

PLANE STRAIN FAILURE CHARACTERISTICS

OF A SATURATED CLAY

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by

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ABSTRACT

Specimens of saturated, remoulded Weald Clay were normally consolidated under K_0 conditions and then sheared in compression under conditions of plane strain. Undrained tests were carried out. When compared with the corresponding data from standard triaxial tests, the average strength of the plane strain specimens is greater by $\Delta \phi' = 1.2^\circ$. Drained tests on "sable de mol" showed a similar strength differential between plane strain and triaxial specimens. It is concluded that this increase in ϕ' in plane strain specimens is due to intermediate values of the second principal stress.

The magnitude of the intermediate principal stress under plane strain conditions is considerably less than one-half the sum of the other two principal stresses during shear for both the sand and clay tested.

In undrained tests on clay specimens, it is shown that both the initial yield and the induced pore water pressures are controlled by the octahedral stresses; failure, however, is better defined by empirical relations.

ACKNOWLEDGEMENTS

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The author is indebted to Dr. Bishop for his many helpful suggestions, and to Dr. D. J. Henkel who guided the programme and helped to interpret the test results. Other members of the staff to give assistance include Dr. R. E. Gibson, Dr. N. N. Ambraseys, Mr. N. Morgenstern, Mr. E. Harris, Mr. D. Evans, Mr. N. Scott, Mr. L. Spall, and Miss J. Brown. Instructive discussions were also held with the writer's colleagues, Messrs. V. A. Sowa, D. L. Webb, E. L. Matyas, and S. W. Smotrych. Mr. Sowa generously provided the standard triaxial data on Weald Clay which was used for comparison with the plane strain test results.

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CHAPTER I

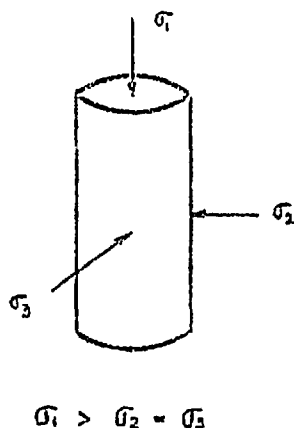
INTRODUCTION

"In certain classes of problems, laboratory tests serve mainly to illustrate the principles on which a judgment may be based or to set broad limits to the probable behaviour of the soil. In many cases, however, the uniformity of the soil conditions or the importance of the project will justify a more accurate analysis ...", for instance, in the "design of water-retaining structures, such as earth dams and embankments, where failure could have catastrophic results, but where an overconservative design is very costly." ... "Current methods of stability and deformation analysis call for a range of test data which can be obtained conveniently only with the triaxial apparatus. This is due to the recognition of the advantages, in routine work as well as in research, of carrying out the analysis in terms of effective stresses and explicitly determined pore pressures." (BISHOP & HENKEL, 1957: Preface to "The Measurement of Soil Properties in the Triaxial Test").

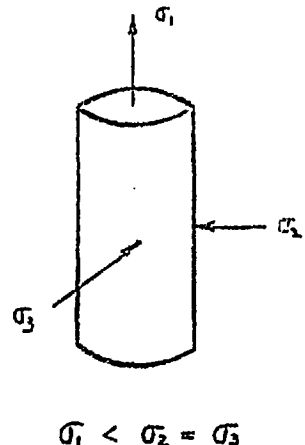
(1.1) The Triaxial Test

The triaxial apparatus was originally developed from a machine designed at the Russian Waterways Experimental Station for the purpose of studying the consolidation of clays under conditions of negligible side friction (SEIFFERT, 1955). It consists fundamentally of a

cylindrical pressure chamber, which can be dismantled, with a movable piston co-linear with its axis. For tests on soil, a cylindrical specimen is placed on the pedestal of the chamber base, capped with a stiff end platen and enclosed in a rubber membrane sleeve to isolate it from the chamber fluid. Depending on what test information is desired, the sample can be either consolidated and/or sheared with various combinations of all round pressure and axial loads. A detailed description of the various loading methods possible in the standard triaxial apparatus is given by BISHOP & HENKEL (1957, 1962). Since the development of the first machine, the present apparatus with modifications and improvements has become firmly established in the field of soil mechanics as the major device used to measure the strength of soils.



(a) Compression Test



(b) Extension Test

Fig. 1-1. Principal Stress System in the Triaxial Test

(1.2) Limitations of the Triaxial Apparatus

However, this apparatus, for all its versatility, has its limitations if the effect of a varying intermediate principal stress on the soil strength is to be investigated. Due to the axial symmetry of the confining pressure (see Fig. 1-1), the intermediate principal stress has a value equal to the minor or major principal stress in the compression and extension tests respectively. In practice, many failures in natural slopes and man-made earth structures occur under stress and deformation conditions that are inadequately represented in the triaxial test model. An early appreciation of this phenomena has initiated the attempts of many experimenters to solve the technical difficulties associated with the independent application of the three principal stresses to soil specimens, in order to define more accurately the failure criteria in three dimensional stress space. BISHOP & HENKEL (1957) state, "Ideally, the triaxial test should permit independent control of the three principal stresses ..., so that generalized states of stress can be examined, including the important special case corresponding to plane strain."

(1.3) The Plane Strain Compression Test

The actual deformations that occur in many earth structure and foundation failures more closely approximate to the plane strain condition than three dimensional deformation. The three principal stresses

are all unequal, and the intermediate principal stress acts along the axis of zero deformation. To simulate this phenomena, an apparatus was constructed at Imperial College to impose two dimensional deformation on a rectangular prism of soil while at the same time measuring the three principal stresses. The soil prism was nominally 16 in. long x 4 in. high x 2 in. thick. The major principal stress, σ_1 , was applied vertically by the testing machine, the minor principal stress, σ_3 , was applied horizontally by the cell pressure, and the intermediate principal stress, σ_2 , which prevented the specimen from deforming lengthwise, acted on the ends of the specimen through rigid end plates. The loading system is shown diagrammatically in Fig. 1-2.

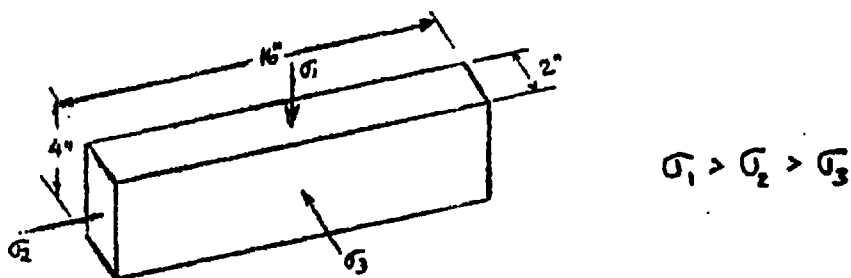


Fig. 1-2. Principal Stress System in the Plane Strain Test

The compression test was carried out by first consolidating the specimen under conditions of no-lateral-yield, and then shearing it by increasing the major principal stress and adjusting the intermediate principal stress to maintain the plane strain condition. Undrained

tests were conducted and by measuring the excess pore water pressures in the saturated samples, the principal effective stresses were obtained.

Although sufficient to determine the plane strain characteristics of a soil, the Imperial College apparatus was not specifically designed to permit exhaustive variation of the intermediate principal stress between the limits of the major and minor principal stresses, and hence the complete failure envelope in stress space cannot be fully established. Nevertheless, in the writer's opinion, the errors associated with the apparatus are small enough to establish with a relatively high degree of accuracy the strength parameters of a soil in plane strain.

(1.4) The Shear Box Test

Another apparatus that imposes plane deformation on a soil specimen is the shear box, which consists basically of a standard box split horizontally into halves. The test is performed on a soil specimen approximately 6 cm. square by 2 cm. deep by moving the top half of the box horizontally in one direction relative to the bottom half. A normal load is usually placed on the top of the specimen, and measurements of vertical and horizontal displacements as well as shearing load are taken. (See Fig. 1-3). This test, advantageous in practice, is simple to operate and can be conducted relatively

quickly even in cohesive soils because of the shortness of the drainage paths. However, the high stress gradients set up in the predetermined zone of shear and the possible rotation of principal stresses tend to make the interpretation of the results controversial; (SKEMPTON & BISHOP, 1954). Thus in the last decade, due to its theoretical and practical limitations, the direct shear test has gone out of favour; (CORNFORTH, 1961).

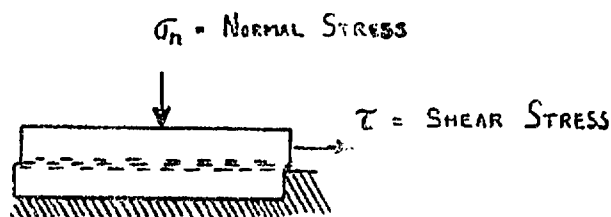


Fig. 1-3. Stress System in the Shear Box Test

(1.5) Aims of the Thesis

The main aspect of the testing programme was to determine the failure characteristics of a remoulded, normally consolidated clay in the plane strain condition. By comparing these characteristics with the complementary standard triaxial failure phenomena, the effect of plane deformation on the strength of the clay could be obtained.

The objects of the present research, then, are:

- (a) to measure the three principal effective stresses, the change in pore water pressure, the axial strain, and the moisture content of the clay in the plane strain, undrained compression test, and to compare these results with the corresponding data obtained from the standard triaxial test;
- (b) to measure the magnitude of the intermediate principal stress during shear, and to assess its influence on the strength of the clay;
- (c) to relate the failure criteria of the plane strain and the standard triaxial tests in a generalized state of stress.

After the clay to be used was selected, preliminary tests to determine the mechanical properties were undertaken, and the relevant triaxial shear tests were performed. The writer was responsible for conducting the plane strain tests, and a colleague, Mr. V. Sowa, carried out the standard triaxial tests on the clay.

A secondary programme on a sand was performed in which both the plane strain and the standard large diameter triaxial tests were carried out by the writer. The results of these tests were compared with each other as above, and also compared with triaxial tests carried out in Belgium (LADANYI, 1960) on the same sand.

CHAPTER II

BRIEF REVIEW OF PREVIOUS WORK

(2.1) General

The point of failure in strength tests on soils is commonly taken as the maximum point on the stress-strain curve, i.e. in the triaxial test, the maximum value of the deviator stress ($\sigma_1' - \sigma_3'$), although some investigators prefer to define failure as the maximum principal stress ratio (σ_1'/σ_3'). In recent years, however, more emphasis has been placed on the stress invariants and the corresponding octahedral stresses; (HVORSLEV, 1960). In terms of principal stresses the first, second and third invariants are:

$$J_1 = \sigma_1 + \sigma_2 + \sigma_3 \quad \dots\dots\dots(2-1 \text{ a})$$

$$J_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 \quad \dots\dots\dots(2-1 \text{ b})$$

$$= 1/6 \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right] \quad \dots\dots\dots(2-1 \text{ c})$$

$$J_3 = \sigma_1\sigma_2\sigma_3 \quad \dots\dots\dots(2-1 \text{ d})$$

(Similar equations are used to define the strain invariants in terms of principal strains; JAEGER, 1962).

The octahedral stresses have the form:

$$\sigma_{oct} = 1/3(J_1) = 1/3(\sigma_1 + \sigma_2 + \sigma_3) = p \quad \dots\dots\dots(2-2 \text{ a})$$

$$\tau_{oct} = 1/3 \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{1/2} \dots\dots(2-2 b)$$

where σ_{oct} is the octrahedral normal stress or the mean normal stress,

and τ_{oct} is the octrahedral shear stress.

Various theories involving the stress invariants and the octrahedral stresses to define failure in soils are discussed in the following pages, as well as simpler theories based on the stress-strain curve.

The apparatus generally used in the investigations of the shear strength of soils is the direct shear box or the triaxial device; the following review will be limited to these apparatus or their modifications.

(2.2) Theories on Shear Strength

The advancement of soil mechanics as an applied science has always been hindered by the lack of an adequate theory which correlates applied stresses and their resulting deformations. The Coulomb Empirical Law (COULOMB, 1776), (See Fig. 2-1):

$$s = c + \sigma \cdot \tan \phi \dots\dots\dots(2-3)$$

which states that the shear strength s is a linear function of the

applied normal stress σ , provided a partial fulfilment to the existing lack of logical hypotheses prevailing up to the late eighteenth century. The relationship between shearing strength and normal stress was further clarified by the graphical representation of MOHR (1882). However, it was not until Terzaghi formulated the concept of effective stress (TERZAGHI, 1936)

$$\sigma' = \sigma - u \quad \dots\dots\dots(2-4)$$

(i.e. the effective normal stress σ' is equal to the applied stress minus the pore pressure u), first used in 1923, that soil mechanics possessed a fundamental unifying principle (SKEMPTON, 1960).

(Although this concept was not previously understood, practising engineers were in fact preloading soft clay deposits "to squeeze out the water and consolidate the mud" as early as 1809; TELFORD, 1830).

The Mohr-Coulomb criterion, since it considered only total stresses, had limited applicability in the general sense; the parameters c and ϕ are not constants of a given soil, but empirical coefficients that depend on the conditions of consolidation, drainage, etc (TAYLOR, 1948). Terzaghi's modified effective stress formula for shear strength,

$$\tau = c' + (\sigma - u) \cdot \tan \phi' \quad \dots\dots\dots(2-5)$$

increased the accuracy of the criterion^{*} immensely. No provision, however, was made for the stress history of the material, and thus discrepancies in c' and ϕ' between normally consolidated and over-consolidated specimens persisted. Soil parameters which would be constant under all testing conditions had yet to be defined.

HVORSLEV (1937) introduced the concept:

$$\tau = K \cdot p_e + \sigma \cdot \tan \rho \quad \text{.....(2-6 a)}$$

which in modern notation can be written:

$$\tau_f = c_e + \sigma'_n \cdot \tan \phi_e \quad \text{.....(2-6 b)}$$

where τ_f is the shear strength of the soil, c_e , the true cohesion, is a function of the water content only, σ'_n is the effective normal stress on the failure plane, and ϕ_e is the true cohesion. This concept, plotted dimensionlessly in Fig. 2-2, takes into consideration the stress history and the void ratio of the specimen by means of the construction shown in Fig. 2-3, and suggests a unique set of parameters for a particular soil. It was not until later that GIBSON (1953) critically examined Hvorslev's hypothesis and pointed out that it should be modified to consider the state of packing or porosity of the soil grains. To make drained tests, in which the specimen volume is allowed to change, comparable with undrained tests,

*The criterion assumes that the shear strength of the soil is independent of the magnitude of the intermediate principal stress.

where no volume change occurs, an energy correction must be applied to the drained strength to account for the volume change effect on the measured shear stress at failure; (BISHOP, 1950). Since Hvorslev's criterion was based on the results of drained tests only, this anomaly did not occur. For the shear box test, Gibson's formula is

$$\tau_f = c_r + \sigma'_n \cdot \tan \phi_r + \sigma'_n \cdot g_f \quad \dots\dots\dots(2-7 a)$$

where g_f is equal to the ratio of the vertical movement of the normal load to the incremental movement of the shear load.

The corresponding general equation for the triaxial test (SKEMPTON & BISHOP, 1954) is:

$$\frac{1}{2}(\sigma_1 - \sigma_3)_f = \frac{c_r \cdot \cos \phi_r + \sigma'_3 \cdot \sin \phi_r}{1 - \sin \phi_r} + \frac{1}{2} \frac{d(\Delta V/V)}{d\epsilon_1} \cdot \sigma'_3 \quad \dots\dots(2-7 b)$$

where $(\sigma_1 - \sigma_3)_f$ is the deviator stress at failure, σ'_3 denotes the minor principal effective stress at failure, c_r and ϕ_r are obtained as in Fig. 2-3, c_r being a function of water content alone, and $\frac{d(\Delta V/V)}{d\epsilon_1}$ is the ratio of an incremental change in volume to the corresponding incremental axial strain in the specimen at failure. (This ratio is zero in an undrained test).

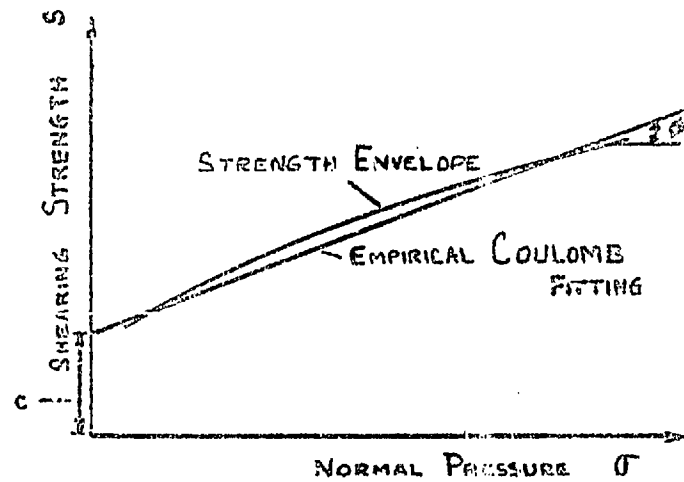


Fig. 2-1. Plot illustrating the rational interpretation of Coulomb's Empirical Law (after TAYLOR, 1948).

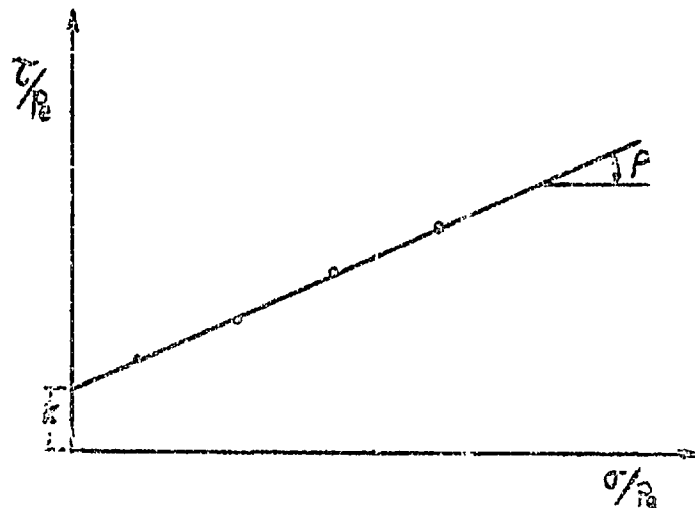


Fig. 2-2. Hvorslev's concept of soil constants, (after BJERRUM, 1960)

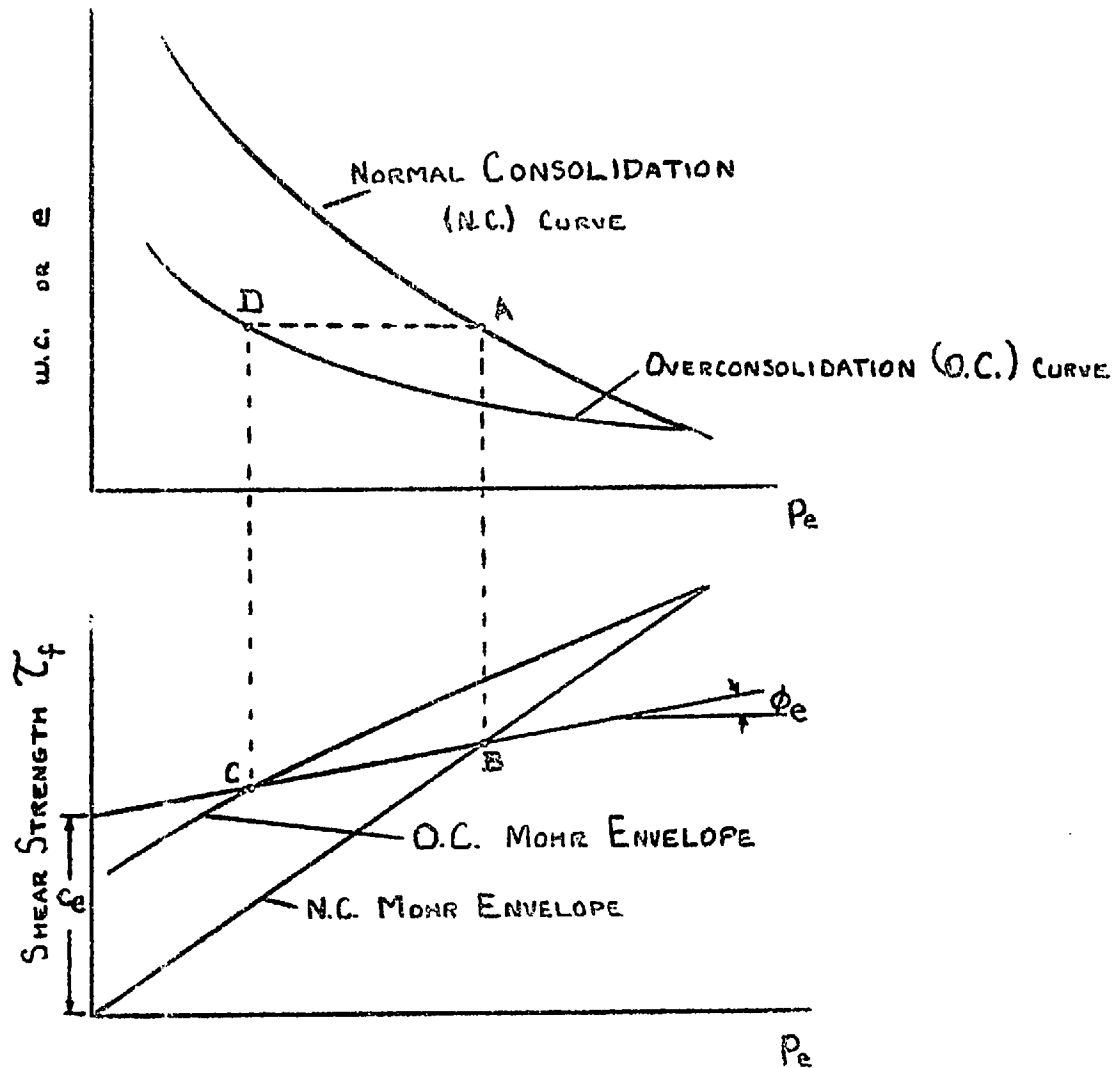


Fig. 2-3. Terzaghi's construction to determine c_e and ϕ_e from drained shear tests (TERZAGHI, 1938).

The construction is as follows: The curves and the envelopes are obtained from a series of drained tests. Arbitrary points A and B are plotted at the same value of p_e ; D is plotted at the same water content as A, and C has the same value of p_e as D. The line joining B and C has an intercept c_e and an angle ϕ_e to the horizontal. Different values of c_e are obtained when different water contents are chosen. The same construction can be used to determine the true parameters in undrained or unsaturated samples if effective stresses are used.

To obtain a satisfactory set of average values for the true cohesion c_r and the true angle of internal friction ϕ_r , BISHOP & HENKEL (1957) proposed that $\frac{(\sigma_1 - \sigma_3)_{\text{f}}}{2p_e}$ be plotted as the ordinate and σ_3'/p_e as the abscissa on a graph, and the best fit straight line be drawn through the points (see Fig. 2-4). Presented in this way, the slope of the line becomes $\sin \phi_r (1 - \sin \phi_r)$ and the intercept $\frac{c_r \cos \phi_r}{p_e (1 - \sin \phi_r)}$, and hence the average true constants of the soil can be determined. (p_e is the all-round consolidation pressure which produces the particular water content in the normally consolidated sample, see Fig. 2-3).

POOROOSHAB & ROSCOE (1961), however, maintain that the boundary energy correction for drained tests given by BISHOP (1950) is valid only for soils which expand in volume at failure (i.e. a dense sand), and should not be applied to specimens that decrease in volume during shear (i.e. normally consolidated clays).

Another extension to Hvorslev's criterion (equation 2-6 b) was provided by ROSCOE, SCHOFIELD & WROTH (1958) in a generalized relationship of the three variables, effective normal stress σ_n' , shearing stress τ , and voids ratio e (or water content w in fully saturated soils). The yield surface in three dimensional space had the equation:

$$\tau = \mu_o \cdot \sigma_n' + \mathcal{V} \cdot \exp.(-B \cdot e) \quad \dots\dots\dots (2-8)$$

where μ_o is a fundamental constant of the soil, $\mathcal{V}$ is a coefficient

which represents the effective cohesion component at zero voids ratio, B is another soil constant, and exp. denotes the exponential function. The paper suggested the existence of a critical void ratio in soils after the ultimate strength had been reached, that is, a condition of continuing deformation of the specimen under constant stress and constant void ratio. This condition exists as a line on the yield surface called the critical void ratio or CVR line, and on the (w, σ) plane has the equation:

$$\sigma = \sigma_h \cdot \exp.\left(\frac{w_h - w}{\lambda}\right) \quad \dots\dots\dots(2-9)$$

where σ and w are the variable normal stress and water contents respectively, σ_h and w_h are known points on the CVR line, and λ is the slope of the w - $\log \sigma$ curve for normally consolidated samples.

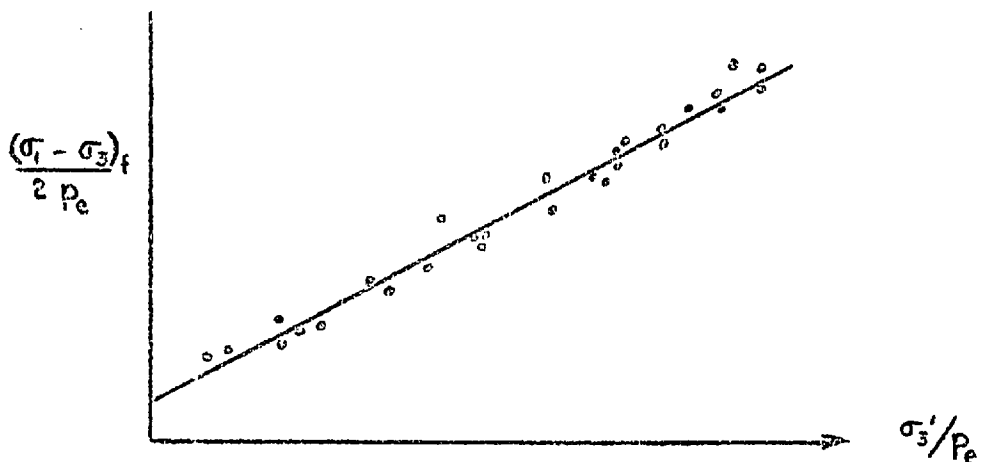


FIG. 2-4. Method of obtaining a satisfactory set of average values for ϕ_p and e_p when many tests are performed, (After BISHOP & HENKEL, 1957).

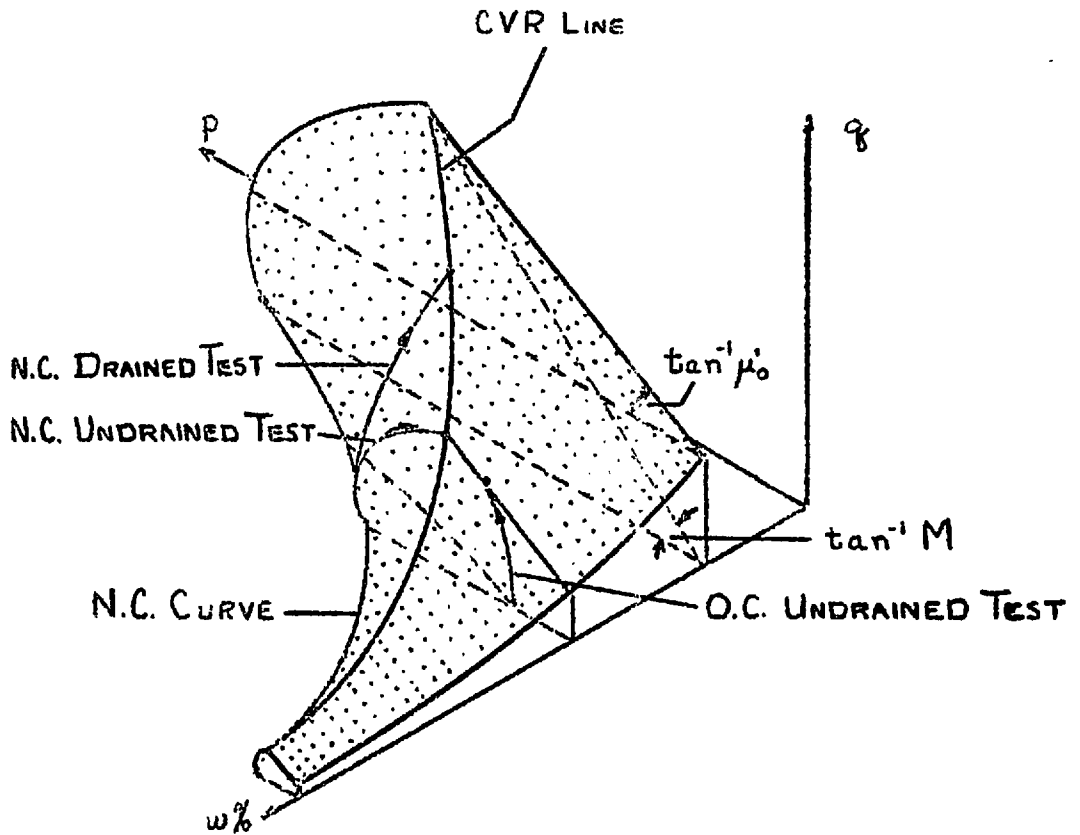


Fig. 2-5. Isometric view of the yield surface for a clay.
(After ROSCOE & POOROOSHASB, 1963).

On the (τ, σ) plane the equation for the CVR line is:

$$\tau = M \cdot \sigma \quad \dots\dots\dots (2-10)$$

where M is the slope of the line. An isometric view of a typical yield surface for a clay is shown in Fig. 2-5, and the two dimensional views are shown in Figs. 2-7 and 2-8.

The surface in space and the CVR line is reputed to hold for all conditions of drainage, and for normally and overconsolidated specimens of cohesive soils. The same general criterion is thought to be valid for cohesionless materials also, but sufficiently accurate test data was not available to allow experimental verification. Most of the confirming data for cohesive soils was published by other experimenters (PENMAN, 1953; PLANT, 1956; HENKEL, 1956; GILBERT, 1954).

In a subsequent paper (ROSCOE & POOROOSHASB, 1963) a further stress-strain relationship to accommodate triaxial test data (only) on normally consolidated clays was proposed. The equation is:

$$\frac{d\varepsilon_v}{d\sigma} = f_1(\eta) \cdot \frac{d\eta}{d\sigma} + f_2(\eta) \quad \dots\dots\dots (2-11)$$

where $f_1(\eta)$ and $f_2(\eta)$ are stress functions of a point in (τ, σ, e)

space, $(d\varepsilon_v/dv)$ is a path dependent variable (i.e. depending on whether the test is drained or undrained), ε_v is the natural or logarithmic axial strain (i.e. the ratio of the incremental height to the current height), v is the natural or logarithmic volumetric strain, and η is equal to $(\sigma_1 - \sigma_3)/p$. The expression (eq. 2-11), due to the complexity of the integration involved, is solved for a given soil by approximate methods; the step-by-step procedure is adequately described in the paper and need not be summarized here. The authors have specified the following conditions for the validity of the relationship: (i) the specimen must be tested in triaxial compression in the conventional way (i.e. $\sigma_1 > \sigma_2 = \sigma_3$); (ii) the state paths must lie on the boundary surface and not within it; thus, tests which have been unloaded or tests on overconsolidated clay are excluded; (iii) the value of η must not decrease at any stage during the test.

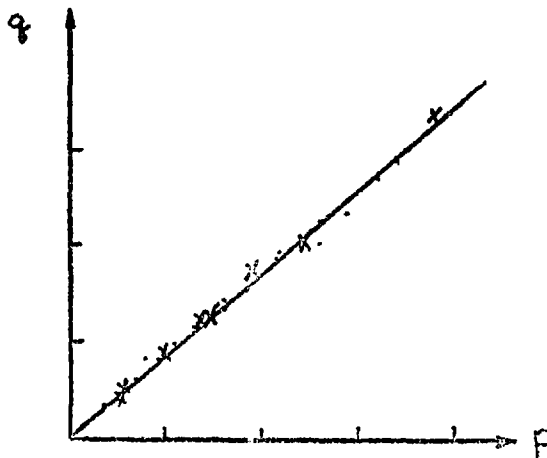


Fig. 2-6

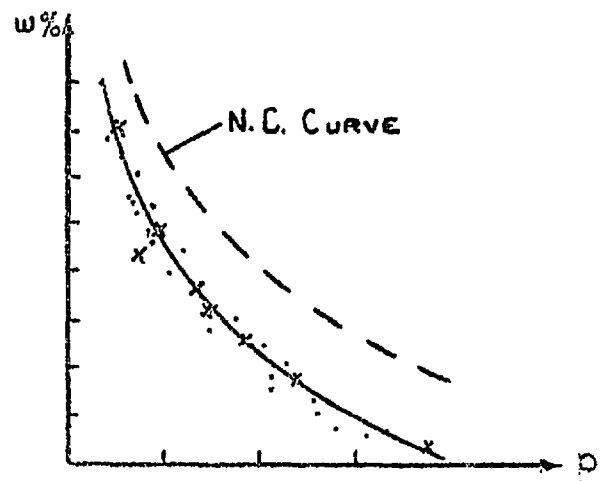


Fig. 2-7

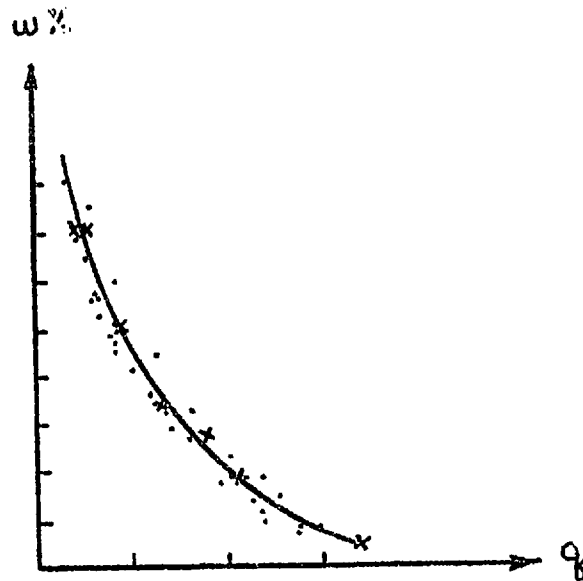


Fig. 2-8

Figs. 2-6 to 2-8: Critical voids ratio line from undrained triaxial tests on a clay (after ROSCOE, SCHOFIELD & WROTH, 1958)

Another approach to the problem of relating applied stresses to the corresponding volume deformations was made by RENDULIC (1936, 1937) when he carried out a series of triaxial tests on normally consolidated Vienna Clay. He established for the first time the

fundamental experimental relationships between axial symmetrical stress systems, void ratios, and pore water pressures. His testing programme consisted of both drained and undrained tests in extension and compression; the undrained tests were conducted with pore pressure measurements so as to obtain the effective stresses. Since no correction was applied to the cross-sectional area, his tests are unreliable at large strains; (HVORSLEV, 1960). Later work by HENKEL (1958, 1959, 1960-a, 1960-b) on Weald Clay improved the experimental relationships induced by the triaxial test but no general formula emerged. By reporting the test data graphically, Henkel was able to depict the three principal stresses with constant water content contours superimposed, so that, in a saturated cohesive material the volume changes could be predicted for a given set of stress increments. (See Fig. 2-9). This plot is known as the unified Rendulic diagram. Henkel, however, did suggest that the change in pore water pressure, Δu , was a function of the octrahedral stresses. His formula is

$$\Delta u = \frac{\Delta \sigma_1 + \Delta \sigma_2 + \Delta \sigma_3}{3} + a \sqrt{\frac{(\Delta \sigma_1 - \Delta \sigma_2)^2 + (\Delta \sigma_2 - \Delta \sigma_3)^2 + (\Delta \sigma_3 - \Delta \sigma_1)^2}{2}}$$

where a is a parameter depending on the soil type.

The foregoing discussion has been concerned, in the main, with empirical theories or hypotheses relating to soil specimens tested in

the shear box, in which the principal stresses are not fully known, or tested under conditions of axial symmetry in the triaxial apparatus. As indicated in Chapter I, stress systems in which the three principal stresses are of unequal magnitude are common in practice, and the theories briefly discussed so far are inadequate to describe these unsymmetrical systems. Comprehensive yield criteria involving the three principal stresses was first established for metals, concrete, crystalline rocks, etc., and the applications of these theories to soils are described by RENDULIC (1938) and KIRKPATRICK (1957). A short summary follows.

The Tresca or Guest failure criterion predicts that the shear strength for a material with $c' = 0$ is equal to the effective cohesion plus some function of the average normal effective stresses, i.e.,

$$(\sigma_1 - \sigma_3) = c' + \mu/3(\sigma_1' + \sigma_2' + \sigma_3') \quad \dots\dots\dots(2-12)$$

When extended to principal effective stress space, the failure envelope is a pyramid with a regular hexagonal right section as is depicted in Fig. 2-10.

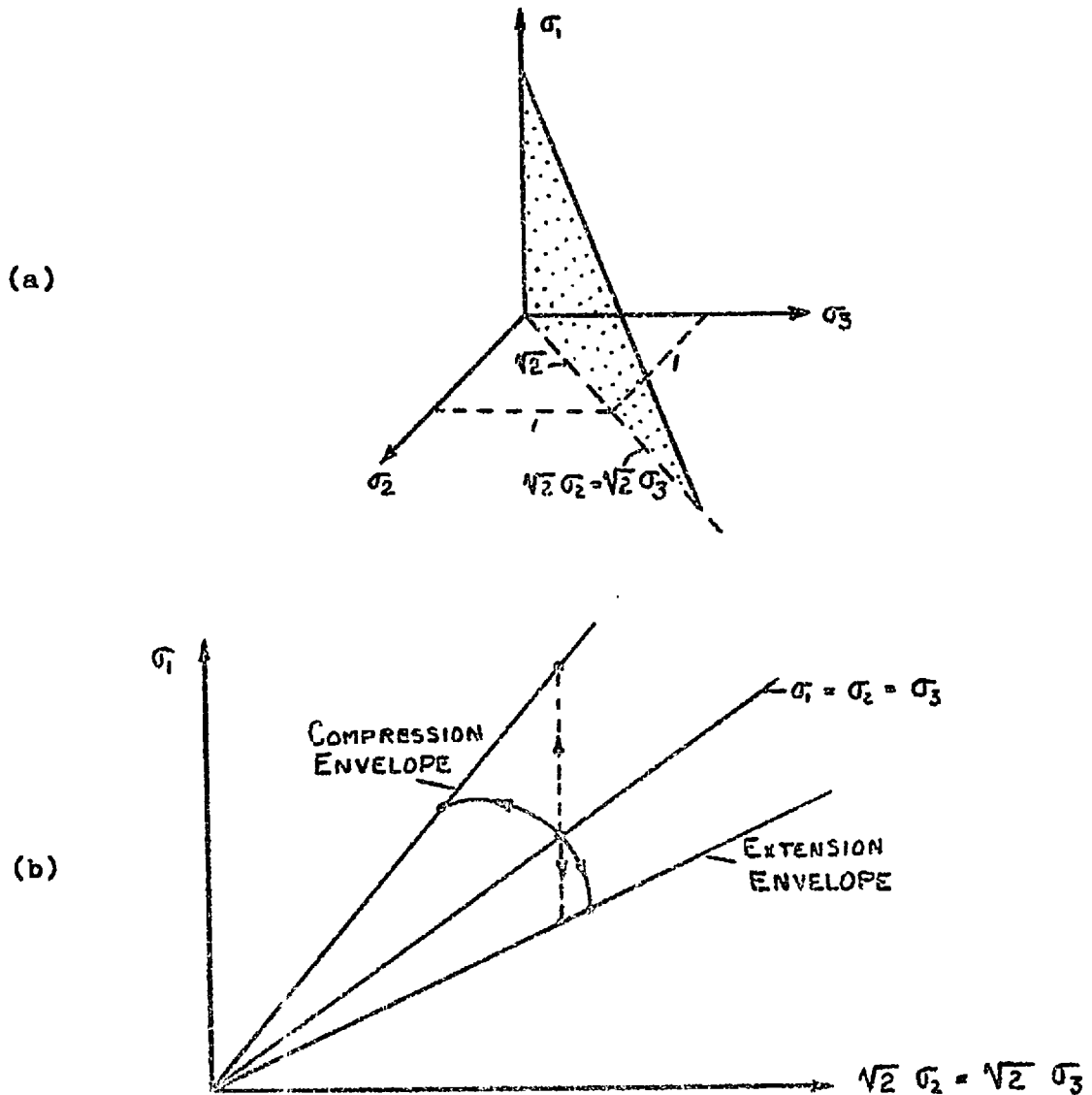
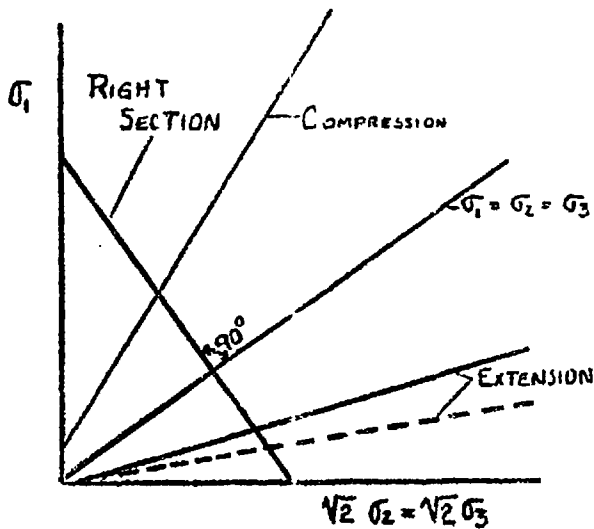
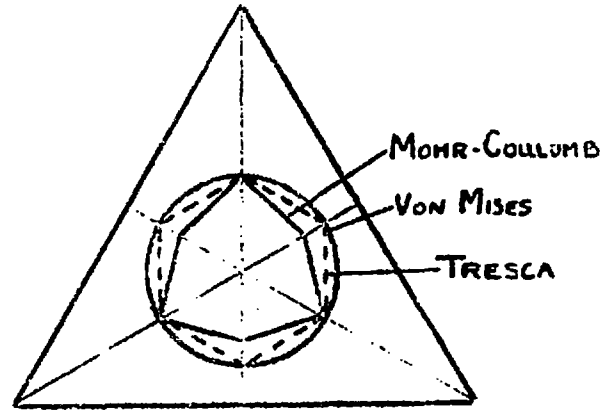


Fig. 2-9.

- (a) Shaded portion is the stress plane that can be investigated in the conventional triaxial test;
- (b) Another view of the stress plane for axially-symmetrical stresses showing the compression and extension envelopes established by the drained tests (dashed) and the undrained tests (solid lines). The undrained stress path is also a constant water content contour. For materials with an effective cohesion c' , the compression and extension failure envelopes have intercepts on the vertical and horizontal axes respectively.



(a) FAILURE ENVELOPES



(b) RIGHT SECTION

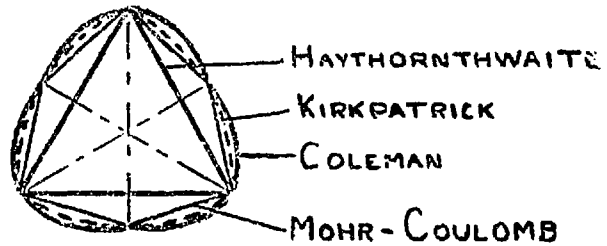


Fig. 2-10. Failure surfaces in principal stress space (after HVORSLEV, 1960; and COLEMAN, 1960).

Note: A point in principal stress space is denoted by the variables $\sigma_1, \sigma_2, \sigma_3$, where the subscripts 1, 2, 3, refer to the line of action of the stresses and are independent of the magnitudes of the stresses; i.e. σ_1 does not necessarily denote the major principal stress.

Theoretical and experimental investigations on many ductile metals and plastic materials by von Mises and others are summarized by the following failure criterion:

$$(\sigma'_1 - \sigma'_3)^2 + (\sigma'_2 - \sigma'_3)^2 + (\sigma'_3 - \sigma'_1)^2 = \left[c' + \mu/3(\sigma'_1 + \sigma'_2 + \sigma'_3) \right]^2 \quad \text{.....(2-13)}$$

The right section is a circle which circumscribes the Tresca hexagon.

COLEMAN (1960) postulates, by considering the failure envelope of an ideal heavy clay in three dimensional principal effective stress space, that the effective angle of internal friction ϕ' is greater than that obtained by the conventional triaxial test when the intermediate principal stress is varied between the major and minor principal stresses. His equation for the envelope in stress space is:

$$-3 \times \Delta'_2 = \left[(2J'_1)/3 + (\Delta'_3/2)^{1/2} \right]^2 \times \sin^2 \phi' \quad \text{.....(2-14)}$$

where J'_1 = the first invariant of the effective stress tensor
 $= \sigma'_1 + \sigma'_2 + \sigma'_3 = 3p'$

Δ'_2 = the second invariant of the effective stress deviator tensor

$$= (\sigma'_1 - p')(\sigma'_2 - p') + (\sigma'_2 - p')(\sigma'_3 - p') + (\sigma'_3 - p')(\sigma'_1 - p')$$

Δ'_3 = the third invariant of the effective stress deviator tensor
 $= (\sigma'_1 - p')(\sigma'_2 - p')(\sigma'_3 - p')$

The Mohr-Coulomb criterion when extended to stress space has the general equation:

$$\begin{aligned} & \left[(\sigma'_1 - \sigma'_2)^2 - (2c' \cos \phi' + (\sigma'_1 + \sigma'_2) \sin \phi')^2 \right] \left[(\sigma'_2 - \sigma'_3)^2 \right. \\ & \quad \left. - (2c' \cos \phi' + (\sigma'_2 + \sigma'_3) \sin \phi')^2 \right] \left[(\sigma'_3 - \sigma'_1)^2 \right. \\ & \quad \left. - (2c' \cos \phi' + (\sigma'_3 + \sigma'_1) \sin \phi')^2 \right] = 0 \\ & \text{.....(2-15)} \end{aligned}$$

In equation 2-15, Hvorslev's true parameters, (c_e and ϕ_e), as well as the parameters suggested by Gibson (c_r and ϕ_r), can be used in place of c' and ϕ' .

For conditions where the actual failure points fall between the Mohr-Coulomb and the Tresca (or von Mises) criteria, unified diagrams or expressions cannot be obtained by use of the principal stresses, octahedral stresses or stress invariants; (see HVORSLEV, 1960). It has been proposed by JOHANSEN (1958) that the stresses used (i.e., the normal and shear stresses) should be in a plane for which the direction cosines of the normal to the plane are $1/(2 + \alpha^2)^{1/2}$, $\alpha/(2 + \alpha^2)^{1/2}$, and $1/(2 + \alpha^2)^{1/2}$. The required stresses τ_α and σ'_α are then defined as

$$\sigma'_\alpha = \frac{\sigma'_1 + \alpha^2 \sigma'_2 + \sigma'_3}{2 + \alpha^2} \text{.....(2-16 a)}$$

$$\text{and } \tau_{\alpha} = \frac{\left[\alpha^2 (\sigma_1' - \sigma_2')^2 + (\sigma_1' - \sigma_3')^2 + \alpha^2 (\sigma_2' - \sigma_3')^2 \right]^{1/2}}{2 + \alpha^2} \dots (2-16 \text{ b})$$

and his suggested failure condition is that the shearing stress τ_{α} on this plane is directly proportional to the normal stress σ_{α}' or,

$$F(\sigma_{\alpha}', \tau_{\alpha}) = k = \text{constant} \dots \dots \dots (2-17)$$

which for small values of α , where τ_{α} is approximately equal to $\tau_m = \frac{1}{2}(\sigma_1' - \sigma_3')$, becomes:

$$F(\sigma_{\alpha}', \tau_m) = k = \text{constant} \dots \dots \dots (2-18)$$

Johansen's criterion, in effect, gives the intermediate principal stress a weighted value. (The direction cosines, say l, m, n, of a line are defined as the cosines of the angles it makes with the mutually perpendicular lines Ox, Oy, Oz. The direction cosines are connected by the relation $l^2 + m^2 + n^2 = 1$) (JAEGER, 1962).

The various yield criteria summarized above will be judged and compared by the way in which their predictions fit the experimental results of this thesis.

(2.3) Experimental Investigations

To avoid unnecessary repetition and to condense the review of experimental results, only tests which approximate to the plane deformation condition or permit the assessment of the three independent principal stresses will be discussed.

KJELLMAN, (1936), tested cubical specimens of dry standard German sand in a triaxial apparatus which permitted independent control of the three principal stresses. His apparatus, shown in Fig. 2-11, consisted of three separate frictionless steel pistons to apply the mutually perpendicular principal stresses to the sample. Reaction plates were positioned around the sample on opposite sides to the loading pistons. The load was transferred to the specimen through the square section (0.6 cm. square) brass rods (100 in number) in contact with each face. The rods, 32 mm. long and with a 0.02 cm. gap between them, were butted against the specimen on all sides with the result that about 3.2% deformation could be effected under conditions of zero side friction. Soils which require larger strains before failure is attained could not be tested in this apparatus without side friction being introduced.

One of the earliest experimenters to obtain data on unsymmetrically stressed soil specimens, Kjellman found that his compression tests (i.e. $\sigma_1 > \sigma_2 = \sigma_3$) gave $\phi' = 35^\circ$, whereas tests approximating to plane strain (in which the intermediate principal stress was varied so that $\sigma_1 > \sigma_2 > \sigma_3$) gave $\phi' = 45^\circ$. The value of the

intermediate stress in the latter series was 0.42 and 0.50 (two tests) times the sum of the major and minor principal stresses. He concluded that the higher strength in the latter case was due to the effect of the intermediate principal stress.

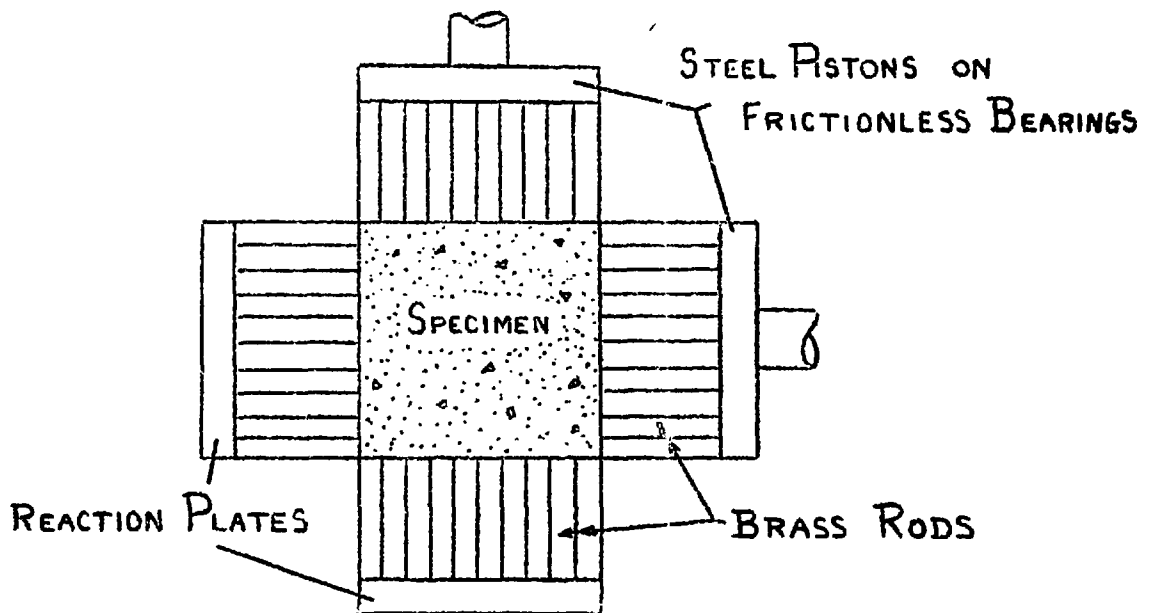


Fig. 2-11. Two dimensional view of Kjellman's triaxial apparatus.

PELTIER (1957) used a modified shear box to carry out tests on various sands. The apparatus consisted fundamentally of a box with two opposing sides horizontally adjustable so that the intermediate principal stress could be varied independently. The major principal stress was applied vertically as in the standard test, and the shearing stress was applied horizontally and perpendicular to the intermediate stress. A sketch of Peltier's apparatus is given in Fig. 2-12. His conclusion was that the angle of internal friction is a maximum of five degrees greater than that predicted by the Mohr-Coulomb criterion when the intermediate principal stress is varied. By conducting other standard triaxial compression and extension tests on the same sands, he obtained a ϕ' in compression in excess of the ϕ' in extension by about seven degrees. In summarizing his work he maintains that the above values of ϕ' are only tentative, and much more research is required to accurately define the strength characteristics of sands. Peltier's apparatus is subject to the same criticism, however, as the standard shear box, namely that the induced failure plane is open to question as to whether it is a plane of rupture or a plane of maximum shear stress (BISHOP, 1957).

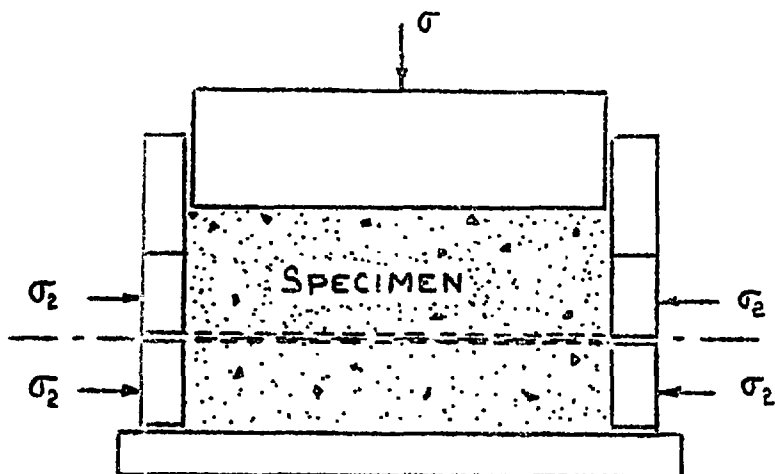


Fig. 2-12. Feltner's modified shear box.

Standard triaxial compression and extension tests as well as thick cylinder tests were performed on a medium fine sand by KIRKPATRICK (1957), to determine the condition of failure for sands. The triaxial tests were used to confirm the Mohr-Coulomb criterion, while the thick cylinder tests were devised to obtain values of the intermediate principal stress different from either the major or minor principal stresses. The specimens in the thick cylinder tests were hollowed out cylinders with an outside diameter of 4 inches and a bore diameter of $2\frac{1}{2}$ inches. The height was approximately 6 inches. Failure was brought about by keeping the outside pressure constant

and increasing the bore pressure until tension cracks appeared in the side of the specimen. Drained tests were carried out, and the cross-sectional area was measured after the specimen had failed. The stress system (see Fig. 2-13) was not uniform throughout the sample, but the simplifying assumption of stress linearity through the sample wall permitted calculations of the major and minor principal stresses. By presenting his results on a three-dimensional stress space diagram (see Fig. 2-10), Kirkpatrick showed that the experimental points lay near to but slightly outside the Mohr-Coulomb surface. He concluded that the angle of internal friction is influenced by the variation of the intermediate principal stress between the limits of the other two principal stresses, and that it is greater by about two degrees than the corresponding ϕ' in the standard triaxial test. His compression and extension tests were in good agreement, when ϕ' 's were compared, over the whole range of obtainable porosities. No measurements were made, however, of the corresponding strains under the stress systems applied in the hollow cylinder tests, and it must be assumed that the condition of plane strain was not definitely achieved. Possibly the slight departure from the Mohr-Coulomb surface might have been increased had the condition of plane deformation been adhered to, since he concedes that the general yield pattern in sand may depend to some extent on strain. The complicating factor is, of course, that the average principal

stresses obtained from the simplified stress analysis are probably not sufficiently accurate to depict the actual failure phenomena under such strain-controlled conditions.

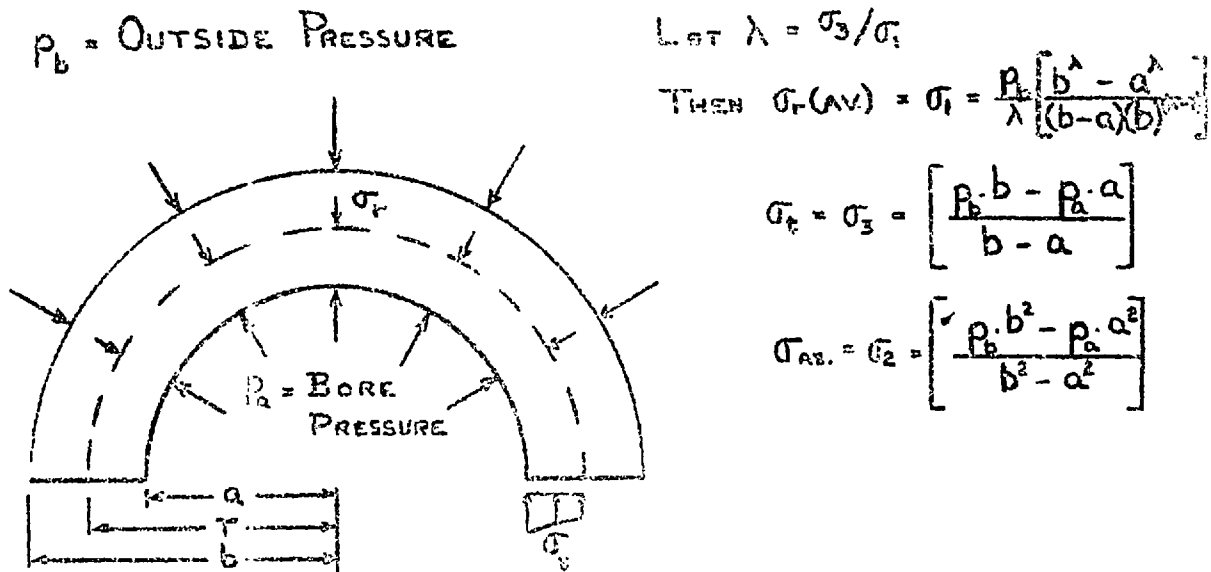


Fig. 2-13. Half plan view of hollow cylinder specimen used by KIRKPATRICK (1957), showing stress system.

CHRISTENSEN (1961) carried out model tests at the Danish Geotechnical Institute to determine the plane active earth pressures in a sand. Although the main object of his programme was to obtain a number of earth pressure coefficients measured for different

densities and other known parameters, the relationship between the void ratio and the angle of internal friction was determined by elimination between formulae. The apparatus used consisted of a wooden box 100 cm. long x 100 cm. wide x 50 cm. deep. One wall of the box was divided into halves horizontally, the top half being able to rotate about a horizontal axis parallel to the wall and located outside and below the wall. The box was filled with sand (level with the top in one series of tests, and 10 cm. from the top in another series, to investigate scale effects), and the motor-driven rotatable wall was rotated away from the sand. The maximum readings on the electric strain gauge type proving rings existed at the beginning of the tests when the horizontal earth pressure corresponded to the "at rest" pressure; the minimum readings, which were approached as the wall rotated, were assumed to correspond to the active earth pressure condition. Zero readings were taken before placing the sand in the apparatus and after the sand was removed. To avoid unwarranted zero drift in the strain gauges the tests were completed within eight hours. Initial void ratios were used in the plot of void ratio versus angle of internal friction. By comparing with a previously determined relationship between e and ϕ , Christensen concludes that "the friction angles found in the present model tests on plane active earth pressures in sand are generally about 4° - 5° greater than those found in triaxial tests carried out at the same initial void ratios" (in the same sand). The results are plotted in

Fig. 2-15.

HAYTHORNTHWAITE (1960) tested a remoulded silt under a system of compound stresses by subjecting hollow cylinder specimens to hydrostatic, axial and torsional stresses. All specimens were consolidated initially at 100 psi. for 14 hours prior to testing. One series of tests was conducted by lowering the hydrostatic pressure to 40 psi. and failing the specimen by increasing the axial and torsional loads in a ratio held constant for each test. (The axial load was applied as a tensile and a compressive force in different tests). The second series consisted of applying particular values of excess axial and torsional loads, and then reducing the hydrostatic pressure until failure occurred. Further series were carried out under axial tension (i.e. the standard triaxial extension test), and simple torsion without axial force; good agreement with the Mohr-Coulomb criterion was achieved in these tests. A substantial departure from this criterion, however, was found in the first two series. Haythornthwaite neglects to show either the tabulated results of his work or the expressions relating the principal stresses to the compound applied stresses. Furthermore, it has been suggested by WU, LOH & MALVERN (1963) that Haythornthwaite considered only total stresses, a fact which would render his comparisons invalid.

The plane strain apparatus at Imperial College, designed by Dr. Bishop with the assistance of Dr. Wood, was used to test a compacted granulite moraine (WOOD, 1958) and a medium-to-fine sand

(CORNFORTH, 1961). The apparatus, a special modification of the standard triaxial machine, is designed to apply the three principal stresses at right angles to each other on a rectangular prism of soil. The machine is limited, if the plane strain condition is to be maintained, to the testing of specimens in compression once they have been consolidated under conditions of no-lateral-yield. More complete details of the plane strain test are given in Chapter 4. Wood carried out a series of undrained tests with pore pressure measurement in both the plane strain apparatus and the standard triaxial machine. Considering failure to occur at the maximum deviator stress, he found that the plane strain tests on moraine gave $c' = 1.9$ psi. and $\phi' = 36.9^\circ$ while the triaxial tests yielded $c' = 0.2$ psi. and $\phi' = 35.1^\circ$, or a difference in the angle of friction of almost two degrees. If the maximum principal stress ratio is considered to be the failure point, the plane strain parameters c' and ϕ' exceeded the corresponding triaxial parameters by 1.0 psi. (approximately) and 3.9° respectively. At failure, the magnitude of the intermediate principal stress was 0.3 times the sum of the other two principal stresses. Cornforth's sand tests showed that the ϕ' in plane strain was $\frac{1}{2}^\circ$ (loose sand) to 4° (dense sand) higher than the corresponding ϕ' for triaxial specimens at the same initial density, and that the intermediate principal stress was 0.27 to 0.36 times the sum of the major and minor principal stresses for loose and dense specimens respectively. The results of both

experimenters showed that the strength of the soils tested in plane strain was about 15% greater than that obtained from axial symmetrical tests. The ratio of the intermediate to the major principal stress was found to be sensibly constant and approximately equal to the coefficient of earth pressure at rest (i.e. K_0), for each soil. With reference to Wood's data, a minor error in the calculated effective stresses could be possible due to the incomplete saturation of the specimens. The actual pressure in the voids of a partly saturated sample consists of some function of the water pressure as well as the air pressure, the latter being greater (BISHOP, ALPAN, BLIGHT & DONALD; 1960). No provision for the measurement of air pressure was made, with the result that the existing effective stresses were probably slightly overestimated.

Recent work at the University of Michigan (WU, LOH & MALVERN; 1963) on both sand and clay has provided weight to the significance of the intermediate principal stress in the failure criteria of soils. Triaxial tests on hollow cylindrical specimens as well as standard compression and extension tests were performed. The hollow cylinder samples, not unlike the ones tested by Kirkpatrick and Haythornthwaite, were nominally of four inches outside diameter and three inches bore diameter. (The height is not given but was probably between five and six inches). They were subjected to independent outside and bore fluid pressures and an axial load. All tests were initially

isotropically consolidated, and then tested under undrained conditions with pore pressure measurement. The remoulded plastic clay used was obtained from a glacial lake deposit and the two batches tested had the following properties: Batch 1 - LL = 52%, PL = 28%, Batch 2 - LL = 56%, PL = 27%. Both batches were tested from an initial moisture content of 40%, and the only obvious difference between them was in the moisture content after consolidation. The sand used was the part of standard Ottawa sand that passes the No. 30 sieve and is retained on the No. 50 sieve. (U.S. standard sieve sizes). On the clay, the conventional triaxial extension and compression tests gave $c' = 0$, and $\phi' = 32^\circ$, with little significant difference in the two types of test. The sand ϕ' values were also equal in extension and compression. The authors conclude that the clay results substantiate the Mohr-Coulomb criterion, better agreement being obtained with the Hvorslev parameters than with the effective stress parameters. As a result, it was proposed that the intermediate principal stress has no influence on the strength of clay. In the sand, however, the opposite was observed; that is, the intermediate principal stress, when it has a value between the other two principal stresses, caused the angle of friction to be increased by about 5 degrees above that obtained in the extension and compression tests. Hence, the Mohr-Coulomb theory does not hold for sand when viewed in stress space.

One shortcoming of the otherwise excellent paper was the omission of data on observed strains or deformations. The only volume change measurement mentioned occurred during consolidation. In the conventional triaxial tests on the clay, the relatively quick rate of testing (3% per hour) could cause inadequate dissipation of pore pressure, which would in turn involve an overestimate of the soil strength.

An apparatus has been constructed at the Norwegian Geotechnical Institute that is similar to the Imperial College plane strain device but much simpler in design. In it rectangular prismatic specimens 4 cm. wide x 12 cm. high x either 30 or 60 cm. long can be subjected to compressive stresses; no end restraint is used on the specimen ends as it is assumed that the frictional forces along the top and bottom platens are sufficient to maintain two-dimensional deformation. Since the apparatus was designed primarily to test cohesionless materials, the specimen is not enclosed in a cell, and the effective lateral pressure is obtained by applying a vacuum to the base of the sample. The sample is, of course, contained in a rubber membrane. The negative internal pressure is held constant for any one test, and its magnitude is determined by a mercury manometer at the top and bottom of the sample.

KUMMENEJE, (1957), conducted tests on a fine-grained quartz sand in this apparatus, and for comparative purposes in a large (8 cm.

diameter x 20 cm. high) and small (3.6 cm. diameter x 8 cm. high) standard triaxial apparatus. He found that the tests approximating to plane strain gave a higher ϕ than the corresponding triaxial tests for the same porosity at the end of consolidation. The largest difference occurred in the medium porosity range and was 3-4 degrees higher. The vertical strains at failure were generally lower in the plane strain specimen.

HABIB, (1953), performed drained torsion tests on loose Fontainebleau sand and compared them with standard triaxial compression and extension tests. The specimens used in the torsion tests were similar in shape to the triaxial specimens but were enlarged at the ends to ensure failure in the central zone. Torque was applied through the top cap so that the sample was twisted relative to the fixed base until failure occurred. An axial load and a confining pressure were applied in addition to the torque. The principal stresses were calculated from the torsion test results by assuming the validity of the plasticity theory. The stress system and the Mohr diagram for the torsion test is shown in Fig. 2-14.

From the triaxial tests, Habib obtained a ϕ in compression greater than the ϕ in extension by 7° (i.e. $\phi_c = \phi_e + 7^\circ$), whereas the torsion tests produced intermediate values of ϕ . No data on the porosities of the respective specimens is given. Typical values of the friction angle are : ϕ (compression) = 32° ; ϕ (extension) = 25° ;

ϕ (torsion) = 36° . These results indicate that the strength of sand is influenced by the magnitude of the intermediate principal stress.

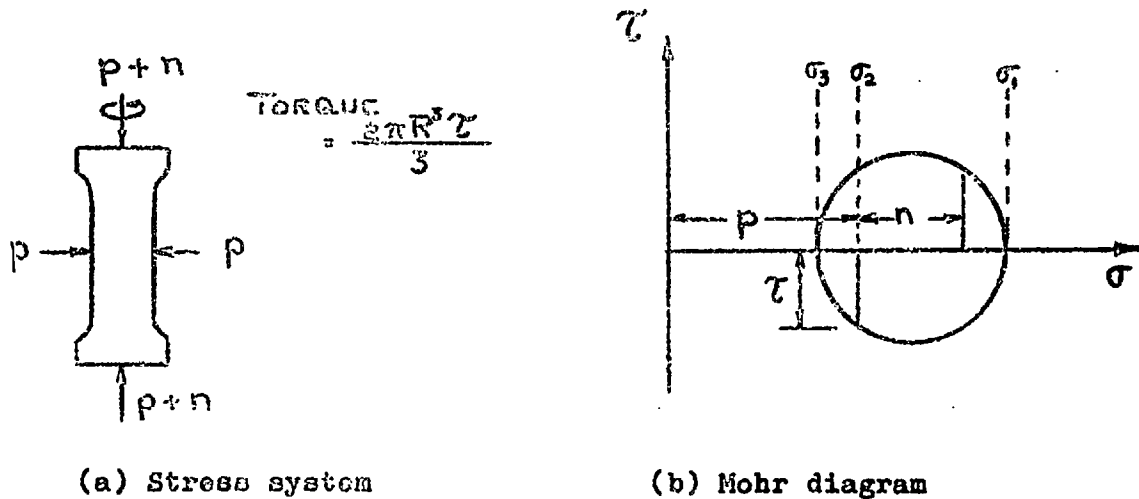
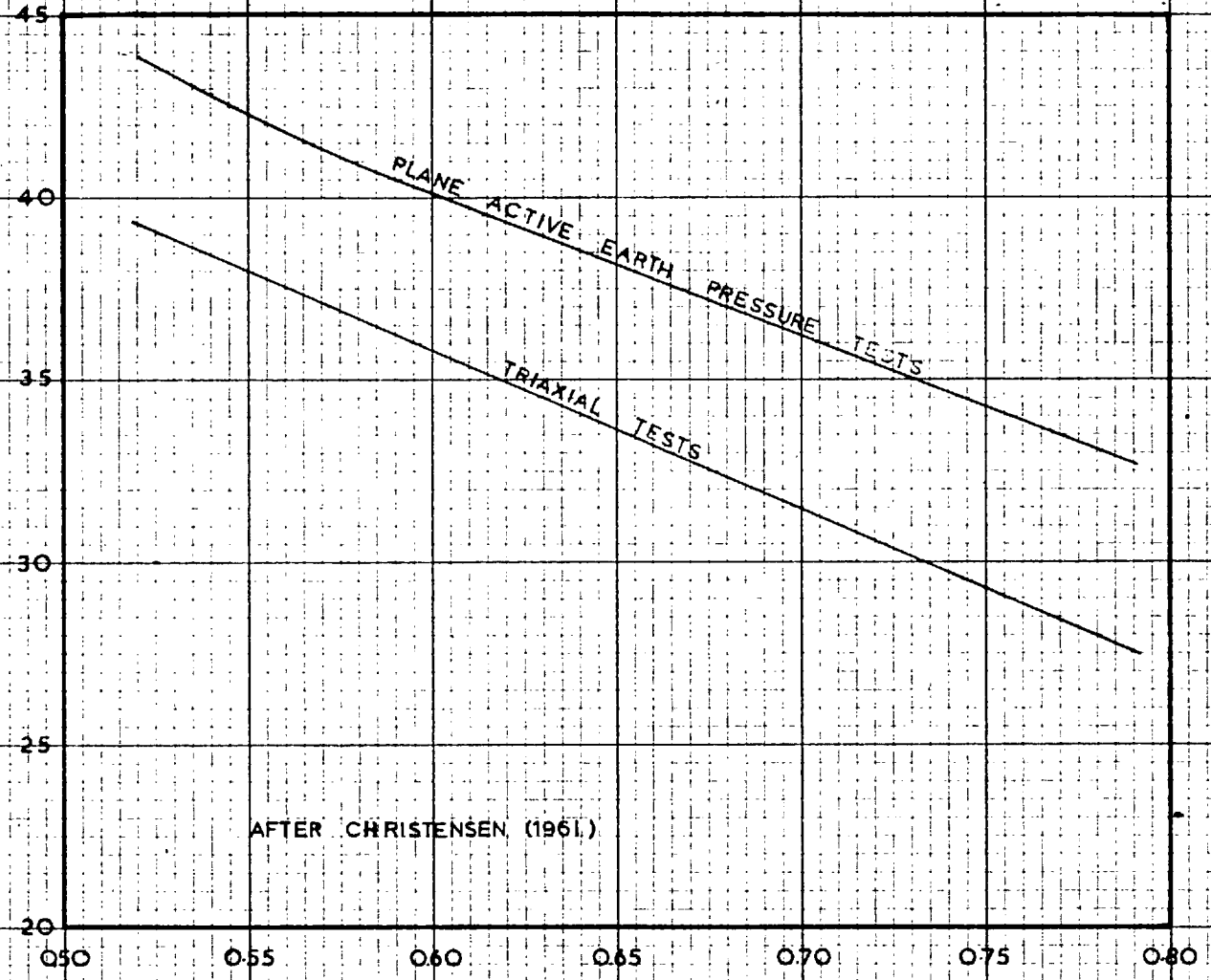


Fig. 2-14. Habib's torsion test.

(2.4) Summary

Published work on the yielding of soils under various loading systems indicates that the strengths of moraines and sands under plane strain conditions are greater than those predicted by the Mohr-Coulomb criterion. Intermediate values of the second principal stress tend to increase the strength of sands, but have little apparent effect on clay strength. Hypotheses are presented to express the generalized yield phenomena of soils in terms of constant parameters.

ϕ°



AFTER CHRISTENSEN (1961.)

INITIAL VOID RATIO

FIG. 2-15

CHAPTER III

DESCRIPTION OF APPARATUS

The plane strain apparatus was designed and constructed by Dr. Bishop with Dr. Wood's assistance, and the detailed apparatus drawings and descriptions are presented elsewhere (WOOD, 1958). It is therefore sufficient to review briefly the main components of the machine and to describe in detail only the auxiliary equipment required for conducting undrained tests on saturated, remoulded cohesive soils.

(3.1) General

The apparatus is basically a cell which contains the specimen and can withstand internal pressure while transmitting a vertical load to the specimen. It consists of three parts, a brass top plate, a bronze body casting and a brass base plate, which are joined by 16 vertical tie bars (see Fig. 3-1). The top plate contains the stainless steel rams and the ram housing enclosing the honed bronze rotatable bushes. The rams, identical twins of $\frac{1}{4}$ in. diameter, load the top bearing plate at the quarter points through hemispherical contacts. Vertical friction between the rams and their bushes is eliminated by rotating the honed sleeves with a motor-driven worm gear continuously during a test. 6 perspex observation windows are

fitted into the cell body. Originally designed to centre on the tri-axial machine pedestal, the base plate in long term tests on cohesive materials sits on a large leaf spring and the spring is centred on the pedestal. Specimen loading is provided hydrostatically by the self-compensating mercury control (as per BISHOP & HENKEL, 1957), and mechanically (i.e. vertically) by the triaxial machine's motor-driven gear train which raises the cell and rams against a fixed head, the load being transmitted through two proving rings and the rams to the specimen. The rams are lubricated by oil leads coming from a pressurized oil reservoir and connected to the ram housings. Anti-rotation links prevent the rams being rotated by the revolving bushes.

The sample, a rectangular prism 2" x 4" x 16", is positioned on a brass platen bolted to the base plate. A similar platen is placed on top of the sample and bolted to the loading head. Immediately in contact with the soil on top and bottom is a strip of perspex sheeting $\frac{1}{4}$ in. thick, which acts as a holder for the four porous stones. A copper tube (bottom) and a nylon tube (top) passing through the platens connect the porous stones with a null indicator or a pressurized volume change measuring unit (see BISHOP & HENKEL, 1957). To isolate it from the cell water, the specimen is enclosed in a rubber membrane which passes under the bottom platen and over the top platen. A machined recess in the bottom platen contains an O-ring which effects a watertight seal with the membrane

around the perimeter of the platen when the platen is bolted to the base plate. A similar seal with the membrane is made between the top platen and the loading head. To prevent cell pressure leakage, further recesses are machined in the top and bottom flanges of the cell body and the seal is completed against the enclosed O-rings around the perimeter when the cell is bolted together with the tie bars.

An outstanding feature of the plane strain apparatus is the σ_2 belt (termed "end clamp" by WOOD, (1958) and CORNFORTH (1961)). It is this device that maintains the plane strain condition during shear and enables the measurement of the intermediate principal stress; a diagram showing the main components is given in Fig. 3-2, and the assembly drawing of Fig. 3-1 depicts its position relative to the sample. The end clamp consists of two anodised aluminium plates held in contact with the ends of the specimen by 4 symmetrically placed tie bars. One end plate has a shallow cylindrical cavity with a fairly stiff rubber diaphragm cover which is held in position by an aluminium ring. A stiffening disc and a loading disc are screwed to the diaphragm such that the rubber forms a slightly flexible annulus. The loading disc transmits pressure to the sample via the removable face plate. The hollowed out end plate, when filled with de-aired water, forms the hydraulic cell which permits the application of the intermediate principal stress with a control

cylinder through the saran lead connected at the top of the end plate. The saran tubing is connected to a null indicator which is calibrated to compensate for the elastic distortion of the tie bars, diaphragm and other parts of the pressure system. The calibration curve is shown in Appendix I. A thin film of silicone grease is applied to the end plate faces in contact with the specimen to minimize end friction as the sample deforms during the test. The coefficient of friction between the greased aluminium and the rubber membrane as determined in the shear box was found to be $\mu = 0.024$; the maximum effect of the end friction on the deviator stress at failure was in the order of 0.3%.

Legs are provided for the plane strain apparatus when it is removed from the triaxial machine pedestal; horizontal and vertical stabilizers are fitted to maintain equilibrium under operating conditions (see Fig. 3-1).

During a test, cell pressure is measured by a Bourdon-type gauge, vertical load is registered on the proving ring dials, and the pressure required to maintain the plane deformation condition is obtained from another Bourdon gauge connected through a null indicator to the σ_2 cell. Changes in specimen height are determined from two dial gauges, one connected to the horizontal arm of each ram. Volume changes in the specimen occurring during consolidation are obtained by recording the change in mercury level in the calibrated volume

change measuring unit, and pore pressures are read from a third Bourdon gauge. The apparatus used for applying and/or measuring the cell pressure, pore pressure, volume change and deviator stress are part of the standard triaxial equipment and more complete details are given in BISHOP & HENKEL (1957); the apparatus for measuring the intermediate principal stress is adequately described by WOOD (1958). Fig. 3-5 gives the pressure system layout.

The additional equipment used in the testing of clay specimens consisted of the following:

- (a) K_0 belt and accessories for indicating departures from the zero-lateral-strain condition during specimen consolidation;
- (b) a servo-mechanism, connected electrically to the lateral strain indicator, to activate the motor for deviator load application (during consolidation);
- (c) a leaf spring positioned between the plane strain cell base and the triaxial machine loading shaft to minimize the relaxation of load on the specimen when the deviator motor was periodically off during consolidation.
- (d) a small motor and gear train arrangement for gradually increasing the cell pressure.
- (e) A Kopp variator.

(3.2) Lateral Strain Indicator

The K_0 apparatus was first designed at Imperial College to maintain the zero-lateral-strain condition in 4-inch diameter tri-axial samples during consolidation. The apparatus used for the plane strain tests is of the same principle but with some modification to suit the specimen shape. With the passing of time a few improvements have been incorporated also.

The lateral strain indicator consists of a belt which surrounds the sample, a perspex unit containing a mercury reservoir and a glass standpipe with a capillary bore, a foot arrangement linking the indicator and the specimen, and a pulley and counterweight system for support. The belt is comprised of two $\frac{1}{4}$ in. square aluminium bars running along each side of the specimen and hinged at one end with two short bars of the same cross-section; the longitudinal bars are each necked to $\frac{1}{8}$ in. wide (plan view, Fig. 3-4) near the σ_2 end plates for additional clearance, and stiffener bars are added adjacent to the specimen. The device containing the mercury reservoir is screwed on one bar at the unhinged end and an adjustable perspex probe is fitted to the opposing bar. In Fig. 3-5, details of the perspex unit for modifying the lateral strain are given; a small movement of the button against the polythene diaphragm produces a large height change in the standpipe mercury column. A rectangular piece of perspex sheeting and a ball bearing, located approximately

at mid-height and mid-length of the specimen, comprises the foot arrangement of the indicator. Even with a light coil spring at the open end holding the belt and feet against the sample, further support was required to keep the indicator in position during the test. This was effected by adding counterweights strung over pulleys positioned on the loading head. The assembled indicator is illustrated in Fig. 3-4.

After the assembly process, final adjustments to the perspex probe are required to ensure that contact is made with the button in front of the mercury reservoir and that the mercury thread is in the proximity of the platinum wire. When the test is in progress, any deviation from the condition of zero-lateral-strain (i.e. any lateral movement) is transferred through the perspex feet and ball bearings to the belt, whence it is registered as a change of height in the mercury thread. The magnification of the lateral movement in the specimen is about 300 to 1 in the device shown in Fig. 3-5; that is, for each 0.001 in. that the sample strains laterally, a change in height of 0.3 in. is registered in the mercury thread. (By observation, it has been established that a mercury thread movement of less than 0.1 in. is all that is required to activate the servo-mechanism; therefore, the actual lateral movement of the sample during consolidation is in the order of 0.0003 in., which is, for all practical purposes, approximately equal to zero lateral strain).

Modifications to the lateral strain indicator centre around the mercury reservoir unit and include the addition of a de-airing screw, the introduction of a raised rim on the front plate to pinch the diaphragm and the addition of a coned hole at the base of the standpipe. The most significant modification was the incorporation of a de-airing screw, without which much time was spent in rendering the chamber air-free. (Entrapped air in the reservoir causes the height of mercury in the standpipe to vary when the device is subjected to an all-round pressure change, the consequences of which impair the accuracy of the K_0 consolidation). The raised rim on the front plate effected a better seal when the reservoir cell was assembled, and the coned hole, by replacing a sharp discontinuity with a gradually reducing cross-section, enabled the mercury column to be less susceptible to breaking.

(3.3) Servo-mechanism

The main components of the servo-mechanism used to automatically control the K_0 consolidation stage of the plane strain test are as follows: a bell transformer, a capacitor, two electro-magnetic switches and a cam-operated switch system. The manual two-way switch, the K_0 (switching) device, and the pilot lamp are the other components shown in the circuit diagram (Fig. 3-6) which are not part of the servo-mechanism unit as such but, with the exception of the lamp, are essential to the operation of the circuit.

The transformer reduces the mains voltage of approximately 240 volts to 5 volts. To prevent excessive sparking in the 5-volt circuit when the K_0 switch is closing, a 1.5 μ f. capacitor is added in parallel to the coil of "A". Electro-magnetic switch "A" when excited completes one circuit, whereas "B" completes two circuits when excited, one being its own and the other being the deviator motor circuit. The switches in "A" and "B" are normally open and are closed only when their respective coils are excited. The cam operated switch system consists of two switches, mechanically joined but electrically insulated, which are activated for about 3 seconds once every minute by a motorised cam being rotated at one revolution per minute. The holding switch is normally closed and is opened for 3 seconds by the cam, while the other switch is normally open and is closed for 3 seconds. The manual two-way switch, which is actually a part of the triaxial machine, is used to stop, start or reverse the deviator motor; for the present discussion it will be assumed that the switch is closed, although the servo-mechanism operation could be explained without reference to the deviator motor circuit at all. The K_0 (switching) device or the K_0 switch is the electrical aspect of the lateral strain indicator described in the previous section. When the consolidation stage of the plane strain test is in progress, an increase in specimen width lowers the mercury in the standpipe and thus opens the K_0 switch; conversely, when the specimen decreases in width the K_0 switch closes. The pilot lamp

was connected into the servo-mechanism circuit to visually indicate when the deviator motor was on.

The operation of the servo-mechanism is as follows: When the cam operated switches are activated for the three second interval, the holding switch is opened and its twin is closed. If the K_0 switch is open, no circuit is completed and the deviator motor remains stationary. If the K_0 switch is closed, the 5-volt circuit is completed, the then excited coil in "A" closes its corresponding switch, and current is allowed to flow to the coil in "B" thus closing the switches there. Consequently, the deviator motor circuit is completed and the motor starts. The motor is kept running after the three second interval because the coil in "B" remains excited. This is accomplished by the holding switch closing again and, due to the inertia of the components in "B", the current flowing through the closed switch in "B" and thus through the coil. As long as the K_0 switch remains closed, the deviator motor will keep running. When the K_0 switch opens, the deviator motor is not stopped until the cam operated switches are activated for the three second interval. The pilot lamp is illuminated only when the coil in "B" is excited, and hence goes on and off with the deviator motor.

This servo-mechanism apparatus was designed and constructed by Dr. Bishop in 1958. The only modification since that time was the addition of the pilot lamp.

(3.4) Leaf Spring

To minimise the relaxation of applied vertical load on the specimen during consolidation when the deviator motor is off, a spring was added between the cell base and the loading shaft of the tri-axial machine. Space limitations required that a leaf spring be used instead of a coil spring, and efficiency considerations necessitated a spring stiffness less than the combined stiffness of the two proving rings. The result was a simply supported leaf spring of three (high tensile steel) leaves that would transfer a 3000-lb. load at mid-span with a maximum total (calculated) deflection of $\frac{1}{4}$ in. The leaf combination is shown in Fig. 3-7.

The length of each 1 in. thick leaf, for the flexibility desired, was calculated as 39.1 in. and the resulting maximum fibre stress under the 3000-lb load was 44,100 psi. The spring constant thus became 4000 lb./in. whereas the combined proving ring constant was 54,600 lb./in. The ratio of the constants is 13.6 which indicates that the spring is 13.6 times as flexible as the twin proving rings under the given load.

The components of the leaf spring (see Fig. 3-7) are: one leaf 4 in. x 1 in. x 42 in., two leaves each 2 in. x 1 in. x 42 in., an end shackle connection, a cap for the loading shaft and a bridge for the cell.

The 4 in. wide leaf is positioned longitudinally under the

centre of the cell and over the loading shaft, with the 2 in. leaves located on each side. Grooves, 39.1 in. apart, are cut for the stainless steel roller bearings in the top of the centre leaf and in the bottom of the outside leaves at each end. Connection between the three leaves is made by the end shackle arrangement depicted in Fig. 3-7. A mild steel cap is used to join the centre leaf to the loading shaft and has a locating pin for the spring. A mild steel bridge, spanning the centre leaf, allows the plane strain cell to be supported by the outside leaves. Flats are machined on the leaves to provide a seating for the loading cap and the bridge.

During a test, load is applied to the bottom of the centre leaf by the triaxial machine and is transferred to the cell via the end connections and the outside leaves through the bridge. When the deviator motor is on, the spring retards the specimen loading; when the motor is off, the spring minimises the load drop-off caused by continuing consolidation of the sample. The spring is intended primarily to prevent excessive load fluctuation during the consolidation stage, and is redundant during the shearing process.

The total deflection of the spring consists of the sum of the centre leaf deflection, which is upward, and the average outside leaf deflection which is downward. Because the maximum clearance between the centre leaf and the cell base was designed as $\frac{1}{8}$ in., loads greater than 3000 lb. were not carried by the spring but were transferred directly through the centre leaf to the cell. The spring

was designed to take only compressive loads. With the addition of the leaf spring to the loading system, the load drop-off during consolidation was reduced by about 93% since the maximum loads attained during this stage never exceeded 3000 lb.

(3.5) Motor and Gear Train Arrangement

Since the consolidation process lasted several days, it was considered expedient to increase the cell pressure at a constant rate instead of incrementally, so as not to cause sudden changes in pore pressure or sample dimensions. This was effected by increasing the height of the mercury pot of the self-compensating mercury control with a small motor-driven gear train. Different rates of cell pressure increase could be obtained by using different gear ratios, the slowest rate being 1.85 psi. per 24 hour day.

(3.6) Kopp Variator

By incorporating a Kopp variable speed unit into the triaxial machine, much finer control of the rate of axial strain was made possible. The advantage of the variator was that the strain rate could be adjusted when the deviator motor was running, a fact which permitted the required decrease in strain rate during consolidation to make the rate of deviatoric stress application comparable to the constant rate of cell pressure increase. (See Chapter 5). The variator could also be used to change the testing rate during the

shear stage if desired. The unit was located between the triaxial machine motor and its gear train.

(3.7) Triaxial Apparatus

For the sand tests in this thesis, standard large diameter triaxial compression and extension test apparatus were used in addition to the plane strain device. The triaxial compression equipment is described in BISHOP & HENKEL (1957) and the extension apparatus is given by CORNFORTH (1961). All triaxial tests were consolidated with the aid of the zero-lateral-strain indicator similar to the one shown in BISHOP & HENKEL.

NOTES TO DRAWINGS

Fig. 3-1 : Plane Strain Apparatus - Assembly Drawing

Section A-A

1. Head of Triaxial Apparatus
2. Head of Adaptor Bar
3. Adjusting screw
4. Frictionless guide
5. Vertical guide bar (or vertical stabilizer)
6. Proving ring and dial gauge
7. Anti-rotation links
8. Vertical loading ram
9. Vertical friction eliminator (rotating bush type)
10. Ram and bush housing
11. Cell top plate casting
12. Cell body casting
13. Perspex observation window (6 No.)
14. Cell base plate casting
15. Tie bar (16 No.)
16. Soil specimen
17. Rubber membrane
18. Top platen
19. Top loading head or bearing plate
20. Porous stones in perspex holder
21. Bottom platen
22. Bridge spanning centre leaf in spring assembly
23. Spring assembly
24. Lead to soil specimen
25. Apparatus for measuring intermediate principal stress
(σ_2 end clamp)
26. Lead to null indicator of intermediate principal stress
27. Cell pressure lead
28. Air release valve
29. K_0 bolt
30. Hydraulic jack
31. Lead to hydraulic jack
32. Loading shaft of triaxial apparatus

Section B-B

- 33. Castor oil lead to ram
- 34. Arm for vertical dial gauge
- 35. Dial gauge for axial deformation
- 36. Horizontal guide bar (in centre of apparatus - added to B-B as an extra)
- 37. Counterweights for K_0 belt
- 38. Pulley and string supporting K_0 belt.

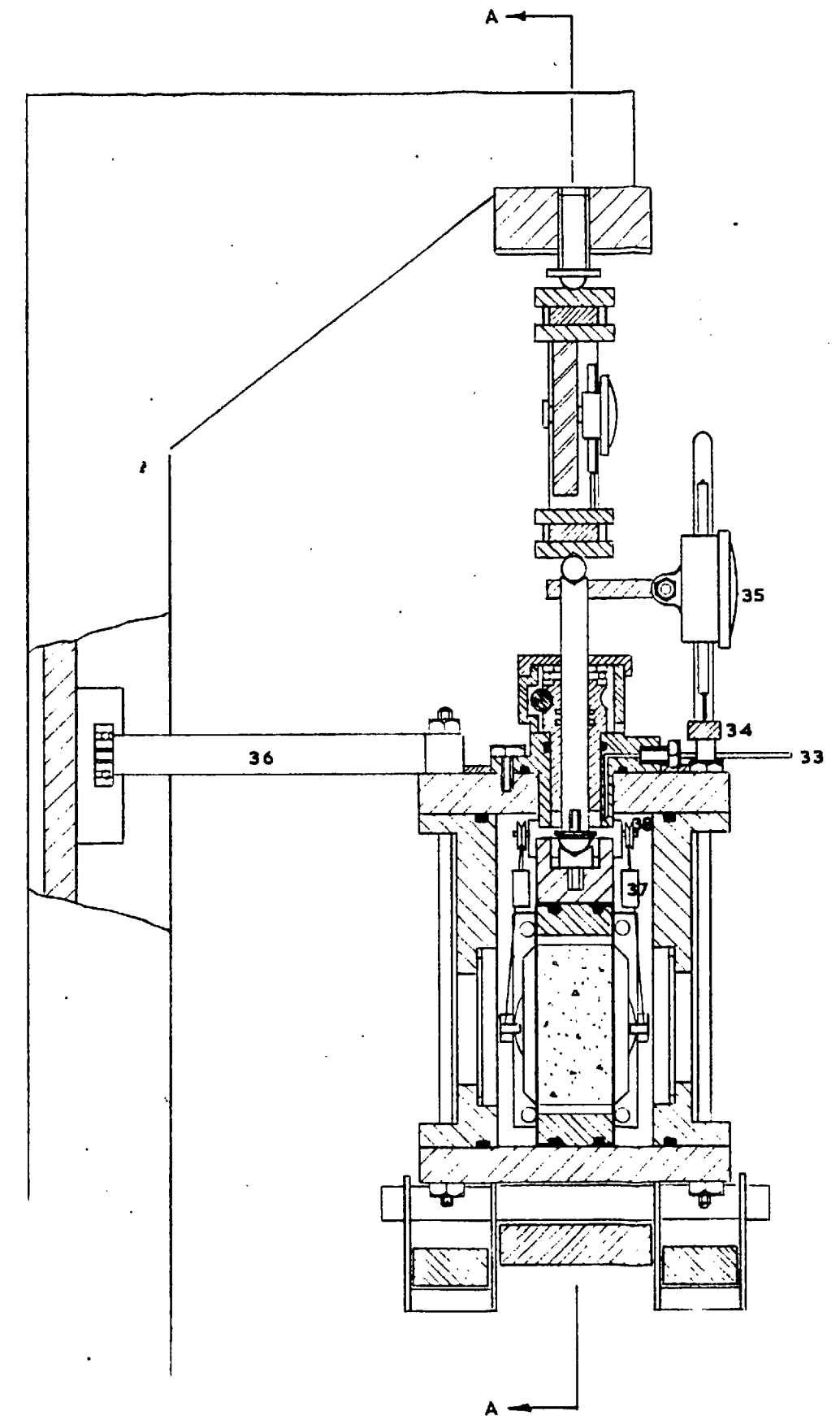
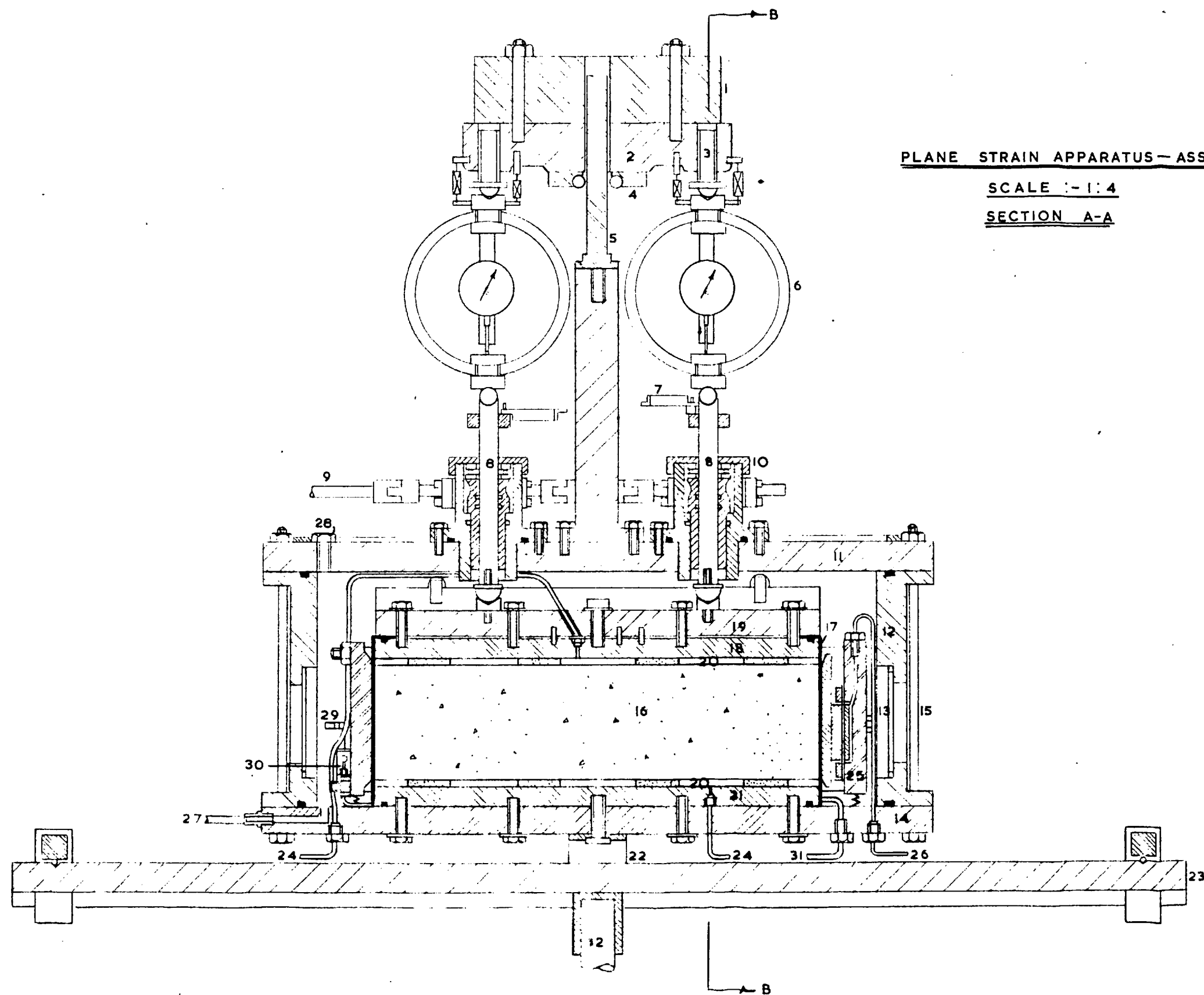
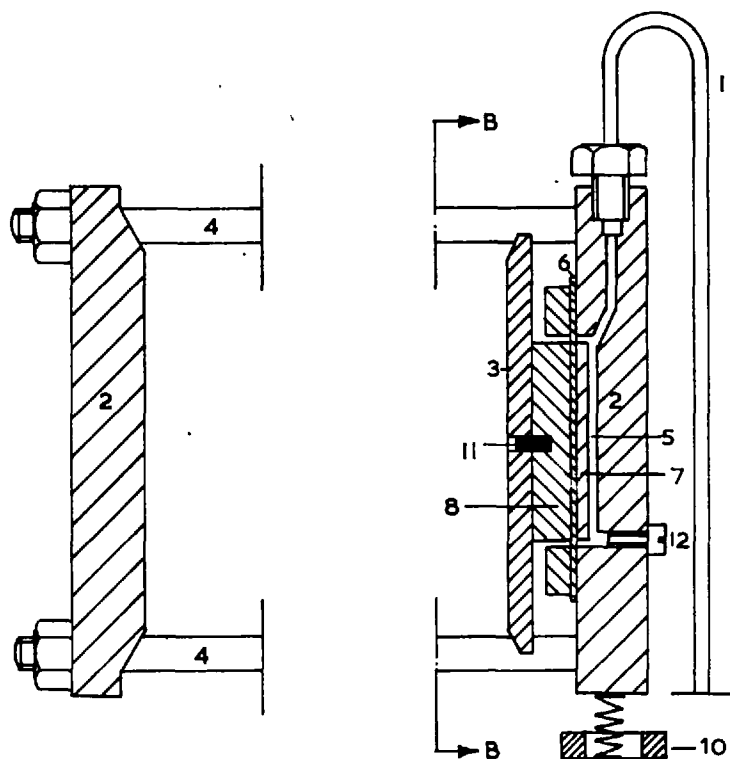


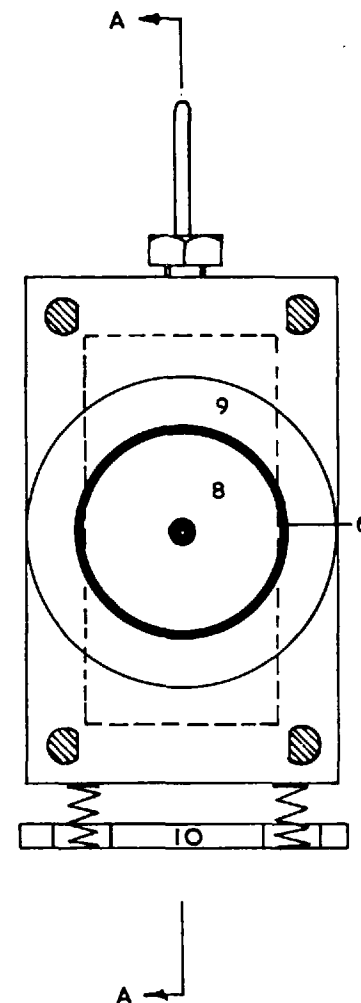
FIG. 3-1

Fig. 3-2 : σ_2 Belt or End Clamp

1. Saran tubing to null indicator of intermediate principal stress
2. End plates
3. Removable face plate
4. Tie bars connecting end plates
5. Hydraulic pressure cell
6. Rubber diaphragm
7. Stiffening plate
8. Loading plate
9. Diaphragm retaining ring
10. End plate support of perspex and springs
11. Locating pin or dowel for face plate
12. De-airing screw.



