

Application of Geometric Nonlinear Control in the Process Industries - A Case Study

by

Shaun David DORE, B.E.(Chem)

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Department of Chemical Engineering and Chemical Technology
Imperial College of Science, Technology and Medicine
London SW7 2BY



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Abstract

Geometric nonlinear control techniques are based on the mathematics of differential geometry. Recently these techniques have been applied to a wide range of simulated process systems examples. Theoretical advances have been made towards obtaining a technique suitable for an industrial application. This thesis concentrates on the issues of applying geometric nonlinear control to the process industries, through the analysis of a CO₂ absorption/desorption process.

Application of the basic geometric nonlinear control technique to a model of the case study results in an unacceptably large control law. It is concluded that an effective method for reducing the size and complexity of the controller is to reduce the relative order of the model. A method is presented to reduce the relative order of the model, using graph theory and a singular perturbation analysis. The result is a reduced order model of differential and algebraic equations. The geometric nonlinear control method is extended to allow the use of algebraic equations as part of the system model.

The effect of the tuning parameters on the servo and regulatory response of the closed loop system with full state measurement is analysed, resulting in general tuning guidelines. It is demonstrated that, for the case study, a suitable regulatory response is not achievable with full state measurement, although it is achievable if the states are obtained from an open loop observer. This results because of differences in the way errors are propagated through the closed loop system.

A reduced order, stable controller is obtained and tested on the simulation of the case study. The controller is also applied to an industrial scale pilot plant CO₂ absorption/desorption system. The geometric nonlinear controller is compared experimentally to a more traditional technique, PID. The geometric nonlinear controller is more effective in tracking set point changes covering a large range of operating conditions. The abilities of the nonlinear and PID controllers to reject unmeasured disturbances are comparable.

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List of Symbols

A	constant matrix (autonomous system)
$\mathcal{A}$	class of disturbances
A_{ant}	Antoine equation coefficient
A_a	cross sectional area of absorber [m^2]
A_d	cross sectional area of desorber [m^2]
a	vector of algebraic variables
a_i	constants ($i=1, \dots, 3$)
a_{int}	interfacial areas per unit volume [$\frac{m^2}{m^3}$]
B	MEA recirculation flowrate through absorber [$\frac{kmol}{s}$]
$\mathcal{B}$	class of disturbances
B_{ant}	Antoine equation coefficient
b	constant column vector (input function)
b	constant
c	constant row vector (output function)
C	liquid phase CO_2 composition in desorber [$\frac{kmol}{m^3}$] subscript r - reboiler, i - tray $1, \dots, n_d$, f - feed tray, F - feed stream to desorber
$\mathcal{C}$	class of disturbances
$C_{a,i}$	liquid phase CO_2 composition on tray i of absorber ($i = 1, \dots, n_a$) [$\frac{kmol}{m^3}$]
C_{ant}	Antoine equation coefficient
C_{bulk}	bulk CO_2 concentration [$\frac{kmol}{m^3}$]
$C_{interface}$	interfacial CO_2 concentration [$\frac{kmol}{m^3}$]
C_{mea}	total MEA concentration in liquid phase [$\frac{kmol}{m^3}$]
$C_{mea,free}$	free MEA concentration in liquid [$\frac{kmol}{m^3}$]
C_{pc}	CO_2 heat capacity [$\frac{kJ}{kmolK}$]
C_{pl}	liquid phase heat capacity [$\frac{kJ}{kmolK}$]
C_{pv}	water vapour heat capacity [$\frac{kJ}{kmolK}$]
$count$	counter used in graph theory algorithms
D	constant matrix related to E
D	dilution rate

D_c	diffusivity of CO_2 [$\frac{\text{m}^2}{\text{s}}$]
D_M	diffusivity of MEA [$\frac{\text{m}^2}{\text{s}}$]
d	vector of disturbances
d_i	disturbance i of vector d ($i = 1, \dots, q$)
E	matrix of eigenvectors of A
E_{st}	stage efficiency
$e(x)$	error term defined in Equation 5.39
$e_i(x)$	error term defined in Equation 5.39 ($i = 1, \dots, r$)
$e_f(x)$	error term defined in Equation 5.38
$F(x)$	state transformation for state space realisation of inverse
F_g	gas phase flowrate through absorber [$\frac{\text{kmol}}{\text{s}}$]
	second subscript i - to cell, o - from cell, u - feed to absorber
$F_i(x)$	state transformation i of $F(x)$ ($i = 1, \dots, n$)
f	MEA flowrate through desorber [$\frac{\text{kmol}}{\text{s}}$]
	subscript r - reboiler, i - tray $1, \dots, n$, f - feed tray, c - condenser
$f(x)$	vector field (autonomous system)
$f(x, a, u)$	vector field (full system with algebraic variables)
$f(x, a, u)$	vector field (algebraic equations)
$f'(x, u)$	implicit scalar function
$f(x, u)$	vector field (full system)
$f(x, u, d, t)$	vector field (full system with disturbances and time varying parameters)
$f_i(x)$	scalar function i of vector field $f(x)$ ($i = 1, \dots, n$)
$f_i(x, a, u)$	scalar function i of vector field $f(x, a, u)$ ($i = 1, \dots, n'$)
f_v	vapour flowrate in desorber [$\frac{\text{kmol}}{\text{s}}$]
	subscript r - reboiler, i - tray $1, \dots, n$
$\hat{f}(x)$	nominal value for $f(x)$
$f'(x, u, \varepsilon, t)$	decomposition of vector field f (parameter set ε)
$f''(x, u, \varepsilon, t)$	decomposition of vector field f (parameter set ε)
$G(x)$	scalar function
$g(x)$	set of vector fields (input function)
$g_i(x)$	vector field i of set $g(x)$ ($i = 1, \dots, m$)
$g_i(x) _k$	scalar function k of vector field $g_i(x)$ ($k = 1, \dots, n$)
$g(x, a, u)$	function (representing output equation)
$g'(x, a, u, v)$	function (representing output equation)
$g''(x, u, v)$	function (representing output equation including algebraic equations)

g_{const}	gravity constant [$\frac{m}{s^2}$]
$\hat{g}(x)$	nominal value for $g(x)$
H	Henry's law coefficient [$\frac{bar m^3}{kmol}$]
H_{vap}	heat of vaporisation [$\frac{kJ}{kmol}$]
H_{rxn}	heat of reaction [$\frac{kJ}{kmol}$]
H_{st}	heat of condensation for water vapour [$\frac{kJ}{kmol}$]
$h(x)$	vector field (output function)
$h(x, a)$	vector field (output function with algebraic variables)
$h_i(x)$	scalar function i of vector field $h(x)$ ($i = 1, \dots, p$)
h_l	height of liquid in reboiler [m]
h_t	height of transfer unit in absorber [m]
h_w	weir height [m]
h_{ow}	liquid height above weir [m]
$\hat{h}(x)$	nominal value for $h(x)$
$h'(x, u, d, t)$	controller equation for GMC
$h^*(x)$	vector field (input contribution to output function)
I	identity matrix
i	square root of -1
K	overall absorption coefficient [$\frac{\sqrt{kmolm}}{bar}$]
K_1	tuning constant matrix for proportional action
K_2	tuning constant matrix for integral action
K_{DH}	dry hole pressure drop constant
k	tuning constant - gain
k_g	gas phase mass transfer coefficient [$\frac{kmol}{m^2 s bar}$]
k_l	liquid phase mass transfer coefficient [$\frac{m}{s}$]
k_r	reaction rate constant [$\frac{m^3}{kmol.s}$]
k''	enhancement factor constant defined in Equation 3.16
L	MEA flowrate through desorber [$\frac{kmol}{s}$] subscript r - reboiler, i - tray $1, \dots, n_d$, f - feed tray, F - feed stream to desorber, c - condenser
M'	dimensionless group defined in Equation 3.20
$\sqrt{M}$	ratio of reaction rate to diffusion rate
m	number of inputs
mw	molecular weight of solution [$\frac{kg}{kmol}$]
n	number of states

n_a	number of trays in absorber
n_d	number of trays in desorber
n_i	scaling factor
n'	number of algebraic variables
P	pressure in desorber [bar] subscript r - reboiler, i - tray $1, \dots, n_d$, f - feed tray, c - condenser
P_a	pressure in absorber [bar]
P^{sat}	saturated H_2O pressure [bar]
p	number of outputs
p_i	closed loop poles ($i = 1, \dots, r(\text{or } r + 1)$)
$\mathbf{p}$	set of unknow parameters
$\hat{\mathbf{p}}$	set of nominal parameters
$p(x)$	scalar function (control law)
p_c^{eq}	equilibrium CO_2 partial pressure
$p_{interface}$	interfacial gas phase CO_2 partial pressure [bar]
p_{bulk}	bulk gas phase CO_2 partial pressure [bulk]
$path(u, y)$	length of minimum path from u to y
$path_i$	defined in Equation 2.44
plt	aggregated stage pressure drop coefficient
$p^*(x)$	scalar function (reference variable contribution to control law)
q	number of disturbances
$q(x)$	output function for state space linearisation
$\bar{R}$	rate of absorption per unit area of interface [$\frac{kmol}{m^2s}$]
R_i	rate of absorption on stage i of absorber ($i = 1, \dots, n_a$) [$\frac{kmol}{s}$]
R_{stage}	total rate of absorption in a stage [$\frac{kmol}{s}$]
$\mathbf{r}$	vector of relative orders
r_i	relative order i of vector $\mathbf{r}$ ($i = 1, \dots, p$)
r_{rxn}	reaction rate [$\frac{kmol}{m^3s}$]
S	steam flowrate [$\frac{kmol}{s}$]
S_{ub}	substrate concentration
$S_{ub,f}$	feed substrate concentration
s	Laplace domain parameter
$ _{ss}$	steady state values
T	temperature in desorber [$^{\circ}C$]

	subscript r - reboiler, i - tray $1, \dots, n_d$, f - feed tray, c - condenser
$T(x)$	state transformation from x to ξ
$T_{l.}$	temperature of liquid to desorber [$^{\circ}\text{C}$]
$T_i(x)$	state transformation i of $T(x)$ ($i = 1, \dots, n$)
t	time
t_d	dead time
U_i	set of inputs corresponding to a particular output i
u	vector of input variables
u_i	input variable i of vector u ($i = 1, \dots, m$)
$V_{g.}$	gas phase volume on each absorber stage [m^3]
V_l	liquid phase volume on each desorber stage [m^3]
$V_{l.}$	liquid phase volume on each absorber stage [m^3]
v	external reference variable
v_n	vapour velocity through sieve holes [$\frac{\text{m}}{\text{s}}$]
W	matrix of weights
w	constant column vector (disturbance input function)
w	constant
$w(x)$	set of vector fields (disturbance function)
$w_i(x)$	vector field i of set $w(x)$ ($i = 1, \dots, q$)
$w^*(x, u, d)$	propagation of unmeasured disturbance error to closed loop system
X	cell concentration
x	vector of states
x_i	state i of vector x ($i = 1, \dots, n$)
x_w	liquid phase H_2O mole fraction in desorber [$\frac{\text{kmol}}{\text{kmol}}$] second subscript r - reboiler, i - tray $1, \dots, n_d$, c - condenser
$\hat{x}$	nominal model state vector
x'	decomposition of state vector, x
x''	decomposition of state vector, x
Y	output variable in case study
y	vector of outputs
$y_{a,i}$	gas phase CO_2 composition on stage i of absorber ($i = 1, \dots, n_a$) [$\frac{\text{kmol}}{\text{kmol}}$]
y_{bulk}	bulk gas phase CO_2 mole fraction [$\frac{\text{kmol}}{\text{kmol}}$] second subscript i - to cell, o - from cell
y_c	vapour phase CO_2 mole fraction in desorber [$\frac{\text{kmol}}{\text{kmol}}$]

	second subscript r - reboiler, i - tray $1, \dots, n_d$, f - feed tray, c - condenser
$y_{c,}$	gas phase CO_2 mole fraction to absorber
y_c^{eq}	equilibrium vapour phase CO_2 mole fraction in desorber $[\frac{\text{kmol}}{\text{kmol}}]$ second subscript r - reboiler, i - tray $1, \dots, n_d$, f - feed tray, c - condenser
y_i	output i of vector y ($i = 1, \dots, p$)
$y_{interface}$	bulk gas phase CO_2 mole fraction $[\frac{\text{kmol}}{\text{kmol}}]$ second subscript i - to cell, o - from cell
y_{sp}	setpoint for output
y_w	vapour phase H_2O mole fraction in desorber $[\frac{\text{kmol}}{\text{kmol}}]$ second subscript r - reboiler, i - tray $1, \dots, n_d$, f - feed tray, c - condenser
$\hat{y}$	nominal model output
y^*	auxiliary output
Z	variables for state space realisation of inverse
z	dimension along length of absorber $[m]$

