Spacecraft observations of solar wind turbulence: an overview

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Abstract

Spacecraft measurements in the solar wind offer the opportunity to study magnetohydrodynamic turbulence in a collisionless plasma in great detail. We review some of the key results of the study of this medium: the presence of large amplitude Alfvén waves propagating predominantly away from the Sun; the existence of an active turbulent cascade; and the presence of intermittency similar to that in neutral fluids. We also discuss the presence of anisotropy in wavevector space relative to the local magnetic field direction. Some models suggest that MHD turbulence can evolve to a state with power predominantly in wavevectors either parallel to the magnetic field ("slab" fluctuations) or approximately perpendicular to it ("2D"). We review the existing evidence for such anisotropy, which has important consequences for the transport of energetic particles. Finally, we present the first results of a new analysis which provides the most accurate measurements to date of the wave-vector anisotropy of wavevector power in solar wind MHD turbulence.

PACS codes: 52.35.Ra Plasma turbulence; 96.50.Ci Solar wind plasma ; 96.50.Ry Waves and discontinuities

1 Introduction

The solar wind is a continuous but highly variable plasma outflow from the Sun that travels at high speed. Embedded within it are structures, waves and turbulent fluctuations on a wide range of scales. In the 1960's the advent of spacecraft that travelled outside the Earth's magnetic field into the solar wind allowed us to measure this plasma directly for the first time. Modern spacecraft can accurately measure many properties of the solar wind, and have travelled through much of the solar system, providing us with a unique resource to study solar wind fluctuations.

There are a number of reasons to study turbulence in the solar wind. First, it is the only collisionless astrophysical plasma in which we can measure turbulence directly, using spacecraft. By studying turbulence in the solar wind we can improve our understanding of this important phenomenon in other astrophysical plasmas, such as around accretion disks or supernovae. Second, turbulence in the solar wind affects the propagation of energetic particles, such as cosmic rays, throughout the solar system: particles scatter off spatial variations in the magnetic field which are caused by the turbulent fluctuations. Third, as we will see, some of the fluctuations in the solar wind are remnants of those in the Sun's corona from which the solar wind originates, so by studying them we can learn more about conditions in the corona. Finally, a collisionless plasma is a rather exotic medium, with wave-wave interactions and anisotropies that are not present in neutral fluids. By comparing the properties of turbulence in plasmas and neutral fluids, we can learn more about turbulence as a universal process.

This paper is in no way a comprehensive account of the state of solar wind turbulence analysis – several good reviews already exist (e.g. Marsch, 1991; Tu and Marsch, 1995; Matthaeus *et al* 1995; Goldstein and Roberts, 1999; Goldstein, 2001; see also Biskamp, 2003) and it is certainly not an introduction to turbulence in general (for which the reader is referred to Frisch, 1995 and Lesieur, 1990). Rather, it is intended to be only a brief introduction to solar wind turbulence with an emphasis on a few of the most important results: the presence of Alfvén waves that originate in the solar corona, an active turbulent cascade, anisotropy of the fluctuations, and the presence of intermittency. Only fluctuations on magnetohydrodynamic (MHD) scales will be discussed. First, however, we must place these measurements in context by discussing the large scale structure of the solar wind in which the fluctuations lie.

2 Large scale structure

The Sun's outer atmosphere – the corona – is hot (~10^6K) and at a considerably higher pressure than the ambient interstellar medium in which the solar system is embedded. The coronal plasma therefore expands into interplanetary space. As the expansion speed rises with altitude, and the plasma wave speeds fall, the expansion becomes supersonic at a few solar radii, reaching speeds of a few hundred km/s at a few tens of solar radii. The radial distance at which the accelerating plasma flows faster than the local Alfvén speed (which decreases with distance as a result of the decreasing magnetic field) is called the Alfvén critical point and lies at around 10 solar radii.

By the distance of the Earth's orbit, at 1AU (Astronomical Unit: 1 AU = 1.5x10^8km, approximately 215 solar radii), the average solar wind density has dropped to only a few particles per cubic centimetre and the medium is effectively collisionless. The Alfvén speed is typically a few tens of km/s and therefore the Alfvén Mach number of the solar wind is typically of order 10. The plasma β - the ratio of the plasma and magnetic pressures – is typically around 1, unlike in the lower corona where it is much smaller. The consequence of this is that in interplanetary space, neither the bulk plasma nor the magnetic field controls the behaviour of fluctuations: rather, they are the result of complex interactions between particle momenta and pressures, and magnetic forces. The corona's dynamics, by contrast, are largely dominated by the magnetic field.

The topology of the coronal magnetic field is crucial in determining solar wind properties, for in some regions it is effectively formed of closed loops, from which no plasma can escape into interplanetary space. In other regions, in contrast, the field is effectively open and plasma can expand along these field lines. The dominance of magnetic field pressure close to the Sun means that plasma flow from open field regions expands to fill all latitudes and longitudes, so that there is solar wind flow in all directions. Beyond a distance of a few solar radii, the flow is nearly radial. The large scale topology of the Sun's magnetic field changes over the 11 year solar cycle, from approximately dipolar to a complex, multi-polar configuration.

Observationally, it is possible to identify three classes of solar wind. The first is fast (~750 km/s) and rather uniform compared to the second, which is very variable in both speed (between ~250/kms and 500 km/s) and density. High speed wind originates deep inside open magnetic field regions. These regions are termed "coronal holes" since they are dark in extreme UV images of the Sun as a consequence of the plasma being slightly colder than that on the surrounding closed field lines. Slow wind appears to originate near or around the boundaries of coronal holes, although the details of this process are poorly understood. The third class of solar wind is that associated with transient ejections of solar material, so-called coronal mass ejections (CMEs), which we will not discuss here at all.

