

Stability and design of steel beams in the strain-hardening range

A thesis submitted to the Imperial College of Science, Technology and Medicine
for the degree of Doctor of Philosophy

By

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Declaration of Originality

I confirm that this thesis is my own work and that any material from published or unpublished work from others is appropriately referenced.

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Abstract

Many of the principal concepts that underpin current metallic structural steel design codes, notably Eurocode 3, were developed on the basis of elastic, perfectly plastic material behaviour, essentially ignoring strain-hardening; such material behaviour lends itself to the concept of discrete cross-section classification. A newly proposed, deformation based approach to structural steel design represents an alternative treatment to cross-section classification that is based upon a continuous relationship between cross-section slenderness and deformation capacity, as well as a rational exploitation of strain-hardening. This method is referred to herein as the Continuous Strength Method. The aim of this research is to develop preliminary guidance for the use of the Continuous Strength Method at the member level, focusing on the behaviour of simply supported and continuous beams. Particular attention will be given to determining the maximum laterally unsupported lengths prior to which the full capacity predictions of the Continuous Strength Method can be achieved, as well as the performance of lateral bracing elements in various structural configurations.

Through a programme of experiments, numerical modelling and parametric studies, the implications of allowing for strain-hardening in the design of laterally restrained steel beams is investigated with particular emphasis on the performance of the bracing elements. A total of fourteen tests on simply supported beams and six tests on continuous beams were performed considering two basic scenarios: discrete rigid restraints and discrete elastic restraints of varying stiffness. In all tests, bending resistances in excess of the plastic moment capacity were observed, but generally it was concluded that closer restraint spacing than specified in current design codes to achieve the cross-section capacity may be required to harness significant benefit from strain-hardening and to develop the full CSM bending resistance. The forces generated in the restraints were within current code requirements although some modifications were suggested. Furthermore, the spacings of the restraints were also considered and a new limiting slenderness and transition curve for the CSM was proposed. The results from the experiments were supplemented by parametric studies conducted using analytical and

numerical models developed as part of this thesis, as well as through the use of proprietary software packages.

A parallel experimental investigation into the material modelling assumptions of the Continuous Strength Method was also conducted, employing an innovative full cross-section tensile test to capture average cross-section material properties. The results from the investigation validated the modelling assumptions of the Continuous Strength Method and improved the accuracy of the predictive capacity equations.

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Chapter 1

Introduction

1.1 Background

Current structural design codes generally represent the material stress-strain characteristics of structural steel by means of an elastic, perfectly plastic model. This leads naturally to the concept of elastic and plastic moment capacities and the process of cross-section classification. Although simple, this treatment can lead to overly conservative designs. A newly proposed, deformation-based approach to structural steel design, referred to herein as the Continuous Strength Method (Gardner, 2008), represents an alternative treatment to cross-section classification that is based upon a continuous relationship between cross-section slenderness and deformation capacity, as well as a rational exploitation of strain-hardening. Strain-hardening can be broadly defined as the additional strength beyond yield arising as a result of plastic deformation; its importance in the design of steel structures has been previously recognised, notably by Horne and Medland (1966) and Davies (2006).

The Continuous Strength Method has been shown to offer increases in member resistance of up to 15% over current European standards, as well as a reduction in scatter when compared with test data. In the context of the trend towards reducing the environmental cost of construction, this represents an attractive enhancement in material efficiency that would provide considerable savings in construction resources.

1.2 The Continuous Strength Method today

To date, the Continuous Strength Method for carbon steel has been developed for a range of cross-sections in compression and bending. Research into the importance of strain-hardening for cross-sections under combined loading, asymmetric sections, as well as member buckling is currently under way. For the introduction of the method into practice across Europe, all structural scenarios need to be examined in detail and this will need to be achieved through a comprehensive programme of laboratory testing and theoretical analysis. A natural extension of the method would be to consider its applicability to the member level behaviour of beams as well as the associated practical considerations.

1.3 Scope of the study and research innovation

The aim of this research is to develop preliminary guidance for the use of the Continuous Strength Method at the member level, focusing on the behaviour of simply supported and continuous beams. Particular attention will be given to determining the maximum laterally unsupported lengths for which the full capacity predictions of the Continuous Strength Method can be achieved. The properties of the discrete lateral restraints used to define these lengths will also be given detailed consideration, with a particular emphasis on their stiffness and strength. To date, these quantities are only prescribed for use in the design of determinate structures and in traditional plastic design.

1.4 Outline of thesis

This chapter places the newly proposed Continuous Strength Method of structural steel design into context, summarising its development to date and identifying the general areas where its implications are yet to be considered. A general overview of the thesis follows.

A general overview of the key theoretical and experimental research reported in the literature that is relevant to this project is provided in Chapter 2. Detailed considerations of particular aspects of the literature are made at the relevant stages in this thesis.

The stability implications of utilising the Continuous Strength Method for statically determinate structural elements with discrete elastic lateral restraints are considered in Chapter 3. The focus is to establish the minimum bracing strength and stiffness requirements to achieve the predicted CSM levels of bending resistance in primary members, comparing the results with codified limits. An extensive laboratory programme is also described, the results of which are used primarily in this chapter and in Chapter 4.

Determining the maximum laterally unsupported length of simply supported beams to achieve the cross-section resistances predicted by the Continuous Strength Method is the subject matter of Chapter 4. A very simple analytical model is developed for the lateral-torsional buckling behaviour of steel I-beams in the strain-hardening range which is then used, in conjunction with numerical modelling and collated test data, to propose some preliminary design recommendations.

The results of a laboratory testing programme comprising tests on continuous steel I-beams with discrete elastic and rigid lateral restraints are described and presented in Chapter 5. The stability implications of forming a collapse mechanism compatible with the Continuous Strength Method are investigated with reference to restraint forces and stiffness requirements. A numerical model is also developed, which is used to determine the maximum laterally unsupported length of continuous beams designed using the Continuous Strength Method.

The strain-hardening characteristics of structural steel are explored in Chapter 6. A new, full cross-section tensile test is used to determine material properties that are more representative of the true material behaviour than an isolated tensile coupon test. Using the results from the tests, certain assumptions originally made in the development of the Continuous Strength Method are validated. The updated material properties are then used to improve the accuracy of the Continuous Strength Method design equations.

A summary of the research and the key findings are presented in Chapter 7, together with suggestions for further work.

Chapter 2

Literature review

2.1 Introduction

The aim of this chapter is to provide an overview of the key theoretical and experimental contributions that underpin the primary topics under examination in this thesis. The review will begin with an account of the treatment of material non-linearity in the design of structural steel elements, summarising early research contributions, current European codes of practice and the most recent advances intended to challenge the current approach to the design of steel beams in the codes of practice. The second section provides a review of beam buckling, identifying the key theoretical contributions and how they relate to current design provisions. The third section examines the effects of restraint conditions on the stability of beams, with particular reference to the stiffness and strength requirements of lateral and torsional restraints. The final section provides an overview of the numerical modelling approaches adopted throughout this research, their assumptions and their specific applications to each part of the thesis.

2.2 Material non-linearity in structural design

Material non-linearity in structural design has been previously considered both experimentally and theoretically. A systematic experimental and theoretical research programme was conducted at the Fritz Engineering Laboratory, Lehigh University during the 1960s to examine the strain-hardening behaviour of steel structures at the material,

cross-section and member levels. Lay (1965a) demonstrated that the flexural stiffness of a uniformly loaded member can only be zero if the parent material possesses zero strain-hardening stiffness. Using the ensuing theoretical model of inelastic stiffness, expressions for the flange local buckling capacity were developed and limiting flange width to thickness ratios were devised (Lay, 1965b). Taking into consideration the local buckling behaviour of beams, a model relating the limiting rotation capacity to the unsupported length of a simply supported beam was then developed by Lay and Galambos (1965). Subsequent work by Byfield and Nethercot (1998) involved the derivation of an expression for the strain-hardened major axis flexural capacity of an I-section beam integrated through the cross-section, making use of an assumed non-linear continuous stress distribution with the maximum outer-fibre strain set at 1.5% strain.

The current European structural steel design code, EN 1993-1-1 (2005), treats material non-linearity through the familiar elastic-plastic or rigid-plastic material modelling approaches, neglecting the influence of strain-hardening. Furthermore, its treatment of the cross-section stability of thin-walled structural elements - the ability of a cross-section to resist local buckling - is achieved via a system of discrete cross-section classification which ascribes four classes of section dependent upon local plate geometry and material strength. Class 1 sections are fully effective under pure compression and are capable of attaining and maintaining their full plastic moment in bending; Class 2 sections have comparatively reduced deformation capacity, but can achieve their yield load in compression and attain their full plastic capacity in bending; Class 3 sections can also achieve their yield load in pure compression, but are limited by first yield (i.e. their elastic moment capacity) in bending; Class 4 sections fail by local buckling prior to yielding and are assigned reduced capacities in both compression and bending dependent upon the slenderness of the plate elements.

A study by Gardner (2008) using stub column test data collected from the literature demonstrated that for stocky sections the cross-section classification system has significant conservatism when the resistance of the cross-sections is limited to the yield load and that its stepwise nature does not reflect the observed physical response. To overcome these limitations, a new deformation-based design method, referred to as the Continuous Strength Method (CSM) was proposed. The Continuous Strength Method

allows for the beneficial influence of strain-hardening by utilising an elastic, linear hardening material model, as well as replacing the system of cross-section classification with a continuous non-dimensional measure of deformation capacity based upon non-dimensional local plate slenderness. To date, design equations for the CSM have been developed for cross-section resistance in bending and compression (Gardner et al., 2011). Previous underpinning research for the CSM was reported by Gardner (2002), Gardner and Nethercot (2004a), and Ashraf et al. (2008).

The extent of strain-hardening that a structural element may exhibit has been shown to be dependent upon the basic form of the cross-section, the material and the forming process. Representative values for the degree of strain-hardening present in different structural steel cross-section types are given in Wang (2011). For hot-rolled I-sections $E_{sh}/E = 0.015$, for hot-rolled hollow sections, $E_{sh}/E = 0.01$ and for cold-formed hollow sections $E_{sh}/E = 0.015$, in which E is the modulus of elasticity and E_{sh} is the strain-hardening modulus were recommended. Previous tests by Kemp et al. (2002) suggested a value of $E_{sh}/E = 0.013$ for hot-rolled I-sections. Material tests reported by Byfield et al. (2005) showed that $E_{sh}/E = 0.0129$ is typical for hot-rolled I- and H-sections.

2.3 Buckling of steel beams

The influence of flexural-torsional buckling is of fundamental importance in the design of steel structures, as its development can seriously limit load carrying capacity. In this section an overview of the key theoretical contributions to the current understanding of flexural-torsional buckling behaviour will be presented, with due consideration made for the effects of structural configuration and material non-linearity.

2.3.1 Elastic lateral torsional buckling

Buckling can be described as the behaviour in which a structure or a structural element suddenly deforms in a plane different to the original plane of loading and response (Trahair, 1993). The current elastic theory of flexural- or lateral-torsional buckling that is widely presented in textbooks (see Bleich (1952) and Timoshenko and Gere (1961)) has its underpinnings in the work presented by Euler (1759) on the flexural buckling anal-

ysis of slender columns, as well as the studies by Saint-Venant (1855) on the twisting response of members subjected to uniform torsion. In two independent studies published in the same year by Prandtl (1899) and Michell (1899), a unified theory considering both the flexural and torsional buckling behaviour of beams of a narrow rectangular cross-section subject to transverse loading was presented, the solution of which was in the form of a second-order differential equation in displacement and twist. Subsequent contributions by Timoshenko (1953a), Timoshenko (1953b) and Wagner (1936) which included the effects of warping torsion in I-sections, led to the development of a general theory of lateral torsional buckling.

Buckling of thin-walled steel beams can be broadly categorised according to the relative wavelengths of the buckling deformations and the presence (or absence) of deformations to the geometry of the cross-section. Where the half wavelength of the buckle is of the same order as the member length, member buckling is said to have occurred and this may be either flexural, torsional or flexural-torsional in nature; in all such cases it is assumed that the cross-section geometry does not change during buckling (Bleich, 1952). Conversely, where the half wavelength of the buckle is of a similar order to the constituent plate element widths, and that during buckling the cross-section undergoes localised distortions, then the buckling is considered to be local. With local buckling, deformations develop in the regions along the member where axial compression is at its greatest; development of local buckling can reduce the capacity of the member to resist flexural-torsional buckling. In between these two cases lies distortional buckling, where unlike member buckling, the cross-section gradually distorts along the member length, but there is also some flexure of the web (Trahair, 1993).

Flexural-torsional buckling most commonly arises in beams, in which context it is generally referred to as lateral-torsional buckling, but may also occur in compression members, where flexural and torsional buckling represent the component modes. Flexural buckling (Fig. 2.1a) concerns itself with the buckling failure of a member in the plane of a principle axis without any rotation of the cross sections (Bleich, 1952). Resistance to flexural buckling is provided solely by the flexural rigidity EI , in which E is the modulus of elasticity and I is the second moment of area about the relevant principal axis and it arises when the second-order moments due to the axial compression forces

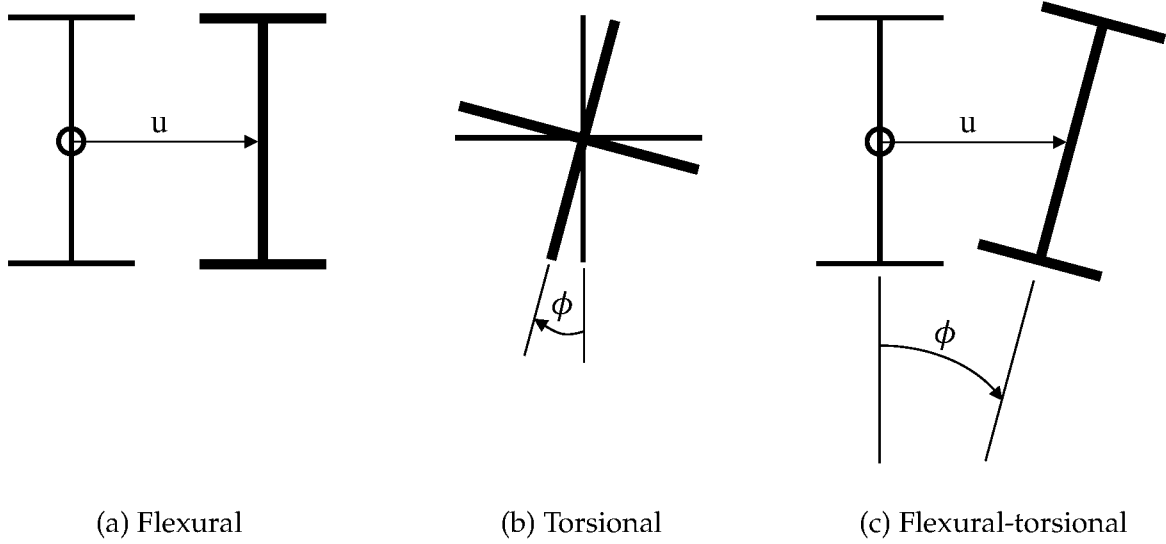


Figure 2.1: Forms of member buckling.

and lateral displacements are equal to the internal resisting moments (Trahair, 1993). Torsional buckling (Fig. 2.1b) on the other hand is concerned with the twisting of the member and it is resisted by the torsional rigidity GI_t and warping rigidity EI_w , in which G is the shear modulus, I_t is the St. Venant's torsion constant and I_w is the warping constant. Torsional buckling occurs when the second-order torques due to axial compression and twist are equal everywhere to the sum of the internal torsion resistances (Trahair, 1993). Flexural-torsional buckling (Fig. 2.1c) is a combination of lateral displacements and twist rotations and is resisted by combinations for flexural, torsional and warping rigidities.

Expressions for the elastic critical moment of a beam are commonly derived using either statics and equilibrium, or through the use of energy principles and the calculus of variations. In either case, for a simply supported beam subject to equal and opposite end moments, the equation relating elastic critical moment M_{cr} to the laterally unsupported span L , as derived by Timoshenko and Gere (1961), is given by Eq. (2.1):

$$\frac{M_{cr}}{M_{pl}} = \frac{\pi}{M_{pl}L} \sqrt{EI_z GI_t \left(1 + \frac{\pi^2 EI_w}{L^2 GI_t} \right)} \quad (2.1)$$

in which I_z is the minor axis second moment of area, I_w is the warping constant and

I_t is the St. Venant's torsion constant. This expression underpins most of the subsequent analysis in the literature on flexural-torsional buckling, with basic variations taking into account factors such as the effect of unequal end moments (Salvadori, 1953), concentrated loading, mono-symmetry (Chwalla, 1939) as well as end and lateral restraint conditions (Nethercot and Rockey, 1971).

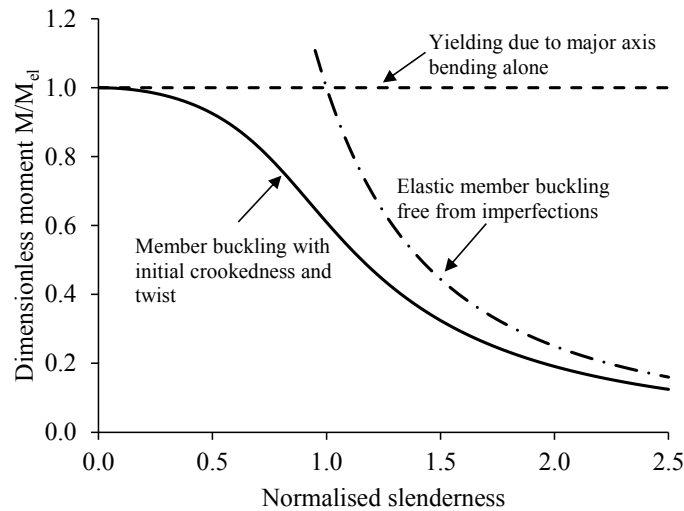


Figure 2.2: Yielding and buckling of beams with initial bow and twist (Trahair, 1993).

Members that are initially straight and free of twist do not deflect laterally or twist until the critical elastic buckling load is attained. Where beams have small initial bow and twist imperfections, lateral deflections and twisting deformations will propagate from the onset of loading leading to a departure from the perfect elastic predictions of behaviour. Fig. 2.2 compares the response given by Eq. 2.1 for a perfect member to that of an imperfect member. For stocky beams, the failure moment approaches the yield moment M_{el} , whilst for long beams the moment approaches the elastic buckling moment M_{cr} (Trahair, 1969a).

Advances in computer technology as well as the development of the finite element method has led to the ability to analyse more complicated cross-section types, statically indeterminate problems as well as complete structural frameworks and systems. Of relevance to this study is the work carried out on continuous beams (Salvadori (1951),

Pettersson (1952), Trahair (1968a) Trahair (1968b), Trahair (1969b), Nethercot and Trahair (1976b)), whose response differs from simply supported beams as a result of the interactions between adjacent spans. Early work by Salvadori (1951) considered a beam of a narrow rectangular section with major axis moments applied at the supports, neglecting longitudinal continuity by treating each span of the beam as being simply supported to establish a lower-bound elastic critical load. Pettersson (1952) introduces the idea of longitudinal continuity by considering a three-span continuous beam subjected to concentrated loading in the central span with the outer spans unloaded; longitudinal continuity is modelled using theoretical predictions from simply supported beams with elastic end restraints.

The first theoretical analyses of continuous beams with all spans loaded were conducted by Trahair (1965, 1966, 1967), demonstrating that for a two-span continuous beam, the behaviour when fully loaded lies in between the cases where one span is unloaded. Fig. 2.3 presents the various buckling modes of a two-span continuous beam that is prevented from twisting and deflecting at its supports, which for a given load position, vary according to the relative magnitudes of the loading intensities Q_{AB} and Q_{BC} . In Fig. 2.3b, only segment AB is loaded and during buckling it is elastically restrained by the unloaded segment BC, with the opposite being the case in Fig. 2.3c. Where the loads are equal (Fig. 2.3c), the two segments are effectively independent of one another and so buckle as two simply supported beams with effective lengths L_{AB} and L_{BC} .

Using a finite element formulation developed by Barsoum and Ghallager (1970) for the buckling analysis of arbitrary open sections, the effect of proportional variations in Q_{BC} relative to Q_{AB} on the critical buckling loads for each span on the continuous beam in Fig. 2.3a is plotted in Fig. 2.4. In between the extreme cases of $Q_{BC} = 0$, $Q_{AB} = 0$ and $Q_{BC} = Q_{AB}$ a series of intermediate buckling loads will develop, producing a complete and non-linear interaction relationship for the critical load combinations on each span. On the basis of this accurate relationship, a simplified linear relationship was proposed by Trahair (1969b), constructed by interpolating between the loads corresponding to the critical buckling modes in Figs. 2.3b - 2.3d. For use in routine design, the critical points corresponding to $Q_{BC} = 0$, $Q_{AB} = 0$ and $Q_{BC} = Q_{AB}$ required for the interpolations must be determined using tabulations provided by Trahair (1968b), but this approach

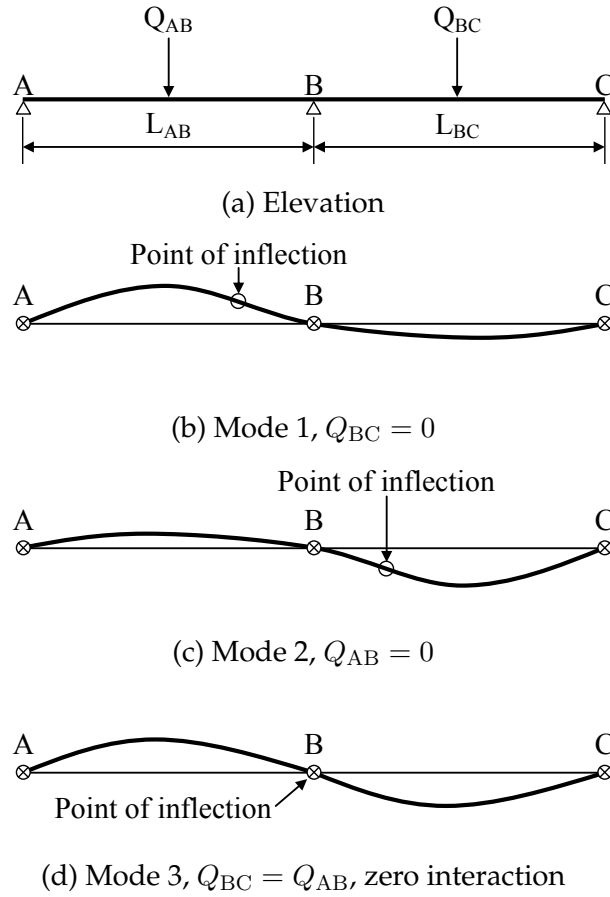


Figure 2.3: Buckling modes for a two-span continuous beam.

has not gained widespread acceptance in practice; instead, the conservative method of neglecting continuity between spans is generally preferred (Trahair et al., 2008), or a numerical analysis of the full system is conducted.

2.3.2 Inelastic lateral torsional buckling

The resistance of beams to elastic lateral torsional buckling increases with reductions in slenderness and in most cases, a certain amount of yielding will occur before ultimate failure by buckling. The effect of yielding is to reduce the major and minor axis rigidities so that the buckling resistance of the member is below its elastic value at a comparable slenderness. Early theories of inelastic flexural, torsional and lateral-torsional buckling were based upon the tangent modulus theory of material behaviour, which assumes a

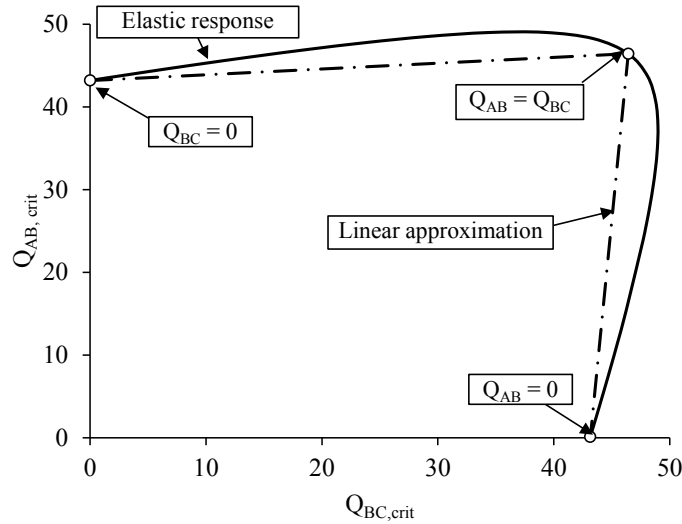


Figure 2.4: Elastic critical load combinations for a two-span continuous beam.

non-linear stress-strain curve that is elastic throughout, with a local slope E_t (Fig. 2.5a). However, inelastic material does not behave in this manner, with unloading paths following the initial slope E of the stress-strain curve (Fig. 2.5b). The latter concept leads to the reduced modulus theory and when applied to buckling it results in E_t being applied to the loaded regions of a member and E being applied to the unloaded regions; the value of the reduced modulus E_r to be used in the buckling analysis is then determined from the geometry of the cross-section (Horne and Merchant, 1965). It is accepted however that the tangent modulus theory yields more accurate results than the reduced modulus theory when compared with experimental data. Resolution of this paradox was provided by Shanley (1947) who demonstrated that the use of the tangent modulus is valid provided that strain reversal is prevented by increasing the applied load during the onset of deflections (Trahair, 1993).

The first attempt to analyse the effect of the spread of plasticity on the lateral-torsional stability of beams was made by Neal (1950). Using theoretical and experimental results to determine reduced rigidities and an application of the reduced modulus theory, a relationship between slenderness and inelastic critical moment was established for a simply supported beam subjected to uniform moment; residual stresses were assumed to be absent and warping was neglected. Subsequent analysis by Flint (1953) introduced

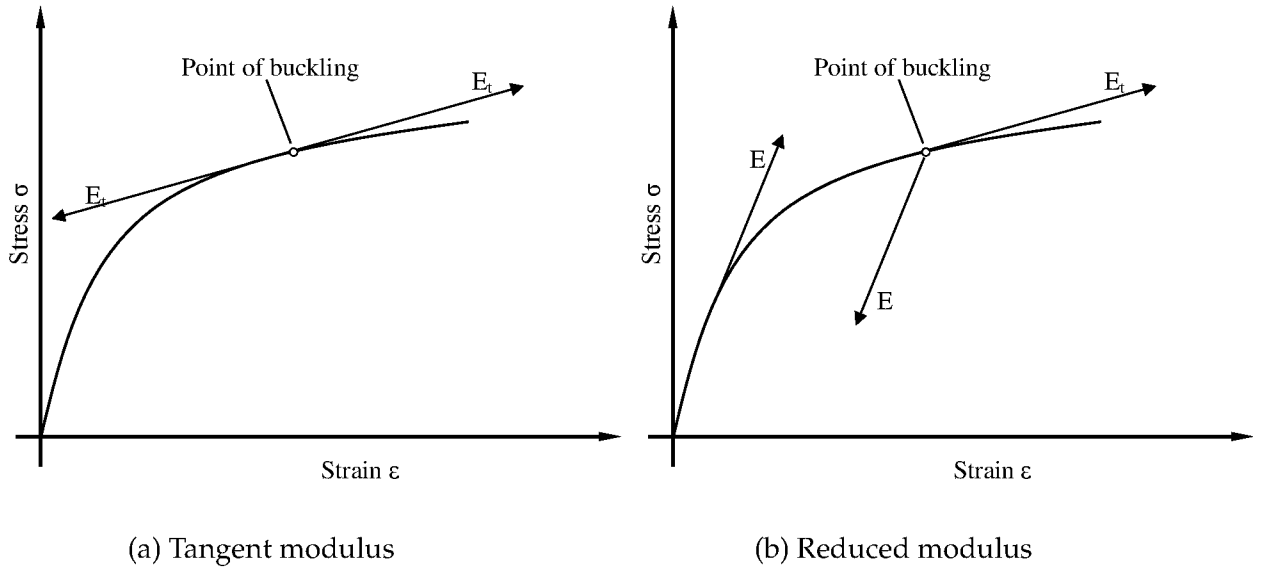


Figure 2.5: Theories of inelastic buckling.

warping rigidity for the application of the theory to I-sections. Using the slip theory of yielding, it was suggested by White (1956) that the material of a steel beam is either elastic or strain-hardened because of the discontinuous process of yielding, suggesting that the strain-hardening modulus E_{sh} should be used instead of $E_t = 0$ in the yielded regions.

The effects of residual stresses on beams subjected to equal end moments were considered by Galambos (1963) who assumed a linearly varying residual stress distribution and $E_t = 0$ in the yielded regions. Subsequent work by Fukumoto and Galambos (1966) extended this analysis to include the effect of unequal end moments. Using the finite-element method to generate accurate buckling curves, simplified inelastic buckling load expressions were proposed by Nethercot (1972) who used a pair of cubic simultaneous equations to describe the relationship between inelastic critical moment and reduced stiffness, factoring in the effect of residual stresses. Various distributions of residual stresses have been suggested, but it was shown by Nethercot (1974) that the chosen distribution is of far less importance than the peak values of the compressive and tensile residual flange tip stresses, suggesting a peak value of $0.3f_y$, where f_y is the material yield stress, to be appropriate for hot-rolled I-sections. An experimental investigation,

primarily looking at the influence of residual stresses, was conducted by Kitipornchai and Trahair (1975b) on six simply supported rolled I-beams of varying length. Half of the specimens were delivered in their hot-rolled condition and the remainder were annealed to relieve the residual stresses. In isolation, it was shown that the annealed beams performed in a similar manner to those with residual stresses, concluding that the effect of geometric imperfections was more significant than that of residual stresses. However, the recorded residual stresses were fairly low, with peak values of only $0.1f_y$.

Much attention has been given to the influence of loading patterns on the inelastic buckling moment of beams. For beams subjected to moment gradients and concentrated loading, it was shown by Kitipornchai and Trahair (1975a), using a numerical model, that as with elastic beams, concentrated loading results in higher buckling loads than for uniform moment - this is due to localised yielding rather than yielding spreading along the entire length of the beam. Subsequent theoretical work by Dux and Kitipornchai (1978) developed an approximate design buckling moment expression by means of a stiffness modification factor to account for yielding and a moment modification factor to account for loading conditions. This formed the basis of the concept of allowing an inelastic beam to be conservatively analysed as an equivalent uniform elastic beam under equivalent uniform elastic moment. A comprehensive study by Nethercot and Trahair (1976a) established the theoretical basis for a family of buckling curves for beams. It was demonstrated that there were three distinct lateral torsional slenderness regions: one corresponding to elastic critical buckling (slender beams), a second for inelastic lateral-torsional buckling (intermediate slenderness) and a final group where the full inelastic flexural capacity can be attained before buckling (stocky beams). A particularly interesting result of this paper is the very large range of critical slenderness values over which the full plastic capacity could be attained, reflecting previous conclusions that concentrated loading is less severe for inelastic stability than uniform moment.

A series of nine beam tests with three variations in moment gradient were conducted by Dux and Kitipornchai (1983), employing a sophisticated testing apparatus to provide a very rigid assembly. It was shown that there are significant variations in member buckling capacity depending upon moment gradient, with beams subjected to steeper gradients sustaining higher moments than the equivalent beam in uniform bending; this is in

very close alignment to the earlier theoretical findings of Nethercot and Trahair (1976a). A major conclusion of this study is the recommendation that multiple design curves should be used, depending upon moment gradient; EN 1993-1-1 (2005) introduces the effect of moment gradient by means of an equivalent uniform moment factor.

A natural extension to the concept of inelastic buckling under a moment gradient is inelastic buckling analysis of continuous beams. When designing a statically determinate I-beam, the slenderness limit, below which the effects of lateral torsional buckling can be ignored, is based upon knowledge of the elastic and inelastic critical moments, which can be determined from the elastic in-plane moment distribution. However, for a continuous beam (and other statically indeterminate structures), the elastic in-plane distribution of moments is not necessarily the same as the final inelastic in-plane distribution and so it is not strictly valid for determining the inelastic buckling load of such a structure (Yoshida et al., 1977).

Much of the research into inelastic buckling of continuous beams has been either numerical or experimental in nature. The first experiments were conducted by Poowanachaiikul and Trahair (1976) comprising 8 different tests, with the primary conclusion that the phenomenon of inelastic moment redistribution in continuous beams may affect buckling strength when compared against simply supported beams. A parallel theoretical investigation demonstrated that in the presence of strain-hardening, the sharp redistribution predicted by rigid plastic theory does not occur. A comprehensive finite element and numerical analysis by Yoshida et al. (1977), partially validated by the experiments of Poowanachaiikul and Trahair (1976), examined the influences of slenderness, load configuration and lateral restraint conditions. It was shown that until the beams are sufficiently stocky for the effects of strain hardening and the eventual redistribution of the in-plane moment to become important, the inelastic buckling loads of continuous beams can be approximated by those for single span simply supported beams with similar patterns of in-plane moment. It was also demonstrated that the prevention of twist at the internal support is the most effective method of bracing, with prevention of lateral deflections alone providing very limited enhancement to the buckling load.

Design formulae and procedures for the inelastic buckling of continuous beams are

somewhat limited owing to the difficulty involved (as with elastic beams) of obtaining closed form general buckling expressions allowing for the interaction between spans. An initial proposal by Nethercot and Trahair (1976a), representing an improvement upon the zero interaction method of Salvadori (1951), takes into account the lateral bending and warping interactions between the critical segment and adjacent segments. The method for inelastic beams mirrors that of elastic beams, but is modified to include an approximate expression for the inelastic moment capacity of simply supported beams and a scaling of elastic interactions by the ratio of the plastic critical moment to the elastic critical moment. An iterative method based upon similar principles was proposed by Dux and Kitipornchai (1984).

2.3.3 Codified approaches

An early approach for the design of beams against lateral-torsional buckling was provided by Nethercot and Trahair (1976a) in which the relationship between the non-dimensional inelastic critical M_I/M_{pl} moment and non-dimensional slenderness $\bar{\lambda}_{LT} = \sqrt{M_{pl}/M_{cr}}$ is given by Eq. (2.2):

$$\frac{M_I}{M_{pl}} = 0.7 + \frac{0.3(1 - 0.7\bar{\lambda}_{LT}^2)}{0.61 - 0.3\beta_m + 0.07\beta_m^2} \quad (2.2)$$

This expression also takes into consideration the effect of in-plane moment distribution via the moment ratio parameter $-1 \leq \beta_m \leq 1$ where $\beta_m = -1$ corresponds to a uniform applied moment and $\beta_m = 1$ corresponds to a linear moment gradient with equal and opposite end moments; inelastic critical moments for various values of β_m are plotted in Fig. 2.6.

The method set out in EN 1993-1-1 (2005) or Eurocode 3, Part 1.1, for the design of laterally unrestrained members against lateral-torsional buckling defines a non-dimensional slenderness (referred to previously) $\bar{\lambda}_{LT}$:

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}} \quad (2.3)$$

in which W_y is the major axis plastic section modulus for Class 1 and 2 cross-sections, the elastic section modulus for Class 3 cross-sections and an effective section modulus

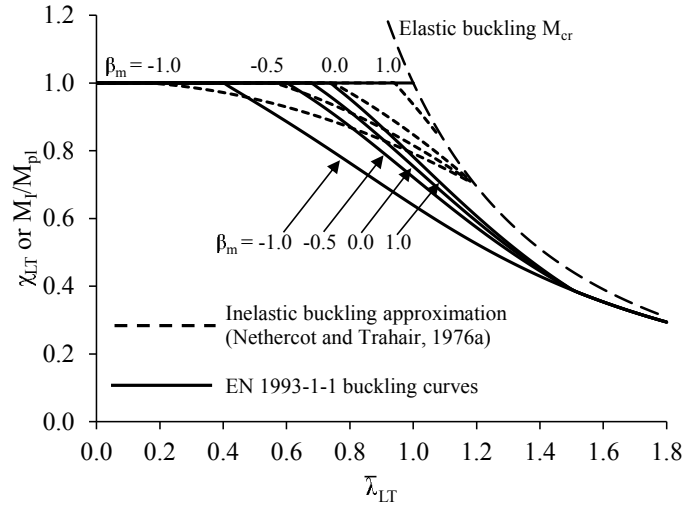


Figure 2.6: Comparison between EN 1993-1-1 and approximated design buckling moment resistances (adapted from Trahair et al. (2008)).

W_{eff} for Class 4 cross-sections, f_y is the material yield strength and M_{cr} is the elastic critical moment for lateral-torsional buckling. The Eurocode 3 method is a general method where the strength predictions of structures are governed by the interactions between yielding and buckling. In contrast to the expression used in Eq.(2.2), the elastic buckling moment M_{cr} must be used, rather than an approximation of the inelastic buckling moment.

EN 1993-1-1 (2005) provides a choice between two sets of lateral-torsional buckling curves, which were determined for a series of empirically validated equivalent imperfections based upon cross-section geometry. The first set of curves is a general case that may be applied to any section type (Clause 6.3.2.2) and the second can be applied to either rolled or equivalent welded sections (Clause 6.2.2.3). For the general case, a reduction factor χ_{LT} , shown in Fig. 2.6, to be applied to M_{pl} for Class 1 and 2 sections, M_{el} for Class 3 sections and $M_{\text{eff}} = W_{\text{eff}} f_y$ for Class 4 sections, is defined as:

$$\chi_{\text{LT}} = \frac{1}{\Phi_{\text{LT}} + \sqrt{\Phi_{\text{LT}}^2 - \bar{\lambda}_{\text{LT}}^2}} \quad \text{but} \quad \chi_{\text{LT}} \leq 1 \quad (2.4)$$

in which:

$$\Phi_{LT} = 0.5 [1 + \alpha_{LT} (\bar{\lambda}_{LT} - 0.2) + \bar{\lambda}_{LT}^2] \quad (2.5)$$

where α_{LT} is the equivalent imperfection factor (given in Table 6.3 of EN 1993-1-1). For rolled sections, an extended slenderness plateau set at $\bar{\lambda}_{LT,0} = 0.4$, below which lateral-torsional buckling may be neglected, is introduced and Eq. (2.4) is replaced by:

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \beta \bar{\lambda}_{LT}^2}} \quad \text{but} \quad \chi_{LT} \leq 1 \quad \text{and} \quad \chi_{LT} \leq \frac{1}{\bar{\lambda}_{LT}^2} \quad (2.6)$$

in which:

$$\Phi_{LT} = 0.5 [1 + \alpha_{LT} (\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}) + \beta \bar{\lambda}_{LT}^2] \quad (2.7)$$

where $\beta = 0.75$. Eq. (2.6) is plotted in Fig. 2.6 for the same distributions of moment as used for Eq. (2.2), using an assumed value of $\alpha_{LT} = 0.49$.

2.4 Elastically restrained steel beams

The resistance of beams to lateral instability can be improved through the provision of effective lateral bracing, either continuously or at intervals along the length of the member. Discrete bracing (for example, secondary beams) provides restraint at nodal points whilst continuous bracing (such as a composite floor deck) resists all lateral movement; the effect of providing bracing is to reduce the critical length of the member and thus increase the critical buckling load. For nodal bracing, it is not possible in practice to have perfect bracing where at the point of restraint there is zero deflection. In all cases, the bracing system will undergo some elastic deformations, but these must be sufficiently small to ensure that buckling of the primary member still occurs around the point of restraint. Furthermore, the elastic deformations will result in the development of internal forces in the bracing system, which it must be able to withstand. Thus two related issues arise with bracing: it must have sufficient stiffness to effectively resist lateral deflections of the primary member and it must have sufficient strength to resist any internal forces developed due to the initial imperfections and lateral deformations.

2.4.1 Restraint stiffness and strength

Flint (1951) provided the first analysis of the effects of intermediate discrete elastic restraints on the elastic buckling stability of a beam. Neglecting the effects of warping rigidity, Flint's analysis examined the effects of end restraint conditions, their torsional rigidity as well as the effects of intermediate lateral restraints. Focusing on the case of discrete intermediate restraints, a solution for the critical buckling load of a beam was provided in terms of the restraint stiffness k_{res} and expressed as an amplification factor c , defined as the ratio of braced to unbraced critical loads (see Fig. 2.7). In so doing, two fundamental results were established: firstly, there exists a threshold restraint stiffness k_{ideal} at which the restraint acts as an effective brace, causing the beam to buckle in the second mode. Secondly, at this stiffness, the effective length of the beam is reduced from L to $L/2$ and further increases in restraint stiffness offer no further enhancement to the critical load factor c and the corresponding critical moment; this is sometimes referred to as the threshold moment. As a generalisation of Flint's initial analysis, further analytical work was conducted by Schmidt (1965) to establish the effect of elastic torsional end restraints and a central elastic lateral restraint on the critical bending moment of a beam.

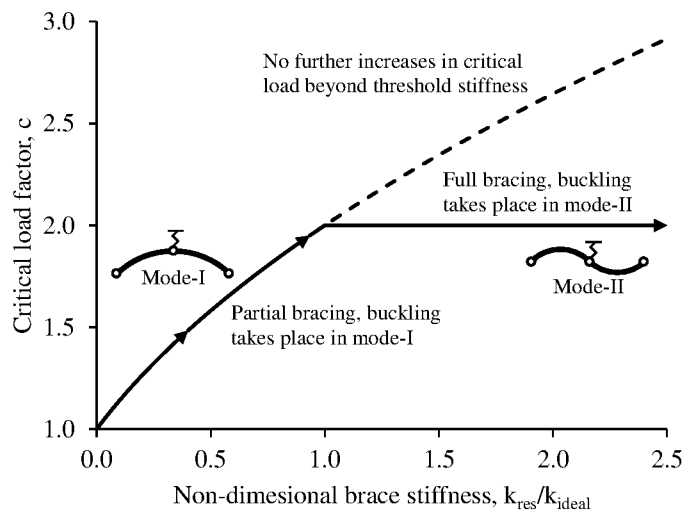


Figure 2.7: Typical theoretical relationship between buckling load enhancement and restraint stiffness for a beam, showing the transition from buckling modes I and II.

The results of Flint (1951) and Schmidt (1965) are useful in design in as far as they allow

the engineer to attain any possible critical load between the first and second buckling modes simply by varying the restraint stiffness. However, to be of full practical use, the forces that the restraining members are subjected to must also be known. The first study into elastic brace forces was conducted by Zuk (1956), who examined various configurations of columns and beams, restrained either at discrete points or continuously. For beams and columns with discrete lateral restraints, the analysis concluded that restraint forces did not generally exceed 0.82% of the axial force in compression flange of the restrained member, which was safely below the value assumed by professional engineers at the time of 2%.

The analysis presented by Zuk (1956) was greatly simplified by Winter (1960) and validated against the results of experiments conducted on scale models. Winter's approach was aimed at providing the design engineer with a relatively simple method for establishing both the transition restraint stiffness and the corresponding restraint force. The method is based on the analysis of a pin-ended column with an initial imperfection amplitude of δ_0 and assumes that the elastic restraint is attached at a fictitious hinge (see Fig. 2.8). Moments about the hinge are evaluated as $M_{\text{res}} = FL/4 - N_E(\delta_0 + \delta)$, where δ is the lateral deflection due to the applied load N and N_E is the Euler load of the segment with length $L/2$. Noting that $F = k_{\text{res}}\delta$, the required bracing stiffness for a column of length L is then $k_{\text{ideal,column}} = \frac{4N_E}{L}(\delta_0/\delta + 1)$ and the corresponding restraint force $F_{\text{res}} = k_{\text{ideal,column}}\delta$. For beams, Winter assumed that the restraint forces developed in equivalent columns can be used conservatively, justified by noting that the portion of the beam that is in tension can provide some restraint against lateral bending. Winter's approach was reviewed by Yura (1996) who presented some extensions that considered the effect of restraint location and unequal restraint spacing.

More recent work by Al-Shawi (1998) considered the forces developed in a restraint located at any location along a column with pinned-pinned, pinned-fixed and fixed-fixed end conditions. Assuming an initial sinusoidal imperfection with a magnitude of $e_0 = L/500$, the column and lateral restraint were modelled as a complete system and it was shown that the restraint force ratio lies between 1% and 2%. The resulting relationship between restraint stiffness and restraint force is presented in Fig 2.9 and shows that at the transition stiffness, restraint forces are relatively large, but restraint forces

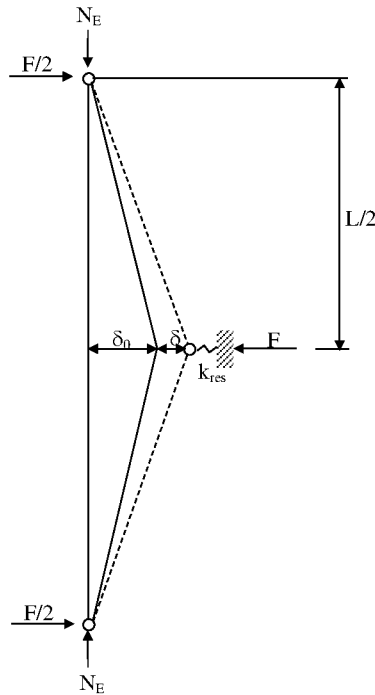


Figure 2.8: Winter's idealisation of a pin-ended column restrained at mid-height by an elastic spring.

stabilise when the bracing stiffness reaches approximately 10-times the ideal level. A related study by Banfi and Feltham (1999) provided basic extensions to account for multiple bracing, whilst Trahair (1999) also considered the influence of lateral-torsional slenderness; a subsequent analysis by Al-Shawi (2001) considered the effect of elastic end restraints. In each case, the consensus was that codified recommendations of restraint force ratios, typically in the region of 2.5%, were too onerous.

The development and proliferation of the finite element method has permitted the analysis of restraint forces in beams with a wide variety of loading and support conditions. Based upon the results of Nethercot and Rockey (1971), a comprehensive analysis of this type was conducted by Nethercot and Rockey (1972) in which the effect of single and multiple torsional and translational restraints was investigated. Where multiple restraints were used, progressive transitions from buckling mode-I to II through to III occur with increasing magnitudes of brace stiffness. The analysis also addressed the limitations of the work of Flint (1951) where the effects of warping were neglected;

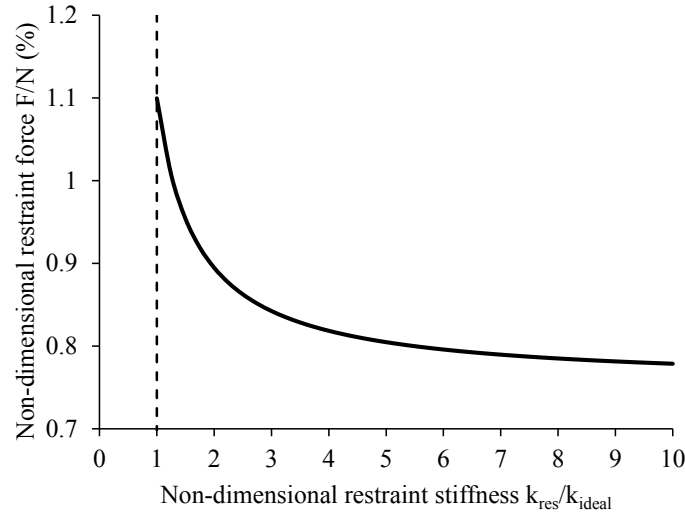


Figure 2.9: Relationship between restraint force and restraint stiffness for various levels of axial load in a centrally restrained column, assuming $e_0 = L/500$.

it was shown that for members where warping is important in resisting the torsional component of lateral-torsional buckling, the critical load of such members is generally more sensitive to variations in restraint stiffness than for members where warping is unimportant - a result also shown to be true in Nethercot (1973b). The finite element method was also used to successfully validate the analytical work of Medland (1980).

Much of the work using the finite element method was carried out in conjunction with full-scale laboratory testing that could be used to partially validate the numerical results. Kitipornchai and Richter (1978) conducted an experimental investigation to examine under different loading conditions the most effective locations for rigid lateral restraints and concluded that they should be located at or above the shear centre and where the in-plane bending moment diagram is at its maximum value. In a similar set of tests, Wong-Chung and Kitipornchai (1987) demonstrated that lateral restraints for simply supported beams provide little restraint when attached to the tension flange, but are effective in either rotational or translational configurations at the shear centre. Wakabayashi and Nakamura (1983) considered the effect of different bracing conditions and conducted tests of complete bracing systems, with elastic lateral restraints provided either as purlins attached to the compression flange or a lateral beam running the full

height of the web. It was demonstrated that both types of bracing greatly enhanced stability, where even for beams with very high slenderness ratios, the full plastic moment capacity of the primary member could be attained before buckling occurred. In the experiments of Wakabayashi and Nakamura (1983), restraint forces were not recorded.

Using a finite element model originally developed by Wang et al. (1987) to determine the ultimate strength of beams with elastic end restraints, the effects of intermediate lateral and torsional restraints were incorporated by Wang and Nethercot (1989) and the model validated against the results of Wakabayashi and Nakamura (1983). By means of a parametric study that examined a single cross-section with various bracing configurations providing either lateral or torsional restraint either at a single point at the midspan, or at multiple points, it was concluded that for single braces a restraint force ratio of 1% is appropriate and for multiple braces 2% is appropriate. It is noted that the key results of this study were also reproduced in a subsequent paper by Wang and Nethercot (1990).

Recent analytical contributions on the subject of the stability of discretely braced steel beams have been made by McCann et al. (2012), where it was shown that two general classes of buckling modes can result: one where the number of buckling modes corresponds to the number of restraints and another where there is an infinite number of buckling modes resulting in no deflection of the lateral restraints. Based on the analytical results of McCann et al. (2012), which also considered the height of the restraint in the cross-section, design expressions for minimum stiffness and strength requirements were presented in McCann et al. (2013).

2.5 Numerical modelling

Numerical modelling can be viewed as an integral component in the analysis of structures, particularly for stability problems that involve complex loading, material properties and geometric considerations. Early studies into the stability of structural elements in the inelastic range (e.g. Horne (1964)) resulted in mathematical expressions that were generally not possible to solve by hand and were thus reliant upon some basic numerical procedures. With improvements in computing power came simultaneous advances

in the development of more computationally demanding and general-purpose analysis techniques, most notably the finite element method and its applications to structural problems (Rajasekaran and Murray, 1973; El-Zanaty and Murray, 1983).

Finite element modelling is a convenient and cost-effective research tool that can overcome some of the constraints that are usually associated with physical testing. Using results obtained from carefully conducted laboratory tests, as well as benchmark analytical results, the ability of the corresponding finite element model to replicate the physical problem can be assessed. The successfully validated model may then be used as the basis for parametric studies, which can reveal the effect of key parameters on the structural response of the element or system under investigation.

Two numerical modelling approaches are adopted in this thesis. Firstly, throughout Chapters 3, 4 and 5 the general purpose finite element package ABAQUS (Simulia, 2010) is used to either develop or substantiate parametric and analytical relationships. ABAQUS has an extensive and sophisticated element library that is capable of modelling elements and systems with considerable realism and detail, making it an extremely useful and flexible tool for reproducing and extending complicated laboratory tests. For the second approach, a component of Chapter 5 uses a numerical model developed as part of this thesis, with similarities to the model reported by Nethercot (1973a). The purpose of developing a model from the ground up is primarily rooted in the desire to understand the modelling process at a fundamental level. It also confers the advantage of having a computer programme that is tailored to its application, thus being able to generate and process results with greater efficiency than a general purpose package. The major limitation of this approach however, is that a considerable investment in time is required to develop each component of the model and as such, its application is restricted to a single aspect of this research.

2.5.1 Element types

Throughout this research both shell and beam elements are used. Both of these are structural finite elements and belong to a general formulation of finite elements referred to as isoparametric elements in which the geometry and displacements of the elements are

described in terms of the same parameters and are of the same order (Zienkiewicz and Taylor, 1989). Furthermore, in the formulation of structural finite elements, displacements are interpolated in terms of mid-surface displacements and rotations and it is assumed that the stresses normal to the mid-surface are zero for beam, plate and shell elements (Bathe, 1996).

Beam elements are used for two components of this thesis and feature in Chapters 4 and 5. In Chapter 4 the B31OS open section, three-dimensional Timoshenko beam element taken from the ABAQUS element library (Simulia, 2010) is used for the purposes of appraising the predictions of an analytical model. In Chapter 5 a two-noded linear Timoshenko beam element is used to model the elastic and inelastic in-plane behaviour of continuous beams as part of a computer programme developed in this thesis. For thin variants of this type of element shear locking can result (Bathe, 1996), so a generalised formulation that assumes linear variations in transverse displacements and section rotations as well as a constant element transverse shear strain is used instead. This is achieved using a generalised version of the basic variational formulation of the finite element method, as proposed by Washizu (1955), Washizu (1975) and Hu (1955), which is implemented in Chapter 5.

For the numerical model developed as part of this thesis in Chapter 5, a buckling analysis is also conducted which makes use of an additional element type developed by Barsoum and Ghallager (1970). This is a thin-walled, open-section, two-noded beam element for which the potential energy expressions segregate pre-buckling deformations from buckling deformations. Two element stiffness matrices arise from this process - that which contains only geometric and pre-buckling force terms (often referred to as either the geometric or incremental stiffness) and that which contains axial, flexural and torsional stiffness terms (often referred to as either the flexural or material stiffness). These matrices form the basis of a subsequent eigenvalue analysis used to determine elastic and inelastic buckling loads.

When cross-section deformations are important, structural members are better represented using shell elements (Bathe, 1996). Shell elements are used throughout this thesis to accurately model the member and local buckling response of thin-walled structures.

To achieve this, the finite element type S4R, which is a four-noded, doubly curved general-purpose shell element with reduced integration and finite membrane strains, selected from the ABAQUS element library, is used. Reduced integration in this element type has much the same effect as employing a generalised variational formulation in that it prevents an overly stiff response as a result of shear locking for thin shells (Owen and Hinton, 1980). This element is suited to the modelling of a range of shell thicknesses and has been successfully implemented in other studies, such as that by Chan and Gardner (2008).

2.5.2 Material modelling

Common to all of the finite element modelling approaches adopted in this research is that in the inelastic range, the material modelling assumptions and procedures are those based upon the Prandtl-Reuss equations that govern the classical incremental theory of plasticity (Hill, 1983). In this theory analysis is conducted in terms of strain increments, rather than total accumulated strain owing to the fact that for a given stress state in the inelastic range there exists no uniquely corresponding strain; the total strain in any given time period is uniquely dependent upon the stress history. However, the current stress can be related to the current increment in strain, from which the total strain can be obtained by accumulating the strain increments (Reddy, 2004).

Calculations of stress increments rely upon a linear decomposition of incremental strains into their elastic and plastic components. To complete the procedure, knowledge of the following is required: (1) a yield function, which specifies the stress state corresponding to the initiation of plastic flow (i.e. the yield stress); (2) a flow rule relating plastic strain increments to current stresses and stress increments; and (3) a hardening rule, which specifies how the yield function is altered to account for plastic flow (Bathe, 1996). The application of this process is discussed in detail in Chapter 5.

For the purposes of this research, two basic material relationships are used. For the numerical model developed in this thesis, an elastic, linear hardening relationship is used, which is the same as that recommended for finite element analysis in EN 1993-1-5 (2006) as well as the material relationship adopted for the Continuous Strength Method.

Where ABAQUS finite element shell models are used, the true material stress-strain behaviour is generated from the engineering stress-strain data obtained from laboratory tensile coupon tests; material non-linearity is then incorporated into the finite element models by means of a piecewise linear discretisation of the true stress-strain response. The relationship between true stress, σ_{true} and engineering stress, σ_{nom} is given by Eq. (2.8), while the relationship between log plastic strain, ϵ_{true} and engineering strain, ϵ_{nom} is given by Eq. (2.9), in which σ_{nom} and ϵ_{nom} are the engineering stress and strain respectively and E is the modulus of elasticity.

$$\sigma_{true} = \sigma_{nom} (1 + \epsilon_{nom}) \quad (2.8)$$

$$\epsilon_{true} = \ln(1 + \epsilon_{nom}) - \frac{\sigma_{true}}{E} \quad (2.9)$$

The expressions in Eq. (2.8) and Eq. (2.9) are used to take into account the change in cross-sectional area during deformation. Engineering stress and strain are defined as $\sigma_{nom} = F/A_0$ and $\epsilon_{nom} = \Delta L/L_0$ in which A_0 is the initial cross-sectional area, L_0 is the initial length, F is the applied force and $\Delta L = L - L_0$ is the change in length. If it is assumed that before and after an axial test there is no change in volume, but the area reduces to A , then:

$$A \cdot L = A_0 \cdot L_0 \quad (2.10)$$

True stress σ_{true} is then analogously defined as:

$$\sigma_{true} = \frac{F}{A} = \frac{F}{A_0} \cdot \frac{L}{L_0} = \sigma_{nom} (1 + \epsilon_{nom}) \quad (2.11)$$

True strain $\epsilon_{true,1}$ is defined as the sum of all infinitesimal engineering strains $d\epsilon = dL/L$, so that:

$$\epsilon_{true,1} = \int d\epsilon = \int_{L_0}^{L_1} \frac{dL}{L} = \ln \frac{L_1}{L_0} = \frac{\ln(L_0 + \Delta L)}{L_0} = \ln(1 + \epsilon_{nom}) \quad (2.12)$$

The true plastic strain ϵ_{true} defined in Eq. (2.9) is simply the plastic component of true strain, which can be readily obtained by subtracting the elastic component of strain σ_{true}/E from Eq. (2.12).

