

Extremely low cycle fatigue tests on structural carbon steel and stainless steel

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Abstract

Cyclic material tests in the low and extremely low cycle fatigue regime were carried out to study the properties of structural carbon steel and stainless steel. A total of 62 experiments were performed in cyclic axial and bending configurations, with strain amplitudes up to $\pm 15\%$. Materials from hot-rolled carbon steel (S355J2H), cold-formed carbon steel (S235JRH) and cold-formed austenitic stainless steel (EN 1.4301 and EN 1.4307) structural sections were tested and the results were compared. The strain-life data from the axial tests were used to derive suitable Coffin-Manson parameters for the three materials; two further extremely low cycle fatigue life prediction models were also considered. The results revealed that the three materials exhibit similar strain-life relationships despite significantly different elongations at fracture measured in monotonic tensile tests. The hysteretic responses of the materials at different strain amplitudes were used to calibrate a combined isotropic/kinematic cyclic material hardening model which can be incorporated into numerical models of structural members. The stainless steel specimens displayed significantly greater levels of cyclic hardening than the corresponding carbon steel samples. A relationship between the results obtained from axial and bending test arrangements was established through consideration of energy dissipation, enabling strain-life models to be derived from either means of testing.

Nomenclature

A_i	cross-sectional area of a small element at a constant strain amplitude $\Delta\epsilon_i/2$
A_0	initial cross-sectional area of the tensile coupon
A_u	minimum cross-sectional area after fracture
b	fatigue strength exponent
b_{iso}	rate at which the size of the yield surface changes as plastic strain increases
C	fatigue ductility exponent
C'	material constant in an extremely low cycle fatigue expression
C_{kin}	constants that can be calibrated by test data from a stabilised cycle
C_m	ductile damage fraction
E	Young's modulus
K'	cyclic strength coefficient
n'	cyclic strain hardening exponent
N_f	number of cycles to failure
Q_∞	maximum change in the size of the yield surface
u_i	plastic hysteresis energy of a small element
u_f	true tensile toughness
V_i	volume of a small element with strain amplitudes large enough to contribute to the fatigue process
V_t	total volume of material subjected to cyclic bending
W_f	total energy to fracture
α	backstress
α'	material constant in an extremely low cycle fatigue expression
ΔW	plastic hysteresis energy per cycle of a material
$\Delta\epsilon/2$	total strain amplitude
$\Delta\epsilon_e/2$	elastic strain amplitude
$\Delta\epsilon_{hyp}/2$	a hypothetical uniform strain amplitude across the section that gives the same plastic hysteresis energy as using the actual strain amplitude at each point

$\Delta\varepsilon_{\max}$	maximum plastic strain range
$\Delta\varepsilon_p/2$	plastic strain amplitude
$\Delta\sigma$	stabilised stress amplitude at half of the number of cycles to failure
ε_p^0	value of plastic strain when the stress-strain curve crosses the strain axis
ε^p	equivalent plastic strain
ε'_f	fatigue ductility coefficient
ε_f	tensile ductility
$\varepsilon_{p,\max}$	maximum tensile plastic strain applied to the specimen
ε_u	ultimate tensile stress from monotonic tensile tests
γ	a material constant that can be calibrated by test data from a stabilised cycle
σ^0	size of the yield surface
σ_i^t	maximum tensile stress in the elastic range
σ_i^c	maximum compressive stress in the elastic range
σ^s	average of the first and the last stress data points
σ'_f	fatigue strength coefficient
$\sigma _0$	yield stress at zero equivalent plastic strain

1 Introduction

Under extreme seismic conditions, structural members, particularly those acting as dissipative elements, experience small numbers of very large displacement cycles. The response of structural members to this form of loading is controlled by their geometry and by the hysteretic behaviour of the constituent material, which can be studied through fatigue testing at high strain amplitudes. This paper focuses on high strain amplitude fatigue testing of structural carbon steel and stainless steel. The results have been used to develop strain-life relationships and cyclic hardening parameters, suitable for incorporation into numerical models to predict fracture of structural members subjected to large amplitude cyclic loading.

Fatigue testing configurations include axial tension-compression, axial tension-tension, torsion and bending. In axial testing arrangements, specimens are subjected to uniform stress and strain amplitudes across the section while maximum stress and strain amplitudes only occur on the surface of specimens in torsion and bending configurations. Axially-loaded fatigue testing has been carried out on many materials, including carbon steel [1,2] and stainless steel (grade EN 1.4301) [3,4]. Stocky cylindrical test specimens are often employed to prevent buckling of specimens under high compressive strains [5], although thin sheet specimens have also been employed when the applied strain amplitudes are small [4]. While axial testing configurations allow easy interpretation of test results owing to the uniform distribution of stress and strain, bending configurations have the advantage of avoiding the issue of buckling and therefore enabling testing of thin plates. Three-point bending [6], four-point bending [7] and uniform bending [8] set-ups are among the possible variations of bending test configurations.

Another way of categorising fatigue tests is to consider the applied strain amplitude, which can be decomposed into elastic and plastic components. Testing regimes in which the elastic strain amplitude is higher than the plastic strain amplitude are generally referred to as high cycle fatigue (HCF); such tests are usually stress-controlled and typically involve more than 10^6 cycles to cause failure. Conversely, the term low cycle fatigue (LCF) is applied when plastic strain dominates, and the number of cycles to failure generally ranges from 10^2 to 10^4 . Low cycle fatigue testing is generally strain-controlled (either by total or plastic strain), since hardening or softening of the material complicates load- or stress-controlled testing.

It has been observed that plastic strain-life data from LCF tests fall approximately on a straight line when plotted on a log-log scale. This observation forms the basis of the Coffin-Manson relationship [9,10], which is expressed as:

$$\frac{\Delta \varepsilon_p}{2} = \varepsilon'_f (2N_f)^c \quad (1)$$

where $\Delta \varepsilon_p/2$ is the plastic strain amplitude, ε'_f is the fatigue ductility coefficient, c is the fatigue ductility exponent and $2N_f$ is the number of reversals to failure.

In the very low cycle regime, often referred to as extremely low cycle fatigue (ELCF), an over-estimation of fatigue life by the Coffin-Manson relationship has been observed [11, 12]. Models accounting for the combined effects of ductile and fatigue damage have been proposed to

establish strain-life relationships that cover both the LCF and ELCF regimes. This has been achieved, for example, by combining an exponential function with the power law of the Coffin-Manson relationship [11], partitioning the damage into ductile and cyclic damage components [12,13] or by employing plastic void growth and coalescence models [14].

In addition to stress- and strain-life relationships, a further means to predict fatigue failure is by consideration of energy dissipation [15]. The relationship between total energy to fracture and fatigue life can be derived from the area under the hysteresis loops of Masing-type materials and is given as:

$$W_f = 4\sigma'_f \varepsilon'_f \left(\frac{c-b}{c+b} \right) (2N_f)^{1+b+c} \quad (2)$$

where W_f is the total energy to fracture, σ'_f is fatigue strength coefficient, ε'_f is the fatigue ductility coefficient, b is the fatigue strength exponent, c is the fatigue ductility exponent and $2N_f$ is the number of reversals to failure. The relationship shows that the total energy to fracture increases as the fatigue life increases. Assessment of energy dissipation is also useful in relating axial and bending fatigue test results. Esin [16] developed a method to correlate fatigue data from axial, bending and rotating bending tests by considering energy dissipation of the cross-section of the specimens. This approach has been applied in the present study.

Existing strain-life relationships, stress-strain hysteretic behaviour and energy criteria reported in the literature have been derived principally from LCF tests at cyclic strain amplitudes of less than $\pm 2.0\%$. However, strain amplitudes encountered by structural members under extreme loading, such as earthquakes, may significantly exceed this value and belong to the ELCF regime. In this paper, ELCF tests on structural steel grades (cold-formed stainless steel and hot-rolled and cold-formed carbon steel) are reported and the results utilised to establish strain-life relationships and cyclic material modelling parameters.

2 Materials and testing arrangements

2.1 Materials and test coupons

LCF and ELCF fatigue tests were carried out to contribute to the limited data currently available on structural materials. Test coupons were machined from the flat faces of square and

rectangular hollow sections to reflect the actual material properties of complete structural members. The location of the extracted coupons was consistently opposite to the weld, as shown in Fig. 1. The materials examined were from hot-rolled and cold-formed carbon steel sections (grades S355 J2H [17] and S235 JRH [18], respectively) and cold-formed stainless steel sections (grades EN 1.4301 and 1.4307). Owing to the similarity in chemical compositions and specified mechanical properties [19], the two stainless steel grades were grouped as the same material in this study, and indeed, exhibited similar behaviour.

The monotonic tensile stress-strain curves of the three materials – hot-rolled carbon steel, cold-formed carbon steel and cold-formed stainless steel – abbreviated herein as ‘hot CS’, ‘cold CS’ and ‘cold SS’, respectively, are known to exhibit distinct behavioural features. Hot-rolled carbon steel displays a sharp yield point, followed by a flat yield plateau before strain hardening occurs, whereas cold-formed carbon steel and stainless steel both exhibit rounded stress-strain characteristics, with the degree of roundedness being significantly more pronounced in the case of stainless steel [20]. Stainless steel also exhibits more substantial strain hardening than carbon steel, with greater ratios of ultimate tensile strength to yield strength, greater levels of elongation at fracture and differences in other physical and thermal properties [21]. Typical stress-strain curves for the three materials (up to 2% strain) are shown in Fig. 2, while a summary of the results of the monotonic tensile coupon tests (performed in accordance with [22]) is given in Table 1. In the table, and throughout the paper, specimens are identified by section size ($B \times D \times t$), material (either carbon steel ‘CS’ or stainless steel ‘SS’), forming process (either hot-rolled ‘HR’ or cold-formed ‘CF’) and finally by test details (either ‘T’ for tensile tests or the strain amplitude for cyclic tests). For both the cold-formed carbon steel and stainless steel, the 0.2% proof strength has been calculated as the equivalent yield strength. The percentage elongation at fracture ε_L is defined as the percentage elongation over the standard gauge length, which is a function of cross-sectional area, as defined in BS EN 10002-1 [22]. The tensile ductility ε_f is defined by Eq. (3) and is determined by means of measuring the reduction in cross-sectional area after fracture.

$$\varepsilon_f = \ln\left(\frac{A_0}{A_u}\right) \times 100\% \quad (3)$$

where A_0 is the initial cross-sectional area of the tensile coupon and A_u is the minimum cross-sectional area after fracture.

The geometry and dimensions of the cyclic material test coupons, which included a central region of reduced cross-sectional area to constrain the location of fracture, are shown in Fig. 3. Drilled and reamed holes 20 mm from each end of the coupons were provided to accommodate the pins in the jaws of the testing machine. As recommended in [23], the dimensions have been chosen to ensure that buckling does not occur under the highest compressive forces anticipated in the test programme, and that premature failure does not occur outside the reduced cross-section. The length and the width of the region of full cross-sectional area are slightly larger than those in the standard in order to provide greater surface area for gripping.

2.2 Axial tests

Axially loaded tests were performed to establish strain-life relationships and to calibrate a hardening model for cyclic plasticity. Coupons were subjected to fully-reversed constant total strain amplitude cycles. Test coupons were gripped by friction in the jaws of the testing apparatus as well as positively held by pins at each end. Relative movement between the top and bottom jaws of the testing apparatus was prevented by guide plates (Fig. 4). Deformation of the coupons was concentrated in the central 3 mm of exposed length of the test coupons; two strain gauges (one on each side of the test coupon, as shown in Fig. 3) covered this exposed region.

Tests were carried out at $\pm 1\%$, $\pm 3\%$, $\pm 5\%$ and $\pm 7\%$ total strain amplitudes, which caused significant plastic deformation and covered both the LCF and ELCF regimes. Generally, three specimens were tested at each strain amplitude. Cyclic loading was applied by means of a 250 kN Instron actuator at strain rates of about $5 \times 10^{-3} \text{ s}^{-1}$, which complied with BS 7270 (2006) [23], to ensure that the heat generated during testing did not significantly affect the results. Initially, amplitudes of applied displacements were slightly adjusted in each cycle according to the readings given by strain gauges in order that the target strain amplitudes were achieved, as in some cases minor slippage was observed. Upon stabilisation, or loss of adhesion of the strain gauges to the specimen, testing proceeded under displacement control. Testing continued until the occurrence of fracture, and failure was defined at the point of a 10% drop in load.

2.3 Bending tests

In axial fatigue tests, specimens are prone to buckling under high strain amplitudes. Preventive measures such as using stockier specimens are not always feasible. For example, in the context of the present study, the thickness of the specimens was restricted to that of the thin-walled

tubular members from which they were extracted. Bending fatigue test specimens are not prone to buckling and thus higher strain amplitudes can be applied.

A four-point minor-axis bending set-up (Fig. 5) was adopted in the current study to convert the axial displacement of the Instron 250 kN actuator into a constant bending moment along the specimen gauge region. This enabled fully reversed constant strain amplitude testing of the coupons under uniform bending. Curvature of the specimens was concentrated in the region of the reduced cross-section and hence the location of fracture was constrained. The end supports of the testing apparatus were free to slide on rollers, minimising any axial force in the coupons.

Under a given applied displacement, a thick coupon will experience higher strain on the surface than a thin coupon. Utilising the full stroke of the actuator, 75 mm, tests were carried out at $\pm 2\%$, $\pm 6\%$, $\pm 10\%$ and $\pm 15\%$ strain amplitudes on the 4 mm thick specimens but at lower strain amplitudes of $\pm 2\%$, $\pm 6\%$, $\pm 8\%$ and $\pm 10\%$ on the 3 mm thick specimens. Tests were conducted at strain rates of about $5 \times 10^{-3} \text{ s}^{-1}$. Strain gauges were attached to both the upper and lower faces of the coupons (Fig. 3). Similar to the axial tests, the amplitudes of applied displacement were initially adjusted according to the readings of the strain gauges to achieve the desired strain amplitudes, after which constant displacement amplitudes were maintained. Failure was defined as in the axial tests.

3 Axial test results

In this section, the results of the constant strain amplitude fully-reversed cyclic axially loaded test specimens are presented and analysed. Cyclic stress-strain curves are provided, strain-life relationships are developed and calibration of a cyclic material hardening model is performed.

3.1 Cyclic stress-strain curves

When a family of stabilised hysteresis loops with various strain amplitudes is plotted on the same graph, a cyclic stress-strain curve can be defined by tracing the locus of the tips of the loops (Fig. 6). The resulting curve may be represented by a form of Ramberg-Osgood relationship, written in terms of stress and strain amplitudes, as given by:

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'} \right)^{1/n'} \quad (4)$$

where $\Delta\varepsilon/2$ is the total strain amplitude, $\Delta\varepsilon_e/2$ is the elastic strain amplitude, $\Delta\varepsilon_p/2$ is the plastic strain amplitude, $\Delta\sigma$ is the stabilised stress amplitude at half of the number of cycles to failure, E is the Young's modulus, K' is the cyclic strength coefficient and n' is the cyclic strain hardening exponent. The material constants K' and n' can be obtained from a power law regression fit to the stress amplitude versus plastic strain amplitude data.

Three approaches for establishing cyclic stress-strain curves have been employed in previous studies, referred to herein as the multiple-step, incremental-step and companion methods. In the multiple step method, a specimen is cycled at constant strain amplitude. Once the material behaviour has stabilised, the strain amplitude is increased and again held constant until the response re-stabilises. This process continues until the full cyclic stress-strain curve has been established from a single specimen. In the incremental step method, a single specimen is subjected to repeated blocks of incrementally increasing and decreasing strain amplitudes. Generally, after a few blocks of loading cycles, the response stabilises and the cyclic stress-strain curve can be obtained from half of a stable block. These two methods have the convenience of using a single specimen to determine the cyclic stress-strain curve, but their use becomes restricted at large strain amplitudes because specimens are likely to fail prematurely before stabilisation is achieved. In the companion method, a series of specimens are required and each specimen is subjected only to a single strain amplitude. The response at half of the number of cycles to failure is used to obtain cyclic stress-strain curves. This method is suitable for establishing cyclic stress-strain curves at high strain amplitudes and has therefore been used in this study.

The extent of cyclic hardening or softening is revealed by cyclic stress-strain curves, and may be measured relative to the monotonic curve. The cyclic and monotonic stress-strain curves from two sections of each material are plotted in Figs 7-9. Stainless steel, shown in Fig. 9, exhibits strong cyclic hardening; as the strain amplitude increases, the stabilised stress amplitude rises substantially to values much higher than the ultimate stress reaches in tensile tests. The hot-rolled and cold-formed carbon steel, shown in Figs 7 and 8, respectively, exhibit less pronounced cyclic hardening than stainless steel. The differences in hardening characteristics between stainless steel and carbon steel may be attributed to the different microstructure (body-centred

cubic for austenitic stainless steel as opposed to face-centred cubic for carbon steel) and alloy content.

From the test results, the four material parameters in Eq. (1) and (4) have been derived and are listed in Table 2. The fatigue ductility coefficient ϵ'_f and the fatigue ductility exponent c are obtained from the plastic strain-life relationships. The cyclic stress-strain curves, as shown graphically in Figs 7-9, are used to determine the material constants K' and n' . Results derived from selected individual sections, together with average values for each material, are presented. It is known that fatigue tests often show considerable scatter even in carefully controlled environments. Some scatter is evident, particularly for the stainless steel specimens, in the present study.

