

HYPersonic FLOW OVER POWER-LAW BODIES

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ABSTRACT

Theoretical solutions based on the expansion scheme for large x and large M_∞ , as proposed by Freeman, are obtained for the asymptotic inviscid flow over plane bodies of the shape $\frac{y}{d} = \left(\frac{x}{d}\right)^m$ in the range $\frac{2}{3} < m < \frac{2}{3}$ where blast wave theory applies as a first approximation. In particular, the second order terms, which are necessary to satisfy the body boundary condition for the normal velocity are computed. The magnitude of the second order terms is found to increase from zero at $m = \frac{2}{3}$ to infinity at $m = \frac{2}{3}$.

As a comparison with theory, experiments at $M_\infty = 8.2$ were made with two plane power-law bodies in the range $\frac{2}{3} < m < \frac{2}{3}$ (at $m = \frac{1}{2}$ and $\frac{5}{8}$) and on a plane parabola with a tangent wedge nose. These consisted of the determination of shock wave shapes, surface pressure distributions and detailed investigations of the distribution of pitot and static pressure across the shock-layer.

The experimental results are in particularly good agreement with the theory in the case $m = \frac{1}{2}$, where the second order effects are small. At $m = \frac{5}{8}$ the region of validity of the theory is limited to much larger distances from the nose of the body and larger Mach numbers. Accordingly, the prediction for the deviation from first order theory, although being correct in sign, is too small. Shock wave shapes on bodies of the same power but of different size are correlated by the similarity theory when scaled with respect to the dimension d .

The experimental results obtained with the wedge-parabola are in very good agreement with a characteristics solution by Lewis.

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NOTATION.

Symbols.

A	-	a constant
a	-	various constants
a_∞	-	speed of sound in the free stream
b	-	a constant
C	-	coefficient
D	-	drag
d	-	length scale of body
I_1, I_2	-	two integrals defined in eqns. 2.33 and 2.34
j	-	integer, = 0 for plane, = 1 for axisymmetric flow
K	=	$\frac{m-2}{1-\frac{1}{4}}$
m	-	exponent of body shape
p	-	pressure
p_{t2}	-	pitot pressure
$P, \mathcal{P}$	-	pressure functions
q	-	speed = $\sqrt{u^2 + v^2}$
$R, \mathcal{R}$	-	density functions
t	-	time
T	-	absolute temperature
u, U	-	velocity in x-direction
v	-	velocity in y-direction
$V, \mathcal{V}$	-	velocity functions
x	-	coordinate in free stream direction
y	-	coordinate normal to free stream direction
Y	-	normal distance function

δ^* - ratio of specific heats

$$\delta = M_{\infty}^{-2}$$

$$\epsilon = \frac{3m}{2} - \frac{1}{8}$$

η - similarity variable in outer layer

η_i - similarity variable in intermediate layer

θ - streamline slope

$$\lambda = \frac{2(1-\mu)}{\mu}$$

μ - exponent of shock wave shape

ν - kinematic viscosity

$$\xi = \eta^{\frac{3m}{2}} - \frac{1}{8}$$

ρ - density

τ - slenderness parameter

ϕ - shock wave slope

$\bar{\chi}$ - viscous interaction parameter

Subscripts.

b - body

s - shock wave

o - first order, also stagnation

l - second order

∞ - free stream, also up to $x = \infty$

1. INTRODUCTION.

The theoretical part of this investigation is concerned with the inviscid asymptotic flow field at large distances from the blunt nose of an asymptotically slender body, when the free stream Mach number is large and where the body still supports a strong shock wave.

The appropriate equations of motion are simplified to the hypersonic small disturbance equations developed by Hayes (1947), Goldsworthy (1952), Van Dyke (1954). This relies on the hypersonic similarity parameter, $M_\infty \tau$, being finite with M_∞ large and τ small, where M_∞ is the free stream Mach number and τ is a slenderness parameter. In deriving the hypersonic small disturbance equations, terms of order τ^2 or $\frac{1}{M_\infty^2}$ are neglected.

The boundary conditions for the hypersonic small disturbance equations are specified along the shock wave by the Rankine-Hugoniot relations, and at the body surface by zero normal velocity. The hypersonic small disturbance equations together with these boundary conditions are identical with those of one-dimensional unsteady flow produced by a moving piston when the longitudinal variation is replaced by time.

When the assumption of a strong shock wave is made, the boundary conditions at the shock wave are simplified considerably. This assumption involves the neglect of terms

of order $(M_\infty^2 \sin^2 \phi)^{-1}$ compared with one. The density is then constant along the shock wave.

Under these conditions the hypersonic small disturbance equations possess the self-similar solutions obtained by Lees and Kubota (1957), provided that the shock wave shape is of the form $y_s \sim x^\mu$. The solutions have direct physical significance in the range $\frac{2}{3+j} < \mu \leq 1$, where $j = 0$ for plane, and one for axisymmetric flow, the body being given by $y_b \sim x^m$, $m = \mu$. Their significance for μ outside this range has been considered by Mirels (1959) but is of no interest here. The solution for $\mu = \frac{2}{3+j}$ is singular, since it does not satisfy the surface boundary condition. This singular solution corresponds to the solution of the one-dimensional unsteady problem of a constant energy explosion obtained by Taylor (1950) and Sedov (1959). This solution had previously been interpreted by Lees (1955) and Cheng and Pallone (1956) as that appropriate for the blunt flat plate or cylinder, and this interpretation is usually referred to as the "blast wave analogy". If the drag of a body for which the similar solutions apply is considered, the pressure at the shock wave is given by the shock conditions as proportional to $x^{2(\mu-1)}$ and thus, since similarity applies, the body pressure will also be proportional to $x^{2(\mu-1)}$. The body shape is y proportional to x^m and with $\mu = m$ the drag will be proportional to $\int x^{3(\mu-1)} x^{j\mu} dx$ which varies as $\text{const.} + x^{(3+j)\mu-2}$. The constant can be interpreted as

the drag of the blunt nose, the other term as the drag of the afterbody. For $\mu > \frac{2}{3+j}$ it is seen that the afterbody drag dominates, whereas for $\mu = \frac{2}{3+j}$ the two terms are of equal order and the drag is independent of x . Hence, the drag of a body with $m < \frac{2}{3+j}$ is finite and dominated by the effect of the blunt nose. This would suggest that the solution for $\mu = \frac{2}{3+j}$ is a first approximation for the flow over all bodies in the range $0 < m < \frac{2}{3+j}$. The effect of the asymptotic body variation then appears as a higher order term. These solutions fail near the nose, where the small disturbance assumptions break down because of the blunt nose. It is possible, however, to treat the effect of the blunt nose as an upstream condition on the asymptotic flow, by using the condition that entropy remains constant along streamlines. This upstream condition is not satisfied by the similar solutions in a layer near the body which has become known as the "entropy layer" or "cool core", because the entropy and temperature are required by the upstream condition to be lower than the predictions of the similar solutions.

A further limitation is the requirement that the shock wave be strong. The error terms due to a weaker shock have been obtained for the one-dimensional unsteady case of a constant energy explosion, by Sakurai (1953,1954) and Swigart (1960), and for blunt bodies in the range $\frac{2}{3+j} < m < 1$ by Kubota (1957).

Attempts to formulate a rational expansion procedure have been made by a number of authors, among them Guiraud (1958), Freeman (1962), Yakura (1962), Vaglio-Laurin (1964), Zolver (1964) and Messiter (1965). It will be sufficient for the purpose of this introduction to describe only a few of the results.

Yakura considered the "inverse problem", in which the shock wave is given and the body shape is to be determined, and thus started with an entropy distribution which was known in terms of the stream function. This yielded an inner (entropy layer) solution which could be joined by matching to an outer similar solution. Although the inverse approach removes many difficulties from the problem, it has often been criticised because, by choosing the shock wave shape, one specifies the transition from the entropy layer to the outer solution and thus applies an artificial constraint to the system.

Guiraud (1958) considered the "direct problem" for the case of the blunt flat plate or cylinder and obtained the order of magnitude of the error term introduced in the Sedov solution by the effect of the entropy layer.

Freeman (1962) considered the direct problem for power law bodies in the range $0 < m < 1$ and obtained the order of magnitude of the error terms introduced in the respective first approximations in expansions in inverse powers of M_∞^2 for large x . He found that, in the range

$0 < m < \frac{2}{3+j}$, where the first approximation takes no account of the body shape, the error is due to the effect of the body or entropy layer, depending on whether m is greater or less than $\frac{2}{(3+j)^{\frac{1}{2}}}$ respectively. In the range $\frac{2}{3+j} < m < 1$ the entropy layer was again found to give the important error, and only for $m > \frac{2(j+1)}{(3+j)^{\frac{1}{2}}+2}$ was the error term as small as that given by Van Dyke (1954) and assumed by Yakura, that is, of order $\frac{1}{M_{\infty}^2}$. Figures 1 and 2 show the essential results of this work.

The approach adopted here will be that used by Freeman. A discussion of other work will be left to section 3. The aim of the theoretical part of the present work is to derive and solve the differential equations for the second order quantities in the range $\frac{2}{(3+j)^{\frac{1}{2}}} < m < \frac{2}{3+j}$, where the first approximation is the Sedov solution and the first perturbation is due to the body.

Compared to the number of theories available for the blunt body problem, experimental data is limited. The most complete set of data is that of Kubota (1957) who obtained surface pressure distributions, shock wave shapes and pitot pressure traverses through the shock layer on three axisymmetric power law bodies, $m = \frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ at $M_{\infty} = 7.7$. All other available experiments are restricted to the determination of shock wave shape and surface pressure distributions and very few deal with power law bodies other than $m = 0$. Among them are the works of

Hammitt and Bogdonoff (1956), Freeman, Cash and Bedder (1962) and Peckham (1965).

As has been pointed out by Freeman et al., the magnitude of the error term depends on the value of μ in such a way, that the region of validity of perturbation solutions will be achieved only for very large values of x for some values of μ . As will be seen in section 3, this value of x is sometimes outside the range of present experimental facilities.

The determination of the flow quantities in the shock layer presents a further difficulty. Kubota relied on the assumptions that the total temperature is constant and that the streamline slope is independent of the distance normal to the body in order to derive all quantities from one measurement (pitot pressure). Although the second assumption becomes more and more accurate as $m \rightarrow 1$, it is certainly not satisfactory for $m < \frac{2}{3+j}$, especially for the plane case. One would therefore like to be able to measure other quantities as well in order to avoid this assumption and, if possible, to achieve some redundancy in the measurements to serve as a check.

The aim of the experimental part of this project was thus to develop a reliable technique for determining the flow parameters in the shock layer under the conditions which could most nearly be compared with the conditions of the theoretical part within the limits of the available facility.

2. THEORY FOR $M_\infty \rightarrow \infty$.

2.1. Equations of Motion.

The equations of motion for two-dimensional, inviscid, compressible flow over an asymptotically slender blunt body are considered in the limiting case $M_\infty \rightarrow \infty$, $x \rightarrow \infty$ under the assumption that rates of change in the x-direction are negligible compared to rates of change in the y-direction. The ratio of these rates of change is of the order of $\frac{1}{M_\infty}$, since small disturbances are propagated across the flow at the free stream speed of sound and convected along the flow direction at the free stream velocity. It is appropriate therefore to choose the following set of dimensionless variables.

$$\left. \begin{array}{ll} \text{pressure} & p' = \frac{p}{p_\infty a_\infty^2} \\ \text{density} & \rho' = \frac{\rho}{\rho_\infty} \\ \text{x-velocity} & u' = \left(\frac{u}{U_\infty} - 1 \right) \delta^{-1} \\ \text{y-velocity} & v' = \frac{v}{a_\infty} \\ & \frac{y'}{x'} = \delta^{-\frac{1}{2}} \frac{y}{x} \end{array} \right\} \text{--- 2.1}$$

where x, y are cartesian coordinates, $\delta = \frac{1}{M_\infty^2}$ and a_∞ is the free stream sound speed.

If the primed variables are defined as of order one and terms of order δ are neglected, the equations of motion become:

$$\left. \begin{aligned} \frac{\partial u'}{\partial x'} + v' \frac{\partial u'}{\partial y'} + \frac{1}{\rho'} \frac{\partial p'}{\partial x'} &= 0 \\ \frac{\partial v'}{\partial x'} + v' \frac{\partial v'}{\partial y'} + \frac{1}{\rho'} \frac{\partial p'}{\partial y'} &= 0 \\ \frac{\partial}{\partial x'} (\rho' y'^j) + \frac{\partial}{\partial y'} (\rho' y'^j v') &= 0 \\ \left(\frac{\partial}{\partial x'} + v' \frac{\partial}{\partial y'} \right) \frac{p'}{\rho'^{\frac{\gamma}{\gamma-1}}} &= 0 \end{aligned} \right\} \text{--- 2.2}$$

and the boundary conditions on the shock wave on which

become

$$\left. \begin{aligned} \frac{dy'}{dx'} &= \tan \phi' \\ p' &= \frac{2}{\gamma+1} \tan^2 \phi' \left[1 - \frac{\gamma-1}{2\gamma} \cot^2 \phi' \right] \\ \rho' &= \left[\frac{\gamma-1}{\gamma+1} + \frac{2}{\gamma+1} \cot^2 \phi' \right]^{-1} \\ u' &= \frac{2}{\gamma+1} \tan^2 \phi' [1 - \cot^2 \phi'] \\ v' &= \frac{2}{\gamma+1} \tan \phi' [1 - \cot^2 \phi'] \end{aligned} \right\} \text{--- 2.3}$$

These are the hypersonic small disturbance equations with the equation for u' decoupled from the remaining three which are identical to those of one-dimensional unsteady flow when the transformation $t = \frac{x}{U_\infty}$ is made. For the case of a strong shock wave, $\tan^2 \phi'$ is large, and the $\cot^2 \phi'$ term in 2.3 may be neglected.

As has been shown by Lees and Kubota (1957), similar solutions exist only for a strong shock wave whose shape is $y'_s \sim x'^{\frac{1}{2}}$. The body boundary condition for v' ,

$$v' = \frac{dy'}{dx'}, \quad \text{on } y' = y'_b \quad \text{--- 2.4}$$

requiring the body to be a streamline, determines the scale for y and x :- If the body shape is given by

$$\frac{y}{d} = \left(\frac{x}{d}\right)^m \quad \text{--- 2.5}$$

2.4 requires that

$$\left(\frac{dy'}{dx'}\right)_{\text{body}} = m \frac{y}{x} \delta^{-\frac{1}{2}} = m \left(\frac{d}{x}\right)^{1-m} \delta^{-\frac{1}{2}} = O(1)$$

and therefore

$$x' = \frac{x}{d} \delta^{\frac{1}{2(1-m)}}$$

$$y' = \frac{y}{d} \delta^{\frac{m}{2(1-m)}}$$

The boundary condition for v' at the strong shock wave requires by an analogous argument that

$$x' \sim \left(\frac{x}{d}\right) \delta^{\frac{1}{2(1-\mu)}}$$

$$y' \sim \left(\frac{y}{d}\right) \delta^{\frac{\mu}{2(1-\mu)}}$$

Thus, if both boundary conditions for v' are to be significant, μ must be equal to m , and the body shape is similar to the shock wave shape.

2.2. The Similar Solutions of Lees and Kubota and Error Terms.

The equations can now be written in terms of a similarity variable $\eta = \frac{\psi'}{\psi_s'}$, where the stream function ψ' is defined by

$$\frac{\partial \psi'}{\partial y'} = \rho' y'^j \quad \text{and} \quad \frac{\partial \psi'}{\partial x'} = -\rho' y'^j v'$$

and $\psi_s' = y_s'^{j+1} = (C_0 x'^m)^{j+1}$ is the value of ψ' on the shock

wave. Considering the plane case, $j = 0$, they become

$$\left. \begin{aligned} \frac{\partial v'}{\partial x'} - \mu \frac{\eta}{x'} \frac{\partial v'}{\partial \eta} + \frac{1}{x'^{\mu}} \frac{\partial p'}{\partial \eta} &= 0 \\ \left[\frac{\partial}{\partial x'} - \mu \frac{\eta}{x'} \frac{\partial}{\partial \eta} \right] \frac{p'}{\rho'^{\frac{1}{\mu}}} &= 0 \\ \frac{\partial \rho'}{\partial x'} - \mu \frac{\eta}{x'} \frac{\partial \rho'}{\partial \eta} + \frac{\rho'^2}{x'^{\mu}} \frac{\partial v'}{\partial \eta} &= 0 \end{aligned} \right\} \text{--- 2.6}$$

With the following expressions for the dependent variables

$$\begin{aligned} p' &= C_0^2 x'^{-2(1-\mu)} P_0(\eta) \\ v' &= C_0 x'^{-(1-\mu)} V_0(\eta) \\ \rho' &= R_0(\eta) \end{aligned}$$

the equations become independent of x' and are

$$\left. \begin{aligned} \frac{dP_0}{d\eta} - (1-\mu)V_0 - \mu\eta \frac{dV_0}{d\eta} &= -\mu(1-\mu) \\ \frac{P_0}{R_0^{\frac{1}{\mu}}} &= \frac{2\mu^2 \left(\frac{\eta-1}{\eta+1}\right)^{\frac{\eta}{\mu}}}{(\eta+1)\eta^{\frac{2(1-\mu)}{\mu}}} \\ R_0^2 \frac{dV_0}{d\eta} - \mu\eta \frac{dR_0}{d\eta} &= 0 \end{aligned} \right\} \text{--- 2.7}$$

Here, the second of equations 2.6 has been integrated using the shock wave shape $y'_s = C_0 x'^{\mu}$ to give the second of equations 2.7. The boundary conditions on the shock wave require that V_0 , P_0 and R_0 are given constants on $\eta = 1$, and the body boundary condition, which requires $V_0 = \frac{\mu}{C_0}$, determines the scale of the shock wave in relation to the scale of the body.

The solution for small η can be obtained directly by

considering P_0 to be constant near the body surface in the last two equations:-

$$\left. \begin{aligned} p' &\propto x'^{-2(1-\mu)} \\ v' &\propto x'^{-(1-\mu)} \left[\eta'^{-\frac{2(1-\mu)}{\mu^2}} + \text{const.} \right] \\ \rho' &\propto \eta'^{\frac{2(1-\mu)}{\mu^2}} \end{aligned} \right\} \text{--- 2.8}$$

It is seen that the velocity boundary condition at $\eta = 0$ can be satisfied if the constant is put equal to μ . However, the density at the body is zero. The value of density can be obtained from the relation $\frac{p}{\rho^{\frac{1}{\mu}}} = \text{constant}$ along streamlines by considering its value from the point where the streamline crosses the shock wave as

$$\rho = \frac{p_s}{p_s^{\frac{1}{\mu}}} p^{\frac{1}{\mu}}$$

But p_s is a constant, and $p_s \propto \frac{1}{1+\cot^2 \phi}$. In the region under consideration $\cot^2 \phi \gg 1$ and

$$\rho' \propto x'^{-\frac{2(1-\mu)}{\mu^2}} \psi'^{\frac{2(1-\mu)}{\mu^2}}$$

as in 2.8, but for streamlines close to the body,

$$\rho' \propto x'^{-\frac{2(1-\mu)}{\mu^2}} \delta^{\frac{1}{\mu}}$$

while it has been assumed to be $O(1)$ everywhere. The similarity solution thus fails in a layer near the body in which $\frac{\psi}{\rho^{\frac{1}{\mu}}} = O(1)$ and not $O\left(\delta^{-\frac{\mu}{2(1-\mu)}}\right)$ as it was assumed. In this layer it fails to satisfy the upstream condition

for the entropy distribution. A new set of variables may now be defined, which remain of order one in this layer. This will, in turn, introduce a perturbation in the outer solution which has to be matched with the inner solution. The expansion in the outer region will then be of the form

$$\left. \begin{aligned} p' &= C_0^2 x'^{-\lambda\mu} P_0(\eta) + C_1 C_0 \delta^{\frac{1}{\lambda}-\frac{1}{8}} x'^{(\frac{\lambda}{8}-\lambda-1)\mu} P_1(\eta) + \dots \\ v' &= C_0 x'^{-\frac{\lambda\mu}{2}} V_0(\eta) + C_1 \delta^{\frac{1}{\lambda}-\frac{1}{8}} x'^{(\frac{\lambda}{8}-\frac{\lambda}{2}-1)\mu} V_1(\eta) + \dots \\ \rho' &= R_0(\eta) + \frac{C_1}{C_0} \delta^{\frac{1}{\lambda}-\frac{1}{8}} x'^{(\frac{\lambda}{8}-1)\mu} R_1(\eta) + \dots \end{aligned} \right\} \quad 2.9$$

where $\lambda = \frac{2(1-\mu)}{\mu}$. If these expansions are substituted into equations 2.2 and terms of order $\delta^{\frac{1}{\lambda}-\frac{1}{8}}$ are collected, a new set of differential equations for V_1 , R_1 and P_1 are obtained, which describe the effect of the inner entropy layer on the similarity solution. The shock wave is then perturbed to the form

$$\eta_s' = C_0 x'^{\mu} + C_1 \delta^{\frac{1}{\lambda}-\frac{1}{8}} x'^{\lambda\mu(\frac{1}{8}-\frac{1}{2})} + \dots$$

and this, substituted into the strong form of 2.3 gives the new shock conditions for the three equations in P_1 , V_1 and R_1 . When $\delta^{\frac{1}{\lambda}-\frac{1}{8}} = \delta$, that is, when $\frac{1}{\lambda}-\frac{1}{8}=1$, or $m = \frac{2(\gamma+1)}{3\gamma+2}$, the error term is of the same order as the terms which have been neglected in deriving equations 2.2. This means that the entropy layer would, for $m > \frac{2(\gamma+1)}{3\gamma+2}$, be so thin that it would lie within a layer in which u can no longer be approximated by U_∞ .

At the other end of the range for which this expansion applies - $m = \frac{2}{3+j}$ or $m = \frac{2}{3}$ for plane flow - the first approximation breaks down at the body surface because then the constant in v' of 2.8 becomes zero. This means that the effect of the body on the outer flow is dominated by the blunt nose.

2.3. The First Order Equations for $m < \frac{2}{3}$.

For $m < \frac{2}{3}$ the shock wave is thus given as $y'_s \propto x'^{\frac{2}{3}}$ or $\mu = \frac{2}{3}$, while the body is $\frac{y}{d} = \left(\frac{x}{d}\right)^m$. The shock condition for v' now requires that $x' = \left(\frac{x}{d}\right) \delta^{\frac{3}{2}}$ and $y' = \frac{y}{d} \delta$ are both $O(1)$ for the purposes of the first approximation, while on the body this would mean that

$$y' = x'^m \delta^{1-\frac{3m}{2}}$$

The effect of the body on the outer flow is thus of the order $\delta^{1-\frac{3m}{2}}$. When m is $\frac{2}{3}$, this becomes

$$y' = x'^{\frac{2}{3}} \delta^{1-\frac{1}{3}}$$

and the perturbation due to the body is again as small as that of the entropy layer.

For the range $0 < m < \frac{2}{3}$ the equations 2.2 become

$$\left. \begin{aligned} \frac{\partial v'}{\partial x'} - \frac{2}{3} \frac{\eta}{x'} \frac{\partial v'}{\partial \eta} + \frac{1}{x'^{\frac{2}{3}}} \frac{\partial p'}{\partial \eta} &= 0 \\ \left(\frac{\partial}{\partial x'} - \frac{2}{3} \frac{\eta}{x'} \frac{\partial}{\partial \eta} \right) \frac{p'}{\rho'^{\frac{2}{3}}} &= 0 \\ \frac{\partial p'}{\partial x'} - \frac{2}{3} \frac{\eta}{x'} \frac{\partial p'}{\partial \eta} + \frac{\rho'^2}{x'^{\frac{2}{3}}} \frac{\partial v'}{\partial \eta} &= 0 \end{aligned} \right\} \quad \text{--- 2.10}$$

where η is again $\frac{\psi}{\psi_s} = \frac{\psi'}{C_0 x'^{\frac{2}{3}}}$ where $\psi' = \frac{\psi}{\rho_0 u_0 d} \delta$, the shock wave being given by $y' = C_0 x'^{\frac{2}{3}}$.

With the expressions

$$\left. \begin{aligned} p' &= C_0^2 x'^{-\frac{2}{3}} P_0(\eta) \\ \rho' &= R_0(\eta) \\ v' &= C_0 x'^{-\frac{1}{3}} V_0(\eta) \end{aligned} \right\} \text{--- 2.11}$$

equations 2.10 become independent of x' and are:-

$$\left. \begin{aligned} \frac{dP_0}{d\eta} - \frac{1}{3} V_0 - \frac{2}{3} \frac{dV_0}{d\eta} &= 0 \\ \frac{P_0}{R_0^\gamma} &= \frac{8}{9} \cdot \frac{\left(\frac{\gamma-1}{\gamma+1}\right)^\gamma}{\gamma+1} \cdot \frac{1}{\eta} \\ \frac{2}{3} \eta \frac{dR_0}{d\eta} - R_0^2 \frac{dV_0}{d\eta} &= 0 \end{aligned} \right\} \text{--- 2.12}$$

The second equation has here been integrated and the constant has been determined from the value of $\frac{p}{\rho^\gamma}$ on the point where a given streamline crossed the shock wave. The boundary conditions on the shock wave ($\eta=1$) are:-

$$\left. \begin{aligned} V_0(1) &= \frac{4}{3(\gamma+1)} \\ P_0(1) &= \frac{8}{9(\gamma+1)} \\ R_0(1) &= \frac{\gamma+1}{\gamma-1} \end{aligned} \right\} \text{--- 2.13}$$

2.4. The Effect of the Body for Small η , $\frac{2}{3\gamma} < m < \frac{2}{3}$.

The behaviour of this solution for small η can be

obtained directly by considering P_0 to be constant:-

$$\left. \begin{aligned} p' &= C_0^2 x'^{\frac{2}{3}} P_0(0) \\ v' &= C_0 x'^{-\frac{1}{3}} \frac{2}{3(\gamma+1)} \left[\frac{8}{9(\gamma+1)P_0(0)} \right]^{\frac{1}{2}} \eta^{1-\frac{1}{\gamma}} \\ \rho' &= \frac{\gamma+1}{\gamma-1} \left[\frac{9}{8}(\gamma+1)P_0(0) \right]^{\frac{1}{2}} \eta^{\frac{1}{\gamma}} \end{aligned} \right\} \quad 2.14$$

But the boundary condition on the body requires

$$v' = m x'^{m-1} \delta^{1-\frac{3m}{2}}$$

This means that when η is so small that $v' \sim O[x'^{-\frac{1}{3}} \eta^{1-\frac{1}{\gamma}}]$, is of the same order as $x'^{m-1} \delta^{1-\frac{3m}{2}}$, that is, when.

$$\psi' = O \left[x'^{\frac{m-\frac{2}{3\gamma}}{1-\frac{1}{\gamma}}} \delta^{\frac{1-\frac{3m}{2}}{1-\frac{1}{\gamma}}} \right]$$

new variables have to be defined as of order one as follows.

$$\left. \begin{aligned} p'' &= p' \\ x'' &= x' \\ v'' &= v' \delta^{-(1-\frac{3m}{2})} \\ y'' &= y' \delta^{-(1-\frac{3m}{2})} \\ \rho'' &= \rho' \delta^{-\frac{1-\frac{3m}{2}}{\gamma-1}} \\ \psi'' &= \psi' \delta^{-\frac{1-\frac{3m}{2}}{1-\frac{1}{\gamma}}} \end{aligned} \right\} \quad 2.15$$

A suitable new similarity variable in this layer could then

be defined as $\eta_1 = \frac{\psi''}{C_0 x''^K}$, where $K = \frac{m - \frac{2}{3}}{1 - \frac{1}{3}}$. Equations 2.10 in x'' and this variable, neglecting terms containing δ to a positive power, become:-

$$\left. \begin{aligned} \frac{\partial p''}{\partial \eta_1} &= 0 \\ \frac{p''}{\rho''^{\frac{1}{3}}} &= \frac{\text{const.}}{\eta_1} \\ \frac{\partial \rho''}{\partial x''} - K \frac{\eta_1}{x''} \frac{\partial \rho''}{\partial \eta_1} + \frac{\rho''^{\frac{2}{3}}}{x''^K} \frac{\partial v''}{\partial \eta_1} &= 0 \end{aligned} \right\} \quad \text{--- 2.16}$$

With the expressions

$$\left. \begin{aligned} v'' &= C_0 x''^{m-1} V_0(\eta_1) \\ p'' &= C_0^2 x''^{-\frac{2}{3}} P_0(\eta_1) \\ \rho'' &= x''^{\frac{1}{3}(K-\frac{2}{3})} R_0(\eta_1) \end{aligned} \right\} \quad \text{--- 2.17}$$

the x'' dependance is again lost:

$$\left. \begin{aligned} \frac{dP_0}{d\eta_1} &= 0 \\ \frac{P_0}{R_0^{\frac{2}{3}}} &= \frac{8}{9} \cdot \frac{(x-1)^x}{(x+1)^x} \cdot \frac{1}{x+1} \cdot \frac{1}{\eta_1} \\ \frac{1}{8} (K - \frac{2}{3}) R_0 - K \eta_1 \frac{dR_0}{d\eta_1} + R_0^{\frac{2}{3}} \frac{dV_0}{d\eta_1} &= 0 \end{aligned} \right\} \quad \text{--- 2.18}$$

and the boundary condition for v'' is

$$V_0(0) = \frac{m}{C_0} \quad \text{--- 2.19}$$

The solution of these equations is immediate, giving

$$\left. \begin{aligned} \mathcal{P}_0 &= P_0(0) \\ \mathcal{R}_0 &= \frac{x+1}{x-1} \left[\frac{9}{8} (x+1) P_0(0) \right]^{\frac{1}{3}} \eta_1^{\frac{1}{3}} \\ \mathcal{U}_0 &= \frac{m}{C_0} + \frac{2}{3(x+1)} \left[\frac{8}{9(x+1) P_0(0)} \right]^{\frac{1}{3}} \eta_1^{1-\frac{1}{3}} \end{aligned} \right\} \quad 2.20$$

Substituting these expressions into 2.17 and re-writing them in terms of $\eta = \eta_1 x^{K-\frac{2}{3}} \delta^{1-\frac{3K}{2}}$, it is seen that the result for large η_1 reproduces the result 2.14 precisely except for the term $m x^{m-1}$ in v'' . This term becomes $\delta^{1-\frac{3m}{2}} x^{m-1}$ in v' and is thus a higher order term in the outer solution. It represents the perturbation produced by the body on the outer solution. The matched solution for v' is now uniformly valid, but the upstream condition is still not satisfied, ρ' being given as zero on $\eta=0$, while an argument similar to that following equations 2.8 gives the true density to be $\rho' = O(\delta^{\frac{1}{3}})$ on $\psi=0$. In the entropy layer, where $\rho' = O(\delta^{\frac{1}{3}})$, it is therefore necessary to define new variables in order to get a uniformly valid solution for the density. It is evident that this procedure fails when $m = \frac{2}{3\gamma}$, because then $K=0$. This means that the effect of the body is so small that the outer solution fails for a value of $\psi' = O(\delta)$ or $\frac{\psi}{\rho_0 u_0 d} = O(1)$, and hence the entropy layer needs to be considered as well.

2.5. The Second Order Equations in the Outer Region, $\frac{2}{3\gamma} < m < \frac{2}{3}$.

The effect of the body can now be determined in the

outer layer in the range $\frac{2}{3}\delta < m < \frac{2}{3}$ by introducing an expansion

$$\left. \begin{aligned} p' &= C_0^2 x'^{\frac{2}{3}} P_0(\eta) + \delta^{1-\frac{3m}{2}} x'^{m-\frac{4}{3}} C_0 C_1 P_1(\eta) + \dots \\ v' &= C_0 x'^{\frac{1}{3}} V_0(\eta) + \delta^{1-\frac{3m}{2}} x'^{m-1} C_1 V_1(\eta) + \dots \\ \rho' &= R_0(\eta) + \delta^{1-\frac{3m}{2}} x'^{m-\frac{2}{3}} \frac{C_1}{C_0} R_1(\eta) + \dots \\ y' &= C_0 x'^{\frac{2}{3}} Y_0(\eta) + \delta^{1-\frac{3m}{2}} x'^m C_1 Y_1(\eta) + \dots \end{aligned} \right\} \quad 2.21$$

in equations 2.10. Collecting terms of order one gives equations 2.12, which are the first order equations. The similarity variable η is taken to be unchanged, such that $\eta=1$ corresponds to the first order shock wave. Collecting terms of order $\delta^{1-\frac{3m}{2}}$ and neglecting any higher order terms, results in

$$\left. \begin{aligned} \frac{dP_1}{d\eta} - (1-m)V_1 - \frac{2}{3}\eta \frac{dV_1}{d\eta} &= 0 \\ \frac{P_1}{P_0} - \frac{\gamma R_1}{R_0} &= \frac{3m}{\eta^{1-\frac{3m}{2}}} \\ R_0^2 \frac{dV_1}{d\eta} - \frac{2}{3}\eta \frac{dR_1}{d\eta} - \left(\frac{2}{3}-m-2R_0 \frac{dV_0}{d\eta}\right) R_1 &= 0 \end{aligned} \right\} \quad 2.22$$

The second of these equations has been obtained from the integrated form of the second of equations 2.10, that is,

$\frac{P}{\rho^{\frac{1}{\gamma}}} = \frac{P_s}{\rho_s^{\frac{1}{\gamma}}}$ along streamlines, to second order, using the perturbed shock wave shape

$$y_s' = C_0 x'^{\frac{2}{3}} + C_1 \delta^{1-\frac{3m}{2}} x'^m \quad 2.23$$

So far, the equation for y' has been omitted because it is

not necessary to consider it in order to solve for p' , v' and ρ' . y' can always be determined afterwards by considering

$$\frac{\partial y'}{\partial \psi'} = \frac{1}{C_0 x'^{\frac{1}{3}}} \frac{\partial y'}{\partial \eta} = \frac{1}{\rho'} \quad \text{--- 2.24}$$

which is a direct result of the definition of ψ' . If the expansion for y' is considered in 2.24, there results

$$\frac{dY_0}{d\eta} = \frac{1}{R_0} \quad \text{--- 2.25}$$

$$\frac{dY_1}{d\eta} = - \frac{R_1}{R_0^2} \quad \text{--- 2.26}$$

for the first and second order respectively.