2.5.3 Geometric imperfections

All structural members contain initial geometric imperfections, but are manufactured to lie within specific geometric tolerances upon delivery. The behaviour of structural members can be significantly influenced by the magnitude and nature of any initial geometric imperfections. In this research, both global lateral imperfections and local plate imperfections are considered in the finite element models. Global imperfections influence the manner and extent to which lateral deflections propagate throughout the loading history and hence the point of initiation of yielding. Local imperfections can influence local buckling capacity, which for partially restrained beams can result in sufficient reductions in stiffness to eventually lead to a member-level failure mechanism.

Both the locally and globally imperfect geometries of all the members were determined using an initial elastic eigenvalue analysis with the resulting local and global eigenmodes being used to define the distribution of local and global imperfections. Scaling of global eigenmodes can be achieved either by using data obtained from laboratory measurements (see Chapter 3) or by using assumed non-dimensional measures, usually as a proportion of unrestrained member length. For local imperfections, amplitudes can again be obtained from laboratory measurements, assumed non-dimensional values (usually as a fraction of local plate thickness) or by using other predictive approaches such as that based on plate slenderness initially proposed by Dawson and Walker (1972) and its subsequent revisions by Gardner and Nethercot (2004c).

2.5.4 Residual stresses

Residual stresses arise in hot-rolled steel sections due to differential cooling after the forming process; these stresses can have a significant effect on the manner in which yield zones develop and have a similar effect upon member buckling capacity as initial geometric imperfections (Lay and Ward, 1969). Notably, the flange tips are typically found to be in residual compression (due to early cooling), whilst the web-to-flange junction is typically in residual tension (due to later cooling). The influence of residual stresses upon the inelastic stability of beams was considered theoretically by White (1956), with a simplified linear distribution proposed by Galambos (1963). Similar to Nethercot (1974), residual stresses are introduced into the ABAQUS finite element mod-

els by partitioning the web and flanges of each cross-section, with each partition representing a stress level corresponding to the mid-point of the linear variation in stress over the partition length.

2.5.5 Solution procedures

For both the ABAQUS models and the finite element model developed as part of this thesis, non-linear solution procedures are required as it is nearly always the case that some element of either the out of balance forces used to check global equilibrium or stiffness matrices are dependent upon the displacements, which are initially unknown. In each case, use is made of various iterative solution algorithms. For the finite element model used in Chapter 5 a very basic, but highly robust direct iteration method, usually referred to as the Picard iteration method, is used (see for example Reddy (2004), Bathe (1996) and Owen and Hinton (1980)). The essence of this procedure is to provide an initial estimate for the first displacement increment, after which the displacements for the system can be solved to update the initial estimate. The subsequent displacement approximations are then used to evaluate updated solutions until a convergence criterion is satisfied; geometrically the Picard method may be interpreted as a secant method since a succession of secants is evaluated until they become tangential to the true solution. This method is well-suited to materially non-linear problems where there is generally no unloading and hence no singularities in the structural stiffness matrices (Reddy, 2004).

For the ABAQUS models, the Riks method is used (Riks (1972), Wempner (1971), Riks (1979)) to allow for the presence of material and geometric non-linearities and their interactions. For most cases, variations of the Newton-Raphson method are used to trace non-linear equilibrium paths. This is a gradient based method that relies upon evaluating tangent values of the global stiffness matrices at given load or displacement increments; however in the vicinity of limit points the stiffness matrix is singular and so cannot be inverted. In the Riks method, the load increment for each step is considered to be an unknown and is solved as part of the solution process (Reddy, 2004).

2.6 Concluding remarks

This chapter has provided an overview of the major subjects to be covered in this thesis. Specific details from the literature will be explained further where they are used in the thesis, and additional sources will be consulted in the relevant chapters.

Recent advances in characterising the flexural behaviour of steel have resulted in a new design method, referred to as the Continuous Strength Method (CSM), which for stocky elements has been shown to lead to more accurate and enhanced design capacity predictions at the cross-sectional level. In order to be fully exploited as a design method, the limitations placed upon the unrestrained lengths of beams at the higher capacities predicted by the CSM need to be established for both statically determinate and indeterminate configurations. Furthermore, realistic bracing conditions need to be considered, with current provisions for stability and strength being re-evaluated in light of the demands of the CSM.

Chapter 3

Bracing requirements for statically determinate structural elements

3.1 Introduction

The resistance of beams to lateral instability can be improved through the provision of effective lateral bracing, either continuously or at intervals along the length of the member. For discrete bracing systems, the spacing of the lateral restraints influences the load bearing capacity of the member. In order to be effective, the restraints should have adequate stiffness to limit the lateral displacements at the point of restraint and have sufficient strength to withstand the forces that arise as a consequence of these displacements as well as any initial imperfections. It was shown by Winter (1960) that, provided the restraint is of adequate stiffness, the bracing forces are small relative to the axial forces in the primary member.

Numerous studies of lateral restraint requirements have been carried out (Flint (1951), Massey (1962), Nethercot and Rockey (1972), Mutton and Trahair (1973), Mutton and Trahair (1975), Wang and Nethercot (1989), Wang and Nethercot (1990), Yura (1996), Al-Shawi (1998), Trahair (1999), Yura (2001), McCann et al. (2013)), typically considering elastic member behaviour. The present research is devoted to examining the lateral stability implications of allowing for strain-hardening in the design of the primary members by means of a newly proposed, deformation-based design procedure that is referred to as the Continuous Strength Method (CSM) (Gardner, 2008). To this

end, a series of experiments on simply-supported beams with variations in restraint spacing and stiffness were conducted. Using a geometrically and materially non-linear finite element model, the test data were reproduced and extended in a parametric study which was then used to inform and develop some basic design equations.

3.2 Key design aspects

3.2.1 Lateral restraint spacing

EN 1993-1-1 (2005) defines a non-dimensional slenderness limit, or plateau length, $\bar{\lambda}_{LT,0} = 0.4$, below which, the effects of lateral torsional buckling can be ignored and the design buckling resistance moment of the member $M_{b,Rd}$ may be taken as the design bending resistance $M_{c,Rd}$ of the cross-section, assuming $\gamma_{M0} = \gamma_{M1}$. $\bar{\lambda}_{LT}$ is defined in Eq. (3.1) as:

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}} \quad (3.1)$$

in which W_y is the major axis plastic section modulus for Class 1 and 2 cross-sections, the elastic section modulus for Class 3 cross-sections and an effective section modulus for Class 4 cross-sections, f_y is the material yield strength and M_{cr} is the elastic critical moment for lateral torsional buckling, which is a function of member length L . For a given set of cross-section and material properties and a fixed value of $\bar{\lambda}_{LT}$, Eq. (3.1) can be solved for L to define the maximum allowable spacing between fully effective lateral restraints before reductions in resistance for lateral torsional buckling are required. For members containing plastic hinges, stable lengths below which lateral torsional buckling can be ignored are given in Annex BB-1 of EN-1993-1-1 (2005).

3.2.2 Restraint forces

Lateral restraints must be of sufficient stiffness to restrict lateral buckling deformations at the point of restraint, whilst also being of sufficient strength to resist the forces generated as a result of the restraining action. In the elastic range, it can be shown that, for a perfect system, there is a threshold level of brace stiffness k_{ideal} that causes a beam

to buckle into the second mode (i.e. between the brace points rather than in an overall mode) - see Fig. 3.1 (Flint, 1951).

For a beam of length L experiencing a force N_{Ed} in the compression flange, EN 1993-1-1 states that the restraint system should be capable of resisting an equivalent stabilising force per unit length q_d (Eq. (3.2)) :

$$q_d = \sum N_{Ed} \frac{8e_0 + \delta_q}{L^2} \quad (3.2)$$

where the assumed initial imperfection amplitude of the restrained member, e_0 , is defined as:

$$e_0 = \alpha_m L / 500 \quad (3.3)$$

in which α_m is reduction factor used for restraining multiple members and δ_q is the lateral deflection of the restrained member into the restraints. Assuming an infinitely stiff restraint system, $\delta_q = 0$, and Eq. (3.2) implies that a restraint must resist 1.6% of N_{Ed} . Eq. (3.2) is derived on the basis of elastic behaviour, but may also be applied when plasticity occurs in the restrained member, allowing for moments up to the full plastic bending capacity, M_{pl} , but not covering the demands of rotating plastic hinges.

3.2.3 The continuous strength method (CSM)

The continuous strength method is a deformation-based design approach for steel elements that allows for the beneficial influence of strain-hardening. To date, design equations for the CSM have been developed for cross-section resistance in bending and compression (Gardner et al., 2011). The CSM bending resistance function $M_{csm,Rd}$, which applies for $\bar{\lambda}_p \leq 0.68$ is defined in Eq. (3.4) as:

$$M_{csm,Rd} = \frac{W_{pl} f_y}{\gamma_{M0}} \left(1 + \frac{E_{sh}}{E} \frac{W_{el}}{W_{pl}} \left(\frac{\epsilon_{csm}}{\epsilon_y} - 1 \right) - \left(1 - \frac{W_{el}}{W_{pl}} \right) \left(\frac{\epsilon_{csm}}{\epsilon_y} \right)^{-2} \right) \quad (3.4)$$

where E is the modulus of elasticity, E_{sh} is the strain-hardening slope taken equal to $E/100$ for structural steel, W_{el} and W_{pl} are the elastic and plastic section moduli and

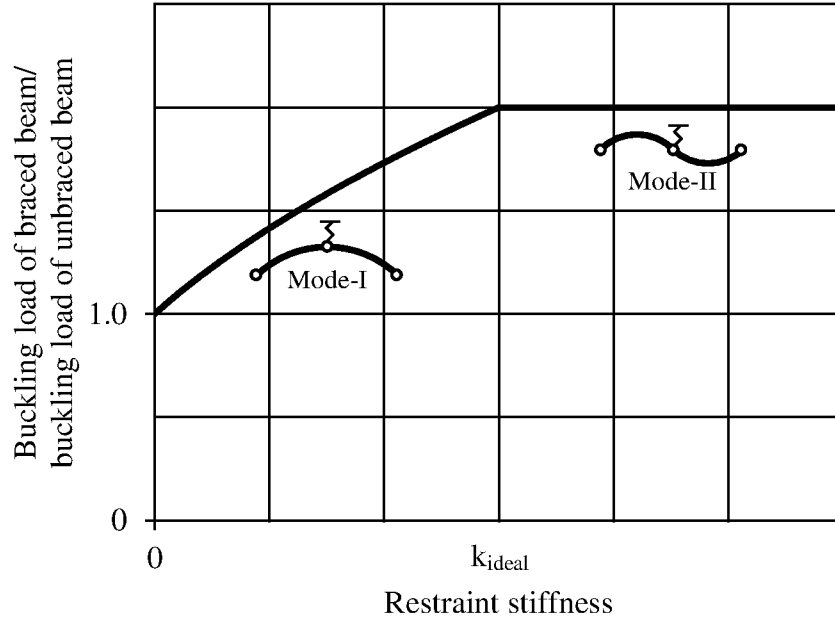


Figure 3.1: Typical theoretical relationship between buckling load enhancement and restraint stiffness for a beam.

$\epsilon_{\text{csm}}/\epsilon_y$ is the strain ratio, defining the limiting strain in the cross-section ϵ_{csm} as a multiple of the yield strain ϵ_y , and given by Eq. (3.5) :

$$\frac{\epsilon_{\text{csm}}}{\epsilon_y} = \frac{0.25}{\bar{\lambda}_p^{3.6}} \quad \text{but} \quad \leq 15 \quad (3.5)$$

in which $\bar{\lambda}_p$ is the local cross-section slenderness, given by Eq. (3.6) as:

$$\bar{\lambda}_p = \sqrt{(f_y/\sigma_{\text{cr}})} \quad (3.6)$$

with σ_{cr} being the elastic buckling stress of the cross-section, or conservatively its most slender constituent plate element.

This study will examine the implications of using the CSM resistance function where moments beyond M_{pl} can be sustained, on the forces developed in lateral restraints, following a series of experiments and numerical simulations.

3.3 Experimental programme

3.3.1 Introduction

A testing programme comprising tensile and compressive material coupon tests, stub column tests and tests on beams with discrete lateral restraints was carried out at the Building Research Establishment and Imperial College London on hot-rolled grade S355 steel I-sections. Two cross-section sizes were chosen: 305×127×48 UB, which had a Class 1 flange ($\bar{\lambda}_p = 0.31$) and a Class 1 web ($\bar{\lambda}_p = 0.30$), and 305×165×40 UB, which had a Class 2 flange ($\bar{\lambda}_p = 0.57$) and a Class 1 web ($\bar{\lambda}_p = 0.44$).

3.3.2 Material properties

Tensile and compressive coupon tests were used to determine the engineering stress-strain material response of the tested specimens; the tests were conducted in the Structures Laboratory of the Department of Civil and Environmental Engineering, Imperial College London.

Tensile and compressive coupons were cut and milled from the web and flanges of two representative UB sections in the longitudinal (rolling) direction only. Testing was carried out in accordance with the provisions of EN 10002-1 (2001). The nominal dimensions of the necked tensile coupons were 320×30 mm and the nominal dimensions of the compressive coupons were 72×16 mm. Prior to testing, half gauge lengths were marked onto the tensile coupons to allow the final strain at fracture, ϵ_f , to be calculated, based on elongation over the standard gauge length $5.65\sqrt{A_c}$ where A_c is the cross-sectional area of the coupon. The tensile coupon tests were carried out in an INSTRON 500 kN hydraulic loading machine with an initial strain rate of 0.001%/s for $\epsilon < 0.5\%$. Between 0.5% and 4% strain, the strain rate was 0.002%/s; between 4% and 17% strain, the strain rate was 0.04%/s. Once the coupon reached $\epsilon = 17\%$ testing switched to displacement control at a constant rate of 0.1 mm/s until failure. Static yield and ultimate strengths were determined by holding the strain constant for two minutes in the yield plateau and at four points near the ultimate stress. Tensile strain was measured using clip gauge and video extensometers.

Compressive coupons were placed in a restraining jig to prevent buckling (Fig. 3.2) with the ends machined flat to ensure they were in a state of pure compression; this was verified by means of strain gauges. Tests were carried out in an INSTRON 500 kN hydraulic loading machine and strain measurements were obtained from strain gauges bonded to the sides of the samples at mid-height. Testing was carried out using displacement control at a constant rate of 0.067 mm/min until the protruding end of the coupon was almost flush with the testing jig.

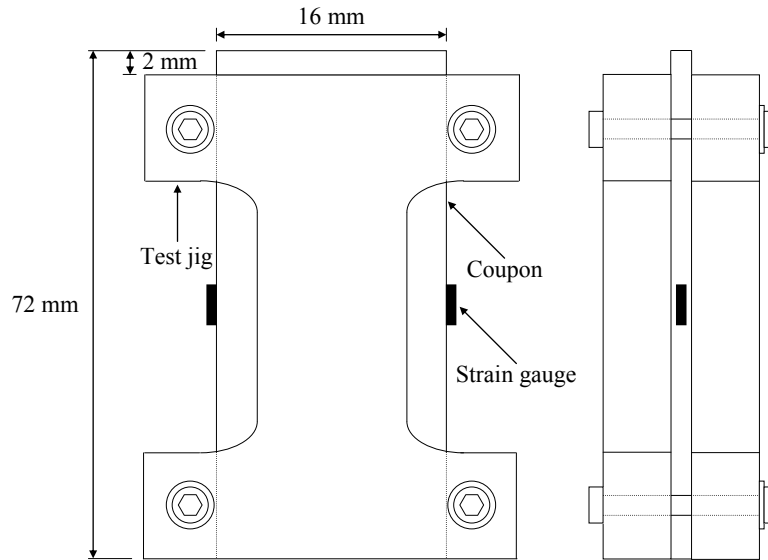


Figure 3.2: Compressive coupon testing jig and nominal coupon dimensions.

All test data, including load, displacement, strain and input voltage were recorded at one-second intervals using the DATASCAN acquisition system. A summary of the results of these tests is provided in Table 3.1. In the coupon designation, T denotes a tensile test, C denotes a compressive test, W denotes a coupon taken from the web and F denotes a coupon taken from the flange. Other symbols are defined as follows: b_c is coupon width, t_c is coupon thickness, A_c is the cross-sectional area of the coupon, E is the modulus of elasticity, f_y is the material yield strength, f_u is the ultimate tensile strength, ϵ_f is the strain at fracture calculated over the standard gauge length set out in EN 10002-1 (2001), and ϵ_u is the strain at the ultimate tensile stress. The stress-strain curves for the tensile and compressive coupons are shown in Figs 3.3-3.6.

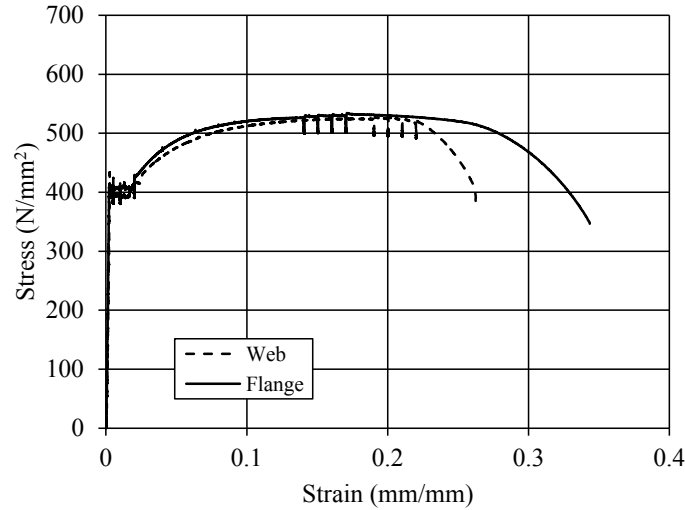


Figure 3.3: Full tensile stress-strain curves for the flange and web of the 305×127×48 UB section.

The tensile and compressive coupons exhibited the anticipated response of a well defined yield point, followed by a plateau before the initiation of strain-hardening. For the compressive coupons, due to the need to prevent buckling, there is a limited amount of material available for deformation; thus, the curves presented in Figs. 3.5 and 3.6 do not present the entire compressive stress-strain response. However, like the tensile coupons, there is a defined yield point followed by a plateau prior to the onset of strain-hardening.

3.3.3 Stub column tests

Compressive stub column tests were performed to investigate the average compressive response of the cross-sections, including the influence of local buckling. The nominal length of the stub columns was selected as twice the overall height of the cross-section but with a global slenderness, $\bar{\lambda}_z = (Af_y/N_{cr})^{1/2}$ where N_{cr} is the elastic buckling load, not greater than 0.1. This ensures that overall buckling does not occur, but sufficient length remains to include a representative distribution of local geometric imperfections and residual stresses. The ends of the specimens were machined flat and true to ensure

Table 3.1: Tensile and compressive coupon test data

Coupon designation	b_c (mm)	t_c (mm)	A_c (mm ²)	E (N/mm ²)	f_y (N/mm ²)	f_u (N/mm ²)	ϵ_f (%)	ϵ_u (%)
305×127×48-TW	19.72	8.48	167.2	198700	402	528	24.2	19.1
305×127×48-TF	19.46	14.41	280.5	191700	391	534	30.8	17.1
305×165×40-TW	20.18	6.36	128.3	201340	459	599	21.6	13.7
305×165×40-TF	19.51	9.97	194.5	204200	436	585	25.1	15.7
305×127×48-CW	15.99	8.48	135.6	207600	408	-	-	-
305×127×48-CF	16.03	14.44	231.4	213100	408	-	-	-
305×165×40-CW	15.78	6.10	96.3	204223	482	-	-	-
305×165×40-CF	15.95	9.97	159.1	218700	454	-	-	-

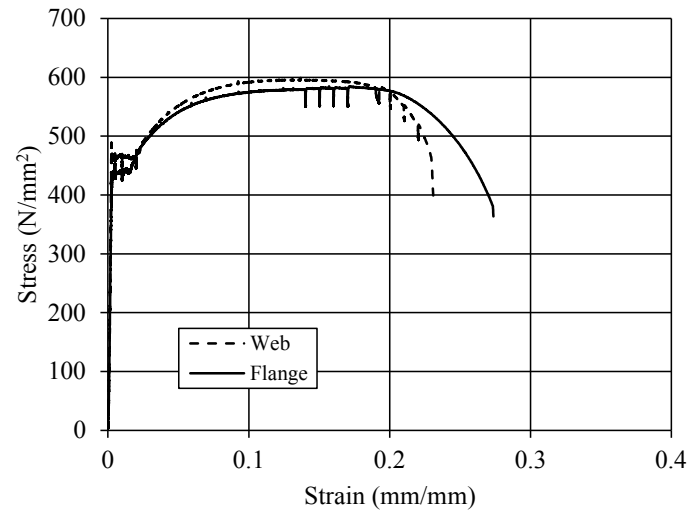


Figure 3.4: Full tensile stress-strain curves for the flange and web of the 305×165×40 UB section.

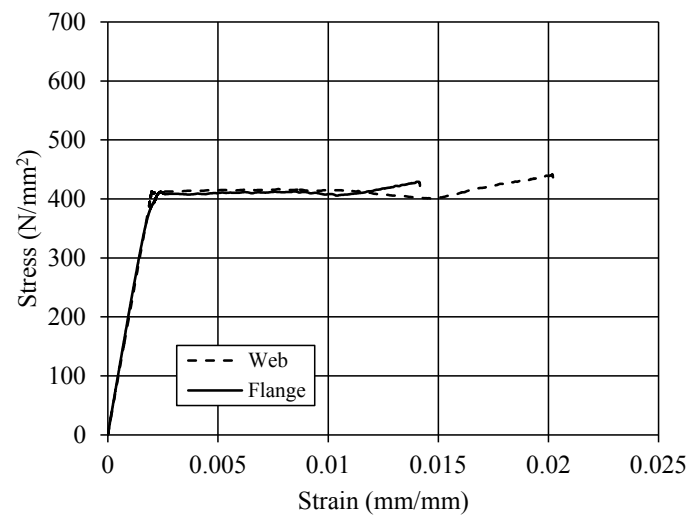


Figure 3.5: Initial portion of the compressive stress-strain curves for the flange and web of the 305×127×48 UB section.

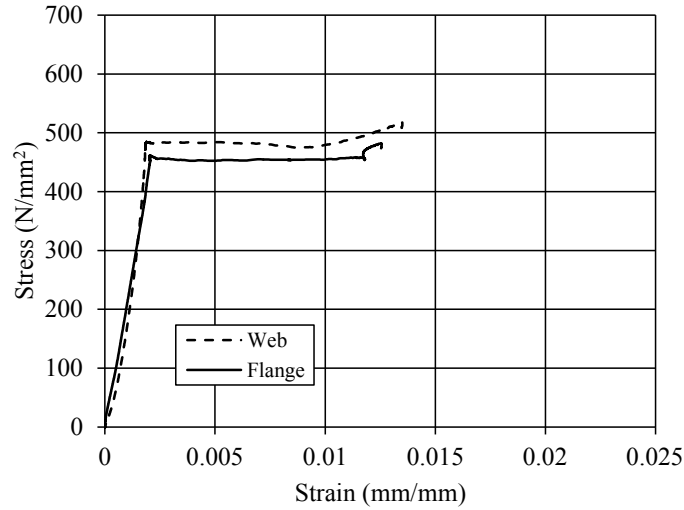


Figure 3.6: Initial portion of the compressive stress-strain curves for the flange and web of the 305×165×40 UB section.

uniform contact with the end platens of the testing machine (Fig. 3.7).

Displacements were recorded by means of two LVDTs in contact with the end platens, the applied load was recorded with a load cell, and four strain gauges attached at the mid-height of the flanges and web were used to ensure concentric load application as well as to eliminate elastic end platen deformations from the end shortening data. Testing was carried out in an INSTRON 3500 kN universal testing machine under displacement-control at a rate of 0.067 mm/min. Results, including load, displacements, strain and input voltage were recorded at one-second intervals using the data acquisition system DATASCAN. Testing was continued beyond the ultimate load-carrying capacity of the stub columns to examine the post-ultimate response. The stub column geometric properties and key test results are presented in Table 3.2. For each section used, SC1 denotes the first stub column test and SC2 denotes the second stub column test, h is the overall section height, b is the overall section width, t_f is the flange thickness, t_w is the web thickness, L_{sc} is the stub column length, and N_u is the ultimate test load.

All specimens failed by local buckling and examples of each cross-section can be seen in

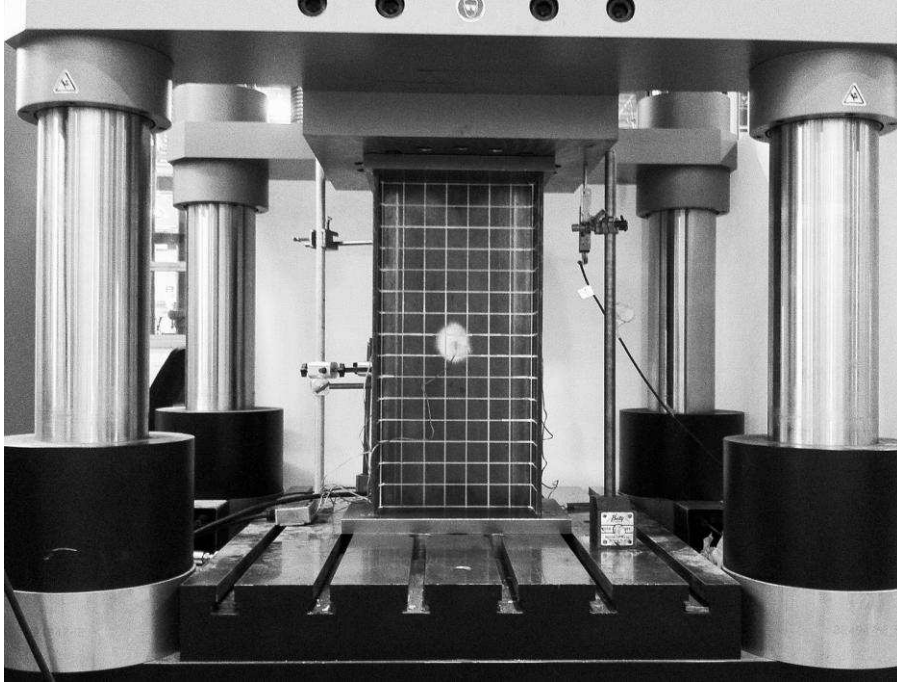


Figure 3.7: Stub column test set-up.

Table 3.2: Stub column test data

Stub column designation	t_f (mm)	t_w (mm)	h (mm)	b (mm)	L_{sc} (mm)	N_u (kN)
305×127×48-SC1	13.93	8.59	311.21	127.54	622	2487
305×127×48-SC2	13.95	8.87	311.56	127.82	622	2487
305×165×40-SC1	9.80	6.11	306.53	166.87	607	2124
305×165×40-SC2	9.78	6.05	305.49	166.52	607	2192

Fig. 3.8. Using load, displacement and strain data obtained from the test, load-true end shortening curves are plotted in Fig. 3.9. During the test, the end platens undergo small elastic deformations that can overstate the displacement readings from the LVDTs. In a procedure developed by C.A.S.E. (1990) the recorded LVDT displacements, δ_{LVDT} , can be modified to obtain true end shortening, δ_{ES} , where:

$$\delta_{\text{ES}} = \delta_{\text{LVDT}} - 2\Delta_{\text{platen}} \quad (3.7)$$

Defining $E_{0,\text{LVDT}}$ as the elastic modulus relating stress, σ , to LVDT strain and $E_{0,\text{true}}$ as the elastic modulus relating stress to strain gauge strains, the the deformation of the platens, Δ_{platen} , can be calculated using Eq. (3.8):

$$\Delta_{\text{platen}} = \frac{L_{sc}}{2} \sigma \left(\frac{1}{E_{0,\text{LVDT}}} - \frac{1}{E_{0,\text{true}}} \right) \quad (3.8)$$

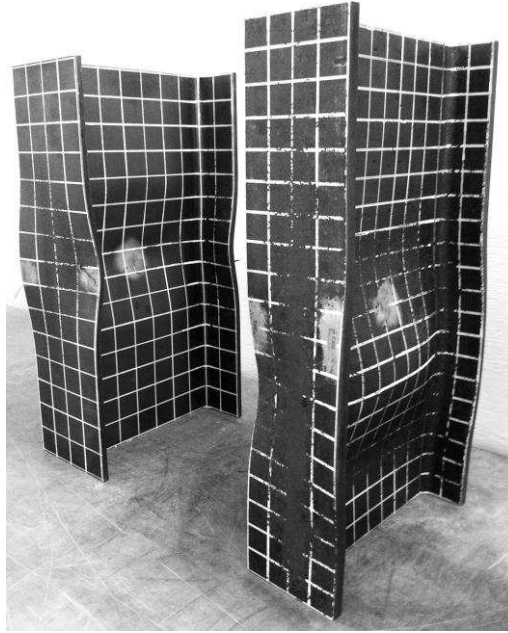


Figure 3.8: Test specimens showing typical stub column mode of failure.

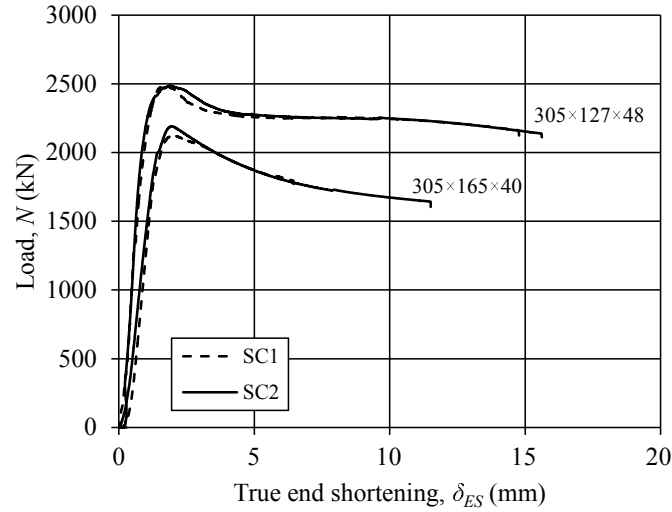


Figure 3.9: Stub columns load-true end shortening curves.

3.3.4 Measurement of geometric imperfections

A total of twelve beam tests were carried out. For each experiment, the basic geometry of the test specimen was measured prior to testing. Global imperfection amplitude measurements were taken for each specimen by holding a fine copper wire at mid-web height taught along the length of the beam and then measuring the distance between the wire and the beam at the mid-length. Local imperfections were measured by placing a representative sample of each section on the bed of a milling machine with an LVDT held securely in the head of the machine. With the LVDT positioned on either the centreline of the web or the tip of the outstand flanges, the specimen was passed up and down and the profile of its surface was recorded. The test set-up is illustrated in Fig. 3.10. The maximum deviation from a straight line connecting the ends of the measured length was taken as the local imperfection magnitude.

The basic geometric measurements, global imperfection magnitudes e_0 , and local flange imperfection magnitudes ω_0 are reported in Table 3.3.

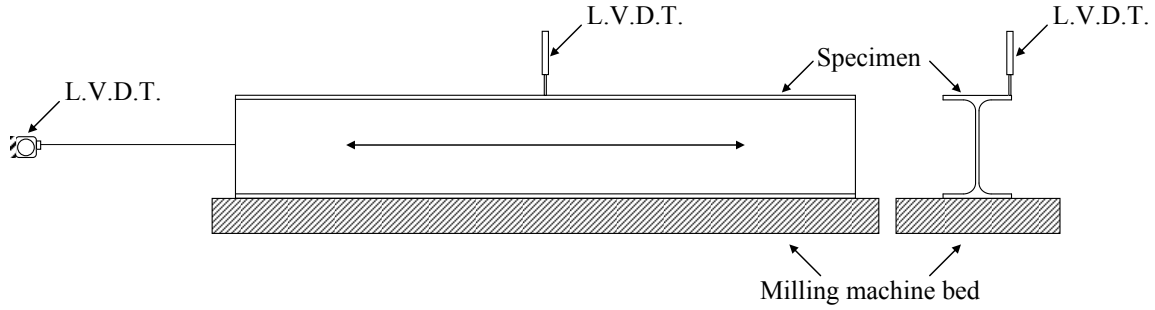


Figure 3.10: Schematic illustration of the local imperfection amplitude measurement procedure.

3.3.5 Four-point bending tests on beams with discrete elastic lateral restraints

A total of twelve four-point bending tests were conducted, four with rigid discrete lateral restraints and eight with the central rigid restraint replaced with elastic restraints of varying stiffness. Four-point bending was chosen because it produces a region of constant bending moment and negligible shear. For the cases in which elastic lateral restraints were used at the mid-span, this configuration also ensured that no frictional restraint was provided by the applied load. Fig. 3.11 shows the general test configurations for the four-point bending tests.

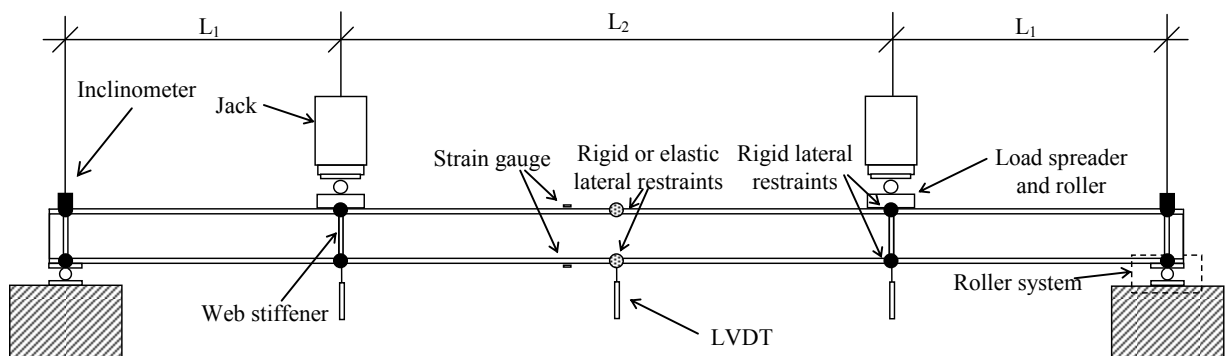


Figure 3.11: Schematic illustration of the four-point bending test configuration.

For all tests, rigid lateral restraints were provided at the loading points and at the supports. The length L_2 was varied to achieve desired values of $\bar{\lambda}_{LT}$ for each test; L_1 was

chosen in relation to L_2 to ensure that buckling takes place in the L_2 region first. Two values of $\bar{\lambda}_{LT}$ were investigated in the tests - 0.3 and 0.4. For the elastically restrained members, threaded rods with a tensile design resistance of approximately 1.6% of the $M_{csm,Rd}$ ($= M_{csm}$ with $\gamma_{M0} = 1.0$, as assumed throughout this study) compression flange force were chosen for the K1 restraints; reduced diameters, using standard threaded rod dimensions, were chosen for the K2 restraints. These elastic restraints were attached to the specimen using an articulated joint connection to minimise bending in the rods. To provide a stable anchorage for the restraints, stanchions with a considerably higher bending stiffness than the lateral stiffness of the specimens were used as fixing points. In order to attain practical restraint stiffnesses, as well as to minimise the effects of bending, the rods were all 2500 mm in length. A schematic plan view of this configuration is illustrated in Fig. 3.12 and a summary of the test parameters, specimen dimensions, and cross-section properties is provided in Table 3.3, where d_{res} is the restraint diameter.

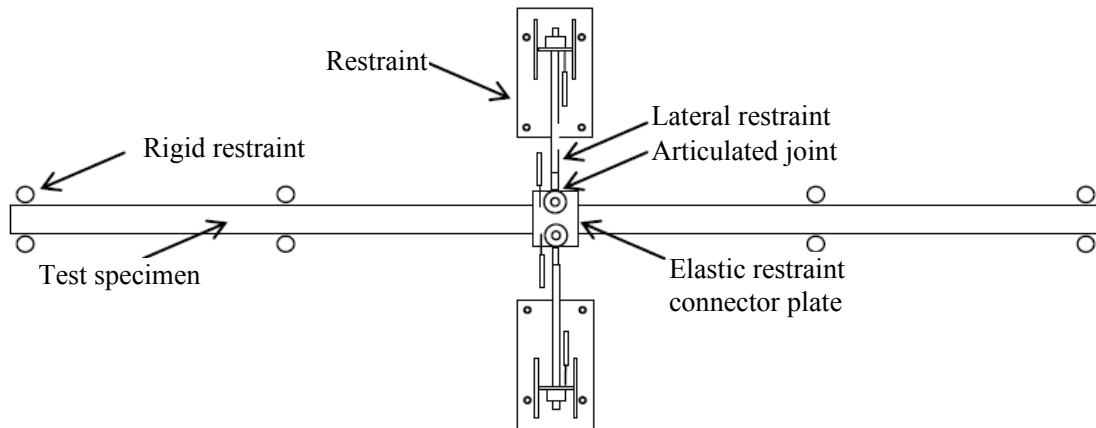


Figure 3.12: Schematic plan view of elastic and rigid lateral restraint systems.

Lateral restraint forces were monitored using a combination of tensile and compressive load cells; as a secondary measure, lateral displacement at the mid-span was also recorded, with corrections being made for any movements of the restraint anchorages - restraint forces can then be calculated knowing the stiffness of the restraint. A schematic section view of the restraint force monitoring approaches is shown in Fig. 3.13.

For each test, web stiffeners were provided at the positions of the supports and the ap-

Table 3.3: Summary of geometric properties of four-point bending test specimens

Test designation	$\bar{\lambda}_p$ (flange)	$\bar{\lambda}_p$ (web)	Restraint type	d_{res} (mm)	Global imperfection magnitude e_0 (mm)	Local imperfection magnitude ω_0 (mm)
305×127×48, $\bar{\lambda}_{LT} = 0.4$, R	0.31	0.30	Rigid	-	0.5	0.100
305×127×48, $\bar{\lambda}_{LT} = 0.3$, R	0.31	0.30	Rigid	-	1.0	0.100
305×165×40, $\bar{\lambda}_{LT} = 0.4$, R	0.57	0.44	Rigid	-	1.0	0.083
305×165×40, $\bar{\lambda}_{LT} = 0.3$, R	0.57	0.44	Rigid	-	1.0	0.083
305×127×48, $\bar{\lambda}_{LT} = 0.4$, K1	0.31	0.30	K1	8.59	0.5	0.100
305×127×48, $\bar{\lambda}_{LT} = 0.3$, K1	0.31	0.30	K1	8.59	1.0	0.100
305×165×40, $\bar{\lambda}_{LT} = 0.4$, K1	0.57	0.44	K1	6.82	6.0	0.083
305×165×40, $\bar{\lambda}_{LT} = 0.3$, K1	0.57	0.44	K1	6.82	1.0	0.083
305×127×48, $\bar{\lambda}_{LT} = 0.4$, K2	0.31	0.30	K2	6.82	0.3	0.100
305×127×48, $\bar{\lambda}_{LT} = 0.3$, K2	0.31	0.30	K2	6.82	0.5	0.100
305×165×40, $\bar{\lambda}_{LT} = 0.4$, K2	0.57	0.44	K2	5.06	2.0	0.083
305×165×40, $\bar{\lambda}_{LT} = 0.3$, K2	0.57	0.44	K2	5.06	2.0	0.083

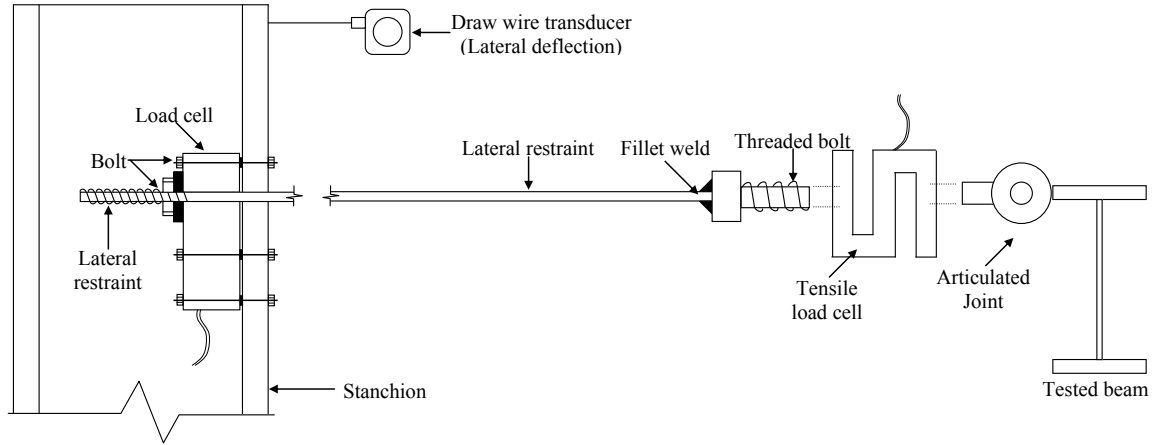


Figure 3.13: Schematic diagram of elastic restraint force measurement system.

plied loads to prevent premature failure through web crippling. Vertical displacements were measured using pull-wire transducers and end rotations were measured using inclinometers; force was applied at each loading point using two hand operated 250 kN load-controlled hydraulic jacks and forces were monitored with load cells. Two linear electrical resistance post-yield strain gauges were bonded to the extreme tensile and compressive fibres of the cross-section at the mid-span (or slightly off-set where necessary). Simple support conditions were provided by a roller system, with longitudinal movement permitted at one end using a sliding plate system. Loads, rotations, displacements and strains were all recorded at one-second intervals using the data acquisition system DATASCAN.

Non-dimensional moment curvature responses (M/M_{pl} versus κ/κ_{pl} , where $\kappa_{pl} = M_{pl}/EI$ is the elastic curvature corresponding to M_{pl}) for all of the four-point bending tests are presented in Figs 3.14 and 3.15. Curvature κ in the L_2 region was determined using measurements from the LVDTs in conjunction with the assumption that the deformed shape of the central span represents a segment of a circular arc (of radius r) (Chan and Gardner, 2008), leading to the expression in Eq. (3.9):

$$\kappa = \frac{1}{r} = \frac{8(u_{2M} - u_{2L})}{4(u_{2M} - u_{2L})^2 + 2L_2^2} \quad (3.9)$$

where u_{2M} is the central vertical displacement and u_{2L} is the average vertical displacement

ment at the loading points.

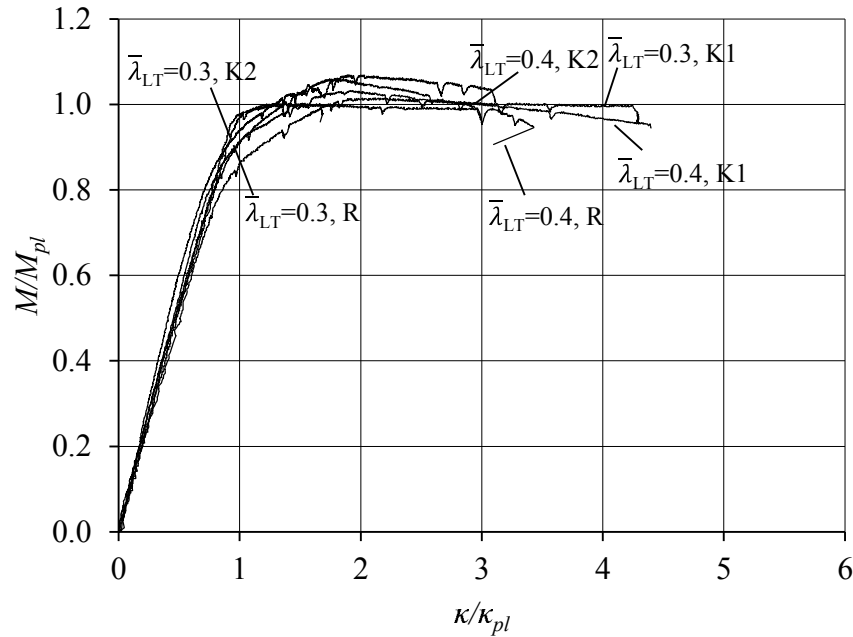


Figure 3.14: Non-dimensional moment-curvature responses for the $305 \times 127 \times 48$ UB sections subjected to four-point bending.

The test results show that substitution of the rigid lateral restraints with the K1 elastic lateral restraints causes negligible variations in capacity. This supports the general principle shown in Fig. 3.1 of a threshold stiffness, beyond which no further gains in ultimate moment from having stiffer restraints are achieved as the beam continues to fail in the second mode, with buckling occurring between points of lateral restraint.

Table 3.4 presents the observed buckling modes and key test results for all of the tests. The predicted restraint force from EN 1993-1-1, $q_d L$, is calculated according to Eq. (3.2), using measured values of initial imperfection amplitude e_0 and successive iterations of δ_q , where δ_q represents the lateral deformation of the elastic brace under load. M_{pl} and M_{csm} are calculated using a weighted average (based on contributions to bending resistance) of flange and web tensile yield stresses. With the exception of the elastically restrained $305 \times 127 \times 48$ specimen with $\bar{\lambda}_{LT} = 0.3$, there is a clear branching of observed buckling mode, with the rigid and stiff (K1) elastic restraints forcing the specimens to

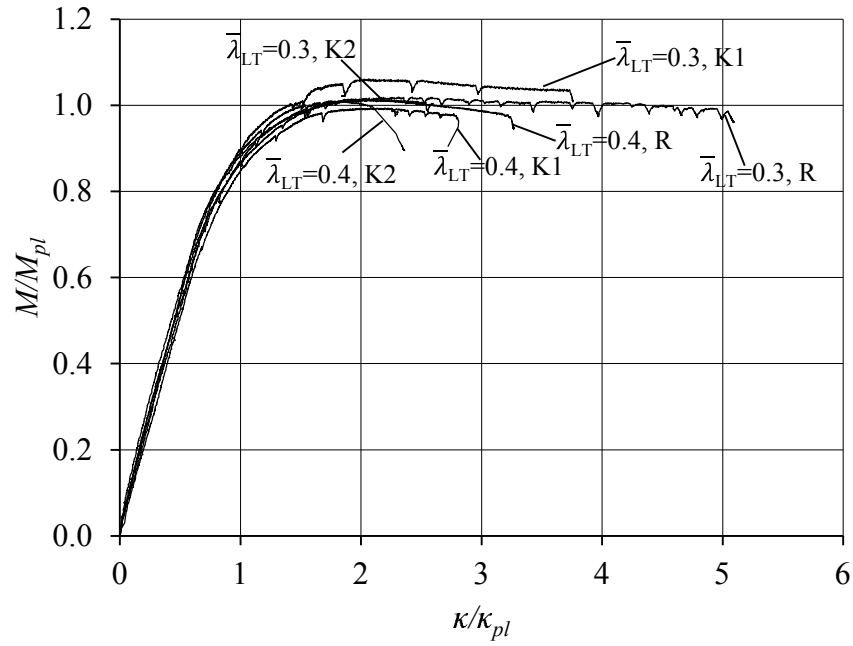


Figure 3.15: Non-dimensional moment-curvature responses for the 305×165×40 UB sections subjected to four-point bending.

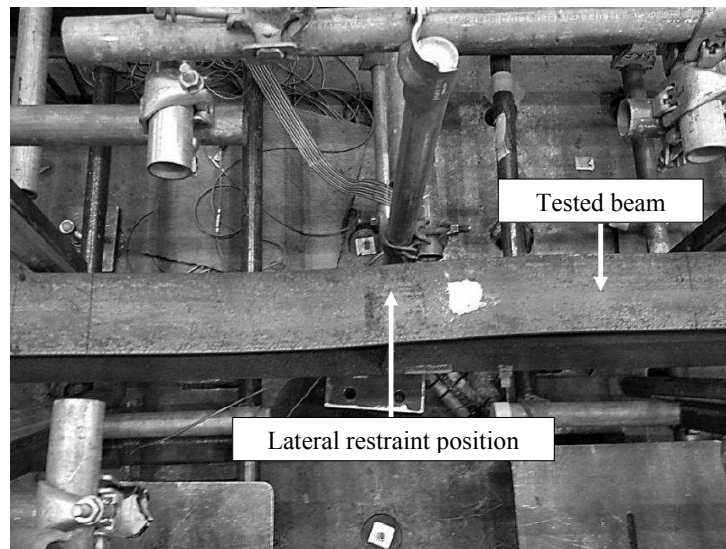


Figure 3.16: Typical mode-II failure observed for a 305×127×48 UB section.

buckle between the central brace (mode-II); the K2 restraints have insufficient stiffness to force the specimen into the second buckling mode, with buckling occurring in an overall mode (mode-I); examples of mode-I and mode-II failures are shown in Figs 3.16 and 3.17. Furthermore, in most cases higher restraint forces are associated with mode-I failures and larger global imperfection magnitudes.

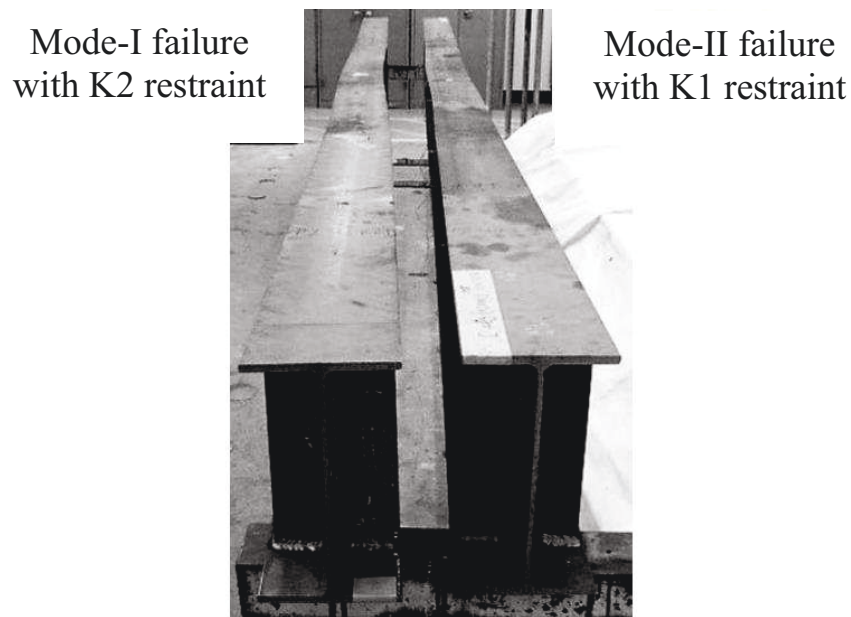


Figure 3.17: Typical mode-I and mode-II failure modes

The implication of this behaviour for restraint forces can be seen Figs 3.18-3.21, which show the tensile forces developed in the restraints for different restraint intervals. For the stiffer K1 restraints, failure occurs in the second mode and bracing forces stabilise at relatively low levels. For the K2 restraints that permitted a first mode failure, restraint forces were higher and increased with increasing deformation (curvature) of the primary member.

A non-dimensional measure of bracing stiffness, k_{ideal} , is presented and defined in Section 3.5.2. The relationship between brace stiffness, k_{ideal} , and brace force, F_{res} , as a percentage of the force developed in the compression flange of the primary member ($F_{res}/(M_u/h)$, where M_u is the ultimate test moment and h is the height of the cross-

Table 3.4: Key test results

Test designation	e_0 (mm)	Ultimate test moment M_u (kNm)	M_u/M_{csm}	M_u/M_{pl}	Restraint force, F_{res} , at M_u (kN)	$q_d L$ (kN)	$F_{res}/(M_u/h)$ (%)	Buckling mode
305×127×48, $\bar{\lambda}_{LT} = 0.4$, R	0.5	286.8	0.93	1.04	-	-	-	Mode-II
305×127×48, $\bar{\lambda}_{LT} = 0.3$, R	1.0	291.1	0.94	1.05	-	-	-	Mode-II
305×165×40, $\bar{\lambda}_{LT} = 0.4$, R	1.0	280.2	1.03	1.02	-	-	-	Mode-II
305×165×40, $\bar{\lambda}_{LT} = 0.3$, R	1.0	281.5	1.03	1.03	-	-	-	Mode-II
305×127×48, $\bar{\lambda}_{LT} = 0.4$, K1	0.5	283.2	0.91	1.03	2.10	2.44	0.23	Mode-II
305×127×48, $\bar{\lambda}_{LT} = 0.3$, K1	1.0	298.6	0.96	1.08	3.52	6.60	0.37	Mode-I
305×165×40, $\bar{\lambda}_{LT} = 0.4$, K1	6.0	274.6	1.01	1.00	4.97	37.83	0.55	Mode-II
305×165×40, $\bar{\lambda}_{LT} = 0.3$, K1	1.0	293.0	1.07	1.07	1.07	8.73	0.11	Mode-II
305×127×48, $\bar{\lambda}_{LT} = 0.4$, K2	0.3	289.1	0.93	1.05	3.69	1.57	0.40	Mode-I
305×127×48, $\bar{\lambda}_{LT} = 0.3$, K2	0.5	294.7	0.95	1.07	1.22	3.30	0.13	Mode-II
305×165×40, $\bar{\lambda}_{LT} = 0.4$, K2	2.0	279.0	1.02	1.02	9.84	12.87	1.07	Mode-I
305×165×40, $\bar{\lambda}_{LT} = 0.3$, K2	2.0	279.9	1.02	1.02	3.31	16.65	0.36	Mode-I

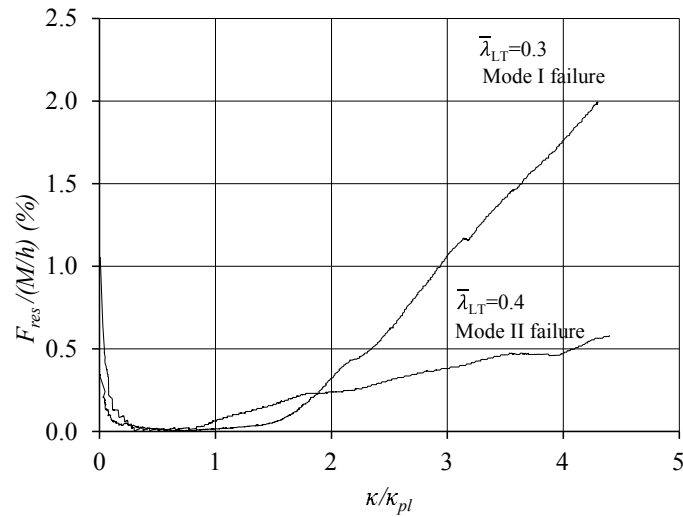


Figure 3.18: Non-dimensional restraint force versus non-dimensional curvature for the 305×127×48 UB section laterally restrained with a K1 elastic restraint and subjected to four-point bending.

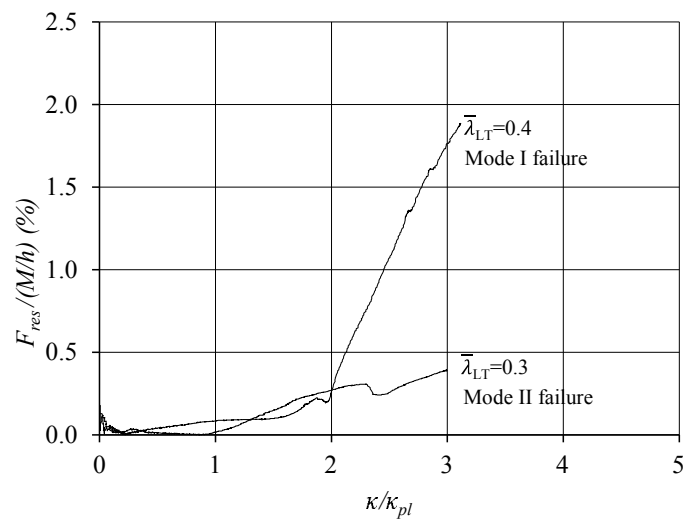


Figure 3.19: Non-dimensional restraint force versus non-dimensional curvature for the 305×127×48 UB section laterally restrained with a K2 elastic restraint and subjected to four-point bending.