The propagation of solar wind of varying speeds at most latitudes, combined with the Sun's rotation, results in the interaction of fast and slow wind streams, producing compressions and rarefactions, including shocks. The solar wind is therefore typically a highly structured medium on scales of days to a month (the solar rotation), and it is this complex medium in which waves and turbulence exist. This can make data analysis, as well as interpretation of the results, significantly harder. There is, however, one location where these effects do not occur, and that is over the poles of the Sun near solar minimum. During these periods, the Sun's magnetic field is roughly dipolar, and so has open magnetic fields and coronal holes over the poles, from which steady, fast solar wind emanates.

While the rest of this paper discusses the properties of waves and turbulence, we must bear in mind that these fluctuations exist in a medium which is highly structured and variable. It is important to take these

effects into account. For example, solar rotation and the limited longitudinal extent of fast and slow solar wind streams at low latitudes means a spacecraft stays within each stream for only a few days. Longer intervals are desirable to study fluctuations in detail – for example, to build up accurate distribution functions to study intermittency. Fortunately, the presence of extended coronal holes over the Sun's poles near solar minimum, and hence high speed solar wind at all longitudes at high latitudes, means that when spacecraft are at these high latitudes, they reside within high speed wind through a solar rotation. In fact, this critical latitude is typically somewhere around 30-40 degrees, and only one spacecraft, Ulysses, has explored above this latitude in the inner solar system. Indeed, Ulysses spent months within flows from a single coronal hole. The steady conditions in this region, which provide the near-stationary data necessary for many analysis techniques, make the Ulysses polar data a unique resource for studying solar wind turbulence.

3 Interpreting spacecraft measurements

The solar wind travels at several hundred km/s, much faster than spacecraft velocities (typically a few km/s) or MHD wave speeds (typically a few tens of km/s at 1 AU). A spacecraft time series can then be considered to be a straight line spatial cut through the plasma, in the (nearly radial) flow direction and because the flow is much faster than the wave speeds, the cut is essentially a "snapshot" of the plasma. This assumption, known as "Taylor's hypothesis," can be used to relate a spacecraft frequency f to a plasma frame wavenumber $k=2\pi f/v_{sw}$, where v_{sw} is the solar wind speed. This makes the interpretation of spacecraft data in some ways rather simple, and similar to that of data from many hydrodynamic experiments. The much higher wave speeds downstream of planetary bowshocks and within magnetospheres means that Taylor's hypothesis cannot be used in such regions, which make their analysis much more difficult.

Just as in hydrodynamics, the power spectrum $P_{ij}(f)$ measured by a single sampling point – here a spacecraft – in a supersonic flow is not the full plasma frame spectrum $P_{ij}(\mathbf{k})$, but the so-called reduced spectrum, integrated over all wavenumbers with the flow-parallel component $k_0=2\pi f/v_{sw}$:

$$P_{ij}(f) = \iiint d^3k P_{ij}(\mathbf{k})\delta(\mathbf{k}_R - \mathbf{k}_0) \qquad (1)$$

where, k_R is the flow-aligned (radial) component of the plasma frame wavevector. This is simply because any fluctuation with a flow-parallel wavelength given by $l_{||}=v_{sw}/f$ will contribute to the observed power at this frequency – in wavevector space, this corresponds to all fluctuations with a flow-parallel wavevector given by $k_0=2\pi f/v_{sw}$. For isotropic turbulence, this does not particularly hinder analysis, but as we will see in section 4.3, solar wind turbulence is significantly anisotropic. The recovery of a full 3D spectrum from the reduced spectrum in this case is in principle impossible without making additional assumptions. We return to these issues in section 4.3.

4 Key results

Since the first measurements of the solar wind, it has been clear that it is pervaded by fluctuations on a very wide range of scales, from fractions of a second to many hours. The study of these fluctuations has revealed a complex interaction between waves, turbulence and structures. The most important results, which we discuss here, are: the presence of Alfvén waves propagating away from the Sun, at least in high speed solar wind; an active turbulence cascade, transferring energy from the Alfvén waves at large scale and heating the plasma; anisotropy of the turbulence relative to the magnetic field direction, an important difference from hydrodynamic turbulence; and intermittency, similar to that found in neutral fluids. We discuss each of these results below.

4.1 Alfvén waves

Magnetohydrodynamic scale fluctuations in the solar wind are often predominantly Alfvénic – that is, variations in the magnetic field **B** and velocity **v** are typically correlated or anti-correlated as expected for an Alfvén wave (e.g. Boyd and Sanderson, 1969):

$$\delta v = \pm \delta b \qquad (2)$$

where $b=B/(\mu_0\rho)^{1/2}$ and ρ is the mass density (temperature anisotropy can slightly alter this relationship, but we do not consider that effect here). This result was first established by Belcher and Davis (1971) and is illustrated in Figure 1, which shows variations in the components of the magnetic field and velocity for two 24 hour periods of solar wind data. The correlation or anti-correlation between magnetic field and velocity variations is visually striking and clearly dominates fluctuations on the scale of hours in the spacecraft frame. This is almost always the case in the high speed solar wind which emanates from the open magnetic field regions (coronal holes) – this Alfvénic correlation is much more variable and typically lower in slow solar wind, which in itself is also much more variable.