The rate and extent of cyclic hardening can also be shown by examining the progression of stress amplitudes at each cycle throughout the tests. This is shown at the four considered strain amplitudes ($\pm 1\%$, $\pm 3\%$, $\pm 5\%$, $\pm 7\%$) for hot-rolled carbon steel in Fig. 10, cold-formed carbon steel in Fig. 11 and cold-formed stainless steel in Fig. 12. The tests on hot-rolled and cold-formed carbon steel show a small degree of cyclic hardening, and the material response stabilises in the first few cycles. For stainless steel, the degree of cyclic hardening is relatively low in the $\pm 1\%$ strain amplitude tests, and the material response stabilises at about half the number of cycles to failure. This accords with the findings of Bernard-Connolly et al. [24] who studied the cyclic properties of stainless steel (grade 1.4301) and observed low levels of cyclic hardening at low strain amplitudes and stated that below $\pm 0.5\%$ strain amplitudes cyclic softening occurs. At the higher strain amplitudes employed in the current study, stainless steel shows strong cyclic hardening (Fig. 12). At $\pm 7\%$ strain amplitude, the stainless steel specimens showed the largest degree of cyclic hardening but failure occurred before the material response had fully stabilised.

3.2 Strain-life relationship

In order to calibrate the material parameters of the Coffin-Manson relationship (Eq. (1)), the total strain amplitudes were resolved into elastic and plastic components. Although the total strain amplitudes were kept constant throughout the tests, the amplitudes of the elastic and plastic components changed due to cyclic hardening of the materials. However, cyclic hardening was transient and the material behaviour generally stabilised quite rapidly, as shown by the strain and

stress measurements (Figs 13 and 14, respectively) from the first ten cycles of a typical $\pm 5\%$ strain amplitude test of a stainless steel specimen. The initial increase in stress amplitude stabilised after 5 cycles of loading. The stabilised stress amplitude $\Delta\sigma$ at half the number of cycles to failure was used to calculate the elastic strain amplitude from the following expression:

$$\frac{\Delta\varepsilon_e}{2} = \frac{\Delta\sigma}{2E} \quad (5)$$

where the terms are defined as in Eq. (4). The plastic strain amplitude was then obtained from the difference between the total and elastic strain amplitudes.

The results of all specimens tested in the axial configuration are summarised in Table 3. Up to four specimens were cut from each section and were tested at different strain amplitudes. The total strain amplitudes and stress amplitudes at half of the number of cycles to failure were measured in the tests while the elastic and plastic strain amplitudes were calculated as described above. The number of reversals to failure is plotted against elastic and plastic strain amplitudes on a log-log scale for each material and shown in Figs 15-17. Tensile test data are included as 0.25 cycle tests at strain amplitudes equal to the tensile ductility (calculated on the basis of area reduction through Eq. (3)). As a characteristic of LCF tests, the plastic strain amplitudes account for the majority of the total strain amplitudes. Variation in fatigue life in LCF largely depends on changes in plastic strain amplitudes. In contrast, elastic strain amplitudes are small and their variation is minor so their influence on fatigue life in LCF is relatively insignificant.

The Coffin-Manson relationship (Eq. (1)) has been fitted to the plastic strain-life data and presented as solid lines in Figs 15-17, while scatter bands at two times the standard deviation are shown as dashed lines. The plastic strain-life data may be seen to be well predicted by the Coffin-Manson relationship though the scatter is larger for the cold-formed material than for the hot-rolled material. This may reflect the greater inhomogeneity in material properties associated with cold-formed material due to variable strain history experienced during sheet and section forming. The test data for the three materials and their respective Coffin-Manson curves are compared in Fig. 18, where the hot-rolled carbon steel displays superior fatigue life relative to the cold-formed carbon steel and cold-formed stainless steel. The difference is partly attributed to the increased level of damage present in the cold-formed material and the reduced ductility due to plastic deformation, which will reduce resistance to failure through the ductility exhaustion mechanism.

The importance of the level of cold-work on strain-life response can be observed by comparing the data from the current study to existing LCF data from other studies [1-5, 25-29], as shown in Figs 19 and 20, though it is worth noting that materials in cold-formed structural members will typically undergo higher levels of cold-work than the cold-worked steel sheets tested in the literature. Although scattered, the results generally indicate the anticipated trend that fatigue performance is degraded in cold-worked material. In the present study, tests were performed on material extracted from the flat regions of the structural sections, though the corner regions of these sections would have undergone a greater degree of cold-work, and would therefore be expected to exhibit inferior characteristics. However, given that the variation in response between hot-rolled and cold-worked flat material is relatively small, the difference in response between the flat and corner material of structural sections may also be expected to be relatively small, though this requires further verification.

Since the range of strain amplitudes applied in the current study cover LCF, ELCF and tensile tests, specimens have failed by various mechanisms. The two failure modes at the highest and lowest strain amplitudes are ductile failure, as observed in the tensile tests, and fatigue failure, as generally seen in the LCF tests, respectively. The ductile damage failure mechanism typically occurs when the number of cycles to failure is below 100. Failure through ductile damage leads to a relatively uneven surface that is dull in appearance, and often exhibits a cup-and-cone profile. Fatigue failure is characterized by a series of ‘beach’ marks that indicate the progressive advancement of a fatigue crack. Crack initiation takes place at the specimen surface where dislocation slip may most easily occur, and propagates inwards creating a relatively flat fracture surface.

The fracture surface of all specimens in the current study have been examined carefully and categorised into different failure modes. Ductile fracture surfaces such as the one shown in Fig. 21(a) and fatigue fracture surface with clear beach marks as shown in Fig. 21(b) have been classed as ductile and fatigue modes respectively. Specimens which do not show distinct ductile or fatigue failure features are classified as mixed mode failure (Fig. 21(c)).

The failure modes of each specimen are also indicated in Fig. 18 – all tensile coupons have failed in a ductile manner while most of the tests which were carried out in the LCF regime, i.e. $\pm 1\%$ strain amplitude, showed fatigue fracture. Specimens that failed in ELCF regime exhibited either

a ductile damage or mixed mode failure. This change in damage mode may partly explain the scatter in the test results.

3.3 Low and extremely low cycle fatigue relationships

It has been shown in previous studies [12,13] that when the number of reversals to failure falls below approximately 200, the strain-life relationship may deviate from the Coffin-Manson model, due to changes in the damage mechanisms. As the strain amplitudes increases from the LCF regime to the ELCF regime, the failure mode changes from fatigue fracture to accumulation of ductile damage. Various models have been proposed to establish a unified strain-life relationship for both the LCF and ELCF regimes. Two of these models are discussed below.

Kuroda [12] considers that ELCF consists of damage due to tensile straining, ductility exhaustion during cyclic straining and crack propagation. The combined effect of these three damage modes determines the fatigue life of a material and the total damage fraction during a cycle is given by the sum of the damage fractions accumulated due to each of these mechanisms. Failure is conceded when the total damage fraction attains unity such that

$$\frac{\varepsilon_{p,max}}{\varepsilon_f} + 4N_f \left(\frac{\Delta\varepsilon_p}{2\varepsilon_f} \right)^{1/\alpha'} + \frac{\Delta\varepsilon_p N_f^{\alpha'}}{C'} = 1 \quad (6)$$

where $\varepsilon_{p,max}$ is the maximum tensile plastic strain applied to the specimen, ε_f is the uniaxial tensile ductility (calculated on the basis of reduction in area at the fracture surface), N_f is the number of cycles to failure, $\Delta\varepsilon_p/2$ is the plastic strain amplitude, and α' and C' are material constants derived from the test data [12].

Tateishi et al. [13] modified the Coffin-Manson relationship to account for ductile damage. The total damage in the material is taken to be the sum of the ductile damage fraction and fatigue damage fraction and again failure is conceded when the total damage attains unity. When the maximum plastic strain range $\Delta\varepsilon_{max}$ exceeds a damage threshold value in pure tension, taken to be the plastic strain corresponding to the ultimate tensile stress from monotonic tensile tests ε_u , then the number of cycles to failure N_f is reduced by a factor linked to the ductile damage fraction C_m , as described by Eq. (7):

$$\frac{\Delta \varepsilon_p}{2} = \varepsilon'_f (2N_f)^c C_m$$

$$C_m = \begin{cases} \left(\frac{\varepsilon_f - \Delta \varepsilon_{\max}}{\varepsilon_f - \varepsilon_u} \right)^{-c} & \text{if } \Delta \varepsilon_{\max} > \varepsilon_u \\ 1.0 & \text{if } \Delta \varepsilon_{\max} \leq \varepsilon_u \end{cases} \quad (7)$$

where ε'_f and c are the Coffin-Manson constants.

These two ELCF life prediction models have been applied to the data generated in this study. Tensile test data are not applicable to the models and thus have not been used in the calibration process, but have been used to determine the fundamental material parameters (ε_u and ε_f). The strain-life curves predicted by these models are compared with the Coffin-Manson curve in Figs 22-24. The two models provide improved agreement with the cyclic test data in the ELCF regime, but do not accurately predict the results of the tensile tests or the results of cyclic tests with higher cycles to failure. The unmodified Coffin-Manson relationship is capable of representing the strain-life relationship of the materials ranging from the tensile tests up to 1000 cycles to failure with reasonable accuracy.

3.4 Calibration of a cyclic material hardening model

Although Eq. (4) provides an overview of how a material behaves under cyclic loading at different stress or strain amplitudes, it only characterises the stabilised hysteretic behaviour. If the behaviour before reaching the stabilised state is required, such as when specifying the material model in a numerical simulation, a constitutive model of cyclic plasticity is needed. The theory of cyclic plasticity provides a mathematical description of stress and strain in plastically deformed solids under cyclic loading. The current study focuses on the constitutive model for cyclic plasticity of metals employed in the non-linear finite element software ABAQUS [30], which is based on the work by Lemaitre and Chaboche [31]. In this model, the von Mises yield criterion and an associative flow rule are assumed. The characteristics of a material are defined by a combined isotropic/kinematic hardening model.

The model consists of a non-linear kinematic hardening component and an isotropic hardening component. The isotropic component defines the change of the size of the yield surface σ^0 as a function of equivalent plastic strain ε^p , and is given by:

$$\sigma^0 = \sigma|_0 + Q_\infty (1 - e^{-b_{iso}\varepsilon^p}) \quad (8)$$

where $\sigma|_0$ is the yield stress at zero equivalent plastic strain (defined in this study as the 0.01% proof stress), Q_∞ is the maximum change in the size of the yield surface and b_{iso} is the rate at which the size of the yield surface changes as plastic strain increases. The size of the yield surface in the i^{th} cycle σ_i^0 can be obtained from:

$$\sigma_i^0 = \frac{\sigma_i^t - \sigma_i^c}{2} \quad (9)$$

where σ_i^t is the maximum tensile stress and σ_i^c is the maximum compressive stress in the elastic range as shown in Fig. 25.

The equivalent plastic strain corresponding to σ_i^0 is:

$$\varepsilon_i^p = \frac{1}{2}(4i-3)\Delta\varepsilon_p \quad (10)$$

where $\Delta\varepsilon_p$ is the plastic strain range and can be approximated as:

$$\Delta\varepsilon_p \approx \Delta\varepsilon - 2\sigma_i^t / E \quad (11)$$

Eq. (8) has been fitted through each data pair $(\sigma_i^0, \varepsilon_i^p)$, including the value $\sigma|_0$ at zero equivalent plastic strain.

The kinematic component of the model defines the change of backstress α , which is expressed as:

$$\alpha = \frac{C_{kin}}{\gamma}(1 - e^{-\gamma\varepsilon^p}) + \alpha_1 e^{-\gamma\varepsilon^p} \quad (12)$$

where C_{kin} and γ are constants that can be calibrated by test data from a stabilised cycle. The ratio C_{kin}/γ is the maximum change in backstress and γ determines the rate at which the backstress varies as the plastic strain increases. Data pairs $(\sigma_i, \varepsilon_i^p)$ are shifted as follows

$$\varepsilon_i^p = \varepsilon_i - \frac{\sigma_i}{E} - \varepsilon_p^0 \quad (13)$$

such that $\varepsilon_1^p = 0$, where ε_p^0 is the value of plastic strain when the curve crosses the strain axis.

For each data pair $(\sigma_i, \varepsilon_i^p)$, values of α_i are obtained as

$$\alpha_i = \sigma_i - \sigma^s \quad (14)$$

where σ^s is the average of the first and the last data points, σ_1 and σ_n , respectively, in a stabilised cycle, which are shown in Fig. 26. Eq. (12) can then be fitted to the pairs of data points $(\alpha_i, \varepsilon_i^p)$.

The method used for fitting Eqs (8) and (12) requires careful consideration. The commonly used least squares regression fits a curve to data points by minimising the sum of the squares of residuals of the dependent variables (i.e. only the vertical distance between data points and the fitted curve is minimised). The general form of the curves resulting from Eqs (8) and (12) comprises an initial steep slope followed by a rounded portion leading onto a more gently sloped region. Any changes in the shape of the curve in this steeply sloped section influence the residuals more significantly than in the flatter region. Effectively, less weighting is given to the flatter region of the curve in the fitting process. Furthermore, if the data points are not evenly spaced along the x-axis, more weighting will be given to the part of the curve where data points are concentrated. Therefore, in order to generate an accurate and representative stress-strain hysteresis curve, the total least squares regression method which can minimise errors on both axes, has been adopted.

The objective of total least squares regression is to minimise the normal distance from the data points to the curve. This method remains unbiased towards any axis if the quantities are measured in the same units or are normalised. A variance of this method is to minimise the area between the data points and the curve instead of the distance between them. This method is units-invariant and is not biased towards any variables but fits the vertical and horizontal residuals with equal weighting. Also, results do not depend on the distribution of the data points. Fig. 27 shows a typical set of data points $(\sigma_i^0, \varepsilon_i^p)$ and the corresponding best fit curve, obtained on the basis of area minimisation.

The values of the parameters calculated for each test are shown in Table 4. The results reveal that stainless steel exhibits a larger degree of cyclic hardening than carbon steel as indicated by the larger values of Q_∞ . The stress-strain curve of stainless steel deviates from linearity at lower

strains than carbon steel and maintains a higher post-yield modulus; this is reflected in higher values of the ratio C_{kin}/γ for stainless steel than for the two carbon steels. Although there is a degree of variability of some of the individual material parameters, it should be noted that the shape of a complete stress-strain hysteresis curves depends on the combination of the isotropic and kinematic components. A more representative assessment of the variability of the parameters is to consider the maximum change in size of a hysteresis loop, which is given by $2(\sigma_{|_0} + Q_{\infty}) + C_{kin} / \gamma$. These values are given in the last column of Table 4 where it can be seen that the coefficients of variance are relatively low. Experimental and FE simulated stress-strain hysteresis curves of cold-formed carbon steel and cold-formed stainless steel in the first 2 cycles at $\pm 1\%$ strain amplitude tests are shown in Figs 28 and 29, respectively, indicating that the cyclic hardening model is capable of capturing the key observed behavioural features.

4 Bending test results

The same materials that were examined in the axial arrangement (Section 3) have also been tested in the bending configuration, to enable higher strain amplitudes, achievable in the absence of instabilities, to be explored. In the bending specimens, the strain distribution is non-linear through the thickness, thus failure is likely to initiate at the specimen surface and develop inwards. The bending test data and surface strain-life relationships are shown in Fig. 30. The results display less scatter than those from the axial tests – this may reflect the benefits of the bending configuration which essentially avoids any issues of buckling or slippage. Similar to the results from the axial tests, the three materials follow a similar trend but the hot-rolled carbon steel generally displays a superior fatigue life for a given strain range.

As for the axial specimens, the fracture surfaces of each of the bending specimens have also been examined to categorise the failure into ductile, mixed and fatigue modes. Specimens that failed in less than 20 reversals generally showed a ductile failure mode, while those that failed after more than 300 reversals exhibited features of fatigue fracture. The progressive change from a ductile failure mode at $\pm 15\%$ strain amplitude to a mixed mode at $\pm 10\%$ and $\pm 6\%$ strain amplitudes and finally to a fatigue failure mode at $\pm 2\%$ strain amplitude is depicted in Fig. 39. The observed failure modes from the bending tests are similar to those from the axial tests, other than the region of final failure, which consistently appears towards the mid-thickness of the

specimens owing to the imposed strain distribution. Despite the different failure modes, the strain-life data from the bending tests is well represented by a simple power-law, indicated by the straight lines in the log-log plot of Fig. 30.

A drawback to the bending configuration is that the results cannot be directly used for obtaining the material parameters in Eq. (1) since the strain amplitude is not uniform through the thickness. For a given surface strain, the number of reversals to failure in a bending test is higher than in an axial test. A method to correlate results from these two testing configurations has been developed [16, 32] and is explored further herein.

Esin (1968) [32] related fatigue damage to plastic hysteresis energy and considered the probability of localised plastic flow in a nominally elastic material. On this basis, Esin (1980) [16] proposed a method to correlate fatigue data from axial, bending and rotating bending tests. The approach was shown to be applicable to elastic strain dominant high cycle fatigue. Esin postulated that material in some parts of a cross-section under smaller stress amplitudes, such as the material towards the mid-thickness of a bending specimen, would not contribute to the fatigue process. Esin's [16] approach, adapted to the strain-based scenario of the current study, is introduced below.