SECTION A-A (Longitudinal Section)

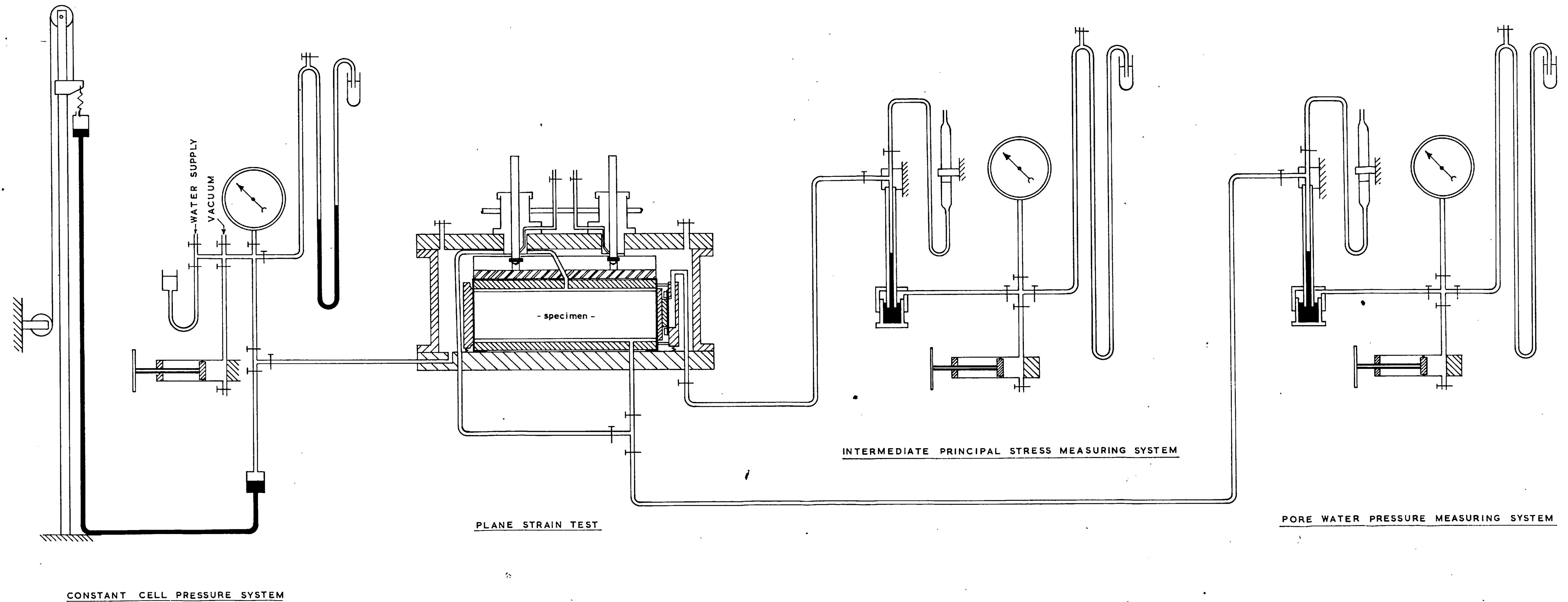


SECTION B-B Without Faceplate

TITLE σ_2 BELT OR END CLAMP

Scale: Half full size

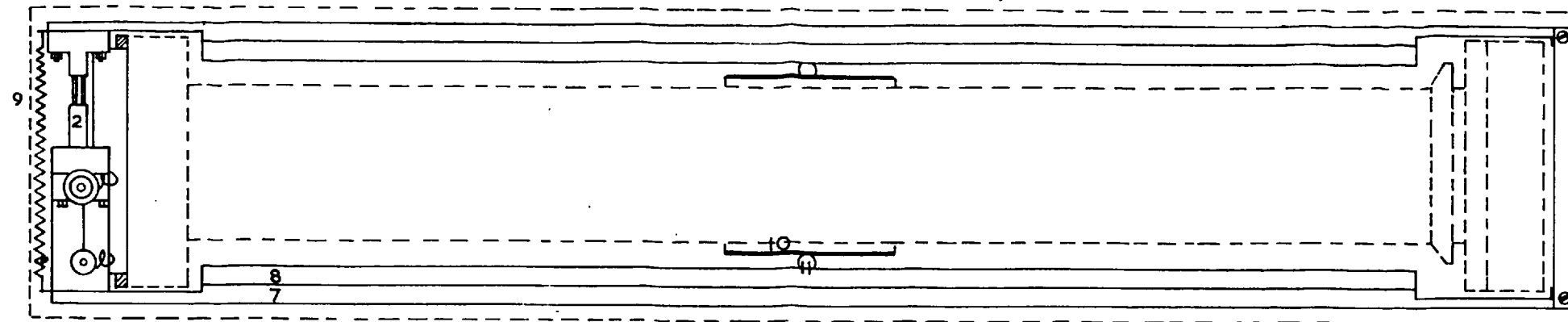
FIG. No. 3-2



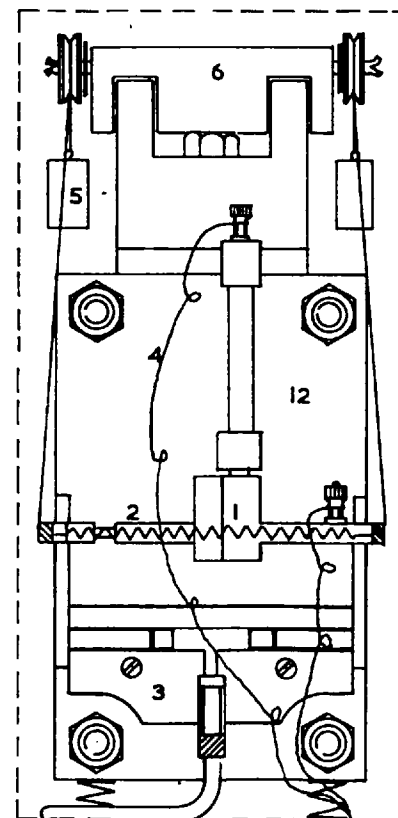
TITLE DIAGRAMMATIC LAYOUT OF APPARATUS FOR UNDRAINED SHEAR TESTS

Fig. 3-4 : K_0 Belt (Lateral Strain Indicator) - Assembly Drawing

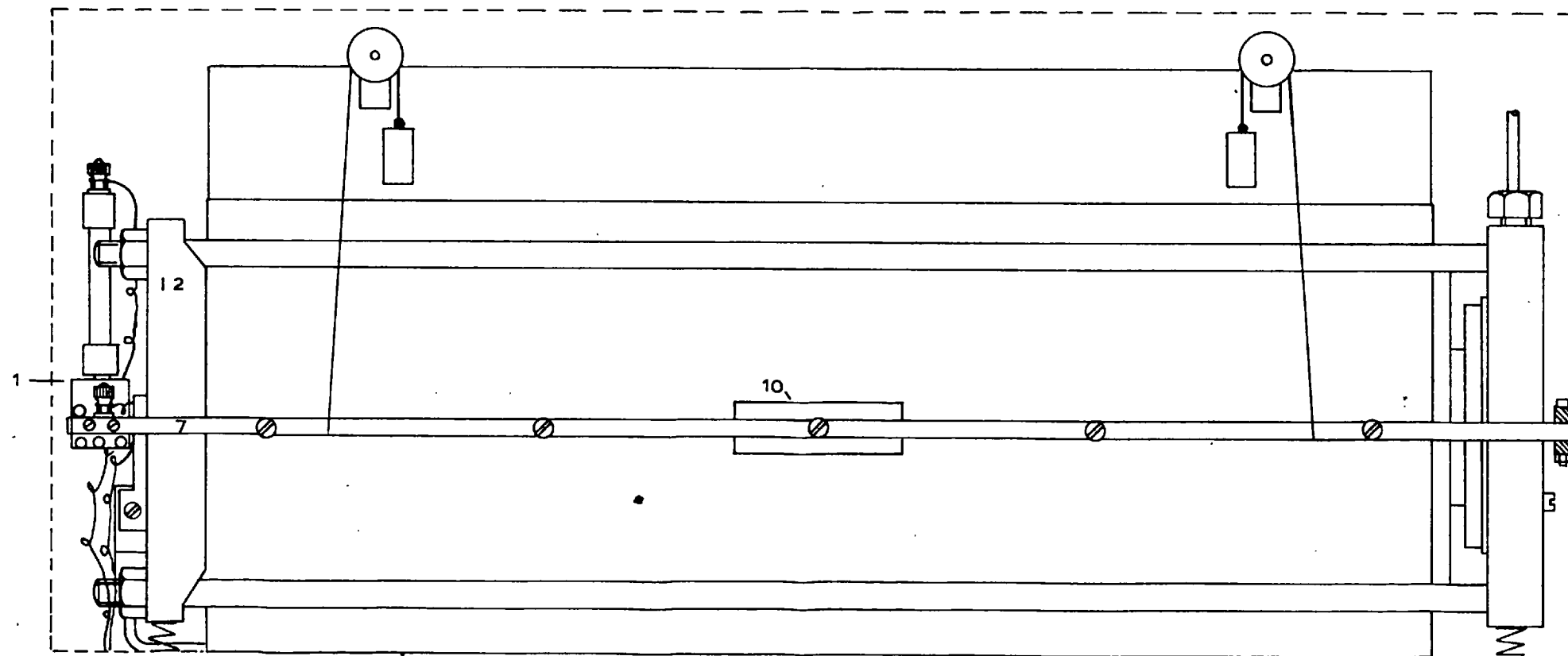
1. K_0 device containing mercury reservoir and capillary standpipe
2. Adjustable probe touching button of K_0 device
3. Manual hydraulic jack of perspex with brass pistons
4. Electrical leads to K_0 device
5. Counterweights for belt assembly
6. Pulleys and mounting assembly
7. K_0 bolt
8. Stiffener bar
9. Light spring to hold the bolt (i.e. the feet) against the sample
10. Perspex foot
11. Stainless steel ball bearing
12. C_2 end clamp.



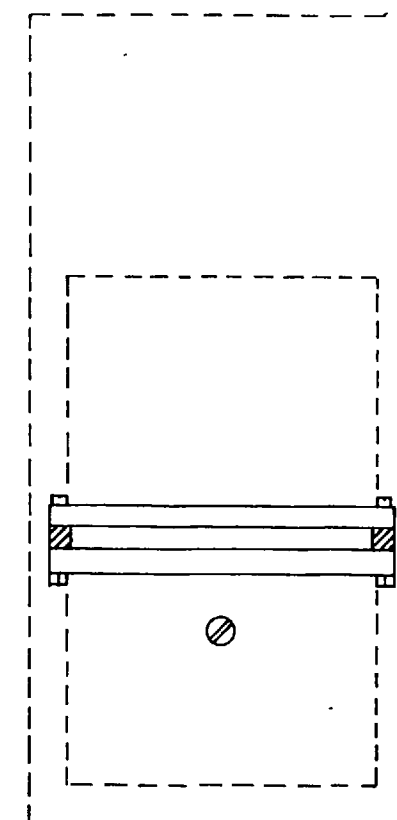
PLAN OF K₆ BELT ONLY



END ELEVATION



FRONT ELEVATION



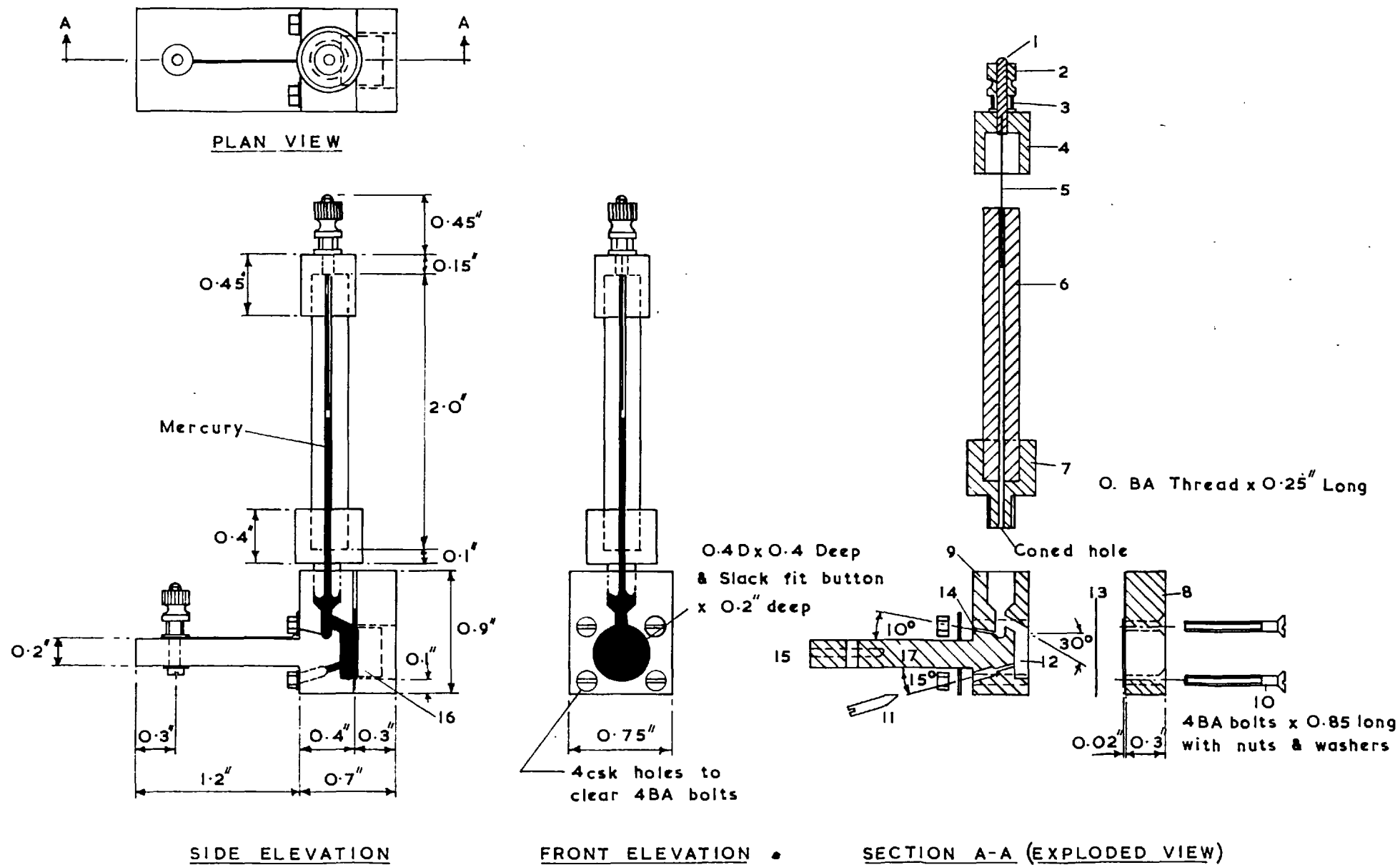
END ELEVATION OF K₆ BELT

TITLE. K₆ BELT ASSEMBLY DRAWING

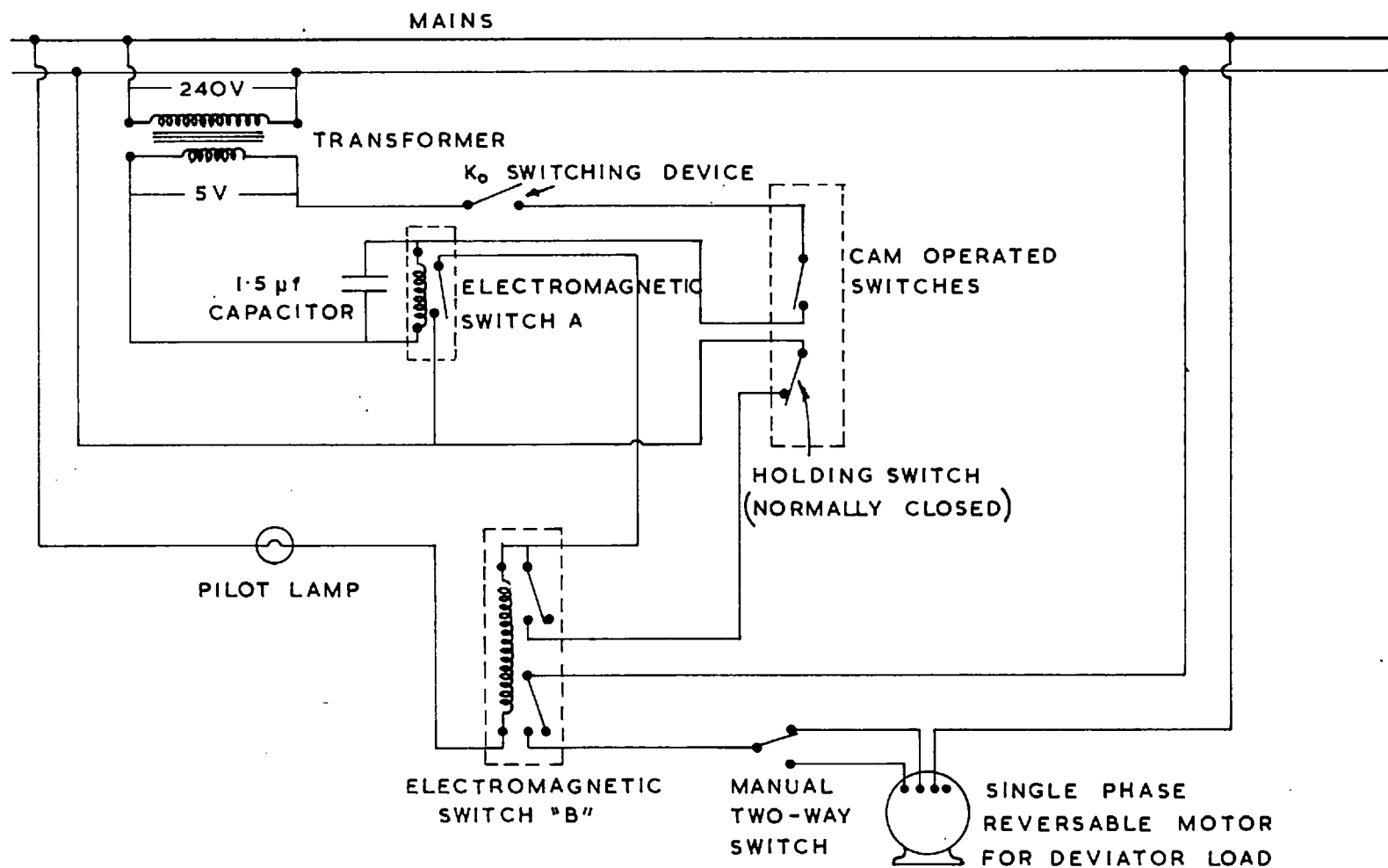
SCALE - HALF FULL SIZE

Fig. 3-5 : K₀ Apparatus

- 1,2,3. Electrical terminal (top and bottom) consisting of
 - (1) 3 B.A. brass stud screwed to perspex cap,
 - (2) knurled brass terminal nut, and
 - (3) brass lock nut and washer
4. Perspex cap
5. Platinum wire soldered into brass stud
6. Glass standpipe (bore 0.9 mm.)
7. Perspex cup to hold standpipe with 0 B.A. thread
8. Perspex front plate with raised rim to pinch diaphragm
9. Perspex body
10. Bolts and nuts to fasten front plate to body (4 B.A.)
11. Stainless steel de-airing screw
12. Reservoir for mercury
13. Diaphragm of polythene sheeting
14. Hole for ground wire
15. Two holes for screwing apparatus to K₀ belt
16. Perspex button
17. Tail or stem of body.



Ko APPARATUS
SCALE - FULL SIZE



SERVO-MECHANISM CIRCUIT DIAGRAM

CHAPTER IV

SOIL PROPERTIES

(4.1) Clay

The Weald Clay used in this programme had the following properties:

Liquid Limit 44% - 49%

Plastic Limit 18% - 20%

Field Moisture (approx.) 20%

Clay Fraction (% < 0.002 mm.) 37% - 39%

Standard Proctor Values:

Max. Dry Density 109 lb./cu.ft.

Optimum Moisture 17%

Specific Gravity of Particles 2.714 ± 0.003

Vertical Permeability - $k_v(i=81.5)$. . . 2.39×10^{-9} cm./sec.

Horizontal Permeability - $k_h(i=81.5)$. . . 1.45×10^{-9} cm./sec.

The scatter in the Atterburg limits over the three year testing period was probably due more to the techniques of the four operators involved than to a real change in the soil properties; non-homogeneity of the large stockpile of material, however, could account for some of the difference in the limits obtained.

The field moisture given is a rough approximation of the water

content in the samples obtained from the field and subsequently selected for the testing programme.

Hydrometer analyses were performed on treated and untreated clay samples according to the standard specifications (BSS 1377/1948) and the resulting sedimentation curves are shown in Fig. 4-1.

The permeability tests were carried out on a cube of soil (1½ inches per side) carved from a plane strain specimen (No. WC 17(o)) that had been anisotropically consolidated and then sheared to failure. It was assumed that the shearing process did not significantly alter the anisotropic structure, and, by passing water through the appropriate cross section, it would be possible to measure the horizontal and vertical permeabilities and detect any difference between them. The cube was subjected to an all-round confining pressure of 100 psi. and an average back pressure of 36 to 38 psi. (depending on the magnitude of the hydraulic gradient). The hydraulic gradient (i) across the cube was varied from 28 to 137, the most reliable values of permeability being obtained at $i = 81.5$ ($k_v = 2.39 \times 10^{-9}$ cm./sec.) and $i = 137$ ($k_h = 2.44 \times 10^{-9}$ cm./sec.). For purposes of comparison a linear relation between k and i was assumed, and hence by interpolation the tabulated values were derived.

At the request of Mr. V. Sowa, a colleague who is conducting the complementary triaxial test programme on Weald Clay, the Building Research Station, Garston, Watford, Herts., performed a mineralogical

examination on the clay and their report is as follows:

SUMMARY.

D.t.a. and X-ray analysis showed the material to consist mainly of illite and fine quartz with a little chlorite and kaolin. The coarser fraction consisted mainly of quartz with a little pyrite.

PROCEDURE

A preliminary d.t.a. of the material passing a 300 mesh sieve indicated the absence of calcite; a small exothermic organic peak at 388°C suggested the presence of only a little organic material, insufficient to warrant hydrogen peroxide treatment.

The material passing a 300 sieve was dispersed in water with the help of a few drops of 0.880 ammonia and the coarser particles allowed to settle. The suspension of fines was filtered and dried in air.

Powder X-ray analysis of the fines indicated the presence of much quartz and illite with a small quantity of chlorite.

Treatment of the material with glycerol produced no 17.7 Å spacing on the X-ray film, so montmorillonite was absent.

Treating the material with hot 2N hydrochloric acid destroyed the chlorite spacings, but a spacing of 7.2 Å was unaffected, showing that kaolin was also present.

Heat treatment below 600°C destroyed the chlorite spacings showing that the chlorite was very poorly crystallized.

Analysis performed by:

Building Research Station,
Garston, Watford, Herts.

27 February, 1963.

(4.2) Sand

The material used in the secondary programme of this thesis was a fine, uniform, quartzite sand from Belgium (sable de Mol) and has been termed simply Belgium Sand throughout. The grading curve is depicted in Fig. 4-2 and shows that most of the sand grains are between 0.1 and 0.5 mm. in diameter. The specific gravity of the grains was taken as 2.65, and the maximum and minimum porosities, reported by LADANYI, (1960), were 47% and 36%. Further sand characteristics and properties can be seen in Ladanyi's paper and will not be described here.

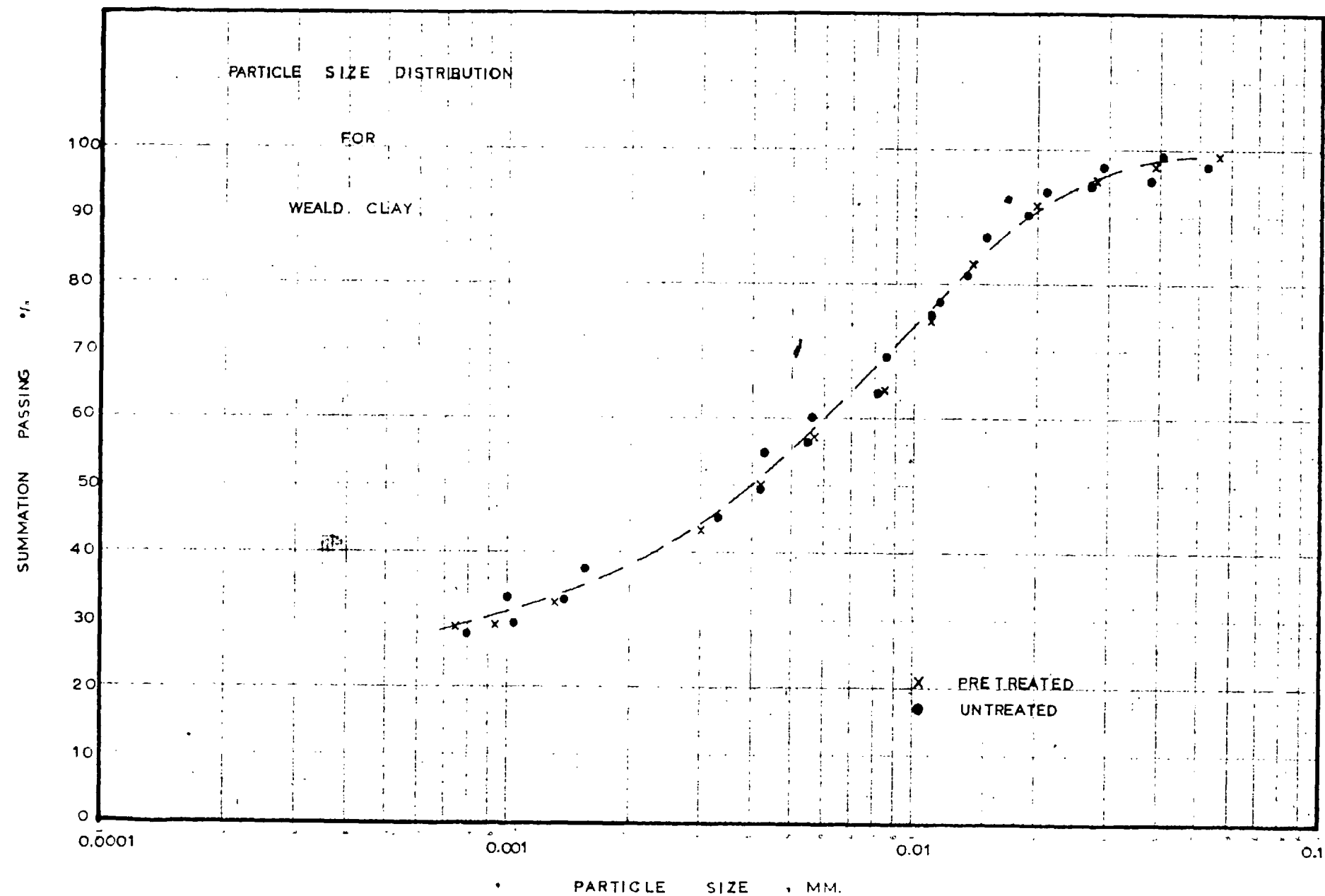


FIG. 4-1

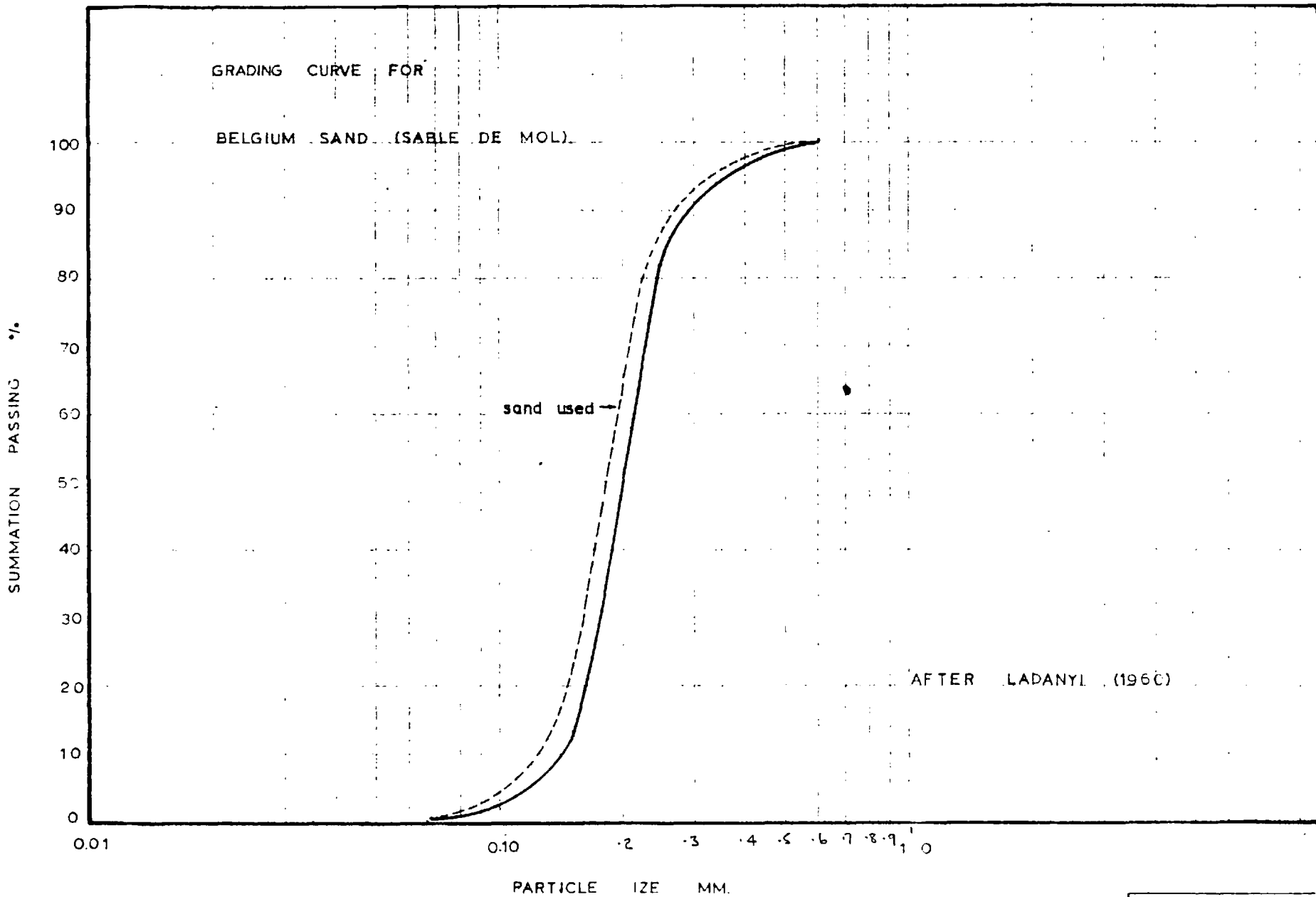


FIG. 4-2

CHAPTER V

DESCRIPTION OF THE TEST and TEST PROGRAMME

(5.1) Description of Test

A sample of remoulded cohesive soil tested in the plane strain apparatus is subjected to a consolidation stage and a compressive shear stage. The consolidation of the specimen is performed by gradually increasing the cell pressure by raising the mercury pot with the small motor and gear train arrangement and allowing the deviator stress, as directed by the servo-mechanism, to maintain the zero-lateral-strain condition. (Previous experience (FRASER, 1957) shows that intermittent deviatoric stress application to effect K_0 consolidation is preferable to intermittent cell pressure variation). Care must be taken to ensure that the rate of increase of deviator stress is greater than the rate of increase of cell pressure as the servo-mechanism is designed to interrupt only the deviator stress circuit; if the cell pressure rate were faster, the deviator stress would never catch up and departures from the K_0 condition would result. The rate of strain is therefore adjusted periodically with the Kopp variator so that the deviator stress is being applied about 70% of the time. Ideally, however, to simulate the natural phenomena of sediment deposition in lakes, oceans, etc., the ratio of the rates of lateral to vertical pressure application should be equal to the coefficient of earth pressure at rest, but this is not readily attained. With

increasing consolidation, the specimen continually gets stiffer, and the deviator stress increases more rapidly with time for the same rate of strain; consequently, continual, not periodic, adjustment of the strain rate is essential to obtain a constant rate of deviator stress application. To more closely approximate the natural phenomena, extensive modifications would have to be incorporated into the existing equipment.

A prerequisite of the plane strain test is that the specimen be consolidated under conditions of zero-lateral-strain, i.e. K_0 conditions. The end clamp, which is positioned when the apparatus is being assembled, cannot be reset after the test has commenced, and hence the sample must retain its initial length during consolidation (as well as during the shear stage) if two dimensional deformation is to be effected when shearing.

The duration of the consolidation stage of a cohesive specimen is a function of its permeability, the effective length of the drainage path and the final consolidation pressure. Considering these variables, consolidation of the plane strain specimen requires one to four weeks for adequate dissipation of pore pressure for this soil.

For all practical purposes, the condition of zero-lateral-strain is adhered to during the consolidation process; however, some strain has to be tolerated to activate the servo-mechanism and thus apply the deviator load. The magnitude of the lateral deformation

depends on the sensitivity of the lateral strain indicator, and is in the order of 0.0003 inches. Information on the value of the coefficient of earth pressure at rest (K_0 value) can easily be obtained from the consolidation stage. The specimen's lateral oscillations provide the limits of the coefficient, and an average value can be calculated. The limiting conditions occur when the deviator motor just comes on and just goes off; it is at these times that readings of axial load and axial deformation, cell pressure, and volume change are taken. When the deviator motor is off, a more or less constant load is maintained on the sample by the leaf spring. At the termination of the consolidation stage, the miniature hydraulic jack (see Fig. 3-4) is utilized to push the longitudinal bars of the lateral strain indicator apart, thus allowing the perspex feet to drop from position.

The compressive shear stage is conducted in the standard way by maintaining the cell pressure constant and increasing the deviator load until failure such that $\sigma'_1 > \sigma'_2 > \sigma'_3$. The duration of the shear stage is of the same order of magnitude as the consolidation stage and varies from one to three weeks depending on the axial strain required. Two or three sets of readings daily are sufficient to define the characteristics of the soil. Readings of pore pressure, vertical load, intermediate principal stress and axial deformation are taken. Pore pressures are measured at the top and bottom of the specimen, and values of the intermediate principal stress are

calculated from predetermined calibration curves (see Appendix 5). The deviator stress and the axial strain are obtained as in the standard triaxial test.

To maintain the plane strain condition, the magnitude of the intermediate principal stress (termed σ_2) is adjusted to keep the mercury thread in the null indicator at a height, previously calibrated, corresponding to no change in specimen length. When σ_2 is manually controlled, the σ_2 cell is isolated from the null indicator between readings and overnight. As the deviator stress increases during this time, the soil tends to deform longitudinally and the pressure in the σ_2 cell increases; as a result, the system expands elastically and some departure from the no-change-in-length condition occurs. The magnitude of the departure depends on the time interval between readings and the rate of deviatoric stress increase. By taking at least two readings a day in a test which is being sheared for 15 days, the incremental change in length is reduced sufficiently, for all practical purposes, to maintain the plane strain condition. Corresponding to this incremental length increase, however, is an increase in the measured value of σ_2 as the soil is forced back to its original length by the requirements of the calibration. The resulting scatter in the calculated σ_2 values can be minimized by decreasing the axial strain rate or shortening the interval between readings. A reduction in the rate of strain during shear unnecessarily adds to the testing time and increases the risk of apparatus breakdown.

due to leaks. More frequent readings is a better solution, the ultimate being continual adjustment or automatic control of σ_2 in a manner similar to the K_0 consolidation technique. By positioning a longitudinal strain indicator between the two opposing end plates of the σ_2 belt and connecting it electrically to a servo-mechanism, a motor-driven control cylinder can be inserted into the σ_2 pressure system to vary the pressure in the σ_2 cell as directed by the indicator. (Automatic control was attempted in several tests with only one success, WC 11; its use was limited, however, as it proved unreliable with the existing apparatus. The slackness of the removable face plate generally caused the mercury in the standpipe to move erratically and sometimes either overflow or disappear from the standpipe altogether). Nevertheless, satisfactory values of the intermediate principal stress can be obtained with relatively frequent manual adjustment during the shear stage.

Sample weight and dimensions are determined before the apparatus are assembled. Saturation of the specimen is effected prior to the consolidation stage by the application of a back pressure to the top and bottom through a volume change measuring unit. Although necessary during consolidation, the leaf spring and lateral strain indicator are not essential for the shear stage.

Plane strain tests on cohesionless materials are essentially similar to tests on cohesive soils. However, due to the high permeability of sands, they can be tested in a few hours compared to a

few weeks for clays. The K_0 consolidation stage can be effected manually without the aid of the leaf spring or the servo-mechanism; the addition of the lateral strain indicator is advantageous but not essential if the values of K_0 are known. The intermediate principal stress during shear is determined in the same manner as above. For further details on the plane strain testing of sands, see CORNFORTH, (1961).

(5.2) Test Programme

(a) Plane Strain Tests

As denoted in Chapter I, both sand and clay tests were performed in the plane strain apparatus. All tests were normally consolidated; the sand specimens were tested under drained conditions after being consolidated to an effective lateral pressure of 40 psi., whereas the clay tests were undrained with pore pressure measurement and were consolidated under various effective cell pressures ranging from 25.0 psi. to 96.7 psi. The clay specimens were saturated by a back pressure of 30 to 35 psi. The sand samples were drained to atmosphere and thus the pore pressure was considered to be zero throughout the test. Moisture contents were determined in the clay specimens after failure had occurred. In the Weald Clay tests, each test is prefixed by the letters WC and the number of the test follows the letters, i.e. Weald Clay test number four is denoted as WC4. Some tests are suffixed (0), which indicates that the sample was initially

consolidated under a small load in the oedometer, such as WC17(0). The Belgium Sand test nos. are prefixed by BS and the triaxial extension tests are suffixed by E.

Fifteen clay tests were completed in the plane strain apparatus and are included in this thesis. All but one were K_0 consolidated; that one was isotropically consolidated to investigate the effects of a different consolidation method on the strength properties. All specimens, except the last four, were subjected to plane strain conditions; the last four were tested without the σ_2 belt or end clamp in position. As mentioned previously, all the clay specimens were sheared in compression under constant cell pressure.

Eighteen plane strain tests were performed on the Belgium Sand. The initial porosity (i.e. the porosity before K_0 consolidation) varied from 37.8% to 42.5%. All specimens were consolidated to 40 psi. effective cell pressure and then sheared in compression with the cell pressure held constant.

(b) Triaxial Tests

No triaxial tests were conducted on clay.

Fifteen standard large diameter triaxial compression tests were performed on the Belgium Sand. The initial porosity varied between 37.6% and 44.4%. To enable comparisons to be made with the plane strain tests, the samples were K_0 consolidated to an effective cell pressure of 40 psi. and sheared with the cell pressure constant.

Four large diameter triaxial extension tests were successfully completed on the Belgium Sand; the specimens were K_0 consolidated before shearing. The initial porosity range was 39.3% to 42.7%. The cell pressure was maintained constant during shear and was 40 psi. for one test and 140 psi. for the other three.

(c) Oedometer Tests

To prepare clay specimens for the plane strain test, it was found advantageous to consolidate a sample of clay in the large oedometer (9 in. dia. x 5 in. deep) under a small effective stress. Seven specimens were prepared in this way.

(d) Other Tests

Classification tests to determine the properties of the Weald Clay used were carried out. These included Atterberg limits tests, hydrometer analyses, specific gravity tests, a standard proctor test, and permeability tests. When a sample was being prepared for the plane strain test, moisture content control was maintained by performing tests in the miniature vane apparatus.

CHAPTER VI

TEST PROCEDURE

(6.1) Specimen Preparation

The material used for the main part of this thesis, Weald Clay, was obtained in May, 1960, from the excavated pits of the London Brick Works in Dorking, Surrey, by Dr. Henkel and the author. Three samples were taken from depths ranging from ten to twenty feet below the original ground surface. Two of the samples were found to have an average liquid limit of about 46%, and these were selected for the test programme because of their similarity to the Weald Clay previously tested at Imperial College.

To obtain a homogeneous batch of material, the samples were air-dried and pulverized sufficiently to pass the No. 8 British Standard sieve, then thoroughly mixed in a large (3-foot diameter) cement blending machine, and finally saturated with liberal quantities of distilled water. The dry weight of the resulting mixture was between 250 and 300 pounds. All plane strain specimens, as well as the tri-axial specimens tested by Mr. V. Sowa, were prepared from this batch.

Two methods of preparing the plane strain specimens were used. One consisted of setting up the mould used for sand tests (see CORNFORTH, 1961), and forming the specimen by placing the material in the mould with a spatula. The other method consisted of slightly pre-consolidating a large sample of the material in the oedometer, and

then carving the plane strain specimen from the oedometer sample.

A description of the two methods is given below.

(a) Mould Method

A batch of wet clay weighing about 16 lb. was obtained from the stockpile and thoroughly mixed to a moisture content of approximately 34%. Earlier work on Weald Clay (GILBERT, 1954) had shown that, for a standard triaxial sample $1\frac{1}{2}$ in. diameter x 3 in. high, a shear strength of $1\frac{1}{2}$ psi. as determined in the vane test was necessary to prevent sample slumping during apparatus assembly. On this premise, a moisture content of 34% in the present Weald Clay was considered adequate to prevent slumping as it corresponded to a vane strength of 1 psi.

The mixing time varied with the initial moisture content, but it generally took about two days and a third day for the equalization of pore pressures that resulted from mixing. Moisture content control was effected with the vane apparatus; random samples from the mixed batch were tested during the remoulding process and just prior to the moulding of the plane strain specimen to ensure a uniform moisture content throughout. The elimination of the larger particles just passing the No. 8 sieve also added to the mixing time.

While waiting for the clay batch to mature, the plane strain apparatus and mould were made ready. The apparatus base with its legs in position was placed on the bench and the bolts joining the bottom

platen to the base were removed. After the rubber membrane was fitted between the platen and the base, the bolts were screwed down securely, the legs were removed and the base positioned on the leaf spring. Since the base was supported by the spring only at its mid-point longitudinally it was necessary to block up the base at its ends to prevent tipping during the moulding process. The perspex holder, which positions the four porous stones, was screwed to the bottom platen. The mould parts were placed around the platen and bolted together so as to firmly pinch the membrane around the periphery of the platen. The membrane was then stretched so that its upper edges could be turned over the recessed edges of the mould. It was held in this position by small clips placed at intervals around the top of the mould. De-aired water was passed through the bottom drainage lead upwards through the porous stones to ensure that no trapped air was present, and then the excess water was mopped up. Filter strips $\frac{3}{8}$ in. wide x 8 in. long cut from Whatman's No. 54 filter paper were placed along the length of the bottom platen so as to interconnect the porous stones. More filter strips were wetted and positioned in a "U" shape vertically, down one side of the membrane wall, across the bottom and up the other wall. The spacing of the strips was approximately $1\frac{1}{4}$ in. from centre to centre. At the ends of the mould, two strips were placed on each in an "L" shape running down the end wall and connecting the strips on the bottom.

The average length of the filter strips on the ends was 6 in. and along the sides 10 in. The overlap was allowed to hang outwards until the specimen was moulded, and then it was folded over the top of the specimen. Whatman's No. 54 filter paper is recommended by BISHOP & HENKEL (1957) because of its wet strength.

When the clay was sufficiently matured, it was placed in the prepared mould with a spatula; considerable care had to be exercised to avoid disturbing the filter strips. The clay was placed in horizontal layers or lifts until a height of about 4 in. was attained. (Because of the addition of the leaf spring beneath the cell, the amount of height available for the sample was limited to $4\frac{1}{2}$ in.). Filter strips were then placed on top of the specimen in the same manner as on the bottom platen, and the overlap from the vertical strips was folded into place. The clips anchoring the membrane to the top of the mould were removed and the membrane was allowed to come back to its normal position over the sample. Water was passed through the top drainage lead to rid the system of entrapped air; however, it was difficult to maintain this de-aired condition as the porous stones affixed to the top platen were face downwards. The coarse stones used throughout the testing programme invariably allowed some water to escape from beneath the perspex holder, thus creating pockets of air in the system. (The amount of air present was reduced to negligible proportions by the application of a back pressure prior to the consolidation stage). The top platen was

positioned on the specimen and the membrane was drawn over it. Then the loading head was put on and screwed down tightly to obtain a good seal with the membrane and the platen. The sample was allowed to stand for a few hours with the mould in position to permit further equalization, and then the mould was removed.

Before the plane strain apparatus was assembled, measurements of the specimen were taken. Six width measurements at each of the three elevations, top quarter-point, mid-height, and bottom quarter-point, were taken to the nearest 0.001 in. The specimen height was measured at five different points along the length to the nearest 1/32 in., and the length was obtained from four measurements with the same accuracy. All readings were respectively arithmetically averaged, the membrane thickness subtracted from the length and width, and the sample dimensions consequently determined. The sample weight was determined and representative moisture content samples were obtained.

The principal objection to the mould method of sample preparation arose from the fact that slumping did occur while the apparatus were being assembled. Contributing factors were the weight of the top platen and loading head, and the time interval between removing the mould and applying the cell pressure.

(b) Oedometer Method

The solution adopted was the oedometer method of preparing plane

strain specimens. This consisted of consolidating a batch of clay, from the initial $34\frac{1}{2}\%$, under a small vertical pressure to make it stiff enough to manipulate without slumping. The required specimen was then carved from the stiff sample.

A batch of wet clay weighing about 20 lb. was obtained from the stockpile and was worked until the moisture content was close to 34% , control being maintained with the vane apparatus. When the mixing and maturing period was completed, the 9 in. diameter oedometer was filled to a height of $4\frac{1}{2} \pm$ in. by spreading the clay in layers with a spatula. The height and weight of the sample were recorded and the oedometer apparatus were assembled. A pressure was applied to the sample by means of the lever arm arrangement, and the sample was allowed to drain to atmosphere. An inverted bottle of water positioned over the oedometer ensured that the top of the sample remained saturated. Most of the oedometer specimens were consolidated under a vertical pressure of 7.7 psi.

When equilibrium was reached, the load was removed from the lever arm and the sample taken from the oedometer ring. The sample was placed on the cutting board and weighed. Next, a thin wire was placed over the sample and in front of two brass uprights on the cutting board, and drawn taut so as to cut from the sample an 8 in. chunk. The wire was worked slowly back and forth along the front of the uprights to obtain a smooth cut surface. The chunk was removed

carefully and the wire was placed in front of the next row of up-rights 2 in. back and a parallel line was cut. By repeating the operation for a third time, two central pieces were obtained which were long enough when placed end to end to form a plane strain specimen. Further trimming was necessary, however, to get rid of the radial portion at each end of the 8 in. length. The centre chunks each had nominal dimensions of 2 in. x 8 in. x $\frac{1}{4}$ in. To be accommodated in the plane strain apparatus, the $\frac{1}{4}$ in. dimension, which was to be the height of the specimen, had to be trimmed to $\frac{1}{8}$ in. The chunks were then weighed and measured. Twenty-four measurements of the width or 2-in dimension were taken; eight readings were noted at each of three different elevations. A total of eight readings of height measurement were made. The height and width were both measured to the nearest 0.001 in. The length was not measured until the two chunks were placed on the bottom platen of the apparatus.

First, however, the chunks were carefully wrapped with filter paper strips. The strips were placed vertically around the specimen and had their loose ends folded over the top to meet each other. The spacing was about $\frac{1}{8}$ in. from centre to centre. One end of each chunk had two strips placed on it with the overlapping ends laid along the top and bottom of the specimen. When finally the wrapping was complete, the chunks were positioned with care on the perspex

holder screwed to the bottom platen with their bare ends adjoining. (Prior to dismantling the oedometer, the plane strain apparatus was prepared by fitting the rubber membrane between the bottom platen and the base, and placing the base on the leaf spring. After locating the porous stone holder, the de-airing process was effected and filter strips were placed over the stones to interconnect them). More filter strips were placed on top of the specimen, and the assembled and de-aired top platen was located on the specimen. For a minimum of sample disturbance, three people were generally required to stretch and hold the membrane and position the sample chunks. The membrane was pulled over the top platen; excess air between the sample and the membrane was removed by stroking the membrane gently upwards before the loading head was fixed in position. The loading head was screwed tightly down to the top platen to make a seal with the membrane. After the two chunks had been firmly pressed together, the length was measured at three different elevations - top, centre, and bottom - and an average length was determined by subtracting the membrane thickness from the average of the readings. Since no time was allowed for pore pressure equalization as in the mould method, the rest of the apparatus could now be assembled.

(6.2) Apparatus Assembly

After the specimen was made ready by either of the two methods described, the end clamp, which remained connected to the base by its

saran pressure lead between tests, was placed around the sample. This was done by bringing the end clamp gently down from the top to span the sample. The left end plate, which had been removed to aid the location process, was replaced on the tie bars and the nuts were screwed on. Two small coil springs with their holders were positioned under each end plate. The removable face plate was put in position on its locating pin, and the tie bar nuts further tightened until the end clamp was just in contact with the ends of the specimen. The saran pressure lead was checked to see if it were damaged or kinked and to make sure that the connection with the hydraulic cell was in order. The valve connecting the O_2 cell with its null indicator was opened, as well as the valve to the burette under atmospheric pressure. The previously de-aired O_2 cell was allowed to stand at atmospheric pressure while the other equipment was being installed.

The K_0 belt was then positioned so that it spanned the sample with its open end at the left end of the specimen. The four pulleys that support the K_0 belt were fixed on the loading head and the counterweight strings were placed over them. The small hydraulic jack was screwed to the end plate. Next, the de-aired and assembled mercury reservoir device was screwed to the K_0 belt in addition to a light coil spring. The perspex feet with the accompanying ball bearings were placed at mid-height and near mid-length of the specimen. While holding the feet and balls in position, the spring at the

reservoir end of the belt was latched. This procedure is somewhat simplified if the membrane is slightly wetted before the feet are positioned, to hold them long enough to get the belt and balls orientated. To prevent interference to the K_0 belt during the test, the flexible nylon top drainage lead was attached to the end plate. The electrical wires coming through the base were connected to the top and bottom terminals of the mercury reservoir. By varying the height of mercury in the standpipe, the servo-mechanism circuit was checked.

The O_2 valve to atmosphere was closed, leaving the null indicator connected to the system. The tie bar nuts were tightened just enough to seat the end plates and exert a pressure on the specimen of approximately 0.5 psi. The mercury thread in the reservoir standpipe was finally adjusted to the proximity of the platinum wire probe, and the complete lateral strain indicator was rocked about its pivot points (i.e. the ball bearings between the feet and the belt) to check that a minimum of movement occurred in the mercury thread. This rocking motion ensured that the indicator was not being obstructed.

An overhead crane running on a monorail was used to bring the plane strain cell body directly over the sample and to lower it onto the base. Since the original design of the plane strain apparatus did not include a lateral strain indicator, a minimum of room was

available between the end clamp and the cell body. As a result, the lowering of the body with the indicator in position proved to be an extremely delicate operation if disturbance of the indicator were to be avoided. In some of the early tests, as many as eight or nine attempts were required to correctly place the cell body, with subsequent adjustments to the indicator each time. Once the body was located, the lateral strain indicator was rocked once more to ensure freedom of movement. Two of the sixteen tie bars for joining the apparatus together were used to guide the cell body down past the specimen.

The overhead crane was used to position the cell top also. Care was taken to place the hemispherical seatings for the rams on the loading head before the cell top was put on. The remaining fourteen tie bars were inserted through the positioned top cap and tightened down securely to minimize vertical expansion of the cell when pressure was applied. The horizontal stabilizer was fixed at the same time as the two central tie bars at the back. The vertical stabilizer was bolted on, and the rotating bushes (or friction eliminators) were connected to their motor. To prevent the rams from rotating as their bushings revolved, two cross arms were fitted. While the cell was being filled with de-aired water, the top and bottom drainage leads were connected to the volume change measuring unit and the castor oil leads were joined to an oil reservoir. When the cell was almost full of water, the water

was turned off and the cell topped up with castor oil. The cell plug was screwed in and the proving rings were placed in position.

(6.3) Back Pressure Application

To effect saturation of the specimen, it was subjected to a back pressure in the following way. The top and bottom drainage valves were kept closed, and the cell pressure was slowly increased manually. At the same time, the pressure in the σ_2 system was increased with the manual control cylinder such that the level of the mercury in the σ_2 null indicator remained unchanged. To prevent departures from the zero-lateral-strain condition, the mercury in the standpipe of the lateral strain indicator was maintained at a constant height by the application of a deviator load to the specimen. When the cell pressure had been built up to about 38 psi., the top and bottom drainage was opened to a mercury pot of the self-compensating mercury control through a volume change measuring unit. The mercury pot had been previously set at a pressure of, say, 35 psi. Zero readings were taken on the proving rings, strain dials, σ_2 gauge, and the volume change measuring unit, before the back pressure was applied. Similar readings, as well as readings of the cell pressure and back pressure, were recorded after the application of the back pressure. Before the consolidation stage was commenced, the specimen was allowed to come to equilibrium under this set of small effective stresses. Automatic control of the specimen width was maintained by

connecting the deviator motor and the servo-mechanism electrically to the lateral strain indicator during the equalization period of one to two days. The clutch of the triaxial machine was engaged to enable the deviator motor to load the specimen when required by the indicator.

During a test, the self-compensating mercury control supplied pressure to the cell through a water-oil interface in a castor oil reservoir, which resulted in oil being introduced into the cell to replenish the "lost" volume caused by specimen consolidation and oil leakage past the rams. This method of applying cell pressure ensured that sufficient oil was present to minimize cell pressure leakage past the rams in long-term tests. The oil reservoir was connected to the oil leads of the plane strain apparatus immediately before the consolidation stage was begun.

(6.4) Consolidation Stage

By trial and error, a rate of cell pressure increase of 5.5 psi. per day was found to be satisfactory in consolidating the specimen. This rate caused no measurable excess pore pressure at the base when the sample was drained from the top, which meant that the pore pressure was being dissipated through the filter paper strips as fast as it was being built up in the sample. However, no indication of the magnitude of the pore pressure in the centre of the specimen could be

determined in this way. Consequently, it was decided to drain the specimen from both top and bottom, and, when the consolidation stage was nearing completion, to slow down further (to 1.85 psi. per day) the rate of cell pressure increase; it was assumed that these measures would help to reduce whatever excess pore pressures that existed in the centre. The rate of cell pressure increase used for each test is given in Table 6-1. The rate of axial strain was periodically adjusted during the consolidation process, as denoted in Chapter 5, to ensure the proper functioning of the lateral strain indicator.

By definition of the K_0 condition, there should be no change in length of the specimen during consolidation; hence, the magnitude of σ_2 should be equal to the value of σ_3 at any given time. σ_2 was measured during consolidation to determine if the specimen were, in fact, undergoing one dimensional consolidation. The σ_2 gauge reading was equal to the cell pressure plus the σ_2 prestress induced during apparatus assembly plus the head of the mercury thread in the null indicator.

Generally, readings of volume change, axial load, cell pressure, axial deformation and σ_2 were taken when the deviator motor just came on or just went off. However, some intermediate readings were noted when the deviator motor was running. The rate of cell pressure increase was usually stopped one or more days before the shearing stage was begun to allow further dissipation of

excess pore pressure from the specimen.

(6.5) Shear Stage

At the completion of the consolidation stage, the top and bottom drainage leads were closed and connected to a null indicator. The pore pressure was noted, and the sample was allowed to rest overnight before the pore pressure was measured again. If the difference between readings were large, the drainage leads were opened again for another day. A small difference between readings indicated that the sample had relatively uniform pore pressure throughout, and thus the shearing stage was begun.

Most tests were sheared at a rate of strain of 0.00002 in. per min. or 0.03% per hour; some were sheared at 0.015% per hour. The duration of the shear stage depended on the % strain required, but, generally the test was terminated after ten to fourteen days of shearing, when the maximum principal stress ratio had been reached. Readings of σ_2 , axial load, top and bottom pore pressure, axial deformation and temperature were taken two or three times daily except when failure was approaching, and then more readings were recorded. The cell pressure was checked each time and the date and time were noted.

When the test was completed, the deviator load was released manually, and σ_2 and σ_3 were brought down to atmospheric pressure. The apparatus was dismantled and the parts cleaned of oil. The

loading head and top platen were removed, and the membrane was stripped to bare the soil. The filter paper strips were taken off and, to minimise drying, the sample was weighed without delay. Moisture content samples were then carved from the specimen to obtain the water content profile. Twelve samples were usually taken, one from each end and one from each third point for each of three elevations. When a failure plane in the specimen occurred, which was rare and usually not clearly defined, moisture content samples were taken along the failure zone and in the two adjacent corners. The remaining portions of the specimen were grated on a cheese grater into a pan to form the basis of another specimen.

The moisture contents were determined in the standard way. After obtaining the wet weights of the samples, they were placed in the constant temperature oven (105° Centigrade) for two days. Upon removal from the oven, the tins containing the samples were allowed to stand in air long enough to come to room temperature and then weighed to get the dry weights.

Dismantling of the apparatus was completed and everything was thoroughly washed in preparation for the next test. The membrane was dusted with French chalk and stored until needed.

(6.6) Sand Tests

The procedure used for preparing and testing the sand specimens was identical to that given by CORNFORTH (1961), and for brevity will not be discussed here.

Test No.	No. Days @ Rate of σ_3 increase psi/da.	$(\sigma_3^1)_c$ psi.
WC4	10 @ 3.85; 1 @ 0	39.1
WC6	8 @ 3.85; 3 @ 0	31.1
WC7	12 @ 3.85; 0 @ 0	47.3
WC8	5 @ 3.85; 1 @ 0	25.0
WC9	5 @ 3.85; 1 @ 0	26.8
WC10	15 @ 3.85; 6 @ 0	69.6
WC11	8½ @ 3.85; 6 @ 0	34.2
WC12	5 @ 3.85; 1 @ 1.85; 2 @ 0	27.6
WC13(0)	12 @ 3.85; 13 @ 0	60.4
WC14(0)	5 @ 5.5 ; 2 @ 3.85; 3 @ 0	48.1
WC15(0)	12 @ 3.85; 3 @ 1.85; 2 @ 0	61.1
WC16(0)	7 @ 11.0; 2 @ 5.5 ; 1 @ 1.85; 1 @ 0	96.7
WC17(0)	11 @ 5.5 ; 3 @ 0	68.5
WC18(0)	6 @ 5.5 ; 5 @ 1.85; 1½ @ 0	53.4
WC19(0)	(Isotropically consolidated)	65.2

TABLE 6-1

The number of days, to the nearest half day, is the length of time that the cell pressure was increasing at the specified rate. When the rate is 0, the number of days represents the time allowed for the sample to come to equilibrium. $(\sigma_3^1)_c$ is the effective cell pressure at the end of the consolidation stage. The difference between $(\sigma_3^1)_c$ and the rate of cell pressure increase times the time, as can be seen in some tests, is due to the initial manually applied cell pressure that was required to overcome specimen slumping or, in the

oedometer consolidated specimens, to simulate conditions in the oedometer.

CHAPTER VII

RESULTS OF TESTS ON WEALD CLAY

(7.1) Plane Strain Tests - General

The first eight specimens tested in the plane strain apparatus were prepared by the mould method (see Chapter VI) from an initial moulding water content of approximately 34%. The test Nos. are WC4 and WC6 to WC12 inclusive. However, since some slumping of these specimens did occur while the apparatus was being assembled, the remaining specimens (in test Nos. WC13(0) to WC19(0) inclusive) were prepared by the oedometer method. All the oedometer-consolidated specimens except two were remoulded from a water content of $34 \pm \%$. To determine the effect of the moulding water content in the failure characteristics, the specimens in test Nos. WC13(0) and WC14(0) were remoulded from water contents of $60 \pm \%$ and $106 \pm \%$ respectively.

For comparison with the tests conducted under plane strain conditions, the last three tests with K_0 consolidation, namely WC16(0), WC17(0), and WC18(0), were conducted with no end restraint in the σ_2 direction (i.e. the end clamp was not used). In test No. WC19(0), the specimen was isotropically consolidated and again the end clamp was not used. These last tests were, therefore, not necessarily subjected to plane strain conditions, although friction along the top and bottom platens would probably have reduced significantly the

longitudinal strain.

For the purpose of defining the average characteristics of the clay when tested in the plane strain apparatus, dimensionless functions of the principal stresses were determined by dividing by the average normal stress at the end of consolidation (i.e. by $\frac{1}{3}(\sigma'_1 + \sigma'_2 + \sigma'_3)_c = P_c$). In calculating P_c it has been assumed that the apparent small prestress in the σ'_2 direction as shown in Fig.7-2 was confined primarily to the specimen ends, and hence it has been neglected. This assumption has in any case only a very small effect on P_c . When a dimensionless basis of plotting was used, three separate groups or families emerged with relatively little scatter in each group. Group A consisted of test Nos. WC6, WC8, WC9 and WC11; group B consisted of WC4, WC10 and WC15(O). Group C included the specimens with no end restraint and consisted of WC16(O), WC17(O) and WC18(O).

(7.2) The Consolidation Stage

Before considering the results in detail, some comments on individual tests must be made. Due to apparatus breakdown, several attempts at starting test Nos. WC4 and WC16(O) were required before the tests were satisfactorily completed. This resulted in each specimen being slightly overconsolidated at the beginning of the consolidation stage. The overconsolidation effect shown in the specimen of test No. WC13(O) is the result of the initial consolidation in the oedometer under a vertical pressure of approximately 15 psi.

Due to excessive leakage from the cell, the cell pressure in test No. WC10 dropped from 86 to 50 psi before the leak could be stopped. The volume change measuring device burst in test WC16(0), with the result that the back pressure dropped from 35 to approximately 22 psi. The measurements of the intermediate principal stress in test WC12 were considered invalid because of the interference of the loading head with the end clamp during the test.

(a) K_0 Path

The principal effective stresses during the K_0 consolidation stage were plotted for each test in graphs of σ'_1 against σ'_2 and σ'_3 . (See Appendix 3, Figs. A3-1 to A3-14). A typical relationship between the major and minor principal stresses is illustrated in Fig. A3-13 for test WC17(0). The total consolidation period for this test was fifteen days. The scatter of the points is due to the intermittent application of the deviator load. The average K_0 value of 0.62 was determined from the mean straight line through the points. For purposes of comparison, the hydrostatic pressure line is also drawn.

The range of average K_0 values for all the tests was 0.545 to 0.62, and the overall average value was $K_0 = 0.59$. The higher K_0 values might be attributed to inadequate dissipation of the excess pore water pressure in the specimens, consolidated at the faster rates^{*}.

^{*} The consolidation rate for each test is given in Table 6-1.

However, as shown in Fig. 7-1, the magnitude of K_0 depends to a limited extent on the initial moulding water content of the specimen. High K_0 values in this diagram occur at the higher water contents. The standard triaxial specimens^{*}, and the oedometer, consolidated plane strain specimens remoulded at c. 34% water content show the same general trend. The average K_0 value for each test is shown on the graph of σ_1' versus σ_3' , and the values are tabulated in Table 7-1.

(b) σ_2' versus σ_3'

A typical relationship between σ_2' and σ_3' during consolidation for the K_0 consolidated tests is given in Fig. 7-2. The straight line drawn at 45 degrees is the ideal stress path, whereas the points lie slightly above this line. This is due in part to the small amount of pre-stressing used during the assembly process to eliminate bedding errors on the end clamp. It can be seen that the pre-stress remains approximately constant during the consolidation period.

In test Nos. WC4 and WC10, the amount of pre-stressing was very small, so that $\sigma_2' \approx \sigma_3'$ during consolidation. Test WC11, in which the intermediate principal stress was automatically controlled (see Chapter V), had a rather higher pre-stress in the σ_2 direction. This is shown in Fig. A3-7. Due to a ruptured pressure line in the σ_2 system, the intermediate principal stress was not measured in test WC15(0). The end clamp was not used in test No. WC16(0),

^{*} After SOWA (1963). Standard triaxial test results on Weald Clay are given later in the Chapter.

WC17(O) and WC18(O).

(c) Volume Change during K_0 Consolidation

The relationships between the change in height and the change in volume of the specimens during K_0 consolidation are given in Figs. A3-15 to A3-27. In each graph, the slope of the dashed line through the origin corresponds to the initial measured area of the sample. The points, which represent the measured values of Δh and ΔV during consolidation, lie on a line approximately parallel to this line. This indicates that the zero-lateral-strain condition was maintained. The curved portion near the origin, however, indicates the presence of air in the specimen before the application of the back pressure. The data from test WC14(O) was not plotted because the volume change device was not properly de-aired during the assembly process.

(d) Water Content - Stress Relations (End of Consolidation)

The graphs of % water content against major principal effective stress during consolidation are illustrated in Figs. A3-28 to A3-31 inclusive. The water contents during consolidation were calculated from the known water contents and specimen heights at the end of consolidation. It was assumed that the specimens were fully saturated and were consolidated with no lateral yield. Except for the early part of the consolidation stage, the w.c. - $\log \sigma_1'$ relationship is a

straight line (approximately) during the consolidation process. The slope of the line depends generally on the moulding water content, with the steeper slopes corresponding to the higher initial water contents. The end of consolidation conditions are shown in Fig. A3-32.

The relationship between water content and average normal stress (i.e. $J_1/3$) at the end of consolidation is given in Fig. 7-3. Test No. WC14(0), in which the specimen was consolidated from a slurry indicates that the moulding water content can have an appreciable effect on the water content-stress relationship. Test No. WC13(0), however, with a moulding water content of c.60%, plots close to the average line. The dashed line represents the results obtained at the end of the consolidation stage for the tests in the standard triaxial cell, and these will be discussed later.

(7.3) The Shear Stage

As indicated earlier, all specimens were tested in compression with the cell pressure held constant, and the pore pressure was measured. The principal effective stresses and the deviator stress were calculated at various points during each test, and the results are presented graphically against the corresponding axial strain during shear in Figs. A3-33 to A3-47 inclusive. A correction of -1.0 psi. was applied to the measured deviator stress to account for the confining effect of the rubber membrane and the filter strips, and

and for the friction on the end plates (See Appendix 2).

For various reasons, however, some tests were considered unrepresentative of the behaviour of the clay when tested in the plane strain apparatus. In test No. WC7 (Fig. A3-35), the pore pressure did not level off after the specimen had failed and it is probable that there was a small leak in the rubber membrane. The σ_2 values obtained from test WC12 were in error because of the interference of the loading head with the end clamp. At the end of the consolidation stage of test No. WC13(O), the room temperature fluctuated between 56° and 76° Fahrenheit over a period of five days. With the high moulding water content used in this specimen (c.60%), high pore pressures were expected. However, the stress path obtained indicates low pore pressures, and the conclusion is drawn that the results were greatly influenced by the temperature variation. The specimen in test WC14(O) was consolidated from a slurry, so that the results are not strictly comparable with the other tests which were remoulded from lower water contents (c.34%). Due to the isotropic consolidation of the specimen in test WC19(O), the stress and strain data of this test differs from that of the K_0 consolidated tests.

(a) Stress-Strain Relations

As illustrated in Fig. A3-33, during an undrained test the three principal effective stresses decrease with increasing strain. In the tests on the K_0 consolidated specimens, the initial value of the

deviator stress is $(1-K_0)(\sigma'_1)_c$, where $(\sigma'_1)_c$ is the magnitude of the major principal effective stress at the end of consolidation. The deviator stress increases to a maximum value and then levels off to a relatively constant value with increasing strain. As the cell pressure is kept constant, the decrease in the minor principal effective stress is a measure of the increase in the pore water pressure. The pore water pressure reaches a maximum after the peak deviator stress has been reached. The maximum value of the principal stress ratio (σ'_1/σ'_3) also occurs after the peak deviator stress has been reached. In Fig. A3-47, the deviator stress for the isotropically consolidated specimen increases from zero to a maximum value before levelling off with increasing strain. The maximum value of the ratio (σ'_1/σ'_3) again occurs after the peak deviator stress.

The stress-strain relations on a dimensionless basis for the A, B, and C groups are given in Figs. 7-4 to 7-6. In Fig. 7-4, group A had $\left[(\sigma_1 - \sigma_3)/P_c\right]_{\max} \doteq 0.82$ at 2.0% axial strain. Group B, shown in Fig. 7-5, had $\left[(\sigma_1 - \sigma_3)/P_c\right]_{\max.} = 0.77$ at $\epsilon_1 = 2.3\%$. The tests conducted with no end restraint comprise group C and have the lowest value for the ratio $\left[(\sigma_1 - \sigma_3)/P_c\right]_{\max.} \doteq 0.73$ at $\epsilon_1 = 3.3\%$ (Fig. 7-6). The test results for the isotropically consolidated specimen are included in Fig. 7-6 for comparison. The stress-strain curves for tests WC12, WC13(0) and WC14(0) were plotted in Fig. 7-7.

A possible explanation for the apparent strength differential between A and B groups is that the B group specimens were consolidated to higher pressures than those in A group as shown in TABLE 7-1. A flattening of the Mohr envelope with increasing normal stress would result in lower strengths for the samples at the higher consolidation pressures.

The variation of the principal stress ratios (σ'_1/σ'_3) , (σ'_2/σ'_1) and $\sigma'_2/(\sigma'_1+\sigma'_3)$ with % axial strain during shear is given for the A and B groups in Figs. 7-8 and 7-9. In the group C tests, σ'_2 was not measured; the variation of (σ'_1/σ'_3) with % strain is shown in Fig. 7-10. The (σ'_1/σ'_3) curves have the same characteristic shape for each family and reach a maximum about one per cent strain after the peak deviator stress occurs. The ratio $\left[\sigma'_1/\sigma'_3 \right]_{\max}$ has the average values of 2.70, 2.70 and 2.55 for groups A, B, and C respectively. The ratio (σ'_2/σ'_1) increases only slightly with % strain after an initial drop at the beginning of the shear stage, and has the average values at maximum deviator stress of 0.58 and 0.54 for groups A and B respectively. These values are approximately equal to the average K_0 values obtained during consolidation. The ratio of the intermediate principal effective stress to the sum of the other two principal effective stresses gradually increases with % strain in much the same manner as (σ'_1/σ'_3) does; the magnitude of $\sigma'_2/(\sigma'_1+\sigma'_3)$ is 0.42 for group A and 0.40 for group B at maximum deviator stress. Also included in Fig. 7-10 is the principal stress ratio for WC19(0).

The difference between isotropically and K_0 consolidated specimens is illustrated by the fact that a higher % strain is required in the isotropically consolidated sample before a maximum ratio is attained. The stress ratios for test Nos. WC12, WC13(0), and WC14(0) were plotted in Fig. 7-11.

(b) The Shear Strength Parameters

The conventional Mohr-Coulomb parameters c' and ϕ' for the plane strain tests were obtained by plotting $(\sigma_1' - \sigma_3')/2$ and $(\sigma_1' + \sigma_3')/2$ as ordinates and abscissae respectively at maximum deviator stress (Fig. 7-12) and at maximum principal stress ratio (Fig. 7-13). The best straight line was drawn through the points corresponding to the specimens under plane strain conditions, and an average cohesion intercept and angle of internal friction in terms of effective stresses were determined. The results of the K_0 consolidated tests conducted with no end restraint were plotted for comparison. It can be seen that these points lie below the straight line envelope in both the maximum deviator stress and maximum principal stress ratio criteria. The results of test WC19(0), in which the specimen was isotropically consolidated, plot slightly above the failure envelope in Fig. 7-12 and on the envelope in Fig. 7-13.

At maximum deviator stress, the average strength was represented by $c' = 0$ and $\phi' = 27.1^\circ$, whereas at maximum principal stress ratio the values were $c' = 0$ and $\phi' = 27.4^\circ$.

(c) c/p Relationship

In Fig. 7-14, the maximum deviator stress was plotted against the major principal effective stress at the end of consolidation. The slope of the average line drawn through the points is 0.56, and the resulting c/p value is 0.28.

(d) Stress Paths

Figs. 7-15 to 7-24 inclusive show in a dimensionless form the stress paths followed in the shear stage of the tests. The tests are grouped as before and the stresses have been divided by the average normal stress at the end of consolidation (i.e. P_c). The average consolidation path for the relevant tests is shown in the graphs relating (σ'_1/P_c) to (σ'_2/P_c) and (σ'_3/P_c) .

Figs. 7-15 to 7-17 inclusive show the stress paths in terms of (σ'_1/P_c) and (σ'_3/P_c) for the three groups A, B, and C. Along the stress path the relative values of (σ'_1/P_c) are highest for group A and lowest for group C. The high values of the A group tests could possibly be explained by the fact that, at the relatively low consolidation pressures used, the samples were not completely normally consolidated, and some effects of the initial stress conditions induced during the remoulding process were still present. Lower relative values of (σ'_1/P_c) would thus be expected with increasing consolidation pressures as these effects then become less significant. This is suggested in the curves of the group B tests.

In Fig. 7-17, the low values of (σ'_1/p_c) obtained in the group C tests could result from the fact that high consolidation pressures were used and the specimens were sheared under conditions of no end restraint.

In Fig. 7-18, the results of test Nos. WC12, WC13(O) and WC14(O) were plotted in terms of (σ'_1/p_c) against (σ'_3/p_c) . It can be seen that the shape of the stress path does not appreciably affect the shear strength parameters of the clay as the paths terminate close to the envelope corresponding to $\phi' = 27.1^\circ$.

In Fig. 7-19, the graph of (σ'_1/p_c) against (σ'_2/p_c) for group A shows relatively good agreement with the average K_o line, while group B in Fig. 7-20 corresponds to a ratio (σ'_2/σ'_1) of 0.54. The low ratio obtained in the group B tests could be the result of insufficient pre-stress in the end clamp at the beginning of the shear stage.

Fig. 7-21 shows the stress paths for tests WC13(O) and WC14(O) in terms of (σ'_1/p_c) and (σ'_2/p_c) . The results of test no. WC14(O) show good agreement with the average $K_o = 0.59$ line which was obtained from the consolidation stage of that test, whereas WC13(O) reflects the high relative values of (σ'_1/p_c) discussed earlier.

The variation of (σ'_2/p_c) with (σ'_3/p_c) for the A and B groups is shown in Figs. 7-22 and 7-23. Fig. 7-24 shows a similar diagram for tests WC13(O) and WC14(O).

(e) Water Content - Stress Relations (at Failure)

The relationship between % water content and average normal stress at failure is given in Fig. 7-25. Failure has been taken to correspond to maximum deviator stress. It was assumed that an approximately uniform water content distribution existed in the specimen at the end of consolidation, but that moisture migration occurred during shear which resulted in a non-uniform water content throughout the specimen as indicated by the water content samples taken at the end of the shear stage. Consequently, the overall average moisture contents were used for the end of consolidation graph (see Fig. 7-3), whereas the failure conditions were represented by the average water contents at mid-height of the specimens. (Typical water content distributions after shearing are given for two specimens in Fig. 7-26). The specimen consolidated from a slurry plots well above the average line as in Fig. 7-3. The average line for tests with axial symmetry will be discussed later. Fig. 7-27 shows the relationship between the water content and the maximum deviator stress.

(7.4) Tests with Axial Symmetry

The tests conducted under conditions of axial symmetry were performed in the standard triaxial apparatus on $1\frac{1}{2}$ in. diameter x 3 in. high specimens. (See SOWA, 1963). All specimens were remoulded from the same batch of clay that was used for the plane strain samples. Only specimens which were K_0 consolidated and then failed in compression

are considered in this thesis.

The K_0 value in the standard triaxial tests varied between 0.58 and 0.61, the overall average being 0.59. The consolidation stage lasted from four to fourteen days depending on the consolidation pressure desired. The rate that the consolidation pressure was increased was generally faster than the rate used for the plane strain specimens.

The relationship between % water content and average normal stress at the end of consolidation for the tests with axial symmetry is shown in Fig. 7-28. Typical stress-strain curves plotted on a dimensionless basis are given in Fig. 7-29. The maximum deviator stress occurs at about 6% axial strain, and the corresponding limits of the ratio $\left[(\sigma_1 - \sigma_3) / P_c \right]$ are 0.68 and 0.75 approximately.

The conventional Mohr-Coulomb parameters c' and ϕ' were determined from the relationship between $(\sigma_1 - \sigma_3) / 2$ and $(\sigma_1' + \sigma_3') / 2$ at both peak deviator stress and the maximum principal stress ratio criteria. In Fig. 7-30, the strength at peak deviator stress is represented by $c' = 0$ and $\phi' = 25.9^\circ$. The values of the parameters at $(\sigma_1' / \sigma_3')_{\max}$ are $c' = 0$ and $\phi' = 26.2^\circ$ as shown in Fig. 7-31. In both failure criteria, the results of tests performed at high consolidation pressures lie below the straight line envelopes. This indicates that some flattening of the envelope may occur at the higher pressures.

Fig. 7-32 shows the relationship between the water content

and the average normal stress at failure for the tests with axial symmetry. As in Fig. 7-25 for the plane strain tests, failure was taken to correspond to the maximum deviator stress. The graph relating the maximum deviator stress and the major principal effective stress at the end of consolidation is illustrated in Fig. 7-33. The slope of the average line through the points is 0.52 and the corresponding c/p value is 0.26.

TEST NO. WC-	GROUP	M.C. _{ij} %	M.C. _o %	M.C. _o %		END OF CONSOLIDATION							PEAK DEVIATOR STRESS					MAXIMUM (σ'_1/σ'_3)				
				AV.	MID-HT.	ϵ_i %	ΔV in. ³	u_c	σ'_1	σ'_2	σ'_3	AV. κ_o	$(\epsilon_1)_s$	Δu	σ'_1	σ'_2	σ'_3	$(\epsilon_1)_e$	Δu	σ'_1	σ'_2	σ'_3
4	B	32.80	-	22.59	22.30	10.50	11.69	36.2	68.9	39.1	39.1	0.57	1.84	16.2	61.0	31.7	22.9	3.18	18.1	57.8	32.1	21.0
6	A	33.05	-	23.32	23.09	14.23	18.76	36.2	54.6	31.4	31.1	0.58	2.02	11.7	50.7	29.5	19.5	2.99	12.5	49.4	28.0	18.7
7	-	33.75	-	23.16	22.98	15.39	21.05	31.2	75.9	48.0	47.3	0.62	1.56	21.3	66.2	39.0	26.2	3.91	27.3	54.4	34.1	20.2
8	A	32.21	-	23.52	23.21	11.25	14.39	35.9	44.5	25.5	25.0	0.57	1.94	9.8	41.1	22.9	15.1	2.66	10.4	40.3	23.2	14.5
9	A	31.00	-	22.75	22.55	9.56	12.44	36.0	48.8	27.8	26.8	0.55	1.55	9.6	45.8	25.9	17.2	2.36	10.3	44.8	25.7	16.5
10	B	34.26	-	20.71	20.46	16.66	23.44	30.8	117.4	69.6	69.6	0.595	1.50	26.4	108.3	59.1	43.2	2.25	29.2	104.9	60.0	40.4
11	A	33.12	-	22.78	22.53	13.71	17.63	29.0	59.4	37.1	34.2	0.57	1.86	14.3	56.8	33.5	23.3	2.31	14.7	53.2	33.1	19.5
12	-	34.39	-	23.68	23.59	12.78	17.47	30.8	43.1	-	27.6	0.64	2.21	12.7	39.6	-	14.9	2.21	12.7	39.6	-	14.9
13(0)	-	60 ⁺	26.79	21.10	20.88	12.90	12.77	30.4	110.0	60.4	60.4	0.545	1.71	22.0	104.6	51.7	38.6	3.08	25.4	99.8	52.2	35.2
14(0)	-	106 ⁺	35.40	25.10	25.05	15.48	-	30.2	81.1	48.1	48.1	0.595	1.15	19.5	71.7	40.8	28.7	2.42	23.9	65.9	38.5	24.3
15(0)	B	35.14	31.30	21.24	20.99	14.70	18.57	30.3	104.6	62.0	61.1	0.59	2.54	26.6	93.4	51.6	34.5	2.54	26.6	93.4	51.6	34.5
16(0)	C	33.92	29.25	19.68	19.50	10.52	11.29	35.2	157.4	-	96.7	0.62	2.80	39.9	141.8	-	56.7	3.51	42.3	137.8	-	54.3
17(0)	C	32.72	28.46	20.60	20.38	14.27	17.37	35.2	110.7	-	68.5	0.62	3.43	28.5	101.5	-	40.0	4.72	30.8	97.9	-	37.7
18(0)	C	33.81	28.98	21.27	21.20	13.23	16.61	36.7	87.5	-	53.4	0.62	2.97	23.5	76.9	-	29.9	3.56	24.4	75.5	-	29.0
19(0)	-	34.32	29.28	21.46	21.29	-	16.43	35.0	65.6	-	65.2	-	5.01	41.3	64.5	-	23.9	5.55	41.4	64.3	-	23.8

TABLE 7-1 SUMMARY OF PLANE STRAIN TESTS ON WEALD CLAY

Notes on TABLE 7-1

$(m.c.)_i$ %	- average initial moulding water content;
$(m.c.)_o$ %	- average water content after material is removed from the oedometer;
$Av.(m.c.)_f$ %	- average overall moisture content after testing;
$Mid-Ht.(m.c.)_f$ %	- average moisture content along mid-height of the specimen after testing;
u_c	- average pore pressure at the end of consolidation in psi., i.e. the back pressure;
(ϵ_1) %	- axial strain at the end of consolidation;
Δu	- is the pore pressure change from the end of consolidation;
$(\epsilon_1)_s$ %	- axial strain during shear.

The principal effective stresses are in psi. units.

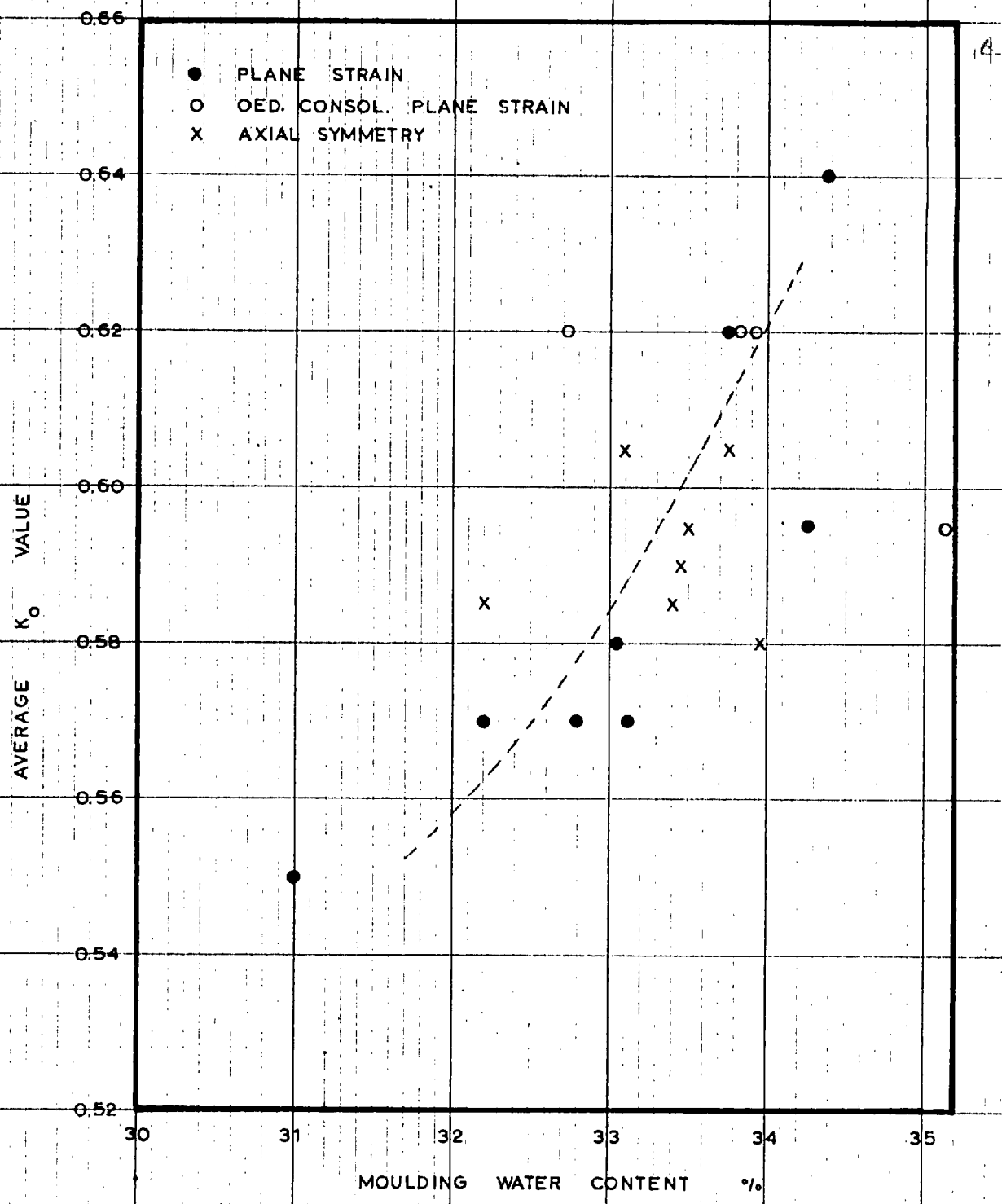


FIG 7-1

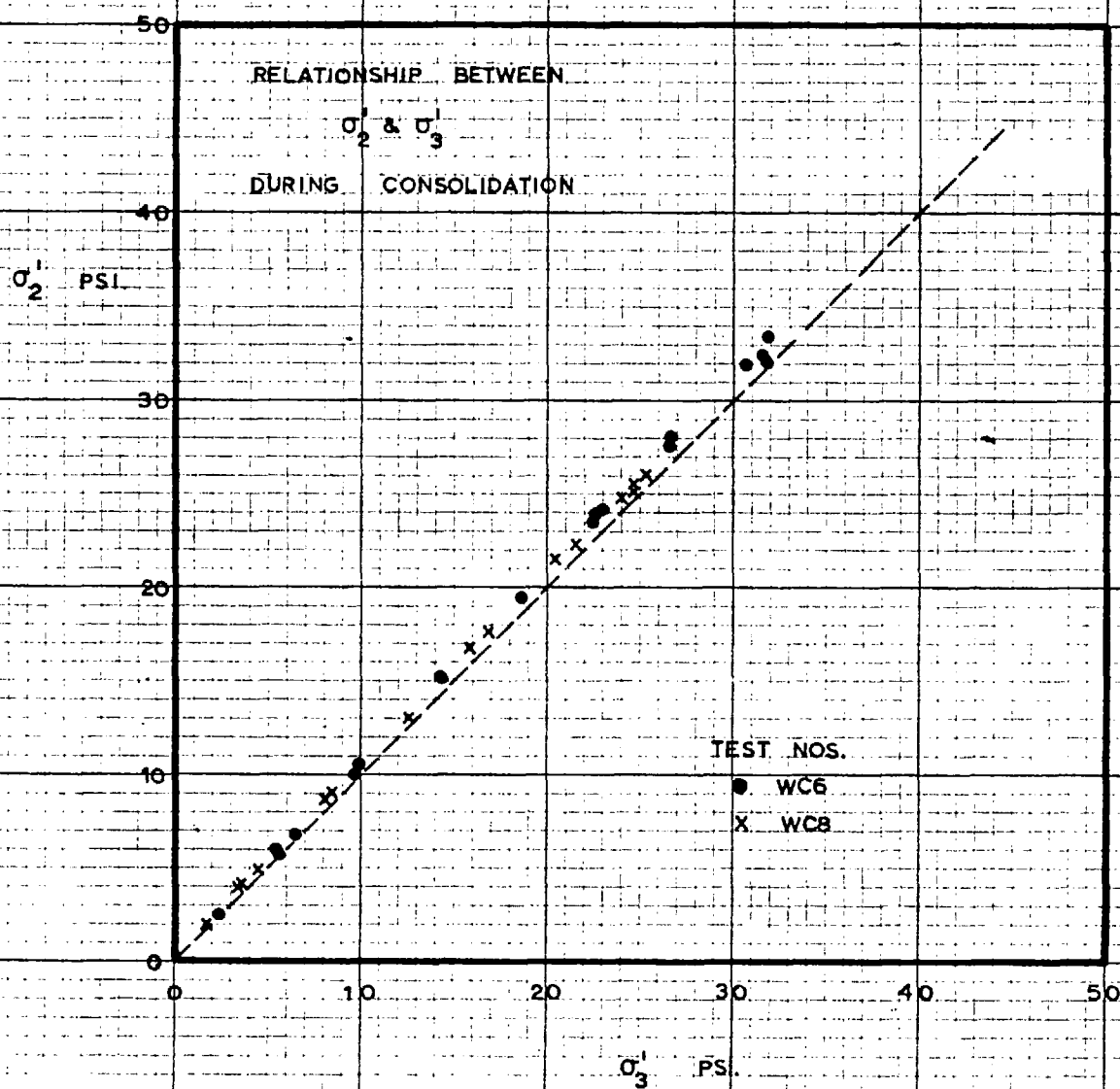


FIG 7-2

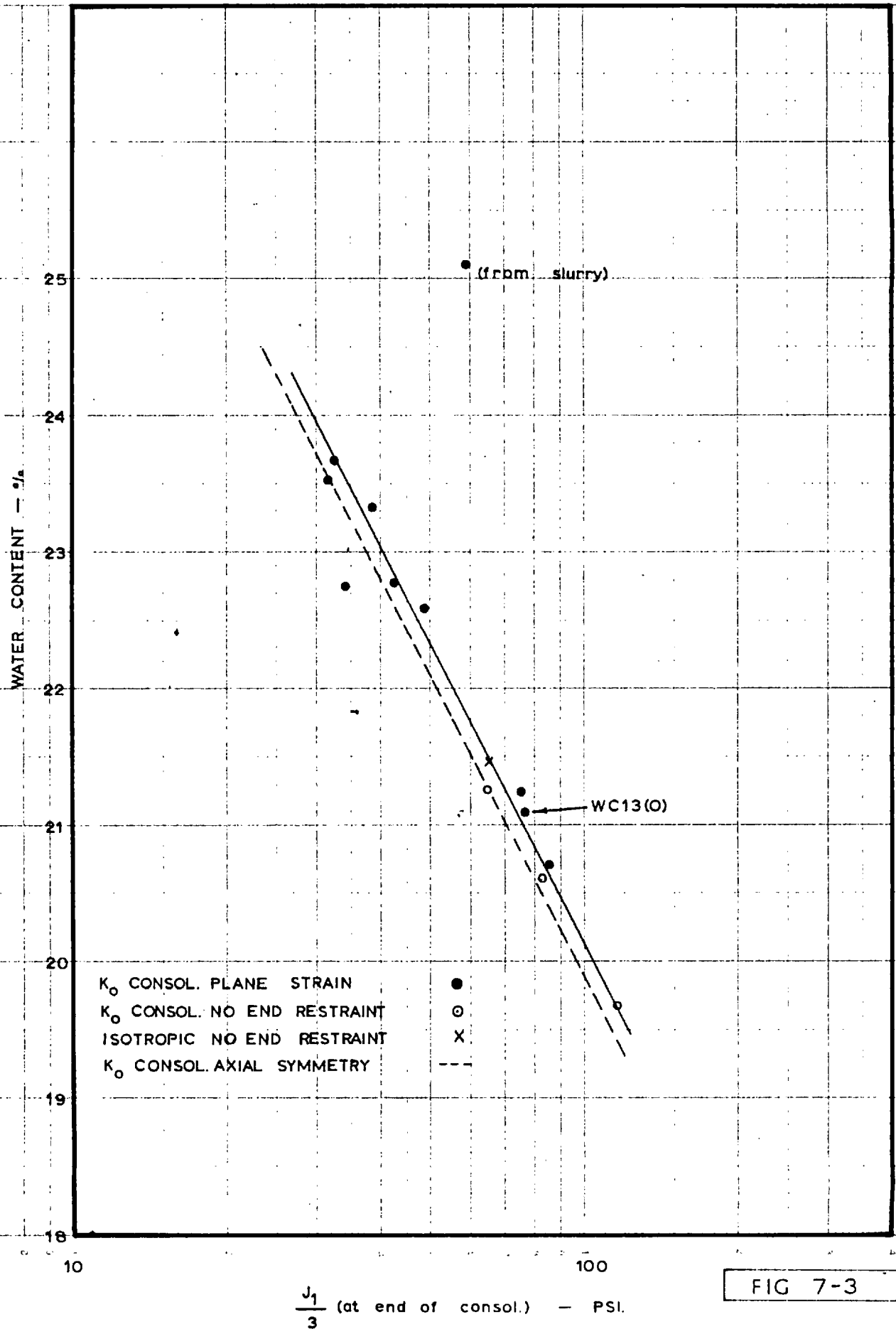


FIG 7-3

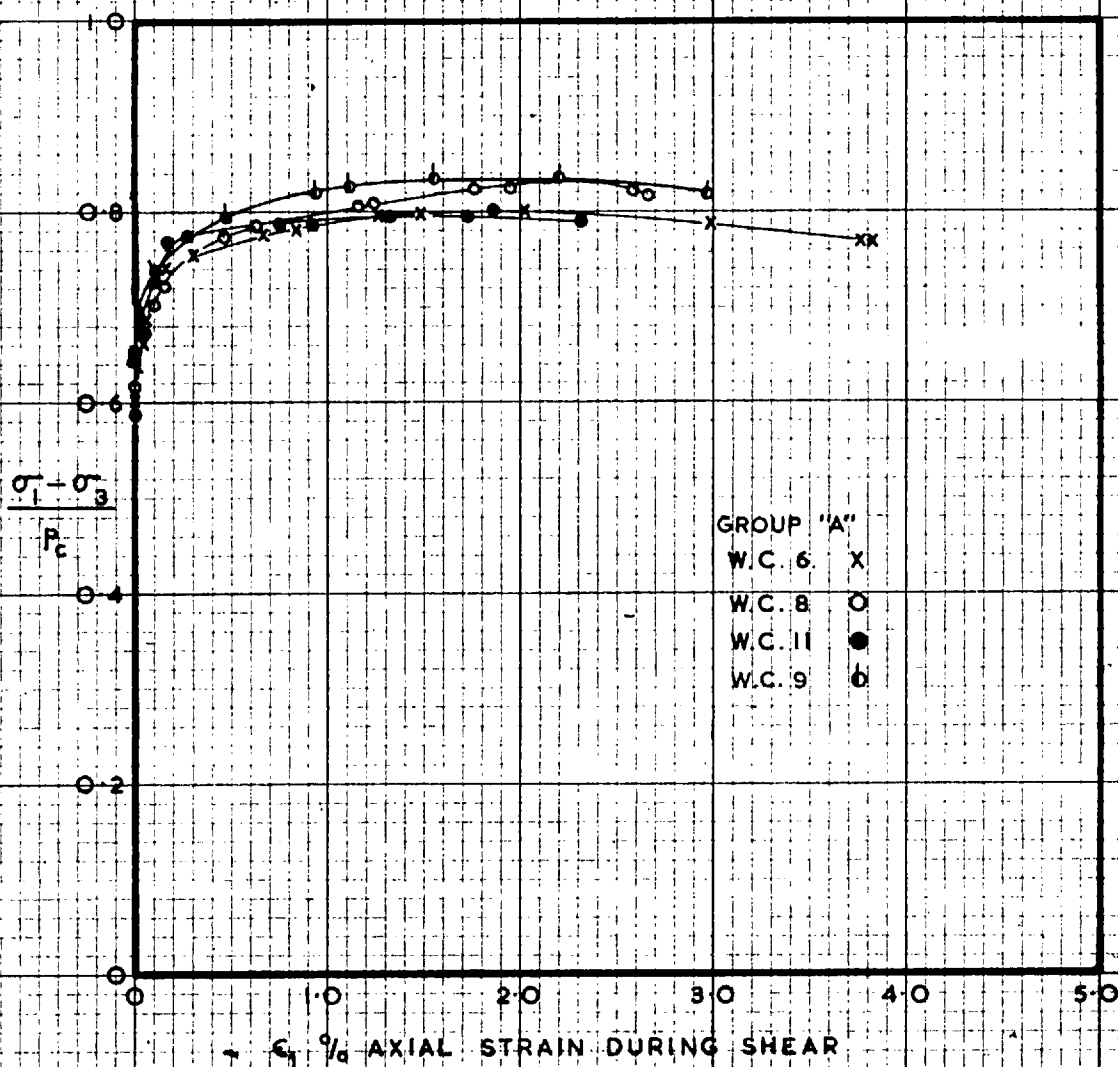


FIG 7-4

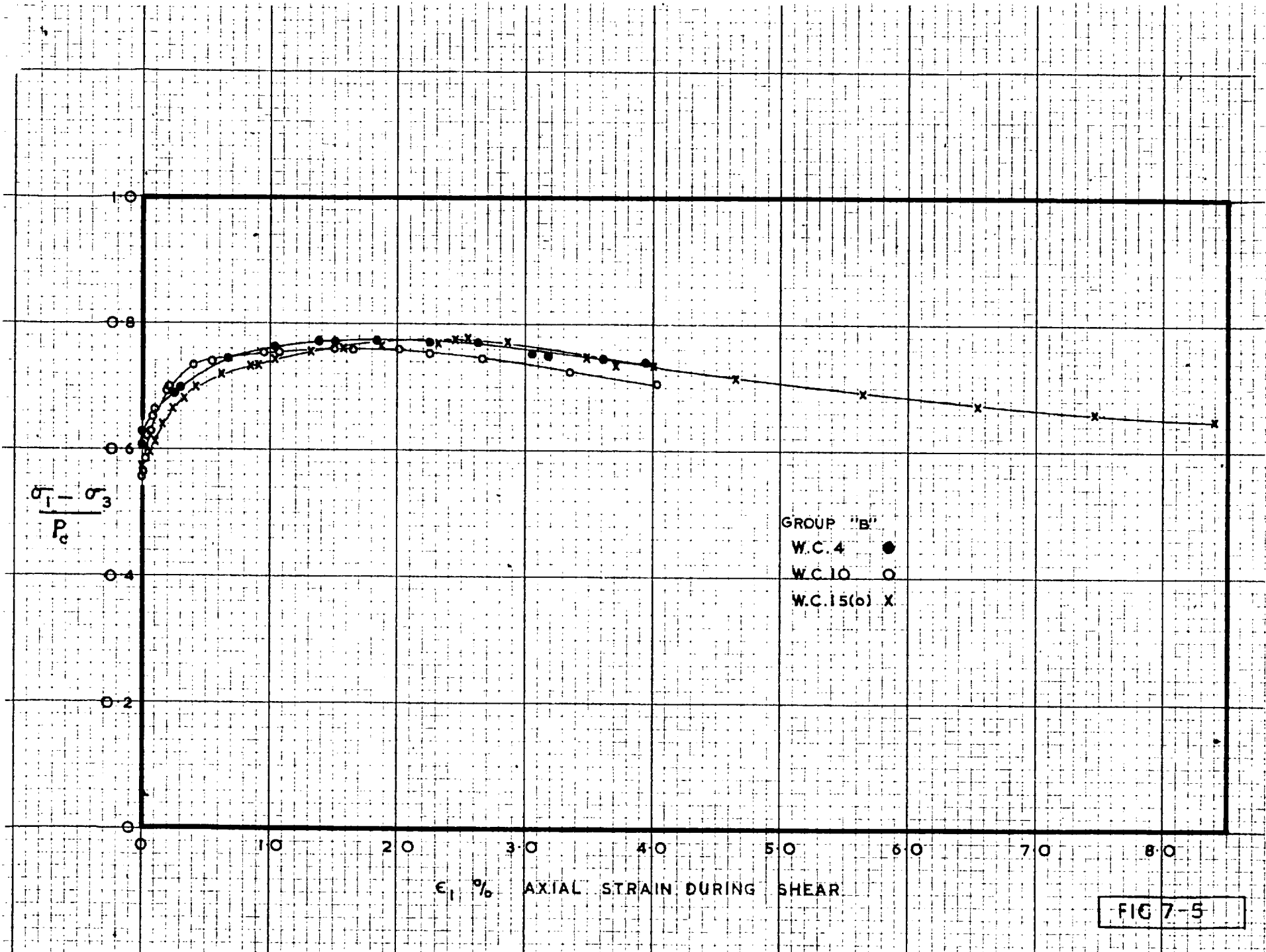


FIG 7-5

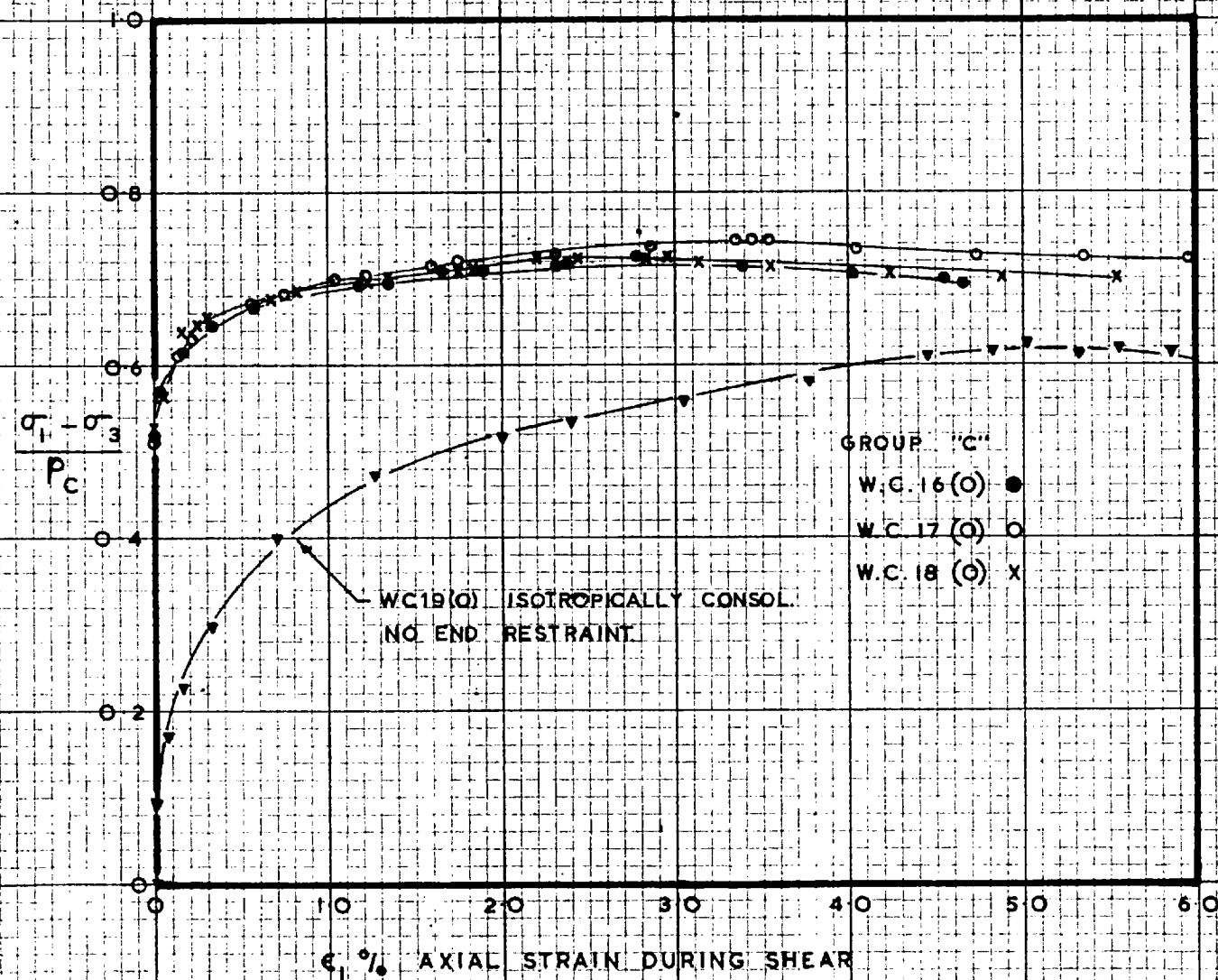


FIG 7-6

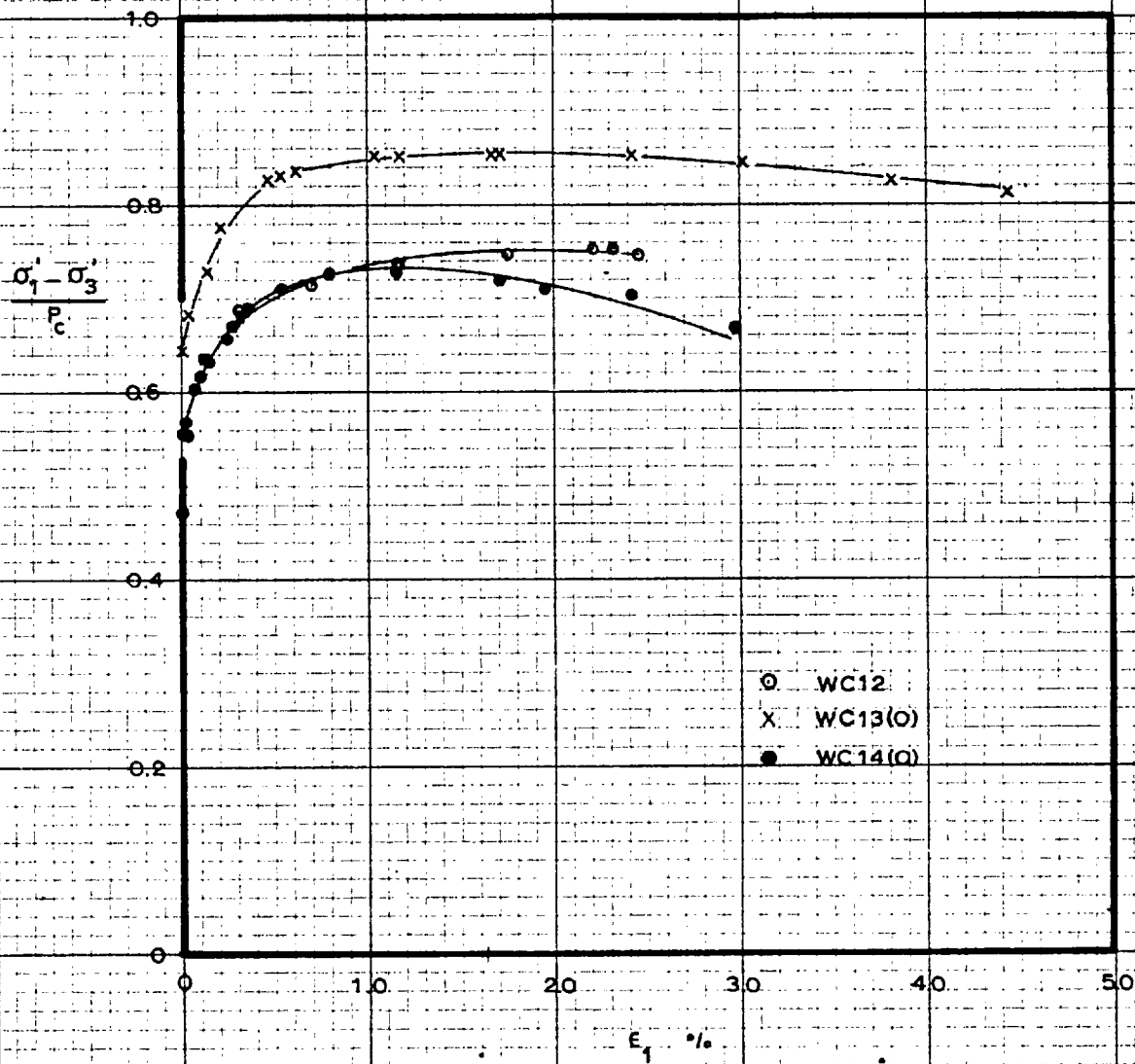


FIG 7-7

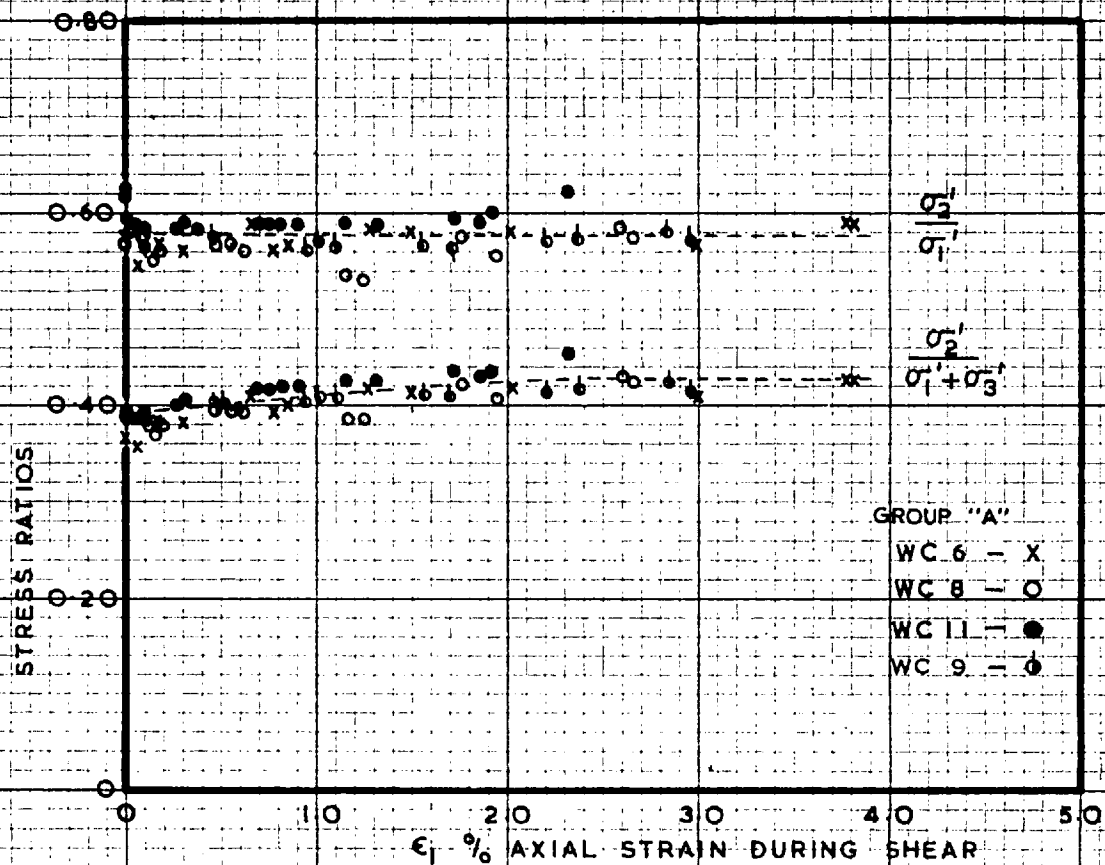
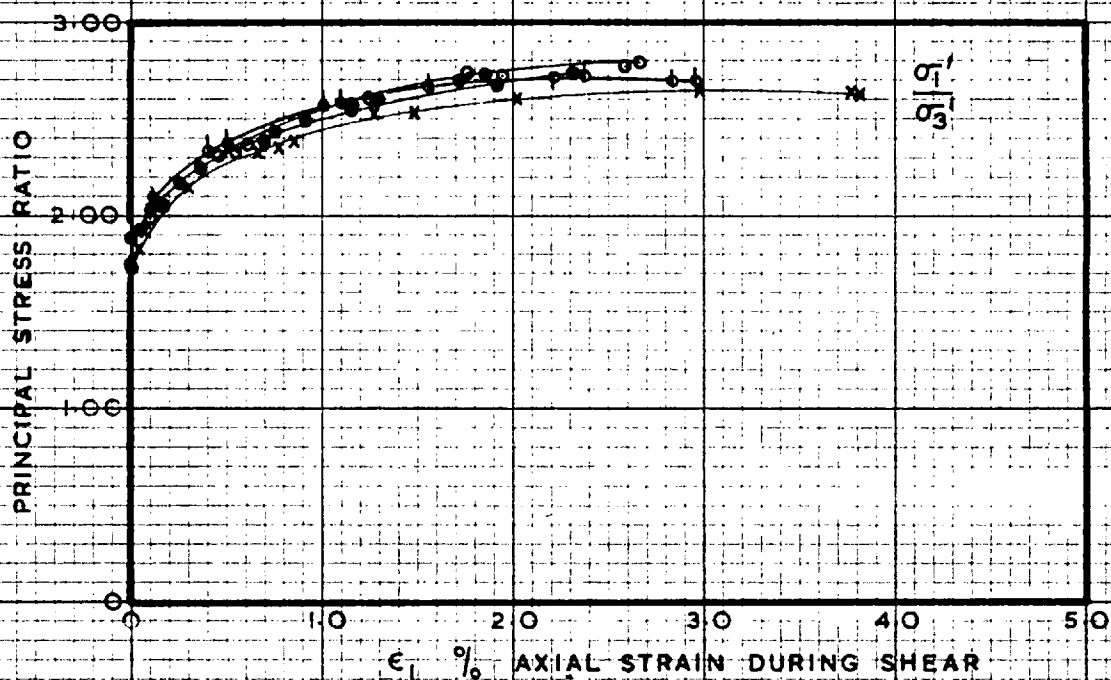


