

Three Dimensional Features of Convective Heat Transfer in Droplet-Based Microchannel Heat Sinks

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Abstract

Convective heat transfer in droplet-based microchannel heat sinks can be enhanced by the recirculating vortices due to the presence of interfaces. In rectangular microchannels, the three dimensional structures of the vortices and the ‘gutters’ (i.e., the space between the curved droplet interface and the corner of the microchannel) can significantly affect the heat transfer process. Numerical simulations of the heat transfer process are performed to study the three dimensional features in droplet-based microchannel heat sinks. The finite volume method and the level set method are employed to simulate the flow dynamics, the evolution of the interface, and the heat transfer. The results show that the ‘gutters’ can hinder the heat transfer process because of its parallel flow, whereas the recirculating flow in droplets and in slug regions between successive droplets can enhance the heat transfer by advecting hot fluid towards the center of the droplets/slugs and advecting fresh fluid towards the wall of the channel. The effects of the length of droplets, the aspect ratio of the channel cross sections, and the Peclet number are analyzed.

Keywords: Droplet, Heat transfer, Thermal boundary layer, Multiphase microfluidics

1. Introduction

High-efficiency cooling is important in numerous applications, such as in microprocessors with very-large-scale integration and in commercial/military high-power optical/electronic systems. The heat generated by such devices must be removed rapidly to ensure the reliability of the devices. Despite many techniques have been developed for high-efficiency cooling, such as heat pipes [1], spraying [2, 3], jet impingement [4], and microchannels [5], it is still attracting researchers’ attention to improve the existing cooling techniques and to develop new techniques for the demand of compact high-heat-flux and high-efficiency heat exchangers [6].

Comparing with other cooling techniques, microchannels can be directly embedded closely to heat sources for compact and efficient designs. Heat transfer in microchannels with multiphase flow can benefit from the recirculating flow induced by fluid interfaces [7], which is difficult to achieve in its single phase counterpart due to the low flow speeds and the small Reynolds numbers in micro-devices. The recirculating flow in multiphase microfluidics can effectively enhance heat transfer by bringing fresh fluid from the center of the channel to the wall, and transporting heated fluid from the wall to the center of the channel [8]. Many experiments have been performed to measure the heat transfer enhancement in microchannels with droplets, slugs, or plugs [9]. Some experiments used cylindrical microtubes [10, 11, 12, 13], while others used rectangular microchannels [14, 15, 16, 17]. Their experimental

results showed significant enhancement of heat transfer when comparing with single phase flow, and the Nusselt numbers are several times higher than the single phase counterpart.

Numerical simulations can serve as a complimentary way to study convective heat transfer in multiphase microchannels without building complex diagnostic systems, and to provide details of the process which are difficult or impossible to measure directly in experiments, and to allow a clear understanding of the physics. To address the effect of interfaces on heat transfer process, some simulations employed assumed interface shapes for the sake of simplicity [8, 18], while others used interface capturing methods to accurately predict interface shapes. The latter category can consider the effect of the complex interface shapes on heat transfer process. Various interface tracking/capturing techniques have been employed to simulate the heat transfer in droplets, slugs, and plugs, such as the volume of fluid method [19], the level set method [20, 21], and the phase field method [22]. Regarding the geometries of microchannels, most simulations were performed with cylindrical microcapillaries [18, 20, 22, 21, 23, 19] or two dimensional (2D) microchannels [8], where the three dimensional (3D) effect on the flow and on the heat transfer process cannot be considered. However, in most microchannel heat exchangers, the microchannels usually have rectangular or other non-circular cross sections [14, 15, 16, 17]. When droplets move in microchannels, liquid films often form beneath the droplets. For non-circular microchannels, ‘gutters’ form in the continuous phase at the corner of the cross sections [24], where ‘gutters’ refer to the space between the curved interface of the droplet and the corner of the microchannel, as shown in Figure 1. The thickness of the liquid film and

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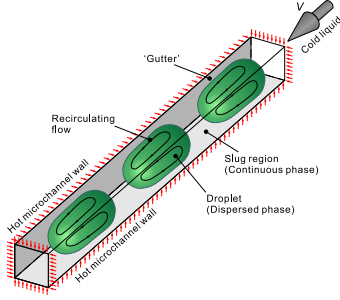


Figure 1: Schematic diagram of droplet heat transfer in microchannels.

the shape of the liquid ‘gutters’ depend on the cross-sectional geometry, the liquid properties, and the flow conditions [25, 26, 27], and can significantly affect the flow dynamics and the heat transfer. This effect, to the best of our knowledge, has not been studied, and will be the focus of this investigation.

In this paper, we performed 3D numerical simulations on the convective heat transfer in droplet flows in the microchannels with rectangular cross sections, as shown in Figure 1. The numerical methods are described in Section 2, including the level set method for interface capturing and the finite volume method for fluid flow and for heat transfer. The results are presented and discussed in Section 3, including the droplet dynamics at different flow conditions, the process of the heat transfer, and the effects of the aspect ratio of the channel cross section, the size of the droplet, and the Peclet number. Finally, concluding remarks are drawn in Section 4.

2. Numerical methods

Simulations of droplet-based heat transfer in microchannels involve the flow field of the fluids, the evolution of the liquid-liquid interface, and the heat transfer process. 3D simulation of these physical processes requires long computation time. To reduce the computation time without sacrificing the resolution with a coarse mesh, we used a frame of reference following each droplet. To do this, the flow fields and the interface shapes of droplets in microchannels are first obtained before implementing them in the heat transfer simulation. This method can significantly reduce the simulation time, and makes it possible to simulate the 3D droplet-based heat transfer in microchannel heat sinks, allowing the systematic parametric investigation of the related effects.

