

# Energy-Efficient Probabilistic Full Coverage in Wireless Sensor Networks

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**Abstract**—It is a common class of applications with wireless sensor network to provide full coverage to the region of interest (ROI), such as environment monitoring, military detection and agricultural observation. Existing literatures on full coverage are mostly based on the binary sensing model to simplify the problem. However, the results are far from the reality since binary sensing model as a coarse approximation is too conservative. The probabilistic sensing model has been proposed as a more realistic model to characterize the sensing region. In this paper, we introduce the concept of  $\varepsilon$ -full coverage based on probabilistic model, i.e., every point in ROI has at least a probability  $\varepsilon$  of being covered by sensors. We explore the mathematic relationship between the probabilities of two adjacent points being covered and transform  $\varepsilon$ -full coverage problem into point coverage problem. Then, we design  $\varepsilon$ -full coverage optimization (FCO) to select a subset of sensors to provide  $\varepsilon$ -full coverage dynamically so that the lifetime of network is prolonged. This algorithm outperforms the state-of-the-art solution significantly, which we have validated by simulations.

## I. INTRODUCTION

Wireless Sensor Networks (WSNs) is one of the most promising technologies to enable applications such as intrusion detection, target location and environment monitoring. In many applications, full coverage is highly required in a region of interest (ROI). Energy efficient sensing therefore attracts significant attention these years [1]–[5], [8], [19]. Previous works mainly focus on how to energy-efficiently provide full coverage to the ROI on limited battery energy. However, the literatures on full coverage are mostly based on the binary sensing model to simplify the problem. A binary sensor can detect any target within a certain range, referred to as its sensing range, with probability 1, but cannot detect anything beyond the range. Binary sensing model is only a coarse approximation to the reality.

Recently, probabilistic sensing model is considered as a more realistic model in the study of sensing coverage [6], [7], [9], [12]. In this model, a sensor is able to detect a target at a distance  $d$  away from the sensor with probability  $\lambda(d)$ , for certain decreasing function  $\lambda(d)$  with  $0 \leq \lambda(d) \leq 1$ . Various functions have been proposed for  $\lambda(d)$  such as the cubic attenuation model [6] and exponential attenuation probabilistic model [10].

Under the probabilistic sensing model, the coverage is defined as the probability of point in the ROI being covered by the network. We say the network can provide  $\varepsilon$ -full coverage if every point can be covered with a probability

larger than  $\varepsilon$ , here  $\varepsilon$ ,  $0 \leq \varepsilon \leq 1$ , is an application dependent threshold. As far as we know, no work can guarantee  $\varepsilon$ -full coverage yet due to the computational difficulty. The amount of points in a ROI is infinite. Xing in [11] considered to cut the ROI into grids and guaranteed the detection probability of vertexes of each grid to provide full coverage. This approach as an approximation solution reduces the computational complexity but it can not actually ensure full coverage. When a more precise coverage is imposed, the number of grids increases significantly, which requires much more computational cost. Probabilistic coverage protocol (PCP) is proposed in [13] to tackle with the full coverage problem based on probabilistic sensing model. It activates a subset of deployed sensors to construct approximate triangular lattices in ROI which guarantees that the points in each lattice are all full covered. However, it also can not ensure full coverage since the deployment of sensors is random and sensor node can not always be found in the desired position, i.e., to form a triangular lattice.

In this paper, we aim to guarantee  $\varepsilon$ -full coverage in the ROI while maximizing the lifetime of network. The main challenge is the amount of points in the ROI is infinite. To tackle with it, we investigate the relationship between probabilities of two points with a distance of  $d$  being covered that if one is  $\varepsilon e^{kd}$ , the other is no less than  $\varepsilon$ , the detail of  $e^{kd}$  can be seen in Section III. By doing so, we can ensure  $\varepsilon$ -full coverage in ROI only by guaranteeing the coverage of some specific points above a certain value greater than  $\varepsilon$ . Based on the theoretical analysis, we further study how to select the specific points to achieve better performance and propose an efficient solution to solve *Minimum Weight  $\varepsilon$ -full Cover Problem*.