FIG 7-8

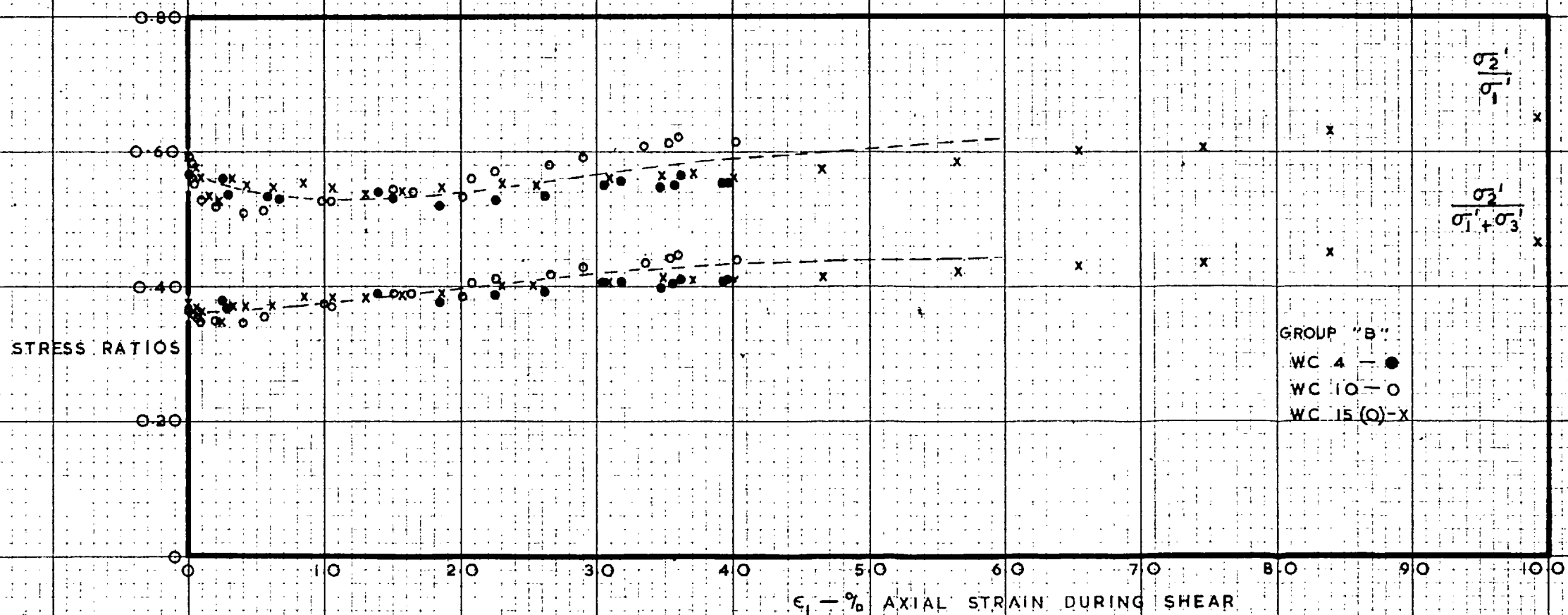
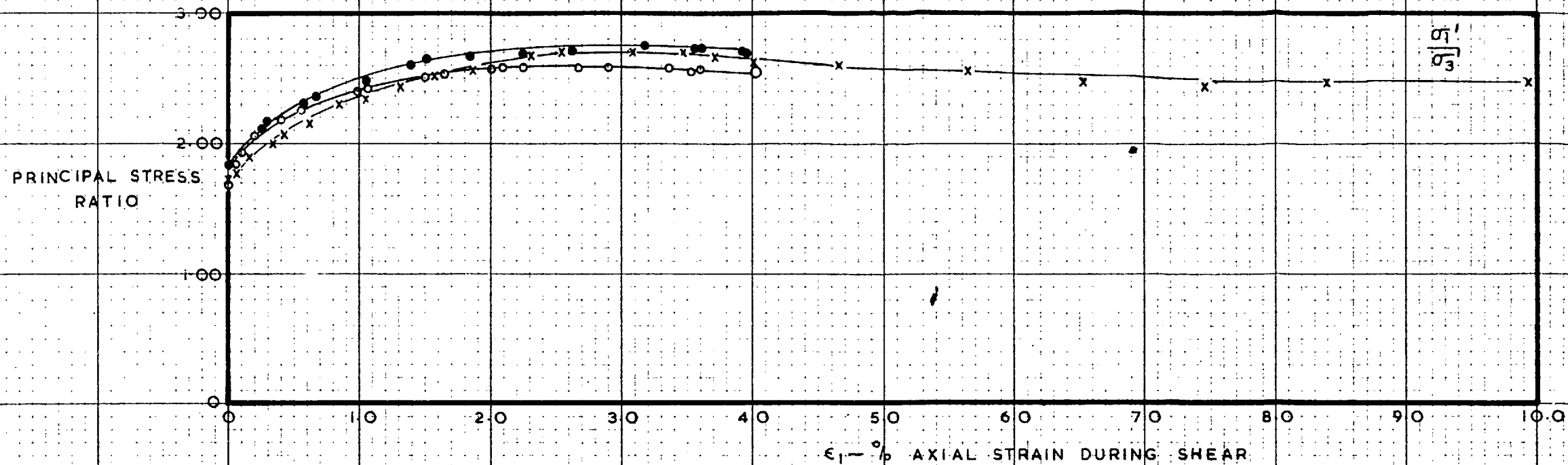


FIG 7-9

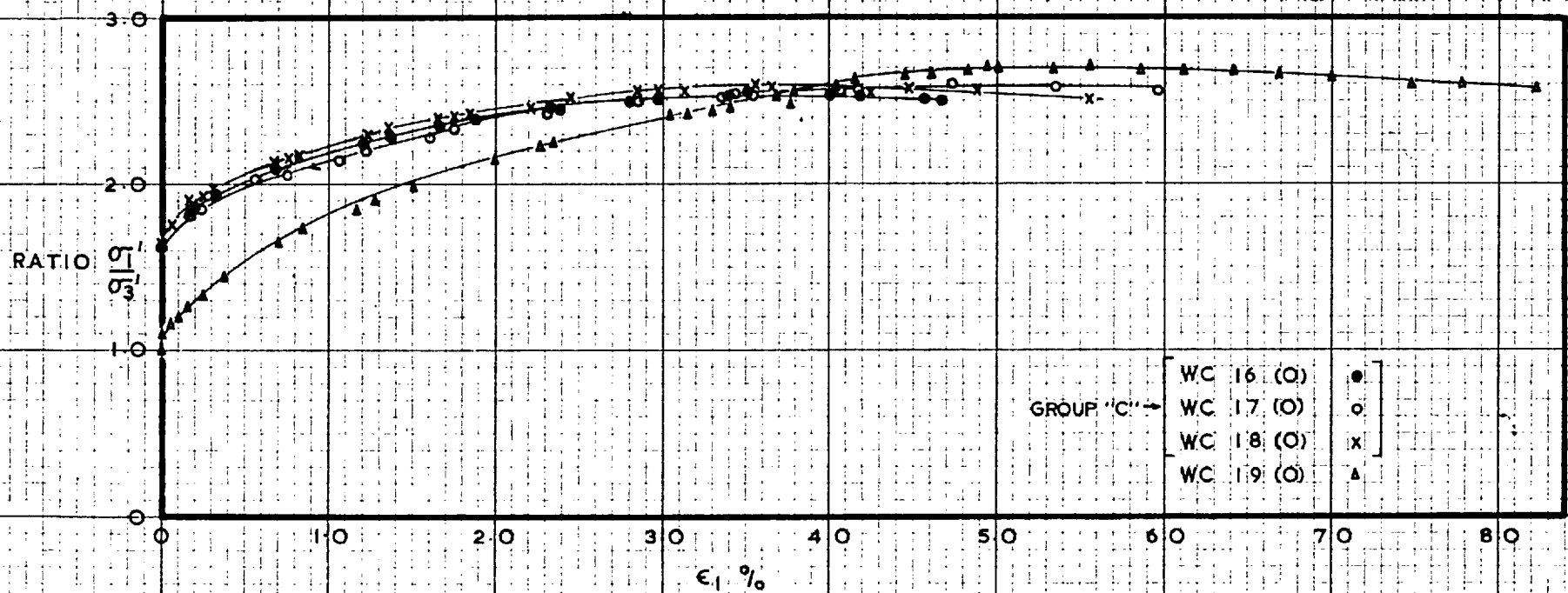


FIG No 7-10

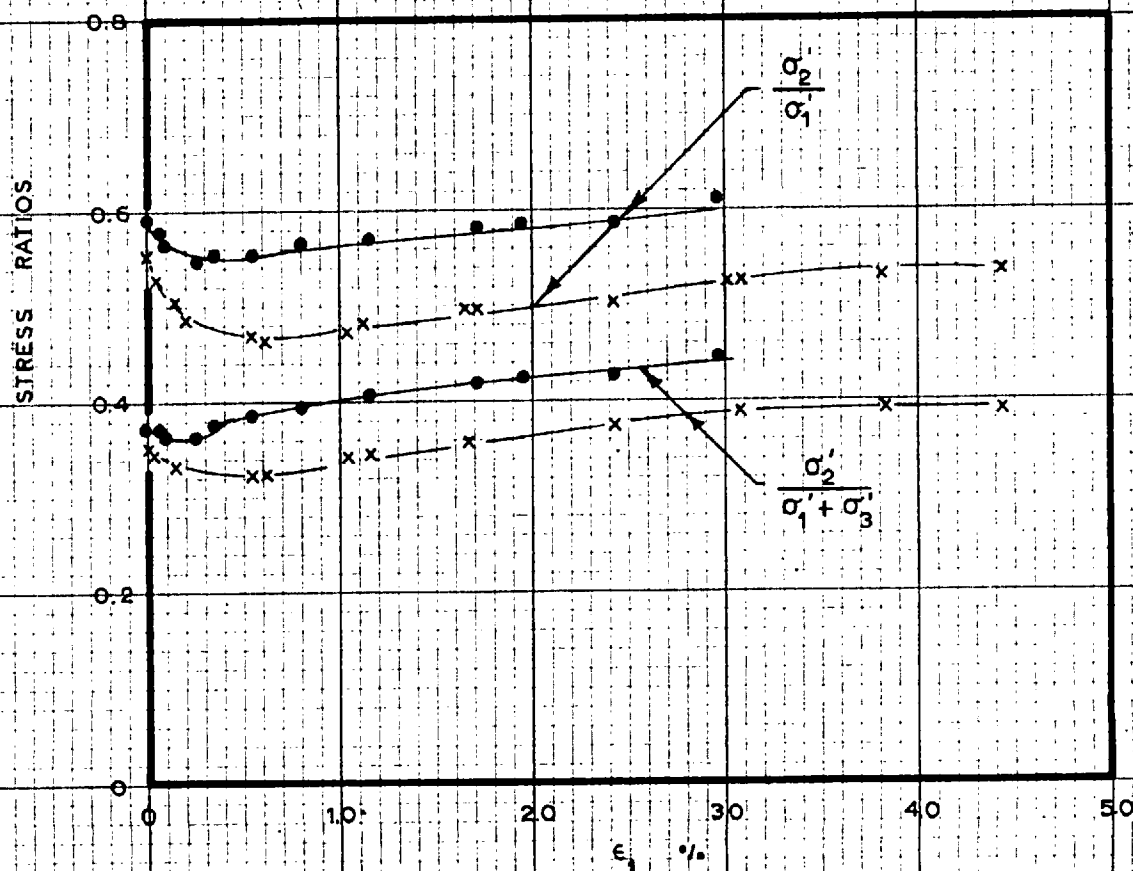
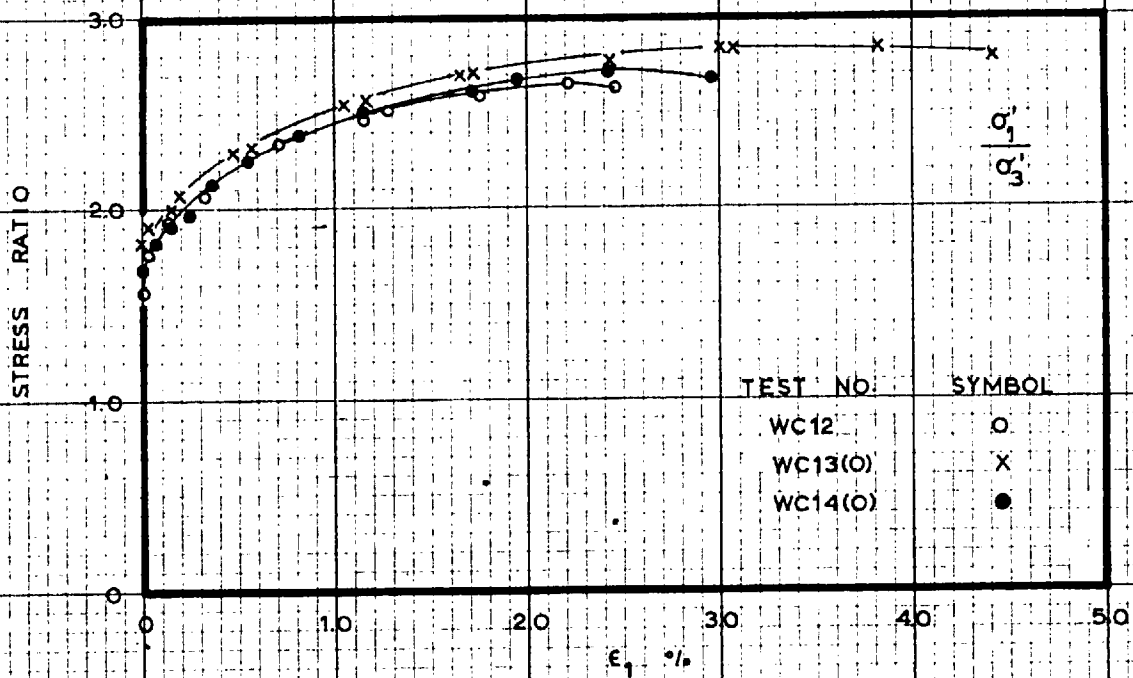


FIG 7-11

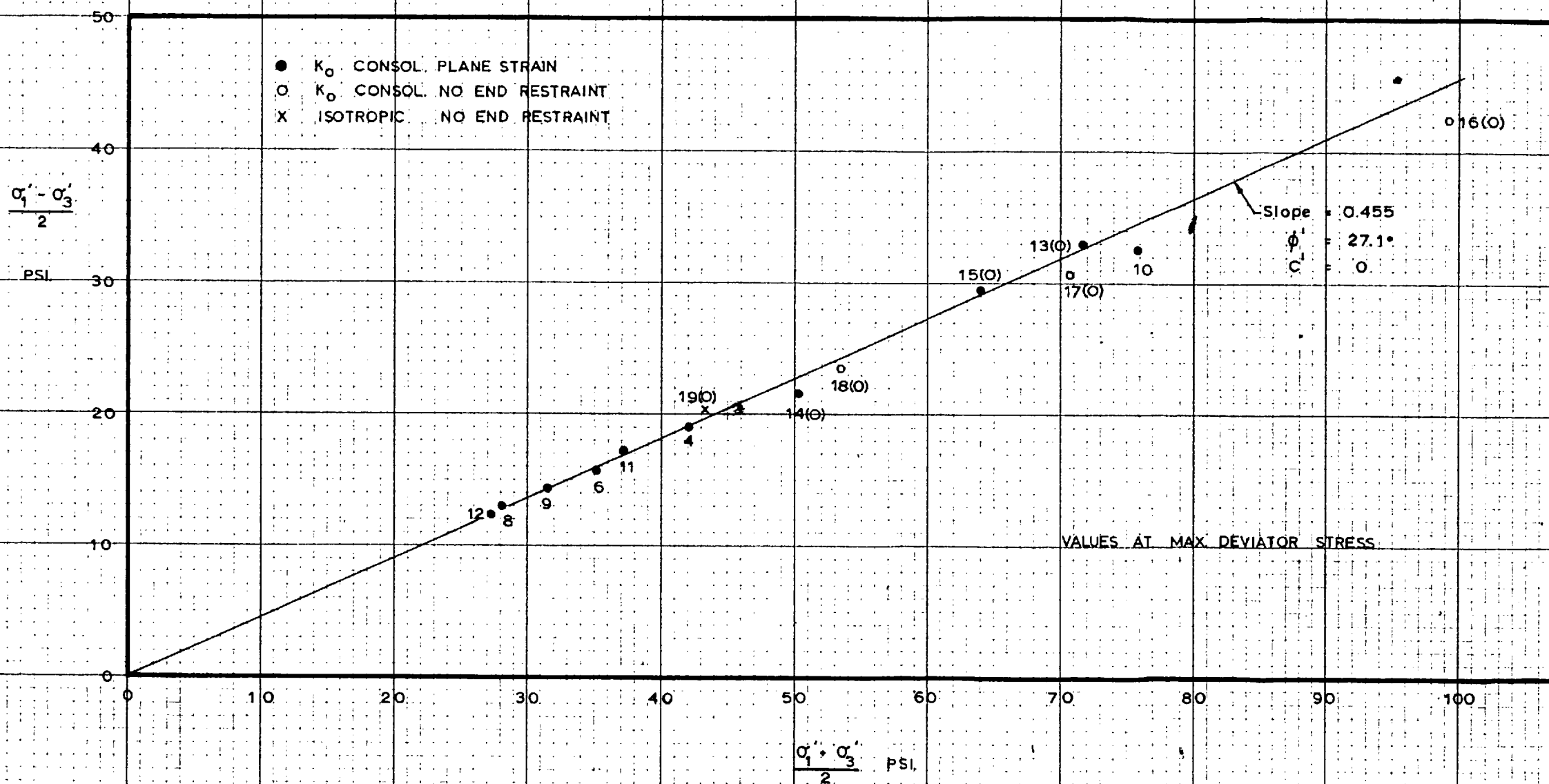


FIG 7-12

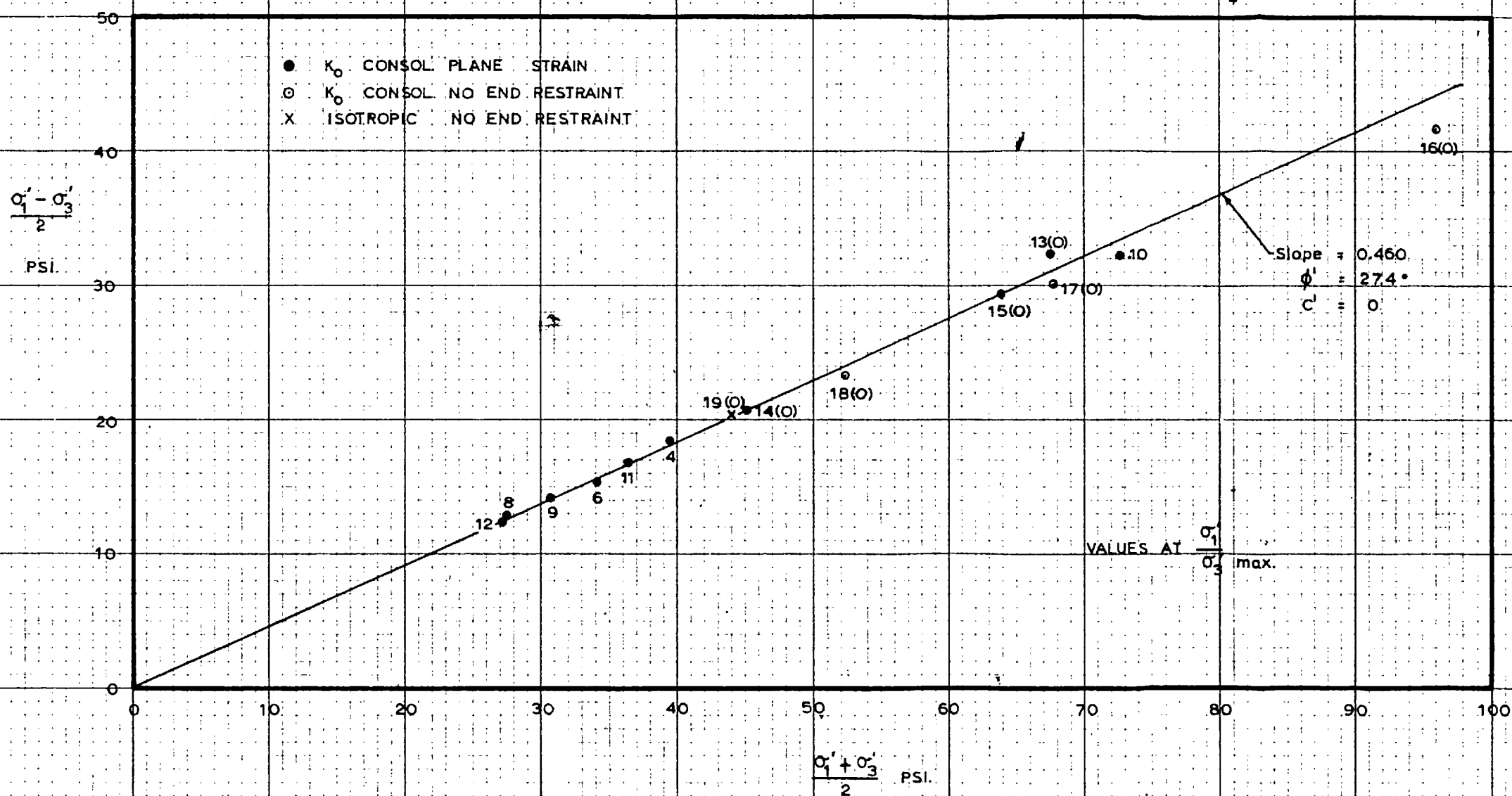


FIG 7-13

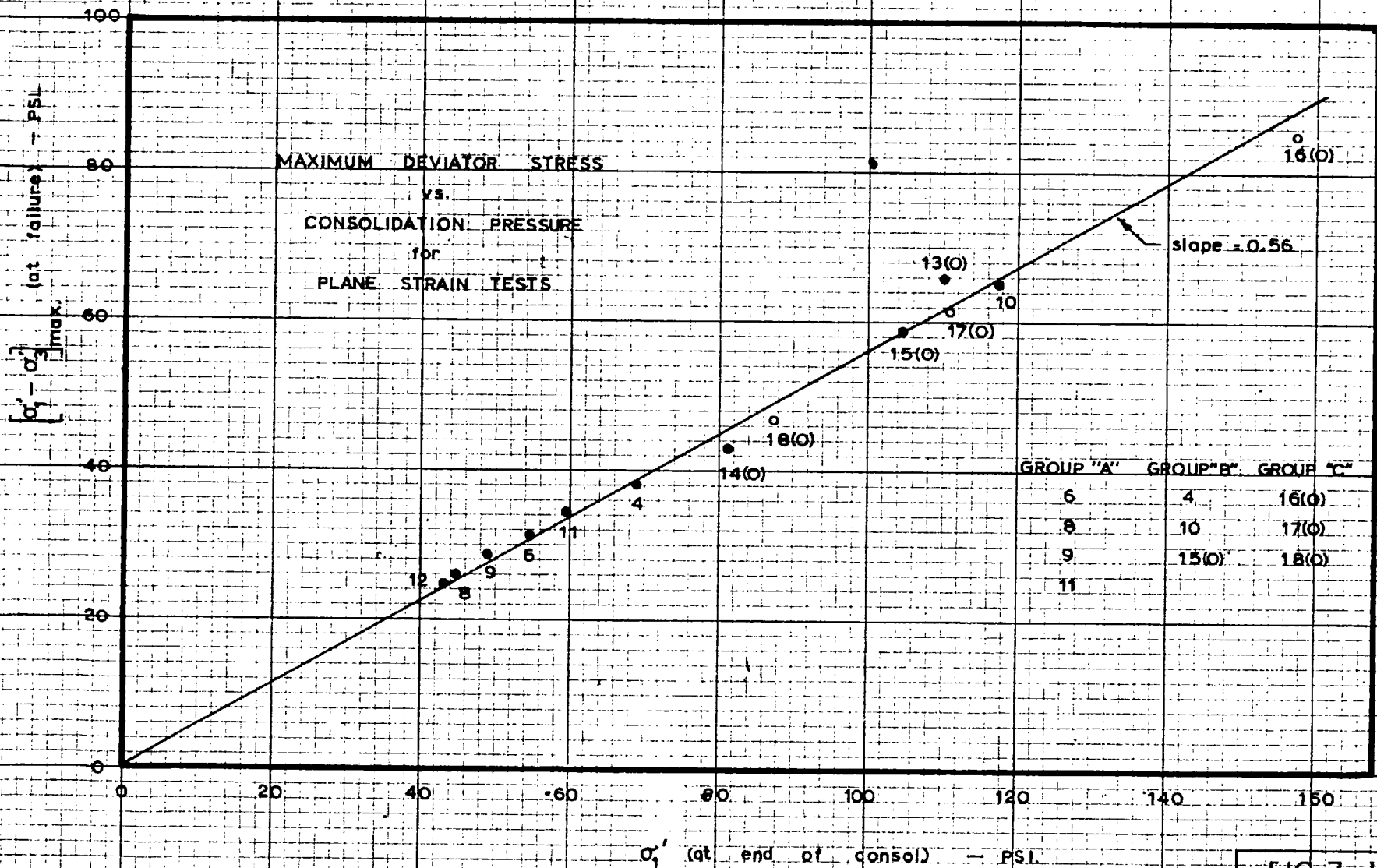


FIG 7-14

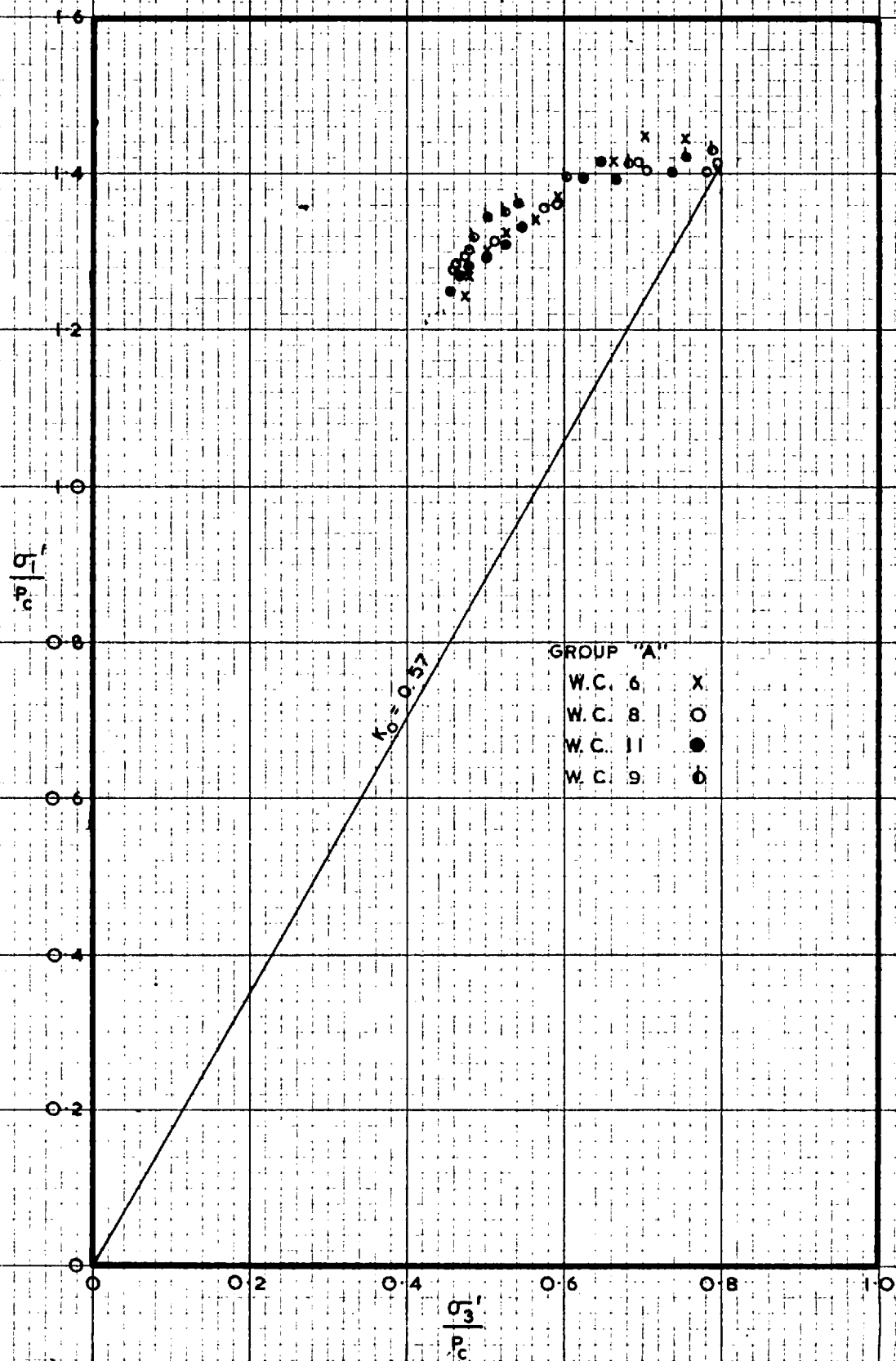


FIG 7-15

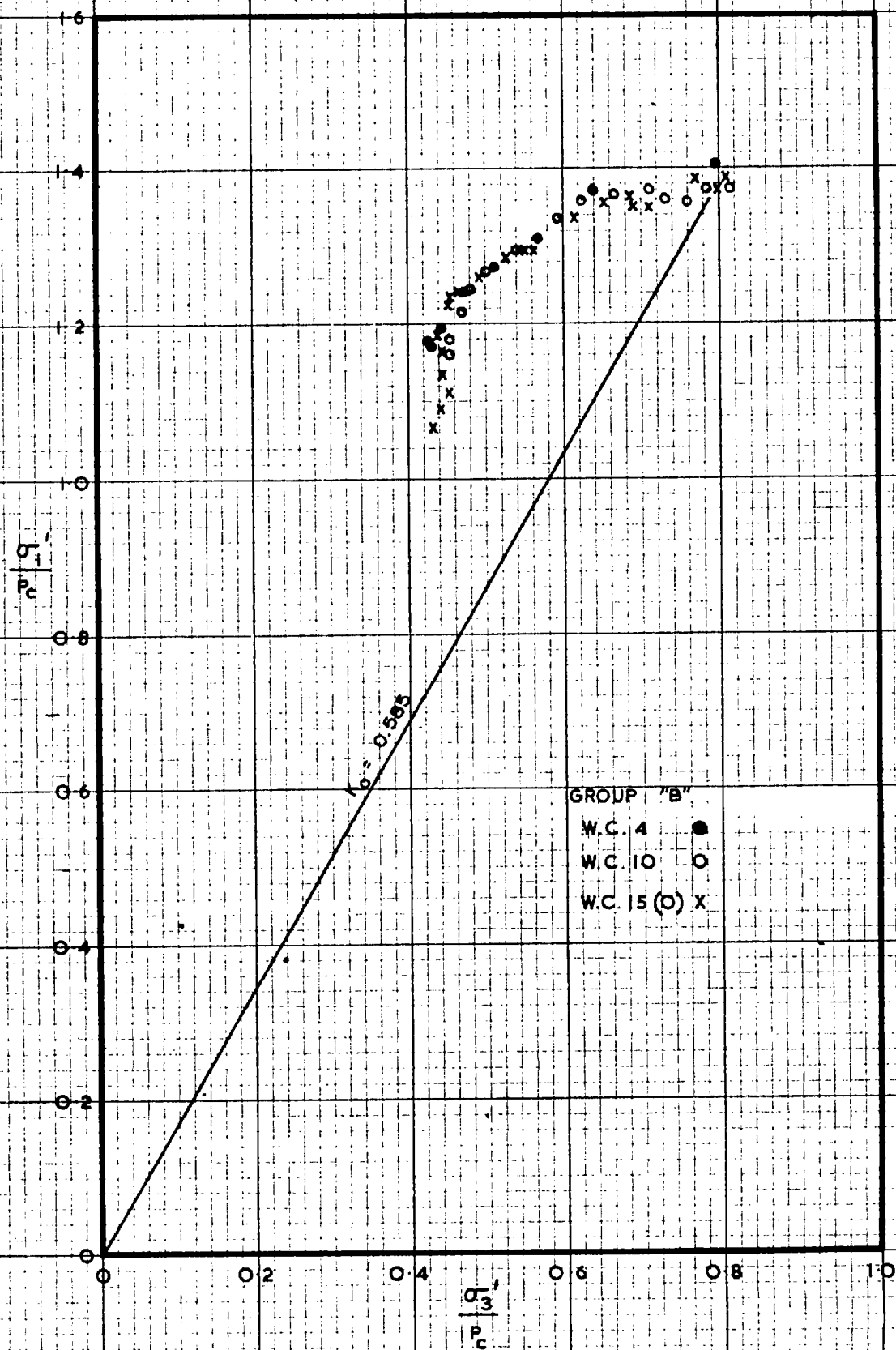


FIG 7-16

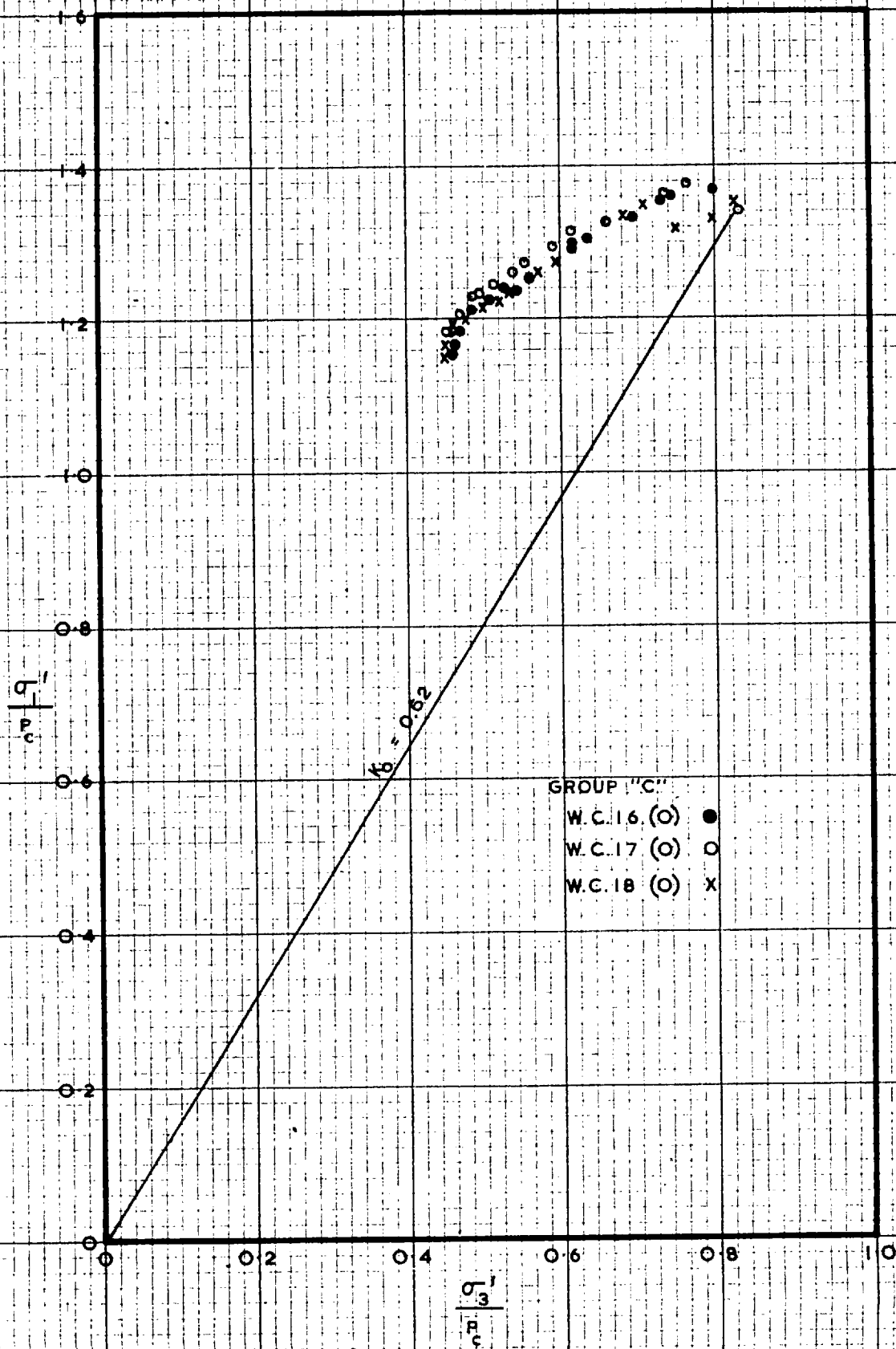


FIG. 7-17

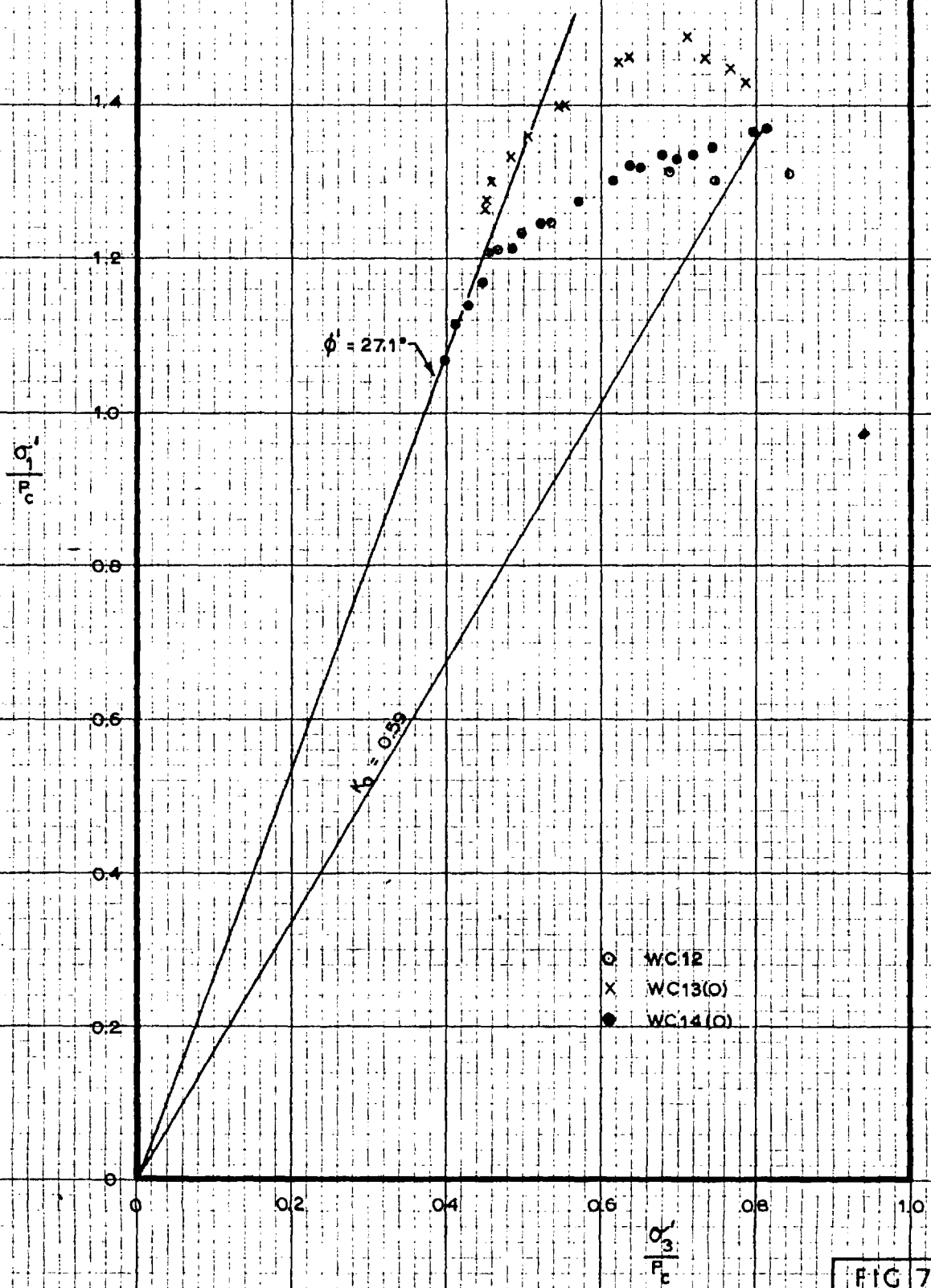


FIG 7-18

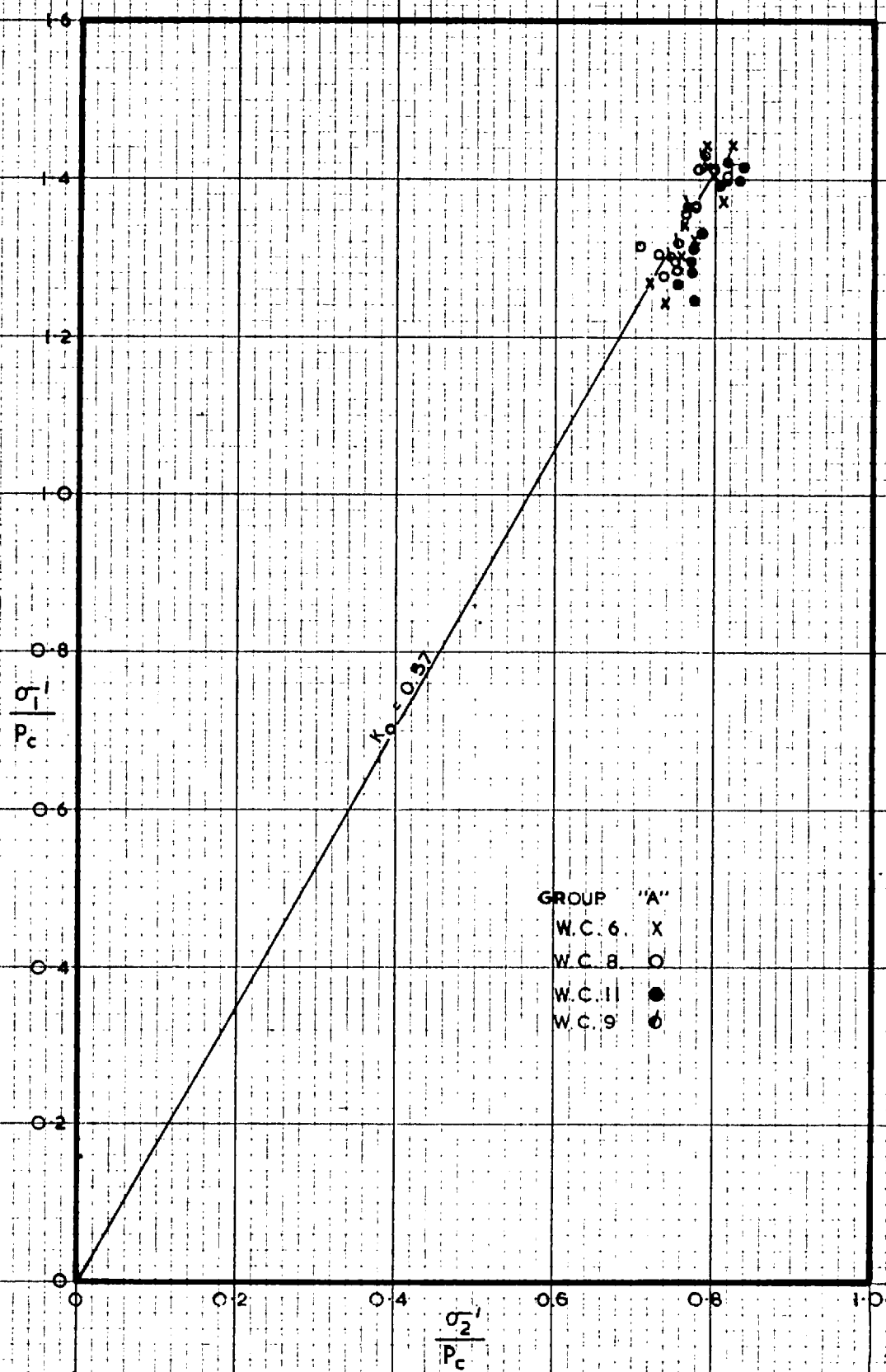


FIG 7-19

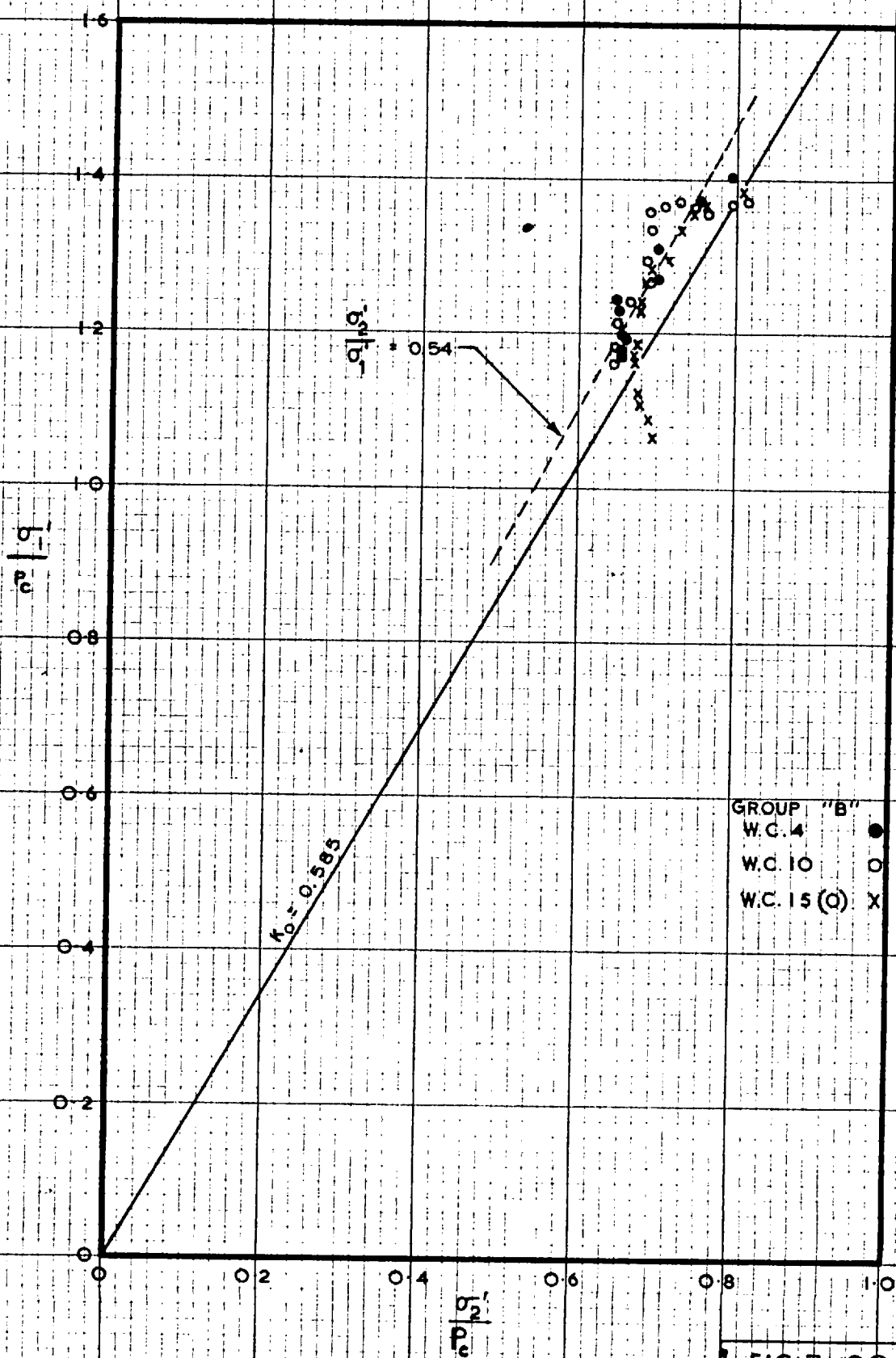
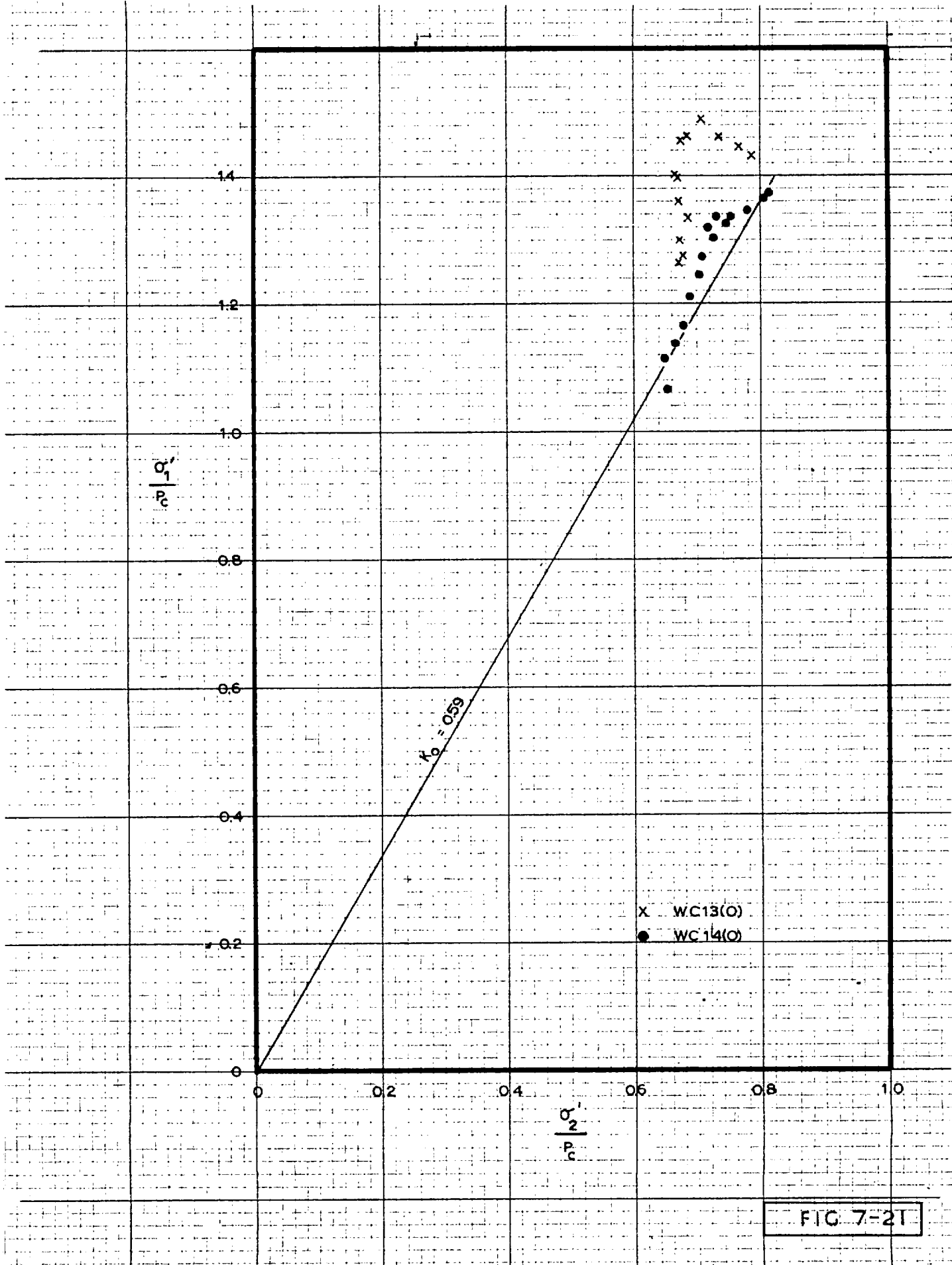
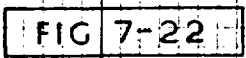
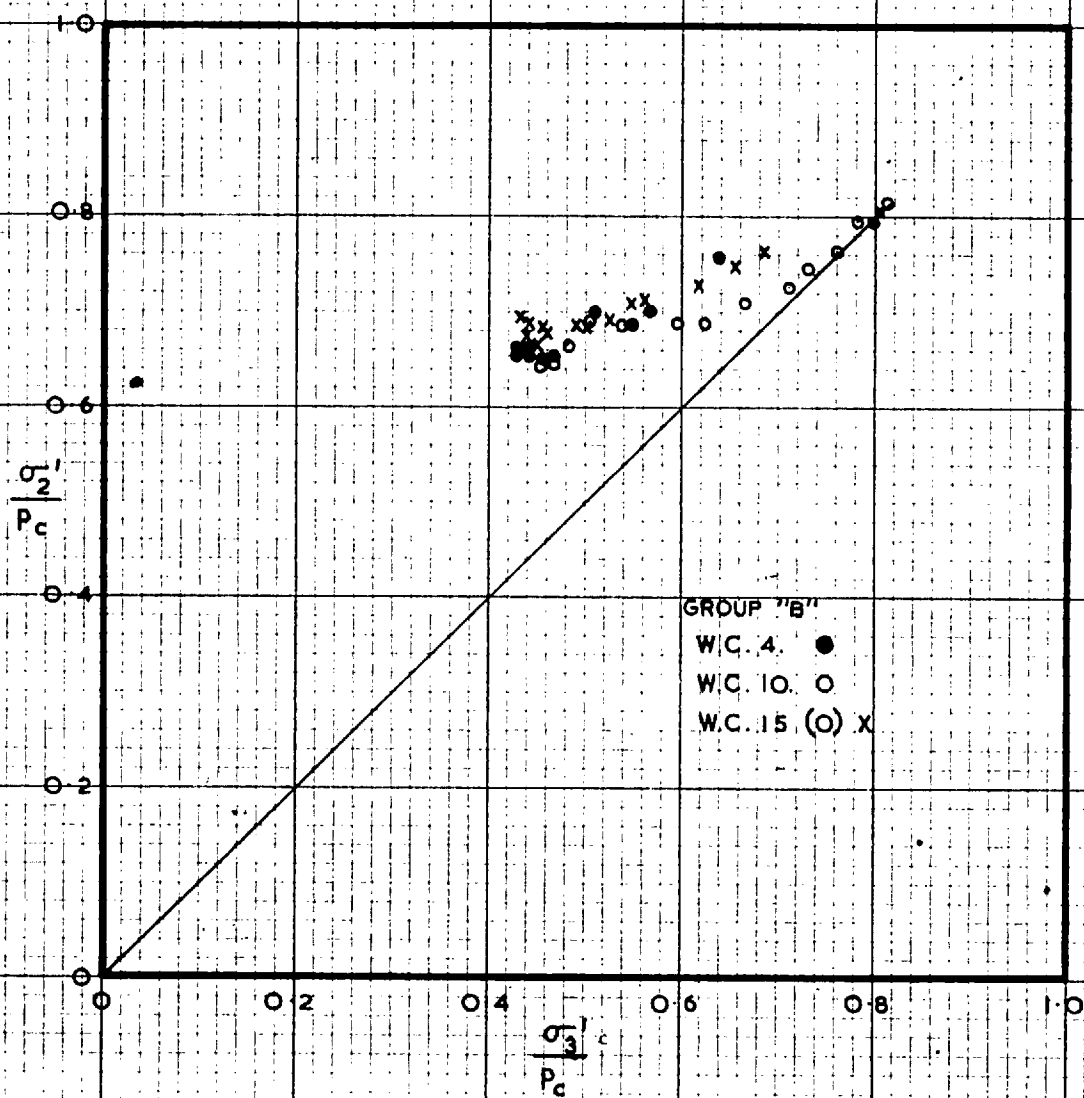


FIG 7-20







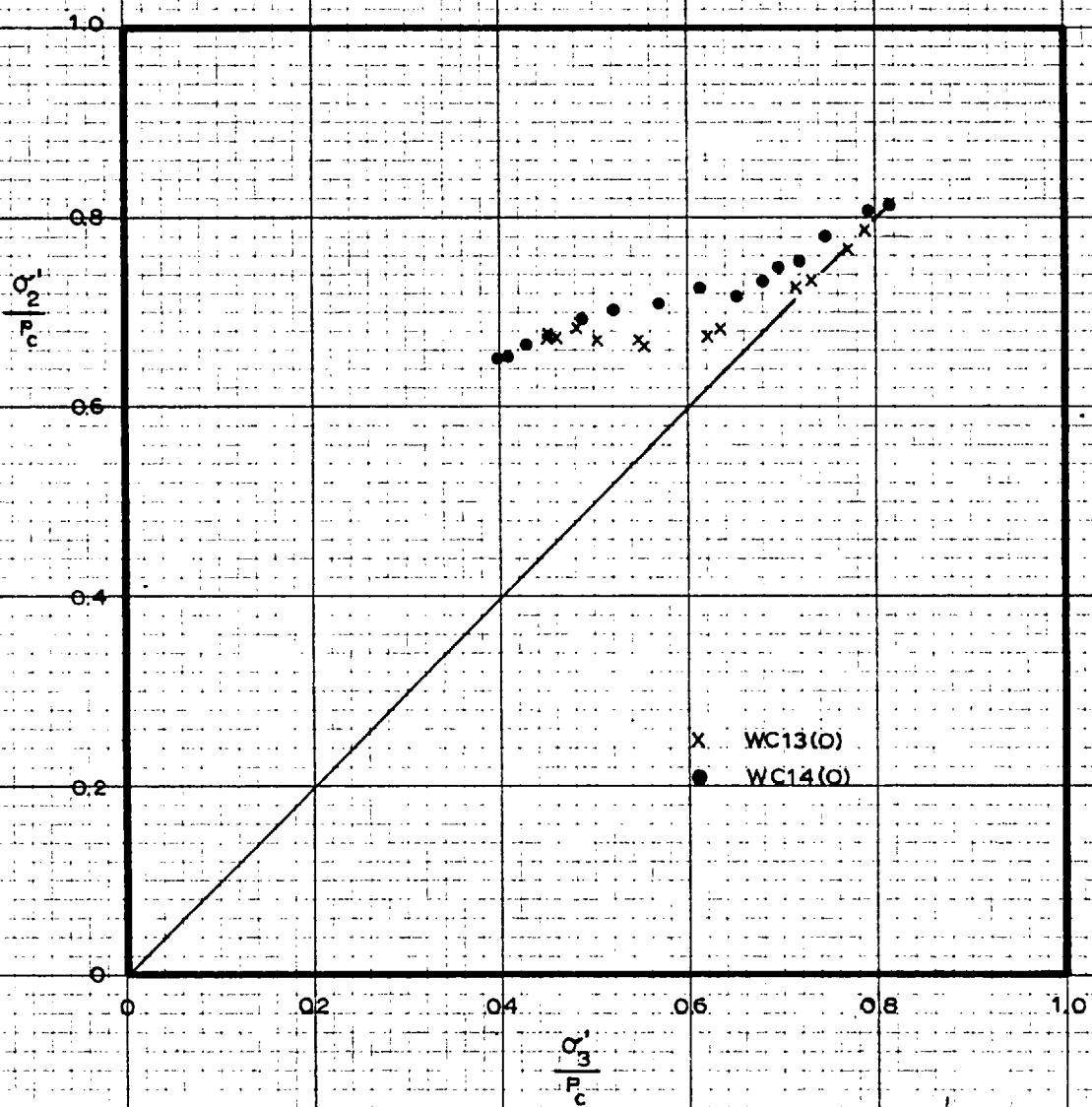


FIG 7-24

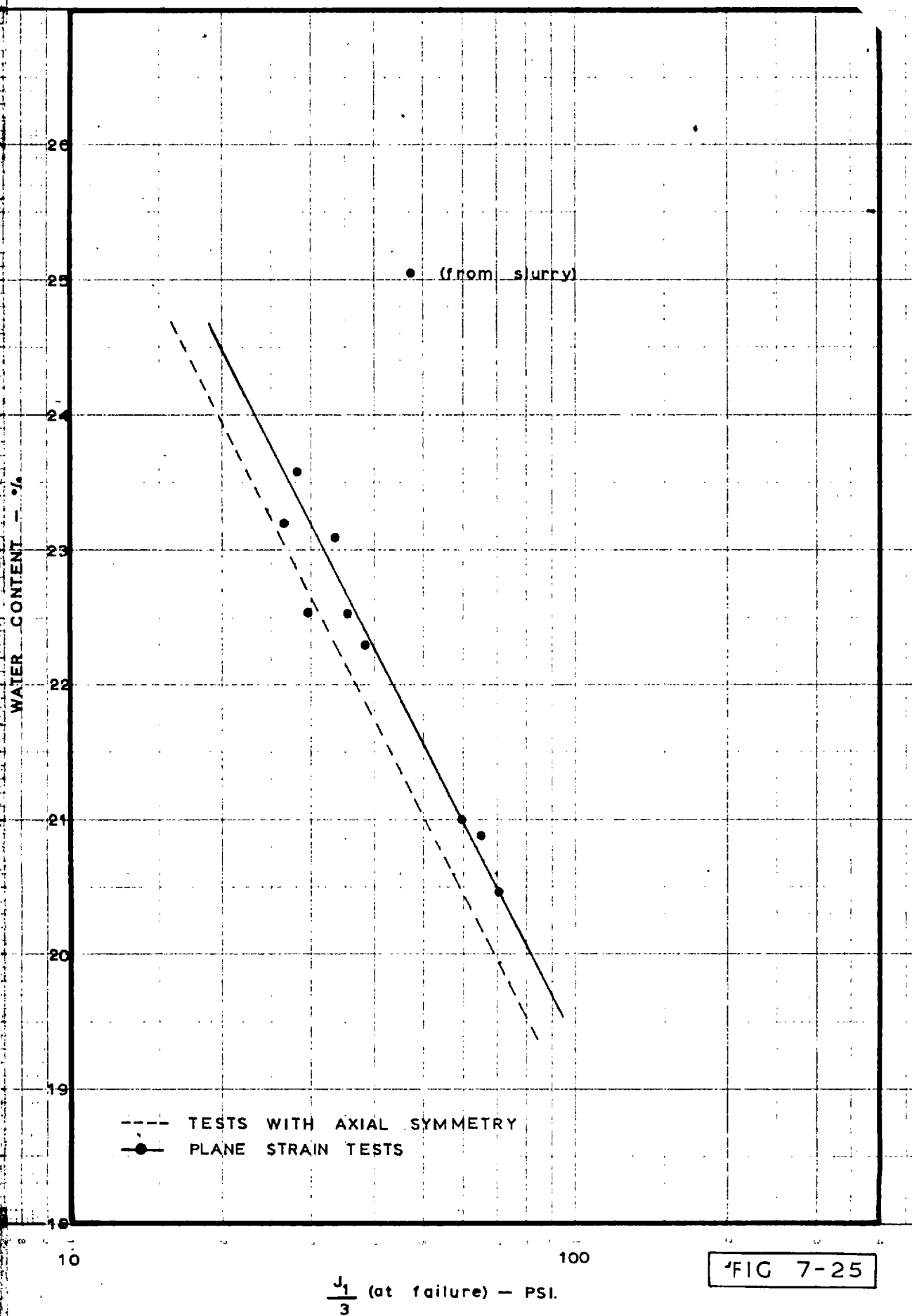
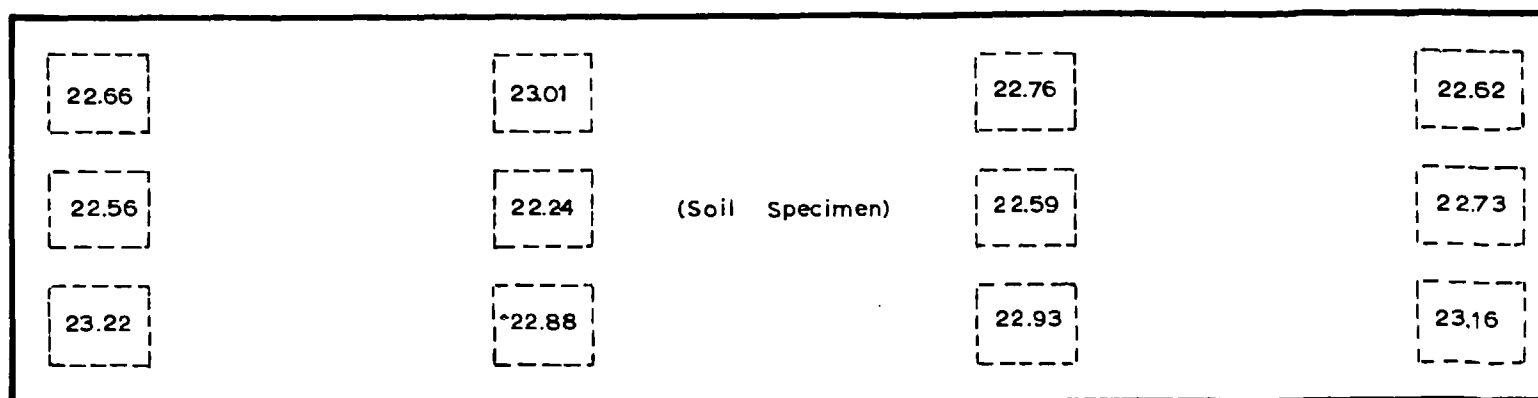
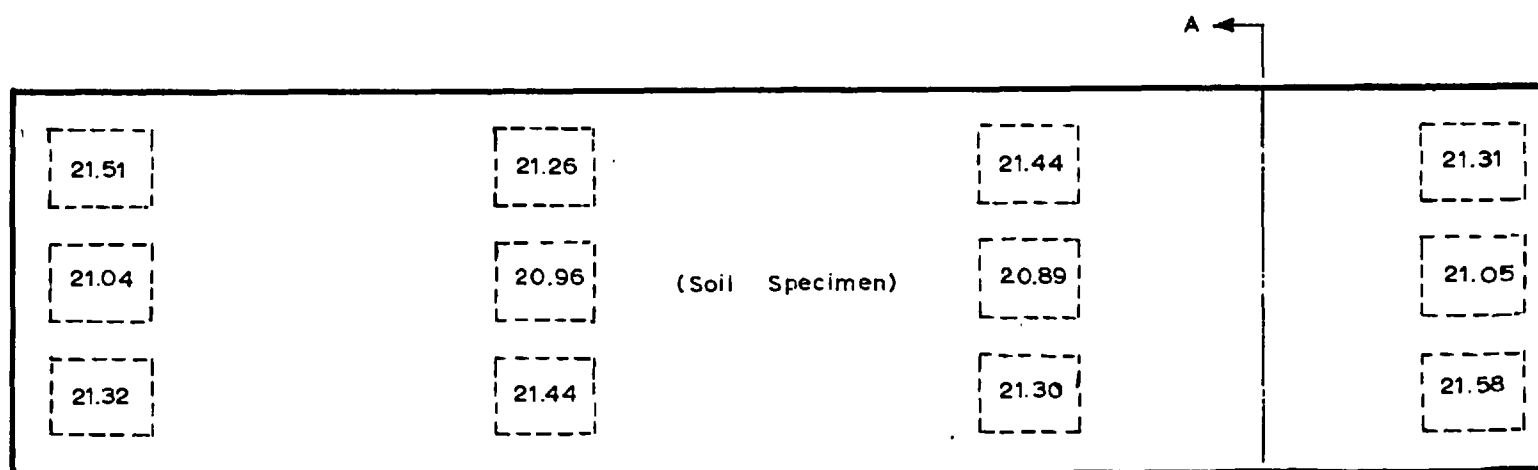


FIG 7-25

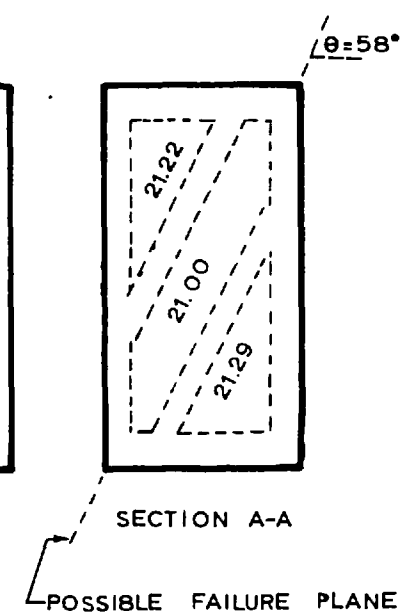


ELEVATION VIEW

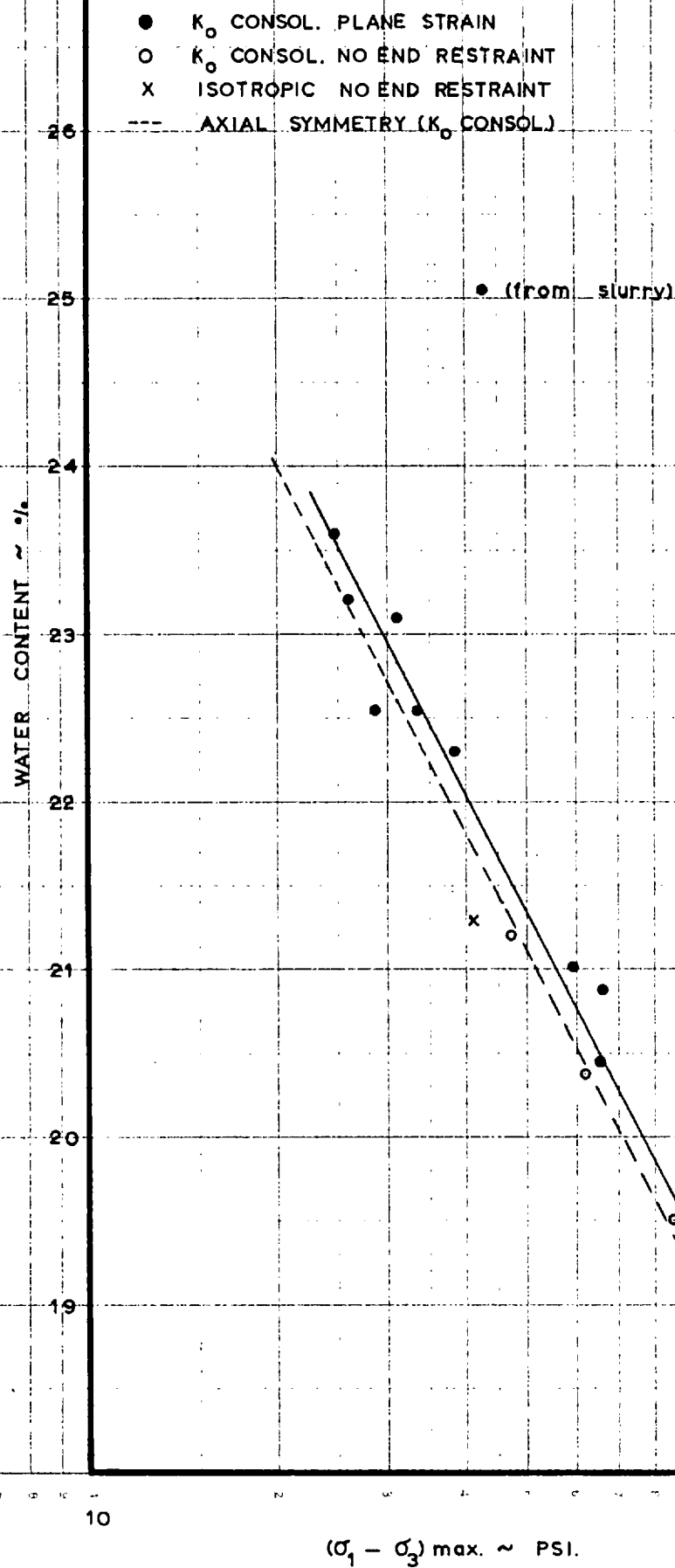
% WATER CONTENT PROFILE AFTER SHEARING
 SPECIMEN FROM TEST NO. WC11
 OVERALL AVERAGE $M.C._f = 22.78\%$
 AV. $M.C._f$ ALONG MID-HT. = 22.53%



ELEVATION VIEW



% WATER CONTENT PROFILE AFTER SHEARING
 SPECIMEN FROM TEST NO. WC15(o)
 OVERALL AVERAGE $M.C._f = 21.24\%$
 AV. $M.C._f$ ALONG MID-HT. = 20.99%



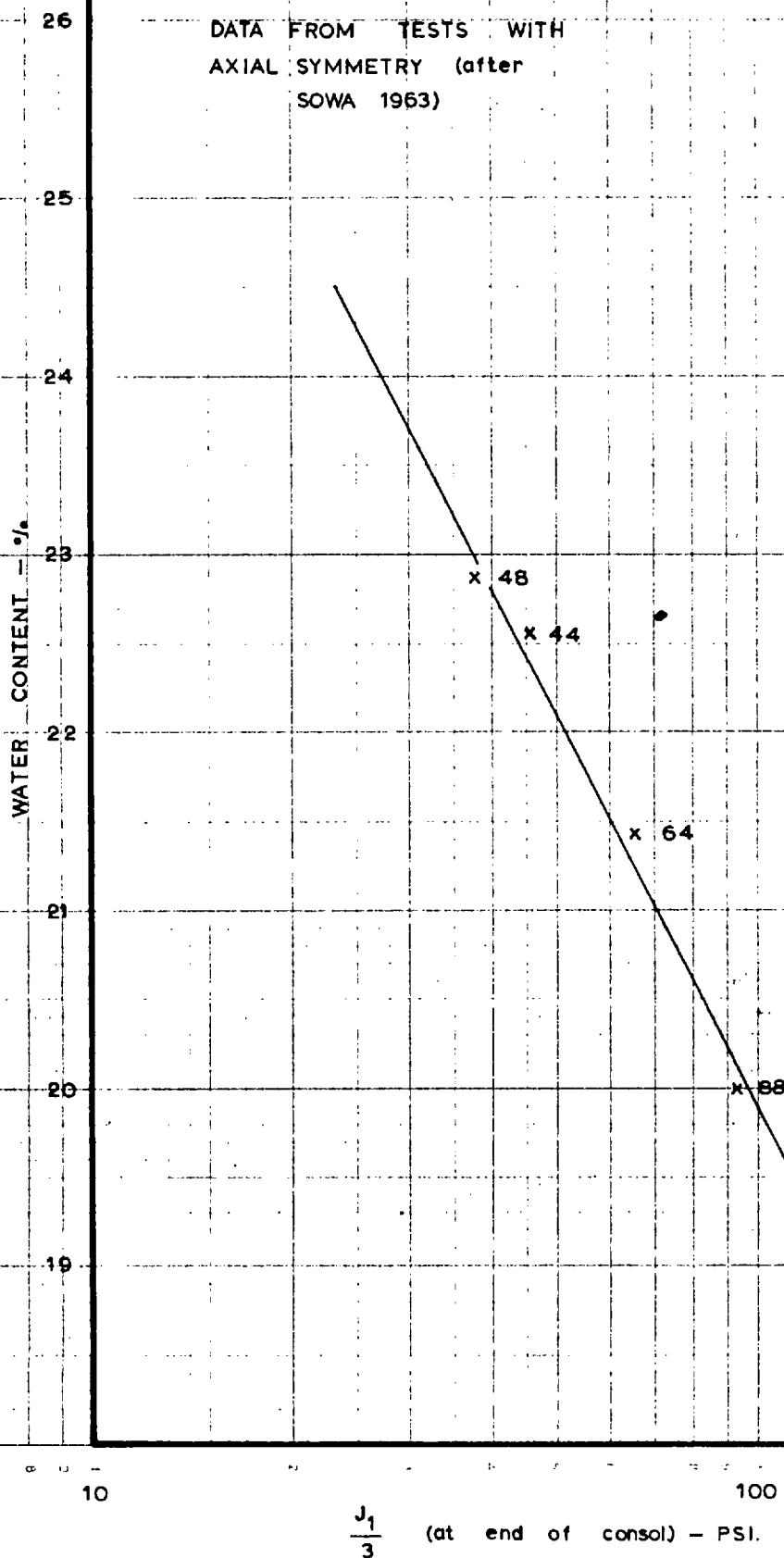


FIG 7-28

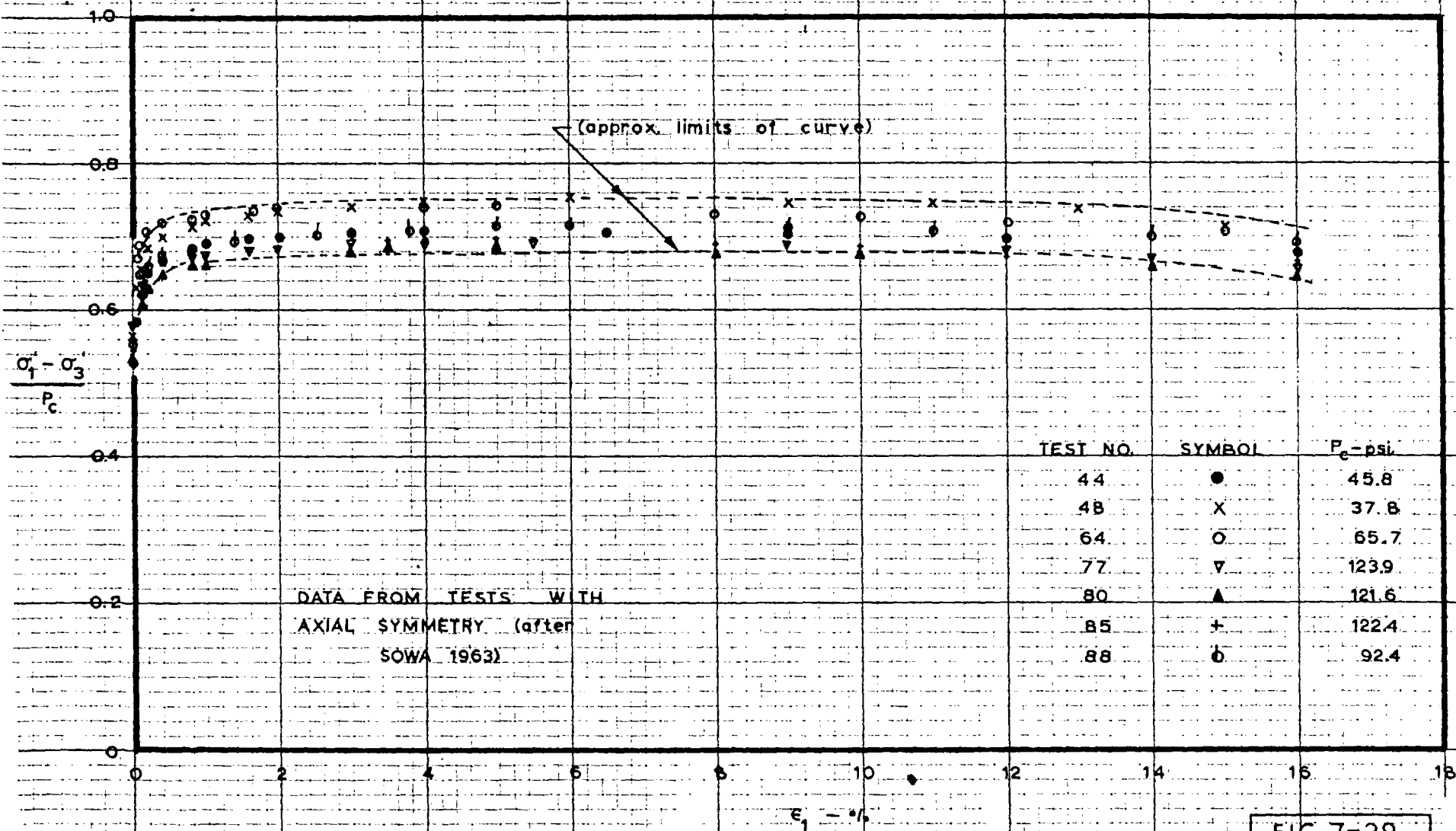


FIG 7-29

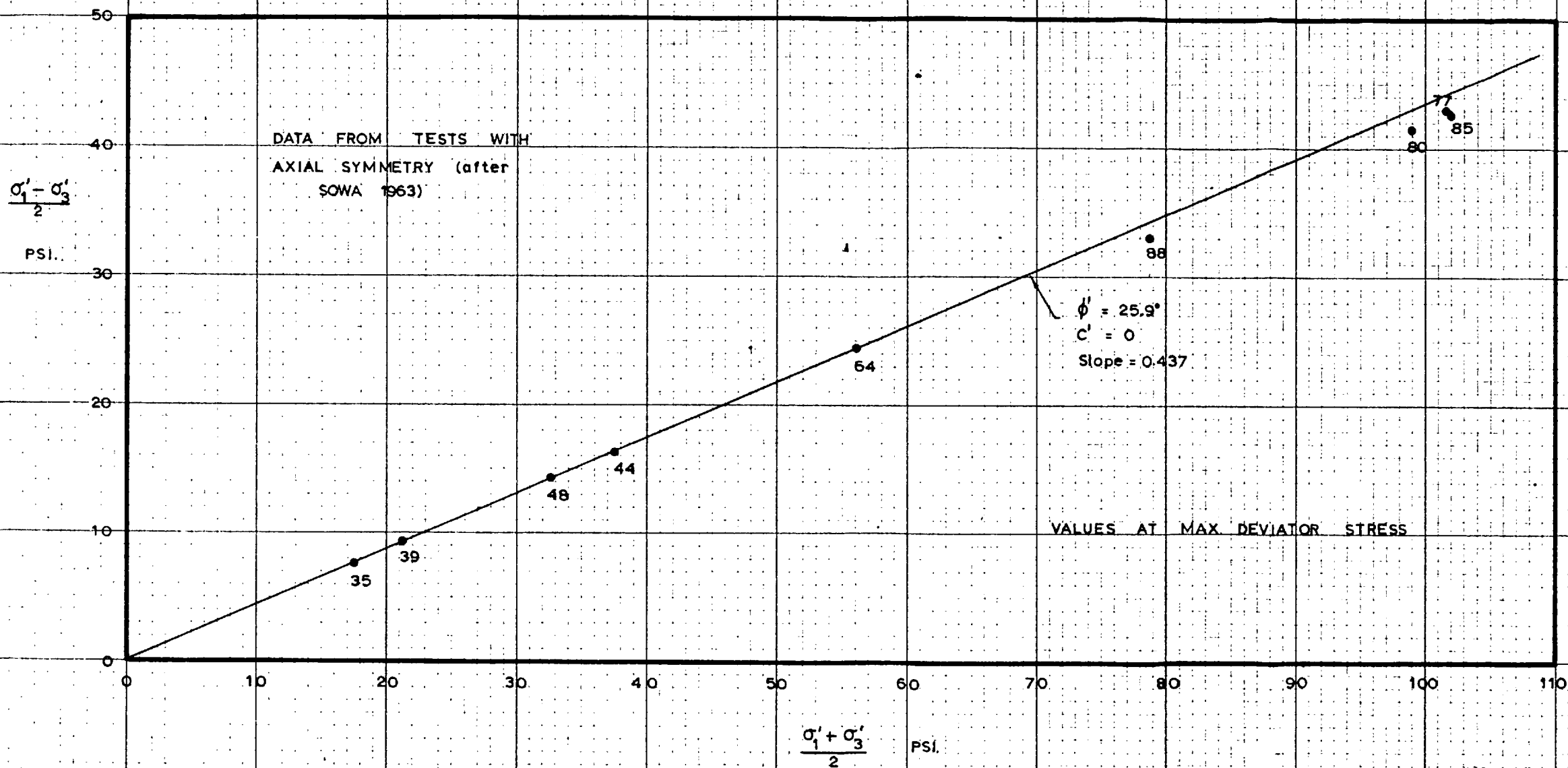


FIG 7-30

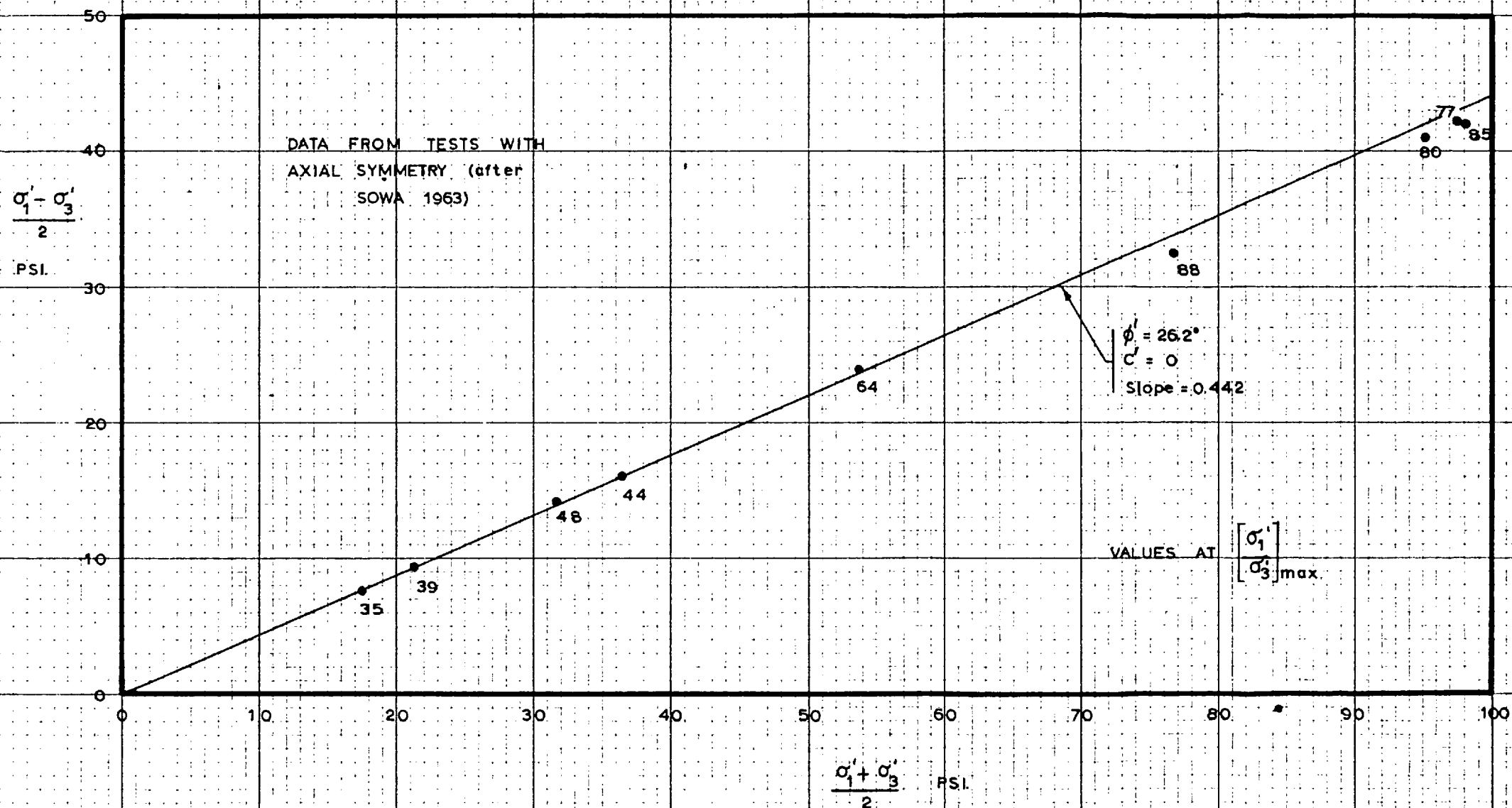


FIG 7-31

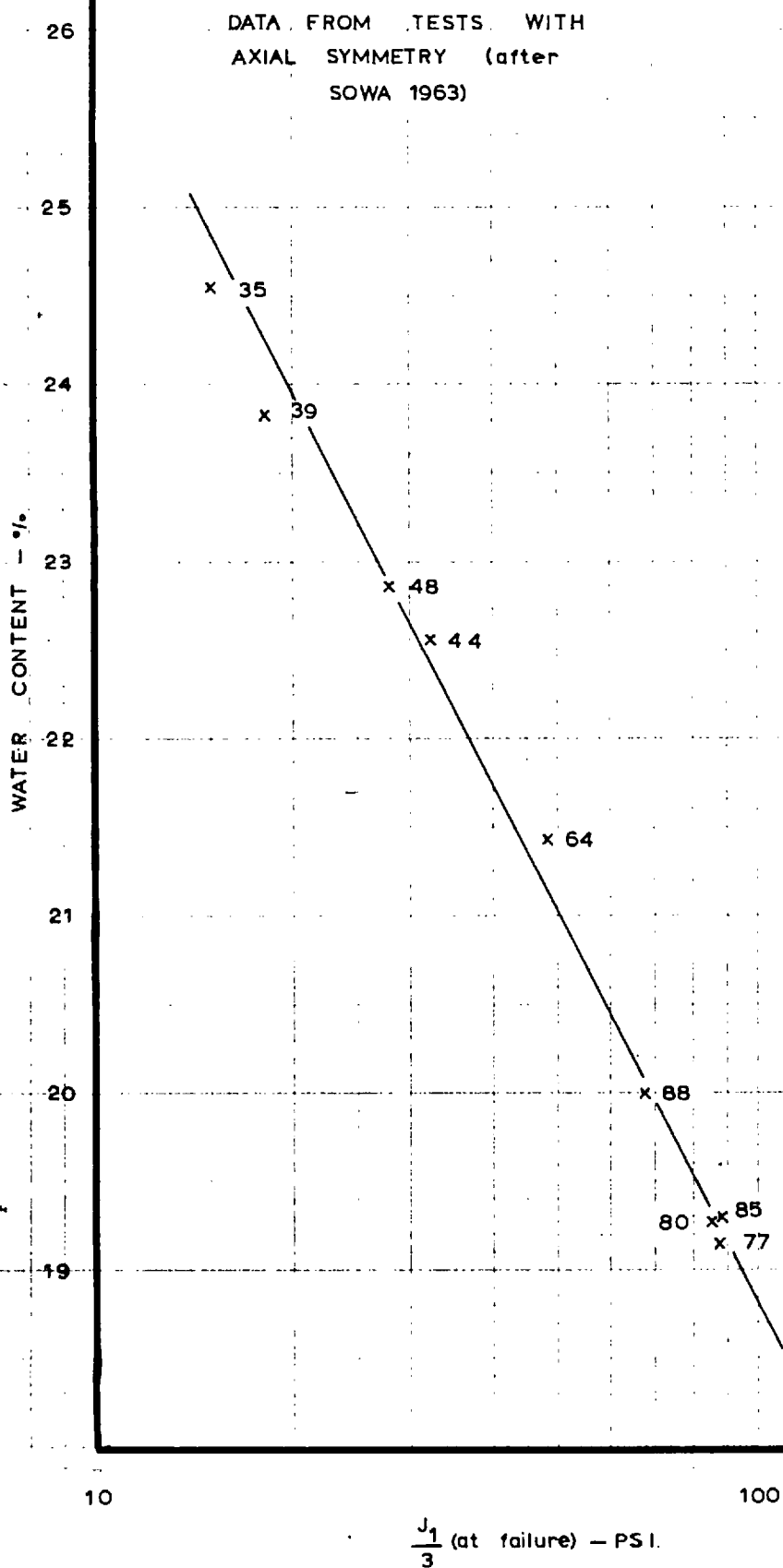


FIG 7-32

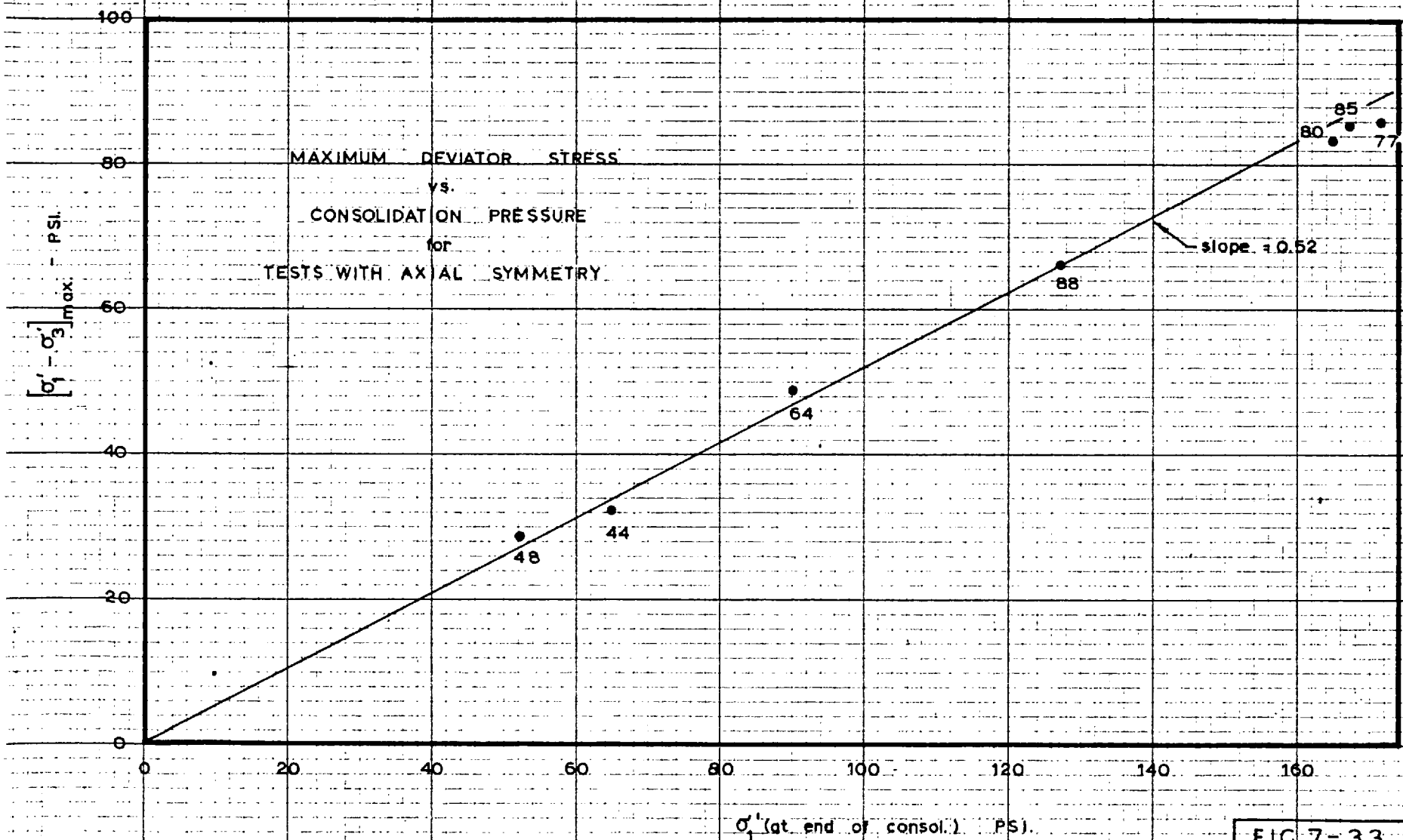


FIG 7-33

CHAPTER VIII

RESULTS OF TESTS ON BELGIUM SAND

(8.1) General

Since the primary objective of the research programme was to determine the characteristics of a clay when subjected to plane strain conditions, the data obtained from the sand tests are examined only briefly to provide a basis for comparison. As indicated in Chapter V, drained compression tests on Belgium Sand were carried out in both the standard triaxial (4 in. diameter x 8 in. high specimen) and plane strain apparatus. Four triaxial extension tests were also conducted. The resulting stress-strain curves were plotted and are included in Appendix 4. Tabulated results are given in Tables 8-1 and 8-2.

