

CALCULATION OF EPICARDIAL POTENTIALS FROM BODY SURFACE  
MEASUREMENTS

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ABSTRACT

The classic difficulty in treating the inverse problem in Electrocardiography is that it is "ill-posed". The large size of the condition number of the forward transfer coefficient matrix indicates this "ill-posedness". In this study, the condition number of the forward transfer coefficient matrix is greatly reduced by using a heart model with unequal-sized epicardial segments. Stability of the solution is obtained by a novel regularisation technique on the forward transfer coefficient matrix. A qualitative comparison between the computed and the measured epicardial electrocardiograms (ECG) shows good and consistent results.

The forward transfer coefficient matrix mentioned above is determined from the configuration and the electrical properties of the human thorax using a finite difference model. A set of inverse transfer coefficients, which relates the body surface to an individual epicardial measurement site is also computed, using both the body and epicardial surface potential measurements from a human subject. Reconstruction of the epicardial ECG using the computed inverse transfer coefficient and the measured body surface potentials shows good results. Epicardial surface ECG are also computed using the inverse transfer coefficient derived from the measurements of a different subject. The results show that it is feasible to use the inverse transfer coefficient derived from

the measurements of one subject for another subject. Although for more accurate results, the inter-subject variation in chest and heart configurations and the internal inhomogeneity effect need to be considered.

Simultaneous measurements of 37 body surface and up to 3 epicardial surface ECG are successfully recorded from each of the 21 human subjects. It is found that despite a large inter-subject variation of chest and heart configurations and ECG voltage amplitudes, the epicardial ECG correlates well during the QRS complex with the body surface measurement. There are also distinct features in the P-wave of the epicardial ECG that are not observed in the body surface measurement indicating that certain information is lost. These findings further show the importance of computing the epicardial surface potentials from non-invasive body surface measurements.

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CONTENTS

|                                                                              |    |
|------------------------------------------------------------------------------|----|
| TITLE PAGE                                                                   | 1  |
| ABSTRACT                                                                     | 2  |
| ACKNOWLEDGEMENTS                                                             | 4  |
| CONTENTS                                                                     | 5  |
| 1. INTRODUCTION                                                              | 9  |
| 1.1 Cardiac anatomy and physiology                                           | 11 |
| 1.1.1 The atria                                                              | 11 |
| 1.1.2 The ventricles                                                         | 11 |
| 1.1.3 Flow of blood through the heart                                        | 12 |
| 1.1.4 The heart valves                                                       | 13 |
| 1.1.5 The excitatory and conducting system of the heart                      | 13 |
| 1.1.5.1 Sino-atrial node                                                     | 13 |
| 1.1.5.2 The atrio-ventricular conducting system                              | 15 |
| 1.2 The normal electrocardiogram                                             | 17 |
| 1.2.1 Excitation sequences of the heart                                      | 20 |
| 1.3 Overview of the forward and inverse problems                             | 22 |
| 1.3.1 The development of scalar ECG lead systems from the dipolar hypothesis | 24 |
| 1.3.2 Body surface mapping                                                   | 31 |
| 1.4 Review of the forward problem                                            | 36 |
| 1.4.1 Physical model approach                                                | 36 |
| 1.4.2 Theoretical (analytical) model approach                                | 37 |
| 1.4.3 Numerical (digital) model approach                                     | 39 |
| 1.4.3.1 Integral equation approach                                           | 39 |
| 1.4.3.2 Differential equation approach                                       | 41 |
| 1.5 Review of the inverse problem                                            | 45 |
| 1.5.1 Equivalent cardiac generators approach                                 | 45 |
| 1.5.2 Epicardial potential approach                                          | 52 |
| 1.6 Objectives of this study                                                 | 56 |

|                                                                                                                                 |         |
|---------------------------------------------------------------------------------------------------------------------------------|---------|
| 2 . FORMULATION AND IMPLEMENTATION OF A COMPUTER MODEL OF<br>THE HUMAN THORAX AS A VOLUME CONDUCTOR                             | 60      |
| 2.1 Introduction                                                                                                                | 60      |
| 2.2 The fundamental equations                                                                                                   | 62      |
| 2.3 Method of finite difference                                                                                                 | 65      |
| 2.3.1 Two-dimensional numerical model                                                                                           | 69      |
| 2.3.2 Three-dimensional numerical model                                                                                         | 74      |
| 2.4 Discrete model of the human thorax                                                                                          | 78      |
| 2.4.1 Introduction                                                                                                              | 78      |
| 2.4.2 Computational technique                                                                                                   | 80      |
| 2.4.2.1 Optimal accelerating factor                                                                                             | 81      |
| 2.4.2.2 Numerical examples on the optimal accelerating<br>factor                                                                | 82      |
| 2.4.3 Verification of technique                                                                                                 | 86      |
| 2.4.3.1 Cylinder model                                                                                                          | 86      |
| 2.4.3.2 Potential measurements on human thorax                                                                                  | 88      |
| 2.5 Programme implementation                                                                                                    | 92      |
| 2.6 Conclusion                                                                                                                  | 95      |
| <br>3 . SOLVING ILL-CONDITIONED MATRIX EQUATIONS IN THE<br>INVERSE PROBLEM IN ELECTROCARDIOGRAPHY                               | <br>96  |
| 3.1 Introduction                                                                                                                | 96      |
| 3.2 Derivation of the forward transfer coefficient                                                                              | 100     |
| 3.2.1 Mathematical formulation of the forward problem                                                                           | 100     |
| 3.2.2 Computational techniques                                                                                                  | 102     |
| 3.2.3 Relation of condition number of the forward<br>transfer coefficient matrix and the<br>configuration of the heart segments | 103     |
| 3.2.4 Programme implementation                                                                                                  | 105     |
| 3.3 Solving ill-conditioned matrix equation $AX=Y$                                                                              | 116     |
| 3.4 Application to inverse problem in electrocardiography                                                                       | 124     |
| 3.4.1 Stability                                                                                                                 | 124     |
| 3.4.2 Correctness                                                                                                               | 129     |
| 3.5 Conclusion                                                                                                                  | 140     |
| <br>4 . INSTRUMENTATION, DATA ACQUISITION AND PRE-PROCESSING                                                                    | <br>141 |
| 4.1 Introduction                                                                                                                | 141     |
| 4.2 Instrumentation                                                                                                             | 143     |
| 4.3 Data acquisition                                                                                                            | 146     |
| 4.4 Data pre-processing                                                                                                         | 149     |
| 4.4.1 Data editing                                                                                                              | 149     |
| 4.4.2 Noise reduction                                                                                                           | 149     |
| 4.4.3 Reconstruction of missing data                                                                                            | 150     |
| 4.4.4 Adjustment for time skew                                                                                                  | 151     |
| 4.4.5 Base line adjustment                                                                                                      | 154     |

|                                                                                                                                           |         |
|-------------------------------------------------------------------------------------------------------------------------------------------|---------|
| 5. COMPUTATION OF EPICARDIAL SURFACE POTENTIALS FROM<br>BODY SURFACE MEASUREMENTS                                                         | 155     |
| 5.1 Introduction                                                                                                                          | 155     |
| 5.2 Reconstruction of surface map from a limited electrode<br>set through interpolation                                                   | 157     |
| 5.2.1 Limited electrode set                                                                                                               | 157     |
| 5.2.2 Interpolation                                                                                                                       | 161     |
| 5.3 Computational procedures                                                                                                              | 166     |
| 5.4 Evaluation of the computed inverse solutions                                                                                          | 168     |
| 5.4.1 Direct comparison of measured and computed<br>epicardial potentials                                                                 | 168     |
| 5.4.2 Reconstruction of surface potentials from the<br>inverse solution                                                                   | 175     |
| 5.5 Effect of omission of back torso potentials<br>on the inverse solution                                                                | 179     |
| 5.5.1 Methods                                                                                                                             | 179     |
| 5.5.2 Results and discussion                                                                                                              | 182     |
| 5.6 Conclusion                                                                                                                            | 188     |
| <br>6. ANALYSIS OF EPICARDIAL AND BODY SURFACE POTENTIAL MEASUREMENTS                                                                     | <br>190 |
| 6.1 Introduction                                                                                                                          | 190     |
| 6.2 Epicardial and body surface electrocardiograms                                                                                        | 192     |
| 6.2.1 Body surface ECG                                                                                                                    | 192     |
| 6.2.2 Epicardial surface ECG                                                                                                              | 194     |
| 6.2.3 Effect of body build on the body surface QRS<br>voltage                                                                             | 198     |
| 6.2.4 Correlation between body and epicardial surface<br>measurements                                                                     | 201     |
| 6.2.5 Discussion                                                                                                                          | 209     |
| 6.3 Computation of transfer coefficients from the<br>measured body and epicardial surface potentials                                      | 211     |
| 6.3.1 Linear Least Squares approximation                                                                                                  | 213     |
| 6.3.2 Definition of singular value decomposition (SVD)                                                                                    | 215     |
| 6.3.3 Least Squares problem and SVD                                                                                                       | 217     |
| 6.3.4 Computation of the inverse transfer coefficient                                                                                     | 219     |
| 6.4 Feasibility of using the inverse transfer coefficient<br>on other patients other than one from which the<br>coefficients were derived | 223     |
| 6.4.1 Methods and results                                                                                                                 | 223     |
| 6.4.2 Discussion                                                                                                                          | 224     |
| 6.5 Conclusion                                                                                                                            | 229     |
| <br>7. CONCLUSION                                                                                                                         | <br>231 |

|                                                                                                                  |     |
|------------------------------------------------------------------------------------------------------------------|-----|
| APPENDICES                                                                                                       | 239 |
| A1 Discretised model of the human thorax                                                                         | 240 |
| A2 Notes on the "condition number"                                                                               | 243 |
| A3 Mathematical derivation (1)                                                                                   | 244 |
| A4 The flow diagrams, source listing and the input data of<br>the derivation of the forward transfer coefficient | 245 |
| A5 Forward transfer coefficient                                                                                  | 261 |
| A6 Mathematical derivation (2)                                                                                   | 262 |
| A7 Numerical examples                                                                                            | 264 |
| A8 Residue norm ( $r^2$ ) of $AX = Y$                                                                            | 267 |

|            |     |
|------------|-----|
| REFERENCES | 269 |
|------------|-----|

## CHAPTER 1

### INTRODUCTION

This work is concerned with the calculation of electrical potentials of the epicardial surface using non-invasive measurements on the body surface. To accomplish this, first an existing computer model of the electrical properties of the human thorax is developed further. From the results of this model, a new stable method of computation of epicardial surface potentials is developed. Epicardial surface potentials measured in human patients after cardiac surgery are used to evaluate the results. The resulting method provides qualitatively a good visualization of epicardial surface potentials from body surface measurements.

This chapter provides a basic introduction to the background and objectives of this study. Section 1.1 gives a brief description of the anatomy and physiology of the heart, of which a more detailed discussion can be found in Spach and Barr (1976) [1]. In section 1.2 the normal electrocardiogram and the excitation sequences of the heart are briefly introduced. Section 1.3 presents a general overview of the two classic problems in electrocardiography, namely the forward and inverse problems. More detailed reviews of the development and the most significant work on the forward and inverse problems are given in sections 1.4 and 1.5 respectively. Finally in section 1.6, the objectives of

this study are outlined in more detail.

## 1.1 Cardiac anatomy and physiology

The heart is a hollow muscular organ, roughly cone shaped. It lies in the middle mediastinum between the lungs in the thoracic cavity. It lies obliquely, a little more to the left than the right and presents a base above and an apex below. Fig.1.1.a and Fig.1.1.b show the exterior and the interior of the heart respectively. The heart is divided into a right and left side by a partition of muscular tissue known as the septum. The right and left sides are divided in turn by partitions known as valves into upper and lower chambers known as the atria and the ventricles.

### 1.1.1 The atria

The simplest representation of the anatomy of the atrial muscle mass (including both right and left atria and the atrial septum) is a hemisphere with a partition in the middle - the atrial septum. The muscular walls of the atria are relatively thinner (ranging from 0.5 millimeter (mm) to 4 mm in thickness) than those of the ventricles (about 4 - 15 mm thick). This is because the atria have merely to pump the blood into the ventricles.

### 1.1.2 The ventricles

The ventricles are separated by the septum into 'right' and 'left' ventricles, anatomically, the right ventricle is quite

flattened and positioned forward and rightward of the left ventricle (see Fig.1.1.a) and the left ventricle is roughly ellipsoid and oriented forward downward and leftward. The ventricles have to pump blood to the lungs (pulmonary circulation) via the right ventricle and into the general (systemic) circulation via the left ventricle. The myocardium of the left ventricle is therefore much thicker and stronger than that of the right ventricle (about 15 and 4 mm thick respectively). The ventricular septum, which separates the two ventricles physically, is slightly thicker than either wall.

#### 1.1.3 Flow of blood through the heart

The two largest veins of the body, the superior and the inferior venae cavae carrying deoxygenated venous blood, empty their contents into the right atrium. This blood passes via the right atrio-ventricular valve into the right ventricle, and from the right ventricle is pumped into the pulmonary artery which is the only artery in the body to carry venous blood. The pulmonary artery is then divided into the left and right pulmonary arteries. These arteries carry the venous blood to the lungs where an interchange of gases occurs. The oxygenated arterial blood is then carried from each lung by two pulmonary veins which are divided into four pulmonary veins and empty their contents into the left atrium. This blood now passes via the left atrio-ventricular valve into the left ventricle. From the left ventricle, the arterial blood is pumped into the aorta and

subsequently into the general circulation.

#### 1.1.4 The heart valves

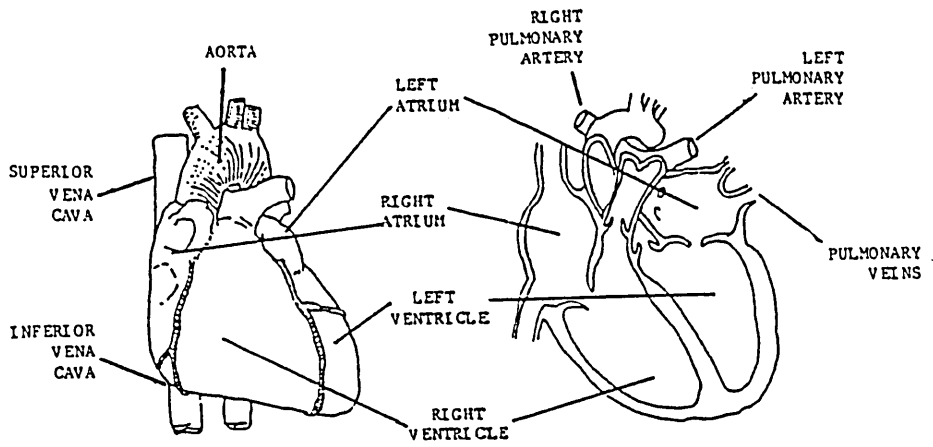
The valves dividing the atria from the ventricles are formed by double folds of endocardium strengthened with fibrous tissues. The valve separating the right atrium from the right ventricle is known as the right atrio-ventricular valve or the tricuspid valve. The valve separating the left atrium from the left ventricle is termed the left atrio-ventricular valve or the mitral valve. The opening of the pulmonary artery is guarded by a valve known as the pulmonary valve. This valve prevents the back flow of blood into the right ventricle. The opening of the aorta is guarded by a valve termed the aortic valve.

#### 1.1.5 The excitatory and conducting system of the heart

The excitatory and conducting systems of the heart is illustrated in Fig.1.2. These comprise the sino-atrial node, the atrio-ventricular node and the atrio-ventricular bundle (bundle of His) and the left and right bundles of Purkinje fibres. They are described in the following sections.

##### 1.1.5.1 Sino-atrial node

Situated at the junction between the superior vena cava and the right atrium is a small strip of specialised neuro-muscular cells known as the sino-atrial (S-A) node. The S-A node is often described as the "pace maker" of the heart because



(a) The exterior view. (b) The interior view.

Fig.1.1 The heart.

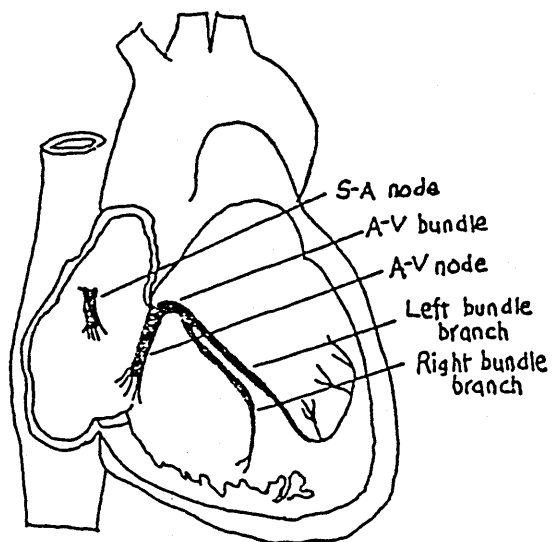


Fig.1.2 The excitatory and conducting systems of the heart.

it is capable of generating rhythmic impulses which stimulate the muscle of the atria to contract. The impulses generated at the S-A node is spread into the surrounding atrial muscle. The atrial muscle in turn conducts the signal in all directions at a velocity of about 0.3 metre/second. It should be noted that it is the atrial muscle fibres themselves that transmit the cardiac impulses and not some specialised conducting system as in the ventricles. This will be discussed in the next section.

The S-A node is innervated with nerves of the autonomic nervous system. These nerves are the vagus nerve (parasympathetic nerve) and a sympathetic nerve which are antagonistic to one another. The vagus nerve tends to slow down the rate of impulses produced by the S-A node, therefore decreasing the force and rate of heart beat. The sympathetic nerve tends to speed up impulses produced by the S-A node thus increasing the rate and force of the heart beat. If stimuli are being received equally at the S-A node the normal heart contraction will be maintained.

#### 1.1.5.2 The atrio-ventricular conducting system

The atrio-ventricular conducting system comprises of (1) the atrio-ventricular (A-V) node, which is a mass of neuromuscular tissue situated near to the septum of the heart close to the atrio-ventricular valve, and (2) the atrio-ventricular bundle (bundle of His) and the left and right bundles of Purkinje fibres spreading over the ventricular myocardium.

The excitation impulse from the atrial tissue propagates quite slowly along the A-V node at a velocity of about 0.1 metre/second. This is about one-fourth the conduction velocity in normal cardiac muscle. This delay in the A-V node allows time for the atrial contraction to move blood into the ventricles before ventricular contraction begins. The impulse then propagates rapidly, at a velocity of about 2 metre/second, along the atrio-ventricular bundles and their branches into the network of Purkinje fibres all over the ventricular myocardium. This rapid and wide propagation over the ventricular myocardium ensures a synchronous contraction and an efficient pumping action of the ventricles.

## 1.2 The normal electrocardiogram

As discussed in the last section, the electrical impulse initiated in the S-A node travels through the atria, the atrio-ventricular bundle and the Purkinje system and finally through the ventricular muscle. As the impulse passes through the heart, electrical currents spread into the tissue surrounding the heart and a small proportion of these spreads all the way to the body surface. These electrical potentials generated by the heart can be recorded. The recording is known as an electrocardiogram (ECG).

The ECG observed in a typical healthy adult is composed of a P wave, a QRS complex and a T wave (Fig.1.3). The P wave is caused by electrical currents generated as the atria depolarize before contraction and the QRS complex is caused by currents generated when the ventricles depolarize before contraction. The T wave is caused by currents generated as the ventricles recovered from the state of depolarization. The T wave is also known as the depolarization wave.

At rest, the membrane potential is normally measured negative inside and positive outside of the muscle fibre. The muscle fibre is said to be depolarized when its normal resting potential (about -90 millivolts) is increased to about +20 millivolts resulting from increased membrane permeability to sodium ions when an action potential occurs. After depolarization, the muscle fibre gradually returns to its resting

state and it is said to be repolarised.

The distinction between the depolarization and repolarization waves is best illustrated in Fig.1.4. This figure shows diagrammatically the four different stages of depolarization and repolarization of a muscle fibre. In Fig.1.4.a the process of depolarization is travelling from left to right and the first half of the fibre is already depolarized, therefore the left electrode is in an area of negativity, while the right electrode is in an area of positivity. This causes the measuring meter to record positively. In Fig.1.4.b, the depolarization has extended over the entire muscle fibre, and the recording has returned to the zero base line as both electrodes are now in an area of equal negativity. The complete wave is called a depolarization wave because it results from the spread of depolarization along the extent of the muscle fibre. Fig.1.4.c shows the repolarization process in the muscle fibre, which has proceeded halfway from left to right. At this point the left electrode is in an area of positivity and the right electrode is in an area of negativity. Consequently, the recording is measured negatively. Finally, in Fig.1.4.d, the muscle fibre has completely repolarized and both electrodes are in an area of equal positivity so no potential is recorded between them. The completed negative wave is the repolarization wave because it results from the spread of the repolarization process over the muscle fibre.

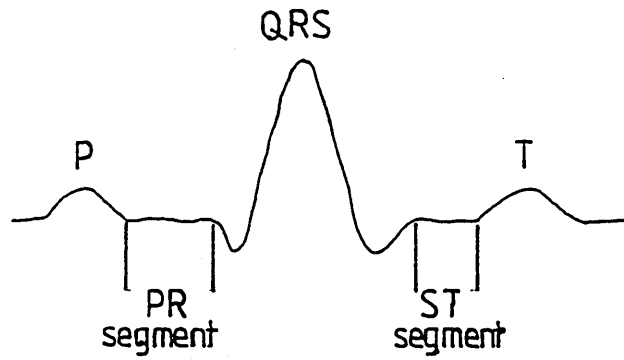


Fig.1.3 The diagrammatic representation of a typical electrocardiogram (ECG) waveform with its main features.

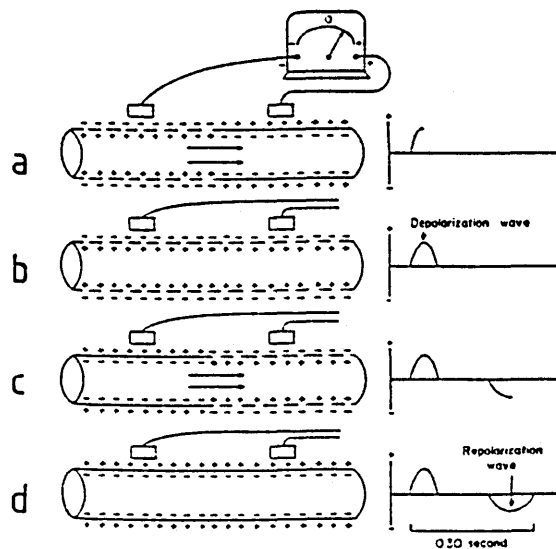


Fig.1.4 The recording of the depolarization and repolarization waves from a cardiac muscle fibre.

### 1.2.1 Excitation sequences of the heart

When the cells are at rest, the relationship of the time sequence of the activation of various tissues to the time sequence of the ECG is approximately represented in Fig.1.5. The first depolarisation occurs in the S-A node. The excitation wave spreads across the atrial myocardium towards the A-V node, and the visible onset of the P-wave occurs as soon as sufficient cells of the S-A node and the surrounding atrial myocardium are actively depolarising to generate a measurable signal at the body surface. The P-wave lasts about 100 millisecond (ms), the time taken for the depolarization to spread throughout the atrial myocardium.

The time taken for the impulse to travel through the atrial ventricular conduction system is highly variable which is indicated by the wide normal range for the interval (the P-Q segment) between the onset of P-wave and the onset of QRS complex (about 120 - 200 ms). The depolarisation of the ventricles takes about 80 ms, which is therefore the duration of the QRS complex. The large number of cells depolarising at any time instant accounts for the large amplitude of the QRS complex observed. The repolarisation of the ventricles is characterised by the T-wave in the ECG. The T-wave is considerably longer in duration and lower in amplitude than the QRS complex because of the prolonged nature of the ventricular repolarisation process (see Fig.1.6).

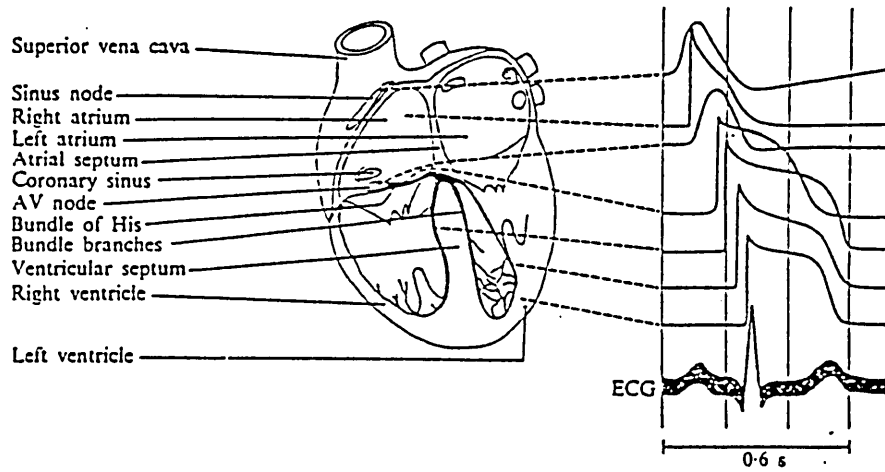


Fig.1.5 The relationship of the time sequences of the activation of various tissues to the time sequence of the electrocardiogram. (From Spach and Barr, 1976 [1].)

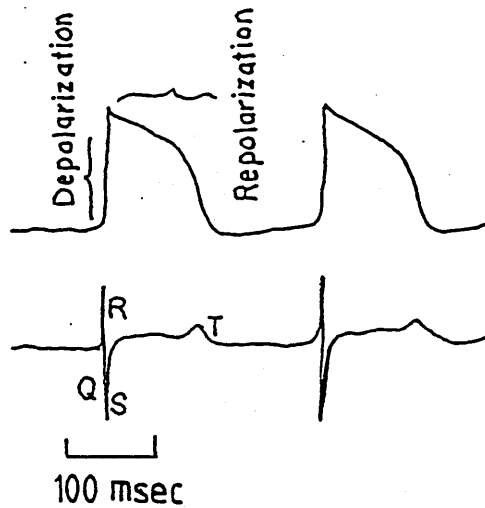


Fig.1.6 The relationship of the unipolar action potential of a ventricular muscle fibre and the electrocardiogram.

### 1.3 Overview of the forward and inverse problems

The two classic problems in electrocardiography, namely the forward problem and the inverse problem have been studied for nearly half a century. The forward problem can be simply defined as the computation of the electrical potentials that would be measured on the body surface, corresponding to known or assumed cardiac sources on or within the heart. The inverse problem can be defined as the determination of the electrical events taking place in the heart at each instant in time, given the corresponding potential distribution on the body surface. The forward problem is normally studied in the form of a model, either physical or numerical, which describes the relationship between the electro-cardio events of the heart and the potential distribution observed on the body surface. Such a model would be of great assistance in studying some of the physiological factors that influence how the body surface potentials are distributed. A general outline of these factors can be found in Lepeschkin (1976) [2]. In particular, with these models, the effects of variation in source and body geometry and volume conductor properties of the thorax on the body surface potentials can be studied. Using a computer model, Lo (1977) [3] showed a significant distortion effect of the simulated body surface potential distribution when various inhomogeneities, such as the lungs, intra-cardiac blood mass, and the anisotropy of the skeletal muscles were introduced. Rudy and Plonsey (1980) [4] used an idealised spherical torso model to study the effects on body and epicardial

surface potential distribution due to body inhomogeneity and displacement of cardiac sources. Although the effect of tissue anisotropy was not included, they produced some interesting results, especially the data that showed that while the body surface potential distribution was very much dependent on the source location, the epicardial potential was almost completely insensitive to variations in the heart locations as occurred in changes of posture. The study by Choi and Pilkington (1981) [5] on the effect of geometrical uncertainty on electrocardiography using a concentric sphere model showed that an uncertainty in geometrical position of  $\pm 0.13$  (this corresponds to an uncertainty of  $1/2 - 1$  centimeter (cm) in man) implied a 19% root mean square error in the computed body surface potentials.

A more significant reason to study the forward problem is its close relationship with the inverse problem. Most approaches to the inverse problem require that the forward problem should first be solved. Apart from its importance in electrophysiology, study of the inverse problem is particularly useful in diagnostic medicine in clinical electrocardiography. The significance of solving the inverse problem is that the features of the potential distribution on the surface over the heart can be related directly to underlying cardiac events without the uncertainty introduced by the intervening tissues. It is known that the ECG obtained from the body surface is a summated and filtered representation of the electrical events occurring in the myocardium. This fact was illustrated when Spach et al. (1977 [6], 1978 [7]) made a

comparison between the isopotential maps of the potential distribution over the body surface and the epicardial surface in chimpanzees. It showed that the surface maps were often much smoother and simpler than the epicardial maps.

#### 1.3.1 The development of scalar ECG lead systems from the dipolar hypothesis

In early electrocardiological studies, the electrical cardiac generators were always regarded as a single dipole vector fixed in position whose magnitude and orientation varied in time. The adoption of this concept was greatly influenced by the work of Waller (1889) [8], who showed that the potential distribution on the body surface was similar to that which would have been generated by a dipole in the chest. By physical laws, two separate measurements perpendicular to each other (Fig.1.7) are required in order to measure the relative magnitude and orientation of this dipole vector. Einthoven, a Dutch physiologist who pioneered many of the electrocardiographic techniques in use today, suggested a new approach to observe the dipole vector [9]. He assumed that the human body could be represented as a flat homogeneous plate in the form of an equilateral triangle (Fig.1.8) where RA, LA and LL corresponded to the potentials at the right arm, left arm and both feet respectively. He then designated the bipolar measurement between RA and LA as lead I, RA and LL as lead II, and LA and LL as lead III (in clinical practice, LL is measured on the left leg for

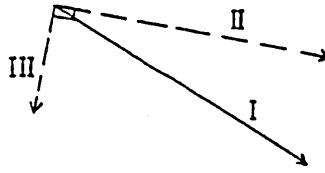


Fig.1.7 Vector summation:  $I = II + III$ .

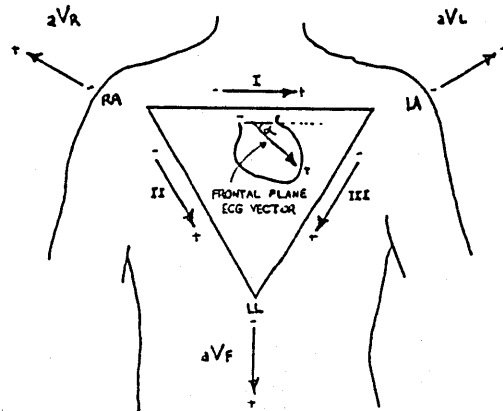


Fig.1.8 The Einthoven triangle, showing the 3 limb leads I, II, III and the 3 augmented limb leads aVR, aVL, aVF.

convenience). The heart was assumed to be at the centre of the triangle and at a certain instant in time the resultant vector of the cardiac electrical potentials assumed a direction with an angle  $\alpha$  referenced horizontally. The convention, facing the triangle,  $\alpha$  has a positive value in a clockwise direction and a negative value in an anti-clockwise direction. This is usually known as the "heart axis". This formed the renowned Einthoven triangle. Einthoven further stated that with the application of physical laws, at any time instant the vector sum of the frontal plane projections of the cardiac vector onto the three axes of the Einthoven triangle will be zero. Mathematically,

$$V_{III} = V_{II} - V_I \quad (1.1)$$

It appears from equation 1.1 that one of the three leads is redundant since it can be derived from any of two known vector measurements. Einthoven carefully illustrated in his study that if both  $V_I$  and  $V_{II}$  happened to be nearly equal, then the calculated value of  $V_{III}$  using equation 1.1 would be very inexact. The practical advantage of adopting this 3-lead measurement is that the limbs can be used for the attachment of measurement electrodes. Burger's distorted triangle (Fig.1.9) was later introduced to compensate for the distortion of the electrical field due to the inhomogeneity effect of the spine and the lungs, a factor which was omitted by Einthoven. It appears that Burger's

distorted triangle is more accurate but since both are rough estimates only, cardiologists prefer Einthoven's approach as a matter of convenience.

As Einthoven's 3-lead system can only reflect the characteristics of the cardiac vector projected on the frontal plane, a new technique was required in order to observe the cardiac vector from the other planes, namely the transverse and the saggital planes (Fig.1.10). This was made possible by the introduction of an "indifferent electrode" by Wilson et al. (1934) [10], (thus it is usually called the Wilson Central Terminal WCT). This terminal was formed by taking the average of the potentials at the right arm, the left arm and the left leg, i.e.:

$$V_{WCT} = (V_{RA} + V_{LA} + V_{LL}) / 3 \quad (1.2)$$

in theory,  $V_{WCT}$  is equal to zero, although in practice a small amplitude can always be measured.

Using the WCT as reference, measurements can be taken with a "chest electrode" placed at various positions on the chest. In common electrocardiographic practice, the six positions (Fig.1.11) are used and they are known as precordial leads ( $V_1$ ,  $V_2$ ,  $V_3$ ,  $V_4$ ,  $V_5$  and  $V_6$ ). These leads measure the transverse plane projection of the cardiac vector.

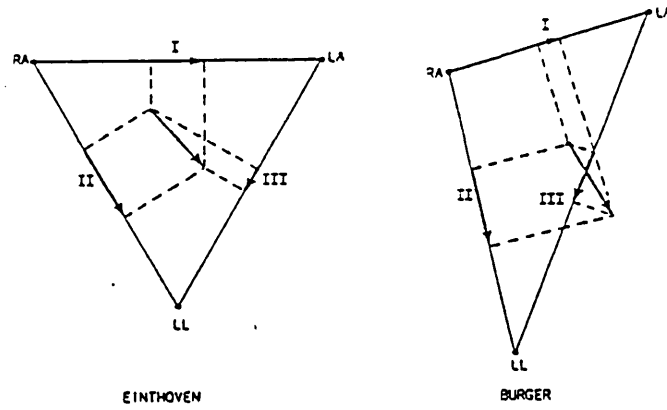


Fig.1.9 The Einthoven triangle and the distorted Burger's triangle.

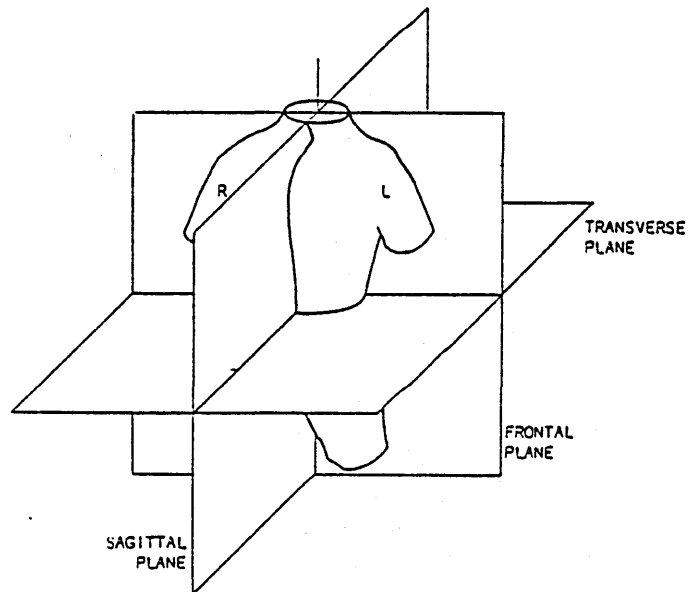


Fig.1.10 The electrocardiographic planes.

Recently, the 9-lead system was developed into the standard 12-lead system by introducing three new measurements known as the augmented unipolar limb lead aVR, aVL and aVF (Fig.1.8). They are the potential measurements of each of the three extremities (right arm, left arm, left leg) and refer to an "indifferent electrode" formed by averaging the potentials of the other extremities. This newly added unipolar limb-lead configuration is simply a rotation by  $30^{\circ}$  of the Einthoven configuration and bears a direct relationship to the three bipolar standard limb leads as follows:

$$aVR = - \frac{I+II}{2}$$

$$aVL = \frac{I-III}{2}$$

$$aVF = \frac{II+III}{2}$$

The projection of the cardiac vector on the sagittal plane can be measured using the unipolar electrode measurement technique. This is sometimes referred to as the "unipolar oesophageal lead", and is done by inserting the measuring electrode into the oesophagus down to the level of the heart (Fig.1.12). This is a very rare clinical practice because the procedure is rather cumbersome and also uncomfortable for the subject.

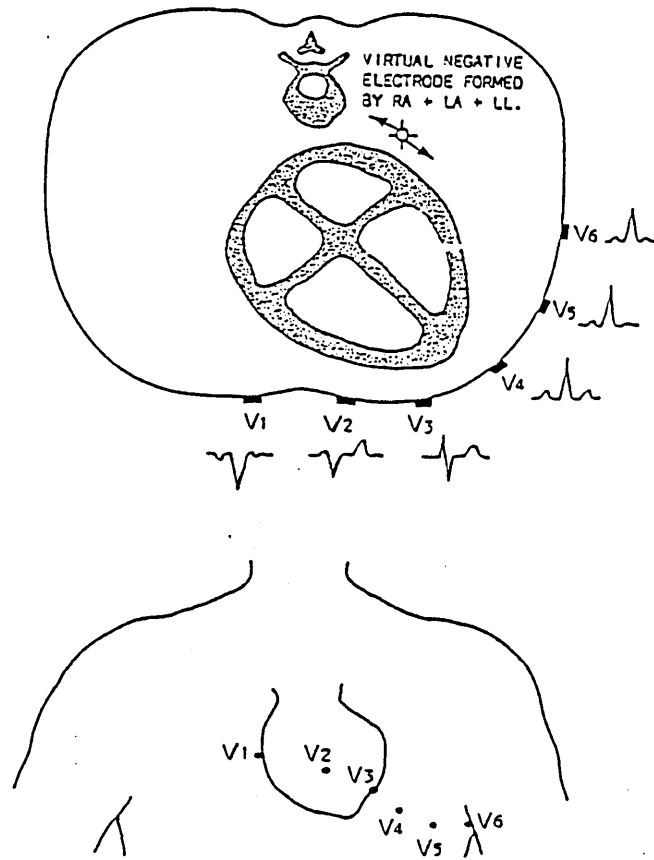


Fig.1.11 The 6 precordial leads. These leads measure the transverse plane projection of the cardiac vector.

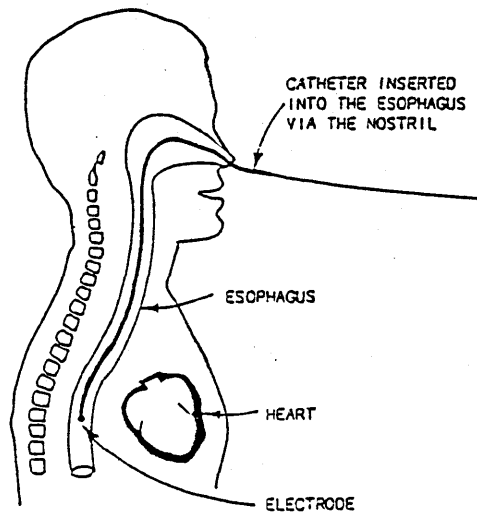


Fig.1.12 The position of the "unipolar oesophageal lead". This measures the sagittal plane projection of the cardiac vector.

### 1.3.2 Body surface mapping

The development of the present standard 12-lead system for electrocardiographic measurement is based on one fundamental assumption that the resultant electrocardial activities can be represented by a centrally located dipole. This over-simplified representation was later disproved by Nahum et al. (1951) [11] and other researchers as discussed further in section 1.5. In order to understand as much as possible of the underlying cardiac activities from body surface measurements, the body surface potential mapping technique is now used extensively for the study of electrocardiology, but has not become widely used clinically. Body surface mapping is not new and ironically was first used by Waller (1889) [8] to demonstrate the dipolar nature of the cardiac electric field, including in his work a diagram (Fig.1.13) showing a doublet distribution of currents and potentials in a human chest.

The conventional 12-lead ECG displays the variation of potential in time due to underlying cardiac activities at the position of interest on the body surface. Body surface mapping differs from this, displaying spatially the potential distribution over the body surface at a certain time instant during the cardiac cycle. The display is normally presented by an iso-potential contouring technique as shown in Fig.1.14.

To generate the body surface map, a large number of body surface potential measurements are required. For example, Nahum

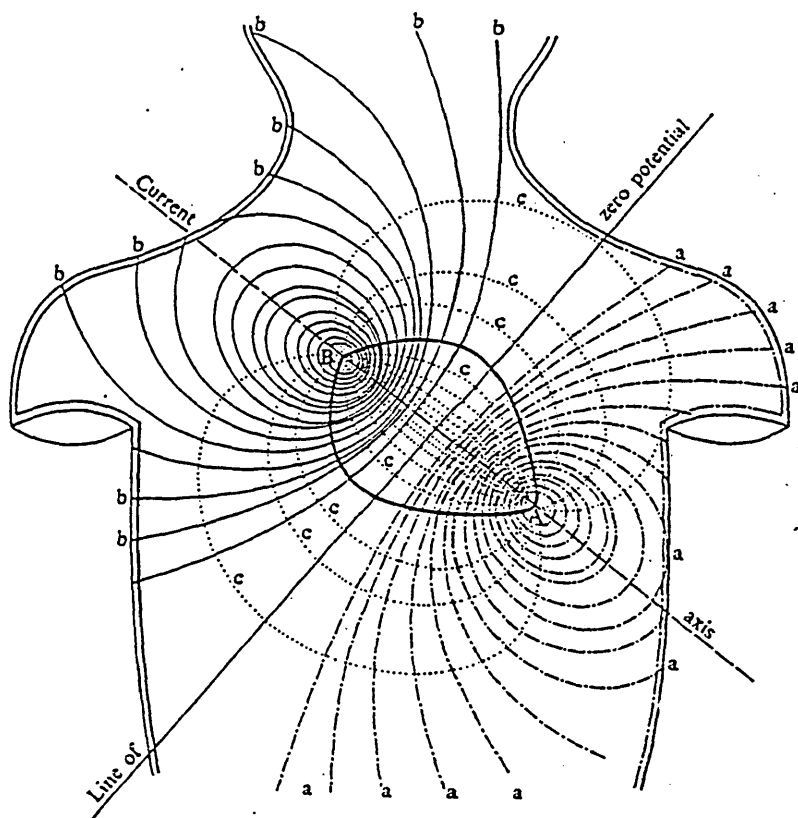


Fig.1.13 The distribution of currents and potentials in a human chest according to Waller's dipolar model of the cardiac generator. (From Waller, 1889 [8].)

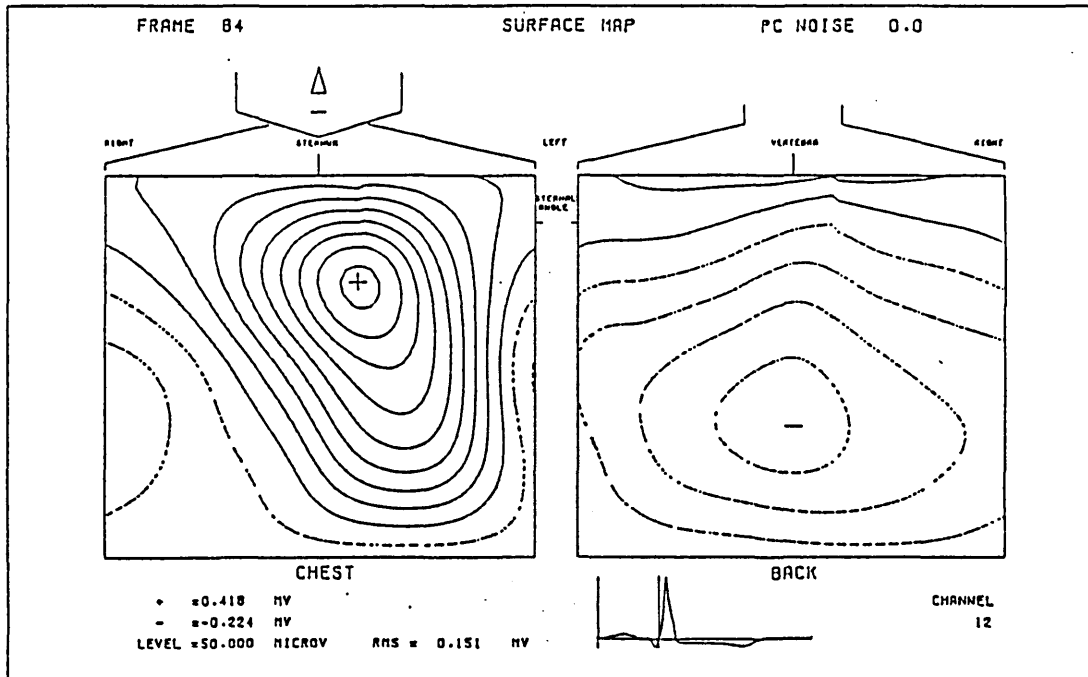


Fig.1.14 The iso-potential contour map. The negative potentials are drawn in dashed lines, and the positive potentials are drawn in solid lines. The first solid line next to the dashed line is zero potential. 'LEVEL' indicates the separation voltage between the iso-potential lines. The positions and magnitudes of the maximum and minimum potentials are drawn as + and - respectively. The scaler plotting of one of the measuring channels is also drawn and a marker indicates the instant in the ECG cycle which corresponds to the map. Each of the 300 time instants (frames) of the whole cardiac cycle can be displayed and they are separated by a 2 milliseconds interval. 'RMS' indicates the root mean square of the spatial potential distribution at the instant.

et al. (1951) [11] used more than 300 measurements on the body surface to generate a surface map for a human subject. This implies a large number of simultaneous recordings, which in terms of instrumentation is very awkward. Alternatively, small subsets of measurements are successively made over one area of the body surface at a time until the whole body surface is measured. This technique reduces the amount of instrumentation but is time consuming and special procedures are required to re-align the timing of all the measurements. Therefore the two methods may be impractical in a clinical situation. This has resulted in a search for a more practical measurement system using fewer recording electrodes which still gives a reliable surface map. This aspect will be further discussed in detail in section 5.2. In this study, the body surface measurements were made using the fully portable 40-channel system developed in the Engineering in Medicine Laboratory at Imperial College. Instrumentation and data acquisition techniques will be described in Chapter 4.

Although current diagnostic techniques like the 12-lead ECG are not adequate in providing details of the cardiac functions and activities, they are still widely used because they are simple and provide crude diagnostic indications in many pathological situations. This is particularly important in view of the ever increasing workload of medical and nursing personnel in present day clinical environment. Any new techniques such as the inverse solution, no matter how sophisticated, has to be tailored for the use of non-engineering personnel. This has always been borne in

mind throughout the development of the equipment and techniques in computing the inverse solution. In the final presentation of the inverse solution, it is necessary not only to adopt the widely used surface potential mapping technique but also retain the more conventional display of the electrocardiograms in time.

#### 1.4 Review of the forward problem

The forward problem has been approached in many different ways in the past, and they can be grouped into three categories as follows:

- (a) Physical model
- (b) Theoretical (analytical) model
- (c) Numerical (digital) model

They are discussed in the following sections.

##### 1.4.1 Physical model approach

The relationship between the electrocardiac source and the body surface potentials has been studied by constructing a physical tank model which simulated the chest configuration and the electrical properties of the thorax in various ways. The advantage of this approach is that the model can be constructed in any shape, but the disadvantages are firstly, it is relatively expensive to build and a future modification implies a reconstruction. Secondly, although inhomogeneity can be and have been simulated to a certain extent by using a tank model, it is still of limited use as it cannot take the anisotropy, such as that of the skeletal muscles, into account.

The first to adopt this approach were Burger and van Milaan (1946) [12] who built a torso-shaped tank with plastic and

other non-conducting materials. The tank was filled with saline solution of a concentration to simulate the electrical properties of the human thorax. Limited inhomogeneity was modelled, such as using sand bags to represent the lungs and insulating corks for the spine. Other inhomogeneities such as liver, blood mass and the anisotropic skeletal muscles were all lumped into one single homogeneous region. Consequently, the model could not be used for more detailed studies, such as the anisotropic effect of the skeletal muscles.

Along this line and supplemented with the resistive network technique of Karplus (1958) [13], Rush (1971) [14] constructed a twice life-size tank model of the human thorax. His model was made up of an array of interlocking insulating plastic rods immersed in electrolytes. The inhomogeneities and anisotropy were simulated by machining the rods in various shape. Rush's model was simpler and more economical than Karplus's resistive network model and overcame the limitations of homogeneity and isotropy in Burger's electrolytic tank model. His model was also much more sophisticated and had a high degree of realism, nevertheless, it still suffered the disadvantage of being difficult to modify.

#### 1.4.2 Theoretical (analytical) model approach

Although a lot of attention had been paid to analytical solutions of electrostatic potential problems [15], they have not

been often used in studying the forward problem in electrocardiography. This is simply because the problem is too complex to be handled analytically. In such studies as have been attempted in nerve physiology for example, Lorente de No, 1947 [16], simplifying assumptions for the conductor being unbounded, homogeneous and isotropic had to be made. In the various attempts to study the electric potential field due to the human heart, similar assumptions have been made to reduce mathematical complexity. The cardiac generators were represented by a centric or eccentric current dipole embedded in the human torso modelled by a sphere (Frank, 1952 [17]), cylinder (Okada, 1956 [18]) and spheroid (Yeh and Martinek, 1957 [19]).

The analytic approach is grossly unsuitable for the purpose of realistic study of the forward and inverse problems, because of the following limitations:

- (i) It is only suitable for unbounded medium.
- (ii) Limited modelling of the inhomogeneity and anisotropy can only be done if they can be described with simple co-ordinate systems such as the cartesian and cylindrical system.
- (iii) Similar to (ii), variation of geometry can only be studied if they are described in the same co-ordinate system. This leads to the limitation to the simple configurations mentioned before.

#### 1.4.3 Numerical (digital) model approach

Since the advent of fast digital computers in the late 1950's, solving the forward problem by numerical techniques has become feasible.

Basically there are two distinct numerical techniques in solving the problem, they are the integral equation and the differential equation approaches. They are discussed in the following sections.

##### 1.4.3.1 Integral equation approach

This method was first introduced and used by Gelernter and Swihart (1964) [20]. They expressed the forward solution in terms of net charges that accumulated at the boundaries of the homogeneous regions. From this the potentials at all points could be computed. This approach reduced a 3-dimensional potential problem into a series of related 2-dimensional ones. Barr et al. (1966) [21] used an alternative approach. By employing Green's theorem, they set up a set of integral equations for the electrical potentials on the boundaries in terms of known generators. In contrast to Gelernter, who had to calculate the desired surface potentials after computing the surface charge, Barr obtained the surface potentials directly.

Following the approach of Gelernter but more vigorously Barnard et al. (1967a [22], 1967b [23]) derived the same set of

integral equations for solving the forward problem and showed explicitly how the time-dependence of the problem can be removed from the solution. They also improved the rate of convergence of the solution by, (i) using a more accurate expression for the electrical interaction between two elements, and (ii) using a deflation technique in the iteration process.

The advantages of the integral equation technique are as follows:

- (i) the integral equations involved only integrals over the boundary surfaces, so a 3-dimensional problem is reduced to a number of related 2-dimensional ones,
- (ii) in solving the equations, it does not require particularly large computer memory and the convergence speed is relatively fast,
- (iii) any chest configuration (surface boundary) can be approximated.

But they suffer the following disadvantages:

- (i) although inhomogeneity can be accounted for, it is limited to a small number of regions only, as the complexity of the problem increases enormously as new regions are added,
- (ii) anisotropy is not included.

These disadvantages can be eliminated by expressing the forward problem in terms of differential equations as will be described next.

#### 1.4.3.2 Differential equation approach

The potential distribution in a conducting medium can be described by Poisson's equation (Plonsey, 1959 [24]). The electrical properties of the physiological tissues found within the human thorax allows us to assume that the capacitive and inductive effects are negligible [24]. In the region external to the myocardium where no electrical source is present, Poisson's equation then reduces to Laplace's equation:

$$\nabla^2 V = 0 \quad (1.3)$$

and can be solved by finite difference or finite element approximation methods. Using the finite difference method to approximate the solution of partial differential equations is common. Klee and Plonsey (1972) [25] employed the method to calculate the three dimensional time-varying bio-potentials of axially symmetric cells. Under the axially symmetric condition (both for the source and the boundaries), the three dimensional problem can then be described by only two dimensions. This reduces the amount of computer storage and execution time in the computational process. Adopting a similar technique, Barker et

al. (1979) [26] studied the action potentials due to active nerve fibre.

Terry (1967) [27] was the first to employ the differential equation approach to the study of electrocardiography. He pointed out that although this procedure involved computing the potentials over the 3-dimensional volume of the human torso, there were certain advantages of this approach which compensated for this increased dimension. Firstly, an individual volume point only interacts with its immediate neighbouring points in contrast to the integral equation approach where each point on the surface interacts with every other point on the surface. Thus although there are more volume points than surface points, the computation time was less because there were fewer interactions between individual points. Secondly, the inhomogeneity and anisotropy of the human thorax could easily be included although to do this a much larger computer memory and execution time are required.

Another popular mathematical technique in solving the partial differential equations is by finite element approximation. Kim et al. (1981) [28] adopted this approach in constructing a three dimensional computer model of the human body.

In the above mentioned approaches, mathematical modelling were used to derive the forward transfer coefficient which describes the relationship between the body surface potentials and the corresponding cardiac events. Because of the recent advances

in experimental techniques, it is possible to measure the body surface and epicardial surface potentials simultaneously in intact dogs. This raises the possibility of studying the forward problem using real data. Hersh et al. (1978) [29] derived the forward transfer coefficient directly from the sequences of epicardial and body surface potential measurements. The result was expected to be better than other techniques, for example, by body geometry measurement (Barr et al. 1977 [30]). This is because the direct measurements should give the most relevant information for the derivation of the forward transfer coefficient. Despite this, two difficulties have to be overcome before this technique could be used clinically, they are:

(i) in order to obtain sufficient data of the epicardial measurement, large numbers of measuring electrodes have to be put over the surface of the heart. It may be feasible in dogs (Hersh used 75 epicardial electrodes), but to extend this experimental protocol to the human subject is out of consideration presently. The first research group known to the author that has successfully measured the body surface potentials and up to three epicardial surface potentials simultaneously on human subjects was Monro et al. (1984) [31],

(ii) even if the epicardial measurement is feasible, the forward transfer coefficient thus derived is only correct for that subject. Inter-subject variation has to be taken into account. The results of the experiments carried out in this study showed

that there was a considerable inter-subject variation in chest and heart configurations as well as the magnitudes of the body and epicardial surface measurements.

In view of the above two obstacles, it appeared that using computer modelling techniques in studying the forward and inverse problems are by all means more practical and advantageous. This approach does not require any in vivo measurement of the epicardial potentials and the inter-subject variation can be studied by modifying the computer model. As real measurements become available, they can be used in evaluating and refining the modelling techniques, as has been done to some extent in this present study.

## 1.5 Review of the inverse problem

Another classic problem in electrocardiography is the inverse problem as introduced in a previous section. Unfortunately, giving the electrical potential distribution on the body surface and the information about the geometry and electrical properties of the medium (i.e. the human thorax) does not necessarily infer the uniqueness of the computed electrical sources in the heart. This is because the electrical potential field due to any configuration of sources can be represented by the potentials on a closed surface enclosing all of the sources. Therefore even if this layer could be uniquely determined from the body surface potentials, there still are infinite numbers of possible source configurations. This well established fact was demonstrated by Helmholtz (1853) [32] over a century ago. This is why it is not feasible to resolve the actual sources. Instead researchers resort to other approaches, including equivalent cardiac generators and more recently epicardial potentials.

### 1.5.1 Equivalent cardiac generators approach

Since the publication of the classic diagram illustrating the distribution of currents and potentials in a human chest by Waller (1889) [8] (Fig.1.13), the concept of representing the cardiac generators by a single dipole has been widely accepted. It appeared that all the information on the electrical activities of the heart could be obtained by determining the strength, orientation and polarity of the single dipole. In this respect,

Einthoven et al. (1950) [9] developed a mathematical function to determine the potentials due to a dipole, assuming that the potential maxima of the dipole were very close together and relatively remote from the recording electrodes i.e. the extremities. They also introduced the renowned Einthoven triangle to calculate the frontal component of the equivalent dipole.

This over simplified representation of the cardiac generators by a single dipole was disproved by Nahum et al. (1951) [11]. They successfully recorded potentials of more than 300 points on the thoracic surface of a human subject and manually produced a series of isopotential maps during the QRS complex of the cardiac cycle. They observed that the body surface potential distribution was much more complicated from that of the surface doublet distribution described by Waller. They also reported the impossibility of employing the simple Einthoven formulation in defining the angle and the direction of the 'heart axis'. Since then, the appearance of many other publications (Taccardi, 1951 [33]; Nelson, 1957 [34]; Taccardi, 1960 [35], 1962 [36], 1963 [37]) strengthened the case against a dipolar interpretation of the body surface potential field and strongly suggested that a more complex electrical model of the heart should be used. This did not only account for the complex surface potential field distribution observed but also indicated that the local activities of different parts of the heart could be determined in the study of inverse problem. The two most common propositions are the multiple dipole and multipole models.