Assuming that fatigue fracture occurs when the plastic hysteresis energy reaches the true tensile toughness, which is measured by the area under the true stress-true strain curve, the number of cycles to failure N_f of a volume V_t of material with a true tensile toughness u_f in a cross-section of unit depth can be expressed as:

$$N_f = \frac{V_t u_f}{\Delta W} \quad (15)$$

where ΔW is the plastic hysteresis energy per cycle of the material. According to Esin's (1980) proposal, a small element in a cross-section has a certain probability to deviate from the theoretically calculated nominal values of stress and strain. Therefore, ΔW can be expressed as

$$\Delta W = \sum V_i u_i \quad (16)$$

where u_i is the plastic hysteresis energy of a small element loaded at a certain strain amplitude and V_i is the volume of that small element with strain amplitudes large enough to contribute to the fatigue process. Let $\Delta \epsilon_{hyp}/2$ be a hypothetical uniform strain amplitude across the section that

gives the same plastic hysteresis energy as using the actual strain amplitude at each point. This leads to:

$$\frac{\Delta \epsilon_{\text{hyp}}}{2} = \frac{\sum \frac{\Delta \epsilon_i}{2} A_i}{\sum A_i} \quad (17)$$

where A_i is the cross-sectional area of a small element at a constant strain amplitude $\Delta \epsilon_i/2$. Under uniform axial loading, the plastic strain energy dissipated by all elements is equal, and by utilising the concept of equivalent strain amplitude, the number of cycles to failure, from Eq. (15), becomes:

$$N_f = \frac{V_t u_f}{u_i \sum V_i} = \left(\frac{u_f}{u_i} \right) \left(\frac{V_t}{\sum V_i} \right) \quad (18)$$

It should be noted that in bending fatigue tests, $V_t \neq \sum V_i$ because elements at lower stress amplitudes do not contribute to the fatigue process; the ratio of $V_t/\sum V_i$ can be determined experimentally. In the current study, the strain amplitudes applied in the bending tests are much higher than those studied by Esin (1980) [16], and it has been assumed that the volume of material not involved in the fatigue process is negligible. Therefore, the ratio $V_t/\sum V_i$ has been taken as unity.

In order to apply the above approach, the total energy to fracture from the axial and bending tests has been calculated. The energy to fracture per unit volume of material in the axial tests was obtained by numerically integrating the areas under the stress-strain hysteresis loops, while in the bending tests the same was calculated by integrating the areas under the moment-curvature hysteresis loops divided by the width and thickness of the specimen. The moment was derived from the load measurements and curvature was calculated from the readings of strain gauges. The length of the specimen under the calculated curvature was defined such that the implied deformed shape was compatible with the actual applied displacement.

The total energy to fracture for the test specimens from the axial and bending configurations has been plotted for the three materials – hot-rolled carbon steel, cold-formed carbon steel and cold-formed stainless steel – in Figs 32-34. It may be seen that the trends of the test data are largely independent of the test configuration, indicating that consideration of total energy is a suitable means of predicting fracture, regardless of the applied strain distribution. Following Esin's

approach, and considering the rectangular cross-section shown in Fig. 35 subjected to a linearly varying bending strain distribution, the hypothetical uniform strain (Eq. (18)) becomes:

$$\frac{\Delta\epsilon_{hyp}}{2} = \frac{\int_{h-x}^h \frac{\Delta\epsilon_s}{2} \frac{y}{h} w dy}{wx} = \frac{\Delta\epsilon_s}{2} \left(1 - \frac{x}{2h}\right) \quad (19)$$

where the symbols are defined in Fig. 35. Assuming all material in the cross-section is subjected to fatigue (i.e. $x = h$), Eq. (19) can be simplified to:

$$\frac{\Delta\epsilon_{hyp}}{2} = 0.5 \left(\frac{\Delta\epsilon_s}{2}\right) \quad (20)$$

This implies that the bending surface-strain life relationship can be determined simply by multiplying the strain amplitudes of axial test data by a factor of two. On this basis, the (surface) strain-life test data from all tests have been plotted in Figures 36-38; the dashed lines represent the prediction of the bending test data (through Eq. (20)) from the axial test data – good agreement may be observed. This finding suggests that calibration of the parameters of the strain-life relationship (Eq. (1)) may be performed on the basis of data obtained from either axial or bending tests.

5 Conclusions

Cyclic material tests have been carried out to study the ELCF and LCF properties of material from hot-rolled carbon steel (S355J2H), cold-formed carbon steel (S235JRH) and cold-formed stainless steel (EN 1.4301 and 1.4307) structural sections. A total of 62 tests were carried out: 31 constant amplitude axial tests were conducted at strain amplitudes from $\pm 1\%$ to $\pm 7\%$, and additional 31 corresponding tests at surface strain amplitudes from $\pm 2\%$ to $\pm 15\%$ were performed in a bending configuration. The test results revealed that the three materials exhibit similar strain-life relationships despite significantly different elongations at fracture measured in monotonic tensile tests. The hot-rolled (annealed) material generally displayed marginally better fatigue performance than the cold-formed material, attributed to the increased level of damage and reduced ductility caused by cold-work. Stainless steel, which displays substantial strain hardening in monotonic tensile tests, also demonstrated a much larger degree of cyclic hardening than the two carbon steel materials. Material parameters for strain-life relationships and cyclic stress-strain curves for the three materials were obtained from the test results.

In order to model stress-strain hysteresis behaviour prior to the attainment of a stabilised state, the non-linear combined isotropic/kinematic hardening model in ABAQUS [30] has been calibrated on the basis of total least squares fits to the generated test data. Although there is some variability in the individual model parameters between different tests, the FE simulated hysteresis loops generated by the combined isotropic/kinematic hardening model (on the basis of average parameters for each material) are able to capture the key observed behavioural features.

Bending tests were carried out without the concerns of slippage and buckling that are possible in axially loaded tests; the scatter of the generated test data was lower in the bending configuration. A method to correlate axial and bending test results through an energy criterion has been established, demonstrating that the strain-life relationship of Eq. (1) can be derived from testing in either configuration.

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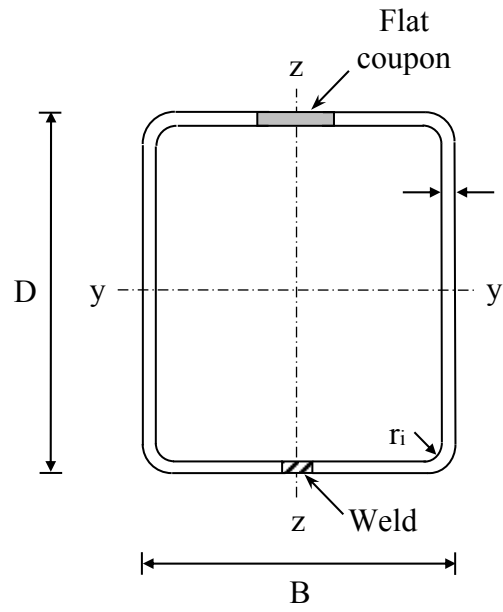


Fig. 1: Location of flat coupons from within structural hollow sections

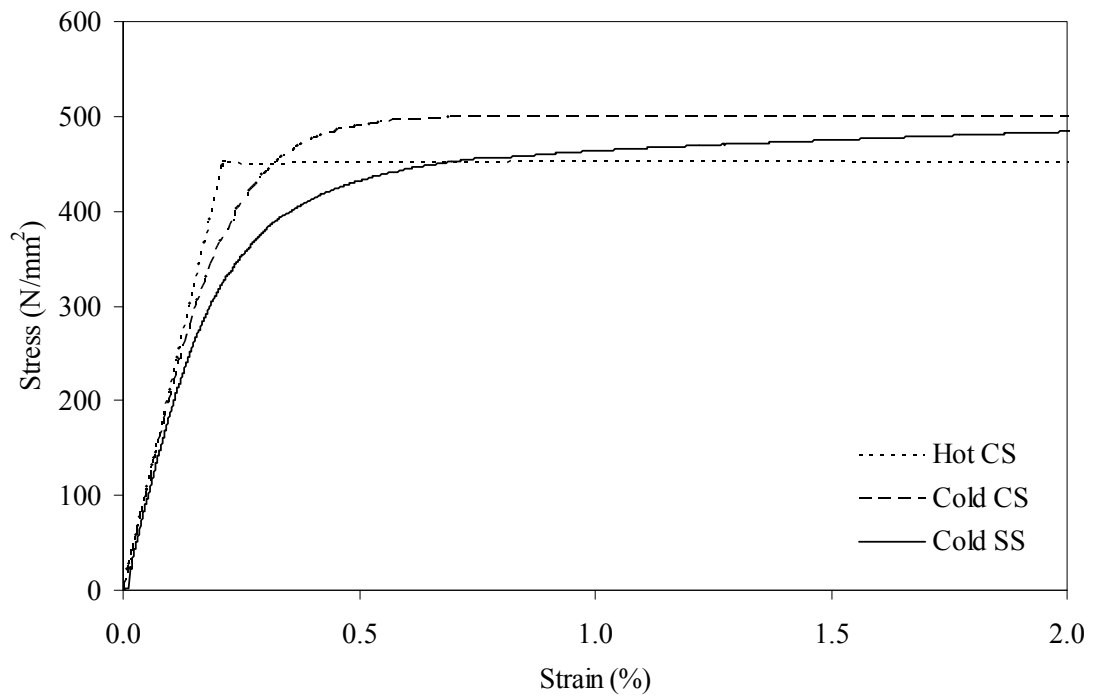


Fig. 2: Monotonic tensile stress-strain curves of hot-rolled carbon steel, cold-formed carbon steel and cold-formed stainless steel

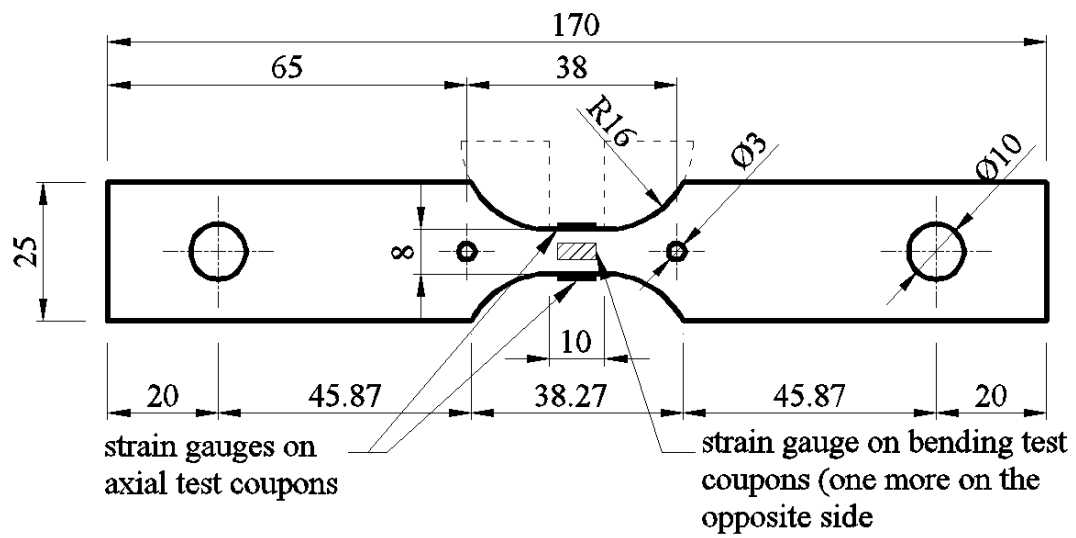


Fig. 3: Dimensions of cyclic material test coupons (in mm)



Fig. 4: Cyclic axial test setup

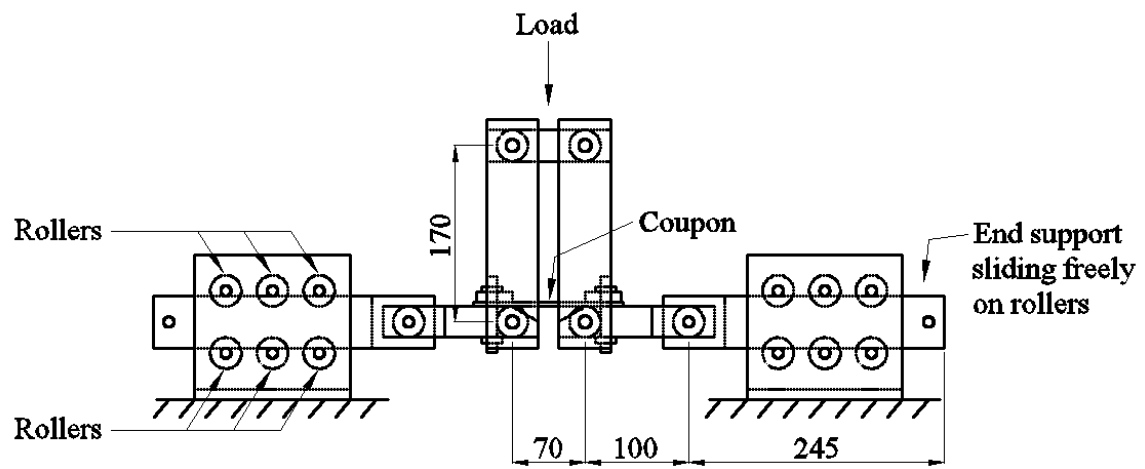


Fig. 5: Cyclic bending test setup

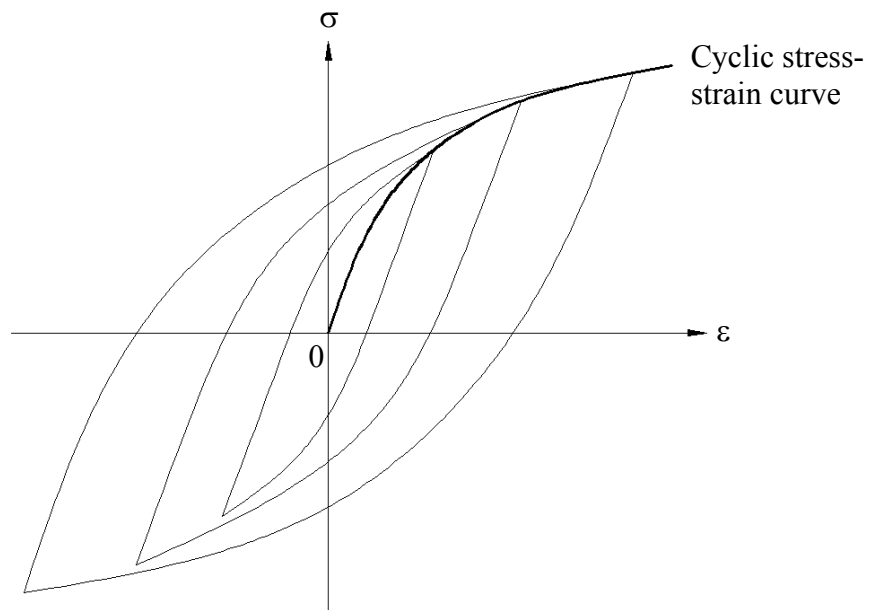


Fig. 6: Stabilised cyclic stress-strain hysteresis loops

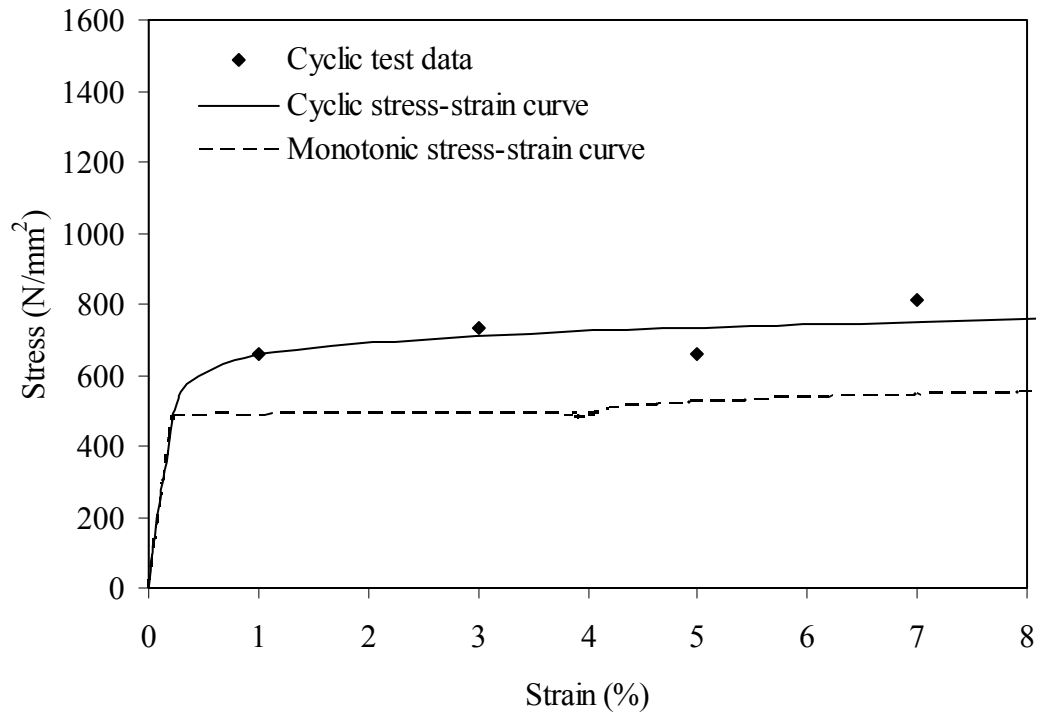


Fig. 7: Comparison of cyclic and monotonic stress-strain curves for hot-rolled carbon steel 40×40×3 specimens

