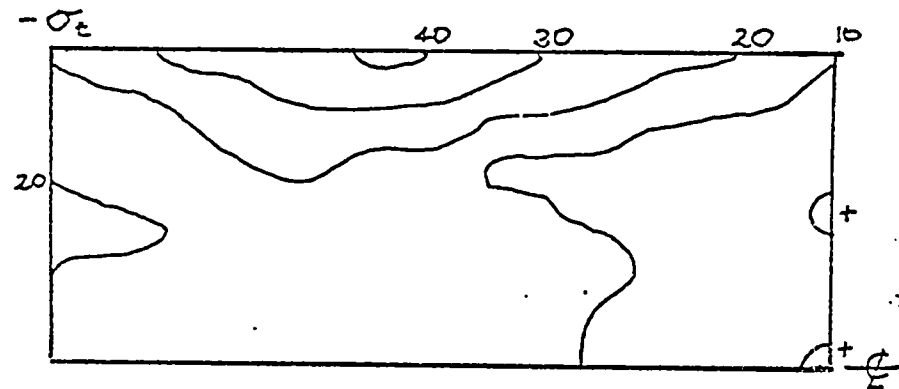
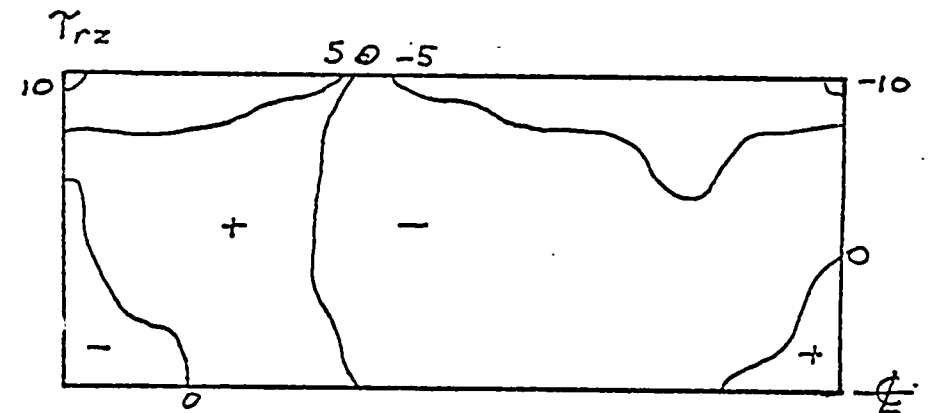
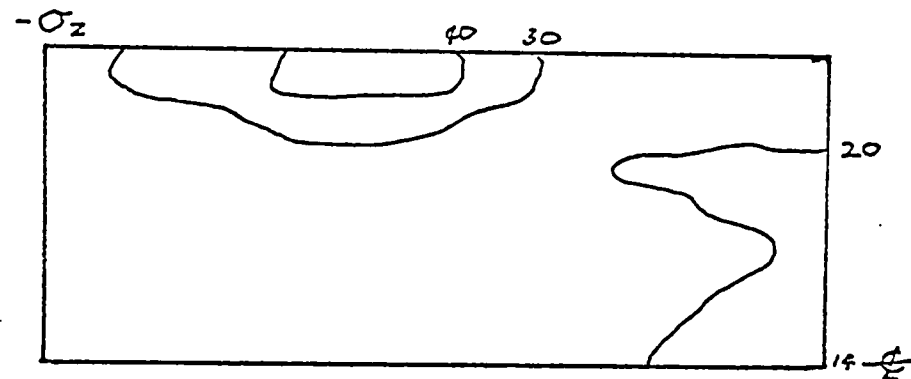
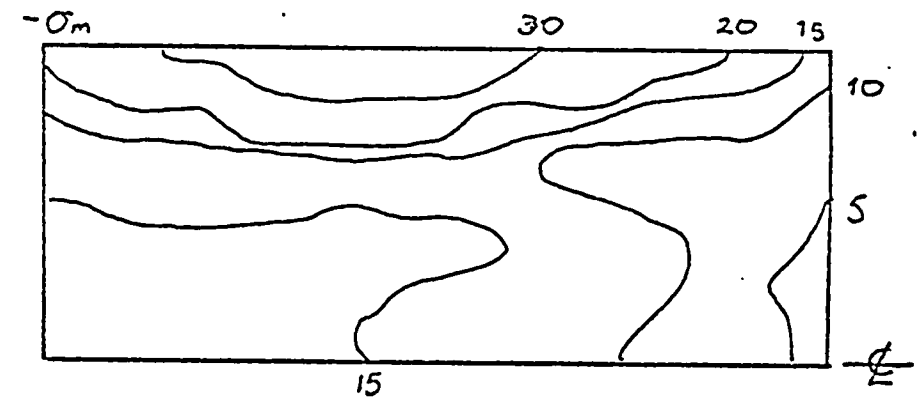
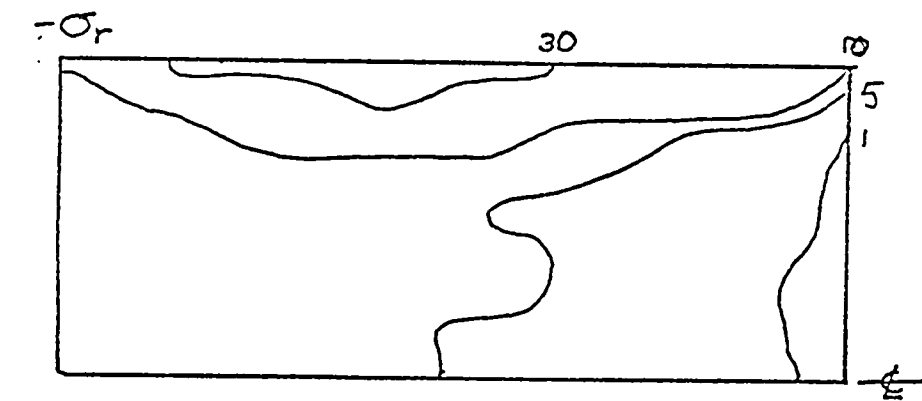


Figure 8.8 Complete stress distributions for a ring test during the first increment
Stresses are in MPa.

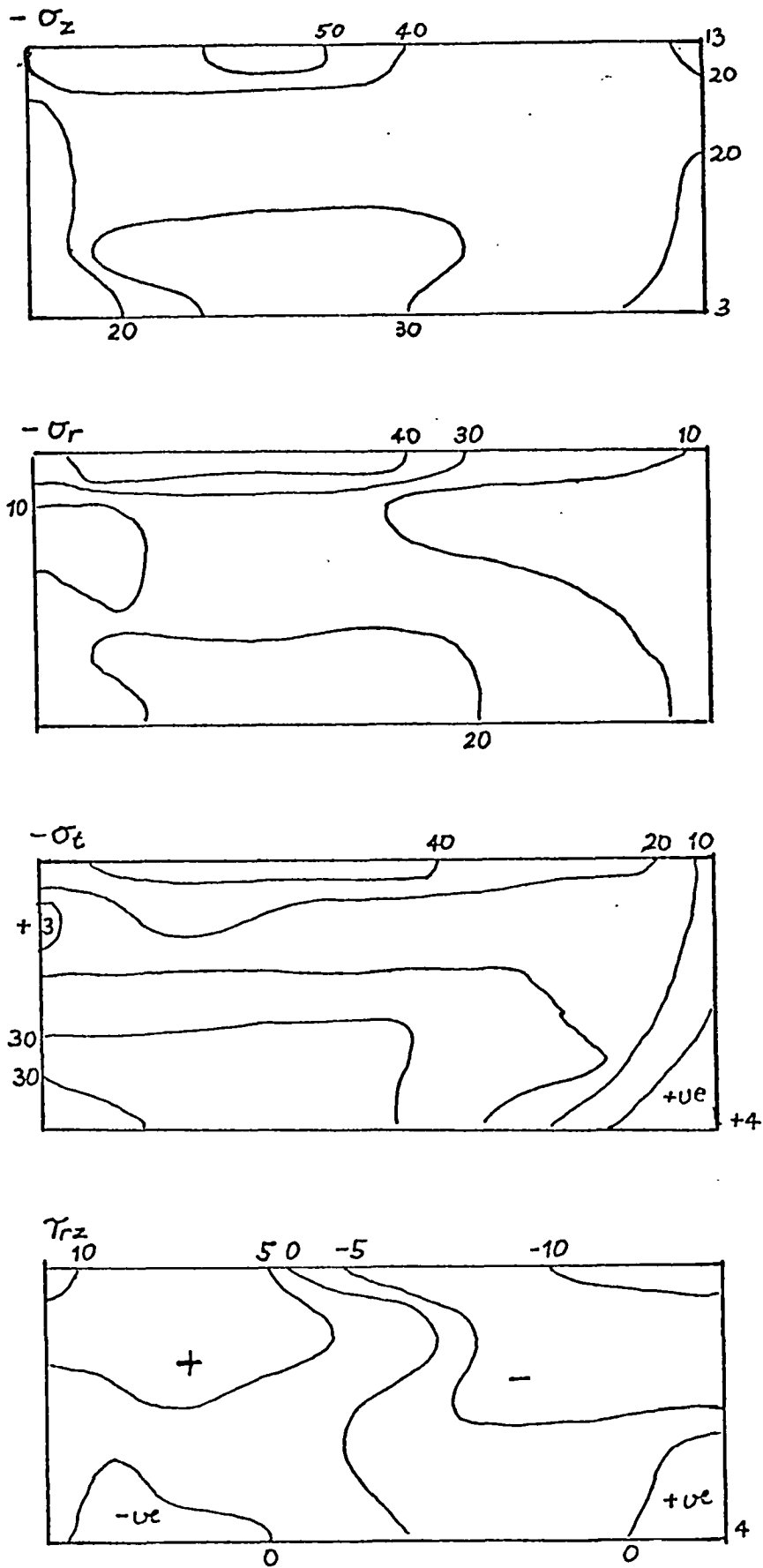


Figure 8.9 Stress distributions in a ring test at 31% compression.
Units are MPa.

continues and the other stress distributions change in detail as well.

The ring test enables an evaluation of the errors inherent in the method of determining the stresses by the method outlined in this Chapter by virtue of the fact that there are two free surfaces on the specimen. It is desired that the radial stresses and tangential stresses at both the inner and outer surfaces should be close to zero. Figure 8.8 illustrates the error that occurs in the integration of the stresses from the outer surface to the inner surface. At the outer surface where integration starts, the value of σ_r at all the nodes except the corner node is less than 10^{-3} MPa, but at the inner surface where the integration ends after traversing ten elements the average value is -5 MPa. This is an indication of the magnitude of error due to the accumulation of errors in nodal deviatoric stresses and Gaussian integration across the elements. On the more highly deformed specimen in figure 8.8, which is bulging due to the effects of sticking friction, the situation has deteriorated. At the outer surface the average radial stress is -5 MPa, the value not being closer to zero due to the fact that, since the specimen is bulging, the radial stress is no longer normal to the surface and thus is likely to be finite. However, on the inner surface the average stress has risen to -15 MPa with a variation of about 5 MPa about this mean. The variation is probably mainly due to effects

of the bulge on the inner surface, but the high mean value can only be explained by increase in errors in nodal stress values and the Gaussian integration that occurs as the elements become more deformed. A study of some of these errors in an element is given in Appendix 4.

8.4 Discussion and Comparison with other Workers

There are a number of solutions in the literature of problems that can be compared approximately with the stress distributions obtained in this work. While no other worker deals specifically with the strain rate sensitive materials that are considered in this thesis, comparisons with strain hardening and elastic-plastic examples of similar geometries can be illuminating.

The first comparisons will be made with the results of some work using the technique of weighted residuals. It is not the intention to describe this method in detail in this thesis, but such descriptions can be found in the work of Steck(79) and Ravindranath(80). The technique of weighted residuals is a method of finding the minimum energy required in a deformation by varying the velocities and often stresses at a number of nodes distributed through a deforming body. In a sense, it can be regarded as a finite element method, involving only one complex element, and vice versa the finite element method can be described as a method of weighted residuals. The use of a penalty function in an

attempt to enforce incompressibility in the manner described in Section 4.5.3 is a particular application of weighted residual type techniques, where a very high weighting is placed on any finite compressibility. This comparison reveals one possible objection to the weighted residual technique because the energy terms tended to be weighted in a rather more arbitrary manner than in the finite element method and the velocity distribution that is allowed inside a body tends, perhaps, to be more restricted. However, the weighted residual technique is accurate and flexible and compares very favourably with the finite element method in terms of computer resources used.

Figure 8.10 gives some results for the compression of a strain hardening cylinder from the work of Steck(79) using the method of weighted residuals. Results from a similarly shaped specimen, calculated using the methods of this thesis and using material data appropriate to Ti6Al4V at 950°C, is shown on figure 8.11. In qualitative terms the shapes of the calculated stress distributions are quite similar. The axial stresses of Steck show a much more vigorous increase towards the axis of the specimen, while the increase in the radial stresses is quite similar. In fact, the agreement between the two figures is quite acceptable once allowance has been made for the fact that the material properties and methods of analysis differ greatly.

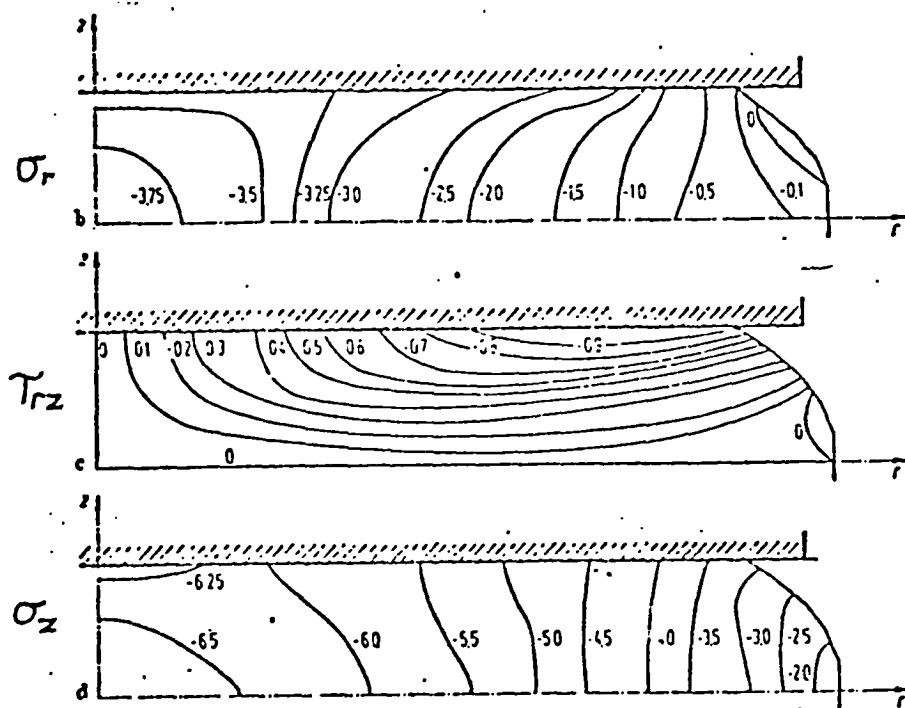


Figure 8.10 Stress distributions for the compression of a cylinder from the work of Steck(79). Figures give the ratio of the stress to the flow stress, k .