Greek Letters

α	carbonation ratio
β	expression representing rate of absorption per unit height defined in Equation 3.19 $[\frac{\text{kmol}}{m \cdot s}]$
β_i	arbitrary constants for geometric nonlinear controller
Δ	distribution
δ	difference operator
ϵ	parameter for extension of controller for ill-defined systems
ε	set of small parameters
$\zeta(x)$	vector field in quasi linear system with unmeasured disturbance
$\theta(x, u)$	vector field (functions of x and u)
θ_{sub}	substrate consumption rate
Λ	diagonal matrix of eigenvalues of A
λ	real valued function
μ	specific growth rate function
ν	vector of lower bounds on relative orders

ν_i	value i of ν ($i = 1, \dots, p$)
$\Xi(x, v)$	implicit function
ξ	set of transformed states
ξ'	set of transformed states
ξ_d	function defined in Equation 5.19
ξ_i	transformed state i of ξ ($i = 1, \dots, n$)
ξ_o	function defined in Equation 5.52
ξ_u	function defined in Equations 5.11 and 5.11
ρ	vector of disturbance inputs relative orders
ρ_g	gas phase density [$\frac{kmol}{m^3}$]
ρ_i	relative order i of ρ ($i = 1, \dots, q$)
ρ_l	liquid phase density [$\frac{kg}{m^3}$]
ρ_v	vapour phase density [$\frac{kg}{m^3}$]
τ	integral time
$\Phi(x)$	scalar function
Φ_{disc}	discrete scalar function
$\Psi(x)$	vector field in quasi linear system with unknown parameters
ψ	enhancement factor
ψ_i	limiting value of enhancement factor

Chapter 1

Introduction

1.1 Control

Control is required in the process industries to ensure that environmental regulations, safety limits and operational requirements are met while meeting quality specifications and maximising profit. The ability to meet all of these requirements is limited by the tightness of the specifications, the characteristics of the system and the control method used.

Control is becoming increasingly important due to tightening of requirements. Most notably, environmental requirements have become much stricter and continue to do so. In addition, safety limits are becoming stricter, tighter product specifications must be met and increasing competition places more importance on maximising profits.

Each system in the process industries has individual characteristics which must be analysed when designing a control system. These characteristics will limit the effectiveness of a control system. In turn, different control strategies have different characteristics which limit the achievable performance from a given system. It is clear that there is a strong interaction between the design of the plant and the design of a control system (Narraway, 1992).

If a plant is designed with little consideration for the effect of control, the ability of the final design to meet the identified requirements may be limited by the control system. On the other hand, if plants are designed with explicit handling of design and control interactions, the best design may be limited by the type of control considered. For control research this highlights the importance of,

1. improving control technology towards achieving the general requirements of control,
2. understanding the benefits and limitations of different control strategies,
3. using as much of the available information about the process as possible in the controller.

1.2 Nonlinear Control

Much of the classical control theory is based on a linear description of the process even though most processes are nonlinear. Linear control techniques will operate effectively in region around the point of linearisation. However, the size of this region strongly depends on the nature of the nonlinearity. Linear control techniques are widely used in the process industries because the systems are often well approximated by a linear description over the required range of operating conditions.

However, there are many cases in the process industries of systems which are poorly controlled by linear control techniques due to nonlinear characteristics (eg. pH control (Waller and Gustafsson, 1983), exothermic CSTR (Chaun and Chen, 1984)). These characteristics are classified as either continuous or discontinuous nonlinearities (Slotine and Li, 1991). Discontinuous nonlinearities are usually associated with process constraints (eg. hysteresis or saturation limits in control valves). In the process industries controllers are often designed to avoid these points of nonlinearity (For example, including positioners on valves or detuning responses to avoid saturation limits). However, the continuous nonlinearities cannot be avoided and these appear as nonlinearities in the gain, nonlinearities in the time constant, multiplicities and limit cycles (other possibilities are given in Slotine and Li, 1991).

The identification of nonlinearities in process systems is not new, nor is the use of nonlinear elements in control structures. Adaptive linear, dual mode and gain scheduling controllers are examples of control strategies which add nonlinear elements to standard linear techniques with the aim of overcoming the deficiencies of these techniques.

More recently, there have appeared two broad classes of nonlinear control techniques which explicitly use a nonlinear model of the process as part of the controller design. These can be classified as nonlinear model predictive controllers and geometric nonlinear controllers. The nonlinear model predictive controllers have developed from the linear model predictive control techniques (eg. dynamic matrix control (Cutler and Ramaker, 1980)). The geometric nonlinear control techniques have developed from the mathematics of differential geometry (eg. state space and input output linearisation) and through independent research (eg. generic model control (GMC) and reference system structures (RSS)).

1.3 Geometric Nonlinear Control

Probably the main reason for the development of geometric nonlinear control techniques from differential geometry (ie. input output linearisation and state space linearisation) is that differential geometry provides a coordinate independent method for analysing the structure of nonlinear, differential systems. This provides the basis for an analogous development of nonlinear control theory with linear control theory.

More importantly, from an implementation perspective geometric nonlinear control is able to account for process nonlinearities and directly uses a model of the process. This is becoming increasingly important because sophisticated dynamic models of process systems are available and there is a need for a transparent method of including the models in the controller design. This has been the driving force for the development of the independent techniques (ie. GMC and RSS).

These reasons make geometric nonlinear control attractive from a theoretical point of view. However, the following questions still remain (cf. Section 1.1).

1. How effective is nonlinear control in achieving the objectives of control?
2. What are the benefits and limitations of geometric nonlinear control?

These issues must be addressed in detail before geometric nonlinear control will be considered as an industrially acceptable technique.

1.4 Thesis Aims

The aim of this thesis is to assess the applicability of geometric nonlinear control in the process industries through the detailed analysis of a case study.

Ultimately this is achieved through an experimental implementation of a geometric nonlinear controller on a pilot plant scale CO₂ absorption/desorption process. The development of the final controller involves,

1. an extensive literature review aimed at geometric nonlinear control,
2. an examination of the structure of the process model and how it affects the complexity of the controller,
3. advancing the theory to allow differential and algebraic descriptions of processes,
4. obtaining basic tuning guidelines for geometric nonlinear controllers,

5. examining problems associated with full state measurement,
6. implementing the controller in simulation, and on the plant.

1.5 Thesis Outline

Chapter 2 provides a literature review of the geometric nonlinear control methods and recent advances towards obtaining a generalised framework. This includes a record of the reviewed simulation and experimental applications of geometric nonlinear controllers in the process industries.

Chapter 3 describes the CO₂ absorption/desorption process, which is the main case study for this thesis. A description of a mathematical model and details of the experimental system are provided. Open loop results are used to estimate the parameters required to complete the specification.

Chapter 4 uses graph theory to examine the complexity of the geometric nonlinear controller and how it is related to the structure of the original model. This leads to a logical approach to model reduction with the aim of reducing controller complexity. An extension of the basic theory is provided to allow the use of differential and algebraic equations in mathematical models. The result is a reduced model with an associated controller which is greatly reduced in complexity.

Chapter 5 considers the tuning of geometric nonlinear controllers for servo and regulatory responses resulting in some general tuning guidelines. The implementation of the geometric nonlinear controller with direct state feedback from the plant does not allow the specification of a suitable regulatory response. The problem arises because the errors due to unmeasured disturbances and model mismatch are propagated through the derivative expressions. The problem is overcome by observing the states with an open loop observer, where errors are propagated through the difference between the observer output and the setpoint.

Chapter 6 demonstrates the implementation of the controller on a simulation of the case study and experimentally on the pilot plant. Tuning of the controller is adjusted on the experimental system to obtain a satisfactory response. The geometric nonlinear controller is compared, both in simulation and experimentation, to a PID controller.

An overview of the thesis is presented in Chapter 7, along with some general conclusions and suggestions for further areas of work.

Chapter 2

Geometric Nonlinear Control

This chapter is a review of the previous work on control techniques based on differential geometry, both from a theoretical and a practical point of view.

Section 2.1 introduces a number of different control techniques which have developed in the literature and presents them from a basis of the realisation of the system inverse. This approach highlights the inter-relationship between the different methods which are referred to as geometric nonlinear control methods. The introduction of the theory is followed by a discussion of the various practical and theoretical limitations of the techniques including the recent modifications which have developed to overcome these limitations. This results in an overall controller structure, incorporating most of the elements required for practical applications.

The geometric nonlinear control techniques have been successfully applied to simulations of a wide range of process systems. A review of these applications is given in Section 2.2. The development of a control method, towards an industrially acceptable method requires experimental verification of its applicability. A number of such applications have appeared in the literature over the last three years and these are reviewed in Section 2.3.

Graph theory is used in the literature to analyse the structure of nonlinear systems. It is particularly applicable as an aid for the design of geometric nonlinear controllers. Section 2.4 gives an introduction to the use of graph theory.

This chapter is concluded by a summary of the present state of the art of geometric nonlinear control.

2.1 Review of Theory

2.1.1 Introduction

The focus of this thesis is the nonlinear controller designs based on differential geometry techniques. These techniques have developed from the abstract mathematical perspective of differential geometry. From the early works of Brockett, 1978, Porter, 1970, Singh and Rugh, 1972, Freund, 1975, has developed an approach to controller design using a state space or an input output description of the process.

One set of methods, which use an input output model, are analogous to linear methods using a transfer function description of the process. Nonlinear operators are used to produce a description directly linking the output to the input (nonlinear analogy of a transfer function for linear systems). The resulting nonlinear controller requires measurement only of the system output. Care must be taken with these techniques as the abstract nonlinear operators effectively hide all the explicit dynamics, which are required to obtain a stable controller. One technique based on this method is the nonlinear IMC structure of Economou *et al.*, 1986.

A second set of methods which uses an input output model are called reference system structures. These methods avoid the use of an internal model, as required for the IMC methods, by using measurements of the output and its derivatives. Developments of these methods are given by Bartee *et al.*, 1989 and Bartusiak *et al.*, 1989.