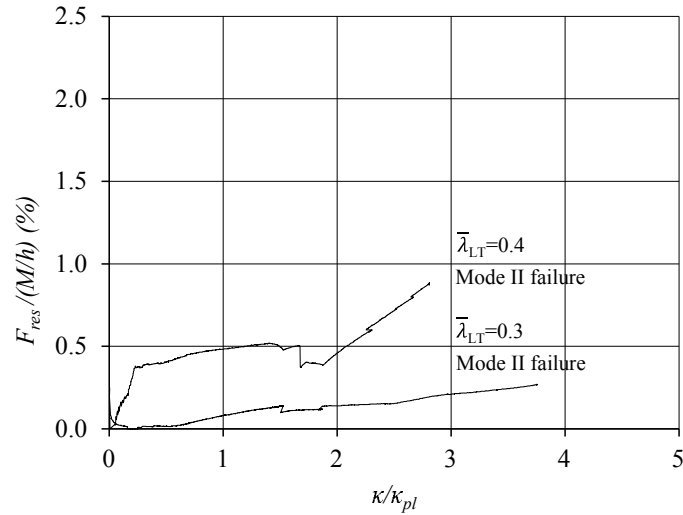


Figure 3.20: Non-dimensional restraint force versus non-dimensional curvature for the 305×165×40 UB section laterally restrained with a K1 elastic restraint and subjected to four-point bending.

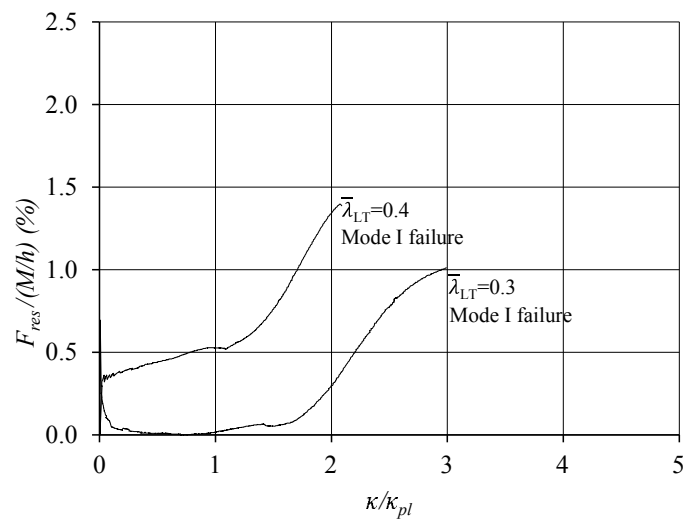


Figure 3.21: Non-dimensional restraint force versus non-dimensional curvature for the 305×165×40 UB section laterally restrained with a K2 elastic restraint and subjected to 4-point bending.

section) at ultimate load is shown in Fig. 3.22, where there is a tendency for brace forces to reduce quite rapidly with increasing stiffness. Additionally, it is apparent that higher brace forces are associated with beams failing in mode-I, as well as those with higher global imperfection magnitudes. In all cases, brace forces are less than 1.5% of the force developed in the compression flange, as well as less than that predicted by Eq. (3.2), based upon measured global imperfections, e_0 , and a second-order analysis for bracing system deflections δ_q .

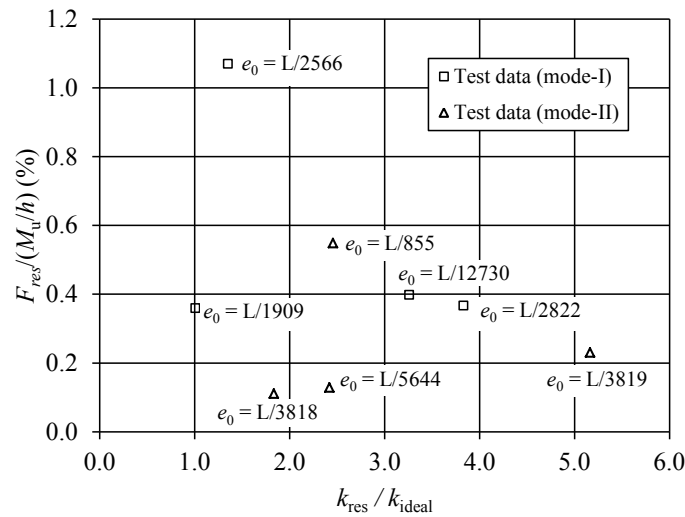


Figure 3.22: Influence of restraint stiffness on restraint force, with measured imperfection amplitude e_0 shown for each data point.

3.3.6 Three-point bending tests

Two three-point bending tests were conducted with rigid lateral restraints placed at intervals such that $\bar{\lambda}_{LT} = 0.1$ and 0.2 ; these tests were included to attain more pronounced strain hardening behaviour in the more concentrated plastic hinge location than was possible with the four-point tests, as well as to examine the effect of having closer restraint spacing. Fig. 3.23 shows the general test configuration for the three-point bending tests and a summary of the test parameters, specimen dimensions, and cross-section properties is provided in Table 3.5; Table 3.6 presents the key test results.

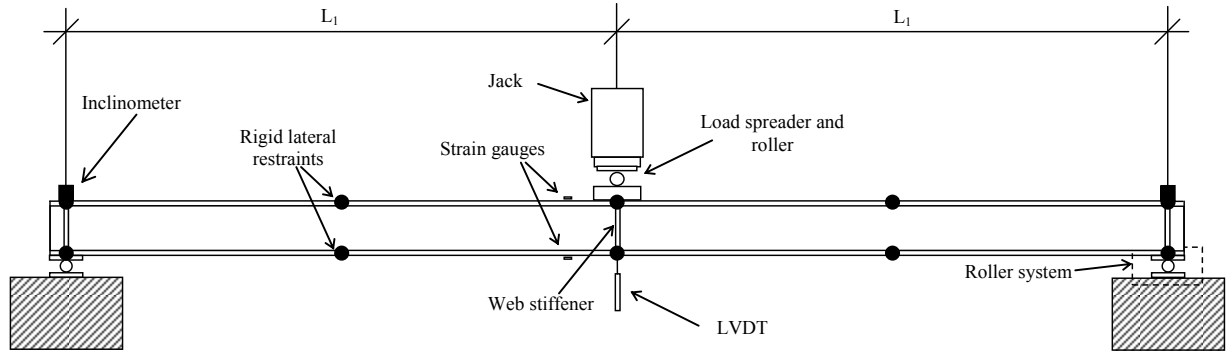


Figure 3.23: Schematic illustration of the three-point bending test configuration.

Table 3.5: Summary of geometric properties of the three-point bending test specimens

Test designation	$\bar{\lambda}_p$ (flange)	$\bar{\lambda}_p$ (web)	Restraint type	e_0 (mm)	ω_0 (mm)
305×127×48, $\bar{\lambda}_{LT} = 0.2$, R	0.57	0.44	Rigid	1.0	0.100
305×127×48, $\bar{\lambda}_{LT} = 0.1$, R	0.57	0.44	Rigid	1.0	0.100

Table 3.6: Key test results for the three-point bending tests.

Test designation	M_u (kNm)	$M_u/M_{csm,Rd}$	$M_u/M_{pl,Rd}$
305×127×48, $\bar{\lambda}_{LT} = 0.2$, R	318.1	1.02	1.15
305×127×48, $\bar{\lambda}_{LT} = 0.1$, R	334.2	1.07	1.21

Non-dimensional moment-rotation responses for the three-point bending tests are presented in Fig. 3.24. Although it is evident that these tests were carried out into the strain-hardening range, the ultimate capacity was not reached as the testing rig did not have sufficient vertical displacement capacity to safely finish the test.

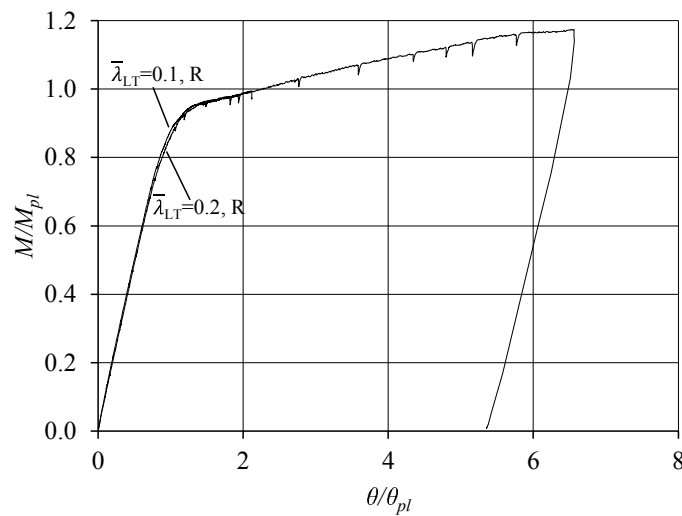


Figure 3.24: Non-dimensional moment-rotation responses for the 305×127×48 UB sections subjected to four-point bending.

The results of the three-point bending tests are combined with existing test data and used to assess a suitable plateau length for beams designed with allowance for strain-hardening in Chapter 4.

3.4 Numerical modelling

3.4.1 Introduction

A numerical study using the general-purpose finite element analysis package ABAQUS (Simulia, 2010) was conducted in parallel with the testing programme. The primary aims of the investigation were to (1) replicate the experimental results using all available measurements, (2) employ a standardised numerical model validated against the experimental data, and (3) undertake parametric studies to examine the influence of key

parameters on the structural response of statically determinate braced steel beams and the corresponding forces in the restraints.

3.4.2 Basic modelling assumptions

The finite element type S4R, a four-noded, doubly curved general-purpose shell element with reduced integration and finite membrane strains, selected from the ABAQUS (Simulia, 2010) element library, was used throughout the study to model the beams. This element is suited to the modelling of a range of shell thicknesses and has been successfully implemented in other studies, such as that by Chan and Gardner (2008). Restraints were modelled using a linear spring element with a defined stiffness in the out-of-plane direction and zero stiffness elsewhere.

To replicate the results from the experimental programme, measured geometric and material properties obtained prior to testing were incorporated into the finite-element models; for the standardised model, measured local and global geometric imperfection magnitudes were replaced with assumed values. Boundary conditions were applied to model simple support conditions at the ends and lateral displacements were prevented at the locations of the rigid restraints; longitudinal displacements were also prevented at the mid-span of the beams. The combined effects of material and geometric non-linearity were allowed for, with the modified Riks (arc-length) algorithm being used to solve the models, as this can accommodate most forms of non-linear load-displacement paths encountered in the analysis of beams.

3.4.3 Material modelling

For the purposes of this investigation, the true material stress-strain behaviour was generated from the engineering stress-strain data obtained from the laboratory tests. The relationship between true stress, σ_{true} , and engineering stress, σ_{nom} was given by Eq. (2.8), while the relationship between log plastic strain, ϵ_{true} , and engineering strain, ϵ_{nom} , was given by Eq. (2.9), in which σ_{nom} and ϵ_{nom} are the engineering stress and strain respectively and E is the modulus of elasticity.

Material non-linearity was incorporated into the finite-element model by means of a piecewise linear discretisation of the true stress-strain response. Fig. 3.25 illustrates the discretised tensile stress-strain relationships for the flanges. In the analysis, both flange and web tensile stress-strain relationships were used.

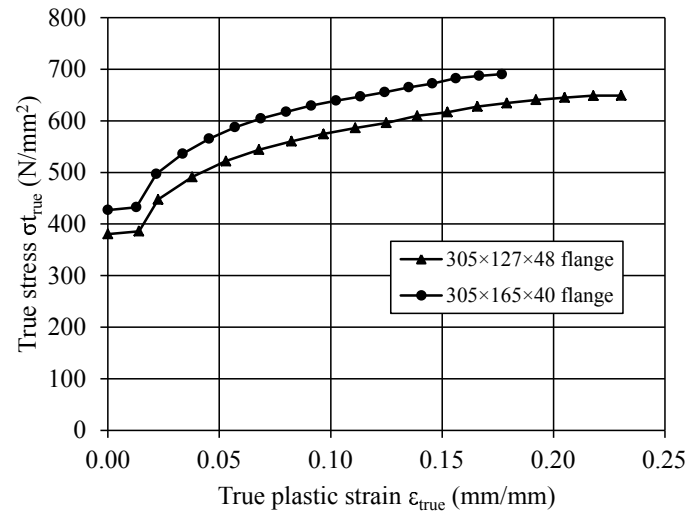


Figure 3.25: Discrete tensile true stress-true strain relationships for flanges used for finite element modelling.

3.4.4 Initial geometric imperfections

The behaviour of structural members can be significantly influenced by the magnitude and nature of any initial geometric imperfections. In this study, both global lateral imperfections and local plate imperfections are considered. Global imperfections influence lateral deflections, the point of initiation of yielding and, of particular interest herein, the forces in the lateral restraints. Lateral restraint forces may be determined as the product of restraint (lateral) displacement and restraint stiffness; it is therefore clear that variations in the magnitude of initial global imperfections will also be influential for restraint forces, since lateral deflections are essentially an amplification of the initial imperfection.

Both the locally and globally imperfect geometries of all the members were determined using an initial elastic eigenvalue analysis with the resulting local and global eigen-

modes being used to define the distribution of local and global imperfections. Global imperfection amplitudes were taken as either measured or assumed ($L/500$) values of e_0 .

Local eigenmodes were scaled using both measured, ω_0 , and assumed peak amplitudes. Assumed peak amplitudes were determined from a fraction of the local flange material thickness ($t/100$) as well as being derived from the modified value, ω_{DW} , of Dawson and Walker (1972) by Gardner and Nethercot (2004c), which is defined in Eq. (3.10).

$$\frac{\omega_{DW}}{t} = 0.028 (f_y/\sigma_{cr})^{1/2} \quad (3.10)$$

in which f_y is the yield stress and σ_{cr} is the elastic critical buckling stress of the most slender constituent plate element in the section; comparisons between all three measures are presented in Table 3.7. For the range of cross-sections considered, finite element results were relatively insensitive to local imperfections; these imperfections were therefore omitted in the parametric studies.

Table 3.7: Measured and assumed initial local imperfection amplitudes.

Specimen	Measured ω_0 (mm)	Dawson and Walker ω_{DW} (mm)	$t/100$ (mm)
305×127×48 UB	0.100	0.059	0.140
305×165×40 UB	0.083	0.062	0.102

3.4.5 Elastic lateral restraints

Elastic lateral restraints can be modelled as a linear spring with a defined stiffness and permitted degrees of freedom. Springs acting as restraints are attached at specific nodes and can be viewed as an additional stiffness matrix at each node; these local matrices are then added to the corresponding element global stiffness matrix, as conceptually explained in Wang and Nethercot (1989). In this investigation, the spring was added to the central node of the top flange, providing only out of plane stiffness. The forces predicted by the linear spring elements compare favourably with the experimental restraint

forces, F_{res} , as illustrated in Fig. 3.26, where P is the axial force in the compression flange of the beam.

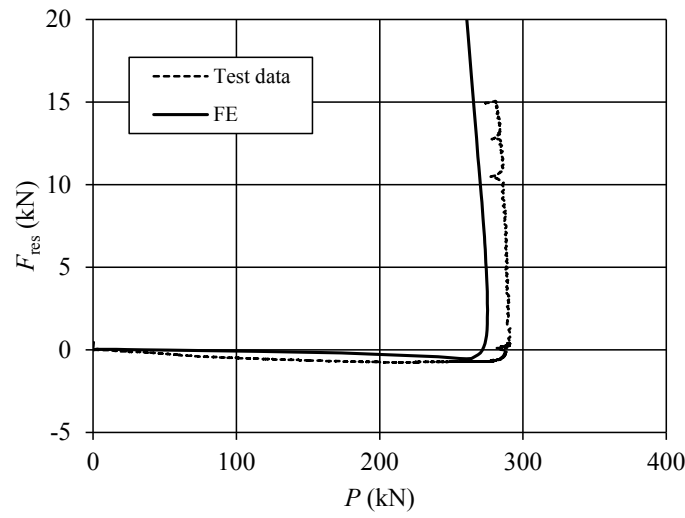


Figure 3.26: Finite element and experimental restraint forces for a $305 \times 127 \times 48$ UB ($\bar{\lambda}_{\text{LT}} = 0.4$) specimen.

3.4.6 Residual stresses

Residual stresses arise in hot-rolled steel sections due to differential cooling after the forming process; these stresses can have a significant effect on the manner in which yield zones develop. Notably, the flange tips are typically found to be in residual compression (due to early cooling), whilst the web-to-flange junction is typically in residual tension (due to later cooling). Residual stresses can be measured by sectioning a member and converting the released strains into stresses; a considerable quantity of data has been gathered in this manner and a survey is presented in Young (1971). The present research utilises an assumed residual stress distribution that is a linearisation of the polynomial distribution used in Nethercot (1974) (Fig. 3.27). As suggested by Galambos (1963), the maximum compressive residual stress, σ_{rcf} , and the maximum tensile residual stress, σ_{rt} , are both set to $0.3f_y$.

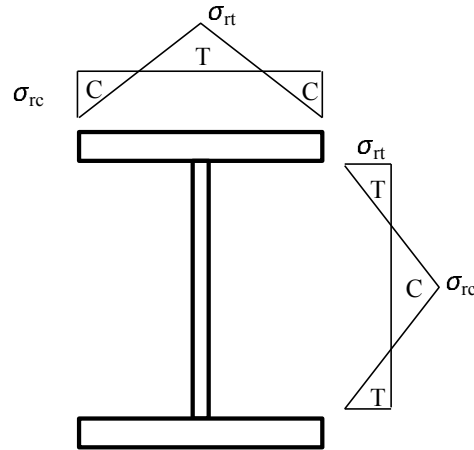


Figure 3.27: Assumed linear residual stress distribution

Similar to Nethercot (1974), residual stresses were introduced into the finite element model by partitioning the web and flanges of each cross-section (16 partitions were used); each partition represents a stress level corresponding to the mid-point of the linear variation in stress over the partition length. The stresses were applied using the *INITIAL CONDITIONS command and equilibration of the stresses was achieved through an initial linear perturbation load step.

An initial calibration study was carried out to examine the relative performance of the non-linear finite element models both with and without residual stresses. For this study, all geometric properties were as measured for the specimen under examination. It is clear that, upon examining the load-displacement relationship plotted in Fig. 3.28, there is an improvement in the accuracy of the model compared to the test data when residual stresses are incorporated, particularly in the regions of first yield and spread of plasticity. Although including residual stresses can create some numerical instabilities, these can be resolved with a sufficiently refined mesh and carefully limited maximum increment sizes; in the present research, residual stresses are incorporated into all of the finite element models.

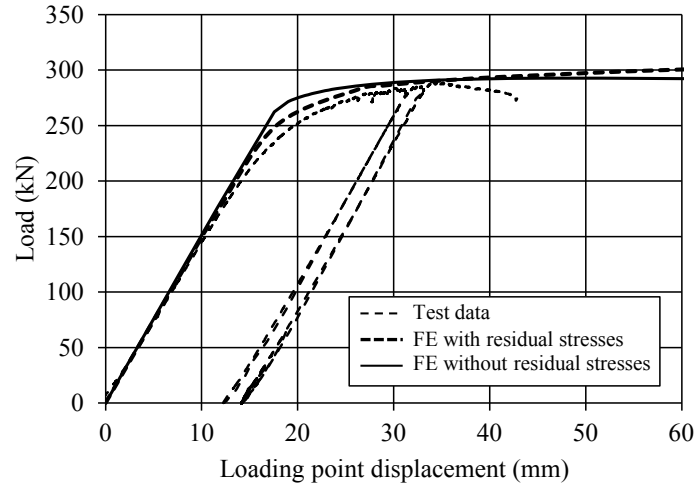


Figure 3.28: Relative performance of finite element models with and without residual stresses for a $305 \times 127 \times 48$, $\bar{\lambda}_{LT} = 0.4$ specimen.

3.4.7 Validation

To validate the FE models, all experiments were simulated using measured material and geometric properties, as well as local and global imperfections and the assumed residual stress pattern. In addition, a second set of finite element models that replaced measured with assumed imperfections were produced; in both cases the ultimate capacities, as well as the general moment-curvature or moment-rotation responses were compared with those obtained in the laboratory tests.

Considering overall moment-curvature and moment-rotation behaviour, there was little variation between the accuracy of the results based on the measured imperfection amplitudes and those based on the assumed local imperfection amplitudes, ω_{DW} , of Dawson and Walker (1972) and global imperfection amplitudes of $e_0 = L/500$. Typical examples of this are given for the $305 \times 127 \times 48$ UB cross-section in Fig. 3.29 for four-point bending. For all comparisons, there was good agreement for initial slope and observed mode of failure; ultimate capacities are compared in Table 3.8.

Comparisons also showed that choosing assumed over measured global imperfection amplitudes had little bearing on the accuracy of restraint force modelling. Fig. 3.30

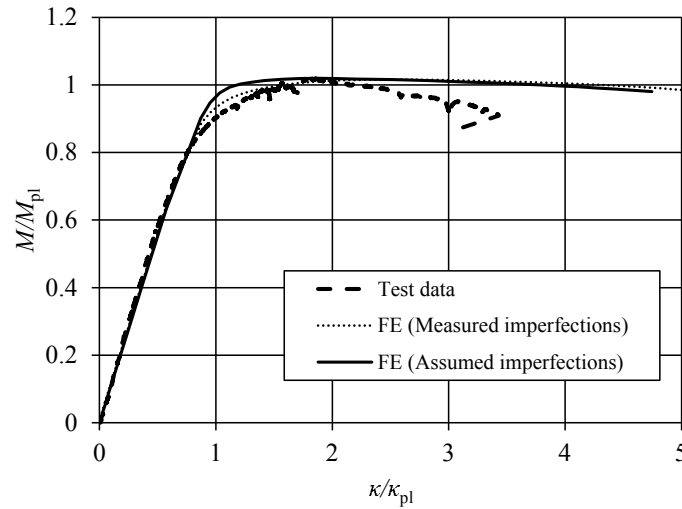


Figure 3.29: Normalised moment-curvature curves for a tested $305 \times 127 \times 48$ UB ($\bar{\lambda}_{LT} = 0.4$) specimen subjected to four-point bending and the corresponding finite element models with measured and assumed initial geometric imperfections.

shows the relationship between non-dimensional bending moment and non-dimensional restraint force as a percentage of the force in the compression flange for a $305 \times 127 \times 48$ ($\bar{\lambda}_{LT} = 0.3$) specimen. In this case, the model using assumed imperfections (which were always greater than the measured imperfections) showed that for a given level of applied moment, slightly higher restraint forces were induced than the model with measured imperfections.

Table 3.8 compares ultimate applied loads obtained from the experiments with those obtained from the two sets of finite element models. The average ratio of test ultimate load to finite element ultimate load with measured and assumed imperfections are 1.01 and 1.00 respectively; the corresponding standard deviations are 0.02 and 0.03. This represents good agreement with minimal dispersion. In the subsequent analysis, the assumed imperfection magnitudes are adopted since they allow a standardised approach that enables a variety of geometries to be considered on a consistent basis.

Table 3.8: Comparison of the laboratory test results with finite element results for measured and assumed imperfection amplitudes

Test designation	Test M_u (kNm)	FE M_u (kNm) (Measured imperfection)	FE M_u (kNm) (Assumed imperfection)	Test/FE (Measured imperfection)	Test/FE (Assumed imperfection)
305×127×48, $\bar{\lambda}_{LT} = 0.4$, R	287	289	291	0.99	0.99
305×127×48, $\bar{\lambda}_{LT} = 0.3$, R	291	283	284	1.03	1.02
305×165×40, $\bar{\lambda}_{LT} = 0.4$, R	280	280	285	1.00	0.98
305×165×40, $\bar{\lambda}_{LT} = 0.3$, R	282	286	291	0.99	0.97
305×127×48, $\bar{\lambda}_{LT} = 0.4$, K1	283	287	289	0.98	0.98
305×127×48, $\bar{\lambda}_{LT} = 0.3$, K1	299	284	283	1.05	1.06
305×165×40, $\bar{\lambda}_{LT} = 0.4$, K1	275	282	282	0.98	0.98
305×165×40, $\bar{\lambda}_{LT} = 0.3$, K1	293	285	287	1.03	1.02
305×127×48, $\bar{\lambda}_{LT} = 0.4$, K2	289	286	286	1.01	1.01
305×127×48, $\bar{\lambda}_{LT} = 0.3$, K2	295	283	282	1.04	1.05
305×165×40, $\bar{\lambda}_{LT} = 0.4$, K2	279	280	278	0.99	1.00
305×165×40, $\bar{\lambda}_{LT} = 0.3$, K2	280	279	282	1.00	0.99
Mean				1.01	1.00
Standard deviation				0.02	0.03

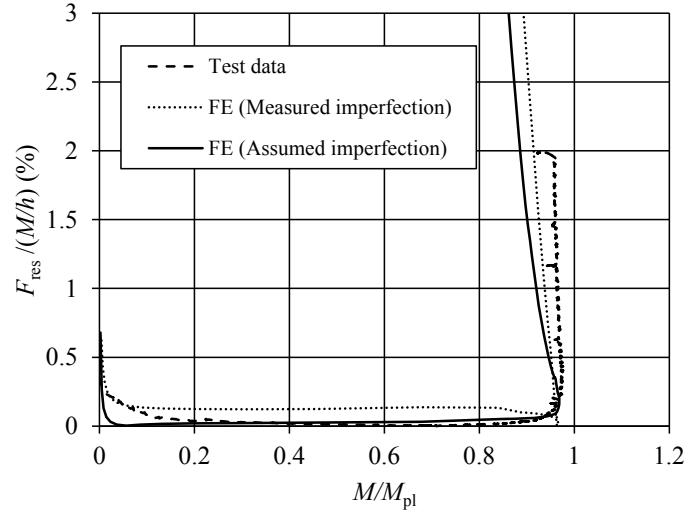


Figure 3.30: Restraint force development throughout the loading history for a $305 \times 127 \times 48$ UB ($\bar{\lambda}_{LT} = 0.3$) specimen.

3.4.8 Parametric studies

Having validated the model against the experimental data, a series of parametric studies was performed, focusing on variations in elastic restraint stiffness as a multiple of k_{ideal} , which is defined in Section 3.5.2. The parametric studies were conducted for both cross-section geometries ($305 \times 127 \times 48$ UB and $305 \times 165 \times 40$ UB) and lateral torsional buckling slendernesses ($\bar{\lambda}_{LT} = 0.4$ and $\bar{\lambda}_{LT} = 0.3$). Table 3.9 presents the multiples of k_{ideal} and corresponding spring stiffness values used for each cross-section and member geometry. Results from the parametric study are analysed and discussed in Section 3.5.

Table 3.9: Restraint stiffness values used in the parametric studies.

Multiples of k_{ideal}	k_{res} (N/mm)			
	305×127×48 UB		305×165×40 UB	
	$\bar{\lambda}_{LT} = 0.4$	$\bar{\lambda}_{LT} = 0.3$	$\bar{\lambda}_{LT} = 0.4$	$\bar{\lambda}_{LT} = 0.3$
0.25	466	722	350	479
0.5	932	1445	700	960
1	1864	2890	1400	1919
2	3728	5780	2800	3838
4	7456	11561	5600	7676
6	11184	17342	8400	11512

3.5 Analysis of results and design implications

In this section, a simplified theoretical model based upon elastic predictions is presented to derive bracing stiffness requirements for members designed with account for strain-hardening using the CSM. The capability of braces of the derived stiffness to provide effective lateral restraint to the main member will be assessed using the finite element methodology described in Section 3.4. The restraint stiffness and spacing parameters used in this section are presented in Table 3.9.

3.5.1 Theoretical background

For the purposes of this analysis, an axially loaded member will be used to represent the forces developed in the compression flange of the beam. This approach has been chosen because it simplifies calculations and it provides a conservative representation of the behaviour of a beam (Flint, 1951). Winter (1960) examined the problem shown in Fig. 3.31 and, by introducing a frictionless hinge at the point of restraint (which is a point of contraflexure in the case of mode-II buckling), derived the minimum stiffness required for the restraint to act as a nodal point, with zero lateral displacement. A study of the full system, without introducing a frictionless hinge was presented by Timoshenko and Gere (1961), and was later described by Galambos and Surovek (2008). The key outcomes of their derivation are summarised below.

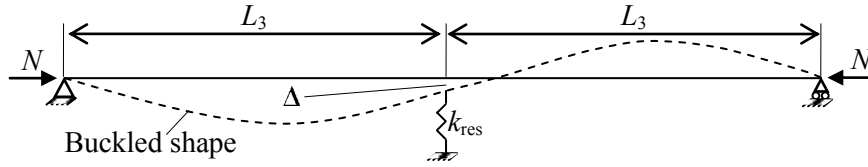


Figure 3.31: A perfect elastic beam-column with an elastic central support.

In Fig. 3.31 the length $L_3 = L_2/2$ (see Fig. 3.11) and Δ is the lateral deflection at the point of the elastic restraint. Upon formulating and solving for the determinant of the appropriate slope-deflection equations for this model, two solutions for the elastic buckling load of the member N_{cr} result, with the first being the Euler load N_E for a column of length L_3 (the mode-II solution) (Eq. (3.11)):

$$N_{cr} = \frac{\pi^2 EI}{L_3^2} = N_E \quad (3.11)$$

and the second defining the relationship between transition levels of critical load and restraint stiffness, k_{res} , between the first and second buckling modes:

$$\frac{k_{res} L_3^3}{2EI} [\alpha L_3 - \tan(\alpha L_3)] - (\alpha L_3)^3 = 0 \quad (3.12)$$

in which $\alpha L_3 = \pi \sqrt{N_{cr}/N_E}$. When $k_{res} = 0$, Eq. (3.12) reduces to an Euler load for a column of length $2L_3$. The resulting relationship between non-dimensional stiffness and critical load is given in Eq. (3.13) and plotted in Fig. 3.32.

$$\frac{k_{res} L_3^3}{EI} = \frac{2\pi^3 \left(\frac{N_{cr}}{N_E} \right)^{3/2}}{\pi \sqrt{\frac{N_{cr}}{N_E}} - \tan \left(\pi \sqrt{\frac{N_{cr}}{N_E}} \right)} \quad (3.13)$$

Setting $N_{cr} = N_E$ in Eq. (3.13) determines the stiffness, $k_{ideal,el}$, at which effective bracing for the primary member (i.e the minimum stiffness required to force buckling in mode-II) in the elastic range is provided:

$$k_{ideal,el} = \frac{2\pi^2 EI}{L_3^3} = \frac{2N_E}{L_3} \quad (3.14)$$

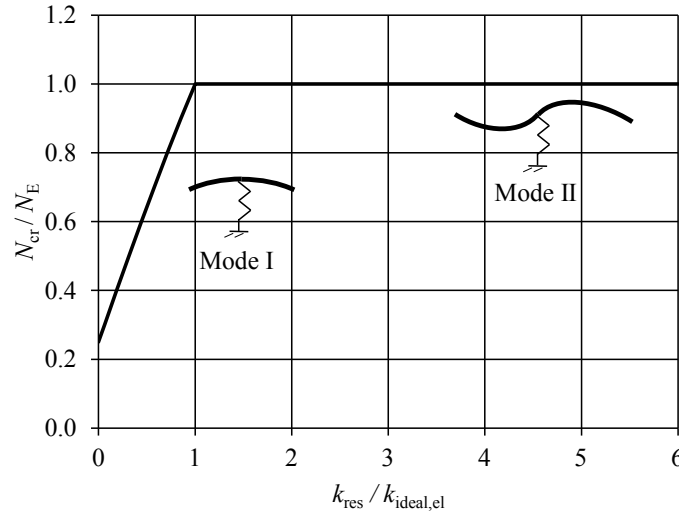


Figure 3.32: Theoretical relationship between restraint stiffness and buckling load for an elastic beam-column.

In Section 3.5.2 it will be shown how this stiffness can be modified to account for plasticity and strain-hardening.

3.5.2 Analysis and design implications

Early studies (Pincus, 1964) initially suggested that inelasticity in the primary member increases bracing stiffness requirements. However, localised inelasticity (i.e. a single plastic hinge) at the point of lateral restraint is much the same as a frictionless hinge at this location, which was shown by Winter (1960) to have a negligible effect on the minimum restraint stiffness required to ensure mode-II buckling. In the present investigation, the members are subjected to substantial regions of uniform moment, leading to extensive zones of plasticity and strain-hardening; bracing stiffness requirements under these circumstances will be established in this section.

For beams of low global slenderness (e.g. in the common case of the lateral restraints spacing being such that $\bar{\lambda}_{LT} \leq 0.4$), the design moment of resistance M_{Rd} will be significantly less than M_{cr} (and hence the design axial load in the beam's compression flange $N_{Rd} \ll N_E$). In such instances, use of N_E in Eq. (3.13) is inappropriate; replacing N_E with the maximum force that can arise in the compression flange of the beam, with

due allowance for the influence of strain-hardening, results in the following stiffness requirement k_{ideal} for the restraint to act as if it were rigid:

$$k_{\text{ideal}} = \frac{2N_{\text{Rd}}}{L_3} \quad (3.15)$$

where $N_{\text{Rd}} = (M_{\text{csm}}/h)$ and h is the overall height of the cross-section.

In order to verify the applicability of this approach, a parametric study was conducted using the finite element package ABAQUS. The general method was to establish first an upper capacity, $M_{\text{u},k=\text{rigid}}$, corresponding to a configuration with a rigid central restraint (Fig. 3.33a) and then set $k_{\text{res}} = 2(M_{\text{u},k=\text{rigid}}/h)/L_3$ to see if the resulting capacity $M_{\text{u},k=\text{ideal}}$ reaches $M_{\text{u},k=\text{rigid}}$, using the configuration in Fig. 3.33b.

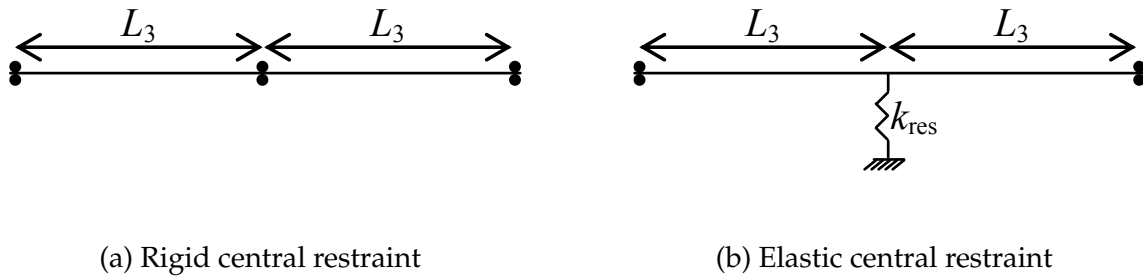


Figure 3.33: Schematic plan view of the lateral restraint configurations and member geometry for the parametric studies.

Implementing the approach outlined above, Table 3.10 presents the relative capacities of the rigid configuration of Fig. 3.33a and the configuration with an elastic restraint of stiffness k_{ideal} (Eq. (3.15)) in Fig. 3.33b. Good agreement between the ultimate moments achieved in the rigid and ideally braced configurations can be seen, with a maximum deviation in capacity of 2%.

Fig. 3.34 shows the relationship between non-dimensional restraint stiffness and moment capacity, where for all but one of the members, there is a small underestimation of the required stiffness to attain the maximum available capacity. Note that for the $305 \times 127 \times 48$ UB section, for which the CSM predicts capacities greater than M_{pl} due to strain-hardening, a slenderness of $\bar{\lambda}_{\text{LT}} = 0.3$ is required for such behaviour to be

Table 3.10: Relative performance of rigid and ideally braced central restraints

Section and member geometry	$M_{u,k=\text{rigid}}/M_{pl}$	$M_{u,k=\text{ideal}}/M_{pl}$	$M_{u,k=\text{ideal}}/M_{u,k=\text{rigid}}$
$305 \times 127 \times 48, \bar{\lambda}_{LT} = 0.4$	1.01	1.00	0.99
$305 \times 127 \times 48, \bar{\lambda}_{LT} = 0.3$	1.07	1.06	0.99
$305 \times 165 \times 40, \bar{\lambda}_{LT} = 0.4$	0.94	0.92	0.99
$305 \times 165 \times 40, \bar{\lambda}_{LT} = 0.3$	0.96	0.94	1.00

seen. For the $305 \times 165 \times 40$ UB, $\bar{\lambda}_{LT} = 0.4$ configuration, a parametric study with more refined intervals of restraint stiffness values was carried out to investigate the performance of the member in the proximity of the transition stiffness k_{ideal} . The results of this extended finite element study are presented in Fig. 3.35 showing a smoother transition between the first and second buckling modes. The discrepancy between the numerical and analytical model may be due to the presence of geometric imperfections; a similar conclusion was reached by Yura (2001) for an elastic column.

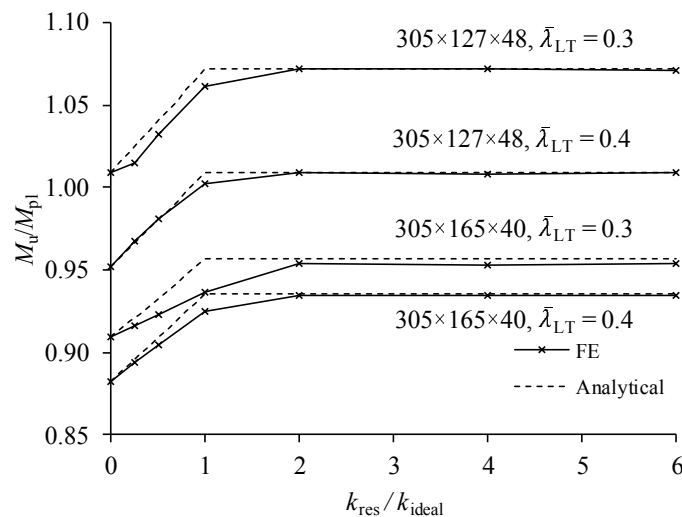


Figure 3.34: Relationship between normalised restraint stiffness and ultimate capacity for inelastic primary members.

Fig. 3.36 shows the relationship between non-dimensional restraint stiffness and restraint force. Below the ideal stiffness, k_{ideal} , a mode-I failure will result, with restraint

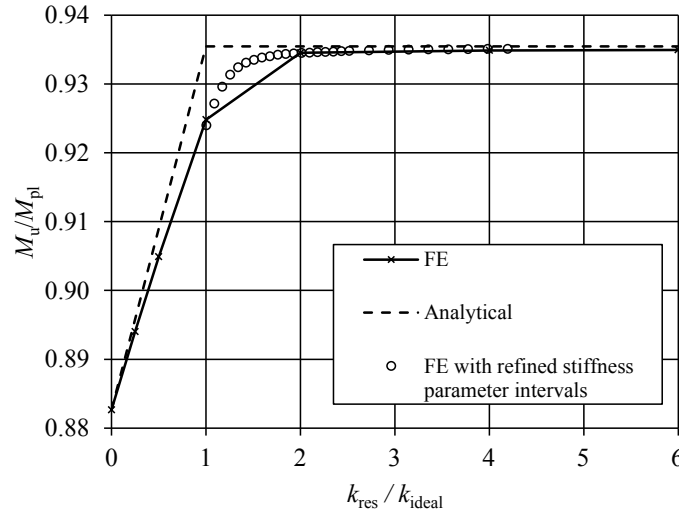


Figure 3.35: A high resolution parametric study for a $305 \times 165 \times 40$ UB, $\bar{\lambda}_{LT} = 0.4$ configuration.

forces at ultimate load increasing rapidly with increasing restraint stiffness, but with proportionate increases in the capacity of the primary member. At, or marginally above k_{ideal} , the member reaches its assumed potential capacity for its material and geometric properties, and it is effectively at the boundary between the first and second buckling modes. As a consequence, the brace force will also be at its maximum level in the region of k_{ideal} . Increases in restraint stiffness beyond k_{ideal} lead to no further increases in capacity, but deflections at the point of lateral restraint reduce with increases in restraint stiffness, resulting in a reduction in restraint force. At approximately $2k_{ideal}$ restraint forces are reduced to 1-3% of the force in the compression flange, and at $4k_{ideal}$ they are all below 2%.

Fig. 3.37 shows restraint forces normalised by equivalent EN 1993-1-1 design restraint force $q_d L$ (Eq. (3.2)), where q_d was determined from a second-order analysis with $e_0 = L/500$ and δ_q being the elastic deflection of the bracing system in Fig. 3.33b. At k_{ideal} restraint forces for beams with restraints spaced such that $\bar{\lambda}_{LT} = 0.4$ are below the required design restraint forces set by EN-1993-1-1 (Eq. (3.2)), whilst at $2k_{ideal}$ restraint forces for all members fall below this value.

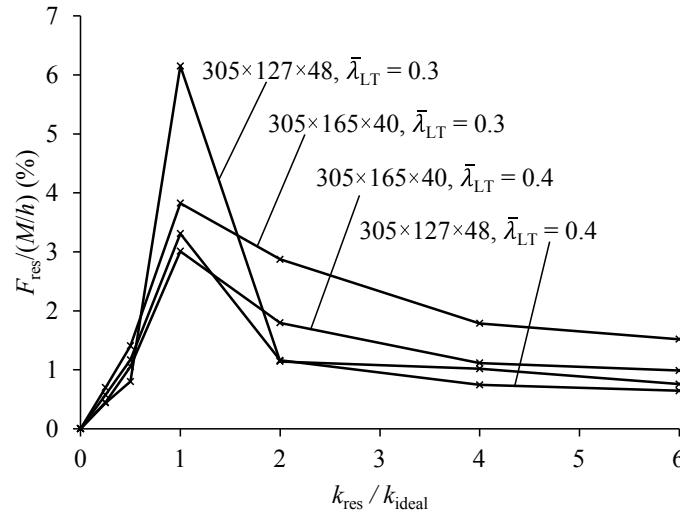


Figure 3.36: Relationship between restraint stiffness and restraint force at ultimate load for inelastic primary members.

For the typical cross-sections considered, it has been found that restraint forces reach their peak values of between about 3% and 6% of the force in the compression flange of the primary member at k_{ideal} . At $2k_{ideal}$, restraint forces are significantly reduced to below the design value set out in EN 1993-1-1 and, for the cases considered, the maximum attainable loads for the restrained members are achieved.

In practice, design decisions for lateral restraints are made on the basis of evaluating the forces developed in a bracing system using Eq. (3.2), which are dependent upon the bracing stiffness by means of δ_q ; designing the primary member using the CSM with allowance for strain-hardening has no bearing on the applicability of this approach. It is recommended, however, that the stiffness of the restraining member is checked against k_{ideal} to ensure it is effective and that an efficient design results, though it has been noted that restraints that satisfy the strength requirements will also typically possess adequate stiffness (Trahair (1999), Gardner (2011)). By means of example, the tested $305 \times 165 \times 40$ UB, with $\bar{\lambda}_{LT} = 0.4$ had an ultimate test moment $M_u = 283$ kNm. Assuming that the brace has a length of 5 m, with assumed values of $E = 210,000$ N/mm² and $f_y = 275$ N/mm², then the required brace area to satisfy the strength requirements (35 kN) is 102 mm² with a corresponding stiffness of 4275 N/mm; the design restraint force,

q_d , according to EN 1993-1-1 is 14.1 kN/m. Using Eq. (3.15) $k_{ideal} = 1833$ N/mm, thus the stiffness provided by satisfying the strength requirement of Eq. (3.2) is $2.3k_{ideal}$.

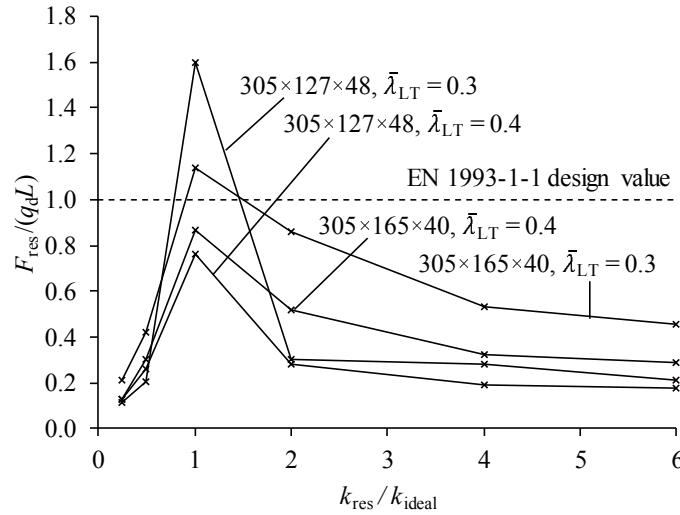


Figure 3.37: Restraint forces from FE models normalised by the EN-1993-1-1 design value for varying restraint stiffness values.

3.6 Conclusions

A programme of twelve four-point bending tests on beams with discrete rigid and elastic lateral restraints has been conducted in order to investigate the implications of utilising strain-hardening in the design of the restrained member on bracing forces and stiffness requirements. Two cross-sections were considered and lateral restraint spacings were varied to achieve two values of $\bar{\lambda}_{LT}$ (0.3 and 0.4) for each restraint configuration; for the elastic restraints, two levels of stiffness were used. Results showed that, for the cases considered, variation in bracing stiffness did not significantly affect the ultimate capacity of the specimens, but to limit the forces developed in the restraints it is necessary to have restraints of sufficient stiffness to ensure that a mode-II failure (i.e. buckling between the restrained points) occurs. Two three-point bending tests were also conducted with closer restraint spacings.

A series of materially and geometrically non-linear finite element models was validated

against the experimental results and shown to be able to capture the observed physical behaviour. On the basis of these models, parametric studies were conducted to investigate the implications of varying the restraint stiffness on the ultimate capacities of the beams and the corresponding forces developed in the restraints.

The specific restraint stiffness values used in the parametric studies were chosen using a simplified analytical model, where it was demonstrated that the minimum stiffness, k_{ideal} , required to ensure an effective brace in the inelastic range is in proportion to the design force in the compression flange of the beam. The approach was validated for restraint spacings that resulted in $\bar{\lambda}_{LT} \leq 0.4$. The results from the parametric studies also showed that at the minimum required stiffness, the restraint forces assumed their peak values, but multiples of this stiffness caused the forces to reduce rapidly, whilst ensuring the full capacity of the restrained member was achieved. Finally, it was shown that consideration could be made for strain-hardening in the design of the primary member through the CSM provided suitable restraint spacings are used, and that this did not result in disproportionate increases in restraint forces; restraint forces derived from EN 1993-1-1 remained appropriate.

Chapter 4

Lateral restraint spacing for statically determinate structural elements

4.1 Introduction

Lateral stability of beams is a key aspect of structural steel design. For beams designed in the inelastic range, the current practice in European design codes is to specify a maximum lateral torsional buckling slenderness ($\bar{\lambda}_{LT}$) below which the effects of lateral instability can be ignored and the full cross-section capacity can be achieved; beyond this limit, reductions in capacity arise.

Currently, inelastic design of structures according to EN 1993-1-1 assumes an elastic-perfectly plastic (EPP) stress-strain curve, and a limiting value of $\bar{\lambda}_{LT} = 0.4$, below which the effects of lateral torsional buckling can be ignored. In a newly proposed design method, referred to as the Continuous Strength Method (CSM) (Gardner, 2008), strain-hardening is allowed for in the material model, with the limiting strain defined as a function of local plate slenderness $\bar{\lambda}_p$. The present chapter is devoted to examining the implications of this new method for the limiting value of $\bar{\lambda}_{LT}$. To this end, the data from a series of experiments on simply supported beams with variations in $\bar{\lambda}_{LT}$, the details of which are presented in Chapter 3, was used in conjunction with data collected from the literature. Using a geometrically and materially non-linear finite element model, the test data were reproduced and extended in a parametric study, which was then used to inform and support some analytically derived design equations.

4.2 Key design aspects

4.2.1 Lateral restraint spacing

EN 1993-1-1 (2005) defines a non-dimensional slenderness limit, or plateau length, $\bar{\lambda}_{LT} = 0.4$, below which, the effects of lateral torsional buckling can be ignored and the design buckling resistance moment of the member $M_{b,Rd}$ may be taken as the design bending resistance $M_{c,Rd}$ of the cross-section, assuming $\gamma_{M0} = \gamma_{M1}$ (see Fig. 4.1, which shows buckling curve a of EN 1993-1-1); $\bar{\lambda}_{LT}$ is defined in Chapter 3 by Eq. (3.1).

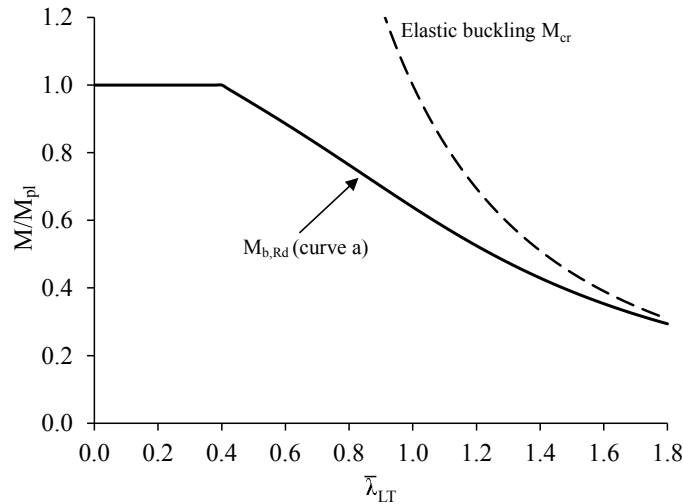


Figure 4.1: Relationship between moment capacity and non-dimensional member slenderness as defined by EN 1993-1-1.

4.2.2 The Continuous Strength Method

The Continuous Strength Method is a deformation-based design approach for steel elements that allows for the beneficial influence of strain-hardening, the details of which are given in Chapter 3 for beams in bending. Equations for this method have been developed on the basis of assuming continuous lateral restraint, or for scenarios where lateral torsional buckling is unimportant (e.g. box sections). Depending upon local plate slenderness values, the capacities of cross-sections in bending designed using the CSM are typically in excess of those predicted by traditional plastic design; this is attributable

to the CSM taking into account the influence of strain hardening. Expressed in terms of $\bar{\lambda}_{LT}$, the current adopted maximum unsupported length for Class 1 and Class 2 cross-sections at which the full plastic capacity $M_{pl,Rd}$ can be attained is $\bar{\lambda}_{LT} = 0.4$. Since the CSM typically predicts higher capacities, then it is likely that this length will need to be reduced.

4.3 Experimental data

A testing programme comprising tensile and compressive material coupon tests, stub column tests and tests on beams with discrete lateral restraints was carried out at the Building Research Establishment and Imperial College London on hot-rolled grade S355 steel I-sections. Two cross-section sizes were chosen: 305×127×48 UB, which had a Class 1 flange ($\bar{\lambda}_p = 0.31$) and a Class 1 web ($\bar{\lambda}_p = 0.30$), and 305×165×40 UB, which had a Class 2 flange ($\bar{\lambda}_p = 0.57$) and a Class 1 web ($\bar{\lambda}_p = 0.44$). In addition, supplementary data from tests on partially restrained beams were sourced from the literature.

4.3.1 Tests performed in the current study

Material testing, as well as a series of three- and four-point bending tests on partially restrained beams have been carried out as part of a related study in Chapter 3. Results from these tests will be used in this chapter.

4.3.2 Test results collected from the literature

Additional experimental data have been collected from tests conducted on I-beams with either continuous or discrete (partial) lateral restraints. The majority of the existing data is in the lateral torsional slenderness range of $\bar{\lambda}_{LT} \geq 0.4$, with a small number of studies investigating behaviour in the range $0 \leq \bar{\lambda}_{LT} \leq 0.4$.

The majority of the collected data comes from a previous survey carried out by Greiner and Kaim (2001), which was used to compare numerically derived lateral torsional buckling curves with data taken from tests on rolled and welded I-sections; these data are denoted in this chapter as ECCS (rolled) and ECCS (welded) respectively. Tests by

Byfield and Nethercot (1998) were conducted to investigate the bending strength enhancements over M_{pl} due to strain-hardening of partially and fully laterally restrained I-sections, with $\bar{\lambda}_{LT} = 0.4$ and 0. For values of $\bar{\lambda}_{LT} \leq 0.4$, test data were obtained from a series of experiments (White, 1956; Lee et al., 1963; Prasad and Galambos, 1963) carried out at Lehigh University between 1956 and 1963. Non-dimensional ultimate moments, with respect to M_{pl} , and corresponding lateral torsional slenderness values from these tests, as well as results from the tests described in Section 4.3.1, are summarised in Fig. 4.2. The results indicate that the current limit to the Eurocode 3 plateau length of $\bar{\lambda}_{LT} = 0.4$ is suitable, since below this limit capacities of at least M_{pl} are typically achieved. These experimental data, supplemented with further numerical results generated in Section 4.4, will be used to assess a suitable limiting slenderness when considering strain-hardening. All of the considered cross-sections from the above sources were either Class 1 or Class 2.

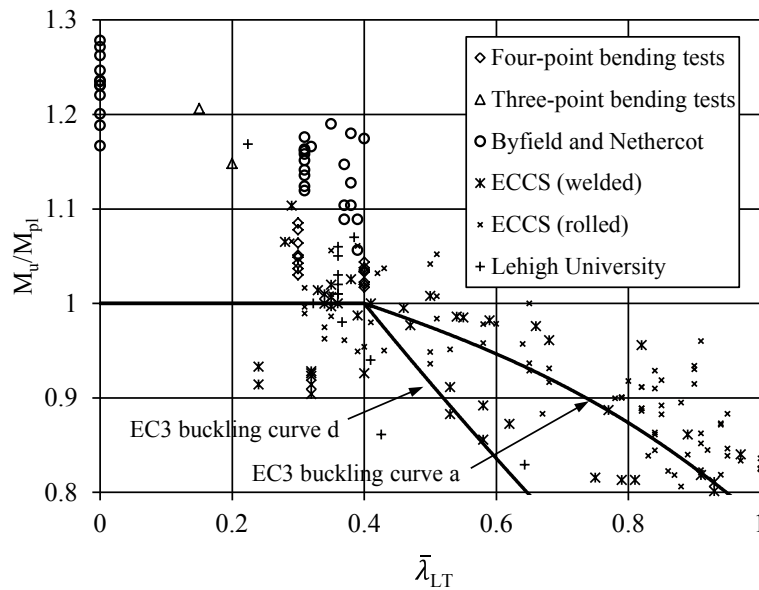


Figure 4.2: Summary non-dimensional ultimate moment capacities versus lateral torsional buckling slenderness for experimental data taken from the literature and produced in this study

4.4 Numerical modelling

4.4.1 Introduction

A numerical study using the general-purpose finite element analysis package ABAQUS (Simulia, 2010) was conducted to supplement the experimental results. A similar model was used in Chapter 3 to model beams subjected to four-point bending and it will be used in this chapter as part of a parametric study. In addition, three-point bending tests were also conducted with additional rigid restraint spacings; replicating the results of these tests will be the focus of the validation in this chapter. In this section, the primary aims are to (1) replicate the three-point bending test results presented in Chapter 3 using all available measurements, (2) employ a standardised numerical model validated against the experimental data that will supplement the model already developed for four-point bending in Chapter 3, and (3) undertake parametric studies to determine the minimum spacing requirements to achieve the predicted CSM capacity in bending. An additional parametric study was carried out to evaluate the performance of an analytical model, which is presented in Section 4.5.

4.4.2 Modelling assumptions and considerations

The finite element type S4R, a four-noded, doubly curved general-purpose shell element with reduced integration and finite membrane strains, selected from the ABAQUS (Simulia, 2010) element library, was used throughout the study to model the beams. This element is suited to the modelling of a range of shell thicknesses and has been successfully implemented in other studies, such as Chan and Gardner (2008). The modelling assumptions used for this element type are identical to those used in Chapter 3, in which reference should be made to Section 3.4.2 for details.

4.4.3 Validation

To validate the finite element models, all experiments were simulated using measured material and geometric properties, as well as local and global imperfections and the assumed residual stress pattern. In addition, a second round of finite element models

that replaced measured with assumed imperfections were produced; in both cases the ultimate capacities, as well as the general moment-rotation responses were compared with those obtained in the laboratory tests.

Fig. 4.3 compares overall non-dimensional moment-rotation behaviour of a three-point bending test specimen against the finite element model (results for the four-point bending tests are presented in Chapter 3, Section 3.4.7). Table 4.1 compares ultimate applied loads obtained from the experiments with those obtained from the two finite element models. The average ratio of test ultimate load to finite element ultimate load where measured and assumed imperfections are used are 0.94 and 0.95 respectively; the corresponding standard deviations are 0.005 for both models. For the three-point bending models, the ultimate FE moments are those that correspond to the ultimate test rotation, as testing was stopped due to a lack of vertical displacement capacity in the testing rig. In the subsequent analysis, the assumed imperfection magnitudes will be adopted as they represent a standardised approach that can be easily reproduced and extended.