It is now necessary to consider the boundary conditions for the second order equations. The similarity variable η was chosen to be unity on the first order shock wave. Since $\psi'_s = y'_s$, however, equation 2.23 gives

$$\eta_s = 1 + \frac{C_1}{C_0} \delta'^{-\frac{3m}{2}} x'^{m-\frac{2}{3}} \quad \text{--- 2.27}$$

It is this value of η , therefore, at which the shock conditions have to be applied. That is, taking the shock condition for the pressure (equation 2.3 in the strong form) as an example,

$$p'_s = C_0^2 x'^{-\frac{2}{3}} P_0(\eta_s) + C_0 C_1 \delta'^{-\frac{3m}{2}} x'^{m-\frac{4}{3}} P_1(\eta_s) = \frac{2}{\gamma+1} \tan^2 \phi'$$

$$\text{but } \tan \phi' = \frac{2C_0}{3} x'^{-\frac{1}{3}} + \delta^{1-\frac{3m}{2}} C_1 m x'^{m-1}$$

$$\tan^2 \phi' = \frac{4}{9} C_0^2 x'^{-\frac{2}{3}} \left[1 + 3m \delta^{1-\frac{3m}{2}} \frac{C_1}{C_0} x'^{m-\frac{2}{3}} \right]$$

Since δ is small, $P_0(\eta_s)$ can be expanded as

$$\begin{aligned} P_0(\eta_s) &= P_0(1) + (\eta_s - 1) \left(\frac{dP_0}{d\eta} \right)_{\eta=1} \\ &= P_0(1) + \frac{C_1}{C_0} \delta^{1-\frac{3m}{2}} x'^{m-\frac{2}{3}} \left(\frac{dP_0}{d\eta} \right)_{\eta=1} \end{aligned}$$

and therefore

$$\begin{aligned} C_0^2 x'^{-\frac{2}{3}} P_0(1) + C_1 C_0 \delta^{1-\frac{3m}{2}} x'^{m-\frac{4}{3}} \left(\frac{dP_0}{d\eta} \right)_{\eta=1} + C_0 C_1 \delta^{1-\frac{3m}{2}} x'^{m-\frac{4}{3}} P_1(1) \\ = \frac{2}{\gamma+1} \left(\frac{4}{9} C_0^2 x'^{-\frac{2}{3}} + \frac{4}{3} m C_1 C_0 \delta^{1-\frac{3m}{2}} x'^{m-\frac{4}{3}} \right) \end{aligned}$$

Collecting like powers of δ ,

$$P_0(1) = \frac{8}{9(\gamma+1)}$$

$$\text{as before, and } P_1(1) = \frac{8m}{3(\gamma+1)} - \left(\frac{dP_0}{d\eta} \right)_{\eta=1}$$

Proceeding in a similar way, the boundary conditions for

V_1 , R_1 and Y_1 can be obtained. There is an additional

boundary condition at $\eta=0$, of course, which requires that

$v'(0, x')$ be matched with $v''(\infty, x'')$. Since the term in $\eta^{1-\frac{1}{2}}$

of the first order solution is already matched (equations

2.20, 2.14) it remains only to satisfy

$$\delta^{1-\frac{3m}{2}} x'^{m-1} C_0 \left(\frac{m}{C_0} \right) = \delta^{1-\frac{3m}{2}} x'^{m-1} C_1 V_1(0)$$

which requires $V_1(0) = \frac{m}{C_1}$. All the arguments leading up

to this statement can be made analogously for y^1 and the parallel result is $Y_1(0) = \frac{1}{C_1}$.

The problem in the outer region, when $\frac{2}{3}\delta < m < \frac{2}{3}$ can now be stated as the solution of the following equations:-

$$\left. \begin{aligned} \frac{dP_0}{d\eta} - \frac{1}{3}V_0 - \frac{2}{3}\eta \frac{dV_0}{d\eta} &= 0 \\ \frac{P_0}{R_0^2} &= \frac{8}{9(\delta+1)} \cdot \frac{(\delta-1)^2}{(\delta+1)} \cdot \frac{1}{\eta} \\ \frac{2}{3}\eta \frac{dR_0}{d\eta} - R_0^2 \frac{dV_0}{d\eta} &= 0 \\ R_0 \frac{dY_0}{d\eta} &= 1 \end{aligned} \right\} \quad 2.28$$

with $P_0(1) = \frac{8}{9} \cdot \frac{1}{\delta+1}$, $V_0(1) = \frac{4}{3} \cdot \frac{1}{\delta+1}$
 $R_0(1) = \frac{\delta+1}{\delta-1}$, $Y_0(1) = 1$

and

$$\left. \begin{aligned} \frac{dP_1}{d\eta} - (1-m)V_1 - \frac{2}{3}\eta \frac{dV_1}{d\eta} &= 0 \\ \frac{P_1}{P_0} - \frac{\delta R_1}{R_0} &= \frac{3m}{\eta^{1-\frac{3m}{2}}} \\ R_0^2 \frac{dV_1}{d\eta} - \frac{2}{3}\eta \frac{dR_1}{d\eta} - \left(\frac{2}{3}-m-2R_0 \frac{dV_0}{d\eta}\right)R_1 &= 0 \\ R_0^2 \frac{dY_1}{d\eta} + R_1 &= 0 \end{aligned} \right\} \quad 2.29$$

with $P_1(1) = \frac{8m}{3(\delta+1)} - \left(\frac{dP_0}{d\eta}\right)_{\eta=1}$, $R_1(1) = -\left(\frac{dR_0}{d\eta}\right)_{\eta=1}$
 $V_1(1) = \frac{2m}{\delta+1} - \left(\frac{dV_0}{d\eta}\right)_{\eta=1}$, $V_1(0) = \frac{m}{C_1}$
 $Y_1(1) = 1 - \left(\frac{dY_0}{d\eta}\right)_{\eta=1}$, $Y_1(0) = \frac{1}{C_1}$

As has been stated earlier, these equations can be solved

without considering the equations and boundary conditions for Y_0 and Y_1 . Furthermore, since the second of equations 2.28 and the second of equations 2.29 have already been integrated using the shock wave conditions for P_0 , R_0 , P_1 and R_1 , the boundary conditions for P_0 and P_1 can not be used again. There remain five boundary conditions, one of which $-V_1(0) = \frac{m}{C_1}$ serves to determine the unknown coefficient C_1 , which is common to all the perturbation quantities. The boundary condition for $Y_1(0)$ also gives C_1 , but this is not an independent condition. It merely leads to the identity $Y_1(0) = \frac{1}{m}V_1(0)$. This is a useful check in the numerical calculations.

2.6. Series Solution for Small η .

The solution for the first order equations, which has been given by Sedov (1959) analytically (see Hayes and Probstein (1959) section 2.6), has the behaviour given by equations 2.14 for small η . Substituting 2.14 into equations 2.22 produces

$$\left. \begin{aligned} \frac{dP}{d\eta} - (1-m)V_1 - \frac{2}{3}\eta \frac{dV_1}{d\eta} &= 0 \\ \frac{P}{P_0(0)} - \frac{\gamma R_1}{A\eta^{\frac{1}{2}}} &= \frac{3m}{\eta^{1-\frac{3m}{2}}} \\ A^2 \eta^{\frac{2}{3}} \frac{dV_1}{d\eta} - \frac{2}{3}\eta \frac{dR_1}{d\eta} + (m + \frac{4}{3\gamma} - \frac{2}{3})R_1 &= 0 \end{aligned} \right\} \quad \text{--- 2.30}$$

where $A = \frac{\gamma+1}{\gamma-1} \left[\frac{9}{8} (\gamma+1) P_0(0) \right]^{\frac{1}{2}}$

A complete solution in terms of hypergeometric functions can

be obtained for equations 2.30. The second of these equations with P_1 assumed constant for small η gives the first two terms in R_1 , and these, substituted into the third, suggest the corresponding terms in V_1 which can then be used in the first equation to get the expansion for P_1 . Proceeding in this manner, the following expressions result.

$$\left. \begin{aligned} R_1 &= -\frac{A}{8} \eta^{\frac{1}{8}} \left[\frac{P_1(0)}{P_0(0)} - 3m \eta^{\frac{3m}{2}-1} \right] + O(\eta^{1+\frac{1}{8}}) \\ V_1 &= V_1(0) + \frac{2m}{A\delta^2} \frac{\eta^{\frac{3m}{2}-\frac{1}{8}}}{\frac{3m}{2}-\frac{1}{8}} - \frac{\frac{2}{3\delta} + m - \frac{2}{3}}{A\delta(\delta-1)} \frac{P_1(0)}{P_0(0)} \eta^{1-\frac{1}{8}} + O(\eta^{\frac{3m}{2}-\frac{2}{8}+2}) \\ P_1 &= P_1(0) - (1-m)V_1(0)\eta + \frac{2m(1-\frac{2}{3\delta})}{A\delta^2(\frac{3m}{2}-\frac{1}{8})} \frac{\eta^{\frac{3m}{2}-\frac{1}{8}+1}}{\frac{3m}{2}-\frac{1}{8}+1} + O(\eta^{2-\frac{1}{8}}) \end{aligned} \right\} \quad 2.31$$

These solutions for small η contain some illuminating information. As is to be expected, they fail when $m = \frac{2}{3\delta}$, as for example in V_1 , where the power of the second term becomes zero and its coefficient becomes infinite. This means that V_1 will behave like $\log \eta$ since, writing $\frac{3m}{2} - \frac{1}{8} = \epsilon$,

$$\begin{aligned} \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} \eta^\epsilon &= \frac{1}{\epsilon} e^{\epsilon \log \eta} = \frac{1}{\epsilon} (1 + \epsilon \log \eta + \dots) \\ &= \frac{1}{\epsilon} + \log \eta + \dots \end{aligned}$$

In P_1 the logarithm appears further down as $\eta \log \eta$ and in R_1 it appears as $\eta^{\frac{1}{8}} \log \eta$. This is the logarithmic term which has been observed by Freeman, Guiraud, Vaglio-Laurin and other authors. Its occurrence is not restricted to the case $m = \frac{2}{3\delta}$ but occurs in all solutions at some stage in the expansion. The difficulties associated with it will

be discussed later, in section 2.9.

The above solutions for small η break down again when $m = \frac{2}{3}$, as for example in R_1 , where the powers of the first two terms become the same. By examining the behaviour of the individual terms in the series solution in terms of ψ and x' , the terms containing m in their power can be traced to perturbations from the entropy layer, and the terms containing m merely in their coefficient are caused by the body itself.

This demonstrates again the change in the relative importance of the perturbations introduced in the outer region by the body and entropy layer when m decreases from $\frac{2}{3}$ through to $\frac{2}{3\gamma}$.

2.7. Energy Considerations and the Coefficients C_0 and C_1 .

If the equations of continuity, momentum and energy are considered for the control volume shown in Fig. 3, the

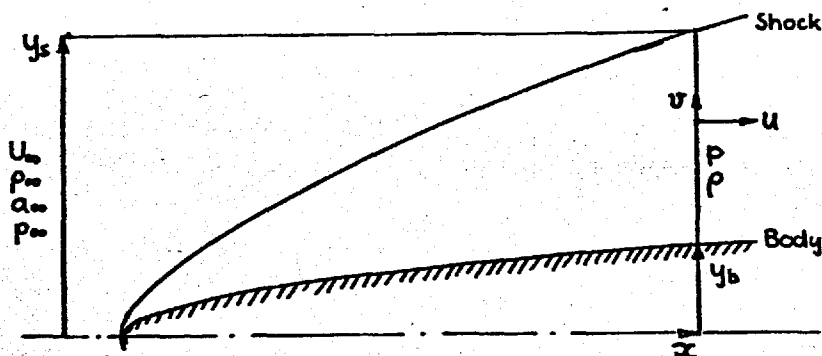


Fig. 3. Control Volume

following result which represents an energy balance is obtained (see Freeman (1962)).

$$\int_0^{y'} \left(\frac{p'}{\rho'(x-1)} + \frac{v'^2}{2} \right) \rho' dy' = \frac{D}{\rho_\infty U_\infty^2 d} \quad \text{--- 2.32}$$

where $D = \int_0^{y_0} p_0 dy$ is the drag of the body up to x . This equation can be expressed in terms of x' and η as follows.

$$C_0 x'^{\frac{2}{3}} \int_0^{\eta} \left[\frac{p'}{(x-1)\rho} + \frac{v'^2}{2} \right] d\eta = \frac{D}{\rho_\infty U_\infty^2 d} = \int_0^{x'} (p' v')_{\eta=0} dx'$$

If the expansions 2.21 are substituted in this equation and terms of equal order in δ are collected, the following two results are obtained.

$$C_0^3 I_1 = \frac{D_\infty}{\rho_\infty U_\infty^2 d} \quad \text{--- 2.33}$$

$$\text{and } C_1 \left[I_1 + \frac{V_0^2(1)}{2} + \frac{P_0(1)}{(x-1)R_0(1)} \right] = \frac{m P_0(0)}{m - \frac{2}{3}} \quad \text{--- 2.34}$$

$$\text{with } I_1 = \int_0^1 \left[\frac{P_0}{(x-1)R_0} + \frac{V_0^2}{2} \right] d\eta = 0.5388$$

$$I_2 = \int_0^1 \left[V_1 V_0 + \frac{1}{x-1} \left(\frac{P_1}{R_0} - \frac{P_0 R_1}{R_0^2} \right) \right] d\eta = I_2(m)$$

$$\text{and } D_\infty = \text{drag to } x' = \infty$$

While equation 2.34 provides yet another means of checking the value of C_1 , the constant C_0 cannot be obtained from 2.33 without the knowledge of D_∞ . This can only be obtained from a knowledge of the pressure distribution on the nose of the body, which is outside the region of validity of the present theory.

2.8. Numerical Solution and Results.

The equations 2.28 and 2.29 were solved numerically on

the London University Mercury Computer for 9 values of m in the range $\frac{2}{3\gamma} < m < \frac{2}{3}$ when $\gamma = \frac{7}{5}$. The integrals I_1 and I_2 were also evaluated and the results are presented in figures 4, 5, 6, 7, 8 and 9. The behaviour of V_1 near $\eta = 0$ made it difficult to determine $V_1(0)$ from the computer results, since the calculations had to be stopped short of $\eta = 0$ to avoid the infinite value of some of the derivatives. In order to avoid this difficulty, the independent variable was transformed to $\xi = \eta^{\frac{3m-1}{2}-\frac{1}{\gamma}}$, and the computer calculations were repeated with ξ as independent variable, giving a straight line for $V_1(\xi)$ near $\xi = 0$ which could be extrapolated accurately to $\xi = 0$ giving $V_1(0) = \frac{m}{C_1}$, and hence C_1 (see fig. 8).

The limit cases of the range, $m = \frac{2}{3}$ and $m = \frac{2}{3\gamma}$, were included in the calculations, although the expansion procedure does not apply for them, and the results obtained in the previous sections are borne out by the behaviour of V_1 obtained by the numerical calculation. When $m = \frac{2}{3\gamma}$, V_1 has a logarithmic singularity at $\eta = 0$, giving $C_1 = 0$ and when $m = \frac{2}{3}$, $V_1(0) = 0$ giving $C_1 = -\infty$. The three methods of obtaining C_1 afforded by the boundary conditions for V_1 and Y_1 , and equation 2.34 give the same result.

2.9. Matching with the Entropy Layer.

The density in the entropy layer, in which $\frac{\psi}{\rho_0 u_0 d} = \psi^m$ is of the order of one, behaves like

$$\rho' \propto \delta^{\frac{1}{8}} x'^{\frac{2}{38}} (1 + a \psi''' + \dots)^{+\frac{1}{8}}$$

$$y' \propto \delta^{1-\frac{1}{8}} x'^{\frac{2}{38}} \int_0^{\psi'''} (1 + a \psi''' + \dots)^{-\frac{1}{8}} d\psi'''$$

where $a = \frac{9}{4C_0^3}$. Introducing the shock wave perturbation of the form x''' as before, and proceeding to get the corresponding result for ρ' and y' ,

$$\left. \begin{aligned} \rho' &\propto \delta^{\frac{1}{8}} x'^{-\frac{2}{38}} \left[1 + a \psi''' (1 - b \psi'''^{\frac{3m}{2}-1} + \dots) \right]^{+\frac{1}{8}} \\ y' &\propto \delta^{1-\frac{1}{8}} x'^{\frac{2}{38}} \int_0^{\psi'''} \left[1 + a \psi''' (1 - b \psi'''^{\frac{3m}{2}-1} + \dots) \right]^{-\frac{1}{8}} d\psi''' \end{aligned} \right\} \text{--- 2.35}$$

where $b = 3m \frac{C_1}{C_0^{\frac{3m}{2}}}$. Expanding the results for large ψ''' ,

$$\left. \begin{aligned} \rho' &\propto \delta^{\frac{1}{8}} x'^{-\frac{2}{38}} \left[\psi'''^{\frac{1}{8}} - \frac{1}{8} b \psi'''^{\frac{1}{8} + \frac{3m}{2}-1} + \dots \right] + \dots \\ y' &\propto \delta^{1-\frac{1}{8}} x'^{\frac{2}{38}} \left[\frac{8}{8-1} \psi'''^{\frac{1}{8}} - \frac{b}{8(\frac{3m}{2}-\frac{1}{8})} \psi'''^{\frac{3m}{2}-\frac{1}{8}} + \dots \right] + \dots \end{aligned} \right\} \text{--- 2.36}$$

It is now necessary to match the result for small η with these expressions. Comparing equations 2.14 and 2.31 with 2.36, it is seen that this would require the form of the functions on the left of the following table to be matched with the form of the functions on the right, without considering matching of the coefficients.

$\eta^{\frac{1}{8}}$	$\longleftrightarrow$	$\delta^{\frac{1}{8}} x'^{-\frac{2}{38}} \psi'''^{\frac{1}{8}}$
$\delta^{1-\frac{3m}{2}} x'^{m-\frac{2}{3}} \eta^{\frac{1}{8}-1+\frac{3m}{2}}$	$\longleftrightarrow$	$\delta^{\frac{1}{8}} x'^{-\frac{2}{38}} \psi'''^{\frac{1}{8}-1+\frac{3m}{2}}$
$x'^{\frac{2}{3}} \eta^{1-\frac{1}{8}}$	$\longleftrightarrow$	$\delta^{1-\frac{1}{8}} x'^{\frac{2}{38}} \psi'''^{1-\frac{1}{8}}$
$\delta^{1-\frac{3m}{2}} x'^m \eta^{\frac{3m}{2}-\frac{1}{8}}$	$\longleftrightarrow$	$\delta^{1-\frac{1}{8}} x'^{\frac{2}{38}} \psi'''^{\frac{3m}{2}-\frac{1}{8}}$

It is seen that these agree throughout except for the case $m = \frac{2}{3\gamma}$, when the last would require

$$x'^{\frac{2}{3\gamma}} \delta^{1-\frac{1}{\gamma}} \left[\log \psi''' + \log \delta - \frac{2}{3} \log x' \right] \longleftrightarrow x'^{\frac{2}{3\gamma}} \delta^{1-\frac{1}{\gamma}} \log \psi'''$$

which is obviously not the case, and a more sophisticated approach is probably necessary to deal with this case.

The problem for $m = 0$ is characterised by a perturbation of the shock wave shape of the form $\delta^{1-\frac{1}{\gamma}} x'^{\frac{2}{3\gamma}}$ in the analysis of Freeman (1962). This will always lead to the difficulty encountered in the above case $m = \frac{2}{3\gamma}$, that the logarithmic term cannot be matched. Guiraud (1958) and Messiter (1965) avoid this difficulty by disallowing a shock wave perturbation of this form and considering the next perturbation to be of the form $\delta^{2(1-\frac{1}{\gamma})} x'^{\frac{4}{3\gamma}-\frac{2}{3}}$. Although this allows the entropy layer to be matched with the outer solution when $m = 0$, it does not remove the difficulty when $m = \frac{2}{3\gamma}$, since then a perturbation of the form $\delta^{1-\frac{1}{\gamma}} x'^{\frac{2}{3\gamma}}$ appears from the body. The reason for the failure of the matching procedure when $m = \frac{2}{3\gamma}$ is not obvious, but it is probable that the full partial nature of the differential equations may then be important.

2.10. General Discussion of Theory and Practical Implications.

In the range $\frac{2}{3\gamma} < m < \frac{2}{3}$ the solutions obtained here are uniformly valid for large x and $M_\infty = \infty$, satisfying the boundary condition for normal velocity on the body and the upstream condition for the entropy distribution. They

demonstrate that the shock layer has to be subdivided into three layers:

a) The outer region in which the Sedov solution is the first order solution, and the effect of the body (terms Ax^m and $x^m \eta^{\frac{3m}{2}-\frac{1}{8}}$ etc.) are of second order.

b) The intermediate region, where the terms Ax^m and $x^m \eta^{1-\frac{1}{8}}$ dominate and the term containing $\eta^{\frac{3m}{2}-\frac{1}{8}}$ is of higher order.

c) The inner region, or entropy layer, in which the term Ax^m dominates and the terms $x^{\frac{2}{3\gamma}} \psi^{1-\frac{1}{8}}$ and $x^{\frac{2}{3\gamma}} \psi^{\frac{3m}{2}-\frac{1}{8}}$ are of higher order.

The way in which these solutions change their order of importance and how they are matched can best be demonstrated in a schematic diagram (Fig. 10) plotting $\log y'$ against $\log \eta$ for particular values of m and x' , where the sum of the three curves, which represent the dominant terms of the three effects, gives the composite solution for y' .

The range of $\frac{x}{d}$ for which this analysis can be expected to describe the flow reasonably accurately can now be evaluated by requiring that

$$\delta^{1-\frac{3m}{2}} x'^{m-\frac{2}{3}} \ll 1, \quad \text{or} \quad \left(\frac{x}{d}\right)^{m-\frac{2}{3}} \ll 1$$

If one specifies the maximum size of this quantity as, say, 0.3, the range of validity is restricted to $\frac{x}{d} > (0.3)^{\frac{1}{m-\frac{2}{3}}}$, which, for $m = \frac{2}{3\gamma}$ and $\gamma = \frac{7}{5}$ is of the order of 500. If m is nearer $\frac{2}{3}$, say, for example $m = \frac{5}{8}$, $\frac{x}{d}$ would have to be larger

than 3×10^{12} and if this is to be applicable on a body of, say, 50 ft length, d would have to be smaller than about 10^{-10} inches, which is well below the limits of the theory anyway, because of restrictions to inviscid and continuum flow.

The practical use of the theory is thus clearly limited. However, it does give an insight into the way in which the blast wave solution is approached asymptotically and the physical reasons for the deviations from it.

The negative value of C_1 for example, shows how the drag D_∞ is approached from below as $x \rightarrow \infty$ and for large x the rate of change of drag reduces rapidly to zero. The form of the drag as a function of x is

$$D_x = D_\infty - \text{const. } \delta^{1-\frac{3m}{2}} x^{m-\frac{2}{3}} + \dots$$

which shows that the drag to be used in order to obtain the coefficient of the first order quantities, C_0 , is D_∞ , the maximum value of D_x as $x \rightarrow \infty$. The shock wave shape will also approach the first order shock asymptotically from below, and thus lie inside the first order shock wave. This appears to be wrong only when the drag caused by the immediate neighbourhood of the nose is considered to be that giving C_0 instead of the asymptotic drag, D_∞ .

If the solutions to second order in the outer region are rewritten in terms of physical quantities, they are

$$\left. \begin{aligned}
 \frac{y}{d} &= C_0 \left(\frac{x}{d}\right)^{\frac{2}{3}} Y_0(\eta) + C_1 \left(\frac{x}{d}\right)^m Y_1(\eta) + \dots \\
 \frac{p}{\rho_\infty U_\infty^2} &= C_0^2 \left(\frac{x}{d}\right)^{-\frac{2}{3}} P_0(\eta) + C_1 C_0 \left(\frac{x}{d}\right)^{m-\frac{4}{3}} P_1(\eta) + \dots \\
 \frac{v}{U_\infty} &= C_0 \left(\frac{x}{d}\right)^{-\frac{1}{3}} V_0(\eta) + C_1 \left(\frac{x}{d}\right)^{m-1} V_1(\eta) + \dots \\
 \frac{\rho}{\rho_\infty} &= R_0(\eta) + \frac{C_1}{C_0} \left(\frac{x}{d}\right)^{m-\frac{2}{3}} R_1(\eta) + \dots
 \end{aligned} \right\} \quad 2.37$$

These have been calculated for $m = \frac{1}{2}$ and $m = \frac{5}{8}$ and various values of $\frac{x}{d}$ and are shown in Figs. 11 and 12 as $\frac{p}{\rho_\infty}$, $\frac{v}{U_\infty}$ and $\frac{\rho}{\rho_\infty}$ versus $\frac{y}{y_s}$. It is seen that as $\frac{x}{d} \rightarrow \infty$ the solutions converge to the first order solutions, and that the deviation from the first order solution is almost undetectable for $m = \frac{1}{2}$ even for large values of $\alpha = \frac{C_1}{C_0} \left(\frac{x}{d}\right)^{m-\frac{2}{3}}$. Taking $\frac{C_1}{C_0}$ to be about 0.1, that is assuming $C_0 = 1.3$ for $m = 0.5$, the deviation from first order theory is less than 5% in normal velocity for as small a value of $\frac{x}{d}$ as 80. This is not the case for $m = \frac{5}{8}$, where an assumed value of $\frac{C_1}{C_0} = 0.5$ ($C_0 = 2.8$) leads to a deviation of about 20% for $\frac{x}{d} = 2^{24} \div 10^7$. In these coordinates the effect of the perturbation is very small in the case of pressure and density, and slightly larger in the case of normal velocity. Pressure and normal velocity are increased while the density is decreased by the perturbation.

All the curves of Figs. 11 and 12 have been cut off at their lower limit of validity in η (where the solutions have to be matched to the inner solution). This corresponds approximately to the point on the body surface, $\frac{y}{y_s} = \frac{y_b}{y_s}$.

3. EXPERIMENTS AT $M_\infty = 8.2$.

3.1. Facility.

The hypersonic gun tunnel at Imperial College was used for the present investigation and it will be described briefly below although a more detailed account is available in a paper by Needham (1963). The gun tunnel uses a light piston, fired along a barrel by the sudden application of a high pressure to compress the working gas to the high stagnation conditions necessary for expansion to high Mach number flows. This expansion is achieved by a convergent-divergent nozzle leading to the working section which is connected to a large evacuated dump tank. The duration of the flow is thus limited by the mass of compressed gas and the mass flow rate through the nozzle, but the steadiness of the flow is interrupted periodically during this time by reflected waves impinging on the back of the piston. A typical uninterrupted steady running time is about 40 msec. The most suitable conditions for the present experiments would be a high Mach number and high Reynolds Number. However, the conical nozzle available for the highest Mach number (15) at Imperial College was considered unsuitable since a non-uniform flow would probably be critical in these experiments, and the $M_\infty = 8.2$ contoured nozzle was chosen. The conditions were chosen to give the highest possible Reynolds number. This meant that the condensation limit

had to be checked. From results collected by Bowman (1965) the lowest total temperature at which condensation effects are undetectable at $M_\infty = 8.2$ and total pressure = 1560 p.s.i.a., is about 640°K . This agrees with some measurements made by Crane (1965) and the total temperature was chosen as 660°K for the present work.

The viscous length scale, $\frac{\gamma_\infty}{U_\infty}$, under these conditions is of the order of 10^{-5} inches and is thus likely to be small compared with a suitable model dimension, since the uniform core of the working section flow is about 6 inches in diameter. The results of a Mach number calibration of the upper half of the vertical centre-plane of the working section are given in Fig. 13.

3.2. Choice of Models.

In view of the short running time of the gun tunnel it is important to have an instrumentation system with a suitable response time. This requires that the pneumatic connections to the pressure transducers must be as short as possible. Hence the most suitable place to house the pressure transducers is inside the model. This in turn suggests a plane two-dimensional configuration. Another point in favour of plane models is the relative ease of mounting which is afforded by the positive division of the "upper" and "lower" surfaces. Previous experimental work on power-law bodies has been restricted almost entirely to axisymmetric flows. Experimental work on plane power-

law bodies would thus usefully fill a relatively large gap in the knowledge of such flows. It was decided that plane models would be used.

In the light of the theoretical work of section 2, the range of interest of powers for the model shapes is here mainly $\frac{2}{3} < m < \frac{2}{3}$. When air is used ($\gamma = 1.4$), this range is $\frac{10}{21} < m < \frac{2}{3}$. It would, of course, be best to test a considerable number of models in this range, but because of the large amount of work involved in making a power-law shape, and because of limited workshop time, this investigation had to be limited to two models. These two models were then chosen to be near the two ends of the above range in m , namely $m = 0.5$ and $m = 0.625$, in order to test the large change in magnitude of the second order quantities predicted by the theory over the range.

It was then necessary to make a compromise regarding the length scale of the body. From the point of view of the theory it would be interesting to look at the flow at very large values of $\frac{x}{d}$. Since x is restricted to about 12 inches by the size of the working section, this would mean a small d . Two factors are against a small d . One is the necessity that d be much larger than the viscous length scale, and the other is the lower limit of the surface pressures imposed by the available instrumentation. An upper limit is also imposed on d by tunnel blockage.

It was decided that some experiments on simple wooden parabolas should be included in the pilot experiments described in the next section in order to determine the viscous effect and the limitation of blockage.

3.3. Pilot Experiments.

3.3.1. Effects of Viscosity and Blockage.

Three wooden parabolas were made, 10 inches long and 5 inches in span, for which d in the relation defining the shape, $\frac{y}{d} = \left(\frac{x}{d}\right)^{\frac{1}{2}}$, was 0.225, 0.156 and 0.100 inches. The smallest of these was tested at two Reynolds numbers corresponding to two viscous length scales in the ratio 2 to 1, by observing the change in shock wave shape with Reynolds number. This was found to be undetectable on two schlieren photographs. A more sensitive check on the effect of Reynolds number is probably afforded by a comparison of boundary layer and shock layer thicknesses. The ratio of these thicknesses is seen to be less than $\frac{1}{20}$ from the schlieren photographs of Fig. 30a. It was concluded that the viscous displacement effect could be neglected.

A test of the largest of the three models (cross-sectional area = 15 sq. in.) showed that the flow was not blocked provided that the diffuser of the tunnel was moved forward to within 5 inches of the nozzle exit plane. This meant that the back part of the model could not be

seen on schlieren photographs. The shock wave shapes on all three of these models are plotted in Fig. 14 as $\log \frac{y}{d}$ vs. $\log \frac{x}{d}$, and it is interesting to note that they collapse onto one curve which is very nearly proportional to $\left(\frac{x}{d}\right)^{\frac{2}{3}}$. This shows that the significant length scale of the shock wave shape is the same as that of the body, namely d , and that the second order effect at $m = 0.5$ is very small even at $\frac{x}{d} = 15$. Compared with the limit of validity of the theory estimated in section 2.10 as $\frac{x}{d} = 500$, this looks very encouraging.

3.3.2. Determination of Test Conditions.

The free stream Mach number was determined by measuring free stream pitot pressure and using the total pressure available from calibrations of the tunnel (Needham 1963). Estimates of stagnation temperature, based on running time and pressure records, were also given by Needham. M_∞ , p_0 and stagnation temperature define the state of the free stream completely. The most unreliable of the measurements needed to obtain these three quantities are those leading to the stagnation temperature. Although the stagnation temperature has been estimated subsequently from stagnation point heat transfer measurements by Holden (1964), an attempt was made in the present investigation to measure stagnation temperature directly. This was done by using the probe

shown in Fig. 15a. This probe functions as follows. The flow is decelerated to about $M = 0.2$ by the normal bow shock wave in front of the probe. The speed of the flow through the probe can be controlled by interchanging the exit throat. The temperature inside the tube is then very nearly the stagnation temperature of the flow. This is measured by using a thin wire (0.001 in. dia.) as a resistance thermometer. If a small constant current is passed through the wire, the voltage drop across it is proportional to the wire resistance and can be monitored on an oscilloscope. The material of the wire was chosen as 90% Platinum - 10% Rhodium for high strength and temperature coefficient of resistivity. In order to minimise radiation losses and errors due to immersion of part of the sensing wire in colder boundary layer flow, the body of the probe which consists of a pyrex glass tube was coated on the inside with a platinum film heater. This film and two silver leads connecting it to wire leads were baked into the glass. The platinum film is heated by discharging a capacitor through it. The temperature-time curve of this film can then be obtained by monitoring current through and voltage across it and using the temperature coefficient of resistance of such films (Holden, 1964). These films could be heated to about 500°C without causing the glass to break. In order to avoid induced currents in the sensing wire from this process, the sensing wire is arranged in a vee shape. It is insulated from the heater by two plastic caps which hold the wire supports.

In order to measure stagnation temperature, it is necessary to choose the capacitor and its charge in such a way that the heater temperature is approximately constant and as near to the stagnation temperature as possible over the period during which the temperature is recorded. It is necessary for this purpose to delay the discharge of the capacitor with respect to the firing of the gun tunnel. This was done by means of a Marconi type TF1415 delay generator.

Stagnation temperature was measured with the tunnel in four configurations: With the conical nozzle at $M_\infty = 7.5$, 10 and 14.5 under conditions for which the estimated stagnation temperature of Needham (1963) was 1300°K , and with the contoured nozzle, $M_\infty = 8.2$, under conditions when the estimated temperature was 660°K . The temperature readings for these cases were, respectively, 1290°K , 1260°K , 1000°K and 665°K . The large discrepancy for $M_\infty = 14.5$ is considered to be due to the fact that the boundary layer is much thicker inside the probe when M_∞ is large and that the heater, which is only capable of functioning up to 800°K , is unable to remove this error.

It is interesting to note the large gradient of stagnation temperature with time during the run, see Fig. 15b, giving the trace for $M_\infty = 10$. This is about $6000^\circ\text{C}/\text{sec}$. and would change the Reynolds number considerably during a run.

When the stagnation temperature is 665°K at $M_{\infty} = 8.2$ this gradient is only about $2000^{\circ}\text{C/sec.}$, and after the passage of the first and second acoustic wave the temperature jumps to the same value again.

The value of stagnation temperature used under the running conditions chosen for the present experiments was that measured by the above probe as 665°K . The free stream conditions were as follows:

$$P_{t2\infty} = 11.5 \text{ p.s.i.a.}$$

$$M_{\infty} = 8.2$$

$$T_0 = 665^{\circ}\text{K.}$$

3.3.3. The Method of Weak Waves.

The fact that small disturbances are propagated at an angle to the local stream direction which depends only on the local Mach number has been used repeatedly in low supersonic flow. It does not seem very promising at high Mach numbers, because the rate of change of Mach angle with Mach number becomes small. However, since the Mach number in the shock layers studied here was not likely to exceed about 6, it was considered worthwhile to make an attempt at using this technique.

The method can best be described with the aid of Fig. 16. A flat plate is placed opposite the body under test and small obstructions are placed on the surfaces of the body and flat plate, such that they generate weak waves.

The family of waves originating on the flat plate are deflected by the curved bow wave of the blunt body and follow a curved path inside the shock layer. The family of weak waves generated from the blunt body travel along curved paths towards the bow wave and intersect the other family. Taking the intersection of two particular waves one from each family, the local flow direction can be found by bisecting the angle 2μ , between the two waves. The local Mach number can then be obtained from

$$M = \sin \mu$$

The method depends critically on the strength of these waves. They have to be weak enough to introduce only negligible disturbances in the flow and to propagate in the direction of the local Mach angle, but strong enough to be seen on a schlieren photograph. It is also important that the bow wave of the flat plate is weak, so that it does not disturb or deflect the flow.

After trying several means of generating the waves, strips of adhesive tape of 0.0015 in. thickness and about $\frac{1}{8}$ th inch width were spaced at $\frac{3}{8}$ inch intervals on both the flat plate and blunt body. Figs. 17, 18 and 19 show a schlieren photograph of one of the models in such a test, the derived Mach number distribution and the streamline slope distribution across the shock layer at a particular station respectively. It is seen that the waves originating

from the body are not always weak enough, since the bow wave and trailing wave from the disturbances converge, instead of being parallel. Also, the bow wave from the flat plate does influence the bow wave of the blunt body slightly.

