

NEUTRAL STRANGE PARTICLES
PRODUCED IN π^-p INTERACTIONS

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ABSTRACT

At incident beam momenta of 4.1 and 6.2 GeV/c total and differential cross-sections and Λ polarisation have been measured in the CERN magnet spark chamber for the baryon exchange channel of the reaction $\pi^- p \rightarrow \Lambda K^0$. The polarisation (0 at $u = u_0$) quickly rises to a value of +1 then appears to fall again to zero at $u \sim 0.7$ (GeV/c)². At this same value of u there is a change of slope in the differential cross-section. Both phenomena can be explained in the Regge pole model by the vanishing of the amplitude for the exchange of the ζ_y trajectory at the wrong signature nonsense point $\alpha = -3/2$.

The differential cross-section for the reaction $\pi^- p \rightarrow K^0 Y^*$ (1520) is presented at 4.1, 5.1, 6.2 and 7.2 GeV/c for the range $t = t_0$ to $t \sim -1.8$ (GeV/c)². It is well fitted by a function of the form $d\sigma/dt \propto \exp(at + bt^2)$ at all energies, and exhibits shrinkage with increasing energy.

PREFACE

Because of the very nature of high energy physics large resources of money and people are necessary for any experiment. The experiment described here was carried out by members of the CERN-ETH-Imperial College collaboration at the CERN proton synchrotron and is therefore the result of the combined efforts of that group of people.

In this thesis I have tried to emphasise those parts of the experiment for which I was particularly responsible - the Monte Carlo programme, the supervision of the events through the data reduction procedure and the analysis of the reactions $\pi^- p \rightarrow \Lambda K^0$ and $\pi^- p \rightarrow K^0 Y^*(1520)$. However, many parts of the first four chapters are not an account of my own work - they are included because they are pertinent to the rest of the thesis.

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CHAPTER 1

Some Experimental and Theoretical Background

1.1 Introduction

This thesis is a description of an experiment carried out at the European Centre for Nuclear Research (CERN).

Two reactions are studied:

- a) $\pi^- + p \rightarrow \Lambda + K^0$ backward scattering at 4.1 and 6.2 GeV/c beam momentum (sometimes abbreviated as 4 and 6 GeV/c); the differential cross-section and Λ polarisation are presented.
- b) $\pi^- + p \rightarrow K^0 + Y^*(1520)$ at 4.1, 5.1, 6.2, and 7.2 GeV/c.

In the light of present knowledge these reactions are best understood in terms of the Regge pole model and most of this chapter is devoted to an introduction to the basic concepts of Regge pole theory. Before that, however, a short section attempts to show the type of experiment for which the various instruments employed in high energy physics research are best suited.

1.2 Instruments used in High Energy Physics

At present there are three major techniques, bubble chambers, spark chambers and scintillation counters; the last two are very often used together.

The most useful attributes of a bubble chamber are its high spatial and momentum resolution, 4π geometry, and its ability to detect events with a complicated topology. However, it is a device which cannot be triggered and in order to collect a useful number of events in a channel having a low production cross-section, many photographs must be taken and selection made of the events to be measured by

scanning each frame - a costly and time-consuming procedure. Typically, an experiment with 3×10^5 photographs produced 10 examples of a channel with cross-section $1\mu\text{b}$.

In an experiment using counters only, selection is made electronically. Ideally, one would be able to select only those events in the channel being studied, and although this is never possible in practice, greatly enriched samples can be obtained at the expense of increased normalisation difficulties. The pure counter method is most suited to channels with very simple topology and total cross-section measurements.

The dead time of the spark chamber is very much greater than that of the scintillation counter and the addition of spark chambers into an array of counters reduces the rate at which events can be collected. However, a spark chamber has a far superior spatial resolution than a scintillation counter array and a combination of the two exploits this while retaining the possibility of triggering the apparatus. Typical experiments with a spark chamber and counter system can study in some detail reactions with cross-section $\leq 1 \mu\text{barn}$.

The spark chamber has evolved considerably since its introduction in 1959¹⁾; gap sizes now vary from a few mm to 50 cm and a number of data recording systems have been developed including optical, vidicon and sonic systems; wire planes are now frequently used so that spark positions can be digitized "on-line", and wide-gap spark chambers are often operated in the "streamer mode"^{2,3)}.

By far the simplest spark chamber to construct and operate is the narrow gap (~ 1 cm) "sampling" type, usually used in conjunction with bending magnets for momentum analysis. Systems of this type are cheap, easy to operate and are usually designed specifically for a given experiment. A good summary of the characteristics of narrow-gap chambers has been given by Cronin⁴⁾.

In a reaction in which a few (≤ 5) high energy secondary particles

are produced in the forward direction, momentum analysis can be more easily carried out by placing spark chambers inside a magnetic field. In such an experiment the secondaries will remain nearly normal to the spark chamber plates over a considerable distance. Most applications of such an instrument (magnet spark chamber) have been for the detection of decays or interactions of neutral particles e.g. K^0 , Λ decays, γ -ray conversions. The essential part of the trigger for such an experiment is an anticoincidence shield around the target.

The CERN magnet spark chamber system, used in this experiment, has been a particularly successful instrument and results have been published on K^-p charge exchange^{5,6}, $\bar{p}p$ charge exchange^{7,8}, C invariance in the three pion decay of the η ⁹ and neutral final states in π^-p interactions^{10,11,12}). It is described in some detail in chapter 2.

1.3 The Theoretical Situation

At present there is no universal theory for the strong interaction in analogy with the quantum electrodynamical theory of the electromagnetic interaction. However, much work has been done on the development of principles and models which hopefully will lead to a theory.

Two developments are of particular interest:-

a) internal symmetry schemes such as $SU(3)$ and the quark model which relate particles with the same spin and different internal quantum numbers.

b) the Regge pole model which relates particles with the same internal quantum numbers but different spins.

Together these two models provide a powerful tool in the classification of particles and are discussed in the succeeding sections.

1.4 The Regge Pole Model

i) Some Two-body Kinematics

The two-body reaction $1 + 2 \rightarrow 3 + 4$ depends upon two independ-

ent kinematic variables; for convenience two of the relativistic invariants (Mandelstam variables) s, t and u are usually taken.

Consider the interaction as four incoming particles with four-momenta P_1, P_2, P_3, P_4 (see diagram).

Conservation of four-momentum gives

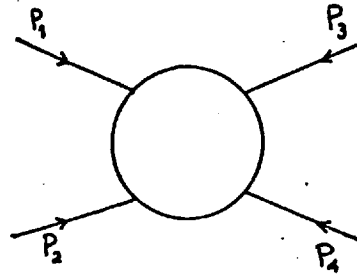
$$P_1 + P_2 + P_3 + P_4 = 0$$

and the variables s, t and u are defined as

$$s = (P_1 + P_2)^2 = (P_3 + P_4)^2$$

$$t = (P_1 + P_3)^2 = (P_2 + P_4)^2$$

$$u = (P_1 + P_4)^2 = (P_2 + P_3)^2$$



Only two of these are independent and in fact

$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$$

If we make use of the particle-antiparticle correspondence principle, i.e. annihilation of particle 2 with momentum p_2 is equivalent to the creation of antiparticle $\bar{2}$ with momentum $-p_2$ then the symmetric diagram can describe three distinct channels, each being named from the variable which is the squared centre of mass energy in that channel.

a) s- channel (direct channel)

$$1 + 2 \rightarrow 3 + 4 \quad \text{eg. } \pi^- p \rightarrow \pi^- p$$

b) t- channel (crossed channel)

$$1 + \bar{3} \rightarrow \bar{2} + 4 \quad \text{eg. } \pi^- \pi^+ \rightarrow \bar{p} p$$

c) u- channel (crossed channel)

$$1 + \bar{4} \rightarrow \bar{2} + 3 \quad \text{eg. } \pi^- \bar{p} \rightarrow \pi^- \bar{p} \\ (= \pi^+ p \rightarrow \pi^+ p)$$

The physical region for each channel is defined by the two conditions:- squared centre of mass energy $\geq (m_i + m_j)^2$ where i and j are the final state particles.

$$\text{and } -1 \leq \cos \theta \leq 1 \quad (\theta = \text{scattering angle})$$

Since $s + t + u = \text{constant}$, we can plot the variables on a triangular diagram (fig 1.1). A given variable has a different meaning in the three physical regions (the hatched areas of fig 1.1); eg. s is a squared energy in region I and is connected to a scattering angle in regions II and III.

The three physical regions have no common point.

ii) Crossing Symmetry

The three physical regions correspond to three different processes each described by a different amplitude $A_i(s, t)$. However, if we extend the amplitudes by the mathematical procedure of analytic continuation to unphysical values of energy and scattering angle we can compare the three amplitudes at any point. The hypothesis of crossing symmetry states that if this is done the three channels are described by one analytic function $A(s, t)$ which represents the amplitudes A_s , A_t and A_u in regions I, II and III respectively.

As a result we conclude that what happens physically in one channel eg. $\pi^- p \rightarrow \pi^- p$ influences the behaviour of the amplitude in what may appear to be a completely unrelated channel eg. $\pi^+ \pi^- \rightarrow p \bar{p}$.

Since s , t and u have been allowed to become complex in the process of analytic continuation it is not a great extension to allow the angular momentum l also to become continuous and complex with the physical regions having the further requirement, $\text{Im} l = 0$, $\text{Re} l = \text{integer}$.

Poles which occur in the amplitude $A(l, s, t)$ are called Regge poles.

iii) Regge Poles in High Energy Interactions

It has not yet been possible to write down the amplitude for a high energy hadronic interaction and the usual practice is to derive certain predictions of the Regge model from potential theory, where a Schrödinger equation can be written down with a potential assumed to be

a sum of Yukawa potentials. The assumption is then made that these predictions are still valid for high energy interactions. The result is a phenomenological model suitable for data fitting.

One important result from the application of the Regge model to potential theory is the prediction of the existence of resonances. If we consider a partial wave of energy E and angular momentum l and allow E and l to become complex, the scattering amplitude has a two dimensional surface of singularities defined by the equation $l = \alpha(E)$ (two real equations). This surface never passes through the physical region (E real and positive); however, if for integer l a pole occurs at $E = E_l - i\Gamma_l/2$ then this pole does affect the amplitude in the physical region and in fact gives rise to a resonance with a mass equal to E_l and width Γ_l .

These resonances are usually displayed on a Chew-Frautschi plot - that section of the surface $l = \alpha(E)$ with $\text{Im}E=0$ is projected onto the plane ($\text{Re}E, \text{Re}l$). The projection is called a Regge trajectory.

Resonances occur on the trajectory at integer l (for bosonic poles); however, because of the presence of exchange forces, the scattering amplitude has to be split into two separate amplitudes for odd or even l giving rise to separate trajectories said to have different signature (signature $\xi = +1$ for even l , $\xi = -1$ for odd l). As a result, on a given trajectory resonances are separated by $\Delta l = 2$. If exchange forces are small the two trajectories lie on top of one another and are said to be exchange degenerate.

By extending the results from potential theory to high energy interactions, it can be shown that the amplitude in the s channel $A(s,t)$ at asymptotic energies is given by

$$A(s,t) = \sum_i \beta_i(t) \zeta(t) \cdot \left(\frac{s}{s_0}\right)^{\alpha_i(t)}$$

where the summation is taken over all Regge trajectories $\alpha_i(t)$ of the t -channel amplitude and $\zeta(t)$ (called the signature factor) is given by

$$\zeta(t) = \frac{1 + \xi e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}$$

$\beta(t)$ is called the residue function and is in general unknown. Each theoretician has his own recipe for writing down $\beta(t)$ when doing a fit to experimental data.

In spite of the difficulties caused by the unknown factor $\beta(t)$, a number of predictions can be obtained from the Regge pole model.

a) The "exchanged" object in an interaction is a Regge trajectory having well-defined quantum numbers and a continuously varying spin $\alpha(t)$ giving rise to an amplitude proportional to $s^{\alpha(t)}$. This gets over the difficulty of the simple exchange model which predicts that when a particle of spin J is exchanged, the amplitude is proportional to s^J , in disagreement with experimental observation for $J > 1$.

b) Forward and backward peaks occur only when a trajectory can be exchanged; forward peaks arise from the exchange of trajectories carrying bosonic quantum numbers, and backward peaks from the exchange of trajectories carrying baryonic quantum numbers (the latter are poles of the u-channel amplitude).

c) If one linear trajectory dominates the amplitude (i.e. $\alpha(t) = \alpha(0) + \alpha'(0)t$)

$$d\sigma/dt \propto e^{at} s^{2\alpha(0)-2} \quad \text{where } a = 2\alpha'(0) \ln(s/s_0)$$

$$\text{Then i) } d\sigma/dt (t=0) \propto s^{2\alpha(0)-2}$$

ii) a shrinking of the peak should be observed for increasing s because of the s -dependence of e^{at} .

iv) Fermion Trajectories

Fermions have half-integer spin and for each angular momentum ℓ of the incident wave there are two partial wave amplitudes corresponding to $J = \ell \pm \frac{1}{2}$ thus giving rise to two series of poles. Furthermore, if exchange forces are non-zero there is splitting of the poles corresponding to even and odd ℓ (signature). As a result there are four

trajectories with lowest poles having $J^P = \frac{1}{2}^+, \frac{1}{2}^-, \frac{3}{2}^-, \frac{3}{2}^+$; these trajectories are labelled $\alpha, \beta, \gamma, \delta$ respectively.

So far most information on the shape of Regge trajectories has come from the nucleon resonances. Interpolation of the resonances indicates that the trajectories are roughly linear with slope $0.9 - 1.0$ $(\text{GeV}/c)^{-2}$ on the Chew-Frautschi plot^{13,14}). There appears to be approximate exchange degeneracy between the α and γ trajectories and between the β and δ trajectories¹⁵).

1.5 SU(3) and the Quark Model

The success of SU(3) and the quark model as a basis for classification of elementary particles has long been recognised. In this scheme, three quarks, labelled p, n and λ , are considered the basic states (or representation) of the unitary symmetry group SU(3). Each quark has a corresponding antiparticle (antiquark).

It is proposed that a baryon is constructed of three quarks (qqq) and a meson of a quark-antiquark pair ($q\bar{q}$). Each quark, therefore, has baryon number $B = \frac{1}{3}$. The p and n quarks form an isotopic spin doublet of zero strangeness ($I = \frac{1}{2}, S = 0$) and the λ quark is an isotopic spin singlet ($I = 0$) with strangeness $S = -1$. Using the definition of hypercharge $Y = S + B$ and the Gell-Mann-Nishijima relation $Q = I_z + \frac{1}{2}Y$, we arrive at the prediction that quarks have fractional charge (see table 1.1 for a summary of quark quantum numbers).

Many attempts have been made to observe quarks and a summary of these attempts has been made by Massam¹⁶). Until recently there had been no suggestions of the discovery of a quark although many upper limits for quark production and existence have been made. Recently however, McCusker claimed that he had discovered the tracks of five quarks in a cloud chamber experiment to look at the core of high energy cosmic ray showers¹⁷). The interpretation of his photographs has been subject to some criticism, but the experiment is bound to stimulate further work to

either confirm or reject his results.

i) The simple quark model

The addition of the constraint that the three quarks make up the triplet representation of $SU(3)$ and can only form qqq or $q\bar{q}$ states implies that the only allowed multiplets of $SU(3)$ are singlets, octets and decuplets. The observation, therefore, of one particle which does not fall into one of these multiplets would destroy the "simple quark model", but to date the existence of such a particle has never been confirmed.

The multiplets of $SU(3)$ group together particles with the same spin and parity which in the limit of exact symmetry would have the same mass also. The symmetry is obviously broken since, for example, the π and the K have different masses. Several multiplets have been completely filled to present.

Further information upon the multiplets of $SU(3)$ existing in nature comes from the presence or absence of peripheral peaks in high energy scattering. In the peripheral model the scattering amplitude is dominated by a limited number of peripheral exchanges (trajectories in the Regge pole model) eg. in meson-baryon scattering meson exchange produces a forward peaking of the differential cross-section and baryon exchange results in a backward peak about an order of magnitude lower (fig 1.2). These peaks are seen when a singlet, octet, or decuplet member can be exchanged but not otherwise.

For example:-

a) $\pi^+ p \rightarrow K^+ \Sigma^+$ has both a forward and a backward peak at high energies, the forward peak coming from K^* exchange, the backward peak from Λ or Σ exchange.

b) $K^- p \rightarrow \bar{K}^0 n$ has only a forward peak; a backward peak would be due to the exchange of an $S = +1$ baryon and is not observed (this may be taken as indirect evidence against the existence of Z^* 's.)

c) $\pi^- p \rightarrow \bar{\Sigma}^- K^+$ has no forward peak since no meson is found with $Q = +2$, $S = +1$, but a backward peak due to Λ or $\bar{\Sigma}$ exchange is observed.

ii) SU(3) and Regge residues

The lack of theoretical predictions for Regge residues is a weakness of the theory at present; however application of SU(3) symmetry results in relations between the residues which can be tested experimentally or used when fitting data from different reactions. These are generally known as "sum rules".

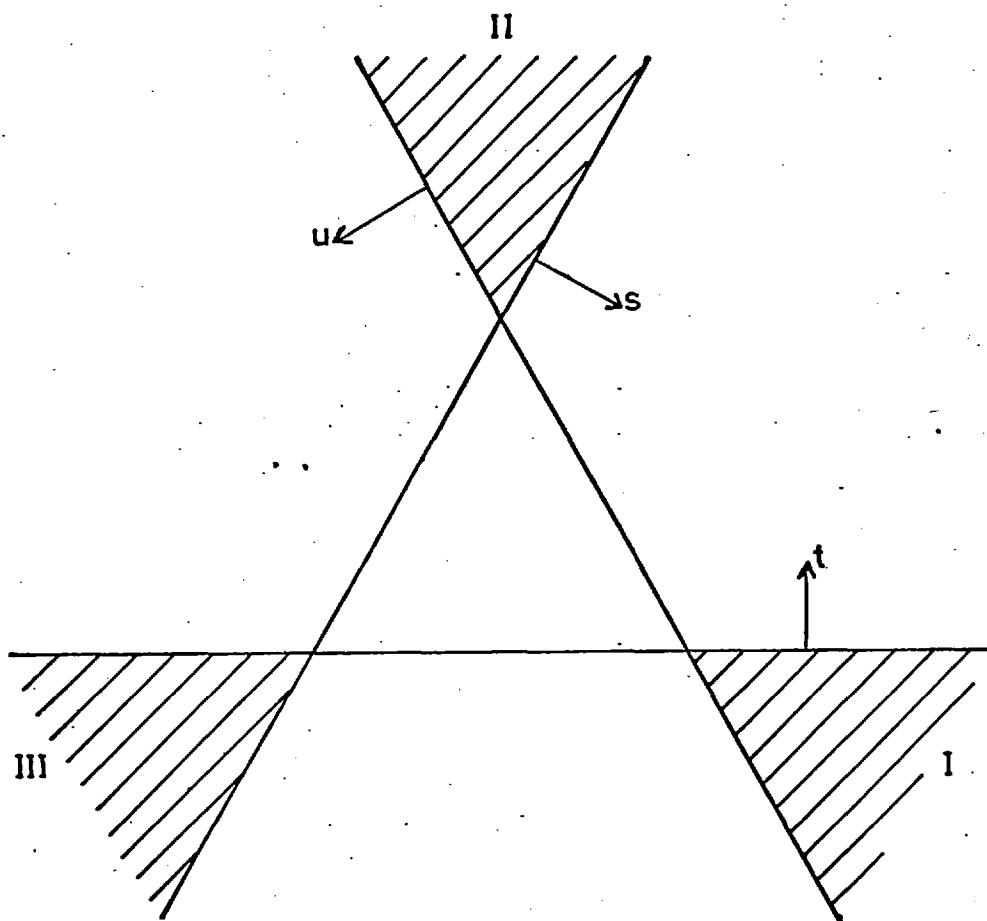
One such sum rule can be expressed in terms of total cross-section and shows remarkable agreement with experiment¹⁸⁾:-

$$\begin{aligned} [\sigma_{\tau}(K^+p) - \sigma_{\tau}(K^+n)] - [\sigma_{\tau}(K^-p) - \sigma_{\tau}(K^-n)] \\ = [\sigma_{\tau}(\pi^+p) - \sigma_{\tau}(\pi^+n)] \end{aligned}$$

TABLE 1.1Quark quantum numbers

	B	I	I_z	Y	S	Q
p	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{2}{3}e$
n	$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{3}$	0	$-\frac{1}{3}e$
λ	$\frac{1}{3}$	0	0	$-\frac{2}{3}$	-1	$-\frac{1}{3}e$

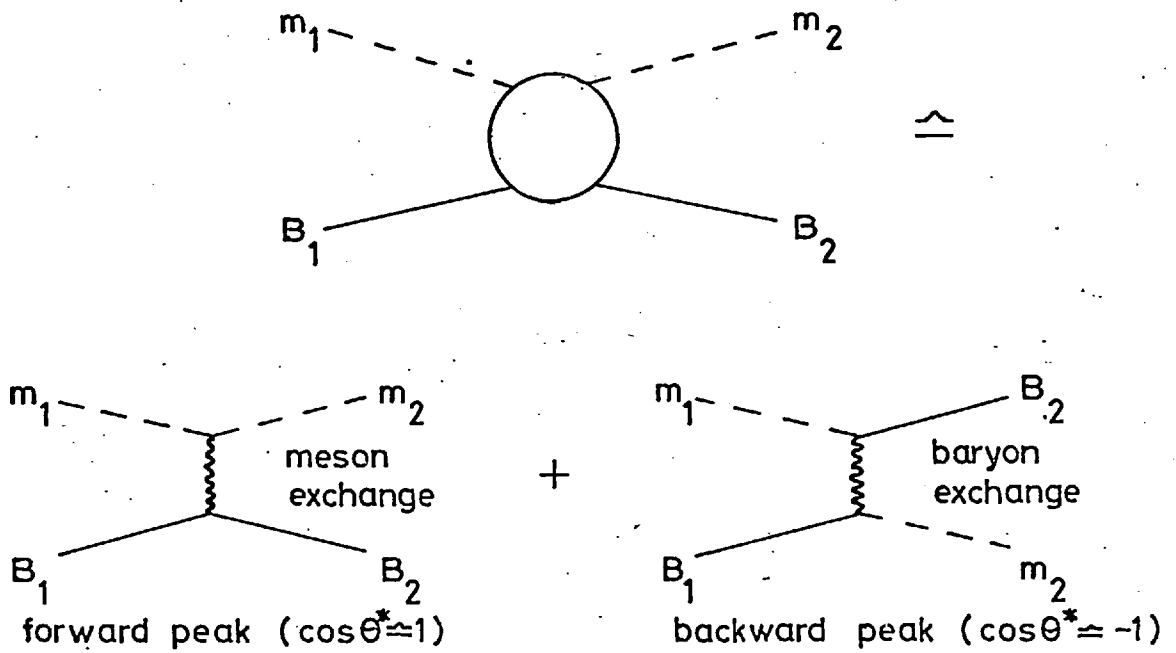
Figure 1.1 The s, t, u Diagram



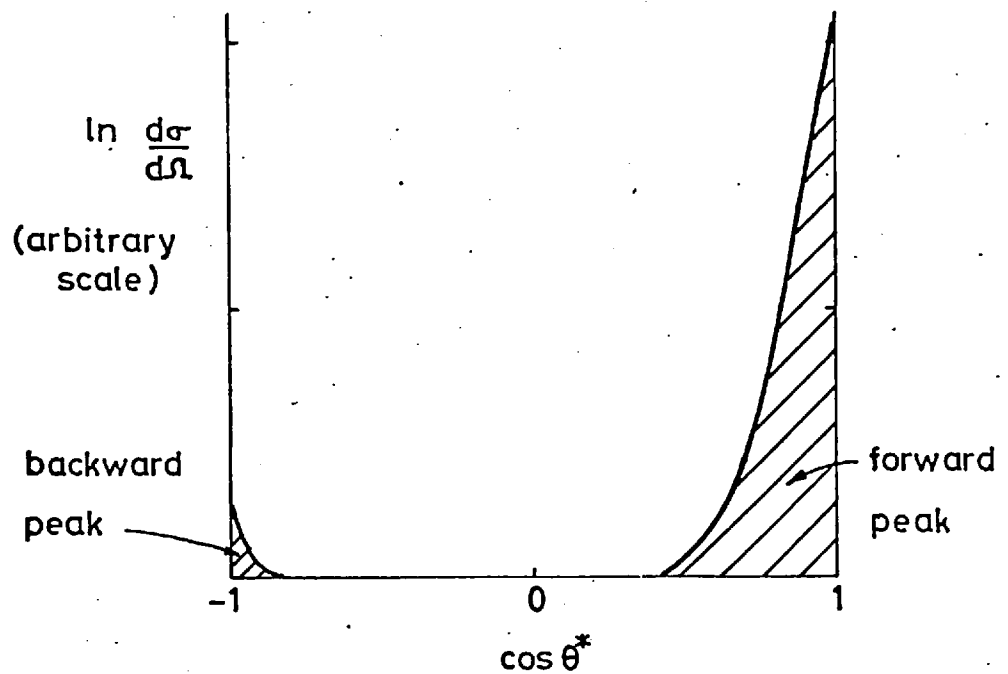
- | | | |
|-----|--------------|-----------------|
| I | s -channel | physical region |
| II | t -channel | " " |
| III | u -channel | " " |

Figure 1.2

a) Peripheral Exchanges at High Energy



b) Typical Angular Distribution



CHAPTER 2

The Apparatus

2.1 Beam and Beam Telescope

The beam of negative pions was produced from an internal target of the CERN proton synchrotron (P.S.); the beam transport and beam telescope counter layout are shown schematically in figs 2.1 and 2.2. The intermediate focus was used by an experimental team with a polarised target whose magnet bent the beam vertically. As a result, two extra bending magnets (C_1 and C_2) were needed to bring the beam back to the horizontal. The beam was focussed about 10 m downstream from the target to reduce the divergence to 3 mrad; its diameter at the target was 5 cms.

Six scintillation counters defined the useable portion of the beam; S_1 , S_2 , S_4 , S_5 were in coincidence and $A_{1,2}$ and A_4 were in anti-coincidence. The momentum slit was adjusted such that about 1.5×10^5 particles passed down the beam line per P.S. burst; of these only 0.8×10^5 passed through the telescope.

The momentum resolution of the beam produced by the momentum slit was 1% at 6 GeV/c and ~2% at 4 GeV/c. However, a counter hodoscope was designed to take advantage of the momentum dispersion at the slit. Four 3mm thick scintillators of 6 or 9mm width, overlapped as in fig 2.3, gave 7 channels of 3mm width. The beam passed through about 4 of these channels and the information from them gave a momentum resolution of 0.5% at 6 GeV/c and 1% at 4 GeV/c.

2.2 Target and Counters after the Target

The target was a hydrogen cell 40 cms long and 8 cms in diameter

containing about 140 gms of liquid hydrogen. Thin mylar windows at the beam entry and exit end resulted in background interactions on nuclei other than hydrogen of less than 10%.

Since the aim of the experiment was to study strange particle neutral final states an essential part of the trigger was an anticoincidence shield around the target. This shield had to have a good detection efficiency for γ -rays and a low scattering probability for neutral secondary particles. The counters used (R,F) produced a compromise solution to these conflicting requirements (fig 2.4), the detection efficiency for a γ -ray was estimated to be at least 95% and the neutral particle mean free path was about 30 cms. The counter R' was a replica of one of the R-counters and was used as a monitor to correct for neutral particle interactions. The correction is discussed in detail in Chapter 6.

The remaining counters E_1 , E_2 and the plane labelled G were in coincidence and the logic demanded that at least one charged particle crossed each plane i.e. that a neutral particle had decayed (or possibly interacted) in the region between F_3 and E. The single particle efficiency was plotted over the whole of each plane during the experiment. It was found to be high (98%) with less than 1% variation. Since in most decays, both secondaries passed through both planes, the correction due to the small inefficiency was ignored.

2.3 Electronics and Counter Logic

The electronic units used in the trigger logic were designed by Beusch and are described in reference 19. The trigger condition demanded that a π^- (or possibly a K^- or $\bar{p}$) interacted in the target, producing no charged secondaries, π^0 's or γ -rays. However, at least one neutral secondary was required to decay in a charged mode to trigger one of the E counters and one of the G counters. Symbolically the trigger was $T \cdot \bar{R}_1 \cdot \bar{F}_1 \cdot E \cdot G$ where T represents the telescope, $S_1 \cdot S_2 \cdot S_4 \cdot S_5$.

$\bar{A}_{1,2} \cdot \bar{A}_4$ and $E(G)$ represents at least one of the counters from that plane.

Fig 2.5 is a block diagram of the logic. The encircled letters on this diagram represent scalars counting the number of coincidences in the various channels. At the end of each roll of film the contents of the scalars were printed and punched onto paper tape; they allowed a running check of the functioning of the apparatus and were later used for the calculation of total cross-sections. In particular, the coincidence T.R is a very sensitive monitor of the density of the liquid hydrogen in the target since the ratio $T.R/T$ is proportional to the density.

2.4 The Magnet and Spark Chambers

This has been fully described in the paper by Astbury et al²⁰). Here we give a brief outline of the system together with the parameters relevant to this experiment.

i) The Magnet was originally designed by M. Morpurgo for a large cloud chamber but has parameters equally suitable for operation with spark chambers. The maximum magnetic field of 10.5 kG at a power of 2 MW varies by less than 10% over a volume $180 \times 90 \times 90 \text{ cm}^3$. A hole in the upper pole piece allows photography of a volume $160 \times 60 \times 60 \text{ cm}^3$, a form particularly well suited to experiments in which the secondaries are strongly peaked forward, as in the present experiment.

ii) The Spark Chambers. The narrow gap spark chamber plates were made by gluing 25 μm Al foil to both sides of a frame of Al bars. Thirteen such plates separated by 8 mm perspex spacers are clamped together in an Al box having thin mylar windows and a glass plate at the top permitting photography. Six of these units form the complete spark chamber (fig 2.6). The gaps between the six units can be clearly seen in the photograph of a typical event (fig 2.7); the smaller gaps between

each spark are the dead spaces of each plate. Alternate plates within each unit are connected to H.T. and to earth, giving a pulsed field of opposite polarity in successive gaps. The H.T. voltage of 9 kV is applied about 400 nsecs after the occurrence of an event; the pulse rise time is 30 nsecs.

The clearing field of 100 V in each gap has polarity opposite to that of the pulsed field and it is this clearing field, when combined with the magnetic field, which produces the "staggering" of the sparks in successive gaps, clearly seen in fig 2.7. The amount of staggering is a measure of the time elapsing between the passage of the particle and the application of the H.T. Thus the "old" beam track in the figure has larger staggering. Details of the staggering and its correction are discussed in section 3.4.