Methods which use the state space description of the plant can be divided into two groups. One group aims to achieve a linearisation in a state space sense, that is, to achieve a linear map between the input and all of the states. This approach is called state space linearisation and has been developed by Jakubczyk and Respondek, 1980, Su, 1982, Hunt *et al.*, 1983. The other approach aims to achieve a linearisation in an input output sense. That is, to achieve a linear map between the input and output. This approach will be referred to as input output linearisation and has been developed by Isidori *et al.*, 1981, Isidori and Krener, 1982, Krener and Isidori, 1983, Byrnes and Isidori, 1984. Both of these approaches use static state feedback to exactly linearise the system and require the measurement of all the states (note: The assumption is generally made in the literature that all the states are measured. However, in some situations, measurement of some of the states is not required).

The type of systems that may be exactly linearised is restricted. A number of alternative linearisation methods exist which do not exactly linearise the system, though partially account for the nonlinearity in the controller design. These methods allow a larger class of problem to be considered. For example, the approximate linearisation method is discussed by Krener, 1984, the uniform systems approximation method is discussed by Hauser *et al.*, 1989, the pseudo linearisation method is discussed by Champetier *et al.*, 1984 and the extended linearisation method is discussed by Baumann and Rugh, 1986. These alternative linearisation methods will not be considered further in this thesis.

In addition to the general approaches a number of more specific techniques have developed independently, for example generic model control of Lee and Sullivan, 1988a. The relationships between the various differential geometric techniques has been reviewed

by a number of authors, including Kravaris and Daoutidis, 1990, Henson and Seborg, 1991a, Kantor, 1986a Kravaris and Arkun, 1991 and McLellan *et al.*, 1990b. These works address the issue of controller design from a chemical engineering perspective and this has led to a more practical interpretation of the differential geometric techniques.

A complete theoretical understanding of differential geometric approaches to controller design requires a substantial grounding in the mathematics of nonlinear systems. Many references, such as Bell, 1990 or Boothby, 1975, present this information. Isidori, 1989 provides a very complete link between the mathematical theory and its use for controller design.

Much of the work on differential geometry uses an abstract language to achieve rigour. However, mathematical rigour is achieved at the expense of ease of understanding. A controller designer requires only a general overview to differential geometry. A short overview of the essential elements of differential geometry and Lie algebra is presented in Appendix A. For a more complete explanation see Slotine and Li, 1991 or Isidori, 1989. A very good introduction to the terminology and theory of differential geometry is given in Bell, 1990.

The following sections introduce the theory and terminology of the control techniques based on differential geometry (geometric nonlinear control). A review of the advances towards a practical control technique is also given. This discussion is structured as follow. In Section 2.1.2 a brief description of the essential mathematical results from differential geometry is presented, including a discussion on system inverses for linear and nonlinear systems. This is followed by a description of five approaches to designing geometric nonlinear controllers, and the relationships between them. A discussion of these methods, particularly focussing on implementation concerns, is given in Section 2.1.4.

2.1.2 System Inverses

The core to geometric nonlinear control is the realisation of the inverse of a nonlinear system. Kantor, 1986a states that the inverse of a system may be defined in two ways, called the left and right inverse. The left inverse reconstructs the input that must have been used to obtain a given output history. The right inverse produces the input required to obtain a particular output trajectory (See Figure 2.1). Both of these inverses are dynamical systems.

The concept of a system inverse for a linear system is intuitive and will be presented first. This will be followed by an analogous development for nonlinear systems.

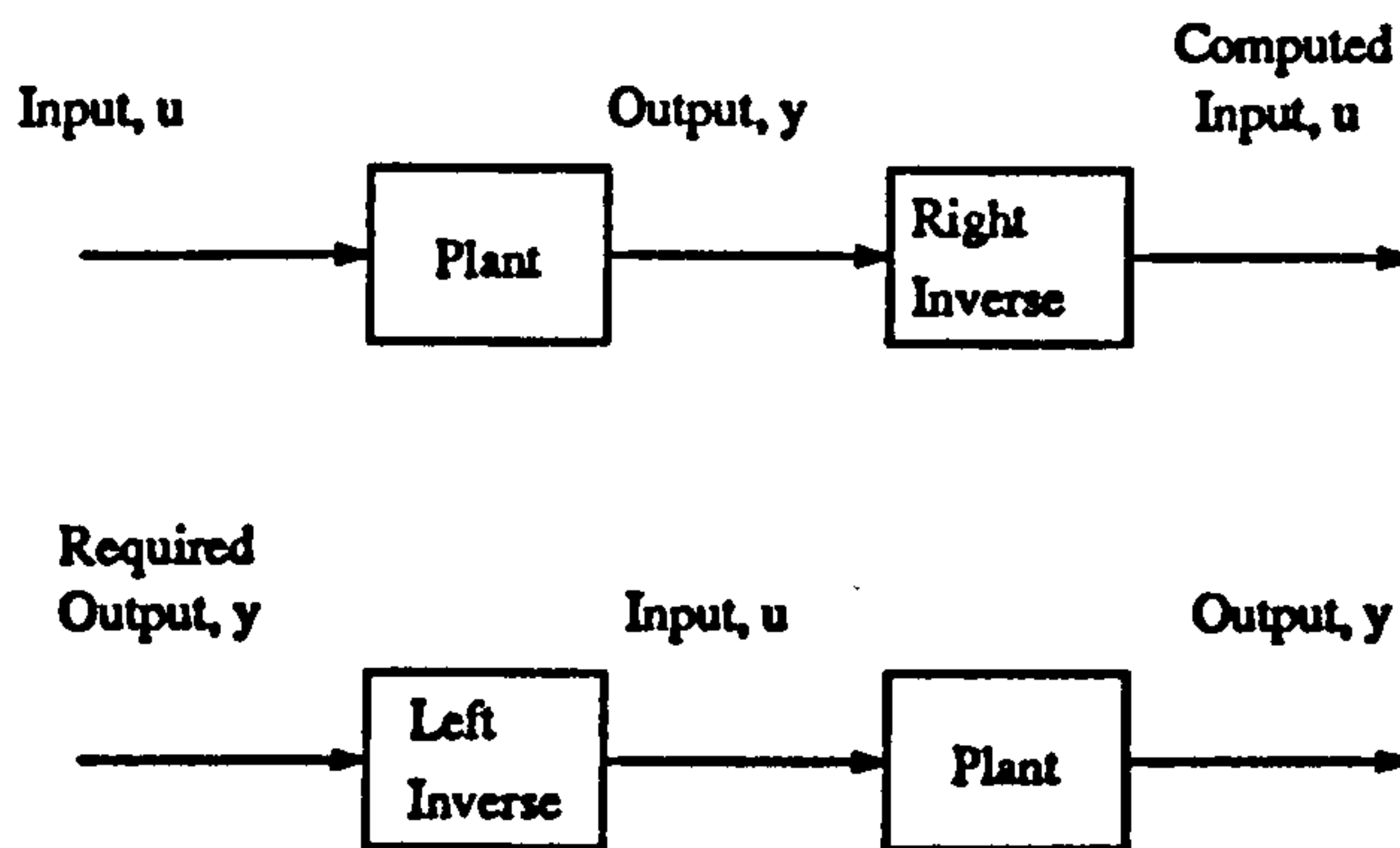


Figure 2.1: Right and left inverses.

Inverse for a Linear System

Consider the following single input single output linear system,

$$\begin{aligned}\dot{x} &= Ax + bu \\ y &= cx\end{aligned}\tag{2.1}$$

where, x is the vector of state variables, $x \in \mathbb{R}^n$,

y is the output variable, $y \in \mathbb{R}$,

u is the input variable, $u \in \mathbb{R}$,

A is a constant matrix, $A \in \mathbb{R}^{n \times n}$,

b is a constant vector, $b \in \mathbb{R}^n$,

c is a constant vector, $c \in \mathbb{R}^{1 \times n}$.

The system may be represented by a transfer function as follows,

$$\frac{y(s)}{u(s)} = \frac{c \text{Adj}(sI - A)b}{\det(sI - A)}\tag{2.2}$$

This transfer function is the input output description of the system in the Laplace domain. The power and compactness of this description results from the representation of the dynamical elements through the s operator. The system inverse is simply described as,

$$\frac{u(s)}{y(s)} = \frac{\det(sI - A)}{c \text{Adj}(sI - A)b}\tag{2.3}$$

This description is commonly used for analysis of linear systems, though it hides the actual complexity of the dynamical system which it describes. To see the full structure

of this system requires a state space description. Kravaris and Daoutidis, 1990 propose that the state space realisation of the inverse is,

$$\begin{aligned}\dot{Z} &= \left(A - \frac{bcA^r}{cA^{r-1}b} \right) Z + \frac{b}{cA^{r-1}b} \frac{d^r y}{dt^r} \\ u &= \frac{1}{cA^{r-1}b} \frac{d^r y}{dt^r} - \frac{cA^r}{cA^{r-1}b} Z\end{aligned}\quad (2.4)$$

where, Z is a vector of transformed variables, $Z \in \mathbb{R}^n$,

r is the relative order of the system.

This is a linear system with the output of the original system, y , as an input, through $\frac{d^r y}{dt^r}$, and the input of the original system, u , as an output.

If the open loop system is minimum phase (all the zeros are in the left half plane) this realisation will be input output stable. However, the order of the inverse, as demonstrated by the transfer function in Equation 2.3, is in general less than the number of states n . The order of the system in Equation 2.4 is n . Therefore, this realisation relies on a number of zero pole cancellations and therefore can be internally unstable (Kravaris and Daoutidis, 1990). To avoid this problem a minimal order and therefore internally stable realisation of the inverse is required.

To obtain a minimal order state space realisation of the inverse requires the definition of the relative order of a system. For a linear system the relative order is defined as the smallest integer r such that,

$$cA^{r-1}b \neq 0 \quad (2.5)$$

This equals the difference between the order of the denominator (always n) and the order of the numerator (ranges from 0 to $n - 1$ for proper systems). It represents the number of times the output must be differentiated to depend explicitly on the manipulated input, u .

A minimal order state space realisation of the inverse of the system must be of order $n - r$. Details about the minimal order realisation of the inverse of a linear system are found in Kravaris and Daoutidis, 1990. In summary, the realisation is a linear dynamical system of order $n - r$, with the output of the original system, y , as an input through the terms, $y, \dots, \frac{d^{r-1}y}{dt^{r-1}}$, and the input of the original system, u , as the output. The derivative of the output is required to add integrators to the inverse as the inverse is not a proper, rational function.

Both realisations of the inverse will give the same input output response and this is equal to the transfer function representation of the inverse in Equation 2.3. The minimal order inverse does not have zeros cancelled by poles and therefore is always internally stable. These issues of stability of the inverse and the relationship with the characteristics of the open loop system and realisation of the inverse are summarised in Figure 2.2.

