
Imperial College London

Department of Mechanical Engineering

Numerical simulation of vertical intermittent flows

Francesco GALLENi, 2017

Supervised by Dr Raad Issa

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7th February 2017

Francesco Galleni

Abstract

The present research extends the application of the “slug capturing” technique, already extensively validated for the prediction of horizontal slug flow, to the case of vertical pipes. In this technique, the one-dimensional two fluid model is solved numerically in order to simulate mechanistically the growth and development of the structures which characterise the vertical intermittent flow, such as slugs and waves.

In this work the technique is first successfully applied to the prediction of vertical slug flow: it is shown here that, when the appropriate correlation for the interfacial friction forces is used, the slug capturing technique is able to simulate correctly – even in vertical configurations - the complete evolution of the slugs and to predict their key characteristics with a notable accuracy when compared against experimental data. This represents certainly the most important achievement of this work.

The one-dimensional two-fluid model is well known to be ill-posed and hence, as the mesh is refined, artificial instabilities may grow so as to render the simulation unreliable. For this reason, a Von Neumann analysis of the discretized form of the model is presented. It is shown that the discretization introduces a cut-off limit for short wavelengths, below which all the perturbations are damped. It is suggested here that this effect, for practical sizes of the mesh, is sufficient to stabilize the system.

Furthermore, this work presents preliminary results of the application of the technique to churn flow. It is shown that, although the results may be considered promising, the model still needs development.

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List of symbols

This section describes the most important symbols used in this work, if not otherwise specified.

ROMAN SYMBOLS

Symbol	Description	Dimension
h	Height of the phase	m
A	Area	m^2
D	Diameter	m
F	Frictional force per unit volume	$\frac{N}{m^3}$
G	Amplification factor	—
I, i	Imaginary Unit	—
M	Mass flux	$\frac{kg}{s}$
P	Pressure	Pa
Re	Reynolds number	—
S	Wetted perimeter	m
U	Superficial phase velocity	$\frac{m}{s}$
f	Friction factor	—
g	Gravitational acceleration	$\frac{m}{s^2}$
k	Wave number	$\frac{1}{m}$

u	Velocity	$\frac{m}{s}$
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GREEK SYMBOLS

Δt	Integration time step	s
Δx	Mesh spacing of the computational grid	m
α	Phase fraction	—
β	Angle of inclination of the pipe	rad
γ	Stratification angle in two-phase stratified flow	rad
λ	Wavelength	m
π	Pi	
ρ	Density	$\frac{kg}{m^3}$
τ	Shear stress	$\frac{N}{m^2}$
ω	Frequency	$\frac{1}{s}$
ϵ	Amplitude of the perturbation	—
ϕ	Phase angle	rad

μ	Dynamic viscosity	$Pa \cdot s$
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SUBSCRIPTS

Symbol	Description
g	Gas phase
l	Liquid phase
wl	Wall-Liquid interface
wg	Wall-Gas interface
i	Interface between phases
k	Generic phase
e	East control volume
s	Superficial velocity
w	West control volume
P	Central control volume
E	East face
W	West face
i	Staggered grid node index

SUPERSCRIPTS

n	Time step index
-----	-----------------

$\wedge$	Perturbation amplitude of specific phase
$'$	Perturbed value
$\sim$	Up-winded value
$-$	Mean value

Chapter 1

Introduction

In a multiphase flow a mixture of gas, liquids and solids flows in complex patterns, with each phase interacting with the others. Multiphase flow systems can be found in many industrial applications: such as in the hydrocarbons industry, where a mixture of oil, water, gas and possibly sand is transported in long pipelines and also in nuclear power generation where the cooling systems are based on liquid-vapour flows. Depending on the flow rates of the phases and on the geometry of the pipeline, one can observe a large variety of flow regimes; under specific conditions, each of these regimes may be particularly detrimental to the lifetime of the facility. Therefore, the ability to predict the occurrence of a specific regime and its behaviour becomes of fundamental importance, in order to improve the efficiency and safety of such facilities.

Two-phase flow systems can be considered of course the simplest among all multiphase configurations; nevertheless, even these systems are characterised by highly complex motion and interaction between the phases (Figure 1 and Figure 2). In slug flow, as shown in Figure 1b and Figure 2e, the gas phase is concentrated in large bubbles which have a size of the same order of the pipe

1.1 Present approach and contribution

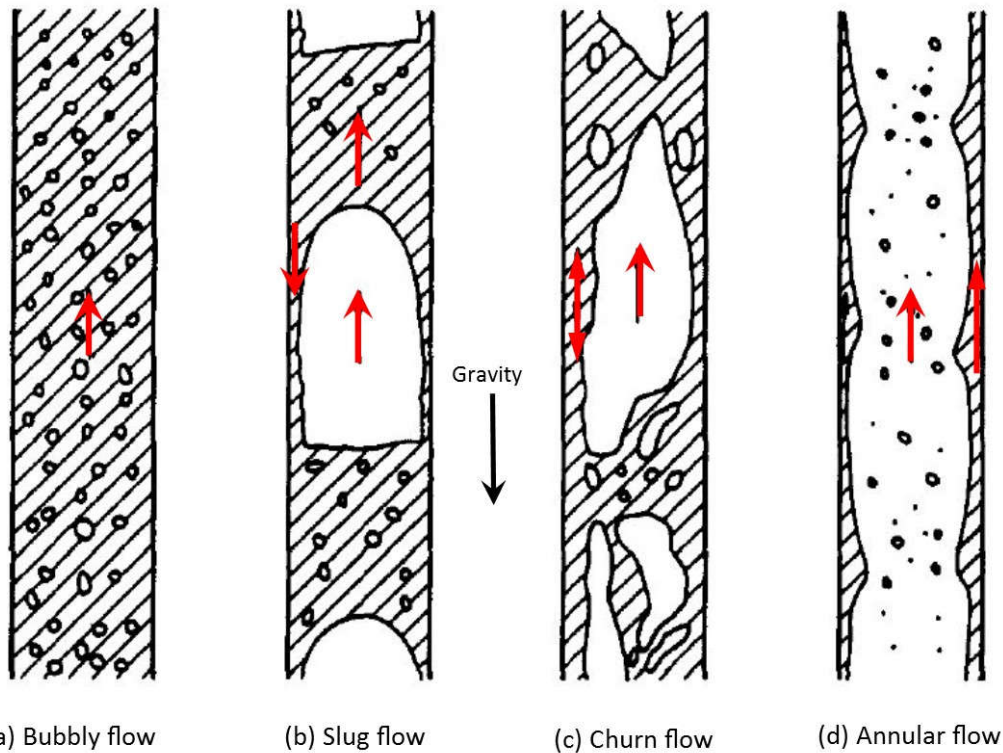


Figure 1 - Flow regimes in upwards two-phase flow in vertical pipes (red arrows indicate flow direction) - McQuillan and Whalley, (1985)

diameter. These bubbles – commonly called Taylor bubbles – are separated by slugs of liquid which may contain dispersed gas. In a vertical configuration, the Taylor bubble is characterised by a typical bullet shape with a hemispherical front and surrounded by a thin liquid film, interposed between it and the wall. The formation of these slugs and, hence, of the Taylor bubbles, is due to different mechanisms, depending on the inclination of the pipe. From vertical slug flow regime, as the gas velocity is increased, a highly chaotic flow regime develops, usually called churn flow, in which the high concentration of gas in the slugs causes the breaking of the Taylor bubbles. The gas plugs become more stretched and irregular, while the liquid phase is characterised by an oscillatory motion.

1.1 Present approach and contribution

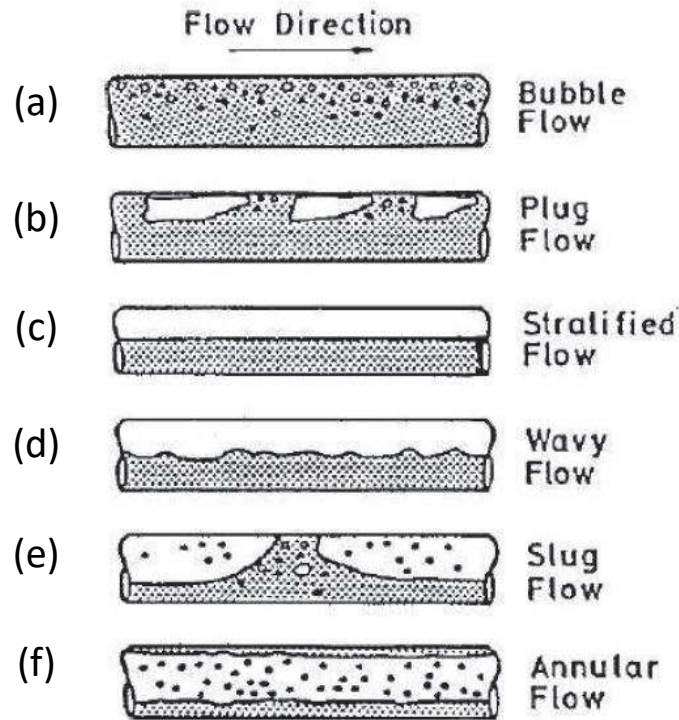


Figure 2- Two-phase flow patterns for a horizontal pipe - Bonizzi, (2003)

In addition to this complexity, the study and modelling of vertical two-phase flows, which is the focus of this work, are made more difficult by the fact that - unlike horizontal flow, where it is possible to find a “stable” stratified regime (such as the wavy and stratified flows in Figure 2d and 2c respectively) - the flow in this case is extremely dynamic and unsteady.

A large amount of research has been dedicated to the modelling and simulation of slug flow in the past thirty years; however, given the complexity described above, there is no model up till now capable of predicting the initiation and development of slug flow in the whole range of configurations (i.e. horizontal, inclined and vertical pipes) while being of practical use for engineering problems. In Chapter 2 an extensive review on the modelling of slug flow will be presented.

1.1 Present approach and contribution

In the last two decades, numerous works (Issa and Woodburn, (1998), Issa and Kempf, (2003), Issa et al., (2006), Kjølås et al., (2013), Shaposhnikov et al., (2015), among others) were carried out to develop the capability of the “slug capturing” technique (based on the numerical solution of the transient one-dimensional two fluid model) for the prediction of slug flow in horizontal and slightly inclined pipes. Numerous computations proved the model to be remarkably accurate in the prediction of the characteristics of slug.

The present thesis describes the progress accomplished in the extension of the “slug capturing” technique to the case of vertical pipes. The starting point of the project was the code called TRIOMPH, developed by Issa and Abrishami, (1986), which solves numerically the discretised form of the one-dimensional two fluid model; as stated above, this code has been already successfully tested for the slug capturing methodology in horizontal flow; an early attempt to extend the method to vertical slug flow concluded that the model was able to reproduce the initiation of the slugs, but wrongly predicted their growth and development and did not yield to results comparable with experimental data, particularly regarding the frequency of the slugs.

The most important achievement of this work is the demonstration that, when the proper correlation for the interfacial friction factor is used, the slug capturing technique has the capability of fully characterise vertical slug flow, simulating the complete development of the slugs - starting from their generation to a semi-stable state (or collapse) - and predicting their main characteristics with a remarkable accuracy when compared against experimental data; in this work the

1.1 Present approach and contribution

predicted characteristics for vertical slug flow, such as frequency and velocity of the slugs and averaged liquid fraction in the pipe, have been validated against experimental data acquired in several experiments performed Escrig Escrig, (2017) at the Chemical Engineering Laboratory of the Nottingham University; a total of 49 experiments were carried out, with flow regimes ranging from bubbly to churn flow. Additionally, the study has been extended to experimental data found in literature, in order to assess the capability of the model in predicting the axial development of slug flow characteristics (Waltrich et al., (2013)) and in simulating different geometrical configurations (Sawai et al., (2004)).

The one dimensional two-fluid model is known to be unconditionally ill-posed in the vertical case, hence this application may raise serious mathematical issues. From a numerical prospective, in the case of an ill-posed system of equations, the artificial perturbations generated by round-off errors may grow unboundedly as the mesh is refined spoiling the solution and making it impossible to reach mesh independence.

A major objective of this work was therefore to study the applicability of this extension, both from a more practical level, validating the predictions against available data from existing literature as well as from more recent experiments, and from a theoretical aspect, analysing the issue of the ill-posedness in the specific vertical case. In fact, as already stated above, the slug capturing technique applied to horizontal cases performs fairly well in the unstable but well-posed region, whereas in the ill-posed region as the mesh is refined the predicted slug frequencies grow exponentially, clearly corrupted by the unbounded growth of spurious instabilities (Issa and Kempf, (2003)). However,

1.1 Present approach and contribution

a mesh sensitivity analysis carried out in this work showed that the predicted frequencies are not affected by the growth of numerical instabilities as the mesh is refined, at least for practical sizes of the mesh and this finding marks an important difference from horizontal ill-posed cases. It is suggested here that the difference with the horizontal case is most likely due to the combination of two factors, namely the different mechanism of slug formation in vertical geometries and the damping effect generated by the numerical diffusion introduced in the system by the differencing scheme. To support this hypothesis, in this thesis a Von Neumann analysis of the discretized form of the one-dimensional two fluid model is presented for slug flow in vertical pipes in order to study the effect of the discretization scheme on the ill-posedness of the system. The resulting growth rate is compared to that obtained from a stability analysis of the parent system of differential equations.

Finally, a secondary objective of the project was to provide a wide basis for further investigations and improvement of the model in the extended intermittent flow region (i.e. slug and churn flow regimes together). First, the capability of the model in mechanistically predicting the transition between slug and churn flow was tested, by simulating experimental cases located in the transition region. Second, a thorough investigation into the performance of the model, in several cases of fully developed churn flow was carried out by comparing predictions with available data in the literature. It was found that the numerical results are, as for the slug flow regime, strongly dependent on the interfacial friction factors chosen to close the model and that the correlations tested in this work were not accurate enough to fully describe the physics behind the development of churn flow. Therefore, although the results were not entirely

satisfactory, the present work provides an important starting point for the extension of the slug capturing technique to the churn flow regime.

1.2 Outline of the thesis

In Chapter 2 a general review of the modelling of slug flow is presented, with particular focus put on the differences between slug flow in horizontal and vertical configurations and hence on the challenges that these differences have in the modelling of vertical slug flow. Also, a review of the available literature on the characteristics and stability analysis of the two fluid model is presented, with particular attention on the issue of ill-posedness and its effect on the two fluid model and the “slug capturing” technique. Furthermore, the main characteristics of churn flow are discussed.

Chapter 3 gives an overview of the “slug capturing” methodology, by presenting the system of equations for the transient one dimensional two-fluid model; geometrical assumptions and closure relations for the shear stresses are also presented. The second section of the chapter is dedicated to the mathematical analysis of the one dimensional two-fluid model: a brief introduction on the characteristic analysis and an extensive discussion of the linear stability analysis of the model are presented,

Chapter 4 details the numerical techniques that are applied in the TRIOMPH code for the discretization of the one dimensional two-fluid model. Furthermore, the effect of the discretization on the stability of the model is discussed, by presenting the results of the Von Neumann analysis of the discretised form of the system and comparing them with the differential analysis.

1.2 Outline of the thesis

Chapter 5 presents the results for the simulation of vertical slug flow; the experimental test cases and the rig setup are described. A qualitative description of the numerical results is first introduced and then a quantitative comparison is made between the predicted slug characteristics and the experimental data. A mesh sensitivity analysis is also presented in the chapter.

Chapter 6 discusses the model for the simulation of churn flow and presents, preliminary results from numerical prediction of churn flow cases as compared with data available in literature.

The last chapter presents the conclusions together with suggestions and recommendations for future work.

Chapter 2

Overview of two phase intermittent flow

2.1 Preamble

As briefly mentioned in Chapter 1 , despite the considerable amount of work – both experimental and numerical - dedicated to slug flow regime, there is not yet a simple model capable of fully characterise the initiation and evolution of the flow in vertical geometries. The complexity in such prediction comes from several characteristics of the flow: it is highly intermittent, irregular and, depending on the inclination of the pipe, it can also be unstable. Furthermore, in the case of vertical slug flow, the initiation process is still not completely understood and therefore difficult to model.

In Section 2.2 a review of the main characteristics of slug flow in horizontal and vertical inclinations is presented, with a particular emphasis on the

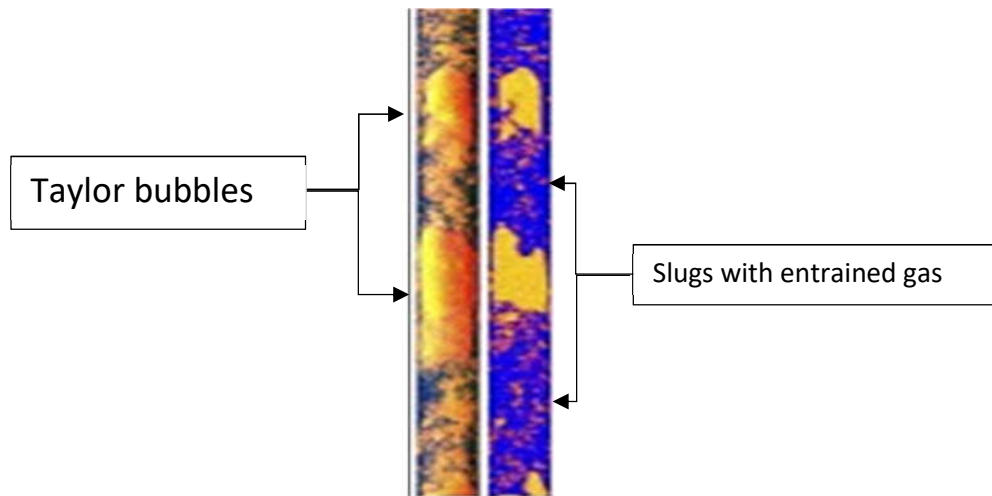


Figure 3- Visualization of vertical slug flow via wire-mesh sensor (Kaji et al, 2009)

differences in the generation of the slugs between the two configurations. Furthermore, in this section, the most important models and techniques for the prediction of slug flow are described. Particularly, the results obtained with the slug capturing technique for horizontal flow are outlined.

Section 2.3 is dedicated to churn flow, another type of intermittent flow, characterised by an even more chaotic behaviour; preliminary results for the simulation of churn flow will be presented in Chapter 5.

2.2 Slug flow

2.2.1 General characteristics

Assuming that slugs have generated in the pipe and they have reached a “stable” condition, a slug unit is generally assumed to be composed of a liquid slug region - where the pipe is filled with aerated liquid - and of a liquid film region, in which the two phases are separated, with the gas phase in horizontal flow occupying the upper part of the pipe. The front and tail of the slugs may move along the

2.2 Slug flow

pipe at different velocities, causing the length of the slug to vary; however, if the slug length has reached an equilibrium state, the propagation velocities of the tail and front must be the same.

In vertical slug flow a stable slug unit consists of a large bullet shaped bubble which fills almost entirely the pipe (Figure 1 and Figure 3); these so called Taylor bubbles are surrounded by a thin liquid film, interposed between them and the wall; in general, the liquid film falls downwards and is recouped by the following slug. Immediately after the Taylor bubble a short region of high turbulent mixing is found (wake region); after this region, in between two bubbles, the pipe is filled with liquid, usually containing entrained gas.

Besides these common generalisations of the configuration of slug flow, one of the most important features which differentiates horizontal and vertical regimes is the mechanism that is behind the formation of slugs and their subsequent development; these issues are often separated when considering slug flow modelling but, as it will be remarked in the following sections, they remain crucial to fully characterise this regime, particularly in vertical slug flow. As a matter of fact, due to the complexity of the flow evolution, the assumption of fully stable vertical slug flow is no longer accepted.

2.2 Slug flow

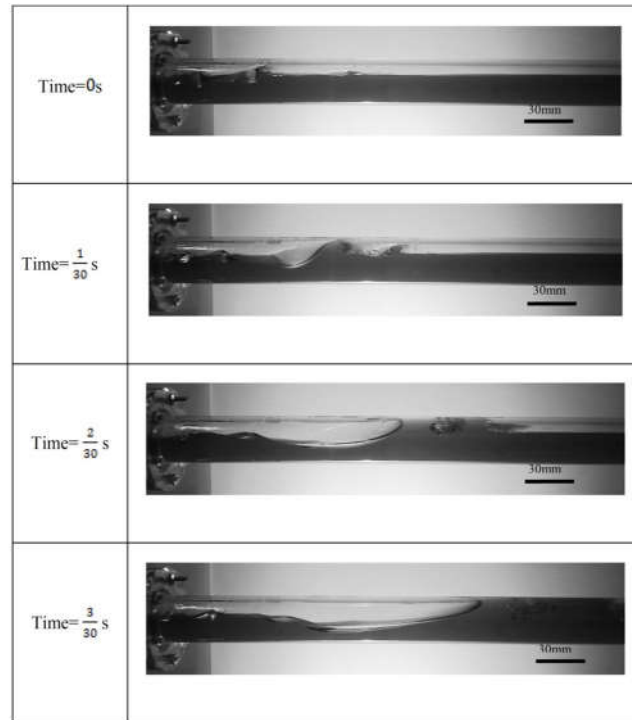


Figure 4 - Slug generation in horizontal pipe (Lu, 2015)

2.2.2 Slug generation and development

Horizontal slug flow

In horizontal geometries it is widely accepted that slug flow is generated by the natural growth of hydrodynamic instabilities – i.e. Kelvin-Helmholtz instability - (Lin and Hanratty, (1986), Fan et al., (1993), Barnea and Taitel, (1994); Ansari, (1998)): starting from a stratified flow condition, if a perturbation of infinitesimal amplitude is generated at the interface between the liquid and the gas, as the gas phase accelerates over the liquid crest, the pressure diminishes because of the Bernoulli effect. If the pressure drop is sufficient to counteract the stabilising effect of gravity, then the wave may grow until it bridges the pipe cross-section, thus forming a slug (Figure 4).

2.2 Slug flow

Applying the Kelvin Helmholtz instability theory, several attempts were made in the past to identify suitable mathematical expressions capable of predicting the transition between stratified and slug flow. Taitel and Dukler, (1976) proposed a mechanistic model derived from a simple extension of the KH theory; they expressed the condition for transition for a circular pipe as:

$$(u_g - u_l) \geq \left(1 - \frac{h_l}{D}\right) \sqrt{\left(\frac{g(\rho_l - \rho_g)A_g}{\rho_g dA_l/dh_l}\right)} \quad (1.1)$$

Lin and Hanratty, (1986) carried out a stability analysis of the viscous form of the one dimensional two fluid model and derived for the first time the Viscous Kelvin Helmholtz limit. Subsequently, Barnea and Taitel, (1993,1994), following a similar approach, found a different expression for the VKH limit and then compared the predictions obtained with both the VKH and IKH (inviscid) criteria: by disturbing the equilibrium level introducing a perturbation of the form $\epsilon \exp[i(\omega t - kx)]$ with frequency ω , wavenumber k and amplitude ϵ their criterion is eventually expressed as:

$$(u_g - u_l) \geq K \sqrt{(\rho_l \alpha_g + \rho_g \alpha_l) \frac{(\rho_l - \rho_g)}{\rho_l \rho_g} g \cos(\beta) \frac{A}{\frac{dA_l}{dh_l}}}, \quad (1.2)$$

where $K = 1$ for the inviscid case, and for the viscous case:

$$K = \sqrt{1 - \frac{(C_v - C_{iv})^2}{\frac{\rho_l - \rho_g}{\alpha_l + \frac{\rho_g}{\alpha_g}} g \cos(\beta) \frac{A}{\frac{dA_l}{dh_l}}}}, \quad (1.3)$$

2.2 Slug flow

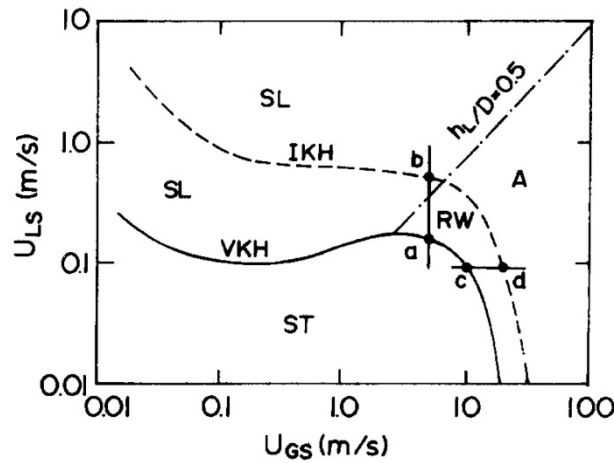


Figure 5 - Flow pattern prediction by the VKH and IKH analyses. Air water, atmospheric pressure, horizontal pipe, $D_i=5\text{cm}$ - SL=slug; ST=stratified; A=annular – Barnea and Taitel (1993)

where C_v and C_{iv} are the critical wave velocities ($C = \omega/k$) at the onset of the instability for the viscous and inviscid analysis respectively. Comparing the curves given by the two limits (Figure 5) with experimental flow patterns, they observed that the two analyses gave different limits: the region below the neutral stability line of the VKH analysis corresponds to a region where the damping effect due to gravity dominates and therefore waves decay and the flow is characterised by a stable stratified regime, whilst in the region in-between the two limits the small perturbations, naturally present in the flow, tend to grow with low amplification and hence they can either reach an equilibrium state (wavy stable flow) or they can bridge the pipe, forming slugs; in the region above the IKH limit, where the wave amplification factor is high, the flow regimes can only be slug flow or annular flow for very high gas velocities.

The flow pattern maps derived from the expressions (Figure 5) have proved to be remarkably accurate (although of limited use), indirectly confirming the KH instability theory as the correct mechanisms behind the formation of slugs.

Vertical slug flow

On the other hand, the situation for vertical slug flow appears to be more complex: firstly, the phenomena that lead to the formation of slugs and Taylor bubbles in vertical pipes are still far from being completely understood; secondly, the classic assumption of a fully developed and stable flow after a well determined length seems to be highly-questionable.

It has been observed that above certain gas velocities, depending on the geometry of the pipes, the small bubbles in bubbly flow begin to coalesce until they almost completely fill the pipe (Radovcich and Moissis, (1962); Griffith and Snyder, (1964); Hasan and Kabir, (1988)), eventually forming Taylor bubbles; this mechanism is more likely triggered by the formation of void waves, which leads to very high local concentrations of the void fraction and hence strongly facilitate the coalescence of the small bubbles (Pickering et al. (2001)); theoretical and experimental works on this subject can be found in Saiz-Jabardo and Bouré, (1989); Song et al., (1995); Cheng et al., (2002), among others.

However, the early suggestion that the onset of vertical slug flow would only result from an instability of bubbly flow is not always accepted (Fabre and Liné, (1992)). In fact, Dukler and Taitel, (1986) reported experimental observations in which they describe slug flow being initiated by entrance conditions similar to churn flow (following the suggestion of Taitel et al. (1980)): at the inlet, after a short region of highly chaotic motion, the two phases begin forming short slugs and Taylor bubbles; since short liquid slugs are unstable, they collapse and fall back, merging with the liquid coming from the bottom; while the slug collapses, the following Taylor bubble coalesces with the leading bubble; the repetition of

2.2 Slug flow

this process along the pipe leads to longer slugs and bubbles and eventually to a region of stable slug flow.

Once stable, independently of the initiation process, the Taylor bubbles rise faster than the liquid and tend to push it upward but, since the interfacial forces are not sufficiently high to counteract the gravitational forces, some of the liquid falls down the wall of the pipe and merges with the subsequent slug (Kaji et al., (2009)). Dukler and Taitel, (1986) also proposed a formula to evaluate the entrance length l_e necessary to reach stable slugs:

$$\frac{l_e}{D} = 42.6 \left(\frac{U_m}{\sqrt{gD}} + 0.29 \right) \quad (1.4)$$

However, the length and characteristics of the development region from a churn-like flow have been analysed by recent experimental works (Van Hout et al., (2001); Kaji et al., (2009); Waltrich et al., (2013)). In all the experimental campaigns the liquid slug and Taylor bubble lengths were seen increasing with the pipe length whilst the frequency of the slugs decreased, indirectly confirming the mechanism described above. It has to be remarked that Van Hout et al., (2001) and Kaji et al., (2009) found that the slug frequency kept decreasing towards the end of the test section, suggesting that the flow was still not completely developed. However, it might be argued that their rigs (with length to diameter ratio ranging from 68 to 416) were not long enough to allow a full development of the flow. Nevertheless, the work of Waltrich et al., (2013) was particularly significant because they performed experiments on a 42 m long pipe with a diameter of 4.8 cm (corresponding to a length to diameter ratio of 875)

2.2 Slug flow

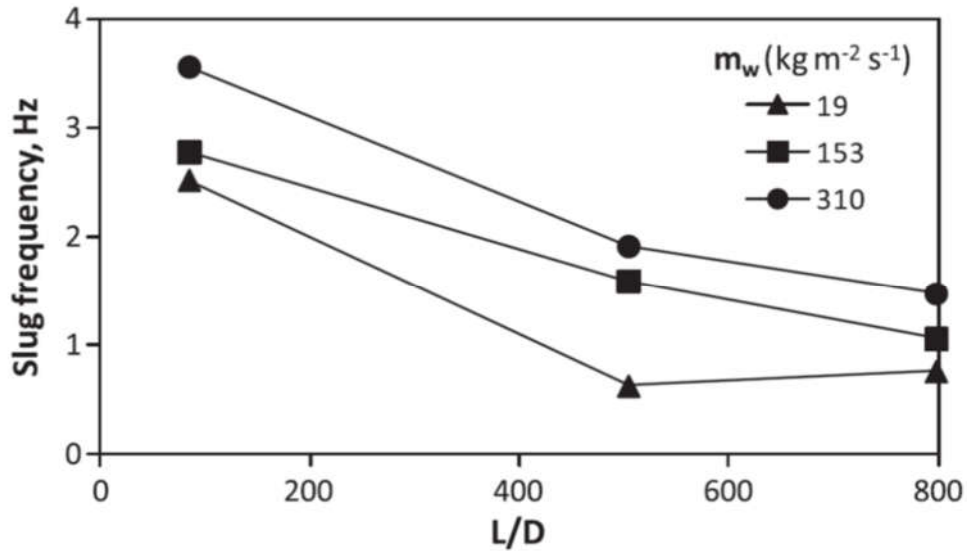


Figure 6 - Slug frequency as a function of axial position in vertical slug flow (Waltrich et al. 2013)

and they found that, at the low gas flow rate at which slug flow was observed, the axial variation of the slug frequency was appreciable and the flow kept developing even after 800 diameters (Figure 6); they attributed these phenomena to the variation of the gas velocity along the pipe.

All these observations confirmed that the assumptions commonly made in the major multiphase models of fully developed, periodic and stable slug flow are not really justified (as already stated by Dukler and Fabre, (1994)) and so the applicability of single slug unit models to properly predict the occurrence and development of the flow is seriously flawed.

2.2.3 Modelling

The first studies on vertical slug flow focused mainly on the dynamics of single Taylor bubbles rising in stagnant and, later, flowing liquid and were limited to

2.2 Slug flow

simple correlations for the calculation of slug properties, such as velocity and pressure drop (Wallis, (1969)). An important example of these correlations is the first reliable relation for the velocity of the Taylor bubbles u_{TB} in a flowing liquid, proposed by Nicklin (1962), and it is a combination of the liquid velocity (u_f) and the bubble velocity in stagnant conditions (u_0):

$$u_{TB} = C_1 u_f + u_0 \quad (1.5)$$

These early approaches led to the idea of focusing only on a system formed by one bubble and one or two liquid slugs, in order to simplify the dynamics studied.

Indeed, as mentioned above, one of the simplest and most common approaches in slug modelling is to ignore the physical phenomena behind the formation and development of Taylor bubbles and slugs and rely on the so called “unit cell” system (Dukler and Fabre, (1994)): a simple control volume based on the assumptions presented in section 2.1, hence formed by one long bubble with a previously assumed shape (surrounded by the liquid film, if appropriate), and one liquid slug; therefore, the slug characteristics are considered periodic and do not vary in time (so called “steady-state” models). The idea of “unit cell” (Figure 7) was introduced by Wallis, (1969) and used first for the study of horizontal flow by Dukler and Hubbard, (1975) and then extended to vertical slug flow by Fernandes et al., (1983). As already remarked, the assumptions of these steady state models are not always strongly justified, especially for vertical flow; furthermore, these models were not capable of capturing transient phenomena, such as changes in flow rates and transitions between different regimes.

2.2 Slug flow

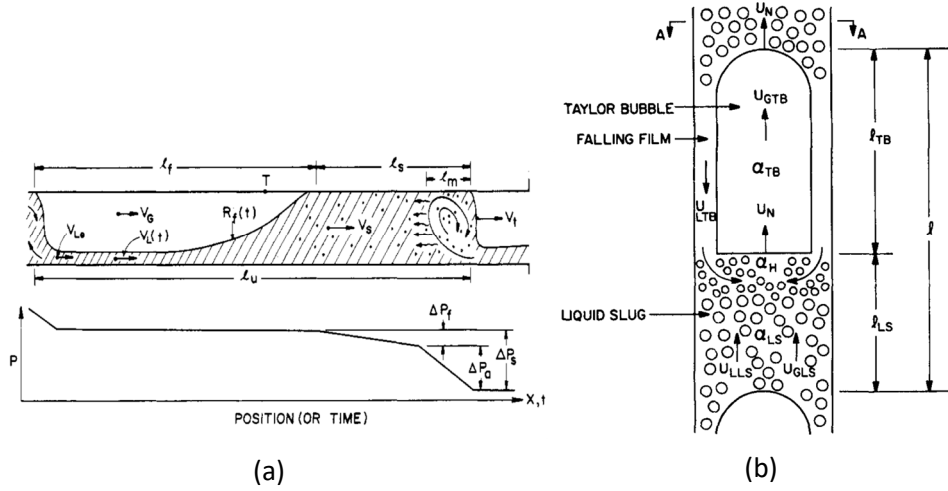


Figure 7 - Unit cells by Dukler and Hubbard (1975) and Fernandes et al. (1983)

Therefore, in order to account for these phenomena, different general transient flow simulation methods have been developed and commercialised. Among others, it is worth mentioning the computer code TACITE (Pauchon et al., (1993)), and OLGA (Bendiksen et al., (1991)). The former is a mechanistic transient code based on an improvement of the “unit cell model” by Fabre and Liné, (1992) - called “statistical cellular model” – which solves only one momentum equation for the gas and liquid mixture (drift-flux model, Zuber and Findlay, (1965)). The OLGA code solves continuity and momentum equations for each phase, instead. These equations require closure models in order to be solved: in addition to the common closure relations for friction factors, a closure law is needed for the velocity of the tail of the bubble and another one for the slug body voidage.

Although these models could acceptably predict the transport of slugs along the pipe even in transient conditions, in order to correctly select the required closure laws they need to assume the initiation of specific regimes according to

2.2 Slug flow

the occurrence of predetermined conditions and therefore they still cannot capture automatically the initiation of slug flow and its consequent development; as already stated above, while these shortcomings may have a lesser impact in simulations of long horizontal pipelines, they do not seem to be negligible for vertical slug flow, due to its highly dynamic nature.

2.2.4 Slug tracking

Another important approach to slug modelling is the so called “slug tracking” model (among others Nydal and Banerjee, (1996); Taitel and Barnea, (1998)); this model is based on the idea of following the development of each single slug, solving equations which describe the pickup process at slug front and the shedding at the slug tail. Slug tracking models are normally implemented in numerical codes using a Lagrangian frame of reference in order to follow the time evolution of the slugs. These codes are claimed to be capable of handling complex systems of long pipelines giving good predictions for the main characteristics of slug flow; the main advantage of the slug tracking method is the possibility to use fairly coarse meshes which might significantly reduce simulation times. Nevertheless, a major shortcoming of using coarse meshes is the loss of the physics related to slug generation; for this reason, slug tracking simulators need to assume the initiation of a fixed slug distribution in the pipe using empirical relations or user inputs (Kjølaas et al., (2013)), hence abandoning any attempt to simulate the initiation of slugs mechanistically.

2.2.5 Multi-dimensional simulations

Given the complexity of the physical phenomena involved it may seem natural

2.2 Slug flow

to use multidimensional numerical computations to properly predict the characteristics of vertical slug flow. In a series of papers, Mao and Dukler, (1989, 1990, 1991) developed and validated a numerical model to predict the shape and velocity of a Taylor bubble in stagnant and flowing liquid. In the early 1990's several researchers started using the VOF method (Volume of Fluid, Hirt and Nichols, (1981)) to simulate vertical slug flow in pipes and Tomiyama et al., (1993) were the first to apply it to the analysis of bubble motion; they showed that the method could correctly predict the main characteristics of a Taylor bubble under different conditions. Their work was followed by those of Kawaji et al., (1997), Bugg et al., (1998) and Bugg and Saad, (2002) among others; all these works confirmed the VOF method to be capable of predicting the shape of the Taylor bubble. Clarke and Issa, (1997) used a finite volume numerical method that predicts the shape of the gas-liquid interface by attaching the mesh to it, for the investigation of the flow field around the Taylor bubble whilst Taha and Cui, (2006) were the first to use a commercial CFD software to simulate the motion of single Taylor bubbles in vertical tubes; they performed a three dimensional simulation without assuming any shape for the bubble. They found their predicted shape and velocity to be in good agreement with experimental data and, furthermore, they obtained a good reproduction of the highly agitated mixing region in the wake of the Taylor bubble (Figure 8).

It is clear from the literature that the majority of the studies were only focused on the prediction of the shape and motion of single or consecutive Taylor bubbles and, where possible, their interaction with the liquid slugs; in addition, most of the numerical computations assumed already formed structures and used a moving frame of reference. To the author's knowledge, few works have been

2.2 Slug flow

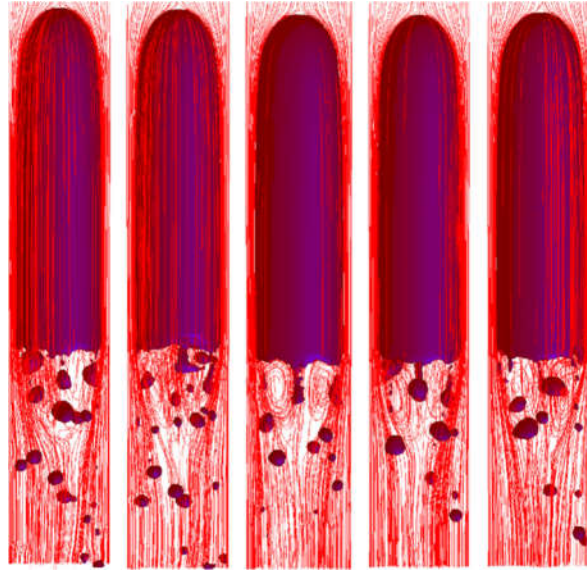


Figure 8 - Progression of wake flow pattern behind a Taylor bubble obtained with 3D CFD simulations Taha and Cui, (2006)

dedicated to the mechanistic simulation of complete systems including slug initiation, growth and subsequent continuous slug generation and movement; Abdulkadir et al., (2015), using a commercial code, performed CFD simulations for a 6 m vertical pipe and compared the numerical results with experimental data; although they found good agreement between the experiments and CFD, they were only able to simulate a period of 16 seconds (compared with 60 seconds of experimental measurements).

Even though the works listed above proved that numerical computations or CFD could be powerful tools for the understanding of the physics behind the dynamics of slug flow, it is clear that they still remain impractical for computations in long pipelines or complete experimental rigs.

2.2.6 Slug capturing

Horizontal flow

In the last decade, a new “slug capturing” numerical method utilising the one-dimensional two-fluid model (Issa and Woodburn, (1998), Issa and Kempf, (2003)) was developed to simulate the initiation, development and movement of trains of slugs in horizontal pipes. Numerous works (Bonizzi and Issa, (2003)(a), (2003)(b); Issa and Kempf, (2003); Montini and Issa, (2010)) were carried out to develop and extend the technique to include three phase flow and gas entrainment into slugs.

In slug capturing, as opposed to slug tracking, it is not necessary to supply an initial slug distribution and no external triggers are needed to initiate slug formation. In this technique the slug flow regime is predicted as a mechanistic and automatic outcome of the growth of physical instabilities. For instance, in horizontal flow, the inception of slugs from stratified condition are due to disturbances in the solution constantly generated by round-off and discretisation errors: these disturbances have the same effect of natural perturbations in the real flow, hence initiating the hydrodynamic instabilities which lead to the formation of slugs.

All the regimes are modelled with the same set of equations (that is, the one-dimensional transient two-fluid model) and closure laws, without the necessity of empirical flow regime models. Hence, the simulated liquid volume fraction can grow and become unity (Figure 9), i.e. forming a slug, from any kind of initial configuration and as a natural outcome of the numerical solution of the system of equations. Furthermore, the development of the slugs depends only on the

2.2 Slug flow

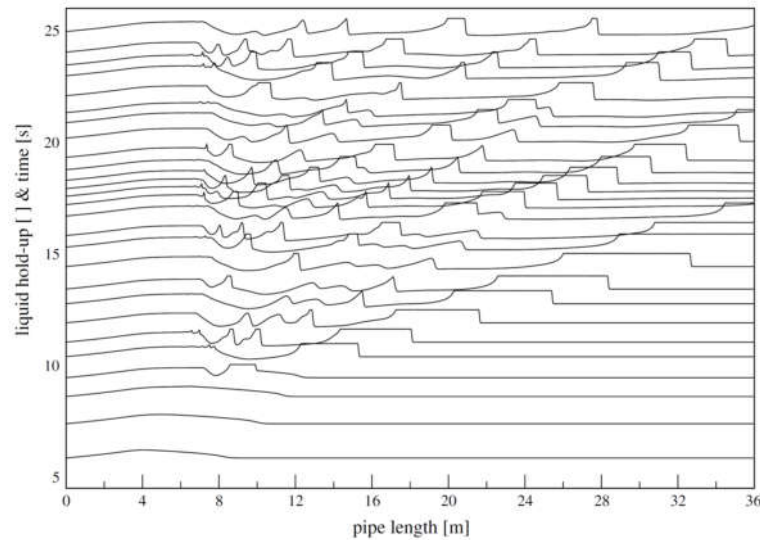


Figure 9 – Mechanistic generation of slugs in horizontal flow with the slug capturing method Issa and Kempf (2003)

solution of the mass and momentum transport equations. The only empirical closure relations needed are those describing the liquid-wall, gas-wall and interfacial shear forces. Numerous computations proved the model to be remarkably accurate in the prediction of the characteristics of slug flow in horizontal and slightly inclined pipes (Figure 10).