Gas (an 80%-20% Ne-He commercial mixture) is passed through the chambers at a rate of about 0.2 litre/minute. Aluminium foil, mylar windows and gas produce in the whole system a total of 1.3 gms/cm^2 of material corresponding to a radiation length of 30 meters.

iii) The Optical System is particularly simple because of the large hole in the upper pole piece (fig 2.6). Two stereo views of the chamber with an optical axis separation of 58 cms are photographed side by side on 70 mm film. A perspex prism on the top of each spark gap allows the camera to "see" straight down into each gap with negligible distortion.

Optical distortions introduced by the lenses and mirrors of the system were evaluated as follows; photographs of wires in space were taken and deviations of the measured y-coordinates from their real values were fitted in each view to a second degree polynomial in x and y. The corrections are small, being typically 0.4 mrad for the azimuth angle and 10^{-5} cm^{-1} for the curvature.

Each picture, as well as having two stereo views of the chamber, has a serial number ("event number") in visual and binary coded decimal

(BCD) form, together with information on the counters triggered in that event. These can be seen at the bottom of the photograph (fig 2.7) and are collectively known as the "data box" information. Also seen on the picture are a series of crosses or "fiducials" whose positions are accurately known in space. These were flashed at each trigger and, measured together with the sparks on the two stereo views, allow a geometrical reconstruction of each event in space.

2.5 Picture Taking

Table 2.1 gives the dates of the picture taking runs together with relevant parameters. Note that pictures were taken at both magnetic field polarities to check for possible asymmetries in the system. With a beam momentum of 6 GeV/c a high magnetic field is preferable for increased momentum resolution of secondaries, however, at times only one generator was available to power the magnet. At 4 GeV/c a field of 7 kG suffices since the lower momentum secondaries would otherwise leave the chamber at the side and hence reduce the measurable track length.

Proton synchrotron parameters during the data taking were as follows:-

Primary beam energy	19.2 GeV/c
Primary beam intensity	10^{12} protons
Repetition rate	$1/2.3 \text{ sec}^{-1}$
Flat top	200 msec

With these parameters a trigger occurred about once per 3 P.S. bursts at both 4 and 6 GeV/c and picture taking proceeded at a rate of about 450 per hour.

TABLE 2.1

Parameters of Data Taking Runs

Run Number	Date	Beam Momentum	Magnetic Field	Polarity	Number of Pictures
1	} Aug. 67	6.2 GeV/c	10 kG	positive	14,400
2				negative	12,800
3	} Sept. 67		7 kG	negative	33,800
4				positive	32,500
5				positive	28,200
6	Nov. 67	4.1 GeV/c	7 kG	negative	55,000
7	Jan. 68			positive	63,700

121,700

118,700

Figure 2.1

Layout of Beam d27

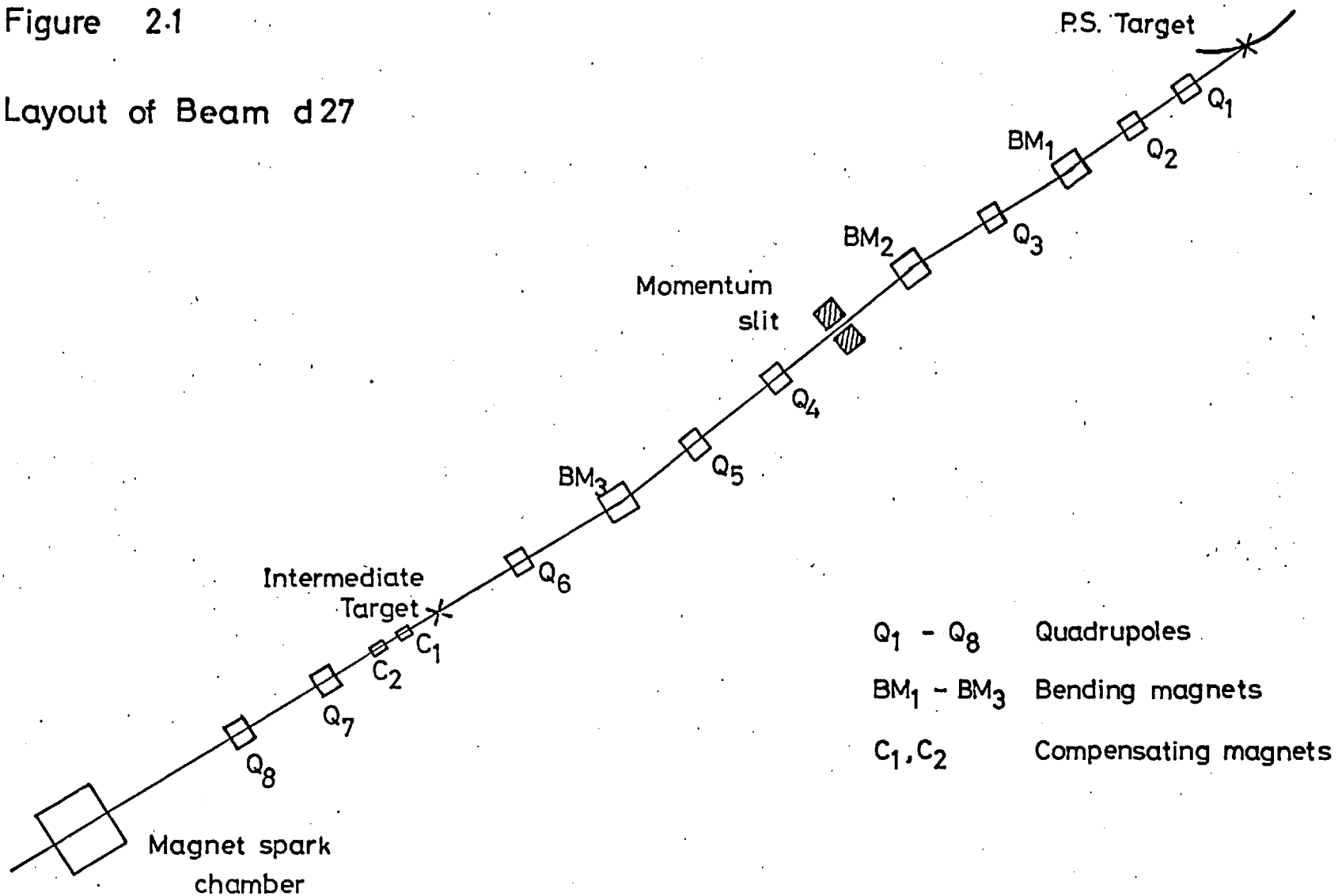
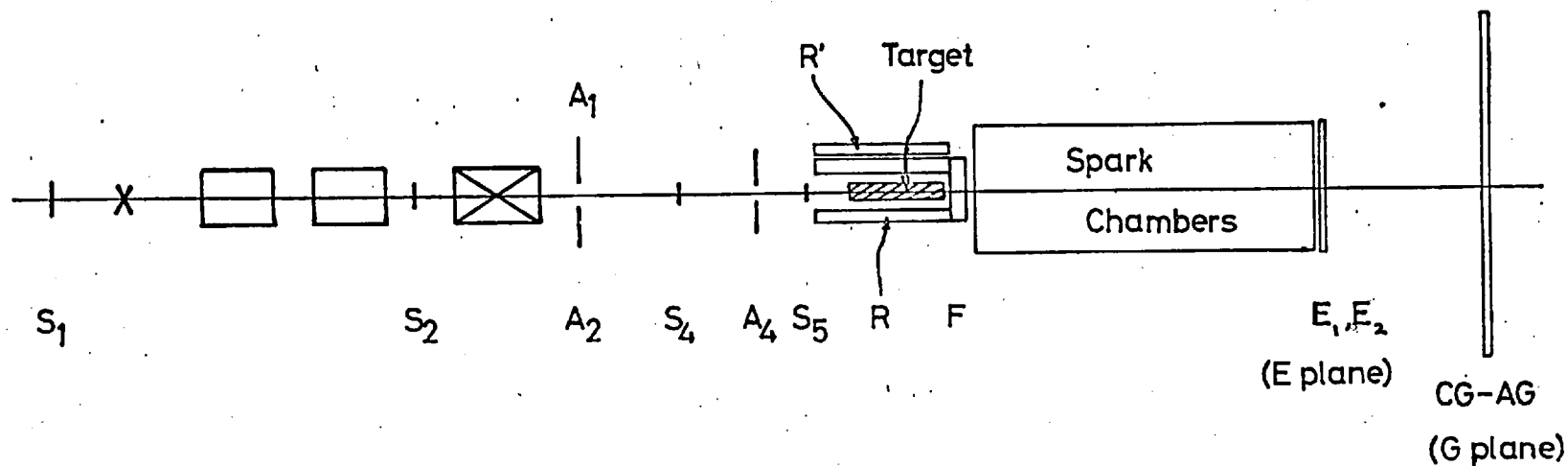


Figure 2.2

Counter Layout



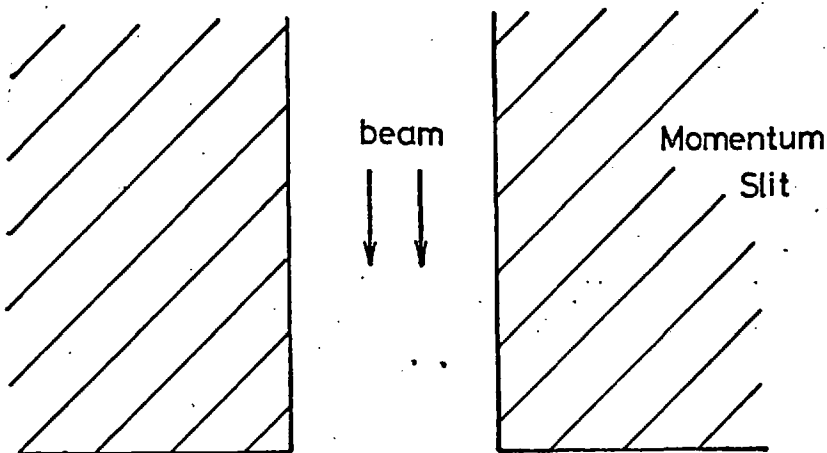
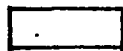
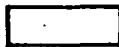
X Beam Stopper

□ Quadrupole

⊗ Bending Magnet

Figure 2.3

Counter Hodoscope

 H_1  H_2  H_3  H_4 

1 2 3 4 5 6 7

Channel Numbers

Channel
Number

Signal

1

 H_1

2

 $H_1 \cdot H_2$

3

 H_2

4

 $H_2 \cdot H_3$

5

 H_3

6

 $H_3 \cdot H_4$

7

 H_4

Figure 24

Counters Around the Target

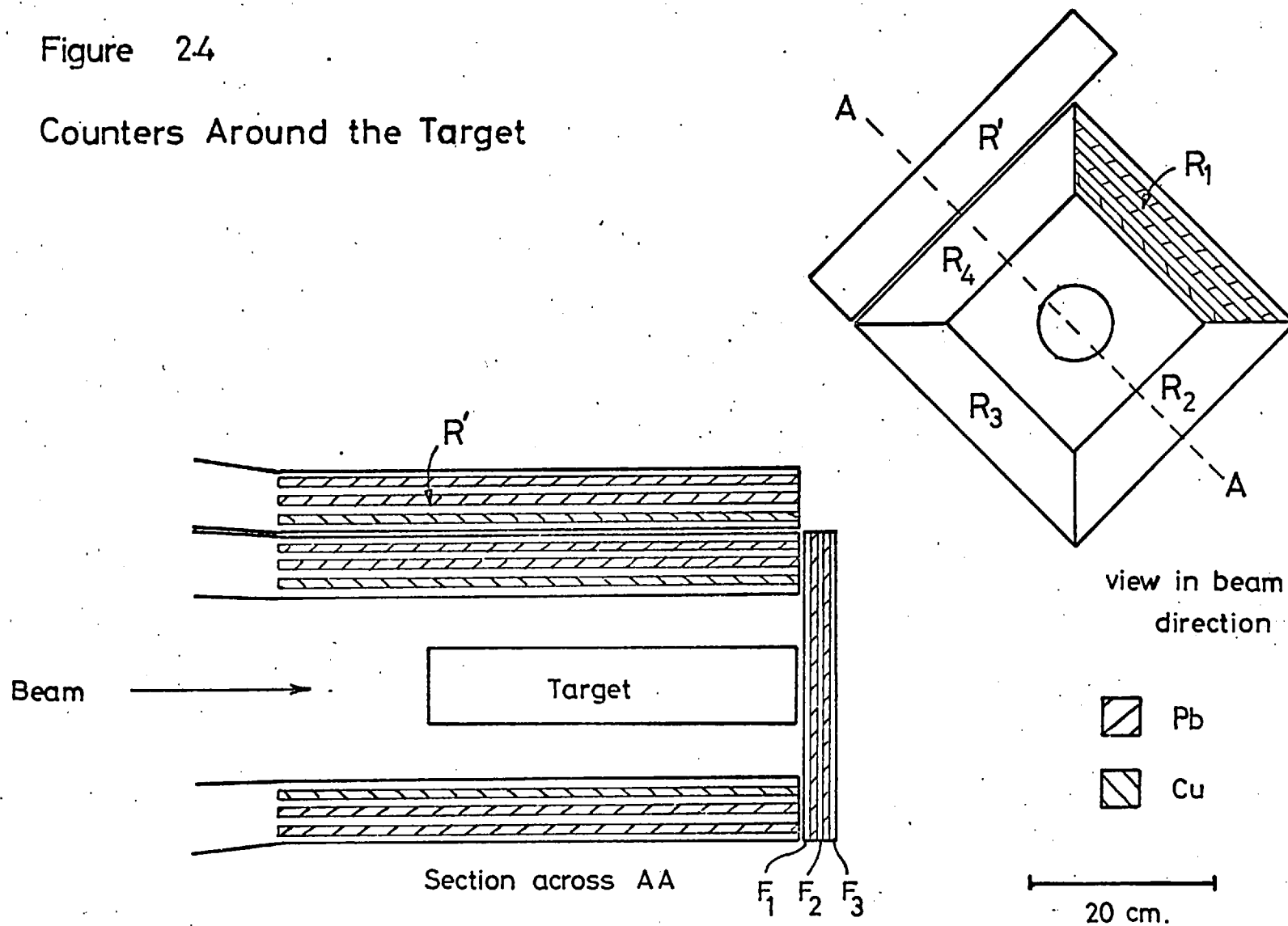


Figure 2.5
Counter Logic

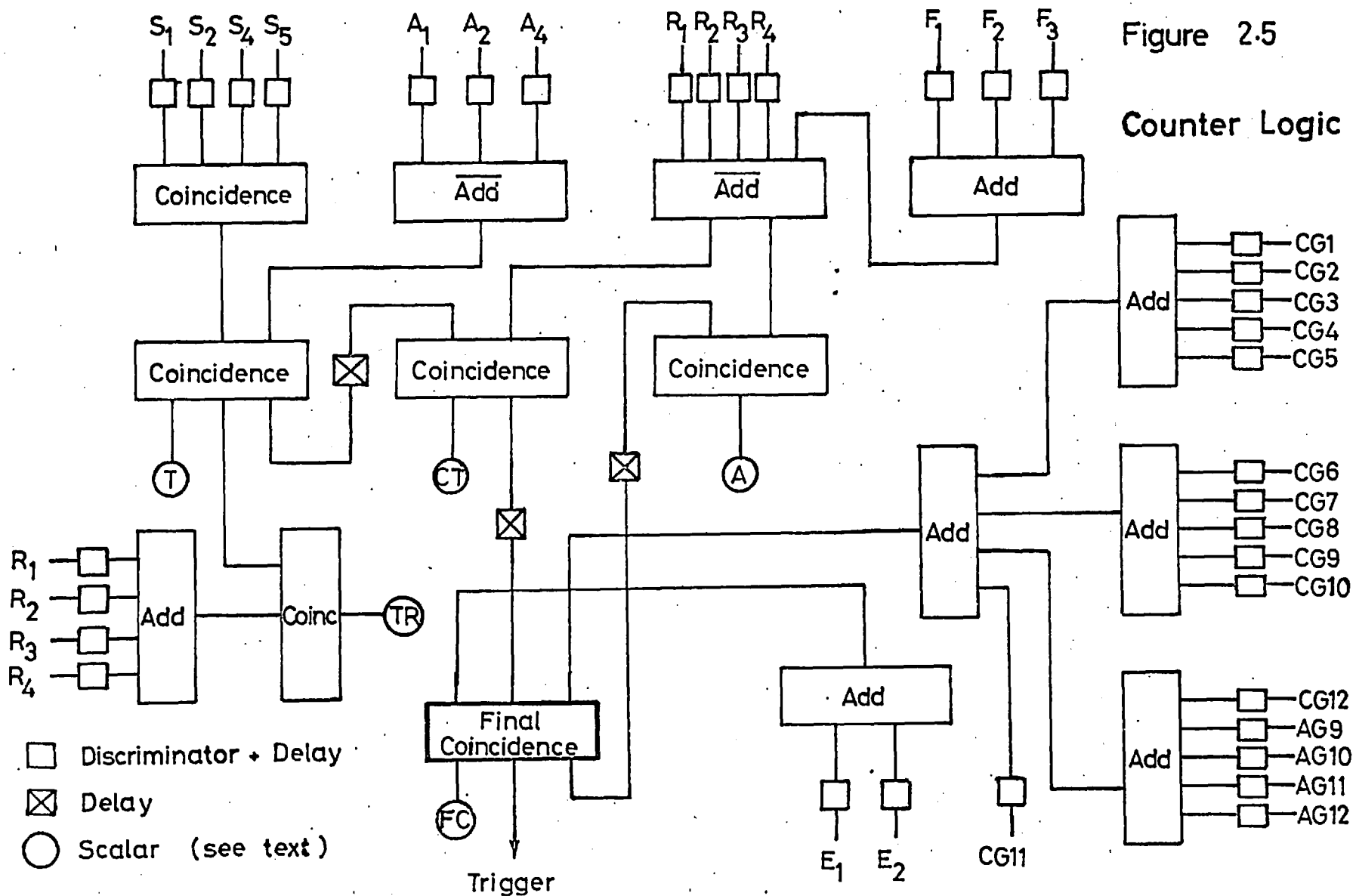


Figure 2.6a Magnet Spark Chamber

Section along beam direction

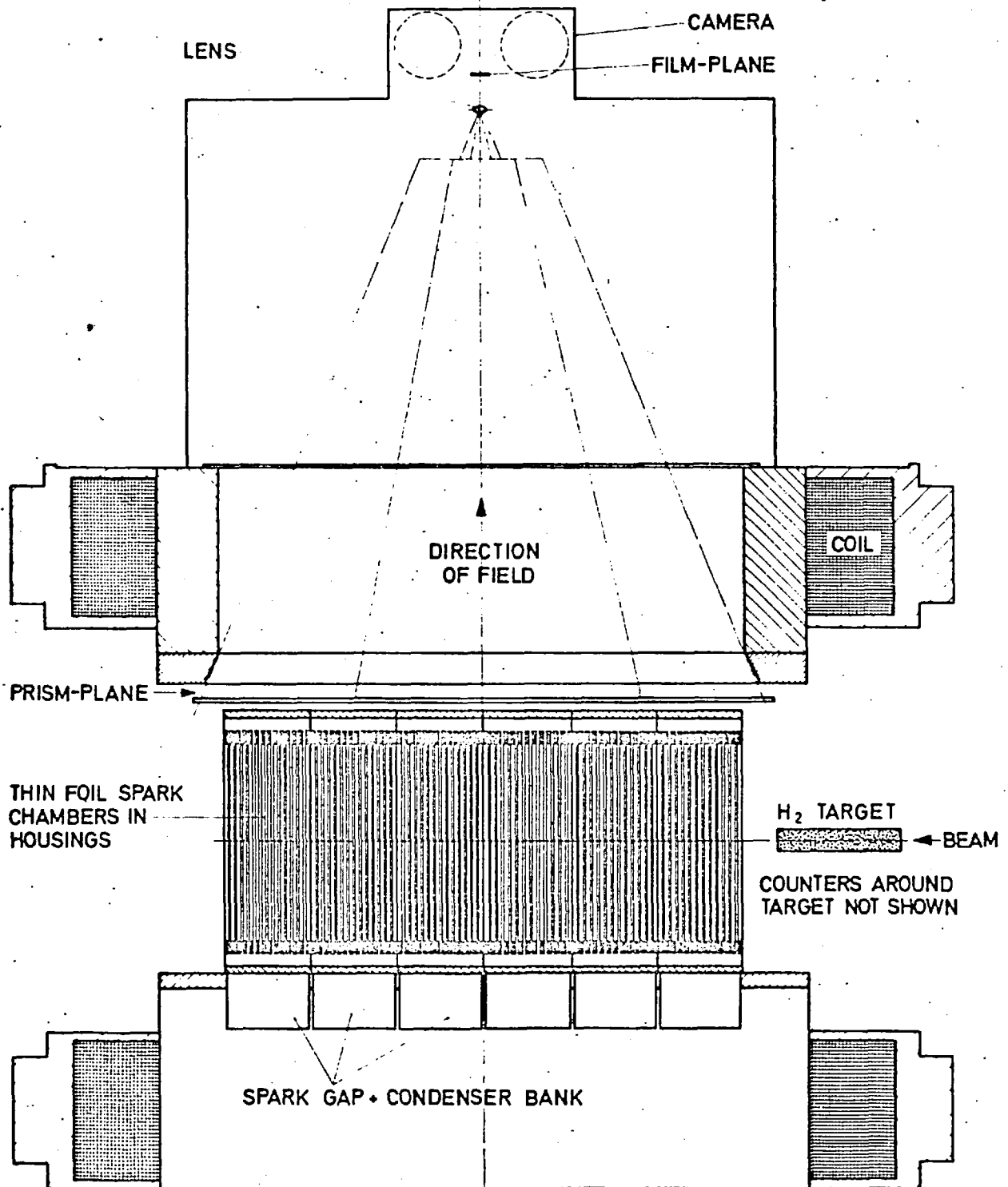


Figure 2.6b Magnet Spark Chamber

Section across
beam direction

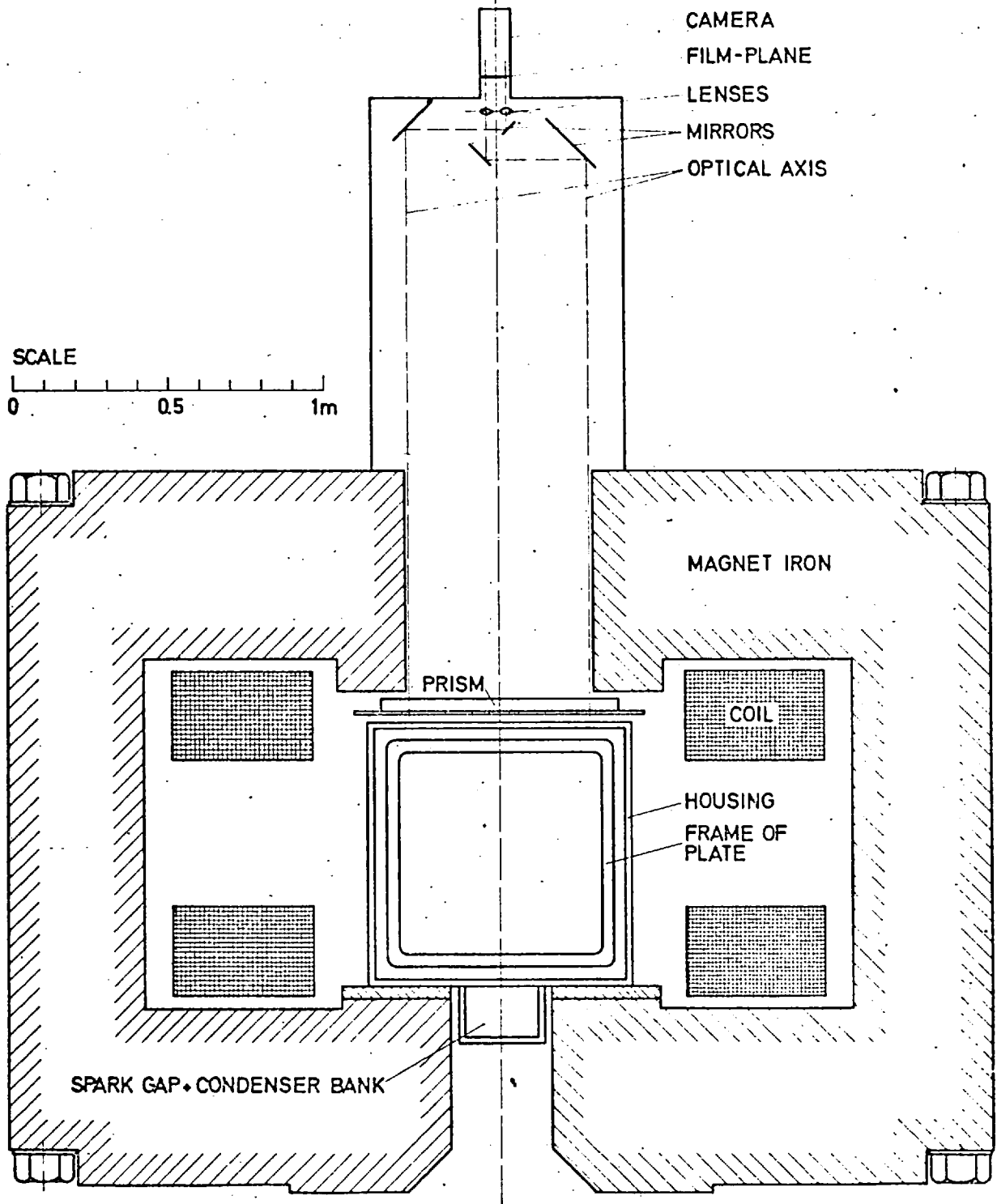
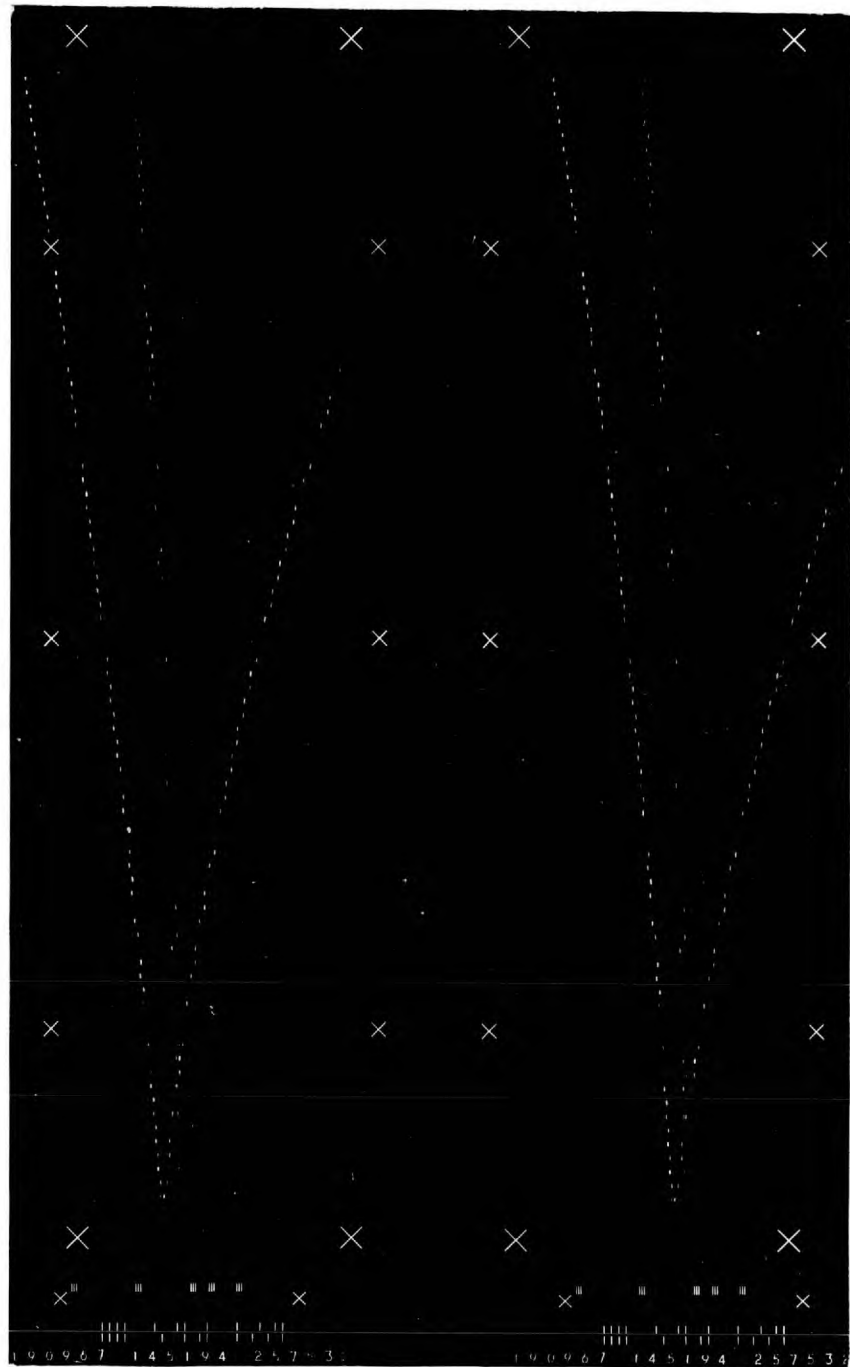


Figure 2.7 A Typical Event



CHAPTER 3

From Photographs to Data Summary Tape

3.1 Scanning and Measuring

The film was scanned before measuring and a punched card ('scan card') prepared for each photograph containing a V^0 decay with an apex in one of the first five units. Each scan-card contained the following information:

- a) the event number
- b) the position of the apex of the V^0 specified by the gap in which sparks were first seen
- c) the number of 'old' and 'new' beam tracks in the picture

The film was measured on CERN HPD I²¹) on-line to a CDC 6600 computer which produced an output tape containing the coordinates of the sparks and fiducial marks found on the two views of each event. This 'sparks tape' is the input tape for the pattern recognition programmes.

3.2 Pattern Recognition

The purpose of the pattern recognition stage of the analysis is to reduce the information on the sparks tape to a few parameters for the two tracks of the V^0 as follows^{22,23)}.

The first step is the simple one of correcting all sparks for the "staggering effect" mentioned in section 2.4. Then, if the scanning girl has indicated that the picture also contains one or more spurious beam tracks, these are sought in a fairly narrow region in the centre of the chamber (such tracks are easily recognisable by their curvature) and the sparks eliminated.

The programme now searches for other tracks among the remaining sparks. Starting at the exit end of the chamber a group of three sparks from up to five successive gaps is found and fitted to a parabola. This fit is used to predict the spark position in the next gap with a certain tolerance and this is sought. When it is found, a new fit is made and the next spark sought within a smaller search area. If at any stage no new spark is found in two successive gaps, an ellipse is fitted to the existing spark coordinates and the search continued using this fit.