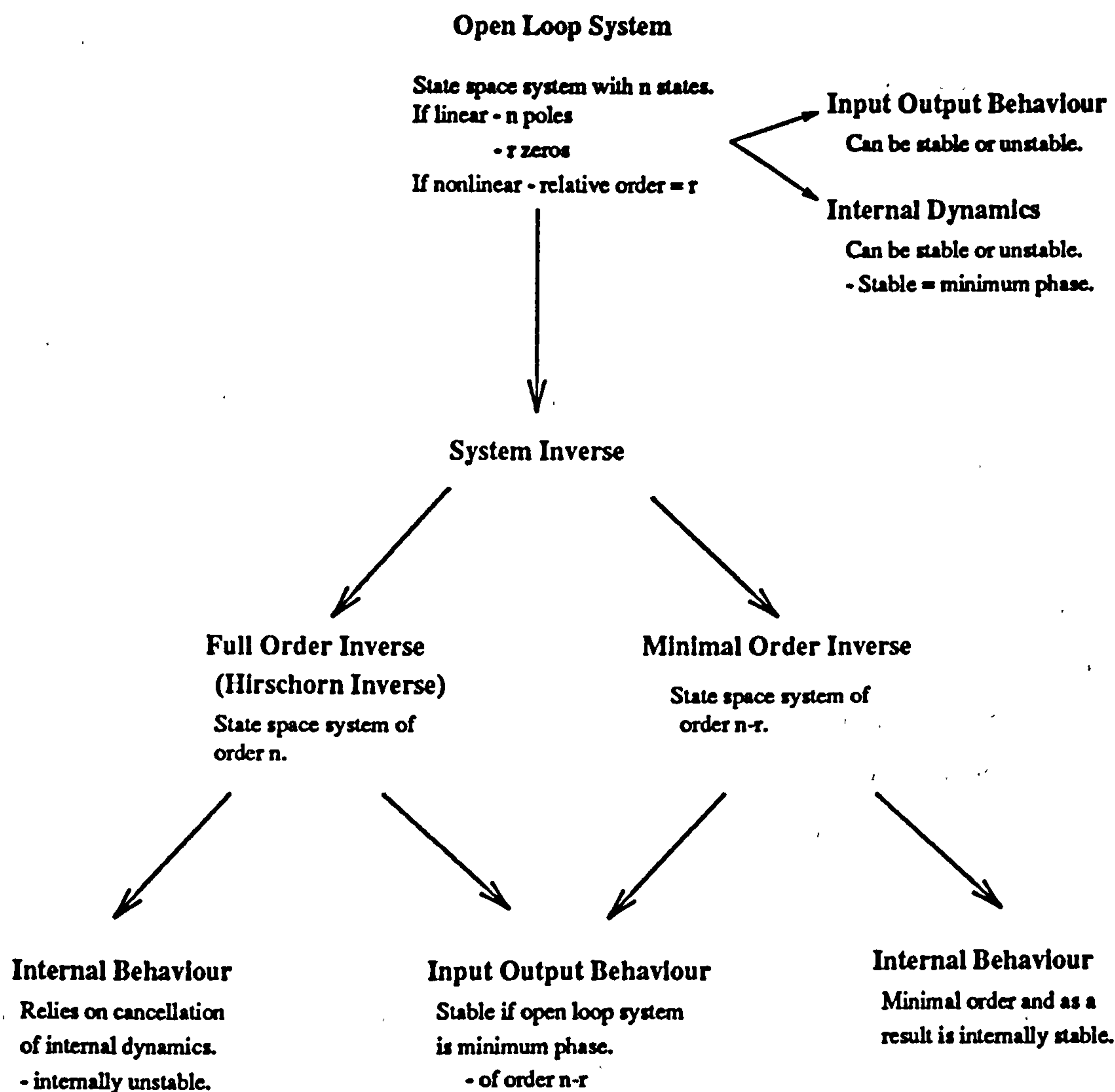


Figure 2.2: Internal and input output stability of the system inverse.

Inverse for a Nonlinear System

Much of the available work on nonlinear control is only applicable to time invariant, control affine state equations, without disturbances.

$$\dot{x} = f(x) + g(x)u$$

$$y = h(x) \quad (2.6)$$

where, x is the vector of state variables, $x \in \mathbb{R}^n$,

u is the vector of manipulated inputs, $u \in \mathbb{R}^m$,

y is the vector of outputs, $y \in \mathbb{R}^p$,

$f(x)$ is a vector field, $f(x) \in \mathbb{R}^n$,

$g(x)$ is a set of vector fields, $g(x) \in \mathbb{R}^{n \times m}$,

$h(x)$ is a vector field, $h(x) \in \mathbb{R}^p$.

Often the analysis is simplified further to consider only single input, single output, control affine systems,

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \quad (2.7)$$

where, x is the vector of state variables, $x \in \mathbb{R}^n$,

u is the manipulated input, $u \in \mathbb{R}$,

y is the output variable, $y \in \mathbb{R}$,

$f(x)$ and $g(x)$ are vector fields, $f(x), g(x) \in \mathbb{R}^n$,

$h(x)$ is a scalar function, $h(x) \in \mathbb{R}$.

The following discussion of the realisation of the inverse for a nonlinear system will be based on this system.

As with a linear system, the aim is to obtain a state space description of the trajectory of the input of the system given a particular output trajectory. Hirschorn, 1979 defined a state space realisation of the inverse of a nonlinear system, which is analogous to the inverse of a linear system. The realisation of the inverse requires the following definition of the system relative order for a nonlinear system. The relative order of a single input single output control affine nonlinear system (See Equation 2.7) is the integer r such that,

$$L_g L_f^{r-1} h(x) \neq 0 \quad (2.8)$$

$$\text{and} \quad L_g L_f^i h(x) = 0 \quad \forall i = 0, \dots, r-2 \quad (2.9)$$

By definition (Freund, 1975) a system has a relative order of zero if the output is explicitly related to the input, for example, if $h^*(x) \neq 0$ in the following output function.

$$y(t) = h(x) + h^*(x)u(t) \quad (2.10)$$

The relative order is said to be ill-defined at a point x^0 if the first non-zero Lie derivative is non-zero in a neighbourhood of x^0 and zero at x^0 . Examples of input/output systems with ill-defined relative orders are given in Hauser *et al.*, 1989 and Henson and Seborg, 1990c.

The relative order represents the number of times the output must be differentiated for it to explicitly depend on the manipulated input, u . A single input single output control affine nonlinear system (Equation 2.7) has the following state space realisation of the inverse,

$$\begin{aligned} \dot{Z} &= f(Z) + g(Z) \frac{\frac{d^r y}{dt^r} - L_f^r h(Z)}{L_g L_f^{r-1} h(Z)} \\ u &= \frac{\frac{d^r y}{dt^r} - L_f^r h(Z)}{L_g L_f^{r-1} h(Z)} \end{aligned} \quad (2.11)$$

where, Z is the vector of transformed variables,

r is the relative order of the system.

This realisation of the inverse was introduced by Hirschorn, 1979 and has become known as the Hirschorn inverse. As stated by Kravaris and Daoutidis, 1990 this is not a minimal order realisation of the inverse. The minimal order inverse of the system is obtained by first transforming the system to the Byrnes-Isidori form (Byrnes and Isidori, 1985) and then finding the Hirschorn inverse of this transformed system. The Byrnes Isidori form is,

$$\begin{aligned} \dot{\xi}_1 &= F_1(\xi) \\ &\vdots \\ \dot{\xi}_{n-r} &= F_{n-r}(\xi) \\ \dot{\xi}_{n-r+1} &= \xi_{n-r+2} \\ &\vdots \\ \dot{\xi}_{n-1} &= \xi_n \\ \dot{\xi}_n &= \Phi(\xi) + G(\xi)u \\ y &= \xi_{n-r+1} \end{aligned} \quad (2.12)$$

The Hirschorn inverse of the transformed system in the Byrnes-Isidori form is the minimal order realisation of the inverse of the system in Equation 2.7. The inverse has the following structure,

$$\begin{aligned} \dot{Z}_1 &= F_1 \left(Z_1, \dots, Z_{n-r}, y, \frac{dy}{dt}, \dots, \frac{d^{r-1}y}{dt^{r-1}} \right) \\ &\vdots \\ \dot{Z}_{n-r} &= F_{n-r} \left(Z_1, \dots, Z_{n-r}, y, \frac{dy}{dt}, \dots, \frac{d^{r-1}y}{dt^{r-1}} \right) \\ u &= \frac{\frac{d^r y}{dt^r} - \Phi \left(Z_1, \dots, Z_{n-r}, y, \frac{dy}{dt}, \dots, \frac{d^{r-1}y}{dt^{r-1}} \right)}{G \left(Z_1, \dots, Z_{n-r}, y, \frac{dy}{dt}, \dots, \frac{d^{r-1}y}{dt^{r-1}} \right)} \end{aligned} \quad (2.13)$$

This realisation of the system inverse, can be used either as a left or right inverse. To be used as a right inverse the state vector must be initialised correctly (Hirschorn, 1979). The importance of obtaining a minimal order realisation of the inverse was highlighted in the discussion in linear systems. This discussion is summarised in Figure 2.2.

The geometric nonlinear control techniques use some form of feedback (state space, dynamic output or output plus derivatives) to obtain a closed loop system which can be transformed to a system with this structure. The key difference is the method of using the feedback information to obtain the inputs to Equation 2.13 ($y, \dots, \frac{d^{r-1}y}{dt^{r-1}}$). These techniques are described below.

2.1.3 Description of Geometric Nonlinear Control

The exact linearisation methods start with a nonlinear system of the form 2.7 and attempt to find a feedback of the form,

$$u = p(x) + p^*(x)v \quad (2.14)$$

where, u is the input to the system, $u \in \mathbb{R}$,

x is the vector of states, $x \in \mathbb{R}^n$,

v is a new external reference variable, $v \in \mathbb{R}$.

to give a closed loop system which can be transformed through the nonlinear change of coordinates,