Among other works it is worth mentioning that of Renault, (2007) which combined the slug capturing and slug tracking techniques using a Lagrangian grid and obtained good agreement between the numerical predictions and experimental data. Furthermore, the slug capturing method has been implemented in the recently developed LedaFlow commercial software; Danielson et al., (2012) presented a comparison between the predictions obtained with the slug capturing technique implemented in LedaFlow and the “slug tracking” technique employed in the software OLGA: the authors remarked that, since a finer mesh was necessary, the slug capturing simulation resulted

2.3 Churn flow

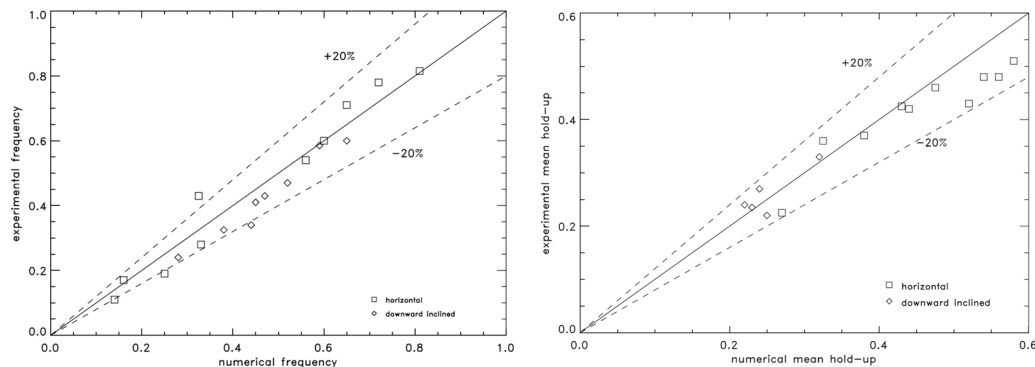


Figure 10 - Prediction of slug flow characteristics in horizontal and inclined pipes with the slug capturing technique - Issa and Kempf (2003)

in longer simulation time but they concluded that the predictions obtained with the slug capturing technique were more accurate than those obtained with the slug tracking methodology. Kjølås et al., (2013) presented a more comprehensive analysis of the performance of the technique as implemented in LedaFlow, concluding that the slug capturing model was able to reproduce well the experimental data both for a large range of flow rates, although, they noted, at high flow rates the numerical solution diverged from the experimental data as the mesh size was decreased. They attributed this effect to the ill-posedness of the equations.

The two-fluid model (and the issues related to the ill-posedness of the system of equations) will be extensively discussed in the Chapter 3 .

2.3 Churn flow

Despite the fact that churn flow occurs over a wide variety of flow conditions, especially in large diameter configurations, it remains probably the least understood of the flow regimes (Azzopardi, 2006). Confusion in the past has

2.3 Churn flow

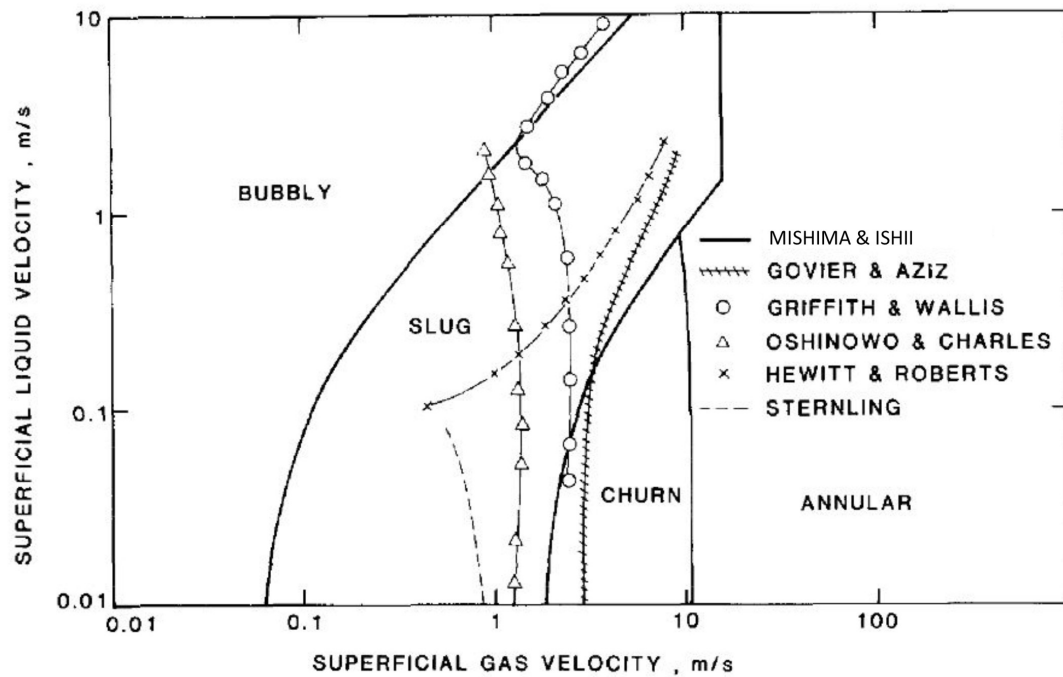


Figure 11- Comparison of slug to churn transition criteria for air-water flow – From Mishima and Ishii, (1984)

been caused by the fact that the term ‘churn’ has been used to describe different conditions in vertical gas liquid flows. First, Zuber and Findlay, (1965) described a “churn-turbulent” bubbly flow regime and later, Taitel et al., (1980) identified churn flow essentially as a developing slug flow; nevertheless, the most accepted definition in recent literature (and the one adopted throughout this work) is the one given by Hewitt and Hall-Taylor, (1970), who described churn flow as an intermediate regime of highly chaotic motion between the regions of slug and annular flow. It is worth noticing, however, that this last definition cannot hold for large diameter pipes, where there is a direct transition from bubbly to churn flow regime (Cheng et al., (1998); Ohnuki and Akimoto, (2000)).

Due to this lack of knowledge, the number of models for fully developed churn flow is extremely limited and most of the literature in the last thirty years is

2.3 Churn flow

focused on experimental studies (see Azzopardi, (2006) for a review of the experimental work) or on the modelling of single physical mechanisms, such as the evolution of the waves on the liquid film (see, for instance, Barbosa et al., (2001); Da Riva and Del Col, (2009)); also, in the 1980s a significant effort has been dedicated to the development of general models for the prediction of the slug to churn flow transition (an extensive review can be found in Jayanti and Brauner, (1994)).

2.3.1 General characteristics

Churn flow is characterised by the presence of a continuous gas core - possibly carrying drops of entrained liquid - and a relatively thick liquid film attached to the wall (Azzopardi, (2006)); the wall film is covered by large waves. Hewitt et al., (1985) observed that the base film flowed mainly downwards, whilst the intermittent large waves swept the liquid upwards, resulting in a net upward flux. As the gas velocity is increased, the occurrence of negative liquid velocity reduces until the region of annular flow is reached. Sawai et al., (2004) confirmed

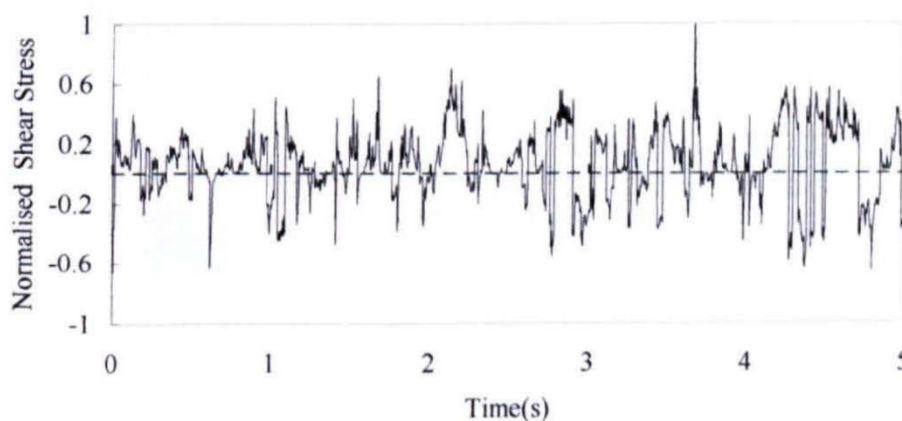


Figure 12 - Normalised wall shear stresses in a churn flow case by Zangana (2011)

this observation; in their work they calculated indirectly the frictional component of the averaged pressure drop, by means of the measured void fraction, and found it to be negative for low gas velocities. A more direct observation has also been carried out by Zangana, (2011) who measured the local and instantaneous values of the wall shear stresses in the occurrence of churn flow and found them fluctuating between positive and negative values (Figure 12).

2.3.2 Churn flow identification

Given the confusion in the classification of churn flow, there is not yet a complete agreement on how to clearly distinguish the boundaries between slug, churn and annular flow; depending on the mechanism identified as the cause of the transitions, several criteria were proposed in the literature (Jayanti and Brauner, (1994) and Figure 11) but none of them has proved to be of general use. Waltrich et al., (2013), presented a comparison of recent experimental maps with different transition models for predicting slug to churn and churn to annular transitions. As can be seen from Figure 13 and Figure 14, they found that, for the transition from slug to churn flow regimes, the transition models which gave better agreement for low outlet pressures (1.4 bar, Figure 13) and lower gas superficial velocities (below 0.3 m/s) were those of Brauner and Barnea, (1986) and Jayanti and Hewitt, (1992). However, these models did not show a good match with the experimental data at higher pressure (3.8 bar, Figure 14); in this case, the transition is better predicted by the map of Hewitt and Roberts, (1969). For the churn to annular flow transition the model of Wallis, (1969) showed good agreement with the experimental data, whilst the criterion of Turner et al., (1969) gave good match only for high superficial velocities.

2.3 Churn flow

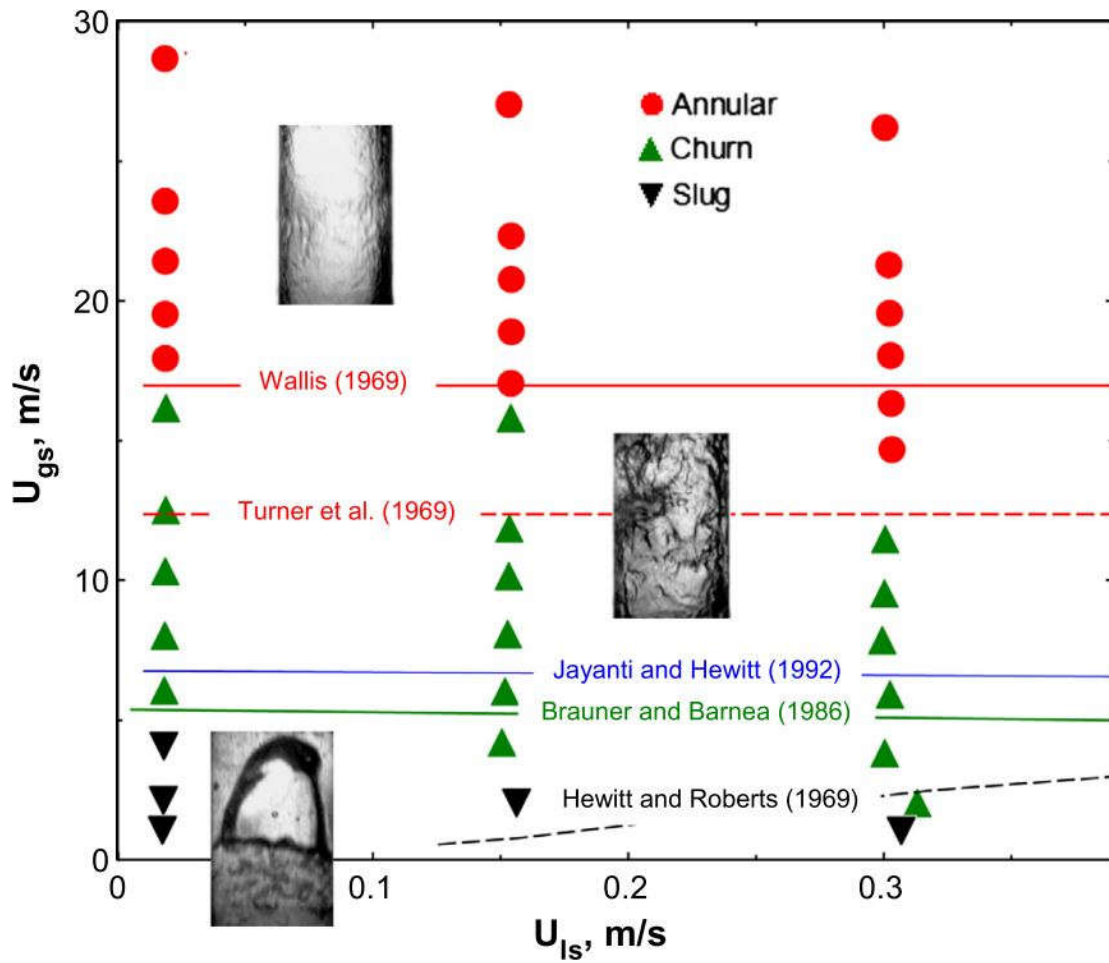


Figure 13 - Comparison between experimental observations and flow regime transition models for vertical air-water flow - Outlet pressure: 1.4 bar - From Waltrich et al, (2013)

However, it is worth mentioning the two most common methods used to identify the occurrence of churn flow in experimental campaigns. In Figure 15a the classification made by Owen, (1986), according to the distribution of the pressure gradient against the gas velocity is shown. The churn flow region is identified as the region where the pressure gradient shows a positive and then a negative gradient as the gas velocity is increased. The second method (Figure 15b), commonly used also for the identification of the slug flow regime, is based on the PDF of the liquid fraction (Costigan and Whalley, (1997)): churn flow is

2.3 Churn flow

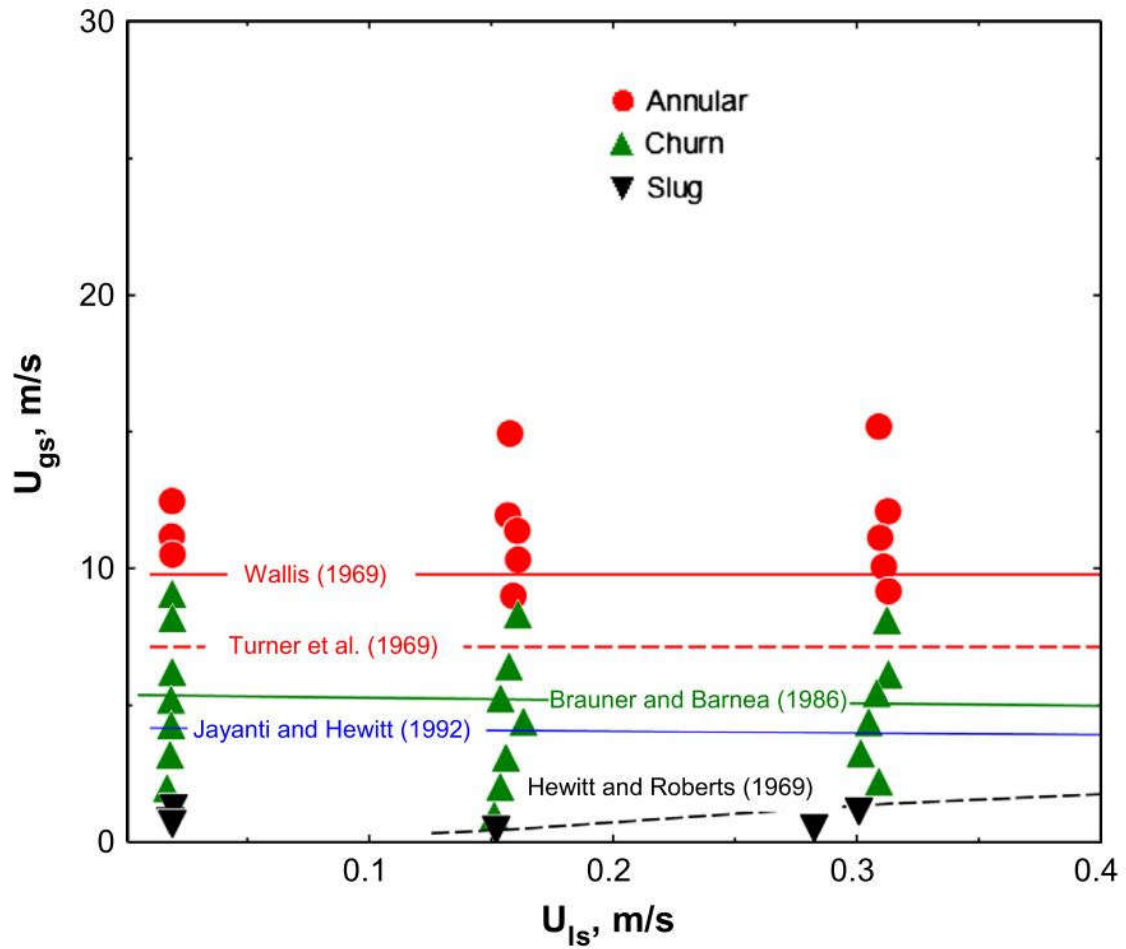


Figure 14- Comparison between experimental observations and flow regime transition models for vertical air-water flow - Outlet pressure: 3.8 bar - From Waltrich et al, (2013)

characterised by a single peak, like annular flow, but with a larger area, indicating a higher fraction of liquid.

2.3 Churn flow

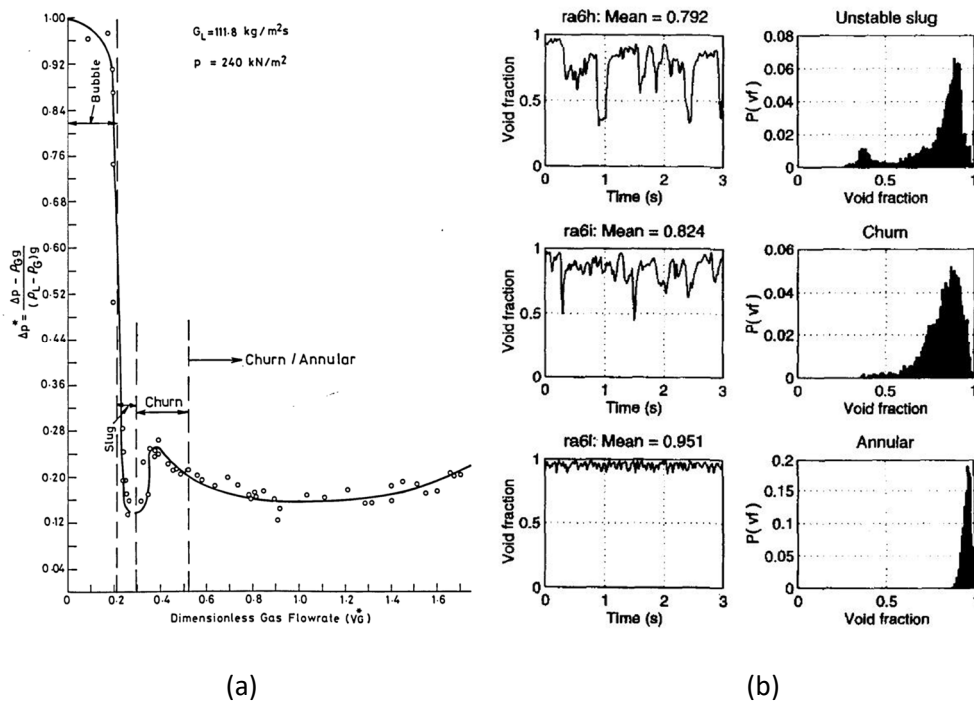


Figure 15 - Classification of flow patterns according to pressure drop (a) and PDF of void fraction (b) - Owen (1986) and Costigan and Wallhey (1996)

Chapter 3

The two-fluid model

3.1 Preamble

The one dimensional two-fluid model (Ishii 1975; Ishii and Hibiki 2006) represents a system of partial differential equations used to mathematically describe the relative motion and interaction of different phases. In the model each phase is considered separately and thus two separate sets of balance equations are specified together with models for the interaction between the phases.

Although two phase flow has a clear three-dimensional nature, being related to phenomena such as turbulence or the presence of slugs, it is a general assumption that - in the case of flows in long pipelines - most of the variations occur in the axial direction, and for this reason the one-dimensional model is considered to be sufficiently accurate and reliable for the simulation of these systems.

The one-dimensional form of the model is obtained by integrating over the cross sectional area of the pipe. By area averaging, the variations occurring in

3.2 One-dimensional equations

the radial direction are obliterated. For this reason, the effect of the interactions (mass and momentum transfer) between the phases must be modelled with appropriate closure relations.

In section 3.2 of this chapter the system of equations forming the transient one-dimensional two-fluid model is presented.

Section 3.3 introduces the relations needed for the closure of the model: the geometric assumptions and the correlations for the calculation of the shear stresses are described.

Section 3.4 finally investigates the mathematical nature of the model, discussing the issue of well-posedness and its relations with the growth of instabilities calculated through a stability analysis of the system; furthermore, the effects of ill-posedness on the slug capturing technique are discussed.

3.2 One-dimensional equations

Assuming isothermal conditions and no mass exchange between the phases, the governing transient equations for the one-dimensional two fluid model are

Continuity

$$\frac{\partial}{\partial t} \alpha_l \rho_l + \frac{\partial}{\partial x} \alpha_l \rho_l u_l = 0 \quad (3.1)$$

$$\frac{\partial}{\partial t} \alpha_g \rho_g + \frac{\partial}{\partial x} \alpha_g \rho_g u_g = 0 \quad (3.2)$$

3.2 One-dimensional equations

Momentum

$$\begin{aligned} \frac{\partial}{\partial t} \alpha_l \rho_l u_l + \frac{\partial}{\partial x} \alpha_l \rho_l u_l^2 \\ = -\alpha_l \frac{\partial P}{\partial x} - \alpha_l \rho_l g \cos(\beta) \frac{\partial h_l}{\partial x} - \alpha_l \rho_l g \sin(\beta) - F_{wl} + F_i \end{aligned} \quad (3.3)$$

$$\frac{\partial}{\partial t} \alpha_g \rho_g u_g + \frac{\partial}{\partial x} \alpha_g \rho_g u_g^2 = -\alpha_g \frac{\partial P}{\partial x} - \alpha_g \rho_g g \sin(\beta) - F_{wg} - F_i \quad (3.4)$$

with the condition

$$\alpha_l + \alpha_g = 1 \quad (3.5)$$

where the l and g subscripts denote the liquid and gas phases respectively, while i denotes quantities at the interface between the phases; denoting $k = l$ or g , α_k represents the volume fraction, ρ_k the phase density and u_k the velocity, whilst P is the cross-sectional averaged pressure. Furthermore, g is the gravitational acceleration and β is the angle between the pipe and the horizontal line ($\sin(\beta) = 1$ in the vertical case). The term containing the liquid height h_l is the hydrostatic pressure term and it is usually neglected in the gas momentum equation, since $\rho_g \ll \rho_l$.

The terms F_{wl} , F_{wg} and F_i represent, respectively, the frictional forces (per unit volume) between the liquid phase and the wall, the gas phase and the wall,

and the liquid-gas interface.

3.3 Closure models

3.3.1 Geometric relations

In vertical flow - as explained in Chapter 2 - in the Taylor bubble region the gas phase occupies the centre of the pipe, forming an axisymmetric bubble having a bullet shape, and surrounded by a liquid film. Furthermore, in the slug region the pipe is filled with pure liquid ($\alpha_l \cong 1$), since entrainment is not modelled; therefore, symmetry is assumed together with no contact between the gas and the wall (Figure 16). Hence, the wetted perimeters are defined as

$$S_l = \pi D \quad (3.6)$$

$$S_g = 0 \quad (3.7)$$

$$S_i = \pi D \sqrt{\alpha_g} \quad (3.8)$$

3.3.2 Shear stresses

The frictional forces (per unit volume) in equations (3.3) and (3.4) are defined as:

$$F_l = \frac{S_l}{A} \tau_{wl} \quad (3.9)$$

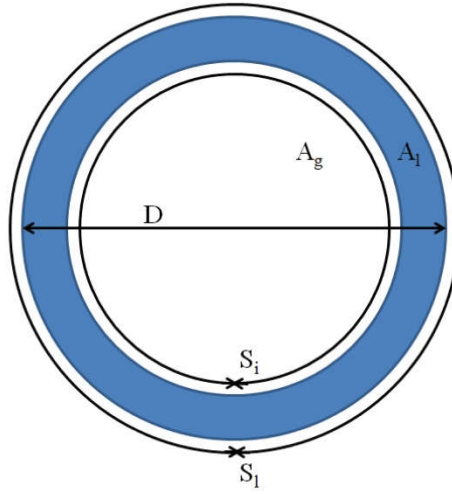


Figure 16 - Geometrical properties for vertical flow – Blue: Liquid

$$F_g = \frac{S_g}{A} \tau_{wg} \quad (3.10)$$

$$F_l = \frac{S_l}{A} \tau_l \quad (3.11)$$

where A is the cross-sectional area of the pipe and S_l , S_g , S_i , are the wetted perimeter (defined in section 3.3.1); τ_{wl} , τ_{wg} are the wall shear stresses, while τ_i is the shear stress between the two phases. They are given by:

$$\tau_{wl} = \frac{1}{2} \rho_l f_l u_l |u_l| \quad (3.12)$$

$$\tau_i = \frac{1}{2} \rho_l f_i (u_g - u_l) |u_g - u_l| \quad (3.13)$$

where f_i and f_l are friction factors and are empirically derived correlations.

The choice of the empirical correlations for the friction factors can have a major impact on the solution of the model as highlighted by Issa and Kempf, (2003) who presented a brief review of the work carried out to study the impact of these correlations on the slug capturing method for horizontal geometries. As for the vertical configurations studied in this work, the friction factors used are presented below.

Slug flow

The correlations for the friction factors are based on the standard Blasius formulation:

$$f_k = \begin{cases} \frac{16}{Re_k}, & \text{for laminar flows} \\ 0.079(Re_k)^{-0.25}, & \text{for turbulent flows} \end{cases} \quad (3.14)$$

Where Re_k represents the Reynolds numbers, defined as

$$Re_i = \frac{D_g |u_g - u_l| \rho_g}{\mu_g} \quad (3.15)$$

for the interfacial correlation, and as

$$Re_l = \frac{D_l |u_l| \rho_l}{\mu_l} \quad (3.16)$$

for the liquid wall correlation.

D_g and D_l are, respectively, the gas and liquid phase hydraulic diameters:

$$D_g = \frac{4A\alpha_g}{S_g + S_i} \quad (3.17)$$

$$D_l = \frac{4A\alpha_l}{S_i} \quad (3.18)$$

The friction forces between the gas and the wall are obviously set to zero, since no contact is assumed as explained earlier,

$$F_{wg} = 0. \quad (3.19)$$

Churn flow

In addition to the standard Blasius formula, which is derived from steady state single-phase pipe flow, another correlation for the interfacial friction factor - specifically derived for churn flow by Bharathan et al., (1978)- has been tested in this work (the following expression is derived from Jayanti and Brauner, (1994)),

$$f_i = 0.005 + 24(\alpha_l)^{2.04} \quad (3.20)$$

This factor is based on a correlation proposed by Wallis, (1969) for annular flow, in order to account for the wavy interface between the phases.

Mixed correlation

The last correlation used takes into account both the relative velocity and the liquid structures (i.e. the wavy interface) and was proposed by Inada et al., (2004).

When the gas phase is laminar flow the interface friction factor is assumed to be

$$f_i = \frac{16}{Re} \left(1 + 75(1 - \alpha_g) \right), \quad (3.21)$$

and when it is turbulent flow,

$$f_i = 0.079 (\overline{Re}_g)^{-0.25} \left(1 + 75(1 - \alpha_g) \right). \quad (3.22)$$

$\overline{Re}_g$ is the gas phase Reynolds number calculated as:

$$\overline{Re}_g = \frac{D_g |u_g - \eta u_l| \rho_g}{\mu_g}, \quad (3.23)$$

where η is a correction factor which depends on the liquid film velocity (the reader may refer to the original paper for further details).

3.4 The issue of well posedness

The two-fluid model represents a system of first order partial differential equations. This kind of systems can be written, in a general form, as

$$\mathbf{A}(\boldsymbol{\phi}) \frac{\partial \boldsymbol{\phi}}{\partial t} + \mathbf{B}(\boldsymbol{\phi}) \frac{\partial \boldsymbol{\phi}}{\partial x} = \mathbf{c} \quad (3.24)$$

where $\mathbf{A}$ and $\mathbf{B}$ are the Jacobian matrices, $\boldsymbol{\phi}$ is the column vector of the independent variables and $\mathbf{c}$ is the vector which contains the algebraic terms.

A general mathematical problem such as the one defined by (3.24), is said to be well-posed when - according to Hadamard (1902) - with proper boundary conditions and initial values,

- A solution exists,
- the solution is unique, and
- the solution depends continuously on the data.

When one of these conditions is not satisfied, the problem is ill-posed. This means that, although it might still exist and be unique, the solution might not actually have any physical meaning.

The application of the one dimensional two-fluid model to the simulation of multiphase systems is therefore generally treated with scepticism, since the system of equations representing the standard one dimensional two-fluid model is well known to be only conditionally well-posed for horizontal geometries and always ill-posed for vertical geometries (Lyczkowski et al., (1978); Ramshaw and Trapp, (1978); Arai, (1980); J. H. Song and Ishii, (2000), among many others).

3.4.1 Characteristics analysis

System (3.24) describes an initial value problem dependent on certain initial conditions and appropriate boundary conditions. In order to verify the

mathematical character (i.e. the “posedness”) of a system of partial differential equation, the characteristics of the system must be analysed. If some or all the characteristics are complex, the equations must be solved as a boundary value problem and, in an initial value problem, to obtain a stable and unique solution information should be theoretically supplied at a future time; this prescription is obviously physically implausible (Anderson, (1995); Prosperetti and Tryggvason, (2007); Fletcher, (2012)).

From numerous works it has been found that the characteristics of the system are real and the system is well-posed if and only if the following condition is satisfied:

$$(u_g - u_l)^2 \leq \left(\frac{\rho_l}{\alpha_l} + \frac{\rho_g}{\alpha_g} \right) (\rho_l - \rho_g) \frac{(\pi D g \cos(\beta))}{4 \sin(\frac{\gamma}{4})} \quad (3.25)$$

Therefore, it is clear from (3.25) that, in case of vertical geometries (i.e. $\cos(\beta) = 0$), the system is well-posed only for the trivial condition $u_g = u_l$.

3.4.2 Linear stability analysis

In addition to this approach, Stuhmiller, (1977) and Ramshaw and Trapp, (1978) noticed that the ill-posedness is associated with the growth of short-wavelength perturbations; indeed, a different definition of well-posedness can be obtained from a stability analysis: in a well-posed system the growth rate of all possible wavelength instabilities must be bounded whilst, in ill-posed systems, the growth rate is unbounded for infinitely small wavelengths (Lin and Hanratty, (1986); Barnea, (1993); Issa and Kempf, (2003)). From a numerical

3.4 The issue of well posedness

prospective, in the case of an ill-posed system the instabilities generated by machine round-off errors may grow to such an extent so as to render the final solution completely meaningless and, in addition, a mesh independent solution not obtainable. For this reason, the numerical results (if any) obtained with the two-fluid model in the ill-posed region are, as already mentioned, considered as invalid.

In order to analyse the stability of a system of equations, a perturbation is imposed to all the variables in the system, and then the evolution of this perturbation is studied by calculating its growth rate (Ramshaw and Trapp, (1978)). Therefore, in equation (3.24), the solution vector $\boldsymbol{\phi}$ is substituted with a perturbed solution such as $\boldsymbol{\phi} + \delta\boldsymbol{\phi}$ and the system is linearized with respect to $\delta\boldsymbol{\phi}$, as follows

$$\begin{aligned} A(\boldsymbol{\phi}) \frac{\partial \delta\boldsymbol{\phi}}{\partial t} + B(\boldsymbol{\phi}) \frac{\partial \delta\boldsymbol{\phi}}{\partial x} + \left(\delta\boldsymbol{\phi} \cdot \frac{\partial A}{\partial \boldsymbol{\phi}} \right) \cdot \frac{\partial \boldsymbol{\phi}}{\partial t} + \\ \left(\delta\boldsymbol{\phi} \cdot \frac{\partial B}{\partial \boldsymbol{\phi}} \right) \cdot \frac{\partial \boldsymbol{\phi}}{\partial x} + \delta\boldsymbol{\phi} \cdot \frac{\partial c}{\partial \boldsymbol{\phi}} = 0 \end{aligned} \quad (3.26)$$

Assuming a solution for equation (3.26) of the form

$$\delta\boldsymbol{\phi} = \delta\boldsymbol{\phi}_0 e^{i(kx - \omega t)} \quad (3.27)$$

where k is the wave number and ω is the angular frequency, equation (3.26) becomes

3.4 The issue of well posedness

$$\begin{aligned}
 & -i\omega \mathbf{A}(\boldsymbol{\phi}) \delta\phi_0 + ik \mathbf{B}(\boldsymbol{\phi}) \delta\phi_0 + \left(\delta\boldsymbol{\phi} \cdot \frac{\partial \mathbf{A}}{\partial \boldsymbol{\phi}} \right) \cdot \frac{\partial \boldsymbol{\phi}}{\partial t} + \\
 & \left(\delta\boldsymbol{\phi} \cdot \frac{\partial \mathbf{B}}{\partial \boldsymbol{\phi}} \right) \cdot \frac{\partial \boldsymbol{\phi}}{\partial x} + \delta\boldsymbol{\phi} \cdot \frac{\partial c}{\partial \boldsymbol{\phi}} = 0
 \end{aligned} \tag{3.28}$$

and, for a nontrivial solution to exist, the determinant of the coefficient matrix must vanish. Therefore, the dispersion relation $\omega(k)$ is given by

$$\det(-i\omega \mathbf{A} + ik \mathbf{B} + \mathbf{D}) = 0 \tag{3.29}$$

where

$$\mathbf{D} = \left(\frac{\partial \mathbf{A}}{\partial \boldsymbol{\phi}} \cdot \frac{\partial \boldsymbol{\phi}}{\partial t} \right) + \left(\frac{\partial \mathbf{B}}{\partial \boldsymbol{\phi}} \cdot \frac{\partial \boldsymbol{\phi}}{\partial x} \right) + \frac{\partial c}{\partial \boldsymbol{\phi}} \tag{3.30}$$

The growth rate for each wavenumber is then given by the imaginary part of ω (namely $\omega_I(k)$). It is clear from the definition of the perturbation in equation (3.27) that the solution will be unstable whenever ω_I is negative.

To study the stability of the two fluid model, following the approaches by Liao et al., (2008) and Vreman, (2011), the perturbation imposed to the system of equations has the following form for each variable:

$$\begin{aligned}
 \alpha'_l &= \hat{\alpha}_l e^{I(\omega t - kx)}, \quad u'_l = \hat{u}_l e^{I(\omega t - kx)}, \\
 u'_g &= \hat{u}_g e^{I(\omega t - kx)}, \quad p' = \hat{p} e^{I(\omega t - kx)}
 \end{aligned} \tag{3.31}$$

where $\hat{\alpha}_l, \hat{u}_l, \hat{u}_g$ and $\hat{p}$ are the amplitudes of the perturbation, ω is the angular frequency, k is the wavenumber and $I = \sqrt{-1}$.

Hence, for instance, for the liquid continuity equation the perturbed form is:

3.4 The issue of well posedness

$$\frac{\partial}{\partial t}(\bar{\alpha}_l + \alpha'_l) + \frac{\partial}{\partial x}[(\bar{\alpha}_l + \alpha'_l)(\bar{u}_l + u'_l)] = 0 \quad (3.32)$$

where $\bar{\alpha}_l$ and $\bar{u}_l$ are the mean values of the liquid fraction and velocity such as:

$$\frac{\partial}{\partial t}(\bar{\alpha}_l) = \frac{\partial}{\partial t}(\bar{u}_l) = \frac{\partial}{\partial x}(\bar{\alpha}_l), \frac{\partial}{\partial x}(\bar{u}_l) = 0. \quad (3.33)$$

And, given that

$$\alpha'_l u'_l = 0, \quad (3.34)$$

substituting (3.31) in (3.32), we obtain

$$\frac{\partial}{\partial t}(\hat{\alpha}_l e^{I(\omega t - kx)}) + \bar{\alpha}_l \frac{\partial}{\partial x}(\hat{u}_l e^{I(\omega t - kx)}) + \bar{u}_l \frac{\partial}{\partial x}(\hat{\alpha}_l e^{I(\omega t - kx)}) = 0 \quad (3.35)$$

Differentiating, yields:

$$\omega I(e^{I(\omega t - kx)})\hat{\alpha}_l - Ik\bar{\alpha}_l(e^{I(\omega t - kx)})\hat{u}_l - Ik\bar{u}_l(e^{I(\omega t - kx)})\hat{\alpha}_l = 0 \quad (3.36)$$

Dividing by $Ie^{I(\omega t - kx)}$ we obtain:

$$(\omega - k\bar{u}_l)\hat{\alpha}_l - (k\bar{\alpha}_l)\hat{u}_l = 0 \quad (3.37)$$

The same procedure can be applied to the remaining equations and, eventually, a system is obtained for the different amplitudes:

3.4 The issue of well posedness

$$\begin{bmatrix}
 \omega - \bar{u}_l k & -\bar{\alpha}_l k & 0 & 0 \\
 \omega - \bar{u}_g k & 0 & \bar{\alpha}_g k & 0 \\
 k H g \cos(\beta) + \frac{I}{\rho_l} \frac{\partial F_l}{\partial \alpha_l} & -(\omega - \bar{u}_l k) + \frac{I}{\rho_l} \frac{\partial F_l}{\partial u_l} & \frac{I}{\rho_l} \frac{\partial F_l}{\partial u_g} & \frac{k}{\rho_l} \\
 \frac{I}{\rho_g} \frac{\partial F_g}{\partial \alpha_l} & \frac{I}{\rho_g} \frac{\partial F_g}{\partial u_l} & -(\omega - \bar{u}_g k) + \frac{I}{\rho_g} \frac{\partial F_g}{\partial u_g} & \frac{k}{\rho_g}
 \end{bmatrix}
 \begin{bmatrix}
 \hat{\alpha}_l \\
 \hat{u}_l \\
 \hat{u}_g \\
 \hat{p}
 \end{bmatrix} = \mathbf{0} \quad (3.38)$$

Where

$$F_l = \frac{S_l \tau_l}{A_l} - \frac{S_i \tau_i}{A_l} + g, \quad F_g = \frac{S_g \tau_g}{A_g} + \frac{S_i \tau_i}{A_g} + g$$

For a non-trivial solution to exist, the determinant of the coefficients must vanish; imposing this condition yields to the following form of the dispersion equation for the angular frequency:

$$\omega^2 - 2(a k - I b) \omega + c k^2 - e k I = 0, \quad (3.40)$$

where, a, b, c and e can be obtained from the determinant of the matrix defined in (3.38) and

$$\omega = (a k - I b) \pm \sqrt{(a^2 - c) k^2 - b^2 + (e k - 2 a b k)} \quad (3.41)$$

The growth rate for each wavelength is then given by the imaginary part of ω .

Barnea and Taitel, (1993,1994) noted that, applying this analysis to the inviscid form of the one-dimensional two fluid model in horizontal flow (i.e. setting the frictional forces to zero), one will find - as stability condition - the same well-posedness limit given by the characteristics analysis in equation

(3.25), whilst, accounting for the frictional forces (VKH analysis) will yield different conditions, as already mentioned in Chapter 2 . The stability line identified with the VKH analysis is at lower values than the curve found with the inviscid analysis, hence the two curves together define three regions (Figure 5, Section 2.2.2): above the IKH limit the system is ill-posed and the flow is unstable, below the VKH limit the equations are well-posed and the flow is stable (e.g. stratified flow in horizontal geometries) and in the region in between the two limits the flow is naturally unstable but the equations are well-posed.

From the stability analysis it is also possible to calculate the growth rate of the instabilities ($-\omega_I$) as a function of the wavenumber k or wavelength λ . This result is particularly important because it clarifies the correlation between the mathematical nature of a system and the growth of instabilities. As shown in Figure 17 it is possible to identify three different behaviours, depending on the values of ω_I , directly associated with the regions identified in Figure 5: for the flow in the well-posed region, positive or zero solution indicates stable conditions (not shown), whilst negative (but bounded) values for all the wavelengths indicate unstable flow (red line in Figure 17); for cases in the ill-posed region, the values of ω_I are negative and unbounded for all the wavelengths ($-\omega_I \rightarrow +\infty$ for $\lambda \rightarrow 0$, blue line in Figure 17).

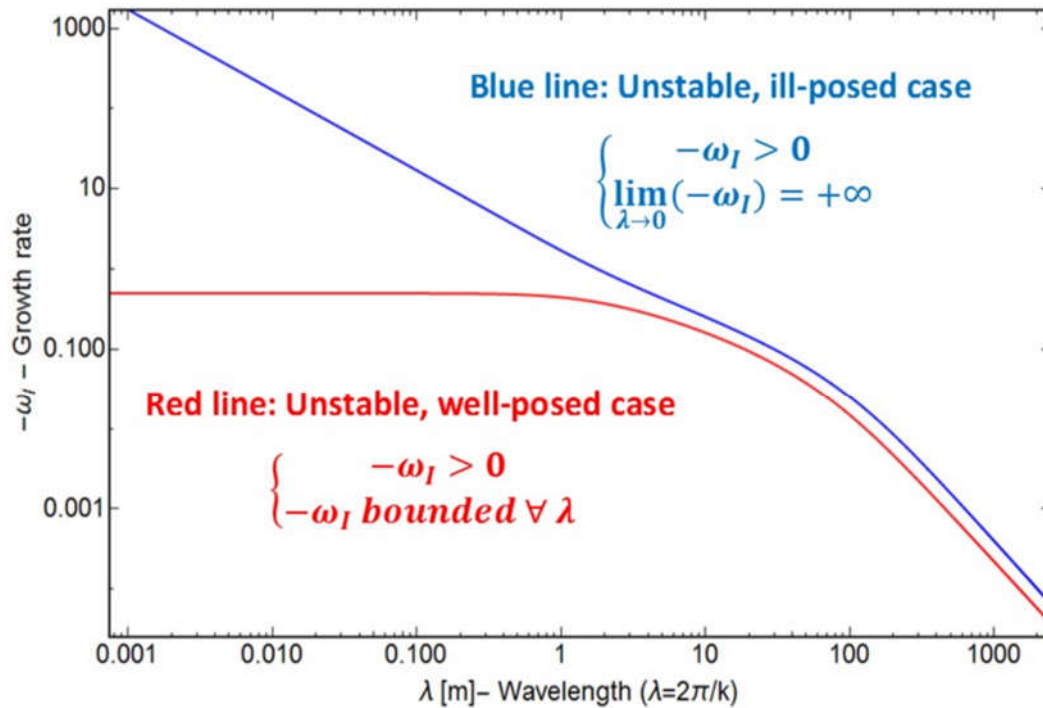


Figure 17 - Stability analysis for ill-posed and well-posed cases – Growth rate of instabilities

3.4.3 Effects on slug capturing

Woodburn and Issa, (1998) demonstrated that the standard two fluid model was able to capture the growth of natural instabilities leading to slug flow in the region in-between the viscous and inviscid limits. In fact the slug capturing technique performs fairly well in the unstable but well-posed region, whereas in the ill-posed region as the mesh is refined the predicted slug frequencies grow exponentially (Figure 18), clearly corrupted by the unbounded growth of spurious instabilities (Issa and Kempf, (2003); Bonizzi(2003)).