Among the first to adopt the multiple dipole heart model in the inverse problem were Bellman et al. (1964) [38]. They did preliminary investigation on determining the current dipole moments from the given surface potentials. In their study they imposed temporal constraints on the dipole moments which corresponded to assumed movements of the depolarization waves over the ventricles. They also assumed the intervening medium to be homogeneous and of infinite extent. Although the reconstruction of the body surface potentials from the computed current dipole moments was good, neither the uniqueness nor the correctness of the inverse solution were shown.

The dipoles were fixed in position and direction in various later studies but allowed to have positive and negative magnitudes. But it was shown that this type of model was very unstable (Rogers, 1967 [39]; Brody, 1966 [40]; Lynn et al., 1967 [41]). Rogers and Pilkington (1968) [42] further investigated the stability of the inverse solution using such a free-moment dipole model and established that the maximum number of free-moment dipoles that could be used without incurring serious solution error was 20. They further concluded that the noise sensitivity of the solution was a function of the model geometry rather than the computational process. They also showed that by reducing the number of dipoles from 20 to 9, the noise sensitivity of the solution could be reduced by a factor of 10. The trade-off was that fewer dipoles would lead to an extremely poor heart representation.

In order to overcome this instability problem, different types of constraints have been used. Lynn et al. (1967) [41] introduced a positive-moment dipole model, i.e. the dipoles were fixed in position and orientation but constrained to a non-negative moment. No constraints in time nor strength were imposed upon the dipoles. They formulated the inverse problem mathematically into a minimization problem, and the inverse solutions were obtained by Wolf's (1959) [43] method of quadratic programming which in essence was similar to least squares fitting but constrained to produce non-negative results only. This positive-moment dipole approach was later vigorously examined by Brody and Hight (1972) [44] with accurately determined model data. It was shown that perturbation of data by contaminating the original input signal with arbitrary random noise and randomly dislocating the equivalent dipoles would degrade the quality of the inverse solution, but would still remain reasonably stable. They further showed that under certain conditions, such as 'ambiguity' and 'competition' between two dipole surfaces, inverse solution would become considerably less acceptable. As would be expected, quantitative evaluation of the results was not possible as no experimentally measured data of the time history of the dipoles was available. Nevertheless it has been shown that by suitably constraining the dipoles, the instability problem could then be overcome.

Attempts at direct comparison of the computed inverse solution with the measured data (in dog) using the multiple dipole

approach was done by Barr et al. (1970) [45]. They imposed even more constraints and produced what they called an on-off model. The equivalent dipoles assigned to the 10 zones of the heart model were either completely active (on) or completely inactive (off) at any one instant in time and when active, the dipole was given a non-negative dipole moment whose magnitude was precalculated to be proportional to the area of heart surface represented. Although the computed solution bore a number of similarities to the activation sequences measured from a dog, there were a few setbacks in this approach. Firstly, as noted by the authors, the resolution of the solution was poor using a 10-dipole heart model and secondly, applying such a severe constraint might produce results in predetermined model subjects but the constraints themselves could prevent the detection of abnormal cardiac activity in clinical application.

While the multiple dipole model was being used extensively in various studies, other researchers investigated the significance of the multipole model in representing the equivalent cardiac generator. In essence, this approach assumed a set of sources, the multipoles, at a point and the potential distribution within a finite medium outside the surface enclosing the set of sources could be expressed with an infinite series. This concept was adopted by Yeh and Martinek (1957) [46] who used a concentric spherical model to approximate the multipolar components from the surface potential measurements. Gelselowitz (1960) [47], using Green's theorem, developed a mathematical representation of the

potential field distribution within an irregularly-shaped, finite, homogeneous conducting medium in terms of higher-order multipole components.

Hlavín and Plonsey (1963) [48] studied the multipolar components using an isolated turtle heart suspended in a spherical electrolytic tank. The results showed that the quadrupolar component of the equivalent heart source made a significant contribution to electrocardiograms measured at a distance of about two times the turtle radius. Heppner (1968) [49] also studied an isolated turtle heart using a different method of analysis and showed similar results. By making use of the orthogonality properties of the Legendre functions it was possible to compute each multipole coefficient to a fairly high degree of accuracy. This eliminated the errors due to truncation of the multipole series in the technique used by Hlavín.

Brody et al. (1971) [50] used a carefully designed data acquisition system, and conducted a series of fourteen separate measurements of the electric field potentials generated by an isolated turtle heart. Their results showed that the contribution of the quadrupolar components towards the surface potentials ranged from 6% to 30% and that of the octapolar components was 6% to 29%.

Despite its mathematical elegance, the multipolar approach was mainly confined to research work in electrocardiography and gained little application in clinical

areas. This was mainly due to the mathematical complexity of the formulation and its clinical usefulness over other techniques has yet to be proved.

The multipolar approach has been used in studying human electrocardiography by other research workers such as Schubert (1968) [51] and Arthur et al. (1971 [52], 1972 [53]). The first real attempts in studying the inverse problem by the multipolar approach, however, were by Guardo (1972) [54], and Arthur (1972) [53]. In all these studies, the equivalent cardiac sources were fixed in location, except for Guardo (1972) [54], whose multipolar series were mobile.

Other researchers have worked with moving sources whose positions, amplitudes and orientations in general vary in time. The parameters are normally optimally fitted to the measured body surface potentials. This moving dipole concept was first introduced by Gabor and Nelson (1954) [55], and since then, has been applied in various studies. These included using a rabbit heart (Ideker et al., 1975 [56], 1977 [57] and Mirvis et al., 1978 [58]) pigs (Hodgkin et al., 1976 [59]), and in intact dogs (Savard et al., 1980 [60]). Generally, these studies have produced plausible means of describing the equivalent cardiac generator, especially for well-localised cardiac sources such as ectopic foci or local cardiac disturbance like bundle branch block and regional ischaemia. In man, measurements which can be used to describe the cardiac sources accurately have not been made. Therefore

investigations of moving dipoles in man have been based on model studies. This has been attempted by Arthur et al., 1971 [52]; Horan and Flowers, 1971 [61]; Guardo, 1972 [54] Kneppo and Titomir, 1979 [62]; Baldwin and Rush, 1979 [63]).

Despite all these developments, the approach to inverse electrocardiography by means of equivalent cardiac generators offers few possibilities for clinical applications. This is not only because the solution obtained cannot be easily evaluated by direct comparison with in-vivo measurement, but also it is in essence an artificial representation and relates only with difficulty to the sequences of depolarisation and repolarisation of the heart. The depolarisation of the heart is distributed spatially throughout the myocardium, and as a distribution is of significance in studying localised cardiac abnormalities such as regional myocardial infarct.

#### 1.5.2 Epicardial potential approach

In the epicardial potential approach, the inverse solution is computed directly in terms of voltage as a function of time at a predetermined position. In this study, this is the determination of the potentials over a surface surrounding the heart. By this approach, some difficulties inherent in the equivalent cardiac generator approach can be overcome. First of all, and most appealingly, it is a form of inverse solution whose uniqueness is theoretically possible (Martin and Pilkington, 1972

[64]; Yamashita, 1982 [65]). Secondly it is feasible to make a direct comparison to measurements in vivo as will be seen. Other advantages of this approach over the more conventional equivalent cardiac generator approach are that the knowledge of the cardiac potential source is not required and the effect of the anisotropy of the tissues like the skeletal muscle, the inhomogeneity and the complex chest configuration can be taken into account into the computation. The inverse solution in terms of potentials still does not infer the complete knowledge of the cardiac generators in the myocardium, nevertheless, the underlying cardiac activities can be adequately visualised (Taccardi and Marchetti, 1965 [66]; Spach et al., 1969 [67]).

Martin and Pilkington (1972) [64] were the first to adopt this approach to the inverse problem. They developed a mathematical model based on Green's theorem. Their model had a disadvantage when the internal inhomogeneities in the human thorax such as the lung regions were included. To do this, a knowledge of the boundary potentials is required, and the computational burden is increased significantly. They further theoretically showed the limitation of accuracy to which epicardial potentials could be deduced from body surface potentials. The results obtained by applying the formulation to a concentric spherical system suggested that without any constraint, as in the equivalent cardiac generator approach, the inverse solution became very unstable even in the presence of a small amount of noise in the body surface data.

This instability problem does not only apply to the inverse problem in electrocardiography, it also occurs in other areas where the characteristics of some physical sources are to be deduced from remote sensing. If it is assumed that the source does not consist only of single oscillatory components, then at distant observation points, because of the probable cancellation effect between close opposite peaks in the source, only the smooth overall characteristics of the source are observed. The noise present in the observed data due to measurement error will then be viewed as the genuine effect of a large oscillatory source component. It is apparent that in order to obtain a reasonably stable solution, additional information or constraints have to be used to suppress the errors introduced by measurement or computation. This approach was adopted in this study and is further discussed in Chapter 3.

Constrained inverse solutions have been used before, Martin et al. (1975) [68] applied a statistical constraint based on the theoretical work by Foster (1961) [69], in an attempt to compute the potential distributions over a spherical surface between the torso and the heart of a dog from the torso surface potentials. The torso surface potentials were obtained from the activation data measured previously [42]. The constraints imposed required a priori knowledge of the characteristics of the signal (epicardial potential) and the noise (body surface noise) which in practice would be difficult to estimate. Martin et al. obtained these characteristics statistically and showed that the error

introduced into the inverse solution due to perturbation from various sources was not unacceptable, although the results have yet to be proved physiologically correct.

Barr and Spach (1978) [70] adopted basically the same approach as Martin et al. to calculate the epicardial potentials during the QRS-T periods from body surface potentials in intact dogs. At the same time epicardial surface potentials were measured so that comparison could be made with the computed solutions. They circumvented the problem of identifying the magnitude of signal and noise by making an assumption that over a long period of time, the potential at each epicardial measurement site had the same RMS value and were uncorrelated. The same assumption was made for the noise on the body surface, and a priori knowledge of the mean square noise to signal ratio is then only required. As a result, the major features of both the computed and measured epicardial maps were similar, for example the presence of multiple maxima and minima. There were, however, many differences in voltage magnitude at individual electrode sites, and the overall pattern of equipotential contours was quite different.

## 1.6 Objectives of this study

This study is concerned with computing the epicardial surface potentials from body surface ECG measurements. This is approached through the forward problem, which must be solved successfully and realistically in order to derive the forward transfer coefficient which describes the relationship between the potential distribution over the body surface and the epicardial surface. From the forward transfer coefficient, an inverse solution is derived.

Mathematically, the forward problem can be expressed as:

$$AE = S$$

where  $E$  = column vector representing the potentials at the epicardial surface

$S$  = column vector representing the potentials at the thoracic surface

$A$  = matrix representing the forward transfer coefficient describing the relationship between  $E$  and  $S$

In this study, the forward transfer coefficient matrix  $A$  is derived using a computer model of the human thorax with known geometry and electrical properties. The technique and the computer model of the human thorax used in this study is basically a further development of the previous studies in the same

laboratory (Lo, 1977 [3]). In this study, the following modifications and improvements have been made.

- (i) A new discrete anatomical model of the human thorax using a 1-cm uniform grid is constructed. This gives a better representation of the human thorax (see section 2.4).
- (ii) A new technique of segmenting the epicardial surface into unequal size segments is used. This reduces the 'condition number' of the forward transfer coefficient matrix (see section 3.2).
- (iii) A new computational technique is implemented. This minimizes the amount of central processor memory and the amount of execution time used (see section 2.5).

The main difficulty in deriving the inverse solution from the forward solution arises from the fact that it is a so called 'ill-posed' problem, i.e. a small perturbation on the body surface data can produce a huge change in the inverse solution. In this study, the instability problem is reduced in two ways, first of all by formulating the forward problem to give a reduced condition number, and secondly by a manipulation of the forward transfer coefficient which stabilizes the inverse solution.

In the past, inverse solutions have been approached by many different techniques either in terms of current generating dipoles or potential distribution over the epicardial surface.

Most of the results reported were either not tested against corresponding experimental measurements or using measured data from dogs or chimpanzees. In no previous study was human data available. In this study, simultaneous measurement of 37 body surface electrocardiograms and 3 epicardial electrocardiograms on 21 human subjects were made. These measurements allow an evaluation of the inverse solution directly.

In Chapter 2, the mathematical formulation and the computer implementation of the human thorax model used in this study is presented.

In Chapter 3, the development of a regularisation technique in solving ill-conditioned matrix equation applicable to the inverse problem is presented. The stability and the correctness of the solutions are also studied.

The protocol and the instrumentation used to collect body and epicardial surface measurements in human subjects and the data pre-processing requirements are described in Chapter 4.

In Chapter 5, the computational techniques developed in Chapter 2 and 3 are used to calculate the epicardial surface potentials from body surface potential measurements of human subjects, and the results are compared with measured data.

A different approach to analysing the simultaneous measurements of epicardial and body surface potentials on human subjects is taken in Chapter 6. These are analysed in a

statistical sense, and their characteristics and correlation with reference to the results predicted by the computer model are discussed.

Finally, Chapter 7 gives the general conclusion of this work.

## CHAPTER 2

### FORMULATION AND IMPLEMENTATION OF A COMPUTER MODEL OF THE HUMAN THORAX AS A VOLUME CONDUCTOR

#### 2.1 Introduction

In the study of the forward problem by simulation of the human thorax with a numerical model, there are in principle two approaches, either a model with differential equations or a model with integral equations. As discussed in section 1.4.3, the differential equation approach is preferred to the integral equation approach. There are several numerical techniques in solving the differential equations and one of the simplest to implement is the finite difference approximation. In this study, a three-dimensional computer model of the human thorax is constructed by describing the potential distribution in the conducting medium (i.e. the human thorax) with known electrical properties by a set of differential equations. This is then approximated by a set of homogeneous linear algebraic equations using the method of finite difference. This set of algebraic equations is then solved by the Gauss-Seidel iterative technique. The inhomogeneities of various internal organs and the anisotropy of the skeletal muscle are taken into account. In section 2.2, a mathematical formulation of the electrical

potential field in the human body is presented and in section 2.3, the method for solving the problem of potential field distribution in a three-dimensional volume conductor by finite difference approximation is described. Section 2.4 presents the construction of the computer model of the human thorax from anatomical data and the computational techniques for solving it. In section 2.5 a brief discussion on the programme implementation is presented.

## 2.2 The fundamental equations

Assuming the electrical field problem in the human body is quasi-stationary (Plonsey and Heppner, 1967 [71]), then the relation between the current density  $J$  ( $A.m^{-2}$ ) and the electrical field  $E$  ( $V.m^{-1}$ ) is given by:

$$J = \sigma E \quad (2.1)$$

where  $\sigma$  is the conductivity ( $S.m^{-1}$ ).

The current density  $J$  is related to the current source density  $I$  ( $A.m^{-3}$ ) by:

$$\nabla \cdot J = I \quad (2.2)$$

where  $\nabla$  is the del operator.

In the stationary case, the potential  $\phi(V)$  obeys the following equation:

$$E = - \nabla \phi \quad (2.3)$$

Combining (2.1) and (2.2),

$$I = \nabla \cdot \sigma E \quad (2.4)$$

and with equation 2.3,

$$I = -\nabla \sigma (\nabla \phi) \quad (2.5)$$

If the conductivity  $\sigma$  can be referred to the principal axes X, Y and Z of the Cartesian coordinate system, and referring J and E to the same principal axes, then expanding (2.1),

$$J_x = \sigma_x E_x$$

$$J_y = \sigma_y E_y$$

$$J_z = \sigma_z E_z$$

and expanding (2.5),

$$I = - \left[ \frac{d}{dx} \left( \sigma_x \frac{d\phi}{dx} \right) + \frac{d}{dy} \left( \sigma_y \frac{d\phi}{dy} \right) + \frac{d}{dz} \left( \sigma_z \frac{d\phi}{dz} \right) \right] \quad (2.6)$$

In order to find the potential distribution in the body, we have to solve (2.6). In the region outside the myocardium (i.e. conductive regions bounded by the body surface and the surface surrounding the heart), one can assume that no electrical source is present, then equation 2.6 reduces to:

$$\sigma_x \frac{d^2\phi}{dx^2} + \sigma_y \frac{d^2\phi}{dy^2} + \sigma_z \frac{d^2\phi}{dz^2} = 0 \quad (2.7)$$

The potential field distribution of the conducting medium between the two surfaces is then described by equation 2.7. Moreover, the potential has to satisfy the following boundary conditions:

- (a) potentials are given on the surface surrounding the heart,
- (b)  $\frac{d\phi}{dN} = 0$  on the body surface,

where N is the outward normal at the body surface.

There are several ways of solving the above stated problem. The finite difference approximation is used in this study, and is described in the following section.

### 2.3 Method of finite difference

The finite difference method is a well known technique for approximating the solution of partial differential equations such as those in the electrical field problem in Electrocardiography. Basically, the method involves the replacement of the original distributed field by discrete elements in such a way that the characteristics of the original field are conserved. The finite difference method has the advantage over the analytical approach in that it can not only model any configuration of the field but also simulate the properties of inhomogeneity and anisotropy. However, it requires extensive computing resources and storage (Heringa et al., 1982 [72]).

In order to illustrate this technique a 2-dimensional potential field problem is considered. The 2-dimensional region  $S$  has conductivities  $\sigma_x$  and  $\sigma_y$  in the two principal axes and the region is discretised with a uniform grid as shown in Fig.2.1.

Assuming that the potential  $V$  at any arbitrary nodes 0,1,2,3 and 4 are known and the voltage gradient is constant throughout the field, then the average voltage gradient at points midway between node 0 and its 4 neighbouring points are approximated as follows:

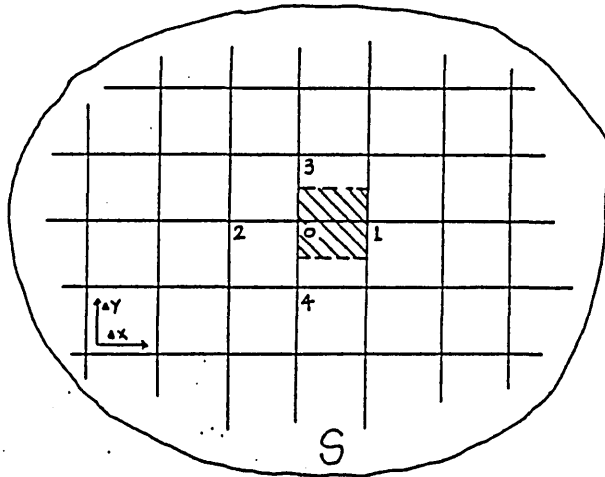


Fig.2.1 Discretization of a 2-dimensional field with a regular grid, showing a discrete approximation of the current paths between 0 and 1 by the shaded block.

$$\sigma_x \frac{dV}{dX}_{1-0} = \sigma_x \frac{V_0 - V_1}{\Delta x} ; \quad \sigma_x \frac{dV}{dX}_{0-2} = \sigma_x \frac{V_2 - V_0}{\Delta x}$$

$$\sigma_y \frac{dV}{dY}_{3-0} = \sigma_y \frac{V_0 - V_3}{\Delta y} ; \quad \sigma_y \frac{dV}{dY}_{0-4} = \sigma_y \frac{V_4 - V_0}{\Delta y}$$

and the second space derivative is approximated as,

$$\sigma_x \frac{d^2V}{dX^2} = \frac{\sigma_x}{\Delta x^2} (V_1 + V_2 - 2V_0) \quad (2.8)$$

$$\sigma_y \frac{d^2V}{dY^2} = \frac{\sigma_y}{\Delta y^2} (V_3 + V_4 - 2V_0) \quad (2.9)$$

Assuming  $\Delta x$ ,  $\Delta y$  are equal and have a distance of unity,  
then adding equations 2.8 and 2.9 gives,

$$\sigma_x \frac{d^2V}{dX^2} + \sigma_y \frac{d^2V}{dY^2} = (\sigma_x V_1 + \sigma_x V_2 + \sigma_y V_3 + \sigma_y V_4 - V_0 2(\sigma_x + \sigma_y)) \quad (2.10)$$

Assuming no electrical sources are present, and applying equations  
2.7 and 2.10 in 2-dimensions, then,

$$\sigma_x V_1 + \sigma_x V_2 + \sigma_y V_3 + \sigma_y V_4 - V_o (2\sigma_x + 2\sigma_y) = 0 \quad (2.11)$$

For a 3-dimensional potential field, it can be easily shown that,

$$\sigma_x V_1 + \sigma_x V_2 + \sigma_y V_3 + \sigma_y V_4 + \sigma_z V_5 + \sigma_z V_6 - V_o (2\sigma_x + 2\sigma_y + 2\sigma_z) = 0 \quad (2.12)$$

The conversion from the distributed problem in (2.7) to a discrete problem in (2.12) is the equivalent of approximating the continuous volume conductor by a network of resistors. In practice, to build a network of resistors to simulate the human thorax would be very tedious and cumbersome (Karplus, 1958 [13]). Instead, the discrete model can be implemented on a computer. The resistivity values are chosen to approximate the real conductor as closely as possible.

Following the above illustration, a three-dimensional numerical model of the human thorax is constructed based on the finite difference approximation of Laplace's equation. In order to understand this concept better, it is best to consider a two-dimensional conducting medium, and then develop on to a three-dimensional type like the human thorax.

### 2.3.1 Two-dimensional numerical model

Let us consider the 2-dimensional uniform field region S, superimposed on top of an interconnecting resistive network as shown in Fig.2.2.

The values of the resistors are chosen to simulate the resistance (associated with a specific vector area e.g. shaded area in Fig.2.1) in the original field to the current flow in the 2 principal directions. Considering any arbitrary node 0 and applying equation 2.11, we have,

$$G_1 V_1 + G_2 V_2 + G_3 V_3 + G_4 V_4 - (G_1 + G_2 + G_3 + G_4) V_0 = 0 \quad (2.13)$$

where  $G_n$  is the conductance in direction n.

A set of homogeneous linear algebraic equations can then be written to describe all the nodal potentials within the field, and the solution of the equations approximates the potential at the corresponding point in the original field.

The inhomogeneity, anisotropy and the boundary condition on the body surface are simulated in the following manner:

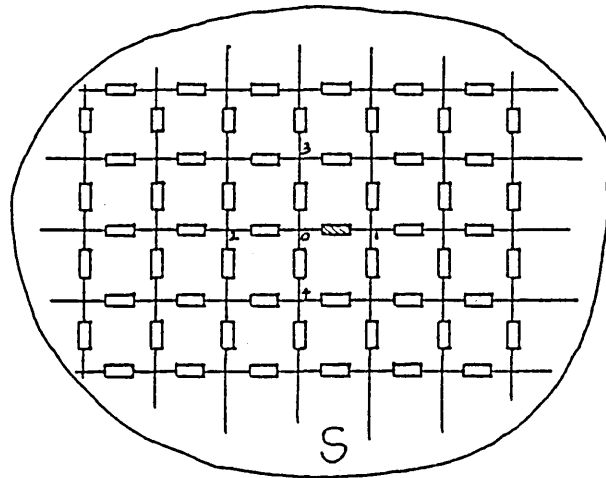


Fig.2.2 A resistive network approximation of a distributed field. The shaded resistor between 0 and 1 is to simulate the resistive pathway of the shaded area in Fig.2.1.

a). Inhomogeneity

As  $G_1$  is chosen to represent the conductance along the current path in the original field between node 1 and 0 (Fig.2.3) which is associated with the vector area LMPQ. This in turn is composed of vector areas LM10 and 01PQ, thus  $G_1$  can be approximated by a parallel combination of  $G_a$  and  $G_b$  which represents the different electrical properties of elements A and B respectively.

Equation 2.13 can be rewritten to simulate the inhomogeneity in the original field as :

$$\begin{aligned} (G_a+G_b) V_1 + (G_d+G_c) V_2 + (G_c+G_a) V_3 \\ + (G_b+G_d) V_4 - GV_0 = 0 \end{aligned} \quad (2.14)$$

where  $G = 2(G_a+G_b+G_c+G_d)$

b). Anisotropy

The anisotropic property of the tissue such as the skeletal muscle is simulated by the following technique:

Consider AB in Fig.2.4.a as the preferred current path along the skeletal muscle fibre. The pathway is discretised with a uniform grid as shown in Fig.2.4.b. In order to simulate the anisotropic effect, 4 resistive values must be individually

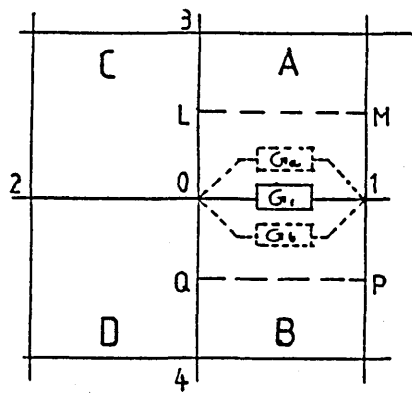


Fig.2.3 Replacement of  $G_1$  by two parallel components  $G_a$  and  $G_b$ .

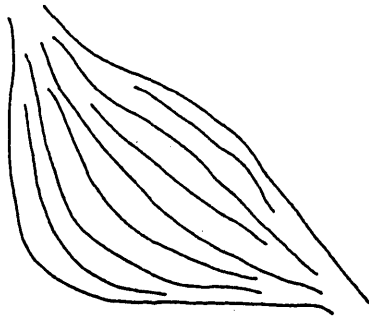


Fig.2.4a A diagrammatic representation of a typical muscle layer. The 'flow lines' indicate high conductive paths.

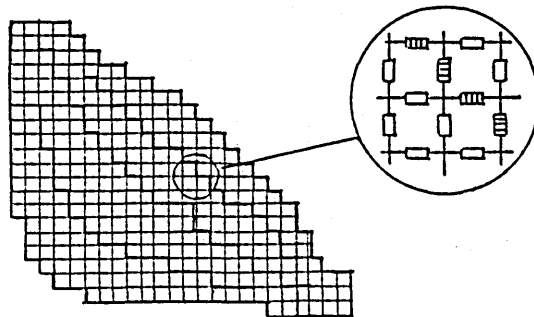


Fig.2.4b A discrete representation of the anisotropic muscle layer. The heavy lines indicate paths connected by high conductive components.

identified for each element as shown in Fig.2.5.

Equation 2.14 can now be rewritten to take the anisotropy into account as follows,

$$\begin{aligned} & (G_a^3 + G_b^1) v_1 + (G_d^1 + G_c^3) v_2 + (G_c^2 + G_a^4) v_3 + \\ & (G_b^4 + G_d^2) v_4 - Gv_0 = 0 \end{aligned} \quad (2.15)$$

where  $G = (G_a^3 + G_b^1) + (G_d^1 + G_c^3) + (G_c^2 + G_a^4) + (G_b^4 + G_d^2)$

c). Boundary condition on the body surface

The body surface boundary is a tissue-air interface in which the air is a non-conducting medium, so no current is allowed to flow through this boundary. This formulates one of the two boundary conditions for the potential field problem of the human thorax as laid down in section 2.1, i.e.  $\frac{dv}{dN} = 0$  on the body surface. In the numerical model, this condition is satisfied by assigning the cubic elements outside the boundary to have a conductance of zero.

### 2.3.2 Three-dimensional numerical model

The approximation of a 2-dimensional field by the finite

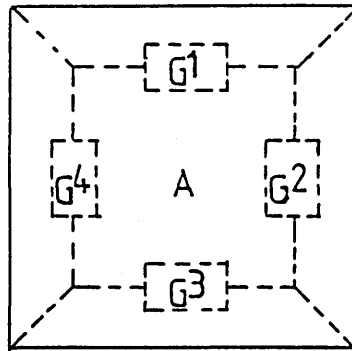


Fig.2.5 The anisotropic elemental conductor is represented by 4 conductive values.

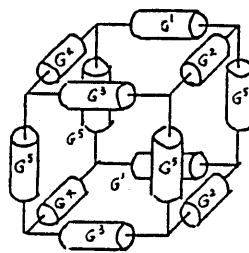


Fig.2.6 Equivalent network for a 3-dimensional elemental conductor. Only 5 resistive values needed to simulate the anisotropic effect of the skeletal muscle.

difference method can be extended to a 3-dimensional field with little modification. In theory, to simulate the anisotropy within the model, 6 different resistive values have to be identified (four in the horizontal direction and two in the vertical direction). In the case of skeletal muscle in the human torso, it can be approximated that the anisotropy only occurs in the horizontal plane (Rush et al., 1963 [73]), thus, five different resistive values are required in the discrete elemental volume conductor (Fig.2.6). Extending equation 2.15, we have the finite difference equation for an arbitrary node 0, associated with its neighbouring nodes as shown in Fig.2.7, in the discretised field as,

$$\begin{aligned} & (G_a^5 + G_b^5 + G_c^5 + G_d^5) V_1 + (G_e^5 + G_f^5 + G_g^5 + G_h^5) V_2 + \\ & (G_a^3 + G_d^1 + G_e^3 + G_h^1) V_3 + (G_b^3 + G_c^1 + G_f^3 + G_g^1) V_4 + \\ & (G_c^4 + G_d^2 + G_g^4 + G_h^2) V_5 + (G_a^2 + G_b^4 + G_e^2 + G_f^4) V_6 - \\ & GV_0 = 0 \end{aligned} \tag{2.16}$$

where  $G$  is the sum of the coefficients of  $V_1, V_2, V_3, V_4, V_5$  and  $V_6$ .

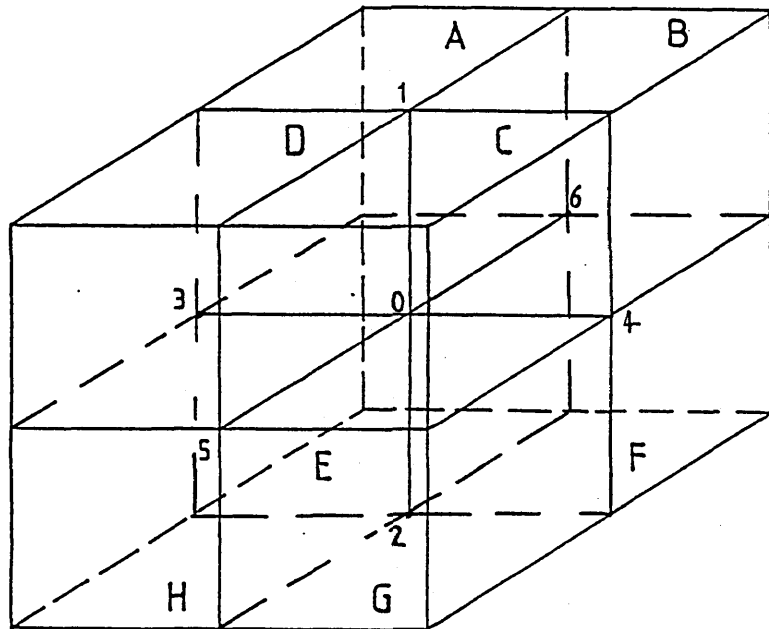


Fig.2.7 An arbitrary node 0 associated with its 5 neighbouring nodes in the 3-dimensional discretised field.

## 2.4 Discrete model of the human thorax

### 2.4.1 Introduction

In any discretisation procedure, the finer the grid interval, the better the approximation to the original by the discretised function. In practice, a finite size of the grid interval has to be chosen due to limitations of computer memory and execution time. In this study, the human thorax was discretised with a regular three-dimensional grid of one centimetre cubes. The size of the model was 19x33x22. This discretised model consisted of 22 slabs covering from the 2<sup>nd</sup> down to 12<sup>th</sup> thoracic vertebra. Each individual cubic element was then assigned an alphanumeric code corresponding to its anatomical structure. Each of these elements is regarded as being homogeneous (not necessarily isotropic), and their conductivity is determined by the electrical properties of the corresponding tissues. Fig.2.8 shows one slab of the discretised thorax. The representation of the human thorax model in discretised form is shown in Appendix A1.

The anatomical data on which the model is based in this study was initially obtained from The Atlas of Anatomical Cross-section of Human Body by Symington (1956)[74]. The model was then modified to be more realistic using additional information of the anatomical data of the human thorax from the Cross-sectional Anatomy by Ledley et al. (1977) [75]. The conductivity values of the tissues used in this study as shown in

| ANATOMY         | CODING | RESISTIVITY<br>(ohm-cm)                    | CONDUCTIVITY RATIO |     |     |     |     |
|-----------------|--------|--------------------------------------------|--------------------|-----|-----|-----|-----|
|                 |        |                                            | G1                 | G2  | G3  | G4  | G5  |
| HUMAN TRUNK     | T      | 463                                        | 1.0                | 1.0 | 1.0 | 1.0 | 1.0 |
| BLOOD           | M      | 162                                        | 2.8                | 2.8 | 2.8 | 2.8 | 2.8 |
| HEART           | H      | 377                                        | 1.2                | 1.2 | 1.2 | 1.2 | 1.2 |
| LUNGS           | L      | 2100                                       | 0.2                | 0.2 | 0.2 | 0.2 | 0.2 |
| LIVER           | R      | 700                                        | 0.6                | 0.6 | 0.6 | 0.6 | 0.6 |
| SKELETAL MUSCLE | 1      | 2100 (high)<br>150 (low)<br>272 (vertical) | 3.0                | 3.0 | 0.2 | 0.2 | 1.7 |
|                 | 2      |                                            | 0.2                | 3.0 | 3.0 | 0.2 | 1.7 |
|                 | 3      |                                            | 0.2                | 0.2 | 3.0 | 3.0 | 1.7 |
|                 | 4      |                                            | 3.0                | 0.2 | 0.2 | 3.0 | 1.7 |
|                 | 5      |                                            | 3.0                | 0.2 | 3.0 | 0.2 | 1.7 |
|                 | 6      |                                            | 0.2                | 3.0 | 0.2 | 3.0 | 1.7 |

TABLE 2.1 Table of codings and the conductivity ratios.

Table 2.1 were obtained from the data prepared by Rush et al. (1963) [73] and Rush and Nelson (1976) [76].

#### 2.4.2 Computational technique

Employing the numerical technique and the discretised model of the human thorax described above, a set of linear algebraic equations representing the whole potential field of the thorax can be written as:

$$\begin{aligned} V_1 &= \frac{1}{G_{11}} \{ E_1 - G_{12}V_2 - \dots - G_{1n}V_n \} \\ &\vdots \\ V_n &= \frac{1}{G_{nn}} \{ E_n - G_{n1}V_1 - \dots - G_{nn-1}V_{n-1} \} \end{aligned}$$

and in matrix form as,

$$GV = E \quad (2.17)$$

where  $G$  = matrix of coefficients (conductivity ratios)

$V$  = column vector of all the nodal potentials

$E$  = column vector of the known nodal potentials

In this study, there are  $20 \times 34 \times 23 = 15640$  simultaneous

equations to solve, each representing the potential value at a node. Due to the large number of equations, the iterative approach is preferred to the direct method in solving the equations. There are several iterative techniques that can be used but the one most commonly used is the Gauss-Seidel iterative process. In this process, the estimate of each nodal point is calculated from the appropriate equation using the most recent estimated values of its neighbouring nodal points. In other words, the previous value at each point will be replaced as soon as the new estimate is computed. The new estimate will then be used for calculating other nodal estimates.

To speed up the rate of convergence, a successive over-relaxation technique is employed which in essence extrapolates a new value of the estimate from the two recently estimated values, thus,

$$Z_i(k) = V_i(k-1) + B\{V_i(k) - V_i(k-1)\}$$

where  $V_i(k)$ ,  $V_i(k-1)$  are the two most recent estimates,

$B$  is the accelerating factor,

and  $V_i(k)$  will have the extrapolated value  $Z_i(k)$ .

#### 2.4.2.1 Optimal accelerating factor

In the Gauss-Seidel iterative process using the

successive over-relaxation technique, the rate of convergence is very much dependent on the value of the accelerating factor used (which lies in the range of 1 and 2). Ideally one should use the optimal value  $B_0$  which will give the greatest convergence rate. One should note that, if  $B$  is to be chosen as unity, the iterative process will reduce to the normal Gauss-Seidel method.

The technique described by B.A.Carre (1961) [77] in obtaining the optimal accelerating factor is employed in this study. The advantage of this method is that the optimum accelerating factor is obtained at the same time as the set of algebraic equations (2.17) is solved.

#### 2.4.2.2 Numerical examples on the optimal accelerating factor

Carre's technique for estimating the optimal accelerating factor was implemented for the set of equations 2.17. The equations were derived for the computer model of the human thorax as described in section 2.4. The vector  $E$  on the right hand side of (2.17) was given, which represented the nodal potentials on a surface surrounding the heart (epicardial surface). In this study, the epicardial surface was divided into a number of segments, (this will be further discussed in detail in Chapter 3). Each segment was related to a certain number of known nodal potentials. For 26 epicardial segments of equal size (i.e. roughly equal number of nodes), the optimal accelerating factor

was found to be 1.9103. If different source configurations are used, the model will be presented with a different source vector E. To investigate the independence of the optimal accelerating factor on vector E, different values are computed using various configurations of the epicardial segments, and the results are shown in Table 2.2.

The results showed that the maximum variation of the value of the optimal accelerating factor was less than 0.3% in these cases, and it is concluded that vector E does not have a significant effect on the value of  $B_0$ .

To study the dependence of the rate of convergence of the iterative process on the value of the accelerating factor, equations 2.17 were solved using different values of the accelerating factor. The results are shown in Fig.2.9. These values were obtained from a 26-unequal sized segment heart model, and it had an optimal value of 1.9153. The terminating criterion for the iterative process was the maximum relative error R (ratio of maximum error of an iterate between two successive iterations and the maximum estimated iterate) was less than 0.0001, i.e.

$$R = \frac{\max [V_j(k) - V_j(k-1)]}{\max [V_i(k)]} < 0.0001$$

The results showed that the number of iterations required was 128 when the optimal value was used, a 10% deviation (decrease) from this value increased the number of iterations by a

| NO. OF HEART SEGMENTS | OPTIMAL ACCELERATING FACTOR |
|-----------------------|-----------------------------|
| 26 (EQUAL SIZE)       | 1.9103                      |
| 26 (UNEQUAL SIZE)     | 1.9153                      |
| 20 (UNEQUAL SIZE)     | 1.9110                      |
| 18 (EQUAL SIZE)       | 1.9132                      |

TABLE 2.2 The optimal accelerating factors for different heart models.

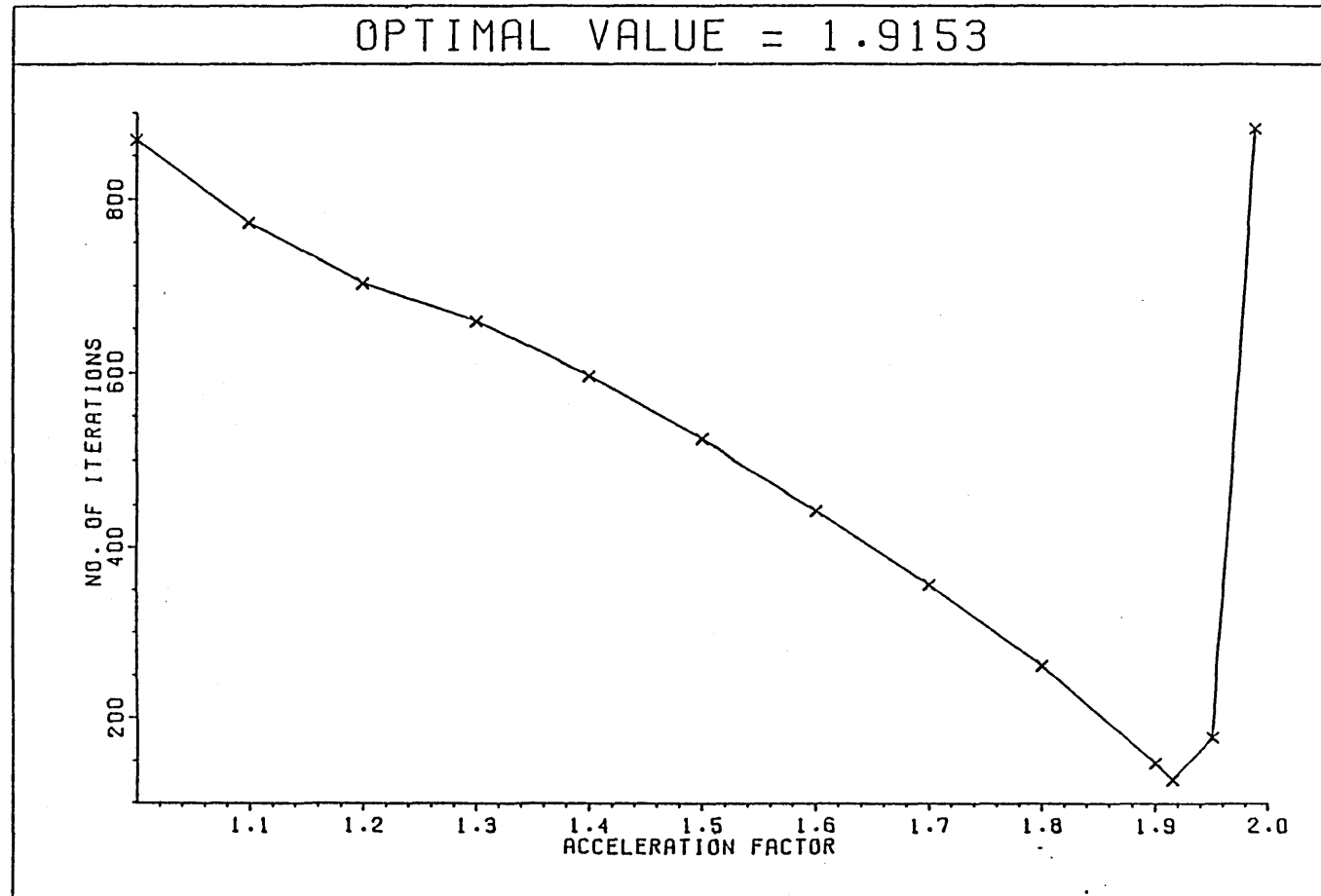


Fig.2.9 The effect of the value of the acceleration factor on the number of iterations.

factor of three. If the accelerating factor was chosen to be unity (i.e. reduced to normal Gauss-Seidel iterative process) the number of iterations increased up to about 800 in order to obtain the same accuracy.

### 2.4.3 Verification of technique

#### 2.4.3.1 Cylinder model

The numerical modelling technique discussed above was used to find the potential distribution on the surface of a finite length circular homogeneous conducting cylinder due to an arbitrary located dipole source within the cylinder (Fig.2.10).

The cylinder has a height ( $h$ ) of 30 cm and a radius ( $r$ ) of 7 cm. It was discretised with a uniform three-dimensional grid of 1 cm into  $14 \times 14 \times 30$  cubic elements. The Z-oriented dipole made an angle ( $\theta$ ) of 45 degrees with the Y-axis and the distance ( $r'$ ) of the centre of the dipole from the axis of the cylinder varied from 0 to 80% of the radius of the cylinder.

The surface potential along the Z-axis was plotted (Fig.2.11) and was compared with the results obtained by the analytical method used by Okada (1956) [18]. In Fig.2.11, the upper row displays the surface potential values obtained using the numerical modelling technique, while the second row contains the

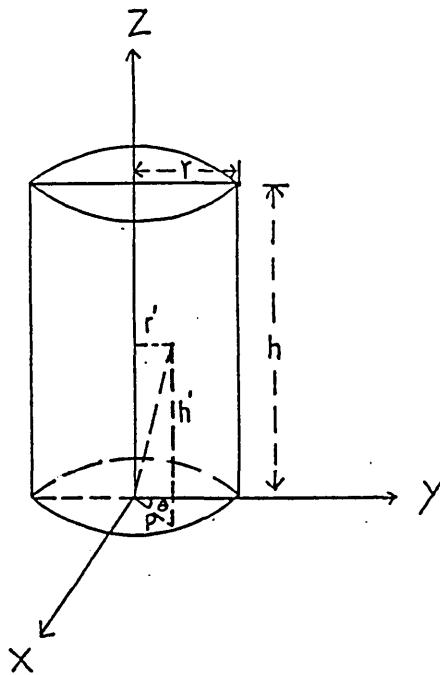


Fig.2.10 Diagram of circular cylinder showing coordinate system and location of dipole source.

results obtained using Okada's method and the bottom row shows the absolute difference between the two solutions.

Okada computed the electrical potential field of a homogeneous conducting cylinder of finite length due to a current dipole by expressing the inverse distance term  $\frac{1}{R}$  in terms of modified Bessel functions in the following potential equation:

$$V = \frac{I}{4\pi\sigma R}$$

where  $\sigma$  = conductance of the medium.

For the case of a finite-length medium, the expression was reduced to a form of double infinite summation. Errors occurred due to the finite number of terms used in the summation process. This was illustrated in Fig.2.11, when only 6 terms were used. The solution appeared to oscillate especially when the source was very close to the surface. The oscillations settled down when a greater number of terms were used in the summation process.

As seen from Fig.2.11, the solutions obtained by the numerical modelling technique were comparable to those obtained by the analytical method.

#### 2.4.3.2 Potential measurements on human thorax

Hamer et al. (1965) [78] measured the surface potentials

CYLINDER(14X14X30)(45 DEGREE H=17.8)(6 TERMS)

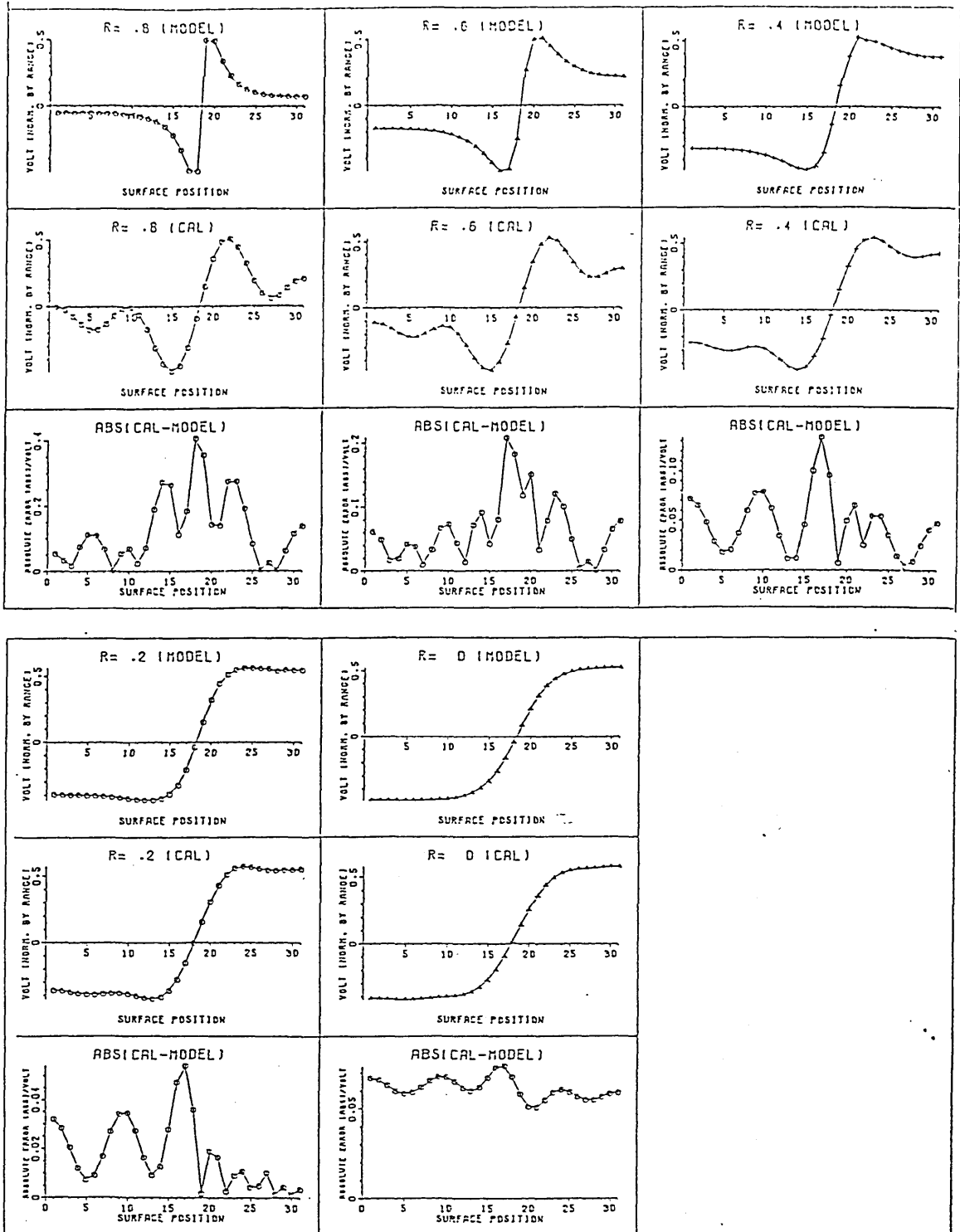


Fig.2.11 Comparison of the surface potentials along the Z-axis.

In each of the diagrams, the upper row displays results obtained using the numerical modelling technique and the second row contains the results obtained using Okada's method 18. The bottom row is the difference between the two results.

CYLINDER(14X14X30)(45 DEGREE H=17.5)(12 TERMS)

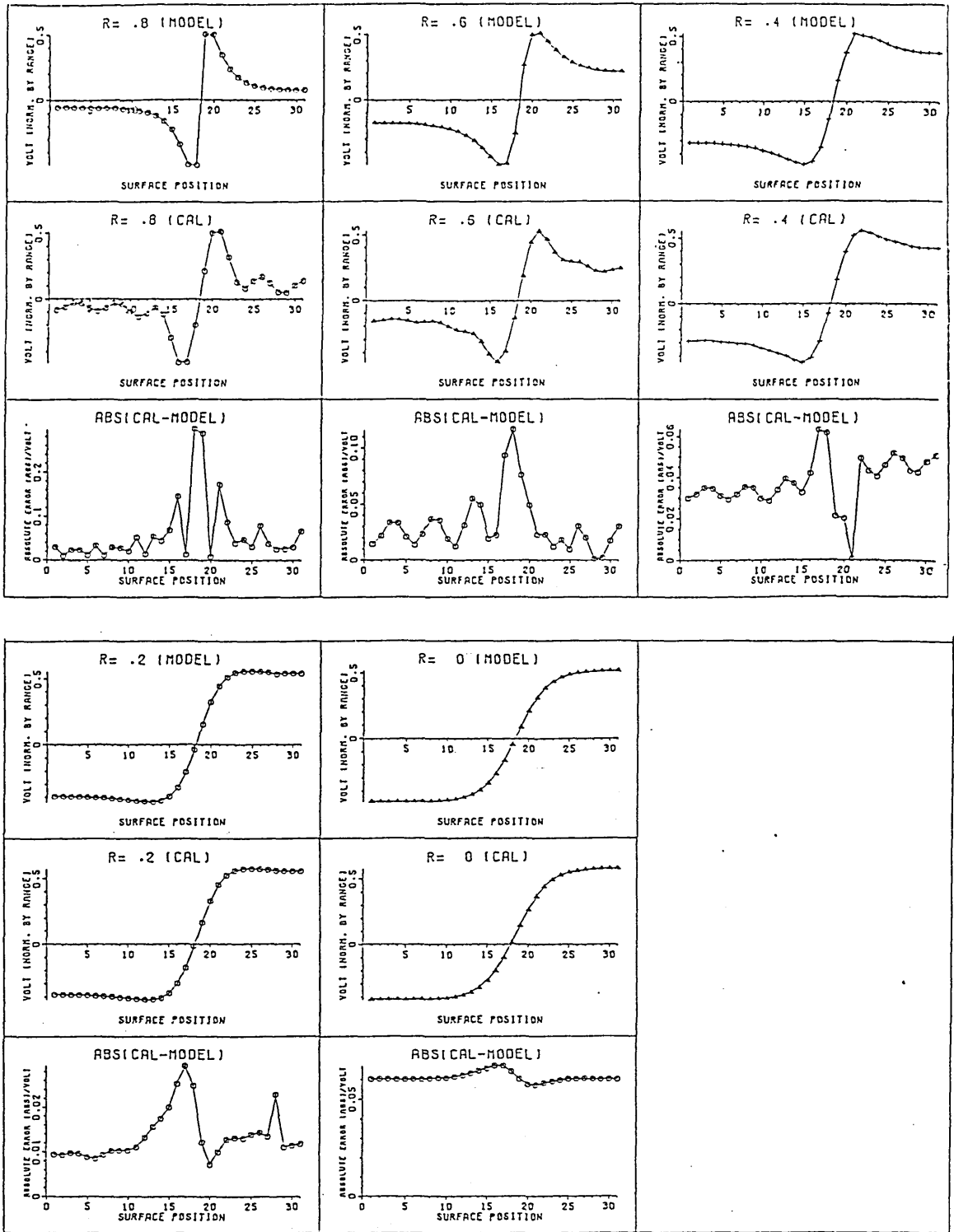


Fig.2.11 (Cont'd)

CYLINDER(14,14,30)(45 DEGREE H=17.5)(24 TERMS)

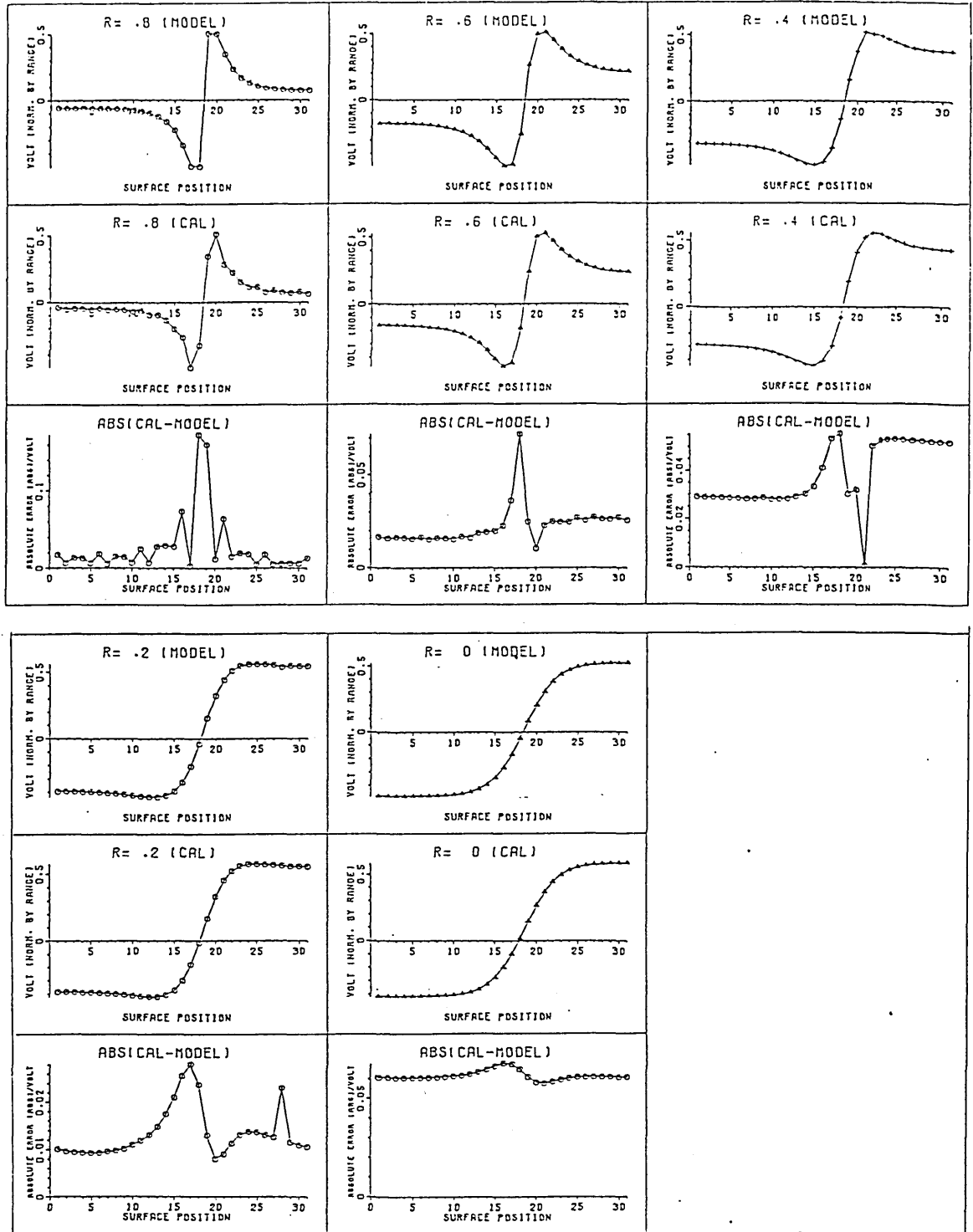


Fig.2.11 (Cont'd)

produced by stimuli applied to a bipolar catheter electrode in the right ventricle of several patients with heart block. From these measurements surface potential maps were constructed (Fig.2.12).