$$\xi = T(x) \quad (2.15)$$

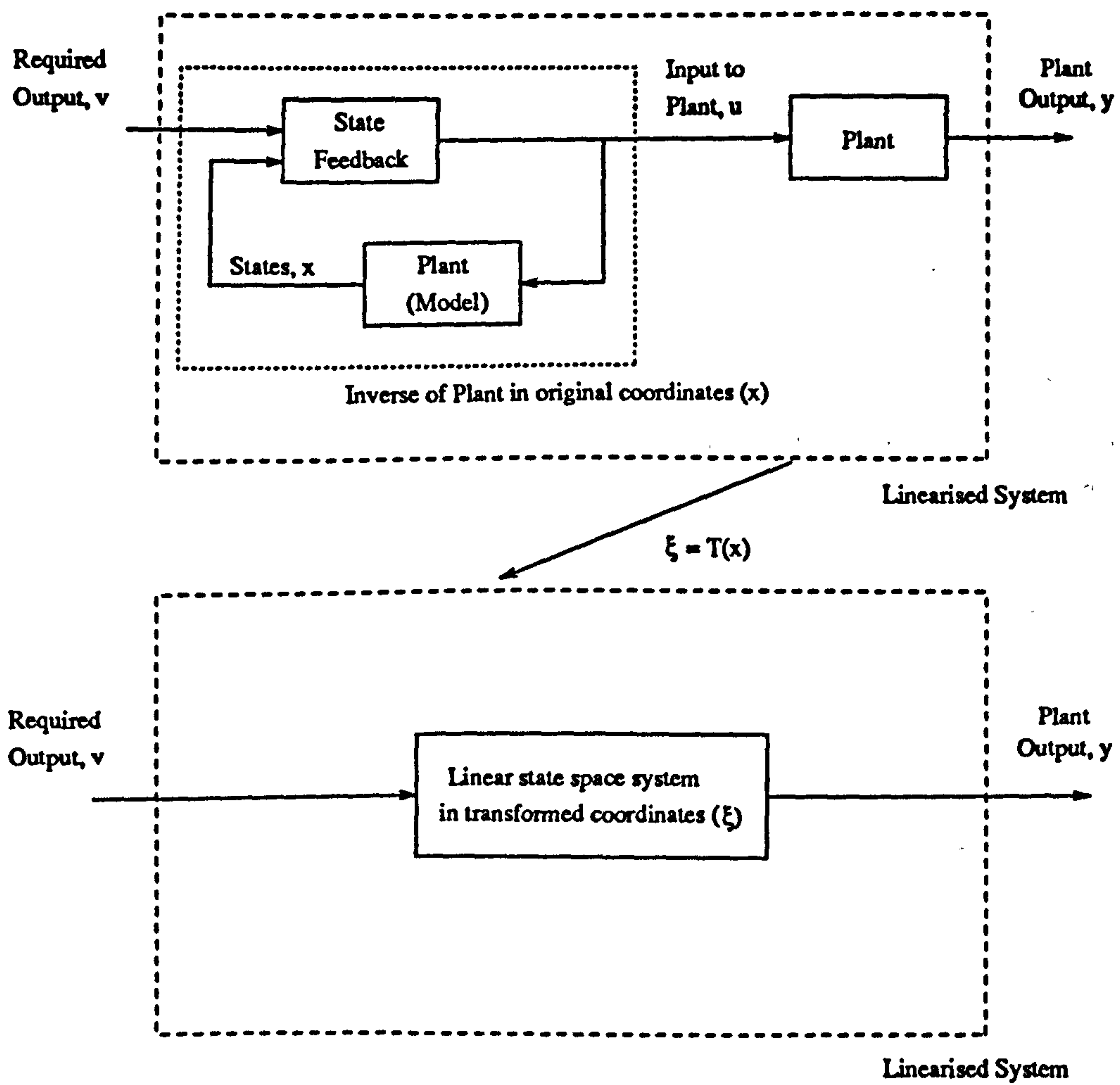


Figure 2.3: Explicit realisation of system inverse through state feedback.

such that the transformed system, in coordinates ξ , is a linear state space system. This is summarised in Figure 2.3.

This structure is known as an explicit realisation of the inverse as the output derivatives of the original system, required to complete the inverse (inputs to Equation 2.13) are obtained from state information using a model of the plant. Under the assumption that the plant and the model are the same, this structure is the same as Figure 2.4. This structure is known as an implicit realisation of the inverse because the output derivatives are obtained directly from the states of the plant.

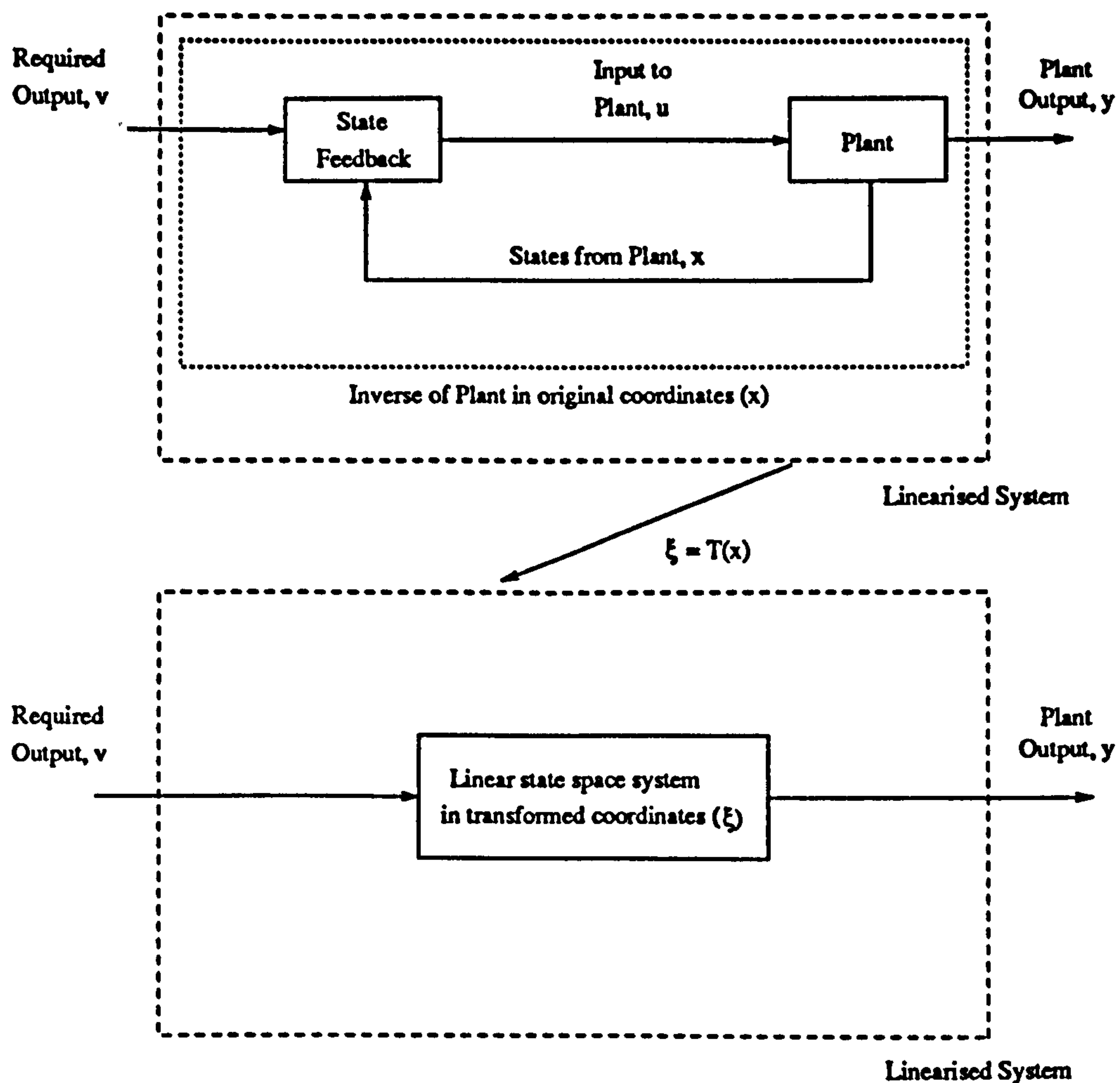


Figure 2.4: Implicit realisation of system inverse through state feedback.

There are four main approaches to solving the exact linearisation problem. The input, output linearisation method uses a state space representation of the system and attempts to linearise the relationship between the inputs and outputs of the closed loop system. The state space linearisation method uses the state equations of a state space representation only and attempts to linearise the relationship between the inputs and state variables. This method is a subset of the input output linearisation method, though due

to historical developments it is generally considered independently in the literature. The third method is called reference system structure and uses derivatives of the output to reconstruct the inverse of the system. The fourth method uses a dynamic output feedback rather than a static state feedback and is the nonlinear extension to the internal model control (IMC) approach. All these methods are discussed below. A final technique called generic model control (GMC) will also be discussed. GMC is essentially a special case of the input output linearisation method, which has been used in many applications in the literature. Therefore, it receives separate treatment in this introduction.

Input Output Linearisation

The general approach to input output linearisation was introduced by Kravaris and Chung, 1987. This paper gives a very clear and concise description of input output linearisation including a consideration of robustness analysis and observers.

A control affine single input, single output system (Equation 2.7) is considered. The following nonlinear state feedback equation will input output linearise this system.

$$u = \frac{v - \sum_{k=0}^r \beta_k L_f^k h(x)}{\beta_r L_g L_f^{r-1} h(x)} \quad (2.16)$$

where, r is the relative order of the system,

$\beta_0, \dots, \beta_r$ are arbitrary constants.

The input output dynamics of the closed loop system may be described by,

$$\sum_{k=0}^r \beta_k \frac{d^k y}{dt^k} = v \quad (2.17)$$

The resulting closed loop system is of minimal order. The linearisation is only local, in the sense that three conditions must be met. First the nonlinear system must have a finite and well defined relative order. This ensures that the input is actually linked to the output and that the state feedback is bounded. Secondly the system must be minimum phase. This requirement ensures that the system is internally stable and therefore stably invertible. Finally, the equations must be sufficiently differentiable. Sufficiency depends on the structure of the system and C^∞ functions are normally assumed.

The transformation for the input output linearisation method is,

$$\xi_{n-r+1} = T_{n-r+1}(x) = h(x) = y$$

$$\begin{aligned}
\xi_{n-r+2} &= T_{n-r+2}(x) = L_f h(x) = L_f T_{n-r+1}(x) \\
&\vdots \\
\xi_n &= T_n(x) = L_f^{-1} h(x) + L_g L_f^{-1} h(x) = L_f T_{n-1}(x) + L_g T_{n-1}(x)u \quad (2.18)
\end{aligned}$$

The closed loop system in the transformed coordinates is the following linear system.

$$\begin{aligned}
\dot{\xi}_{n-r+1} &= \xi_{n-r+2} \\
\dot{\xi}_{n-r+2} &= \xi_{n-r+3} \\
&\vdots
\end{aligned} \quad (2.19)$$

$$\begin{aligned}
\dot{\xi}_{n-1} &= \xi_n \\
\dot{\xi}_n &= v - \beta_1 \xi_{n-1} \dots - \beta_r \xi_{n-r+1}
\end{aligned} \quad (2.20)$$

$$y = \xi_{n-r+1} \quad (2.21)$$

To complete a diffeomorphic transformation requires the specification of a further $n-r$ transformations, $T_i(x)$. Isidori, 1989 states that it is possible to find the $n-r$ extra transformations which are independent of the transformed input, v .

$$\begin{aligned}
\dot{\xi}_1 &= T_1(\xi) \\
&\vdots \\
\dot{\xi}_{n-r} &= T_{n-r}(\xi)
\end{aligned} \quad (2.22)$$

The coordinates $\xi_{n-r+1}, \dots, \xi_n$ determine the input output behaviour of the closed loop system. In the closed loop system the first $n-r$ coordinates have no effect on the last r coordinates and are therefore unobservable. They represent the internal dynamics of the closed loop system and are equal to the internal dynamics of the open loop system. For the closed loop system these dynamics must be stable. It has been noted by Kravaris, 1988 that input output linearisation is equivalent to zero pole cancellation in linear controller design. Through state feedback the internal dynamics of the original system are rendered unobservable. In the limit that the magnitudes of the closed loop poles tend to infinity an ISE optimal response for step changes in the new reference input, v is obtained (Kravaris, 1988).

The internal dynamics of the closed loop system will be locally asymptotically stable if the zero dynamics are locally asymptotically stable. The unforced zero dynamics of the open loop system, are defined as follows.

$$\begin{aligned}
\dot{\xi}_1 &= T_1(\xi_1, \dots, \xi_{n-r}, 0, \dots, 0) \\
&\vdots \\
\dot{\xi}_{n-r} &= T_{n-r}(\xi_1, \dots, \xi_{n-r}, 0, \dots, 0)
\end{aligned} \tag{2.23}$$

Analogous to linear control theory, a nonlinear system with stable unforced zero dynamics is said to be minimum phase, otherwise it is nonminimum phase (Byrnes and Isidori, 1985). Kravaris and Daoutidis, 1990 make the point that more or less stringent stability requirements may be necessary, depending on the situation. One almost universally relevant example is when there is a disturbance (Henson and Seborg, 1990b). In this case it is necessary to test the forced internal dynamics. Checking the location of the zeros of the linear approximation of the the open loop nonlinear system at a nominal operating point is equivalent to testing the stability of the unforced zero dynamics, at the same point. Computing and testing the stability of the internal dynamics of a large system is very difficult. For many examples in the process industries it will suffice to compute the zeros of the linear approximation of the nonlinear system at a number of operating points, over the full range of operating conditions.