The finite volume method (FVM) and the level set method (LSM) are employed to simulate the heat transfer process of droplets moving in microchannels. The FVM for fluid flow and LSM for interface prediction are described in Ref. [28] and are explained briefly here.

2.1. Finite volume method for flow field

The finite volume method [29] is used to discretize the continuity equation and the momentum equation,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \quad (1)$$

$$\frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p + \nabla \cdot [\mu (\nabla \mathbf{u} + \nabla \mathbf{u}^T)], \quad (2)$$

where ρ and μ are the density and the viscosity of the fluid, respectively. For multiphase flow, fluid properties ρ and μ in each control volume are calculated using a smoothed Heaviside function H as follows:

$$\rho = H\rho_d + (1 - H)\rho_c, \quad (3)$$

$$\frac{1}{\mu} = \frac{H}{\mu_d} + \frac{1 - H}{\mu_c}, \quad (4)$$

where the subscripts ‘d’ and ‘c’ refer to the dispersed and continuous phases, respectively. The exact form of H will be presented in Section 2.2. The ghost fluid method [30, 31] is used to take account of the surface tension through the pressure term,

$$[p] = -\kappa\sigma, \quad (5)$$

where $[p]$ indicates the pressure jump across the interface and $\kappa \equiv \nabla \cdot \mathbf{n}$ is the curvature of the interface. The symbol $\mathbf{n}$ denotes the unit direction vector normal to the interface, which will be presented in Eq. (9). No-slip boundary conditions are imposed on the walls. The inlet and outlet are set to be periodic to consider monodisperse droplets in microchannels. Since the frame of reference is following the droplets, the flow can gradually achieve a steady state. Therefore, instead of the whole microchannel, only a period of the flow is used in the simulations, which can significantly reduce the computation time.

In the simulations, the width of the microchannel is set to be 200 μm , the depth is varied from 200 to 800 μm , and the length of the simulation domain is 800 μm , which is used to represent a period of the flow. The fluid properties are set based on the system of water droplets in mineral oil with surfactant Span 80 at a concentration of 2% by weight, which is widely used in droplet-based microfluidic experiments to facilitate the formation of monodisperse droplets and to stabilise the droplets against coalescence [32, 33]. The high surfactant concentration can guarantee a complete coverage of the interface by surfactant molecules, and the low speed of droplets can allow rapid replenishment of surfactant molecules from the bulk. Therefore, the surfactant distribution and the interfacial tension can be regarded as uniform. The dynamic viscosities and the densities are $2.39 \times 10^{-2} \text{ Pa} \cdot \text{s}$ and 840 kg/m^3 for the continuous phase, and $8.9 \times 10^{-4} \text{ Pa} \cdot \text{s}$ and 997 kg/m^3 for the dispersed phase. The interfacial tension between the continuous phase and the dispersed phase is 3.65 mN/m.

2.2. Level set method for interface capturing

The evolution of the droplet interface is captured using the level set method [34]. The level set function ϕ is a signed distance from the interface. The level set equation and the re-initialization equation for the distance function ϕ are, respectively,

$$\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = 0, \quad (6)$$

$$\frac{\partial \phi}{\partial \tau} = \text{sign}(\phi)(1 - |\nabla \phi|), \quad (7)$$

where τ is the pseudo-time for the re-initialization. The spatial terms in Eqs. (6) and (7) are discretized using the fifth-order WENO scheme [34], and the temporal terms are integrated using the third-order Runge-Kutta (RK) scheme with the total variation diminishing (TVD) property [34].

The smooth Heaviside function H in Eqs. (3) and (4) is defined using the distance function ϕ as follows,

$$H \equiv \begin{cases} 0, & \phi < -\varepsilon \\ \frac{\phi + \varepsilon}{2\varepsilon} + \frac{1}{2\pi} \sin\left(\frac{\pi\phi}{\varepsilon}\right), & -\varepsilon < \phi < \varepsilon \\ 1, & \phi > \varepsilon \end{cases} \quad (8)$$

where ε is set to be 1.5 times of the mesh size. The curvature in (5) can be obtained from the distance function ϕ as follows,

$$\kappa \equiv \nabla \cdot \mathbf{n} = \nabla \cdot \frac{\nabla \phi}{|\nabla \phi|}. \quad (9)$$

2.2.1. Mass correction

Ideally, the interface should remain stationary during the re-initialization procedure. However, numerical errors could move the interface to some degree, which causes mass loss of the droplet, even though the fifth order WENO scheme and the third order RK scheme are used. To resolve this issue, a local constraint term [34] is added into Eq. (7)

$$\frac{\partial \phi}{\partial \tau} = \text{sign}(\phi)(1 - |\nabla \phi|) + \lambda \delta |\nabla \phi|, \quad (10)$$

where δ is the smoothed delta function, which takes the form

$$\delta(\phi) \equiv \begin{cases} 0, & \phi < -\varepsilon \\ \frac{1}{2\varepsilon} + \frac{1}{2\varepsilon} \cos\left(\frac{\pi\phi}{\varepsilon}\right), & -\varepsilon < \phi < \varepsilon \\ 0, & \phi > \varepsilon \end{cases} \quad (11)$$