3.4 The issue of well posedness

To overcome the negative effects of the ill-posedness, a number of extensions to the standard model have been proposed in the literature, such as the addition of closure relations for surface tension or artificial diffusion, among others (see for example Ramshaw and Trapp, (1978); Arai, (1980); J. H. Song, (2003); Holmas et al., (2008); Vreman, (2011)).

However, these modifications have proved to have small or even negative impact on the mechanistic prediction of slug flow. This is because these added terms are generally poorly justified from a physical point of view: for instance, Pokharna et al., (1997) analysed the effects of closure terms added to the code CATHARE with the aim of making the system unconditionally well-posed and concluded that these terms wrongly introduced nonphysical behaviour in the model, rather than an effective regularisation.

Therefore, within the area of the slug capturing technique, the addition of new terms capable of rendering the system unconditionally well-posed must be considered with extreme caution, since such terms are liable to suppress the growth of natural instabilities altogether or lead to the calculation of the wrong

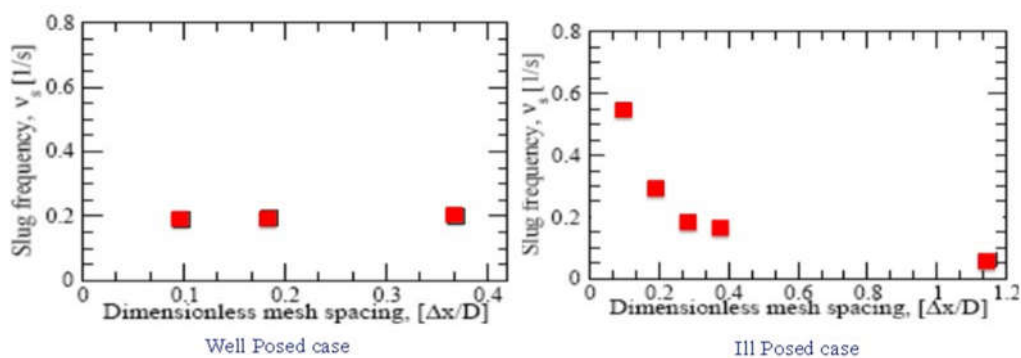


Figure 18 - Predicted slug frequency in horizontal flow with the slug capturing technique in ill-posed and well-posed cases. (Bonizzi, 2003)

growth rate, hence giving incorrect prediction of wave/slug growth and evolution.

3.5 Summary

The main characteristics of the general form of the one-dimensional two fluid model was presented in this Chapter.

A special focus is put on presenting the crucial issue of well-posedness, since the model is well known to be unconditionally ill-posed for vertical geometries and this attribute is generally considered to undermine the results obtained in the ill-posed region (and this is in fact the case for horizontal geometries).

Nevertheless, it is suggested in this work that it is misleading to judge the numerical solutions obtained with the model by the analyses of the original differential equations; instead, numerical discretisation introduces its own well-conditioning of the system which might result in valid solutions, without the necessity of adding new concocted terms to the model. A Von Neumann analysis of the discretised system is presented in the next Chapter to further support this postulate together with a study of the effects of mesh refinement on the prediction of slug frequency for vertical slug flow in Chapter 5 .

Chapter 4

Numerical Solution

4.1 Preamble

The one dimensional two-fluid model presented in Chapter 3 is implemented and solved in a computer code called TRIOMPH (TRansient Implicit One-dimensional Multi-Phase) by Issa and Abrishami, (1986).

The equations of the system are discretised on a staggered grid using the finite volume method; this kind of grid is employed in order to avoid the odd-even pressure-velocity decoupling.

The TRIOMPH code utilises the well-known two-step pressure-correction scheme PISO (Issa, (1986)) to solve the system of transient equations in an iterative manner at each time step, until a converged solution to a given tolerance is reached.

In Section 4.2 of this chapter the finite volume method used is briefly introduced together with the discrete form of the equations.

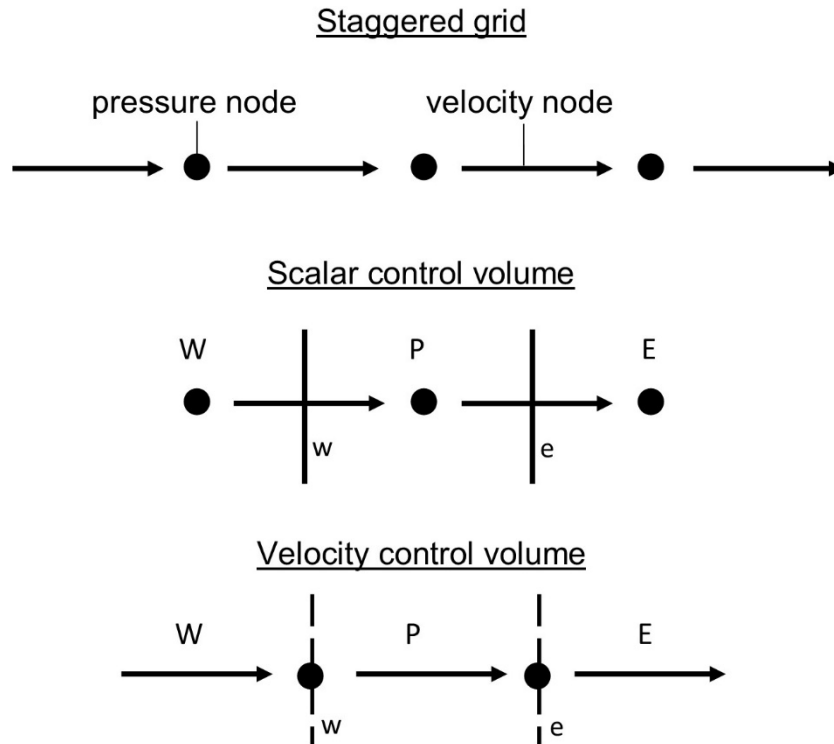


Figure 19 - Staggered grid scheme

Section 4.3 presents a study on the effects of the discretisation on the stability of the two-fluid model, comparing the results from the Von Neumann analysis of the system with those from the differential stability analysis introduced in Chapter 3 . It is shown that the discretisation has a positive effect on the model, introducing an artificial diffusion capable of rendering the finite system well-posed.

4.2 Finite volume formulation

In the present finite volume discretisation of the system an upwind difference scheme of the first order is used for the spatial derivatives, whilst a fully implicit scheme (backward Euler) is utilised for the temporal integration.

4.2 Finite volume formulation

As mentioned in the preamble the equations are discretised on a staggered grid; in this configuration the scalar variables (such as the phase fractions and the pressure) and the velocities are stored at different locations, with the latter stored at the faces of the scalar control volume; Figure 19 shows the distribution of the one-dimensional staggered grid used for the discretisation of the model.

4.2.1 Continuity

The discrete form of the phase continuity equations is obtained by integration over a control volume which contains the nodes where the scalar variables are stored. For node P the integral form is

$$\int_P \frac{(\alpha\rho)^{n+1} - (\alpha\rho)^n}{\delta t} A_P \delta x + \int_E (\alpha\rho u)_E^{n+1} \delta A - \int_W (\alpha\rho u)_W^{n+1} \delta A = 0 \quad (4.1)$$

and the time derivative term becomes

$$\int_P \frac{(\alpha\rho)^{n+1} - (\alpha\rho)^n}{\delta t} A_P \delta x = A_P \frac{\delta x}{\delta t} [(\alpha_P \rho_P)^{n+1} - (\alpha_P \rho_P)^n] \quad (4.2)$$

where P indicates values at the centre of the control volume, E and W values at the east and west faces respectively and the superscripts n and $n + 1$ refer to two successive time steps. From here onwards the time step indication $n + 1$ will be ignored for the convective terms for the sake of simplicity of presentation.

The density and phase fraction values in the spatial convective fluxes (last two terms in (4.2)) are approximated with a standard upwind differencing scheme (Versteeg and Malalasekera, (2007)). For instance, the mass flux M_E at the east

4.2 Finite volume formulation

face is

$$M_E = (\alpha \rho u)_E \delta A = m_E [\beta_E \alpha_E \rho_E + (1 - \beta_E) \alpha_P \rho_P] \quad (4.3)$$

where $m_E = (\rho u A)_E$ and the term β_E depends on the direction of the flux as

$$\beta_E = \begin{cases} 0 & \text{if } m_E > 0 \\ 1 & \text{if } m_E \leq 0 \end{cases} \quad (4.4)$$

Defining the coefficient $a_E = -\min(m_E, 0)$, the east flux M_E can be rewritten in a more compact form:

$$M_E = a_E (\alpha_P - \alpha_E) + m_E \alpha_P \quad (4.5)$$

and the west flux M_W can be expressed as

$$M_W = (\alpha \rho u)_W \delta A = a_W (\alpha_W - \alpha_P) + m_W \alpha_P \quad (4.6)$$

where $a_W = \max(m_W, 0)$.

Therefore, the finite volume formulation for the continuity equation is

$$\begin{aligned} \left(\rho^{n+1} A_p \frac{\delta x}{\delta t} + a_W + a_E + m_W - m_E \right) \alpha_P^{n+1} \\ = a_W \alpha_W^{n+1} + a_E \alpha_E^{n+1} + \frac{\delta x}{\delta t} (\rho \alpha A)_P^n \end{aligned} \quad (4.7)$$

This formulation can be compactly rewritten as

4.2 Finite volume formulation

$$a_P \alpha_P^{n+1} = H(\alpha^{n+1}) + \frac{\delta x}{\delta t} (\rho \alpha A)_P^n \quad (4.8)$$

where the coefficient a_P and the operator H are defined as

$$a_P = \left(\rho A_P \frac{\delta x}{\delta t} + a_W + a_E + m_W - m_E \right)^{n+1}, \quad (4.9)$$

$$H(\alpha^{n+1}) = a_E \alpha_E^{n+1} + a_W \alpha_W^{n+1}. \quad (4.10)$$

4.2.2 Momentum

The equations of momentum conservation ((3.3),(3.4)) can be discretised following the same procedure used for the continuity equations, with the difference that, in this case, the control volume is centred on the velocity node (as in the staggered grid shown in Figure 19).

Therefore, integrating the generic form of the equation we obtain

$$\begin{aligned} \int_P \frac{(\alpha \rho u)^{n+1} - (\alpha \rho u)^n}{\delta t} A_P \delta x + \int_E (\alpha \rho u^2)_E^{n+1} \delta A - \int_W (\alpha \rho u^2)_W^{n+1} \delta A \\ = \int_P \alpha_P \frac{\partial p}{\partial x} A_P \delta x + \int_P \Psi A_P \delta x, \end{aligned} \quad (4.11)$$

where Ψ is a source term including the hydrostatic pressure term, the gravitational term and the friction forces:

$$\Psi = \alpha \frac{\partial h}{\partial x} - \alpha \rho g \sin(\beta) - F_k - F_i. \quad (4.12)$$

The time derivative is treated in the same way as in the continuity equation,

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$$\int_P \frac{(\alpha \rho u)^{n+1} - (\alpha \rho u)^n}{\delta t} A_P \delta x = A_P \frac{\delta x}{\delta t} [(\alpha \rho u)_P^{n+1} - (\alpha \rho u)_P^n] \quad (4.13)$$

while the spatial convective fluxes are approximated using a standard upwind differencing scheme,

$$M_E = (\alpha \rho u^2)_E \delta A = m_E [\beta_E u_E + (1 - \beta_E) u_P], \quad (4.14)$$

where β_E is defined as in (4.4).

The weighted mass flux m_E and m_W are

$$m_E = f_W (\alpha \rho u A)_W + f_P (\alpha \rho u A)_P \quad (4.15)$$

$$m_W = f_P (\alpha \rho u A)_P + f_E (\alpha \rho u A)_E \quad (4.16)$$

where f_W, f_P and f_E are interpolation factors which depend on the grid spacing (they are all equal to 0.5 for a homogenous grid).

Defining again $a_E = -\min(m_E, 0)$ and $a_W = \max(m_W, 0)$ we can write the fluxes in the compact form

$$M_E = (\alpha \rho u^2)_E \delta A = a_E (u_P - u_E) + m_E u_P \quad (4.17)$$

$$M_W = (\alpha \rho u^2)_W \delta A = a_W (u_W - u_P) + m_W u_P \quad (4.18)$$

Finally, the pressure term and the source term can be discretised as

$$\int_P \alpha_P \frac{\partial p}{\partial x} A_P \delta x = A_P \alpha^{n+1} (p_E - p_W)^{n+1} \quad (4.19)$$

4.2 Finite volume formulation

$$\int_P \Psi A_P \delta x = \bar{\Psi} \quad (4.20)$$

Therefore, the difference equation obtained for (4.11) is

$$\begin{aligned} & \left(\alpha^{n+1} \rho^{n+1} A_P \frac{\delta x}{\delta t} + a_W + a_E + m_W - m_E \right) u_P^{n+1} = \\ & a_W u_W^{n+1} + a_E u_E^{n+1} + (\alpha^n \rho^n A)_P \frac{\delta x}{\delta t} u_P^n - \alpha_P^{n+1} A_P (p_E - p_W)^{n+1} + \bar{\Psi} \end{aligned} \quad (4.21)$$

and, in a more compact form

$$a_P u_P^{n+1} = H(u^{n+1}) + (\rho^n \alpha^n A)_P \frac{\delta x}{\delta t} u_P^n - \alpha_P^{n+1} A_P (p_E - p_W)^{n+1} + \Psi \quad (4.22)$$

where again the coefficient a_P and the operator H are defined as

$$a_P = \left(\alpha \rho A_P \frac{\delta x}{\delta t} + a_W + a_E + m_W - m_E \right)^{n+1} \quad (4.23)$$

$$H(u) = a_E u_E^{n+1} + a_W u_W^{n+1} \quad (4.24)$$

4.2.3 Pressure

The equation for the pressure is obtained from a combination of both the momentum and the continuity equations of the two phases. By summing the liquid and gas continuity equations, it is possible to obtain an overall density weighted continuity equation,

$$\begin{aligned} & A_P \frac{\delta x}{\delta t} (\alpha_{Pl}^{n+1} + \alpha_{Pl}^n) + \frac{A_P \delta x}{\rho_g \delta t} [(\alpha \rho)_{Pg}^{n+1} + (\alpha \rho)_{Pg}^n] \\ & + [(\tilde{\alpha} u A)_{El} - (\tilde{\alpha} u A)_{Wl}] + \frac{1}{\rho_g} [(\tilde{\alpha} \tilde{\rho} u A)_{Eg} - (\hat{\alpha} \hat{\rho} u A)_{Wg}] \end{aligned} \quad (4.25)$$

where $\tilde{\rho}$ and $\tilde{\alpha}$ are the up-winded values of density and volume fraction

respectively.

To derive equation (4.25) the two continuity equations are divided by two reference phase densities (e.g. the constant liquid density for the liquid phase and the density at atmospheric pressure for the gas phase). The velocity fields in the equation must be replaced by those obtained from the discretised momentum equation (4.21).

Equation (4.25) can eventually be rearranged into the compact form

$$a_p p_p^{n+1} = H(p^{n+1}) + \bar{\Psi}. \quad (4.26)$$

4.2.4 Solution algorithm

The algorithm used to solve the discretised system of equations is based on an adapted version of the well-known PISO method. The equations are solved sequentially and an iterative procedure is employed to advance at each time step in order to handle the strong non-linearity of the equations. The iterative steps are as following:

1. Time step is calculated based on the maximum phase velocity and the Courant number.
2. The friction forces are calculated.
3. First the gas and then the liquid momentum equations are solved using the most updated value for the pressure (i.e. values from previous iteration or time step).
4. The pressure is corrected twice solving equation (4.25).
5. The phase velocities are updated with the new values for the pressure.
6. The volume fraction equation is solved.

4.2 Finite volume formulation

7. The boundary conditions are imposed.

Figure 20 presents the flow chart of the algorithm as implemented in the code TRIOMPH.

4.2 Finite volume formulation

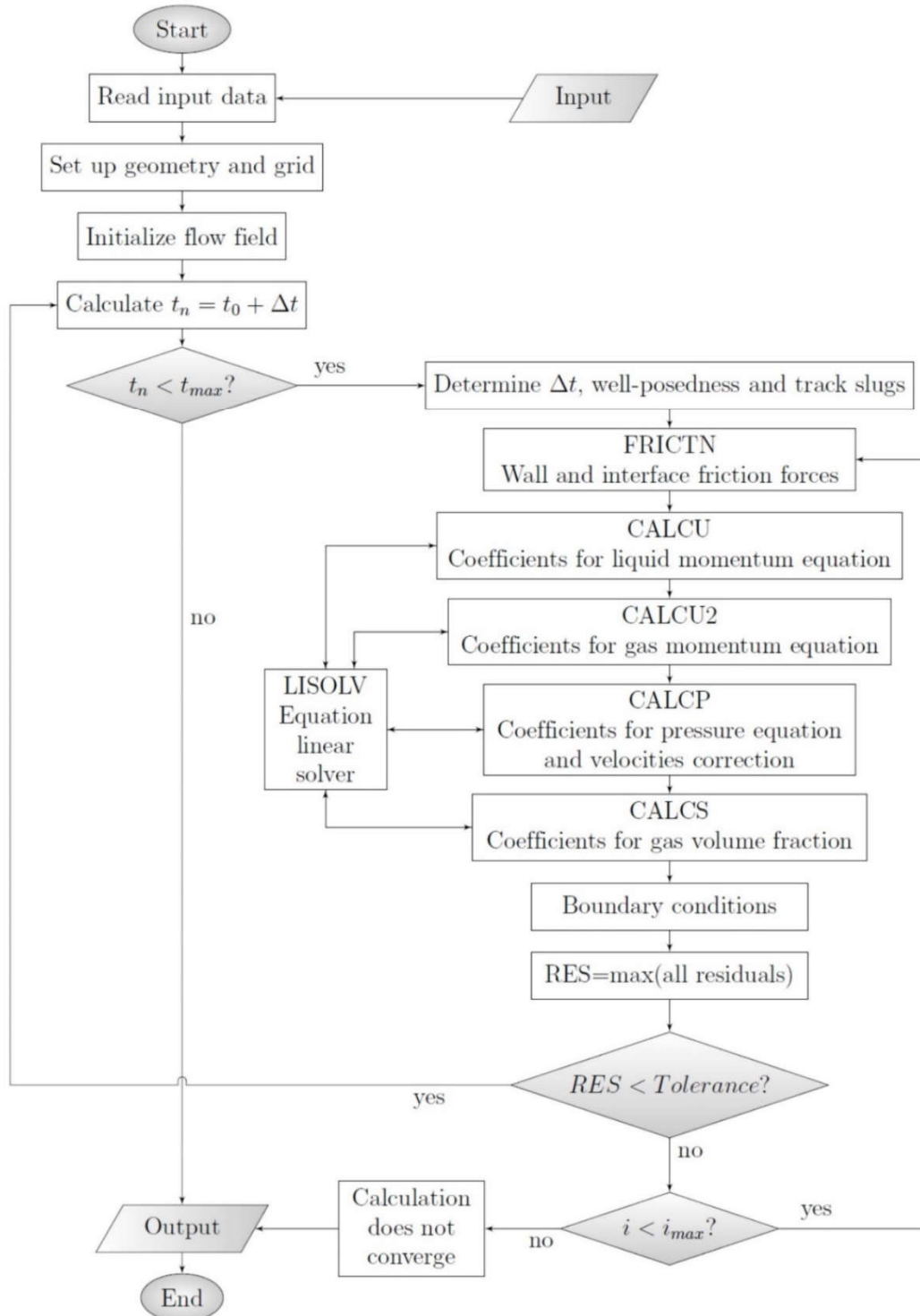


Figure 20 - Flow chart of the TRIOMPH code (Montini, 2010)

4.3 Effects of numerical discretisation

The first order upwind differencing scheme that has been used in the present work for the discretisation of the differential equations is well known to introduce into the system of discretised equations an implicit artificial viscosity (or numerical diffusion; Tannehill et al., (1997), Hirsch, (2007)). In order to investigate the effects of this numerical viscosity on the stability of the one-dimensional two-fluid model, a Von Neumann analysis of the discretised form of the system of equations is presented in the following sections and compared with the results of the stability analysis (as presented in section 3.4) for real cases in vertical geometries.

4.3.1 Von Neumann stability analysis

To analyse the stability of a system of discretised equations it is possible to follow a procedure similar to the one presented above for the differential stability analysis, imposing a disturbance to the discretised form of the equation set; this is commonly known as Von Neumann stability analysis (Hirsch, (2007)).

To illustrate how this is accomplished, an example is presented below for the discretised continuity equation for the liquid phase using a fully implicit First Order Upwind scheme with a staggered grid leads to (for the sake of simplicity the velocity is assumed positive everywhere):

$$\frac{(\alpha_l)_i^{n+1} - (\alpha_l)_i^n}{\Delta t} \Delta x + \left((u_l)_{i+\frac{1}{2}}^{n+1} (\alpha_l)_i^{n+1} - (u_l)_{i-\frac{1}{2}}^{n+1} (\alpha_l)_{i-1}^{n+1} \right) = 0 \quad (4.27)$$

Where the indices n and i denote respectively the time step and the nodal

4.3 Effects of numerical discretisation

position on the grid (see Figure 21).

The disturbances can be expressed as:

$$\begin{aligned} \alpha'_l &= \hat{\alpha}_l \epsilon^n e^{I(kx)}, u'_l = \hat{u}_l \epsilon^n e^{I(kx)}, \alpha'_g = \hat{\alpha}_g \epsilon^n e^{I(kx)}, u'_g = \hat{u}_g \epsilon^n e^{I(kx)}, \\ p' &= \hat{p} \epsilon^n e^{I(kx)} \end{aligned} \quad (4.28)$$

Where ϵ^n is the common amplitude factor at the time step n , k is the wave number, I is equal to $\sqrt{-1}$ and superscript $\wedge$ denotes the amplitude of perturbation for each variable.

Substituting the variables in (4.27) with their relative disturbances (4.28) and after some simplifications we obtain:

$$\hat{\alpha}_l \left(\frac{\Delta x}{\Delta t} \left(1 - \frac{1}{G} \right) + \bar{u}_l (1 - e^{-I\phi}) \right) + \hat{u}_l e^{-I\frac{\phi}{2}} (-1 + e^{-I\frac{\phi}{2}}) \bar{\alpha}_l = 0 \quad (4.29)$$

where

$G = \frac{\epsilon^{n+1}}{\epsilon^n}$ is the amplification factor and $\phi = k \Delta x$ is the phase angle.

The term ϕ is defined over the interval $[0, \pi]$ and characterises all the resolvable wavelengths in the computational domain for the given grid.

Applying the same procedure to the entire system (further details can be found in the Appendix) it is possible to obtain an amplification matrix, B:

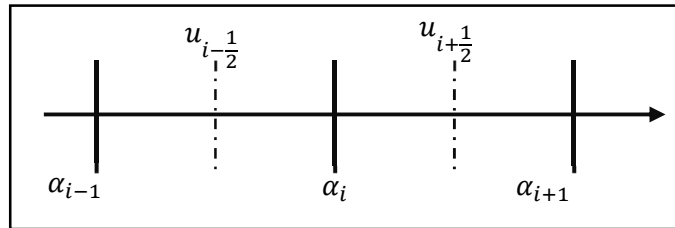


Figure 21 - Node index on a staggered grid

4.3 Effects of numerical discretisation

$$\begin{pmatrix} \left(\frac{(1 - \frac{1}{G}) \Delta x}{\Delta t} + (1 - e^{-I\phi}) \bar{u}_l \right. & e^{-\frac{I\phi}{2}} (-1 + e^{I\phi}) \bar{\alpha}_l \\ \left. \frac{(1 - \frac{1}{G}) \Delta x}{\Delta t} + (1 - e^{-I\phi}) \bar{u}_g \right. & 0 \\ e^{-\frac{I\phi}{2}} \frac{\partial F_l}{\partial \alpha_l} \Delta x & \frac{\partial F_l}{\partial u_l} \Delta x + (1 - G^{-1}) \frac{\Delta x}{\Delta t} + (1 - e^{-I\phi}) u_l \\ e^{-\frac{I\phi}{2}} \frac{\partial F_g}{\partial \alpha_l} \Delta x & \frac{\partial F_g}{\partial u_l} \Delta x \end{pmatrix} \begin{pmatrix} 0 \\ -e^{-\frac{I\phi}{2}} (-1 + e^{I\phi}) \bar{\alpha}_g \\ \frac{\partial F_l}{\partial u_g} \Delta x \\ \frac{\partial F_g}{\partial u_g} \Delta x + (1 - G^{-1}) \frac{\Delta x}{\Delta t} + (1 - e^{-I\phi}) \bar{u}_g \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \frac{e^{I\frac{\phi}{2}} - e^{-I\frac{\phi}{2}}}{\rho_l} \\ \frac{e^{I\frac{\phi}{2}} - e^{-I\frac{\phi}{2}}}{\rho_g} \end{pmatrix} \quad (4.30)$$

The disturbed system can be rewritten in the form $B \cdot v = 0$ where again $v^T = [\hat{\alpha}_l \quad \hat{u}_l \quad \hat{u}_g \quad \hat{p}]$ is the vector of the amplitude of the perturbations.

Imposing $\text{Det}(B) = 0$ gives a dispersion equation for the discretised system and the solution of that equation provides the amplification factor G . A difference scheme is then considered to be unconditionally stable when condition $G \leq 1$. However, in an unstable flow to allow the growth of natural instabilities, the condition for G must be $G > 1$ for a finite interval of wavelengths. However, G must be upper-bounded for the solution to possess physical meaning with the correct growth rate.

To properly compare the Von Neumann with the differential analysis, the

4.3 Effects of numerical discretisation

growth rates obtained from both analyses must be compared against each other; the discrete growth rate can be obtained from the change in amplitudes between successive time steps and then related to the amplification factor G simply as follows:

$$\frac{\left(\frac{\epsilon^{n+1} - \epsilon^n}{\epsilon^n}\right)}{\Delta t} = \frac{G - 1}{\Delta t} \quad (4.31)$$

The values obtained for each wavelength from the solution of the dispersion equation can then be compared with the growth rate $-\omega_I$ calculated with the VKH stability analysis.

It should be noted that the stability analysis was applied to both the raw form of the continuity and momentum equations as shown here, as well as the combined equations in which a pressure equation is derived (as is solved by the numerical procedure in Issa and Kempf, (2003)). The resulting growth rates were identical in both cases.

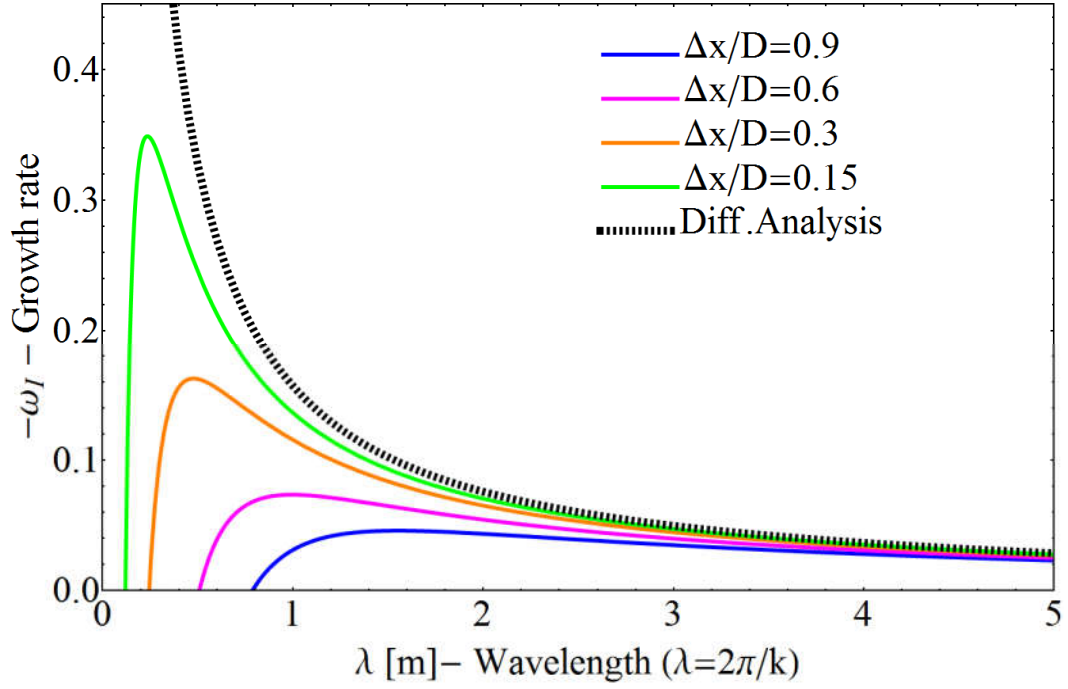


Figure 22 - Von Neumann analysis for different sizes of the mesh

$$U_{gs} = 0.43, U_{ls} = 0.05$$

4.3.2 Comparison with VKH analysis

Figures from Figure 22 to Figure 24 show the growth rates obtained with the Von Neumann analysis for three different vertical cases compared with the results from the differential VKH analysis. Further details on the cases analysed here can be found in Chapter 5 . The different curves are for different sizes of the mesh and a constant Courant number (equal to 0.1).

It is clear that the numerical diffusion does not affect the growth rate at long wave lengths: for $\lambda \rightarrow +\infty$ as all the curves collapse on the one obtained from the differential analysis, independently of the mesh size.

4.3 Effects of numerical discretisation

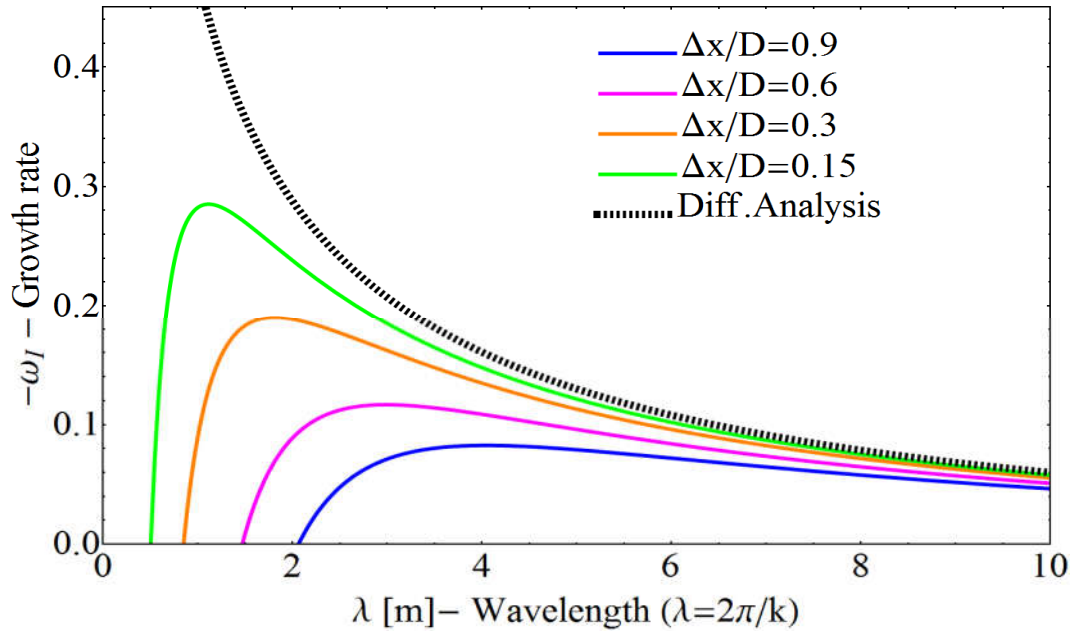


Figure 23-- Von Neumann analysis for different sizes of the mesh -

$$U_{gs} = 0.43, U_{ls} = 0.21$$

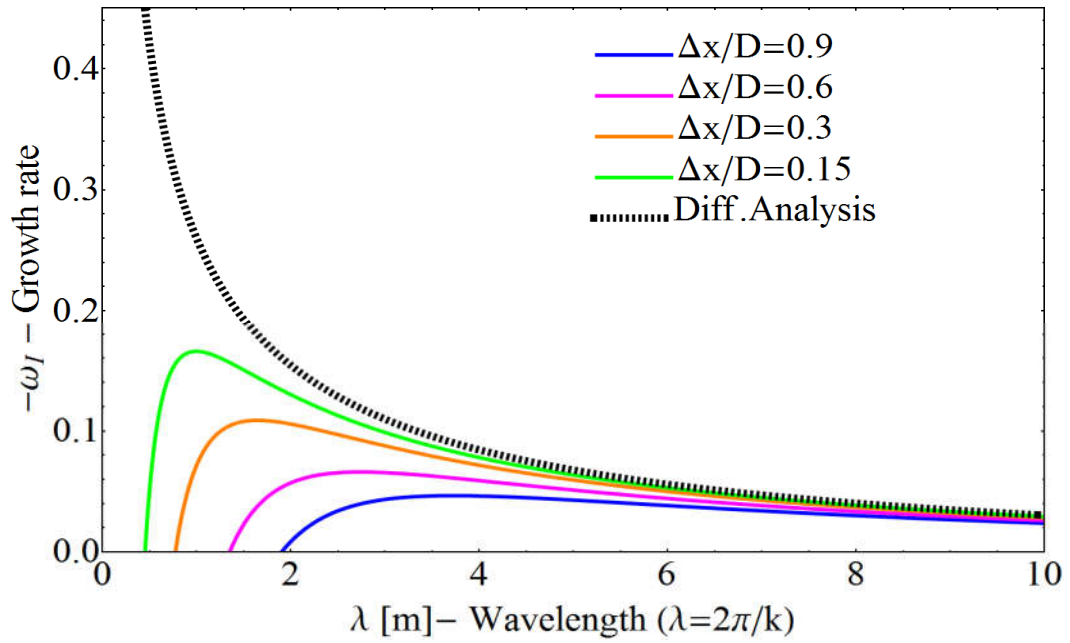


Figure 24 - Von Neumann analysis for different sizes of the mesh -

$$U_{gs} = 0.43, U_{ls} = 0.41$$

4.4 Summary

On the other hand, the discretised system shows a maximum for the growth rate (similar to the well-posed unstable case in horizontal geometries) and, furthermore, a cut-off wavelength for the growth rate; this means that all wavelengths shorter than this cut-off value will be damped (negative growth rate) by the numerical diffusion introduced by the discretisation scheme. As the mesh is refined ($\Delta x \rightarrow 0$), this cut-off limit becomes smaller and, eventually, the curve is expected to correspond to the differential one as it should.

4.4 Summary

In this Chapter the discretised form of the one-dimensional two-fluid model - as implemented in the computer code TRIOMPH - is presented together with a Von Neumann stability analysis of the finite system of equations.

Through the comparison between the stability analysis of the differential and discretised forms of the model it was found that the discretisation of the system introduces a cut-off limit for the growth rate of short wavelengths instabilities, rendering the system well posed from a numerical point of view; this effect should be enough to stabilize the solution and at the same time it should not affect the numerical prediction of the generation of vertical slugs, since the formation of slugs in vertical geometries does not depend on the growth of instabilities (as explained in Chapter 2).

In addition to this, it is worth noticing that, as expected, the discretisation of the model does not affect the growth rate at long wavelengths and therefore would not affect the prediction of the natural generation of slugs in horizontal geometries in the well-posed region. However, in the region where the system is

4.4 Summary

ill-posed - in the horizontal configuration - the results are corrupted by the spurious growth of numerical instabilities.

It is important to remark that, since the value of the cut-off wavelength still depends on the mesh cell size, the ill-posedness will eventually manifest itself for highly refined meshes ($\Delta x \rightarrow 0$); but, such small cell sizes (i.e. $\Delta x/D \ll 0.1$, see Chapter 5 for further details) are of no practical relevance, hence it is suggested that computations for intermittent vertical flow using the two-fluid model can be treated as valid for real-world applications; the results presented in Chapter 5 will confirm this conclusion.

Chapter 5

Prediction of vertical slug flow

5.1 Preamble

In Chapter 4 the Von Neumann analysis carried out on the finite form of the one-dimensional two fluid model demonstrated that the numerical discretisation is capable of rendering the system well-posed, by means of the inherent numerical diffusion that has the effect of damping the instabilities at short wavelengths; these results might mean that the simulation of vertical slug flow with the two-fluid model could lead to meaningful predictions even if the differential form of the system is ill-posed. Obviously, this conclusion needs to be supported by a validation of the predicted characteristics with real experimental data.

This chapter is mainly dedicated to the comparison of the results obtained from the numerical simulations with data acquired in several experiments performed in the Chemical Engineering laboratory of the University of Nottingham (Escrig Escrig, (2017)). In addition to this evaluation, a study of the capability of the

5.1 Preamble

model in predicting the axial development of slug flow is presented, comparing the predicted axial distribution of the main characteristics of the flow with data from the work of Waltrich et al., (2013).

In Section 5.2 the qualitative aspects of the numerical simulation of vertical slug flow will be outlined, showing that the model is able to correctly predict the distribution of the phases and the mechanisms behind the formation and development of structures such as slugs and Taylor bubbles.

In Section 5.3 the experimental setup for the campaign carried out at the University of Nottingham and the classification of the observed flow patterns from the experimental campaign are introduced firstly, together with a description of the flow characteristics used for the comparison; then the actual comparison between the predictions and the experimental data for slug flow is presented. Firstly, a qualitative comparison to assess the capability of the model in predicting the evolution of the flow regimes is discussed via visualisation of both the axial distribution and the Probability Density Function of the liquid fraction. Secondly, a quantitative comparison of the main characteristics of the slug flow regime (such as the average liquid holdup, slug frequency and velocity) is presented. The comparison is also extended to the slug/churn transition region. Finally, the effects of the boundary conditions on the predicted characteristics are analysed and discussed.

Section 5.4 presents a mesh sensitivity analysis for a real case in order to understand the effects of mesh refinement on the prediction of the slug characteristics, with a particular focus on slug frequency.

In Section 5.5 the predicted axial distribution of the phases is compared with

5.2 Qualitative prediction

experimental data from literature Waltrich et al., (2013), in order to assess the capability of the model to simulate the axial development of the flow along the pipe.

In Section 5.6 the effects of three different correlations used for the interfacial friction factors are presented and discussed.

In Section 5.7 a few results for the simulation of slug flow in small diameters pipe are presented and compared with data from literature. These results are to be considered as preliminary but they give important indications to the need for further studies.

Finally, in Section 5.8 the results presented in this Chapter are summarised and put in the general context of this work.

5.2 Qualitative prediction

Figure 25 shows a snapshot of the instantaneous distribution of the liquid fraction (blue area) and liquid velocity (red line) as predicted by the model for vertical slug flow (in order to better understand the flow direction the gravity force is indicated in the figure).

As can be seen from the figure all the main structures of slug flow are well captured: it is possible to clearly distinguish the slugs - where the liquid fraction is close to unity - the Taylor bubbles (white areas) and the liquid film. In the liquid film region, the liquid velocity can become negative (with the liquid flowing downwards) leading to accumulation of liquid that bridges the pipe, thereby forming slugs.

5.2 Qualitative prediction

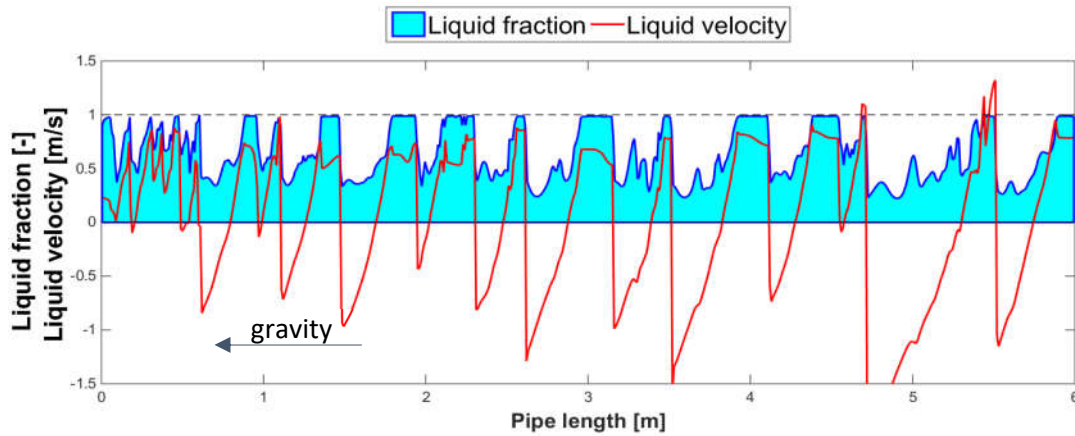


Figure 25- Liquid fraction and liquid velocity, predicted instantaneous distribution for slug flow – $U_{sg} = 0.28 \text{ m/s}$ $U_{sl} = 0.2$ – Pipe ID = 6.7 cm .

In addition to this, the flow evolution is evidently predicted as already described above: after a chaotic “churn like” region in which the velocities of the fluids are both mainly positive, short slugs are formed close to the inlet; moving upwards, some of them collapse and merge until a sufficiently stable length is reached. This collapsing mechanism is seen to occur, although less frequently, even towards the end of the pipe where some of the shorter slugs are overtaken by bigger and faster ones.

Figure 26 and Figure 27 show an instantaneous distribution of the phases along the whole pipe where slug generation, collapse and merger are evident. In Figure 27 the prediction of the collapsing phenomenon is clearly highlighted showing different snapshots of the process.

Furthermore, the development of the Taylor bubbles is ~~clearly well~~ captured, with the bubbles growing in length as they move towards the outlet, because of the pressure decrease.