A check on the grading curve of the sand, as reported by LADANYI (1960) and shown in Fig. 4-2, was made* and relatively good agreement with the published curve was obtained. Rapid tilt tests on dry sand* (see KOLBUSZEWSKI 1948) gave an average maximum porosity of 46.8% which agrees with Ladanyi's value of 47%.

(8.2) Consolidation Stage

All specimens of Belgium Sand were consolidated under K_0 con-

* The tests were performed by M. Jean Normand.

ditions by utilizing the lateral strain indicators described in Chapter III. In both the plane strain and standard triaxial tests, the consolidation pressures were applied to the specimens manually, the time taken to reach a cell pressure of 40 psi. being approximately 30 minutes. The specimens were then allowed to rest for an hour before being sheared.

In Fig. 8-1, the average K_0 values obtained from the plane strain tests and the tests with axial symmetry were plotted against the initial porosities of the respective specimens. The plane strain results show the same trend as the data from the triaxial tests, i.e., the magnitude of K_0 increases with increasing porosity. Average K_0 values are 0.4 for $n_i=37\%$ and 0.5 for $n_i=43\%$.

(8.3) The Shear Stage

The results of the plane strain compression tests are given in Figs. A4-1 to A4-13. In a typical medium dense specimen (see Fig. A4-1), the major principal stress, which has an initial value of σ_3^0/K_0 , increases with increasing strain during shear to a maximum value before dropping off sharply to a relatively constant value. In the loose specimens (say, Fig. A4-4), on the other hand, the peak value of σ_1^0 is not as high, and, in some cases, more than one peak occurs* before the curve levels off.

* A similar phenomenon occurred, but to a lesser extent, in the plane strain tests on Brasted Sand (see CORNFORTH, 1961).

The intermediate principal effective stress has an initial value equal to the cell pressure plus the small amount of pre-stress applied during the assembly process. σ_2' increases to a maximum value, which is lower than the σ_1' peak value, and then levels off with increasing strain in both the loose and dense specimens.

Since the minor principal effective stress is maintained constant during shear, the magnitude of the deviator stress is implicit in the curve of σ_1' . The peak deviator stress in the dense specimens occurs at a lower % strain than in the loose specimens, the overall range for all plane strain specimens being 1½ to 3½ %.

The relationship between the % volume change and the % strain during shear is also shown in each Fig. More consolidation occurs in the loose specimens, and the rate of dilation [i.e. $(\frac{dV}{V_c})/d\epsilon_1$] at peak deviator stress is smaller than in the dense specimens.

The results of the triaxial compression tests are shown in Figs. A4-14 to A4-24 inclusive. In a typical medium dense specimen ($n_1 = 38.43\%$), the relationship between the major principal effective stress and the % strain is similar to that obtained from the plane strain data, but, for the same values of K_0 and n_1 the relative magnitude of σ_1' at failure is smaller in the standard triaxial test. This can be seen by comparing the curves in Figs. A4-1 and A4-22. The maximum value for $(\sigma_1' - \sigma_3')$ for the triaxial specimens occurs between 4% (dense specimens) and 10% (loose specimens) axial strain.

The graphs of the triaxial extension tests, in which the

specimens were also K_0 consolidated and sheared under drained conditions, are illustrated in Figs. A4-25 to A4-28. In calculating the results of these tests, it was assumed that, after necking of the specimen began, the strains were concentrated in the necked zone. The area used, therefore, was the average cross-sectional area of the necked zone in each test.

In Fig. 8-2, the relationship between the maximum principal stress ratio $(\sigma'_1/\sigma'_3)^*$ and the initial porosity is given for both the plane strain and standard triaxial tests. The relative values of (σ'_1/σ'_3) are higher for conditions of plane strain in the range of porosities used. Good agreement between triaxial extension and compression results was obtained. As would be expected, loose specimens fail at the lower values of the ratio, and the difference between the plane strain and triaxial ratios becomes less for the higher porosities.

Fig. 8-3 shows the graph of $[\sigma'_2/(\sigma'_1 + \sigma'_3)]$ at failure against initial porosity for the plane strain tests. The ratio is almost of constant magnitude over the porosity range obtained, the variation being 0.34 to 0.29 for loose and dense specimens respectively. By assuming that $\sigma'_2 = K_0 \sigma'_1$ ** and $K_0 = (1 - \sin \phi')$ ***, it can be shown that:

* The maximum (σ'_1/σ'_3) ratio occurs at the peak deviator stress in drained tests.

** After WOOD (1958).

Proposed by JAKY.

$$\frac{\sigma_2'}{\sigma_1' + \sigma_3'} = \frac{\cos^2 \phi'}{2} \dots\dots\dots(8-1)$$

This relation was suggested by Dr. A.W. Bishop, ~~and inferred by~~
~~IADANYI (1960).~~ When plotted in Fig. 8-3, the $\left[(\cos^2 \phi')/2 \right]$ values
follow the same trend and lie only slightly below the points re-
presenting the $\left[\sigma_2' / (\sigma_1' + \sigma_3') \right]$ values.

The ratio (σ_2' / σ_1') at failure, the average K_0 values determined
during consolidation, and the function $(1 - \sin \phi')$ were all plotted
against initial porosity in Fig. 8-4. As illustrated in the diagram,
the relation

$$\frac{\sigma_2'}{\sigma_1'} = (1 - \sin \phi') \dots\dots\dots(8-2)$$

is a better approximation than the one suggested by Wood, (i.e.
 $\sigma_2' / \sigma_1' = K_0$).

Figs. 8-5 and 8-6 show various principal stress ratios plotted
against % strain during shear for a loose and a dense specimen,
respectively, under conditions of plane strain. The ratios (σ_1' / σ_3') and
 (σ_2' / σ_3') in each graph, since σ_3' is maintained constant, have the
same characteristic shape as the corresponding σ_1' and σ_2' versus %

* Private discussion.

strain curves (see Figs. A4-3 and A4-11). The (σ_1'/σ_3') curves have an initial value of $1/K_0$, and with increasing strain they increase to a maximum value before decreasing to a relatively constant value. The initial value of (σ_2'/σ_3') is approximately 1, and its magnitude increases to a maximum value and remains fairly constant as the strain increases. The ratio $(\frac{\sigma_2'}{\sigma_1 + \sigma_3})$, however, is almost constant during shear in both the loose and dense specimens, with average values at peak deviator stress of 0.33 and 0.27 respectively.

The variation of ϕ_d with initial porosity (Fig. 8-7) illustrates that, for the range of porosities used, the strength** of specimens subjected to plane strain conditions is greater in terms of ϕ_d than that of specimens with axial symmetry. For dense specimens ($n_1=38\%$) the average values of ϕ_d are 42° and 39.2° respectively. This difference in the ϕ_d value, however, becomes less with increasing porosity, such that at $n_1=43\%$, the values are 35.2° and 34.2° . The results of triaxial extension tests generally agree with the triaxial compression data but some scatter exists.

Fig. 8-8 shows the relationship between ϕ_d and the maximum rate of dilation (i.e. $\left[\frac{d(\frac{\Delta V}{V_c})}{d\epsilon_1} \right]$ at peak deviator stress) for both plane strain and triaxial specimens. For the same amount of dilation, the relative magnitude of ϕ_d is larger for the plane strain case.

* $\phi_d = \phi'$ in drained tests.

**

At maximum deviator stress rather than the ultimate strength at large strains.

When the rate of dilation is zero, the specimens can be considered to be in the loosest possible state (i.e. maximum porosity). Due to the lack of data on loose specimens, it is difficult to predict from Figs. 8-7 and 8-8 the minimum angle of shearing resistance. However, by extrapolation, approximate values of $\phi_d = 33^\circ$ and $\phi_d = 32^\circ$ can be obtained for the plane strain and standard triaxial specimens respectively.

Fig. 8-9 indicates that the rate of dilation increases with increasing specimen density, and, for a given density, more dilation occurs in specimens with axial symmetry when tested in compression. When the dilation is zero, the two curves should meet at $n_1 = 47\%$.

The relationship between ϕ_d and the porosity after consolidation is given in Fig. 8-10. Superimposed on this diagram are approximate contours of the mean normal stress at the end of consolidation (i.e. σ_m) (after LADANYI 1960). The numbers beside the experimental points are the values of σ_m for that test. The results of the standard triaxial compression tests are in relatively good agreement with Ladanyi's curves, whereas the plane strain data shows a higher ϕ_d for the same values of n_c and σ_m .

TABLE 8-1. SUMMARY OF PLANE STRAIN TESTS ON BELGIUM SAND

TEST	POROSITY%		END OF CONSOL.				@ PEAK DEVIATOR STRESS					
	INIT.	CONSOL.	σ'_1	σ'_2	σ'_3	K_o	σ'_1	σ'_2	σ'_3	$\epsilon_1\%$	ϕ°	$d(\frac{\Delta V/V_c}{d\epsilon_1})$
1	39.66	39.20	89.2	41.2	40.4	0.453	191.8	72.1	40.4	1.78	40.7	0.67
2	37.76	37.24	96.2	40.4	40.2	0.418	203.0	73.9	40.2	1.89	42.0	0.62
3	38.44	37.93	85.1	40.3	40.2	0.472	200.7	70.0	40.2	1.95	41.8	0.59
4	38.34	37.83	92.7	40.7	40.2	0.434	196.5	67.8	40.2	2.11	41.3	0.62
5	41.26	40.69	96.0	41.8	40.2	0.419	165.4	68.0	40.2	1.76	37.5	0.30
6	42.41	41.78	85.9	40.5	40.2	0.468	156.5	58.2	40.2	2.19	36.2	0.17
7	42.14	41.53	79.9	40.3	40.2	0.503	158.4	62.5	40.2	2.13	36.5	0.22
8	42.00	41.40	85.6	40.5	40.2	0.470	163.4	66.4	40.2	2.68	37.2	0.25
9	42.13	41.53	88.6	40.7	40.2	0.454	157.9	66.0	40.2	2.23	36.5	0.23
10	39.55	39.02	98.0	41.8	40.2	0.410	189.2	69.9	40.2	1.94	40.5	0.51
11	39.22	38.71	98.3	40.5	40.2	0.409	187.2	66.3	40.2	1.92	40.3	0.44
12	41.08	40.44	93.9	42.2	40.2	0.428	163.1	67.5	40.2	2.21	37.2	0.24
14	42.27	42.72	82.9	40.5	40.2	0.485	157.9	66.0	40.2	3.12	36.5	0.17
15	42.20	41.57	83.4	40.6	40.2	0.482	158.1	65.2	40.2	2.65	36.5	0.21
16	42.52	41.88	76.2	40.5	40.2	0.528	150.9	61.7	40.2	3.63	35.4	0.16
18	39.89	39.39	89.2	40.6	40.2	0.451	176.5	68.4	40.2	2.18	39.0	0.40
19	41.09	40.54	86.5	40.5	40.2	0.465	167.8	66.4	40.2	2.47	37.9	0.28
20	39.79	39.30	91.5	40.5	40.2	0.439	178.0	73.8	40.2	2.01	39.2	0.43

TABLE 8-2. SUMMARY OF STANDARD TRIAXIAL TESTS ON BELGIUM SAND

TEST: POROSITY %			END OF CONSOL.			@ PEAK DEVIATOR STRESS				
BS-	INIT.	CONSOL.	σ'_1	σ'_3	K_o	σ'_1	σ'_3	$\epsilon_1\%$	ϕ°	$\frac{d\epsilon_v/\epsilon}{d\epsilon_1}$
2	37.60	37.35	94.4	40.2	0.426	189.8	40.2	3.95	40.6	0.79
3	39.27	39.27	83.2	40.2	0.483	167.0	40.2	4.53	37.7	0.55
4	39.76	39.76	103.7	40.2	0.387	163.3	40.2	4.21	37.2	0.48
5	40.40	40.06	84.9	40.2	0.473	161.6	40.2	5.53	37.0	0.44
6	43.47	42.99	81.2	40.2	0.495	141.3	40.2	7.80	33.8	0.17
7	38.49	38.20	93.9	40.2	0.428	175.1	40.2	4.25	38.8	0.65
8	38.43	38.16	89.5	40.2	0.449	169.6	40.2	5.11	38.1	0.61
9	38.63	38.31	90.6	40.2	0.442	169.7	40.2	4.94	38.1	0.60
10	43.17	42.73	77.6	40.2	0.518	141.1	40.2	9.46	33.8	0.18
11	42.52	42.03	87.2	40.2	0.461	145.4	40.2	7.18	34.5	0.24
12	43.72	43.18	85.1	40.2	0.472	139.8	40.2	9.45	33.6	0.14
13	38.47	38.21	98.4	40.2	0.408	181.3	40.2	4.23	39.6	0.64
14	44.35	43.81	80.4	40.2	0.500	136.4	40.2	8.87	33.0	0.11
15	39.75	39.40	92.3	40.2	0.436	164.5	40.2	5.29	37.4	0.49
16	43.05	42.54	84.1	40.2	0.478	140.4	40.2	7.46	33.7	0.17
17	43.09	42.58	84.2	40.2	0.477	142.5	40.2	8.51	34.0	0.18
15E*	42.66	42.21	79.8	40.3	0.505	10.8	40.3	-8.41	35.3	0.14
27E	39.45	38.73	321.1	139.8	0.435	32.5	139.8	-7.29	38.5	0.22
28E	39.34	38.66	320.3	139.8	0.436	33.2	139.8	-7.02	38.0	0.23
29E	39.74	38.98	316.0	139.8	0.442	36.0	139.8	-7.34	36.0	0.24

* E denotes extension test.

** ϵ_1 is negative in extension.

BELGIUM SAND
X PLANE STRAIN TESTS
• TRIAXIAL TESTS

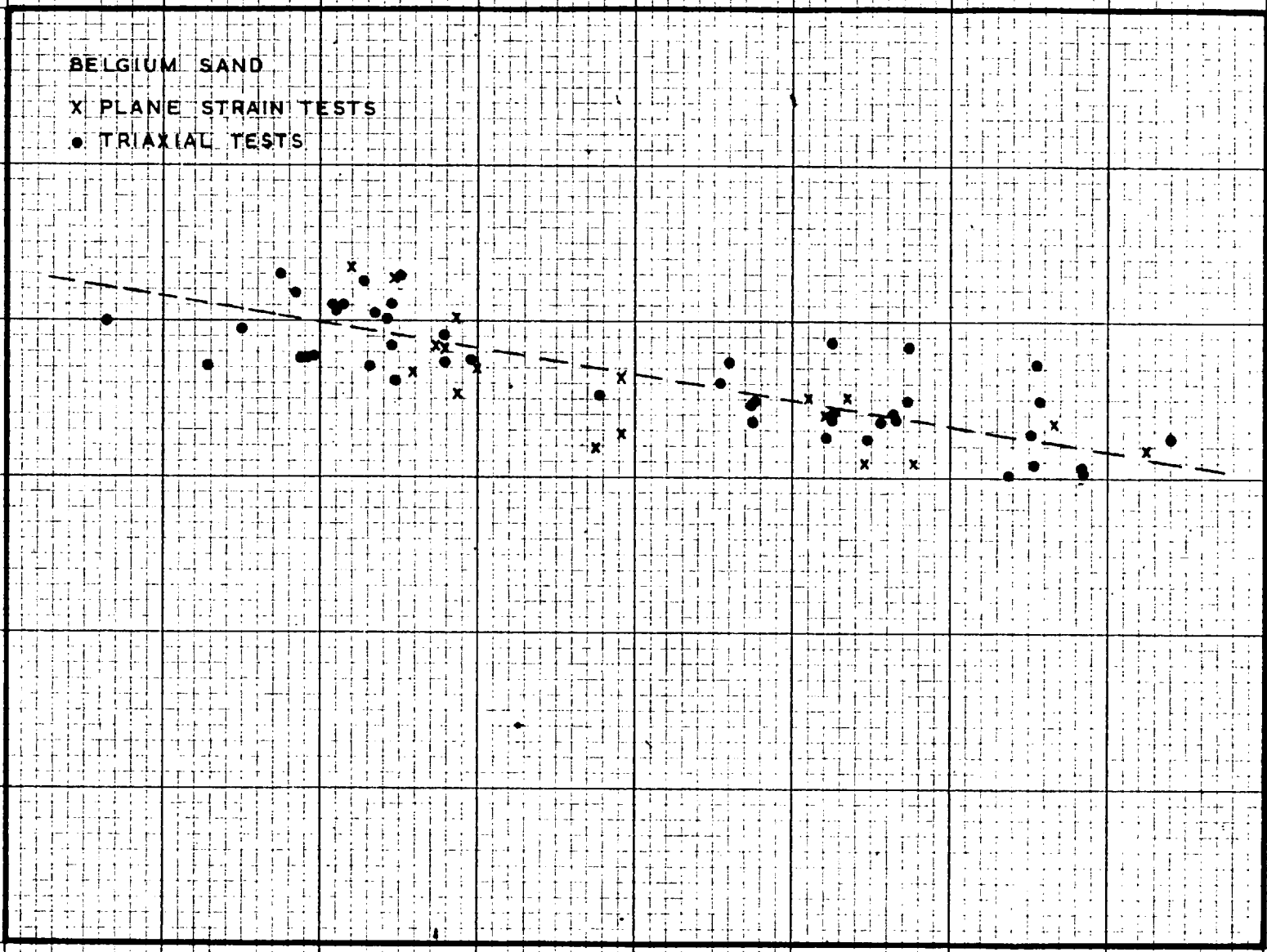
K_0 VALUES

0.6
0.5
0.4
0.3
0.2
0.1

45 44 43 42 41 40 39 38 37

n_1 %

FIG B-1



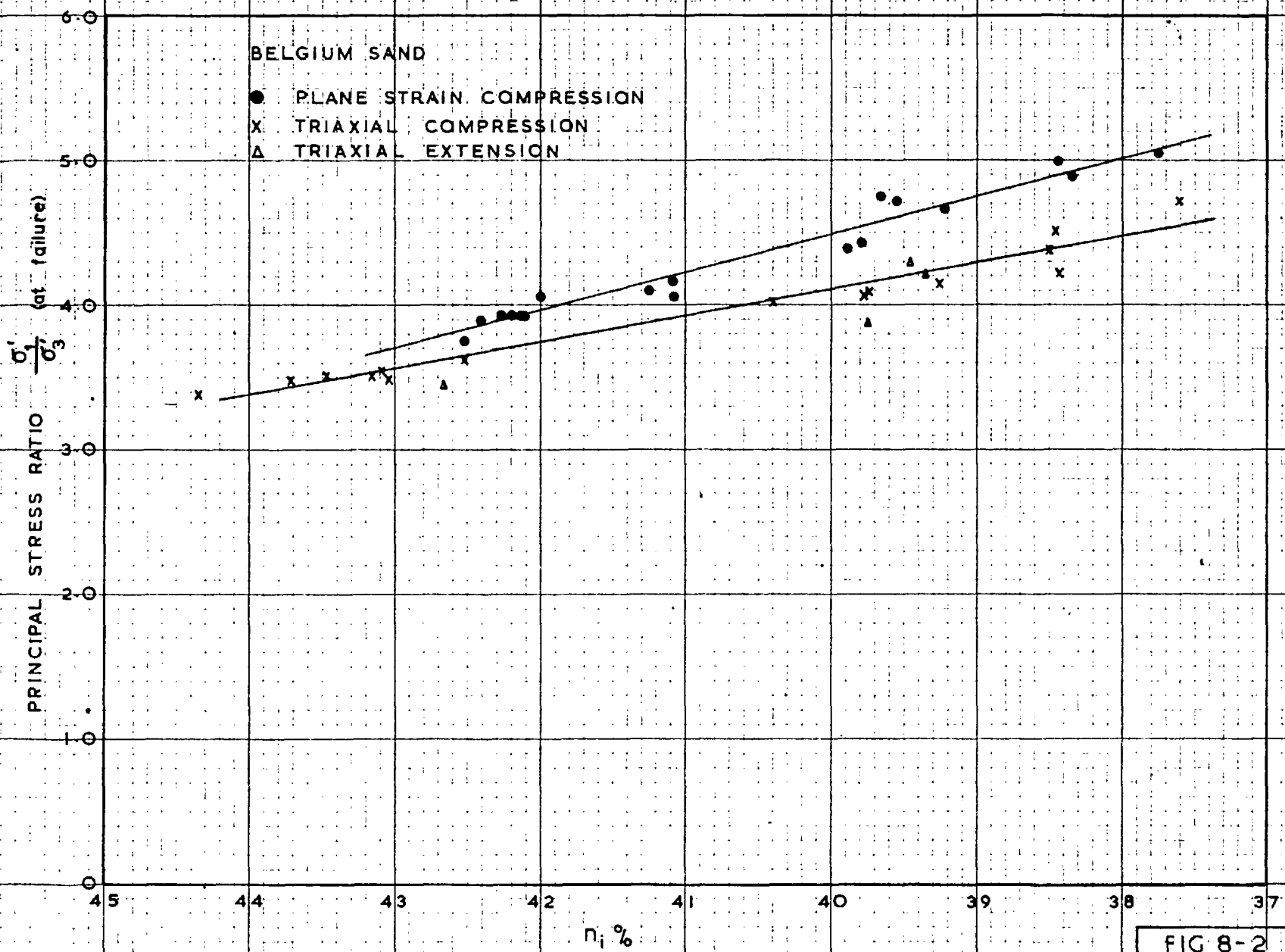


FIG 8-2

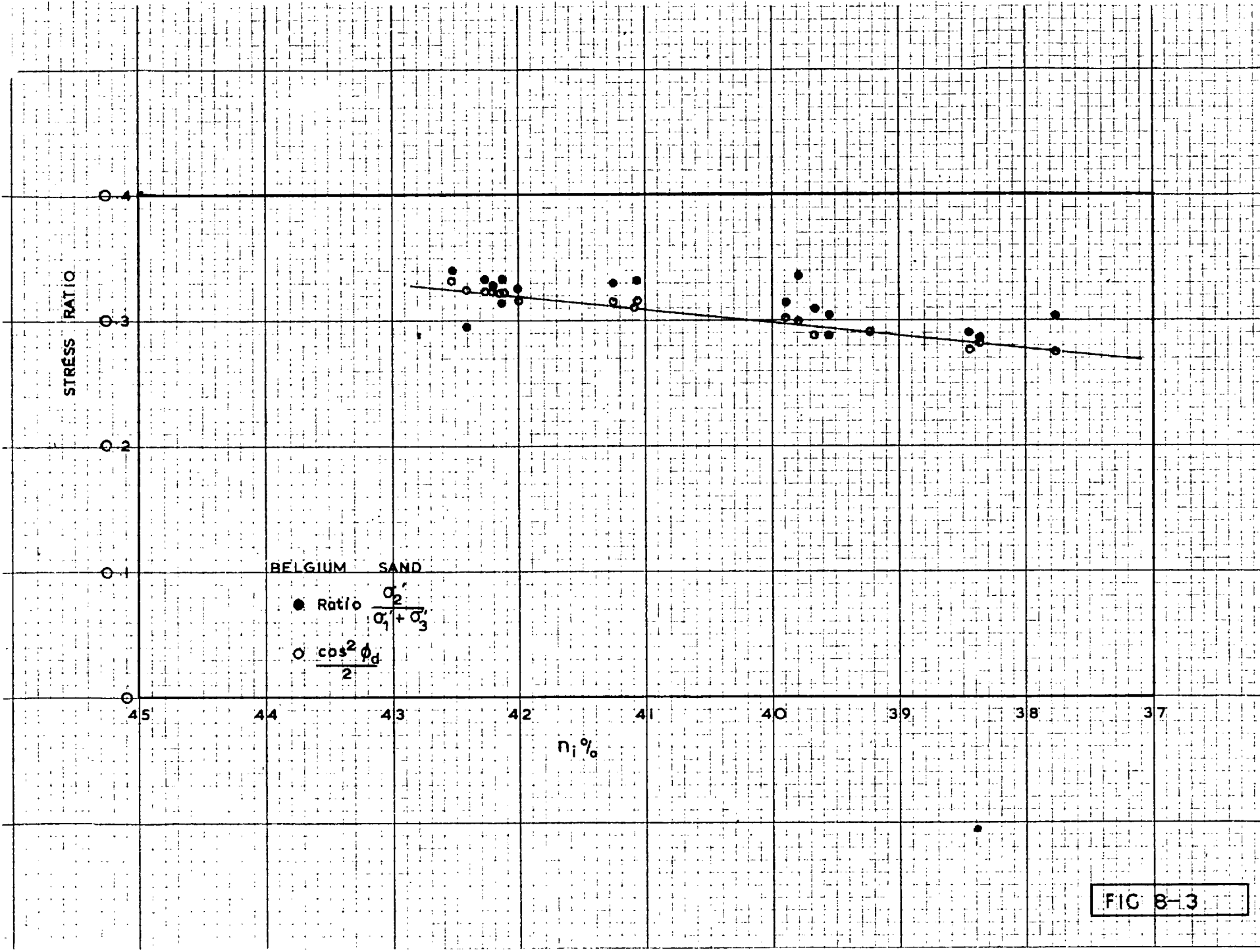


FIG 8-3

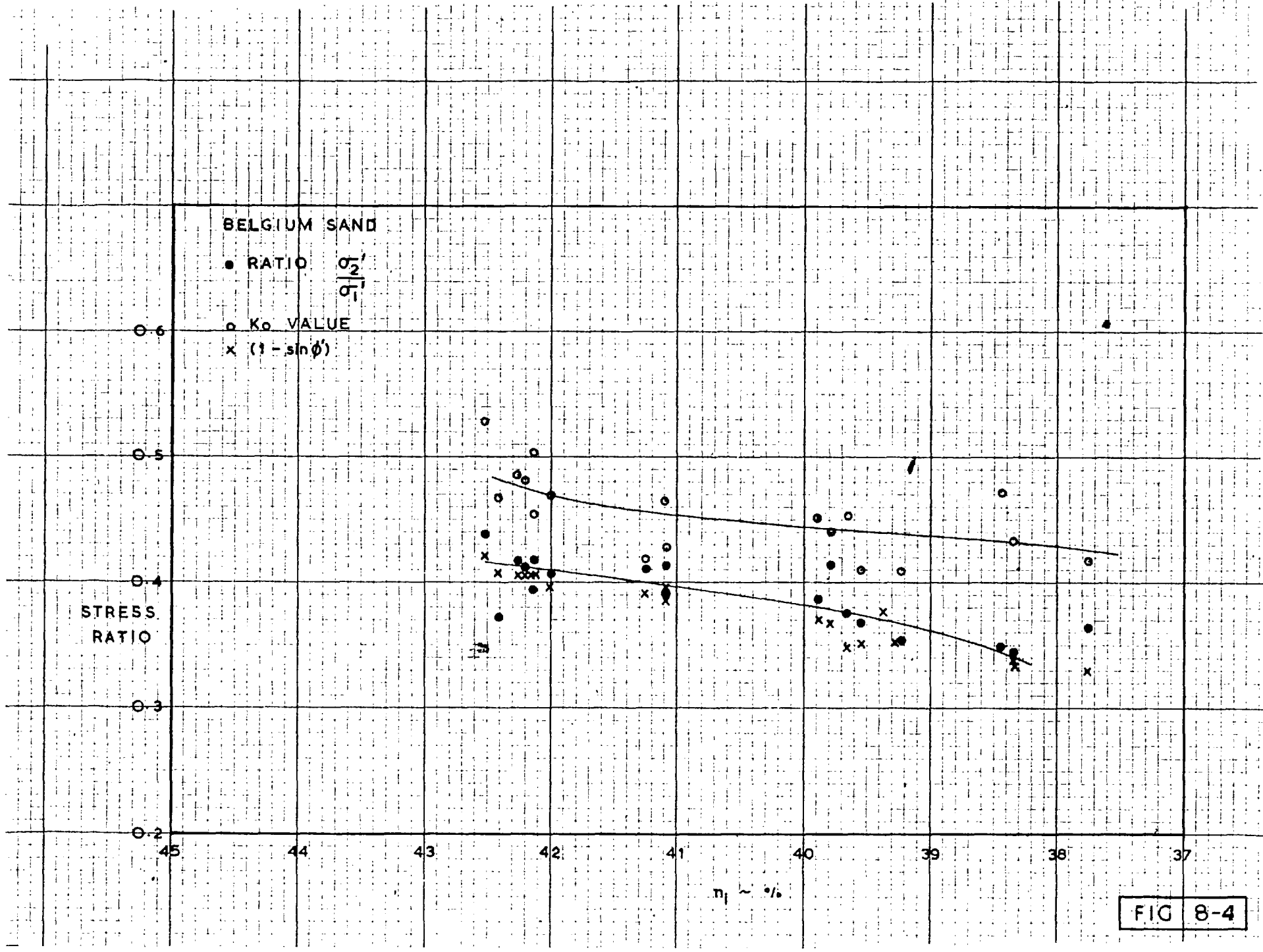


FIG 8-4

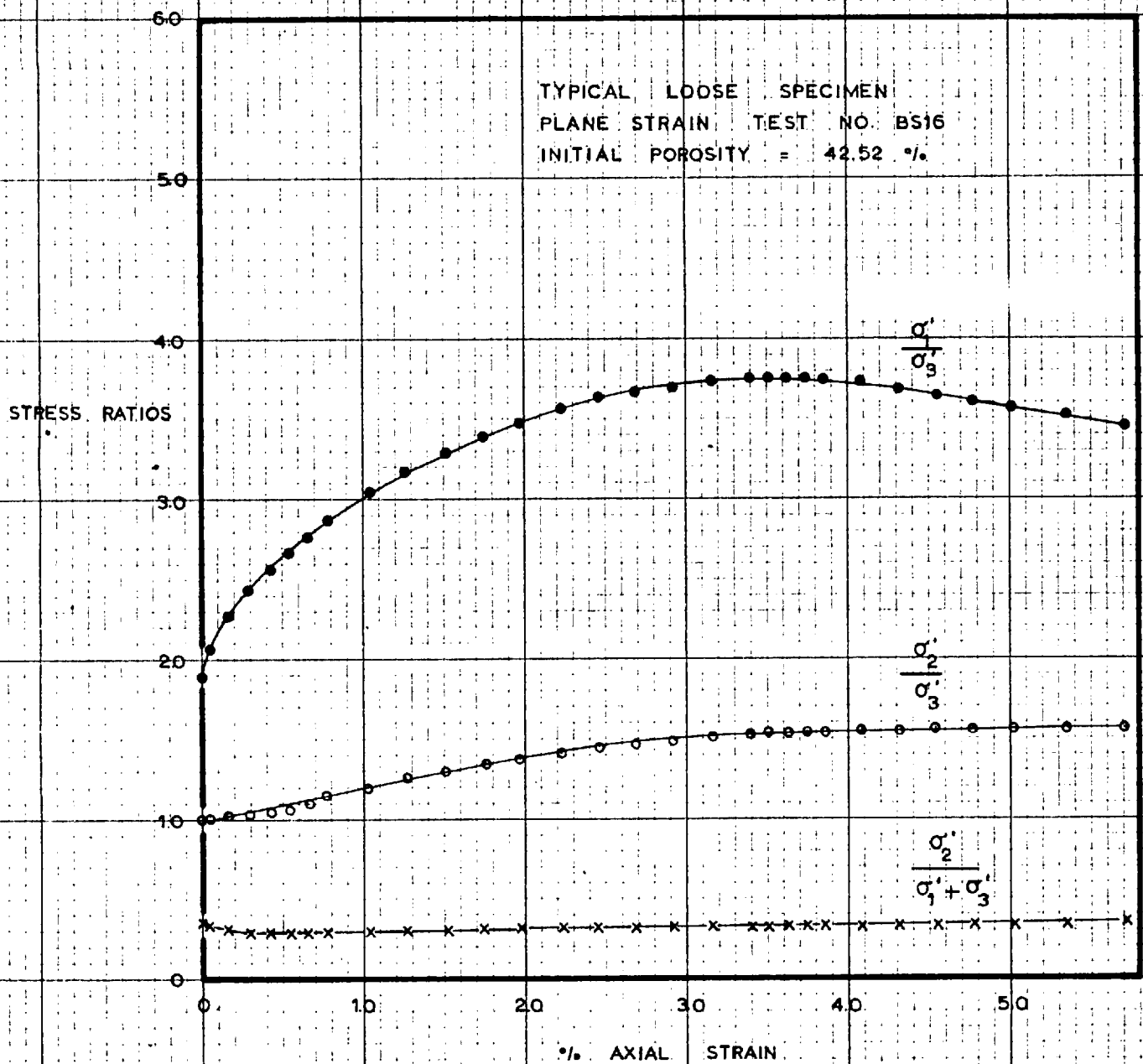


FIG B-5

TYPICAL DENSE SPECIMEN
 PLANE STRAIN TEST NO. BS4
 INITIAL POROSITY = 38.34 %

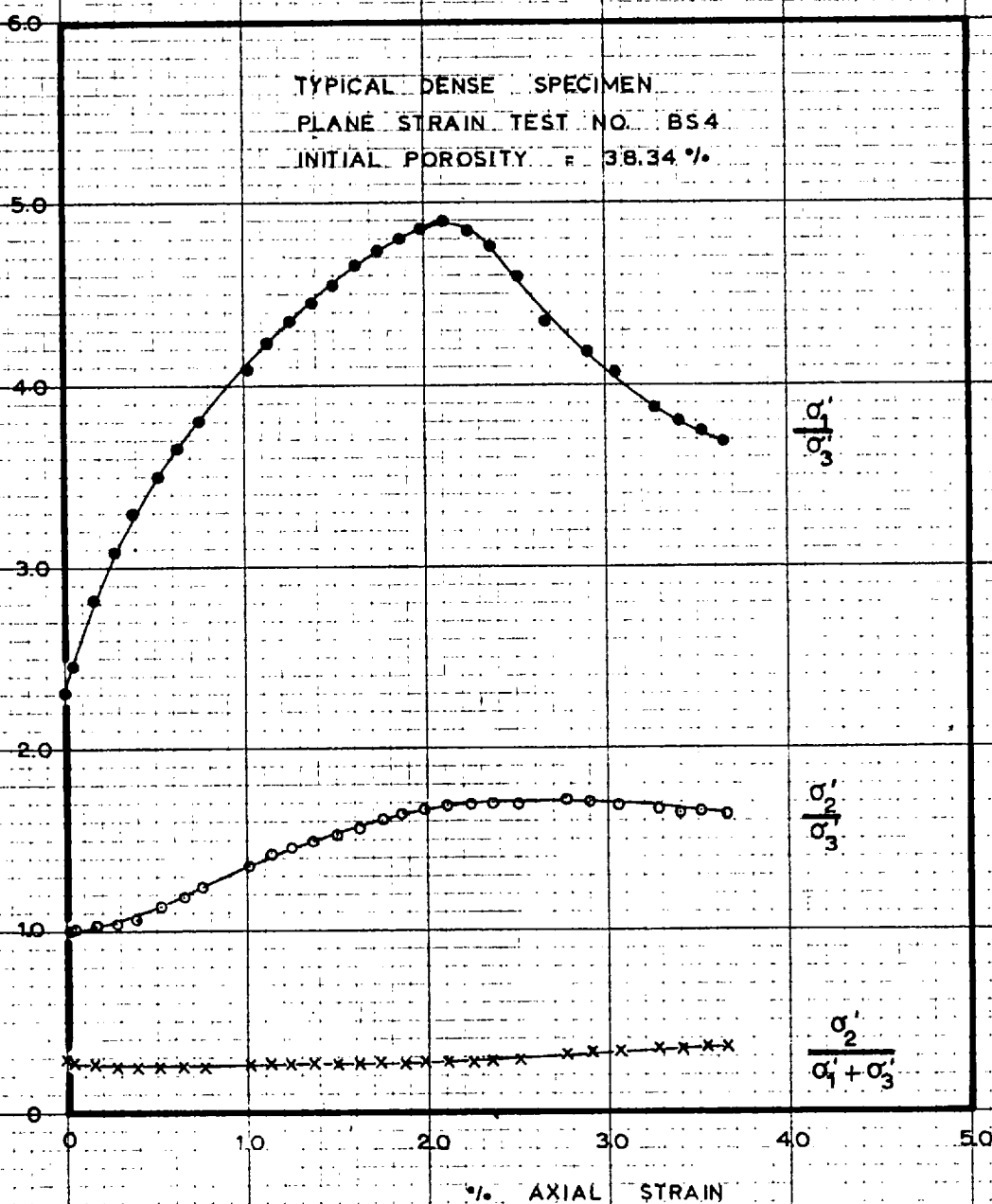
STRESS RATIOS

$\frac{\sigma_3}{\sigma_1}$

$\frac{\sigma_2}{\sigma_1}$

$\frac{\sigma_2'}{\sigma_1' + \sigma_3'}$

% AXIAL STRAIN



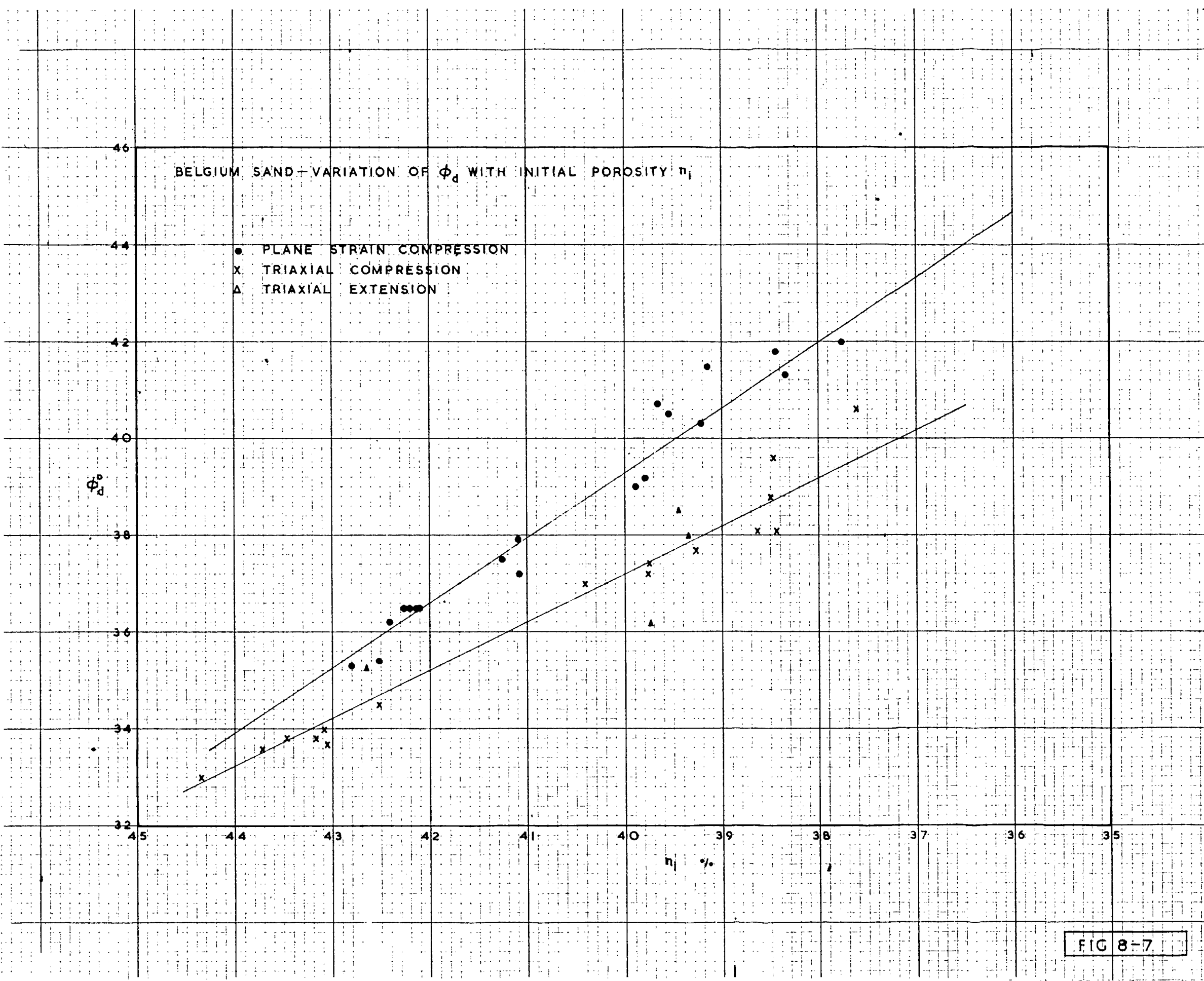


FIG 8-7

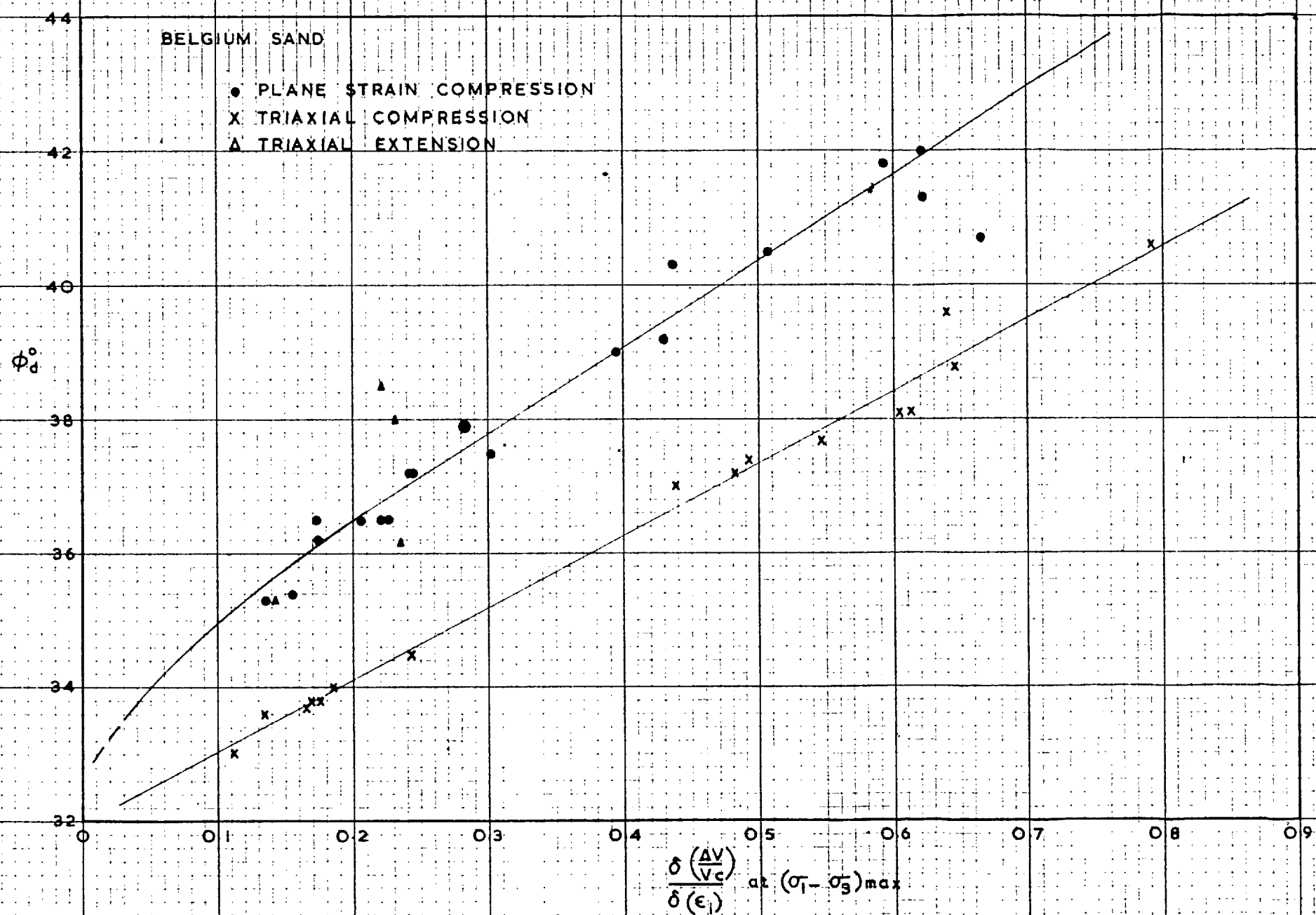


FIG 8-8

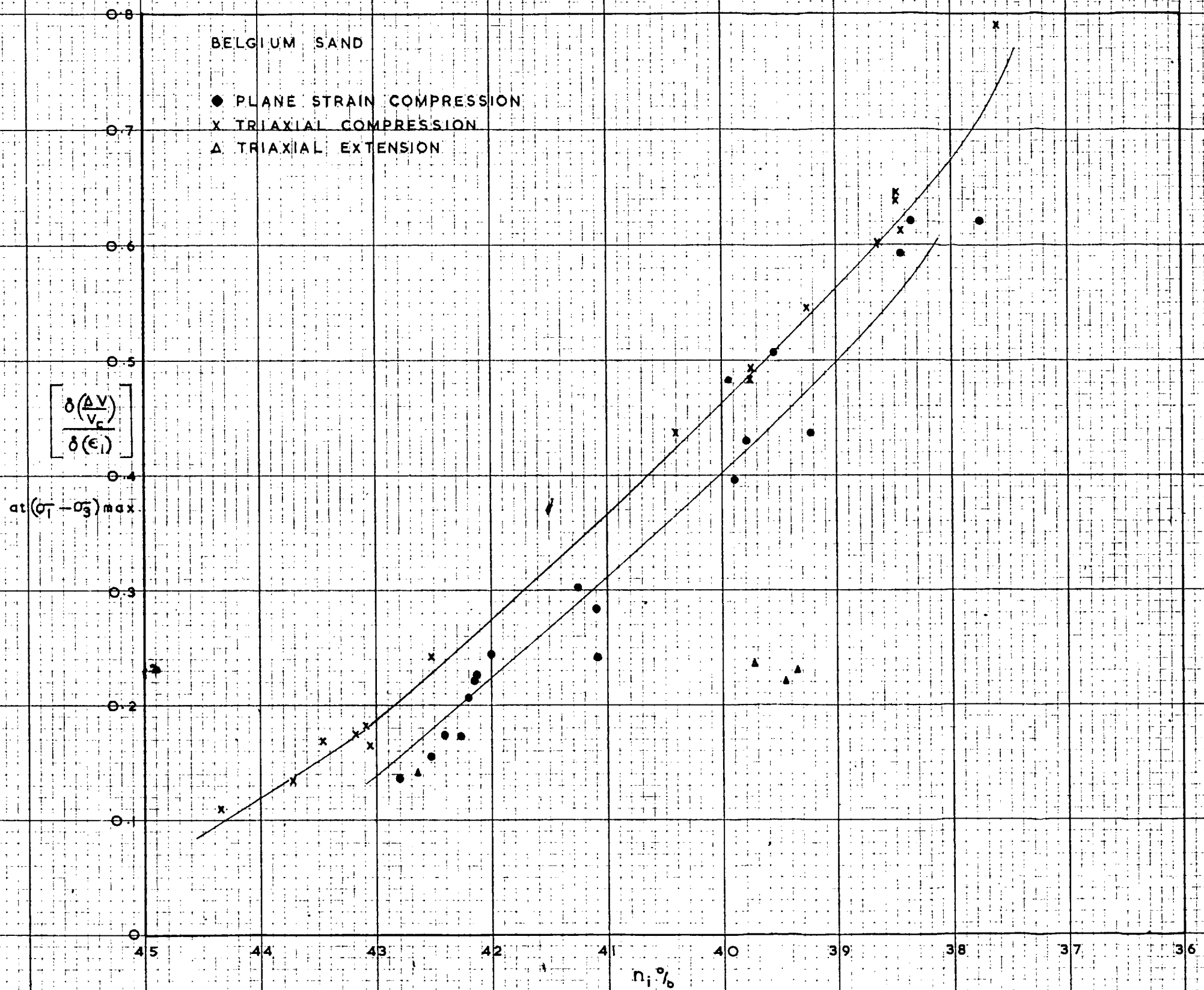
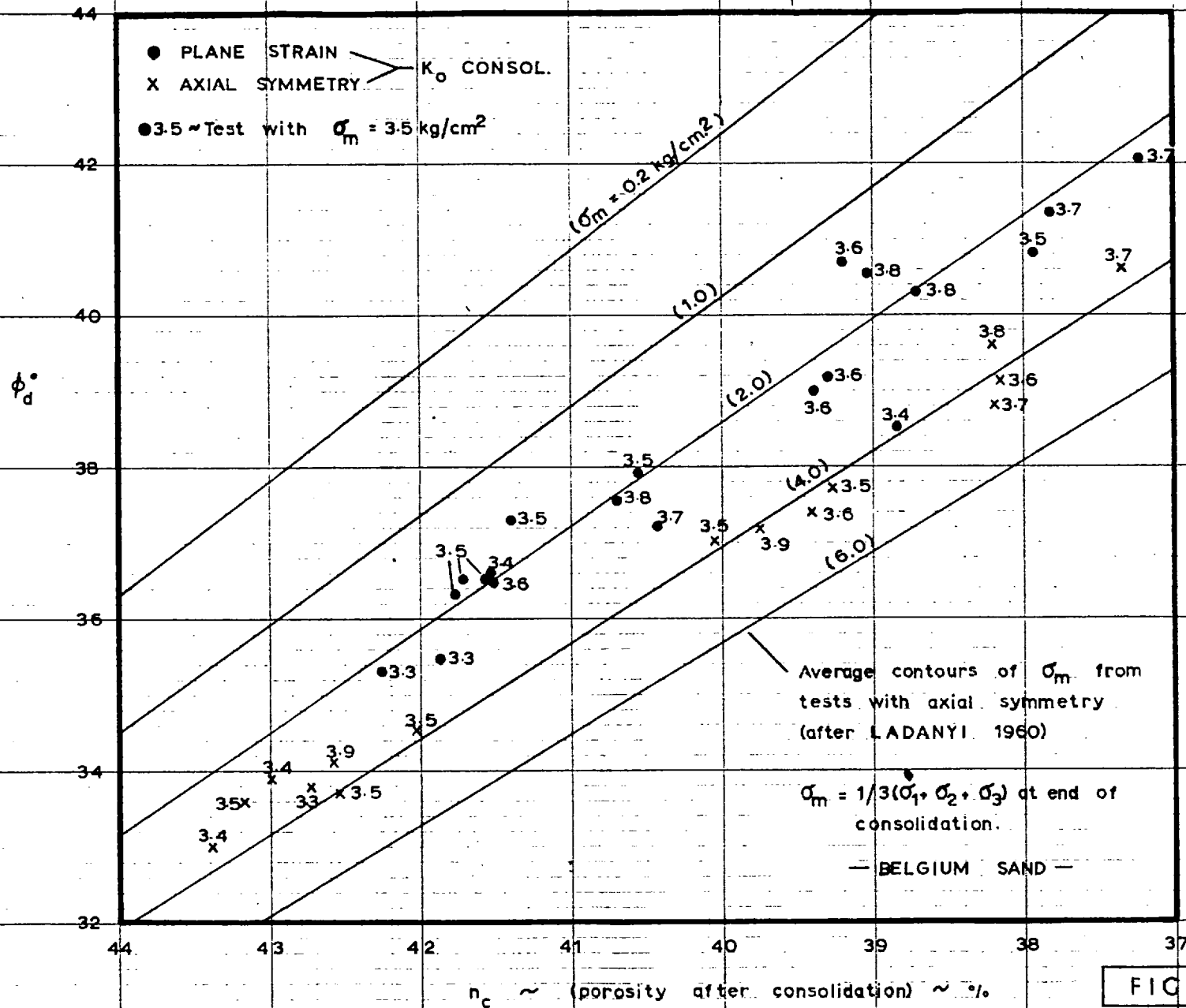


FIG 8-9



CHAPTER IX

DISCUSSION OF TEST RESULTS

To illustrate the characteristics of the Weald Clay tested, the data obtained from specimens subjected to plane strain conditions and those under conditions of axial symmetry are examined and compared with reference to the hypotheses reviewed earlier (see Chapter II). When required, further illustration of the clay behaviour is provided by introducing comparative data from the tests on Belgium Sand as well as published data on other soils.

(9.1) Water Content - Normal Stress at End of Consolidation

In the water content versus average normal stress relationship at the end of consolidation (Fig. 7-3), the results from the plane strain tests lie slightly above the average line corresponding to the standard triaxial test data (shown dashed). For the same value of $(J_{1/3})$, the difference in water content is about 0.25%. This phenomenon could possibly result from the respective initial remoulding processes. Due to their relative sizes, the standard triaxial specimen is probably more thoroughly remoulded than the plane strain specimen, and, as a result, it could be assumed that the clay particles in the triaxial specimen have a denser packing. On this hypothesis, it is conceivable that a triaxial specimen would require a smaller relative consolidation

pressure to obtain a given water content. (However, the effect of the particle orientation or specimen structure* on the strength of a soil remains controversial. SEED & CHAN (1959) suggested that the initial structure of a specimen would probably have relatively little influence on the strength since a re-orientation of the particles in the zone of shear may occur. On the other hand, HENKEL & SOWA (1963) found an appreciable difference in the w.c.- $\log (\sigma_1 - \sigma_3)$ relations at failure between isotropic and K_0 consolidated standard triaxial specimens). Structural effects may also account for some of the variation in the K_0 values obtained from both the axial symmetry and plane strain tests.

(9.2) Stress-Strain Relations

As indicated in the $\left[(\sigma_1 - \sigma_3) / p_c \right]$ versus % strain graphs, (Figs. 7-4, 7-5 and 7-29), representative stress-strain curves for specimens subjected to plane strain conditions have a higher peak deviator stress ratio at a lower % strain than the corresponding curves for specimens with axial symmetry. The relative values of the axial strain at maximum deviator stress are 2-3% for the plane strain specimens and 5-6% for the standard triaxial specimens.

In the specimens with no end restraint (i.e. group C, Fig. 7-6), however, the ratio of the peak deviator stress to p_c lies within the limits of the curves of the triaxial specimens. The Group C specimens

* Structure refers to the geometry or arrangement of the particles.

fail at 2-4% axial strain. Average values of the maximum deviator stress divided by P_c , as well as other stress ratios, are given in TABLE 9-1 for the A, B, and C group plane strain tests, and for the standard triaxial tests.

This phenomenon of a higher peak deviator stress ratio at a lower % strain under conditions of plane strain when compared to the corresponding conditions of axial symmetry at the same relative density is substantiated by the plane strain tests on Belgium Sand, Brasted Sand (CORNFORTH, 1961) and Glen Shira compacted moraine (WOOD, 1958).

(9.3) Maximum Deviator Stress versus Consolidation Pressure

As is suggested in the dimensionless stress-strain curves of the previous section, the ratio of the maximum deviator stress to the major principal effective stress at the end of consolidation is larger for the plane strain specimens. The relative values are 0.56 and 0.52 for the plane strain (Fig. 7-14) and triaxial (Fig. 7-33) specimens respectively. In both diagrams, the results of specimens consolidated to $\sigma'_1 > 120$ psi. plot below the average line, thus indicating that the respective ratios may decrease slightly at high consolidation pressures.

(9.4) ϕ' at Failure

The respective strengths of plane strain and standard triaxial

specimens were compared, in Figs. 9-1 and 9-2, with their average normal stresses at failure by plotting $(\sigma_1 - \sigma_3)/(\sigma_1 + \sigma_3)$ against $(\sigma_1 + \sigma_3)/2$ at peak deviator stress and at maximum principal stress ratio. The average horizontal line through each set of points represents a constant ϕ . For a normal stress range $10 < (\sigma_1 + \sigma_3)/2 < 70$ psi., the average strength of specimens subjected to plane strain conditions is greater than the strength of triaxial samples by $\Delta \phi' = 1.2^\circ$ (approximately) for both failure criteria. The strength of Weald Clay under plane strain conditions is represented by $\phi' = 27.1^\circ$ for peak deviator stress and $\phi = 27.4^\circ$ for $(\sigma_1/\sigma_3)_{\max}$. When $(\sigma_1 + \sigma_3)/2 > 70$ psi., the ratio $(\sigma_1 - \sigma_3)/(\sigma_1 + \sigma_3)$ and hence the strength decreases from the average value. This could indicate a flattening of the Mohr envelope with increasing normal pressure.

As shown in the diagrams, the results of the specimens tested with no end restraint are in better agreement with the conventional triaxial test data. However, tests on Brasted Sand (CORNFORTH, 1961) showed that the strength of sand in the plane strain apparatus was little affected by the omission of the end clamp. It can be assumed that the frictional forces along the top and bottom platens in Cornforth's tests were sufficient for all practical purposes to maintain the plane strain condition during shear. A similar assumption was made at the Norwegian Geotechnical Institute when tests on rectangular prismatic specimens of sand with no end restraint were carried out (See chapter II). In the clay tests, on the other hand,

relatively smaller frictional forces could be expected^{*} along the platens which would result in a departure of the specimen from the plane strain condition. The specimens in test Nos. WC16(O) and WC18(O) were measured after being sheared with no end restraint, and average longitudinal strains of 1.4 and 1.0% respectively, were observed. Consequently, the evidence on both sand and clay suggests that a higher effective angle of shearing resistance is obtained when a specimen is subjected to plane strain conditions.

Further evidence of this increase in ϕ' under conditions of plane strain is given in Fig. 9-3. In this diagram, the sands tested by Kjellman, Kummeneji and Christensen were subjected to conditions approximating to plane strain, and the remaining soils were tested in the Imperial College plane strain apparatus. The dashed line corresponds to equal ϕ' values in the plane strain and standard triaxial tests; all the soils tested lie above this line.

Investigations on sands by HABIB (1953), KIRKPATRICK (1957) and WU, LOH & MALVERN (1963) tend to indicate that the strength of sands depends on the magnitude of the intermediate principal stress (See Chapter II), whereas tests on hollow cylinders of clay by Wu, Loh and Malvern indicated to the authors that σ_2 had little influence on the strength of clay. In Fig. 9-4, the hollow cylinder data on this clay, as well as the results of conventional triaxial compression and extension tests, were plotted as $(\sigma_1' - \sigma_3')/(\sigma_1' + \sigma_3')$ against $(\sigma_1' + \sigma_3')/2$.

^{*} Due to the lower coefficient of friction between clay and porepaper.

The average horizontal line corresponds to $\phi' = 32^\circ$, which was obtained from the standard triaxial test results. The magnitude of the ratio $(\frac{\sigma'_2}{\sigma'_1 + \sigma'_3})$ at failure for the hollow cylinder tests is denoted beside each plotted point. Generally, the tests with the larger values of σ'_2 lie below the average line, but, due to the scatter of the results, no definite conclusions regarding the significance of the intermediate principal stress on the strength characteristics can be drawn.

(9.5) Stress Paths

Dimensionless stress paths in terms of $(\sigma'_1 - \sigma'_3)/(\sigma'_1 + \sigma'_3)_c$ versus $(\sigma'_1 + \sigma'_3)/(\sigma'_1 + \sigma'_3)_c$ were plotted for the tests with axial symmetry in Fig. 9-5, and for the A and B groups of the plane strain tests in Figs. 9-6 and 9-7 respectively. In all three graphs, the stress paths of the specimens consolidated to higher pressures are generally flatter and have a lower value of $(\sigma'_1 - \sigma'_3)/(\sigma'_1 + \sigma'_3)_c$ at failure. Approximate limits of the triaxial stress paths were determined from Fig. 9-5 and superimposed on the graphs for the plane strain specimens. It can be seen that the respective triaxial and plane strain stress paths are not unique, the relative values of $(\sigma'_1 - \sigma'_3)/(\sigma'_1 + \sigma'_3)_c$ being higher in the plane strain tests, except at the beginning of the shear stage, over the length of the paths. This phenomenon indicates that the ϕ' mobilized^{cc} during shear and the strength at failure in a

^a The subscript c refers to the values at the end of consolidation.
^{cc} i.e. the ratio $(\sigma'_1 - \sigma'_3)/(\sigma'_1 + \sigma'_3)$.

specimen subjected to plane strain conditions is greater than that predicted by the Mohr-Coulomb hypothesis.

When the intermediate principal stress is considered the dimensionless stress paths during shear are expressed in terms of τ_{oct}/P_c and $(J_1/3)/P_c$, where τ_{oct} and $J_1/3$ are the octahedral shear and octahedral normal stresses respectively, and P_c is the magnitude of $J_1/3$ at the end of consolidation. Fig. 9-8 shows the relevant stress paths followed by specimens with axial symmetry; the A and B group plane strain tests are illustrated in Figs. 9-9 and 9-10. By superimposing the approximate boundaries of the triaxial stress paths on the plane strain graphs, the paths of both the A and B group tests show reasonable agreement with the triaxial data. However, the stress paths of the plane strain specimens terminate sooner, i.e. the relative values of $(J_1/3)/P_c$ at failure are higher in the plane strain specimens. This phenomena accounts for the difference in the relative $J_1/3$ values shown in the water content versus normal stress graphs (Fig. 7-25), since along each stress path the water content is constant.

(9.6) Pore Water Pressure Relations

A relationship between the change in pore water pressure (in saturated specimens) and the change in applied (total) stresses can be derived from elastic theory. The equation* is

* after ~~NICHOL & EIDEN (1950)~~. SKEMPTON (1954).

$$\Delta u = B \cdot \frac{1}{3} (\Delta \sigma_1 + \Delta \sigma_2 + \Delta \sigma_3) \quad \dots\dots\dots 9-1a$$

where

$$B = \frac{1}{1 + n \cdot \frac{C_w}{C_c}} \approx 1 \quad \dots\dots\dots 9-1b$$

and n = specimen porosity
 C_w = compressibility of water
 C_c = compressibility of soil structure

Computation of the results of a typical plane strain test on Weald Clay shows that, at maximum deviator stress, the predicted pore water pressure is equal to 5.7 psi, whereas the measured value is 16.2 psi. Consequently, the elastic theory does not describe the pore pressure phenomenon as observed in the tests.

The fact that the relative values of both $(\sigma'_1 - \sigma'_3)/(\sigma'_1 + \sigma'_3)_c$ and $(\sigma'_1 + \sigma'_3)/(\sigma'_1 + \sigma'_3)_c$ are larger in the plane strain specimens (Figs. 9-6 and 9-7) indicates that the induced pore water pressures are smaller in specimens under conditions of plane strain (see Table 9-1), and that the major and minor principal stresses alone do not control the pore pressure behaviour.

The octahedral stress paths (Figs. 9-9 and 9-10), on the other hand, show reasonable agreement in the two types of test, and this

indicates that a unique relationship may exist during shear between the pore water pressure and the octrahedral stresses. The possibility of this relationship was suggested by HENKEL (1960-b), who hypothesized that the change in pore pressure during shear is related to the change in principal stresses by the following:

$$\Delta u = \frac{\Delta \sigma_1 + \Delta \sigma_2 + \Delta \sigma_3}{3} + a \sqrt{(\Delta \sigma_1 - \Delta \sigma_2)^2 + (\Delta \sigma_2 - \Delta \sigma_3)^2 + (\Delta \sigma_3 - \Delta \sigma_1)^2} \dots\dots\dots(9-2)$$

where $\Delta \sigma_1$, $\Delta \sigma_2$ and $\Delta \sigma_3$ are the changes in the total stresses, and a is a pore pressure parameter depending on the soil type.

The experimental results indicate, however, that the relationship should be expressed as:

$$\Delta u = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} + \frac{a}{3} \cdot \Delta \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \dots\dots(9-3a)$$

or in octrahedral nomenclature

$$\Delta u = \Delta p + a \cdot \Delta \tau_{oct.} \dots\dots\dots(9-3b)$$

where, in terms of effective stresses, a is defined as

$$a = - \frac{\Delta p'}{\Delta \tau_{oct.}} = - \frac{\Delta \sigma_3/3}{\Delta \tau_{oct.}} \dots\dots\dots(9-3c)$$

Expressed in these terms, the empirical parameter a is thought to describe, approximately, the pore pressure behaviour during shear in both plane strain and standard triaxial specimens, providing, of course, the relative values of τ_{oct} and $J_1/3$ at the end of consolidation, are equal in the respective specimens. Since the plane strain specimen reaches failure before the triaxial specimen (i.e. the stress path is shorter), the values of a at failure will not be the same in the two tests. Typical values of a at peak deviator stress are 2.9 and 3.7 for the plane strain and triaxial specimens respectively. Average values of a for the A and B group plane strain tests as well as for the triaxial tests are given in Table 9-1. These average values, however, are not strictly comparable because of the differing conditions at the end of consolidation.

(9.7) Octahedral Failure Conditions

In the (τ_{oct}/p_c) versus $(J_1/3)/p_c$ diagrams (Figs. 9-9 and 9-10), the coincidence of the octahedral stress paths prior to failure for the plane strain and triaxial specimens shows that the initial yield of the specimens is controlled by the octahedral stresses (c.f. von Mises material). Failure, however, is not controlled by the octahedral stresses. This is illustrated in Figs. 9-11 and 9-12, in which $\left[\tau_{oct} / (J_1/3) \right]$ was plotted against $(J_1/3)$ for the maximum deviator stress and maximum principal stress ratio criteria respectively. In both diagrams, the failure points for the

plane strain specimens lie below the points representing the standard triaxial data, and hence the von Mises hypothesis for failure is not valid for the clay tested.

In Fig. 9-13, a graphical representation of the plane strain results on the octrahedral plane shows the relative position of the failure points to various failure envelopes. All envelopes were determined from $\phi' = 25.9^\circ$, as obtained from the standard triaxial test data. In the Mohr-Coulomb and Coleman^{**} criteria, it was assumed that ϕ' in extension is equal to ϕ' in compression^{***}. Also illustrated on the diagram are the stress paths for the group B tests, and, in another sector, the average stress pack for the group A tests. The starting point of each stress is determined by the magnitude of the octrahedral shear stress at the end of consolidation and, since $\sigma'_2 = \sigma'_3$, this point lies close to the major principal axis. During shear, as the difference between σ'_2 and σ'_3 increases, the path departs from the proximity of the major principal axis and approaches the failure point in approximately a straight line. The failure points for the

^{*} Coleman's criteria in terms of effective stresses is:

$$- 3\Delta_2 = \left[(2J_1)/3 + (\Delta_3/2)^{1/2} \right]^2 \sin^2 \phi'.$$

where $\Delta_2 = (\sigma'_1 - p')(\sigma'_2 - p') + (\sigma'_2 - p')(\sigma'_3 - p') + (\sigma'_3 - p')(\sigma'_1 - p')$

$$p' = J_1/3$$

$$\Delta_3 = (\sigma'_1 - p')(\sigma'_2 - p')(\sigma'_3 - p')$$

^{***} SOWA (1963) found ϕ' (extension) $\neq$ ϕ' (compression) for the Weald Clay used in this thesis.

nine plane strain tests shown do not agree with any of the yield criteria drawn on the diagram. The hypotheses of von Mises, Tresca and Coleman each overestimate the strength of clay when subjected to plane strain conditions, and the Mohr-Coulomb criteria, as illustrated earlier, underestimates the strength.

JOHANSEN (1958) proposed, in cases where the actual failure points lie between the von Mises and Mohr-Coulomb envelopes in stress space, that a unique criteria might be obtained by using weighted values of the intermediate principal stress. He suggests that the normal and shear stresses in a plane for which the direction cosines to the normal are $[(1, \alpha, 1)/\sqrt{2+\alpha^2}]$ be used as variables (See Chapter II). The stresses are defined by the following:

$$\sigma_{\alpha} = \frac{\sigma_1' + \alpha^2 \sigma_2' + \sigma_3'}{2 + \alpha^2} \quad \dots\dots\dots(9-4a)$$

and

$$\tau_{\alpha} = \frac{\alpha^2 (\sigma_1' - \sigma_2')^2 + (\sigma_1' - \sigma_3') + \alpha^2 (\sigma_2' - \sigma_3')^2}{2 + \alpha^2} \quad \dots\dots\dots(9-4b)$$

and the proposed failure criterion is:

$$F(\sigma_{\alpha}, \tau_{\alpha}) = \text{constant.} \quad \dots\dots\dots(9-4c)$$

The results of both the plane strain tests and the tests with

axial symmetry were plotted using Johansen's variables as $\frac{\tau_{\alpha}}{\sigma_{\alpha}}$ against σ_{α} in Fig. 9-14. Although the plane strain data plots slightly below the standard triaxial results, this empirical hypothesis provides a more unique correlation between the two types of test than any of the other criteria investigated.

(9.8) Intermediate Principal Stress

The magnitude of the intermediate principal stress under plane strain conditions can be predicted from elastic theory if it is assumed that the soil behaves as an elastic body and that the soil structure is isotropic. The principal strains can be related to the principal effective stresses by the following:

$$e_1 = \frac{1}{E} \left[\Delta \sigma_1' - \mu (\Delta \sigma_2' + \Delta \sigma_3') \right] \quad \dots\dots\dots (9-5a)$$

$$e_2 = \frac{1}{E} \left[\Delta \sigma_2' - \mu (\Delta \sigma_1' + \Delta \sigma_3') \right] \quad \dots\dots\dots (9-5b)$$

$$e_3 = \frac{1}{E} \left[\Delta \sigma_3' - \mu (\Delta \sigma_1' + \Delta \sigma_2') \right] \quad \dots\dots\dots (9-5c)$$

where E = Young's modulus and μ = Poisson's ratio.

For K_0 consolidation, $e_2 = e_3 = 0$, and $\Delta \sigma_2' = \Delta \sigma_3'$. Substituting in equation (9-5c) and simplifying, we get:

$$\frac{\Delta \sigma_2'}{\Delta \sigma_1'} = \frac{\mu}{1 - \mu} = K_0 \quad \dots\dots\dots (9-5d)$$

or
$$\mu = \frac{K_o}{1 + K_o} \dots\dots\dots(9-5e)$$

Since $\epsilon_2=0$ for the condition of plane strain as well as for K_o consolidation, equation (9-5b) is valid from the beginning of the test and by substitution the equation reduces to

$$\Delta \sigma_2^i = \mu (\Delta \sigma_1^i + \Delta \sigma_3^i) \dots\dots\dots(9-5f)$$

or
$$\sigma_2^i = \mu (\sigma_1^i + \sigma_3^i) \dots\dots\dots(9-5g)$$

substituting (9-5e)

$$\sigma_2^i = \frac{K_o}{1+K_o} (\sigma_1^i + \sigma_3^i) \dots\dots\dots(9-5h)$$

Taking the average K_o value of 0.59 for Weald Clay, elastic theory predicts $\mu = \left[\sigma_2^i / (\sigma_1^i + \sigma_3^i) \right] = 0.37$ which is slightly smaller than the measured values (see Figs. 7-8 and 7-9). The theory, therefore, underestimates the value of the intermediate principal stress. This is substantiated by the tests on Belgium Sand, Brasted Sand (CORNFORTH, 1961) and Glen Shira compacted moraine (WOOD, 1958).

An alternative method of determining σ_2^i is to assume that the clay behaves as a perfectly plastic material. The relation between the plastic principal strains and the principal stresses for the

condition of incompressibility (i.e. $\Delta V=0$) is given by*

$$\frac{2 \sigma'_2 - \sigma'_1 - \sigma'_3}{\sigma'_1 - \sigma'_3} = \frac{2 de_2 - de_1 - de_3}{de_1 - de_3} \quad \dots\dots\dots(9-6a)$$

where de_1 , de_2 , and de_3 are small increments of plastic strain.

Under conditions of plane strain $de_2 = 0$ and $de_1 = -de_3$, therefore equation (9-6a) reduces to

$$\sigma'_2 = \frac{1}{2}(\sigma'_1 - \sigma'_3) \quad \dots\dots\dots(9-6b)$$

which overestimates the intermediate principal stress.

Dr. Bishop's relationship for obtaining σ'_2 , i.e.

$$\frac{\sigma'_2}{\sigma'_1 + \sigma'_3} = \frac{\cos^2 \phi'}{2} \quad \dots\dots\dots(8-1)$$

can be derived as follows. If we assume that $\sigma'_2 = K_o \sigma'_1$ and $K_o = (1 - \sin \phi')$, and we know that, for $c' = 0$, $\sigma'_1 / \sigma'_3 = \frac{1 + \sin \phi'}{1 - \sin \phi'}$, then

$$\frac{\sigma'_2}{\sigma'_1 + \sigma'_3} = \frac{K_o \sigma'_1}{\sigma'_1 + \sigma'_3} = \frac{(1 - \sin \phi') \sigma'_1}{\sigma'_1 + \sigma'_3}$$

* See JAEGER 1962, p.99.

dividing by σ_3' and substituting for σ_1'/σ_3' ,

$$\frac{\sigma_2'}{\sigma_1' + \sigma_3'} = \frac{(1 - \sin \phi') \frac{(1 + \sin \phi')}{(1 - \sin \phi')}}{\frac{1 + \sin \phi'}{1 - \sin \phi'} + 1}$$

which reduces to

$$\frac{\sigma_2'}{\sigma_1' + \sigma_3'} = \frac{(1 - \sin^2 \phi')}{2} = \frac{\cos^2 \phi'}{2} \quad \dots \dots (8-1)$$

The results of both Wood's and Cornforth's tests on moraine and Brasted Sand respectively, as well as the data from the Weald Clay (Figs. 7-8 and 7-9) and Belgium Sand tests (Figs. 8-3) indicate that equation 8-1 approximately describes the relationship between the three principal stresses and the angle of shearing resistance. The values of the ratio $\left[\frac{\sigma_2'}{(\sigma_1' + \sigma_3')} \right]$ are only slightly higher than the values predicted by $\frac{\cos^2 \phi'}{2}$ for all four materials.

(9.9) Yield Surface in (e, p', q) Space

ROSCOE, SCHOFFIELD & WROTH (1958) proposed that a soil specimen, when sheared, ultimately reached a critical voids ratio at which large deformations occurred with no change in volume or applied

e = voids ratio; $q = (\sigma_1' - \sigma_3')$; $p' = \text{mean normal effective pressure} = \sigma_1'/3$.

stress. Also it was suggested that the continuous yielding of the specimen during shear could be represented by a loading path which rises to a yield surface in (e, p, q) space and then remains on that surface. In a subsequent paper (ROSCOE & POOROOSHASB, 1963), a method of representing the yield surface diagrammatically in two dimensions was suggested. The variables required are η and ξ , where

$$\eta = \frac{(\sigma_1 - \sigma_3)}{p'} = q/p' \quad \dots\dots(9-7a)$$

and

$$\xi = \frac{1}{p'} \cdot \exp. \left(\frac{w_a - w}{\lambda_1} \right) \quad \dots\dots(9-7b)$$

and where

- w_a = water content of a normally consolidated sample under unit anisotropic^a stress;
- w = water content at the end of consolidation;
- λ_1 = slope of the $(w.c. - \log_e p')$ curve.

As pointed out in the paper, for an undrained test,

$$\xi = \frac{p'_c}{p'}$$

^a Roscoe & Poorooshasb defined w_a as the water content under unit isotropic stress. However, since K_o consolidation was used in this case, no anomaly would be expected in defining w_a as above.

where p_c is the equilibrium pressure at the end of consolidation.

In Figs. 9-15 and 9-16, the triaxial data and the results of the plane strain tests on Weald Clay, respectively, are shown with these variables. When the approximate boundaries of the triaxial curves are superimposed on the plane strain graph, it can be seen that the surfaces* defined by the plane strain curves lie above those defined by the triaxial data.

However, when the octahedral shear stress is used instead of the deviator stress, the curves become coincident and a unique surface can be obtained for both the standard triaxial and plane strain data. The triaxial results plotted on this basis are shown in Fig. 9-17, and the plane strain results are given in Fig. 9-18.

(9.10) Summary and Conclusions

For both the clay and the sand tested, the strength in terms of ϕ' of specimens subjected to plane strain conditions is greater than that of specimens under conditions of axial symmetry. The angles of shearing resistance for Weald Clay as determined from the plane strain data are $\phi' = 27.1^\circ$ and $\phi' = 27.4^\circ$ for the maximum deviator stress and maximum principal stress ratio criteria respectively. These values are c.4½% larger than the ϕ' values obtained from the standard triaxial

* More than one surface exists because, for this clay, the relative $\left(\frac{\sigma_1 - \sigma_3}{p}\right)$ values depend on the consolidation pressure.

test results. The data from the Belgium Sand tests showed that the plane strain condition increased the angle of shearing resistance by c.3 to 4 $\frac{3}{4}$ % depending on whether the specimens were loose or dense, respectively. The results of the sand tests are substantiated by the works of Kjellman, Kummeneje, Christensen and Cornforth.

The % axial strain at failure is smaller in the plane strain specimens than in the standard triaxial specimens for both soils.

In the clay tests, the pore water pressures appear to be controlled by the octrahedral stresses (as hypothesized by Henkel), and, at failure, are smaller in the plane strain specimens for a given consolidation pressure.

The intermediate principal effective stress for the plane strain condition is slightly under-estimated if predicted by elastic theory and over-estimated if the plastic theory is used. The magnitude of σ_2^0 for Weald Clay as measured was 0.40 to 0.42 times the sum of the other two principal stresses. The relationship suggested by Dr. Bishop between the three principal stresses and the angle of shearing resistance in terms of effective stresses shows good agreement with the measured values.

When expressed in terms of octrahedral stresses, the strength of the Weald Clay specimens is over-estimated by the criteria of von Mises, Tresca and Coleman and under-estimated by the extended Mohr-Coulomb hypothesis. The empirical criterion proposed by Johansen, however, provides an approximate correlation between the octrahedral

stresses of the plane strain and standard triaxial data.

The coincidence of the octahedral stress paths up to failure in the plane strain specimens indicates that a unique yield surface exists in $(e, p', \bar{\epsilon}_{oct})$ space for the two types of test.

TABLE 9-1 STRESS RATIOS FOR PLANE STRAIN $\bar{\epsilon}$ AXIAL SYMMETRY

Ratio	Group A	Group B	Group C	Axial Symmetry
$\frac{\sigma_3'}{\sigma_1'} = K_0$	0.568	0.585	0.62	0.59
$\frac{(\sigma_1')_c}{P_c}$	1.410	1.387	1.346	1.378
$\frac{(\sigma_2')_c}{P_c}$	0.795	0.802	0.827	0.824
$\frac{(\sigma_3')_c}{P_c}$	0.795	0.802	0.827	0.824
$\frac{(\sigma_1' - \sigma_3')_c}{P_c}$	0.620	0.585	0.519	0.554
$\frac{(\sigma_1' - \sigma_3')_{\max.}}{P_c}$	0.820	0.772	0.733	0.714
$\frac{(\sigma_1' - \sigma_3')_{\max.}}{(\sigma_1')_c}$	0.580	0.557	0.544	0.518
$\frac{(\sigma_1')_f}{P_c}$	1.308	1.248	1.211	1.194
$\frac{(\sigma_2')_f}{P_c}$	0.749	0.673	-	-
$\frac{(\sigma_3')_f}{P_c}$	0.488	0.476	0.478	0.480
$A = \frac{\Delta U_f}{\Delta(\sigma_1' - \sigma_3')_f}$	1.54	1.74	1.63	2.15
$a = -\left(\frac{\Delta J_1/3}{\Delta \tau_{oct}}\right)$	2.83	3.76	-	3.83

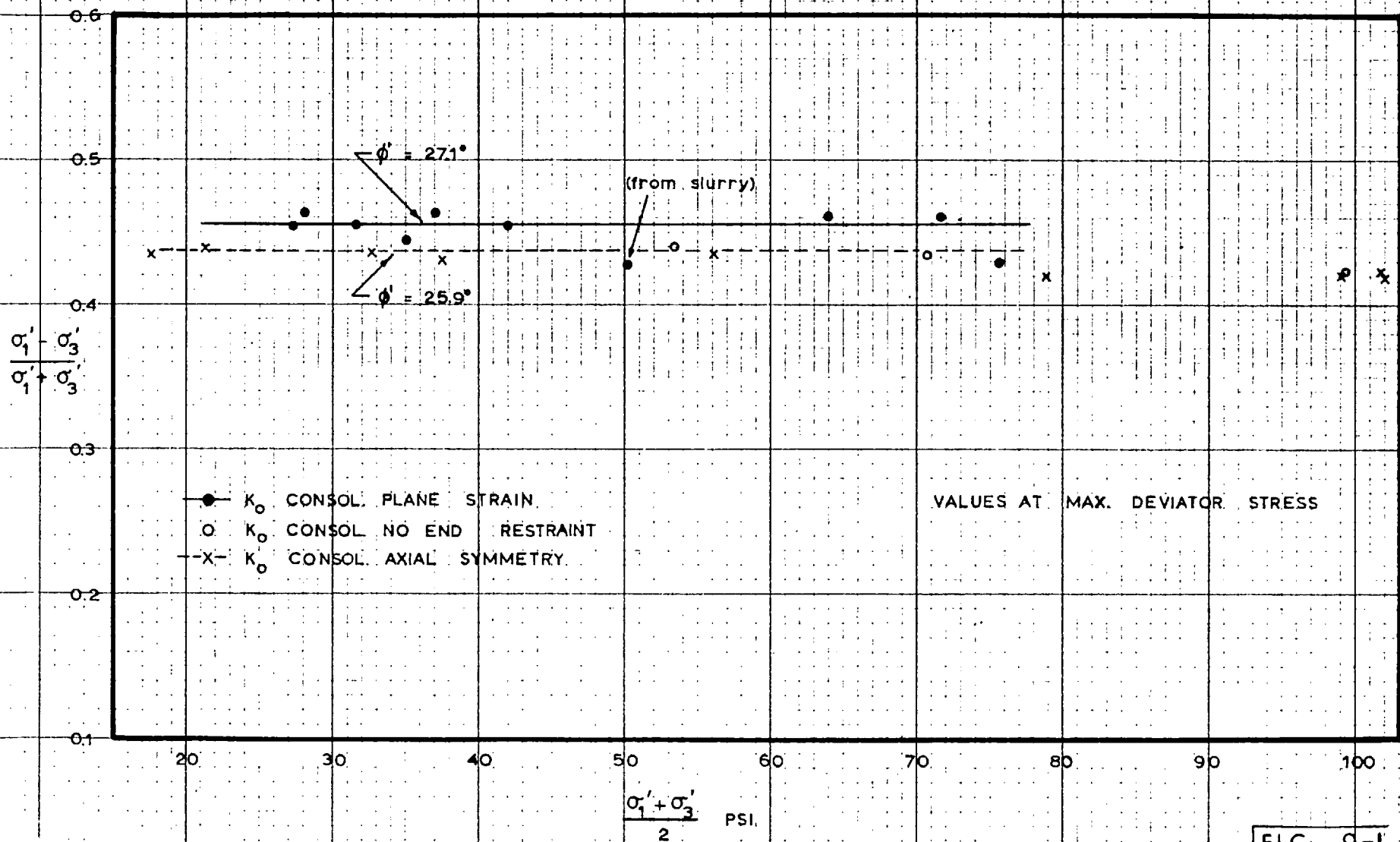


FIG 9-1

$$\frac{\sigma'_1 + \sigma'_3}{\sigma'_1 + \sigma'_3}$$

0.6

0.5

x

0.4

0.3

0.2

0.1

- K_0 CONSOL. PLANE STRAIN
- K_0 CONSOL. NO END RESTRAINT
- x- K_0 CONSOL. AXIAL SYMMETRY

(from slurry)

 $\phi' = 27.4^\circ$ $\phi' = 26.2^\circ$ VALUES AT $\frac{\sigma'_1}{\sigma'_3} \max.$

$$\frac{\sigma'_1 + \sigma'_3}{2}$$

PSI

FIG 9-2

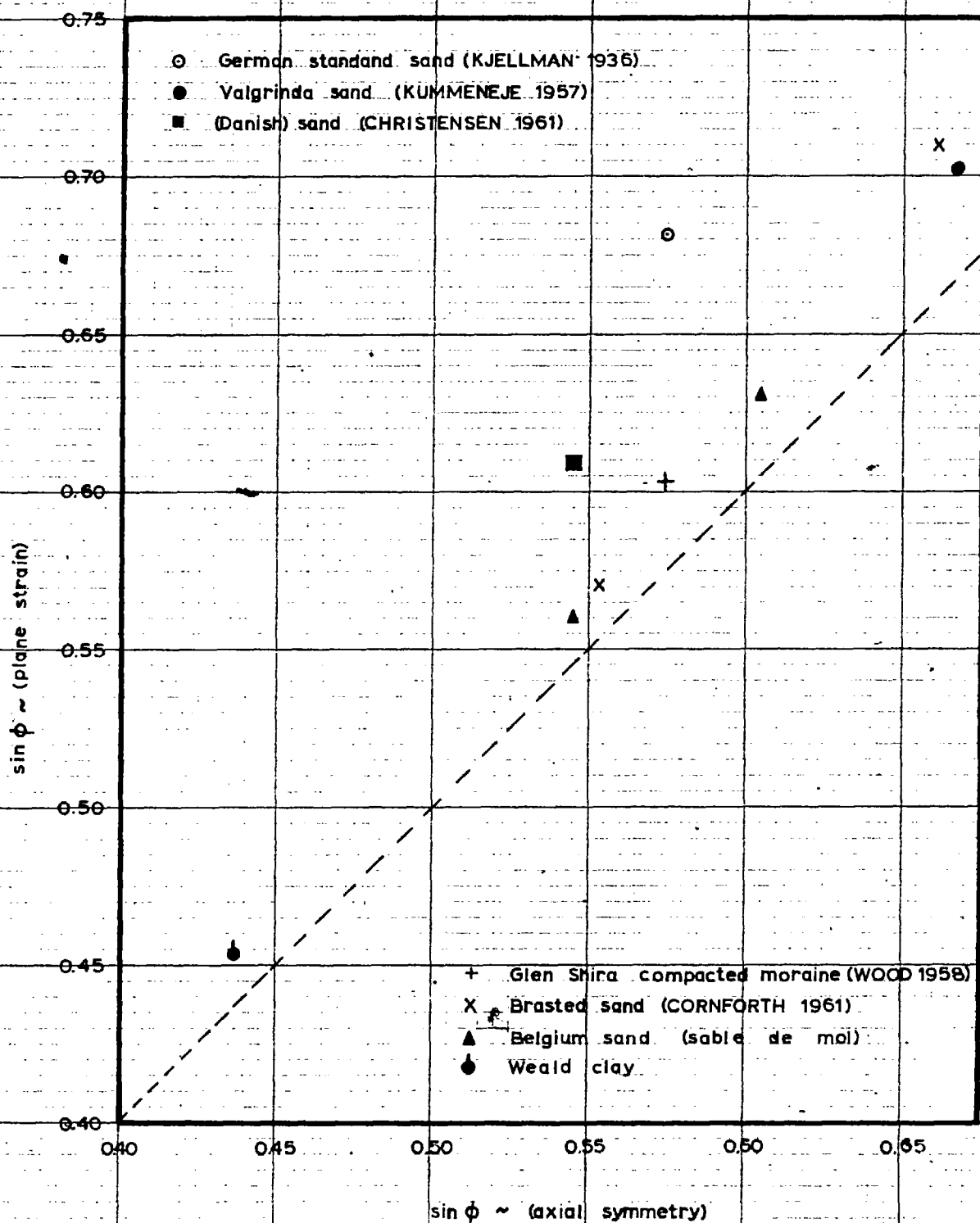


FIG 9-3

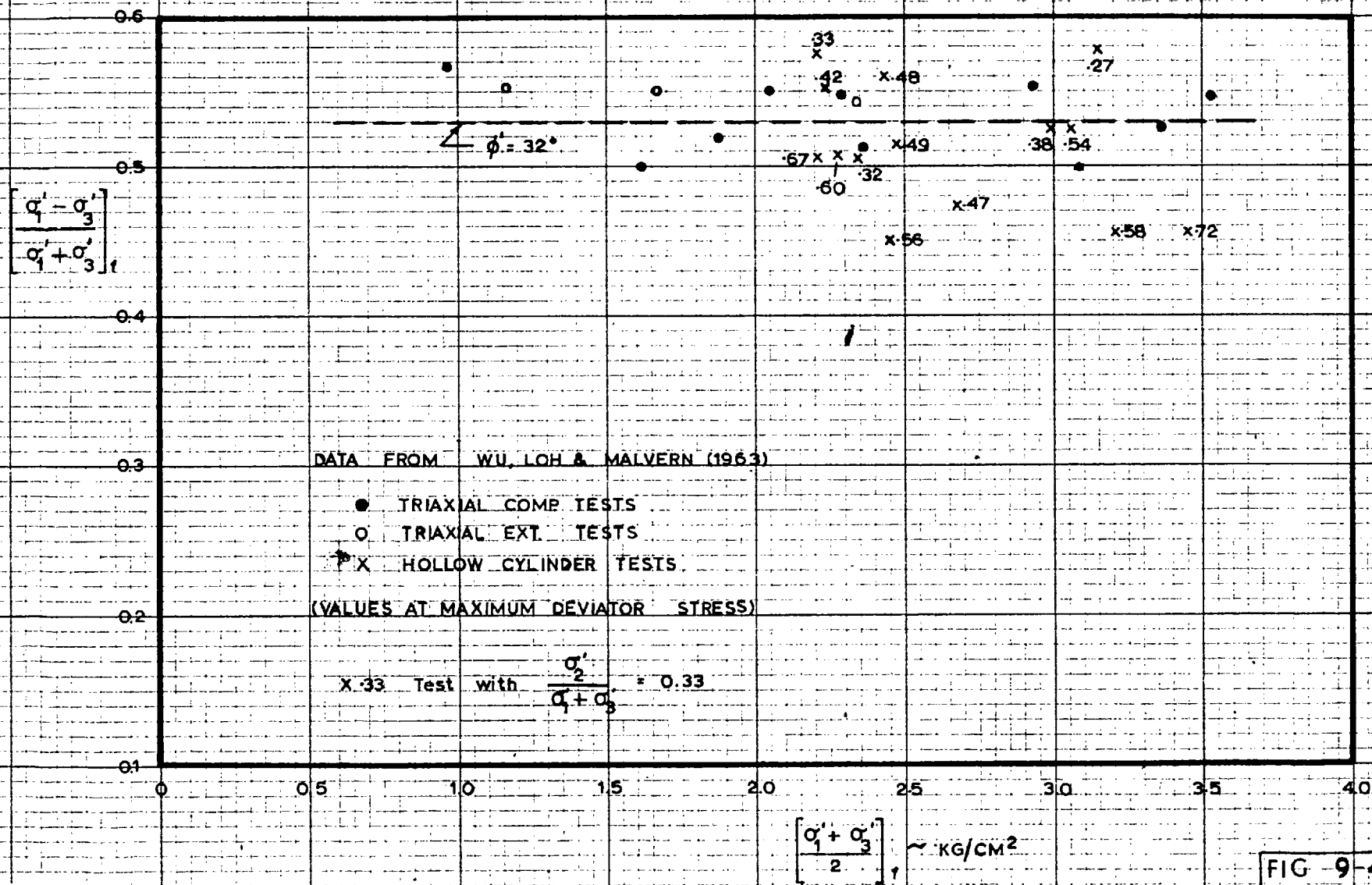


FIG 9-4

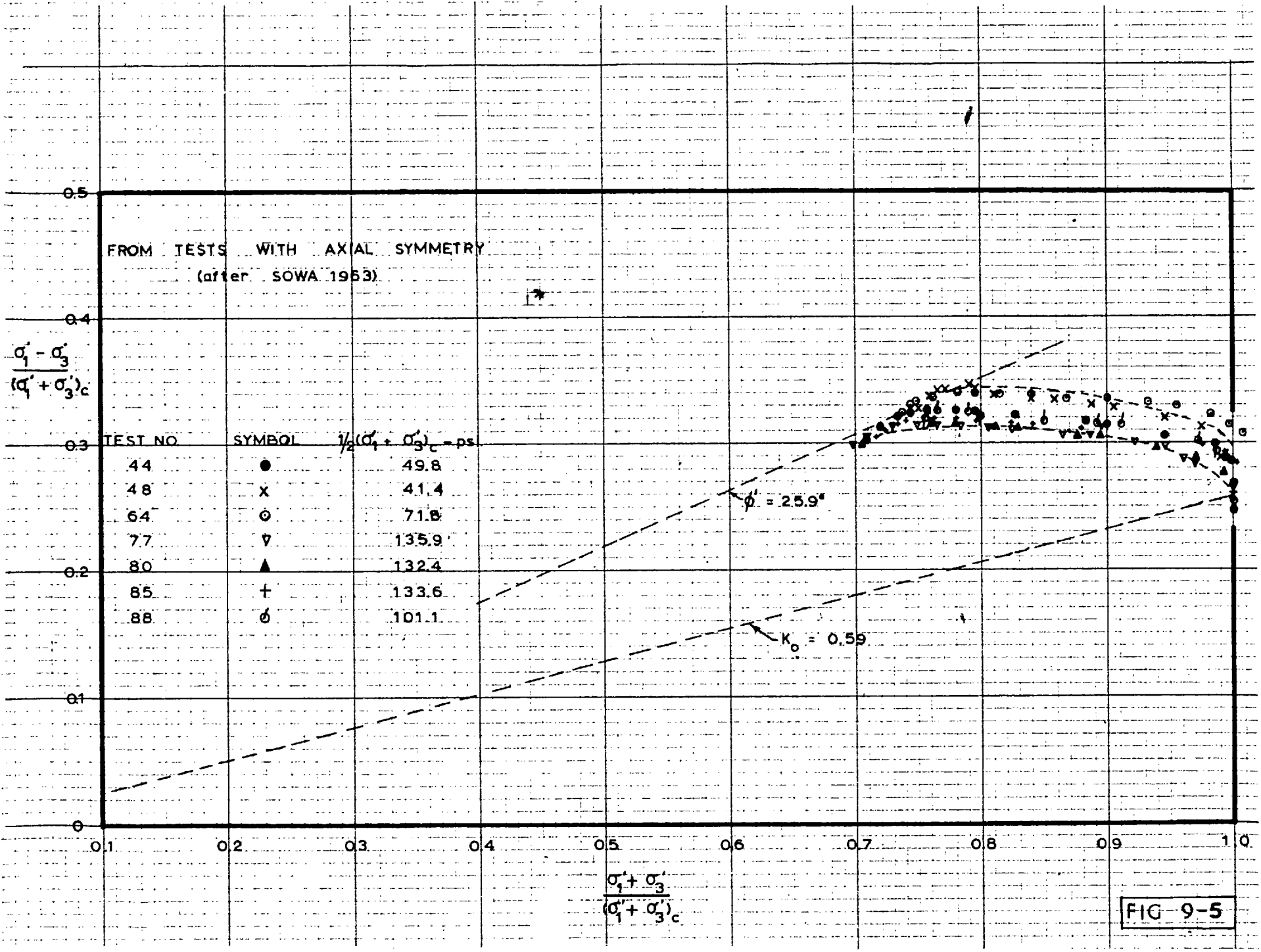


FIG 9-5

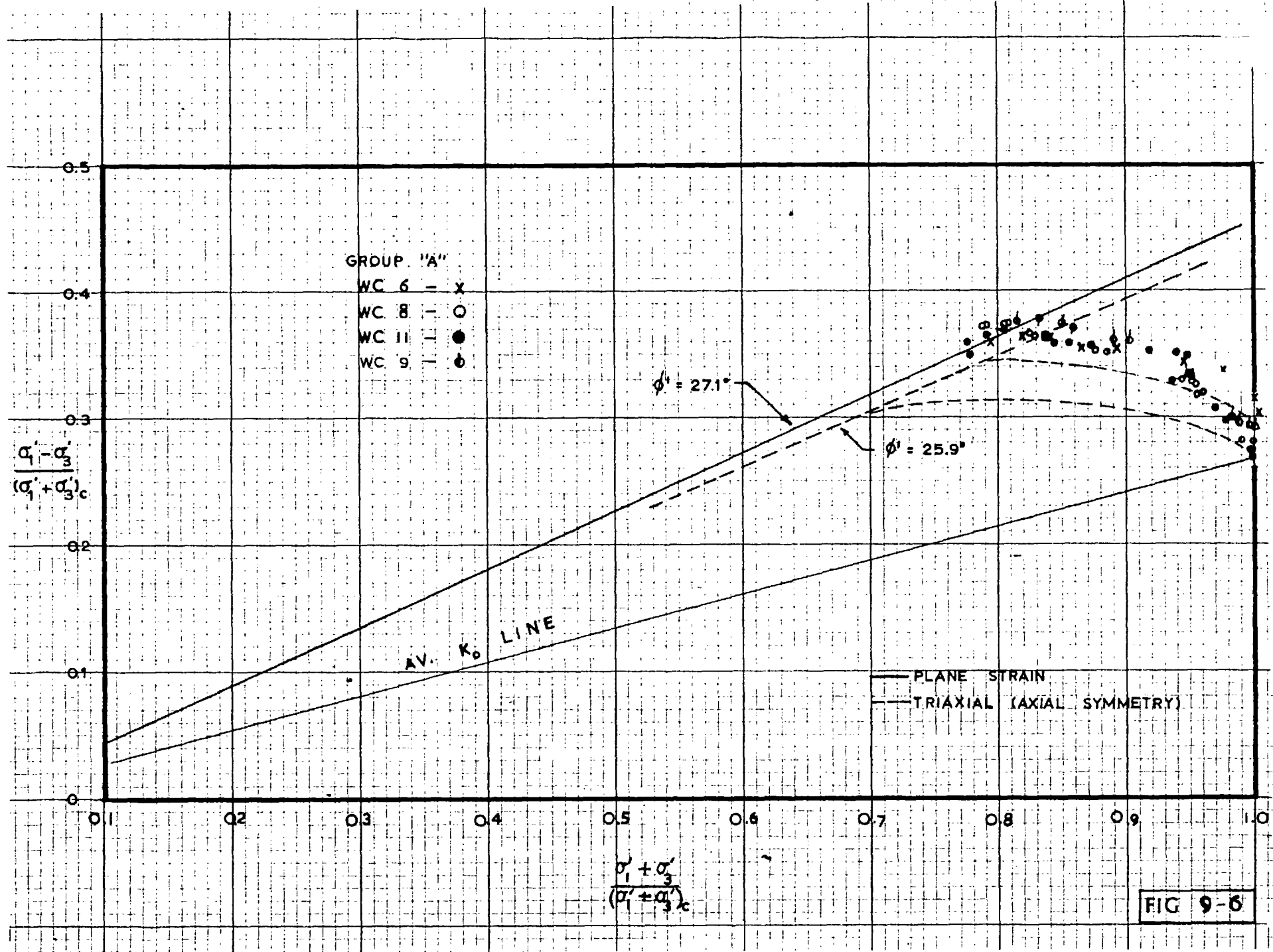
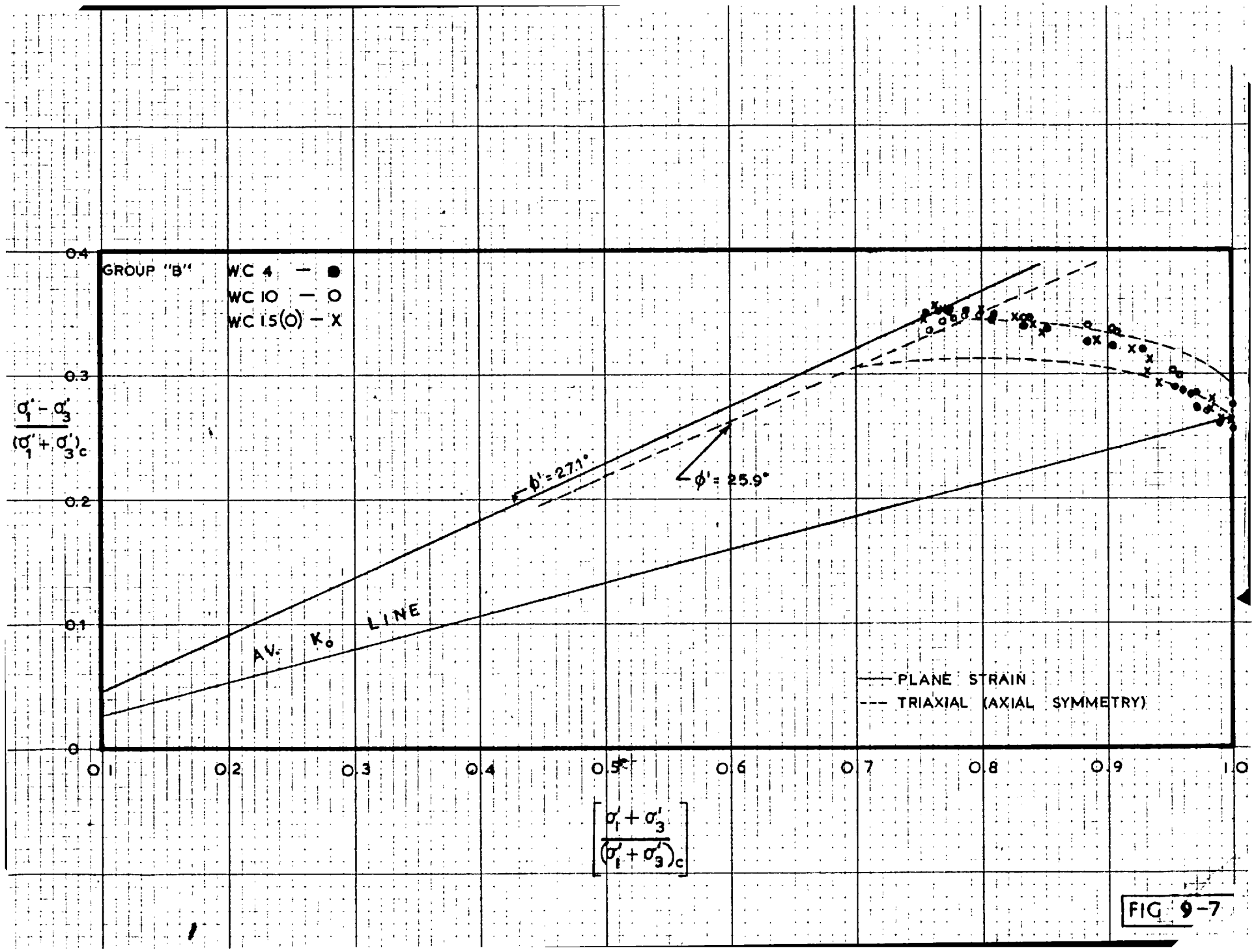


