

A GENERAL MODEL FOR THE ANALYSIS OF
NUCLEAR PROCESS INSTRUMENTATION ^{DATA} ~~SYSTEMS~~

A thesis submitted for the degree of
Doctor of Philosophy
by

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To my wife and family.

ABSTRACT

This work addresses the twin problems of designing an instrumentation system to make measurements on a general nuclear process and then interpreting the results to recognise the required process features. The earlier chapters describe the motivation behind the approach and define the mapping from the output of a nuclear process to the observed signals in terms of practical hardware and real system examples. A modular electronic configuration is proposed, which incorporates a number of new circuits, and this may be used to collect data for the majority of nuclear processes. The second part of the work is the construction of a general mathematical model. This commences with the definition of an abstract signal space in which the measurement results from all of the above instrumentation may be represented. An examination of the 'naive' measurement concept traditionally associated with nuclear instrumentation is undertaken and a number of weak areas are identified. It is proposed that this concept is replaced with an 'axiomatic' concept based on the mathematical model. By the derivation of the model from the axiomatic base it is shown that this approach avoids the problems associated with the traditional concept.

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NOTATION

x	observed tracking peak centre voltage
μ	mean (set) voltage for tracking peak
k	'k-gain' constant
A	count in upper tracking window
B	count in lower tracking window
K	spectrum stabilisation threshold
λ	mean input pulse rate
q	probability of a count in the upper tracking window
p	probability of a count in the lower tracking window
E_k	k th energy interval
t_n	n th time interval
V_m	m th volume element
Γ_L	signal geometry
Ω_L	spectral image range
s	spectral image function
K	number of energy channels
N	number of time channels
M	number of detectors or scanning positions

τ	dead time period
π	prior density function
p	posterior/conditional density function
$\mathcal{L}$	likelihood function
θ	isotope activity vector $\theta = (\theta_1, \theta_2, \dots, \theta_n)$
ρ	bias parameter vector $\rho = (\rho_1, \rho_2, \dots, \rho_{m-n})$
s	signal vector $s = (s_1, s_2, \dots, s_r)$
k	constant/normalisation factor
R^m	m-dimensional Euclidian space
w	composite vector $w = (\theta, \rho)$ s.t. $w \in R^m$
R^r	r-dimensional Euclidian space
μ	mean signal vector $\mu = (\mu_1, \mu_2, \dots, \mu_r)$ s.t. $\mu \in R^r$
e	basis vector of R^m
f	basis vector of R^r
g	linear transformation $g: R^m \rightarrow R^r$ s.t. $\mu = Gw$
G	transformation matrix of g
g_{ij}	element of G
Ω	sample space s.t. $w \in \Omega \in R^m$
F	σ -field
P	probability measure
X	random process (for sample configuration)
$X(t)$	random vector in continuous time
X_t	random vector in discrete time
x	sample path of X
t	index
T	index set
Σ	state space s.t. $x \in \Sigma_X, s \in \Sigma_S$
f	probability density/mass function
F	finite dimension distribution function
E	expected value
cov	autocovariance
ψ	conditional expectation operator
h	arbitrary function
S	random process (for signal configuration)
s	sample path of S
$\hat{}$	estimated value
$(\dots)^T$	transposed vector

V, U	covariance matrices
μ	mean vector of prior density
λ	selection parameters
L_C	decision level
L_D	detection level
L_Q	determination level
α	error of the first kind
β	error of the second kind
σ	standard deviation
k_α, k_β	abscissa of the standardised normal distribution
A	photopeak area
B	background signal
v, w	spectral window widths
r	spatial position $r = (x, y, z)$
t	time
Ω	direction vector $\Omega = (\alpha, \beta, \gamma)$
α, β, γ	direction cosines
E	energy
i	type of isotope
j	particle type
I	number of isotopes
J	number of particle types
L	number of signals
θ	mass density of sample
λ	radioactive decay constant
ρ	corrected mass density of sample
T	duration of a measurement period
a	radioactivity of a sample
Γ_4	source geometry
Γ_7	flux geometry
Γ	general geometry
Ω_I	range of source density I-tuple
Ω_J	range of flux density J-tuple
ϕ	flux density
g_1, g_2	composite mappings
R	reaction rate
σ	microscopic cross section/particle emission rate

$\bar{v}$	mean particle yeild
S	source strength
p	particle emission coefficient
δ	Dirac/Krondecker delta functions
Σ	macroscopic cross section
G	Green's function
$\hat{K}$	Integral operator
ξ	detection efficiency function
$\langle . . \rangle$	inner product
E	detection efficiency matrix
P	emission coefficient matrix
e	element of E
ρ	correlation coefficient
$T_k(E)$	Chebyshev polynominal
A	coefficient matrix
T	polynomial value matrix
$\mathbf{v}$	velocity vector of a particle
v	speed of a particle
$\mathbf{k}$	spatial frequency vector
ω	angular temporal frequency
ν	temporal frequency
x	spatial Fourier transform
ψ	temporal Fourier transform
a	first order residual
θ	polar angle
ψ	azimuth angle
ψ^+	forward current density vector
ψ^-	backward current density vector
P^{++}, P^{--}	forward tensors
P^{+-}, P^{-+}	backward tensors
α^+	mean forward particle production cosine
α^-	mean backward particle production cosine
β^+, β^-	mean orthogonal particle production cosines
$\mathbf{x}$	even parity current density vector
ψ	odd parity current density vector
P	even parity tensor
Q	odd parity tensor

ξ^+	forward detection efficiency
ξ^-	backward detection efficiency
η	even parity detection efficiency
ζ	odd parity detection efficiency
H_m	quadrature weight
a_m	quadrature coefficient
h	inversion function
a	inversion coefficient
E^*	fixed geometry detection efficiency matrix
b^*	fixed geometry background vector
η^*, ζ^*	fixed geometry detection efficiency vectors

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CHAPTER 1

INTRODUCTION AND RESEARCH REQUIREMENTS

1.1 NUCLEAR PROCESS MONITORING AND CONTROL

From the development of the first nuclear reactors to the present day the commercial use of nuclear energy to generate electricity has been subjected to adverse criticism. In part, this is because of its association with nuclear weapons, and also because of major incidents such as the Windscale fire and those at Three-mile Island and Chernobyl, which have done much to destroy public faith in the technology. The primary safety consideration is the fail-safe design of nuclear reactors. This is achieved for reactors with negative void coefficients, secondary containment and other features laid down in the various codes of practice for reactor design. However, even with intrinsically safe reactors, public concern still exists over issues such as the radiological protection of both people and the environment and the safe disposal of radioactive waste.

It is reasonable to suppose that nuclear electricity will continue as an important supply of energy since, despite its well-publicised

shortcomings, detrimental effects are also associated with competing energy sources. For example, over the past few years, problems have arisen in the burning of fossil fuels to produce electricity. These include not only climatic conditions such as the greenhouse effect and acid rain but also the possible short supply of these materials which are extremely useful for many other purposes and will become increasingly expensive as the present accessible reserves are used. In addition, it will be many years, if ever, before renewable sources of energy such as the wind, waves or sun can provide similar generating capacity to conventional or nuclear power plants, and then only with considerable destruction of the countryside.

It has been forecast that, in England and Wales, the demand for electricity will exceed supply by 1993 if no new generating capacity is added to the National Grid and that, by the year 2000, there will be a short-fall of some 13 GW [1]. Therefore, new power plants are essential. It is the policy of the CEGB to continue the development of a nuclear generating capacity together with new coal-fired plants. In particular, a programme has commenced to build six large pressurised water reactor (PWR) power plants. Sizewell B is already under construction and work was scheduled to start on plants at Hinkley Point, Wylfa and Trawsfynydd, and two at Dungeness, at one year intervals over the next five years, although these plants are now threatened by the impending privatisation of the CEGB. While this may cause a delay, it is unlikely to lead to complete cancellation of the programme when this is reviewed in 1994.

With the increasingly widespread development of nuclear power plants and support operations, such as fuel reprocessing, the radiological protection of people and the environment and the safe disposal of radioactive waste must be considered. In general, the record of the UK nuclear industry is good with respect to these issues but incidents of pollution or irregular waste disposal have occurred and, although these incidents are usually sensationalised by the media, it is possible that a real danger could be presented to the public in some cases.

Conceptually, the nuclear fuel cycle can be regarded as a collection of

interrelated processes. These can be combined in series or in parallel and large processes associated with complex operations can usually be subdivided into a combination of smaller processes with simpler operations. At each stage of the fuel cycle, desired products such as energy or nuclear fuel are produced. To obtain the correct process output, the normal method of control is to monitor the output products and then adjust inputs to the process so that the desired products are always observed. For example, in the case of a PWR, the operating power is dependent upon the neutron flux density in the reactor. Therefore, to control the process, the flux density is regulated by adjusting the boron concentration in the coolant water and the height of control rods in the reactor so that the required power output is maintained.

In addition to the desired products, nuclear processes also produce unwanted by-products such as radioactive waste. These are usually fed into separate waste-treatment processes and then stored or sent to a permanent disposal site. However, the by-products are often in an awkward form, for example, as trace contaminants in reactor coolant or storage pond water, in which case the extraction of the waste materials can involve complex filter networks. Also, for used fuel elements, the waste material will be in chemical compound with re-usable fuel. Therefore, to extract the waste materials, decladding and chemical separation operations are necessary prior to their stabilisation and packaging for storage or disposal. In all of these processes, potential exists for the development of faults which may lead to an incident.

It is possible that the occurrence of many incidents could be avoided by the closer monitoring of the waste materials which are output from the processes *1 since detected irregularities in the composition or radioactivity of the waste can indicate that a fault has occurred. If a fault is detected early, appropriate control procedures can be initiated to halt the chain of events which could lead to an incident. Furthermore, the monitoring may provide data which could suggest ways of refining a process so as to reduce the probability of a fault occurring.

*1 Here the waste materials themselves are considered only insofar as they represent the output of a process.

This method of control can be applied to any process of the nuclear fuel cycle and for most of the waste streams output from these processes, subject only to the availability of suitable measuring apparatus and the achievability of the necessary system or operator response to a fault condition.

A schematic diagram of a general process monitoring and control system is shown in Figure 1.1. It can be seen that there are three high level stages to such a system, which are data collection, data processing and nuclear process control. The operation of these stages is not independent.

To a large extent, the basic hardware associated with process monitoring systems - for example, detectors, nuclear materials, electronics, computers etc - is at a state of development which would be sufficient for most plant applications. In fact, in many cases, to obtain even a marginal improvement in the performance of a system by the further optimisation of the hardware, a large capital investment would be required. However, there are still basic problems associated with the overall system design and the recognition of required features of the source (nuclear process output) from the observed data spectrum.

The features of a source which need to be quantified are governed by the characteristics of the nuclear process and these remain fixed for a particular application. The recognition problem to quantify these features may only be soluble under certain specific conditions. Therefore, to design a data processing system, it is necessary first to design a data collection system in such a way that a solution to this problem exists.

The operating concept of conventional nuclear instrumentation systems is to reduce the observed data to a small number of statistics which can be related directly to the radionuclide activities of the source by one or more calibration constants. However, a data spectrum may not allow statistics to be formed without a significant loss of information concerning these parameters.

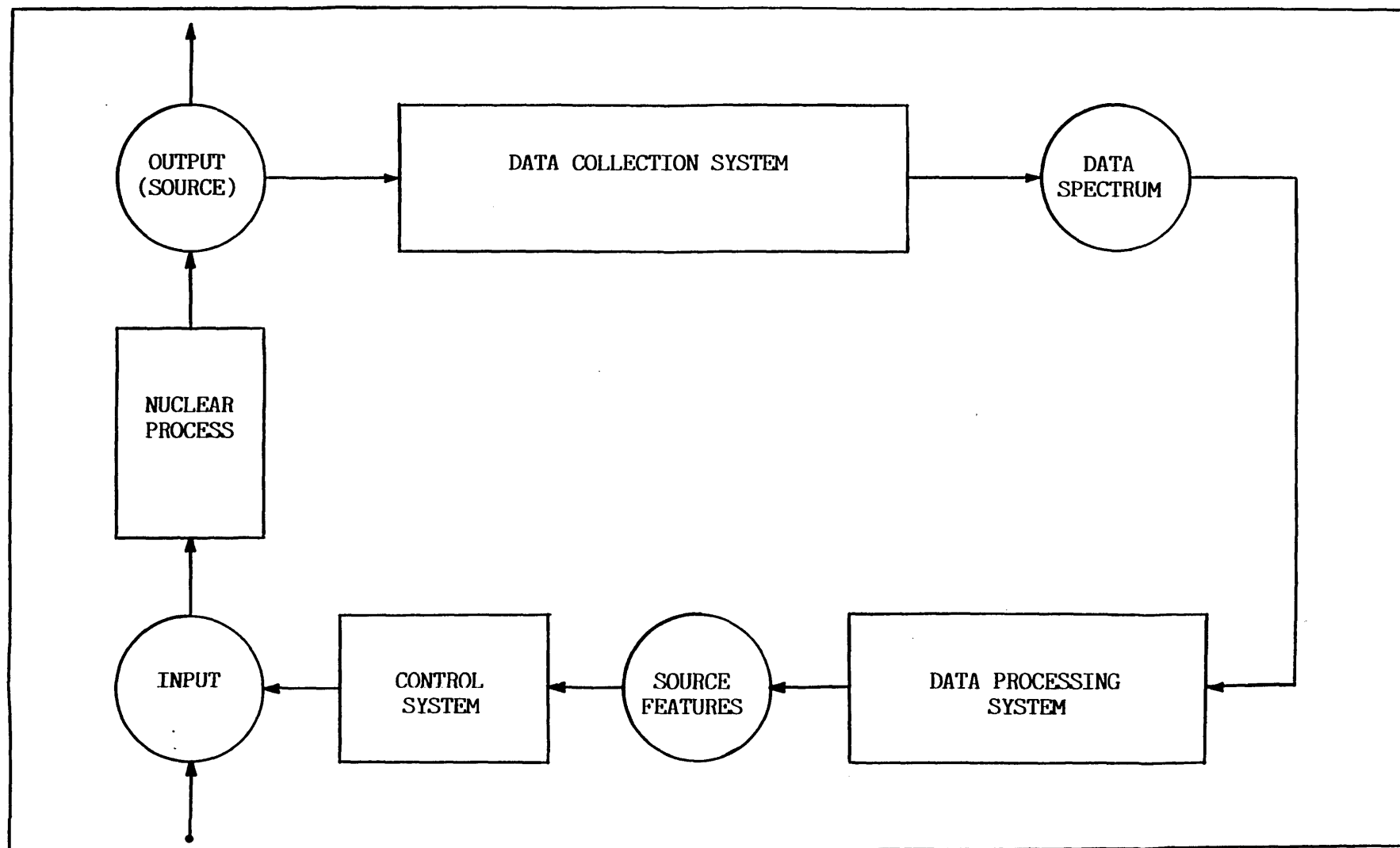


FIG 1.1 NUCLEAR PROCESS MONITORING AND CONTROL SYSTEM

Furthermore, for measurements on a nuclear process, the observed data could depend upon many features other than the composition and radioactivity of the source and these additional features may vary with the observations. Therefore, even if the statistics can be formed, exact calibration relationships cannot be established and the best that can be achieved is calibration of the system in terms of some typical output of the nuclear process. This leads to a fundamental uncertainty in all of the measurement results. Indeed, for many applications, the uncertainty may not be bounded and this severely limits the usefulness of monitoring for these processes and the effectiveness with which they can be controlled.

To improve upon this situation, it is proposed to undertake a study of the way the measurements can be made if they are treated explicitly as measurements on the collection of all possible process outputs, rather than the individual outputs. For, with this apparently trivial observation, it is possible to formulate a statistical treatment for the measurement data, based on existing general theory, which could offer a significant improvement in both performance and applicability of the process monitoring systems. This approach will be illustrated by a small number of real applications. In particular, it is intended to address both the problems of system design and source recognition for the nuclear process monitoring systems. The problem of nuclear process control will only be considered insofar as it directly affects the design and recognition problems.

1.2 PRODUCT DEVELOPMENT

The requirement for this research has arisen from the needs of a process instrumentation company (Plessey Controls Ltd) with respect to the development of on-line instrumentation for the monitoring of nuclear processes. In this section, an outline of the industrial considerations which have played an important role in directing this work will be given.

Until fairly recently, contractors such as Plessey Controls Ltd have

operated primarily as design and manufacturing houses. That is, using the design and manufacturing capabilities of the contractor, the end user of the process monitoring equipment would take responsibility for the development of a system, outlining the operations step by step and funding the work on a cost-plus-profit basis. Today, design and manufacturing contracts of this kind are rarely obtainable. The current trend is for end users to require customised systems, with short lead times and at low cost, expecting industry to fund its own research and development. This places all the responsibility for a system upon the contractor and requires that contractors adapt their operations to the development and competitive marketing of complete products and systems rather than supplying a design and manufacturing service to the end user.

While the market for nuclear process monitoring equipment is wide in the sense that many diverse applications are encountered, it is not particularly deep in terms of system quantities for each application. However, because of the complexity of the systems, if each new design is started from a clean sheet of paper, a long research and development phase will normally be required to establish a final design for the system. This situation presents a serious problem in that industry has no way of funding research and development other than by selling products and if each development project leads only to a small number of production units, these systems will be disproportionately expensive and hence unsaleable.

The solution to this problem is to produce more generally marketable products. However, since this must be achieved without losing the capacity to build customised systems, a new approach to development must be considered.

In a 'cost-plus' environment, the development of a system is directed towards the funder's specific needs and evolves independently of other systems. This tends to result in the direct empirical development of each system whereby a prototype is proposed, built, tested and modified until a final design is established. The design is then frozen and the production units manufactured.

The normal development procedure is to partition the system into a number of subsystems, for example, mechanical assemblies, power supply and distribution, data collection, data processing, control and data output. Each of these is divided into functional blocks, or modules, which are specified by their inputs and outputs and the ranges these can take. The modules are then designed in detail and combined (or integrated) to produce the prototype. Since each module is designed empirically, the system integration will invariably highlight problems. In fact, if all of the requirements of a measurement are not correctly identified at the system specification stage, a fundamentally defective prototype can be produced. With 'cost-plus', this is of no great consequence, since project setbacks are accepted as a normal part of development and the correction of design problems is fully funded by the end user.

To supply systems at a cost and time-scale which are acceptable to the modern user, it is essential to standardise system modules so that a complete system need not be developed for each application. To obtain the necessary range of application, the modules must be adaptable to a wider range of requirements than the modules developed for a specific system. This may make the individual modules more complicated; however, since the major cost of a system is in the development, increasing the applicability of the modules will result in a large reduction in the overall cost of each production unit.

The main areas where standardisation is possible are the detectors, data collection electronics and the processing hardware and software. A possible hardware configuration of such a standard system is shown in Figure 1.2.

The radiation detectors incorporated into process monitoring systems are usually of the ion chamber, proportional counter, Geiger Muller, scintillator or germanium type. With the exception of ion chambers, each of these detectors outputs charge pulses and for ion chambers, the continuous current output can be converted into pulses by use of a suitable pre-amplifier. Therefore, it is possible to choose a standard range of detector and pre-amplifier pairs with the same output format.

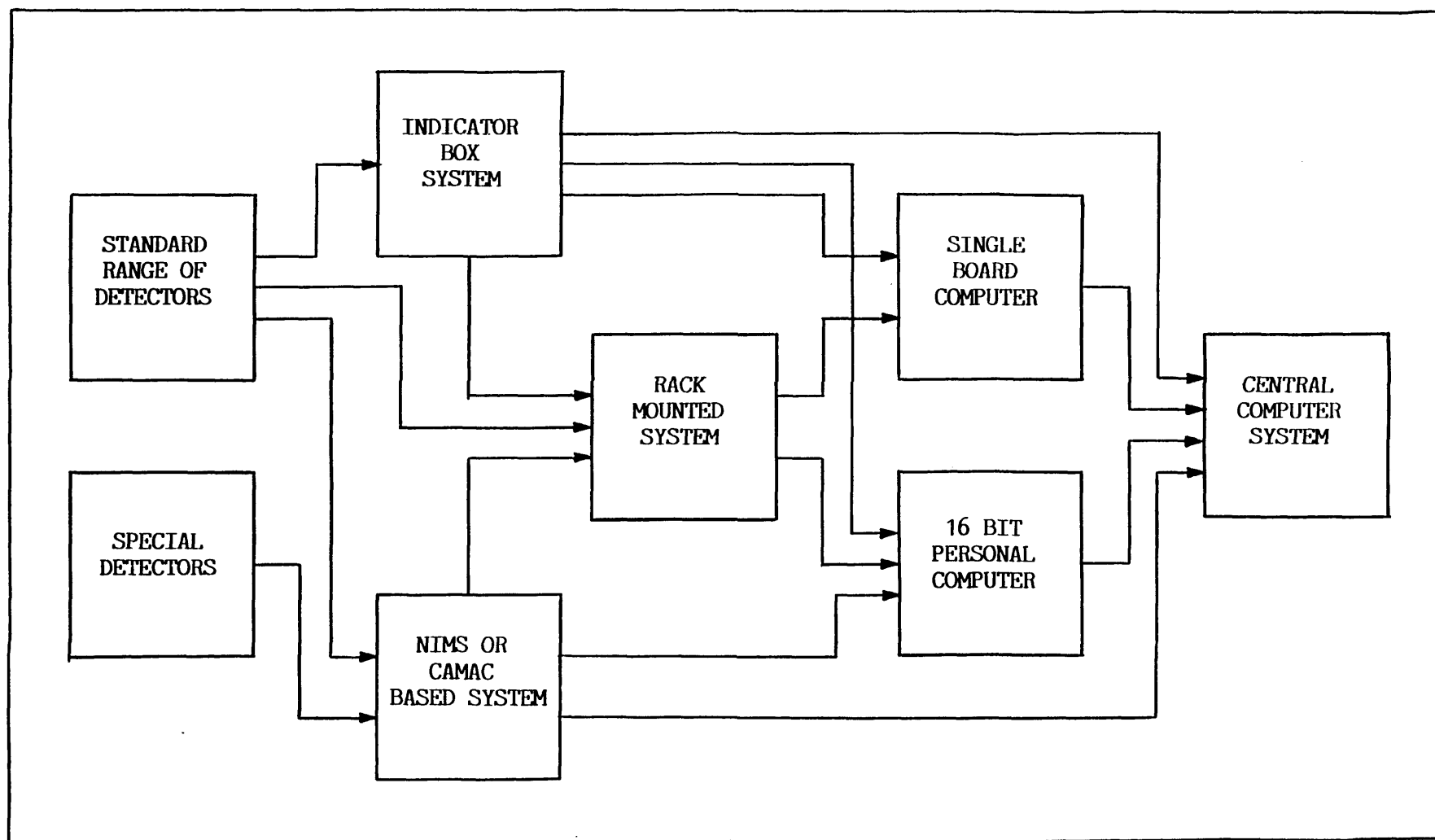


FIG 1.2 HARDWARE CONFIGURATION FOR STANDARD MODULAR SYSTEMS

However, this does not necessarily imply that it will be practical to interchange the detector types, keeping the subsequent electronic pulse shaping and data collection circuits the same. This is because the detectors have different energy resolutions and high resolution detectors, such as high purity germanium, require more complicated electronics than lower resolution detectors. In terms of electronic modulisation, the best that may be achievable is to adopt a small number of module formats which together cover the range of detectors.

The concept of modular electronic hardware is not new within the nuclear industry. In fact, nuclear instrumentation modules (NIMS) [2] are standard throughout the world and, although these are primarily designed for laboratory use, application in a plant environment is made possible by enclosing the electronics in a suitable industrial package. The NIMS are generally high performance and precision systems and can be used with any pulsed type of detector. However, they necessarily contain a large redundancy of components to obtain their wide range of application and each module performs only a single operation. This makes them bulky and expensive. Therefore, NIMS are best used for high resolution systems where the complexity of the application ensures that many of the built-in features are required and the cost of the modules and industrial packaging of the system is justified.

Another electronic format used within the nuclear and high energy physics fields, is CAMAC [3]. Apart from the mechanics of the systems, and the fact that NIMS originated in North America while CAMAC is European, these formats are very similar. Today NIMS seems to be more widely used.

For integrating or low resolution systems, NIMS or CAMAC represent an overkill; therefore, modulisation is best achieved by another route. Here, the objective is to simplify and combine operations which may be carried out by several NIMS or CAMAC modules on to a single printed circuit board (pcb) which can be mounted in a standard indicator box together with power supplies and other peripheral devices. By configuring the input stage of the pcb in such a way that components can be interchanged to account for different pulse shapes, the same pcb

can be used with different types of detector.

Furthermore, if the data collection circuit is designed to cope with the most complicated requirement expected for this system, the pcb can be configured for simpler applications either by omission of the components which are not needed or by software selection.

A third possibility for the modularisation of electronics, which is especially useful for multiple detector systems, involves pcb's of a standard size which can be mounted directly into a rack. Several pcb's of the same kind can be included to provide input lines for the detectors together with data collection and microprocessor cards. Communication between pcb's is via a digital bus incorporated into the backplane of the rack. Several versions of each data input and collection card can be kept, thereby allowing the redundancy of the NIMS or CAMAC modules to be avoided. For applications where all of the detectors form part of a single measurement system, these will be directly connected. Alternatively, this configuration can be used to collect data from several of the indicator box systems which may be distributed about an industrial plant.

Both NIMS and CAMAC modules are peculiar to the nuclear industry and this excludes the use of general electronics designed for wider industrial applications. The Eurocard format [4] is standard throughout the whole electronics industry. Furthermore, this format allows standard sized pcb's to be configured to produce racked systems, in which the pcb's can be inserted directly or as modules, or as single units which can be mounted in an indicator box. Therefore, Eurocard may be superior to both NIMS and CAMAC, although these formats may have to be adopted for certain applications to maintain compatibility with existing (proprietary) equipment.

The flexibility of the Eurocard format can be further enhanced by the effective use of computer aided design (CAD) and the automatic testing (ATE) of the pcbs. For example, it is possible to design a standard motherboard with blank areas in which different blocks of circuitry can be inserted by the computer. The motherboard would be configured with

the components needed for communication, together with an arrangement of standard tracks to which the input and output tracks of the circuit blocks can be connected, and these provide the connections between the circuit blocks and with the outside world. The circuit blocks themselves can be configured to perform the function of standard modules. Hence, all manner of pcbs can be produced simply by arranging the standard blocks of circuit on the motherboards.

Today, it is common practice to separate the front-end hardware (i.e. detectors, amplifiers and data collection electronics) and the data processing system. With this approach, the electronic modules are controlled by an on-board microprocessor while the data processing is carried out using standard "state of the art" computers plus applications software. This strategy can be implemented with all of the above systems and offers many advantages over conventional systems where an embedded computer is configured specifically for the application and performs all of the control and processing operations. These advantages include:

- 1) Computer technology is advancing too quickly for instrumentation contractors to compete with organisations such as IBM and DEC. Therefore, by the time an embedded processing computer is developed by a contractor, it may be obsolete. By configuring separate front-end hardware, it is possible to upgrade the processing system without having to replace the data collection system and it becomes possible to keep up to date with processing hardware.
- 2) The on-board microprocessor software can be made reasonably straight-forward, while the additional processing power offered by standard computers will enable high-level modular software with a wide range of applications to be constructed. Furthermore, the processing techniques which can be implemented may offer considerable improvement in system performance over the conventional techniques.
- 3) The use of modular electronics together with standard computers offers considerable cost savings since the development costs of the

computer are covered by the large production runs of the manufacturer.

- 4) Networks involving many measurement stations can be constructed by connecting local computers into a central processing system using standard network software.

To use this strategy effectively, it is necessary to adopt a particular family of computers and to design hardware and software with this family in mind.

Presently, IBM computers are supported by many instrumentation manufacturers. Therefore, if this family is adopted, it is possible to incorporate proprietary electronic modules and software within one's own systems.

1.3 THE SYSTEM DESIGN PROBLEM

It is common practice to consider a nucleonic measurement system as a sensor (or detector) coupled to electronic data collection equipment which inputs signals from the sensor and then stores or displays them in some pre-specified format. This instrument concept is adequate for the purpose of system design only if the following conditions are met:

- 1) The input signals from the sensor are not corrupted by noise, losses or contamination.
- 2) The quantities about which information is required are the same as those measured by the sensor.

These ideal conditions may be encountered in practice for many process control applications. However, for the measurement of radioactivity these conditions do not hold. In this circumstance, it is necessary to extend the above instrument concept to include the configuration of the radioactive source and the transport of radiation from the source to the detector.

This introduces a new step into the system design process, which is to design a source and detector arrangement which leads to the observation of meaningful signals. Since radioisotopes are rarely pure and the radiation can interact with many materials (including the source itself), it is unlikely that this design will be achieved without signal corruption. Therefore, it is necessary to consider methods for data collection and processing which allow the different components of the signals to be separated and estimates of the isotope activities formed.

A useful concept for visualising all of the factors which affect a measurement is that of a measurement process. This can be thought of as a sequence of stages linking the source to the observed signals. Then, the high level design of a measurement system can be thought of as the design of a measurement process. The particular advantage of this approach is that it focusses attention on the operational aspects of the system and allows the designer to consider these independently of their detailed hardware or software implementations. For example, typical stages of a measurement process may be the radioactive source, the radiation field between the source and detector, the analogue signals output from the detector and the digital signals (or data) collected by the system. To design a measurement process, it is necessary to construct a process model, which is simply a specification of the process configuration at each of its stages together with the possible variation associated with these configurations. Providing this task is carried out correctly, the propagation of signal corruption can be traced through the process and any problem areas defined.

Once all of the factors which affect a measurement have been identified, the measurement process can be refined so as to avoid problems and optimise performance. There are three ways to approach this:

- 1) Optimise the front-end system so as to reduce the signal corruption by the use of filters or by arranging for error factors to cancel as far as this is possible.

- 2) Design the source and detector arrangement and the data collection electronics so that the observed signals carry sufficient information regarding both the corruption and the required output parameters to allow corrections to be computed from the observed data.
- 3) Quantify the effects of the corruption in advance of the measurement and use this information to correct the observed signals.

A practical measurement system generally uses a combination of all three approaches and there will be trade-offs between them. However, because the first approach is naturally limited by the application while the second and third are limited by the instrumentation, it is important to specify the measurement process in detail before commitment to particular hardware.

To construct the process model for a particular system, the first step is to define initial conditions such as the possible distributions of radionuclides within the source. Then, using these initial conditions, the process configuration at each of its higher stages must be predicted. To do this it is necessary to define the operations which map one stage into the next.

For applications with a fixed source configuration (for example, a point source), these operations can be defined experimentally and will be expressed in terms of a calibration constant. However, for measurements on a nuclear process which is evolving in time, it is unlikely that a simple experiment can be designed to accurately simulate the real situation. Therefore, a superior approach may be to define only the fixed characteristics of the instrumentation experimentally and then incorporate this data into a theoretical model of the measurement process, where the initial conditions and the operations which map one stage of the process into the next are defined mathematically.

A strong case can be made for producing a realistic process model,

since this will eliminate the high risks associated with the empirical approach to development, allow reliable system optimisation and form a basis for the data processing system. However, a rigorous mathematical analysis of a measurement process is a time consuming task and will usually lead to equations in a large number of variables which can only be solved by computer simulation. Hence, the costs and time-scales associated with producing such a model will normally be prohibitive within the framework of a one-off development project.

The solution to this problem is to produce a general mathematical model of a measurement process which can be configured to describe any particular process as a specific case *2. This is possible because, in abstract terms, the measurement processes have many common properties. Therefore, by initially disregarding the specific details of a process which tend to obscure the common properties, it is possible to define general factors upon which the observed data depends for all of the processes.

Furthermore, since the majority of design and data processing problems are common to all systems, these can be solved once only for the general case and the solutions applied to all the specific cases. In fact, using the model, it is possible to develop a number of standard measurement processes which are mathematically consistent and can be taken off the shelf and implemented within a specific system, quickly and easily, using standard hardware and software modules.

The adoption of standard measurement processes will allow each process to be thoroughly tested and optimised before it is implemented within a system, with the final testing of the process being carried out experimentally. The collection of measurement processes also provides a basis for the design of the standard hardware and software modules. That is, a measurement process defines the sequence of operations to be implemented within the modules and these can be specified in terms of the required operation and the method by which the operation is to be

*2 The construction of such a model will form a major portion of the present work.

carried out in addition to the ranges of their inputs and outputs. Providing the operations are successfully implemented, the modules will always fit together to produce a consistent system because the measurement process upon which they are based is consistent.

This methodology can be taken further by implementing the general model on a computer and using this to simulate the operation of each specific measurement system. With this development tool, it will be possible to select appropriate methods for the data collection and processing from the family of standard measurement processes and system modules, and to confirm that a proposed system will work before it is necessary to finalise the design. Also, there are certain aspects of a process monitoring system such as the source and detector arrangement which may be specific to an application. The computer simulation technique can be used to try out possibilities in this area and to define an optimum design. Even though the front-end geometry of a system may be unique to the application, the design problem will always be of the same form. Therefore, the computer program can be applied to all the cases by changing the data and boundary conditions used in the simulation.

1.4 QUALITY IMPROVEMENT PROCESS

It is a fact that the most successful companies are those who supply quality products that customers want to buy. In everyday use, "quality" has an unclear meaning which is somehow associated with goodness. This definition is not very helpful to companies struggling with the problems of the development, manufacture and marketing of products. Therefore, companies tend to use a formal system to define and improve quality. Plessey Controls Ltd. use an American system supplied by Philip Crosby Associates, Inc. [5]. It will be shown that a formal product development system, incorporating the ideas of the previous sections, follows naturally from the principles of quality improvement.

According to Crosby, quality is defined as "conformance to a customer's requirements" and the basis of quality improvement is to understand

exactly what these requirements are and how to meet them. Here, "customer" can refer to an internal customer within one's own organisation in addition to the external customer to whom monitoring systems are supplied. A key concept is that all work is a process. The system used to achieve an improvement in quality is one of defect prevention by the design of the work process rather than the appraisal and correction of non-conforming process outputs. This is implemented by monitoring the outputs of the work process and testing these against a quality performance standard of zero defects. If defects (or non-conformances) are found, these must be corrected and the cost incurred is used to evaluate the seriousness of the fault in the work process which led to the non-conformance. The work process is then redefined (or regulated) so as to permanently eliminate the fault.

For nuclear process monitoring applications, there are two fundamental requirements which are common to all systems. These are:

- 1) To make decisions regarding the state of the nuclear process so as to determine whether the process is operating normally or if a fault condition has occurred.
- 2) To collect and log information regarding the nuclear process.

To meet these requirements it is necessary to make reliable measurements on the outputs of the nuclear process so as to quantify the parameters which define its state. Therefore, correctly designed measurement systems are essential to meeting the customer's requirements.

If an empirical (cost-plus) development methodology is applied within a fixed price or product environment, then there are two possible outcomes. Firstly, the systems will be expensive so few orders will be won, or, secondly, projects will be underfunded which leads to development short-cuts and defective systems. Clearly, there is need to regulate this process and the adoption of standard modules and measurement processes, discussed in the previous sections, is offered as a possible solution.

There are actually four different levels of processes within the present investigation. These are the nuclear processes upon which measurements are made, the measurement processes which define the sequence of operations performed by the monitoring systems, the process of system development and the quality improvement process itself. Here, the objective is to define a new work system for the development of measurement processes, together with the hardware and software modules necessary for their implementation, which will eliminate the current problems.

The installation of a new work system requires two preliminary steps, which are:

- 1) The definition of the work process.
- 2) The proving of the work process by trial applications.

To ensure that consistent measurement processes which will lead to conforming systems are constructed, it is essential to define the work process in such a way that the correct definition of the measurement and nuclear processes follows automatically. Crosby suggests a pictorial process model as a tool for defining a work process. The inputs to the process model are all of the requirements necessary to complete the work successfully and the process outputs are finished components, products or services which conform to the customer's requirements. The model is applied to the present situation in figure 1.3, and discussed in the remainder of this section.

The quality performance standard is particularly important since this asserts that it is only possible to achieve zero defects, in the sense of measurement processes which can be implemented to produce conforming systems, if all of the input requirements to the work process are correctly defined and supplied.

The requirements under the "inputs" heading of figure 1.3 are specific requirements for the measurement process and system modules under development. These include the definition of the nuclear process in

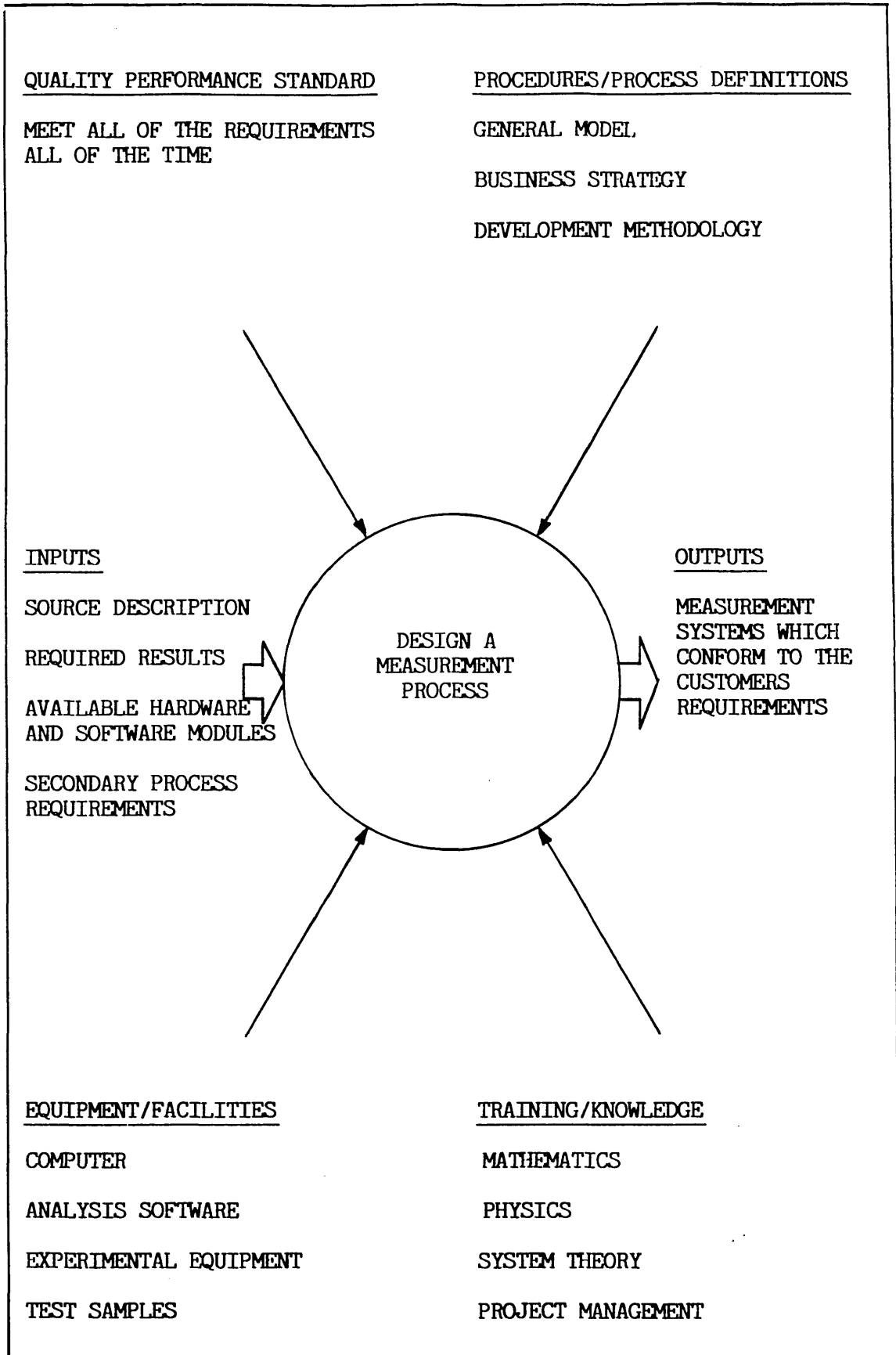


FIG 1.3 PROCESS DIAGRAM FOR THE DESIGN OF A MEASUREMENT PROCESS

terms of a source description and the measurement requirement in terms of the parameters which must be quantified to adequately control the nuclear process. The above specify the inputs and outputs of the measurement process and it is essential that they take into account all of the relevant factors over the expected application range of the systems.

To minimise new design work in the implementation of the measurement process, this should be developed around existing hardware and software modules as far as possible. If new modules are required, these should be compatible with all those existing so as to add to the standard range of products. Secondary process requirements such as the need for spectrum stabilisation should be identified and these should be developed in parallel with the main process.

It is helpful to distinguish between the "mechanics" and the "dynamics" of the measurement processes. The mechanics are the essential components to make the process work such as the detection of radiation, the formation of spectra and the calculation of results. To a large extent, these are taken for granted by customers who expect that the instrumentation supplier will have sufficient knowledge to automatically conform to these requirements. On the other hand the process dynamics are features such as nice graphic displays, ease of operation and the legibility of documentation. It is the dynamics in which the customer is interested and these sell the systems.

The present work is aimed primarily at the process mechanics for the following reasons:

- 1) The dynamics look silly if the fundamental operation of a process is defective.
- 2) If the mechanics are correctly defined by a standard measurement process, a greater proportion of time can be spent making the dynamics conform to the customer's requirements.

As far as possible the system modules should be portable. This means

that features designed to meet requirements which may not be the same for each customer, can easily be changed. For example, the modules should be designed so that any size system can be configured simply by adding in more modules, or features, such as display or data transmission formats, can be altered by changing permanent data rather than hardware or software.

The inputs to the model under the "procedures and process definitions" heading are used to guide the work outlined by the process model. In the present case, these are the general mathematical model of a nucleonic measurement system (see section 1.3), a development methodology and a business strategy.

In the present context the general model can be thought of as a knowledge base which provides the means to produce standard high level designs for a range of measurement systems which can be implemented with a reasonable cost and time overhead, and with confidence that the system will work. This is possible because the development does not start from a clean sheet of paper but from the existing state of the general model which may already be well advanced towards the final measurement process.

The development methodology (figure 1.4) states how to use the general model together with published data and reports to produce a theoretical concept for the range of measurement instruments and a specification for the associated hardware and software. This concept is then tested and modified until it is demonstrated to be satisfactory.

Proof-of-principle testing can be carried out experimentally where a simplified (experimental) system is constructed and measurements are made on a range of sources which are designed to represent outputs from a nuclear process. Adjustments are then made to the experimental equipment and the testing repeated until a suitable process configuration is defined. To extrapolate from the simple experiment to the real application it is necessary to simulate the operation of the measurement process using a computer. This generates results using a combination of theoretical predictions and experimental data. To test

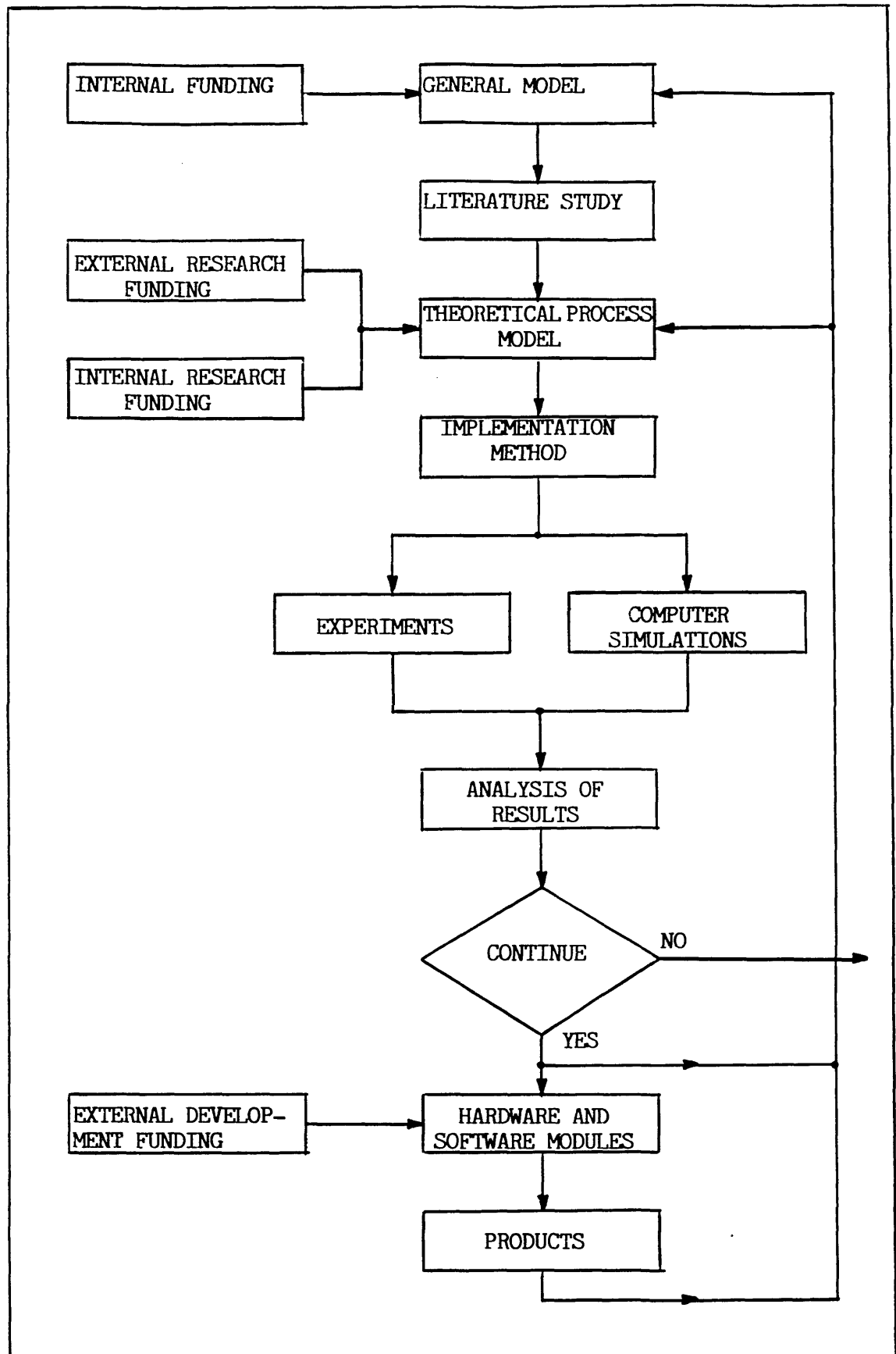


FIG 1.4 DEVELOPMENT METHODOLOGY

the simulation results, a number of specific cases can be constructed experimentally which are different from those used to calibrate the model. If these results agree with the simulation then confidence will be gained in the computer program and, once this program validation has been carried out for the range of applications, little or no further experimental input will be required when the process is implemented.

To follow the above methodology requires both the equipment and skills indicated in figure 1.3.

The third procedure is a business strategy, which is a statement as to the market areas of interest and the products best suited for this market, together with the commitment to support the work necessary to develop these products.

In particular, the business strategy should be chosen to maximise the profit - or, equivalently, to minimise the loss - associated with the exploitation of each market area. In practice, this is achieved by choosing the product development path which minimises "risk". In quantitative terms, risk is defined as the potential loss associated with an action, multiplied by the probability of that loss occurring. Therefore, the development path with the minimum risk is the path for which the total sequence of actions needed to develop a product represents an optimum balance between development investment and potential loss. This path is not necessarily the one involving the smallest capital outlay.

This completes the outline of the Crosby process model. The development and proving of the model are considered in the next section.

1.5 RESEARCH OBJECTIVES

In the previous four sections of this chapter it has been established that a formal method for the development of process monitoring systems may be feasible. The key to this method is the recognition that only a

limited number of measurement processes exist and that these have sufficient common properties to allow a general mathematical model to be produced. This model can be used to formulate solutions to both the problems of design and the recognition of nuclear process outputs, for a wide range of systems.

The above statement could suggest that a model is merely a convenient design tool. However, in physics, models are more fundamental. This is because observations of a physical system can only be made by experiment but experiments by themselves are meaningless since, without existing knowledge of the system, an experiment cannot be interpreted. Therefore, an essential prerequisite to any experiment is to construct a model which may be regarded as a plausible representation of some required feature of the system if it successfully predicts the outcome of the experiment.

Essentially, a model is a set of mathematical parameters which represent physical concepts such as length, energy or radioactivity, together with any relationships which are proposed to exist between these parameters. Each parameter can be represented by numbers which are regarded as multiples of a basic unit. Then, the relationships can be represented graphically by plotting corresponding numerical values of the parameters, where a point on the graph denotes a possible state of the underlying physical system and the relationship specifies the set of all states that a system can take. The validity of the relationships can now be checked by experimental observation, since the experiments can be interpreted in terms of the model. If they are not disproved, the relationships are accepted as laws, while the model is taken to represent the physical system over the range of parameter values for which it has been tested.

The graphical representation is limited to denoting relationships involving, at most, three parameters per graph. Therefore, although this method of representation is extremely useful, it is rather cumbersome for use with larger systems. Here, an alternative approach would be to use the tools of mathematical abstraction, which involves working directly with the equations for the geometrical configurations

which denote the relationships rather than their graphs. This method has at least two advantages over the graphical approach:

- generalisation to relationships between many parameters is relatively easy;
- given certain fundamental relationships or axioms, it is possible to derive new relationships typographically.

For the present application, the objective is to specify the radioactivity of certain radionuclides output from a nuclear process, from the results of measurement. Therefore, it is necessary to construct a model which embodies the relationship between these quantities.

More specifically this work will include:

- 1) An examination of current electronic signal processing techniques to determine methods for collecting data which result in a minimum loss of information concerning the sample, but do not produce more data than can be processed with a reasonable computational overhead. These techniques will be illustrated by considering, as examples, systems which have recently been investigated by Plessey Controls Ltd. In particular, a number of new circuits will be presented and the problems of spectrum stabilisation and dead time correction will be considered.
- 2) The construction of the general model which involves an examination of the intuitive concepts upon which instrumentation systems tend to be based and the development of an axiomatic formalisation of these concepts. This will lead to a Bayesian procedure for the interpretation of measurements and an investigation into the underlying properties of the nuclear processes which will be defined in terms of a Markov random process. The result will be a scheme which allows information concerning the nuclear process to be built up during the course of system operation and this information used to stabilise the measurement interpretation. An

example of this scheme's implementation using the multivariate normal family of distributions will be considered.

- 3) The general model will define the high level statistical aspects of measurements on the nuclear processes. In fact, this model could be applied to any source which is described by a Markov process which also includes many non-nuclear processes. However, a pure statistical approach may lead to a large number of parameters to be quantified if every dependence of the measurement is to be taken into account. Therefore, a deterministic model which provides a system of constraints will be considered. By nature, such a model will be more specific to a particular class of processes than the general model. For the present application, this will be based on the branch of statistical mechanics known as Transport theory, which is concerned mainly with neutron and gamma radiations. Even so, these radiations form the basis of almost all nuclear instrumentation systems and these provide a wide application field for the model.
- 4) Through out their development, both models will be applied by way of examples to the design and recognition problems associated with the systems illustrated.

This work establishes the theoretical basis of new techniques for the processing of data from nucleonic measurement systems, together with a design procedure which will eliminate the current high risk associated with the development of such systems. Furthermore, by adopting the procedures outlined, it is expected that an improvement in the technical quality and a reduction in the costs and lead times of the products will be achieved.

1.6 REFERENCES

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CHAPTER 2

ON-LINE MEASUREMENT PROCESSES

In this chapter the problem of collecting measurement data will be addressed and the hardware requirements of a typical data processing system identified. This will involve a brief discussion of the detection instrumentation and associated electronics together with methods for representing the observed signal data. It is hoped that this will provide a basis for the specification of standard electronic modules and a starting point for the construction of the general system model. To illustrate the concepts, the application to a number of real systems will be examined.