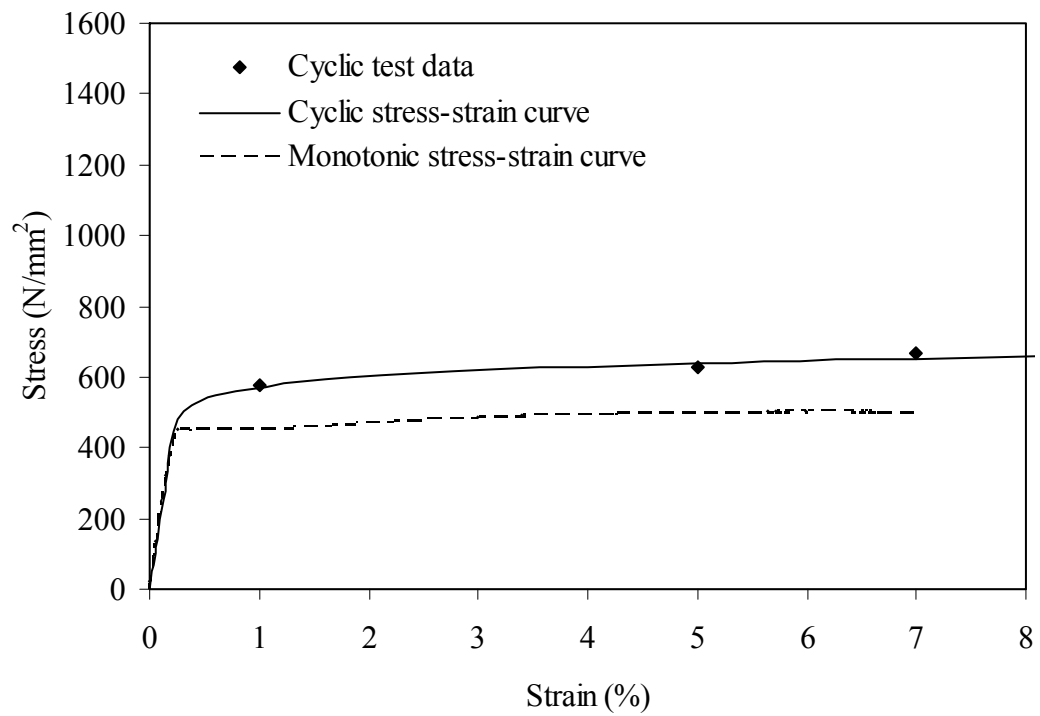


Fig. 8: Comparison of cyclic and monotonic stress-strain curves for cold-formed carbon steel 40×40×3 specimens

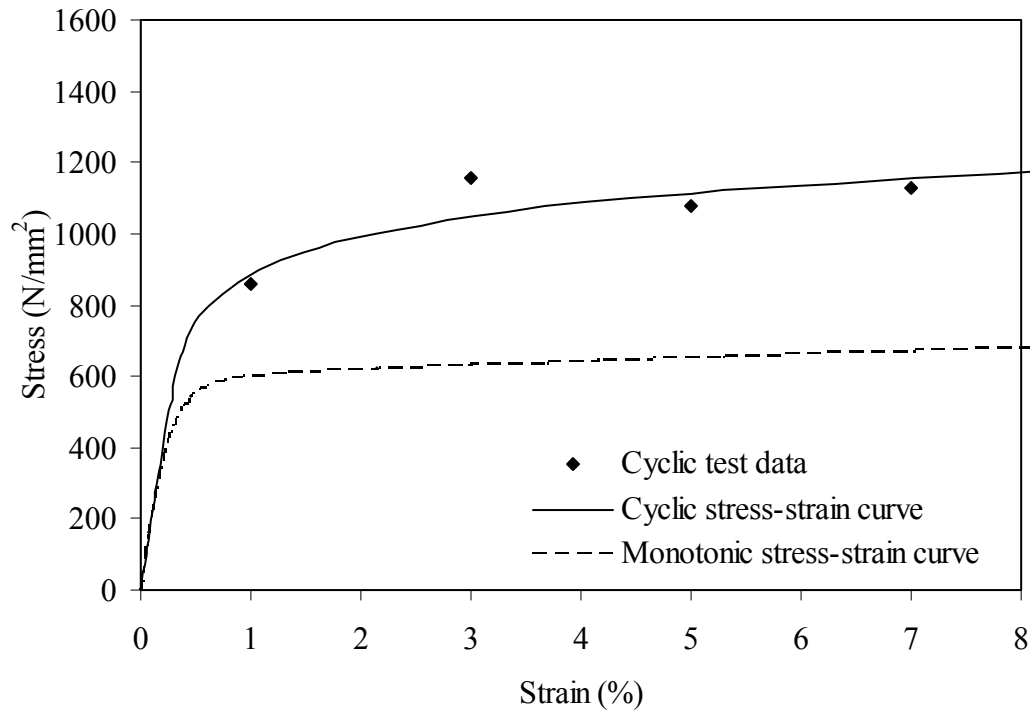


Fig. 9: Comparison of cyclic and monotonic stress-strain curves for cold-formed stainless steel 50×50×3 specimens

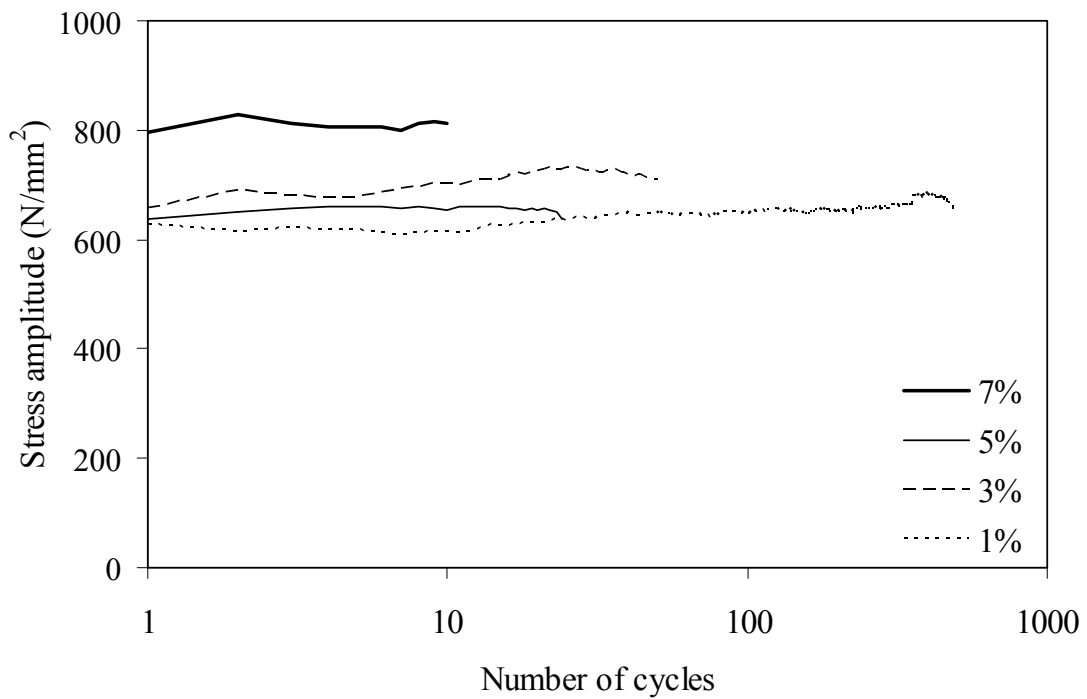


Fig. 10: Variation of stress amplitudes during tests at different strain amplitudes of hot-rolled carbon steel 40×40×3 specimens

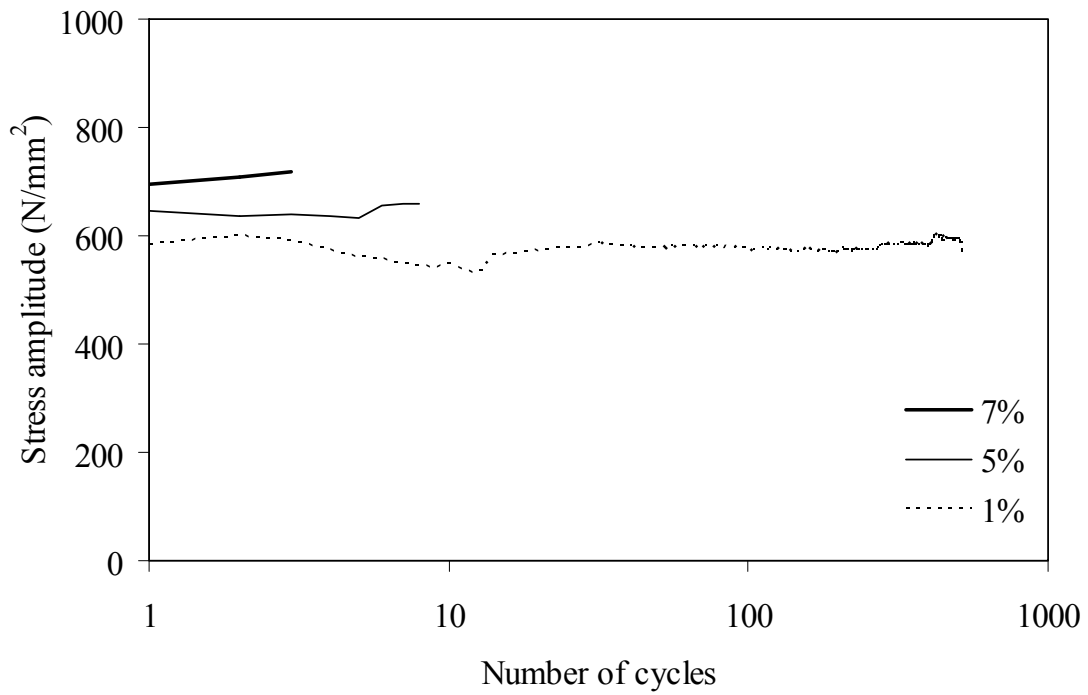


Fig.11: Variation of stress amplitudes during tests at different strain amplitudes of cold-formed carbon steel 40×40×3 specimens

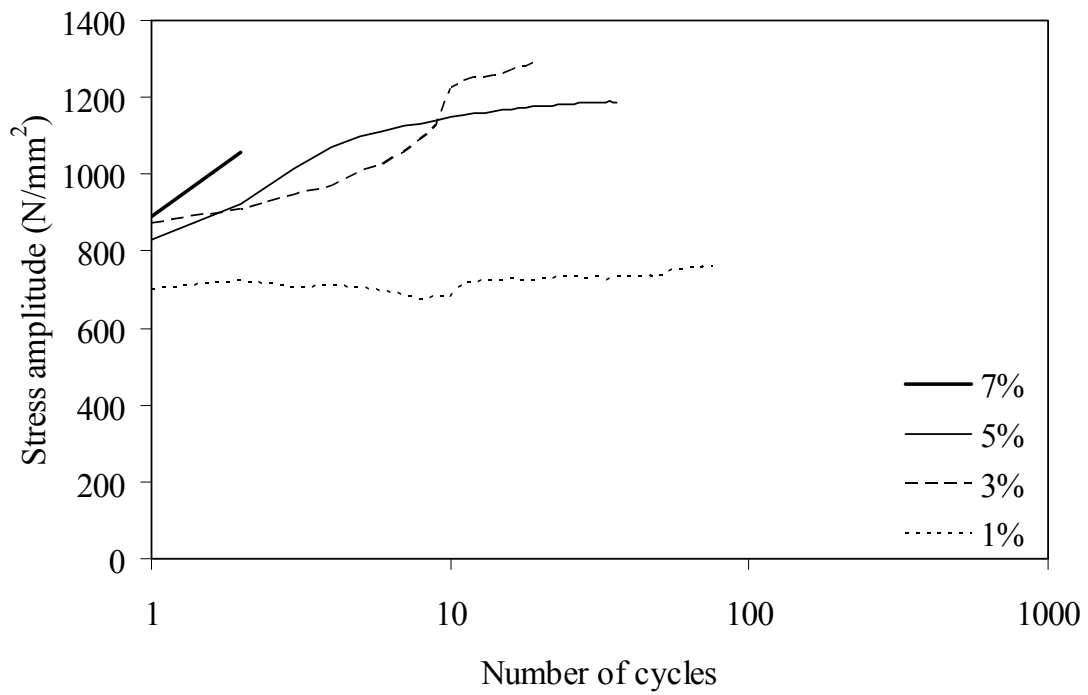


Fig.12: Variation of stress amplitudes during tests at different strain amplitudes of cold-formed stainless steel 50×50×3 specimens

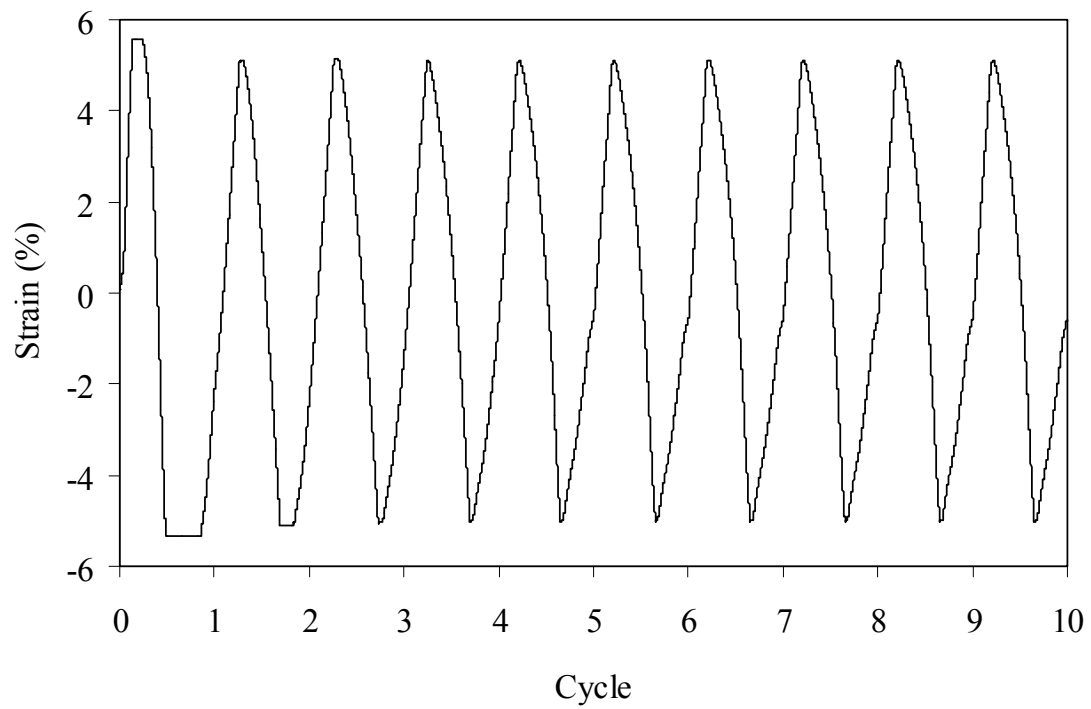


Fig. 13: Measurement of strain during the first few cycles of a typical test at $\pm 5\%$ amplitude (60 $\times$ 40 $\times$ 3-SS-CF-5% (a))

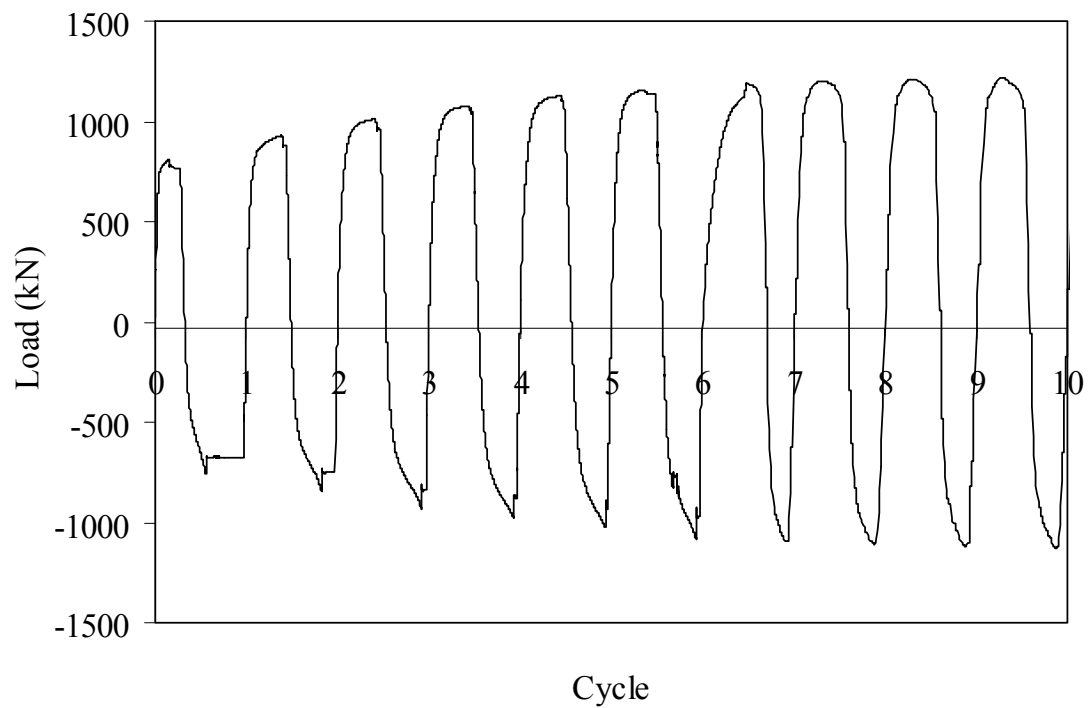


Fig. 14: Measurement of load during a typical test at $\pm 5\%$ amplitude (60 $\times$ 40 $\times$ 3-SS-CF-5% (a))