The sign of the correlation in equation (2) determines whether an Alfvénic fluctuation propagates parallel or anti-parallel to the local magnetic field direction. The two cases in Figure 1 show examples of both possibilities. However, the sense of the observed correlation is, in general, related to the polarity (inward or outward) of the local magnetic field. The left plot in Figure 1, with a positive correlation between magnetic field and velocity variations, is dominated by waves propagating anti-parallel to the magnetic field. During this interval, the background magnetic field direction was sunward – therefore, in the plasma frame, these waves were propagating away from the Sun. The waves in the right plot of Figure 1 were propagating parallel to the local magnetic field – but in this case, the background field was directed away from the Sun. In both cases, therefore, the waves were propagating anti-sunward in the plasma frame. This is the dominant behaviour in high speed solar wind streams, and is consistent with these fluctuations originating in the corona below the Alfvén critical point. Any inwardly-propagating waves generated below this point will travel towards the Sun, while outwardly-propagating waves will travel into the solar wind. Above the critical point, however, inwardly-propagating waves will be swept away from the Sun by the faster moving solar wind plasma. The dominance of outwardly-propagating waves, therefore, suggests that the Alfvénic fluctuations on these scales were generated in the Sun's corona and any inwardly-propagating waves travelled sunwards, which is why they are not present in the solar wind. Indeed, these Alfvén waves in the solar wind may be remnants of fluctuations which are partially responsible for heating the corona.

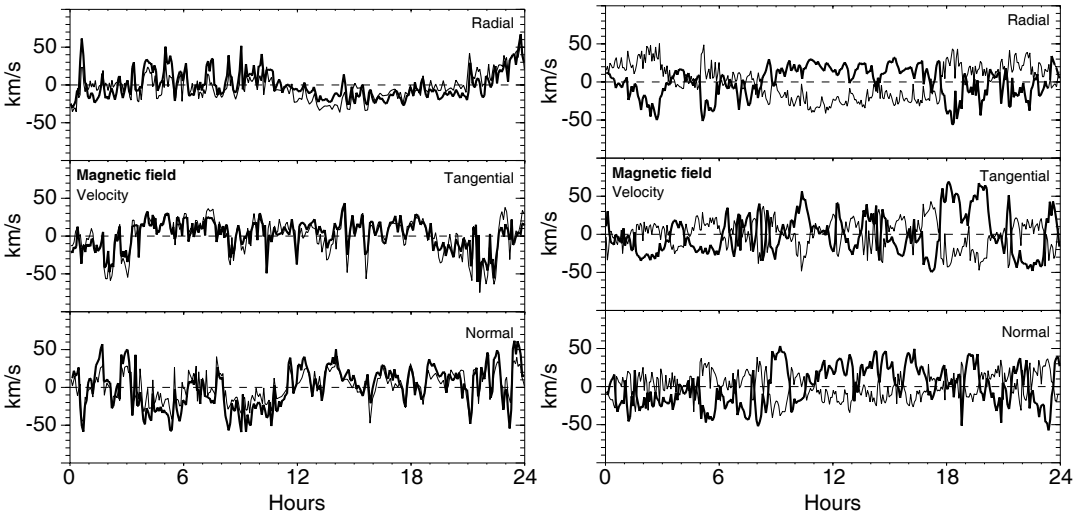


Figure 1. Variations in the components of the velocity (thin lines) and magnetic field (thick lines) during two days of data taken within high speed solar wind streams. Depending on the polarity of the ambient magnetic field, the field and velocity variations are either correlated or anti-correlated, indicating the presence of Alfvén waves.

4.2 Active turbulent cascade

We have established that much of the solar wind, at least within high speed solar wind streams, is filled with Alfvénic fluctuations. As we saw in section 4.1, the predominantly anti-sunward sense of propagation of these fluctuations is consistent with them being generated close to the Sun, in the corona. However, not all fluctuations in the solar wind are simply remnants of processes in the corona, carried out unaffected into the heliosphere. Instead, there is an active turbulent cascade of energy between scales. Perhaps the best evidence of this cascade is provided by examining the power spectrum of solar wind fluctuations.

A typical magnetic field power spectrum is shown in Figure 2. Recall that, as discussed in section 3, the spacecraft frame frequency power spectrum is essentially a plasma frame spatial (albeit reduced) spectrum,

so one should be careful not to interpret frequencies as being in the plasma frame. The spectrum reveals broadband fluctuations over all measured scales, but with two distinct domains in frequency each with power law variations of power with scale: one, at low frequencies (large scales) with a spectral index near 1 (that is, $P(f) \propto f^{-1}$) and one at higher frequencies with a spectral index near 5/3. The highly Alfvénic fluctuations in Figure 1 are on the scale of hours (below 1 mHz) in the spacecraft frame and therefore are in the range of frequencies in Figure 2 where there is a f^{-1} spectrum.

The presence of a broad range of scales with a $f^{-5/3}$ spectrum is suggestive of a turbulent cascade, since 5/3 is the familiar Kolmogorov (1941) value of the spectral index in hydrodynamic turbulence. However, the measurement of a 5/3 spectral index alone is not sufficient to prove the existence of an active turbulent cascade – the fluctuations could, for example, be the remnants of turbulent processes in the corona, and be carried into the heliosphere as a "remnant," non-interacting population. However, it can be shown that these fluctuations do, indeed, reflect an active turbulent cascade, with a transfer of energy between scales, by considering how the power spectrum changes with distance from the Sun.