This method has the following advantages. It gives the flow direction and Mach number relatively directly and supplies a distribution of these quantities over the whole flow field at the expense of one experimental run. Its disadvantages are that it is not very accurate at high Mach numbers and that a considerable amount of work has to be done on the photograph to obtain the above distributions. It would also be more difficult to use the method in axially symmetric flow.

In order to obtain all the quantities specifying the flow field it is necessary in addition to measure the pitot pressure and total temperature. The latter can, however, be assumed constant in the non-dissipative flow outside the boundary layer.

3.3.4. Static Pressure Probe.

In relation to the low degree of precision obtained by the method of weak waves it seemed worthwhile to try to measure the static pressure as a means of obtaining Mach number distributions. Some experiments on the viscous interaction effects of a static pressure probe at $M = 6$ were reported by Behrens (1963). An estimate of the viscous

interaction parameter $\bar{X}$ was made for the typical shock layer under the experimental conditions of the present work. The low value (≈ 0.08) of this parameter based on a suitable dimension from probe tip to orifice (about 0.5 inch) suggested that the viscous interaction effect would be very small, causing an error of less than 1% according to the empirical formula given by Behrens.

A probe was then designed on the same lines as that used by Behrens but using the response time as a criterion for size. The method of Kendall (1958) for optimising pneumatic connections was used to relate the probe size to the necessary length of pneumatic connections and transducer volume. The available running time (about 40 msec.), the pressure levels to be measured (about 0.6 p.s.i.a.) and the geometry of pneumatic connections made it necessary to choose a larger probe than that used by Behrens. The final design of the probe is shown in Fig. 20.

Three such probes were made and mounted on a rake, together with three pitot pressure probes, such that the streamwise position of the measuring point was the same for all six probes and the spacing between them was sufficient to avoid interference for Mach numbers above about 2.5. In order to test the static probes, this rake was then used to measure the static pressure in the shock layer of a 20° wedge at $M_\infty = 8.2$ with the three static pressure probes at

the same distance from the wedge surface. The true inclination of the wedge was 18.8° and the shock wave angle measured from the schlieren photograph (Fig. 21) was 25.3° . The static pressure in the shock layer corresponding to these conditions with $p_{\infty} = 0.135$ p.s.i.a. and $M_{\infty} = 8.2$ was calculated to be 1.95 p.s.i.a. from the oblique shock relations. The pressures as measured by the probes were 1.935, 1.925 and 1.955 p.s.i.a., that is, within 2% of each other and the calculated value. This is also well within the accuracy of the instrumentation and recording system which will be discussed later. Although this pressure was considerably higher than the pressures expected in the shock layer of the blunt models, it looked very encouraging. The oscilloscope traces showed, however, that the response times were slightly longer than those estimated from Kendall (1958). One of the traces is shown in Fig. 22. The pressure was higher and the length of the pneumatic connections longer than those used in the estimate. The effect of these changes on the response time oppose each other and it was to be expected that the response time for the case of the estimate would be about the same as that encountered in the wedge test. This was about 60 msec., which made it necessary to take the reading after the interruption caused by the first acoustic wave. Some later measurements, in which the response time was reduced by shortening the pneumatic

connections showed that the pressure settles to the same value after the first acoustic wave.

It was decided that the static and pitot pressure method would be used to measure the local Mach number in the shock layer and that the total temperature would be assumed constant throughout the flow field in order to determine the remaining flow parameters.

3.4. Design of Models and Traversing Gear.

The results of section 3 suggest that the value of d in the body shape $\frac{y}{d} = \left(\frac{x}{d}\right)^m$ should be varied with m in order to obtain comparable second order effects on models of similar overall length. Shock wave shapes obtained during the pilot experiments showed that the second order effect is small for parabolic bodies. The largest value of d used in the parabolas of the pilot experiments (0.225 inches) was thus chosen for the fully instrumented metal parabola. In contrast, the second order effect can be expected to be large for the $\frac{5}{8}$ -power body and the value of d for this model was chosen as 0.010 inches. This meant that the base thicknesses were 3 inches for the parabola and $1\frac{1}{2}$ inches for the $\frac{5}{8}$ -power body at $x = 10$ inches. The maximum values of $\frac{x}{d}$ were thus 44.4 and 1000 respectively.

With the space available in the $\frac{5}{8}$ -power model it was not feasible to house the pressure transducers inside the model. However, a sting mounting capable of housing four

transducers which was available from another project (see Needham, 1965) could be used in conjunction with this model without excessive lengthening of the pneumatic connections. The $\frac{5}{8}$ -power model was thus not made hollow. Details of this model are shown in Fig. 23.

The parabola was designed in two sections: The main afterbody which was machined hollow out of solid duralumin and open at the bottom. The nose which was solid and attached to the afterbody by two screws. The nose and afterbody surfaces were finished on assembly and two locating surfaces machined on the bottom of the two sections ensured proper assembly. The afterbody was stiffened by a central web of $\frac{5}{8}$ inch width which served also as a mounting surface for the traversing mechanism. Twentyeight pressure orifices were suitably distributed along the centreline, and to check the two-dimensionality of the flow, three pairs of orifices at $x = 0, 1.25$ and 7 inches were spaced symmetrically about the centreline at $\frac{1}{2}$ inch from the edges. In addition, an orifice was positioned on the bottom of the nose, such that it was directly opposite one of the top orifices in order to check that the model was at zero incidence. The space in the model allowed for the mounting of 6 pressure transducers sleeved with foam rubber and held in place by spring clips.

The main purpose of making this model in two parts was

to enable the ratio of the magnitudes of first and second order quantities to be modified by substituting a different nose shape. One additional nose was made. This started with a 30° half-angle wedge which had two common tangents with the parabola and whose apex was on the axis of symmetry of the parabola. In this configuration the model is referred to as the "wedge-parabola" below. Fig. 24 shows the parabola with the traversing mechanism mounted in the rear position. It could also be mounted at the centre of the parabola, in which case the probe support protruded through a rectangular hole in the surface. This hole was sealed with plasticine during runs.

The traversing mechanism (see Fig. 23) has three degrees of freedom. The main slide body rotates about an axis about $\frac{1}{8}$ inch below the model surface which enables the incidence of a probe to be changed by up to 27° . The probe support which is vee shaped in section engages with the slide and this constitutes the traversing motion. It can be held in position by a clamp and its position is recorded by a micrometer screw mounted on the slide. The anvil of the micrometer system can be moved by 1 inch, thus enabling a full traverse of 2 inches to be made. At the top of the probe support there is a clamp which holds the tubular probe sting in position at any streamwise station. On the $\frac{5}{8}$ -power model the traversing mechanism was mounted

only on the base of the model by means of a tee block which could also be attached to the wooden parabolas.

3.5. Experimental Technique and Accuracy.

3.5.1. Instrumentation.

The pressure measurements were made using the transducers described in the table below.

Pressure transducers:

No. off	Make	Type	Range, p.s.i.a.
3	Solartron	NT4-313	0-30
1	Solartron	NT4-313	0-15
2	C.E.C.	4-326	0-10

None of these is very suitable for the lowest pressures measured, but the transducers were calibrated statically over the full range. The signals from the transducers were measured by Tektronix 502 oscilloscopes and recorded photographically using Land-Polaroid cameras.

Schlieren photography was used to observe flow details and shock wave shapes. The light source for the schlieren system is a spark which can be delayed to any desired time during the run.

3.5.2. Surface Pressure Measurements.

The support for the metal parabola was ground parallel to its base on assembly such that the incidence of this

model with respect to the base of the working section was zero to within $\pm 0.05^\circ$. This was checked by dial gauge. The alignment of the nozzle centreline with the working section base was then checked by tightening a string accurately along the axis of symmetry of the contoured nozzle and measuring its height from the base at two points. The alignment was within 0.2° . A further check was obtained from two pressures measured on the parabola both at $x = 1.25$ inches but on the upper and lower surface. Since these agreed to within 2%, alignment with the base only was checked for the other models by dial gauge. The two-dimensionality check showed that the side pressures at $x = 1.25$ inches were within 2% of each other and the pressure on the centreline. So also were those at $x = 0$ while those at $x = 7$ inches were both 8% lower than those on the centreline. At any time during the experiments all the pressure orifices which were not connected to transducers were sealed off with plasticine or rivets inserted in the plastic tubing fixed to them.

Taking into account the repeatability of conditions, the mode of reading pressures off a photograph and the calibrations of the transducers and oscilloscopes, the error in the pressure measurements was estimated to be about $\pm 5\%$ at pressure levels down to about 0.7 p.s.i.a., and about $\pm 8\%$ at pressure levels down to about 0.2 p.s.i.a.

3.5.3. Shock Wave Shapes and Shock-Layer Traverses.

Shock wave shapes were measured from schlieren photographs enlarged to twice full size. The well-defined shock front was taken to be the position of the shock wave and its position was measured from the axis of symmetry by a pair of dividers, transferred to a rule. The error in this measurement was estimated to be ± 0.01 inch, which corresponds to ± 0.005 inch on true size. The shock wave position could be checked in three isolated cases of the shock layer traverses, when the position of the pitot probe happened to coincide with the shock wave. These checks agreed with the photographic measurements to within ± 0.015 inch.

The static and pitot rake was traversed along a line which was slightly inclined upstream from the normal to the body surface, and its incidence was maintained throughout each traverse. That is, no account was taken of the change in streamline slope from body to shock wave. This change in slope amounted to a maximum of about $\pm 3^\circ$. The variation of the static pressure reading with probe incidence was found to be smaller than 2% over a change of incidence from 0° to 3° in the shock layer of a 20° wedge.

The spacing of the three sets of probes was measured accurately and the first reading of each traverse was taken with the lowest probe touching the surface of the model. The probe positions were known to an accuracy of $\pm .005$ inch.

When the uppermost static probe was touching the bow shock wave of the model, strong interference resulted and the static pressure was unreliable (see Fig. 25). All such readings were discarded. It was hoped that the shock wave slope could be used in conjunction with the free stream conditions to calculate conditions on the shock wave. However, since the local Mach number outside the shock can vary with position from about 8 to 8.5 (see Fig. 13) and conditions on the shock wave are very sensitive to shock wave slope, the static pressure behind the shock could only be determined to within $\pm 15\%$ by this method.

4. EXPERIMENTAL RESULTS AND COMPARISON WITH THEORY.

4.1. Surface Pressure Distribution.

The surface distributions on the parabola, wedge-parabola and $\frac{5}{8}$ -power-law body are plotted in logarithmic coordinates as $\frac{p}{\rho_{\infty} U_{\infty}^2}$ versus $\frac{x}{d}$ in Fig. 26. They are compared with the first and second order theories, modified Newtonian and tangent wedge approximations and, in the case of the wedge-parabola, with a characteristics solution, which will be discussed in section 4.4.

In order to obtain the coefficient C_0 of the first order theory, the experimental pressure distributions were integrated numerically to give the drag to x , D_x , as a function of x . D_x was plotted against $\frac{x}{d}$ in Fig. 27 and the value of D_{∞} , the asymptotic drag, was obtained by fitting the expression

$$D_x = D_{\infty} - a \left(\frac{x}{d} \right)^{m-\frac{2}{3}}$$

to the experimental curve. D_{∞} , substituted into equation 2.33, yields the coefficient C_0 . These values of C_0 are given in Fig. 27. Although this method of obtaining C_0 is only an estimate, it is the result of an integration of experimental quantities and thus likely to be more accurate than the alternative method of comparing the magnitude of first and second order quantities directly. It also does not involve the use of any second order quantities apart from the actual body shape.

With this value of C_0 it is now possible to calculate the theoretical behaviour of the flow variables, and all the theoretical curves in the illustrations use C_0 as obtained in this fashion. The pressure distributions of the parabola and wedge-parabola agree reasonably well with the present theory although the effect of the finite free stream pressure manifests itself at large values of $\frac{x}{d}$. This effect is partially cancelled by the lack of two-dimensionality near the tail end of the model, where the pressure drops off again. These two effects are best observed in the pressure distribution of the wedge-parabola: The experimental pressure distribution falls off from the characteristics solution near the end of the model. This demonstrates the effect of the loss of two-dimensionality, since the effect of finite p_∞ is taken into account by the characteristics solution. The experimental and characteristics pressures deviate upwards from the present theory at large $\frac{x}{d}$ because of the finite free stream pressure effect. The lack of two-dimensionality at the rear end of the model makes it necessary to treat all experimental results downstream of $\frac{x}{d} \doteq 30$ in the case of the parabola and wedge-parabola with some suspicion.

The pressure distribution of the $\frac{5}{8}$ -power-law body differs widely from the theoretical prediction and is in rather better agreement with the tangent wedge approximation

if plotted as $\frac{p}{\rho_{\infty} U_{\infty}^2}$. However, since the pressures on this body are considerably lower than those on the other two, an attempt has been made to correct the error due to finite free stream pressure by comparing theory with experimental values of $\frac{p - p_{\infty}}{\rho_{\infty} U_{\infty}^2}$. The agreement is quite good.

All three pressure distributions show a consistent "dip" in pressure in the region between $x = 1$ and $x = 3$ inches. This has also been observed by Needham (1965) in the pressure distribution over a flat plate and is considered to be a characteristic of the tunnel.

4.2. Shock Wave Shape.

The pilot experiments (see section 3.3.1. and Fig. 14) have already shown that the significant length scale of the shock wave shape on a two-dimensional parabola is the same as d , the scale of the parabola. Since only one model was tested at $m = \frac{5}{8}$, a similar test of correlation could not be made for this value. However, this correlation was tried in the experimental shock wave shapes obtained by Peckham (1965) and Kubota (1957) on axisymmetric power-law bodies. The result is shown in Fig. 28 for $m = \frac{3}{4}$ and $m = \frac{1}{2}$. Unfortunately, the curvature of the shock shape (in logarithmic coordinates) could not always be taken into account, since most of the results were drawn as straight lines in Peckham's report. However, the correlation is quite good and it can be expected that shock wave shapes on a body of

the shape $\frac{y}{d} = \left(\frac{x}{d}\right)^m$ collapse for a particular m onto a single curve in these coordinates for large Mach numbers. If the stand-off distance of the bow wave is proportional to d which is true for $m = \frac{1}{2}$, this can be expected to be true right down to the nose. At the other end of the scale - $\frac{x}{d} \rightarrow \infty$ - the shock wave must, of course, become weak eventually and shrink to a Mach wave. This behaviour would exhibit itself in the shock wave shapes as a "tailing-off" of the shock wave to a line of the form $\frac{y_s}{d} = a + \frac{1}{\sqrt{M_\infty^2 - 1}} \frac{x}{d}$ where a is presumably a function of M_∞ and m (see Fig. 29). The tailing-off of the shock wave can be detected in the shape for $m = \frac{1}{2}$ and $M_\infty = 6.8$ in Fig. 28 as a deviation of the shock shape from that for $M_\infty = 7.7$.

The shock shapes obtained in the present experiments are shown as schlieren photographs in Fig. 30a and plotted in logarithmic coordinates in Figs. 30b and 14, where they are compared with the present theory using the values of C_0 obtained from the drag. The magnitude of the second order quantities is again seen to be much larger for $m = \frac{5}{8}$ than for $m = \frac{1}{2}$, and the way in which the second order effect modifies the first order theory is in the right direction, except in the case of the parabola, where the theoretical prediction for y_s is 10% less than the experimental result. Although a better fit could have been achieved by choosing C_0 and C_1 for fit, this was not done here in order to give a

true comparison of theory and experiment.

4.3. Shock Layer Traverses.

Shock layer traverses of pitot and static pressures were made at five stations on the parabola $d = 0.225$ inch, 3 stations on the wedge parabola, two stations on the 0.625 power body and two stations on the smallest wooden parabola ($d = 0.100$ inch). The location of the stations is shown in Fig. 31, and the measured pitot and static pressures are presented in Fig. 32. The length coordinate in these plots is measured from the body surface. Although the two-dimensionality of the flow is doubtful downstream of $\frac{x}{d} = 30$ on some of the bodies, such traverses have been included in Fig. 32 to show the general trend. The quantities M , q , ρ and T derived from these measurements and from the assumption that the total temperature is constant at 665°K are shown in a similar form in Fig. 33.

The static pressure as measured on the surface of the metal models opposite the traversing station is included in Fig. 32 as a point on the vertical axis. It is seen that the surface pressure agrees well with the extrapolation of the static pressure traverses, giving additional confidence in the static pressure probe.

The assumption that the velocity in the x -direction is constant and equal to U_∞ is seen to be a good first approximation except when $\frac{y-y_b}{d}$ is smaller than 5.

It is seen from the density profiles that the assumption of a strong shock wave is quite drastic for the conditions of the present experiments, the density on the shock wave being less than 50% of the value on a strong shock wave for large $\frac{x}{d}$. It can therefore be expected that the density profiles will not be in good agreement with the theory. This is evident even from a cursory inspection of the density profiles on the $\frac{5}{8}$ -power-law body, whose shape is considerably different from the density profile of Fig. 12. The general shape of the density profiles on the parabola, however, is similar to the theoretical prediction.

The strong shock assumption is not so serious in the case of pressure and velocity and in the hope that the effect of the error in density on the pressure and normal velocity is not too strong, these were compared with theory in Figs. 34a and 34b. The normal velocity v was calculated from the streamline slope distributions of Fig. 19 and the q -distributions of Fig. 33. It is seen that in both cases the experiments for the parabola are in very good agreement with theory. In the case of $m = \frac{5}{8}$ the pressure distribution is of the right general shape but considerably higher than the theoretical prediction. The weak shock effect (finite free stream pressure) would account for some of this discrepancy. The normal velocity for $m = \frac{5}{8}$ is also underestimated by the theoretical prediction. However, from a close inspection of

the shock wave shape for $m = \frac{5}{8}$, Fig. 30b, it is seen that the shock wave begins to "tail off" at about $\frac{x}{d} = 300$. For a reasonable comparison with the present theory, $\frac{x}{d}$ should be at least about 500. The two regions in which the theory breaks down (large $\frac{x}{d}$ - weak shock, small $\frac{x}{d}$ - slow convergence) thus overlap, and a comparison with experiment becomes somewhat marginal unless M_∞ is larger than in the present experiment.

The Mach number distribution as obtained from static and pitot pressure measurements on the $\frac{5}{8}$ -power model at $\frac{x}{d} = 560$ is superimposed on that obtained by the method of weak waves at the same station in Fig. 18. The scatter of the points derived from pressure measurements in this traverse amounts approximately to that of the weak waves method. Within this scatter of about $\pm 6\%$ the agreement is quite good.

Fig. 36 shows streamline shapes derived from the weak waves method in the case of the wooden parabola, $d = 0.100$ inch.

4.4. Comparison with Characteristics Solution.

Lewis (1965) calculated the flow over the wedge-parabola of the present investigation under the same conditions by the method of characteristics. These data are compared with the experimental results in the appropriate parts of Figs. 26, 30b and 35. It is seen in Fig. 26 that the experimental surface pressure distribution is in good agreement with the characteristics solution up to about $\frac{x}{d} = 30$, where the experimental

pressure drops off due to loss of two-dimensionality. The comparison also lends support to the statement made in section 4.1., that the consistent "dip" in pressure is a characteristic of the tunnel.

The shock wave shape as obtained by the characteristics method is also in good agreement with experiment as is seen in Fig. 30b. The discrepancy between experiment and characteristics solution for small $\frac{x}{d}$ amounts only to about 0.020 inch and is largely attributable to experimental error.

Two interpolations of Lewis's data coincided with two of the traverses of the shock-layer of the wedge-parabola. The position of these interpolations is marked on Fig. 31. The results of the interpolations are compared with the experimental traverses in Figs. 35a and 35c. (Note that the full line of Fig. 35b is a mean line through the experimental points.) An important result is brought out by the comparison in Fig. 35a where the experimental static pressure deviates considerably from the characteristics prediction when the uppermost probe protrudes through the shock wave. The discrepancy starts at $y-y_b = 0.44$ inch and this is corroborated by the position at which it should occur according to the shock wave and probe geometry (see Fig. 25 which is taken at the same station). This gives an indication of the importance of probe-shock interference and has been used in assessing the validity of static pressure measured

near the shock wave in other traverses. At the downstream station, which is in the region where the two-dimensionality of the flow is doubtful, ($\frac{x}{d} = 36$), the static pressure readings are consistently lower than the pressure obtained by the method of characteristics. This discrepancy amounts to a maximum of about 10%.

Apart from these discrepancies the agreement between experiment and characteristics theory is remarkably good, the pitot pressures being almost identical. The distinct kink in the profiles at the upstream station which results from the discontinuous curvature of the body at the junction of wedge and parabola is shown by both experiment and characteristics method and coincides with the change in density gradient shown in the schlieren photograph Fig. 30a.

The relatively large error of the quantities on the shock wave is due to the fact that they were determined from the shock wave slope and free stream conditions. This is a relatively inaccurate technique.

It is more difficult to start characteristics solutions when the flow is locally subsonic, as it is near a blunt nose, especially in plane flow. The computer program of Lewis (1965) is unfortunately unable to deal with such cases at present. However, the comparisons of Fig. 35 give an indication of the potential of this method as a criterion for other theoretical methods. They also indicate the extent to which one can trust

the static pressure measurements in the blunt body shock layers of the present investigation.

The experimental and theoretical results are available in the form of detailed tables at the Imperial College Aeronautics Department.

5. FUTURE WORK.

The experiments conducted in this investigation were insufficient in number to determine the complete behaviour of the coefficients $C_1(m)$ and $C_0(m)$ over the range $\frac{2}{3} < m < \frac{2}{3}$, two points only having been obtained. A more complete investigation, say, of about five models in this range would demonstrate more clearly the strong and weak points of the theory. The similarity of the shock wave shapes in $\frac{y}{d}$ and $\frac{x}{d}$ coordinates could be verified more fully by testing models with several different length scales for each value of m .

The same could be repeated for other ranges of m , and for axisymmetric flow. Theoretical solutions for other ranges of m could be obtained in the manner suggested by Freeman (1962), (see also section 2.2.).

Ideally, the aim would be to obtain theoretical solutions for the full range of m from 0 to 1 with a full range of experimental results to check them in both plane and axisymmetric flow. It would be better in a comparison of theoretical and experimental shock wave shapes to plot $C_0(m)$ and $C_1(m)$ rather than $\mu(m)$, since $\mu(m)$ is more properly $\mu(m, \frac{x}{d})$.

The experimental verification of theoretical work could very suitably be complemented by numerical solutions such as those presented here for the wedge-parabola. These could serve to check the extent to which the assumptions of both

theory and experimental technique can be made safely, and to give a more accurate comparison with the theory.

It would also be interesting to see how the theoretical solutions depend on γ and this would need only a simple extension of the computer programme so long as $\gamma - 1$ is not small. The most difficult problem of the theory, however, remains the determination of matched solutions for the particular values of m , for which the solution remains singular near the wall to second order. These are $m = \frac{2}{(3+j)\gamma}$, $\frac{2}{3+j}$, $\frac{2(\gamma+1)}{(3+j)\gamma+2}$.

6. CONCLUSIONS.

Theoretical solutions, based on the expansion scheme for large x and large M_∞ as proposed by Freeman (1962), were obtained for the asymptotic inviscid flow over plane bodies of the shape $\frac{y}{d} = \left(\frac{x}{d}\right)^m$ in the range $\frac{2}{3} < m < \frac{2}{3}$. These second order solutions enabled the condition that the body surface be a streamline to be satisfied.

The magnitude of the second order term was found to increase from zero at the singular point $m = \frac{2}{3}$ to infinity at the singular point $m = \frac{2}{3}$. At these two points it is probably necessary to adopt a more sophisticated approach since, at $m = \frac{2}{3}$, a matching condition cannot be satisfied, and at $m = \frac{2}{3}$ the expansion does not converge in the present approach.

The coefficient of the second order quantities was found to be negative, such that the shock wave of the second order theory lies closer to the body than that of the first term in the expansion. Similarly, the surface pressure approaches the first order value from below as $\frac{x}{d} \rightarrow \infty$.

Experiments at $M_\infty = 8.2$ were performed on two plane power-law bodies in the range $\frac{2}{3} < m < \frac{2}{3}$ (at $m = \frac{1}{2}$ and $m = \frac{5}{8}$) and on a plane parabola with a 30 degree half-angle tangent wedge nose. These included the determination of shock wave shapes, surface pressure distributions and detailed investigations of the distribution of pitot and static pressures

across the shock-layer.

A comparison of shock wave shapes on plane parabolas of different length scales showed that the significant length scale of the shock wave shape is the same as that for the body, the shock waves for all parabolas collapsing onto a single curve in the body coordinates $\frac{y}{d}$ versus $\frac{x}{d}$. The same was found to apply to the results given by Peckham (1965) for axisymmetric bodies with $m = \frac{1}{2}$ and $\frac{3}{4}$.

The deviation of the experimental results from first order theory was generally in the same direction as predicted by second order theory and the magnitude of the deviation was found to be much larger in the case $m = \frac{5}{8}$ than at $m = \frac{1}{2}$ where it was very small. This again agrees with the prediction of the theory. The accuracy of the experimental results was insufficient for a conclusive comparison of the shape of the deviations with the second order quantities.

The extent of the range of validity of the theory was shown to vary with m . When the ratio of first and second order coefficients, $\frac{C_1}{C_0}$, is taken equal to one, the range of validity is restricted to very large values of $\frac{x}{d}$. The experimental determination of C_0 showed, however, that this ratio is smaller than one, thus reducing the lower limit of the range of validity of the theory to about $\frac{x}{d} = 10$ at $m = \frac{1}{2}$ and $\frac{x}{d} = 500$ for $m = \frac{5}{8}$.

The experimental results were compared with a

characteristics solution by Lewis (1965) in the case of the wedge-parabola with good agreement. The maximum discrepancy of the static pressure in this comparison was about $\pm 8\%$ in the region where the static probe is free from interference. A similar discrepancy of the pitot pressure amounts to about $\pm 5\%$. This comparison lends strong support to the gun tunnel as a good facility for testing steady flows. It also shows that numerical solutions could be used with confidence as a test of the theory for values of $\frac{x}{d}$ outside the range of experimental facilities.

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conducteur de la chaleur sur un
obstacle en loi de puissance.
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ILLUSTRATIONS

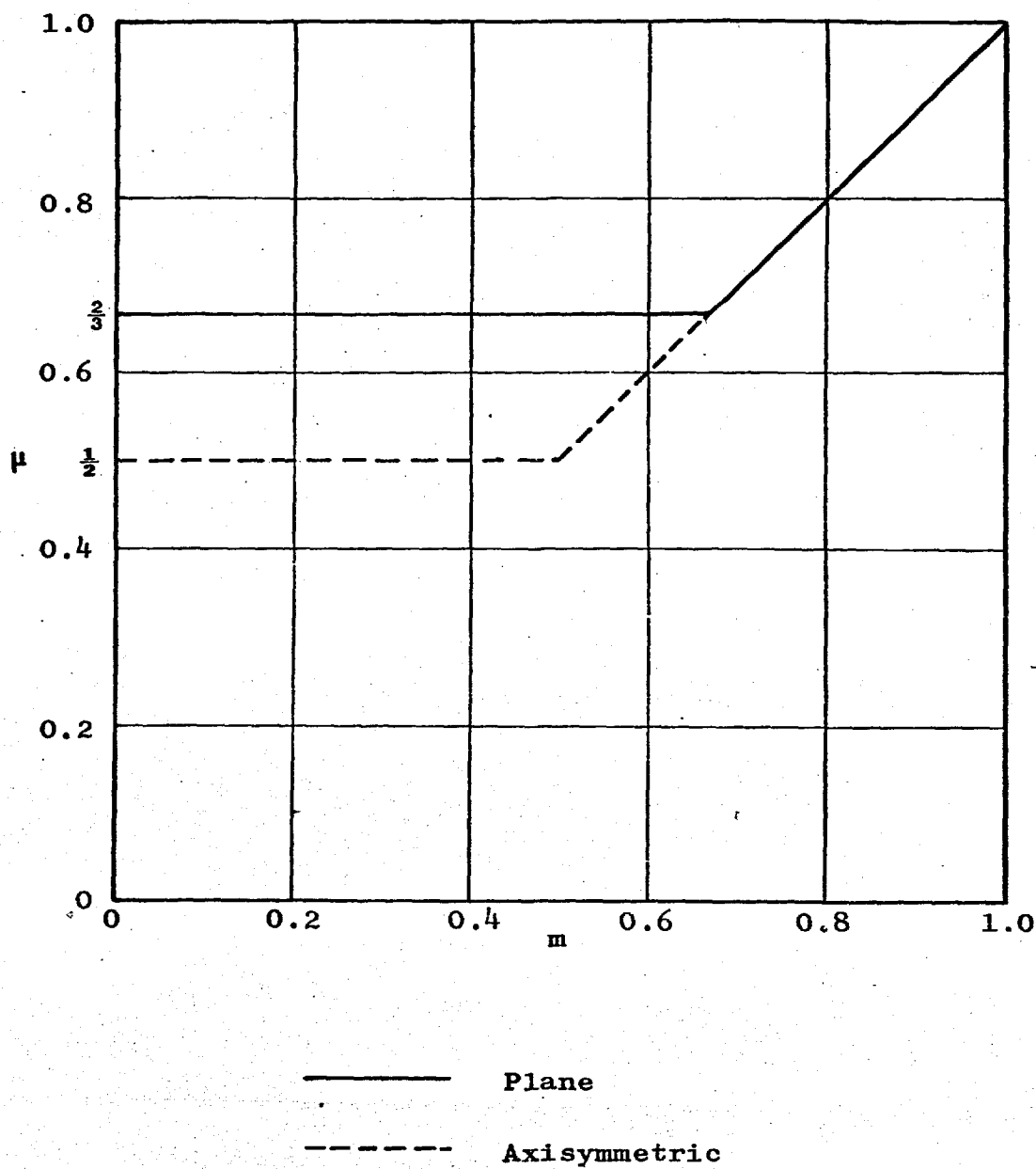


Fig.1. THE RELATION BETWEEN THE EXPONENTS μ AND m OF THE SHOCK WAVE AND BODY SHAPES IN AXISYMMETRIC AND PLANE FLOW, $\gamma = 1.4$.

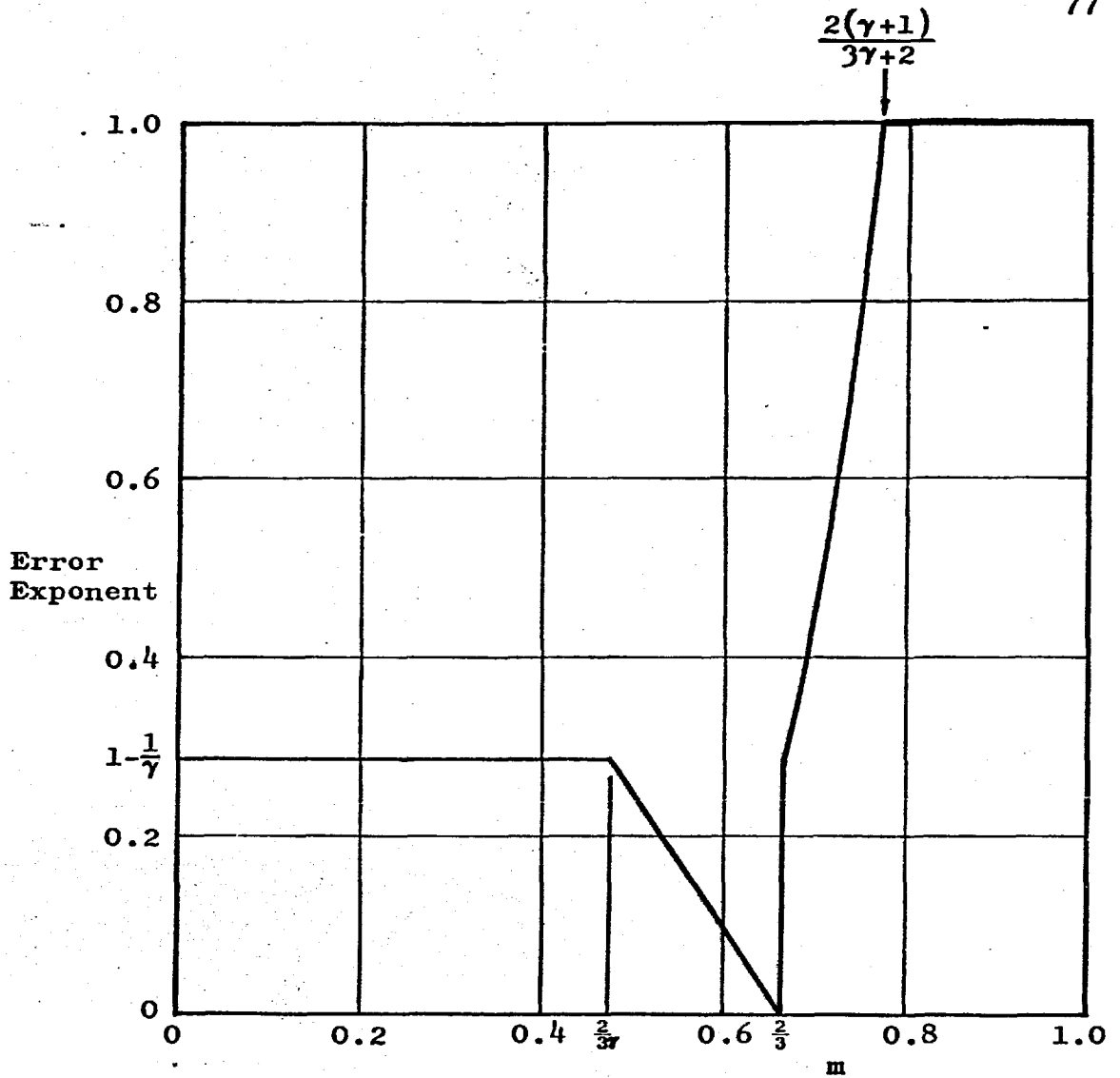


Fig.2. ORDER OF MAGNITUDE OF ERROR OF FIRST ORDER THEORY PLOTTED AS THE EXPONENT OF M_∞^{-2} VERSUS m , AS OBTAINED BY FREEMAN. PLANE FLOW, $\gamma = 1.4$.