2.1 RADIATION DETECTORS

When one refers to "radioactivity", this normally means the primary (alpha or beta) decay-rate of a sample. Because of the non-penetrating nature of alpha and beta radiation, these quantities cannot usually be measured directly for nuclear process monitoring applications, but must be inferred from neutron or gamma-ray measurements.

There are three basic types of detector used for neutrons or gamma-rays; which are gas-filled, scintillators and semiconductor detectors.

The gas-filled detectors include ion chambers, proportional counters and Geiger-Muller tubes (GMT's). These are essentially an outer metal case which is earthed together with a central wire anode or cathode. This assembly contains a gas with which incident radiation interacts to produce ionization. The electron-ion pairs are swept from the active volume of the detector by the electric field between the outer case and the central wire, and collected to produce an output pulse. For slow charge collection, as in some ion chambers, the pulses build up to produce a d.c. output which is proportional to the total energy of the absorbed particles. Such detectors are referred to as "integrating". Fast charge collection results in distinct pulses which can be counted to produce an estimate of the incident flux. These are referred to as "pulse type" detectors and this is the normal operating mode of proportional counters and GMT's.

For the detection of gamma-rays with any of the three detectors, the filling gas is typically a mixture of argon and nitrogen. Here, electron-ion pairs are produced by Compton scattering or photoelectric absorption of the gamma-rays by gas atoms and the liberated electrons are collected by a central wire anode. As the electrons and positive ions are swept through the gas, they may ionize further atoms causing a multiplication of the charge deposited in the detector. In a GMT, this effect dominates and similar height pulses are produced by gamma-rays of all energies. For a proportional counter the multiplication is constant while, in an ion chamber, the effect is negligible so only the original charge contributes to the output signal.

Because of the constant charge multiplication, proportional counters output pulses with height proportional to that energy of the original gamma-rays which is deposited in the gas. Therefore, these detectors can be used for low resolution spectroscopy (although scintillators are now more common for this application). Neither of the other gas detectors allows the discrimination between gamma energies. However, by suitable choice of the argon to nitrogen ratio, and various

mechanical adjustments, it is possible to compensate these detectors so that the product of the gamma detection probability and the charge deposited by an event is approximately constant. Hence, they will be relatively insensitive to changes in the gamma energy distribution and the measurements can usually be interpreted in terms of the original flux using an experimentally defined calibration constant.

For neutron detection, proportional counters and ion chambers can be filled with boron tri-fluoride where detection takes place by the $^{10}\text{B}(n,\alpha)^7\text{Li}$ reaction, or with helium-3 where detection is by $^3\text{He}(n,p)^3\text{H}$. The charge resulting from ionization by the alphas or protons is collected by a central wire cathode. The cross-sections for both reactions are highest for thermal neutrons. However, since the overall detection efficiency is greater for the ^3He detectors, their energy range is extended into the epi-thermal region. For proportional counters, this introduces the possibility of energy discrimination. On the other hand, the BF_3 detectors give superior natural discrimination against gamma backgrounds.

A scintillation detector consists of a scintillation phosphor, which is a material which interacts with ionizing radiation to produce light pulses, together with a device such as a photomultiplier tube (PMT) or a photodiode for converting the light pulses into charge.

There are a large number of materials which will scintillate, including inorganic crystals, plastics, liquid and glass, and phosphors are available for the detection of gamma-rays, neutrons and alpha and beta particles [1], [2]. The main application within nuclear process monitoring is for inorganic crystals to detect gamma-rays. Of these thallium activated sodium iodide ($\text{NaI}(\text{Tl})$) is most common.

For inorganic crystal scintillators, the incident gamma photons interact with bound electrons in the crystal lattice by photoelectric absorption and Compton scattering or by the production of electron-positron pairs in the electro-static field of a nucleus. In this case, the positron will rapidly annihilate with an atomic electron to produce secondary gamma-rays which may undergo Compton or photoelectric

interactions. All of these processes liberate electrons from the crystal lattice and these dissipate kinetic energy in collisions with further electrons which are raised to higher energy states within their atoms.

When an electron is excited it leaves a positively charged hole in the valence band of the crystal. If the electron is not given sufficient energy to completely ionize the atom or, if an electron is subsequently captured, it will remain electrostatically bound to the hole. These bound electrons occupy a band called the exciton band, just below the conduction band, and the bound electron-hole pairs are called excitons. Since these are electrically neutral they can drift freely through the crystal. There are three important mechanisms by which the electrons return to the valence band:

- 1) Fluorescence - when the exciton is captured by an activation centre, exciting this atom, and the activator rapidly decays with the emission of a visible photon.
- 2) Quenching - when the electron returns to the valence band with the dissipation of heat but no photon emission.
- 3) Phosphorescence - when the electron is trapped in a metastable state of an impurity atom and this eventually decays with the emission of a photon.

Of the three decay modes, fluorescence is the desired mechanism, while quenching reduces the efficiency of the crystal and phosphorescence produces an unwanted afterglow effect. The activator is introduced into the crystal to optimise the fluorescence. The mean time for the electrons to return to the valence band is a constant known as the decay constant and for NaI(Tl) this is of order 250ns.

The scintillation photons collectively form a pulse of light with intensity proportional to the energy deposited in the detector by the original gamma-ray. For NaI(Tl), these photons have wavelength of order 415nm and this is particularly well matched to a bi-alkali type

PMT [3]. To detect the visible light, the scintillation crystal is mounted in an aluminium case which is lined with a reflective material such as MgO and contains a glass or quartz exit window. This is optically coupled, directly or via a light guide, onto the photo-cathode of the PMT. The light emitted by the scintillator liberates electrons from the photo-cathode and this charge is swept along the PMT, being multiplied at each dynode stage, until it is collected on the anode to produce an output pulse.

The observed variation in the output pulse height distribution will depend upon all of the events which lead to the formation of the pulses. Typically, for incident radiation at 660keV, the width of the distribution (at half height) is of order 55keV or 8%.

A semiconductor detector is essentially a p-n junction operated with a reverse bias electric field so that it acts like an ion chamber where charge created in the depletion region is swept out by the electric field to form an output pulse. The main difference between these detectors is in the energy resolution of the observed signals which arises from the mean energies to create a pair of charge carriers. This is of order 30eV to ionize a gas atom compared with 3 to 4eV to create an electron-hole pair in silicon or germanium [4].

A p-type semiconductor is doped with a trivalent impurity to introduce an excess of holes in the covalent bond structure. These act like positively charged particles, and can drift through the crystal carrying a current. Likewise, for pentavalent doped (n-type) semiconductors, the impurity atoms introduce extra electrons which act as the charge carriers. In the neighbourhood of a p-n junction, holes drift into the n region and combine with electrons, while electrons drift into the p region and combine with holes. This introduces a net negative charge in the p region and a net positive charge in the n region, which prevents the further combination of electrons and holes. If a reverse bias potential difference is applied to the crystal, the depletion region where the electron-hole combination occurs, grows wider but no current flows across the junction. However, if extra electron-hole pairs are created in this region by thermal excitation or

ionizing radiation, these are swept across the junction by the electric field and a small current flows, producing an output pulse.

There are a number of basic types of semiconductor detectors, including surface barrier and diffused junction detectors for charged particles, lithium drifted silicon (Si(Li)) detectors for X-rays and lithium drifted germanium (Ge(Li)) and high-purity germanium (HPGe) detectors for gamma-rays.

A drifted detector is made by diffusing lithium into the surface of a p-doped crystal of silicon or germanium, then heating and applying an electric field to drift the lithium through the crystal forming a large intrinsic region between the p and n layers. This allows the depletion region to be expanded to almost the size of the crystal by countering the effects of impurities in the crystal which act as electron trapping centres *1. To counter the effects of thermal noise and also because the lithium becomes mobile at room temperatures, drifted detectors must be operated and permanently stored at cryogenic temperatures (about 77 Kelvin).

In recent years, germanium crystals have been grown with a sufficiently low density of trapping centres to allow gamma-ray detectors to be produced without drifting. These HPGe detectors can be made from p-type germanium with lithium diffused contacts or from n-type material with boron contacts. In either case, the detectors can be stored at room temperature. However, because of the thermally produced carriers, liquid nitrogen cooling is still required for operation.

Typically for a Ge(Li) or HPGe detector, the variation in the pulse height distribution is of order 2keV (0.3%) at 660 keV. Comparing with an NaI(Tl) detector, this represents an improvement in energy resolution by a factor greater than 20. The importance of the energy resolution is that it allows the identification of photopeaks from

*1 These capture electrons for a time of similar order or greater than the mean charge collection time, and hence cause the loss of these electrons to the detection process.

individual isotopes in a mixed gamma emitting source, thus providing a simple method for measuring the isotopic composition in addition to the radioactivity of a sample. On the other hand, NaI(Tl) has advantages over Ge(Li) and HPGe with respect to gross detection efficiency, room temperature operation, the geometry with which the crystals can be manufactured and neutron damage resistance.

All of the above detectors produce small quantities of charge in response to the incident photons or neutrons. To be of use, this charge must be amplified and transmitted to electronic modules where the signals are processed and represented in the required form. A pre-amplifier is therefore an essential component of any detector.

Two pre-amplifier circuits are shown in figure 2.1. For a pulse type detector, the pre-amplifier is usually of the integrating (or charge) type. The output from this circuit is a voltage pulse with height proportional to the total charge output from the detector and an exponential tail of length governed by the RC time constant of the circuit. To obtain an accurate representation of the charge, the time constant should be large compared to the width of the input pulses (ie, of order 50 μ s).

The second pre-amplifier is used with integrating detectors to convert currents in the region of a few nanoamps to a sequence of pulses with frequency proportional to the detector output. This approach yields a more accurate representation of the charge deposited in the detector than the traditional approach where the output current is amplified directly. However, the amplification of the tiny signals from these detectors still represents a difficult problem and careful design and tight component tolerances are essential. Since the pulses from a GMT carry no information regarding the energy of the detected particles, there is little benefit to be gained from using a pre-amplifier which preserves the pulse height information. Therefore a simple voltage to frequency converter along the same lines as circuit B is all that is required, although an electronic quenching circuit may be added to the basic circuit. For these detectors the design and component tolerances are much greater than for the corresponding case of an ion chamber.

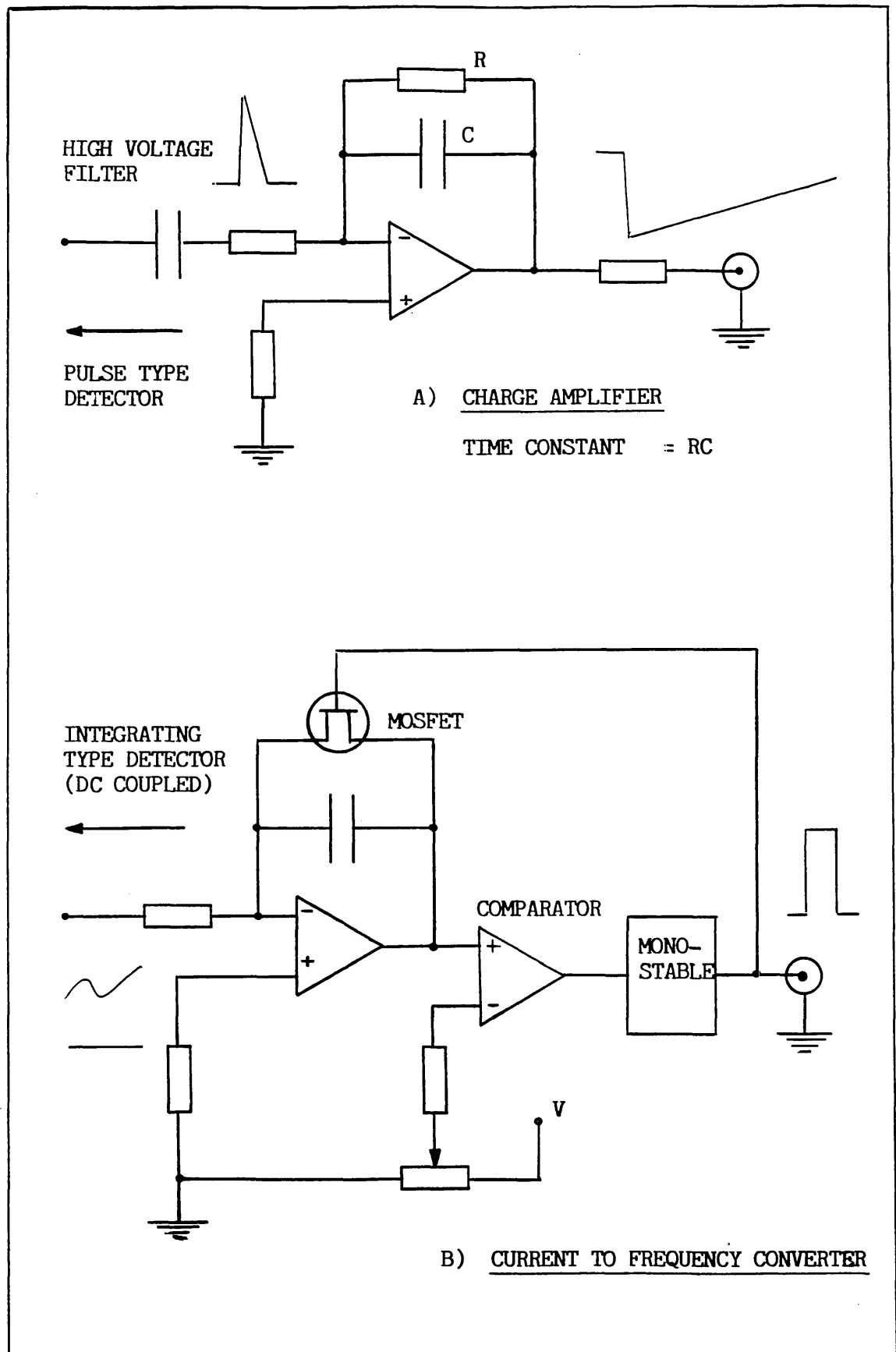


FIG 2.1 PRE-AMPLIFIER CIRCUITS

Between them these pre-amplifiers will cover the entire range of detectors. Therefore they can be adopted as standard interfaces between the detectors and the data collection electronics.

2.2 FRONT-END SIGNAL PROCESSING ELECTRONICS

For measurements on a nuclear process, the collection of data involves the six operations shown in figure 2.2. Of these, the pulse shaping, quantisation and stabilisation are essentially hardware operations which represent the input pulses in a digital form suitable for scaling. Since the output of a current to frequency converter is already in this form, these hardware operations are necessary for pulse type detectors only. It will be assumed that a charge sensitive pre-amplifier is incorporated into the detector assembly.

The pre-amplifier output is typically a pulse with a sharp rise time and a slow decay. For example, with a NaI(Tl) detector, these values are of order 250ns and 50 μ s respectively. At high count-rates the long tails of the pulses will cause base-line instability. Therefore, it is necessary to shape the pulses to remove the long tails.

A simple shaping amplifier is shown in figure 2.3. This differentiates each input pulse, adjusts the d.c level to remove the negative tail and re-integrates to produce a semi-gaussian shaped pulse. The d.c. adjustment is carried out by adding a proportion of the input pulse train to the differentiated pulses so as to cancel the negative component. This is known as pole-zero cancellation. The simple circuit is often adequate for proportional counters and NaI(Tl) detectors. However, for high-resolution systems the circuit is usually modified by the addition of active base-line restoration, gated integration and other features designed to minimise the loss of information regarding the energy of the absorbed particles. The basic modified circuit is shown in figure 2.4 *2. Alternatively, shaping circuits which minimise the loss of information concerning the timing

*2 See also figures 2.7, 2.8 and 2.9.

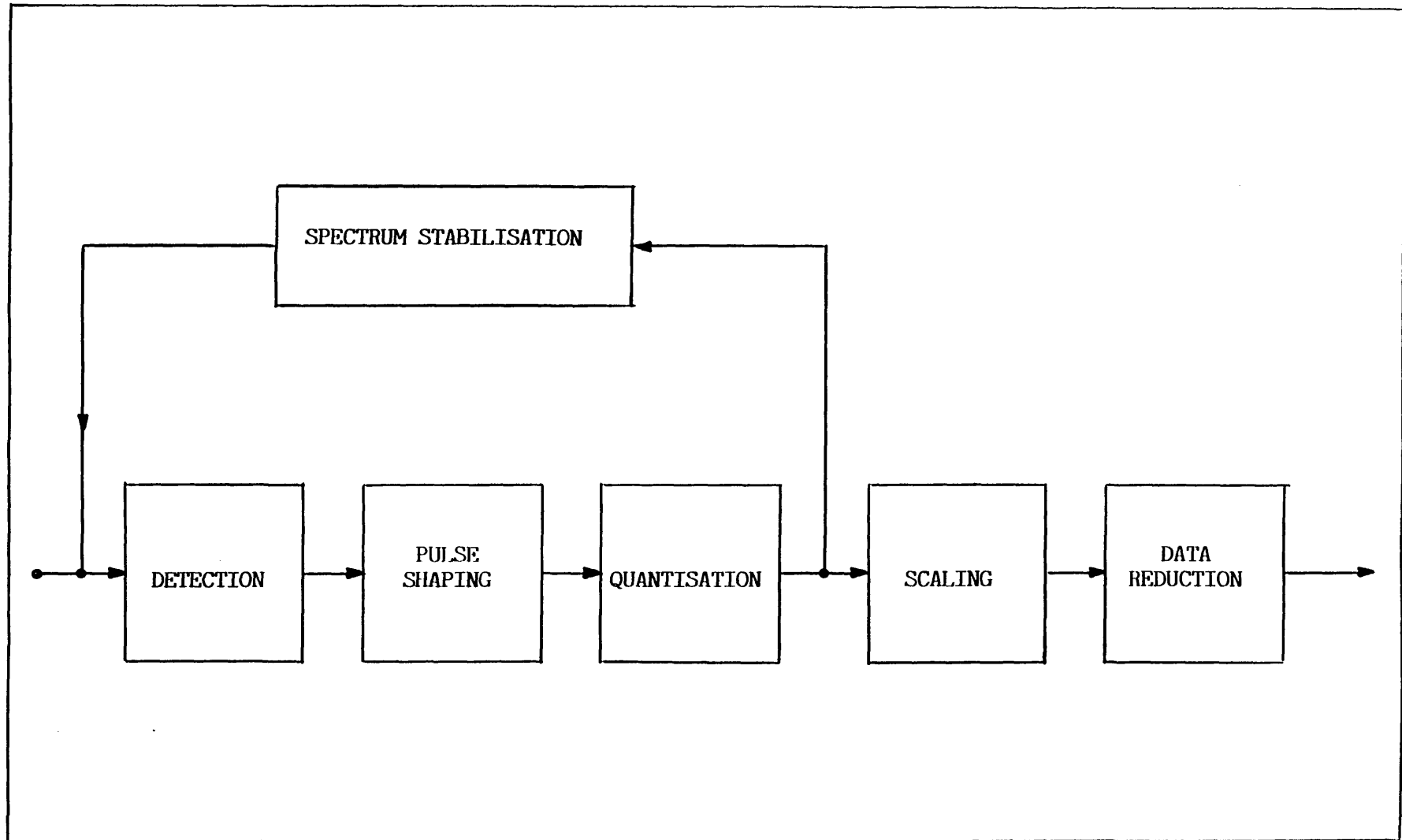


FIG 2.2 SCHEMATIC DIAGRAM OF A DATA COLLECTION SYSTEM

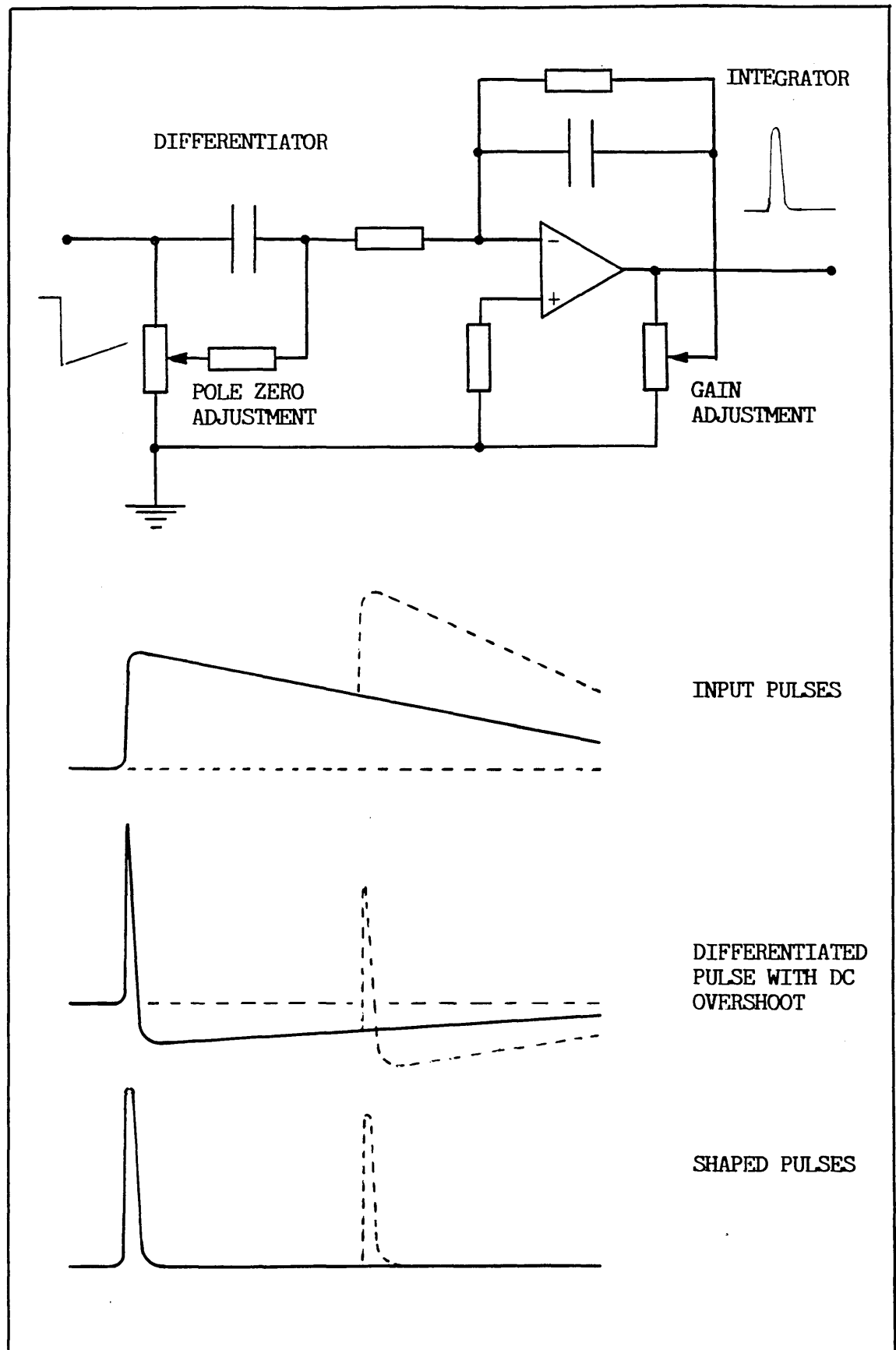


FIG 2.3 SIMPLE (SEMI-GAUSSIAN) SHAPING AMPLIFIER

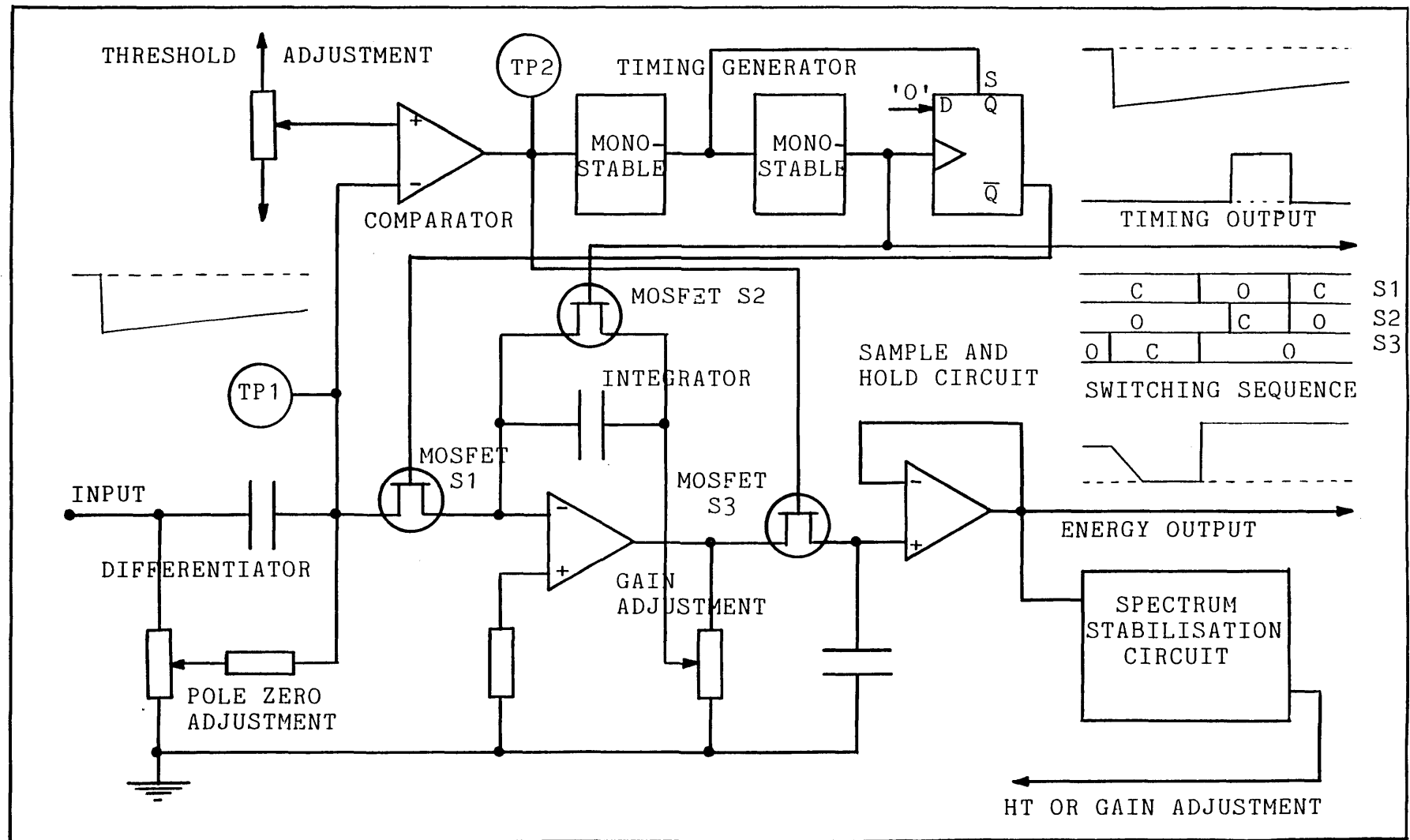


FIG 2.4 GATED INTEGRATOR SHAPING AMPLIFIER

of the pulses may be used [5]. Such a circuit, based on a delay line, is shown in figure 2.5.

To enable the subsequent scaling and data processing operations to be carried out it is necessary to assign numerical values to the analogue pulses output from the shaping amplifier. This process is known as quantisation. A schematic representation of a quantiser's operation is shown in figure 2.6. Here the height of each input pulse is tested against a set of decision levels and if the input falls into the interval between two decision levels it will be assigned a value associated with that interval. These values are called reconstruction levels. One possibility is to allow the k th reconstruction level to represent the centre of the interval between the $(k-1)$ th and the k th decision levels. If the intervals between the decision levels are equal, the quantiser is said to be uniform.

In certain systems, the voltage amplitudes may correspond to one of a set of expected values. For these systems it may be possible to arrange the decision levels so that they are more closely spaced about the regions where the most probable values occur. Furthermore, in some situations, improvement in system speed and efficiency may be obtained by assigning a separate quantizer to individual or groups of inputs. In recent years, intelligent, non-uniform quantisers which adapt themselves to a particular input have been developed. By optimum quantisation these systems offer reduction in processing overhead or improvement in accuracy over conventional systems.

If a quantiser consists of a single reconstruction level and one or two decision levels, then it is known as a threshold filter or a differential discriminator respectively. This circuit consists of one or two comparators and logic to output a digital signal if a pulse falls into a region of interest defined by the comparators.

Applications are for gross counting in a broad spectral region, distinguishing pulses from different types of radiation, for example, neutron pulses amid a gamma background and as an input stage to a timing or spectrum stabilisation circuit. Two differential discriminator circuits are shown in figure 2.7.

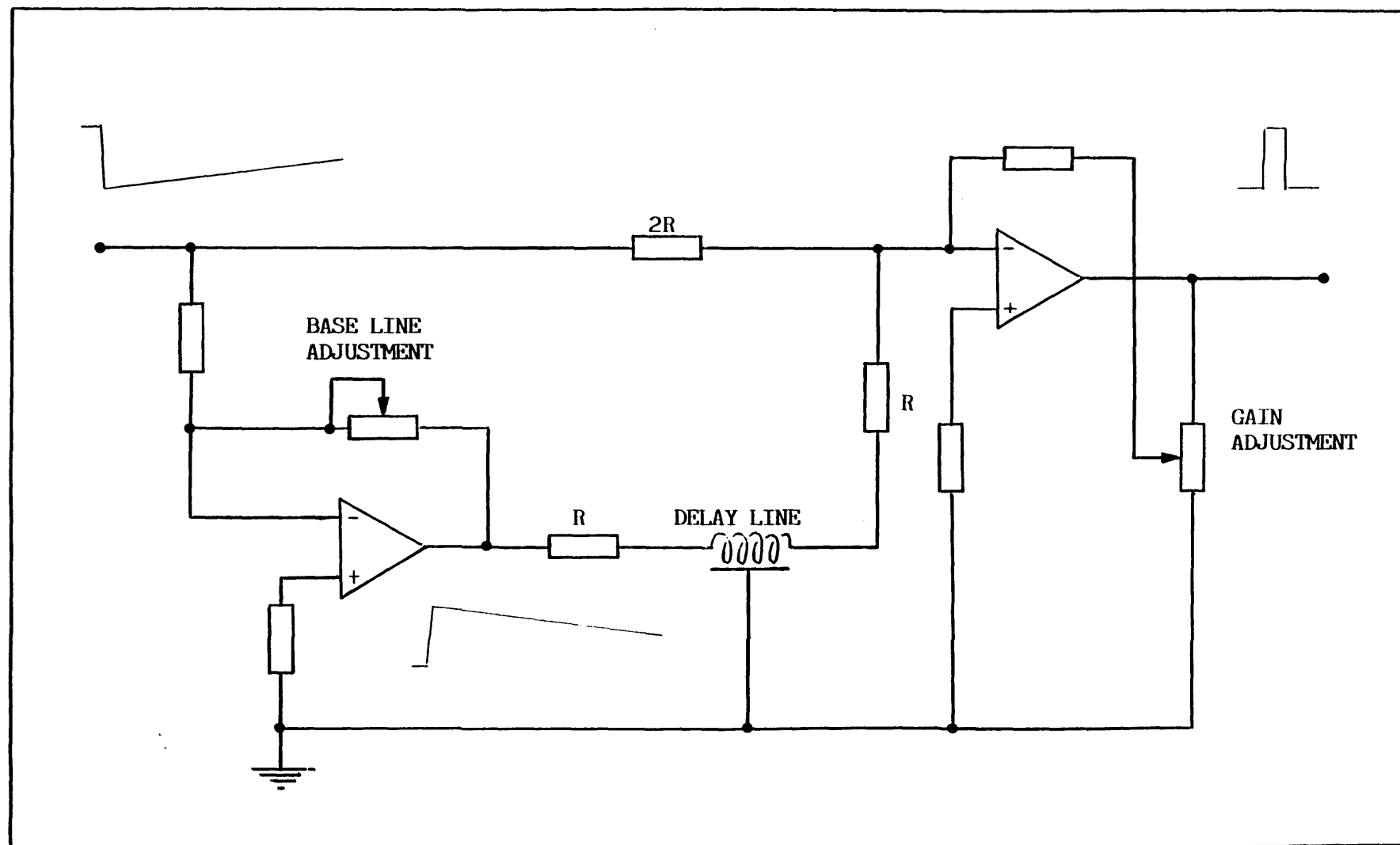


FIG 2.5 DELAY LINE SHAPING CIRCUIT

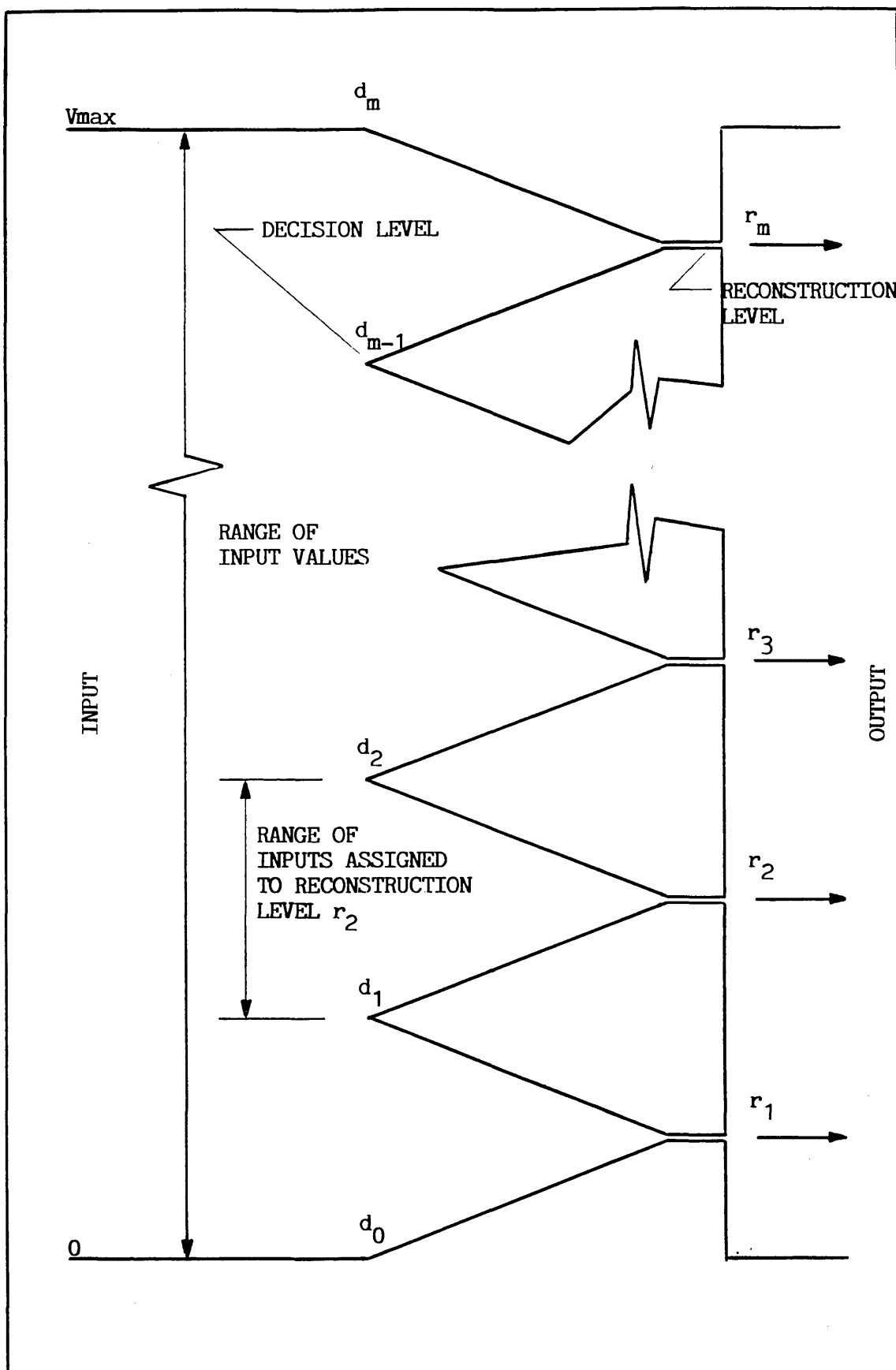


FIG 2.6 SCHEMATIC REPRESENTATION OF A QUANTISER

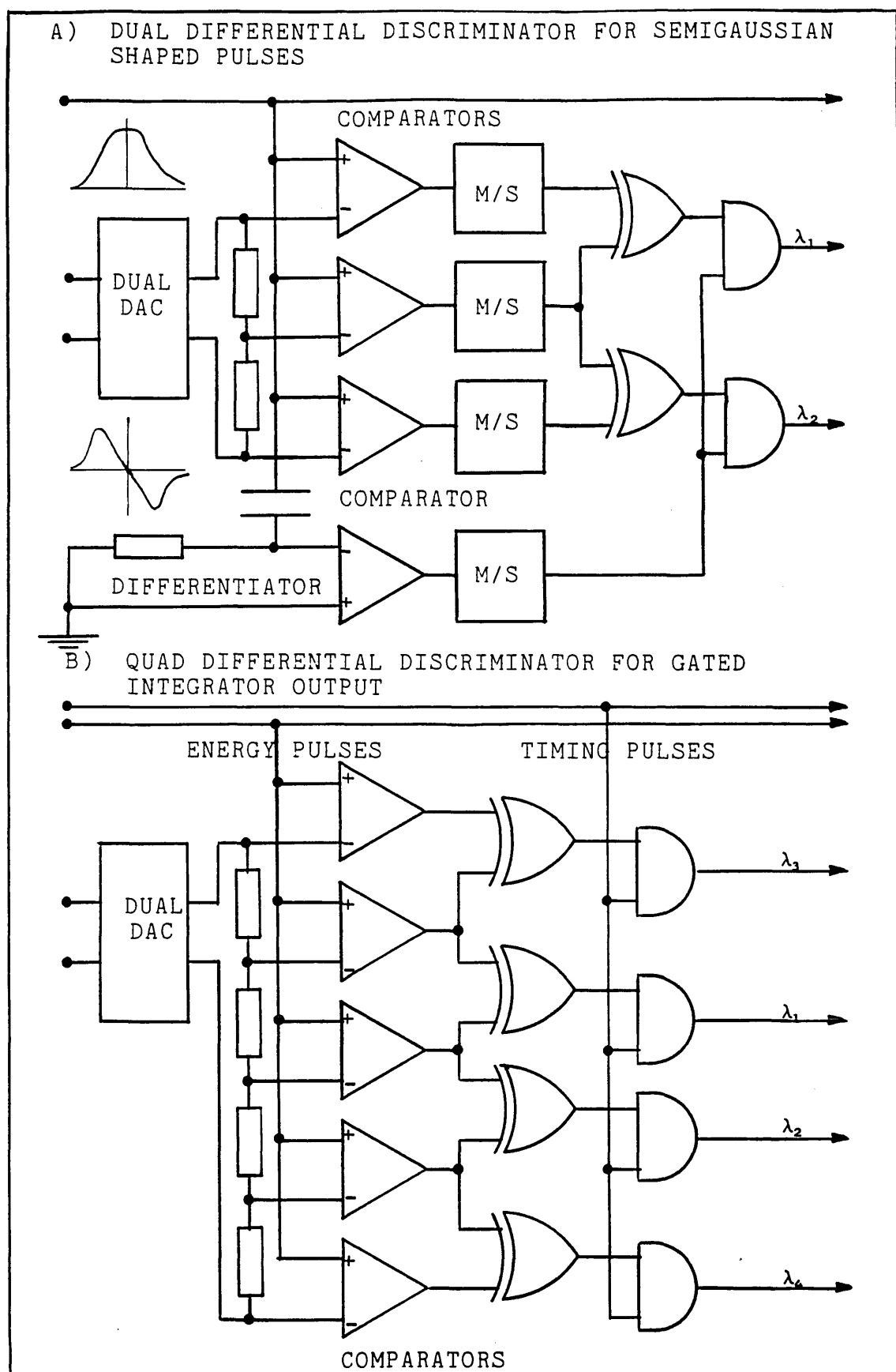


FIG 2.7 DIFFERENTIAL DISCRIMINATOR CIRCUITS

An analogue-to-digital converter (ADC) forms a multiple channel uniform quantiser. Of the three types commercially available, a dual-slope converter operates in tens of milliseconds *3, the successive-approximation type in a few microseconds and a flash converter in the nanosecond region. However, flash converters tend to be expensive; thus, for fast systems, a useful alternative is a hybrid successive approximation and flash converter. This has performance and cost between the two basic types. The main application of an ADC is for the collection of gamma-ray or neutron spectra where typically, a twelve bit (4096 channel) ADC is used; although 8-bit (256 channel) ADC's are used for many applications.

To obtain non-uniform quantisation, it is usual practice to use an ADC with uniform decision levels spaced at the smallest interval required, in conjunction with a further quantisation stage which may be implemented in hardware or software. For non-uniform quantisers, the optimum arrangement of the decision and reconstruction levels is defined using a model of the measurement process.

The detection of a particle involves a spatial integration over the energy deposited in the active volume of the detector, followed by amplification and signal processing. The three types of signal processing circuit - that is, pre-amplifier shaping as with a current to frequency converter and energy optimised and time optimised shaping combined with a quantisation stage - map the detector output into a discrete phase space with the dimensions of time and voltage, where a point in this space represents the observed particle(s).

As the measurement process evolves in time a random sequence of pulses is mapped into the phase space and the collection of these points embodies the total information available from the measurement. However, the quantities about which information is required are generally deterministic quantities such as mass, radioactivity and isotope type. The random point sequence cannot be related directly to

*3 Dual-slope converters are generally too slow for nuclear instrumentation purposes.

these quantities. Furthermore, the required information is encoded into a very large number of data items. Therefore, it is necessary to reduce the gross quantity of data and at the same time to strengthen the correlation between the observations and the required features of the source.

This is achieved by forming the energy and time distributions (or spectra) of the observed data as these distributions contain (almost) the same information as the raw data but in a much more useable form. To successfully collect an energy spectrum, it is essential that the height of the voltage pulses bears a constant relationship to the energy of the absorbed particles. In fact, to allow interpretation of the spectrum, this relationship must be the same for all of the measurements made by the system. The only way this can be guaranteed (to within reasonable limits) is to actively stabilise the spectrum throughout the measurements.

Spectrum stabilisation involves the definition of points at the bottom and top of the spectra and keeping the voltages corresponding to these points fixed. This will allow only second order variations due to spectral non-linearity to affect the measurement. With reasonable pulse shaping, these variations will normally be small.

The bottom point can be taken as zero and this is fixed electronically using a base-line restoration circuit. The top point is defined by implanting a radioactive source near to or within the active volume of the detector which emits at a fixed (high) energy. The pulses from this source are tracked and their height adjusted to keep the mean height constant.

The source used may be an alpha emitter such as Am-241 or a high energy gamma emitter such as Co-60.

For an NaI(Tl) detector *4, an Am-241 doped pellet of NaI may be

*4 In this case it may also be necessary to temperature stabilise the detector.

mounted inside the main crystal. As the Am decays the alpha particle produces a pulse of light, which is collected to form a tracking pulse. The voltage corresponding to the mean pulse height can be adjusted by varying the position of the source within the crystal to give a pulse height equivalent to a gamma energy in the range of one to four MeV.

Alpha sources can also be used to provide tracking pulses for He-3 and BF₃ proportional counters. Here, the Am-241 is mounted inside the detector and the alpha particles are detected in the same manner as the charged particles produced by incident neutrons. For Ar/N₂ filled detectors, it is normal practice to install a small gamma source into the detector. For proportional counters, this can be used for tracking while for non-energy discriminating detectors, the source provides a constant background signal which indicates the detector is working.

Since alphas may damage a germanium gamma-ray detector, the tracking pulses must be provided by a gamma source. The background produced by down-scattered gammas will be greater than the background produced by an alpha source and the energies corresponding to the tracking pulses will generally be lower. Therefore, spectrum stabilisation is more difficult for germanium detectors.

Having established the reference source, there are a number of approaches to the tracking. One method uses the dual differential discriminator of figure 2.7A to set two adjacent windows over the voltage region where the pulses are expected such that pulses with height above the centre threshold are observed in one window while pulses below the centre are observed in the other. For a scintillator or gas detector the HT voltage is stepped up or down to keep the counts in the windows the same while for a germanium detector this adjustment must be carried out by varying the pre-amplifier gain since the pulse height is almost independent of the HT. One way of doing this is to reduce the HT or gain by one step for each pulse observed in the upper window and increase the HT or gain by one step for each pulse observed in the lower window. While this method keeps the mean counts in the windows the same, there will be an oscillation about the mean value. This can be kept reasonable by choosing a small step length.

Alternatively, since spectral drift is usually gradual, it is not necessary to adjust the HT or gain with every pulse and there are two basic methods for determining an optimum adjustment. The first keeps the time between adjustments constant then sets the steplength to match the drift over that period. This requires minimum hardware but has the disadvantage of a software calculation of the steplength. Therefore, the method is best suited to simple systems where the need for minimum hardware outweighs the disadvantage of the off-line calculation.

For a gaussian tracking peak, an approximate formula for the step length is:

$$x - \mu = k \ln \frac{B}{A} \quad (2.1)$$

where A is the count in the upper window and B the count in the lower window. The quantity x is the voltage corresponding to the centre of the offset peak, μ is the constant voltage at which the peak centre is to be set and k is an empirically defined constant. Equation (2.1) will automatically give the correct sign to the step length.

This formula has become known locally as the "k-gain" formula and is a consequence of the limit relationship:

$$\lim_{a \rightarrow 0} \frac{\int_z^{z+a} \exp(y^2/2) dy}{\int_{z-a}^z \exp(y^2/2) dy} = \exp(az) = 1 \quad (2.2)$$

This may be proved as follows.

Let:

$$P(z) = \int_{-\infty}^z \exp(y^2/2) dy \quad (2.3)$$

then equation (2.2) may be written:

$$\lim_{a \rightarrow 0} \frac{P(z+a) - P(z)}{P(z) - P(z-a)} = 1 \quad (2.4)$$

But:

$$\frac{P(z+a) - P(z)}{P(z) - P(z-a)} = \frac{(P(z+a) - P(z))/a}{(P(z) - P(z-a))/a} + \frac{dP(z)/dz}{dP(z)/dz} = 1$$

$$\text{as } a \rightarrow 0 \quad (2.5)$$

since $P(z)$ is a differentiable function and the direction from which the limit is approached is immaterial. This concludes the proof.

The k-gain formula has been shown to be almost exact for a true gaussian tracking peak and offsets up to two standard deviations. However, when the peak deviates from a gaussian, a tendency to overshoot or undershoot, depending on the direction of the adjustment, has been observed. This effect may be countered using the rational approximation:

$$\frac{x - \mu}{1 - a(x-\mu) + \frac{1}{2}a^2(x-\mu)^2} = k \ln \frac{B}{A} \quad (2.6)$$

which may be rearranged to give:

$$x - \mu = \frac{1 + ay}{a^2y} + \left[\left(\frac{1 + ay}{a^2y} \right)^2 - \frac{2}{a^2y} \right]^{\frac{1}{2}} \quad (2.7)$$

where y is the initial approximation given by the k-gain formula and a is an empirical constant. Both constants can be found by plotting $\ln((z-\mu)/\ln(B/A))$ against $(x-\mu)$ which will give an approximate straight line with gradient $-a$ (at $x-\mu = 0$) and intercept $\ln k$.

For the application of both equations (2.1) and (2.7) it has been found beneficial to set k to a slightly lower value than given by the experiment which expresses a preference for taking more than one iteration to recentre the peak from a large distance rather than risk overshooting and a subsequent oscillation.

The second method of adjustment involves fixing a small step length then setting the time between adjustments to that taken to drift this distance. The method can be implemented completely in hardware which provides real time stabilisation. A schematic circuit diagram is shown

in figure 2.8. This includes an up-down counter such that pulses in one window add to the count and pulses in the other subtract from the count. This smooths the input sequence of pulses and reduces oscillation. Then an adjustment is made only when the counter overflows in the positive or negative directions.

The counting process may be modelled by a simple random walk with absorbing boundaries at the overflow threshold levels [see ref 2 of chapter 3]. It can be shown that the maximum mean time to overflow is obtained when the probabilities for pulses in either window are equal and this time is K^2/λ where K is the overflow threshold and λ is the mean input frequency of the pulses. Likewise, the minimum time to overflow is when pulses are only observed in one window and in this case the mean time is K/λ . Between these extreme values, the overflow time varies as the smooth function:

$$\tau = \frac{K((q/p)^K - 1)^2}{\lambda(q - p)((q/p)^{2K} - 1)} \quad (2.8)$$

where p is the probability for an up-count and $q = 1 - p$ is the probability for a down count. For a gaussian tracking peak, these probabilities may be related to the drift using the k -gain formula (where we use the fact that $p/q = B/A$), which gives:

$$\tau = \frac{K(e^y + 1)(e^{-Ky} - 1)}{\lambda(e^y - 1)(e^{-2Ky} - 1)} \quad (2.9)$$

where $y = (x - \mu)/k$ is a standardised variable. Therefore, the circuit will naturally make more frequent adjustments when the window is off centre than when the system is correctly stabilised.

Providing the step length is kept reasonably small to prevent large oscillations at high frequencies of adjustment, the above property of the counting system can be used to provide an optimum mean frequency of adjustment for a particular rate of spectral drift. The circuit achieves this by comparing the actual adjustment frequency to the trial frequency $\lambda/K(K-1)$ which is one over the difference between the minimum and maximum adjustment periods. If the actual frequency is less than

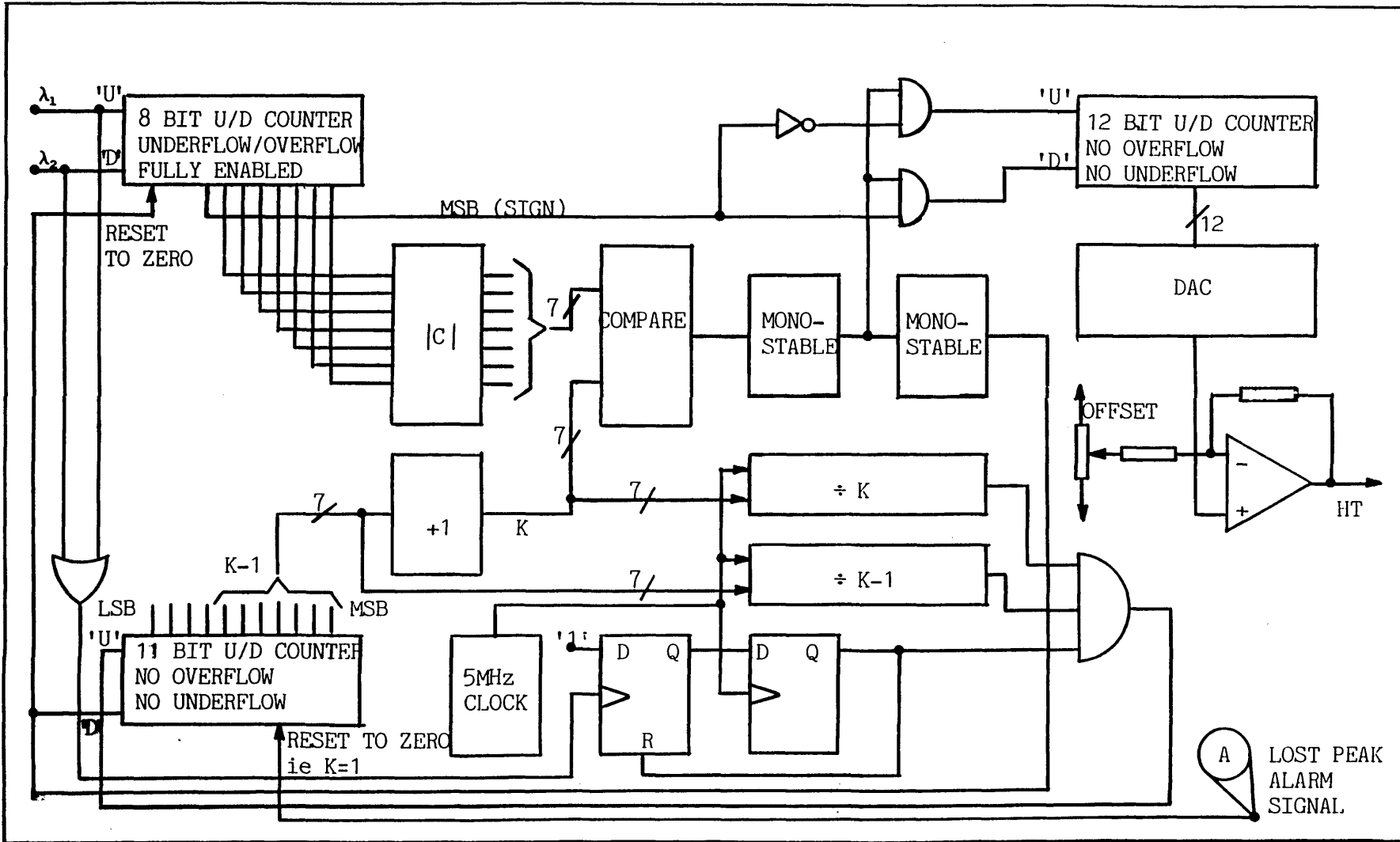


FIG 2.8 SPECTRUM STABILISATION CIRCUIT

the trial frequency, this indicates a stable situation so the circuit slows down the rate of adjustment by adding one to the threshold level. On the other hand, if the actual frequency is greater than the trial this indicates instability, therefore the circuit increases the rate of adjustment by subtracting one from the threshold level. With each adjustment a new trial frequency is generated, and the process repeated until it converges to a threshold level where the actual frequency equals the trial frequency.