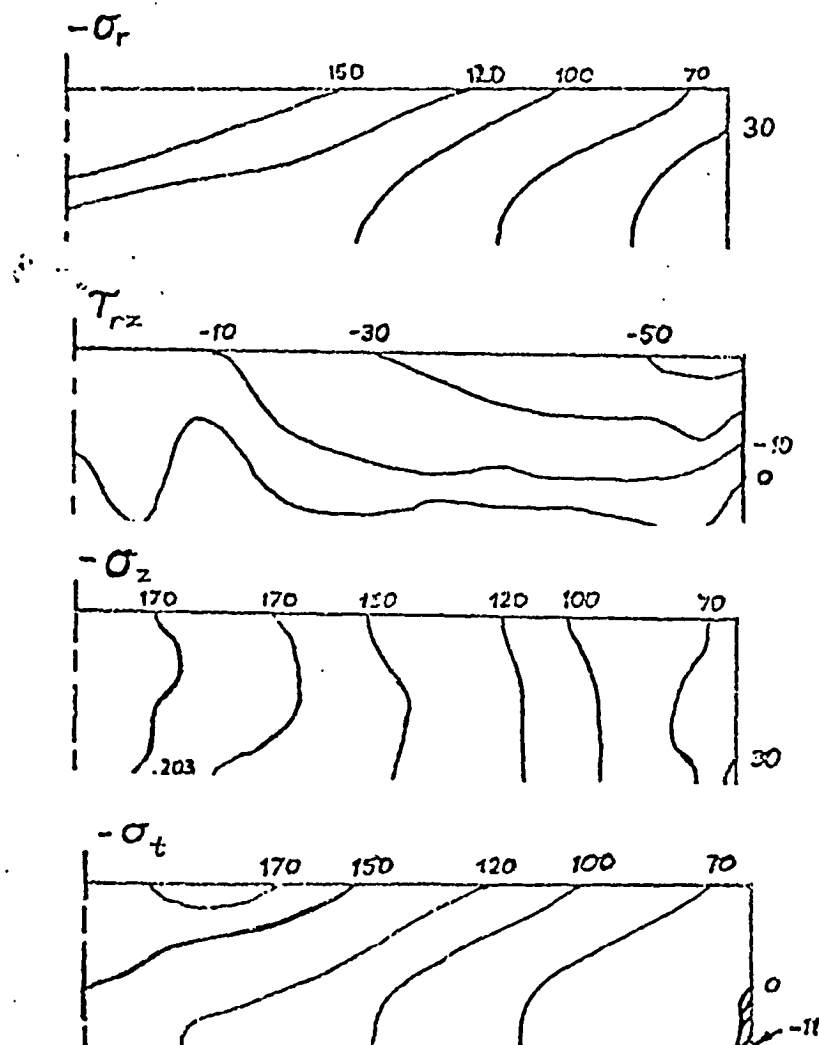


Figure 8.11 Stress distributions from this work. Stresses in MPa. (Cf. figure 8.10)

The second comparisons are with the work of Shah and Kobayashi(43), some results from which are shown in figures 8.12 and 8.13. The method of calculation of this work has been reviewed in Section 4.5.2; the method does not allow for strain rate sensitivity. Figure 8.12 shows the analysis of a cold heading operation which can be considered as being approximately the same as a cylindrical compression test if only the top half is considered. At 30% reduction the cylinder virtually has the proportions 1:1 and can be compared to figures 8.2-8.4. The effective strains on figures 8.11 show qualitatively the same distribution as the strain rates of figure 8.4. On the question of the mean stress distribution the 30% reduction on figure 8.12 can be compared with figure 8.2 and the 60% reduction with figure 8.11. In the bulk of the specimen, the agreement is again satisfactory in a qualitative sense, but some disagreement does appear at the die/specimen interface. Figure 8.12 gives the axial stresses along this interface for the cold heading operation, and it can be seen that a reverse friction hill is predicted at the lighter reductions and very high values of axial stress are reported at the circumference of the specimen. These results contrast with the results in this thesis which always show a low axial stress at the circumferential node on the top surface, and always tend to show a normal friction hill.

Another contribution to the study of axial stresses at the die/specimen interface is given by Nagamatsu et al.(29) and figures 8.14-8.16 are reproduced from this work. Figure

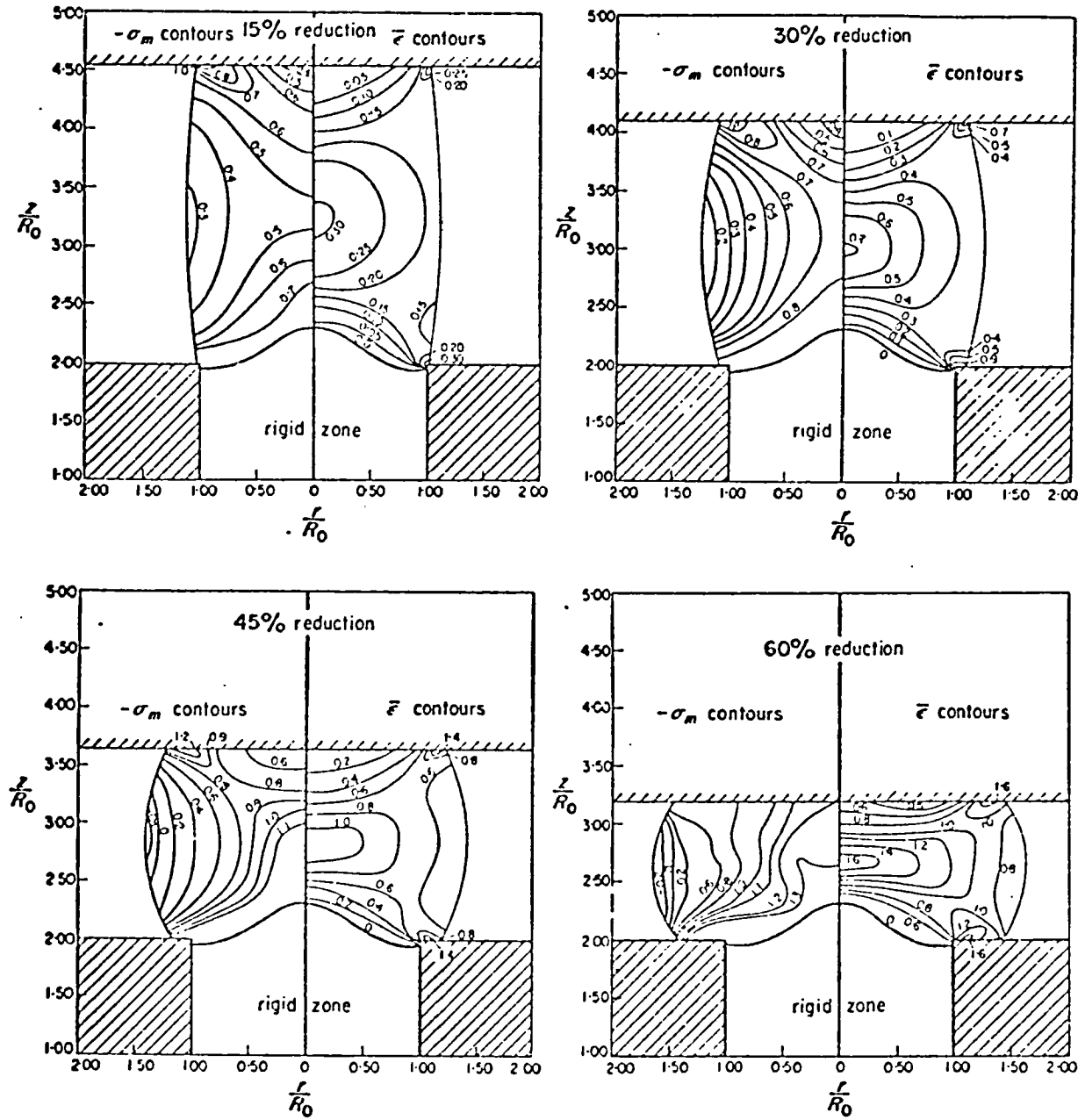


Figure 8.12 Mean stress and effective strain distributions from Shah and Kobayashi(43)

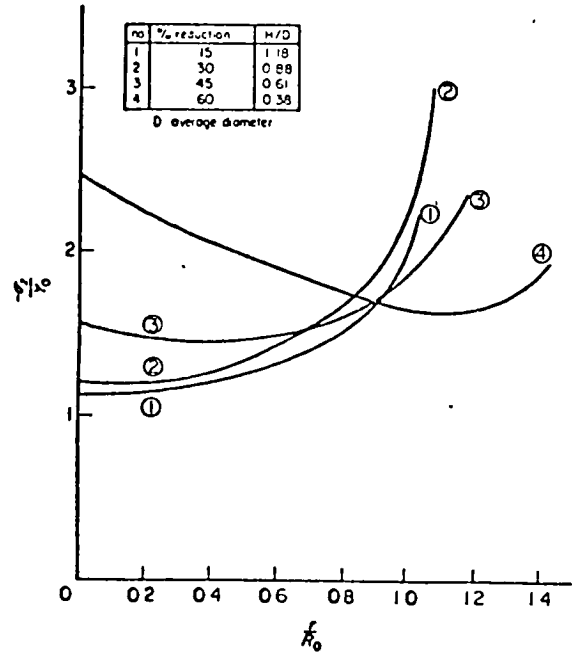


Figure 8.13 Axial stress distribution at the die/specimen interface from Shah and Kobayashi (43).

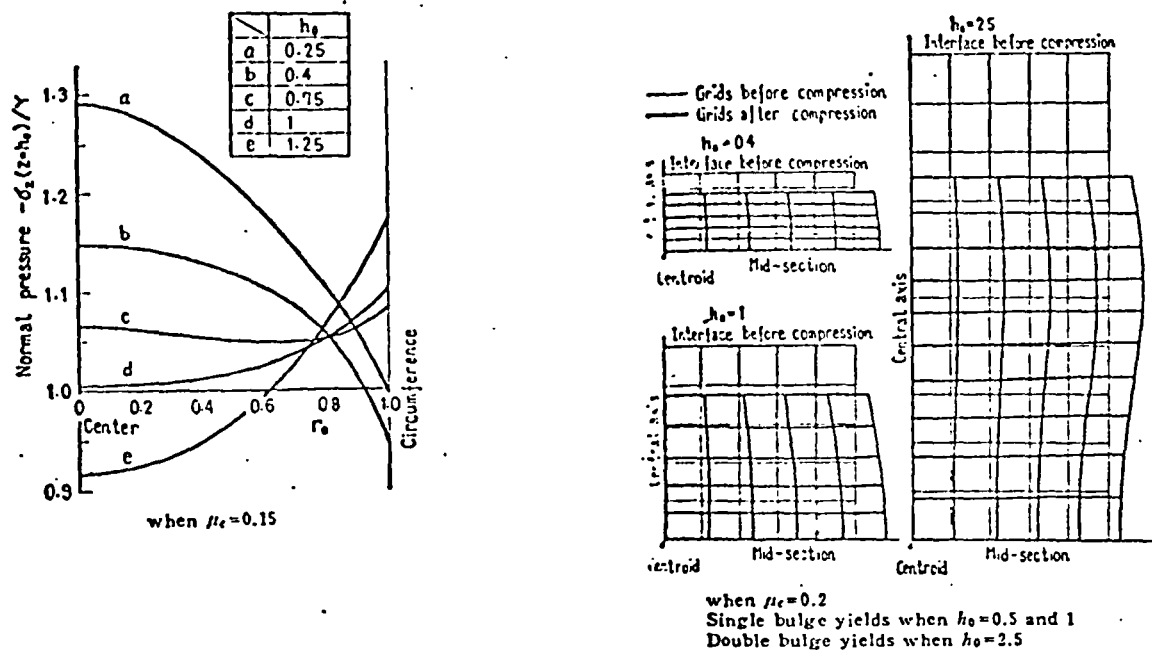


Figure 8.14 Axial stresses at die/specimen interface and deformed grids for cylindrical compression from Nagamatsu et al.(29).

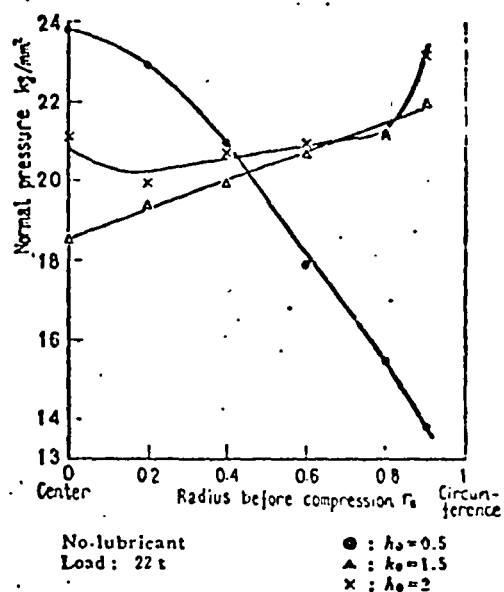


Figure 8.15 Experimental axial stress distributions for cylindrical compression again from Nagamatsu et al.(29).

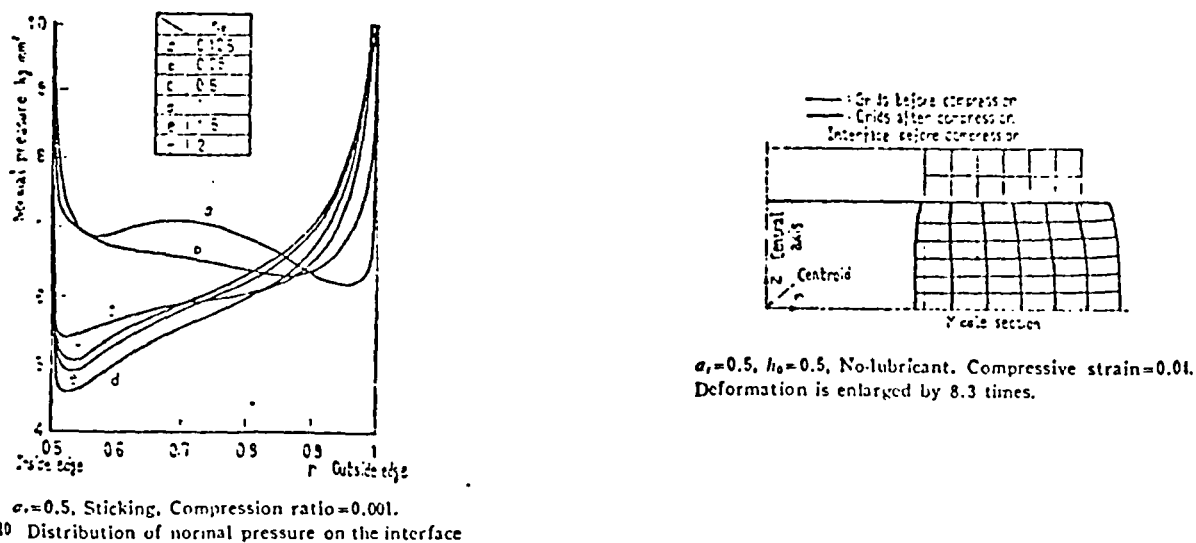


Figure 8.16 Axial stress distributions and deformed grids from ring tests from Nagamatsu et al.(29).