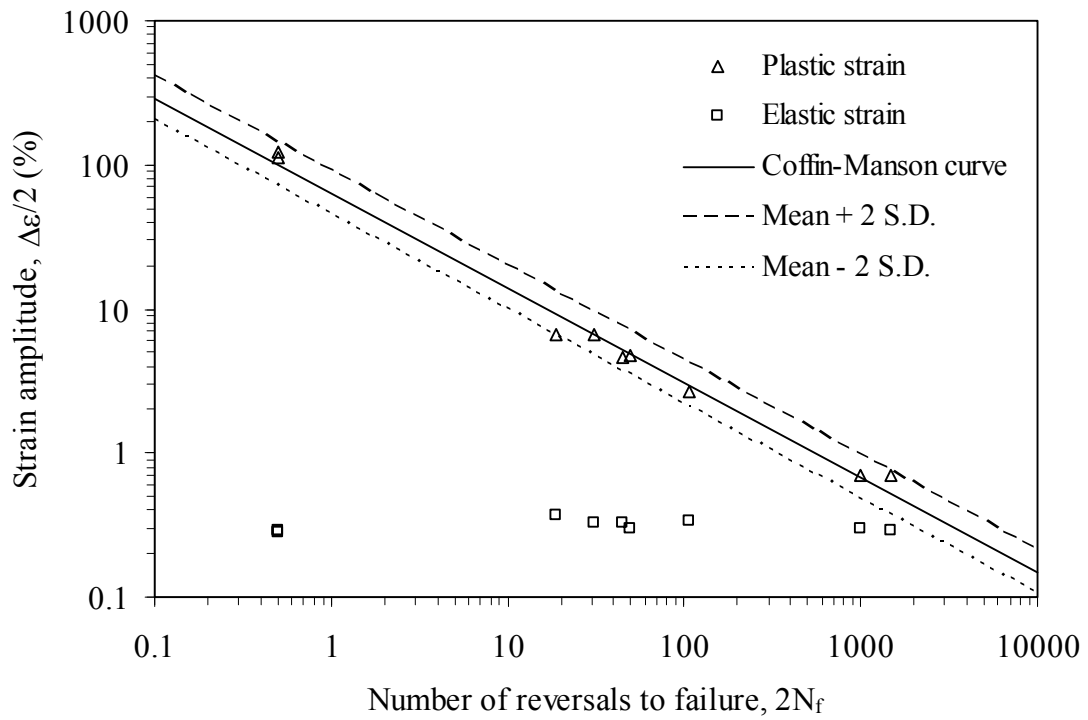


Fig. 15: Strain-life relationship in ELCF and LCF regimes for hot-rolled carbon steel specimens

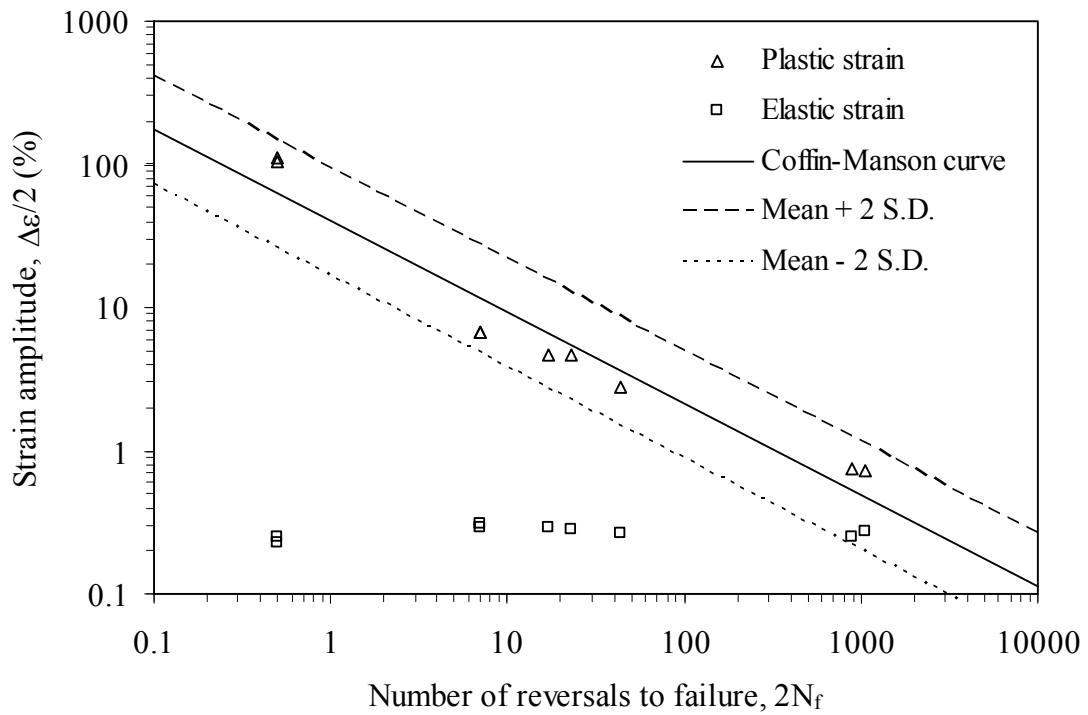


Fig. 16: Strain-life relationship in ELCF and LCF regimes for cold-formed carbon steel specimens

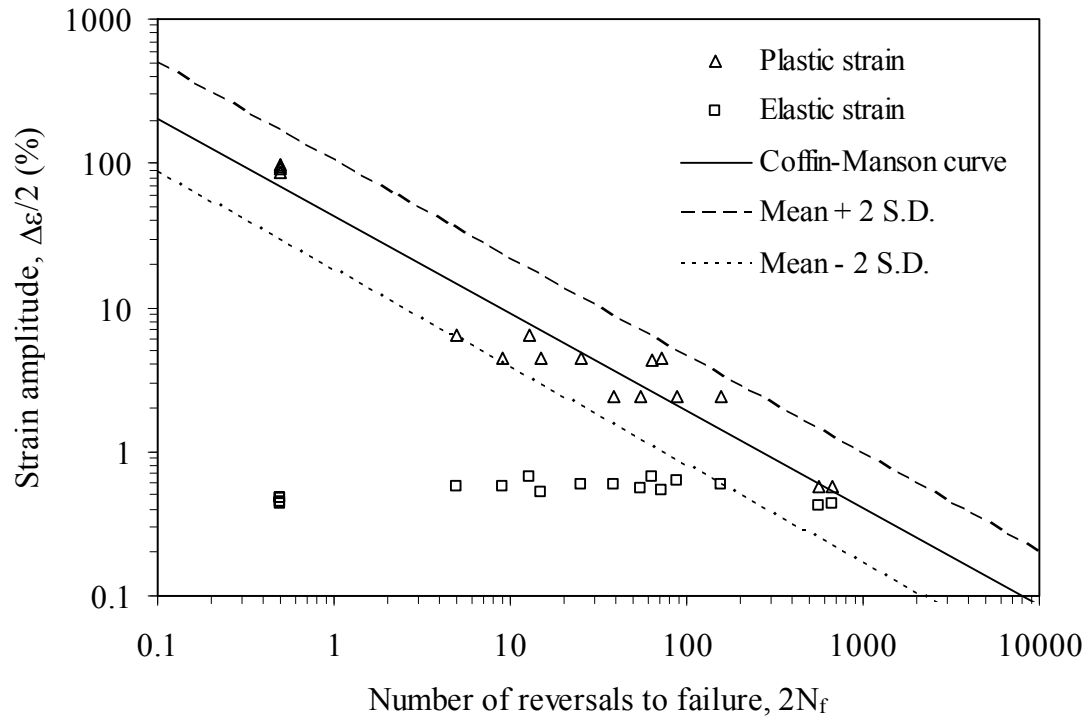


Fig. 17: Strain-life relationship in ELCF and LCF regimes for cold-formed stainless steel specimens

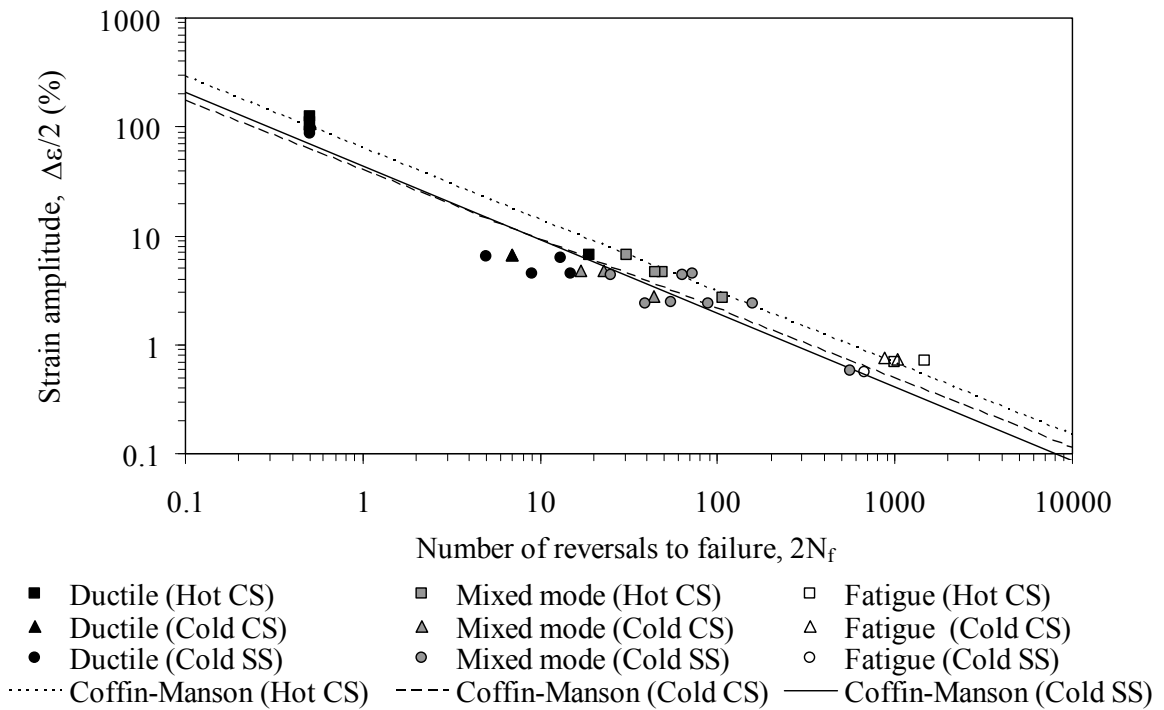


Fig 18: Comparison of the strain-life relationships for hot-rolled carbon steel, cold-formed carbon steel and cold-formed stainless steel, and their failure modes

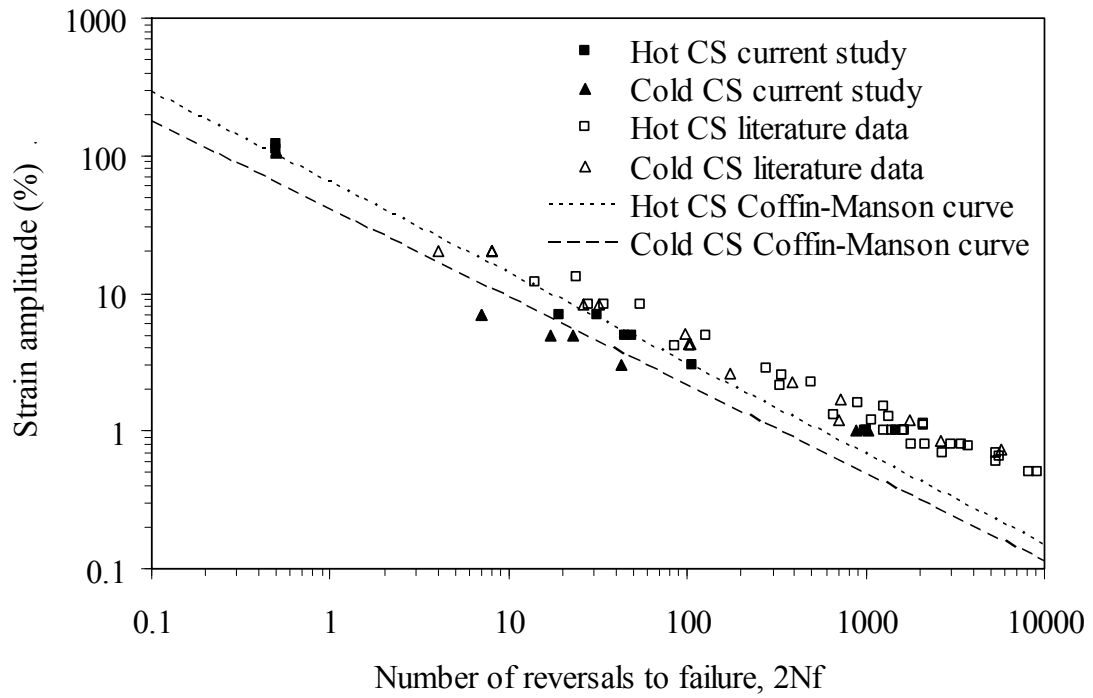


Fig. 19: Strain-life relationships of low and high cycle fatigue data, and monotonic tensile data, of carbon steel (CS) material from tests performed in the current study and in the literature

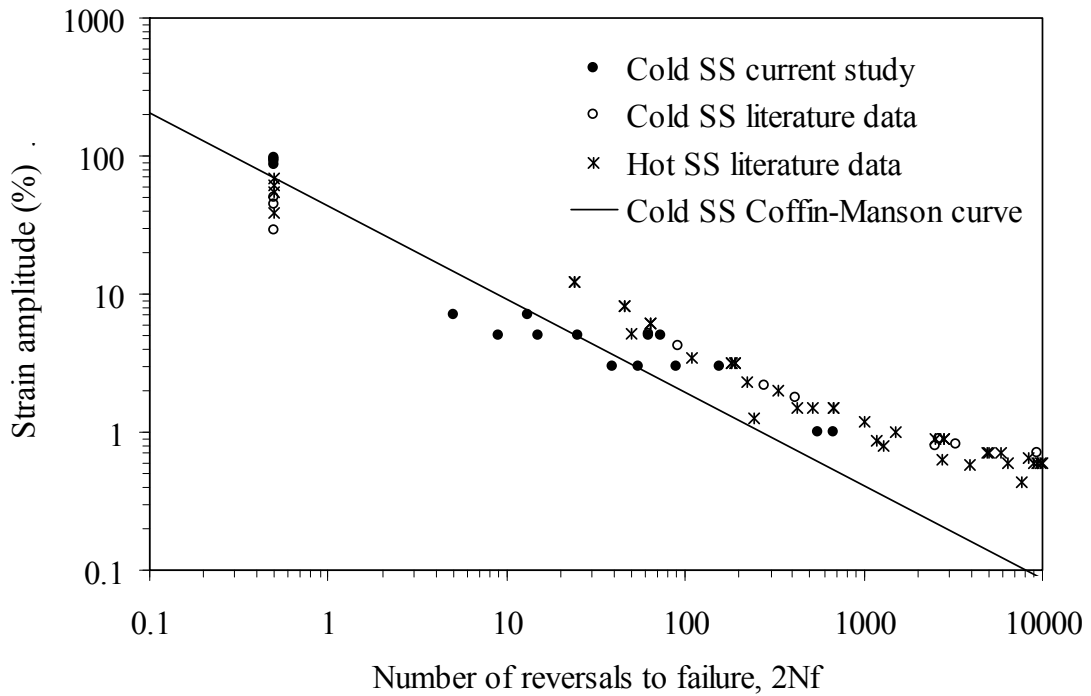


Fig. 20: Strain-life relationships of low and high cycle fatigue data, and monotonic tensile data, of stainless steel (SS) material from tests performed in the current study and in the literature