It is possible for more than two tracks per view to be found since some of the pictures contain spurious tracks; however, since it is known that two of the tracks having opposite curvature form a V with an apex at a position given on the scan-card, it is not difficult to decide which are the tracks being sought. The two tracks in one view are now associated with the corresponding tracks in the other view and track and fiducial coordinates, together with the identifying information contained in the data box, are written onto magnetic tape.

Events could fail the above analysis for a number of reasons, among them:-

- a) not enough fiducials found
- b) many background tracks confuse the picture
- c) a short, large-angle track at the side of the chamber cannot be found
- d) the scanning girl gave an incorrect vertex position

The first could not possibly introduce a bias into an analysis and can be ignored, although the event must be counted when calculating cross-sections. Other failures could, however, introduce biases eg. c) produces a possible loss of events with a large scattering angle.

About 16% of the events at 4 GeV/c and 8% at 6 GeV/c failed during the pattern recognition process. Each failing event was written onto a separate tape and later inspected as described in the next section.

3.3 Rejects from the Pattern Recognition

Normally a small sample of rejected events are remeasured by hand on conventional measuring machines at a rate of about 3 per hour. In the HPD measurement sequence, spark coordinates are almost always found correctly; most rejects are caused by the failure to associate sparks into tracks, to combine two tracks into a V^0 or to correlate the tracks in the two views. Thus remeasurement of sparks is a waste of effort; it is better to use as much as possible of the information previously gained.

The following system, developed by Lassalle and Zanella²⁴⁾, was used to process a sample (about 50%) of the rejected events at a rate of about 30 per hour, none of the precision of the HPD being lost..

An operator was shown the results of the unsuccessful attempt on a CRT display console connected to a CDC 3100 computer, and by answering a programmed series of questions he could supply the pattern recognition programme with any extra information required. Using a light pen and keyboard he could, for example, add sparks to a track, inform the programme which two tracks formed the V^0 , or match the corresponding tracks in the two views. At least 70% of the rejects could be rescued in this manner.

The rescued events were processed in exactly the same way as the main sample and the two groups were later compared to search for possible biases (see section 6.2).

3.4 Geometrical Reconstruction

Spark and fiducial coordinates from the two views could now be combined to give a three dimensional reconstruction of each event in space, each track being described by a helix with axis parallel to the magnetic field. The parameters used to describe each helix are the interaction (or decay) point of the track (x,y,z) and the curvature ρ and

angles λ and ϕ of the helix at the centre of the track (fig 3.1).

The least squares fitting of the helices was performed by a modified version of the CERN geometry programme THRESH²⁵). The modifications to this programme, written originally for the analysis of bubble chamber film, were necessary because of different problems associated with the magnet spark chamber. They are discussed briefly below.

i) Systematic Spark Displacement

This effect, mentioned in section 2.4 arises from the force on a moving ion or electron in electric and magnetic fields,

$$\underline{F} = q (\underline{E} + \underline{v} \wedge \underline{H}) \quad 3.4.1$$

where the field $\underline{E}$ is the clearing field (the pulsed field has some small effect before the spark forms, but is ignored here.)

The first term in this expression has an effect only on inclined tracks where the drift of the electrons towards the positive plate by an amount Δx results in a displacement of the centre of gravity of the electrons remaining by an amount $\frac{1}{2}\Delta x \tan\theta$ in the y direction, θ being the angle that the track makes to the normal to the plates (see fig 3.2). The spark, normal to the plates and through the centre of gravity of the electrons (point C of fig 3.2) is thus displaced by the same amount.

The second term in 3.4.1 results in an electron drift in the direction of $\underline{E} \wedge \underline{H}$, and since $\underline{E}$ is of opposite polarity in adjacent gaps a "staggering" effect is produced.

The "staggering correction" is made during the pattern recognition procedure; the "inclined track correction" can only be made when θ is known, i.e. during THRESH.

ii) Apex Position

The apex position cannot be measured accurately since measurements in the x -direction are quantized by an amount equal to the gap separation. A better estimate is therefore obtained by extrapolating the

helix fits to the tracks and calculating the separation of the helices at 5 mm intervals. The apex is defined to be the point where minimum separation was found; its position is known to ± 2.5 mm in space.

iii) Beam Tracks

The momentum distribution of the beam in the different channels of the hodoscope was found by measuring beam tracks in the chambers. Then for each event the number of the hodoscope channel containing the beam particle could be obtained from the data box information to give the appropriate parameters for that event.

3.5 Kinematic Fitting

This task was carried out by the CERN programme GRIND²⁶⁾. The principles involved are outlined below.

At each interaction or decay point assume that four-momentum is conserved. Then for each such point there are four equations (the constraint equations):-

$$\begin{aligned}\sum_i p_{x_i} &= \sum_i g_i p_i \cos \varphi_i \cos \lambda_i = 0 \\ \sum_i p_{y_i} &= \sum_i g_i p_i \sin \varphi_i \cos \lambda_i = 0 \\ \sum_i p_{z_i} &= \sum_i g_i p_i \sin \lambda_i = 0 \\ \sum_i E_i &= \sum_i g_i (p_i^2 + m_i^2)^{\frac{1}{2}} = 0\end{aligned}$$

where $g_i = +1$ or -1 according as the i th track is in the initial or final state. (Note that p_i , λ_i , and φ_i are now the values of these quantities at the vertex and not at the centre of the track as defined in THRESH).

To each track a mass m_i is assigned to give a "hypothesis" for the event. The assigned masses m_i are, of course, limited to those of the known particles and the event must conserve such quantum numbers as charge, hypercharge and baryon number. In general there will be several

hypotheses possible and for each event it must be decided which is the most likely.

The constraint equations can be solved for each vertex if there are zero or one missing neutral particles, giving respectively a four- and a one-constraint fit. In general, to obtain a solution the measured values of p_i , λ_i , ϕ_i must be adjusted somewhat to give "fitted" values. This is done as follows:-

For each hypothesis a quantity χ^2 is defined

$$\chi^2 = \sum_{i,j} (X_i^m - X_i^f) G_{ij} (X_j^m - X_j^f)$$

where $X_{i,j}$ are the variables $1/p, \lambda, \phi$ i.e. $i, j = 3 \times$ number of tracks and G is the inverse error matrix i.e. $(G_{ij})^{-1} = \Delta X_i^m \Delta X_j^m$. Superscripts m, f denote measured and fitted values respectively. Then the fitted variables are calculated by minimising

$$\chi^2 + 2 \sum \lambda_\alpha f_\alpha$$

where $f_\alpha = 0$ are the constraint equations and λ_α are Lagrangian multipliers. Since the constraint equations are non-linear, the solution involves iteration until convergence is obtained.

The final value of χ^2 for each hypothesis is a measure of the amount by which the variables have to be adjusted to obtain a fit to the equations. χ^2 is converted to a number called the χ^2 -probability by means of the equation

$$P(\chi^2) = \int_{\chi^2}^{\infty} \frac{\chi'^2 (N/2 - 1)}{2^{N/2} \Gamma(N/2)} e^{-\chi'^2/2} d\chi'^2$$

where N is the number of degrees of freedom. If the measurement errors have been correctly estimated, then for a large number of events $P(\chi^2)$ should be uniformly distributed between 0 and 1.

In general, for the wrong hypothesis $P(\chi^2)$ will be very small, and the attempt at a fit can be abandoned at a very early stage. In this experiment a fit was retained if it gave a χ^2 -probability $> 10^{-5}$. Even at this level, ambiguities, i.e. fits to more than one hypothesis, were rare ($< 8\%$).

Hypotheses attempted for each V^0 decay were:-

$$K^0 \rightarrow \pi^+ + \pi^-$$

$$\Lambda \rightarrow p + \pi^-$$

$$\bar{\Lambda} \rightarrow \pi^+ + \bar{p}$$

$$\gamma \rightarrow e^+ + e^-$$

The last hypothesis cannot in fact satisfy four-momentum conservation; the actual physical process is

$$\gamma + \text{Nucleus} \rightarrow e^+ + e^- + \text{Nucleus}$$

but since the magnitude of the non-conservation of four-momentum is less than the amount by which the constraint equations had to be satisfied, it is justified to treat the interaction as a simple decay.

If a fit to one of the first three hypotheses was obtained, an attempt was made to fit corresponding interactions in the target, namely

$$\pi^- + p \rightarrow K^0 + \Lambda$$

$$\pi^- + p \rightarrow \Lambda + K^0$$

$$\pi^- + p \rightarrow \bar{\Lambda} + \bar{K}_0$$

where the first particle on the right hand side was that fitting at the decay vertex. The third of these reactions is not physically possible - it does not conserve baryon number - but it is useful since we can expect the same number of events which fit this reaction also to wrongly fit the second. A more detailed discussion of this point is given in section 6.2.

In the bubble chamber analysis chain any ambiguous event can be further inspected on film to see if any hypothesis can be eliminated on the basis of ionization (given by the bubble density). The magnet spark chamber does not allow this possibility and the data summary tape (D.S.T.) containing quantities of physical interest was written by GRIND. One extra routine in the standard version of GRIND made this possible.

3.6 Statistical Analysis

The total information on the single V events was contained on 14 separate magnetic tapes and moreover, not all quantities of physical interest were on these tapes; some of them had to be derived. Therefore, the 14 tapes were processed and the information compressed so that all relevant physical quantities were contained on 4 tapes.

The statistical analysis of the compressed D.S.T.'s was performed using SUMX²⁷⁾, a programme specifically developed for the analysis of high energy physics data. Among the facilities included in SUMX are definition of tests, data editing, display of one- and two-dimensional histogrammes, calculation of moments and listing of specified words from events.

A summary of the whole of the data processing chain is given in Figure 3.3.

Figure 3.1

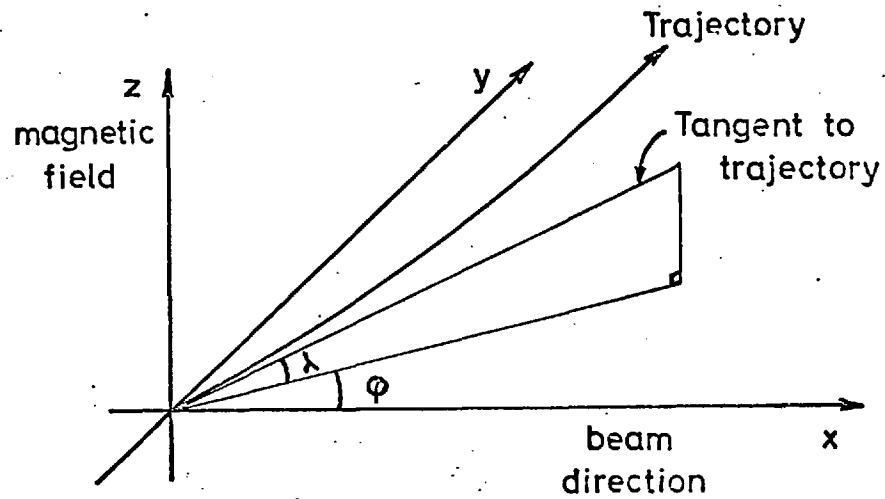
Coordinate System and
Definition of Angles

Figure 3.2

Inclined Track Effect

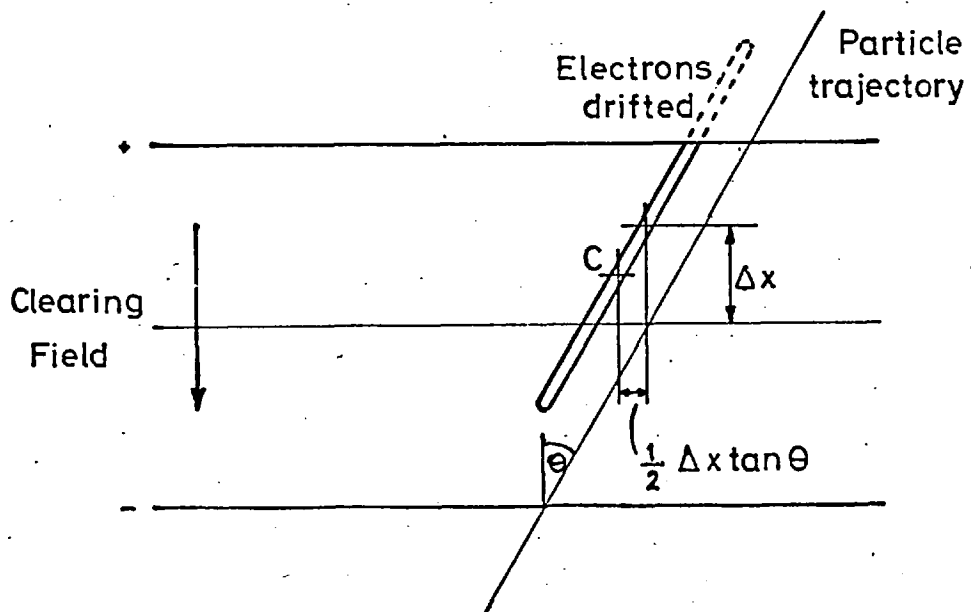
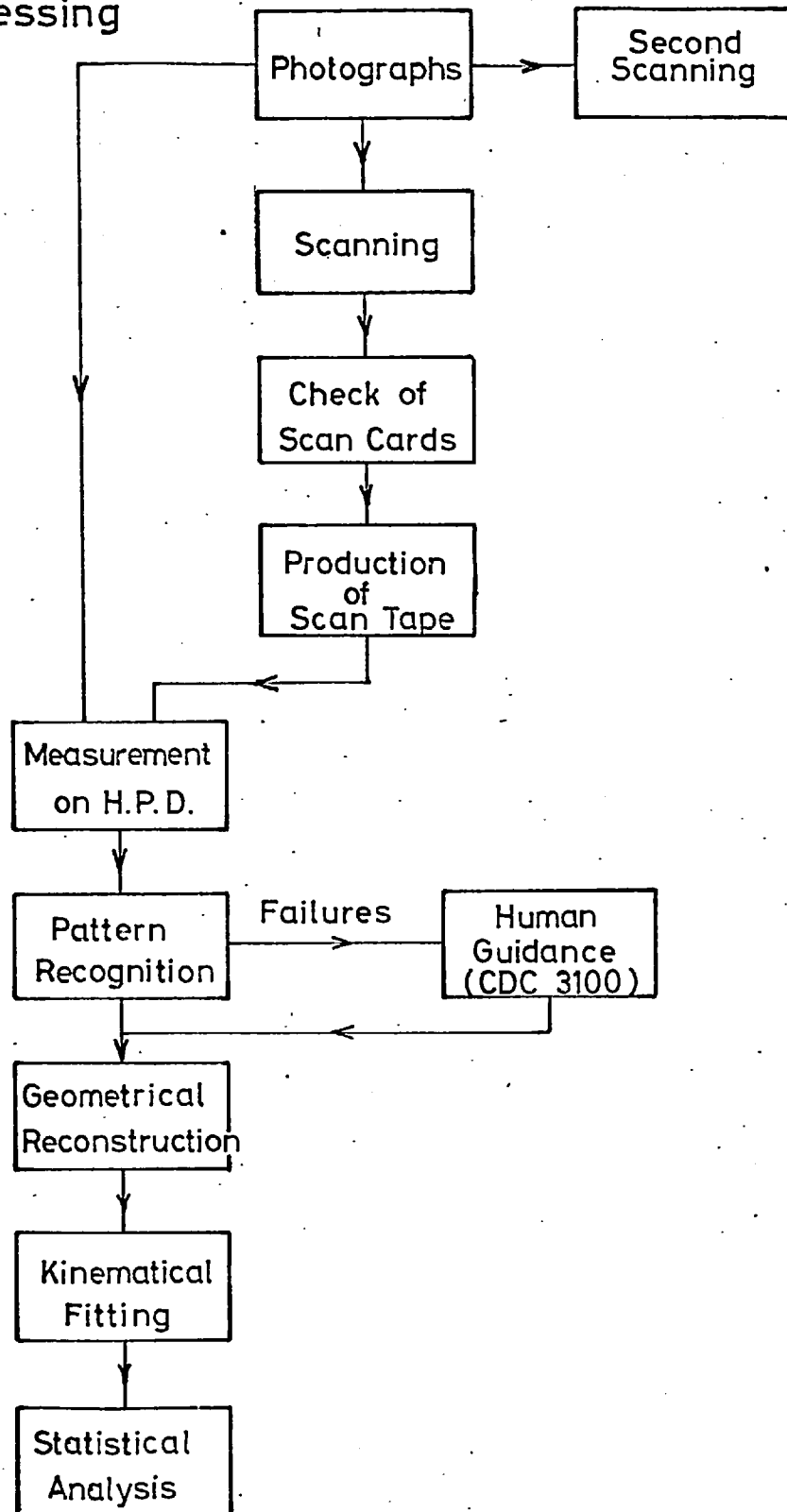


Figure 3-3

Data Processing
Chain

CHAPTER 4

Instrumental Resolution

4.1 Spatial Resolution

The spatial resolution of the instrument is determined by the random scatter of measured spark coordinates about the true particle trajectory. The error in the fit is small compared to the random deviations of individual sparks and thus the difference between measured and fitted spark coordinates is a good estimate of individual spark deviation.

There are three main sources of error in the spark coordinates;

- a) spark formation mechanism
- b) optical distortions
- c) measurement of photographs

Optical distortions and their correction have been discussed in section 2.4 and the two systematic deviations in spark position in section 3.4. Residual random fluctuations in spark position have been shown to be about 0.18 mms in space²⁰⁾. In the same paper the measurement error on the y-coordinate was shown to be 0.09 mms in space (3.7 μ m on film). The spark image on film is about 80 μ m wide so that the centre of the spark is found to within 5% of its width. The two random effects, measurement error and random fluctuation in spark position, thus produce an error in the y-coordinate of $\sigma_y = 0.20$ mm.

4.2 Momentum Resolution

The accuracy in measurement of the momentum p (p, λ, ϕ) of a particle depends upon multiple coulomb scattering, the spatial resolution, knowledge of the magnetic field and the apex reconstruction.

The radiation length due to all the material in the chambers is 30 metres (cf 8 metres for a hydrogen bubble chamber). Coulomb scattering is thus small and gives a contribution to $\Delta p/p$ of about 0.008 at all momenta²⁸⁾.

The magnetic field is known to $\pm 0.3\%$. Since $\Delta p/p = \Delta H/H$, the contribution to $\Delta p/p$ from this effect is also small. (A recent remeasurement of the magnetic field suggests that the values used in this experiment may have been incorrect by up to 1% in certain regions near the edges of the chambers).

An error in the apex position leads to an error in the azimuth angle ϕ . Since the apex position is known only to 0.05 cms $\Delta\phi = 0.05/\rho$ where ρ , the radius of curvature of the track, is measured in cms.

The spatial resolution limits the precision on the sagitta s . Websdale²⁸⁾ shows that $\Delta s = 4\sigma_y/\sqrt{N}$, where N is the number of sparks on the track. Typically there are about 30 sparks per track and hence $\Delta s = 0.15$ mm. In order to take into account the larger fluctuations in spark position as the angle between the track and the normal to the plates increases, it has been found empirically that the error in the sagitta should be given by

$$\Delta s' = \Delta s + 0.7 \tan(0.6L/\rho + \phi) \text{ mm.}$$

where L is the length of the track.

The external errors used in GRIND are

$$\Delta 1/\rho = 8\Delta s'/L^2 \cos^2 \lambda$$

$$\Delta \phi = 0.5/\rho$$

$$\Delta \lambda = 6\Delta s'(\cos^2 \lambda + 0.01)^2/L$$

$$\Delta \phi \Delta 1/\rho = -32 (\Delta s')^2/L^3 \cos^3 \lambda$$

Thus for a 3 GeV/c track of length 1 metre with $\lambda, \phi = 0$

$$\Delta s' = 0.20 \text{ mm}$$

$$\Delta 1/\rho = 1.6 \times 10^{-5}$$

$$\Delta \phi = 0.5 \text{ mrad}$$

$$\Delta \lambda = 1.2 \text{ mrad}$$

A similar track with momentum 2 GeV/c has $\Delta s'$, and hence $\Delta 1/p$ and $\Delta \phi$, increased by 20% and $\Delta \lambda$ increased by 50%.

A check on error assignment are the stretch functions, P_i , defined by

$$P_i = (X_i^m - X_i^f) / \Delta(X_i^m - X_i^f) \\ = (X_i^m - X_i^f) / [(\Delta X_i^m)^2 - (\Delta X_i^f)^2]^{\frac{1}{2}}$$

where X_i^m are the measured variables

X_i^f are the fitted variables

$i = 1, 2, 3$; $X_i = 1/p, \lambda, \phi$.

The P_i should be normally distributed with zero mean and unit variance if errors are correctly assigned. A non-zero mean indicates a systematic error; a narrow (broad) distribution indicates an overestimated (underestimated) error. Fig 4.1 is an example of stretch functions at the decay vertex $K_1^0 \rightarrow \pi^+ + \pi^-$. We can feel confident that there are no large systematic effects and that errors have been correctly assigned.

4.3 Mass Resolution

When a particle of mass m decays into two particles of equal mass μ (eg. $K_1^0 \rightarrow \pi^+ + \pi^-$) conservation of four-momentum gives

$$m^2 = 2\mu^2 + 2(E_1 E_2 - p_1 p_2 \cos \theta)$$

where θ is the opening angle.

If we assume errors Δp_1 in p_1 , Δp_2 in p_2 and $\Delta \theta$ in θ we find that the error Δm on m is given by

$$\frac{\Delta m}{m} = \left[\frac{1}{2} - \frac{\mu^2}{m^2} - \frac{E_2 \mu^2}{E_1 m^2} \right] \frac{\Delta p_1}{p_1} + \left[\frac{1}{2} - \frac{\mu^2}{m^2} - \frac{E_1 \mu^2}{E_2 m^2} \right] \frac{\Delta p_2}{p_2} \\ + \frac{p_1 p_2 \sin \theta \Delta \theta}{m^2} \quad \dots\dots 4.3.1$$

Consider a K_1^0 of momentum 4 GeV/c decaying into $\pi^+ + \pi^-$ each having momentum 2 GeV/c and track length 1 metre in a 7 kG field. Then

using the equations of section 4.2 we find $\Delta p_1/p_1 = \Delta p_2/p_2 = 1.7\%$, and hence the error due to each $\Delta p/p$ term in 4.3.1 is 0.6% .

The symmetric decay of a 4 GeV/c K_1^0 produces an opening angle of about 20° in the laboratory, and with $\Delta\phi \sim 0.6$ mrad and $\Delta\lambda \sim 1.8$ mrad the last term in 4.3.1 gives an error of 0.8% in $\Delta m/m$.

Hence $\Delta m/m \approx 1.2\%$

Now $m \approx 500$ MeV/c²

$\therefore \Delta m \approx 6$ MeV/c²

Since $\Delta m^2 = 2m\Delta m$ and $m \approx 0.5$ GeV/c² for the K^0 then $\Delta m^2 = \Delta m$.

Figure 4.2 is a histogramme of the K_1^0 mass squared for all fitted K_1^0 decays at 4 GeV/c. Δm^2 for this histogramme is 5.5 (MeV/c²)². This compares well with typical values from bubble chamber experiments.

4.4 Resolution in Missing Mass

The resolution in missing mass for the magnet spark chamber is not good. We have seen that typically the error on the momentum of a track from the decay of a particle is 1.7% leading to an error on the momentum of the decaying particle of $1\% - 1.5\%$. The error in the beam momentum in the present experiment was typically 1% .

These numbers result in an error on the missing mass to the Λ in the reaction $\pi^- + p \rightarrow \Lambda + K^0$ of about 150 MeV/c². In the reaction $\pi^- + p \rightarrow K^0 + \gamma^*(1520)$ the error in the missing mass to the K^0 is about 60 MeV/c².

4.5 Resolution in Four-Momentum Transfer

For the backward peak reaction $\pi^- + p \rightarrow \Lambda + K^0$ we define $u = (P_1 - P_3)^2$ where P_1 and P_3 are the four-momenta of the π^- and Λ respectively.

$$\text{ie. } u = m_1^2 + m_3^2 - 2E_1E_3 + 2p_1p_3\cos\theta$$

Then errors in p_1, p_3 and ϑ give

$$\Delta u \approx (2p_3 \cos \vartheta - 2E_3) \Delta p_1 + (2p_1 \cos \vartheta - 2E_1) \Delta p_3 - 2p_1 p_3 \sin \vartheta \Delta \vartheta$$

We have already discussed the errors on p_1 and p_3 . The error in p_1 would normally arise from the bending of the beam as it passes through the target and would give an error in u varying from $\Delta u = 0.002$ at $\vartheta = 0^\circ$ to $\Delta u = 0.07$ at $u = -1.0 \text{ (GeV/c)}^2$.

However it was discovered that the value used in THRESH for the azimuth angle (φ) of the beam for the 4 GeV/c positive polarity data was incorrect by 13 mrad thus increasing the error in u . The maximum error occurs when the production plane is horizontal, when the error in u due to the error in $\vartheta = 2p_1 p_3 \sin \vartheta \Delta \vartheta$

$$\approx 0.35 \sin \vartheta$$

For $u = -0.1$, $\vartheta = 0.07 \text{ rad}$, $\Delta u = 0.025 \text{ (GeV/c)}^2$.

Figure 4.1

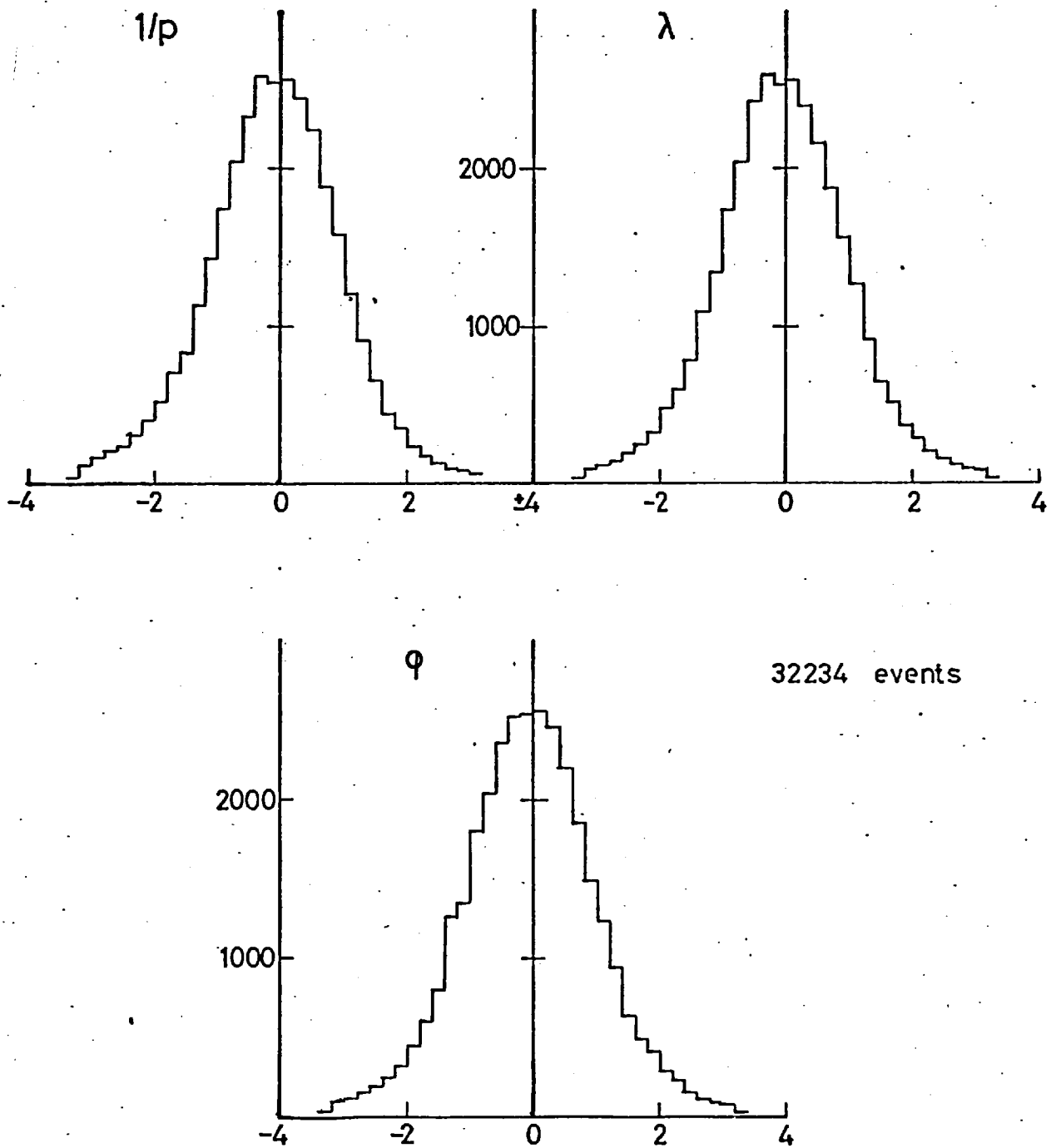
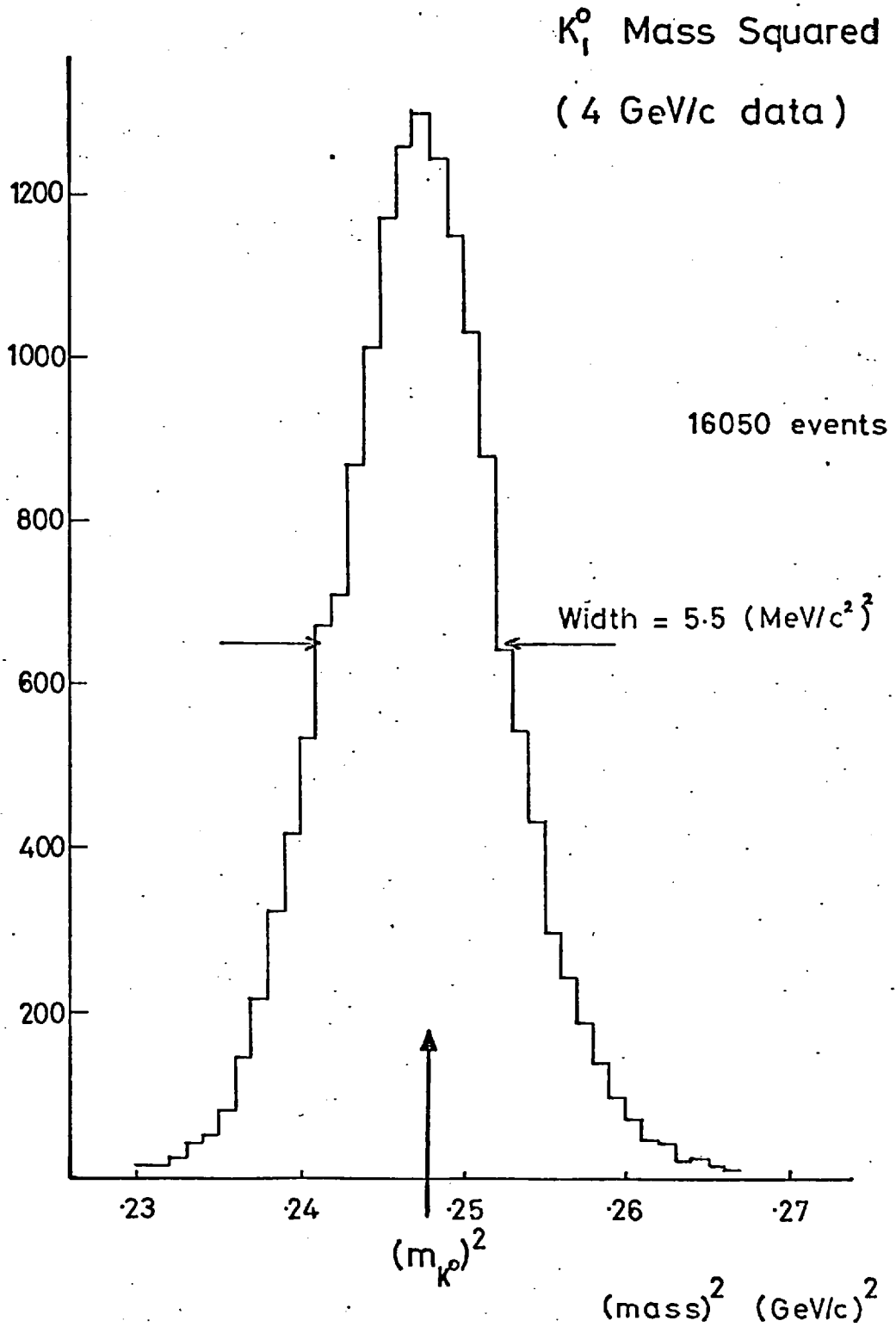
Stretch Functions at 6 GeV/c for $K_L^0 \rightarrow \pi^+ + \pi^-$ 

Figure 4.2



CHAPTER 5

The Monte Carlo Programme "SPRUCE"

5.1 Introduction

The detection efficiency of a spark chamber and counter configuration is rarely 100% and normally varies considerably with the production angle of the secondary particles. It is, therefore, essential to determine the detection efficiency as a function of all relevant parameters - a problem in integration over a many-dimensional space. Monte Carlo is the most convenient method of carrying out this integration.