8.14 shows some deformed grids and axial stresses at the interface for a variety of cylindrical compression specimens which were calculated using an elastic-plastic finite element program of the type described in Section 4.2. The frictional conditions are not those of sticking friction but a 'slip-ratio' is used which is, in fact, a kind of geometrically determined boundary condition and does not invoke the use of the techniques which require the calculation of shear stresses at the interface such as described in Section 7.2. The deformed grids of figure 8.14 compare favourably with the sticking friction examples of figure 7.3. However, the axial stress distributions at the die/specimen interface again show evidence of the reverse friction hill, an effect that is reduced and eventually reversed as the specimens become more squat. Nagamatsu et al. also present some experimental data for axial stress at the interface which is shown on figure 8.15. This data does suggest that a reverse friction hill may well exist, but the concavity is not as strong as suggested by the theoretical results.

Figure 8.16 presents axial stress data at the top surface of a ring specimen and the deformed grids obtained from the finite element work. Once again the deformed grid compares favourably with the results of this thesis shown on figure 7.7, but the axial stresses are in conflict with figures 8.7 to 8.9. Once again, and as in Shah and Kobayashi's work, there is a distinct tendency for the stresses at the

edges of the specimen to be higher than the stresses found in the interior of the specimen. The feature of a high stress at the edges of the die/specimen interface are not detected in the experimental work of van Rooyen and Backofen (74) or in the theoretical determinations of Steck(79), or of this thesis.

8.5 Conclusions

The comparison of all these results is difficult because of the variety of methods and material models used. However, there are some agreements which might be emphasized.

(a) Strain or strain rate distributions and the deformed grids determined by all the methods seem to be in satisfactory agreement with experiments.

(b) The general form of the stress distributions are in reasonable agreement for all the methods in most regions.

(c) There seems room for improvement in theoretical and experimental determinations of stresses along the die/surface interface. However, all methods do agree in predicting that the friction hill is lowered as the specimens become taller. It is only the reversal of the friction hill which is in question, since this is not predicted by all methods. Moreover, it is not clear what particular features of the test can cause such a reversal, and until these features are identified, proper comparison between

results is hampered. It is the opinion of this author that, in the particular case of the high axial stresses determined at the very edges on the die/specimen interface by Kobayashi and Nagamatsu et al., the results could in fact turn out to be a consequence of the finite element method and the restraints placed on nodes in this region. This effect could be similar to the problems experienced in this thesis at the specimen axis (see Appendix 4).

CHAPTER 9

CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

- 9.1 Dieless Drawing
- 9.2 Isothermal Forming
- 9.3 Application of the Analyses to Different
 Materials and Situations

Chapter 9 . - CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

9.1 Dieless Drawing

Dieless drawing is a difficult process from the point of view of control and prediction. It is hoped that the theoretical techniques presented in this thesis provide a method for determining the parameters required for successful application of the process. The analytical solution should be of particular interest in this regard, but the additional accuracy of the two computational techniques should also be useful.

A general appraisal of the two computational techniques suggests that the one-dimensional slab method appears to be almost as accurate as the finite element method. However, there were problems encountered with the slab technique when the reduction was altered, while the finite element method seemed to be more tolerant to changes of parameters in the process. Both methods produced satisfactory results, both giving good predictions of the outside profile of the forming zone and the load for various changes in conditions.

From a more general point of view, it is difficult to see whether dieless drawing will ever find wide application as an industrial process. This is chiefly because most of the deformation is done under conditions of hydrostatic tension, and defects, such as voids and surface cracks, will tend to increase both in number and in size. The main

emphasis of further work on the process will probably concentrate on this particular problem and practical solutions, such as the introduction of shaping rolls and dies, will be considered. This, of course, tends to remove most of the advantages of the process, which lie in the absence of tooling, and the similarities with the process considered in this thesis will consist in the localised heating and reduction augmented by crosshead movements.

Theoretical studies of the process may still yield some interesting results, and probably the most interesting area is a study of the free shape changes that occur in specimens that are not axisymmetric. The analysis would essentially study a single cross-section as it passes through the forming zone and this is probably amenable to a two-dimensional finite element analysis. This kind of work would be analogous to the study of changes seen in a frictionless compression test when viewed axially.

9.2 Isothermal Forming

The experiments in isothermal forming showed how very large deformations could be achieved in a single forging stroke at quite low stress levels and that the material conformed very accurately to the dies. As has been noted, practical application of the technique to aerospace alloys has reached a high degree of sophistication and the method is now extensively used in production.

The method of finite elements has been demonstrated to give a satisfactory modelling of the flow of the material during the process of isothermal forming, and with the penalty function approach and limited convergence of the results, the computing times are quite moderate. It has been pointed out that in the case of cylindrical compression and ring tests the classical methods produce useful results, and without so much computing effort as involved in finite elements. However, the range of geometries and conditions to which these simpler methods can be applied is limited, and as the tests become more complicated the amount of ingenuity and number of assumptions required lessens the usefulness of the classical approaches. In such circumstances, the finite element method presents a generalised and thorough approach.

In the work for this thesis, the preparation time for each class of solution with the finite element method was quite extensive; each new profile die, geometry of specimen or boundary condition requiring new programming and introducing new difficulties. However, it has been demonstrated that each of these problems can in turn be overcome and thus it is possible to claim that further work on the finite element method as an approach to metal forming is well justified. This further work will probably centre on the development and improvement of subsidiary programming which will enable each new situation to be considered with a minimum of involvement on the part of the researcher.

This type of development is analogous to the work that has produced the kind of finite element package that is routinely applied to a wide variety of stress, vibration or heat transfer problems in industry. It is envisaged that the penalty function approach could be the system chosen for such applications because of its economy, simplicity and the fact that it can be applied to three dimensions. (This is in contrast to the stream function approach which is limited to two dimensions.) One particularly attractive application of the finite element method to metal forming is in cold forming where it could be used for the design of cold forming processes assessing the extent, nature and number of steps required to produce the final shape.

The technique of determining mean stresses outlined in Chapter 8 is attractive from the point of view of its being a peripheral tack-on type of program which requires quite low computing times. The whole question of the determination of stresses inside plastically deforming bodies is still open to further investigation, however, and any further experimental evidence about stresses in the interior of such bodies would be very useful.

9.3 Application of the Analyses to Different Materials and Situations

The principal concerns of this thesis were the processes of dieless drawing and isothermal forming and it has been stated these processes exhibit strong strain rate and temperature

dependence but negligible effects due to strain hardening of the material. If the analyses in the thesis were to be extended to study other processes, it is crucial that a realistic constitution be incorporated to represent the flow behaviour of the material involved. For many metal working processes, such as conventional hot or cold forging of steel, for example, the total effective strain at any point is of paramount importance, since it determines the work hardening of the material. To allow for work hardening to be involved the formulation relating deviatoric stress to strain rate used in this thesis, viz.

$$\sigma'_{ij} = 2\eta\dot{\epsilon}_{ij} \quad (4.8)$$

should be replaced by an equation relating total stress increment to strain increment, such as

$$d\sigma_{ij} = D_{ijkl}d\epsilon_{kl} \quad (4.2)$$

or

$$\dot{\sigma}_{ij} = D_{ijkl}\dot{\epsilon}_{kl} \quad (4.3)$$

(which are given in the most general anisotropic formulations for non-linear materials). The functional which is to be minimised will include a term of the type

$$d\sigma d\epsilon dV$$

or

$$\dot{\sigma}\dot{\epsilon}dV$$

which are not energy rate or total energy terms.

These formulations have been used by several workers and some of the difficulties have been discussed in Section 4.2. There is still scope to investigate the utilisation of these formulations to rigid plastic deformations and to include accurate constitutive relations. In this connection it may well be worthwhile to extend some techniques of this thesis, which are characterised by economical computer times, to enable application to more situations and materials.

APPENDICES

APPENDIX 1

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Most Common Symbols

APPENDIX 3

Details of Gaussian Elimination Solution Method

APPENDIX 4

A Discussion of some Errors encountered in the Finite
Element Formulation

APPENDIX 5

A Study of Isothermal Forming -
Reprint of a paper with J.M. Alexander

APPENDIX 6

The Application of the Finite Element Method to two
High temperature Metal Deformation Processes -
Reprint of a paper with J.M. Alexander

APPENDIX 1

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APPENDIX 2

MOST COMMON SYMBOLS

Most symbols are explained near where they first occur in the text. Below is a list of the most commonly recurring symbols.

b_o	Element length in the slab approach (Chapter 3)
B_{ijkN}	An element of the shape function tensor for node N
C	A constant
D_{ijkl}	An element of the constitutive matrix
E	Young's modulus
f	Indicates a function
J_{iJ}	Element of the Jacobian tensor
L	Load in dieless drawing
L_{iJ}	Element of the inverse Jacobian tensor
m	Shear coefficient of friction
n	A constant (normally strain rate sensitivity index)
P, P_{ij}	local coordinate
q	Local coordinate
Q	A constant (in heat equations)
r	Radius, as a subscript indicates 'radial'
r'	Radial gradient, $\frac{\partial r}{\partial z}$

R_1, R_2	Outside radius: entry, exit (for dieless drawing)
t	Time, as a subscript indicates 'tangential' or 'circumferential'
T	Temperature variable
T_c, T_m	Temperature constants (Chapter 2)
T'	Temperature gradient (Chapter 2) = $\frac{\partial T}{\partial z}$
u	A velocity
v	A velocity
v_{iN}	Velocity in direction x_i
v_1, v_2	Velocities entry, and exit (in dieless drawing)
v_h	Velocity of heat source
V	Volume
W	Total work
$\dot{W}$	Total work rate
x	A cartesian direction
y	A cartesian direction
z	A cartesian direction, as a subscript indicates 'axial'
z_1, a_2	Axial position of start and end of deforming zone (in dieless drawing)
δ_{ij}	Kroneka delta (= 0 if $i \neq j$, = 1 if $i = j$)
∇	del operator, $\nabla^4 \psi = \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right)^2$
ϵ_{ij}	Element of the strain tensor

$\dot{\epsilon}_{ij}$	Elements of the strain rate tensor
$\bar{\epsilon}, \dot{\bar{\epsilon}}$	Effective strain, effective strain rate
η	Viscosity
$d\lambda$	Incremental constant in Levy Mises relationship (equation (3.1))
ν	Poisson's ratio
σ_{ij}	Element of the stress tensor
σ'_{ij}	Element of the deviatoric stress tensor
σ	Mean stress
$\bar{\sigma}$	Effective stress
$\bar{\sigma}_0$	Effective flow or yield stress

APPENDIX 3

DETAILS OF THE GAUSSIAN ELIMINATION SOLUTION METHOD
USED IN THE PROGRAM FOR ISOTHERMAL FORMING

The Gaussian elimination solution technique is a method for solving matrix equations of the form

$$\underline{K} \underline{v} = \underline{F} \quad (4.23)$$

The technique is divided into two stages, the forward elimination and the back substitution. The approach used in forward elimination can be illustrated by the equation where the matrices are partitioned as follows:

$$\left[\begin{array}{c|c} K_{11} & K_{r1} \\ \hline K_{1c} & K \end{array} \right] \left[\begin{array}{c} v_1 \\ \hline v \end{array} \right] = \left[\begin{array}{c} F_1 \\ \hline F \end{array} \right] \quad (3A.1)$$

where K_{11} , v_1 and F_1 are single element matrices. The elimination essentially operates on the system so that K_{1c} is equal to a zero vector. This is achieved as follows for a row n where the relevant terms in the K matrix are

$$\left[K_{1n} \mid K_n \right]$$

from which is subtracted the terms

$$\left[K_{11} \mid K_{r1} \right] \times \frac{K_{1n}}{K_{11}}$$

which results in row n being replaced by

$$\left[0 \mid K'_n \right]$$

In order to maintain v at the same value F_n must also be modified so

$$F'_n = F_n \times \frac{K_{1n}}{K_{11}}$$

Hence the equation 3A.1 becomes

$$\left[\begin{array}{c|c} K_{11} & K_{r1} \\ \hline 0 & K' \end{array} \right] \left[\begin{array}{c} v_1 \\ \vdots \\ v \end{array} \right] = \left[\begin{array}{c} F_1 \\ \vdots \\ F' \end{array} \right]$$

The terms in the vector $K_{11} \mid K_{r1}$ are now available for storage since they will no longer be altered by any subsequent operation in the forward elimination. Eventually in store there is a matrix $\underline{K}'$ for which all terms are zero below the leading diagonal. At this point back substitution can occur, starting by solving for the lowest term of the v vector. This can be illustrated for node 1 in equation (3A.1) as follows

$$v_{11} = \frac{F_1}{K_{11}} - \frac{[v]}{K_{11}} [K_{r1}] \quad (3A.2)$$

The implementation of the elimination scheme in the program is designed to make as few demands on active central processor memory as possible. The first stage in the preparation of values for the matrix $\underline{K}$ is to prepare the terms on the left hand side of the equation

$$2\eta \int_V v_{1M}^B B_{KN1M} dV + \alpha \int_V v_{1M}^B B_{qqkN} B_{rr1M} dV = f_{kN} + t_{kN} \quad (4.27)$$

The integrations are carried out by Gaussian integration at the Gauss points (normally 2×2 are used) in each element and then the values for each element are stored on disc. The summation over M indicated in equation (4.27) is over all nodes in the mesh, but this is only finalized as necessary in the forward elimination stage.

Another very significant saving that can be achieved in the solution is due to the fact that matrix $\underline{K}$ is highly banded around the leading diagonal. As a result only a small proportion of the $\underline{K}$ matrix is affected by the calculations during the forward elimination for any one node. The program takes full advantage of this fact and the method is illustrated on figure 3A.1, which illustrates the situation when elimination is proceeding at row n (the 'front' of the 'frontal solution'). Clearly the equations in the region B are not involved in this step of the elimination and so they are not drawn into the central processor. However, all the equations in region C may be involved so these must all be accessible. The size of region C is determined by the spacial distribution and numbering of the elements in the original mesh, and hence the size (or semi-bandwidth) is determined in a separate sub-routine. This sub-routine essentially determines the greatest difference in nodal numbers for two nodes that happen to reside on any single element. The sub-routine also includes a procedure whereby the nodal numbers can be re-ordered so as to reduce the

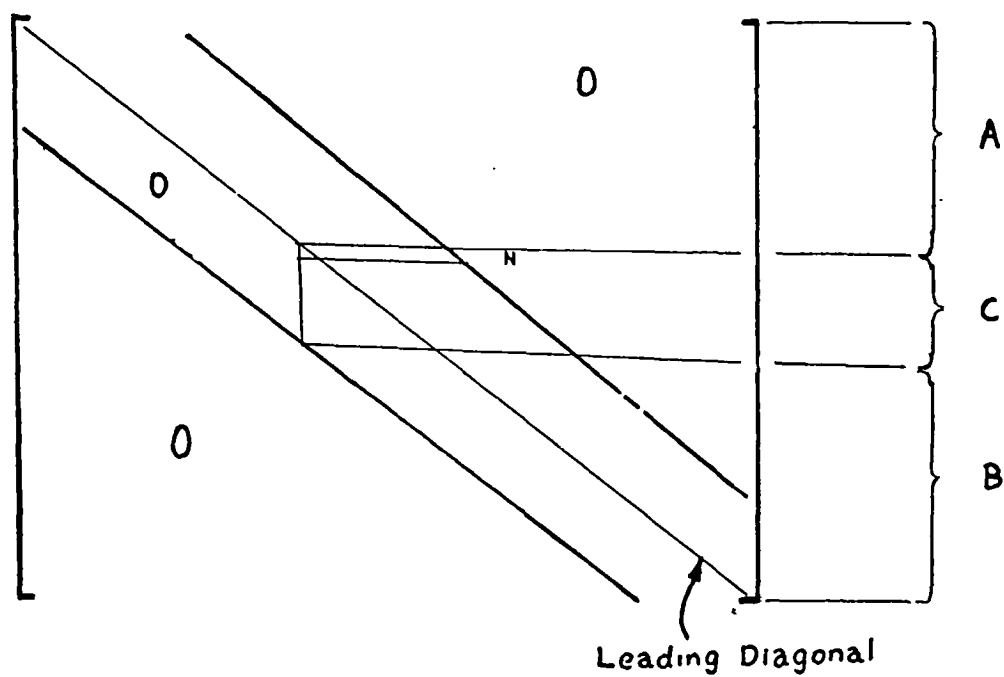


Figure 3A.1 Diagram of spacial distribution of values in a matrix which is highly banded around the leading diagonal. Only region C is involved in a Gaussian forward elimination at row N.

semi-bandwidth, though this re-ordering does not affect the original mesh it just alters the order of terms in vector y. After a certain number of eliminations or nodes have been processed, then a large amount of information is dumped into disc store, and this is essentially represented by regions A on the figure.