and

$$\lambda_{i,j} = -\frac{\int_{\Omega_{i,j}} \delta\left(\frac{\phi^{n+1} - \phi^n}{\Delta t}\right) d\mathbf{x}}{\int_{\Omega_{i,j}} \delta^2 |\nabla \phi| d\mathbf{x}}, \quad (12)$$

where $\Omega_{i,j}$ refers to the control volume. The integration in Eq. (12) is obtained using 27-point stencil ω_{ijk} ($-1 \leq i, j, k \leq 1$). With a uniform mesh for the 3D domain, the 27-point stencil is obtained from the one dimensional stencil ω_i ($-1 \leq i \leq 1$) following the production rule [35]

$$\omega_{ijk} = \omega_i \omega_j \omega_k, \quad -1 \leq i, j, k \leq 1, \quad (13)$$

where ω_i ($-1 \leq i \leq 1$) could be obtained by integrating the Lagrange interpolating polynomial over the control volume based on the information from the three adjacent nodes, x_{n-1} , x_n , and x_{n+1} ,

$$\omega_{-1} = \frac{1}{24}, \quad \omega_0 = \frac{22}{24}, \quad \omega_1 = \frac{1}{24}. \quad (14)$$

Therefore, the integration in Eq. (12) could be numerically approximated by

$$\int_{\Omega_{ijk}} f d\mathbf{x} = \sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 \omega_{ijk} f_{ijk} \Delta x \Delta y \Delta z, \quad (15)$$

where Δ_x , Δ_y , and Δ_z are the mesh size in x , y , and z directions of the simulation domain.

The simulation of the flow fields and the interface shapes is performed until the results reached a steady state. The flow fields and interface shapes are then used to model the heat transfer process.

2.3. Finite volume method for heat transfer

The heat transfer process is simulated using the finite volume method [29] solving the energy equation,

$$\frac{\partial(\rho c_p T)}{\partial t} + \nabla \cdot (\mathbf{u} \rho c_p T) = \nabla \cdot (k \nabla T), \quad (16)$$

where T is the temperature of fluid, c_p and k are the specific heat capacity and the thermal conductivity of the fluid, respectively. c_p and k in each control volume are treated using the smoothed Heaviside function in Eq. (8),

$$c_p = H c_{p,d} + (1 - H) c_{p,c}, \quad (17)$$

$$\frac{1}{k} = \frac{H}{k_d} + \frac{1 - H}{k_c}. \quad (18)$$

The Peclet number is used to characterize the relative effect between the advection and the diffusion in the domain, and it is defined as

$$Pe \equiv \frac{D_H V_{\text{droplet}}}{\alpha}, \quad (19)$$

where $D_H \equiv 4A/P = 2wd/(w+d)$ is the hydraulic diameter of the microchannel, V_{droplet} is the speed of the droplet, and α is the characteristic thermal diffusivity of the fluid,

$$\alpha \equiv \frac{k}{\rho c_p}. \quad (20)$$

A constant surface temperature is imposed on the wall of the microchannel. Without losing generality, the wall temperature is set to be $T_w = 1$, and the initial temperature of the fluid is $T_0 = 0$.

Two dimensionless values are adopted to quantify the heat transfer performance, the Nusselt number (Nu) and the heat transfer index (η). Their definitions are similar to those in Ref. [8], but in a 3D manner and taking both the continuous and the dispersed phases into consideration. The Nusselt number is defined as

$$Nu \equiv \frac{h D_H}{k_c}, \quad (21)$$

where h is the heat transfer coefficient defined as

$$h \equiv \frac{Q}{A(T_w - \bar{T})}, \quad (22)$$

where $\bar{T}$ is the mean temperature of the liquid defined as

$$\bar{T} \equiv \frac{\int \rho c_p u_x T dV}{\int \rho c_p u_x dV}, \quad (23)$$

and Q is the heat flux on the wall of the microchannel. The fluid properties ρ and c_p in Eq. (23) follow the definitions in Eqs.

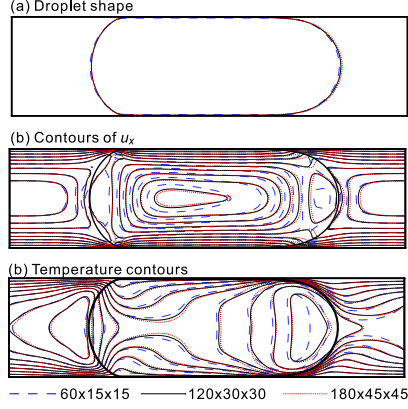


Figure 2: Mesh independence study for droplet heat transfer in microchannels. (a) Droplet shape on $z = 0$ plane. (b) Contours of the velocity component u_x on $z = 0$ plane. (c) Contours of the fluid temperature on $z = 0$ plane. Three different mesh densities with $60 \times 15 \times 15$, $120 \times 30 \times 30$, and $180 \times 45 \times 45$ control volumes are used and compared. The comparison shows that the mesh with $120 \times 30 \times 30$ control volumes is sufficient for the simulation.

(3) and (17) respectively to consider the contribution of the two phases to the mean temperature. For non-square microchannels, since the value of Q on the side walls ($y = \pm 0.5w$) is different from that on the top/bottom walls ($z = \pm 0.5d$), two Nusselt numbers, Nu_y and Nu_z , are used to represent the heat transfer over the side walls and the top/bottom walls, respectively.