Since the solar wind travels away from the Sun, the distance from the Sun is approximately proportional to the time that the fluctuations within it have had to interact with each other. Therefore, by measuring how the properties of the fluctuations vary with distance, we are essentially considering how they evolve with time. Bavassano *et al* (1982) provided the first clear evidence of this evolution, by considering how the power spectrum of fluctuations in high speed solar wind changes shape with distance from the Sun.

The key data from Bavassano *et al* (1982) are replotted in Figure 3. These show the spectral index of magnetic field fluctuations as a function of frequency in high speed solar wind at three distances from the Sun. The figure reveals a consistent trend, with the $f^{-5/3}$ regime extending to lower frequencies with increasing solar distance, or equivalently that the breakpoint or transition scale between the f^{-1} and $f^{-5/3}$ regimes moves to lower frequencies (i.e. larger scales) with distance. Such a change in the shape of the power spectrum strongly suggests energy transfer between scales.

The results in Figure 3 can be interpreted in a straightforward fashion. Assume an initial population of f^{-1} Alfvén waves is present in the solar wind after it passes the Alfvén critical point. These gradually decay and transfer energy to smaller scales. However, the higher frequency waves decay more rapidly than those at lower frequencies. At any given distance, some waves have not transferred significant energy and at these scales, the spectral index is still 1. At higher frequencies, significant energy transfer has occurred, producing a turbulent cascade and hence an inertial range with a spectral index of 5/3. As the fluctuations are carried further from the Sun, progressively lower frequencies decay and transfer energy to smaller scales, extending the inertial range to progressively lower frequencies, as seen in Figure 3. In this interpretation, the transition or breakpoint scale between the f^{-1} and $f^{-5/3}$ regimes is the scale of the highest frequency Alfvén waves which have yet to transfer significant energy into the cascade, and therefore represents the so-called "energy-containing scale" – and frequencies below this contain waves which are dynamically "young." Indeed, fluctuations on these large scales decrease in amplitude with solar distance as expected from the WKB theory for non-interacting waves in the expanding solar wind, while those at higher frequencies decay more rapidly, again demonstrating that there is a transfer of energy to smaller scales.

In addition to changes in the power spectrum with distance, there is an associated decrease in the Alfvénicity (essentially, the *B*-*V* correlation in Figure 2) of the fluctuations (e.g. Roberts *et al* 1987) which seems to be related to the turbulent evolution. Indeed, the Alfvénicity of the fluctuations is generally lower at higher frequencies within the inertial range, with no dominant sense of propagation at kinetic scales. It seems, therefore, that as the initially rather pure, outwardly propagating Alfvén waves decay, the resulting turbulent fluctuations are progressively less Alfvénic.

The above discussion has considered only high speed solar wind. The spectrum in slow wind is rather different, with a 5/3 spectral index over a wide range of scales and no discernable range of frequencies with a f^{-1} spectrum. It is therefore not possible to study the evolution of the spectrum in the same was as done by Bavassano *et al* (1982), because there is no spectral breakpoint. However, the broadband $f^{-5/3}$ spectrum, combined with a generally low Alfvénicity (e.g. Tu *et al* 1989), suggests that fluctuations in slow wind represent fully developed turbulence.

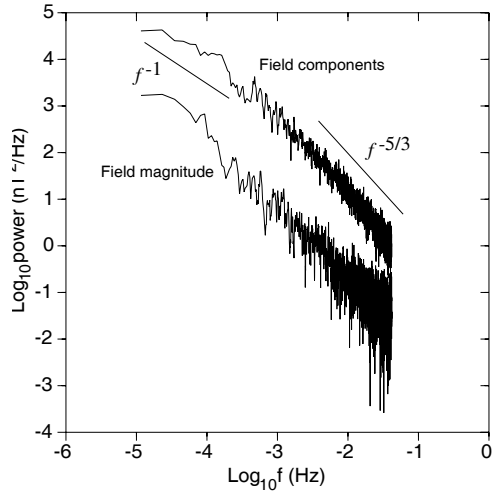


Figure 2. A typical power spectrum of fluctuations in the components of the magnetic field (strictly, the trace of the spectral tensor) and magnitude in high speed solar wind. Note that frequencies here are in the spacecraft, not the plasma, frame.

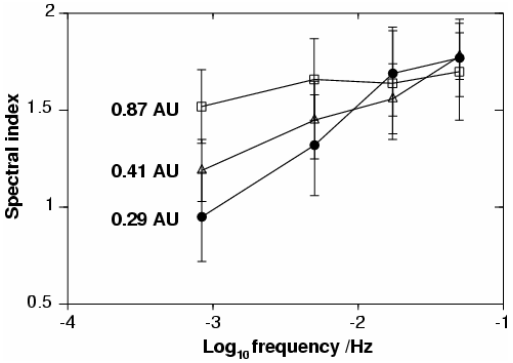


Figure 3. Measurements of the frequency dependent spectral index of magnetic field fluctuations in high speed solar wind, at 0.3 AU (circles), 0.4 AU (triangles) and 0.9 AU (squares). The extension of the small scale $f^{-5/3}$ region to larger scales (lower frequencies) with distance is evidence of energy transfer between scales and hence a turbulent cascade. Data from Bavassano *et al* (1982).