Our mainly contribution can be summarized as below:

- 1) To the best of our knowledge, we are the first to study the mathematics relationship between the coverage of two points in ROI. This mathematics relationship implies that if one of these two points is sensed with a probability greater than a certain value  $\varepsilon e^{kd}$ , the sensing probability of the other one is no less than  $\varepsilon$ .
- 2) We formulate the *Minimum Weight  $\varepsilon$ -full Cover Problem* and propose an efficient solution called  *$\varepsilon$ -full coverage optimization (FCO)* to the problem. Theoretical analysis validates that FCO can fully provide  $\varepsilon$ -full coverage to the monitored area and efficiently prolong

the lifetime of network.

- 3) Extensive simulations are conducted to validate the effectiveness of our proposed algorithm in terms of both coverage maintenance and the lifetime of network.

The rest of the paper is organized as follows. We present preliminary and problem formulation in Section II, followed by the problem transformation and the design of algorithm  $\varepsilon$ -full cover optimization in Section III. We perform extensive simulations to evaluate the algorithm in Section IV. Finally, we conclude the paper in Section V.

## II. PRELIMINARY AND PROBLEM FORMULATION

### A. Network Model

Consider a large region of interest (ROI)  $R$  where  $N$  sensors are deployed to cover the ROI. We assume that  $R$  is a rectangle of size  $S = l_1 \times l_2$  for simplicity and sensor nodes are randomly deployed in  $R$ . Considering  $R$  as a two dimensional graph, any position in  $R$  is denoted by its coordinate  $(x, y)$ .

In this paper, we adopt the probabilistic model which is more realistic compared with the binary model. This model considers the uncertainty in sensing process and utilizes all information including what the binary model ignores. Let  $\lambda(S_i, P)$  denote the probability of point  $P$  covered by sensor node  $S_i$ . We use the exponential attenuation probabilistic model introduced in [14] for our problem formulation<sup>1</sup>. The coverage decays with the sensing distance denoted by  $d(S_i, P)$  as shown in the following equation:

$$\lambda(S_i, P) = \begin{cases} 1, & \text{if } d(S_i, P) \leq r_s, \\ e^{-ka}, & \text{if } d(S_i, P) > r_s, \end{cases} \quad (1)$$

where  $a = d(S_i, P) - r_s$  and  $k$  is a decay factor. As the signal the sensor node receives is strong enough when event occurs within the  $r_s$  disc of it, sensor node can definitely detect the occurrence and the probability is set to be 1. On the contrary, when the event occurs far from the sensor node, the sensor node can not distinguish the noise and the received signal. In such a case, the probability is asymptotic to be 0.  $k$  and  $r_s$  vary with the type of sensor node and environment and these parameters can be obtained through experiments.

The coverage performance can be improved using data fusion as proved in [6]. There are three classes of data fusion protocols commonly used: data level fusion, feature level fusion and decision level fusion. We adopt the decision level fusion: each sensor node makes local decision and global decision is achieved by fusing local decisions. The WSN definitively detects the occurrence of event if at least one sensor node does. Therefore, this fusion rule can be illustrated by:

$$\gamma(\mathbf{S}, P) = \bigvee_{S_i \in \mathbf{S}} \gamma(S_i, p), \quad (2)$$

<sup>1</sup>The other probabilistic models can also be adopted in a similar way.

where  $\gamma(\mathbf{S}, p)$  denotes the result of the network detecting the event occurring at the point  $p$  and  $\gamma(S_i, p)$  denotes the result of individual sensor  $S_i$  and  $\mathbf{S}$  is the fusion set of sensors which  $S_i$  belongs to.

### B. Problem Statement

Based on the probabilistic model and the aforementioned fusion rule, the coverage of an arbitrary point  $p$  which denotes the probability of event occurring in  $p$  detected by operative sensors can be described as follows:

$$\lambda_p = 1 - \prod_{S_i \in C} (1 - \lambda(S_i, p)), \quad (3)$$

where  $C$  is the set of operative sensors. With above equation, we define  $\varepsilon$ -full coverage.