The heat transfer index is defined to represent the progress of heat transfer as

$$\eta \equiv \frac{Q(t)}{Q_{\max}}, \quad (24)$$

where $Q_{\max}$ is the maximum amount of heat to be transferred if the process continues to $t \rightarrow \infty$, while $Q(t)$ is the total energy transferred from the wall to the liquid over the time interval 0 to t . With a constant temperature T_w maintained on the wall, $Q_{\max}$ corresponds to the state when the liquid temperature becomes T_w . Therefore, the heat transfer index η can be calculated as

$$\eta = \frac{\int \rho c_p (T - T_0) dV}{\int \rho c_p (T_w - T_0) dV}. \quad (25)$$

Therefore, it varies from 0 to unity as heat transfer progresses.

2.4. Mesh independence study

The mesh independence of the simulation is tested by progressively refining the mesh used for the simulations. The results of the droplet shape, the contours of u_x and the temperature at a typical instant at the three different mesh densities ($60 \times 15 \times 15$, $120 \times 30 \times 30$, $180 \times 45 \times 45$) are plotted in Figure 2. The comparison shows that the results from the mesh with $120 \times 30 \times 30$ control volumes are mesh independent. Therefore, this mesh density is used for other simulations.

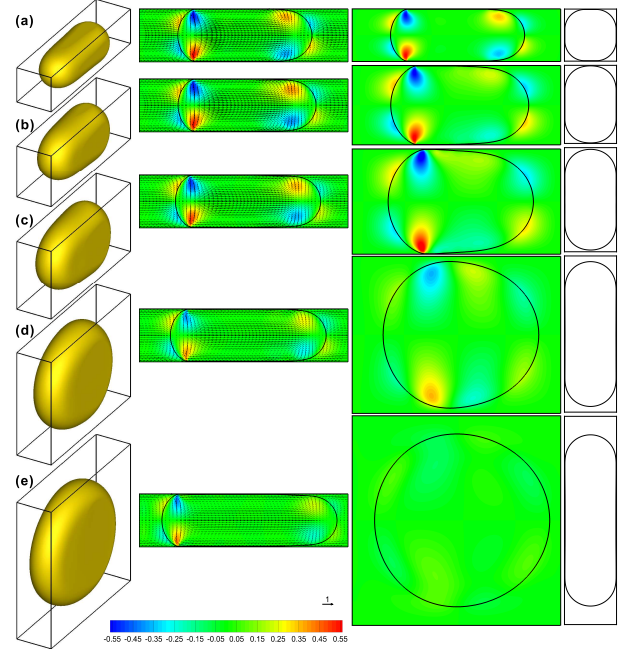


Figure 3: Droplet shapes and flow fields for droplet flow in microchannels with different aspect ratios. The aspect ratios of the channels are (a) $AR = 1$ (b) $AR = 1.5$, (c) $AR = 2$, (d) $AR = 3$, and (e) $AR = 4$, respectively. The figures in the first column are the 3D views of the droplet shapes; the second column shows the cross sectional views ($z = 0$) of the droplet shapes and the contours of the dimensionless velocity component u_y/V_{droplet} ; the third column shows the cross sectional views ($y = 0$) of the droplet shapes and the contours of the dimensionless velocity component u_z/V_{droplet} ; and in the last column are the cross sectional views of the droplets at $x = x_{\text{mc}}$ where the subscript 'mc' refers to the mass center of the droplets.

3. Results and discussions

3.1. Droplet shapes and flow fields

3.1.1. Droplet shapes and flow fields in microchannels with different aspect ratios

For a droplet immersed in another phase without confining effect or other external forces, the surface tension restores droplets into spherical shape by minimizing the interfacial areas. The confining effect of microchannel walls deforms the droplets away from their spherical shapes, and this effect is significant for the droplets whose diameters are larger than the channel width. The relative size of the droplet can be characterized by the dimensionless length of the droplet and the aspect ratio of the channel cross section,

$$\hat{L} = \frac{\text{Vol}}{w^2 d}, \quad (26)$$

$$AR = \frac{d}{w}, \quad (27)$$

where Vol is the volume of the droplet, and w and d are the width and the depth of the microchannel.

For microchannels with large aspect ratios, as shown in Figure 3(e), the droplets are not confined in the depth direction (z direction), and take a drum shape. The interfaces near the side walls ($y = \pm w/2$) are flat while the interfaces near the

top/bottom walls ($z = \pm d/2$) adopt a curved shape due to the strong effect of surface tension and the weak effect of wall confinement. As the aspect ratio decreases, the droplets are squeezed by the top/bottom wall, and gradually change into a bullet shape, where the interfaces close to the top/bottom walls gradually become flat, as shown in Figure 3(a). At the front and the rear of the droplet, the interfaces take a spherical-cap shape. This is due to the strong effect of surface tension at small scales. The front cap of the interface has a larger curvature than the rear cap. This is because the flow within the droplet pushes the interface at the front cap and increases the local pressure. At the corner of the microchannel, ‘gutters’ form between the droplet interface and the corner of the wall. The size of the ‘gutter’ increases as the aspect ratio of the microchannel increases, as shown in the last column of Figure 3. The details of the flow in the ‘gutter’ will be discussed in Section 3.1.3.