The above trial frequency has two attractive properties. Firstly, it is reasonably straightforward to generate in hardware which is achieved by dividing a clock signal by K and by $K-1$, then synchronizing the gross input pulses within the tracking window to the clock and adding the three pulse trains. Secondly, the rate of drift tolerated by the circuit decreases with increasing K . For example, at $K=10$, the circuit will add one to K if the drift is less than approximately $k/10$ whereas at $K = 100$, the corresponding figure is $k/1000$. This counters the tendency for oscillation instabilities caused by over adjustment to increase the adjustment frequency still further and forces the mean peak position towards the centre of the window as the circuit settles to an optimum threshold level.

With some radiation detectors, in particular NaI(Tl), there exists the possibility for rapid gain changes caused by mechanical or thermal shock. To counter this effect it is advisable to include a lost-peak-alarm circuit in the system. A schematic diagram of this circuit is shown in figure 2.9. This compares the input pulse frequency in each window to half of its expected value, using two 4-bit up-down counters. If the peak moves rapidly in the downwards direction this should create an alarm from the upper window before the peak is lost completely and conversely for a movement in the upwards direction. In this case, the circuit overrides the threshold optimisation circuit and sets K to one so that an adjustment is carried out with every input pulse and the rapid gain change corrected. If the default trial frequency with K equal to one is set to 2λ , then the optimisation circuit will reset the K value in the course of its normal operation, unless this is again overridden. If the gain change is not corrected,

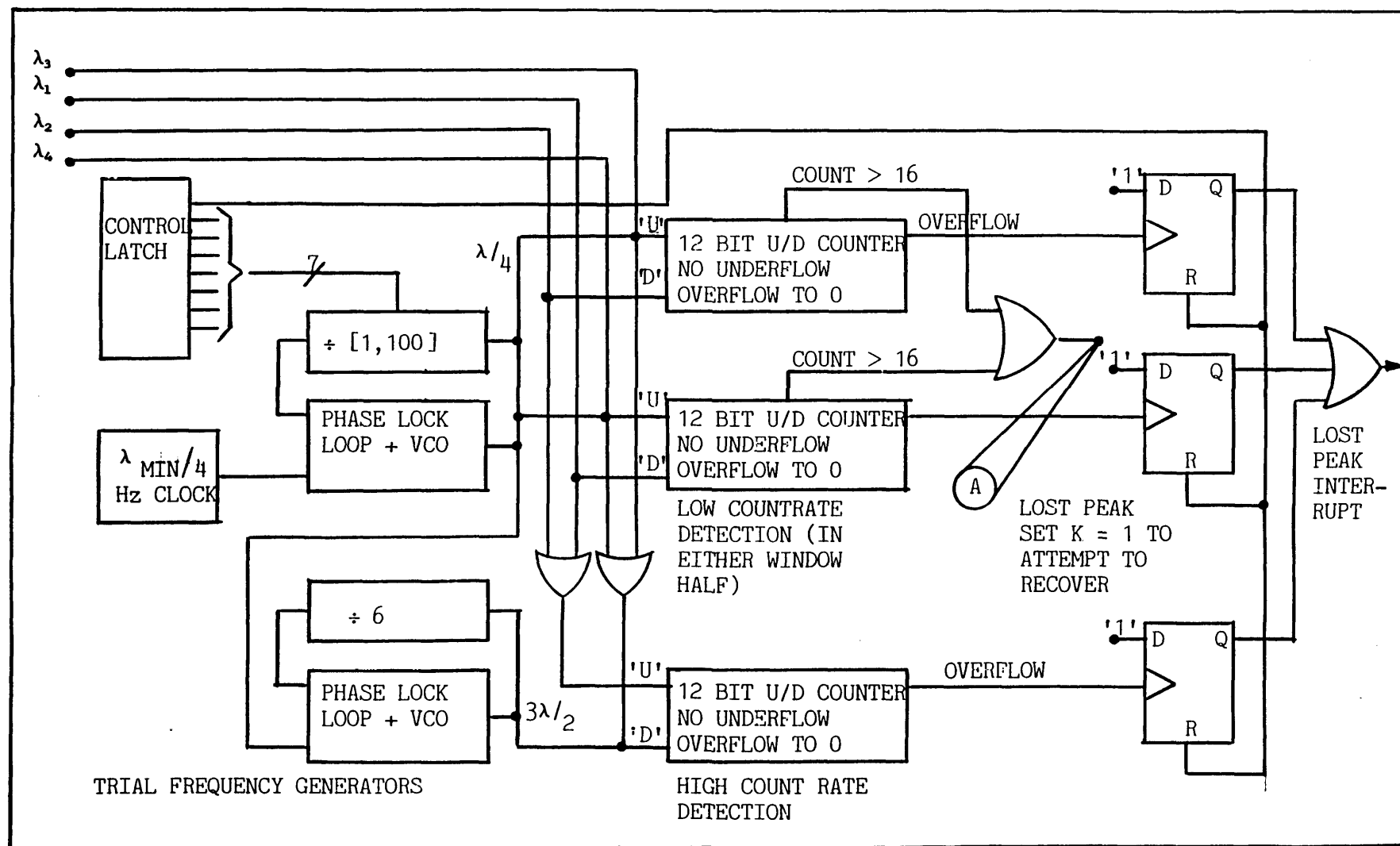


FIG 2.9 LOST TRACKING PEAK ALARM CIRCUIT

the circuit continues counting in a further 8-bits of the up-down counters, and if one of these counters overflows an external lost peak alarm is actuated.

For many systems, the tracking peak will be situated among a background from other peaks in the spectrum. This will not affect the primary stabilisation circuit since the backgrounds in each window will cancel. However, the background must be taken into account for the generation of the trial frequency and in the peak alarm circuit. This may be estimated by setting extra windows at either side of the tracking windows using the quad differential discriminator circuit of figure 2.7B. For the generation of the trial frequency the pulses from these windows are subtracted from the pulse train from the tracking windows and generation of the frequency proceeds as previously. For the peak alarm, the pulses are added into the counters as shown in figure 2.9 and this configuration will actually improve the performance of the system since the background subtracted count becomes negative if the peak shifts by a significant distance.

It is also beneficial to test the background subtracted gross pulse train against a higher frequency of $3/2\lambda$ since if this level is exceeded it may indicate that the system has stabilised on the wrong peak following a step gain change. In the same spirit limits could be placed on the possible range of gain adjustment and a fault alarm raised if any of the above conditions are violated.

2.3 SCALING AND DATA REPRESENTATION

Having obtained a sequence of digital outputs from a quantiser, these must be encoded with a minimum loss of information concerning features of the source. Since this information is conveyed by the radiation field, the required details must be inferred from its observed configuration where the useful properties correspond to distributions of the constituent particles with respect to space, time or energy. The basic strategy for data collection is to form spectra which can be related to the underlying radiation distribution and then to reduce

these spectra so as to provide an optimum data set with respect to the required source features.

A signal space is defined as a phase space with components corresponding to the set of variables over which the radiation field is distributed together with the observed distributions themselves. By configuring the data collection system so that several measurements are made simultaneously or in sequence, the spectra with respect to one or more of these variables may be constructed. This operation is known as scaling and maps the quantiser output into the signal space. A system is said to resolve the variables for which spectra are collected and the resolution of a system reflects the detail with which a spectrum represents the underlying distribution of the field.

A spectrum is constructed by dividing the range of values taken by each variable into a number of intervals and assigning an address in a microcomputer memory to each interval. For example, in the case of energy, a memory address would be assigned to each reconstruction level of the quantiser. Then, the spectrum is collected by adding unity into an address each time the value associated with that address is observed. The result of the process is a histogram stored in the memory of the microcomputer, where the histogram bins correspond to the intervals over which the radiation field is sampled. The memory addresses together with the sub-systems serving them are commonly referred to as channels.

Let $E_k = (E_{1k}, E_{2k})$ be the k th energy interval and $t_n = (t_{1n}, t_{2n})$ be the n th period of time. Then, the spectral function, $s: \Gamma_L \rightarrow \Omega_L$, has domain:

$$\Gamma_L = \{E_k \times t_n : k = 1, \dots, K; n = 1, \dots, N\} \quad (2.10)$$

where Γ_L is known as the signal geometry *5 (next page), and image:

$$\Omega_L = \{s_{kn} : k = 1, \dots, K; n = 1, \dots, N\} \quad (2.11)$$

where s_{kn} is the count obtained on the k th energy interval in the n th

period of time. The pair (Γ_L, Ω_L) form a signal space. The general subscript L denotes the size of the signal space which in this case is the number of combinations of k and n (i.e. $L = KN$).

If the input pulses are uncorrelated the observed count will be Poisson distributed. That is:

$$p(s_{kn}) = \frac{(\lambda_{kn} t_n)^{s_{kn}}}{s_{kn}!} e^{-\lambda_{kn} t_n} \quad (2.12)$$

where $p(s_{kn})$ is the probability that the count, s_{kn} , will be observed when the mean input rate of the pulses is λ_{kn} .

For systems with a Poisson input from a single pulse type detector, the configuration of the data collection system can be defined by stating the number and end-points of the energy and time intervals.

If λ_{kn} is constant for all time periods, t_n , there is no benefit to be gained by segmenting the measurement period into more than one interval. In this case, the spectral image, s_{kn} , reduces to a traditional energy spectrum which is denoted s_k . For uniform quantisation, this involves adding unity into the memory addresses which appear on the output lines of the ADC. After a suitable counting time, the spectrum will be formed in the memory. Such a data collection circuit is referred to as a multichannel buffer (MCB) and a schematic diagram of this circuit is shown in figure 2.10A.

For a gross counting system, the output from a differential discriminator is counted into a single register. The result of this operation is a number, s , which corresponds to the particles observed in a spectral window defined by the discriminator settings. For a resolving system this number can be computed by adding together all of the counts in the individual channels.

Resolution with respect to time is achieved by collecting data over

*5 The operation $A \times B$ is a cartesian product and defines the set of all ordered pairs (a,b) , $a \in A$, $b \in B$ (see previous page).

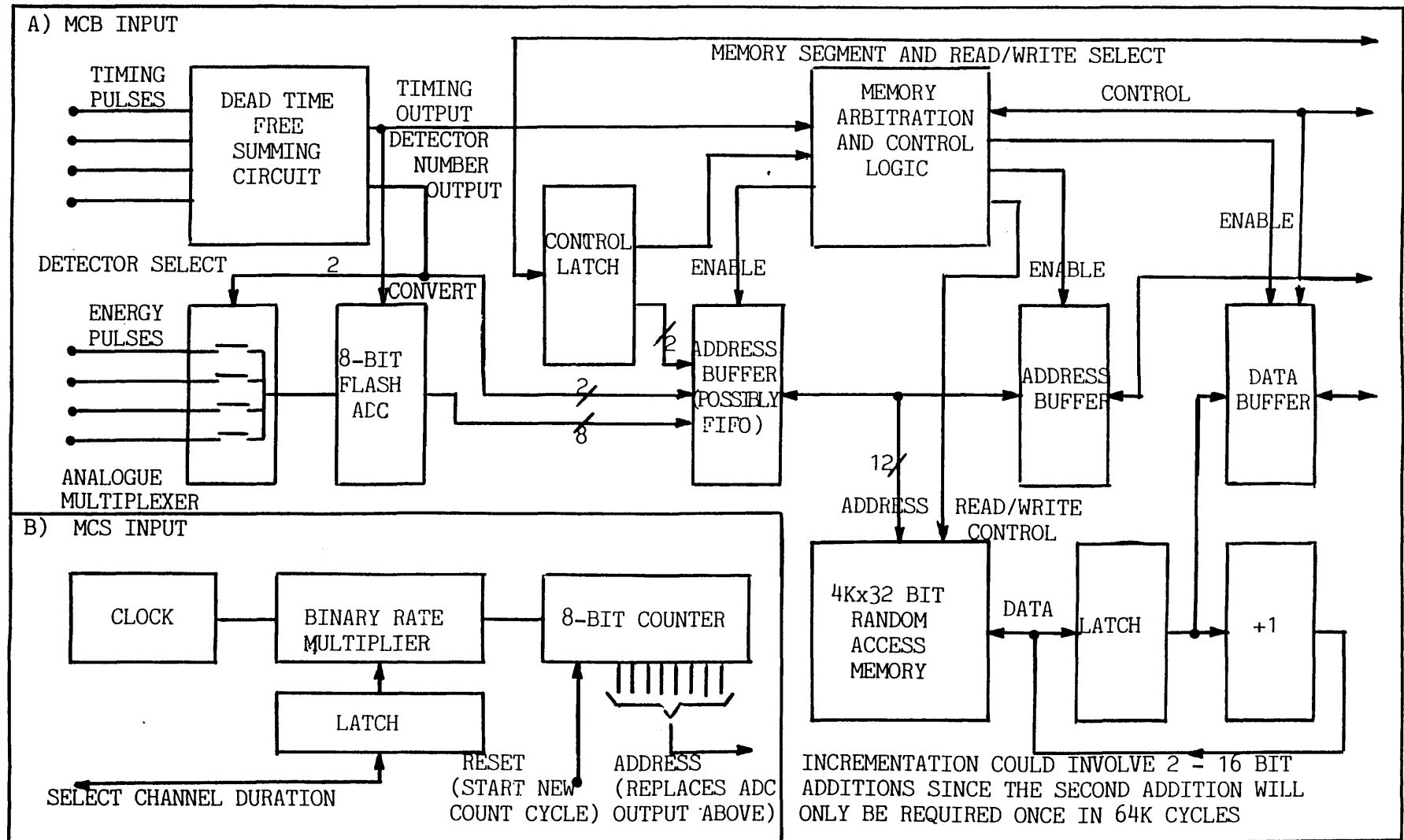


FIG 2.10 MULTICHANNEL BUFFER AND MULTICHANNEL SCALER CIRCUITS

several time periods. In the case of a non-energy-resolving detector and signal processing system - for example, delay line shaping together with a threshold filter or a differential discriminator to remove noise - the spectral response function, s_{kn} , reduces to the time spectrum, s_n . If the periods, t_n , are selected to obtain a significant number of counts in each channel, time variations in the mean input rate, λ_n , can be quantified. This technique is particularly useful with systems where the observed signal is induced by pulses of interrogating radiation from a source such as a neutron tube, LINAC or X-ray generator. Here, the system can be reset so that counting recommences at the first channel each time the radiation source is fired. Then a spectrum will be built up in the memory which represents the evolution of the process following an interrogating pulse. This method of data collection is sometimes called multichannel scaling (MCS). An MCS circuit is identical to the MCB circuit shown in figure 2.10A except that the memory address is generated by a counter incremented with a fixed frequency rather than an ADC. This input circuit is shown in figure 2.10B.

In the case where λ_{kn} is changing with time, it is possible to collect energy spectra over several time periods so as to follow the build-up or decay of λ_{kn} . However, since this approach may yield a large quantity of data which may rapidly exceed the memory capacity of a small computer, the possible resolution with respect to both energy and time is limited. Furthermore, a sufficient number of counts must be collected in each channel to ensure meaningful results, therefore, the minimum width of the energy channels or the duration of the periods is also restricted. However, this method is still useful with slowly varying sources, for pulsed interrogation where the system is reset after a small number of periods or for systems where only a coarse energy resolution is required.

An alternative approach is to collect only the marginal distributions with respect to energy and time. If these are statistically independent, that is the spectral function factorizes as $s_{kn} \propto s_k s_n$, then this involves no information loss. On the other hand, if the energy distribution changes as the system evolves in time, information

will be lost and this can only be recovered if precise prior knowledge of the time or energy dependence of each spectral component is available.

To collect marginal distributions, the signal from the pre-amplifier is split and fed to both a time shaping amplifier and an energy shaping amplifier. The outputs from these are treated independently and the time and energy spectra collected in the normal manner.

Since the frequency of pulses from an integrating detector is proportional to the energy deposited in the detector, the observed count over a period, t_n , is of the form:

$$I_n = \int_0^{\infty} E s_n(E) dE \quad (2.13)$$

Here, E is a continuous variable denoting the energy of the absorbed particles and $s_n(E)dE$ is the spectral function defined on the intervals $E_k = (E, E+dE)$ for a pulsed type detector with the same efficiency as the integrating detector.

Every real counting system has a characteristic deadtime such that pulses arriving within a deadtime period of the previous pulse cannot be counted. There are actually two types of deadtime.

- 1) Extended deadtime whereby each input pulse creates a deadtime period regardless of whether that pulse is counted.
- 2) Non-extended deadtime whereby only those pulses which are counted create deadtime periods.

Typically, deadtime periods are of order one μs .

Providing the deadtime is one of the above pure types, a statistical model is available. For extended deadtime, the model may be constructed directly from the waiting time distribution of the pulses (that is, the probability distribution for the times between their

arrival). For a Poisson distribution of pulses, the waiting time distribution is exponential with mean $1/\lambda$ where λ is the mean input rate of the pulses. Since all of the pulses create deadtime, but only those separated from the previous pulse by $t > \tau$ (where τ is the deadtime) are seen, the probability of observing a pulse is given by:

$$P(t > \tau) = \int_{\tau}^{\infty} \lambda e^{-\lambda t} dt = e^{-\lambda \tau} \quad (2.14)$$

Hence, the observed mean count rate is:

$$s = \lambda e^{-\lambda \tau} \quad (2.15)$$

Note that as λ tends towards infinity, the observed count tends towards zero.

In general, the natural deadtime associated with the pulses directly output from a detector is of the extended type. However, the natural deadtime is rarely constant and is difficult to define accurately. Therefore, on the grounds that an accurately known deadtime which can be corrected is superior to a shorter unknown deadtime, it is common practice to introduce a longer artificial deadtime of the non-extended type by the use of a gated integrator or a quantiser with a fixed operation time.

For non-extended deadtime the model is constructed using expected values. That is, the expected proportion of the time for which the system is dead is given by the product $s\tau$, since only the observed counts create deadtime and the deadtime periods are disjoint. But then, the expected observed count in one unit of time is the total input counts minus those which arrive when the system is dead. Or:

$$s = \lambda - \lambda(s\tau) \quad (2.16)$$

which can be rearranged to give:

$$s = \frac{\lambda}{1 + \lambda \tau} \quad (2.17)$$

and, inversely:

$$\lambda_{kn} = \frac{s_{kn}}{1 - s\tau} \quad (2.18)$$

Equation (2.18) denotes the input pulse rate to a channel given the gross observed countrate, s , and the countrate on the channel, s_{kn} .

Note in this case, that as $\lambda \rightarrow \infty$, $s \rightarrow 1/\tau$ and that (2.17) is a monotonically increasing function. However, in practice, high count rate experiments reveal that the count drops after reaching a certain maximum value. The explanation for this is that after a certain point the natural extended deadline overtakes the artificial periods and the deadline is no longer of a pure form. The above correction is usually good for up to 25% deadline, and this adds at least one decade to the range of accurate counting.

Further dimensions may be added to the signal spaces by using more than one detector or by scanning. These dimensions are used to represent the spatial distribution of the radiation field.

The output of each detector is representative of the radiation field over the region of space defined by the active detector volume. This volume may be considered as the cartesian product of three intervals:

$$V_m = (x_{1m}, x_{2m}) \times (y_{1m}, y_{2m}) \times (z_{1m}, z_{2m}) \quad (2.19)$$

where the integer, m , denotes a specific detector or scanning position. The signal space may now be defined by the signal geometry:

$$\Gamma_L = [E_k \times t_n \times V_m : k = 1, \dots, K; n = 1, \dots, N; m = 1, \dots, M] \quad (2.20)$$

together with the spectral image:

$$\Omega_L = \{s_{knm} : k = 1, \dots, K; n = 1, \dots, N; m = 1, \dots, M\} \quad (2.21)$$

where s_{knm} denotes the observed count on the k th energy channel, in the n th period of time, from the m th detector (or scanning position).

The spatial partitioning of the radiation field is of the same form as the energy and time partitions in the sense that the field is sampled over finite regions. However, in the cases of energy or time, the sampling regions are dense in the range of measurement. For spatial resolution, this can only be achieved if the volume of interest is completely filled with detectors, which is not usually possible. Therefore, it is necessary to choose the M spatial positions of the detectors so that information concerning the spatial distribution of the field can be obtained from a sparse array which may be remote from the source.

Consider, first, a sparse array of detectors throughout the volume of interest. There are two possible treatments for the results. Firstly, using the continuity of the radiation field, the measurements are interpreted by interpolating between the field sampling points defined by the detector positions. The result of this process is a continuous density function which describes the spatial distribution of the field. The accuracy of this representation will depend upon the number of sampling points and the variation of the field between points. The second approach is to distribute the detectors throughout the volume to obtain a sample from each small region. Then, the output of each detector is treated independently and taken to represent a constant field in that region. It is also possible to add the detector outputs to produce a single average value of the field for the volume as a whole.

In the case where the detectors are remote from the source, it is necessary to extrapolate from the observed counts to the field in the volume of interest. If the volume is small compared with the distance between the source and detector or if the spatial distribution of the source is fixed (see section 4.3), then only one detector is needed since the possible variation in the field due to the distribution of the source may be neglected. By increasing the distance between the source and detector, the system geometry may often be improved but the efficiency of detection will be correspondingly reduced. Therefore, for many systems with a single detector, there will be a trade off between the geometry and the efficiency of the measurement.

If the source volume is larger, then a sufficient number of detectors or scanning positions must be used to allow inferences concerning the radiation field to be made over regions within the volume which are small. The standard detector configurations include surrounding the volume with detectors to form a marginal spatial distribution near to the boundary of the source. Alternatively, similar results can be obtained by using one detector, scanning up and down with this through several discrete positions and rotating the sample. The rotation can be continuous, in which case the measurement results will be integrated over the azimuthal dependence of the field. It is also possible to add the results from the vertical scanning positions so the different detector positions are used simply to reduce the effect of spatial variations on the measurement rather than provide information concerning the variations. In the case of no signal absorption, by Gauss's law, the total radiation flux through a surface is independent of the spatial distribution of the source within the enclosed volume. Therefore, the signal averaging will produce good results. However, if the signal is partly absorbed within the sample, Gauss's law does not hold and it is necessary to rotate the sample through several discrete positions to resolve the azimuthal distribution of the field in addition to the vertical distribution.

For both a detector array surrounding the source and vertical and azimuthal scanning, the field sampling regions are dense on the vertical and azimuthal axes. However, in the radial direction, only one (remote) sampling interval is obtained. Since the three spatial coordinates of the field are not independent, the resulting marginal distribution is insufficient to fully quantify the distribution of the field within the enclosed volume. Further information can be obtained by scanning the detector through discrete positions on the radial axis. This is because, ideally, the field from a point source falls off as $1/R^2$ (where R is the radius) and deviations from this distribution are representative of the source distribution. Alternatively, a detector arrangement used with transmission tomography systems is a flat array perpendicular to the direction of the interrogating radiation. This arrangement has properties which allow the measurements to be interpreted using Fourier transforms.

If the field passes through solid material, then the energy and time distributions may carry (phase) information concerning the spatial distribution of the field. For each detector or scanning position, it is possible to collect data by all of the methods previously discussed and hence approximate the joint space, energy and time distributions of the radiation field, or any combination of these which proves useful in a particular situation.

When correlations exist between the input pulses, the observed counts will no longer be Poisson. This situation can occur, for example, with actinide sources where spontaneous fission events may be encountered. Here, two to three neutrons together with seven to ten gamma-rays are produced simultaneously, which introduces correlations between pairs of neutron or gamma-ray pulses. For such sources, it is useful to quantify the number of neutron or gamma-ray pairs observed as these relate directly to the fissions. This is achieved by coincidence counting.

In the case of gamma measurements, the gamma-rays are transmitted almost instantaneously. Therefore, the gamma pairs can be detected by opening a time-gate immediately after each pulse is detected and if a second pulse is observed in the time-gate, these are counted as a pair. Because the time-gate can be made very short, the probability of accidental coincidences from unrelated events is small. This method is known as fast coincidence counting.

For neutron detection, the time-gates must be longer to allow for the finite speed of the neutrons. This significantly increases the probability of observing accidental coincidences. These can be distinguished from true coincidences by counting the pulses singly, in addition to the pairs, and using this information to correct the results statistically. This method is known as slow coincidence counting. With regard to the data collection, the accuracy of the results can be improved by using more than one time-gate. A possible circuit configuration, based on shift registers, is shown in figure 2.11. Here, neutron pulses entering the circuit are held on the input latch for one clock pulse. They are then clocked through the chain of

shift registers moving one bit with each clock pulse. When a pulse enters a shift register, one is added to the corresponding up/down counter and when a pulse leaves the register, one is subtracted. Hence, each up/down counter contains the current number of pulses held in the shift register. Whenever a neutron pulse is held on the latch, the adders are enabled so that unity is added to the S scaler and the contents of the up/down counters are added to the A scalars. It can be shown that in the absence of dead-time an exponential distribution will build up in the A scalars and the area under this curve can be related to the fission rate of the source.

For both fast and slow coincidence counting, the efficiency with which pairs of neutrons or gamma-rays are detected is the square of that for the single particles so it is essential to optimise the detection efficiency of the measurement. Therefore, the best counting geometry is to completely surround the source with detectors and to sum the outputs of these in front of the data collection circuit *6.

For systems involving many detectors the requirement exists to form either the spatial distribution of the radiation field or to sum the detector outputs without introducing extra deadtime. The former requirement may be met using the same circuit as the MCB or MCS but with the ADC or timing counter replaced by a priority encoder. This converts pulses from each detector into a binary address which denotes a channel where a count is accumulated. Alternatively the priority encoder can be used to form the high bits of an address while the ADC or timing counter denote the lower bits. This allows separate time or energy spectra to be collected for each detector using a common data collection system. To sum together the detector outputs, no extra deadtime is introduced if the input pulses are held on a latch until they can be processed and the summing operation is carried out in less than one deadtime period. A circuit for implementing this is shown in figure 2.12, where each latch is examined in turn and a pulse output if the latch is set. The circuit scans through all of the latches in less

*6 An example of such a system is given in section 2.7.

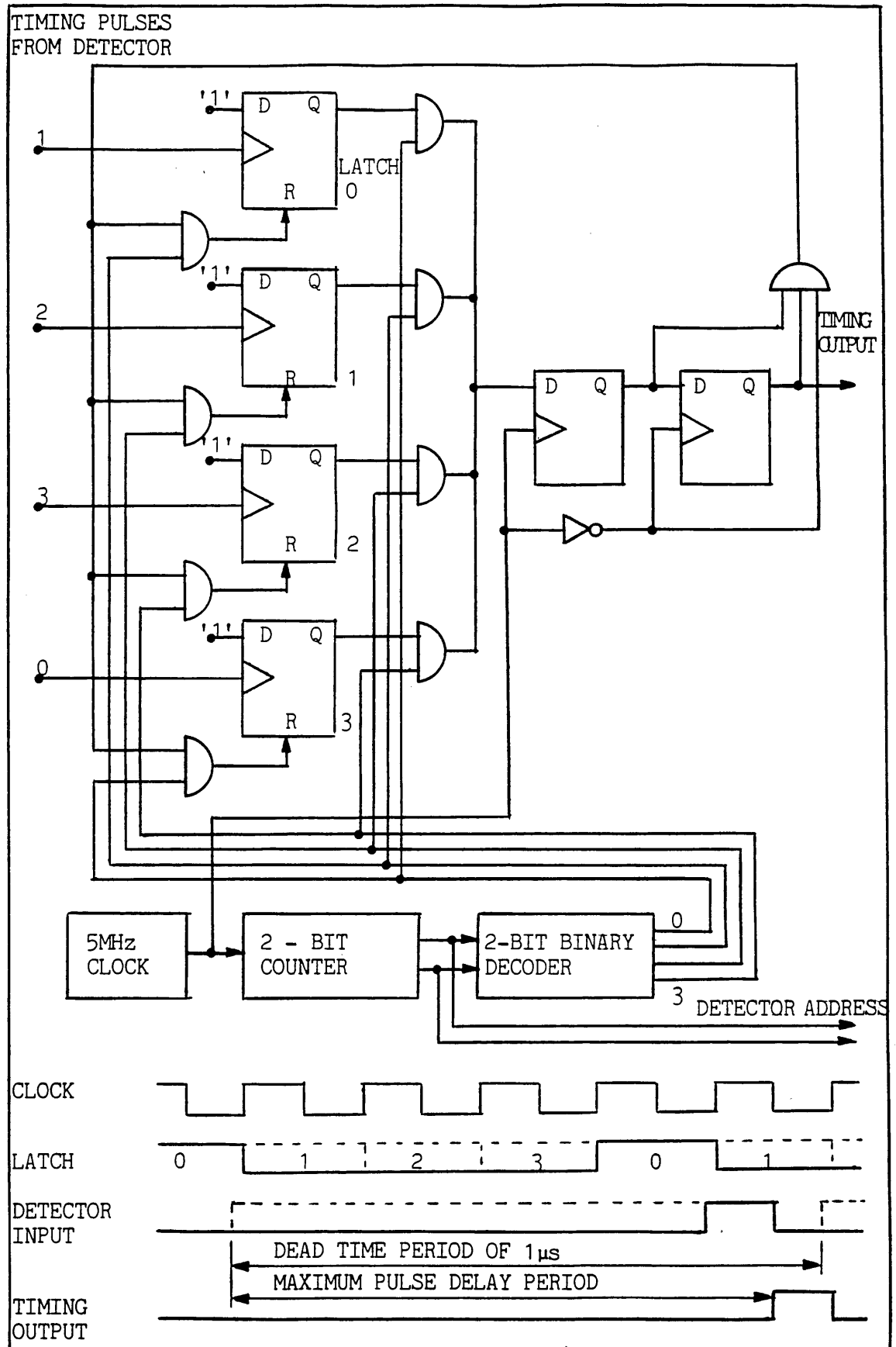


FIG 2.12 DEAD TIME FREE SUMMING CIRCUIT

than one μ s and hence introduces no extra deadtime. This approach is particularly suited for use with the shift register coincidence counting circuit, however in conjunction with the appropriate address generator it can also be applied to the collection of simultaneous time or energy spectra from several detectors as shown in figure 2.10.

In all of the above methods of data collection the observed signals are resolved in terms of space by using more than one detector or by scanning with a single detector, energy using pulse height analysis or time by multichannel scaling, coincidence counting or repeated measurements. Other than gross counting, there are no other practical possibilities. This limited number of data collection techniques suggest that a signal space, (Γ_L, Ω_L) , with the dimensions of space, energy and time (or multiples of these) is the most general signal space in the sense that the results of measurements by all nucleonic instruments can be defined in this space.

With this observation, the design problem to define a measurement process reduces to the configuration of a specific case of this signal space which mirrors the source or sample as closely as possible. Furthermore, the limited number of data collection possibilities supports the standard module methodology in that it may be feasible to configure systems for the majority of applications, from a small number of electronic modules. Such a standard system, based on the techniques defined in this chapter, is illustrated in figure 2.13.

Furthermore, the proposed standard interface can be used with radiation detectors such as surface barrier diodes which are not explicitly denoted in figure 2.13 and can also be used for non-radiation measurements. An example of this is given in section 2.6.

The recognition problem to reconstruct the required features of the source from the incomplete image in the signal space and to design the signal space to make this possible, requires that the relationship between the source and observed signals is defined in detail. This, however, is the purpose of the general system model which will be developed in chapters 3 and 4.

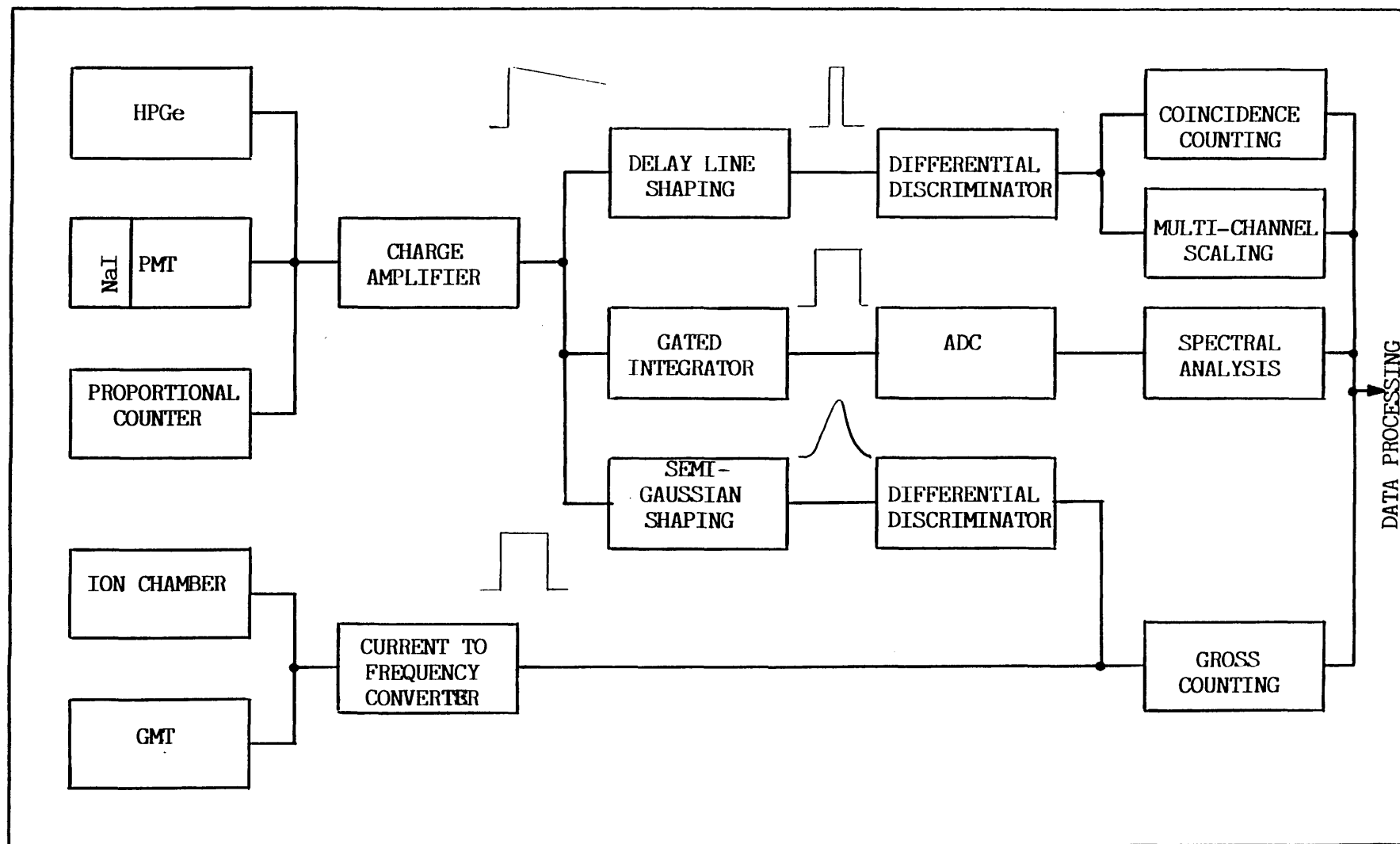


FIG 2.13 STANDARD DATA COLLECTION SYSTEM

The data collection techniques can yield large quantities of data, particularly if resolution with respect to several variables is obtained. The cost of processing this data may be substantial. Therefore, in many cases it is desirable to reduce the bulk data prior to the information processing stage of the system. There are two main techniques for data reduction which are non-linear quantisation and fitting parametric functions to the data. The implications and trade-offs of these methods will be discussed in chapter 3 while the mechanics of data reduction will be discussed in the context of the nucleonic system examples in the following sections of this chapter.

However, in certain cases such as reactor guardline equipment (see section 2.6), the data reduction must be implemented in hardware. This is possible when the parameter can be represented by an inner product of the signals with some fixed vector, $\mathbf{a}$, such that:

$$\theta = \sum_{\mathcal{Q}=1}^L a_{\mathcal{Q}} s_{\mathcal{Q}} \quad (2.22)$$

where the subscript $\mathcal{Q}$ is used to denote the particular combination of k , m and n which define the signal space for this application. The above equation may be written:

$$\theta = \sum_{\mathcal{Q}=1}^L \sum_{\mathcal{Q}'=1}^{s_{\mathcal{Q}}} a_{\mathcal{Q}} \quad (2.23)$$

which implies that the parameter θ can be constructed by adding the $\mathcal{Q}$ th coefficient, $a_{\mathcal{Q}}$, into an accumulator each time a count is observed on the $\mathcal{Q}$ th channel, hence building up the estimate in the accumulator.

The implementation of this circuit is shown in figure 2.14 where the coefficients are stored in the E^2 -PROM and the ADC, timing counter and/or priority encoder outputs denote the address of the coefficient to be added into the accumulator on each occasion. The logic behind this circuit can be extended to give a real time test on the parameter if the accumulator is configured so that an overflow indicates that the parameter takes the value one and an underflow indicates the value minus one. Then, as shown in the figure, the accumulator outputs are

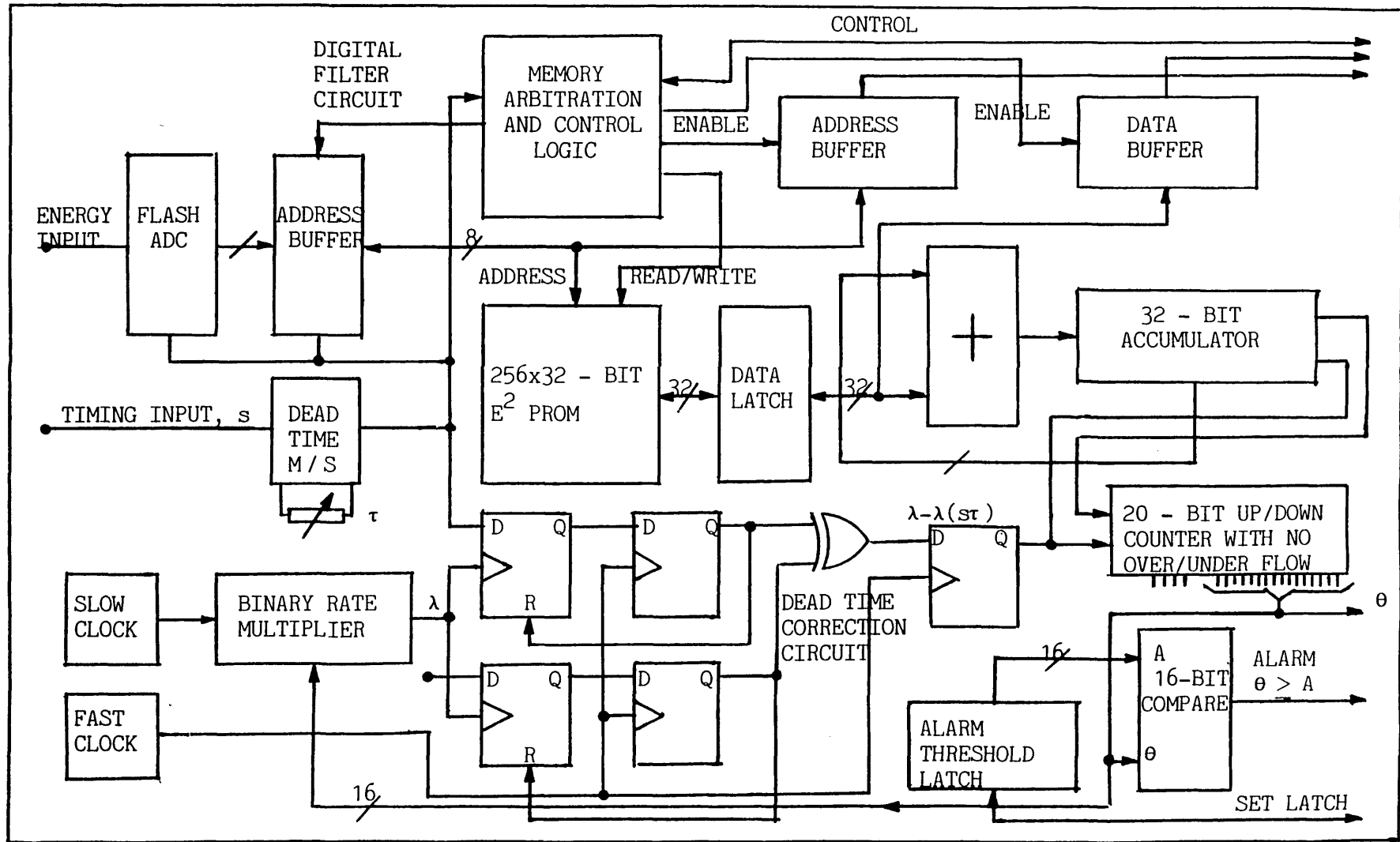


FIG 2.14 DIGITAL FILTER CIRCUIT CONFIGURED AS A RATEMETER

counted in an up-down counter and compared against a pulse train with frequency representing an alarm level. The comparison pulses may be configured to incorporate an on-line deadline correction.

This circuit may be applied to both radiation dose measurements (see section 2.5) and activity measurements in the specific case of a fixed geometry system (see sections 2.6 and 4.3).

2.4 DATA PROCESSING HARDWARE

A schematic representation of a possible data processing hardware configuration is shown in figure 2.15. This system includes several measurement stations, where data is collected using the standard electronics, and these are networked into a central computer. The stations may be distributed throughout an industrial plant, or operated as a local group making several measurements on the same source.

The data processing is carried out on two levels. These are local processing which is performed by processors dedicated to each measurement station, and central processing carried out simultaneously for all of the measurement stations, by the central system.

The role of the local processor is to remove from the observed data the distortion and corruption introduced by the measuring instrumentation and, if appropriate, to reduce this data by quantisation or parametric representation. The distortion factors introduced by the measuring operation are constant since the instrumentation is fixed for all measurements. Furthermore, the measurement stations operate independently of each other, therefore, there is no benefit to be gained by correcting the data for distortion on the higher central processing level. On the other hand, if the data is reduced before it is corrected, a significant information loss may occur. Hence, the local pre-processing represents an optimum configuration.

Once the dependence upon the measurement stations has been removed, the data from different measurements will relate directly to the radiation

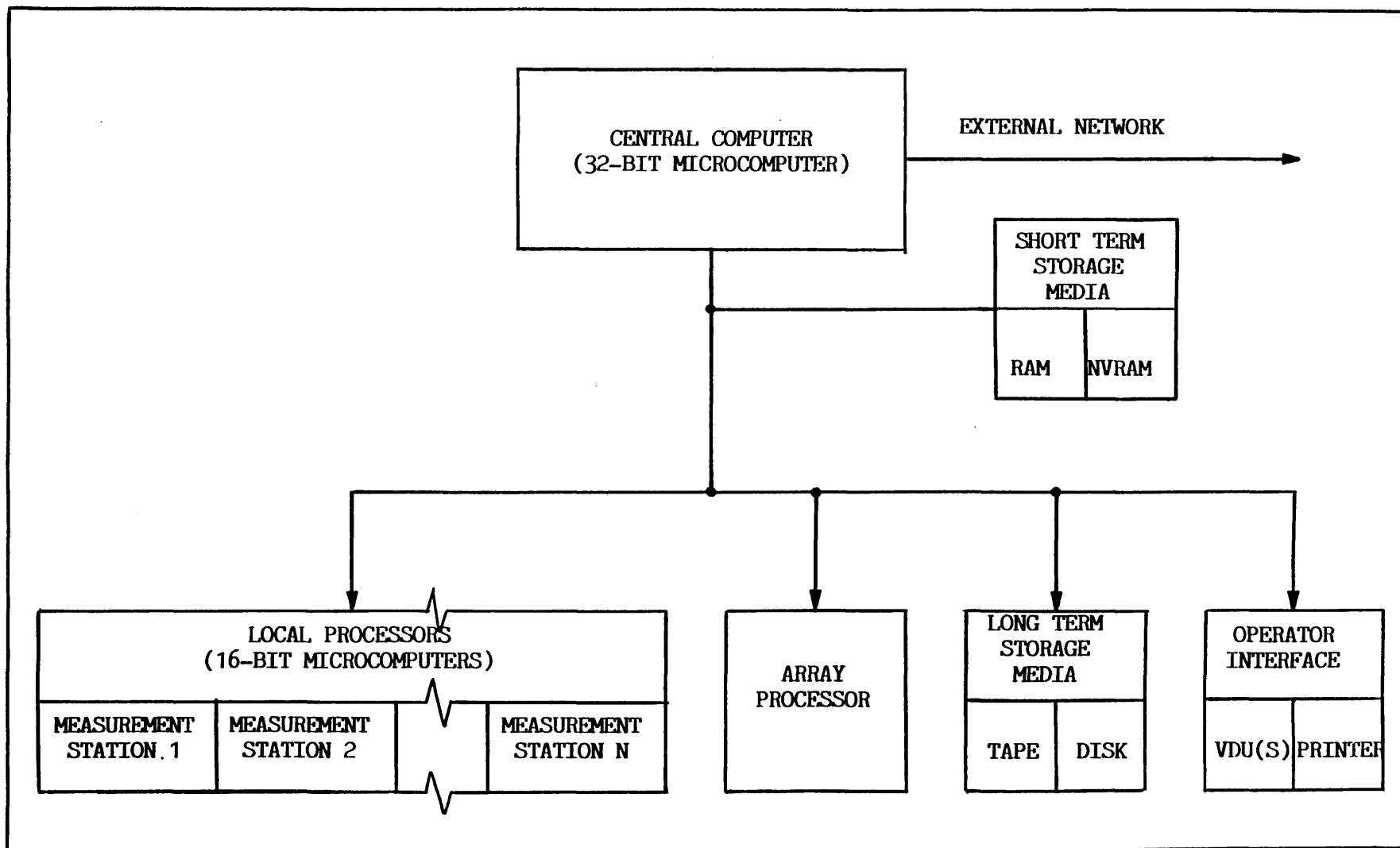


FIG 2.15 POSSIBLE HARDWARE CONFIGURATION FOR DATA PROCESSING SYSTEM

field or in some cases to the activity of a remote sample. Therefore, this data can be combined at the central processing level to produce estimates of the required features of the nuclear process.

An advantage of this approach is that the local processors can be configured to suit the requirements of the measurement station. In general, there are two formats which can be adopted. These are, firstly, to use a standard PC which can be mounted in a cubicle together with NIMS or CAMAC modules; or, secondly, to use a single board computer which can be mounted in a Eurocard rack (see section 1.2). In either case, the main processor can be backed up with a maths co-processor, a transputer array or extra memory cards as necessary. To obtain maximum flexibility, each measurement station should have the capacity for standalone operation in addition to its operation as part of a network.

For many systems, the central processing will involve a large number of arithmetic operations. Therefore, a powerful (although dedicated) processor is required. Thirty-two-bit array processors with an appropriate performance specification are readily available at a cost suitable for most applications. These include the Multiputer 190 [6], containing 19 T800 transputers together with an INTEL 80386 processor for input/output. Such an array processor will not support multi-tasking. Therefore, there is a requirement for a central computer to act as a data collection facility for the data transmitted from the local processors, to perform operator interface and data logging functions and (if required) to communicate with any external networks. The central computer will feed data to the array processor as necessary, with the array processor being used only for calculations.

The operator interface is the subsystem which displays the results of measurement together with any system status reports to a human operator. This could involve a number of graphics terminals to display a pictorial representation of the source, where more than one terminal may be used to display different representations at the same time. Quantitative information could be displayed on an alpha-numeric terminal or recorded on a hard copy device such as a line-printer.

In addition to the requirement to make a decision regarding the correct operation of a nuclear process, a monitoring system may be required to keep records of the activities and isotopic composition of the observed nuclear process outputs, so as to form an inventory.

An important consideration of the inventory is the security of the information storage media, since, if this information is lost, a complete record of the operation of the nuclear process may no longer exist. The principal inventory information should be stored on-line so that it can be accessed quickly and easily, perhaps in non-volatile RAM. This should be backed up regularly on magnetic media such as tape or hard disk, for long-term, off line storage and to prevent loss of information in the event of a hardware crash or other system failure. For the purposes of data logging, the long term media could be 1600 bpi, 12mm tape, which is inexpensive and can be used to store large quantities of data. For storing programs and shorter term data, a winchester fixed disk is more suitable. These are available with capacity from 10 to 150 Mbytes.

2.5 FLUX MONITORING SYSTEMS

The installed measurement systems associated with nuclear process monitoring fall into three broad categories:

- a) Radiation flux monitoring systems;
- b) Contaminated fluid sampling and analysis systems;
- c) Radioactive substance assay systems.

For the measurement of radioactivity, the assay and analysis systems are of primary interest. However, the other systems are related and could be based on similar hardware. Therefore, these will also be considered.

Typical applications of the flux monitoring systems are neutron flux monitoring for a nuclear reactor and area radiation dose monitoring for a power station or other nuclear establishment.

Neutron flux monitoring systems may be based on ion or fission chambers which are distributed in and about the reactor core, [7]. Here, the measurement requirement is to construct a spatial flux map which provides an (almost) instantaneous indication of changes to the thermal power and operating stability of the reactor. Neither of these detector types differentiate between neutron energies. However, providing the neutron energy distribution remains approximately constant, the measurements can usually be interpreted using an experimentally defined calibration constant to correct for detection efficiency. The flux map is constructed by interpolating between the sampling points.

Because of the harsh environmental conditions encountered in or about a reactor core, the detectors and cables must be fabricated from materials which will withstand these conditions and resist neutron activation. Also, the in-core detectors must produce a sufficiently strong signal to allow transmission to the first amplification stage outside the reactor without excessively degrading the detector lifetime.

Although the front-end detection system is very specialised, the data can be collected using standard rack-mounted electronics and then transmitted to a remote computer for processing.

The radiation dose monitoring systems are usually based on ion chambers or Geiger-Muller tubes. The normal implementation is to compensate the detectors so that their energy response is flat or, more recently, characteristic of the radiation damage effects in human tissue, [8]. Hence, these systems can be calibrated directly in Grays (for the flat response) or Sieverts.

An alternative approach is to use a resolving detector such as a scintillator or a proportional counter. To obtain a measurement of dose, the observed signals must be corrected for detector efficiency and adjusted to give the required response. The standard method is to partition the energy range into a number of regions (channels) so that the required adjustments are approximately constant over each channel.