During back substitution once again only parts of the K matrix need be drawn into the central processor at any one time. If the node has been geometrically specified, then the operations indicated in equation (3A.2) are not carried out but instead

$$V_1 = v_s$$

is performed where v_s is the specified velocity at this point. Only the values of y need be retained from the back substitution stage.

One other saving in storage space is achieved by using a single dimensioned array to store all the values of K and even F. Since the program is designed to be used with quadrilaterals and it is not normal to have more than four quadrilaterals sharing a single node, the number of positions allocated to each node in this array can be limited. Moreover, since the original matrix K is symmetrical, the terms in the region of the matrix below the leading diagonal do not need to be duplicated.

Some features that are implicit in the Gaussian elimination scheme are that the original matrix $\underline{K}$ must be symmetrical and have no zero terms on its leading diagonal. These are the reasons why this powerful and efficient method cannot be used to solve the Lagrangian multiplier formulations described in Section 4.5.2, and thus such approaches are more complicated. The program includes checks to ensure that the diagonal terms are, in fact, non-zero and also that a specified semi-band width is not exceeded.

APPENDIX 4

A DISCUSSION OF SOME ERRORS ENCOUNTERED IN THE
FINITE ELEMENT FORMULATION

As has been mentioned in Section 7.3.1, there are certain errors involved in the penalty function formulation in that the volumetric strain rate is not exactly zero throughout an element. This appendix presents some tables which show how these errors are affected by strain rate size, deformation of the elements and the position of the elements.

Table 4A.1 presents the volumetric strain rates (in units of sec^{-1}) found at the nodes of a typical element (element 21) in the 6 x 6 element mesh which produced the results for the 25mm high by 25mm diameter specimen illustrated in figures 8.1 - 8.4. The results here are reported for the first four iterations at the first increment of deformation and for a crosshead speed of .3mm/sec. As can be seen, few of the results at the nodes are exactly zero, while the figures for the volumetric strain rates at the Gauss points are very nearly zero throughout (less than 10^{-7}). The figures show no sign of changing in magnitude with repeated iterations at the same increment (line 2 is the second increment; line 3 the third; line 4 the fourth). These volumetric strain rates are, however, two or three orders of magnitude smaller than the strain rates in the various coordinate directions as reported in table 4A.2.

TABLE 4A.1 ELEMENT 21 CROSSHEAD SPEED -0.3
VOLUMETRIC STRAIN RATES AT NODES

NODE	1	2	3	4	5	6	7	8
FIRST INCREMENT FIRST ITERATION								
1	-.00016	-.00008	-.00013	.00000	-.00013	-.00008	-.00015	.00002
FIRST INCREMENT SECOND ITERATION								
2	-.00010	-.00016	-.00011	.00011	-.00009	-.00016	-.00009	.00011
FIRST INCREMENT THIRD ITERATION								
3	-.00007	-.00019	-.00010	.00015	-.00008	-.00019	-.00006	.00015
FIRST INCREMENT FOURTH ITERATION								
4	-.00007	-.00022	-.00011	.00019	-.00008	-.00023	-.00005	.00017

TABLE 4A.2 ELEMENT 21 CROSSHEAD SPEED -0.3
VALUES OF STRAIN RATES AT NODES. ED Z = AXIAL STRAIN RATE ETC.

NODE	1	2	3	4	5	6	7	8
ED R	.07153	.07353	.07554	.07083	.06469	.05870	.05271	.06284
ED Z	-.14063	-.14374	-.14590	-.13002	-.11414	-.10697	-.09886	-.11974
ED T	.06812	.06875	.06947	.06019	.04863	.04616	.04527	.05797
ED RZ	-.02094	-.02502	-.02711	-.04829	-.07197	-.06889	-.06382	-.04113

Table 4A.3 shows the same element, but in this case the crosshead rate is ten times quicker (3.0mm/sec). The volumetric strain rates reported at the first increment (line 1) are exactly ten times larger than the volumetric strain rates reported on line 1 for the first increment on table 4A.1. Similarly, the strain rates at the nodes (which are not reported) are also ten times larger. This is not surprising for the non-linear material properties cannot be introduced during the first iteration. In the second iteration small, second order, departures from this relation start to occur (compare lines 2 on 4A.1 and 4A.3). Lines 3 and 4 on table 4A.3 introduce new increments of deformation. The specimen is 22mm high at line 3 and 19mm high at line 4. These are quite significant deformations and it is to be noted that the errors, represented by the finite values of the volumetric strain rates at the nodes, tend to increase as the element deforms. Highly deformed elements also start to show slight errors at the Gauss points, but these are still at least two orders of magnitude smaller than the errors at the nodes. These are the chief reasons why less reliance can be placed on the calculated results at the nodes as the elements become deformed and increasingly irregularly shaped. It is to be noted that this element is in the centre of the specimen and the errors at all the nodes are of the same magnitude.

TABLE 4A.3 ELEMENT 21 CROSSHEAD SPEED -3.0
VOLUMETRIC STRAIN RATE AT NODES

NODE	1	2	3	4	5	6	7	8
FIRST INCREMENT FIRST ITERATION								
1	-.00159	-.00080	-.00134	.00003	-.00130	-.00052	-.00154	.00018
FIRST INCREMENT SECOND ITERATION								
2	-.00098	-.00146	-.00090	.00101	-.00082	-.00150	-.00087	.00108
SECOND INCREMENT FIRST ITERATION								
3	-.00360	-.00324	-.00411	.00142	-.00410	-.00339	-.00364	.00127
THIRD INCREMENT FIRST ITERATION								
4	-.01405	-.00227	-.01520	-.00464	-.01515	-.00274	-.01378	-.00470

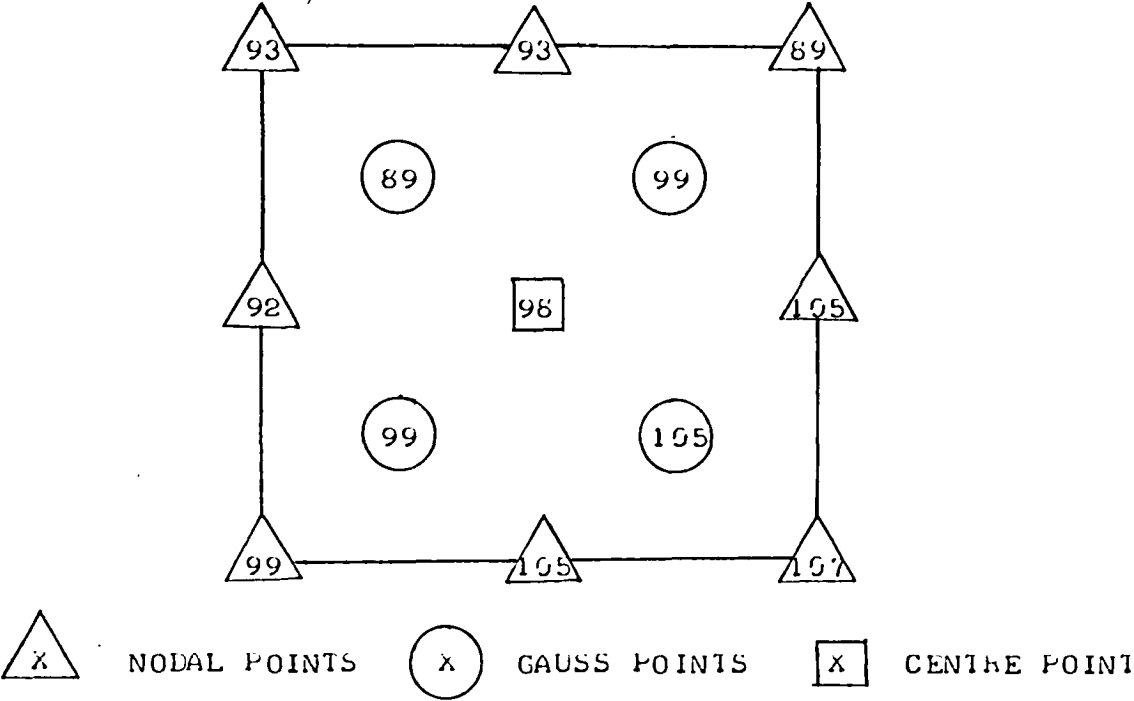
TABLE 4A.4 ELEMENT 1 CROSSHEAD SPEED -3.0
VOLUMETRIC STRAIN RATES AT NODES

NODE	1	2	3	4	5	6	7	8
FIRST INCREMENT FIRST ITERATION								
1	-.04796	.01365	-.00876	-.01802	-.00876	.01365	-.00876	-.03299
FIRST INCREMENT SECOND ITERATION								
2	-.03485	.01507	-.00592	-.01806	-.00601	.01507	-.00591	-.02643
SECOND INCREMENT FIRST ITERATION								
3	-.03842	.01218	-.00811	-.01844	-.00894	.01303	-.01056	-.02773
THIRD INCREMENT FIRST ITERATION								
4	-.05175	.01283	-.01322	-.02424	-.01504	.01438	-.01888	-.03635

An element of particular interest is element 1, some results for which are shown on table 4A.4. In this element, nodes 8 and 1 are on the axis and nodes, 5, 6 and 7 fixed to the upper die. Here in contrast to the element discussed previously, the errors at the nodes in this highly restrained element are larger. As shown on figure 8.4, the strain rates in the region of this element are generally quite low, so in this element the errors in volumetric strain rates are of the same order of magnitude as the strain rates themselves. Moreover, the corner nodes 3, 5 and 7 appear to have lower errors determined at them than the mid-side nodes 2,4,6,8. (Node 1 has an error as large as any of the mid-side nodes.) This effect may start to explain the kind of oscillations that are clearly element dependent that are reported on figure 8.3. This kind of oscillation between midside and corner nodes appears to be a feature of the more restrained elements only, these elements occurring particularly along the die/specimen interface and at the central axis.

One final point is illustrated in table 4A.5, which shows mean stress values for points in an element (in a different test from those formerly mentioned). The values of mean stress do not appear to vary greatly around the element and thus the kind of oscillations discussed by Naylor(72) are not a feature of the mean stress calculations.

TABLE 4A.5 MEAN STRESSES CALCULATED AT VARIOUS POINTS AROUND AN ELEMENT; STRESSES REPORTED IN MPA.



APPENDIX 5

1976

PROCEEDINGS

May 17, 18, 19, 1976

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A STUDY OF THE ISOTHERMAL FORMING OR CREEP FORMING OF A TITANIUM ALLOY

by

J.W.H. Price and J.M. Alexander, Department of Mechanical Engineering, Imperial College, London

The basic mechanisms of isothermal forming with particular reference to the titanium alloy 6 Al 4V (6% Aluminium 4% Vanadium remainder Titanium) are examined by means of compression and die filling experiments. Lubrication at 800°C-980°C is studied for the forming of this alloy and the load-displacement relation for the tests is predicted from upper-bound calculations. A steel is tested under isothermal forming conditions as well. A finite element solution suitable for studies of the high deformations encountered in the process is presented.

INTRODUCTION

Isothermal forming (or creep forming) is a technique of forging in which use is made of the fact that metals generally are softer the higher the temperature and the lower the strain-rate imposed upon them. In isothermal forming the dies are heated to the same temperature as the specimen and loading is at a slow speed. As a result the material has time for extensive deformation, and can be made to conform very accurately with the shape of the dies. The dimensional accuracy of the product is thus very good and only a minimum of further working or machining is required to obtain a finished product. The process has been used in particular for the production of difficult shapes in titanium alloys and is discussed and examples given in numerous recent references (1).

The study undertaken here was to examine isothermal forming from a more fundamental viewpoint, since researches so far have been concerned with complicated, commercially important, practical forgings. The present research has thus been directed towards compression tests and die filling experiments using dies of simple axisymmetric shape, and also with obtaining some theoretical predictions. This paper is mainly devoted to a description of the experimental section of this work. There is also an outline of a finite element solution that is suitable for application in a situation having the high strains involved in the process. The method enables accurate solutions to be obtained without an unreasonable use of computer resources.

EXPERIMENTS

i Equipment

The experimental equipment was designed to model industrial isothermal forging. An induction furnace of 13kWatt and 3kHz frequency was used which enabled fairly even heating of axisymmetric shapes up to 75mm diameter. The time for heating from room temperature up to 950°C (1740°F) was about 15 minutes. A 500kN (50 ton) standard Denison testing machine with controllable speeds was used and a continuous autographic recording of load, temperature of dies and displacement of crosshead was made. The dies were made from a nickel alloy designated IN-100 which has the chemical composition 10Cr, 15Co, 3Mo, 7Ti, 5.5Al, 1.0V, 0.06Zr, 0.015B and balance Ni. These dies are extremely hard up to and beyond the temperature of 950°C and as a result their main machining was carried out by EDM (spark erosion) with the final finish being obtained from conventional methods (i.e. turning with tungsten carbide tipped tools then 'fine ground'). The experiments were carried out using specimens of titanium 6 Al 4V unless otherwise noted.

ii Cylindrical Compression Tests

Figures 1 and 2 are typical results from the compression of a cylinder of titanium alloy to a very high strain, the graphs being the output of a computer program which processes the data. The material was held at a constant temperature of about 800°C, a graphite lubricant (in water base) was applied lightly to it and the crosshead speed was set at 3mm/sec. The original cylinder was

was only 1.6mm high and about 45mm in diameter. This represents a uniform logarithmic strain of -2.4 along the axis or a reduction of 91.5% in height.