The stability of the internal dynamics is totally dependent on the structure of the open loop input, output system. By redefining the output it is possible to alter the internal dynamics, though state feedback alone does not alter them. Kantor, 1986b presents an example demonstrating how the choice of output can effect the range of stable internal dynamics for a system.

A more detailed description of the stability of internal dynamics is presented in Slotine and Li, 1991 and Isidori, 1989.

Input output linearisation of single input, single output systems is discussed by a large number of authors. Those dealing with this method in detail include Isidori, 1989, Kravaris and Daoutidis, 1990 and Kravaris and Chung, 1987. A less mathematical approach to input output linearisation is presented by Kravaris and Arkun, 1991, Henson and Seborg, 1991a, Kantor, 1986a and a very readable account is given by Slotine and Li, 1991.

The input output linearisation method is also valid for control affine multi-input, multi-output systems (Equation 2.6). The extension to multi-input, multi-output systems is presented by Andrea and Levine, 1986 and is called the decoupling problem. This produces a linearised closed loop system with noninteracting control loops. A more general and less restrictive extension for multiple input, multiple output systems, which does not

necessarily decouple the control loops, has been discussed by Isidori, 1989 and Kravaris and Soroush, 1990. As for single input, single output systems, the open loop system must have stable internal dynamics. The realisation of the inverse of a multi-input, multi-output system, and hence the internal dynamics, is given by Daoutidis and Kravaris, 1991. Charlet *et al.*, and Slotine and Li, 1991 discuss a method for solving the decoupling problem for a larger class of systems by using a dynamic state feedback.

State Space Linearisation

The state space linearisation method, also known as Su-Hunt-Meyer linearisation, is described in Su, 1982 and Hunt *et al.*, 1983.

In Su, 1982 and Hunt *et al.*, 1983 a control affine single input, single output system (Equation 2.7) is considered. This method attempts to find a real valued function, $q(x)$ satisfying,

$$\begin{aligned} L_g L_f^i q(x) &= 0 & i = 0, \dots, n-2 \\ L_g L_f^{n-1} q(x) &\neq 0 \end{aligned} \quad (2.24)$$

Determining $q(x)$ requires the solution of a set of partial differential equations. This is computationally very difficult and an analytical solution is not always available.

The following static state feedback will fully linearise the nonlinear system.

$$u = \frac{v - \sum_{k=0}^{n-1} \beta_k L_f^k q(x)}{\beta_n L_g L_f^{n-1} q(x)} \quad (2.25)$$

The similarity between this equation and the static state feedback required by the input output linearisation in Equation 2.16 is obvious. The state space linearisation differs because an output function producing a system with a relative order of n must be available. It can be seen that a nonlinear system with an output of relative order n will have a solution to the input output linearisation problem which will also solve the state space linearisation problem. Such a system is said to have a distinguished process output (Kantor, 1986a) and the linearisation is called a full linearisation (Kravaris and Kantor, 1990).

The coordinate transformation for the state space linearisation method is,

$$\xi_1 = T_1(x) = q(x)$$

$$\begin{aligned}
\xi_2 &= T_2(x) = L_f T_1(x) \\
\xi_3 &= T_3(x) = L_f T_2(x) \\
&\vdots \\
\xi_n &= T_n(x) = L_f T_{n-1}(x) + (L_g T_{n-1}(x)) u
\end{aligned} \tag{2.26}$$

The closed loop system in the transformed coordinates is the following linear system, of order n .

$$\begin{aligned}
\dot{\xi}_1 &= \xi_2 \\
\dot{\xi}_2 &= \xi_3 \\
&\vdots \\
\dot{\xi}_{n-1} &= \xi_n \\
\dot{\xi}_n &= v - \beta_1 \xi_1 - \dots - \beta_n \xi_n
\end{aligned} \tag{2.27}$$

The closed loop system is linear in two senses. Firstly the relationship between the new reference point, v , and each of the transformed states, ξ , is linear. Secondly, the relationship between the new reference point, v and the new output function, $q(x)$, is linear. For systems without a distinguished output the relationship between the new reference point, v , and any prespecified output, y , may be nonlinear. An output function which is a linear combination of the transformed states will also be linear with respect to v .

For a set of state equations to be state space linearisable two conditions must be met.

1. $g(x), ad_f^1 g(x), \dots, ad_f^{n-1} g(x)$ are linearly independent.
2. The set of vector fields $g(x), ad_f^1 g(x), \dots, ad_f^{n-2} g(x)$ is involutive.

The first condition is equivalent to the controllability condition for linear systems. The second condition is a requirement on the structure of the nonlinear state space system. The involutivity condition is very restrictive for a general set of nonlinear equations. However, many simple chemical engineering examples have an underlying structure which makes them involutive and all second order systems are automatically involutive (Kravaris and Arkun, 1991).

The problem of requiring stable internal dynamics for the input output linearisation method is not applicable to the state space linearisation problem because this method

finds an output function which has an associated relative order, r , equal to the system relative order, n . Therefore, there are no internal dynamics.

A complete theoretical approach to state space linearisation may be found in Su, 1982 and Hunt *et al.*, 1983. Isidori, 1989 also gives a very complete and concise development of state space linearisation. All of the review articles, Kravaris and Arkun, 1991, Henson and Seborg, 1991a, Kantor, 1986a give a less mathematical approach to state space linearisation, and a very readable account is given by Slotine and Li, 1991.

Reference System Structures

The method of reference system structures (RSS) was originally presented by Bartusiak *et al.*, 1989 for a general nonlinear system of relative order one. This method allows the specification of a general reference trajectory (linear or nonlinear) for the first derivative of the output, leading to an implicit relationship for the manipulated variable. The method does not require the original model to be control affine and incorporates feed forward action for relative order one disturbances. This method is a direct subset of the input output linearisation method for relative order one systems, with an external PI regulator.

The reference system structure has been extended to a general nonlinear system with the relative order equal to the system order ($r = n$) by Bartee *et al.*, 1989. By analogy with the input output linearisation and state space linearisation methods, the RSS uses a realisation of the system inverse. The input output and state space linearisation methods use state feedback to compute the inputs to the inverse (in the form of the derivatives of the output variable) and the RSS uses measurements of the output and its derivatives directly as the inputs to the inverse. The resulting control law is directly analogous to the state space linearisation control law with derivatives of the output replacing the algebraic relationships of the state variables.

A similar approach to RSS based on an error trajectory description is described by Bhat *et al.*, 1990. The error trajectory description simply considers the trajectory of the error between the output and the setpoint rather than considering the trajectory of the output directly (See McLellan *et al.*, 1990b). This method is the same as the method presented by Bartusiak *et al.*, 1989 and is relevant only to systems of relative order one. An extension of this method for systems of any relative order is presented in Chidambaram and Malleswararao, 1992, resulting in a control law which is analogous to the input output linearisation method with the state relationships replaced by derivatives of the output.

The RSS method can be seen to be a direct subset of the input output lineari-

sation method in principle. The fact that the plant information is obtained through the output and its derivatives appears to be a disadvantage for practical applications.

IMC Based (Dynamic Output) Methods

The set of nonlinear controllers which are based on the differential geometric approach to be considered are extensions to the linear internal model control (IMC) approach. The use of the basic IMC structure for nonlinear systems was originally presented by Economou *et al.*, 1986. This development used abstract operators to represent the input output nonlinear system and hence derive the nonlinear IMC approach. As with the linear IMC approach, an inverse of the nonlinear system is required and the approach of Hirschorn, 1979 was originally used. As has been stated this inverse is nonminimal and is not internally stable (Kravaris and Kantor, 1990). Therefore, a numerical approach to computing the inverse based on Newton's method was adopted. This was originally suggested in Economou and Morari, 1985 and later used by Li and Biegler, 1990. However, the method is computationally intensive and difficult to analyse (Henson and Seborg, 1991a).

The extension of the IMC approach using a minimal order realisation of the inverse is presented by Henson and Seborg, 1990b (and again later by Henson and Seborg, 1991a). The structure of the IMC controller is shown in Figure 2.5.

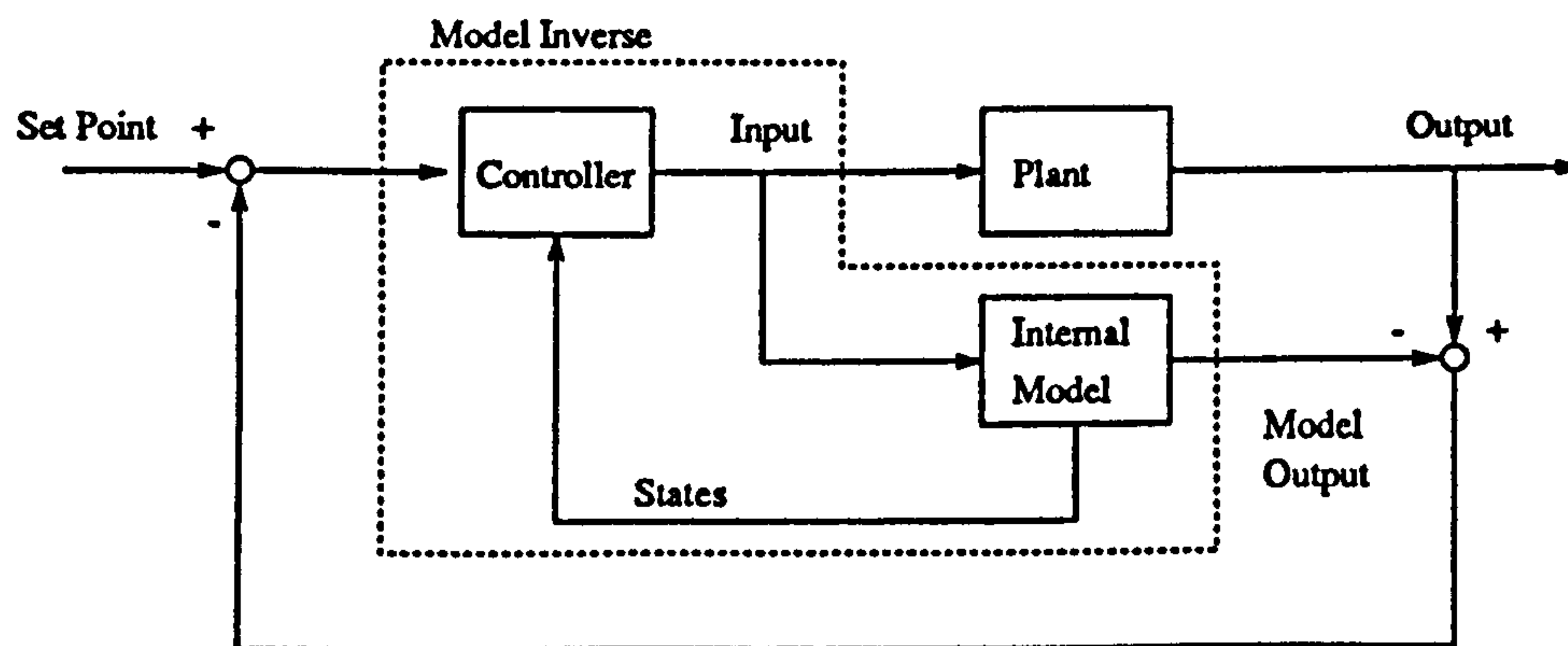


Figure 2.5: Nonlinear IMC controller structure.