Then counts are collected separately and multiplied by the appropriate correction factors before being summed to produce the final output. This scheme can be implemented in hardware using the digital filter of section 2.3 and figure 2.14 where the results can be interpreted directly in Grays or Sieverts, although a constant factor to allow for the gross detection efficiency could be scaled from both the interpreted signals and the trip frequency against which they are compared.

For an area monitoring system, several detectors may be located throughout a plant with each connected to a local indicator box which contains data collection electronics and the trips which test the input signals against a pre-set threshold level. The indicator box may also contain a local display and be connected to an alarm unit to warn personnel of a high radiation level in the area. The local systems are connected via a rack-mounted multiplexer to a central computer which displays all of the radiation levels. For large systems, the detectors and indicators may be operated in groups with each group having its own computer and these, in turn, networked to the central system.

2.6 SAMPLING SYSTEMS

Contaminated fluid sampling systems are designed to obtain a representative sample of gas or liquid from a volume such as a reactor core, pipeline or storage pond. The sample is fed in a regulated manner to an analysis vessel and may be analysed on-line, using an assay instrument such as a gamma spectrometer, or off-line using chemical analysis techniques. In either case, an estimate of the activity and isotopic composition of the trace contaminants is obtained. From these values, inferences are made regarding the activity of the contaminants in the original volume.

The configuration of the sampling system depends upon the nature of the fluid and the volume from which the sample is taken. In general, this will be either a static mass of fluid or a flow. To allow

interpretation of the measurement, the sample must be a known mass fraction of the original fluid. In the case of a fluid volume, such as the coolant in a reactor core or water in a storage pond, the overall mass of fluid will usually be known. Therefore, to regulate the sample, the system need only draw fluid at a constant (known) mass flow rate. For measurements on a pipeline or stack, the instantaneous mass of fluid may vary considerably. To obtain a known mass fraction, the mass flow rate must be estimated and the sample drawn at a known proportion of this estimated flow rate.

The mass flow rate in a stack or pipeline can be inferred from a measurement of the differential pressure across a pitot tube and, in the case of a compressible fluid, separate measurements of the absolute pressure and temperature. The pressure measurement can be made using pressure transducers while the temperature is best measured with a fast-acting electrical thermometer such as a platinum resistance or a thermocouple. The output from these devices will be a continuous current or voltage which can be treated in exactly the same manner as the output from an ion chamber. Therefore, the data can be collected using a standard indicator box system. The results of the measurements are processed and a signal is returned to control the sample flow regulator.

The configuration of the analysis vessel used to contain the fluid sample will depend upon the mode of system operation, the measurement requirement and the concentration and nature of contaminants in the fluid. If assessment of all of the significant elements is required, a simple bottle or flow-through vessel may be suitable. Alternatively if the contaminants are present in low concentration or information concerning a particular element is required, a filter which separates and concentrates the contaminants by preferential absorption may be needed.

A high resolution gamma spectrometer located near to the analysis vessel could be used to obtain an on-line assay of the sample. We shall illustrate this using the example of a system known as the Blowdown Monitor.

Among the trace contaminants present in the CO₂ coolant gas used in advanced gas-cooled reactors, the radioactive isotope, iodine 131, would represent a health hazard if released to atmosphere. Iodine tends to accumulate in the thyroid gland when taken into the body and the iodine 131, with a half-life of 8.05 days, would remain in the atmosphere long enough for human damage to occur. The Blowdown Iodine Spectrometers, now installed in the new Heysham II and Torness nuclear power plants, will detect significant levels of iodine 131 in a sample of the coolant gas which is discharged to atmosphere during reactor blowdown.

The gas sample is taken from the blowdown stack where it is representative of the discharged coolant after the majority of the radioactive contaminants have been removed by an effluent filter network.

A schematic diagram of the sampling system is shown in figure 2.16. This has been designed to take off a constant mass fraction of the CO₂ flowing to atmosphere by measuring temperature, differential pressure and absolute pressure in the stack and using this information to estimate the instantaneous mass flow of gas to atmosphere [9]. The sample flow is then regulated so as to keep it at a constant fraction of the estimated stack mass flow.

The mass of reactor gas flowing to atmosphere changes throughout a blowdown as the gas pressure in the reactor drops to atmospheric pressure and the last few tonnes are removed by dry nitrogen purging. The stack acts as a critical restriction during the initial stages of the blowdown. This means that the mass flow of gas to atmosphere is restricted to the physical limit that can be transported through the stack under any pressure and the flow is approximately constant for this time. Once the differential pressure in the stack drops below the critical level, the flow ideally falls exponentially.

Once collected from the stack, the gas sample is drawn through a charcoal filter to concentrate and separate the iodine component of the gas. The absorption of the iodine occurs by a surface effect in that

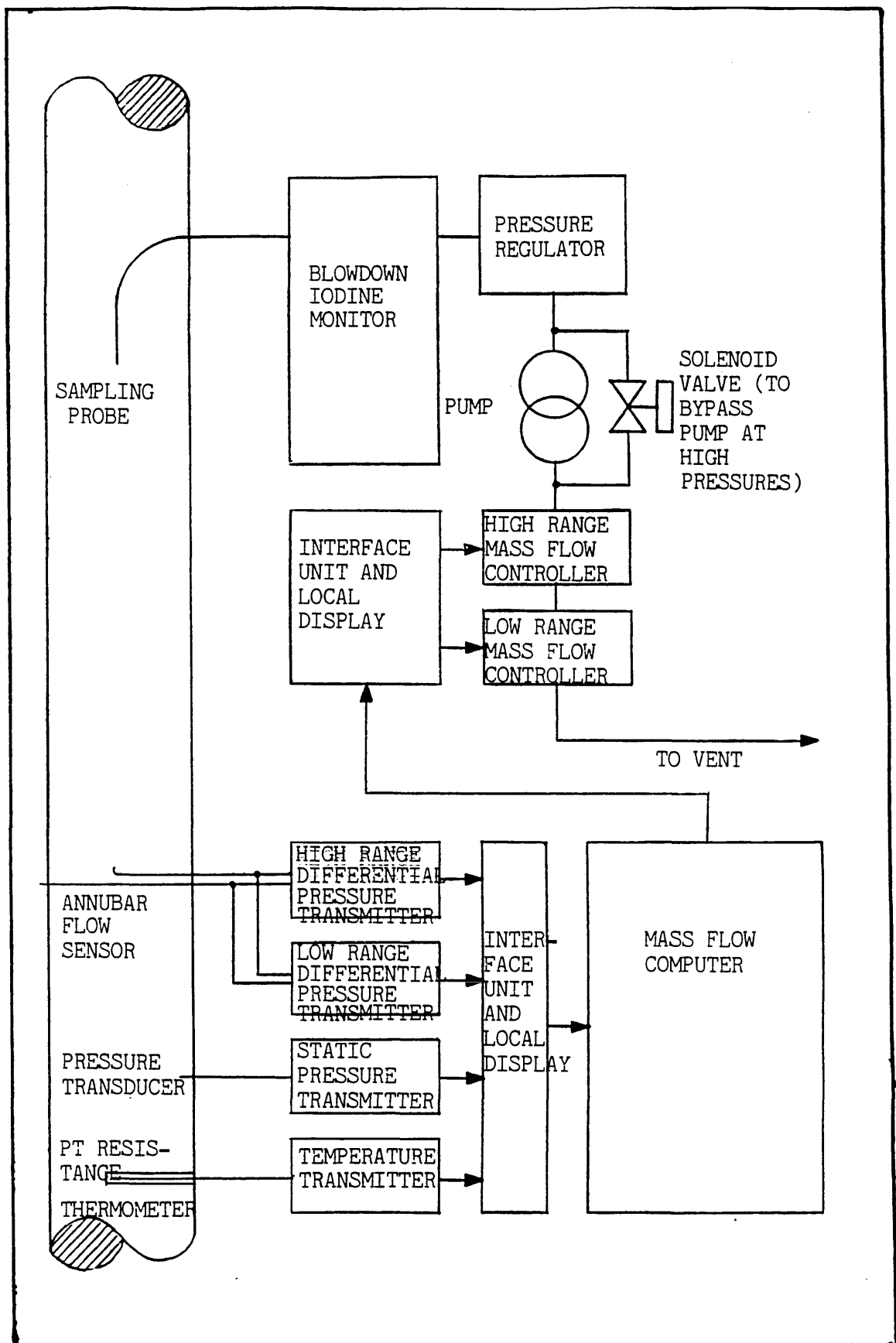


FIG 2.16 GAS SAMPLING SYSTEM FOR BLOWDOWN IODINE MONITOR

methyl iodide lodges in cavities in the lattice structure of the charcoal and is retained by surface tension. The efficiency of the absorption is increased by activating the filter with non-radioactive iodine which will be exchanged with the radioactive iodine as the latter diffuses through the filter. To avoid the absorption of Ar41 and other noble gases, the filter is heated to a steady state temperature of approximately 170°C so that sufficient thermal agitation will be imparted to the system to prevent the noble gases being absorbed, while the more retentive iodine is still held on the filter. The efficiency of the filter is of the order of 85%. However, this is dependent upon the chemical form of the iodine, the moisture content of the charcoal and the diffusion properties of the filter assembly.

The gamma photons emitted by the radioactive iodine nuclides in the collection filter are monitored by a liquid-nitrogen-cooled, high-purity, germanium (HPGe) detector, mounted in a lead collimator assembly beneath the filter. A schematic diagram of the collection filter, the detector and the collimator is shown in Figure 2.17. The output of the detector is a sequence of charge pulses with the total charge in each pulse proportional to the energy deposited by a detected photon. The pulses are amplified and integrated by an EG & G Ortec Model 673 spectroscopy amplifier and gated integrator.

The main requirement of the system is to collect a gamma spectrum and to interpret this to yield estimates of the total released activity from the iodine isotopes over several, equal-length, measurement periods, covering the entire blowdown. The subsystem which performs the data collection has been supplied by Nuclear Data, Inc.

The output from the spectroscopy amplifier is digitised by an ND580 analogue-to-digital converter (ADC) and the spectrum is collected by a MICRO MCA multichannel buffer. At the end of each measurement period, the spectrum is transmitted, via a modem link, to an ND6700 computer and deconvoluted using standard Nuclear Data software. The results of this process are estimated peak areas for iodine spectral lines; an average count-rate is taken by division of the observed count by the measurement period.

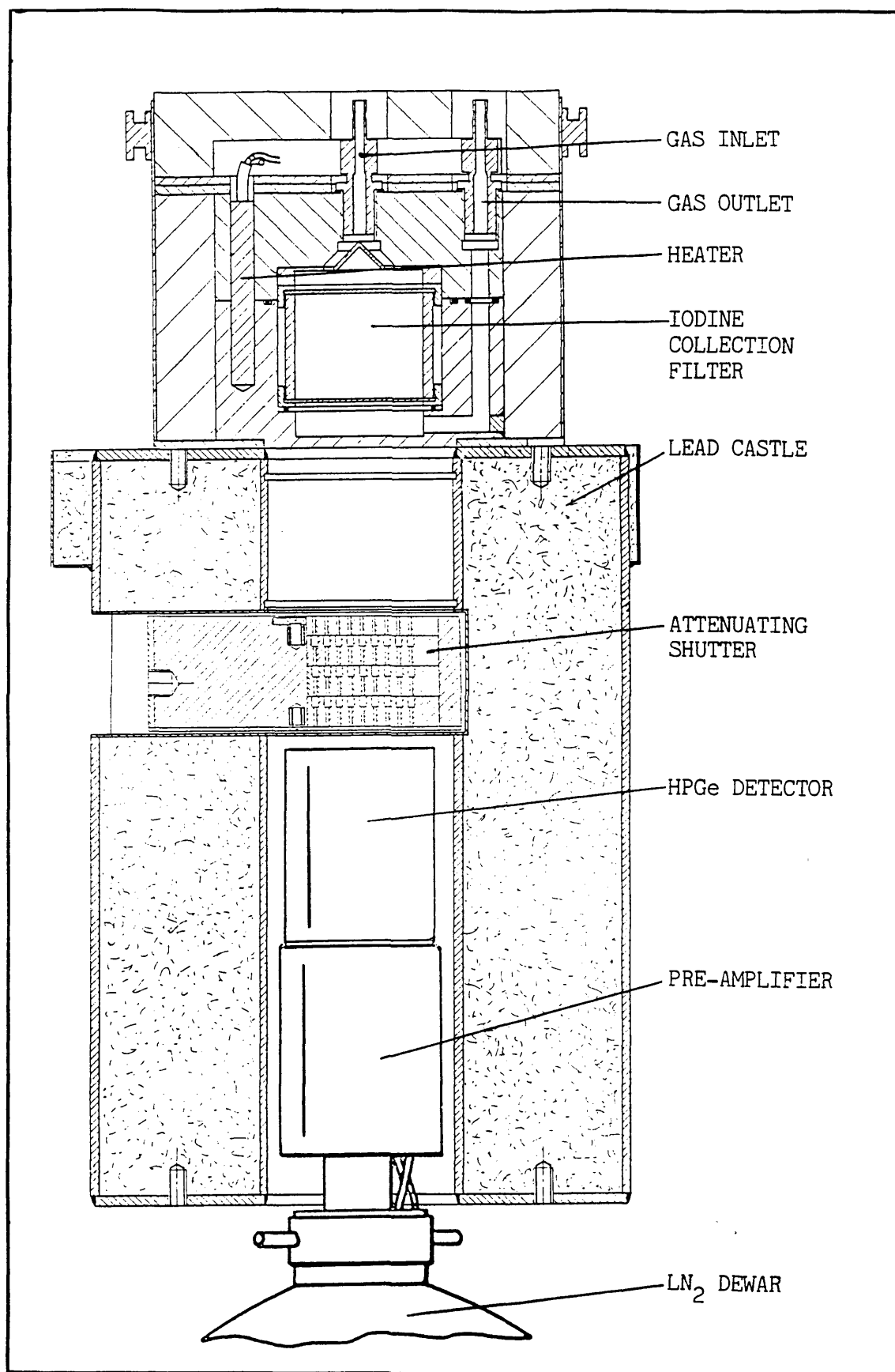


FIG 2.17 BLOWDOWN IODINE MONITOR

The software included in the Nuclear Data package for the purpose of calculating the peak areas is a FORTRAN IV program called APS PEAK, [10]. This program first finds the peaks in a designated spectral region by testing a numerical approximation to the second derivative of the spectrum against a threshold value and tentatively identifying peak boundaries where the second derivative approximation crosses the threshold. Once a peak has been located, the program refines the peak boundaries by testing pairs of channel counts in the neighbourhood of the initial boundary positions against three empirical criteria until a pair of channels are found which satisfy the criteria or come closest to satisfying the criteria after 100 comparisons. With the boundaries established, the program estimates outer backgrounds at either side of the peak by averaging the counts on a small number of channels adjacent to the peak boundaries. The inner background in the region beneath the photopeak is defined by linearly interpolating between the outer background estimates. The interpolated inner background is then subtracted from all the channels in the photo-peak region and, in the case of a singlet peak, the sum of the background-subtracted channel counts is returned as the peak area. In the case of a multiplet, a sum of gaussians function is fitted to the channels and the areas of the gaussians are returned as the peak areas for the multiplet.

The ADC has a conversion time of $14\mu\text{s}$ for each input pulse. This will cause significant dead time at high count-rates. A correction is available for the counts lost during dead time in that a record is kept of the total time lost and this is subtracted from the overall measurement period. The correction is applied by normalising the peak areas with the corrected live time for the measurement (see equation (2.16)). This correction method will produce the correct distribution of count-rates providing the correction periods are chosen so that the source magnitude remains approximately constant throughout the measurement. For the present system, it is possible for high dead time to be encountered at the top end of the measurement range.

In the event of a discharge-filter failure in the stack, the maximum gross count-rate observed by the system will exceed the design tolerance of $3 \times 10^5 \text{ cps}$. This situation will be indicated on a

ratemeter, which is connected directly to a second output of the spectroscopy amplifier, and the instrument will be switched to a fault condition counting mode. This involves the automatic insertion of an attenuating shutter between the iodine collection filter and the detector which will prevent system overload by reducing the gross count-rate to the design limit. Because a solid shutter with the required attenuating factor would absorb the lower energy photons to a much greater extent than those at higher energies and introduce excessive Compton down-scatter into the main 364keV peak from I-131, the shutter incorporates a hole and flame box arrangement (see figure 2.18) to reduce the signal magnitude evenly across its energy range.

The shutter has been designed to attenuate the net signal by a factor of 40 and introduce noise of similar magnitude from scattered radiation, thus giving a gross attenuation factor of 20. All the data produced during this period is interpreted for the fault condition geometry.

In certain circumstances the monitoring of a fluid sample may be carried out using a lower resolution NaI(Tl) detector based system. A typical detector arrangement for making measurements on a liquid sample is shown in figure 2.19. Such a system may be used to detect burst fuel pins by monitoring reactor coolant and a system block diagram for this application is shown in figure 2.20. This system is configured to make independent measurements on four samples taken from different points in the reactor. Each of these measurements is interpreted to yield the activity of an isotope such as Kr-88 where a high concentration indicates a burst pin. If two detectors out of the four detect Kr-88 then an alarm is raised. Voting logic is included in the system to perform this function.

The detector outputs are divided into two paths the first is the guardline path and this must be independent of the computer system. To meet this requirement the measurements are interpreted using the digital filter of figure 2.14, where the coefficients are obtained by taking the appropriate row of a matrix which relates the isotope activities to the observed signals. This matrix is developed in the

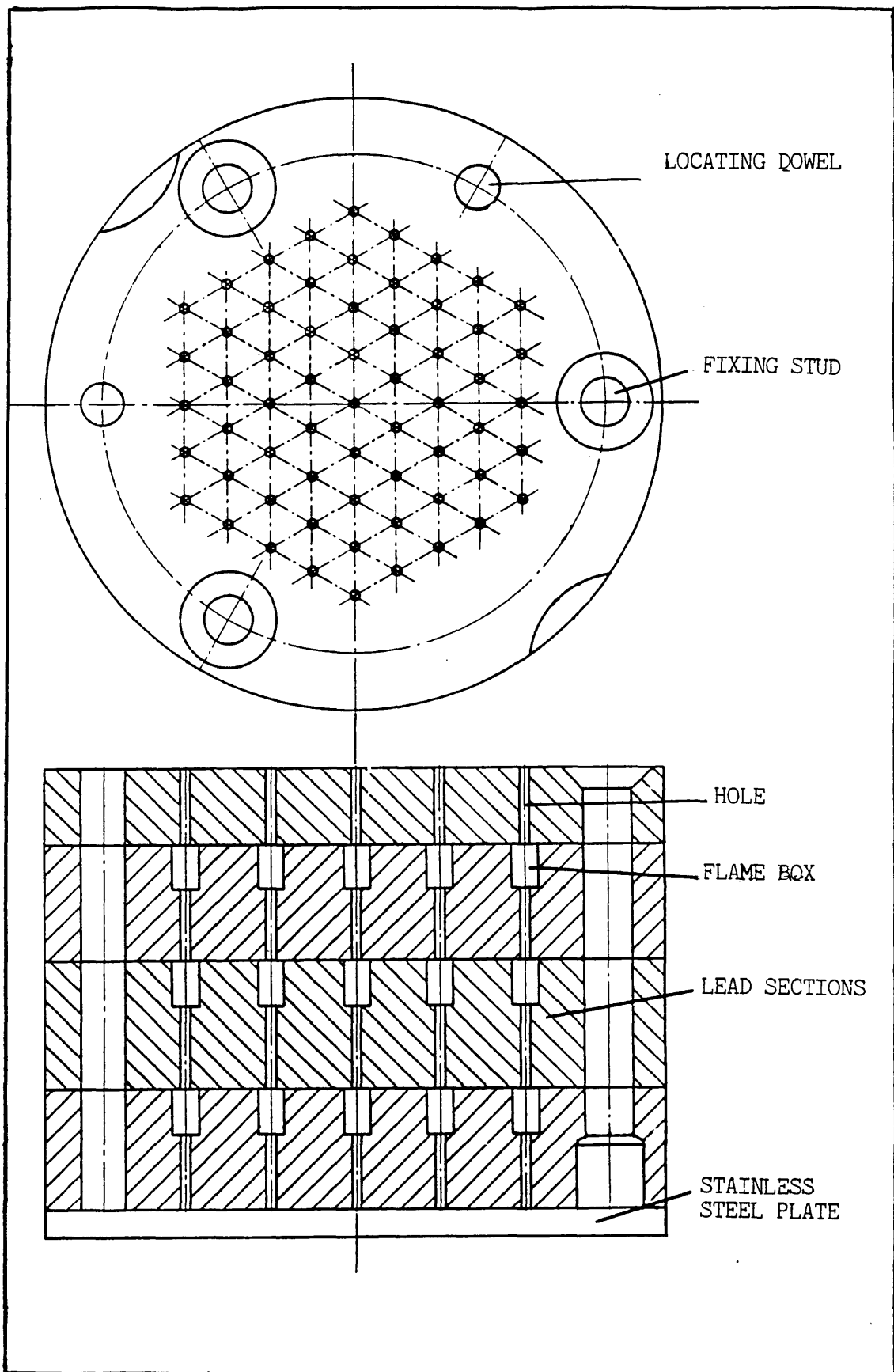


FIG 2.18 ATTENUATING SHUTTER FOR THE BLOWDOWN MONITOR

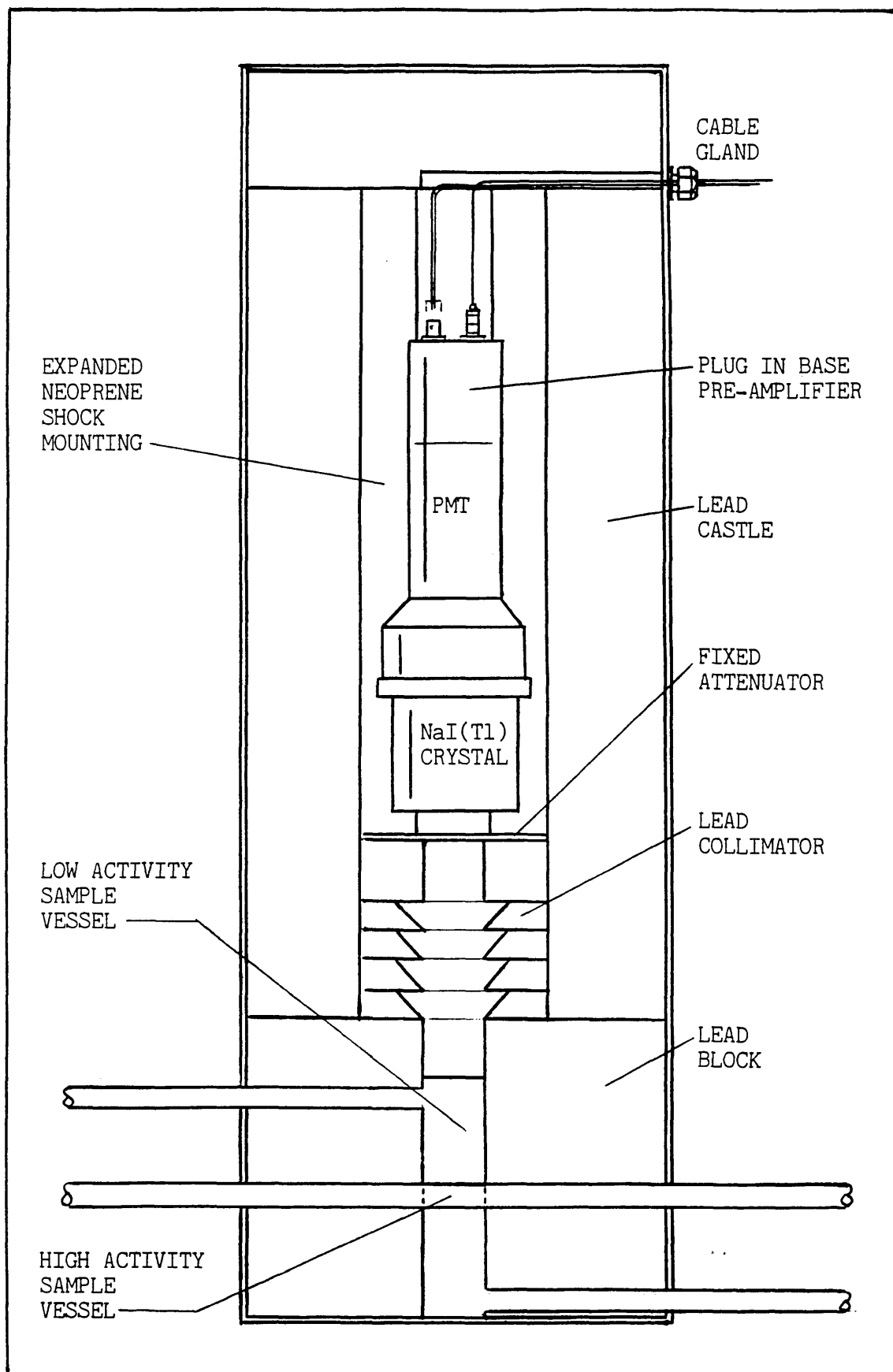


FIG 2.19 NaI(Tl) DETECTOR GEOMETRY FOR MEASUREMENTS ON A LIQUID SAMPLE

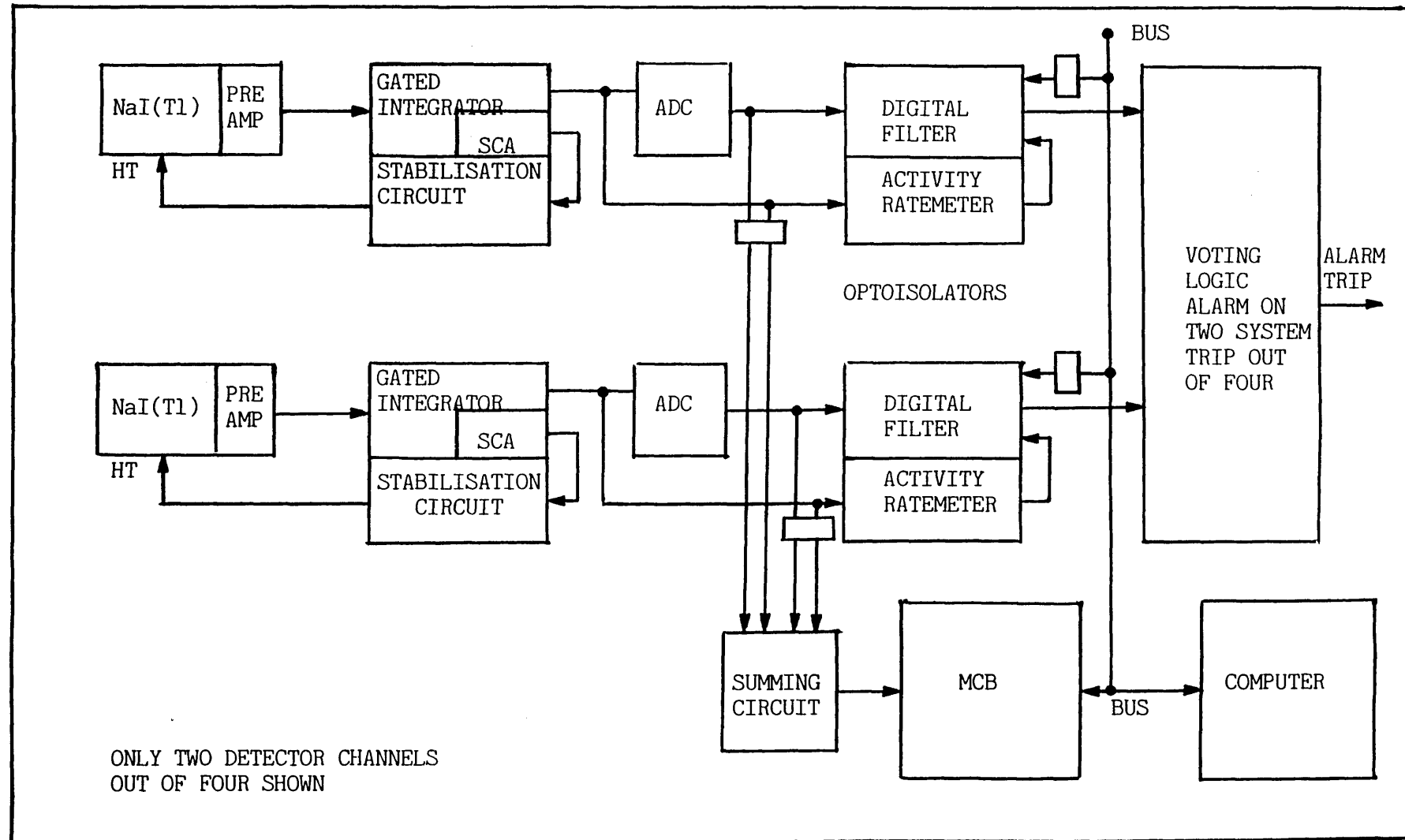


FIG 2.20 SYSTEM BLOCK DIAGRAM FOR A SUITE OF REACTOR GUARDLINE EQUIPMENT

context of the fixed geometry problem in section 4.3. The output of the digital filter will be a pulse train with frequency proportional to the Kr-88 activity and the alarm from this detector system is raised when the frequency exceeds the pre-set trip frequency. Almost real time monitoring is achieved on this path.

The second path is a conventional multichannel buffer together with a software interpretation system and this is configured to make measurements over an extended period and give a complete assay of the sample. The purpose of this is to form an inventory of the contaminants in the coolant, to provide a comparative estimate of the Kr-88 activity to demonstrate the correct operation of the guardline path and to provide the means to calibrate the system. During operation, the analysis and guardline paths must be physically separated by means of opto-isolating circuits, however, when the system is off-line the computer can be used to interrogate and update the calibration data.

2.7 ASSAY SYSTEMS

In addition to the analysis of sampled fluids, assay systems are used for the analysis of materials in radiochemical processes such as uranium or plutonium refinement and in the management of radioactive waste.

There are two distinct types of radionuclide commonly encountered, which are beta/gamma-emitting nuclides and alpha-emitting nuclides. Beta/gamma-emission is normally associated with fission, spallation or neutron activation products such as Co-60 or Cs-137 while the majority of alpha emitters are concentrated about the actinides. In general, assay samples can contain many different nuclides of either or both types in a wide variety of containment. Specific examples are gas or liquid samples, loose material contained in open trays or moving along a conveyor belt and sealed containers including light storage drums containing actinide metal, oxide or fluoride and radioactive waste packages.

Assay samples arise at most of the stages of the nuclear fuel cycle, including the extraction of natural uranium and thorium from their ores, enrichment of uranium in U-235 by isotopic separation, the fabrication of fuel rods, the operation of nuclear reactors and the reprocessing of spent fuel. At each of these stages, there is a need to assay the samples to ensure the correct operation of the process and to keep and audit inventories of fissile material.

Radioactive waste is categorised by its specific activity into low-, intermediate- and high-level waste. In the UK, the disposal of low- and intermediate-level waste is the responsibility of the Nuclear Industry Radioactive Waste Executive (NIREX). Low-level waste such as contaminated clothing, wipe-test swabs and hospital waste is compacted into nominal 200 litre drums and may be disposed of by shallow land burial in soil covered trenches, for example, at Drigg, Cumbria [11]. No disposal routes presently exist for intermediate level waste such as ion-exchange resins, used fuel claddings and filter materials. However, the recommended route for future use is encapsulation with materials such as cement or thermosetting polymers [12], in nominal 500 litre drums and deep land or undersea burial. Such a deep repository is expected to be under construction by the end of this century.

Small amounts of high-level waste arise from the reprocessing of fuel. This is presently stored as a liquid in cooled vessels. The recommended future disposal route is vitrification and dry storage initially in a CO₂ atmosphere and then in air for a period of at least 50 years prior to disposal in a deep repository.

The main applications for assay systems within waste management are in the preparation of waste packages and the independent checking of packages prior to disposal. Samples encountered at the preparatory stage will be loose or lightly contained material while the waste packages encountered by an independent checking system could be fully encapsulated.

Assay systems are usually based upon the detection of either gamma-rays or neutrons and are categorised into emission (or passive) and

transmission (or active) systems. In an emission system radiation which is spontaneously emitted by the sample is detected, while in a transmission system, the sample is interrogated with radiation from an external source and the primary radiation or radiation from secondary reactions in the sample detected.

Most beta/gamma-emitters do not spontaneously emit neutrons, therefore the systems used to assay samples containing these nuclides are based upon the detection of gamma-rays. On the other hand, actinides are susceptible to fission which may be spontaneous or induced by neutrons or gamma-rays. Also certain alpha-emitters such as Am-241 emit gamma-rays in transitions following the alpha decay. Hence, systems based upon the detection of either neutrons or gamma-rays can be used for the assay of actinides.

Measurement techniques available for the assay of light beta/gamma or gamma-emitting actinide samples include gamma spectroscopy and emission tomography. These techniques involve forming a representation of the joint space and energy distribution of the gamma field around the sample. In spectroscopy systems this data is reduced to eliminate the spatial dependence of the measurement and inferences are made regarding the gross activity of the sample. For a tomography system the energy distribution is eliminated and inferences made regarding the sample's spatial distribution.

These techniques can be implemented by segmented scanning with an HPGe detector or an NaI(Tl) scintillator and PMT combination, which are collimated to observe a horizontal segment of a sample. Here, the detector and collimator assembly is scanned through a number of vertical positions, while the package is rotated about its central axis. Data is collected with the detector and package stationary in each position. This requires a motorized turntable to rotate the sample and a linear actuator to position the detector.

A system block diagram for a typical segmented scanner is shown in figure 2.21 while the mechanical arrangement is illustrated in figure 2.22.

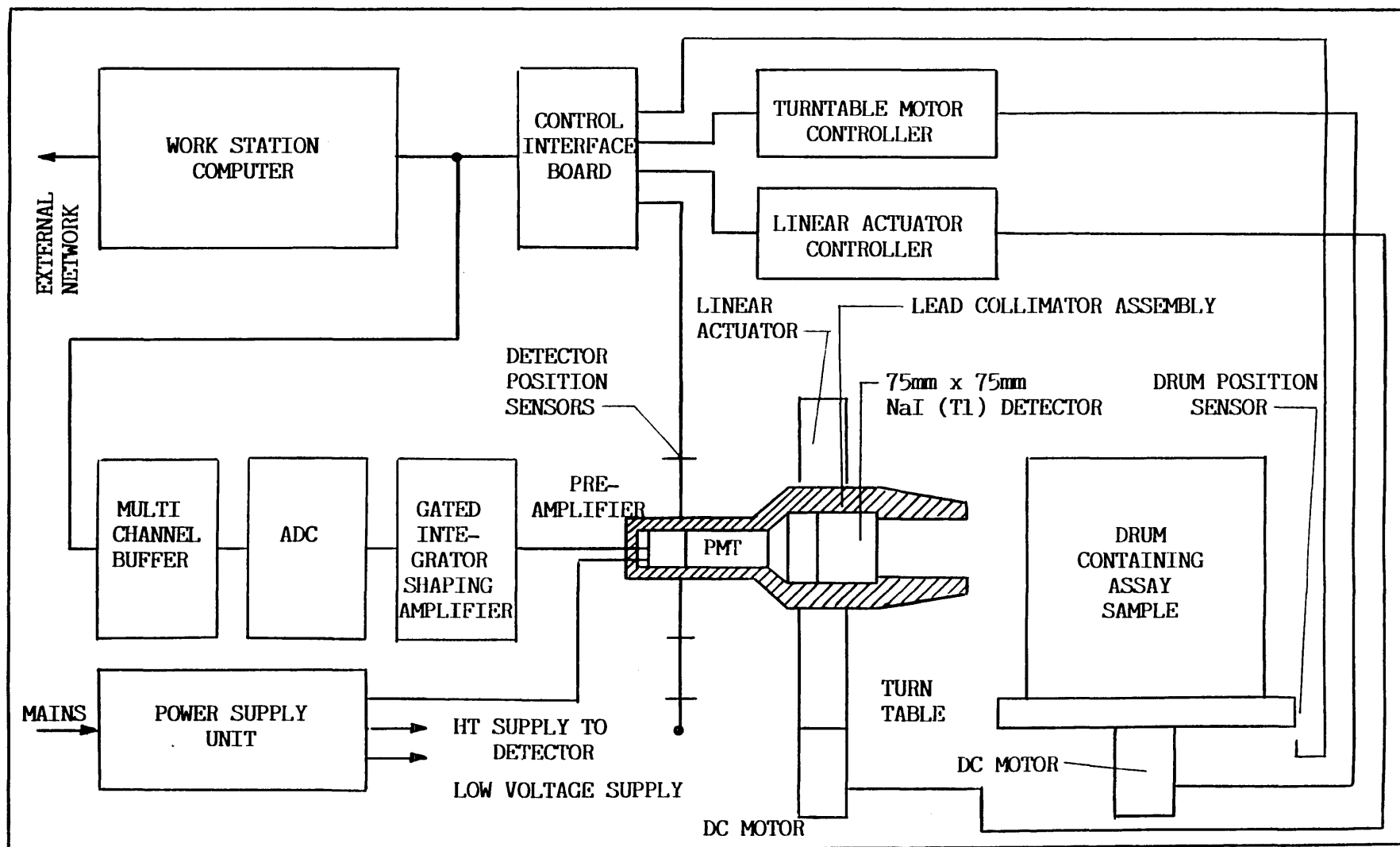


FIG 2.21 SYSTEM BLOCK DIAGRAM FOR A SEGMENTED GAMMA SCANNER

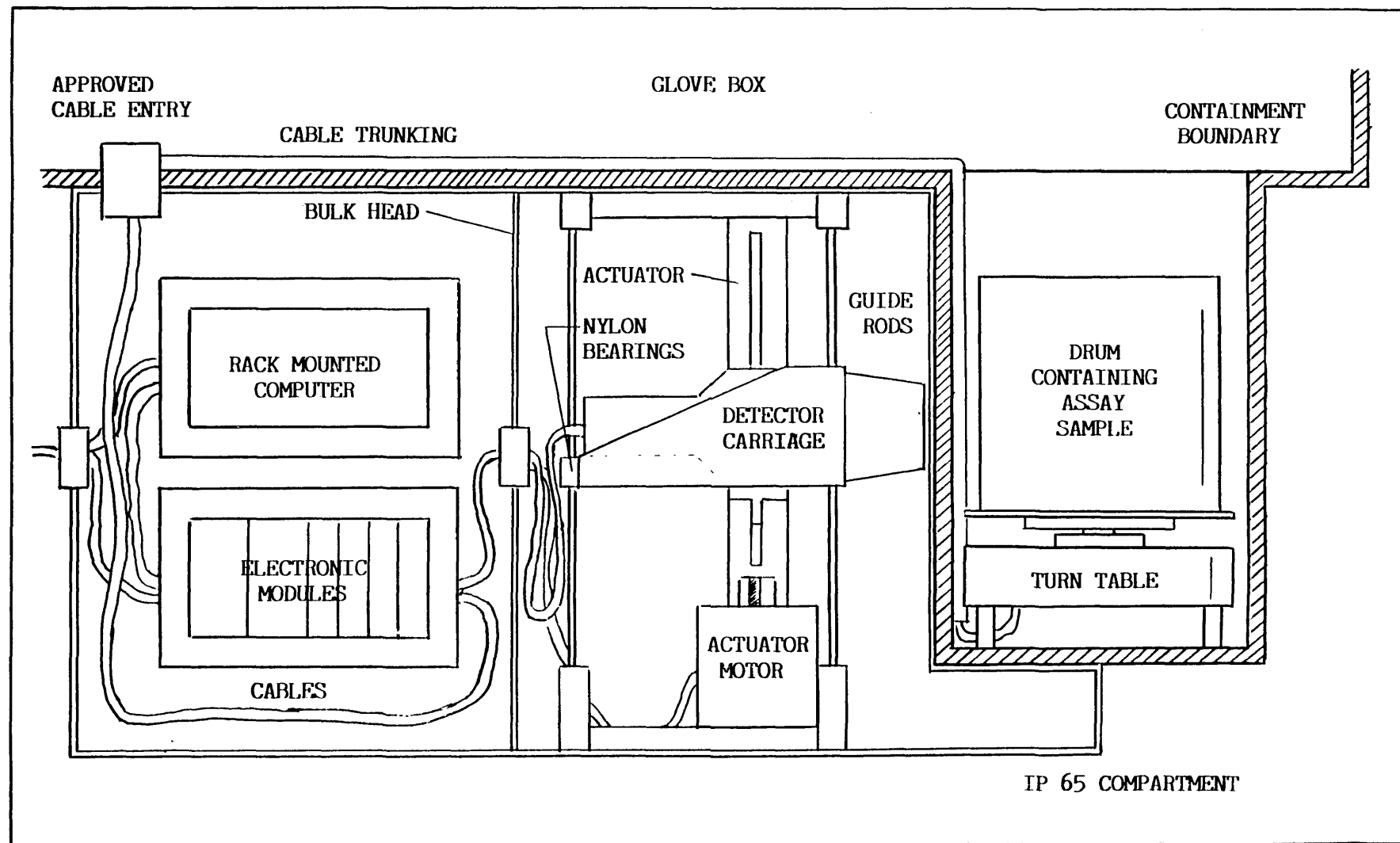


FIG 2.22 MECHANICAL CONFIGURATION OF A SEGMENTED GAMMA SCANNER

The electronics package is divided into two parts, which are data collection and control. The data collection system is based on the techniques described in sections 2.2 and 2.3, and could be implemented with NIMS, CAMAC or Eurocard. The control circuit provides signals to start and stop the turntable and to position the actuator. This could be based on proprietary controllers which are interfaced to a computer.

For even-numbered actinides, such as Pu-240 and U-232, which are susceptible to spontaneous fission, neutron coincidence counting can be used to quantify the fission rate of the sample. An instrument using this technique is a neutron well counter. As shown in figure 2.23, this could consist of an annular polyethylene moderator surrounding the sample. He-3 or BF₃ proportional counters are embedded in the polyethylene and detect neutrons which are emitted by the sample and thermalised in the moderator. The shift register circuit discussed in section 2.3 is used for data collection. This system is illustrated by the block diagram shown in figure 2.24.

The quality checking of encapsulated radioactive waste packages, prior to disposal, is an important feature of waste management. For, in order to gain the acceptance of a disposal site, not only must waste packages destined for that site conform to a rigorous specification, but this conformance must be actively demonstrated both to the regulatory authorities and to the public at large.

Measurement techniques available for the assay of encapsulated packages include gamma spectroscopy and gamma tomography for beta/gamma emitting waste, and passive neutron counting and active neutron [13][14] and gamma interrogation [15] together with neutron detection for alpha-emitting waste. While adequate for 200 litre packages, these techniques applied in isolation may not be sufficiently selective to produce satisfactory results for a cement-encapsulated 500-litre drum containing mixed waste such as actinides together with fission products. One method of increasing the measurement sensitivity is to apply several measurement techniques in sequence to build up information concerning the contents of a waste package.

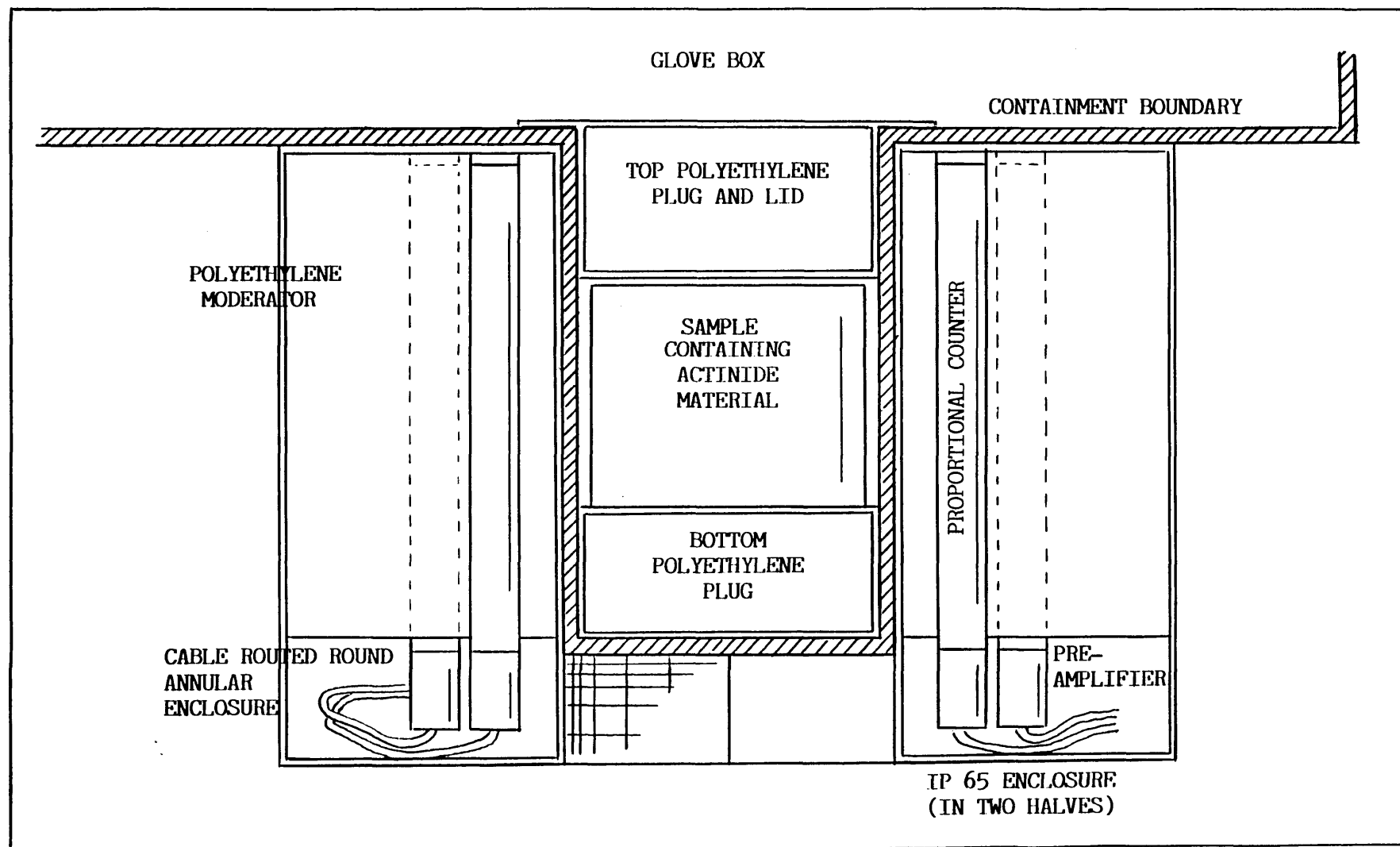


FIG 2.23 MECHANICAL CONFIGURATION OF NEUTRON WELL COUNTER

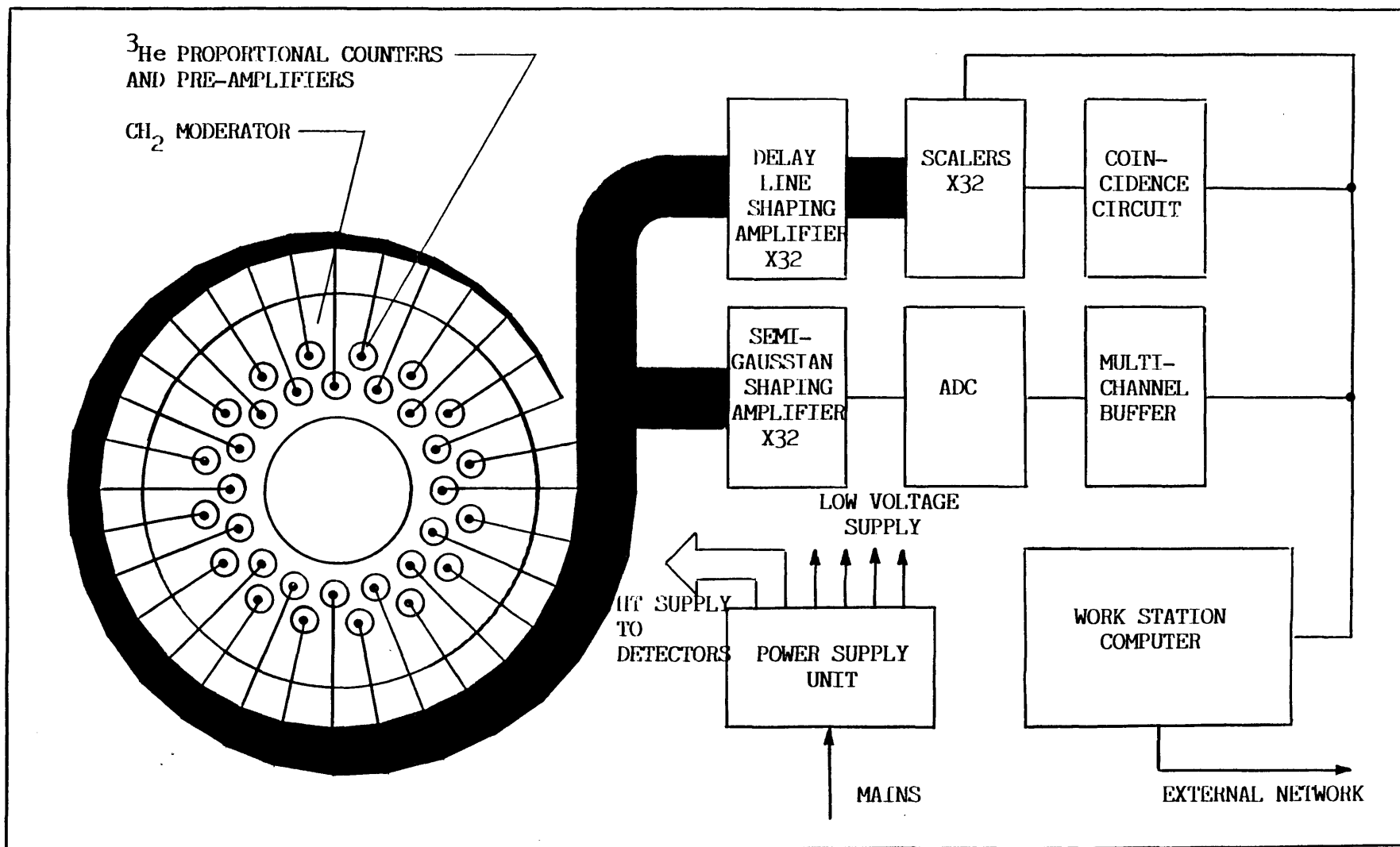


FIG 2.24 SYSTEM BLOCK DIAGRAM OF A NEUTRON WELL COUNTER

A proposal has been forwarded for the development of a non-destructive assay facility [16], based on this sequential assay process, which is capable of examining all of the waste packages on a disposal route. An artist's impression of this facility is shown in figure 2.25.

The waste package assay facility is to incorporate a commercially available LINAC with suitable targets, which is used to provide interrogating radiation fluxes, and a number of measurement stations. A typical arrangement may include four such stations. The first measurement station would be an entry stage where the incoming packages are weighed and checked for surface contamination. The second would be the main system for the assay of beta/gamma waste and might be a segmented gamma scanner or an emission tomography system (ie, passive gamma counting). The third would be a gamma radiography set used mainly to check the physical soundness of waste packages and the fourth a neutron assay system. This is considered to be the main system for the assay of alpha-emitting waste and makes use of three measurement techniques, namely:

- passive neutron counting
- active gamma interrogation
- active neutron interrogation.

The primary requirement of the radioactive waste assay facility is to provide a verification of the contents of encapsulated 500 litre waste packages and to show that these packages conform to their specifications. However, because information concerning a waste form is relatively easily obtained at the point of production, before the packages are encapsulated, the need for a system such as the assay facility is negated unless its measurements are independent of the producer's own quality assurance. It is recognised that the majority of waste packages will conform to their specifications. However, interspersed among a stream of conforming packages, there may be a small number of "rogue" packages which, in some sense, do not comply with the requirements of a package which is suitable for disposal. The assay facility may also encounter old waste packages about which very little is known.