**Definition 1:** ( $\varepsilon$ -full coverage) Sensor set  $C$  provides  $\varepsilon$ -full coverage to ROI  $R$  if coverage of any point in  $R$  is not less than  $\varepsilon$ , i.e.,  $\forall p \in A, \lambda_p \geq \varepsilon$ .

The value of  $\varepsilon$  is decided by the requirements of practical applications that satisfies  $0 \leq \varepsilon \leq 1$ . Larger  $\varepsilon$  appeals for more working sensor nodes. When  $\varepsilon = 1$ , this coverage model reduces to *full coverage* defined under the boolean sensing model. Due to the random deployment, the number of deployed sensors is usually much larger than the optimal [15], [19]. Therefore, to save energy and prolong network lifetime, sensors should operate alternatively between active mode and sleep mode. The operation duration of the WSN is divided into time slots and in each time slot we try to activate a small set of sensors to reduce the energy consumption [15], [16]. We formulate the problem to energy-efficiently provide  $\varepsilon$ -full coverage below:

**Minimum Weight  $\varepsilon$ -full Cover Problem:** Given a ROI  $R$ , a set of  $N$  sensors, the weight coefficient  $\omega_i$  associated with each sensor  $S_i$ ,  $i = 1, 2, \dots, N$ , *Minimum Weight  $\varepsilon$ -full Cover Problem* is to find a subset of sensors with the minimum weight to provide  $\varepsilon$ -full coverage to ROI  $A$ .

The weight coefficient is a parameter which is negatively correlated to the residual energy of sensor, i.e., a sensor with more residual energy has lower weight coefficient. Activating a set with minimum aggregate weight during each time slot can efficiently reduce energy and balance the energy consumption among deployed sensors to improve the lifetime of network.

## III. FULL COVERAGE OPTIMIZATION

It is extremely difficult to solve the *Minimum Weight  $\varepsilon$ -full Cover Problem* directly as the full coverage can be regarded as a point coverage problem with uncountable points. In this section, we first study mathematic relationship between the coverage of a point and its neighbor points. By exploring and utilizing these relationships, we transform the full coverage problem into the discrete point coverage problem and design  *$\varepsilon$ -full coverage optimization algorithm (FCO)* to solve the *Minimum Weight  $\varepsilon$ -full Cover Problem*.

### A. Problem Transformation

Different from the binary model, the probabilistic model given by (1) is a continuous function. Therefore, with (3), the coverage of points in  $R$  must be continuous on coordinates. Exploiting this property, we try to provide  $\varepsilon$  coverage for every point in the neighborhood of a certain point  $p$  by ensuring  $p$  is covered with a higher probability than  $\varepsilon$  to simplify the full coverage problem into the point coverage problem.

To fulfill this transformation, we explore the mathematic relationship between the coverage of two points nearby  $P_1$  and  $P_2$  as shown in Fig. 1. The distance between  $P_1$  and  $P_2$  is  $L$ . The coverage is assumed as  $\rho_1$  of  $P_1$  and  $\rho_2$  of  $P_2$ . We aim to obtain the lower bound of  $\rho_2$  while the  $\rho_1$  is provided to be  $\tau$  by operative sensor nodes. The number and the positions of active sensors that minimize  $\rho_2$  while fixing  $\rho_1$  as  $\tau$  are given by the following two lemmas. To simplify description, let  $P_1A$  denote the part of line  $AB$  left to  $P_1$ .

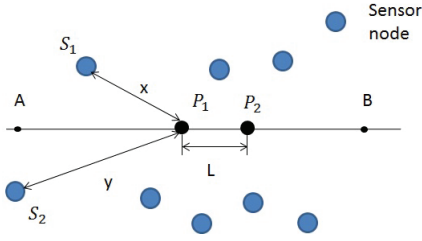


Figure 1. Illustration of two adjacent points

**Lemma 1:** Assume there are  $n$  active sensor nodes and  $\rho_1$  is fixed as  $\tau$ . Then, the coverage  $\rho_2$  is the minimum value when these  $n$  active sensor nodes are in the same position on  $P_1A$ .