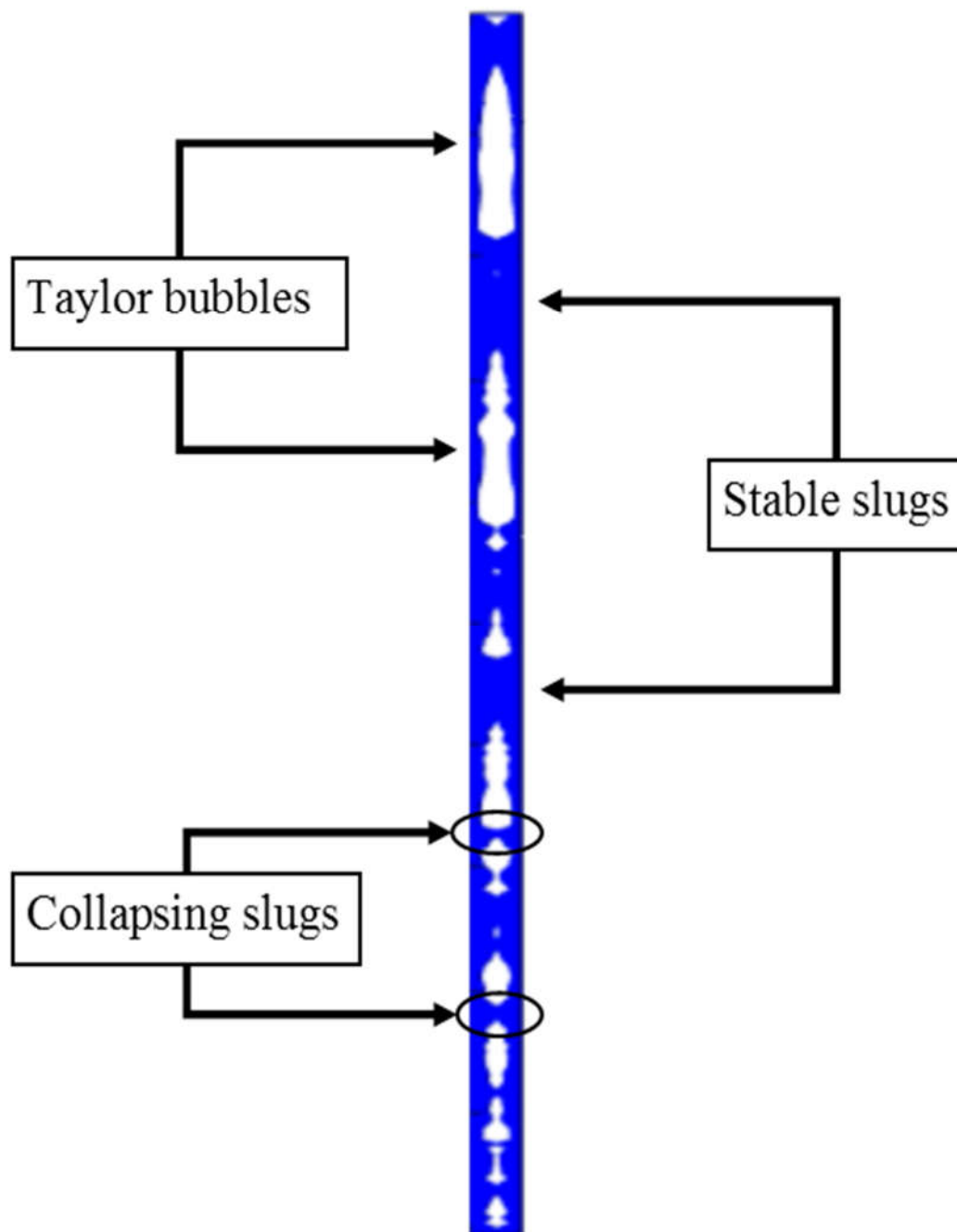


Figure 26 - 2D representation of vertical slug flow obtained with the model

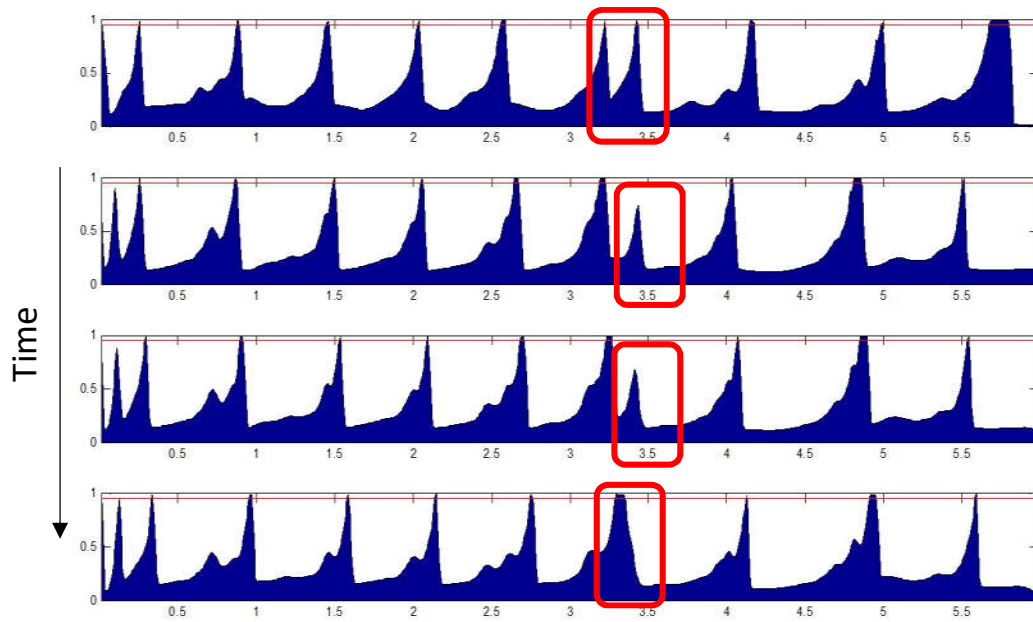


Figure 27 - Instant snapshots of liquid fraction distribution - The slug collapsing process is highlighted

5.3 Validation of the model

5.3.1 Experimental test case

The experiments were carried out on a rig which consisted of a 6 meters long vertical pipe mounted on a long rigid steel frame; the pipe internal diameter is 67 mm; Figure 28 shows a flow diagram of the rig. In the experiments a mixture of air and silicon oil was used, and all the tests were performed at room temperature. The properties of the fluids and the geometrical settings are summarized in Table 1.

5.3 Validation of the model

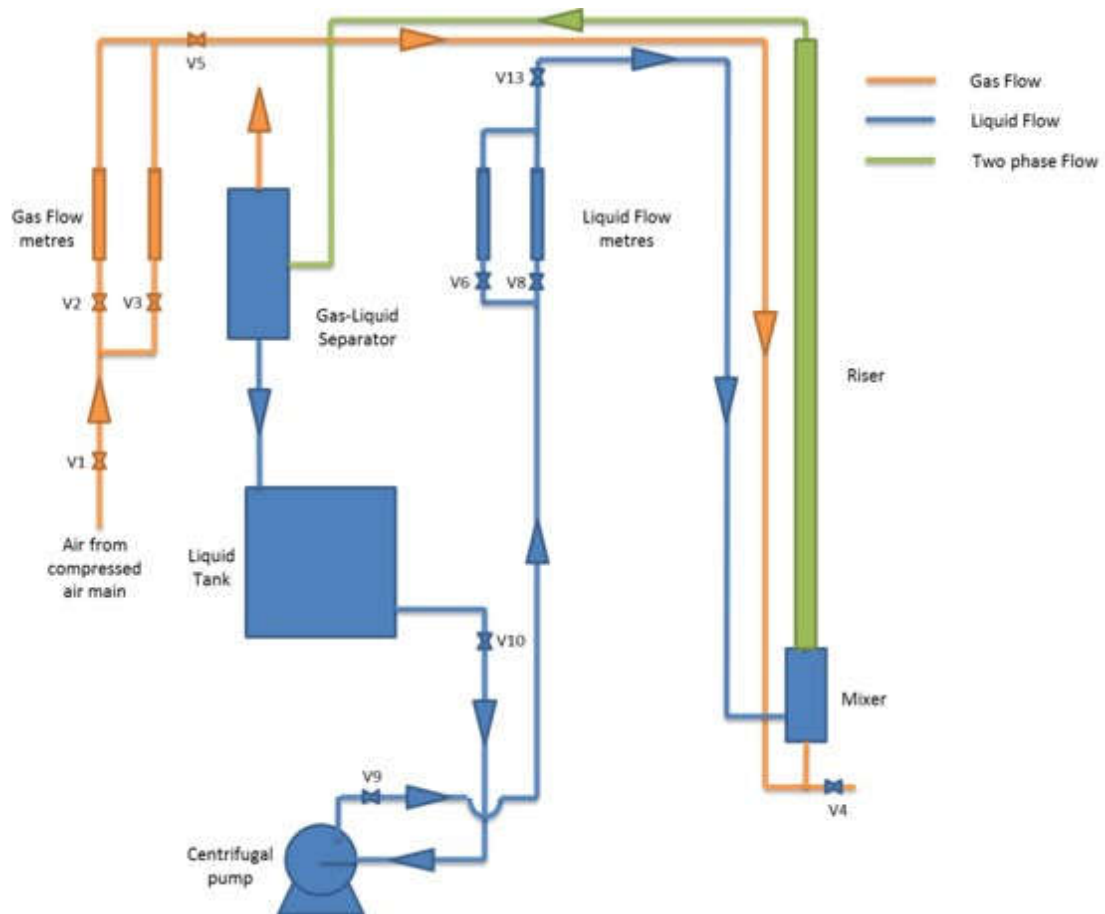


Figure 28 - Scheme of the experimental rig for vertical two-phase flow - University of Nottingham

Air was introduced into the liquid phase through the inner pipe. The liquid fraction was then measured at about 5 meters above the mixing region. Since the computational model is only one-dimensional it is not possible to simulate the inlet-mixing region precisely; however, in the experimental campaign it was found that the injection geometry does not affect the characteristics of the flow (measured at 5 meters from the inlet); this issue will be further discussed in sections 5.3.2 and 5.3.6.

5.3 Validation of the model

Length (m)	Diameter (m)	Air density (kg/m ³)	Liquid density (kg/m ³)	Air viscosity (Pa s)	Liquid viscosity (Pa s)
6.0	0.067	1.24	913.39	1.8x10-5	5x10-3

Table 1 - Summary of geometric and physical characteristics of the fluids for the Nottingham experimental campaign

A total of 49 experiments were carried out, with superficial velocities varying between 0.1 m/s and 1 m/s for the gas and between 0 and 0.4 m/s for the liquid. All the cases are summarized in Figure 29. The patterns were classified in the experiments according to the Probability Density Function of the recorded liquid fraction combined with a visualization of the flow (Figure 29).

For the comparison between measurements and numerical results the most important characteristics predicted are:

Slug/structures frequency: This is simply obtained by counting the number of slugs passing through each monitored node. It is worth noticing that there is a difference between the experimental and numerical procedures employed to discriminate between slugs and waves: in the experiments a threshold for the liquid fraction is set according to the average value of the fraction calculated from the recorded signal, following the method developed by Hazuku et al., (2008), whereas in the simulation the threshold to identify a slugged cell is fixed for all the cases and set to $\alpha_l = 0.98$ (since no gas entrainment is assumed in the

5.3 Validation of the model

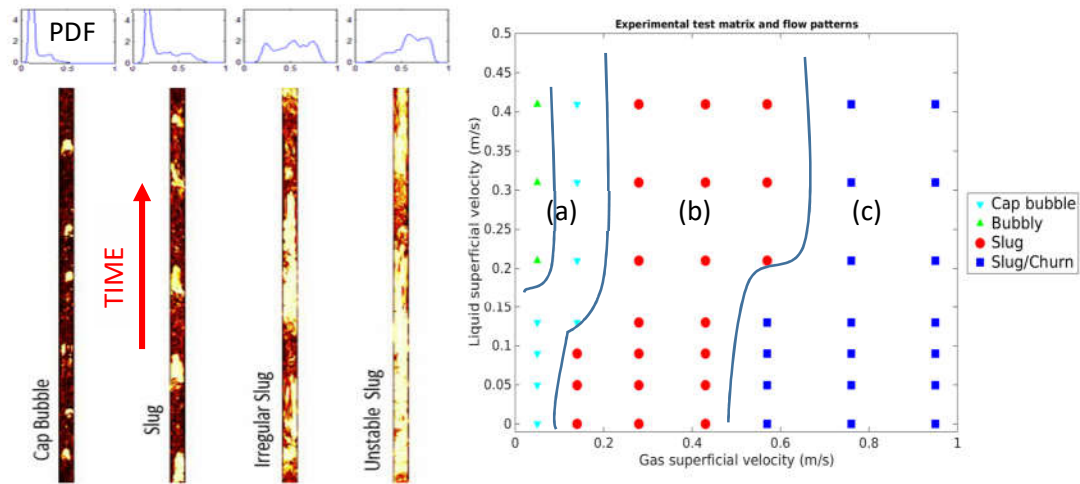


Figure 29- Experimental cases, flow patterns as classified from PDF : a) Bubbly/cap bubble flow; b) Slug flow; c) Irregular/unstable slug flow

numerical model). Furthermore, predicted slugs that are shorter than two diameters are discarded from the slug count. This filter in slug counting is applied only in the fully developed slug region (namely gas velocities of 0.28 and 0.43 m/s).

Average global liquid hold-up: This is obtained by time averaging the instantaneous liquid volumetric fraction (α_L) recorded at one probe position.

Structures velocity: This is determined from cross-correlating two signals (liquid fraction) at two different positions.

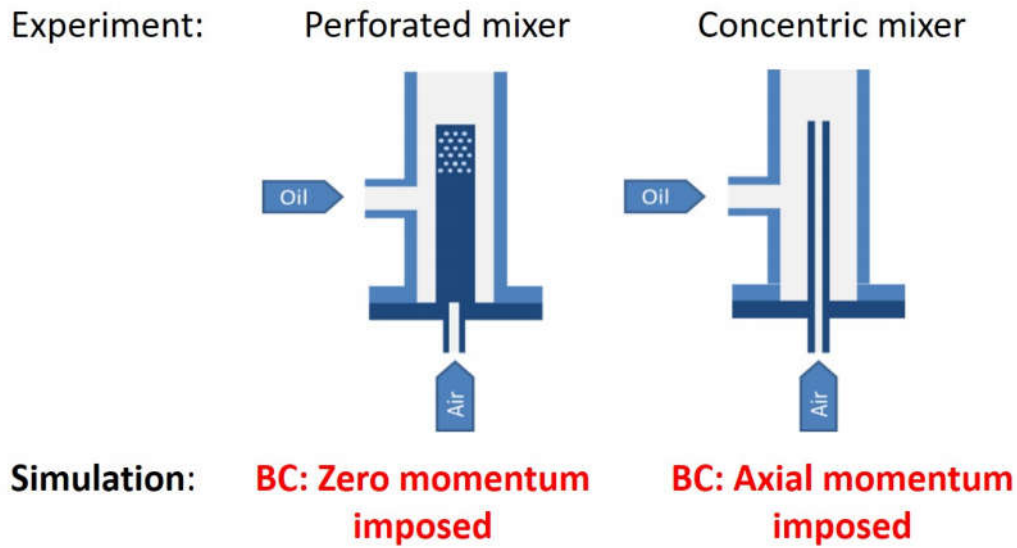


Figure 30 - Mixer geometries scheme and related boundary conditions

5.3.2 Effects of mixer geometry

In the experimental campaign, in order to investigate the influence of the injection geometry on the characteristics of the flow the same set of experiments was repeated with two different gas and liquid mixers at the inlet. The first mixer had two concentric pipes; the air was introduced to the liquid phase through the inner one. The second one had a perforated cylinder and the air was injected through the holes (see Figure 30). Therefore, in the first case the gas velocity had only an axial component, whilst in the second case it had only a radial component.

A significant conclusion from the experimental campaign was that the injection geometry does not affect the characteristics of the flow (measured at 5 meters from the inlet) as it is shown in Figure 31 (the experimental error was not provided). Also this aspect has been investigated in the simulations, imposing

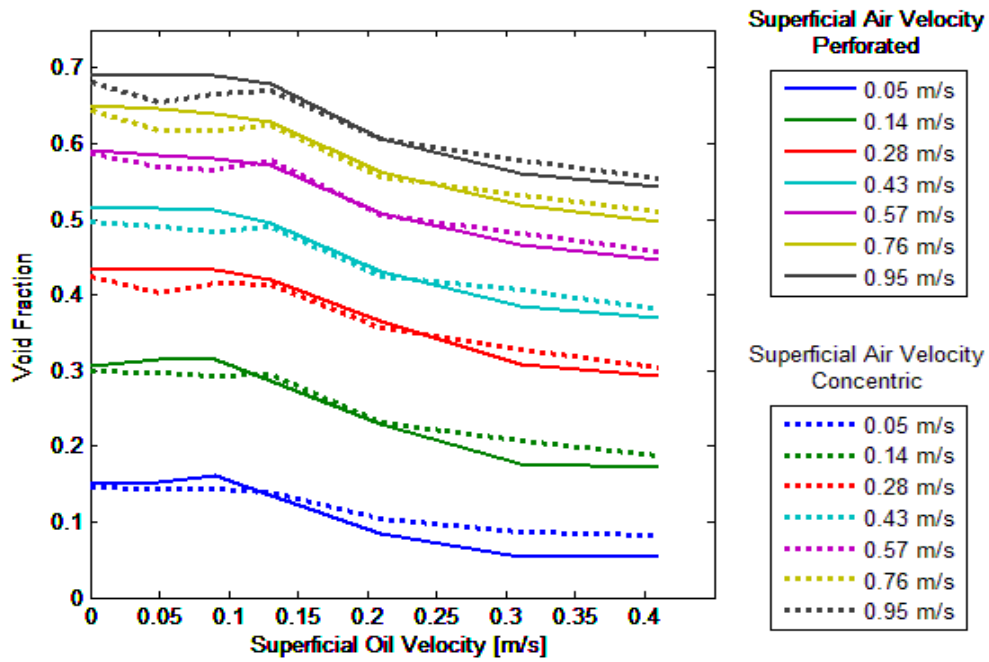


Figure 31- Measured void fraction at 5m from inlet for different inlet configurations (Escrig Escrig, 2017)

two different boundary conditions to mimic the diverse geometries (see Figure 30 and the next section).

5.3.3 Numerical setup

Boundary conditions and friction factors

In the simulations the liquid fraction and the gas and liquid mass flow rates were imposed at the inlet whereas the pressure was imposed at the outlet. The characteristics of the slugs at the measuring point were found to be insensitive to the liquid fraction value imposed at the inlet, hence the value of 0.5 was used throughout. In addition to these conditions, a zero gradient condition is imposed at the outlet for liquid fraction and velocities.

Furthermore, in order to simulate the two different injection geometries, the

gas flow rate was specified in two different ways. For the perforated mixer, a zero momentum condition was imposed, whilst for the concentric injection the axial component of the velocity was specified (Figure 30).

Additionally, during this work several interfacial friction factors were tested, and the predicted characteristics were found to be remarkably sensitive to the different correlations. The results presented in this section were all obtained applying the Blasius type correlation for the interfacial friction factor based on the slip velocity, which yielded the best results for vertical slug capturing. For the sake of completeness, the predictions obtained with two other factors are presented in section 5.6.

Initial conditions

In the experiment the pipe was initially filled with only liquid and the gas velocity was slowly increased up to the desired value. When a reasonably stabilised condition for the flow was reached, the signal was measured for one minute. The same practice was applied in the numerical simulations starting with an initial liquid fraction distribution equal to 0.99 throughout the entire computational domain.

5.3.4 Prediction of regime development

Visual observation

As explained in section 5.3.1, in the experiments the flow structures were found to vary significantly as the mass flow rate of the gas increased, starting from a cap bubble flow regime at low gas velocities and reaching a state of highly

5.3 Validation of the model

elongated Taylor bubbles and short and irregular slugs at the highest velocity (see Figure 29).

As a preliminary step, it is important to understand if the model is capable of predicting automatically these changes of regime and the consequent variations of the structures, at least qualitatively; as discussed in Chapter 2, this would be a significant achievement, since most of existing models are not able to mechanistically capture the regime transition.

Figure 32 shows instantaneous snapshots of the axial distribution of liquid fraction for six different cases with the same liquid superficial velocity (equal to 0.3 m/s) and gas velocities increasing from 0.05 m/s to 0.95 m/s, hence covering all the flow regimes detected in the experimental campaign. As can be seen the transition of the flow regimes is very well predicted.

At the lowest gas velocity, a sort of bubbly flow is predicted, with the pipe almost completely filled with water and the gas fraction homogeneously distributed along the pipe. As the gas superficial velocity is increased to 0.14 m/s the gas fraction accumulates in larger bubbles (cap bubbles) and also starts showing a weak axial development, with the bubbles becoming larger as they approach the outlet of the pipe. For the velocities in the slug flow region (0.28 m/s and 0.43 m/s) the slugs and the Taylor bubbles are well captured as already presented in section 5.2.

At the highest superficial velocities, the model still predicts few long slugs for the case at 0.73 m/s but the Taylor bubbles are longer and stretched. For the case at 0.95 m/s the slugs are short and the model mostly predicts the occurrence of waves with long sections of the pipe being filled with gas, as in

5.3 Validation of the model

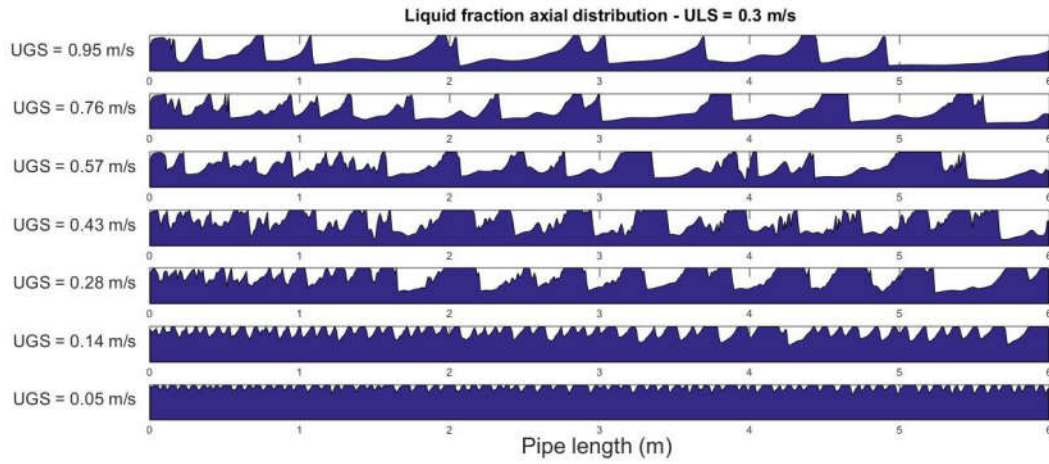


Figure 32 - Numerical prediction of phases axial distribution at different gas flow rates and liquid superficial velocity equal to 0.3 m/s - $\Delta x/D \approx 0.15$ ($\Delta x = 1$ cm)

churn flow; all these results are in very close agreement with the experimental observations.

Probability Density Function

In Chapter 2, besides the visual observation, another approach was presented to study the occurrence of different flow regimes, that is the analysis of the Probability Density Function of the recorded gas fraction at one position. For the sake of completeness, this analysis was also carried out and is presented below.

In Figure 33 the PDF for the cases presented in the previous section are shown; also from this analysis it is clear that the model captures the transition and the change in structures.

For the bubbly flow regime (0.5 m/s) there is only one peak, corresponding to high fraction of liquid, with 50% of the values above 0.98, and there are no values of the liquid fraction lower than 0.7; as the gas velocity is increased (0.14 m/s) the area below the graph corresponding to higher concentrations of gas becomes

5.3 Validation of the model

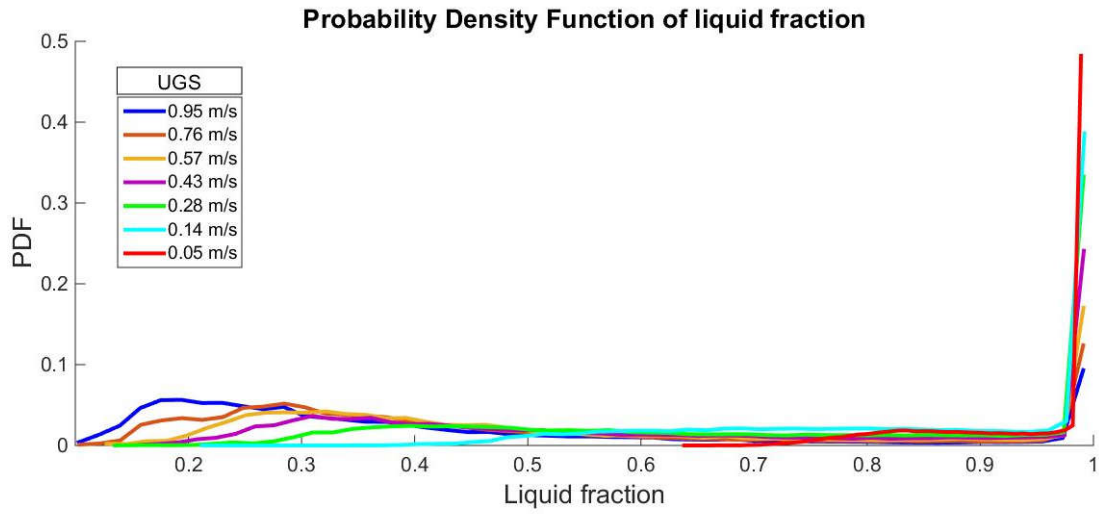


Figure 33 - Predicted Probability Density Function of liquid fraction for different gas velocities and liquid superficial velocity equal to 0.3 m/s - $\Delta x/D \approx 0.15$ ($\Delta x = 1$ cm)

larger, but no clear peaks are seen, meaning that no large gas structures are formed.

At 0.28 m/s, corresponding to slug flow regime, the prediction of gas fraction higher than 0.6 (i.e. liquid fraction lower than 0.4) shows a first peak that becomes more pronounced at 0.43 m/s and 0.57 m/s and this is obviously due to the formation of the Taylor bubbles; furthermore, the peaks at high liquid fraction remain significant indicating that the probability to find sections of the pipe filled with liquid (that is, slugs) is still significant.

In the unstable/irregular slug region (0.76 and 0.95 m/s) the probability of finding slugs decreases substantially as the gas velocity is increased and the peaks at higher gas fractions reach the level of those at high liquid fraction.

An attempt can be made to compare the PDF obtained from the simulations and those obtained from the experimental measurements; however, as can be seen for three different cases in Figure 34, although the trend is qualitatively

5.3 Validation of the model

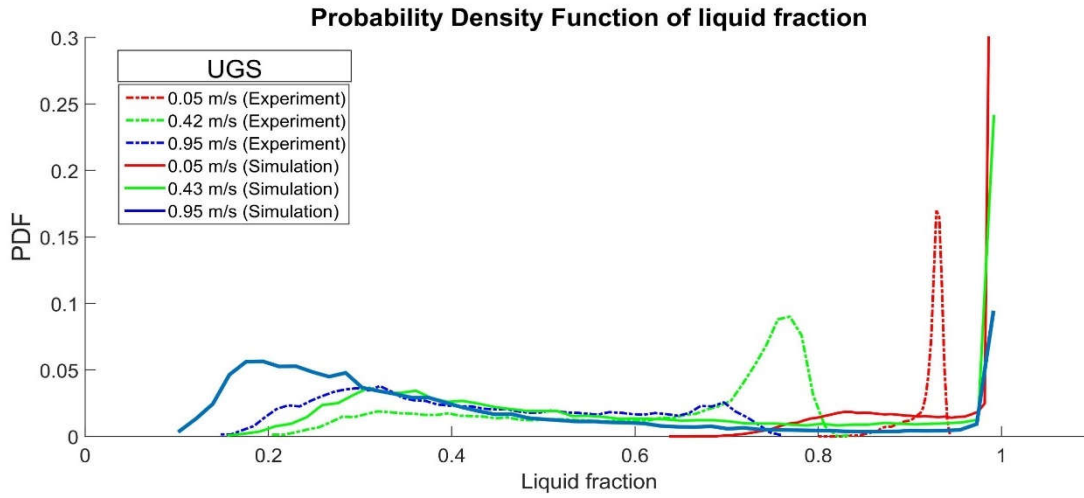


Figure 34 - Probability Density Function of liquid fraction for different gas velocities and liquid superficial velocity equal to 0.3 m/s – Comparison between predicted and experimental data (Escrig Escrig, 2017) - $\Delta x/D \approx 0.15$ ($\Delta x = 1$ cm)

well-captured, the distributions do not correspond quantitatively. This is due to the fact that in the simulation the slugs are composed of pure liquid, since the gas entrainment in the liquid is not modelled; therefore, the peaks corresponding to predictions of slugs are considerably closer to 1 (see Figure 35) than those found in the experiments. For this reason, this comparison of the absolute magnitudes is not particularly useful and only the trends would serve the

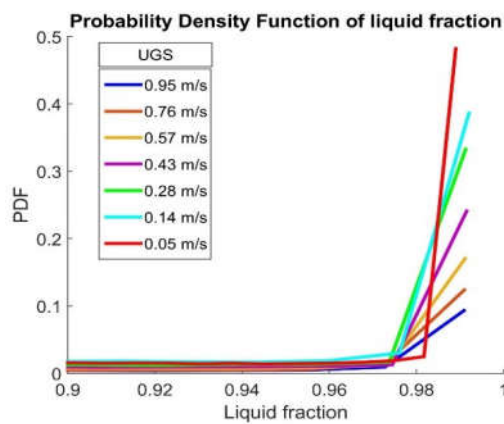


Figure 35 - Predicted Probability Density Function of liquid fraction for different gas velocities and liquid superficial velocity equal to 0.3 - Detail of the slug region. - $\Delta x/D \approx 0.15$ ($\Delta x = 1$ cm)

purpose.

5.3.5 Quantitative comparison

Figures from Figure 36 to Figure 41 show quantitative comparison between the experimental and predicted data in the slug region (i.e. for gas velocities equal to 0.28 and 0.43 m/s) at about 5 meters from the inlet; all the numerical results shown in the figures were obtained with a mesh cell size equal to 1 cm (corresponding to $\Delta x/D=0.15$).

As can be seen, the predicted results compare fairly well with the experimental ones, with almost all values falling within $\pm 30\%$ discrepancy for all flow characteristics. This benchmark value was taken from previous works (Bonizzi, (2003)(a), Montini, (2010)) and given the simplicity of the model (i.e. one-dimensional approximation and the use of correlations for the friction factors that are assumed to apply locally and instantaneously when they are actually obtained from steady-state, fully developed flow) can be considered an acceptable offset from the experimental data. Moreover, the measurements themselves (gas flow rate in particular) contain inaccuracies that are estimated to be up to 10%.

Furthermore, it is important to underline that the model is able to capture quantitatively the main characteristics of the flow even in the region where the transition from slug to churn occurs (irregular/unstable slug flow region in Figure 29), with an acceptable accuracy as it is shown in figures Figure 42, Figure 43 and Figure 44. It must be remarked that in this case the prediction and experimental data for the frequency and velocity of the structures take also into account the large waves of liquid generated in the pipe.

5.3 Validation of the model

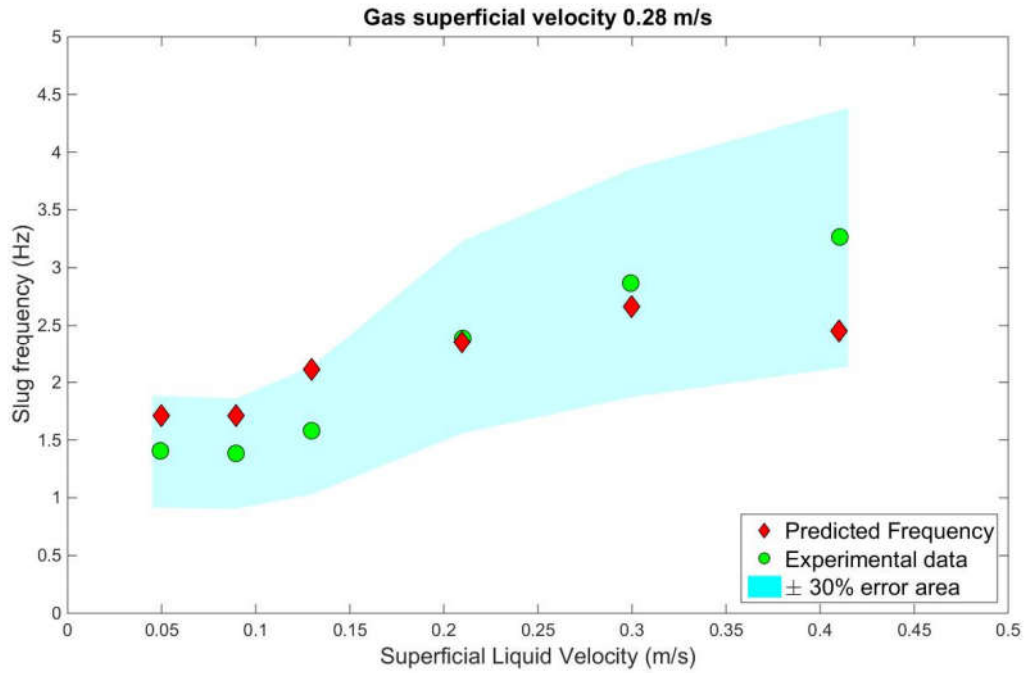


Figure 37 - Slug frequency - Comparison between predicted and experimental data – Gas superficial velocity: 0.28 m/s – Exp. data: Escrig Escrig (2017) - $\Delta x/D \approx 0.15$ ($\Delta x = 1$ cm)

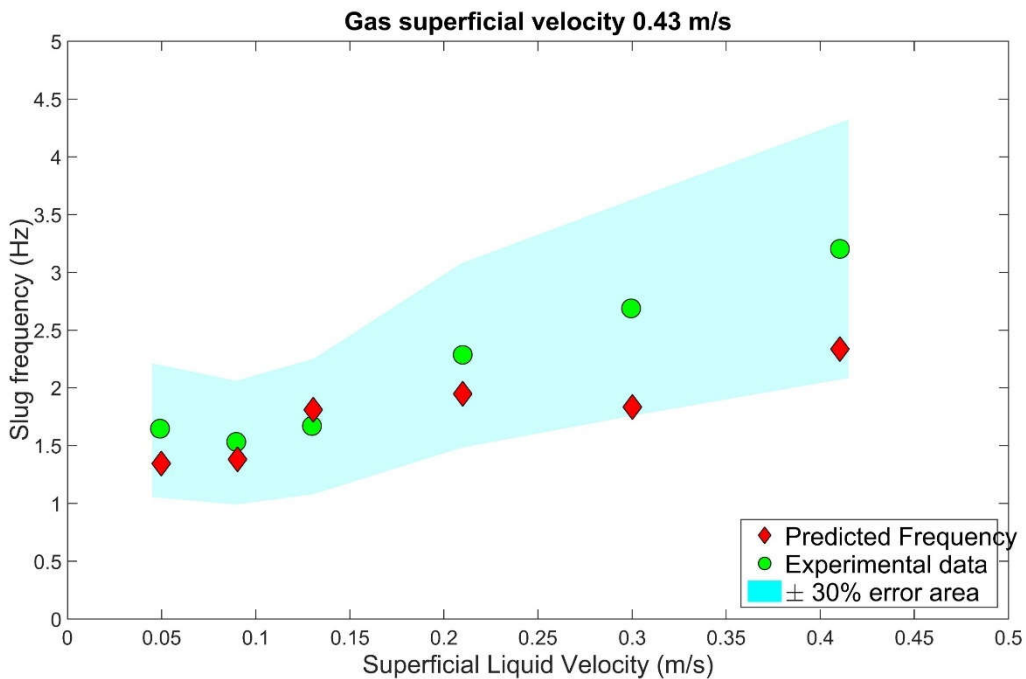


Figure 36 - Slug frequency - Comparison between predicted and experimental data – Gas superficial velocity: 0.43 m/s – Exp. data: Escrig Escrig (2017) - $\Delta x/D \approx 0.15$ ($\Delta x = 1$ cm)

5.3 Validation of the model

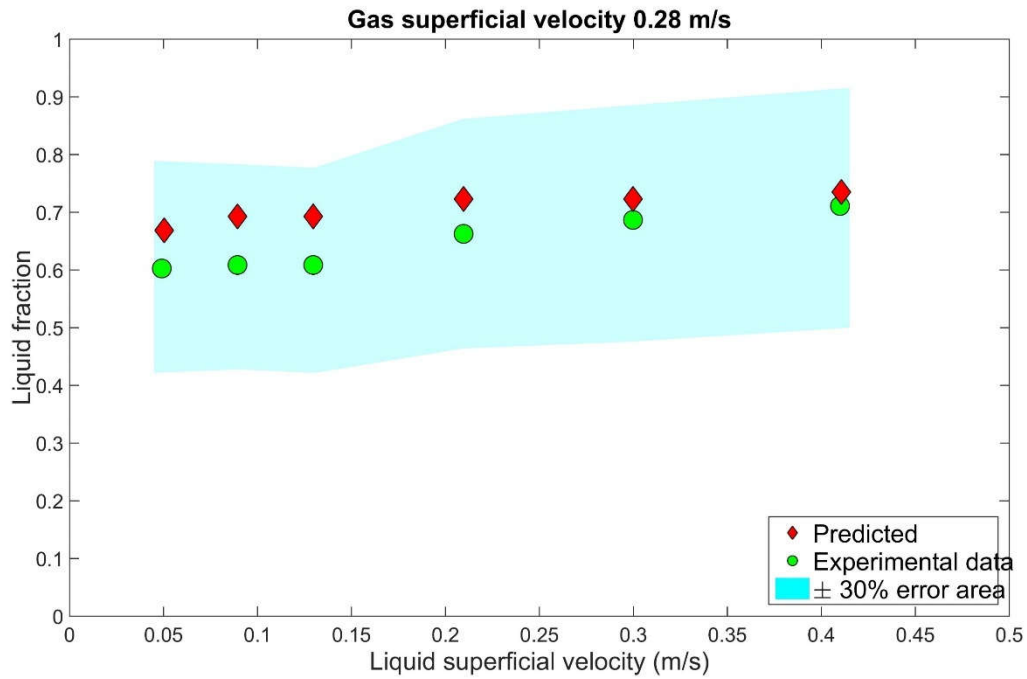


Figure 38 - Averaged liquid fraction - Comparison between predicted and experimental data – Gas superficial velocity: 0.28 m/s – Exp. data: Escrig Escrig (2017) - $\Delta x/D \approx 0.15$ ($\Delta x = 1$ cm)

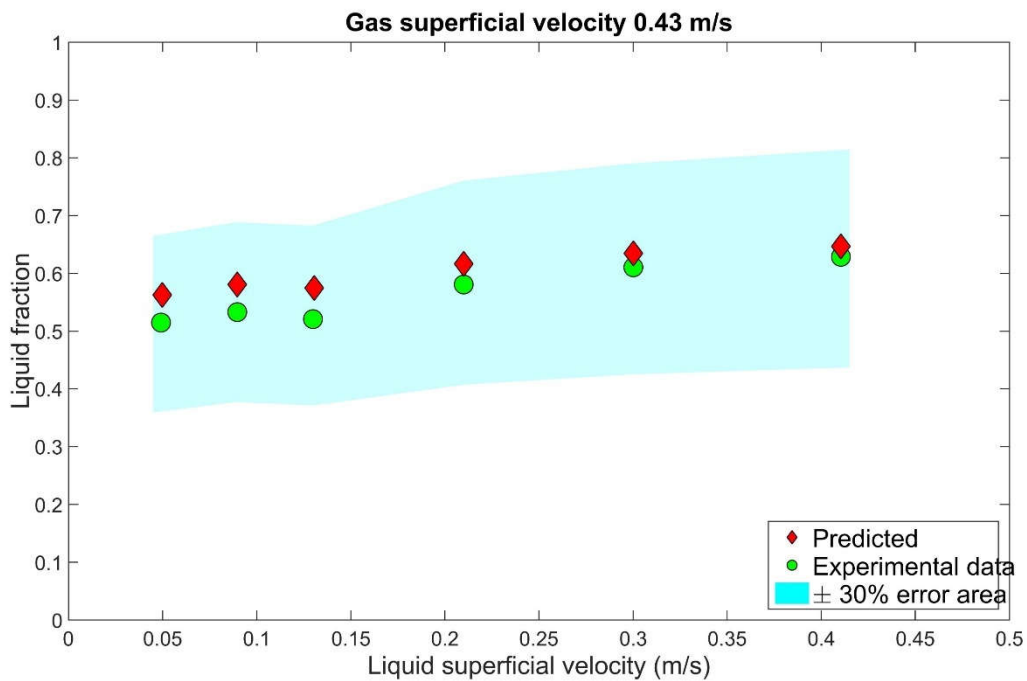


Figure 39 - Averaged liquid fraction - Comparison between predicted and experimental data – Gas superficial velocity: 0.43 m/s – Exp. data: Escrig Escrig (2017) - $\Delta x/D \approx 0.15$ ($\Delta x = 1$ cm)

5.3 Validation of the model

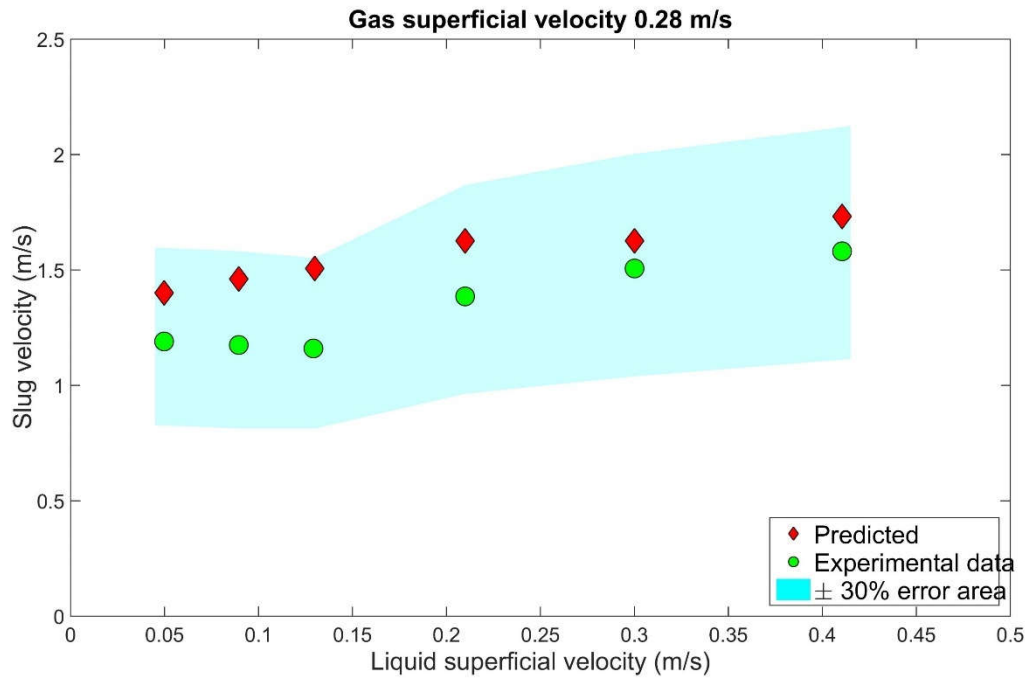


Figure 40 – Slug velocity - Comparison between predicted and experimental data – Gas superficial velocity: 0.28 m/s – Exp. data: Escrig Escrig (2017) - $\Delta x/D \approx 0.15$ ($\Delta x = 1$ cm)