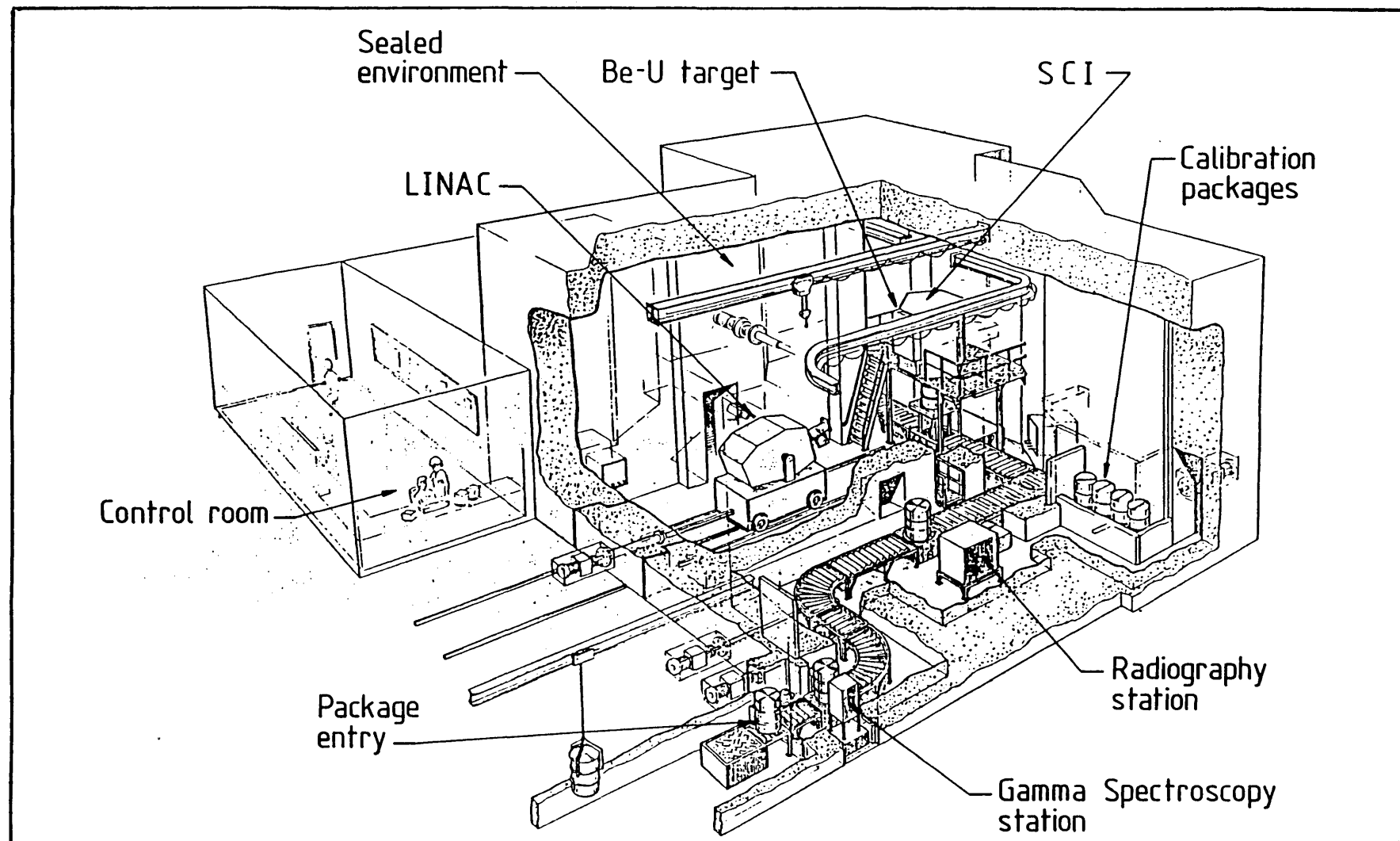


FIG 2.25 ARTISTS IMPRESSION OF THE IRIS SYSTEM

In view of the cost of recycling packages (if a rogue package is found, the whole waste stream may be stopped) or destructive testing, it is important that the number of false alarms and border-line cases is reduced to a minimum. This requires that the efficiency of measurement is optimum and that the decision criteria against which the packages are tested are objective and powerful enough to show clearly whether almost all of the packages are within or out of specification.

The cement-encapsulated waste packages are approximately 1.2m high by 0.8m diameter and weigh up to 1.5 tonnes. Moreover, it is possible for the cement matrices of different (conforming) packages from the same waste streams to vary in form and consistency, and because of the size and density of the waste packages, even a small change in the matrix may greatly affect a measurement. In addition to the matrix variations, the distribution of the radionuclides in such a waste package (or even the type of radionuclides) cannot be known exactly. This problem is compounded by the existence of many different waste streams.

For the waste forms considered, the normal criterion for the assessment of the suitability of a waste package for disposal is the primary activity (ie, alpha or beta activity) per unit mass of the package. Therefore, under present disposal recommendations, the output required by an operator of the waste assay facility will be a decision as to whether the gross alpha and beta activities contained in a waste package are greater or less than the specified disposal limits for a site. This requires two separate hypothesis tests to be performed, one on the alpha activity and one on the beta activity. There may also be requirements to specify the activity of particular radionuclides such as plutonium or americium.

To obtain independent verification of the contents of a waste package, we must preclude the use of information such as the documented history and contents of the package, and other claims by the producer, in the formation of the test statistics. Hence, all information used to establish these results must stem from direct measurement, general physical laws or the results of unconnected measurements using, for

example, calibration packages.

The above requirement alone more than justifies the development of the general system design and data processing model considered in the following chapters.

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CHAPTER 3

GENERAL MODEL

In this chapter, the design and recognition problems will be addressed for a system to monitor the radioactive samples output from a general nuclear process which is evolving in time. Two system concepts are examined. The first, termed the "naive concept", is based on certain simplifying assumptions which make the design and recognition problems tractable. However, it is shown that this approach leads to difficulties which may cause unbounded variations in the measurement results. Therefore, an attempt is made to form a second concept, called the "axiomatic concept", which avoids these difficulties. It is shown that the axiomatic concept forms a natural generalisation of the naive concept, which is obtained by avoiding the simplifying assumptions. To make the problems tractable, it is proposed that the method of top-down development is adopted.

It is considered that the best approach to this development is to construct first a mathematical model of a general sample and measurement process. The need for this model is self-evident if one recognises the measurement process as a sequence of operations which

encompass measurements on the collection of all samples encountered by the system, and that this process must be explicitly designed in exactly the same manner as individual instrument modules. The model is constructed in two parts. The first is the subject of the present chapter. This provides an optimum (statistical) method for interpreting a measurement and establishes conditions for the existence of the probability density functions which make this possible. The second part is considered in the next chapter and investigates the asymptotic (deterministic) relationships between the outputs of a nuclear process and the observed signals. These relationships establish a set of parameters which are used to describe a sample and provide the means to simplify the problem in a manner which does not lead to difficulties.

3.1 NAIVE CONCEPT

With the naive concept of an instrumentation system, data from each measurement is treated in isolation to determine "main features" of the sample; for example, the radioactivity of particular isotopes, the density of attenuating material or the spatial distribution of material within the sample.

In addition to these targeted main features, each of the measurements will always be dependent upon a collection of bias parameters which describe the effects of all variable influences upon the measurement other than the specific main features to be quantified by that measurement *1. The set of bias parameters may be different for each measurement but could include some of the following:

- radiation interactions with stable materials within the sample;
- the spatial distribution of radionuclides within the sample;

*1 By a parameter, we mean a quantity which has a fixed but possibly unknown value or set of values for a particular sample. Parameters may be functions of time, space, energy etc. By this definition, the main features will also be represented by parameters.

- background radiation fluxes;
- interrogating radiation fluxes;
- radiation interactions with source radionuclides themselves.

The effect of a single bias parameter on a measurement is illustrated in figure 3.1 using the example of self shielding of the 375 and 415 keV gamma rays emitted by Pu-239. It can be seen that in the presence of self shielding a wide range of counts can be observed from the same mass of Pu-239 depending on the spatial form of the material. That is, for a concentrated lump of plutonium only the gamma-rays emitted from the surface layers will be seen, with those emitted from the interior being absorbed; while, for a distributed source, a much higher percentage of the gamma-rays can be detected. Moreover, this fundamental uncertainty in the observed counts translates into an uncertainty in the mass value when a particular set of counts is observed by measurement. In the present example, the self shielding can cause an underestimation of the mass by up to approximately 84% of the true value.

The data generated by each measurement must be corrected for the detection efficiency of the measuring instrumentation together with the combined effect of the bias parameters. The corrections may necessarily require the use of information from other measurements or from a priori assumptions that the bias parameters which are not quantifiable by measurement remain fixed for all of the samples encountered by the system - in which case they can be defined by calibration - or that they influence the measurement by an insignificant amount.

With the above assumptions, simple processing methodology can be applied to the individual measurements where the objective is to reduce the measurement data to a small number of statistics with minimum variance and which can be directly related to the required main features by one or more constants. These constants embody the bias correction and, using the assumptions, are formed by the composition of radiological data, main features of the sample defined by earlier measurements and experimental calibration data. To define the

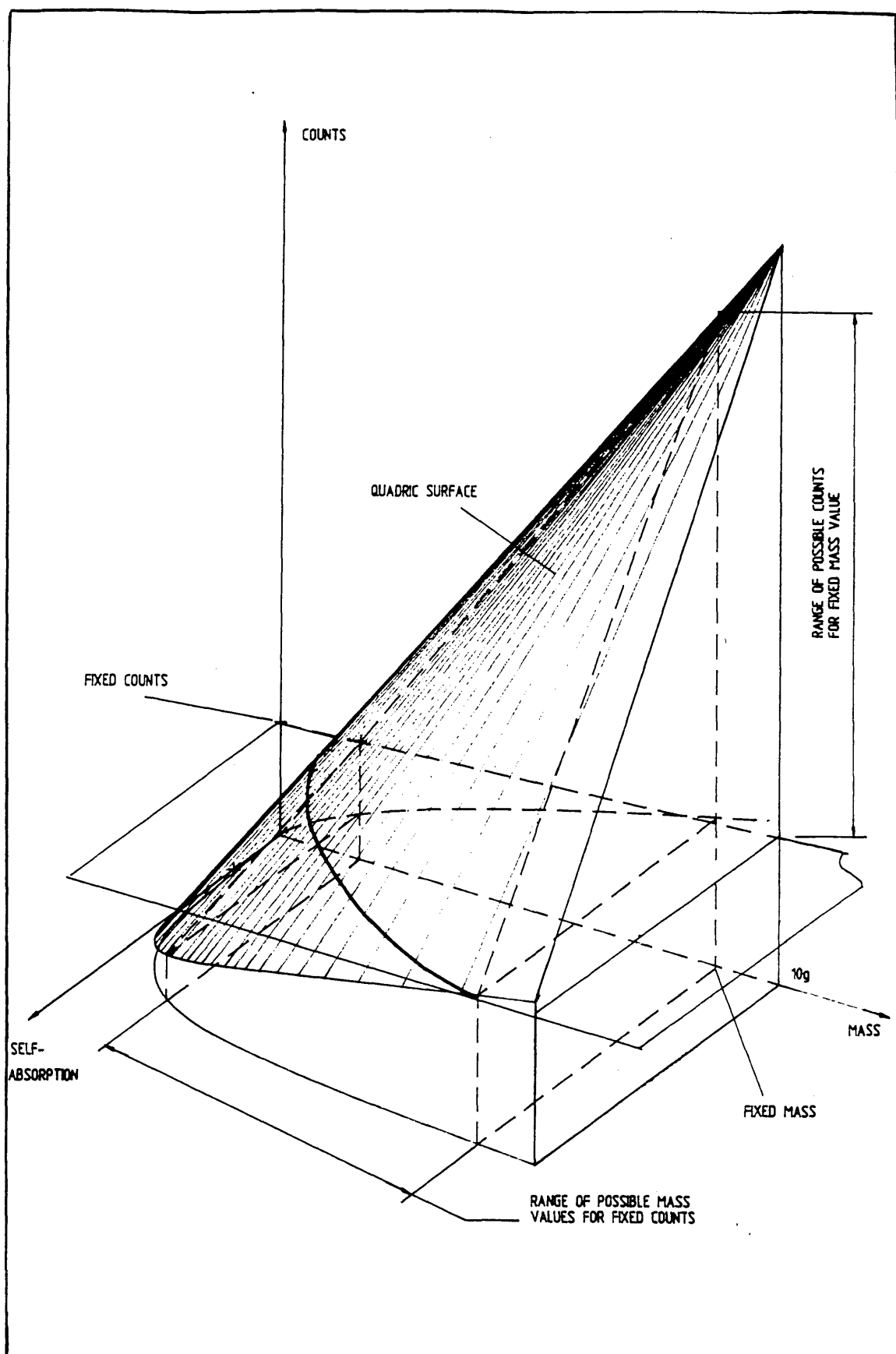


FIG 3.1 THE EFFECT OF SELF-ABSORPTION ON A MASS MEASUREMENT

calibration data, it is necessary to construct a (representative) standard calibration sample.

However, the above approach leads to difficulties. Firstly this is because the observed signals are the result of a process which can involve many complex interactions between the radiation flux, the source and the measurement system. Therefore, a neatly partitioned section of the signal set which relates to each main feature will not exist. Furthermore, it may not be clear how to deconvolute the signal set to obtain statistics which do relate to each feature.

In certain cases these statistics can be defined. For example, the photopeaks in a high resolution gamma spectrum relate to the activity of a specific isotope. Even in this case, however, difficulties can still be encountered. This is because the calibration constants, which are central to the naive concept, rely upon assumptions which could break down for some samples and this possibility introduces unquantifiable uncertainties into the measurement results.

To see this, consider the implications of the standard sample used to define the calibration constants. By definition, such a sample must be constructed to the same specification as the samples output from the nuclear process, while conversely, the use of the calibration constants requires the assumption that the outputs being tested are of the same form as the representative standard. But, the bias parameters may be different for each measurement. Therefore, the assumption breaks down and this introduces an unknown uncertainty into the measurements which may be unbounded.

In the above cases, it would be possible to argue that it is always possible to define empirical statistics which relate to combinations of source features and that it may be possible to deconvolute these using the results of other measurements. Likewise, it could be argued that the calibration constants will be an approximate representation of the effects of the bias parameters on a measurement for most of the samples. However, the resulting estimates of the main features will certainly be dependent upon both the compositions and the calibration

constants used in the calculation and, even if these factors are a true representation for most of the samples, who can say what the composition and bias effects will be under a fault condition? Furthermore, the naive concept is not well suited to the use of information obtained by other measurements for correction purposes, since, by the definition of the naive concept, the results of these measurements have been reduced to a combination of main features and exclude the effects of the bias parameters on the measurements. Also, one may encounter the practical difficulty that correction information is required to define the correction information and it is not clear at which point one should start.

Hence, rather than pursue the naive concept and try to solve its intrinsic problems, an attempt will be made to formulate an axiomatic concept which avoids (rather than solves) the above problems.

3.2 AXIOMATIC CONCEPT - THE KEY IDEA

The naive concept is loosely based upon a classical, information theory approach [1], where the objective is to reduce the measurement data to a small number of statistics with the minimum loss of information concerning the main features of a sample although minimum variance of the statistics is not necessarily the same thing as minimum loss of information. Then, these statistics are interpreted in terms of the main features using a pre-defined numerical relationship between these quantities. This approach breaks down because the underlying parameters (main features) do not define an adequate statistical model of the sample in the sense that other factors affect a measurement (eg, the bias parameters) and the model does not account for these extra factors. Therefore, the statistics cannot be related directly to the main features.

Within the information theory approach, an adequate statistical model can be defined by combining the set of isotope activities about which information is required and the set of bias parameters, into a single parameter vector. Here, the bias parameters are explicitly included in

the model for analysis purposes only, so as to take them properly into account, and they will be eliminated from the final results. It is important that the set of bias parameters, together with the isotopes, totally describe a sample. In other words, if two samples have the same sets of values for the isotopes and bias parameters, then, in the absence of random noise *2, the samples will yield identical signals for each of the measurements. Having defined an adequate model, the objective is to estimate values for the isotopes and bias parameters which are mutually consistent and best fit the measurement data. In principle, to eliminate the bias parameters, one need only integrate them out of the final probability distribution for the estimates. However, if the measurement data does not contain sufficient information concerning both the isotopes and the bias parameters, this integral or the estimator itself can diverge. Therefore, the axiomatic concept includes an extension of this approach to incorporate Bayesian statistics, which form a natural generalisation of the classical approach and (within reasonable limits) can be used to stabilise the estimation and the integration over the bias parameters.

This involves the construction of a prior probability distribution which contains general information concerning the samples which are output from the nuclear process, such as the isotopes which could be encountered and the correlations between them. Then, we can take this distribution as the starting point of a measurement interpretation, rather than starting from the position of no information at all.

With this starting point, the key proposition of the axiomatic concept is that the measurement data can be used to restrict the isotopes and bias parameters to a smaller range of possible values than the original prior distribution and, in particular, eliminate some of the isotopes altogether. Furthermore, if several different measurements are made on the same sample, for example, in a scanning system, these measurements will be representative of the sample in different states and, therefore, will not restrict the prior distribution in the same way.

*2 Random variations in the signals for identical measurements on the same sample.

Thus - and this is the key point - the intersection of all the restrictions to the prior distribution will be much smaller and contain fewer isotopes than any of the restrictions themselves.

It is proposed that the prior distribution can be defined by starting from an initial approximation and refining this initial distribution using the results of measurements on all of the samples which are output from the nuclear process. Thus, once the system is operating, it will gradually "learn" about the nuclear process and the measurement results will continually improve until a limiting distribution is reached.

The above method of measurement interpretation can be used for making "decisions" concerning the operation of a nuclear process. For, with information concerning activity and isotopic composition represented by a probability distribution (ie, the prior distribution combined with the measurement data), hypothesis tests can be constructed to check the correctness of the process operation. These tests will be subject to two types of error which are:

- error of the first kind, rejecting a sample which conforms to its requirements,
- error of the second kind, accepting a sample which does not conform to its requirements

The results of these tests will be probabilities that a sample is within or out of specification, together with the probability for the corresponding type of error.

If a sample is well within specification, as will generally be the case, then the measured probability that the sample is out of specification will be small, even with noisy data. However, for border-line samples more accurate measurements will be required to reduce the probability of an error to an acceptable level.

To optimise the efficiency of the process monitoring system, data

adaptive processing can be used. This involves making measurements for enough time to obtain sufficient accuracy to test most of the well conforming samples but not the border-line cases. Then, when a non-conclusive result is obtained, the measurements are repeated until sufficient accuracy for a conclusive result is achieved. Providing the sets of isotopes and bias parameters totally describe the samples, the probability of an error can be made as small as desired by repeating the measurements in this manner, while no more time than is necessary is spent testing the easier samples.

3.3 MEASUREMENT INTERPRETATION

The axiomatic concept is based on assumptions that certain features of the measurement process remain fixed for all of the samples (including those observed under a fault condition). In fact, these fixed features involve the generalisation of ideas from the naive concept in three specific areas.

- 1) The variable sets of isotopes output from a nuclear process are all replaced by the fixed set of isotopes which could be output from the process, where the possibility that certain isotopes have zero mass is not excluded. Also, it is recognised that certain combinations of isotopes have higher probability than others.
- 2) All that can be known about the bias parameters prior to a measurement is the probability that they will take a particular set of values. Therefore, the fixed calibration constants of the naive concept are replaced by a distribution of bias parameters, with each combination of the parameters having a fixed probability. Correlations may exist between the isotope sets and the bias parameters.

It is desirable that the set of bias parameters are concerned purely with features of the samples and, in particular, exclude the distortion effects of the measuring apparatus from the set. This is because these effects will be unique to a measurement

station and problems with the combination of data from different measurement stations may be encountered if the bias parameters are not independent of the apparatus.

- 3) The naive concept of several isolated measurements of different parameters (main features) is replaced by the concept of several observations of the same (fixed) set of isotopes and bias parameters but under different physical states. That is, each signal observed by the system is equivalent to a combination of these fixed parameters, with different observations giving rise to different combinations.

Since the above assumptions relate to features of all samples output from a nuclear process they do not violate the conditions for unbiased measurement results. Indeed, the assumptions state no more than the fact that we are making measurements on the nuclear process, as opposed to something else. However, the first of the assumptions specifies all that can be known concerning the radionuclide content of the process outputs prior to a measurement, while the second assumption concerns all the prior information regarding other features of a sample which can affect a measurement.

Conceptually, all of the prior information concerning the isotopes can be summarised by a (Bayesian) prior (joint probability) density function which, in particular, contains information from the correlations between isotopes. To define the effects of the bias parameters on a measurement, the prior density can be extended to incorporate the bias parameters by adding extra dimensions. Then, in addition to the information concerning the isotopes, this augmented function will contain information from correlations between the bias parameters and isotopes, and restrictions to the ranges of the bias parameters. It is traditional to denote the prior density function by $\pi(\theta, \rho)$ where $\theta = (\theta_1, \theta_2, \dots, \theta_n)$ is a vector of isotope activities and $\rho = (\rho_1, \rho_2, \dots, \rho_{m-n})$ is a vector of bias parameters.

With the above conditions, a sample can be completely characterised by a specific vector (θ, ρ) and further, the probability associated with

that vector will be the probability that the sample will be encountered by the system.

The third of the assumptions is concerned with the information which is available by measurement. For a given set of isotopes and bias parameters, it would be possible to form a probability density function for the signals observed from this sample. This is known as a conditional density function and is regarded as a function of the signal data for fixed isotopes and bias parameters. The conditional density is usually denoted $p(s|\theta, \rho)$, where $s = (s_1, s_2, \dots, s_r)$ is regarded as a sample from a random vector of observed signals. If (and only if) the parameters (θ, ρ) represent an adequate model of the samples output from a nuclear process, no factors other than random noise will contribute to the variation of s , and a well-defined relationship exists between the parameters (θ, ρ) and the means of the conditional density, which, in particular, accounts for the (constant) distortion of the measuring apparatus.

For the present application, this relationship could take the form of a linear transformation. For example, let R^m be the set of all m -dimensional vectors such that the vector $w = (\theta, \rho)$ belongs to R^m (written $w \in R^m$), then R^m defines an m -dimensional Euclidian space in which a point specifies the configuration of a sample. Likewise, if $\mu = (\mu_1, \mu_2, \dots, \mu_r)$ is a vector of the signal means, then μ belongs to the space R^r . The linear transformation, g , is a mapping from R^m to R^r (written $g: R^m \rightarrow R^r$) which takes the form $Gw = \mu$, where G is an $m \times r$ matrix.

To define the matrix, G , let $\{e_1, e_2, \dots, e_m\}$ be a base for R^m and $\{f_1, f_2, \dots, f_r\}$ be a base for R^r . Then, the image under the transformation of each basis vector, $g(e_i)$, is a vector in R^r . Therefore, we may write down the system of equations:

$$\begin{aligned} g(e_1) &= g_{11}f_1 + g_{21}f_2 + \dots + g_{r1}f_r \\ g(e_2) &= g_{12}f_1 + g_{22}f_2 + \dots + g_{r2}f_r \\ &\dots \quad \dots \quad \dots \quad \dots \quad \dots \\ g(e_m) &= g_{1m}f_1 + g_{2m}f_2 + \dots + g_{rm}f_r \end{aligned} \tag{3.1}$$

and thus construct the matrix, such that:

$$\begin{pmatrix} g_{11} & g_{12} & \cdots & g_{1m} \\ g_{21} & g_{22} & \cdots & g_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ g_{r1} & g_{r2} & \cdots & g_{rm} \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ \cdots \\ w_r \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \cdots \\ \mu_r \end{pmatrix} \quad (3.2)$$

With this relationship, the conditional density function will have the form:

$$p(s|\theta, \rho) = h(s - g(\theta, \rho)) \quad (3.3)$$

which represents the possible distribution of observed signals about a fixed mean value which is related to the isotopes and bias parameters by the transformation (3.2).

In practice, the above formulation cannot be used directly since the fixed values for the isotopes and bias parameters are unknown, while the signal values are known, having been observed by measurement. Therefore, we define a likelihood function as a function of the isotopes and bias parameters which takes identical numerical values to the conditional density, for fixed (observed) signal values.

For example, if the likelihood function is denoted by $\mathcal{L}(\theta, \rho|s)$, then:

$$\mathcal{L}(\theta, \rho|s) = p(s|\theta, \rho) \quad (3.4)$$

for all θ and ρ , with fixed s . Note that $\mathcal{L}(\theta, \rho|s)$ is not a probability density function and the area under the curve is not necessarily unity.

A likelihood function, $\mathcal{L}_i(\theta, \rho|s_i)$, can be constructed for each of the measurements on the same sample and these can be combined as a product:

$$\mathcal{L}(\theta, \rho|s_1, \dots, s_N) = \prod_{i=1}^N \mathcal{L}_i(\theta, \rho|s_i) \quad (3.5)$$

to summarise all of the information which is available by measurement, where the s_i are independent observations.

The total information concerning a sample is contained in the likelihood function (3.5) together with the prior density function. These can be combined using Bayes Theorem:

$$p(\theta, \rho | s_1, \dots, s_N) = k \mathcal{L}(\theta, \rho | s_1, \dots, s_N) \pi(\theta, \rho) \quad (3.6)$$

where k is a normalisation factor. The resulting probability density is known as a (Bayesian) posterior density function. Since knowledge is only required of the isotopes, the bias parameters must be eliminated from the posterior density by integration:

$$p(\theta | s_1, \dots, s_N) = \int p(\theta, \rho | s_1, \dots, s_N) d\rho \quad (3.7)$$

Then, $p(\theta | s_1, \dots, s_N)$ will be the best available representation of the activities, given the prior knowledge and the measurement data.

Equation (3.6) can be interpreted as the prior density function reducing the ranges of isotope activity and bias parameter values on which the likelihood function can be defined. Therefore, if an informative prior density is constructed, this will tend to counter variations of the likelihood function resulting from noisy data which contains little information, and limit the increase in the variation of the isotope activity arising from the elimination of the bias parameters, which may otherwise cause the integral (3.7) to diverge.

Moreover, providing the prior density relates to all samples it can be freely used in the interpretation of the measurement data without violating the conditions for unbiased measurement.

Equation (3.6) can also be regarded as a linear operation which maps the prior density to the posterior density, with the mapping representing the change in the quantity of information concerning a sample brought about by measurements on that sample.

The interpretation process is illustrated in one dimension, using simple rectangular functions to indicate the range of the parameter, in Fig 3.2. In particular, if the intersection of the restrictions to

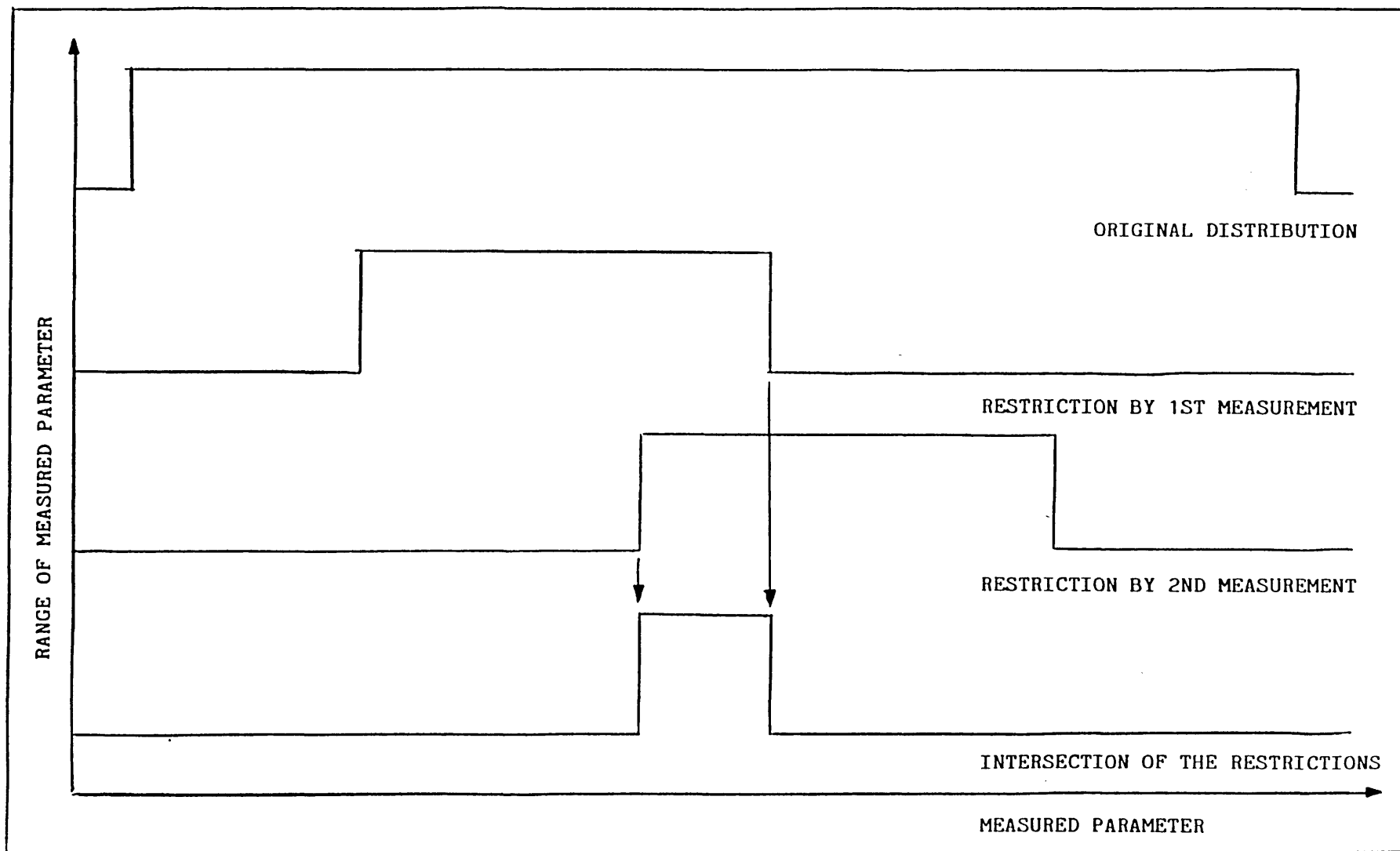


FIG 3.2 : SCHEMATIC OPERATION OF THE PROCESSING ALGORITHM

these ranges is zero, the probability that the isotope is contained in the sample is also zero.

In more than one dimension, this method is potentially more powerful than in the single dimensional case. For example, each likelihood function will contain strong negative correlations between the isotopes and bias parameters. That is, with a likelihood function arising from a single measurement, the product of all the isotopes with the bias parameters could be accurately known having been defined by the measurement, but individually, all of the parameters would still be able to vary over wide ranges. However, the isotopes and bias parameters will not necessarily affect each measurement in the same way and when the product of all the likelihood functions is taken, the resulting ranges of the isotopes and bias parameters will be reduced to those values which are consistent with all of the measurement data. So, if reasonably different correlation is obtained for each of the likelihoods, taking the product of the likelihoods will go some way towards decorrelating the sets of isotopes and bias parameters. However several different combinations of isotopes may still be able to produce the same signals; hence, strong negative correlations may still exist between certain isotopes, and this will be reflected in their possible ranges.

On the other hand, the prior density can be configured so that it contains strong positive correlations between the isotopes. This is because the information concerning the isotopes arises from knowledge of the composition of the different waste streams, rather than knowledge of the gross nuclear reaction rates within the sample. Therefore, when the product of the likelihoods and prior density is taken, this complementary information will be combined and may decorrelate the isotopes to a very large extent, thus reducing their ranges to an acceptable level. This concept is illustrated in Figure 3.3 for a three dimensional space.

We can conclude from this that optimum performance of a process monitoring system will be obtained from an informative prior density together with signal sets which contain a maximum number of different

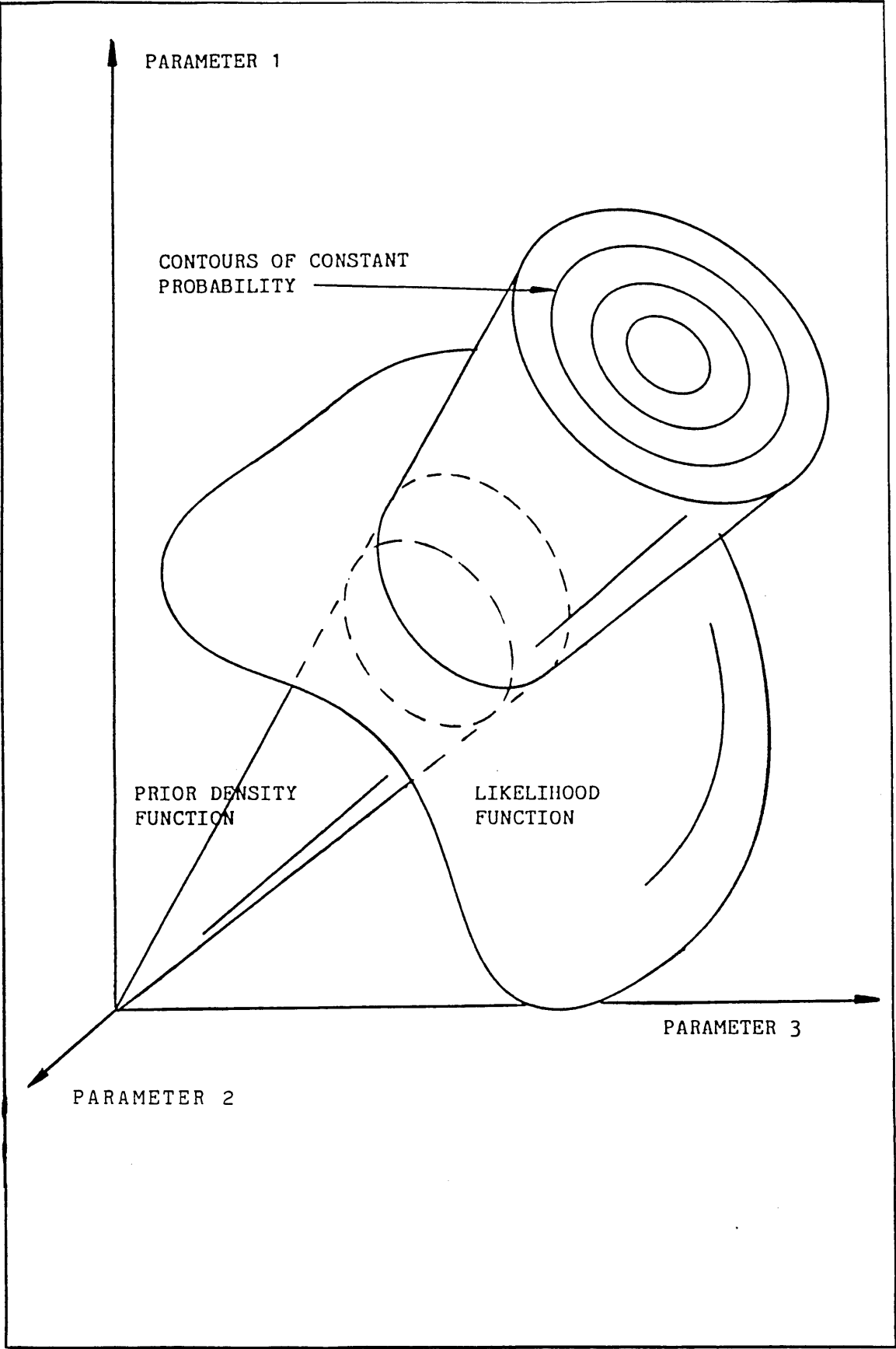


FIG 3.3 INTERSECTION OF THE PRIOR DENSITY AND LIKELIHOOD FUNCTIONS

correlations between the isotopes and the bias parameters. These, almost certainly, will not be the same as the signals required by the naive concept for which the bias parameters introduce minimum variance and the correspondence with the isotopes is assumed to be one-one.

3.4 RANDOM PROCESSES

The formulation of the model in the previous sections has concentrated on the mapping from the source to the observed signals and its inverse. In particular, little mention has been made of the underlying processes and the existence of probability density functions has been assumed rather than demonstrated. Therefore, in this section an attempt will be made to present an argument which may go some way towards justifying the assumptions. However, in keeping with most Physics, standard mathematical results will be used without proof. Such proofs can be found in many mathematical texts, for example, references [2] and [3].

To form the probability density functions it is necessary to consider the evolution of the system configuration with time. There are three main areas of interest.

- 1) The sequence of samples output from the nuclear process.
- 2) The evolution of the radiation flux in and around the source containment.
- 3) The build-up of counts on the signal space.

In this section we shall consider the output of the nuclear process together with the definition of the Bayesian prior density function used to describe these outputs. The evolution of the flux and the build-up of counts will be considered in chapter 4.

The Bayesian prior density contains all of the information concerning a sample which is available prior to measurements being made, with the two conditions that:

- 1) A point in the (domain of the) prior density must totally describe the sample.
- 2) No prior distinction can be made between any of the samples encountered by the measurement system in the interpretation of the measurements.

It has been proposed that the prior density can be defined in two steps by first forming an initial approximation to the density function from calibration measurements and then refining this density using the measurement information from all of the samples output from the nuclear process, including those under fault conditions.

In the previous sections, only measurements on a single (although arbitrary) sample have been considered. There, the isotopes and bias parameters were defined as fixed but unknown parameters relating to this sample while the prior density was interpreted as a distribution of possible values these fixed parameters could take. Here, however, the variations throughout all of the samples output from the process must be considered. To do this, it is necessary to introduce some new notation.

Let Ω be the set of all possible sample configurations. Then Ω is called the sample space and the parameter vector $\mathbf{w} = (\theta, \rho)$ can be defined as a fixed but arbitrary sample configuration belonging to Ω .

Since the actual values of a parameter vector are unknown and the most that may be known in practice concerning a vector is that it belongs to some subset of Ω , it is necessary to introduce the concept of a σ -field. A σ -field, F , is defined as a collection of subsets of Ω , such that:

- 1) The empty set, $\emptyset$, belongs to F .
- 2) All countably infinite unions of the sets of F belong to F .
- 3) All complements of the sets of F belong to F .

The above conditions ensure that the subsets of Ω belonging to a σ -field are sufficiently nice in the sense that they exclude pathological cases, such as the subsets of the empty set, which can lead to technical difficulties.

A probability measure, P , on (Ω, F) is a function $P: \Omega \rightarrow [0, 1]$, such that:

$$1. \quad P(\emptyset) = 0, \quad P(\Omega) = 1 \quad (3.8).$$

2. If $A_1, A_2, \dots$ is any collection of disjoint sets belonging to F , then:

$$P\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} P(A_n) \quad (3.9).$$

The probability measure of a set, A , can be interpreted as the probability that a sample, w , belongs to A ; where, in particular, the probability that a sample belongs to the empty set is zero while the probability that a sample belongs to Ω is one.

The triple (Ω, F, P) , is called a probability space.

A suitable model for the samples which are encountered by the monitoring system is a random process, denoted:

$$X = \{X(t) : t \in T\} \quad (3.10)$$

where each $X(t): \Omega \rightarrow \Sigma_X$ is a random vector which is defined as a function mapping the sample space onto a state space, Σ_X *3, with the condition that:

$$\{w \in \Omega : X(t; w) \in x(t)\} \in F \quad (3.11)$$

*3 The subscript, X , is used to highlight the role of X in the definition of Σ . This will be omitted when discussing arbitrary state spaces.

The technical condition asserts that the subsets of Ω for which the random vectors are defined belong to the σ -field. If this condition holds for all of the elements $X(t) \in \Sigma_X$ the random vector is said to be F -measurable. The specification of these subsets is often abbreviated to $\{X(t) \leq x(t)\}$, and we shall conform to this practice here.

A state space Σ can be constructed to represent any consequences of an experiment in which we are interested. For the present application, $X(t)$ could be interpreted as the identity function since we are interested in the sample configuration itself. However, it would be equally feasible to define a random vector to map the sample configuration onto a radiation flux or a count on the signal space. In fact we shall do this shortly. A state space is called discrete if the random vectors take discrete values or continuous if the random vectors are continuous. The elements of the state space Σ_X denoted:

$$x = \{X(t;w) : t \in T\} \quad (3.12)$$

are called the sample paths of the process. These can be regarded as specific numerical values, each corresponding to a sample, w , which are taken by the random process, X , as this evolves in time.

The form of the index set, T , depends upon the system under investigation. For example, in the case of discrete nuclear process outputs, such as the radioactive waste packages encountered by the assay facility of section 2.7, the index set can be put into correspondence with the natural numbers, such that $t \in \{0,1,2,\dots\}$. Then the random process is called a discrete time process and it is common practice to denote the random vectors by X_t . In the case of continuous outputs of a nuclear process such as the iodine building up on the collection filter of the blowdown monitor (see section 2.6), then $t \in [0,\infty)$. Such a random process is called a continuous time process and the random vectors are denoted by $X(t)$.

The joint probability distribution functions:

$$F_t(x) = P(X_1 \leq x_1, X_2 \leq x_2, \dots, X_n \leq x_n) \quad (3.13)$$

of the collection of the random vectors $\{X_1, X_2, \dots, X_n\}$ for a discrete time process, are called the finite dimensional distributions (fdds). In the case of a continuous state space, the corresponding probability density functions (pdfs) can be defined:

$$f_t(x)dx = dF_t(x) \quad (3.14)$$

while for a discrete state space, the probability mass functions (pmfs) are defined:

$$f_t(k) = F_t(k) - \lim_{x \uparrow k} F_t(x) \quad (3.15).$$

In particular, the "one" dimensional distribution, $F_t(x)$, is the distribution function of the single random vector, X_t , and is referred to as the distribution of the process at the point t , with the density or mass of the process at the point t being defined similarly. The fdds, pdfs and pmfs can be defined for a continuous time process, by considering the process at discrete points in time.

It is possible to study the evolution of a random process in terms of either its sample paths or its fdds. In fact, the problem of updating the prior density function can be considered as the construction of the density function of the process at the next point in time, given information concerning a sample path up to the present. However, in both cases, this analysis is difficult unless extra conditions are imposed on the random process. In practice, it is usually possible to define the state space in such a way that one or more of these conditions hold and the applicability of the theory to real physical systems is little diminished. In particular, the sequence of outputs from a nuclear process can be modelled by a discrete time Markov process with a continuous state space *4. Subject to an appropriate definition of the state space, this is guaranteed to converge to a unique stationary distribution from which the prior density can be defined.

*4 See section 4.3 for an example of how this model can be applied to a nuclear process with continuous outputs.

A discrete time random process is called a Markov chain if it has the property:

$$P(X_n \leq x | X_0, X_1, \dots, X_{n-1}) = P(X_n \leq x | X_{n-1}) \quad (3.16).$$

This asserts that the future values taken by the process depend only on its present value and not the past.

With the Markov property it is possible to define the transition probabilities:

$$p_{t,t-1}(x|y)dx = dP(X_t \leq x | X_{t-1} = y) \quad (3.17)$$

which gives the probability of the next value of the chain being x given that the present value is y . If:

$$p_{t,t-1}(x|y) = p_{10}(x|y) \quad (3.18)$$

for all t , x and y , the Markov chain is said to be homogeneous and the transition probabilities are written $p(x|y)$. Let $f_{t-1}(y)$ be the pdf of the $(t-1)^{th}$ random vector. Then:

$$f_t(x) = \int_{\Sigma_X} p(x|y) f_{t-1}(y) dy \quad (3.19)$$

maps $f_{t-1}(y)$ to the pdf of the t^{th} random vector, $f_t(x)$. Equation (3.19) can be expanded to give:

$$\begin{aligned} f_t(u) &= \int_{\Sigma_X} p(u|v) \dots \int_{\Sigma_X} p(y|z) f_{t-n}(z) dz \dots dv \\ &= \int_{\Sigma_X} p_n(u|z) f_{t-n}(z) dz \end{aligned} \quad (3.20)$$

where the $p_n(x|y)$ are called the n -step transition probabilities and map a pdf n places higher up the chain.

A pdf π is called a stationary distribution of the chain if it has the property:

$$\pi(x) = \int_{\Sigma_X} p(x|y)\pi(y)dy \quad (3.21)$$

which implies:

$$\pi(x) = \int_{\Sigma_X} p_n(x|y)\pi(y)dy \quad (3.22)$$

Equations (3.21) and (3.22) assert that if any random vector has distribution π , then all subsequent random vectors will have the same distribution.

Therefore, π can be regarded as a limiting distribution of the process. In fact, it can be shown that:

$$p_n(x|y) \rightarrow \pi(x) \text{ as } n \rightarrow \infty \quad (3.23)$$

for all starting values, y , and this implies:

$$\int_{\Sigma_X} p_n(x|y)f(y)dy \rightarrow \pi(x) \text{ as } n \rightarrow \infty \quad (3.24)$$

regardless of the starting distribution $f(y)$; providing the stationary distribution exists. However, necessary and sufficient conditions for the existence of the stationary distribution are that the Markov chain is irreducible and non-null persistent. These conditions hold if Σ_X is closed and bounded *5, and:

$$\lim_{n \rightarrow \infty} p_n(x|x) > 0 \quad (3.25)$$

*5 Since $\Sigma_X \subset \mathbb{R}^n$ this is simply the assertion that the state space is compact.

for all $x \in \Sigma_X$. Equation (3.25) asserts that the probability for obtaining a particular sample does not tend towards zero as the process evolves in time or, equivalently, that the range of the nuclear process outputs does not change. Therefore, if the state space is defined so that these conditions hold, the existence of the prior density is guaranteed.

This argument can be taken further if the collections of random vectors $\{X_{t_1}, X_{t_2}, \dots, X_{t_n}\}$ and $\{X_{t_1+h}, X_{t_2+h}, \dots, X_{t_n+h}\}$ have the same fdds, then the process is strongly stationary. Or, if:

$$E(X_{t_1}) = E(X_{t_2}) \quad (3.26)$$

and

$$\text{cov}(X_{t_1}, X_{t_2}) = \text{cov}(X_{t_1+h}, X_{t_2+h}) \quad (3.27)$$

where $E(X)$ denotes the expected value of X and $\text{cov}(X_1, X_2)$ is the autocovariance of X , then the process is stationary. It should be noted that strong stationarity implies stationarity but not conversely.

Clearly, a Markov chain is stationary if and only if the starting distribution of the process is the stationary distribution, $\pi(x)$, and in this case the process is strongly stationary. However, a stationary process is not necessarily Markov.

With the outputs of a nuclear process modelled by a stationary random process, X , the random process:

$$S = \{S_t : t = 0, 1, 2, \dots\} \quad (3.28)$$

where $S_t: \Omega \rightarrow \Sigma_S$ is a random vector mapping the sample space onto the observed signals, will also be a stationary process. It is a requirement of the present work to construct the prior density, $\pi(x)$, from the observed sample path:

$$s = \{S_t(w) : t = 0, 1, 2, \dots\} \quad (3.29)$$

This can be done using the conditional expectation operator, $\psi(S)$, *6 where:

$$\begin{aligned}\psi(s) &= E(X|S = s) \\ &= \int_{\Sigma_X} xp(x|s)dx\end{aligned}\quad (3.30)$$

and ψ is a function mapping Σ_S onto Σ_X . Note, however, that the conditional density function $p(x|s)$ is just the Bayesian posterior density function defined in section 3.3. Furthermore, the expected value of a function, h , of X is given by:

$$\begin{aligned}E(h(X)) &= E(h(\psi(S))) \\ &= \int_{\Sigma_S} h(\psi(s))f(s)ds \\ &\sim \frac{1}{n} \sum_{j=1}^n h(\psi(s_j))\end{aligned}\quad (3.31)$$

hence:

$$E(h(X)) \sim \frac{1}{n} \sum_{j=1}^n h(\hat{x}_j) \quad (3.32)$$

where $\hat{x}_j = \psi(s_j)$ is the estimate for the configuration of the i th sample.

Equation (3.32) can be used to estimate the moments of the prior distribution, from which this distribution can be constructed.

It remains to demonstrate the convergence of (3.31) to a unique stationary value since this does not follow from the law of large

*6 In the case of a stationary process we can drop the subscript, t , from the random vectors X_t and S_t .

numbers in the case of dependent random vectors. However, by restricting our attention to Markov chains, this convergence is guaranteed by virtue of (3.23).

Finally, a few words concerning the function ψ are in order. In section 3.3 a function, g , was constructed. This was assumed to be a transformation which mapped the sample configuration onto the observed signals, such that:

$$s = g(x) \quad (3.33)$$

It can be seen that ψ is the inverse function to g . With this identification Bayes theorem can be written as:

$$p(x|\psi(s)) = k \mathfrak{L}(g(x)|s)\pi(x) \quad (3.34)$$

where x and $\psi(s)$ are defined on the state space Σ_x , while s and $g(x)$ are defined on the state space Σ_s . Hence Bayes theorem provides a mapping, ψ , from Σ_s to Σ_x given a mapping, g , from Σ_x to Σ_s .

Furthermore, since x and $\psi(s)$ are defined on the same space, a specific value of ψ given the signals, s , can be computed using standard estimating methods.

The problem of defining the forward mapping, g , in detail will be considered in chapter 4. Note, however, that equation (3.34) is completely general in that g is not necessarily linear, although if linearity is achieved this will reduce the computational overhead associated with the implementation of the model.

3.5 THE NORMAL BAYESIAN FAMILY

It is important that the model includes consideration of the practical limitations on computer power, memory capacity and the possible processing time for each sample output from a nuclear process. To a large extent these requirements are governed by the form of the

transformation between the source and the observed signals, the number of parameters and signals, and the method of representing the likelihood, prior and posterior density functions by the computer system.

With the above constraints, the obvious numerical method of representing the functions as multi-dimensional histograms is clearly impractical, simply because of the gross quantity of data involved. Therefore, the likelihoods, prior and posterior densities are best represented by analytic functions. These are represented in the computer by auxiliary parameters such as means and covariances, which can be used to construct the required functions providing their form is known.

For the practical application of Bayes theorem, it is important that the likelihood, prior and posterior densities are functions of the same type. Such a collection of probability densities is known as a Bayesian family [4]. In this section the general model will be examined in the particularly simple and practically useful case of the Normal Bayesian family.

Let x be an element of Σ_x such that:

$$\begin{aligned} x &= X(\theta, \rho) \\ &= X(\theta_1, \theta_2, \dots, \theta_n, \rho_1, \rho_2, \dots, \rho_{m-n}) \\ &= (x_1, x_2, \dots, x_m)^T \end{aligned} \quad (3.35)$$

where θ is a vector of isotope activities and ρ is a vector of bias parameters. Assume that k measurements are made on the same sample and that for the i th measurement x is related to the vector of observed signals by the linear transformation:

$$s_i = G_i x \quad (3.36)$$

Then the likelihood functions can be defined, as previously, by

equating it to the values taken by the conditional pdf for the signals:

$$\mathcal{L}_i(x|s_i) \equiv p_i(s_i|x), \quad i = 1, 2, \dots, k \quad (3.37)$$

where $p_i(s_i|x)$ is a multivariate normal distribution, such that:

$$\mathcal{L}_i(x|s_i) = c_i \exp\left[-\frac{1}{2}(s_i - G_i x)^T V_i^{-1}(s_i - G_i x)\right] \quad (3.38)$$

The collection of likelihoods can be combined with a multivariate normal prior density:

$$\pi(x) = c \exp\left[-\frac{1}{2}(x - \mu)^T U^{-1}(x - \mu)\right] \quad (3.39)$$

using Bayes theorem to yield a posterior density of the form:

$$p(x|s) = k c c_1 \dots c_k \exp\left[-\frac{1}{2} \sum_{i=1}^k (s_i - G_i x)^T V_i^{-1}(s_i - G_i x) - \frac{1}{2}(x - \mu)^T U^{-1}(x - \mu)\right] \quad (3.40)$$

This equation can be tidied up as follows.