Using the location and orientation of the catheter electrode in the patients, the surface potentials were computed using the model described above and surface maps were plotted (Fig.2.12). A close agreement in all the main features between the measured and the simulated surface potential distribution can still be seen, despite the lack of information about the chest configuration of the patients and using the same model configuration for all the patients.

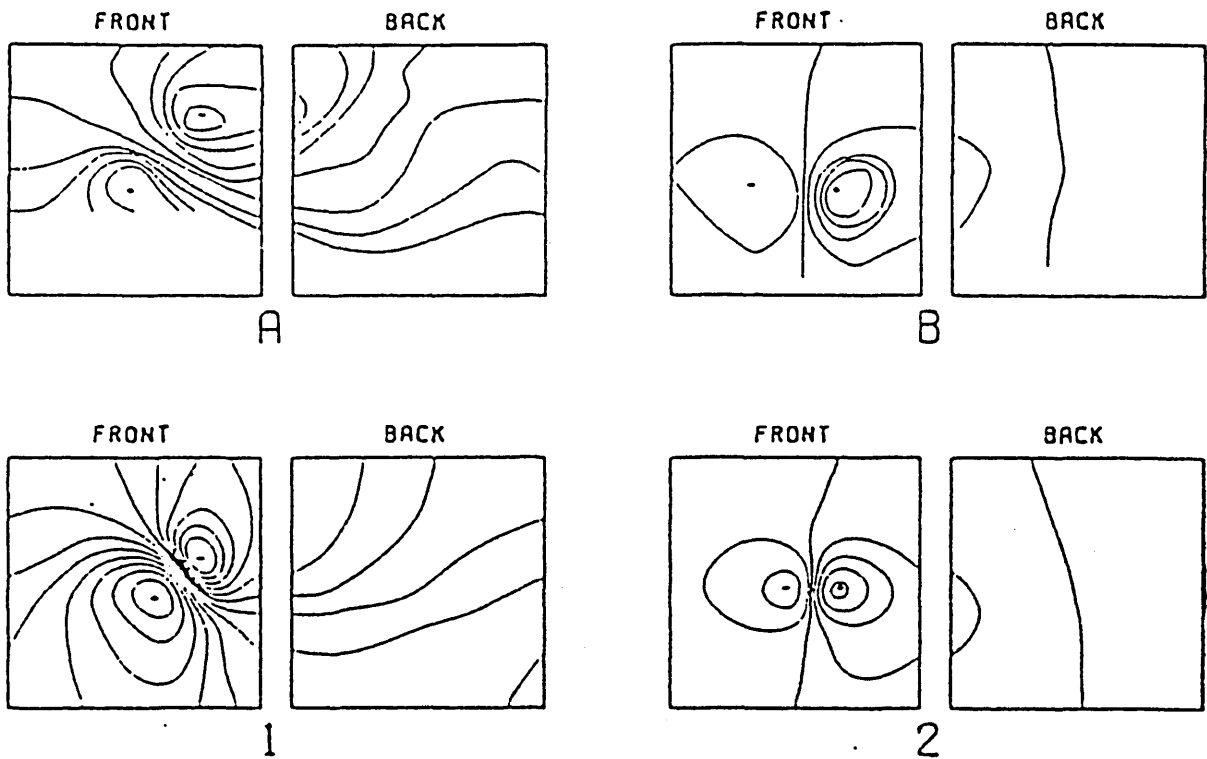


Fig.2.12 Comparison of body surface potentials.

A and B are the measured body surface potentials produced by catheter impulses (Redrawn from Hamer et al. [78]).

1 and 2 are obtained by model simulation.

## 2.5 Programme implementation

As described in section 2.4.2, the computation of the potential field of the human thorax model required the solving of 15640 simultaneous algebraic equations, and the Gauss-Seidel iterative process was used. In order to speed up the rate of convergence, the successive over-relaxation technique was employed in this process. In developing the computing programmes, two aims have to be borne in mind, they are :

- (1) to minimize the amount of central processor memory.
- (2) to minimize the amount of execution time.

As described in section 2.3.2, in order to simulate the anisotropic property of the skeletal muscles, five different conductivity values have to be identified for each discrete elemental block. Storing these values along with the nodal potential values during the programme execution required at least 100,000 central memory addresses. This can be dealt with by either running the programme in a machine with sufficient central memory or using temporary storage (such as magnetic tape). The former method depends on the availability of computing facilities and the latter requires an enormous amount of execution time. In this study the amount of central memory required was reduced by using a scanning technique. The coding of each discrete elemental block was stored instead of all the conductivity values. In the process of computing the nodal potentials, the conductivity values (maximum number for each node was six) required for the solving of

that equation were then generated from scanning the codings of the neighbouring elemental blocks. Execution time was reduced by identifying the nodes of which the potential values were already known thus avoiding redundant computation. These nodes included those which have been assigned on the epicardial surface, those in the insulating regions such as the bones and the air space and those enclosed by the defined epicardial surface. Table 2.3 shows the amount of central processor (CP) time and memory required to run the programme by the direct method (i.e. all the conductivity values were computed and stored) and by the scanning method (i.e. the conductivity values were computed only when required). It is shown that by using the scanning method, the amount of central memory required was reduced by half although the execution time was increased by about 20%.

| METHOD | C.P. TIME<br>(sec/iteration) | C.P. MEMORY<br>(Kilo-byte) |
|--------|------------------------------|----------------------------|
| direct | 0.63                         | 865                        |
| scan   | 0.76                         | 477                        |

Note: These were run on AMDAHL 470/V8 computer.

TABLE 2.3 Computational time and memory required for the direct and scan method.

## 2.6 Conclusion

The validity of the numerical modelling technique in computing the potential distribution of the human thorax was demonstrated by comparing the solutions obtained with the analytical method and the measured data in human subjects.

In the computational process, using the Gauss-Seidel successive over-relaxation technique, the rate of convergence of the iterative process is greatly improved by using the optimal value of the accelerating factor. It is also demonstrated that this optimal value is independent of the potential values and the configurations of the epicardial segments. This implies that once the optimal value is obtained, provided that the electrical properties of the conducting medium remains the same, it can be used for future computation with variations of the epicardial potential parameters.

## CHAPTER 3

### SOLVING ILL-CONDITIONED MATRIX EQUATIONS IN THE INVERSE PROBLEM IN ELECTROCARDIOGRAPHY

#### 3.1 Introduction

As stated in Chapter one, another classic problem in electrocardiography is the inverse problem, which is to determine the electrical activity of the heart through body surface potential measurements. This must take into account the information about the geometry and electrical properties of the thorax, as derived from the forward problem for example. However this still does not infer a unique computed solution for the electrical sources in the heart. This is because the electric field generated by any configuration of sources can be replaced by a double layer on a closed surface enclosing all of the sources. In other words, the number of possible configurations for the cardiac sources will be infinite in the case of any one body surface potential map. The equivalent cardiac generators approach, usually in terms of dipoles, reduces the possible solutions to a finite number especially when heavy constraints on the characteristics and behaviour of the dipoles are imposed. There are three difficulties in using this approach. First, the solutions cannot be compared directly with experimentally measured

data. Secondly, interpretation of the cardiac activities in terms of a collection of dipoles appears to be conceptually vague for clinicians as well as being physiologically unrealistic. Finally, even if the solutions obtained with heavy constraints may be plausible for normal or limited abnormal heart conditions, they may not be suitable for studying many other heart conditions.

The alternative approach in terms of epicardial potentials, allows us to compare the computed solutions directly with those measured as discussed in detail in section 1.5. Basically, this approach is to estimate the epicardial potentials directly from the body surface measurements. This estimation requires the knowledge of the transformation matrix, known as the forward transfer coefficient in this study, which describes the relationship between the body surface and epicardial surface potentials. Mathematically, it can be stated as:

$$AE = S \quad (3.1)$$

where A = forward transfer coefficients,

E = epicardial potentials,

S = body surface potentials.

In theory, if the forward transfer coefficient matrix A and the body surface potentials S are known, the epicardial surface potentials E can be calculated according to equation 3.1.

The main difficulty in the inverse problem in electrocardiography is its ill-posedness, i.e. a small perturbation in S will give rise to a huge oscillatory error in E. Mathematically, this is characterised by matrix A being ill-conditioned, and the degree of difficulty can be described by an index called the condition number (CN). A large condition number indicates a more ill-conditioned matrix. There are different descriptions for the condition number of a matrix and they are outlined in Appendix A2. Throughout this study, the P- condition number as defined in Appendix A2 is used. It is also shown in Appendix A3 that the condition number is related to the error in the computation of equation (3.1) in the following manner:

$$\frac{\|\Delta E\|}{\|E\|} \leq CN \times \frac{\|\Delta S\|}{\|S\|}$$

where  $\Delta E$  and  $\Delta S$  are the associated errors with E and S respectively, and  $\|\cdot\|$  represents the Euclidean norm.

In theory, any error  $\Delta S$  associated with S will be amplified in the solution E by the factor which is the condition number of the system matrix A. In other words, if we have a system matrix of large condition number, a small perturbation on S will give rise to a huge solution error. In order to reduce the condition number for the forward transfer coefficient, Lo (1977) [3] suggested that optimal surface sites, i.e. the sites where each epicardial segment is maximally represented, should be used in the derivation of the matrix.

This phenomenon of instability is common to both the equivalent cardiac generator approach and the epicardial potential approach. In this chapter, an attempt to obtain a more stable solution is made by firstly reducing the condition number of the transfer coefficient matrix as much as possible, and secondly applying a regularisation technique for solving ill-conditioned matrix equations in the procedure for calculating the epicardial potentials.

In section 3.2, the derivation of the forward transfer coefficient matrix is described. In section 3.3, the ill-conditioned matrix equation  $AX=Y$  is solved using a regularisation technique. The stability and correctness of the solutions using the regularisation technique applied to the inverse problem of electrocardiography will be investigated in section 3.4.

### 3.2 Derivation of the forward transfer coefficient

#### 3.2.1 Mathematical formulation of the forward problem

Let us denote  $u$  to be the potential in the region of the volume conductor  $V$  (i.e. the human body), excluding the electrical cardiac generators within the heart region, so volume  $V$  is bounded by the body surface  $Y$  and epicardial surface  $X$  (Fig.3.1). The formulation of the forward problem is to compute the potentials  $u$  in  $V$ , from which the body surface potentials on  $Y$  are obtained, given the following:

$$(1) \nabla^2 u = 0 \text{ in } V$$

$$(2) u = v \text{ on } X$$

$$(3) \frac{du}{dn} = 0 \text{ on } Y$$

Condition (1) imposes that the voltage gradients everywhere between the two surfaces must be equal, this implies the exclusion of any electrical sources within this region. Condition (2) simply implies that the epicardial potentials have to be given; which is the initial value condition, and condition (3) is the boundary condition for the surface  $Y$ . Surface  $Y$  is the body-air interface. Since the air is a non-conducting medium, no current is allowed to flow from the body out into the air, and this means that the current must flow tangentially on the body surface. In other words, the normal voltage gradient at the boundary must be zero.

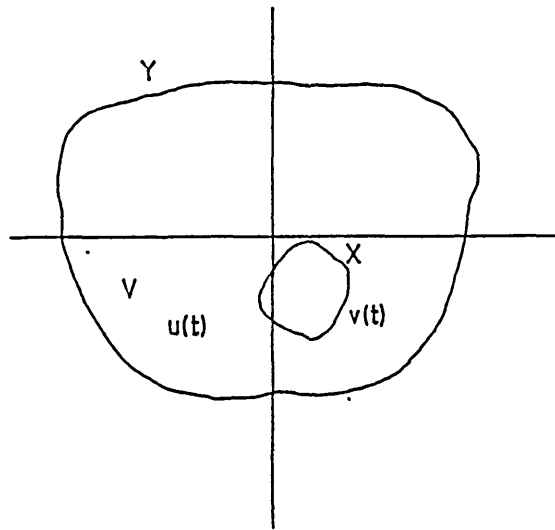


Fig.3.1 Cross-sectional view of the volume conductor V and the heart region X.  $u(t)$  represents the potential distribution between surfaces X and Y within the volume conductor V and  $v(t)$  represents the potential distribution on surface X.

The above stated problem, can be represented mathematically as:

$$av = u \quad (3.2)$$

where  $a$  is a linear operator which reflects the function  $v$  on  $X$  to  $Y$ . In discretised form, equation 3.2 can be approximated as:

$$AV = U \quad (3.3)$$

where  $A = \text{mxn matrix}$ , the forward transfer coefficients

$V = \text{n-vector}$ , the epicardial potentials

$U = \text{m-vector}$ , the body surface potentials

This formulation of the forward problem is mathematically known as the mixed boundary problem for Laplace's equation and the solution is unique [79].

### 3.2.2 Computational techniques

Using the numerical model of the human thorax and the computational technique described in Chapter 2, the forward transfer coefficient matrix  $A$  was computed in the following manner. The epicardial surface was segmented into  $N$  areas which

could be of equal or unequal size. The reference level for potentials was the zero potential of the cardiac field which persisted everywhere outside the thorax. Taking each segment in turn, a unit potential was assigned over its entire surface as represented in the model. With all other segments clamped at zero potential, the potential distribution was then computed throughout the thorax. The values then obtained at the N surface sites gave one column of the  $N \times N$  forward transfer coefficient matrix. Repeating this procedure for all the remaining segments, the whole matrix was constructed.

### 3.2.3 Relation of condition number of the forward transfer coefficient matrix and the configuration of the heart segments

Five different heart models were used to investigate the effect of the epicardial segment configuration on the condition number of the forward transfer coefficient matrix. The models included 14-, 18- and 26-equal size segments, and 20- and 26-unequal size segments. The associated condition numbers were calculated as shown in Table 3.1

It is very clear that the condition number of the transfer coefficient matrix is very much dependent on the number of epicardial segments used. A model with 26 equal size segments has a condition number of 1356; this is drastically reduced when

| NO. OF HEART SEGMENTS | EQUAL/UNEQUAL SIZE | CONDITION NO. |
|-----------------------|--------------------|---------------|
| 26                    | EQUAL              | 1356          |
| 18                    | EQUAL              | 708           |
| 14                    | EQUAL              | 84            |
| 26                    | UNEQUAL            | 457           |
| 20                    | UNEQUAL            | 117           |

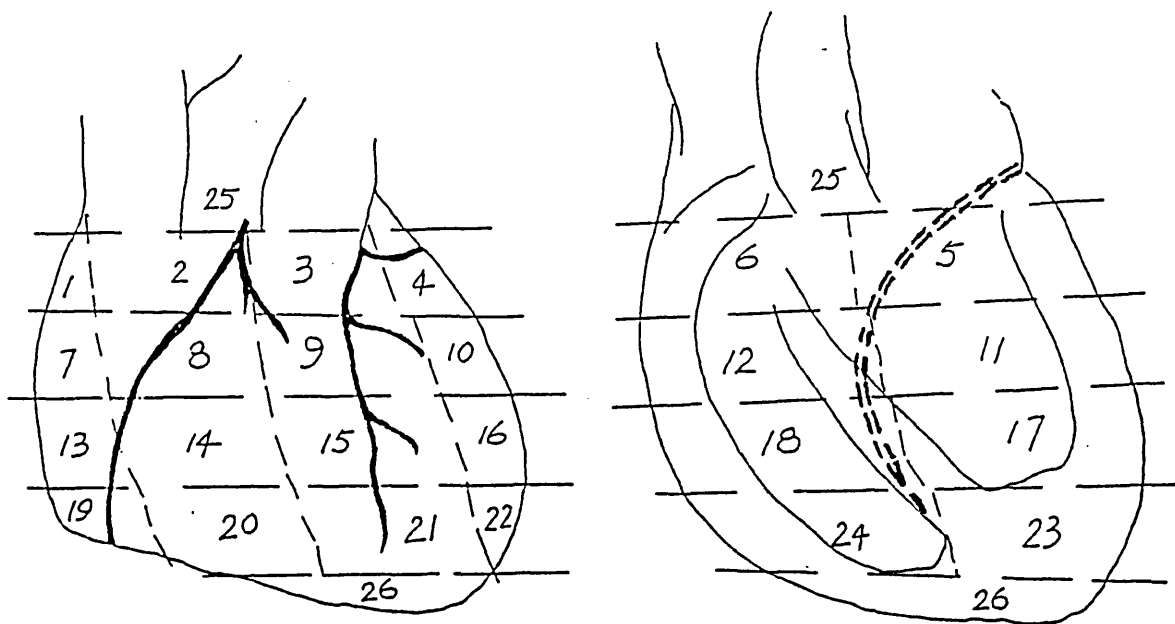
TABLE 3.1 The condition number of the transfer coefficient matrix associated with different number and size of heart segments.

fewer epicardial segments are used. A reduction of less than half in the number of segments (from 26 to 14) resulted in a reduction of a factor of 14 (from 1356 to 84) in the condition number.

The reduction of condition number is even more significant when unequal size epicardial segment models are used. It can be seen that with the same number of segments (i.e. 26), the condition number is reduced from 1356 to 457 when the epicardial surface was divided using an unequal size grid. Since reducing the number of epicardial segments will give poorer resolution, it is better to use a model with a larger number of epicardial segments of unequal size. Therefore, for the remainder of this work, the model with 26 unequal segments as shown in Fig.3.2 will be used. The corresponding matrix of the forward transfer coefficient is listed in Table 3.2. All the input data is given in Appendix A4 which is required to carry out the computation of this transfer coefficient matrix.

#### 3.2.4 Programme implementation

To derive a forward transfer coefficient for a 26 epicardial segment model, it is necessary to compute the body surface potential distribution by solving the forward problem 26 times. The optimal sites could then be identified and the corresponding potential values were extracted to form the matrix manually. This would be very laborious and time consuming. In



|    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|
| 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 |
| 1  | 2  | 3  | 4  | 5  | 5  | 6  | 6  |
| 7  | 8  | 9  | 10 | 11 | 11 | 12 | 12 |
| 13 | 14 | 15 | 16 | 17 | 17 | 18 | 18 |
| 19 | 20 | 21 | 22 | 23 | 23 | 24 | 24 |
| 26 | 26 | 26 | 26 | 26 | 26 | 26 | 26 |

Fig.3.2 Sketch of the human heart.  
The heart is divided into 26 unequal sized segments.  
The corresponding grid shown below is used in the  
display of the epicardial iso-potential contour map.

|      |       |       |      |       |       |      |       |       |       |      |      |      |       |       |       |      |      |       |       |       |       |       |       |       |       |
|------|-------|-------|------|-------|-------|------|-------|-------|-------|------|------|------|-------|-------|-------|------|------|-------|-------|-------|-------|-------|-------|-------|-------|
| 5810 | 12314 | 2072  | 2152 | 5548  | 7505  | 2124 | 7810  | 4630  | 1647  | 1406 | 3200 | 1589 | 2921  | 975   | 1492  | 1220 | 3143 | 2074  | 2331  | 1129  | 788   | 2248  | 4218  | 15668 | 911   |
| 4435 | 16401 | 8262  | 2810 | 4128  | 4638  | 1869 | 11912 | 8538  | 2121  | 1168 | 1956 | 1147 | 2777  | 1272  | 1632  | 919  | 1949 | 1451  | 1329  | 1117  | 658   | 1612  | 2697  | 11876 | 681   |
| 1972 | 9819  | 11151 | 8115 | 1275  | 1042  | 861  | 9918  | 14597 | 4811  | 1456 | 1289 | 574  | 2032  | 2104  | 3123  | 1106 | 1213 | 816   | 1377  | 1347  | 844   | 1684  | 1787  | 8625  | 591   |
| 1658 | 6156  | 11546 | 9706 | 7093  | 3829  | 576  | 4439  | 11172 | 7099  | 3045 | 1753 | 460  | 1290  | 1696  | 4647  | 1960 | 1532 | 733   | 1017  | 1277  | 1304  | 2794  | 2268  | 10188 | 721   |
| 1577 | 2464  | 2377  | 2905 | 12309 | 8304  | 714  | 2195  | 2904  | 3389  | 5598 | 6097 | 898  | 1284  | 843   | 4340  | 4707 | 4943 | 1629  | 1493  | 1574  | 2570  | 7523  | 6775  | 8868  | 1721  |
| 2138 | 3119  | 1956  | 1888 | 10285 | 10007 | 1171 | 3075  | 2405  | 2145  | 3885 | 7404 | 1470 | 1974  | 784   | 2808  | 3553 | 6237 | 2506  | 2213  | 1457  | 1899  | 6159  | 8157  | 9453  | 1703  |
| 3727 | 10367 | 3217  | 1230 | 2808  | 4025  | 3768 | 17376 | 6098  | 1236  | 918  | 2573 | 2699 | 7579  | 1959  | 1241  | 866  | 3121 | 3355  | 4913  | 2077  | 764   | 1849  | 4558  | 6176  | 1249  |
| 2468 | 11707 | 5024  | 1638 | 1953  | 2299  | 2404 | 24542 | 12234 | 1757  | 690  | 1320 | 1446 | 6454  | 2875  | 1509  | 575  | 1584 | 1949  | 3662  | 2198  | 654   | 1182  | 2446  | 4491  | 908   |
| 463  | 2217  | 6502  | 4100 | 2042  | 1319  | 680  | 10157 | 26788 | 6608  | 1150 | 730  | 555  | 4597  | 7322  | 4966  | 770  | 817  | 1071  | 2996  | 3913  | 1140  | 1322  | 1441  | 2981  | 941   |
| 423  | 1547  | 3051  | 4411 | 1337  | 1464  | 216  | 1958  | 11756 | 20350 | 3599 | 962  | 216  | 989   | 5161  | 21426 | 2584 | 916  | 546   | 890   | 3261  | 2961  | 3144  | 1584  | 2615  | 791   |
| 872  | 1885  | 2733  | 3988 | 8361  | 4069  | 376  | 1655  | 4214  | 7364  | 7811 | 3096 | 452  | 841   | 1386  | 10852 | 6626 | 2906 | 912   | 998   | 2375  | 4569  | 9246  | 4880  | 5644  | 1831  |
| 1607 | 2413  | 1555  | 1576 | 8134  | 7703  | 961  | 2545  | 2020  | 1930  | 4148 | 8455 | 1375 | 1785  | 716   | 2396  | 4511 | 8385 | 2643  | 2208  | 1608  | 2317  | 8604  | 11002 | 6598  | 2188  |
| 2054 | 3439  | 1275  | 724  | 2473  | 3501  | 2563 | 8831  | 3124  | 855   | 954  | 3080 | 4445 | 10139 | 1915  | 1107  | 1027 | 5456 | 8777  | 10806 | 3204  | 1056  | 2634  | 9539  | 3956  | 2508  |
| 1025 | 2808  | 1388  | 673  | 1092  | 1386  | 1596 | 12835 | 6928  | 1188  | 463  | 1100 | 2135 | 17307 | 6245  | 1633  | 471  | 1905 | 5097  | 13548 | 7306  | 1410  | 1626  | 4071  | 1886  | 2845  |
| 264  | 954   | 1093  | 886  | 664   | 481   | 305  | 3817  | 8655  | 3796  | 558  | 352  | 372  | 4586  | 23438 | 11684 | 586  | 519  | 1263  | 5336  | 19419 | 4199  | 1943  | 1396  | 796   | 2398  |
| 187  | 520   | 858   | 1153 | 1376  | 689   | 111  | 943   | 3451  | 6705  | 1747 | 545  | 133  | 806   | 8108  | 36925 | 2343 | 598  | 370   | 1023  | 12426 | 10505 | 4478  | 1326  | 1010  | 1243  |
| 575  | 1174  | 1577  | 2263 | 5233  | 2776  | 272  | 1137  | 2808  | 5684  | 5948 | 2406 | 351  | 676   | 1474  | 16054 | 8200 | 2536 | 798   | 959   | 3827  | 8611  | 13534 | 5062  | 3454  | 2574  |
| 1814 | 2711  | 1350  | 1110 | 5368  | 6186  | 1303 | 3342  | 1890  | 1335  | 2640 | 6842 | 2239 | 2759  | 790   | 2028  | 3254 | 9942 | 4702  | 3639  | 1744  | 1857  | 7504  | 15461 | 5623  | 2460  |
| 1032 | 1966  | 801   | 454  | 1475  | 1971  | 1310 | 5337  | 2393  | 636   | 626  | 1887 | 2997 | 8379  | 2368  | 985   | 730  | 4331 | 13190 | 18127 | 6113  | 1598  | 2799  | 10821 | 2140  | 5301  |
| 608  | 1346  | 646   | 364  | 844   | 1057  | 839  | 4774  | 2890  | 671   | 394  | 974  | 1701 | 8507  | 4112  | 1243  | 491  | 2200 | 9370  | 22813 | 12491 | 2721  | 2961  | 6839  | 1237  | 7881  |
| 151  | 431   | 392   | 324  | 458   | 343   | 174  | 1670  | 2544  | 1172  | 396  | 302  | 263  | 2597  | 7376  | 4847  | 628  | 503  | 1262  | 5416  | 42101 | 14674 | 4602  | 1854  | 455   | 5047  |
| 104  | 241   | 242   | 265  | 585   | 379   | 87   | 629   | 1011  | 830   | 571  | 369  | 135  | 898   | 2555  | 4507  | 1061 | 545  | 663   | 2277  | 31795 | 32566 | 9424  | 2084  | 442   | 5716  |
| 297  | 546   | 561   | 720  | 2240  | 1428  | 171  | 680   | 1039  | 1465  | 2100 | 1494 | 259  | 644   | 980   | 5551  | 4020 | 2083 | 947   | 1752  | 8936  | 17021 | 27482 | 6952  | 1519  | 9083  |
| 1263 | 1924  | 1194  | 1193 | 5376  | 5295  | 829  | 2202  | 1643  | 1580  | 3444 | 6469 | 1359 | 1711  | 668   | 2812  | 4969 | 9197 | 2994  | 2377  | 1904  | 2903  | 12896 | 15697 | 4677  | 3154  |
| 4630 | 9258  | 6110  | 3408 | 8188  | 8642  | 1135 | 4700  | 4484  | 2313  | 1753 | 2507 | 886  | 1656  | 818   | 1927  | 1331 | 2170 | 1225  | 1369  | 883   | 825   | 2180  | 2942  | 23904 | 617   |
| 177  | 406   | 290   | 243  | 573   | 465   | 199  | 1351  | 1360  | 586   | 461  | 464  | 369  | 2322  | 2839  | 1975  | 860  | 908  | 2488  | 8537  | 26329 | 13874 | 10466 | 4705  | 531   | 17202 |

TABLE 3.2 The forward transfer coefficient matrix for a heart model with 26 unequal-sized segments.

this study, therefore, it was carried out automatically. This required fractionally more central processor memory, but this was easily justified by the amount of labour saved.

In the course of studying the forward and inverse problems, it is expected that different sets of transfer coefficients will have to be computed. It is therefore essential to develop an efficient and convenient algorithm. The organisation of the programme developed for use in this study is indicated by the flow diagram and programme listing in Appendix A4.

As the forward solution is obtained by an iterative method, the executing time is very much dependent on the criterion used to terminate the iterative process. Two terminating criteria can be used, (1) the absolute error (ABS) which is defined as :

$$ABS = \max [X_i(k) - X_i(k-1)] < 0.0001$$

and (2) the relative error (REL), defined as :

$$REL = \frac{\max [(X_i(k) - X_i(k-1))]}{\max [X_i(k)]} < 0.0001$$

where  $0 < [X_i(k), X_i(k-1)] < 1$  are the solution in the  $i^{th}$  position after  $k^{th}$ ,  $(k-1)^{th}$  iteration respectively.

The forward transfer coefficients obtained using the two

different terminating criteria are listed in Appendix A5. The execution time required for each procedure was shown in Table 3.3. It can be seen that there was a reduction of about 50% in execution time when the relative error criterion was used. To justify the use of the relative error criterion, the difference between the two sets of transfer coefficients was computed (Table 3.4). There were  $26 \times 26 = 676$  entries in each set of matrix and the maximum magnitude of difference between the two sets was  $6.99 \times 10^{-4}$  (Note: the maximum magnitude for all the entries in both sets was 1). The maximum relative error was less than 5% with a mean value of 0.359%. Although these results showed that the difference between the two sets of transfer coefficients was very small, it is necessary to consider the effect this has on the inverse solution before accepting this as a stopping criterion.

To test the effect on the inverse solution using these two sets of transfer coefficients, the epicardial surface potentials of the whole cardiac cycle were computed from the measured body surface potentials of a human subject using each set of coefficients. Fig.3.3 shows sampled epicardial surface maps in the P-wave, QRS complex and T-wave of the two solutions. By visual comparison, there was little difference between the two sets of solutions. This was also reflected in the values of cross-correlation coefficient between the two solutions which were very close to 1.0 (which is the maximum value). Table 3.5 shows the error in a more quantitative manner, and that the maximum relative spatial error was about 1.7%. These results indicate that

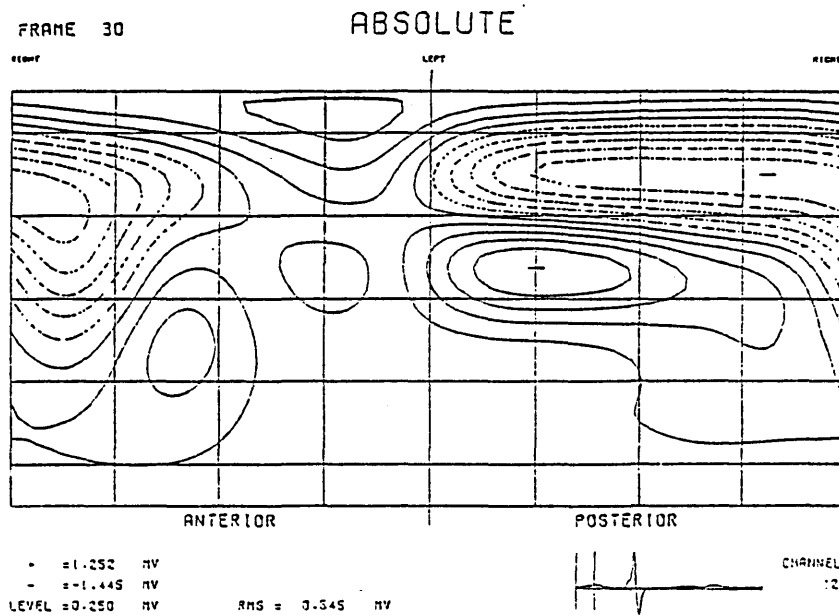
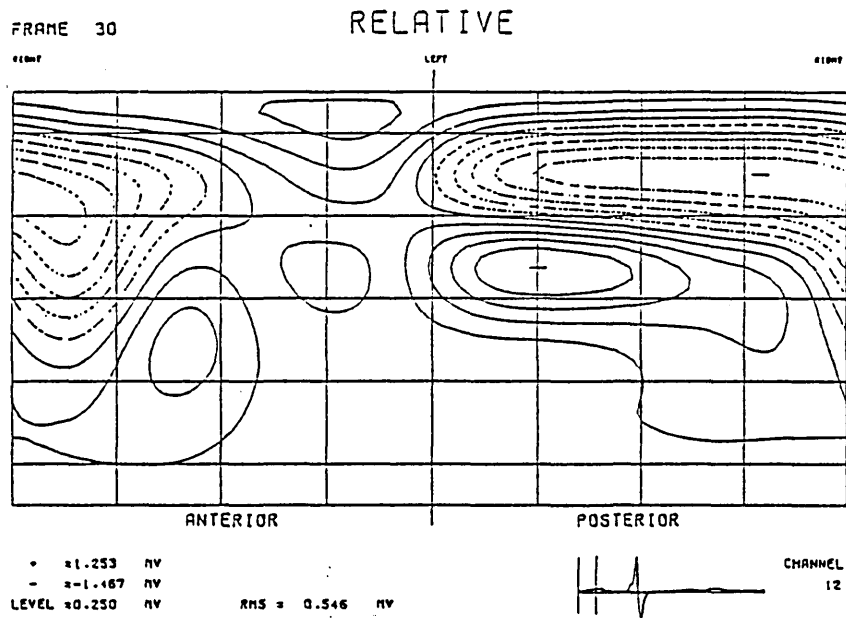
| METHOD       | C.P. TIME       | C.P. MEMORY<br>(Kilo-byte) |
|--------------|-----------------|----------------------------|
| DIRECT (ABS) | 42 min. 34 sec. | 900                        |
| DIRECT (REL) | 21 min. 45 sec. | 900                        |
| SCAN (ABS)   | 50 min. 40 sec. | 490                        |
| SCAN (REL)   | 24 min. 30 sec. | 490                        |

Table 3.3 Computational time and memory used in different methods. ABS and REL indicate the absolute error and the relative error is used respectively as the terminating criterion in the iterative process.

| ERROR CRITERIA |      | ABSOLUTE   | RELATIVE |
|----------------|------|------------|----------|
| F- NORM        |      | 2 .61922   | 2 .61785 |
| ERROR- NORM    |      | 1 .46 E- 8 |          |
| ABSOLUTE       | MAX  | 6 .99E-4   |          |
| ERROR          | MEAN | 9 .18E-6   |          |
| RELATIVE       | MAX  | 4 .55E-2   |          |
| ERROR          | MEAN | 3 .59E-3   |          |

$$\begin{aligned}
 * \text{ F-NORM} & : \sum_{i=1}^n \sum_{j=1}^n \left( w_{ij} \right)^2 \\
 \text{ERROR- NORM} & : \sum_{i=1}^n \sum_{j=1}^n \left( w_{abs,ij} - w_{rel,ij} \right)^2
 \end{aligned}$$

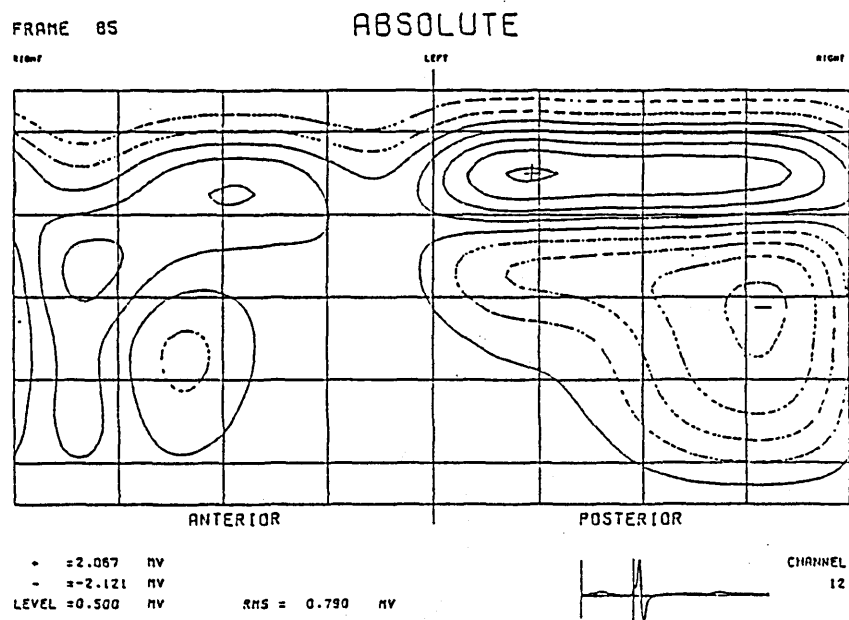
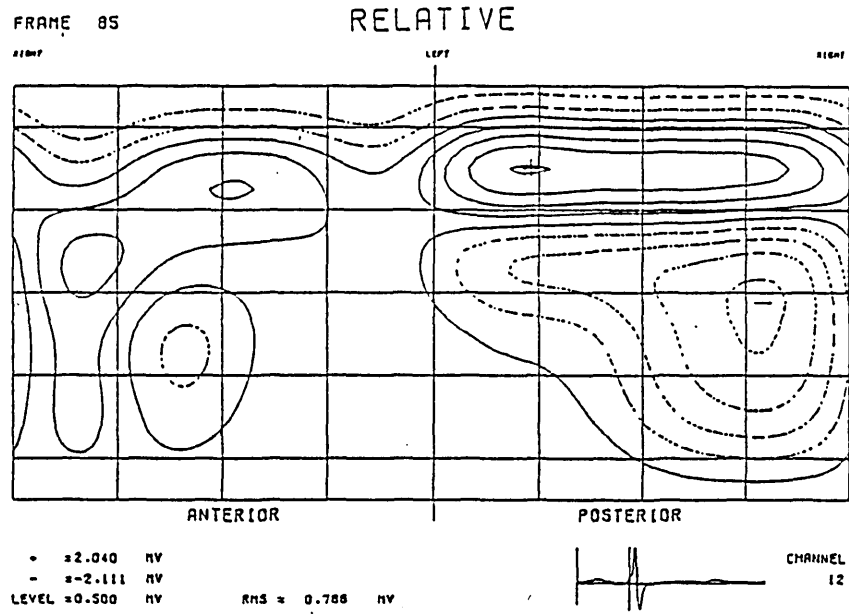
TABLE 3.4 Comparison between two sets of transfer coefficient matrices which are obtained by using absolute and relative error as the terminating criteria.



(a)

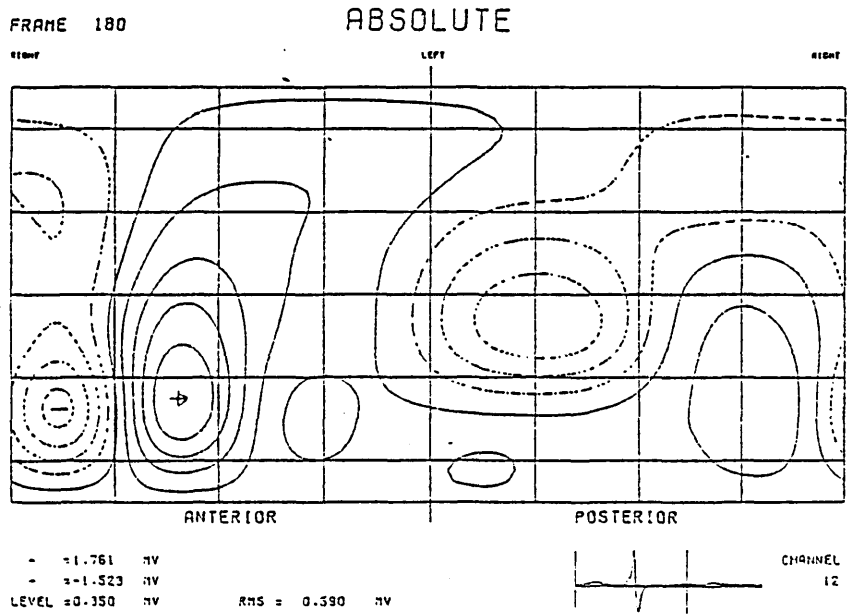
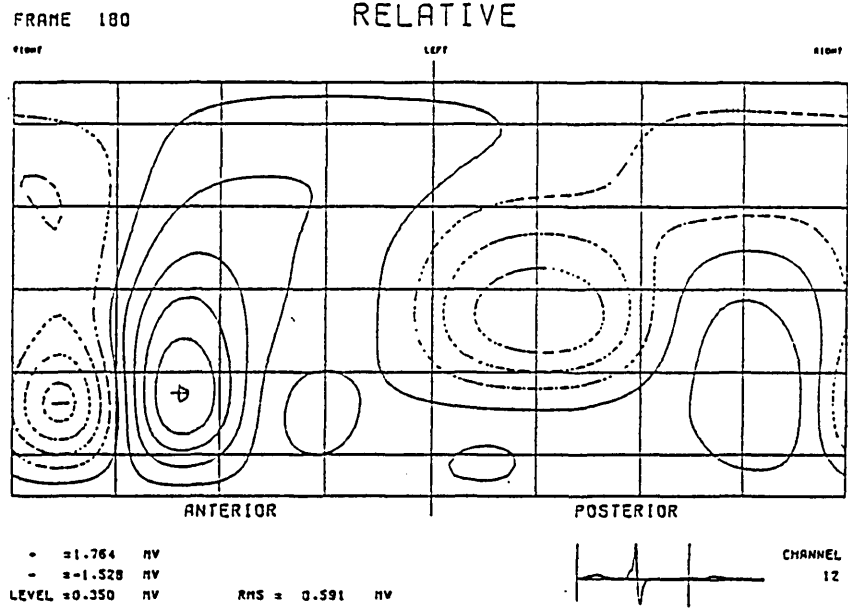
Fig.3.3 The epicardial surface maps in the (a) P-wave, (b) QRS complex and (c) T-wave.

'RELATIVE' and 'ABSOLUTE' indicate the solutions are obtained by using the relative and absolute error as the terminating criteria in the iterative process respectively. The display format of the iso-potential maps is the same as in Fig.1.14. The grid used in the display corresponds to Fig.3.2.



(b)

Fig.3.3 (Cont'd)



(C)

Fig.3.3 (Cont'd)

|                            |                    |                            |
|----------------------------|--------------------|----------------------------|
| SEGMENT<br>ERROR           | MAX<br>MIN<br>MEAN | 0.034<br>0.0007<br>0.0067  |
| FRAME<br>ERROR             | MAX<br>MIN<br>MEAN | 0.036<br>0.00003<br>0.0037 |
| CROSS-<br>CORRELATION      | MAX<br>MIN<br>MEAN | 0.9999<br>0.9998<br>0.999  |
| RELATIVE<br>FRAME<br>ERROR | MAX<br>MIN<br>MEAN | 0.0166<br>0.0019<br>0.0037 |
| SOLUTION NORM              | 4.43               |                            |

TABLE 3.5 Quantitative comparison of the inverse solutions using two different sets of transfer coefficient.

using relative error as the terminating criterion in the iterative process is justified.

The computer used in this study was the Amdahl 470V/8. The execution speed is estimated to be about 1.5 millions floating point operations per second (MFLOPS). In solving the forward problem as described in Chapter 2, it required about 35 MFLOPS and the execution time on the Amdahl 470V/8 would be about 25 seconds. To obtain a set of forward transfer coefficient using a 26-segment heart model required about 900 MFLOPS. This implied a 30 minute execution time on the Amdahl 470V/8. Using other existing computers with faster execution speed, such as the Cray X-MP [80], the set of forward transfer coefficient could be computed in less than 2 minute execution time.

In the model study of forward and inverse problems, it will be more convenient and practical if the computational process can be carried out on a portable mini-computer. At the present stage of development in computer hardwares, the speed of floating point operation in most of the mini-computers is about 50 to 100 times slower than the main frame computers. Nevertheless, in view of the history of development of computers, this discrepancy can easily be rectified within a decade.

### 3.3 Solving ill-conditioned matrix equation $AX=Y$

In this section the techniques applied to obtain the inverse solution are described. In particular it is shown how the stability of the inverse solution can be improved by using a regularisation method in which a stabilizing factor is introduced in the matrix.

Given the matrix equation

$$AX = Y \quad (3.4)$$

$X$  can be found by the direct inverse method provided that  $A$  is non-singular, which would give

$$X = A^{-1}Y \quad (3.5)$$

where  $A^{-1}$  is the inverse of matrix  $A$ .

If  $Y$  is subject to an error  $\Delta Y$ , solution  $X$  will have an absolute error  $\Delta X$ , i.e.,

$$(X + \Delta X) = A^{-1} (Y + \Delta Y) \quad (3.6)$$

and it can be shown (Appendix A3) that

$$\frac{\|\Delta X\|}{\|X\|} = \|A\| \cdot \|A^{-1}\| \frac{\|\Delta Y\|}{\|Y\|} \quad (3.7)$$

where  $\|A\| \cdot \|A^{-1}\|$  is defined as the condition number of matrix A.

So a small perturbation in Y will be amplified in the solution vector X by the factor of the associated condition number of matrix A.

Instead, if we consider equation 3.5 as,

$$X = (A^T A + kI)^{-1} A^T Y \quad (3.8)$$

where  $k > 0$  and  $A^T$  is the transpose of A, then it may be possible to improve the conditioning by a suitable choice of k.

Solving (3.8) in the presence of noise  $\Delta Y$  in Y, we have

$$X' = (A^T A + kI)^{-1} A^T Y' \quad (3.9)$$

where  $X'$  is the approximation of X, and

$$Y' = (Y + \Delta Y)$$

There are now two problems namely,

1. to choose k so that  $X'$  is a good approximation of X
2. to define the error bound for  $(X - X')$  due to  $(Y - Y')$

Considering first the error bound and introducing a few terms as follows:

$$\begin{aligned} \|Y - Y'\| &\leq e_1 \\ \|X - X_c\| &\leq e_2 \end{aligned}$$

where  $X_c$  is a function assumed to closely approximate the desired solution vector  $X$  (extensive work can be found in Kockler (1974) [81] and Ribiere (1967) [82]).

Then from equations 3.4 and 3.9, we have

$$\begin{aligned} X - X' &= (X - X_c) + (A^T A + kI)^{-1} (A^T (Y - Y') + \\ &\quad A^T A (X_c - X) + k(A^T A + kI)^{-1} X_c \quad (\text{see Appendix A6}) \\ &= (X - X_c) + (A^T A + kI)^{-1} A^T A (X_c - X) + \\ &\quad (A^T A + kI)^{-1} A^T (Y - Y') + k(A^T A + kI)^{-1} X_c \end{aligned}$$

collecting terms,

$$X - X' = (X - X_c) + (A^T A + kI)^{-1} A^T (A(X_c - X) + (Y - Y')) + k(A^T A + kI)^{-1} X_c$$

and using the property of triangular inequality of norm, then

$$\begin{aligned} \|X - X'\| &= \|X - X_c\| \\ &\quad + \|(A^T A + kI)^{-1} A^T (A(X_c - X) + (Y - Y'))\| \\ &\quad + k\|(A^T A + kI)^{-1} X_c\| \end{aligned}$$

$$\begin{aligned}
 & \leq \|X - X_c\| \\
 & \quad + \left\| (A^T_{A+kI})^{-1} A^T \right\| \cdot \left\| A(X_c - X) + (Y - Y') \right\| \\
 & \quad + k \left\| (A^T_{A+kI})^{-1} X_c \right\| \\
 & \leq \|X - X_c\| \\
 & \quad + \left\| (A^T_{A+kI})^{-1} A^T \right\| \cdot (\|A\| \cdot \|X_c - X\| + \|Y - Y'\|) \\
 & \quad + k \left\| (A^T_{A+kI})^{-1} X_c \right\|
 \end{aligned}$$

substituting  $e_1$  and  $e_2$

$$\begin{aligned}
 \|X - X'\| & \leq e_2 \\
 & \quad + \left\| (A^T_{A+kI})^{-1} A^T \right\| \cdot (e_2 \|A\| + e_1) \\
 & \quad + k \left\| (A^T_{A+kI})^{-1} X_c \right\|
 \end{aligned} \tag{3.10}$$

Equation 3.10 shows the effect of different components on the error bound  $\|X - X'\|$  and has to be considered individually as follows:

1. The first term  $e_2$  is assumed to be small and is independent of  $\|Y - Y'\|$ .
2. The second term consists of  $e_1$  which is defined as the error norm  $\|Y - Y'\|$  and this term is amplified by the term  $\left\| (A^T_{A+kI})^{-1} A^T \right\|$  which appears in equation (3.9). Thus as the

factor  $k$  becomes smaller, this term becomes bigger and the error norm  $\|Y - Y'\|$  will be amplified in the solution  $X'$  and vice versa.

3. In the third term, factor  $k$  appears twice and it has a counterbalancing effect, besides this term is also independent of the error norm  $\|Y - Y'\|$ .

For a good approximation, equation 3.10 can be simplified to,

$$\|X - X'\| \leq \|(A^T A + kI)^{-1} A^T\| \cdot \|Y - Y'\| \quad (3.11)$$

and it can be seen that the solution error norm  $\|X - X'\|$  due to the vector error norm  $\|Y - Y'\|$  is very much dependent on the choice of factor  $k$ . If  $k$  is chosen to be zero, then it reduces to the error in the direct inverse solution as shown in equation 3.6.

The remaining problem is the choice of factor  $k$  so that  $X'$  of equation 3.9 can be obtained to be a good approximation of  $X$  of equation 3.5. Mathematically, it is to find  $X'$  such that,

$$\|X - X'\| = \text{minimum} \quad (3.12)$$

For simplicity in the following derivation, it is denoted that,

$$R = \|X - X'\|$$

from equations 3.5 and 3.9,

$$R = \left\| (A^{-1}Y) - [(A^T A + kI)^{-1} A^T (Y + \Delta Y)] \right\| \quad (3.13)$$

from the Casey-Hamilton theorem,  $(A^T A + kI)^{-1}$  can be expanded as:

$$(A^T A + kI)^{-1} = (A^T A)^{-1} - k(A^T A)^{-2} + k^2(A^T A)^{-3} - \dots \quad (3.14)$$

provided that all the eigenvalues of  $(A^T A)$  have moduli  $< 1$ .

Substituting from (3.14) to (3.13), then,

$$R = \left\| (A^{-1}Y) - [(A^T A)^{-1} - k(A^T A)^{-2} + k^2(A^T A)^{-3} - \dots] A^T (Y + \Delta Y) \right\| \quad (3.15)$$

Expanding equation 3.15,

$$\begin{aligned} R &= \left\| (A^{-1}Y) - [(A^T A)^{-1} A^T (Y + \Delta Y) - k(A^T A)^{-2} A^T (Y + \Delta Y) \right. \\ &\quad \left. + k^2(A^T A)^{-3} A^T (Y + \Delta Y) - \dots] \right\| \\ &= \left\| -(A^T A)^{-1} A^T \Delta Y + k(A^T A)^{-2} A^T (Y + \Delta Y) \right. \\ &\quad \left. - k^2(A^T A)^{-3} A^T (Y + \Delta Y) + \dots \right\| \end{aligned} \quad (3.16)$$

and assuming that  $\left\| k^2(A^T A)^{-3} \right\| \ll 1$ , then the higher order terms in equation 3.16 can be neglected. Since the nature of the

derivation of the forward transfer coefficient matrix  $A$  in this study,  $\|A^T A\|$  is always  $> 1$ , so  $\|(A^T A)^{-1}\|$  will be always  $< 1$ . If factor  $k$  is chosen to be small, then assumption  $\|k^2(A^T A)^{-3}\| \ll 1$  can be satisfied and higher order terms can be omitted, equation 3.16 becomes,

$$R = \left\| k(A^T A)^{-2} A^T (Y + \Delta Y) - (A^T A)^{-1} A^T \Delta Y \right\| \quad (3.17)$$

It is obvious that  $R$  will be minimum if,

$$\left\| k(A^T A)^{-2} A^T (Y + \Delta Y) \right\| = \left\| (A^T A)^{-1} A^T \Delta Y \right\|$$

i.e. if,

$$k = \frac{\left\| \Delta Y \right\|}{\left\| (A^T A)^{-1} A^T (Y + \Delta Y) \right\|} \quad (3.18)$$

In the above derivation, an ill-conditioned matrix equation was replaced with a better conditioned one and by the proper choice of factor  $k$ , a good approximation of the solution vector was obtained.

Qualitatively, factor  $k$  is limited to be small and positive, and equation 3.18 gives a quantitative representation of  $k$ . In its application to electrocardiography studies, equation 3.18 simply implies that factor  $k$  is related to the power ratio of the body surface noise and the epicardial signals.

Numerical examples of solving ill-conditioned matrix equations of order 2, 3 and 4 using the direct inverse method and regularisation techniques are given in Appendix A7. This shows that using the direct inverse method, the error in the solutions are directly related with the condition number of the system matrix. Solutions become very unstable if the condition number is large. Using the regularisation technique, more stable solutions were obtained despite the relatively large condition number. A typical example for a matrix with a condition number of 796; 10% error gave an incredible  $\pm 8000\%$  solution error in the direct inverse method and only  $\pm 40\%$  when the regularisation technique was used.

### 3.4 Application to inverse problem in electrocardiography

The regularisation technique for solving the ill-conditioned matrix equation discussed previously will be used to solve the inverse problem in electrocardiography. In assessing its usefulness, two important aspects have to be examined. These are the stability and the correctness of the solution.

#### 3.4.1 Stability

As discussed in section 1.4, in the unconstrained cases, both the epicardial potential approach and the equivalent cardiac generator approach (in terms of current dipoles) suffer the same instability problem. The best way to test the stability of the solution is by contaminating the surface measurements with noise to see whether the error introduced in the solutions remain acceptable.

In this study, the measured body surface potentials  $S(t)$  were contaminated with a certain percent (PC) of noise (N) which was Gaussian distributed with standard deviation  $\sigma$  and mean zero, i.e.

$$S_i'(t) = S_i(t) + N(\sigma(t), 0)$$

where  $S_i(t)$  is the measured surface potential at the  $i^{\text{th}}$  position at time  $t$ , and

$$\sigma(t) = PC \times \left[ \frac{\sum_{i=1}^I s_i(t)^2}{I} \right]^{0.5}$$

where I is the number of surface measurement sites.

Three sets of noise were added, which were 1%, 5% and 10% respectively. The inverse solutions were computed from the measured body surface potentials of a human subject. The results were compared with reference to those with no extra noise added. Fig.3.4 shows the sum squared epicardial potentials and the corresponding sum squared residues in P-wave, QRS complex and ST-T segment when different amounts of noise were added. Fig.3.5 shows the cross-correlation coefficients throughout the whole cardiac cycle. Fig.3.6 shows the relative squared residues which are defined as:

$$\frac{\sum_{i=1}^I (E_i - E'_i)^2}{\sum_{i=1}^I (E_i)^2}$$

where  $E'_i$  and  $E_i$  are the computed epicardial potentials with and without noise added to the body surface measurements, and I is the number of epicardial segments.

The results are summarised in Table 3.6. It can be seen that with 1% of Gaussian noise added to the body surface measurements, the computed epicardial surface potentials remained

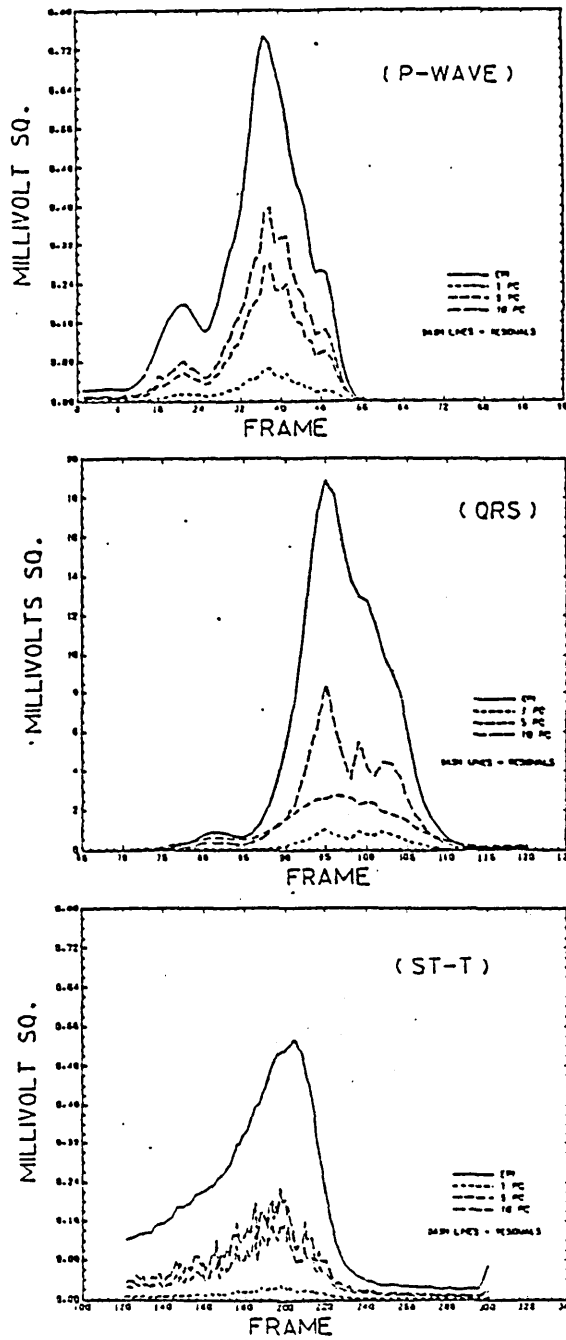


Fig.3.4 The sum squared epicardial potentials (EPI) and the corresponding sum squared residues in the P-wave, QRS complex and T-wave with different amount (1%, 5% and 10%) of noise added.