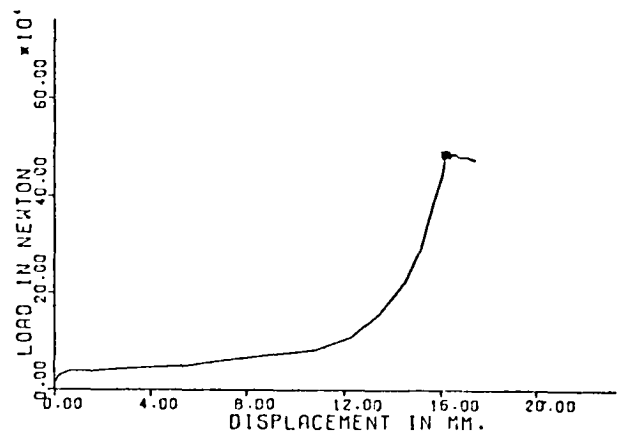


Figure 1. Compression Test; Load versus displacement

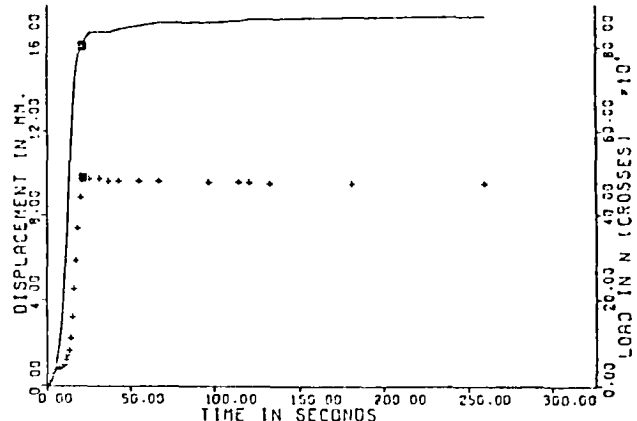


Figure 2. Compression Test; Load (Crosses) and displacement versus Time

On figures 1 and 2 two stages of deformation might be discerned separated by the squares on the figures. The first stage is a smooth curve during which the crosshead moves with an approximately constant speed of 3mm/sec. Most of the deformation was achieved in the lower 10% of the load range. In the region of the square the machine approached its maximum load of 500kN (50 tons) and the crosshead speed slowed down so that it was barely moving. In this region it can be considered that the deformation continues by creeping. On figure 2 it can be seen that the creeping regime occupies most of the time of the experiment but in fact there is very little further decrease in height of the specimen during this period.

On inspection of the specimen **after** the experiment it was found that there had been almost no relative movement between specimen and dies. Marks on the ends of the original specimen did not change **their radius** during the experiment. The very large increase in area of the ends that occurs **during the experiment** is provided by the material from the sides stretching and rolling onto the dies. The graphite thus serves very little purpose in promoting sliding along the die-metal interface.

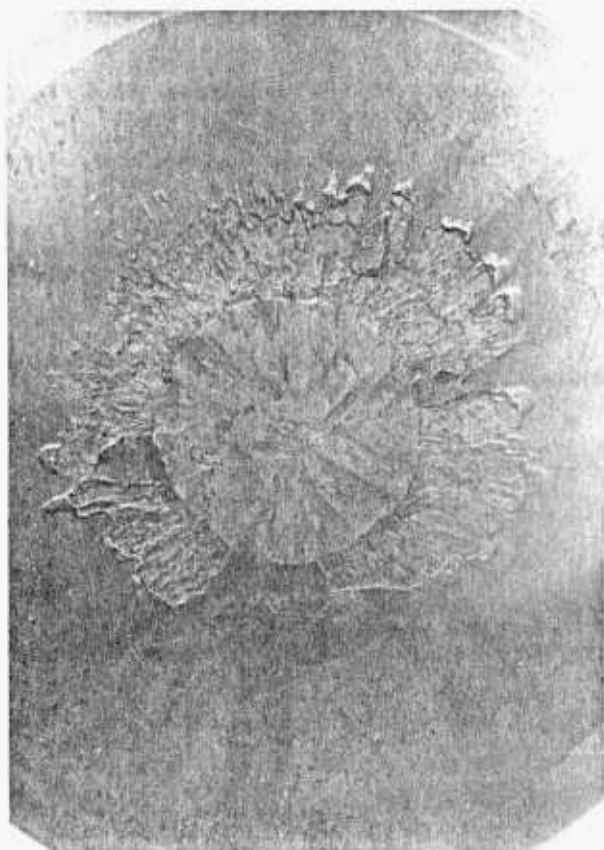


Plate 1. A specimen over-lubricated with graphite

Application of too much graphite can, however, cause considerable damage to a specimen as illustrated on Plate 1. This plate illustrates a specimen originally 12mm high and 12.8mm in diameter now deformed to 2.1mm thick and 4)mm in diameter. There are several features of this features of this picture that require explanation.

The top of the specimen is marked by a 'sunburst' appearance with lines radiating from the centre of the specimen. This feature is apparently caused by the layer of graphite lubricant being extruded radially out between specimen and die. The lubricant forms a layer of comparatively low viscosity between specimen and die and the amount of it extruding increases with pressure. This phenomenon is probably a measure of the 'friction hill' on these specimens, as discussed by Avitzur (2). In the centre of the specimen there is a circular area which comprises its original top end. This area has basically not changed in size, though movement of lubricant across it is obvious. Outside this is a region where the surfaces that were originally on the outside of the cylinder have rolled up onto the die. Towards the centre of this region the outflow of lubricant has been so strong that in most areas it has torn up the oxide layer on the specimen and forced it outwards. Near the outside circumference of the specimen there is a lighter bend. This is probably a surface, most of which has been newly created. Figure 3 illustrates an experiment with plasticine which could explain this phenomenon, 3(a) illustrates the initial, two colour cylinder. Figure 3(b) illustrates a cross-section of the cylinder after having imposed a heavy deformation without lubricant whilst figure 3(c)

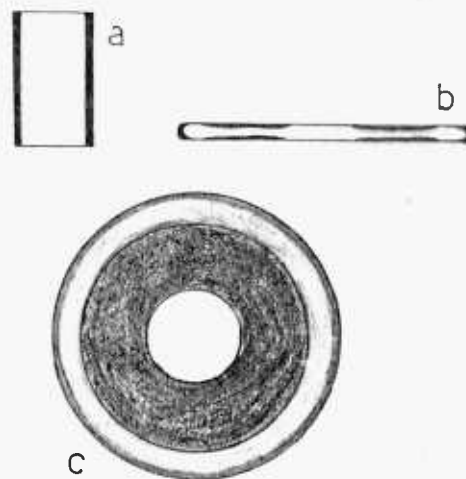


Figure 3. Compression of a plasticine cylinder

shows a plan view. These diagrams illustrate that the light band could be caused by material from the interior of the specimen approaching the surface very closely thus forming a band of mainly fresh surface. The original outside surface is mainly trapped onto the dies at this high strain.

In situations where damage occurs, such as that illustrated here, the die surface is quite likely to be damaged as well as the specimen and die may require regrounding. The amount of damage that occurs depends on the relative strengths of the die and the specimen. The strengths of the nickel alloy dies and the titanium alloy specimen both vary with temperature, but in the range 800-980°C it appears that the nickel alloy changes at a much slower rate than the titanium alloy. In experiments it was found that the die was much more likely to be marked by the tests at 800°C than at 980°C.

There was no need to proceed to a load as high as 500kN to achieve a large strain in the titanium cylinders. In some experiments the load was held at a much lower level and almost as much strain was achieved although in a longer time. A strain of -2.0 was obtained after holding a specimen, similar to the one reported in figures 1 and 2, for 3 minutes at 50kN (5 tons). Stresses can also be reduced by raising the temperature of the tests but in titanium 6 Al 4V the usual maximum temperature for hotworking is 980°C, for above this temperature the alloy becomes single phase and this can cause deleterious effects on properties when the specimen is cooled.

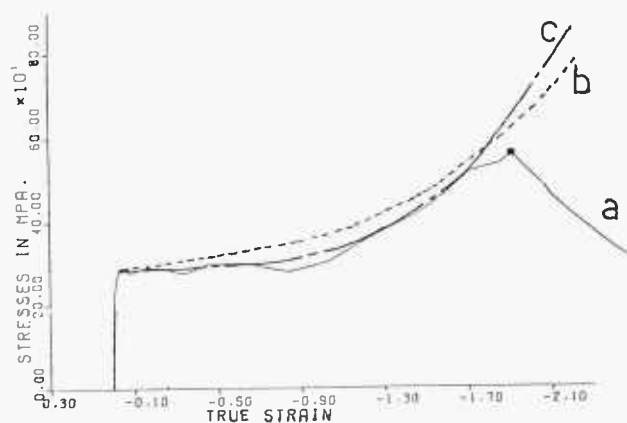


Figure 4. Stress-strain curves for a compression Test
Curve a. Experimental results
Curves b and c. Upper bound solutions

iii The Stress Strain Curve

Figure 4 is a plot of the average axial stress (pressure at interface) versus average logarithmic axial strain (curve A). The period of creeping which starts in the region indicated by the square is seen to involve a considerable strain, although the specimen does not decrease in height greatly. This is because the specimen is now a thin disc with a height to diameter ratio of less than 1:20. This curve is better understood (at least up to the region of the square) when it is plotted in conjunction with an upper bound solution for axisymmetric compression. There is an upper bound solution which assumes no barreling and friction governed by a shear coefficient at the boundary (see for example reference (2)). The velocity field for this situation is

$$u = \frac{r v_0}{2a} \quad \& \quad v = \frac{z v_0}{a} \quad (1)$$

where the symbols are described on figure 5.

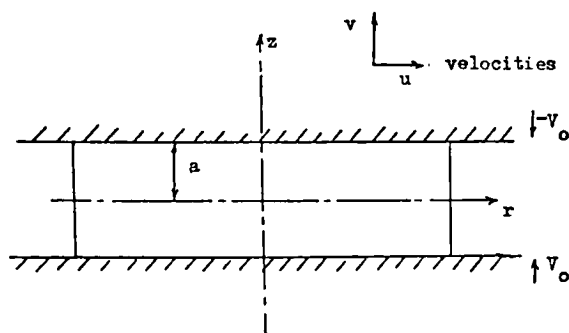


Figure 5. Notation for upper bound solutions

A shear coefficient, m , is used such that

$$T = m \frac{\bar{\sigma}_0}{\sqrt{3}} \quad (2)$$

where T is shear stress at the boundary and $\bar{\sigma}_0$ is the flow stress of the material (which is assumed to be constant). For the curve illustrated m was chosen as its maximum possible value of 1.

However the curve for this solution (curve B) does not conform very closely to the data. In view of the fact that there is no sliding at the boundary and that there is extreme barreling during the test this is not a very satisfactory velocity field. A special case of a velocity field proposed by Kobayashi and Thomsen (3) can be used. In this special case there is no relative movement between the platens and the specimen and the barreling can be taken to be parabolic in form. This is written

$$\begin{aligned} u &= \frac{3 v_0}{4a} r \left(1 - \frac{z^2}{a^2}\right), \\ v &= -\frac{3 v_0}{2a} \left(z - \frac{z^3}{3a^2}\right). \end{aligned} \quad (3)$$

The solution is much better as shown by curve C on figure 4. In the region of the square, however, the solution fails. This means that in this region the basic assumptions of the upper bound technique must no longer be applicable. Thus either the velocity field is no longer accurate or the flow stress of the material is changing. Of these the latter seems most relevant for, in the region of the square, the speed of the crosshead is decreasing considerably and it has been shown by Lee and Backofen(4) that the material has a flow stress that is strongly strain-rate dependent. This region can thus be termed the region of creep forming.

iv Lubrication

The graphite lubricant used in the compression tests described above serves no apparent function in the process. In fact, isothermal forming of titanium can be performed without any lubricant without any change in loads or behaviour of the specimen. However, in these experiments a thin film of graphite was normally applied to the specimens and dies because it is possible that it might aid the removal of the specimen from the dies or reduce die wear (though no direct evidence was obtained to these effects). Thus, with graphite lubricant the contact between the specimen and the dies is not characterised by the normal processes of metal to metal lubrication seen at lower temperatures but is probably dominated by the hard oxide coatings on both the dies and specimens. Glass is another lubricant which is readily available and suitable for metal working at these high temperatures. In contrast to graphite, glass has a profound effect on the type of deformation that is seen. If a thick layer is applied to the ends of a cylinder in a compression test it can even produce a reverse barrel or hour-glass shaped specimen as shown by Douglas and Altan (5). In that case the layer of molten glass has such a low viscosity that its radial extrusion during the compression test causes the specimen to deform outwards more at the die-specimen interface than at the centre of the specimen.

Comparison of ring tests of the lubricants used in these studies is made on Plate 2. At the top is the starting ring which is 6.45mm high, with an outer diameter of 25.5mm and an inner diameter of 12.9mm. The two specimens on the left are not lubricated and in this case the friction is so high that the hole in the middle eventually closes up at high reductions. Using the calibrations of Hawkyard and Johnson (6) the shear coefficient, m in equation 2 above, appears equal to one (actually the points fall above the line for $m=1$ but this value for m is the maximum according to the theory). The centre specimen was lubricated by a thick layer of graphite. Apart from the damage on the surface of the specimen there is no apparent difference in performance from the specimens with no lubricant. The two specimens are not so circular as those without lubricant, an effect which is caused by the uneven flow of the glass lubricant. This flow is often limited preferentially to small areas and the material in these areas can be distorted more than in others. This effect is also seen to a lesser extent on thickly lubricated graphite specimens. For the glass lubricated rings Hawkyard and Johnson's graphs give a shear coefficient of 0.06.

In general the glass lubricant is probably not satisfactory for isothermal forming when compared to a thin film of graphite. The surface finish on the specimen can be rougher with glass lubricants and the specimens are less likely to be circular after forming. The glass solidifies as the specimen cools and thus has to be removed physically from the specimen. The specimens are also more difficult to remove from the dies for at high temperatures the molten glass forms an adhesive mass which can cause an air lock and retard the removal of the specimen, while at low temperatures the solid glass must be fractured.

v Profile Dies

The main profile die experiments have been carried out on two different dies. Both dies have similar dimensions but one die has a more generous fillet radius and taper angle than the other. The basic dimensions of the two dies are illustrated on figure 6. The final product thus has a truncated conical region, hereafter termed the 'cap' around which there is formed a flange as the dies close. There was no noticeable difference in performance between these two dies (P die and R die). In view of the fact that there is no slipping of the titanium along the horizontal surfaces of the dies the fillet radius makes virtually no difference. There was no problem extracting the finished product from the dies when using a thin film of graphite lubricant, though glass provided considerable

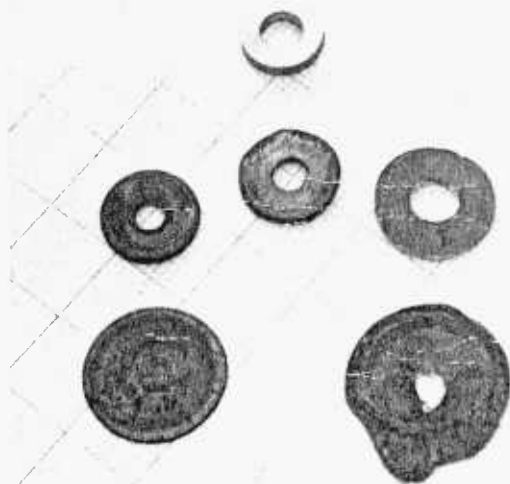


Plate 2. Rings for lubricant tests.

Top - Original specimen, ratio 1:4:4

Left - No lubricant

Centre - Graphite

Right - Glass

Speed - 3mm/sec; Material - Titanium 6Al 4V

Temperature - 950°C.

Grid on plate 2 is inches.

Plates 3-5

Profile Die (P die).

Speed - 3mm/sec

Metal - Titanium 6Al 4V

Lubricant - Graphite

Temperature - 950°C.

Specimen volume - 16,500mm³

(1 cubic inch)

Original specimen diameters -

Plate 3 19.2mm (¾ in.)

Plate 4 25.4mm (1 in.)

Plate 5 35.0mm (1.38 in.)