Let $K = k c c_1 \dots c_k$ be a new normalisation constant and define:

$$s = \begin{pmatrix} s_1 \\ s_2 \\ \dots \\ s_k \\ \mu \end{pmatrix} \quad G = \begin{pmatrix} G_1 \\ G_2 \\ \dots \\ G_k \\ I \end{pmatrix} \quad V = \begin{pmatrix} V_1 & & 0 \\ & V_2 & \\ & & \dots \\ 0 & & V_k & U \end{pmatrix} \quad (3.41)$$

where I is the identity matrix. Then equation (3.40) can be written as:

$$\begin{aligned} p(x|s) &= K \exp\left[-\frac{1}{2}(s - Gx)^T V^{-1}(s - Gx)\right] \\ &= K \exp\left[-\frac{1}{2}(x - \psi(s))^T G^T V^{-1} G (x - \psi(s))\right] \end{aligned} \quad (3.42)$$

which is multivariate normal with covariance matrix:

$$W = [G^T V^{-1} G]^{-1}$$

and mean:

$$\hat{x} = \psi(s) = [G^T V^{-1} G]^{-1} G^T V^{-1} s \quad (3.44)$$

Equation (3.44) may be identified in this case as the linear least square estimator for x .

The above estimator is actually more general than implied by the analysis presented here. In fact, it can be shown that equation (3.44) defines the optimum linear estimator when the shape of the distribution is unknown [1]. Because of this the linear least square estimator is often referred to as the general linear model and its robustness is of great value in practical problems.

The inverse of the covariance matrix may have the form:

$$V^{-1} = \begin{pmatrix} X & & & & & & & 0 \\ & X & & & & & & \\ & & \ddots & & & & & \\ & & & X & & & & \\ & & & & 0 & X & \dots & X \\ & & & & X & 0 & \dots & X \\ & & & & & & \ddots & \ddots \\ 0 & & & & & & \dots & \dots & \dots \\ & & & & X & X & \dots & 0 \end{pmatrix} \quad (3.45)$$

where X denotes a non-zero element. Note that in this case the signals are mutually independent which results in the diagonal upper portion of the matrix. The diagonal elements of the lower portion have been set to zero so that the means of the prior density are given infinite variance, or equivalently, zero weight, in the estimation of parameter values. The estimator therefore takes into account only the relative proportions of the prior parameter values and as a result is not biased towards any particular value by the inclusion of the prior knowledge. On the other hand, the non-zero off-diagonal elements will restrict the values which can be taken according to the strength of the correlation between these parameters. The resulting estimate will represent an optimum balance between the a priori and measurement information *7.

*7 See also figure 3.3 and section 4.3

To define the prior density it is only necessary to specify the means and covariances of the appropriate multivariate normal distribution function. Using equation (3.32) these are given by:

$$\hat{\mu}_{1N} = \frac{1}{N} \sum_{j=1}^N \hat{x}_{1j} \quad (3.46)$$

and

$$\hat{v}_{1kN} = \frac{1}{N} \sum_{j=1}^N (\hat{x}_{1j} - \hat{\mu}_{1j})(\hat{x}_{kj} - \hat{\mu}_{kj}) \quad (3.47)$$

where $\hat{x}_{ij}$ is the estimate of the i th component of x for the j th sample and $\hat{\mu}_{1N}$ and $\hat{v}_{1kN}$ are respectively the N th estimates for the elements of the mean vector and the covariance matrix.

In practice there is no need to recompute the entire partial sums (3.46) and (3.47) for each sample as these can simply be updated, by:

$$\hat{\mu}_{1N} = \frac{1}{N}(\hat{x}_{1N} + (N-1)\hat{\mu}_{1N-1}) \quad (3.48)$$

and

$$\hat{v}_{1kN} = \frac{1}{N}((\hat{x}_{1N} - \hat{\mu}_{1N})(\hat{x}_{kN} - \hat{\mu}_{kN}) + (N-1)\hat{v}_{1kN-1}) \quad (3.49)$$

However, with the multivariate normal family, there is a problem in that the estimate of the variance is based on the estimated mean, $\hat{\mu}$, rather than the true mean, μ . Therefore an accurate starting value is required for this parameter, which may be difficult in reality. The alternative is to use a family of distributions which take this uncertainty into account [5].

If the multivariate normal family is used an improvement can be made to (3.48) and (3.49) by fixing the parameter N at some suitably large value. Then, the latest terms of the sum will be given highest weight and since these are based on the later (more accurate) values of $\hat{\mu}$, the uncertainties in the early terms will slowly be weighted out. Furthermore, the rounding errors which would be introduced by adding smaller and smaller terms to the partial sums will be avoided. This approach also has the advantage that (long term) changes in the form of

the nuclear process outputs encountered by the system can be taken into account and the prior density made to follow trends, hence relaxing the requirement for a pure stationary process. These possibilities are examined in greater detail in section 4.3.

It is useful to estimate the number of operations required to compute the parameter values since this will give an indication of the computation overhead associated with each method. In the case of the normal family with an embedded linear transformation, g , it can be shown that the total number of operations required to compute parameter estimates of the form (3.44) is approximately:

$$\frac{4}{3}m^3 + 3m^2 + n(m+1)^2 \quad (3.50)$$

where m and n are the number of parameters and signals respectively.

In practical applications, the situation may arise where samples of several different types may be encountered by the same monitoring system. For example, for the waste package assay facility, radioactive waste may arrive from several locations each producing waste of a unique form. In this case it is unlikely that the prior density function will be well represented by a multivariate normal distribution. However, it would not be unreasonable to assume that the waste arising from each source was normally distributed.

The above analysis can be extended to incorporate this possibility by forming a separate multivariate normal prior density for each sample type. These can be combined to construct a prior density of the form:

$$\pi(x) = c \exp \left[-\frac{1}{2} \sum_{j=1}^Q (\mu_j - F_j x)^T U_j^{-1} (\mu_j - F_j x) \right] \quad (3.51)$$

where:

$$x = \bigcup_{j=1}^Q x_j \quad (3.52)$$

and μ_j and x_j are the mean and sample path values for the j th sample type. The matrices F_j have elements zero and one such that $F_j: x \rightarrow x_j$.

This prior density can be used in place of equation (3.39) in the interpretation of measurements.

To update the prior density it is necessary to test the result to determine which sample type has been observed. Then only the prior density for this sample type is updated. This may involve setting those parameter values below a particular critical level to zero, then testing to see which prior density best fits the remaining data. One method for performing this test is to transform the data to a set of statistics which are in one to one correspondence with the sample types, then simply read off the observed sample type.

Let x be represented by a linear combination of the prior density means such that:

$$\begin{aligned} x &= \lambda_1 \mu_1 + \lambda_2 \mu_2 + \dots + \lambda_L \mu_L \\ &= M\lambda \end{aligned} \tag{3.53}$$

where M is an $L \times m$ matrix with columns λ_j . Equation (3.53) can be solved for λ using the general linear model to give:

$$\lambda = [M^T V^{-1} M]^{-1} M^T V^{-1} x \tag{3.54}$$

where V is the covariance matrix of the posterior density for x .

The statistics λ_j should all be near zero except the one corresponding to the sample type and this should be near to unity. If this is not the case, then either mixed samples have been observed or the prior densities are not a sufficiently good representation of the sample types to be able to distinguish between them. In the former instance clear results should still be obtained with more than one λ_j being non-zero. In the latter instance, the situation may be improved by giving the zero parameters higher weight, repeating measurements to obtain more accurate data or using the results of the next section to determine which sample types are significant and repeating the test with a smaller number of possible outcomes.

3.6 HYPOTHESIS TESTING

In this section we shall specify criteria to indicate the minimum detectable activity (MDA) of isotopes within a sample, together with the activity of these isotopes which can be determined to a particular accuracy [6].

Within classical statistics, the criteria concerning detection and the determination of the isotope activity apply to the observed signals rather than the activity directly. These factors are related by a linear transformation and, given the minimum detected and determined count rates, the required activities can be calculated using the method of section 3.5.

The specification of the detection limits for a measurement of activity requires the construction of three specific threshold levels:

1. An a posteriori decision, L_D , which establishes whether an observed signal yields a significant result in the sense that the presence of an isotope is indicated.
2. An a priori detection level, L_D , which defines the signal magnitude which must be exceeded to yield a significant result in the sense that the presence of an isotope will be indicated when a measurement is made.
3. A determination level, L_D , where an expected signal yields a significant result relating to the standard deviation of the signal falling within a quoted accuracy for the measurement process.

In statistical hypothesis testing, a hypothesis that a parameter takes a particular value (called the null-hypothesis) is tested against the hypothesis that the parameter does not take that value (called the alternative-hypothesis). A significant result occurs when a variate takes a value in the tail of the underlying probability distribution

for which the parameter is defined. In this circumstance the alternative-hypothesis is accepted with the added proviso that the conclusion has a fixed probability of being wrong. This probability can be made as small as desired by choice of the threshold (critical) levels used in the test. Conversely, a non-significant result occurs when a variate takes a value outside the tail of the distribution. Here, it would be appropriate to reject the alternative-hypothesis and conclude that the true value of the parameter under test is in a region defined by the threshold levels, about the null-hypothesis value for the parameter.

Decisions of this nature are subject to the two kinds of errors introduced in section 3.2, which we may now define as:

- a) error of the first kind, accepting an alternative-hypothesis which is false.
- b) error of the second kind, rejecting an alternative-hypothesis which is true.

The probabilities for the errors of the first and second kinds are conventionally denoted by α and β respectively.

For the present application only one-sided hypothesis tests are of interest, for example, we need only establish whether the limiting (population) mean μ_s , for the signals, s , from a particular isotope, is greater than the null-hypothesis value used in the test.

In the a posteriori case an actual experimental observation is available. It must be established, therefore, if this signal arises exclusively from the background radiation of isotopes other than the one of particular interest, or whether the signal deviates sufficiently from the limiting mean μ_b , for the background signals, b , to indicate the presence of this isotope. The above requirement amounts to testing the null-hypothesis $\mu_s = 0$ against the alternative hypothesis $\mu_s > 0$, using the decision level, L_D . Given a significant result, it would be appropriate to conclude that a non-zero signal has been observed from

the isotope of interest and to assign a confidence interval to this result. Alternatively, we accept L_C as an upper limit to the mean signal from the isotope.

The decision level L_C , is established from the maximum acceptable value of α and the standard deviation, σ_0 , of the net signals when $\mu_s = 0$, such that the area in the tail of the distribution, greater than L_C , is equal to the maximum value of α . Hence:

$$L_C = k_\alpha \sigma_0 \quad (3.55)$$

where k_α is the abscissa for the standardised normal distribution corresponding to the probability level $1 - \alpha$.

In the a priori case, an experimental observation is not yet available. Instead the requirement is to define a limiting mean, μ_s , to the signal distribution for which a random sample, drawn from the distribution (i.e. when a measurement is made), yields a significant result to an (a posteriori) test of the null-hypothesis $\mu_s = 0$, with a probability of $1 - \beta$. The mean μ_s , is taken as the detection limit, L_D for the measurement process, such that:

$$L_D = L_C + k_\beta \sigma_D \quad (3.56)$$

where σ_D is the standard deviation of the signal, s , at the detection limit and k_β is the abscissa for the standardised normal distribution corresponding to the probability level $1 - \beta$.

The relationship between L_C , L_D , α and β is shown in figure 3.4. L_D may be considered to be the smallest mean count which will lead to the detection of an isotope with the probability $1 - \beta$, where the a posteriori decision criterion has a built-in protection level, α , against concluding (falsely) that a background observation represents a significant signal.

In the case of a sequence of measurements the accuracy of the hypothesis test may be improved by using two decision levels, as shown

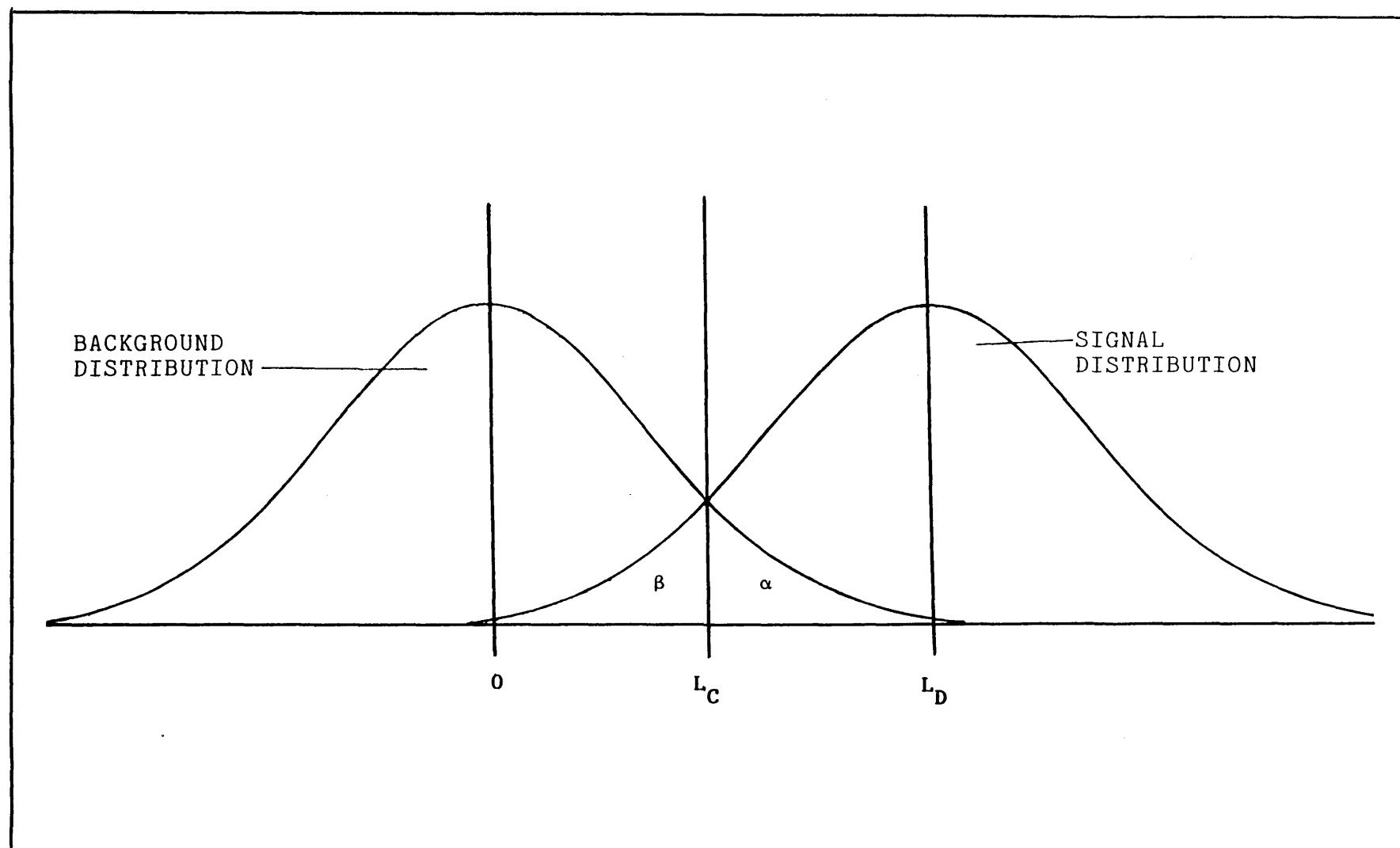


FIG 3.4 THE RELATIONSHIP BETWEEN α , β , L_C AND L_D

in figure 3.5, and drawing conclusions according to the following scheme.

1. If the gross signal is less than L_{C_1} then the alternative hypothesis is rejected with the (small) probability β , for the decision being incorrect.
2. If the gross signal is above the level L_{C_2} then the alternative hypothesis is accepted with the (small) probability α_2 , for the decision being incorrect.
- 3) If the gross signal is greater than L_{C_1} but less than L_{C_2} , we refrain from drawing any conclusion and repeat the measurement, but recognize that, were the alternative hypothesis accepted, we would have a probability α_1 (much greater than α_2) for being incorrect.
- 4) If subsequent measurements yield a gross signal less than L_{C_1} or greater than L_{C_2} then the appropriate action indicated in 1) or 2) is taken. However, if option 3) arises a predefined number of times in succession we accept the alternative hypothesis.

The reasoning behind this approach is that conditions 1) and 2) need be met once only and may be met by any of the measurements, therefore the probabilities β and α_2 combine as sums. However, condition 3) must be met by all of the measurements. Therefore the probabilities α_1 combine as a product. But, if β and α_2 are made suitably small initially, then $n\beta$ and $n\alpha_2$ may also be small while, since α_1 must be less than one, α_1^n can be made as small as desired by the choice of n . For example, if $\beta = \alpha_2 = 2 \times 10^{-6}$, $\alpha_1 = 10^{-1}$ and $n = 5$, then $n\beta = n\alpha_2 = 10^{-5}$ while $\alpha_1^n = 10^{-5}$ so that we have equal (small) probabilities for both kinds of error.

The above procedure is an example of data adaptive processing (see section 3.2) and this will converge to a particular accuracy more quickly than a conventional hypothesis test.

The a priori determination level, L_Q , is the value of the signal mean,

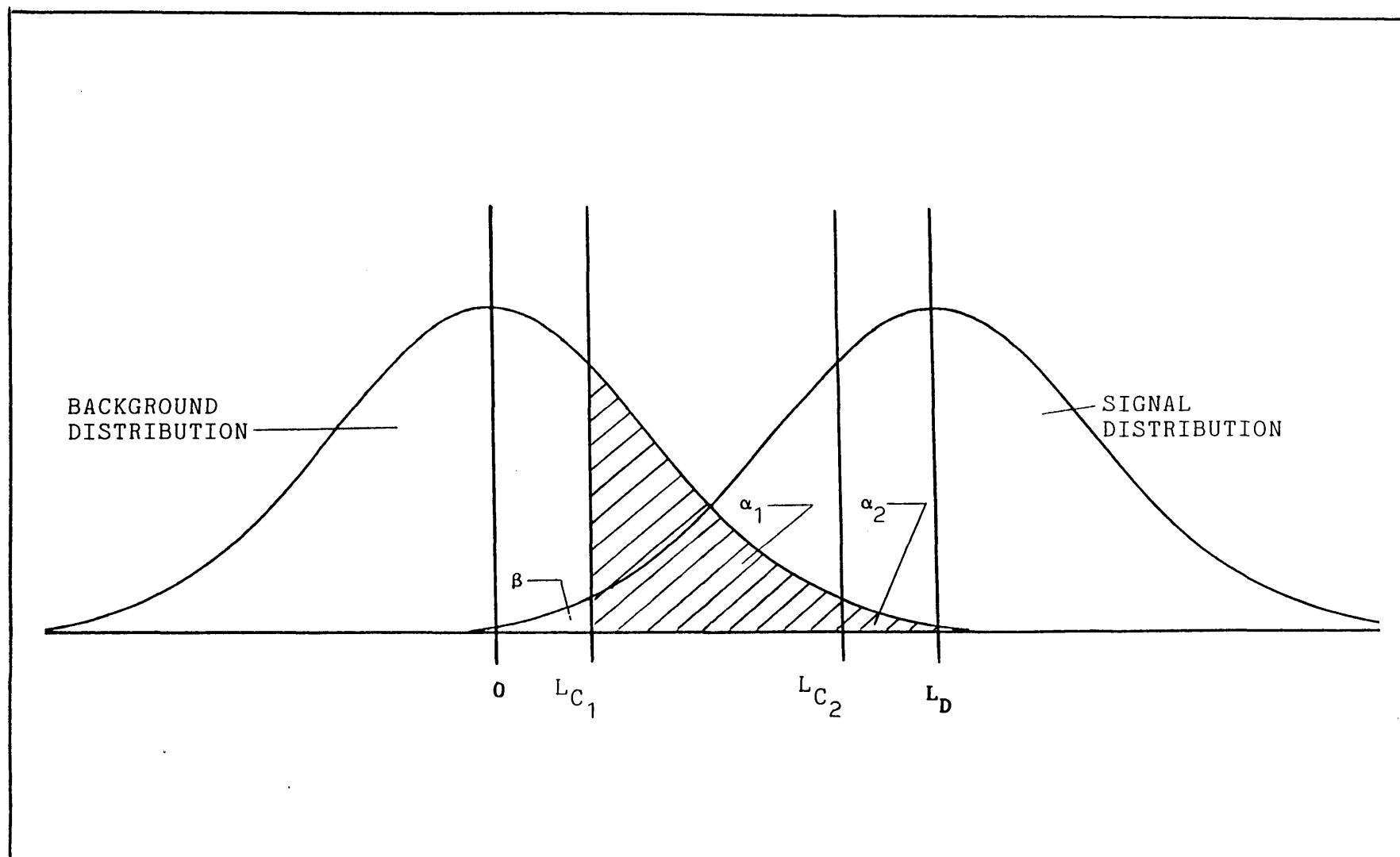


FIG 3.5 PROBABILITY DISTRIBUTIONS FOR DATA ADAPTIVE MEASUREMENT

μ_s , when a particular accuracy of data is achieved. This is given by:

$$L_Q = k_Q \sigma_Q \quad (3.57)$$

where σ_Q is the standard deviation of the net signal when $\mu_s = L_Q$ and $1/k_Q$ is the required relative accuracy of the measurement. For example, for 10% accuracy, $1/k_Q = 0.1$ and $k_Q = 10$.

We shall now consider an application of this methodology to a spectroscopy system such as the Blowdown iodine monitor (see section 2.6).

A schematic representation of the observed signal is given in figure 3.6, showing a region of interest about a gaussian photopeak on the channels of a multichannel buffer. The photopeak is assumed to rest on a linear background signal. If the centroid channel is denoted by c and σ is the standard deviation of the gaussian (in units of channel width), the expected count on the i th channel is given by:

$$\begin{aligned} \mu_i &= \frac{A}{Q(c, \sigma)} \exp \left[- \left(\frac{i - c}{2\sigma} \right)^2 \right] + D.(i - c) + \frac{B}{w} \\ Q(c, \sigma) &= \sum_{i=-\infty}^{\infty} \exp \left[- \left(\frac{i - c}{2\sigma} \right)^2 \right] \end{aligned} \quad (3.58)$$

Here, A is the count within the photopeak and B is the total background. The background may be divided into inner and outer components denoted by B_1 and B_0 respectively, where B_1 is the background under the photopeak, given by:

$$B_1 = \frac{v}{w} B = \frac{v}{w - v} B_0 \quad (3.59)$$

Let T represent the gross count in the region v , then T is Poisson with mean $A + B_1$ ($v > 2\sigma$) and if R represents the counts outside v , then R is Poisson with mean B_0 . For large T and R , the net signal:

$$s = T - \frac{v}{w - v} R \quad (3.60)$$

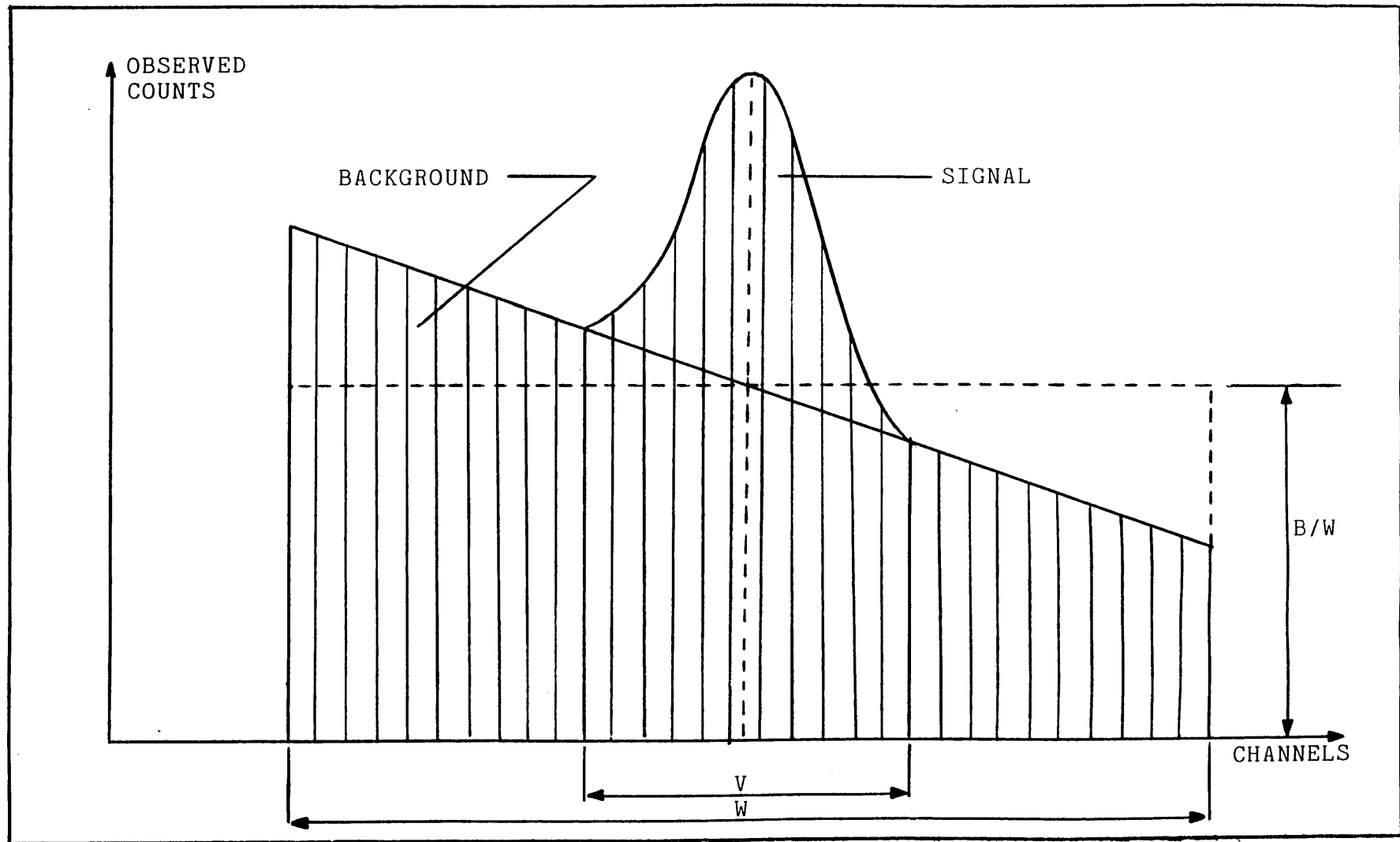


FIG 3.6 SCHEMATIC REPRESENTATION OF A GAUSSIAN PHOTOPEAK PLUS BACKGROUND

will be normally distributed with mean:

$$\mu_s = (A + B_1) - \frac{v}{w - v} B_0 = A \quad (3.61)$$

and variance

$$\sigma_s^2 = \sigma_T^2 + \left(\frac{v}{w - v} \right)^2 \sigma_R^2 = A + \left(\frac{w}{w - v} \right)^2 B_1 \quad (3.62)$$

where B_1 is estimated from the outer background B_0 . In the specific case where $w = 2v$, (3.61) reduces to the familiar form $\sigma_s^2 = 2B_1$.

The detection limits L_D and L_Q are functions only of the background under the photopeak. To determine these, set A in (3.62) to zero and substitute for σ_0 in (3.55) to obtain:

$$L_C = k_\alpha \left(\frac{w}{w - v} \right) B_1^{\frac{1}{2}} \quad (3.63)$$

Then, setting A to L_D in (3.62) and substituting for σ_D in (3.56) gives:

$$L_D = L_C + k_\beta \left(L_D + \frac{w}{w - v} B_1 \right)^{\frac{1}{2}} \quad (3.64)$$

These equations may be solved for L_D to yield:

$$L_D = k_\alpha \frac{w}{w - v} B_1^{\frac{1}{2}} + k_\beta^2 \left[\frac{1}{2} + \left(\frac{1}{4} + \frac{k_\alpha}{k_\beta^2} \left(\frac{w}{w - v} B_1 \right)^{\frac{1}{2}} + \frac{1}{k_\beta^2} \left(\frac{w}{w - v} B_1 \right) \right)^{\frac{1}{2}} \right] \quad (3.65)$$

For the specific case of $k_\alpha = k_\beta = k$, this reduces to:

$$L_D = k^2 + 2k \left(\frac{w}{w - v} B_1 \right)^{\frac{1}{2}} \quad (3.66)$$

Finally, to define L_Q , we substitute for σ_Q in equation (3.57) to give:

$$\begin{aligned} L_Q &= k_Q \left(L_Q + \frac{w}{w - v} B_1 \right)^{\frac{1}{2}} \\ &= \frac{k_Q^2}{2} \left[1 + \left(1 + \frac{4w}{w - v} B_1 \right)^{\frac{1}{2}} \right] \end{aligned} \quad (3.67)$$

Hence, the detection limits are found by substituting the appropriate values for B_1 into equations (3.65) or (3.66) and (3.67).

A similar analysis may be carried out within Bayesian statistics except in this case the test is carried out directly on the parameters of interest using the posterior density function. This may be applied to both the problems of deciding on a significant isotope set and the sample type best fitting the data which were discussed in section 3.5.

3.7 REFERENCES

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CHAPTER 4

DETERMINISTIC MODEL

The deterministic model involves the construction of a discrete transformation, g , from the isotope activities of a sample to the observed signals. For this transformation to exist, it is necessary to design a parameter set which incorporates bias parameters. These will be defined by expressing the activities as density functions on the source and radiation flux geometries. Then, starting from these functions, the required mapping will be configured so that the transformation matrices contain only a combination of known factors while all of the unknown influences on a measurement are combined into the set of inference parameters together with a minimum number of bias parameters.

With the forward mapping from the source to the observed signals defined in this manner, it can be inverted to give the mapping, ψ , from the observed signals to the source using the general techniques established in chapter 3.

4.1 BASIC SPACES AND THE DECOMPOSITION OF g .

The radionuclides of interest are distributed throughout a sample. Therefore it is necessary to construct a representation of the source which takes into account this distribution. The parameters which describe a radionuclide are:

- spatial position: $r = (x, y, z)$,
- time: t ,
- type of isotope: i .

The collection of all points, (r, t) , in space and time, define the geometry of the source. This is a four dimensional Euclidian geometry which will be denoted by Γ_4 . The index, i , defines the type of isotope and hence the interaction properties of a radionuclide at a point of this geometry.

For each isotope, the primary radioactivity is proportional to the number of nuclei of that type. Let $\theta_i(r|t)$ be the number of nuclei of the i th isotope per unit volume of the sample at the position, r , and time, t . Then, the expected number of these nuclei which decay in unit time is given by:

$$\frac{d}{dt} \theta_i(r|t) = -\lambda_i \theta_i(r|t) \quad (4.1)$$

where λ_i is the radioactive decay constant for the i th isotope. The function $\theta_i(r|t)$ will be referred to as the source density. The vertical line in the argument is used to indicate that it has been normalised to unit volume only and not to unit time.

For sources which are not decaying rapidly, θ_i may be taken to be approximately constant over the time scales of a measurement. In this case, there is no need to specify the precise point in time at which the source is considered and the parameter, t , may be dropped. For many applications, it will be possible to assume that only slowly

decaying radionuclides are present within a sample; or, if this is not exactly true, it may be a simple matter to formulate a decay correction using equation (4.1). With these conditions, the source geometry reduces to normal three dimensional space which will be denoted by Γ_3 . For the present we shall continue working with Γ_4 because the analysis will then be applicable to systems such as the Blowdown Monitor (see section 2.6) where radioactive material is taken from a stack and builds up on a collection filter in addition to the decay by the normal radioactive process. Even in this case, however, it is still possible to correct for decay. For example, let $\rho_i(t)$ be the total quantity of radioactive material which flows onto the collection filter in the time interval $[0, t]$. Then, if $\theta_i(t)$ is the instantaneous quantity of material (see equation (4.5)), the above equation may be extended to:

$$\frac{d}{dt} \theta_i(t) = -\lambda_i \theta_i(t) + \frac{d}{dt} \rho_i(t) \quad (4.2)$$

and this rearranged to give:

$$\rho_i(t) = \theta_i(t) + \lambda_i \int_0^t dt' \theta_i(t') \quad (4.3)$$

which simply says that the total quantity of material which has entered the filter is equal to the quantity currently there plus the quantity which has decayed. However, if measurements are made over N periods of duration T the integral is exactly the sum over these periods of all of the (previously) observed disintegrations, regardless of how the sample changes with time. Therefore, if an estimate of $\theta_i(t_n)$ is constructed by dividing the observed disintegrations by the measurement period, an estimate of $\rho_i(t_n)$ can be constructed by:

$$\hat{\rho}_i(t_n) = \hat{\theta}_i(t_n) + \lambda_i T \sum_{n=1}^N \hat{\theta}_i(t_n) \quad (4.4)$$

This formulation may also be applied to cases such as the rapid decay of daughter nuclides where, if the parent is uncorrected, near perfect negative correlation exists between $\rho_i(t)$ and the change in activity of the parent, independently of its decay mode (see section 4.2).

If there are I different isotopes within a sample which contribute to the primary activity, the radioactive decay constants for these nuclei can be defined as an ordered set, or I -tuple, of real numbers, $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_I)$. Likewise, the source densities can be defined as an I -tuple of real functions, $\theta(r|t) = (\theta_1(r|t), \theta_2(r|t), \dots, \theta_I(r|t))$. If Ω_I is the set of all I -tuples over $\Gamma_4 \times 1$, then $\theta(r|t) \in \Omega_I$ and, furthermore, since λ_i can be regarded as a constant function over Γ_4 , $\lambda \in \Omega_I$. The pair (Γ_4, Ω_I) will be referred to as the source space.

Because λ and θ are defined on the same space, it is possible to represent the primary activity, $a_\theta(t)$, of a sample by the inner product:

$$\begin{aligned} a_\theta(t) &= \langle \lambda | \theta(t) \rangle = \sum_{i=1}^I \lambda_i \int_V dr \theta_i(r|t) \\ &= \sum_{i=1}^I \lambda_i \theta_i(t) \int_V dr f_i(r|t) = \sum_{i=1}^I \lambda_i \theta_i(t) \end{aligned} \quad (4.5)$$

where $f_i(r|t)$ is a normalised distribution function for the i th isotope, which may be regarded as a conditional probability density for the spatial position of a radionuclide, and as defined above, $\theta_i(t)$ is the number of these nuclei in the sample at the time t . These functions will be referred to as the distribution and magnitude of the source respectively.

Equation (4.5) suggests that the primary activity is dependent only upon the isotopic composition and the magnitude of the source and not upon its distribution. Therefore, if the measurement requirement is to quantify the primary activity, the source magnitudes can be taken to define the set of inference parameters since these are sufficient to define the activity. Note that the total activity itself cannot be taken as an inference parameter since this depends on the isotopic composition of the sample and the source magnitudes or the activities

*1 Alternatively Γ_3

of the individual isotopes must be defined before a meaningful estimate of the total activity can be made.

In most non-destructive assay processes, details concerning the source are inferred by the observation of the gamma or neutron fields emitted by the sample where the observation may be under passive conditions, when radiation is spontaneously emitted, or under active conditions where nuclear decays leading to emission are induced by interrogating radiation from an external source. The observed fields will depend on the distribution of radionuclides within the sample, therefore, the spatial positions of the radionuclides must be included in the set of bias parameters. Notice, however, that the interpretation of the source distribution (defined by equation (4.5)) as a conditional probability density, implies that the spatial dependence may be treated as a parametric function in the same manner as the probability density for the source magnitudes (see section 3.3).

The particles which are detected may be produced by nuclear reactions in many materials other than those which contribute to the primary activity. Let l^+ denote the (augmented) number of isotopes which by some mechanism influence the radiation fields observed by the detectors. This collection will include most of the stable nuclides in addition to the radionuclides which contribute to the primary activity, hence l^+ may be much greater than l . The set Ω_{l^+} contains the collection of all l^+ -tuples of source density functions, $\theta_i(r|t)$, which describe these isotopes and the source space may be extended to incorporate this augmented isotope set. Both the source magnitudes and distributions of the additional isotopes (ie. for which $i = l+1, \dots, l^+$) belong to the set of bias parameters. In fact, since the augmented set of source density functions totally describe the sample (see section 3.2), ideally these functions constitute a full (ie. inference plus bias) parameter set for a measurement of the primary activity.

In the above particular case, the set of inference parameters therefore includes the source magnitudes, $\theta = \{\theta_i; i = 1, 2, \dots, l\}$, while the set of bias parameters includes the source magnitudes $\theta^+ = \{\theta_i; i = l+1, l+2, \dots, l^+\}$, together with the spatial positions,

$r = \{r_i; i = 1, 2, \dots, l^+\}$. These can be combined into the composite parameter vector which denotes elements of the sample space (see section 3.4) and if the real time, t , belongs to an index set $*2$, the general theory of chapter 3 may be applied in principle to this parameter configuration. In practice we would find the pdf for the spatial distribution almost impossible to define without making gross assumptions. Therefore we have need to give greater structure to the sample space and this is the purpose of the deterministic model. It will be shown that the parameter sets are by no means unique and for a particular application practical sets become apparent when the transformation which relates these parameters to the observed signals is considered. In fact, constructing this transformation will introduce further bias parameters associated with the transport of radiation from the point of emission to the point of detection. We shall eliminate these parameters using the deterministic model and reduce the bias parameters to a set similar to that indicated above.

For notational simplicity the "+" will henceforth be dropped from references to the augmented isotope set and the source space will continue to be denoted by (Γ_4, Ω_1) except in instances when it is necessary to distinguish between these sets.

To allow the construction of the transformation. It is useful to extend the source geometry to incorporate the geometrical properties of the particles produced by reactions within the sample. The parameters which describe a particle are:

- spatial position: r ,
- direction of travel: $\Omega = (\alpha, \beta, \gamma)$,
- energy: E ,
- time: t ,
- particle type: j .

The collection of all points, (r, Ω, E, t) , define a seven dimensional

*2 Alternatively, t could be included in the parameter set and the sample number $n = 1, 2, \dots$ taken as the index.

Euclidian geometry which will be denoted by Γ_7 . The coefficients α , β and γ are direction cosines *3 while the index, j , is a label attached to each particle to denote its type. The variables j , Ω , and E are said to define the phase of the particle.

A suitable model for the collection of particles within a system is the angular flux density function, $\phi_j(r, \Omega, E, t)$. This is defined as the number of particles of type, j , which pass through unit area at the position r , in unit solid angle in the direction, Ω , per unit energy at the energy, E , and per unit time at the time, t .

Let J be the number of different particle types which contribute to the observed signals either by direct detection or by playing a role in the creation of directly detectable particles. Then, the flux density functions can be defined as a J -tuple, $\phi(r, \Omega, E, t,) = (\phi_1(r, \Omega, E, t), \phi_2(r, \Omega, E, t), \dots, \phi_J(r, \Omega, E, t)) \in \Omega_J$, where Ω_J is the set of all J -tuples over Γ_7 . The pair (Γ_7, Ω_J) will be referred to as the flux space.

The geometries Γ_3 and Γ_4 may be considered as (smaller) geometries which are contained in Γ_7 , and will be referred to as sub-geometries of Γ_7 . Likewise any Euclidian geometry constructed from points defined by a combination of the parameters r , Ω , E or t will also be a sub-geometry of Γ_7 . Since the analysis involves the construction of functions over Γ_7 and many of its sub-geometries, the more general symbol, Γ , will be used to denote the geometry, Γ_7 , together with all of its sub-geometries. For completeness, Γ will include the geometry Γ_0 where the functions over Γ_0 are the set of all constant functions over Γ_7 , such as λ_i , the radioactive decay constant for nuclei of type i . With this generalisation the source and flux spaces may be denoted by (Γ, Ω_1) and (Γ, Ω_J) respectively.

In chapter 2 the signal geometry, Γ_L , was defined as a collection of

*3 By virtue of the constraint $\alpha^2 + \beta^2 + \gamma^2 = 1$, the three direction cosines denote a point on the unit sphere and hence span a two dimensional space.

multi-dimensional parallelepipeds of the form, $[E_k \times t_n \times V_m]$, where E_k is an energy interval, t_n a time interval and V_m an element of volume. However, each of these parallelepipeds defines a subregion of Γ . Therefore, Γ_L may be regarded as a sub-geometry of Γ and the general signal space can be written as the pair (Γ, Ω_L) where Ω_L is the collection of spectral image functions of the form, s_{knm} .

The transformation, g , from the source to the observed signals may now be identified as a mapping from the source space onto the signal space:

$$g: (\Gamma, \Omega_I) \rightarrow (\Gamma, \Omega_L) \quad (4.6)$$

such that:

$$s = g(\theta(r|t)) \quad (4.7).$$

The objective is to define this function. However, in the above formulation the source space bears little resemblance to the signal space. Therefore, it is necessary to consider an intermediate step.

Let g_1 be a transformation which maps the source space onto the flux space, such that:

$$g_1: (\Gamma, \Omega_I) \rightarrow (\Gamma, \Omega_J) \quad (4.8)$$

and

$$\phi(r, \Omega, E, t) = g_1(\theta(r|t)) \quad (4.9)$$

Then, let g_2 be a transformation which maps the flux space onto the signal space, such that:

$$g_2: (\Gamma, \Omega_J) \rightarrow (\Gamma, \Omega_L) \quad (4.10)$$

and

$$s = g_2(\phi(r, \Omega, E, t)) \quad (4.11)$$

Therefore, the transformation, g , can be written as:

$$\begin{aligned} g(\theta(r|t)) &= g_2(g_1(\theta(r|t))) \\ &= (g_2 \circ g_1)(\theta(r|t)) \end{aligned} \quad (4.12)$$

The symbol "o" denotes the composition of two functions and this operation is defined by equation (4.12). Hence, g can be constructed by defining g_1 and g_2 separately and combining these according to the above rule. Note that by the above procedure we have both introduced and eliminated the set of bias parameters $\{\phi(r, \Omega, E, t)\}$.

The transformation, g_1 , can be constructed using radiation transport theory while g_2 may be regarded as the constant distortion introduced by the measuring apparatus and it is intended to define this function experimentally.

The definition of g_1 will be considered first where the basic strategy is to construct a source term to describe the mapping from the masses of the sample nuclides onto the flux space and then to generalise this by incorporating the movement of particles about the flux space.

4.2 MAPPING ONTO THE FLUX SPACE

In this section expressions for the particles produced by reactions within the sample or from an external source will be considered. We shall then introduce the concept of a Green's function and write down the general integral form of the Boltzmann Transport Equation.

The following reactions are of major interest.

1) Passive reactions

- Primary decay
- Gamma transitions accompanying a primary decay event
- Spontaneous fission.

2) Induced reactions

- Absorption
- Induced fission
- Elastic and inelastic scattering
- Secondary particle production accompanying an absorption or inelastic scattering event.

3) The production of an interrogating flux

- Neutron interrogation
- Gamma interrogation.

Consider the observation of a sample under passive conditions. Let $R_{ik}(r,t)$ denote the number of reactions of the k th type for the i th isotope, per unit volume of the sample at the position, r , and per unit time at the time, t . Then if each nuclei has a probability σ_{ik} of decaying by the k th reaction in unit time:

$$R_{ik}(r,t) = \theta_i(r|t) \sigma_{ik} \quad (4.13)$$

If $\bar{v}_{ijk}(\Omega, E)$ is the expected yield of particles of the j th type per unit solid angle in the direction, Ω , and per unit energy at the energy, E , then the rate of production of these particles is given by:

$$\begin{aligned} S_j^P(r, \Omega, E, t) &= \sum_{i=1}^I \sum_{k=1}^K R_{ik}(r, t) \bar{v}_{ijk}(\Omega, E) \\ &= \sum_{i=1}^I \theta_i(r|t) \sum_{k=1}^K \sigma_{ik} \bar{v}_{ijk}(\Omega, E) \end{aligned} \quad (4.14).$$

With the passive reactions, the production of particles may be taken to be isotropic *4. Therefore, the particle yield may be written:

$$\bar{v}_{ijk}(\Omega, E) = \frac{1}{4\pi} \bar{v}_{ijk}(E) \quad (4.15)$$

*4 In the absence of external fields the angular and linear momentum vectors of a nucleus will have random orientation.

where $\bar{v}_{ijk}(E)$ is the expected yield of particles of the j th type, from the k th reaction and the i th isotope, per unit energy at the energy, E . Hence, by multiplying both sides of equation (4.14) by the factor 4π it is possible to eliminate the angular dependence so that:

$$S_j^P(r, E, t) = \sum_{i=1}^I \theta_i(r|t) \sum_{k=1}^K \sigma_{ik} \bar{v}_{ijk}(E) \quad (4.16)$$

Since σ_{ik} and $\bar{v}_{ijk}(E)$ are generally available from tables of radiological data, it is possible to further simplify the passive source term by computing the sum over k . Hence:

$$S_j^P(r, E, t) = \sum_{i=1}^I \theta_i(r|t) p_{ij}(E) \quad (4.17)$$

or:

$$S_j^P(r, E) = \sum_{i=1}^I \theta_i(r) p_{ij}(E) \quad (4.18)$$

if the source is of constant activity.

In the specific case of a primary decay event, for example alpha decay, beta minus and beta plus decay, and electron capture, the decay probability, σ_{ik} , is simply the radioactive decay constant, λ_i . The particle yield is a function of energy which, for alpha decay is a discrete function of the form:

$$\bar{v}_{i\alpha k}(E) = p_{i\alpha k} \delta(E_k - E) \quad (4.19)$$

where $\delta(E_k - E)$ is a Dirac delta function with E_k being the energy of the emitted alpha particle, while $p_{i\alpha k}$ is the probability for an emission at that energy. For the beta decay processes, the energy of the reaction is split between an electron and an antineutrino for the beta minus decay, a positron and a neutrino for the beta plus decay and carried off by the neutrino for electron capture. Since neutrinos and antineutrinos are not detectable (within the scope of the applications considered) only the electron and positron spectra need be considered. These will be continuous functions of energy with a fixed end point corresponding to the total energy of the reaction for the beta minus

and plus decays and zero for electron capture. The positrons produced in beta plus decay will usually be annihilated to produce pairs of photons.

Following a primary decay the nucleus may be left in an excited state and this energy is dissipated by a gamma transition. The probability for one of these reactions occurring is the radioactive decay constant for the primary decay mode while the reactions themselves include gamma-ray emission at one of a number of discrete energies, internal conversion resulting in the emission of an orbital electron and internal pair production. These reactions may be followed by the emission of characteristic X-rays, Auger electrons or annihilation photons.

In the case of spontaneous fission, the energy is carried away by the fission fragments which subsequently decay to yield two or three prompt neutrons and up to ten gamma-rays. The probability for a nucleus decaying by this mode is constant for each isotope while the yields are functions of energy which are normalised to the mean number of neutrons or gamma-rays produced. Following a fission, the fragments may emit a delayed neutron, or undergo an alpha or beta decay as described above.

For the observation of a sample under active conditions, the reaction rates will depend upon both the source density and the intensity of the interrogating flux. The probability for an interaction of a nucleus with a unit flux of energy, E , and particle type, j , is called a microscopic cross-section and is denoted by $\sigma_{ijk}(E)$, where, again, i refers to the isotope and k refers to the type of nuclear reaction. The product of a source density and a microscopic cross-section is usually called a macroscopic cross-section, and denoted:

$$\Sigma_{ijk}(r, E, t) = \theta_i(r, t) \sigma_{ijk}(E) \quad (4.20)$$

Then, the reaction rate, $R_{ijk}(r, \Omega, E, t)$, which is induced by the angular flux density, $\phi_j(r, \Omega, E, t)$, is given by:

$$R_{ijk}(r, \Omega, E, t) = \Sigma_{ijk}(r, E, t) \phi_j(r, \Omega, E, t) \quad (4.21)$$

Comparison of equations (4.13) and (4.21) shows that, unlike passive reactions, the rates of reactions induced by interrogating radiation have an energy dependence they take from the product of the microscopic cross-section and the flux. The energy dependence of a microscopic cross-section for a particular isotope and reaction is unique.

For the induced reactions, the expected yield depends upon the energy and angle of incidence *5 of the interrogating particle which induced the reaction. Let $\bar{v}_{ijj',k}(\Omega, E | \Omega', E')$ denote the expected yield of particles of the j th type created in the k th reaction, per unit solid angle in the direction, Ω , per unit energy at the energy, E , and for the angle of incidence, Ω' , and energy, E' , of the particle of type, j' , which induced the reaction.

In all the stated cases of induced reactions, the macroscopic cross-sections govern the probability of the reactions occurring, while the particle yields determine the configuration of the product particles. If it is specified that all of the particles which enter a reaction disappear from the system and are replaced by the product particles, then all of the specific cases, including absorption and scattering, can be incorporated into the model by the appropriate definition of the particle yields. For example, for induced fission and absorption or inelastic scattering followed by secondary particle production interactions, the particle yields are the mean number of neutrons, gamma-rays or charged particles which are produced in a region of the source geometry by the reactions, minus the original particle. For elastic scattering the particle yield is a single new particle moving in a different direction and with a different energy from the original particle (which disappears from the system). For pure particle absorption the yield is minus one.

The loss of the original particle can be taken into account by subtracting the factor $\delta_{jj'} \delta(\Omega - \Omega') \delta(E - E')$ from the positive yield function $\bar{v}_{ijj',k}(\Omega, E | \Omega', E')$ which describes the particles created in the

*5 This is the reason for the inclusion of an angular dependence in (4.21) which is taken solely from the interrogating flux.

the reaction, where $\delta_{jj'}$ is a Kronecker delta function and $\delta(\Omega - \Omega')$ and $\delta(E - E')$ are Dirac delta functions.

The particle production rate is then given by:

$$\begin{aligned}
 S_j^A(r, \Omega, E, t) &= \sum_{i=1}^I \sum_{j'=1}^J \sum_{k=1}^K \int_{4\pi} d\Omega' \int_0^{E_{\max}} dE' \{ \bar{v}_{ijj'k}(\Omega, E | \Omega', E') \\
 &\quad - \delta_{jj'} \delta(\Omega - \Omega') \delta(E - E') \} R_{ijk}(r, \Omega', E', t) \\
 &= \sum_{i=1}^I \theta_i(r|t) \sum_{j'=1}^J \left[- \sum_{k=1}^K \sigma_{ijk}(E) \phi_j(r, \Omega, E, t) + \int_{4\pi} d\Omega' \int_0^{E_{\max}} dE' \right. \\
 &\quad \left. \sum_{k=1}^K \bar{v}_{ijj'k}(\Omega, E | \Omega', E') \sigma_{ijk}(E') \phi_j(r, \Omega', E', t) \right] \\
 &= \sum_{i=1}^I \theta_i(r|t) \left[- \sigma_{ij}(E) \phi_j(r, \Omega, E, t) + \sum_{j'=1}^J \int_{4\pi} d\Omega' \int_0^{E_{\max}} dE' \right. \\
 &\quad \left. p_{ijj'}(\Omega, E | \Omega', E') \phi_j(r, \Omega', E', t) \right] \quad (4.22)
 \end{aligned}$$

$E_{\max}$ denotes the maximum energy of the particles which are created within the system or enter the system from outside.

The main absorption mechanism for gamma-rays of low to intermediate energies (i.e. 10keV to 1MeV) is the photoelectric effect. Here, the incident photons interact with bound atomic electrons passing all their energy to the electrons which are ejected from the atom with kinetic energy equal to the energy of the initial gamma-ray less their own binding energy. At low energies the cross-section for this reaction exhibits a number of sharp edges, called absorption edges. These occur when the photon energy equals the binding energy of an electron shell and the absorption edge indicates that gamma-rays with energy less than the binding energy do not interact with the electrons of that shell while those with higher energy do. Since momentum must be conserved by the recoil of the atom the photoelectric interaction can only occur with bound electrons.

For higher energy gamma-rays, photonuclear absorption may be significant. These reactions can result in the production of lower energy gamma-rays, neutrons or protons.