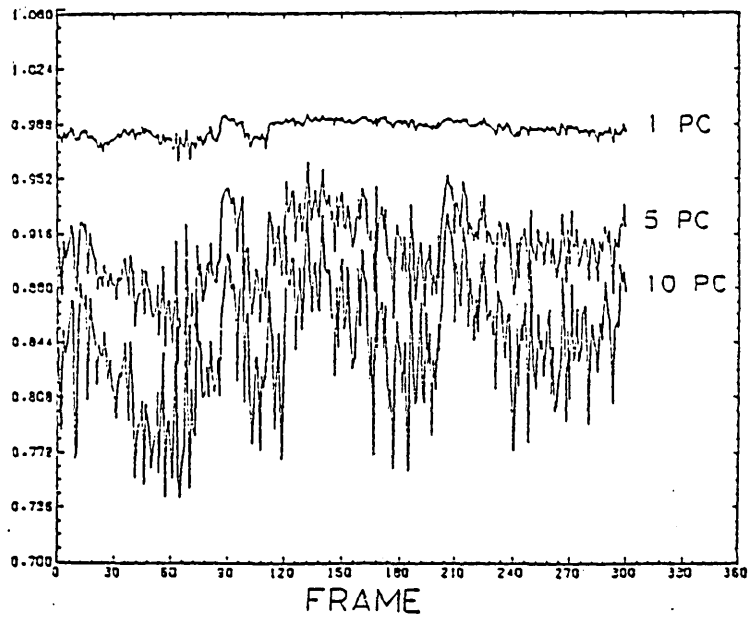


Fig.3.5 Cross correlation coefficient.

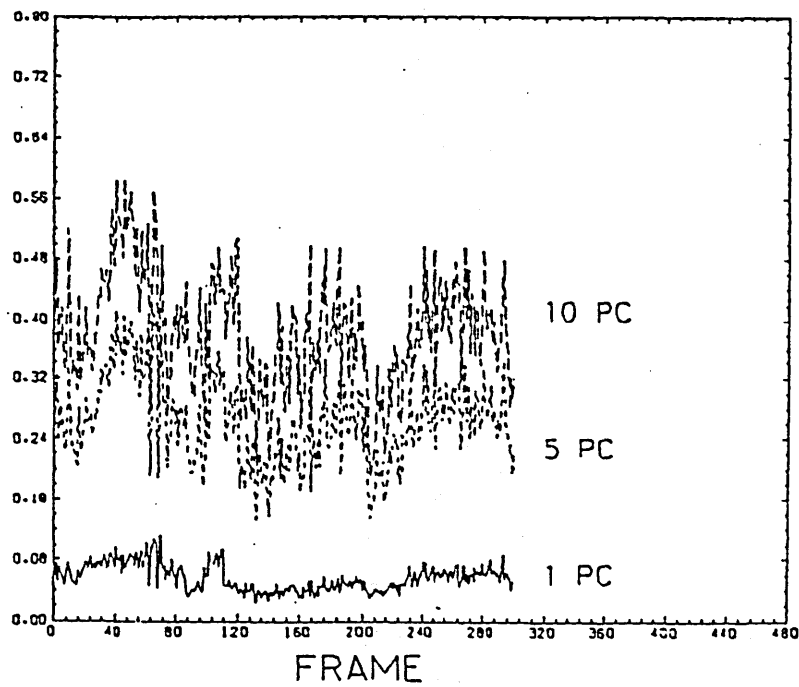


Fig.3.6 Relative squared residue. (See text.)

| NOISE ADDED<br>(percent)      |      | 1     | 5     | 10    |
|-------------------------------|------|-------|-------|-------|
| SEGMENT<br>ERROR              | MIN  | 0.032 | 0.120 | 0.173 |
|                               | MAX  | 0.476 | 0.911 | 1.232 |
|                               | MEAN | 0.204 | 0.468 | 0.583 |
| FRAME<br>ERROR                | MIN  | 0.002 | 0.003 | 0.004 |
|                               | MAX  | 1.000 | 2.320 | 2.891 |
|                               | MEAN | 0.130 | 0.285 | 0.842 |
| CROSS-<br>CORRELATION         | MIN  | 0.963 | 0.829 | 0.742 |
|                               | MAX  | 0.994 | 0.962 | 0.742 |
|                               | MEAN | 0.984 | 0.906 | 0.842 |
| RELATIVE<br>RESIDUE<br>SQUARE | MIN  | 0.024 | 0.133 | 0.189 |
|                               | MAX  | 0.111 | 0.412 | 0.587 |
|                               | MEAN | 0.056 | 0.263 | 0.382 |

TABLE 3.6 Error study of the inverse solutions with noise added on the body surface measurements.  
The definition of percentage of noise added is found in text.

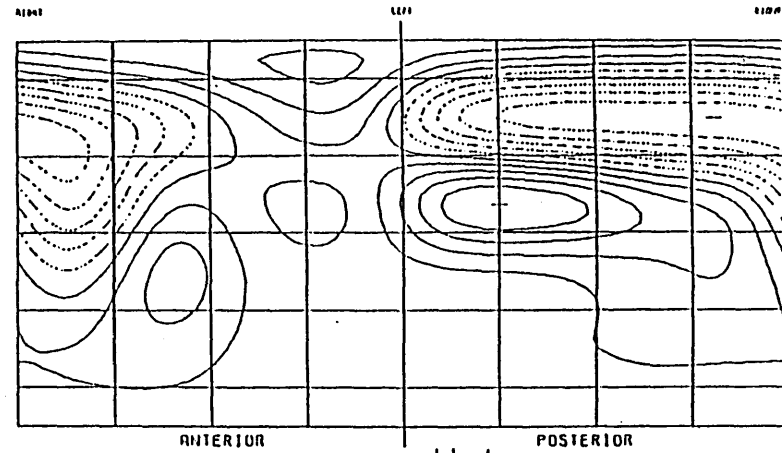
very stable and the maximum relative squared residue was about 11% with a mean value of 5.6%. Even with 10% noise added, the error introduced to the solution was not as high as predicted by the condition number of the system matrix (which was 457). The maximum relative squared residue was only 58% and a mean value of 38%. Despite such an error, the main features could still be seen on the isopotential maps in Fig.3.7.

#### 3.4.2 Correctness

Even if the model is stable with a reasonable amount of noise present, it is still necessary to consider whether the computed solution agrees with the actual sequence of the cardiac cycle.

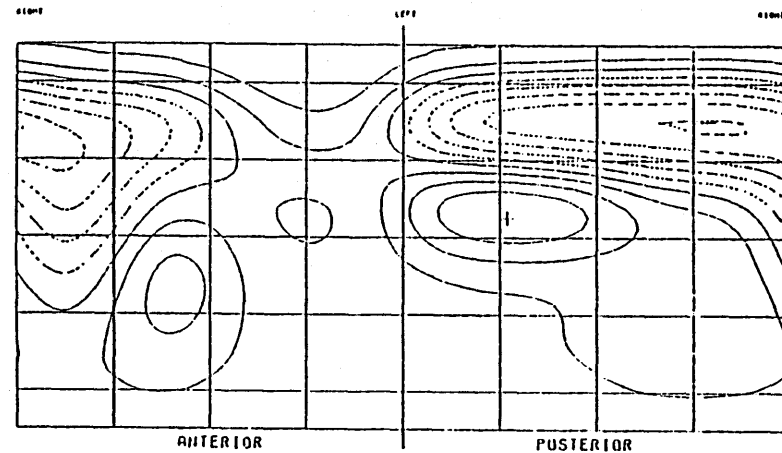
In order to verify the correctness of the computed epicardial potential, it is necessary to have epicardial measurements. Ideally, simultaneous measurements of the body surface and epicardial surface potential have to be done. Using the present methods, it is not technically or ethically feasible to put a large number of electrodes over the human epicardium in the closed chest.

In Chapter 4 some direct measurements of the human epicardial surface potentials are described, but as a maximum number of 3 epicardial measurements from each subject were taken, it is obviously inadequate to generate an epicardial surface map.



•  $\pm 1.251$  mV  
 -  $\pm 1.449$  mV  
 LEVEL  $\pm 0.260$  mV  
 RMS = 0.643 mV

CHANNEL  
12

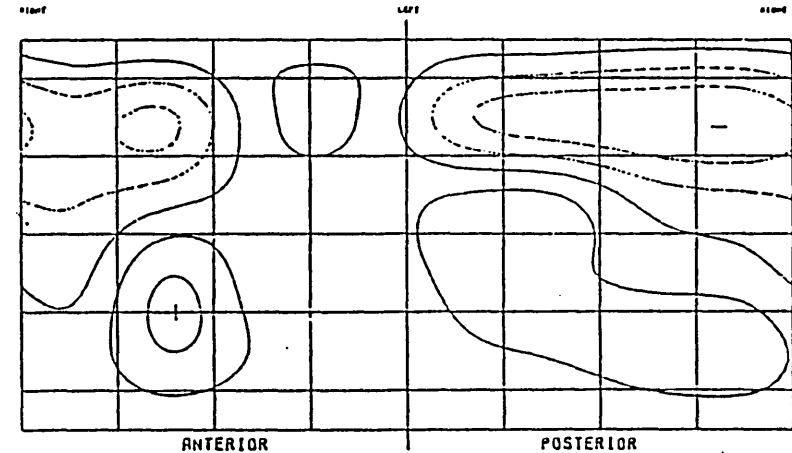


•  $\pm 0.875$  mV  
 -  $\pm 1.274$  mV  
 LEVEL  $\pm 0.250$  mV  
 RMS = 0.461 mV

CHANNEL  
12

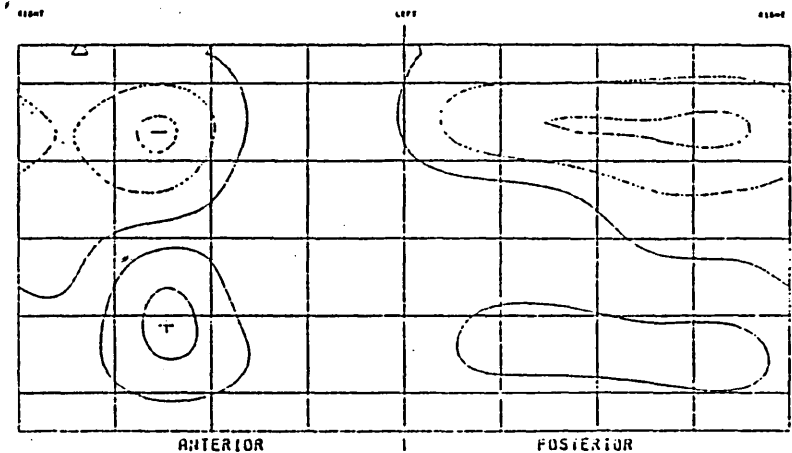
0%

1%



•  $\pm 0.576$  mV  
 -  $\pm 0.745$  mV  
 LEVEL  $\pm 0.250$  mV  
 RMS = 0.296 mV

CHANNEL  
12



•  $\pm 0.571$  mV  
 -  $\pm 0.542$  mV  
 LEVEL  $\pm 0.250$  mV  
 RMS = 0.216 mV

CHANNEL  
12

5%

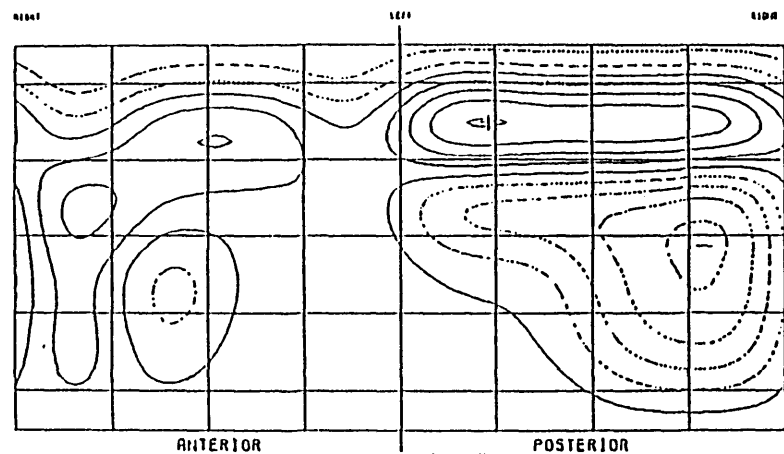
10%

130

(a)

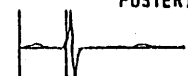
Fig.3.7 Epicardial contour maps in the (a) P-wave, (b) QRS complex and (c) T-wave. 0%, 1%, 5% and 10% noise added on the body surface measurements. The display format of the iso-potential maps is the same as Fig.1.14. The grid used corresponds to Fig.3.2.

FRAME 85

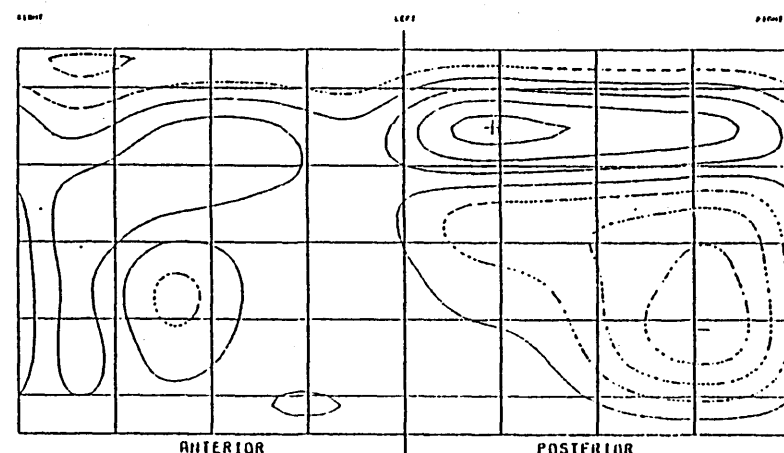


0%

•  $\pm 2.046$  mV  
 -  $\pm 2.132$  mV  
 LEVEL  $\pm 0.600$  mV  
 RMS = 0.787 mV

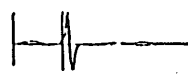


CHANNEL 12

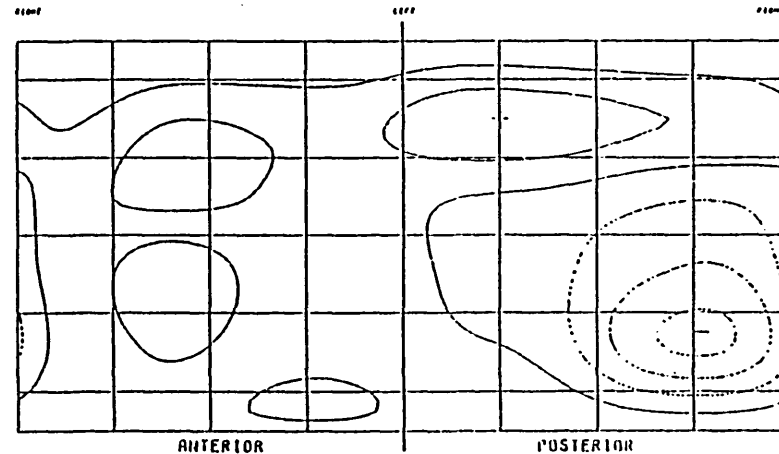


1%

•  $\pm 1.689$  mV  
 -  $\pm 1.840$  mV  
 LEVEL  $\pm 0.500$  mV  
 RMS = 0.656 mV

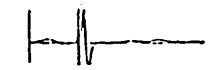


CHANNEL 12

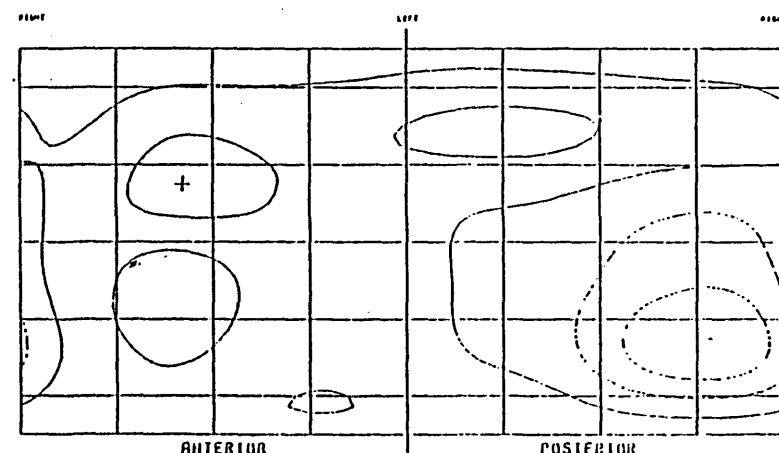


5%

•  $\pm 0.890$  mV  
 -  $\pm 1.658$  mV  
 LEVEL  $\pm 0.500$  mV  
 RMS = 0.457 mV

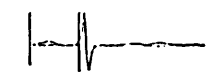


CHANNEL 12



10%

•  $\pm 0.702$  mV  
 -  $\pm 1.418$  mV  
 LEVEL  $\pm 0.500$  mV  
 RMS = 0.407 mV

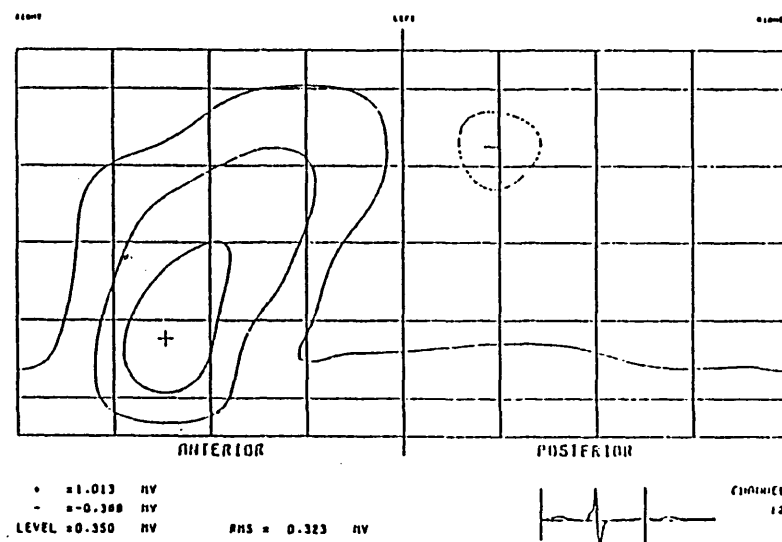
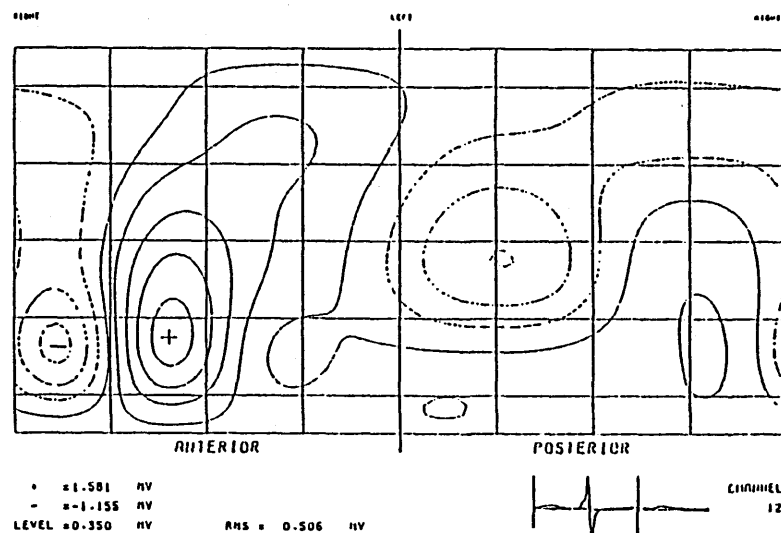
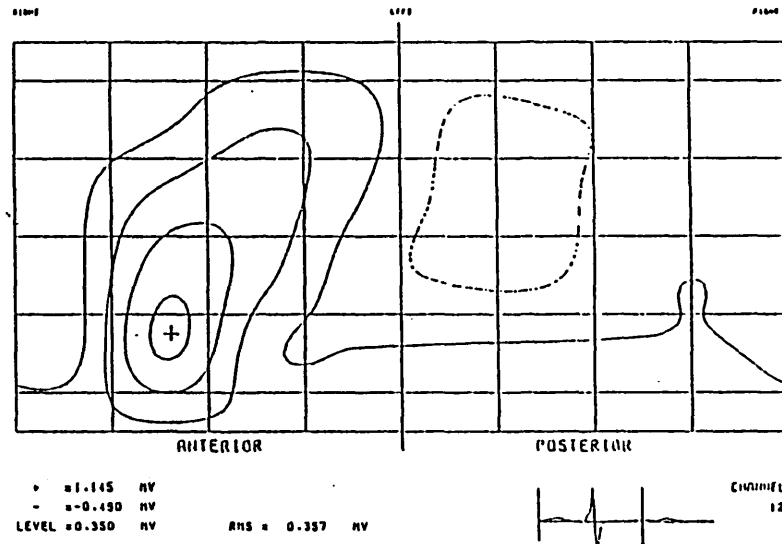
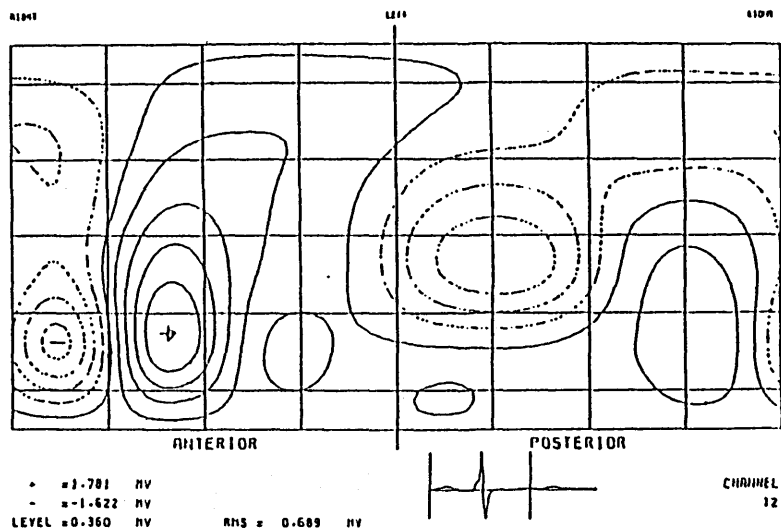


CHANNEL 12

(b)

Fig.3.7 (Cont'd)

FRAME 180



(c)

Fig.3.7 (Cont'd)

In order to have a general idea of how close the computed inverse solution in this study was to the physiological sequence during the whole cardiac cycle, the observations done by Spach et al. (1977) [6] on chimpanzees were used. In terms of torso shape and cardiac geometry, the chimpanzee has the closest similarity among other animals to humans.

The following descriptions are the characteristics of the epicardial and body surface map observed by Spach on chimpanzees during the QRS complex. The corresponding isopotential contour maps in the QRS complex of a subject are displayed in Fig.3.8. In this figure, the top one shows the measured body surface maps while the bottom are the computed epicardial surface maps.

a. At the beginning of the QRS:

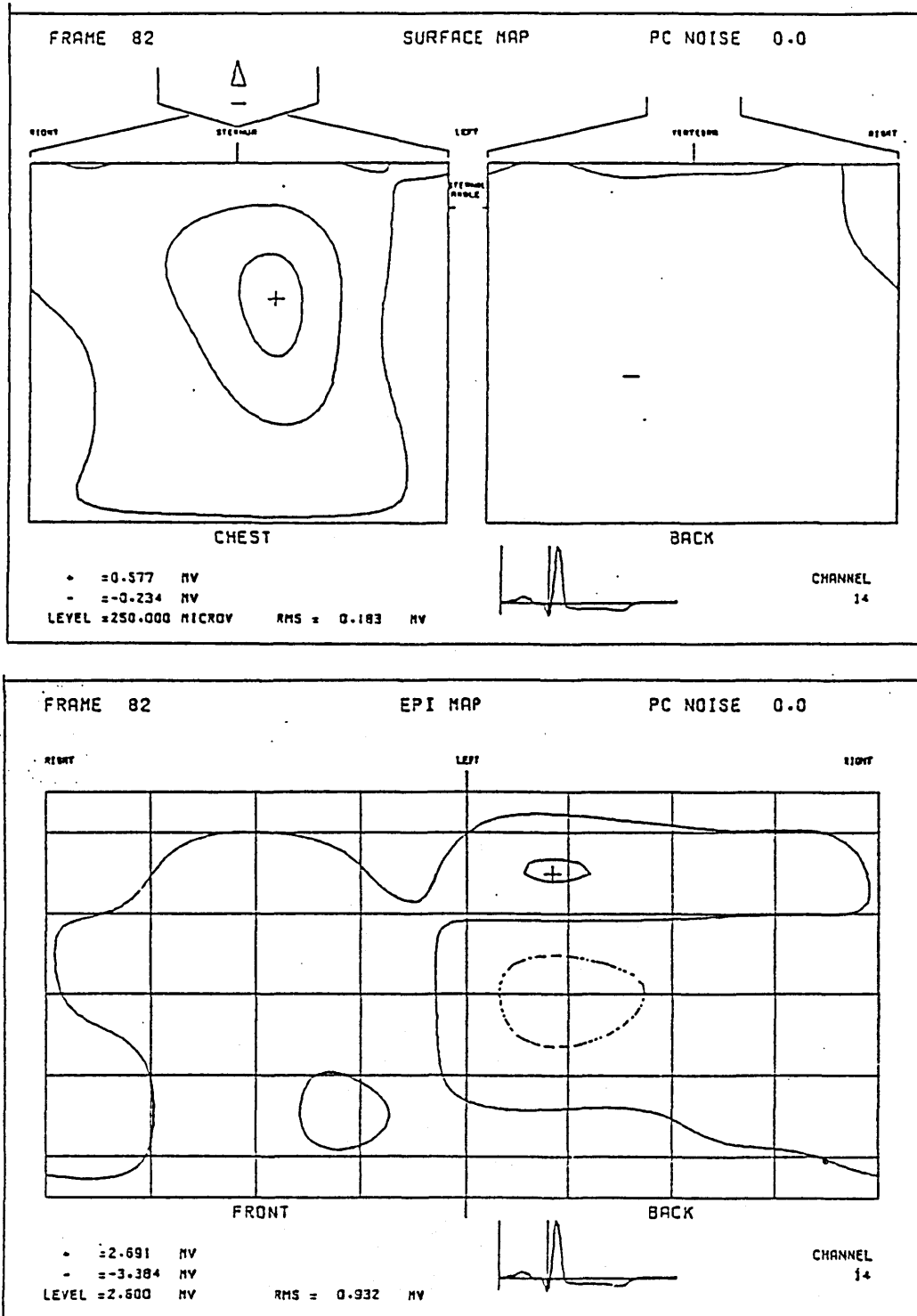
Positive potentials covered most of the ventricles except for negative potentials on the lateral left ventricle (LV). On the body surface, this produced an anterior maximum and a posterior minimum.

b. At the early stage of the QRS:

On the epicardium, a maximum was present over the septum anteriorly and a minimum at the LV base. The body surface pattern reflected this with an anterior maximum and a minimum on the back.

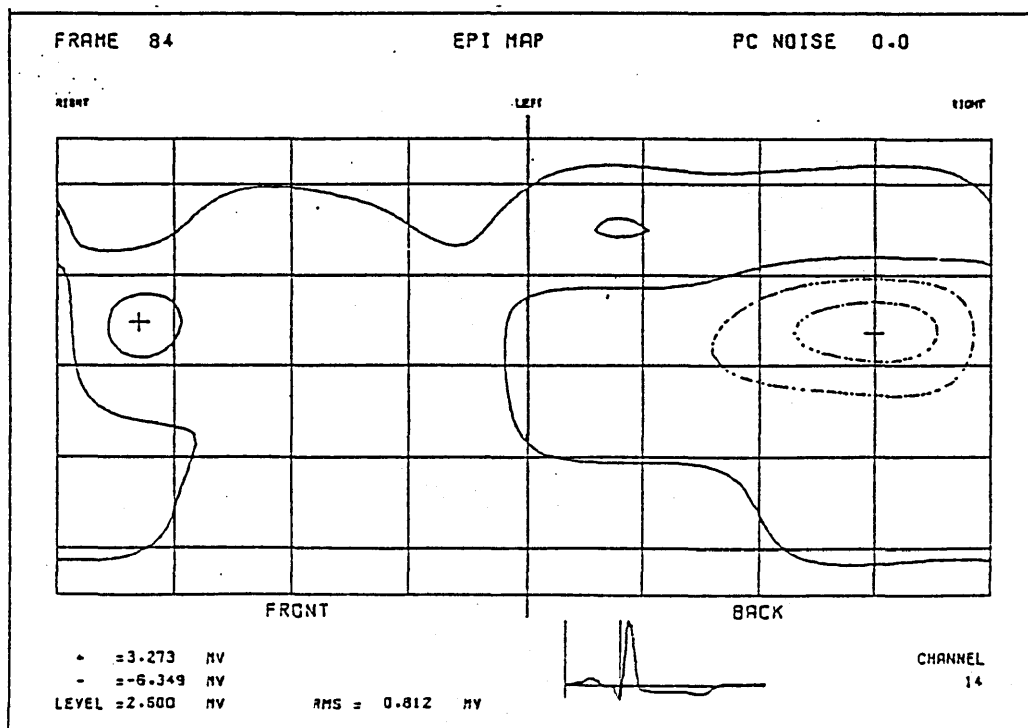
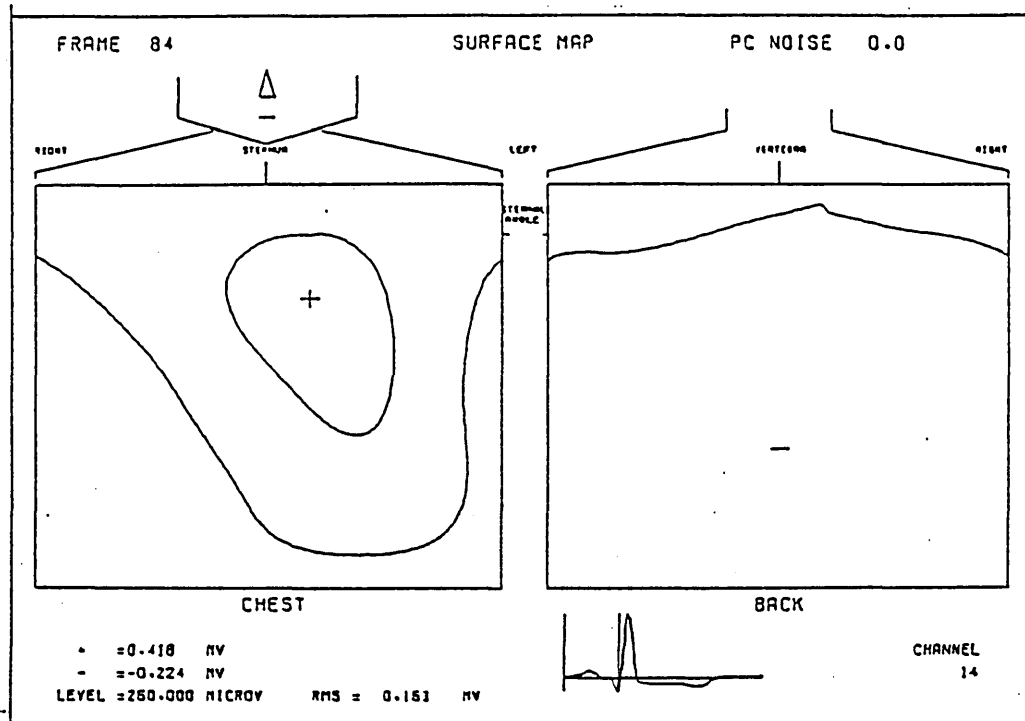
c. Mid-way through the Q-R:

Negative potentials had developed on the anterior right ventricle (RV) due to epicardial breakthrough. A maximum was



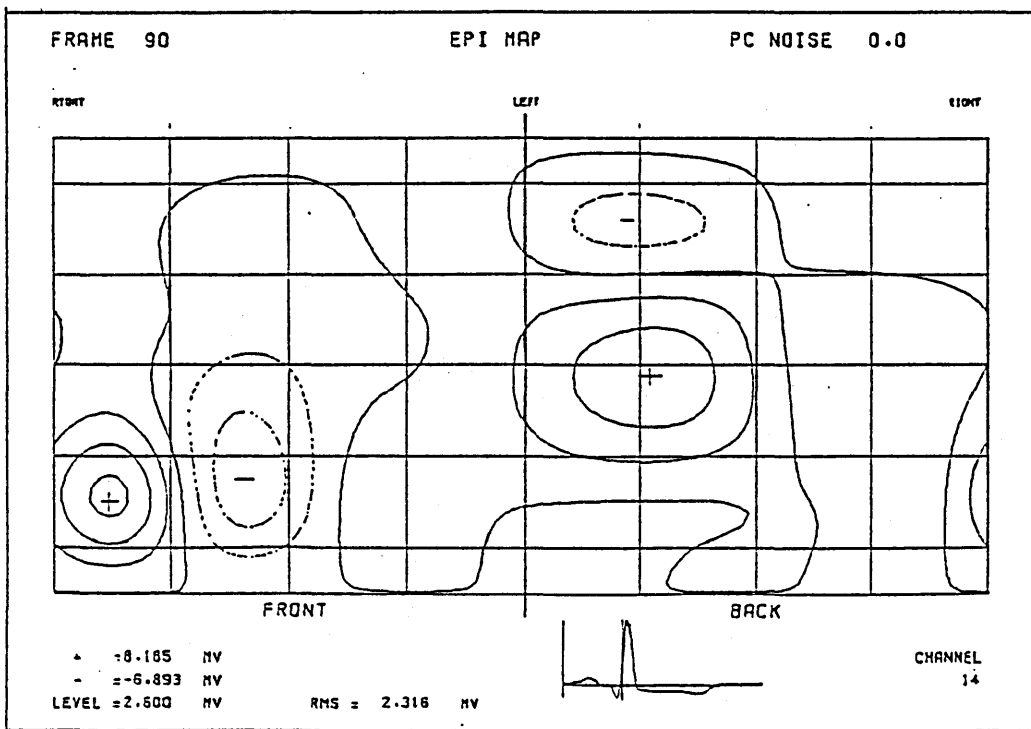
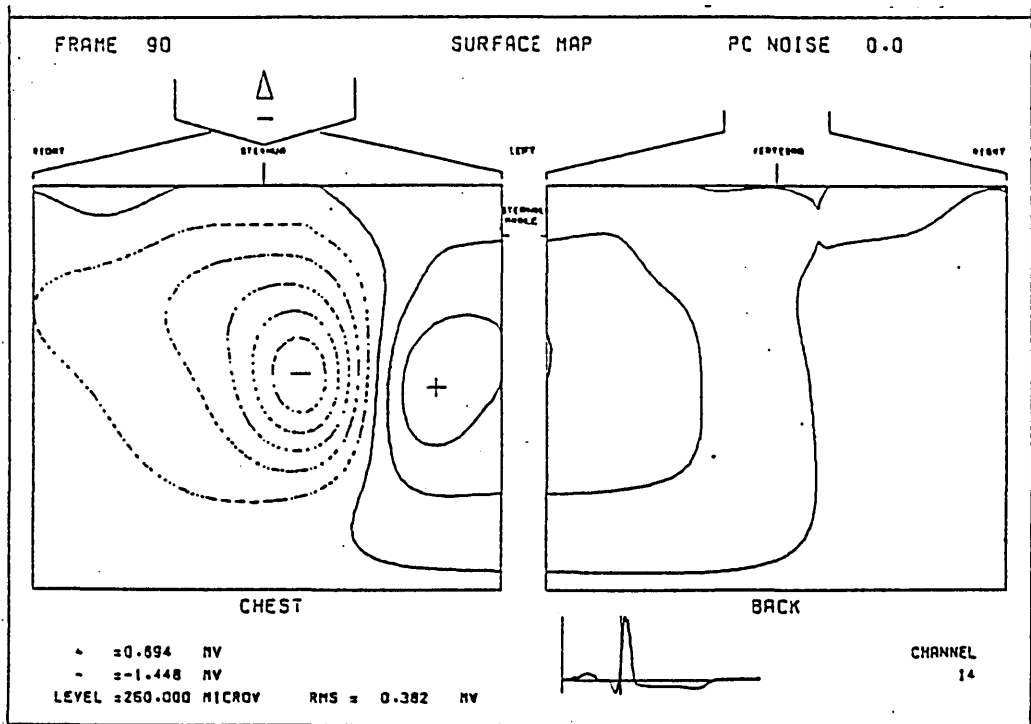
a.

Fig.3.8 The iso-potential contour maps in the QRS complex. The top one shows the measured body surface maps and the bottom one shows the computed epicardial surface maps. The grid used in the epicardial maps corresponds to Fig.3.2.



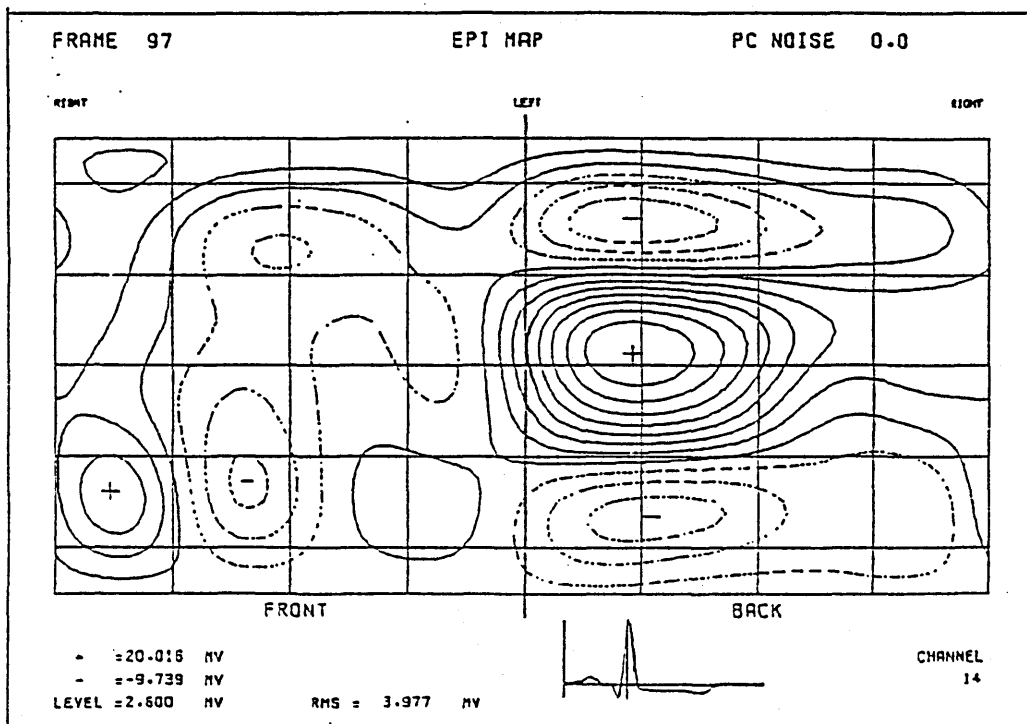
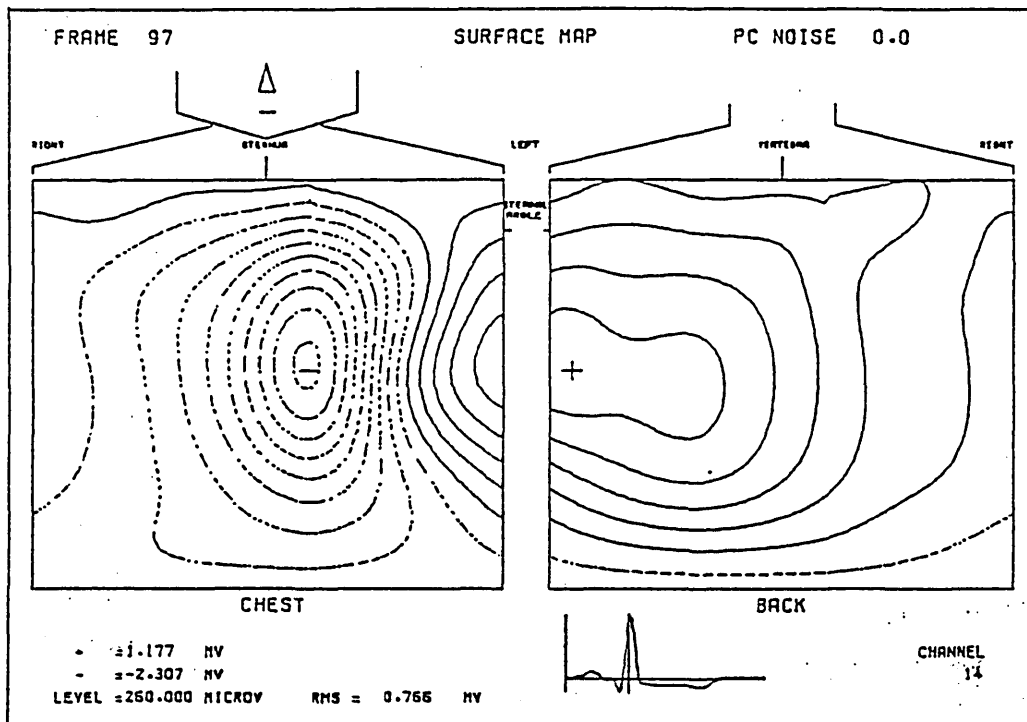
b.

Fig.3.8 (Cont'd)



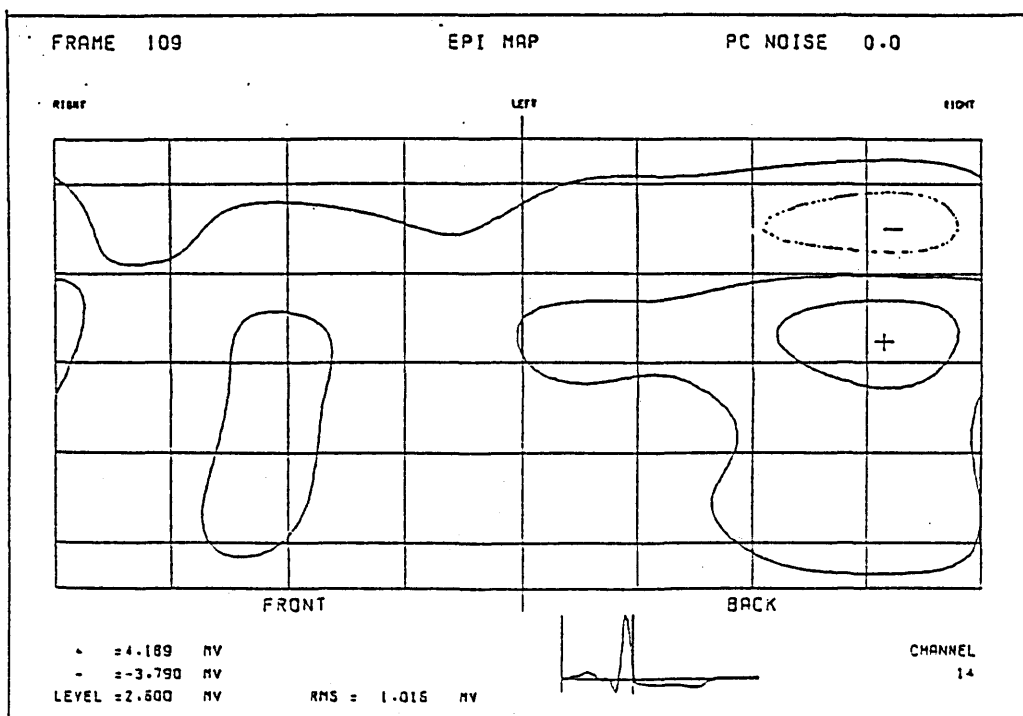
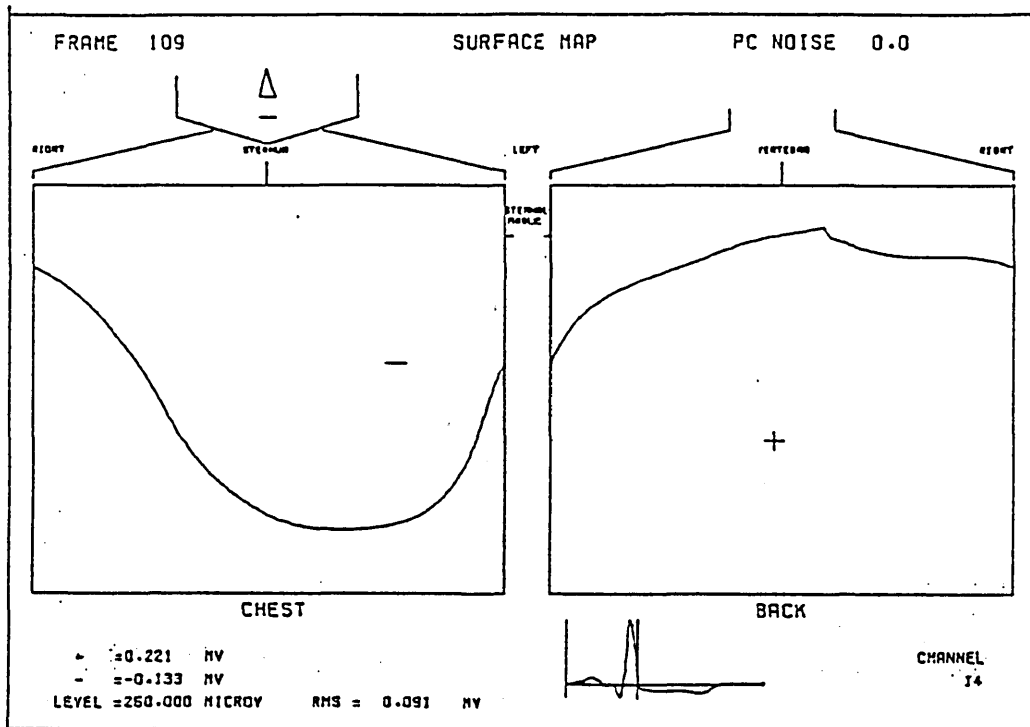
C.

Fig.3 .8 (Cont'd)



d.

Fig.3.8 (Cont'd)



e.

Fig.3.8 (Cont'd)

on the LV.

On the body surface, there is a distinctive saddle distribution. This is a region such that an observer staying in it can see growing potentials in front of and behind himself, decreasing potentials at his left and right.

d. R-peak:

Epicardial breakthrough on the diaphragmatic surface of LV produced a minimum there and the RV was enveloped in negative potential. Positive potentials covered most of the LV on which there was a single maximum.

The body surface distribution was simpler, the minimum was over the lower sternum and the maximum was on the left precordium.

e. Towards the end of the QRS:

Isolated positive potential areas superimposed on a predominantly negative epicardial pattern. Scattered positive potentials due to repolarisation appeared.

On the body surface, there was little change of the maximum and minimum.

As can be seen from the descriptions and the plots, all the main features present are in agreement, although this is only a very crude comparison. In Chapter 5 the time sequence of epicardial potentials computed from body surface maps in humans are compared directly with epicardial measurements, but only at a limited number of sites.

### 3.5 Conclusion

The forward transfer coefficient matrix which describes the relationship between the body and epicardial surface potentials was derived using a numerical model of the human thorax. It was shown that the condition number of the matrix was very much dependent on the configuration of the epicardial segments and was greatly reduced by dividing the epicardial surface into unequal segments. The condition number could also be reduced with a smaller number of segments, but it would give a poorer resolution so it was decided that a larger number of segments of unequal size should be used.

In order to minimise programme execution time in deriving the forward transfer coefficient, relative error was used as a terminating criterion for the iterative process and it was found that it did not degrade the stability nor the accuracy of the inverse solution significantly.

A regularisation technique was developed for the calculation of epicardial potentials from body surface measurements. It was shown that despite the large condition number of the system matrix, the solutions remained very stable when the body surface measurement was contaminated with a reasonable amount of noise. A comparison with published measured data in intact chimpanzees showed the solutions were physiologically reasonable.

## CHAPTER 4

### INSTRUMENTATION, DATA ACQUISITION AND PRE-PROCESSING

#### 4.1 Introduction

The fully portable dedicated system developed in the Engineering in Medicine Laboratory at Imperial College, was used for the simultaneous measurement of body and epicardial surface potentials in this study. This portable system has the capacity of taking measurements from 40 electrode sites simultaneously. In this study, 37 of the 40 channels available were used for the surface ECG measurement and the remaining 3 for the epicardial ECG measurement. The experiment was carried out with the co-operation of the Cardiology Department, St. Mary's Hospital, Paddington. Over a period of six months, a total number of 21 patients (20 males and 1 female) who were undergoing coronary bypass graft operations were selected for measurement in this study. All patients included had angiographic evidence of good left ventricular function and no evidence of extensive infarction. At the completion of the surgical procedure and before cardio-pulmonary bypass was completed, three epicardial stainless steel pacing wires (Ethicon F EP 13) were placed on the epicardium. The terminal end of each wire (about 2 cm long) was bared of insulation and bent in the middle to form a hook. The

"elbow" of the hook was drawn against the epicardium by means of a looped suture. The proximal ends of the epicardial wires were passed through the chest wall and secured at the body surface by sutures. Each was labelled and charted appropriately. About 7-10 days after the operation, if the patient's recovery was uncomplicated, recordings were made from the epicardial wires and from the body surface. In section 4.2, essential features of the data acquisition system are described (details of the system hardware and the accompanying software are given by Clarke (1984) [83]). Section 4.3 gives a detailed description of the data acquisition procedure and section 4.4 describes how the measured data are pre-processed.

#### 4.2 Instrumentation

A block diagram of the data acquisition system is given in Fig.4.1. The electrodes used for the body surface measurement were commercially available silver/silver chloride disposable electrodes. The distal ends of the 3 epicardial wires were connected directly to the input of the system. All body surface and epicardial signals were measured with respect to a Wilson Central Terminal. "Active" electrode, i.e. an active amplifier of unit gain was connected directly to each electrode before the input of the system, was used to improve the signal to noise ratio (Guardo, 1972 [54]; Monro et al., 1974 [84]).

The input amplifiers for each of the 40 channels were identical and calibrated to have the same gain except those three connected to the epicardial wires which have approximately one fourth of the gain of those connected to the surface electrodes.

The bandwidth of the amplifiers was 0.01 to 138 Hz. (-3 dB). This was sufficient to minimise low frequency baseline drifts and to avoid aliasing during the sampling process. The amplified signals were multiplexed and sampled at a rate of 500 Hz per channel (total sampling rate was  $40 \times 500 = 20$  kHz). The analogue to digital conversion (A/D) was done with a 12-bit precision. The beginning of the data conversion was triggered by the LSI 11/23 processor, and the sampled data transferred to a Winchester Hard Disk. Typically about 90 seconds (real time) of data were stored. The system also provided the facility for

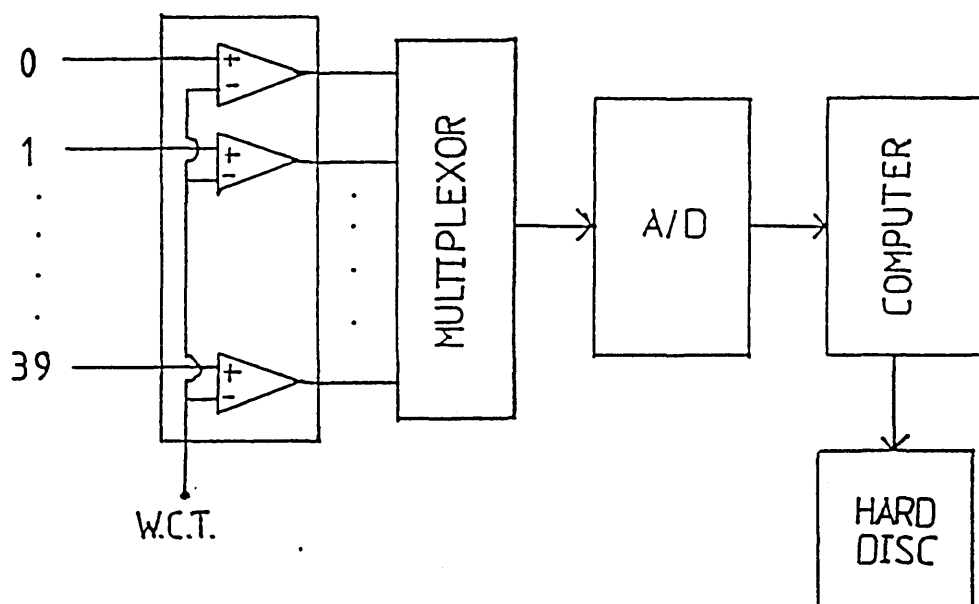


Fig.4.1 Schematic diagram of the data acquisition system.

inspection of all channels on an analogue oscilloscope before recording commenced. After recording, data could be displayed on a digital graphic screen (Tektronix 4006) for inspection. Any segment of any of the recorded channels could be displayed.

All the connections to the patient were isolated from the power supply and earth to meet the electrical safety regulations laid down for hospitals [85]. The American Heart Association (Gelselowitz et al., 1980 [86]) recommends that all electrocardiographic equipment be designed should have a leakage current to the patient of less than 10 microamps. In this study, it is assured by using a unit gain buffer amplifier on all leads which features high input impedance and low leakage current. Resistors of high values were also incorporated in all leads connected to the patient as a second safeguard to limit the current flow in the event of the breakdown of the buffer amplifier.

#### 4.3 Data acquisition

The 40 channels were numbered 0 to 39, channels 0 to 36 were used for the body surface measurement and channels 37 to 39 were assigned for the epicardial surface measurement. The sites of body surface measurement were carefully determined so that errors due to electrode misplacement were minimised. The circumference and width of the patient's chest were measured and the measurement sites were marked before attaching the electrodes. The skin at each of the electrode sites was prepared by rubbing with an alcohol swab to give a good electrical contact.

Three rows of eleven equally-spaced electrodes were positioned horizontally around the chest (with the patient sitting upright). The vertical position of the row was found by dividing the distance between the umbilicus and the suprasternal notch by four (Fig.4.2). One electrode in each row was placed on the spine. The two reasons for doing this were: (1) the spine is anatomically easy to locate thus used as the reference for placing other equally-spaced electrodes, and (2) this would avoid the placing of electrodes over the surgical wound at the front of the chest (because the number of electrodes in a row is odd). In addition to the 33 equally-spaced electrodes, four extra electrodes were placed just next to the umbilicus (no.33), on the right (no.34) and the left (no.36) supraclavicular fossa and on the left side of the neck (no.35). Channel nos.37, 38 and 39 were connected to the epicardial wires and were systematically

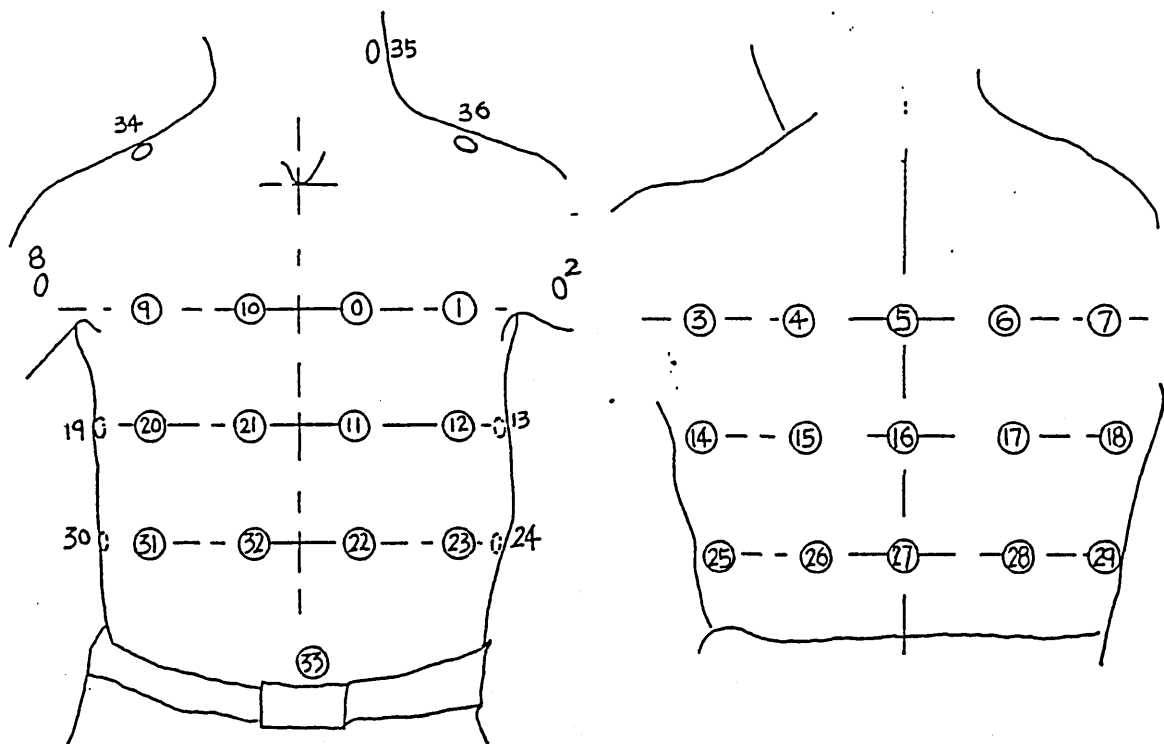


Fig.4.2 Arrangement of the measuring electrodes.

identified with the diagram drawn by the surgeon during the operation. Three extra electrodes were connected to the right arm, left arm and left leg to form the Wilson Central Terminal.