4.3 Anisotropy

In inertial range hydrodynamic turbulence, there is in general no preferred direction and the turbulence is therefore isotropic. In a plasma, in contrast, the presence of a background magnetic field breaks the isotropic symmetry and results in an anisotropy of the fluctuations relative to this direction. Since it represents a fundamental difference between plasma and hydrodynamic behaviour, anisotropy is therefore a particularly interesting aspect of MHD turbulence. In addition, anisotropy is vitally important in understanding the transport of particles in turbulence plasmas, in a wide range of applications, from cosmic ray propagation within the galaxy to heavy ion diffusion in tokamaks. As we will see, field-aligned anisotropy in turn results in anisotropies in this transport.

The amplitude (or variance) anisotropy of fluctuations in the solar wind is well established (e.g. Belcher and Davis, 1971): they tend to be perpendicular to the local magnetic field direction. An example interval is shown in Figure 4, where the small dots point along the local minimum variance direction – the direction in which the fluctuations vary least. This tracks slow changes in the magnetic field direction (caused in turn by the large amplitude Alfvén waves discussed in section 4.1), showing that small scale, turbulent fluctuations in the magnetic field direction – in fact, on such a small scale as to be invisible in Figure 4 – tend to be perpendicular to the local magnetic field. This important result demonstrates the significance of the magnetic field as a direction which orders the fluctuations.

Of more interest here than variance anisotropy, however, is anisotropy in the distribution of turbulent energy in wavevector space (also known as spectral anisotropy). As we will see, there is clear evidence that, within the inertial range of the solar wind turbulent cascade, energy tends to lie in wavevectors at large angles to the local magnetic field, rather than being distributed isotropically.

The data analysis, interpretation and theory regarding field-aligned wavevector anisotropy in MHD turbulence is a complex and unsolved problem which we cannot address in any detail here. In particular, we only consider incompressible MHD. Oughton and Matthaeus (2005) have recently provided a comprehensive description of the concepts and results regarding anisotropy in MHD turbulence, particularly in the solar wind, and we refer the reader to that paper for more detail than we can present here – and a more rigorous derivation of results that we merely quote.

To appreciate the effect of the magnetic field on plasma turbulence, it is helpful briefly to consider the hydrodynamic case. Kolmogorov's (1941) theory can be considered in terms of the timescale τ_{NL} for energy to be transferred nonlinearly from one scale to the next, which scales as $\tau_{NL}(k) \propto 1/(ku_k)$, where u_k is the characteristic amplitude of fluctuations at wavenumber k. This expression for τ_{NL}, combined with a constant rate of energy transfer between scales, leads in a straightforward way to the celebrated $f^{-5/3}$ inertial range hydrodynamic spectrum.

In a plasma, Iroshnikov (1964) and Kraichnan (1965) pointed out that, in contrast to the non-propagating eddies of hydrodynamic turbulence, incompressible MHD turbulence is comprised of Alfvénic fluctuations, which can propagate. Since only oppositely propagating waves interact, any two such waves will decorrelate as a result of their motion. Two waves of typical wavenumber k, propagating (strictly) parallel and antiparallel to the magnetic field (so-called "slab" fluctuations), will decorrelate on a timescale given by $\tau_A(k) \propto 1(kV_A)$ where V_A is the Alfvén speed. This decorrelation has the effect, if $\tau_A << \tau_{NL}$, of slowing the transfer of energy between scales in the turbulence cascade and flattens the spectrum, leading to a spectral index of 3/2.

On the basis of the above, one might expect to observe a $f^{-3/2}$ power spectrum in the solar wind – but it is a $f^{-5/3}$ spectrum that is seen. This puzzling result can, it seems, be resolved by considering wavevector anisotropy. In general turbulent, Alfvénic fluctuations will have wavevectors which are not simply parallel or antiparallel to the magnetic field, but lie at a range of angle θ_{kB} to it. In this case, their phase speed will be $V_A \cos\theta_{kB}$ and the decorrelation time will be increased, to $\tau_A(k) \propto 1/(k\theta_{kB})$. If θ_{kB} is sufficiently large (see Oughton *et al* 2004 and Oughton and Matthaeus, 2005 for a detailed discussion of the term "sufficiently" in this context), the decorrelation time τ_A will be larger than τ_{NL} and decorrelations will begin to be dominated by nonlinear effects (as in hydrodynamics) rather than wave propagation effects. One might then expect to observe a K41-like, $f^{-5/3}$, spectrum. So, if the energy in solar wind fluctuations tends to be mostly in wavevectors which are approximately perpendicular to the magnetic field, and thus have relatively small phase propagation speeds, one can explain the observed 5/3 spectral index. Thankfully, there is both theoretical and experimental evidence that this is indeed the case.

Shebalin *et al* (1983) showed, using resonance arguments, that the turbulent transfer of energy in wavevector space is anisotropic relative to the magnetic field. In particular, transfer parallel to the field is weak compared to that in the perpendicular direction. Therefore, even if one begins with most power in slab fluctuations (which wavevectors parallel to the field), the turbulent cascade will tend to transfer this power to progressively higher perpendicular wavenumbers – in other words, larger θ_{kB}. Eventually, most power would lie in wavevectors with $\theta_{kB} \sim 90°$, the decorrelation effect would not be significant, and a K41-like cascade would result. These results have since been extended to the three-dimensional case (e.g. Oughton *et al* 1994; Cho *et al* 2002).