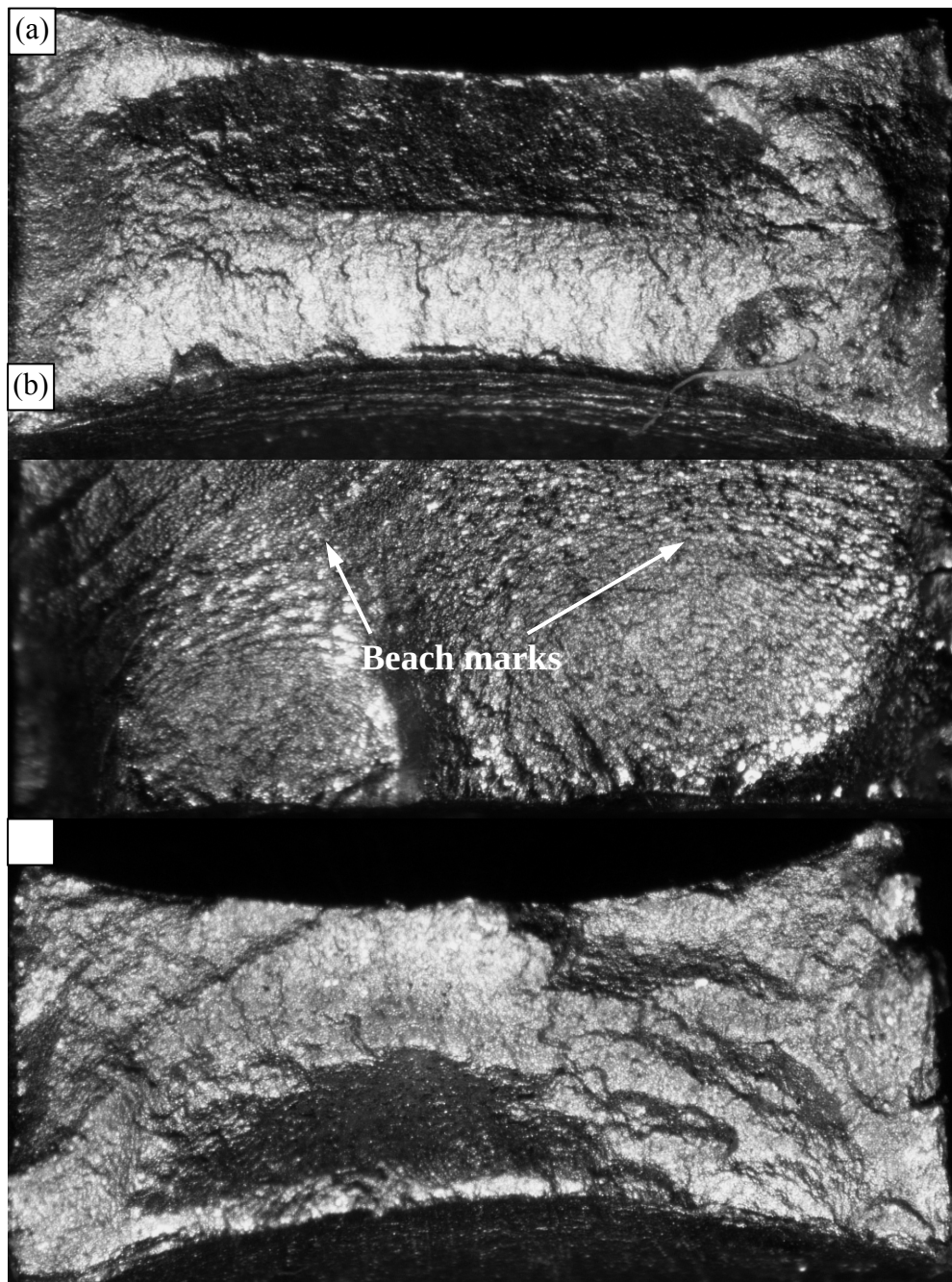


Fig. 21: (a) Ductile, (b) fatigue, and (c) mixed mode fracture surface shown on axially loaded specimens tested at $\pm 7\%$, $\pm 1\%$ and $\pm 3\%$ strain amplitudes respectively.

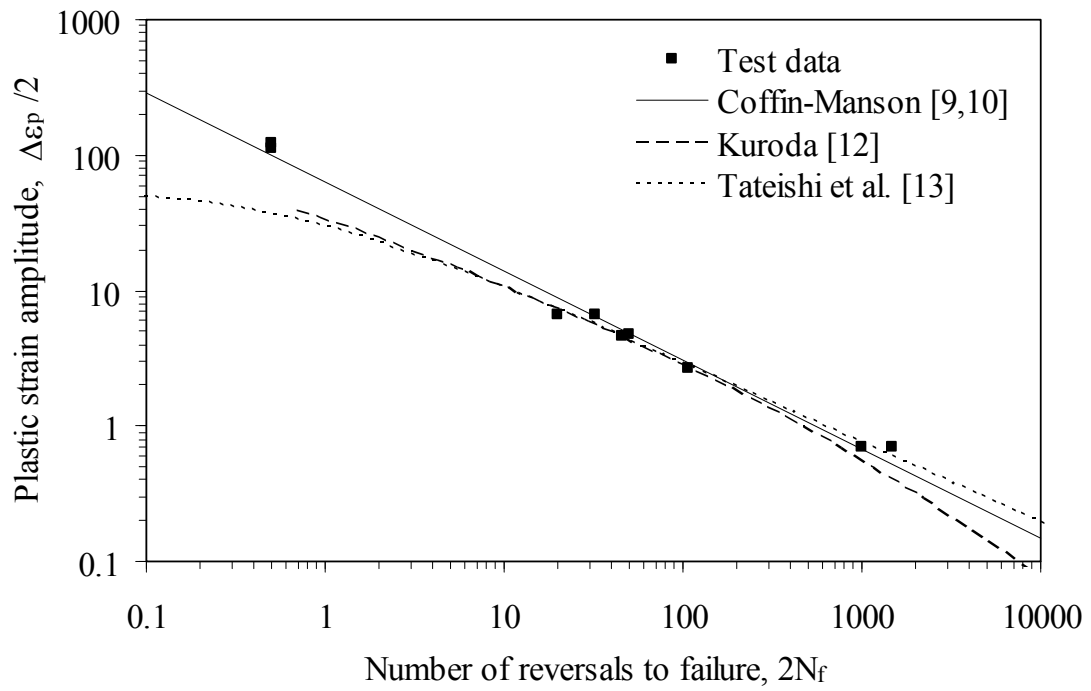


Fig. 22: Comparison between strain-life models and hot-rolled carbon steel data

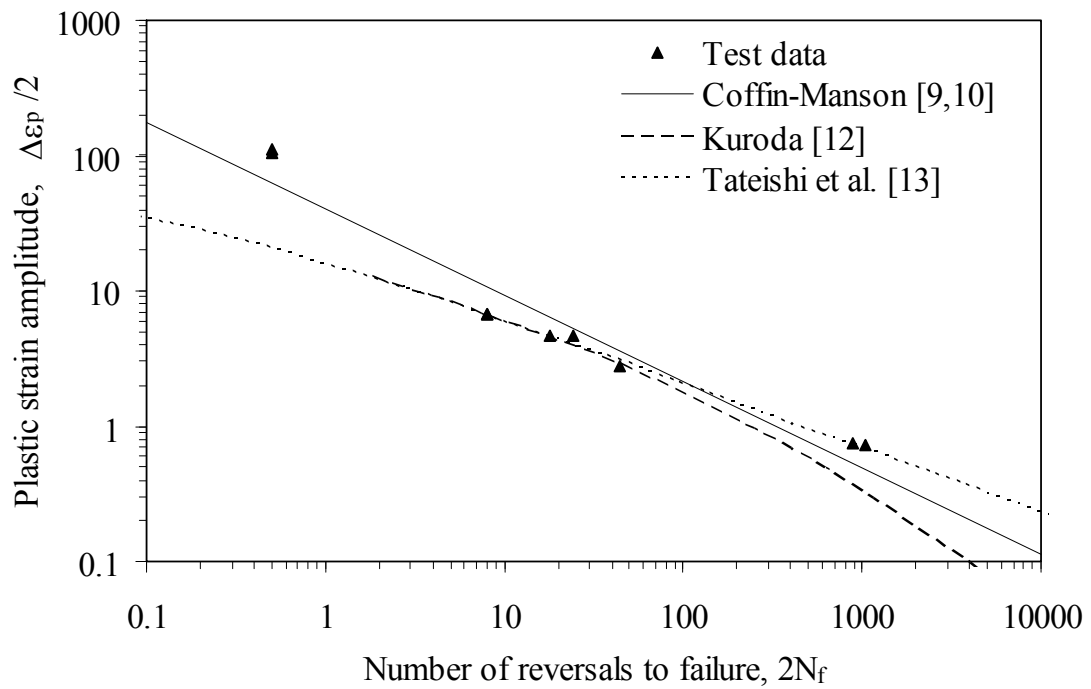


Fig. 23: Comparison between strain-life models and cold-formed carbon steel data

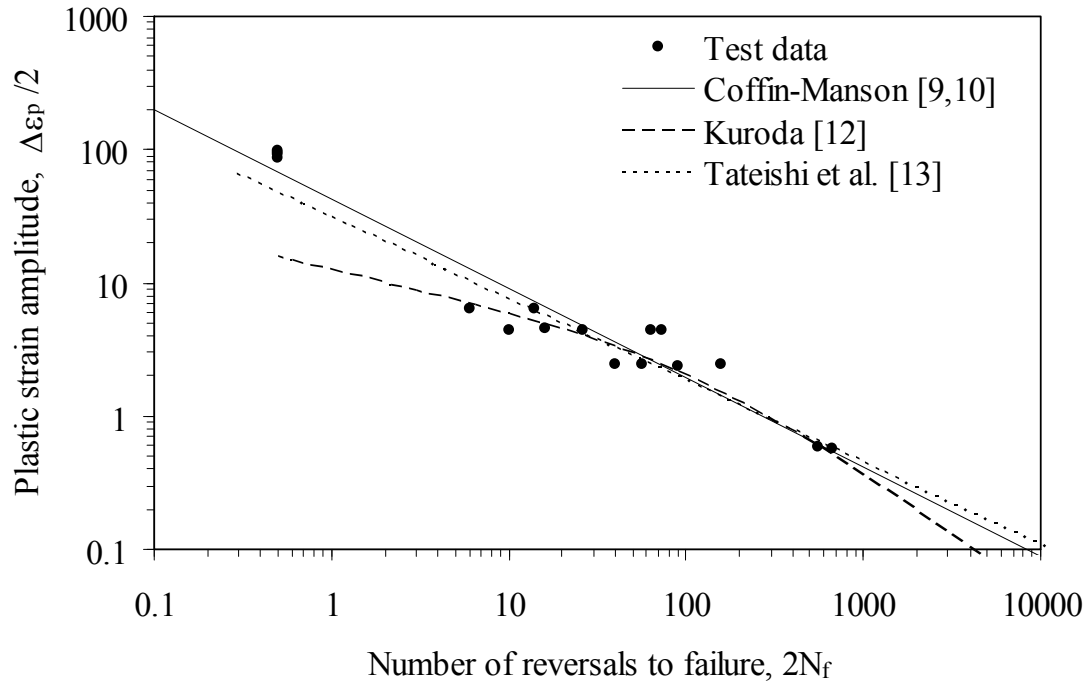


Fig. 24: Comparison between strain-life models and cold-formed stainless steel data