After all the connections were made, the patient was placed in a supine position as comfortably as possible and was told to relax in order to minimise the amount of noise on the recording due to skeletal muscle tremors. Each channel was then checked on the oscilloscope to confirm the presence of electro-cardio signals before commencement of recording. About 90 seconds (real time) of ECG data was taken and stored on the Winchester Disk. The recorded data was then displayed on a digital graphic screen, and the recording was repeated if necessary. The whole recording session lasted about 60 to 90 minutes.

#### 4.4 Data pre-processing

Due to the limitation of the instrumentation and a non-ideal measuring environment, data pre-processing is required before the data can be used for clinical or experimental studies. The data pre-processing for this work includes data editing, noise reduction, reconstruction of missing data, adjustment for time skew and base line correction.

##### 4.4.1 Data editing

Once the experimental recording was initiated, ECG data were collected continuously up to the amount of data required. These recorded data were then displayed on the screen and were reviewed in order to assess the quality of the the recorded signals. Any non-cardiac generated signals due to electrical artefact were then discarded. In addition, any physiologically significant events such as transient ectopics were noted.

##### 4.4.2 Noise reduction

Noise may be introduced into the ECG data due to various sources. The magnetic and electrical field interferences were reduced to a minimal effect by careful design of the data acquisition system. The noise due to the physiological effect of muscle movement was minimised by placing the subject at ease in a quiet and comfortable environment. As this type of noise is non-periodic and the ECG signals recorded can be assumed to be

repetitive, thus the coherent averaging technique can be used to further improve the signal-to-noise ratio.

The coherent averaging technique has been widely used for other applications in biomedical engineering, such as in the study of evoked cortical response in Electroencephalography (EEG). In this case a trigger pulse, usually a light strobe, is used which has a fixed time relationship with the evoked signals, and the wanted signals are extracted from a background of unwanted spontaneous EEG signals by coherent averaging. In many circumstances, averaging can be carried out in the absence of the external triggering pulse if it is possible to use part of the physiological signal itself as the reference. This aspect of averaging is currently applied to ECG studies in noise reduction and extraction of the small His Bundle signal [87,88] from body surface measurements.

In this work up to 64 ECG complexes were averaged. This would improve the signal to noise ratio by a factor of eight. Averaging was done interactively to remove bad complexes.

#### 4.4.3 Reconstruction of missing data

Despite careful attention paid to the data acquisition procedure, there is always a possibility that part of the recordings have to be discarded due to erroneous measurements, or the whole recording of a channel is missing due to falling off of the electrode or failure of recording amplifier. Reconstruction

of the missing data is required to minimise the artefact introduced in the body surface potential distribution. In this study, the cubic spline interpolation technique was employed to reconstruct the missing data. The procedure is described as follows. Initially the recorded ECG waveforms are displayed on a graphic screen to confirm the electrode (channel) number of missing data. Fig.4.3 shows a row of measuring electrodes with channel 13 missing, the estimate of the missing data on channel 13 is made by:

1. An initial value was found by linear interpolation between the nearest neighbours.
2. The estimate was used to find an improved estimate for the missing channel employing the cubic spline interpolation technique (further discussion in section 5.2).
3. Step 2 was repeated using the improved estimate until the value found converged to a given error tolerance.

The whole procedure was repeated using the column data as shown in Fig.4.3. Then the final estimated value for the missing channel was found by averaging the row and column estimates.

#### 4.4.4 Adjustment for time skew

Time skew error was introduced in the measured data due to the multiplexing technique used in the recording process. The data of different channels were not sampled and recorded at the same time instant, but separated successively by a time interval

$1/(N.F)$  where  $N$  is the number of channels being sampled and  $F$  is the sampling frequency for each channel (Fig.4.4). In this study, 40 channels were multiplexed each with a sampling frequency of 500 Hz., thus the separation interval between two successive channels was 50 microsecond (us) and the maximum separation error was 1.95 millisecond (ms). This time skew error is appreciable in the surface mapping especially during depolarisation when potentials are rapidly changing in amplitude and distribution (Lux, 1983 [89]). Time resynchronisation is needed to minimise this error and it was achieved by interpolating the original signal sequence by a factor of  $N$  and the sample corresponding to the time  $k/(N.F)$  was chosen, where  $k=1,2,\dots,39$ , and are the number of channels with respect to a reference channel for resynchronisation.

Various interpolating techniques are available, Monro (1980) [90] introduced the frequency domain interpolating technique using the advantage of the Fast Fourier Transform algorithm. Monro also recommended the much simpler cubic spline technique. Unlike the former method which limited the data sequence to be periodic and the number of data points to be a power of 2, the cubic spline technique can be used for aperiodic signals of any length. Detailed treatment of the interpolating techniques applied in ECG studies are found in Ribiero (1984) [88]. His results suggested that the cubic spline technique was satisfactory.

350

 $20_0$ 

11  
-0

12  
0

13  
X

14  
0

15  
0

 $24_0$ 

330

Fig.4.3 Interpolation of missing electrodes.

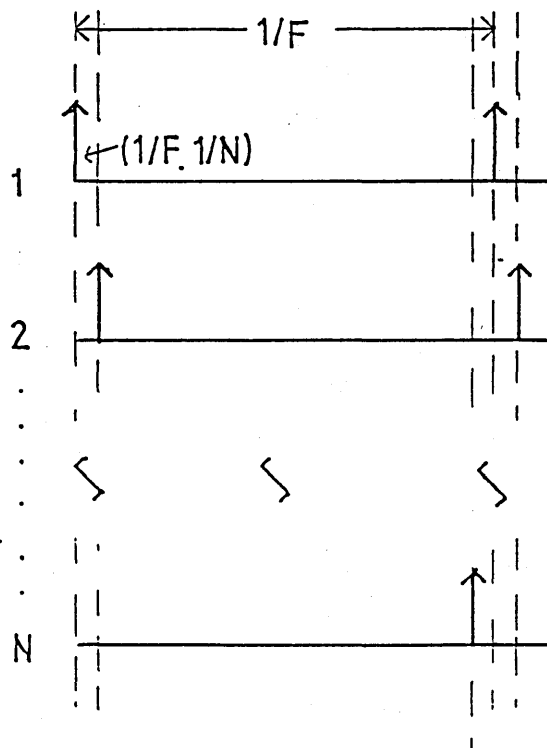


Fig.4.4 Time skew of sampling channels.

#### 4.4.5 Base line adjustment

The low frequency drift of the base-line is mainly due to movement of the subject during respiration or with positional changes, and has no relation to cardiac physiology whatsoever. Most of the procedures used to correct the base-line drift in ECG mapping assume that during the ECG cycle there is a period, e.g. the T-P segment, when the total body was isoelectric. Frequently however the T-P segment is not isoelectric, as in exercise ECG mapping in which the T and P waves might be fused together. So alternatively the PQ segment could be used, although here prominent potentials due to atrial depolarisation could be present. In fact, it is very difficult to define the "true" base-line technically. However for the purpose of minimising error due to baseline drift, both of the above approaches are probably satisfactory (Lux, 1983 [89]). In this study, the base-line was adjusted by removing the D.C. level in the P-Q segment, this was done by defining the mean value of 10 data points in the P-Q segment (selected visibly by the operator) as zero and the data sequence were adjusted accordingly, this procedure was followed for all channels.

## CHAPTER 5

### COMPUTATION OF EPICARDIAL SURFACE POTENTIALS FROM BODY SURFACE MEASUREMENTS

#### 5.1 Introduction

The forward transfer coefficient and the computational techniques developed in Chapter 3 are used to calculate the epicardial surface potentials from the body surface measurements on human subjects (detailed descriptions of the data acquisition were given in Chapter 4). The forward transfer coefficient was derived from a computer model with 26 unequal-sized epicardial segments, and they describe the relationship between the potentials of the 26 epicardial segments and of the corresponding optimal surface sites only. Consequently the potentials at these sites have to be known for the calculation of the epicardial potentials. The fact that these optimal sites are scattered on the body surface in an irregular pattern makes it very cumbersome for placing the measuring electrodes on these sites. Another method which can be used is to place a large number of measuring electrodes in a regular pattern on the body surface in order to cover the optimal sites. This proves to be impractical in clinical practice because of the large number of electrodes to be

handled. Alternatively body surface measurements can be made with a limited electrode set (i.e. with a fewer number of electrodes) placed regularly on the body surface and then the potentials at the optimal sites could be recovered through an interpolation method.

In section 5.2, the adequacy of using a limited electrode set to sample the body surface potential and various interpolating techniques are discussed. In section 5.3 the computational procedures of calculating the epicardial potentials are presented and in section 5.4 the evaluation of the computed inverse solution from the measured data are made. Finally in section 5.5, the effect of omission of the potential measurements on the back of the body surface on the inverse solution is investigated.

## 5.2 Reconstruction of surface map from a limited electrode set through interpolation

### 5.2.1 Limited electrode set

In the early studies in surface mapping, as many as 300 body surface measurements have been made [11], and the iso-potential contour display of surface maps were drawn by hand. This proved to be very time consuming and impractical for any body surface mapping study at all. With the advent of micro-electronics and high speed digital computers, the simultaneous measurements and display of large numbers of body surface sites has become feasible (e.g. the 240-lead system by Cottini et al., 1972 [91] and the 192-lead system by Wyatt and Lux, 1974 [92]). However, to refine the data acquisition and mapping technique to a more practical and reliable form, the problem of redundancy of information contained in a large number of leads has to be looked into. The question here is how many and what locations on the body surface must be measured to give reliable information about the body surface potential distribution. Barr et al. (1971) [93] examined this in a narrower manner, they investigated on the number and position which should be measured to be able to compute consistently and with an acceptable accuracy the potential values at 150 particular positions over the body surface during the QRS complex. They studied a group of 41 abnormal and 4 normal paediatric patients and concluded that a minimum of 24 properly placed electrodes were

required to sample the body surface potential distribution to an acceptable degree of accuracy which was defined by the authors as the average mean square error to be less than 4%. Lux et al. (1978) [94] reported similar results in their study using 192 surface measurements on 70 normal subjects and 62 patients with old myocardial infarctions. The results showed that 30 optimally placed leads were required to reproduce body surface maps up to an accuracy of 32  $\mu$ V. The two findings were encouraging which suggested that limited lead mapping of the body surface potential distribution was possible. However, their electrode sites are scattered irregularly on the body surface and this caused inconvenience in placing the electrodes in the correct locations in actual clinical practice.

For practical convenience, it would clearly be better to use a regular placing of electrodes. This leads to the question, would it be possible to sample the body surface potentials with a regular-grid and with the values at optimal sites then derived from the measured data ? Before going into the second part of the question which leads to the consideration of the accuracy of the reconstruction technique, it is best to initially consider whether a limited lead set of regular grid for sampling is adequate. In the same study, Barr et al., 1971 [93] showed a deterioration of results by a factor of 2 to 4 when regular grid measurement was used (Fig.5.1). But the regular grid lead sets they used were by no means ideal since the electrodes were either placed clustering on the front or just along two narrow strips around the torso.

Monro et al. (1974) [84] used a 24 regularly spaced electrode set, arranged into 3 rows to cover most of the body surface (Fig.5.2), to sample the body surface potentials. They validated the use of this system by means of Fourier analysis of the power spectrum of the sampled signals. The results using the surface potential measurements of 15 normal subjects showed that the measurement represented at least 93% of the power of the dipole field and at least 83% for the quadrupolar fields. Although no comparable result was shown, their regular grid system appeared to be adequate. The 24 electrode system was later improved into a 26 lead system (Attwood and Monro, 1979 [95]) to ensure the upper and lower closure of the torso surface, which is a necessary condition for the use of Fast Fourier Transform algorithm (FFT) in the reconstruction and analysis of body surface map. Ribeiro (1984) [88], based on Monro's lead set, employed a spectral analysis technique in estimating the effect of the number of measuring electrodes used on the recovery of the power content of the ECG signals during P wave, PR segment, QRS complex, ST segment and T wave of the cardiac cycle on 38 normal subjects (5 females and 33 males). His results showed that the maximum power error with the 26-lead set was 4.8%. He further showed that an increase to 40 electrodes, (i.e. 3 rows of 12 equally spaced electrodes with 3 neck and 1 umbilicus electrodes), would reduce the maximum power error to 1.5%. From the various results and analyses, it can be concluded that reconstruction of body surface potential contour maps from a limited number of regularly spaced electrodes is

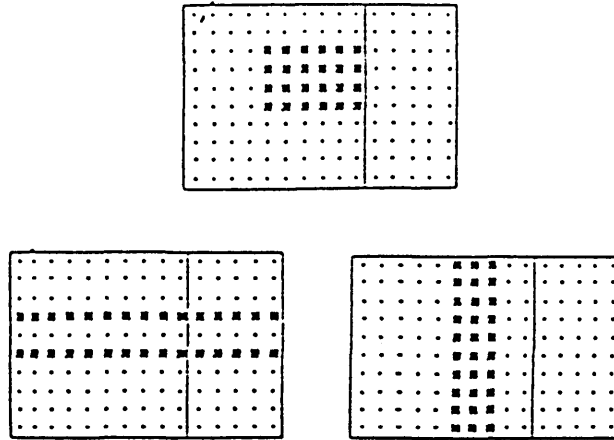


Fig.5.1 Location of the measuring electrodes using regular grid by Barr et al. (1971) [93].

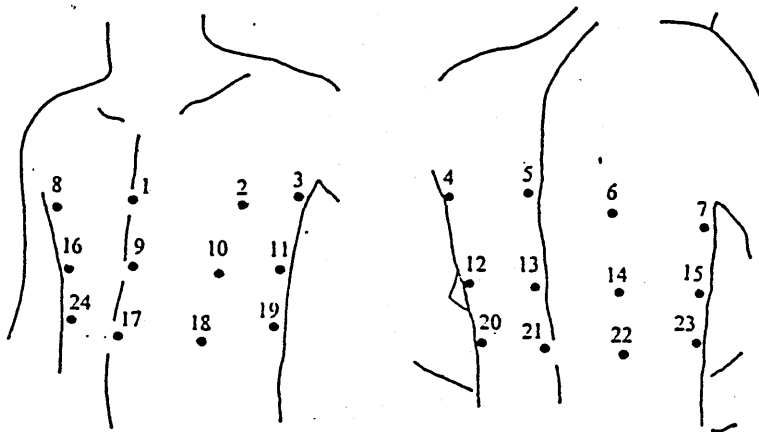


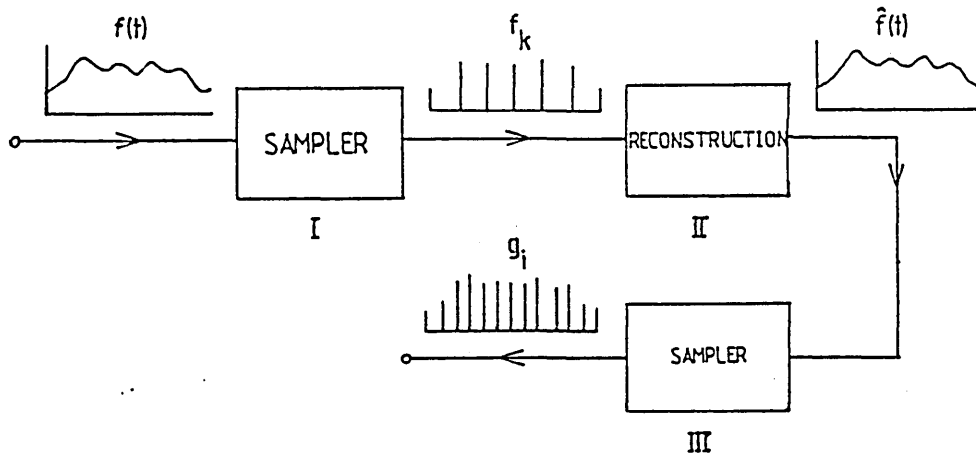
Fig.5.2 Arrangement of 24 electrodes using a regular grid by Monro et al. (1974) [84].

possible, with an acceptable error, with as few as 26 electrodes. In this study, a modified 37-electrode set is used. A more detail description is in Chapter 4.

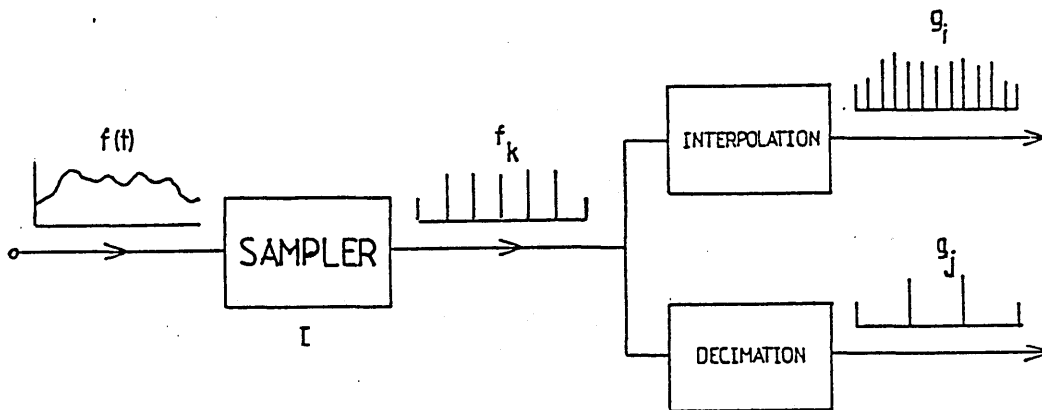
### 5.2.2 Interpolation

The process of reconstructing a body surface map from limited lead sampling to obtain values at other sites of interest is basically equivalent to the image resolution conversion process commonly used in the signal and image processing field. This is illustrated in Fig.5.3.a. The continuous function  $f(t)$  is sampled in stage I (assuming it is adequately sampled according to Shannon's sampling theorem) to give the discrete data  $f_k$ . In stage II, a continuous function  $\hat{f}(t)$  is reconstructed from  $f_k$  and then in stage III, it is resampled with a different sampling rate to give a new set of discrete data  $g_i$ . However, in real digital processing, stage II and III are done in one operation (as no continuous function can exist in the digital equipment). The two in one digital process is called interpolation when the number of data samples increases, and is called decimation when it decreases as shown in Fig.5.3.b. Nevertheless, the two processes are basically the same.

There are many interpolating techniques, but some are strictly not suitable for ECG studies, such as the classical polynomial interpolation approach e.g. Lagrange interpolation. Hou and Andrews (1978) [96] outlined the limitations of this



(a)



(b)

Fig.5.3 Image resolution conversion process.

approach, the particular one concerning ECG studies was its "non-local" effect, i.e., if the function to be interpolated varies rapidly in some part of the region of interest, the effect of this on the interpolation would be felt everywhere. This is practically useless for ECG studies because the local variation of voltage gradient on the body surface is considerable, especially during the QRS complex in the cardiac cycle. Monroe et al. (1974) [84] introduced the spatial two dimensional Fourier transform for interpolation applicable to ECG studies. The theory and implementation can be found in his two other papers (Monro, 1979 [97], 1980 [90]). In these papers, he also described the Chebyshev transform interpolation technique using the FFT algorithm. Besides these, another suitable candidate for interpolation of body surface maps is the spline approximation.

Spline approximations are basically piecewise polynomials approximation. If we are given a continuous function  $f(x)|_{a < x < b}$ , the approximation function  $g(x)$  of  $f(x)$  is obtained by partitioning the interval  $a < x < b$  into  $n$  subintervals with common end points (called nodes or knots) as

$$a = x_0 < x_1 < \dots < x_n = b$$

and the  $m^{\text{th}}$  order polynomial approximations are fitted for each of the  $n$  subintervals. In other words, we approximate the continuous function  $f(x)|_{a < x < b}$  by  $n$  piecewise  $m^{\text{th}}$  order polynomials instead of one single polynomial on  $a < x < b$ . This has the advantage of minimizing oscillations between nodes which is very common when a

single polynomial, especially of lower order, is used. Hou and Andrews (1978) [96] illustrated that for satisfactory results, a third order polynomial is recommended as it requires only five nodes to compute and the piecewise polynomials join at the nodes continuously together with their slopes. This is usually called the cubic spline function. Monro (1980) [90] implemented the cubic spline approximation technique for interpolation of body surface map. The feature of his method is that all the originally sampled data values are preserved (i.e a uniquely determined cubic spline function).

The first direct comparison of the above mentioned interpolation techniques, namely the Fourier transform, the Chebyshev transform and the cubic spline, applicable to ECG studies found in the recent literature was by Ribeiro (1984) [88]. Using body surface potential measurements on normal human subjects, he investigated the errors introduced in reconstructing the signals during all the relevant periods in the cardiac cycle using the three different methods. Ribeiro concluded that although the individual techniques appeared to be better than each other in different periods in the cardiac cycle, overall, the cubic spline method is generally recommended. Also technically the cubic spline interpolation is the simplest of the three methods and it is easy to implement. Therefore the cubic spline method is preferred in this study to the Fourier transform and the Chebyshev transform methods.

In order to give a quantitative measure of the error introduced at the optimal sites in the interpolation process, experimental measurements have to be made at these sites. This requires an addition of 26 measuring channels in the data acquisition system and at the present moment it is not practical in our Laboratory. Nevertheless, the error figures reported by Ribeiro would give an indication of the order of error introduced at the optimal sites. Ribeiro reported that the maximum averaged power error of reconstruction using the cubic spline interpolation method during the QRS complex was 4.5%. This was obtained using a 26-electrode set. Since the figure was obtained from the row and column that were most difficult to reconstruct, it can be confidently assumed that the error introduced at the optimal sites should be equal to or less than this figure.

### 5.3 Computational procedures

For each of the 21 subjects, the potential of the 26 epicardial segments are computed from the body surface measurements throughout the whole cardiac cycle. The procedures (Fig.5.4) consist of the following stages:

- (I) Reconstruction of the body surface map from the 37 measured body surface potentials using the cubic spline interpolation method throughout the whole cardiac cycle.
- (II) Extraction of potential values at the optimal sites from the grid of interpolation.
- (III) Inverse calculation of the epicardial potentials using a regularisation technique as described in Chapter 3.

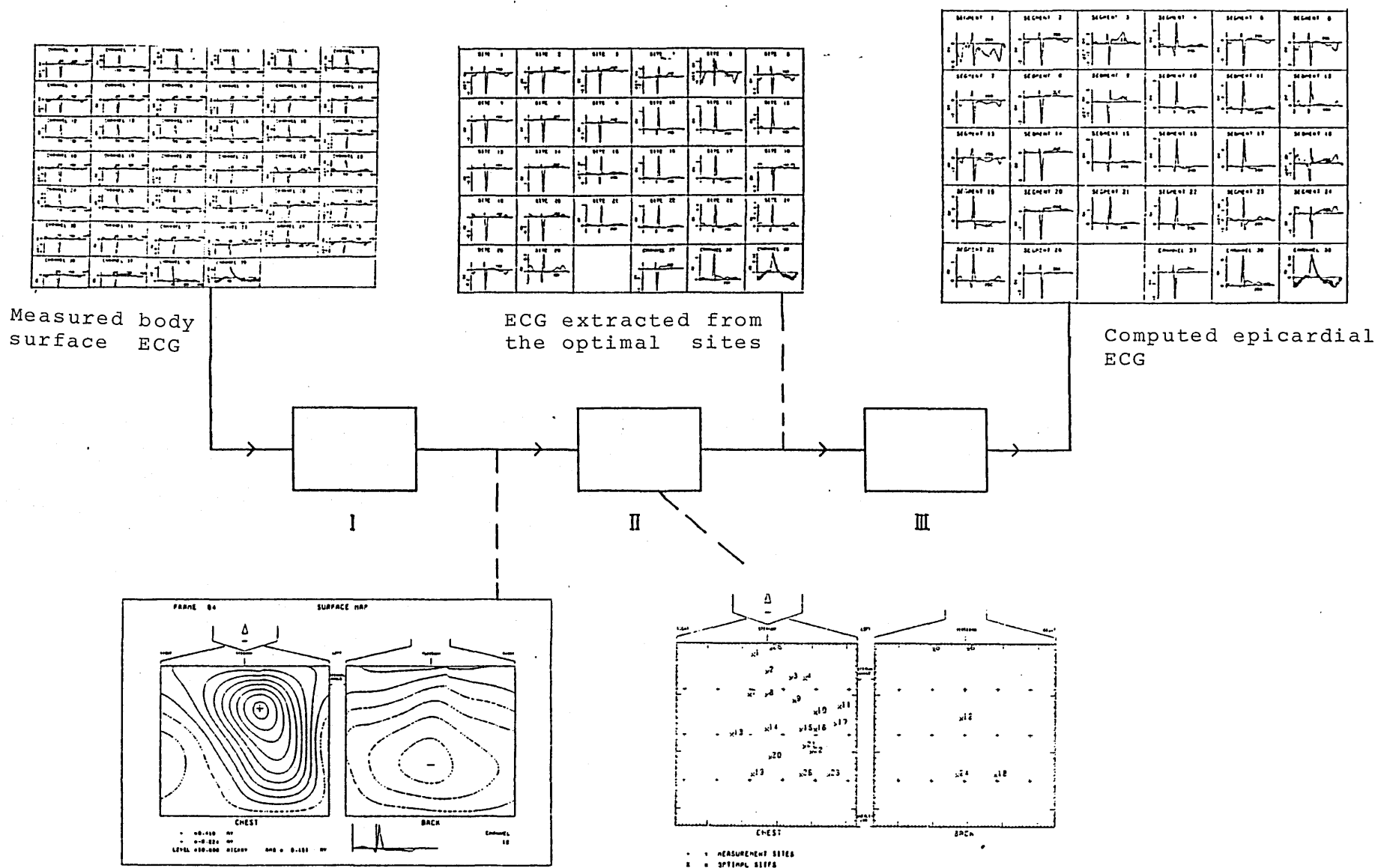


Fig.5.4 Schematic diagram of the computational process of the epicardial potentials from body surface measurements.

#### 5.4 Evaluation of the computed inverse solutions

The epicardial electrocardiograms however they are computed have to be verified before they can be introduced for clinical use. The most direct and simple way of verification is by comparing the computed epicardial potentials with those measured in situ. Alternatively, an indirect method could be used, by reconstructing the body surface potentials from the computed inverse solutions and then comparing them with the body surface measurements. A good comparison of results obtained in the indirect method does not necessarily infer the correctness of the computed inverse solutions, but at least the stability of the solutions can be implied.

##### 5.4.1 Direct comparison of measured and computed epicardial potentials

The simplest technique is spatial comparison in the form of an isopotential map. This qualitative comparison will immediately indicate whether the main features, such as the maxima and minima, are comparable on both maps. This technique, although is simple, requires a considerable number of measuring electrodes to be put on the surface of the heart which is not feasible for human subjects at the present stage of development.

An alternative approach is by temporal comparison, i.e. the measured epicardial electrocardiogram is compared with the computed solution at the corresponding site in time throughout the

whole cardiac cycle. In this aspect, there are several comparison criteria which can be used and perhaps the most meaningful qualitative comparison is the product moment correlation coefficient (r) which is defined as:

$$r = \frac{\sum_{i=1}^n (V_i - V)(E_i - E)}{[\sum_{i=1}^n (V_i - V)^2 \cdot \sum_{i=1}^n (E_i - E)^2]^{0.5}}$$

where  $V_i$  and  $E_i$  are the measured and estimated values,

$V$  and  $E$  are the averaged values of  $V_i$  and  $E_i$  respectively  
and  $n$  is the number of observations.

This criterion is used as an indicator of differences in patterns and would not differentiate any amplitude discrepancy. For quantitative comparison, the most commonly used criteria are the root mean square (RMS) error which is defined as:

$$\text{RMS error} = \left[ \frac{1}{n} \sum_{i=1}^n (V_i - E_i)^2 \right]^{0.5}$$

and the relative percentage error (REL) and is defined as:

$$\text{REL} = \left[ \frac{\sum_{i=1}^n (V_i - E_i)^2}{\sum_{i=1}^n (V_i)^2} \right]^{0.5} \times 100 \%$$

In this study, 48 epicardial measurement sites from 21 subjects were available. Out of the 48, 7 sites did not have sufficient information to have their locations identified, and

consequently cannot be used for comparison. The position of the epicardial measurement sites was obtained from the diagram drawn by the surgeon after the surgical operation. Fig.5.5 shows the positions of the 41 epicardial measurement sites (the 2 characters are the patient's initials and the integers differentiate the 3 epicardial measurement sites of each subject).

For each subject, the epicardial electrocardiogram of the heart segment which corresponds to the measurement site was computed from the measured body surface potentials. In Fig.5.6, the upper waveforms are the measured epicardial electrocardiograms (only showing the QRS complex) and the lower waveforms are the corresponding computed solutions. The product moment correlation coefficients were computed between each of the 41 pairs of the electrocardiograms. The results are shown in Fig.5.7, and show that about 50 % have a correlation coefficient above 0.8 and only 17 % of them have a correlation coefficient less than 0.6.

In the above study, absolute magnitude was not taken into consideration. Firstly, this is because in the computing of the inverse solution, a standard computer model of the human thorax was used for all the subjects (whose chest configurations varied considerably). In the 21 subjects done, the chest circumference ranged from 88 to 114 centimeters (cm), the chest width ranged from 29 to 37 cm and the chest depth from 21 to 29 cm. Also from the information derived from chest X-rays, the configuration and orientation of the heart varied considerably between subjects.

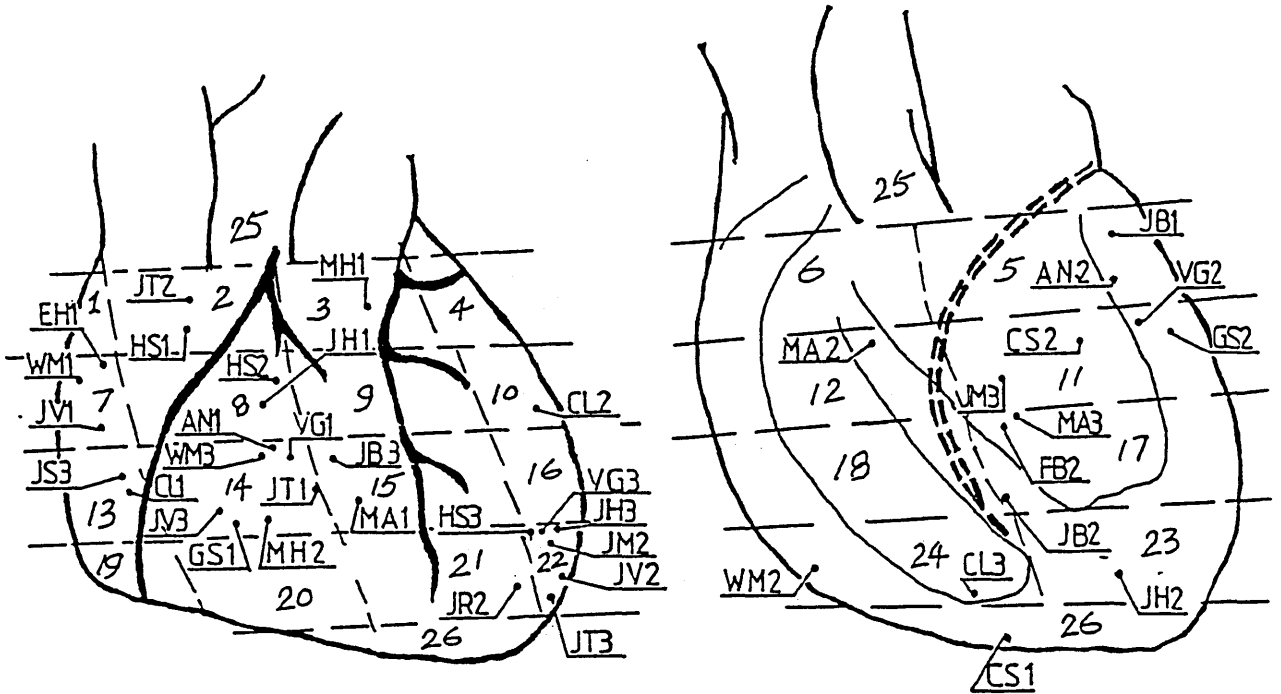


Fig.5.5 Location of the 41 epicardial measurement sites.

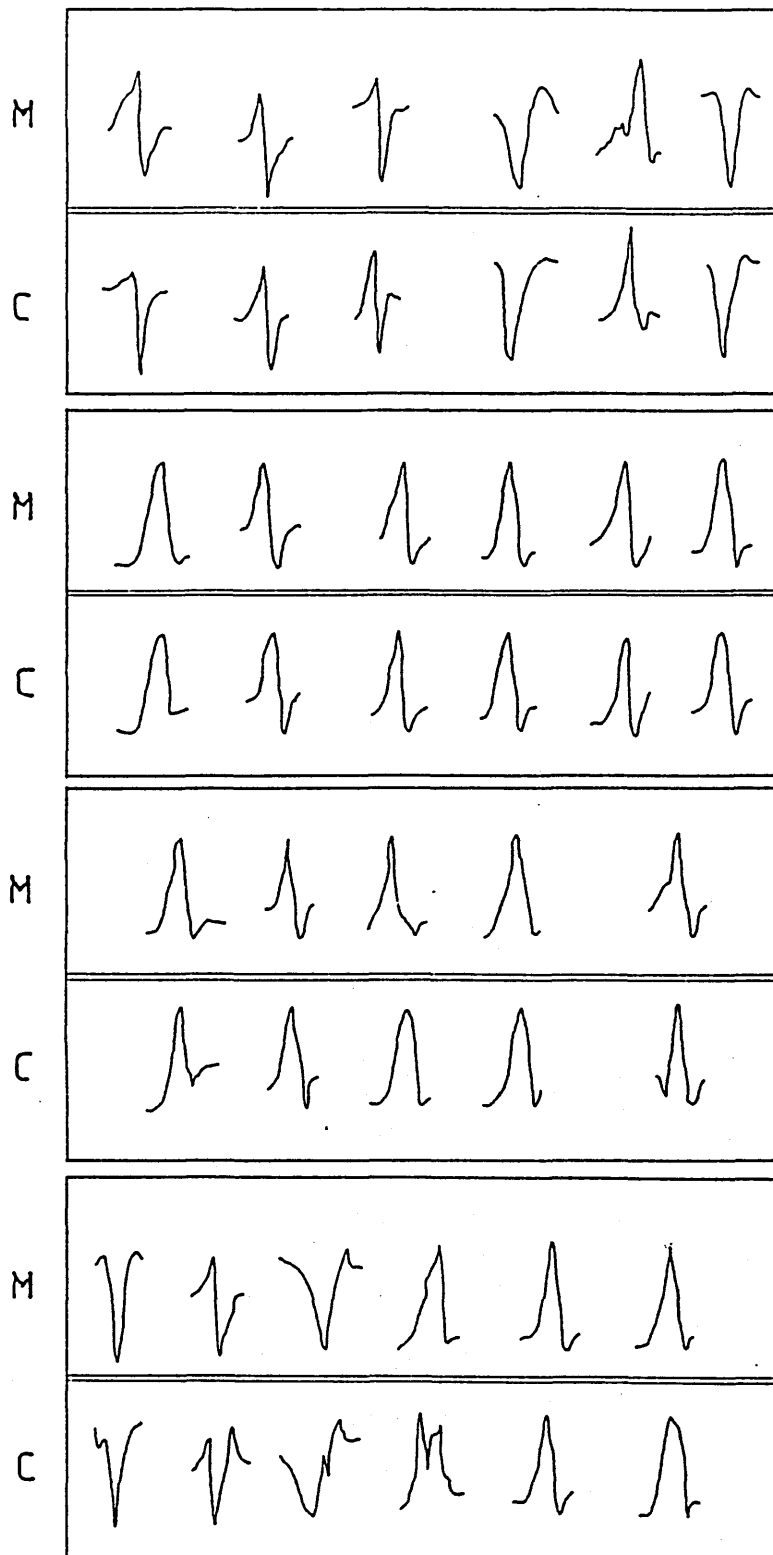


Fig.5.6 The epicardial waveforms.  
The rows marked with 'M' are the measured epicardial electrocardiograms and those marked with 'C' are the corresponding computed solutions.

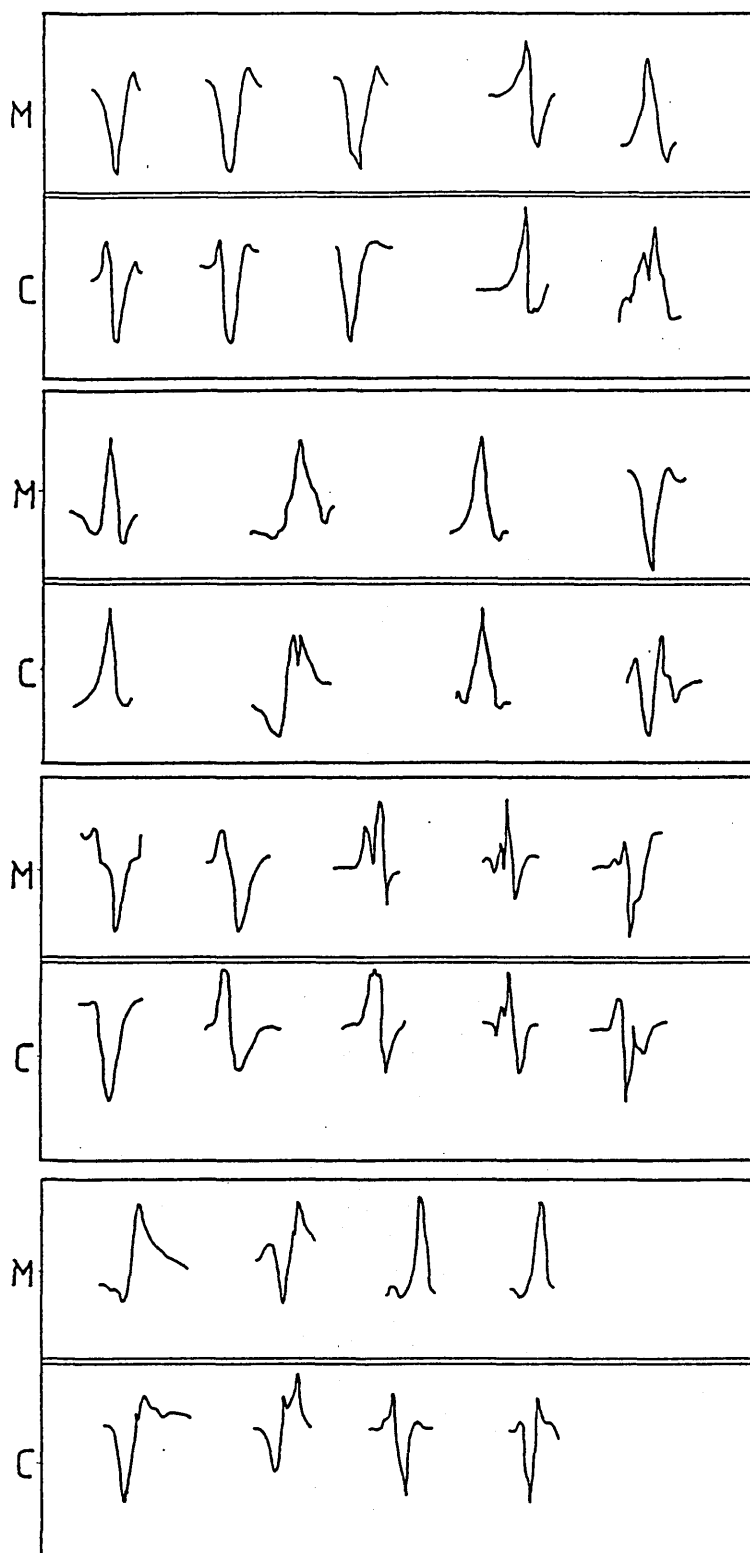


Fig.5.6 (Cont'd)

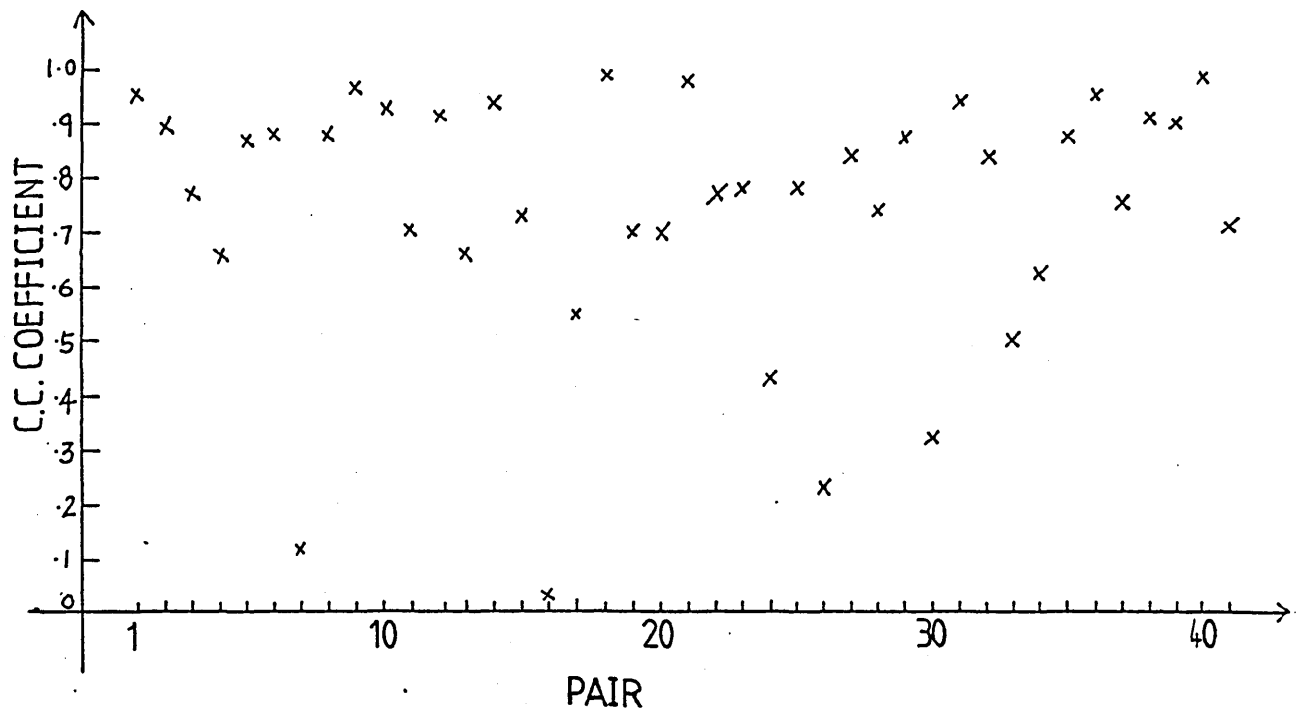


Fig.5.7 The cross correlation coefficient of the 41 measured and computed pairs of epicardial electrocardiograms.

Thus it would be meaningless trying to compare the measured and the computed solutions quantitatively unless a different computer model is set up for each individual subject or correction is made for the difference in chest and heart configurations. Secondly, in the computer model, the epicardial surface was segmented into 26 segments only, this limited the resolution of local cardiac activities and the computed solution was a summation of the activities in a relatively larger area compared with the actual measured electrocardiogram. Despite all these unfavourable conditions, there was still a remarkable resemblance in the main features between the measured and computed epicardial waveforms, especially of those around the ventricular sites. This will be further discussed in Chapter 6.

#### 5.4.2 Reconstruction of surface potentials from the inverse solution

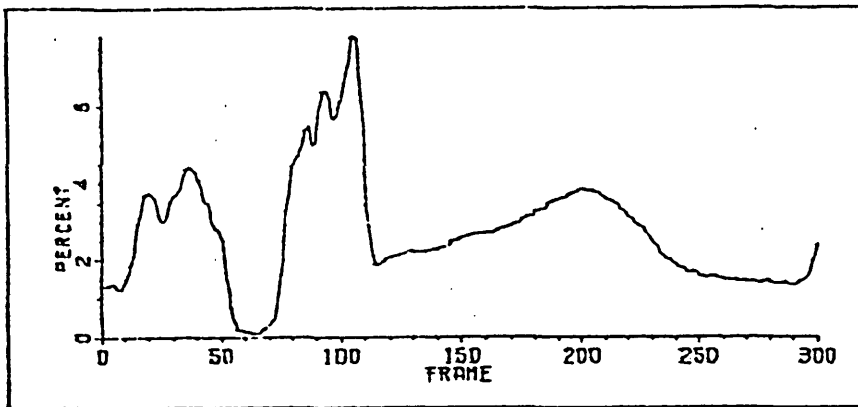
The computer model used in this study consists of 26 epicardial segments which are related with the 26 optimal surface measurement sites by the transfer coefficient matrix  $A$ , therefore given the epicardial potentials  $E$  of the heart segments and the transfer coefficient  $A$ , the surface potentials  $S$  can be obtained as follows:

$$S = A E \quad (5.1)$$

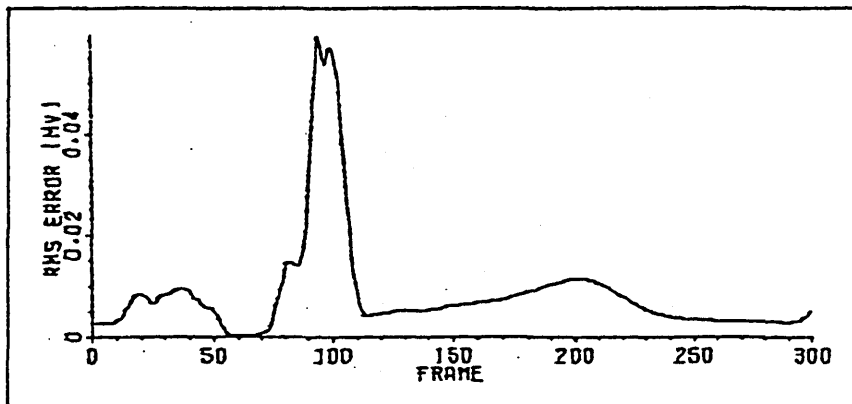
Using the inverse solutions obtained by the computational technique described in Chapter 3 and section 5.3, the surface potentials at the 26 optimal surface measurement sites were reconstructed through equation 5.1.

300 time instants (separated with a 2-millisecond interval) of the body surface potentials were reconstructed which covered the whole cardiac cycle. Fig.5.8 shows the errors between the measured and the reconstructed body surface potentials of one subject and the error criteria RMS error and relative percentage error used were as defined in section 5.4. The maximum RMS error was found at the QRS complex which had a value of 53  $\mu\text{V}$ . The relative percentage error was consistently less than 4% throughout the whole cardiac cycle except at the QRS complex which had an error of about 7.5%.

The procedure of reconstruction of the body surface potentials from the inverse solution were done for all the 21 human subjects. The maximum RMS error ranged from 22.95  $\mu\text{V}$  to 67.70  $\mu\text{V}$  (Fig.5.9) with a mean of 47.05  $\mu\text{V}$ , and the relative percentage error for all patients was consistently less than 10%.



(a) The relative percentage error.



(b) The RMS error.

Fig.5.8 The errors between the measured and the reconstructed body surface potentials.

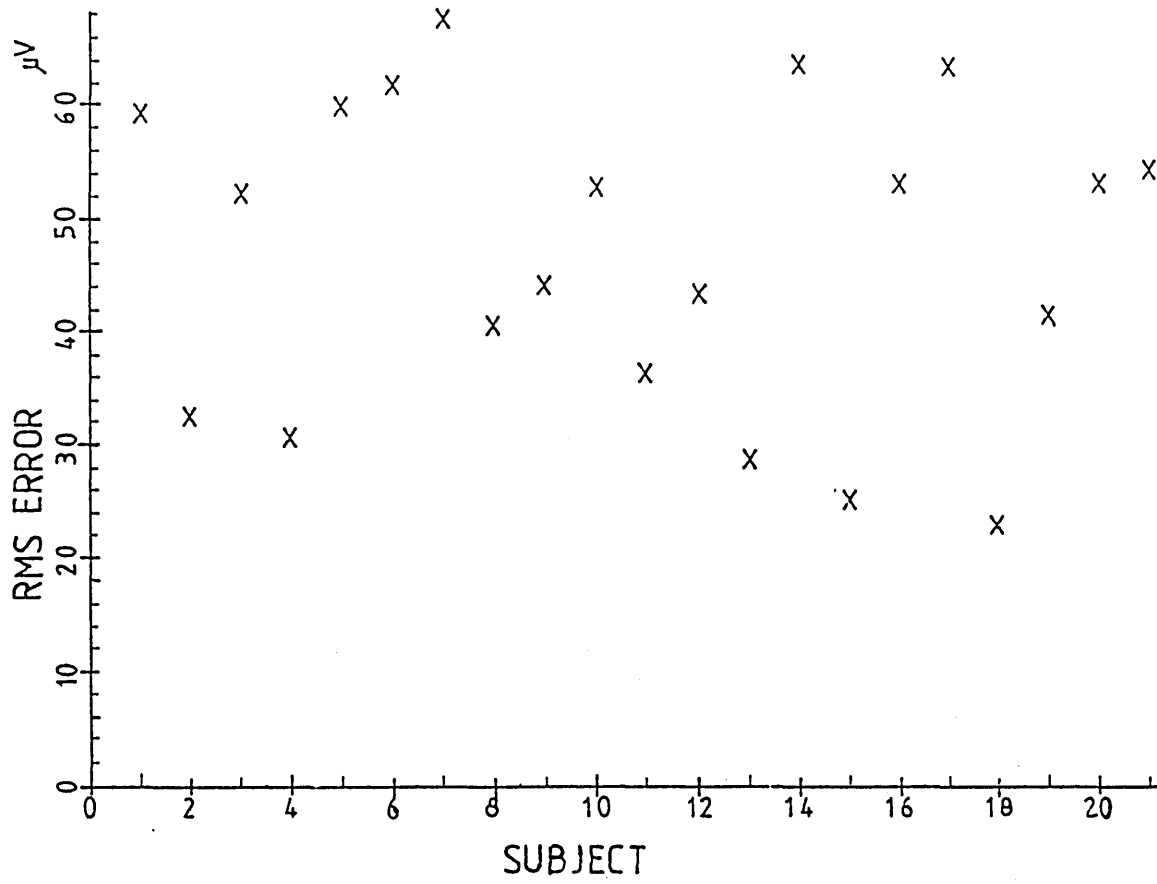


Fig.5.9 The maximum RMS error between the measured and reconstructed body surface potentials of each of the 21 subjects.

### 5.5 Effect of omission of back torso potentials on the inverse solution

In body surface mapping various number of surface electrodes ranging from 9 to 200 are used either arranged regularly over the body surface or arranged irregularly in an optimal pattern. In both cases certain numbers of electrodes have to be put on the back of the patient. This is sometimes not practical in a clinical situation, e.g. for bed-ridden patients.

From the information in the prediction of the optimal surface measurement sites (Fig.5.10) using the computer model in this study, it can be seen that the majority of the measurement sites (21 out of 26) were clustered at the front. It may be concluded from these observations that it is worth investigating the effect on the inverse solution if the information from posterior measurements was omitted in the computation. If the results were accepted clinically, then in future computations, only anterior measurements would be required and this would simplify the measuring procedure.

#### 5.5.1 Methods

Two sets of inverse solutions were obtained, the waveforms are displayed in Fig.5.11. The first set was computed using all the body surface potentials available at the 26 optimal measurement sites, while the second set was computed with the omission of the posterior measurements (i.e. measurements at

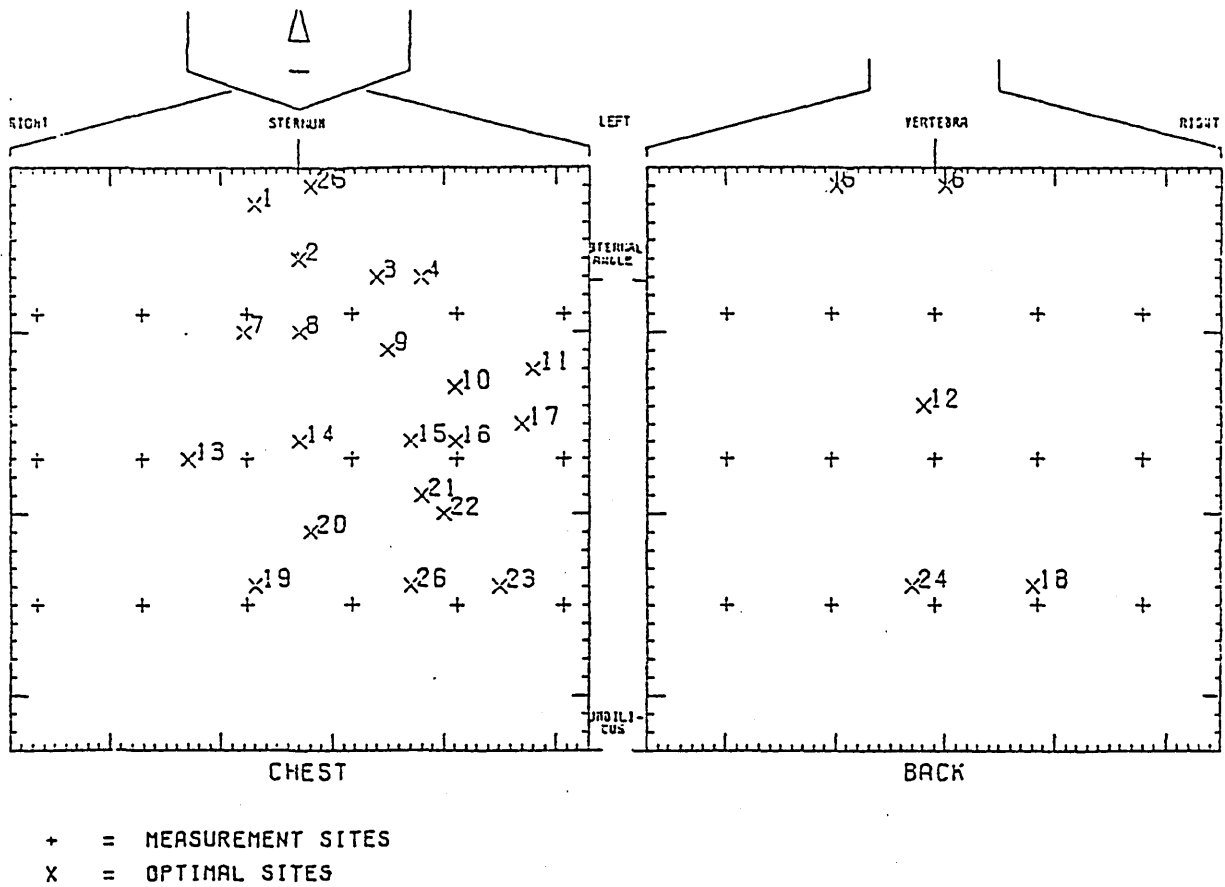


Fig.5.10 The position of the optimal surface measurement sites predicted by the computer model used in this study.

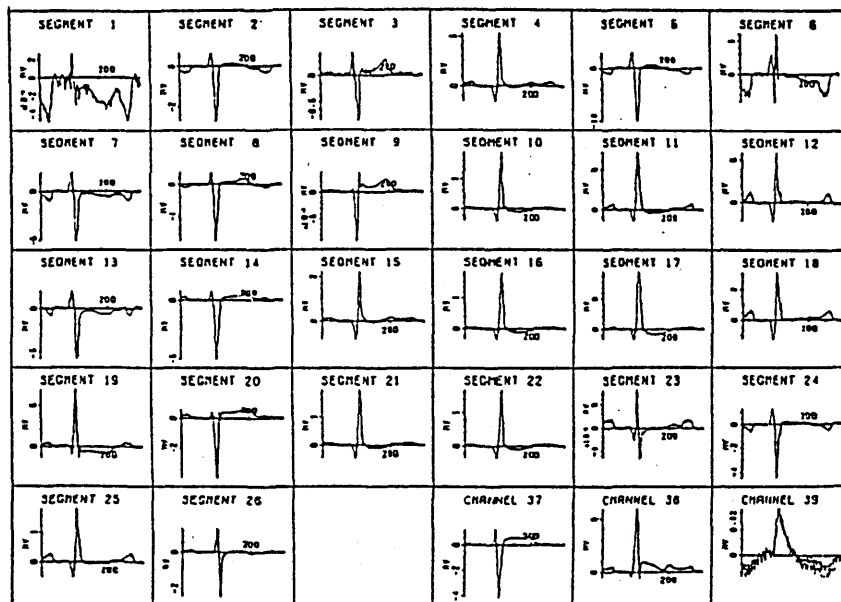
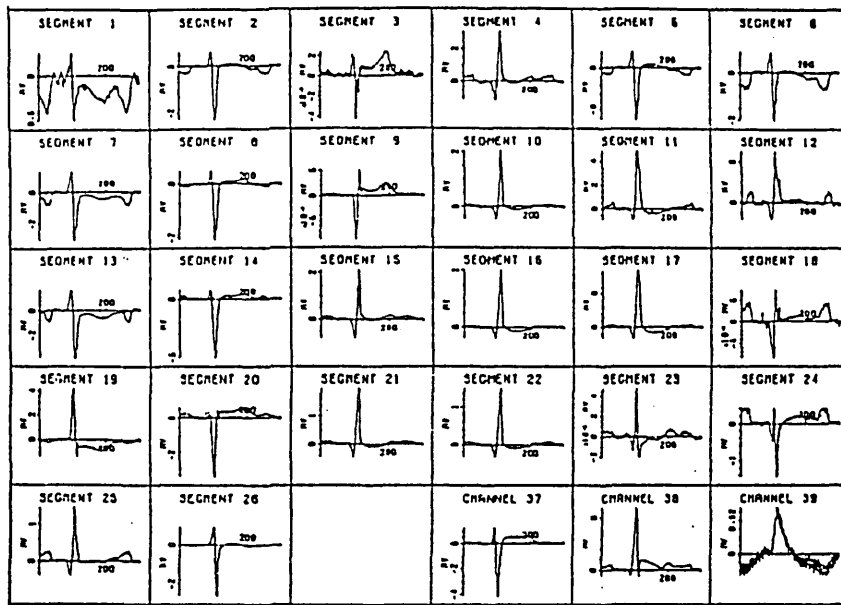


Fig.5.ii Two sets of inverse solutions.