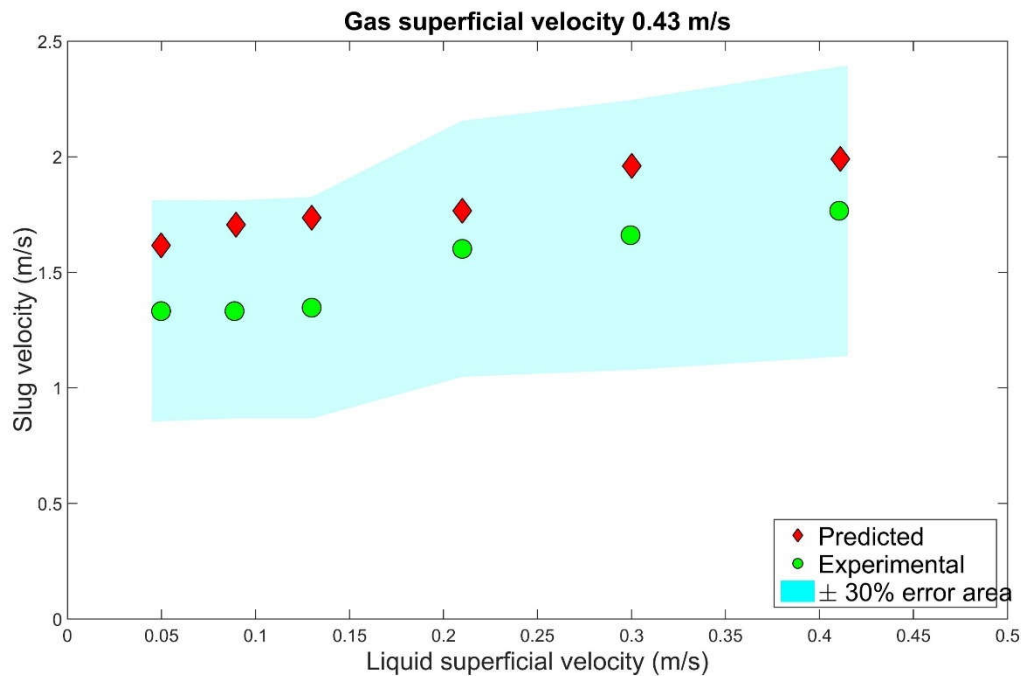


Figure 41 – Slug velocity - Comparison between predicted and experimental data – Gas superficial velocity: 0.43 m/s – Exp. data: Escrig Escrig (2017) - $\Delta x/D \approx 0.15$ ($\Delta x = 1$ cm)

5.3 Validation of the model

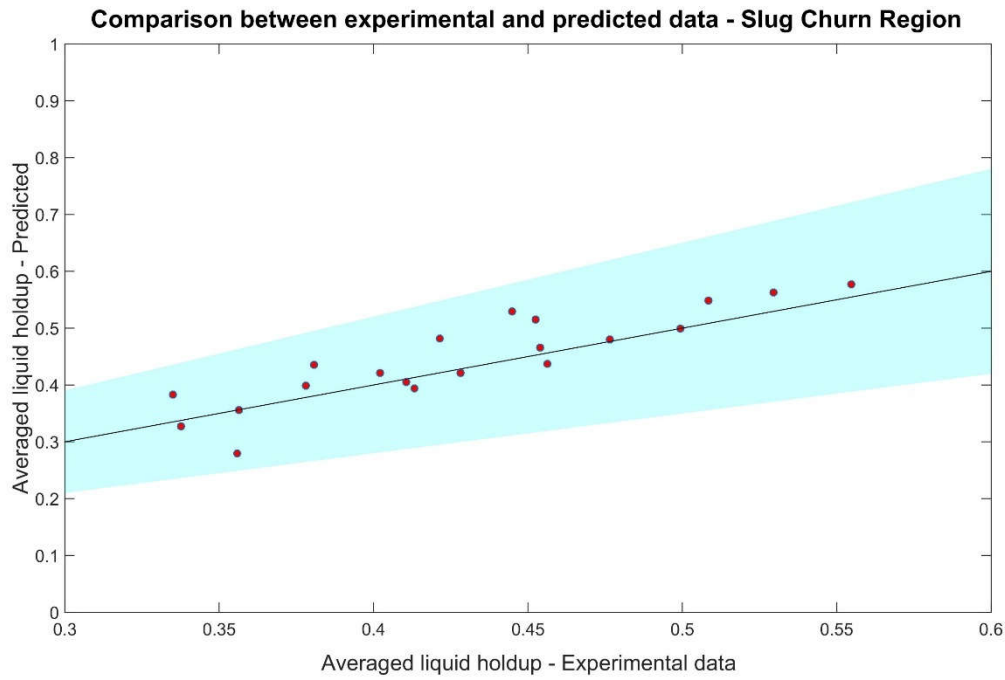


Figure 42 – Averaged liquid fraction - Comparison between predicted and experimental data - Slug/Churn region – Exp. data: Escrig Escrig (2017) - $\Delta x/D \approx 0.15$ ($\Delta x = 1$ cm)

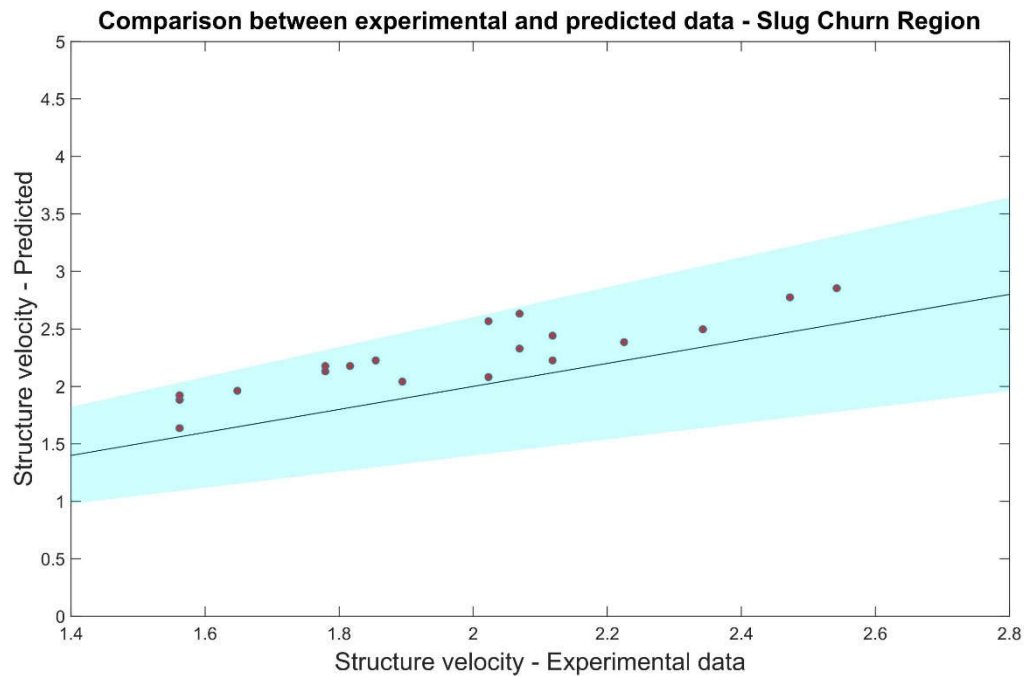


Figure 43 – Structures (slugs and waves) velocity - Comparison between predicted and experimental data - Slug/Churn region – Exp. data: Escrig Escrig (2017) - $\Delta x/D \approx 0.15$ ($\Delta x = 1$ cm)

5.3 Validation of the model

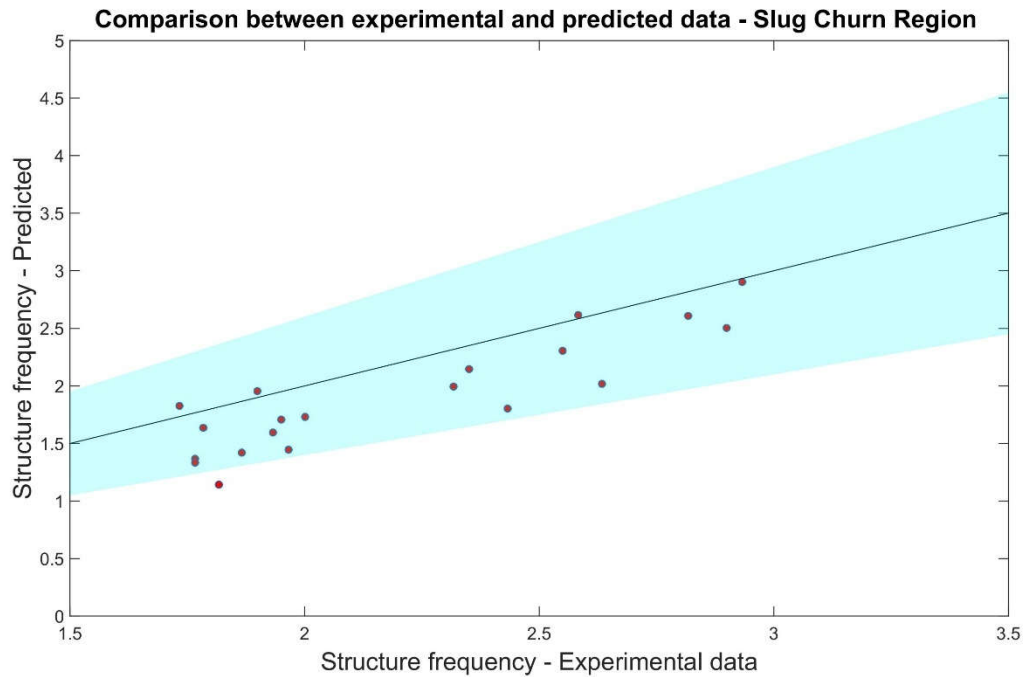


Figure 44 – Structures (slugs and waves) frequency - Comparison between predicted and experimental data - Slug/Churn region – Exp. data: Escrig Escrig (2017) - $\Delta x/D \approx 0.15$ ($\Delta x = 1$ cm)

5.3.6 Influence of boundary conditions

In Figure 45 to Figure 46 a comparison between the results obtained with the two different boundary conditions described in section 5.3.3 (zero momentum and axial momentum imposed at the inlet) is shown for the cases with gas superficial velocity equal to 0.28 m/s and different liquid velocities. It is clear that, as for the experimental cases, the results are the same for both the conditions.

Furthermore, as a preliminary study, the effects of the liquid fraction imposed at the inlet were also analysed. In Figure 49 to Figure 50 the results for six different cases in the slug flow regime region are presented. For each case, three different boundary conditions were imposed, corresponding to inlet liquid

5.3 Validation of the model

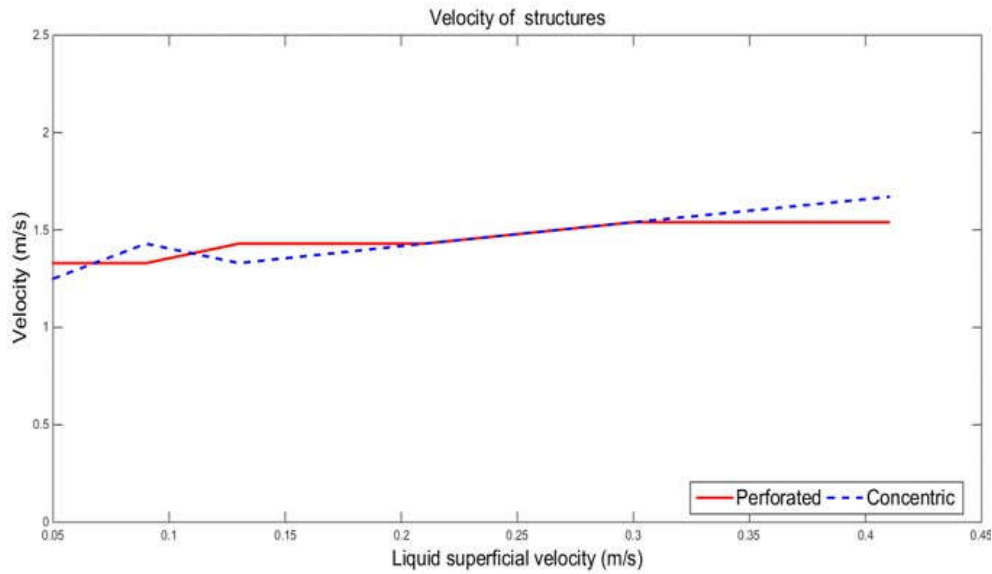


Figure 45 - Velocity of the structures (slugs) - Comparison between different boundary conditions – Imposed axial momentum (blue) and zero momentum (red) conditions - $\Delta x/D \approx 0.15$ ($\Delta x = 1$ cm)

fraction equal to 0.25, 0.5 and 0.75, while the mass flow rate at the inlet was kept constant (therefore varying the imposed gas and liquid superficial velocities). As can be seen the predicted characteristics are independent of those boundary conditions.

As a summary, the results presented above confirm that the main characteristics of slug flow calculated at a sufficient distance from the bottom depend only on the mass flow rate imposed at the inlet and not on the type of boundary conditions or on the imposed values of liquid fraction or phase velocities.

5.3 Validation of the model

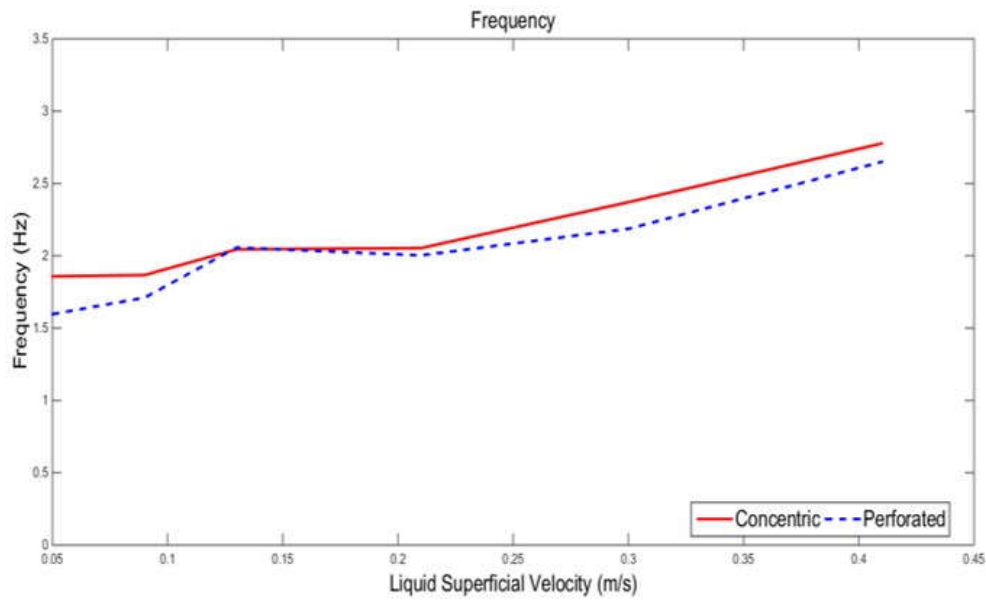


Figure 46 - Slug frequency - Comparison between different boundary conditions – Imposed axial momentum (blue) and zero momentum (red) conditions - $\Delta x/D \approx 0.15$ ($\Delta x = 1$ cm)

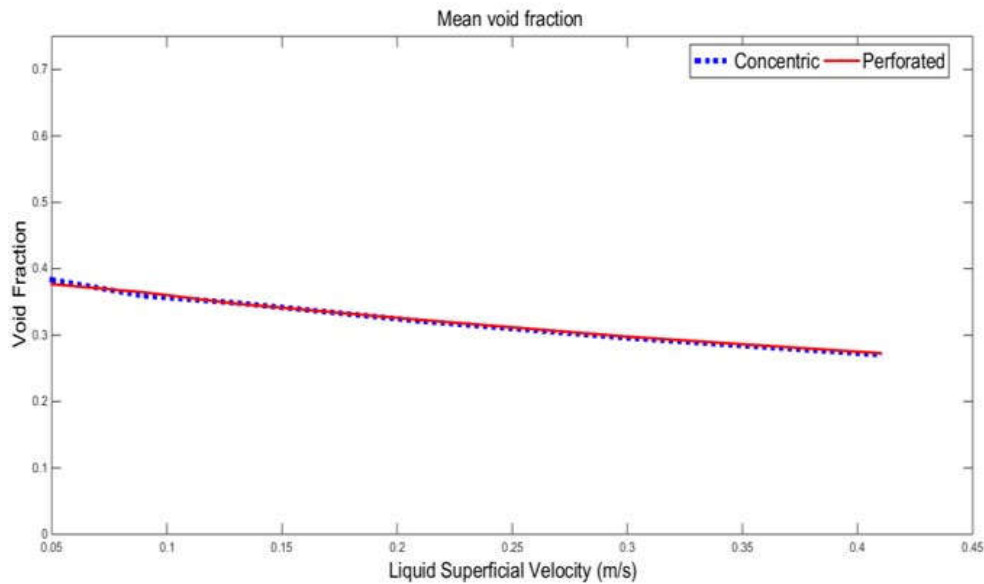


Figure 47 - Average void fraction - Comparison between different boundary conditions – Imposed axial momentum (blue) and zero momentum (red) conditions - $\Delta x/D \approx 0.15$ ($\Delta x = 1$ cm)

5.3 Validation of the model

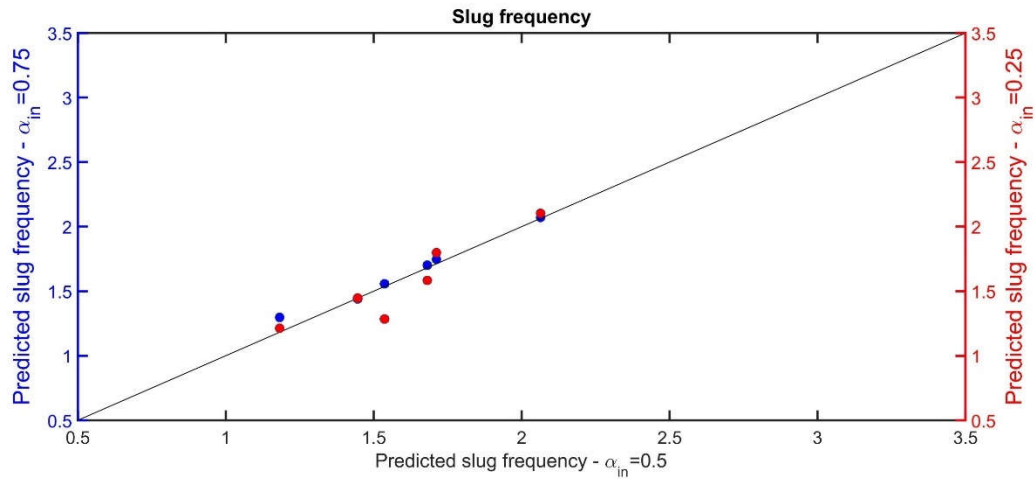


Figure 48-- Averaged liquid fraction - Comparison between predicted values with different boundary conditions for the liquid fraction imposed at the inlet (α_{in}). - $\Delta x/D \approx 0.15$ ($\Delta x = 1$ cm)

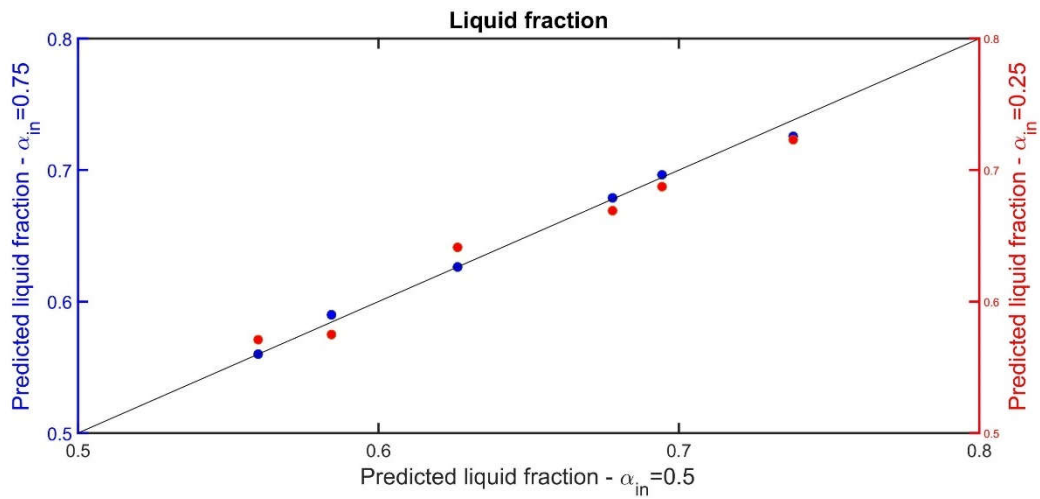


Figure 49 - Averaged liquid fraction - Comparison between predicted values with different boundary conditions for the liquid fraction imposed at the inlet (α_{in}). - $\Delta x/D \approx 0.15$ ($\Delta x = 1$ cm)

5.4 Mesh sensitivity study

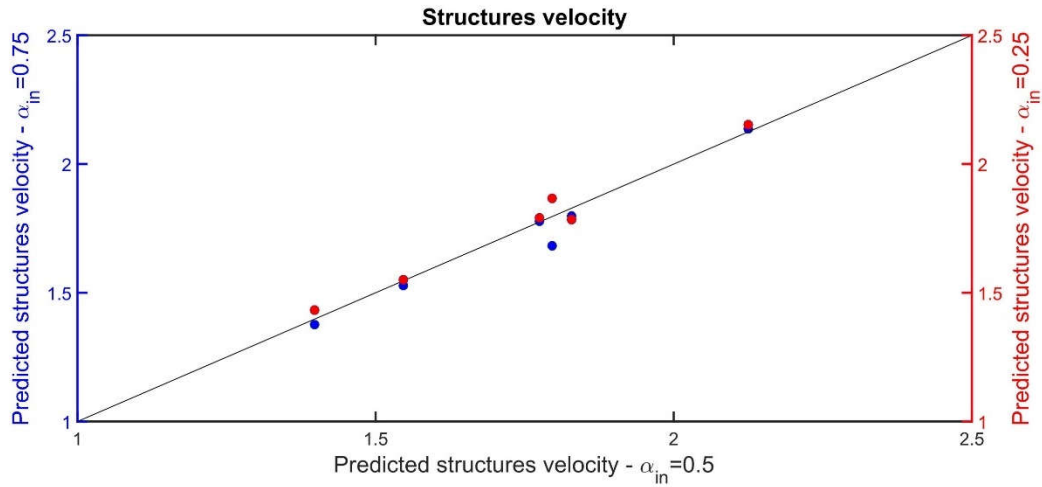


Figure 50 - Structures velocity - Comparison between predicted values with different boundary conditions for the liquid fraction imposed at the inlet (α_{in}) - $\Delta x/D \approx 0.15$ ($\Delta x = 1$ cm).

5.4 Mesh sensitivity study

In a well-posed model successive refinement of the mesh will lead to a mesh independent solution, but this would not occur when the system is ill posed. This was indeed found to be the case in horizontal flow (Issa, (2007); Montini and Issa, (2010)). Theoretically in the case for vertical flow the two-fluid model is unconditionally ill-posed and mesh independence should not in principle be attainable.

However, as it was shown in Chapter 4 , since the model differential equations are only solved numerically, discretisation errors (namely numerical diffusion) on practical mesh sizes that are normally employed can introduce their own well-conditioning of the system. Indeed, a mesh sensitivity study was carried out to establish the mesh independence of the solutions and to determine the minimum mesh size that would correctly capture the physical phenomena being simulated. The result of this study for one test case (for the sake of brevity only this case

5.4 Mesh sensitivity study

is presented here) is shown in Figure 51.

As can be seen, the variation of the computed characteristics still depends on the mesh refinement but the change is small and the simulation gives reasonable results (discrepancy below 30%) when the grid size is smaller than 2 centimetres which is equivalent to $\frac{\Delta x}{D} \approx 0.3$.

The finding that the predicted slug frequency is not affected by the refinement of the mesh is particularly important and will be briefly discussed in the summary section 5.8 and in Chapter 7 , where the general conclusions are drawn and recommendations for future work are suggested.

5.4 Mesh sensitivity study

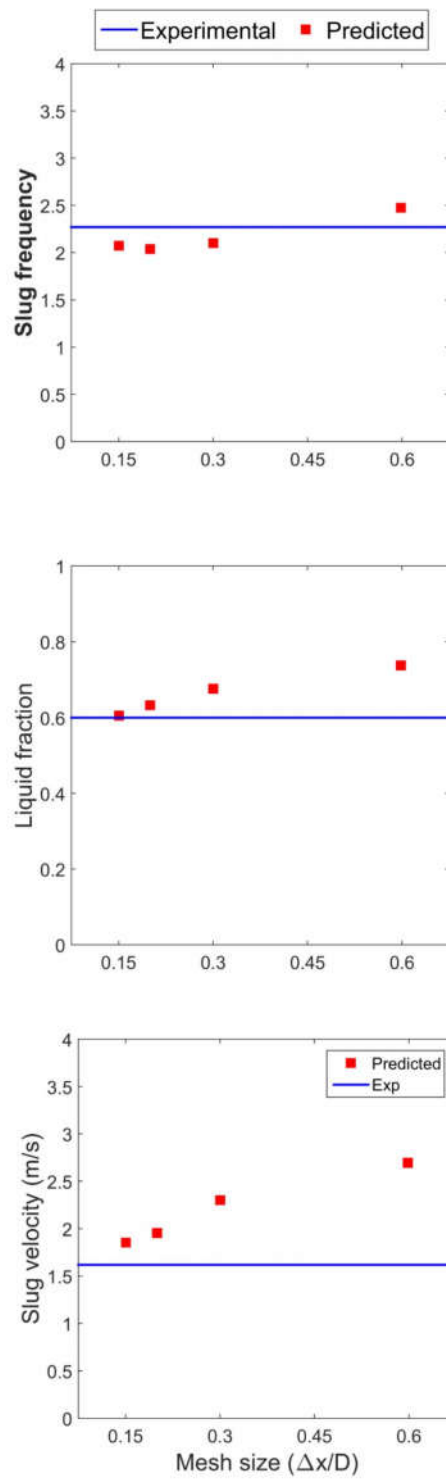


Figure 51- Predicted slug flow characteristics at different mesh sizes – Exp. data: Escrig Escrig (2017).

5.5 Axial development

As already mentioned in Chapter 2 the slug flow regime is also characterised by a strong axial development of the flow characteristics along the pipe; therefore, in addition to a good agreement with the experimental data measured at a single point, in order to completely and accurately characterise the flow regime the model should be able to capture also the axial variation of properties such as the liquid fraction, the frequency of the slugs or the absolute pressure distribution, at least where the slug regime starts to develop.

It has been shown above (Figure 26) that the model predicts qualitatively the growth in length of the Taylor bubbles (due to the coalescence of smaller cap bubbles and to the axial decrease in gas density) and the collapse of shorter slugs which leads to the formation of longer and more stable slugs with the attendant decrease in slug frequency along the pipe. This demonstrates that the model is able to capture, at least qualitatively, the highly dynamic nature of vertical slug flow.

To obtain a quantitative evaluation for the prediction of the evolution of the flow and the axial distribution of its characteristics along the pipe a comparison has been made with experimental data from the work of Waltrich et al., (2013); in that work, carried out on a 42 meters long vertical pipe of internal diameter of 48 mm, the liquid fraction was measured at three different axial positions whilst the pressure was measured at five locations. Further details on the experimental and numerical set-up can be found in Chapter 6.

Although the work was primarily focused on churn and annular flow, a few cases were classified as slug flow; the numerical results and experimental data

5.5 Axial development

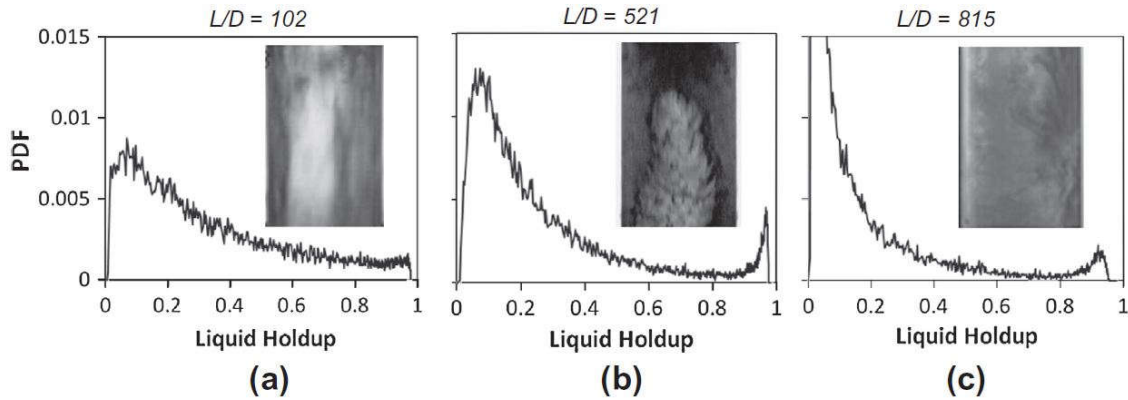


Figure 52 - Axial evolution of the flow regime identified by the PDF of the liquid fraction at different positions along the pipe - (a) is distribution commonly associated to churn flow, whilst (b) and (c) to slug flow - From Waltrich et al. 2013.

presented in the following sections and shown in figures Figure 53, Figure 54 and Figure 55 are for a case with a superficial gas velocity equal to 1.7 m/s and a liquid superficial velocity of 0.15 m/s; the pressure at the outlet was atmospheric. By means of visual observation and analysis of the Probability Density Function (PDF) of the liquid holdup at different axial positions, Waltrich et al., (2013) found a region of underdeveloped slug flow (or churn-like region) at a distance equal to 102 diameters from the inlet (around 5 metres from the bottom) of the pipe and a “fully developed” slug flow at 521 diameters (Figure 52); furthermore, in their work all the measured characteristics showed a significant axial variation.

5.5.1 Slug Frequency

Figure 53 shows the predicted results for the frequency of the slugs compared with experimental data from Waltrich et al., (2013); as can be seen the evolution of the frequency is qualitatively well captured by the model and furthermore, the

5.5 Axial development

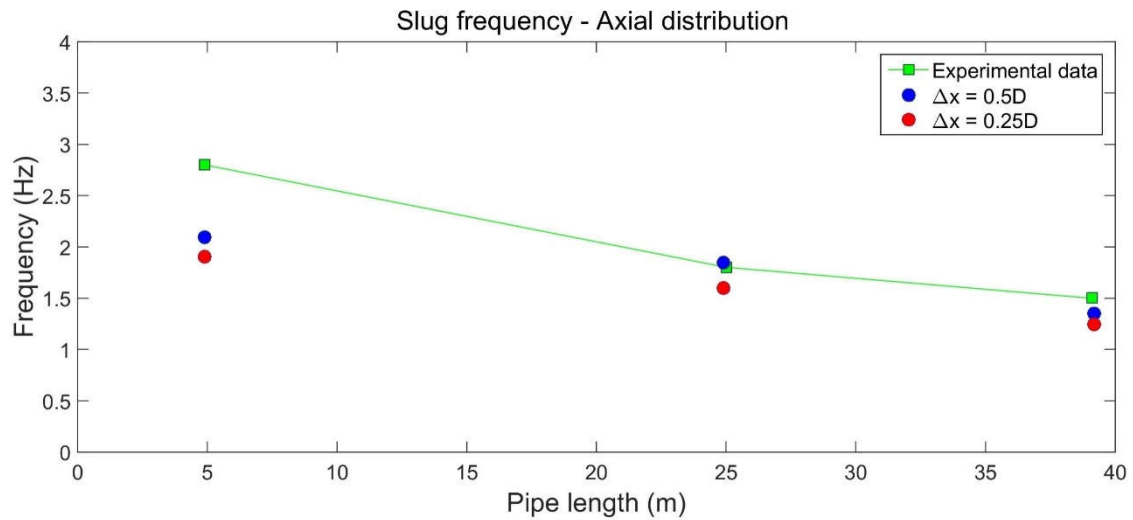


Figure 53 - Axial distribution of slug frequency for different sizes of the mesh - Exp. data: Waltrich et al. (2013)

predicted values are in good agreement with the experimental data in the region where the slug flow is considered to be fully developed (25 and 39 meters from the bottom of the pipe).

A significant under prediction is seen for the first axial position (5 meters); however, in this region, as reported in the paper, the flow is characterised by a churn-like regime and therefore the comparison of the calculated frequency of the slugs with the measured frequency of the structures might be misleading (for instance, the filter of two diameters applied to count the slugs in the simulation might need to be tuned).

In Figure 53, two different results are presented, for two different sizes of the mesh (2.5 cm and 1.2 cm, corresponding respectively to $\frac{\Delta x}{D} \approx 0.5$ and $\frac{\Delta x}{D} \approx 0.25$). It is clear from the figure that the predicted slug frequency is found to be only weakly dependent on the mesh size and this is an important confirmation of the

5.5 Axial development

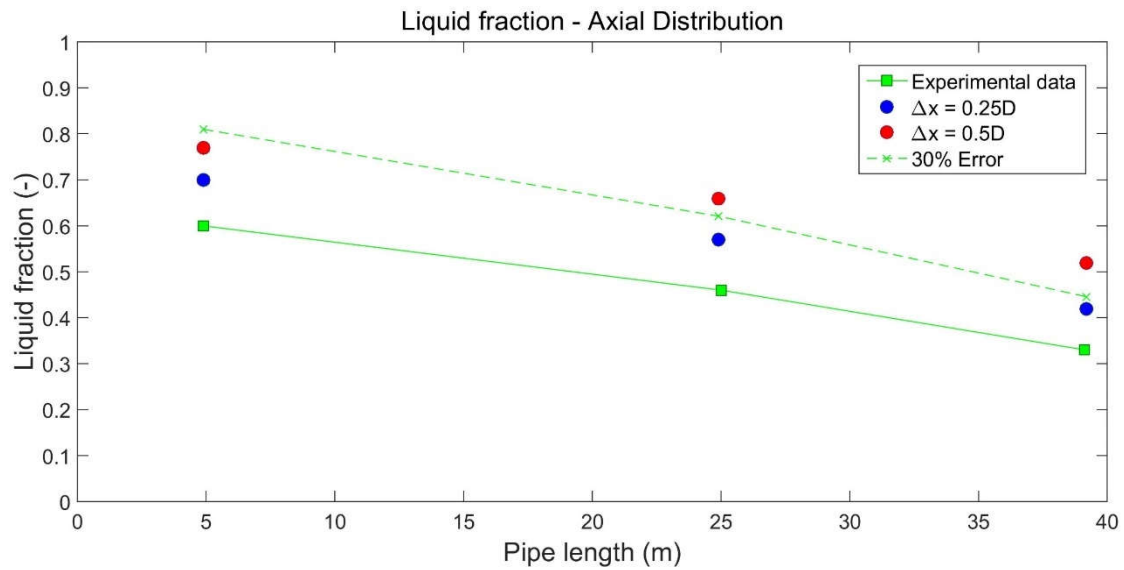


Figure 54 - Axial distribution of liquid fraction for different sizes of the mesh - Exp. data: Waltrich et al. (2013)

findings reported above in section 0 for the Nottingham cases.

5.5.2 Liquid fraction

A comparison between the predicted values and the experimental data for the axial development of the liquid holdup along the pipe is shown in Figure 54. As can be seen the development of the liquid fraction is again fairly well predicted by the model, with all the computed values inside a 30% error range for the mesh size equal to 1.2 cm ($\frac{\Delta x}{D} \approx 0.25$).

It is nevertheless important to notice that there is a general over-prediction of the global liquid fraction. However, in this case a significant change in the prediction is seen as the mesh is refined, with a decrease of the predicted liquid fraction by about 15% when refining the mesh from a cell size of 2.5 cm to a cell size of 1.2 cm; it is highly likely that the computed fraction would further decrease

5.5 Axial development

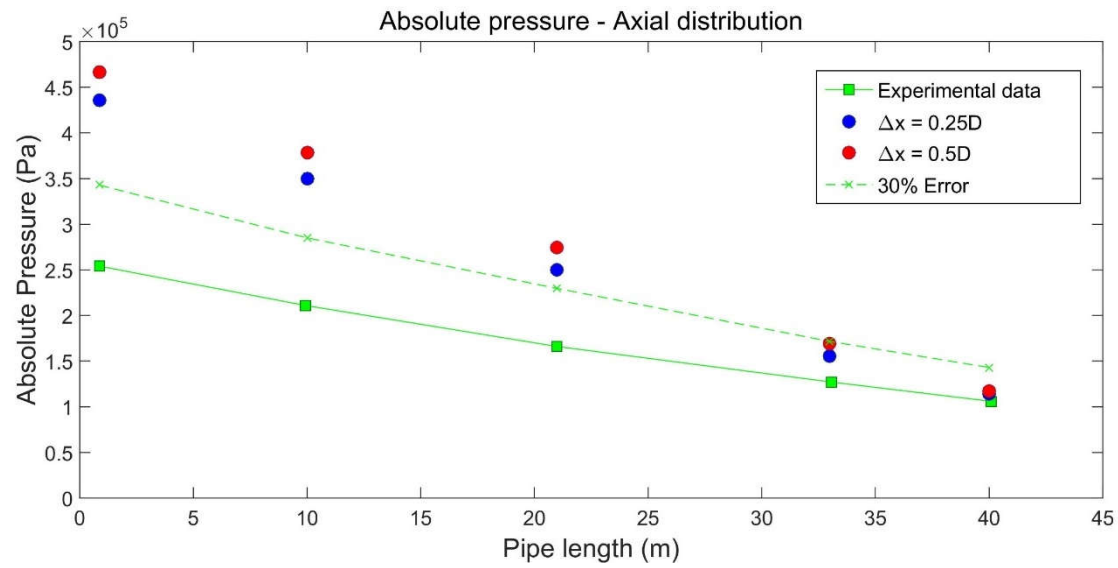


Figure 55 - Axial distribution of absolute pressure for different sizes of the mesh - Exp. data: Waltrich et al. (2013)

with a finer mesh. This result follows the trend found for the cases presented above, where the prediction of the liquid fraction was found to be acceptable but still dependent on the mesh for cell sizes equal to roughly $0.3D$; however, in this case it has not been possible to reduce the cell size below this value, due to computational issues: since the pipe is 42 meters long, a mesh of more than 6000 points would be required to decrease the cell size to $0.1D$, resulting in a computational time of several weeks with the current code. A discussion on the computational issues encountered in vertical slug capturing will be presented in Chapter 7 .

5.5.3 Pressure

Figure 55 presents the results for the predicted axial distribution of pressure, compared with experimental data. It is clear that there is a strong over-prediction

of the pressure in the churn-like developing region, with the error well above 50% both at 1 meter and 10 meters from the inlet. The error significantly reduces at farther positions along the pipe, being around 35% at 21 meters and below 30% at 33 and 40 meters from the inlet.

As stated in the previous section, the error in the last three positions (slug flow region) is likely to further decrease as the mesh is refined and the liquid fraction is predicted with higher accuracy, since in a vertical configuration the pressure drop due to the gravitational component is dominant.

The over-prediction in the first 10 meters is, on the other hand, probably due to an incorrect choice of the wall friction factors used in the highly chaotic developing region; as explained in Chapter 3 , the wall and interfacial friction factors are the same throughout the pipe and are those found to yield the best results for the prediction of vertical slug flow. An attempt was made to apply interfacial friction factors specifically developed for churn flow in a region close to the inlet but this application resulted in a significant reduction of the predicted slug frequency; this issue will be briefly discussed in the following section.

Nevertheless, it is important to remark that this over-prediction does not seem to invalidate the results in the “fully developed” slug flow region.

5.6 Effect of interfacial friction factors

In order to study the effects of the interfacial shear forces on the predicted characteristics, three different correlations were implemented and tested: (i) a “full pipe” Blasius type correlation, based on the slip velocity between the phases, (ii) a correlation specifically developed for churn flow proposed by

Bharathan, which depends only on the local liquid fraction and (iii) a factor which takes into account both the relative velocity and the liquid structures, suggested in Inada et al; these correlations are were presented in Chapter 3 . The attempt of using a correlation developed for churn flow is justified by the fact that, as explained in Chapter 2 and in the previous section, this kind of regime is often indicated as being a precursor to slug flow and it is therefore important to understand if even this correlation will lead to the formation of slugs.

Figure 56, Figure 57 and Figure 58 present the predictions obtained for the case introduced in section 5.5 (Waltrich et al., (2013)) with the different friction factors. According to the results for the axial development of the liquid fraction and the absolute pressure, the correlation suggested by Bharathan seems to give the best prediction throughout the pipe, with a relatively low error even in the developing region. However, looking at Figure 58, it is clear that the predicted slug frequency with this correlation is zero at all the positions, meaning that no slugs were formed during the simulation. As for the Inada correlation, the predicted characteristics are similar to those obtained with the Blasius formulation, but there is a significant under-prediction of the slug frequency at the end of the pipe, probably due to an underdevelopment of the slugs.

To further investigate this issue, it is possible to plot a snapshot of the instantaneous distribution of the liquid fraction in the pipe, as in Figure 59. From this picture it is evident that no slugs are formed with the Bharathan correlation, whereas for the Inada correlation slugs are formed but appear to be shorter and less stable than those predicted with the Blasius formulation.