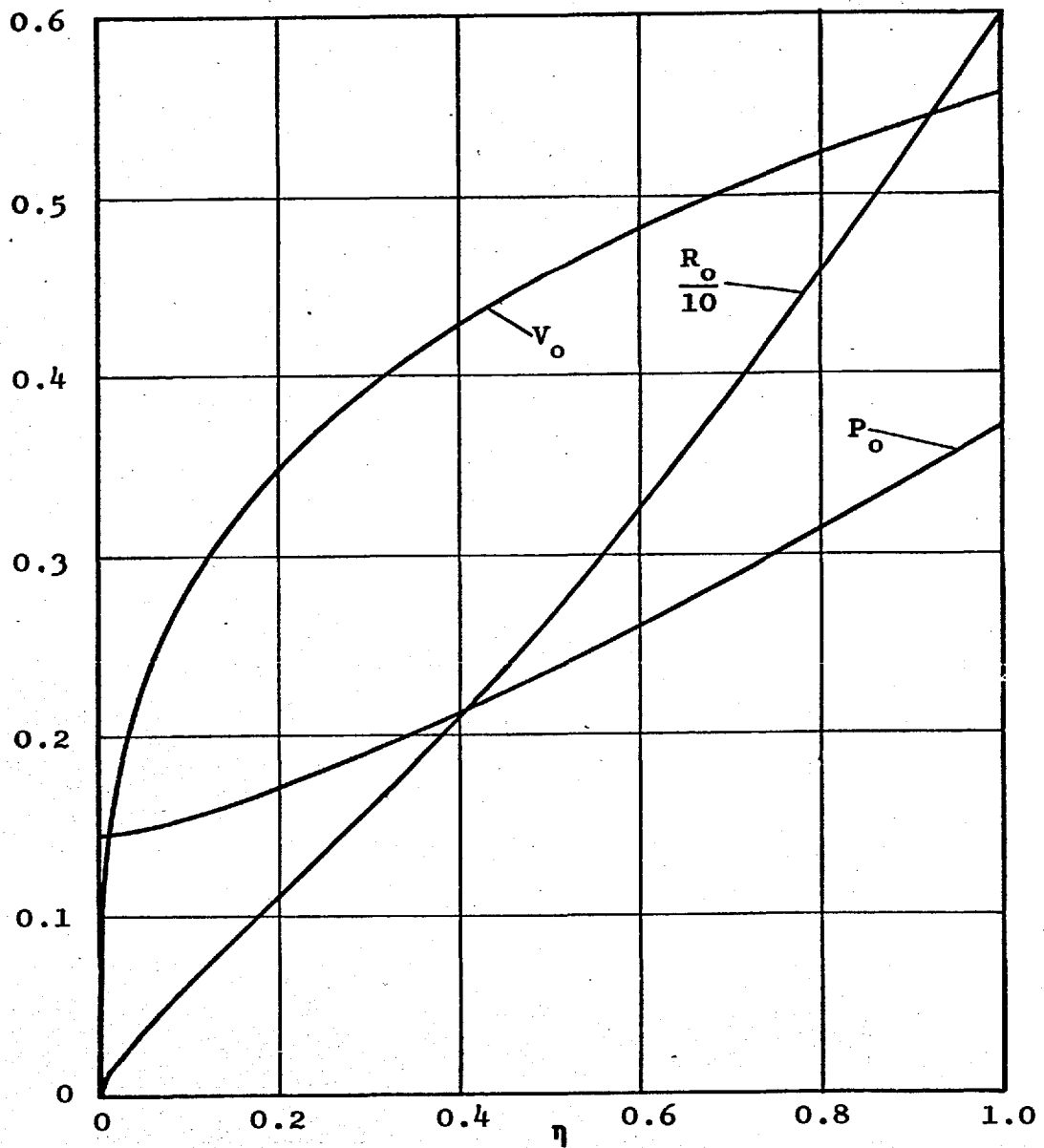


Fig.4. SOLUTIONS OF EQUATIONS 2.12.

FIRST ORDER QUANTITIES, $m < \frac{2}{3}$, $\gamma = 1.4$.

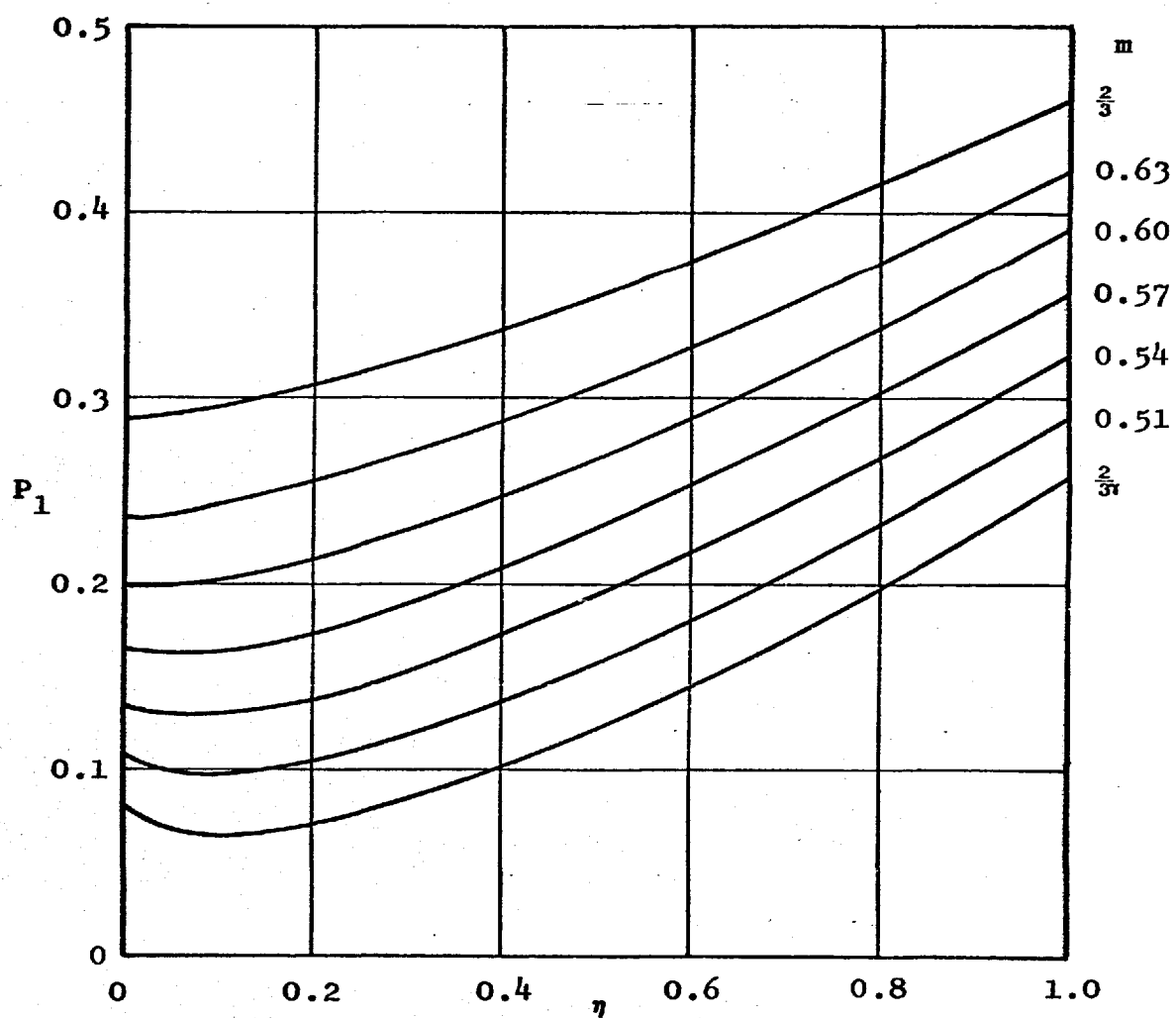


Fig.5. SOLUTION OF EQUATIONS 2.22 FOR $P_1(m, \eta)$.
SECOND ORDER PRESSURE, PLANE FLOW, $\gamma = 1.4$.

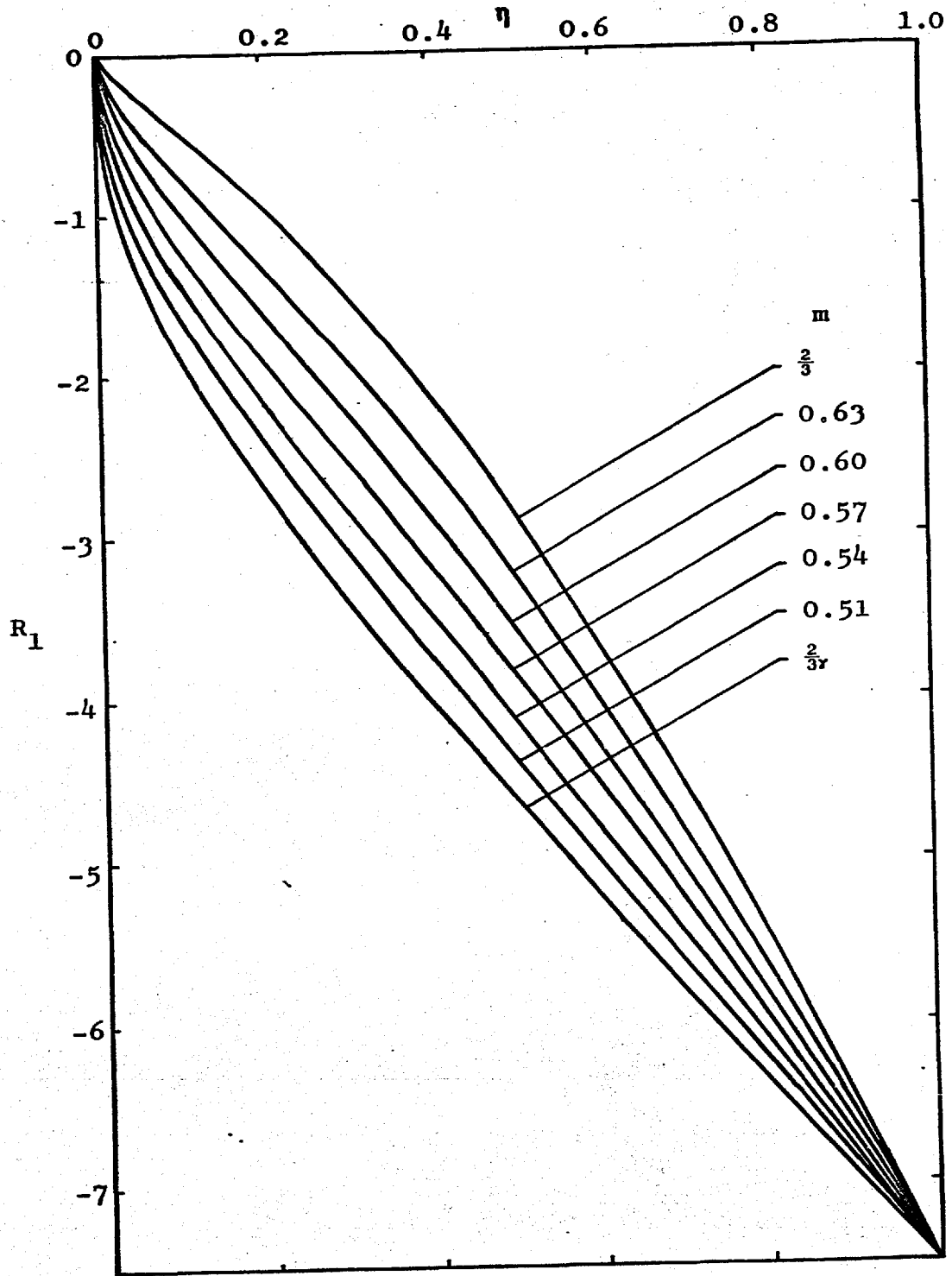


Fig.6. SOLUTIONS OF EQUATIONS 2.22 FOR $R_1(m, \eta)$.
SECOND ORDER DENSITY, PLANE FLOW, $\gamma = 1.4$.

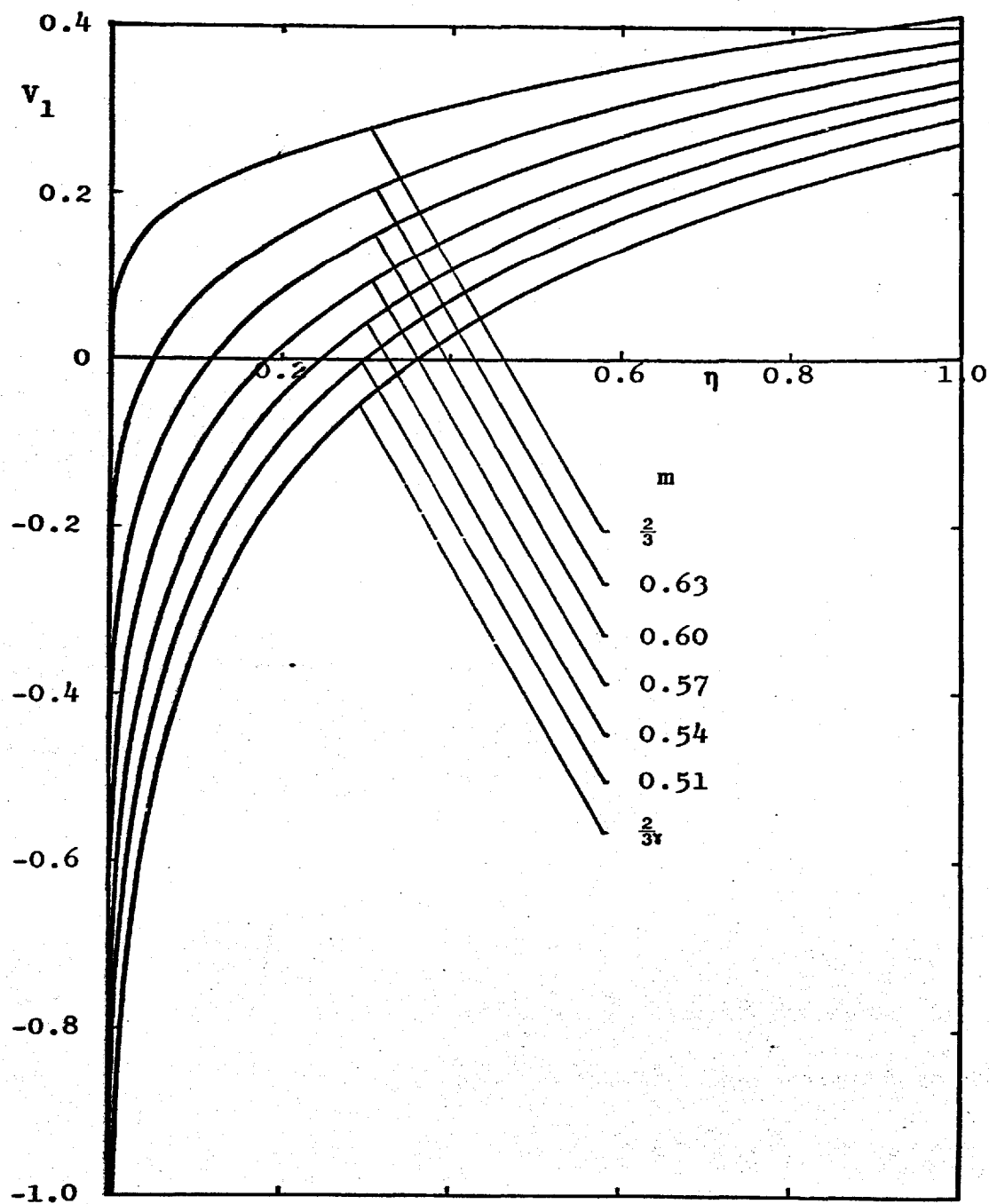


Fig.7. SOLUTION OF EQUATIONS 2.22 FOR $V_1(m, \eta)$.
SECOND ORDER NORMAL VELOCITY, PLANE FLOW, $\gamma = 1.4$.

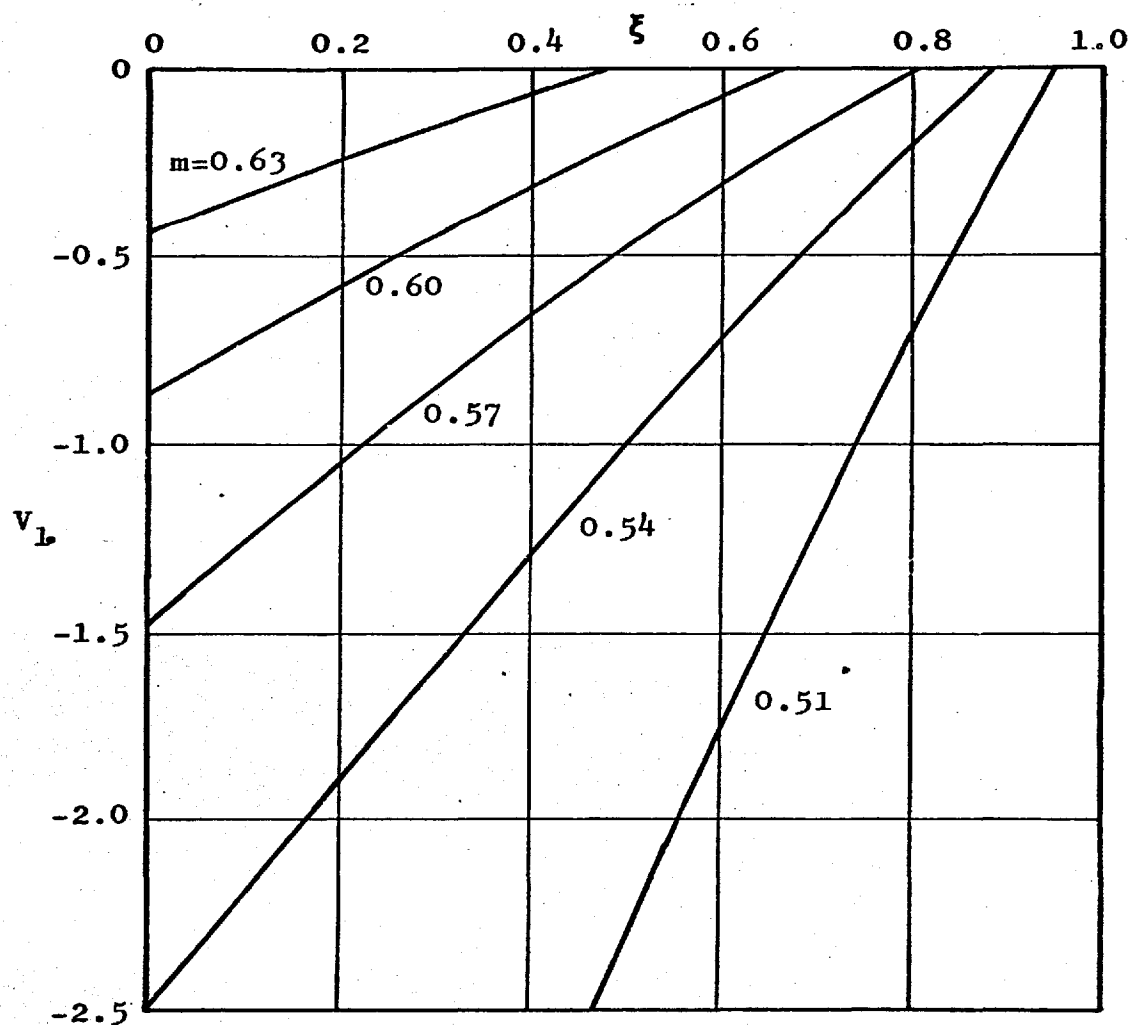


Fig.8. BEHAVIOUR OF v_1 FOR SMALL η . v_1 PLOTTED AGAINST $\xi = \eta^{\frac{3m}{2} - \frac{1}{2}}$. PLANE FLOW, $\gamma = 1.4$.

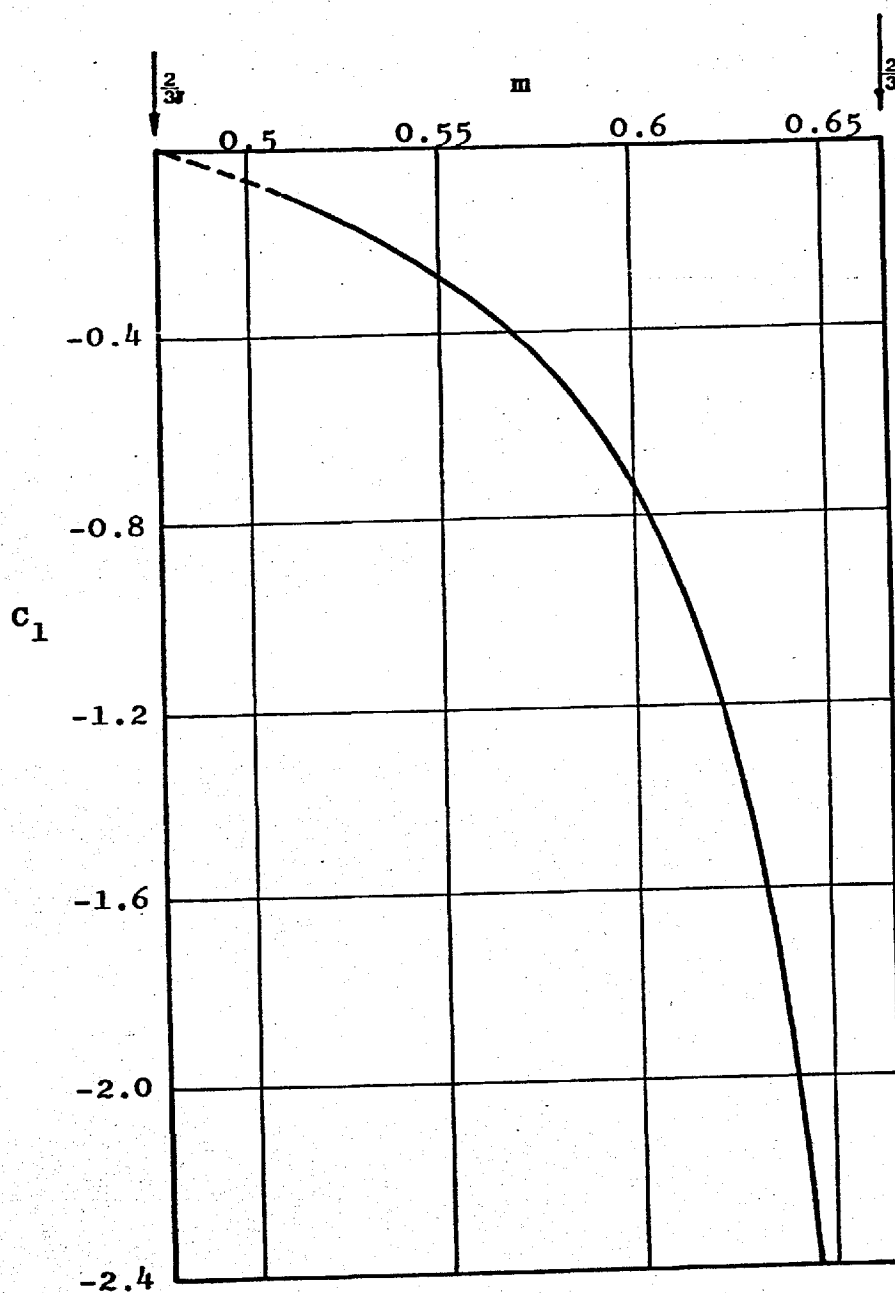


Fig.9. VARIATION OF COEFFICIENT OF SECOND ORDER QUANTITIES WITH m . PLANE FLOW, $\gamma = 1.4$.

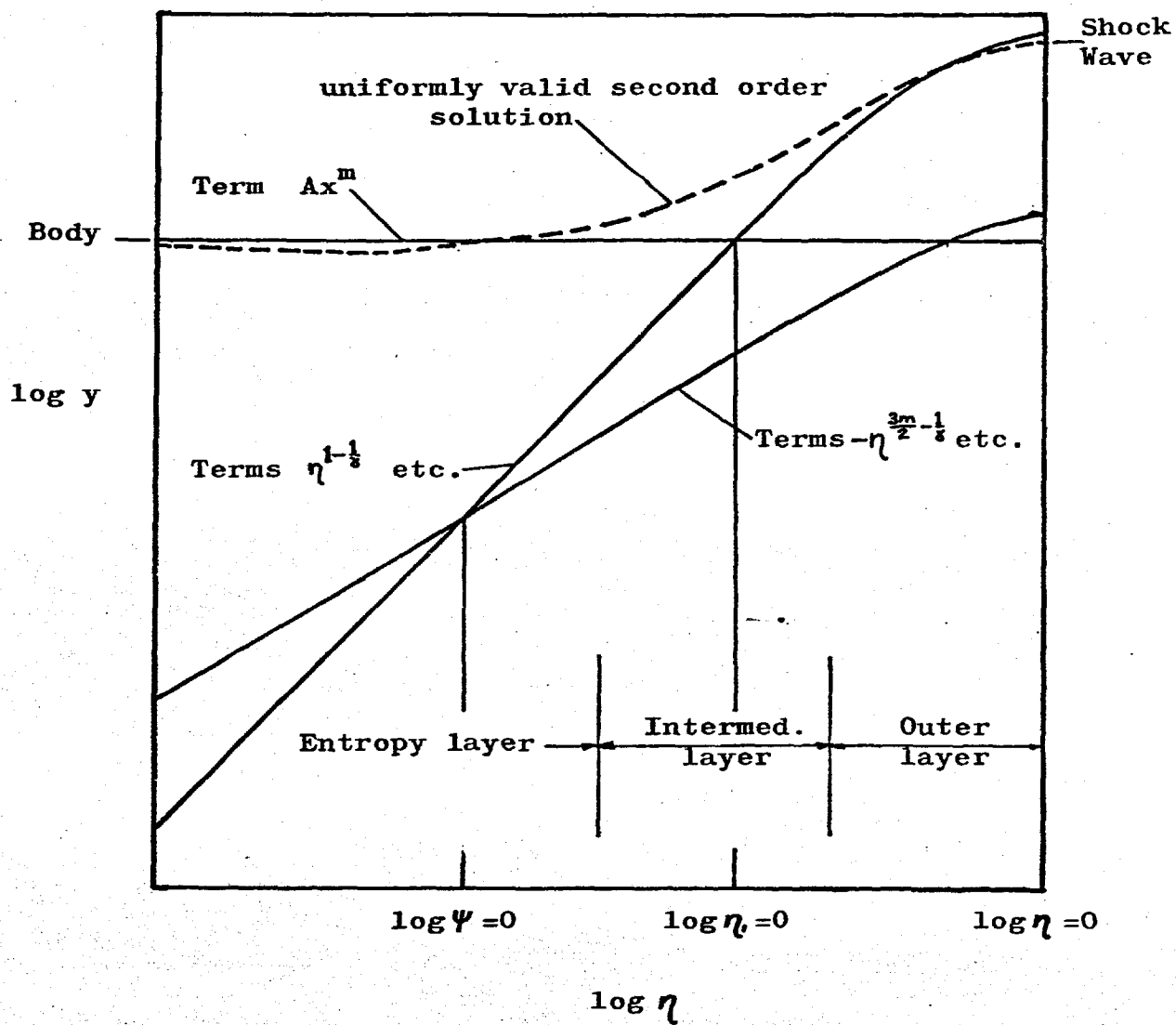


Fig.10. RELATIVE IMPORTANCE OF VARIOUS TERMS IN THE EXPANSION FOR y IN DIFFERENT LAYERS AND MATCHING. SCHEMATIC DIAGRAM FOR PARTICULAR VALUES OF x AND m .

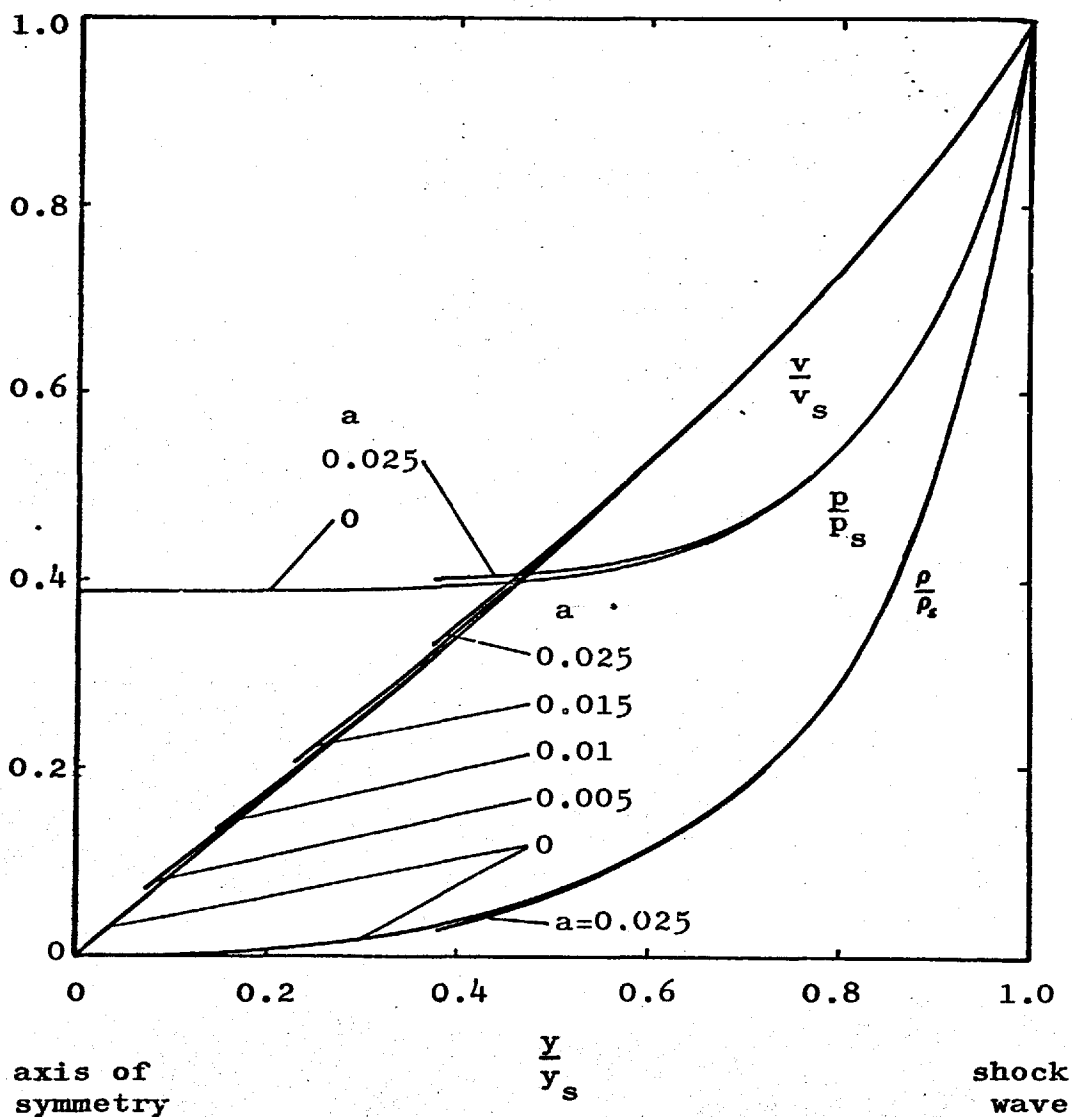


Fig.11. THE BEHAVIOUR OF THE SOLUTION OF EQUATIONS 2.12 AND 2.22 NORMALISED WITH RESPECT TO VALUES ON THE SHOCK. PLANE FLOW, $m=0.5$, $a=C_1/C_0 x'^{m-\frac{2}{3}}$, $\gamma=1.4$.

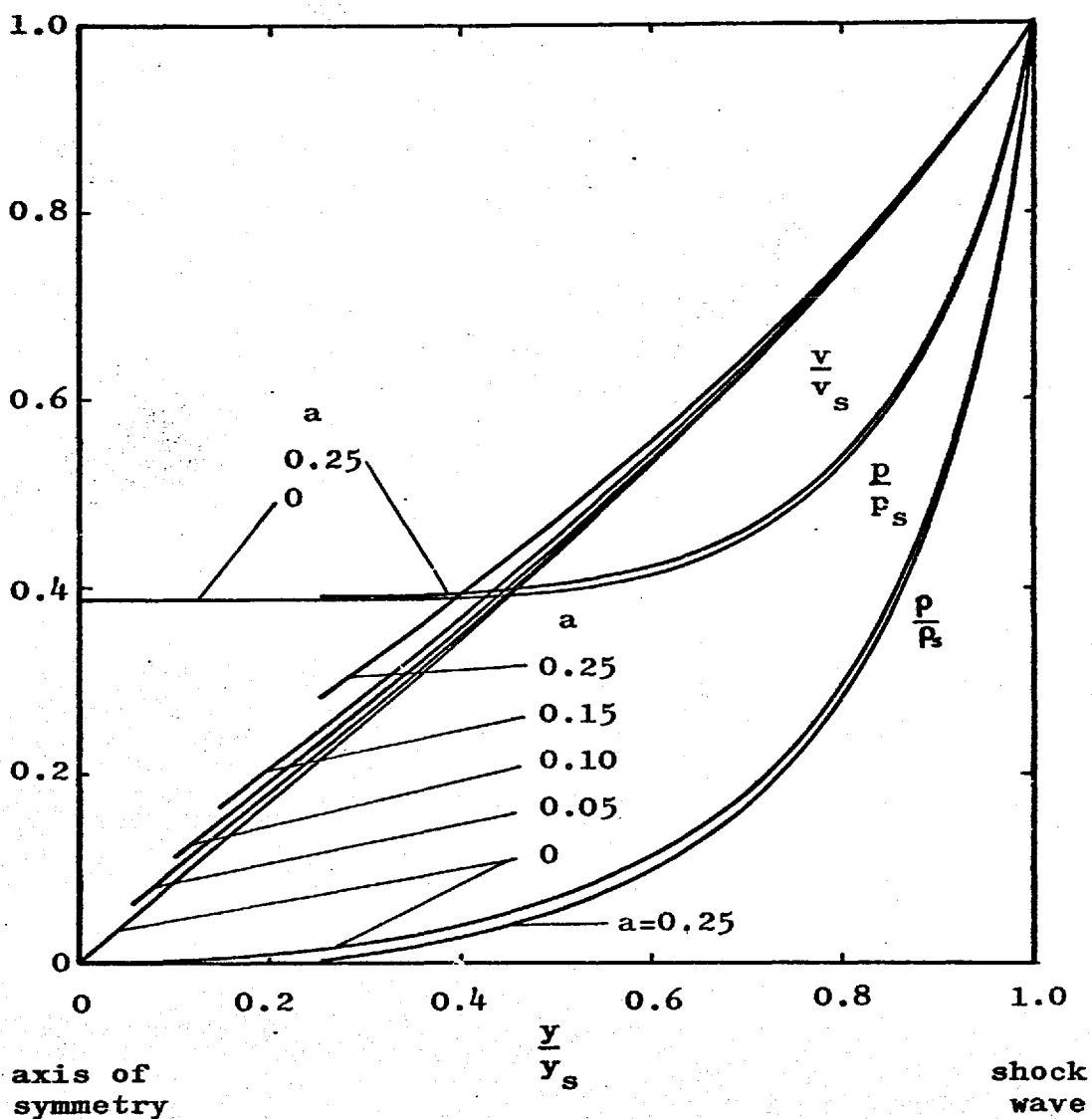


Fig.12. THE BEHAVIOUR OF SOLUTIONS OF EQUATIONS 2.12 AND 2.22 NORMALISED WITH RESPECT TO VALUES ON THE SHOCK. PLANE FLOW, $m=0.625$, $a=C_1/C_0 x^{m-\frac{2}{3}}$, $\gamma=1.4$.