Gamma-ray scattering can occur with bound or free electrons. However, if the energy of the incident photons is much greater than the binding energy, an atomic electron will behave as if it were free. Therefore, it is usually only necessary to consider incoherent (or Compton) scattering with free electrons. For the energy range of interest this mechanism will dominate the coherent and nuclear scattering reactions.

For gamma-rays with energy above 1.022MeV the production of electron-positron pairs (ie. external pair production) within the electro-static potential field of a nucleus or an orbital electron is possible. The resulting positron is extremely reactive and may quickly annihilate with an atomic electron to produce a pair of 511keV gamma-rays.

Neutron absorption occurs by radiative capture which has a continuous cross-section but with sharp resonances occurring at the energies corresponding to nuclear energy levels. These reactions result in the emission of prompt, high energy gamma-rays along with lower energy delayed gamma-rays if the daughter nucleus is radioactive. It is also possible for neutron capture to result in the production of charged particles such as protons or alpha particles although there is usually a threshold energy associated with these reactions. If materials such as U-238 and Th-232 capture a neutron these can decay by the beta minus reaction to yield a fissile nucleus (see next paragraph). These isotopes are therefore called fertile.

There are three types of induced fission reactions. These are thermal neutron induced fission, fast neutron induced fission and photofission. Thermal fissions can occur in a small number of fissile isotopes, where only U-233, U-235 and Pu-239 are sufficiently stable to exist for a reasonable period of time and these have half-lives of many thousands of years. Other nuclides such as Th-232 and U-238 will fission when interrogated by a neutron or gamma-ray above a certain threshold energy which is characteristic of the isotope. These are known as fissionable

nuclides. As with spontaneous fission the energy produced in the induced reactions is carried away by the fission fragments which decay to yield prompt neutrons and gamma-rays and may also emit a delayed neutron.

Neutrons undergo two forms of scattering which are elastic scattering and inelastic scattering. Elastic scattering is by far the most common reaction for slow neutrons and predominates when the neutrons are in thermal equilibrium with their surroundings. In these reactions the mean energy of the thermal neutrons remains unchanged while the neutrons rapidly lose any directional preference and for a homogeneous material will diffuse isotropically through space. Faster neutrons give up part of their energy to the scattering nucleus and are therefore slowed down until they achieve the thermal equilibrium state where the probability of being speeded up in a reaction equals the probability of being slowed down.

For inelastic scattering part of the collision energy is given to excite a target nucleus which may decay to produce further particles. For example, in certain heavy materials, it is possible for neutrons of about 10MeV or greater to excite a target nucleus so that it emits a second neutron thus giving rise to $(n,2n)$ interactions.

In general, all of the above reactions commence with the absorption or scattering of a gamma-ray or neutron which may leave the atom or nucleus in an unstable state and this subsequently decays (or fissions) to yield secondary particles. A similar pattern is observed with charged particles except that in this case the main interactions at the energies of interest occur with atomic electrons since the charged particles must overcome the electrostatic fields of the atom to interact with a nucleus. Hence the primary mechanism by which a charged particle loses energy is by scattering with electrons and subsequent ionization of the atom. In the case of a proton or alpha particle, once the kinetic energy has been lost, these can capture one or two electrons to produce elemental hydrogen or helium, respectively. Likewise, an electron may lose its kinetic energy and be captured by an atom.

Charged particles (particularly electrons) also lose energy when passing through the Coulomb field of a nucleus. This is converted into electromagnetic energy and bremsstrahlung photons are produced. These have a continuous energy spectrum and may take any energy from zero to that of the charged particle.

One important exception to the rule that charged particles interact mainly with the atomic electrons are alpha-neutron interactions. Here, alpha particles produced by the decay of heavy nuclei interact with light nuclei to produce neutrons. Since the heavy metals naturally form oxides and fluorides, these reactions are common and cannot be neglected.

Since the production of the interrogating particles is independent of the sample, there is some simplification to be gained by treating the interrogating flux as a time-dependent background. This introduces an extra flux term of the form $\phi_j^0(r, \Omega, E, t)$ which must be defined by calibration.

The flux density function $\phi_j^0(r, \Omega, E, t)$ may be interpreted as the flux that is observed throughout an empty sample container. When material is present within a container, $\phi_j^0(r, \Omega, E, t)$ is modified by nuclear and atomic interactions as described above and defined by the composite passive and active source term:

$$S_j(r, \Omega, E, t) = \sum_{i=1}^I \theta_i(r|t) \left[\frac{1}{4\pi} p_{i,j}(E) - \sigma_{i,j}(E) \phi_j(r, \Omega, E, t) \right. \\ \left. + \sum_{j'=1}^J \int_{4\pi} d\Omega' \int_0^{E_{\max}} dE' p_{i,j,j'}(\Omega, E|\Omega', E') \phi_{j'}(r, \Omega', E', t) \right] \quad (4.23)$$

These interactions always occur at a point in space and time (although, which particular point cannot be known exactly) and change only the phase (ie. the variables j, Ω and E) of the particle. However, between interactions, the particles propagate through space and time and to ascertain the extent by which the initial flux is modified for a particular sample configuration, it is necessary to take this

propagation into account. This amounts to defining an operator kernel, G , known as a Green's function, such that:

$$\phi_j(r, \Omega, E, t) = \phi_j^0(r, \Omega, E, t) + \int_0^t dt' \int_V dr' G_j(r, t | r', \Omega, E, t') S_j(r', \Omega, E, t') \quad (4.24)$$

or using (4.23) to expand the source term:

$$\begin{aligned} \phi_j(r, \Omega, E, t) = \phi_j^0(r, \Omega, E, t) + \int_0^t dt' \int_V dr' G_j(r, t | r', \Omega, E, t') \sum_{i=1}^I \theta_i(r' | t') \\ \left[\frac{1}{4\pi} p_{ij}(E) - \sigma_{ij}(E) \phi_j(r', \Omega, E, t') + \sum_{j'=1}^J \int_{4\pi} d\Omega \int_0^{E_{\max}} dE' \right. \\ \left. p_{ijj'}(\Omega, E | \Omega', E') \phi_{j'}(r', \Omega', E', t') \right] \end{aligned} \quad (4.25)$$

Equation (4.25) is the general integral form of the Boltzmann transport equation and this corresponds to the mapping g_1 .

The problem of defining the kernel will be considered in the following sections. However, first, we divert to consider a particular class of systems for which its detailed, theoretical definition can be avoided.

4.3 APPLICATION TO FIXED GEOMETRY SYSTEMS

A fixed geometry system is one for which the detection efficiency of a measurement is constant for all of the samples encountered by the system. That is, the observed signals are directly proportional to the particle emission rate of the source and the detection efficiency can be defined by calibration.

For example, the class of fixed geometry systems includes assay systems to make measurements on a sample of contaminated fluid where the fluid is drawn through a sample vessel at constant density and only the levels of contamination change. Of course, the assumption that secondary reactions with the contaminants themselves are negligible,

is implicit to the above definition. This class of systems also includes the majority of flux monitoring instruments since, here, the observed signals are dependent only upon the efficiency of the detector and this remains fixed for all measurements.

In certain circumstances, for example, in the case of the Blowdown monitor, the definition of a fixed geometry system can be extended to include small variations in the consistency of an absorption filter. This is because weights can be used to correct variations (up to approximately 10%) in these using the correlation between different spectral lines from the same isotope. For the Blowdown monitor the weights may be applied directly to the photopeak areas resulting from the use of a high resolution HPGe detector. However, it will be shown that this method may also be applied when statistics of this form are not available such as for a system based on an NaI(Tl) detector.

To formulate the transformation between the source and the observed signals, the first step is to discretize the energy and time dependence of equation (4.25). This is achieved using a multi-group approximation [1] which involves integrating over small regions and summing the resulting terms to yield an equation of the form:

$$\begin{aligned} \phi_{jkn}(r, \Omega) = & \phi_{jkn}^0(r, \Omega) + \sum_{n'=1}^n \int_V dr' G_{jkn n'}(r|r', \Omega) \left[\sum_{i=1}^I \theta_{in'}(r') \left[\frac{1}{4\pi} p_{ijk} \right. \right. \\ & \left. \left. - \sigma_{ijk} \phi_{jkn'}(r, \Omega) + \sum_{j'=1}^J \sum_{k=1}^K \int_{4\pi} d\Omega' p_{ijj'kk'}(\Omega|\Omega') \phi_{j'k'n'}(r', \Omega') \right] \right] \end{aligned} \quad (4.26)$$

where k denotes a small energy interval and n denotes a small time interval which are chosen to match the partition of the signal space.

Next, let:

$$\phi_{jkn}^1(r, \Omega) = \phi_{jkn}^0(r, \Omega) + \sum_{n'=1}^n \int_V dr' G_{jkn n'}(r|r', \Omega) \left[\frac{1}{4\pi} \sum_{i=1}^I p_{ijk} \theta_{in'}(r') \right] \quad (4.27)$$

which using the decomposition of the source density into its magnitude

Because of the contraction property of the mapping, it can be shown:

$$\phi = \lim_{q \rightarrow \infty} \phi^q = \{1 + \hat{K} + \hat{K}\hat{K} + \dots + (\hat{K})^q + \dots\} \phi^1 \quad (4.32)$$

The first term of the series may be interpreted as the direct flux at the detector from the passive sources while the second, third and fourth terms (etc) correspond to the flux incident on the detector after respectively one, two or three secondary reactions. The powers of $\hat{K}$ may be interpreted as the q -step transition probabilities for a discrete time, continuous state space Markov process with starting distribution ϕ^1 (see section 3.4).

Summing the corresponding terms of the Neumann series in n' , j' and k' and integrating over the primed angular and spatial variables, gives:

$$\phi_{jkn}(r, \Omega) = q_{jkn}(r, \Omega) + \sum_{n'=1}^n \sum_{k'=1}^K \sum_{j'=1}^J g_{nn'kk'jj'}(r, \Omega) \sum_{i=1}^I p_{ij'k'} \theta_{in'} \quad (4.33)$$

where the background function, q , depends on ϕ^0 , but, otherwise, both of the functions g and q depend only on the configuration of the measurement system which is constant and not upon the activity or isotopic composition of the contaminants within the sample.

The observed signals are related to the flux within a detector volume by the inner product:

$$\begin{aligned} s_{jknm} &= \sum_{n'=1}^n \sum_{k'=1}^K \int_V d\mathbf{r} \int_{4\pi} d\Omega \xi_{jkk'nn'm}(r, \Omega) \phi_{jk'n'}(r, \Omega) \\ &= \langle \xi_{jknm} | \phi_j \rangle \end{aligned} \quad (4.34)$$

where this equation defines the mapping g_2 for the case of a fixed geometry system. Substituting for $\phi_{jk'n'}(r, \Omega)$ from (4.33) to expand the inner product gives:

$$s_{jknm} = \langle \xi_{jknm} | q_j \rangle + \sum_{n'=1}^n \sum_{k'=1}^K \sum_{j'=1}^J \langle \xi_{jknm} | g_{jj'kk'nn'} \rangle \sum_{i=1}^I p_{ij'k'} \theta_{in'} \quad (4.35)$$

which is the required composite mapping between the source density magnitudes and the observed signals.

The detectors discussed in section 2.1 respond primarily to one particle type, therefore, it is usually possible to filter out any background from other particles. For the majority of applications, the detected particles will be of the type emitted by the source. In these cases the index j does not play an active part in the interpretation of the measurements and may be dropped. In the cases where reactions such as (α, n) are significant whereby particles of one type are transformed into particles of a detectable type in their passage through the sample, these can only be included in the fixed geometry model if they can be incorporated into an augmented passive source term and treated as a primary source of detectable particles or interact only with the fixed isotopes of the sample so that they bear a constant relationship to the primary source. For the remainder of this section we shall assume that such reactions are neglectable and ignore this index.

Each combination of the discrete variables k , n and m , which denote the energy and time channels and the detector position, leads to a linear equation in the source density magnitudes. Let $Q = 1, 2, \dots, L$ denote the set of these combinations which are chosen to yield one of the signal space configurations discussed in section 2.3. Then equation (4.35) may be written:

$$s_Q = \langle \xi_Q | q_Q \rangle + \sum_{Q'=1}^L \langle \xi_Q | g_{QQ'} \rangle \sum_{i=1}^I p_{Q' i} \theta_i \quad (4.36)$$

where the time dependence of ϕ has been included in $p_{Q' i}$ or leads to several separate measurements each yielding its own set of equations of the above form. In matrix notation, equation (4.36) becomes:

$$s - b = EP\theta \quad (4.37)$$

where, writing equation (4.12) as:

$$s - b = (g_2 \circ g_1)(\theta) \quad (4.38)$$

it can be seen that P is the matrix of the transformation g_1 , and E the matrix of g_2 . Notice that the flux and signal spaces have now merged into a single space, which could be referred to as the measurement space. Hence, g_1 maps the source space onto the measurement space while g_2 maps the measurement space onto itself to account for the constant distortion introduced by the measuring apparatus.

For the fixed geometry systems only the source magnitudes of the radioactive isotopes need be considered. Therefore the set of inference parameters consists of those radioactive isotopes in which we are interested while the remainder are bias parameters. Note, however, that the total parameter set is fixed by the application and while isotopes may be exchanged between the parameter sets and only those of interest reported, the system must be configured to make measurements on the total parameter set and this cannot be changed.

Equation (4.37) may be solved for θ using the general techniques established in chapter 3. For many sampling systems, the change in the activity of the contaminants in the fluid will be continuous. In these cases the best apriori estimate of the current activity would be the configuration of the nuclear process at the end of the previous measurement. Therefore, rather than constructing the prior density from the stationary distribution of the process, it is possible to simplify this procedure by taking the posterior density for one measurement as the prior density for the next. In this way the measurement data is used to up-date the previous estimate rather than restricting the more general prior density to a specific case independently of previous estimates. This method of estimating is equivalent to the construction of a moving average, or trend, to represent the levels of contaminant activity.

Using the general linear model of section 3.5 to solve equation (4.37), and taking the posterior density as the prior density for the next measurement, results in an estimator of the form:

$$\theta_{n+1} = \left[\begin{pmatrix} EP \\ I \end{pmatrix}^T \begin{pmatrix} v_{n+1} \\ u_n \end{pmatrix}^{-1} \begin{pmatrix} EP \\ I \end{pmatrix} \right]^{-1} \begin{pmatrix} EP \\ I \end{pmatrix}^T \begin{pmatrix} v_{n+1} \\ u_n \end{pmatrix}^{-1} \begin{pmatrix} s_{n+1} - b_{n+1} \\ \theta_n \end{pmatrix} \quad (4.39)$$

where:

$$U_{n+1} = \left[\begin{pmatrix} EP \\ I \end{pmatrix}^T \begin{pmatrix} V_{n+1} \\ U_n \end{pmatrix} \begin{pmatrix} EP \\ I \end{pmatrix} \right]^{-1} \quad (4.40)$$

gives the variance of the next prior density.

The matrix P may define the emission of particles at (say) different energies from the same isotope. In this case positive correlation will exist between certain of the energy channels, and this may be used to define non-zero, off-diagonal elements of the covariance matrix, V_n . In general, these are given by:

$$v_{ll'} = \rho_{ll'} \sigma_l \sigma_{l'} = \rho_{ll'} (s_l + 2b_l)^{1/2} (s_{l'} + 2b_{l'})^{1/2} \quad (4.41)$$

where ρ is the correlation coefficient between the l th and l' th channels and σ_l and $\sigma_{l'}$ are the standard deviations of these channels. In the absence of fission, the signals are Poisson distributed (see section 2.3) and the best estimate of the standard deviations are given by their square roots where a correction for the subtracted background is made as necessary (see section 3.6).

The correlation coefficients may be defined by considering the inner products:

$$\phi_l = \sum_{i=1}^I p_{li} \theta_i \quad \text{and} \quad \phi_{l'} = \sum_{i=1}^I p_{l'i} \theta_i \quad (4.42)$$

which represent the multiplication of rows of the matrix, P , and the activity vector, θ . It is assumed that the matrix elements are known accurately.

Expand ϕ_l and $\phi_{l'}$ in a Taylor series about the mean activities, such that:

$$\phi_l(\theta_1, \dots, \theta_I) = \phi_l(\mu_1, \dots, \mu_I) + \sum_{i=1}^I (\theta_i - \mu_i) \frac{\partial \phi_l}{\partial \mu_i} + o(\theta_i^2) \quad (4.43)$$

and similarly for $\phi_{l'}$. Then, multiplying these two series together and

taking expected values, gives:

$$\text{cov}(\phi_{\mathcal{L}}, \phi_{\mathcal{L}'}) = \sum_{i=1}^I \text{var}(\theta_i) \frac{\partial \phi_{\mathcal{L}}}{\partial \mu_i} \frac{\partial \phi_{\mathcal{L}'}}{\partial \mu_i} \quad (4.44)$$

if the θ_i are independent. Hence, it is possible to construct a covariance matrix with elements:

$$v_{\mathcal{L}\mathcal{L}'} = \sum_{i=1}^I \sigma_i^2 \rho_{\mathcal{L}i} \rho_{\mathcal{L}'i} \quad (4.45)$$

or, writing (4.45) in matrix notation:

$$\mathbf{V}_{\phi} = \mathbf{P} \Sigma \mathbf{P}^T \quad (4.46)$$

where Σ is a diagonal matrix with elements σ_i^2 . If a sufficient number of channels are used to ensure that no more than one isotope emits on each channel, then there is no loss of generality in taking Σ to be the identity matrix, $\mathbf{I}$. Alternatively Σ may be taken to be the identity matrix so as to give the isotopes equal weight or constructed from the variances of the prior density.

The above procedure may be repeated for the inner products of the rows of the detector efficiency matrix and the particle emission rate:

$$s_{\mathcal{L}} = \sum_{\mathcal{L}'=1}^L e_{\mathcal{L}\mathcal{L}'} \phi_{\mathcal{L}'} \quad \text{and} \quad s_m = \sum_{\mathcal{L}'=1}^L e_{m\mathcal{L}'} \phi_{\mathcal{L}'} \quad (4.47)$$

where s denotes the sample mean of the signals. This results in a covariance matrix of the form:

$$\mathbf{V}_s = (\mathbf{E}\mathbf{P}) \Sigma (\mathbf{E}\mathbf{P})^T \quad (4.48)$$

Finally, the matrix of correlation coefficients has elements:

$$\rho_{\mathcal{L}\mathcal{L}'} = \frac{v_{\mathcal{L}\mathcal{L}'}}{(v_{\mathcal{L}\mathcal{L}} v_{\mathcal{L}'\mathcal{L}'})^{1/2}} \quad (4.49)$$

where the $v_{\mathcal{L}\mathcal{L}'}$ are elements of $\mathbf{V}_s$.

The matrix P is easily constructed from published tables of radiological data [3], while the background vector may be quantified by a simple preliminary measurement without a sample. It remains to define the detection efficiency matrix E , which is constructed by a calibration experiment.

The first step is to use the system to collect spectra for a number of different isotope configurations. It is not necessary for these to be the same as those encountered in practice. However, the emissions from the calibration isotopes should occur in the same range as the intended samples and contain sufficient variety to provide a well conditioned emission probability matrix.

For each spectrum, the observed signals are related to the source emissions by:

$$E\phi_U = s_U \quad (4.50)$$

where the emissions may be computed from the known source activities by:

$$\phi_U = P\theta_U \quad (4.51)$$

The equations for individual spectra can be combined into a single expansion of the form:

$$E\Phi = S \quad (4.52)$$

where S and Φ are $L \times U$ matrices, with L being the number of channels (ie, the dimension of the square matrix, E) and U the number of calibration spectra.

For a large number of channels it is inconvenient to collect enough spectra to make (4.52) over-determined. However, the probability of detecting a particle at one point of the signal geometry given its emission at another is a continuous function of these points.

Therefore, the matrix elements may be treated as samples from this

function and represented by series of orthogonal polynomials. For example, for the energy resolving system, this takes the form:

$$e_{\Omega\Omega'} = \sum_{j=0}^{R-1} \sum_{k=0}^{R-1} a_{jk} T_j(E_{\Omega}) T_k(E_{\Omega'}) \quad (4.53)$$

where E_{Ω} and $E_{\Omega'}$ may be taken as the energies corresponding to the centre of each channel, normalised to the interval $[0,1]$. The maximum order and type of polynomial are best chosen to match a specific application. However, for numerical work, the Chebyshev polynomials defined by:

$$\begin{aligned} T_0(E) &= 1, & T_1(E) &= E \\ T_{k+1}(E) &= 2E T_k(E) - T_{k-1}(E) \end{aligned} \quad (4.54)$$

have several optimal properties [4].

Equation (4.53) may be written as

$$E = TAT^T \quad (4.55)$$

where T is the matrix of polynomial values and A is an $R \times R$ matrix of coefficients where $R-1$ represents the highest polynomial order. With this decomposition, equation (4.52) becomes:

$$TA(T^T\Phi) = S \quad (4.56)$$

where the unknown detection efficiency matrix, E , is reduced to the much smaller unknown coefficient matrix, A . This makes the experimental construction of E a viable proposition.

Multiplying both sides of (4.56) by $(T^T T)^{-1} T^T$ on the left, gives:

$$A(T^T\Phi) = (T^T T)^{-1} T^T S \quad (4.57)$$

which may be transposed to move the coefficient matrix to the right of the matrix $(T^T\Phi)^T$.

The matrices $G = (T^T Q)^T$ and $F = ((T^T T)^{-1} T^T S)^T$ are both of order $R \times U$. Therefore, if the number of spectra, U , is greater than the number of polynomial terms, R , the equation:

$$GA = F \quad (4.58)$$

may be solved for A using the general linear model. However this solution is more convenient if we proceed as follows.

Let a_j and f_j denote the j th columns of A and F , respectively. Then (4.58) may be written:

$$\begin{pmatrix} G & 0 \\ & G & 0 \\ 0 & \dots & G \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_R \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_R \end{pmatrix} \quad (4.59)$$

or more succinctly as:

$$Ga = f \quad (4.60)$$

Then we may use the earlier procedure to define the covariance matrix of f and the equation can be solved using the general linear model as formulated in chapter 3.

Once an initial estimate for the coefficients has been obtained, this can be refined using periodic calibration measurements throughout the lifetime of the system. In particular, there is no need for a complete set of calibration spectra to be defined since the old coefficient estimates can be updated using a small number of spectra together with an estimator of the form:

$$a_{n+1} = \left[\begin{pmatrix} G_{n+1} \\ I \end{pmatrix}^T \begin{pmatrix} V_{n+1} \\ U_n \end{pmatrix}^{-1} \begin{pmatrix} G_{n+1} \\ I \end{pmatrix} \right]^{-1} \begin{pmatrix} G_{n+1} \\ I \end{pmatrix}^T \begin{pmatrix} V_{n+1} \\ U_n \end{pmatrix}^{-1} \begin{pmatrix} f_{n+1} \\ a_n \end{pmatrix} \quad (4.61)$$

where the terms of (4.61) are defined in an almost identical manner to those of (4.39). It may even be possible to install a shine through source in the sample container and use its known activity together with the estimated sample activities to refine the coefficients on-line.

4.4 SPATIAL DEPENDENCE AND TRANSPORT THEORY

We now return to the detailed definition of the Integral Boltzmann transport equation. The usual derivation, obtained by considering the balance of particles in a region of the flux geometry [1], leads to an integro-differential equation of the form:

$$\begin{aligned} \frac{1}{v_j(E)} \frac{\partial}{\partial t} \phi_j(r, \Omega, E, t) + \Omega \cdot \nabla \phi_j(r, \Omega, E, t) = & \sum_{i=1}^I \theta_i(r|t) \left[\frac{1}{4\pi} p_{ij}(E) \right. \\ & \left. - \sigma_{ij}(E) \phi_j(r, \Omega, E, t) + \sum_{j'=1}^J \int_{4\pi} d\Omega' \int_0^{E_{\max}} dE' p_{ij'j}(E, \Omega | \Omega', E') \phi_{j'}(r, \Omega', E', t) \right] \end{aligned} \quad (4.62)$$

where $v_j(E)$ is the speed of a particle of type, j , and energy, E .

The right hand side of this equation is the source term of equation (4.23). This may be positive or negative by virtue of the total interaction term, $-\sigma_{ij}(E)\phi_j(r, \Omega, E, t)$. This is traditionally included in the left hand side of the equation where it describes the loss of propagating particles by interaction. Its incorporation into the source has the advantage of removing the dependence of the left hand side upon the unknown source density, $\theta_i(r|t)$. The importance of this lies in the fact that even with a known source density, it is difficult to define a Green's function (see equation 4.25) unless assumptions such as time independence, homogeneity and isotropic scattering are made - which rather defeats the objective of the present exercise. However, the incorporation of the total interaction term in the source requires the conceptual block of negative sources and their effects propagating through space to be overcome. In fact, there is no reason why this should not be accepted if the arguments of section 4.2 are accepted. With the above stipulation, equation (4.62) may be written:

$$\frac{\partial}{\partial t} \phi_j(r, \Omega, E, t) + \mathbf{v} \cdot \nabla \phi_j(r, \Omega, E, t) = v_j(E) S_j(r, \Omega, E, t) \quad (4.63)$$

Where $\mathbf{v} = v_j(E)\Omega$ is the velocity of a particle. This equation may be regarded as a pure partial differential equation.

The characteristics of a partial differential equation are the paths on which it reduces to an ordinary differential equation. As shown in figure 4.1, the characteristics of the transport equation are the straight lines defined by the point, r , and direction, Ω . We wish to construct a Green's function to describe the propagation of a particle along a characteristic so as to express equation (4.63) in the form of equation (4.24).

Writing $G(r, t)$ as a shorthand notation for the Green's function $G_j(r, t | r' \Omega, E, t')$, this function is a solution of the equation:

$$\frac{\partial}{\partial t} G(r, t) + \mathbf{v} \cdot \nabla G(r, t) = 2v_j(E) \delta(r - r') \delta(t - t') \quad (4.64)$$

where the Dirac delta functions, $\delta(r - r') \delta(t - t')$, define a unit point source at the position r' and time t' . The factor 2 allows for the fact that a particle can propagate in either direction along a characteristic.

Taking the Fourier transform of equation (4.64) with respect to the spatial variables gives:

$$\begin{aligned} \iiint_{-\infty}^{\infty} \frac{\partial}{\partial t} G(r, t) e^{i\mathbf{k} \cdot \mathbf{r}} d\mathbf{r} + \mathbf{v} \cdot \iiint_{-\infty}^{\infty} \nabla G(r, t) e^{i\mathbf{k} \cdot \mathbf{r}} d\mathbf{r} \\ = 2v_j(E) \delta(t - t') \iiint_{-\infty}^{\infty} \delta(r - r') e^{i\mathbf{k} \cdot \mathbf{r}} d\mathbf{r} \end{aligned} \quad (4.65)$$

which reduces to:

$$\frac{\partial}{\partial t} x(\mathbf{k}, t) - i(\mathbf{v} \cdot \mathbf{k}) x(\mathbf{k}, t) = 2v_j(E) \delta(t - t') e^{i\mathbf{k} \cdot \mathbf{r}'} \quad (4.66)$$

where $x(\mathbf{k}, t)$ is the spatial Fourier transform of $G(r, t)$. Then, taking the Fourier transform of (4.66) with respect to time gives:

$$\begin{aligned} \int_{-\infty}^{\infty} \frac{\partial}{\partial t} x(\mathbf{k}, t) e^{i\omega t} dt - i(\mathbf{v} \cdot \mathbf{k}) \int_{-\infty}^{\infty} x(\mathbf{k}, t) e^{i\omega t} dt \\ = 2v_j(E) e^{i\mathbf{k} \cdot \mathbf{r}'} \int_{-\infty}^{\infty} \delta(t - t') e^{i\omega t} dt \end{aligned} \quad (4.67)$$

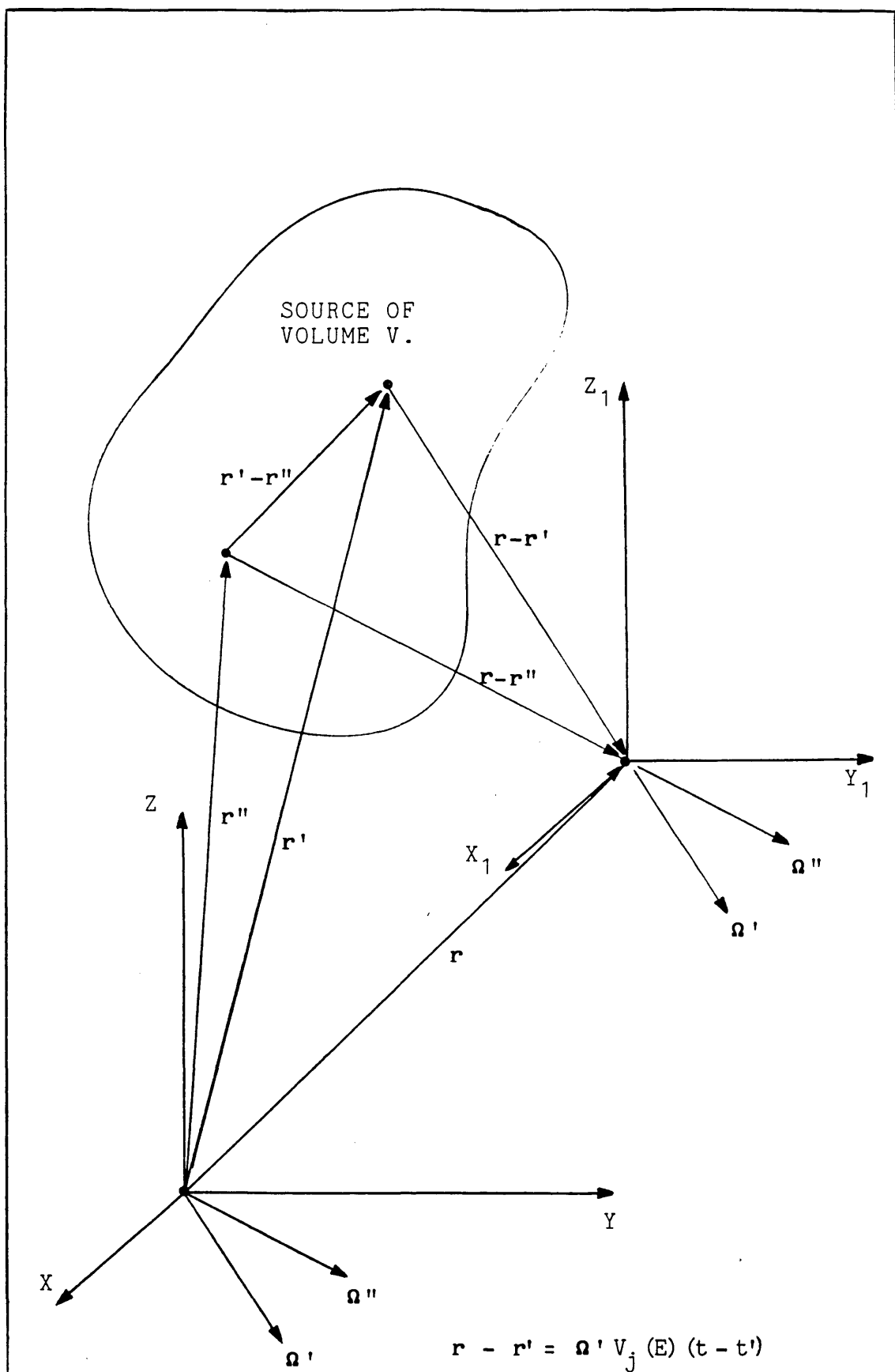


FIG 4.1 SOURCE GEOMETRY

which reduces to:

$$i\omega\psi(\mathbf{k},\omega) + i(\mathbf{v}\cdot\mathbf{k})\psi(\mathbf{k},\omega) = -2v_{\perp}(E) e^{i(\mathbf{k}\cdot\mathbf{r}' + \omega t')} \quad (4.68)$$

where $\psi(\mathbf{k},\omega)$ is the temporal Fourier transform of $\chi(\mathbf{k},t)$.

We have succeeded in reducing the partial differential equation (4.64) to an algebraic equation in the complex plane (4.68). This can be rearranged to give:

$$\psi(\mathbf{k},\omega) = -\frac{2v_{\perp}(E)}{i} \frac{e^{i(\mathbf{k}\cdot\mathbf{r}' + \omega t')}}{\omega + \mathbf{v}\cdot\mathbf{k}} \quad (4.69)$$

To reconstruct the Greens function it is necessary to take the inverse spatial and temporal Fourier transforms of $\psi(\mathbf{k},\omega)$. These can be combined to produce an expression of the form:

$$\begin{aligned} G(\mathbf{r},t) &= \frac{2v_{\perp}(E)}{(2\pi)^4} \iiint_{-\infty}^{\infty} \frac{e^{i(\mathbf{k}\cdot\mathbf{r}' + \omega t')}}{\omega + \mathbf{v}\cdot\mathbf{k}} e^{-i(\mathbf{k}\cdot\mathbf{r} + \omega t)} d\omega d\mathbf{k} \\ &= \frac{v_{\perp}(E)}{(2\pi)^3} \iiint_{-\infty}^{\infty} \left[-\frac{1}{\pi i} \int_{-\infty}^{\infty} \frac{e^{-i(\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}') + \omega(t-t'))}}{\omega + \mathbf{v}\cdot\mathbf{k}} d\omega \right] d\mathbf{k} \quad (4.70) \end{aligned}$$

The inner integral, with respect to ω , has a simple pole on the real axis at $\omega_0 = -\mathbf{v}\cdot\mathbf{k}$. This integral can be evaluated by completing the contour through the complex plain at $R = \infty$ as shown in figure 4.2, then applying the calculus of residuals. Since the closed contour does not enclose any poles it can be shown [4] that the only contribution to the integral is the Cauchy principal value of the pole on the real axis.

Hence we may write:

$$\begin{aligned} &-\frac{1}{\pi i} \int_{-\infty}^{\infty} \frac{e^{-i(\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}') + \omega(t-t'))}}{\omega - \omega_0} d\omega = a_{-1} \\ &= \frac{1}{2\pi i} \oint_C \frac{e^{-i(\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}') + \omega(t-t'))}}{\omega - \omega_0} d\omega \\ &= e^{-i(\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}') + \omega_0(t-t'))} \quad (4.71) \end{aligned}$$

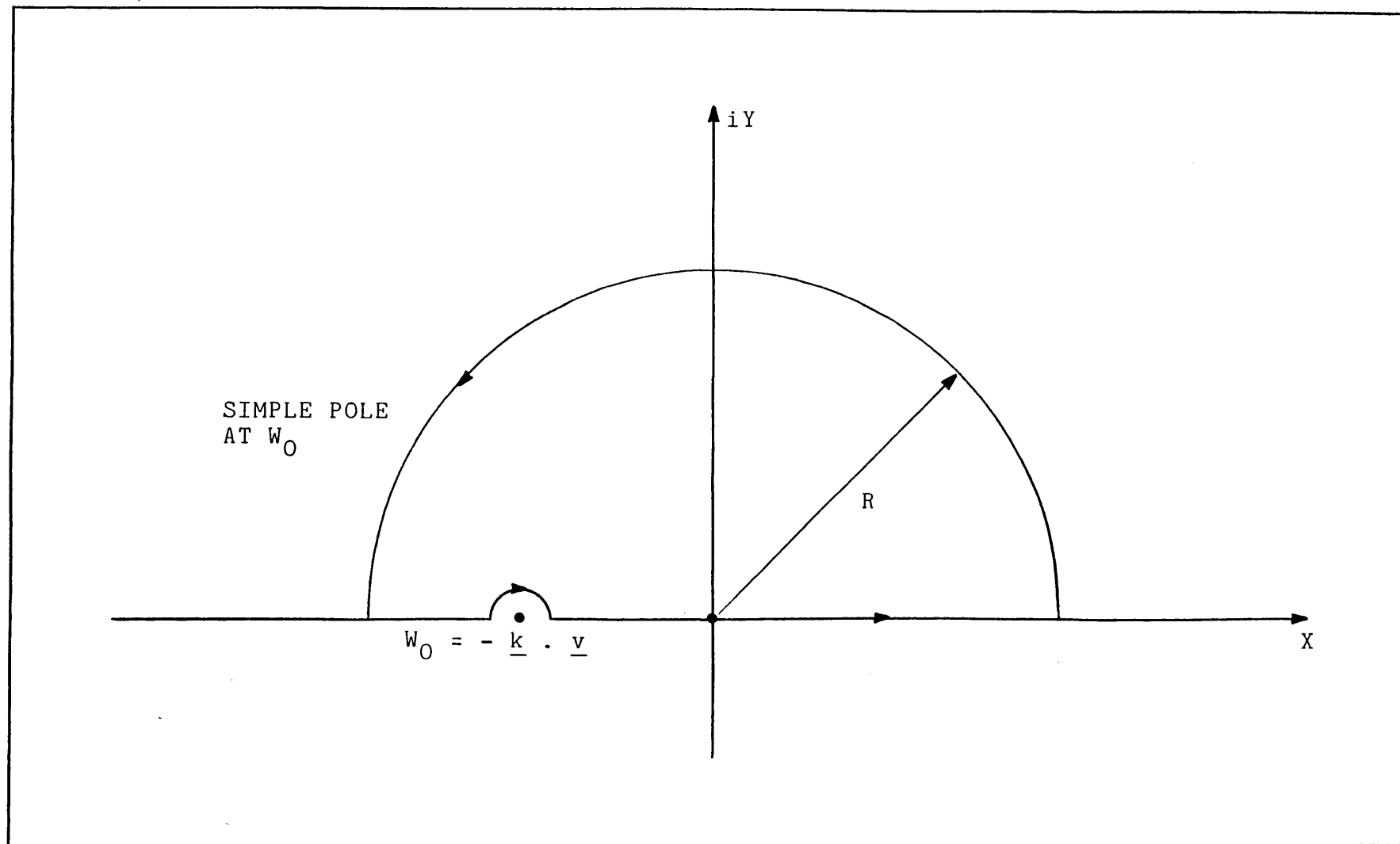


FIG 4.2 INTEGRATION CONTOUR ON THE COMPLEX PLANE

where a_{-1} is the first order residual at ω_0 and the last step arises from the direct application of the Cauchy integral formula.

Substituting for ω_0 in (4.71) the expression for the Greens function (4.70) becomes:

$$G(r, t) = \frac{v_j(E)}{(2\pi)^3} \iiint_{-\infty}^{\infty} e^{-i(k \cdot (r-r') - v(t-t'))} dk \quad (4.72)$$

which is an alternative expression for the Dirac delta function:

$$G(r, t) = v_j(E) \delta((r-r') - v(t-t')) \quad (4.73)$$

In fact, for the present application (4.72) is the more useful expression. Substituting this into (4.24) and rearranging gives:

$$\begin{aligned} \phi_j(r, \Omega, E, t) &= \phi_j^0(r, \Omega, E, t) + v_j(E) \int_0^t dt' \exp[-i\omega_0(t-t')] \\ &\quad \frac{1}{(2\pi)^3} \int_U dk \exp[-ik \cdot r] \int_V dr' \exp[ik \cdot r'] S_j(r', \Omega, E, t') \end{aligned} \quad (4.74)$$

Since the integrals over r' and k represent a forward and inverse Fourier transform pair, equation (4.74) reduces to:

$$\phi_j(r, \Omega, E, t) = \phi_j^0(r, \Omega, E, t) + v_j(E) \int_0^t dt' \exp[-i\omega_0(t-t')] S_j(r, \Omega, E, t') \quad (4.75)$$

The above transformation has the effect of evaluating the flux at an interior point of the source and this may be of great value in the processing of data from nuclear instrumentation systems.

The Greens function must be real, therefore equation (4.75) only makes sense at the discrete points in time.

$$(t - t') = \frac{2n\pi}{\omega_0} = \frac{n}{v} \quad (4.76)$$

where v is interpreted as an interaction frequency at the point, r .

But this implies:

$$t' = t - \frac{n}{v} = t_n \quad (4.77)$$

and when $t_n = 0$

$$n = [vt] \quad (4.78)$$

where $[]$ denotes the integer part of the real number, vt . This denotes the total number of interactions at the point, r , in the time interval, $[0, t]$. Hence, the time integral may be written as a sum and equation (4.75) becomes:

$$\phi_j(r, \Omega, E, t) = \phi_j^0(r, \Omega, E, t) + \frac{v_j(E)}{v} \sum_{n=1}^{[vt]} S_j(r, \Omega, E, t_n) \quad (4.79)$$

Since $S_j(r, \Omega, E, t)$ is the number of particles produced in unit time and v is the number of interactions per unit time, the product $\frac{1}{v} S_j(r, \Omega, E, t)$ denotes the particles produced per interaction and the sum is taken over all of the interactions in the time interval, $[0, t]$.

This result is not totally unexpected because of the phase relationship embodied in the Dirac delta function of equation (4.73). It implies that the initial flux at the point, r , is modified only by the series of discrete reactions at that point. This situation arises because the movement of particles about the source volume is independent of the source density and it has been demonstrated that in free space the divergence of the angular flux is zero.

Because the frequency v is unknown, equation (4.79) is best expressed in another form. Consider the change in the flux over a time interval, h , small enough for the probability of two reactions occurring in that period to be neglectable and for the probability of one reaction to be proportional to h . Then equation (4.79) may be written:

$$\begin{aligned} \phi_j(r, \Omega, E, t+h) - \phi_j(r, \Omega, E, t) &= \phi_j^0(r, \Omega, E, t+h) - \phi_j^0(r, \Omega, E, t) \\ &\quad + hv_j(E)S_j(r, \Omega, E, t) \end{aligned} \quad (4.80)$$

where the frequency, ν , cancels. Dividing through by h , then taking the limit as h tends towards zero, gives:

$$\frac{d}{dt}(\phi_j(r, \Omega, E, t) - \phi_j^0(r, \Omega, E, t)) = \nu_j(E) S_j(r, \Omega, E, t) \quad (4.81)$$

Integrating both sides with respect to t and using the fact that $\phi_j(r, \Omega, E, 0) = \phi_j^0(r, \Omega, E, 0)$, we have:

$$\phi_j(r, \Omega, E, t) = \phi_j^0(r, \Omega, E, t) + \int_0^t dt' \nu_j(E) S_j(r, \Omega, E, t') \quad (4.82)$$

Finally, expanding the source term gives:

$$\begin{aligned} \phi_j(r, \Omega, E, t) = & \phi_j^0(r, \Omega, E, t) + \int_0^t dt' \nu_j(E) \sum_{i=1}^I \theta_i(r|t') \left[\frac{1}{4\pi} p_{ij}(E) \right. \\ & - \sigma_{ij}(E) \phi_j(r, \Omega, E, t') + \sum_{j'=1}^J \int_0^{E_{\max}} dE' \int_{4\pi} d\Omega' \\ & \left. p_{ijj'}(\Omega, E|\Omega', E') \phi_{j'}(r, \Omega', E', t') \right] \end{aligned} \quad (4.83)$$

which is the required integral form of the transport equation and defines the mapping, g_1 , from the source space onto the flux space in detail.

4.5 APPLICATION TO GENERAL SYSTEMS

As with the fixed geometry solution the mapping g_2 which relates the observed signals to the flux takes the form of the inner product:

$$\begin{aligned} s_{\Omega_j} &= \int_0^{t_n} dt \int_V dr \int_{E_k}^{E_{\max}} dE \int_{4\pi} d\Omega \xi_{\Omega_j}(r, \Omega, E, t) \phi_j(r, \Omega, E, t) \\ &= \langle \xi_{\Omega_j} | \phi_j \rangle, \quad \Omega = 1, 2, \dots, L \end{aligned} \quad (4.84)$$

where each Ω represents a combination of the parameters k , n and m

which define the signal space. The function ξ is independent of the source density.

Equation (4.84) may be expanded using (4.83) to give the composite mapping:

$$\begin{aligned}
 s_{\mathcal{Q}_j} = & b_{\mathcal{Q}_j} + \int_0^{t_n} dt \int_V dr \int_{E_k}^{E_{\max}} dE \int_{4\pi} d\Omega \xi_{\mathcal{Q}_j}(r, \mathcal{Q}, E, t) v_j(E) \int_0^t dt' \sum_{i=1}^I \theta_i(r|t') \\
 & \left[\frac{1}{4\pi} p_{ij}(E) - \sigma_{ij}(E) \phi_j(r, \mathcal{Q}, E, t') + \sum_{j'=1}^J \int_0^{E_{\max}} dE' \int_{4\pi} d\Omega' \right. \\
 & \left. p_{ijj'}(\mathcal{Q}, E | \mathcal{Q}', E') \phi_{j'}(r, \mathcal{Q}', E', t') \right] \quad (4.85)
 \end{aligned}$$

where the background function, $b_{\mathcal{Q}_j}$, is defined by:

$$\begin{aligned}
 b_{\mathcal{Q}_j} &= \int_0^{t_n} dt \int_V dr \int_{E_k}^{E_{\max}} dE \int_{4\pi} d\Omega \xi_{\mathcal{Q}_j}(r, \mathcal{Q}, E, t) \phi_j^0(r, \mathcal{Q}, E, t) \\
 &= \langle \xi_{\mathcal{Q}_j} | \phi_j \rangle, \quad I = 1, 2, \dots, L \quad (4.86)
 \end{aligned}$$

By interchanging the order of the time integrals in equation (4.85) and factorizing out the source density, this equation may be written:

$$\begin{aligned}
 s_{\mathcal{Q}_j} &= b_{\mathcal{Q}_j} + \sum_{i=1}^I \int_0^{t_n} dt \int_V dr g_{\mathcal{Q}_{ij}}(r, t) \theta_i(r|t) \\
 &= b_{\mathcal{Q}_j} + \sum_{i=1}^I \langle g_{\mathcal{Q}_{ij}} | \theta_i \rangle, \quad I = 1, 2, \dots, L \quad (4.87)
 \end{aligned}$$

where the function $g_{\mathcal{Q}_{ij}}(r, t)$ is defined by:

$$\begin{aligned}
 g_{\mathcal{Q}_{ij}}(r, t) &= \int_{E_k}^{E_{\max}} dE \int_{4\pi} d\Omega \int_0^t dt' \xi_{\mathcal{Q}_j}(r, \mathcal{Q}, E, t') v_j(E) \left[\frac{1}{4\pi} p_{ij}(E) \right. \\
 &\quad \left. - \sigma_{ij}(E) \phi_j(r, \mathcal{Q}, E, t) + \sum_{j'=1}^J \int_0^{E_{\max}} dE' \int_{4\pi} d\Omega' p_{ijj'}(\mathcal{Q}, E | \mathcal{Q}', E') \phi_{j'}(r, \mathcal{Q}', E', t') \right] \quad (4.88)
 \end{aligned}$$

The general procedure for the solution of equation (4.87) is to first solve equation (4.84) for the flux, then substitute this solution into equation (4.88) so as to define the function, g . Equation (4.87) can then be solved for the source density using the results of chapter 3.

With the present formulation, the above procedure requires that the function $\xi_{ij}(r, \Omega, E, t)$ be defined by experiment and that the function $p_{ijj}(\Omega, E | \Omega', E')$ be defined from published tables of data. However, because of the number of variables, both of these requirements present difficult problems. Therefore, considerable benefit would be obtained by re-formulating the equations so as to reduce the gross quantity of data required to define these functions.

Returning to equation (4.83), the angular dependence is necessarily included in this equation to obtain the correct solution in the presence of source inhomogeneities and anisotropic scattering and particle production, but the angular distribution cannot be resolved by any of the measurement systems we have discussed. Therefore, any simpler method of incorporating the angular dependence which satisfactorily accounts for this behaviour would be suitable for the present requirement. It is intuitively apparent that the operation of mapping the angular flux from one state to the next by the direction dependent particle yields, should be equivalent to the operation of a tensor on a vector representation of the flux. Furthermore, if these formulations are equivalent, then it should be possible to derive one from the other. In fact, we shall show that this is the case.

The first step is to multiply equation (4.83) through by Ω , so as to produce a vector equation of the form:

$$\begin{aligned} \Omega \phi_j(r, \Omega, E, t) = & \Omega \phi_j^0(r, \Omega, E, t) + \int_0^t dt' v_j(E) \sum_{i=1}^I \theta_i(r | t') \left[\frac{1}{4\pi} \Omega p_{ij}(E) \right. \\ & - \sigma_{ij}(E) \Omega \phi_j(r, \Omega, E, t') + \sum_{j'=1}^J \int_0^{E_{\max}} dE' \Omega \int_{4\pi} d\Omega' \\ & \left. p_{ijj}(\Omega, E | \Omega', E') \phi_{j'}(r, \Omega', E', t') \right] \quad (4.89) \end{aligned}$$

Next, we seek to replace the vector flux, $\Omega\phi_j(r, \Omega, E, t)$, by a vector current density which implicitly contains the angular dependence of the particles as they propagate through a sample. By the conventional definition [5], a current density has the form:

$$\psi_j(r, E, t) = \int_{4\pi} \Omega\phi_j(r, \Omega, E, t) d\Omega \quad (4.90)$$

Where $\psi_j(r, E, t)$ describes the net flow rate of particles across unit area of a surface perpendicular to the mean flow direction of the particles. This quantity is in many ways similar to an electrical current density. In fact the above definition reduces to the electrical case when the particles under consideration are electrons. However, it is unsuitable for the present requirement since the conventional current disappears when the flux is isotropic. Therefore, we have need to modify the definition to avoid this problem.

In cartesian coordinates, the unit direction vector is given by:

$$\Omega = i\alpha + j\beta + k\gamma \quad (4.91)$$

where α , β and γ are direction cosines and i , j and k are the cartesian unit vectors in the x , y and z directions. In the spherical polar coordinate system the direction cosines are given by:

$$\begin{aligned} \alpha &= \sin\theta \cos\psi \\ \beta &= \sin\theta \sin\psi \\ \gamma &= \cos\theta \end{aligned} \quad (4.92)$$

and, also:

$$d\Omega = \sin\theta d\theta d\psi \quad (4.93)$$

where θ is the polar angle and ψ the azimuth angle. We have expressed the three interdependent direction cosines in terms of two independent variables which, by virtue of (4.92), automatically contain the constraint of footnote 3. With these definitions, equation (4.90) may

be written:

$$\begin{aligned} \psi = & i \int_0^{2\pi} \int_0^{\pi} \sin^2 \theta \cos \psi \phi \, d\theta d\psi + j \int_0^{2\pi} \int_0^{\pi} \sin^2 \theta \sin \psi \phi \, d\theta d\psi \\ & + k \int_0^{2\pi} \int_0^{\pi} \sin \theta \cos \theta \phi \, d\theta d\psi \quad (4.94) \end{aligned}$$

or, equivalently:

$$\begin{aligned} \psi = & i \int_{-\pi/2}^{\pi/2} \int_0^{\pi} \sin^2 \theta \cos \psi \phi \, d\theta d\psi - i \int_{\pi/2}^{3\pi/2} \int_0^{\pi} \sin^2 \theta \cos \psi \phi \, d\theta d\psi \\ & + j \int_0^{\pi} \int_{-\pi/2}^{\pi/2} \sin^2 \theta \sin \psi \phi \, d\theta d\psi - j \int_{\pi/2}^{3\pi/2} \int_0^{\pi} \sin^2 \theta \sin \psi \phi \, d\theta d\psi \\ & + k \int_0^{2\pi} \int_0^{\pi/2} \sin \theta \cos \theta \phi \, d\theta d\psi - k \int_0^{2\pi} \int_{\pi/2}^{\pi} \sin \theta \cos \theta \phi \, d\theta d\psi \quad (4.95) \end{aligned}$$

where equation (4.94) has been separated into component integrals over the forward and backward facing hemispheres centred about each of the cartesian coordinate axis. These component integrals will be denoted more succinctly by, for example:

$$\psi_x^+ = \int_{2\pi_x^+} \alpha \phi \, d\Omega \quad (4.96)$$

where (4.96) denotes the integral over the hemisphere about the positive x-axis.