One can imagine a simple idealised model of this scenario, with power only in wavevectors parallel to the magnetic field (the "slab" fluctuations) and in perpendicular wavevectors (so-called "2D" fluctuations), but not in between. However, even this simple model is in fact rather difficult to test with solar wind data. To understand why this is the case, we must return to the subject of the interpretation of a spacecraft time series, which we discussed briefly in section 3. As we saw, the power measured by a spacecraft at a given frequency is in fact the so-called "reduced" spectrum, given by equation (1), which is an integral of the full spectrum over a plane in wavevector space which is perpendicular to the flow. Therefore, a power spectrum

from a spacecraft time series contains power from wavevectors covering a very wide range of amplitudes and directions.

A number of attempts have been made to disentangle this information and deduce properties of the anisotropy of solar wind turbulence, most of which are based on considering how the power spectrum changes between different intervals of data, when the magnetic field lies at various angles to the solar wind flow (e.g. Matthaeus *et al* 1990; Carbone *et al* 1995). This is because the relative contributions of slab and 2D turbulence fluctuations to the observed reduced spectrum depends on the angle between the magnetic field vector and the sampling, or flow, direction: in this paper, we call this angle θ_{BV}. The most convincing evidence for slab and 2D fluctuations in the solar wind was presented by Bieber *et al* (1996), who showed that, if the turbulence was composed of just slab and 2D fluctuations, observed power levels should vary like

$$P(\theta_{BV}) = C_{slab}(\cos\theta_{BV})^{\gamma-1} + C_{2D}(\sin\theta_{BV})^{\gamma-1} \qquad (3)$$

where γ is the spectral index of the fluctuations and C_{slab} and C_{2D} are the respective amplitudes of the slab and 2D components. Bieber et al. analysed many intervals of data and examined how power levels varied with θ_{BV}. The results of their analysis are shown in Figure 5. It is clear that power levels increase with increasing field/flow angle, a result which is inconsistent with most power being in field-parallel (slab) wavevectors. Indeed, Bieber *et al* (1996) concluded, on the basis of this analysis and two other techniques, that only about 20% of the power was in slab fluctuations, while around 80% was in 2D. Leamon et al. (1998) have more recently showed that the dominance of 2D fluctuations over slab also extends into the kinetic dissipation range.

This result has important consequences for the propagation of energetic particles in the solar system and beyond. Such particles, for example cosmic rays, can be scattered by slab fluctuations, but not 2D. The dominance of 2D fluctuations therefore increases the particle mean free paths compared to that expected if the power were slab or isotropic (Bieber *et al* 1996). However, 2D modes have the effect of "braiding" the magnetic field, leading to enhanced field-perpendicular diffusion (Shalchi and Schlickeiser, 2004). An understanding of these effects is vital for understanding cosmic ray propagation, but also in other applications, such as tokamaks. Precise measurements of turbulence in space plasmas can therefore help us to quantify these effects.

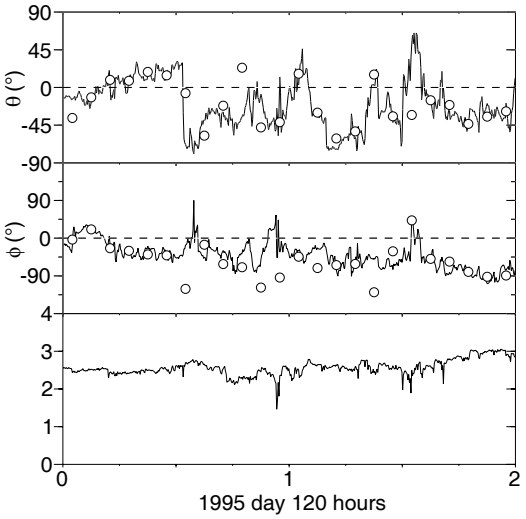


Figure 4. Two hours of magnetic field data in a high speed solar wind stream: the field direction is shown as the solid lines in the top two panels and the magnitude in the bottom panel. The minimum variance direction of successive 5 minute intervals of data is shown as open circles in the top two panels. This direction closely follows the local magnetic field, demonstrating that small scale turbulent fluctuations are predominantly perpendicular to the local magnetic field.

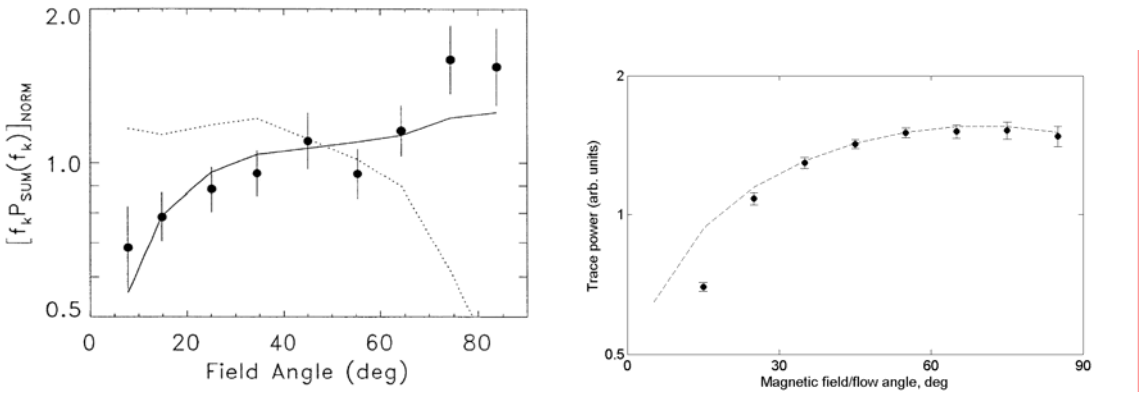


Figure 5 (left): Magnetic field power levels as a function of the angle between the magnetic field and the flow direction. Increasing power levels with angle demonstrate that the turbulence is dominated by power in wavevectors at large angles to the magnetic field direction. Figure reproduced from Bieber *et al* 1996. Figure copyright American Geophysical Union. (right): A similar estimate, made using a wavelet method, which suffers from much lower noise and scatter. The dashed line would be expected for 20% slab, 80% 2D fluctuations. This agrees well with the data, except for small field/flow angles, where observed power levels are lower.