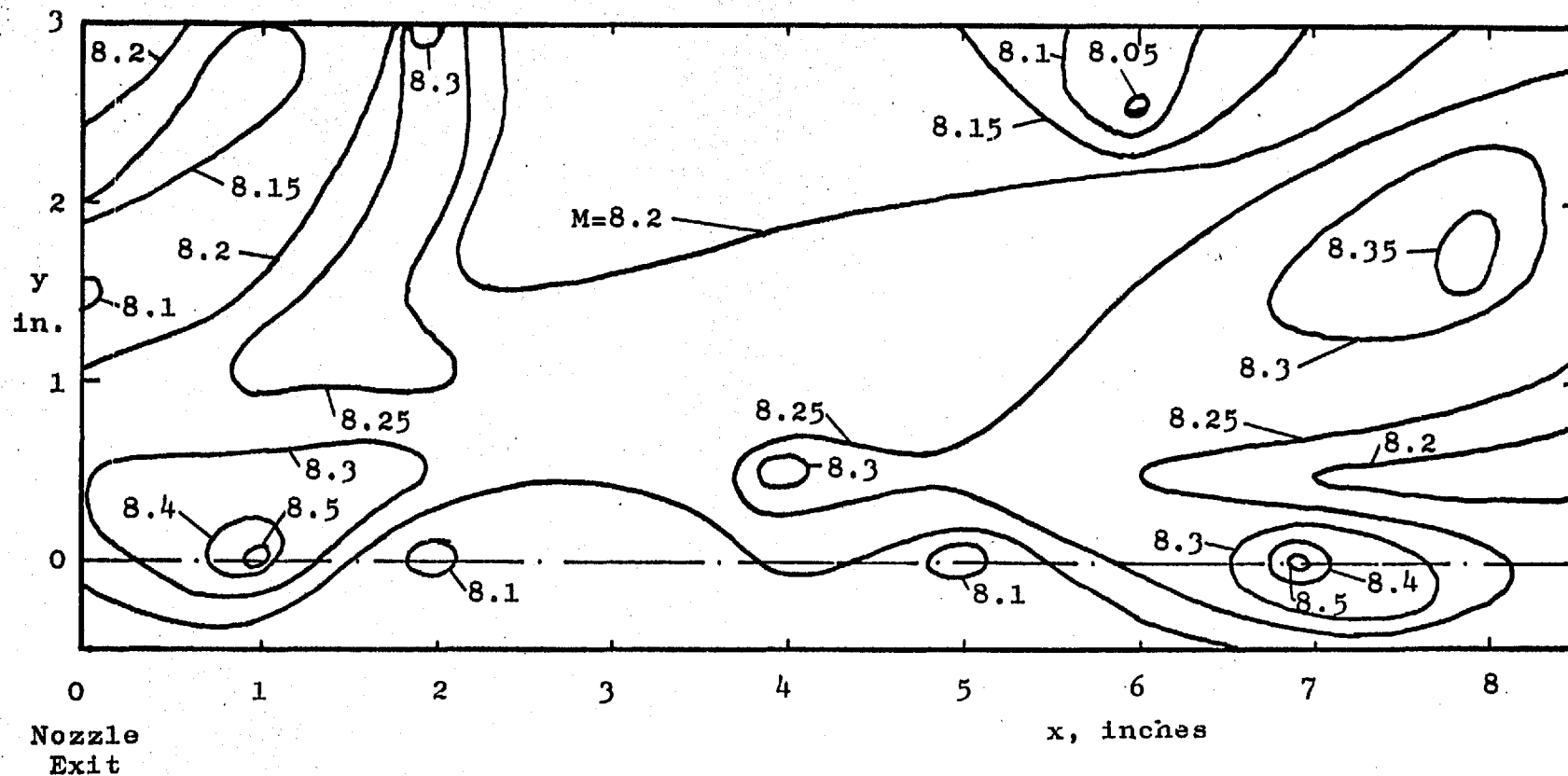


Fig.13. CONTOUR PLOT OF MACH NUMBER ON UPPER HALF OF VERTICAL CENTRE-
PLANE OF CONTOURED NOZZLE.

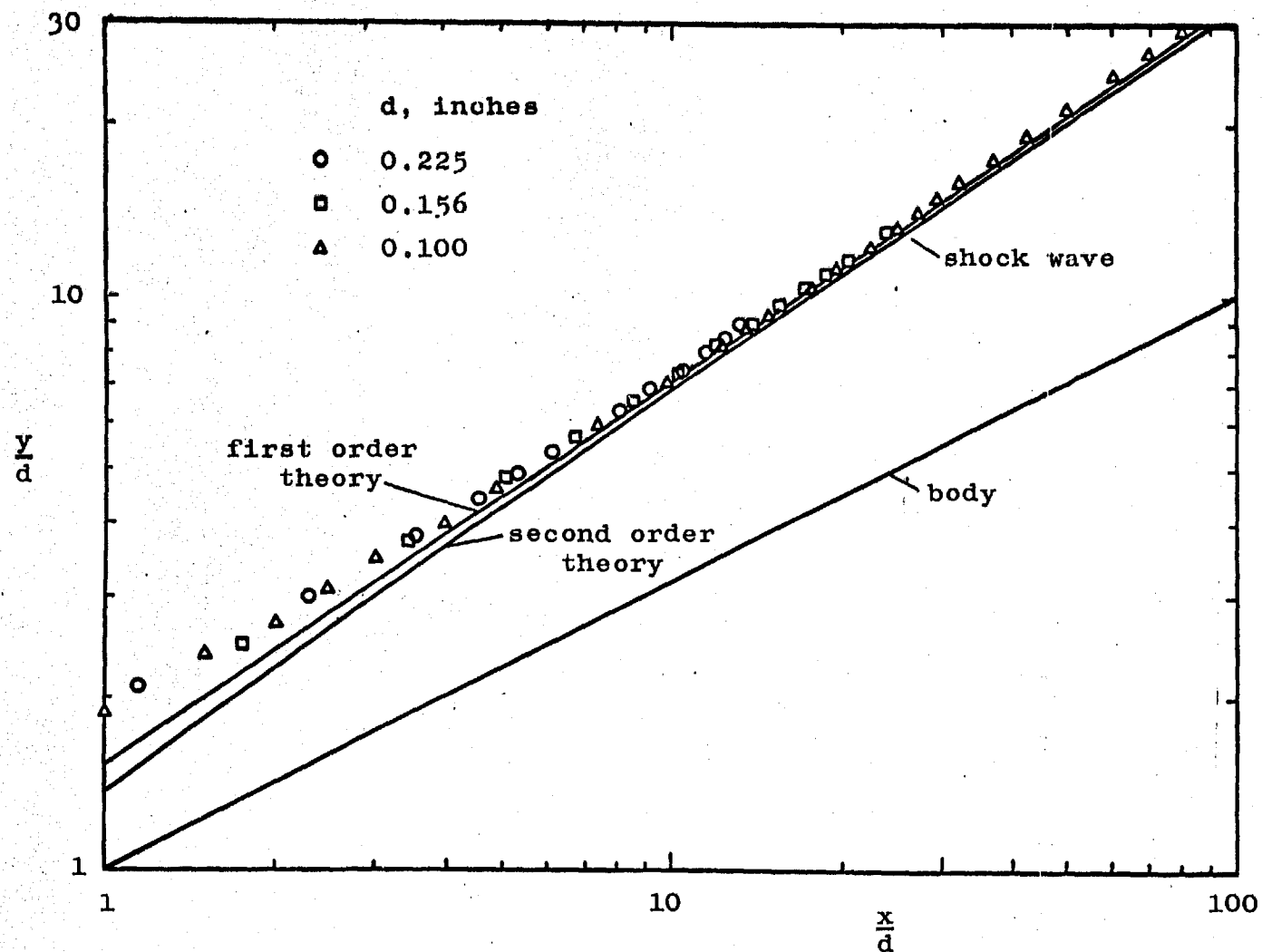


Fig.14. SHOCK WAVE SHAPE ON THREE PLANE PARABOLAS. $M_{\infty} = 8.2$.

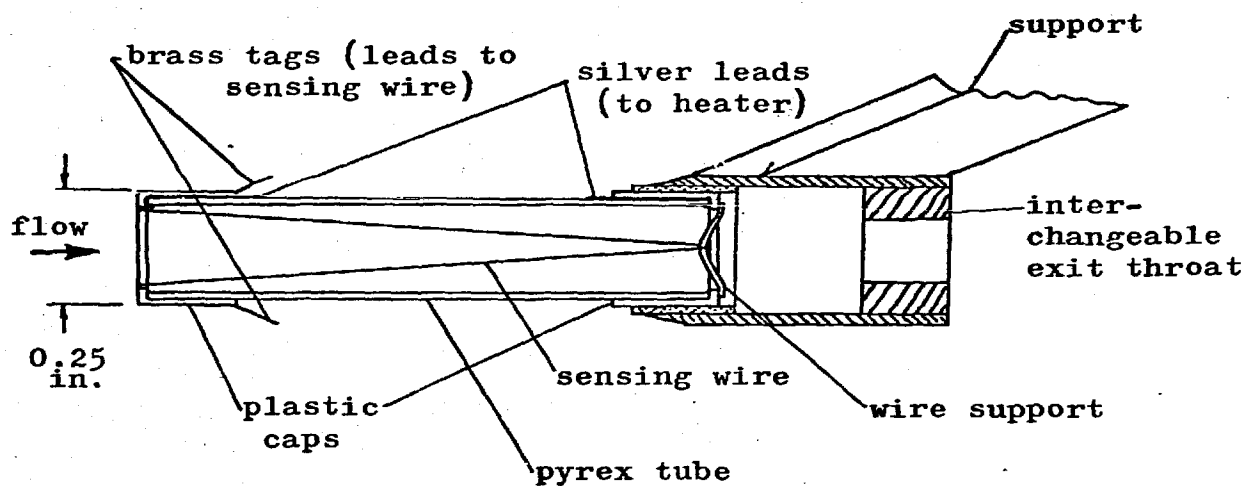


Fig.15a. CROSS-SECTIONAL VIEW OF STAGNATION TEMPERATURE PROBE.

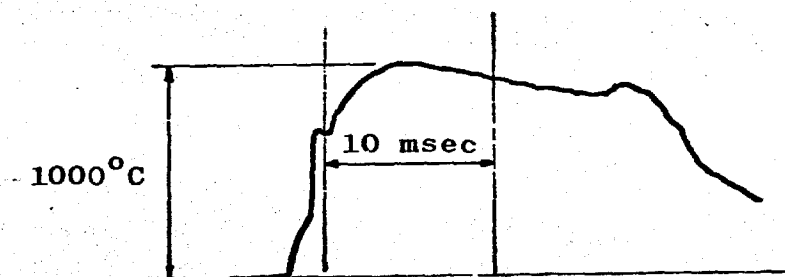


Fig.15b. OSCILLOSCOPE TRACE OF STAGNATION TEMPERATURE AT $M_{\infty} = 10$.

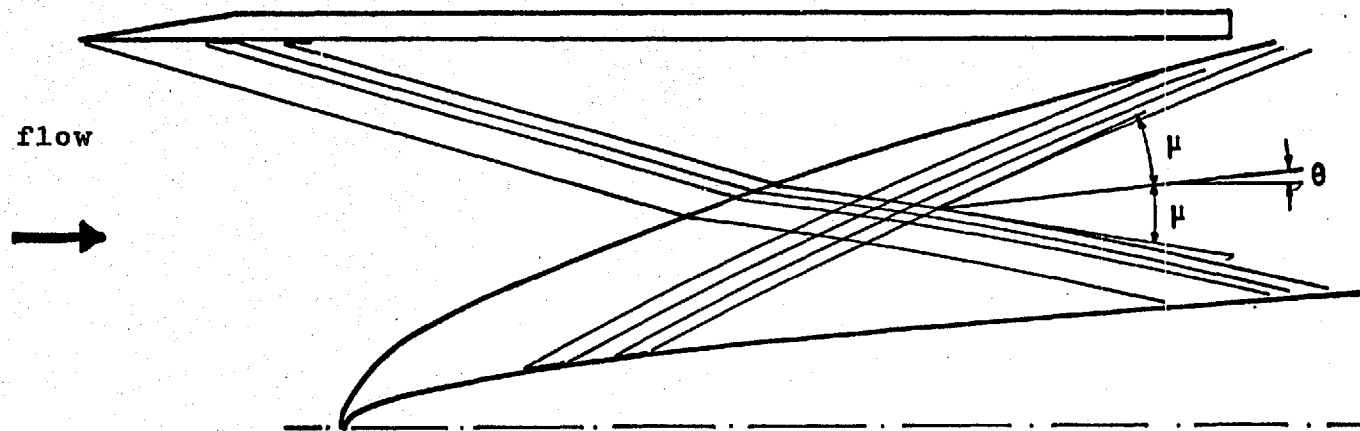


Fig. 16. ILLUSTRATING THE METHOD OF WEAK WAVES.



Fig.17. SCHLIEREN PHOTOGRAPH OF METHOD OF WEAK WAVES, $m = \frac{5}{8}$.

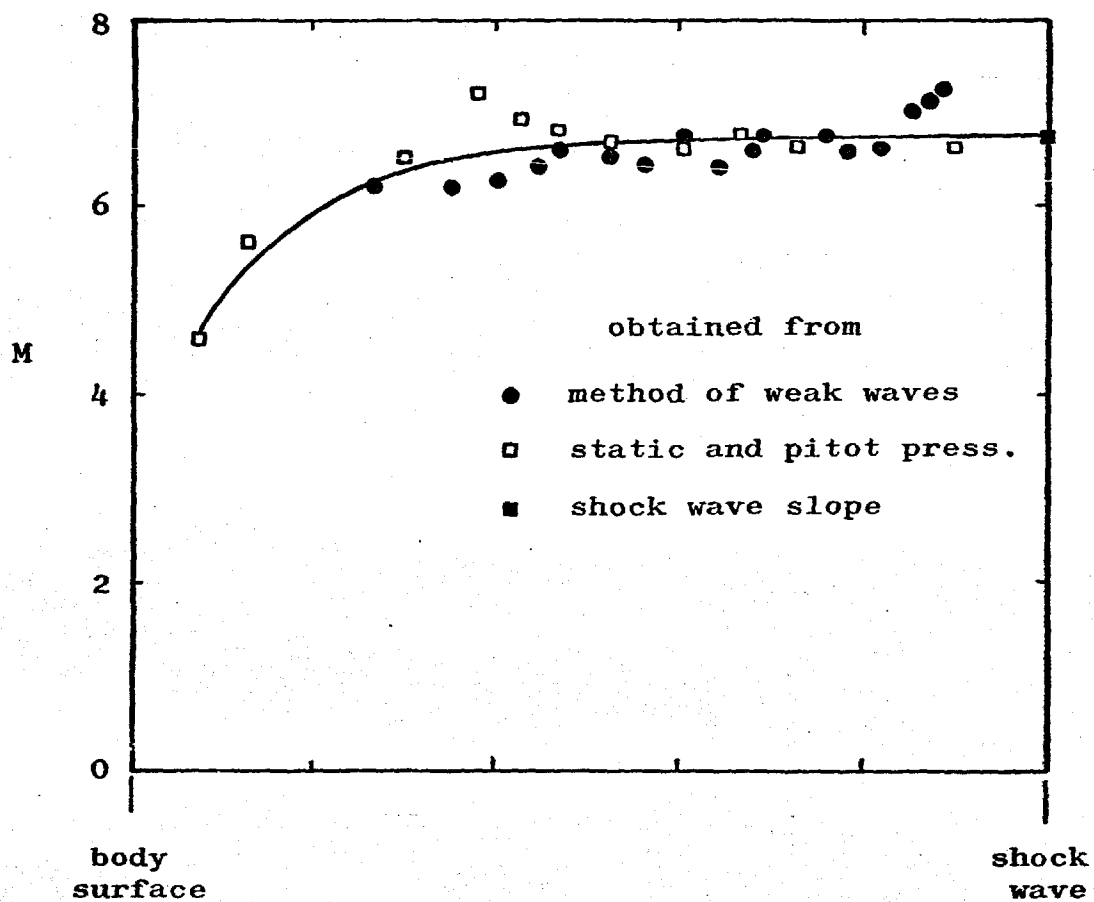


Fig.18. COMPARISON OF MACH NUMBER AS OBTAINED BY VARIOUS METHODS. $m=\frac{5}{8}$, $\frac{x}{d}=560$, $d=0.010$ INCH.

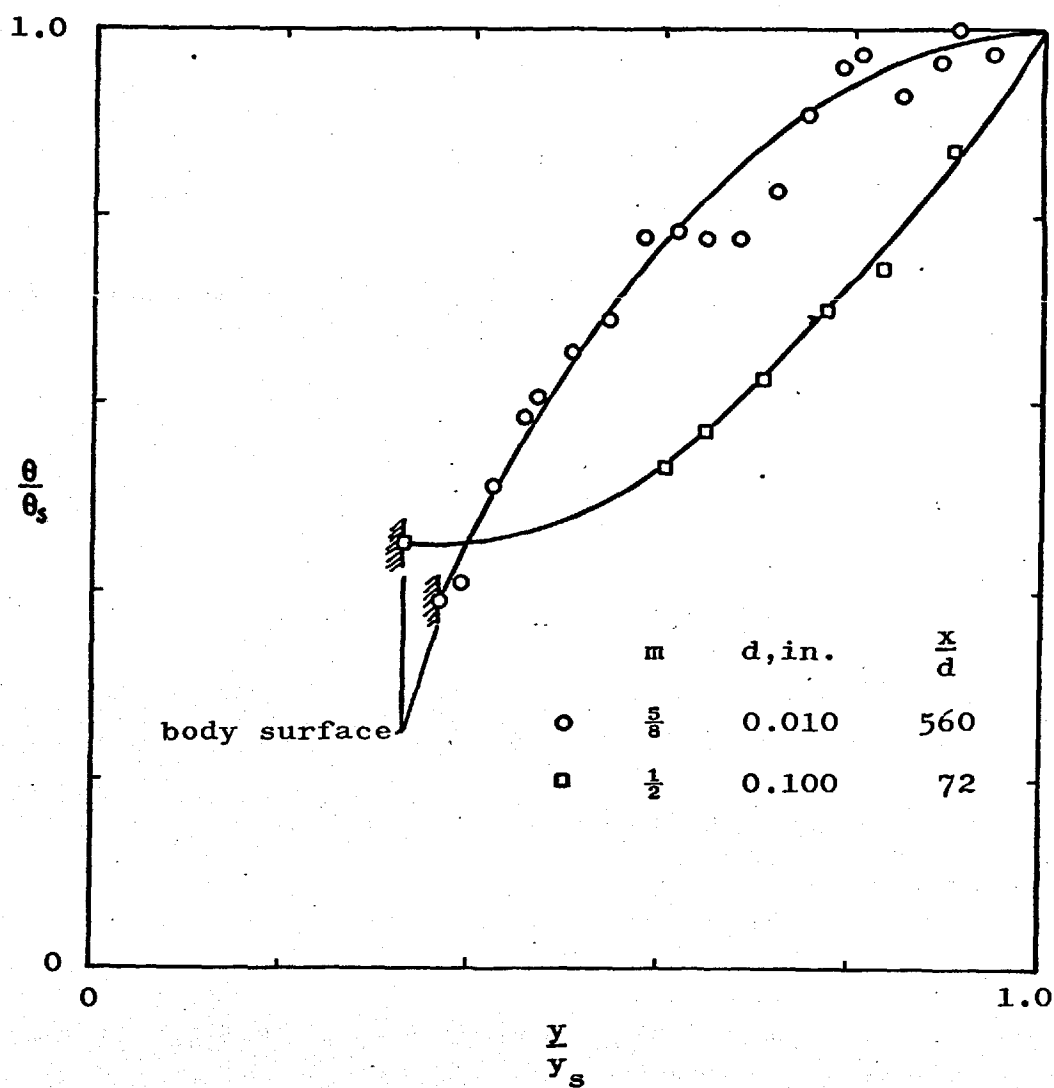


Fig.19. DISTRIBUTION OF STREAMLINE SLOPE IN SHOCK-LAYER. FROM METHOD OF WEAK WAVES.

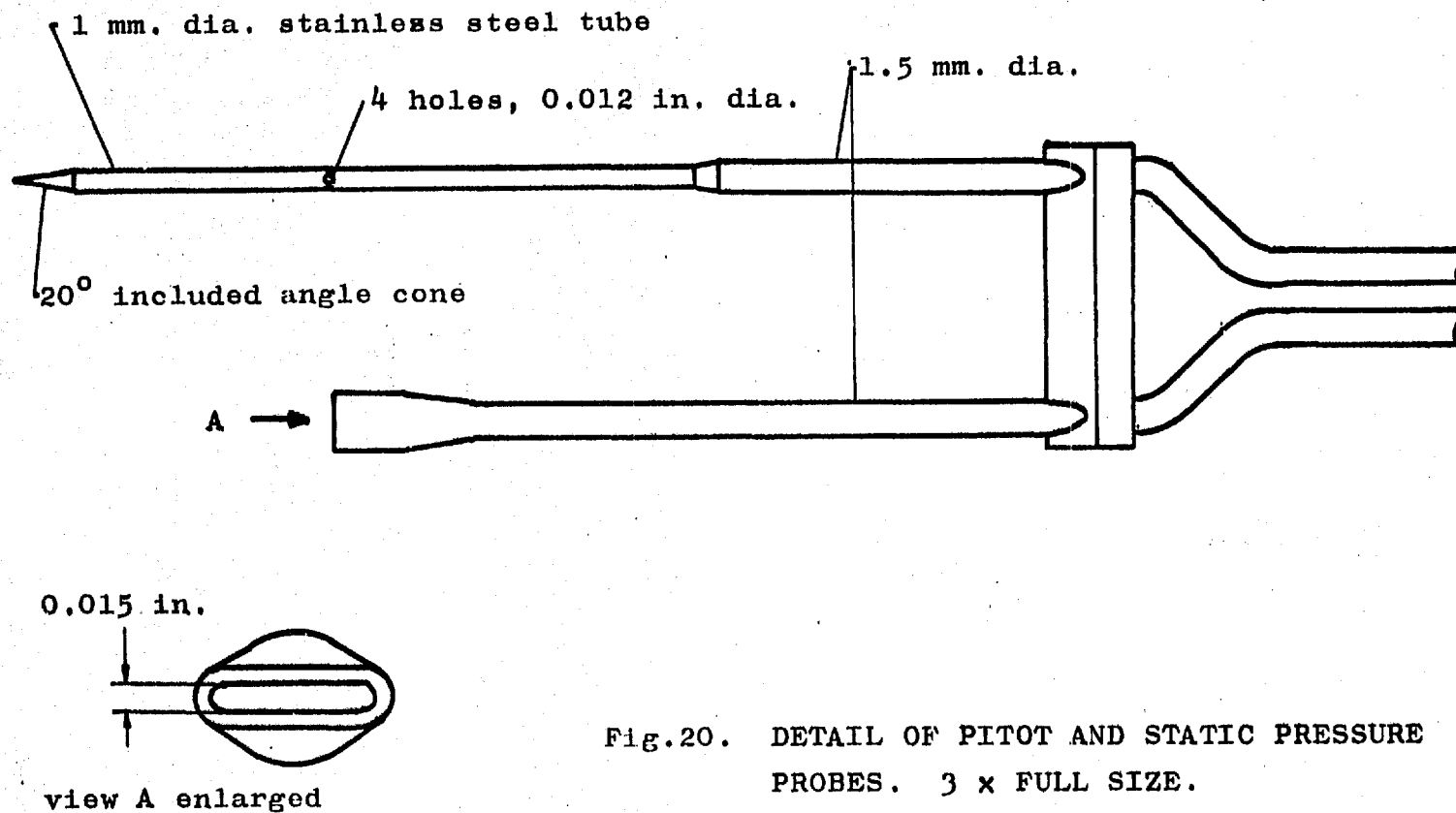


Fig.20. DETAIL OF PITOT AND STATIC PRESSURE PROBES. 3 x FULL SIZE.

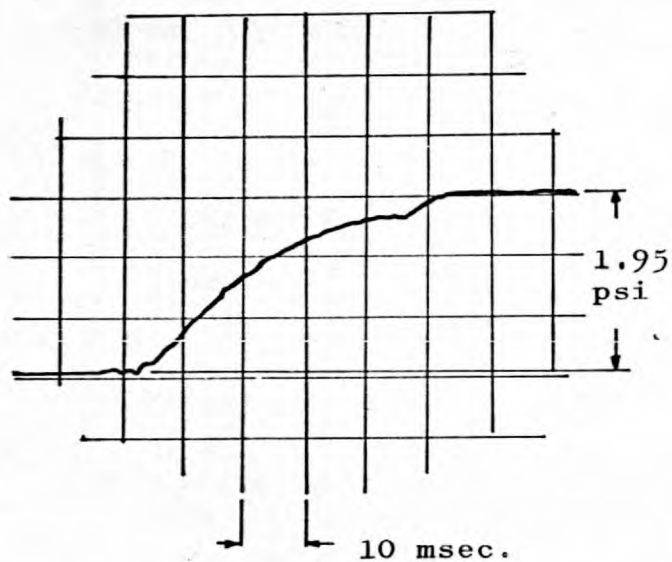


Fig.22. OSCILLOSCOPE TRACE OF STATIC PRESSURE.

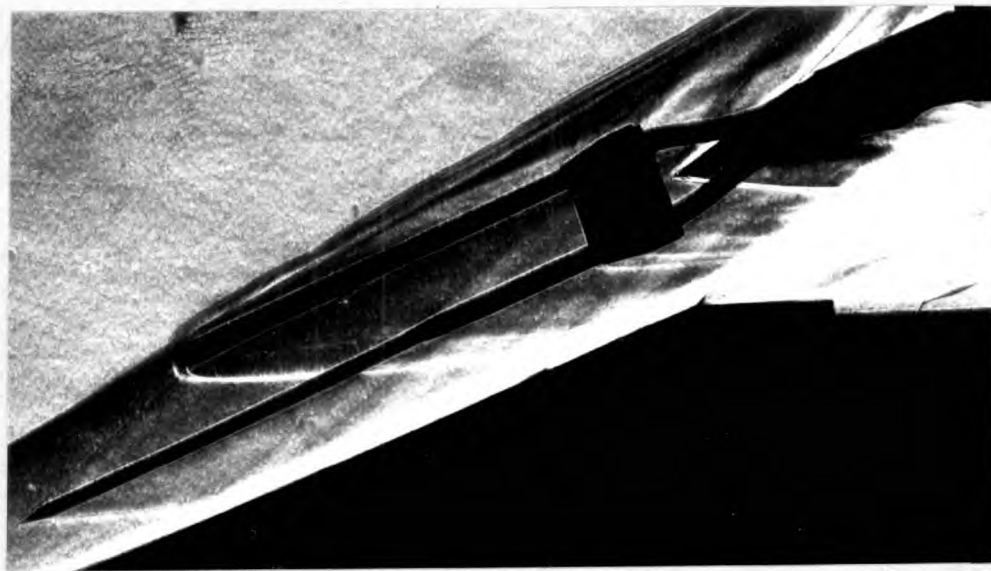


Fig.21. STATIC PRESSURE PROBE IN SHOCK-LAYER
OF 20° WEDGE.

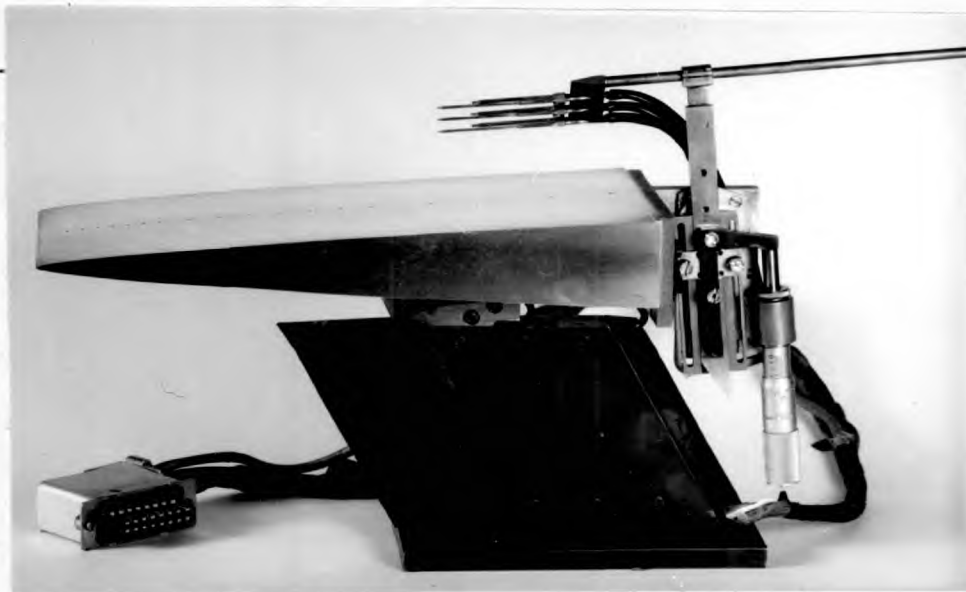


Fig.23. $m=\frac{5}{8}$ -MODEL MOUNTED ON STING.

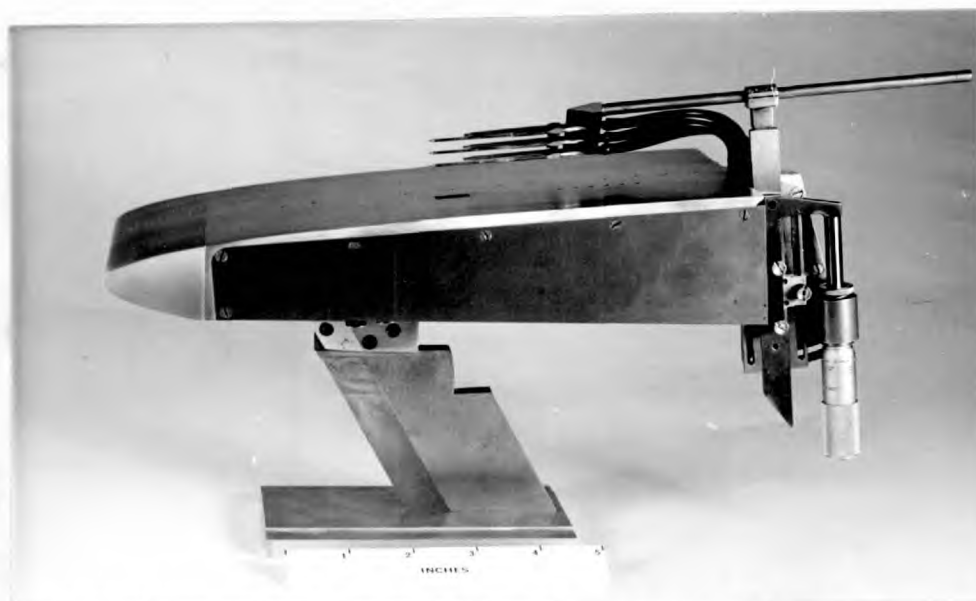


Fig.24. PARABOLA MOUNTED ON STING WITH TRAVERSE GEAR AND PROBE.

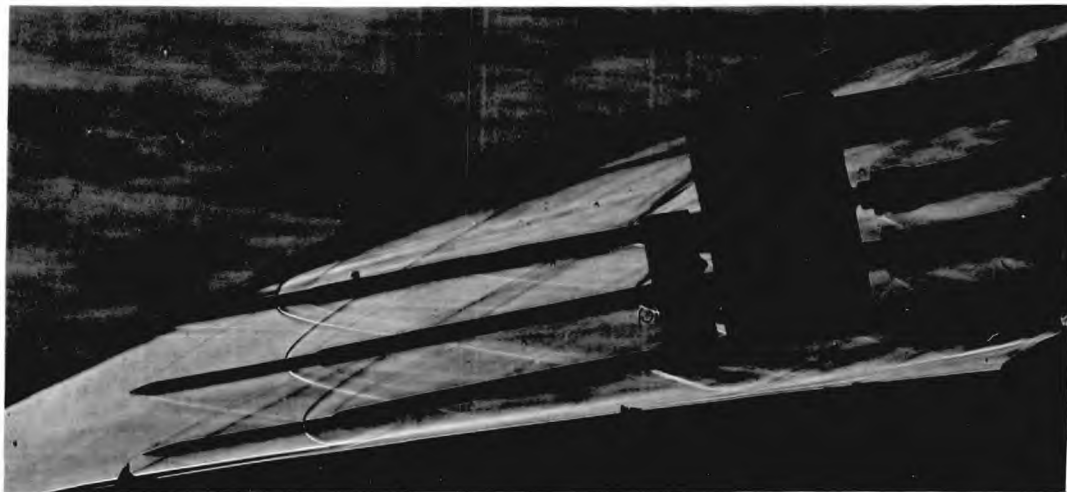


Fig.25. STATIC AND PITOT PRESSURE RAKE IN
SHOCK-LAYER OF WEDGE-PARABOLA.

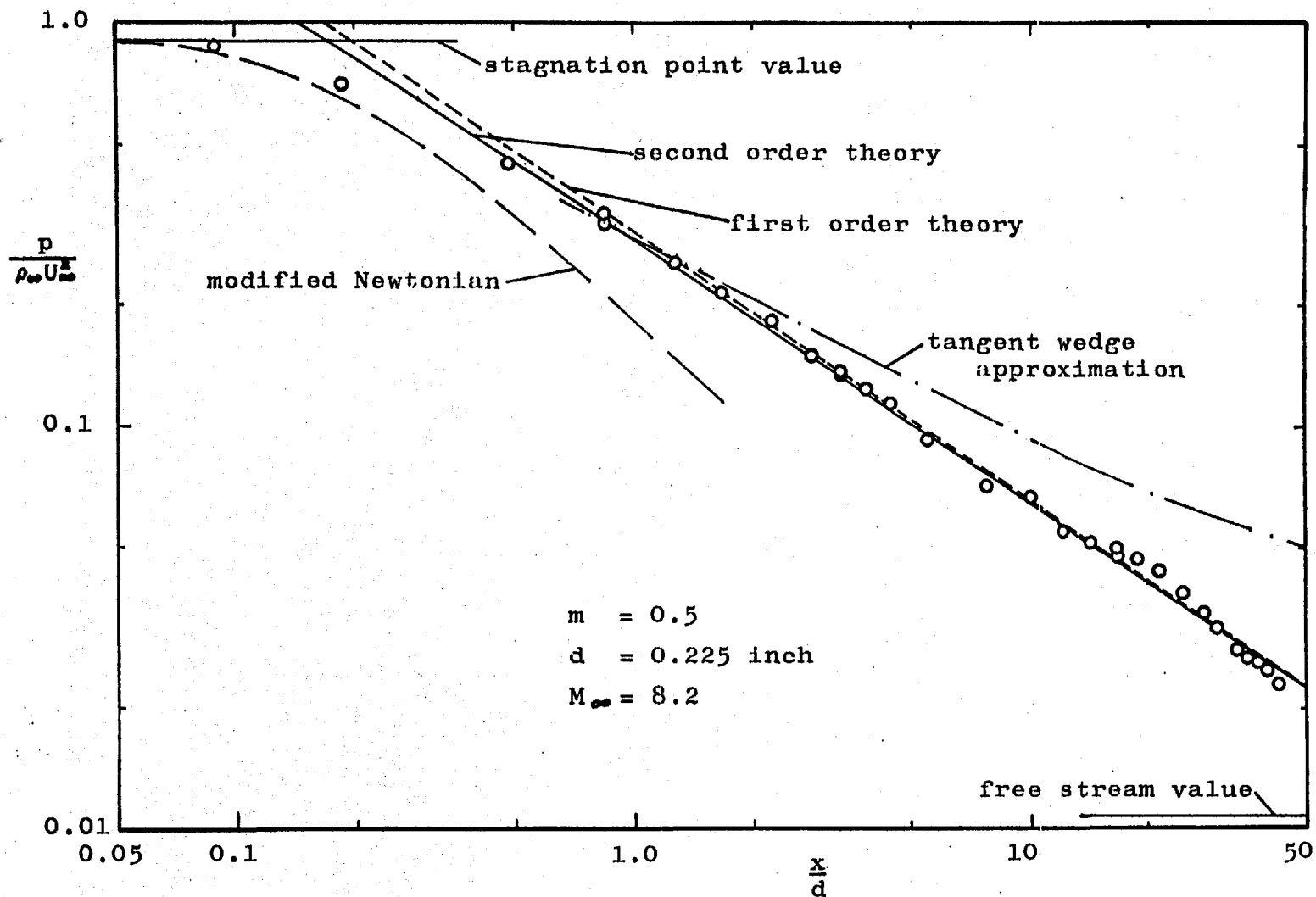


Fig. 26. SURFACE PRESSURE DISTRIBUTION ON PARABOLA.