It is noted by Henson and Seborg, 1990b that the resulting controller is essentially the same as the input output linearisation method with an open loop observer. The important difference is that the integral action is implicit in the nonlinear IMC approach (through feeding back the difference between the plant and internal model) and must be added explicitly to the input output linearisation method. The only measurement required from the plant is the output measurement and the IMC structure (and input

output linearisation with open loop observer) can be viewed as a dynamic feedback.

It is pointed out by Henson and Seborg, 1990b that as the presented IMC approach implicitly uses an open loop observer (as an internal model) it is only valid for open loop stable systems. Daoutidis and Kravaris, 1992a have more recently noted that closed loop instability for open loop unstable systems results from a nonminimal order realisation of the controller. They then extended the analogy of the IMC approach being a dynamic feedback by specifying a reduced order observer. This observer requires measurement of the output and its derivatives (up to $r - 1$) to reconstruct the zero dynamics resulting in a minimal order realisation of the controller. As a result, open loop unstable systems can be considered at the expense of requiring derivatives of the output.

Generic Model Control

Generic model control (GMC) is a specific nonlinear control method which appeared independently of the linearisation methods. However, the basic structure of GMC is related to the input output linearisation technique. The method was originally presented by Lee and Sullivan, 1988b and has since been developed and used in many applications (eg. Lee, 1991 and Lee *et al.*, 1989).

The basic GMC controller as presented in Lee, 1991 is,

$$\text{Given} \quad \dot{x} = f(x, u, d, t)$$

$$y = h(x)$$

$$\text{Choose} \quad u(t)$$

$$\text{s.t.} \quad \|u\| \leq \text{bounds}$$

minimising,

$$\int_0^{t_f} [h'(x, u, d, t)^T W h'(x, u, d, t)] dt$$

$$\text{where,} \quad h'(x, u, d, t) = \frac{dh(x)}{dx} f(x, u, d, t) - K_1(y_p - y) - K_2 \int (y_p - y) dt$$

In this form GMC can be viewed as an optimal control problem. However, applications of GMC use models which are of relative order one (in multi input multi output cases the relative order with respect to each of the outputs is one). In this case the optimal control problem reduces to an implicit algebraic relationship for the input, which is the same as the input output linearisation method with an external PI regulator. In this respect GMC can be seen as a special case of the input output linearisation method. By forming the controller using an optimisation problem consideration can be given to problems with relative orders greater than one, though the resulting closed loop system will no longer be an exact linearisation.

2.1.4 Discussion of Geometric Nonlinear Control

The previous section has reviewed the basic theory behind the nonlinear control methods based on differential geometry. This discussion has highlighted the following theoretical limitations for obtaining a geometric nonlinear controller.

1. The system equations must be sufficiently differentiable (sufficiency depends on the structure of the system).
2. The output (or each output for the multiple input, multiple output case) must have a well defined relative order.
3. The system must be minimum phase.
4. The system descriptions in the previous section have not allowed the inclusion of disturbances.
5. There must be no modelling errors or unmeasured disturbances.
6. The complete state vector must be available for the input output linearisation and state space linearisation methods.
7. The system model must not include dead time.
8. Continuous control is assumed.
9. Unbounded control action is assumed.

This list highlights not only a number of theoretical limitations, but also a number of practical limitations. This section reviews the literature which addresses these limitations with the aim of summarising the present state of the art. These issues will be discussed in the following order: ill-defined relative order, disturbance decoupling, internal stability, robustness, state observers/estimators, dead time compensation, sampled data implementation and bounded control. The section is concluded by an overall discussion of the available methods leading to a general structure for a nonlinear controller.

Ill-defined Relative Order

The development of geometric nonlinear control methods relies on the fact that the system has a finite relative order that is well defined over the region of operation. It is possible that at specific combinations of state variable values the relative order of the system may increase. In many cases these points correspond to extreme conditions

within the model and will not cause a problem for normal operation. However, there have been cases reported where a point of ill-defined relative order occurs in the region of normal operation (eg. Lien and Wang, 1990, Henson and Seborg, 1990c and Daoutidis and Kravaris, 1992a). In fact, many of these cases represent an extreme in the sense that they correspond to a point such as the maximum conversion in a chemical reactor. At a point of ill-defined relative order a singularity occurs in the control law and effectively infinite control action is required. As the system approaches this point progressively larger control action is required. The simulation example in Henson and Seborg, 1990c demonstrates that very large moves are required in the control action, though the output response is relatively smooth.

Two ad hoc methods for handling these singularities have been found in the literature. The first method presented by Henson and Seborg, 1990c holds the control action constant when the plant is operating in a certain region (as determined from the state information). The resulting controller requires much less action with only a slight degradation in performance.

The second method presented by Lien and Wang, 1990 augments the denominator and numerator of the input output control law so that zero control action is required at the singularity point.

$$u = \frac{(v - L_f h(x))L_g h(x)}{\epsilon + (L_g h(x))^2} \quad (2.28)$$

Away from the point of singularity, the control law tends to the original input output control law. The new parameter ϵ can be adjusted to allow operation closer to the point of singularity and to alter the maximum control action required. The problem with this method is that the controller is forced to do the opposite of what should be done. That is, when the controller is trying to use a lot of control action it is forced to use none.

Disturbance Decoupling

The basic geometric nonlinear control methods will achieve a linear output response if unaffected by a disturbance. The disturbance decoupling problem attempts to find a feedforward plus feedback controller which decouples the effect of the disturbances from the output. The problem was originally considered by Calvet and Arkun, 1988 who presented a solution based on the state space linearisation method. This solution is very restrictive as it is limited to the same conditions as state space linearisation and considers only a limited set of disturbances.

The disturbance decoupling problem as presented by Daoutidis and Kravaris, 1989 considers a system affine in the control and disturbance variables, as follows.

$$\begin{aligned}\dot{x} &= f(x) + g(x)u + \sum_{i=1}^q w_i(x)d_i \\ y &= h(x)\end{aligned}\tag{2.29}$$

where, x is the vector of state variables, $x \in \mathbb{R}^n$,

u is the manipulated inputs variable, $u \in \mathbb{R}$,

d is the vector of disturbance variables, $d \in \mathbb{R}^q$,

y is the output variable, $y \in \mathbb{R}$.

By analogy with the definition of the system relative order a disturbance relative order (ρ_i) is defined in Daoutidis and Kravaris, 1989. The system relative order and the disturbance relative order represent how directly the manipulated input and the disturbances affect the output. This has led to the characterisation of disturbances into three categories (Daoutidis and Kravaris, 1989). These categories are discussed below.

Class $\mathcal{A}$

A disturbance, d_i , is defined as a class $\mathcal{A}$ disturbance if $\rho_i > r$. In this case the effect of the disturbance on the output is less direct than the effect of the manipulated input. Therefore, the static state feedback calculated for the input output linearisation will also decouple all class $\mathcal{A}$ disturbances.

Class $\mathcal{B}$

A disturbance, d_i , is defined as a class $\mathcal{B}$ disturbance if $\rho_i = r$. In this situation the input output linearisation state feedback must be modified with a term involving the measurement of the disturbance variable.

Class $\mathcal{C}$

A disturbance, d_i , is defined as a class $\mathcal{C}$ disturbance if $\rho_i < r$. In this situation the output is affected more directly by the disturbance than by the input. The input output linearisation state feedback must be modified with terms involving the measurement of the disturbance variable and its derivatives (up to order $r - \rho_i$).

For complete disturbance decoupling of class B and C disturbances the model must include the effect of these disturbances and they must be measured. The required state feedback is given by,

$$u = \frac{v - \sum_{j=0}^r \beta_j L_j^j h(x) - \sum_{d_i \in B} \beta_r L_w L_j^{r-1} h(x) d_i - \sum_{d_i \in C} \sum_{l=0}^{r-\rho} \sum_{j=\rho+1}^r \beta_j \frac{d^l (L_w L_j^{j-l-1} h(x) d_i)}{dt^l}}{\beta_r L_g L_j^{r-1} h(x)} \quad (2.30)$$

The closed loop system is completely decoupled from the disturbances and is equivalent to Equation 2.17. However, the internal dynamics are forced by the disturbances. Therefore, characterisation of a minimum phase system requires stability of the forced zero dynamics. Details of this process are given Daoutidis and Kravaris, 1989.

Disturbance decoupling has been generalised for multi input multi output systems with a nonsingular characteristic matrix by Daoutidis *et al.*, 1990 and for dynamic state feedback in Huijberts *et al.*, 1992.

Internal Stability

A theoretical limitation of all the geometric nonlinear controller design techniques addressed so far is that the model of the system must be minimum phase. Models which are nonminimum phase due to the existence of dead time are treated below. The controller design for nonminimum phase systems resulting from unstable internal dynamics is treated here.

The design of controllers for nonlinear, nonminimum phase systems remains an unsolved problem requiring further research. The methods presented in the literature, based on the geometric nonlinear controller technique, to date fall into three categories. The first category uses an extension of the controller design for nonminimum phase linear systems. This approach decomposes the original system into two parts: a minimum phase and a nonminimum phase part. Decompositions for a very limited class of nonlinear systems into a nonlinear part and a linear nonminimum phase part are presented by Nikolaou and Manousiouthakis, 1990. Another decomposition is presented by Kravaris and Daoutidis, 1990 for nonlinear second order systems. The resulting closed loop system is transformable into a first order linear system in series with a first order nonlinear all-pass. The definition and properties of a first order nonlinear all-pass system are given in Kravaris and Daoutidis, 1990. It is analogous to a first order linear all-pass system (a system with one pole and one zero with the pole at the mirror image of the zero) and the linear approximation around a steady state point is equal to a first order linear all-pass.

More recently Wright and Kravaris, 1992 have consider a general nonlinear non-minimum phase system. This approach avoids decomposing the system into a minimum phase and a nonminimum phase part by defining the concept of a statically equivalent minimum phase output, y^* (This approach has independently been developed by Tornambe, 1991 and Tornambe, 1992). Such an output is minimum phase and is equal to the required output at all operating conditions. An input output linearising controller with an open loop observer is then designed for the equivalent output, y^* . This is augmented with a Smith predictor for the nonminimum phase element. The output of the closed loop system then asymptotically stable (See Figure 2.6). The choice of the equivalent output is arbitrary, though is the determining factor for the dynamic response of the system. By analogy with the approach of Kravaris and Daoutidis, 1990 a characterisation of an ISE optimal equivalent output is obtained for a limited class of systems.