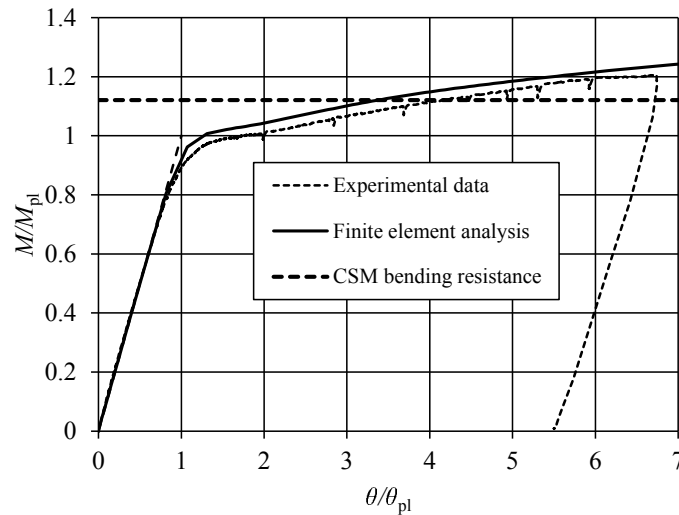


Figure 4.3: Normalised moment-rotation curves for a tested 305×127×48 UB ($\bar{\lambda}_{LT} = 0.1$) specimen subjected to three-point bending and the corresponding finite element model with measured initial geometric imperfections and an assumed residual stress distribution.

Table 4.1: Comparison of the three-point bending test results with finite element results for measured and assumed imperfection amplitudes

Test designation	Test M_u (kNm)	FE M_u (kNm) (Measured imperfection)	FE M_u (kNm) (Assumed imperfection)	Test/FE (Measured imperfection)	Test/FE (Assumed imperfection)
$305 \times 127 \times 48, \bar{\lambda}_{LT} = 0.2, R$	318*	343	334	0.93	0.95
$305 \times 127 \times 48, \bar{\lambda}_{LT} = 0.15, R$	334*	356	354	0.94	0.94
Mean				0.94	0.95
Standard deviation				0.005	0.005

*Ultimate test load not achieved

4.4.4 Parametric studies

Having validated the model against the additional three-point experimental data, in which the restraint spacing was varied, a series of parametric studies was performed, focusing on variations in $\bar{\lambda}_{LT}$, as defined in Eq. (3.1). The parametric studies were performed on both of the cross-sections used in the testing programme (305×127×48 UB and 305×165×40 UB) corresponding to local flange plate slenderness values of $\bar{\lambda}_p = 0.31$ and 0.57 respectively. An further section based on the 305×127×48 UB was also created with a local flange plate slenderness of $\bar{\lambda}_p = 0.45$. A summary of the lateral torsional slenderness values used in the parametric studies are reported in Table 4.2. It is evident from collecting available experimental data that few tests have been performed on beams whose non-dimensional slenderness values lie in the region $0 < \bar{\lambda}_{LT} < 0.4$; the purpose of the parametric study is to bridge this gap for the sections considered.

Table 4.2: Values of $\bar{\lambda}_{LT}$ used in the parametric studies.

$\bar{\lambda}_{LT}$		
$\bar{\lambda}_p = 0.31$	$\bar{\lambda}_p = 0.45$	$\bar{\lambda}_p = 0.57$
0.05	0.05	0.05
0.10	0.10	0.10
0.15	0.14	0.13
0.19	0.18	0.16
0.24	0.23	0.24
0.29	0.27	0.27
0.38	0.40	0.29
0.43	0.45	0.32
0.48	0.49	0.42

4.5 Lateral restraint spacing

Currently EN 1993-1-1 defines a non-dimensional slenderness limit, or plateau length, $\bar{\lambda}_{LT} = 0.4$, below which, the effects of lateral torsional buckling can be ignored. Tests

conducted in this study, as well as additional tests from the literature, have demonstrated that this limit is reasonable when the moment capacity is limited to M_{pl} , as in current design approaches, but may need to be reduced when strain-hardening is exploited in design, as is the case in the CSM. In this section, equations are developed to form the preliminary basis for a new CSM lateral torsional buckling slenderness limit, taking into consideration the key CSM parameters of cross-section geometry and the rate of strain hardening E/E_{sh} .

4.5.1 Analytical study

An analytical study to consider the effect of exploiting strain-hardening in beams upon the lateral restraint spacing requirements, as characterised by $\bar{\lambda}_{LT}$, is conducted to gain insight into the key underlying parameters controlling the physical behaviour. The analysis will be of a simply supported beam subjected to a uniform bending moment which will result in a region of constant curvature along the length of the beam and no shearing forces. In so doing, the following assumptions are made:

- (i) The beam is initially straight and free of imperfections.
- (ii) Residual stresses are neglected.
- (iii) The ends of the flanges are free to warp but cannot displace or twist (simple support end conditions).
- (iv) The material is assumed to behave elastically with a Young's modulus E up until the yield stress f_y , immediately after which work hardening will commence with a hardening modulus E_{sh} until the limiting (local buckling) strain ϵ_{csm} is reached, and because of this, local buckling is assumed to be absent.

4.5.2 Elastic buckling equation

The equation relating the elastic critical buckling moment M_{cr} of a beam to the laterally unsupported span L , as derived by Timoshenko and Gere (1961), is given by Eq. (4.1):

$$\frac{M_{cr}}{M_{pl}} = \frac{\pi}{M_{pl}L} \sqrt{EI_z GI_t \left(1 + \frac{\pi^2 EI_w}{L^2 GI_t} \right)} \quad (4.1)$$

in which E and G are the elastic and shear moduli, I_z is the minor axis second moment of area, I_w is the warping constant and I_t is the St. Venant's torsion constant. Eq. (4.1) represents the positive eigenvalue of the differential equation for lateral torsional buckling under pure moment with the following boundary conditions:

$$u = \frac{d^2u}{dx^2} = \phi = \frac{d^2\phi}{dx^2} = 0 \quad \text{at} \quad x = 0 \quad \text{and} \quad x = L \quad (4.2)$$

where u is the lateral displacement from the longitudinal centroidal axis x and ϕ is the angle of twist about the centroid. Defining the general stiffness values $B_z = EI_z$, $C_t = GI_t$, and where h is the overall height of the cross-section, $C_w = EI_w = (1/4)EI_z h^2$, Eq. (4.1) can be generalised to account for inelastic buckling with the introduction of bending and warping stiffness reduction parameters τ_1 and τ_2 respectively. The parameters $0 \leq \tau_1 \leq 1$ and $0 \leq \tau_2 \leq 1$, which are dependent upon the degree of yield penetration α and hence the yielded area η , serve as a means of reducing bending and warping stiffness values below their elastic values as yield progresses through the cross-section.

4.5.3 Cross-section resistance dependent upon yield penetration

The design expression for the CSM moment of resistance (Eq. (3.4)) can be parameterised in terms of yield penetration, α . In light of previous work on the CSM (Gardner et al., 2011), it is assumed that the material is elastic, linear hardening, with a strain-hardening slope $E_{sh} = E/100$ and with a limiting strain, ϵ_{CSM} . ϵ_{CSM} is defined using Eq. (3.5), which is derived from an empirical relationship between local buckling and non-dimensional plate slenderness. The constitutive relationship used in the CSM is summarised in Fig. 4.4.

In a similar approach to previous work (Gardner, 2008; Gardner et al., 2011), moment equilibrium can be considered by integrating the stress through the depth of the section, derived from the adopted material model and assuming plane sections remain plane. In the present work, analytical expressions for moment capacity are derived in terms of the degree of yield penetration α , where $\alpha = 1$ denotes first yield and $\alpha = 0$ denotes full cross-section yield penetration (see Fig. 4.5). Table 4.3 presents expressions for moment capacities for the cases when (a) the flange is partially yielded and (b) when the flange

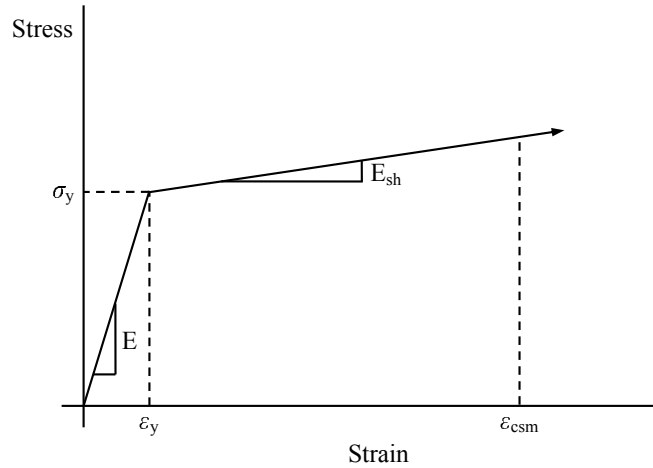


Figure 4.4: The constitutive model used for the CSM.

is fully yielded and the web is partially yielded; these are based upon the definitions shown in Fig. 4.5, which also depicts the assumed linear variation in strain throughout the cross-section. In this table, the superscript *I* denotes expressions that correspond with a partially yielded flange and an elastic web, whilst the superscript *II* denotes expressions that correspond with a fully yielded flange and a partially yielded web; all expressions can be defined from Fig. 4.5.

When $E_{sh} = \alpha = 0$, the fully plastic moment is defined and this will be used as a numeraire in subsequent analysis. For a typical I-section ($305 \times 127 \times 48$ UB), the relationship between yielded area η and moment capacity, allowing for strain-hardening, is shown in Fig. 4.6. This shows that beyond M_{pl} , all of the minor axis rigidity is provided by the web.

4.5.4 Stiffness reduction parameters

In order to develop an expression for the inelastic buckling moment, a relationship needs to be developed between cross-section moment, M , yield proportion, α , and out of plane flexural and warping stiffness. This will be achieved through developing non-dimensional stiffness reduction parameters for out of plane flexural stiffness (τ_1) and warping (τ_2), which can then be applied to a generalised version of Eq. (4.1).

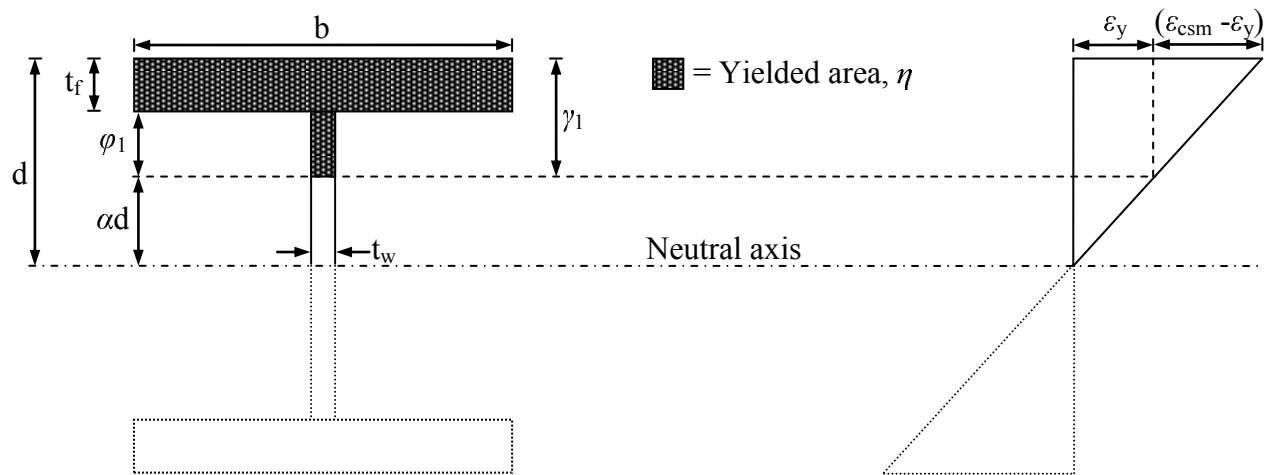


Figure 4.5: Cross-section definitions and strain distribution for determining cross-section bending resistance.

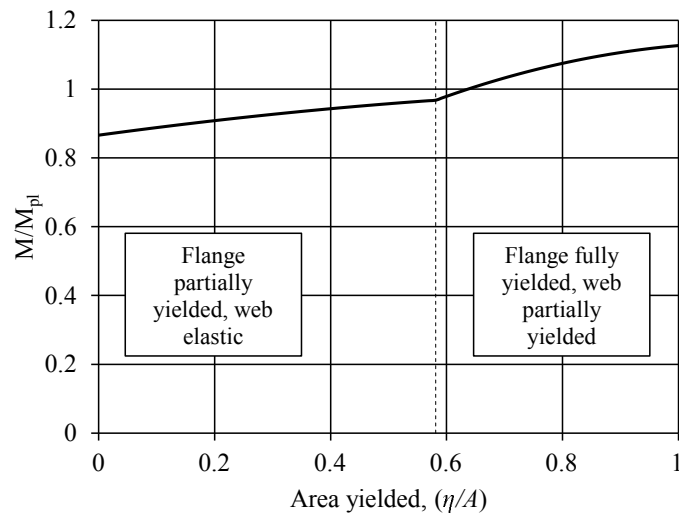


Figure 4.6: Typical relationship between moment capacity and yielded area for a 305 x 127 x 48 UB.

Table 4.3: Parameter definitions for the yield-dependent CSM moment capacity expressions.

$1 > \alpha > \alpha_{\text{lim}}$	$\alpha_{\text{lim}} > \alpha > 0$
$\gamma_1 = (1 - \alpha)$	$\varphi_1 = \gamma_1 - t_f$
$\gamma_2 = t_f - \gamma_1$	$\varphi_2 = \alpha d$
$\beta_1^I = \epsilon_y (1 - t_f) / \varphi_2$	$\beta_1^{II} = (\epsilon_{\text{csm}} / d) \varphi_1$
$\beta_2^I = \epsilon_y - \beta_1^I$	$\beta_2^{II} = (\epsilon_{\text{csm}} - \epsilon_y) - \beta_1^{II}$
$\mu_1^I = \alpha d + (1/2)\gamma_1$	$\mu_1^{II} = d - (1/2)t_f$
$\mu_2^I = \alpha d + (2/3)\gamma_1$	$\mu_2^{II} = d - (1/3)t_f$
$\mu_3^I = (d - t_f) + (1/2)\gamma_2$	$\mu_3^{II} = \varphi_2 + (1/2)\varphi_1$
$\mu_4^I = (d - t_f) + (2/3)\gamma_2$	$\mu_4^{II} = \varphi_2 + (2/3)\varphi_1$
$\mu_5^I = (2/3)(d - t_f)$	$\mu_5^{II} = (2/3)\varphi_2$
$M_1^I = \gamma_1 \epsilon_y \mu_1^I b E$	$M_1^{II} = t_f \epsilon_y \mu_1^{II} E b$
$M_2^I = (1/2)\gamma_1 (\epsilon_{\text{csm}} - \epsilon_y) \mu_2^I b E_{\text{sh}}$	$M_2^{II} = t_f (\beta_1^{II} \mu_1^{II} + (1/2)\beta_2^{II} \mu_2^{II}) b E_{\text{sh}}$
$M_3^I = \gamma_2 \beta_1^I \mu_3^I b E$	$M_3^{II} = \varphi_1 \epsilon_y \mu_3^{II} t_w E$
$M_4^I = (1/2)\gamma_2 \beta_2^I \mu_4^I b E$	$M_4^{II} = (1/2)\varphi_1 (\epsilon_{\text{csm}} - \epsilon_y) \mu_4^{II} t_w E_{\text{sh}}$
$M_5^I = (1/2)(d - t_f) \beta_1^I \mu_5^I t_w E$	$M_5^{II} = (1/2)\varphi_2 \epsilon_y \mu_5^{II} t_w E$
$M^I = \sum_{i=1}^5 M_i^I$	$M^{II} = \sum_{i=1}^5 M_i^{II}$

4.5.4.1 Minor axis bending stiffness

Assuming that no reversal of stress occurs anywhere in the cross-section (Flint, 1953), the material stressed beyond yield may be assumed to have a reduced modulus of $E_{sh} = E/100$, with the unyielded material possessing its full elastic stiffness E . Noting that for any rectangular section of depth t and width b , $I_y = bt^3/12$. Thus for the partially yielded flange the minor axis bending stiffness will be:

$$B^I = \frac{1}{12} [(A_f - \eta) b^2 + A_w t_w^2] E + \frac{1}{12} \eta b^2 E_{sh} \quad (4.3)$$

in which A_f is the area of the flange and A_w is the area of the web. For the partially yielded web and fully yielded flange, the minor axis stiffness will be:

$$B^{II} = \frac{1}{12} [A_w - (\eta - A_f)] t_w^2 E + \frac{1}{12} [A_f b^2 + (\eta - A_f) t_w^2] E_{sh} \quad (4.4)$$

Reduced minor axis bending stiffness can then be obtained by comparing B^I and B^{II} with B_z over the relevant range of yield penetration:

$$\tau_1(\alpha) = \begin{cases} \frac{B^I}{B_z} & 1 > \alpha > \alpha_{lim} \\ \frac{B^{II}}{B_z} & \alpha_{lim} > \alpha > 0 \end{cases} \quad (4.5)$$

The relationship between yielded area η normalised by full cross-section area and normalised minor axis bending stiffness is illustrated in Fig. 4.7 which shows that once the flange has yielded, the available minor axis rigidity provided by the web is approximately 1% of the minor axis rigidity available at first yield.

The relationship between normalised minor axis bending stiffness and non-dimensional moment capacity is illustrated in Fig. 4.8 which shows that the minor axis flexural rigidity is reduced considerably as M_{pl} is approached.

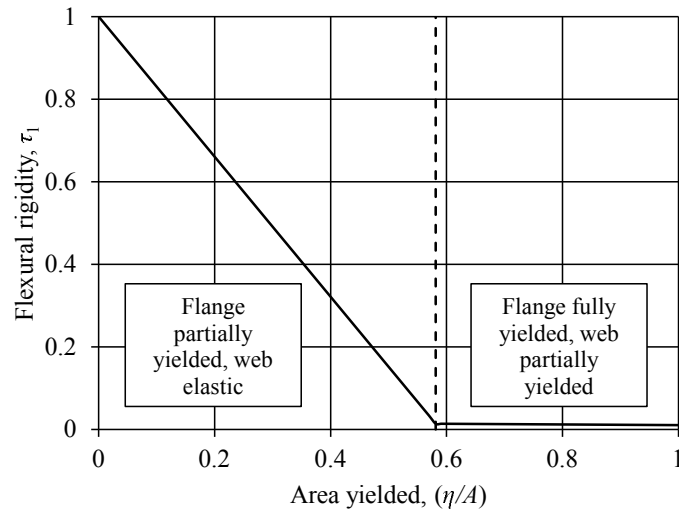


Figure 4.7: Typical relationship between yielded area and bending stiffness for a $305 \times 127 \times 48$ UB.

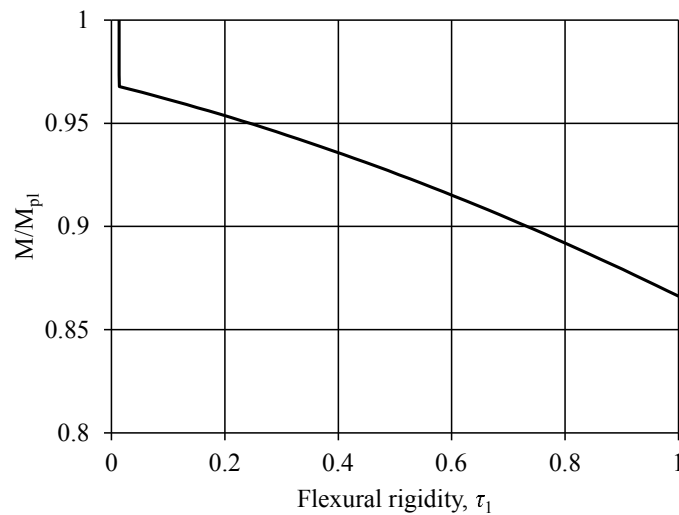


Figure 4.8: Typical relationship between bending stiffness and cross-section moment capacity for a $305 \times 127 \times 48$ UB.

4.5.4.2 Torsional stiffness

Neal (1950) demonstrated both experimentally and theoretically that, for partially plastic narrow beams, the initial torsional rigidity maintains its elastic value; subsequent experimental work by Morrison and Shepherd (1950) on a tube strained into the inelastic range has shown similar results. On this basis, no reduction factor for torsional rigidity C_t will be provided and it will be assumed that $C_t = GI_t$ will remain independent of α .

4.5.4.3 Warping stiffness

It has been shown that warping rigidity is an influential factor on the critical moment for thin-walled open sections (Nethercot and Rockey, 1972). Assuming that all of the warping resistance is provided by the flanges, the relationship between non-dimensional warping stiffness, τ_2 , and yield penetration, α , is identical to τ_1 in the range $1 > \alpha > \alpha_{\text{lim}}$ and zero thereafter:

$$\tau_2(\alpha) = \begin{cases} \tau_1(\alpha) & 1 > \alpha > \alpha_{\text{lim}} \\ 0 & \alpha_{\text{lim}} > \alpha > 0 \end{cases} \quad (4.6)$$

4.5.5 The CSM buckling equation

Introducing the generalised terms for stiffness defined in Section 4.5.2 and applying the non-dimensional stiffness reduction factors defined in Eqs. (4.5) and (4.6), the critical non-dimensional bending moments for an inelastic beam with strain-hardening over the range $(\epsilon_{\text{csm}} - \epsilon_y)$ are given by Eq. (4.7):

$$\frac{M_{\text{cr}}}{M_{\text{pl}}} = \frac{\pi}{M_{\text{pl}}L} \sqrt{B_z C_t \tau_1 \left(1 + \frac{\pi^2 C_w \tau_2}{L^2 C_t} \right)} \quad (4.7)$$

This can be re-written in terms of the critical buckling length L_{cr} :

$$L = \sqrt{\sqrt{D} - \frac{B}{2A}} \quad (4.8)$$

where

$$A = \left(\frac{M_{cr}}{M_{pl}} \right)^2, \quad B = -\frac{\pi^2 B_z C_t \tau_1}{M_{pl}^2}, \quad C = -\frac{\pi^4 B_z C_w \tau_1 \tau_2}{M_{pl}^2} \quad \text{and}$$

$$D = \left(\frac{B}{2A} \right)^2 - \frac{C}{A}$$

Having developed relationships between M , α , τ_1 and τ_2 , assumed values of M_{cr}/M_{pl} can now be used to obtain corresponding critical buckling lengths for the CSM.

4.5.5.1 Comparison to experimental and numerical data

Using the experimental data obtained from the tests and the literature described in Section 4.3, as well as the numerical data generated in the main parametric study (Section 4.4.4), the performance of Eq. (4.7) for a range of cross-sections is assessed (Fig. 4.9).

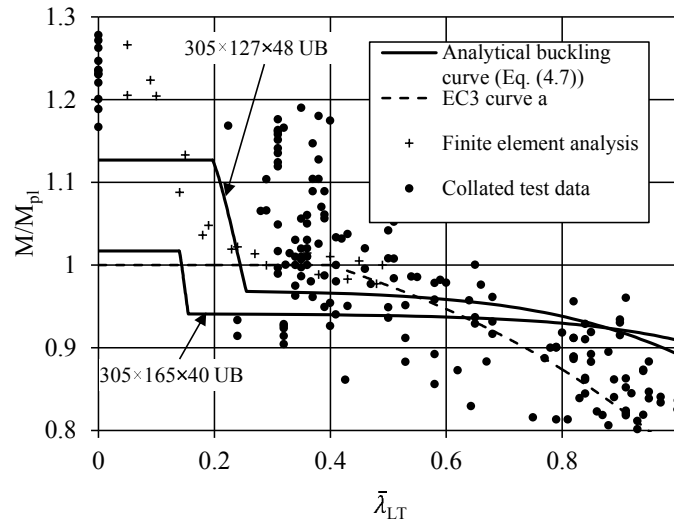


Figure 4.9: The CSM member buckling equation, experimental and numerical data.

Although there is considerable scatter in the data, experimental results typically lie above Eq. (4.7) particularly in the low slenderness region of primary interest, making it a conservative prediction for limiting CSM lateral torsional buckling slenderness.

An additional parametric study was performed using the B31OS open section, which is a three-dimensional beam element taken from the ABAQUS element library. Using a very small imperfection magnitude, a materially and geometrically non-linear finite element analysis was performed using the same material model and cross-section definitions used to develop the analytical model in Section 4.5.5. Lateral torsional slenderness values were varied in equal steps between 0.05 and 0.35. Fig. 4.10 compares Eq. (4.7) to the results generated using the B31OS elements. The finite element analysis suggests a similar response to that predicted by the analytical model (Eq. (4.7)) but shows a slightly earlier and more gradual transition in lateral torsional buckling slenderness from M_{pl} to M_{csm} . This may be attributable to discretisation, the element type admitting shear deformations and geometric non-linearity, as well as the presence of imperfections promoting the slightly earlier onset of yielding.

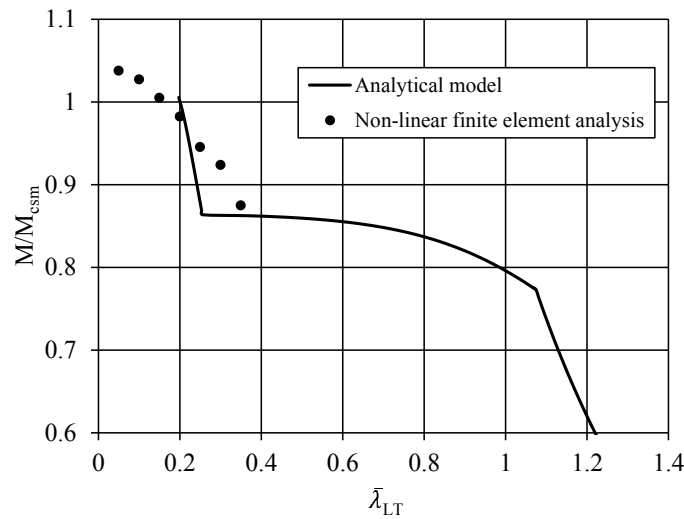


Figure 4.10: Variation of buckling resistance with slenderness according to Eq. (4.7) and a geometrically and materially non-linear finite element beam model.

4.5.6 Response to key design parameters

The Continuous Strength Method is dependent upon two key design parameters: cross-section geometry and the rate of strain-hardening (E/E_{sh}). This section will examine the sensitivity of Eq. (4.7) to these parameters and with the aim to provide an indicative

range of limiting $\bar{\lambda}_{LT}$ values for the CSM, below which the full CSM bending resistance M_{csm} can be achieved. The transition to M_{pl} will also be examined.

4.5.6.1 Cross-section geometry

Eq. (4.7) is uniquely defined by cross-section properties. To investigate the influence of cross-section geometry on the limiting CSM slenderness, a practical range of cross-sections was selected; this was achieved by considering all of the commercially available rolled I-sections (SCI, 2013). For each case, the limiting values of $\bar{\lambda}_{LT}$ at which the CSM capacities of each cross-section can be attained were determined. The values are plotted against b/t_f (where b is the flange width and t_f is the flange thickness) in Fig. 4.11. It may be observed that the limiting values of $\bar{\lambda}_{LT}^{csm}$ for M_{csm} lie in the range $0.13 \leq \bar{\lambda}_{LT} \leq 0.26$.

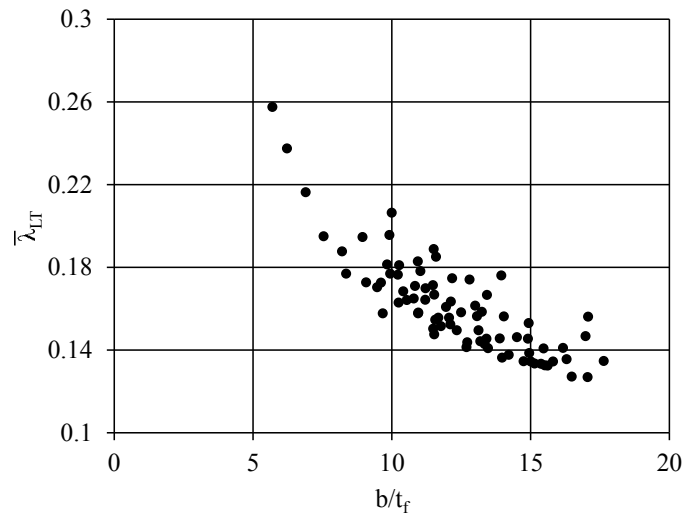


Figure 4.11: Influence of cross-section geometry (expressed in terms of b/t_f) on limiting slenderness $\bar{\lambda}_{LT}^{csm}$ for commercially available cross-sections.

4.5.6.2 Rate of strain-hardening

The ratio of Young's modulus to strain-hardening modulus E/E_{sh} is currently assumed to be fixed at $E/E_{sh} = 100$, in line with EN 1993-1-5. Introducing increased rates of E_{sh} not only enhances the CSM capacity at the cross-section level, but also permits a small increase in restraint spacing. Fig 4.12 shows the effect of varying values of E/E_{sh}

in equally spaced increments between $50 \leq E/E_{sh} \leq 100$ on the maximum value of $\bar{\lambda}_{LT}$ at which M_{csm} can be attained for the same cross-section geometry; for the range $50 \leq E/E_{sh} \leq 100$, maximum lateral torsional buckling slenderness values lie in the range $0.19 \leq \bar{\lambda}_{LT} \leq 0.23$.

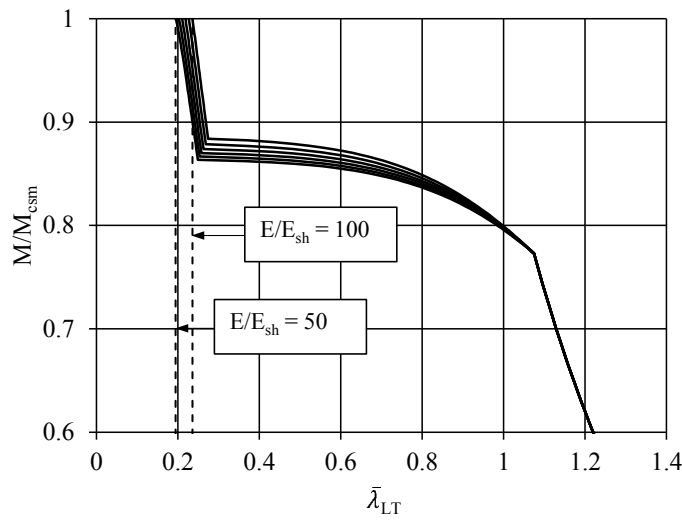


Figure 4.12: Influence of E_{sh} on limiting slenderness below which M_{csm} can be achieved.

From this analysis, it is clear that cross-section geometry is the most significant parameter of the CSM for the purposes of lateral stability.

4.6 Preliminary design recommendations

4.6.1 Proposal for limiting slenderness and a transition from M_{pl} to M_{csm}

The transition from the plastic moment capacity to the CSM moment capacity, as well as the limiting lateral torsional buckling slenderness, should reflect the basic strain-hardening properties of the material (see Chapter 6) as well as lie in reasonable proximity to numerical and experimental data. A previous equivalent proposal by Trahair (1998) for the limiting slenderness $\bar{\lambda}_{LT}^{csm}$ may be expressed as:

$$\bar{\lambda}_{LT}^{csm} = \sqrt{E_{sh}/E} \quad (4.9)$$

This may be coupled with a linear transition from M_{pl} to M_{csm} , given by:

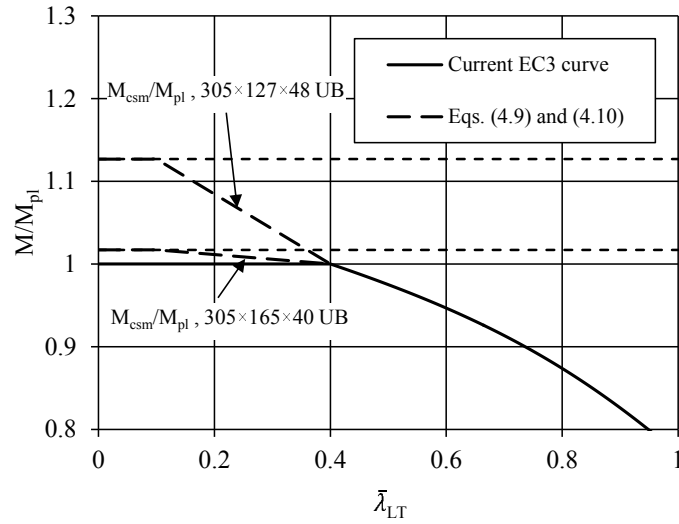
$$\frac{M}{M_{pl}} = \frac{(\bar{\lambda}_{LT} - \bar{\lambda}_{LT}^{csm})}{0.4 - \bar{\lambda}_{LT}^{csm}} \left(1 - \frac{M_{csm}}{M_{pl}} \right) + \frac{M_{csm}}{M_{pl}} \quad (4.10)$$

This relationship is plotted for two cross-sections in Fig. 4.13. Compared with numerical data, there is a degree of over-prediction during the transition phase, which could be eliminated by a more sophisticated non-linear transition. For conservative design, it would be sufficient to assume M_{pl} between $\bar{\lambda}_{LT} = 0.4$ and $\bar{\lambda}_{LT}^{csm}$.

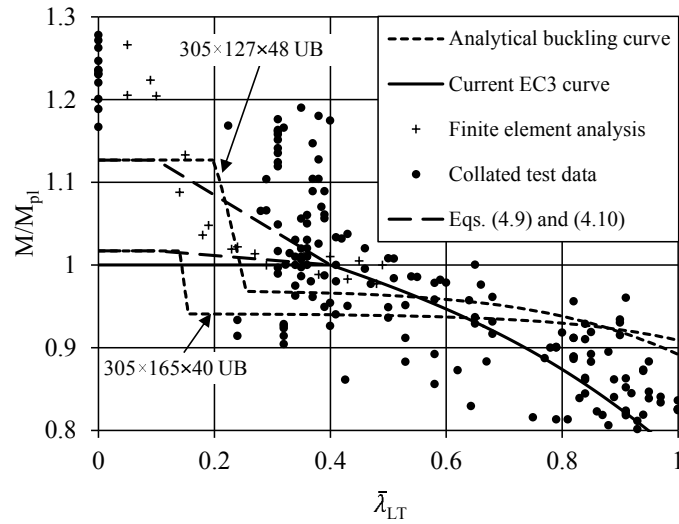
4.6.2 Refinement using analytical, numerical and test data collected from the literature

Fig. 4.14 shows the same experimental and numerical data as Fig. 4.9, but normalised by M_{csm} rather than M_{pl} . Additional data points were generated using the analytical model presented in this chapter for commercially available I-sections (SCI, 2013). From this data, the first observation that can be made is that there is a broad range of $\bar{\lambda}_{LT}$ values at which M_{csm} can be attained, typically in the range $0.15 \leq \bar{\lambda}_{LT} \leq 0.4$ (although the main concentration of data is in the range $0.15 \leq \bar{\lambda}_{LT} \leq 0.25$); all of these values exceed the value conservatively suggested by $\bar{\lambda}_{LT}^{csm}$ in Eq. (4.9). From this observation, and the collected dataset, it may be reasonable to suggest that the practical slenderness limit for attaining M_{csm} at the member level can be revised to $\bar{\lambda}_{LT} = 0.2$.

The second observation is that between $0 \leq \bar{\lambda}_{LT} \leq 0.2$, the majority of the available data exceeds M_{csm} , which supports setting the limiting slenderness to $\bar{\lambda}_{LT} = 0.2$. Indeed, making further reference to Fig. 4.9, a proportion of the specimens between $0.2 \leq \bar{\lambda}_{LT} \leq 0.4$ lie below M_{pl} , which suggests that the current plateau limit of $\bar{\lambda}_{LT} = 0.4$ for traditional plastic design set out in EN 1993-1-1 also caters for most, but not all of the test data. By this analogy, a general plateau length of $\bar{\lambda}_{LT} = 0.2$ for the CSM can be considered to be an acceptable initial proposal.



(a) Proposed transition curve assuming $E/E_{sh} = 100$ for cross-sections with $\bar{\lambda}_p = 0.31$ and $\bar{\lambda}_p = 0.57$



(b) Comparison of transition curves to analytical, finite element and experimental data.

Figure 4.13: Example M_{pl} to M_{csm} transition curves.

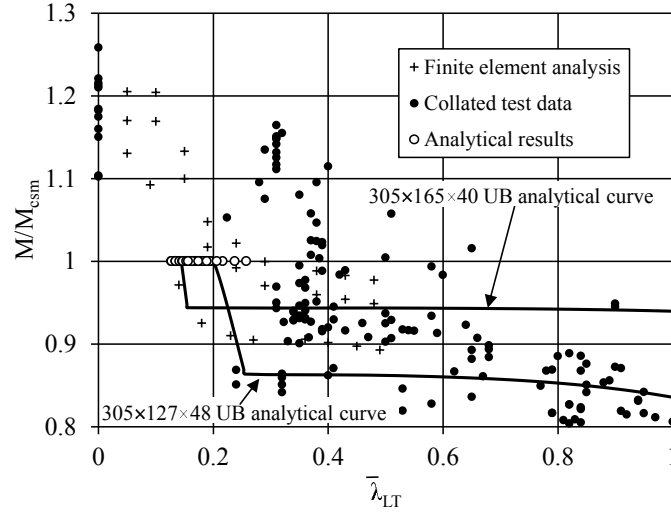


Figure 4.14: Collated analytical, numerical and experimental data normalised by M_{csm} .

In the light of these observations the following design equation is recommended (Eq. (4.11)):

$$\chi_{LT,csm} = \frac{(\bar{\lambda}_{LT} - 0.2)}{0.2} \left(1 - \frac{M_{csm}}{M_{pl}} \right) + \frac{M_{csm}}{M_{pl}} \quad \text{for } 0.2 \leq \bar{\lambda}_{LT} \leq 0.4 \quad (4.11)$$

in which $\chi_{LT,csm}$ is a factor applied to M_{pl} to obtain M_{csm} in the region $0.2 \leq \bar{\lambda}_{LT} \leq 0.4$. For $\bar{\lambda}_{LT} \leq 0.2$, the full value of M_{csm} may be used. Furthermore, if $M_{csm} < M_{pl}$, then $\bar{\lambda}_{LT}^{csm} = 0.2$ remains, but the transition should be from M_{el} rather than M_{pl} at $\bar{\lambda}_{LT} = 0.4$. These design expressions are illustrated in Fig. 4.15 for a typical range of values of M_{csm} .

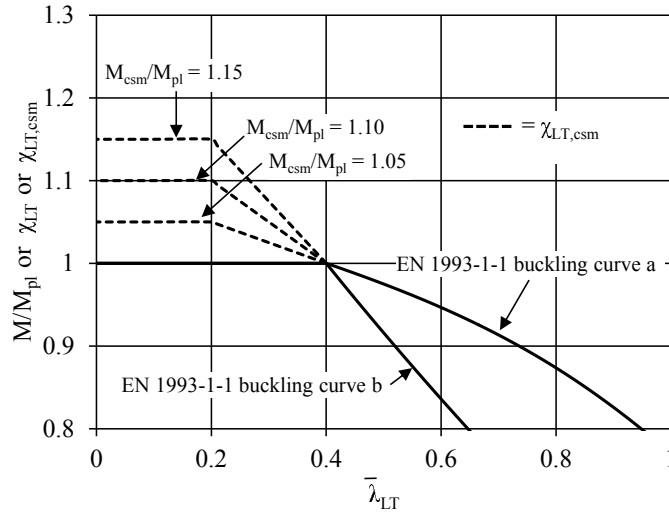


Figure 4.15: Illustration of the application of the factor $\chi_{LT,CSM}$ for a typical range of values of M_{CSM} .

4.7 Conclusions and design recommendations

Using the insights obtained from an analytical model in conjunction with data generated either from tests performed in this thesis, collected from the literature, or from a numerical model, the lateral-torsional buckling instability implications of allowing for strain-hardening in the design of structural steel members have been considered.

A simplified analytical model was developed to investigate the influence of the key parameters of the CSM on the limiting values of $\bar{\lambda}_{LT}$, where it was demonstrated that cross-section geometry was the most significant parameter. Using a simple methodology, a conservative relationship between $\bar{\lambda}_{LT}$ and M_{CSM} , which is entirely based upon strain-hardening material properties, was reviewed and shown to be safe but conservative. Using additional data generated as part of an analytical and numerical study, as well as test data collected from the literature, a basic design approach was presented that incorporated a limiting CSM slenderness of $\bar{\lambda}_{LT} \leq 0.2$ as well as a transition function from M_{pl} to M_{CSM} via the factor $\chi_{LT,CSM}$.

Chapter 5

Lateral restraint conditions for statically indeterminate structural elements

5.1 Introduction

In the traditional plastic design of statically indeterminate structures, the final collapse mechanism develops through the sequential formation of plastic hinges. In order for subsequent hinges to form, the preceding plastic hinges are required to rotate. Once the hinge has formed, there is a reduction in stiffness and no further spread of yield. At these rotating plastic hinges, additional demands will be placed upon the restraints compared with statically determinate structures, which will not contain rotating plastic hinges. Provisions in EN 1993-1-1 reflect this increased demand and minimum restraint force resistances have been stipulated for the case of traditional plastic design. However, this provision is yet to be verified for the Continuous Strength Method (CSM) where moments beyond M_{pl} can be achieved; this will be the first topic of investigation in this chapter.

When designing an unbraced statically determinate I-beam in the inelastic range, the limiting value of non-dimensional slenderness $\bar{\lambda}_{LT}$, below which the effects of lateral torsional buckling can be ignored, is based upon knowledge of the inelastic critical moment. The inelastic buckling load of statically determinate structures can be determined

using the relatively straightforward analysis described in Chapter 4, which is based upon modifications of elastic predictions. Key to this simplified analysis is that there is no plastic redistribution, meaning that the elastic and inelastic moment distributions are the same. However, for statically indeterminate structures, the elastic in-plane distribution of moment is not necessarily the same as the final inelastic in-plane distribution and so is not strictly valid for determining the inelastic buckling load of such a structure (Yoshida et al., 1977).

Numerous investigations into the critical buckling loads of elastic continuous beams (Salvadori (1951), Trahair (1968a), Trahair (1968b), Trahair (1969b), Nethercot and Trahair (1976b), Dux and Kitipornchai (1982), Trahair (1983)) and inelastic continuous beams (Yoshida and Imoto (1973), Poowannachaikul and Trahair (1976), Yoshida et al. (1977), Dux and Kitipornchai (1984)) have been carried out, typically considering behaviour up to the plastic design moment. The second component of this chapter is devoted to examining the lateral stability implications of the Continuous Strength Method (CSM) for statically indeterminate structures through the stability analysis of continuous beams in the strain-hardening range. The objective is to establish practical limitations on lateral restraint spacing such that the effects of lateral torsional buckling can be ignored and the full indeterminate CSM cross-section capacity can be achieved.

To these ends, a series of experiments on continuous beams with variations in restraint spacing and stiffness were conducted. Using a numerical model for inelastic buckling, as well as a geometrically and materially non-linear finite element model, the test data were reproduced and extended in a parametric study which was then used to inform and develop some basic design recommendations.

5.2 Key design aspects

For statically indeterminate structures to produce a collapse mechanism, multiple plastic hinges must form in sequence, with the first plastic hinge (the location of which is determined by the static theorem) undergoing further rotations until all of the subsequent plastic hinges have formed. To eliminate the effects of lateral torsional buckling, EN 1993-1-1 introduces more stringent requirements for restraint spacing and restraint

forces for members that contain rotated plastic hinges compared with those summarised in Chapter 3, Section 3.2 for statically determinate structures.

5.2.1 Restraint forces

The additional deformation demands at rotating plastic hinges will place additional demands upon the bracing system. For non-rotating plastic hinges at M_{csm} , it has been previously established in Chapter 3 that no modifications are necessary to the current provisions of EN 1993-1-1. For members that do contain rotated plastic hinges, the additional requirements in EN 1993-1-1 are:

- (i) At each plastic hinge location, the cross-section should have an effective lateral and torsional restraint, provided at both the tension and compression flanges.
- (ii) The braces at the compression flange should be designed to resist a local force of at least 2.5% of N_{Ed} , where $N_{Ed} = M_{Ed}/h$ is the force in the compression flange, M_{Ed} is the moment in the beam at the plastic hinge location and h is the overall depth of the beam.

5.2.2 Restraint spacing

The yielded zones that characterise the formation of plastic hinges weaken the cross-section and member with regard to lateral torsional buckling (Davies, 2006). As such, EN 1993-1-1 stipulates that the length between lateral restraints must not exceed the stable length L_{stable} . For uniform beam segments with I- or H-sections with $h/t_f < 40\epsilon$ under linear moment without significant axial compression, the stable length may be taken as:

$$L_{stable} = \begin{cases} 35\epsilon i_z & \text{for } 0.625 \leq \psi \leq 1 \\ (60 - 40\psi)\epsilon i_z & \text{for } -1 \leq \psi \leq 0.625 \end{cases} \quad (5.1)$$

in which $\epsilon = \sqrt{235/f_y}$, ψ is the ratio of end moments in the segment, h is the overall height of the cross section and t_f is the plate thickness of the flange. More detailed provisions exist for members with axial force and this is defined as L_m in Annex BB.3 of EN 1993-1-1.

5.2.3 The continuous strength method for statically indeterminate structures

Traditional plastic design methods for indeterminate steel structures are based upon the formation and subsequent rotation of plastic hinges at their full plastic moment capacity. Depending upon the degree of statical indeterminacy, a sequence of plastic hinges forms until there is a sufficient reduction in structural stiffness to form a collapse mechanism. A key assumption of this method is rigid-plastic behaviour, whereby upon attaining the plastic moment capacity no further increases in capacity occur with deformation and infinite rotations can be achieved. Introducing strain-hardening precludes the notion that plastic hinges may rotate at a constant moment, and with stocky sections (low $\bar{\lambda}_p$ values) significant increases in capacity beyond M_{pl} are possible (Gardner et al., 2011).

For statically determinate structures, increases in cross-section capacity beyond M_{pl} can be safely predicted and used for a basis of design using the Continuous Strength Method (CSM) (Gardner (2002), Wang (2011), Gardner et al. (2011)). For statically indeterminate structures, the basic features of traditional plastic design (equilibrium, mechanism and yield) can be combined with the more refined approach for determining cross-section capacity using the CSM (Wang (2011), Gardner et al. (2011)) by modifying the CSM moment capacity predictions at individual plastic hinge locations based upon relative deformation demands. The procedure can be summarised in the following steps:

- (i) Identify the locations of the plastic hinges and where necessary determine the critical collapse mechanism.
- (ii) Using the theorem of virtual work, evaluate the rotations θ_i at each plastic hinge location i .
- (iii) Based upon cross-section slenderness $\bar{\lambda}_p$, determine the deformation capacity $\frac{\epsilon_{csm}}{\epsilon_y}$ using Eq. (3.5).
- (iv) For each plastic hinge, evaluate the ratio of deformation demand to deformation capacity α_i , using Eq. (5.2):

$$\alpha_i = \frac{\theta_i}{\left(\frac{\epsilon_{\text{csm}}}{\epsilon_y} \right)_i} \quad (5.2)$$

and then evaluate $\alpha_{\text{crit}} = \max\{\alpha_i\}$, which is the location of greatest deformation demand relative to the deformation capacity at the hinge location.

(v) Evaluate the local deformation demands using Eq. (5.3):

$$\left(\frac{\epsilon_{\text{csm}}}{\epsilon_y} \right)_{\text{hinge},i} = \frac{\alpha_i}{\alpha_{\text{crit}}} \left(\frac{\epsilon_{\text{csm}}}{\epsilon_y} \right)_{\text{hinge,crit}} \quad (5.3)$$

(vi) Evaluate the respective CSM capacities $M_{\text{csm,crit}}$ and $M_{\text{csm},i}$ at each hinge location and then determine the collapse load of the structure using the usual kinematic theorem.

In a recent study by Zhao (2012), it has been shown that the CSM for statically indeterminate structures can provide a mean increase in capacity of 9% above the collapse load predicted by the traditional plastic design method, while maintaining safe side predictions of test data. In the same study, it was also concluded that: (1) reductions in cross-section plate slenderness reduce the hinge moment ratio (the ratio of the moment at the support to the moment at the span in a two (equal) span continuous arrangement); (2) decreases in plate slenderness reduce the capacity for moment redistribution, as the deformation capacity of the cross-section reduces with increasing slenderness. These aspects will be further explored in this study.

5.3 Experimental investigation

A testing programme comprising tensile and compressive material coupon tests, stub column tests and tests on continuous beams with discrete lateral restraints was carried out at the Building Research Establishment and Imperial College London on hot-rolled grade S355 steel I-sections. Two cross-section sizes were chosen: 305×127×48 UB, which had a Class 1 flange ($\bar{\lambda}_p = 0.31$) and a Class 1 web ($\bar{\lambda}_p = 0.30$), and 305×165×40 UB,

which had a Class 2 flange ($\bar{\lambda}_p = 0.57$) and a Class 1 web ($\bar{\lambda}_p = 0.44$). These cross-sections, which were tested in determinate configurations in Chapter 3, are now studied in indeterminate configurations.

5.3.1 Material properties

The cross-sections examined in this chapter are the same as those tested in Chapters 3 and 4. As such, the material property data obtained from those tensile and compressive coupon tests will also be utilised in this chapter.

5.3.2 Tests on continuous beams

A total of six continuous beam tests (two rigidly restrained and four elastically restrained) were conducted on two cross sections ($305 \times 127 \times 48$ UB and $305 \times 165 \times 40$ UB). The lateral restraints were placed at varying intervals to achieve unrestrained lengths between bracing points with $\bar{\lambda}_{LT} = 0.4$ or $\bar{\lambda}_{LT} = 0.3$, in which $\bar{\lambda}_{LT}$ is the non-dimensional lateral torsional buckling slenderness as defined in Eq. (3.1) and EN-1993-1-1 (2005). Restraints were either rigid or elastic.

5.3.2.1 Testing conditions

Fig. 5.1a shows the general test configuration for the continuous beam tests, with the lateral restraint conditions illustrated in Fig. 5.1b. For all tests, rigid lateral restraints were provided at the loading points and at the supports. The length L_2 was varied to achieve desired values of $\bar{\lambda}_{LT}$ for each test; L_1 was chosen in relation to L_2 to ensure that buckling takes place in the L_2 region first.

For each test, web stiffeners were provided at the positions of the supports and the applied loads to prevent premature failure through web crippling. Vertical displacements were measured using pull-wire transducers and end rotations were measured using inclinometers; force was applied at each loading point using two hand operated 1000 kN load-controlled hydraulic jacks and forces were monitored with load cells. Two linear electrical resistance post-yield strain gauges were bonded to the extreme tensile and compressive fibres of the cross-section at the mid-span (or slightly off-set where

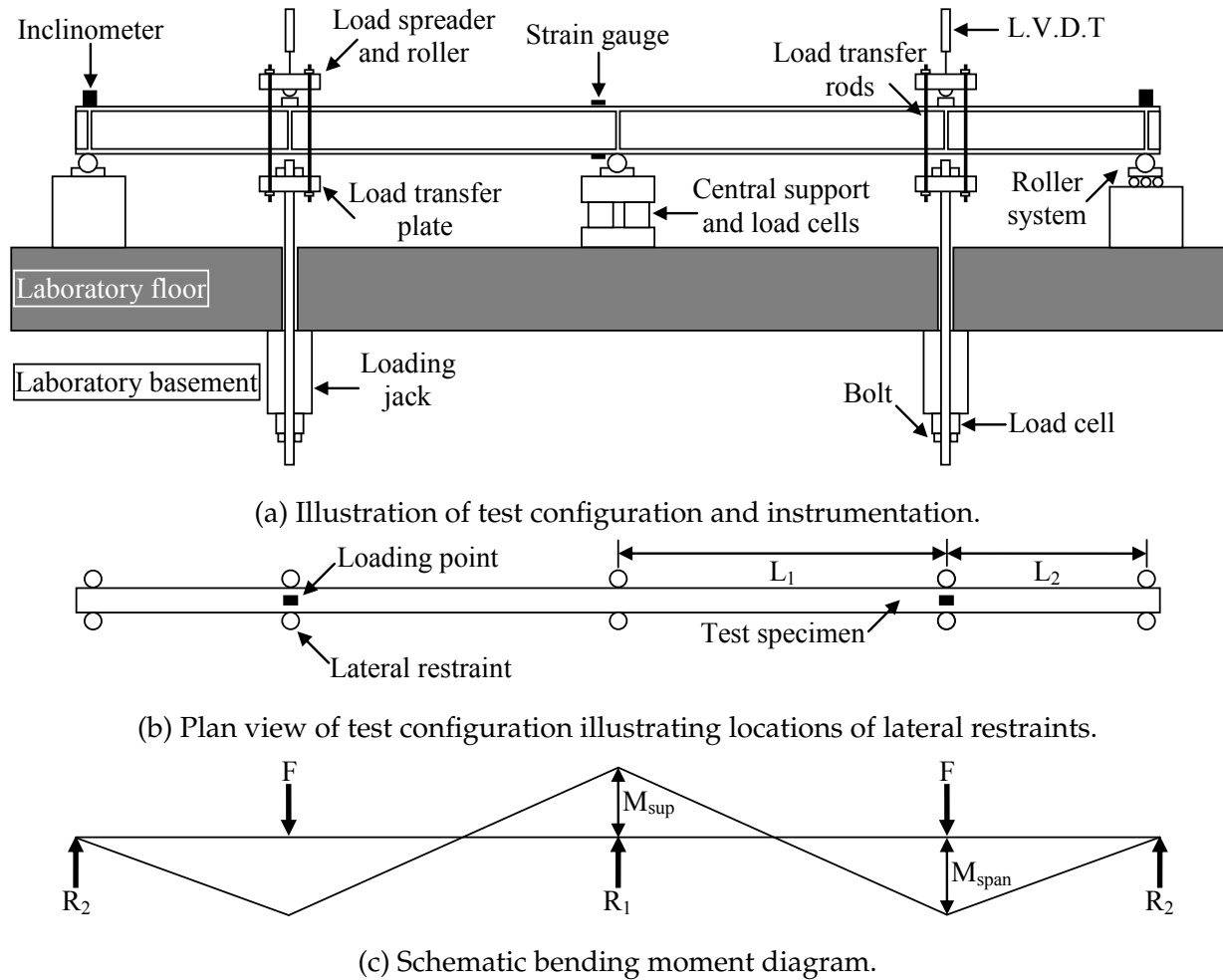


Figure 5.1: Schematic illustrations of continuous beam tests.

necessary) to monitor the progression of strain throughout the test. Simple support conditions were provided by a roller system, with longitudinal movement permitted at one end using a sliding plate system. Loads, rotations, displacements and strains were all recorded at one-second intervals using the data acquisition system DATASCAN.

5.3.2.2 Geometric properties of the test specimens

Prior to testing, the cross-section properties of each specimen were measured and recorded. Global imperfection amplitude measurements (e_0) were taken for each specimen by holding a fine copper wire at mid-web height taught along the length of the beam and

then measuring the distance between the wire and the beam at the mid-length. Table 5.1 provides summary measured geometric properties of the continuous beam test specimens, including global imperfections measurements (for local imperfections ω_0 , the representative measurements made in Chapter 3 will be used in this chapter); all symbols and notation used are defined identically to those used in Table 3.3, Chapter 3.

5.3.2.3 Test results

Non-dimensional moment-rotation responses (M/M_{pl} versus θ/θ_{pl} at the end supports, where θ_{pl} is the elastic component of the rotation when M_{pl} is reached) for all of the continuous beam tests are presented in Figs. 5.2a and 5.2b. Summary ultimate test moments at the loading points ($M_{u,span}$) at the ultimate load of the system normalised by M_{pl} and M_{csm} , as well as the ultimate system load F_u normalised by the plastic collapse load F_{col} and CSM collapse load $F_{col,csm}$ are presented in Table 5.2, where F refers to the value of each of the two point loads.

The test results show that the full CSM bending resistance can be achieved at both the loading points and at the central support for all specimens of both cross-sections with the exception of the $305 \times 127 \times 48$, $\bar{\lambda}_{LT} = 0.4$ R test (see Fig. 5.2a). For all but two of the tests, it was not possible to attain the ultimate load of the specimens due to instabilities in the testing rig. Despite considerable efforts being made to provide effective rigid lateral restraints, there was evidence of either restraint deformation or displacement in every test, effectively increasing the $\bar{\lambda}_{LT}$ values of the specimens to a higher value. In most cases, it was necessary to terminate the testing prior to attaining ultimate capacity to prevent a sudden and uncontrolled failure.