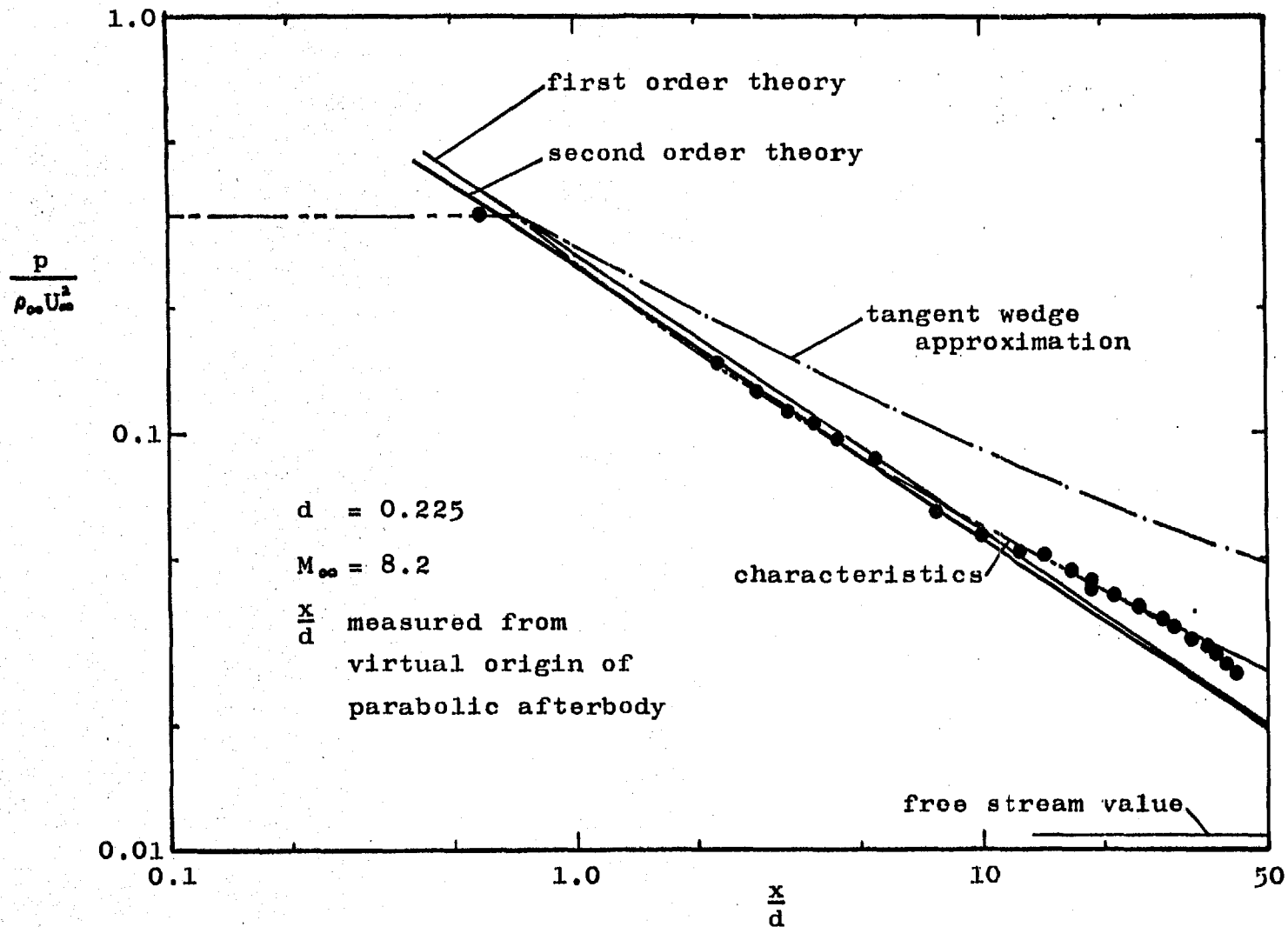


Fig.26. (cont.) SURFACE PRESSURE DISTRIBUTION. WEDGE-PARABOLA.

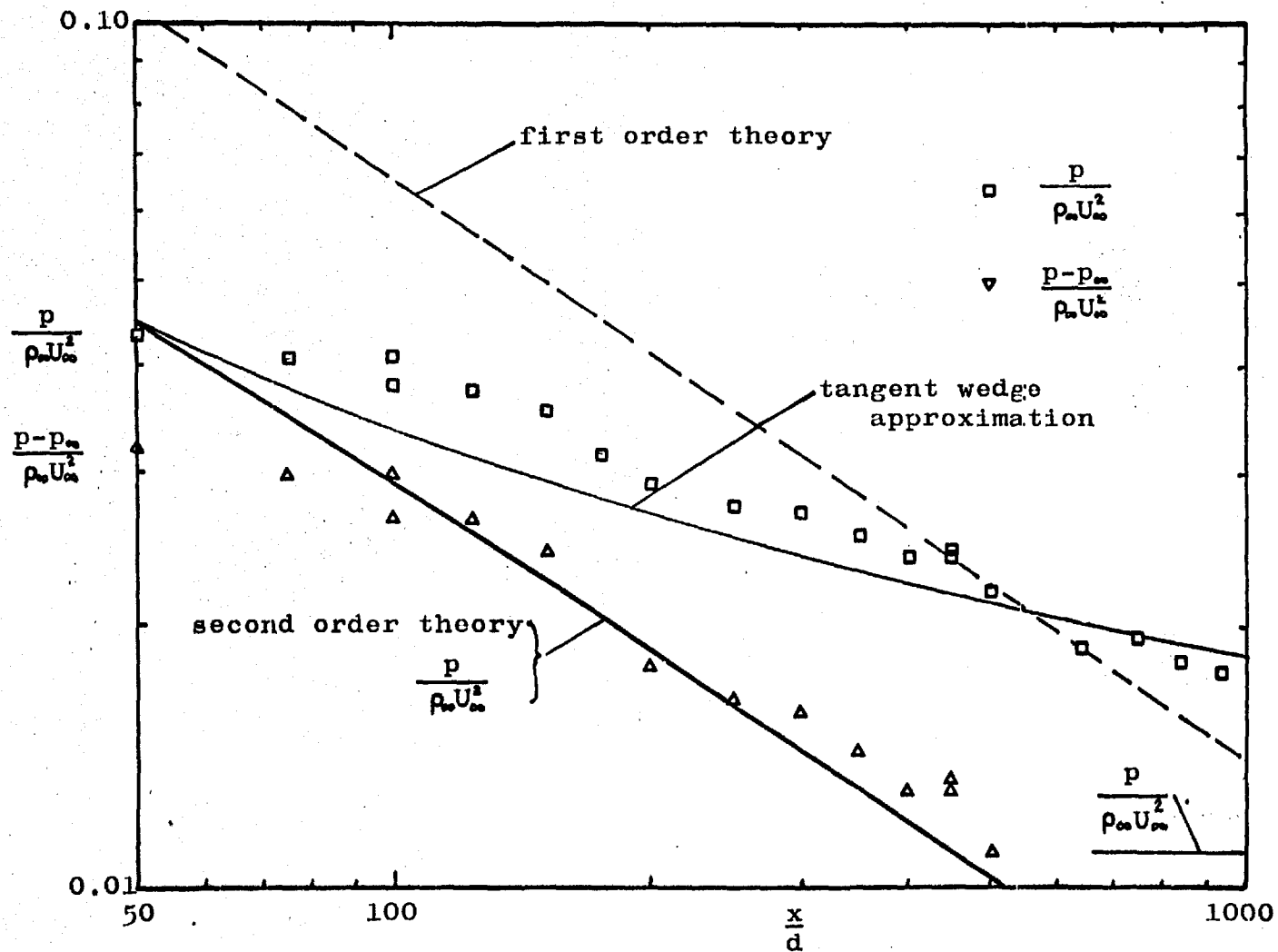


Fig.26. (concluded) SURFACE PRESSURE DISTRIBUTION. $m=\frac{5}{8}$, $d=0.010$ in.

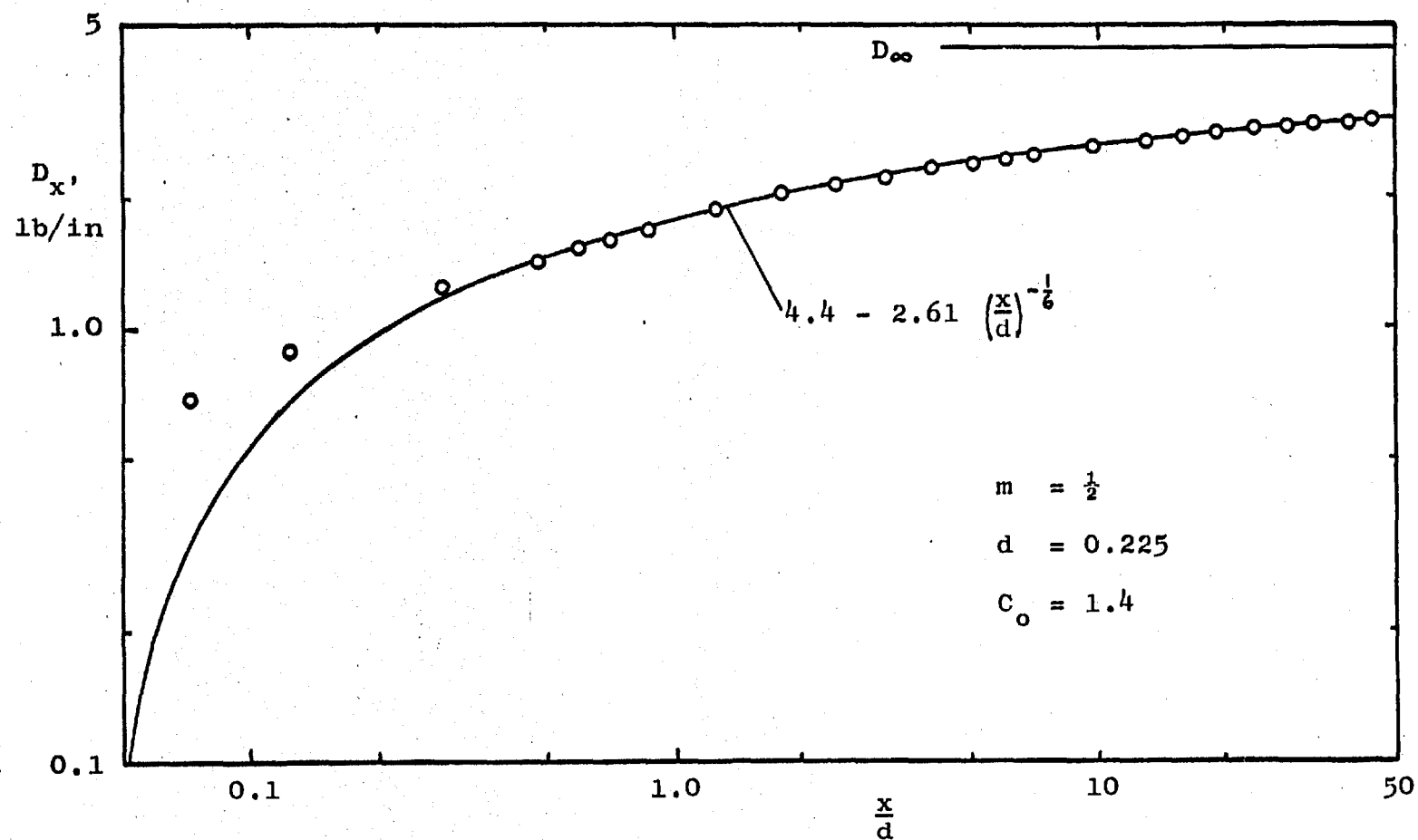


Fig.27. FIT OF $D_\infty - a\left(\frac{x}{d}\right)^{m-\frac{2}{3}}$ TO EXPERIMENTAL DRAG.

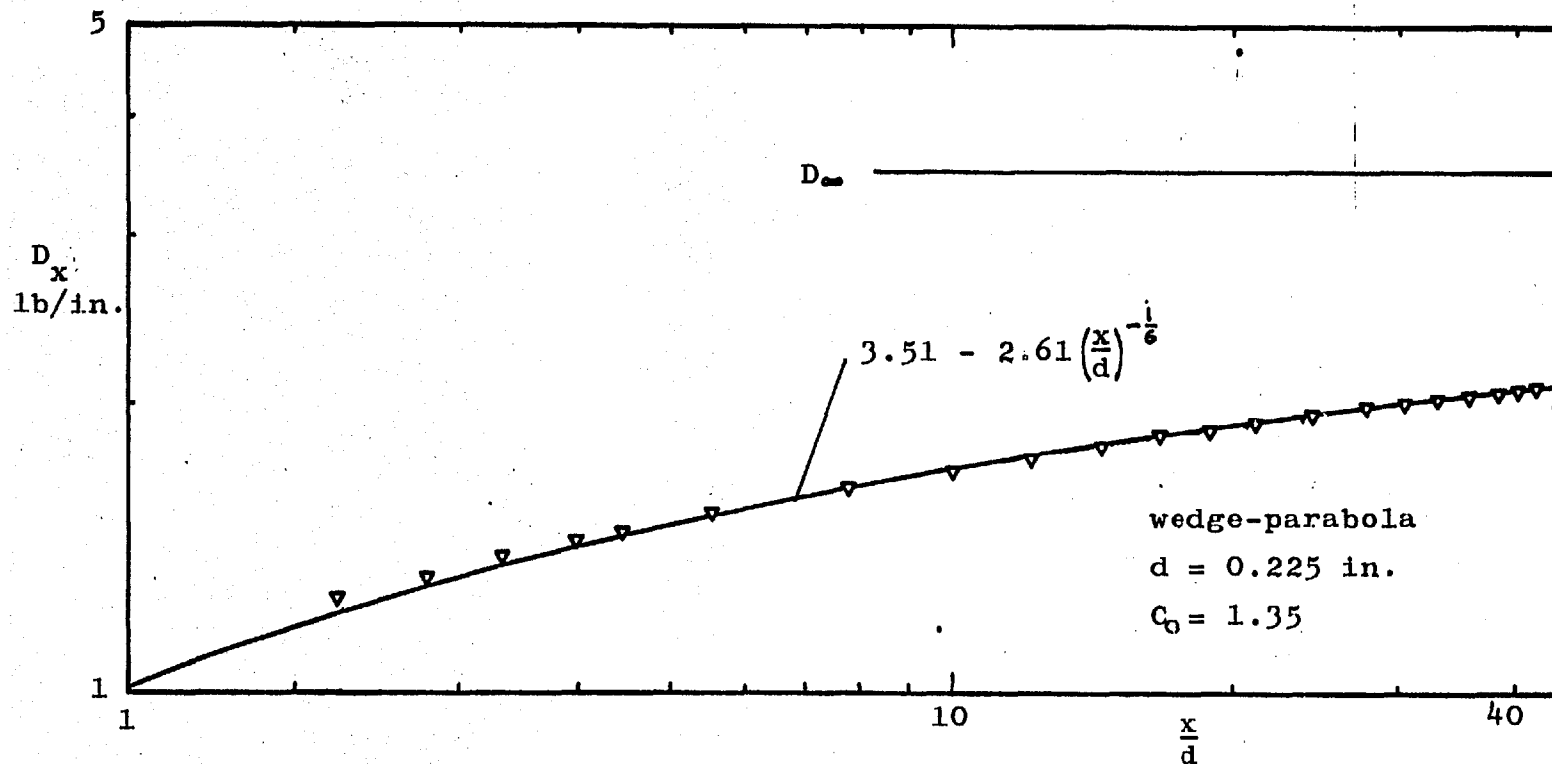


Fig.27.(cont.) FIT OF $D_\infty - a \left(\frac{x}{d} \right)^{m - \frac{2}{3}}$ TO EXPERIMENTAL DRAG.

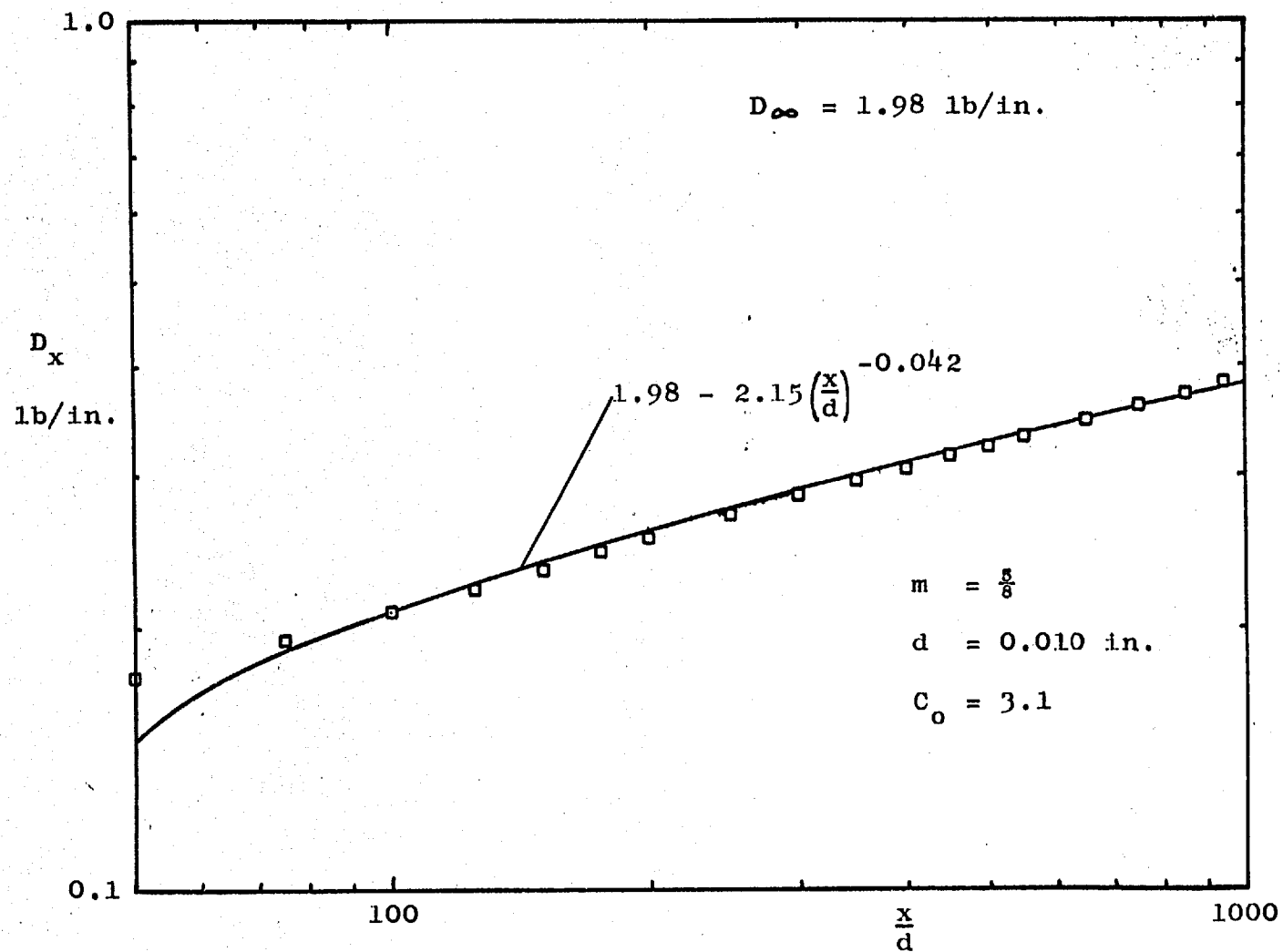


Fig.27.(concluded) FIT OF $D_\infty - a \left(\frac{x}{d} \right)^{m-\frac{2}{3}}$ TO EXPERIMENTAL DRAG.

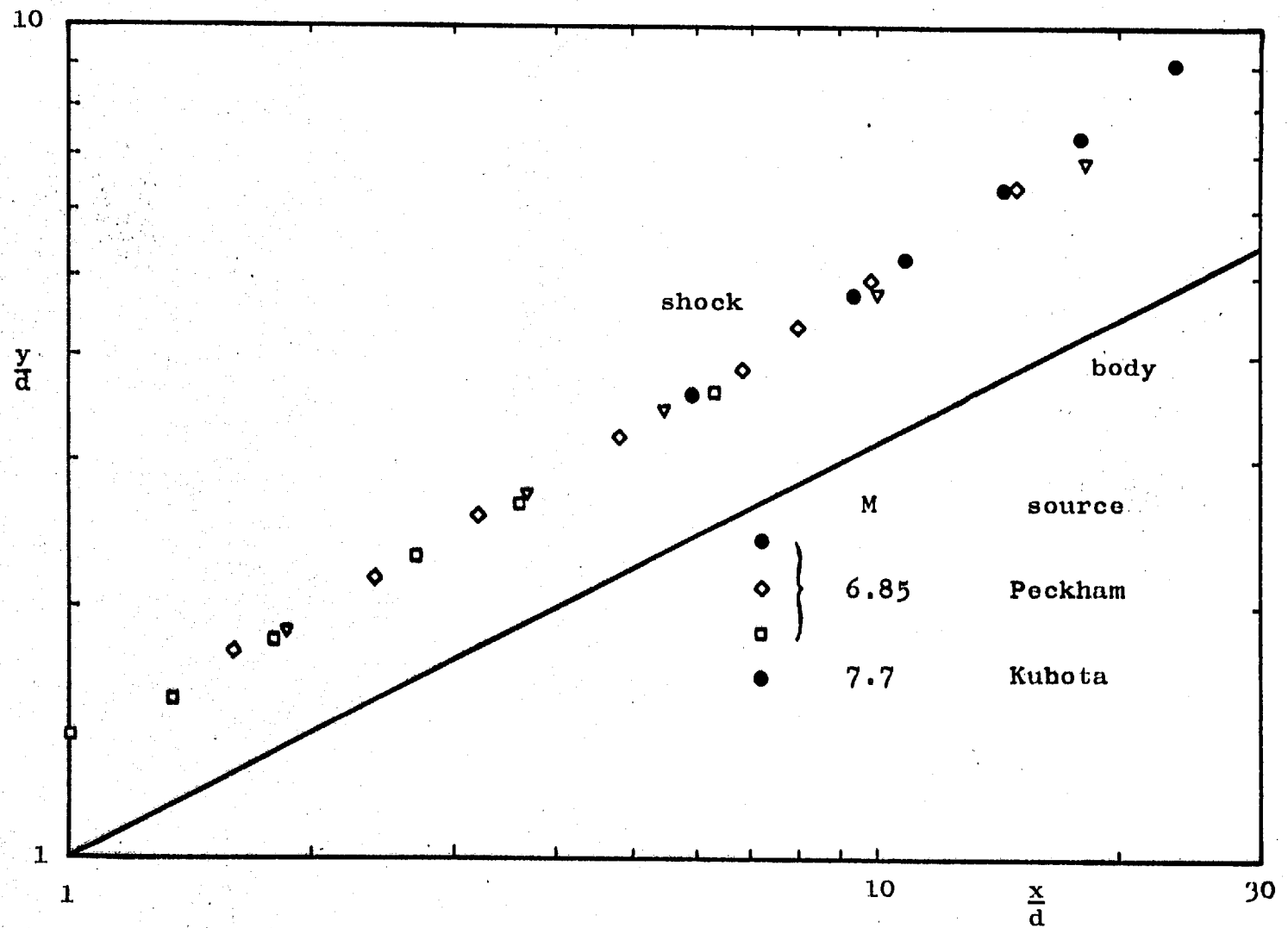


Fig.28. CORRELATION OF SHOCK WAVE SHAPE ON AXISYMMETRIC PARABOLA.

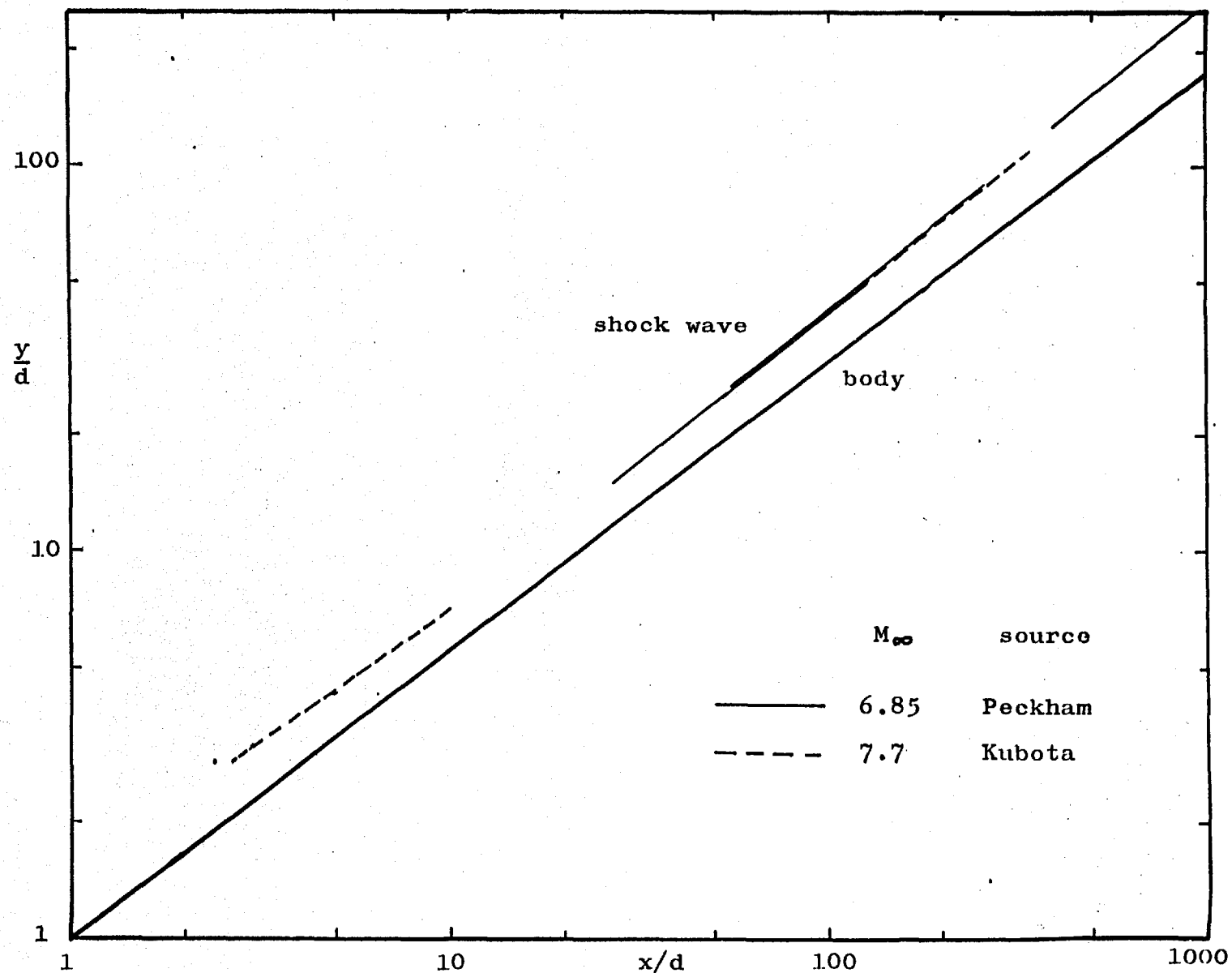


Fig.28.(cont.) CORRELATION OF SHOCK SHAPES ON AXISYMMETRIC BODIES, $m=\frac{3}{4}$.

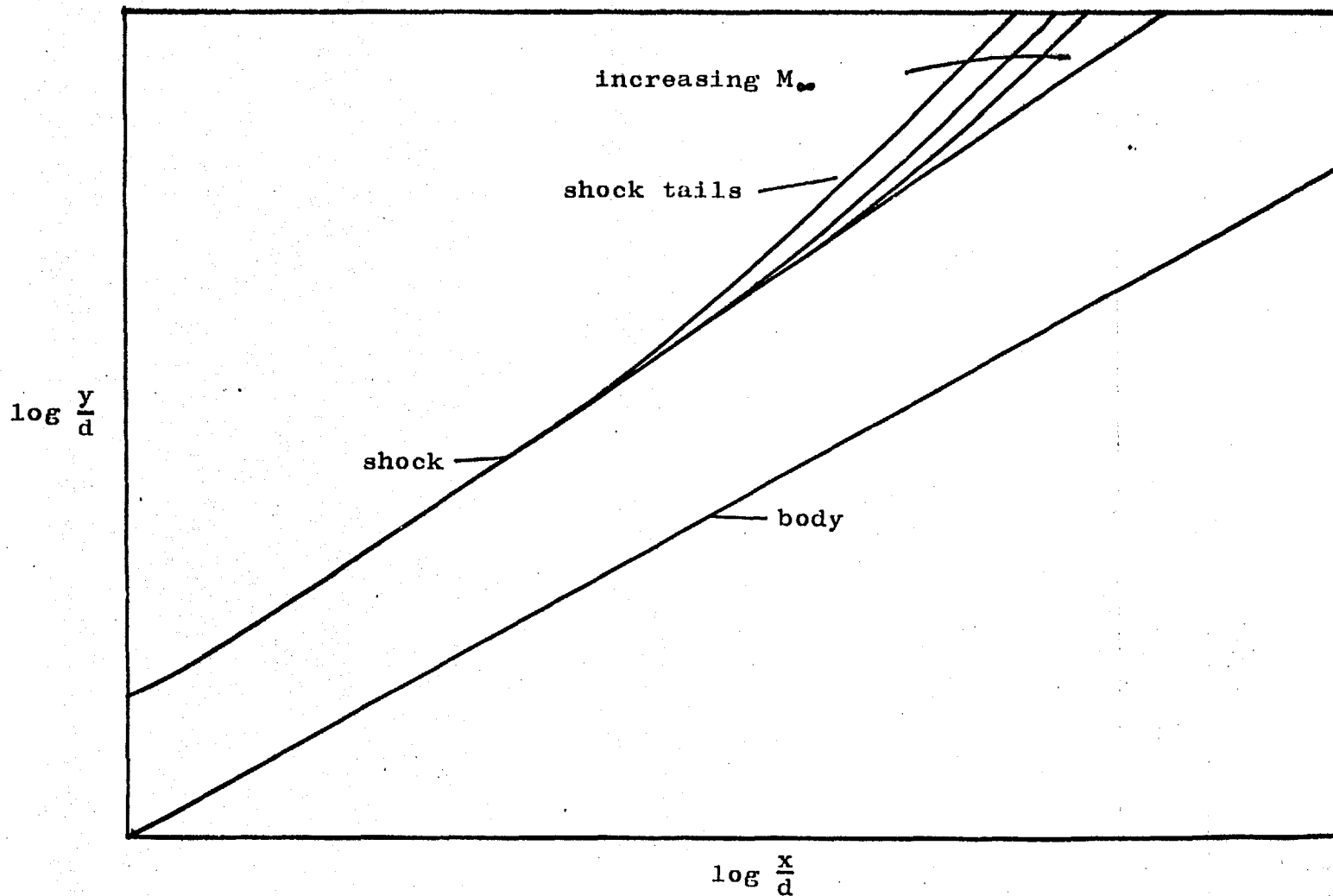


Fig.29. SKETCH SHOWING WEAKENING OF SHOCK WAVE AT LARGE $\frac{x}{d}$ AND FINITE M_∞ .

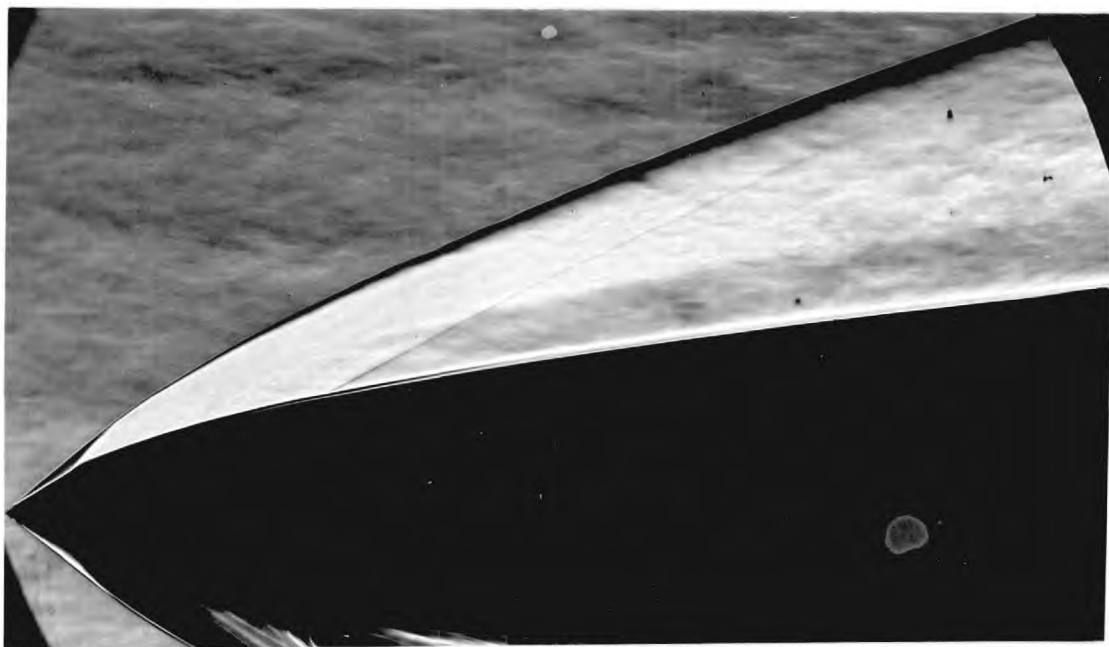


Parabola, $d = 0.225$ in.