The top one was computed using all the available body surface measurements and the bottom one was computed with the posterior measuring sites omitted.

optimal site 5, 6, 12, 18 and 24 were not used).

#### 5.5.2 Results and discussion

Fig.5.12 depicts the error between the two sets of solutions. The cross correlation coefficients were always above 0.9 in the QRS complex, the minimum and the maximum values were 0.943 and 0.999 respectively and had a mean value of 0.983.

The minimum and maximum values of the relative RMS error (REL) were 0.019 and 0.33 respectively, the mean value was 0.18. The relative RMS error (REL) was defined as:

$$REL = \frac{\sum_{i=1}^n (v_i - v_{oi})^2}{\sum_{i=1}^n (v_i)^2}$$

where  $v_i$  and  $v_{oi}$  are the inverse solutions with and without the posterior measurements in the computation, and  $n$  is the number of optimal sites.

Fig.5.13 showed the contour maps of two frames (frame 96 and 115) from each of the two sets of inverse solutions. Frame 96 and 115 were the frames within the QRS complex that had the largest and smallest discrepancies in terms of relative RMS error between the two inverse solutions respectively. The upper map was

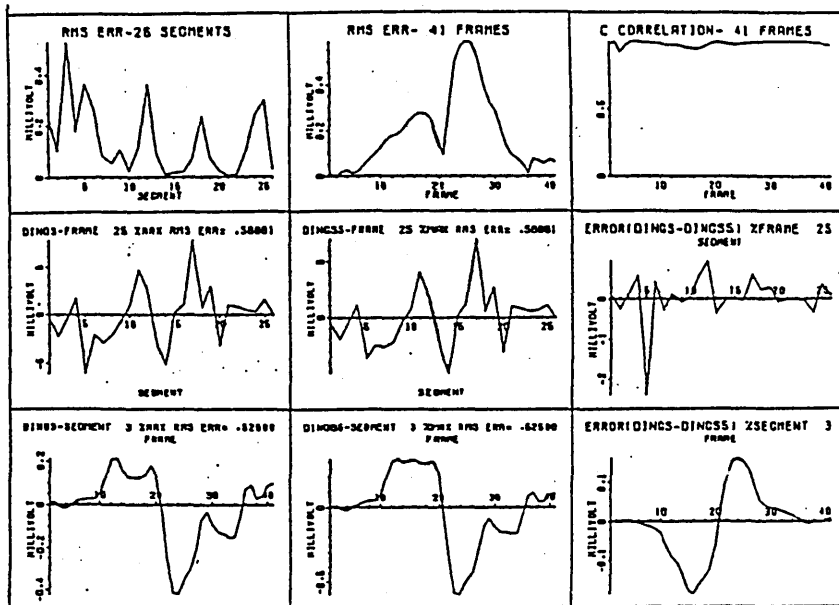


Fig.5.12 The errors between the 2 sets of inverse solutions.

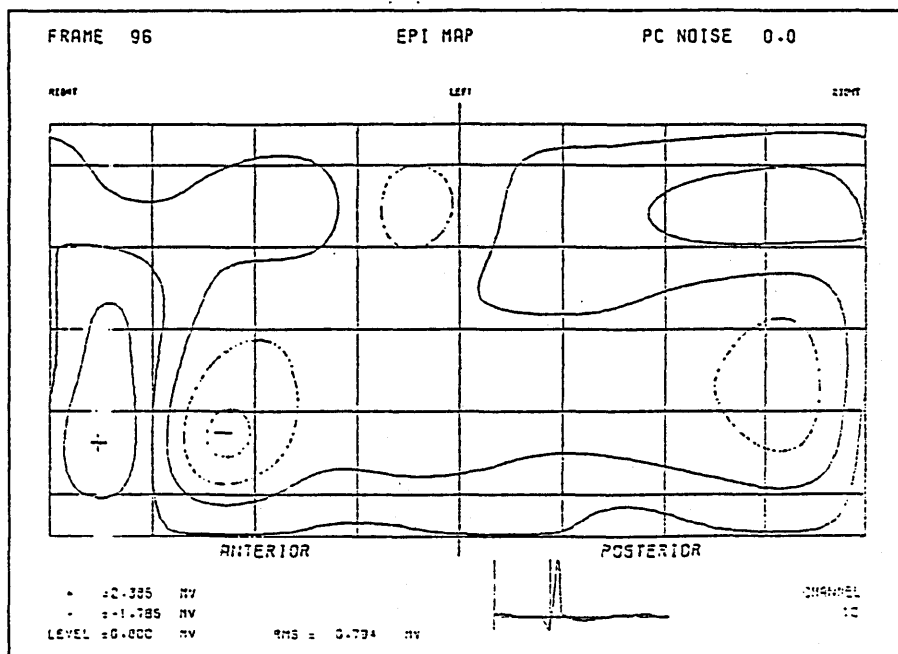
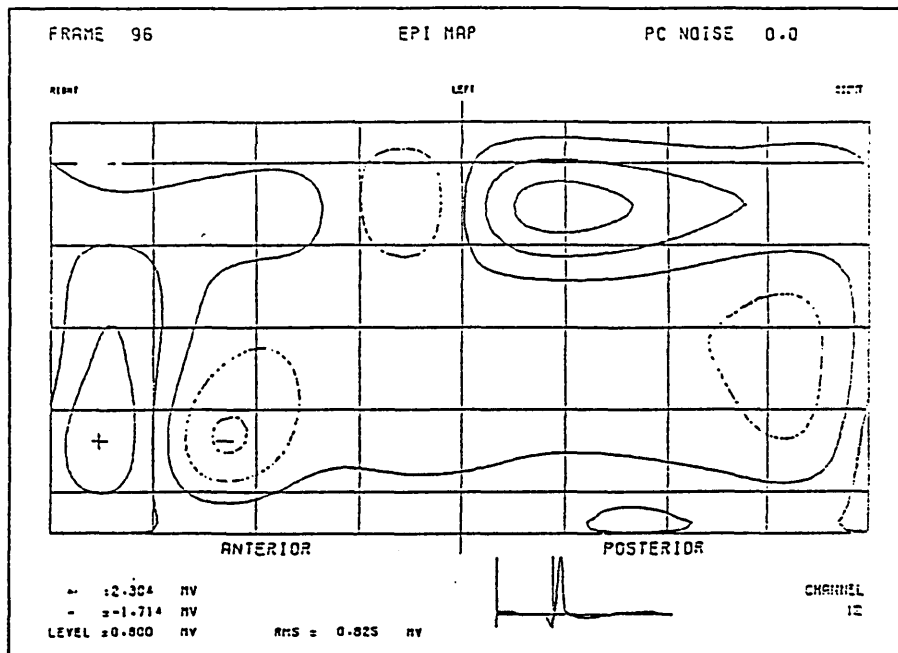


Fig.5.13 The epicardial potential maps. (See text.)

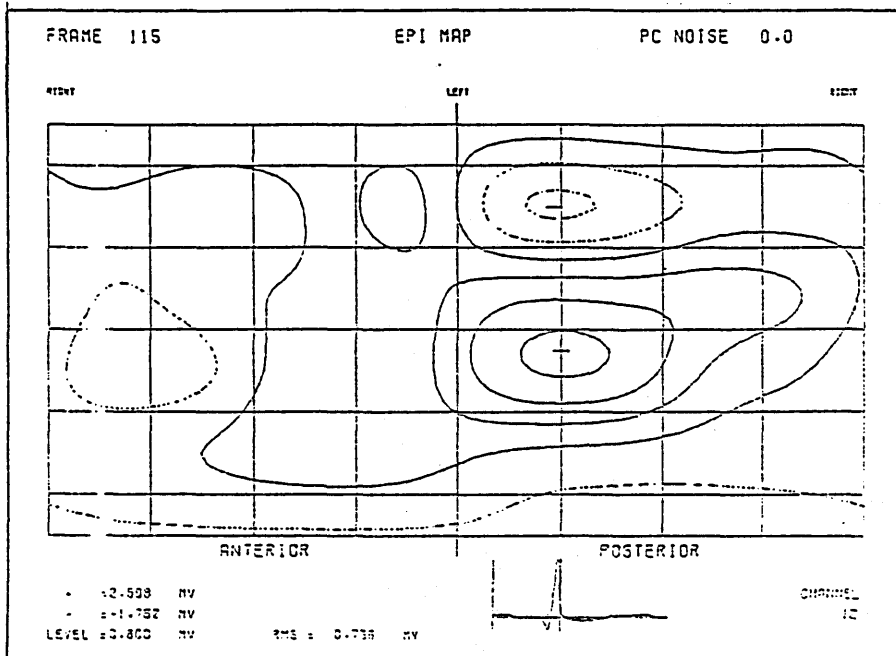
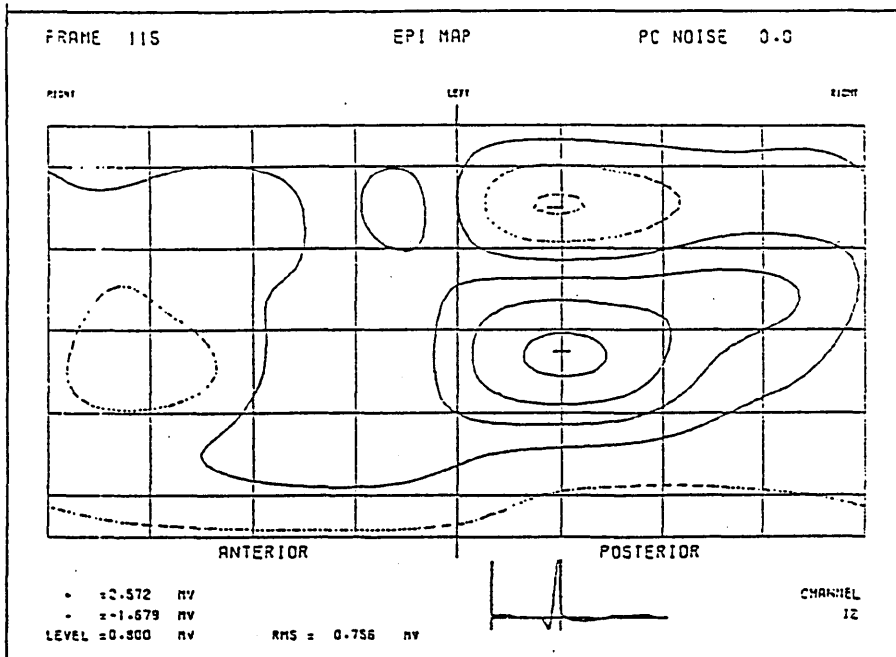


Fig.5.13 (Cont'd)

the inverse solution with all the 26 optimal site measurements used in the computation while the lower map was the one with the potentials of the optimal sites on the back omitted.

In frame 96 (near the Q-wave), despite the large relative RMS error which was 0.33, the main features were still comparable on both contour maps. On the two contour maps, the ventricular areas were predominantly negative with the maximum negative potential over the right ventricular base. The atrial regions were covered with positive potentials. It can be seen that the largest discrepancies between the two maps were at the upper region on the back which was expected since the posterior measurements were omitted in the computation. There was a decrease in magnitude in this area on the contour map of the inverse solution computed without the information of the posterior measurements. Nevertheless the computed epicardial potential map does indicate the existence and position of the same gross features.

In frame 115 (towards the end of QRS), the relative RMS error between the two maps was 0.019 and the cross-correlation coefficient was 0.999. It can be seen in Fig.5.13 that the features of the potential distribution on both maps were very similar, although there was a slight discrepancy in magnitude (the absolute error in the maximum positive and negative potential regions were 0.01 mV and 0.08 mV respectively).

It was shown that the error introduced in the inverse

solution due to the lack of information of the posterior measurements was reasonably acceptable. Although there were discrepancies in magnitude, the general features on the contour maps were maintained. This type of error may not be significant if we were only interested in observing the pattern of the depolarisation sequence of the heart. But it may cause significant errors in detecting activities of low level signals such as the His bundle signals.

Although the above results showed that the inverse solutions were not grossly distorted by omission of the information on the back, one point which should be noted was that the anterior potential measurements used in the computation were measured at the optimal sites which sometimes could be cumbersome in placing the electrodes. If the regular measurement grid is used, as in this study, and the potentials at the optimal sites are then obtained through the interpolation technique, electrodes then have to be put on the back and this will defeat the whole purpose of the exercise. Thus investigation of the feasibility of reconstructing the body surface map by making measurements only on the anterior body surface is needed.

## 5.6 Conclusion

In this chapter, the epicardial potentials were computed from the measured body surface potentials of 21 human subjects. The solutions were evaluated using the measured data. It showed that despite the discrepancy in magnitudes, the computed solutions closely resembled the measured data. The discrepancy in magnitudes is due to:

1. In the computational process only one computer model was used for all subjects. Since there was a large inter-subject variation in the position of the heart, the position of the computed solution need not correspond to the point of measurement. Also the physical size of the chest varied considerably between subjects, discrepancy in magnitudes between the computed solutions and the measurements was expected as only one computer model was used.
2. The heart model was divided into 26 segments each representing a summation of activities of a relatively large area. It was not possible to resolve local activities.

The results of the reconstruction of the body surface map from the computed inverse solution showed that the computational process is stable.

It was shown that the inverse solution obtained without the information of the posterior measurements was generally acceptable. This implies a simplification of measurement

procedure because it is not necessary to put measuring electrodes on the back of the subject. However, the adequacy of reconstructing the body surface map from anterior measurements only needs to be investigated.

## CHAPTER 6

### ANALYSIS OF EPICARDIAL AND BODY SURFACE POTENTIAL MEASUREMENTS

#### 6.1 Introduction

Since there are no cardiac sources between the epicardial and body surfaces, the potentials on the two surfaces should bear a very close relationship reflecting the properties of the volume conductor (i.e. the intervening tissues between the two surfaces). In this work the forward and inverse problems have been formulated in terms of epicardial and body surface potentials and because of this, a comparison with direct measurements is particularly relevant. Attention will be given to the inter-subject variation in the hope of standardising the modelling data used in the procedures to calculate the epicardial surface potentials.

The forward transfer coefficient used in this study for calculating the epicardial surface potentials from body surface measurements was derived from a knowledge of the electrical properties of the medium between the two surfaces and the configuration of the human torso and epicardial segments. An alternative technique is to obtain the transfer coefficient directly from the sequences of epicardial and body surface

measurements. The advantage of this approach is that a knowledge of the electrical properties of the intervening tissues is not required. Hersh et al. (1978) [29] calculated the forward transfer coefficient from measurements on intact dogs using 75 epicardial and 150 body surface electrodes. They showed that this set of transfer coefficients produced comparable results with a second set of coefficients determined from geometry measurements. In this study, the 3 epicardial measurements available with the 37 body surface measurements are used to compute the inverse transfer coefficient instead of the forward transfer coefficient, because of the limitation of the number of epicardial electrodes used in the human subject.

In section 6.2, the measured body and epicardial surface electrocardiograms are compared by cross-correlation, which is sensitive to the shape but not the amplitude of the waveforms. The correlation between the two surfaces is also studied with reference to the prediction from the computer model. In section 6.3, the computation of the inverse transfer coefficient from the measured body and epicardial surface potentials is presented, and the feasibility of using the inverse transfer coefficient on other subjects other than the one from which the coefficient was derived is considered in section 6.4.

## 6.2 Epicardial and body surface electrocardiograms

A total of 21 human subjects were measured. Each measurement consisted of 37 body surface and a maximum of 3 epicardial surface ECG as described in Chapter 4. They will be studied separately in the following sections.

### 6.2.1 Body surface ECG

For each of the 21 subjects, 37 body surface ECG were recorded (total 777). Fig.6.1 shows a typical measurement from one subject.

The maximum peak-to-peak amplitude of the body surface ECG in each subject generally occurred at the left anterior measurement sites, i.e. channels 1,11,12 or 13, which lie just above the heart. (Refer to Fig.4.2 for the arrangement of electrode sites). The maximum peak-to-peak amplitude in the 21 subjects ranged from 1.375 mV to 3.546 mV with a mean of 2.077 mV and a standard deviation of 0.593 mV. The minimum peak-to-peak amplitude was recorded mostly from the measurement sites on the neck or at the back. This ranged from 0.047 mV to 0.271 mV with a mean of 0.174 mV and a standard deviation of 0.053 mV.

In order to show the inter-subject variation, the measurements on electrode site 11 (equivalent to lead  $V_2$  in the standard 12-lead ECG) of all the 21 subjects were compared. The maximum peak-to-peak magnitude ranged from 0.866 mV to 3.547 mV

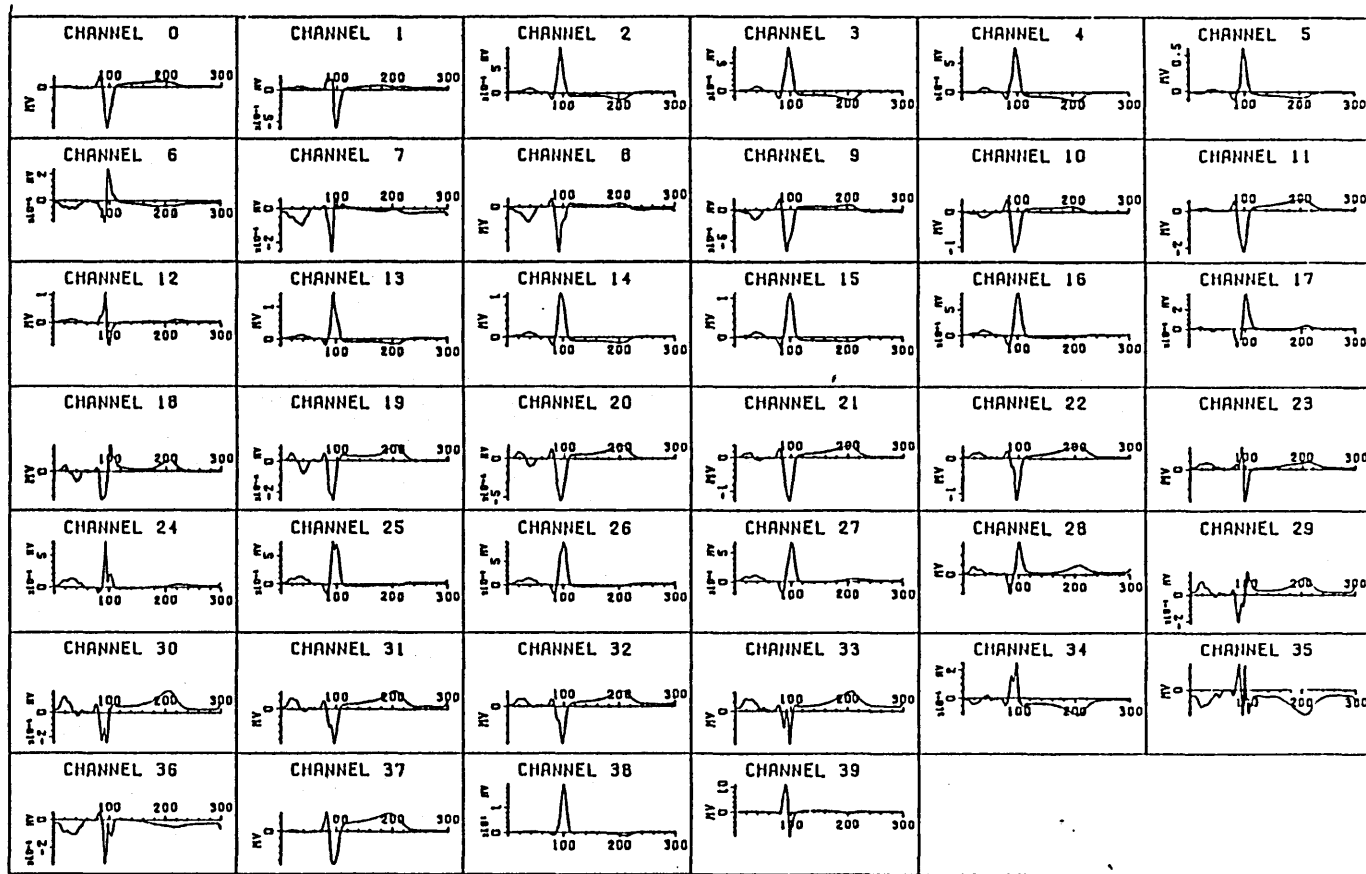


Fig.6.1 The measured 37 body surface and 3 epicardial surface electrocardiograms of one subject.

with a mean of 1.760 mV and a standard deviation of 0.714 mV. Different factors can be attributed to the variation of amplitude of the body surface measurement between subjects. The amplitude of the potential recorded on the body surface not only depends on the distance to the source (Kootsey and Johnson, 1976 [98]), but also depends on other physiological factors that include the respiratory movement of the chest and alternations of body position (Ruttkey-Nedecky, 1976 [99]), the heart position and the tissues situated between the heart and the body surface such as the lung and adipose tissues (Lepeschkin, 1976 [2]).

#### 6.2.2 Epicardial surface ECG

A total number of 48 epicardial ECG have been successfully recorded from the 61 epicardial wires available for this study (ranging from 1 to 3 measurements from each subject). The causes of failure included the malfunction of the input amplifier, the epicardial wires becoming displaced and in one case, there was a total absence of epicardial signal (this could be due to the proximal end of the epicardial wire being embedded in fibrous tissue). Fig.5.5 shows the position of the electrodes on the epicardial surface, 7 of the 48 electrode sites were not shown because their positions could not be correctly identified from the information given by the surgeon. The electrodes covered most of the epicardial surface segments except 1,4,6,9,16,19,20 and 25. Fig.6.2 shows the measurements grouped according to their

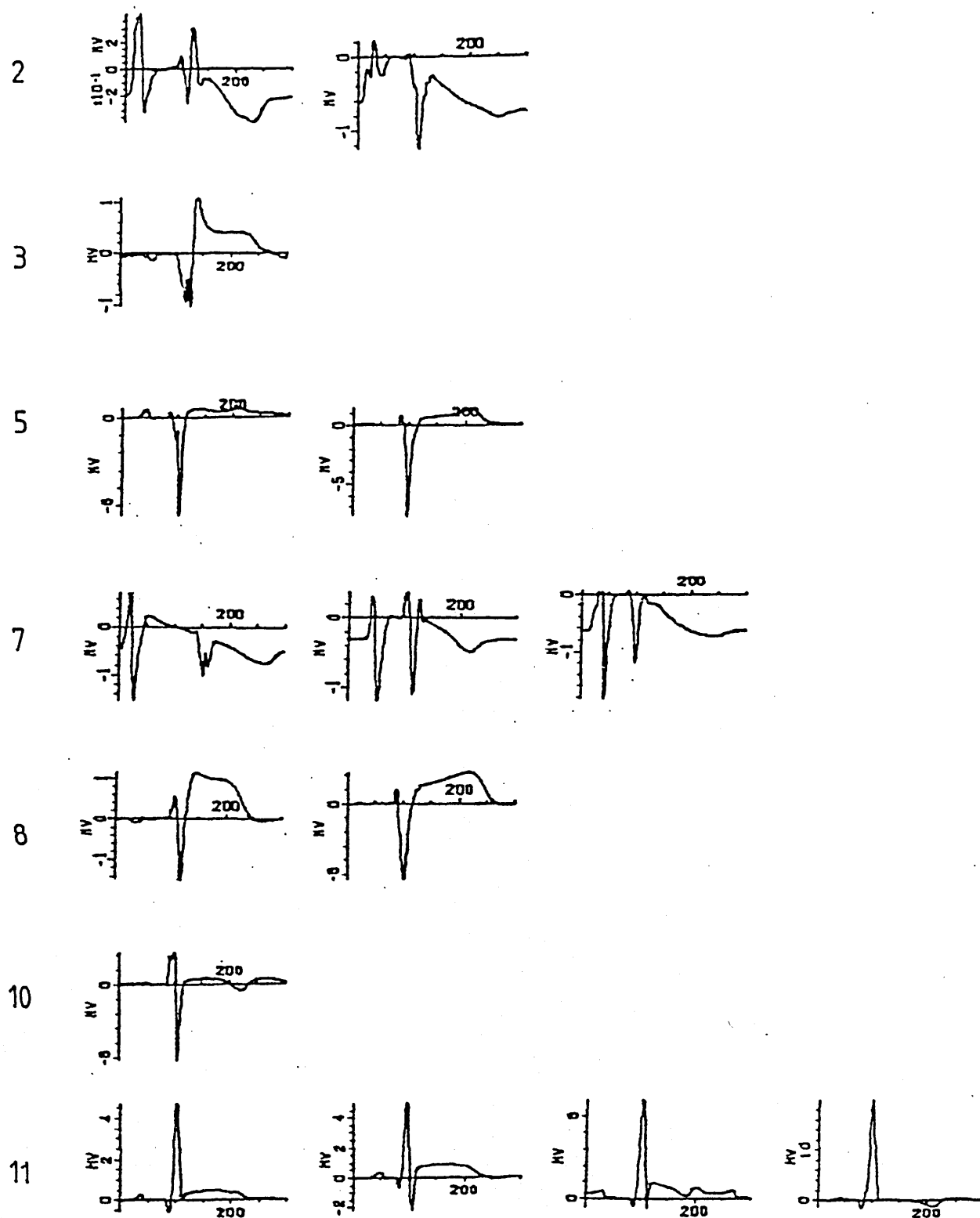


Fig.6.2 The measured epicardial electrocardiograms are grouped corresponding to the epicardial segments of the heart model used in this study. The numerals on the L.H.S. indicate the corresponding heart segment as shown in Fig.3.2.

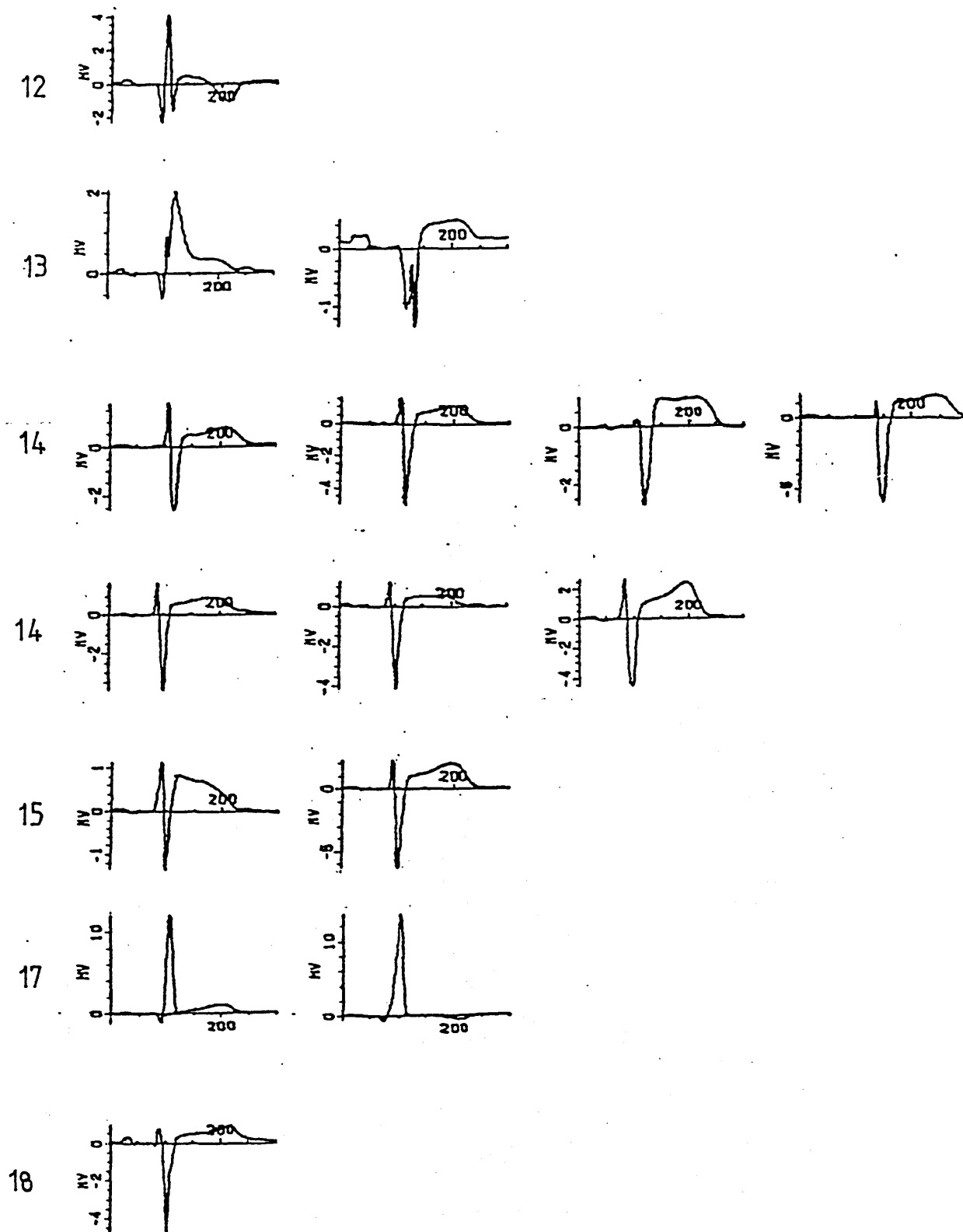


Fig.6.2 (Cont'd)

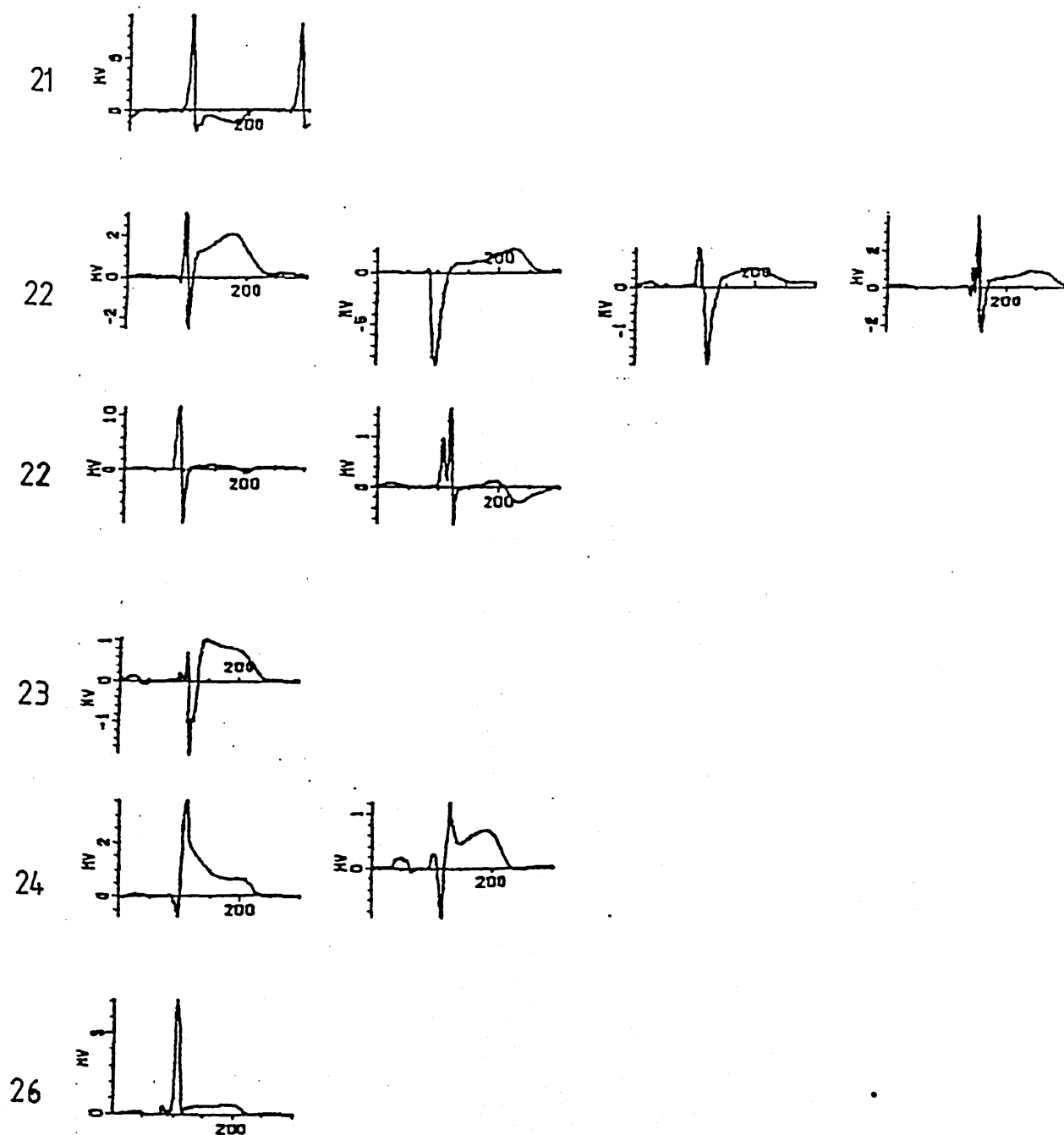


Fig.6.2 (Cont'd)

corresponding epicardial segments.

The epicardial electrocardiograms showed considerable variation in amplitude. The peak-to-peak amplitude ranged from 1.261 mV to 21.758 mV and had a mean value of 7.49 mV and a standard deviation of 4.69 mV. The variation in amplitude of electrocardiograms measured at similar sites of different subjects was not so large. For example, there were 7 subjects in whom the epicardial wires were placed on the anterior right ventricle (corresponded to segment 14 of the computer model). In this group the peak-to-peak amplitude ranged from 2.61 mV to 8.45 mV and had a mean value of 5.81 mV and a standard deviation of 2.08 mV.

#### 6.2.3 Effect of body build on the body surface QRS voltage

It is expected that the amplitude of the body surface ECG will be affected by the size of the chest [2] although one still has to take into account other physiological factors described in the previous section. Fig.6.3.a indicates the variation of chest size of the 21 subjects measured in this study. Among them the chest circumference ranged from 88 to 114 cm. and the chest width ranged from 27.5 to 37 cm.. Fig.6.3.b shows the maximum peak to peak voltage of the QRS complex measured on either electrode site 11 or 12 against the chest circumference. It can be seen that there is a trend of reduced QRS voltage with increased chest size.

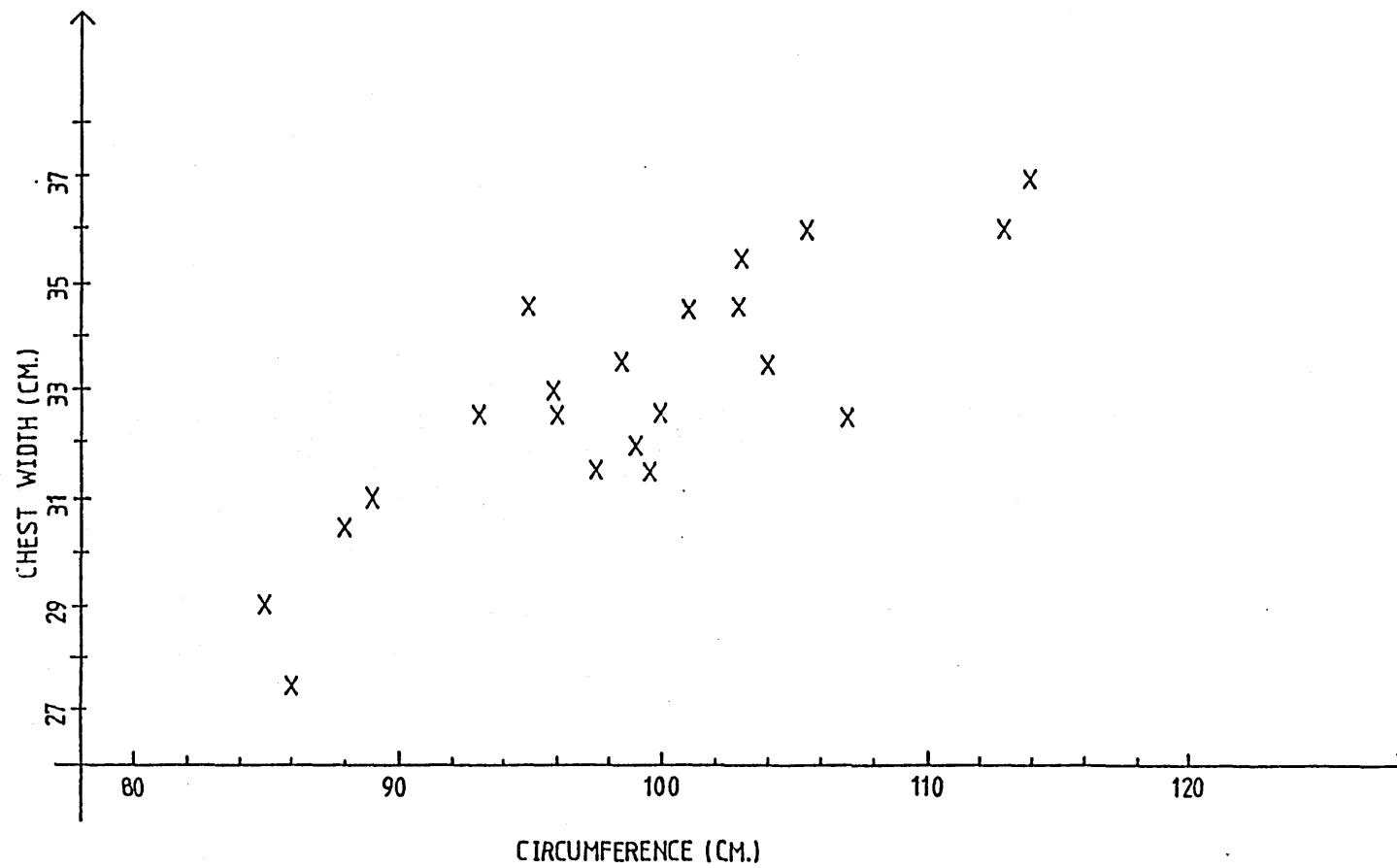


Fig.6.3.a Variation of chest size.

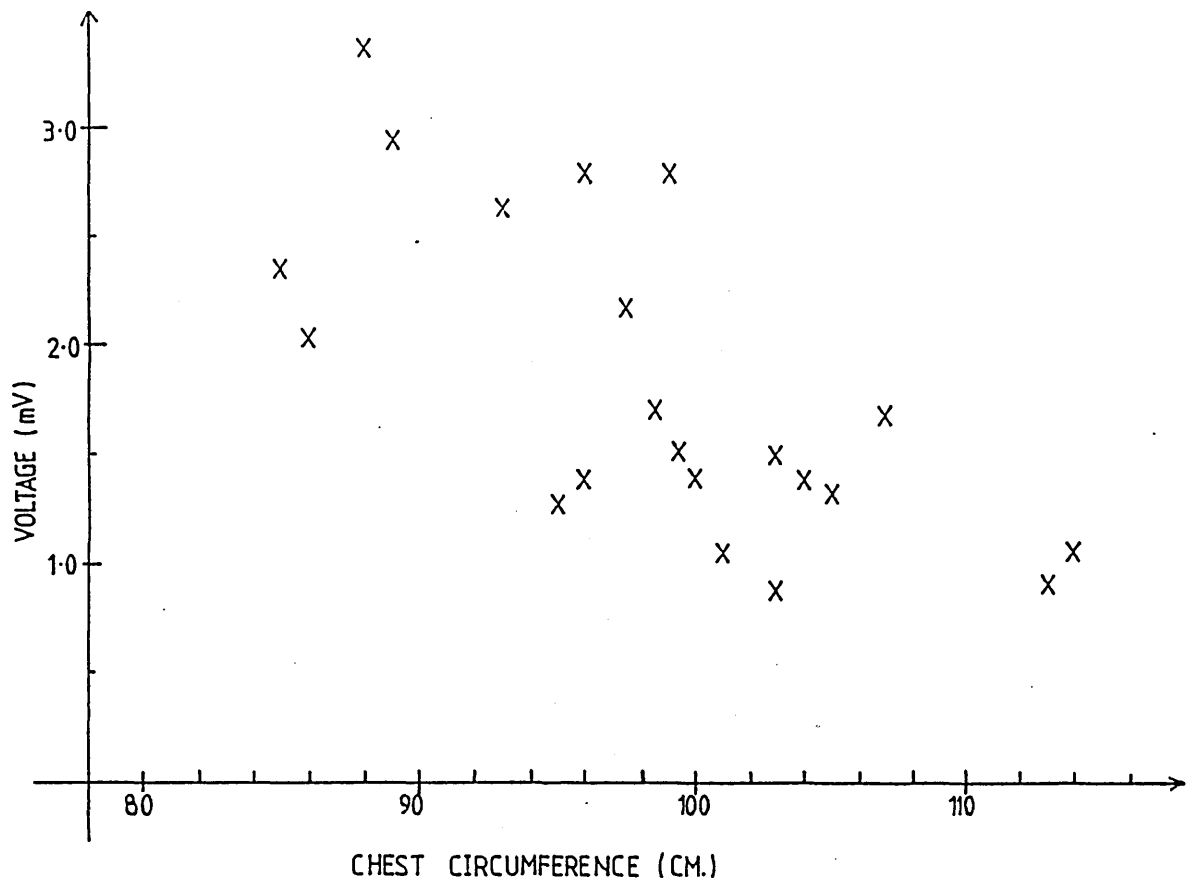


Fig.6.3.b The peak to peak voltage of the body surface QRS in relation to chest circumference.

#### 6.2.4 Correlation between body and epicardial surface measurements

The product moment coefficient as defined in section 5.4 was used as a measure of similarity between the epicardial and body surface electrocardiograms. Each of the 41 epicardial electrocardiograms was cross-correlated with its 37 corresponding body surface electrocardiograms and the best correlation found. Cross-correlation was done using the QRS complex. The results are shown in Table 6.1. It can be seen that good correlation was found between the epicardial electrocardiogram and at least one of the body surface electrocardiograms. In the comparison using only the QRS complex, 81% of the electrocardiograms had a correlation coefficient above 0.8 and the lowest value was 0.6. This similarity between the epicardial and body surface electrocardiograms is illustrated in Fig.6.4 which shows data for the 6 subjects in whom the epicardial electrocardiogram was measured at the anterior right ventricle. In the figure, the upper waveforms are the epicardial electrocardiograms, the lower are the best correlated body surface electrocardiograms, which were found in all cases to be either surface electrode site 0, 10, 11 or 22. The correlation coefficients are all above 0.9 except one which has a value of 0.84. The close similarity of features can be observed in all pairs. Fig.6.5 displays the mean of the correlation coefficients of the 6 subjects in the form of a contour map.

Fig.6.6 shows the electrocardiograms of four subjects in

| EPI CARDIAL<br>SEGMENT | SUBJECT<br>IDENTITY | CROSS-CORRELATION<br>COEFFICIENT |
|------------------------|---------------------|----------------------------------|
| 2                      | HS1                 | 0.96                             |
|                        | JT2                 | 0.83                             |
| 3                      | MH1                 | 0.67                             |
| 5                      | JB1                 | 0.94                             |
|                        | AN2                 | 0.87                             |
| 7                      | EH1                 | 0.99                             |
|                        | WM1                 | 0.97                             |
|                        | JV1                 | 0.88                             |
| 8                      | HS2                 | 0.94                             |
|                        | JH1                 | 0.60                             |
| 10                     | CL2                 | 0.91                             |
| 11                     | VG2                 | 0.98                             |
|                        | GS2                 | 0.85                             |
|                        | CS2                 | 0.95                             |
|                        | JM3                 | 0.95                             |
| 12                     | MH2                 | 0.70                             |
| 13                     | CL1                 | 0.75                             |
|                        | JS3                 | 0.88                             |
| 14                     | VG1                 | 0.97                             |
|                        | GS1                 | 0.99                             |
|                        | AN1                 | 0.98                             |
|                        | WM3                 | 0.99                             |
|                        | JT1                 | 0.92                             |
|                        | MH2                 | 0.93                             |
|                        | JV3                 | 0.93                             |
|                        | MA1                 | 0.96                             |
|                        | JB3                 | 0.85                             |
|                        | FB2                 | 0.88                             |
| 17                     | MA3                 | 0.98                             |
|                        | JB2                 | 0.94                             |
| 18                     | JR2                 | 0.94                             |
| 21                     | VG3                 | 0.81                             |
| 22                     | HS3                 | 0.86                             |
|                        | JM2                 | 0.80                             |
|                        | JH3                 | 0.98                             |
|                        | JT3                 | 0.69                             |
|                        | JV2                 | 0.87                             |
|                        | JH2                 | 0.75                             |
| 23                     | CL3                 | 0.96                             |
| 24                     | WM2                 | 0.80                             |
| 26                     | CS1                 | 0.89                             |

Table 6.1 Cross correlation coefficients between  
the epicardial ECG and the body surface ECG.

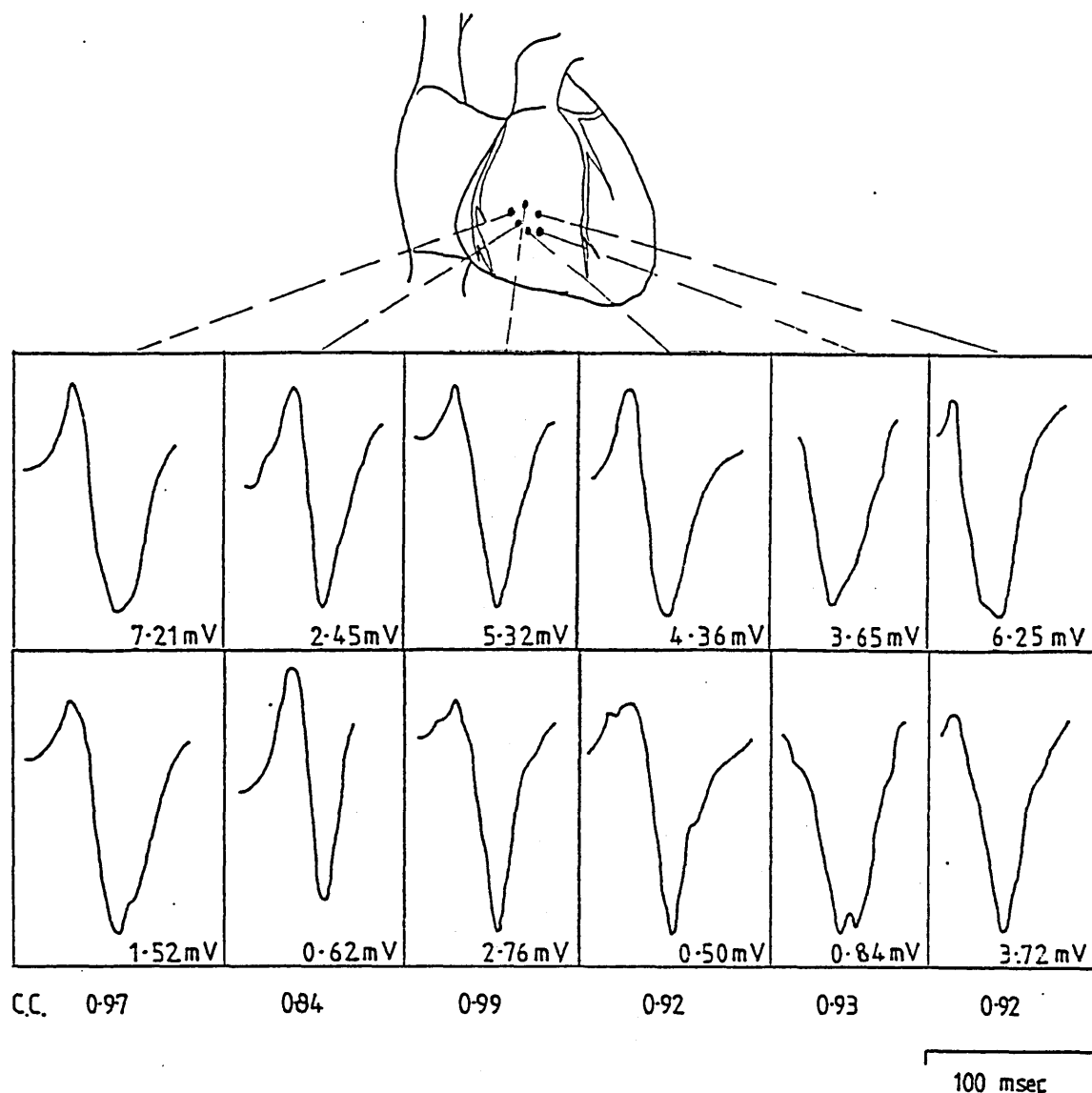
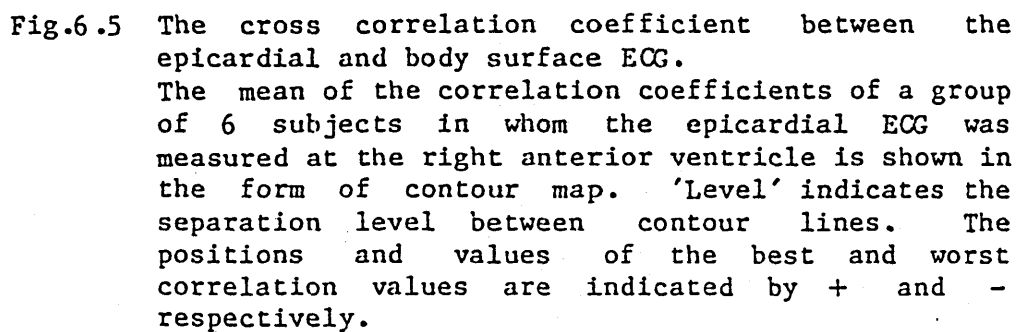


Fig.6.4 The typical degree of similarity between epicardial and body surface electrocardiograms. The upper waveforms are the epicardial ECG from 6 subjects in whom the measurement was made on the right anterior ventricular region. The sites are shown above. The lower waveforms are the corresponding body surface measurements of the best correlation. All measurements are scaled in millivolt and the correlation coefficient of each pair is also shown.



whom the epicardial wires were positioned on the right atrial region. The P-wave of the epicardial electrocardiograms and their corresponding best correlated body surface electrocardiograms are shown for each subject. It can be seen that the correlation coefficients are generally lower than those of the QRS complex. The amplitude of the P-wave of the epicardial electrocardiograms were all considerably larger than those on the body surface. They also featured the rise of potential to a positive maximum, changing rapidly to a negative value as described by Kootsey and Johnson (1976) [98]. This feature could not be observed in the body surface electrocardiograms. The contour display of the mean value of the correlation coefficients of this group is shown in Fig.6.7.

Fig.6.8 summarises the correlation between the epicardial electrocardiograms and the body surface measurements. The epicardial electrocardiograms were grouped into 7 sets (at least 2 electrocardiograms per set) and Fig.6.8 depicts the 7 regions on the body surface that were best correlated with the corresponding sets of epicardial measurements. The results show that the epicardial sets match the body surface regions although some regions cover relatively large areas, this is due to the fact that the epicardial electrocardiograms correlated equally well with more than one body surface site and there was a considerable inter-subject variation in the position of best correlation. Superimposed on Fig.6.8 the positions of the optimal surface sites (i.e. the sites which were most sensitive to its corresponding

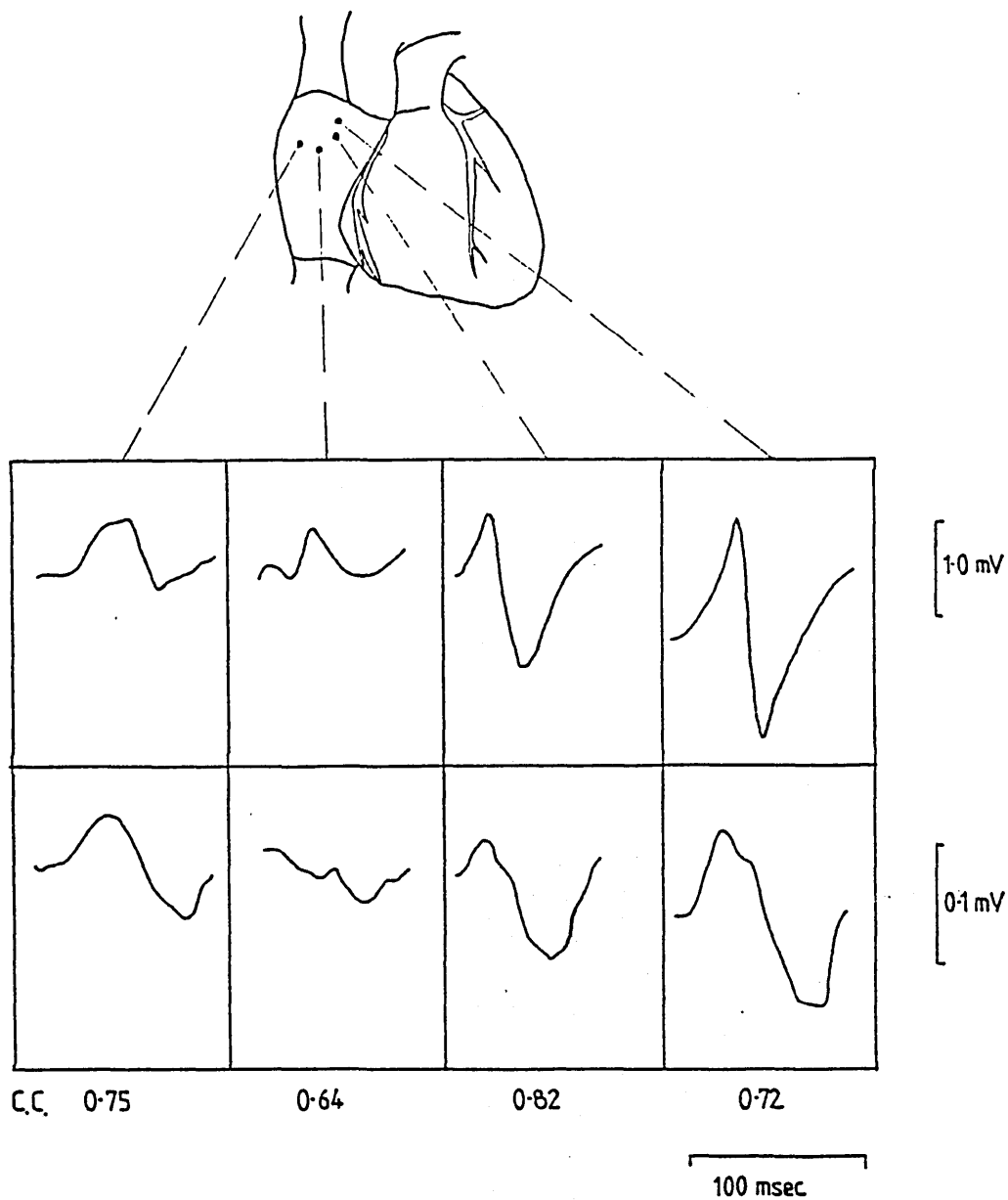


Fig.6.6 The typical degree of similarity between epicardial and body surface electrocardiograms. The upper waveforms are the epicardial ECG from 4 subjects in whom the measurement was made at the anterior atrial region. The display format is the same as Fig.6.4.