FIG 9-6



FROM TESTS WITH AXIAL SYMMETRY
(after SOWA 1963)

$\frac{\tau_{oct}}{P_c}$

TEST NO.	SYMBOL	P_c - psi
44	●	45.8
77	▽	123.9
48	X	37.8
64	○	65.7
80	▲	121.6
85	+	122.4
88	⊙	92.4

$K_0 = 0.59$

$\frac{J_1/3}{P_c}$

FIG 9-8

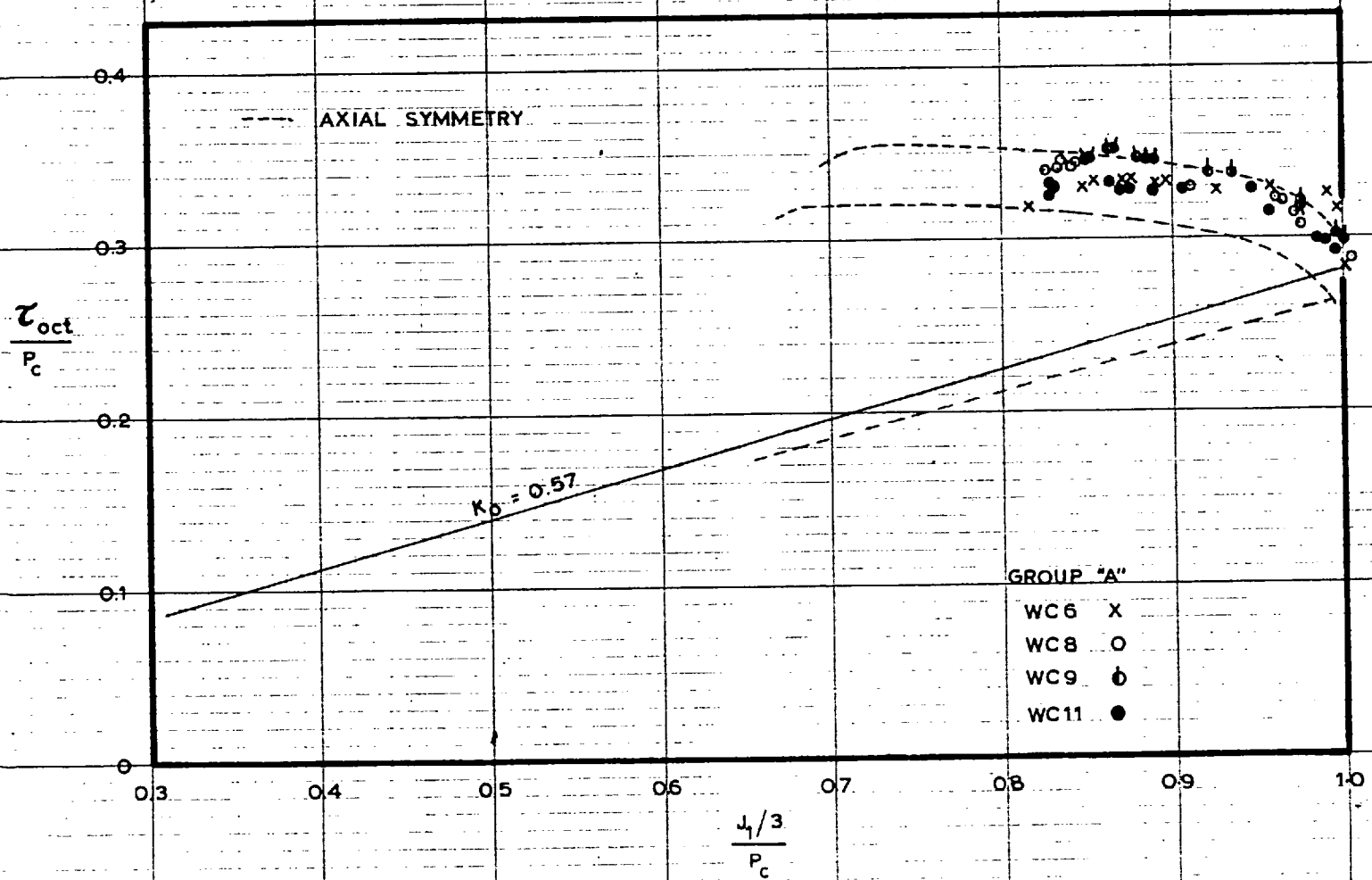


FIG 9-6

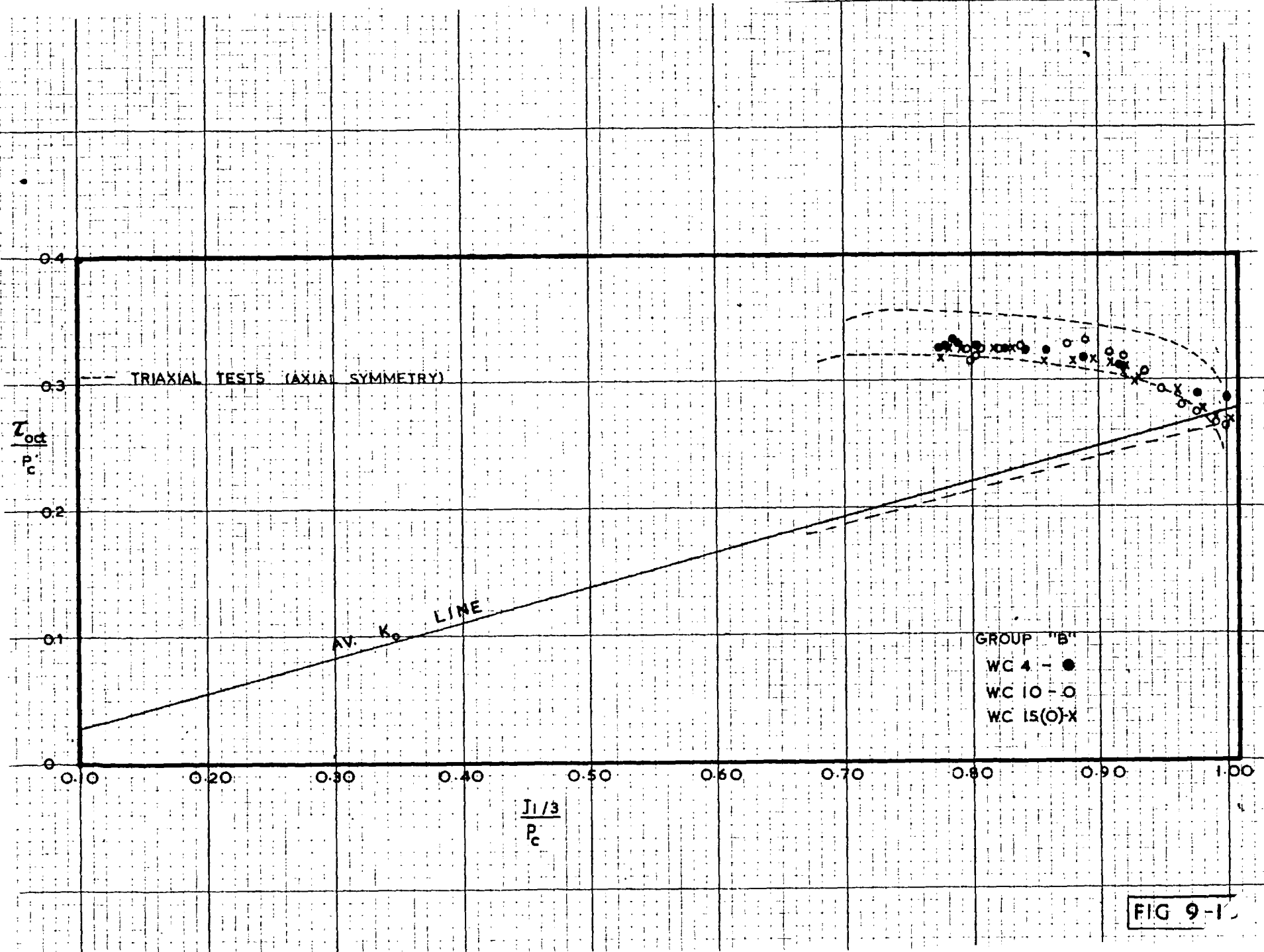


FIG 9-1

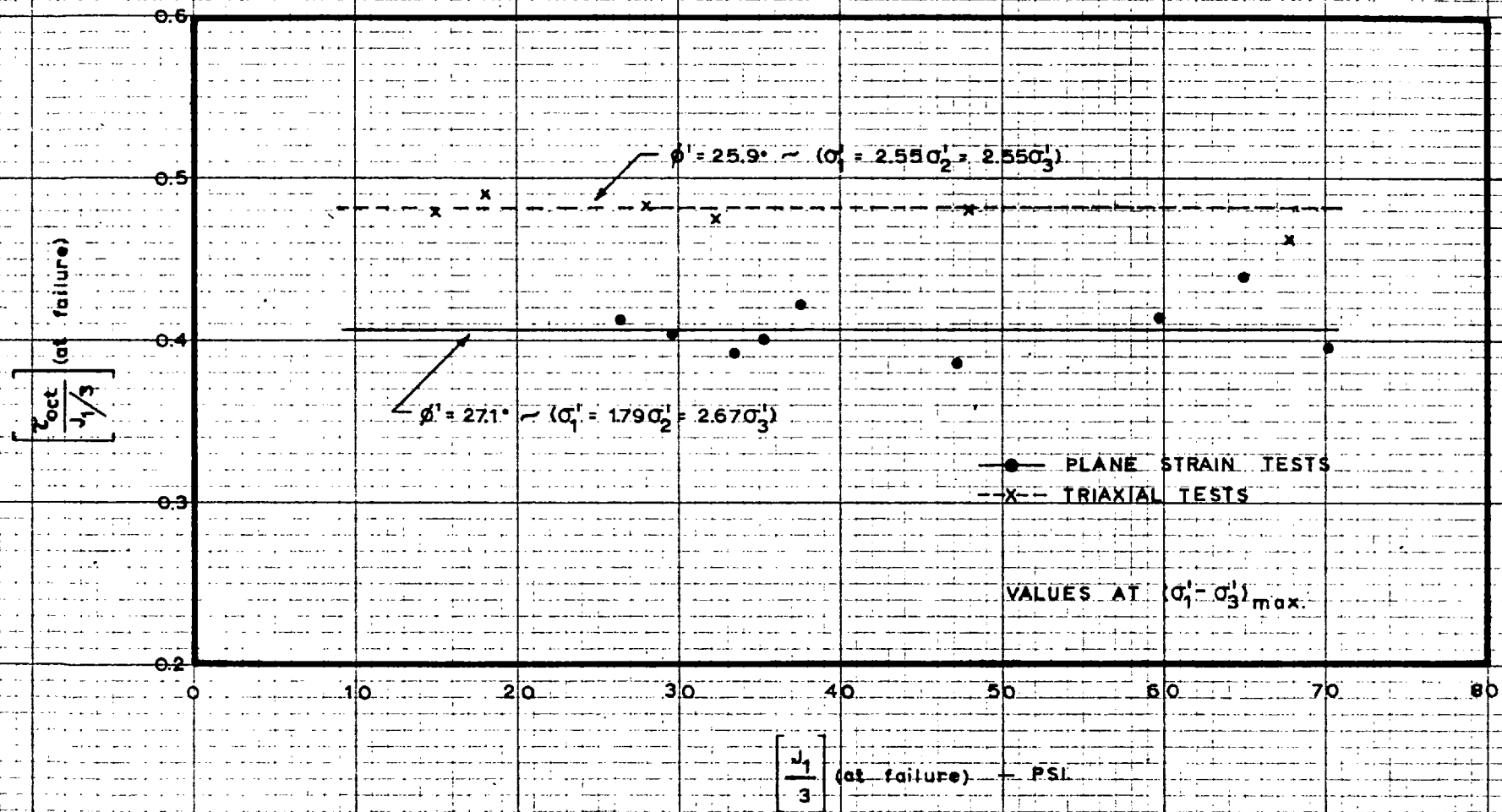


FIG 9-11

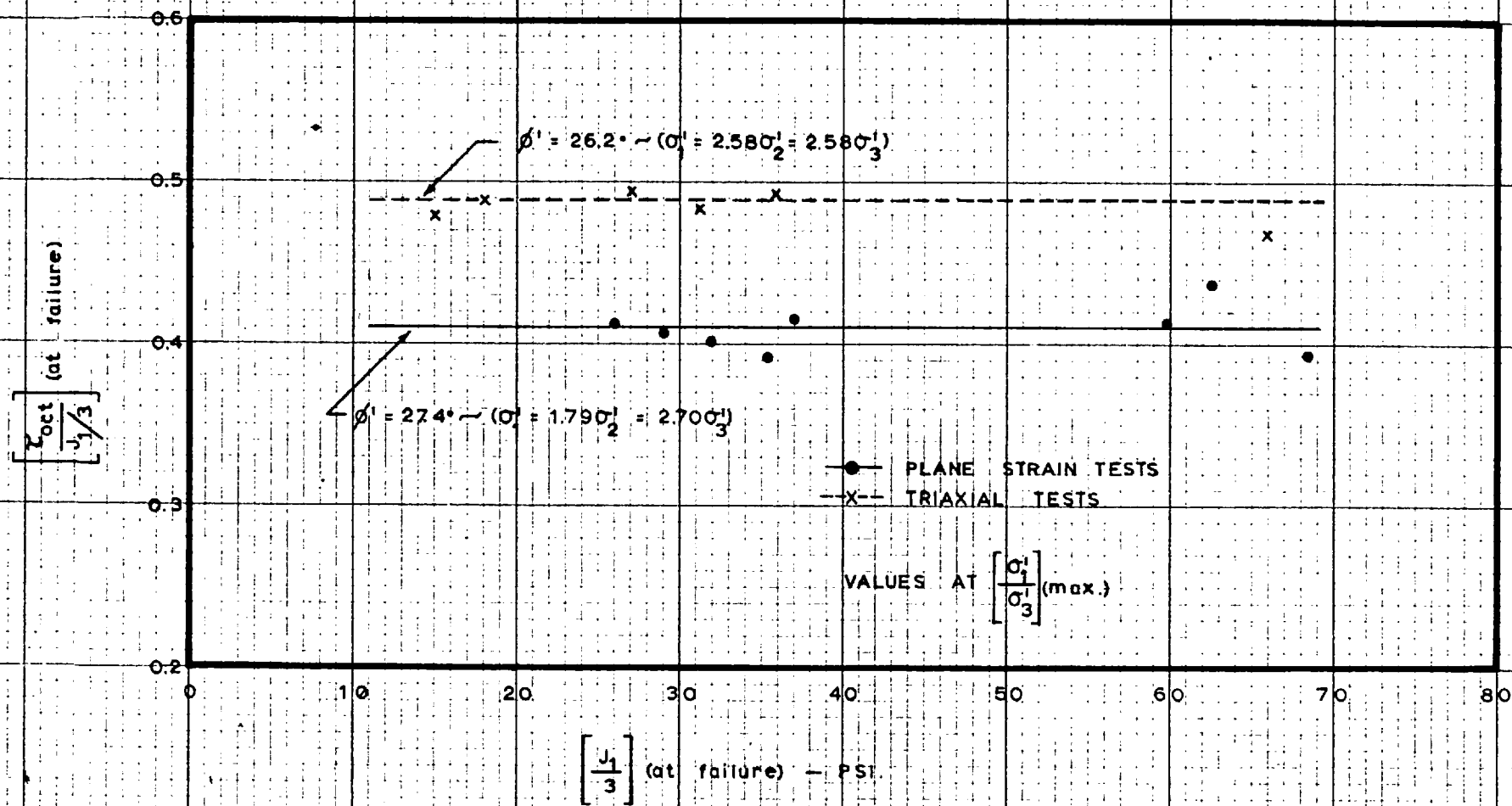


FIG 9-12

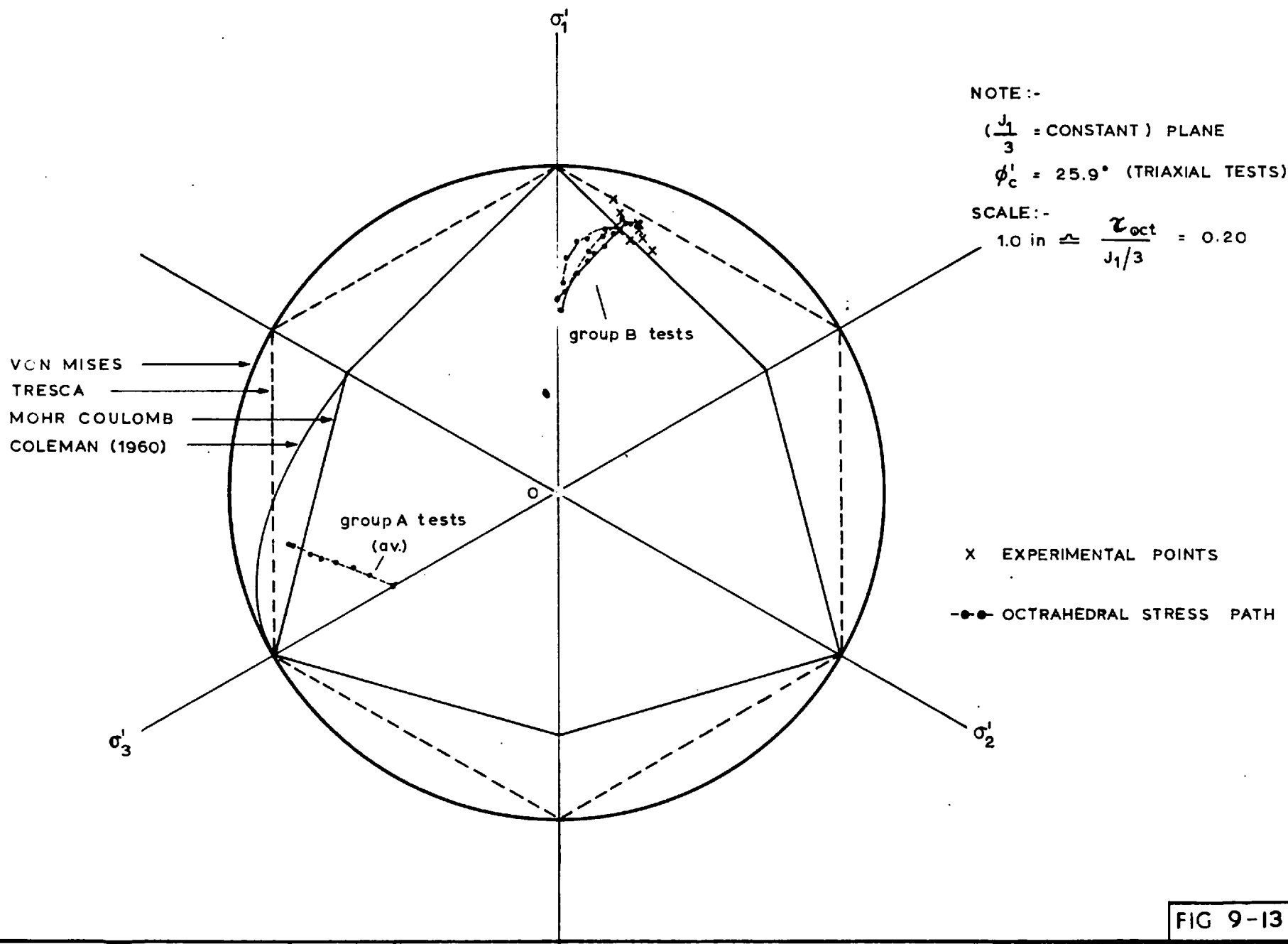


FIG 9-13

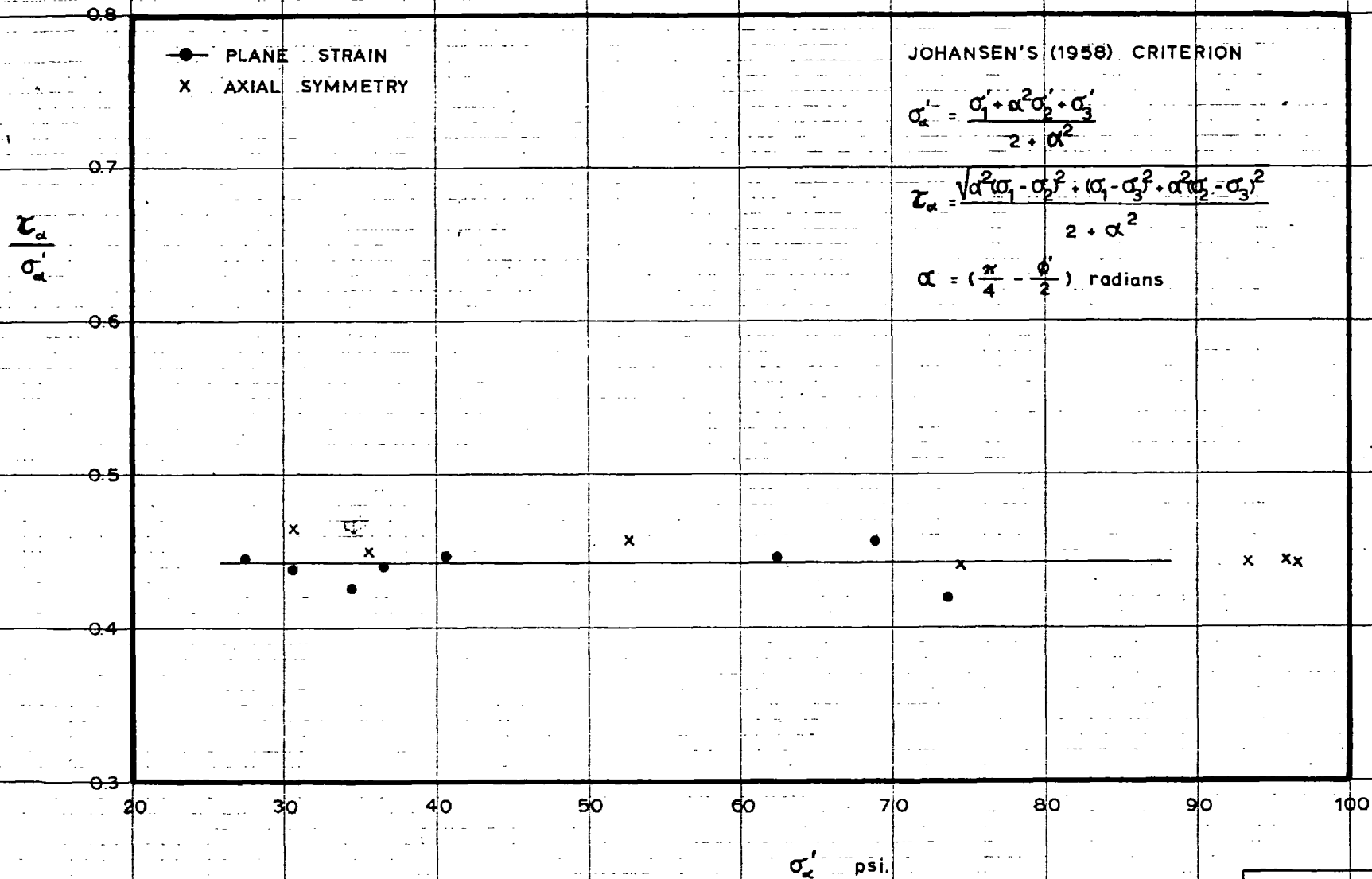


FIG 9-14

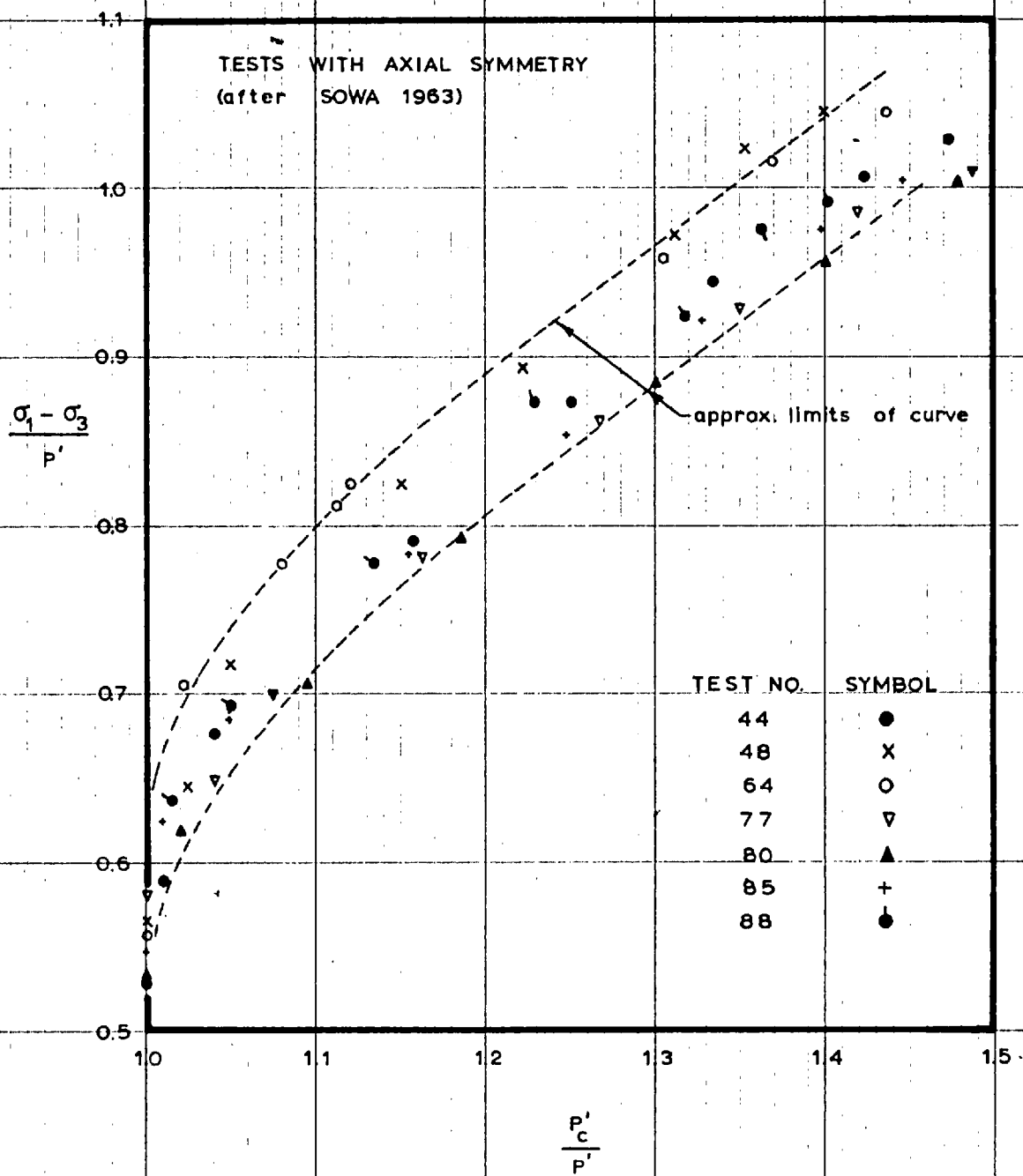


FIG 9-15

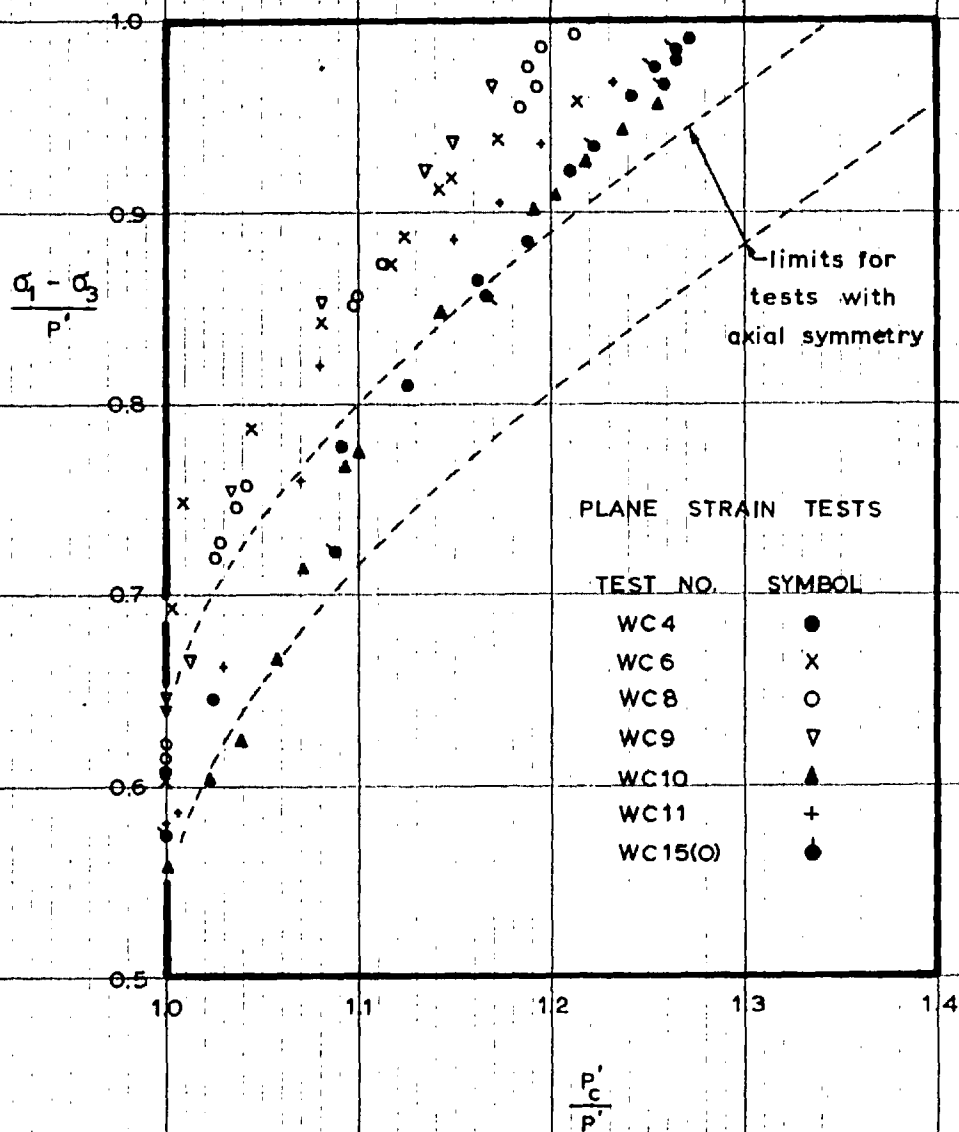


FIG 9-16

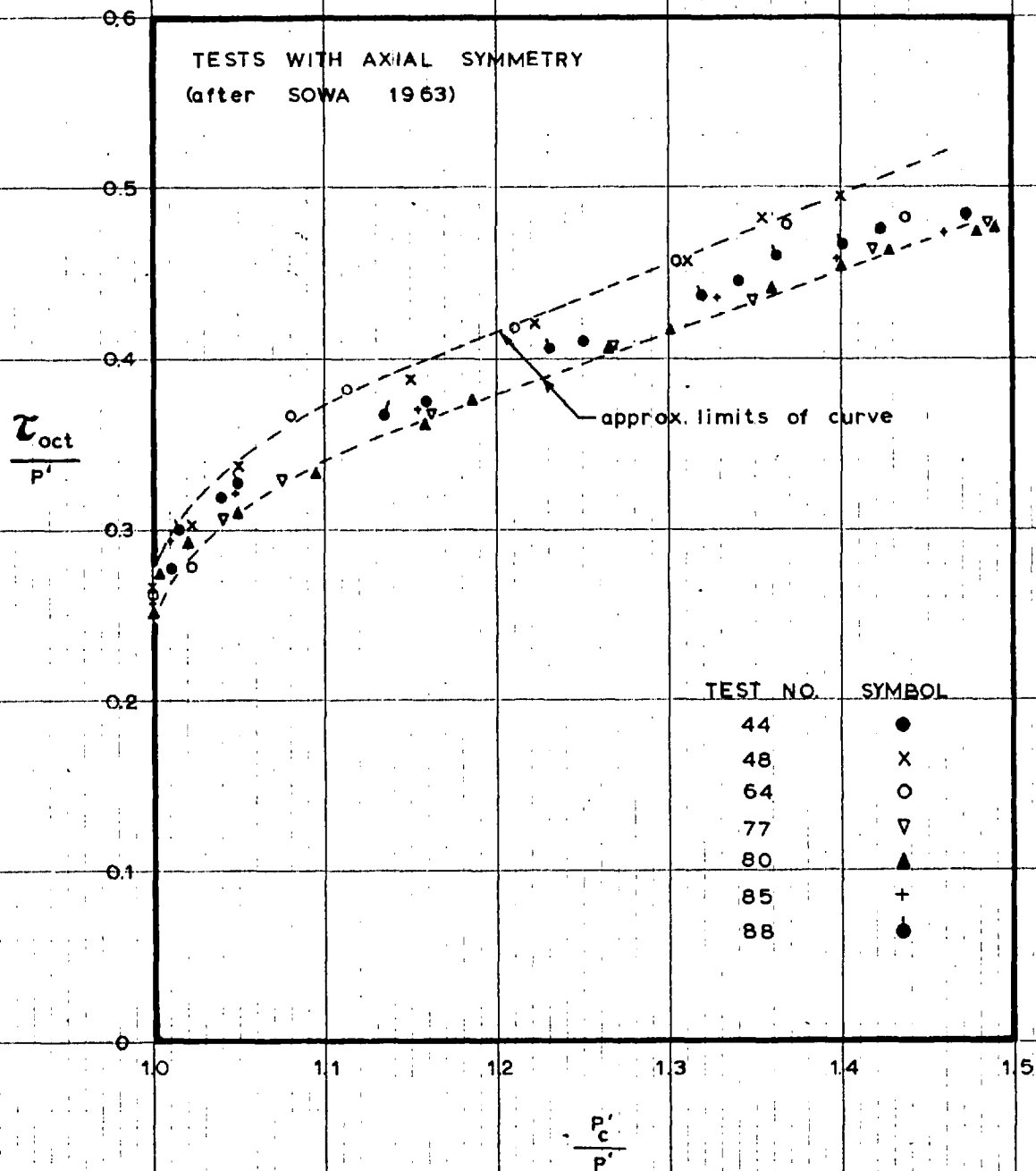


FIG 9-17

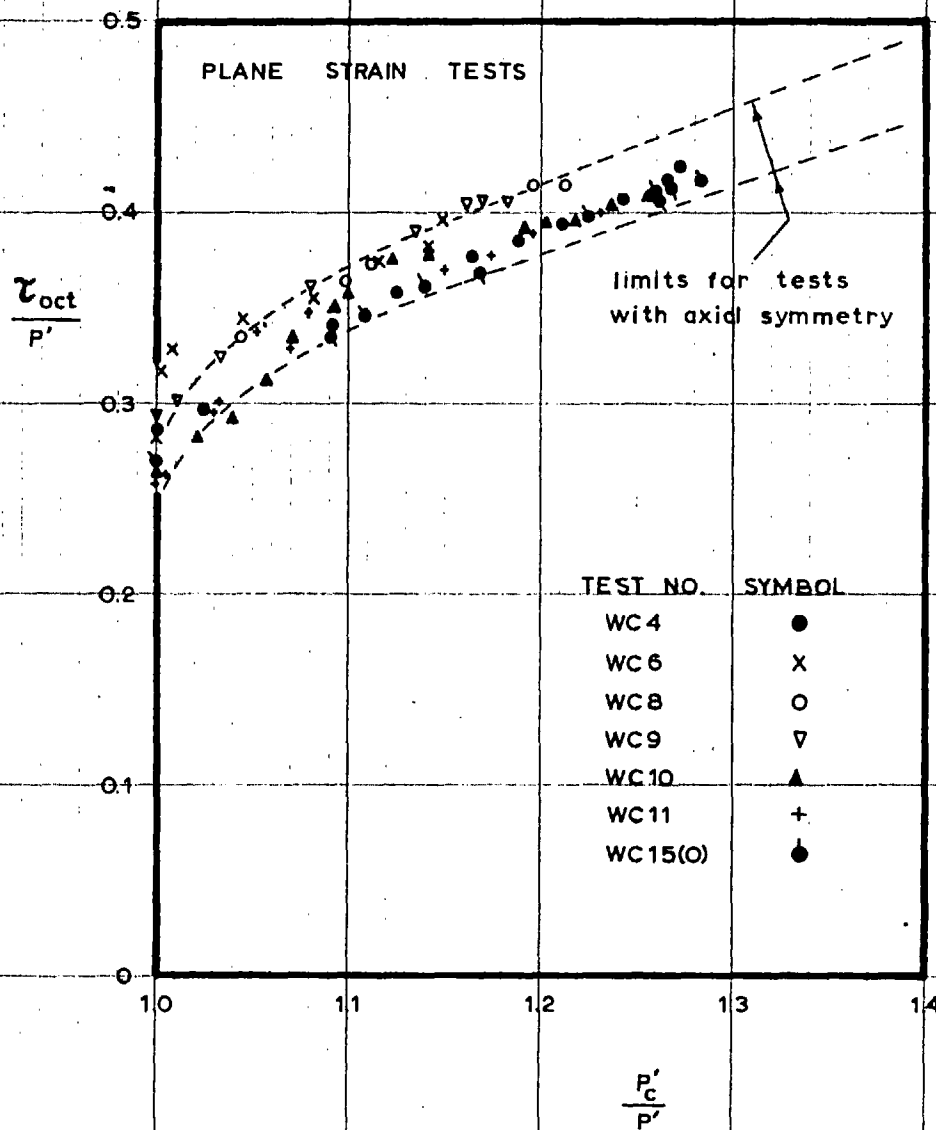


FIG 9-18

APPENDIX 1

PROCEDURE FOR MAKING RUBBER MEMBRANES

As with the previous experimenters on the plane strain apparatus, the author found it more convenient to produce rubber membranes in the laboratory from a latex solution than to obtain commercially made replicas. The procedure adopted, due to the requirements of the long-term tests, differed slightly from the previous procedures. To avoid excessive moisture migration through the membrane during the three to five week testing period, it was decided to make the membrane at least 0.02 in. thick. This was accomplished by dipping the former, which had been scrubbed clean in warm water and detergent, into the latex bath approximately eight times.

The dipping process consisted of slowly immersing the former into the bath so as not to create air bubbles, allowing about a minute to elapse, and then slowly withdrawing it until the top was just above the surface of the latex. Further time was permitted for the excess solution to run off the top of the former before it was removed from the bath. The process took about three minutes, after which the former was hung up to dry for a minimum of two hours. To obtain an even membrane thickness, the former was inverted for alternate dippings, and extra latex was applied to the corners between dippings.

When the final dipping was completed, the membrane was allowed

to "cure" on the former for 24 hours. Then French chalk was dusted on the surface, the upper and lower surfaces were cut away to fit the platens of the plane strain apparatus, and the membrane was carefully stripped from the former as more chalk was applied to its inner surface. After being checked for holes or imperfections, the membrane was put away to cure for another two days before use.

A membrane made in the above manner would have a useful life of three to four months if it were handled with care. For more detailed information about the equipment used in this process, the reader is referred to WOOD (1958).

APPENDIX 2

CORRECTIONS TO THE MEASURED STRENGTH

(a) Clay Tests:

Corrections to the measured deviator stress in the plane strain test are necessary to account for the restraint imposed by the rubber membrane, the filter paper strips and the σ_2 end effect. Due to the complexity of the factors involved, a suitable correction can be found only by empirical means. In standard triaxial specimens, HENKEL & GILBERT (1952) obtained an average correction to the deviator stress by comparing the results of unconfined tests with tests in which a rubber membrane and filter strips were used. However, time did not permit the corresponding series of plane strain tests, and thus the adopted average correction was determined from the approximate methods described below.

(1) LA ROCHELLE (1960) - Modified Method:

For 1½ in. diameter standard triaxial specimens, La Rochelle proposed the following deviator stress correction to account for the membrane and filter strip effects:

$$\sigma_r' = - \left[1.00 + 0.06 \varepsilon_1 \% + 0.06 + 0.04 \sigma_3' \right] \text{ psi.(A2-1a)}$$

where σ_r' = total deviator stress correction
 ε_1 = axial strain

1.00 = filter drain correction in psi.

$0.06\epsilon_1\%$ = membrane correction due to specimen bulging

0.06 = correction for rigidity of membrane

$0.04\sigma_3^0$ = friction component between membrane and specimen.

To account for the difference in the relative sizes of the plane strain and triaxial specimens, and for the difference in the thickness of the rubber membranes used in the two types of test, equation (A2-1a) is expressed for the plane strain test in the form:

$$\sigma_r^0 = - \left[0.375 + 0.0225\epsilon_1\% + 0.087 + 0.015\sigma_3^0 + 0.148\mu\sigma_2 \right] \text{ psi. (A2-1b)}$$

where the last term in the expression is the correction arising from the friction at the specimen ends caused by the σ_2 end clamp. (see WOOD, 1958).

(ii) BISHOP & HENKEL (1957) - Modified Method:

$$\sigma_r^0 = - \left[0.53 + \frac{(P \times M \times \epsilon_1(1 - \epsilon_1))}{A_0} + 0.148\sigma_2/\mu \right] \text{ psi. (A2-2)}$$

where σ_r^0 is the total correction to be applied to the deviator stress;

0.53 = $1.4 \times 1.5/4$ is the filter drain correction and is independent of % strain and the effective cell pressure;

the second term is the correction for the stiffness of the membrane (P is the perimeter of the specimen, M is the

extension modulus of the rubber membrane in lb./in,
 A_0 is the initial cross-sectional area of the specimen
and ϵ_1 is the % strain);

the last term is the σ_2 effect as above.

Both Wood and Cornforth used a form of equation A2-2 without the filter strip correction. In the hypotheses of Bishop & Henkel and La Rochelle, however, no consideration of K_0 consolidation is given; hence it is uncertain whether the % strain should be taken from the beginning of the test or from the start of the shear stage. In the graph of Fig. A2-1, it has been assumed that the rubber membrane and filter paper restraint is effective from the beginning of the consolidation stage and is fully operative when the shear stage starts. It has been further assumed that the corrections are proportional to % strain until buckling occurs and then the strain dependent correction remains constant. Plotted in Fig. A2-1 against the % strain during the shear stage are the above equations and the results of rubber sample tested in the plane strain apparatus with and without filter strips and a membrane. The curve for the rubber sample goes through zero as the sample was not K_0 consolidated before stressing. The adopted correction to the deviator stress in the shear stage of all clay tests was 1.0 psi. throughout.

The correction to the deviator stress due to the σ_2 end effect which was described by Wood for the undrained test, can be modified to conform to the requirements of the drained test. A brief review of

the undrained case will be given first.

The additional amount of deviator stress required to overcome the friction between the end plates and the greased membrane at the specimen ends can be calculated from the work necessary to move the specimen along the end plates. By considering a point with the coordinates x, y , on the end of the specimen as in Fig. A2-2, the x component of movement will be $\epsilon_x \cdot x$, where ϵ_x is the lateral strain, and the y component of movement will be $\epsilon_y \cdot y$, where ϵ_y is the axial strain.

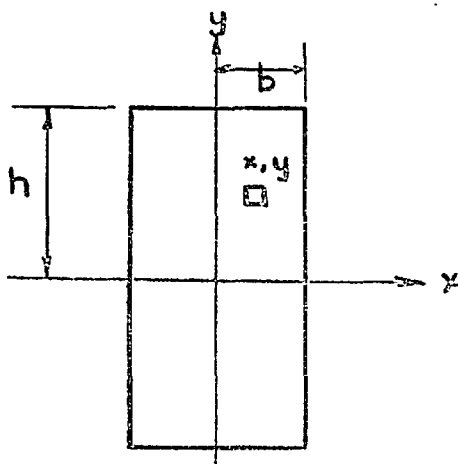


Fig. A2-2. Element on the end of the specimen.

For the undrained test, $\epsilon_y = -\epsilon_x$ and the resultant displacement of the point is:

$$\epsilon_y = \sqrt{x^2 + y^2}$$

The work done by the friction force acting on the area $dx \cdot dy$ will be

$$dW_c = \epsilon_y \cdot \sqrt{x^2 + y^2} \cdot \mu \cdot C \cdot dx \cdot dy$$

where μ is the coefficient of friction and σ is the normal force. By integrating twice the work done on one-quarter of the end of the specimen becomes

$$W_e = \frac{h^3}{6} \left[\sinh^{-1} \frac{b}{h} + \frac{b^3}{h^3} \cdot \sinh^{-1} \frac{h}{b} + 2 \frac{b}{h} \cdot \sqrt{1 + \frac{b^2}{h^2}} \right] \epsilon_y \cdot \sigma \cdot \mu$$

Substituting the numerical values of $b = 4$ units and $h = 8$ units, the expression reduces to

$$W = 151.3 \epsilon_y \cdot \sigma \cdot \mu$$

Now if the input of work by the deviator stress to overcome this σ_2 effect is W_t , then

$$W_t = L \times 2b \times 2h \times \epsilon_y \times (\sigma_1 - \sigma_3)_r$$

where L is the specimen length, b is one-half the width, h is one-half the height and $(\sigma_1 - \sigma_3)_r$ is the extra deviator stress to overcome the friction.

Since $W_e = W_t$, then

$$(\sigma_1 - \sigma_3)_r = 0.148 \sigma \mu \quad \dots\dots\dots (A2-3)$$

In the drained test, $\epsilon_x + \epsilon_y = \Delta V/V_e$, and by a similar analysis the extra deviator stress required is determined, as:

$$W_o = \frac{8 \sigma_m}{\epsilon_x \cdot \epsilon_y} \int_0^{h \epsilon_y} \int_0^{b \epsilon_x} \sqrt{x^2 + y^2} dy dx$$

or

$$W_o = \frac{8 \sigma_m}{\epsilon_x \cdot \epsilon_y} \left[\frac{(h \epsilon_y)^3}{6} \left(\frac{2 b \epsilon_x}{h \epsilon_y} \sqrt{1 + \left(\frac{b \epsilon_x}{h \epsilon_y} \right)^2} + \left(\frac{b \epsilon_x}{h \epsilon_y} \right) \sinh^{-1} \frac{h \epsilon_y}{b \epsilon_x} + \sinh^{-1} \frac{b \epsilon_x}{h \epsilon_y} \right) \right]$$

$$\therefore (\sigma_1 - \sigma_3)_r = \frac{W_o}{\epsilon_y \times L \times 2b \times 2h} = \frac{W_o}{128 \epsilon_y} \dots\dots\dots(A2-4)$$

(b) SAND TESTS:

The corrections to the deviator stress for plane strain tests on sands consist of the rubber membrane correction and the σ_2 end effect as described above. Fig. A2-3 shows the typical corrections for loose, medium and dense sand.

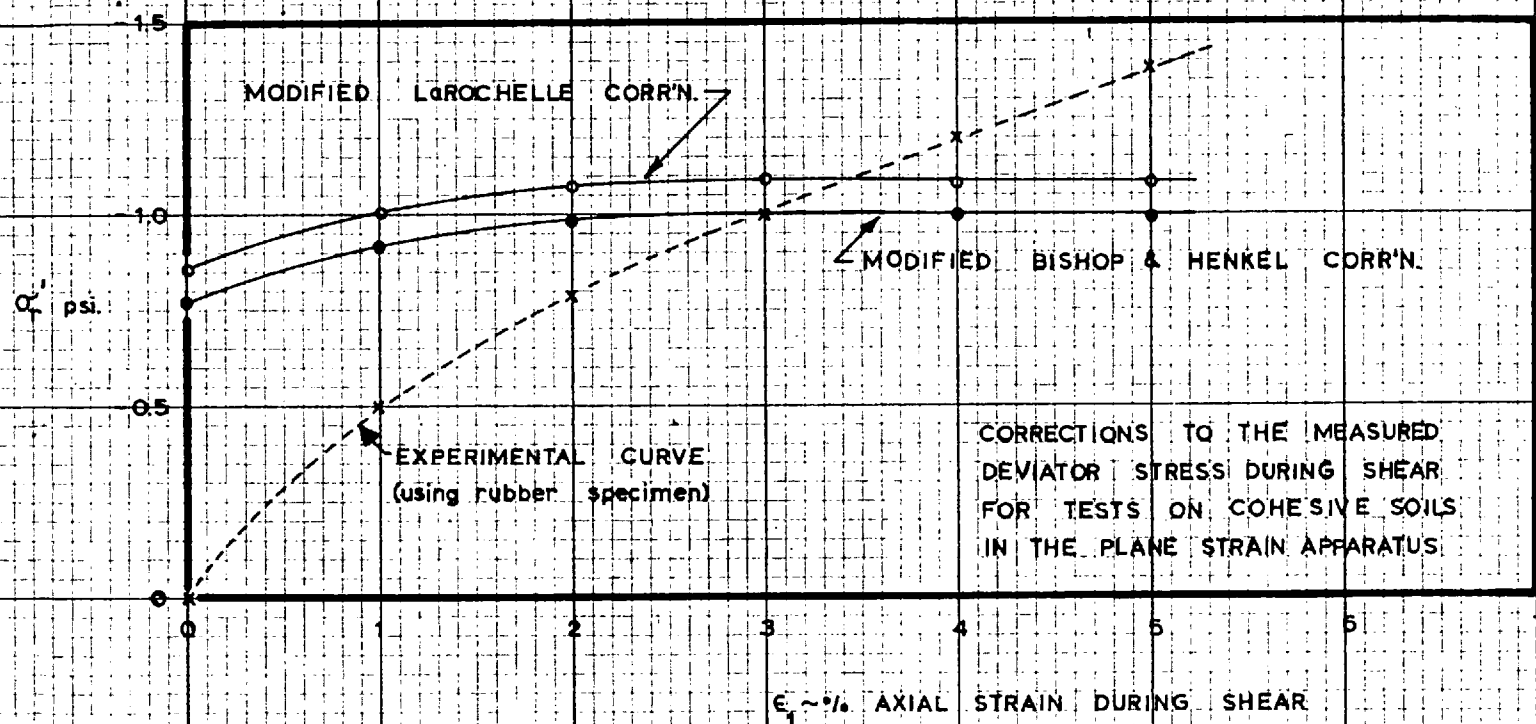
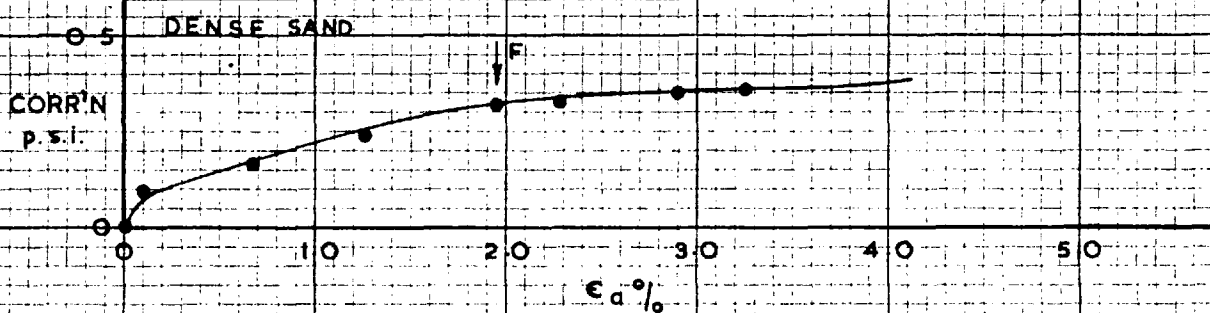
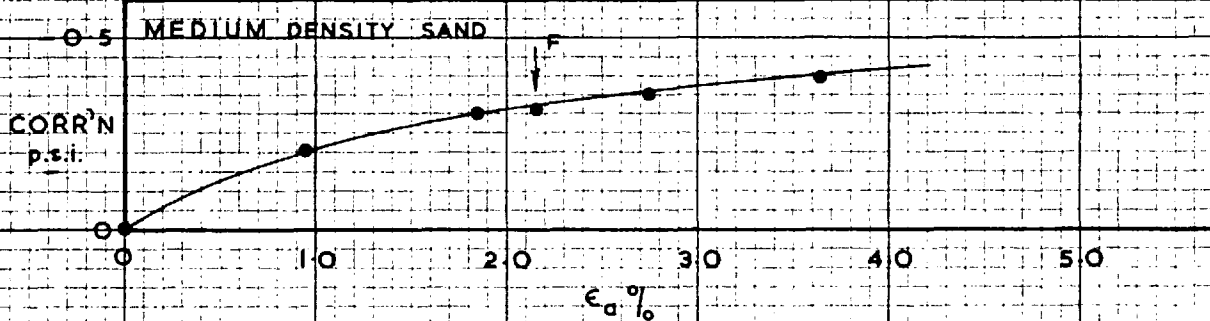
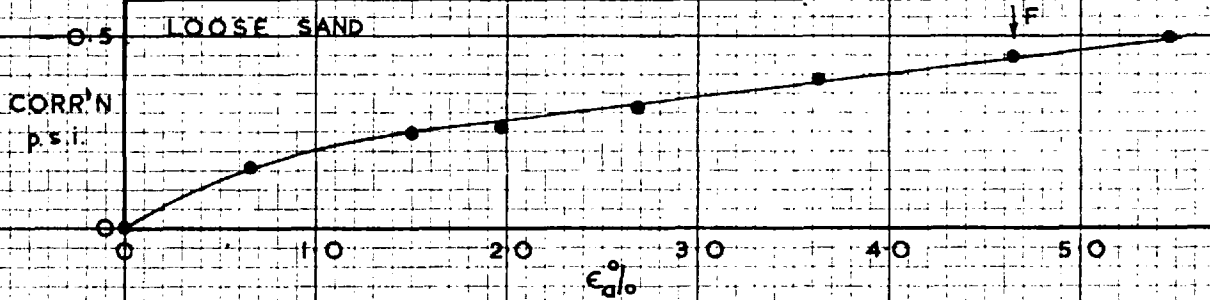


FIG A2-11



CORRECTIONS TO THE MEASURED DEVIATOR STRESS
to account for membrane & σ_2 end (friction) effects

FIG No A2-3

APPENDIX 3

GRAPHICAL PRESENTATION OF THE RESULTS OF THE PLANE STRAIN
UNDRAINED TESTS ON WEALD CLAY

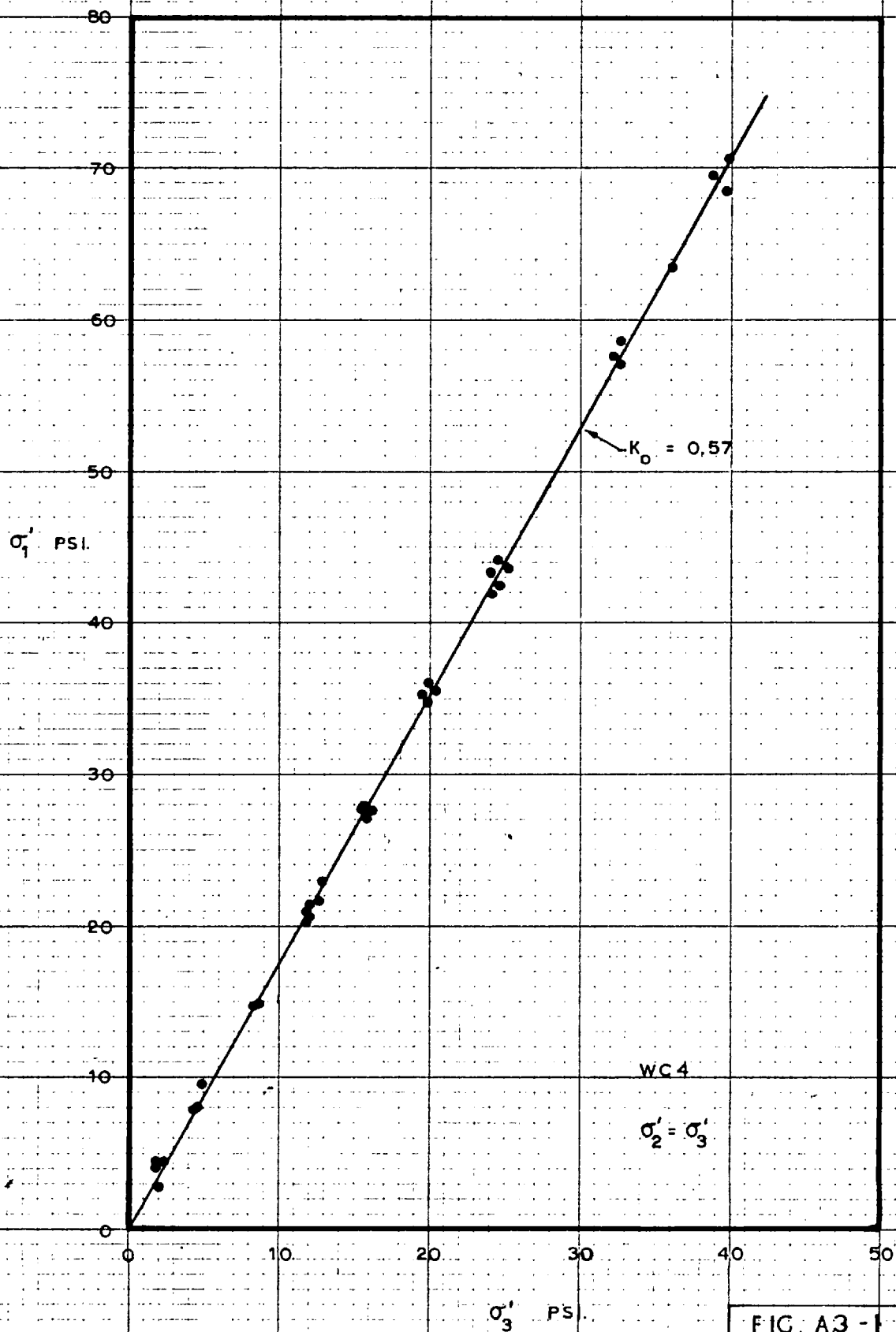


FIG. A3 - 1

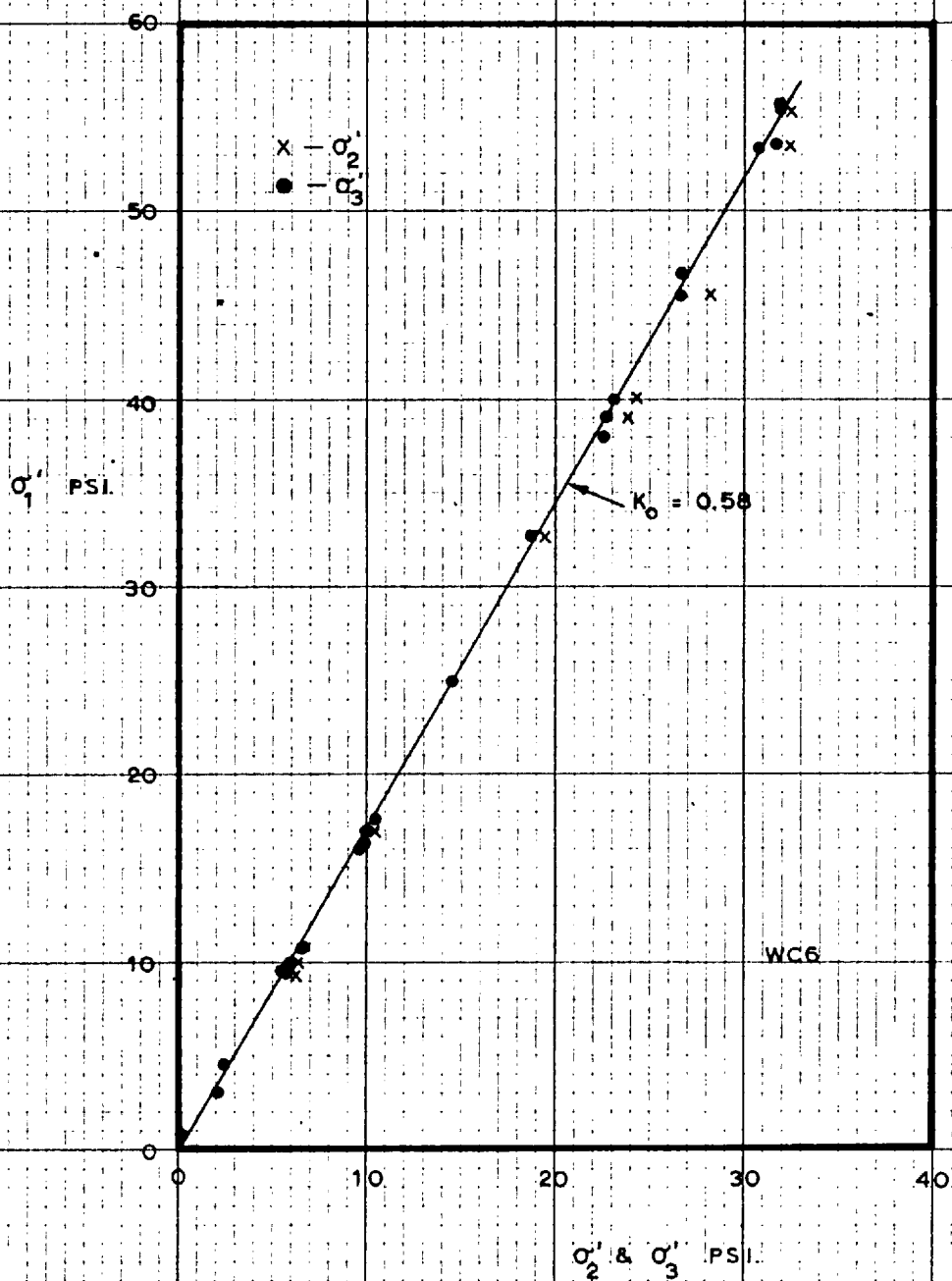


FIG A3-2

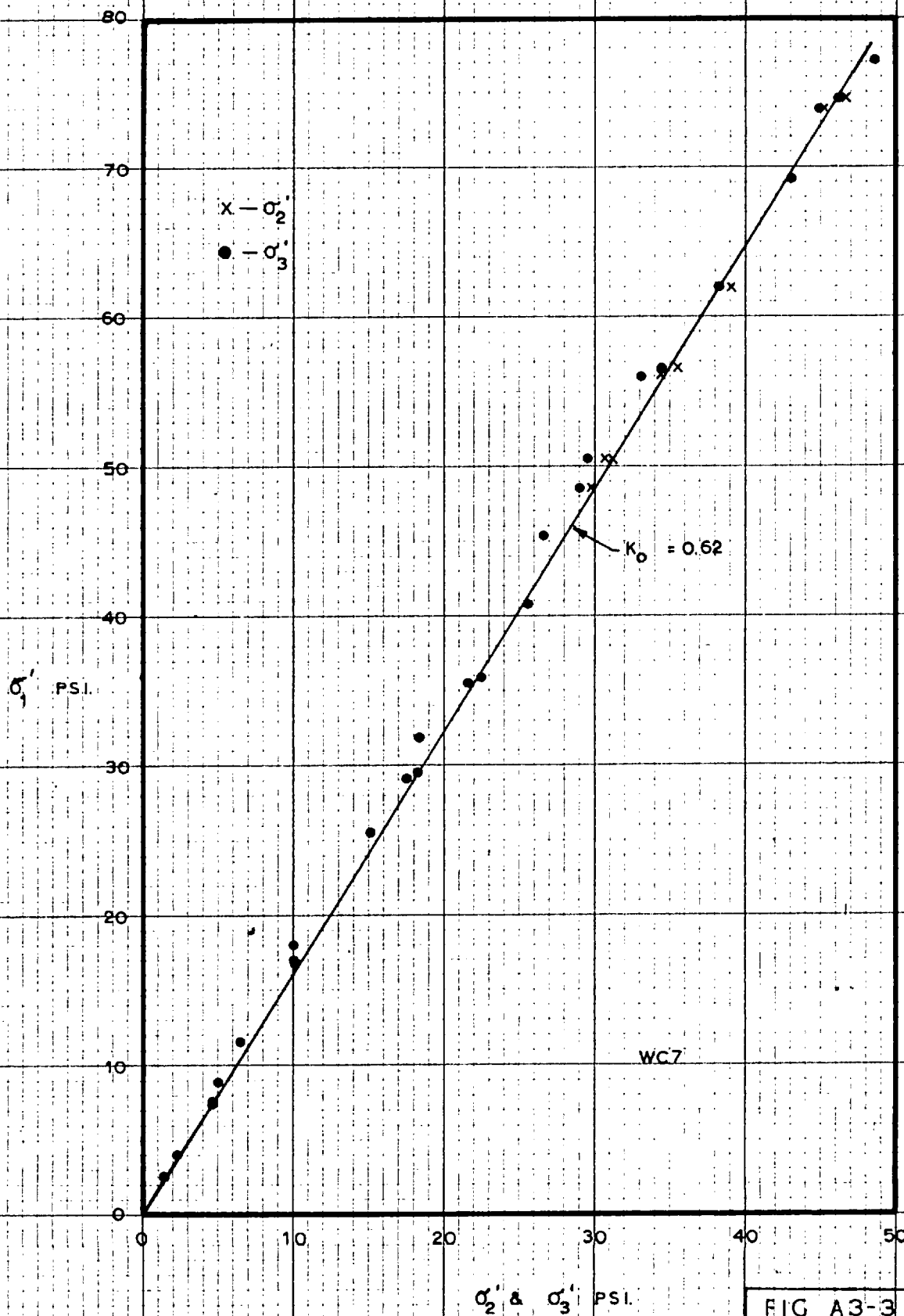


FIG A3-3

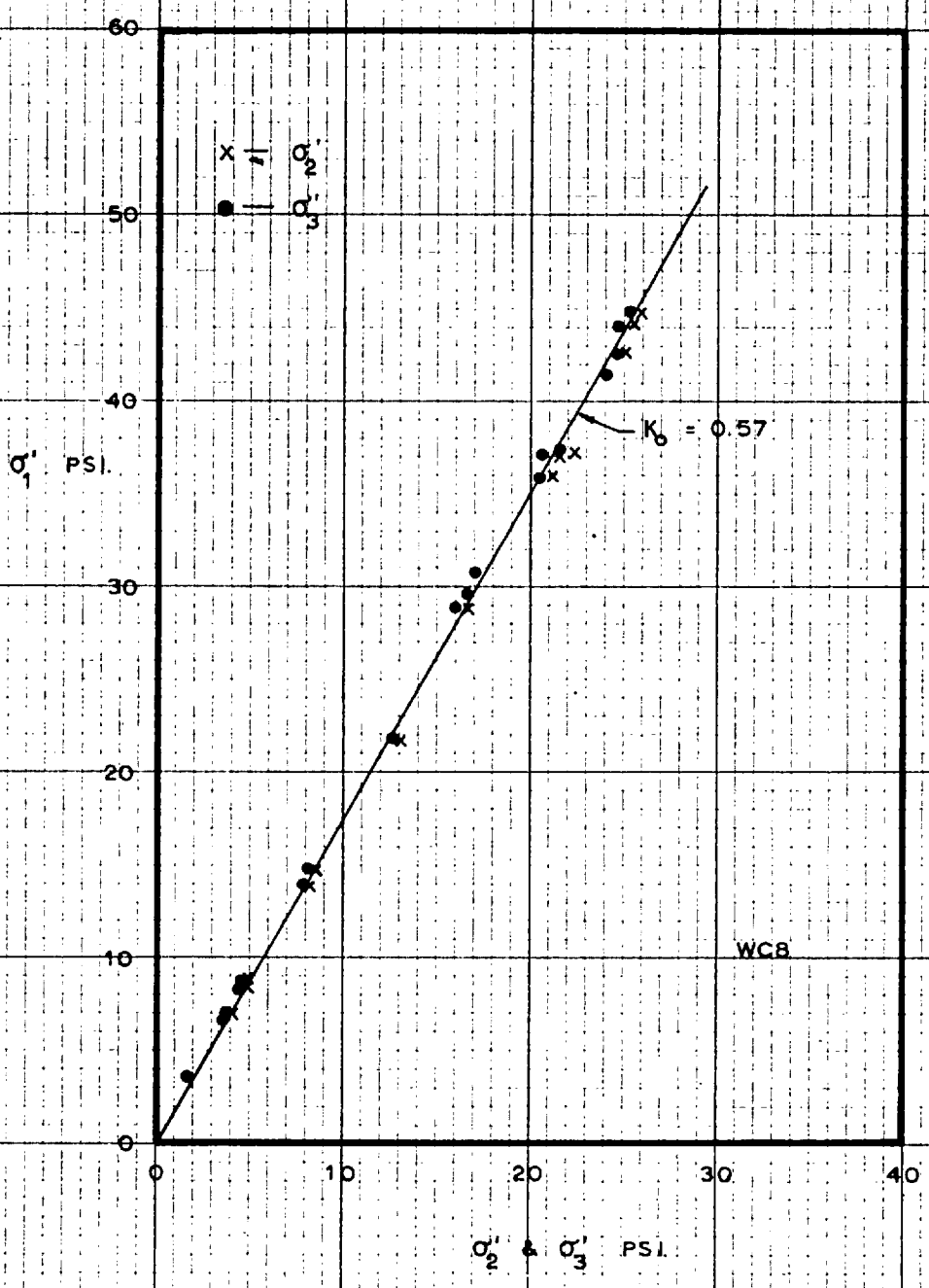


FIG. A3-4

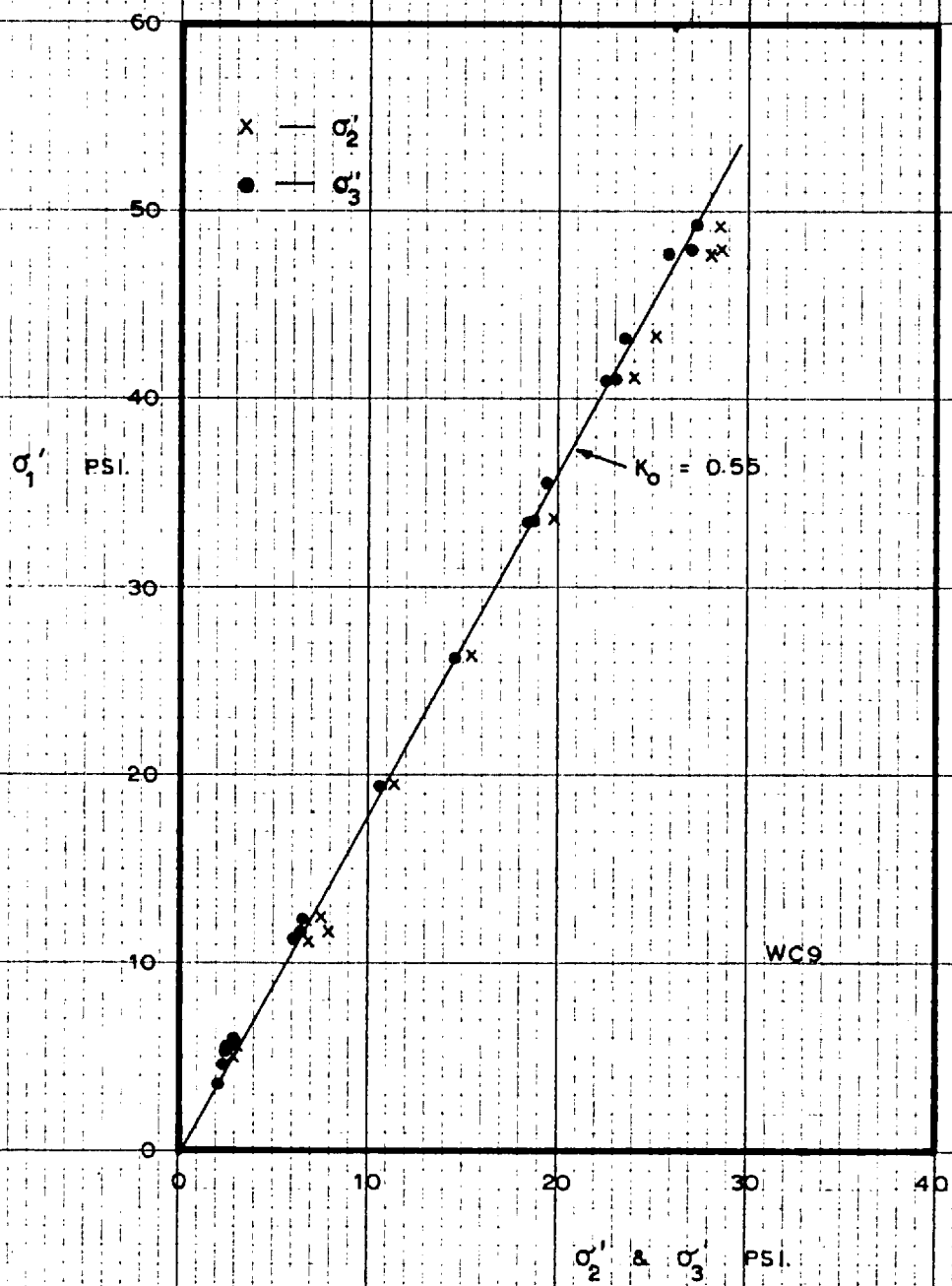


FIG A 3-5

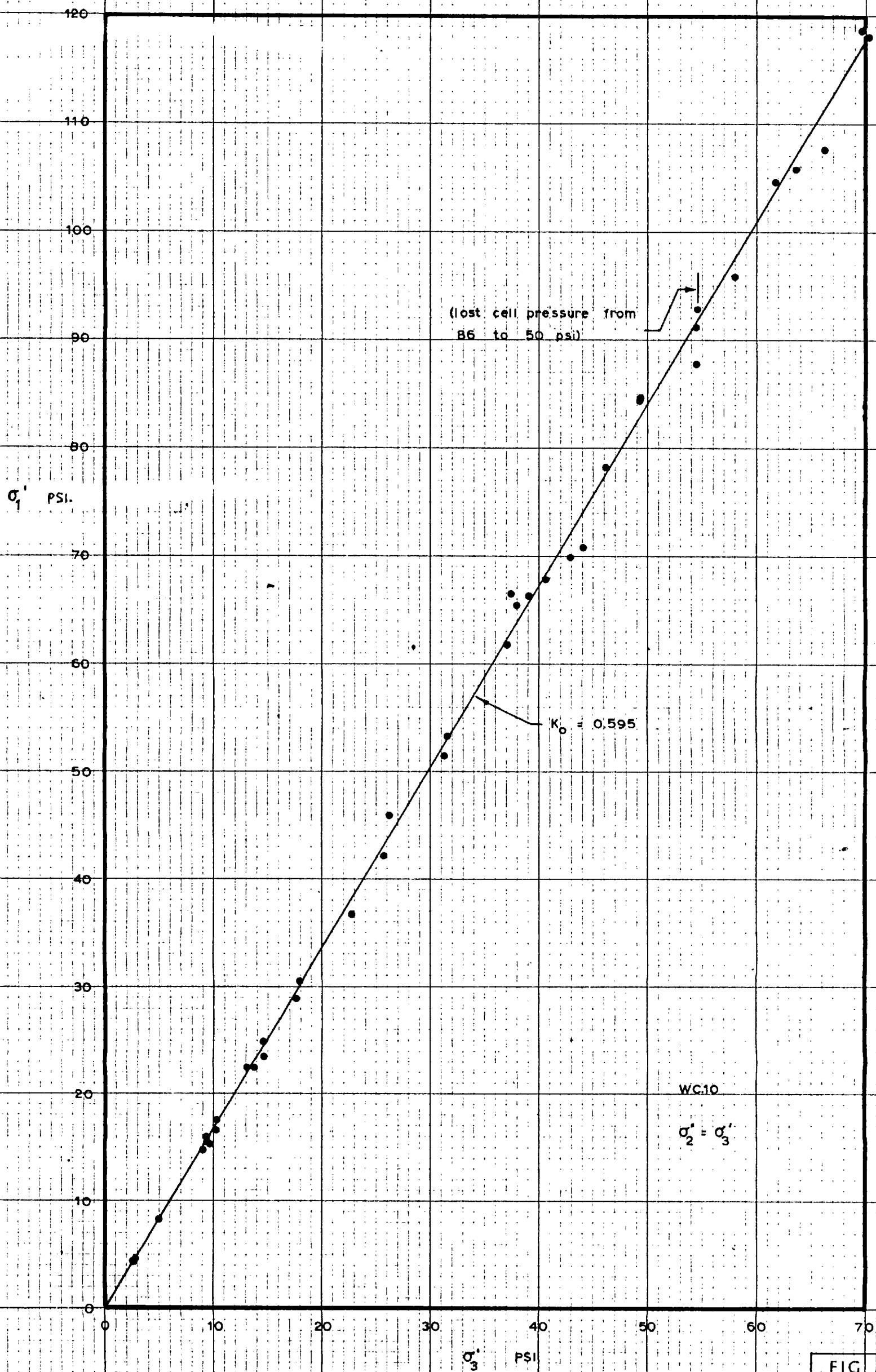


FIG A3-6

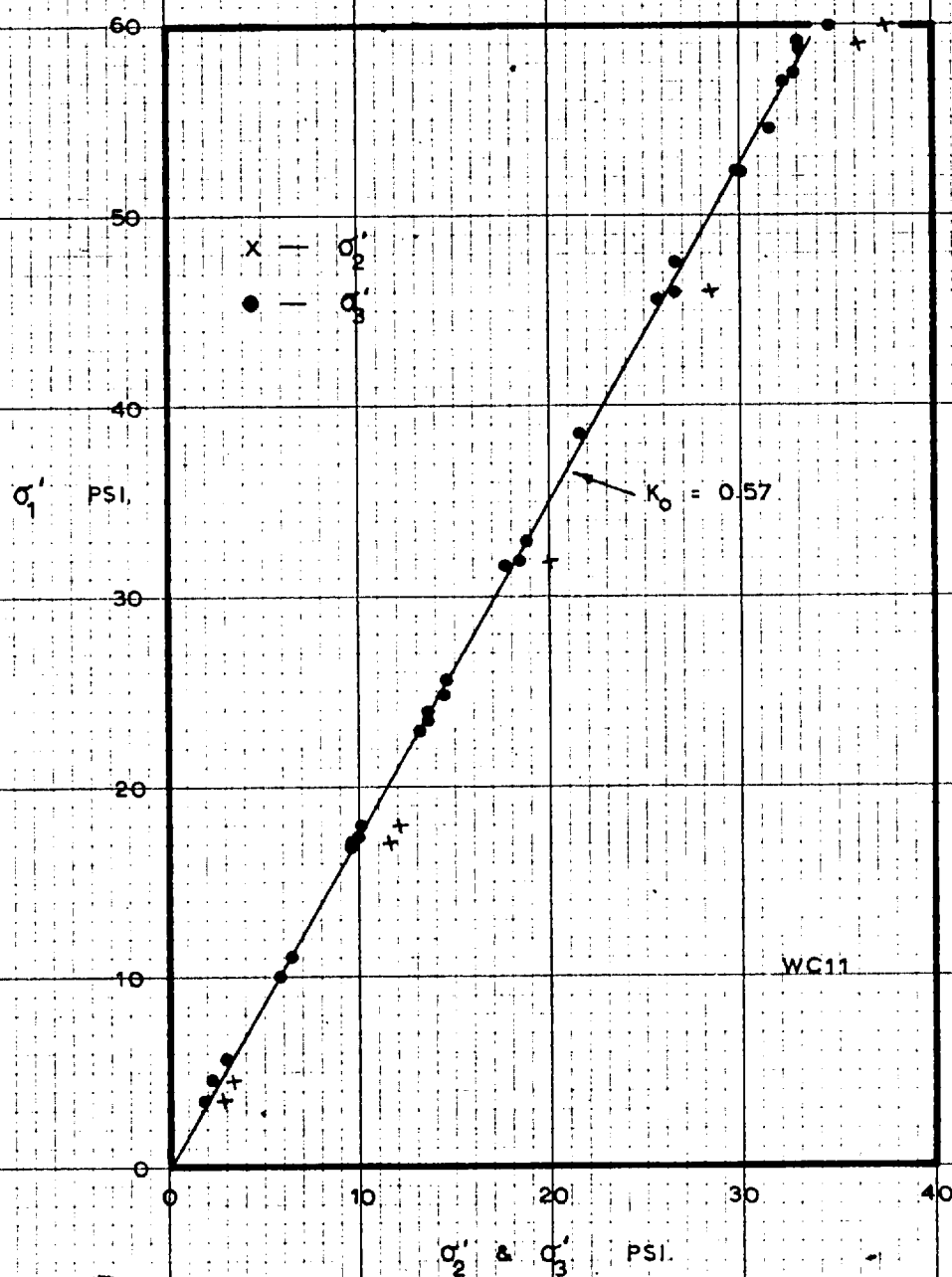


FIG A3-7

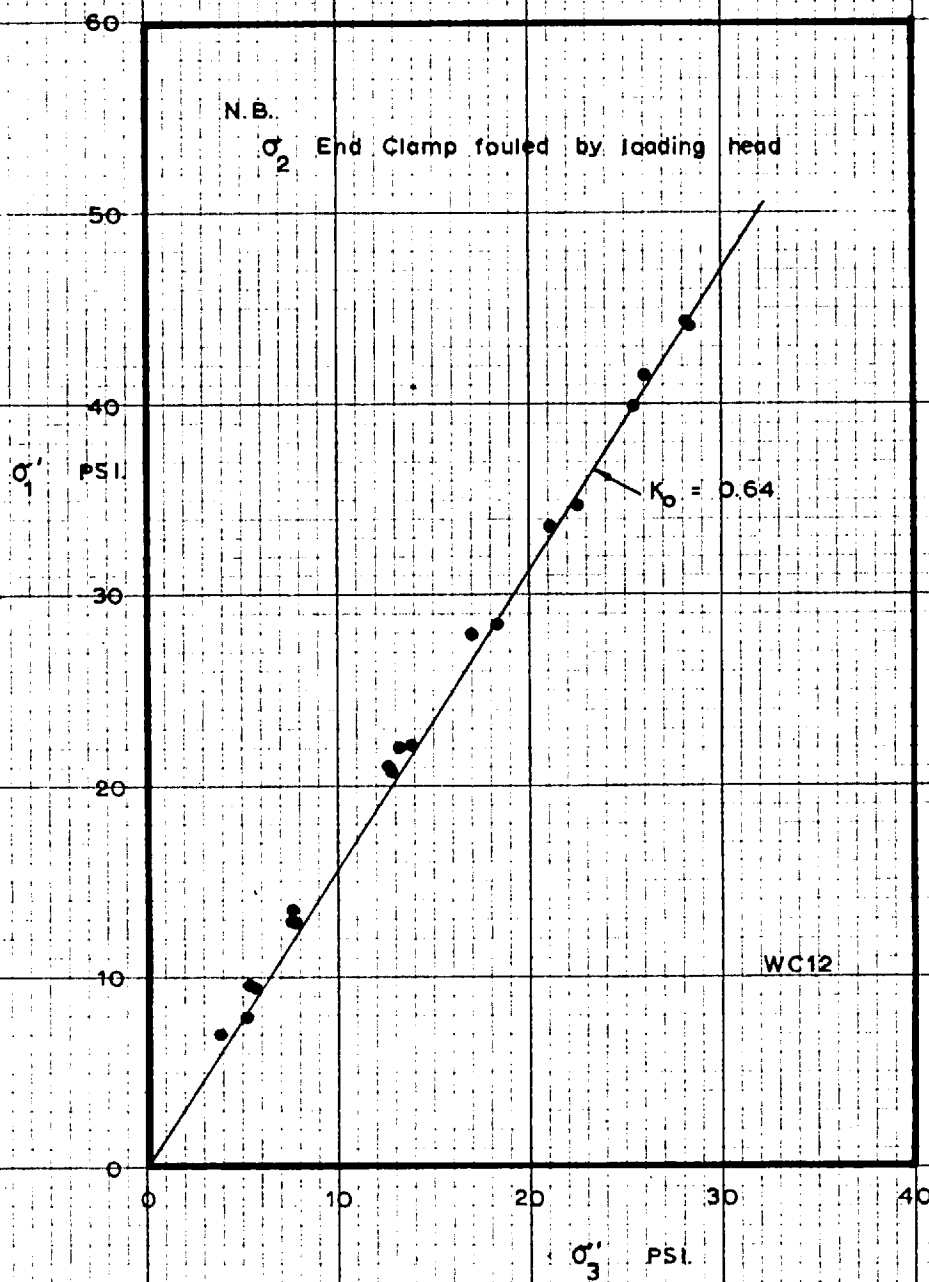


FIG A3-8

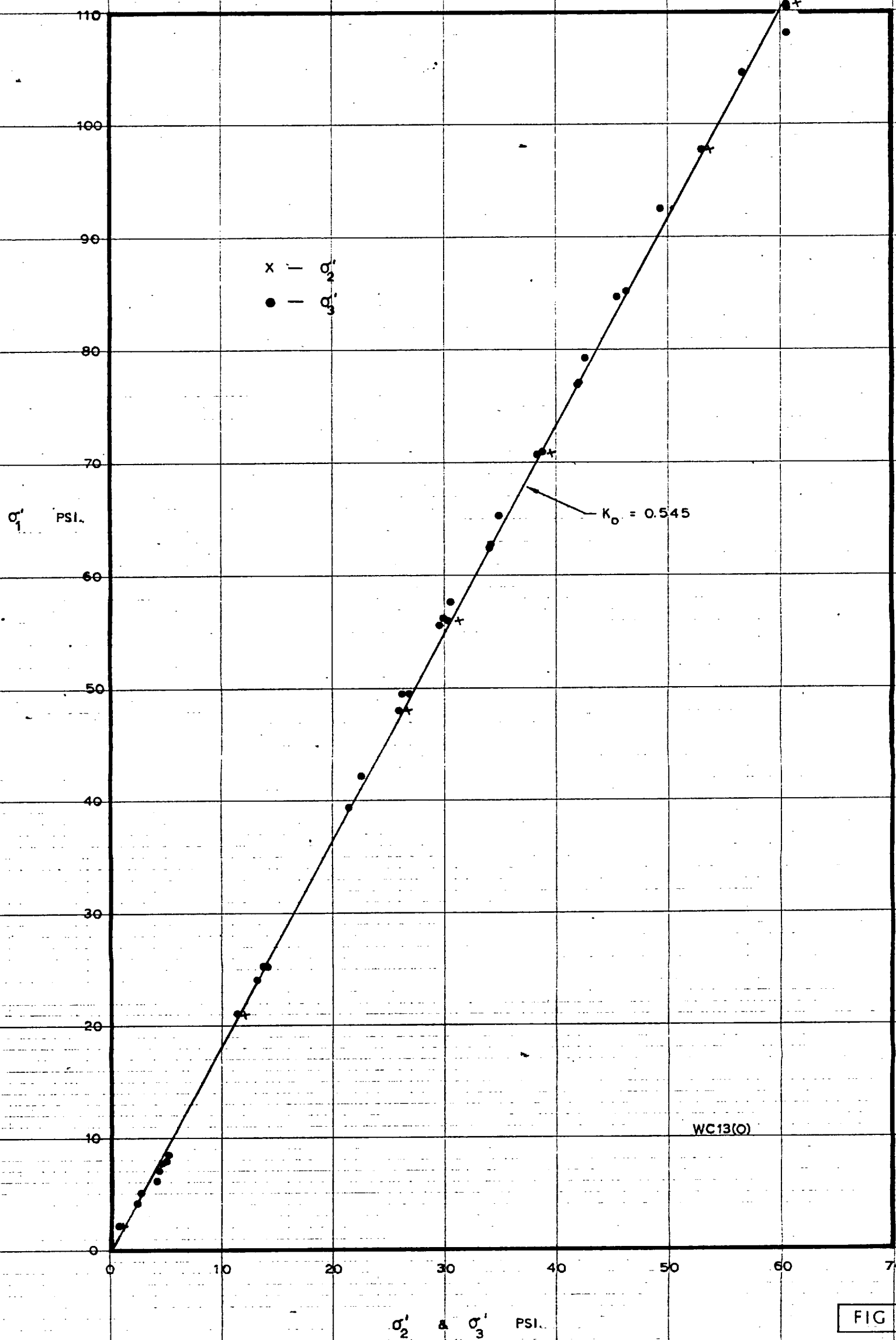


FIG A3-9

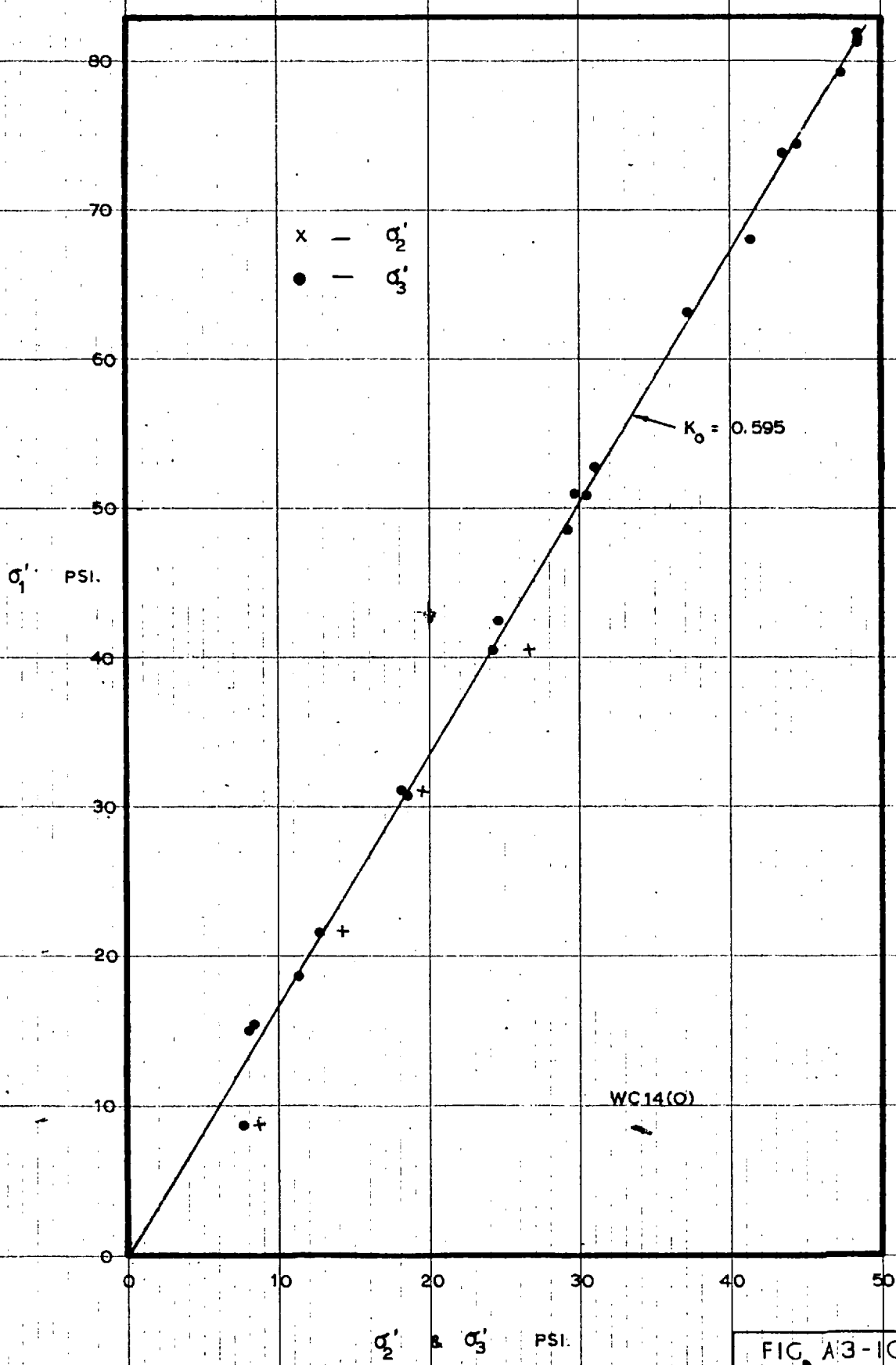


FIG. A3-10

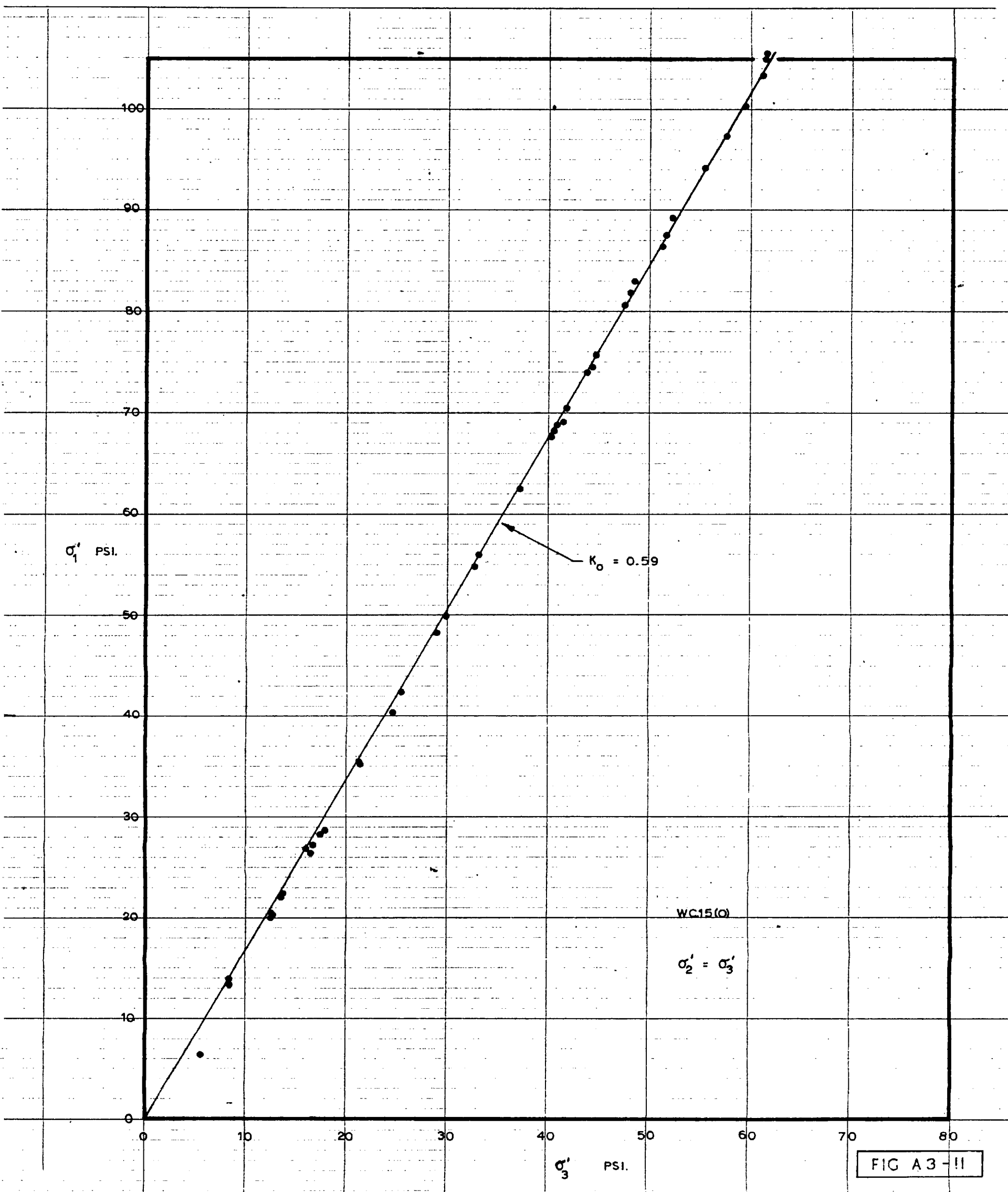


FIG A3-II

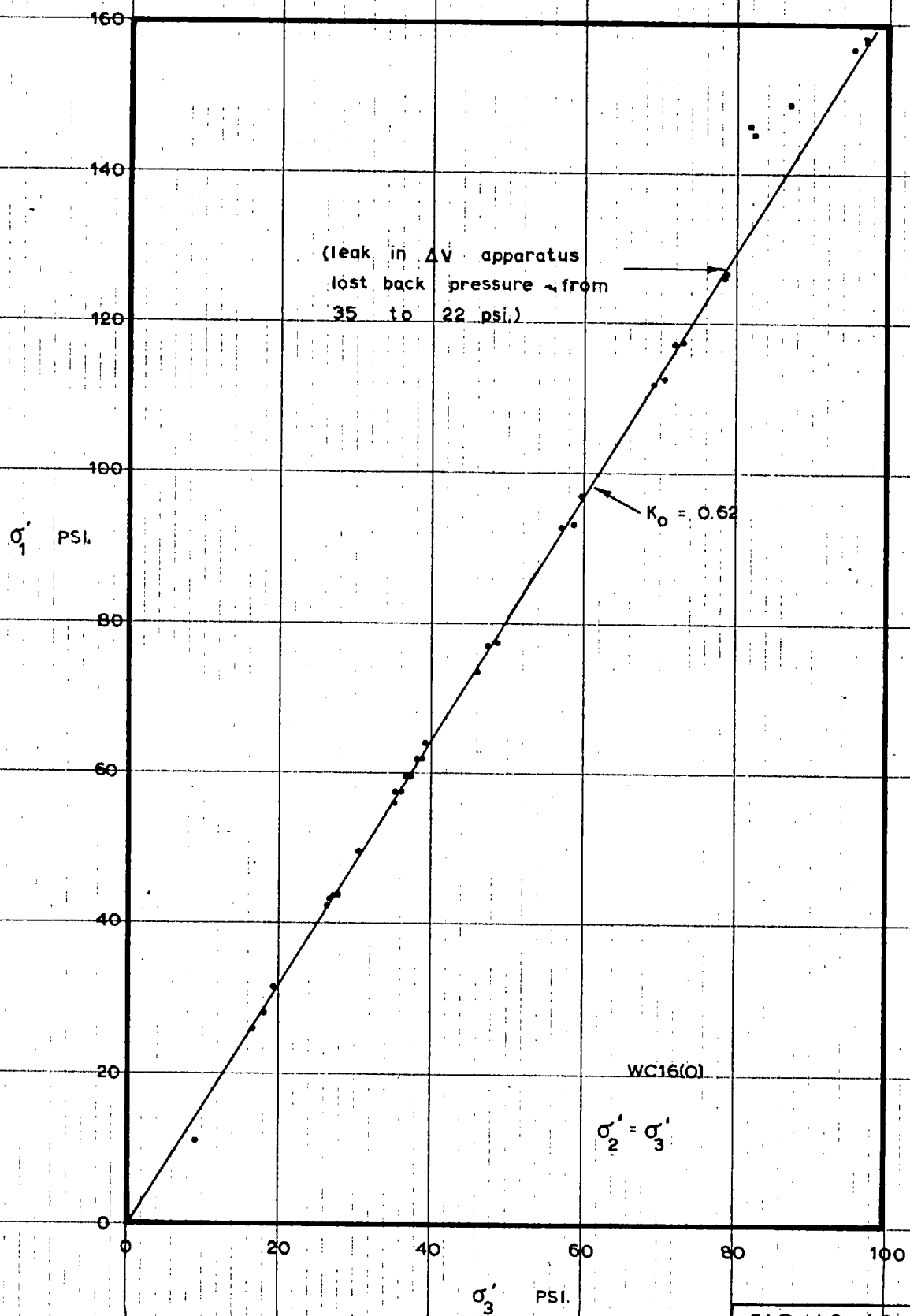


FIG A3-12

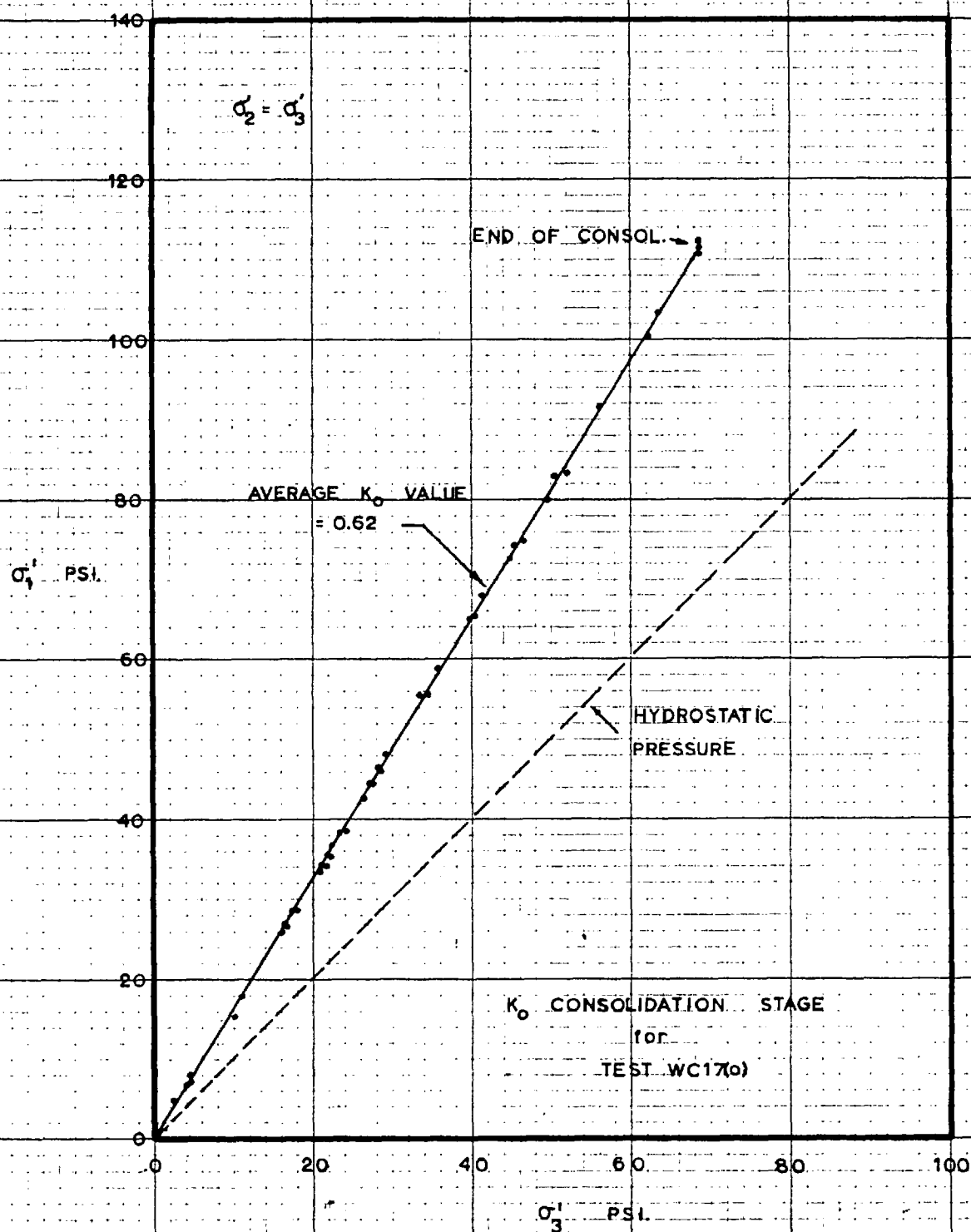
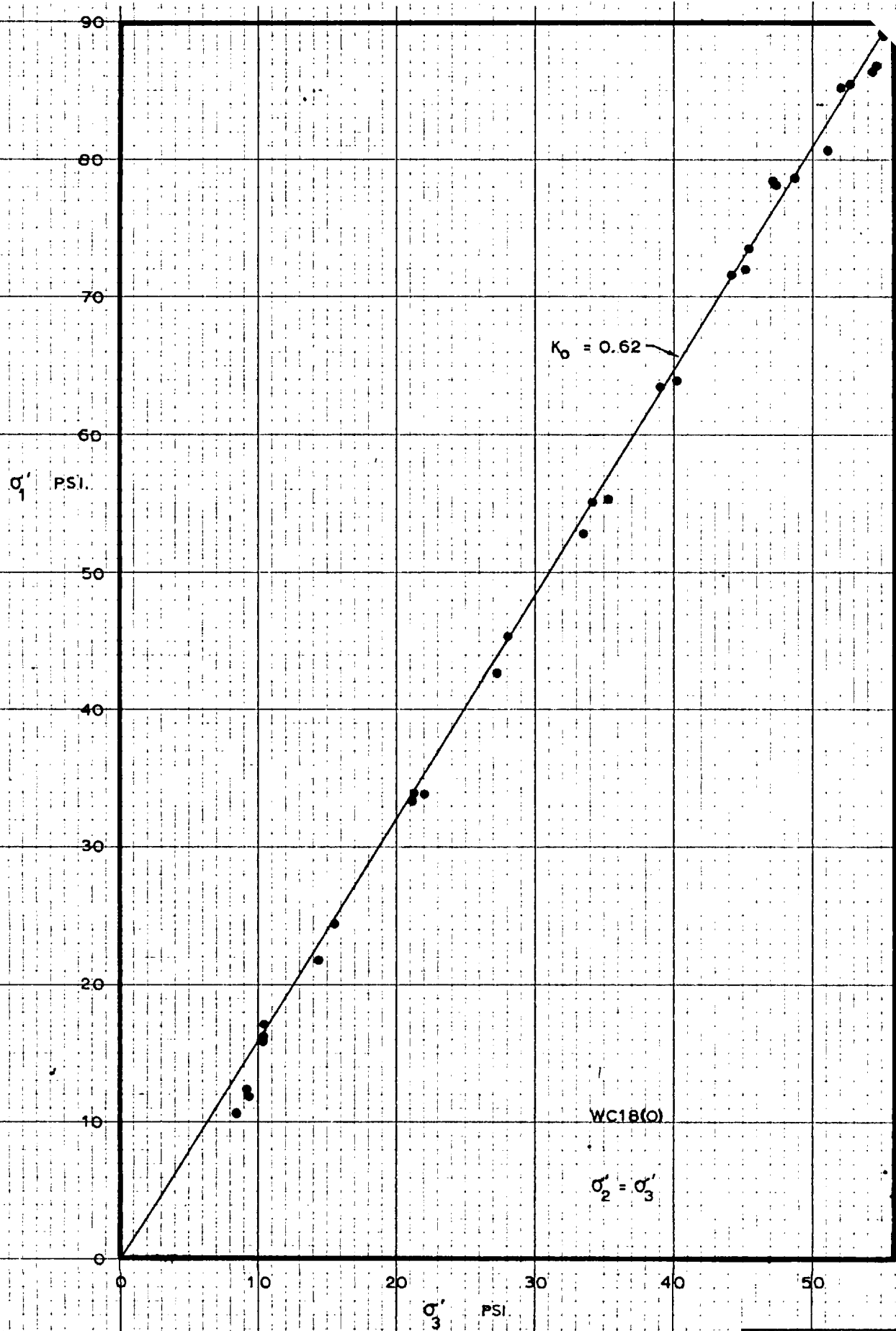