*Proof:* We first look into the case when the number of active sensor nodes is 2. These two sensor nodes are denoted by  $S_1$  and  $S_2$  respectively.  $x$  is the Euclidean distance between the position of  $S_1$  and point  $P_1$ , while  $y$  is the Euclidean distance between the position of  $S_2$  and point  $P_1$ .

To minimize  $\rho_2$  when fix  $\rho_1$ , based on the probabilistic model, it is obvious that the positions of sensors should be on  $P_1A$ , as the distance between sensor and  $P_2$  is the largest when distance between sensor and  $P_1$  stays the same.

$\rho_1$  and  $\rho_2$  can be calculated respectively by the equations  $\rho_1 = 1 - (1 - e^{-ka_1})(1 - e^{-ka_2})$  and

$$\rho_2 = 1 - (1 - e^{-k(a_1+L)})(1 - e^{-k(a_2+L)}),$$

where  $a_1 = x - r_s$ ,  $a_2 = y - r_s$ . Let  $x_1 = e^{-ka_1}$ ,  $x_2 = e^{-ka_2}$ ,  $C = e^{-kL}$ . These values vary between 0 and 1. Then,

$$\rho_2 = 1 - (1 - Cx_1)(1 - Cx_2). \quad (4)$$

Note that  $\rho_1$  is fixed as  $\tau$ , which means that

$$\tau = 1 - (1 - x_1)(1 - x_2). \quad (5)$$

Then, (4) can be re-written as

$$\rho_2 = 1 - (1 - Cx_1) \left( 1 - C \frac{x_1 - \tau}{x_1 - 1} \right).$$

Taking derivation with regard to  $x_2$  for the above equation yields

$$\begin{aligned} \frac{d\rho_2}{dx_2} = & C(1 - C \frac{x_1 - \tau}{x_1 - 1}) \\ & + (\frac{C}{x_1 - 1} - C \frac{x_1 - \tau}{(x_1 - 1)^2})(1 - Cx_1). \end{aligned} \quad (6)$$

Since  $\frac{d\rho_2}{dx_2} < 0$  for  $x_1 < 1 - (1 - \tau)^{1/2}$  and  $\frac{d\rho_2}{dx_2} > 0$  for  $x_1 > 1 - (1 - \tau)^{1/2}$ , one obtains that  $x_1 = 1 - (1 - \tau)^{1/2}$  is the minimum point of  $\rho_2$ . From (5), we have  $x_2 = 1 - (1 - \tau)^{1/2}$  when  $x_1 = 1 - (1 - \tau)^{1/2}$ , which means  $\rho_2$  gets the minimal value when  $x_1 = x_2$ , namely  $x = y$ . This result for two sensor nodes case will be easily extended into  $n$  sensor nodes case through proof by contradiction. Thus, the proof is completed. ■

As Lemma 1 has been proved, we wonder that what is the value of  $n$  that minimizing  $\rho_2$  while  $\rho_1$  remains  $\tau$ . We have lemma 2.

**Lemma 2:**  $\rho_1$  is fixed as  $\tau$  by  $n$  sensor nodes in the same position on  $P_1A$ ,  $\rho_2$  is the minimal value when  $n = 1$ .