Plate 3



Plate 4



Plate 5



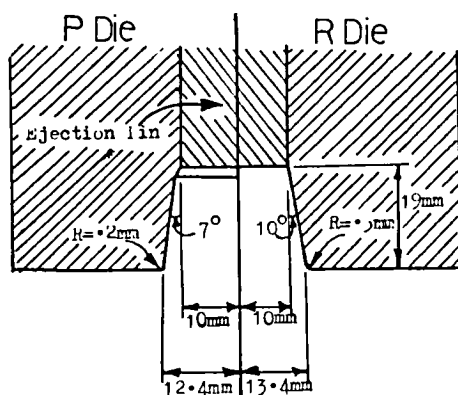


Figure 6. Dimensions of the two profile dies

problems. A die with a taper angle of 2° was also used but removal of the specimens from this die was more difficult requiring somewhat more force and even heat expansion of the die. With dies P and R a complete fill of the cap was always possible and a thin flash (0.05-0.2mm) was extruded up where the ejection pin did not fit exactly inside the die.

Plates 3-5 demonstrate the differences in forming stages when starting with three different billets. Die P was used with graphite lubricant at 950°C , and the three starting shapes all have the same volume ($16,500\text{mm}^3$). The first billet (on plate 3) has a diameter that is a little smaller than the ejection pin (19.2mm) and thus it fits into the bottom of the cap of the die. In the first stage the cap is completely filled and the bottom of the specimen is compressed and is thickening. At this stage the load begins to rise and the rest of the test is occupied by what is virtually the compression of a cylinder to a thin disk, since the material in the cap barely deforms. Plate 4 illustrates a specimen of starting diameter 25.4mm which is only slightly smaller than the diameter of the bottom of the cap. As the load is applied this specimen slips at the edges where it contacts the inside of the cap in the die. Such a situation causes a quite severe situation for die wear, and the press shudders a little at this time which possibly indicates that a stick-slip friction situation is occurring. As deformation continues a lip is formed on the specimen which indicates a region of quite severe shearing. The lip disappears in the final specimen which is also the only specimen to show complete filling of the cap. Plate 5 illustrates a specimen of 35mm in diameter. In this specimen there is no rising into the cap during the early stages of the experiment, in fact, the total height of the specimen slightly decreases. Finally, however, the flange becomes sufficiently thin for the material to be forced up into the cap, and the height of the specimen increases. At constant crosshead velocity this billet is the quickest of the three to be finished for it requires the least travel of the crosshead.

vi The Isothermal Forming of Steel

A steel of composition 0.23C, 2V, 4Cr, 5.1Mo, 5.2W was used to test the possibilities of the isothermal forming of steel. The basic purpose of the additions is to provide a durable oxide that could withstand the conditions of isothermal forming. It is probable that a steel with 4% Chromium as its only addition would probably form satisfactorily in an air environment as discussed by Alexander and Turner (7) and, of course, steels with different additions may also prove useful.

Compression tests at 980°C with thin film graphite lubrication using billets 25.4mm height and 25.4mm diameter were performed with the steel alloy and the titanium alloy. The two experiments are compared in Table 1.

TABLE 1

Alloy	Steel	Titanium
Final load	500kN	400kN
Final thickness	4mm	3mm $\pm$.2mm
Final diameter	63mm	72mm
Reduction	84%	88%
Axial strain	-1.9	-2.1
Stress at -.18 strain	51.5MPa (7.46ksi)	18.1MPa (2.62ksi)
At maximum load	No apparent creeping	Appreciable creeping

The main conclusion from these tests is that for steel the stress levels were about 2.8 times higher and that very little useful deformation occurs by 'creep forming'.

Profile die P was also used successfully for forming the steel. The billet diameter of 25.4mm is smaller than the opening of the cap so that filling of the cap was easily achieved. However the minimum flange thickness of 3.25mm that was obtained at 500kN was not sufficiently thin to ensure that good rising into cavities could be obtained as illustrated for titanium on plate 5. By comparison the titanium easily achieves a flange thickness of 2.75-2.50mm and filling of the cap at 400kN. The steel was unable to form a vertical flash between the ejector pin and die as thin as that of the titanium, the minimum flash achieved having a thickness of 0.5mm. The steel conformed well to little marks on the dies and also to the sharp fillet radii. However the oxide coat was not quite as good as that of the titanium, being less tough and slightly friable, so the steel does not fit quite so sharply into the detailed features. The steel specimen was easy to extract from the die, but presumably because the oxide coat is not so strong as titanium oxide there was more evidence of the specimen slipping over the die during forming.

FINITE ELEMENT STUDIES OF ISOTHERMAL FORMING

An outline of the approach used for obtaining finite element solutions for large deformations is given here. A fuller description of the method used in this work will be given elsewhere (8). Several features of the isothermal forming of graphite lubricated titanium alloy must be considered in the approach.

(i) The material is strongly strain-rate sensitive but at the temperature of forming there is little strain hardening. The constitutive relationship must therefore be such that flow stress is a function of both strain rate and temperature but not necessarily of strain.

(ii) There are high deformations obtained in the process and in most cases all the material is substantially deformed. The plastic strains that occur will be many times larger than any elastic strains. Hence a method in which the elastic strains are neglected will be satisfactory.

(iii) The die-specimen interface normally exhibits no slipping but some boundaries that originally are free eventually come into contact with the die.

(iv) The dies can be considered to be rigid.

From consideration of these properties a finite element scheme was chosen that would give a satisfactory modelling of the process without too great a use of computer time.

(a) An increment of plastic work per unit volume, dW_p , can be written, following Hill (9), as

$$dW_p = \sigma'_{ij} d\epsilon^p_{ij} \quad (4)$$

where σ'_{ij} are the deviatoric stresses, $d\epsilon^p_{ij}$ are the increments of plastic strain, and the summation convention applies. This can be rewritten in terms of a plastic strain rate per unit volume

$$\dot{W}_p = \sigma'_{ij} \dot{\epsilon}^p_{ij} \quad (5)$$

The normal elastic finite element formulation can be converted to using strain rates and deviatoric stresses instead of strains and stresses respectively (11). The nodal variables are now considered to be velocities and the increments of deformation are really time steps. It is a work rate that is minimized throughout the volume.*

(b) For the constitutive relationship the Levy-Mises flow rule can be used (9)

$$d\epsilon^p_{ij} = \sigma'_{ij} d\lambda \quad (6)$$

where $d\lambda$ is an instantaneous constant. In order to calculate $d\lambda$ the equation must be written to involve time:

$$\dot{\epsilon}^p_{ij} = \sigma'_{ij} / 2\eta \quad (7)$$

It is written in this form in order to emphasize the connection between this formulation and that for the flow of viscous fluids at very low Reynolds numbers. Thus η can be interpreted as an instantaneous viscosity. In the present program provision was made for the 'viscosity' to be estimated at each iteration for each gauss point in each element. The value was found from interpolation directly from experimentally determined stress-strain rate-temperature data (as is provided in ref 4) or from an equation giving viscosity in terms of temperature and strain rate. The strain rate and temperature from the last iteration is used to estimate the viscosity for the current iteration.

(c) Incompressibility must be ensured. Of the techniques which may be used to achieve incompressibility the most efficient and simplest is the penalty function approach (10). In this approach a large positive number, α , is multiplied by the volume integral of the square of the volumetric strain rate:

$$\alpha \int_V \dot{\epsilon}^2 dv \quad (8)$$

and this is added to the work rate functional, X , for the finite element virtual work rate formulation

$$X = \int_V \dot{W}_p dv + \alpha \int_V \dot{\epsilon}^2 dv + B \quad (9)$$

B represents the contributions of body forces (normally considered to be unimportant in slow speed plasticity problems) and surface tractions.

(d) The boundaries of the specimen are either free or geometrically determined by the dies if sticking friction, as described in the experiments above, is assumed. Frictional conditions have been allowed for in the present program but are not normally useful for isothermal forming. Provision is made for boundary nodes that were originally free to roll onto the dies and become fixed.

(e) During the deformation the mesh must be continuously updated to reduce the problem as far as is possible to one of infinitesimal strain. This is achieved by using small time or displacement steps and each node is moved after each increment. The movement of each node is calculated by multiplying the velocity calculated at that node in the previous increment by the time elapsing between increments. The distorted elements that are produced after a number of increments represent the only memory that the model material has of its previous shape.

As a result the mesh must be arbitrarily reformed only when it is absolutely necessary. This occurs at times when elements become excessively distorted or negative areas are created. An example of this is shown on figure 7.

* The original program was kindly provided by Professor O.C. Zienkiewicz. It is an elastic program with isoparametric quadrilateral elements and a high speed elimination solution for banded matrices. Technical details of this program are described by Zienkiewicz (10).

(f) A subroutine was developed to calculate the mean stress, σ_m , at the nodes using the deviatoric stresses and shear stresses calculated in the main program. This is achieved by use of the equilibrium equations and this can be demonstrated by reference to the x direction as follows:

$$\text{First} \quad \sigma_m = \sigma_x - \sigma'_x \quad (10)$$

where σ_x is stress in the x direction and the dash indicates a deviatoric stress. The equilibrium equation, for the case where the only shear stress (T) is in the x,y plane, is given by

$$\frac{\partial \sigma_x}{\partial x} = - \frac{\partial T_{xy}}{\partial y} \quad (11)$$

Thus using equation (10)

$$\frac{\partial \sigma_m}{\partial x} = - \left(\frac{\partial \sigma_x}{\partial x} + \frac{\partial T_{xy}}{\partial y} \right) \quad (12)$$

Similarly it can be shown

$$\frac{\partial \sigma_m}{\partial y} = - \left(\frac{\partial \sigma_y}{\partial y} + \frac{\partial T_{xy}}{\partial x} \right) \quad (13)$$

Hence an integration in the x,y plane can be carried out to find σ_m throughout that plane. The initial values of σ_m are calculated at a free boundary, for there the deviatoric stress normal to the boundary must be equal to the negative of the mean stress.

This method of mean stress calculation has the chief advantage over other methods (12) that the stiffness matrix still requires only two variables at each node, and that the mean stress calculation need only be performed at times during the calculation when it is specifically required. However, the method relies on the values of deviatoric stresses calculated at the nodes in order that the integration can be carried from one element to another. In the formulation used the stresses calculated at the nodes are not so accurate as the stresses calculated at the four (2×2) gauss points in each element. As a result a small but cumulative error occurs during integration across the specimen and the method is not suited to regions of extreme strains where nodal stresses are often unsatisfactory.

Some example results from this program are presented in figures 7-9. The results compare well with experiments. The computer time taken for a single increment is about 20 seconds on a CDC 6400 computer and this is sufficiently economic to consider for routine use. The largest practical increment considering the conflicting demands of economy and accuracy of assumptions and difficulties of mesh and boundary distortions is probably about 10-15% reduction in height.

CONCLUSIONS

The experimental evidence presented leads to several main conclusions about isothermal forming.

- i The strains that can be obtained are very high and the loads are low. High strains can be achieved by holding the material for extended periods at low loads. A successful upper-bound solution is possible.
- ii Satisfactory results are best obtained with a thin layer of graphite lubricant or no lubricant at all. However there is no slipping between dies and specimens except in extreme circumstances.
- iii Sharp fillet radii and low taper angles can be used without difficulty on profile dies. Die filling is easy, even into sharp corners if a suitably shaped starting billet is used.

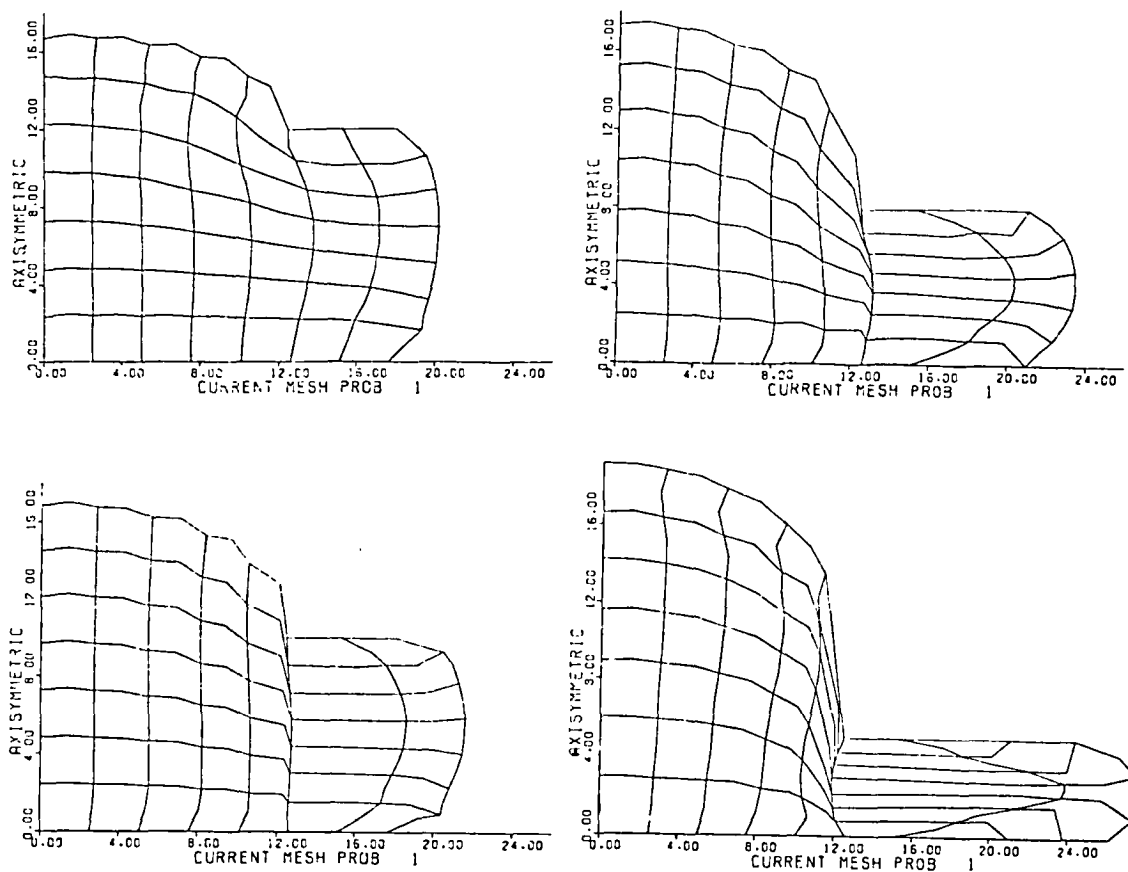


Figure 7. The isothermal forming of Titanium 6Al 4V with a profile die as calculated by the finite element method. Compare with Plate 5.