A restriction of the method used by Bieber *et al* (1996) is that it requires long (of order days) intervals of data, during which the magnetic field direction is assumed to be stable. Unfortunately the ubiquitous presence of low frequency fluctuations and structures means that there will be inevitable variations in the field direction during each interval. In addition, although Bieber *et al* (1996) performed some normalisation, the method assumes that plasma-frame power levels do not vary between successive, sometimes rather long, intervals. These assumptions are inevitably not perfectly satisfied, and this can result in noise and errors in the final results and which probably explains the scatter in the results of Bieber *et al* (1996) shown in Figure 5(left). However, the scatter and errors are sufficiently large that significant deviations from the slab/2D model may be present, but undetectable.

We have recently implemented a wavelet-based method to track the local field direction in a scale-dependent manner: this removes the assumption of a constant magnetic field direction and thus dramatically reduces noise in the measurements. Preliminary results are shown in the right panel of Figure 5: the smaller errors and lower scatter are clear. Also shown as a dashed line are the expectations from the slab/2D model with $f^{-5/3}$ scaling, with 20% slab as found by Bieber *et al.* The model fits remarkably well, providing more support for the slab/2D paradigm. The data are rather lower than the model at small flow/field angles. This likely represents a smaller slab fraction than found by Bieber *et al* (not surprising given the different solar wind conditions and scales considered in the two analyses) but may also reflect deviations from the simple slab/2D picture – our analysis of these data is ongoing, but their higher precision offers the prospect of a more detailed analysis of the anisotropy of solar wind MHD turbulence than has previously been possible.

4.4 Intermittency

Intermittency, the spatial inhomogeneity of fluctuation power levels in a turbulent medium, is well established in neutral fluids (e.g. Frisch, 1995) and arises spontaneously from the fluctuations. Intermittency is manifested as non-Gaussian probability distributions of increments of quantities such as the fluid velocity: larger numbers of large jumps are observed, compared with that expected from a normal distribution. These deviations are typically larger at higher frequencies, further down the inertial range. In hydrodynamics, velocity shear (which is responsible for the observed power levels) is directly related to energy transfer and the observed intermittency is therefore interpreted as spatial variability in this energy transfer rate.

Intermittency can be analysed in a number of ways, which are all aimed at quantifying the deviation from Gaussianity of the increments, and in particular how these deviations vary with scale. A commonly used technique is that of structure functions, which are calculated from a stationary time series $x(t)$ (for example, a component of the velocity) as:

$$S(\tau, p) = \left\langle \left| x(t+\tau) - x(t) \right|^{p} \right\rangle \quad (4)$$

$S(\tau,p)$ is a function of a time scale τ and a moment p – it is essentially just the moment of the increments of the time series at a time lag τ. In the inertial range, S is a power law function of τ,

$$S(\tau, p) \propto \tau^{\zeta(p)} \quad (5)$$

It is the $\zeta(p)$ functions that are of interest. These measure how different moments of the increments of the time series vary with scale. For a Kolmogorov (1941) fluid with no intermittency – that is, where the energy transfer rate is spatially homogeneous – one expects $\zeta(p)=p/3$. However, the observed values (see, e.g. Borgas, 1992 for a review of hydrodynamic observations) do not agree with this prediction: they are larger than $p/3$ for $p<3$, and smaller for $p>3$. However, $\zeta(3)=1$ is observed experimentally. In fact, $\zeta(3)=1$ is an exact result for isotropic hydrodynamics independent of intermittency; see Frisch (1995) for more discussion of this point.

Various models have been proposed to explain the observed intermittency in hydrodynamic turbulence, although they are generally rather ad hoc in their approach. Meneveau and Sreenivasan (1987) proposed the so-called '*p* model' which has one adjustable parameter, *p*, which varies from 0.5 (no intermittency) to 1 (most intermittent). The *p* model generates intermittency by distributing energy unequally in space when a turbulent eddy decays and it is possible to predict $S(\tau,p)$ from the model: fits to observed values are generally good for $p\sim0.8$. A model was introduced by She and Leveque (1994) which has the advantage of having a parameter which can be interpreted physically: the dimension of the dissipative surfaces in the fluid. This model also agrees well with hydrodynamic observations, when the dissipative structures are assumed to be one dimensional, consistent with them being vortex tubes.

Intermittency is now a well established property of solar wind turbulence (see, e.g. Marsch and Tu, 1997 for a review). However the anisotropy of MHD turbulence means that there is no simple way to relate velocity or magnetic field fluctuations to energy transfer as there is in hydrodynamics. The interpretation of intermittency is therefore more difficult in MHD. However, accurate measurements of structure functions in the solar wind show remarkable similarities with terrestrial measurements – see, for example, Figure 6, where the *p* model replicates the observed structure function scalings very accurately, with an intermittency parameter close to that found for hydrodynamic turbulence. The She and Leveque (1994) model also agrees well, when dissipative structures are defined to be two dimensional (for example, current sheets). It is also clear from Figure 6 that the data are not consistent with an absence of intermittency (which would result in straight lines), for either a K41 or K65 cascade. In addition, it can be seen that $\zeta(3)=1$, as in hydrodynamics. Carbone (1994) noted that, just as $\zeta(3)=1$ in a K41 cascade independent of intermittency, so $\zeta(4)=1$ in a K65 cascade with just parallel and anti-parallel propagating fluctuations. The fact that we observe $\zeta(3)=1$, and not $\zeta(4)$, again confirms the hydrodynamic-like nature of the cascade in MHD turbulence in the solar wind.