5.6 Effect of interfacial friction factors

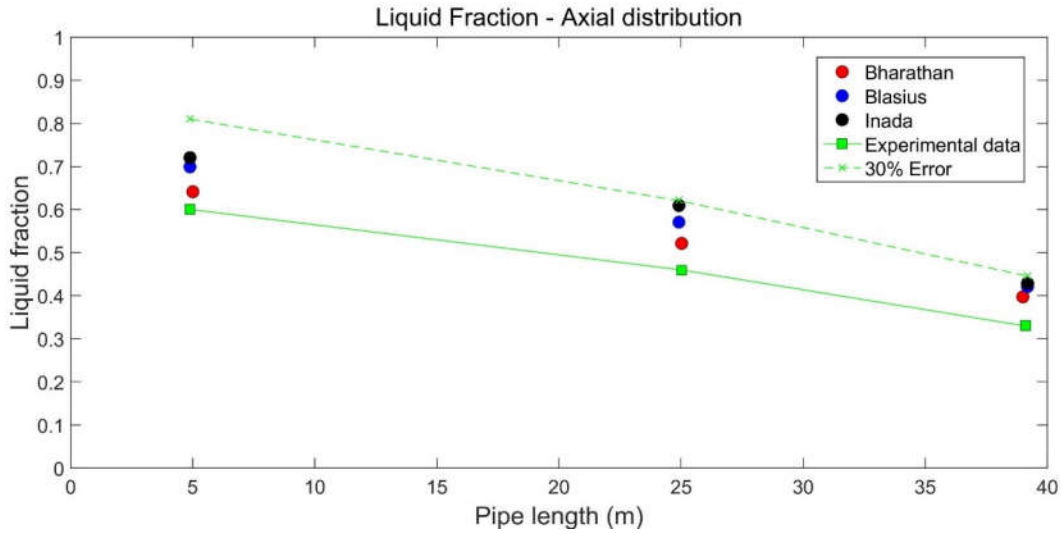


Figure 56 - Predicted axial distribution of the liquid fraction for different correlations of the interfacial friction factor, compared with experimental data - Exp. data: Waltrich et al. (2013) - $\Delta x/D \approx 0.25$ ($\Delta x = 1.2$ cm)

These results can be explained by comparing the time-averaged values of the liquid velocity, as shown in Figure 60; as can be seen, for the simulation with the Blasius friction factor the velocity is mainly negative for a large section of the pipe (approximately starting 10 meters from the bottom). This means that, since the Blasius correlation yields to relatively weak interfacial forces, the gas is unable to lift the liquid which mainly falls down flooding the pipe and this effect triggers the formation and facilitates the development of slugs, as explained above. On the other hand, in the Bharathan correlation case, the interfacial forces are too strong and the liquid velocity never becomes negative; therefore, wave-like structures are formed but they never develop into slugs, because of the absence of the flooding mechanism. Finally, because the magnitude of the forces given by the Inada correlation is closer to the Blasius one but still higher, it allows the liquid to fall and initiate slugs but the flooding effect is not sufficient to permit their full development.

5.6 Effect of interfacial friction factors

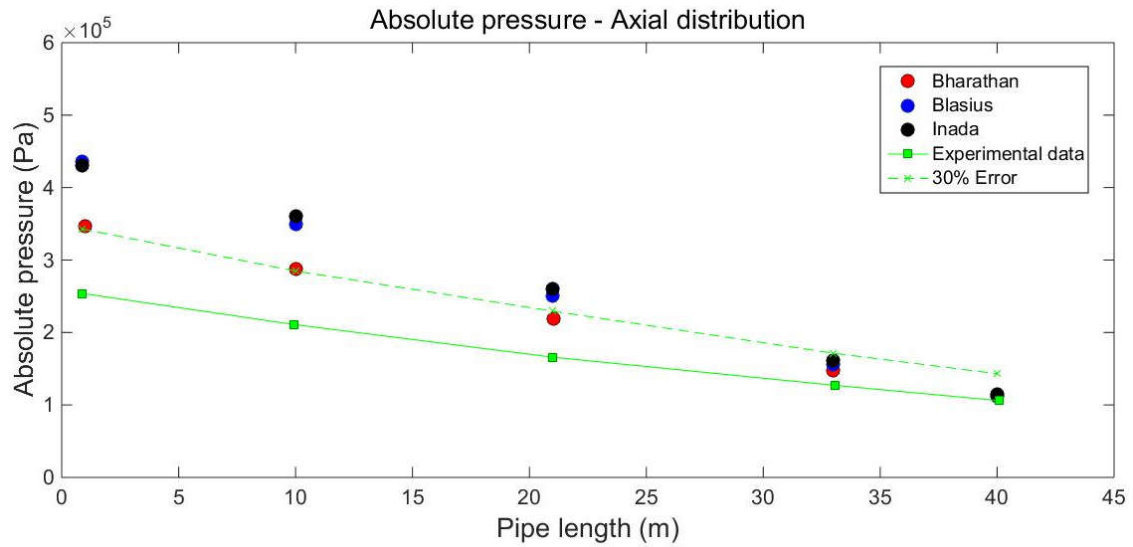


Figure 57 - Predicted axial distribution of the absolute pressure for different correlations of the interfacial friction factor, compared with experimental data - Exp. data: Waltrich et al. (2013) - $\Delta x/D \approx 0.25$ ($\Delta x = 1.2$ cm)

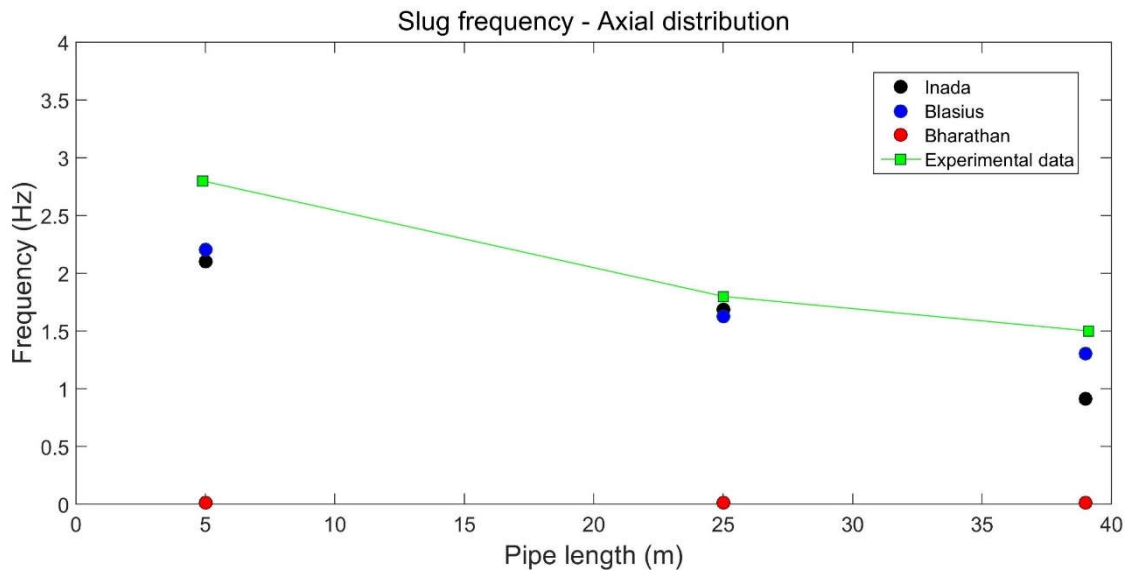


Figure 58 - Predicted axial distribution of the slug frequency for different correlations of the interfacial friction factor, compared with experimental data - Exp. data: Waltrich et al. (2013) - $\Delta x/D \approx 0.25$ ($\Delta x = 1.2$ cm)

5.6 Effect of interfacial friction factors

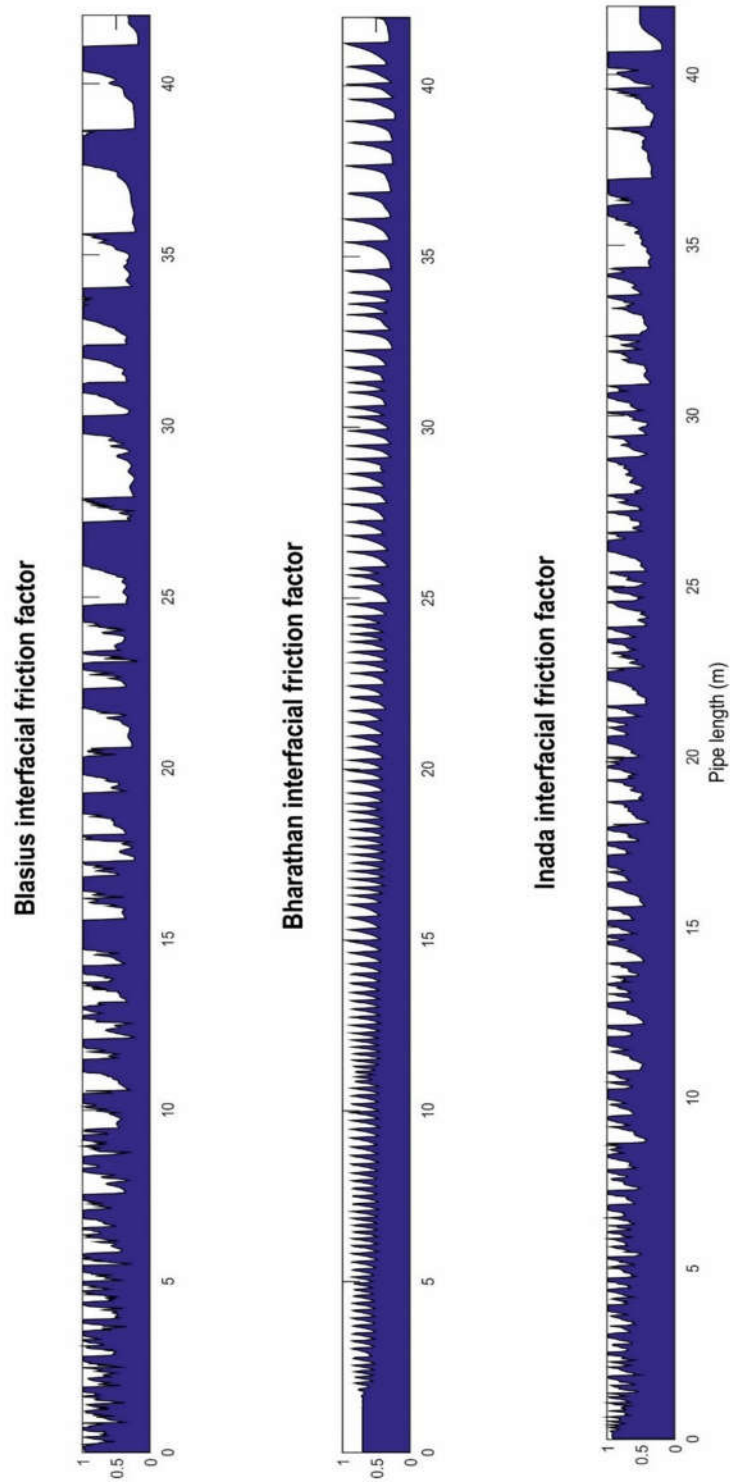


Figure 59 - Snapshot of the instantaneous distribution of the phases with different correlations for the friction factors $\Delta x/D \approx 0.25$ ($\Delta x = 1.2$ cm)

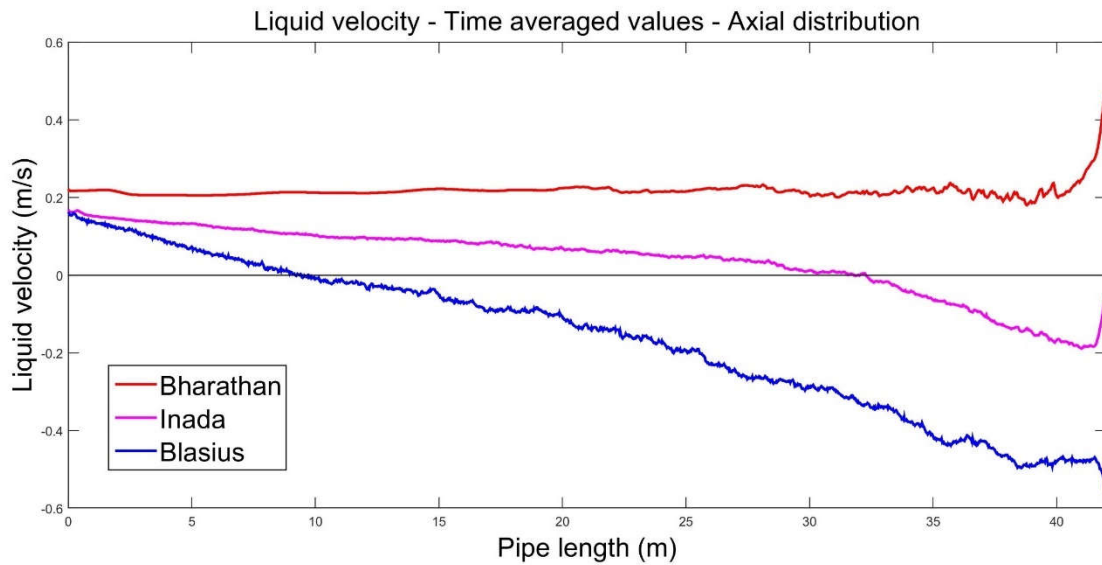


Figure 60 – Time averaged liquid velocity - Predicted axial distribution for different correlations of the interfacial friction factor $\Delta x/D \approx 0.25$ ($\Delta x = 1.2$ cm)

5.7 Small diameter

In Chapter 6 several results will be presented in order to assess the capability of the model in predicting the main characteristics of churn flow and these results will be compared with experimental campaigns described in literature; in one of these campaigns, Sawai et al., (2004) gave also numerous measurements of the pressure gradient in slug flow in a pipe with small diameter (i.e. $D < \sim 3$ cm), as shown in Figure 13.

Although the present work is mainly focused on slug flow in medium and large diameter configurations, as a preliminary study also the predictions of the pressure gradients obtained for the cases of Sawai et al., (2004) in the slug flow region are presented and discussed below, together with the experimental setup.

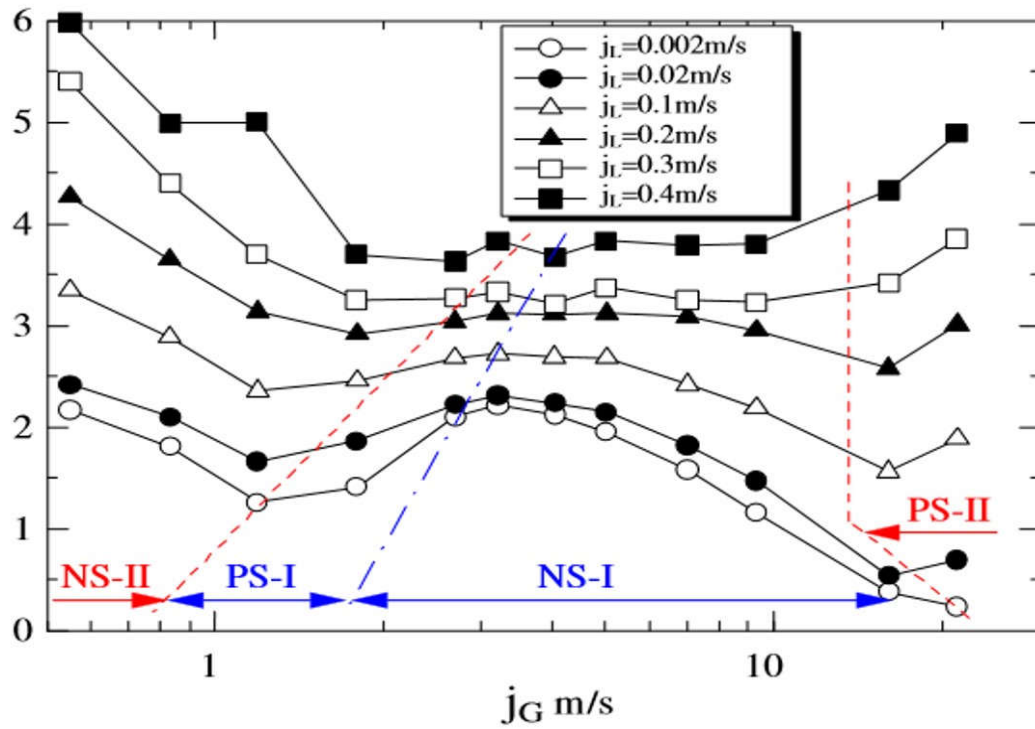


Figure 61 - Variation of pressure gradient with gas superficial velocity and classification of the flow patterns - NSII: Slug flow, PSI: Transition region, NSI: Churn flow, PSII: Annular flow – From Sawai et al. 2004

5.7.1 Experimental setup

The experimental rig used by Sawai et al., (2004) consisted of a vertical tube pipe 4.2 meters long with an internal diameter of 25.8 mm. The pressure drop was measured by a pressure difference transducer over a length of 1422 mm. The fluids used were air and water with superficial velocities ranging from 0.6 to 21.3 m/s and from 0.002 and 0.4 m/s, respectively. Further details can be found in Chapter 6 .

The flow regimes obtained during the experiments were identified according to the variations of the time averaged pressure gradients with the gas volumetric flux, as explained in Chapter 2 . In Figure 61 it is possible to see this

classification. In the present section, only the slug flow region is under investigation.

5.7.2 Results

In total, 18 cases were simulated and for each case three different correlations for the interfacial friction factors were used; the numerical predictions obtained for the pressure gradient are presented in Figure 62 and compared with the experimental data.

From Figure 62, it is evident that none of the correlations is able to give acceptable results. The pressure gradient is strongly over-predicted, especially for the cases at low liquid velocities (0.6 and 1.2 m/s); for higher gas and liquid flow rates the errors decrease but it remains significant.

Such a high over-prediction is likely due to an incorrect prediction of the liquid holdup since, as already remarked, in vertical slug flow the pressure drop caused by the hydrostatic pressure is dominant. To clarify this point, in Figure 63 it is possible to see a bar chart -from both experimental data and numerical simulation - highlighting the ratio between the hydrostatic/gravity pressure gradient and the frictional pressure gradient term due to the wall shear. It is clear that the gravity term is much higher than the frictional term, and this dominance is even more accentuated for the simulated cases.

5.7 Small diameter

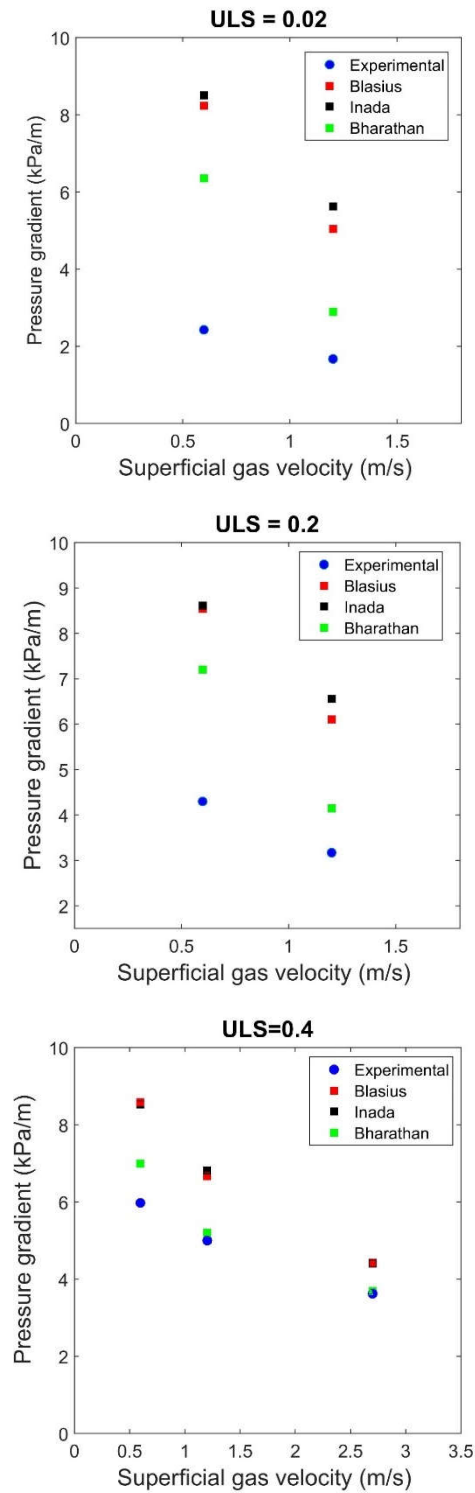


Figure 62 - Pressure gradient - Comparison between predicted and experimental data in the slug flow regime region from Sawai et al. 2004 - $\Delta x/D \approx 0.2$ ($\Delta x = 0.5$ cm)

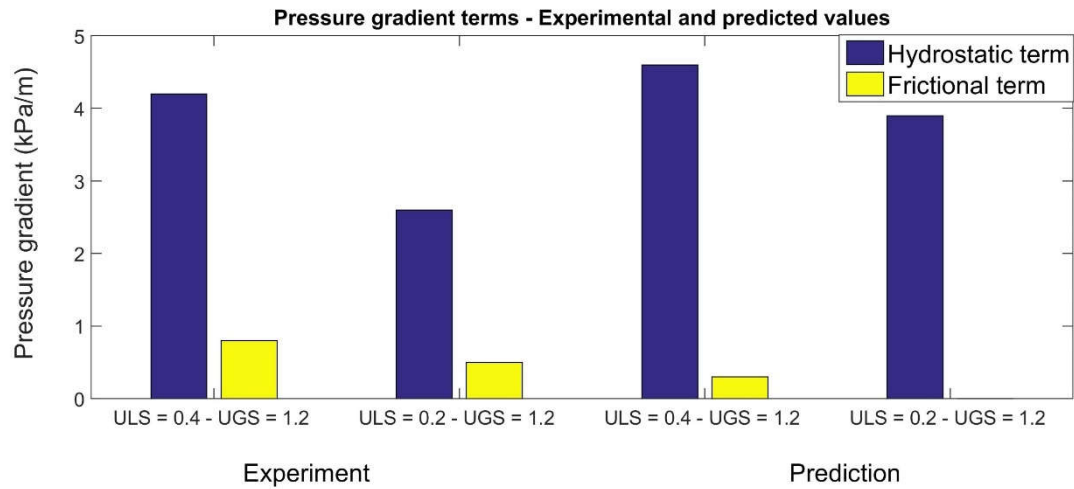


Figure 63 – Comparison between the hydrostatic and frictional terms of the pressure gradient - Both in the experimental data (Sawai et al. 2004) and in the predicted results the hydrostatic term is strongly dominant - $\Delta x/D \approx 0.2$ ($\Delta x = 0.5$ cm)

Unfortunately, it has not been possible to perform a meaningful and complete comparison of the components of the predicted and the experimental values of the liquid fraction because neither the measurements of the liquid fraction nor those of the wall shear stresses in the slug flow region were presented in the paper comprehensively.

5.8 Summary

In this Chapter it has been proved that the two fluid model is capable of predicting the phenomena behind the initiation and evolution of vertical slug flow and its main characteristics, both from qualitative and quantitative points of view.

The model successfully predicts the evolution of the flow regime and the development of the flow structures (i.e. cap bubbles, Taylor bubbles and slugs) which naturally happen as the flow rate of the phases is increased; this finding

is confirmed both from simple visualisations of the axial distribution of the liquid fraction and from the calculation of the liquid fraction Probability Density Function at different flow rates. Moreover, all the predicted characteristics - such as slug frequency and velocities and the averaged liquid fraction - compare fairly well with the experimental data.

Furthermore, a mesh sensitivity analysis showed that the predicted slug frequency is not affected by the refinement of the mesh, at least for practical grid sizes. This is an important finding showing significant difference from the horizontal case. As explained, the driving mechanism for the generation of slugs in vertical flow is the flooding of the pipe caused by the falling liquid, and not the growth of natural instabilities such as in the horizontal case; therefore, the formation of slugs is significantly less affected by numerical perturbations and the numerical diffusivity introduced by the discretization of the system -as showed in Chapter 4 - is sufficient to damp those for practical mesh sizes.

It is also worth remarking that the model performed well also in the transition region between slug and churn flow regimes. This is a significant achievement, considering the important objective of developing a model capable of automatically predicting flow transition. However, further investigation would be necessary to correctly validate the model in this region. A preliminary study on the capability of the model to predict the main features of fully developed churn flow regime will be presented in the next Chapter.

The model was also validated in predicting the axial development of the flow along the pipe, comparing the predicted characteristics against data from the literature. It has been shown that the model is able to give reasonable results

for all the characteristics in the region where the slug regime starts to develop. On the other hand, in the chaotic region before the slug regime, the results were poor: significant over-prediction for the absolute pressure and the liquid fraction were obtained and also the frequency of the slugs was under-predicted, although this latter result is less important, given the uncertainties in the definition of slug in this churn-like region. However, it is important to remark that the error in this initial section did not affect the predictions in the slug region, confirming once more the weak dependence on the flow boundary conditions. These poor predictions are likely to be caused by wrong correlations for the interfacial friction factors in the initial region, which might need to be adjusted for the specific flow regime, and further investigation should be conducted on this issue. Indeed, the correlations used for the interfacial friction factors were found to have a crucial impact not only on the slug characteristics, but even in the prediction of the initiation of the regime. It was shown in this chapter that using a correlation developed for churn flow did not allow the liquid to fall down in the pipe and therefore prevented the formation of slugs. The best results were obtained with a weak Blasius correlation for the interfacial forces, which permitted the flooding of the pipe and the development of slugs. Further results and analyses on the simulation of churn flow with the one dimensional two fluid model will be presented in the next chapter.

Also preliminary results on the capability of the slug capturing technique with pipes of small diameters were presented; the predictions were found to be poor for the pressure gradient but, due to the lack of experimental data, it has not been possible to understand the source of error. It might be speculated that the model used is too simple in this case and it is not properly modelling the

5.8 Summary

interaction between the phases in such a small diameter, resulting in less liquid pushed out by the gas and therefore in a higher contribution to the pressure drop of the hydrostatic component. Further investigations on this issue are certainly needed and this will be pointed out in Chapter 7 .

Chapter 6

Simulation of churn flow

6.1 Preamble

Churn flow is a highly chaotic flow regime characterised by the presence of an irregular central gas core and the oscillatory motion of the liquid phase; the occurrence of liquid fall-back – in a thin liquid film attached to the wall, as in slug flow – leads to the subsequent formation of large wave structures due to flooding but, due to the high gas flow rates at which it occurs, these wave structures rarely develop into slugs. In addition to this, the central gas core might also carry a large amount of entrained liquid (Azzopardi, (2006)). Churn flow can either develops from slug flow or from bubbly regime as the flow rate of the gas phase is increased depending on the diameter of the pipes (in large diameter pipes slug flow tends not to occur).

In the present work a preliminary attempt has been made to extend the slug capturing technique to the prediction of the churn flow regime, also in the light of the good results obtained through the simulation of the “transition region” presented in Chapter 5 .

Due to this chaotic nature, it is indeed yet not clear if one-dimensional codes can be suitable tools for the simulation of this regime; to the author's knowledge there are no works reported in literature on the numerical prediction of the main characteristics of churn flow with the slug capturing technique, for instance. However, as already remarked above, it would be of high importance to develop a model capable of mechanistically predict the whole range of flow regimes occurring in vertical pipes. Therefore, the work presented here should be seen as a preliminary assessment of the capability of the one-dimensional two-fluid model of simulating churn flow regime.

In Section 6.2 the results obtained for the simulation of churn flow in a medium diameter pipe are presented. All the cases are taken from the work of Waltrich et al., (2013) which was carried out in a 42 meters long vertical pipe with an inner diameter of 48 mm. It is first shown that the main features of churn flow such as the phases distribution and the characteristic Probability Density Function are well captured before a more quantitative evaluation is presented, comparing the predicted values with the experimental data. As seen for the slug flow simulations in Chapter 5 it is pointed out that the predictions strongly depend on the interfacial friction factors chosen.

In section 6.3 the same evaluations are presented for a different geometry taken from Sawai et al., (2004); in this experimental campaign the rig consisted of a vertical pipe 4.2 metres long and an internal diameter of 25 mm. In this case, besides others, the most important comparison is the evolution of the pressure gradient with the superficial gas velocity, since it gives a strong indication of how the model captures the transition between the regimes in this specific

configuration (i.e. small diameter pipes).

In section 6.4 a few predictions are presented for a large diameter pipe (127 mm) with experimental data obtained by Zangana, (2011). Although the predictions presented in this work are limited, this case is interesting because in the experimental campaign the instantaneous wall shear stresses were directly recorded.

Finally, in section 6.5 a summary of the work is given.

6.2 Churn flow in medium diameter pipes

6.2.1 Experimental test case - Tower LAB facility

The experimental results presented in this section were obtained at the TowerLAB facility of the Texas A&M University and the whole experimental campaign is described in Waltrich et al., (2013).

The facility consisted of a 42 m vertical pipe with an inner diameter of 0.048 m; the length to diameter ratio was therefore equal to 875. The fluids used in the experiments were water and air with the liquid mass fluxes ranging from 15 to 315 kg/m²s and the gas mass fluxes from 2 to 80 kg/m²s (corresponding to superficial gas velocities ranging from 0.05 to 1.5 m/s). The uncertainties were estimated to be ± 2 kg/h and ± 2.5 kg/h for the liquid and gas flow rates respectively. The experiments were carried out over a range of absolute pressures at the outlet between 1 and 5.2 bar.

A main feature of the facility is the possibility to measure the flow properties

6.2 Churn flow in medium diameter pipes

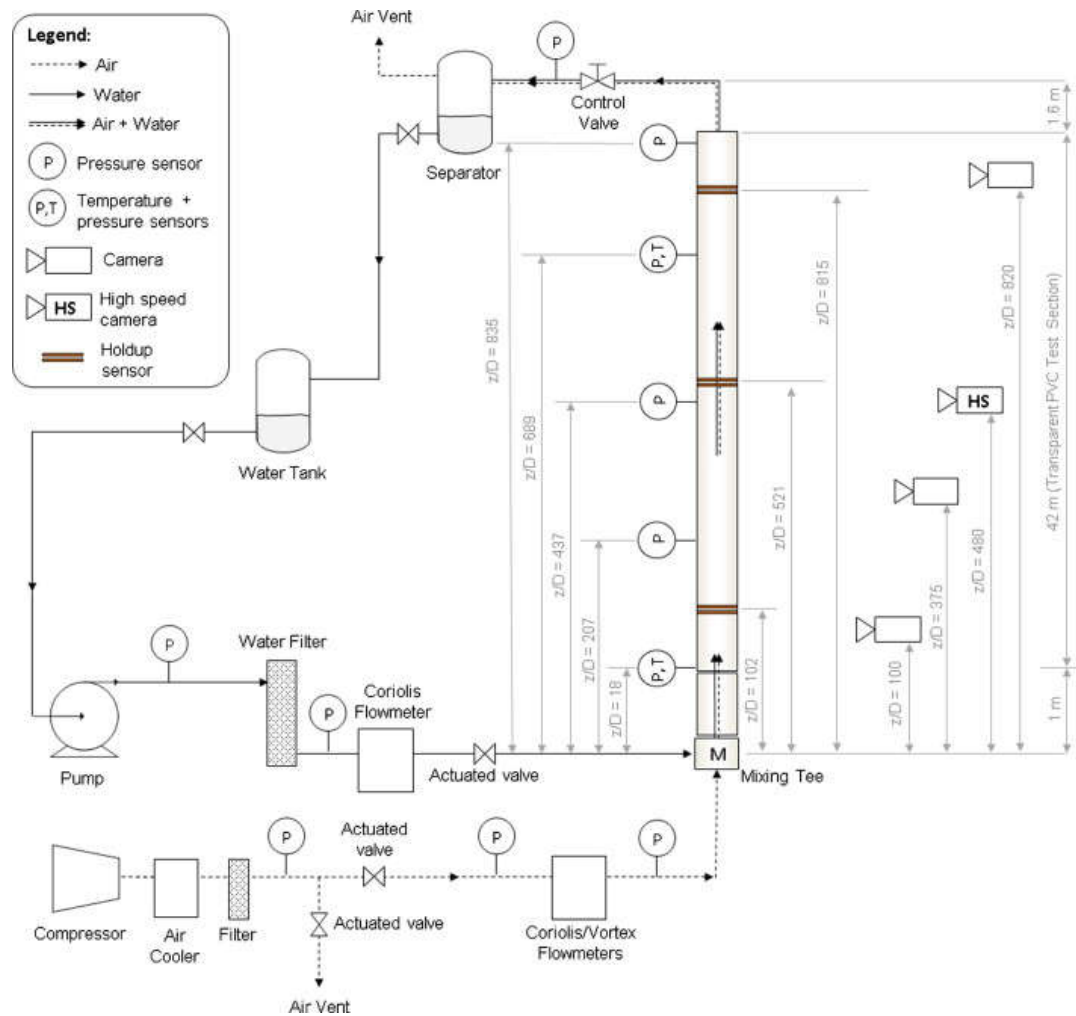


Figure 64 - TowerLAB facility - Scheme of the rig, from Waltrich et al. 2013

at different locations along the pipe; Figure 64 shows an overview of the rig and the different positions of the flow monitoring instrumentation. The liquid holdup was measured using two-wire conductivity probes located at distances of 102D, 521D and 815D (i.e. around 5, 25 and 39 meters respectively) from the bottom of the pipe whilst absolute pressure measurements were carried out at five different locations corresponding to 18D, 207D 437D, 689D and 835D (that is 1, 10, 21, 33 and 40 meters).

6.2.2 Numerical simulations

The numerical results presented below were obtained for a liquid superficial velocity equal to 0.1 m/s and four different gas velocities equal to 6, 13, 15 and 22 m/s with a pressure of 1.4 bar imposed at the outlet; it is worth noticing that the highest gas velocity corresponded to a flow regime closer to annular flow, according to the experimental classification.

The numerical setup for the boundary conditions and the initial configurations for the simulations of all the cases from Waltrich et al., (2013) is identical to the setup presented in the previous Chapter.

6.2.3 Results

Visualization

Figures from 65 to 67 present a visualization of the axial distribution of the liquid fraction along the pipe, as obtained with the three different correlations for the interfacial friction factors.

It is clear that the predicted structures are quite different from those found in slug flow simulations and their characteristics – such as size and frequency - seem to strongly depend on the correlation used for the friction factors, at least qualitatively. Apparently, no slugs are predicted and the main liquid structures consist of small waves which are formed on a thin liquid film; furthermore, the centre of the pipe is continuously occupied by a gas core. The height of the waves and the film thickness decrease as the gas velocity is increased.

6.2 Churn flow in medium diameter pipes

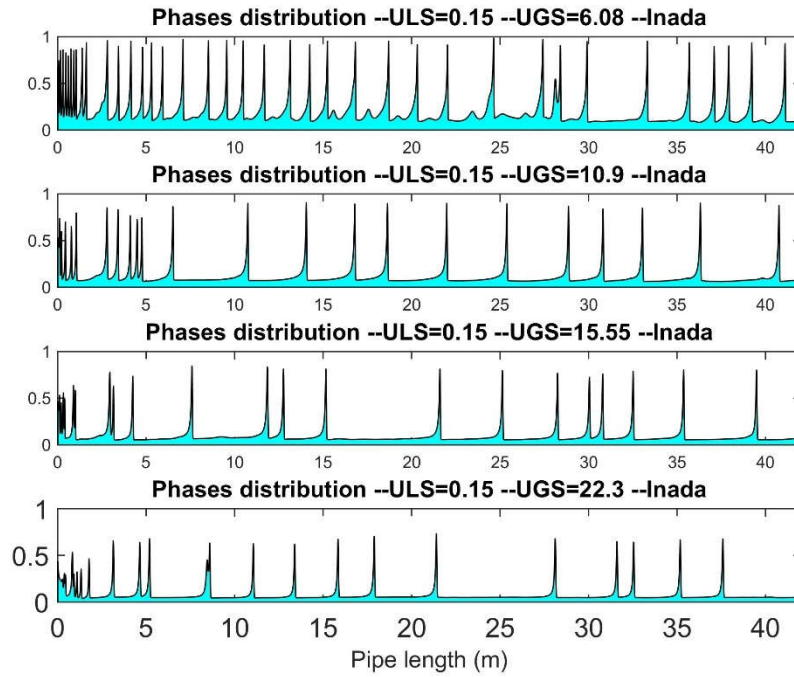


Figure 65 - Instantaneous axial distribution of the liquid fraction for different gas velocities. Prediction with the Inada correlation - $\Delta x/D \approx 0.25$ ($\Delta x = 1.2$ cm)

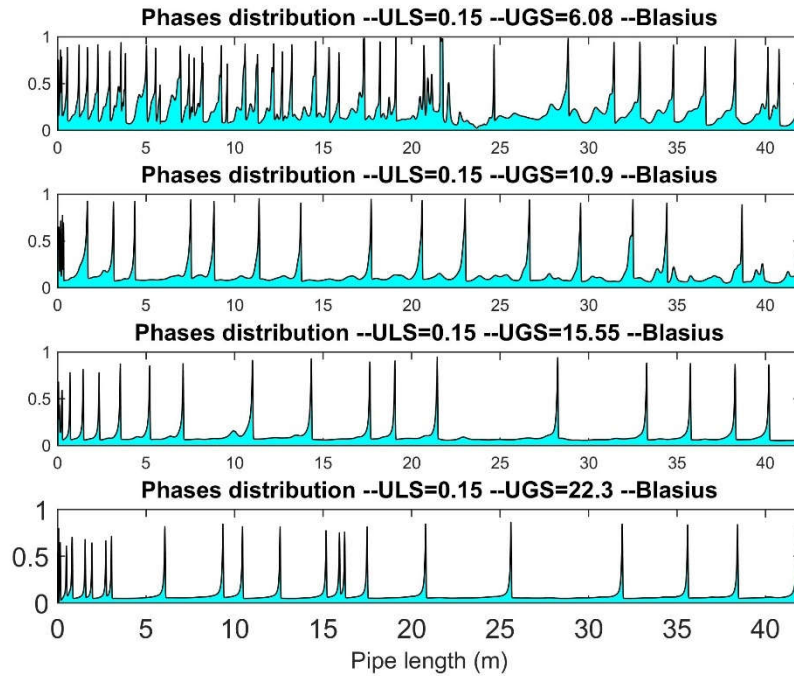


Figure 66 – Instantaneous axial distribution of the liquid fraction for different gas velocities. Prediction with the Blasius correlation - $\Delta x/D \approx 0.25$ ($\Delta x = 1.2$ cm)

6.2 Churn flow in medium diameter pipes

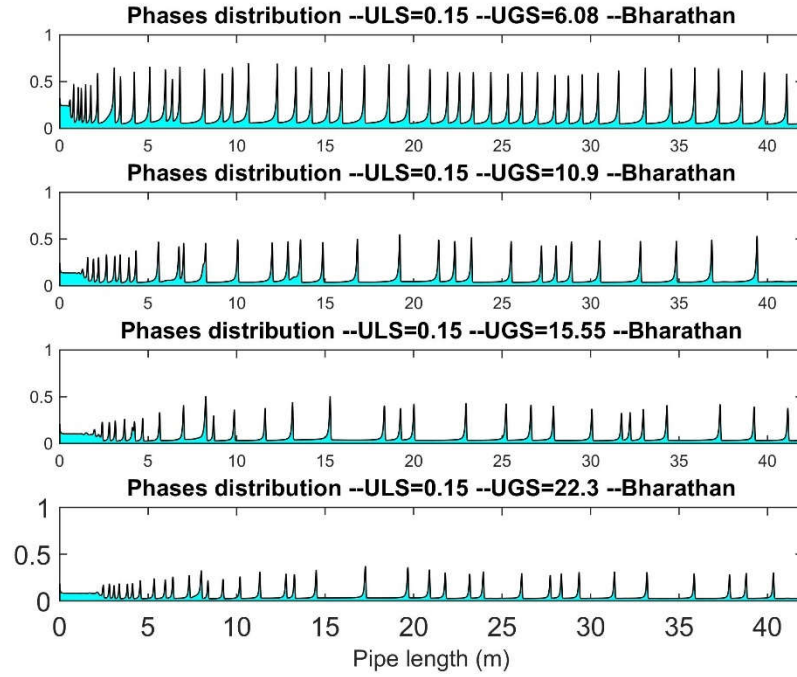


Figure 67 - Instantaneous axial distribution of the liquid fraction for different gas velocities. Prediction with the Bharathan correlation - $\Delta x/D \approx 0.25$ ($\Delta x = 1.2$ cm)

All these observations are confirmed also by the analyses of the PDF of the liquid fraction at 35 metres from the bottom, as presented in figure 68 , which show a single high peak at low values of the liquid fraction and very small or zero probability for higher values; interestingly, from this analysis it can be seen that for the Blasius correlation at low gas velocity (6 m/s) there is a low peak close to liquid fraction equal to 1 indicating that, with this correlation, a few slugs are actually still predicted (see details in Figure 69)

6.2 Churn flow in medium diameter pipes

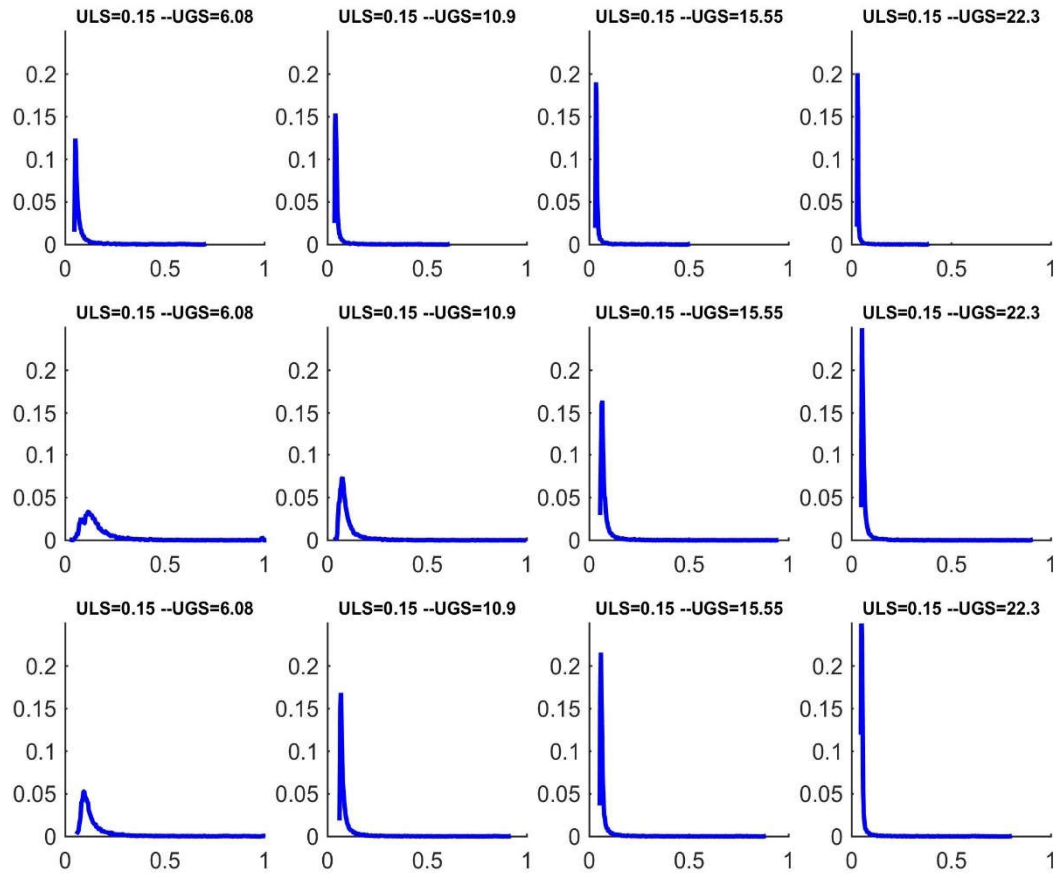


Figure 68 - Probability Density Function of the predicted liquid fraction at 35 metres from the bottom of the pipe - $\Delta x/D \approx 0.25$ ($\Delta x = 1.2$ cm)

6.2 Churn flow in medium diameter pipes

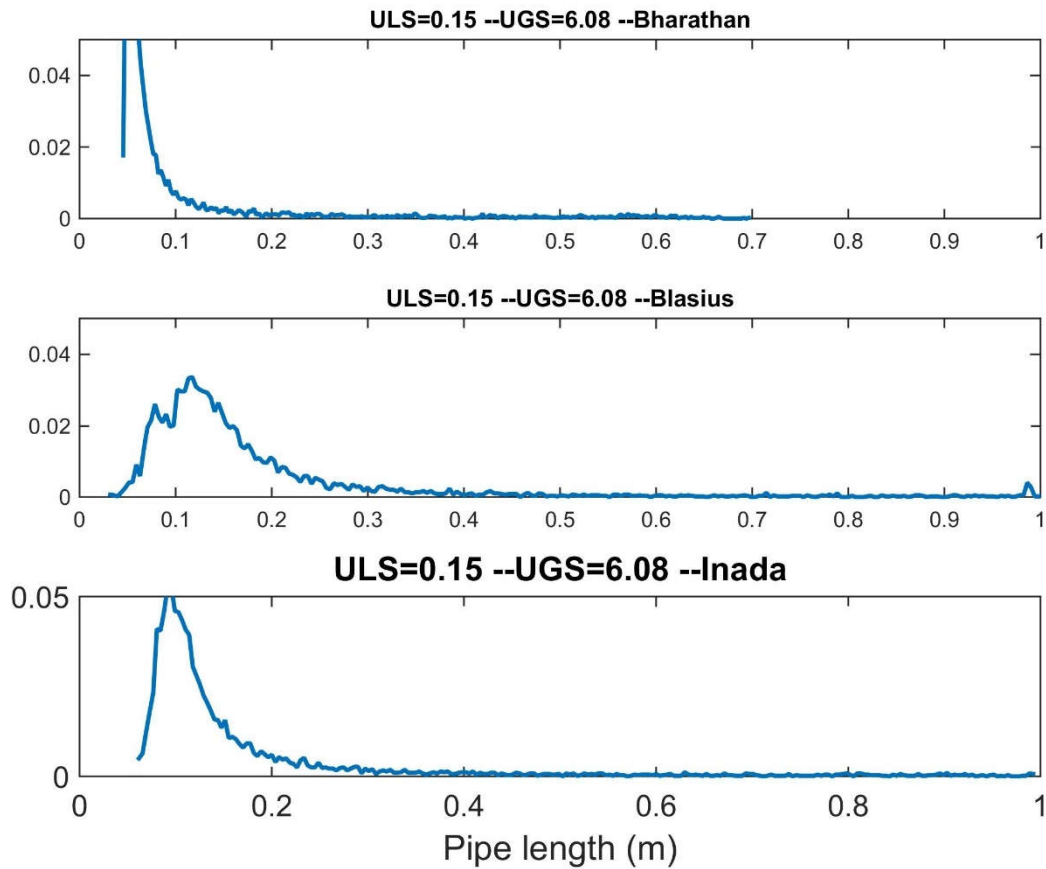


Figure 69 - Probability Density Function of the predicted liquid fraction at 35 metres from the bottom of the pipe - $UGS = 6.0 \text{ m/s}$ - $\Delta x/D \approx 0.25$ ($\Delta x = 1.2 \text{ cm}$)

Quantitative comparison

Figures 70 and 72 show a comparison between the axial distribution of the predicted flow characteristics and the experimental data for the absolute pressure and for the liquid fraction, respectively.