The principle of the Monte Carlo method is to calculate the integral by determining the average value of the integrand over a large number of random points in the space. At first sight it seems strange that the points at which the values of the integrand are determined should be scattered at random, however it can be shown that such an integration always converges as $n^{-\frac{1}{2}}$ (n is the number of points taken) whatever the dimensionality of the space, whereas the trapezoidal rule, perhaps the simplest method of numerical integration, converges only as $n^{-2/K}$ where K is the dimensionality of the space. Clearly the Monte Carlo method is more efficient for $K > 4$.

Detailed accounts of the Monte Carlo method, together with extensive bibliographies can be found in references 29 and 30.

5.2 Purpose and Application of the Programme

The essential purpose of the programme SPRUCE is to generate, in a computer, "typical" events in a target and to follow each secondary particle through the magnet spark chamber system. Relevant parameters

can then be written onto paper and/or magnetic tape and a statistical analysis carried out as if they were "real" events.

The programme is intended for use at two stages of an experiment - during the planning stage to optimise the detection efficiency by good design of the counter configuration, and during the data analysis for a detailed study of biases for correction of the experimental distributions.

5.3 Outline of the Programme

Since several people would be using SPRUCE the following considerations had to be borne in mind during the planning and writing:-

- a) The programme should not have to be changed to cater for different experiments - data cards only should be needed.
- b) There should be as few data cards as possible and these should be as simple as possible.
- c) Extensions should be possible without major re-writing.

The programme has two main segments, one dealing with the kinematics of interactions and decays, the other following the particles through the magnetic field and counter configuration.

As an example, consider the reaction $\pi^- + p \rightarrow \Lambda + K^0 \rightarrow \pi^+ + \pi^- + \pi^- + p$

Then for each event (fig 5.1) SPRUCE must do the following:-

- a) produce the interaction in the target and do the kinematics
- b) choose a decay time for the Λ , go to the point 2 given by this decay time and the momentum of the Λ and do the kinematics of the decay.
- c) repeat b) for the K^0 .
- d) follow the decay particles (π^- , p) in turn through the magnetic field, checking for the triggering of counters, until the particles leave the 'fiducial volume'.
- e) repeat d) for the π^+ and π^- from the decay of the K^0 .
- f) write out various words onto magnetic tape and/or paper and start a new event.

5.4 Kinematics

In its present form SPRUCE produces two- or three-body final states only - the more complex four- or more-body state is unlikely to be the subject of a spark chamber experiment.

i) Two-Body Final State

The energies of the two final state particles are determined by the four-momenta of the initial state particles and it is only necessary to produce an orientation in the centre-of-mass system.

A value for the polar angle θ can be chosen at random from a given distribution as explained in Appendix A. Alternatively a value for the four-momentum transfer t of the interaction can be chosen at random from a given distribution and from this the polar angle calculated. The azimuth angle ϕ is chosen at random from a uniform distribution between 0 and 2π .

ii) Three-Body Final State

As the number of final state particles increases, so the problem becomes more involved; at present the programme produces only a phase space distribution when there are three final state particles. Non-phase space distributions have been considered recently by Van Hove and others³¹⁾.

The principle used is the prediction of Lorentz invariant phase space that the T_1, T_2 plane is uniformly populated in the allowed kinematical region (T_i is the kinetic energy of particle i). Thus the kinetic energies of particles 1 and 2 are chosen at random from a uniform distribution between 0 and $T_{\max}$, the maximum possible kinetic energy of the particle, corresponding to the given centre of mass energy of the interaction. The energy of the third particle is given by energy conservation, and a check is made to see that each of the three particles has a momentum smaller than the algebraic sum of the other two, ie. that the event lies within the kinematic boundary for the interaction. If not a new attempt is made until this condition is satisfied.

The angles between the tracks are now determined by momentum conservation and the event is randomly oriented in space by three rotations performed on all three tracks; an 'azimuth-like' rotation about the x-axis, a 'polar angle-like' rotation about the z-axis, and finally an 'azimuth-like' rotation about the new x-axis.

A Lorentz transform performed on all particles (for both the two- and three-body final state) takes them from the centre-of-mass to the laboratory frame of reference.

5.5 Particle Following

After a particle has been produced, its progress through the configuration of magnetic field, spark chambers and counters must be plotted. In general four checks are made on each particle as outlined below, however first of all we must explain how the trajectory of the particle through the field is found.

Neutral particles do not interact with a magnetic field, and thus the problem is trivial. The path of a charged particle is approximated as follows: since the magnetic field varies over the region of interest, the volume is divided into segments in the x (beam) direction with the field constant in each segment, but in general varying from one segment to the next (see fig 5.2). Then for the segment i, given $\underline{x}_{i-1}$ ($x_{i-1}, y_{i-1}, z_{i-1}$), H_i and p_{i-1} , both $\underline{x}_i$ and p_i are completely determined. The four tests can now be carried out.

a) Decay check

When a particle of mass m is destined to decay it is given a 'decay time' t according to the formula $t = -\tau \ln(1-r)$ where τ is the proper lifetime of the particle and r is a random number. The distance to the point of decay is then given by $d = pct/m$, where the symbols have their usual meaning. If d is larger than 1000 cm or if the particle does not decay then d is set equal to 1000 cm. If the particle is charged, the total distance from the production point is checked after each segment.

When this distance becomes larger than d we move back along the track to the point at which it became equal to d . This point is then defined as the decay point.

A neutral particle can be immediately extrapolated the distance d to the decay point, given its production point and its momentum.

Any particle with d equal to 1000 cm will have a decay point outside the region of interest and will be forgotten once it gets outside that region (see c) below).

b) Counter check

Either planar or cylindrical counters may be defined on data cards and their coordinates given. Then if a particle passes through a segment containing one or more counters a test is made to see if the trajectory passes through any of the counters. If it does that counter is considered to have been triggered for the event, and at the end of each event the overall trigger for the event is recorded.

c) Volume check

Two geometrical volumes are defined - the actual volume of the spark chambers (ie. that volume in which sparks would be observed in an actual experiment), and a 'fiducial volume' (the total region of interest). When a particle passes outside the latter it is no longer followed.

A check is constantly kept of the position of each particle relative to these two volumes and when a charged particle leaves the chamber the exit coordinates are calculated and stored.

d) Reference plane check

Up to four x-coordinates can be fed into the programme as data, and when any particle crosses one of the planes the y- and z-coordinates are calculated and stored. This facility allows distributions at various points down the chamber, or even outside the chamber, to be displayed.

5.6 Extensions

SPRUCE was written in such a way that future extensions would be relatively simple. Although the version outlined above was adequate for the work described in this thesis, several modifications have been either implemented or planned for possible future experiments. They include the following:-

a) Routines to simulate multiple scattering in the target and chamber and to introduce 'measurement errors', allowing these effects to be studied using the kinematical fitting programme GRIND. Associated therefore, with these routines is another which produces a tape for input to GRIND.

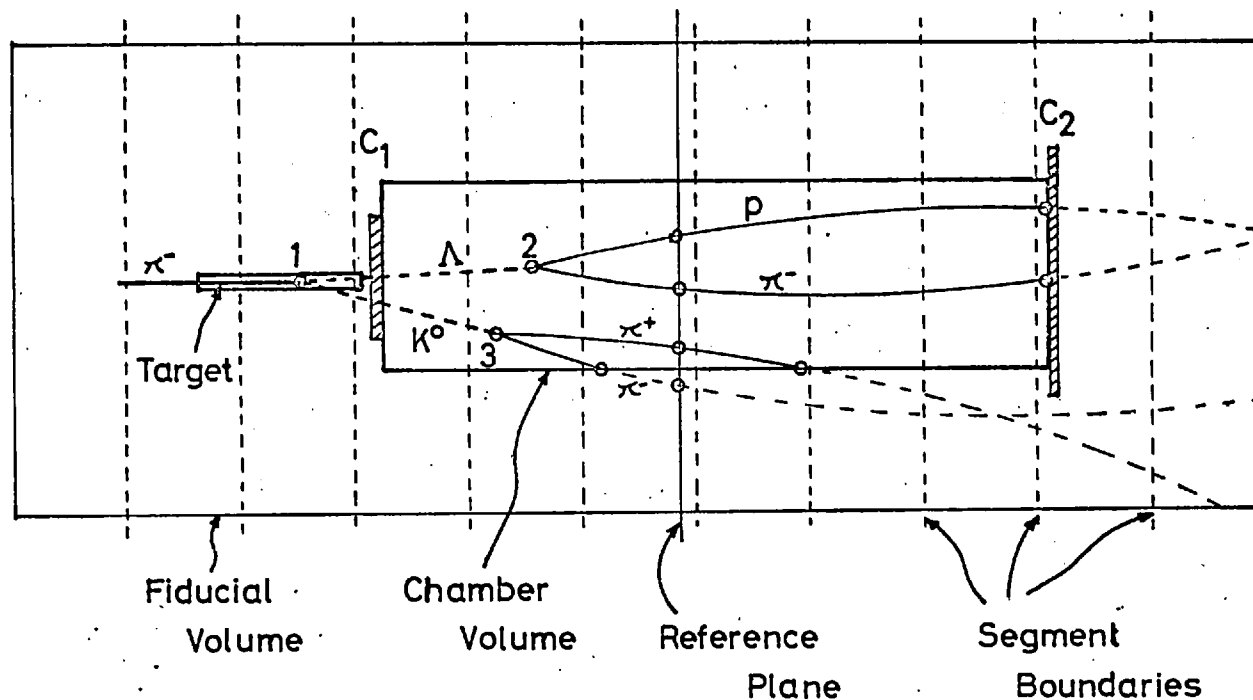
b) A plotting routine which prints the projection onto the x-y plane of a series of events, thus simulating typical photographs for visual inspection.

c) A routine to deal with gamma conversion in the chambers if some of the spark gaps have heavy (Pb or thick Al) plates.

d) Routines to allow gamma counters to be defined.

Figure 5.1

The Interaction $\pi^- p \rightarrow \Lambda K^0$ as seen by SPRUCE



C_1, C_2

Counters

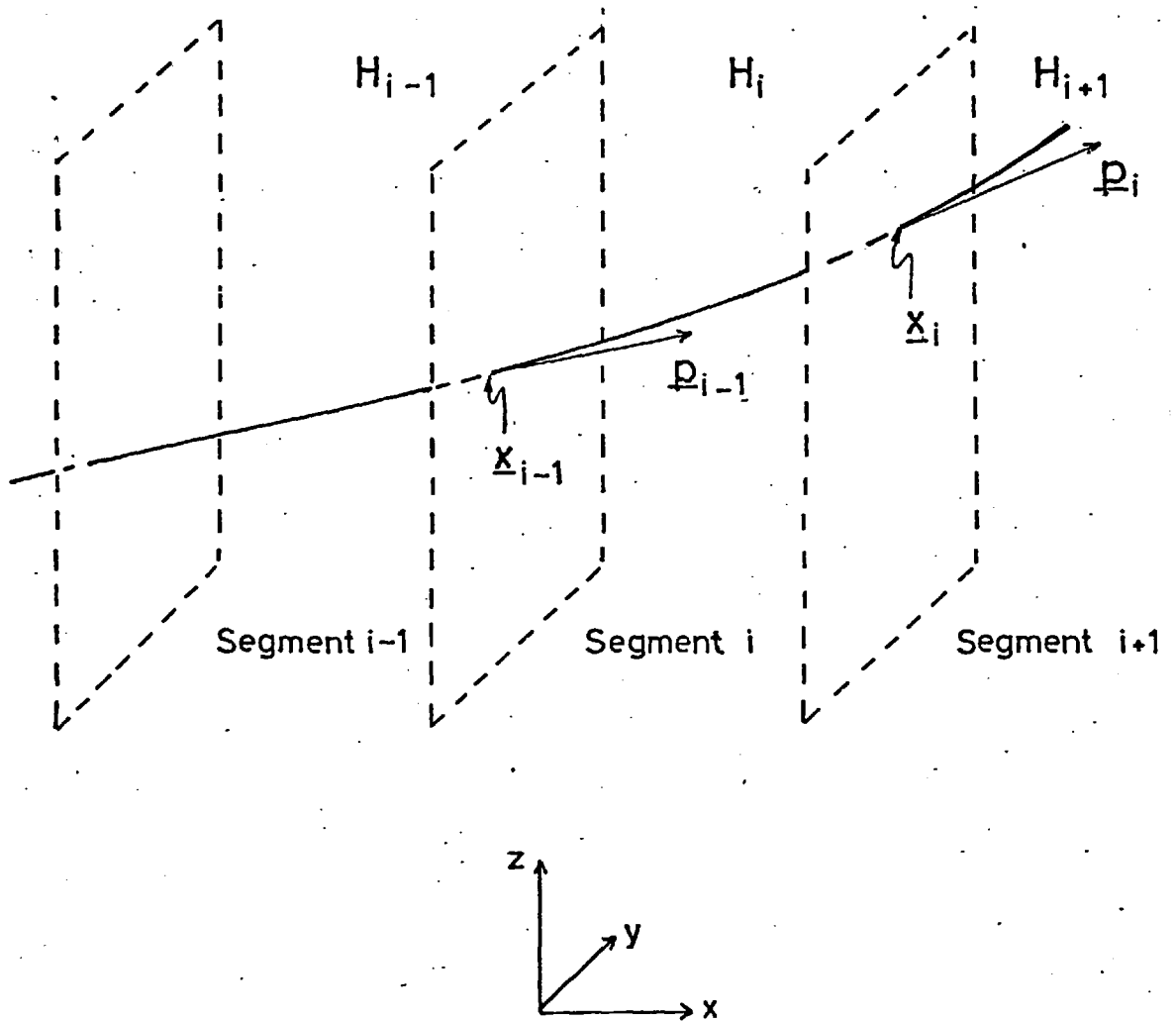
1 Interaction point $\pi^- p \rightarrow \Lambda K$

2 Decay point $\Lambda \rightarrow \pi^- p$

3 Decay point $K \rightarrow \pi^+ \pi^-$

o Coordinates calculated and stored

Figure 5.2 Particle Following in SPRUCE



CHAPTER 6

The Backward Peak in $\pi^- p \rightarrow \Lambda K^0$ - Experimental Results

6.1 Introduction

From the data summary tape it is possible to find the number of events seen and the polarisation of those events as a function of the four momentum transfer. However, before these can be presented as a differential cross-section, each event must be multiplied by the inverse of the probability that such an event should be detected. This number is called the "weight" of an event and is in general a function of u .

The first part of this chapter is a discussion of the various factors which make up this weight. Except for the u -dependent effect discussed in 6.2 (ii), they are summarised in table 6.1.

6.2 The Weight of an Event

i) The Counter R'

At the time that the experiment was planned no data existed on K_2^0 interactions on various nuclei and in order to estimate and correct for K_2^0 interactions in the R counters the duplicate counter R' was used as a monitor. For those events with the K_2^0 in the quadrant containing the counter R', the fraction in which the counter didn't trigger was plotted as a function of u (fig 6.1 a). All events in this quadrant with $0.1 \leq u_0 - u \leq 0.4$ must have had the K_2^0 passing through R', and from this region of fig 6.1a the mean free path, λ , of K_2^0 's in R' was derived as a function of momentum (fig 6.1 b). It appears to be flat with a value of 30 cms.

Using this value the curve shown on fig 6.1a was obtained. It agrees well with the experimental data in the extrapolated regions $u_0 - u < 0.1$ and $u_0 - u > 0.4$ (GeV/c)².

ii) Geometry and Absorption Effects

The correction due to the geometry of the spark chambers and counters and for K_2^0 absorption was calculated by Monte Carlo. A fiducial volume of the following dimensions was defined (in cm)

$$-84 \leq x \leq 53$$

$$-20 \leq y \leq 20$$

$$-20 \leq z \leq 20$$

and the corrections calculated from 10000 Monte Carlo events at the three experimental conditions. These are shown in fig 6.2.

The dominant correction at small $u_0 - u$ is produced by the Λ decaying before the F counters and by K_2^0 interactions in the R counters. At higher $u_0 - u$, the probability that the charged particles from the Λ decay will trigger the downstream counters drops rapidly, particularly at 4 GeV/c.

As the beam passes down the target it is attenuated and there is also a small probability that the Λ will be absorbed after production. The π^-p total cross-section is ~ 30 mbarns in the region of 5 GeV/c, the Λp total cross-section should be similar to that for proton-proton, i.e. ~ 40 mbarns. Total absorption due to the two effects is about 6%.

Absorption of Λ 's passing through the F counters was calculated assuming the same mean free path as for K_2^0 's resulting in a correction of 1.09.

iii) Analysis Losses

Throughout the analysis chain from photographs to D.S.T. events can be lost. To correct for this we must determine both the total number lost and the distribution of event types.

a) Scanning Efficiency. About 10% of the film was rescanned and the efficiency determined by comparison of the two scans. The loss

was small - 3%. The events lost were most likely to be those obscured by a number of background tracks or events occurring close to the edges of the chambers. The latter effect was eliminated by choosing a fiducial volume well inside the physical limits of the chamber. The first effect produces a very small depletion of events in the region where the great majority of events occur anyway and is unlikely to result in any bias.

b) H.P.D. Losses. At 4 GeV/c 84% and at 6 GeV/c 92% of scanned events were successfully measured by the H.P.D. A sample of 50% of the rejected events were examined on a display console attached to a CDC 3100 computer (see section 3.3), and of these about 70% were "recovered" by the intervention of the operator. These events were analysed normally and at both energies it was found that a Λ decay was less likely to have failed the pattern recognition at the first attempt. Those rescued events fitting the $\pi^- p \rightarrow \Lambda K$ reaction showed the same features as the main sample i.e. a sharp backward peak and a high positive polarisation. We therefore conclude that the rejected events are unbiased. The correction to the backward peak was 1.07 and 1.05 at 4 GeV/c and 6 GeV/c respectively.

c) Losses in THRESH and GRIND. Occasionally THRESH was unable to reconstruct a track or GRIND to converge during kinematic fitting and the event failed to give a fit. Failures in THRESH were rare (<1%) but at 4 GeV/c and 6 GeV/c 16% and 10% of events respectively, failed to give any kinematic fit with a confidence level $> 0.1\%$. The distribution of the x-coordinate of the decay point indicates that many of these events were χ conversions or K_2^0 interactions. However, some of those within the exponential region could have been Λ decays; the number was determined as follows.

For all "no fit" events the histogramme of the measured momentum of the positive track was subtracted from that of the measured momentum of the negative track. This compound histogramme showed excess of events with a high momentum positive track and low momentum negative track, i.e. Λ 's, and if we assume that these Λ events contain the same proportion of backward peak events as the fitted Λ 's, we obtain correction factors of 1.02 at both 4 and 6 GeV/c.

Four per cent of the events on the D.S.T. were removed by cuts on the error in the measurement of the decay point. As well as removing badly reconstructed events these cuts will also remove "non-events" of two tracks crossing close to each other and a correction of 4% will be an upper limit.

iv) Contaminations

There are three major sources of contamination of the backward peak channel; spurious fits, the reactions $\pi^- p \rightarrow \Sigma^0 K^0$ and $\pi^- p \rightarrow \Lambda K^0 \pi^0$, and interactions in the walls of the target.

Fits to the non-baryon conserving reaction $\pi^- p \rightarrow \bar{\Lambda} K$, attempted by GRIND must be spurious. Spurious fits to the reaction of interest $\pi^- p \rightarrow \Lambda K$ are equally likely, and the number of fits to the former reaction indicated a contamination to the latter of 1% and 6% at 4 and 6 GeV/c respectively. However, the "differential cross-section" for the $\pi^- p \rightarrow \bar{\Lambda} K$ fits was rather flat and to eliminate any possible bias the correction was made by subtraction of histogrammes.

A source of contamination from true Λ 's are the reactions $\pi^- p \rightarrow \Sigma K^0$ and $\pi^- p \rightarrow \Lambda K \pi^0$ when the γ -rays were not detected by the $\rightarrow \Lambda \gamma$ $\rightarrow 2 \gamma$ anticoincidence. If the missing mass to the Λ was in the region of the K^0 mass, then a fit to the backward peak could be obtained.

Figure 6.3 shows the missing mass squared to the Λ at both energies for those events having a unique Λ fit. Background is seen to the right of the K^0 peak. Some of this is due to spurious Λ fits at the decay vertex and is removed by subtracting the equivalent histogramme for $\bar{\Lambda}$ fits. A small background remains giving an estimated contamination under the peak of 2% at 4 GeV/c and 1% at 6 GeV/c.

Most of those events at 6 GeV/c with missing mass squared between 4 and 6 $(\text{GeV}/c^2)^2$ are from the reaction $\pi^- p \rightarrow \Lambda \bar{\Lambda} n$. This reaction cannot give a fit to $\pi^- p \rightarrow \Lambda K$.

Several rolls of film were taken with an empty target. The

number of backward peak events collected was small (~ 20 at each energy) but allowed an estimate to be made of the background from interactions on the target walls.

v) Decay Mode Correction

Backward peak events are only seen when the Λ decays in a charged mode and when the K^0 produced in the interaction decays outside the R counter. In general the latter occurred when the K^0 decayed as a K_2^0 ; however, the K_2^0 could decay within the R counters and a K_1^0 could in fact escape from the anticoincidence. These effects were small and are shown in fig 6.4 together with the necessary total correction. These curves are valid for both 4 GeV/c and 6 GeV/c since the K^0 angle and momentum is a very weak function of beam momentum for a given value of u .

A K^0 decays 50% of the time as a K_2^0 ; the branching fraction $\Lambda \rightarrow p \pi^-$ is $0.624 \pm 0.012^{32)}$. The total correction for decay modes is thus 3.06 ± 0.06 .

vi) Electronics Losses

If a pulse occurs in any of the anticoincidence counter electronics during the gate of the coincidence electronics no trigger occurs. Normally the anticoincidence would be triggered because charged particles produced in an interaction in the target went through it. However, it is possible for the anticoincidence to produce a noise pulse or a pulse due to scattered particles unconnected with the event, resulting in a loss of events.

This loss was determined by mistiming the anticoincidence electronics by 100 nsecs and counting the number of accidental triggers. The loss with the anticoincidence in time will be the same.

6.3 Differential and Total Cross-Sections

The differential cross-sections after the application of all correction factors are shown in fig 6.5 and tabulated in table 6.2. They

are derived from a total of ~ 3000 events at 4 GeV/c and ~ 1080 events at 6 GeV/c with $0 \leq u_0 - u \leq 1.6$ and with χ^2 probabilities $> 3\%$ at decay and interaction vertices. The normalisation was determined from the weighted number of events and the number of beam particles to give the total cross-section:-

$$\sigma_T = \frac{N}{A \rho l n_\chi} \text{ cm}^2 \quad (1 \text{ cm}^2 \equiv 10^{30} \text{ } \mu\text{barns})$$

where N = number of events

n_χ = number of beam particles

A = Avogadro's number ($= 6.0 \times 10^{23}$)

ρ = density of liquid hydrogen ($= 0.070 \text{ gm/cc}$)

l = length of target ($= 40 \text{ cm}$)

The values obtained for σ_T are:-

$$1.77 \pm 0.23 \text{ } \mu\text{barns} \quad \text{at } 4.1 \text{ GeV/c}$$

$$0.44 \pm 0.055 \text{ } \mu\text{barns} \quad \text{at } 6.2 \text{ GeV/c} \quad \text{for } 0 \leq u_0 - u \leq 1.6 \text{ (GeV/c)}^2$$

A number of checks were made on the self-consistency of the data. The values obtained for the total cross-section from the different beam conditions (table 2.1) were compared to the value from all data at both energies and are listed in table 6.3.

In particular there is no evidence for asymmetry which could show up in differences between the positive and negative field polarity data, and the fact that there is consistency between the numbers for the two fields at 6 GeV/c gives confidence in the Monte Carlo corrections.

As a further check the differential cross-sections were plotted for a fiducial volume defined by:

$$-80 \leq x \leq 0$$

$$-15 \leq y \leq 15$$

$$-15 \leq z \leq 15$$

They showed no significant variations from those given in fig 6.5. The values obtained for σ_T for this second fiducial volume were:-

$$1.71 \pm 0.25 \text{ } \mu\text{barns} \quad \text{at } 4 \text{ GeV/c}$$

$$0.425 \pm 0.06 \text{ } \mu\text{barns} \quad \text{at } 6 \text{ GeV/c} \quad \text{for } 0 \leq u_0 - u \leq 1.6 \text{ (GeV/c)}^2$$

6.4 Lambda Polarisation

Since the decay of the Λ is observed in the spark chambers its polarisation can be measured. In the Λ centre of mass system, the angular distribution of the decay proton with respect to the normal to the production plane is given by

$$W(\vartheta) = 1 + \alpha P \cos \vartheta \quad 6.4.1$$

where

$$\cos \vartheta = \frac{\underline{n} \cdot \underline{p}_p}{|\underline{p}_p|} ; \quad \underline{n} = \frac{\underline{p}_\pi \wedge \underline{p}_k}{|\underline{p}_\pi \wedge \underline{p}_k|}$$

and α , the decay asymmetry parameter of the $\Lambda = 0.646^{32)}$.

The distributions in $\cos \vartheta$ for all events with $0 \leq u_0 - u \leq 1$ are shown in fig 6.6. They are clearly consistent with the linear form of equation 6.4.1 and show a high positive value of P . To obtain the polarisation as a function of u the events were divided into intervals in u and P calculated for each of these intervals by the method of moments³³⁾

$$P = \frac{3}{\alpha N} \sum_{i=1}^N \cos \vartheta_i = \frac{3}{\alpha} \overline{\cos \vartheta}$$

$$\sigma = \frac{3}{\alpha \sqrt{N}} \sum_{i=1}^N \left[\cos^2 \vartheta_i - \overline{\cos^2 \vartheta} \right]$$

The results for the two energies are shown in fig 6.7 and tabulated in table 6.4.

To search for possible biases several checks were made.

a) The data for positive and negative magnetic field polarity were consistent at both energies.

b) Events identified as K_1^0 decays showed zero polarisation over the whole range of scattering angle.

c) Monte Carlo calculations showed that no bias in $\cos \vartheta$ was introduced by loss of events due to counter and chamber geometry. However, a small deviation from a flat distribution in the angle describing the

polarisation along the line of flight of the Λ (an angle which must have a flat distribution by parity conservation) at 4 GeV/c, was reproduced by the Monte Carlo calculations. Also reproduced by Monte Carlo was the loss of events at higher $u_0 - u$ values at 4 GeV/c when the production plane was vertical. We can, therefore, feel confident that any instrumental bias in ϕ would have been reproduced by the Monte Carlo programme.

d) There is no evidence for increased contamination in the region $u_0 - u \geq 0.6 \text{ (GeV/c)}^2$ where the polarisation appears to drop from around a value of +1.