We may now to define the forward current density vector.

$$\psi^+ = (\psi_x^+, \psi_y^+, \psi_z^+) \quad (4.97)$$

as the vector with components given by the integrals over the forward facing hemispheres about each of the coordinate axes, together with the backward current density vector:

$$\psi^- = (\psi_x^-, \psi_y^-, \psi_z^-) \quad (4.98)$$

with components given by the integrals over the backward facing hemispheres.

The above procedure may be carried out on each term of equation (4.89) to yield a set of six coupled integral equations in the components of the forward and backward currents of the form:

$$\begin{aligned} \psi_{xj}^+(r, E, t) = & \psi_{xj}^{0+}(r, E, t) + \int_0^t dt' v_j(E) \sum_{i=1}^I \theta_i(r|t') \left[\frac{1}{6} p_{ij}(E) \right. \\ & - \sigma_{ij}(E) \psi_{xj}^+(r, E, t') + \sum_{j'=1}^J \int_0^{E_{\max}} dE' \int_{2\pi_x^+} d\Omega \alpha \int_{4\pi} d\Omega' \\ & \left. p_{ijj'}(\Omega, E|\Omega', E') \phi_{j'}(r, \Omega', E', t') \right] \end{aligned} \quad (4.99)$$

We now turn our attention to the angular dependence of the induced source term. This part of the equation may be written:

$$A_x^+ = \int_{2\pi_x^+} \alpha \int_{4\pi} p \phi d\Omega' d\Omega \quad (4.100)$$

If Ω' has cartesian coordinates α' , β' and γ' , then:

$$\begin{aligned} A_x^+ = & \int_{2\pi_x^+} \alpha \int_{2\pi_x^+} \alpha'^2 p \phi d\Omega' d\Omega + \int_{2\pi_x^+} \alpha \int_{2\pi_x^-} \alpha'^2 p \phi d\Omega' d\Omega \\ & + \int_{2\pi_x^+} \alpha \int_{2\pi_y^+} \beta'^2 p \phi d\Omega' d\Omega + \int_{2\pi_x^+} \alpha \int_{2\pi_y^-} \beta'^2 p \phi d\Omega' d\Omega \\ & + \int_{2\pi_x^+} \alpha \int_{2\pi_z^+} \gamma'^2 p \phi d\Omega' d\Omega + \int_{2\pi_x^+} \alpha \int_{2\pi_z^-} \gamma'^2 p \phi d\Omega' d\Omega \end{aligned} \quad (4.101)$$

since $\alpha'^2 + \beta'^2 + \gamma'^2 = 1$ and the integral (4.100) may be separated into component integrals over the forward and backward hemispheres. Similar relationships exist for the integrals mapping the angular flux into the other components of the forward and backward currents.

We may now define the forward tensors, P^{++} and P^{--} , and the backward tensors, P^{+-} and P^{-+} , such that each term of equation (4.101), and the

equations for the other current components, gives rise to a component of one of these tensors. That is, for example:

$$\int_{2\pi_x^+} \alpha \int_{2\pi_x^+} \alpha'^2 p \phi d\Omega' d\Omega = P_{xx}^{++} \psi_x^+ = P_{xx}^{++} \int_{2\pi_x^+} \alpha' \phi d\Omega' \quad (4.102)$$

where the last step uses the definition of the forward current.

Equation (4.102) must be true for all angular fluxes, ϕ . Furthermore, the forward and backward tensors must depend only on the propagation medium and, in particular, they must be independent of the flux. Therefore, without loss of generality, the flux may be taken to be isotropic. But then the above equation reduces to:

$$P_{xx}^{++} = \frac{1}{\pi} \int_{2\pi_x^+} \alpha \int_{2\pi_x^+} \alpha'^2 p d\Omega' d\Omega \quad (4.103)$$

since the isotropic fluxes cancel and:

$$\int_{2\pi_x^+} \alpha' d\Omega' = \pi \quad (4.104)$$

Equation (4.103) may be further reduced if the angular interaction coefficients are written as:

$$p_{ijj}(\Omega, E | \Omega', E) = p_{ijj}(E | E') f_{ijj}(\Omega | \Omega', E') \quad (4.105)$$

where $f_{ijj}(\Omega | \Omega', E')$ is a probability density for a particle's final direction given its initial configuration and the type of interacting nucleus. This decomposition is always possible since the final energy and angle of the particle are independent variables. Interchanging the order of integration, equation (4.103) may now be written:

$$\begin{aligned} P_{xx, ij}^{++}(E | E') &= \frac{1}{\pi} p_{ijj}(E | E') \int_{2\pi_x^+} \alpha'^2 \int_{2\pi_x^+} \alpha f_{ijj}(\Omega | \Omega', E') d\Omega d\Omega' \\ &= \frac{1}{\pi} p_{ijj}(E | E') \int_{2\pi_x^+} \alpha'^2 E_{ijj}(\alpha | \Omega', E') d\Omega' \end{aligned} \quad (4.106)$$

where $E_{ijj}(\alpha|\Omega', E')$ is a conditional expectation (see section 3.4). However as shown in figure 4.3, this may be written as:

$$E_{ijj}(\alpha|\Omega', E') = \alpha' E_{ijj}(\alpha|\Omega'=i, E') = \alpha' \alpha_{ijj}^+(E') \quad (4.107)$$

where $\alpha_{ijj}^+(E')$ is the cosine of the expected forward scattering or particle production angle. Hence:

$$\begin{aligned} P_{xxijj}^{++}(E|E') &= \frac{1}{\pi} \alpha_{ijj}^+(E') P_{ijj}(E|E') \int_{2\pi_x^+} \alpha'^3 d\Omega' \\ &= \frac{\alpha_{ijj}^+(E')}{2} P_{ijj}(E|E') \end{aligned} \quad (4.108)$$

since:

$$\int_{2\pi_x^+} \alpha'^3 d\Omega' = \frac{\pi}{2} \quad (4.109)$$

The remaining thirty five components of the four tensors are defined similarly. In fact, all of the diagonal components of the forward tensors, that is P_{xx}^{++} , P_{yy}^{++} , and P_{zz}^{++} , and P_{xx}^{--} , P_{yy}^{--} and P_{zz}^{--} are identical and are given by equation (4.107). Likewise, the diagonal components of the backward tensors, P^{+-} and P^{-+} are given by:

$$P_{xxijj}^{+-}(E|E') = \frac{\alpha_{ijj}^-(E')}{2} P_{ijj}(E|E') \quad (4.110)$$

where $\alpha_{ijj}^-(E')$ is the cosine of the expected backward scattering or particle production angle. All other (off-diagonal) components take identical values of the form:

$$P_{xyijj}^{++}(E|E') = P_{xyijj}^{+-}(E|E') = \frac{\beta_{ijj}^+(E')}{2} P_{ijj}(E|E') \quad (4.111)$$

where $\beta_{ijj}^+(E')$ is the cosine of the expected scattering or particle production angle into a hemisphere about a coordinate axis orthogonal to the direction of particle incidence.

The formulation of equation (4.102) may be substituted into equation

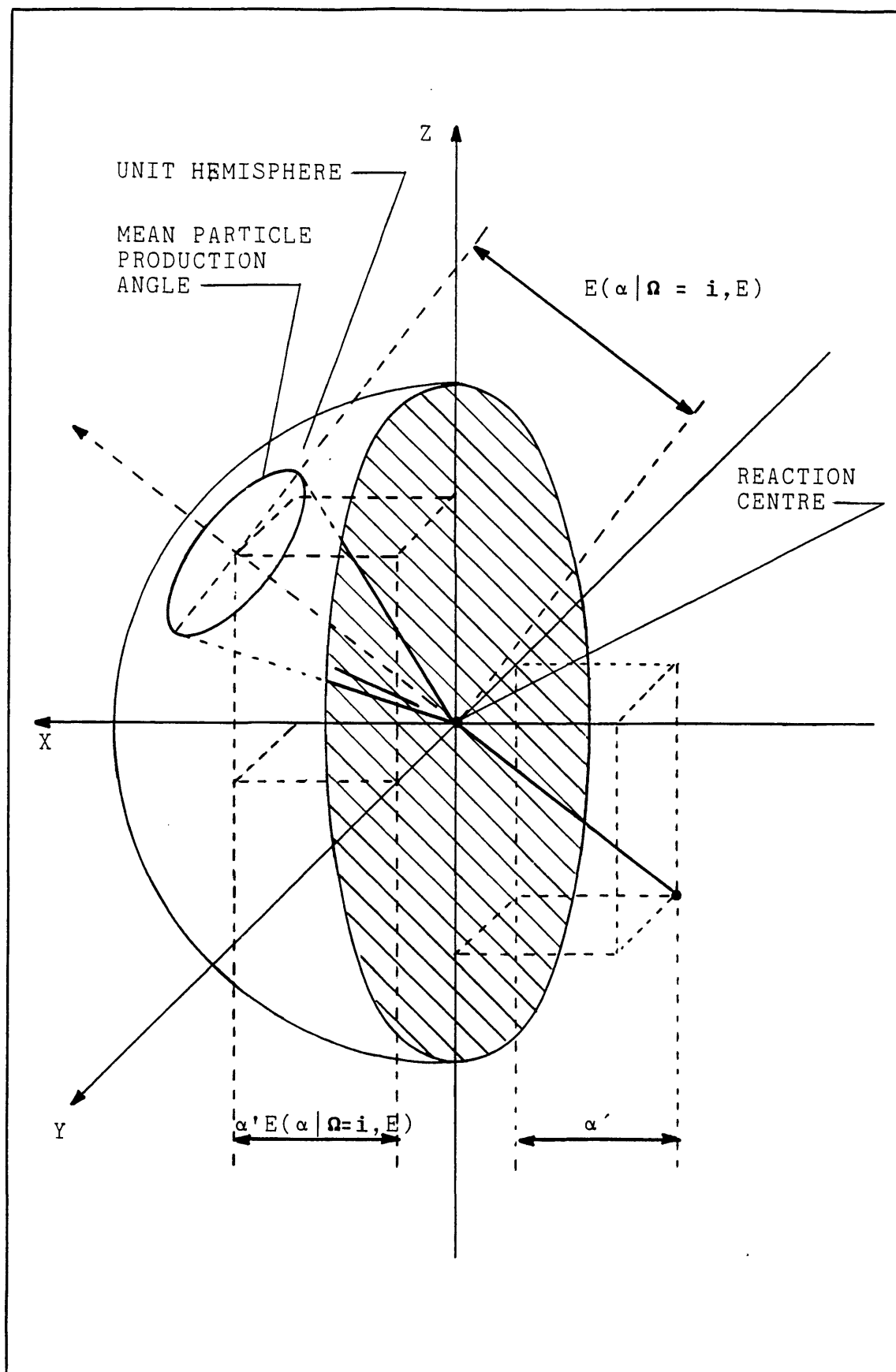


FIG 4.3 PARTICLE PRODUCTION GEOMETRY

(4.99) and collecting together the components of the current density vectors gives:

$$\begin{aligned}\psi_j^+(r, E, t) = & \psi_j^{0+}(r, E, t) + v_j(E) \int_0^t dt' \sum_{i=1}^I \theta_i(r|t') \left[\frac{1}{2} p_{ij}(E) \right. \\ & - \sigma_{ij}(E) \psi_j^+(r, E, t') + \sum_{j'=1}^J \int_0^{E_{\max}} dE' (P_{ijj'}^{++}(E|E') \psi_j^+(r, E', t') \\ & \left. + P_{ijj'}^{+-}(E|E') \psi_j^-(r, E', t')) \right] \quad (4.112)\end{aligned}$$

for the forward current, and:

$$\begin{aligned}\psi_j^-(r, E, t) = & \psi_j^{0-}(r, E, t) + v_j(E) \int_0^t dt' \sum_{i=1}^I \theta_i(r|t') \left[\frac{1}{2} p_{ij}(E) \right. \\ & - \sigma_{ij}(E) \psi_j^-(r, E, t') + \sum_{j'=1}^J \int_0^{E_{\max}} dE' (P_{ijj'}^{--}(E|E') \psi_j^-(r, E', t') \\ & \left. + P_{ijj'}^{-+}(E|E') \psi_j^+(r, E', t')) \right] \quad (4.113)\end{aligned}$$

for the backward current. The passive source term, $p_{ij}(E)$, is a three dimensional vector with components, $\frac{1}{3} p_{ij}(E)$.

The above formulation has the disadvantage that both the forward and backward currents depend on the orientation of the coordinate system. This situation can be rectified by adding together equations (4.112) and (4.113), and using the fact that $P^{++} = P^{--}$ and $P^{+-} = P^{-+}$ to produce an equation of the form:

$$\begin{aligned}x_j(r, E, t) = & x_j^0(r, E, t) + v_j(E) \int_0^t dt' \sum_{i=1}^I \theta_i(r|t') \left[p_{ij}(E) \right. \\ & \left. - \sigma_{ij}(E) x_j(r, E, t') + \sum_{j'=1}^J \int_0^{E_{\max}} dE' Q_{ijj'}(E|E') x_{j'}(r, E', t') \right] \quad (4.114)\end{aligned}$$

where $x_j(r, E, t)$ is known as the even parity current and is defined by:

$$x_j(r, E, t) = \psi_j^+(r, E, t) + \psi_j^-(r, E, t) \quad (4.115)$$

while $Q_{ijjj'}(E|E')$ is the even parity tensor which is defined by:

$$Q_{ijjj'}(E|E') = P_{ijjj'}^{++}(E|E') + P_{ijjj'}^{+-}(E|E') \quad (4.116)$$

Alternatively, we may subtract equation (4.113) from (4.112) to give:

$$\begin{aligned} \psi_j(r, E, t) = & \psi_j^0(r, E, t) + \sum_{i=1}^I v_j(E) \int_0^t dt' \theta_i(r|t') \left[-\sigma_{ij}(E) \psi_j(r, E, t') \right. \\ & \left. - \sum_{j'=1}^J \int_0^{E_{\max}} dE' P_{ijjj'}(E|E') \psi_{j'}(r, E', t') \right] \end{aligned} \quad (4.117)$$

where $\psi_j(r, E, t)$ is known as the odd parity current, which is defined:

$$\psi_j(r, E, t) = \psi_j^+(r, E, t) - \psi_j^-(r, E, t) \quad (4.118)$$

and $P_{ijjj'}(E|E')$ is the odd parity tensor, defined:

$$P_{ijjj'}(E|E') = P_{ijjj'}^{++}(E|E') - P_{ijjj'}^{+-}(E|E') \quad (4.119)$$

The odd parity current is actually the conventional current defined by equation (4.90). Note that since the passive sources emit particles isotropically, this term is of even parity and disappears from the odd parity equation.

The above analysis yields two uncoupled vector equations. The even parity flux describes the spreading out of particles in space while the odd parity flux relates to the movement of the centre of mass of the particles.

The main quantity of interest is the angular independent flux:

$$\phi_j(r, E, t) = \langle \psi_j(r, E, t) | X_j(r, E, t) \rangle^{1/2} \quad (4.120)$$

Furthermore, we have shown that this is identical to the function:

$$\phi_j(r, E, t) = \int_{4\pi} d\Omega \phi_j(r, \Omega, E, t) \quad (4.121)$$

where $\phi_j(r, \Omega, E, t)$ is the solution to equation (4.83). Hence, we have shown that both formulations of the theory produce equivalent results for the angular independent flux.

Having simplified the angular distribution the time and energy distributions can be discretized by integrating over the small regions corresponding to the partition of the signal space, in the same manner as for the fixed geometry solution. Hence equations (4.114) and (4.117) may be written as:

$$\begin{aligned} x_{jkn}(r) = & x_{jkn}^0(r) + v_{jk} \sum_{n'=1}^n \sum_{i=1}^I \theta_{in'}(r) \left[p_{ijk} - \sigma_{ijk} x_{jkn'}(r) \right. \\ & \left. + \sum_{j'=1}^J \sum_{k'=1}^K Q_{ijj'kk'} x_{j'k'n'}(r) \right] \quad (4.122) \end{aligned}$$

and

$$\begin{aligned} \psi_{jkn}(r) = & \psi_{jkn}^0(r) + v_{jk} \sum_{n'=1}^n \sum_{i=1}^I \theta_{in'}(r) \left[-\sigma_{ijk} \psi_{jkn'}(r) \right. \\ & \left. + \sum_{j'=1}^J \sum_{k'=1}^K P_{ijj'kk'} \psi_{j'k'n'}(r) \right] \quad (4.123) \end{aligned}$$

The angular dependence of the detection efficiency may be treated in a similar manner to the induced reaction coefficients. That is, using the constraint of footnote 3 and defining component integrals over the forward and backward facing hemispheres, the angular integral of equation (4.84) may be written:

$$\begin{aligned} \int_{4\pi} \xi \phi \, d\Omega = & \int_{2\pi_x^+} \alpha^2 \xi \phi \, d\Omega + \int_{2\pi_x^-} \alpha^2 \xi \phi \, d\Omega \\ & + \int_{2\pi_y^+} \beta^2 \xi \phi \, d\Omega + \int_{2\pi_y^-} \beta^2 \xi \phi \, d\Omega \\ & + \int_{2\pi_z^+} \gamma^2 \xi \phi \, d\Omega + \int_{2\pi_z^-} \gamma^2 \xi \phi \, d\Omega \\ = & \xi^+ \cdot \psi^+ + \xi^- \cdot \psi^- \quad (4.124) \end{aligned}$$

Then allowing the flux to become isotropic, gives:

$$\xi_x^+ = \frac{1}{\pi} \int_{2\pi_x^+} \alpha^2 \xi \, d\Omega = \frac{\xi}{\pi} \int_{2\pi_x^+} \alpha^2 \, d\Omega \quad (4.125)$$

and similarly for the other components of ξ^+ and ξ^- . The direction cosines may be interpreted as the cosines of the angles between the coordinate axis and the line joining the absorption centre of the detector to the point, r , and ξ is the gross detection efficiency.

Equation (4.124) may be expressed in terms of the even and odd parity currents using the decompositions:

$$\psi^+ = \frac{1}{2}(\chi + \psi) \quad \text{and} \quad \psi^- = \frac{1}{2}(\chi - \psi) \quad (4.126)$$

Then, by rearranging this equation we may define the even parity detection efficiency:

$$\eta = \frac{1}{2}(\xi^+ + \xi^-) \quad (4.127)$$

and the odd parity detection efficiency:

$$\zeta = \frac{1}{2}(\xi^+ - \xi^-) \quad (4.128)$$

so that equation (4.84) may be written:

$$s_{\ell j} = \sum_{n'=1}^n \sum_{k'=k}^k \int_V dr (\eta_{\ell j n' k'}(r) \cdot \chi_{j n' k'}(r) + \zeta_{\ell j n' k'}(r) \cdot \psi_{j n' k'}(r)) \quad (4.129)$$

Substituting the even and odd parity current terms of equations (4.122) and (4.123) into the above equation gives *6:

$$s_{\ell j} = b_{\ell j} + \sum_{i=1}^I \sum_{n'=1}^n \int_V dr g_{\ell i j n'}(r) \theta_{i n'}(r) \quad (4.130)$$

*6 If θ is constant in time the sum over n' may be incorporated into equation (4.132) and hence eliminated from equation (4.130)

where:

$$b_{lj} = \sum_{n'=1}^n \sum_{k'=k}^K \int_V dr (\eta_{ljn'k'}(r) \cdot x_{jn'k'}^0(r) + \zeta_{ljn'k'}(r) \cdot \psi_{jn'k'}^0(r)) \quad (4.131)$$

and:

$$g_{lijn'}(r) = \sum_{k'=k}^K v_{jk'} \left[\eta_{ljn'k'}(r) \cdot \left[p_{ijk'} + \sum_{j'=1}^J \sum_{k''=1}^K Q_{ijj'k'k''} x_{jn'k''}(r) \right] + \zeta_{ljn'k'}(r) \cdot \left[\sum_{j'=1}^J \sum_{k''=1}^K P_{ijj'k'k''} \psi_{jn'k''}(r) \right] \right] \quad (4.132)$$

Equation (4.132) has been tidied up by subtracting $\sigma_{ijk'}$ from each diagonal term of the tensors $Q_{ijj'k'k''}$ and $P_{ijj'k'k''}$ so as to absorb the total reaction term into the sums over j' and k'' . Also the detection efficiencies have been redefined so that:

$$\eta_{ljn'k'}(r) = \sum_{n''=n'}^n \eta_{ljn''k'}(r) \quad (4.133)$$

and likewise for $\zeta_{ljn'k'}(r)$.

It remains to discretize the spatial distribution of equation (4.130). In section 4.1 it was established that the spatial distribution of the source could be represented by a probability density function. For the fixed geometry solution, it was convenient to assume that this function was common to all of the radioactive isotopes and this allowed it to be incorporated into the detection efficiency and defined by experimental calibration. In the general case, the spatial distribution must be explicitly included in the model. However, its shape is usually unknown and may not correspond to the shapes available within any standard family of probability distributions. Therefore, we seek a representation which allows non-standard shaped functions to be computed from the observed data.

This can be achieved by numerical quadrature. For example suppose the source volume is cylindrical with height $2Z$ and diameter $2R$. Then, the integral of a function, $f(r)$, over this volume may be expressed in

cylindrical polar coordinates:

$$\int_V dr f(r) = \int_0^\pi \int_{-Z}^Z \int_{-R}^R f(r, z, \rho) r dr dz d\rho \quad (4.134)$$

where r is the radial coordinate, z the height and ρ the azimuth angle. In numerical quadrature it is normal practice to transform the range of each variable into the interval $[-1, 1]$, using the transformations:

$$\begin{aligned} r(x_1) &= Rx_1, & dr &= Rdx_1 \\ z(x_2) &= Zx_2, & dz &= Zdx_2 \\ \rho(x_3) &= \frac{1}{2}\pi(x_3 + 1), & d\rho &= \frac{1}{2}\pi dx_3 \end{aligned} \quad (4.135)$$

Then equation (4.134) may be written:

$$\begin{aligned} \frac{\pi R^2 Z}{2} \int_{-1}^1 \int_{-1}^1 \int_{-1}^1 f(r(x_1), z(x_2), \rho(x_3)) x_1 dx_1 dx_2 dx_3 \\ = \frac{\pi R^2 Z}{2} \sum_{w=1}^W \sum_{v=1}^V \sum_{u=1}^U (H_u H_v H_w a_u) f(r(a_u), z(a_v), \rho(a_w)) \end{aligned} \quad (4.136)$$

where the a 's are selected points in the interval $[-1, 1]$ and the H 's are weights associated with these points. If the points are chosen at equal intervals, we obtain the Newton-Cotes formulae of which the second order formula (Simpson's rule) has optimum properties. If an odd number of points are chosen, this will be exact when f is a polynomial of degree n , where n is the number of points. Better accuracy can be obtained if the points are chosen to coincide with the zeros of an n th degree orthogonal polynomial (for example one of the Legendre polynomials). Then a Gaussian quadrature solution will be obtained which is exact for a polynomial of degree $2n-1$, [1].

In the above example, a Gaussian quadrature solution could be obtained for the radial and height distributions while Simpson's rule could be applied to the azimuthal distribution so that the equal intervals coincide with the steps of a source which is discretely rotated as

measurements are made. This scheme is illustrated in figure 4.4 for a radioactive waste package and one of the measurement stations of the waste package assay facility described in section 2.7.

If a single index, $m = 1, \dots, M$, is used to denote the combinations of u, v and w , then equation (4.136) may be written in the simpler form:

$$\int_V dr f(r) = \sum_{m=1}^M H_m f_m \quad (4.137)$$

The above formulation may be applied to equation (4.129) to obtain:

$$\begin{aligned} s_{\mathcal{Q}j} &= \sum_{m=1}^M \sum_{n=1}^N \sum_{k'=k}^K H_m (\eta_{\mathcal{Q}jmnk'} \cdot x_{jmnk'} + \zeta_{\mathcal{Q}jmnk'} \cdot \psi_{jmnk'}) \\ &= \langle \eta_{\mathcal{Q}j} | x_j \rangle + \langle \zeta_{\mathcal{Q}j} | \psi_j \rangle \end{aligned} \quad (4.138)$$

which implies:

$$s_{\mathcal{Q}j} = b_{\mathcal{Q}j} + \sum_{i=1}^I \sum_{m=1}^M \sum_{n=1}^N g_{\mathcal{Q}jimn} \theta_{imn} \quad (4.139)$$

where:

$$b_{\mathcal{Q}j} = \sum_{m=1}^M \sum_{n=1}^N \sum_{k'=k}^K H_m (\eta_{\mathcal{Q}jmnk'}^0 \cdot x_{jmnk'}^0 + \zeta_{\mathcal{Q}jmnk'}^0 \cdot \psi_{jmnk'}^0) \quad (4.140)$$

and:

$$\begin{aligned} g_{\mathcal{Q}jimn} &= H_m \sum_{k'=k}^K v_{jk'} \left[\eta_{\mathcal{Q}jmnk'} \cdot \left[p_{ijk'} + \sum_{j'=1}^J \sum_{k''=1}^K q_{ijj'k'k''} x_{j'mnk''} \right] \right. \\ &\quad \left. + \zeta_{\mathcal{Q}jmnk'} \cdot \sum_{j'=1}^J \sum_{k''=1}^K p_{ijj'k'k''} \psi_{j'mnk''} \right] \end{aligned} \quad (4.141)$$

This gives the required discrete form of the transformation.

The expression for $g_{\mathcal{Q}jimn}$ may be tidied up if we let:

$$f_{\mathcal{Q}jimn} = H_m \sum_{k'=k}^K v_{jk'} (\eta_{\mathcal{Q}imnk'} \cdot p_{ijk'}) \quad (4.142)$$

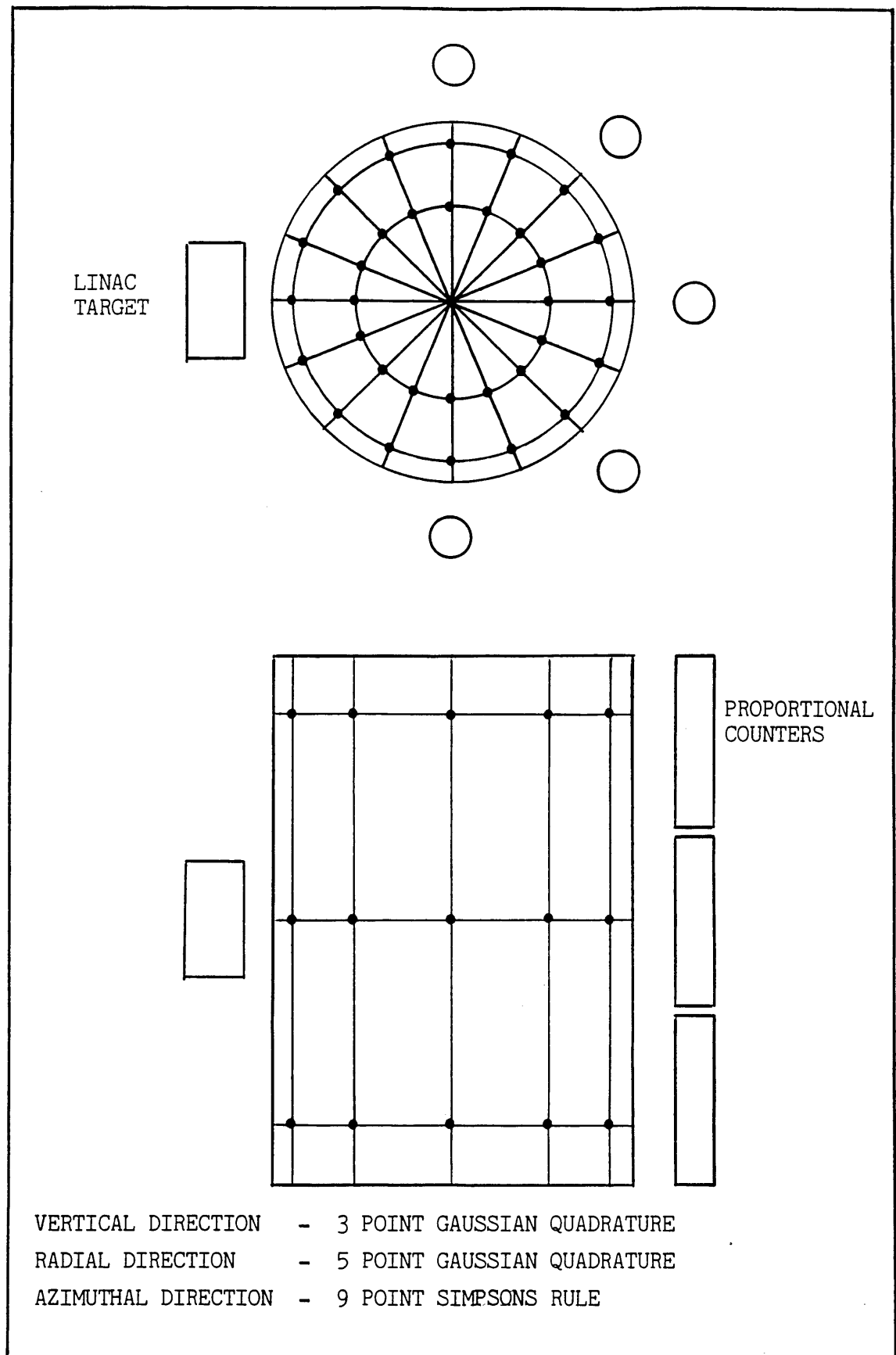


FIG 4.4 QUADRATURE POINTS FOR A RADIOACTIVE WASTE PACKAGE

and write the induced reaction terms as a bilinear form, so that:

$$g_{\ell jmn} = f_{\ell jmn} + H_m \sum_{j'=1}^J \sum_{k'=k}^K \sum_{k''=1}^K v_{jk'} \\ (\eta_{\ell jmnk'}, \zeta_{\ell jmnk'}) \begin{pmatrix} Q_{\ell jj'k'k''} & 0 \\ 0 & P_{\ell jj'k'k''} \end{pmatrix} \begin{pmatrix} x_{j'mnk''} \\ \psi_{j'mnk''} \end{pmatrix} \quad (4.143)$$

To solve equation (4.139) for the source density, it is necessary to first form estimates for the even and odd parity currents.

Equation (4.138) is an inner product on the flux space, therefore, the set of L detection efficiency functions, $(\eta_{\ell jmnk'}, \zeta_{\ell jmnk'})$, span a subspace which is closed and convex on the flux space. Hence, there exists a unique element of this subspace which has minimum norm on the flux space and solves equation (4.138). This element is called the normal solution and, since it is defined on the space spanned by the detection efficiencies, it may be represented by a linear combination of these functions:

$$(x_{jmnk'}, \psi_{jmnk'}) = \sum_{\ell'=1}^L a_{\ell'j} (\eta_{\ell'jmnk'}, \zeta_{\ell'jmnk'}) \quad (4.144)$$

It is implicit to the above argument that the measurement subspace is arranged so that it is sufficiently large with respect to the flux space so as to obtain a reasonable representation of the currents.

The $a_{\ell'j}$ may be found by substituting (4.144) into (4.138) to give:

$$s_{\ell j} = \sum_{\ell'=1}^L a_{\ell'j} (\langle \eta_{\ell j} | \eta_{\ell'j} \rangle + \langle \zeta_{\ell j} | \zeta_{\ell'j} \rangle) \\ = \sum_{\ell'=1}^L a_{\ell'j} h_{\ell \ell'j} \quad (4.145)$$

Then inverting the matrix $(h_{\ell \ell'j})_j$ to obtain a solution of the form:

$$(x_{jmnk'}, \psi_{jmnk'}) = \sum_{\ell=1}^L s_{\ell j} \sum_{\ell'=1}^{L-1} h_{\ell \ell'j}^{-1} (\eta_{\ell'jmnk'}, \zeta_{\ell'jmnk'}) \quad (4.146)$$

where $h_{\ell\ell',j}^{-1}$ is an element of the inverse matrix. This solution for the currents may be substituted into equation (4.143) to obtain an expression for $g_{\ell jmn}$ of the form:

$$g_{\ell jmn} = f_{\ell jmn} + \sum_{\ell'=1}^L \sum_{j'=1}^J s_{\ell'j'} q_{\ell\ell'jj'mn} \quad (4.147)$$

where:

$$q_{\ell\ell'jj'mn} = H_m \sum_{\ell''=1}^L \sum_{k'=k}^K \sum_{k''=1}^K v_{jk'} h_{\ell\ell''j'}^{-1} \begin{pmatrix} \eta_{\ell jmnk'}, \zeta_{\ell jmnk'} \\ 0 \end{pmatrix} \begin{pmatrix} Q_{\ell jj'k'k''} & 0 \\ 0 & P_{\ell jj'k'k''} \end{pmatrix} \begin{pmatrix} \eta_{\ell''j'mnk''} \\ \zeta_{\ell''j'mnk''} \end{pmatrix} \quad (4.148)$$

These results may be substituted into equation (4.139) and this may be solved for the source densities using the general linear model of chapter 3. The function, q , contains only factors which are constant for all measurements. Therefore the values of this function need be computed once only.

Equations (4.139), (4.142), (4.147) and (4.148) define a general solution which gives the complete isotopic configuration of a sample using the detection efficiency of the system for an empty sample container. It is worthwhile to explore the connection between this solution and the fixed geometry solution of section 4.3.

Let (ρ_{inm}) denote the source densities of a calibration sample. Then the configuration of an unknown sample can be denoted $(\rho_{inm} + \theta_{inm}^*)$ where (θ_{inm}^*) are the differences between the samples. The equations for the even and odd parity currents may now be written:

$$x_{jknm} = x_{jknm}^* + \sum_{n'=1}^n \sum_{j'=1}^J \sum_{k'=1}^K v_{jj'k'} \sum_{i=1}^I \rho_{in'm} Q_{\ell jj'kk'} x_{j'k'n'm} \quad (4.149)$$

and

$$\psi_{jknm} = \psi_{jknm}^* + \sum_{n'=1}^n \sum_{j'=1}^J \sum_{k'=1}^K v_{jj'k'} \sum_{i=1}^I \rho_{in'm} P_{\ell jj'kk'} \psi_{j'k'n'm} \quad (4.150)$$

where:

$$x_{jknm}^* = x_{jknm}^0 + v_{jk} \sum_{i=1}^I \sum_{n'=1}^n \theta_{in'm}^* \left[p_{ijk} + \sum_{j'=1}^J \sum_{k'=1}^K q_{ijj'kk'} x_{j'k'n'm} \right] \quad (4.151)$$

and:

$$\psi_{jknm}^* = \psi_{jknm}^0 + v_{jk} \sum_{i=1}^I \sum_{n'=1}^n \theta_{in'm}^* \sum_{j'=1}^J \sum_{k'=1}^K p_{ijj'kk'} \psi_{j'k'n'm} \quad (4.152)$$

It has been assumed that none of the isotopes (ρ_{inm}) are radioactive.

Equations (4.149) and (4.150) may be written in matrix form, as:

$$x_m = x_m^* + K_m x_m \quad (4.153)$$

and:

$$\psi_m = \psi_m^* + J_m \psi_m \quad (4.154)$$

where we have a pair of equations for each point in space. These are of the same form as a Fredholm integral equation (see equation (4.30)). Therefore, using the contraction properties of the mapping, we can obtain the Neumann series:

$$x_m = I + K_m + K_m K_m + \dots x_m^* \quad (4.155)$$

and:

$$\psi_m = I + J_m + J_m J_m + \dots \psi_m^* \quad (4.156)$$

If the series in K_m and J_m have limits G_m and F_m , respectively, then the elements of these matrices may be combined with the detection efficiencies of equation (4.138) so that:

$$(\eta_{jknm}^*, \zeta_{jknm}^*) = \sum_{n'=1}^n \sum_{j'=1}^J \sum_{k'=1}^K (\eta_{jj'k'n'm}, \zeta_{jj'k'n'm}) \begin{pmatrix} G_{jj'kk'nn'm} & 0 \\ 0 & F_{jj'kk'nn'm} \end{pmatrix} \quad (4.157)$$

Equation (4.139) may be modified to incorporate the new detection efficiencies, which gives:

$$s_{\ell j} = b_{\ell j}^* + \sum_{i=1}^I \sum_{m=1}^M \sum_{n=1}^N g_{\ell j i n m}^* \theta_{i n m}^* \quad (4.158)$$

where:

$$b_{\ell j}^* = \sum_{m=1}^M \sum_{n=1}^N \sum_{k'=k}^K H_m (\eta_{\ell j k' n m}^*, \zeta_{\ell j k' n m}^*) \begin{pmatrix} 0 \\ x_{j k' n m} \\ 0 \\ \psi_{j k' n m} \end{pmatrix} \quad (4.159)$$

$$g_{\ell j i n m}^* = f_{\ell j i n m}^* + H_m \sum_{j'=1}^J \sum_{k'=k}^K \sum_{k''=1}^K v_{j k'} (\eta_{\ell j k' n m}^*, \zeta_{\ell j k' n m}^*) \begin{pmatrix} Q_{i j j' k' k''} & 0 \\ 0 & P_{i j j' k' k''} \end{pmatrix} \begin{pmatrix} x_{j' k'' n m} \\ \psi_{j' k'' n m} \end{pmatrix} \quad (4.160)$$

and

$$f_{\ell j i n m}^* = H_m \sum_{k'=k}^K v_{j k'} (\eta_{\ell j k' n m}^* \cdot p_{i j k'}) \quad (4.161)$$

Note that equation (4.160) contains the current vector (x, ψ) which relates to the particles created by interactions with all of the nuclides within a sample, rather than (x^*, ψ^*) which relates only to the isotope set (θ^*) . This result is not surprising since it follows from the basic postulate of section 4.4 which allowed us to obtain a linear general solution. However, with the present method of solution the current vector must be estimated by solving equation (4.138) rather than the corresponding equation involving η^* and ζ^* . This results in a formulation with the current vector given by equation (1.145) and this being related to the observed signals by the set of equations (1.158) to (1.161). Hence, the formulation includes both types of detection efficiency, which must be quantified by calculation or experiment. This limitation may only be considered worthwhile in practice if the change in the isotope set from (θ) to (θ^*) offers a large reduction in the number of isotopes which need be considered (which will most often be the case). Alternatively, more elegant formulations involving only one detection efficiency may be obtained by implementing the general

solution within the Bayesian framework so as to ensure stability *7, or by seeking another method of solution to the set of equations, (4.158) to (4.161), although it is not guaranteed that such a solution would be linear.

The above argument aside, the present formulation is of both practical and theoretical interest since, letting $\mathbf{x}$ denote a combination of the indices x, j, k, n , and m , equations (4.158) to (4.161) may be written in matrix form as:

$$\mathbf{s} = \mathbf{b}^* + \mathbf{E}^*[\mathbf{P} + \mathbf{Q}\Psi] \theta^* \quad (4.162)$$

where $\mathbf{E}$ is a detector efficiency matrix, $\mathbf{P}$ is a passive emission coefficient matrix, $\mathbf{Q}$ a matrix of induced reaction coefficients and Ψ is a matrix representation of the composite current densities which is arranged to give the correct row dimension to the combined reaction coefficient matrix. Then, since equation (4.146) may be written:

$$\Psi = \mathbf{E}\mathbf{H}^{-1}\mathbf{S} \quad (4.163)$$

so that (4.162) becomes:

$$\mathbf{s} = \mathbf{b}^* + \mathbf{E}^*[\mathbf{P} + \mathbf{Q}\mathbf{E}\mathbf{H}^{-1}\mathbf{S}] \theta^* \quad (4.164)$$

where $\mathbf{S}$ is a matrix of signals arranged in the same manner as Ψ , this system of equations may be solved using the general linear model as formulated in section 3.5.

It can be seen by comparison with equation (4.37), that the general solution may be regarded as a perturbation about the fixed geometry solution, where the fixed geometry solution is obtained by assuming the secondary reactions with the sample itself can be neglected.

The above result also implies that the detection efficiency matrix for the general solution can be defined in a similar manner to the fixed

*7 Although note equation (4.133).

geometry matrix, which was considered in section 4.3. However, in the general case, it is essential to consider the spatial distribution of the source along with the other measurement dependences. This may lead to large matrix which must be constructed piece by piece with each dependence of the measurement defined by a separate experiment.

Using the decomposition of the source density into its magnitude and distribution (see section 4.1), we may write:

$$\theta_{inm} = \theta_{in} f_{inm} \quad (4.165)$$

where for a measurement of the primary activity, the θ_{in} may be regarded as inference parameters while the f_{inm} are bias parameters. Since θ_{in} and f_{inm} are combined as a product, they must be estimated together. However once numerical values have been obtained, the bias parameters may be eliminated by numerical quadrature to produce a solution of the form:

$$\theta_{in} = \sum_{m=1}^M H_m \theta_{imn} \quad (4.166)$$

where the weights and abscissa values are the same as used above.

4.6 REFERENCES

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CHAPTER 5

CONCLUSION

Two general requirements for a nuclear process instrumentation system have been established. These are firstly to make decisions concerning the activity and composition of the radioactive material output from the process and secondly to form an inventory of the sequence of process states taken. These requirements introduce two further problems which are to design a data collection system so as to obtain sufficient information concerning the source to enable the reliable interpretation of the measurements, and then to design a data processing system capable of recognizing the required source features from the observed data.

The design of the data collection system amounts to the definition of a specific case of a general signal space with the dimensions of space, time and energy or multiples of these. Then to configure the radiation detectors and system electronics, which could be chosen from a collection of standard system modules, so as to construct this space in reality. To this end an examination of the standard radiation detectors together with a number of electronic data collection circuits

has been undertaken. In particular, three new circuits are presented which perform the functions of adaptive, real time spectrum stabilisation, data reduction using a digital filter and enhanced coincidence counting using a circuit which gives an increased resolution over existing circuits and a direct estimate of the Poisson distributed, "accidental" coincidence rate. It has been proposed that these, together with other already established circuits, are combined into the collection of standard modules from which the majority of nucleareonic data collection systems can be configured.

Two concepts for the interpretation of nuclear process measurements have been examined. The first, called the "naive" concept, has been rejected because it required the introduction of assumptions which could compromise the reliability of the measurements and prevented an optimum resolution being achieved. The second concept called the "axiomatic" concept avoids these problems and provides a more rigorous, analytic approach to the data processing.

The axiomatic concept is based on Bayesian Statistics where a priori information concerning the activity and isotopic composition of the nuclear process outputs, together with information concerning other factors which affect a measurement such as the spatial distribution of radionuclides within a sample, is contained in a prior probability density function. This can be used to restrict the range of values which can be taken by estimates of the activity and composition of the process outputs, to those values which are consistent with the "physics" of the processes. Furthermore, the method allows data from several different measurements to be combined so as to produce a simultaneous estimate which is consistent with all of the measurement data and makes full use of the extra information available from correlations between different measurements on the same process. Hence, leading to optimum measurement resolution.

The major criticism of the Bayesian approach is that the definition of the prior density function is subjective in the sense that this must be formed using limited information and may not accurately represent the true distribution of the nuclear process outputs. In an attempt to

overcome this problem it was proposed that the prior density could be refined using the measurement information from the samples which have been previously output from the nuclear process.

The first step is to show that the proposed technique is theoretically possible and to this end a statistical analysis has been undertaken. It was found that the sequence of nuclear process outputs can be modelled by a discrete time Markov random process and that an estimator can be constructed which is guaranteed to converge to a unique stationary distribution of the process from which the prior density can be defined. This result may be applied to any nuclear process, even if its outputs change continually in time, since the measurements are always made over finite periods and in the case of a continuous time process we may consider the cumulative activities output over each measurement period. Therefore, for the present application, it is possible to formulate an objective method for defining the prior density which overcomes the main criticism of the approach.

This works (in the present situation) because the method of defining the prior density is rooted in classical sampling theory. Therefore, the measurement interpretation method may be considered as an estimator for the activities and composition of a sample which uses appropriately weighted information from all of the samples previously encountered by the system. The net result is an interpretation algorithm which is locally Bayesian and which possesses the power of a Bayesian method, but globally classical and possesses the objectivity of a classical method. Hence, achieving the best of both approaches.

For the practical definition of the prior density it is convenient to use a Bayesian family of distributions although this restricts the number of shapes which can be generated. However, for many applications, a multivariate normal prior density may be a reasonable approximation to the true shape of the distribution and the algebraic convenience of this particular distribution family has to be examined. Furthermore, it was shown that the analysis could be extended to incorporate the case of many different sample types being encountered by the measurement system whereby each sample type is described by a

multivariate normal distribution and these are combined to construct the prior density.

It has been shown that given a mapping, g , from the parameters which describe the sample configuration to the observed signals and a conditional density function for the signals, Bayes Theorem provides a method for switching to the space on which the parameters are defined which automatically inverts g to construct a mapping, ψ , from the observed signals to the source. This observation provides a connection between the statistical theory of measurement and the deterministic models of theoretical physics, which may be interpreted as relationships on the sample space of the Markov process. Moreover, this gives a structure to the sample space which can be used to construct a system of constraints on the values which are taken by estimates of the nuclear process configuration. For the present application, the practical value of the observation lies in the fact that the mapping ψ is very difficult to define directly, whereas the function g can be constructed by using transport theory to predict the radiation flux within a detector for a given source configuration and combining this mapping with an experimental detection efficiency.

Formally, the deterministic model which relates the isotope activities within a sample to the observed signals involved the definition of two further mathematical spaces called the source space and the flux space. The required transformation is then defined as a composite mapping from the source space onto the flux space then from the flux space onto the signal space.

The first mapping (designated g_1) is constructed by using the properties of the nuclear reactions within a sample to define a generalised source term, then incorporating this into the integro-differential form of the Boltzmann transport equation to define the movement of gamma-rays and neutrons about the flux space. This requires the introduction of a set of "bias" parameters to describe the angular flux density within the sample. These parameters are eliminated by transforming the general Boltzmann equation into a pure integral equation using Fourier transform techniques. Then, expressing

the second mapping (designated g_2) as an inner product on the flux space of the angular flux with the detection efficiency function, this mapping is inverted and used to eliminate the angular flux from the integral Boltzmann equation.

A surprising consequence of the transformation of the Boltzmann equation to its integral form is that the integral over the spatial variables disappears. This is a result of the fact that the time, angular and energy variables are sufficient to define the spatial distribution of the angular flux and we are left with a general integral equation in these phase variables, which is linear with respect to both the isotope activities and the angular flux. This result may only be derived from the general time dependent form of the integro-differential equation and, in particular, it cannot be derived when the simpler and more widely used steady-state equation is taken as the starting point for the analysis.

Furthermore, the above formulation arises only when all of the nuclear reaction effects (including absorption) are incorporated into the source term. However, since the only connection between the angular flux and the sample is by the nuclear reactions, the mapping g_2 is effectively independent of the sample. A side effect of this is that a detection efficiency relating to a calibration sample, which may considerably reduce the number of isotopes which need to be considered, is different from the detection efficiency used to eliminate the fluxes, which embodies only geometrical properties and relates to an empty sample container. Therefore, introducing two detection efficiency functions to be defined by experiment or calculation. These arise because the detection efficiency for the calibration sample maps the particle production rate onto the observed signals while the second detection efficiency maps the flux which induces these reactions onto the signals. In practice this is not expected to present a serious problem since both detection efficiencies are represented by series of functions such as orthogonal polynomials and the differences between the detection efficiencies can be incorporated into the coefficient of these functions. In fact, the role of the detection efficiencies seems to be analogous to that of the contravariant and covariant vectors

within the theory of differential manifolds. It may therefore be possible to develop this correspondence at least to yield a deeper insight into the construction of the deterministic model, but also a practical method for defining the detection efficiencies from a common basis.

The motivation behind basing the model on transport theory, which leads to an equation in the angular flux, rather than diffusion theory which leads to an equation in the scalar (angular independent) flux, is that the diffusion equation is non-linear in the source density.

Furthermore, the diffusion kernel incorporates both absorption and scattering and represents the propagation of particles as the limit of a random walk, as the step length tends towards zero, on a five dimensional phase space with components space, energy and time. Therefore, the theory is only applied easily to the case of a homogeneous sample, since, otherwise, integrals over all of these sample paths must be computed for each detected particle. On the other hand the transport kernel represents only particles moving in straight lines, and if the absorption (e.g. the total interaction term) is incorporated into the source rather than the kernel, this takes a particularly simple form which can be applied to a general nuclear process. However, the cost of this approach is that the seven dimensional angular flux must now be considered.

Within the usual derivation of the diffusion equation a simplifying assumption known as Fick's Law is made. This reduces the result to an approximate equation which, in particular, breaks down near intermedium boundaries. If this derivation is carried out from first principles (e.g. from a diffusion process) then Fick's Law is unnecessary and the resulting equation will have a common solution with the Boltzmann transport equation. However, in general this derivation results in a tensor rather than a scalar equation.

The above common solution implies that it should be possible to reduce the seven dimensional scalar transport equation to a simpler five dimensional tensor equation of the same form as the general diffusion equation, except with the particles propagating in straight lines as

opposed to along the sample paths of a random walk. In fact, it was shown that this is indeed the case. Moreover, it was found that the angular dependence could be incorporated into the deterministic model using only the cosines of the mean forward, backward and orthogonal scattering and particle production angles.

The energy and time dependence of the transport equation were discretized using a multigroup approximation with the groups chosen to coincide with the partition of the signal space. The spatial dependence of the measurement was similarly discretized using numerical quadrature. Therefore, in conclusion, the deterministic model now defines a discrete transformation, g , which relates the source density to the observed signal. This can be inverted using the techniques of the general model to produce an estimate of the source density where the estimator is stabilised using the prior density function defined from the results of earlier measurements on the nuclear process, and this completes the construction of the model.

It has been emphasised that the construction of the theoretical model is only the first stage in the development of a formal method for designing the nuclear process instrumentation systems and insuring they operate in an optimum manner. The next stage is the detailed application of the model to the design and data processing of specific instruments. These may include gamma spectrometers for both fixed geometry and general applications, neutron well counters, tomography systems and active gamma and neutron interrogation systems. The implementation will include the construction of both hardware and software together with the integration of these modules to produce prototype instruments. These are then tested and the component parts optimised. With this work complete it will be possible to produce a detailed engineering specification for the unified modular system.

To apply the model to the nuclear process measurements it is necessary to produce a radiological data base containing the nuclear and atomic reaction cross-sections for the radionuclides which will be encountered by the systems.

To design the front end system geometrics it is necessary to construct a computer program to simulate the operation of the systems for various known source configurations. The program could achieve this using the Newmann series expansion of the transport equation and computing the integrals using monte-carlo techniques, to estimate the flux at the detectors for each source configuration. The system geometry is then adjusted to obtain an optimum flux for the range of sources modelled. With this development tool it may be possible to produce a standard range of system geometrics for application to particular nuclear processes.

Having obtained some practical experience with the implementation of the model it may be beneficial to undertake further theoretical work. In particular group theory and topological interpretations of the spaces and models should be undertaken as these will give greater insight into their structure. Then the general model could be extended to incorporate Bayesian hypothesis testing and also implemented with different distribution families. It may be helpful to consider the properties of these distributions and how they are related to Markov and other random processes. To extend the model to application areas other than nuclear processes a deterministic model could be constructed for the appropriate equations of theoretical physics. This could include further work with the diffusion equation, the classical wave equation, the Helmholtz equation and the Schrodinger equation, to name but a few. Once this work is completed it may be possible to give a physical interpretation and unify these models into a general theory of measurement which might be applicable to a large number of physical processes.

The final application of the theory is to the quality improvement process proposed by Crosby since modelling the development of products on a work process opens up possibilities for the description of this work in terms of a random process. Furthermore, the quality improvement process itself is an application of the same methodology since this is a work process to control a company by monitoring financial loss then modifying the inputs to the process so as to minimise the probability of the loss occurring. But now we are in

danger of encountering Russell's paradox. Alternatively we could recognise that the highest level of development will always be empirical, but then the Bayesian model emulates this process by sampling output products and using this information to modify the prior density. This is probably a good point to end the discussion. ■