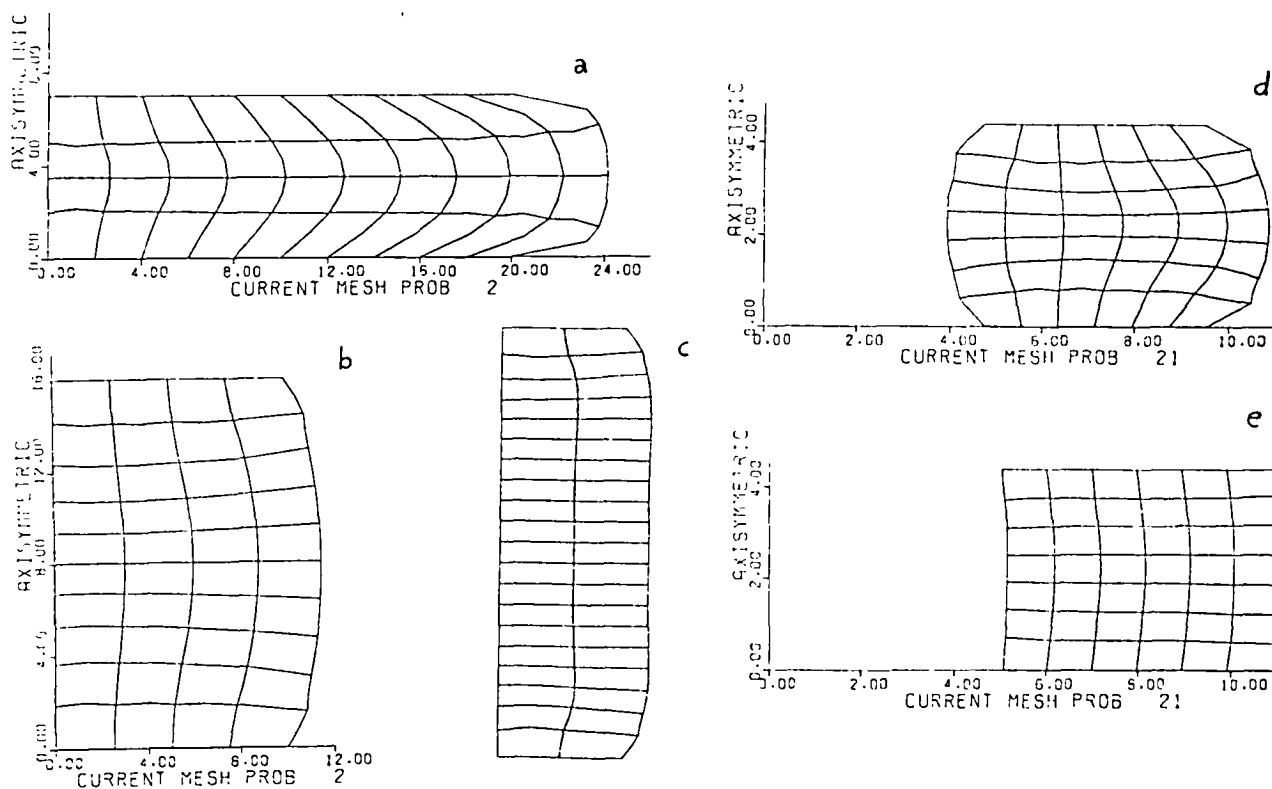


Figure 8. Five examples of forging between flat dies as calculated by the finite element method. a, b and c are cylindrical compression specimens of height to diameter ratios of 4:1:4 respectively. Note double bulging in specimen c. d and e are ring test, d with sticking friction, e with a friction coefficient of 0.16. Original sizes were 6.37mm high, 4.77mm inner and 9.54mm outer radii.

- iv Forming to quite high strains is possible even if the material does not exhibit any creeping or super-plastic properties, as illustrated by the isothermal forming of steel.

With the finite element method described it is possible to obtain a theoretical representation of the forming process with less time taken than with many finite element solutions to non-linear problems.

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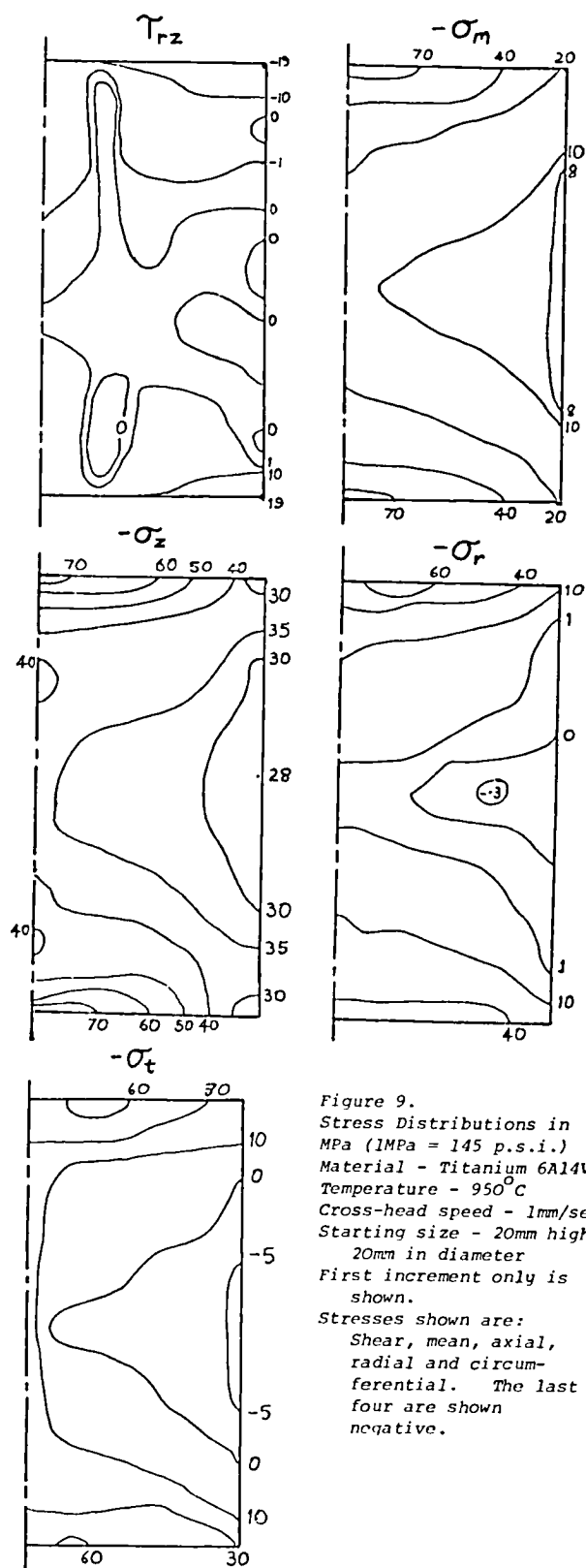


Figure 9.
Stress Distributions in MPa (1MPa = 145 p.s.i.)
Material - Titanium 6Al4V
Temperature - 950°C
Cross-head speed - 1mm/sec
Starting size - 20mm high
20mm in diameter
First increment only is shown.
Stresses shown are:
Shear, mean, axial,
radial and circumferential. The last four are shown negative.

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FINITE ELEMENT METHODS

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THE FINITE ELEMENT ANALYSIS OF TWO HIGH
TEMPERATURE METAL DEFORMATION PROCESSES

J.W.H. Price, J.M. Alexander
(U.K.)

SESSION 8: SPECIAL FLOW AND APPLICATIONS

THE FINITE ELEMENT ANALYSIS OF TWO
HIGH TEMPERATURE METAL DEFORMATION PROCESSES

by

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Department of Mechanical Engineering, Imperial College, London

A finite element technique for obtaining solutions to problems in plasticity is presented which makes use of the Levy Mises equation to describe the material and a penalty function to ensure constant volume. A method of calculating the mean stress throughout the body is also given, since the finite element analysis only allows deviatoric stresses to be calculated. The material is considered to have a 'viscosity' which is dependent on temperature and strain rate. These approaches are applied to two metal forming processes called die-less drawing and isothermal forming which both require several unusual features in the programs.

Introduction

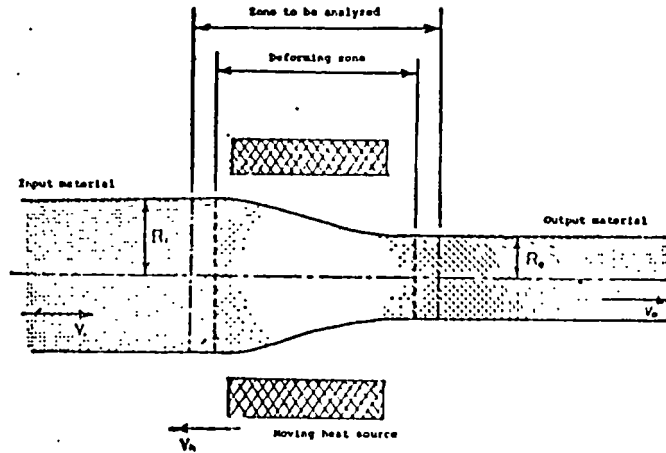
1. The Processes

In this paper two metal deformation processes are analysed using finite element techniques. Both processes are rather unusual and are not at present widely applied in industry.

The first process considered is die-less drawing which is one of the few useful metal forming processes in which there are no tools actually contacting the deforming part of the specimen and thus there are no friction boundaries. The process consists essentially of a localized heated zone which is moved along a long prismatic specimen subjected to tension provided by a moving crosshead. The metal undergoes a controlled deformation in the heated region where it is much softer than in the surrounding unheated regions (see figure 1). This region is slowly moved along the entire length of the specimen, the progression of the heating source being controlled by a mechanism activated by the moving crosshead. The resulting deformation on an axisymmetric billet is analogous to wire drawing or extrusion.

The second process considered is isothermal forming which is a technique of metal forging in which use is made of the fact that metals are generally softer the higher the temperature and the lower the strain rate imposed on them. In isothermal forming the dies are heated to the same temperature as the specimen and then the specimen is compressed between them at a slow speed. This contrasts with normal hot forging in which the dies are much colder than the specimen and the operation is carried out with repeated quick blows. Both the above processes are described more fully in other papers (1-3).

Figure 1.
Diagram of dieless
drawing, indicating
meaning of symbols
and expressions.



The chief similarities between these two processes are that they require high temperatures and slow strain rates for success. The titanium alloy, Titanium 6Al 4V (6% aluminium 4% vanadium remainder titanium), has been used satisfactorily with both processes and this is the material used as the example for the calculations in this paper.

2. The Method of Analysis

Whilst it is desirable to model the materials as accurately as possible, there are many technologically important problems in which the elastic properties of the metal may not be of great significance. Into this category fall many metal working processes in which the deformation imposed upon the material is very large. In such cases the treatment of the problem with an elastic plastic approach may not only be unnecessary to achieve the required results but can cause a considerable encumbrance on the calculations, especially if there are both plastic and elastic regions present (4). These considerations have led to a growth of interest in the rigid-plastic approach because it is simpler (5,6).

In the hot-working of metals, other considerations apply because elastic phenomena may be considered to be non-existent. At hot working temperatures metals are normally considered to creep even if an infinitesimal stress is imposed upon them (7). In hot forming the stresses that are required are dependent mainly on the temperature and strain rate at which the forming takes place, but in elastic calculations these two factors are usually ignored. In the plastic working of metal at high temperatures there is not necessarily any influence of strain hardening or previous history on the stresses required (2). In these circumstances the rigid-plastic formulation is ideal for the material but its 'viscosity' must be dependent on strain rate and temperature.

The Finite Element Formulation for both Processes

1. Work Rate per Unit Volume

Following from the arguments presented in the introduction, the only internal work in the hot forming situation is that due to plastic strains. An increment of plastic work per unit volume, dW , can be written (as shown by Hill (8)), using the usual notation

$$dW = \sigma_{ij}' d\epsilon_{ij}$$

where $d\epsilon_{ij}$ are the increments of plastic strain, σ_{ij}' are the deviatoric stresses and the summation convention applies. The deviatoric stresses are defined as follows

$$\sigma_{ij}' = \sigma_{ij} - \delta_{ij} \sigma \quad (1a)$$

where σ_{ij} are the true or total stresses, δ_{ij} is the kroneka delta and σ is the mean stress such that

$$\sigma = \frac{1}{3}(\sigma_{ii}) \quad (1b)$$

This can be rewritten in terms of plastic strain rate per unit volume,

$$\dot{W} = \sigma_{ij}' \dot{\epsilon}_{ij} \quad (2)$$

A strain rate $\dot{\epsilon}_{ij}$ is defined (for infinitesimal strains) as

$$\dot{\epsilon}_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (3)$$

The constitutive relationship that is applicable is the Levy Mises flow rule which is given by (see Hill (8))

$$d\epsilon_{ij} = \sigma_{ij}' d\lambda$$

where $d\lambda$ is an instantaneous constant. In order to calculate $d\lambda$ the equation is rewritten to involve time:

$$\dot{\epsilon}_{ij} = \sigma_{ij}' / 2\eta \quad (4)$$

It is written in this form to emphasize the connection between this formulation and the flow of viscous fluids at very low Reynolds numbers. Thus η can be interpreted as an instantaneous viscosity. Combining equations (2) and (4)

$$\dot{W} = 2\eta \dot{\epsilon}_{ij} \dot{\epsilon}_{ij} \quad (5)$$

Since both the processes are normally axisymmetric, all the above equations are rendered in terms of cylindrical coordinates in this work.

2. The Viscosity

As noted in the Introduction, the material will be considered as being sensitive to strain rate and temperature but that it is insensitive to strain hardening. Thus at a particular point the viscosity (η in equation (4)) will have to be estimated from a knowledge of the temperature at that point. In these particular deformation processes the temperature at a point can be considered to be unaltered during the deformation but the strain rate can only be estimated from the strain rate that occurred at that point in the previous increment. (The first increment, of course, requires some 'dummy' strain rates.) This is a non-linearity in the problems, but very little difficulty was experienced in obtaining a quick convergence for the viscosities. The best stress, strain rate and temperature data that is available for Titanium 6Al 4V is that of Lee and Backofen (9), who present it as a series of data triplets (on graphs). They show that it is difficult to represent this data by any simple equation that will be valid for all conditions, so the approach used in this work was to store the data in the program and interpolate the required viscosities directly from this data.

A formula is required to govern the interpolation and it may be considered that over small ranges of temperature and strain rate a single equation will be satisfactory. For the situation that occurs in the two processes being considered, a suitable equation is the one suggested by Schoek (7):

$$\sigma_z = C \exp\left(\frac{\Delta H}{kT}\right) (\dot{\epsilon}_z)^m \quad (6)$$

3. Incompressibility

In the formulation so far described there is no provision to ensure that the material is incompressible. The approach used in this paper is the penalty function approach which is one of the simplest to apply. In this technique a large positive number, α , is multiplied by the volume integral of the square of the volumetric strain rate, i.e.

$$\alpha \int \dot{\epsilon}_v^2 dv \quad (7)$$

This integral is added to the work rate functional. Thus if at any point at which it is evaluated the volumetric strain rate deviates from zero a large penalty is applied to the functional. Hence when the functional is minimized the volumetric strain rate will be made to be close to zero. This method requires only two variables at each node and only a small change to an elastic program (as illustrated by Zienkiewicz and Godbole (6)). It suffers from the disadvantage of not enabling a direct calculation of σ , the mean stress. As will be described later, a separate technique was devised for calculating the mean stress from the known deviatoric stresses.

4. The Complete Functional

The complete functional, χ , that was minimized using the finite element method was therefore

$$\chi = \int_V \dot{w} dv + \alpha \int_V \dot{\epsilon}_v^2 dv + \int_S \dot{T}_{ST} dS + \int_V \dot{w}_{BF} dv \quad (8)$$

where V is the volume, S is the outside surface of the body, and $\dot{T}_{ST}$ is the work rate per unit area due to tractions over S and $\dot{w}_{BF}$ is the work rate per unit volume due to body forces. $\dot{w}_{BF}$ is normally taken as zero in slow speed plasticity since the body forces are much smaller than the other forces involved. The boundaries in the two deformation processes described in this paper are normally free or geometrically determined. However, provision has been made to use the surface traction term for frictional situations in the case of isothermal forming.

The two deformation processes were analysed using two different finite element programs. For dieless drawing the program employed triangular constant strain ring elements and a Gaussian iterative solution. The isothermal program was based on a program with quadrilateral isoparametric elements and had a high speed elimination solution for banded matrices.* The elements were normally used in an axisymmetric form with 8 nodes and 2×2 gauss integration points for each element.