MEAN

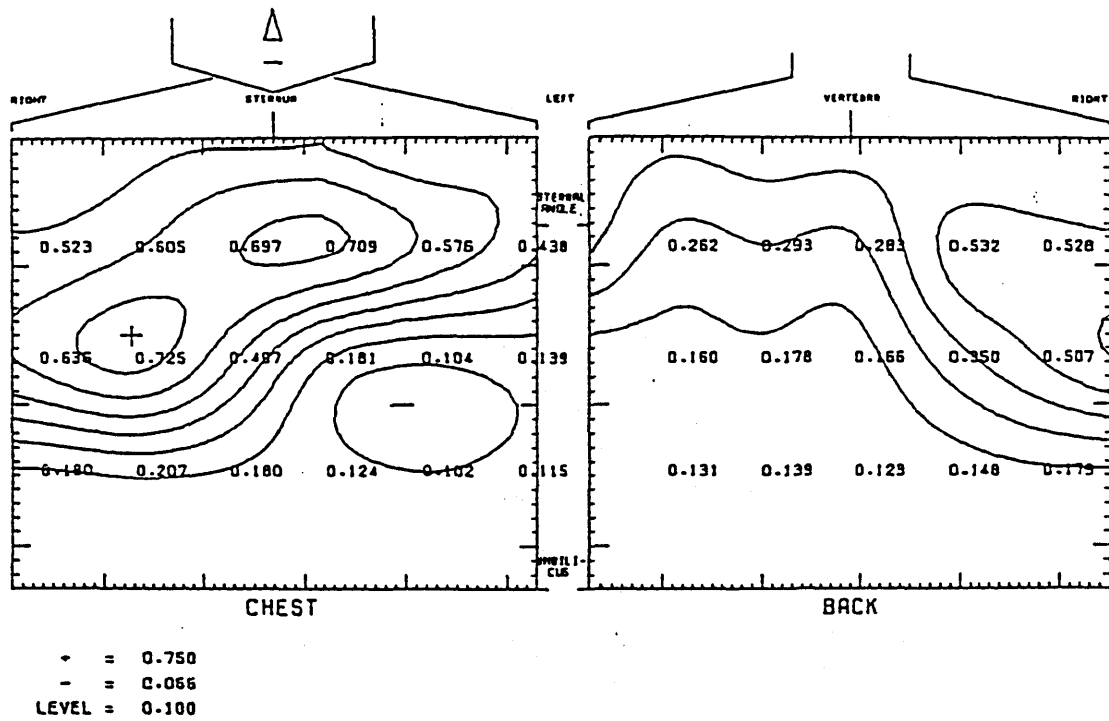


Fig.6.7 The cross correlation coefficient between the epicardial and body surface ECG. The mean of the correlation coefficients of a group of 4 subjects in whom the epicardial ECG was measured at the right atrial region is shown in the form of contour map. The display format is the same as described in Fig.6.5.

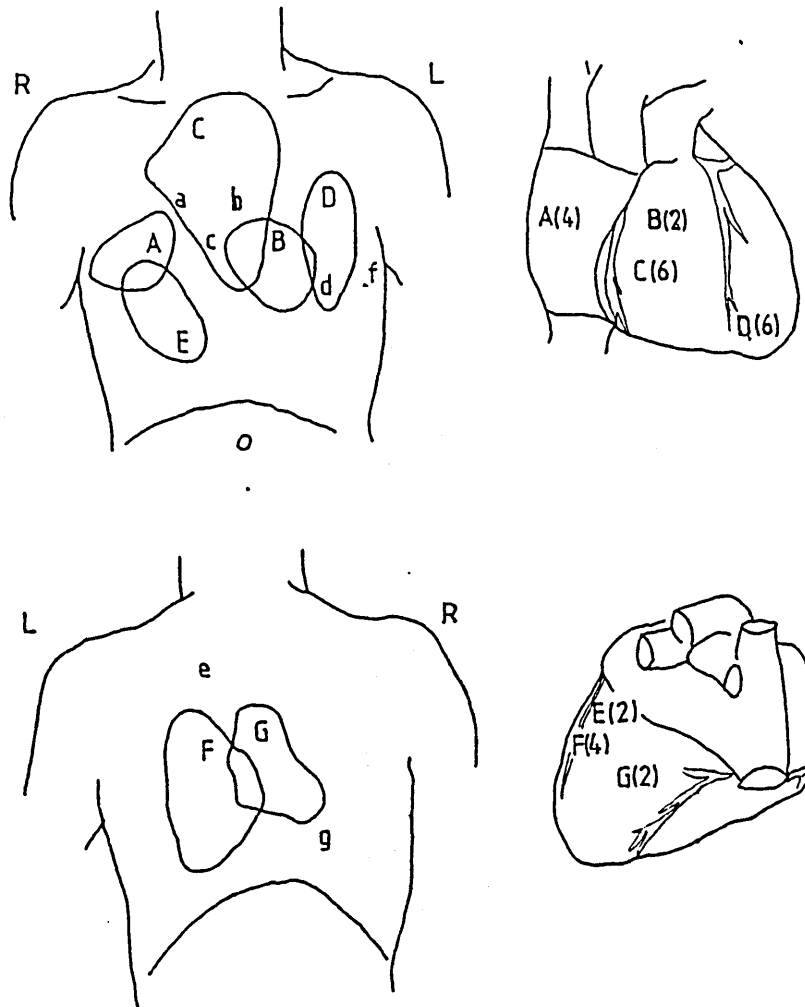


Fig.6.8 The relationship between the epicardial measurement sites and the regions on the body surface at which the QRS was best correlated. 7 groups of epicardial sites are identified on the R.H.S. of the figure by the uppercase letter A to G. The number of individual measurements in each group is enclosed in brackets. On the L.H.S. of the figure, the outlines on the body surface are the mean values of the best correlation coefficients between the body surface ECG and the given epicardial ECG in each group. The lowercase letter indicate the body surface sites predicted by the computer model of the human thorax used in this study.

epicardial segment) predicted by the computer torso model. It can be seen that there was a very close resemblance between the experimental results and the prediction.

#### 6.2.5 Discussion

It can be seen from the results that the epicardial electrocardiograms closely resemble the body surface electrocardiograms during the QRS complex. This is not the case in the atrial activation period, when the P-wave of the epicardial electrocardiograms show distinct features which are not observed in the body surface measurements. In both cases (i.e in the period of QRS and P-wave) the magnitude of the body surface measurements were smaller than the corresponding best correlated epicardial electrocardiograms. This is because the amplitude of the potentials recorded depends on the distance between the source and the measuring sites (Kootsey and Johnson, 1976 [98]).

The significant difference in features observed in the P-wave implies that the information of the underlying cardiac events is either lost or smoothed in the transfer process between the two surfaces. This fact was also illustrated by the measurements on intact chimpanzees [6] and the simulated model study by Rudy and Plonsey (1980) [4]. This effect is less significant during the QRS complex, perhaps because of the different anatomical structures and the depolarisation patterns

between the atrium and the ventricles. The ventricular myocardium is thicker and the depolarisation sequences spread mainly from the inside (Scher, 1976 [100]). As a result the unipolar electrode placed on the epicardial surface near the ventricular area only reflects the gross direction of the depolarisation wave front rather than its actual position. In the thinner atrial myocardium which can reasonably be approximated as a thin mass of cells surrounded by an extensive volume conductor (Kootsey and Johnson, 1976) [98], the depolarisation wave front spreads across the atrial epicardium like that produced when dropping a stone into still water (Scher, 1976) [100]. Placing a unipolar electrode at the atrial epicardium will actually sense the passing of the depolarisation wave front.

Although there were some discrepancies between the position of the optimal surface sites predicted by the computer model and the body surface positions that were best correlated with the corresponding epicardial regions in terms of measured potentials, in general they were quite close together. The differences are likely to be due to the fact that only one configuration of the thorax and the heart model was used for all the subjects, and, it has been shown that, there was a considerable inter-subject variation in size and configuration of the body and the heart.

### 6.3 Computation of transfer coefficients from the measured body and epicardial surface potentials

In this study, the forward transfer coefficient was derived based on the knowledge of the electrical properties of the medium between the epicardial and body surfaces. But this is not the only way the forward transfer coefficient can be derived. If the information of the potential distribution of the 2 surfaces is given, in theory it is possible to derive the forward transfer coefficient which describes the relationship between these two surfaces. Mathematically, it can be written as:

$$AE = S \quad (6.1)$$

where  $A$  = matrix of the transfer coefficients

$E$  = column vector of the epicardial potentials

$S$  = column vector of the body surface potentials

In practice when the potentials  $E$  and  $S$  are given, matrix  $A$  cannot be obtained directly from equation 6.1 because it cannot be satisfied exactly in the presence of measurement error. Thus some form of estimation technique is required to compute the forward transfer coefficient matrix  $A$ .

In this study, a maximum of 3 epicardial measurements were available and obviously could not give a complete picture of the epicardial potential distribution, hence it was impossible to

derive the forward transfer coefficient matrix A described above.

Alternatively, one can consider the inverse transfer coefficient B which describes the relationship between the surface potentials with each individual epicardial measurement site. Mathematically,

$$B_{11}S_1 + B_{12}S_2 + \dots + B_{1n}S_n = E_1 \quad (6.2)$$

where  $B_{1n}$  = inverse transfer coefficients

$S_n$  = body surface potentials

$E_1$  = epicardial potentials at a particular site

and  $n = 1, 2, \dots, N$

$N$  = number of body surface measurements.

In the above case, it is obviously impossible to find  $n$  unknowns from one equation, but one can make use of the fact that the inverse transfer coefficient is time invariant throughout the whole cardiac cycle (as the transfer coefficient only depends on the properties of the medium which does not change in time), then,

$$B_{11}S_{1t_1} + B_{12}S_{2t_1} + \dots + B_{1n}S_{nt_1} = E_{1t_1}$$

.

.

.

$$B_{11}S_{1t_m} + B_{12}S_{2t_m} + \dots + B_{1n}S_{nt_m} = E_{1t_n}$$

in matrix form,

$$B_{1N}S_{NM} = E_{1M} \quad (6.3)$$

where  $N$  = number of surface sites

$M$  = number of time instants

In this study,  $N$  has a maximum value of 37 while  $M$  has a maximum value of 300 and we have a so called over-determined problem to solve which often requires some form of approximation. The Least Squares approximation technique is one of these and was used in this study.

### 6.3.1 Linear Least Squares approximation

Given a set of  $M$  linear equations involving  $N$  variables

$x_i$  as follows:

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \quad (6.4)$$

where  $i = 1, 2, \dots, N$

$m = 1, 2, \dots, M$

$n = 1, 2, \dots, N$

and  $M > N$

It will not normally be possible to find a solution that will satisfy all these equations simultaneously, and hence for any particular proposed solution, one or more equations are likely to be in error.

Least Squares approximation defines the most acceptable solution to be the one with the minimum length of the residue norm  $r$  which can be expressed mathematically as,

$$r = \|AX - B\|^2 = \text{minimum} \quad (6.5)$$

In the simplest case when  $N = 1$ , we then have

$$a_m x_1 = b_m \quad (6.6)$$

Solving the above by Least Squares approximation is equivalent to fitting a best straight line from the data given under the condition that the sum of the square error  $r$  is minimum (Fig.6.9).

As solving the Least Squares problem is an approximating process, it is best to derive some quantities and use them to select a useful and reasonable solution. In this study the singular value decomposition analysis technique is used to provide these determining quantities.

#### 6.3.2 Definition of singular value decomposition (SVD)

It is an orthogonal decomposition of the  $m \times n$  matrix  $A$  of rank  $k$ , such that,

$$A = USV^T \quad (6.7)$$

where  $U = m \times m$  orthogonal matrix

$V = n \times n$  orthogonal matrix

$S = m \times n$  diagonal matrix with  $k$  nonzero entries

and  $V^T$  is the transpose of  $V$ .

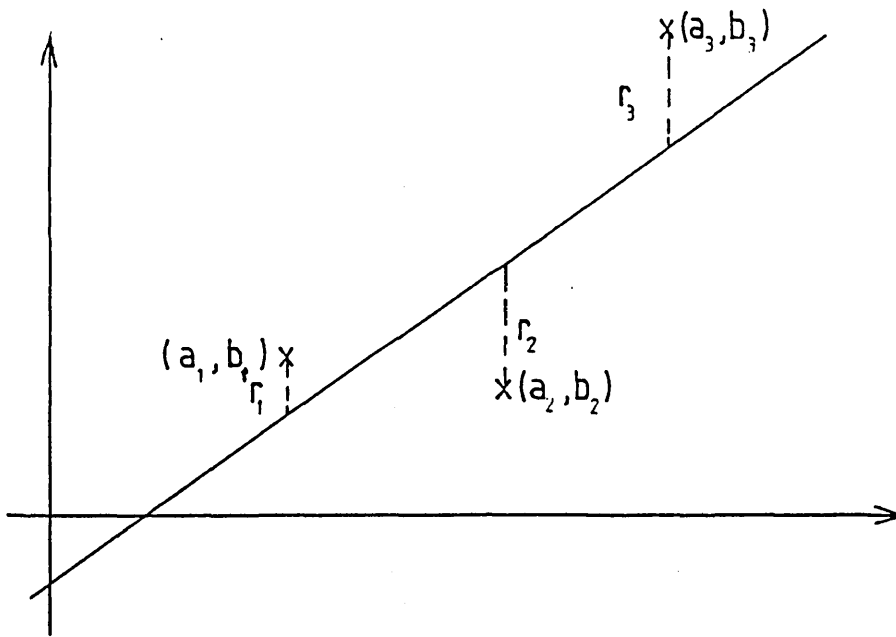


Fig.6.9 Fitting a best straight line using Least Squares approximation.

The diagonal entries of  $S$  are called the singular values of  $A$ . Detailed treatment of the SVD can be found in Lawson and Hanson (1974) [101].

### 6.3.3 Least Squares problem and SVD

Reconsider the Least Squares problem defined in 6.3.1 where it is required to find the solution vector  $X$  of the equation

$$AX = B \quad (6.8)$$

so that,

$$r = \|AX - B\|^2 = \text{minimum} \quad (6.9)$$

where  $r$  is the residue norm.

Employing singular value decomposition and let,

$$A = USV^T \text{ (see equation 6.7)}$$

substituting equation 6.8 for  $A$ , we have

$$USV^T X = B$$

then,

$$SV^T X = U^T B \quad (6.10)$$

letting,

$$P = V^T X \quad (6.11)$$

and,

$$G = U^T B \quad (6.12)$$

equation 6.10 becomes,

$$SP = G \quad (6.13)$$

It can be shown that (Appendix A8),

$$r^2 = \|(g_2)\|^2 = \text{minimum when } g_1 = S_k p_1$$

where  $k$  is the rank of matrix  $A$ .

The index  $k$  can be determined by inspecting the diagonal matrix  $S$  which contains the singular values of matrix  $A$ . After  $k$  is defined,  $P$  can be found by solving equation 6.13 as,

$$\begin{bmatrix} S_k & 0 \\ 0 & 0 \end{bmatrix} \cdot P = G$$

and the solution vector  $X$  is given by equation 6.11.

#### 6.3.4 Computation of the inverse transfer coefficient

The numerical technique derived above was used to compute the inverse transfer coefficients from the body and epicardial surface measurements. As the inverse solutions prepared by computer modelling does not produce inverse transfer coefficients, they cannot be directly compared. However, the epicardial electrocardiograms were reconstructed using the computed inverse transfer coefficient and the body surface measurement for all 21 subjects. Fig.6.10 shows the results of an arbitrarily chosen subject. The top and the second rows show the measured and the reconstructed epicardial electrocardiograms respectively (the epicardial electrocardiograms were reconstructed from the measured body surface potentials and the computed inverse transfer coefficient). The bottom row is the absolute error between the measured and reconstructed signals.

Fig.6.11 summarises the results for all 48 epicardial electrocardiograms available. The top row is the mean square values of all the epicardial measurements. The second and third row show the mean square error and the cross correlation coefficient between the measured and reconstructed epicardial electrocardiograms respectively. The relative mean square error

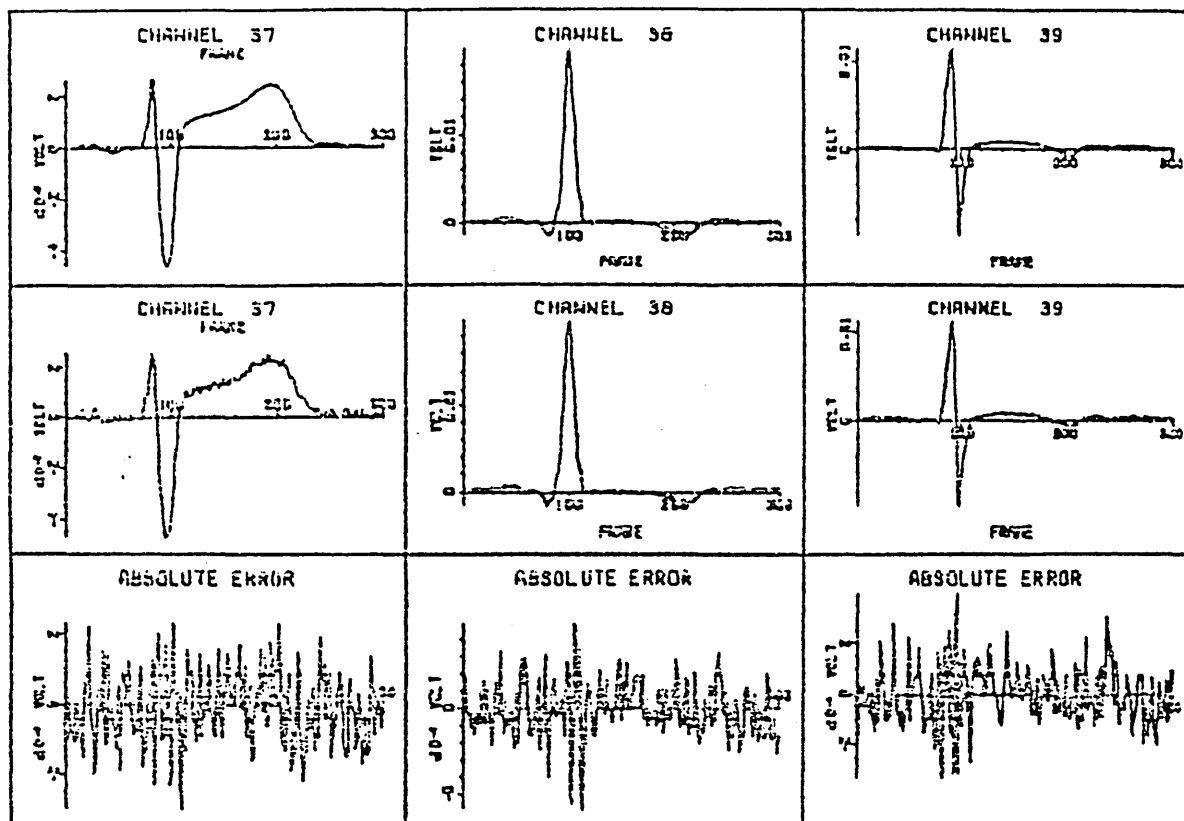


Fig.6.10 Comparison between measured and reconstructed epicardial ECG.  
The top row shows the measured epicardial ECG and the second row shows the reconstructed ones. The bottom row is the absolute error between the measured and reconstructed ECG.

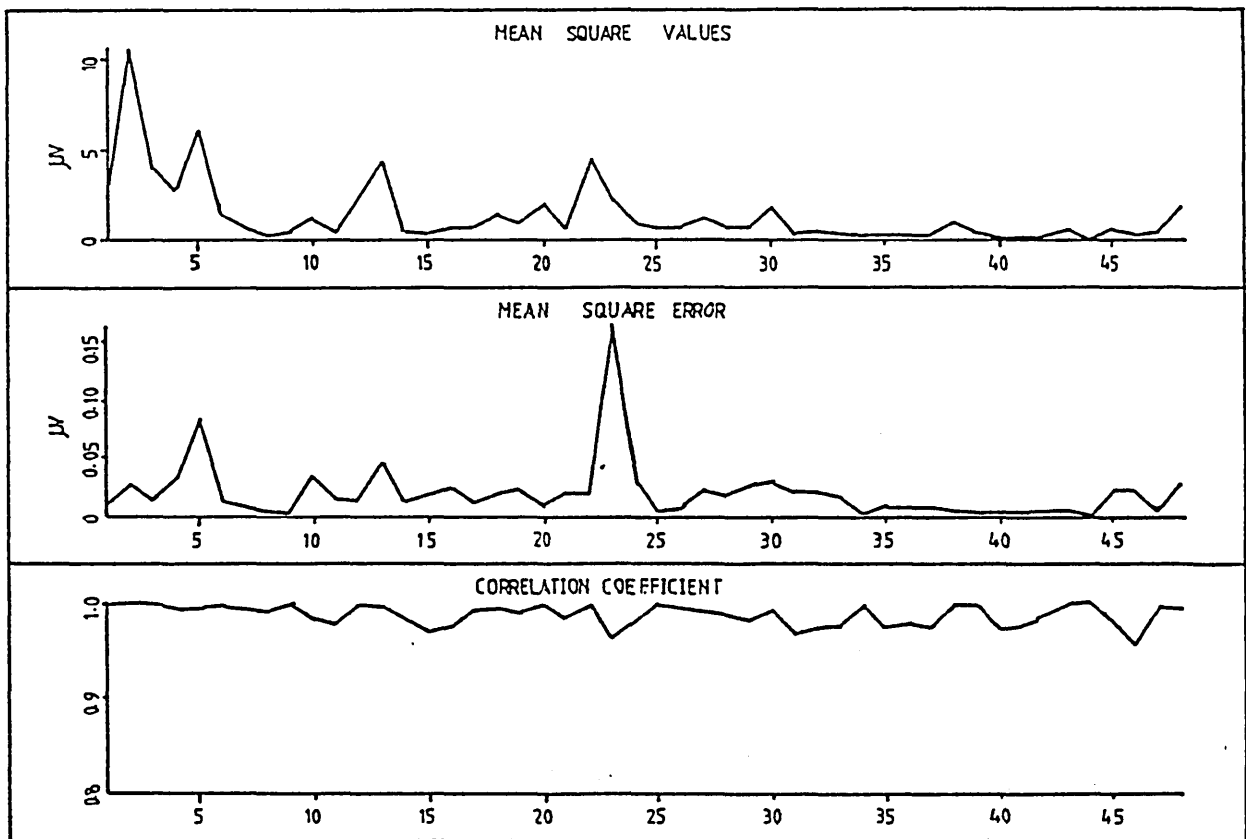


Fig.6.11 The error of reconstruction of 48 epicardial ECG. The top row is the mean square values of the epicardial measurements. The second and bottom row show the mean square error and the cross correlation coefficient between the measured and reconstructed epicardial ECG respectively.

ranged from 0.00027 to 0.046 and had a mean value of 0.017. The correlation coefficients all had a value above 0.98. The results suggested that the above computational process, to derive the inverse transfer coefficient from the measured surface potentials, was stable.

#### 6.4 Feasibility of using the inverse transfer coefficients on other patients other than one from which the coefficients were derived

In studying the forward problem or inverse problem in Electrocardiography, it is always necessary to have a knowledge of the set of transfer coefficients which describe the relationship between the body and epicardial surfaces. The transfer coefficient can be obtained from either a knowledge of the geometry and electrical properties of the human thorax or directly from the measurement of the body and epicardial surface potentials. The former method is cumbersome and time consuming while the latter requires an invasive operation which is not always practical, so it is helpful to see if a set of transfer coefficients can be obtained and applied to other subjects.

As discussed in the above section there is insufficient measured epicardial data for the derivation of the forward transfer coefficient and instead the inverse transfer coefficient was computed. In the following section, the feasibility of using the inverse transfer coefficient on other subjects other than the one from which the coefficient is derived was studied.

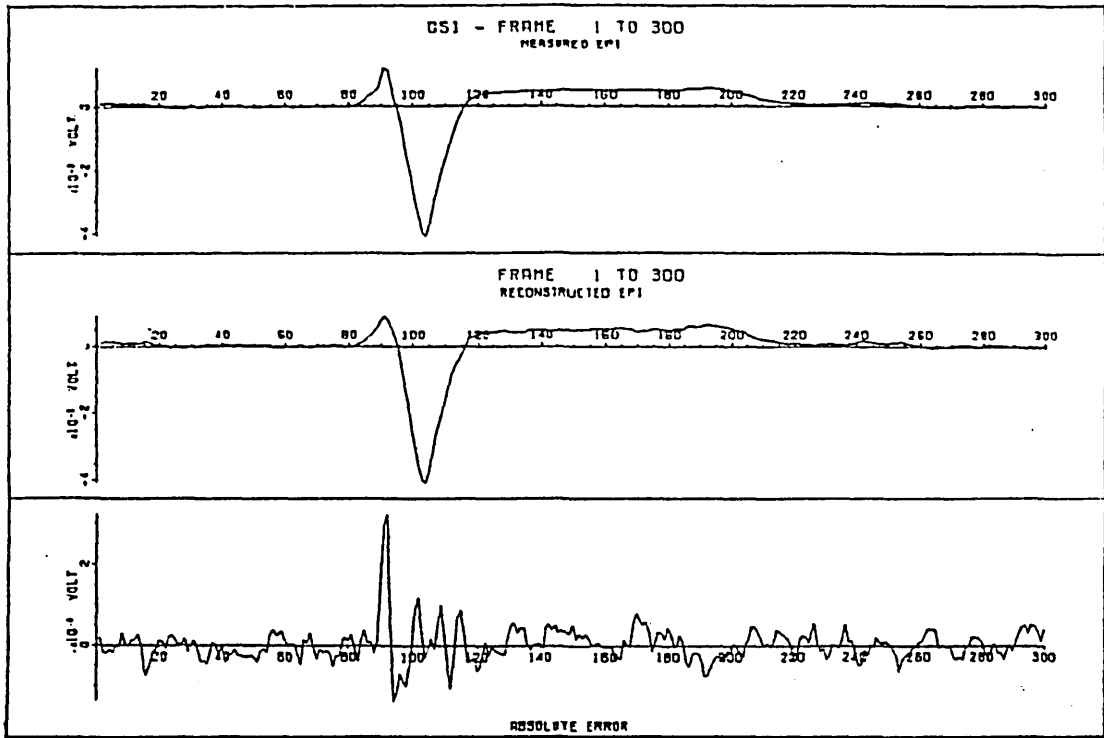
##### 6.4.1 Methods and results

A group of six subjects with the measuring electrodes on the anterior right ventricle (segment 14 on the computer model) were used in this study. The inverse transfer coefficient of one

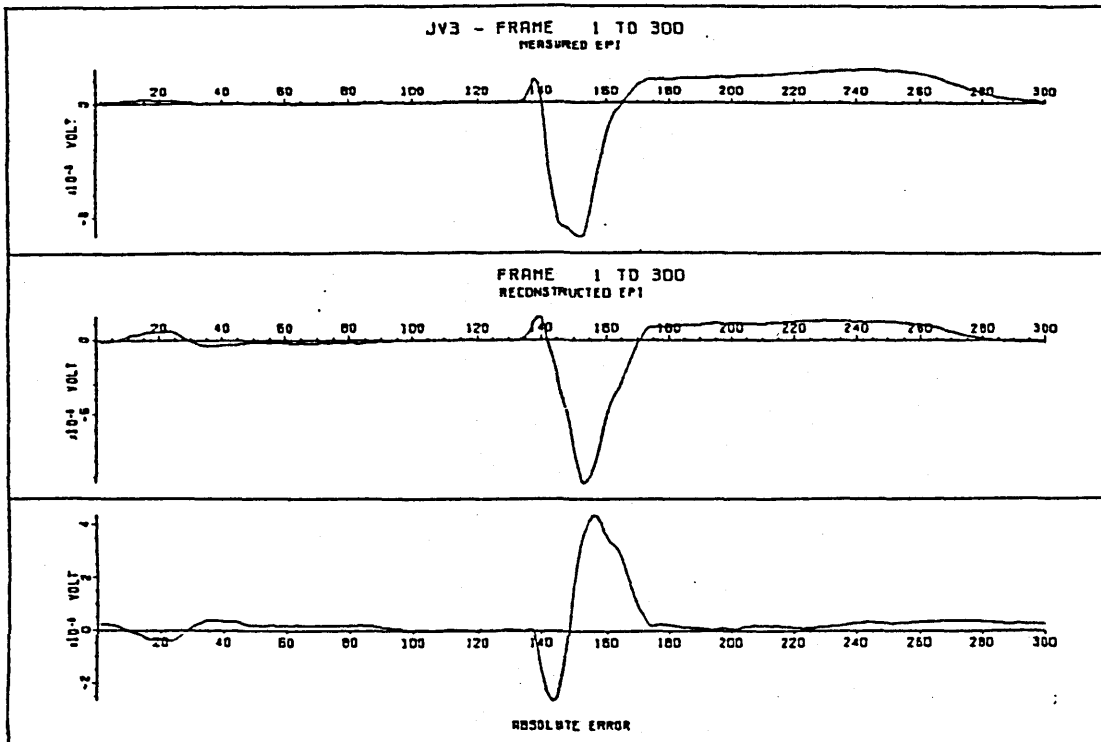
arbitrarily chosen subject was computed using the method described above. The inverse transfer coefficient was then used to reconstruct the epicardial potentials from the body surface measurement of that subject as well as other subjects in the same group. The results are shown in Fig.6.12 (a-f). On each plot, the first and second row is the measured and reconstructed epicardial ECG waveforms respectively and the bottom row shows the absolute error. Fig.6.12.a shows the waveforms of the subject from whom the inverse transfer coefficient was derived. The correlation coefficient and the relative mean square error between the measured and reconstructed waveforms were 0.999 and 0.0029 respectively. In the remaining 5 subjects in this group whose epicardial ECG waveforms were reconstructed using the inverse transfer coefficient from the first subject, the correlation coefficients between the measured and reconstructed waveforms ranged from 0.84 to 0.97 and the relative mean square error ranged from 0.09 to 0.58. The measured and reconstructed ECG waveforms of these 5 subjects are shown in Fig.6.12 (b-f).

#### 6.4.2 Discussion

It was expected that the quality of the reconstruction of the epicardial potentials in other subjects would be poorer than the one from which the inverse transfer coefficient was derived. Although the error between the measured and reconstructed data was considerable, the cross-correlation coefficient however always had



(a)

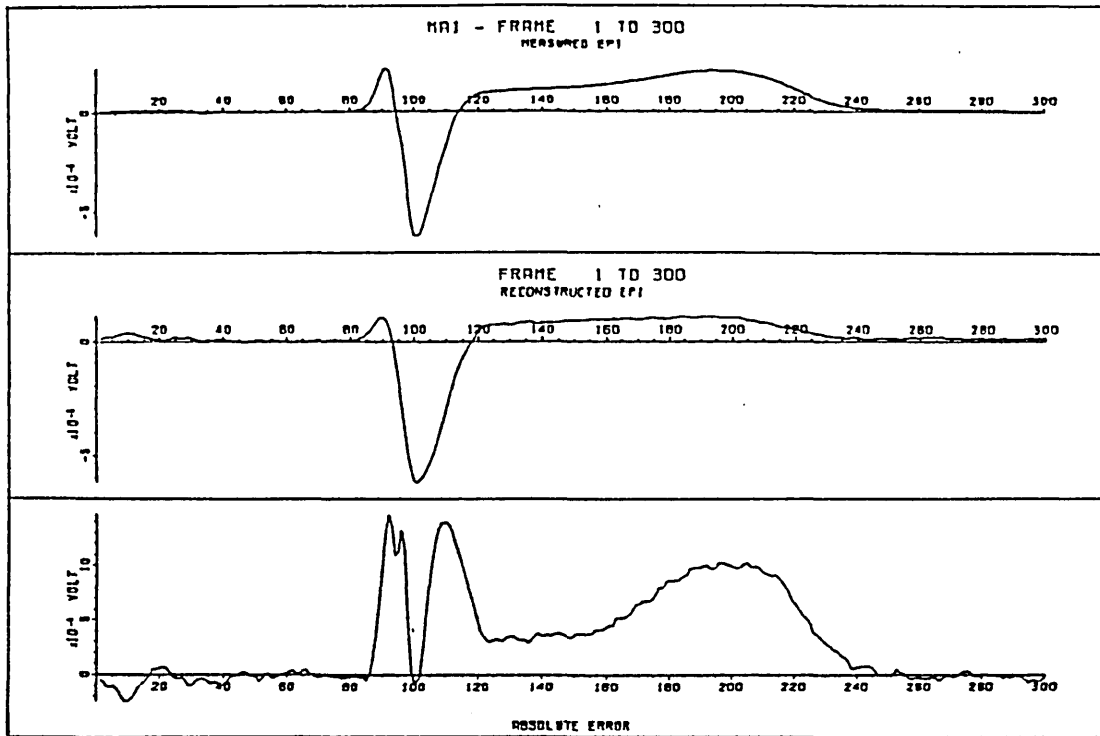


(b)

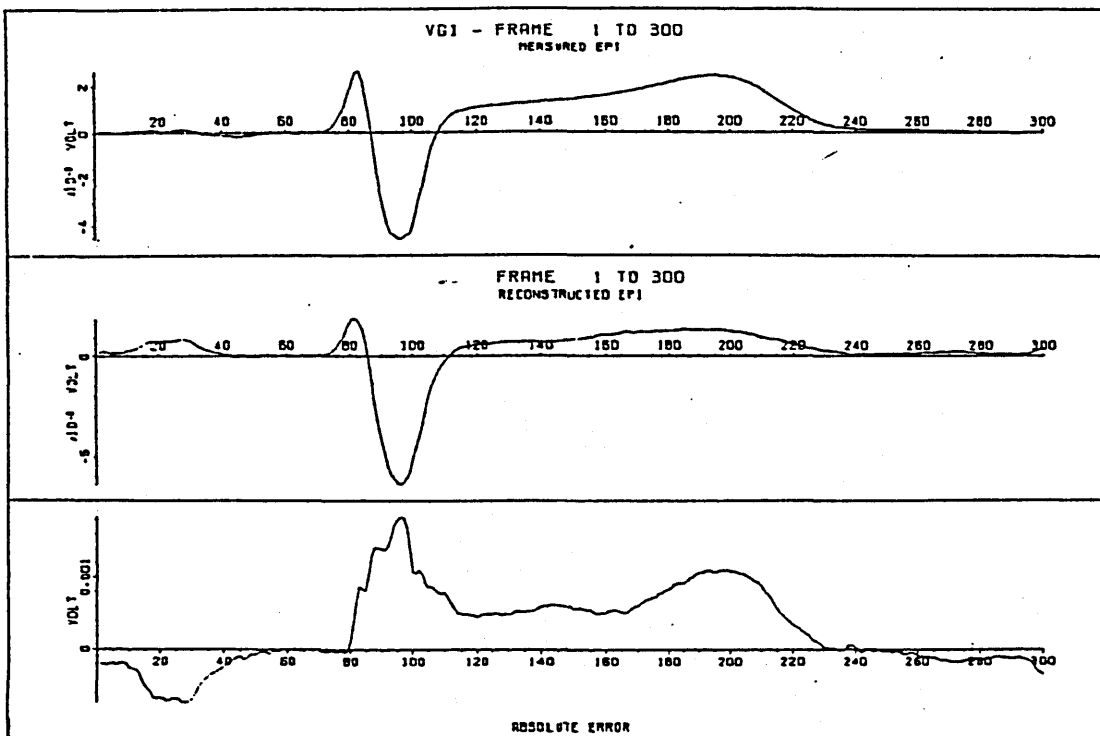
Fig.6.12 Comparison between the measured and the reconstructed epicardial ECGs.

(a) Subject whose epicardial ECG was reconstructed using the inverse transfer coefficient derived from the same subject.

(b)-(f) Subjects whose epicardial ECGs were reconstructed using the inverse transfer coefficient used in (a).

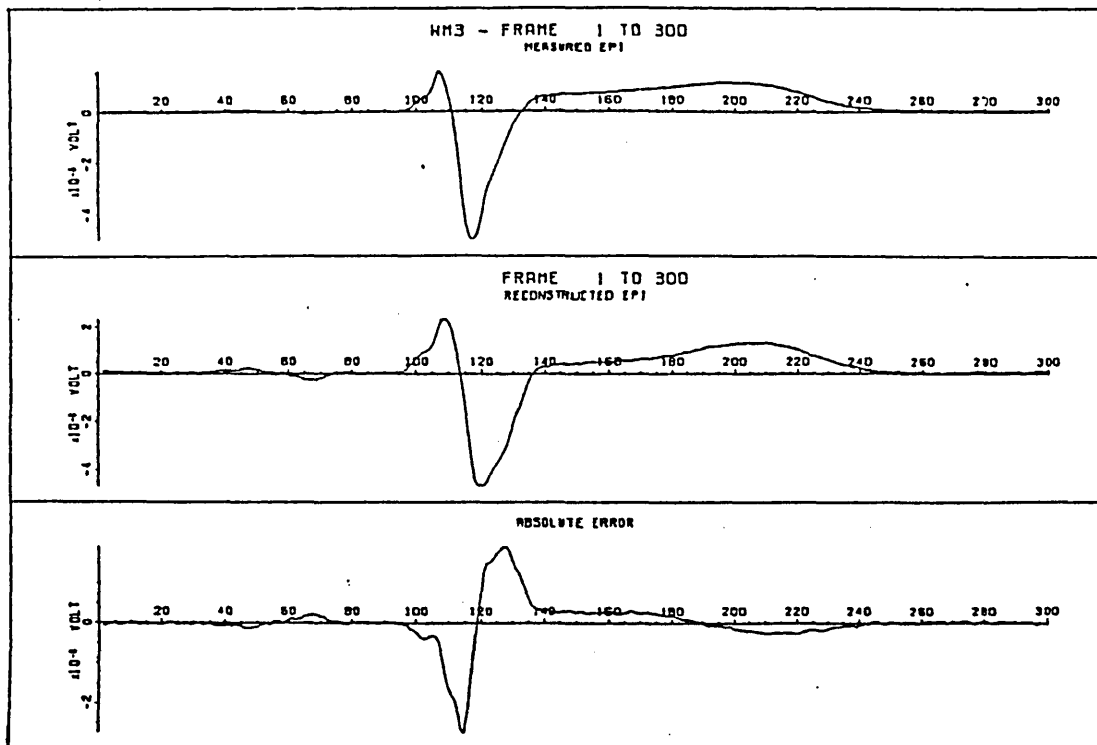


(c)

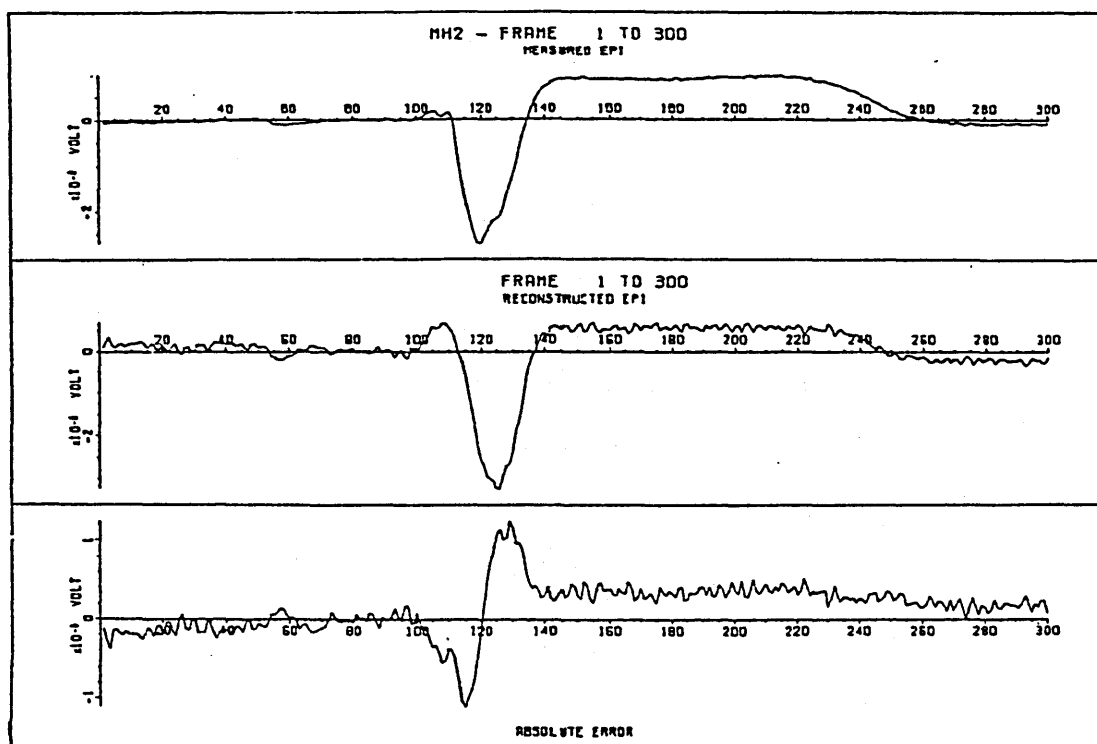


(d)

Fig.6.12 (Cont'd)



(e)



(f)

Fig.6.12 (Cont'd)

a value greater than 0.8, indicating that the differences were primarily in the voltage magnitude rather than in the pattern. The discrepancy in the voltage amplitude can be due to the fact that there was a large inter-subject variation of voltage amplitude on similar sites of measurement especially of the body surface potentials which are governed by many factors such as body build, the internal inhomogeneity of various organs and the physiological factors of respiration. Since the inverse transfer coefficient is very much dependent on the values of the measured potentials, error in the reconstruction is expected unless some form of correction for the inter-subject variation is made.

## 6.5 Conclusion

1. There was a large inter-subject variation in amplitude of the body surface electrocardiograms, one of the likely factors was the body build. It was shown that there was a trend of reduced QRS voltage with increased chest size.

2. Unipolar epicardial electrocardiograms correlated well during the QRS complex with the body surface measurements but not so well in the P-wave.

3. Distinct features in the P-wave of the epicardial electrocardiograms not observed in the body surface measurements indicated that a certain amount of information is lost in the transfer process of the ECG signals between the two surfaces.

4. Optimal surface sites predicted by the computer torso model were quite close to the best correlated surface sites calculated from the measurements. Inter-subject variation has to be taken into account to minimise errors.

5. Inverse transfer coefficient can be obtained from the measured body and epicardial surface ECG. The reconstruction of the epicardial ECG from the computed coefficient implied the process is stable. In order to compute the forward transfer coefficient, more epicardial surface measurements are required which is not ethically feasible at the present moment.

6. It is feasible to use the inverse transfer

coefficient derived from the measurements of one subject for another subject. To obtain more accurate results, the inter-subject variability of human thorax and heart configurations and the internal inhomogeneity effect need to be considered.

## CHAPTER 7

### CONCLUSION

When using models, solving the forward problem is an essential step towards the goal of solving the inverse problem in Electrocardiography. There have been numerous approaches to studying these problems and among them, the surface potential approach has the most advantages. It is not only easier to understand the problems conceptually, but also the results can be interpreted and compared directly with reference to clinical measurements. In this study, the forward problem was solved by setting up a computer model of the human thorax and then approximating the electrical field distribution in the human torso by the finite difference method. The mathematical formulation and the computer implementation of the human thorax was presented in Chapter 2. In this study a discrete anatomical model of the human thorax was constructed using a 1-cm regular grid, and this gave a total number of 15640 nodal points. In order to solve this potential field problem, a considerable amount of computer memory and lengthy execution time are required. In this study, the amount of computer memory required was vastly reduced by introducing a scanning technique in the computation algorithm and the execution time was reduced by implementing the Gauss-Siedel successive over-relaxation technique. The optimal accelerating

factor was used, and in Chapter 2 it was shown that the convergence rate of the Gauss-Siedel iterative process was very much dependent on the value of the accelerating factor used. A 10% deviation (decrease) from the optimal value increased the number of iterations by a factor of three. It was also demonstrated that the optimal accelerating factor remained the same as long as the electrical properties of the conducting medium remained unchanged. This implies that once the optimal accelerating factor is obtained, it can be used for future computation with the variation of the epicardial segment parameters. By comparing the solutions of the potential distribution of a cylindrical model obtained by analytical technique with those computed using the numerical modelling technique in this study showed a valid methodology was established. A close resemblance between the simulated body surface potential distribution, computed with the numerical modelling technique in this study, and those actually observed on human subjects was shown proving the validity of the technique.

It is felt that this study has been taken as far as is currently possible using present computers. In future it will be desirable to carry out the computation on specific geometries, perhaps not one per subject, but certainly to cover certain classes of thoracic geometry. As discussed in Chapter 3, this requires only an order of magnitude increase in the power of computers to make it feasible on either laboratory or centralized computers. In view of the history of the development of

computers, this should be possible within 10 years.

In studying the inverse problem, whatever methods are used, there is always the problem of "ill-posedness", i.e. a small amount of perturbation will lead to a large oscillatory error in the inverse solution. This ill-posedness is characterised by the large "condition number" of the system equations. This instability problem was dealt with in this study by two approaches. Firstly, the condition number of the system equations was reduced by using a model with unequal size epicardial segments. Secondly, a regularisation technique, using the a priori knowledge of the power ratio of the body surface noise and the epicardial signals, was used in the computation. It was shown that despite a relatively large condition number of the system equations, the solutions remained very stable when the body surface measurement was contaminated with a reasonable amount of noise.

In solving the inverse problem using the surface potential approach, the body surface potential distribution has to be known. This implies the requirement of a large number of body surface measurements which often proved to be too cumbersome and impractical. So it will be very useful if the body surface potential distribution can be reconstructed from a limited electrode set. This aspect was discussed in Chapter 5. The results from various studies showed that it is adequate to sample the body surface potentials with a limited electrode set.

The body surface and epicardial surface potentials were measured simultaneously on 21 human subjects. Using the computational technique described in Chapter 3, the epicardial electrocardiograms were computed from the measured body surface potentials. The results were compared directly with the measured data and it was shown that despite some discrepancies in amplitude, the computed solutions resembled closely the measured ones in most of the subjects. The discrepancy in magnitude is due to the fact that only one computer model was used for all the 21 subjects and that there was a large inter-subject variation of the heart position, so the position of the computed solution need not correspond to the point of measurement. Another contributing factor in the voltage discrepancy is the limited resolution of the heart model. In this study a 26 epicardial segment model was used, each segment representing a summation of activities over a relatively large area. Although better resolution can be achieved by using a larger number of heart segments, this will increase the amount of computer memory and execution time. The compromise depends on the facilities available and the quality of results required.

In this study due to the limitation of the number of measuring electrodes on the epicardial surface, the inverse solutions can only be compared with the measured data in terms of temporal waveforms. Perhaps of more significance than the temporal waveforms at points on the epicardial surface is the variation of potential over the epicardial surface at different

time instants in the cardiac cycle. This epicardial surface mapping technique can not only present a clear view of localised activation events but can also give a full picture of the time events of the depolarisation wavefronts. In view of the success in the measurement of the epicardial ECG using three transthoracic pacing wires on human subjects in this study, it can be confidently said that epicardial surface mapping on human subject is within reach. In view of the inter-subject variation in ECG voltage amplitude, chest and heart configuration etc., it is even more important to collect more data of simultaneous measurement of body and epicardial surface potentials so the effect of these inter-subject variations on the relationship between the body and epicardial surface ECG can be studied and used to improve the method of estimating epicardial surface potentials from body surface measurements.

It was shown in chapter 5 that the epicardial potentials could be computed without the information of the back torso and the results were generally acceptable. This implies a simplification of measurement procedure as it may not be necessary to put measuring electrodes on the back of the subject which is sometimes impractical e.g. for bed-ridden patients. However the adequacy of reconstructing the body surface potentials from anterior body surface measurements alone needs to be investigated further.

The forward transfer coefficient which describes the

relationship between the epicardial and body surface potentials was obtained in this study from the knowledge of the configuration of the human thorax and the heart as well as the electrical properties of the intervening tissues between the two surfaces. An alternative approach using the measured epicardial and body surface potentials can be used to derive the forward transfer coefficient. But this requires a large number of measuring electrodes on the epicardial surface which is not feasible at the present stage of development. Alternatively, in this study the three epicardial measurements available with the body surface measurements were used to derive the inverse transfer coefficient which describes the relationship between the body surface potentials and the potentials of individual epicardial measuring sites. Although there is no comparable data available for the computed inverse transfer coefficient, the results of the reconstruction of the epicardial electrocardiograms using the coefficient suggested that the computation process is correct and very stable. The advantage of deriving the transfer coefficient directly from measured surface potentials is that it will not be affected by the variation of body geometry nor by the electrical properties of the tissues between the two surfaces. The only disadvantage of this approach is that it requires a large number of measurements on the body and epicardial surfaces.

Whether the transfer coefficient is derived from the knowledge of the geometry and electrical properties of the human thorax or directly from the measurements of the body and

epicardial surface potentials, the process is very cumbersome and time consuming. So it is very helpful if only one set of transfer coefficients is derived and then used in other subjects rather than just on a single subject. In Chapter 5 an inverse transfer coefficient was derived from the potential measurements from one subject, and this coefficient was used for reconstruction of the epicardial electrocardiograms of other subjects. Although the error between the measured and reconstructed data was large they quite closely resembled each other. The discrepancy in voltage amplitude is because the inverse transfer coefficient is very much dependent on the values of the measured potentials. As it has been shown that there is a large inter-subject variation of voltage amplitude on similar sites of measurement, so error in the reconstruction is unavoidable unless some form of correction for the inter-subject variability is made. This needs further investigation.

It is evident that in the study of forward and inverse problems in Electrocardiography, the inter-subject variation of thorax and heart configuration and voltage magnitude is an important area. The study in Chapter 6 showed that one of the likely factors of the variation in voltage amplitude of the body surface ECG in different subjects was the body build, it was shown that there was a trend of reduced QRS voltage with the increase in chest size. Despite the large inter-subject variations, in all the 21 subjects studied, the unipolar epicardial electrocardiograms correlated well during the QRS complex with the

body surface measurements. In the atrial activation period, there were distinct features in the P-waves of the epicardial electrocardiograms that were not observed on the body surface measurements, this indicates that certain information is lost in the transfer process of the ECG signals between the two surfaces. These findings further show the importance of computing the epicardial potential from the non-invasive measurement of the body surface potentials.

## APPENDICES

A1 Discretised model of the human thorax

A2 Notes on the "condition number"

A3 Mathematical derivation (1)

A4 The flow diagrams, source listing and the input data of  
the derivation of the forward transfer coefficient

A5 Forward transfer coefficient

A6 Mathematical derivation (2)

A7 Numerical examples

A8 Residue norm ( $r^2$ ) of  $AX = Y$

A1. Discretised model of the human thorax.





A2. Notes on the "condition number".

There are various measures of the ill-conditioning of a matrix. The most common one has been the relative size of the determinant. A matrix is regarded to be ill-conditioned if its determinant is "small". As the measure of "smallness" of the determinant is relative, more adequate proposals for the measure of the condition of a matrix have been made, among these are :

$$(1) \quad P\text{-condition number} = \frac{|\lambda_{\max}|}{|\lambda_{\min}|}$$

where  $\lambda_{\max}$ ,  $\lambda_{\min}$  are the largest and smallest eigenvalues.

$$(2) \quad N\text{-condition number} = [N(A) N(A^{-1})]/n$$

$$\text{where } N(A) = \left[ \sum_{i,k=1}^n (a_{i,k})^2 \right]^{0.5}$$

and  $n$  is the order of the matrix.