Table 5.2 also reports values for ultimate rotation capacity R , which is defined as:

$$R = \frac{\theta_{ur}}{\theta_{pl}} - 1 \quad (5.4)$$

where θ_{ur} is normally defined as the total rotation at the midspan until the moment falls below M_{pl} (see also Fig. 5.3). Considering the moment rotation responses in Figs. 5.2a and 5.2b, such a definition is only strictly applicable to the $305 \times 127 \times 48$, $\bar{\lambda}_{LT} = 0.4$ (K1),

Table 5.1: Summary geometric properties for the continuous beam test specimens

Test designation	$\bar{\lambda}_p$ (flange)	$\bar{\lambda}_p$ (web)	e_0 (mm)	b (mm)	h_w (mm)	t_f (mm)	t_w (mm)
$305 \times 127 \times 48, \bar{\lambda}_{LT} = 0.4, R$	0.31	0.30	1.0	127.51	283	14.33	8.73
$305 \times 127 \times 48, \bar{\lambda}_{LT} = 0.4, K1$	0.31	0.30	1.0	127.29	283	14.46	8.68
$305 \times 127 \times 48, \bar{\lambda}_{LT} = 0.4, K2$	0.31	0.30	1.0	127.59	282	14.77	9.07
$305 \times 165 \times 40, \bar{\lambda}_{LT} = 0.4, R$	0.57	0.44	10.5	165.86	284	9.93	6.54
$305 \times 165 \times 40, \bar{\lambda}_{LT} = 0.4, K1$	0.57	0.44	2.5	166.81	286	10.44	6.40
$305 \times 165 \times 40, \bar{\lambda}_{LT} = 0.3, K1$	0.57	0.44	6.0	165.49	287	10.05	6.53

Table 5.2: Ultimate moments, rotations and load capacities of the tested continuous beams.

Test designation	$\frac{M_{u,sup}}{M_{pl}}$	$\frac{M_{u,sup}}{M_{csm}}$	$\frac{M_{u,span}}{M_{pl}}$	$\frac{M_{u,span}}{M_{csm}}$	$\frac{F_u}{F_{col}}$	$\frac{F_u}{F_{col,csm}}$	R
$305 \times 127 \times 48, \bar{\lambda}_{LT} = 0.4, R$	1.11	0.99	1.16	1.03	1.17	1.05	2.71
$305 \times 127 \times 48, \bar{\lambda}_{LT} = 0.4, K1$	1.13	1.01	1.18	1.05	1.25	1.12	3.99
$305 \times 127 \times 48, \bar{\lambda}_{LT} = 0.4, K2$	1.16	1.04	1.21	1.08	1.21	1.09	3.11
$305 \times 165 \times 40, \bar{\lambda}_{LT} = 0.4, R$	1.07	1.08	1.11	1.13	1.13	1.14	1.50
$305 \times 165 \times 40, \bar{\lambda}_{LT} = 0.4, K1$	1.09	1.10	1.14	1.15	1.14	1.15	3.38
$305 \times 165 \times 40, \bar{\lambda}_{LT} = 0.3, K1$	1.09	1.10	1.14	1.15	1.13	1.15	2.97
				Mean	1.17	1.12	

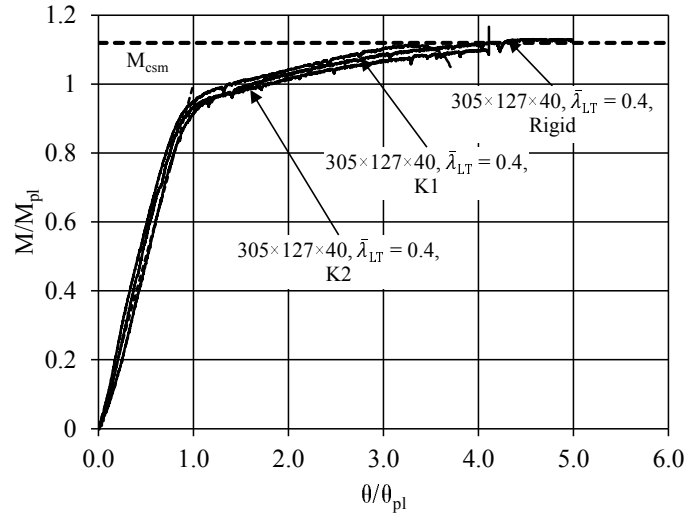
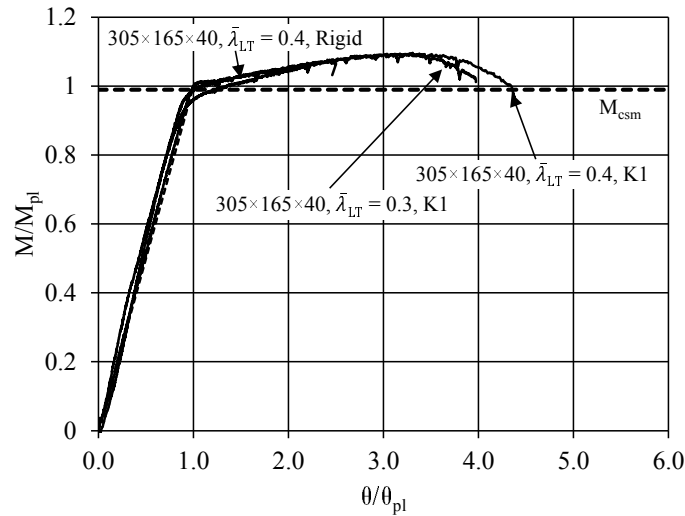

 (a) $305 \times 127 \times 48$ UB (Note: $M_{csm} = 1.12M_{pl}$)

 (b) $305 \times 165 \times 40$ UB (Note: $M_{csm} = 0.99M_{pl}$)

Figure 5.2: Non-dimensional support moment versus support rotation curves for the tested continuous beams.

$305 \times 165 \times 40$, $\bar{\lambda}_{LT} = 0.4$ (K1), and $305 \times 165 \times 40$, $\bar{\lambda}_{LT} = 0.3$ (K2) test data. In the cases where θ_{ur} cannot be determined, the terminal test rotation is used instead; this is taken as the last recorded value. For statically indeterminate carbon steel structures, EN-1993-1-1 requires a minimum rotation capacity of $R = 3$ for a cross section to be considered Class 1. The results show that this requirement (where achieved) is just sufficient to

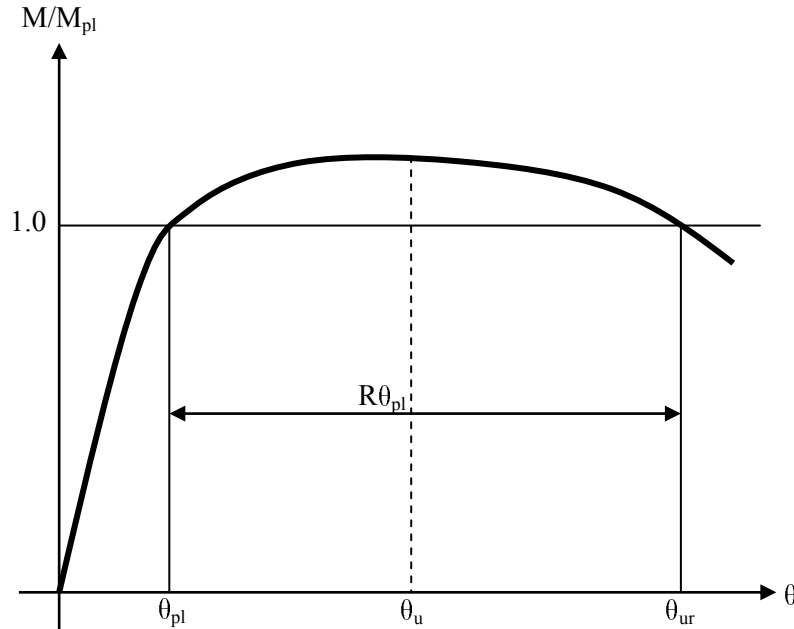
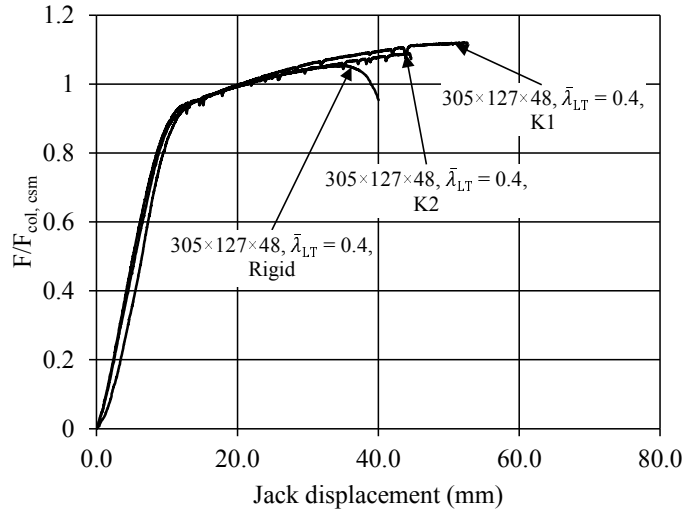


Figure 5.3: Definition of rotation capacity.

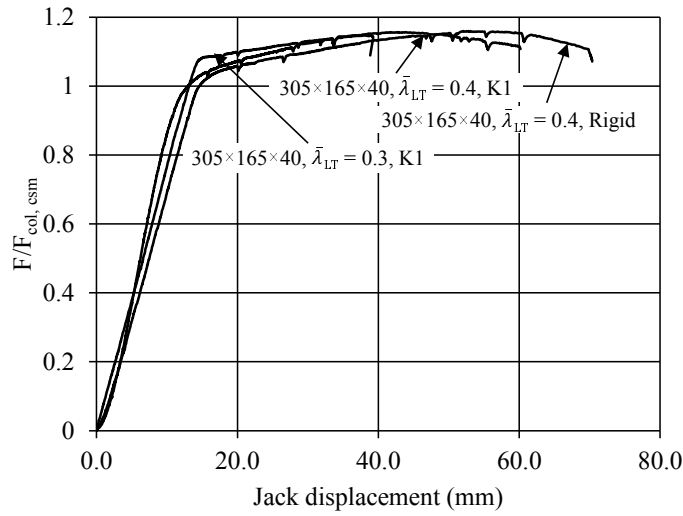
attain M_{csm} and with adequate lateral restraint, higher values of R could be attainable. Plots in terms of the CSM normalised collapse load $F/F_{\text{col, csm}}$ in Figs. 5.4a and 5.4b show that the collapse loads as predicted by the CSM for indeterminate structures were achieved. With reference to Table 5.2, compared with the conventional collapse loads derived from a plastic analysis F_{col} , the CSM collapse load shows an improvement when compared with ultimate test loads, with mean values of $\frac{F_u}{F_{\text{col, csm}}} = 1.12$ and $\frac{F_u}{F_{\text{col}}} = 1.17$ respectively.

5.3.3 Elastically restrained beams

For the four elastically restrained members, threaded rods with a tensile design resistance of approximately 2.5% of the $M_{\text{csm, Rd}}$ compression flange force were chosen for the K1 restraints; reduced diameters, using standard threaded rod dimensions, were chosen for the K2 restraints. These elastic restraints were attached to the tension and compression flanges of the specimens using articulated joint connections to minimise bending in the rods. To provide a stable anchorage for the restraints, stanchions with a considerably higher bending stiffness than the lateral stiffness of the specimens were



(a) $305 \times 127 \times 48$ UB.



(b) $305 \times 165 \times 40$ UB.

Figure 5.4: Normalised load versus displacement curves for the tested specimens.

used as fixing points. In order to attain practical restraint stiffness values, as well as to minimise the effects of bending, the rods were all 2500 mm in length. A schematic plan view of the restraint configuration is illustrated in Fig. 3.12 and an illustration of the restraint system in-situ at a plastic hinge location is shown in Fig. 5.5.

The recorded compression flange restraint forces and the ultimate moments at the load-

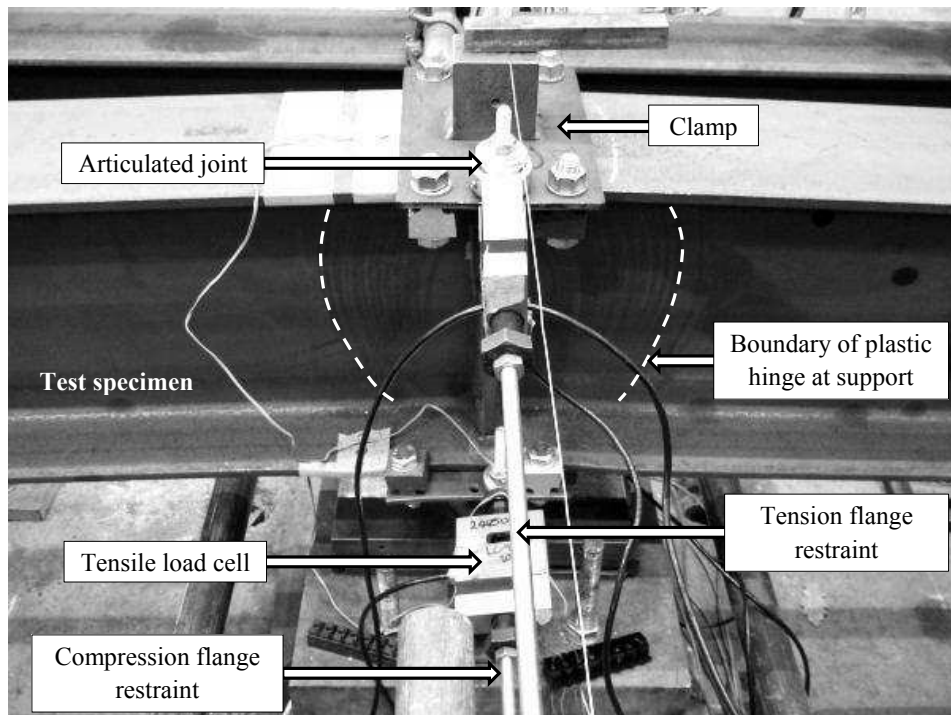


Figure 5.5: Elastic tension and compression flange restraints attached to the central rotated plastic hinge of a $305 \times 165 \times 40$ UB continuous beam.

ing points are reported in Table 5.3. None of the recorded restraint forces exceed the maximum level of 2.5% N_{Ed} required by EN 1993-1-1, though several factors in the tests prevented the ultimate capacity of the specimens from being attained, most notably the local deformations and displacements of the rigid lateral restraints. These issues contributed to the testing rig becoming unstable, with the tests being stopped early as a safety precaution. Despite this, all of the specimens attained their CSM capacities. Tension flange restraint forces for all tests were less than 0.1% N_{Ed} .

Table 5.3: Experimental compression flange restraint forces and corresponding ultimate moment capacities.

Test designation	$F_{res}/(M_u/h)$ %	$M_{u,sup}/M_{pl}$
$305 \times 127 \times 48, \bar{\lambda}_{LT} = 0.4, K1$	0.78	1.13
$305 \times 127 \times 48, \bar{\lambda}_{LT} = 0.4, K2$	0.13	1.16
$305 \times 165 \times 40, \bar{\lambda}_{LT} = 0.4, K1$	1.02	1.09
$305 \times 165 \times 40, \bar{\lambda}_{LT} = 0.3, K1$	1.56	1.09

This section has presented the details and results of an experimental investigation using continuous beams with both rigid and elastic lateral restraint conditions. The results from these experiments will be reproduced and extended in the forthcoming sections as part of two parametric studies, the first examining restraint forces and the second examining restraint spacing.

5.4 Lateral restraint forces

Restraint to the primary member is often provided in the form of a secondary bracing member that will undergo elastic deformations. The extent of the elastic deformations is governed by the stiffness of the brace (or bracing system). At the very least, there should be sufficient stiffness to force the primary member to buckle around the restraint (i.e. fail in the second mode - see Chapter 3); increased bracing stiffness beyond this threshold value will further limit the deformations of the bracing and correspondingly, the internal forces produced.

The Continuous Strength Method places very specific deformation demands at plastic hinge locations, which may be in excess of those encountered during normal plastic design approaches. The purpose of this section is to examine the implications of these deformations for continuous beams elastically braced at rotating plastic hinges. Specifically, the following will be examined: (i) the stiffness requirements for bracing to be effective and (ii) the additional strength demands placed upon the bracing system at rotating plastic hinges upon reaching a CSM-compatible collapse mechanism.

5.4.1 Design requirements

For non-rotating plastic hinges at M_{csm} , it has been previously established in Chapter 3 that no modifications are necessary to the current provisions of EN 1993-1-1. For members that do contain rotated plastic hinges, the additional requirements in EN 1993-1-1 are summarised in Section 5.2.1.

5.4.2 Numerical model

A numerical study using the general-purpose finite element analysis package ABAQUS (Simulia, 2010) was conducted in parallel with the testing programme. The primary aims of the investigation were to (1) replicate the experimental results using all available measurements, (2) employ a standardised numerical model validated against the experimental data, and (3) undertake parametric studies to examine the influence of key parameters on the structural response of statically indeterminate braced steel beams and the corresponding forces in the restraints. The basic modelling assumptions made in this chapter mirror those used in Chapter 3 and so will not be repeated here. The key differences lie with the boundary conditions, which were modified to model continuous beam support conditions, and the elastic lateral restraints at the central support, where a spring element was attached to the tension flange as well as the compression flange to provide torsional as well as lateral restraint.

5.4.2.1 Validation

To validate the FE models, all continuous beam experiments were simulated using measured material and geometric properties, as well as local and global imperfections and the assumed residual stress pattern (see Chapter 3 section 3.4.6). The ultimate capacities, as well as the general moment-rotation responses were then compared with those obtained in the laboratory tests. Typical comparative responses are shown in Figure 5.6.

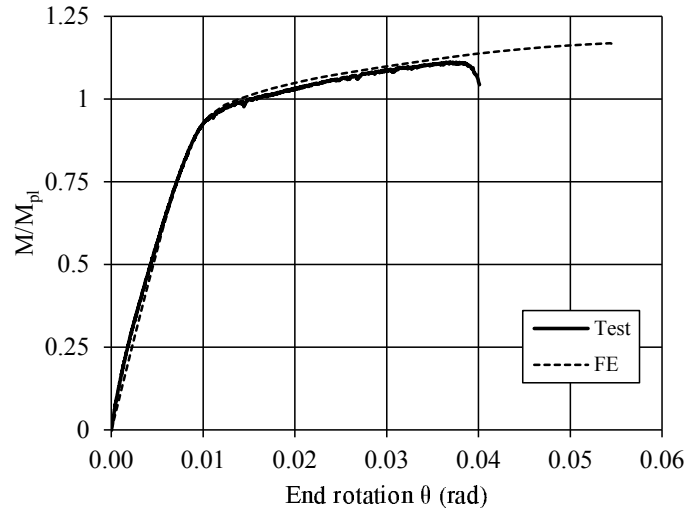
Table 5.4 compares ultimate applied loads obtained from the experiments with those obtained from the finite element models. The average ratio of test ultimate load to finite element ultimate load with measured imperfections is 1.02 and the corresponding coefficient of variation is 0.02. This represents good agreement with minimal dispersion.

Table 5.4: Comparison of the laboratory continuous beam test results with FE results using measured imperfection amplitudes.

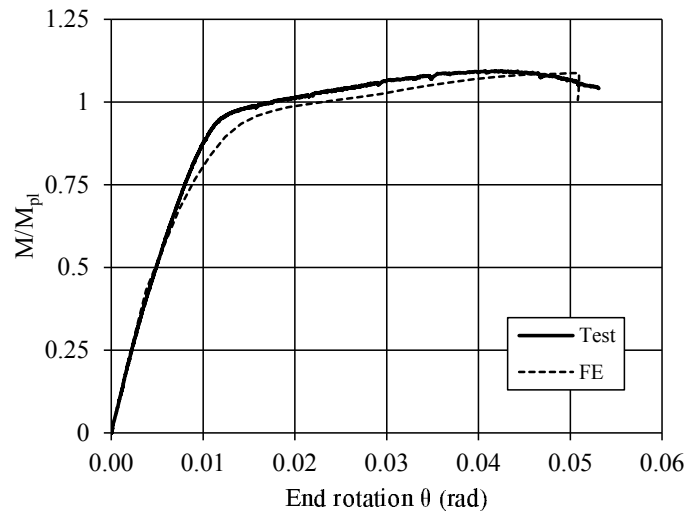
Test designation	Test M_u / FE M_u at Test θ_u
$305 \times 127 \times 48, \bar{\lambda}_{LT} = 0.4, K1$	1.02
$305 \times 127 \times 48, \bar{\lambda}_{LT} = 0.4, K2$	1.03
$305 \times 127 \times 48, \bar{\lambda}_{LT} = 0.4, \text{Rigid}$	1.01
$305 \times 165 \times 40, \bar{\lambda}_{LT} = 0.4, K1$	1.03
$305 \times 165 \times 40, \bar{\lambda}_{LT} = 0.3, K1$	0.98
$305 \times 165 \times 40, \bar{\lambda}_{LT} = 0.4, \text{Rigid}$	1.03

5.4.3 Parametric studies

Having obtained good overall agreement between the numerical model and the experimental data (for both continuous and simple beams), a series of parametric studies was performed, focusing on variations in elastic restraint stiffness as a multiple of k_{ideal} , which is defined in Section 3.5.2. The parametric studies were conducted for both cross-section geometries ($305 \times 127 \times 48$ UB and $305 \times 165 \times 40$ UB) and a lateral torsional buckling slenderness of $\bar{\lambda}_{LT} = 0.2$. Table 5.5 presents the multiples of k_{ideal} and corresponding



(a) $305 \times 127 \times 48$, $\bar{\lambda}_{LT} = 0.4$, Rigid



(b) $305 \times 165 \times 40$, $\bar{\lambda}_{LT} = 0.3$, K1

Figure 5.6: Typical experimental and numerical moment versus end rotation curves for the continuous beams under investigation.

spring stiffness values used for each cross-section. Throughout, an assumed global bow imperfection amplitude e_0 of $L/500$ was used and assumed peak local imperfection ω_0 amplitudes were determined using ω_{DW} , as defined in Eq. (3.10).

Table 5.5: Restraint stiffness values used in the parametric studies.

Multiples of k_{ideal}	k_{res} (N/mm)	
	$305 \times 127 \times 48$	$305 \times 165 \times 40$
0.25	483	275
0.5	966	549
1	1931	1098
2	3862	2196
3	5793	3294
4	7724	4392
5	9655	5491
6	11586	6589
8	15448	8785
10	19310	10981
12	23172	13177
14	27034	15373
16	30897	17570
18	34759	19766
20	38621	21962

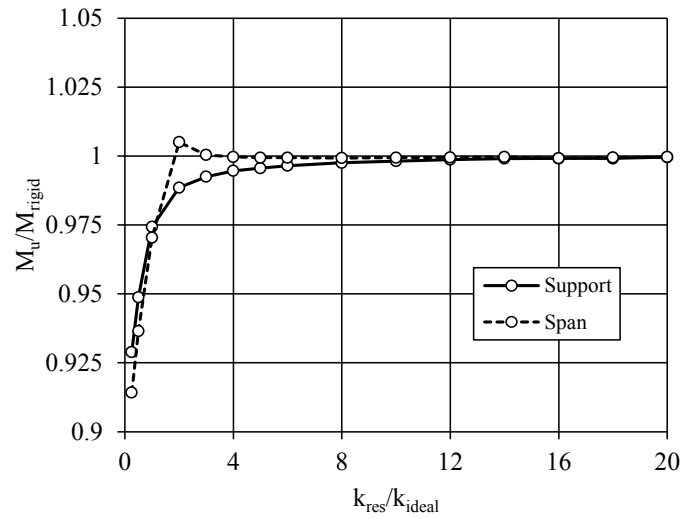
Fig. 5.7 shows the relationship between non-dimensional restraint stiffness and ultimate moments attained, normalised by the ultimate moments attained in the equivalent rigidly restrained beam at both the support and at the loading points in the span. In both locations, effective lateral restraint is provided at a level of stiffness of $8k_{ideal}$ for both cross-section types; this is considerably higher than the stiffness required for the effective restraint of simply supported beams, which is typically $2k_{ideal}$ (see Fig. 3.34). Such a discrepancy is likely to be attributable to the need to resist both lateral and torsional forces at the point of restraint, as the restraining action of the tension flange is reduced in continuous beams (Yoshida et al., 1977).

Fig. 5.8 shows the relationship between restraint stiffness and restraint force as a percentage of the force in the compression flange. For both sections, compression flange restraint forces decrease with increments in restraint stiffness, stabilising at a restraint force of approximately 3% N_{Ed} . Tension flange restraint forces show a tendency to increase with restraint stiffness, with quite pronounced differences in magnitudes between the two cross-section types: restraints attached to the tension flange of the $305 \times 127 \times 48$ UB tend towards 3% N_{Ed} , whilst those for the $305 \times 165 \times 40$ UB tend towards 0.5% N_{Ed} . The tension flange restraint is responsible for providing the additional component of resistance to prevent torsional failure of the beam at the central support; compared with the $305 \times 165 \times 40$ UB, the higher demands placed upon the tension flange restraints of the $305 \times 127 \times 48$ UB may be attributable to the lower minor axis second moment of area of the cross-section ($I_z = 461 \text{ cm}^4$ for the $305 \times 127 \times 48$ UB and $I_z = 764 \text{ cm}^4$ for the $305 \times 165 \times 40$ UB), as well as the higher I_y/I_z ratio ($I_y/I_z = 20.8$ for the $305 \times 127 \times 48$ UB and $I_y/I_z = 11.2$ for the $305 \times 165 \times 40$ UB).

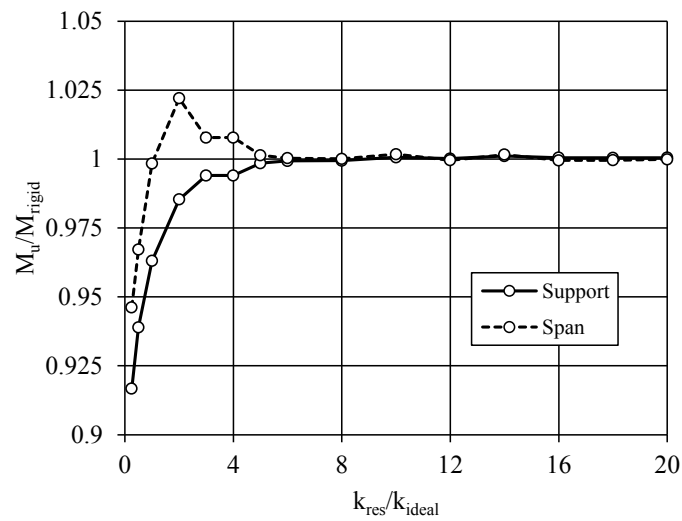
5.4.4 Design implications for the CSM

The forces in the restraints at the ultimate collapse loads of the considered continuous beams are typically higher than the 2.5% N_{Ed} prescribed by EN-1993-1-1, with values tending towards 3% N_{Ed} , even at very high restraint stiffness values. However at these ultimate loads, significant rotations at the support have occurred, as well as a significant amount of plastic redistribution. It is likely that the restraint forces for both the tension and compression flanges at support rotations sufficient for a CSM mechanism to form will be somewhat lower than this. Figs. 5.9 and 5.10 show the evolution of plastic redistribution (defined herein as the ratio of the support moment M_{sup} to the span moment M_{span}) with rotations normalised by ultimate end rotation θ_u , which is taken as the rotation corresponding to the ultimate moment. The corresponding propagation of restraint forces for the compression (Fig. 5.9) and tension (Fig. 5.10) flange restraints are superimposed onto the same plot.

Examination of Fig. 5.9 shows the ratio of the support moment M_{sup} to the span moment

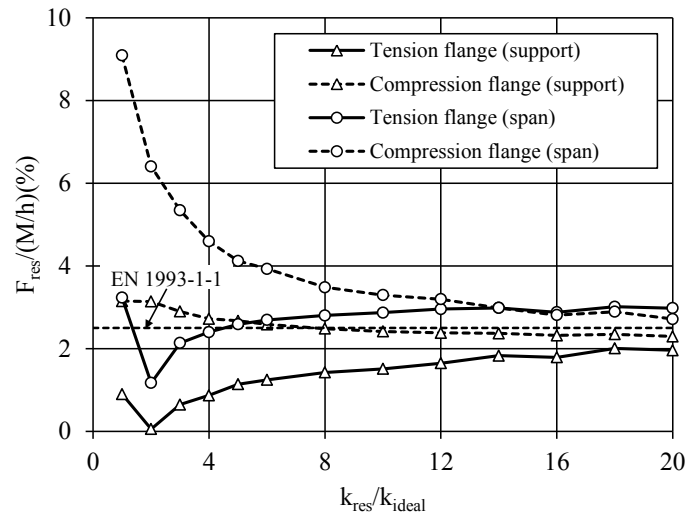


(a) 305×127×48 UB

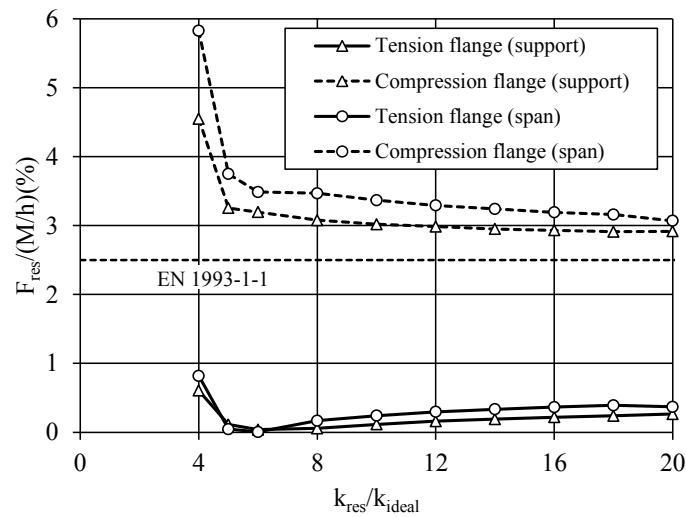


(b) 305×165×40 UB

Figure 5.7: Relationship between normalised restraint stiffness and ultimate moments attained in the continuous beams with elastic restraints, relative to those with rigid restraints.

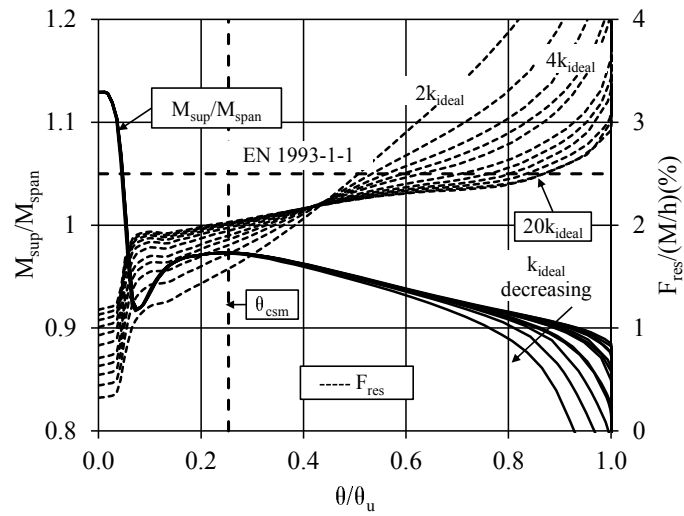


(a) 305×127×48 UB

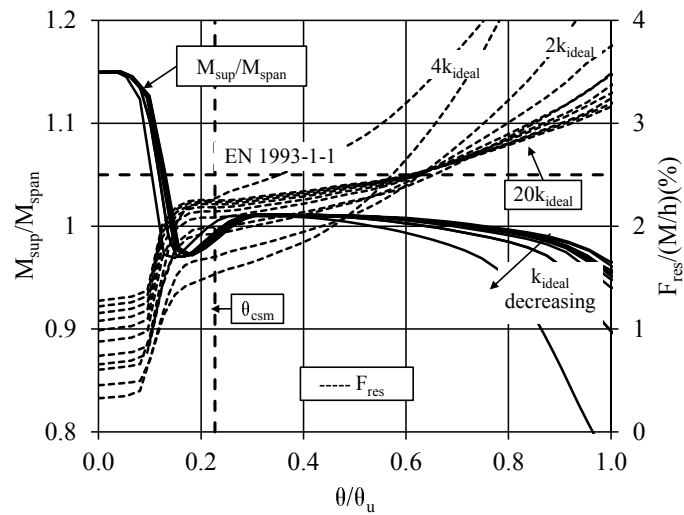


(b) 305×165×40 UB (Note: restraint stiffness values less than $4k_{ideal}$ did not permit M_{csm} to be attained)

Figure 5.8: Relationship between restraint stiffness and restraint force at ultimate load for modelled continuous beams.

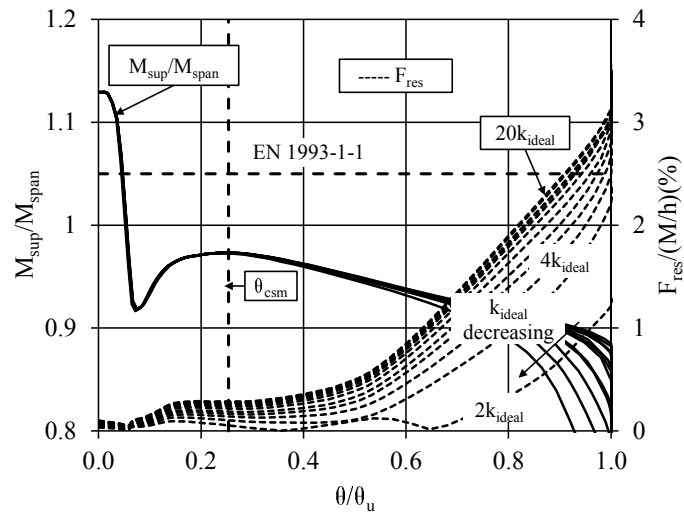


(a) 305×127×48 UB

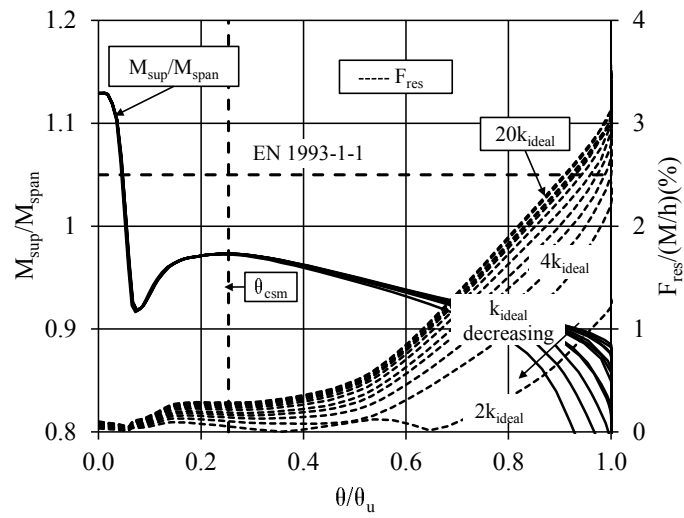


(b) 305×165×40 UB

Figure 5.9: Evolution of compression flange restraint forces with plastic redistribution.



(a) 305×127×48 UB



(b) 305×165×40 UB

Figure 5.10: Evolution of tension flange restraint forces with plastic redistribution.

M_{span} starting at its initial elastic value before reducing to approximately $M_{\text{sup}}/M_{\text{span}} = 0.92$ after which it settles to approximately $M_{\text{sup}}/M_{\text{span}} = 0.97$; similar behaviour can be seen in Fig. 5.10. Firstly it is observed that the initial value of $M_{\text{sup}}/M_{\text{span}} = 1.13$ is 6% below the elastic value of $M_{\text{sup}}/M_{\text{span}} = 1.2$ as determined from beam theory. This discrepancy may be explained by the presence of local and global imperfections in the finite element model, as well as the fact that because shell-elements are used, cross-section and shear deformations are admitted. Neither of these factors are taken into account in the elastic beam theory calculation of $M_{\text{sup}}/M_{\text{span}}$. Secondly, the observed evolution of moment distribution can be explained with the aid of Fig. 5.11 which superimposes $M_{\text{sup}}/M_{\text{span}}$ onto the non-dimensional moment-rotation curves for the support and span. For the region where $M_{\text{sup}}/M_{\text{span}}$ is less than the stable value of approximately 0.97, the moment-rotation curve of the span exhibits a small plateau, whilst the response at the support continues to increase.

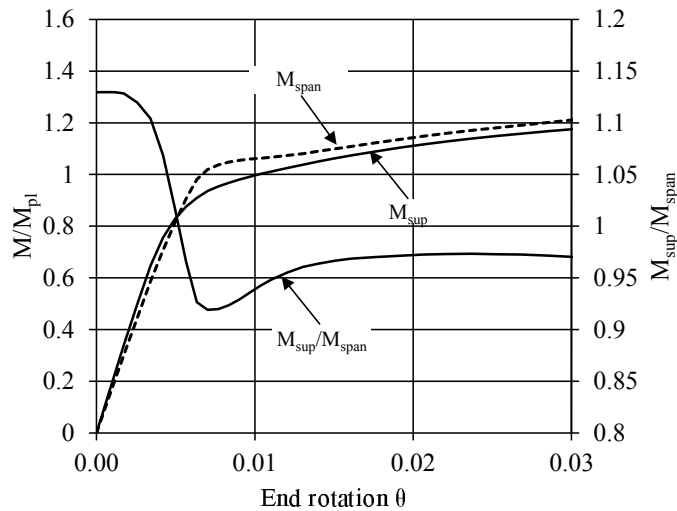


Figure 5.11: Decomposition of central support and span moments for a $305 \times 127 \times 48$ UB continuous beam with continuous and rigid lateral restraints.

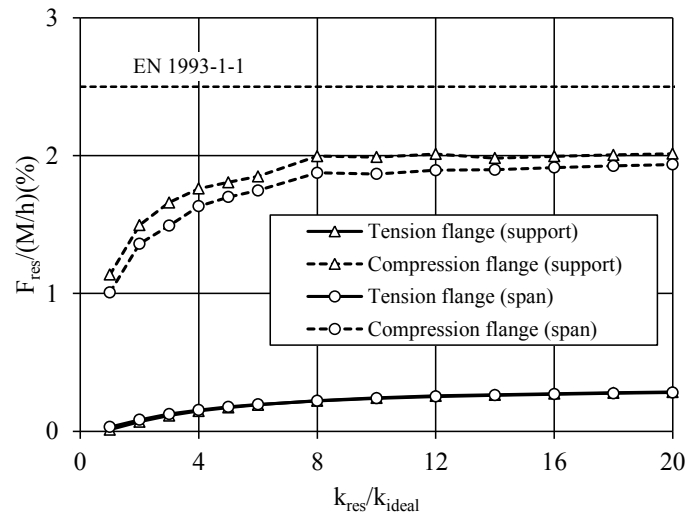
During the plastic redistribution phase, the compression flange restraints rapidly attract forces up to around 2% N_{Ed} . By θ_{csm} (the value of end rotation corresponding to M_{csm}), compression flange restraint forces do not generally exceed 2.5% N_{Ed} (with tension flange forces being considerably lower - typically 0.5% N_{Ed}), which is within the requirements of EN-1993-1-1. The relationships between restraint stiffness and restraint

forces at the full CSM rotations for both cross-sections are summarised in Fig. 5.12.

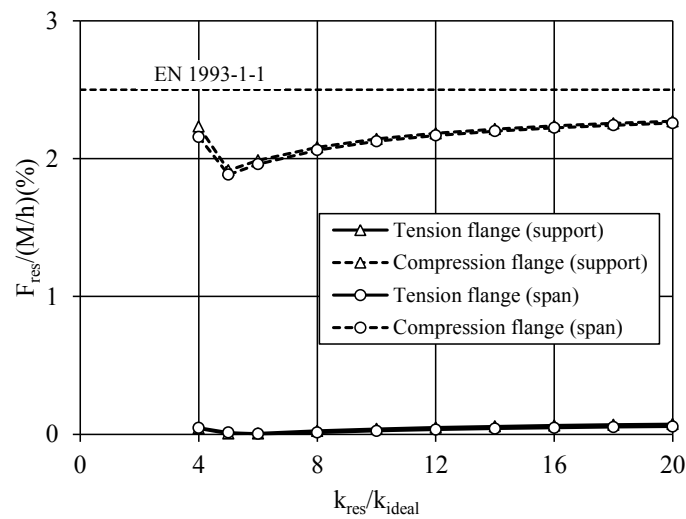
Comparing Figs. 5.8 with 5.12 it is apparent that at the ultimate load, restraint forces for restraints attached to the compression flange reduce with increments in restraint stiffness, but at the CSM collapse load restraint forces increase with restraint stiffness. This discrepancy can be explained with reference to Fig. 5.13a, which shows the relationship between major axis bending moment at the support and lateral displacement, as well as the relationship between compression flange restraint force and lateral displacement; these relationships are plotted for various multiples of restraint stiffness k_{ideal} . Assuming a restraint that is acting in uniaxial tension (or compression if it is modelled as a spring), then restraint forces vary linearly with lateral displacement at the restraint location and restraint stiffness. However, the lateral displacements of the beam vary non-linearly with restraint stiffness. This means that for the small lateral displacements encountered early in the loading history, restraint forces tend to increase with restraint stiffness, since the proportional reductions in lateral displacements are less than the proportional increases in restraint stiffness. However, as loading progresses, the effect of stiffer restraints on lateral displacements becomes more pronounced, with ultimate lateral displacements being reduced by a greater proportion than the corresponding increase in restraint stiffness.

Further examination of the moment-lateral displacement curves in Fig. 5.13a reveals a distinctive stiffening in the response between the values $0.8 \leq M/M_{pl} \leq 1.0$. This is due an initial tendency for the primary member to buckle in mode-I (due to it having a mode-I initial global imperfection), after which the elastic lateral restraint promotes buckling in mode-II. This tendency is illustrated in Fig 5.13a, which compares the response of an unrestrained beam with that of an elastically restrained beam; the unrestrained beam exhibits a load-deformation path with a continuous progression in lateral displacement, whilst the restrained beam clearly shows a stiffening of the response.

Although it would appear that for the two cross-sections studied the requirements of EN 1993-1-1 have been met, there is an appreciable amount of variation in restraint force between the cross-sections. Fig. 5.14 presents the evolution of restraint forces for both the cross-section types with a restraint stiffness of $20k_{ideal}$. It is apparent that the restraint

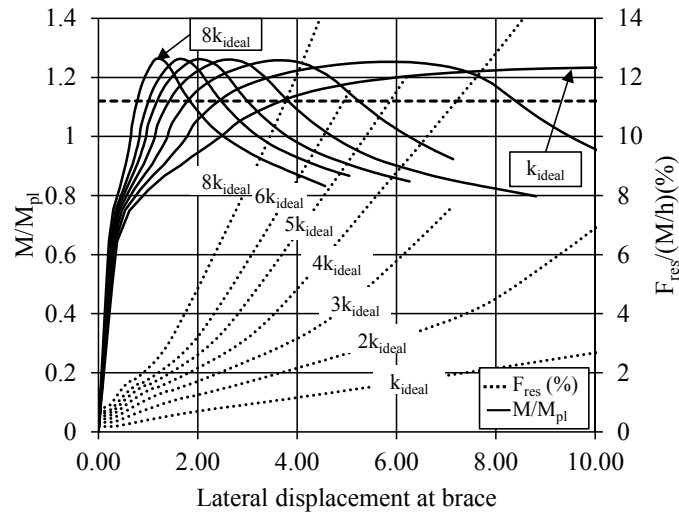


(a) 305×127×48 UB

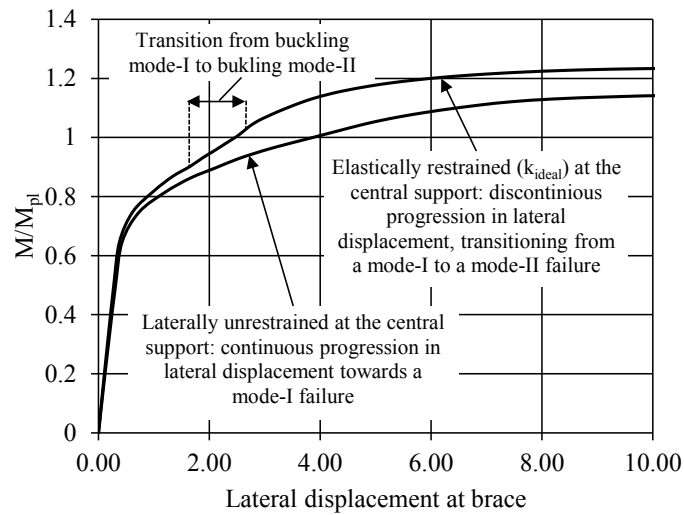


(b) 305×165×40 UB (Note: restraint stiffness values less than $4k_{ideal}$ did not permit M_{csm} to be attained)

Figure 5.12: Relationship between restraint stiffness and restraint force at the CSM predicted collapse load for continuous beams.



(a) The influence of restraint stiffness on restraint force.



(b) The effect of bracing on lateral displacements.

Figure 5.13: Lateral displacement and restraint force behaviour throughout the loading history of a $305 \times 127 \times 48$ UB continuous beam .

forces at θ_{csm} for the $305 \times 127 \times 48$ UB cross-section is approximately 0.5% lower than for the $305 \times 165 \times 40$ UB cross-section. At ultimate load, however, both sections converge at approximately 3% N_{Ed} . Unlike the specific requirements for bracing systems that assign a variable value of tolerable restraint force dependent upon imperfections and deflections (see Section 3.2.2), the bracing requirement for members containing rotated plastic hinges is a fixed value in EN 1993-1-1. Therefore, in the light of this and the observed variation in restraint forces between cross-section types at θ_{csm} , it is suggested that when using the CSM, bracing for members containing plastic hinges should be able to withstand forces of at least 3% N_{Ed} .

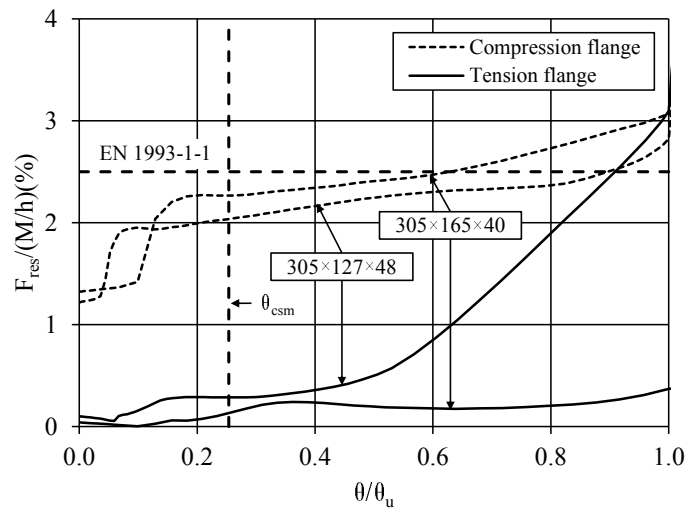


Figure 5.14: Variation of tension and compression flange restraint forces during plastic redistribution.

5.5 Lateral restraint spacing

Some basic analytical expressions for the critical moment of resistance for beams in the strain-hardening range were introduced in Chapter 4, which formed the basis of the analysis of the CSM at the member level for statically determinate elements. This analysis was applied to simply supported beams subjected to uniform moment. For statically indeterminate structures, the in-plane distribution of bending moment is more complicated and this is further complicated by plastic redistribution; as a result of this,

determining the distribution of minor axis rigidities required for a buckling analysis is complex. To overcome these difficulties a numerical model is developed in this section for use in parametric studies to investigate restraint spacings for the CSM when applied to statically indeterminate structures. This section will present an overview of the model, after which each of the components will be discussed in detail.

5.5.1 Overview of the numerical model

The primary aim of the numerical model is to generate critical buckling loads in the inelastic range for statically indeterminate structures. There are several approaches that may be taken to achieve this and they include the finite element method, the method of finite integrals, the finite difference method and the transfer matrix method (Trahair, 1983). For the purposes of this study, the finite element method will be employed, largely for its flexibility and widespread use in the literature; for its application to the problems considered in this chapter, the following general steps will be taken:

- (i) Firstly a static in-plane pre-buckling analysis needs to be performed to determine the distribution of bending and axial forces along the length of the structure for use in the incremental (geometric) component of the buckling analysis.
- (ii) Knowing these forces (and stresses), the points of transition from elastic, to yielded to strain-hardened material properties can be established and introduced into the flexural stiffness component of the buckling analysis (Trahair, 1993).

For statically indeterminate inelastic structures, these steps cannot be separated as stresses and yield distributions are interrelated by a system of non-linear equilibrium equations (Owen and Hinton, 1980). In practice, the procedure for arriving at an inelastic load prediction using the finite element method can be subdivided into the following stages (adapted from Nethercot (1973a) and Yoshida et al. (1977)) :

- (i) Perform an in-plane (pre-buckling) analysis to establish the distribution of internal forces due to a nominal distribution of concentrated and distributed applied loads.

- (ii) Using the information obtained from the pre-buckling analysis, perform an elastic buckling analysis to determine a critical load factor λ_{cr} corresponding to the applied loads.
- (iii) Evaluate the subsequent distribution of internal forces with a second in-plane analysis. If any internal stresses exceed the material yield stress σ_y , proceed to an inelastic analysis, otherwise accept the critical load from the elastic buckling analysis.
- (iv) Where stresses exceed the yield stress, initiate a materially non-linear in-plane analysis with a trial value of the inelastic load factor. This will be some value that results in the maximum moment being slightly in excess of the yield moment.
- (v) Extract the distribution of yielded and strain-hardened material from the materially non-linear analysis. Using this information, perform a linear eigenvalue analysis to obtain an inelastic critical load and corresponding inelastic critical moment.
- (vi) If the trial and inelastic critical values for the load factor are sufficiently close, accept the solution, otherwise introduce another trial load.

The relationships between these steps are summarised in Fig. 5.15.

5.5.2 In-plane analysis - common principles

In this model, there are two in-plane finite element programmes: the first is a linear elastic mixed-formulation Timoshenko beam model and the second is a materially non-linear mixed formulation layered Timoshenko beam model. Common to both models is the Timoshenko beam element - the assumptions of this element and the specific variant used are the subject of this section.

5.5.2.1 Element formulation

For analyses that exclude shear deformations, a normal to the neutral axis of the beam remains straight as the element deforms; its rotation is the slope of the neutral axis, implying that all subsequent deformations (i.e. axial strain) can be characterised by a

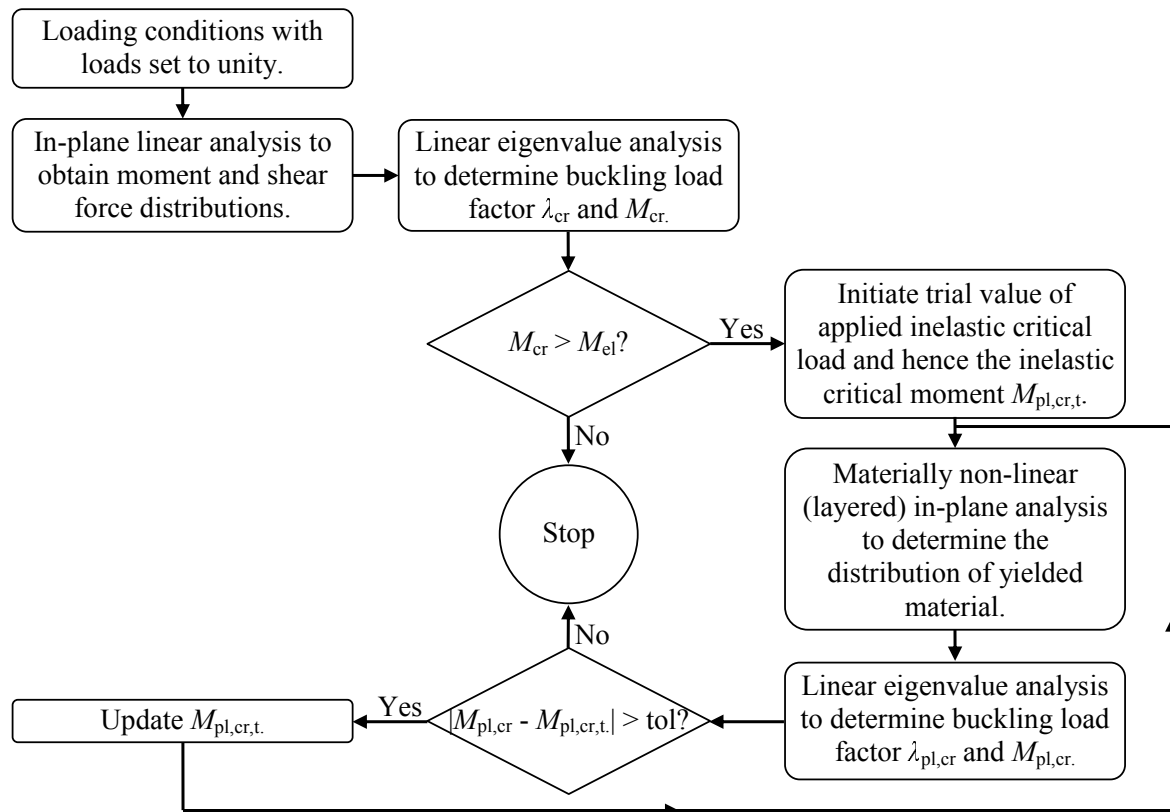


Figure 5.15: Overall computational regime for the inelastic buckling analysis of continuous beams.

single variable, w , the transverse displacement (see Fig.5.16) (Bathe, 1996).

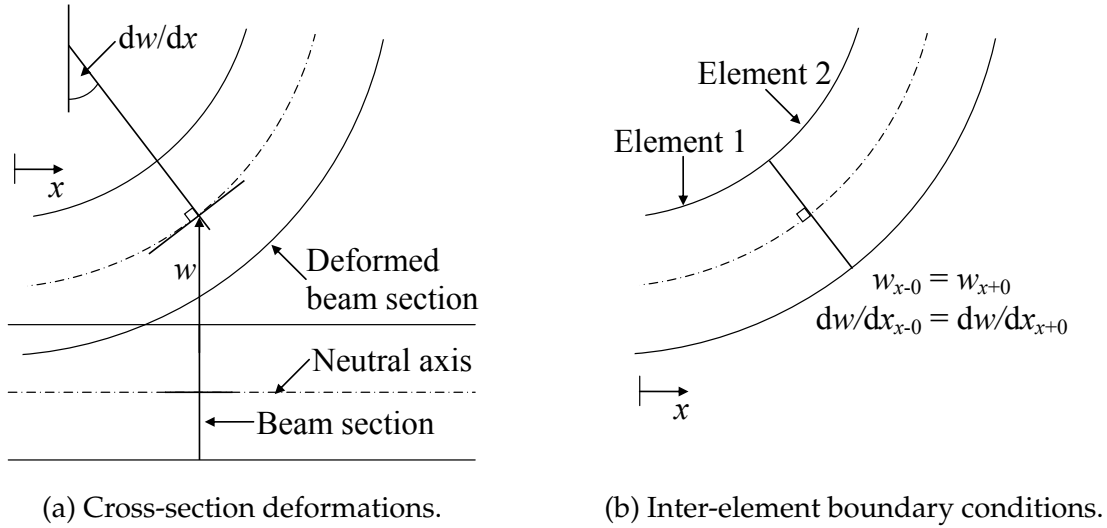


Figure 5.16: Beam element kinematics without shear deformations (adapted from Bathe (1996)).

For analyses that include shear deformations based on Timoshenko beam theory (see Fig. 5.17), plane sections still remain plane, but they no longer remain normal to the neutral axis. The total rotation is now the rotation of the tangent to the neutral axis and the additional shear deformation β , where:

$$\beta = \frac{dw}{dx} - \gamma \quad (5.5)$$

in which γ is a constant shear strain across the beam section. In reality, shear deformations vary through the cross-section, thus it is assumed that γ is an equivalent constant strain on a corresponding shear area A_s and:

$$\tau_s = \frac{V}{A_s}; \quad \gamma = \frac{\tau_s}{G}; \quad \kappa_s = \frac{A_s}{A} \quad (5.6)$$

in which V is the shear force in the section under consideration and $G = E/[2(1 + \nu)]$ is the shear modulus for an isotropic material. It is shown by Crandall et al. (1978) that for rectangular sections, $\kappa_s = 5/6$.