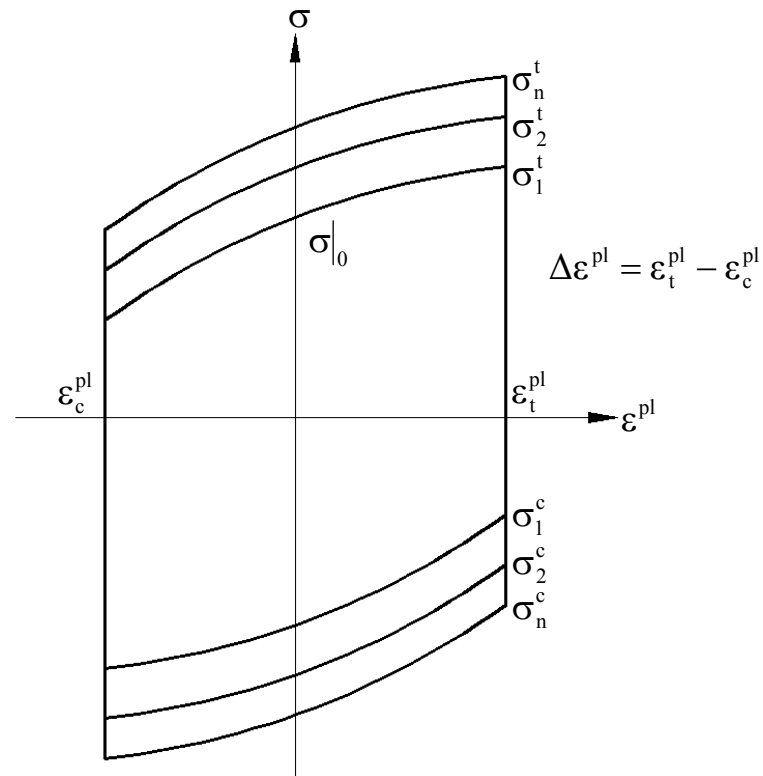


Fig. 25: Calibration of isotropic hardening component

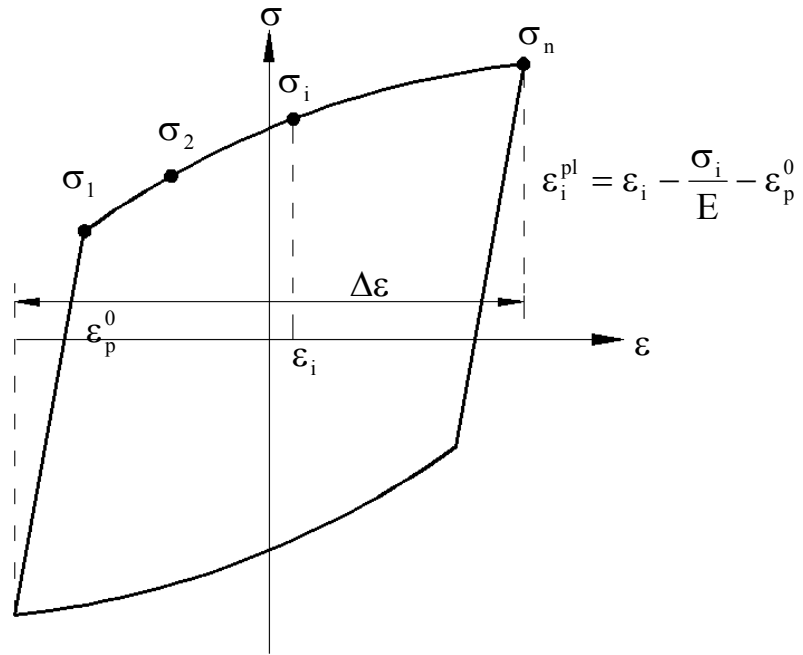


Fig. 26: Calibration of kinematic hardening component

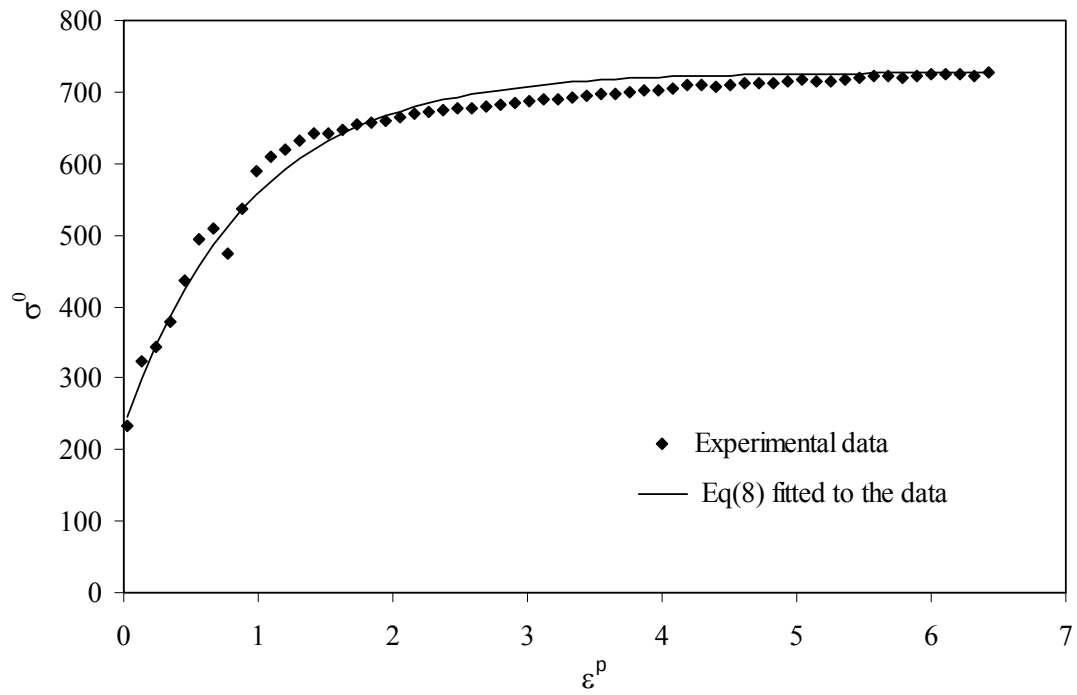


Fig. 27: Best fit curve of Eq. (8) to experimental data

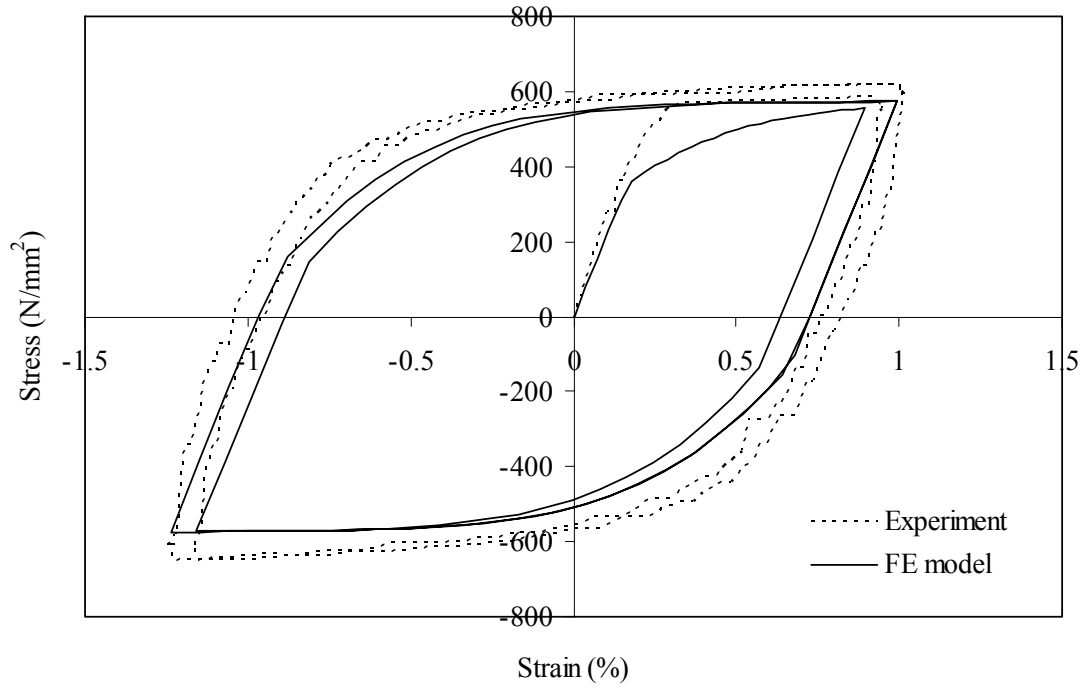


Fig. 28 Experimental and FE simulated stress-strain hysteresis curves of a cold-formed carbon steel specimen in the first 2 cycles of $\pm 1\%$ strain amplitude tests

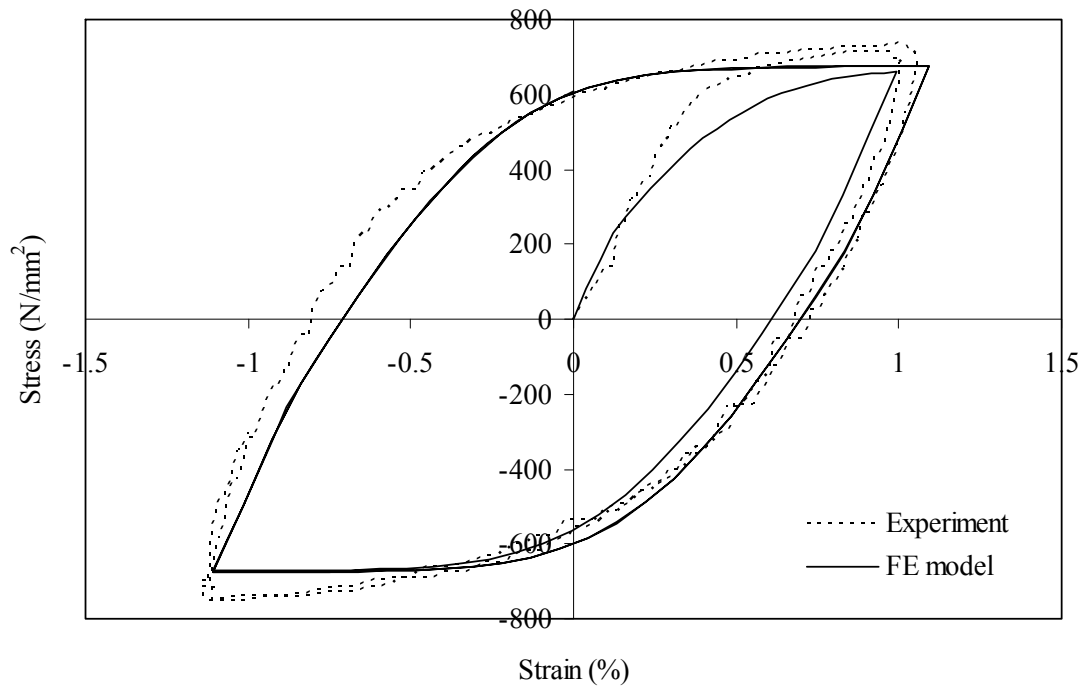


Fig. 29: Experimental and FE simulated stress strain hysteresis curves of a cold-formed stainless steel specimen in the first 2 cycles of $\pm 1\%$ strain amplitude tests

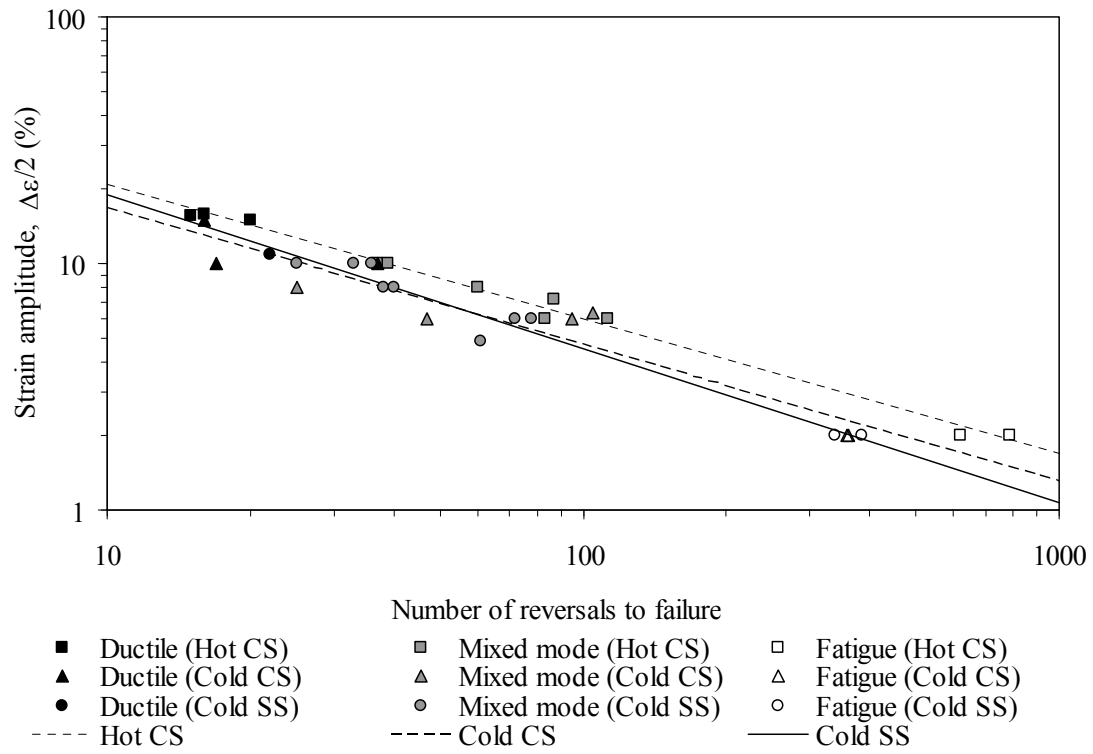


Fig. 30: Strain-life relationships and fracture modes of bending test specimens

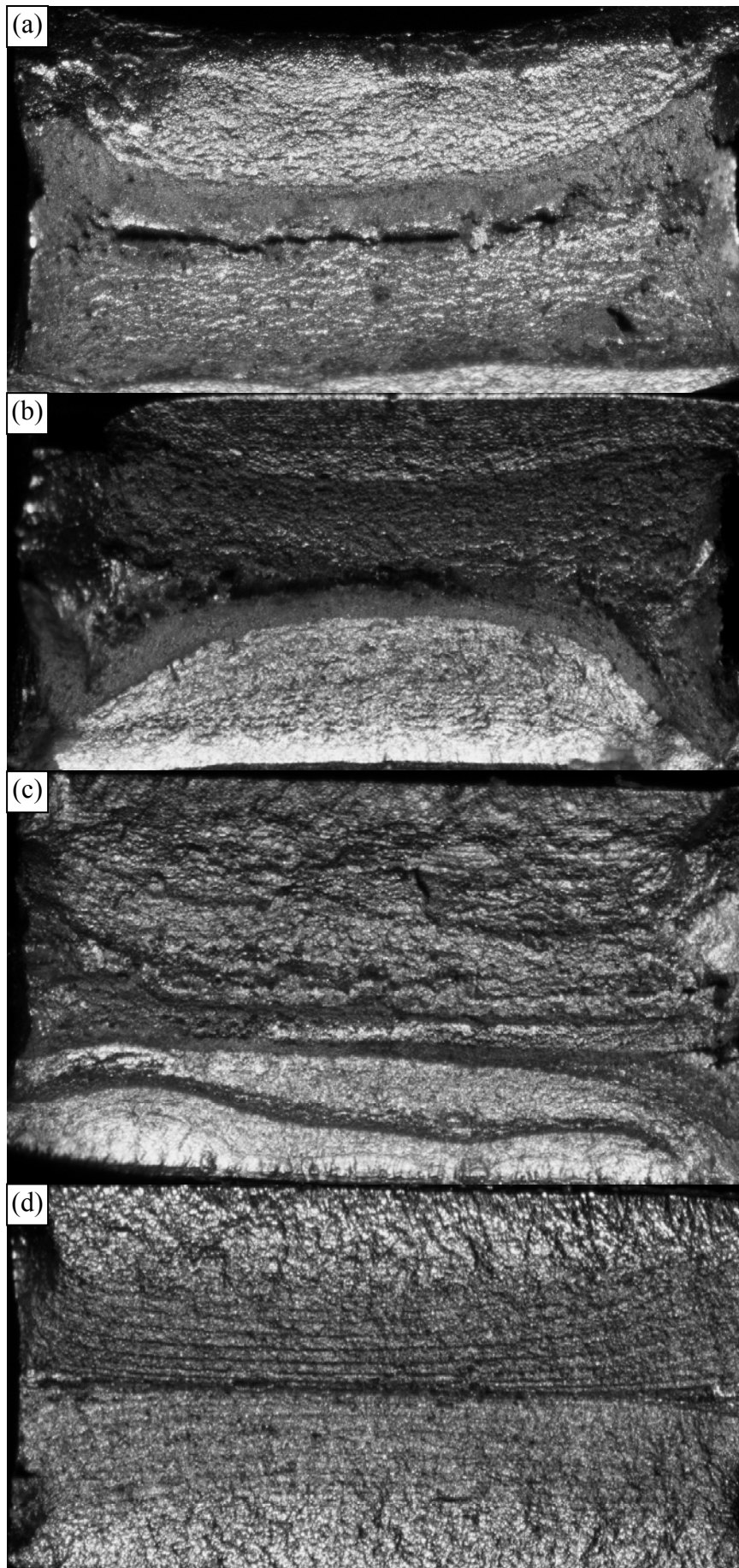


Fig. 31: Fracture surfaces of bending specimens tested at (a) $\pm 15\%$, (b) $\pm 10\%$, (c) $\pm 6\%$ and (d) $\pm 2\%$ respectively

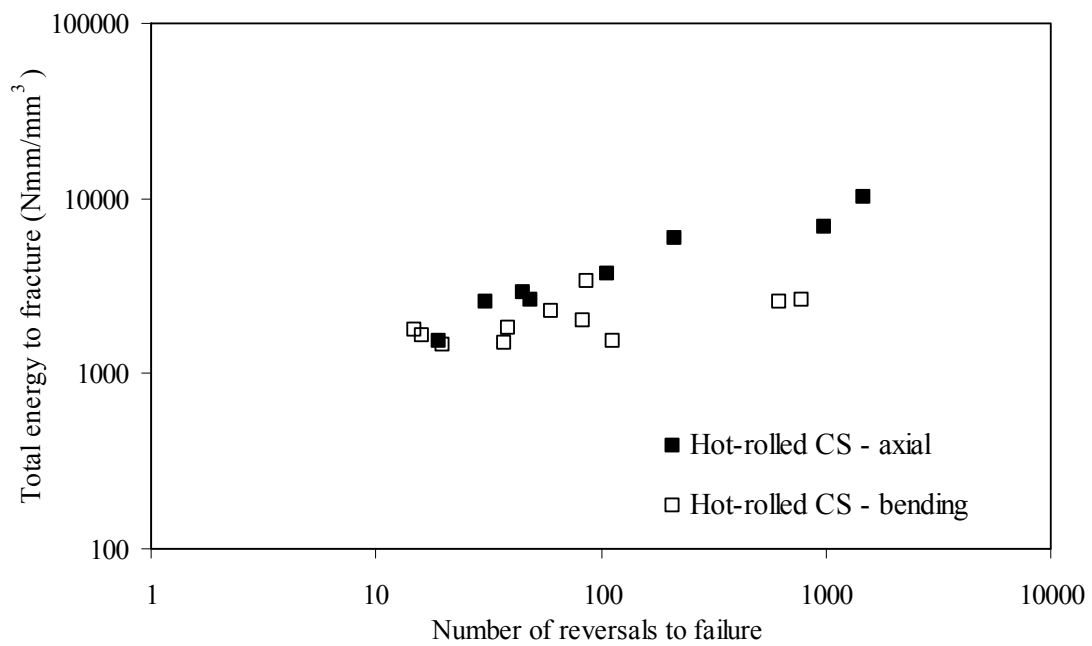


Fig. 32: Relationship between total energy to fracture and number of reversals to failure of hot-rolled carbon steel specimens

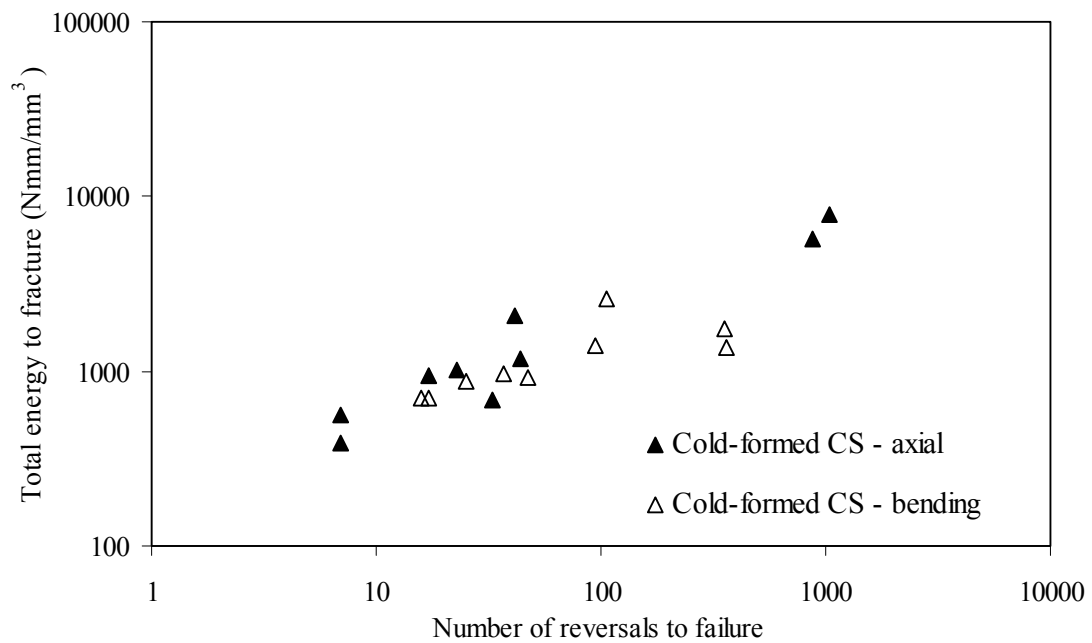


Fig. 33: Relationship between total energy to fracture and number of reversals to failure of cold-formed carbon steel specimens