TABLE 6.1Correction Factors

	4 GeV/c	6 GeV/c
Anticoincidence losses	1.04 ± 0.02	1.08 ± 0.02
Scanning efficiency	1.03 ± 0.01	1.03 ± 0.01
Cuts on χ^2	1.06	1.06
Cut on apex	1.02 ± 0.02	1.02 ± 0.02
H.P.D. losses	1.07 ± 0.02	1.05 ± 0.02
Losses in GRIND	1.02 ± 0.02	1.02 ± 0.02
Beam attenuation and Λ interactions in hydrogen	1.06 ± 0.02	1.06 ± 0.02
Λ interactions in F counters	1.09 ± 0.05	1.09 ± 0.05
Contaminations from Σ^0, π^0	0.98 ± 0.02	0.99 ± 0.01
Interactions on target walls	0.88 ± 0.03	0.92 ± 0.04
Decay mode correction	3.06 ± 0.06	3.06 ± 0.06
Product	3.84 ± 0.31	4.14 ± 0.35

TABLE 6.2Differential Cross-Section

u interval	$d\sigma/du (\mu\text{b}/\text{GeV}/c)^2$	
	4 GeV/c	6 GeV/c
0.0 - 0.025	8.30 ± 0.43	1.99 ± 0.20
0.025 - 0.05	8.15 ± 0.42	1.96 ± 0.20
0.05 - 0.075	7.15 ± 0.38	2.05 ± 0.20
0.075 - 0.10	6.67 ± 0.38	1.47 ± 0.16
0.10 - 0.15	5.03 ± 0.23	1.09 ± 0.10
0.15 - 0.20	4.08 ± 0.20	0.89 ± 0.09
0.20 - 0.25	2.79 ± 0.17	0.85 ± 0.08
0.25 - 0.30	2.20 ± 0.15	0.58 ± 0.07
0.3 - 0.4	1.08 ± 0.07	0.33 ± 0.04
0.4 - 0.5	0.66 ± 0.06	0.20 ± 0.03
0.5 - 0.6	0.34 ± 0.05	0.075 ± 0.02
0.6 - 0.7	0.12 ± 0.03	} 0.04 ± 0.01
0.7 - 0.8	0.11 ± 0.03	
0.8 - 1.0	0.10 ± 0.025	0.015 ± 0.007
1.0 - 1.2	0.085 ± 0.025	0.02 ± 0.008
1.2 - 1.4	0.075 ± 0.02	0.02 ± 0.01
1.4 - 1.6	0.12 ± 0.04	0.02 ± 0.015

TABLE 6.3Consistency check on Total Cross-Sections

Run number (i)	σ_T^i / σ_T	Energy
1	0.97 ± 0.11	6 GeV/c
2	0.86 ± 0.11	
3	1.04 ± 0.08	
4	1.14 ± 0.08	
5	0.96 ± 0.08	
6	0.98 ± 0.04	4 GeV/c
7	1.01 ± 0.03	

TABLE 6.4Polarisation as a Function of $u_0 - u$

$u_0 - u$ interval	4.1 GeV/c	6.2 GeV/c
0.0 - 0.025	0.57 ± 0.14	0.30 ± 0.25
0.025- 0.05	0.44 ± 0.14	0.86 ± 0.21
0.05 - 0.075	0.59 ± 0.15	0.93 ± 0.22
0.075- 0.10	0.50 ± 0.16	0.70 ± 0.26
0.10 - 0.125	0.70 ± 0.16	} 0.81 ± 0.20
0.125- 0.15	1.14 ± 0.17	
0.15 - 0.20	1.07 ± 0.13	0.91 ± 0.23
0.20 - 0.25	0.98 ± 0.16	} 1.28 ± 0.16
0.25 - 0.30	1.08 ± 0.18	
0.30 - 0.35	1.07 ± 0.24	} 1.49 ± 0.25
0.35 - 0.40	1.33 ± 0.26	
0.4 - 0.5	1.18 ± 0.24	0.86 ± 0.33
0.5 - 0.7	0.45 ± 0.32	$- 0.03 \pm 0.44$
0.7 - 1.0	$- 1.13 \pm 0.59$	0.14 ± 0.56

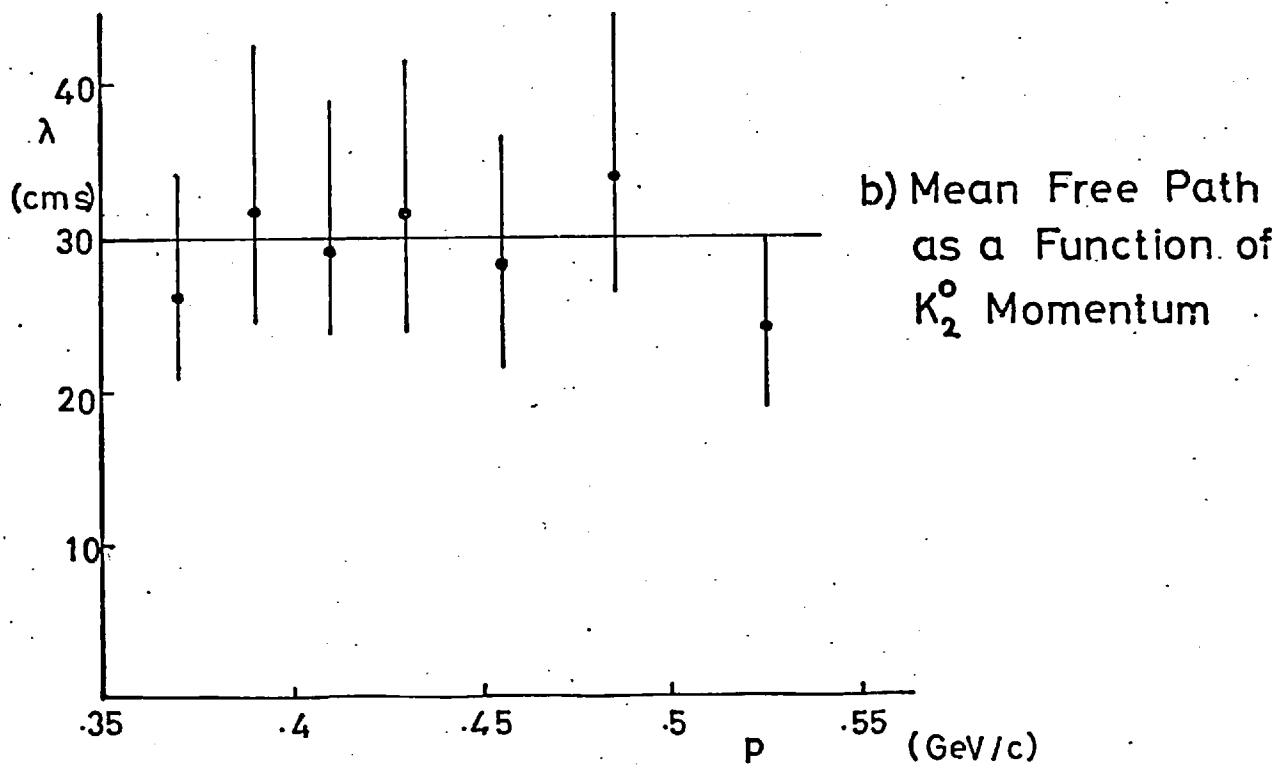
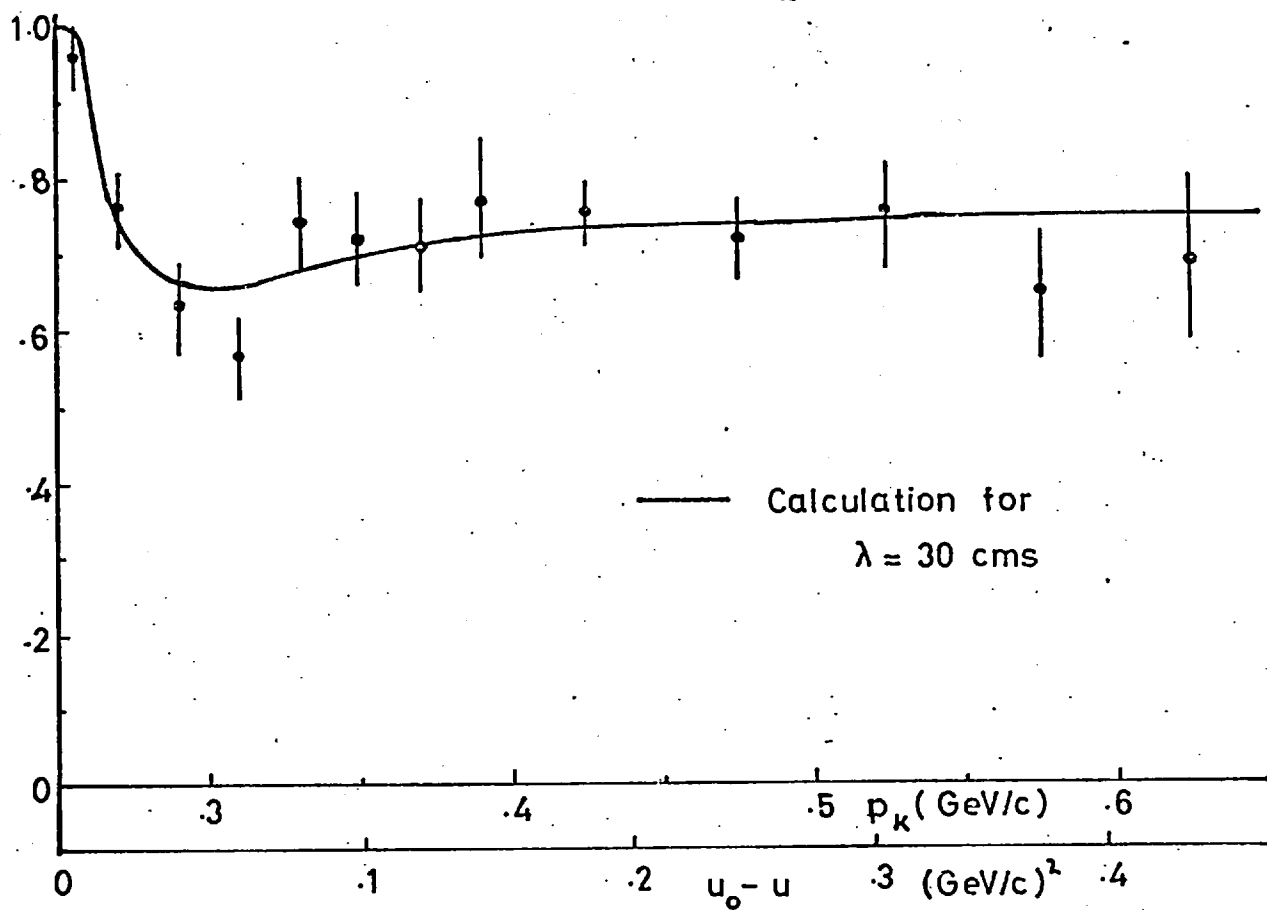
Figure 6.1 a) Transmission of K_2^0 Through R' 

Figure 6.2 Geometrical and Counter Weight

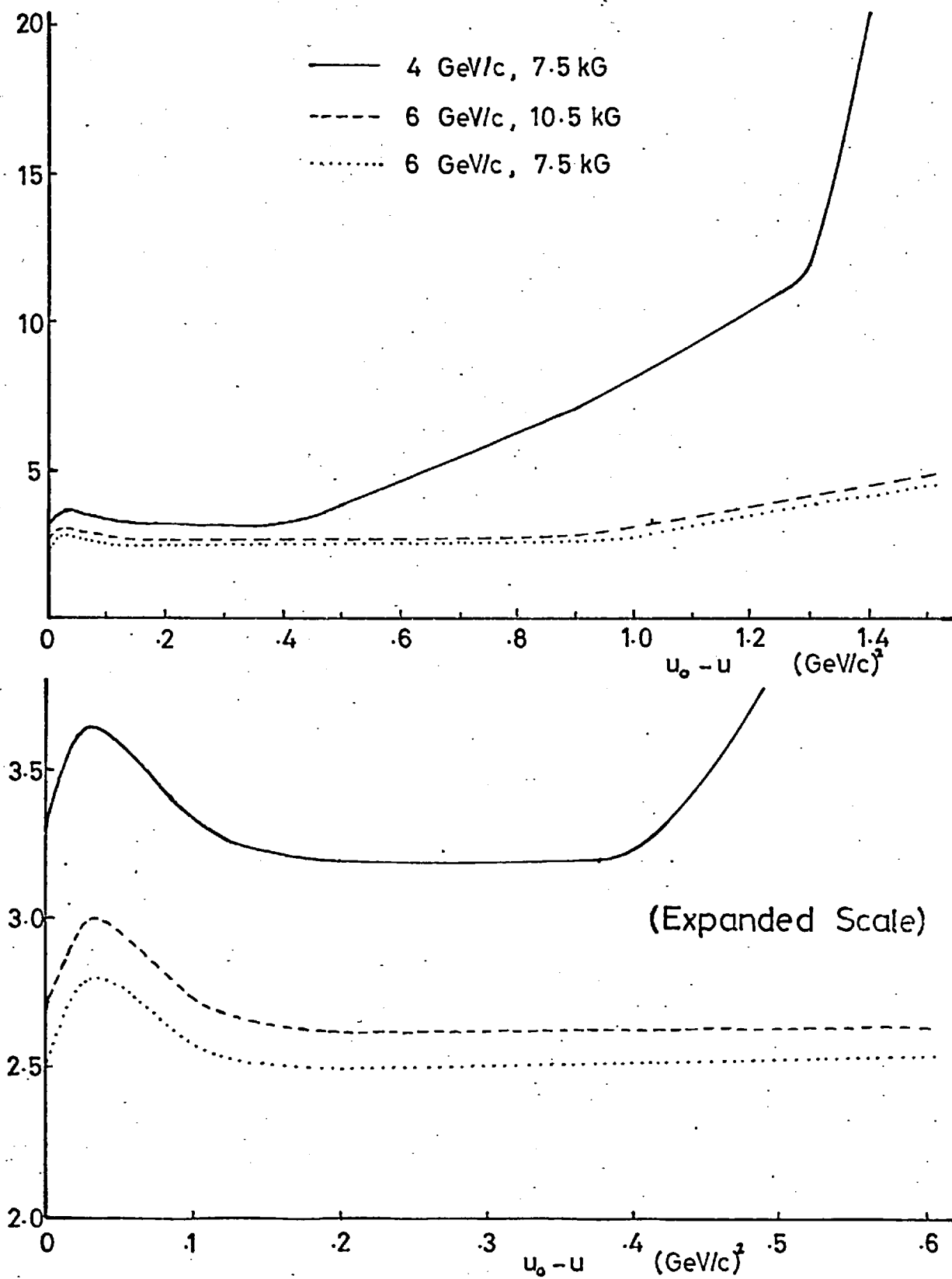


Figure 6.3

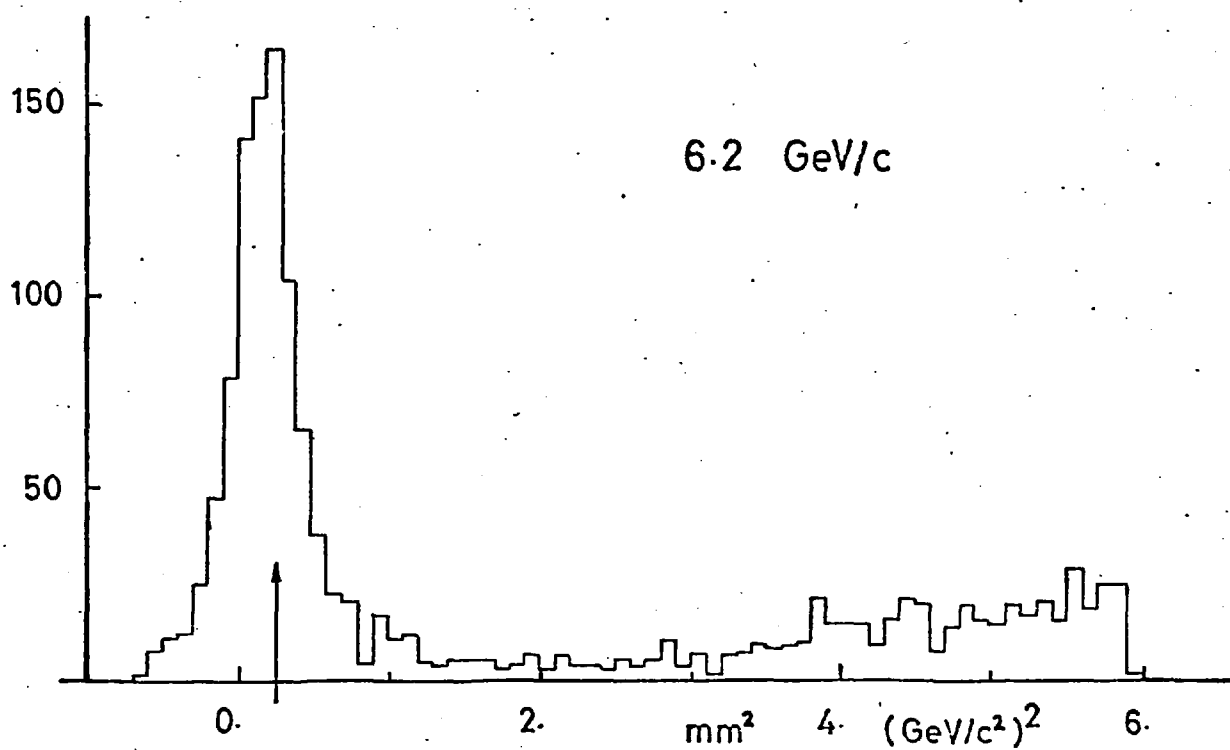
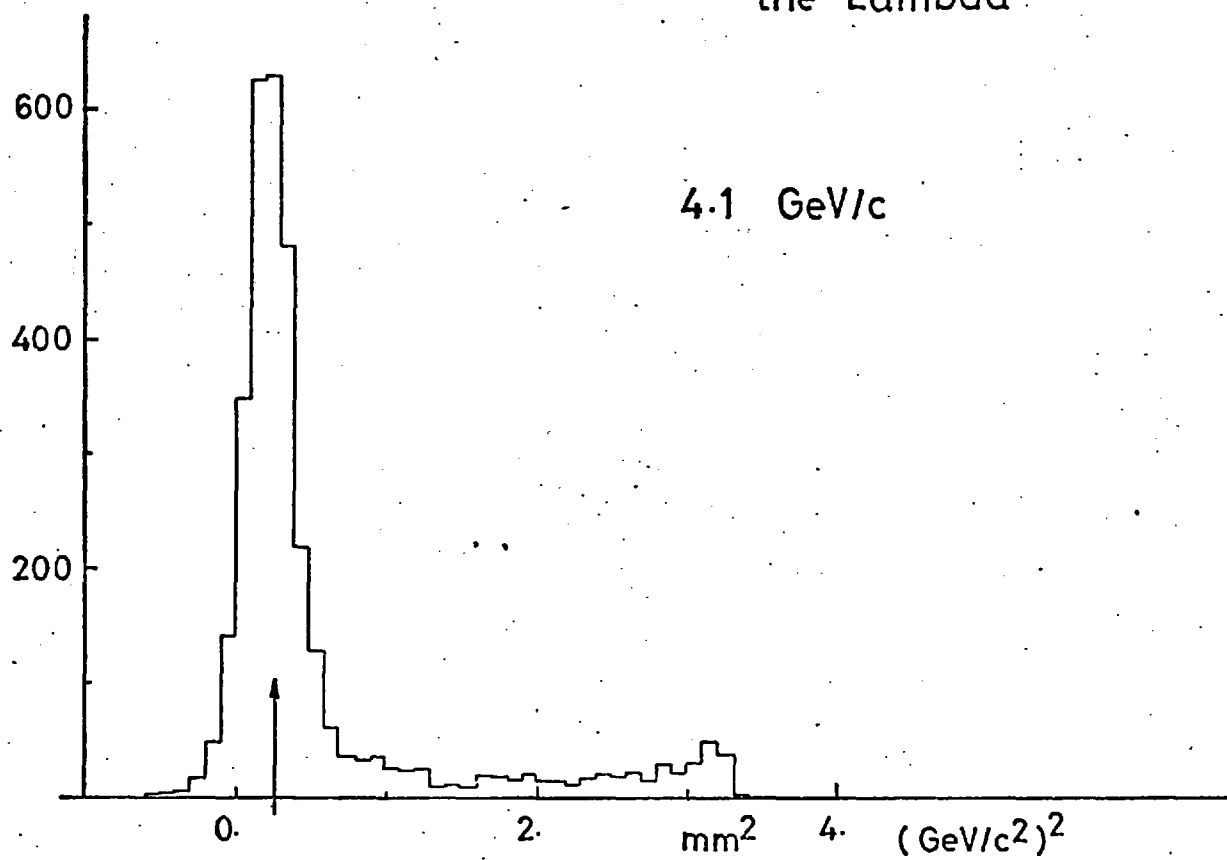
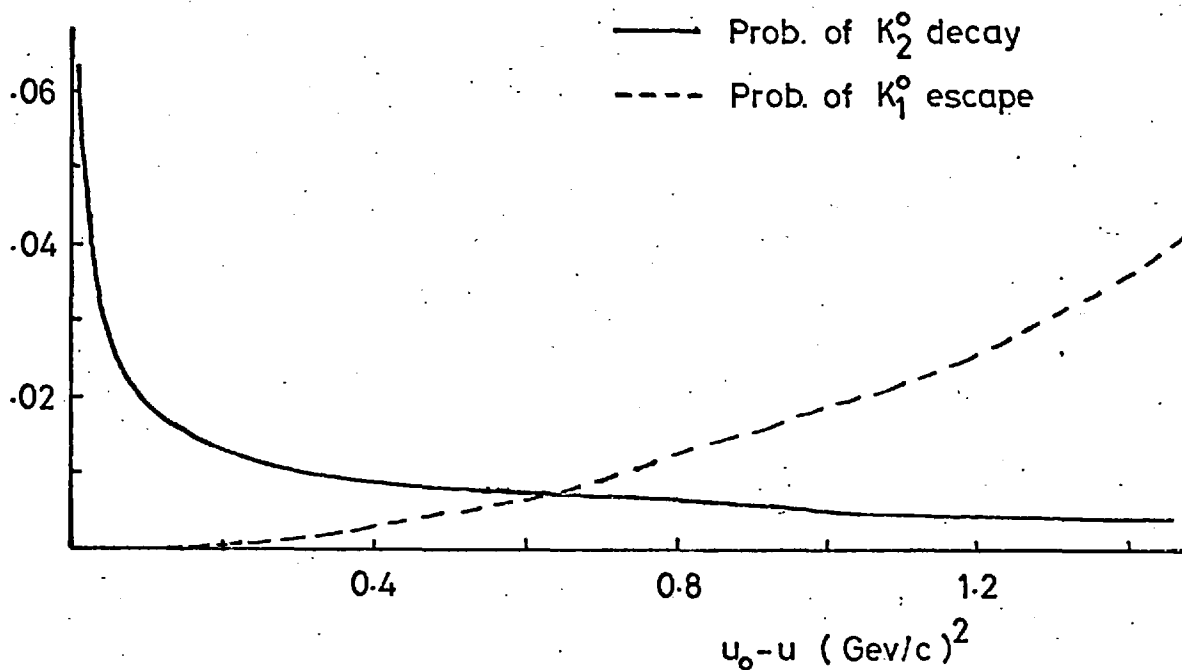
Missing Mass Squared to
the Lambda

Figure 6.4

Effect of K_1^0 escape and K_2^0 decay in anticoincidence

Total Correction

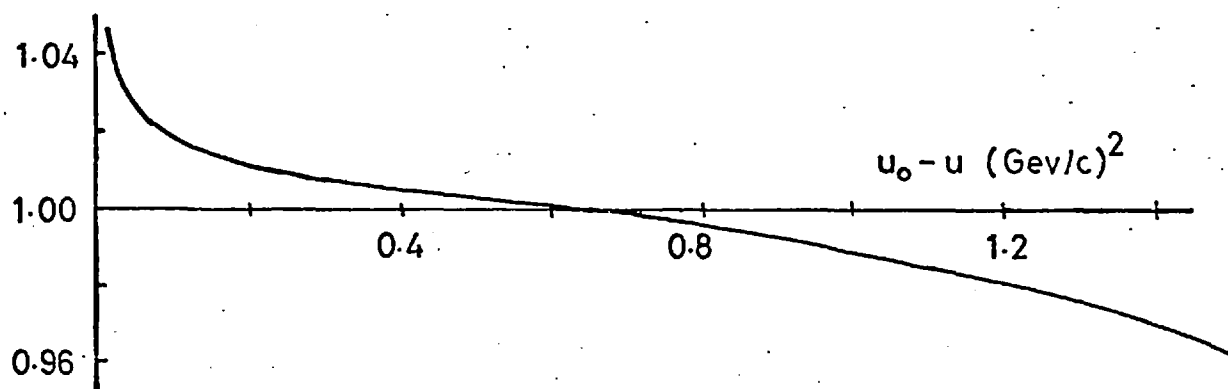


Figure 6.5 Differential Cross-Section for
 $\pi^- p \rightarrow \Lambda K^0$

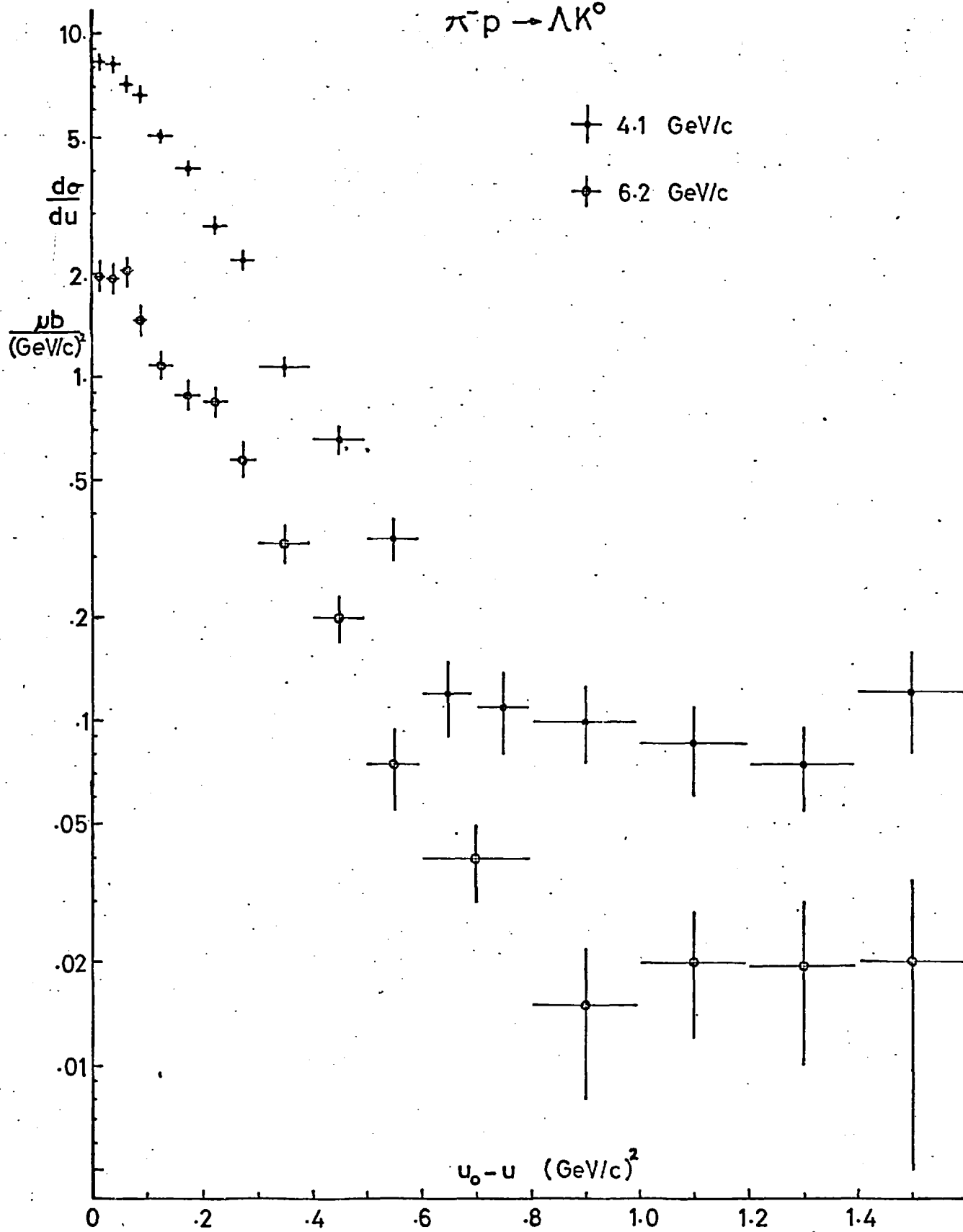
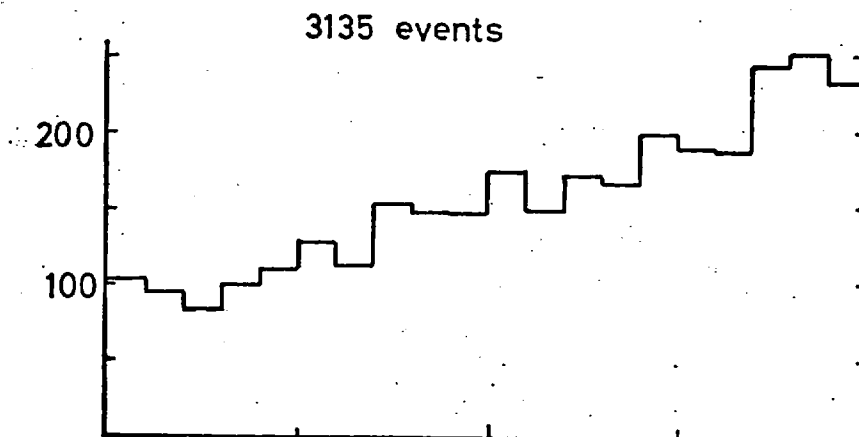