More recently, analysis of intermittency has shifted to a consideration of the probability distribution functions of increments themselves, rather than their moments. In particular, the Castaing distribution (Castaing *et al* 1990), which consists of a hierarchy of log-normal distributions, has been used successfully to describe hydrodynamic intermittency. Recently, Castaing distributions have also been shown to describe solar wind turbulent fluctuations (Sorriso-Valvo *et al* 2001): see figure 7.

In the light of the highly anisotropic distribution of energy in the turbulent fluctuations (section 4.3), it is worth considering the interpretation of these intermittency measurements. We have seen that most power is contained in wavevectors that are at large angles to the local magnetic field. The intermittency analyses presented here, therefore, which do not attempt to distinguish between contributions from different wavevector directions, are dominated by those at large angles to the field, the "quasi-2D" fluctuations and therefore reflect the intermittency only of the quasi-2D component of the turbulence.

The close agreement of hydrodynamic models of intermittency with MHD turbulence observations is remarkable and perhaps rather surprising. It is not clear why these models describe the measurements so well. This may be the result of the universality of the spontaneous generation of intermittency by turbulent flows, whatever the medium. On the other hand, it may simply reflect the crude nature of existing

intermittency diagnostics, and future analyses may be better able to distinguish between, and perhaps even explain, intermittency in neutral and plasma fluid turbulence.

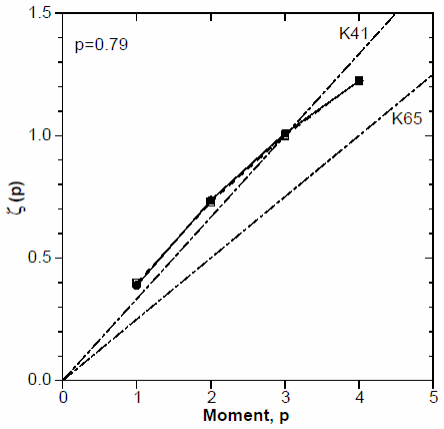


Figure 6. Scaling exponents of structure functions, ζ(p), of intertial range MHD turbulence in the solar wind. Filled circles and solid lines are measured values; errors are smaller than the symbols. Values calculated from the p model (Meneveau and Sreenivasan, 1987) with p=0.79 are shown as open squares linked by dashed lines; the agreement is very good. Values for K41 and K65 turbulence, with no intermittency, are shown as dash-dotted lines.

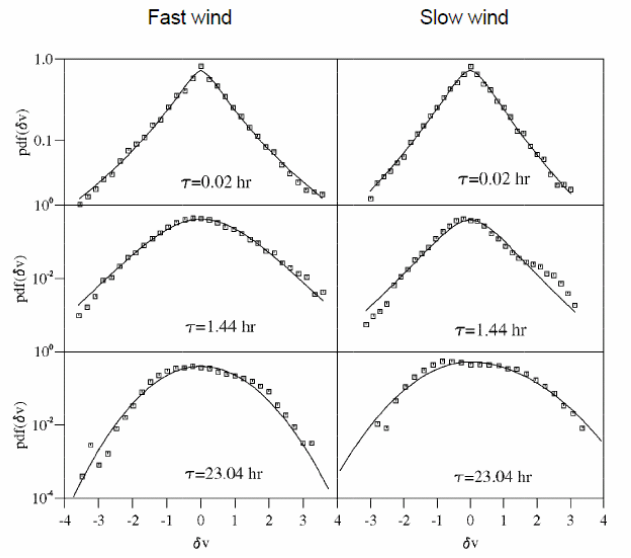


Figure 7. Distributions of velocity increments at three scales in the solar wind. The distributions change from Gaussian at large scales to having much higher 'wings' at smaller scales, and are well described by Castaing distributions. Figure reproduced from Sorriso-Valvo *et al* (2001), copyright American Geophysical Union.

5 Summary

While much has been learned about turbulence in the solar wind, many puzzles remain. For example, what is the nature of intermittency in MHD? How does the turbulent cascade develop? How is turbulent energy dissipated at kinetic scales? How do coronal conditions affect fluctuations injected in the solar wind as it leaves the Sun, and what is the origin of the f^{-1} spectrum of Alfvén waves in high speed wind? What is the 3D structure of the magnetic field fluctuations and how does this affect particle propagation?

Many of these questions are being addressed, and it is likely that considerable progress will be made in the next few years, particularly via the availability of new data sets. The twin STEREO spacecraft, due to launch in early 2006, will study the large scale structure of the solar wind. Solar Orbiter will hopefully launch in 2013 and travel to 0.21 AU, closer than any previous spacecraft, to study the links between the corona and the solar wind. A number of missions have been proposed to take high time resolution plasma

measurements, which will help in the study of kinetic phenomena. Finally multi-spacecraft analysis, using recently-recorded data from missions such as Cluster and other near-Earth spacecraft, will allow us to study the 3D structure of solar wind turbulence in detail for the first time.

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