For heat transfer process in plug flows in 2D microchannels [8] or in cylindrical microchannels [18], the radial velocity component is important because it carries the heated fluid from the wall to the center of the channel, and the fresh fluid from the center of the channel back to the wall. Similarly, for heat transfer process in 3D droplets, the velocity components in the radial directions (u_y , u_z) are important, and the details of the effects of the radial components on heat transfer process will be discussed in Section 3.2. Here, the velocity components are plotted in the second column (u_y at $z = 0$) and the third column (u_z at $y = 0$) of Figure 3, which shows that u_y and u_z are large near the front and the rear of the droplets, in both the continuous and the dispersed phases. This is because the radial velocity is induced by the interface which blocks the liquid flow in the axial direction. For droplets in square microchannels ($AR = 1$), the flow on $z = 0$ and $y = 0$ planes are identical due to the symmetrical nature of the geometry. With increasing AR , the velocity component u_z decreases. This is because as AR increases, the cross-sectional area of the ‘gutter’ increases, and the blockage effect of the interface to produce flow in the radial direction and the confining effect of the top/bottom walls are reduced, and the top/bottom walls consequently have reduced shear effect on the droplet. As less fluid arrives at the rear of the droplet and then flows in the radial direction, the magnitude of u_z decreases.

3.1.2. Droplet shapes and flow fields at different droplet length ratios

The effect of the droplet length ratio between the dispersed phase and the continuous phase, $\frac{\hat{L}}{\hat{L}_{\text{total}} - \hat{L}}$, is studied by changing the length of the droplet from $\hat{L} = 0.45$ to 2.22, while the length of the continuous phase is decreased correspondingly with a fixed total dimensionless length of $\hat{L}_{\text{total}} = 4$, as shown in Figure 4. For droplets with small dimensionless lengths (such as $\hat{L} = 0.45$ in Figure 4), they are almost spherical due to the strong effect of surface tension and the weak confining effect by the channel walls. With increasing the droplet length, the confining effect becomes strong, and the interface near the wall becomes flat (such as for $\hat{L} = 2.22$ in Figure 4). In addition, the ‘gutter’ between the wall and the droplet becomes thinner as the droplet

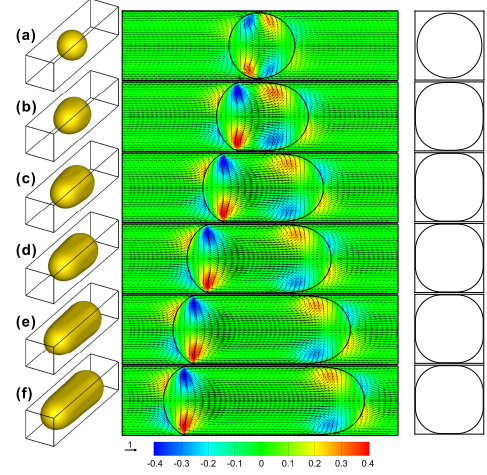


Figure 4: Droplet shapes and flow fields for droplets with different lengths. The dimensionless lengths of the droplets are (a) 0.45, (b) 0.80, (c) 1.16, (d) 1.51, (e) 1.87, and (f) 2.22, respectively. The aspect ratio of the channels is fixed at $AR = 1$. The figures in the first column are the 3D views of the droplet shape; the second column shows the cross sectional views ($z = 0$) of the droplet and the contours of the dimensionless velocity component u_y/V_{droplet} ; and the last column shows the cross sectional views of the droplet at $x = x_{\text{mc}}$ where the subscript ‘mc’ refers to the mass center of the droplet.

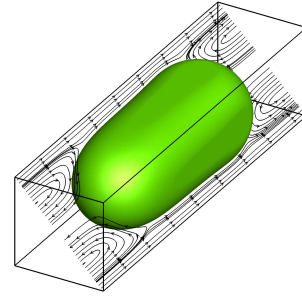


Figure 5: Flow in the diagonal plane for a typical square microchannel. The droplet length is $\hat{L} = 1.87$, and the aspect ratio of the channel is $AR = 1$.

length increases, as shown in the third column of Figure 4.

The contours of the velocity component u_y are plotted in the second column of Figure 4. With increasing the droplet length, the magnitude of the velocity component u_y , which is mainly located near the front and the rear of the droplets, does not change significantly. This is because for droplets with significant confining effect, the interface near the wall is almost flat, and the liquid film between the wall and the dispersed phase is very thin. The liquid in the droplet near the interface has almost the same speed with that of the wall for different droplet lengths. Therefore, the magnitudes of the velocity component u_y are similar for different droplet lengths.

3.1.3. Flow in ‘gutters’

Because of the non-axisymmetric structure of the microchannels, ‘gutters’ form at the corner of the microchannel for the flow of the continuous phase. The flow in a typical square microchannel is plotted in Figure 5, which shows the streamline of the continuous phase along a diagonal plane. In the slug region of the continuous phase, i.e., the region

between successive droplets, recirculating flow forms due to the presence of the droplet interface. In the ‘gutter’ beneath a droplet, the flow in the continuous phase is approximately in parallel. The stream lines are slightly curved only near the front and the rear of the droplet. The parallel flow in the ‘gutter’ could hinder the heat transfer because the flow direction is perpendicular to the direction of heat transfer, which is normal to the wall.