Figure 6.6 $\cos \theta_p$ for $0 \leq u_0 - u \leq 1.0 \text{ (GeV/c)}^2$

4.1 GeV/c



6.2 GeV/c

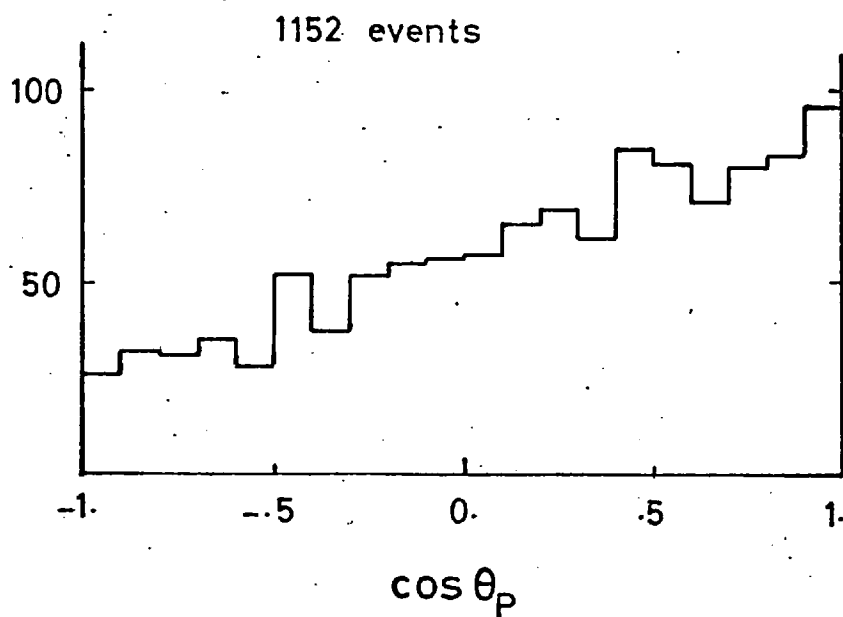
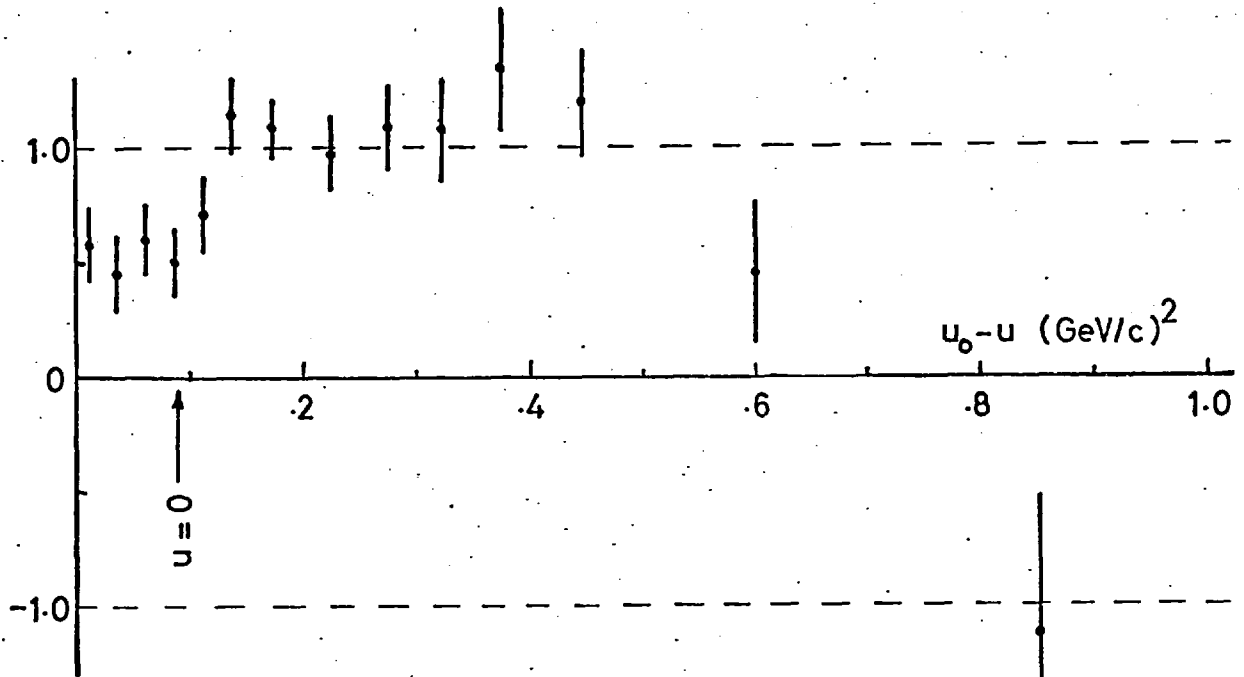
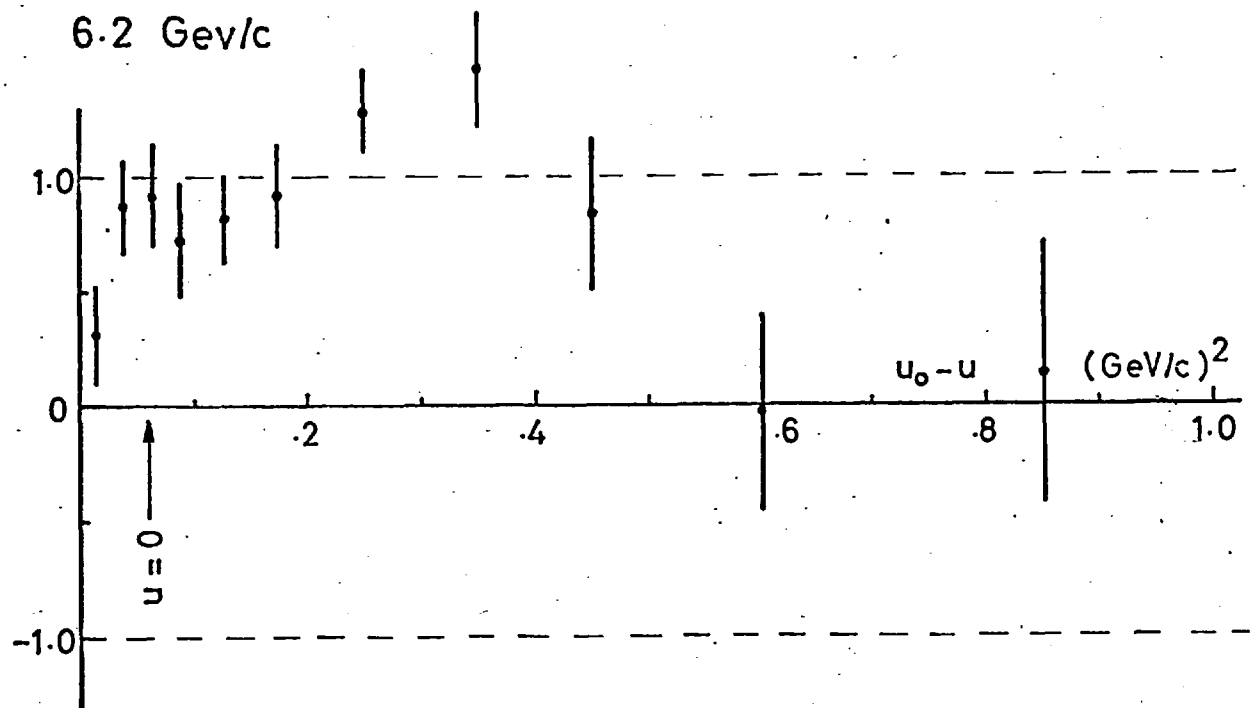


Figure 6.7 Λ Polarisation

4.1 GeV/c



6.2 GeV/c



CHAPTER 7

The Backward Peak in $\pi^- p \rightarrow \Lambda K^0$ — Some Theoretical Considerations7.1 Introduction

In contrast to the large amount of data published on elastic scattering in the backward direction in $\pi^- p$ and $K^- p$ interactions very little has been published on inelastic two body backward peaks, presumably due to the difficulty of carrying out these low cross-section experiments. (The situation is improving however; recently results were reported on $\pi^- p \rightarrow \pi^0 n$ backward scattering³⁴⁾). The only other work on the backward peak of $\pi^- p \rightarrow \Lambda K^0$ at energies ≥ 3 GeV known to the author is as follows:-

a) An early experiment using the magnet spark chamber at 5, 7 and 12 GeV/c resulted in 113, 90 and 25 events respectively¹¹⁾. This experiment indicated a high Λ polarisation, but there were too few events to give the u -dependence. Total and differential cross-sections were given and will be considered in this chapter together with the present results.

b) An experiment by Hoang et al³⁵⁾ using a spark chamber spectrometer at 4 GeV/c produced 131 events. However they could not differentiate between $\pi^- p \rightarrow \Lambda K^0$ and $\pi^- p \rightarrow \Sigma^0 K^0$ and their results are for the sum of these two reactions. Parameterizing the backward peak by $\frac{d\sigma}{du} \propto e^{a(u_0 - u)}$ they found $a = 2.7 \pm 0.6$ for $0 \leq u_0 - u \leq .9$ (GeV/c)².

They also found $\left. \frac{d\sigma}{du} \right|_{u_0} \sim 0.5 \mu\text{b}/(\text{GeV}/c)^2$ and Λ polarisation consistent with zero.

None of these results is consistent with the present experiment.

c) Crennell et al.³⁶⁾ found 10 events in a bubble chamber experiment at 6 GeV/c of which they suspect 6 were ΛK^0 , 3 were $\Xi^0 K^0$ and one was undetermined. They report an experimental slope for the backward peak of 5.7 ± 2.0 (GeV/c)⁻² and their data indicate a total cross-section of $\sim 0.5 \mu\text{b}$. Both these numbers are consistent with the present experiment.

7.2 The Cross-sections

In 1966 Morrison suggested³⁷⁾ that for two-body reactions at high energy, the total cross-section could be parameterised by

$$\sigma_T \propto p_{\text{lab}}^{-n}$$

and that the value of n was characteristic of the particle exchanged in the reaction; for elastic-like reactions $n \sim 0.0 - 0.5$, for non-strange meson exchange $n \sim 1.5$, for strange meson exchange $n \sim 2$ and for baryon exchange $n \sim 3 - 4$.

This behaviour can be readily understood in terms of the Regge pole model. The optical theorem states that $\sigma_T = 4\pi/k \text{Im}f(0)$ where $f(0)$ is the total amplitude in the exact forward (or backward) direction. In the Regge model, at high energies one trajectory only is assumed to dominate (the highest lying trajectory on the Chew-Frautschi plot) and the optical theorem leads to the relationship $\sigma_T \propto (s/s_0)^{\alpha(0)-1}$. In this energy region $s \propto p_{\text{lab}}^2$ and since, as we have seen in chapter 1, $d\sigma/dt(t=t_0) \propto s^{2\alpha(0)-2}$ then it would be expected that $n = -(2\alpha(0)-2)$ in the relationship $\sigma_T \propto p_{\text{lab}}^{-n}$. Inspection of the trajectories for the various classes of exchanged particle mentioned above indicates values of n in agreement with those determined from total cross-section measurements.

In figures 7.1 and 7.2 σ_T and $d\sigma/du$ ($u=u_0$) are plotted as a function of p_{lab} and s respectively for the data from the present experiment and reference 11. Linear relationships on the log-log plots are clearly seen and the fitted values of n and α are :-

$$\begin{aligned} n &= 3.22 \pm 0.29 \\ &\quad - 0.10 \\ \alpha &= -0.82 \pm 0.13 \end{aligned}$$

7.3 The u-dependence of Differential Cross-sections and Polarisation

In general, because of the unknown nature of the residue function $\beta(u)$, the Regge pole model says very little about the u-dependence of an amplitude. It does however say something about the behaviour of amplitudes at certain values of u, since it can be shown from the analyticity and factorisation properties of Regge residues that an amplitude must vanish at wrong signature nonsense values of α .*

Thus if only one amplitude contributes to a reaction the differential cross-section must go to zero at wrong signature nonsense points; if more than one amplitude contributes, a dip or change of slope in the cross-section would be expected at these points on the trajectories of the dominant amplitudes.

A well known example of such a dip occurs at $u = -0.2 \text{ (GeV/c)}^2$ in π^+p backward elastic scattering. Both N_α and Δ_s exchange may contribute to this reaction, and the dip occurs at the point at which the N_α trajectory passes through the wrong signature nonsense point $\alpha = -\frac{1}{2}$. Moreover since in π^-p backward elastic scattering only Δ_s exchange is expected, at $u = -0.2$ Clebsch-Gordon coefficients should lead to the relation

$$\frac{d\sigma}{du} (\pi^-p) : \frac{d\sigma}{du} (\pi^+p) = 9 : 1$$

This ratio is observed experimentally.

7.4 A Regge Pole Fit

Barger, Cline and Matos³⁸⁾ attempted a Regge pole fit to preliminary results from the present experiment³⁹⁾ based on the following considerations:-

a) The known trajectories available for exchange in the u-channel of $\pi^-p \rightarrow \Lambda K$ are ξ_ω , ξ_ρ , ξ_γ , ξ_s . Under the assumption of

* Some of the jargon used in this paragraph is explained briefly in Appendix B.

linear parallel trajectories, two approximately exchange degenerate combinations can be seen on the Chew-Frautschi plot, $(\underline{z}_\omega, \underline{z}_\chi)$ and $(\underline{z}_\rho, \underline{z}_\delta)$ (fig 7.3). Treating them as exchange degenerate gives the following parameters for the trajectories:-

$$(\underline{z}_\omega, \underline{z}_\chi) : \alpha = -0.86 + 0.9u$$

$$(\underline{z}_\rho, \underline{z}_\delta) : \alpha = -0.30 + 0.9u$$

b) The highest lying trajectory, $\underline{z}_\delta$, would be expected to be dominant at high energies but would lead to an energy dependence of $d\sigma/du(u=u_0)$ of $s^{-2.5}$ corresponding to $\alpha(0) = -0.25$. As we have seen the energy dependence results in $\alpha(0) \sim -0.8$, a value consistent with that expected for $\underline{z}_\omega$ or $\underline{z}_\chi$ exchange.

c) At $u \sim -0.6$ (GeV/c)² the experimental data indicate a change of slope in the differential cross-section and a zero in the polarisation at both 4 and 6 GeV/c. Such an effect would be expected if the reaction is dominated by the $\underline{z}_\omega$ and $\underline{z}_\chi$ amplitudes, since a linear $\underline{z}_\delta$ trajectory would pass through the wrong signature nonsense point $\alpha = -3/2$ at that value of u .

Assuming, therefore that only the $\underline{z}_\omega$ and $\underline{z}_\chi$ amplitudes are present in the reaction, and that the trajectories are exchange degenerate they parameterised the corresponding residue functions according to the formulation of Igi³⁸⁾

$$\beta_i(\sqrt{u}) = [A_i(\sqrt{u} + M_i) + B_i(u + M_i^2)] e^{C_i u}$$

where i labels the amplitude ($i = 1$ for $\underline{z}_\omega$, $i = 2$ for $\underline{z}_\chi$) $M_1 = 1.189$, $M_2 = 1.660$ (the masses of the lowest lying particles on the two trajectories) and where A_i , B_i , C_i are 6 parameters to be fitted.

In performing the fit, Barger et al. considered only the data at 5, 6, 7 and 12 GeV/c and then extrapolated to 4 GeV/c. The result was a good fit to the polarisation data (which in both the experiment and the theoretical model is independent of energy) but a very poor fit to

the differential cross-section at 4 GeV/c where theory predicted a broader peak than was observed and also an increase in the value of $d\sigma/du(u=u_0)$ of $\sim 30\%$.

The fit gave trajectory parameters $\alpha = -0.84 + 1.0u$, in excellent agreement with the values in the resonance region and good evidence for linear trajectories.

The discrepancy in the shape of the 4.1 GeV/c backward peak has now been resolved; the preliminary results of ref.39 did not include the u -dependent geometry correction discussed in chapter 6 (this effect is much less important at 6.2 GeV/c), and it therefore seemed worthwhile to repeat the fit of Barger et al on the final data.

Since there was essentially no change between the values of $d\sigma/du(u=u_0)$ for the preliminary and final analyses at 4 GeV/c two approaches were made to the fitting of the data:

FIT I : 4 GeV/c differential cross-section not included in the fit and the extrapolation of the fit at higher energies compared with the experimental results.

FIT II : 4 GeV/c differential cross-section fitted with the rest of the data.

The results of FIT I are shown in fig 7.4. (58 data points and 8 parameters give a χ^2 of 62). The trajectory α obtained from this fit is

$$\alpha = -0.85 + 1.05u$$

in reasonable agreement with the parameters of α in the resonance region. However, the prediction for the differential cross-section at 4 GeV/c is not in agreement with the data; the curve lies $\sim 50\%$ above the data over most of the backward peak.

When the 4 GeV/c differential cross-section is included the fit becomes much worse (fig 7.5) ; 75 data points and 8 parameters lead to a χ^2 of 188 (the fit to the polarisation data is not shown; it is very

similar to the fit in fig 7.4b). The parameters for α for FIT II are $\alpha = -0.88 + 1.09u$.

Both these fits indicate a normalisation difficulty between the different momenta which cannot be explained by the parameterisation chosen by Barger et al. An attempt was made to change the parameterisation by allowing:-

$$\begin{aligned} \text{a) non-degenerate trajectories i.e. } \alpha(\xi_u) &= a_1 + b_1 u \\ \alpha(\xi_v) &= a_2 + b_2 u \end{aligned}$$

b) non-linear trajectories of the form

$$\alpha = (a + bu)/(1 - cu)$$

Neither device led to a significantly improved fit.

7.5 Conclusions

What conclusions can therefore be drawn? Two possibilities exist:-

a) There is a normalisation error $\sim 50\%$ in the 4 GeV/c experimental data.

b) The theoretical parameterisation of Barger et al. is inadequate.

For a number of reasons the first possibility seems unlikely: a relative normalisation error between the two groups of data (4 and 6 GeV/c) and (5, 7 and 12 GeV/c) could possibly be explained by the fact that the apparatus was modified between the two experiments, but it is the 4 GeV/c data only which appears to be inconsistent. Since all corrections to the 4 and 6 GeV/c data are similar except for the Monte Carlo calculations, the latter might seem to be a possible source of error. There are a number of indirect arguments which make this unlikely;

i) The corrections are reasonable. Near the backward direction the correction due to the R counters is the same at both 4 and 6 GeV/c; the principal effect is due to the different laboratory life-times of the

λ at the two energies and can easily be calculated. The result is in agreement with that found from the Monte Carlo calculations.

ii) Although for $u_0 - u \approx 0.4 \text{ (GeV/c)}^2$ the corrections for the two momenta differ widely (fig 6.2) the differential cross-sections for the two momenta are very similar and the change of slope occurs at the same point.

iii) As reported in section 6.4 the Monte Carlo programme reproduced well the angular effects observed in the experiment indicating that the effect of the downstream counter planes on the detection efficiency is reliably estimated.

It is therefore likely that the parameterisation of the amplitude chosen by Barger et al. is unable to explain the energy dependence, the differential cross-section and the polarisation observed in this experiment. It does, however, explain the experimental data very much better than the calculations of Høgaasen⁴¹⁾, who assumed a dominant ξ_s trajectory. He predicted a smooth variation of polarisation with $u_0 - u$ and gave an energy dependence of the total cross-section of $s^{-2.5}$.

An independent experimental check of this reaction in the region of 4 GeV/c is obviously desirable.

Figure 7.1

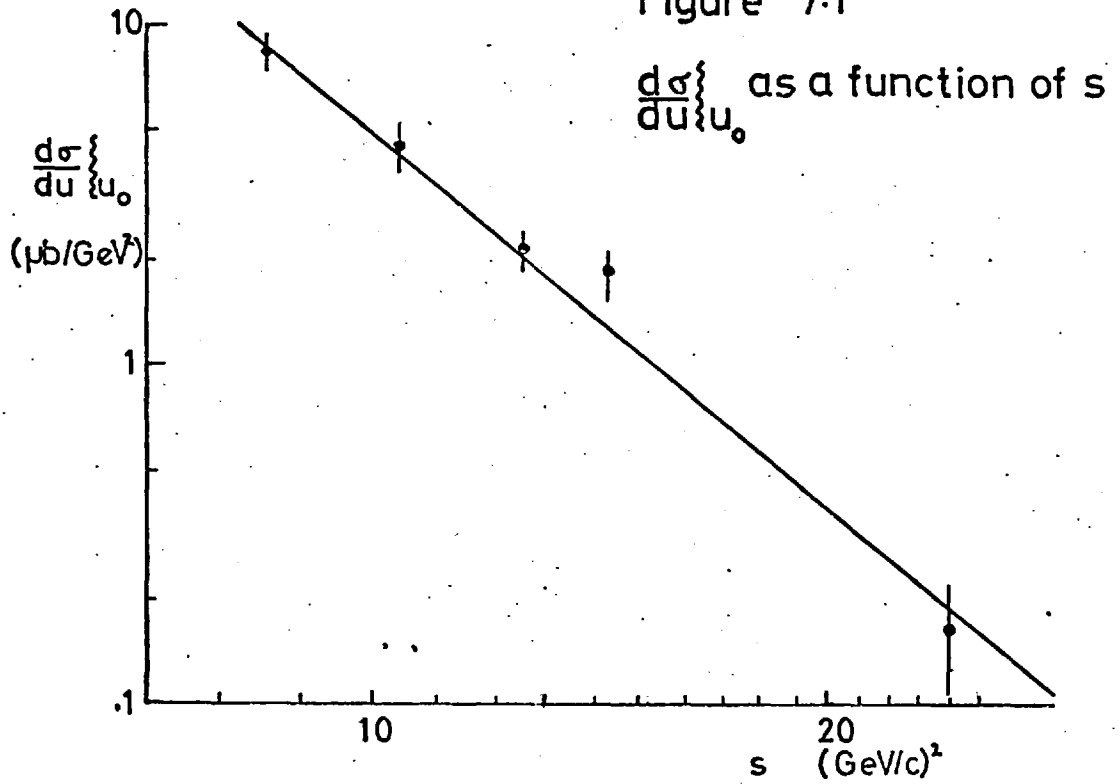


Figure 7.2

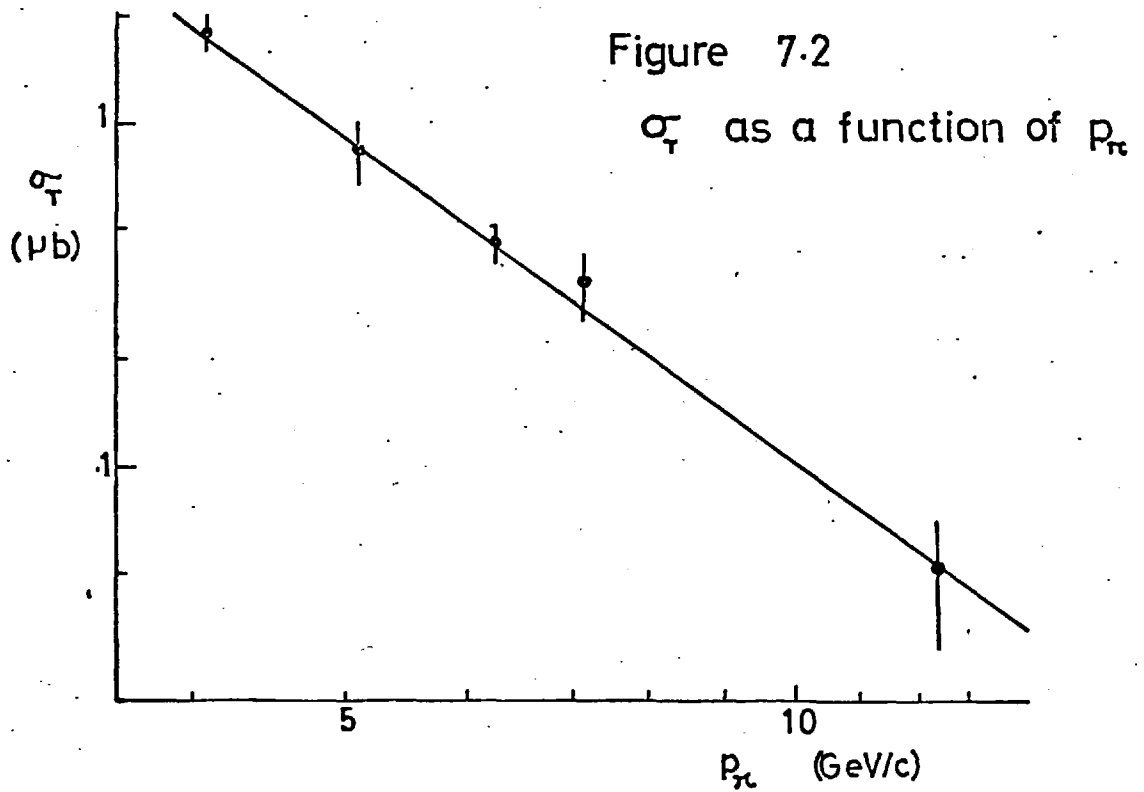


Figure 7.3

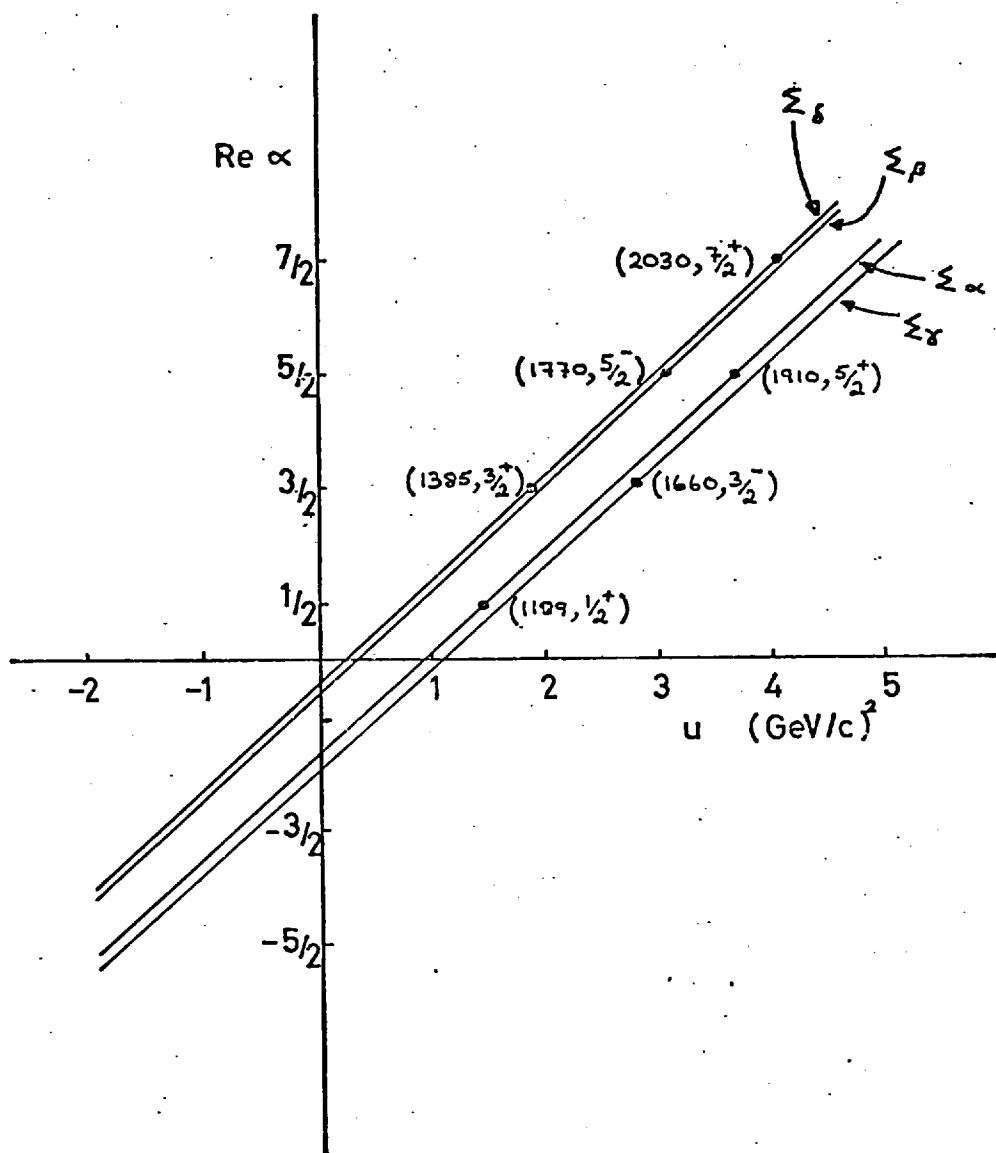
 Σ Trajectories

Figure 7.4a

Differential cross-section

FIT I

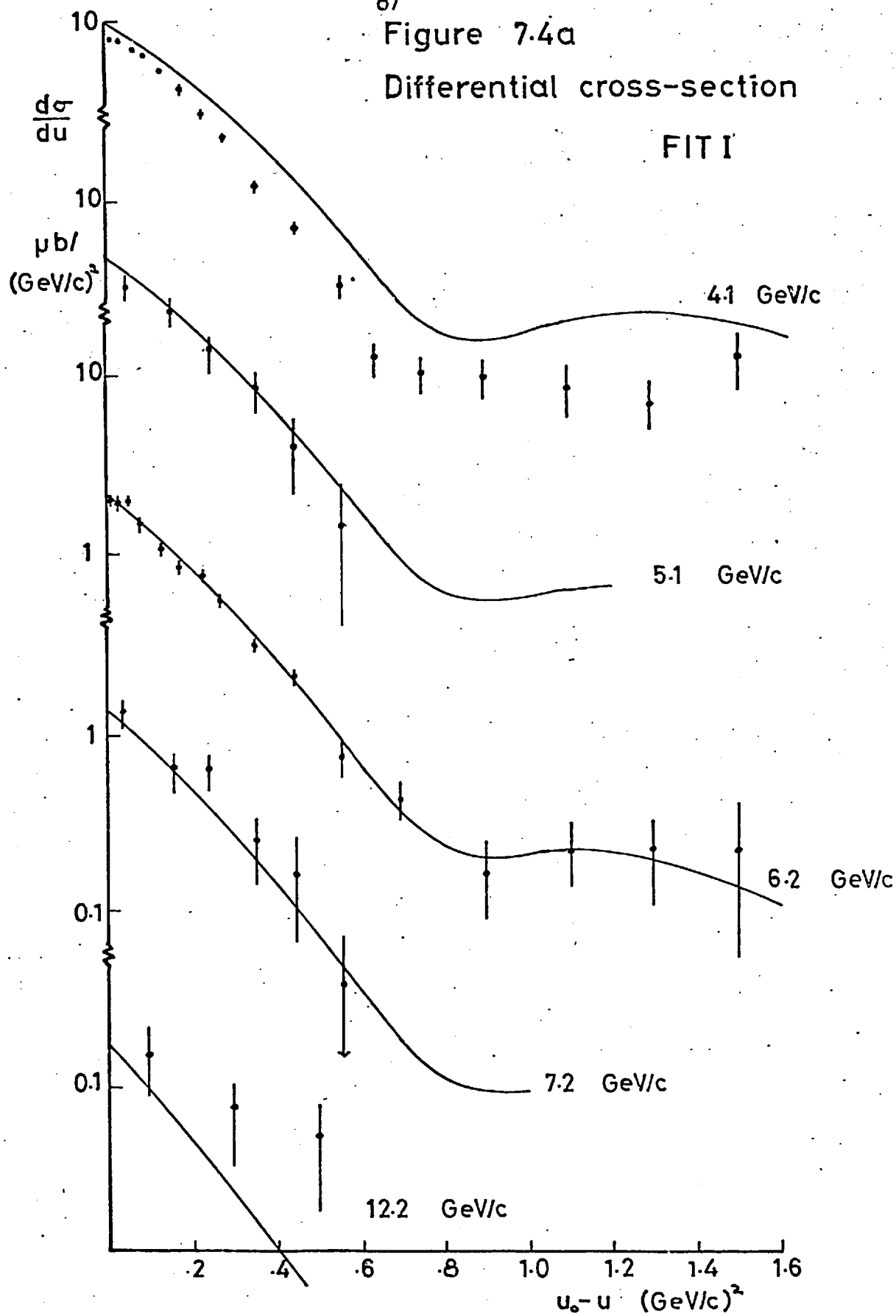
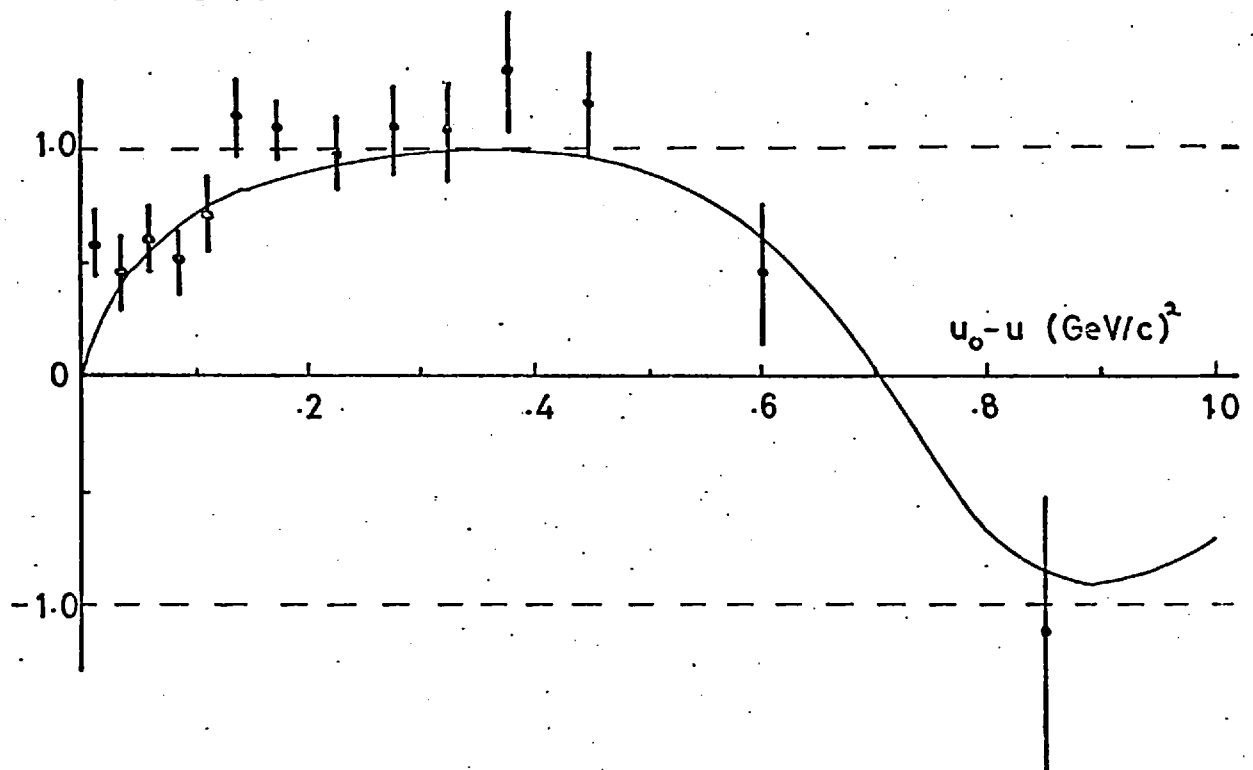