In order to obtain a clearer analysis in Figure 74 the relative error obtained with the different correlations is presented for each position in the pipe. For the correlations from Blasius and Inada, the error in the churn regime region (that is for superficial gas velocities below 20 m/s) is acceptable for all the cases (inside a $\pm 30\%$ error range for the liquid fraction and a $\pm 15\%$ for the pressure) with the

6.2 Churn flow in medium diameter pipes

Inada factor performing slightly better particularly in the prediction of the liquid fraction. The Bharathan correlation, on the other hand, yields under-predictions of the liquid fraction up to 45%.

For the case with the highest gas velocity (22 m/s), it can be seen that there is a very large error at all axial positions and with all the correlations for the prediction of the liquid fraction. However, this large error does not affect the prediction of the absolute pressure which is captured fairly well; this is because the actual value of the liquid fraction for this case is very low, with an axial average of 0.05 and therefore, the pressure drop is weakly affected by the hydrostatic component.

6.2 Churn flow in medium diameter pipes

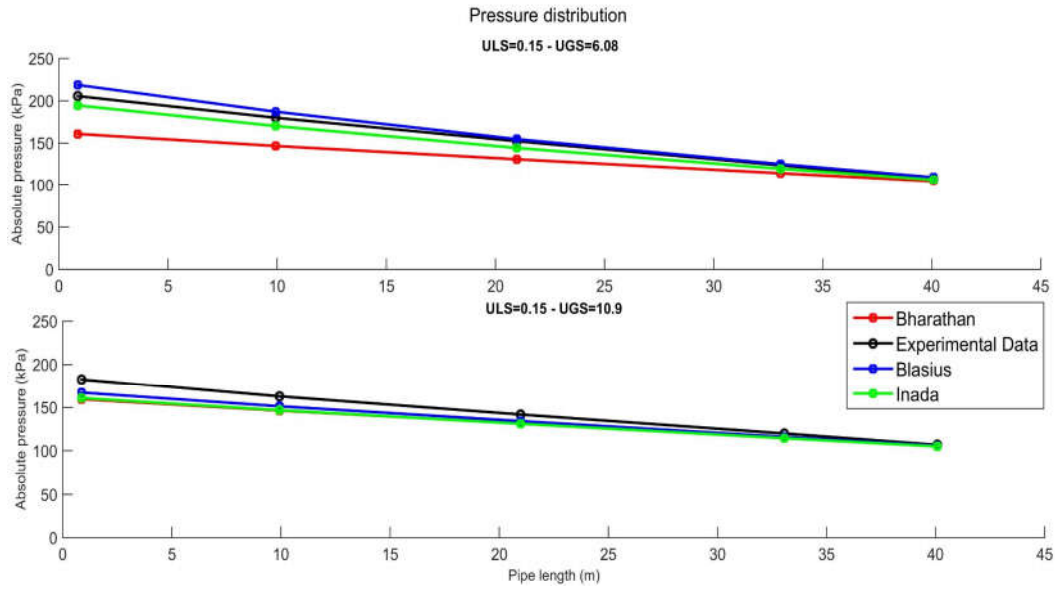


Figure 70 - Comparison between experimental data and numerical results for different gas velocities: axial distribution of the absolute pressure - $U_{ls} = 0.15 \frac{m}{s}$; $U_{gs} = 6.08 \frac{m}{s}$ (top); $U_{gs} = 10.9 \frac{m}{s}$ (bottom) - Exp. data: Waltrich et al. (2013) - $\Delta x/D \approx 0.25$ ($\Delta x = 1.2$ cm)

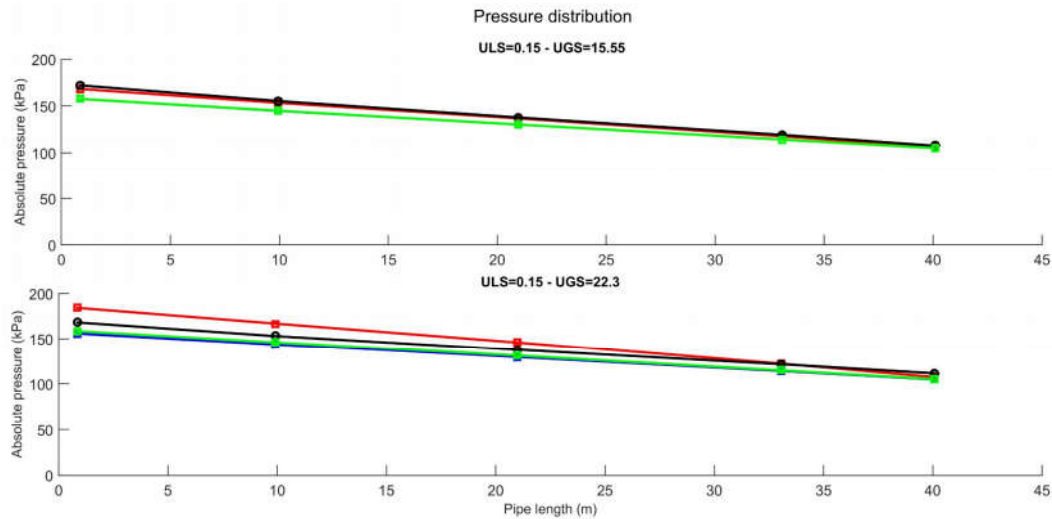


Figure 71 - Comparison between experimental data and numerical results for different gas velocities: axial distribution of the absolute pressure - $U_{ls} = 0.15 \frac{m}{s}$; $U_{gs} = 15.55 \frac{m}{s}$ (top); $U_{gs} = 22.3 \frac{m}{s}$ (bottom) - Exp. data: Waltrich et al. (2013) - $\Delta x/D \approx 0.25$ ($\Delta x = 1.2$ cm)

6.2 Churn flow in medium diameter pipes

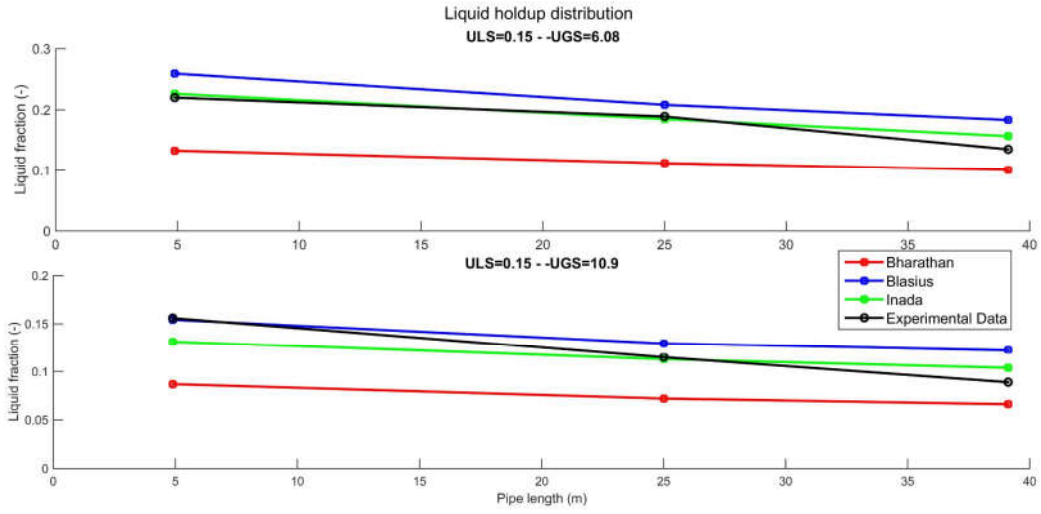


Figure 72 - Comparison between experimental data and numerical results for different gas velocities: axial distribution of the liquid fraction: - $U_{ls} = 0.15 \frac{m}{s}$; $U_{gs} = 6.08 \frac{m}{s}$ (top); $U_{gs} = 10.9 \frac{m}{s}$ (bottom) - Exp. data: Waltrich et al. (2013) - $\Delta x/D \approx 0.25$ ($\Delta x = 1.2$ cm)

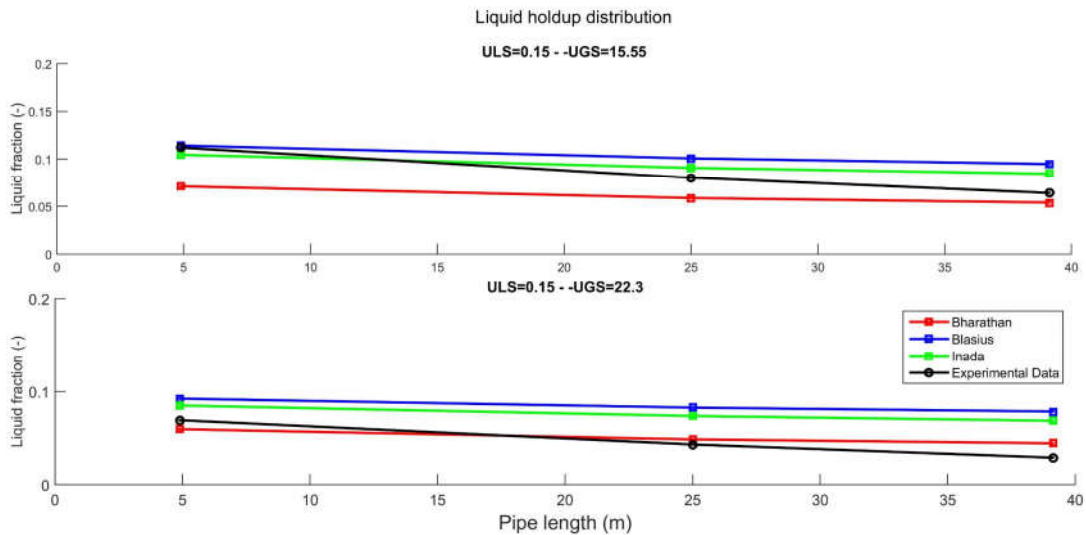


Figure 73- Comparison between experimental data and numerical results for different gas velocities: axial distribution of the liquid fraction: - $U_{ls} = 0.15 \frac{m}{s}$; $U_{gs} = 15.55 \frac{m}{s}$ (top); $U_{gs} = 22.3 \frac{m}{s}$ (bottom) - Exp. data: Waltrich et al. (2013) - $\Delta x/D \approx 0.25$ ($\Delta x = 1.2$ cm)

6.2 Churn flow in medium diameter pipes

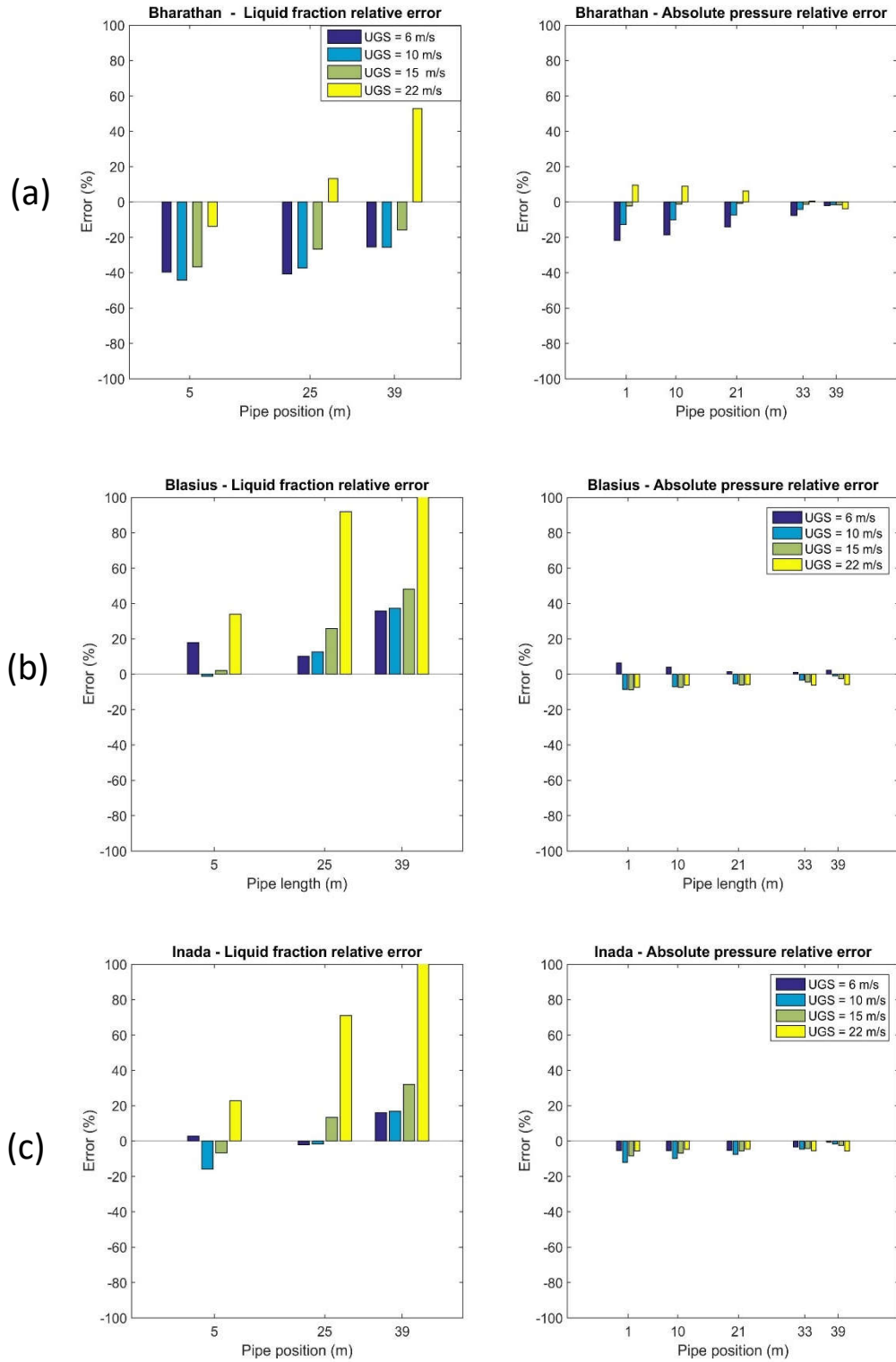


Figure 74 - Relative error at different axial positions along the pipe for different gas velocities and friction factors: Liquid fraction (left) and absolute pressure (right); Bharathan (a), Blasius (b) and Inada (c) friction factors. (bottom) - Exp. data: Waltrich et al. (2013) - $\Delta x/D \approx 0.25$ ($\Delta x = 1.2$ cm)

6.3 Small Diameter pipes

6.3.1 Experimental test case

In order to make a preliminary assessment of the capability of the model in the churn flow region for small diameters, a set of experimental data was chosen from the literature (Sawai et al., (2004)) and a comparison between the predicted pressure gradient and liquid hold up was made.

The experimental rig consisted of a vertical pipe 4.2 metres long, with an internal diameter of 25.8 mm. The section along which the liquid holdup was measured was 775 mm long and was located 2315 mm above the inlet mixing device. The pressure drop measurement section was 1422 mm long and started 1931 mm from the inlet. In Figure 75 it is possible to see a scheme of the rig and

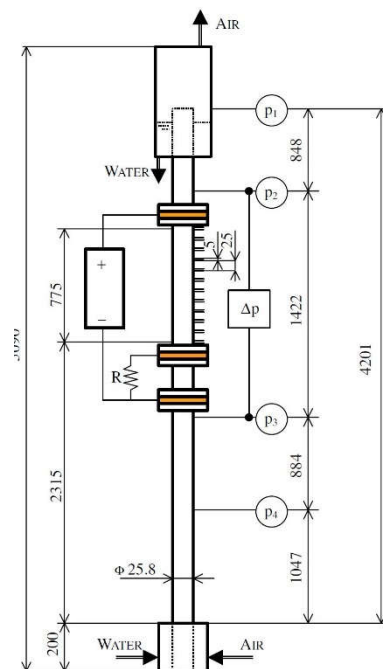


Figure 75 - Scheme of the experimental rig - From Sawai et al. 2004

6.3 Small Diameter pipes

the position of the measurement devices. The experiments were conducted with water and air at room temperature with the outlet pressure kept constant at about 100 kPa; the gas and liquid superficial velocities ranged from 0.6 to 21.3 m/s and from 0.002 to 0.4 m/s respectively.

The flow patterns obtained were identified in the experiments according to the variations of the time averaged pressure gradients against the gas volumetric flux. In Figure 61 (Section 5.7) it is possible to see the regions identified for this classification: two negative slope regions, “NS-I” and “NS-II”, and two positive slope regions, “PS-I” and “PS-II”. According to the paper, PS-II and NS-II regions correspond to annular flow and slug flow respectively, whilst “NS-I” corresponds to churn regime region; the region called “PS-I” is a transition region between slug and churn flow.

In the paper the averaged frictional pressure drop was also provided, calculated indirectly from the measured void fraction.

6.3 Small Diameter pipes

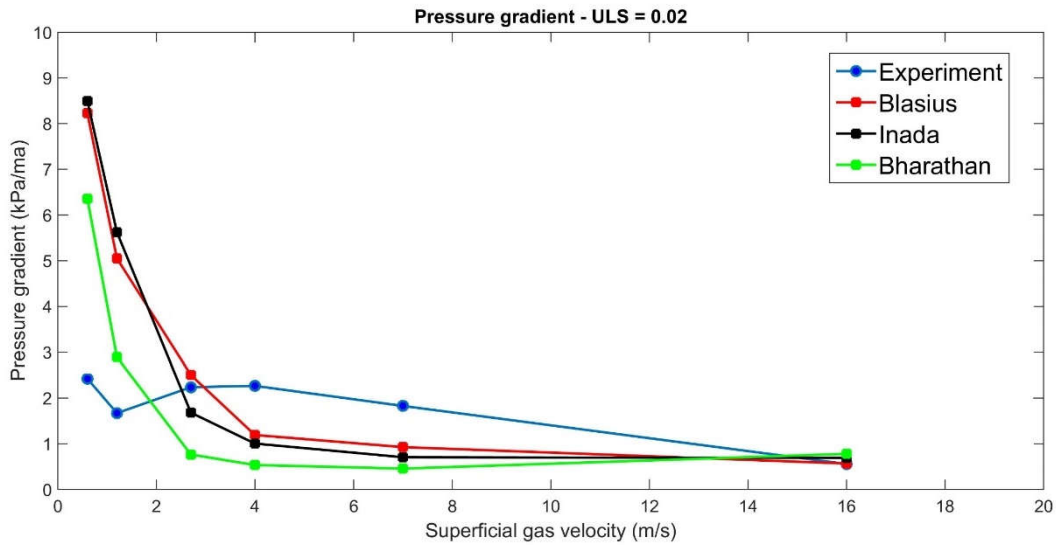


Figure 76 - Variation of pressure gradient with gas superficial velocity - Comparison between experimental data (Sawai et al. (2004)) and predicted values with three different interfacial friction factor correlation for liquid superficial velocity equal to 0.02 m/s - $\Delta x/D \approx 0.2$ ($\Delta x = 0.5$ cm)

6.3.2 Results

Qualitative comparison

A first, qualitative, comparison between the numerical results and the experimental data can be done in order to see whether the model is able to capture the change of sign of the pressure gradient slope against the gas superficial velocity. This comparison is presented in Figures 76, 77 and 78 for three different liquid superficial velocities (0.02, 0.2 and 0.4 m/s) and six gas velocities (0.6, 1.2, 2.7, 4.0, 7.0 and 16 m/s); it is worth noticing that in the figures the cases classified as slug flow are also included, to obtain a more comprehensive picture.

From the figures it is evident that the pressure gradient predicted by the model

6.3 Small Diameter pipes

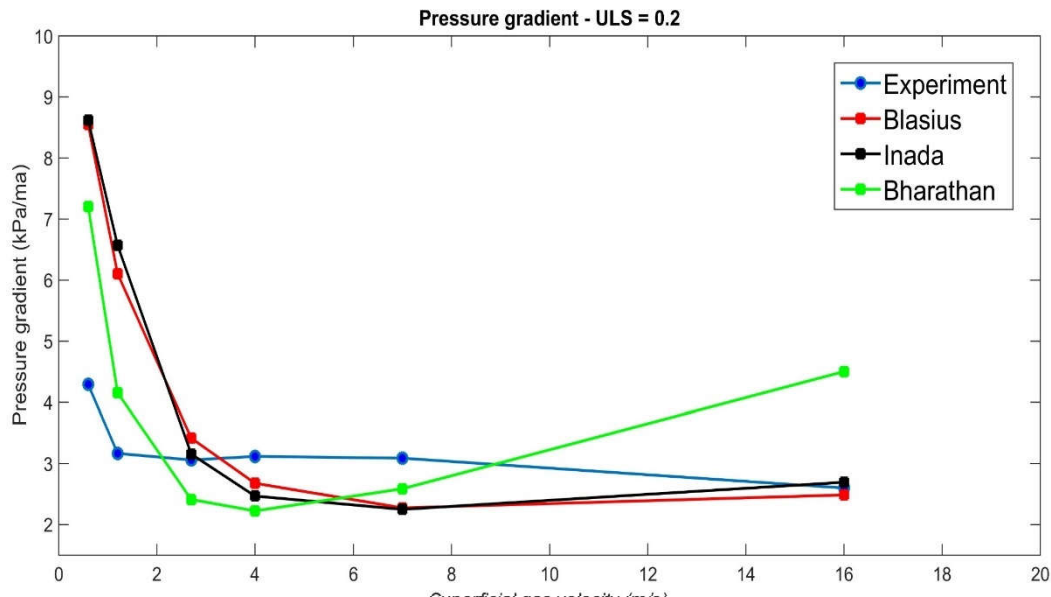


Figure 77 - Variation of pressure gradient with gas superficial velocity - Comparison between experimental data (Sawai et al. (2004)) and predicted values with three different interfacial friction factor correlation for liquid superficial velocity equal to 0.2 m/s - $\Delta x/D \approx 0.2$ ($\Delta x = 0.5$ cm)

is monotonically decreasing as the gas velocity is increased for velocities below 7 m/s; therefore, the change in slope (from negative to positive and then again to negative) seen in the experimental data, which marks the transition between slug and churn flow, is not predicted. For higher gas velocities (above 7 m/s) the experimental trend is well captured both with the Inada and Blasius correlations only for liquid velocity equal to 0.4 m/s.

It is worth noting that the trend predicted by the model is similar to the one seen for the medium size diameter, where the pressure gradient is constantly decreasing with the gas velocities; in Figure 79 it is possible to see this kind of trend as measured by Waltrich et al., (2013) for a pipe with an internal diameter

6.3 Small Diameter pipes

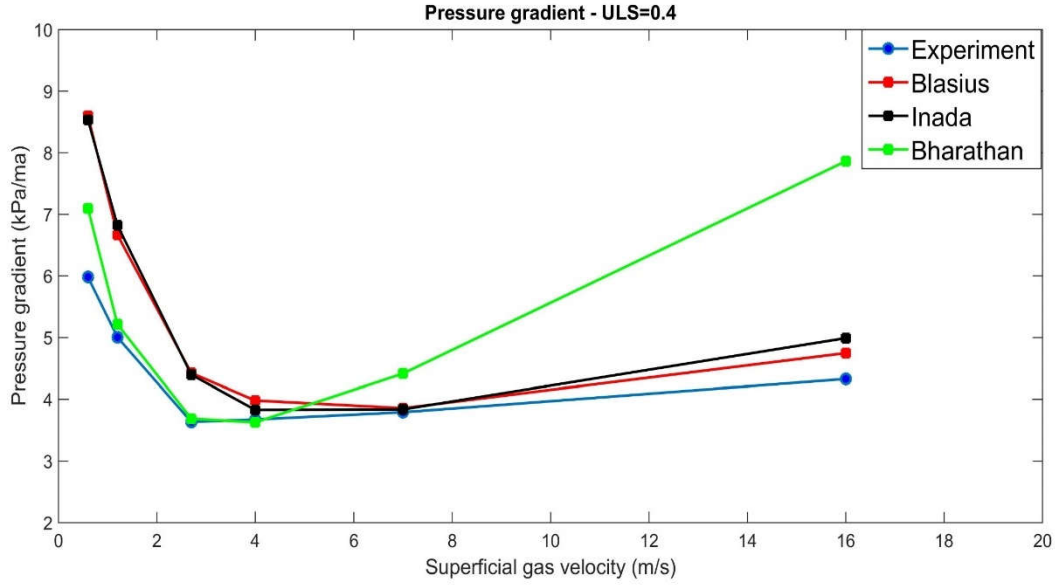


Figure 78 - Variation of pressure gradient with gas superficial velocity - Comparison between experimental data (Sawai et al. (2004)) and predicted values with three different interfacial friction factor correlation for liquid superficial velocity equal to 0.4 m/s - $\Delta x/D \approx 0.2$ ($\Delta x = 0.5$ cm)

of 48 mm. This observation corroborates the conclusion drawn in Chapter 5 that the model appears to be missing some physical effects necessary to properly capture the flow behaviour in the slug regime and transition regions in small

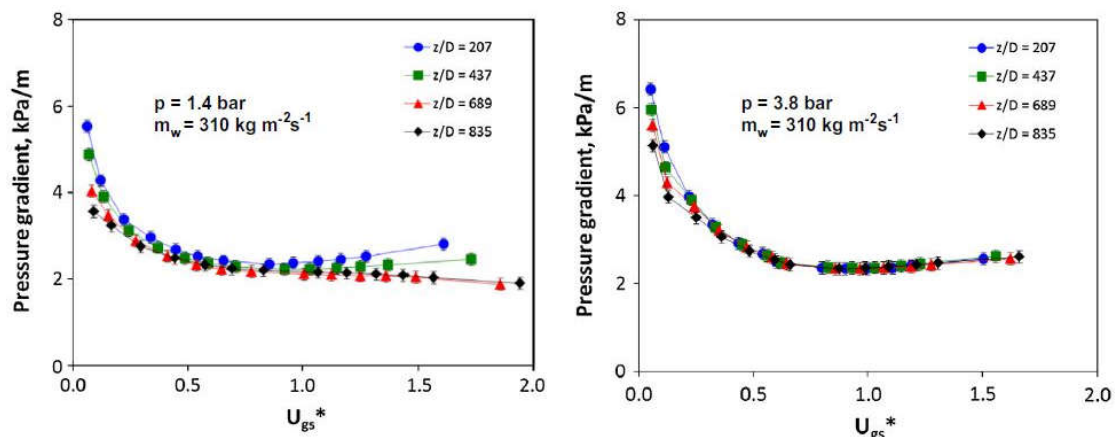


Figure 79 - Pressure gradient variation with the dimensionless superficial gas velocities and liquid superficial velocity equal to 0.15 m/s - Different axial positions are shown (from Waltrich et al. 2013)

6.3 Small Diameter pipes

diameter pipes.

To further investigate this issue a quantitative analysis of the numerical results in the churn region is also presented below.

Quantitative comparison

Figures 80, 81 and 82 present a comparison of the predicted pressure gradient against the experimental data for all the velocities and grouped according to the correlation used. From Figure 80 it is possible to see that the correlation by Bharathan gives quite poor results, strongly over-predicting the pressure gradient at high gas velocities and liquid velocities equal to 0.2 and 0.4 m/s and under-predicting all the cases with liquid velocity equal to 0.02 m/s.

The correlations by Inada and Blasius yield very similar results (81 and 82), both giving fairly good comparison against the experimental data; almost all the results are inside a $\pm 30\%$ error area and only a larger under-prediction - around -40% - is seen at very low liquid velocity.

Figures 83 to Figure 85 present the results for the prediction of the averaged liquid holdup. The correlations by Inada and Blasius confirm the good performance reported above whilst the Bharathan correlation yields strong underestimations for all the cases. Given this under prediction it is evident that the strong over prediction found for the pressure gradient can only be due to the liquid-wall shear forces.

This is confirmed through a comparison between the predicted frictional pressure gradients and those calculated in the experiment, as shown in figures 86 a, b and c (in these pictures the cases are grouped according to the liquid

6.3 Small Diameter pipes

velocity). For liquid velocities equal to 0.2 m/s and 0.4 m/s and gas velocities above 7 m/s the frictional pressure gradient obtained with the Bharathan correlation is much higher than that found in the experiments (from +50% to 200%), whereas at lower gas velocities the gradients are generally under estimated but in much closer agreement. At very low liquid velocity (0.02 m/s, Figure 86c) the captured trend is completely incorrect with the model predicting negative or close to zero shear forces.

It is interesting to notice that the Blasius and Inada correlations also predict, for liquid velocity equal to 0.02 m/s, too small or even negative shear forces (Figure 86c) and this might explain the under prediction of the pressure gradient. The occurrence of negative shears means that the computed time-averaged liquid velocity is negative and the model is predicting the liquid to mainly fall down; this was not seen in the experiments. Inada and Blasius correlations yield good results only for the cases with liquid velocity equal to 0.4 m/s.

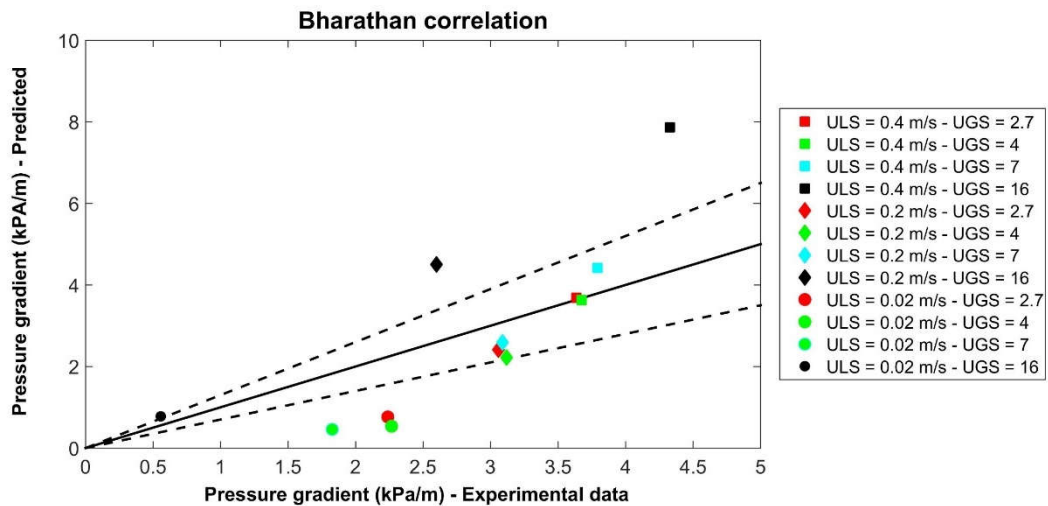


Figure 80 – Pressure gradient - Comparison between predicted and experimental data for Bharathan correlation - Exp. data: Sawai et al. (2004) - $\Delta x/D \approx 0.2$ ($\Delta x = 0.5$ cm)

6.3 Small Diameter pipes

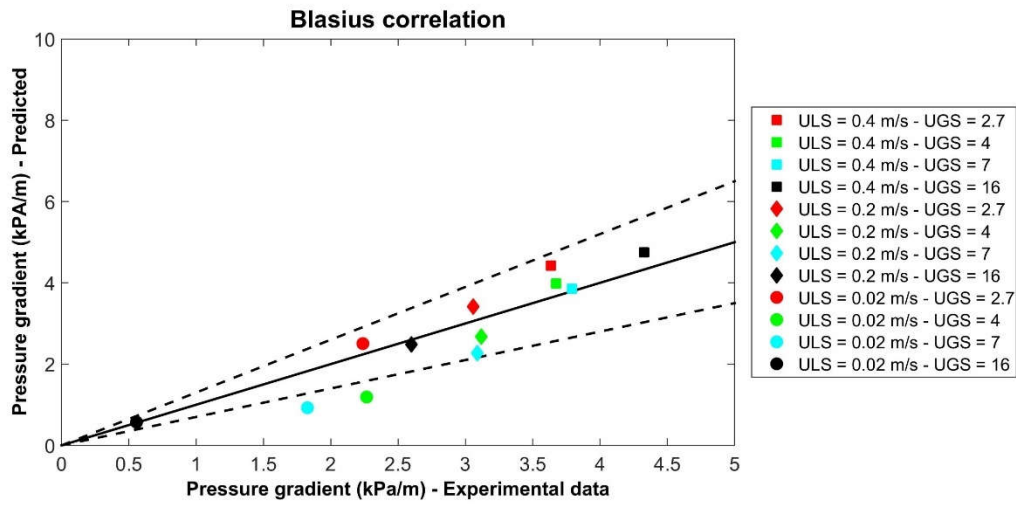


Figure 81 – Pressure gradient - Comparison between predicted and experimental data for Blasius correlation - Exp. data: Sawai et al. (2004) - $\Delta x/D \approx 0.2$ ($\Delta x = 0.5$ cm)

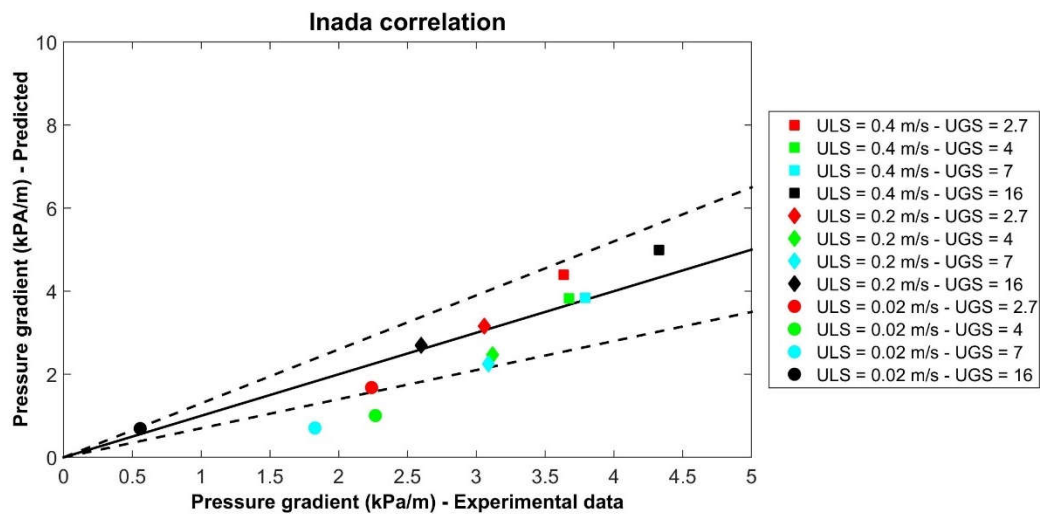


Figure 82 – Pressure gradient - Comparison between predicted and experimental data for Inada correlation - Exp. data: Sawai et al. (2004) - $\Delta x/D \approx 0.2$ ($\Delta x = 0.5$ cm)

6.3 Small Diameter pipes

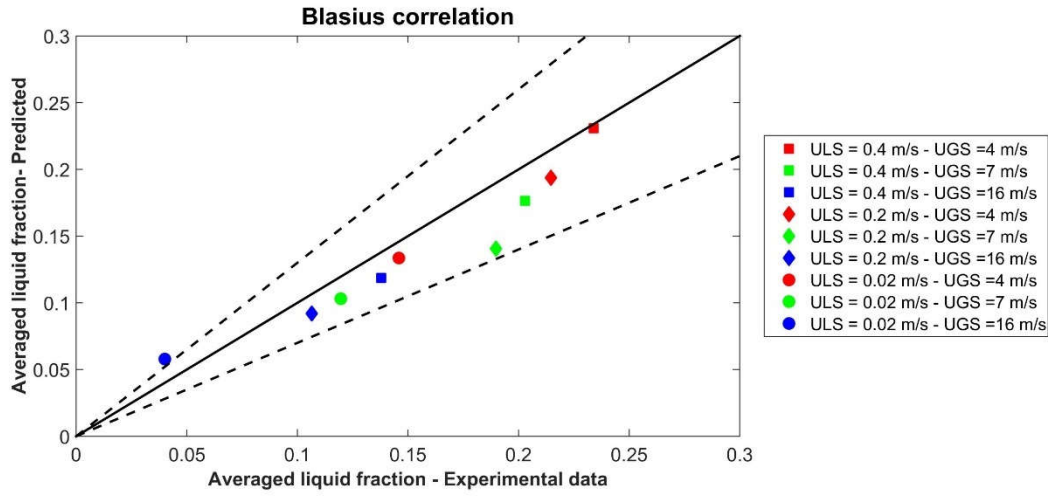


Figure 83 – Averaged liquid fraction - Comparison between predicted and experimental data for Blasius correlation - Exp. data: Sawai et al. (2004) - $\Delta x/D \approx 0.2$ ($\Delta x = 0.5$ cm)

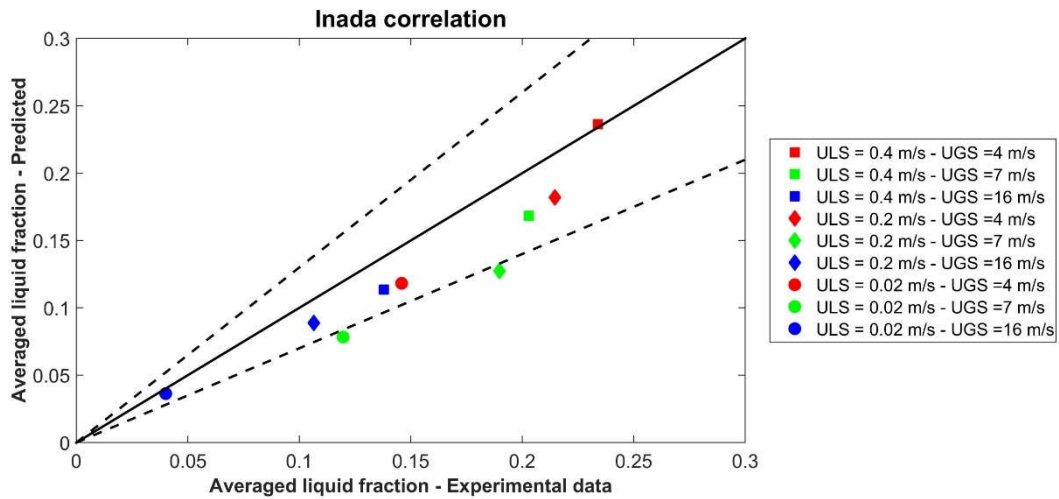


Figure 84 – Averaged liquid fraction - Comparison between predicted and experimental data for Inada correlation - Exp. data: Sawai et al. (2004) - $\Delta x/D \approx 0.2$ ($\Delta x = 0.5$ cm)