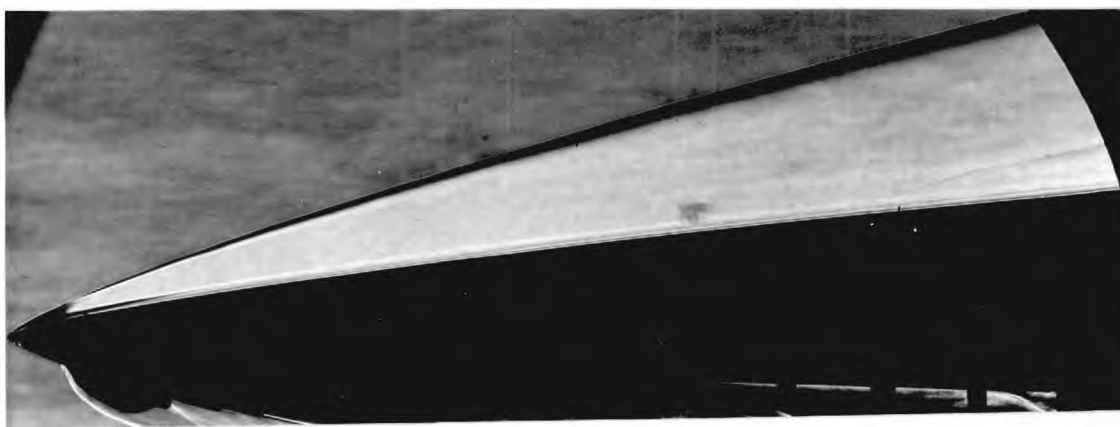


Parabola, $d = 0.100$ in.

Fig.30a. SCHLIEREN PHOTOGRAPHS. $M_{\infty} = 8.2$.



Wedge-parabola



$m = 0.625$

Fig.30a. (cont.) SCHLIEREN PHOTOGRAPHS. $M_{\infty} = 8.2$.

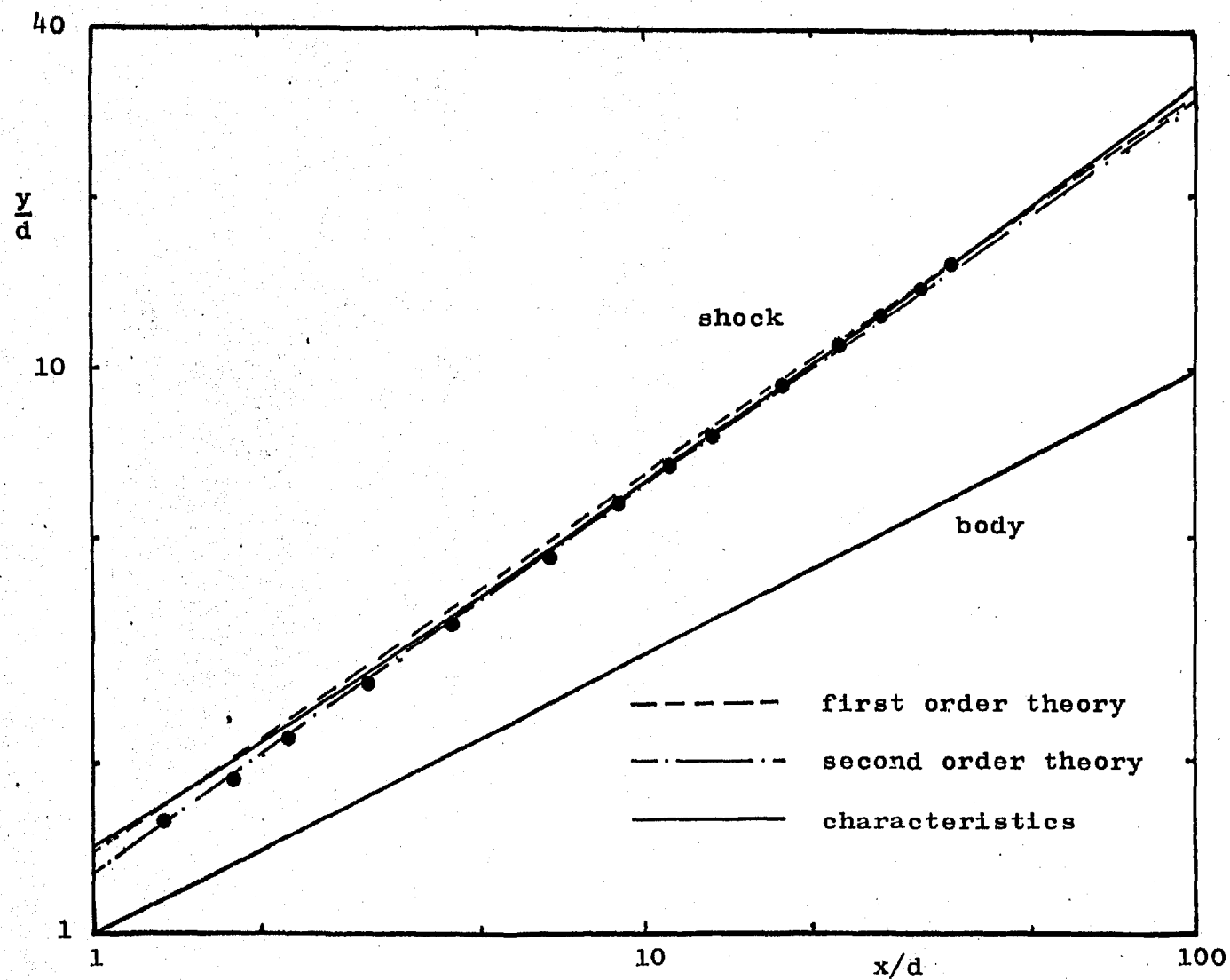


Fig.30b. SHOCK WAVE SHAPE ON WEDGE-PARABOLA.

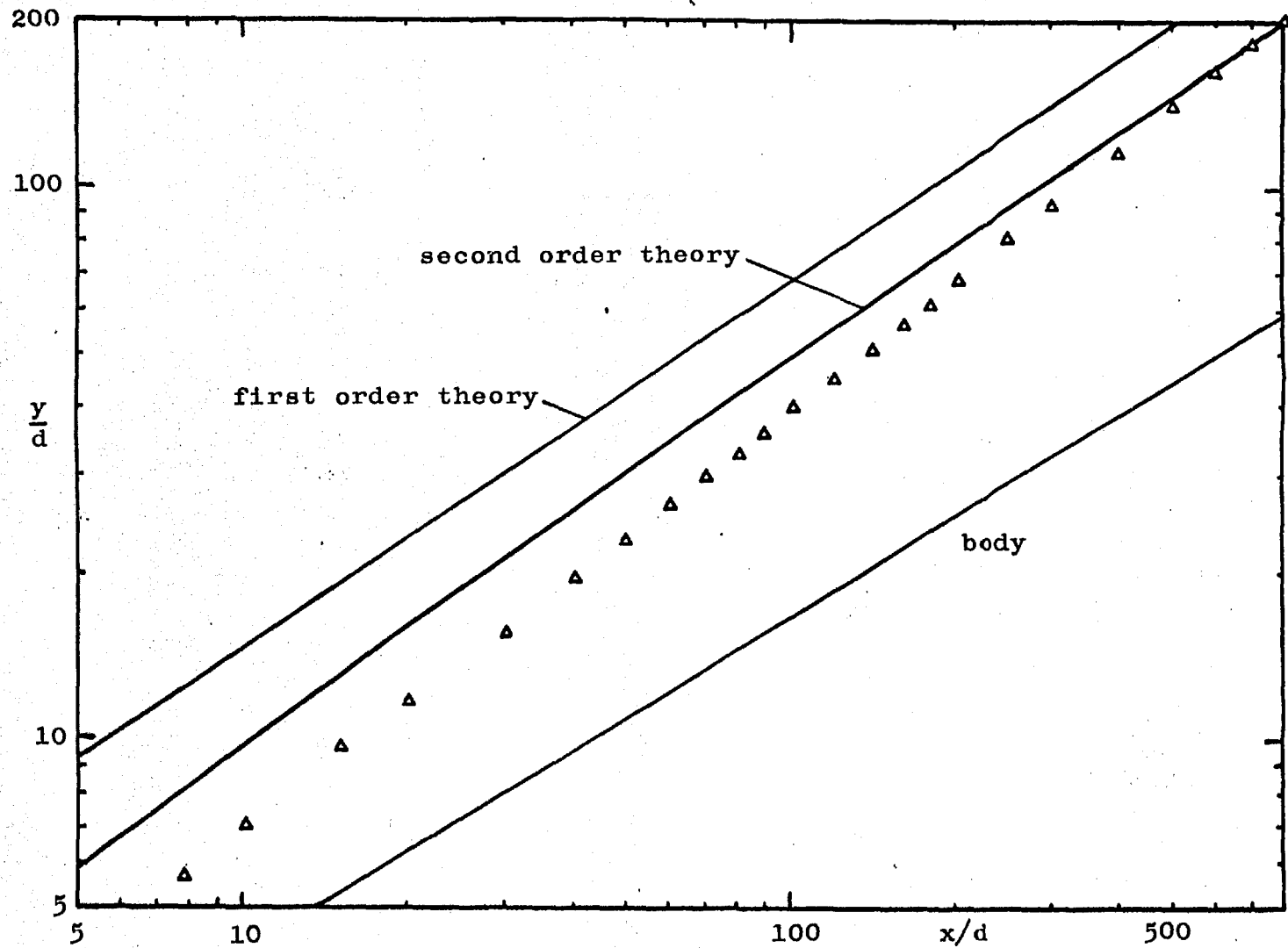


Fig.30b.(cont.) SHOCK WAVE SHAPE COMPARED WITH THEORY, $m = \frac{5}{8}$.

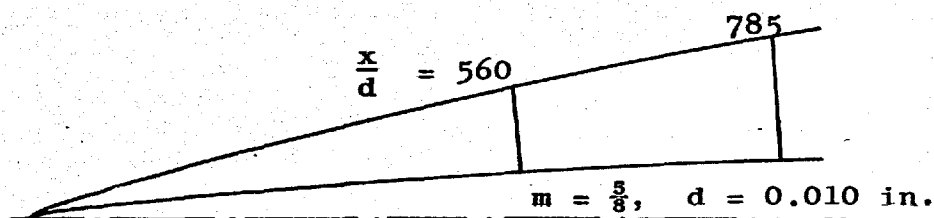
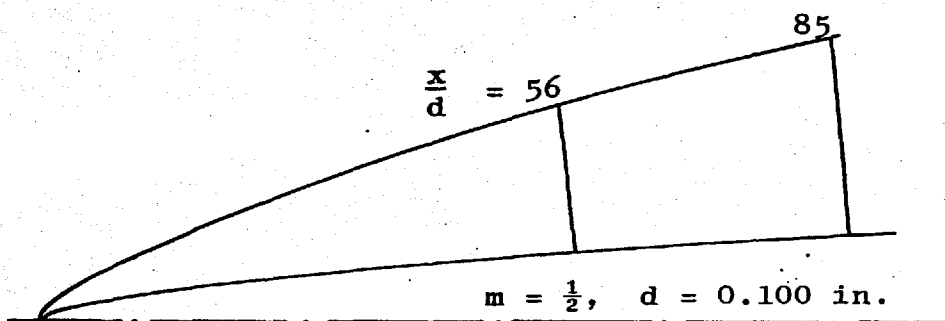
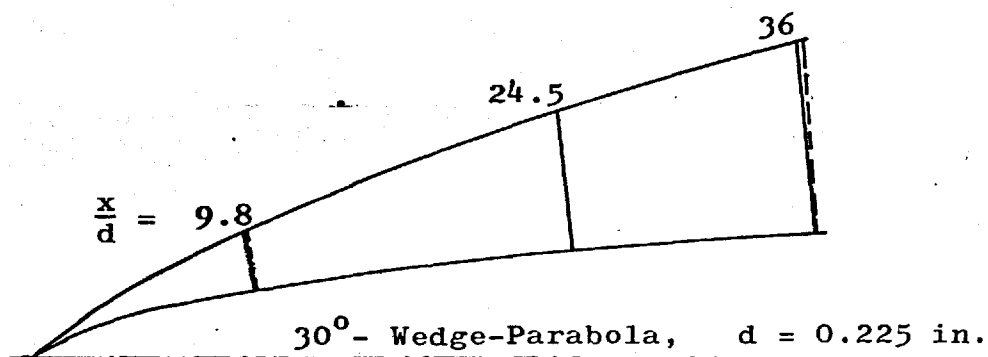
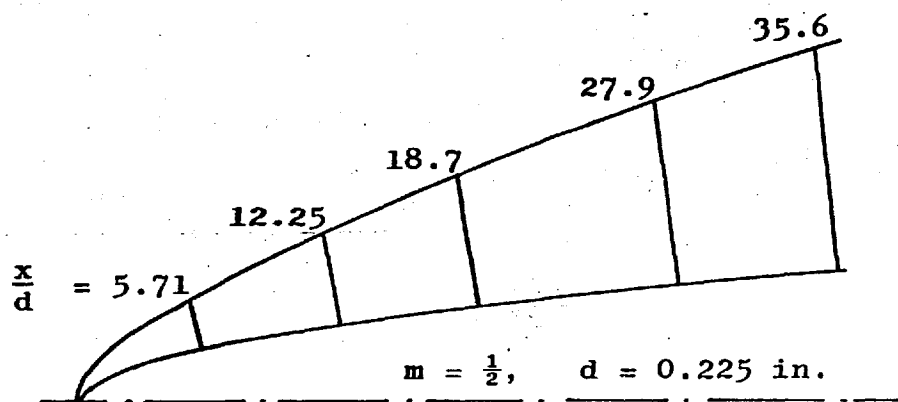


Fig.31. TRAVERSE STATIONS. ——— EXPERIMENTS,
 ----- CHARACTERISTICS.

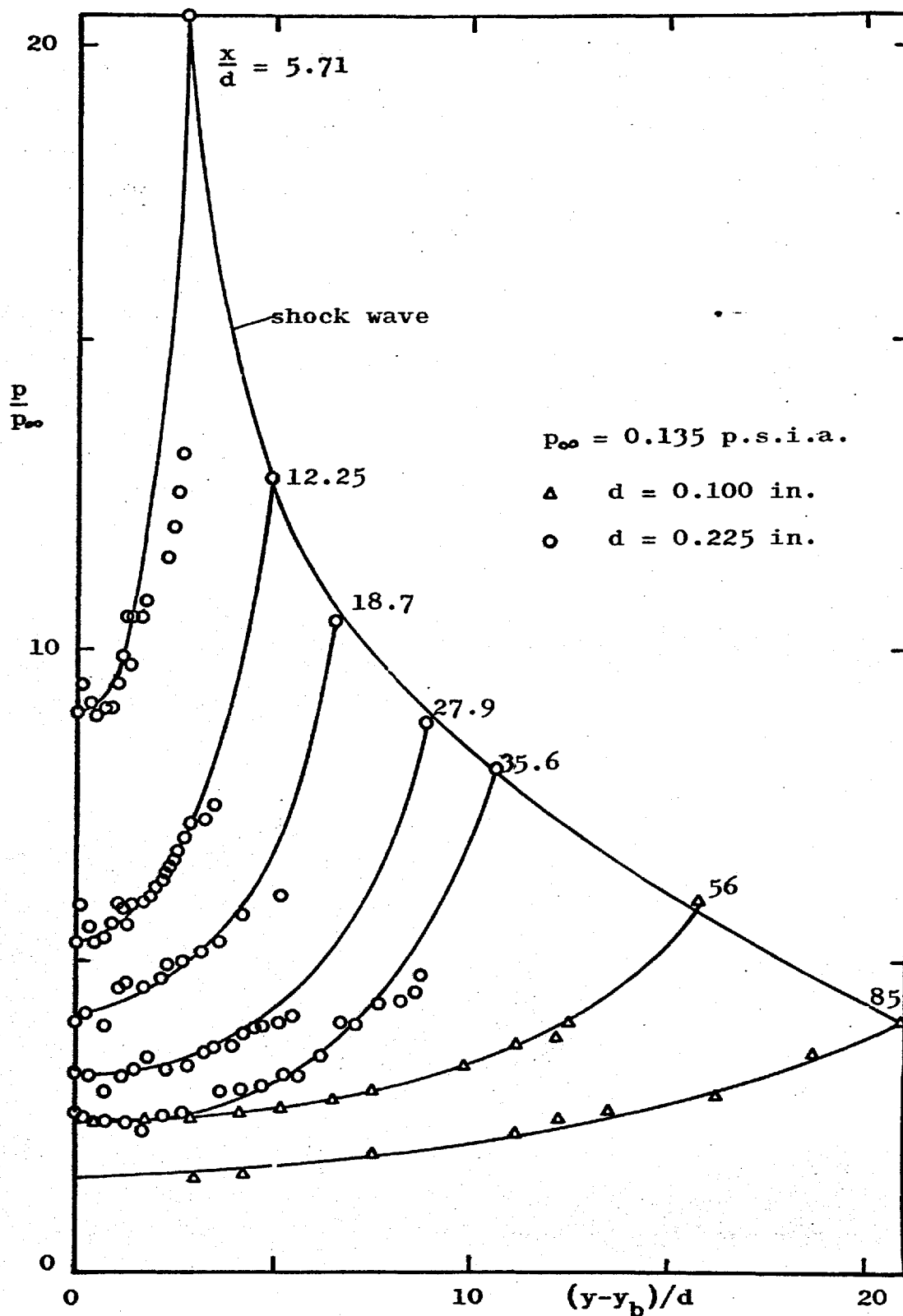


Fig.32. STATIC PRESSURE DISTRIBUTION THROUGH SHOCK-LAYER, $m=\frac{1}{2}$.

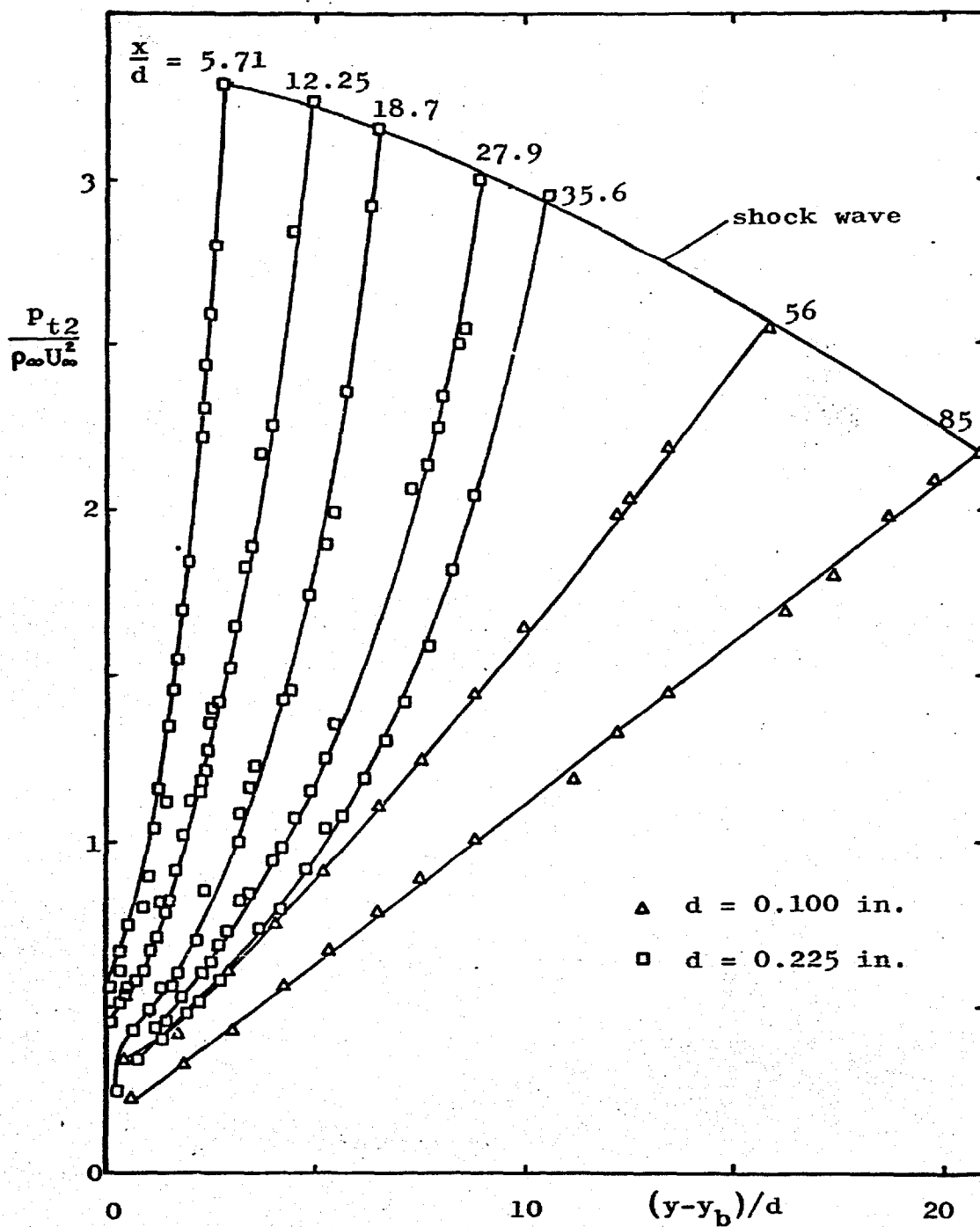


Fig.32.(cont.) EXPERIMENTAL PITOT PRESSURE DISTRIBUTION
THROUGH SHOCK-LAYER, $m = 0.5$.

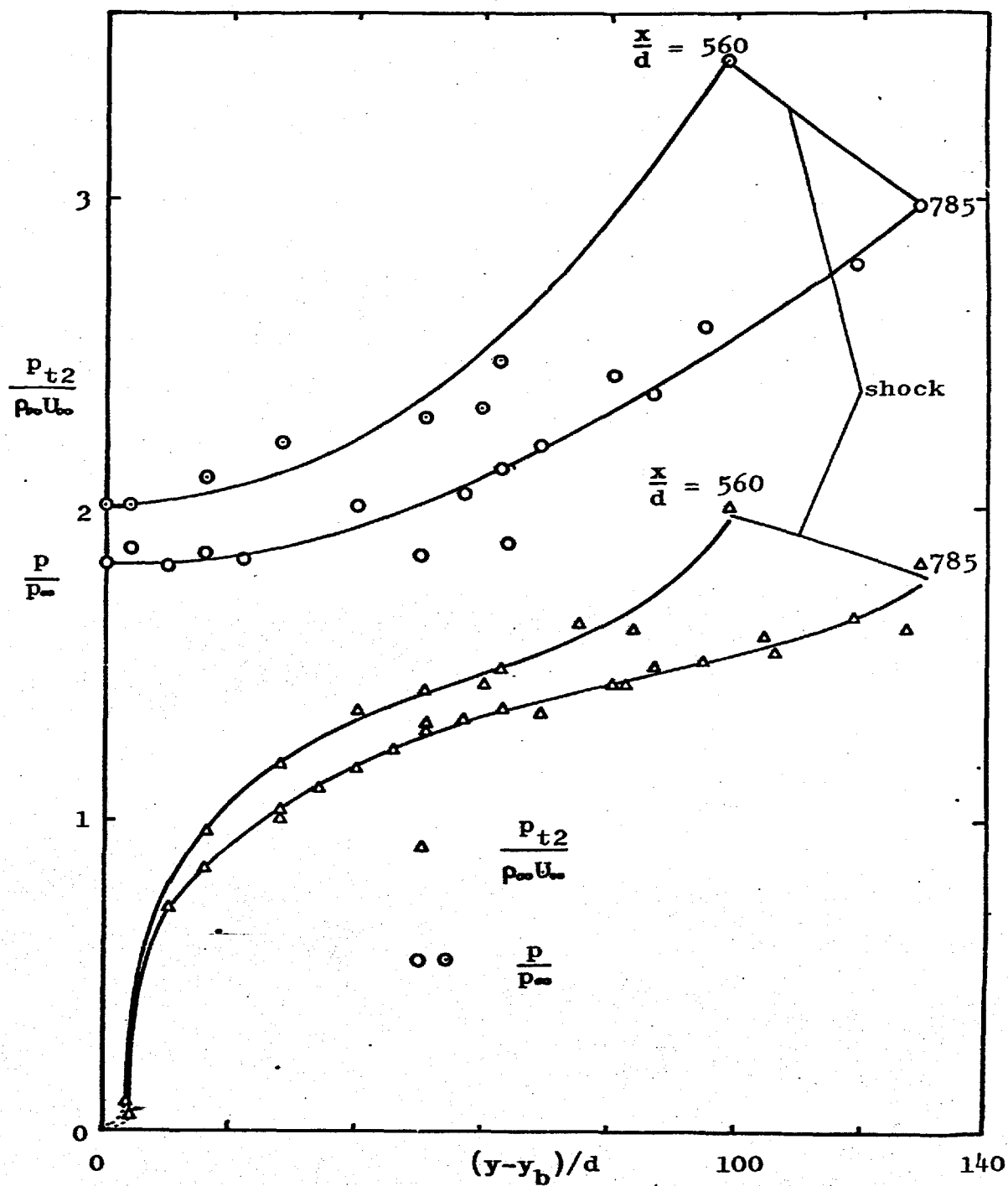


Fig.32.(contl.) EXPERIMENTAL PITOT AND STATIC PRESSURE DISTRIBUTION THROUGH SHOCK-LAYER, $m = \frac{5}{8}$.

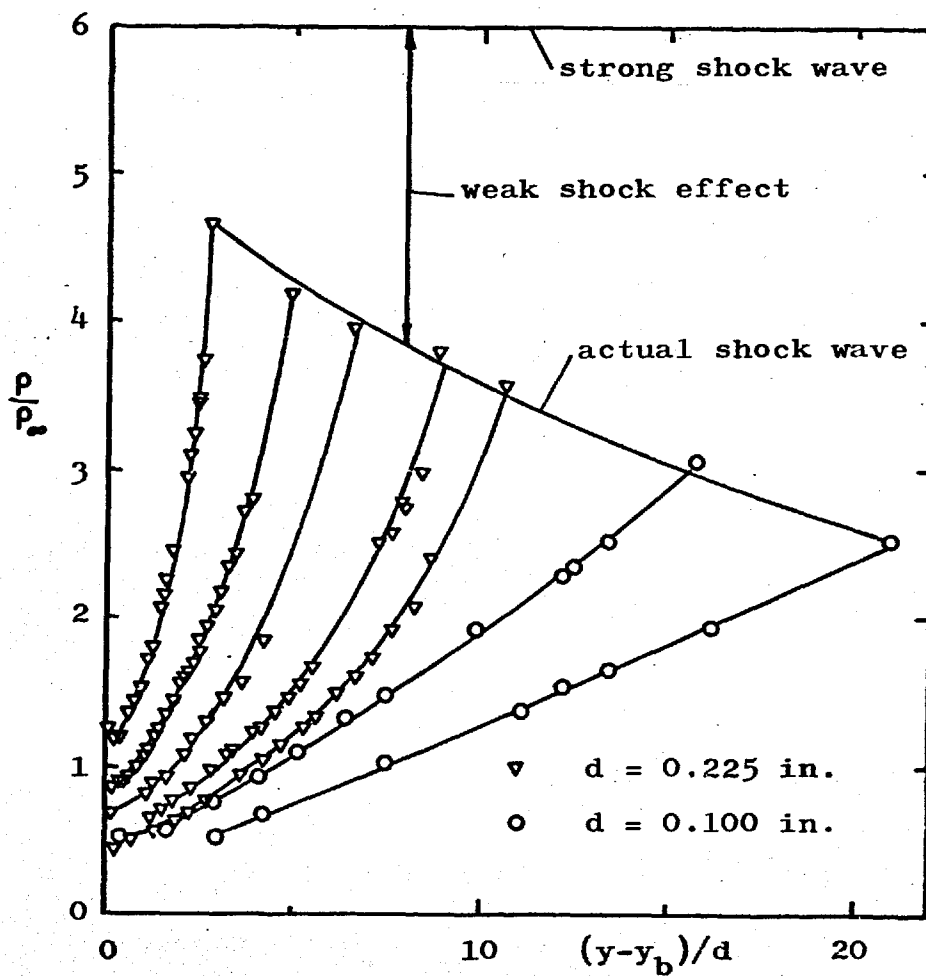


Fig.33a. EXPERIMENTAL DENSITY DISTRIBUTION
THROUGH SHOCK-LAYER, $m = \frac{1}{2}$.

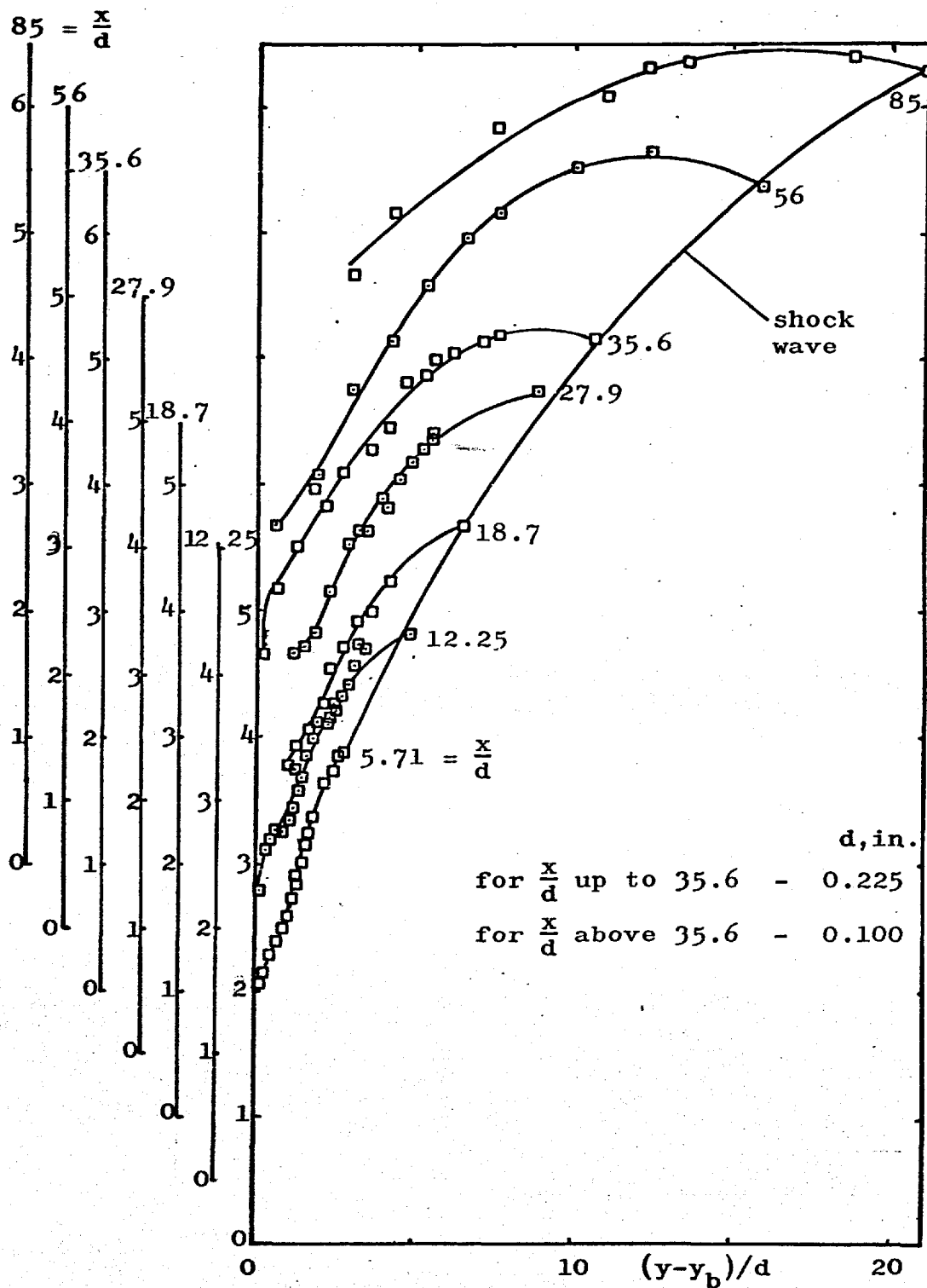


Fig.33a. (cont.) EXPERIMENTAL MACH NUMBER DISTRIBUTION THROUGH SHOCK-LAYER, $m = \frac{1}{2}$.

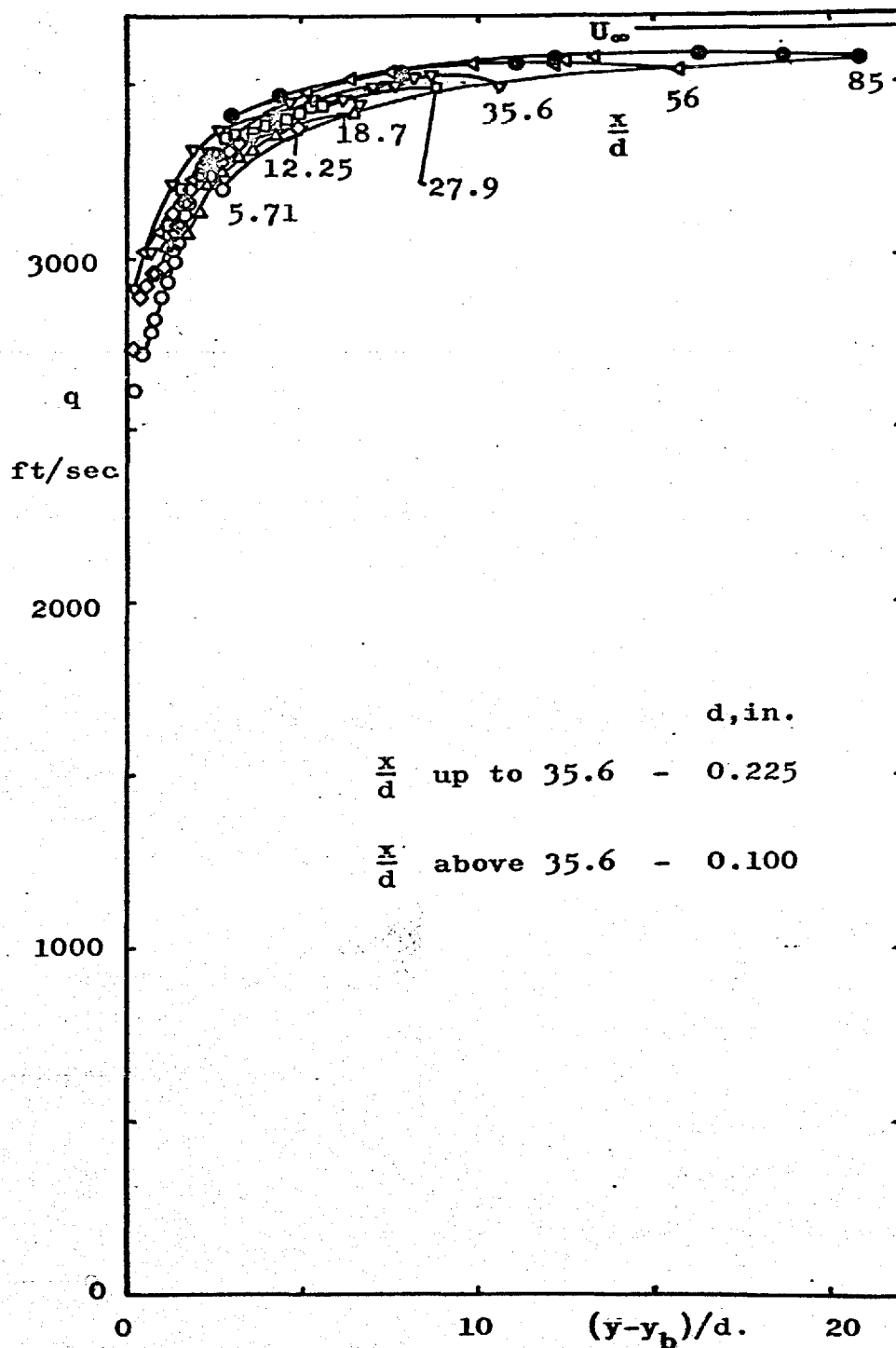


Fig. 33a. (cont.) EXPERIMENTAL DISTRIBUTION OF VELOCITY THROUGH SHOCK-LAYER, $m = \frac{1}{2}$.