FIG A3-13



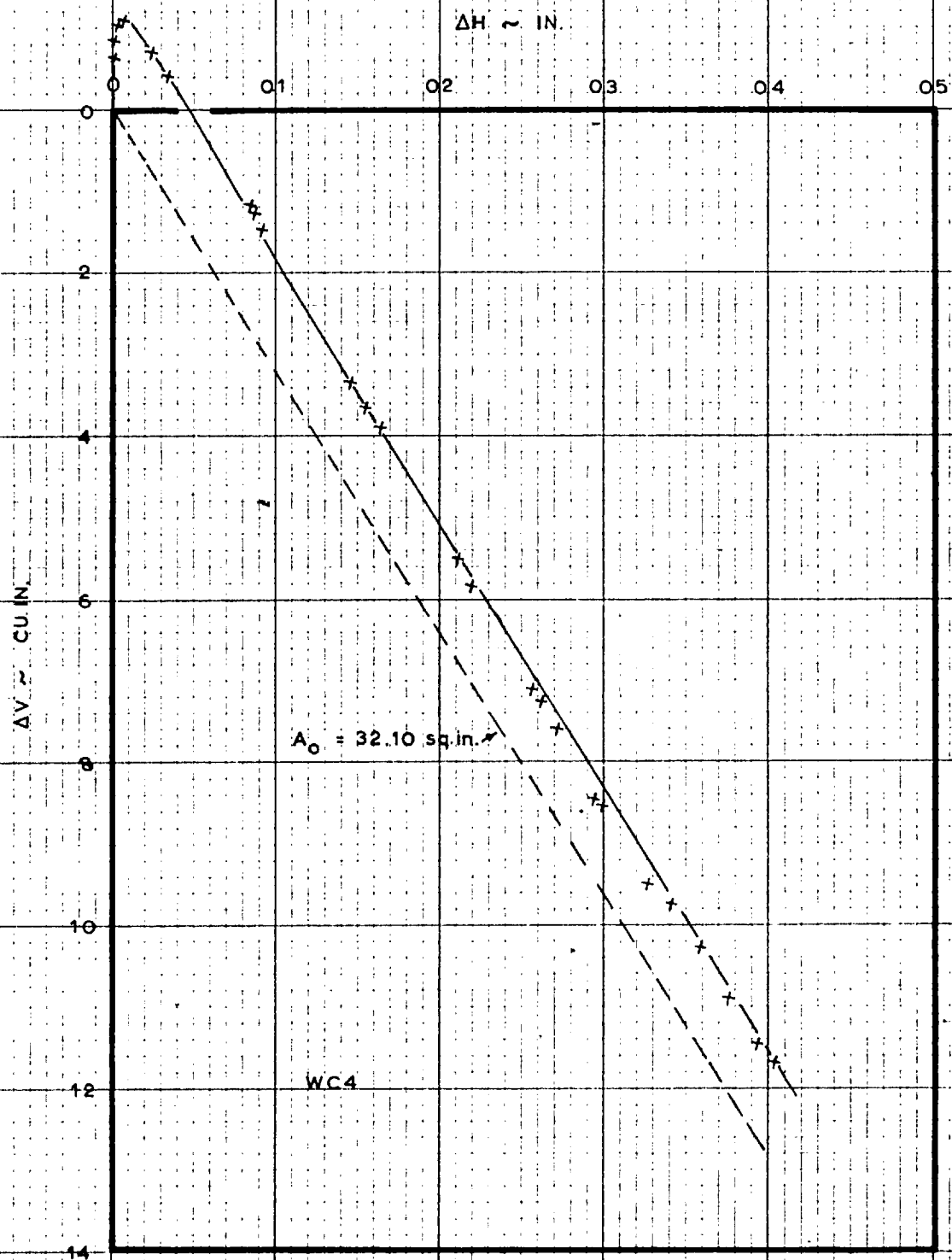


FIG A3-15

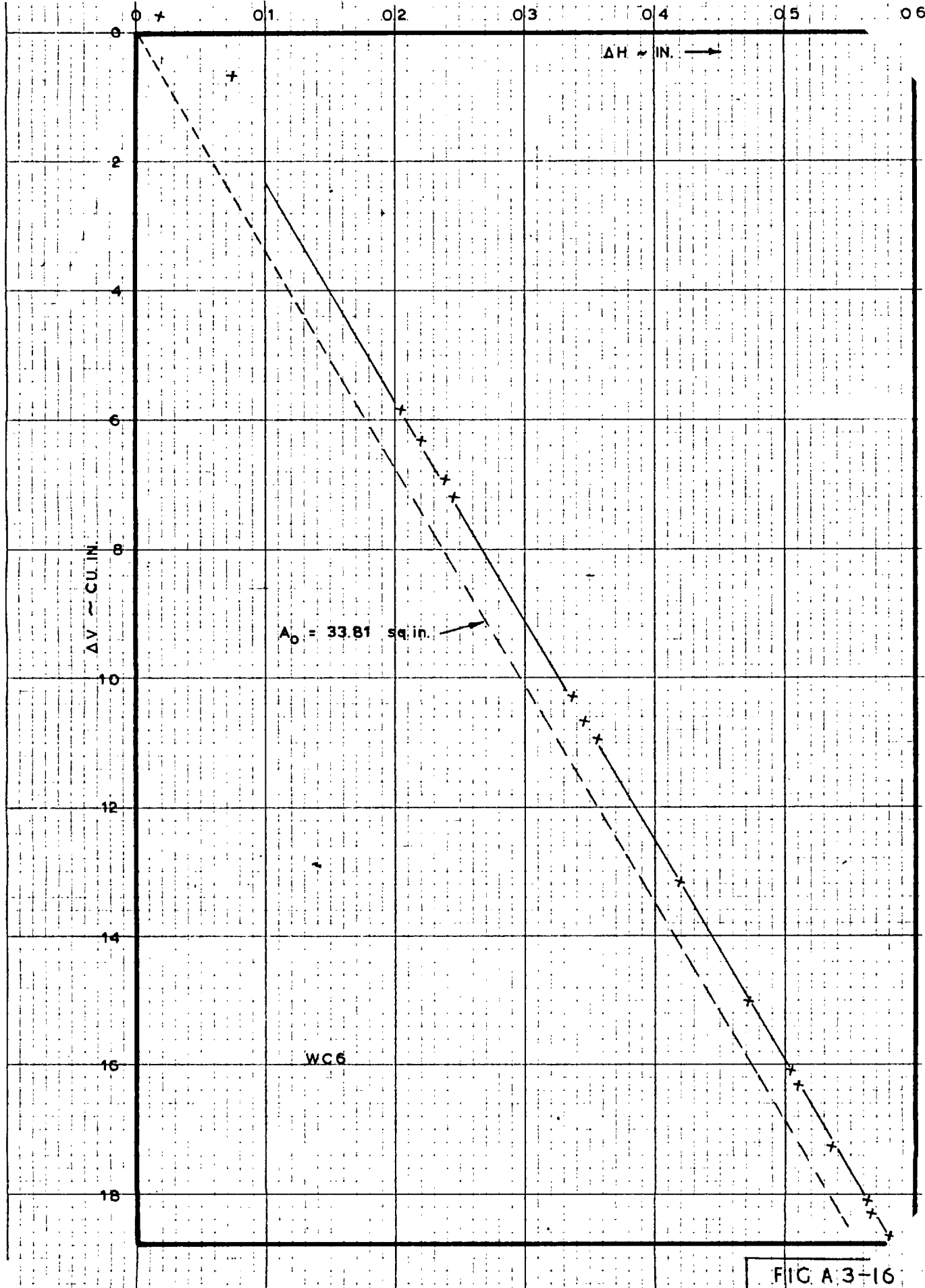


FIG A 3-16

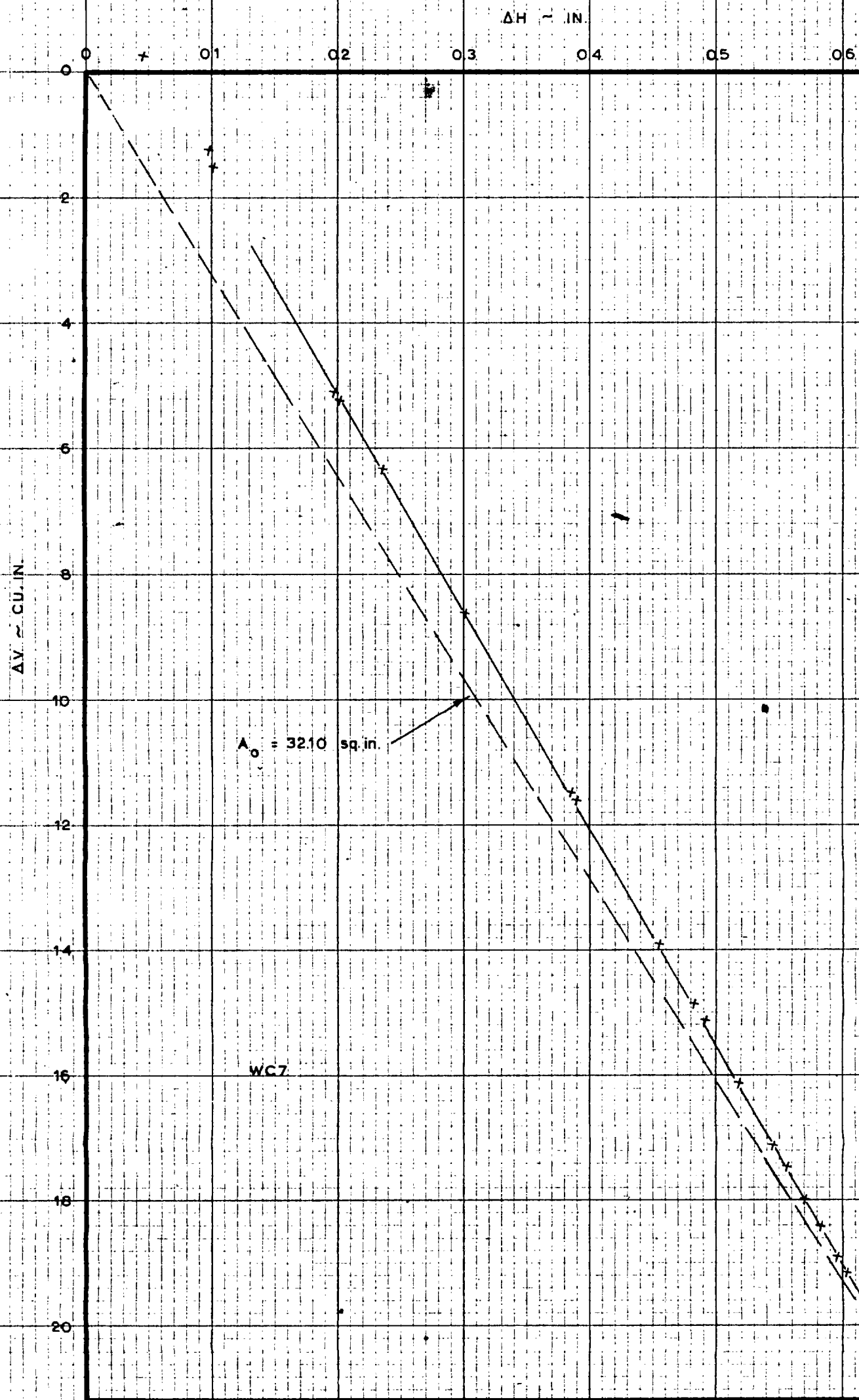


FIG A3-17

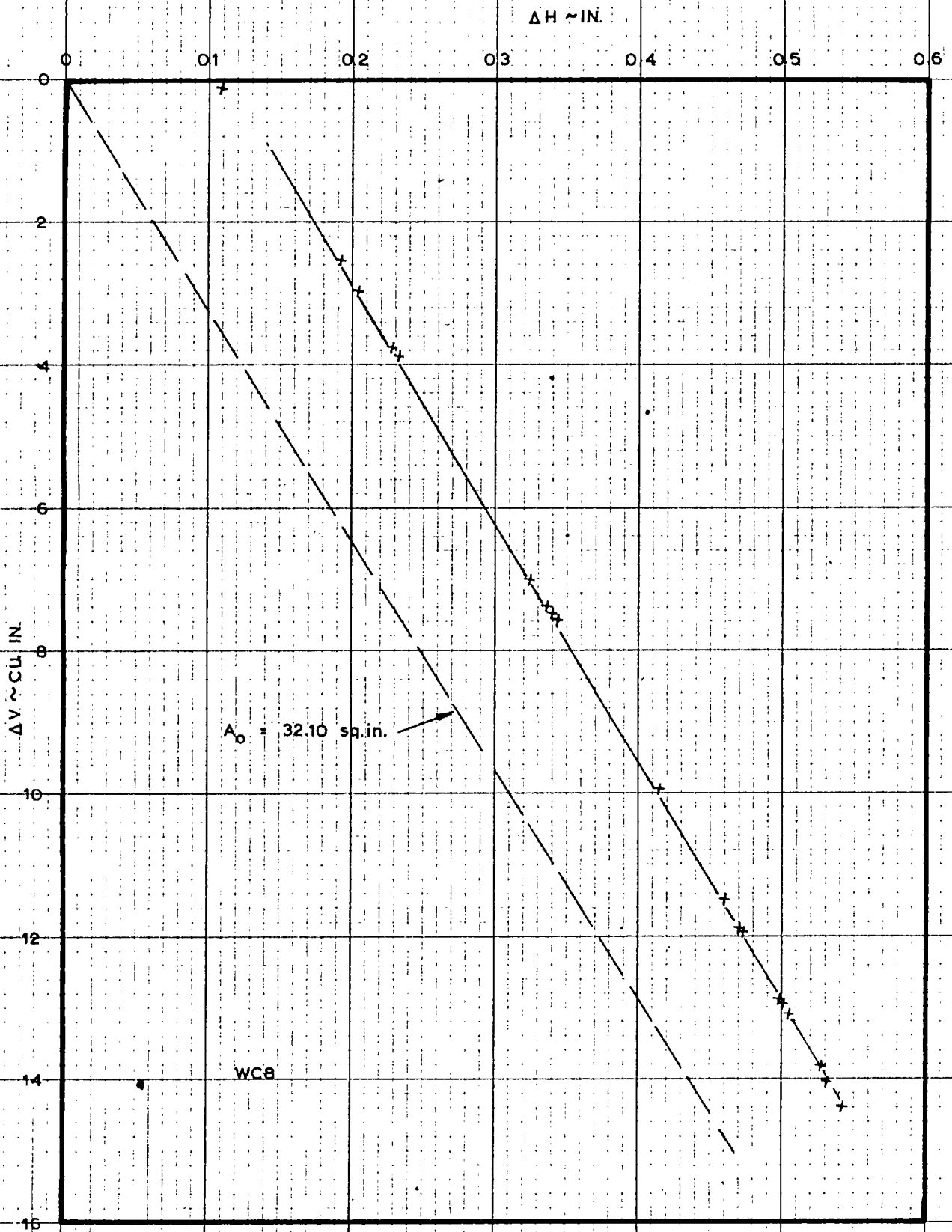


FIG A3-18

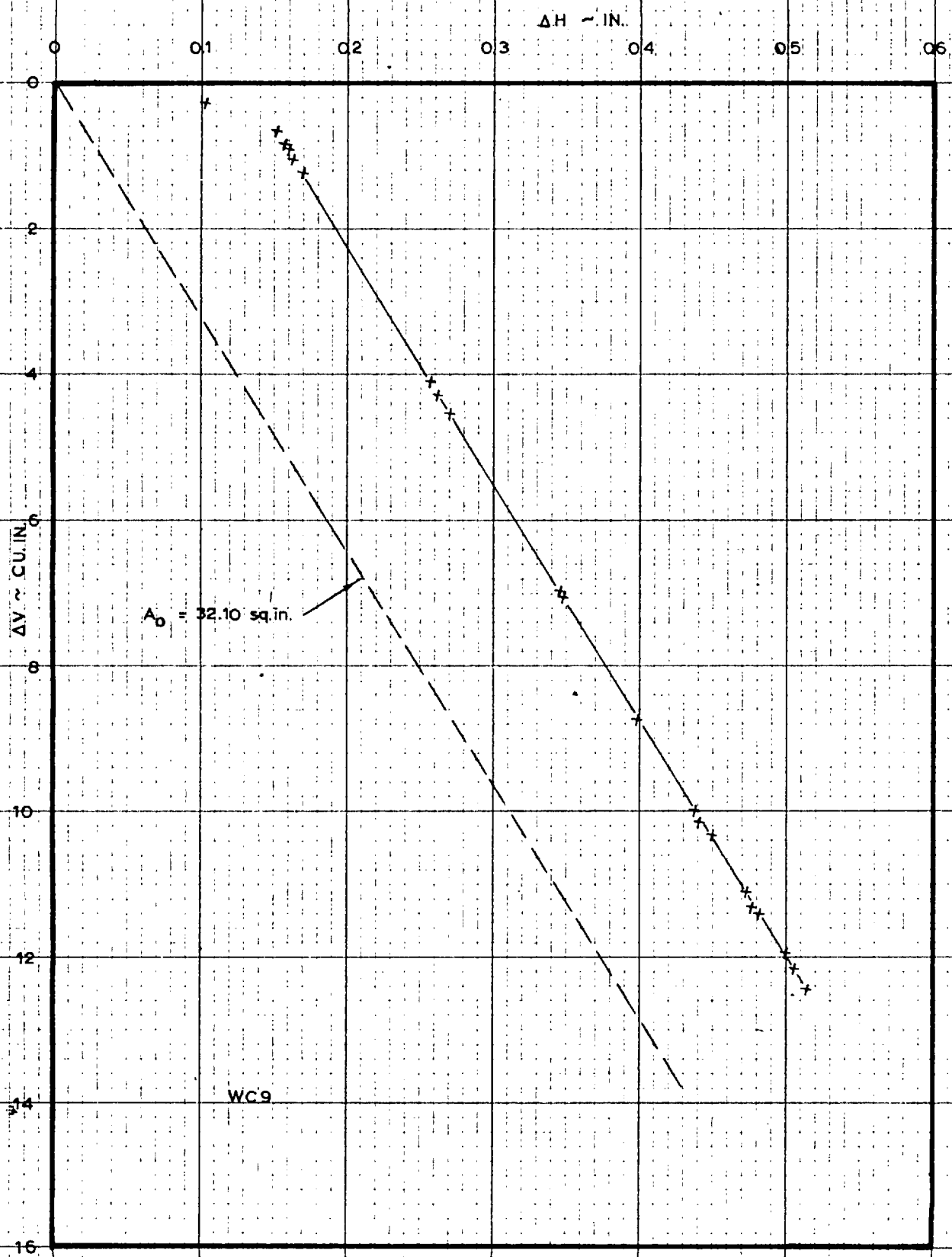


FIG A3-19

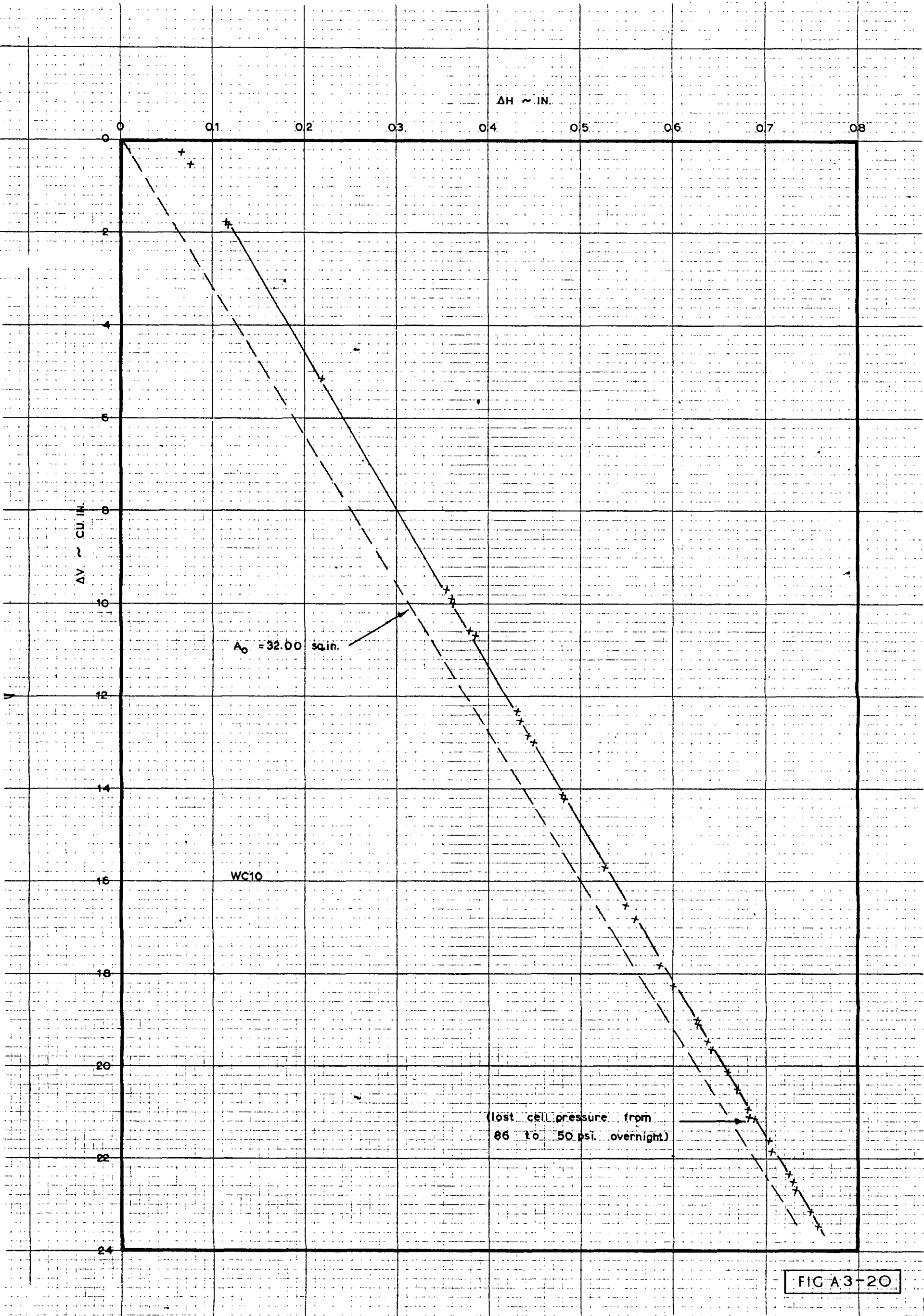


FIG A3-20

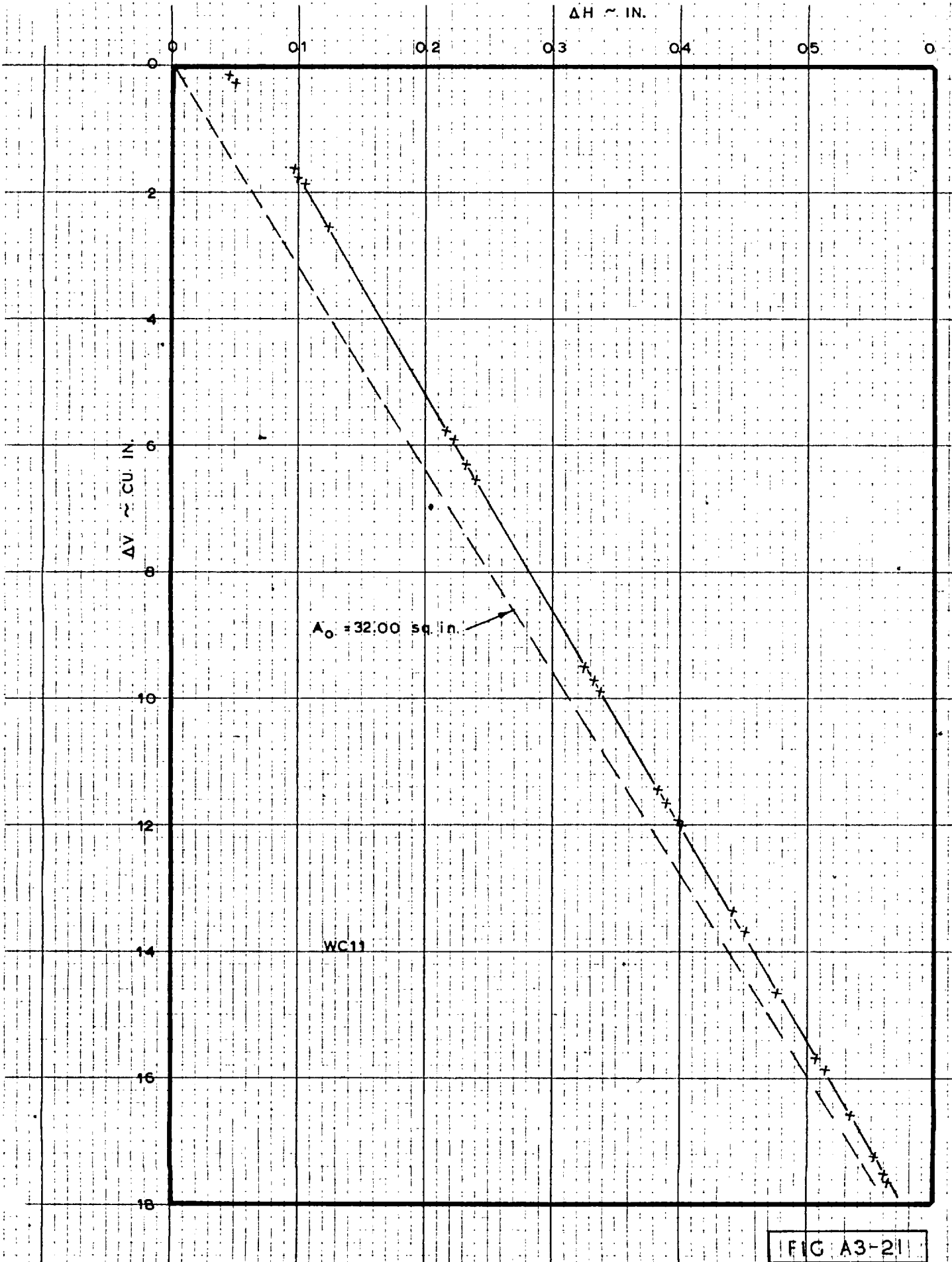


FIG A3-21

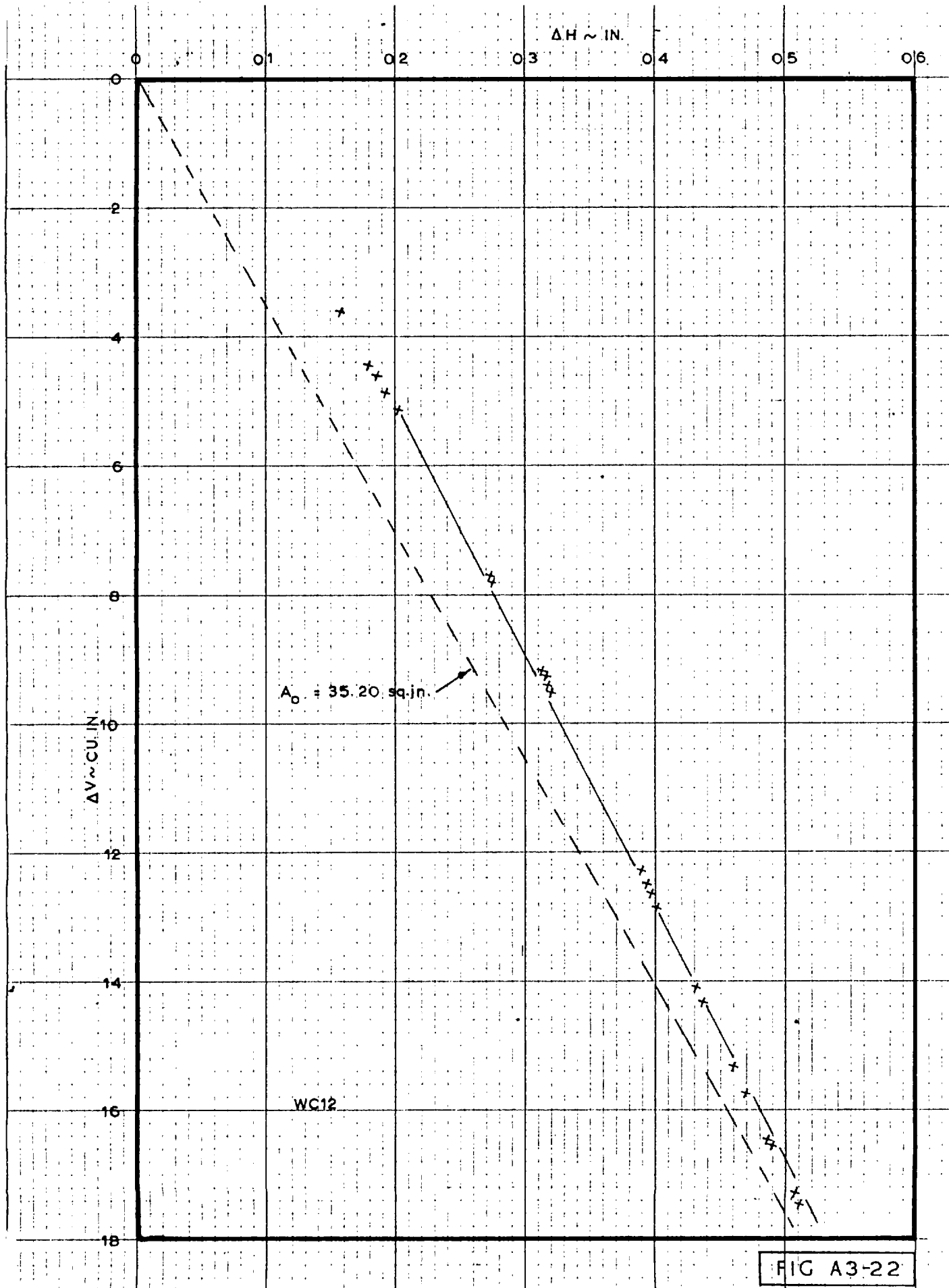


FIG A3-22

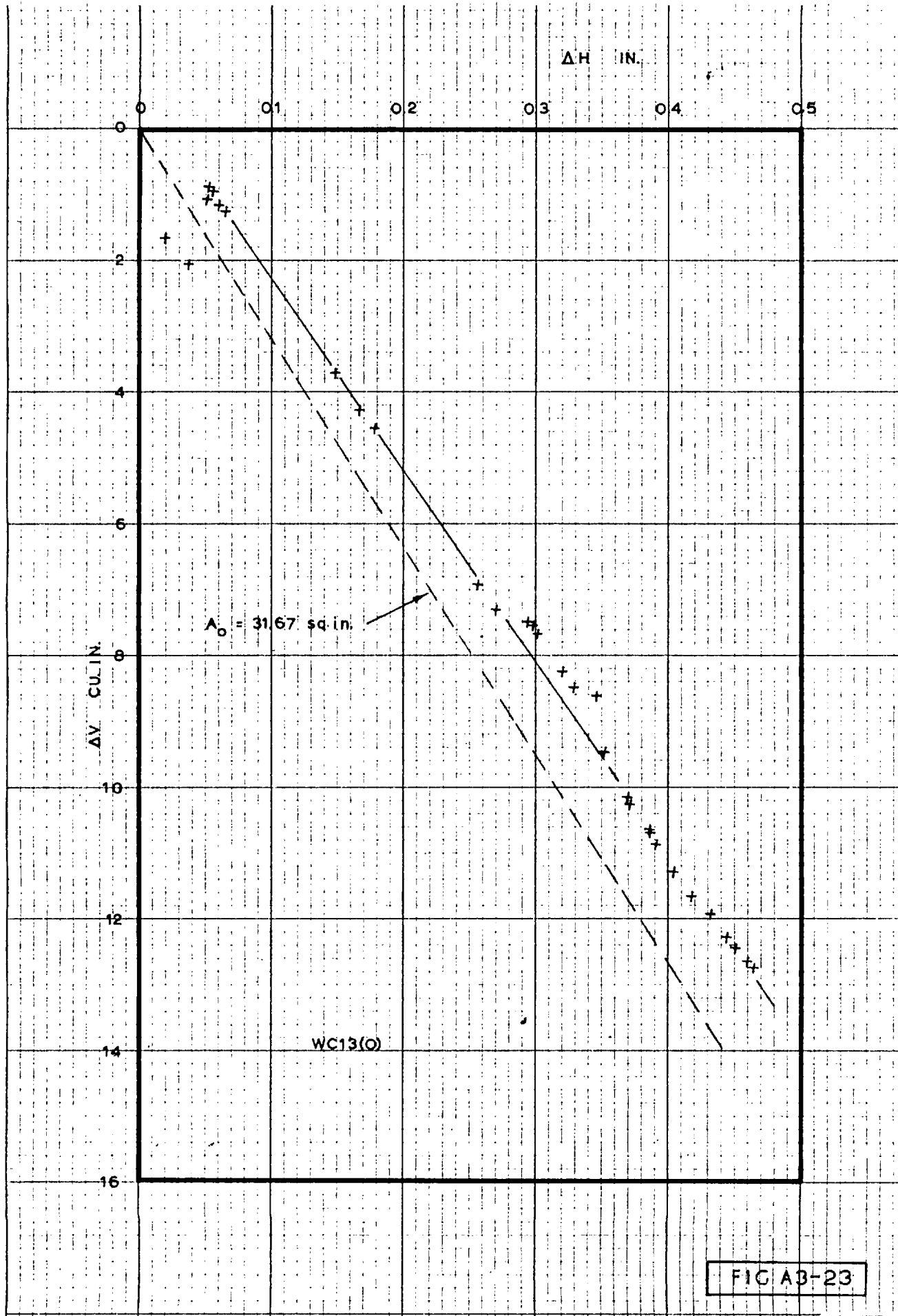


FIG A3-23

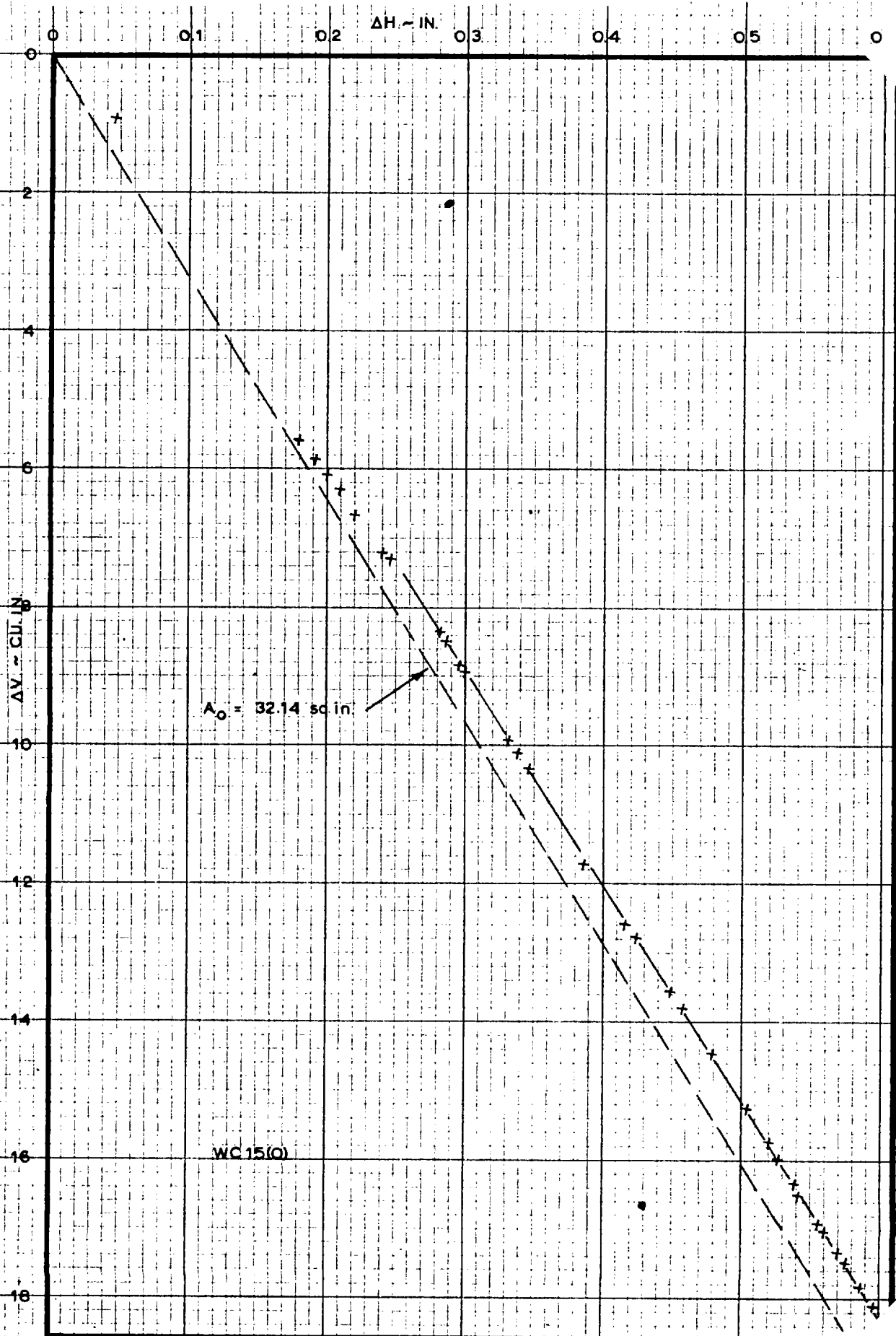


FIG A3-24

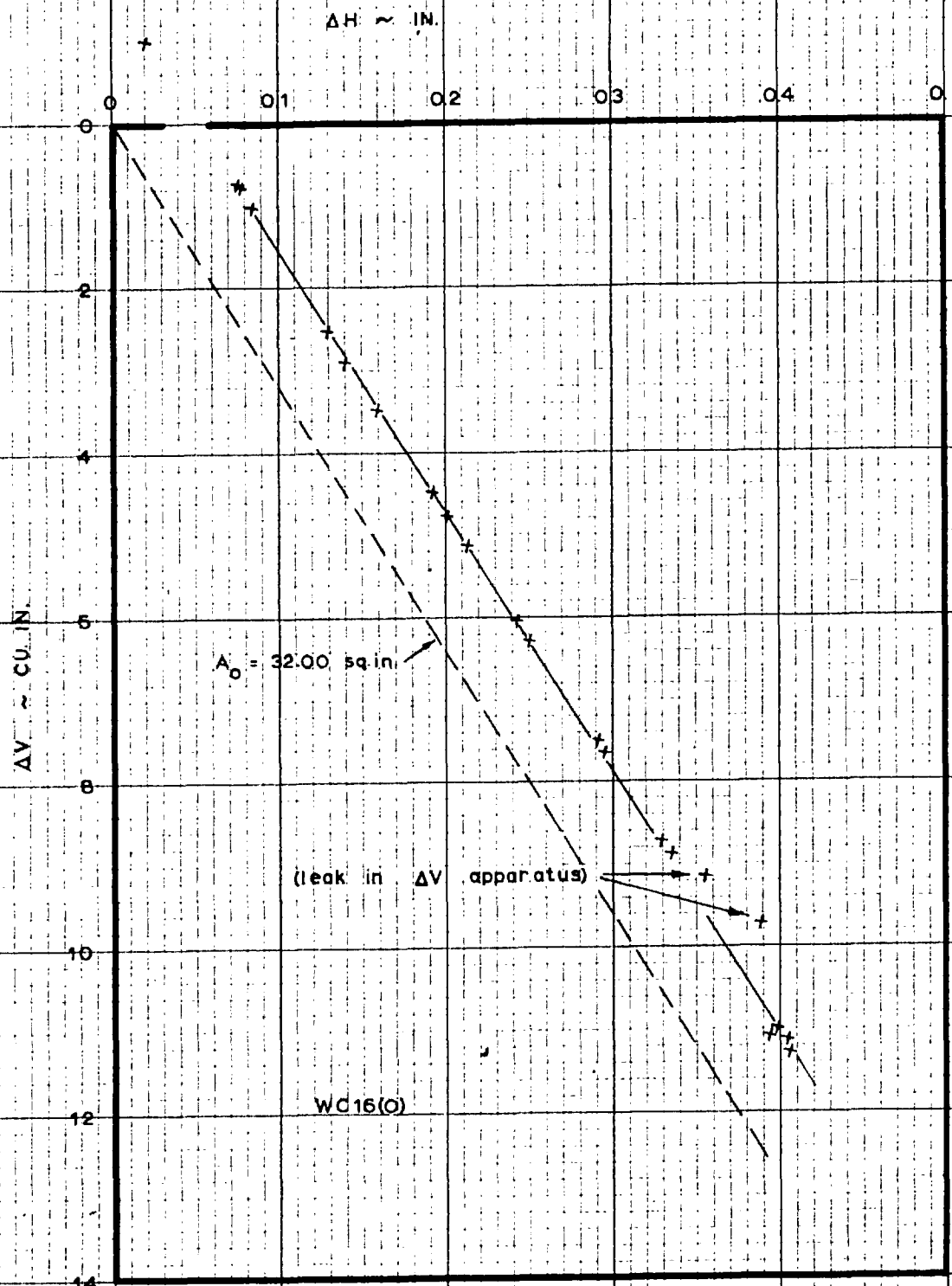
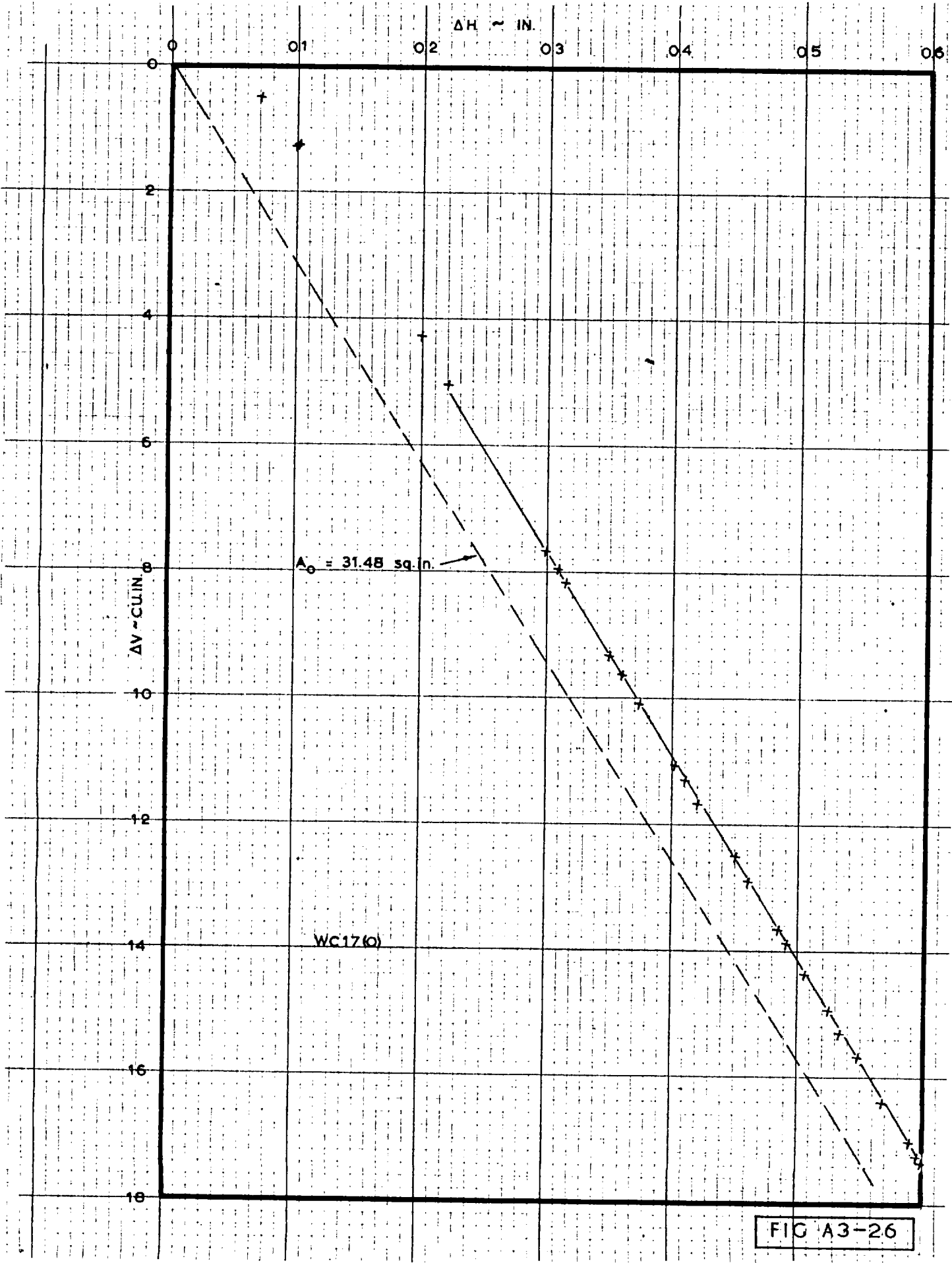


FIG A3-25



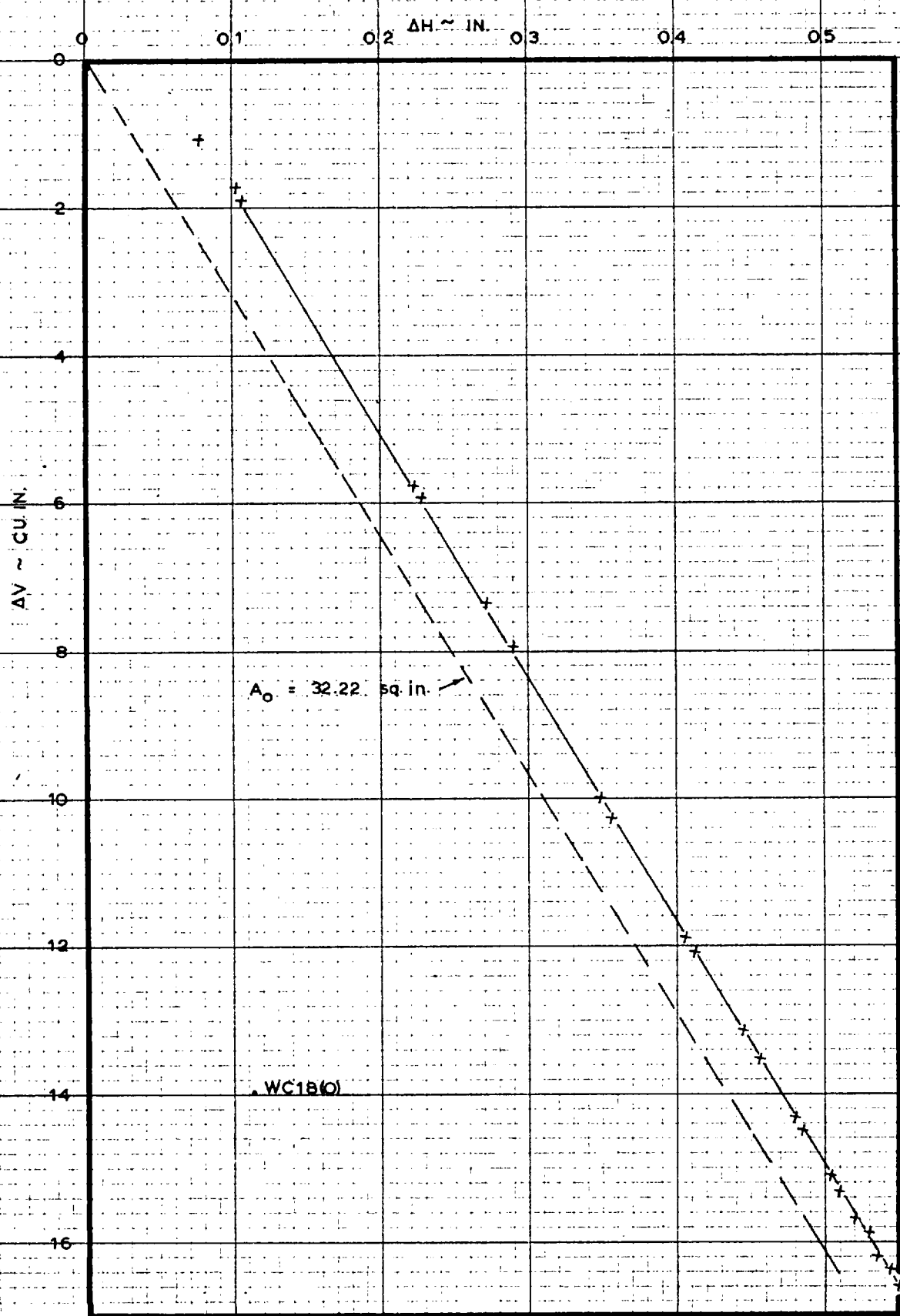
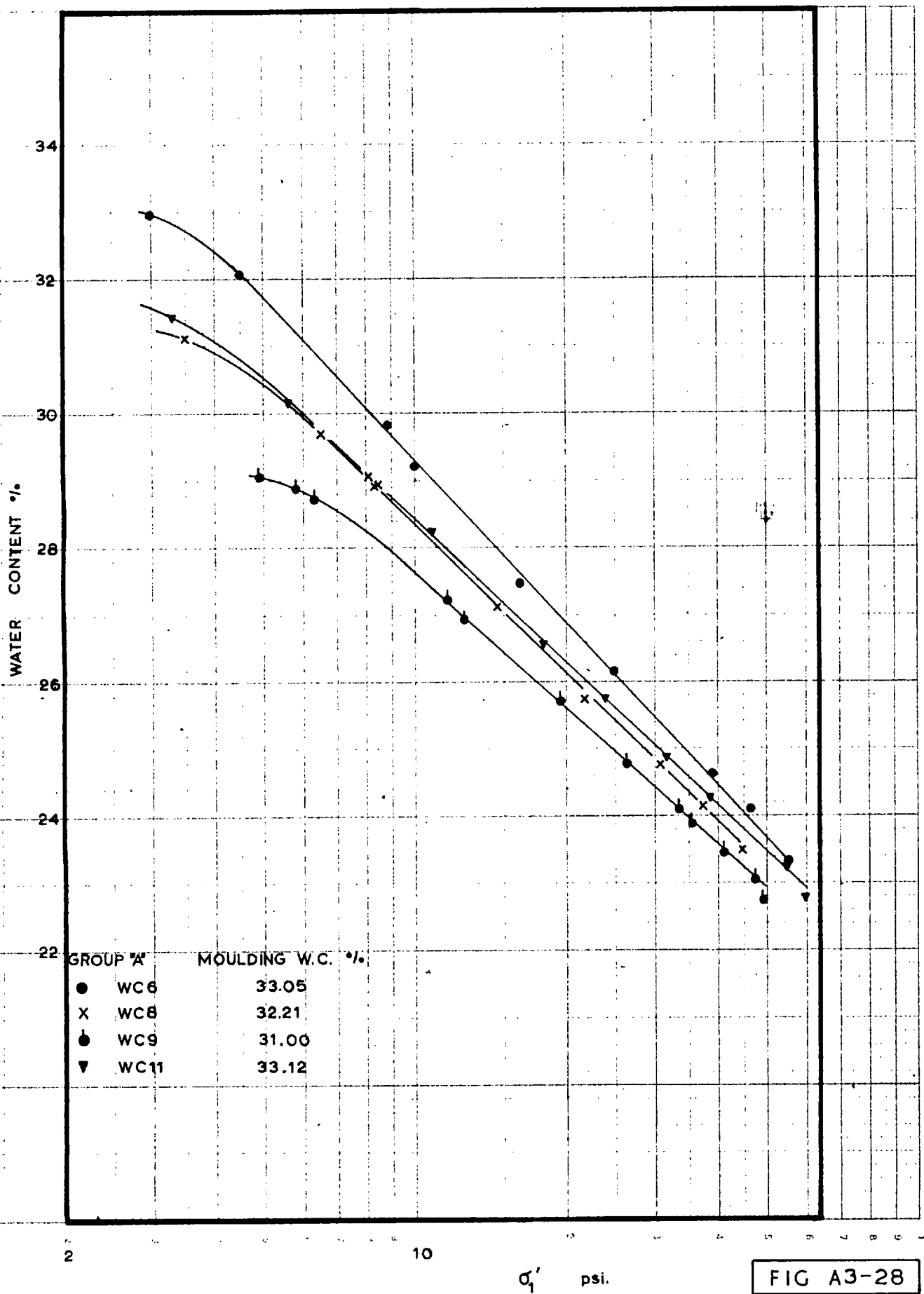


FIG A3-27



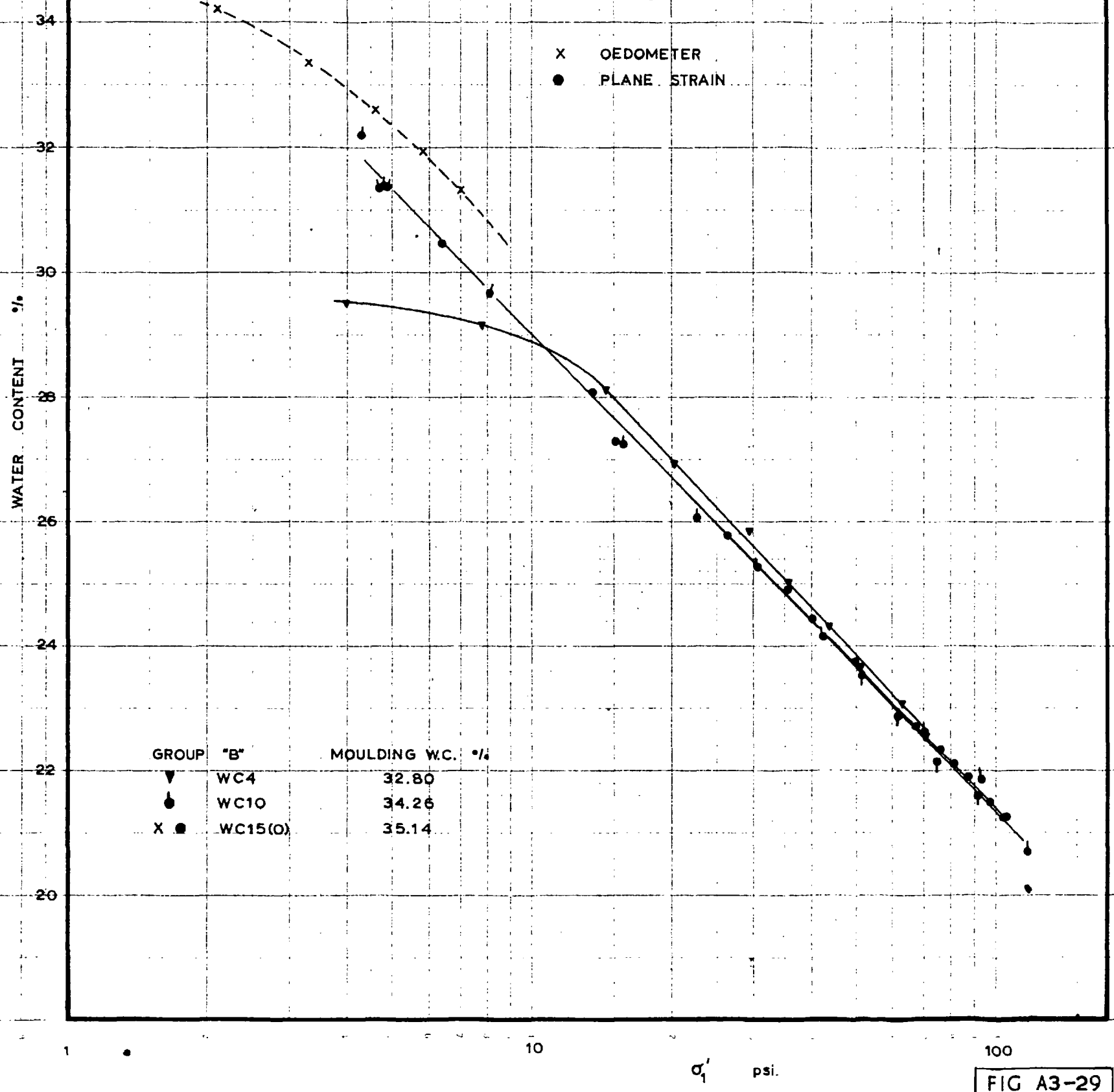


FIG A3-29

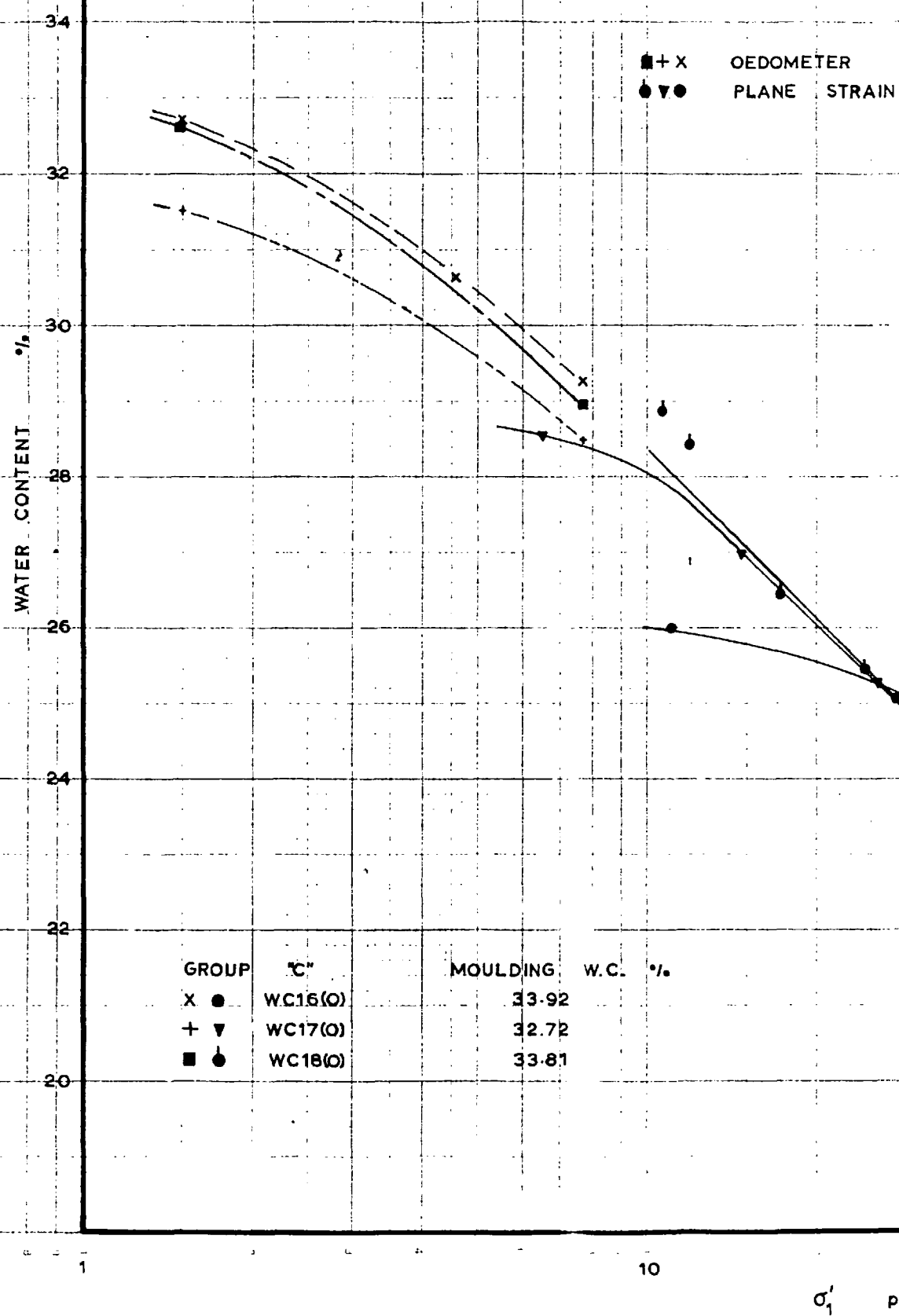
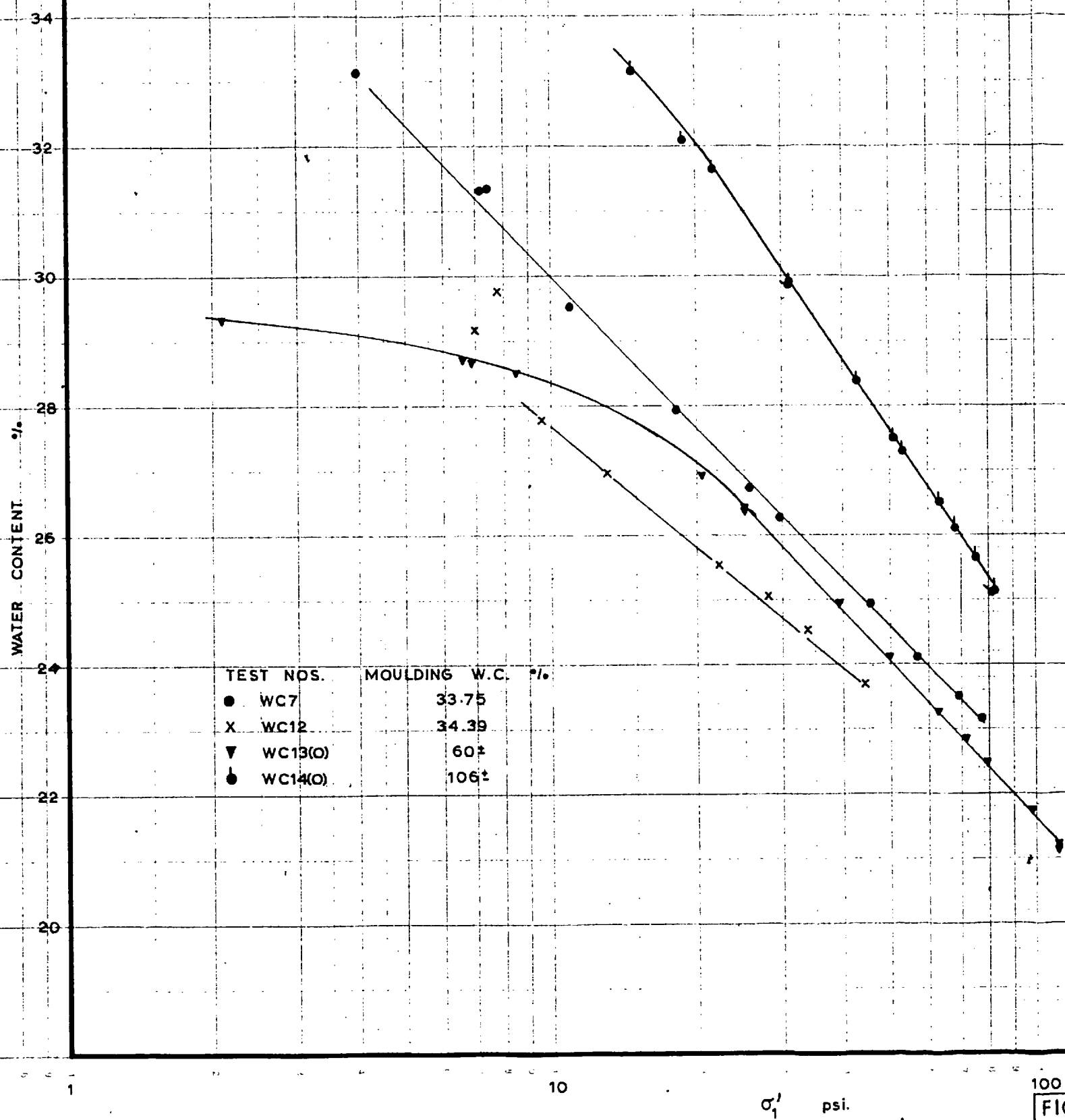


FIG A3-30



100
FIG A3-31

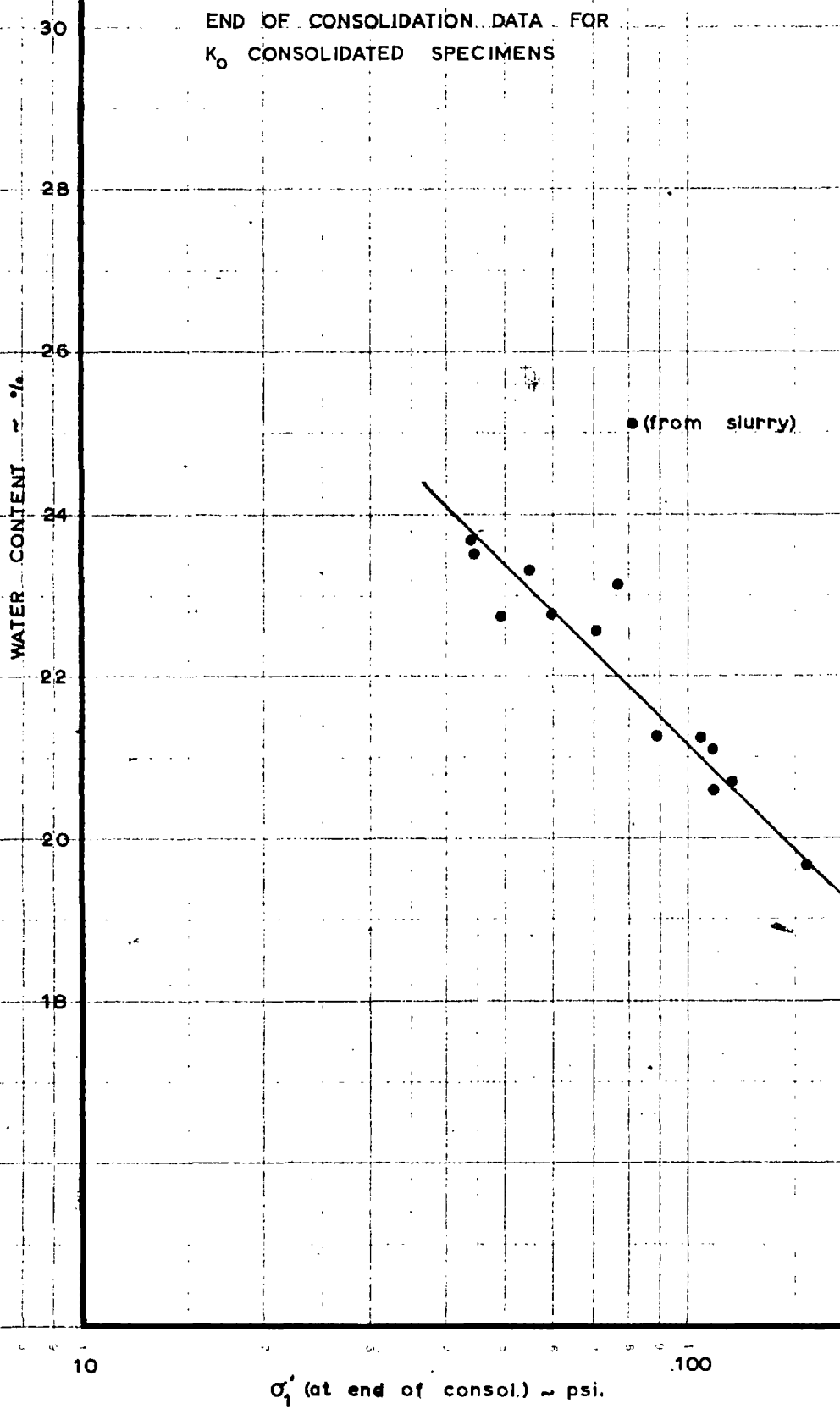


FIG A3-32

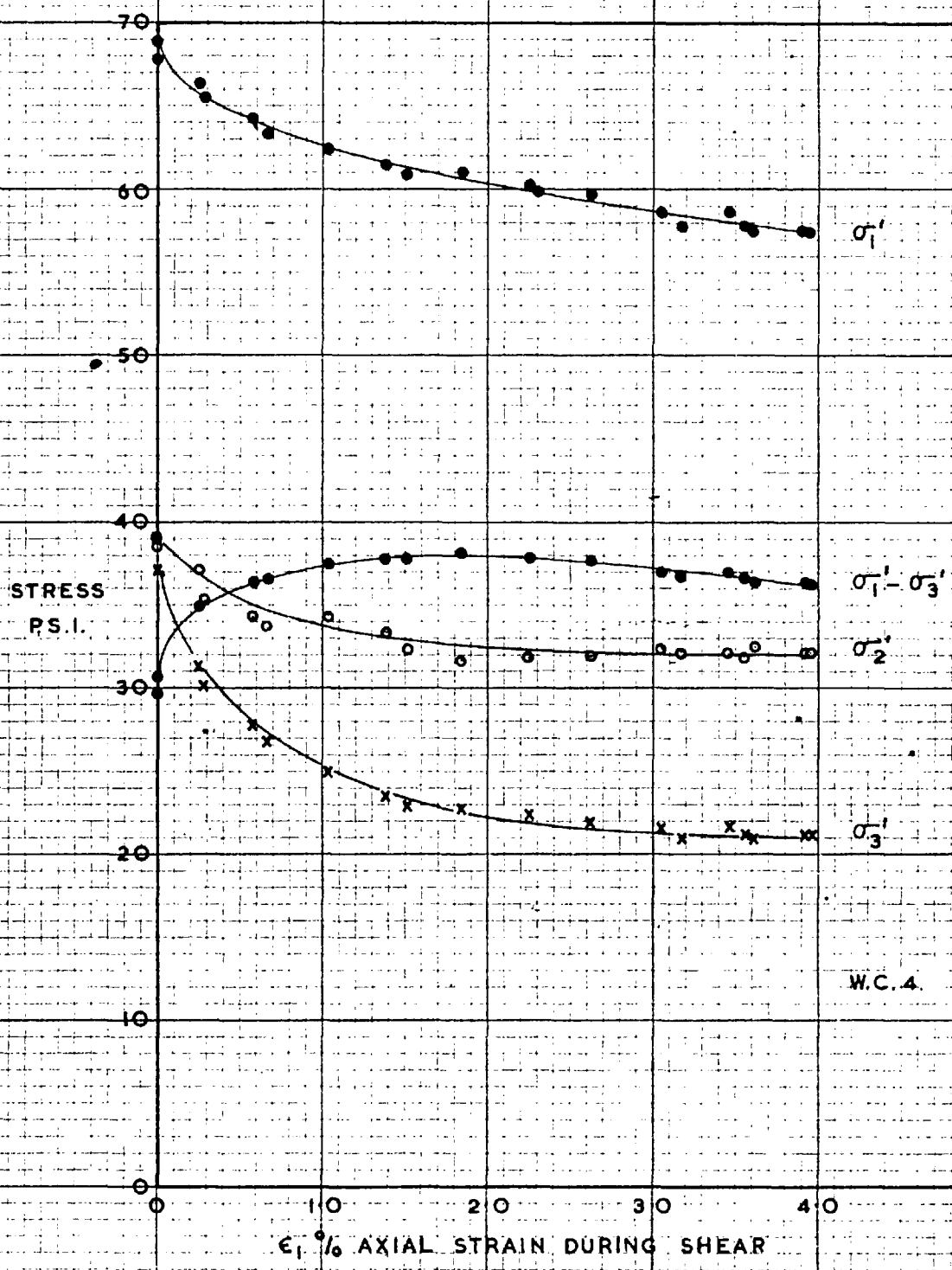
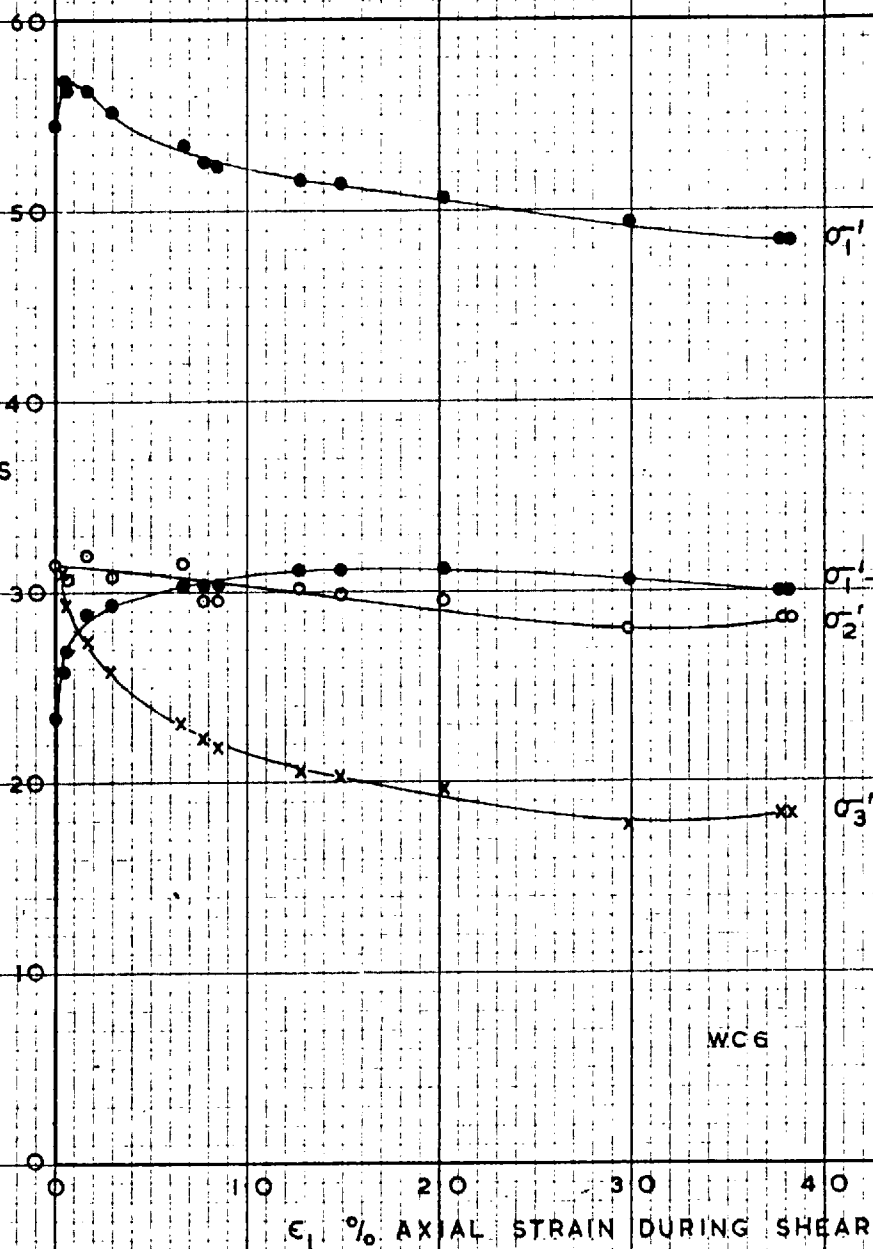


FIG A3-33

STRESS
P.S.I.



WC6

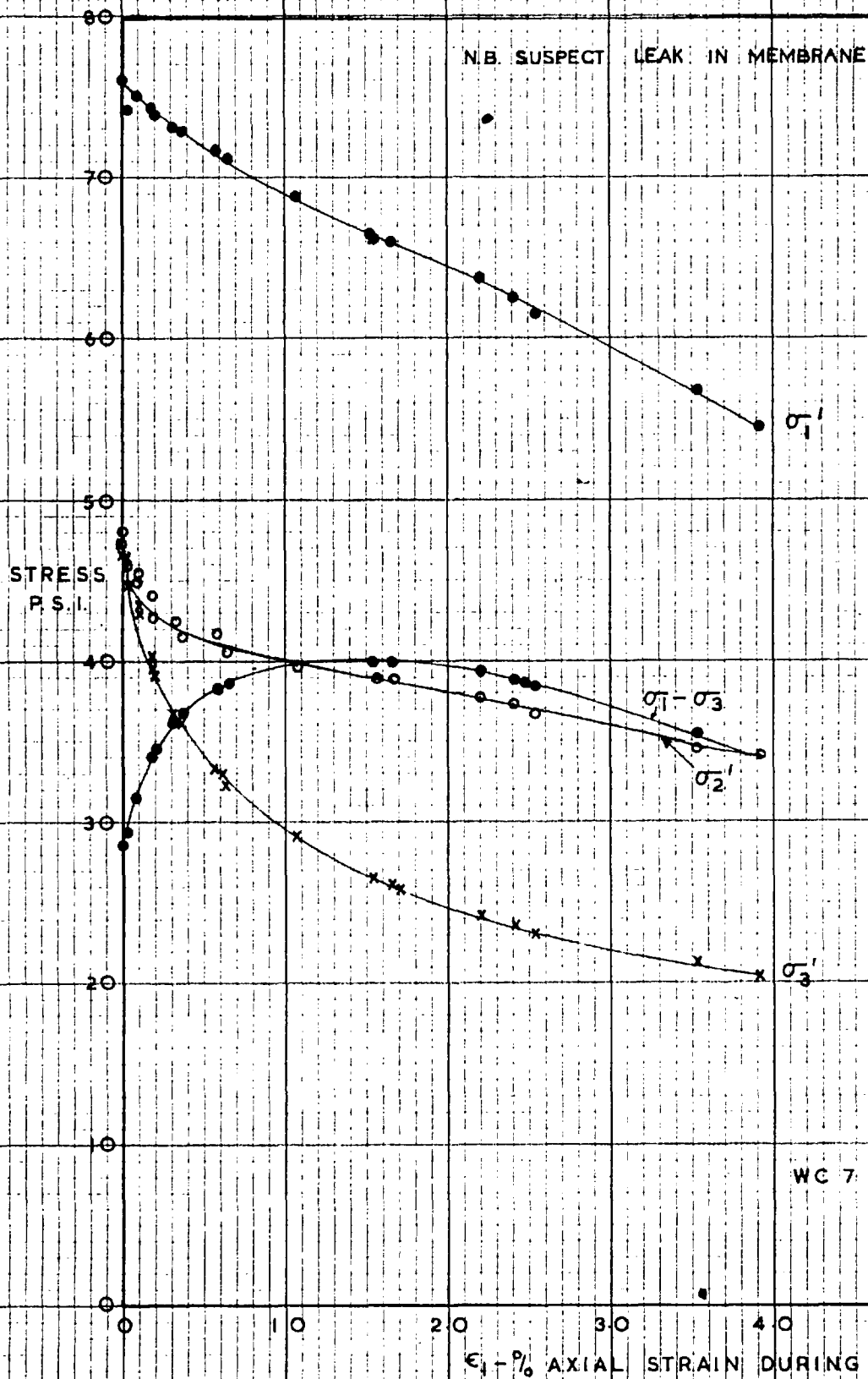


FIG. A3-35

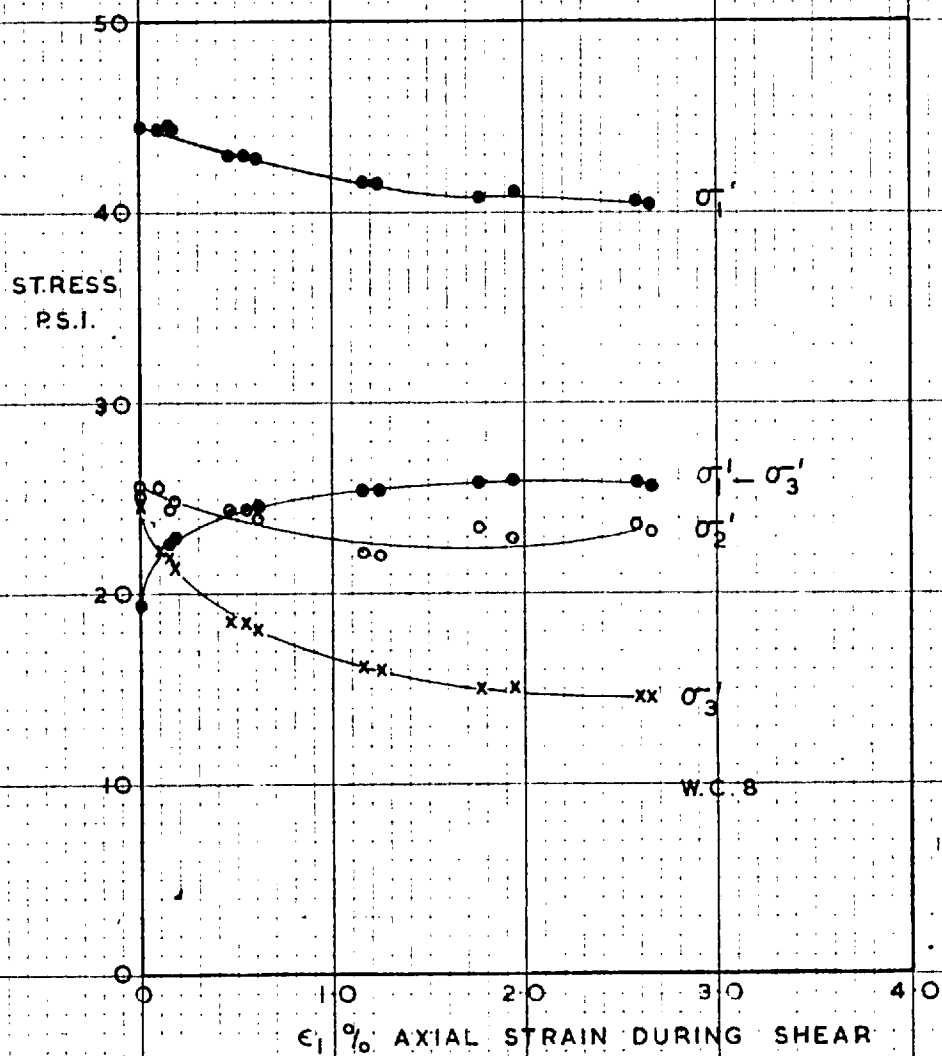


FIG. A3-36

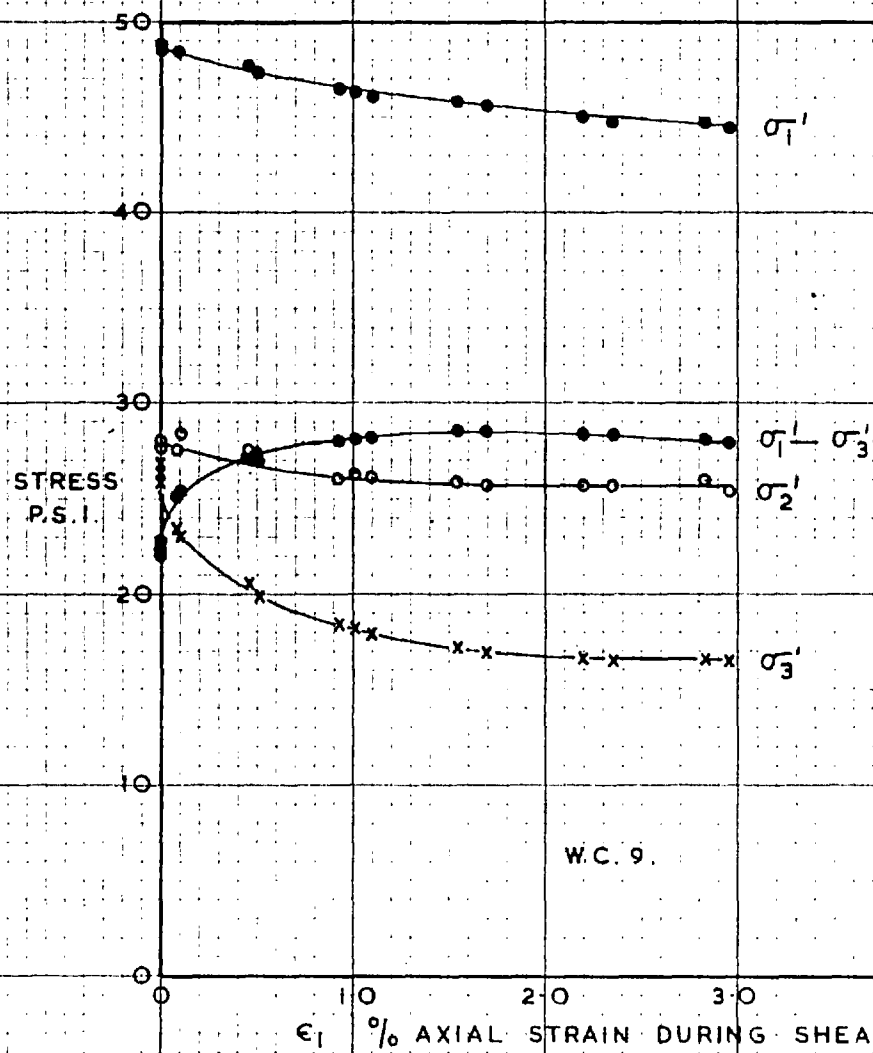
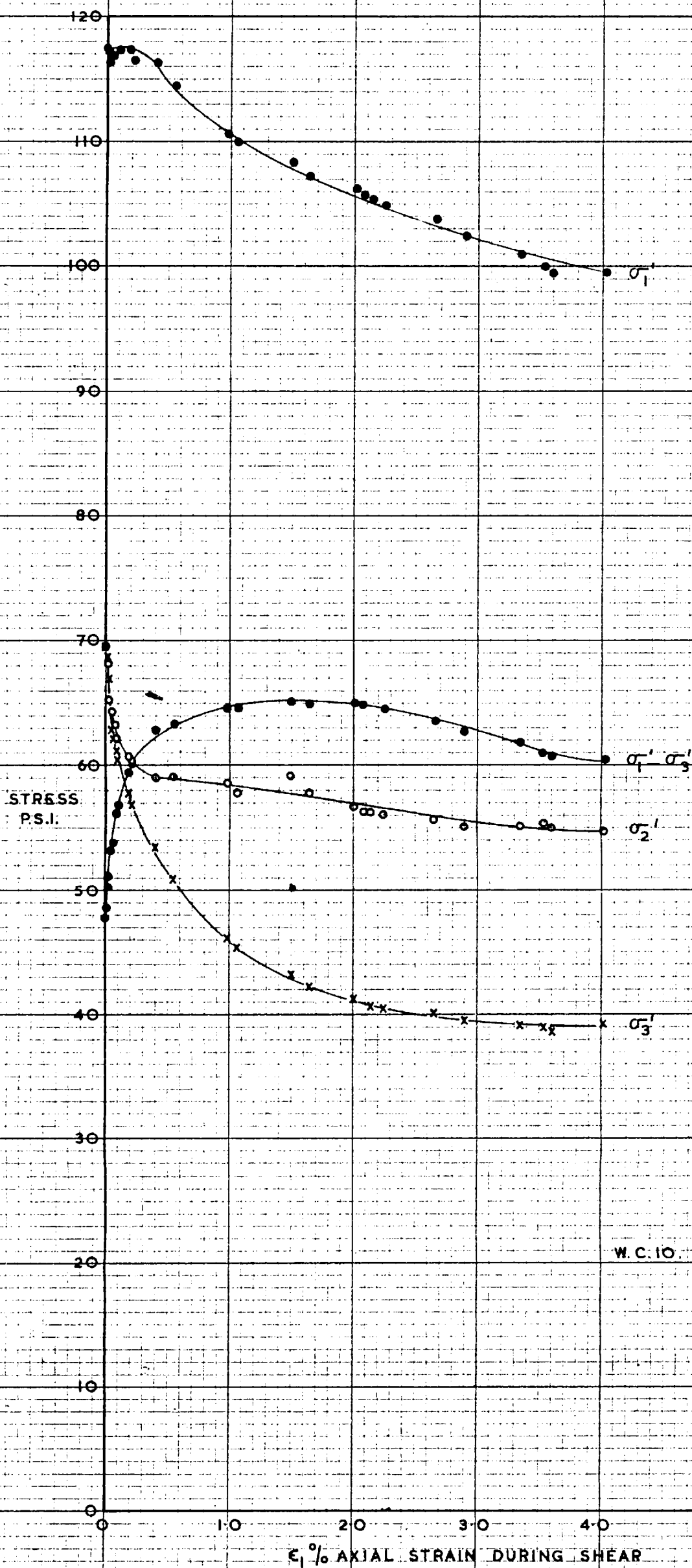


FIG. A3-37



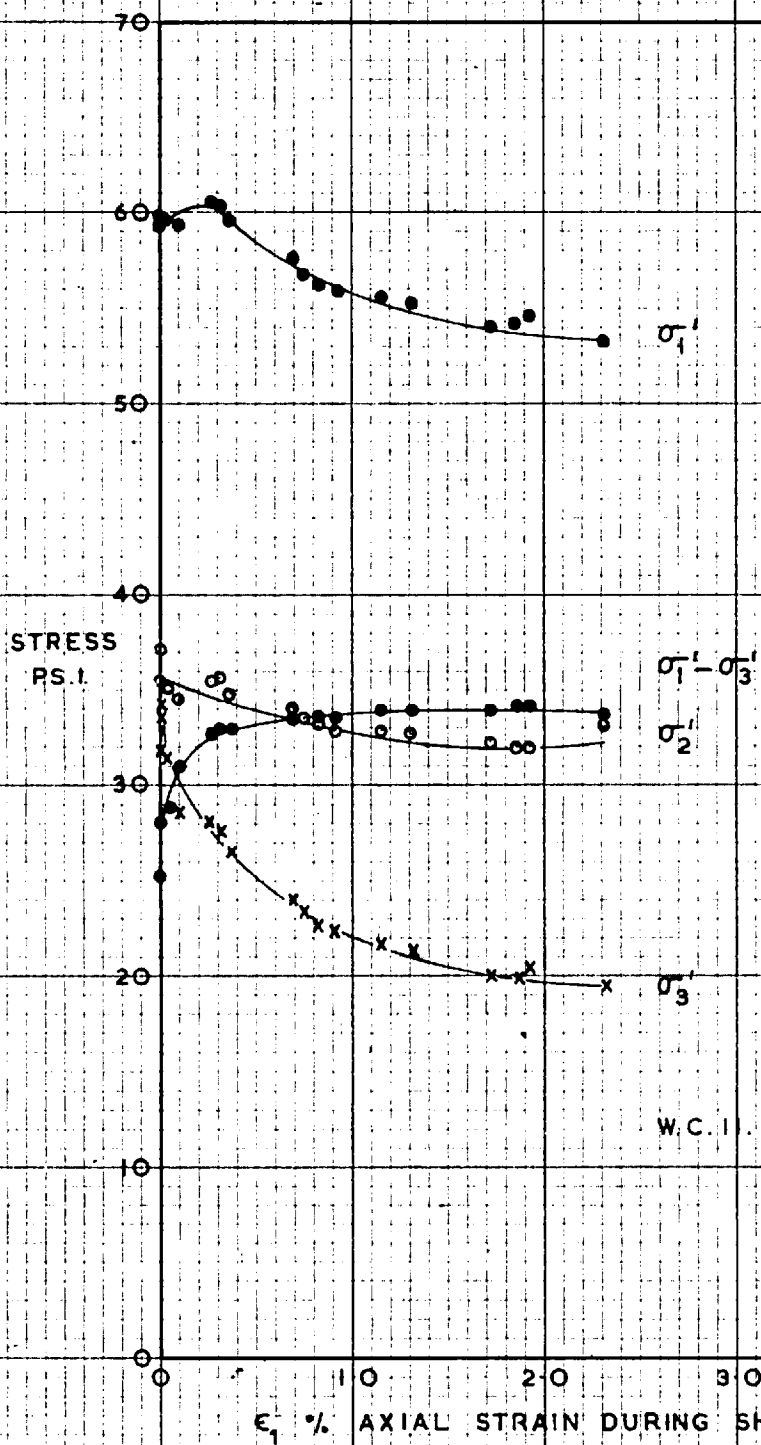


FIG. A3-39

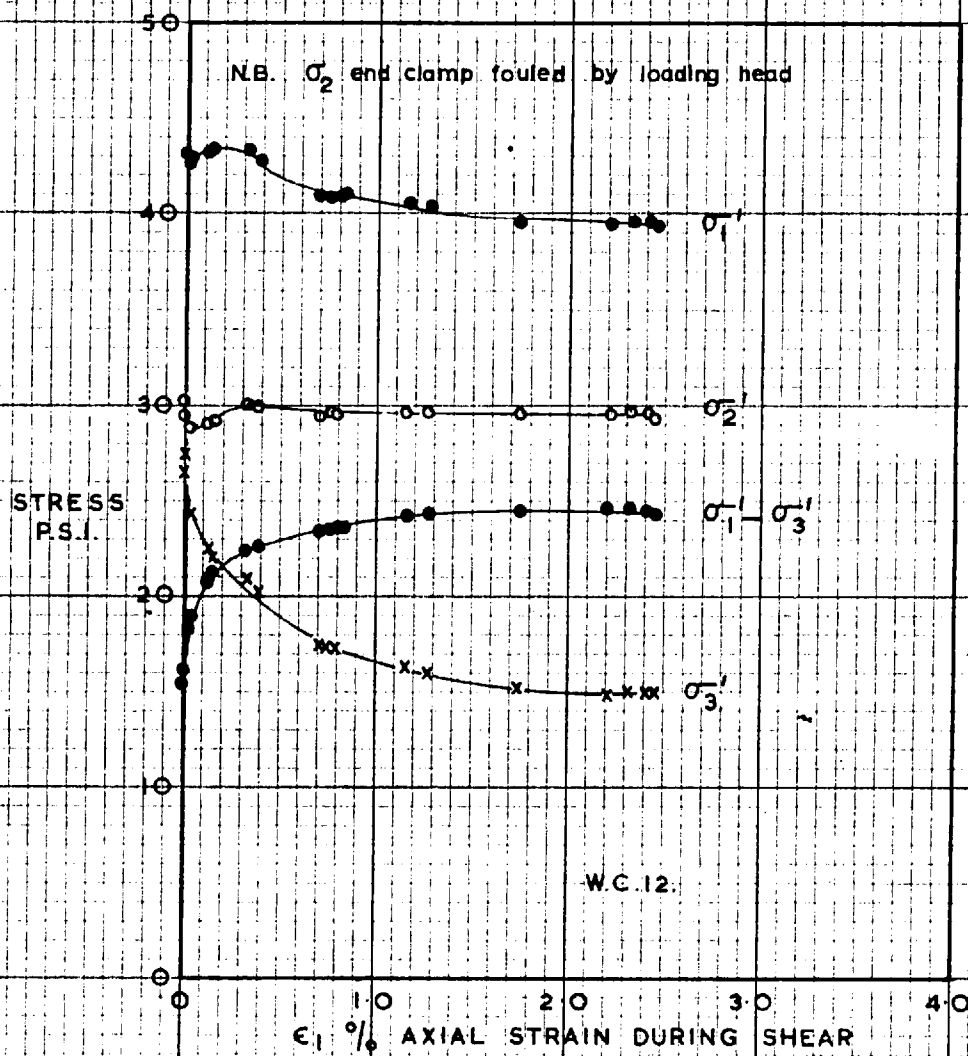


FIG. A3-40

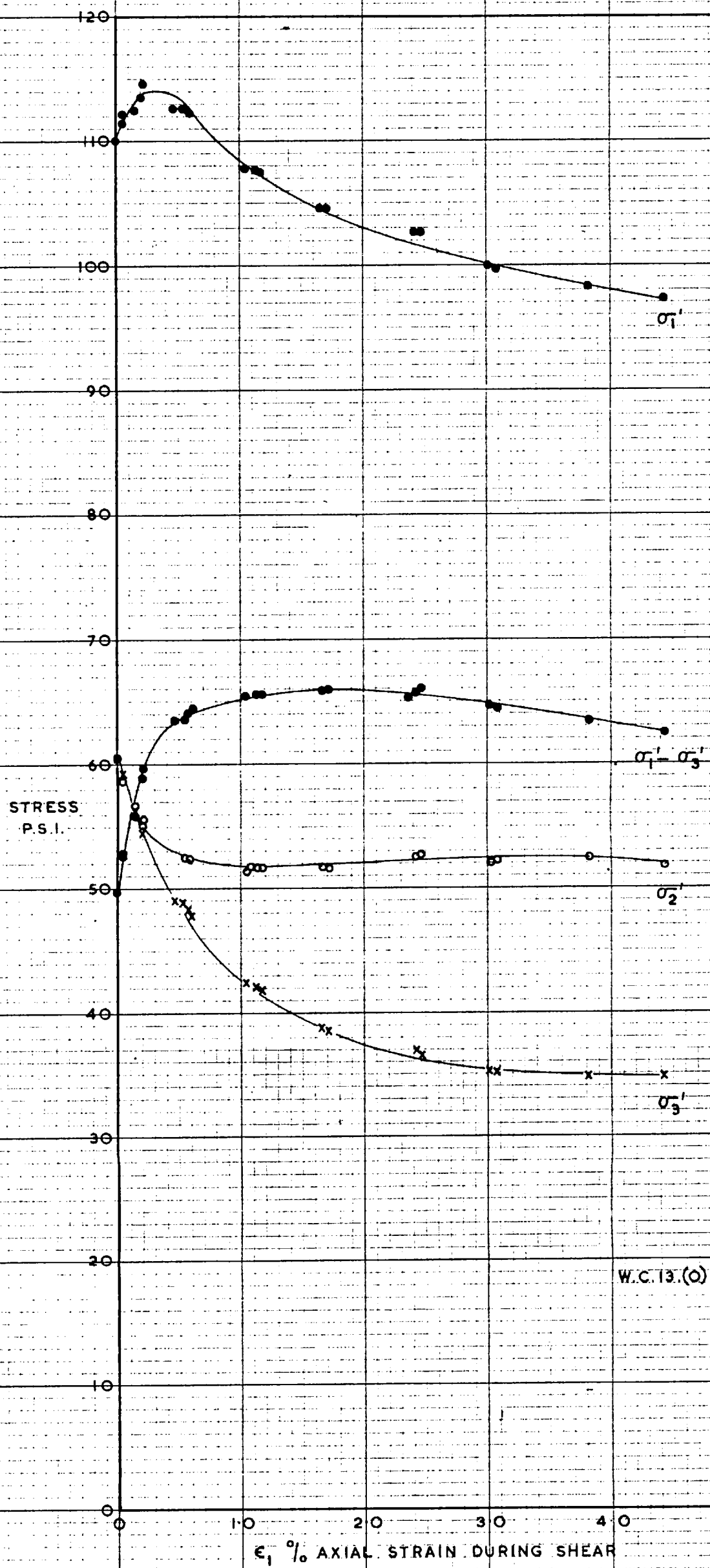


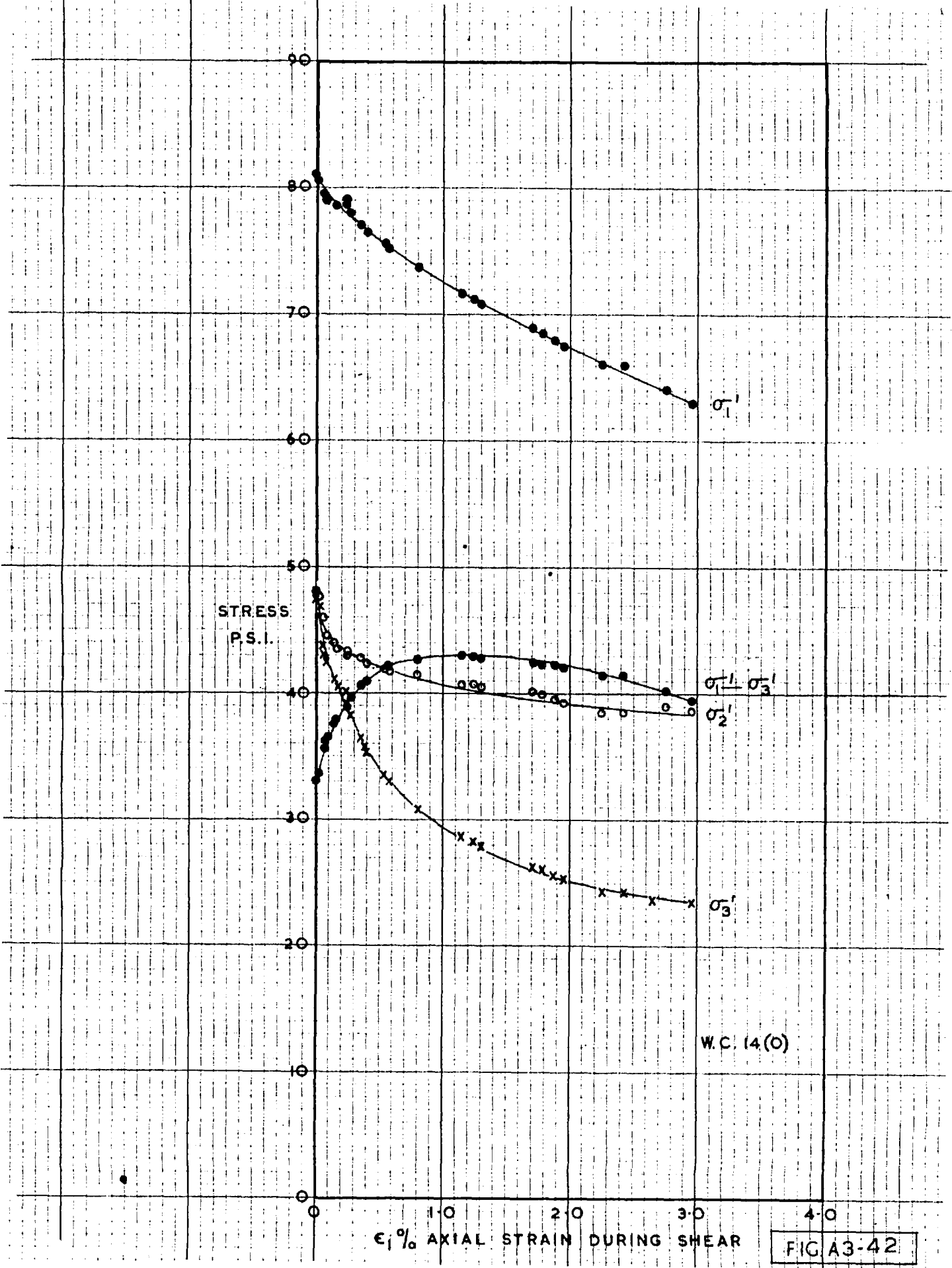
FIG. A3-41

STRESS
P.S.I.