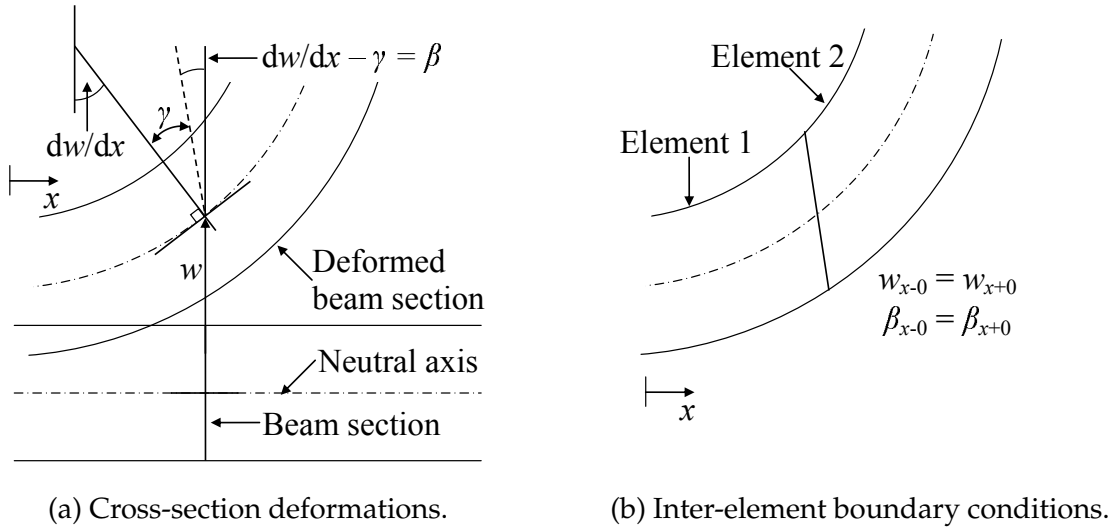


Figure 5.17: Beam element kinematics with shear deformations (adapted from Bathe (1996)).

It is shown in Owen and Hinton (1980) that the total potential Π for a Timoshenko beam element of length L_e is:

$$\Pi = \int_0^{L_e} \left(\frac{d\beta}{dx} \right)^2 dx + \alpha \int_0^{L_e} \left(\frac{dw}{dx} - \beta \right)^2 dx \quad \text{where} \quad \alpha = \frac{GA\kappa_s}{EI} \quad (5.7)$$

With reference to Fig. 5.18, which is a two-node beam element, h is the overall depth of the element, upon which the term EI depends. As $h \rightarrow 0$ then $EI \rightarrow 0$ too and as a result $\alpha \rightarrow \infty$, meaning that as the element becomes thin, shear deformations will tend to zero, resulting in the phenomenon known as element shear locking (Bathe, 1996). The implication of this is a solution that is far too stiff.

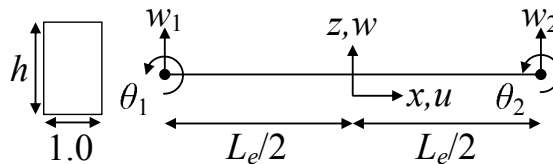


Figure 5.18: A two-node Timoshenko beam element.

The standard approach to formulating finite element equations is referred to as the pure-displacement method, where displacements are the primary variables (Eq. (5.8)):

$$\Pi(\mathbf{u}) = \frac{1}{2} \int_V \epsilon^T \mathbf{C} \epsilon dV - \int_V \mathbf{u}^T \mathbf{f} dV \quad \text{where} \quad \epsilon = \partial_\epsilon \mathbf{u} \quad (5.8)$$

in which $\mathbf{u}$ is a vector of element displacements, ϵ is a vector of element strains derived from element displacements, $\mathbf{C}$ is the constitutive matrix and $\mathbf{f}$ is the applied loading vector. Knowledge of the displacements then allows strains and stresses to be formulated as functions of the displacements.

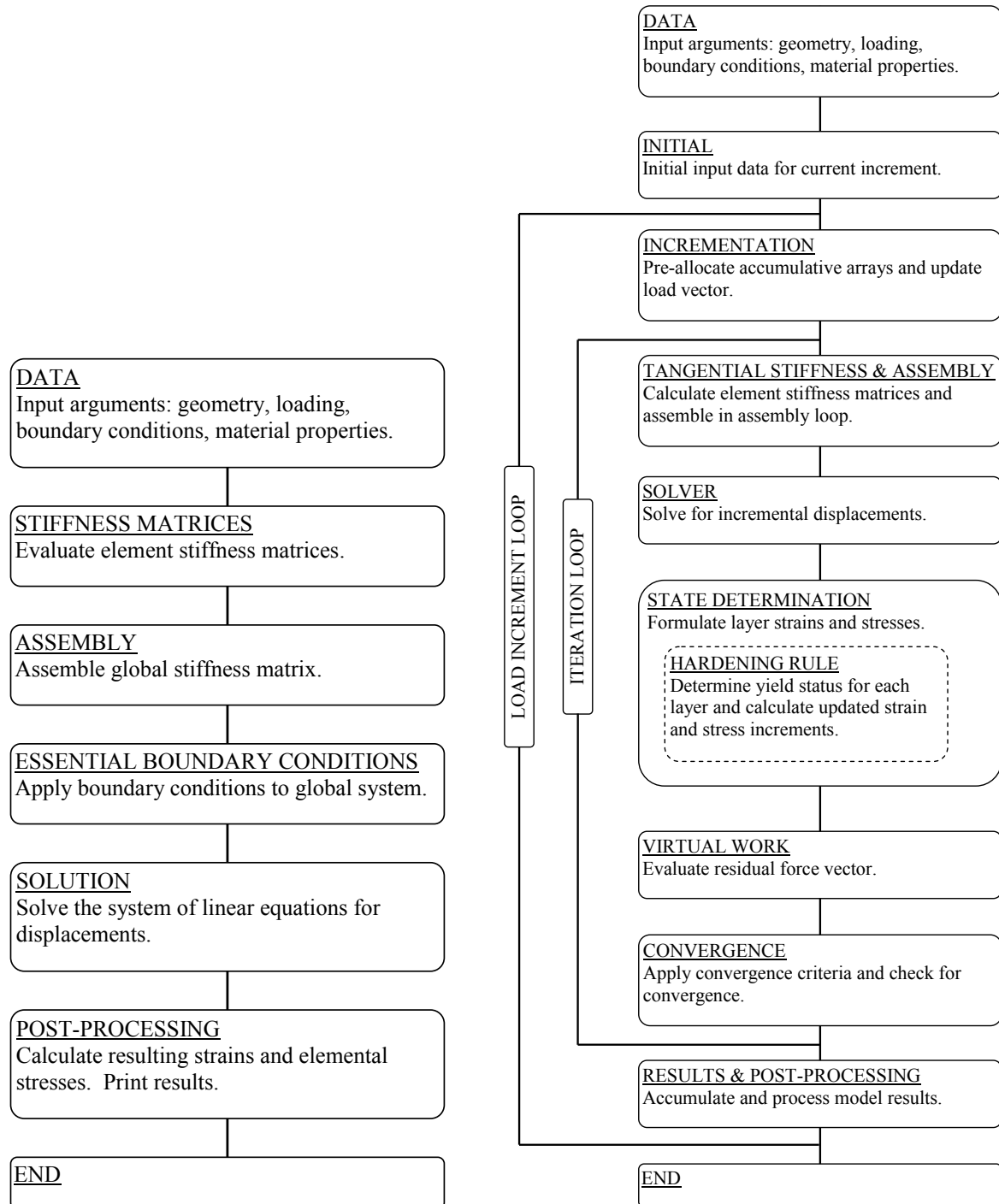
The two-node Timoshenko beam element, when used with a pure-displacement formulation, will result in the shear locking behaviour described above (Bathe, 1996). If numerical integration is to be used, then 1-point Gauss Legendre integration will prevent shear locking (Owen and Hinton, 1980). Alternatively, by invoking a mixed formulation, shear locking can be avoided if it is assumed that both displacements and strains are unknown variables in the total potential and it is assumed that there is a constant element shear strain γ (Bathe, 1996). Adopting a linear displacement representation (Hughes et al., 1977), the resulting element flexural stiffness matrix $\mathbf{k}$ is given by Eq. (5.9):

$$\mathbf{k} = \begin{bmatrix} \frac{Gh}{L_e} & \frac{Gh}{2} & -\frac{Gh}{L_e} & \frac{Gh}{2} \\ \frac{Gh}{2} & \left(\frac{GhL_e}{4} + \frac{Eh^3}{12L_e} \right) & -\frac{Gh}{2} & \left(-\frac{GhL_e}{4} - \frac{Eh^3}{12L_e} \right) \\ -\frac{Gh}{L_e} & -\frac{Gh}{2} & \frac{Gh}{L_e} & -\frac{Gh}{2} \\ \frac{Gh}{2} & \left(\frac{GhL_e}{4} - \frac{Eh^3}{12L_e} \right) & -\frac{Gh}{2} & \left(\frac{GhL_e}{4} + \frac{Eh^3}{12L_e} \right) \end{bmatrix} \quad (5.9)$$

Using the procedure outlined in Fig. 5.19a, this matrix can be assembled into a global stiffness matrix and solved for the global displacements $\mathbf{U}$ for any given set of boundary conditions, global geometry, material and cross-section properties.

5.5.3 Inelastic in-plane analysis

In order to determine the spread of plasticity along the beam and through its cross-section, material non-linearity needs to be introduced. For a beam, the general expression for element stiffness $\mathbf{k}^e$ is given by Eq. (5.10):



(a) Programme structure for the elastic in-plane Timoshenko beam analysis.

(b) Programme structure for the inelastic in-plane Timoshenko beam analysis.

Figure 5.19: Programme structures for in-plane analyses.

$$\mathbf{k}^e = \int_0^{L_e} \mathbf{B}^T \mathbf{C} \mathbf{B} dx \quad (5.10)$$

in which $\mathbf{C}$ is the constitutive matrix and $\mathbf{B}$ is the strain-displacement matrix. With reference to Fig. 5.20, in the elastic range $\mathbf{C} = EI$ (where EI is the flexural stiffness) and during plastic deformations, $\mathbf{C} = EI_t$ (where EI_t is the tangent flexural stiffness).

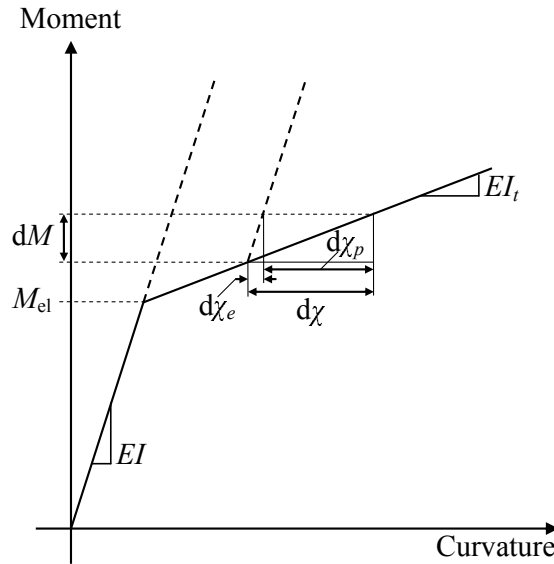


Figure 5.20: Schematic moment-curvature behaviour with strain-hardening.

It is assumed that in the post-yield range that increments in curvature can be decomposed into elastic and plastic components. For an increment dM in bending moment in the post-yield range, the resulting change in curvature $d\chi$ is represented as:

$$d\chi = d\chi_e + d\chi_p \quad (5.11)$$

in which χ_e and χ_p are the respective elastic and plastic curvature components. In the strain-hardening range, plastic deformations are dependent upon the current degree of plastic deformation, the extent to which is characterised by the strain-hardening parameter H :

$$H = \frac{dM}{d\chi_p} = \frac{dM}{d\chi - d\chi_e} = \frac{EI_t}{1 - EI_t/EI} \quad (5.12)$$

The curvature increment (Eq. (5.11)) may then be re-written as (Owen and Hinton, 1980):

$$d\chi = \frac{dM}{EI} + \frac{dM}{H} = \frac{dM(H + EI)}{EIH} \quad (5.13)$$

which results in an incremental moment-curvature relationship:

$$dM = \frac{EIH}{EI + H} d\chi = EI \left(1 - \frac{EI}{EI + H} \right) d\chi \quad (5.14)$$

Thus in the post-yield range $C = EI \left(1 - \frac{EI}{EI + H} \right)$. The shear force-shear strain relationship is assumed to remain elastic (Owen and Hinton, 1980).

In this analysis a layered cross-section will be used, where the depth of the cross-section is subdivided into k layers (see Fig. 5.21); this is an important consideration for continuous beams as the degree of moment redistribution is closely linked to the degree of plastic penetration at different locations along the member.

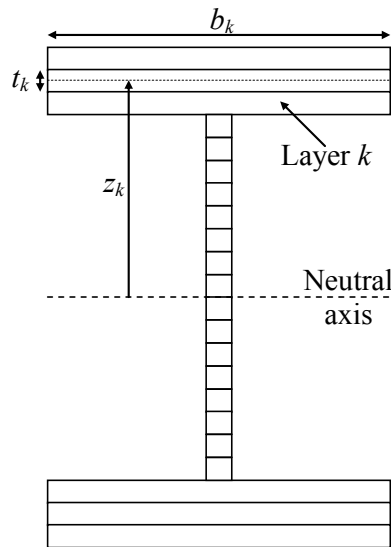


Figure 5.21: Cross-section discretisation for the layered model.

In the layered formulation, individual layer stresses (rather than cross-section stress resultants) are used, implying that bending moments M and shear forces Q are determined as:

$$M = \sum_k b_k \sigma_{x,k} z_k t_k; \quad Q = \sum_k b_k \tau_{x,k} t_k \quad (5.15)$$

in which (with reference to Fig. 5.21) b is the layer breadth, t is the layer thickness, z distance of the mid-surface of layer k from the neutral axis, σ is the uniaxial stress, and τ is the shear stress, and x denotes the longitudinal direction. Along similar lines, flexural rigidity EI is evaluated as:

$$EI = \sum_k E_k b_k z_k^2 t_k \quad (5.16)$$

If the stress at layer k exceeds the uniaxial yield stress σ_y , then the Young's modulus E_k at that layer is replaced with $E_k(1 - E_k/(E_k + H))$.

5.5.3.1 Computational procedure

The overall computational scheme for the inelastic in-plane analysis is summarised in Fig. 5.19b. The specific steps that need to be followed in the analysis are:

- (i) Update stresses.
- (ii) Update plastic strains.
- (iii) Update the yield stress limit.
- (iv) Identify the loading/unloading paths.
- (v) Satisfy equilibrium for virtual displacements.

These are discussed in the following sub-sections.

5.5.3.1.1 Update stresses With reference to Fig. 5.22, at load step r the stress in layer k of element i with a strain increment $\Delta\epsilon_{i,k}^r$ is $\sigma_{i,k}^r = \sigma_{i,k}^{(r-1)} + E_{i,k}\Delta\epsilon_{i,k}^r$. Up until the yield stress σ_y this approach requires no further modification. However, when making the transition from point A (in the elastic range) to Point B (in the strain-hardening range), a predictor-correction approach is necessary (Reddy, 2004). This has three steps:

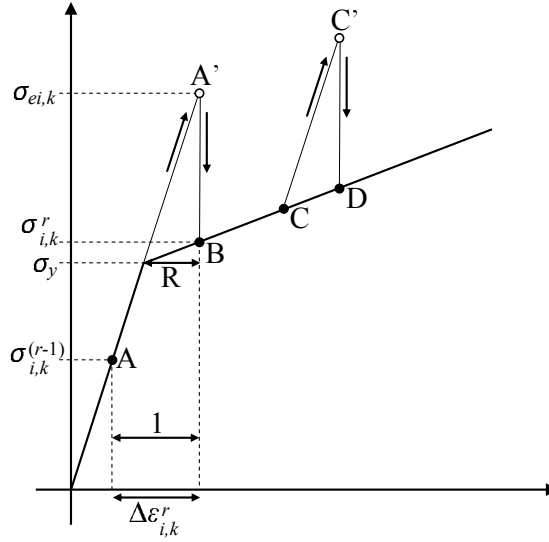


Figure 5.22: Predictor-corrector algorithm for inelastic behaviour.

- (i) Compute the elastic stress predictor $\sigma_{ei,k}$ using Eq. (5.17) to get to point A' in Fig. 5.22:

$$\sigma_{ei,k} = \sigma_{i,k}^{(r-1)} + E_{i,k} \Delta \epsilon_{i,k}^r \quad (5.17)$$

- (ii) Formulate the correction factor R for the load increment using Eq. (5.18):

$$R = \frac{\sigma_{ei,k} - \sigma_y}{\sigma_{i,k}^{(r-1)} - \sigma_{ei,k}} \quad (5.18)$$

which can be derived from similar triangles in Fig. 5.22. R determines the proportion of the strain increment that is plastic.

- (iii) Calculate the corrected stress at point B using Eq. (5.19):

$$\sigma_{i,k}^r = \sigma_{i,k}^{(r-1)} + [(1 - R)E_{i,k} + RE_{ti,k}] \Delta \epsilon_{i,k}^r \quad (5.19)$$

If the material has previously yielded (i.e. the transition from point C to point D), the same steps are followed, but with $R = 1$ in Eq. (5.19).

5.5.3.1.2 Update plastic strains In the inelastic range, permanent plastic deformations are experienced and so the plastic strains must be computed for each load increment. Re-writing Eq. (5.19):

$$\sigma_{i,k}^r = \sigma_y + E_{ti,k} R \Delta \epsilon_{i,k}^r \equiv \sigma_y + \Delta \sigma_{i,k}^r \quad (5.20)$$

in which $\Delta \sigma_{i,k}^r$ and $R \Delta \epsilon_{i,k}^r$ are the stress and strain components involved in plastic flow. The plastic strain increment is then:

$$\Delta \epsilon_{pi,k}^r = R \Delta \epsilon_{i,k}^r - \frac{\sigma_{i,k}^r}{E_{i,k}} \quad (5.21)$$

5.5.3.1.3 Update the yield stress limit During plastic straining, the yield limit of the material changes as a result of plastic flow. The updated yield stress $\sigma_{yi,k}^r$ is calculated using Eq. (5.22):

$$\sigma_{yi,k}^r = \sigma_y + H \Delta \epsilon_{pi,k}^r \quad (5.22)$$

5.5.3.1.4 Identify the loading/unloading paths The correct application of Eqs. (5.19) and (5.21) relies upon identifying the correct deformation modes, and these are:

- (i) Elastic loading
- (ii) Elastic-plastic loading
- (iii) Plastic loading
- (iv) Elastic unloading

5.5.3.1.5 Satisfy equilibrium for virtual displacements In the linear elastic range, the principle of virtual displacements is satisfied; however in the inelastic range, stress adjustments produce out of balance forces that may not satisfy equilibrium. At the element interface level, forces between neighbouring yielded and elastic elements will initially be out of equilibrium; overall equilibrium must be satisfied by redistributing

the force imbalance to all other elements in the system. This is achieved using a corrective displacement increment within the loading step. Equilibrium will then be satisfied approximately once a given tolerance has been achieved within the desired number of iterations.

5.5.4 Linear buckling analysis

The finite element model used in the linear buckling analysis subroutine is based upon the stiffness and stability matrices derived by Barsoum and Ghallager (1970). These matrices were developed for an arbitrary open section subjected to any combination of distributed, concentrated, axial, moment and torsional loading.

In a similar manner to regular displacement-based finite element formulations, expressions for strain energy and the total potentials for the applied loads are formulated. Within the strain energy expression, buckling deformations can be separated from pre-buckling deformations, with the latter being excluded from the buckling analysis and instead dealt with in the pre-buckling analysis stage outlined in Section 5.5.2. Interpolation functions may then be substituted for the displacement terms in the out of plane strain energy expression - in this model, cubic interpolation functions are used. The resulting functions can be arranged into two matrices: the first contains terms for axial, flexural and torsional stiffness (referred to as the element flexural stiffness matrix $\mathbf{k}_f^e$); the second contains terms that are purely dependent upon geometric terms (i.e. no material properties) and the element forces determined in the pre-buckling analysis (this is referred to as the element incremental or geometric stiffness matrix $\mathbf{k}_g^e$).

Bringing together the potential energy of the applied loads $\mathbf{f}$ and the strain energy terms results in an expression for total potential energy Π . For stable equilibrium, the principle of stationary potential energy is invoked:

$$\delta\Pi = 0 : \quad \mathbf{f} = \mathbf{k}_f^e \mathbf{u} + \mathbf{k}_g^e \mathbf{u} \quad (5.23)$$

For elastic instability problems, the intensity of the element forces is the unknown. Hence, the geometric stiffness matrix needs to be numerically evaluated for an arbitrary

load intensity λ for any distribution of element forces (Gallagher, 1975). The stability problem is then written as:

$$\delta\Pi = 0 : \quad \mathbf{k}_f^e \mathbf{u} + \lambda \mathbf{k}_g^e \mathbf{u} - \mathbf{f} = 0 \quad (5.24)$$

Considering the second variation of Π :

$$\begin{aligned} \delta^2\Pi > 0 & \Rightarrow \text{stable equilibrium} \\ \delta^2\Pi = 0 & \Rightarrow \text{bifurcation of equilibrium} \end{aligned}$$

Hence:

$$[\mathbf{k}_f^e + \lambda \mathbf{k}_g^e] \mathbf{u} = 0 \quad (5.25)$$

The non-trivial solution requires:

$$\det(\mathbf{k}_f^e + \lambda \mathbf{k}_g^e) = 0 \quad (5.26)$$

which is a linear eigenvalue problem with the critical load being the lowest eigenvalue $\lambda = \lambda_{cr}$. The specific expressions for the terms of $\mathbf{k}_f^e$ and $\mathbf{k}_g^e$ are reported in Barsoum and Ghallager (1970).

In the usual manner, element stiffness matrices are assembled so that the potential energy for the system is the sum of the potential energies of the component elements. The eigenvalue analysis is then performed on the system, rather than the elemental equations. A description of the implementation of the buckling analysis is provided in Fig. 5.23.

This model has been systematically validated against known analytical results by Barsoum and Ghallager (1970). In all cases, convergence to the analytical solution was achieved with negligible error using no more than six elements. An example for a beam subject to uniform moment is provided in Fig 5.24, in which the error is defined as:

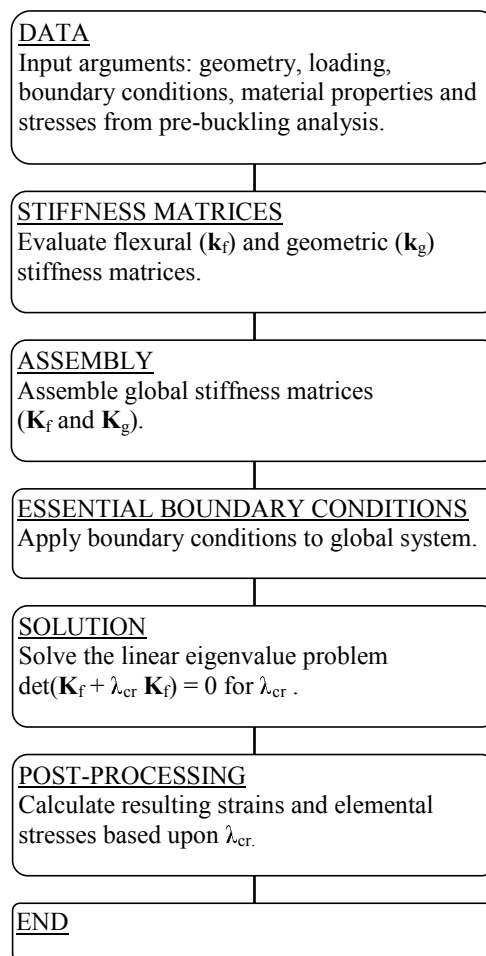


Figure 5.23: Programme structure for the elastic buckling analysis.

$$\text{Error}(\%) = M_{\text{cr,numerical}} - M_{\text{cr,analytical}} \quad (5.27)$$

in which $M_{\text{cr,numerical}}$ is the critical moment as determined from the numerical buckling analysis and $M_{\text{cr,analytical}}$ is the elastic critical moment of a simply supported beam subjected to a uniform moment (see Eq. (4.1)).

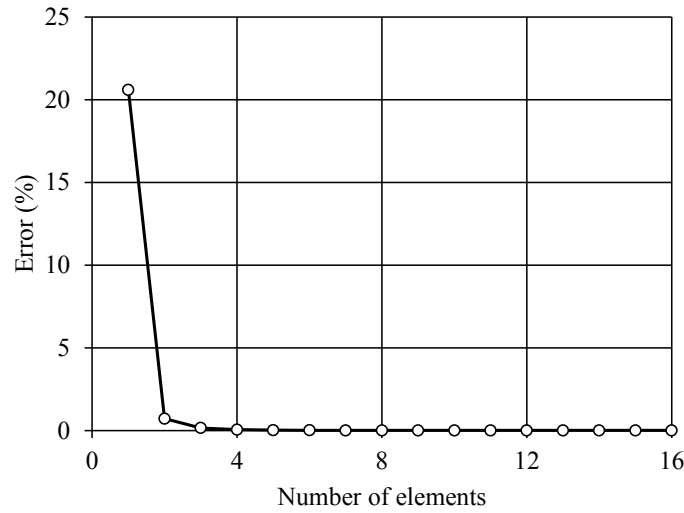


Figure 5.24: Comparison of finite element results with analytical result for the elastic buckling moment of a beam subject to uniform moment.

5.5.4.1 The inelastic buckling load

The final step in the analysis is to iterate towards the inelastic buckling load. Using a trial load factor λ_{trial} , a corresponding maximum trial moment $M_{\text{trial,pl}}$ is determined from an inelastic analysis (Section 5.5.3). Using the material properties from this analysis, a critical buckling load factor $\lambda_{\text{pl,cr}}$ and corresponding moment $M_{\text{pl,cr}}$ are evaluated. The trial load λ_{trial} is then adjusted until the difference between $M_{\text{trial,pl}}$ and $M_{\text{pl,cr}}$ satisfies the specified tolerance (1%) for a given number of iterations. This procedure is carried out using the bisection method. Defining tolerance as $|M_{\text{trial,pl}} - M_{\text{cr,pl}}|$, then the method proceeds by evaluating $\alpha_{\text{tol}}(\lambda_{\text{trial}}^a)$, $\alpha_{\text{tol}}(\lambda_{\text{trial}}^b)$ and $\alpha_{\text{tol}}(\lambda_{\text{trial}}^c)$ where λ_{trial}^a and λ_{trial}^b are chosen such that the corresponding α_{tol} have opposite signs and $\lambda_{\text{trial}}^c = (\lambda_{\text{trial}}^a + \lambda_{\text{trial}}^b)/2$. If λ_{trial}^a and λ_{trial}^b result in different signs in the first iteration, λ_{trial}^c is

selected instead of λ_{trial}^b as the new trial load factor in the next iteration. This process is allowed to continue for a specified number of iterations until the error is less than or equal to the specified tolerance. This process has been implemented as part of the overall computer programme.

5.5.5 Validation and convergence

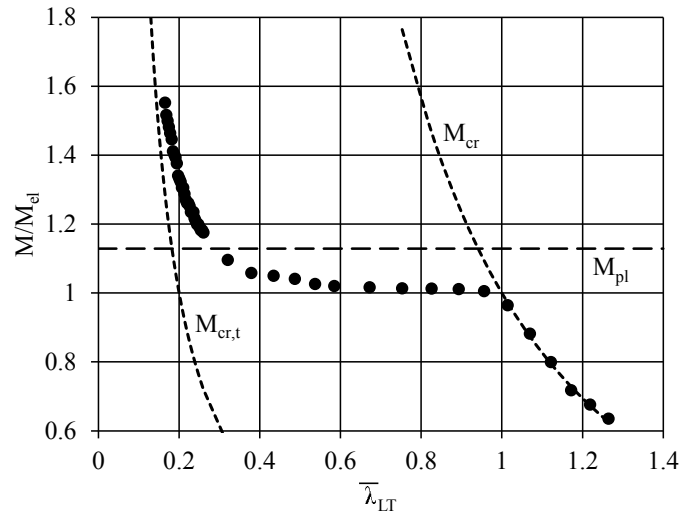
To validate the numerical model, two approaches were taken: firstly a comparison of the numerical model results against two standard analytical cases and secondly, a comparison to laboratory test data.

For the first approach, two analytical expressions for the critical buckling moment are formulated. The first is the elastic critical moment M_{cr} with all material properties (E and G) assumed to be in the elastic range. The second is the inelastic critical moment $M_{cr,t}$ which uses effective values of material properties (E_t and G_t) according to the tangent modulus theory. All data is plotted against the non-dimensional slenderness parameter $\bar{\lambda}_{LT}$ with reference to the elastic section modulus W_{el} :

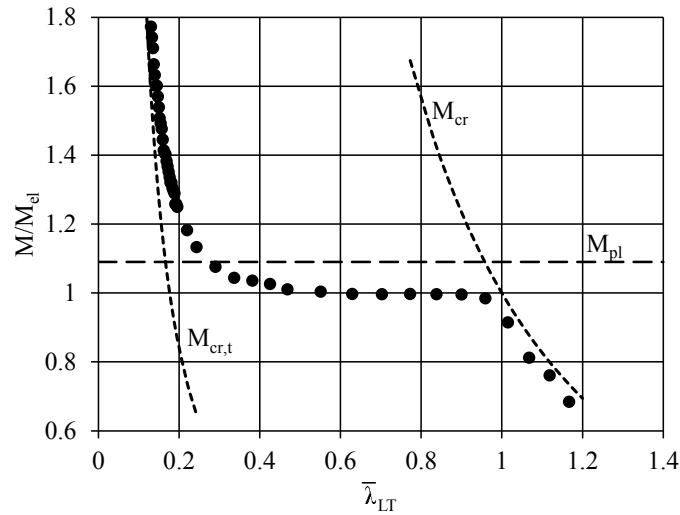
$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{el} f_y}{M_{cr}}} \quad (5.28)$$

Fig. 5.25 plots these curves for the two cross-sections used in this study alongside the data generated by the numerical model. The configuration is a two-span continuous beam with concentrated loads applied at the two mid-spans in equal proportions (zero interaction). The results show that the numerical model satisfies two theoretical expectations: firstly, where $\bar{\lambda}_{LT} = M/M_{el} = 1$ the analytical and numerical expressions coincide; secondly, the numerical model results approach $M_{cr,t}$ as full plasticity is able to develop through the cross-section at low values of $\bar{\lambda}_{LT}$.

For the second approach, moment rotation data obtained from the laboratory tests are compared with data generated by the numerical model using the measured material and geometric properties of the test specimens. For reasons discussed in Section 5.3, the tests did not attain their ultimate loads, so instead, comparisons between the numerical and test data are made at the test rotations corresponding to the maximum test



(a) Comparison of numerical model results with analytical buckling curves for a $305 \times 127 \times 48$ UB cross-section.



(b) Comparison of numerical model results with analytical buckling curves for a $305 \times 165 \times 40$ UB cross-section.

Figure 5.25: Validation of the numerical model against analytical results.

moments. These are summarised in Table 5.6 and show acceptable agreement between the tests and numerical model.

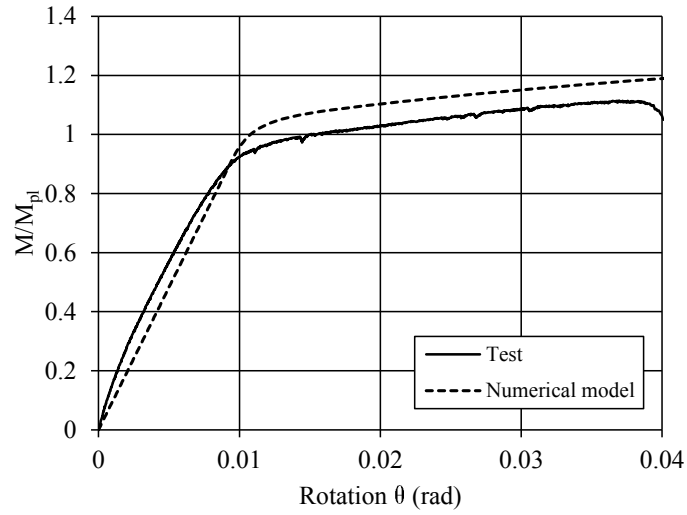
Table 5.6: Comparison of the laboratory test results on continuous beams with the numerical model.

Test designation	Test $M_{u,span}$ / FE M_u at test θ_u
$305 \times 127 \times 48$, $\bar{\lambda}_{LT} = 0.4$, Rigid	0.95
$305 \times 165 \times 40$, $\bar{\lambda}_{LT} = 0.4$, Rigid	0.96

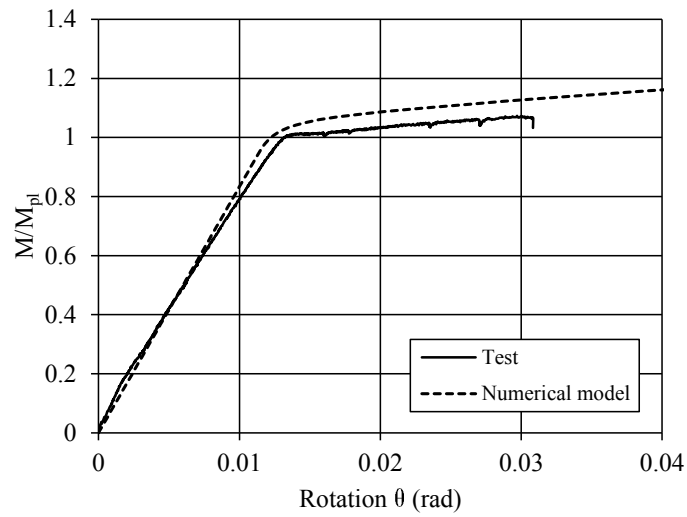
Figure 5.26 plots the comparative moment-rotation responses of the tests and the numerical model from which the key observations are:

- (i) The initial slope of the numerical model is perfectly straight, whilst the experimental data shows some rounding throughout - this is likely to be caused by elastic deformations of the load transfer apparatus, as well as friction between the specimen and the lateral restraints.
- (ii) For the numerical model, yielding commences at the yield moment, which is an intrinsic assumption of the computer model. The marginally earlier yielding shown by the test data is likely to be due to residual stresses.
- (iii) For the $305 \times 165 \times 40$ UB specimen, there is evidence of a small plateau in the test data, which is not apparent in the numerical data due to the assumption of a bi-linear material model (the material model in the numerical analysis employs only the measured yield stress from the tests).

The overall buckling model is composed of three distinct numerical models, each with separate convergence demands. Through a series of trial studies, overall convergence with significantly less than 1% error between $M_{trial,pl}$ and $M_{cr,pl}$ was achieved using 60 cross-section layers and 200 elements. For a stable solution in the materially non-linear model, it was found that 100 load increments were sufficient.



(a) 305 × 127 × 48 UB



(b) 305 × 165 × 40 UB

Figure 5.26: Experimental and numerical moment versus end rotation curves for continuous beams.

5.5.6 Parametric studies

A series of parametric studies were performed, focusing on variations in $\bar{\lambda}_{LT}$. The aim of these studies was to arrive at a limiting value of $\bar{\lambda}_{LT}$ for the CSM that would satisfy a variety of basic structural and loading configurations. For these studies, two cross-sections were used ($305 \times 127 \times 48$ UB and $305 \times 165 \times 40$ UB) and three basic loading configurations were chosen: (i) concentrated loads at the mid-spans; (ii) concentrated loads at the $1/3$ spans; and (iii) concentrated loads at the $2/3$ spans - these are illustrated in Fig. 5.27.

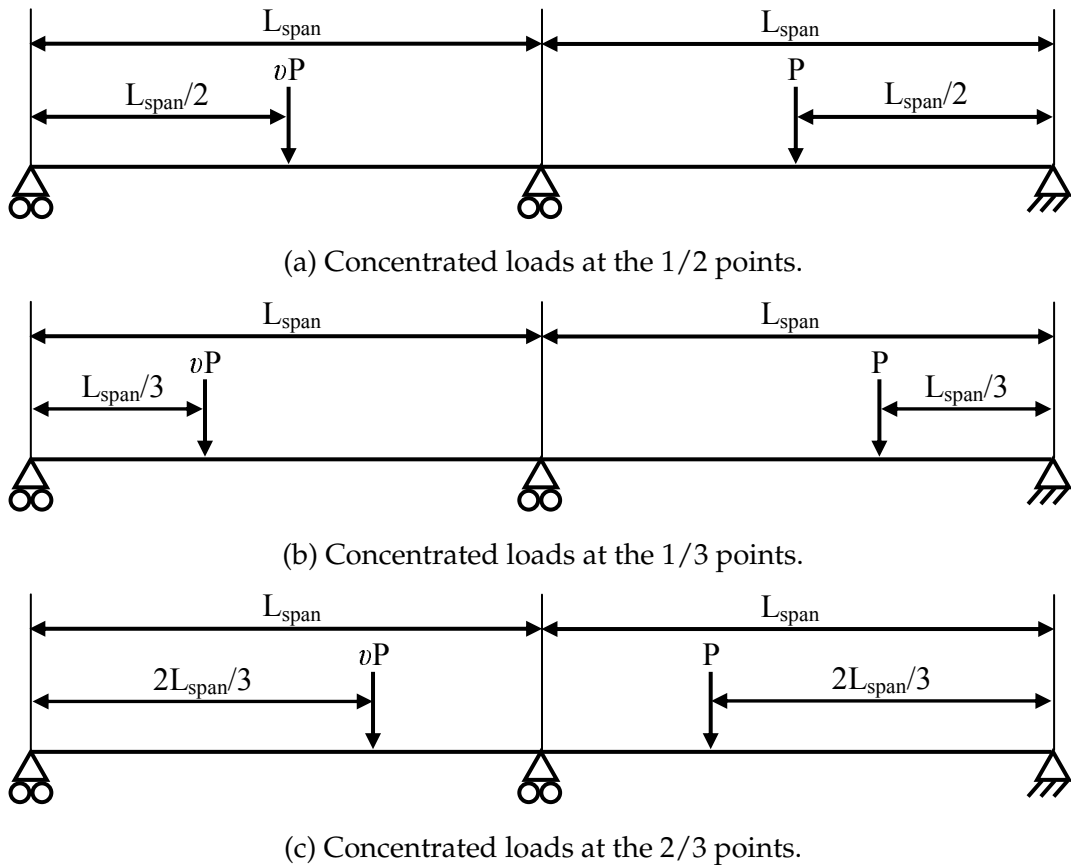


Figure 5.27: Loading configurations for the parametric studies.

The parametric studies are organised into two groups. The first focuses on interaction relationships and the second assumes zero interactions, instead focusing upon a wide range of $\bar{\lambda}_{LT}$ values for the three structural configurations under consideration. In all

cases, a bi-linear material model is assumed with $E/E_{sh} = 100$.

5.5.6.1 Interaction relationships

For this component of the study, interaction relationships are constructed by holding the load in one span at a constant value of P and allowing the load in the other span to take a different value vP where $0 \leq v \leq 1$. Three variations in lateral torsional buckling slenderness $\bar{\lambda}_{LT}$ are considered for each cross-section and loading configuration - $\bar{\lambda}_{LT} = 0.25, 0.20, 0.18$.

The parametric study was performed using the computer model discussed in Section 5.5. The inelastic critical moments at the loading points non-dimensionalised by the corresponding CSM moment are shown in the interaction diagrams of Figs. 5.28 and 5.29. For the low slenderness values used, in most cases sufficient strain-hardening takes place to attain critical buckling moments at or above the in-plane CSM capacities. For spans whose loads are placed at the 2/3 loading point, the full CSM moment capacity is generally not attained at the loading points, but sufficient yielding still occurs for a full CSM mechanism to develop.

At high and low ratios of M_{LP1}/M_{LP2} , extensive strain-hardening can be observed, and this develops due to extensive pre-buckling redistribution of bending moments (Yoshida et al., 1977). This redistribution is favourable with respect to lateral torsional buckling as it tends to occur at the interior support where reductions in rigidities have minimal effect upon member stability. This effect can be observed with greater clarity in Figs. 5.30a and 5.30b which plot the CSM collapse load interaction relationships for the three load configurations. For loads positioned at the 1/3 and 1/2 spans there is little deviation from the zero interaction load, whilst for loading at the 2/3 points, there is considerable deviation. This difference is due to the fact that yielding is concentrated in the support in the latter case, with the majority of the beam remaining elastic and able to provide restraint to the more heavily loaded span. Where loading is applied away ($> 0.5L_{span}$) from the support, two separate regions of yielding develop, reducing the available material for restraint and the capacity for interaction.

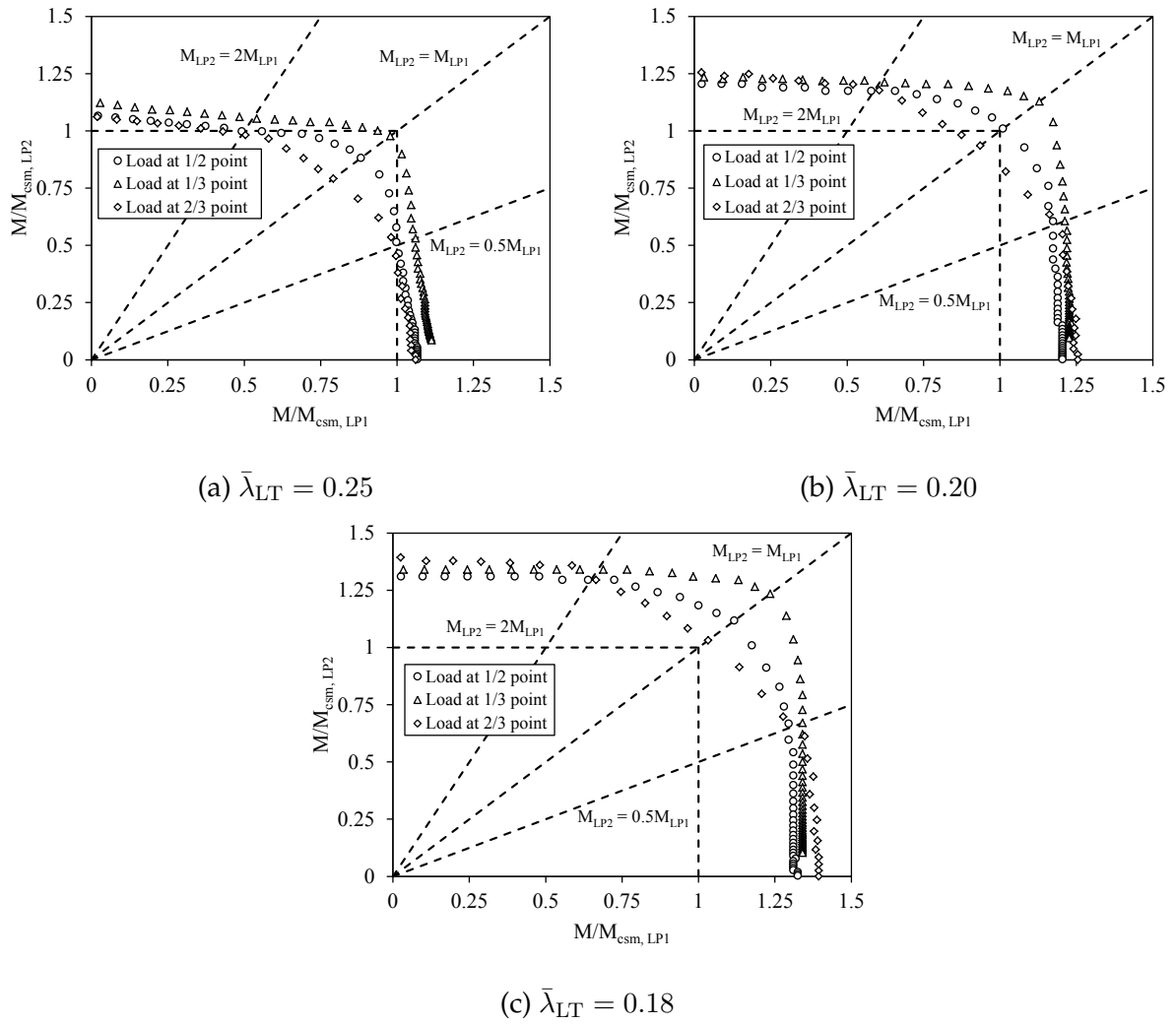


Figure 5.28: Interaction buckling relationships for a two span $305 \times 127 \times 48$ UB continuous beam.

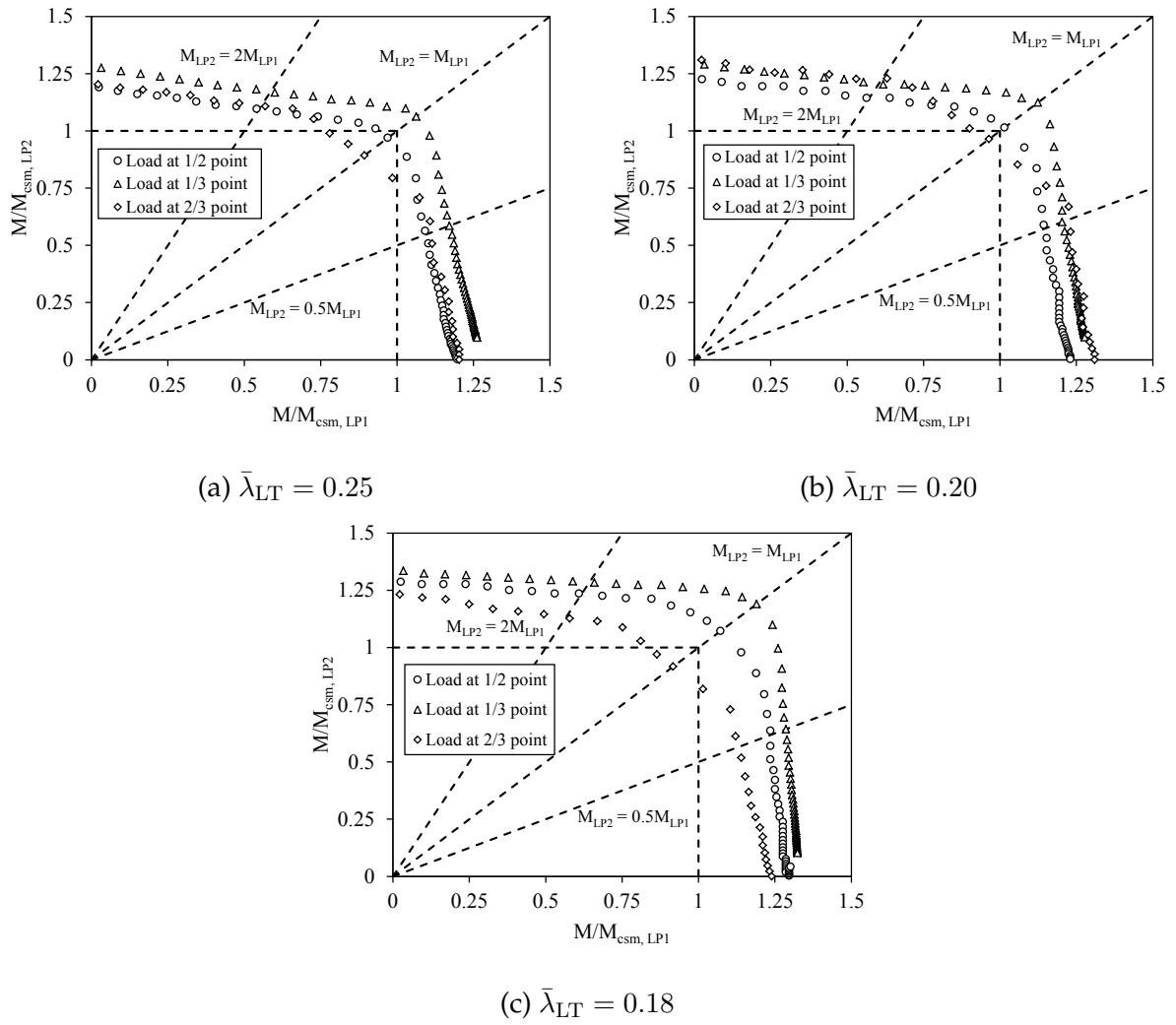


Figure 5.29: Interaction buckling relationships for a two span $305 \times 165 \times 40$ UB continuous beam.

Overall, it may be concluded that for the load cases and cross-section geometries considered, a limiting slenderness of $\bar{\lambda}_{LT} \leq 0.20$ is sufficient to attain CSM compatible collapse loads.

5.5.6.2 Restraint spacing

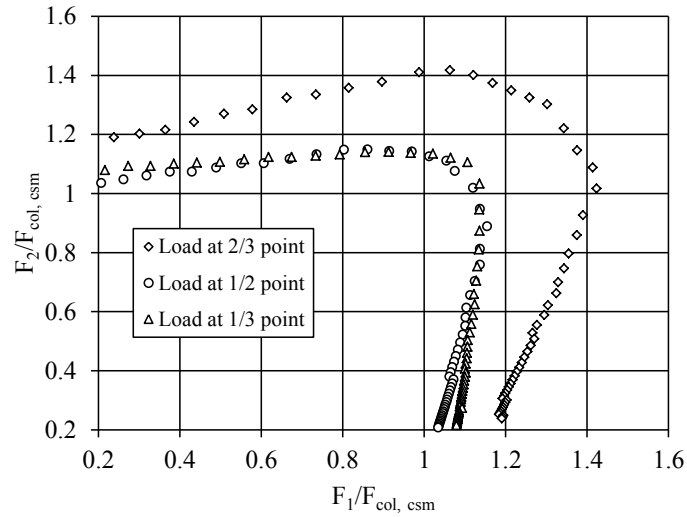
The variations in non-dimensional critical moment with the lateral torsional buckling slenderness parameter $\bar{\lambda}_{LT}$ are plotted for each section type and the corresponding loading configurations in Figs. 5.31a and 5.31b. In the transition from elastic to plastic behaviour, there are some fairly pronounced differences in the critical moments for the section types and their various configurations. However, at low slenderness values, these disparities are greatly reduced as extensive strain-hardening is able to occur before the onset of inelastic buckling. In all cases, CSM levels of bending resistance are attained for both section types and all configurations at a slenderness $\bar{\lambda}_{LT} = 0.18$, which is in line with the analytical and numerical results presented in Chapter 4 for statically determinate beams.

Fig. 5.32 plots the lateral torsional buckling slenderness against the critical load, normalised by the CSM collapse load. This clearly shows a more favourable spacing, with all cross-sections attaining $F_{col,csm}$ at $\bar{\lambda}_{LT} = 0.2$; for loading in close proximity to the support at the 2/3 span, $F_{col,csm}$ is achieved at $\bar{\lambda}_{LT} = 0.3$.

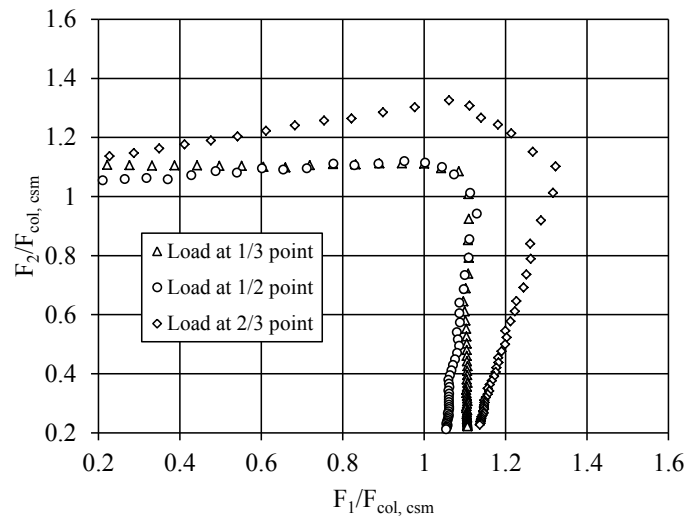
Table 5.7 compares the critical lengths at which the CSM collapse load is attainable L_{csm} with the EN 1993-1-1 stable length L_{stable} (Eq. (5.1)) and shows that in cases where loading is at 1/3 of the span, L_{stable} is approximately 25% too long for the CSM; where loading is concentrated about the central support, there is approximate agreement.

5.5.6.3 Influence of imperfections

As with simply supported beams, in the low slenderness range, imperfections do not significantly alter the failure load. To demonstrate this, a materially and geometrically non-linear beam element model was constructed to model a continuous beam with a lateral torsional buckling slenderness of $\bar{\lambda}_{LT} = 0.2$, with concentrated loads at the mid-

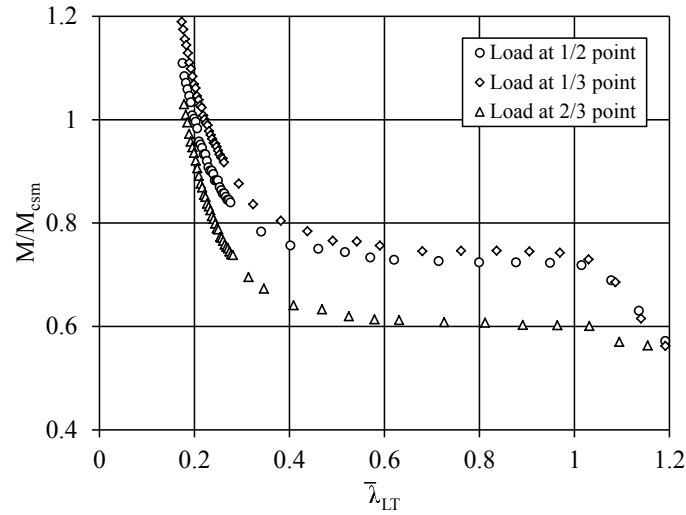


(a) 305 × 127 × 48 UB

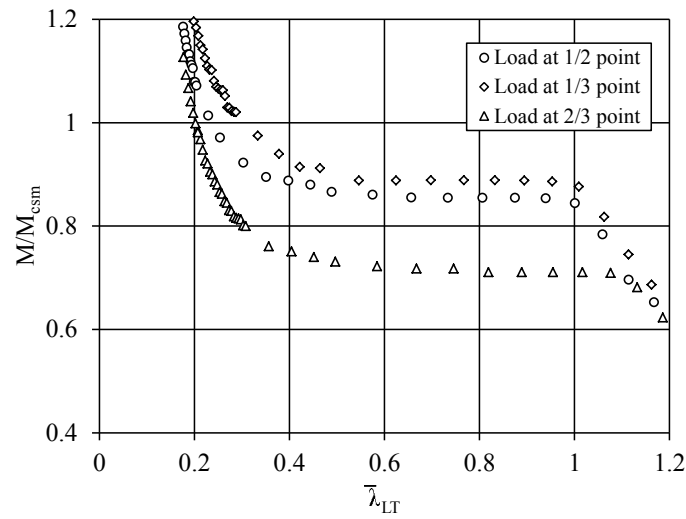


(b) 305 × 165 × 40 UB

Figure 5.30: Collapse load interaction relationships for continuous beams with $\bar{\lambda}_{LT} = 0.20$ critical spans.



(a) $305 \times 127 \times 48$ UB



(b) $305 \times 165 \times 40$ UB

Figure 5.31: Lateral torsional buckling slenderness relationships for two-span continuous beams.

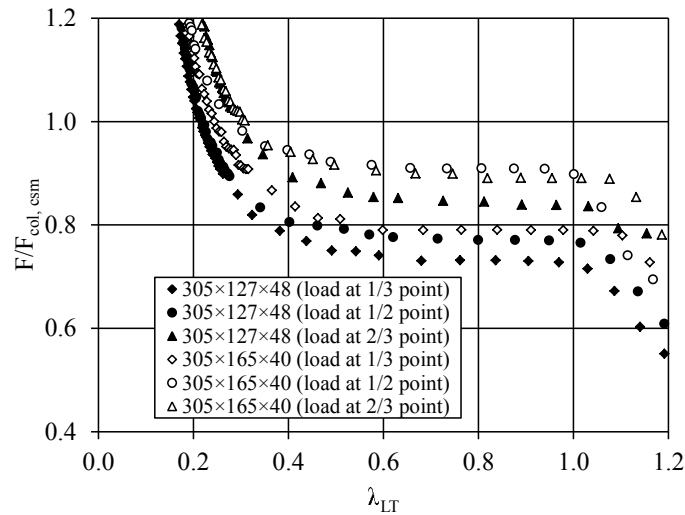


Figure 5.32: CSM collapse loads versus lateral torsional buckling slenderness for all sections and load configurations.

Table 5.7: Critical CSM restraint spacings compared with EN-1993-1-1 restraint spacings.

	L_{csm}/L_{stable}	
	305 × 127 × 48 UB	305 × 165 × 40 UB
Load at the 1/3 span	0.81	0.76
Load at the 1/2 span	0.80	0.76
Load at the 2/3 span	1.09	1.08

span. The analysis was performed using the commercial software package ABAQUS. Five global imperfection amplitudes e_0 were considered, as well as a base case with no imperfections - their effect upon the failure moment is compared in Table 5.8.

Table 5.8: The influence of global imperfections on ultimate moment in the low slenderness range.