6.3 Small Diameter pipes

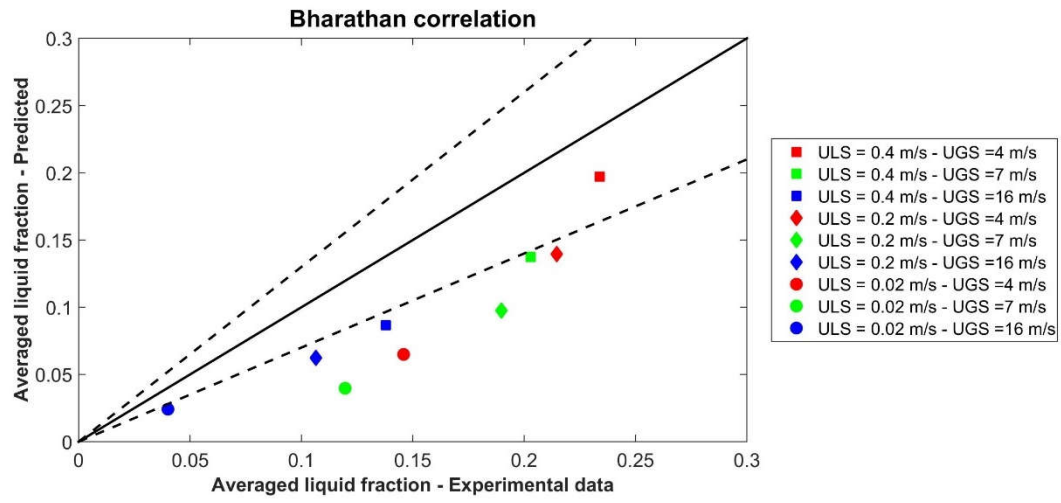


Figure 85 – Averaged liquid fraction - Comparison between predicted and experimental data for Bharathan correlation - Exp. data: Sawai et al. (2004) - $\Delta x/D \approx 0.2$ ($\Delta x = 0.5$ cm)

6.3 Small Diameter pipes

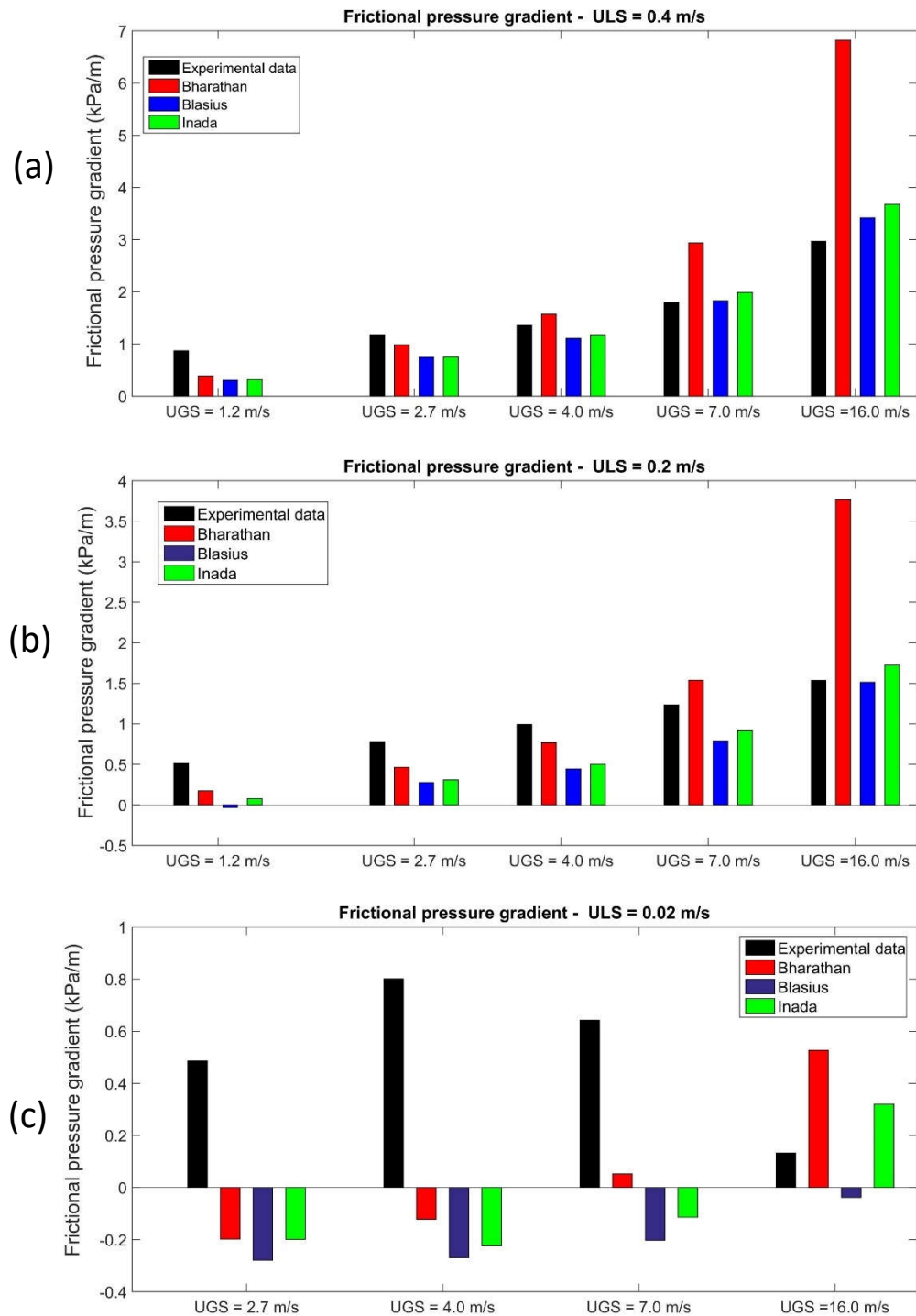


Figure 86 - Comparison between the predicted frictional pressure gradient and the experimental data from Sawai et al. (2004) for several gas and liquid superficial velocities and different interfacial friction factors correlations - $\Delta x/D \approx 0.2$ ($\Delta x = 0.5$ cm)

6.4 Large diameter pipes

6.4.1 Experimental test case

With the intention of obtaining a comprehensive introductory analysis on all the possible configurations of churn flow regime in pipes, results obtained from the simulations of a few cases with a large diameter pipe are presented in this section.

The experimental test cases are taken from Zangana, (2011); this work was carried out in a 11 metres long vertical pipe with an inner diameter of 127 mm. In the experiments air was used as the gas phase and pure water as liquid phase, both at ambient temperature; the pressure at the outlet was around 3 bara. The gas superficial velocity ranged from 3 m/s to 16 m/s, whilst the liquid superficial velocity from 0.008 m/s to 0.7 m/s; however, in the present work only four cases are presented, corresponding to one value for the liquid velocity of 0.1 m/s and four different gas velocities, of 6.6, 8.5 10 and 12 m/s.

The most important flow characteristics measured were the pressure gradient, the liquid film thickness (calculated from the recorded liquid fraction) and the instantaneous wall shear stresses, of which also the direction was extrapolated from the signal. It is interesting to notice that the instantaneous directional signal showed that the flow direction of the liquid film is changing and it is oscillating on the pipe wall; for this reason, the time averaged directional wall shear stresses were found to be always smaller than the averages calculated

6.4 Large diameter pipes

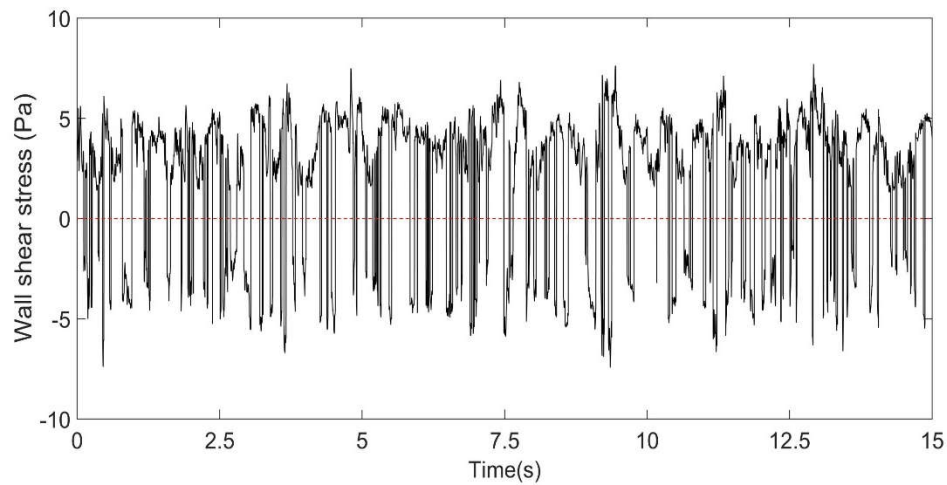


Figure 87 - Instantaneous directional wall shear stress as recorded in the experiments for a gas velocity of 6 m/s and liquid velocity equal to 0.1 m/s - From Zangana (2011)

according to the absolute values. Figures 87 and 88 show, both from experimental data, the recorded signal and a comparison between the directional and absolute averages.

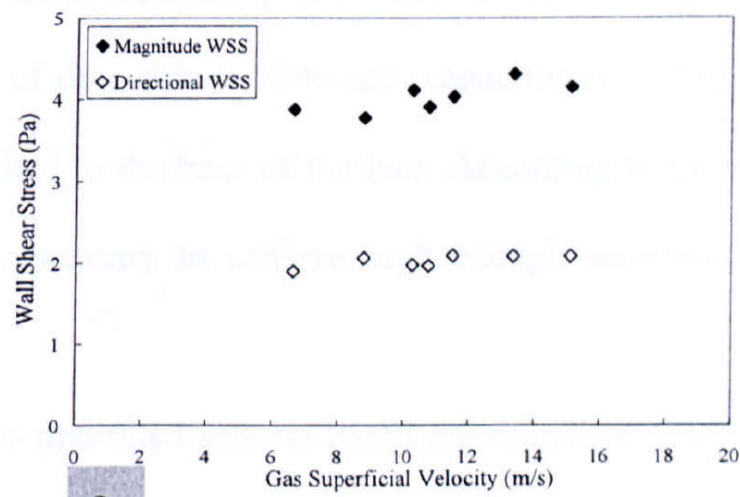


Figure 88 - Magnitude and directional mean wall shear stress as a function of gas superficial velocity and at liquid superficial velocity of 0.1 m/s – From Zangana (2011)

6.4 Large diameter pipes

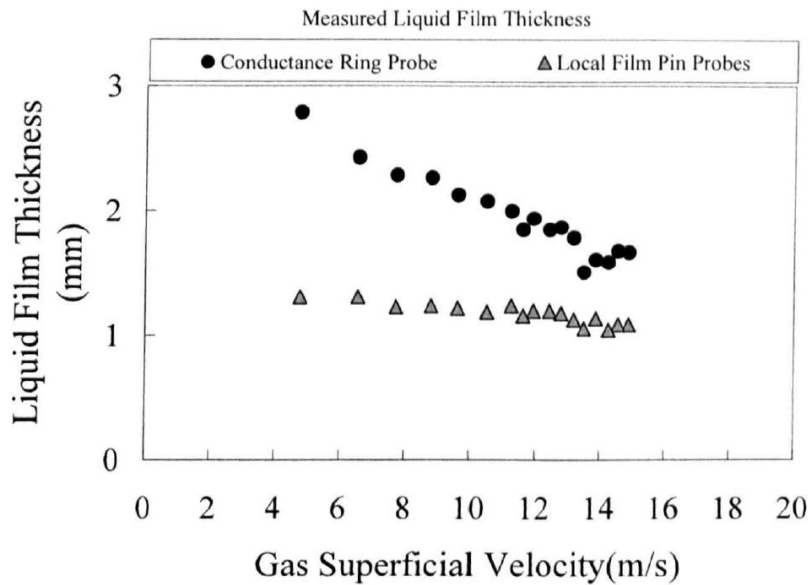


Figure 89 - Measured liquid film thickness from conductance ring probes and local film pin probes at liquid superficial velocity of 0.1 m/s – From Zangana (2011)

The liquid fraction was measured at three different positions in the pipe, placed at 8, 8.1 and 8.3 metres from the mixer (it corresponds to 62.7, 63.8 and 65.5 diameters respectively). It must be pointed out that two different approaches were used to measure the liquid fraction (pin probes and ring probes, see Zangana, (2011) for further details) which gave quite different results, with discrepancies even close to 100% (see Figure 89). The pressure drop was measured on a 1.64 metres long section starting 6.69 m above the mixer and the shear stress probes were placed at 8.05 m.

6.4.2 Results

Comparisons are shown between the experimental data and predicted values obtained with the three different correlations for the interfacial friction factors in Figures 90, 91 and 92, for the pressure gradient, the liquid fraction, and the

directional wall shear stresses.

The pressure gradient (Figure 90) is under predicted for all the cases, with the Inada and Blasius correlations giving very similar results and slightly better predictions than the Bharathan one at low gas velocities.

For the liquid fraction the numerical results are compared with both the experimental data from the pin probes and the ring probes and they give a reasonable comparison for all the friction factors used, with all the predictions essentially inside the experimental range (Figure 91). However, this comparison does not seem to be particularly meaningful, given the strong difference in the experimental values obtained with the probes.

Figure 92 shows the comparison of the directional wall shear stresses (i.e. calculated by averaging in time the signal taking into account the direction of the liquid flow). In this case, all the correlations wrongly predict even the trend of the shear stresses, showing a strong dependency on the gas velocity: the predicted values increase with the gas velocity whilst the experimental data remain constant. It is worth noticing that for the Blasius cases the same conclusion as in 7 can be drawn: the time averaged wall shear stresses are mainly negative, meaning that, with this correlation, the liquid is predominantly falling downward. On the contrary, the Bharathan correlation gives wall shear stresses which are always positive and strongly over predicted. All these results are obviously not satisfactory.

6.4 Large diameter pipes

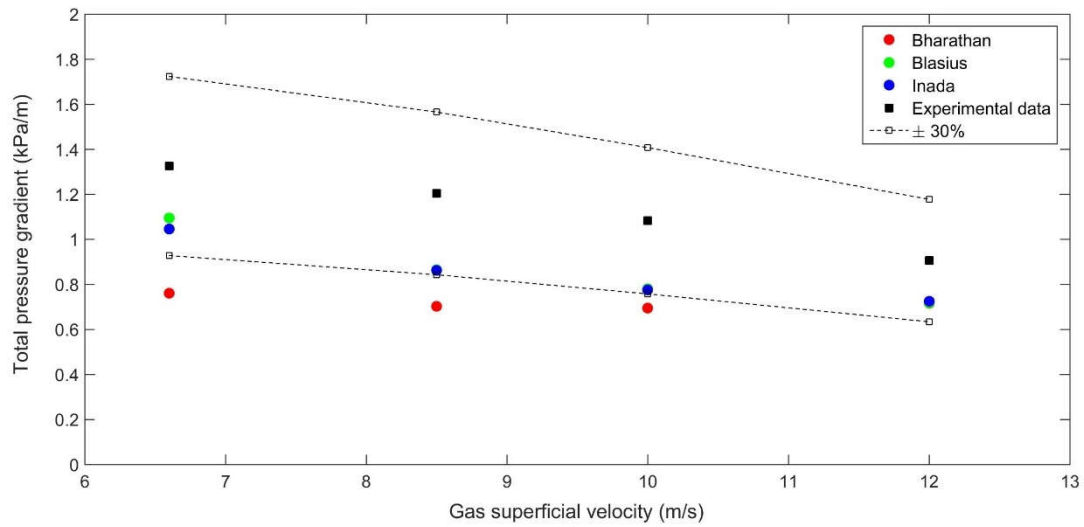


Figure 90 - Pressure gradient - Comparison between predicted and experimental data from Zangana (2011) for three different interfacial friction factor correlations.

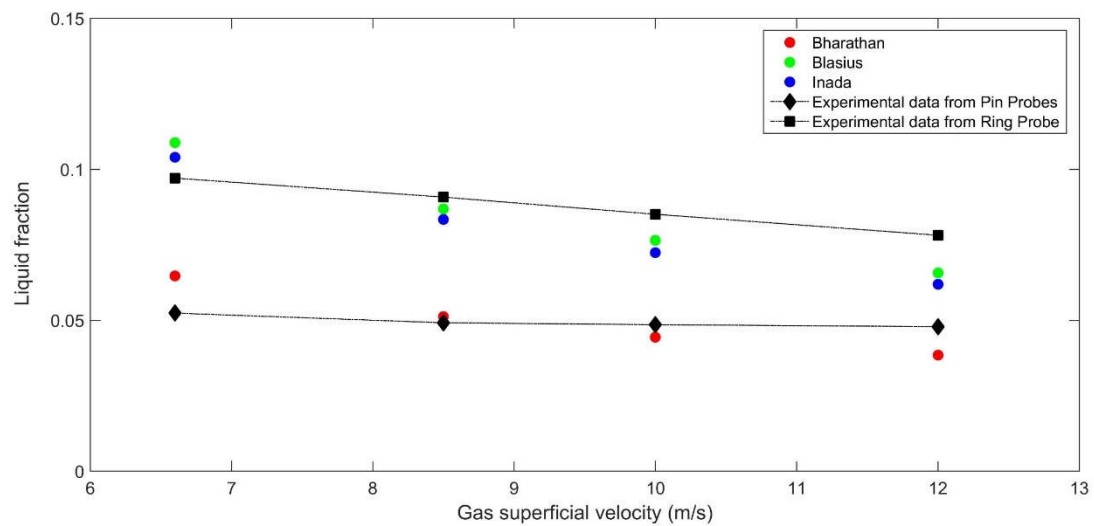


Figure 91 - Averaged liquid holdup - Comparison between predicted and experimental data from Zangana (2011) for three different interfacial friction factor correlations.

6.5 Summary

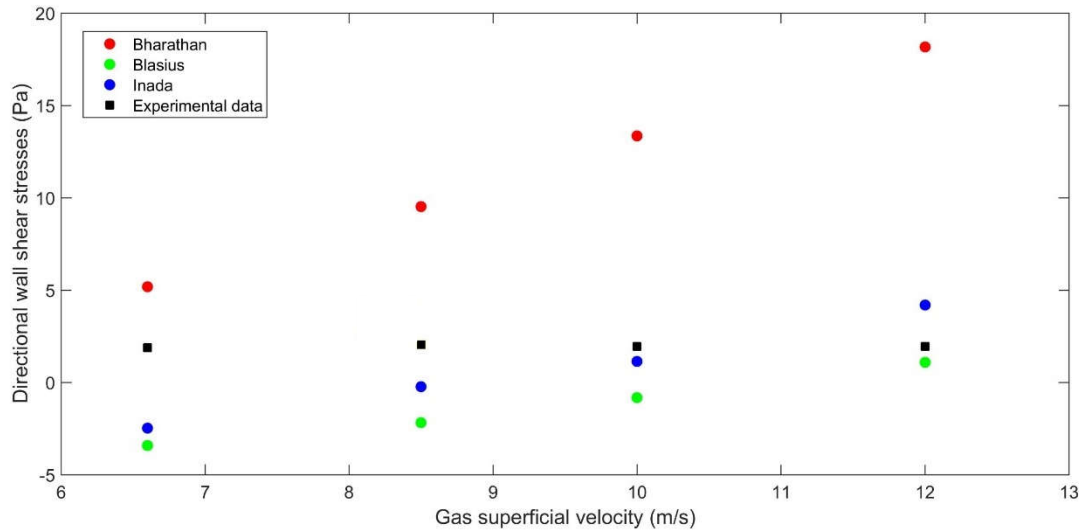


Figure 92– Averaged directional wall shear stresses - Comparison between predicted and experimental data from Zangana (2011) for three different interfacial friction factor correlations.

6.5 Summary

In this Chapter, a preliminary analysis of the applicability of the slug capturing technique to the churn flow region was presented for an extended range of pipe diameters from 28 mm to 127 mm. The main objective of this chapter was to create the basis for further studies and development of the technique -which has never been applied to churn flow - to the intermittent flow region, highlighting the most important issues to be investigated. Indeed, to extend the analysis as much as possible, the three correlations for the interfacial friction factors presented in Chapter 3 and used for the simulation of slug flow were separately applied and their results were compared.

From a qualitative point of view the model is able to capture the main characteristics of churn flow such as the presence of a continuous central gas

core (without the occurrence of slugs) and the formation of small waves – due to interfacial friction forces - on the surface of the thin liquid film along the wall of the pipe; furthermore, the intermittency of the flow is well captured with the liquid velocity (or the wall shear stresses) fluctuating between positive and negative values and seldom leading to the formation of large disturbance waves which do not develop into slugs.

However, a significant discrepancy was found when comparing the prediction of the variation of the pressure gradient slope with the gas velocity for the small diameter pipe; this might be a serious issue, because it might mean that the model is not properly capturing the regime transition between slug and churn flow in this particular configuration.

Furthermore, the quantitative results showed very different errors according to size of the pipe diameter and, more significantly, to the friction factors used. The predicted characteristics such as the pressure gradient (or directly the absolute axial pressure distribution, where possible), the averaged liquid fraction and the averaged wall shear stresses were compared against the available experimental data. The cases with the medium and small size diameters gave fairly good results for the pressure gradient and the liquid fraction using the Blasius and Inada correlations, whilst the predictions with the Bharathan correlation were quite poor. For all the the large diameter pipe cases all the correlations gave incorrect results.

Where the wall shear stresses were compared (small and large diameters) the errors were fairly large for all the cases; this is most likely due to wrong predictions of the liquid velocity and, therefore, to an incorrect modelling of the

6.5 Summary

interfacial friction factors. Moreover, at low gas and liquid velocities the averaged shear stresses computed by the model with both Blasius and Inada correlations were negative, meaning that the model was predicting that the liquid was mainly falling down even in churn flow; this further indicates a wrong estimation of the interfacial forces.

It is clear that the model is not appropriately capturing the physics behind the evolution of churn flow and further investigation is necessary to better understand where improvements are needed. General conclusions and suggestions for future work will be extensively given in the next Chapter.

Chapter 7

Conclusions and recommendations for future work

7.1 Preamble

An extension of the slug capturing technique to intermittent flow regimes in vertical pipes - such as slug and churn flow – has been presented in this work. This method entails the numerical solution of the one-dimensional two-fluid model, in which all flow quantities are averaged integrating over the cross-sectional area of the pipe, under the assumption that the variations mostly occur in the axial direction; this operation yields a simple but reasonably representative model, particularly useful for the simulation of complex flow in long pipelines, where multi-dimensional models would be expensive and time consuming. Furthermore, this method has the advantage of being almost fully mechanistic and it is therefore able to capture automatically the initiation, growth and

development of slugs and wave structures in the pipe. The slug capturing technique had already been validated for horizontal slug flow, but the application to vertical configurations is new.

7.2 Stability analyses

The one-dimensional two-fluid model is well known to be unconditionally ill-posed in vertical geometries; this results in an unbounded growth of the short wave length instabilities which might corrupt the numerical solution and make the method unreliable. Indeed, in horizontal flow it was found that, in the region where the model is ill-posed, the predicted slug frequency grows exponentially as the mesh is refined, consequently rendering it impossible to obtain a mesh independent solution; however, it is also well known that the First Order Upwind discretisation scheme introduces in the finite system of equations an artificial diffusion that might have a damping effect on the spurious instabilities. Therefore, a major objective of this work has been to study the effects of the discretisation on the growth rate of the instabilities, performing a Von Neumann analysis of the discretised system and comparing the results with those of a stability analysis of the differential form. As main conclusion from this analyses it has been shown here that the artificial diffusion originated by the discretisation scheme effectively introduces a cut-off wavelength below which all the shorter wavelengths instabilities are damped (i.e. it presents negative values of the growth rate) rendering the discretised system well-posed de facto from a numerical point of view. Since the value of the cut-off wavelength depends on the size of the mesh cell, the ill-posedness will be recovered eventually for highly

refined mesh, but these mesh densities are however of no practical use in real world applications.

7.3 Vertical slug flow simulation

In the second part of this work, results from simulations of experiments carried out at the University of Nottingham are presented. It is firstly shown that the method is able to correctly capture the mechanism for slug generation in vertical flow, namely the liquid fall-back and the subsequent flooding of the pipe which leads to the formation of successive slugs. Furthermore, to achieve a quantitative validation, the results obtained from the numerical simulations were compared with experimental data. All the predicted values (averaged liquid holdup, structure velocity and slug frequency) compare fairly well with the data.

The correlations used to calculate the interfacial shear were found to have a critical effect on the predicted characteristics of the slugs and other structures, and even in the prediction of the initiation of the regime. For instance, it was found that using a correlation which had been developed for churn flow did not allow the liquid to fall back and hence prevented the formation of slugs. The best results were obtained with the weaker Blasius correlation for the interfacial forces, which permitted the flooding of the pipe and the development of slugs.

To address the problem of ill-posedness, a mesh sensitivity study for vertical slug flow was conducted and it was verified that, differently from simulations of horizontal slug flow, the predicted slug frequency is not so dependent on the mesh size for practical mesh densities (down to $\frac{\Delta x}{D} = 0.15$). It is suggested here that this important difference with the horizontal case is due to two factors: first,

7.3 Vertical slug flow simulation

as stated above, the numerical diffusion introduced by the discretisation scheme has the effect of stabilising the model for practical mesh sizes; second, since the mechanism behind the slug formation is different for vertical geometries and it does not depend on the growth of natural instabilities, the mesh refinement has a substantially weaker effect on the prediction of slug initiation.

Furthermore, in order to fully assess the capability of the model in the prediction of the axial development of the characteristics and structures of slug flow, the simulated properties obtained at different positions along the pipe were compared with experimental data available in literature. The model gave acceptable results for all the characteristics in the pipe section where the slug regime is initiated, but failed to correctly predict the initial development of the flow in the region close to the inlet; where the flow is highly chaotic and more similar to the churn regime. Although this error is not significant in the assessment of the simulation of slug flow, it is already an indication of some issues in the correct characterisation of churn flow, most likely due to inaccurate correlations for the interfacial friction factors.

Also a few results from simulations of pipes of small diameters were presented; the main finding was that the model was unable to correctly predict the pressure gradient. It may be speculated that the physical model implemented in the code is too simple to properly capture all the interactions between the two phases in such a small diameter, resulting in less liquid pushed out by the gas and consequently in a higher contribution to the pressure gradient of the hydrostatic term.

However, it is important to stress that, even when the simulated results from

this work do not completely agree with the experimental data – such as for some specific geometries - the demonstration that the slug capturing technique has the capability to fully characterize vertical slug flow remains certainly the most important achievement presented in this work. As a matter of fact, the application of the two-fluid model to vertical slug flow has always been viewed with suspicion, because of its unconditional ill-posedness and, in the past, it has often been speculated that obtaining any meaningful result would have been impossible; this work proves the contrary.

7.4 Churn flow simulation

To provide a solid starting point for further investigations and improvement of the model on the whole intermittent flow region (i.e. the region consisting of slug and churn flow) a preliminary analysis of the capability of the slug capturing technique in the churn flow region was also included in this work for an extended range of pipe diameters. The results in this case are not intended to give a complete validation for the simulation of churn flow but rather to highlight the most important issues to be studied in future works.

It was found that the model is able to qualitatively capture the main characteristics of the churn regime, such as the formation of small waves - generated by the interfacial friction forces - on the liquid film attached to the wall. Moreover, the intermittency of the flow is correctly predicted: the simulated liquid velocity fluctuates between positive and negative values and this motion leads to the generation of large disturbance waves which, however, do not develop into slugs.

Nevertheless, the quantitative results showed significant discrepancies when compared with experimental data, depending on the pipe diameter and, more importantly, the friction factors. The cases with the medium and small size diameters gave fairly good results for the pressure gradient and the liquid fraction, using “weak correlations” such as the ones by Blasius and Inada, whilst the predictions with the Bharathan correlation gave relatively large errors. For all the cases with the large diameter all of correlations implemented yielded incorrect results.

The results were also poor for all the cases for which it was possible to compare the wall shear stresses (small and large diameters) and this is presumably due to the wrong estimation of the liquid velocity and, hence, to the inadequate modelling of the interfacial friction factors. In addition to this, at low velocities the averaged shear stresses predicted by the Blasius and Inada correlations were negative and this means that the model still predicts the down flow of the liquid, like in slug flow; this behaviour strongly indicates a wrong estimation of the interfacial forces.

7.5 Future works

Some suggestions and recommendations for a future development are proposed in what follows.

7.5.1 General recommendations

First, a general observation emerged during the execution of the project; this work largely benefitted from the close collaboration with the scientists and

engineers from the University of Nottingham who conducted the experiments necessary for the validation of the model. This approach gave the possibility to gain a full understanding of the experimental setup – which is often quite hard to achieve from the literature - and to have fruitful discussions on the definition of the characteristics and properties of the flow that were significant in characterising each regime. This has a prime impact on determining what and how to properly compare the numerical data with the measured quantities. In the light of this, the author strongly recommends pursuing this kind of collaboration whenever possible.

To give a practical and significant example of the necessary collaboration between the researchers who conduct the experiments and those implementing the models and performing the simulations is the agreement on the definition of slugs. The fact that an important amount of gas might be entrained in the liquid has lead in the past to some confusion on the threshold to be set for the liquid fraction to detect a slug and therefore on what should be counted as slugs and what are counted as waves when analysing the recorded signal of the phase fraction; furthermore, some researchers attempted to measure the frequency of slugs through the FFT transform of the signal, finding significant differences in the measurement when compared to simply counting the slugs (Hernandez-Perez 2008).

7.5.2 Slug flow

The assessment of the slug capturing technique in the simulation of vertical slug flow has been thoroughly and carefully conducted in the present work; however, additional studies can be done for a further improvement.

A more extended comparison with experimental data for the pressure gradient could fully complete the validation for the medium size diameter pipes. Indeed, this was the only important characteristic which was not properly compared with recent data, since the pressure drop was not measured in the experimental campaign at the University of Nottingham and, in the comparison with the Waltrich et al. (2013) data, it was not possible to refine the mesh as much as necessary to achieve the most accurate predictions. Nevertheless, as already remarked above, this should not affect the good results already found here, because the liquid fraction was predicted correctly and in slug flow the hydrostatic component has a much stronger effect on the pressure drop than the frictional one, as it was largely shown in the present work.

The results with the small diameter pipe were uncertain, and more work is certainly needed. It would be important to extend the analysis and comparison to other data from dedicated experiments or, where not possible, from literature. However, already from the results shown in the present work it is fairly clear that the model is lacking some physical terms that are essential to properly model the interaction between the phases. Hence besides further comparisons, a better insight into the physics of the formation and, more importantly, of the development of slugs and Taylor bubbles in configurations with small diameters would be necessary. Several closure terms were tested in the past with the aim of better describing the physics of the flow, such as surface tension, virtual mass and effects associated with turbulence (Montini, 2010); although those were more related to the physics of horizontal slug flow, they might represent a good starting point for the improvement of the model in vertical flow.

Another issue that definitely requires further investigation is the understanding of the numerical limits of the method. The stability analysis of the discretised system of equations which was performed in this work showed that, whilst the discretised model introduces its own well-posedness and therefore yields physically meaningful results, the intrinsic disruptive instability of the differential form of the system might be recovered if the mesh is excessively refined. On the other hand, the present work also showed that the mesh size strongly affects the accuracy of the predictions of the model, especially for the liquid fraction and, consequently, for the pressure drop. Some indications on the required refinement are already given here, but a dedicated analysis to define the best balance between accuracy of the results and stability of the model would be important, considering that the final objective of developing a technique is to produce a flexible tool to simulate real industrial flows.

Finally, regarding the general modelling and analysis of vertical slug flow, a less important but still interesting matter would be to attempt a detailed simulation of the initiation and formation of vertical slugs, starting from a small section of the pipe, with the computational domain including the geometry of the inlet and by means of two or three-dimensional CFD calculations. This was not attempted within this project, since it was found both experimentally and numerically that the inlet geometry and the precursor regime do not affect the characteristics of slug flow far from the inlet. These kinds of simulations – which attempt to address directly the full complexity of two-phase systems - are rarely attempted in the multiphase flow field, given the extreme complexity of the physics involved and therefore the computational effort required, but they could give an insight into the mechanisms behind the initiation of vertical slug flow

regime which are still not fully understood.

7.5.3 Churn flow

The recommendations on churn flow must start with the consideration that even the two correlations for the interfacial friction factors that were specifically developed for churn flow - such as those from Inada and Bharathan - gave quite poor results. It is suggested here that the lack of reliable and physically accurate correlations to model the interfacial shear forces might be one of the main causes of the bad performance of the numerical model. Indeed, these friction factors are derived from steady and fully developed pipe flow but they are applied, in the simulation, locally and instantaneously; the author is aware of the weakness of this application but no other correlations are currently available in literature, probably due to the extreme difficulty in measuring and modelling these forces in such a chaotic regime. Therefore, the suggestion here is to perform CFD calculations to better understand the interaction between the two phases in fully developed churn flow, especially regarding the evolution of small waves on the liquid film and the formation and growth of the larger disturbance waves. From these CFD calculations it should be possible to extrapolate interfacial correlations more fitted for the slug capturing technique, and obtain a better understanding of the role of the various forces involved in highly intermittent flows, such as form and wave drag.

Additionally, to improve the prediction of the pressure gradient, it might be important to model the entrainment of the liquid in the gas core which has hitherto been ignored in this work. A model for the entrainment of the gas phase in the liquid for horizontal slug flow was successfully introduced in the past

(Bonizzi, 2003) and this could provide base for the extension of the model to vertical flow.

7.5.4 Code improvements

The code TRIOMPH was used to perform all the simulations presented in this work. This code was created in the 1980s using the now extremely old language Fortran77 and even though it has been used and modified by a considerable number of students in the last thirty years, it has never been properly updated, and it is largely retaining obsolete and strongly deprecated features (such as the awkward implicit variables). Moving the code to a more modern and well supported language (such as the new versions of Fortran or C) together with a cleaning up and reorganisation of the code would largely facilitate the training and workflow of future researchers.

Another consideration that comes from the specific work on vertical flow - which is strictly related to the updating of the code - involves the improvement of its performance. As already remarked in the previous chapters, the slug capturing technique requires much finer meshes than those needed by other one-dimensional methods - with the cell size often being of the order of few centimetres - hence requiring high computational resources. In addition to this, it has been found in this work that, differently from the horizontal case, the time step required to obtain stable calculations must be extremely small (of the order of 10^{-5} for some calculations versus 10^{-2} for most of the horizontal cases); this is likely due to the strong and very fast fluctuations of the gas properties, such as the velocity and density. This issue becomes particularly detrimental when the simulations involve large domains (i.e., with a number of cells higher than 2000),

7.5 Future works

because the computational time becomes quite large, reaching, for instance, almost 5 days to obtain around 200 seconds of simulated time for a mesh of 3000 points. This severely limits the research that can be done on configurations which involve larger geometries, such as large scale rigs or entire pipelines. Therefore, it would be worthwhile to develop a parallelised version of the TRIOMPH code.

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APPENDIX A

Further geometrical relations

The geometrical relations for horizontal flow are presented here, in order to provide a more comprehensive view of the geometrical assumptions of the one dimensional two-fluid model.

For a separated flow in a horizontal pipe where D is the diameter and γ is the stratification angle, the wetted perimeters are defined as (see Figure 93):

$$S_l = \frac{D}{2} \gamma \quad (4.32)$$

$$S_g = \frac{D}{2} (2\pi - \gamma) \quad (4.33)$$

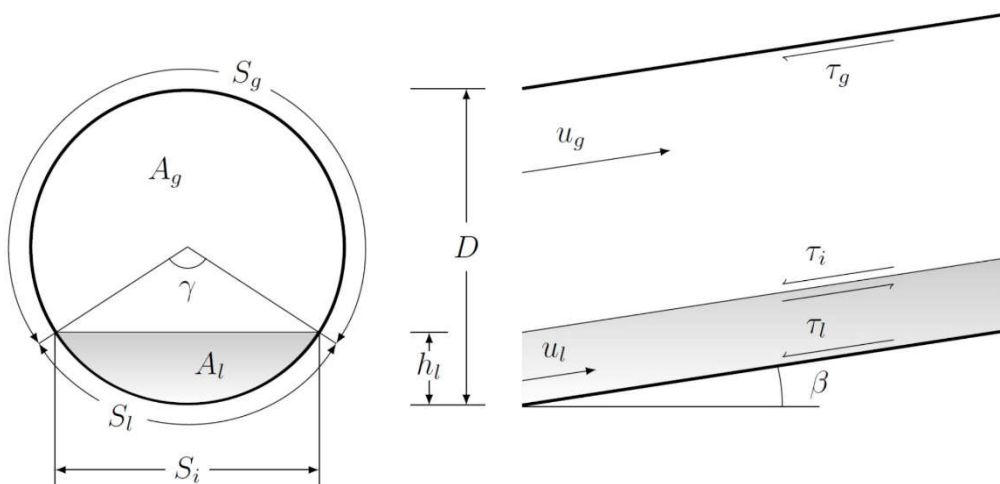


Figure 93 - Geometrical properties for horizontal and inclined flow

$$S_i = D \sin\left(\frac{\gamma}{2}\right) \quad (4.34)$$

The height of the gas liquid interface from the bottom of the pipe is

$$h_l = \frac{D}{2} (1 - \cos\left(\frac{\gamma}{2}\right)) \quad (4.35)$$

From geometric considerations - not shown here - it is also possible to determine a relation between the spatial derivative of the height of the gas-liquid interface h_l and the derivative of the liquid fraction α_l ,

$$\frac{\partial h_l}{\partial x} = \frac{\pi D}{4 \sin\left(\frac{\gamma}{2}\right)} \frac{\partial \alpha_l}{\partial x} = H \frac{\partial \alpha_l}{\partial x}. \quad (4.36)$$

APPENDIX B

Linearised form of the discretised Momentum

Equations

The discretised form of the momentum equations for a fluid with constant density, employing the upwind differencing scheme and assuming the phase velocities as positive is (Issa and Kempf, (2003)):

$$\begin{aligned} \frac{\Delta x}{\Delta t} \left(\alpha_{l_i}^{n+1} u_{l_{i+\frac{1}{2}}}^{n+1} - \alpha_{l_i}^n u_{l_{i+\frac{1}{2}}}^n \right) &+ \left(\alpha_{l_i}^{n+1} u_{l_{i+\frac{1}{2}}}^{n+1} \right) u_{l_{i+\frac{1}{2}}}^{n+1} \\ &- \left(\alpha_{l_{i-1}}^{n+1} u_{l_{i-\frac{1}{2}}}^{n+1} \right) u_{l_{i-\frac{1}{2}}}^{n+1} + \frac{\alpha_{l_i}^{n+1}}{\rho_l} (p_{i+1}^{n+1} - p_i^{n+1}) \\ &- \Delta x \left[\left(-\frac{S_l \tau_l}{A \rho_l} + \frac{S_i \tau_i}{A \rho_l} \right)_{i+\frac{1}{2}}^{n+1} + \alpha_{l_i}^{n+1} g \right] = 0 \end{aligned} \quad (C.1)$$

$$\begin{aligned} \frac{\Delta x}{\Delta t} \left(\alpha_{g_i}^{n+1} u_{g_{i+\frac{1}{2}}}^{n+1} - \alpha_{g_i}^n u_{g_{i+\frac{1}{2}}}^n \right) &+ \left(\alpha_{g_i}^{n+1} u_{g_{i+\frac{1}{2}}}^{n+1} \right) u_{g_{i+\frac{1}{2}}}^{n+1} \\ &- \left(\alpha_{g_{i-1}}^{n+1} u_{g_{i-\frac{1}{2}}}^{n+1} \right) u_{g_{i-\frac{1}{2}}}^{n+1} + \frac{\alpha_{g_i}^{n+1}}{\rho} (p_{i+1}^{n+1} - p_i^{n+1}) \\ &- \Delta x \left[\left(-\frac{S_g \tau_g}{A \rho_g} - \frac{S_i \tau_i}{A \rho_g} \right)_{i+\frac{1}{2}}^{n+1} + \alpha_{g_i}^{n+1} g \right] = 0 \end{aligned} \quad (C.2)$$

To obtain a linearized form we substitute equations (4.28) in the above equations and after some simplifications we obtain:

$$\begin{aligned}
 \left(e^{-\frac{i\phi}{2}} \frac{\partial F_l}{\partial \alpha_l} \Delta x \right) \hat{\alpha}_l + \left(\frac{\partial F_l}{\partial u_l} \Delta x + (1 - G^{-1}) \frac{\Delta x}{\Delta t} + (1 - e^{-i\phi}) u_l \right) \hat{u}_l \\
 + \left(\frac{\partial F_l}{\partial u_g} \Delta x \right) \hat{u}_g + \left(\frac{e^{i\frac{\phi}{2}} - e^{-i\frac{\phi}{2}}}{\rho_l} \right) \hat{p} = 0
 \end{aligned} \tag{C.3}$$

$$\begin{aligned}
 \left(e^{-\frac{i\phi}{2}} \frac{\partial F_g}{\partial \alpha_l} \Delta x \right) \hat{\alpha}_l + \left(\frac{\partial F_g}{\partial u_l} \Delta x \right) \hat{u}_g \\
 + \left(\frac{\partial F_g}{\partial u_g} \Delta x + (1 - G^{-1}) \frac{\Delta x}{\Delta t} + (1 - e^{-i\phi}) u_g \right) \hat{u}_g \\
 + \left(\frac{e^{i\frac{\phi}{2}} - e^{-i\frac{\phi}{2}}}{\rho_g} \right) \hat{p} = 0
 \end{aligned} \tag{C.4}$$

where F_l and F_g are defined in Chapter 3.157