Application to Dieless Drawing

1. The Shape of the Deformation Zone

Dieless drawing can be considered as a continuous process once the operation is under way. To start the analysis, the deformation zone is set up in the program in some arbitrary manner from which iterations will remove all the conditions of non-equilibrium. There are two main unknowns that have to be found, the viscosity distribution throughout the deformation zone, and the shape of the external surface of this zone. The viscosities of the elements can be successfully estimated by using temperatures and strain rates from the previous iteration as has been described above. The external surface presents new problems.

To simplify the situation the deformation zone is considered to be fixed in space and the heat source is thus stationary even though this may not be the experimental arrangement. This is equivalent to putting $v_h = 0$ on figure 1. The reduction that should occur in the process is given by

$$\frac{R_o}{R_i} = \sqrt{\frac{(v_h + v_i)}{(v_h + v_o)}} = \sqrt{\frac{v_i}{v_o}} \quad (\text{if } v_h = 0) \quad (9)$$

using the condition of constant volume. The criterion for determining

* The original elastic program from which the program for isothermal forming was developed was kindly provided by Professor O.C. Zienkiewicz. Technical details of the programs are described by Zienkiewicz in his standard text (10).

the position of the external surface is that the velocities at the nodes along the surface should be parallel to the surface.

Many schemes can be envisaged which will estimate the position of the nodes on the surface for an iteration from values calculated during the previous iteration and several methods were tried in the course of this study. A quick method that is almost certain to converge is that described by Alexander and Turner (1). If r_n is the radius of a node n , $(v_r/v_z)_n$ is the ratio of velocities at node n and h is the axial distance between node n and node $n+1$ (both nodes being on the external boundary) then the new estimate of r_{n+1} is given by

$$r_{n+1} = r_n + \frac{[(v_r/v_z)_n + (v_r/v_z)_{n+1}]h}{2} \quad (10)$$

This new estimate of r_{n+1} is used to estimate r_{n+2} and so on. The radial position of all the nodes in the deforming zone is subsequently adjusted to ensure that grossly distorted elements are not created. The axial position of the nodes is never changed and hence the temperature in the elements can always be considered to be the same throughout the iterations since there are only small changes in the radial positions of the nodes with respect to the heat source. The temperature distribution that was used was drawn from the work of Turner (11).

Figure 2a illustrates the movement of the outside profile as the iterations proceed. Two profiles from experiments are illustrated on figure 2b and it can be seen that the agreement is quite satisfactory. Exact agreement is not to be expected, not only because of approximations inherent in the finite element method but also because the temperature distribution in the experiments cannot be subject to exact control and thus may not correspond entirely with the temperature distribution that was used for the calculations.

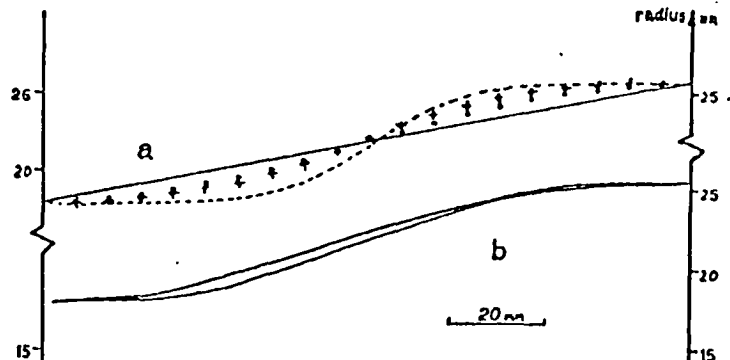


Figure 2. a. Progress of iterations in solution for dieless drawing. Continuous line, starting profile; dots/position of outside nodes/ after first iteration; crosses, after second iteration; dashed line, acceptable convergence (ninth iteration).
b. Two experimental profiles for comparison.

2. Optimum Penalty Function

As presented above the penalty function, α , can be given no specific value except that to ensure accurate attainment of constant volume it should be as large as possible. However, as the penalty function becomes larger the terms in the stiffness matrix due to it will be much greater than those due to the energy rate terms (see equation (8)). The maximum allowable value of the penalty function will be determined by the precision obtainable on a particular computer, for the magnitude of the energy rate terms must be much greater than the rounding off errors on the largest terms.

The solution technique used to determine the nodal variables also has an influence. In the case of dieless drawing, the Gauss-Seidel iterative technique was used and the number of iterations used in this technique to obtain a satisfactory convergence is increased if terms vary substantially in magnitude. Figure 3a illustrates that at $\alpha = 10^5$ a much faster solution is obtained than for $\alpha = 10^6$ (α is expressed in units MPa-sec).

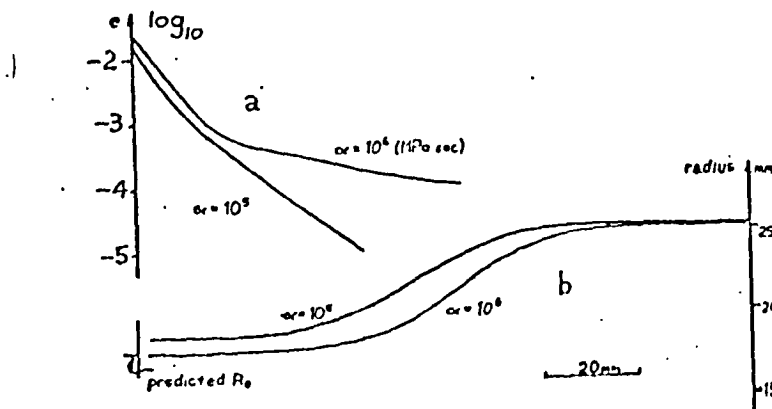


Figure 3. a. e is the error in the gaussidal iteration for two values of penalty function, α , shown.

b. Corresponding outside profiles for the two values of α for dieless drawing.

An unusual feature of the method of estimating the position of the outer surface devised by Alexander and Turner (1) is that the only outer radius that need be specified throughout the calculations is the input radius (R_1 on figure 1). As a result if the calculations predict a correct exit radius (R_0) for the particular velocity conditions as calculated by equation 9, then the method is obeying the law of constant volume. This is a sensitive method for confirming both that the Alexander and Turner method and the penalty function method of obtaining constant volume elements are satisfactory. As can be seen from figure 3b, the constant volume condition is not obtained very well with $\alpha = 10^5$ but is obtained much better for $\alpha = 10^6$. Better prediction of constant

Application to Isothermal Forming

1. Boundary Conditions and Mesh Regeneration

The authors have described isothermal forming from an experimental point of view in another paper (2) and the way these findings were applied to the finite element analysis.

a. Since isothermal forming consists in a specimen being forged between dies it cannot be considered as a continuous process. To analyse the deformation it is broken up into a series of increments representing increments of movement of the tool into the specimen.

b. In a typical situation where the specimen is Titanium 6Al 4V and the lubricant is graphite it can be considered that there is no relative movement between the dies and the specimen. The dies are hard and elastic at the temperatures being used and can be considered to be rigid.

c. The mesh is continuously updated so that the infinitesimal strain formulation of equation 3 can be used for the large deformation that occurs. If the time increment is Δt , then each node is normally moved according to the formula

$$\Delta x_i = u_i \Delta t \quad (11)$$

where Δx_i is the movement in direction i .

d. The viscosities are estimated by use of the strain rate from the previous increment as has been previously discussed. As the process is isothermal and slow the temperature at all points is constant and the same.

e. As the forging proceeds provision may have to be made for boundaries that were originally free to roll onto the dies where they then become attached.

Some increments of an example of an application of these approaches are shown on figure 5. Plate 1 illustrates the experimental results for the same situation and as can be seen there is considerable agreement.

2. Mean Stress Calculations

In the calculations described so far only the deviatoric stresses have been calculated. However, it is much more useful to have the total stresses and so a method has been developed to estimate the mean stress from the deviatoric stress results of the program.

The mean stress, σ , in one direction can be found from equation 1a, and the equilibrium equations are

$$\sigma_{ij,j} = 0 \quad (12a)$$

volume is obtained with higher values of α but the improvement is not significant and the computing time must be increased because convergence is much slower. Because of these conflicting demands the value of α was set at 10^6 for the dieless drawing work. In the solution of isothermal forming it was found that the elimination scheme of solution was not greatly influenced by larger values of α and a value of 10^8 was used without difficulty.

3. Rigid Regions

Outside the deforming region where there is no heating, the temperature decreases sharply. The colder material has a much higher yield stress than the heated material and so that no plastic deformation occurs in this region. The program has provisions which enable it to obtain results in the cold regions which do not invalidate the analysis but avoid the complexity of removing elastic or rigid elements from the analysis. Using equation 6, the following relationship is obtained for viscosity:

$$\eta = \frac{1}{3}C \exp\left(\frac{\Delta H}{kT}\right) \dot{\epsilon}_z^{m-1} \quad (6)$$

When extrapolation from Lee and Backofen's data (9) are carried out in the program to find viscosities at points with low strain rates or low temperatures, the extrapolations follow equation (6). Hence, as the temperature or strain rate decreases, the viscosity increases (m being less than unity). As a result the elements that lie outside the hot region are modelled as having strain rates close to zero. Elements which have a strain rate falling below a certain value can be identified as being rigid (as was done by Shah and Kobayashi (12)). The rigid regions that were identified on the same calculations as illustrated on figure 2a are shown on figure 4. The shape of these regions are determined both by the temperature and stress distributions.

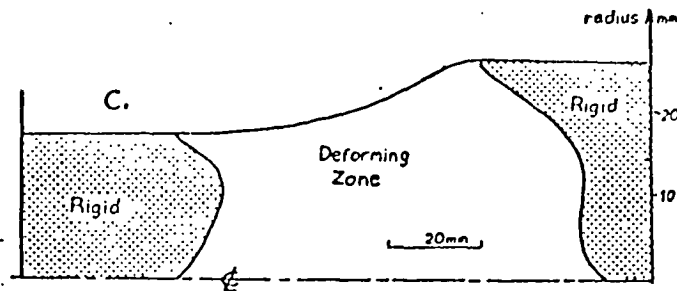


Figure 4. The rigid regions predicted by the solution for the same case as illustrated on figure 2a.

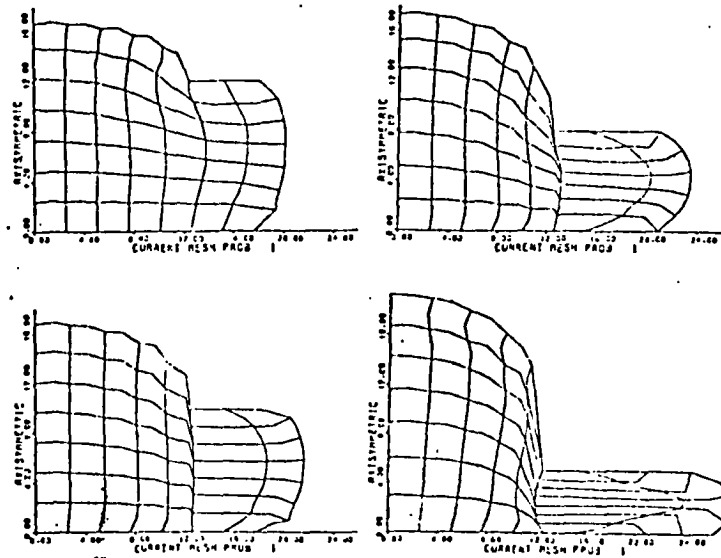


Figure 5. Solution of isothermal forming of Titanium 6Al 4V at 950°C and a crosshead velocity of 1mm/sec. Starting dimensions are 35mm diameter and 17mm high. Compare these with Plate 1. Because of the high deformations, some arbitrary mesh reforming, not obeying equation 11 is required.

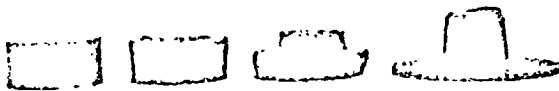
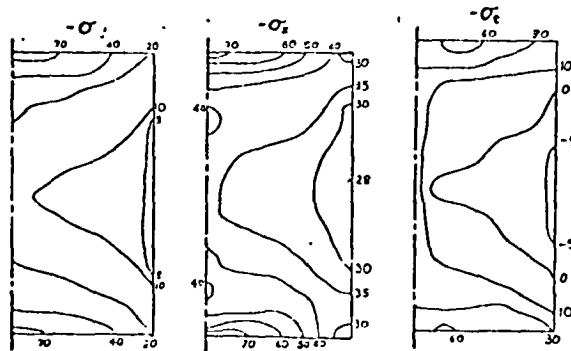


Plate 1. Some isothermally formed specimens at different stages of compression.

Figure 6. The calculated distributions of stress. Stresses shown are mean stress, axial stress, hoop stress (in MPa)



where the body forces are assumed to be zero and the comma followed by a subscript indicates partial differentiation (by x_j). Equations (1a) and (12a) can be reformulated

$$\sigma_{,i} = -\sigma'_{ij,j} \quad (12b)$$

This can be integrated as a line integral within a single element and to do this the equations 12a and 12b must be rewritten in local coordinates which will be written as X_I where capital letters indicate the local coordinates.

If L_{IJ} are the elements of the inverse Jacobian matrix, equation 12b can be rewritten in terms of local coordinates

$$\sigma_{,i} = L_{iJ}\sigma_{,J} = -L_{iK}\sigma'_{il,K} \quad (12c)$$

Now the increment of mean stress is

$$d\sigma = \sigma_{,K}dx_K \quad (13a)$$

Thus if the mean stress at a point a is known and equal to σ_a , then the mean stress at point b is

$$\sigma_b = \int_a^b d\sigma + \sigma_a = \int_a^b \sigma_{,i}dx_i + \sigma_a \quad (13b)$$

which is a line integral. It is necessary to convert to local coordinates and if J_{iK} is an element of the Jacobian equation, 13b can be expanded

$$\sigma_b = \int_a^b L_{iJ}\sigma'_{il,J}J_{iK}dx_K + \sigma_a \quad (13c)$$

This integral can be easily calculated by gaussian integration if the term $\sigma'_{ij,J}$ can be evaluated at points within an element. Considering that these calculations are going to be inserted into a finite element program they can be most easily achieved if the values that have already been calculated at nodes and shape functions can be used. Hence if $\sigma'_{ij}{}^N$ is one of the deviatoric stress components at node N, then the deviatoric stresses at a point X can be written

$$\sigma'_{ij} = P(\bar{X})^N \sigma'_{ij}{}^N \quad (14a)$$

where $P(\bar{X})^N$ is the value of the shape function for node N at a point $\bar{X}$ and there is a summation on N over all the nodes. This can be differentiated

$$\sigma'_{ij,J} = P(\bar{X})^N_{,J} \sigma'_{ij}{}^N \quad (14b)$$

The terms $P(\bar{X})^N_{,J}$ are normally easily formed for a point $\bar{X}$, for they are usually available in a program in which they are used to calculate strains or strain rates from the nodal variables. The values of the

deviatoric stresses at the nodes have already been calculated. L_{ij} and J_{ij} are also normally available in the program.

To start the calculations of mean stress, it is necessary to know the mean stress somewhere in the body. This is possible along free boundaries where the deviatoric stress normal to the boundary must be equal to minus the mean stress since the total stress normal to such a boundary is zero.

Figure 6 shows some stress results from a compression test during its first increment.

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