W.C. 14(O)

ϵ_1 % AXIAL STRAIN DURING SHEAR

FIG. A3-42



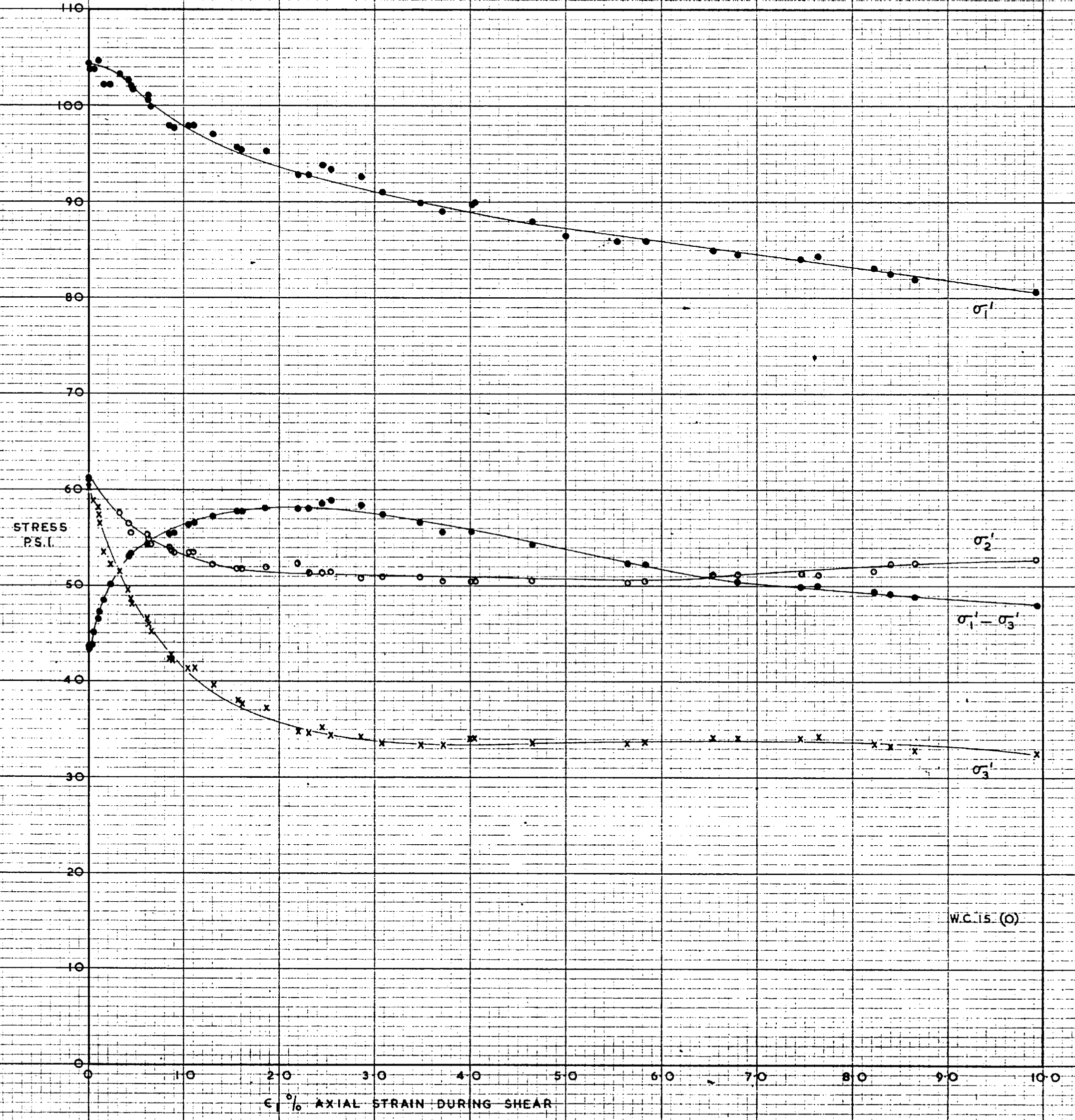


FIG. A3-43

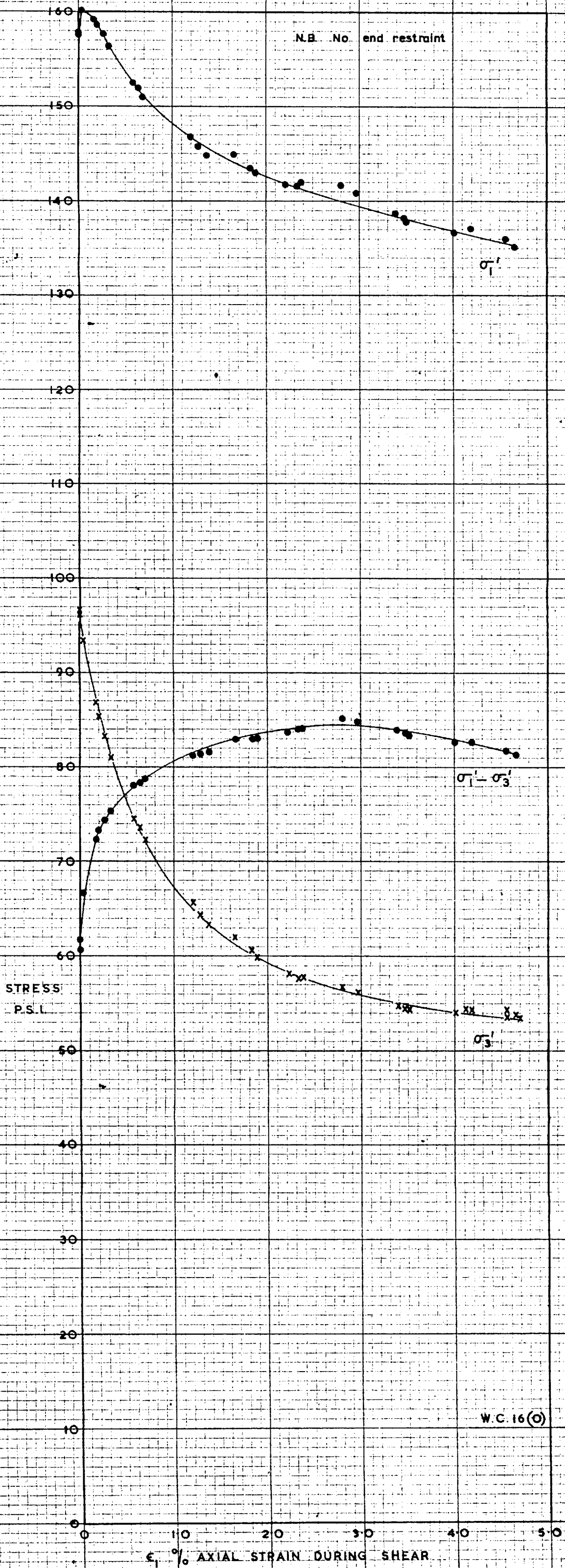


FIG. A3-44

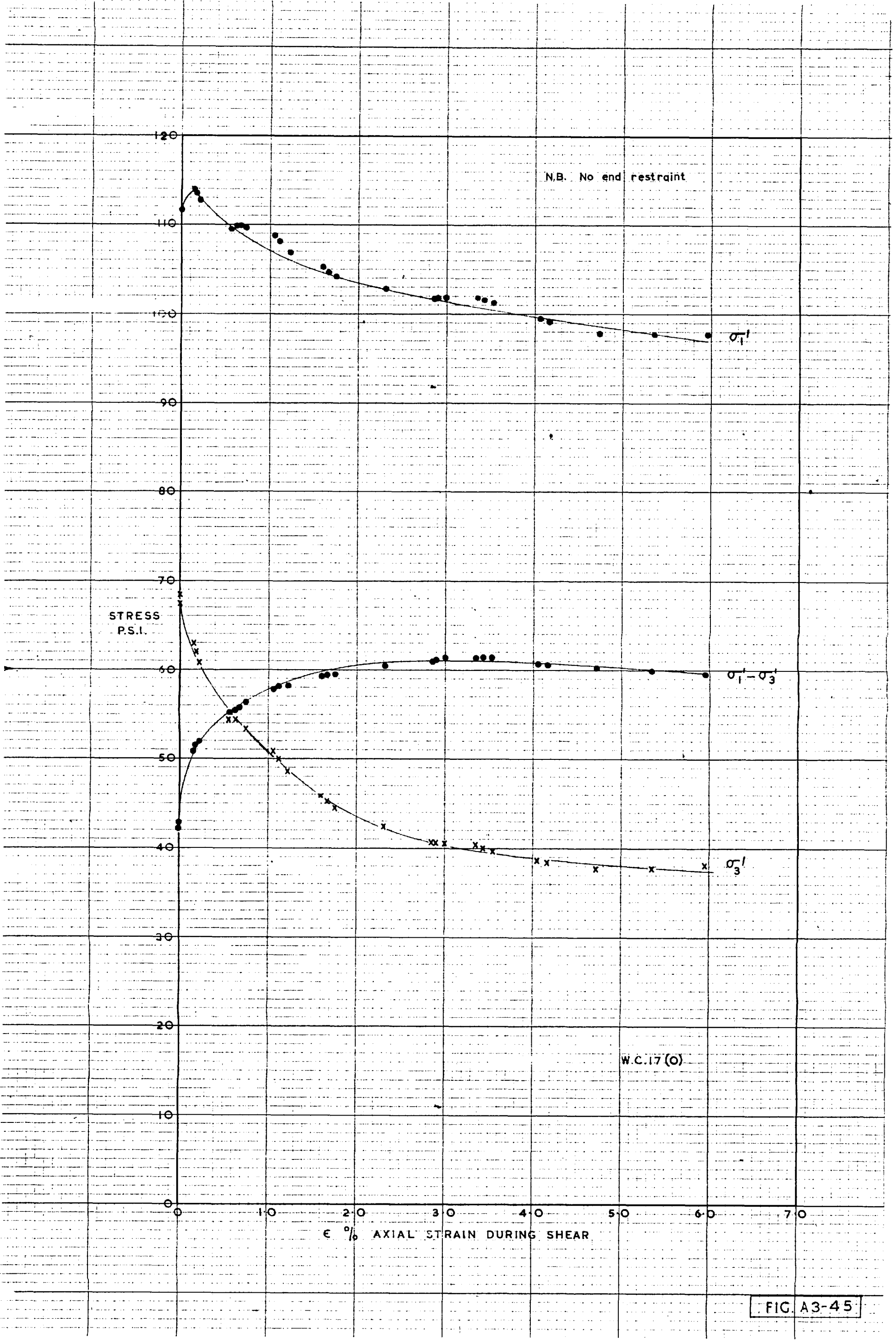
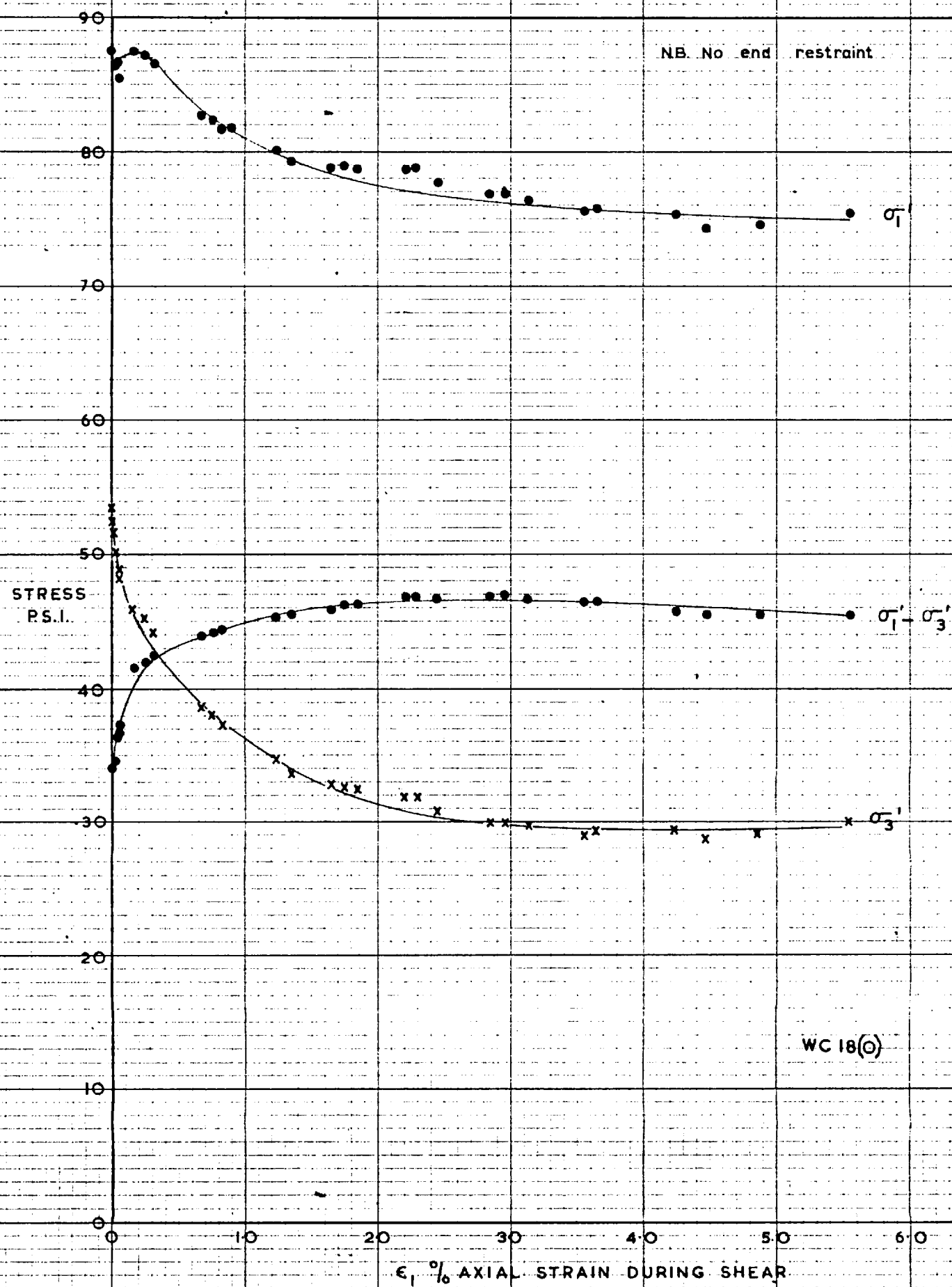


FIG. A3-45



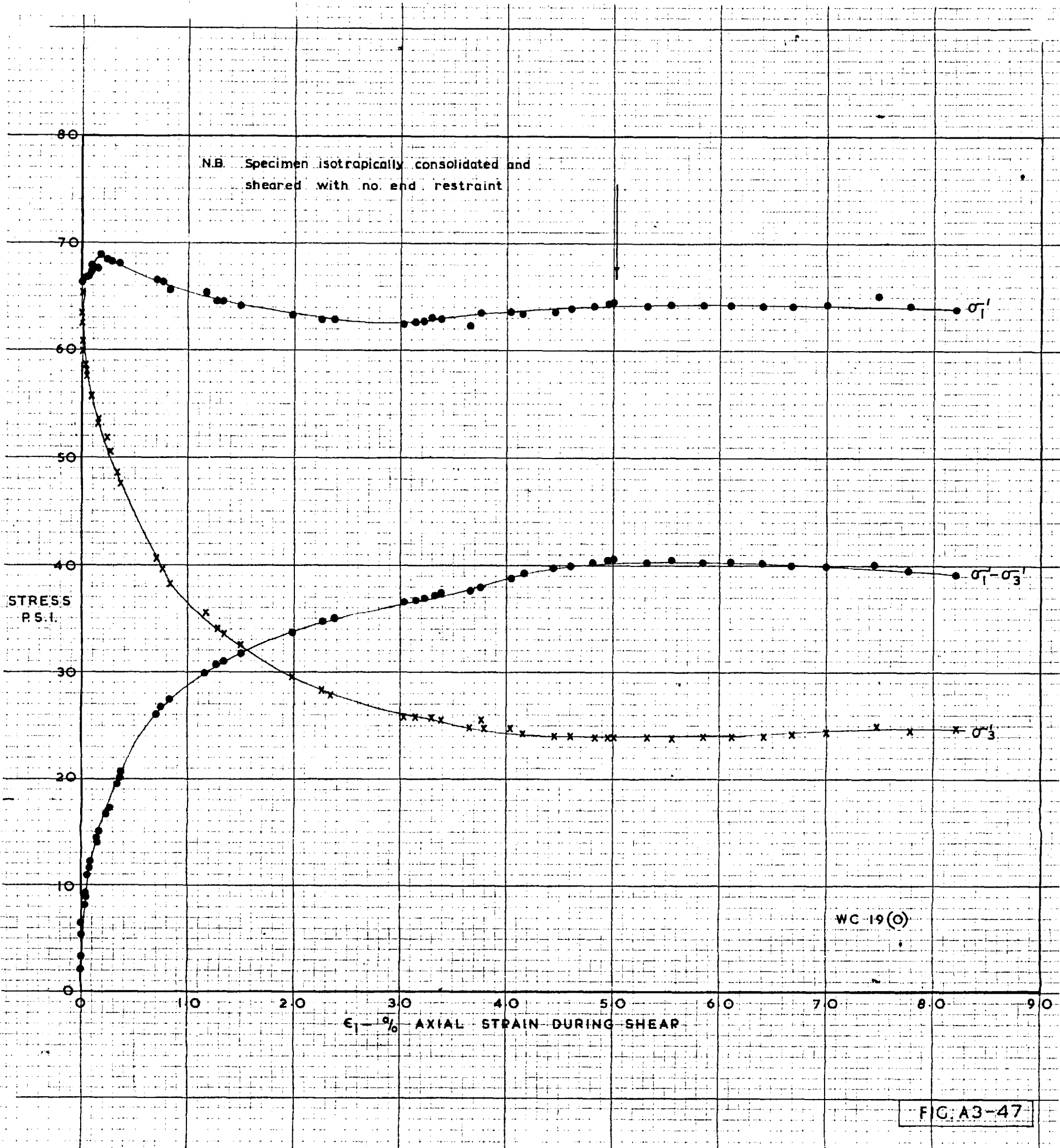


FIG. A3-47

APPENDIX 4

GRAPHICAL PRESENTATION OF THE RESULTS OF THE PLANE STRAIN AND
STANDARD TRIAXIAL DRAINED TESTS ON BELGIUM SAND (SABLE DE MOL)

N.B. The energy correction suggested by BISHOP (1950)
has not been applied to the test results.

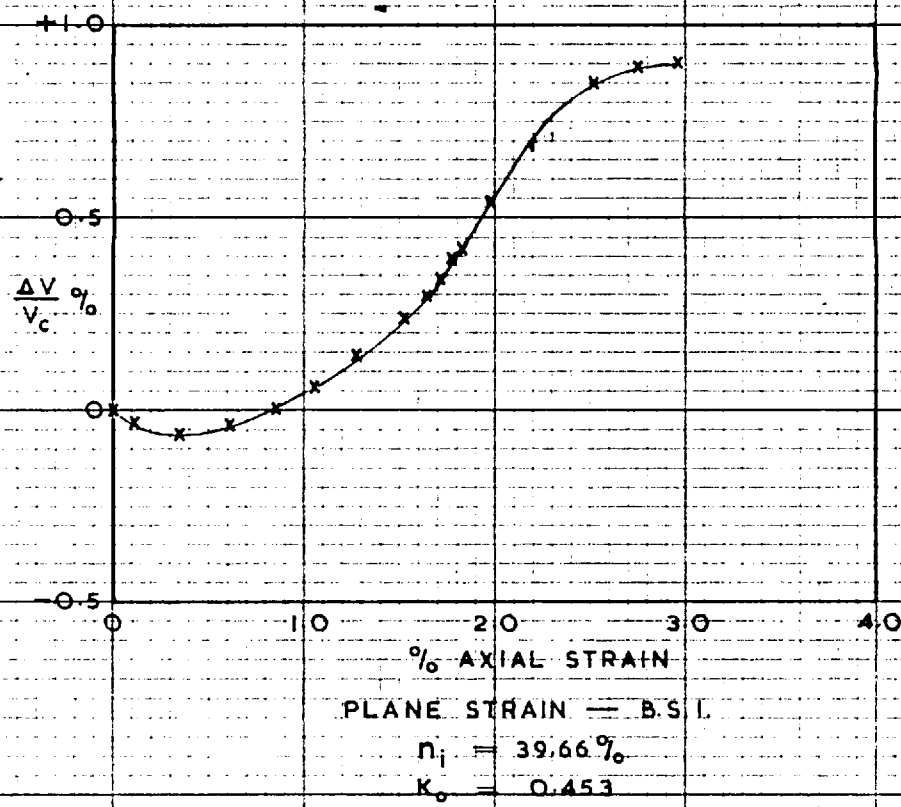
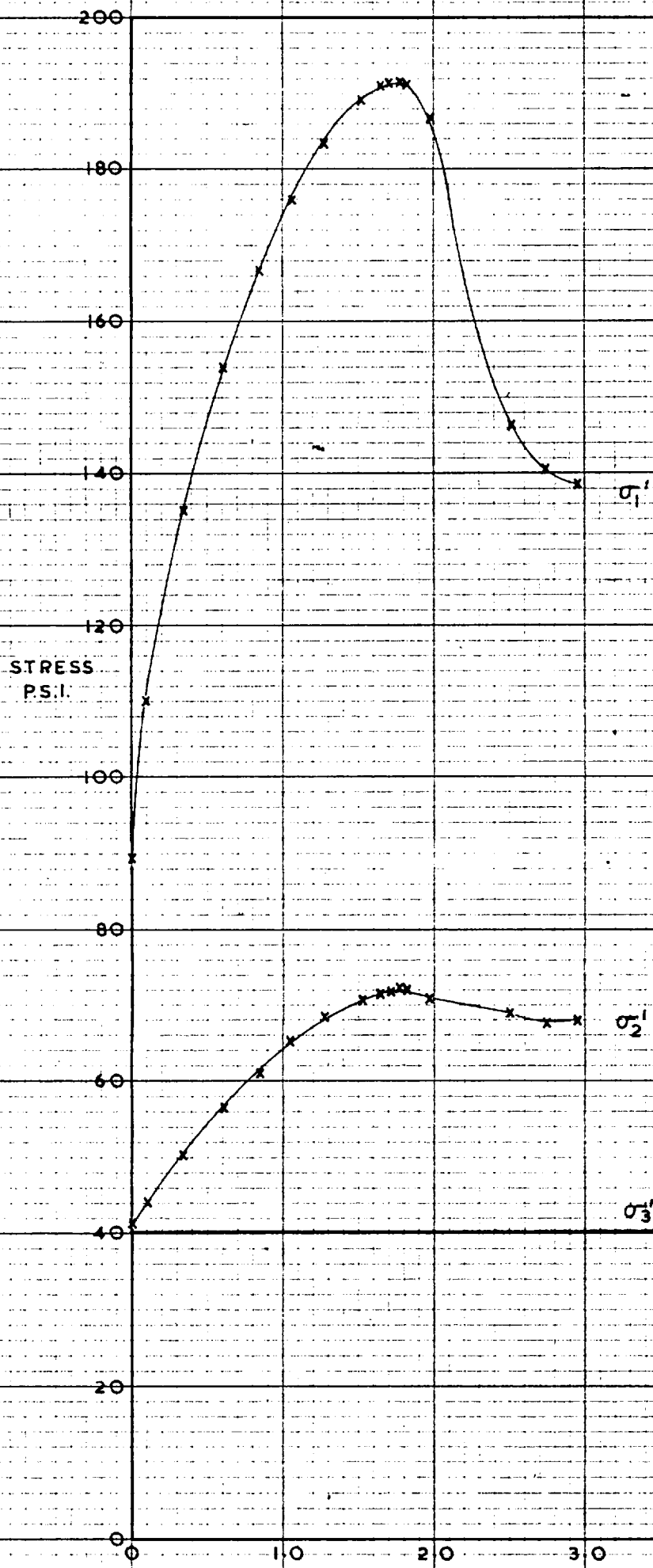


FIG A4-1

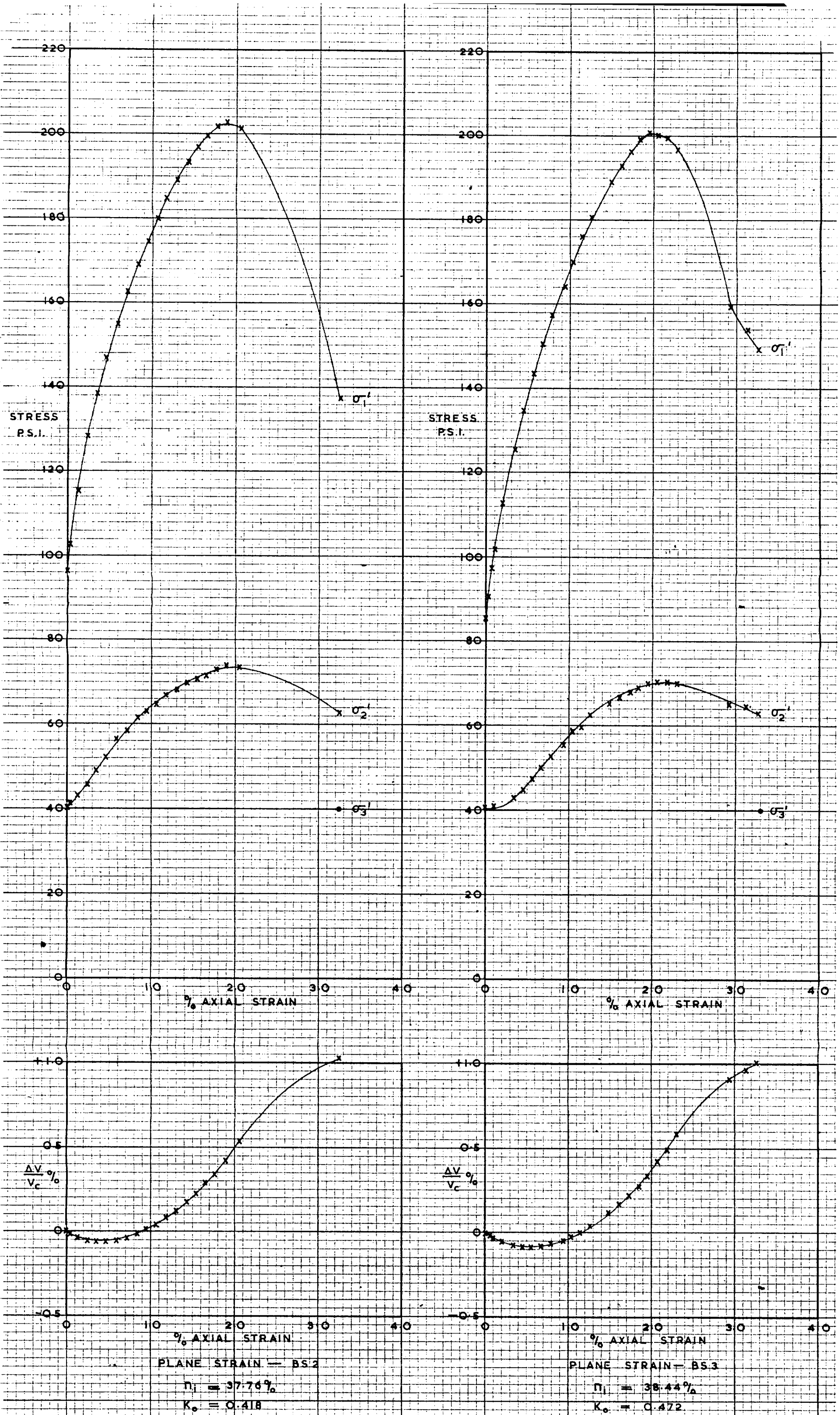


FIG A4-2

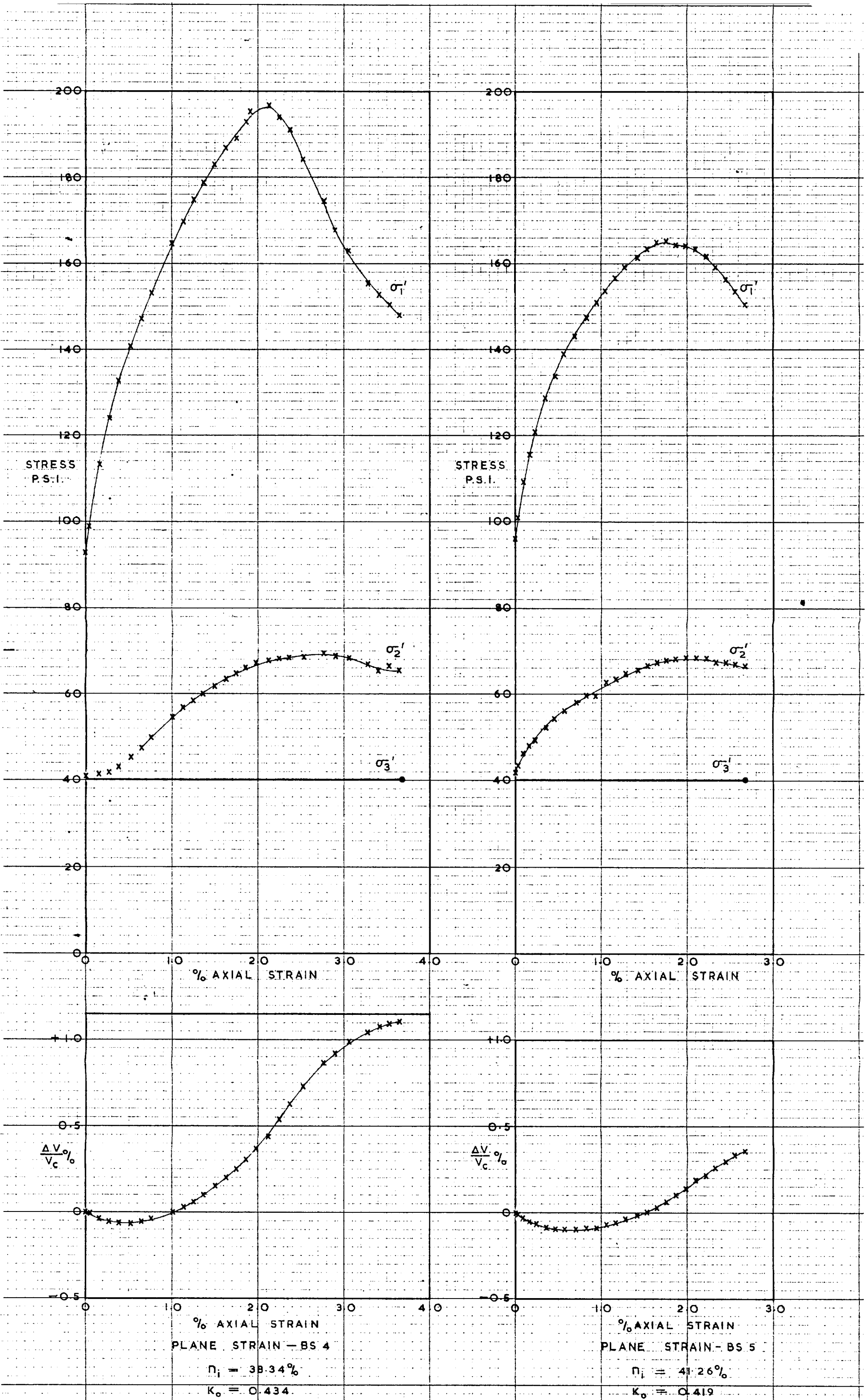


FIG A4-3

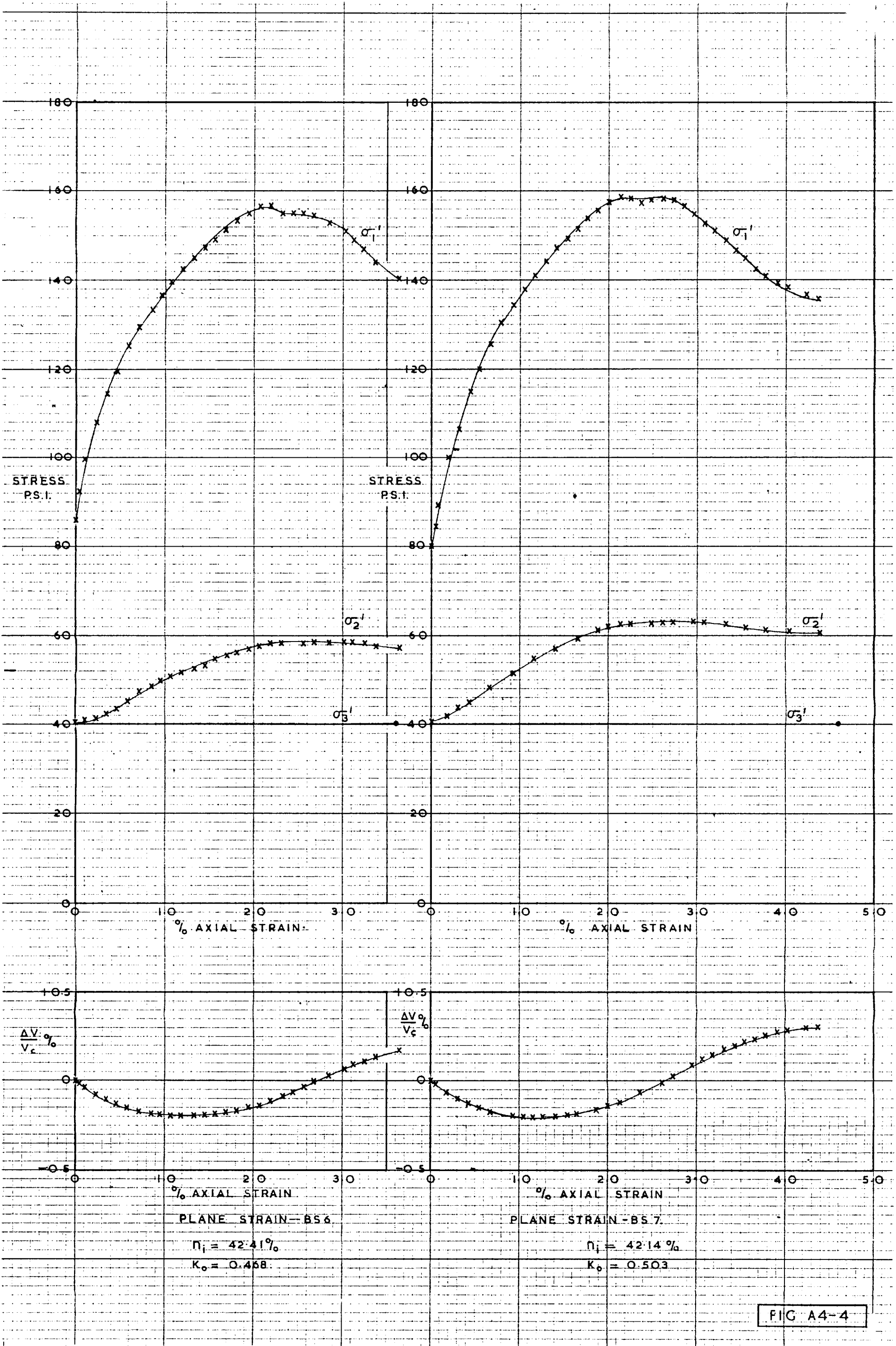


FIG A4-4

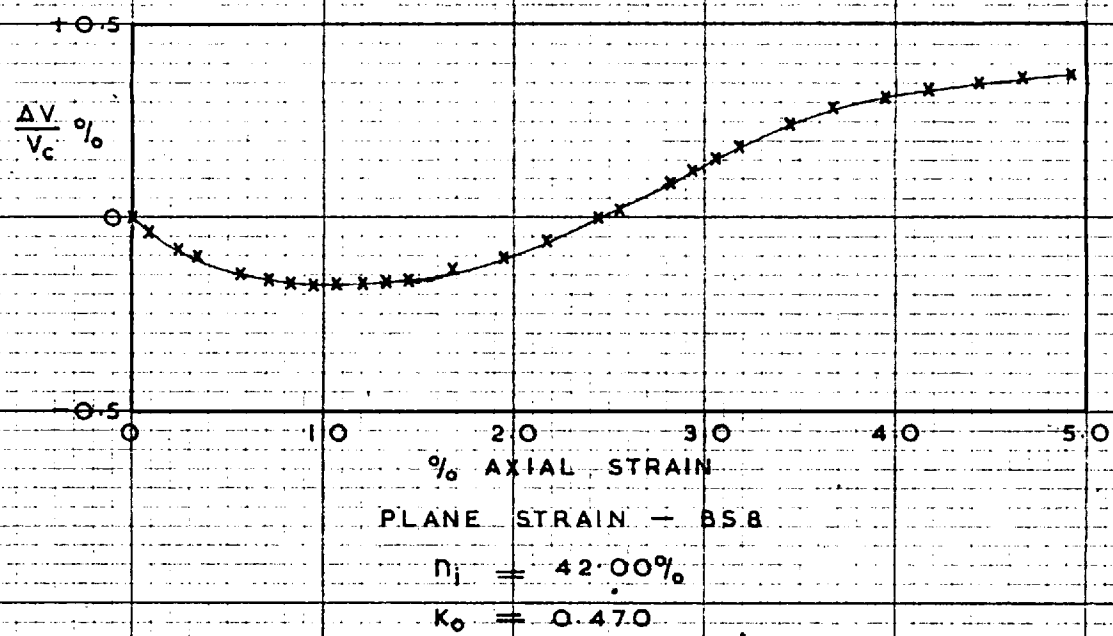
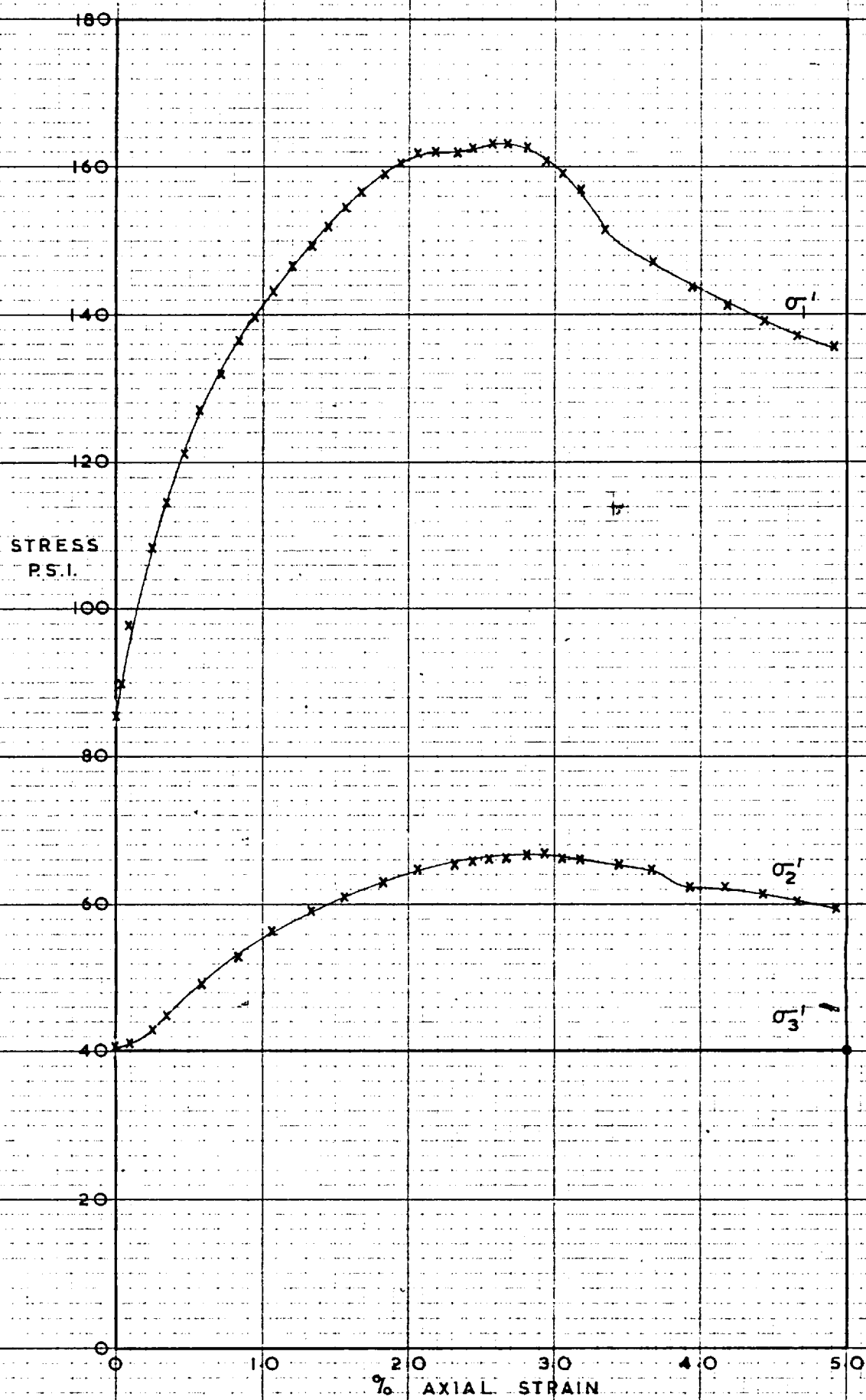
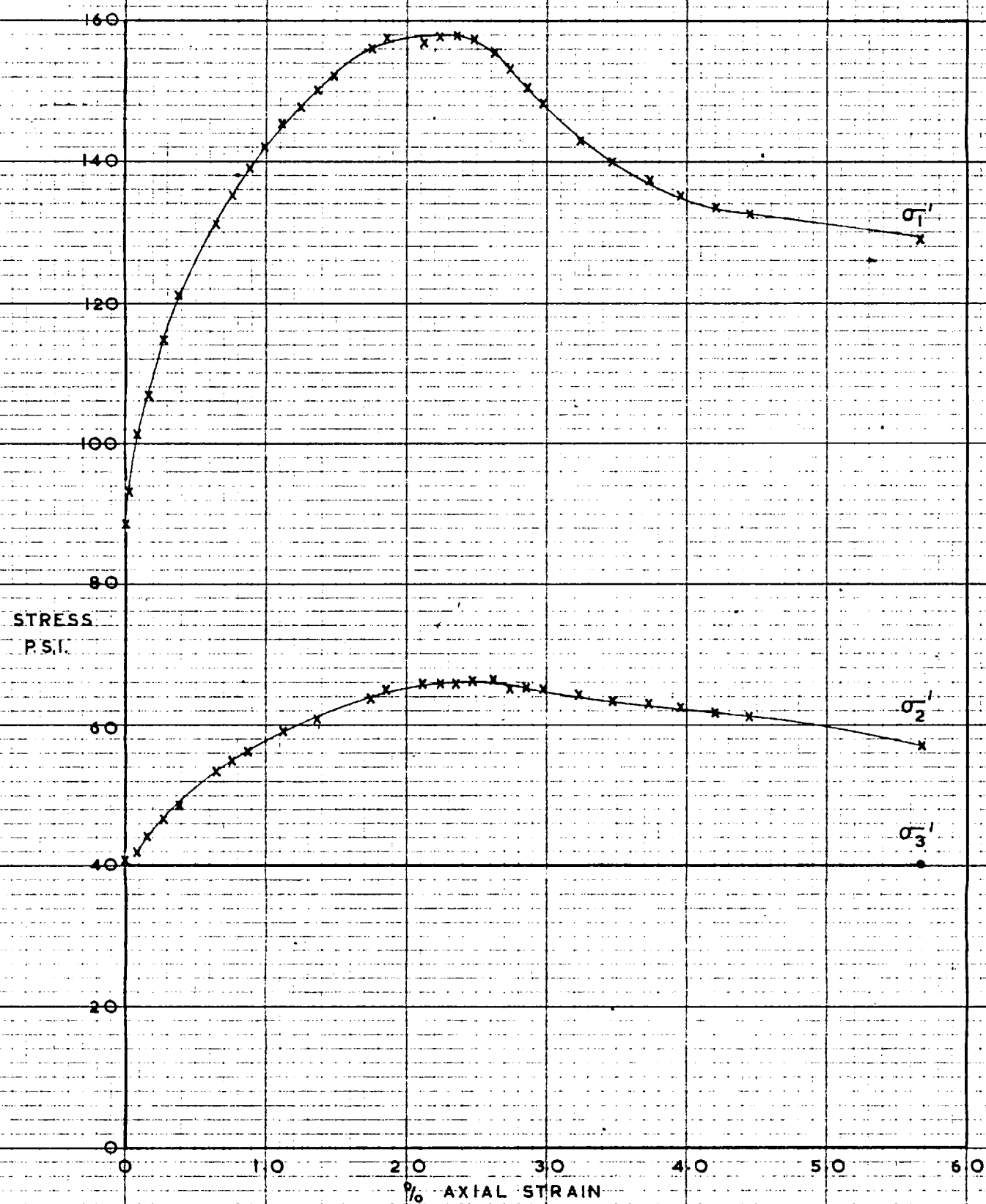


FIG A4-5



PLANE STRAIN — B.S. 9.
 $n_i = 42.13\%$
 $K_0 = 0.454$

FIG A4-6

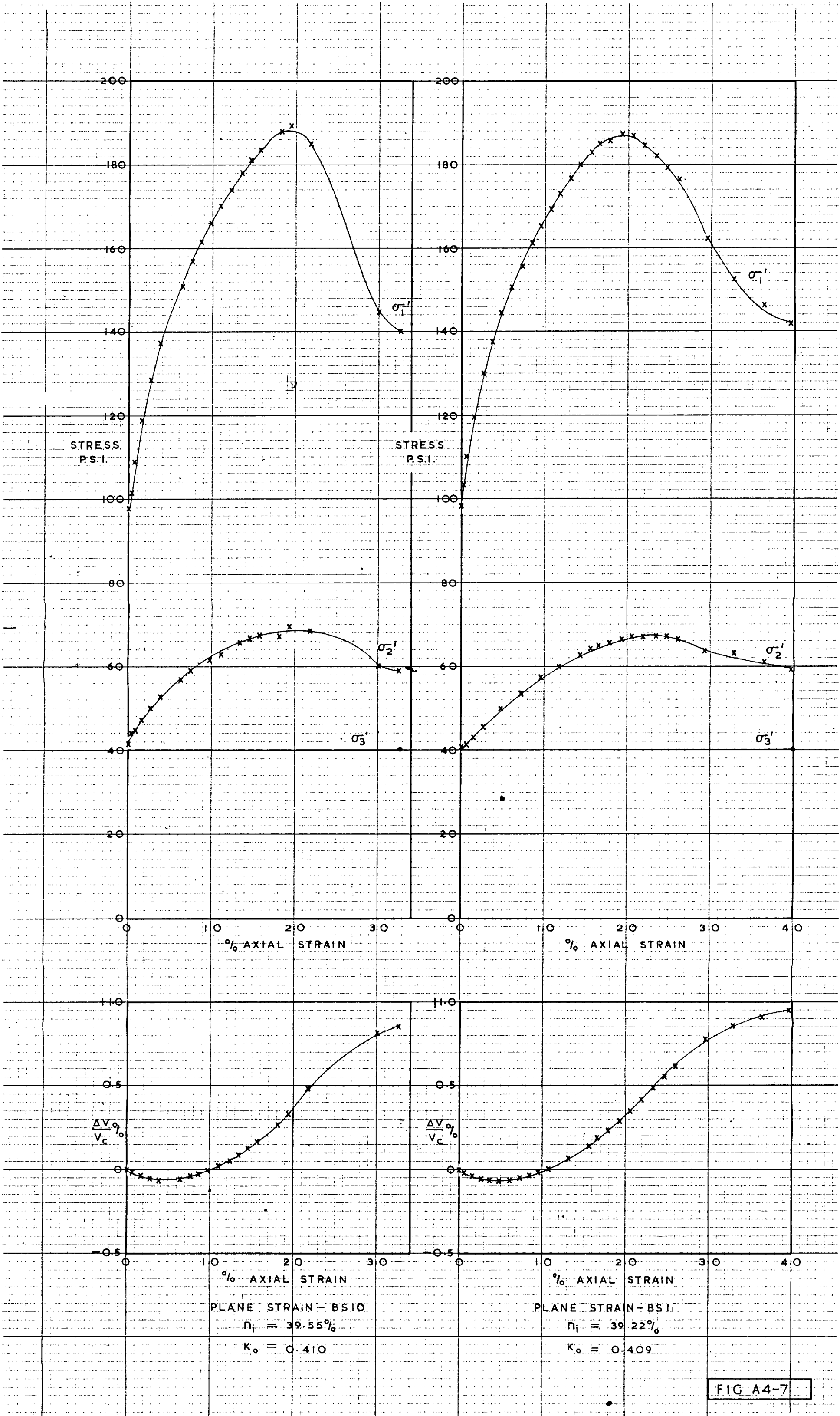


FIG A4-7

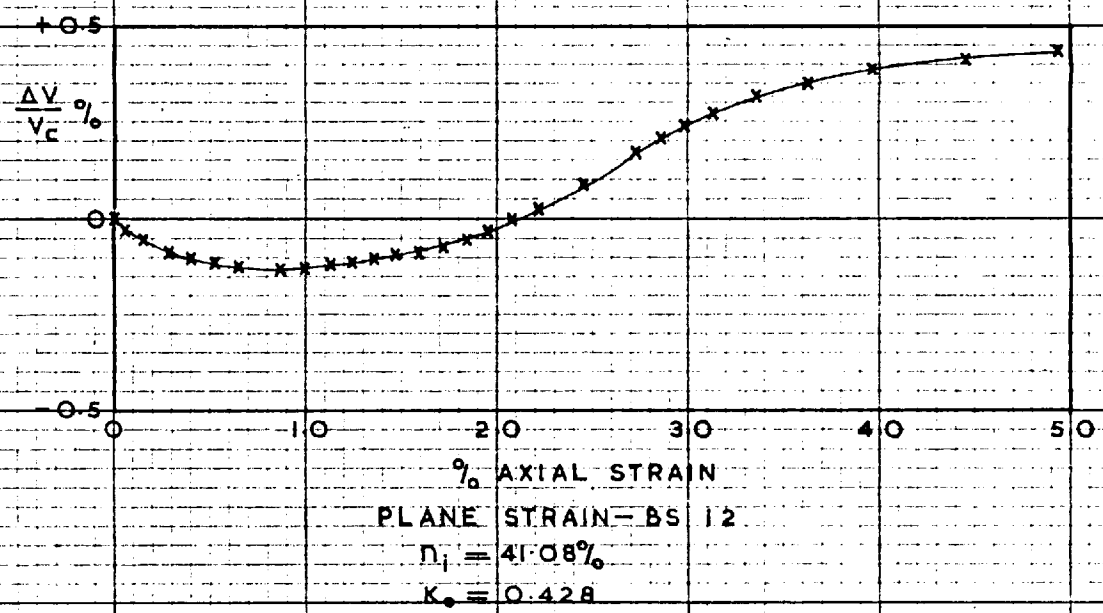
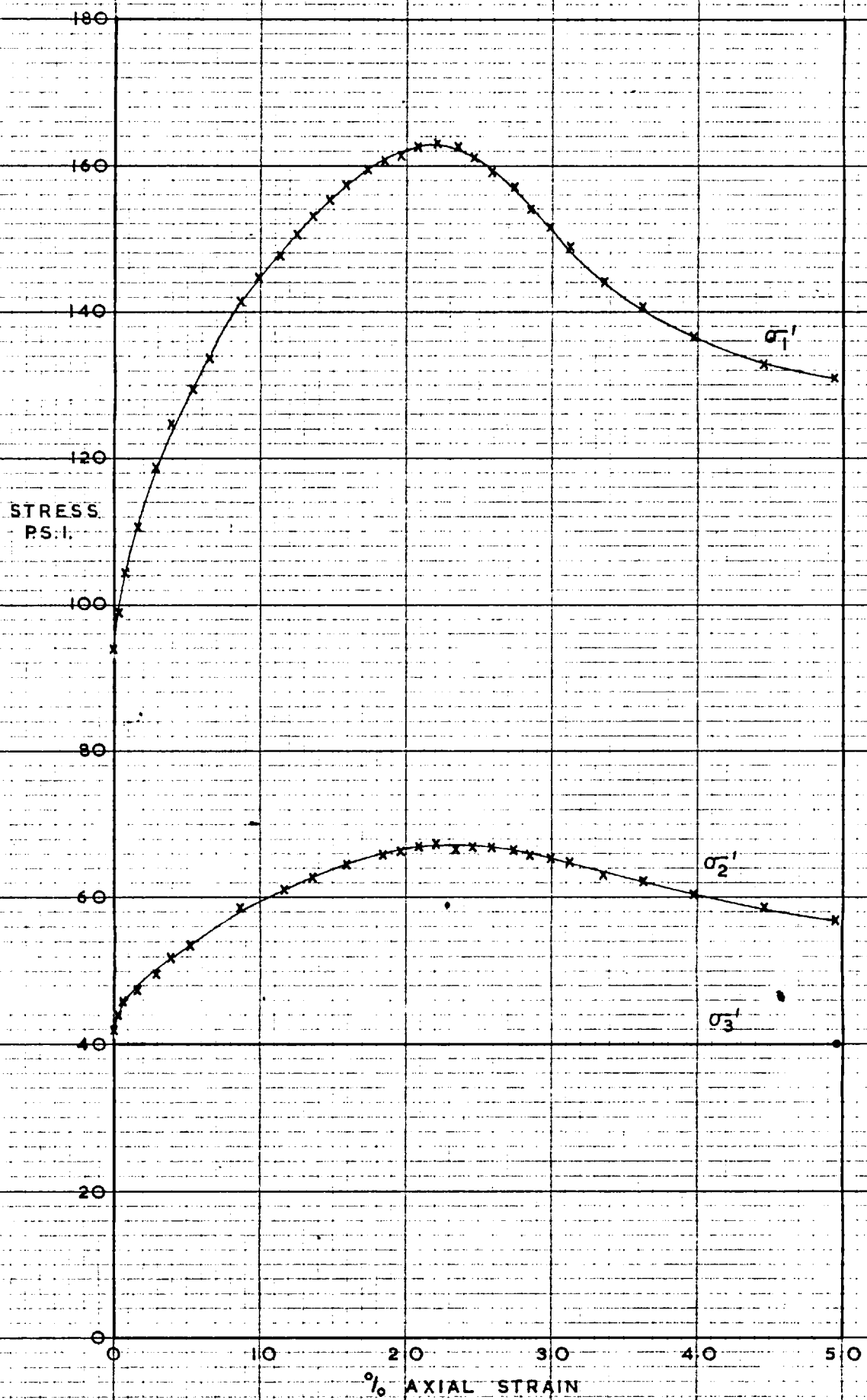
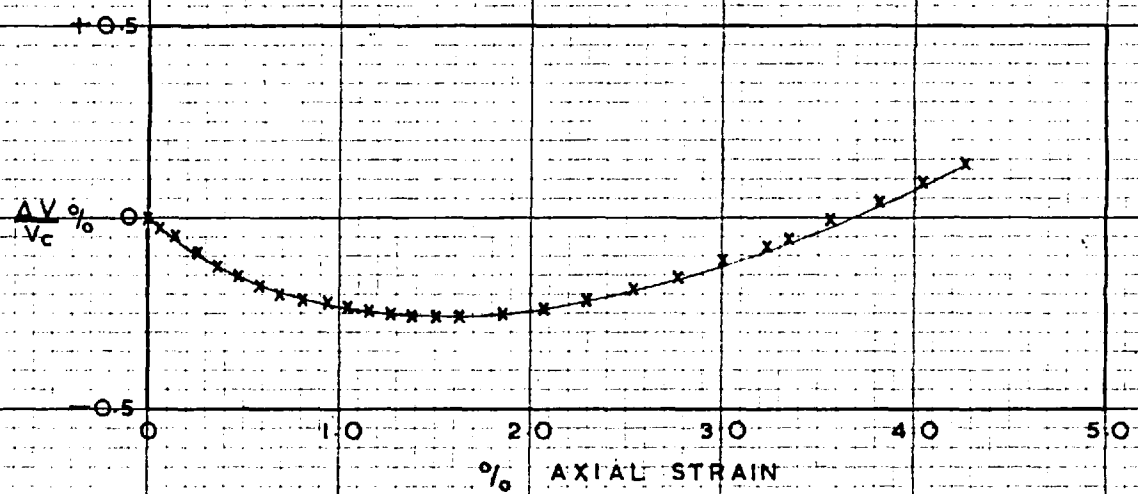
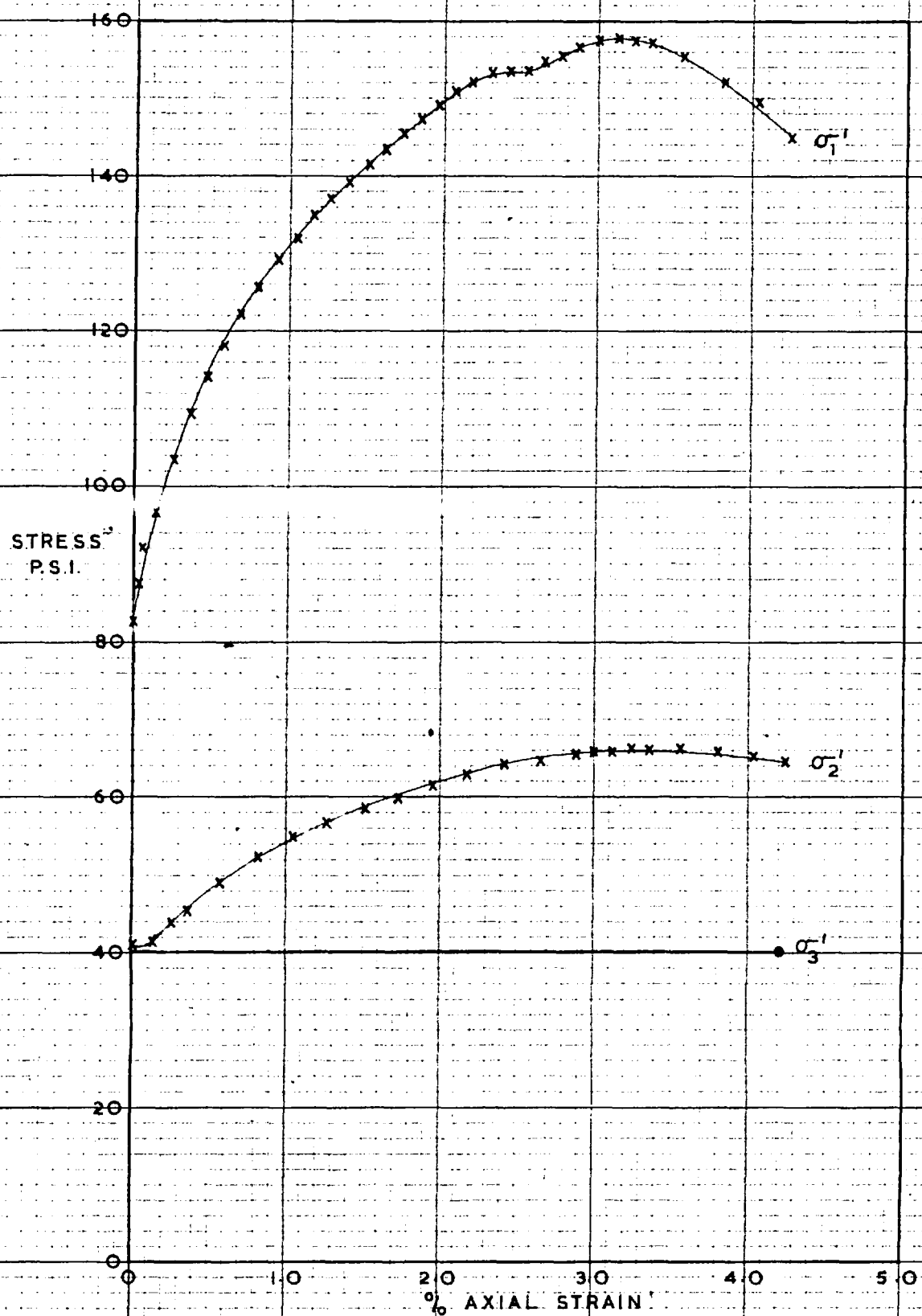


FIG A4-8



PLANE STRAIN — B.S. 14

$n_1 = 42.27\%$

$K_0 = 0.485$

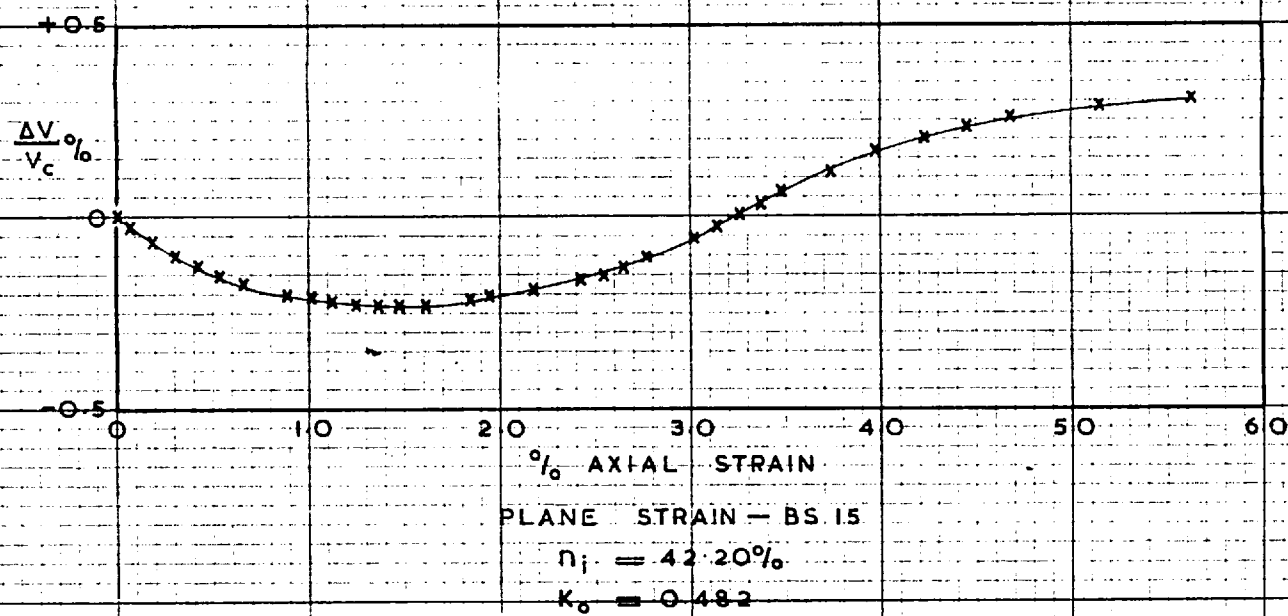
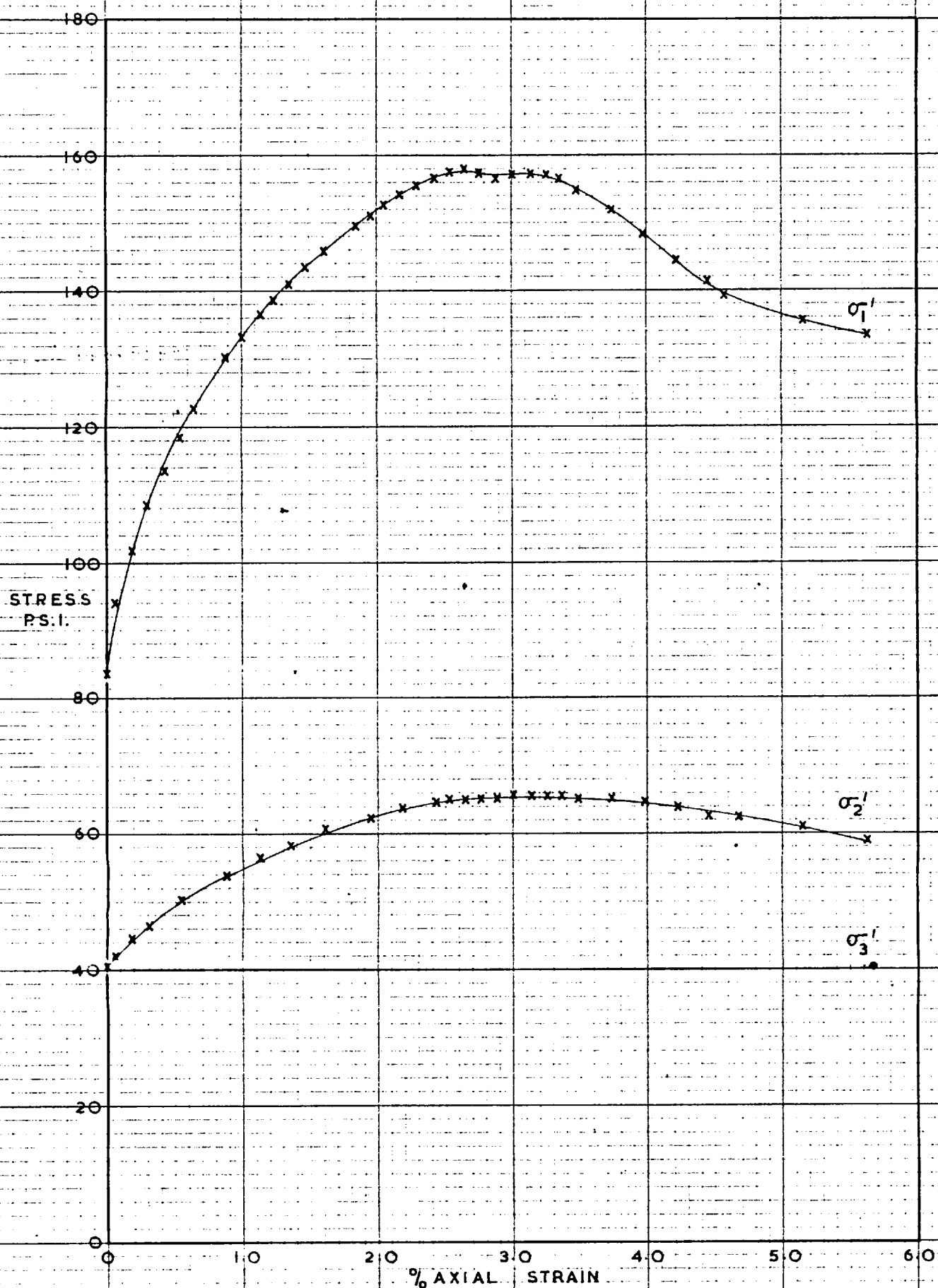
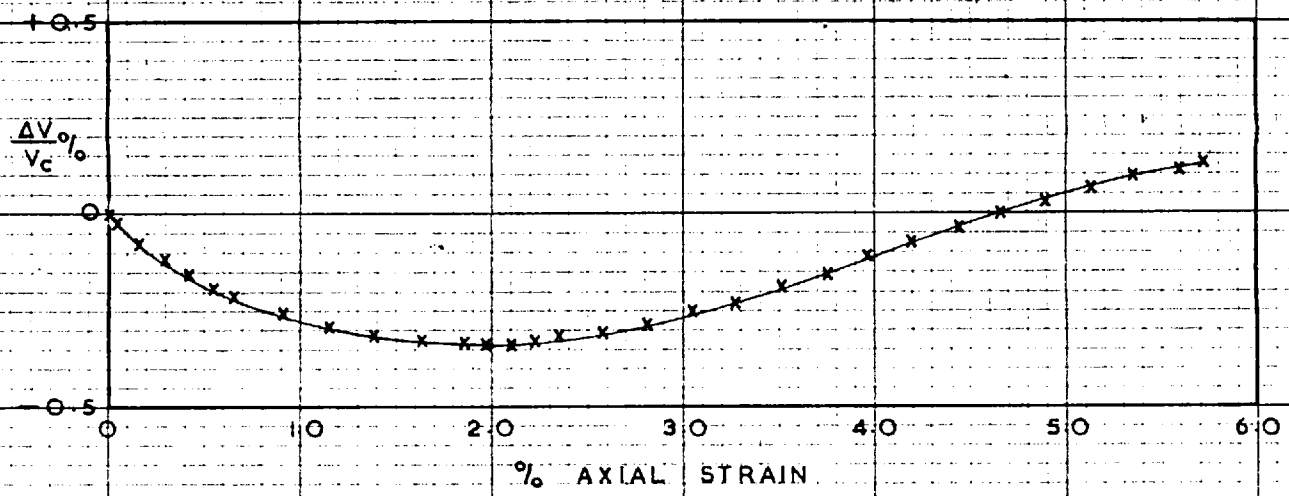
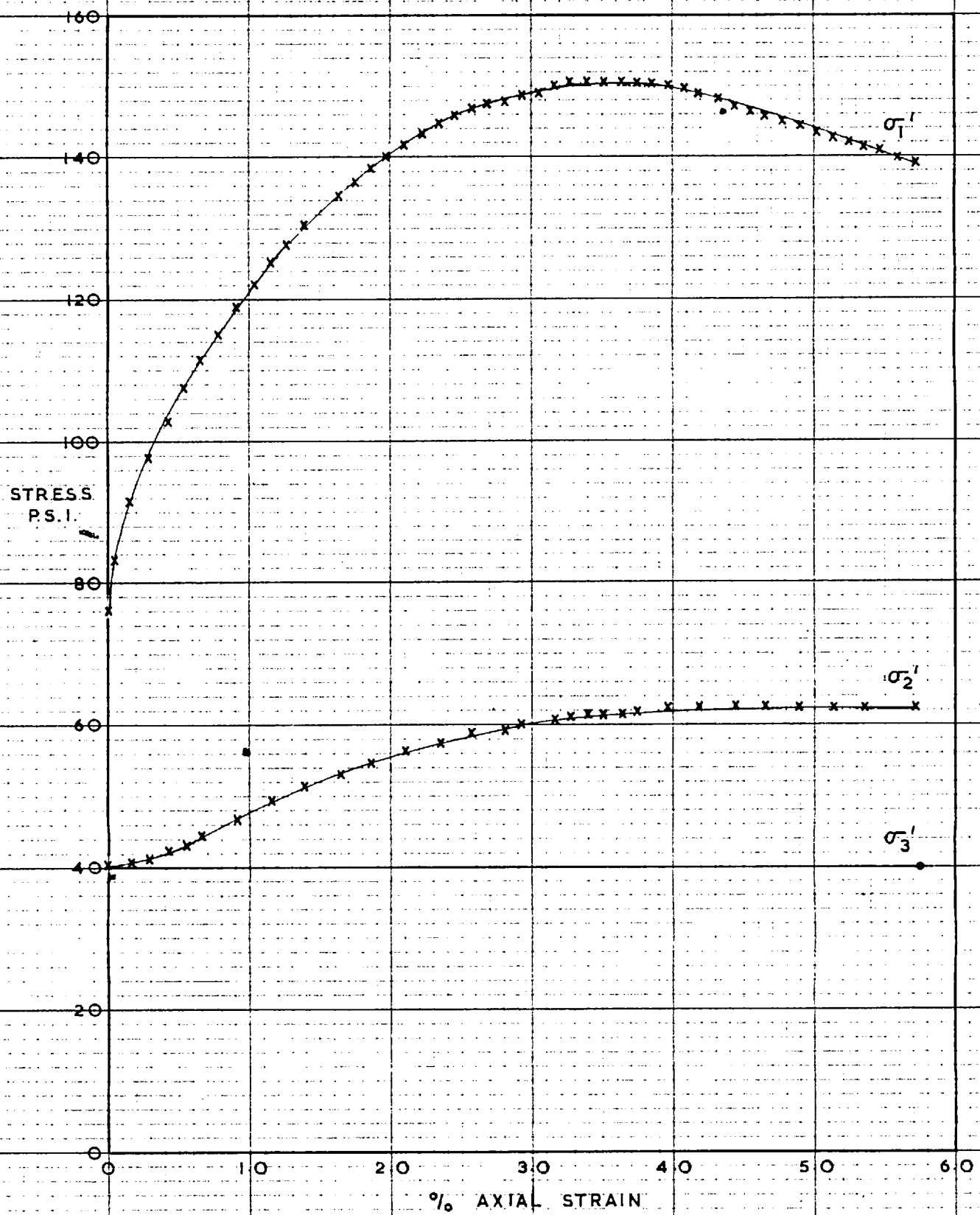


FIG A4-10



PLANE STRAIN - BS 16

$$\eta_1 = 42.52\%$$

$$K_0 = 0.528$$

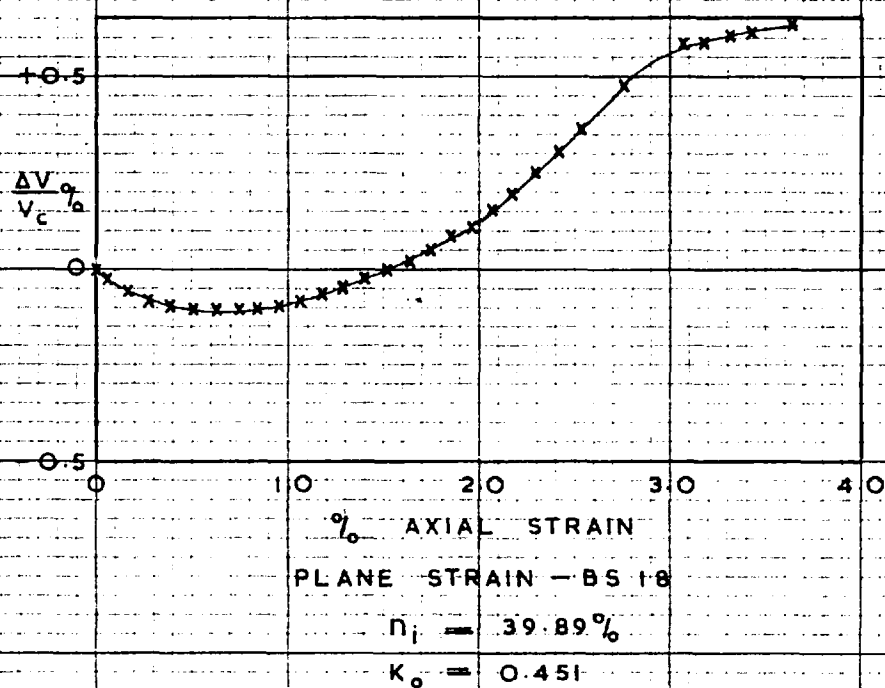
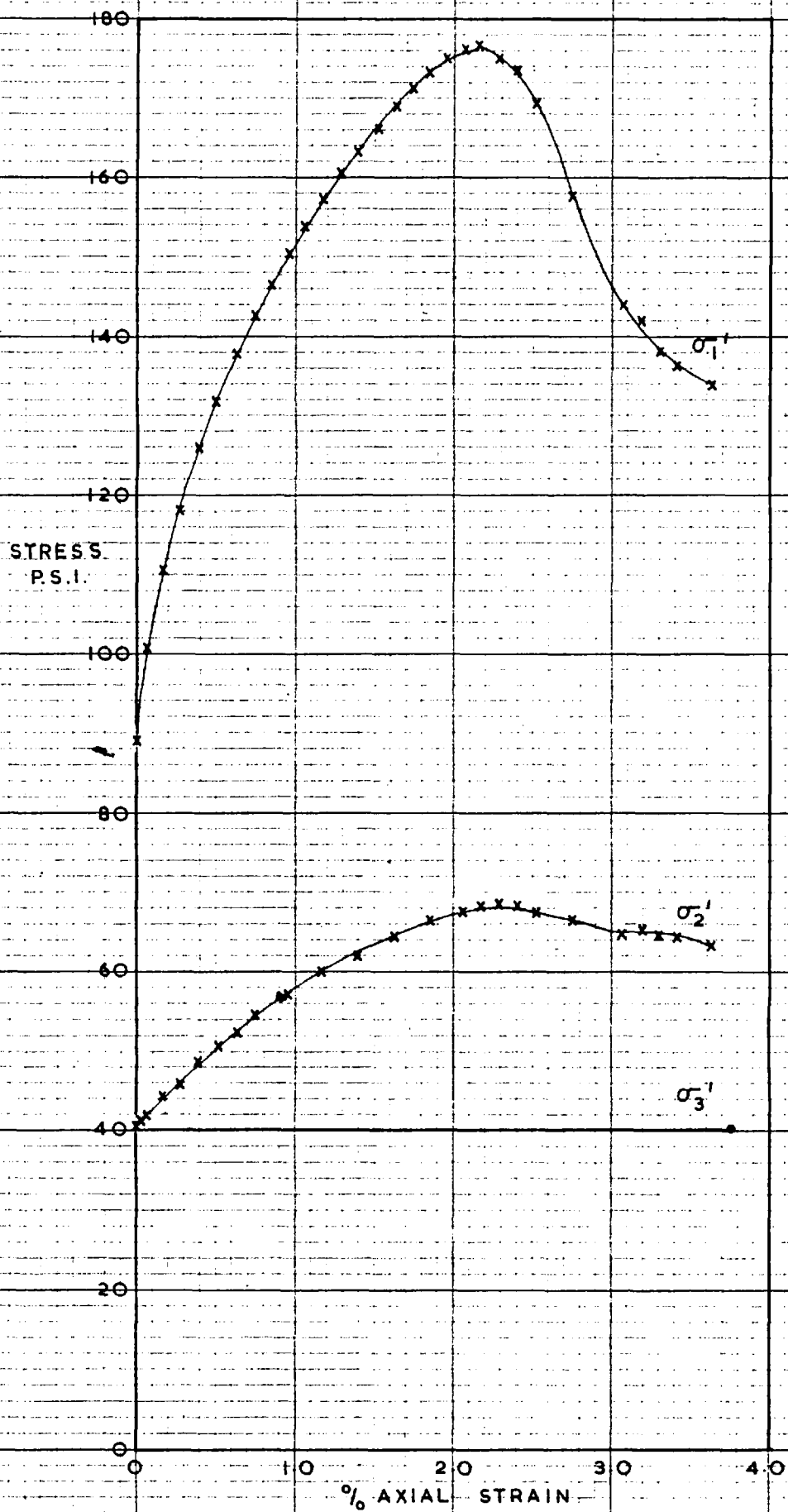


FIG. A4-12

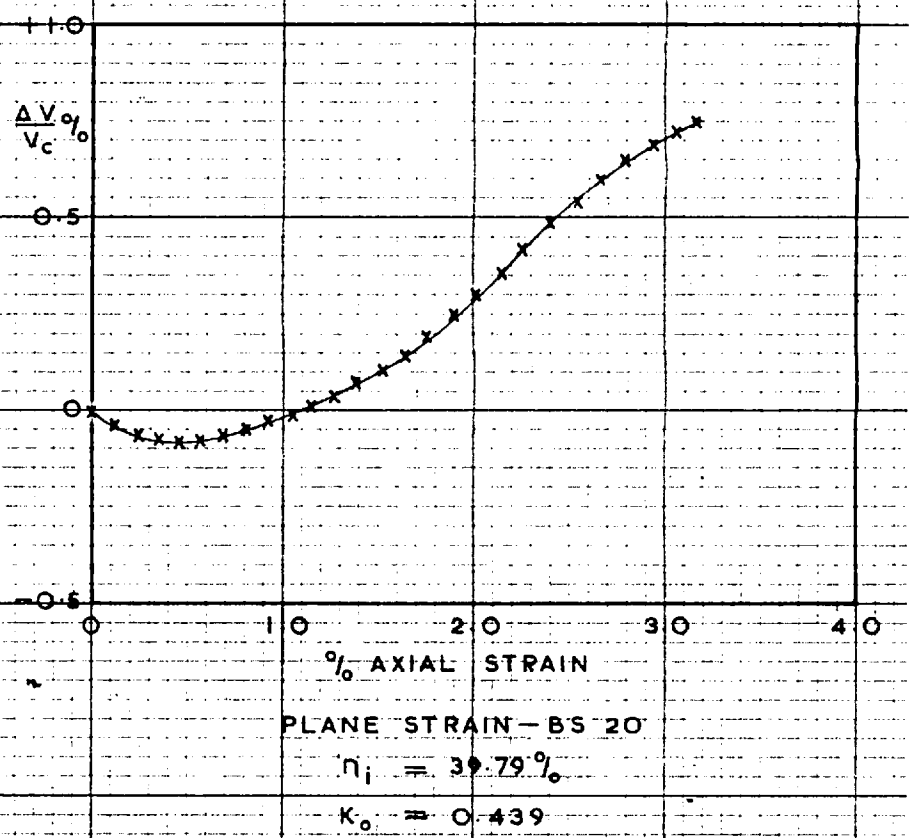
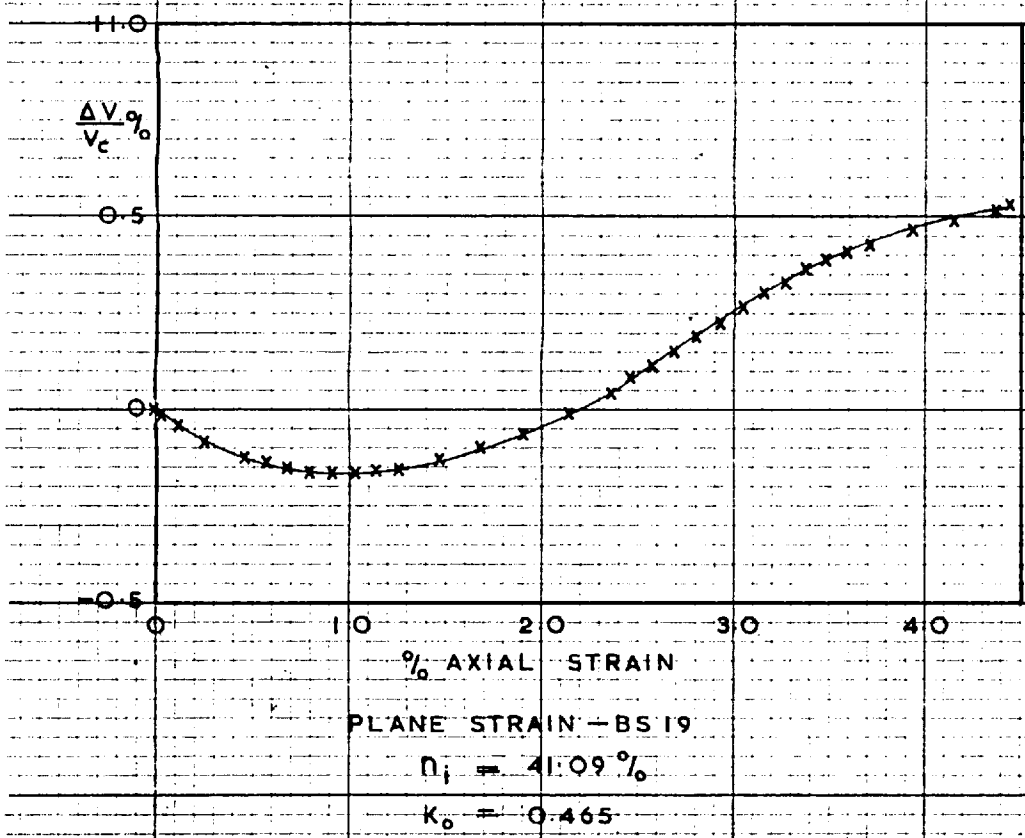
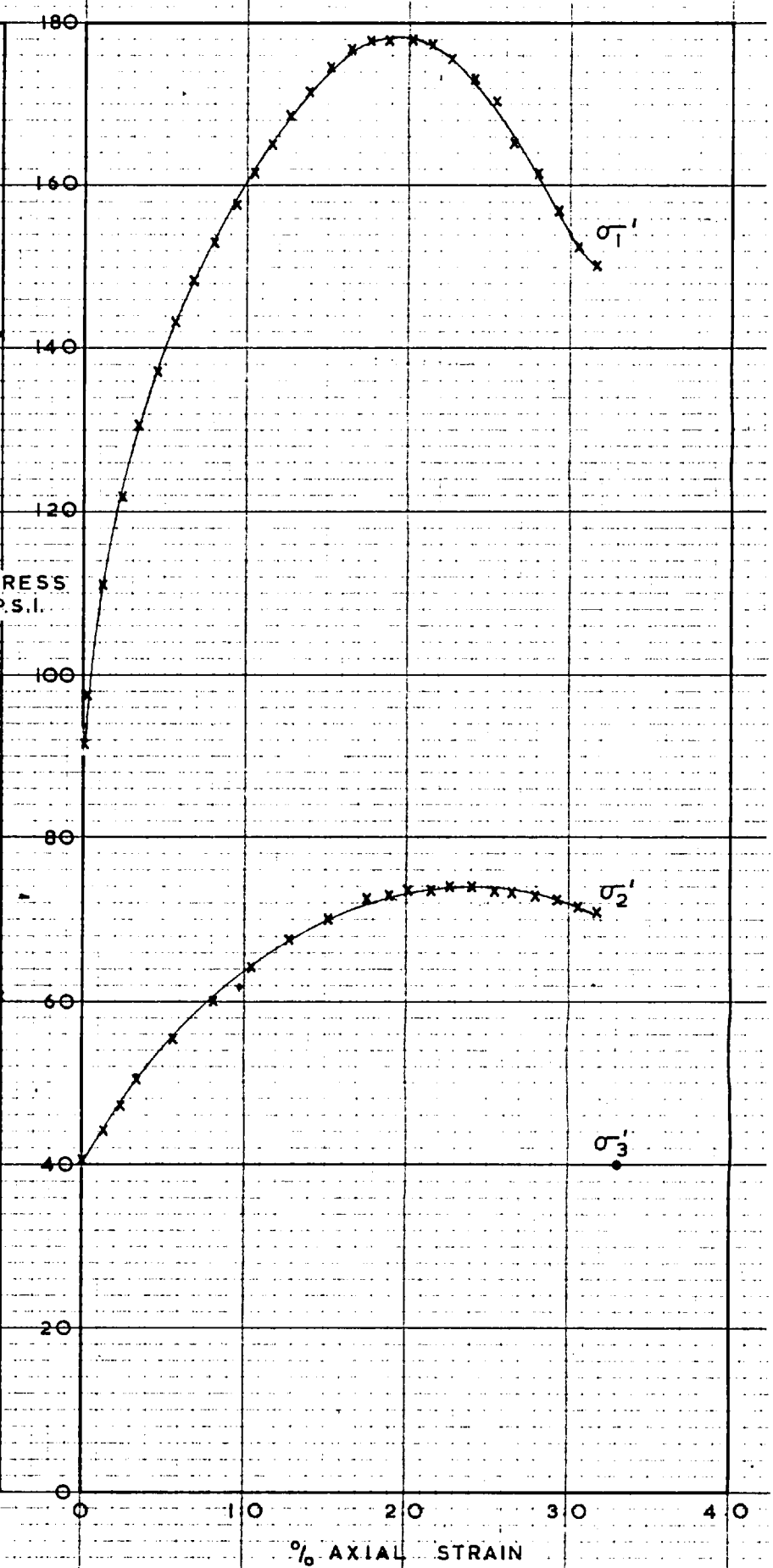
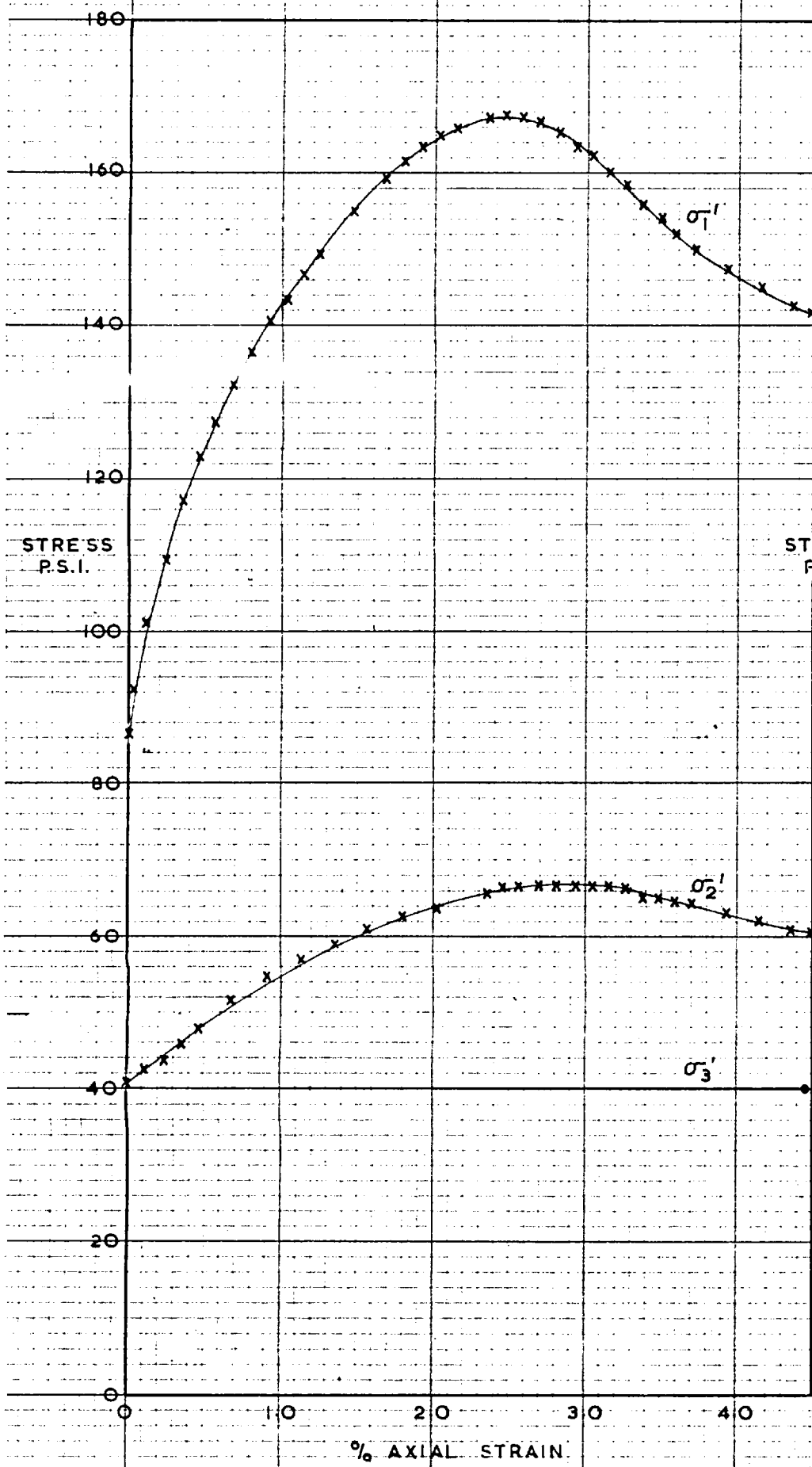


FIG A4-13

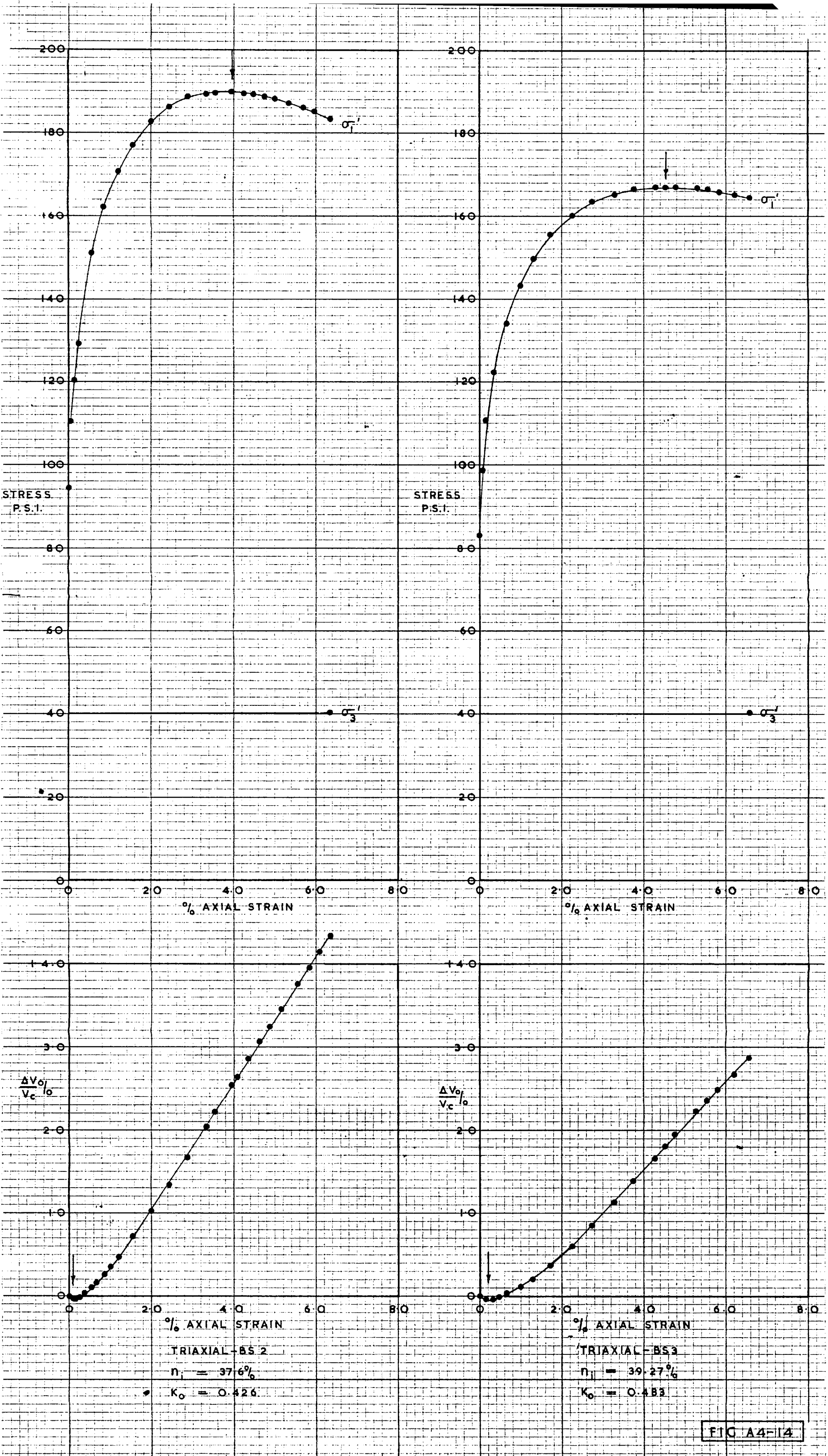
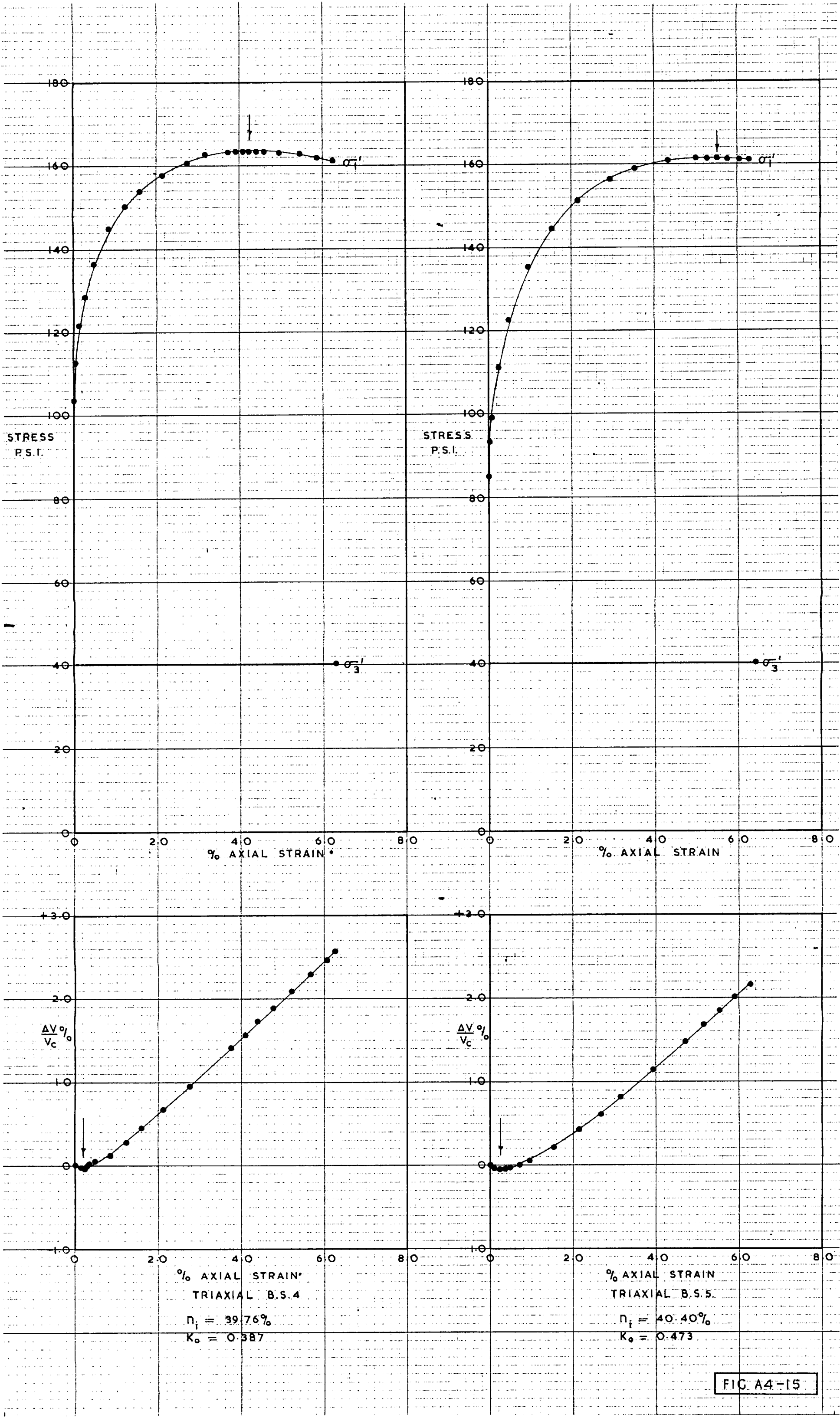