*Proof:* With  $n$  active sensor nodes in same position on  $P_1A$ , we have  $\rho_1 = 1 - (1 - e^{-ka})^n$  and  $\rho_2 = 1 - (1 - e^{-k(a+L)})^n$ , where  $a = x - r_s$ . Let  $X = e^{-ka}$  and  $C = e^{-kL}$ . Since  $\rho_1$  is fixed as  $\tau$ , we have  $\tau = 1 - (1 - X)^n$  and  $\rho_2 = 1 - (1 - CX)^n$ , and then  $\rho_2$  satisfies

$$\rho_2 = 1 - (1 - C(1 - (1 - \tau)^{1/n}))^n.$$

Let  $f(n)$  be a function of  $n$ , which satisfies  $f(n) = (1 - C(1 - \sigma^{\frac{1}{n}}))^n$ . Let  $\sigma = 1 - \tau$  and  $\sigma_1 = f(n + 1)$ . We thus have

$$1 - \sigma_1^{\frac{1}{n+1}} = C(1 - \sigma^{\frac{1}{n+1}}). \quad (7)$$

Since  $0 < C < 1$ , we have  $\sigma_1 > \sigma$ .

Define a function  $g(x)$  as

$$g(x) = \frac{1 - x^{\frac{1}{n}}}{1 - x^{\frac{1}{n+1}}}. \quad (8)$$

Taking derivation yields (8) that

$$\begin{aligned} g'(x) &= \frac{\frac{1}{n(n+1)}x^{\frac{1}{n} + \frac{1}{n+1} - 1} + \frac{1}{n+1}x^{-\frac{n}{n+1} - 1} - \frac{1}{n}x^{-\frac{n-1}{n}}}{\left(1 - x^{\frac{1}{n+1}}\right)^2} \\ &= \frac{\frac{1}{n(n+1)}x^{\frac{n+1-n^2}{n(n+1)}} + \frac{n}{n(n+1)}x^{-\frac{n}{n+1} - 1} - \frac{1}{n}x^{-\frac{n-1}{n}}}{\left(1 - x^{\frac{1}{n+1}}\right)^2} \\ &\geq \frac{(n+1)^{n+1} \sqrt{\left(\frac{1}{n(n+1)}\right)^{n+1} x^{-\frac{n \times n}{n+1} - \frac{n+1-n^2}{n(n+1)}} - \frac{1}{n}x^{-\frac{n-1}{n}}}}{\left(1 - x^{\frac{1}{n+1}}\right)^2} \\ &= 0. \end{aligned} \quad (9)$$

Note that  $g(x)$  is monotonically increasing as  $g'(x) \geq 0$ , which means  $g(\sigma_1) \geq g(\sigma)$ . Since  $\sigma \in (0, 1)$ , it follows

from the definition of  $g(x)$  that  $g(\sigma) > 0$ . From (7), we obtain

$$\left(1 - \sigma_1^{\frac{1}{n+1}}\right) g(\sigma_1) \geq C \left(1 - \sigma^{\frac{1}{n+1}}\right) g(\sigma),$$

i.e.,  $1 - \sigma_1^{\frac{1}{n}} \geq C(1 - \sigma^{\frac{1}{n}})$ . Hence, we have  $\sigma_1 \leq (1 - C(1 - \sigma^{\frac{1}{n}}))^n$ , which means that  $f(n+1) \leq f(n)$  and thus  $f(n)$  is monotonically non-increasing. Thus,  $f(1)$  gets the maximum value, i.e.,  $\rho_2$  gets the minimum value when  $n = 1$ . Thus, the proof is completed. ■

Lemma 1 and Lemma 2 suggest that when the number of active sensors is 1 and the position of this sensor is on  $P_1A$ ,  $\rho_2$  gets the minimal value  $\tau e^{-kL}$  while fixing  $\rho_1$  as  $\tau$ . The mathematical relationship between coverage of two adjacent points with distance  $L$  can be concluded as that if one is  $\tau$ , the lower bound of the other is  $\tau e^{-kL}$ . Thus, we have the following theorem:

**Theorem 1:** Let  $P_i$  be an arbitrary point in ROI, if the coverage of  $P_i$  is  $\varepsilon e^{-kR_i}$ , then any point with  $R_i$  disc centered at  $P_i$  will be covered at a probability no less than  $\varepsilon$

Thus, we divide *Minimum Weight  $\varepsilon$ -full Cover Problem* into two subproblems: Firstly, find a set of anchor points (APoint) set  $PS$  in ROI that circles centered in  $P_i$  with radius  $R_i$  can full cover  $A$ ; Secondly, find *Minimum Weight Cover* that each point  $P_i$  can be detected at a probability no less than  $\varepsilon_i$ , where  $\varepsilon_i = \varepsilon e^{kR_i}$ . Hence, the coverage of an arbitrary point  $p$  in  $A$  is guaranteed to be at least  $\varepsilon$  according to theorem 1.