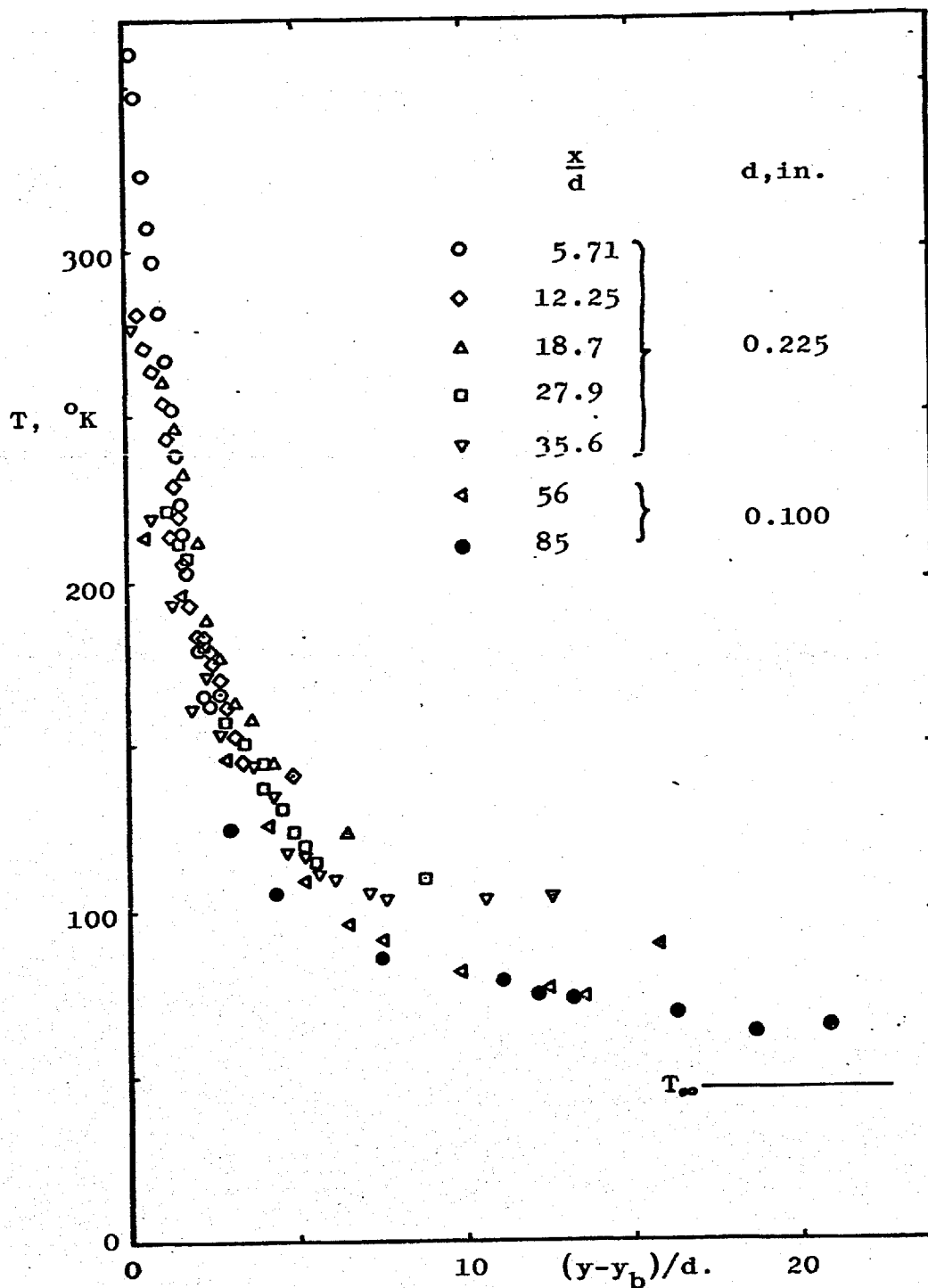


Fig.33a. (contl) EXPERIMENTAL TEMPERATURE DISTRIBUTION THROUGH SHOCK-LAYER, $m = \frac{1}{2}$.

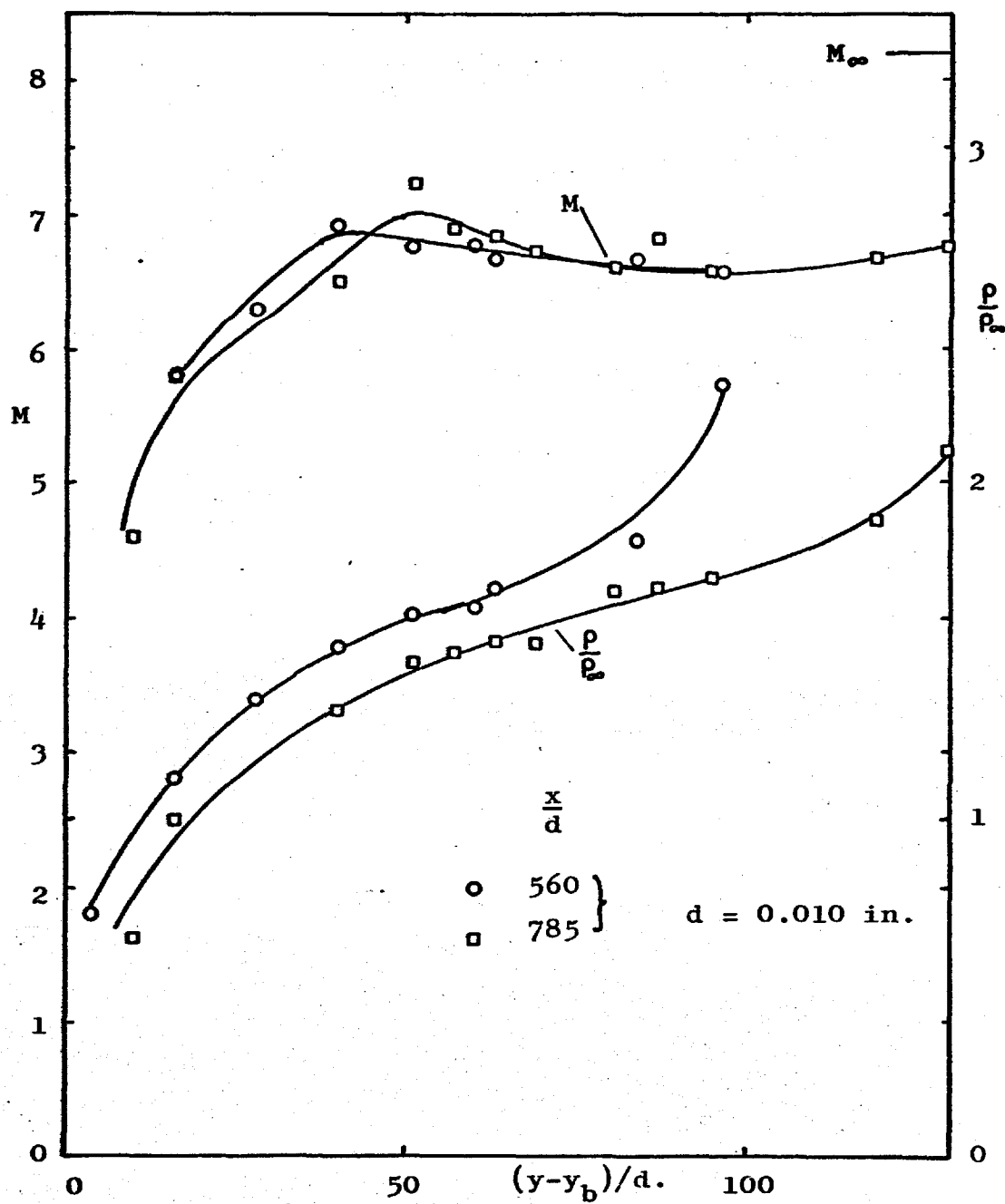


Fig.33b. EXPERIMENTAL MACH NUMBER AND DENSITY DISTRIBUTIONS THROUGH SHOCK-LAYER, $m = \frac{5}{8}$.

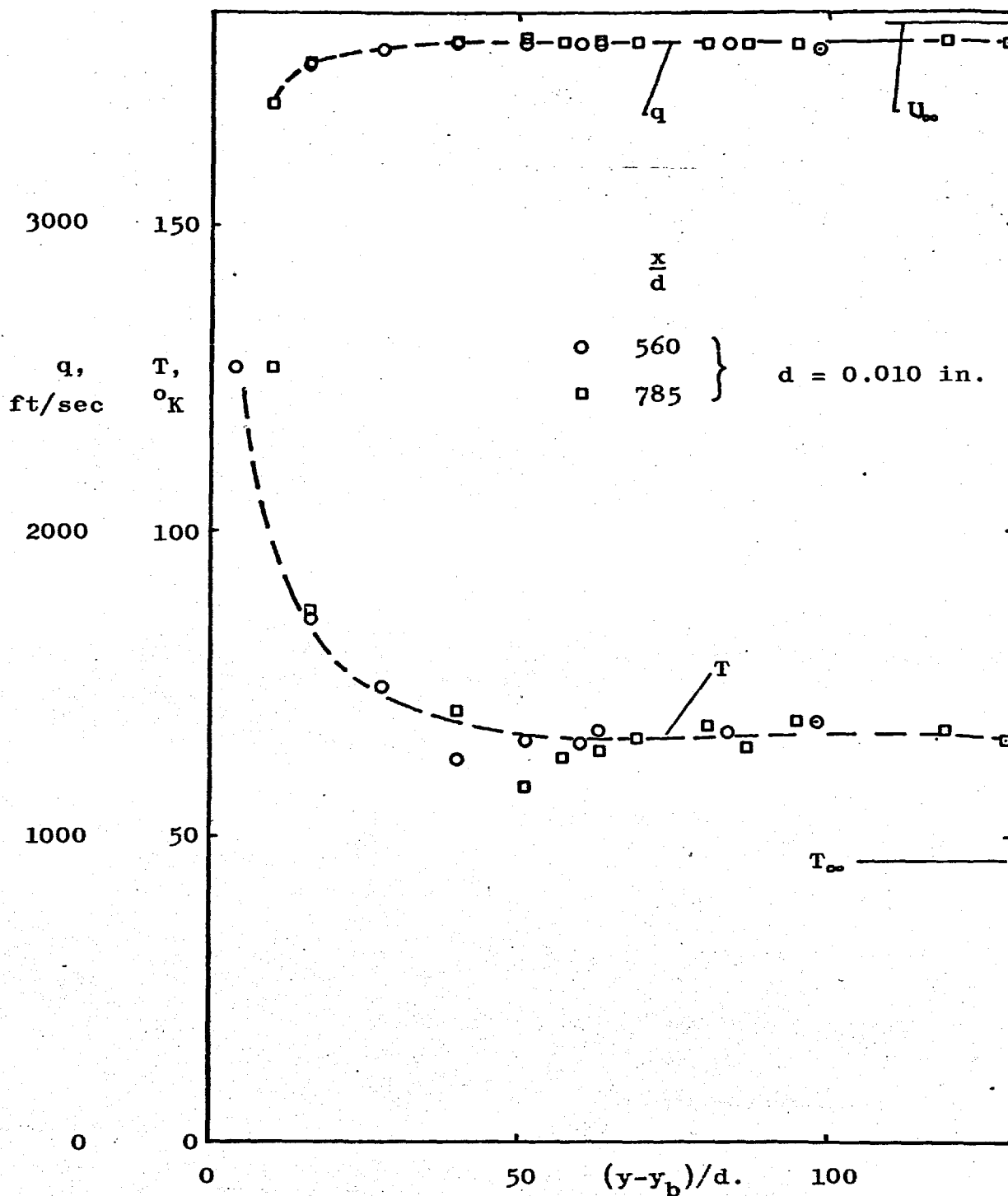


Fig.33b. (cont.) EXPERIMENTAL TEMPERATURE AND VELOCITY DISTRIBUTION THROUGH SHOCK-LAYER, $m = \frac{5}{8}$.

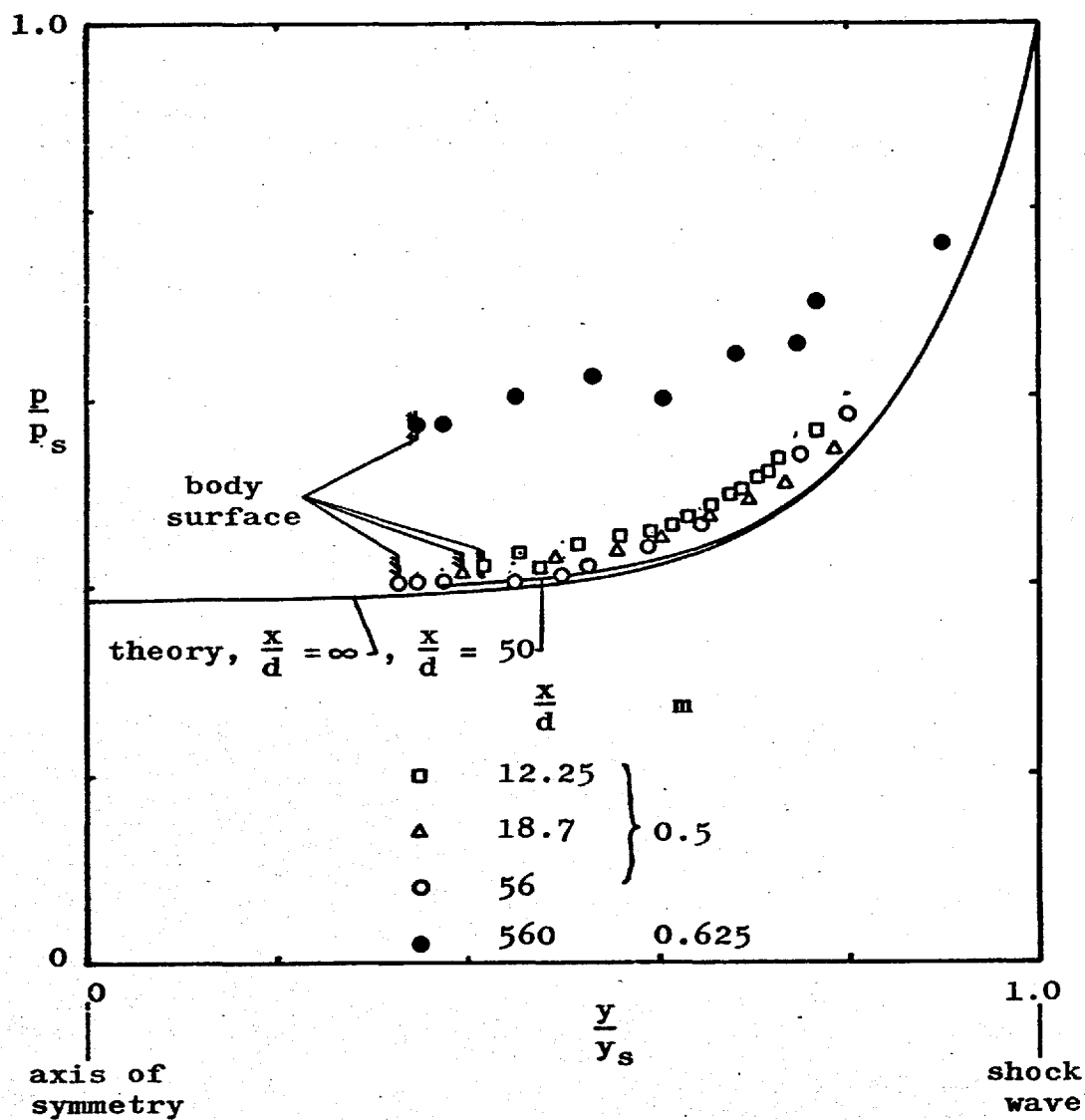


Fig.34a. PRESSURE DISTRIBUTION THROUGH SHOCK-LAYER.
COMPARISON WITH THEORY.

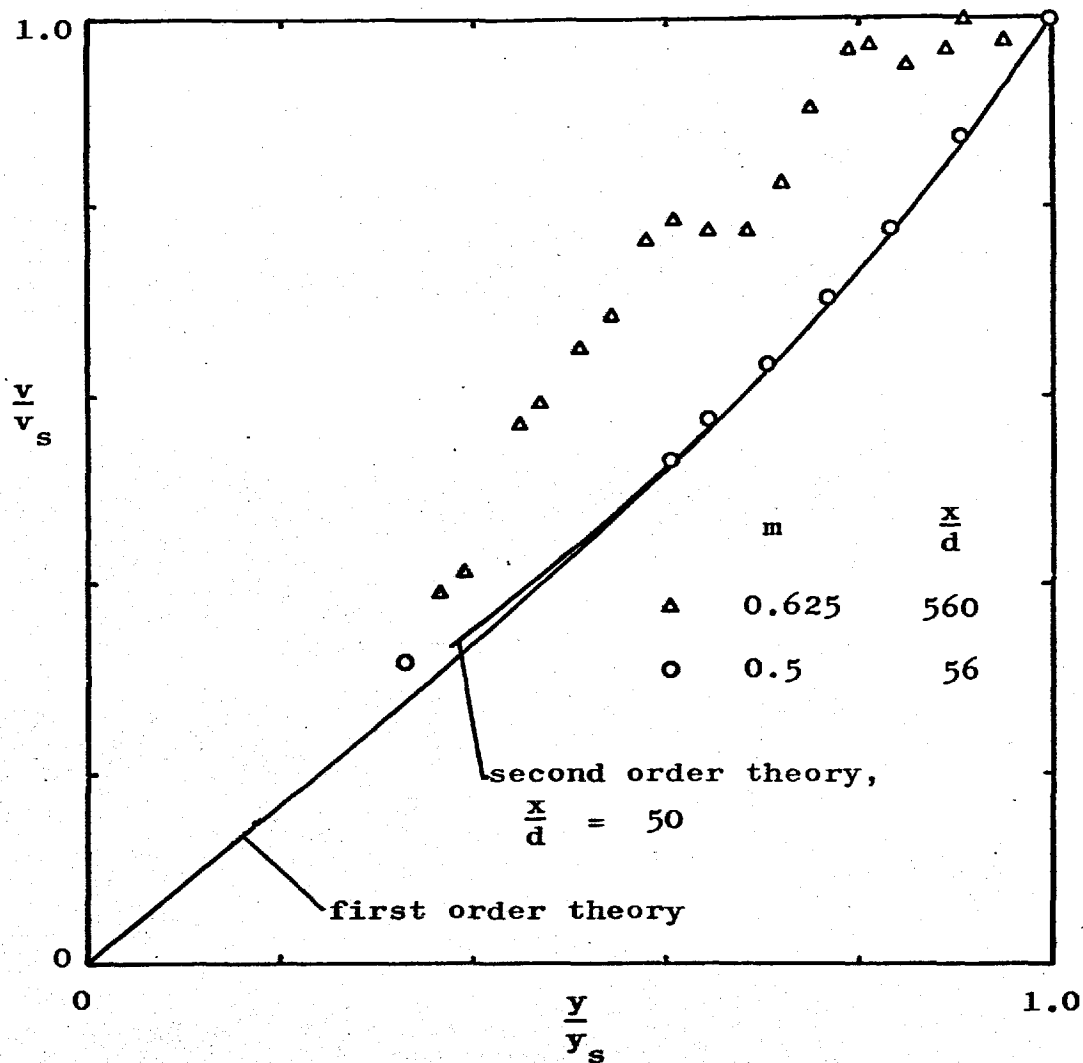


Fig.34b. NORMAL VELOCITY DISTRIBUTION THROUGH SHOCK-LAYER. COMPARISON WITH THEORY.

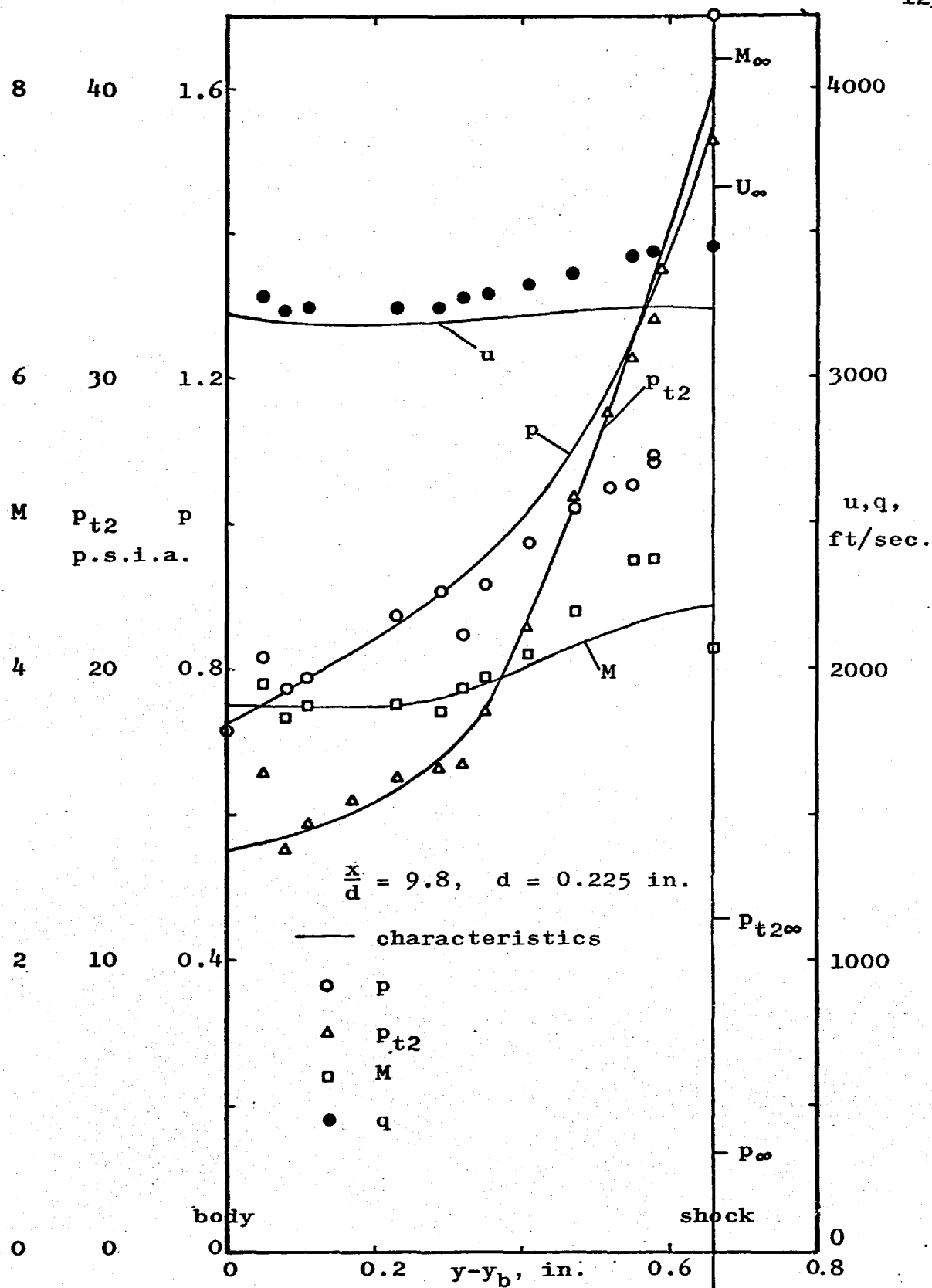


Fig.35a. SHOCK-LAYER TRAVERSE, WEDGE-PARABOLA.

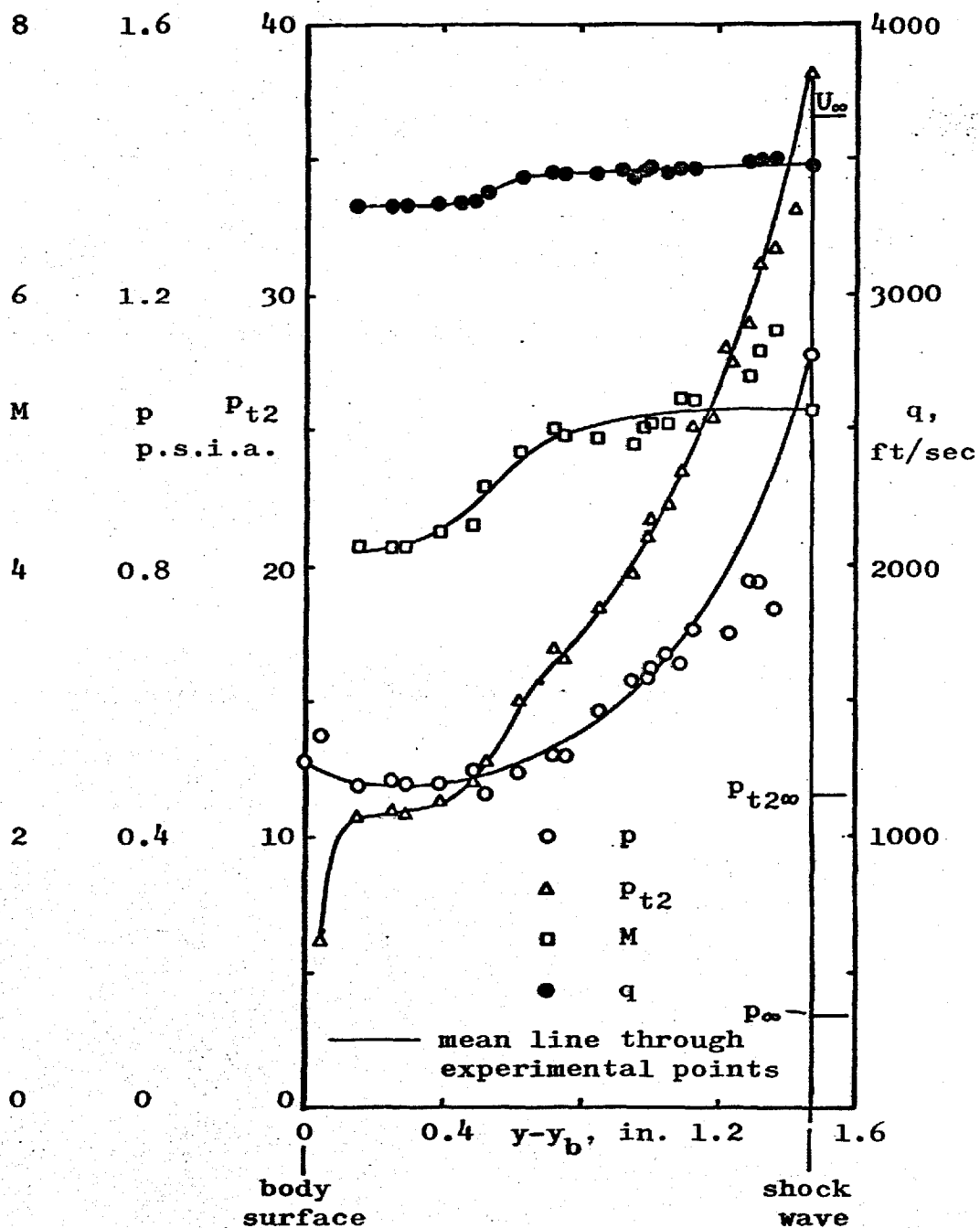


Fig.35b. SHOCK-LAYER TRAVERSE, WEDGE-PARABOLA. $\frac{x}{d} = 24.5$.

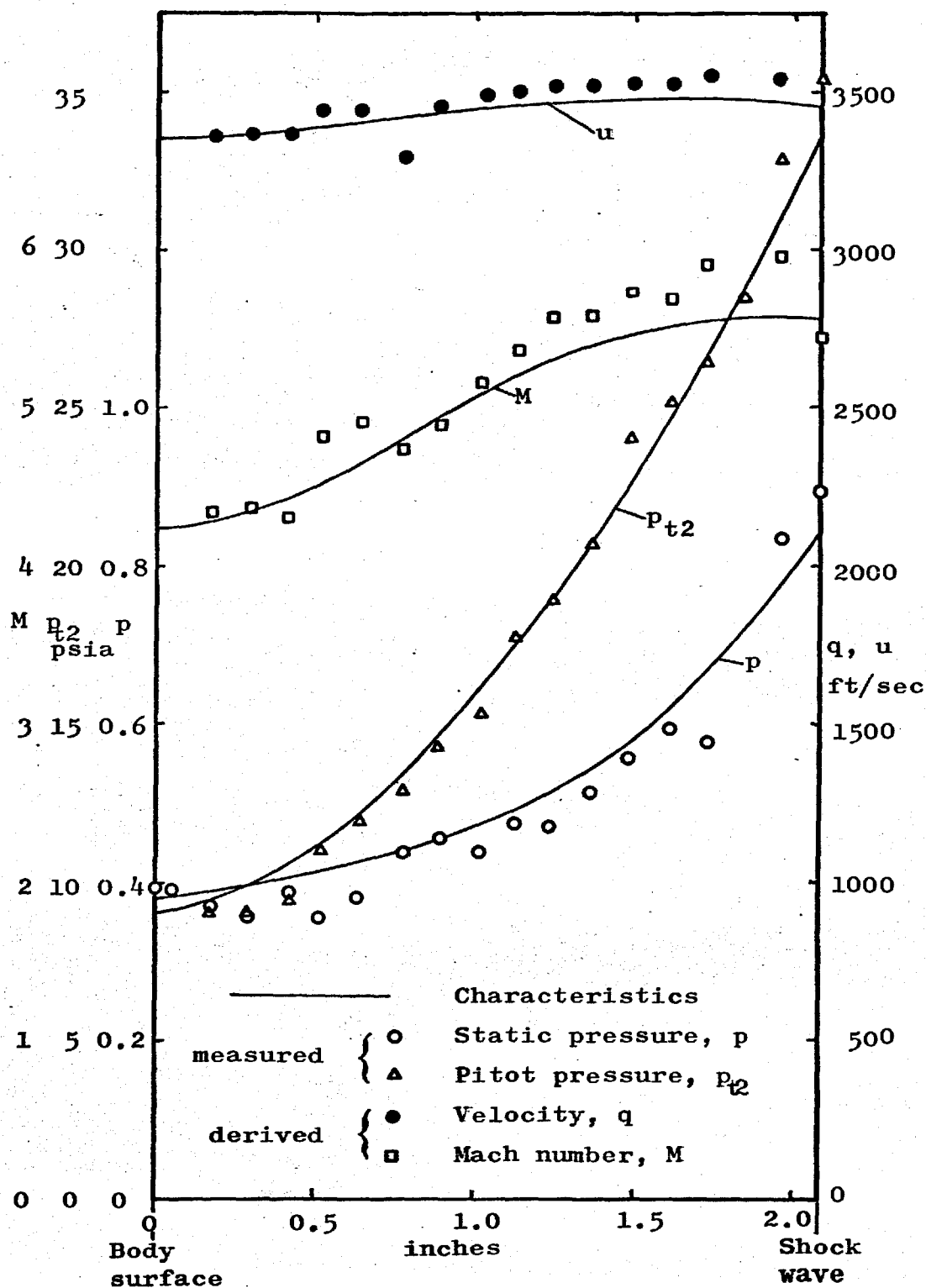


Fig.35c. SHOCK-LAYER TRAVERSE, WEDGE-PARABOLA. $\frac{x}{d} = 36$.

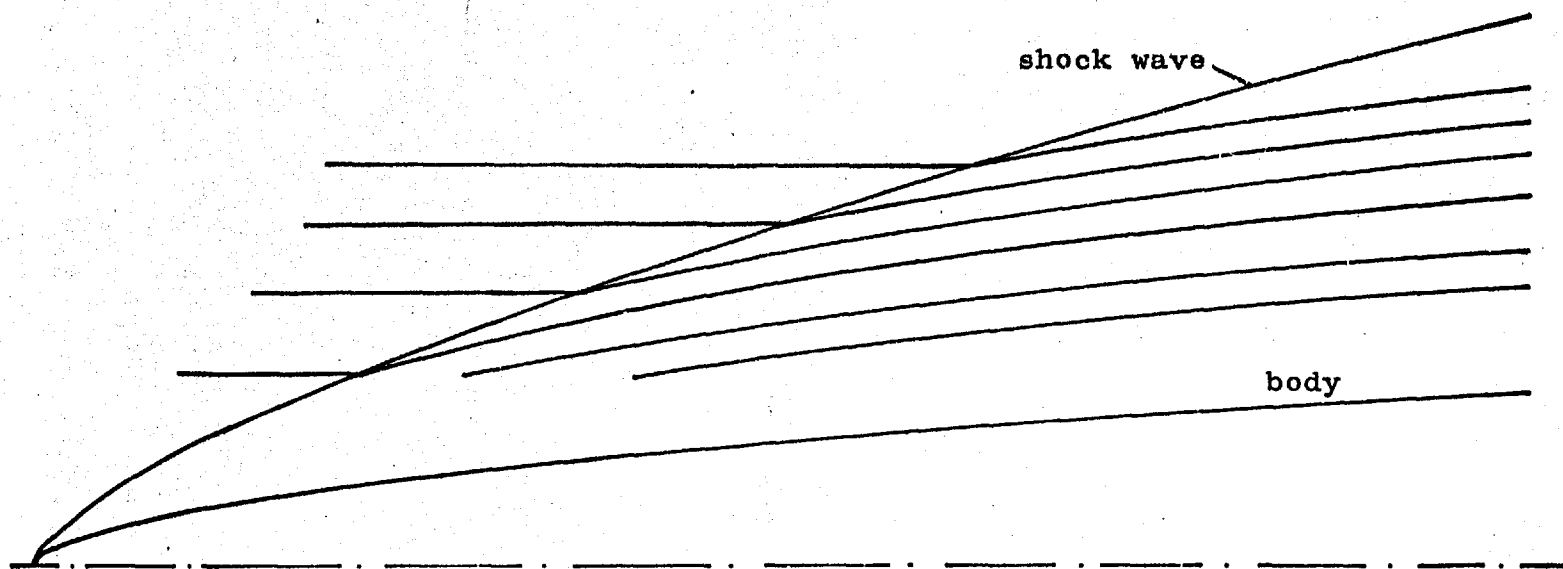


Fig.36. STREAMLINE SHAPES ON PARABOLA ($d = 0.100$ in.) AS OBTAINED BY THE METHOD OF WEAK WAVES.