Designation	Imperfection	e_0 (mm)	$M_u/M_{u, \text{straight}}$
M_{5000}	$L_{\text{span}}/5000$	0.79	0.99
M_{2500}	$L_{\text{span}}/2500$	1.58	0.98
M_{1250}	$L_{\text{span}}/1250$	3.16	0.97
M_{500}	$L_{\text{span}}/500$	7.90	0.96
M_{250}	$L_{\text{span}}/250$	15.80	0.95

The quantity $M_u/M_{u, \text{straight}}$ is the ultimate moment of a beam M_u with one of the considered global imperfections divided by the ultimate moment of the beam $M_{u, \text{straight}}$ without a global imperfection. The results show that for typical imperfection magnitudes (usually observed to be around $L_{\text{span}}/1250$) there is no more than a 3% reduction in capacity compared to the perfect case. For the more extreme case of $L_{\text{span}}/250$, the reduction in capacity is still only 5%. Thus, the results from the preceding analysis without imperfections can be considered representative for slenderness values less than $\bar{\lambda}_{\text{LT}} = 0.2$.

5.5.7 Design recommendations

For restraint spacing along continuous beams, it has been shown that the minimum slenderness required to achieve CSM levels of buckling resistance is similar to that required for simply supported beams. As a consequence, the same transition relationship from $\bar{\lambda}_{\text{LT}} = 0.4$ to the CSM slenderness will be adopted. Recalling from Chapter 4, the simple limiting CSM slenderness $\bar{\lambda}_{\text{LT}}^{\text{CSM}}$ equivalent to the proposal of Trahair (1998) is:

$$\bar{\lambda}_{\text{LT}}^{\text{CSM}} = \sqrt{E_{sh}/E} \quad (5.29)$$

with an initial proposal for a suitable linear transition from M_{pl} to M_{csm} given by:

$$\frac{M}{M_{pl}} = \frac{(\bar{\lambda}_{LT} - \bar{\lambda}_{LT}^{csm})}{0.4 - \bar{\lambda}_{LT}^{csm}} \left(1 - \frac{M_{csm}}{M_{pl}} \right) + \frac{M_{csm}}{M_{pl}} \quad (5.30)$$

This relationship is plotted for each cross-section in Fig. 5.33. Furthermore, the empirical analysis conducted in Chapter 4 that prescribes a limiting member slenderness of $\bar{\lambda}_{LT} = 0.2$ can also be used in the same manner for continuous beams, employing the same factor to transition from M_{pl} to M_{csm} , as defined by Eq. (4.11).

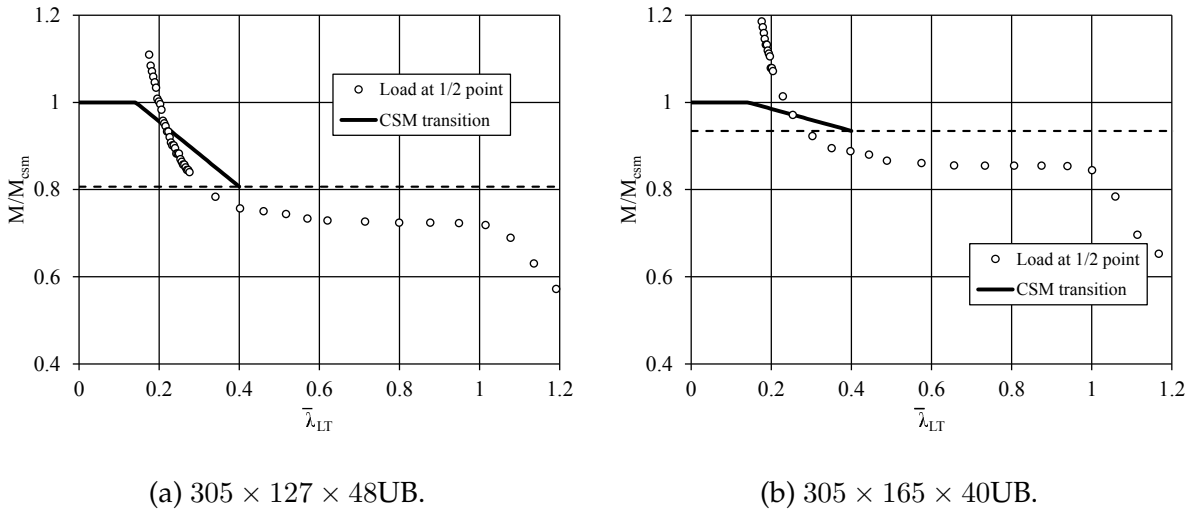


Figure 5.33: M_{pl} to M_{csm} transition relationships.

5.6 Concluding remarks

A programme of six tests on continuous beams with discrete rigid and elastic lateral restraints has been conducted in order to investigate the global stability implications of utilising the Continuous Strength Method in the design of statically indeterminate structures. Two cross-sections were considered and lateral restraint spacings were varied to achieve two values of $\bar{\lambda}_{LT}$ (0.3 and 0.4) for each restraint configuration; for the elastic restraints, two levels of stiffness were used. Results showed that, for the cases considered, the CSM-compatible collapse load was either met or exceeded, with the CSM on average

providing better predictions of capacity than conventional plastic methods. Variation in bracing stiffness did not significantly affect the ultimate capacity of the specimens, but as with statically determinate structures, to limit the forces developed in the restraints it is necessary to have restraints of sufficient stiffness to ensure that a mode-II failure (i.e. buckling between the restrained points) occurs.

A series of materially and geometrically non-linear finite element models was validated against the experimental results and shown to be able to capture the observed physical behaviour. On the basis of these models, parametric studies were conducted to investigate the implications of varying the restraint spacing and stiffness, as well as the loading configurations on the ultimate capacities of the beams and the corresponding forces developed in the restraints.

The results from the restraint spacing parametric studies showed that, in line with statically determinate structures, distances between lateral restraints need to be reduced to attain the full CSM capacity compared with the requirements of EN 1993-1-1 to achieve M_{pl} . Restraint stiffness requirements were higher than required for statically determinate structures owing to the need to resist lateral forces during plastic rotation. At CSM levels of load, restraint forces were well within the limit of 2.5% N_{Ed} set out by EN 1993-1-1; however at ultimate load, forces of 3% of the compression flange force N_{Ed} were typically observed. To reflect the increased deformation demands of the CSM, it is therefore recommended that restraints should be designed to withstand 3% N_{Ed} when using the CSM.

Chapter 6

Strain-hardening material properties of structural steel

6.1 Introduction

Amongst the key parameters required to develop and use the Continuous Strength Method (CSM), material properties are of fundamental importance. For the CSM, two basic assumptions are made: (1) the underlying material model is elastic, linear hardening and (2) in the elastic range the relationship between stress and strain is defined by the Young's modulus E and beyond the yield stress f_y this relationship is defined by a strain-hardening modulus, taken as $E_{sh} = E/100$ as recommended by EN 1993-1-5. An elastic, linear hardening model is able to represent strain-hardening effects, and the slope can be adjusted to suit the grade, section type and forming method. It is worth noting that, as demonstrated further in this chapter, the yield plateau observed in coupon tests on hot-rolled sections is substantially eroded when considering the stress-strain response of the full cross-section, due to it encompassing variable plate thickness, residual stresses and localised strain-hardening due to cold-forming.

The purpose of this chapter is to make use of a new testing method in which the whole cross-section is tested in tension with the aim to:

- (i) Examine the strain-hardening behaviour of various hot-rolled and cold-formed carbon steel sections to determine section dependent values of E_{sh} ;

- (ii) Propose a suite of material models suitable for the CSM as an enhancement to the general provisions made by EN 1993-1-5;
- (iii) Validate the assumption of using a elastic, linear hardening stress strain curve with immediate strain-hardening in the post-yield range;
- (iv) Quantify the accuracy enhancements to the CSM resulting from the improved material models.

A wide range of parameters (steel grade, cross-section shape, forming process, loading conditions and local plate thickness) affecting the stress-strain response of structural steel have been identified and their relevance to the CSM has been discussed by Wang (2011) whose conclusions were drawn from the analysis of tensile and compressive coupon test data.

Assuming the same general properties (steel grade, cross-section shape, forming process and loading conditions), the stress-strain response of any given specimen will encounter location specific variations in material properties (i.e the material properties will vary depending on the location from which the coupon is extracted), which are determined by factors such as plate thickness, work hardening due to forming and the distribution of residual stresses due to differential cooling rates through the cross-section. A coupon test will only provide a representative stress-strain response for the area from which it has been cut; coupons taken from multiple locations will provide a family and hence a range of stress-strain responses, but these will still neglect any interactions that develop when the full cross-section is stressed.

This chapter will present (1) a brief summary of the most widely adopted material modelling approaches; (2) a summary of previous studies and proposals for material models based on local coupon test data; (3) the results of an experimental programme carried out at Imperial College London; (4) an updated proposal for the material models to be used in the CSM, taking into account the average tensile cross-section stress-strain properties determined in the experimental investigation.

6.2 Overview of material modelling approaches

6.2.1 General

The typical mechanical properties of hot-finished structural steel subjected to static uni-axial load are illustrated in Fig. 6.1. In the elastic range the slope is linear and defined by the modulus of elasticity (Young's modulus) E , which is valued at $210,000 \text{ N/mm}^2$ in EN-1993-1-1. The elastic range is limited by the yield stress, f_y , and corresponding yield strain ϵ_y . Beyond ϵ_y a plateau forms with no increases in stress until ϵ_{sh} is reached, which is the strain at which strain-hardening initiates. At this point, stress accumulation recommences at a reduced rate E_{sh} which is the tangent modulus of the slope at the onset of strain-hardening.

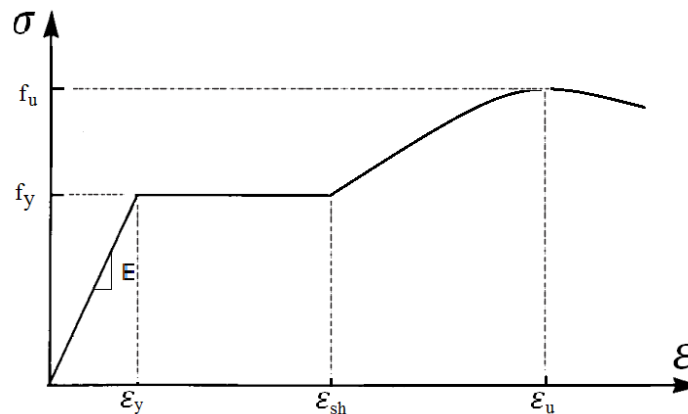


Figure 6.1: Typical stress-strain curve for hot-rolled carbon steel

Various idealisations of this relationship exist and can be grouped as (1) elastic or rigid, perfectly plastic; (2) elastic, linear hardening; or (3) elastic, multi-linear hardening or non-linear hardening. The rigid plastic model is illustrated in Fig. 6.2a and forms the basis of current plastic design methods that neglect strain-hardening.

Linear hardening models have at least two distinct phases of stress accumulation characterised by the initial slopes at each transition strain. For the elastic, linear hardening model illustrated in 6.2b there is an initial elastic phase where stress and strain are related by E , followed by a strain-hardening phase whose rate of stress accumulation is

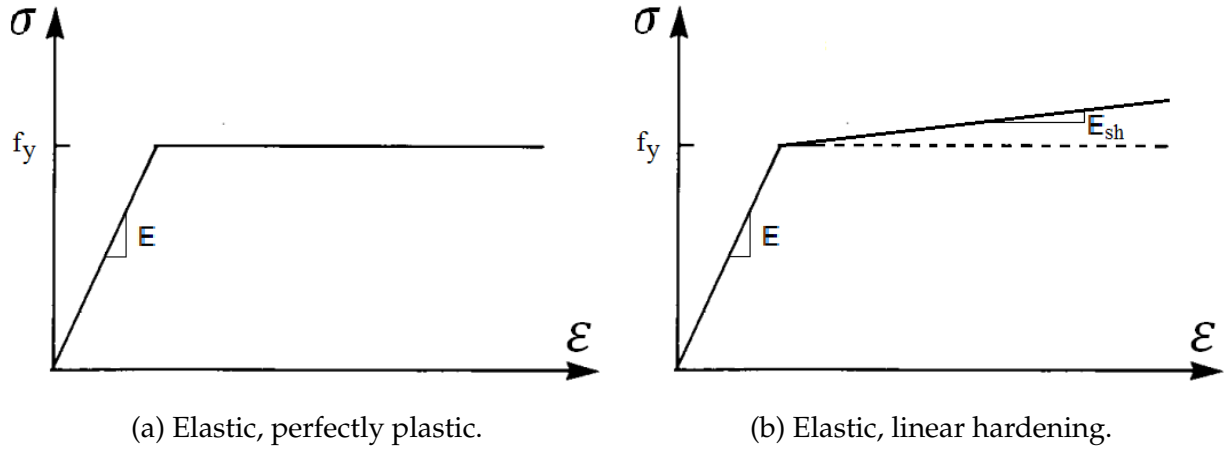


Figure 6.2: Material models with and without strain-hardening.

reduced by some proportion of E to give E_{sh} .

Tri-linear material models typically incorporate a yield plateau before the onset of strain-hardening, after which stress accumulation commences at a rate E_{sh} . Additional phases may be included to represent different rates of strain-hardening as the material approaches the ultimate stress f_u ; piecewise linear models are often used in inelastic finite-element simulations.

Some materials have rounded stress-strain characteristics (e.g. aluminium and stainless steel), while other materials (e.g. steel) can exhibit a similar response following cold-forming. For such materials, a continuous non-linear relationship is generally considered more suitable; the most prevalent is that proposed by Ramberg and Osgood (1943) (eq. (6.1)):

$$\epsilon = \epsilon_e + \epsilon_p = \frac{f}{E} + K \left(\frac{f}{E} \right)^n \quad (6.1)$$

in which ϵ is the total strain, f is the stress level, ϵ_e is the elastic strain, ϵ_p is the plastic strain, and K and n are constants determined empirically.

A comprehensive review of the form of material model to be adopted by the CSM is given in Wang (2011) where it was identified that the following criteria should be satisfied:

- (i) A minimal number of parameters;
- (ii) Overall accuracy of the stress-strain description for mechanical behaviour;
- (iii) Consideration of strain-hardening;
- (iv) Stress can be solved for explicitly;
- (v) Consistency with the current design code (Eurocode 3).

It was concluded that the elastic, linear hardening model best satisfied the criteria and has since been used throughout the development of the CSM elsewhere (Gardner et al., 2011).

6.2.2 Current modelling approaches adopted by the CSM

Early work on the CSM concentrated on applications to stainless steel (Gardner, 2002; Gardner and Nethercot, 2004b; Gardner and Theofanous, 2008), and as such employed the Ramberg-Osgood material model. Subsequent extensions to carbon steel (Gardner, 2008) have led to the general application of the elastic, linear hardening material model; this model has recently been applied to stainless steel design (Afshan and Gardner, 2013), motivated by its simplicity and consistency with current design codes.

EN 1995-1-1-5 suggests a value of $E_{sh} = E/100$ for all types and grades of steel section. Previous work conducted by Wang (2011) on carbon steel properties demonstrated that such a generalised measure of strain-hardening behaviour can be refined. In particular, following a survey of experimental data collected from the literature, it was shown that:

- (i) The degree of strain-hardening is a function of f_u/f_y ;
- (ii) The yield plateau for hollow sections exceeds that of I-sections;
- (iii) For a given value of f_u/f_y , I-sections exhibit a higher degree of strain-hardening than hollow sections;
- (iv) The ultimate non-dimensional stress f_u/f_y of hot-rolled sections is higher than it is for cold-formed sections, but hot-rolled sections exhibit a lower initial degree of strain-hardening;

- (v) Stub column tests exhibit shorter yield plateaus and a higher initial degree of strain-hardening than coupon tests.

Expressing the degree of strain-hardening in non-dimensional form (E_{sh}/E), the proposed models for the CSM suggested by Wang (2011) are presented in Table 6.1.

6.2.3 Methods to calculate the strain-hardening modulus

For all considered section types and materials, the value of the strain-hardening modulus varies with progression along the stress-strain curve. Selecting a representative value of E_{sh} for an elastic, linear hardening material model needs some care if it is to be representative of overall material behaviour. Broadly, E_{sh} can be determined according to the following methods:

- (i) Basing E_{sh} on the initial slope at the onset of strain-hardening;
- (ii) The equal energy dissipation method;
- (iii) Direct linear method.

The initial slope method is generally only used for tri-linear representations of material behaviour (Galambos (1998), Kato (1990)) and assumes a constant value of E_{sh} based on the initial post-yield tangent slope taken at ϵ_{sh} (see Fig. 6.1). This model is unsuitable for elastic, linear hardening models as it often provides a value of E_{sh} that is too high to be used for the entire post yield range of the stress-strain curve.

The concept of equal energy dissipation is illustrated schematically in Fig. 6.3. Here, the post-yield portion of the stress-strain curve is represented by a straight line connecting ϵ_y to ϵ_u via a value of E_{sh} that ensures the area of the shaded portions between the model and actual curves are the same (Bruneau et al., 1998). This method can be applied generally, but requires some effort to compute.

The direct method involves fitting a straight line through two values obtained from the experimental stress-strain curve. In a survey of 50 mill tests taken from hot-rolled I- and

Table 6.1: Summary of existing material models for the CSM (Wang, 2011).

Hot-rolled I-sections		
$E_{sh}/E = 0.015 \frac{(f_u/f_y - 1.0)}{1.7 - 1.0}$	for	$f_u/f_y \leq 1.7$
$E_{sh}/E = 0.015$	for	$f_u/f_y > 1.7$
Hot-rolled hollow sections		
$E_{sh}/E = 0.003 \frac{(f_u/f_y - 1.0)}{1.3 - 1.0}$	for	$f_u/f_y \leq 1.3$
$E_{sh}/E = 0.003 + 0.007 \frac{(f_u/f_y - 1.0)}{1.6 - 1.3}$	for	$1.3 < f_u/f_y \leq 1.6$
$E_{sh}/E = 0.01$	for	$f_u/f_y > 1.6$
Cold-formed hollow sections		
$E_{sh}/E = 0.01 \frac{(f_u/f_y - 1.0)}{1.25 - 1.0}$	for	$f_u/f_y \leq 1.25$
$E_{sh}/E = 0.015$	for	$f_u/f_y > 1.25$

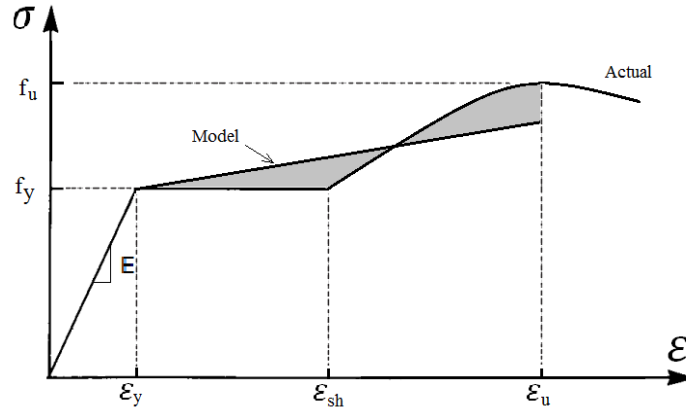


Figure 6.3: Elastic linear-hardening model based upon equal energy dissipation.

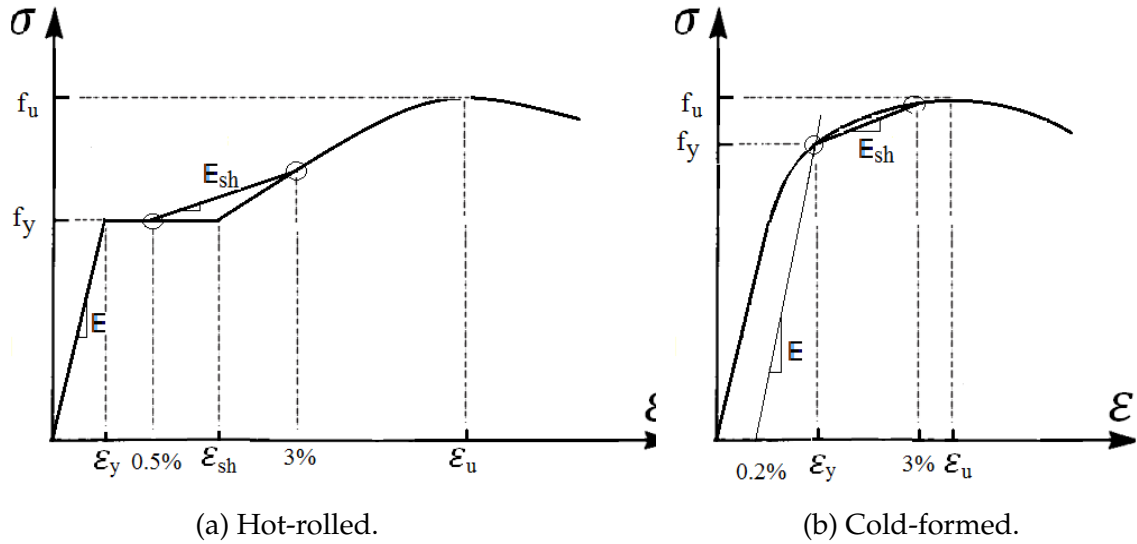
H-sections, Byfield et al. (2005) examined strain-hardening behaviour by calculating the slope of the lines connecting 1% and 4% strains (or $6\epsilon_y$).

The approach developed by Wang (2011) and that will be adopted in this research draws upon methods used for material properties analysis of stainless steel (Afshan and Gardner, 2013), as well as the elements of the direct approach outlined above; this method will then be applicable to both hot-rolled and cold-formed carbon steel sections. The particular elements of the method are as follows:

- (i) For hot-rolled I- and hollow sections, a line is constructed between 0.5% and 3% strain (see Fig. 6.4a).
- (ii) For cold-formed sections, a line is constructed between the 0.2% offset strain and 3% strain (see Fig. 6.4b).
- (iii) If the ultimate stress is reached at a strain less than 3%, then the ultimate strain ϵ_u is used.

6.2.4 Previous experimental procedures

Although the tensile coupon test is the most prevalent method for extracting fundamental information on material behaviour, it faces a number of limitations. Firstly, residual

Figure 6.4: Methods to determine E_{sh} .

stresses present in the parent material due to non-uniform cooling and plastic deformations are released upon cutting the coupons (Alpsten, 1968). Secondly, the stress-strain curve will vary according to the location on the section from which the coupon is cut (Roderick (1954), Wang (2011)). Compression coupon tests face similar limitations and because of the need to prevent buckling, the range of strain over which data can be collected is limited.

Average cross-section behaviour can partially be taken into account using stub column tests - these encompass both residual stresses and the interactions between varying local stress-strain relationships. However, their primary role is for evaluating local buckling capacity (Gardner et al., 2010) and not E_{sh} as often local buckling occurs at low strain levels. Hence, performing a tensile full cross-section test will confer the benefits of stub column tests, but they will be observed over the ranges of strain typically associated with a tensile coupon test, as well as permitting a more meaningful comparison between the two. This type of test will be discussed in detail in Section 6.3.3.

6.3 Experimental programme

6.3.1 Introduction

A testing programme comprising tensile material coupon tests and full cross-section tension tests was carried out on hot-rolled and cold-formed steel I - and hollow sections in the structures laboratory at Imperial College London. Full cross-section tensile tests were carried out on the following sections:

- (i) Hot-rolled grade S355JR I-sections in seven sizes - $305 \times 127 \times 48$ UB, $305 \times 165 \times 40$ UB, $305 \times 102 \times 28$ UB, $254 \times 102 \times 28$ UB, $203 \times 133 \times 25$ UB, $152 \times 152 \times 23$ UC, and $152 \times 152 \times 30$ UC;
- (ii) Hot-rolled grade S355J2H square and rectangular hollow sections in four sizes - $40 \times 40 \times 3$ SHS, $40 \times 40 \times 4$ SHS, $60 \times 40 \times 4$ RHS, and $60 \times 60 \times 3$ SHS;
- (iii) Cold-formed grade S235JRH square and rectangular hollow sections in four sizes - $40 \times 40 \times 3$ SHS, $40 \times 40 \times 4$ SHS, $60 \times 40 \times 4$ RHS, and $60 \times 60 \times 3$ SHS.

Tensile material coupon tests were carried out for the I-sections as part of this programme to determine the local engineering stress-strain material response of the full cross-section tensile specimens; complementary tensile coupons for the hollow sections were tested and reported by Wang (2011).

6.3.2 Local tensile material properties

The tensile coupons were cut and milled from the web and flanges of the I-sections in the longitudinal (rolling) direction only (Fig. 6.5a). Testing was carried out in accordance with the provisions of EN 10002-1 (2001). The nominal dimensions of the necked tensile coupons were 320×30 mm with the parallel necked region of nominal width 20mm (Fig. 6.5b).

Prior to testing, half gauge lengths were marked onto the tensile coupons to allow the final strain at fracture, ϵ_f , to be calculated, based on elongation over the standard gauge length $5.65\sqrt{A_c}$ where A_c is the cross-sectional area of the coupon. A summary of the

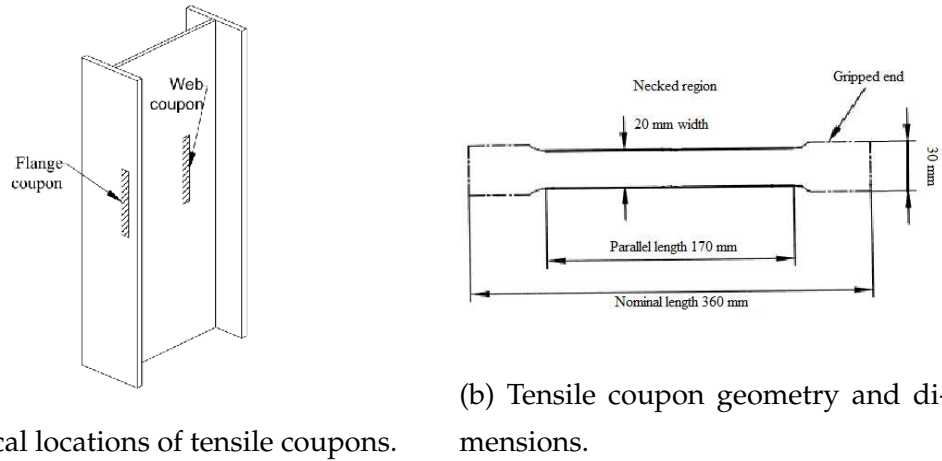


Figure 6.5: Details of tensile coupon specimens.

measured coupon dimensions is provided in Table 6.2, in which F denotes material extracted from the flange and W denotes material extracted from the web, and b_c and t_c are the measured width and thickness of the coupons.

6.3.2.1 Testing conditions

The tensile coupon tests were carried out in an INSTRON 500 kN hydraulic loading machine with an initial strain rate of 0.001%/s for $\epsilon < 0.5\%$. Between 0.5% and 4% strain, the strain rate was 0.002%/s; between 4% and 17% strain, the strain rate was 0.04%/s. These test rates are summarised in Table Fig. 6.3.

Once the coupon reached $\epsilon = 17\%$ testing switched to displacement control at a constant rate of 0.1 mm/s until failure. Static yield and ultimate strengths were determined by holding the strain constant for two minutes in the yield plateau and at four points near the ultimate stress.

Pausing the tests permits the stress relaxation associated with plastic straining to take place. In the plastic range, total strain can be decomposed into elastic and plastic components; in order to maintain a constant level of strain, plastic deformations must in-

Table 6.2: Basic tensile coupon specimen geometry.

Tensile coupon designation	b_c (mm)	t_c (mm)	A_c (mm ²)
305 × 127 × 48 F	19.5	14.4	280.5
305 × 127 × 48 W	19.7	8.5	167.2
305 × 165 × 40 F	19.5	10.0	194.5
305 × 165 × 40 W	20.2	6.4	128.3
305 × 102 × 28 F	20.0	8.1	163.1
305 × 102 × 28 W	19.9	6.3	124.5
254 × 102 × 28 F	20.1	9.7	194.8
254 × 102 × 28 W	20.1	6.3	126.4
203 × 133 × 25 F	20.0	7.1	141.5
203 × 133 × 25 W	20.1	6.4	128.7
152 × 152 × 23 F	20.0	6.8	136.6
152 × 152 × 23 W	20.0	6.2	123.3
152 × 152 × 30 F	19.9	9.1	180.6
152 × 152 × 30 W	20.1	6.7	133.9

Table 6.3: Summary of tensile coupon testing rate.

Strain	Test rate
0 - 0.5 %	0.001 %/s
0.5 - 4 %	0.002 %/s
4 - 17 %	0.04 %/s
> 17 %	0.1 mm/s

crease relative to elastic deformations (Jeong et al., 1999). Reductions in the elastic strain component results in a reduction in the stress under constant total strain; this is known as stress relaxation. Underpinning this process is the observation by Lay (1965a) that when the shear stresses at the structural level of the material reach a critical value, slip planes form in the direction of the shear stresses; these are unstable and will progress until a change of surface is encountered. It was also shown by Lay (1965a) that the stress required to initiate a slip plane is greater than that required to maintain it. Thus, when slip planes are forming under a critical stress, stopping any further strain increments will cause the stress to fall to a static level, the level at which a change in surface is encountered. It is shown by Krapf (2010) that stress decreases logarithmically with time during pausing, suggesting that relatively brief pauses will be sufficient to obtain reliable static values during tensile coupon tests.

Tensile strain was measured using a clip gauge extensometer and strain gauges affixed to the face of the specimen (Fig. 6.6). The clip gauge was used to control the testing machine and the strain gauges were used to provide accurate readings in the elastic range, against which any necessary adjustments to the clip gauge data can be made. Preliminary analysis in Wang (2013) demonstrated that in the elastic range, strain-gauges are more accurate and so are used for determining the modulus of elasticity, but the full stress-strain response is based upon the corrected readings from the clip-gauge extensometer.

6.3.2.2 Test results and discussion

All test data, including load, displacement, strain and input voltage were recorded at one-second intervals using the DATASCAN acquisition system. A summary of the results of these tests is provided in Table 6.4 together with the test results of Wang (2011) in Table 6.5. In the coupon designation system HR denotes hot-rolled, CF denotes cold-formed and C denotes material taken from the corners of the cross-section.

Summary stress-strain curves for all of the coupon tests conducted in this programme are shown in Fig 6.7a, with a second plot (Fig 6.7b) focusing on the region up until the

Table 6.4: Measured tensile coupon test results.

Tensile coupon designation	f_y (dynamic) (N/mm ²)	f_u (dynamic) (N/mm ²)	ϵ_{sh} (%)	ϵ_u (%)	ϵ_f (%)	Area Reduction (%)	E (N/mm ²)	E_{sh} (N/mm ²)
UB 305 $\times$ 127 $\times$ 48 F	381 (392)	499 (534)	1.6	17.1	30.8	62.8	191700	2700
UB 305 $\times$ 127 $\times$ 48 W	396 (403)	494 (528)	2.0	19.1	24.2	56.1	198700	2100
UB 305 $\times$ 165 $\times$ 40 F	428 (438)	579 (585)	1.6	15.7	25.1	67.0	204200	2900
UB 305 $\times$ 165 $\times$ 40 W	452 (459)	569 (599)	2.0	13.7	21.6	47.9	201340	2900
UB 305 $\times$ 102 $\times$ 28 F	363 (375)	488 (518)	1.0	16.0	33.9	63.8	207400	2600
UB 305 $\times$ 102 $\times$ 28 W	357 (366)	492 (523)	1.1	17.5	32.1	59.4	188500	2600
UB 254 $\times$ 102 $\times$ 28 F	382 (386)	501 (530)	1.1	12.0	26.7	65.0	198800	2700
UB 254 $\times$ 102 $\times$ 28 W	386 (401)	513 (549)	1.2	16.3	32.5	58.0	212500	2400
UB 203 $\times$ 133 $\times$ 25 F	354 (364)	492 (522)	0.7	14.4	28.1	61.4	204100	3200
UB 203 $\times$ 133 $\times$ 25 W	379 (385)	490 (519)	1.6	16.2	32.9	61.9	203100	1900
UC 152 $\times$ 152 $\times$ 23 F	345 (355)	471 (500)	0.8	15.5	30.0	60.9	201300	2900
UC 152 $\times$ 152 $\times$ 23 W	362 (371)	476 (507)	1.2	16.0	32.9	54.0	211400	2300
UC 152 $\times$ 152 $\times$ 30 F	374 (382)	498 (528)	0.7	14.3	28.3	57.5	210000	3000
UC 152 $\times$ 152 $\times$ 30 W	376 (383)	490 (521)	1.4	15.5	30.8	57.0	205300	2300

Table 6.5: Collated tensile coupon test data (Wang, 2011).

Tensile coupon designation	f_y (N/mm ²)	f_u (N/mm ²)	ϵ_{sh} (%)	ϵ_u (%)	ϵ_f (%)	E (N/mm ²)	E_{sh} (N/mm ²)
SHS 40 × 40 × 3 HR	504	581	3.9	9.1	36.0	219600	0
SHS 40 × 40 × 3 CF	451	502	1.2	5.9	24.0	212900	1280
SHS 40 × 40 × 3 CF-C	534	589	0.4	3.3	16.0	196700	2380
SHS 40 × 40 × 4 HR	496	572	4.3	14.2	34.0	212300	0
SHS 40 × 40 × 4 HR-C	499	578	3.9	8.5	37.0	215500	0
SHS 40 × 40 × 4 CF	410	430	0.4	5.1	38.0	201600	730
SHS 40 × 40 × 4 CF-C	479	507	0.4	1.5	17.0	210900	2870
RHS 60 × 40 × 4 HR	468	554	4.7	9.8	37.0	213800	0
RHS 60 × 40 × 4 CF	400	452	0.4	5.0	21.0	212000	1880
RHS 60 × 40 × 4 CF-C	480	570	0.4	2.4	15.0	202400	3910
SHS 60 × 60 × 3 HR	449	555	3.8	9.9	31.0	215200	0
SHS 60 × 60 × 3 CF	361	402	0.4	12.0	49.0	207400	1030
SHS 60 × 60 × 3 CF-C	442	471	0.4	1.4	21.0	208000	2550

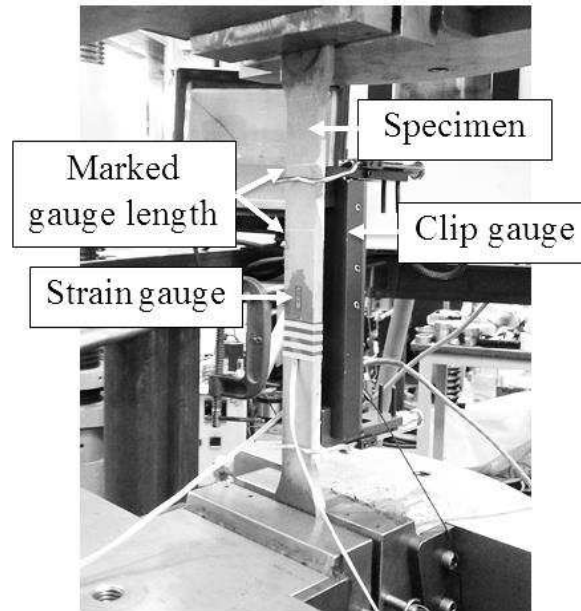
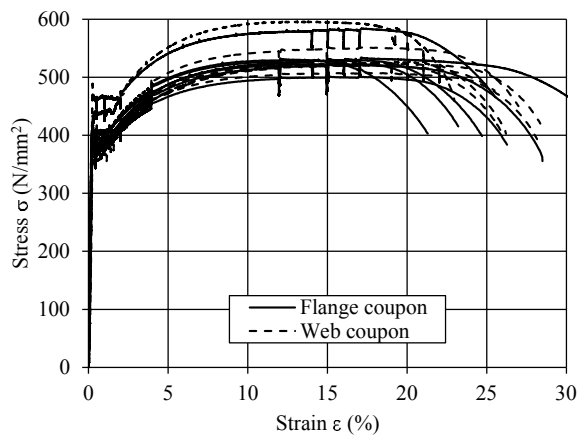
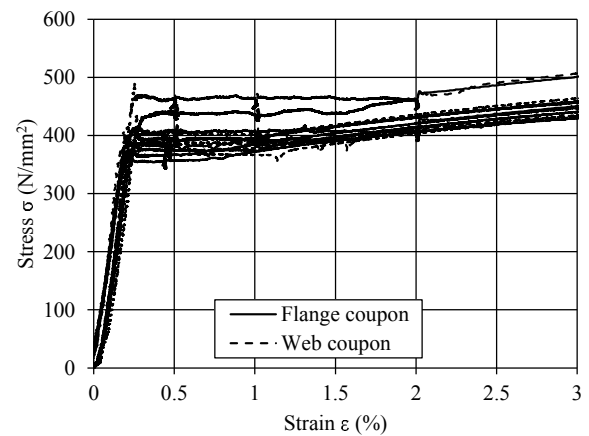


Figure 6.6: Tensile coupon test set-up.

onset of strain-hardening.



(a) Complete stress-strain curves.

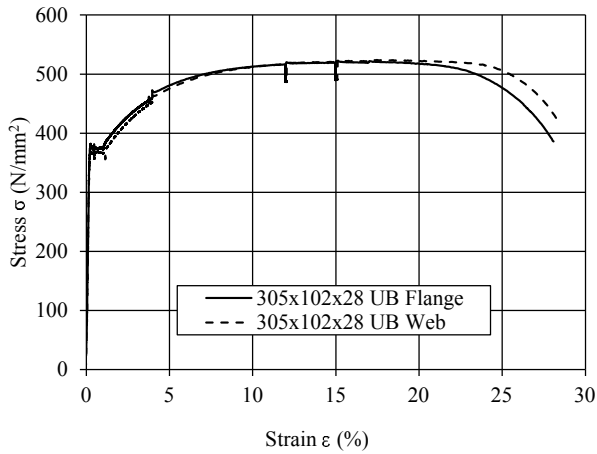


(b) Initial portion of stress-strain curve.

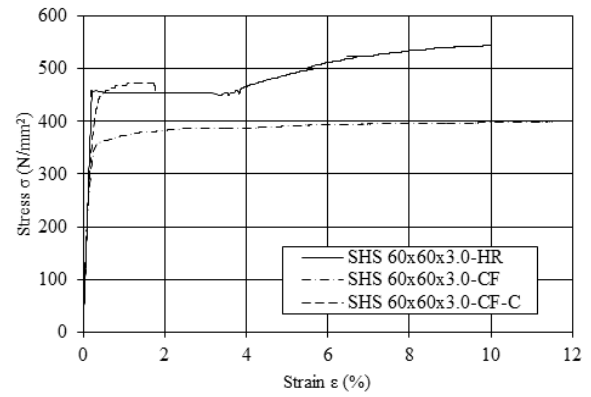
Figure 6.7: Summary of tensile stress strain curves for I-sections.

The tensile coupons for the hot-rolled sections exhibited the anticipated response of a well defined yield point, followed by a plateau before the initiation of strain-hardening.

Comparing Figs. 6.8a and 6.8b it is noted that material taken from hot rolled hollow sections exhibit a longer yield plateau than material taken from I-sections. The cold-formed material does not exhibit a well defined yield point, with a progressive and rounded transition from elastic to inelastic behaviour. Examination of Fig. 6.8a also illustrates the differences in response of material taken from the flanges and webs, with the flange material generally experiencing a shorter yield plateau than the web.



(a) I-sections.



(b) Hot-rolled hollow sections.

Figure 6.8: Comparative stress-strain response for different cross-section types.

Examination of the results presented in Table 6.4 indicates that the value of E_{sh} is generally higher for thicker material. Studies have shown that whilst this pattern holds true in some cases, it is not a general relationship (Byfield et al., 2005), but it has been argued elsewhere that the ratio f_u/f_y decreases with decreasing thickness and that E_{sh} increases with increases in f_u/f_y ; thus thickness is implicitly related to the strain-hardening modulus via the ratio of ultimate stress to yield stress (Wang, 2011).

6.3.3 Full cross-section tensile tests

In order to account for the interactions between varying material and geometric properties of the different section types that are otherwise overlooked by tensile coupon tests, a new, full cross-section tensile test is proposed. As previously discussed, taking into account average cross-section properties will give a more accurate representation of the

uniaxial tensile behaviour of the different section types. In this section, the details of the test procedure will be discussed, along with a presentation of the basic results and how they compare to data obtained from local tensile coupon tests.

6.3.3.1 Test specimens

Full cross-section tensile tests were performed on hot-rolled (S355JR) I-sections, hot-rolled (S355J2H) hollow sections and cold-formed (S235JRH) hollow sections. All of the test specimens were cut to a nominal length of 600 mm. The basic measured geometric properties of the specimens are reported in Tables 6.6 and 6.7. For the $305 \times 127 \times 48$ UB and $305 \times 165 \times 40$ UB specimens, two tests of each section were carried out for initial assessment purposes and these are denoted S1 and S2. Notation for the geometric properties is defined in Fig. 6.9. Tolerance compliance was confirmed according to EN 10310-2 (2006) and EN 10319-1 (2006).

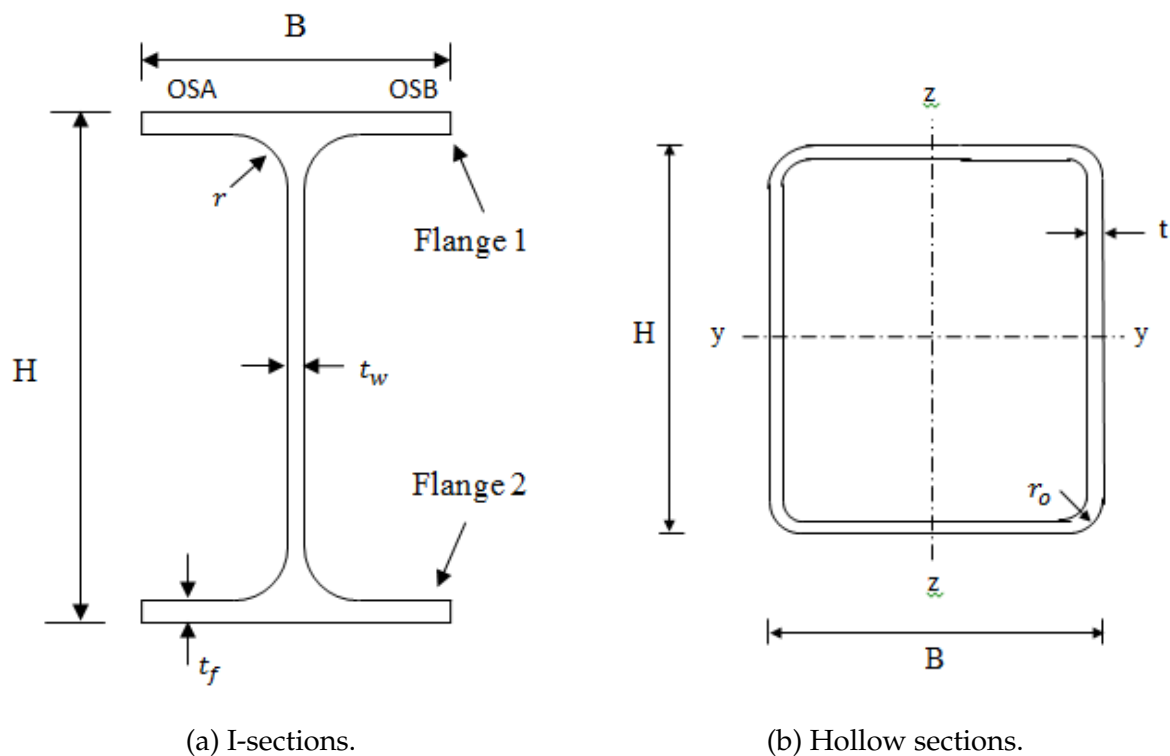


Figure 6.9: Notation for full section tensile test specimens.

Table 6.6: Basic geometric properties of tensile I-sections.

Cross-section tensile test designation	H (mm)	B (mm)	t_f (mm)	t_w (mm)	r (mm)	A (mm ²)
305 × 127 × 48 UB S1	311.48	127.39	13.93	8.60	15.00	6183
305 × 127 × 48 UB S2	311.48	127.39	13.93	8.60	15.00	6183
305 × 165 × 40 UB S1	305.88	166.78	10.01	6.54	12.00	5331
305 × 165 × 40 UB S2	306.15	166.89	9.96	6.30	12.00	5249
305 × 102 × 28 UB	307.67	105.08	8.44	6.35	9.40	3697
254 × 102 × 28 UB	260.25	102.74	9.36	6.39	12.40	3599
203 × 133 × 25 UB	201.49	133.95	6.96	6.34	12.00	3178
152 × 152 × 23 UC	153.77	152.38	6.53	5.94	14.50	3006
152 × 152 × 30 UC	155.91	154.28	8.75	6.57	14.40	3788

Table 6.7: Basic geometric properties of tensile hollow sections.

Cross-section tensile test designation	H (mm)	B (mm)	t (mm)	r_o (mm)	A (mm ²)
SHS 40 × 40 × 3 HR	40.00	40.44	3.13	3.80	453
SHS 40 × 40 × 3 CF	40.24	40.34	2.83	5.50	404
SHS 40 × 40 × 4 HR	40.13	40.08	3.99	4.00	563
SHS 40 × 40 × 4 CF	40.69	40.45	3.82	7.50	525
RHS 60 × 40 × 4 HR	40.50	60.47	3.92	5.30	708
RHS 60 × 40 × 4 CF	40.27	60.33	4.01	5.60	718
SHS 60 × 60 × 3 HR	60.40	60.59	3.29	5.50	731
SHS 60 × 60 × 3 CF	60.11	59.86	2.80	7.50	611

6.3.3.2 Testing conditions

The following conditions were sought from the testing arrangement:

- (i) An even distribution of load across the cross-section; this was achieved by welding end plates to the specimen.
- (ii) Minimal deformation of the end plates under load; this is achieved by using thick end plates (25mm) with a symmetrical distribution of bolts at sufficiently close intervals to spread the load evenly.
- (iii) Sufficiently strong welds between the end plates and the specimen; vertical stiffener fins were used with the primary function of increasing the welded area between the specimen and the end plates.

A summary of these design features is provided in Fig. 6.10.

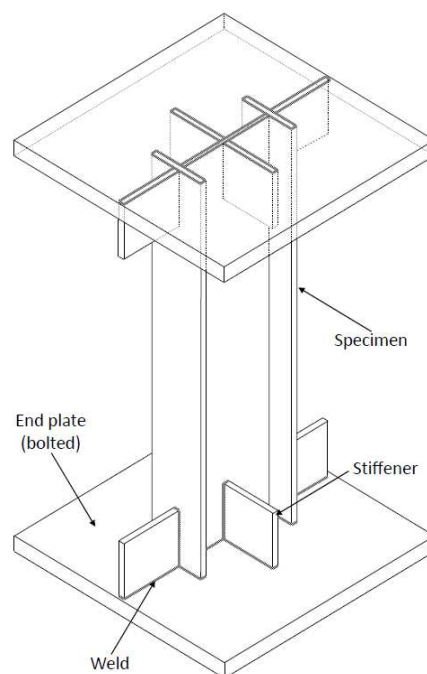


Figure 6.10: Typical details of fabricated specimens for full cross-section tension tests.

Prior to testing, half gauge lengths were scribed along the length of the specimens to allow the final strain at fracture, ϵ_f , to be calculated. The standard gauge length $L_0 =$

$5.65\sqrt{A_c}$ (where A_c is the pre-test cross-sectional area of the specimen) suggested by EN 10002-1 (2001) for tensile coupons is too long for this type of test. The following modifications to the gauge length were therefore introduced:

- (i) For the hollow sections, the local plate cross-sectional area was used for A_c .
- (ii) For the I-sections, a non-proportional gauge length $L_0 = 80$ mm was used. Elongation values based on this gauge length are comparable to those based upon $L_0 = 5.65\sqrt{A_c}$ by means of a conversion provided in Table 4 of EN 2566-1 (1999).

The full cross-section tensile tests were carried out using an INSTRON 3500 kN hydraulic universal testing machine. The end plates of the specimen were securely bolted to the end platens of the testing machine prior to testing. Displacements were recorded by 8 L.V.D.T.s positioned at uniform locations on both ends of the specimen (see Fig. 6.11a). The applied load was recorded with a load cell and strain gauges were attached to each face of the specimen at the mid-height (6 for I-sections and 4 for hollow sections) - see Fig. 6.11b.

Testing was carried out under displacement control with similar relative variations in the testing rate to the coupon tests, which are summarised in Table 6.8. Unlike the coupon tests, displacement holding points were not introduced into the test profile as firstly, the testing machine was not capable of reliably introducing such holds and secondly, by having a very slow test rate, the difference between static and dynamic stresses is likely to be negligible (Krempf and Nakamura, 1998). All test data, including load, displacement, strain and input voltage were recorded at one-second intervals using the DATASCAN acquisition system.

6.3.3.3 Test results

All specimens failed by ductile fracture, which was the anticipated mode of failure. In most, but not all cases, fracture occurred at the mid-height of the specimens. Examples of this mode of failure are presented in Fig. 6.12.

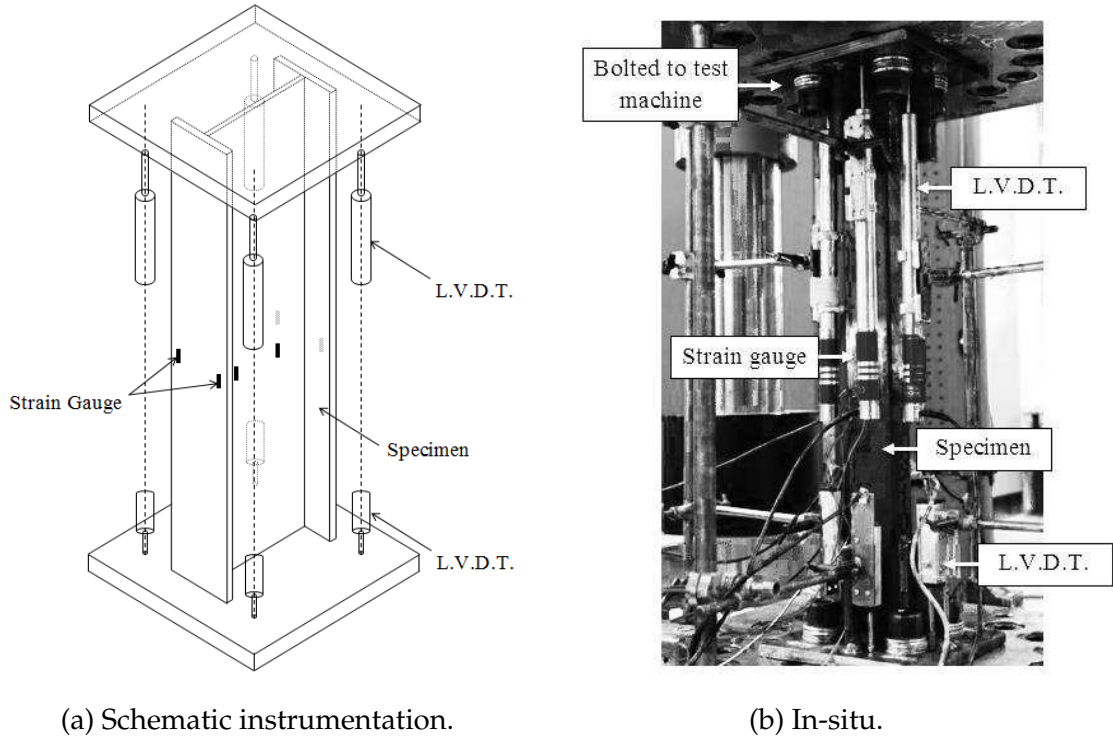


Figure 6.11: Test set-up for full cross-section tension tests.

6.3.3.3.1 Data processing During the tests, measurements based upon L.V.D.T., strain gauge, load cell and machine displacement readings were collected. Prior to analysis, certain considerations need to be taken into account. Firstly, unlike a tensile coupon test, inclusion of the additional stiffening material at the ends of the specimens resulted in non-uniform strains developing along the length of the test member. Secondly, like the stub column tests, the end plates experienced small elastic deformations which resulted in small discrepancies between L.V.D.T. and strain gauge initial stiffness readings.

The procedure for accounting for non-uniform strain along the length of the test member was developed by Wang (2013) and the key steps involved are presented in this section. The first step is to construct the basic stress-strain (σ, ϵ) curve directly from the load-displacement readings obtained from the tests, where:

$$\sigma = F/A \quad \text{and} \quad \epsilon = \delta/L \quad (6.2)$$

in which δ is the axial displacement recorded by the L.V.D.T.s, L is the specimen length,

Table 6.8: Test rate for full tensile cross-section tests.

Displacement (mm)	Test rate (mm/s)
0 - 3	0.004
3 - 14.5	0.008
14.5 - 20	0.016
20 - 24	0.08
> 24	0.16

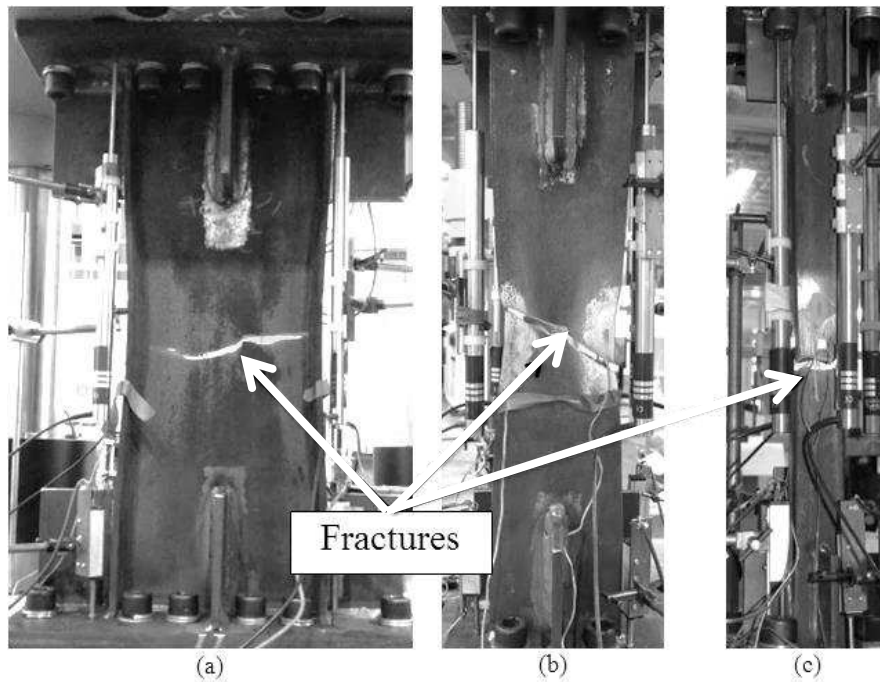


Figure 6.12: Typical failure modes for all cross-section tensile tests.

F is the recorded load and A is the specimen cross-section area. Using this data, the initial tangent modulus E' is calculated. This value is an average for the whole specimen and includes the additional rigidity of the stiffeners.

The second step is to calculate a corrected modulus of elasticity from the L.V.D.T. data, E_{LVDT} , allowing for the influence of the stiffeners, which is achieved using Eq. (6.3):

$$E_{\text{LVDT}} = k_{\text{LVDT}} E' \quad (6.3)$$

in which:

$$k_{\text{LVDT}} = \left[\frac{\beta}{\alpha} + (1 - \beta) \right] \quad (6.4)$$

where $\alpha = \frac{(A + A_{\text{stiff}})}{A}$ and $\beta = \frac{2L_{\text{end}}}{L}$ and A_{stiff} is the area of the stiffeners. With reference to Fig. 6.13a this area is defined by the shaded triangle formed by the welded edges (the triangle defined by the free edges is assumed to have a zero contribution), which in turn provides a rigid body connection between the stiffeners and the specimen. With reference to Fig. 6.13b, the triangular area is simplified to an equivalent rectangular region. This region, in conjunction with Fig. 6.14 defines the length of the specimen, L_{end} , over which the stiffeners influence the elastic rigidity of the specimen.

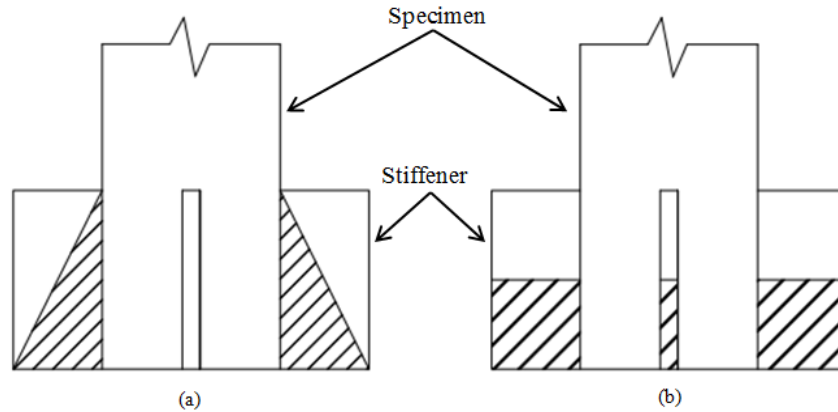


Figure 6.13: (a) Area contributions of stiffeners and (b) simplification of effective stiffener geometry.