### B. Solutions for Minimum Weight $\varepsilon$ -full Cover Problem

We formulate the *Minimum Weight  $\varepsilon$ -full Cover Problem* as the following 0 – 1 integer programming problem:

$$\begin{aligned} & \text{Minimize} \quad \sum_{i=1}^N \omega_i x_i \\ & \text{subject to} \quad \forall p \in A, \bigvee_{i=1}^M (D_{ip} < R_i) = 1, \\ & \quad \forall P_j \in PS, 1 - \prod_{i=1}^N (1 - \lambda_{ij} x_i) > \varepsilon_i, \\ & \quad x_i \in \{0, 1\}, \end{aligned} \quad (10)$$

where the value 1 and 0 of  $x_i$  denote the active and sleep mode of sensor  $S_i$ ,  $D_{ip}$  is the distance between APoint  $P_i$  and point  $p$ ,  $\lambda_{ij}$  is the probability of sensor  $S_i$  sensing  $P_j$ . The first constraint maintains that any point in  $R$  is covered by at least one disc centered in an APoint of corresponding radius. The second constraint holds every APoint in  $PS$  is covered at a probability no less than  $\varepsilon_i$ , where  $\varepsilon_i = \varepsilon e^{kR_i}$ . Therefore, these two constraints guarantee the monitored area  $R$  is  $\varepsilon$ -full covered. Based on this coverage maintenance, we minimize the weight sum of activated sensors as described by the objective function.

This problem has already been proved to be a NP hard problem in previous work [17]. We develop  $\varepsilon$ -full coverage

*optimization (FCO)* to solve this *Minimum Weight  $\varepsilon$ -full Cover Problem*. The FCO is composed of two steps as described in detail following.

**Step 1:** Determine the A-Point set  $PS$ . The selected  $PS$  should meet the first constraints in (10). we present two methods to determine the  $PS$  as following.

**homo-APoint:** In this method, points in  $PS$  are homogeneous that the values of the radius  $R_i$  and probabilities  $\varepsilon_i$  accordingly of all points are identical as  $R$  and  $\varepsilon^*$ . These APoints are evenly spaced on the area in any uniform distribution, e.g., vertices of triangular lattices or grids. Without loss of generality, we cut ROI into congruent  $l \times l$  grids and select vertexes of each grid as A Points. Then, to provide  $\varepsilon$ -full coverage, we have

$$\varepsilon^* = \varepsilon e^{kR}, \quad (11)$$

where  $R = \sqrt{2}/2l$ . Note that larger  $l$  results in raising up the coverage requirement due to higher  $\varepsilon^*$  but reducing computing time as less APoints needed. The value of  $l$  is decided depending on the practical demand.

Consider that points with more sensors surrounded are easier to obtain high detection probability and sensors with more energy are preferred to be activated, we improve the APoints selection by *hetero-APoint*.

**hetero-APoint:** Select heterogeneous APoints of various radius  $R_i$  associated with various detection probability threshold  $\varepsilon_i$ . We cut the whole area into several subareas and employ the *homo-APoint* method for each subarea  $A_j$  independently. And the number of POIs in  $A_j$  is decided according to the energy sum of sensor nodes in surrounding area of  $A_j$ .

**Step 2:** Determine the sensor set  $C$  to satisfy the second constraint in 10. There are a bunch of algorithms presented in literature we can employ to deal with this point coverage problem, e.g., greedy method [14] and intelligent algorithms [18]. Although intelligent algorithms are proved to be much more efficient [18], we adopt the simple Greedy algorithm [14] to estimate the performance of our method.