FIG A4-14



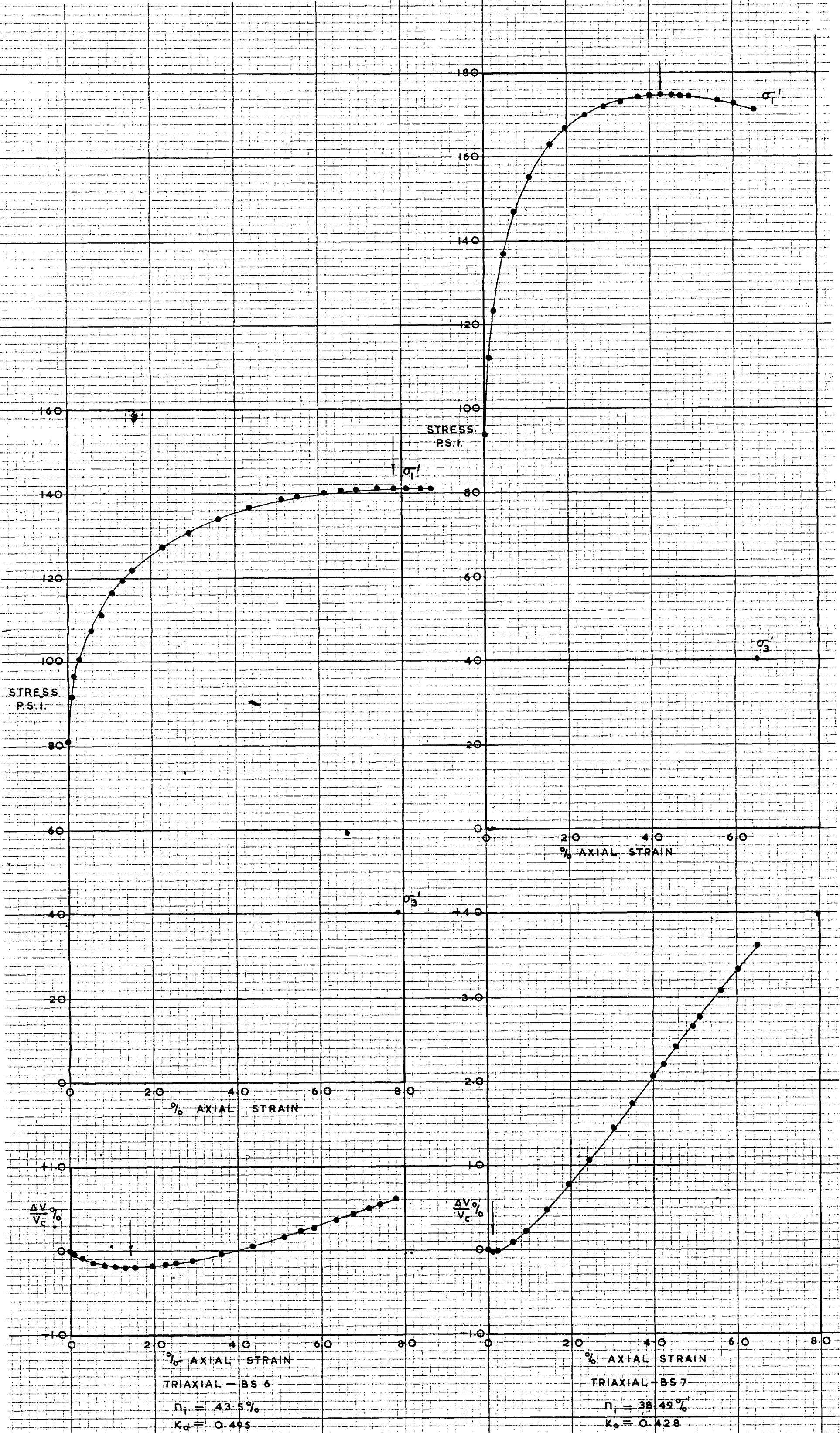


FIG A4-16

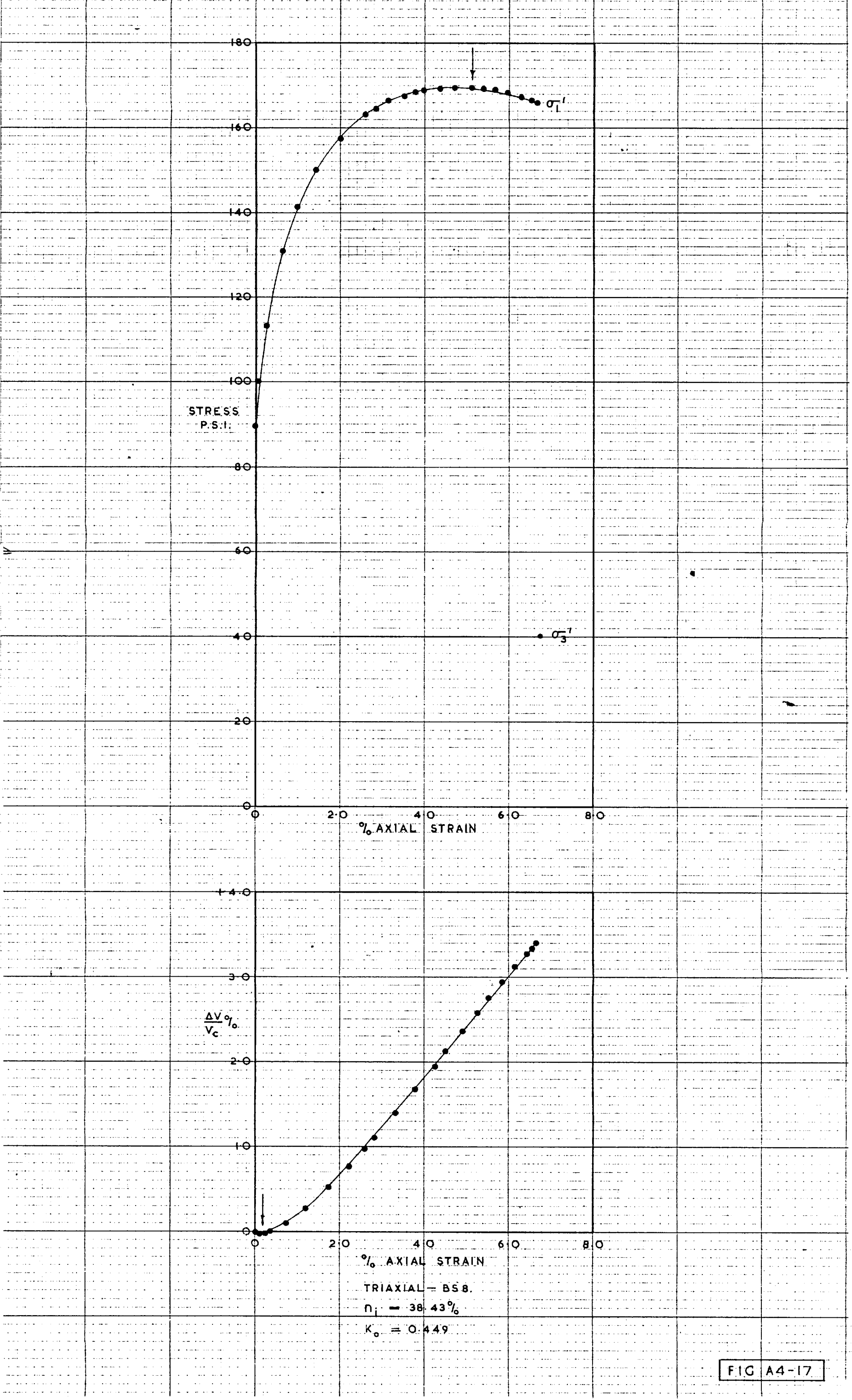


FIG A4-17

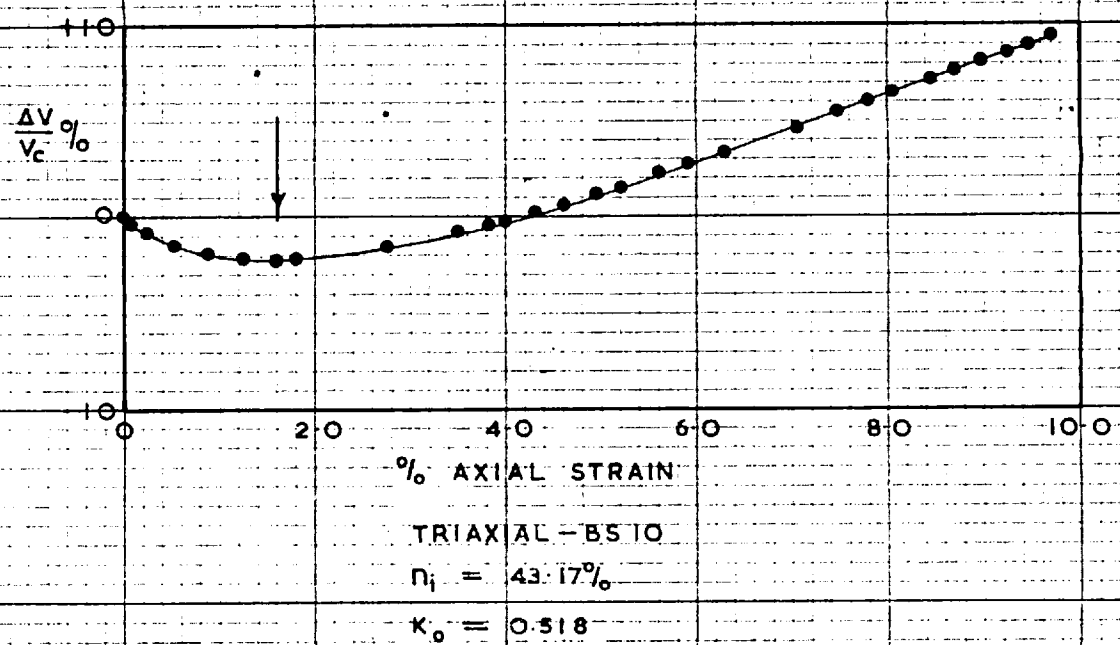
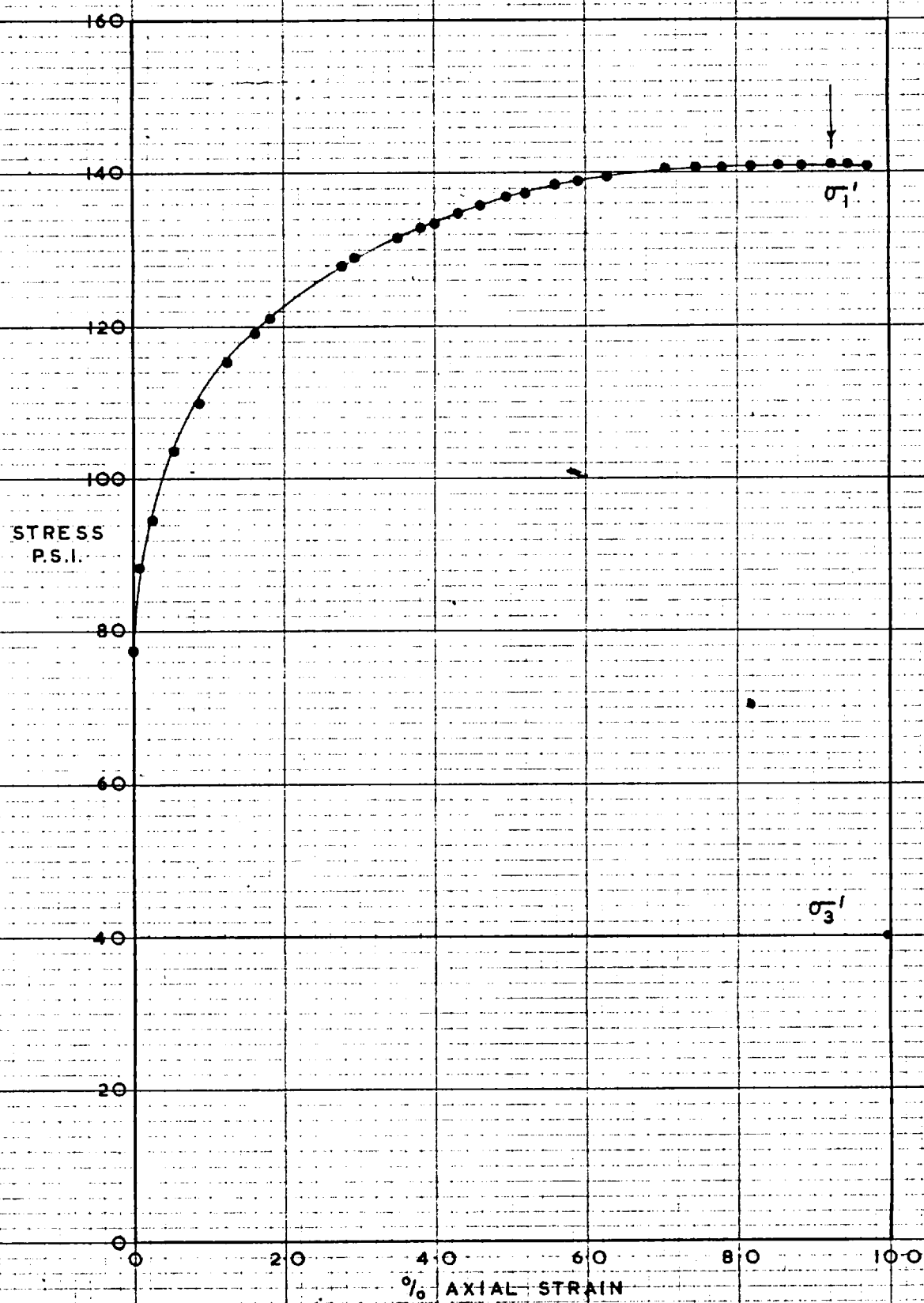


FIG A4-18

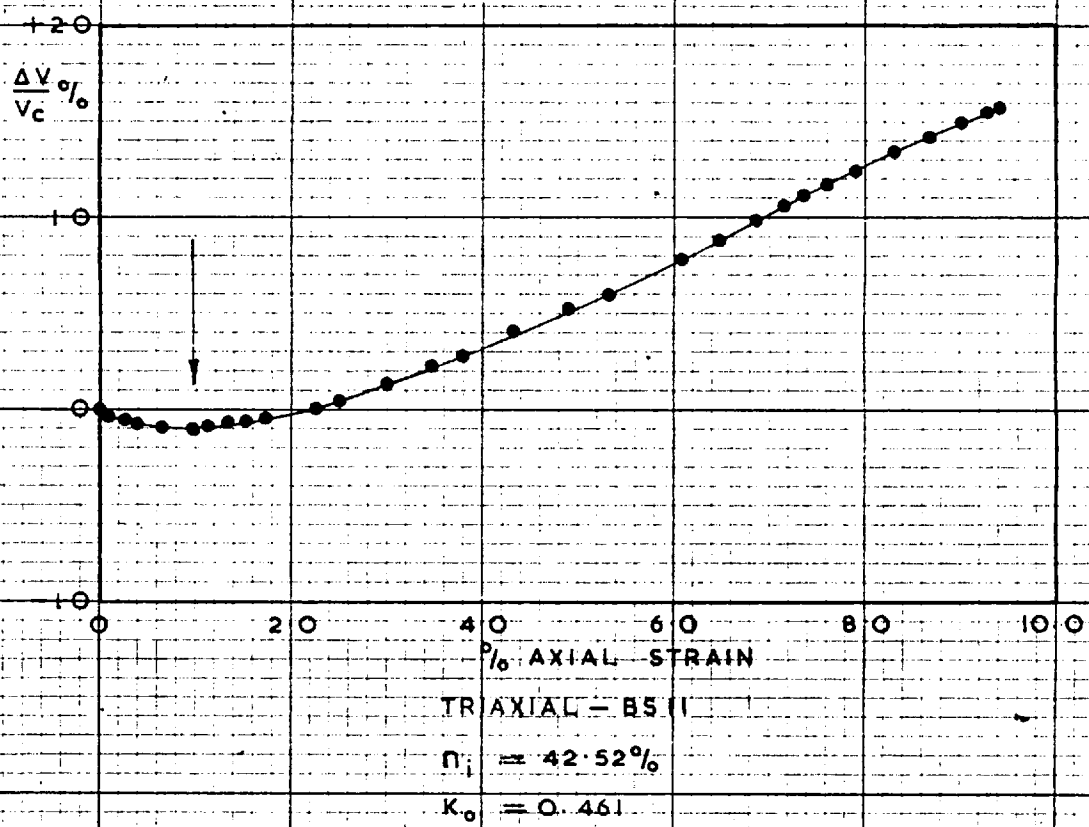
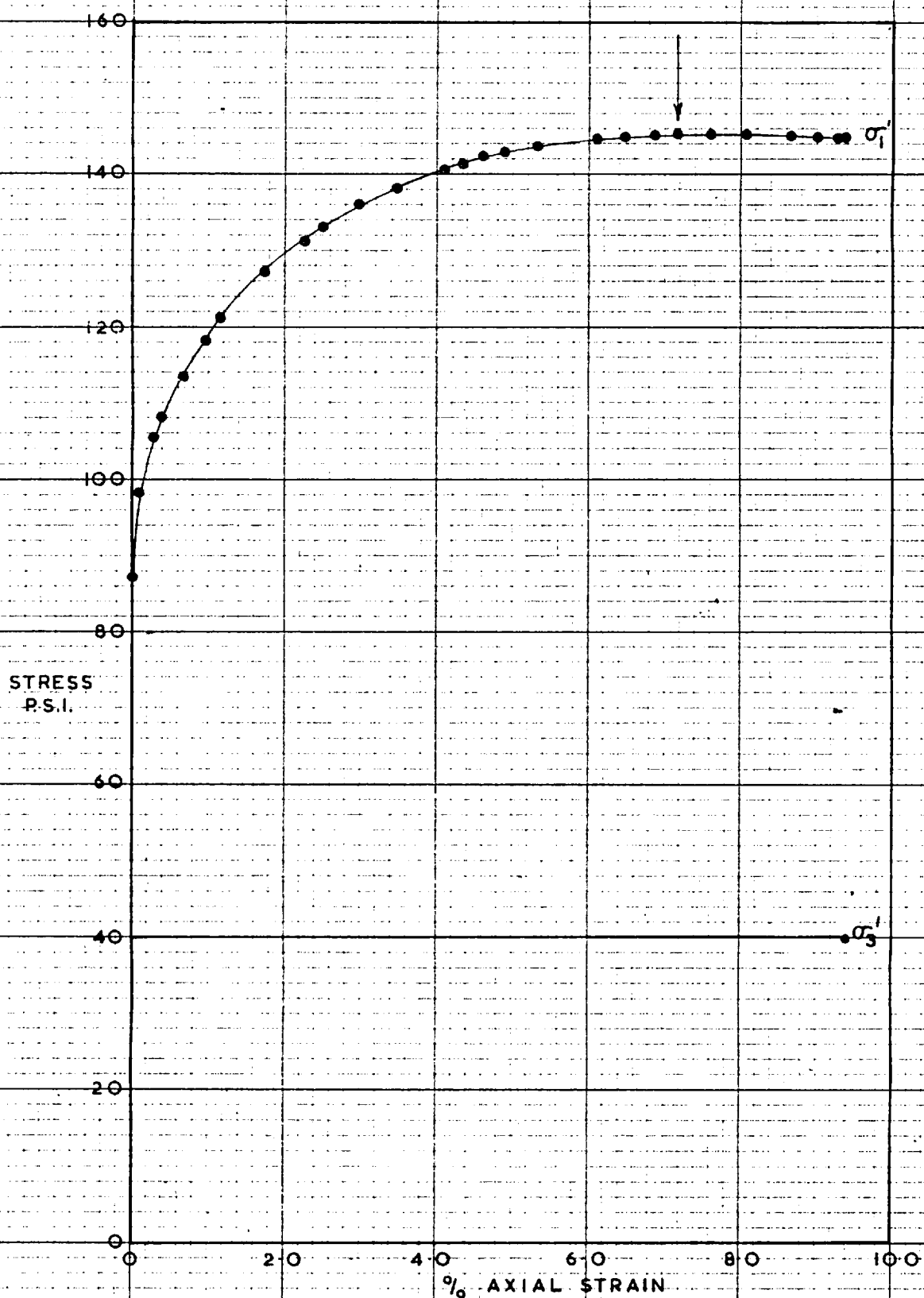
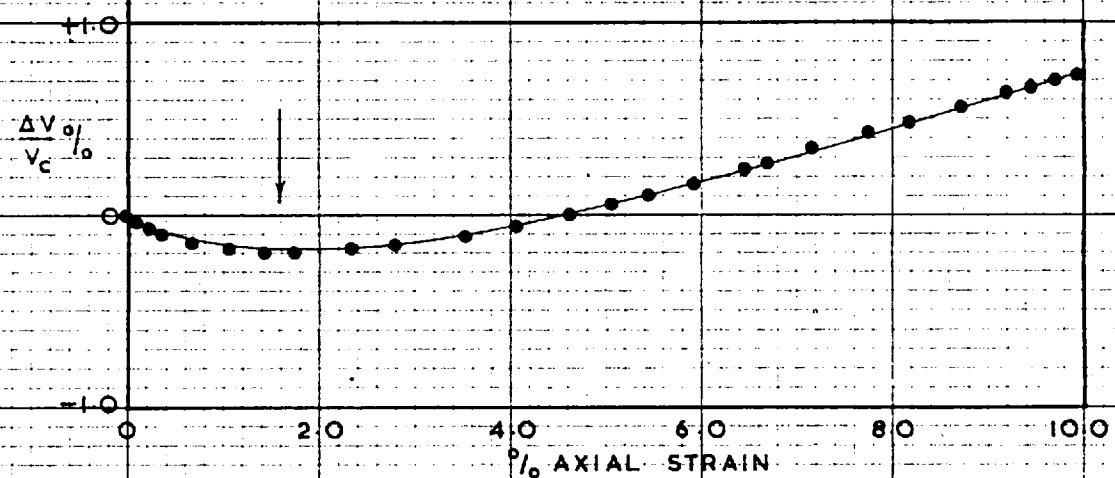
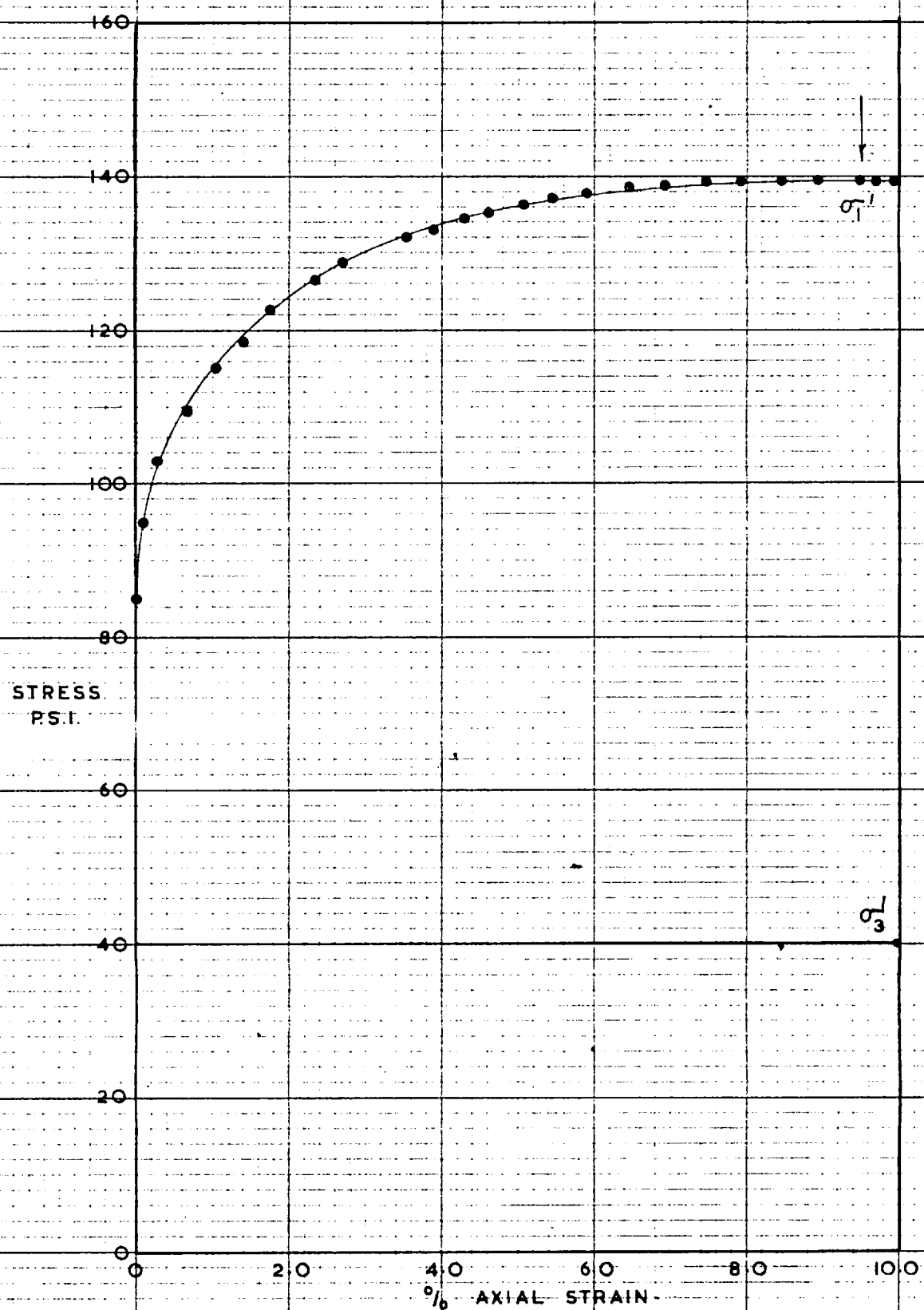


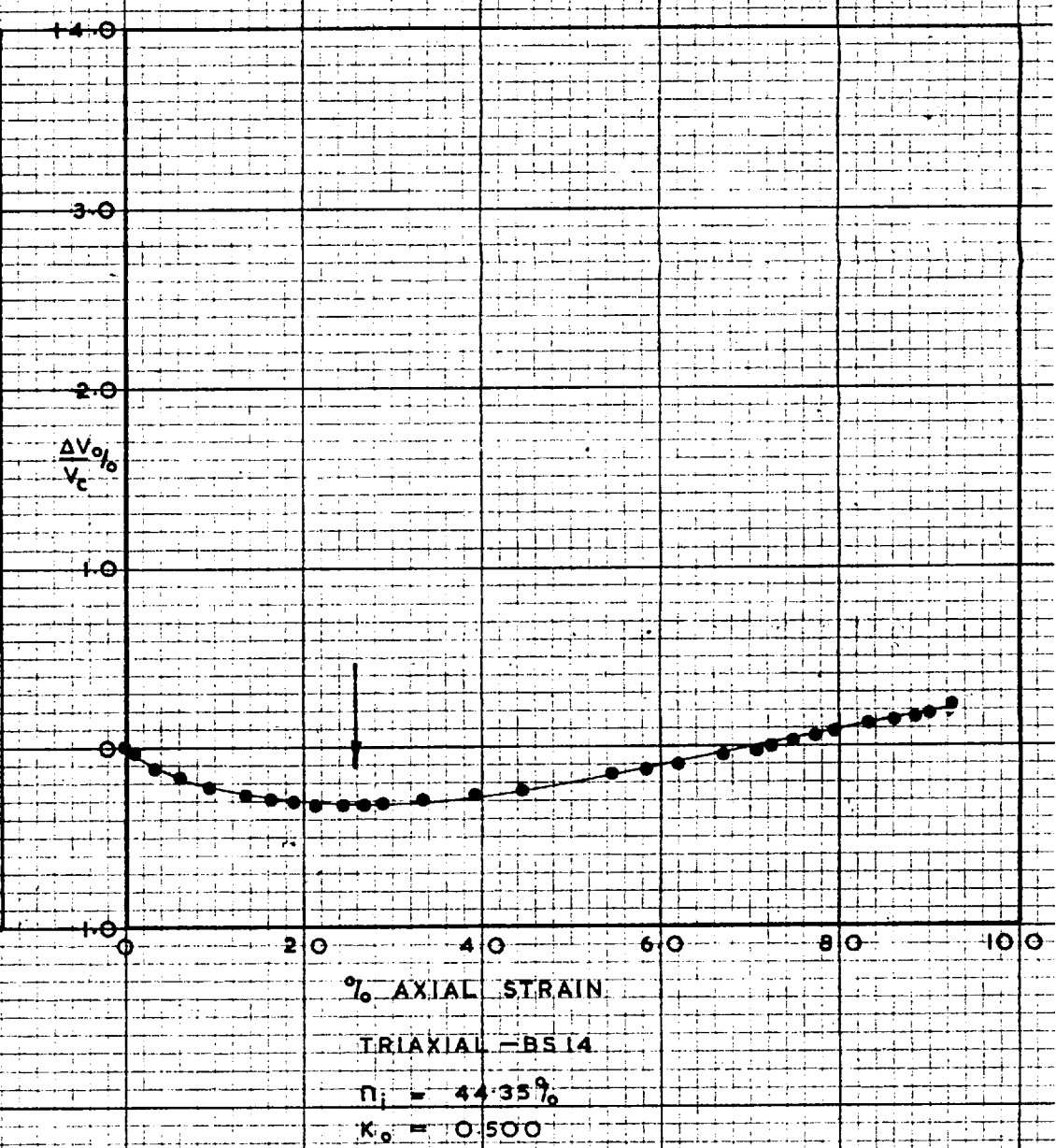
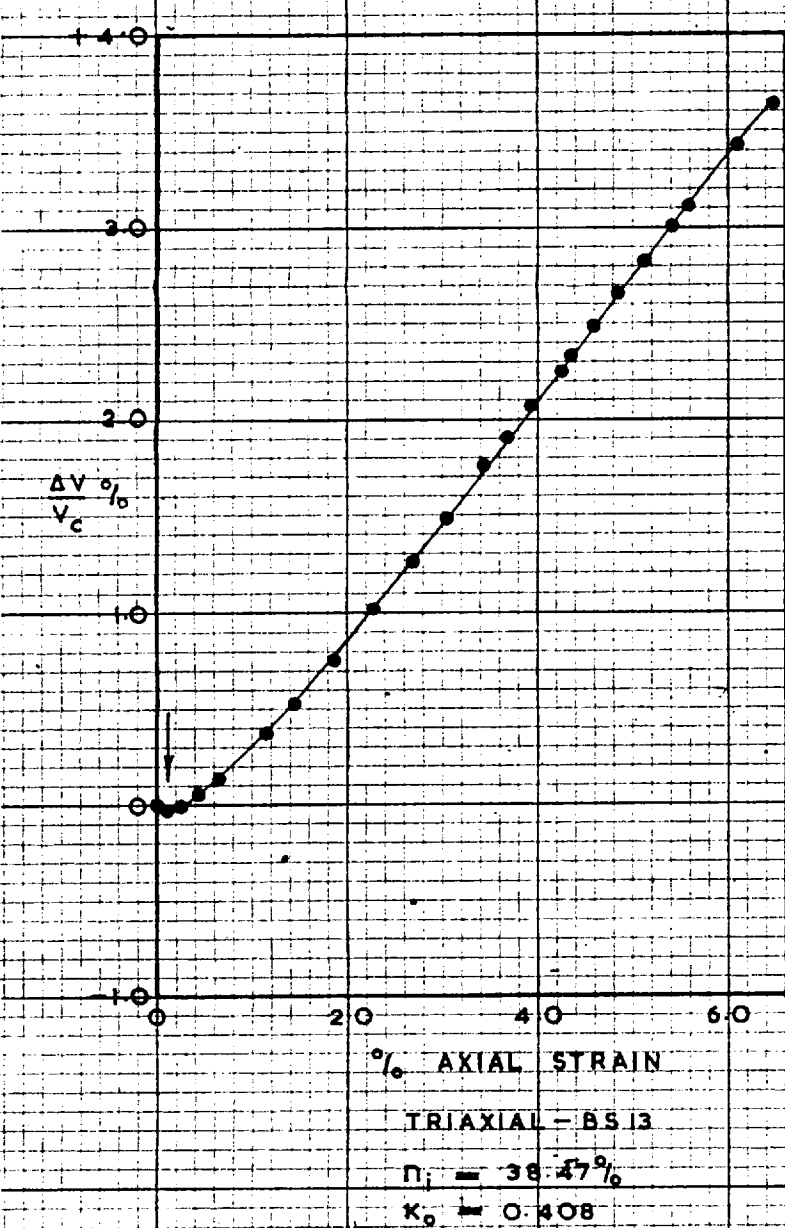
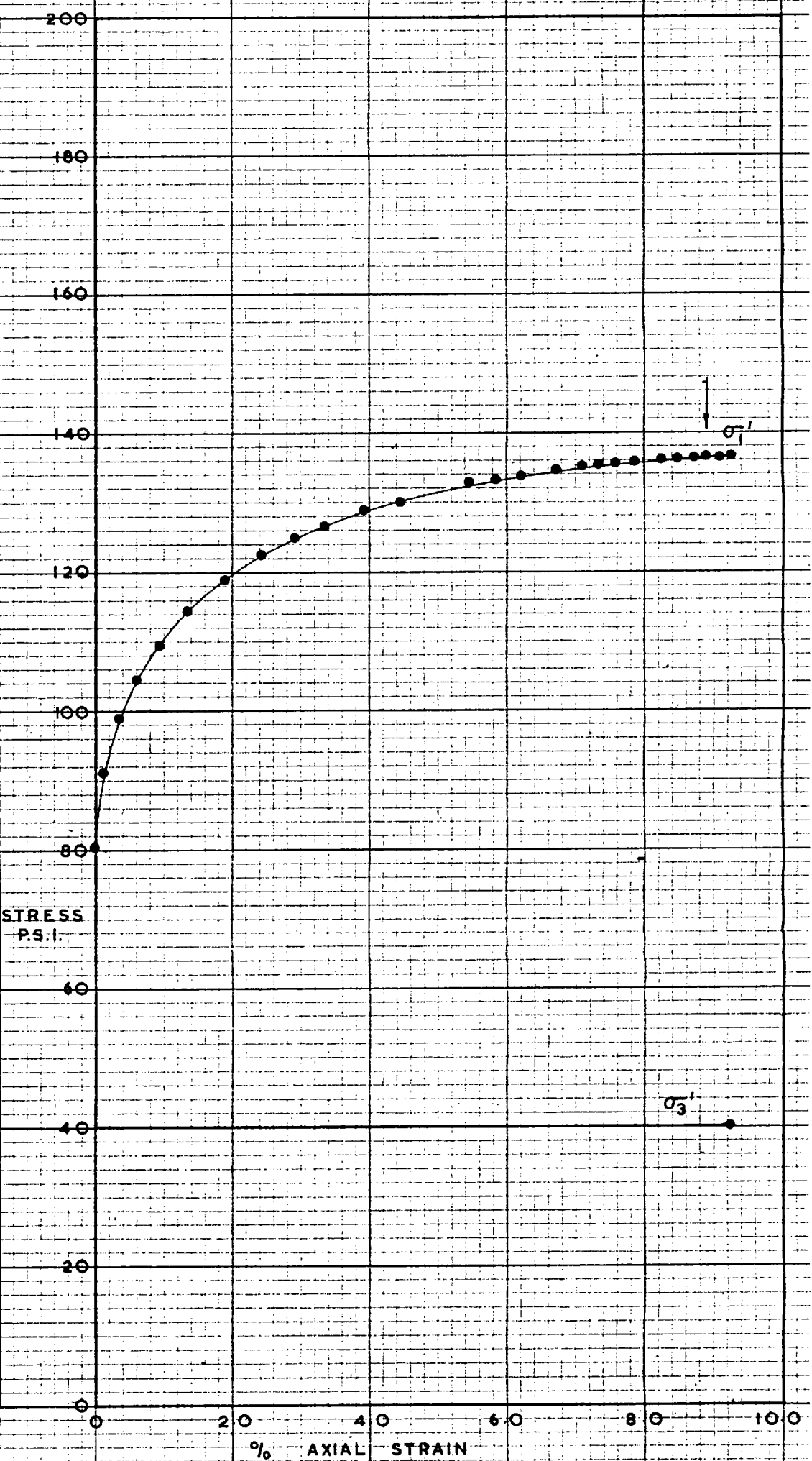
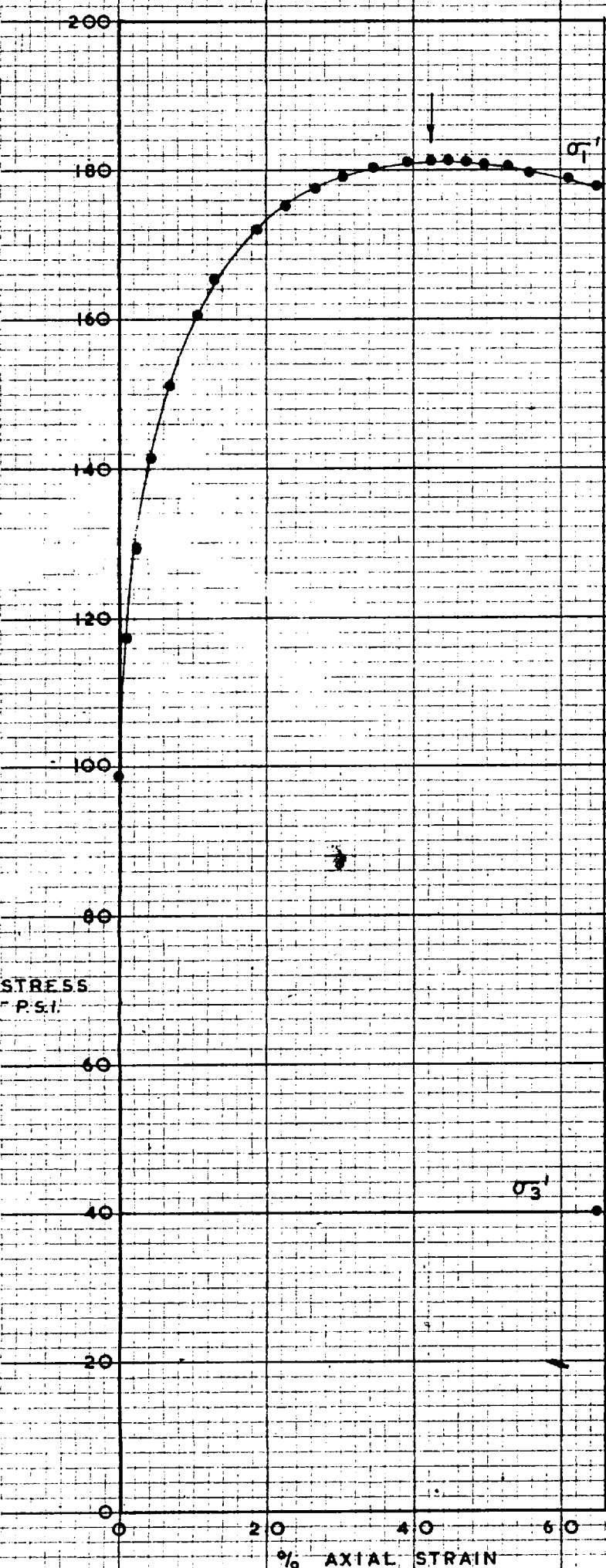
FIG A4-19

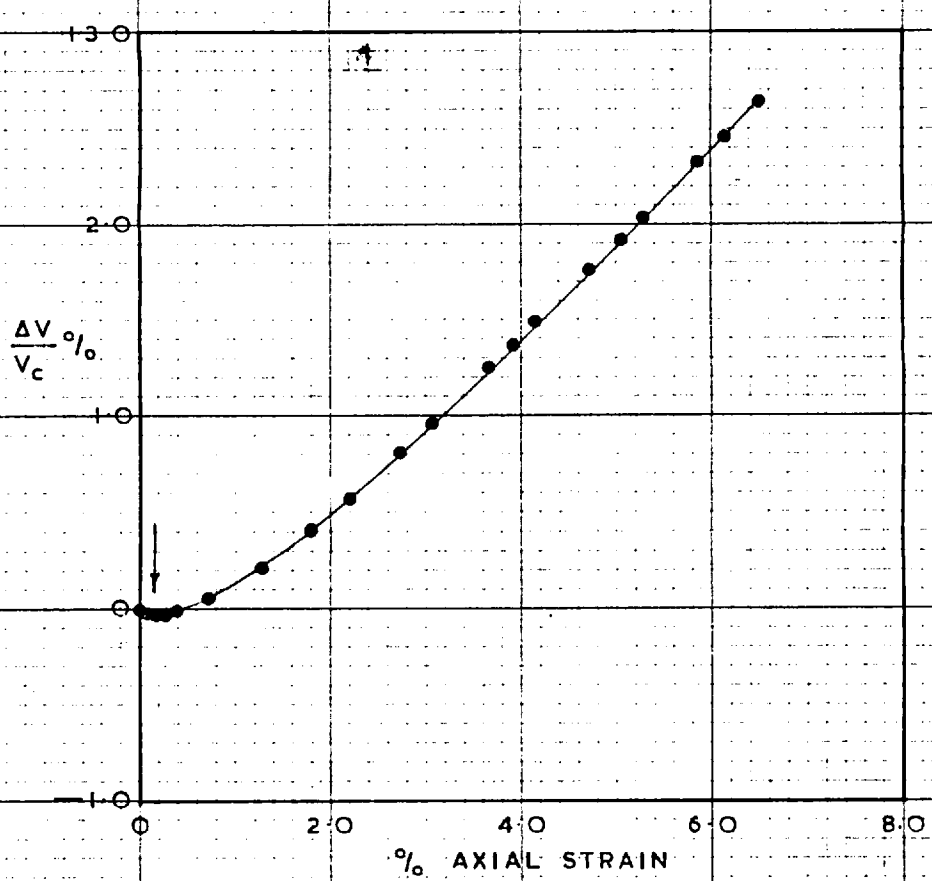
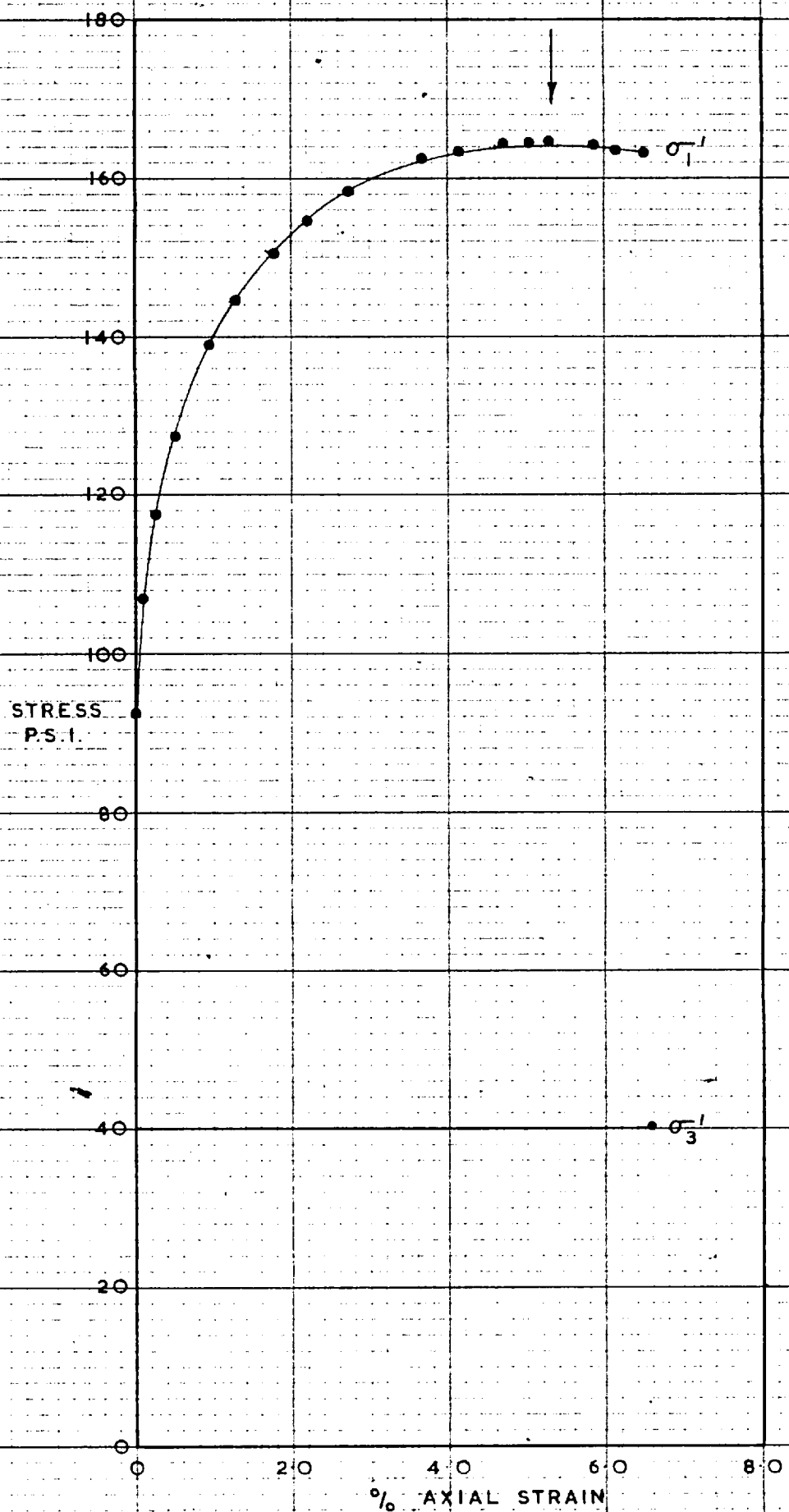


TRIAXIAL - BS 12

$\Pi_1 = 43.72\%$

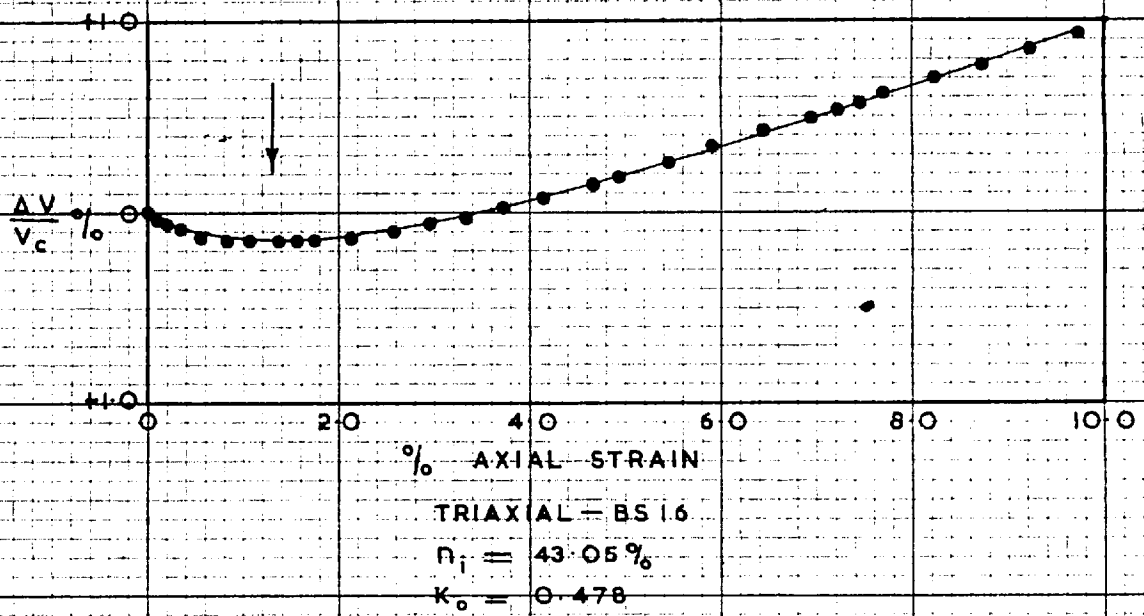
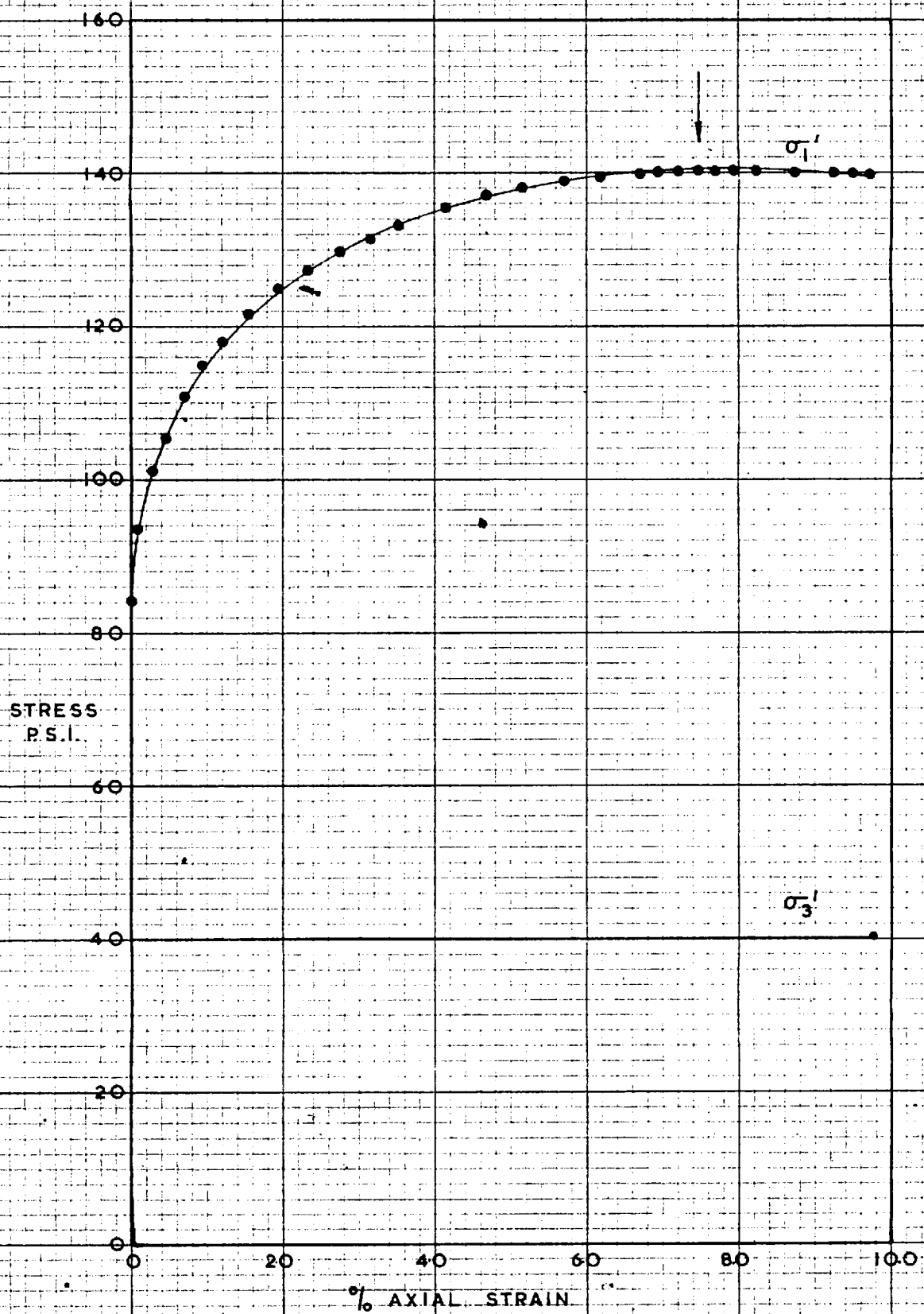
$K_0 = 0.472$





TRIAXIAL-BS 15
 $\eta_i = 39.75\%$
 $K_0 = 0.436$

FIG. A4-2.2



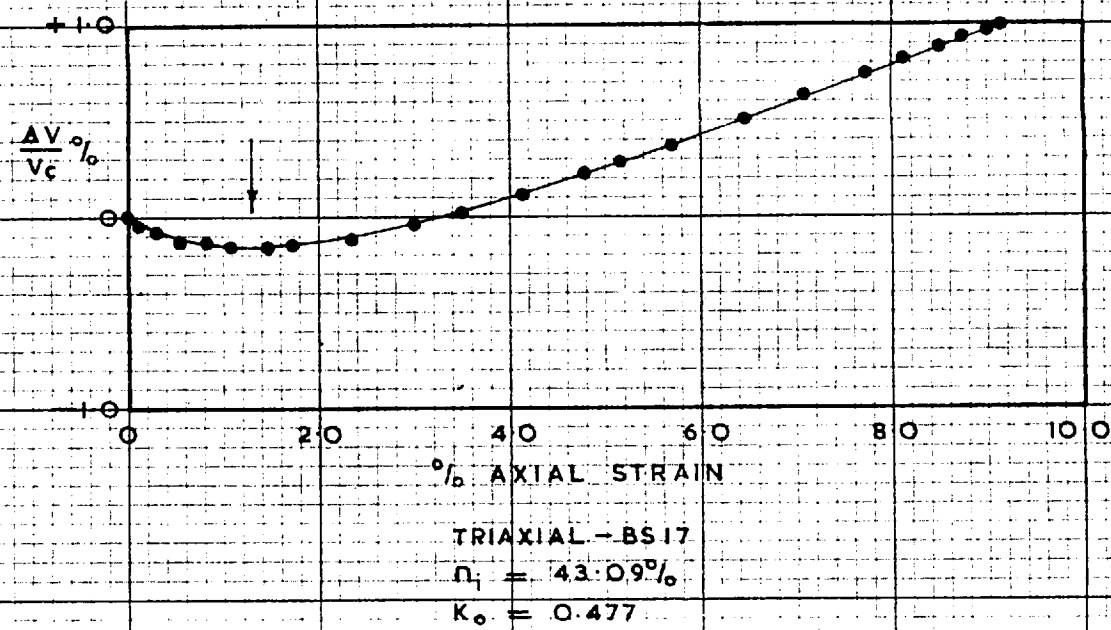
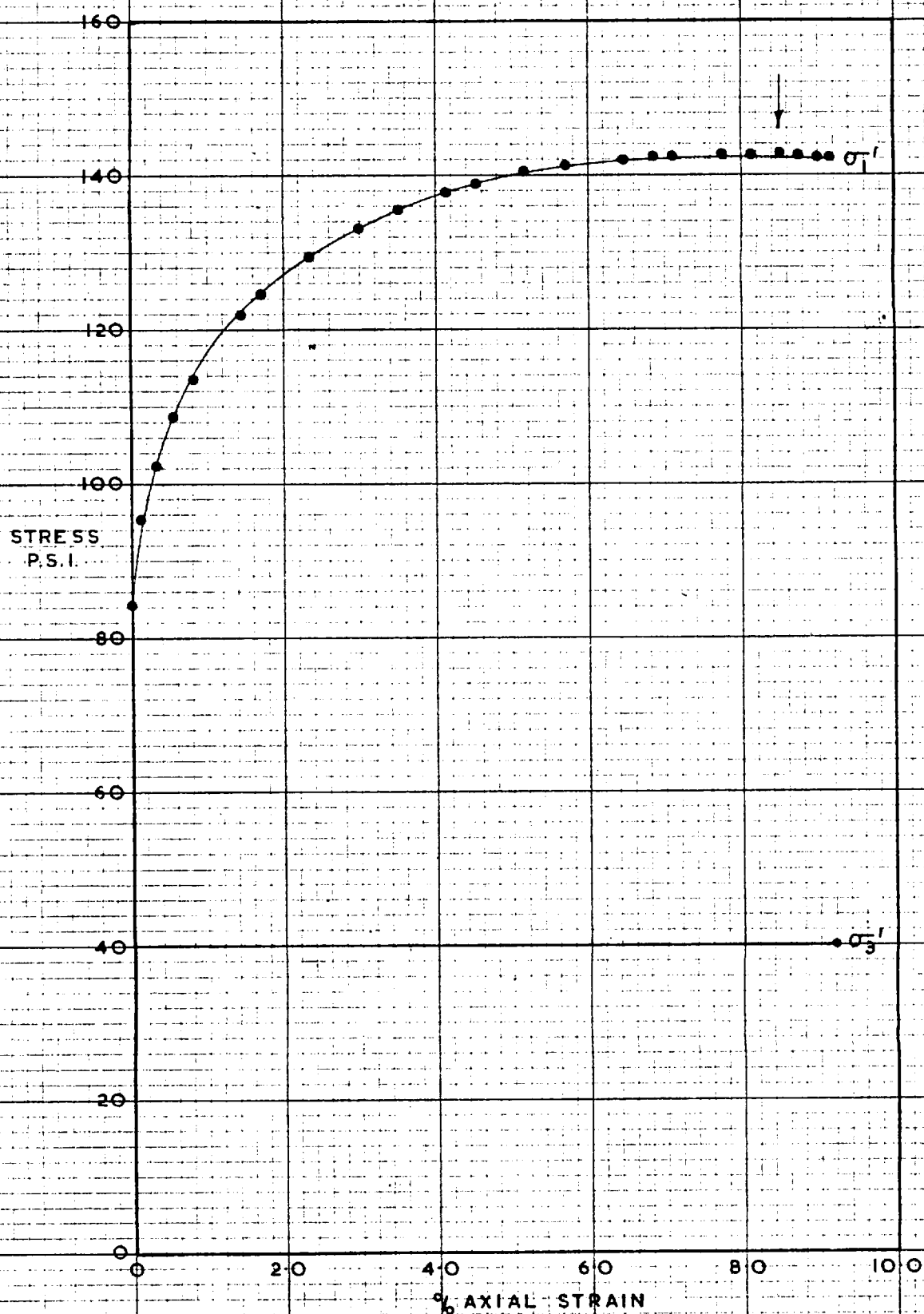


FIG A4-24

STRESS
P.S.I.

% AXIAL STRAIN

$\frac{\Delta V}{V_c} \%$

% AXIAL STRAIN

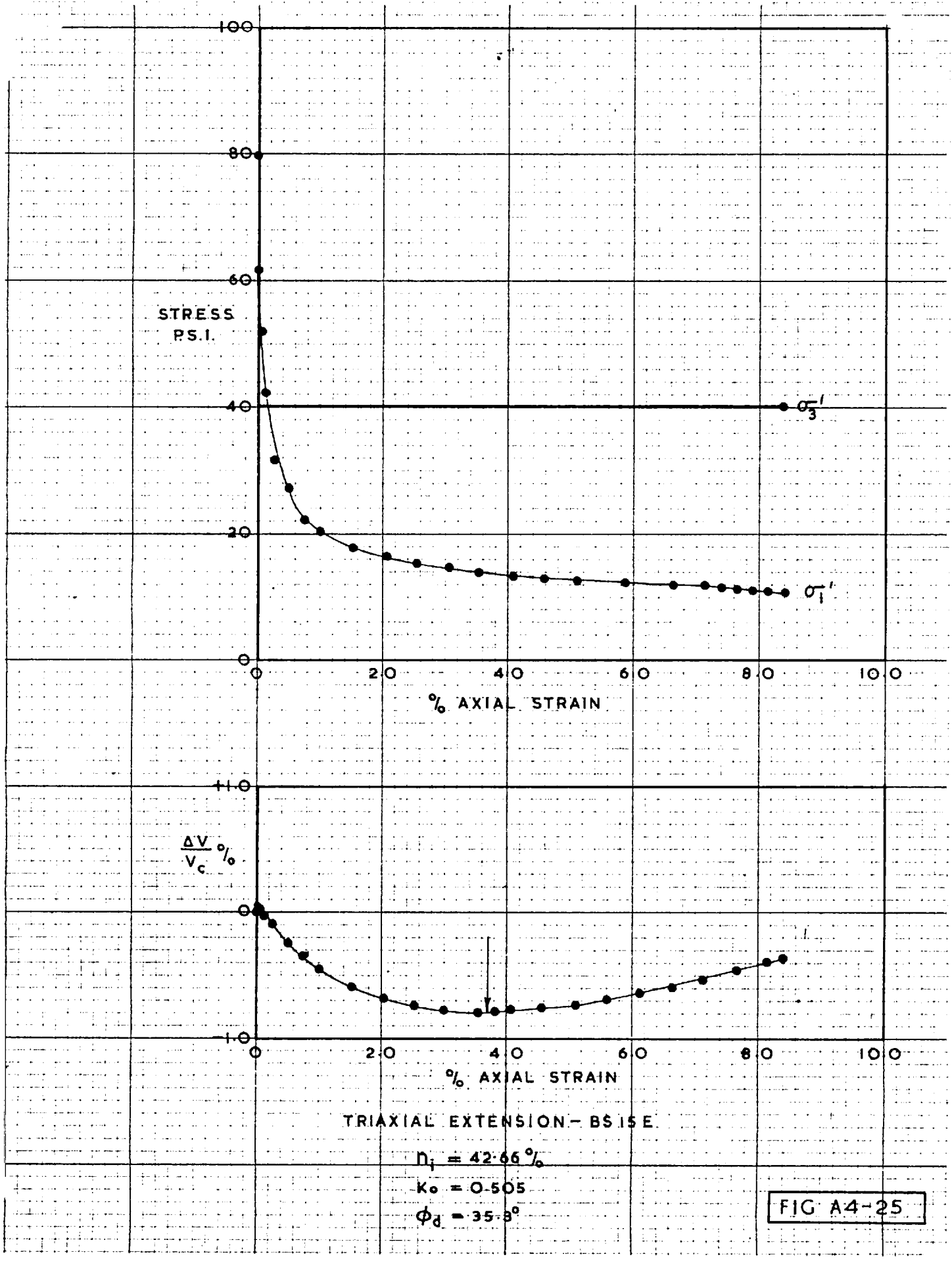
TRIAXIAL EXTENSION - BS 15 E

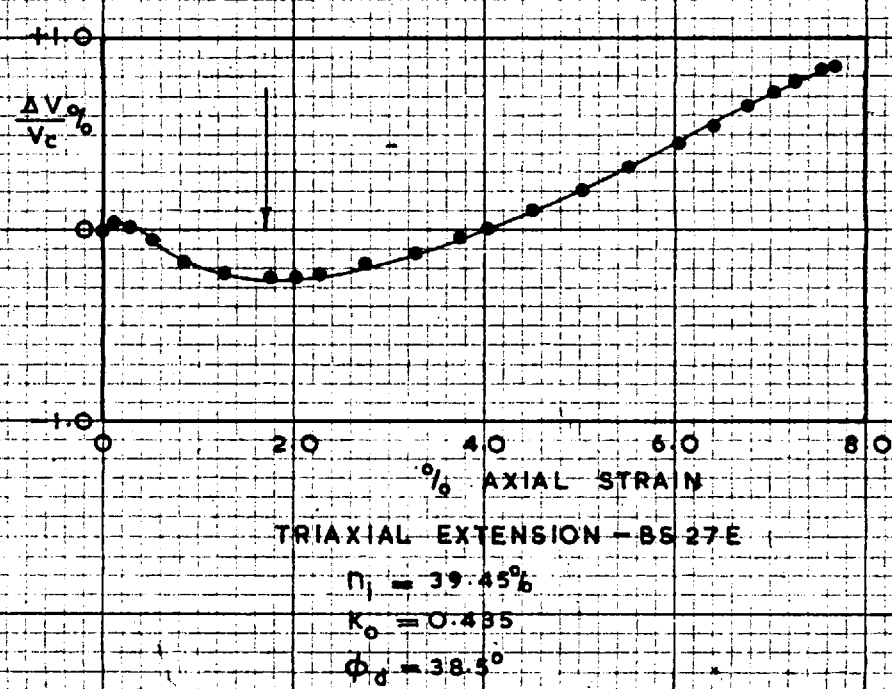
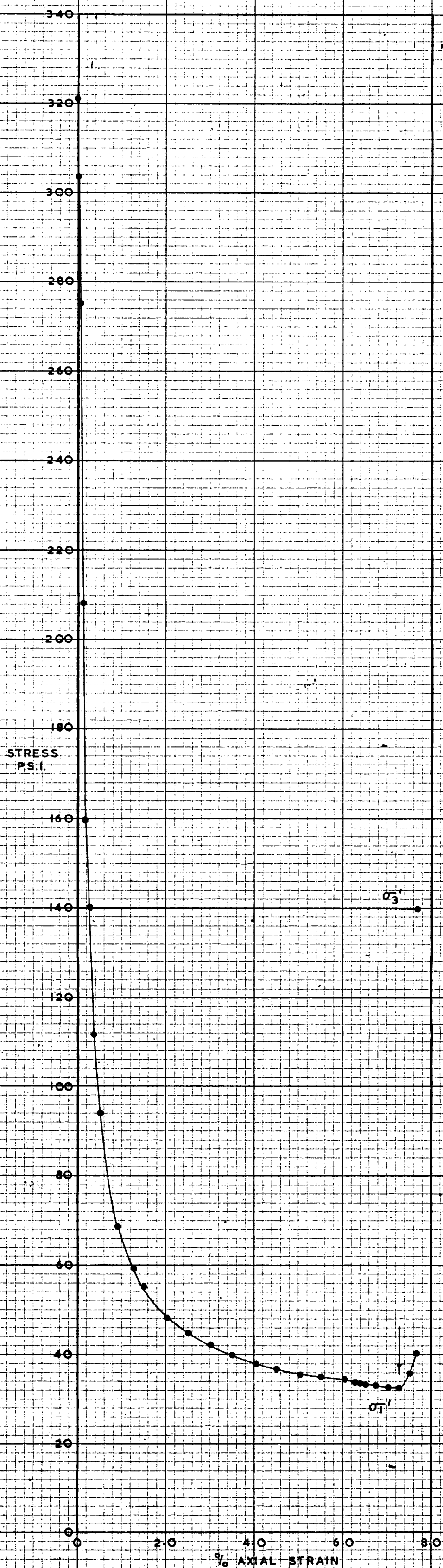
$$\eta_i = 42.66\%$$

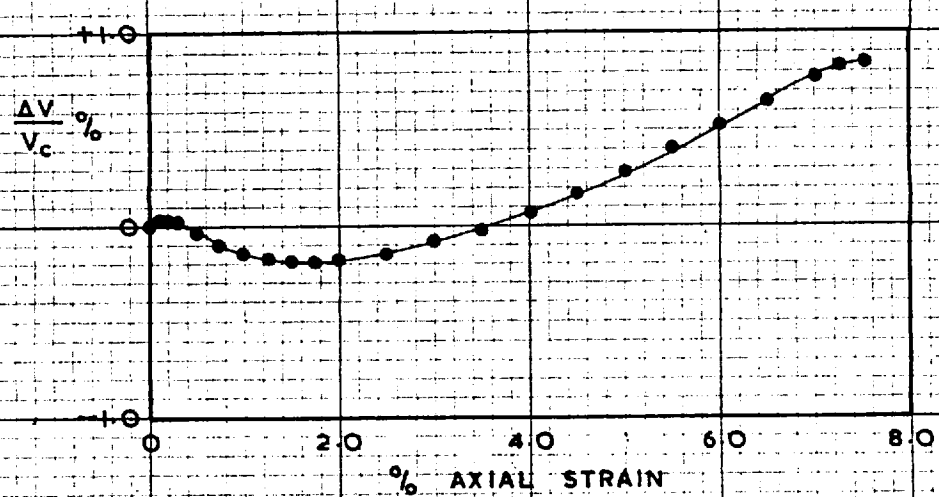
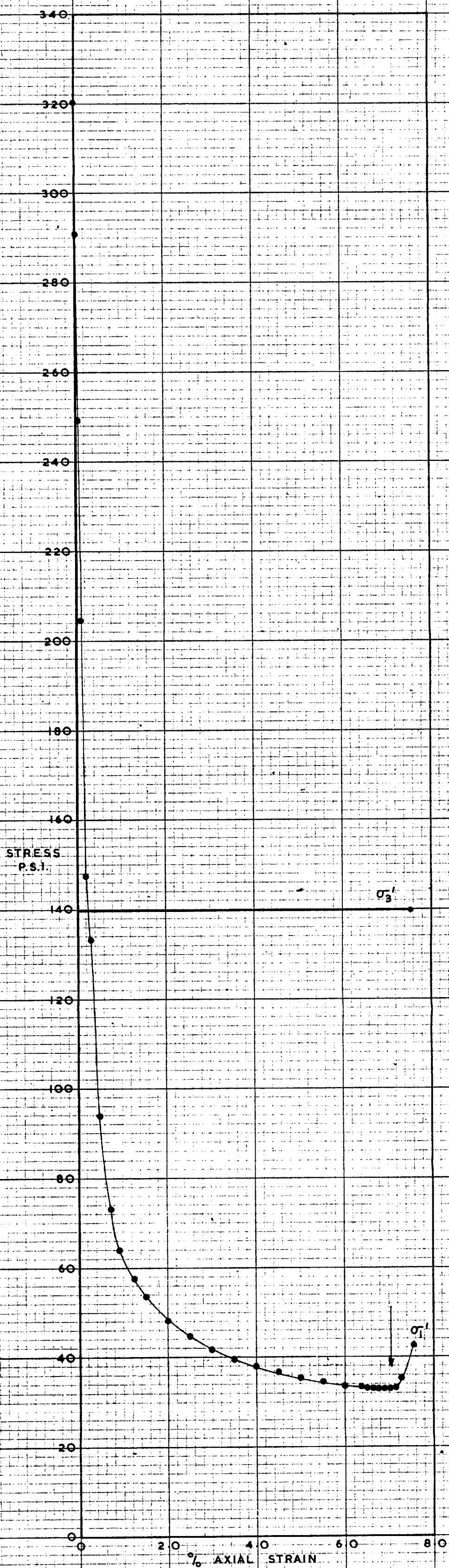
$$K_o = 0.505$$

$$\phi_d = 35.3^\circ$$

FIG A4-25





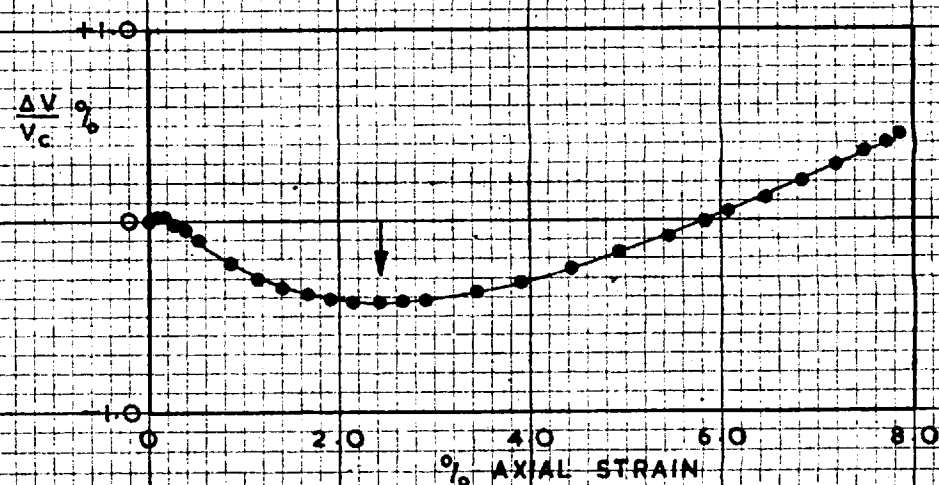
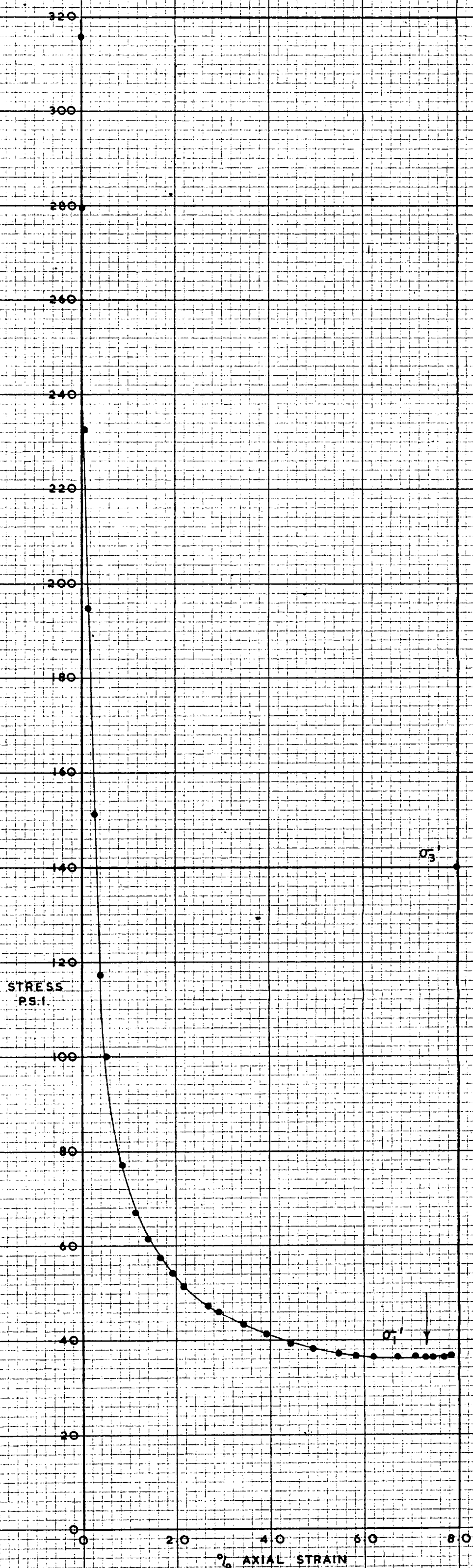


TRIAXIAL EXTENSION - BS 28 E

$\eta_1 = 39.34\%$

$K_0 = 0.496$

$\phi_d = 38.0^\circ$



$\eta_i = 39.74\%$
 $K_0 = 0.443$
 $\phi_0 = 31.6^\circ$

APPENDIX 5

CALIBRATION OF MEASURING DEVICES

(a) Proving Rings

The proving rings used each had a maximum capacity of 3000 lb. and were calibrated by three different methods depending on the load range required. For loads up to 120 lb., dead weights were placed on each ring through a piston running in a ball race. Intermediate loads up to about 1500 lb. were applied by utilizing the lever system of the shear box apparatus; the lever arm was measured with a vernier telescope to the nearest 0.01 mm. to ensure an accurate assessment of the applied load. The Loehenhausenwerk hydraulic testing machine in the Aeronautical Engineering Department was used for the heavy loading of the rings. Each calibration was carried out with ball bearings at the top and bottom of the rings to simulate actual testing conditions.

The hydraulic testing machine calibration for ring No. 130 was higher than either of the other calibrations. (See Fig. A5-1). This could possibly be the result of incorrect alignment when initially positioning the ring in the machine. Since the rings were never loaded beyond 50% capacity in the plane strain tests, the hydraulic machine calibrations were rarely used.

(b) Pressure Gauges

The cell pressure, pore pressure and σ_2 gauges were calibrated by connecting them to a triaxial cell fitted with a rotating bush. The cell was filled with water and topped with castor oil, and dead weights were placed on the ram to increase the pressure in the system. The magnitude of the pressure was determined by dividing the weights by the cross-sectional area of the ram. The calibration curves are plotted in Fig. A5-2.

(c) Plane Strain End Clamp

Before any calibrations could be performed on the end clamp, it had to be de-aired as completely as possible. This was effected by disconnecting the saran lead from the null indicator and loosening the de-airing screw. (See Fig. 3-2). The entire end clamp was then immersed in a pan of de-aired water and the de-airing screw taken out altogether. A vacuum was applied to the cell so that water passed through it; a gentle pumping action on the diaphragm usually helped to dislodge trapped air bubbles. When this was completed, the de-airing screw was replaced and tightened securely, and the end clamp taken from the pan of water. The saran tubing was reconnected, care being taken to prevent air getting into the system. To determine whether the de-airing operation were successful, the plane strain cell was assembled around the end clamp and filled with water. The pressure in both the cell and the end clamp was increased by an equal amount,

the σ_2 gauge reading being equal to the cell pressure plus the pressure head of the mercury column in the null indicator. If no air were present, the mercury column moved only by an amount corresponding to the expansion of the saran tubing and other components in the system. The presence of air was indicated by a large increase in the height of the mercury. (In the adequately de-aired system under equal internal and external pressures, the increase in mercury height to account for the expansion of the saran tubing, etc., was approximately 1/10 in. or 1 division on the scale for a pressure of 150 psi.).

For all tests in which the intermediate principal stress was manually controlled, calibrations were required to ensure the maintenance of the plane strain condition and to enable the calculation of the σ_2 values to be made. During the shear stage when the pressure in the end clamp cell is greater than the cell pressure, a correction to the mercury height in the null indicator has to be made to compensate for the extension of the tie bars, the compression of the rubber diaphragm and further expansion of the σ_2 system. This was determined by placing a dummy steel sample between the end plates of the σ_2 belt, incrementally increasing the pressure in the system with the hand-operated screw ram, and noting the corresponding rise of the mercury thread in the null indicator. The resulting curve is given in Fig. A5-3. The elastic compression of the dummy sample is subtracted from the observed curve, and the σ_2 net pressure is defined

as the difference between the pressure in the σ_2 cell and the confining pressure. To simplify readings, the head of mercury was added to the net pressure, thus giving the net σ_2 gauge reading. The end clamp was re-calibrated several times during the test programme to check the original calibration. By replacing the previous nylon tubing with a saran lead, the loss of water from the system through the tubing walls was reduced to negligible quantities for the duration of each test. Although the tests lasted for approximately one month each, the stresses in the σ_2 system were considered insufficient to produce significant creep phenomena; variations in temperature, especially in the winter months, did produce small fluctuations in the mercury thread of the null indicator as well as in pore pressure measurements. However, both creep and temperature were assumed to exhibit only second order effects on the measured stresses.

The second part of the calibration was carried out to correlate the observed σ_2 gauge pressure with the load transmitted to the specimen ends during shear. Wood's procedure of applying known loads to the face of the end clamp and noting the resulting gauge pressure was adopted. By placing the end clamp on its "back" with the tie bars in the air, dead weights were applied via a hanger system supported on a ball bearing in the centre of the face plate. The mercury level in the null indicator was adjusted by increasing the pressure with the screw ram, and the gauge pressure was noted. Fig. A5-4 shows the load-pressure relationship for the mercury thread kept at

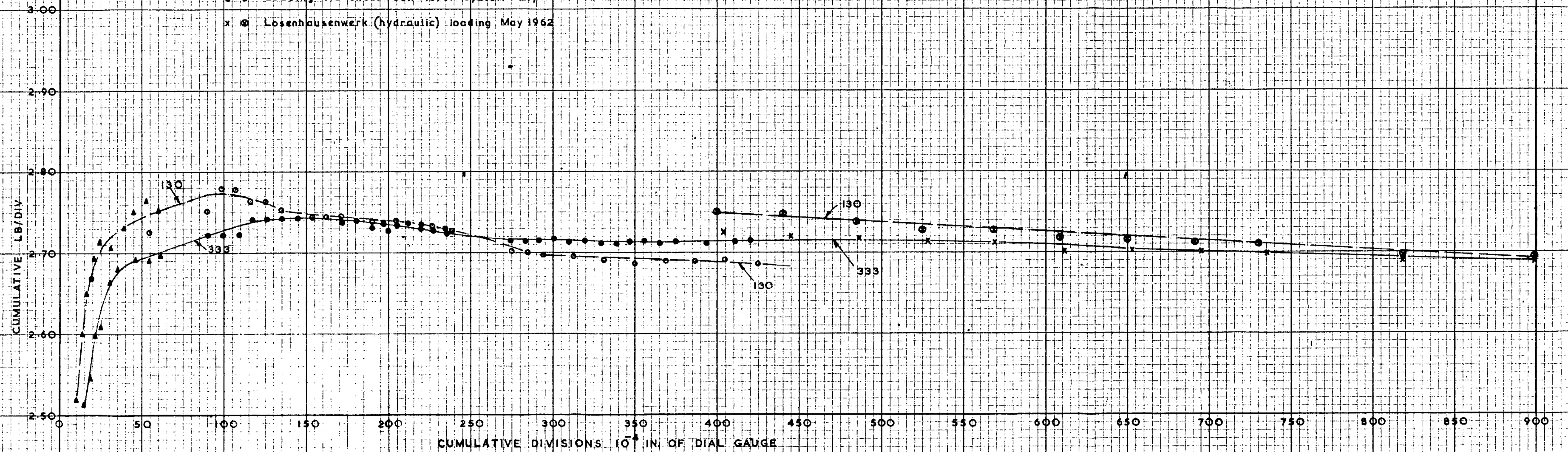
the zero mark and for the mercury level varied to compensate for tie bar extension, etc. as indicated in Fig. A5-3. This method avoided the necessity of determining the effective area of the diaphragm to correlate the measured σ_2 pressure with the σ_2 acting on the specimen.

For the test (WC11) in which the intermediate stress was automatically controlled, it was assumed that the no-length-change condition was maintained by the longitudinal strain indicator's activating via a servo-mechanism, the motor-driven screw ram connected to the end clamp pressure system. Consequently, the calibration in Fig. A5-3 was not used. However, a correction to the measured σ_2 pressure was necessary to account for the diaphragm restraint when it was being stretched by the pressure in the σ_2 cell. The amount of stretching during the shear stage of the test was registered as an increase in height of the mercury level in the null indicator. The calibration was conducted with the end clamp sitting under atmospheric pressure; it consisted of increasing the pressure in the σ_2 cell with the screw ram, allowing the system to come to equilibrium, and noting the null indicator mercury height. A similar procedure was followed when the pressure was decreased. The pressure was measured on a mercury manometer and included the diaphragm restraint plus the head of mercury in the null indicator. (See Fig. A5-5). The diaphragm restraint is the difference between the curve and the straight line. After correcting the gauge pressure for the

diaphragm restraint, the force on the sample ends was obtained from the load-pressure curve in Fig. A5-4.

CALIBRATION CURVE OF PROVING RING NOS 130 & 333

- ▲ ▲ Direct loading April 1962
- ○ Loading via shear box lever system July 1961
- x ⊗ Losenhausenwerk (hydraulic) loading May 1962



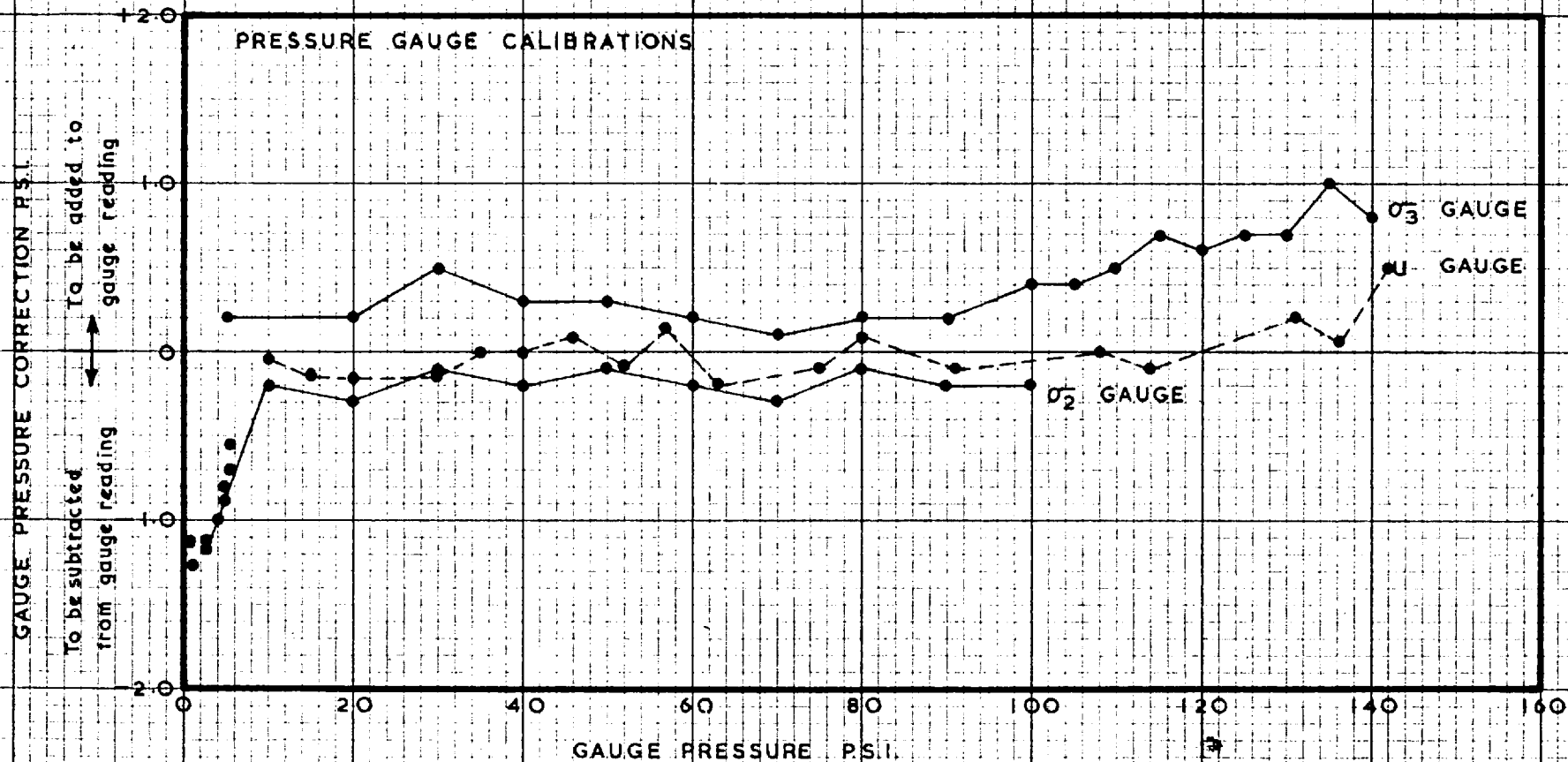


FIG A5-2

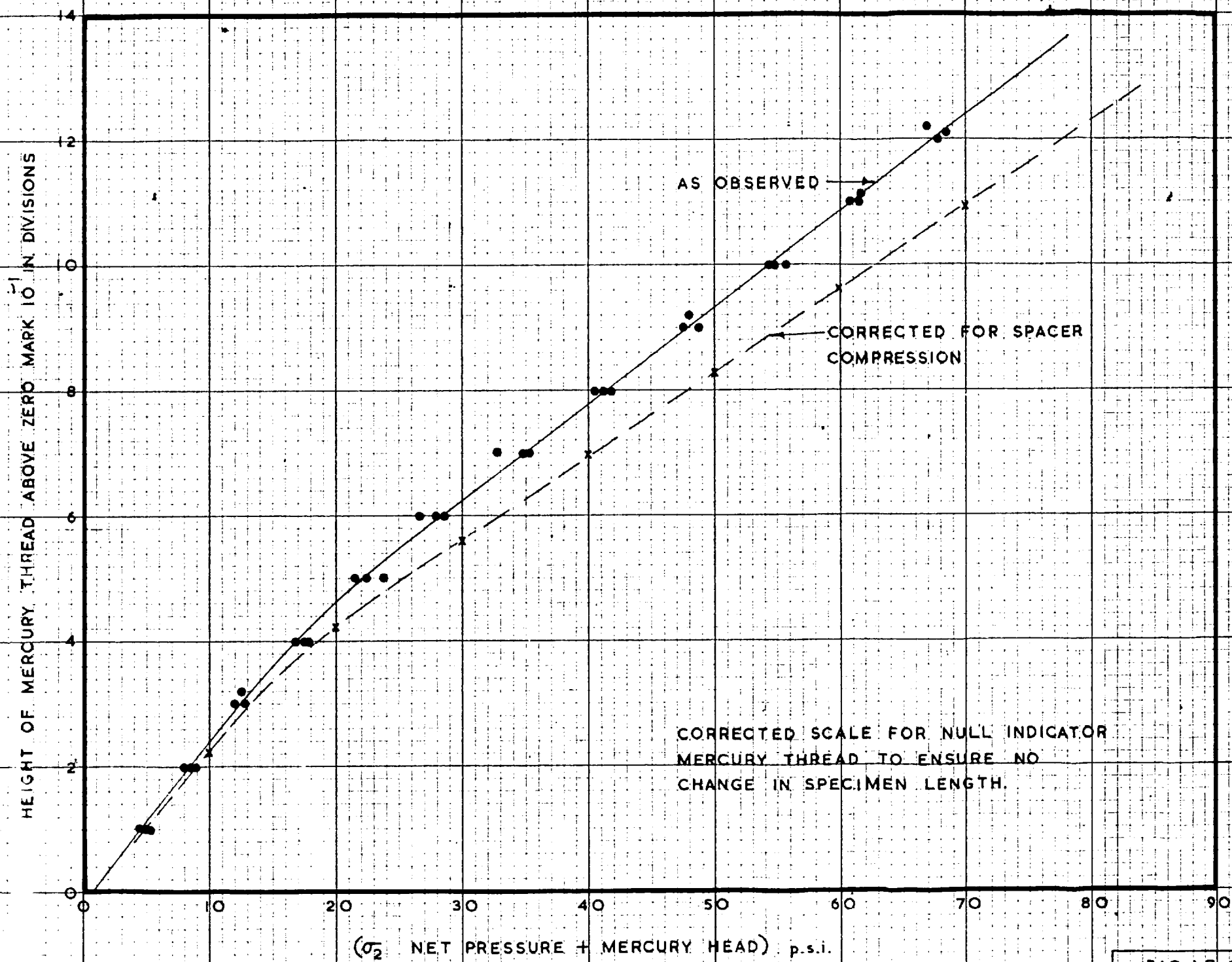


FIG.A5-3

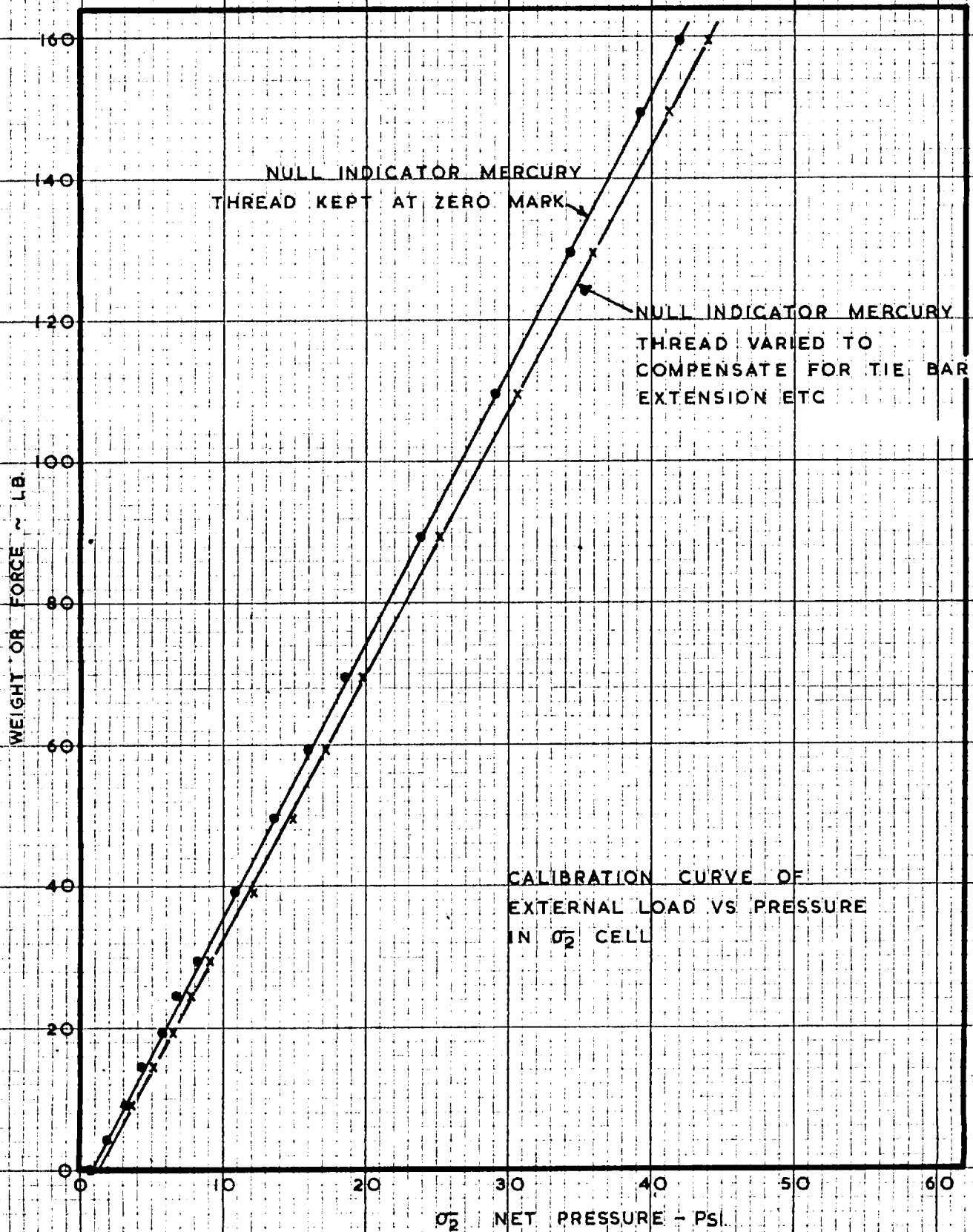
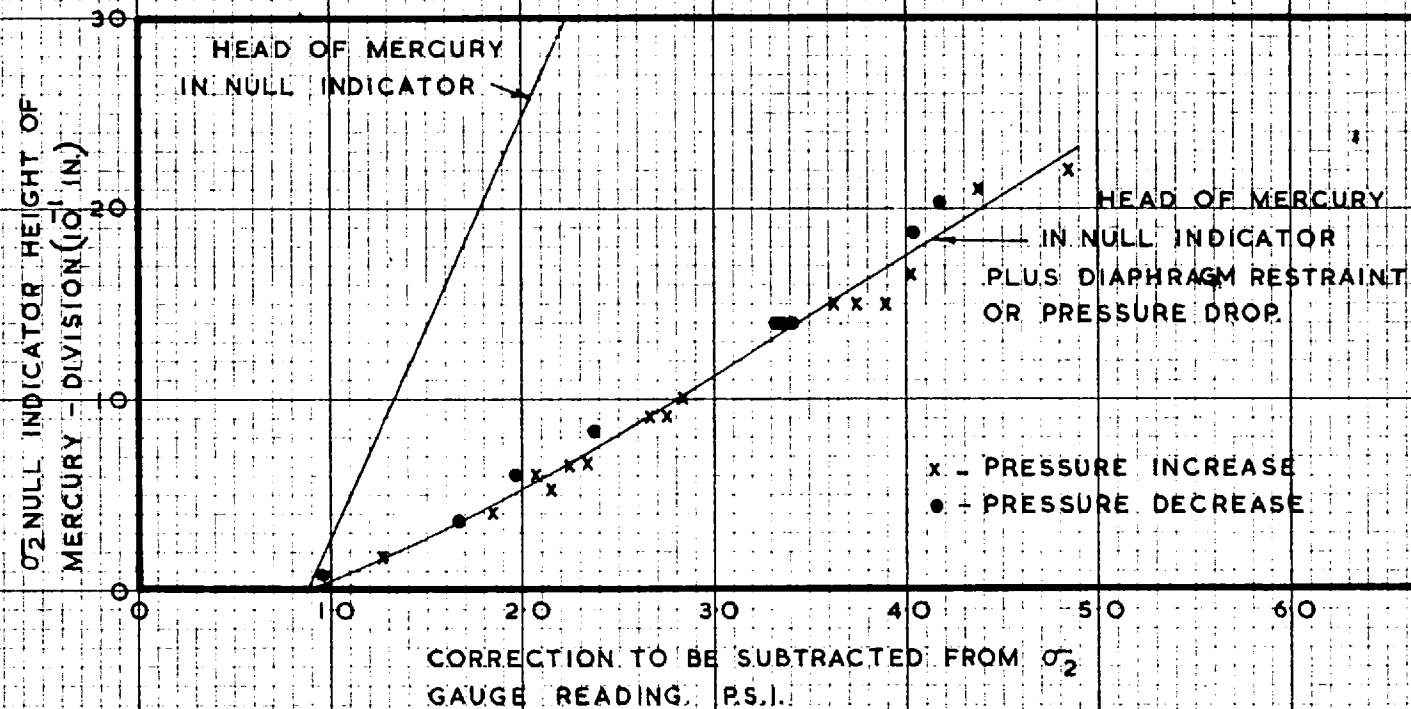


FIG A5-4



CALIBRATION CURVE OF CHANGE IN HEIGHT OF MERCURY COLUMN
VS
PRESSURE LOSS ACROSS RUBBER DIAPHRAGM
OF σ_2 MEASURING DEVICE

APPENDIX 6

THE RELATIONSHIP BETWEEN THE WATER CONTENT AND THE UNDRAINED SHEAR
STRENGTH OF WEALD CLAY AS DETERMINED IN THE MINIATURE VANE APPARATUS

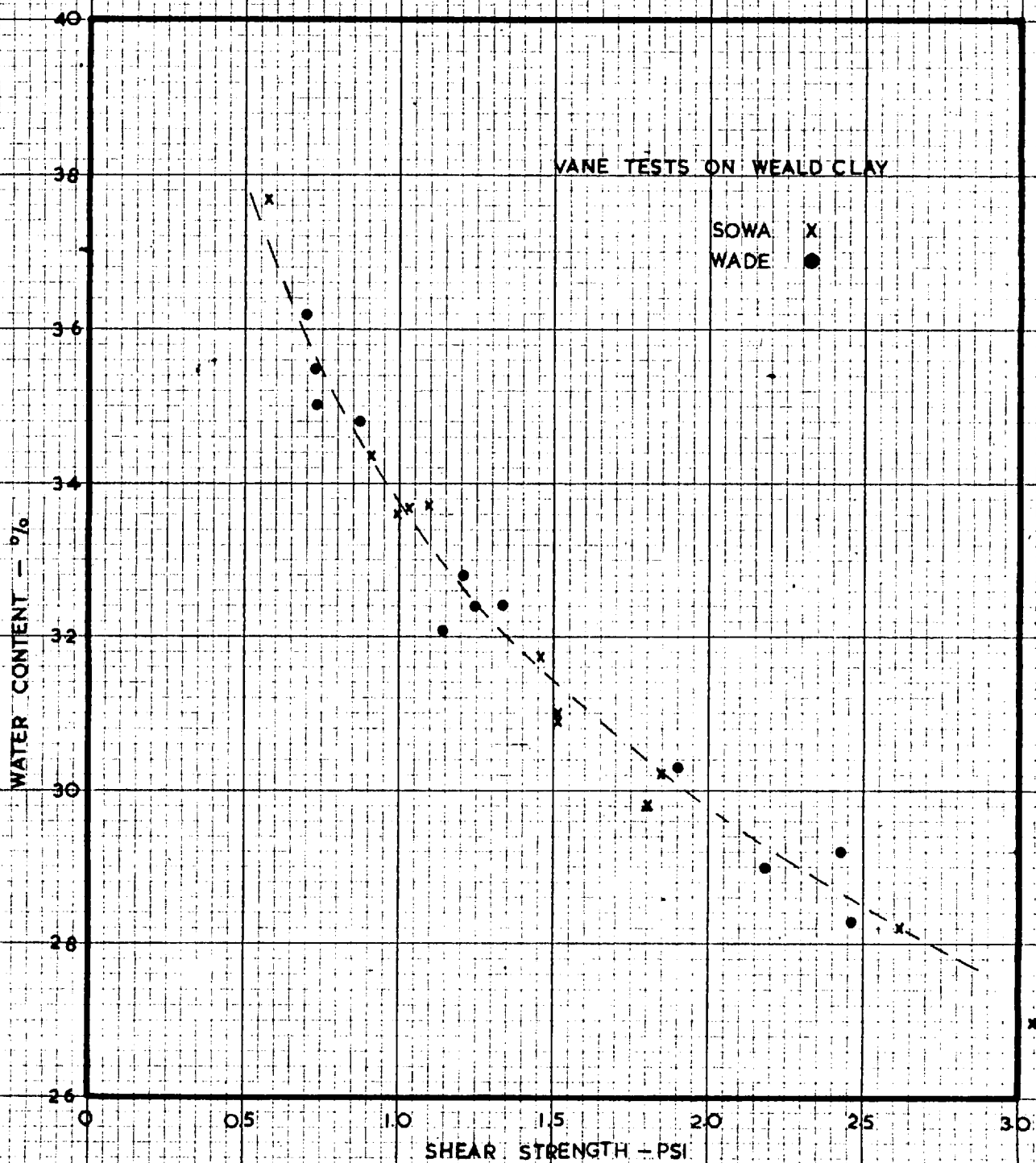


FIG A6-1

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