In the above context, the matrix is assumed symmetrical, for asymmetrical case,

$$P\text{-condition number} = \frac{|\mu_{\max}|}{|\mu_{\min}|}$$

where  $\mu^2 = \lambda$

and  $\lambda$  is the eigenvalue of  $(AA^T)$

A3. To show,

$$\frac{\|\Delta X\|}{\|X\|} \leq \|A\| \cdot \|A^{-1}\| \cdot \frac{\|\Delta Y\|}{\|Y\|}$$

Consider equation,

$$AX = Y \quad (A3.1)$$

where  $\det(A) \neq 0$

Let's suppose vector Y is subjected to uncertainty, then

$$A(X + \Delta X) = (Y + \Delta Y) \quad (A3.2)$$

From (A3.1) and (A3.2),

$$\Delta X = A^{-1} \Delta Y$$

Using the inequality property of Euclidean norm,

$$\|\Delta X\| \leq \|A^{-1}\| \cdot \|\Delta Y\| \quad (A3.3)$$

also for (A3.1),

$$\|Y\| \leq \|A\| \cdot \|X\| \quad (A3.4)$$

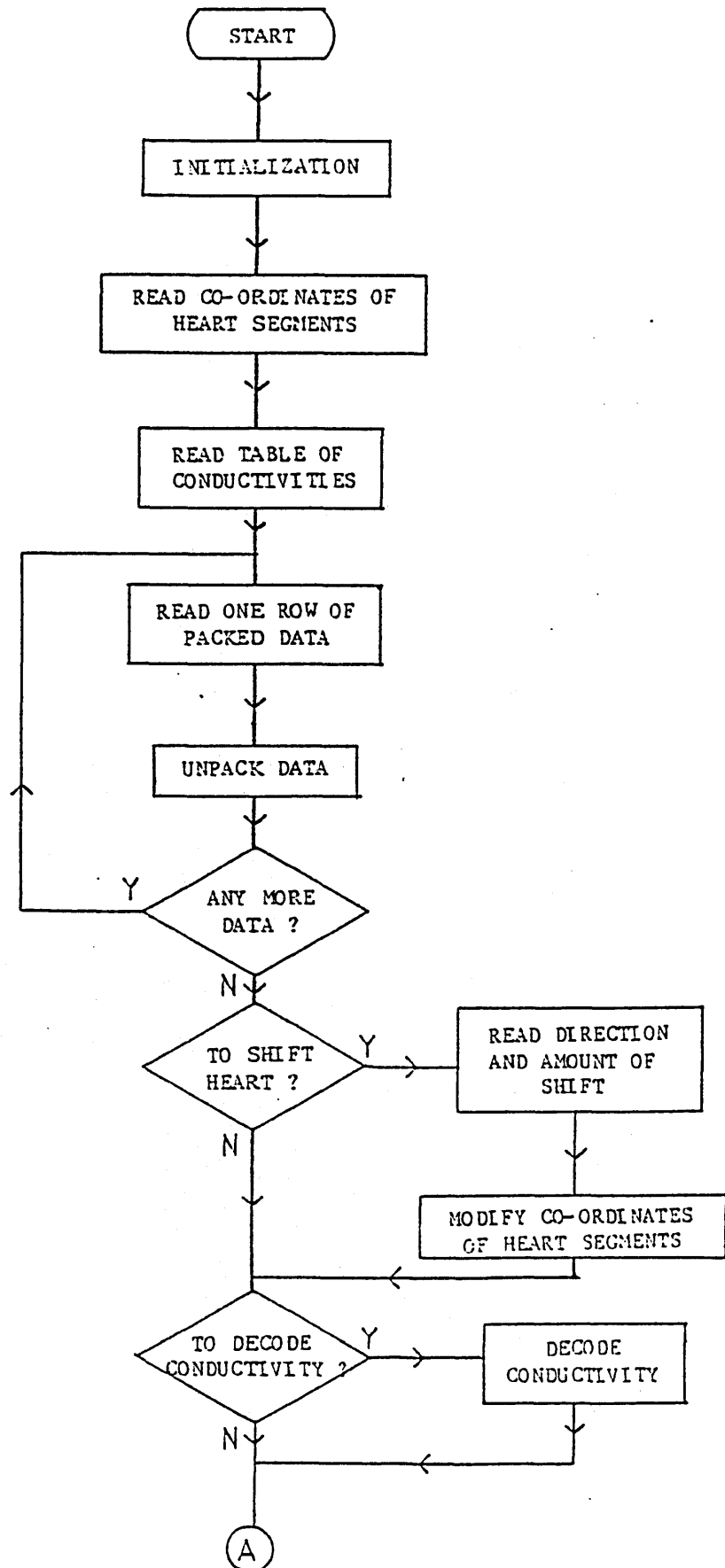
Multiplying (A3.3) and (A3.4),

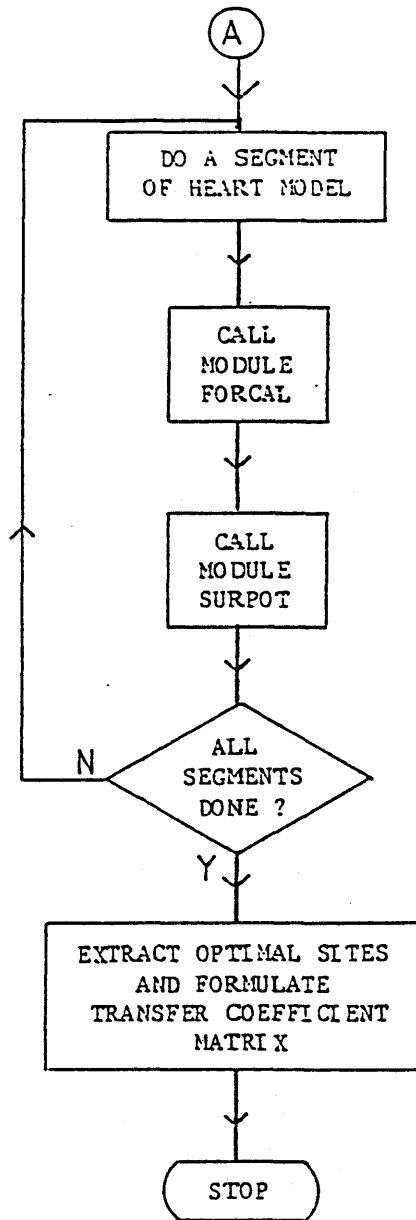
$$\|\Delta X\| \cdot \|Y\| \leq \|A\| \cdot \|A^{-1}\| \cdot \|X\| \cdot \|\Delta Y\| \quad (A3.5)$$

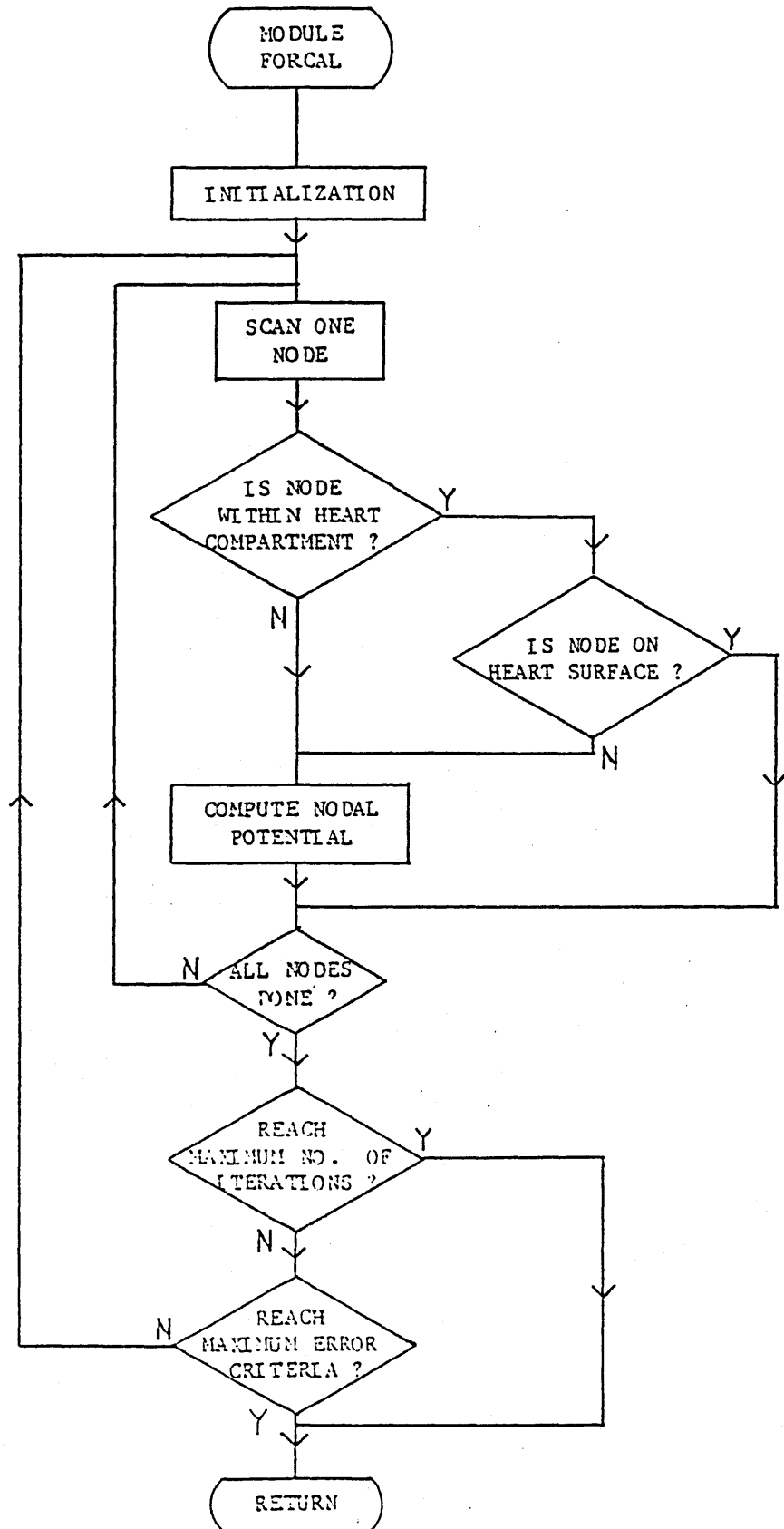
rearranging (A3.5),

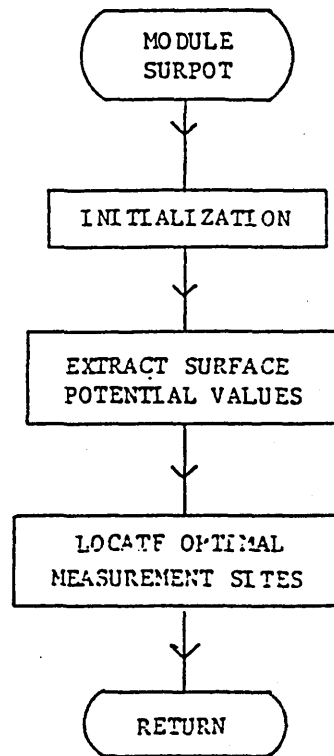
$$\frac{\|\Delta X\|}{\|X\|} \leq \|A\| \cdot \|A^{-1}\| \cdot \frac{\|\Delta Y\|}{\|Y\|}$$

A4. The flow diagrams, source listing and the input data of the  
derivation of the forward transfer coefficient.









|                                       |                                                                                                 |
|---------------------------------------|-------------------------------------------------------------------------------------------------|
| ICUBE(I,J,K)                          | INTEGER ARRAY, NUMERICALLY CODED DATA OF HUMAN THORAX                                           |
| KODE(I)                               | INTEGER ARRAY, WORKSPACE FOR READING IN ONE ROW OF ALPHA-NUMERICALLY CODED DATA OF HUMAN THORAX |
| N(I)                                  | INTEGER ARRAY, WORKSPACE CONTAINS 8 NUMERICALLY CODED DATA IN CONDUCTIVITY DECODING PHASE       |
| ICODE(I)                              | CHARACTER ARRAY, WORKSPACE CONTAINS THE ALPHA-NUMERIC DATA IN THE DATA INPUT PHASE              |
| I23(I,J)                              | INTEGER ARRAY, CONTAINS THE TRANSFER COEFFICIENTS                                               |
| ITMAX                                 | INTEGER VARIABLE, MAXIMUM NUMBER OF INTERATION                                                  |
| IBIT                                  | INTEGER VARIABLE, NUMBER OF ELEMENTS PER ROW OF THE MODEL                                       |
| JUIT                                  | INTEGER VARIABLE, NUMBER OF ROWS OF THE MODEL                                                   |
| KSLAB                                 | INTEGER VARIABLE, NUMBER OF SLABS OF THE MODEL                                                  |
| I1                                    | INTEGER VARIABLE, =IBIT+1                                                                       |
| J1                                    | INTEGER VARIABLE, =JUIT+1                                                                       |
| K1                                    | INTEGER VARIABLE, =KSLAB+1                                                                      |
| IGEN(I)                               | INTEGER ARRAY, CONTAINS THE NUMBER OF CARDIAC GENERATORS IN EACH SEGMENT                        |
| ICOLUMN(I)                            | INTEGER ARRAY, CONTAINS THE I-COORDINATES OF THE OPTIMAL SITES                                  |
| IROW(I)                               | INTEGER ARRAY, CONTAINS THE J-COORDINATES OF THE OPTIMAL SITES                                  |
| CON(I,J)                              | REAL ARRAY, CONTAINS THE CONDUCTIVITY TABLES                                                    |
| IC(I,J,K),<br>JC(I,J,K),<br>KC(I,J,K) | INTEGER ARRAYS, CONTAIN THE I, J AND K COORDINATES OF THE CARDIAC GENERATORS                    |
| ISH(I)                                | CHARACTER ARRAY, CONTAINS THE DIRECTION OF EACH HEART SLIFT                                     |
| HL(K)                                 | INTEGER ARRAY, CONTAINS THE NUMBER OF UNITS SHIFTED                                             |
| U(I,J,K)                              | REAL ARRAY, CONTAINS THE POTENTIAL VALUES OF THE WHOLE POTENTIAL FIELD                          |
| ISS,JSS,KSS                           | INTEGER VARIABLES, TOTAL NUMBER OF UNITS SHIFTED IN I, J AND K DIRECTION                        |
| G(I,J)                                | REAL ARRAY, CONTAINS THE 6-CONDUCTIVITY VALUES RELATED TO EACH ELEMENT                          |
| BTEMP(I,J,K)                          | REAL ARRAY, WORKSPACE CONTAINS THE SURFACE POTENTIALS DUE TO EACH EXCITED HEART SEGMENT         |

CCC

```

C
C      READ IN THE POSITION OF EACH SEGMENT
C
CCC
      READ (5,1000) HSEG
      DO 10 N=1,HSEG
      READ (5,1100) HGEN(N)
      JJ=HGEN(N)
      READ (5,1000) ((IC(N,J),JC(N,J),KC(N,J)),J=1,JJ)
10  CONTINUE
CCC
C
C      READ IN A TABLE OF CONDUCTIVITY VALUES
C
CCC
      READ (5,1200) (ICODE(I),I=1,12)
      DO 30 J=1,5
      READ (5,1350) (CON(I,J),I=1,12)
30  CONTINUE
      WRITE (6,1400)
      WRITE (6,1500) ((ICODE(I),(CON(I,J),J=1,5)),I=1,12)
CCC
C
C      READ IN THE UNPACKED CODE DATA
C
CCC
      DO 60 K=1,KSLAB
      DO 60 I=1,I1
      READ (5,1300) (KODE(IJ),IJ=1,JBIT)
      DO 100 J=1,JBIT
      DO 90 L=1,I3
      IF (KODE(J).EQ.ICODE(L)) GOTO 35
90  CONTINUE
35  ICUBE(I,J+1,K+1)=L
100 CONTINUE
60  CONTINUE
CCC
C
C      TO SHIFT POSITION OF HEART
C
CCC
      ISS=0
      JSS=0
      KSS=0
      DO 110 K=1,HSHIFT
      IF (H(K).EQ.0) GOTO 110
      IF (ISH(K).EQ.I1) GOTO 701
      IF (ISH(K).EQ.IR) GOTO 702
      IF (ISH(K).EQ.IFO) GOTO 703
      IF (ISH(K).EQ.IR) GOTO 704
      IF (ISH(K).EQ.IU) GOTO 705
      IF (ISH(K).EQ.ID) GOTO 706
701 CONTINUE
      JSS=JSS+H(K)
      CALL SLEFT (K)
      GOTO 110
702 CONTINUE
      JSS=JSS-H(K)
      CALL SRIGHT (K)
      GOTO 110

```

```

703 CONTINUE
      ISS=ISS+H(K)
      CALL SFOR (K)
      GOTO 110
704 CONTINUE
      ISS=ISS-H(K)
      CALL SBACK (K)
      GOTO 110
705 CONTINUE
      KSS=KSS-H(K)
      CALL SUP (K)
      GOTO 110
706 CONTINUE
      KSS=KSS+H(K)
      CALL SDOWN (K)
110 CONTINUE
CCC
C
C      CORRECT COORDINATE OF CARDIAC INDEX
C
CCC
      DO 70 L=1,NSEG
      JJ=HGEN(L)
      DO 70 N=1,JJ
      IC(L,N)=IC(L,N)+ISS
      JC(L,N)=JC(L,N)+JSS
      KC(L,N)=KC(L,N)+KSS
70  CONTINUE
      WRITE (6,1700) ISS,JSS,KSS
CCC
C
C      DECODE CONDUCTIVITY VALUE STORE IN ARRAY G(6,15640).
C
CCC
      N=1
      DO 300 K=1,K1
      DO 300 J=1,J1
      DO 301 L=1,8
301  H(L)=12
      DO 300 I=1,I1
      H(5)=ICUBE(I+1,J,K)
      H(6)=ICUBE(I+1,J+1,K)
      H(7)=ICUBE(I+1,J,K+1)
      H(8)=ICUBE(I+1,J+1,K+1)
      G(1,N)=CON(H(1),5)+CON(H(2),5)+CON(H(5),5)+CON(H(6),5)
      G(2,N)=CON(H(3),5)+CON(H(4),5)+CON(H(7),5)+CON(H(8),5)
      G(3,N)=CON(H(1),3)+CON(H(3),3)+CON(H(5),1)+CON(H(7),1)
      G(4,N)=CON(H(2),3)+CON(H(4),3)+CON(H(6),1)+CON(H(8),1)
      G(5,N)=CON(H(5),2)+CON(H(6),4)+CON(H(7),2)+CON(H(8),4)
      G(6,N)=CON(H(1),2)+CON(H(2),4)+CON(H(3),2)+CON(H(4),4)
      N=H(1)
      DO 302 L=1,4
302  H(L)=H(4+L)
300 CONTINUE
CCC
C
C      COMPUTE THE FORWARD TRANSFER COEFFICIENTS
C
CCC

```

```

DO 200 IS=1,NSEG
CALL FORCAL (IS)
CALL SURPOT (IS)
200 CONTINUE
DO 140 IH=1,NSEG
DO 140 IS=1,NSEG
HC=ICOLUH(IS)
HR=IROW(IS)
XX=BTEMP(HC,HR,IH)*10000000.0
I23(IS,IH)=IFIX(XX)
140 CONTINUE
WRITE (6,2000)
WRITE (6,2400)
DO 145 H=1,NSEG
WRITE (6,2500) IROW(H),ICOLUH(H)
145 CONTINUE
WRITE (6,2000)
WRITE (6,2100)
WRITE (6,2200) ((I23(I,J),J=1,13),I=1,NSEG)
WRITE (6,2200) ((I23(I,J),J=14,NSEG),I=1,NSEG)
STOP
END

```

```

C
C
C*****
C
C
C      SUBROUTINE FORCAL (IS)
CCC
C
C      COMPUTE THE MODAL POTENTIALS
C
CCC
      DIMENSION U1(19800)
      COMMON/SHALL/ ITIAX,NSHX,I1,J1,K1,NGEN(26),ICOLUH(26),IROW(26)
      COMMON/SHALL/ COH(12,5),IC(26,45),JC(26,45),KC(26,45),ISS(10)
      COMMON/SHALL/ ICUBE(21,35,24),U(22,36,25),ISS,JSS,KSS,HI(10)
      COMMON/MEDIUM/ G(6,15640)
      COMMON/LARGE/ BTEMP(104,23,26)
      EQUIVALENCE (U(1,1,1),U1(1))
CCC
C
C      INITIALISATION
C
CCC
      DO 20 I=1,19800
      U1(I)=0.0
20 CONTINUE
      IGIS=NGEN(IS)
      DO 30 L=1,NGIS
      I=IC(IS,L)+1
      J=JC(IS,L)+1
      K=KC(IS,L)+1
      U(I,J,K)=100.0
30 CONTINUE
CCC
C
C      SCAN MODES
C
CCC
      FACT=1.0
      NHI=1
      ZIAX=0.0
      ERRHX=0.0
      DO 940 K=1,K1
      DO 940 J=1,J1
      DO 940 I=1,I1
      IF (K.LT.(8+KSS).OR.K.GT.(19+KSS)) GOTO 935
      IF (J.LT.(15+JSS).OR.J.GT.(29+JSS)) GOTO 935
      IF (I.LT.(9+ISS).OR.I.GT.(18+ISS)) GOTO 935
      DO 950 NS=1,NSEG
      IGNS=NGEN(NS)
      DO 950 NG=1,IGNS
      IF (KC(NS,NG).EQ.K.AND.IC(NS,NG).EQ.I.AND.JC(NS,NG).EQ.J)
      +GOTO 955
950 CONTINUE
935 CONTINUE

```

```

GG=G(1,HH)+G(2,HH)+G(3,HH)+G(4,HH)+G(5,HH)+G(6,HH)
IF (GG.EQ.0.0) GOTO 955
Z=(G(1,HH)*U(I+1,J+1,K)+G(2,HH)*U(I+1,J+1,K+2)+G(3,HH)*
+U(I+1,J,K+1)+G(4,HH)*U(I+1,J+2,K+1)+G(5,HH)*U(I+2,J+1,K+1))+
+G(6,HH)*U(I,J+1,K+1))/GG
Z=FACT*(Z-U(I+1,J+1,K+1))+U(I+1,J+1,K+1)
IF (Z.EQ.0.0) GOTO 955
ER=ABS(Z-U(I+1,J+1,K+1))
IF (ER.GT.ERRHX) ERRHX=ER
IF (ABS(Z).GT.ZMAX) ZMAX=ABS(Z)
U(I+1,J+1,K+1)=Z
955 HH=HH+1
940 CONTINUE

C
C
C
FACT=1.91553
DO 10 IT=1,ITMAX
HH=1
ZMAX=0.0
ERRHX=0.0
DO 40 K=1,K1
DO 40 J=1,J1
DO 40 I=1,I1
IF (K.LT.(8+KSS).OR.K.GT.(19+KSS)) GOTO 35
IF (J.LT.(15+JSS).OR.J.GT.(29+JSS)) GOTO 35
IF (I.LT.(9+ISS).OR.I.GT.(18+ISS)) GOTO 35
DO 50 HS=1,NSEG
NGHS=NGEN(HS)
DO 50 IG=1,IGNS
IF (KC(HS,IG).EQ.K.AND.IC(HS,IG).EQ.I.AND.JC(HS,IG).EQ.J)
+GOTO 55
50 CONTINUE
35 CONTINUE
GG=G(1,HH)+G(2,HH)+G(3,HH)+G(4,HH)+G(5,HH)+G(6,HH)
IF (GG.EQ.0.0) GOTO 55
Z=(G(1,HH)*U(I+1,J+1,K)+G(2,HH)*U(I+1,J+1,K+2)+G(3,HH)*
+U(I+1,J,K+1)+G(4,HH)*U(I+1,J+2,K+1)+G(5,HH)*U(I+2,J+1,K+1))+
+G(6,HH)*U(I,J+1,K+1))/GG
Z=FACT*(Z-U(I+1,J+1,K+1))+U(I+1,J+1,K+1)
IF (Z.EQ.0.0) GOTO 55
ER=ABS(Z-U(I+1,J+1,K+1))
IF (ER.GT.ERRHX) ERRHX=ER
IF (ABS(Z).GT.ZMAX) ZMAX=ABS(Z)
U(I+1,J+1,K+1)=Z
55 HH=HH+1
40 CONTINUE
IF (ERRHX.LE.(1.0E-4*ZMAX)) GOTO 15
10 CONTINUE
15 CONTINUE
RERRHX=(ERRHX/ZMAX)
WRITE (6,3200) IS,IT,RERRHX
RETURN
END

```

```

C
C
C*****
C
C
C
C
SUBROUTINE SURPOT (IS)
C
C
C
C
C
C
COMPUTE THE SURFACE POTENTIALS
C
C
C
C
DIMENSION FRONT(60,23),BACK(60,23),TEMP1(30)
DIMENSION TEMP2(30),TEMP3(30),TEMP4(30)
COMMON/SMALL/ ITMAX,NSFG,I1,J1,K1,NGEN(26),ICOLUH(26),IROW(26)
COMMON/SMALL/CON(12,5),IC(26,45),JC(26,45),KC(26,45),ISII(10)
COMMON/SMALL/ ICUBE(21,35,24),U(22,36,25),ISS,JSS,KSS,NI(10)
COMMON/MEDIUM/ G(6,15640)
COMMON/LARGE/ BTEMP(104,23,26)
CCC
C
C
C
C
INITIALISATION
CCC
I3=I+2
K2=K+1
DO 200 J=1,60
DO 200 K=1,K1
FRONT(J,K)=0.0
BACK(J,K)=0.0
200 CONTINUE
CCC
C
C
C
START
CCC
DO 210 K=2,K2
J=18
I=13
205 I=I-1
IF (U(I,J,K).NE.0) GOTO 215
IF (I.GT.2) GOTO 205
GOTO 210
215 TEMP1(1)=U(I,J,K)
I=I-2
225 J=J+1
IF (U(I,J,K).EQ.0) GOTO 235
TEMP1(II)=U(I,J,K)
II=II+1
GOTO 225
235 J=J-1
I=I-1
236 IF (U(I,J,K).NE.0) GOTO 237
J=J-1
GOTO 236

```

```

237 TEMP1(HI)=U(I,J,K)
   IF (I.EQ.11) GOTO 240
   HI=HI+1
   GOTO 225
240 I2=1
245 IF (U(I,J,K).EQ.0) GOTO 255
   TEMP2(I2)=U(I,J,K)
   I2=I2+1
   J=J+1
   GOTO 245
255 J=J-1
   I=I-1
   IF (U(I,J,K).EQ.0) GOTO 265
   TEMP2(I2)=U(I,J,K)
   IF (J.EQ.18) GOTO 270
   I2=I2+1
   J=J+1
   GOTO 245
265 J=J-1
   I=I+1
275 TEMP2(I2)=U(I,J,K)
   IF (J.EQ.18) GOTO 270
   I2=I2+1
   I=I-1
   IF (U(I,J,K).NE.0) GOTO 275
   GOTO 265
270 I3=1
285 IF (U(I,J,K).EQ.0) GOTO 295
   TEMP3(I3)=U(I,J,K)
   I3=I3+1
   I=I-1
   GOTO 285
295 J=J-1
   I=I+1
   IF (U(I,J,K).EQ.0) GOTO 305
   TEMP3(I3)=U(I,J,K)
   IF (I.EQ.11) GOTO 310
   I3=I3+1
   I=I-1
   GOTO 285
305 J=J+1
   I=I+1
306 IF (U(I,J,K).NE.0) GOTO 315
   J=J+1
   GOTO 306
315 TEMP3(I3)=U(I,J,K)
   IF (I.EQ.11) GOTO 310
   I3=I3+1
   J=J-1
   IF (U(I,J,K).NE.0) GOTO 315
   GOTO 305
310 I4=1
325 IF (U(I,J,K).EQ.0) GOTO 335
   TEMP4(I4)=U(I,J,K)
   I4=I4+1
   J=J-1
   GOTO 325

```

```

335 J=J+1
   I=I+1
   IF (U(I,J,K).EQ.0) GOTO 345
   TEMP4(I4)=U(I,J,K)
   IF (J.EQ.18) GOTO 400
   I4=I4+1
   J=J-1
   GOTO 325
345 J=J+1
   I=I-1
355 TEMP4(I4)=U(I,J,K)
   IF (J.EQ.18) GOTO 400
   I4=I4+1
   I=I+1
   IF (U(I,J,K).NE.0) GOTO 355
   GOTO 345
400 KK4=30-I4
   DO 410 KK=1,I4
   KK4=KK4+KK
410 FRONT(KK4,K-1)=TEMP4(KK)
   DO 420 KK=1,III
   KKK1=29+KK
420 FRONT(KKK1,K-1)=TEMP1(KK)
   KK2=30-I2
   DO 430 KK=1,I2
   KKK2=KK2+KK
430 BACK(KKK2,K-1)=TEMP2(KK)
   DO 440 KK=1,I3
   KKK3=29+KK
440 BACK(KKK3,K-1)=TEMP3(KK)
210 CONTINUE
   DO 460 K=1,K1
   DO 460 J=5,56
   BTEMP(J-4,K,IS)=FRONT(J,K)
   BTEMP(J+48,K,IS)=BACK(J-1,K)
460 CONTINUE
CCC
C
C
C
CCC
J4=104
BIG=0.0
DO 10 J=1,J4
   DO 10 K=1,K1
   IF (BTEMP(J,K,IS).LE.BIG) GOTO 10
   BIG=BTEMP(J,K,IS)
   ICOLUH(IS)=J
   IROW(IS)=K
10 CONTINUE
   RETURN
   END

```

```

C
C
C*****
C
C
C      SUBROUTINE SLEFT (IK)
C
C      SHIFT HEART TO LEFT
C
C
C      DIMENSION ICUBE1(21,35,24)
COMMON/SHALL/ ITIAX,HSEG,I1,J1,K1,NGEN(26),ICOLUH(26),IROW(26)
COMMON/SHALL/CON(12,5),IC(26,45),JC(26,45),KC(26,45),ISH(10)
COMMON/SHALL/ ICUBE(21,35,24),U(22,36,25),ISS,JSS,KSS,NI(10)
COMMON/MEDIUM/ G(6,15640)
EQUIVALENCE (G(1,1),ICUBE1(1,1,1))
NI K=NI (IK)
DO 10 KK=1,NI K
DO 15 K=2,K1
DO 15 J=2,J1
DO 15 I=2,I1
15 ICUBE1(I,J,K)=ICUBE(I,J,K)
DO 10 K=2,K1
DO 10 I=2,I1
DO 10 J=2,J1
IF (ICUBE1(I,J,K).EQ.3.AND.ICUBE1(I,J-1,K).NE.3) GOTO 20
GOTO 30
20 CONTINUE
ICUBE(I,J+1,K)=ICUBE1(I,J,K)
ICUBE(I,J,K)=ICUBE1(I,J-1,K)
IF (ICUBE(I,J,K).NE.1.AND.ICUBE(I,J,K).NE.2.AND.
+ ICUBE(I,J,K).NE.3.AND.ICUBE(I,J,K).NE.4.AND.
+ ICUBE(I,J,K).NE.5) ICUBE(I,J,K)=1
30 CONTINUE
IF (ICUBE1(I,J,K).NE.3.AND.ICUBE1(I,J-1,K).EQ.3)
+ ICUBE(I,J,K)=ICUBE1(I,J-1,K)
IF (ICUBE1(I,J,K).EQ.12) ICUBE(I,J,K)=12
10 CONTINUE
RETURN
END

```

```

C
C
C*****
C
C
C      SUBROUTINE SRIGHT (IK)
C
C      SHIFT HEART TO RIGHT
C
C
C      DIMENSION ICUBE1(21,35,24)
COMMON/SHALL/ ITIAX,HSEG,I1,J1,K1,NGEN(26),ICOLUH(26),IROW(26)
COMMON/SHALL/CON(12,5),IC(26,45),JC(26,45),KC(26,45),ISH(10)
COMMON/SHALL/ ICUBE(21,35,24),U(22,36,25),ISS,JSS,KSS,NI(10)
COMMON/MEDIUM/ G(6,15640)
EQUIVALENCE (G(1,1),ICUBE1(1,1,1))
NI K=NI (IK)
DO 10 KK=1,NI K
DO 15 K=2,K1
DO 15 J=2,J1
DO 15 I=2,I1
15 ICUBE1(I,J,K)=ICUBE(I,J,K)
DO 10 K=2,K1
DO 10 I=2,I1
DO 10 J=2,J1
IF (ICUBE1(I,J,K).EQ.3.AND.ICUBE1(I,J-1,K).NE.3) GOTO 20
GOTO 30
20 CONTINUE
ICUBE(I,J-1,K)=ICUBE1(I,J,K)
ICUBE(I,J,K)=ICUBE1(I,J+1,K)
IF (ICUBE1(I,J-1,K).EQ.12) ICUBE(I,J-1,K)=12
IF (ICUBE1(I,J,K).NE.1.AND.ICUBE1(I,J,K).NE.2.AND.
+ ICUBE1(I,J,K).NE.3.AND.ICUBE1(I,J,K).NE.4.AND.
+ ICUBE1(I,J,K).NE.5) ICUBE(I,J,K)=1
30 CONTINUE
IF (ICUBE1(I,J,K).NE.3.AND.ICUBE1(I,J-1,K).EQ.3) GOTO 40
GOTO 10
40 CONTINUE
ICUBE(I,J-1,K)=ICUBE1(I,J,K)
IF (ICUBE1(I,J,K).NE.1.AND.ICUBE1(I,J,K).NE.2.AND.
+ ICUBE1(I,J,K).NE.3.AND.ICUBE1(I,J,K).NE.4.AND.
+ ICUBE1(I,J,K).NE.5) ICUBE(I,J-1,K)=1
10 CONTINUE
RETURN
END

```





# INPUT DATA:

```

1
K 0
193322
200
26
16
1416 8 131610 131710 1516 8 1517 8 1518 8 1416 9 1516 9 1517 9 1518 9 1617 9
141610 151610 151710 161610 161710
15
161810 171710 1618 8 1619 8 1620 8 1618 9 1619 9 1620 9 1718 9 1719 9 1720 9
171810 171910 172010 172110
14
1621 8 1622 8 1623 8 1621 9 1622 9 1623 9 1721 9 1722 9 152310 162210 162310
162410 172210 172310
15
142410 142510 1424 8 1523 8 1524 8 1324 9 1325 9 1424 9 1425 9 1523 9 1524 9
142610 152410 152510 162510
33
1222 8 1223 8 1224 8 102210 1324 8 1124 9 1125 9 1223 9 1224 9 1225 9 102510
112410 112510 112610 122510 122610 132510 132610 1120 8 1121 8 1122 8 102310
102410 1022 9 1023 9 1024 9 1122 9 1123 9 1222 9 92110 92210 92310 102110
30
1117 8 1118 8 1119 8 121710 121810 1019 9 1020 9 1021 9 1119 9 1120 9 1121 9
92010 101810 101910 102010 111910 1216 8 1217 8 1316 8 111810 121610 1117 9
1118 9 1216 9 1217 9 1218 9 1316 9 1317 9 111610 111710
21
141611 151611 161611 161711 141612 141712 151612 151712 161612 161712 161812
171612 171712 141613 151613 161613 161713 171613 171713 171813 171711
17
171811 171911 172011 181811 181911 182011 171812 171912 181812 181912 182012
182112 181713 181813 181913 182013 182113
18
162311 172111 172211 172311 172411 182111 182211 172312 172412 172512 182212
182312 172313 172413 172513 182213 182313 182413
19
142511 142611 152511 152611 162411 162511 142712 152612 152712 162512 162612
142713 142813 152613 152713 162513 162613 162713 172613
32
102511 112511 112611 122611 132611 92211 92311 92411 102311 102411 102612
112612 112712 122712 132712 102613 112613 112713 122713 122813 132713 132813
92212 92312 92412 92512 102512 92213 92313 92413 92513 102513
39
92011 92111 101811 101911 102011 102111 111811 101611 101711 111611 111711
121611 131611 92012 92112 101812 101912 102012 92013 92113 101913 102013
102113 101612 101712 111612 121612 121712 131612 131712 101613 101713 101813
111613 111713 121613 131613 102112 111712
20
141514 141614 151514 151614 161614 161714 171614 171714 141515 151515 161615
171615 141516 141616 151516 151616 161616 171616 171716 151615
17
181714 181814 181914 182014 182114 171715 181715 181815 181915 182015 182115
181716 181816 181916 182016 182116 182216
18
172414 172514 182214 182314 182414 172415 172515 182215 182315 182415 182515
182615 172616 172716 182316 182416 182516 182616
21
142714 142814 152714 152814 162614 162714 172614 142815 152715 152815 162615
162715 162815 172615 172715 142816 142916 152816 152916 162716 162816

```

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38
102614 112614 112714 122714 122814 132814 102615 102715 112615 112715 112815
122715 122815 132815 92214 92314 92414 92514 102514 92215 92315 92415
92515 102415 102515 102716 112616 112716 112816 122816 132816 132916 92316
92416 102316 102416 102516 102616
44
92014 92114 101914 102014 102114 111914 92115 102015 102115 111915 112015
101714 101814 111614 111714 111814 121514 121614 131514 131614 111815 121715
121815 121515 121615 131515 92116 92216 102016 102116 102216 112016 111916
121916 122016 111816 121516 121616 121716 121816 131516 131616 131716 131816
20
141617 141717 151617 161617 171617 151618 151718 151818 151918 161618 161718
161818 161918 162018 171618 171718 171818 151919 161919 162019
20
171717 171817 171917 181717 181817 181917 182017 182117 171918 172018 172118
181918 182018 182118 182218 182318 172019 172119 182119 182219
20
182217 182317 182417 182517 182617 182717 172617 172717 172817 172918 172818
182418 182518 182618 182718 182319 182419 182519 182619 182719
18
142817 142917 152817 152917 162717 162817 142418 142518 142618 142718 142818
152718 152818 162818 162918 162919 172719 172819
35
112717 112817 122717 122817 132717 132817 132917 102417 102517 102617 112417
112517 112617 132517 132617 122418 122518 132418 132518 132618 132718 132818
122218 122318 132218 132318 142819 152819 152919 142519 142619 142719 122417
122517 122617
39
131617 131717 131817 121817 121917 122017 122117 122217 122317 112017 112117
112217 112317 102017 102117 102217 102317 141618 141718 141818 141918 142018
131718 131818 131918 132018 132118 121818 121918 122018 122118 132019 132119
132219 132319 132419 141919 142019 142419
22
1218 8 1219 8 1220 8 1221 8 1317 8 1318 8 1319 8 1320 8 1321 8 1322 8 1323 8
1417 8 1418 8 1419 8 1420 8 1421 8 1422 8 1423 8 1519 8 1520 8 1521 8 1522 8
23
142119 142219 142319 152019 152119 152219 152319 152419 152519 152619 152719
162119 162219 162319 162419 162519 162619 162719 172219 172319 172419 172519
172619
THHLR123456
1.02.81.20.20.63.00.20.23.03.00.20.0
1.02.81.20.20.63.03.00.20.20.23.00.0
1.02.81.20.20.60.23.03.00.23.00.20.0
1.02.81.20.20.60.20.23.03.00.23.00.0
1.02.81.20.20.61.71.71.71.71.71.70.0

```



[illegible][illegible]

4555555555555555551  
455555553665555TTTT551  
42455555555566555TTTT5341  
4242 RRRRRJ TTTTTT16  
642 RRRRRRJ TTTTTTT361  
462 RRRRRRRRRR TTTTTTTTTT66  
66RRRRRRRRRRR TTTTTTTTTT66  
66RRRRRRRRRRR THTTTTTTTT66  
66RRRRRRRRRRRT THTTTTTTTT66  
66RRRRRRRRRRRWT THTTTTTTTT66  
66RRRRRRRRRRRRRTTTTTTTTTT66  
36RRRRRRRRRRRRRTTTTTTTTT66  
61RRRRRRRRRRRRRTTTTTTTTT66  
36RRRRRRRRRRRRRTTTTTTTTT66  
13RRRRRRRRRRRRRTTTTTT462  
13RRRRRRRRRRRRRTTTTT432  
351RRRRRRRRRRRTT4552  
35555555555555552

43555555555555555555  
43555555555555555555  
435555555T T555555556I  
662RRRRT TTTTTTTJ6I  
462RRRRRTT TTTTTTTT66  
66RRRRARRTT TTTTTTTTJ6  
462RRRRRRRTT TTTTTTTT76  
66RRRRRRARRRT TTTTTTTT76  
66RRRRRRRRRIITTTTTTTTTTTT6  
66RRRRRRRRRIITTTTTTTTTTT6  
66RRRRRRRRRIITTTTTTTTTTT6  
36RRRRRRRRRRRTTTTTTTTT86  
62RRRRRRRRRRRTTTTTTTTT462  
36IRRRRRRRRRRTTTTTTTTT462  
36IRRRRRRRRRRTTTTTTT62  
3655IRRRRRRRRTTTTTTT453  
355555555555555555

# A5. Forward transfer coefficient.

|      |       |       |      |       |       |      |       |       |       |      |      |      |       |       |       |      |      |       |       |       |       |       |       |       |       |
|------|-------|-------|------|-------|-------|------|-------|-------|-------|------|------|------|-------|-------|-------|------|------|-------|-------|-------|-------|-------|-------|-------|-------|
| 5810 | 12316 | 5072  | 2152 | 5548  | 7505  | 2124 | 7810  | 4630  | 1647  | 1504 | 3200 | 1589 | 2921  | 975   | 1492  | 1220 | 3143 | 2074  | 2331  | 1129  | 786   | 2248  | 4218  | 15668 | 912   |
| 4435 | 16401 | 8262  | 2810 | 4126  | 4638  | 1869 | 11912 | 8538  | 2121  | 1168 | 1956 | 1147 | 2777  | 1272  | 1631  | 919  | 1949 | 1451  | 1929  | 1117  | 658   | 1612  | 2697  | 11876 | 688   |
| 1972 | 9899  | 17351 | 6118 | 5295  | 7092  | 851  | 5918  | 10597 | 4811  | 1656 | 1289 | 574  | 2032  | 2104  | 3123  | 1106 | 1213 | 936   | 1377  | 1347  | 844   | 1684  | 1787  | 8625  | 598   |
| 4658 | 1456  | 11546 | 9706 | 7093  | 1829  | 576  | 4439  | 11172 | 7099  | 3045 | 1753 | 460  | 1290  | 1696  | 4647  | 1960 | 1532 | 733   | 1017  | 1277  | 1304  | 2796  | 2268  | 10188 | 724   |
| 1477 | 2464  | 2377  | 2995 | 12709 | 8104  | 714  | 2195  | 2904  | 3789  | 5598 | 6097 | 898  | 1284  | 863   | 4340  | 4707 | 4943 | 1629  | 1493  | 1574  | 2570  | 7523  | 6775  | 8864  | 1726  |
| 2138 | 3119  | 1956  | 1488 | 10285 | 10007 | 1171 | 3075  | 2405  | 2145  | 3885 | 7404 | 1470 | 1974  | 786   | 2808  | 3553 | 6237 | 2506  | 2213  | 1457  | 1899  | 6159  | 8157  | 9453  | 1705  |
| 3229 | 10367 | 3219  | 1230 | 2808  | 4025  | 3768 | 17376 | 4098  | 1236  | 918  | 2573 | 2699 | 7579  | 1959  | 1241  | 866  | 3121 | 3355  | 4913  | 2077  | 766   | 1849  | 4558  | 6176  | 1249  |
| 2468 | 11797 | 5074  | 1638 | 1951  | 2294  | 2404 | 26542 | 12214 | 1757  | 690  | 1320 | 1446 | 6454  | 2875  | 1509  | 575  | 1584 | 1949  | 3662  | 2198  | 634   | 1182  | 2446  | 4491  | 908   |
| 453  | 217   | 6502  | 4100 | 2042  | 1319  | 680  | 10157 | 24788 | 6708  | 1150 | 730  | 555  | 4597  | 7322  | 4966  | 770  | 817  | 1071  | 2996  | 3913  | 1140  | 1322  | 1441  | 2981  | 944   |
| 443  | 1447  | 3051  | 4411 | 7117  | 1464  | 216  | 1958  | 11756 | 20350 | 3599 | 962  | 216  | 989   | 5161  | 21426 | 2584 | 916  | 546   | 890   | 3261  | 2961  | 3144  | 1584  | 2615  | 791   |
| 872  | 1885  | 2733  | 3988 | 8361  | 4069  | 376  | 1655  | 2216  | 7364  | 7811 | 3096 | 452  | 841   | 1386  | 10852 | 6626 | 2906 | 912   | 998   | 2375  | 4569  | 9246  | 4880  | 5646  | 1834  |
| 1609 | 2413  | 1555  | 1576 | 8154  | 7703  | 961  | 2545  | 2020  | 1930  | 4148 | 8655 | 1375 | 1785  | 716   | 2396  | 4511 | 8385 | 2643  | 2208  | 1608  | 2317  | 8604  | 11002 | 6598  | 1188  |
| 2654 | 3819  | 1378  | 724  | 2473  | 3501  | 2563 | 8831  | 3124  | 855   | 954  | 3080 | 4445 | 10139 | 1915  | 1107  | 1027 | 5456 | 8777  | 10806 | 3204  | 1056  | 2634  | 9539  | 3956  | 1508  |
| 1025 | 2808  | 1388  | 678  | 1094  | 1386  | 1594 | 12835 | 6928  | 1188  | 463  | 1100 | 1135 | 17307 | 6245  | 1633  | 471  | 1905 | 5097  | 13548 | 7306  | 1410  | 1626  | 4071  | 1886  | 1845  |
| 266  | 954   | 1093  | 886  | 684   | 481   | 305  | 1817  | 8655  | 3996  | 558  | 332  | 372  | 4586  | 23438 | 11684 | 586  | 519  | 1263  | 5326  | 19419 | 4199  | 1943  | 1396  | 796   | 1398  |
| 137  | 520   | 858   | 1153 | 1376  | 689   | 111  | 943   | 3451  | 6705  | 1747 | 545  | 133  | 806   | 8108  | 36925 | 2343 | 598  | 370   | 1023  | 12426 | 10505 | 4478  | 1326  | 1010  | 1243  |
| 575  | 1174  | 1577  | 2243 | 5233  | 2776  | 272  | 1137  | 2808  | 5684  | 5948 | 2406 | 351  | 676   | 1474  | 16054 | 8200 | 2536 | 798   | 959   | 3827  | 8611  | 13534 | 5062  | 3454  | 1574  |
| 1814 | 2711  | 1350  | 1110 | 5768  | 4186  | 1303 | 3342  | 1890  | 1335  | 2640 | 6842 | 2239 | 2759  | 790   | 2028  | 3254 | 9942 | 4702  | 3639  | 1744  | 1857  | 7504  | 15461 | 5623  | 1460  |
| 1032 | 1966  | 801   | 454  | 1475  | 1471  | 1310 | 5337  | 2193  | 636   | 626  | 1887 | 2997 | 8379  | 2368  | 985   | 730  | 4331 | 13190 | 18127 | 6313  | 1598  | 2799  | 10821 | 2140  | 1501  |
| 608  | 1346  | 646   | 364  | 864   | 1059  | 819  | 4774  | 2840  | 671   | 394  | 974  | 1701 | 8507  | 4112  | 1243  | 491  | 2200 | 9370  | 12813 | 12491 | 2721  | 2961  | 6839  | 1237  | 1881  |
| 151  | 431   | 392   | 324  | 458   | 343   | 174  | 1670  | 2544  | 1172  | 396  | 302  | 263  | 2597  | 7376  | 4847  | 628  | 503  | 1262  | 5416  | 42101 | 14674 | 4602  | 1854  | 455   | 1047  |
| 104  | 241   | 242   | 265  | 585   | 379   | 87   | 629   | 1011  | 810   | 571  | 369  | 135  | 898   | 2555  | 4507  | 1081 | 545  | 663   | 2277  | 31795 | 32566 | 9424  | 2084  | 442   | 5716  |
| 297  | 546   | 561   | 720  | 2240  | 1428  | 171  | 680   | 1019  | 1465  | 2100 | 1496 | 259  | 644   | 980   | 5551  | 4020 | 2083 | 947   | 1752  | 8936  | 17021 | 27482 | 6952  | 1519  | 9083  |
| 1263 | 1924  | 1194  | 1193 | 5576  | 5295  | 829  | 2202  | 1641  | 1580  | 3444 | 6469 | 1359 | 1711  | 668   | 2812  | 4969 | 9197 | 2994  | 2377  | 1904  | 2903  | 12896 | 15697 | 4677  | 1354  |
| 4630 | 9258  | 6110  | 3408 | 8188  | 8642  | 1135 | 4700  | 4484  | 2313  | 1753 | 2507 | 886  | 1656  | 818   | 1927  | 1331 | 2170 | 1225  | 1369  | 883   | 825   | 2180  | 2942  | 23904 | 697   |
| 177  | 406   | 290   | 243  | 573   | 465   | 199  | 1351  | 1360  | 586   | 461  | 464  | 369  | 2322  | 2839  | 1975  | 860  | 908  | 2488  | 8537  | 26329 | 13874 | 10466 | 4705  | 531   | 11202 |

Transfer coefficient obtained by using relative error terminating criterion.

|      |       |       |      |       |      |      |       |       |       |      |      |      |       |       |       |      |      |       |       |       |       |       |       |       |       |
|------|-------|-------|------|-------|------|------|-------|-------|-------|------|------|------|-------|-------|-------|------|------|-------|-------|-------|-------|-------|-------|-------|-------|
| 4829 | 12327 | 5060  | 2149 | 5575  | 7499 | 2104 | 7830  | 4623  | 1642  | 1415 | 3213 | 1567 | 2930  | 962   | 1508  | 1220 | 3159 | 2060  | 2330  | 1121  | 790   | 2274  | 4227  | 15651 | 898   |
| 4444 | 16407 | 8257  | 2809 | 4138  | 4636 | 1860 | 11921 | 8534  | 2119  | 1172 | 1963 | 1137 | 2782  | 1265  | 1639  | 918  | 1957 | 1444  | 1928  | 1113  | 658   | 1624  | 2701  | 11868 | 680   |
| 1991 | 9813  | 17348 | 6118 | 4302  | 3001 | 858  | 8943  | 16594 | 4811  | 1658 | 1293 | 569  | 2035  | 2101  | 3126  | 1106 | 1218 | 933   | 1377  | 1347  | 844   | 1690  | 1790  | 8620  | 594   |
| 1665 | 6160  | 11541 | 9706 | 7101  | 1827 | 569  | 4446  | 11169 | 7097  | 3047 | 1757 | 553  | 1293  | 1691  | 4652  | 1960 | 1538 | 729   | 1018  | 1275  | 1305  | 2802  | 2272  | 10183 | 719   |
| 1508 | 2487  | 2356  | 2901 | 12351 | 8295 | 682  | 2226  | 2891  | 3381  | 5608 | 6116 | 864  | 1000  | 823   | 4364  | 4709 | 4969 | 1607  | 1492  | 1561  | 2572  | 7562  | 6788  | 8840  | 1702  |
| 2190 | 3157  | 1919  | 1882 | 10355 | 9988 | 1118 | 3129  | 2382  | 2129  | 3910 | 7438 | 1413 | 2002  | 751   | 2849  | 3548 | 6281 | 2471  | 2215  | 1437  | 1905  | 6225  | 8181  | 9406  | 1663  |
| 3932 | 10375 | 3212  | 1228 | 2825  | 4021 | 3756 | 17389 | 6093  | 1233  | 924  | 2582 | 2686 | 7584  | 1952  | 1252  | 865  | 3131 | 3347  | 4912  | 2073  | 768   | 1865  | 4562  | 6165  | 1239  |
| 2474 | 11711 | 5021  | 1637 | 1960  | 2296 | 2399 | 24548 | 12231 | 1757  | 693  | 1324 | 1440 | 6457  | 2871  | 1513  | 574  | 1588 | 1945  | 3661  | 2196  | 654   | 1189  | 2448  | 4486  | 903   |
| 850  | 4219  | 6500  | 4100 | 2045  | 1319 | 678  | 10159 | 26787 | 6607  | 1151 | 731  | 553  | 4599  | 7320  | 4968  | 770  | 819  | 1069  | 2997  | 3912  | 1141  | 1325  | 1443  | 2978  | 143   |
| 446  | 1449  | 3049  | 4410 | 7340  | 1463 | 214  | 1960  | 11755 | 20349 | 3600 | 964  | 213  | 990   | 5159  | 21428 | 2584 | 918  | 444   | 890   | 3259  | 2962  | 3147  | 1585  | 2612  | 789   |
| 881  | 1891  | 2727  | 3987 | 8374  | 4067 | 368  | 1664  | 4210  | 7361  | 7815 | 3102 | 443  | 846   | 1380  | 10859 | 6626 | 2914 | 906   | 999   | 2370  | 4571  | 9257  | 4885  | 5639  | 1827  |
| 1639 | 2433  | 1536  | 1571 | 8195  | 7692 | 931  | 2575  | 2006  | 1923  | 4159 | 8473 | 1343 | 1800  | 696   | 2920  | 4510 | 8411 | 2622  | 2208  | 1596  | 2321  | 8644  | 11016 | 6571  | 2163  |
| 2067 | 3848  | 1370  | 722  | 2491  | 3496 | 2549 | 8845  | 3119  | 852   | 959  | 3088 | 4430 | 10145 | 1906  | 1118  | 1026 | 5467 | 8768  | 10805 | 3198  | 1058  | 2651  | 9545  | 3945  | 2198  |
| 1029 | 2811  | 1386  | 678  | 1100  | 1354 | 1592 | 12839 | 6926  | 1187  | 465  | 1102 | 2131 | 17310 | 6242  | 1635  | 470  | 1909 | 5095  | 13548 | 7304  | 1409  | 1631  | 4074  | 1882  | 2842  |
| 267  | 955   | 1092  | 886  | 685   | 481  | 304  | 3818  | 8655  | 3996  | 558  | 353  | 371  | 4587  | 23438 | 11684 | 586  | 520  | 1262  | 5336  | 19418 | 4199  | 1944  | 1397  | 795   | 2197  |
| 188  | 521   | 857   | 1153 | 1378  | 688  | 110  | 944   | 3850  | 6705  | 1748 | 546  | 131  | 807   | 8107  | 36925 | 2343 | 599  | 369   | 1023  | 12425 | 10505 | 4479  | 1327  | 1008  | 1242  |
| 580  | 1178  | 1574  | 2262 | 5241  | 2774 | 266  | 1143  | 2805  | 5682  | 5951 | 2410 | 344  | 679   | 1470  | 16059 | 8200 | 2541 | 794   | 959   | 3824  | 8612  | 13541 | 5066  | 3448  | 2569  |
| 1846 | 2733  | 1328  | 1106 | 5413  | 6177 | 1271 | 3374  | 1877  | 1325  | 2653 | 6864 | 2205 | 2776  | 768   | 2053  | 3253 | 9969 | 4680  | 3639  | 1732  | 1860  | 7545  | 15474 | 5596  | 2134  |
| 1037 | 1970  | 798   | 453  | 1483  | 1968 | 1304 | 5343  | 2390  | 634   | 628  | 1890 | 2991 | 8382  | 2364  | 990   | 730  | 4337 | 13186 | 18126 | 6309  | 1598  | 2806  | 10824 | 2134  | 5197  |
| 611  | 1348  | 643   | 363  | 848   | 1058 | 836  | 4777  | 2888  | 670   | 395  | 976  | 1698 | 8508  | 4110  | 1245  | 491  | 2203 | 9368  | 12813 | 12489 | 2721  | 2964  | 6841  | 1235  | 7879  |
| 152  | 432   | 392   | 324  | 459   | 342  | 174  | 1671  | 2543  | 1172  | 397  | 303  | 263  | 2598  | 7376  | 4848  | 628  | 503  | 1262  | 5416  | 42101 | 14674 | 4603  | 1854  | 454   | 5047  |
| 105  | 242   | 242   | 265  | 586   | 378  | 86   | 630   | 1011  | 829   | 571  | 369  | 135  | 898   | 2555  | 4507  | 1061 | 546  | 663   | 2277  | 31795 | 32567 | 9425  | 2084  | 441   | 5716  |
| 306  | 548   | 559   | 719  | 2243  | 1428 | 168  | 683   | 1038  | 1464  | 2101 | 1496 | 256  | 645   | 978   | 5553  | 4020 | 2085 | 945   | 1752  | 8935  | 17021 | 27486 | 6954  | 1517  | 9081  |
| 1282 | 1938  | 1180  | 1190 | 5602  | 5289 | 809  | 2222  | 1635  | 1575  | 3453 | 6482 | 1338 | 1720  | 654   | 2828  | 4969 | 9213 | 2979  | 2378  | 1895  | 2905  | 12970 | 15705 | 4660  | 3138  |
| 4644 | 9267  | 6101  | 3407 | 8205  | 8639 | 1122 | 4714  | 4478  | 2308  | 1761 | 2516 | 872  | 1663  | 809   | 1938  | 1329 | 2182 | 1216  | 1371  | 877   | 826   | 2196  | 2948  | 23893 | 687   |
| 178  | 407   | 289   | 243  | 575   | 465  | 198  | 1352  | 1360  | 585   | 462  | 464  | 368  | 2323  | 2838  | 1975  | 860  | 908  | 2488  | 8537  | 26329 | 13874 | 10467 | 4706  | 530   | 17022 |

A6. To show,

$$(X - X') = A^{-1} - (A^T A + kI)^{-1} A^T T_Y, \quad (A6.1)$$

Given:

$$X = A^{-1} Y \quad (A6.2)$$

$$X' = (A^T A + kI)^{-1} A^T T_Y, \quad (A6.3)$$

Proof:

assuming (A6.1) is correct, expanding R.H.S. of (A6.1),

$$\begin{aligned} \text{R.H.S.} &= (X - X_c) + (A^T A + kI)^{-1} A^T T_Y - (A^T A + kI)^{-1} A^T T_Y \\ &\quad + (A^T A + kI)^{-1} A^T A (X_c - X) \\ &\quad + k(A^T A + kI)^{-1} X_c \\ &= (X - X_c) + (A^T A + kI)^{-1} A^T T_Y - (A^T A + kI)^{-1} A^T T_Y \\ &\quad + (A^T A + kI)^{-1} A^T A X_c - (A^T A + kI)^{-1} A^T A X \\ &\quad + k(A^T A + kI)^{-1} X_c \\ &= (X - X_c) - (A^T A + kI)^{-1} A^T T_Y \\ &\quad + [(A^T A + kI)^{-1} A^T A + k(A^T A + kI)^{-1}] X_c \\ &= (X - X_c) - (A^T A + kI)^{-1} A^T T_Y \end{aligned}$$

$$+ [(A^T A + kI)^{-1}(A^T A + kI)] X_c$$

$$= X - (A^T A + kI)^{-1} A^T Y'$$

from (A6.3),

$$= (X - X')$$

$$= \text{L.H.S. of (A6.1)}$$

A7. Numerical examples.

Solving  $AX=Y$  using direct and optimum inverse method.

$$A = \begin{pmatrix} 0.025 & 0.020 \\ 0.020 & 0.025 \end{pmatrix}$$

condition number of  $A = 9$

$$Y = \begin{pmatrix} 0.045 \\ 0.045 \end{pmatrix}$$

$$X = \begin{pmatrix} 1.0 \\ 1.0 \end{pmatrix}$$

contaminated with + 22% error  $E = \begin{pmatrix} 0.01 \\ -0.01 \end{pmatrix}$

|                 | <u>solution X-vector</u>                       | <u>% error</u> |
|-----------------|------------------------------------------------|----------------|
| direct inverse  | $\begin{pmatrix} 3.0 \\ -1.0 \end{pmatrix}$    | +200<br>-200   |
| optimal inverse | $\begin{pmatrix} 0.322 \\ 0.300 \end{pmatrix}$ | -67<br>-70     |

Two more examples:

$$A = \begin{bmatrix} 1.00 & 1.01 & 0.99 \\ 1.01 & 1.00 & 0.99 \\ 0.99 & 0.99 & 1.00 \end{bmatrix}$$

condition number of A = 300

$$Y = \begin{bmatrix} 3.0 \\ 3.0 \\ 2.98 \end{bmatrix}$$

$$X = \begin{bmatrix} 1.0 \\ 1.0 \\ 1.0 \end{bmatrix}$$

contaminated with 10% noise

|                 | <u>solution X-vector</u>                                     | <u>% error</u>             |
|-----------------|--------------------------------------------------------------|----------------------------|
| direct inverse  | $\begin{bmatrix} -22.956 \\ 36.844 \\ -11.068 \end{bmatrix}$ | <br>-2400<br>3584<br>-1200 |
| optimal inverse | $\begin{bmatrix} 0.851 \\ 1.364 \\ 0.680 \end{bmatrix}$      | <br>-15<br>36<br>-32       |

$$A = \begin{bmatrix} 1.00 & 0.99 & 1.01 & 1.01 \\ 1.01 & 0.99 & 1.00 & 1.00 \\ 1.00 & 1.00 & 0.99 & 1.00 \\ 1.01 & 1.00 & 0.99 & 0.99 \end{bmatrix}$$

condition number of A = 796

$$Y = \begin{bmatrix} 4.01 \\ 4.00 \\ 3.98 \\ 3.99 \end{bmatrix}$$

$$X = \begin{bmatrix} 1.0 \\ 1.0 \\ 1.0 \\ 1.0 \end{bmatrix}$$

contaminated with 10% noise

|                 | <u>solution X-vector</u> | <u>% error</u> |
|-----------------|--------------------------|----------------|
| direct inverse  | -78.5                    | -7950          |
|                 | 81.3                     | 8000           |
|                 | 80.5                     | 7950           |
|                 | -78.1                    | 7910           |
| optimal inverse | 0.61                     | -39            |
|                 | 1.39                     | 39             |
|                 | 1.40                     | 40             |
|                 | 0.61                     | -39            |

A8. Residue norm ( $r^2$ ) of  $AX = Y$

Given

$$AX = Y \quad (A8.1)$$

and

$$AX - Y = r$$

where  $r$  is the residue.

Using SVD for matrix  $A$ ,

$$A = USV^T \quad (A8.2)$$

where  $T$  is the transpose.

Substituting  $A$  in (A8.1),

$$\begin{aligned} USV^T X &= Y \\ SV^T X &= U^T Y \end{aligned} \quad (A8.3)$$

where

$$S = \begin{pmatrix} S_k & 0 \\ 0 & 0 \end{pmatrix}$$

and  $k$  is the rank of  $A$ .

Let

$$V^T X = P$$

$$U^T Y = G$$

Substituting (A8.3),

$$\begin{bmatrix} s_k & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}$$

Then, the residue norm is given as,

$$r^2 = \left\| \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} - \begin{bmatrix} s_k & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \right\|^2$$

$$= \left\| (g_1 - s_k p_1) + (g_2) \right\|^2$$

and  $r^2$  has a minimum value of  $\|g_2\|^2$  when  $g_1 = s_k p_1$

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