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### Algorithm 1 Full Cover Optimization

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- 1) Find a APoint set  $PS$  using *homo-APoint* or *hetero-APoint* method and determine the radius  $R_i$  associated with the coverage threshold  $\varepsilon_i$  for each APoint  $P_i$ .
  - 2) Find a *minimum weight cover set*  $C$  using Greedy algorithm in [14] that ensures the coverage of each APoint  $P_i$  in  $PS$  no less than  $\varepsilon_i$ .
  - 3) If using *homo-APoint* in 1, back to step 2 in next time slot; If using *hetero-APoint* in 1, back to step 1 in next time slot.
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#### IV. PERFORMANCE ANALYSIS

In this section, we evaluate our proposed FCO by extensive simulations using Matlab. We compare the performance of *homo-APoint* and *hetero-APoint* in Sec. IV-A and then verify the correctness of our algorithm and compare our FCO with other probabilistic protocols, i.e., *Probabilistic Coverage Protocol* (PCP) [13] and *Greedy-MSC Heuristic* [17].

The parameters for experiments are set as follows unless otherwise specified. Assume that the ROI  $A$  is a square with the size of  $100\text{m} \times 100\text{m}$  and the value of  $\varepsilon$  is 0.9. The parameters in probabilistic model are chosen the same as in the simulation of [13], i.e.,  $r_s = 5$  and  $k = 0.05$ . The initial energy of a sensor node is assumed to be 1 units and it cost 0.1 unit energy when a sensor node is active in a time slot.

##### A. Comparing FCO with different APoint set

We estimate the impact of APoint selection on the performance of FCO and compare the performance of *homo-APoint* and *hetero-APoint*. In this simulation, we randomly deploy 300 sensor nodes in  $A$ . The area  $A$  is cut into  $25 \times 25$  grid subareas for *hetero-APoint*.

We focus on the lifetime of network perform by FCO with different amounts of APoInts using *homo-APoint* or *hetero-APoint*. We present the results about the lifetime of network in Fig. 2. The  $X$  axis represent the number of APoInt in PS. We can summarize that the lifetime of network increased with the number of APoInt and *hetero-APoint* clearly outperform *homo-APoint*.

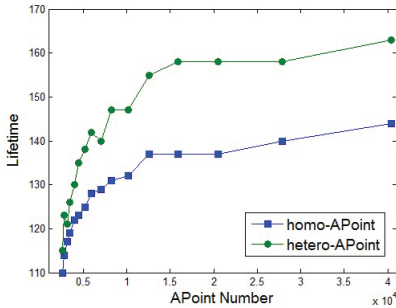


Figure 2. The lifetime of network

##### B. Comparing FCO versus other probabilistic coverage protocol

We compare the performance our FCO with the *Probabilistic Coverage Protocol* (PCP) and the *Greedy-MSC Heuristic*. The idea of PCP is to activate a subset of deployed sensors to construct an approximate triangular lattice on top of the area to be covered. PCP has been validated to be more efficient than other probabilistic area coverage protocol in the literature of [13]. Meanwhile, *Greedy-MSC Heuristic* [17] is the best algorithm to maximize the lifetime

of network under full coverage model based on the binary model of sensors, to our best knowledge.