Figure 7.4b

Polarisation FIT I

4.1 GeV/c



6.2 GeV/c

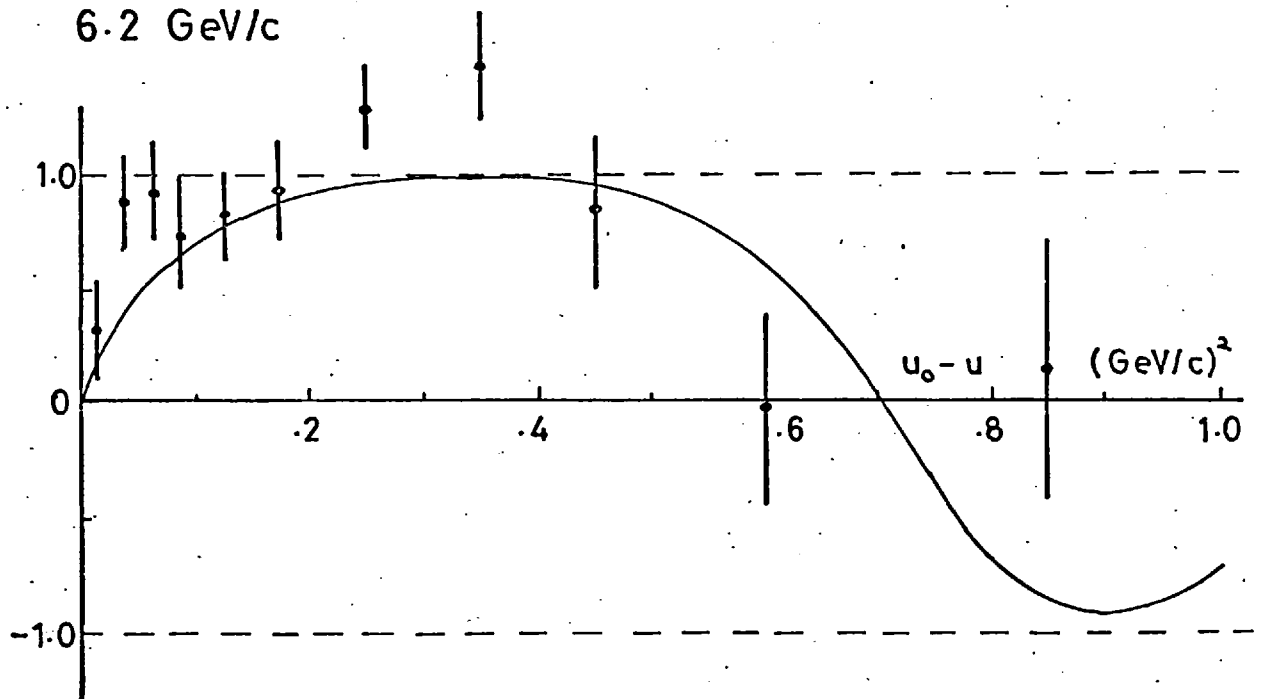
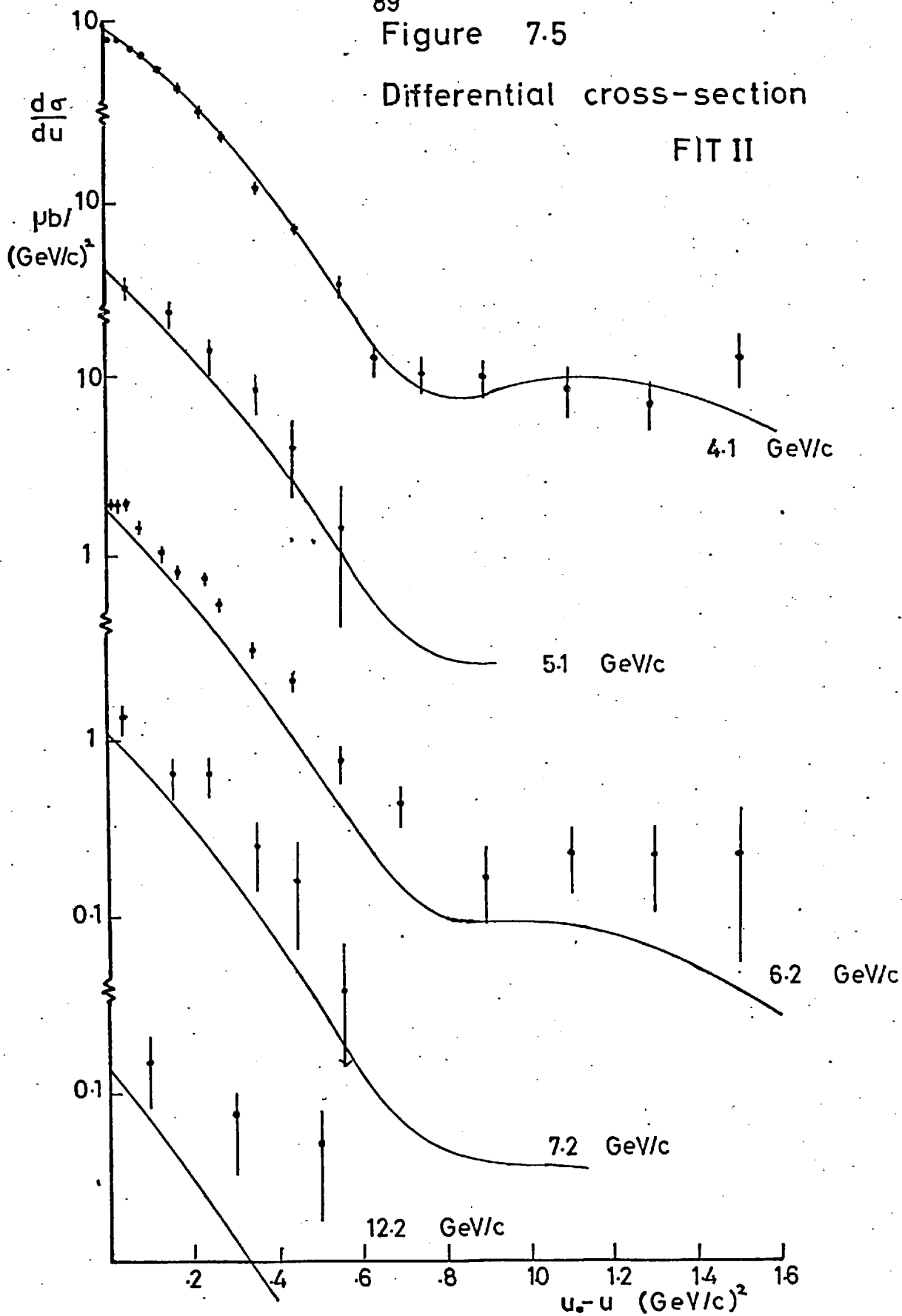


Figure 7.5



CHAPTER 8

The Reaction $\pi^- p \rightarrow K^0 Y^*(1520)$

8.1 Introduction

Most of the events observed in the experiment described here were from the reaction $\pi^- + p \rightarrow K^0 +$ missing mass. The condition that this reaction should be seen is that the particles constituting the "missing mass" should be uncharged and should decay outside the anti-coincidence.

This condition is satisfied if the missing particles are a $\bar{K}^0$ and a neutron with the $\bar{K}^0$ decaying as a K_2^0 . (The reaction $\pi^- + p \rightarrow K^0 + \Lambda$ would also be expected for the decay $\Lambda \rightarrow \pi^0 + n$ since occasionally the two γ -rays from the π^0 decay can escape detection by the anti-coincidence). Resonances in the $\bar{K} n$ system are therefore to be expected in the missing mass plots.

These are indeed seen, unfortunately over a considerable background. One resonance, the $Y_0^*(1520)$, dominates the low-mass end of the missing mass plot, and it has been possible to find the differential cross-section and to estimate the total cross-section for the production of this resonance in the region 4-7 GeV/c.

8.2 Some Remarks on the Decay Angular Distribution of the $Y^*(1520)$

The decay products of the $Y^*(1520)$ were not observed and therefore its decay angular distribution could not be measured. Indeed, in order to calculate the corrections for K_2^0 and neutron interactions in the R-counters the density matrix elements must at first sight be known. As

far as is known to the author they have not been measured at the energies discussed here (measurements at 2 GeV/c favour $\rho_{33} \approx \text{Re}\rho_{3,-1} \approx \text{Re}\rho_{31} \approx 0$)⁴²) and it was necessary to investigate the behaviour of the detection efficiency as a function of the density matrix elements.

The $Y^*(1520)$ is a spin 3/2 particle and thus has a decay angular distribution given by:-

$$W(\cos\theta, \varphi) = \frac{3}{4\pi} \left[\rho_{33} \sin^2\theta + \rho_{11} \left(\frac{1}{3} + \cos^2\theta \right) - \frac{2}{\sqrt{3}} \text{Re}\rho_{3,-1} \sin^2\theta \cos 2\varphi - \frac{2}{\sqrt{3}} \text{Re}\rho_{31} \sin 2\theta \cos \varphi \right]$$

where θ and φ are defined as shown in fig 8.1.

Now the density matrix elements obey the following relationships:-

$$\begin{aligned} 2\rho_{33} + 2\rho_{11} &= 1 \\ \rho_{31} + \rho_{3,-1} &\leq (2\rho_{33}\rho_{11})^{\frac{1}{2}} \end{aligned}$$

thus for the extreme cases $\rho_{11} = 0$ and $\rho_{33} = 0$ we have

$$W(\cos\theta, \varphi) = \frac{3}{8\pi} \sin^2\theta$$

$$\text{and } W(\cos\theta, \varphi) = \frac{3}{8\pi} \left(\frac{1}{3} + \cos^2\theta \right) \text{ respectively.}$$

Monte Carlo calculations of the effect of K_2^0 and neutron absorption in the R counters for these two decay angular distributions and for the uniform distribution $W(\cos\theta, \varphi) \propto \cos\theta$ were made.

The results are shown in fig 8.2 as a function of the four momentum transfer t in the reaction $\pi^- p \rightarrow K^0 Y^*(1520)$.

The amount of absorption for small $t_0 - t$ varies by less than about 5% for the three cases and the distributions appear to be flat in t except for a suggestion of increased absorption at higher values of $t_0 - t$ for a uniform decay distribution.

We therefore conclude that it is not necessary to know the density matrix elements for the decay of the $Y^*(1520)$ to be able to find the total and differential cross-sections.

8.3 Missing Mass Plots

Fig 8.3 shows the histogrammes of missing mass squared to the K_1^0 at beam momenta 4.1, 5.1, 6.2, 7.2 GeV/c for all events giving a fit (with χ^2 probability ≥ 0.03) at the decay vertex. Peaks corresponding to the Λ and $Y^*(1520)$ are clearly visible at all energies. (The shoulder on the low mass side of the Λ peak at 4 and 6 GeV/c is due to a small K^- beam contamination producing the reaction $K^- p \rightarrow \bar{K}^0 n$).

In order to calculate the amount of resonance production it is necessary to consider all possible resonances that can be produced as well as the non-resonant background. This latter is the most difficult; it is impossible to calculate since the production mechanism is unknown and it is therefore necessary to choose an appropriate function which behaves as "a smooth hand-drawn background". Such a function is

$$n = n_0 (m^2 - m_0^2) e^{-\lambda m^2}$$

where n_0 , m_0^2 and λ are constants to be fitted at each energy.

There are four possible Y^* resonances with mass ≤ 2 GeV/c² known to decay to $\bar{K}n$ with appreciable branching ratio³²⁾. Table 8.1 lists relevant parameters of these four. All have a half width ($\Gamma/2$) smaller than the experimental resolution and a Breit-Wigner parameterisation is therefore inappropriate. Instead, a Gaussian parameterisation in mass squared was chosen for the $Y^*(1520)$ and $Y^*(1815)$ and since the $Y^*(1670)$ and $Y^*(1690)$ lie very close together they were parameterised as a single Gaussian.

The fit was therefore made to the missing mass plots in the region $1.7 \leq (\text{missing mass})^2 \leq 4.0 (\text{GeV}/c^2)^2$ with the function:-

$$N(m^2) = n_0 (m^2 - m_0^2) e^{-\lambda m^2} + \sum_{i=1}^3 A_i \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left[-(m_{0i}^2 - m_i^2)^2 / 2\sigma_i^2\right]$$

There are 11 free parameters in this expression (12 parameters connected by the condition that the area under the curve is equal to the observed

number of events). The fitted curves and the fitted smooth background are shown in the missing mass plots of fig 8.3; the parameters of the fits are given in table 8.2.

The values obtained for these parameters are all reasonable; some of the masses are somewhat low because of small errors in the beam momenta used in the kinematical fitting programme. At all energies the three "bumps" are necessary. Attempts to fit without a resonance at about $1680 \text{ MeV}/c^2$ resulted in a poor fit and a very low mass and large width for the $Y^*(1815)$.

8.4 Detection Efficiency

The data at beam momenta of 5.1 and 7.2 GeV/c had been taken earlier and with a somewhat different trigger to that used at 4.1 and 6.2 GeV/c. The anticoincidence shield was the same but the counters downstream of the target consisted of a large counter covering the end of the chambers and another at one side of the chambers inside the magnet. While this arrangement ensured that essentially all decays occurring within the spark chambers resulted in triggers it also caused a large background - only one picture in ten contained a useful event.

The detection efficiency for these two momenta was determined as follows. If an interaction occurs at the point x in the target such that the K^0 is produced at an angle θ to the axis (fig 8.4) then the probability that it will decay within the fiducial volume if it decays as a K_1^0 is:-

$$\text{prob}(x, \theta) = e^{-l_1/l_0} - e^{-l_2/l_0}$$

where $l_0 = pc\tau/m$ (p , τ and m are the parameters of the K_1^0) and l_1 and l_2 are functions of x and θ .

The probability that such a decay will occur wherever the interaction point occurs in the target can be simply calculated by integ-

rating the above expression over the length of the target. This calculation can be made for each event and the correction thus made without need for Monte Carlo calculations. This procedure was adopted, the correction for interactions in the R-counters being made assuming that the two corrections were independent. This R-counter correction has already been discussed in section 8.2.

The "geometry correction" to the 4 and 6 GeV/c data could not be calculated in this manner and Monte Carlo calculations were necessary. The procedure was similar to that already discussed in section 6.2 except that there was an additional particle, the neutron, to be considered. West⁴³⁾ has shown that over a wide range of neutron momenta the probability of interaction for a neutron passing normally through the R-counters is 0.09, thus this correction was easily included in the calculations.

The results of the Monte Carlo calculations for 10000 events at each energy are shown in fig 8.5. Note that these curves do not show structure at low four-momentum transfer of the type seen in fig 6.2 since the $\bar{K}^0$ from the Y^* decay is not produced at a unique angle. The statistical accuracy on these curves is $\pm 5\%$.

Most of the corrections independent of four-momentum transfer have already been discussed in chapter 6 (although the actual value of the correction may be different for the channel $\pi^- p \rightarrow K^0 Y^*$). Table 8.3 gives the values of the various corrections. The following remarks are necessary.

a) No empty target events existed at 5 and 7 GeV/c. The correction was estimated (and assigned a correspondingly larger error) from the corrections at 4 and 6 GeV/c. This procedure is justified by the fact that most of the empty events seen in the spark chambers arise from the mylar window at the end of the target nearest the chambers, and the same thickness of mylar was used in the separate experiments. The target empty events at 4 and 6 GeV/c showed the same features in missing

mass and t distributions as the target full events. The correction is larger for this channel compared with $\pi^-p \rightarrow \Lambda K^0$ since the K_1^0 has a shorter proper lifetime than the Λ (and also a lower momentum) so that events originating in the upstream end of the target are very much more suppressed compared to those events originating near to or in the downstream end window.

b) The correction for branching ratios is made up as follows:-

$K^0 \rightarrow K_1^0$	2
$K_1^0 \rightarrow \pi^+\pi^-$	1.46
$Y^* \rightarrow \bar{K}^0 n$	4.44
$\bar{K}^0 \rightarrow K_2^0$	2

c) The $\bar{K}^0$ from the decay of the Y^* has, in general, a somewhat larger momentum than the K^0 from the reaction $\pi^-p \rightarrow \Lambda K^0$. The correction for K_1^0 escape from the anticoincidence and K_2^0 decay within it is thus smaller than that shown in fig 6.4 and is ignored.

8.5 Differential and Total Cross-Sections

Unlike the reaction $\pi^-p \rightarrow \Lambda K$ where the determination of the differential cross-section was simply a matter of correcting for the detection efficiency, in the reaction $\pi^-p \rightarrow K^0 Y^*(1520)$ it is necessary to be more careful since there is a large background which may show a different t -dependence to the resonance production. There are two arguments to show that it has in fact approximately the same t -dependence.

a) Bartsch et al⁴⁴⁾ have shown that this is true for 8 GeV/c π^+p and 10 GeV/c K^-p reactions. In their paper they state;

" A study of differential cross-sections for 8 GeV/c π^+p and 10 GeV/c K^-p reactions, treated as two body processes, has indicated that as the mass M of one of the "bodies" varies the small angle differential cross-section is described by $d\sigma/dt' = K \exp(-At')$ for all values of M ,

whether corresponding to a resonance or "background" region. In general similar values of A are obtained for a resonance region and for its neighbouring mass bands."

b) In the present reaction this has been shown to be true over a limited mass range in the region of the $Y^*(1520)$. The limited mass range ensures that corrections for detection efficiency are independent of mass, but in the same region the background/signal ratio varies considerably and hence if there is a large difference in the t -dependence for background and resonance it should be clearly seen. In fact no such difference is observed.

Thus it is justifiable to take all events within the $Y^*(1520)$ mass region and plot the differential cross-section after correction for detection efficiency, the normalisation being determined by the number of events under the fitted Gaussian.

The differential cross-sections so obtained are plotted in fig 8.6 and tabulated in table 8.5 as a function of $t_0 - t$. These values were obtained for a mass cut $2.0 \leq \text{mass squared} \leq 2.5 \text{ (GeV/c}^2)^2$. The plots were repeated for a mass cut $2.1 \leq \text{mass squared} \leq 2.4 \text{ (GeV/c}^2)^2$; they showed no difference either in absolute normalisation or shape from those of fig 8.6.

The total cross-sections were determined by integration of the differential cross-sections over the range $0 \leq t_0 - t \leq 1.8 \text{ (GeV/c)}^2$.

The values are:-

beam momentum (GeV/c)	σ_T (μb)
4.1	10.6 ± 5.3
5.1	25.6 ± 12.8
6.2	7.0 ± 3.5
7.2	12.3 ± 6.2

These are plotted in fig 8.7 together with other measurements of this reaction at lower momenta 42, 45). The point at 2.7 GeV/c is for the charge symmetric reaction $\pi^+ n \rightarrow K^+ Y^*(1520)$.

It is not realistic to assign errors to σ_T by combination of statistical errors, errors from table 8.3 and errors due to Monte Carlo calculations. By far the greatest uncertainty in the analysis results from the rather arbitrary shape for the hand drawn background. Inspection of fig 8.3 leads to the conclusion that a realistic error on the number of events estimated from the fit is 50%.

Such a systematic error, if present, would be expected to occur at all energies, whereas it is obvious that σ_T at 4 and 6 GeV/c is hardly consistent with the values at 5 and 7 GeV/c if there is an absence of structure in σ_T at these energies (as would be expected). Moreover the 4 GeV/c value is not consistent with Dahl's measurement at the same energy. In view of the difficulty on fitting resonances plus an unknown background this is not too surprising; the background at 4 and 6 GeV/c is not expected to have the same shape as that at 5 and 7 GeV/c since the mass dependence of the detection efficiency depends upon the counter geometry which was different for the two sets of data.

8.6 Comparison with $\pi^- p \rightarrow K^0 \Lambda$ Associated Production

In the Regge pole model the reaction $\pi^- p \rightarrow K^0 Y^*(1520)$ is mediated by the exchange of the same trajectories as the reaction $\pi^- p \rightarrow K^0 \Lambda$, namely those trajectories passing through the $K^*(890)$ and $K^*(1420)$. These two trajectories are approximately exchange degenerate and it is therefore expected that these two reactions will show similar features.

This is indeed the case. Reports of the reaction $\pi^- p \rightarrow K^0 \Lambda$ at energies in the range 2-11 GeV/c 35, 42, 46) indicate that the differential cross-section for this reaction has a smooth t -dependence with an

absence of large dips or bumps. In the region $0 \leq t_0 - t \leq 0.4 \text{ (GeV/c)}^2$ the differential cross-section has generally been fitted to an expression of the form

$$\frac{d\sigma}{dt} \propto e^{a(t_0 - t)}$$

with $a \sim -7.5 \text{ (GeV/c)}^{-2}$ over the whole energy range. For $t_0 - t > 0.4 \text{ (GeV/c)}^2$ there are deviations from this simple exponential behaviour but Hoang et al ³⁵⁾ get a good fit to the expression

$$\frac{d\sigma}{dt} \propto e^{a|t|+b|t|^2} \quad \text{for} \quad |t| \leq 1.0 \text{ (GeV/c)}^2$$

The curves on fig 8.6 are of a similar fit to

$$\frac{d\sigma}{dt} \propto e^{at'+bt'^2} \quad (t' = t_0 - t)$$

for the reaction $\pi^- p \rightarrow K^0 Y^*(1520)$. The fit at all energies is good and the parameters of the fits are given in table 8.5. The parameter a gives the behaviour at small momentum transfer and indicates that shrinkage occurs in this reaction. This parameter is also somewhat smaller than the value of -7.5 found in the reaction $\pi^- p \rightarrow K^0 \Lambda$ confirming the observations of Bartsch et al ⁴⁴⁾ that the angular distribution generally becomes broader as the mass of one of the produced bodies increases (in this case the strange baryon mass increases from 1115 to 1520 MeV/c^2).

TABLE 8.1

Resonance	Mass (MeV/c ²)	Width (MeV/c ²)	Principle Decay Modes and fractions	
$\Upsilon^*(1520)$	1519	16	$N\bar{K}$	0.45 ± 0.04
			$\Sigma\pi$	0.45 ± 0.04
			$\Lambda\pi\pi$	0.10
$\Upsilon^*(1670)$	1670	25	$N\bar{K}$	0.14
			$\Lambda\eta$	0.33
			$\Sigma\pi$	0.45
$\Upsilon^*(1700)$	1690	40	$N\bar{K}$	0.25
			$\Sigma\pi$	0.35
			$\Lambda\pi\pi$	0.20
			$\Sigma\pi\pi$	0.20
$\Upsilon^*(1815)$	1815	75	$N\bar{K}$	0.65
			$\Sigma\pi$	0.11
			$\Sigma(1385)\pi$	0.09

TABLE 8.2Fitted Parameters of the Mass Plots

		4.1 GeV/c	5.1 GeV/c	6.2 GeV/c	7.2 GeV/c
Background	m_0^2	1.71	1.59	1.72	1.68
	λ	0.86	0.12	0.79	0.41
$Y^*(1520)$	m^2	2.24 ± 0.01	2.33 ± 0.02	2.27 ± 0.01	2.29 ± 0.02
	σ	0.13 ± 0.01	0.16 ± 0.02	0.13 ± 0.01	0.16 ± 0.02
$Y^*(1680)$	m^2	2.56 ± 0.16	2.84 ± 0.10	2.77 ± 0.17	2.78 ± 0.12
	σ	0.35 ± 0.05	0.14 ± 0.05	0.53 ± 0.05	0.15 ± 0.05
$Y^*(1815)$	m^2	3.24 ± 0.03	3.30 ± 0.05	3.29 ± 0.03	3.28 ± 0.07
	σ	0.16 ± 0.025	0.21 ± 0.05	0.13 ± 0.03	0.11 ± 0.03
χ^2 for fit (35 d.f.)		41.8	20.7	46.6	33.4

TABLE 8.3Correction Factors

	4.1 GeV/c	5.1 GeV/c	6.2 GeV/c	7.2 GeV/c
Anti-coincidence losses	1.04 $\pm$ 0.02	1.07 $\pm$ 0.03	1.08 $\pm$ 0.02	1.07 $\pm$ 0.03
Scanning Efficiency	1.03 $\pm$ 0.01		1.03 $\pm$ 0.01	
π^- , K^0 interactions in hydrogen	1.06 $\pm$ 0.02	1.06 $\pm$ 0.02	1.06 $\pm$ 0.02	1.06 $\pm$ 0.02
K_1^0 interactions in F-counters	1.06 $\pm$ 0.03	1.06 $\pm$ 0.03	1.06 $\pm$ 0.03	1.06 $\pm$ 0.03
χ^2 probability cut	1.03	1.03	1.03	1.03
Target empty rate	0.76 $\pm$ 0.06	0.78 $\pm$ 0.10	0.80 $\pm$ 0.04	0.82 $\pm$ 0.10
H.P.D. losses	1.12 $\pm$ 0.03	1.32 $\pm$ 0.18	1.07 $\pm$ 0.02	1.32 $\pm$ 0.18
No fit events	1.05 $\pm$ 0.03		1.03 $\pm$ 0.02	
Cuts on decay point	1.02 $\pm$ 0.02		1.02 $\pm$ 0.02	
Branching ratios	26.0 $\pm$ 1.1	26.0 $\pm$ 1.1	26.0 $\pm$ 1.1	26.0 $\pm$ 1.1
Total Correction	29.4 $\pm$ 3.4	33.2 $\pm$ 6.5	30.2 $\pm$ 2.8	34.8 $\pm$ 6.8

TABLE 8.4

Differential Cross-Sections ($\mu\text{b}/(\text{GeV}/c)^2$)

$t_0 - t$	4.1 GeV/c	5.1 GeV/c	6.2 GeV/c	7.2 GeV/c
0.025	21.4 $\pm$ 1.1	} 70.0 $\pm$ 7.5	30.2 $\pm$ 1.4	} 50.0 $\pm$ 3.5
0.075	21.2 $\pm$ 1.1		22.2 $\pm$ 1.2	
0.125	16.4 $\pm$ 0.9	} 41.0 $\pm$ 5.7	14.4 $\pm$ 1.0	} 27.3 $\pm$ 2.5
0.175	14.8 $\pm$ 0.9		12.1 $\pm$ 0.9	
0.225	11.9 $\pm$ 0.8	} 31.2 $\pm$ 4.9	10.4 $\pm$ 0.9	} 11.5 $\pm$ 1.7
0.275	11.3 $\pm$ 0.8		8.6 $\pm$ 0.8	
0.35	9.6 $\pm$ 0.5	25.2 $\pm$ 4.3	6.4 $\pm$ 0.5	8.9 $\pm$ 1.5
0.45	7.2 $\pm$ 0.5	16.1 $\pm$ 3.4	4.0 $\pm$ 0.4	9.3 $\pm$ 1.5
0.55	7.0 $\pm$ 0.6	11.3 $\pm$ 3.0	2.3 $\pm$ 0.3	3.5 $\pm$ 1.0
0.65	5.6 $\pm$ 0.6	9.8 $\pm$ 2.7	2.1 $\pm$ 0.3	2.9 $\pm$ 0.9
0.75	4.0 $\pm$ 0.5	10.4 $\pm$ 2.8	1.3 $\pm$ 0.2	1.6 $\pm$ 0.65
0.85	3.7 $\pm$ 0.6	5.1 $\pm$ 2.0	1.0 $\pm$ 0.2	1.1 $\pm$ 0.55
0.95	3.3 $\pm$ 0.6	7.1 $\pm$ 2.3	0.9 $\pm$ 0.2	1.9 $\pm$ 0.7
1.1	4.1 $\pm$ 0.6	3.9 $\pm$ 1.3	0.5 $\pm$ 0.1	1.1 $\pm$ 0.55
1.3	2.3 $\pm$ 0.3	2.5 $\pm$ 1.0	0.5 $\pm$ 0.1	2.0 $\pm$ 0.75
1.5	2.6 $\pm$ 0.6	5.0 $\pm$ 1.4	0.3 $\pm$ 0.1	0.9 $\pm$ 0.5
1.7		3.1 $\pm$ 1.1	0.25 $\pm$ 0.1	1.2 $\pm$ 0.6

TABLE 8.5

Parameters of the Fit $d\sigma/dt \propto e^{at'} + bt'^2$

	4.1 GeV/c	5.1 GeV/c	6.2 GeV/c	7.2 GeV/c
a	-2.97 $\pm$ 0.18	-4.19 $\pm$ 0.43	-5.45 $\pm$ 0.21	-6.70 $\pm$ 0.37
b	0.97 $\pm$ 0.15	1.35 $\pm$ 0.28	1.59 $\pm$ 0.16	2.60 $\pm$ 0.28
Degrees of freedom	13	11	14	11
χ^2	13.7	6.0	14.7	14.6