In deriving the stiffness correction factor k_{LVDT} the following assumptions are made:

- (i) The whole specimen consists of three regions: two identical end regions defined over the lengths L_{end} and a single middle region L_{mid} (Fig. 6.14).
- (ii) The material properties of the stiffener and the test specimen are homogeneous.

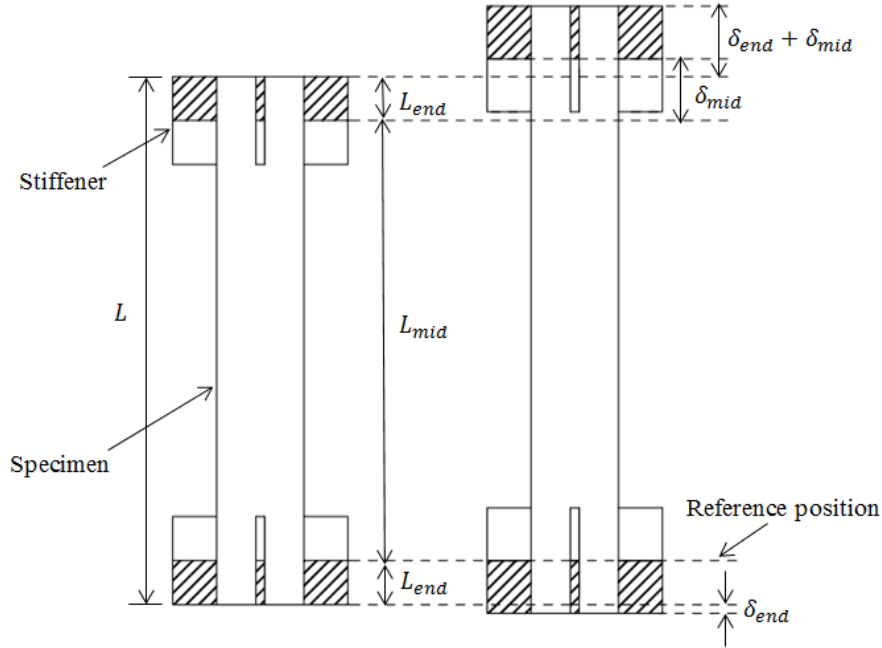


Figure 6.14: Nomenclature for stiffness reduction factor calculations.

- (iii) The welded edges of each stiffener provide a rigid body connection with the specimen.
- (iv) The free edges of the stiffener and the triangular region that they enclose have no contribution to the stiffness of the segment L_{end} .
- (v) The effective material area can be simplified into equivalent rectangular regions (Fig. 6.13b).

For equilibrium, the corrected initial modulus E_{LVDT} must satisfy:

$$E_{LVDT} = k_{LVDT} E' \quad (6.5)$$

Correspondingly:

$$\frac{\sigma_{end}}{\epsilon_{end}} = \frac{\sigma_{mid}}{\epsilon_{mid}} \iff \frac{F/(A + A_{stiff})}{\delta_{end}/L_{end}} = \frac{F/A}{\delta_{mid}/L_{mid}} \quad (6.6)$$

To derive an expression for k_{LVDT} it is necessary to solve for ϵ_{mid} in Eq. (6.6). Solving initially for ϵ_{end} :

$$\epsilon_{\text{end}} = \frac{\epsilon_{\text{mid}} A}{(A + A_{\text{stiff}})} = \frac{1}{\alpha} \epsilon_{\text{mid}} \quad (6.7)$$

Axial displacement δ can be expressed as:

$$\delta = \delta_{\text{mid}} + 2\delta_{\text{end}} = \epsilon_{\text{mid}} L_{\text{mid}} + 2\epsilon_{\text{end}} L_{\text{end}} \quad (6.8)$$

Noting, with reference to Fig. 6.14, $L_{\text{mid}} = L - 2L_{\text{end}}$ and by definition, $\epsilon = \delta/L$, Eq. (6.8) becomes (upon substituting Eq. (6.7)):

$$\epsilon = \epsilon_{\text{mid}} \left(\frac{L - 2L_{\text{end}}}{L} \right) + \frac{1}{\alpha} \epsilon_{\text{mid}} \frac{2L_{\text{end}}}{L} \quad (6.9)$$

From the earlier definition of β , Eq. (6.9) becomes:

$$\epsilon = \epsilon_{\text{mid}} (1 - \beta) + \frac{\beta}{\alpha} \epsilon_{\text{mid}} \quad \text{or} \quad \epsilon = \left[(1 - \beta) + \frac{\beta}{\alpha} \right] \epsilon_{\text{mid}} \quad (6.10)$$

Rearranging:

$$\epsilon_{\text{mid}} = \frac{\epsilon}{\left[(1 - \beta) + \frac{\beta}{\alpha} \right]} \quad (6.11)$$

With reference to Eqs. (6.5) and (6.6):

$$E_{\text{LVDT}} = \frac{\sigma_{\text{mid}}}{\epsilon_{\text{mid}}} = \frac{F/A [(1 - \beta) + \beta/\alpha]}{\epsilon} \quad (6.12)$$

For Eq. (6.5) to hold, $E' = (F/A)/\epsilon$, which is the initial slope of the stress-strain curve derived using Eq. (6.2), and $k_{\text{LVDT}} = [(1 - \beta) + \beta/\alpha]$. The values α and β can be readily evaluated from measurements obtained from the test specimens.

By introducing the correction in Eq. (6.4) it is assumed that over the length L_{end} , the material remains in the elastic range throughout the test. Defining the stress in this region as $f_{\text{stiff}} = F_u/(A + A_{\text{stiff}})$, Tables 6.9 and 6.10 show that all stresses in this region

are indeed below the respective material yield stress values (assuming that the material properties of the stiffeners are the same as those of the specimen), indicating that this is a reasonable assumption.

Table 6.9: Summary stress calculations for stiffeners (I-sections).

Cross-section tensile test designation	A (mm ²)	F_u (kN)	A_{stiff} (mm ²)	f_{stiff} (N/mm ²)
305 × 127 × 48 UB S1	6183	3071	4000	302
305 × 127 × 48 UB S2	6183	3063	4000	301
305 × 165 × 40 UB S1	5331	2772	4000	297
305 × 165 × 40 UB S2	5249	3059	4000	331
305 × 102 × 28 UB	3697	1983	4000	258
254 × 102 × 28 UB	3599	1957	4000	257
203 × 133 × 25 UB	3178	1697	4000	236
152 × 152 × 23 UC	3006	1535	4000	219
152 × 152 × 30 UC	3788	1946	4000	250

Assuming that the region L_{end} deforms elastically confines all of the inelastic deformations to the region L_{mid} (Fig. 6.14). The third step is then to construct the appropriate stress-strain relationship ($\sigma_{\text{mid}}, \epsilon_{\text{mid}}$). Noting that:

$$\epsilon_{\text{mid}} = \frac{\delta_{\text{mid}}}{L_{\text{mid}}} \quad (6.13)$$

it is necessary to determine δ_{mid} . With reference to Fig. 6.14, the total axial displacement δ can be decomposed as:

$$\delta = \delta_{\text{mid}} + 2\delta_{\text{end}} \quad \Longleftrightarrow \quad \delta_{\text{mid}} = \delta - 2\delta_{\text{end}} \quad (6.14)$$

Evaluating the modulus of elasticity from the strain gauges, E_{SG} , and invoking the previous assumption of elastic behaviour in the region L_{end} , the quantity $\delta_{\text{end}} = \epsilon_{\text{end}} L_{\text{end}}$ can be readily evaluated using:

Table 6.10: Summary stress calculations for stiffeners (hollow sections).

Cross-section tensile test designation	A (mm ²)	F_u (kN)	A_{stiff} (mm ²)	f_{stiff} (N/mm ²)
SHS 40 × 40 × 3 HR	453	242	2000	99
SHS 40 × 40 × 3 CF	404	212	2000	88
SHS 40 × 40 × 4 HR	563	300	2000	117
SHS 40 × 40 × 4 CF	525	247	2000	98
RHS 60 × 40 × 4 HR	708	372	2000	137
RHS 60 × 40 × 4 CF	718	350	2000	129
SHS 60 × 60 × 3 HR	731	406	2000	149
SHS 60 × 60 × 3 CF	611	256	2000	98

$$\epsilon_{\text{end}} = \frac{\sigma_{\text{end}}}{E_{\text{SG}}} = \frac{F/(A + A_{\text{stiff}})}{E_{\text{SG}}} \quad (6.15)$$

The fourth step compensates for the small elastic deformations of the end plates. Using a procedure developed by (C.A.S.E., 1990) for stub columns, the recorded L.V.D.T. displacements δ_{mid} (adjusted from δ using Eq. (6.15)) can be modified to obtain true axial displacement $\delta_{\text{mid,true}}$, where:

$$\delta_{\text{mid,true}} = \delta_{\text{mid}} - 2\Delta_{\text{end}} \quad (6.16)$$

The deformation of the end plates Δ_{end} can be calculated using Eq. (6.17):

$$\Delta_{\text{end}} = \frac{L_{\text{mid}}}{2} \sigma \left(\frac{1}{E_{0,\text{LVDT}}} - \frac{1}{E_{\text{SG}}} \right) \quad (6.17)$$

The final corrected value for strain in the region L_{mid} is then:

$$\epsilon_{\text{mid,true}} = \frac{\delta_{\text{mid,true}}}{L_{\text{mid}}} \quad (6.18)$$

6.3.3.3.2 Summary results Plots for all test specimens are presented in Figs. 6.15-6.19. For each case load displacement plots are presented alongside the corresponding stress-strain curves after allowing for the adjustments discussed previously. For all load-displacement curves, load measurements are obtained directly from the test and displacements are calculated as the difference between averaged top and bottom L.V.D.T. readings.

Tables 6.11 and 6.12 present the key test results for all of the tests; all notation is as previously defined for the tensile coupon tests. The final strain at fracture ϵ_f , is based upon the weighted average area contributions of each component of the specimen cross-sections. Attempts to quantify area reduction for some of the I-sections are also presented, though in general this proved difficult to calculate with any reliability. The modulus of elasticity was obtained from strain gauge readings and the strain-hardening modulus was estimated using the method described in the preceding section. For cold-formed sections, f_y and ϵ_y are taken as the 0.2% proof stress and 0.2% offset strain.

The key observations from the test results are:

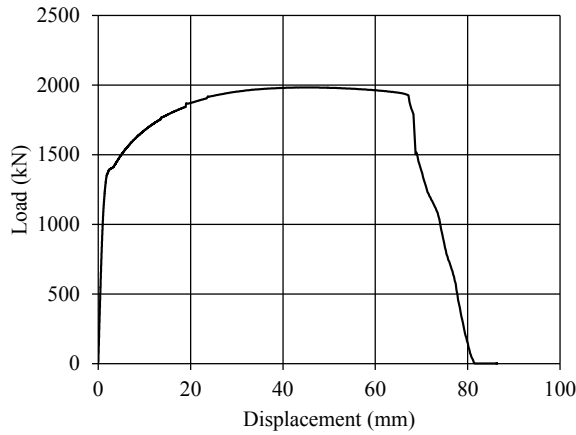
- (i) All hot-rolled cross-sections still exhibit a yield plateau, though this is significantly reduced when compared to the results from the coupon tests. For I-sections, the average $\epsilon_{sh} = 1.28\%$ for the coupons, whilst for the full section tensile tests the average $\epsilon_{sh} = 0.72\%$ - a 43% reduction in the length of the yield plateau. For hot-rolled hollow sections, the average $\epsilon_{sh} = 4.16\%$ for the coupons, whilst for the full tensile tests the average $\epsilon_{sh} = 2.91\%$ - a 30% reduction in the length of the yield plateau. For cold-formed sections, the average $\epsilon_{sh} = 0.59\%$ for the flat coupons (average $\epsilon_{sh} = 0.43\%$ for corner coupons), whilst for the full section tensile tests the average $\epsilon_{sh} = 0.42\%$.
- (ii) In all cases there is an absence of a well-defined yield point, reflecting the interactions of varying material properties around the cross-section and residual stresses.
- (iii) Comparing values of E_{sh} to those derived from the coupon tests, higher values for E_{sh} are observed in all cases, with the exception of the hot-rolled hollow sections which typically exhibit zero strain-hardening (up to 3% strain).

Table 6.11: Key cross-section tensile test results (hot-rolled I-sections).

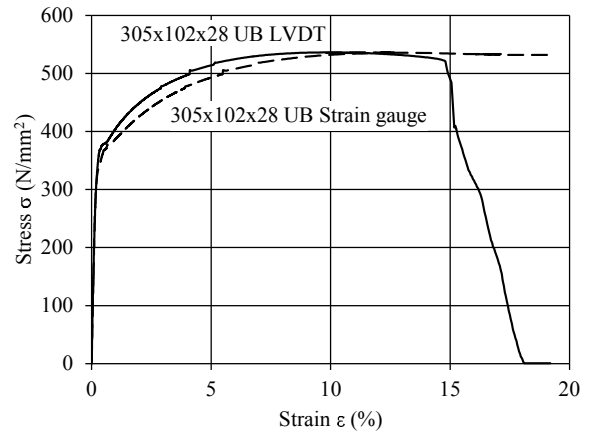
Cross-section tensile test designation	A (mm ²)	f_y (N/mm ²)	f_u (N/mm ²)	ϵ_{sh} (%)	ϵ_u (%)	ϵ_f (%)	Area reduction at fracture (%)	E (N/mm ²)	E_{sh} (N/mm ²)
305 × 127 × 48 UB S1	6183	392	-	0.94	-	-	-	218000	3100
305 × 127 × 48 UB S2	6183	392	-	0.92	-	-	-	214900	3100
305 × 165 × 40 UB S1	5331	427	-	0.95	-	-	-	211100	-
305 × 165 × 40 UB S2	5249	451	580	0.92	9.70	-	-	217300	3400
305 × 102 × 28 UB	3697	375	536	0.61	10.00	24.50	48.10	196000	4000
254 × 102 × 28 UB	3599	392	544	0.66	9.50	26.20	41.70	202700	3800
203 × 133 × 25 UB	3178	372	534	0.40	9.20	22.90	46.70	212000	3900
152 × 152 × 23 UC	3006	373	511	0.44	8.50	28.90	44.60	205200	3700
152 × 152 × 30 UC	3788	403	-	0.68	-	-	-	209700	3400

Table 6.12: Key cross-section tensile test results (hot-rolled and cold-formed hollow sections).

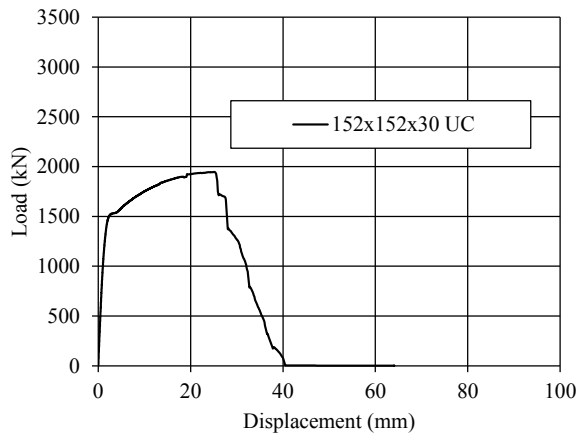
Cross-section tensile test designation	A (mm ²)	f_y (N/mm ²)	f_u (N/mm ²)	ϵ_{sh} (%)	ϵ_u (%)	ϵ_f (%)	E (N/mm ²)	E_{sh} (N/mm ²)
SHS 40 × 40 × 3 HR	453	471	535	3.46	10	45	208200	0
SHS 40 × 40 × 3 CF	404	474	521	0.43	4.2	21.9	211900	1600
SHS 40 × 40 × 4 HR	563	450	532	2.57	10.4	45.9	207100	0
SHS 40 × 40 × 4 CF	525	450	467	0.43	1.2	26.8	193700	2200
RHS 60 × 40 × 4 HR	708	443	525	2.98	11.5	45.2	206600	0
RHS 60 × 40 × 4 CF	718	433	486	0.41	2.7	20	204700	2300
SHS 60 × 60 × 3 HR	731	456	555	2.61	12.3	41.3	212300	300
SHS 60 × 60 × 3 CF	611	375	418	0.39	4.6	38.5	197800	1600



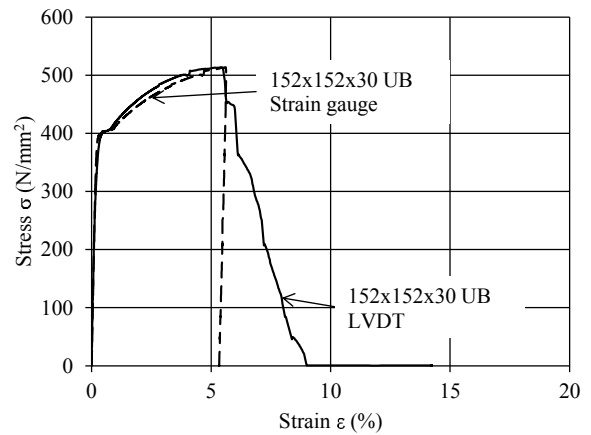
(a) 305 × 102 × 28 UB load-displacement plot.



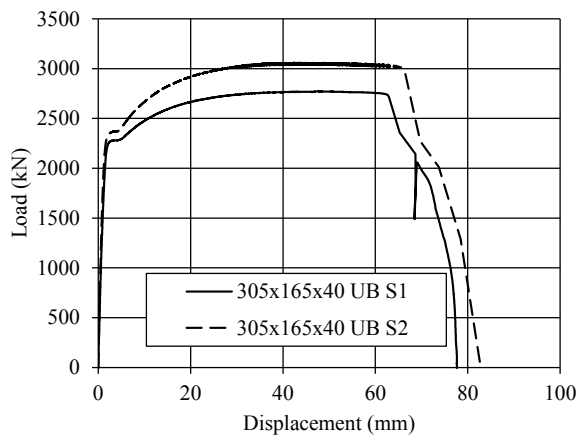
(b) 305 × 102 × 28 UB stress-strain curves.



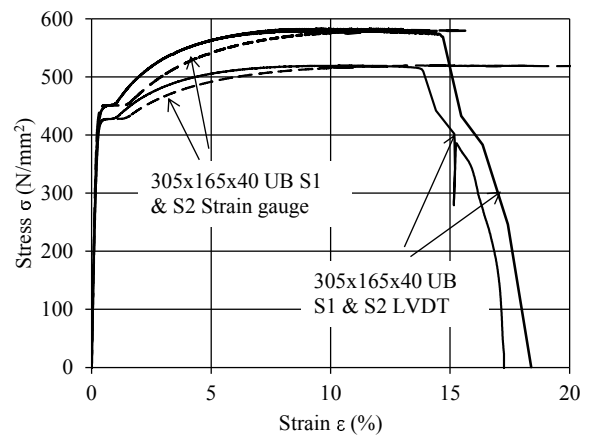
(c) 152 × 152 × 30 UC load-displacement plot.



(d) 152 × 152 × 30 UC stress-strain curves.

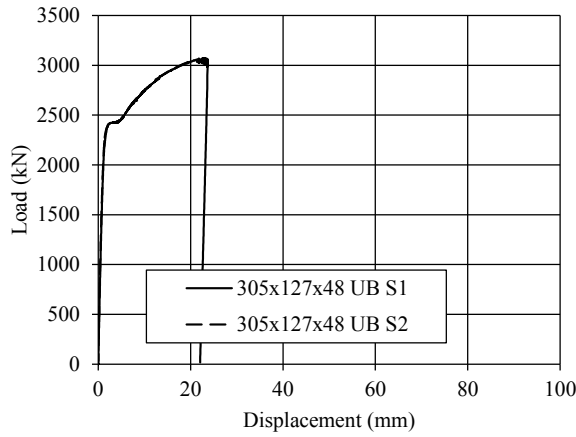


(e) 305 × 165 × 40 UB load-displacement plot.

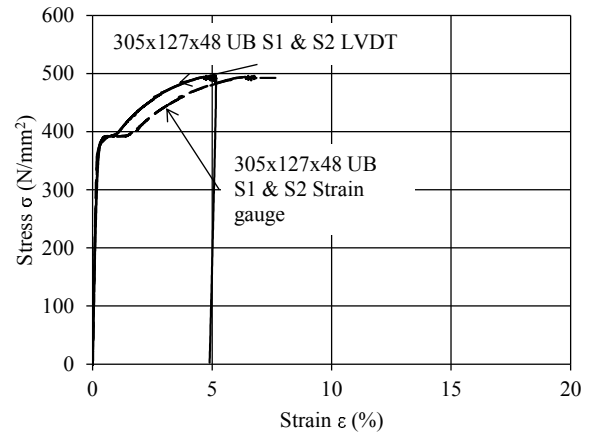


(f) 305 × 165 × 40 UB stress-strain curves.

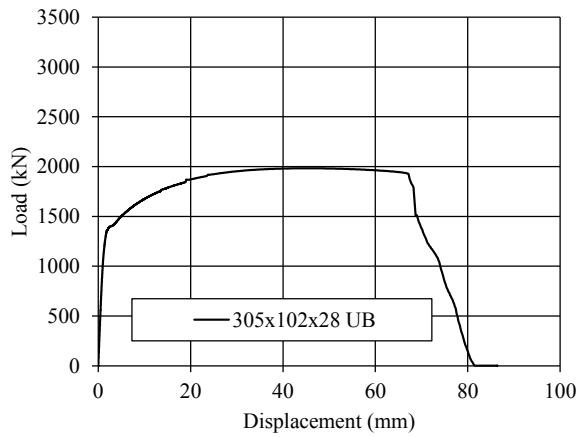
Figure 6.15: Summary test results for tensile cross-section tests (hot-rolled I-sections)



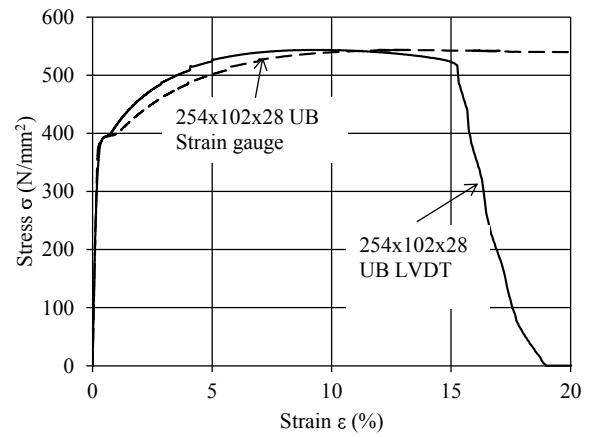
(a) $305 \times 127 \times 48$ UB load-displacement plot.



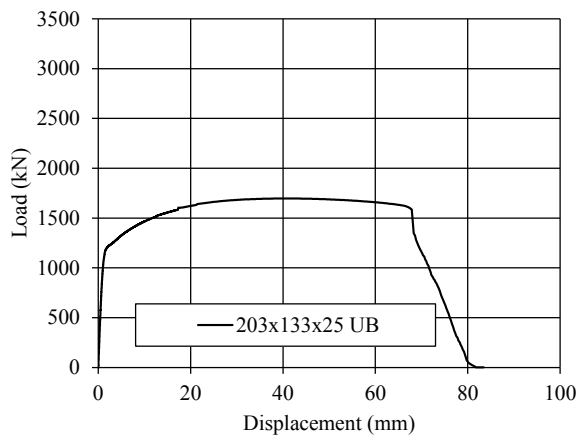
(b) $305 \times 127 \times 48$ UB stress-strain curves.



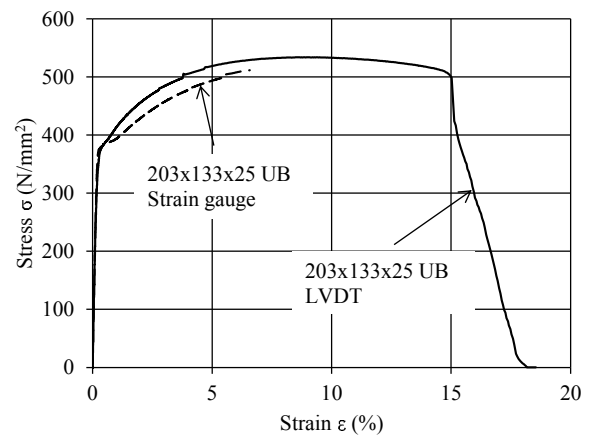
(c) $305 \times 102 \times 28$ UB load-displacement plot.



(d) $305 \times 102 \times 28$ UB stress-strain curves.

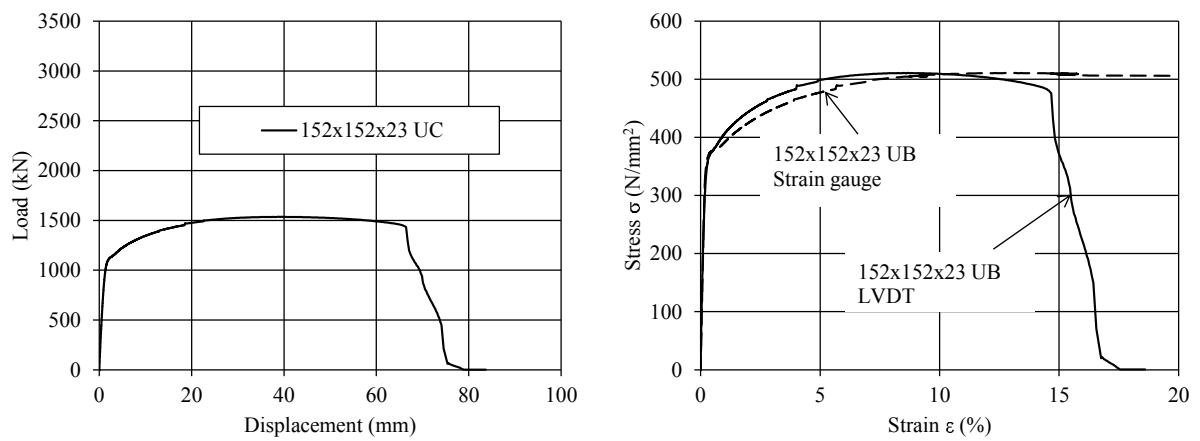


(e) $203 \times 133 \times 25$ UB load-displacement plot.



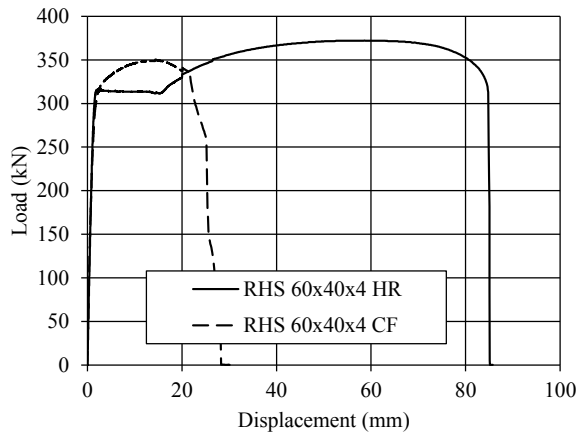
(f) $203 \times 133 \times 25$ UB stress-strain curves.

Figure 6.16: Summary test results for full cross-section tensile tests (hot-rolled I-sections)

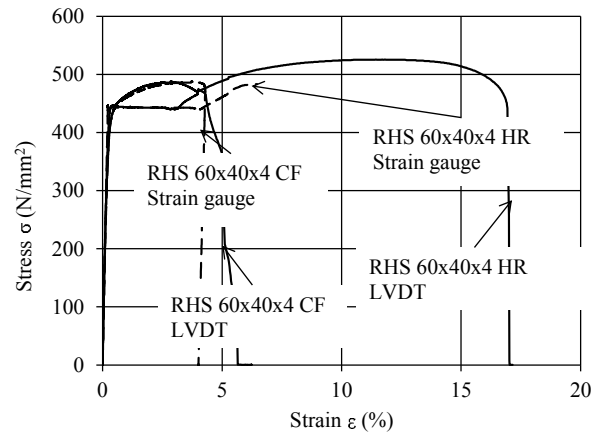


(a) $152 \times 152 \times 23$ UB load-displacement plot. (b) $152 \times 152 \times 23$ UB stress-strain curves.

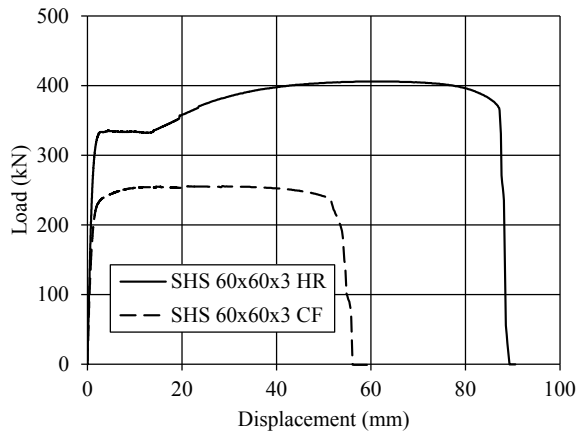
Figure 6.17: Summary test results for full cross-section tensile tests (hot-rolled I-sections)



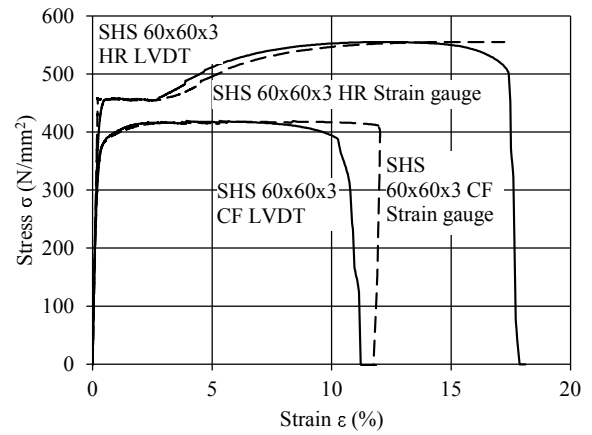
(a) $60 \times 40 \times 4$ RHS load-displacement plot.



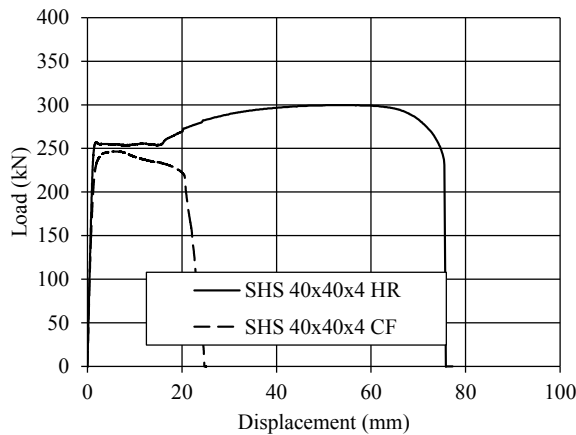
(b) $60 \times 40 \times 4$ RHS stress-strain curves.



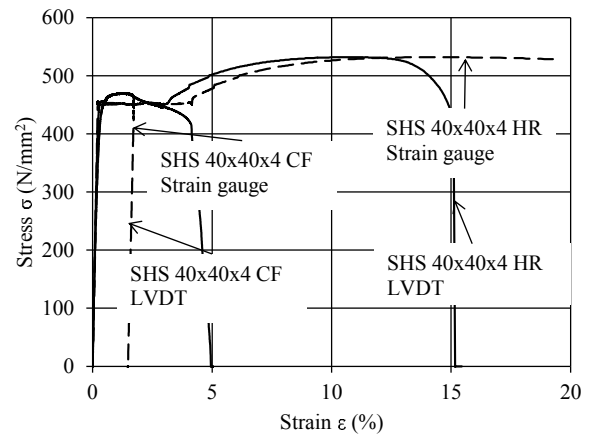
(c) $60 \times 60 \times 3$ SHS load-displacement plot.



(d) $60 \times 60 \times 3$ SHS stress-strain curves.

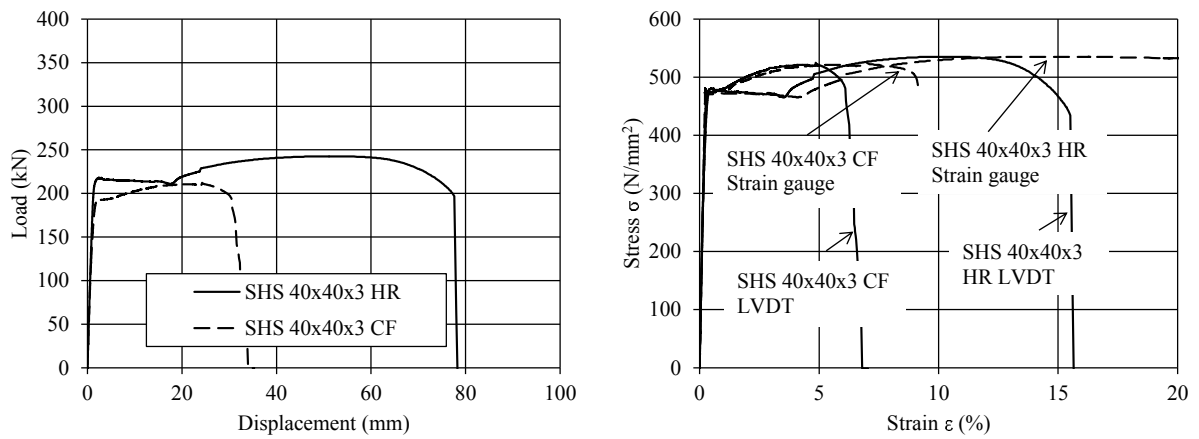


(e) $40 \times 40 \times 4$ SHS load-displacement plot.



(f) $40 \times 40 \times 4$ SHS stress-strain curves.

Figure 6.18: Summary full cross-section tensile test results (hot-rolled and cold-formed hollow sections).



(a) $40 \times 40 \times 3$ SHS load-displacement plot.

(b) $40 \times 40 \times 3$ SHS stress-strain curves.

Figure 6.19: Summary full cross-section tensile test results (hot-rolled and cold-formed hollow sections).

6.4 Analysis of test data

In this section, the results of the full cross-section tensile tests for each section type are compared and discussed. Thereafter, the relative attributes of the full cross-section tensile tests are examined alongside those of the tensile coupon tests. Summary non-dimensional E/E_{sh} values derived from the tensile full cross-section tests and the coupon tests are plotted against f_u/f_y in Fig. 6.20. These will be used in conjunction with the results presented in Tables 6.11 and 6.12.

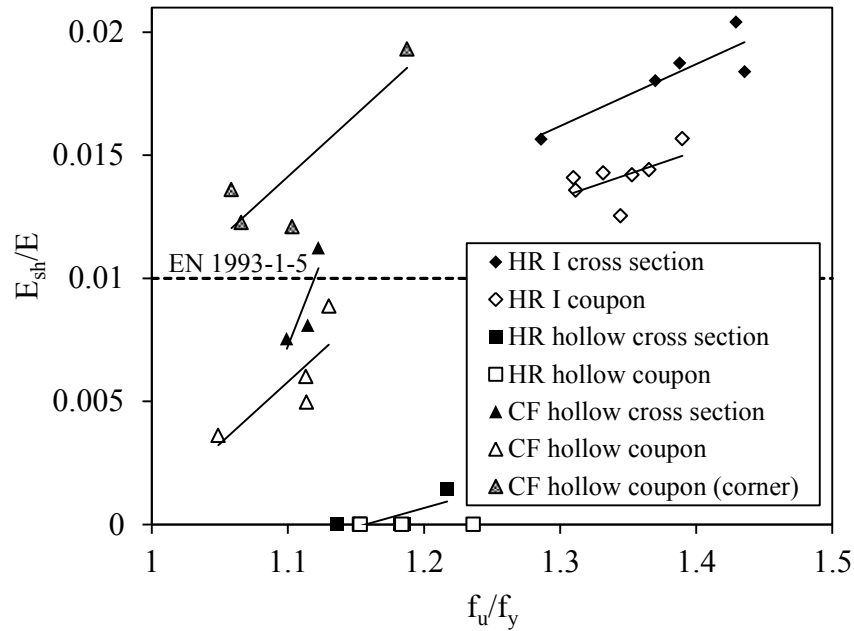


Figure 6.20: Summary relationship between E_{sh}/E and f_u/f_y for all full cross-section and tensile coupon tests.

6.4.1 Full cross-section tensile tests: hot-rolled I-sections vs. hot-rolled hollow sections

Comparing E_{sh}/E values obtained for the hot-rolled I-sections with those of the hot-rolled hollow sections in Fig. 6.20, it is clear that the hot-rolled I-sections exhibit a higher degree of strain-hardening. Furthermore, they also show a higher f_u/f_y ratio, which is in contrast to a similar comparison made by Wang (2011) for equivalent coupon test

data where the range of f_u/f_y differed for the two cross-section types. This discrepancy is most likely to be explained by the comparatively small sample size of the current experimental programme rather than any specific deterministic relationship.

6.4.2 Full cross-section tensile tests: hot-rolled vs. cold-formed hollow sections

For the cross-section tensile tests, the hot-rolled hollow sections have higher f_u/f_y ratios than their cold-formed counterparts. This is in line with the observations made in Wang (2011) for tensile coupon tests. As to be expected, cold-formed hollow sections exhibit a far higher degree of strain-hardening in the initial stage than their hot-rolled equivalents, due to the rounding effect of the cold-forming process on the shape of the stress-strain curve.

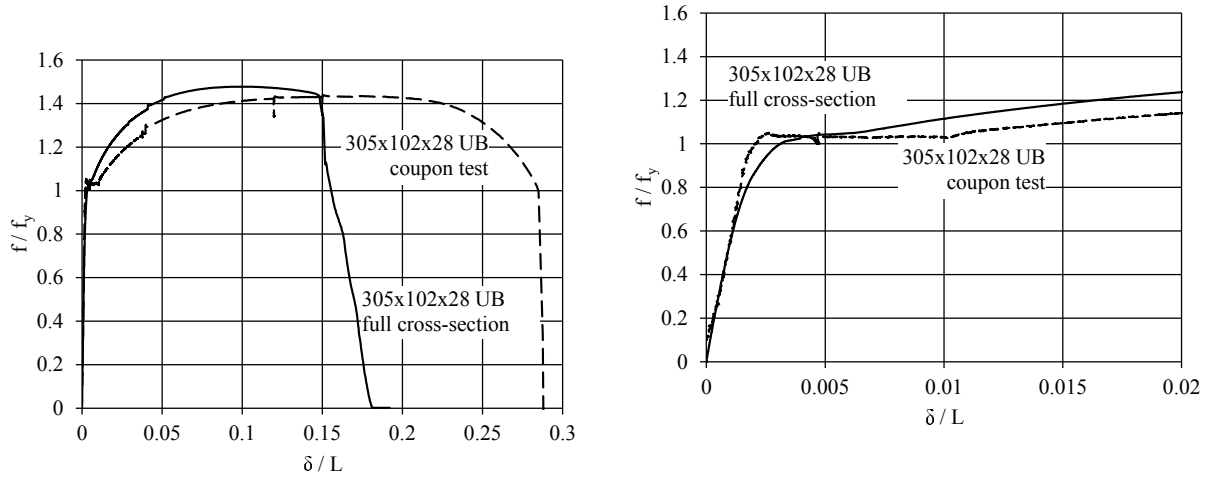
6.4.3 Full cross-section tensile test vs. tensile coupon test

6.4.3.1 Yield plateau

With reference to Tables 6.11 and 6.12 the following observations may be made in relation to the length of the yield plateau:

- (i) For hot-rolled I-sections, the length of the yield plateau, as defined by ϵ_{sh} , is reduced considerably when a full cross-section tensile test is carried out. On average, the reduction in plateau length from the equivalent coupon test is 43%. It is also noted (with reference to Fig. 6.21b) that the response around the yield stress is more rounded - similar observations can be made for the hot-rolled hollow sections.
- (ii) For hot-rolled hollow sections, the length of the yield plateau, as defined by ϵ_{sh} , is reduced in a similar manner to the I-sections in the full cross-section tensile tests. On average, the reduction in plateau length from the equivalent coupon test is 30%.
- (iii) For cold-formed hollow sections, there is no clear yield plateau, but the general yield response from the results of the full cross-section tensile tests differ from

those of the flat coupons due to the influence of residual stresses and the effect of the corner regions.



(a) Non-dimensional full stress-strain response. (b) Relative length of yield plateaus and proper-ties of the yield point.

Figure 6.21: Comparative responses of full cross-section and tensile coupon tests.

Overall, it can be concluded that a full cross-section tensile test suggests a shorter yield plateau than derived from a tensile coupon test, which supports the assumption of the elastic, linear hardening material model currently employed by the continuous strength method.

6.4.3.2 Strain-hardening modulus

With reference to Tables 6.11 and 6.12 and Fig. 6.20 the following observations can be made in relation to the strain-hardening modulus:

- (i) I-sections: compared with data obtained from coupon tests, E/E_{sh} derived from the full cross-section tests is higher in every case - on average for the coupon tests, $E_{sh}/E = 0.014$ whilst for the full cross-section tests, $E_{sh}/E = 0.017$.
- (ii) Hot-rolled hollow sections: in a similar manner to the coupon tests, hot-rolled hollow sections do not experience any strain-hardening within the admissible range of strains (up to 3%), as discussed in Section 6.2.3.

- (iii) Cold-formed hollow sections: as expected, the E_{sh}/E values for the full cross-section tests lie between the flat and corner coupon values. On average, $E_{sh}/E = 0.006$ for the coupon flats, $E_{sh}/E = 0.014$ for the coupon corners and $E_{sh}/E = 0.009$ for the full cross-section tensile tests.

Overall, the full cross-section tensile tests have demonstrated that the ratio E_{sh}/E as defined by the tensile coupon tests is somewhat conservative for I-sections and suggests the type of compromise between corner and flat material behaviour expected of cold-formed hollow sections.

6.5 Revisions to the CSM material model for carbon steel

Figure 6.22a is a graphical representation of the cross-section dependent suite of material models based upon coupon test data as proposed by Wang (2011), against which the data obtained from the three sets of full cross-section tensile tests are plotted. Based upon earlier observations, the following revisions are proposed:

- (i) For I-sections: enhancements to the degree of strain-hardening from $E/E_{sh} = 0.01$ to $E/E_{sh} = 0.015$ and a reduction in the limiting value of f_u/f_y over which the maximum value of E/E_{sh} applies from $f_u/f_y = 1.7$ to $f_u/f_y = 1.3$.
- (ii) For cold-formed hollow sections: the degree of strain-hardening shall remain unchanged from Wang (2011), but the range over which $E/E_{sh} = 0.015$ applies shall commence from $f_u/f_y = 1.15$ rather than $f_u/f_y = 1.25$.
- (iii) For hot-rolled hollow sections the strain-hardening modulus proposed by Wang (2011) shall be replaced with $E_{sh}/E = 0$.

A summary of the revised material models is provided in Table 6.13 and plotted in Fig. 6.22b.

6.5.1 Associated improvements to the CSM

To assess the benefits of an updated material model to the CSM, the predictive capacity of the CSM design equations will be compared with data obtained from a collection of

Table 6.13: Summary of revised material models for the CSM.

Hot-rolled I-sections		
$E_{sh}/E = 0.015 \frac{(f_u/f_y - 1.0)}{1.3 - 1.0}$	for	$f_u/f_y \leq 1.3$
$E_{sh}/E = 0.015$	for	$f_u/f_y > 1.3$
Hot-rolled hollow sections		
$E_{sh}/E = 0$		
Cold-formed hollow sections		
$E_{sh}/E = 0.015 \frac{(f_u/f_y - 1.0)}{1.15 - 1.0}$	for	$f_u/f_y \leq 1.15$
$E_{sh}/E = 0.015$	for	$f_u/f_y > 1.15$

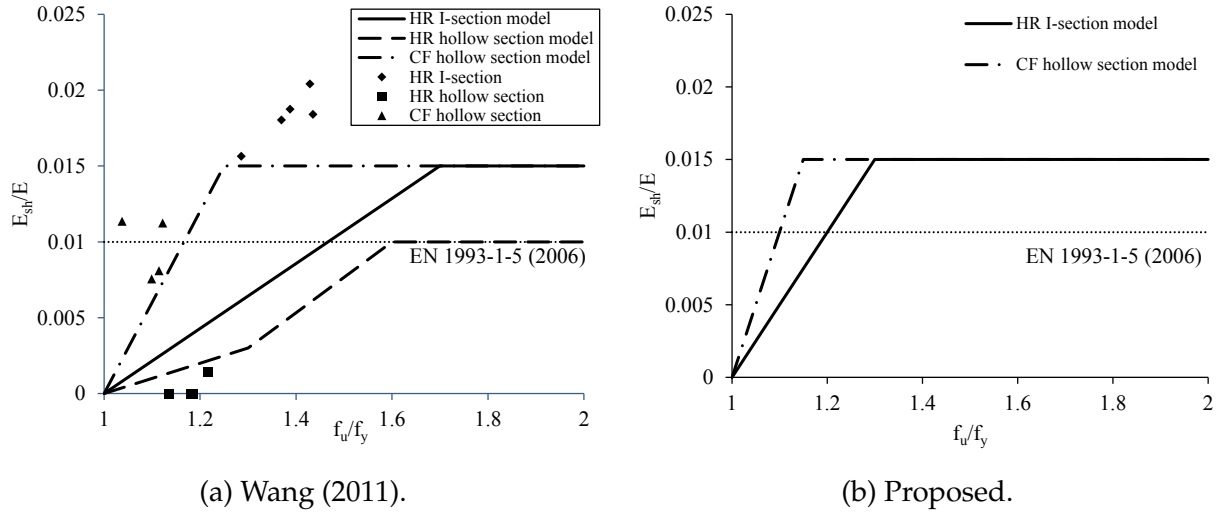


Figure 6.22: Previous and proposed material models for the CSM.

laboratory tests in the literature. Expressions for compression and bending resistance (see Gardner et al. (2011) for details) shall be evaluated. For compression, stub-column tests data was obtained for hot-rolled and cold-formed hollow sections (Gardner et al. (2010), Gao et al. (2009), Greiner et al. (2008), Hu et al. (2011), Rasmussen and Hancock (1992), Sakino et al. (2004), Elchalakani et al. (2002)); for bending resistance, test data was obtained for hot-rolled I-sections, as well as hot-rolled and cold-formed hollow sections (Gardner et al. (2010), Wilkinson and Hancock (1998), Zhao and Hancock (1991), Byfield and Nethercot (1998)).

Tables 6.14 and 6.15 present comparisons between ultimate test values and predicted capacity values using the CSM equations for compression and bending. The first comparisons are between the multi-linear material models reported in Tables 6.1 and 6.13. There is a small improvement in the accuracy of the CSM when the updated material model (Table 6.13) is used; this holds both within groupings for each cross-section type and over the entire sample; the coefficient of variation (C.O.V.) values for the updated material models show very slight decreases in scatter.

The second comparison assumes a single value of E_{sh} that is independent of f_u/f_y . Here the comparisons are between the previously assumed $E_{sh} = E/100$ by Wang (2011) and the values derived from the tests conducted in this chapter. In pure compression,

there is a slight reduction in accuracy compared with assuming $E_{sh} = E/100$, but it is associated with a slight reduction in scatter. In bending there is a slight improvement in accuracy, but an increase in scatter. Such inconsistencies in the results at this stage are not entirely surprising given the relatively small sample sizes of (i) the test results used to derive new E_{sh} values and (ii) the highly variable sample sizes of the bending and axial test data, with considerably more relevant data being available for cold-formed hollow sections than for other types of material or cross-section type.

Table 6.14: Comparison between previous and updated CSM predictions in compression (test/predicted).

	$E_{sh} = f(f_u)$		Constant E_{sh}	
	Wang (2011)	Updated	Wang (2011)	Updated
Hot-rolled hollow sections				
Mean	1.067	1.082	1.009	1.082
C.O.V.	0.076	0.081	0.054	0.081
Cold-formed hollow sections				
Mean	1.091	1.074	1.087	1.056
C.O.V.	0.085	0.080	0.085	0.081
All section types				
Mean	1.083	1.077	1.061	1.065
C.O.V.	0.084	0.080	0.087	0.080

Table 6.15: Comparison between previous and updated CSM predictions in bending (test/predicted).

	$E_{sh} = f(f_u)$		Constant E_{sh}	
	Wang (2011)	Updated	Wang (2011)	Updated
Hot-rolled hollow sections				
Mean	1.111	1.131	1.017	1.131
C.O.V.	0.026	0.026	0.025	0.026
Cold-formed hollow sections				
Mean	1.180	1.151	1.101	1.106
C.O.V.	0.071	0.070	0.074	0.074
Hot-rolled I-sections				
Mean	1.072	1.045	1.090	1.045
C.O.V.	0.021	0.026	0.024	0.026
All section types				
Mean	1.140	1.112	1.095	1.096
C.O.V.	0.070	0.070	0.068	0.071

6.6 Conclusions and design recommendations

A programme of seventeen full cross-section tensile tests on hot-rolled I-sections, hollow sections and cold-formed hollow sections has been conducted in order to investigate the influence of average cross-section properties on the constitutive relationships for carbon steel. Using data obtained from the literature, as well as a supplementary programme of fourteen tensile coupon tests, the overall behavioural response of the cross-section tensile tests demonstrated that assuming an elastic, linear hardening material model for the Continuous Strength Method is a reasonable assumption. Furthermore, it was shown that the values obtained for the strain-hardening modulus based upon the tensile coupon tests are overly conservative, except in the case of hot-rolled hollow sections, where for the range of validity, it is zero. Revising the suite of material models originally proposed by Wang (2011) in the light of these findings has been shown to have the potential to furnish the CSM capacity equations with a higher degree of accuracy when compared with experimental data. With additional full cross-section tensile tests, there is also potential for a reduction in scatter in the data. These improvements apply to all cross-section types investigated, both in compression and in bending.

Chapter 7

Conclusions and suggestions for further work

This section summarises the key findings of the present research and draws together overall conclusions. More detailed concluding remarks are given at the ends of each individual chapter. Suggestions are also made for further work.

7.1 Conclusions

The Continuous Strength Method has been shown to offer increases in member resistance of up to 15% over current European standards, as well as a reduction in scatter of capacity predictions when compared with test data. The work carried out to date has predominantly focused upon complete development of the method at the cross-section level. The primary subject of this research has been to examine the implications of the higher resistances predicted by the Continuous Strength Method for member-level behaviour, focusing in particular on the validity of the current code rules on restraint spacing and load-bearing requirements. Additional work has also been conducted to validate the basic material behaviour assumptions of the Continuous Strength Method and improve their accuracy.

The investigation carried out in this research involved reviewing the existing guidance for member stability in Eurocode 3, collecting all relevant and carefully reported ex-

perimental data and conducting a survey of the pertinent literature. An extensive programme of laboratory tests was performed to examine the lateral-torsional stability of various configurations of steel I-beams with different lateral restraint conditions in the strain-hardening range. A secondary experimental investigation was also conducted, which introduced a new, full cross-section tensile test to supplement traditional tensile coupon tests for the purpose of more realistically modelling material behaviour. Using both proprietary software and a model developed as part of this thesis, the finite element method was used to generate further data as well as reinforce conceptual understanding of the physical relationships observed. On the basis of these analyses and experimental investigations, simple design recommendations were made.

Chapter 3 outlined and presented the results of a laboratory testing programme of twelve four-point and two three-point bending tests on beams with discrete rigid and elastic lateral restraints to investigate the implications of utilising strain-hardening in the design of the restrained member on bracing forces and stiffness requirements. A parallel numerical investigation was performed in which finite element models were initially validated against the experimental results. Good correlation with the test data was achieved and on the basis of these models, parametric studies were conducted to investigate the implications of varying the restraint stiffness on the ultimate capacities of the beams and the corresponding forces developed in the restraints. The results from the parametric studies showed that at the minimum required stiffness, the restraint forces assumed their peak values, but multiples of this stiffness caused the forces to reduce rapidly, whilst ensuring the full capacity of the restrained member was achieved. It was concluded that provided suitable restraint spacings are used, consideration for strain-hardening in the design of the primary member does not result in restraint forces in excess of those derived from EN 1993-1-1.

A simplified analytical model was developed in Chapter 4 to investigate the influence of the key parameters of the CSM on the limiting values of $\bar{\lambda}_{LT}$, where it was demonstrated that cross-section geometry was the most significant parameter. Using a simple methodology, a conservative relationship between $\bar{\lambda}_{LT}$ and M_{csm} , which is entirely based upon strain-hardening material properties, was reviewed and shown to be safe but conservative. Using additional data generated as part of an analytical and numerical study,

as well as test data collected from the literature, a basic design approach was presented that incorporated a limiting CSM slenderness of $\bar{\lambda}_{LT} \leq 0.2$ as well as a transition function from M_{pl} to M_{csm} via the factor $\chi_{LT,csm}$.

Chapter 5 outlined and presented the results of a programme of six tests on continuous beams with discrete rigid and elastic lateral restraints, which were conducted to investigate the member stability implications of utilising the Continuous Strength Method in the design of statically indeterminate structures. Results showed that, for the cases considered, the CSM-compatible collapse load was either met or exceeded, with the CSM on average providing better predictions of capacity than conventional plastic methods. Variation in bracing stiffness did not significantly affect the ultimate capacity of the specimens, but as with statically determinate structures, to limit the forces developed in the restraints it is necessary to have restraints of sufficient stiffness to ensure that a mode-II failure (i.e. buckling between the restrained points) occurs.

Two numerical modelling approaches were chosen: one that used proprietary software (ABAQUS), which was used as part of a parametric study into the effects of restraint forces and the other, which was developed as part of this research, was used to conduct a parametric study into restraint spacing. Both of these models were successfully validated against the data obtained from the laboratory tests on continuous beams. The results from the restraint spacing parametric studies showed that, in line with statically determinate structures, distances between lateral restraints need to be reduced to attain the full CSM capacity compared with the requirements of EN 1993-1-1 for reaching M_{pl} . Restraint stiffness requirements were higher than required for statically determinate structures owing to the need to resist lateral and torsional forces at the support whilst undergoing plastic rotations. At CSM levels of load, restraint forces were well within the limits set out by EN 1993-1-1; however at ultimate load, forces of 3% of the compression flange force N_{Ed} were typically observed.

A programme of seventeen full cross-section tensile tests on hot-rolled I-sections and hollow sections, and cold-formed hollow sections was conducted in Chapter 6 as part of an investigation into the strain-hardening properties of different cross-sections and materials for use in the Continuous Strength Method. The overall behavioural response

of the cross-section tensile tests demonstrated that assuming an elastic, linear hardening material model for the Continuous Strength Method is a reasonable assumption and the values obtained for the strain-hardening modulus based upon the tensile coupon tests are overly conservative. A revised suite of material models was proposed and incorporated into the CSM capacity equations, resulting in a higher degree of accuracy when compared with experimental data.

7.2 Suggestions for further work

The findings of this research are based upon the analysis of experimental data, supplemented by numerical models, from which relationships are drawn by means of parametric studies. An initial first round of further research would be to develop rigorous analytical models for all of the stability topics covered in this thesis (Chapters 3 - 5); their predictions can then be compared against the data developed in this research. Possessing an understanding of this nature would permit the development of sophisticated design expressions that rely more heavily upon deterministic, rather than statistical relationships. This is particularly advantageous where testing can result in considerable scatter.

A second avenue of future research is the extension of the analysis to cover different cross-section geometries and material types (e.g. stainless steel, high-strength steel and aluminium). With the finite element model that has been developed as part of this research, material properties can be readily changed, whilst the incorporation of different geometries is also possible, provided that aspects such as movement of the plastic neutral axis are accounted for. Parallel analytical studies, in line with the previous recommendation would also be of considerable value.

For the full incorporation of the Continuous Strength Method into international design standards, a broader range of stability topics will need examination. A natural departure from beam stability would be that of beam-column stability to complement the CSM design expressions already developed for combined loading at the cross-section level. Knowledge of this, preferably from an analytical perspective, but also supported

with good quality numerical and experimental data, can then lead to the development of design guidance for using the CSM for the stability design of frame structures.

The Continuous Strength Method is a refinement of existing structural design methods, and in many respects it also represents a more streamlined approach to design. This combination may have wider application to classes of structures that may have traditionally resorted to more performance-based approaches to design, or where the design codes are less prescriptive, such as marine or offshore structures. Understanding the design requirements of these types of structures could lead to very interesting applications of the Continuous Strength Method, perhaps offering guidance where none was previously offered.

Overall, this thesis has provided future researchers with a good collection of numerical and experimental data, as well as a computer model, which can all be used as a departure point for wider applications of the Continuous Strength Method.

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