Due to the parallel flow in the ‘gutter’, the velocity components in the radial direction and the velocity gradient in the flow direction are negligible, i.e.,

$$\frac{\partial u_x}{\partial x} \sim 0, u_y \sim 0, u_z \sim 0. \quad (28)$$

In addition, as the wall is maintained at a constant temperature, the temperature gradient in the flow direction is negligible, i.e.,

$$\frac{\partial T}{\partial x} \sim 0. \quad (29)$$

Therefore, the full governing equation for the heat transfer in the ‘gutter’

$$\begin{aligned} \frac{\partial(\rho c_p T)}{\partial t} + \frac{\partial}{\partial x}(u_x \rho c_p T) + \frac{\partial}{\partial y}(u_y \rho c_p T) + \frac{\partial}{\partial z}(u_z \rho c_p T) \\ = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) \end{aligned} \quad (30)$$

is reduced to

$$\frac{\partial(\rho c_p T)}{\partial t} = \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right), \quad (31)$$

which indicates that the heat transfer relies on the heat conduction in the radial direction, and the convection does not contribute the process. Therefore, in the ‘gutter’ beneath the droplet, heat transfer from the wall to the central region of the liquid could only be via conduction through the ‘gutter’, which is much slower than heat advection happening in the droplet and in the slug region resulted from the recirculating vortices.

3.2. Process of droplet heat transfer

A typical process of droplet heat transfer in a rectangular microchannel at a constant surface temperature is shown in Figure 6. The droplet moves in the positive x direction, and the frame of reference is following the droplet. Initially, the liquid has a uniform temperature $T_0 = 0$, while the wall has a constant temperature $T_w = 1$, as shown in Figure 6(a). A thin thermal boundary layer forms immediately after the heat transfer process starts (see Figure 6(b)). The thickness of the thermal boundary layer increases as more heat is transferred to the liquid (see Figure 6(c)). The thickness is also affected by the flow field and the interface shape, which causes the thermal boundary layer to be thin at the front and thick at the rear of the droplet.

When the thermal boundary layer is sufficiently thick, the heated fluid is advected by the recirculating vortices in both the

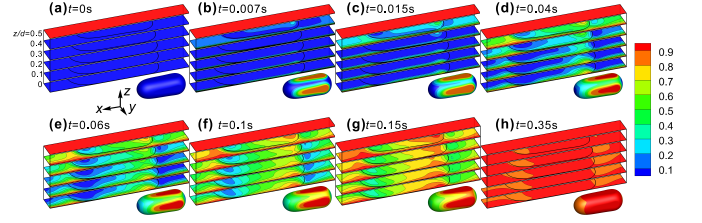


Figure 6: Temperature contours for droplet heat transfer in a microchannel. The droplet length is $\hat{L} = 1.87$, and the aspect ratio of the channel is $AR = 1$. The Peclet number is $Pe = 200$. The temperature contours at different depths ($z/d = 0$ (mid depth plane), 0.1, 0.2, 0.3, 0.4, 0.5 (wall)) are shown as well as the temperature contours on the surface of the droplet.

continuous and the dispersed phases. In the dispersed phase, the liquid at the rear of the droplet has a higher temperature; while in the slug region of the continuous phase, the liquid at the rear also has a higher temperature, as shown in Figure 6(c-d). After the temperature gradient in the two phases gradually forms, the process of heat transfer goes to the stage of a thermally fully developed flow (see Figure 6(f)). During this stage, the temperature gradually increases, but the pattern of the temperature contours remains unchanged (see Figure 6(g)). The process of heat transfer continues as more heat is transferred from the wall to the liquid, and the liquid temperature gradually approaches the uniform wall temperature (see Figure 6(h)).

The temperature contours on the droplet surface in Figure 6 show that the liquid near the four walls increases its temperature more rapidly than near the ‘gutter’. Due to the short distance to the wall, the liquid temperature near the wall increases rapidly during the formation of the thermal boundary layer. In contrast, the liquid near the ‘gutter’ has a relatively lower temperature, due to the fact that the liquid in the ‘gutter’ hinders the heat transfer from the wall to the droplet. In addition, the liquid at the front cap of the droplet has a lower temperature than at other parts. This is because the recirculating flow can advect fresh fluid to this region.

The process of heat transfer is characterized by the heat transfer index and the Nusselt number, which are plotted in Figure 7. Initially, the Nusselt number is very high due to the large temperature gradient and the large heat flux near the wall, while the heat transfer index is 0. With the growth of the thermal boundary layer, the Nusselt number decrease dramatically, and the heat transfer index η increases. The Nusselt number fluctuates because the recirculating vortices could intermittently bring fresh fluid to the wall. (The fluctuation is affected by the Peclet number and will be discussed in Section 3.5) The Nusselt number reaches a constant value when the heat transfer process approaches the thermally fully developed stage. The heat transfer index gradually approaches $\eta=1$ as the liquid temperature reaches the wall temperature.

3.3. Effect of the aspect ratio of microchannels

The effect of the aspect ratio of the channel cross section is studied by varying the aspect ratio from 1 to 4, while fixing the droplet length and the Peclet number at $\hat{L} = 1.87$ and $Pe = 200$, respectively. For microchannels with a square cross section

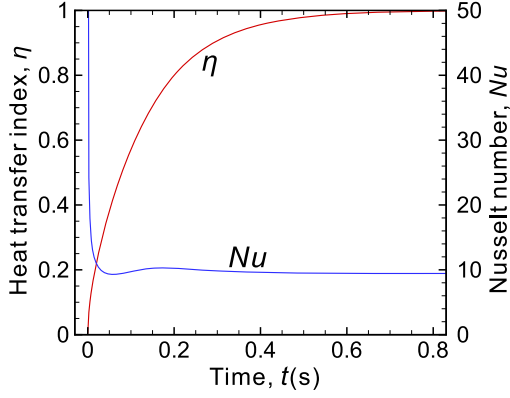


Figure 7: Heat transfer index (η) and the Nusselt number (Nu) for droplet heat transfer in a microchannel. The droplet length is $\hat{L} = 1.87$, and the aspect ratio of the channel is $AR = 1$. The Peclet number is $Pe = 200$.