Figure 8.1 Definition of Decay Angles

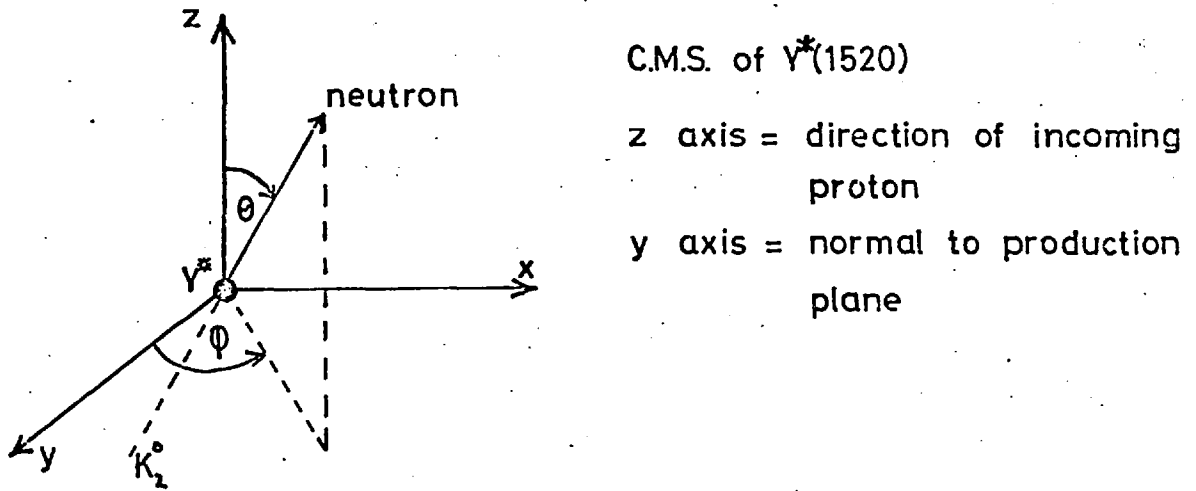
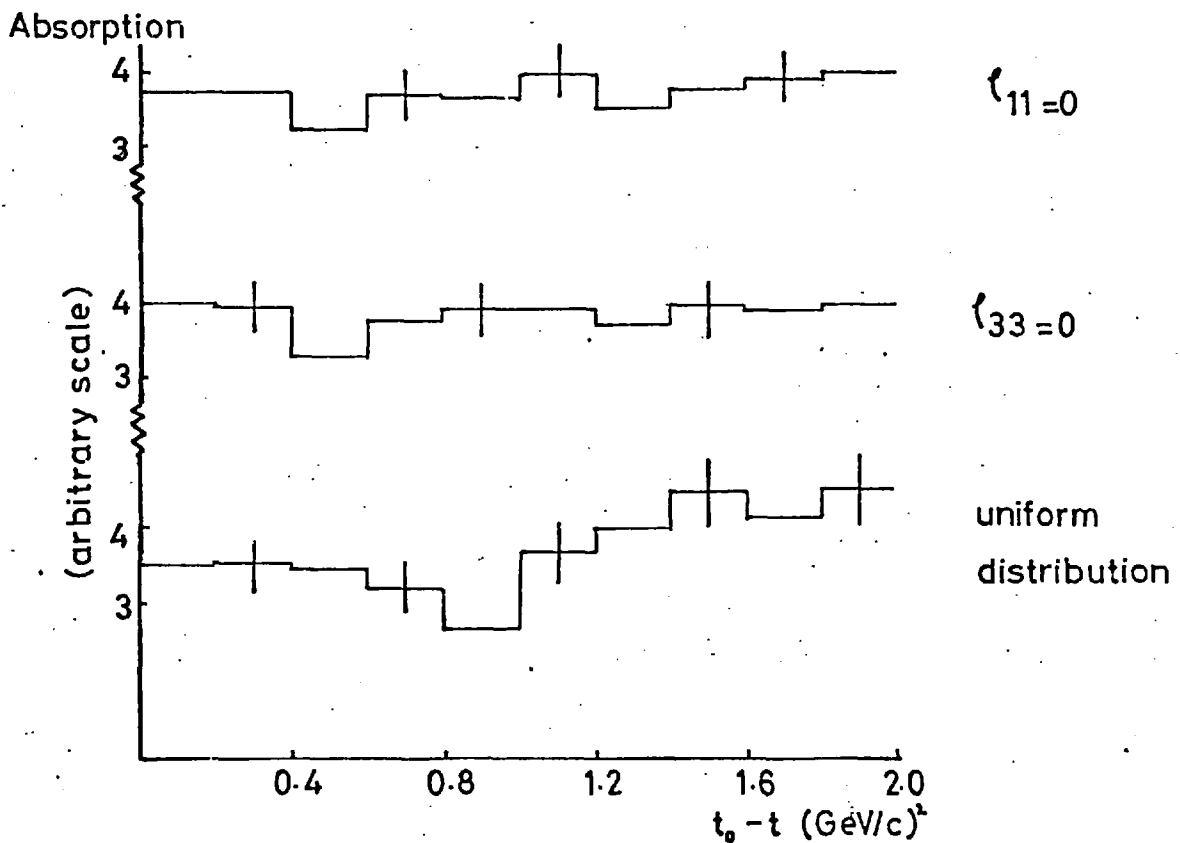
Figure 8.2 Monte Carlo Calculations for K_1^0 Absorption

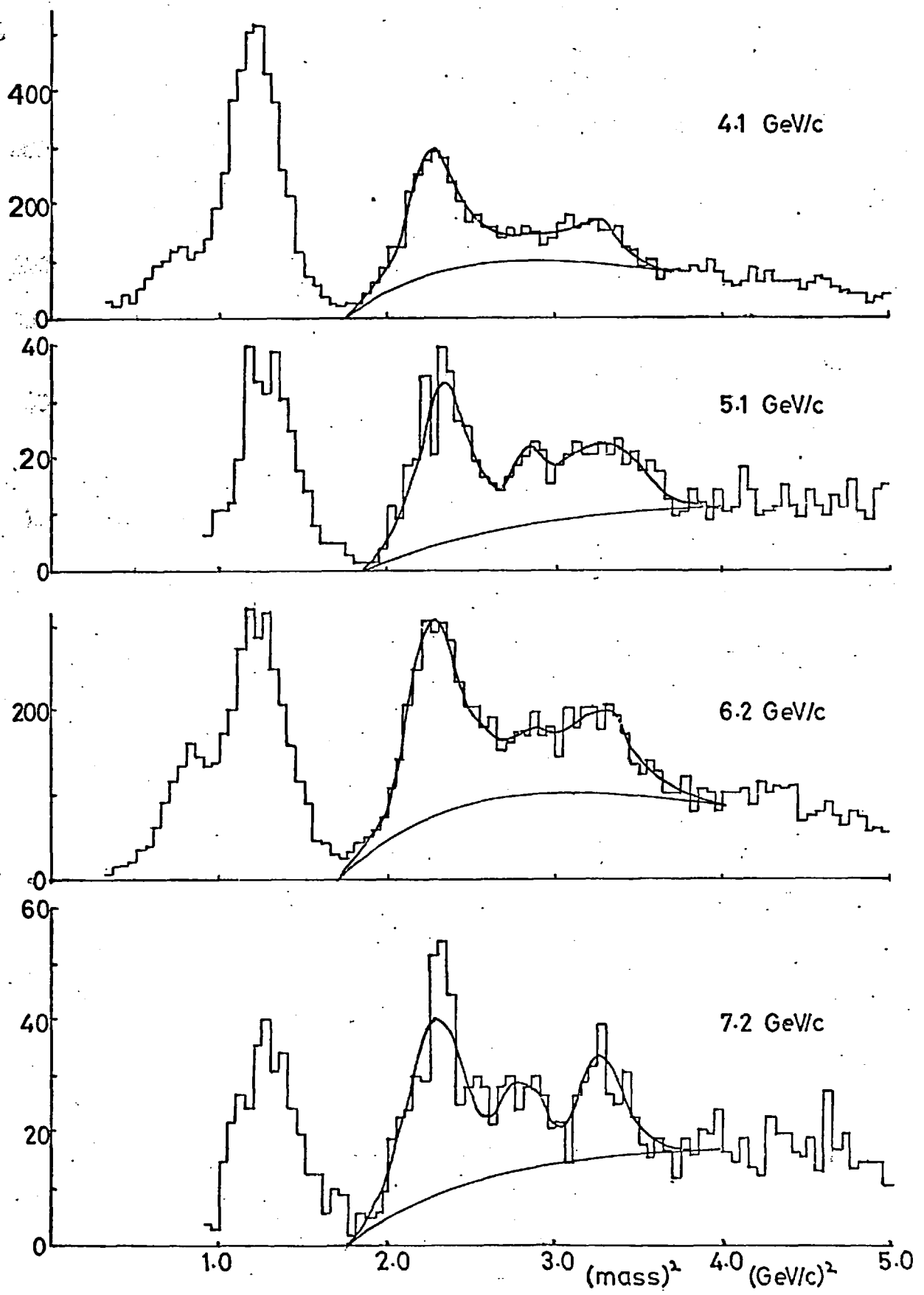
Figure 8.3 Missing Mass to the K^0 

Figure 8.4

Geometry Correction

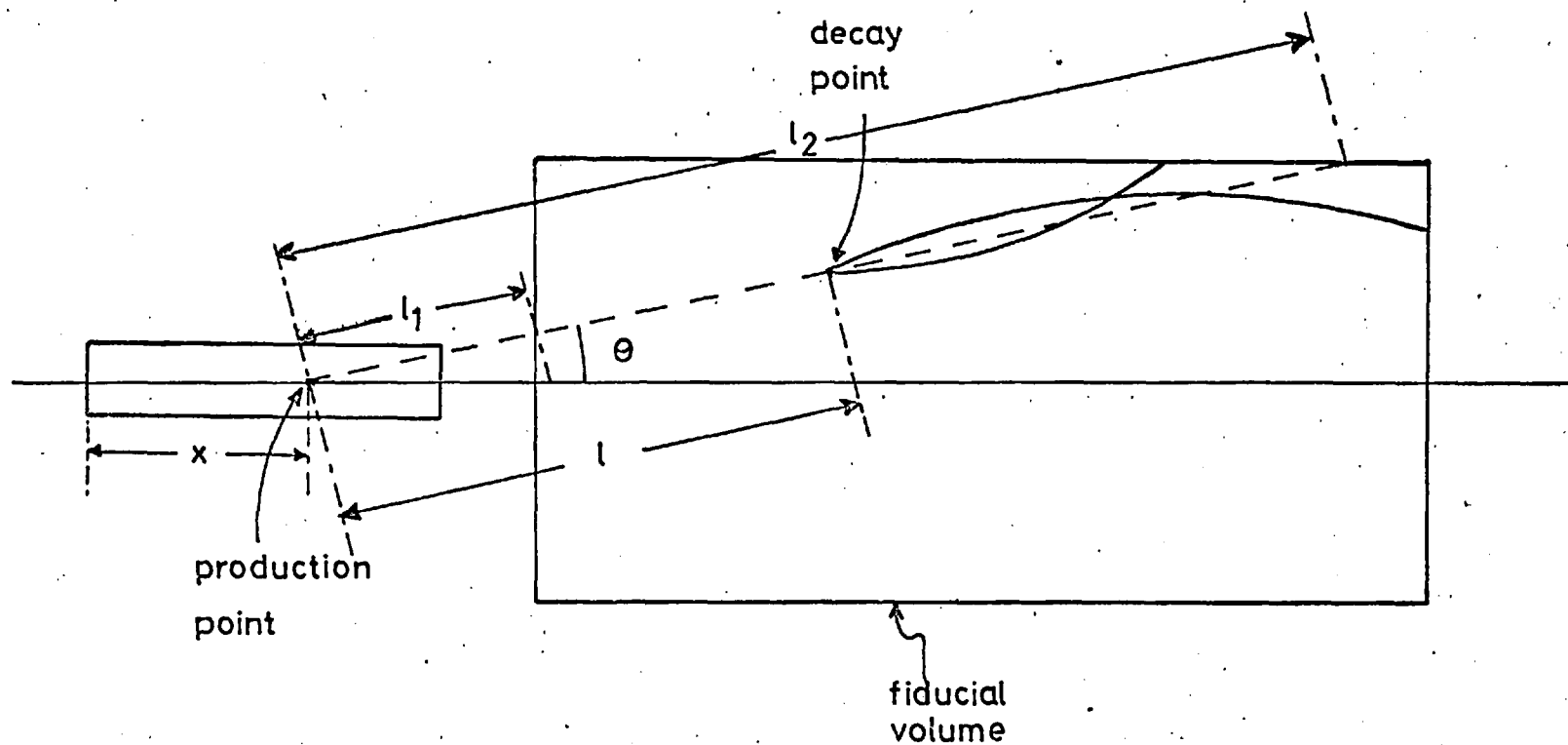


Figure 8.5

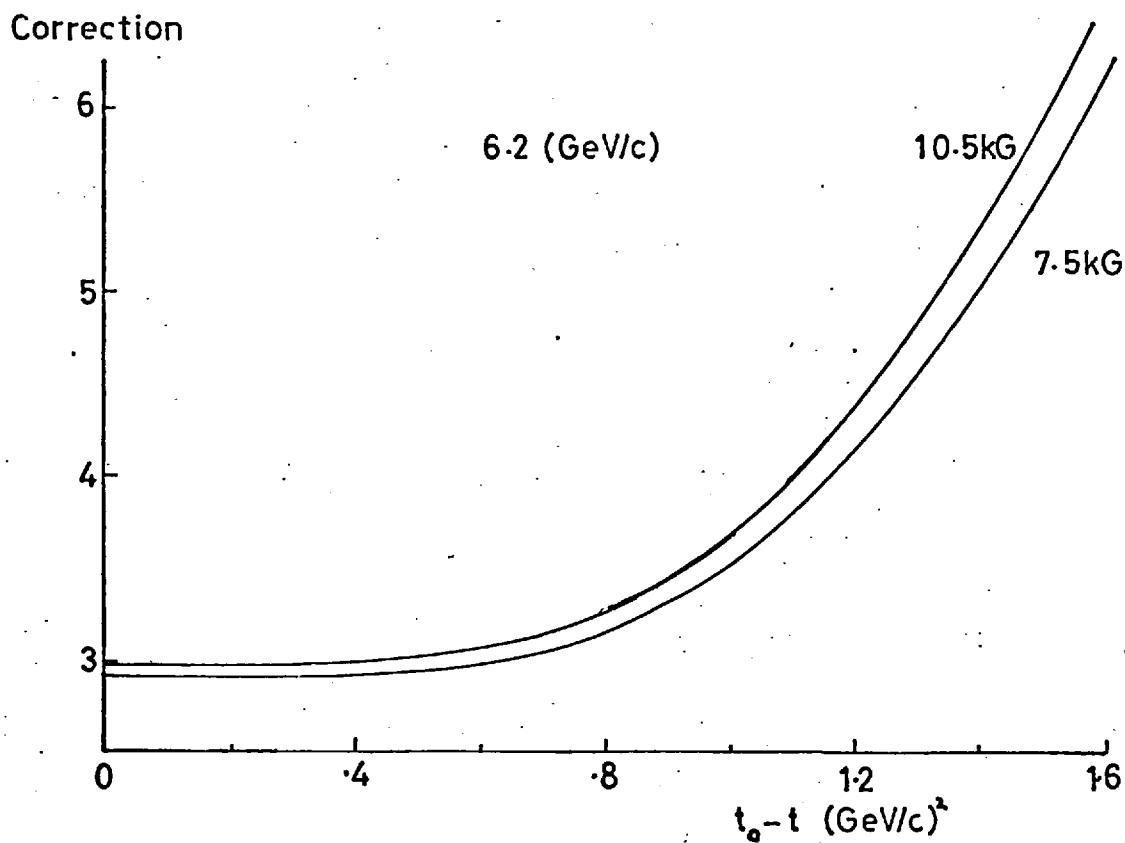
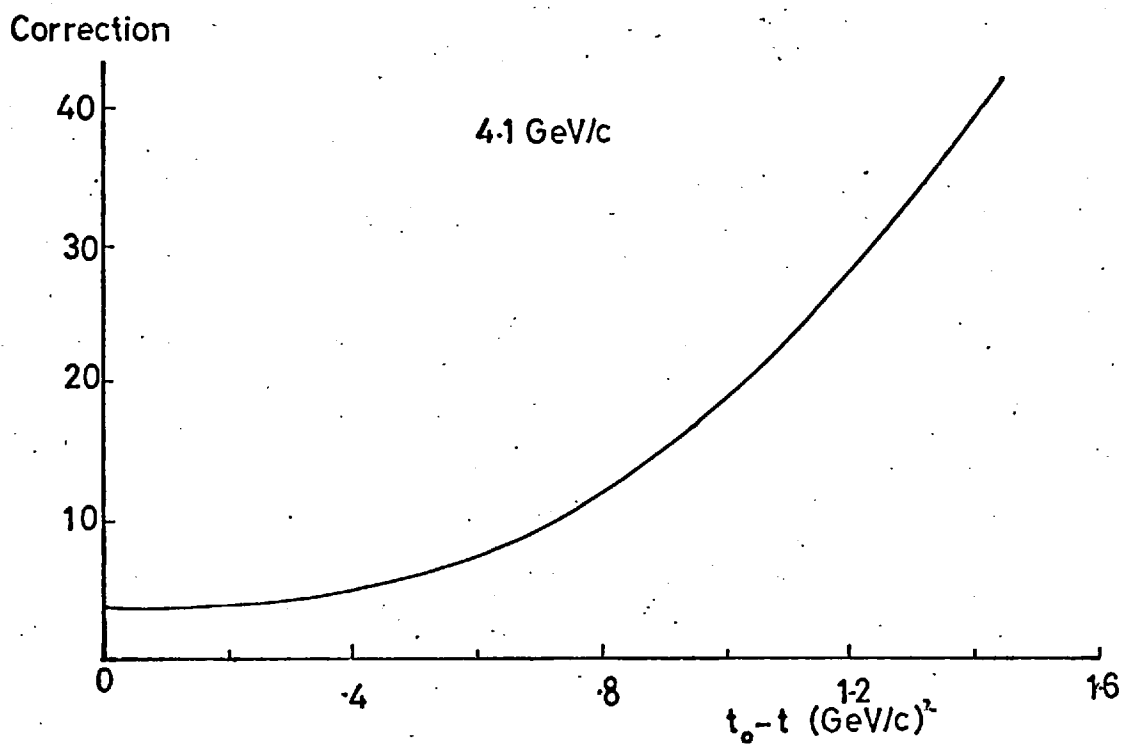
 u -dependent Correction at 4 and 6 GeV/c

Figure 8.6

Differential Cross Sections

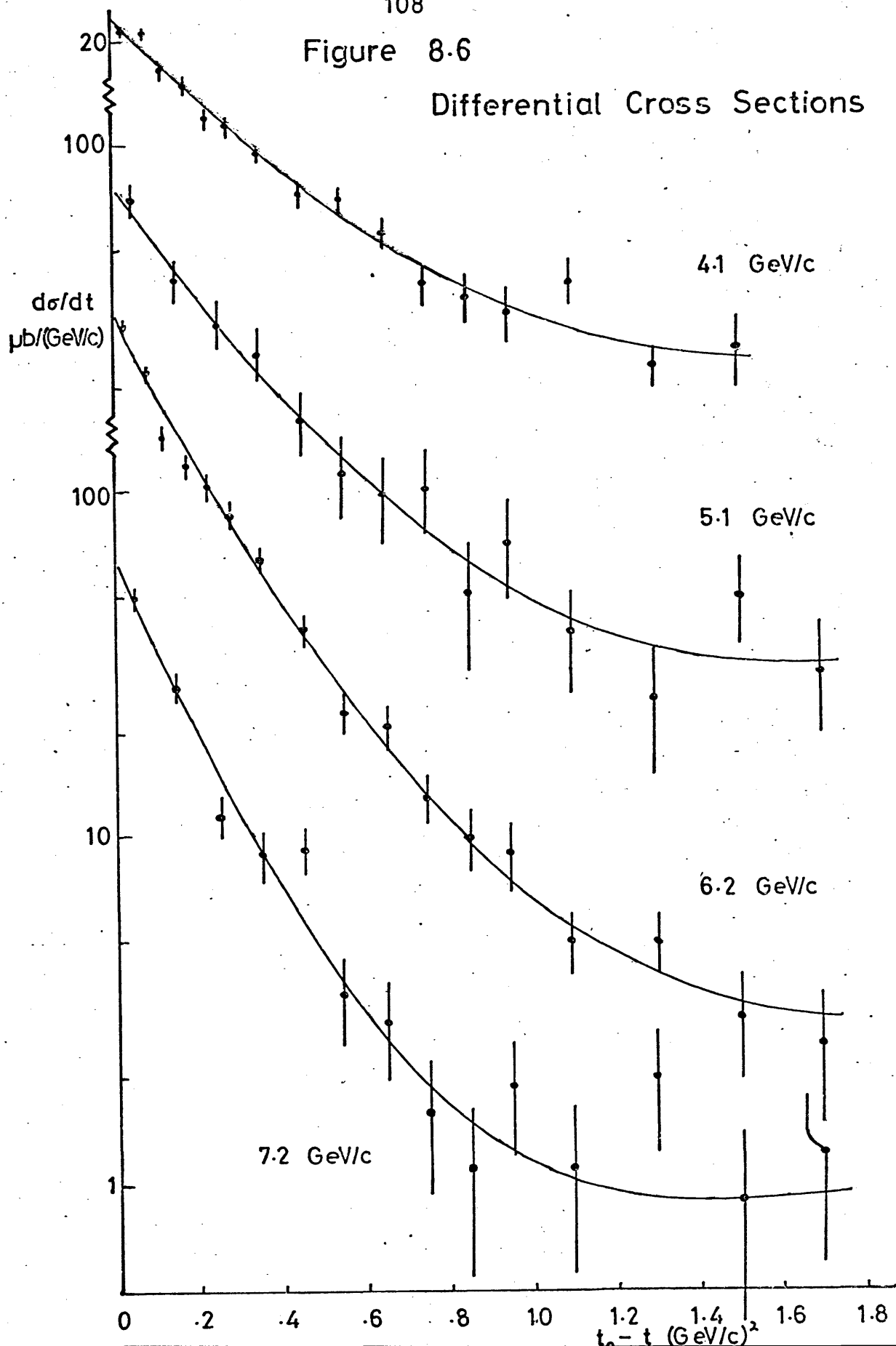
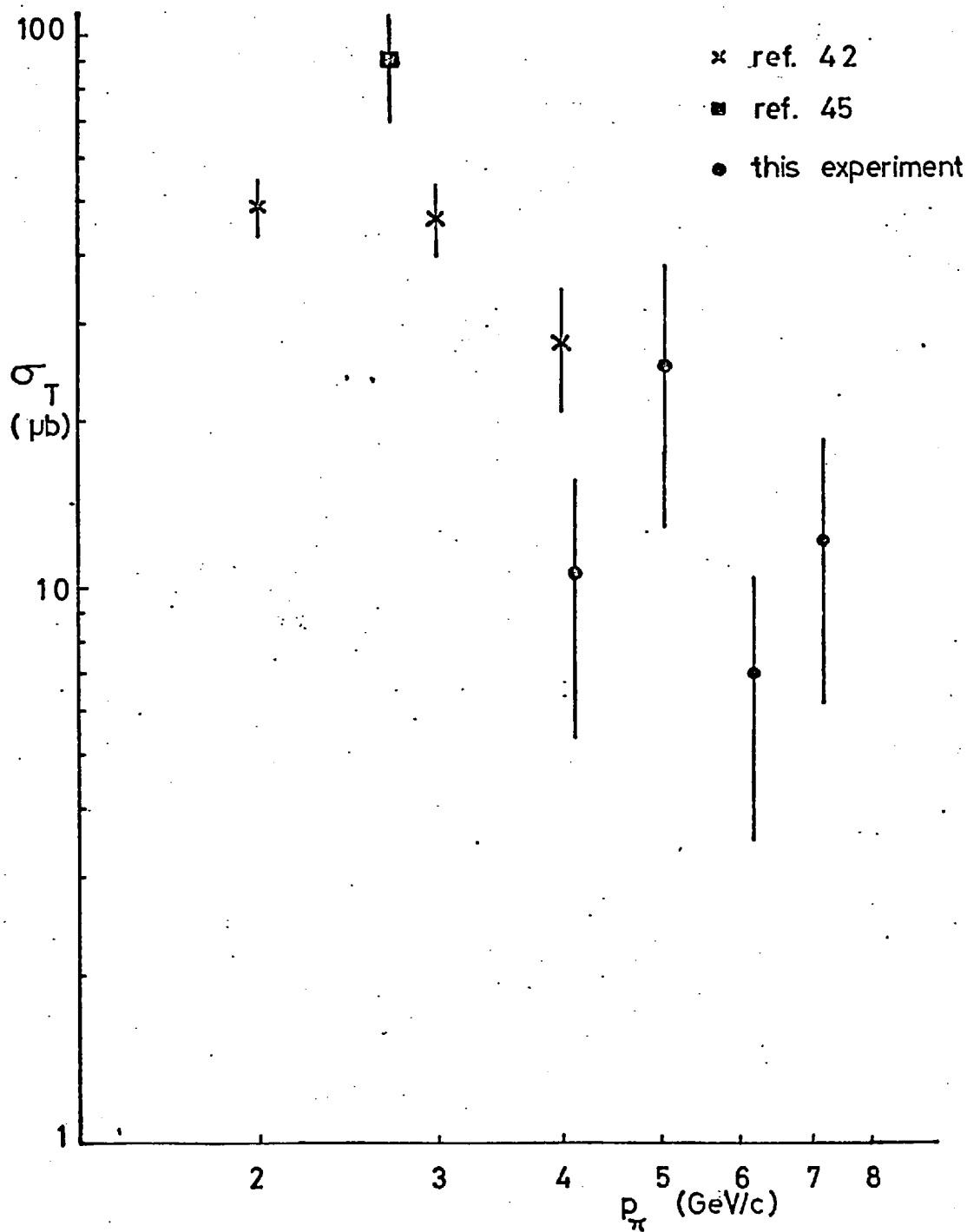


Figure 8.7 Total Cross-section as a function of p 

APPENDIX A

Choice of a Number at Random from a Distribution $f(x)$

In this appendix and chapter 5, unless stated to the contrary the expression 'random number' is understood to be a number chosen at random from a uniform distribution between 0 and 1. Such a distribution is, by definition, one such that, if we choose from it a number x , then the probability of choosing another number less than x is just x .

Consider a distribution function $f(x)$, well-behaved in the interval $a \leq x \leq b$, and from which we wish to choose a number at random. Then, by definition, the number chosen must be such that if we repeat the procedure a large number of times we obtain the original distribution.

This is true if we have

$$\int_a^{x_R} f(x) dx = R \int_a^b f(x) dx$$

where R is a random number, i.e. the (normalised) area under the curve between a & $x_R = R$, and that between

x_R & $b = 1 - R$ (see figure). This equation must be solved for x_R .

Consider as an example $f(x) = e^{-ax}$.

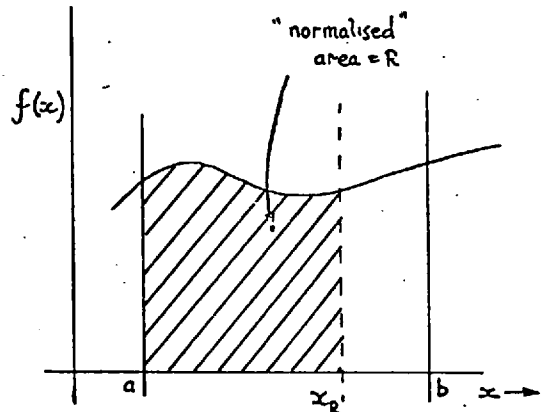
Then we have:

$$\int_0^{x_R} e^{-ax} dx = R \int_0^{\infty} e^{-ax} dx$$

$$\text{i.e.} \quad (1 - e^{-ax_R})/a = R/a$$

Then

$$\underline{x_R = -\frac{1}{a} \ln(1 - R)}$$



If the distribution is given in the form of a histogramme of N bins each of width h and lower limit x_n , then the procedure is as follows.

Choose a random number R and find the bin n such that

$$\sum_{i=1}^{n-1} c_i \leq R \sum_{i=1}^N c_i < \sum_{i=1}^n c_i$$

where c_i is the contents of the i th bin

If $n = 1$ then $\sum_{i=1}^{n-1} c_i = 0$

Then the required value x_R is given by the equation

$$\underline{\sum_{i=1}^{n-1} c_i + \left(\frac{x_n - x_R}{h} \right) c_n = R \sum_{i=1}^N c_i}$$

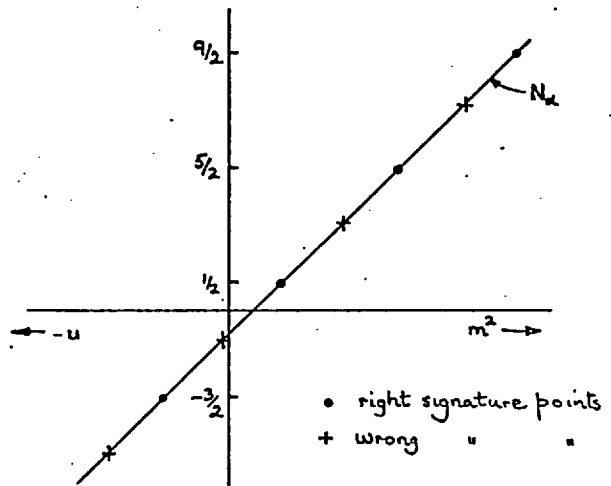
APPENDIX B

Some Regge Pole Jargon

Right and Wrong Signature Points

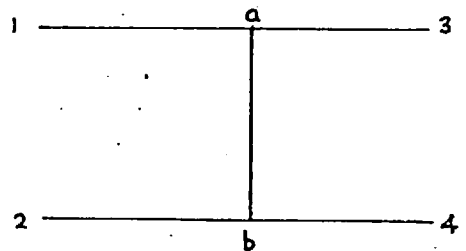
Points spaced by $\Delta\alpha = 2$ from the lowest lying particle on a trajectory are known as right signature points. Thus on the N trajectory (see diagram) right signature points occur at $\alpha = (\dots -7/2, -3/2, 1/2, 5/2, \dots)$.

Wrong signature points are the intermediate half integer values of α between right signature points. On the N_α trajectory they occur at $\alpha = (\dots -5/2, -1/2, 3/2, 7/2, \dots)$.



Sense and Nonsense Points

Consider the exchange diagram for the reaction $1 + 2 \rightarrow 3 + 4$. Then the point α on the exchanged trajectory is said to be a nonsense point if $|\lambda_1 - \lambda_3| > \alpha$ or $|\lambda_2 - \lambda_4| > \alpha$ where λ_i is the helicity of particle i .



If $|\lambda_1 - \lambda_3| > \alpha$ vertex a is said to be a nonsense vertex and if $|\lambda_2 - \lambda_4| > \alpha$ vertex b is said to be a nonsense vertex.

If $|\lambda_1 - \lambda_3| \leq \alpha$ vertex a is a sense vertex and similarly for vertex b . Both vertices must be sense for the point α to be a sense point.

Factorisation

The factorisation property of Regge residues can be stated algebraically as

$$\beta(t) = \beta_a(t)\beta_b(t)$$

where the partial residues $\beta_a(t), \beta_b(t)$ characterise the coupling of the trajectory $\alpha(t)$ to vertices a and b (in rather the same manner that the vertex form factors $F_a(t), F_b(t)$ characterise vertex couplings in the particle exchange model).

This property has the effect of reducing the number of parameters when making a fit to several reactions if the same vertex occurs more than once.

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