To conduct fair comparisons, we use the same probabilistic model (1) with same parameters for FCO and PCP as aforementioned. Under this experimental setup, the maximum separation between any two active sensors on the triangular lattice is calculated to be 30.227m. The sensing range  $R_s$  of sensors for *Greedy-MSC Heuristic* is chosen as the value below which the sensing can be achieved with the probability of 0.9, i.e.,  $R_s = 17.1072$ . We employ *homo-POI* for FCO in this comparison, the performance of which can be improve by using the *hetero-POI* as validated in Sec. IV-A. In the first experiment, we randomly deploy 400 sensor nodes in  $A$ . The POI set is selected as  $101 \times 101$  points on the vertices of the grids of the side length  $L = 1\text{m}$  and then the detection threshold calculated by (11) is 0.9659. The  $\delta$  for PCP is calculated as  $\delta = 2\sqrt{2} \times 100/(\sqrt{500} - 1) \approx 14$ . We plot the performance comparisons provided by these three protocols in terms of area coverage, amount of activated sensors and average residual energy of activated sensors respectively in Fig. 3(a), Fig. 3(b), Fig. 3(c). These simulation results can be summarized as follows: FCO and *Greedy-MSC Heuristic* can guarantee  $\varepsilon$ -full coverage to ROI  $A$  while PCP not due to randomly deployment of sensor nodes; compared to the *Greedy-MSC*, our algorithm can significantly reduce the energy consumption since fewer sensors are activated during each time slot as showed in Fig. 3(b) while providing  $\varepsilon$ -full coverage; it can be seen from Fig. 3(c) that the  $\varepsilon$ -FCO activate sensors with more residual energy to balance the energy consumption among all sensors.

In the second experiment, we present the simulation results about the network lifetime performed by FCO and *Greedy-MSC Heuristic*. The simulation of PCP is not conducted as PCP can not provide  $\varepsilon$ -full coverage to  $A$ . We define the network lifetime as the amount of time slots that  $\varepsilon$ -full coverage is provided. The lifetimes show in 4 are the averages of 10 times simulations under different initial sensor number, which indicate the remarkable outperformance of  $\varepsilon$ -full coverage in lifetime prolonging.

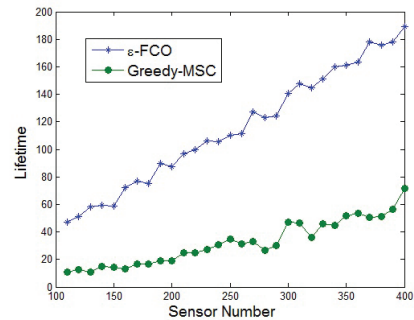
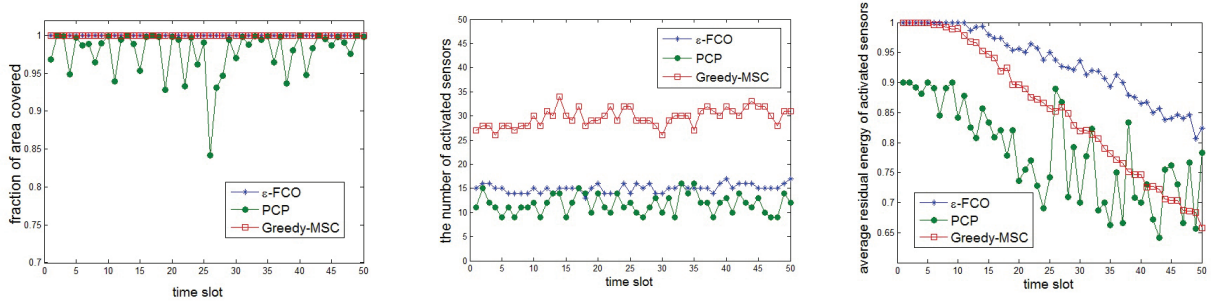


Figure 4. The lifetime of network



(a) Fraction of the area sensed with probability high than  $\varepsilon$ . (b) The number of activated sensors in each time slot. (c) Average residual energy of sensors in activated sensor cover.

Figure 3. Simulation results.

## V. CONCLUSION

In this paper, we investigated full coverage problem under the probabilistic sensing model in the sensor networks. We have studied that mathematics relation between the coverage of two adjacent points and transformed the full coverage problem into the point coverage problem. Based on these results, we proposed FCO algorithm to solve the *Minimum Weight  $\varepsilon$ -full Cover Problem*, and put forward two methods to select the APoint set. Our theoretical analysis and simulation results demonstrate that  $\varepsilon$ -FCO algorithm can fully ensure  $\varepsilon$ -full area coverage and outperform other probabilistic coverage protocols in term of the number of active nodes, average residual energy and network lifetime. Moreover, the results are not limited to the exponential sensing model, but can be applied to other probabilistic sensing models.

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