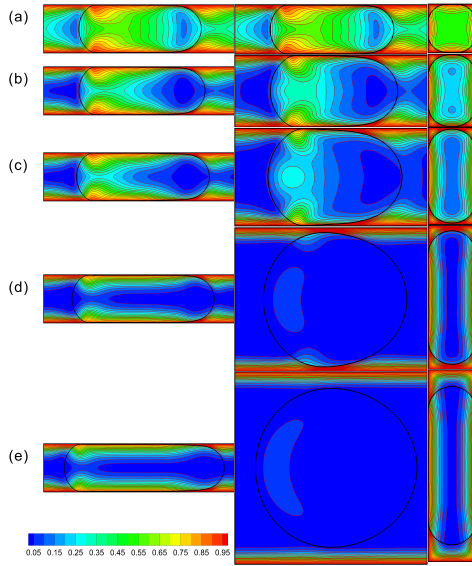


Figure 8: Temperature contours at the instant of $\hat{t} = 0.1$ for droplet heat transfer at different aspect ratios of the microchannel. The aspect ratios are, respectively, (a) 1, (b) 1.5, (c) 2, (d) 3, and (e) 4. The droplet length and the Peclet number are fixed at $\hat{L} = 1.87$ and $Pe = 200$. The first column shows the cross sectional views at $z = 0$; the second column shows the cross sectional views at $y = 0$; the third column shows the cross sectional views at $x = x_{mc}$.

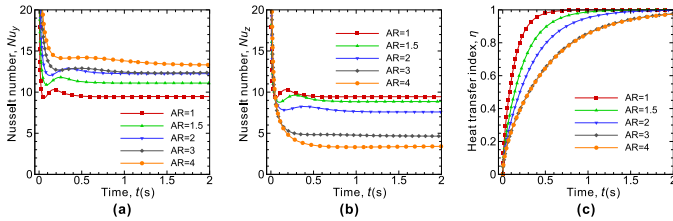


Figure 9: Nusselt number and heat transfer index for droplet heat transfer in microchannels with different aspect ratios. The droplet length and the Peclet number are fixed at $\hat{L} = 1.87$ and $Pe = 200$. (a) Nusselt number Nu_y on the $y = w/2$ surface, (b) Nusselt number Nu_z on the $z = h/2$ surface, (c) Heat transfer index η .

($AR = 1$ as shown in Figure 8(a)), the temperature contours on the $y = 0$ plane and on the $z = 0$ plane are the same due to the symmetry of the flow and the heat transfer. With increasing the

aspect ratio, the velocity component u_z decreases, as discussed in Section 3.1.1. In addition, a larger channel depth requires more time for both diffusion and advection in z direction. Therefore, the heat transfer in z direction is inhibited by the large aspect ratio. Therefore, the Nusselt number is lower for large aspect ratios than that at low aspect ratios, as shown in Figure 9(b).

For the heat transfer in y direction, the velocity component u_y is not significantly affected by the channel depth. Therefore, the contributions of u_y to the heat transfer in y direction are not significantly affected. As shown in Figure 8, the temperature contours are the combining effect of the heat transfer in y and z directions; the temperature on the plane of $z = 0$ is also lower for microchannels with high aspect ratios than those with low aspect ratios.

3.4. Effect of droplet length ratio

The effect of droplet length ratios between the dispersed phase and the continuous phase on the heat transfer process is studied by fixing the aspect ratio of the channel at $AR = 1$ and the Peclet number at $Pe = 200$. The dimensionless droplet length is increased from $\hat{L}=0.45$ to 2.22, while the length of the continuous phase is decreased correspondingly with a fixed total length of $\hat{L}_{total} = 4$. The results show that the heat transfer process is not significantly affected by the droplet length ratio. As discussed in Section 3.1.2, the radial flow is mainly located near the front and rear interfaces of the droplet, and the magnitude of the radial velocity does not change significantly with the droplet length. For short droplets, the area controlled by the radial flow in the droplet is relatively large, while the flow in the continuous phase is dominated by the parallel flow. Therefore, the recirculating vortices in the droplet could advect the droplet phase significantly, while the recirculating effect in the continuous phase is not remarkable, which relies mainly on the heat conduction to transfer the heat to the continuous phase. With the droplet length increases, the recirculating effect in droplets becomes weak, while the recirculating effect in the continuous phase becomes relatively strong, as shown in Figure 10(f). Therefore, with increasing the droplet length ratio, the heat transfer in the continuous phase is promoted, while the heat transfer in the dispersed phase is inhibited. Consequently, the overall effect on the heat transfer process is insignificant, as shown in the heat transfer index in Figure 11(b). The Nusselt number increases with the droplet length ratio, as shown in Figure 11(a). This is because the Nusselt number characterizes the heat transfer on the surface of channel wall, which is in direct contact with the continuous phase. Therefore, the effect of the droplet length ratio on the continuous phase is directly counted into the Nusselt number calculation. Therefore, the Nusselt number at large droplet length ratios is higher than that at small droplet length ratios.