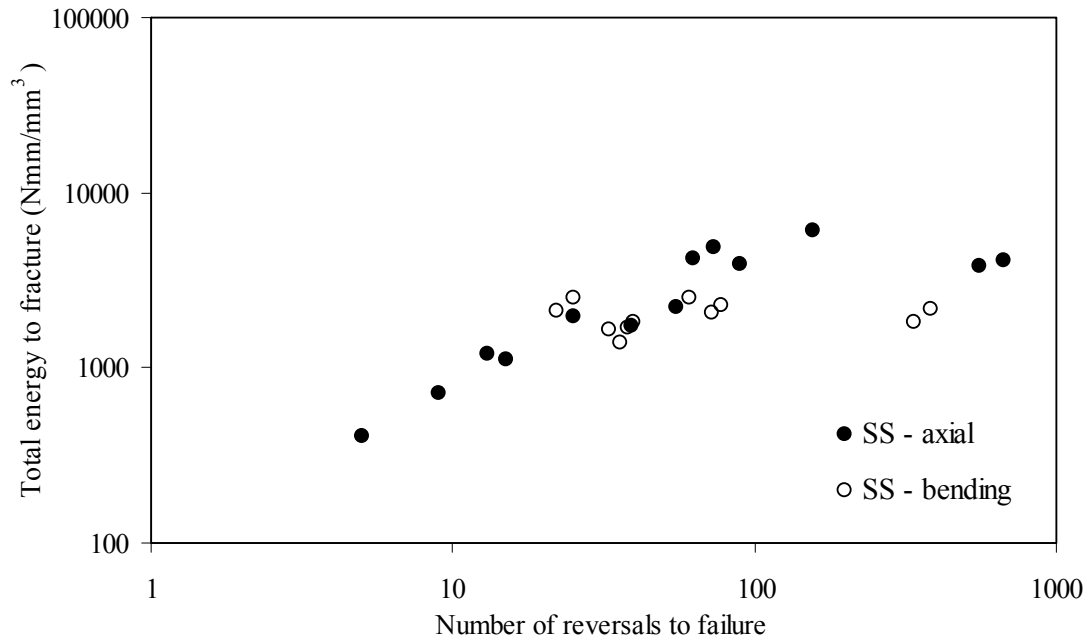


Fig. 34: Relationship between total energy to fracture and number of reversals to failure of cold-formed stainless steel specimens

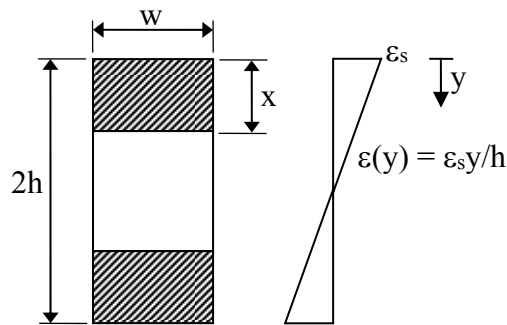


Fig. 35: Distribution of strain through a rectangular cross-section subjected to bending

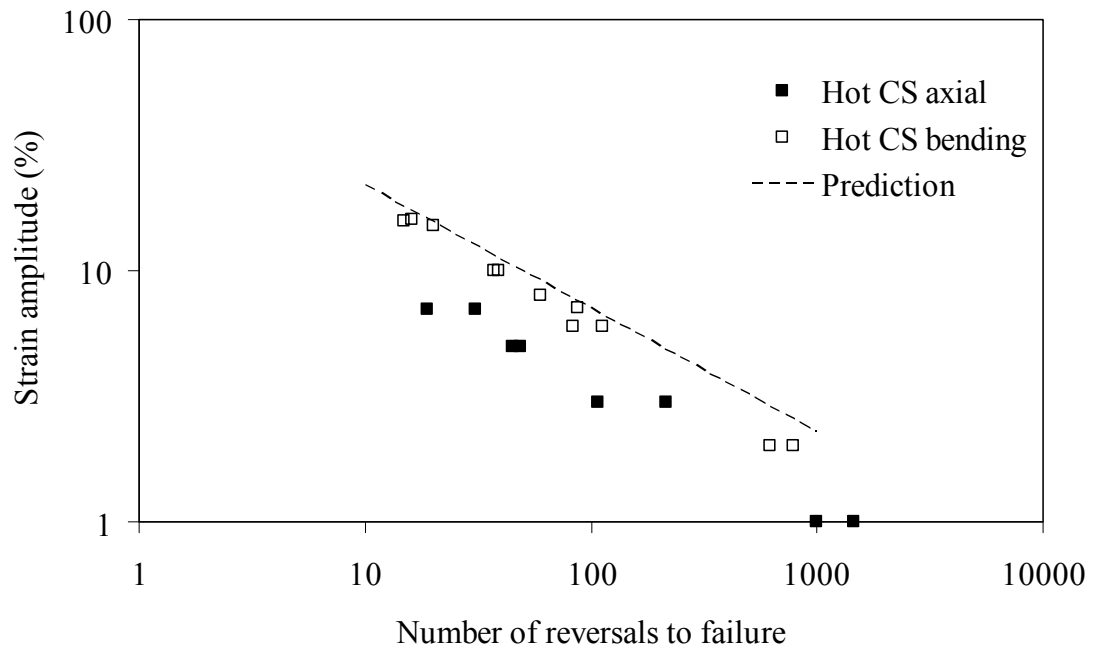


Fig. 36: Prediction of strain-life relationship of bending tests from axial test data of hot-rolled carbon steel specimens

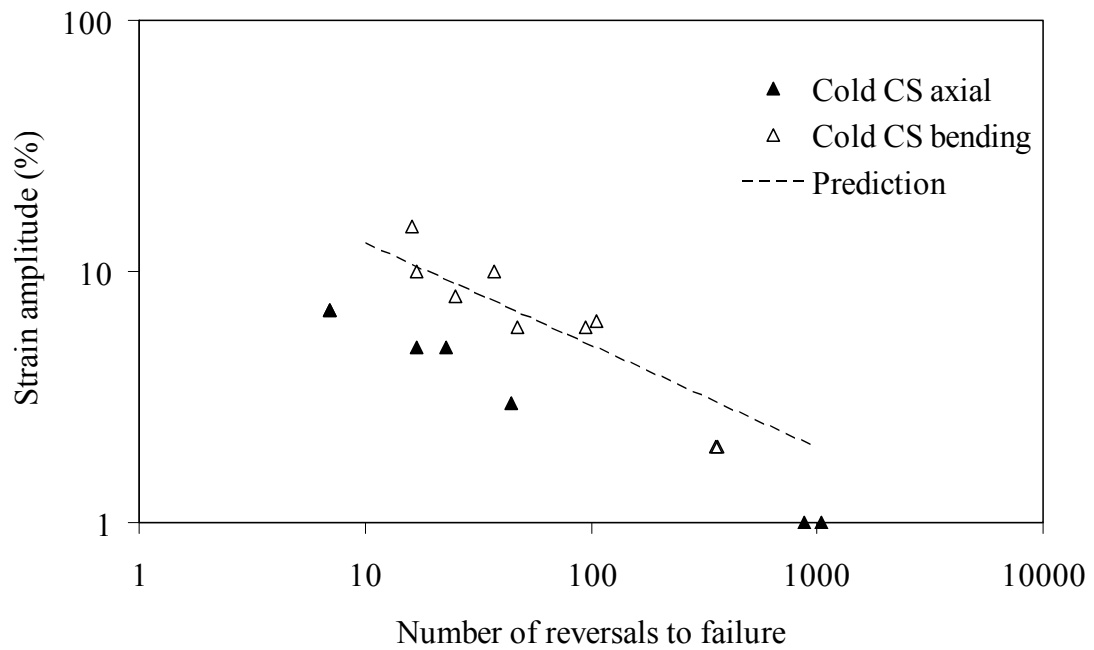


Fig. 37: Prediction of strain-life relationship of bending tests from axial test data of cold-formed carbon steel specimens

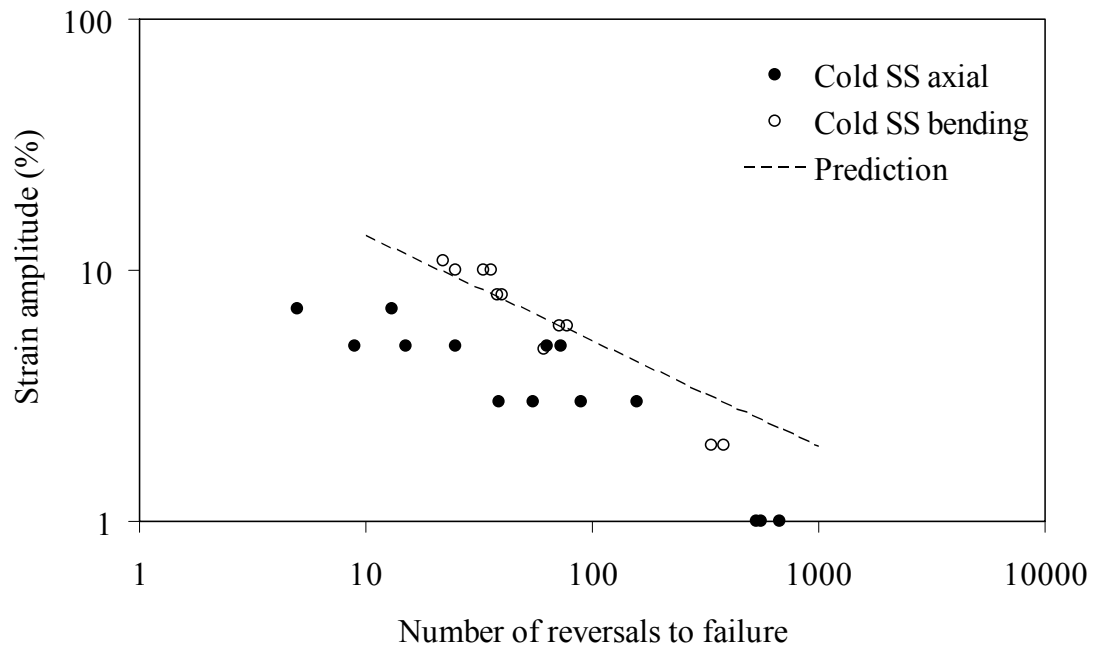


Fig. 38: Prediction of strain-life relationship of bending tests from axial test data of cold-formed stainless steel specimens

Table 1. Monotonic tensile material properties

Specimen	Young's modulus, E (N/mm ²)	Yield strength / 0.2% proof stress, f_y (N/mm ²)	Ultimate tensile strength, f_u (N/mm ²)	Percentage elongation at fracture, ϵ_L (%)	Tensile ductility, ϵ_f (%)
40×40×4-CS-HR-T	212310	443	531	41	123
40×40×3-CS-HR-T	219610	478	555	31	111
40×40×4-CS-CF-T	201640	410	430	38	110
40×40×3-CS-CF-T	212910	451	502	24	106
60×40×4-CS-CF-T	211950	400	452	21	65
100×100×3-SS-CF-T	201300	419	725	66	96
60×60×3-SS-CF-T	197730	483	745	58	88
50×50×3-SS-CF-T	197960	552	798	59	97
60×40×3-SS-CF-T	191690	538	753	58	92

Table 2. Parameters of Coffin-Manson law and cyclic stress-strain curve

Specimen	K'	n'	ϵ'_f	c
40×40×3-CS-HR	890	0.0622	0.6115	-0.6658
40×40×4-CS-HR	807	0.0564	0.6823	-0.6496
All hot CS	847	0.0591	0.6406	-0.6581
40×40×4-CS-CF	702	0.0688	0.4143	-0.6532
40×40×3-CS-CF	764	0.0580	0.3937	-0.525
All cold CS	726	0.0634	0.4027	-0.6392
60×60×3-SS-CF	2348	0.1975	0.5343	-0.6742
50×50×3-SS-CF	1562	0.1081	0.3875	-0.6534
All cold SS	1768	0.1333	0.4352	-0.6766

Table 3. Results of cyclic axial material tests.

Specimen	Total strain amplitude, $\Delta\epsilon/2$ (%)	Elastic strain amplitude, $\Delta\epsilon_e/2$ (%)	Plastic strain amplitude, $\Delta\epsilon_p/2$ (%)	Stress amplitude at half-life, $\Delta\sigma/2$ (N/mm ²)	Number of reversals to failure, $2N_f$
40×40×4-CS-HR-1%	1	0.3	0.7	621	1465
40×40×4-CS-HR-3% *	3	0.3	2.7	623	215
40×40×4-CS-HR-5%	5	0.3	4.7	705	45
40×40×4-CS-HR-7%	7	0.3	6.7	695	31
40×40×3-CS-HR-1%	1	0.3	0.7	657	991
40×40×3-CS-HR-3%	3	0.3	2.7	732	107
40×40×3-CS-HR-5%	5	0.3	4.7	660	49
40×40×3-CS-HR-7%	7	0.4	6.6	810	19
40×40×4-CS-CF-1%	1	0.3	0.7	508	879
40×40×4-CS-CF-3%	3	0.3	2.7	531	43
40×40×4-CS-CF-5%	5	0.3	4.7	572	23
40×40×4-CS-CF-7%	7	0.3	6.7	592	7
40×40×3-CS-CF-1%	1	0.3	0.7	576	1043
40×40×3-CS-CF-3% *	3	0.2	2.8	333	33
40×40×3-CS-CF-5%	5	0.3	4.7	627	17
40×40×3-CS-CF-7%	7	0.3	6.7	664	7
60×40×4-CS-CF-5% *	5	0.5	4.5	1160	41
100×100×3-SS-CF-1% *	1	0.2	0.8	402	533
100×100×3-SS-CF-3%	3	0.6	2.4	1251	89
100×100×3-SS-CF-5%	5	0.5	4.5	1062	15
60×60×3-SS-CF-1%	1	0.4	0.6	835	557
60×60×3-SS-CF-3%	3	0.6	2.4	1152	157
60×60×3-SS-CF-5%	5	0.7	4.3	1315	63
60×60×3-SS-CF-7%	7	0.7	6.3	1299	13
50×50×3-SS-CF-1%	1	0.4	0.6	861	671
50×50×3-SS-CF-3%	3	0.6	2.4	1156	39
50×50×3-SS-CF-5%	5	0.5	4.5	1076	73
50×50×3-SS-CF-7%	7	0.6	6.4	1126	5
60×40×3-SS-CF-3%	3	0.6	2.4	1067	55
60×40×3-SS-CF-5% (a)	5	0.6	4.4	1127	25
60×40×3-SS-CF-5% (b)	5	0.6	4.4	1083	9

* Data point rejected due to abnormal load values.

Table 4. Parameters of non-linear combined isotropic/kinematic hardening model of cyclic plasticity

Specimen	Total strain amplitude, $\Delta\varepsilon/2$ (%)	$\sigma _0$ (N/mm ²)	Q_∞ (N/mm ²)	b_{iso}	C_{kin} (N/mm ²)	γ	C_{kin}/γ (N/mm ²)	$2(\sigma _0 + Q_\infty) + C_{kin}/\gamma$
40×40×4-CS-HR-1%	1	448	33	0.17	29700	229	130	1092
40×40×3-CS-HR-1%	1	478	87	1.50	30200	188	161	1290
40×40×3-CS-HR-3%	3	478	97	1.53	14300	120	119	1269
40×40×4-CS-HR-5%	5	448	30	6.63	11600	68	171	1126
40×40×3-CS-HR-5%	5	478	51	3.21	4278	40	107	1166
40×40×4-CS-HR-7%	7	448	46	3.46	16800	129	130	1119
40×40×3-CS-HR-7%	7	478	38	0.18	58000	202	287	1319
Mean CS-HR		465	55	2.38	23554	139	169	1197
COV CS-HR		0.03	0.45	0.89	0.70	0.47	0.36	0.07
40×40×4-CS-CF-1%	1	229	5.08	0.131	221000	798	277	745
40×40×3-CS-CF-1%	1	353	32.9	0.133	87100	394	221	993
40×40×4-CS-CF-3%	3	229	34.8	0.900	127000	413	308	835
40×40×4-CS-CF-5%	5	229	43.4	0.707	202000	635	318	863
40×40×3-CS-CF-5%	5	353	37.7	0.289	47000	150	313	1095
40×40×4-CS-CF-7%	7	229	58	4.23	90200	318	284	858
40×40×3-CS-CF-7%	7	353	35.9	1.47	82500	303	272	1050
Mean CS-CF		282	35	1.12	122400	430	285	920
COV CS-CF		0.22	0.41	1.20	0.49	0.47	0.11	0.13
60×60×3-SS-CF-1%	1	232	544	0.155	96900	283	342	1894
50×50×3-SS-CF-1%	1	217	319	0.170	254000	558	455	1527
100×100×3-SS-CF-3%	3	215	648	1.16	59200	156	379	2105
60×60×3-SS-CF-3%	3	232	495	1.08	73900	181	408	1862
50×50×3-SS-CF-3%	3	217	529	1.21	127000	228	557	2049
60×40×3-SS-CF-3%	3	283	395	1.21	70600	177	399	1755
100×100×3-SS-CF-5%	5	215	422	2.33	69700	146	477	1751
60×60×3-SS-CF-5%	5	232	550	1.92	121000	236	513	2077
50×50×3-SS-CF-5%	5	217	410	1.50	95000	187	508	1762
60×40×3-SS-CF-5% (a)	5	283	428	1.51	66000	157	420	1842
60×40×3-SS-CF-5% (b)	5	283	282	3.56	90900	172	528	1658
60×60×3-SS-CF-7%	7	232	470	2.71	152000	262	580	1984
50×50×3-SS-CF-7%	7	217	717	0.815	99200	169	587	2455
Mean SS-CF		237	478	1.49	105800	224	472	1902
COV SS-CF		0.11	0.25	0.62	0.47	0.47	0.16	0.12