3.5. Effect of Peclet number

The Peclet number significantly affects the heat transfer process, as shown in Figures 12 and 13, which are the temperature contours, the Nusselt number, and the heat transfer index at

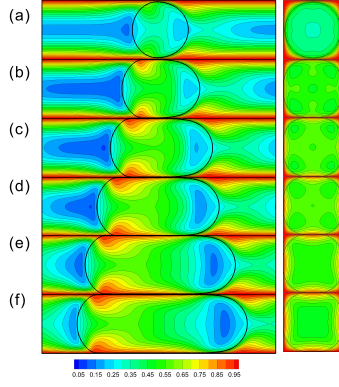


Figure 10: Droplet heat transfer with different droplet length ratios. The dimensionless lengths of the droplets, $\hat{L}$, are respectively, (a) 0.45, (b) 0.80, (c) 1.16, (d) 1.51, (e) 1.87, and (f) 2.22. The aspect ratio of the channel is fixed at $AR = 1$, and the Peclet number is fixed at $Pe = 200$. The first column shows the cross sectional views at $z = 0$; the second column shows the cross sectional views at $x = x_{mc}$.

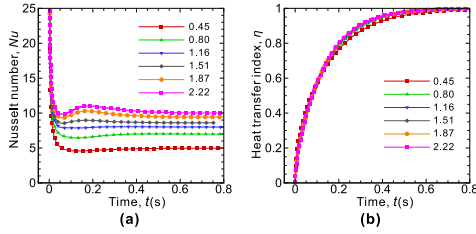


Figure 11: (a) Nusselt number Nu and (b) heat transfer index η for droplet heat transfer with different droplet length ratios. The aspect ratio of the channel are fixed at $AR = 1$, and the Peclet number is fixed at $Pe = 200$.

different Peclet numbers. The droplet length and the aspect ratio are fixed at $\hat{L} = 1.87$ and $AR = 1$, respectively, while the Peclet number is varied from 25 to 800. At a low Peclet number such as $Pe = 25$, the liquid could immediately achieve the wall temperature due to the strong effect of thermal diffusion. The corresponding Nusselt number, which characterizes the relative effect between advection and conduction at the wall, is low. Increasing Peclet number leads to a slower heat transfer process. For a high Peclet number such as $Pe = 800$, the advection effect is strong, and the clear front of the heated fluid could be easily distinguished, and the fluctuation of the Nusselt number during the process becomes more significant.

4. Conclusions

The droplet heat transfer in the microchannels with rectangular cross sections has been simulated numerically using the finite volume method for the flow field and the heat transfer and the level set method for interface capturing. A frame of reference following the droplets was employed to reduce the simulation time without reducing the simulation accuracy. Steady states of the flow fields were obtained first, and then they were used to simulate the heat transfer process of droplets moving in three dimensional microchannels with constant surface temperature. The results showed that the heat transfer was significantly affected by the three dimensional flow

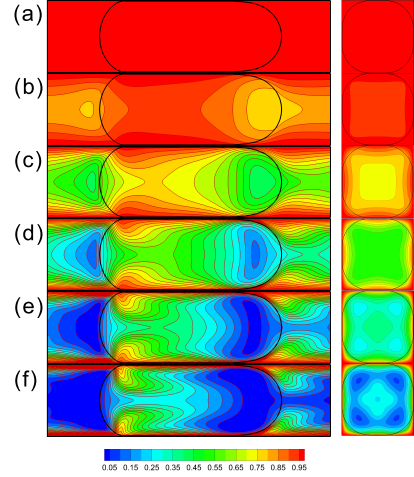


Figure 12: Temperature contours at the instant of $\hat{t} = 0.1$ for droplet heat transfer at different Peclet numbers. The Peclet numbers are, respectively, (a) 25, (b) 50, (c) 100, (d) 200, (e) 400, and (f) 800. The droplet length and the aspect ratio are fixed at $\hat{L} = 1.87$ and $AR = 1$, respectively. The first column shows the cross sectional views at $z = 0$; the second column shows the cross sectional views at $x = x_{mc}$.

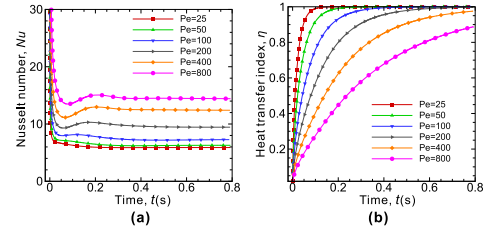


Figure 13: (a) Nusselt number Nu and (b) heat transfer index η for droplet heat transfer at different Peclet numbers. The droplet length and the aspect ratio are fixed at $\hat{L} = 1.87$ and $AR = 1$, respectively.

structure. The liquid ‘gutters’ between the droplet interface and the corner of the microchannel hindered the heat transfer process because of its parallel flow, whereas the recirculating flow in the droplets and in the slug regions between successive droplets can enhance the heat transfer by advecting hot fluid towards the center of the droplets/slugs and advecting fresh fluid towards the wall of the channel. The effects of the length of the droplets, the aspect ratio of the channel cross sections, and the Peclet number of the flow were studied. The Nusselt number and the heat transfer index were used to quantitatively describe the heat transfer process and to facilitate the comparison between different working parameters. This study can deepen our understanding of the flow and the heat transfer process in droplet-based microchannels, and guide the corresponding design of multiphase microchannel heat sinks.

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