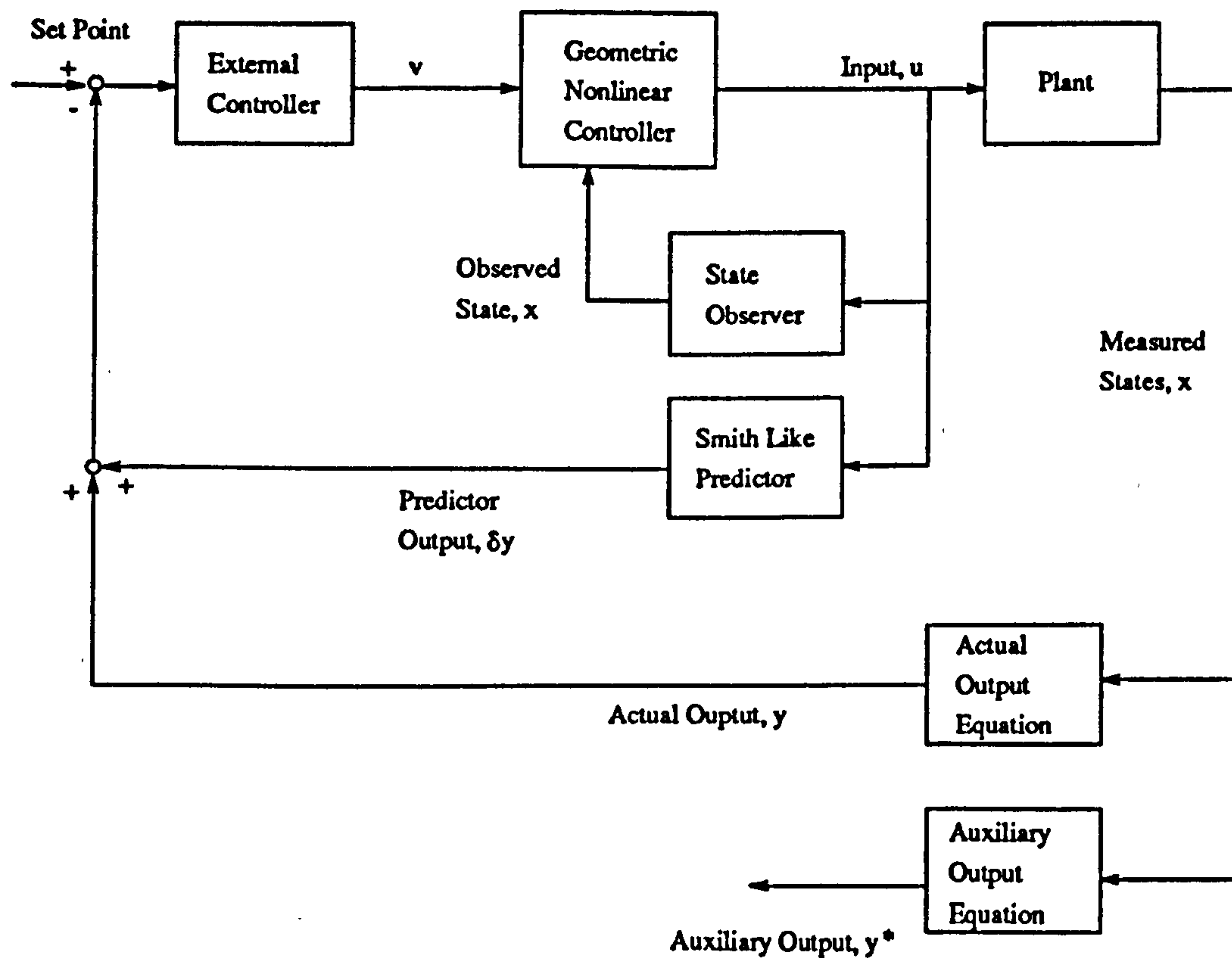


Figure 2.6: Structure of nonminimum-phase output predictor.

The final category of controllers based on the differential geometric approach for nonlinear nonminimum phase systems is dealt with in Dochain, 1992. This approach designs an adaptive linearising controller based on a minimum phase, reduced order model of the process. The reduced order model is obtained using singular perturbation arguments. However, a method for relating nonminimum phase elements to particular states is not

presented.

The basis for a general technique for nonlinear, nonminimum phase processes, based on the geometric nonlinear controller technique is available through the work of Wright and Kravaris, 1992. However, work is still required to generalise the choice of statically equivalent output, before this method becomes useful. For most systems the adaptive approach of Dochain, 1992 appears to be the most convenient, particularly if the nonminimum phase effect is relatively minor.

Robustness

As discussed in Section 2.1.3 the dynamics of the closed loop linearised system are determined by the choice of $r + 1$ arbitrary constants $(\beta_0, \dots, \beta_r)$. The resulting input output dynamics are represented by equation 2.17.

This equation is only true in an ideal situation and in practice the dynamics are also affected by process model mismatch and unmeasured disturbances. A number of methods have been developed to overcome these problems and these will be discussed next.

Process Model Mismatch

One type of uncertain nonlinear system has the following structure.

$$\begin{aligned}\dot{x} &= f(x; p) + g(x; p)\theta(x, u) \\ y &= h(x)\end{aligned}\tag{2.31}$$

where, p is a bounded set of uncertain parameters.

To design a state transformation a nominal set of the parameters, $\hat{p}$, is assumed. This leads to a closed loop system with the following structure.

$$\dot{\xi} = A\xi + Bv + \Psi(\xi, \delta f, \delta g)\tag{2.32}$$

$$\begin{aligned}\text{where, } \delta f &= f(x; p) - f(x; \hat{p}), \\ \delta g &= g(x; p) - g(x; \hat{p}).\end{aligned}$$

The factors δf and δg represent the model mismatch between the actual and nominal systems. The structure shown in the above equation is called a quasi linear system (QLS) (See Calvet and Arkun, 1989b), and reduces to a standard linear system for the case of no model mismatch.

Kravaris and Chung, 1987 characterised the model mismatch as an error in the state feedback components required for an input output linearisation. They used perturbation theory to quantify the stability of the closed loop system. However, this approach can be used for analysis only and is at best viewed as a qualitative measure, due to the complexity of defining the error terms.

Calvet and Arkun, 1989b present a method for designing a static state feedback controller for state space linearisable systems to ensure robust stabilisation of the nonlinear system. The method uses a state space linearisation of a nominal system. The process model mismatch is expressed as an uncertainty in the vector fields f and g (ie. δf and δg). For the basic method, both of the uncertainties must be structured and meet strict matching conditions. Then an explicit expression for the closed loop input, v , is obtained based on the solution of an algebraic Riccati equation. The feedback is a simple static feedback (P regulator for the QLS) and has the following form.

$$v = -K_1 \xi \quad (2.33)$$

where, K_1 is a constant matrix,

ξ is a vector of transformed linear state variables,

v is the input to the transformed (closed loop) system.

This provides a regulator which meets sufficiency conditions for stabilisation and ultimate boundedness, based on an adaptation of the second method of Lyapunov. The method includes a procedure for obtaining the least conservative gains. Calvet and Arkun, 1989a extended the basic method to consider unstructured uncertainty in the vector field f . An equivalent explicit expression for v is obtained, though asymptotic stability cannot be guaranteed. These results are generalised to input output linearisable systems in Calvet and Arkun, 1990 and Arkun and Calvet, 1992.

Kravaris and Palanki, 1988a and Kravaris and Palanki, 1988b presented another method for the design of a static state feedback controller to ensure robust stabilisation of the nonlinear system. This method uses an input output linearisation of the nominal system. The process model mismatch is expressed as uncertainties in the vector fields f and g and in the output equation h . An explicit nonlinear expression for the closed loop input, v , is obtained, which ensures that the tracking error is uniformly ultimately bounded. The uncertainties must be structured and meet a strict matching condition (Kravaris and Palanki, 1988a state that this matching condition is less restrictive than the conicity condition of the small gain theorem). The method is also based on the second

method of Lyapunov and involves finding a Lyapunov function, which is a non trivial problem. Kravaris and Palanki, 1988b suggest a method of Vannelli and Vidyasagar for numerical generation of Lyapunov functions.

The above methods for robust control of uncertain nonlinear systems are too restrictive to be of general use. Kravaris and Arkun, 1991 have highlighted the following problems as the main limitations.

1. How can parametric and dynamic uncertainties be combined?
2. What is the best uncertainty description?
3. What is the best structure for the correcting control law?
4. How can the conservatism of the present methods be best reduced?

For systems where the model mismatch is due purely to parameter uncertainty, adaptive control techniques have been developed to ensure robust control. These have been reviewed in Henson and Seborg, 1991a, but will not be considered in this review.

A strict analysis of robustness is performed in very few applications of nonlinear differential techniques. Many articles simply consider what effect step changes in the simulated parameters have on the stability of the system. This provides a qualitative measure for trial and error pole placement of the transformed system.

Unmeasured Disturbances

Calvet and Arkun, 1989b considered the effect of unmeasured disturbances on the stability of a linear system. A state space linearised system affected by unmeasured disturbances may be represented as a quasi linear system (QLS) as follows.

$$\dot{\xi} = A\xi + Bv + \zeta(z)d \quad (2.34)$$

where, $\zeta(z)$ is a state dependent perturbation term.

Calvet and Arkun, 1989b and Calvet and Arkun, 1990 presented a method for robust stabilisation of a nonlinear system with a bounded set of unmeasured disturbances. The disturbances must be modelled, and for the basic method, are required to meet a strict matching condition which ensures that the perturbation term of the QLS lies in the control input space, which effectively ensures that all the disturbances are of class B . This method designs a proportional regulator for the QLS and is analogous to the method for robust controller design of uncertain nonlinear systems presented in Calvet and

Arkun, 1989a. The proportional controller meets the sufficiency condition for stability and ultimate boundedness and is therefore conservative. The method presented in Calvet and Arkun, 1989b includes a procedure for obtaining the least conservative gains. An extension to this method is also presented which considers bounded perturbations which do not meet the matching condition. In this case the resulting design can not guarantee asymptotic stability of all of the states. Finally, Calvet and Arkun, 1990 consider a proportional and integral controller for the QLS which ensures zero steady state offset for the controlled output.

This method for robust design under unmeasured disturbances was applied to a simulated continuously stirred tank reactor system in Calvet and Arkun, 1990. No other applications of this method have been found.

Most applications of nonlinear differential geometric techniques design the external regulator by simulating the process and introducing step changes in the disturbance variables. The tuning of an external regulator is then done on a trial and error basis.

In summary, the limitations of the current theory for handling robustness issues for geometric nonlinear control are the following.

1. Difficulty in characterising uncertainty.
2. Necessity to model unmeasured disturbances.
3. Strict matching conditions must be met to guarantee asymptotic stability.
4. Complex procedure to obtain solution (especially for large systems).
5. Conservativeness of results.

Given these constraints (especially the difficulty in characterising uncertainty) these methods are not suitable for a practical application.

State Observers/Estimators

The basic geometric nonlinear control method uses state feedback and therefore require values for some of the state variables. In most developments of geometric nonlinear control it is assumed that all of the states are measurable. The actual states that have to be measured depend on the structure of the state equations. For many practical applications on-line measurement of all states is not possible. This has led to the development of a number of state observers to estimate the unmeasurable states.

One approach to obtaining unmeasured states for a nonlinear system is to use a state estimation technique. For example, the extended Kalman filter has been used by Hoo and Kantor, 1986a, Hoo and Kantor, 1986b and Niranjana and San, 1988. This method is shown to work well and this is expected as long as the operation is maintained around the nominal design point. However, the extended Kalman filter is not able to provide good estimates for a large range of operating conditions of a nonlinear system because it is based on a linearised approximation of the model (Dootin et al., 1992). Therefore, the achievable closed loop response will always be limited by the fact that a linear estimation of the unmeasured states is used. Furthermore, exponential convergence of this method is only guaranteed for systems which are locally observable (Dochain, 1992).

For asymptotically stable open loop systems an open loop observer may be sufficient. Given initial estimates of the states and the manipulated variables, an open loop observer uses the process model to estimate the states. The design of this type of observer is simple and convergent state estimation is guaranteed for asymptotically stable open loop systems with a perfect model. However, the dynamics of the error are limited by the dynamics of the open loop system.

In some applications, an open loop observer has been simplified to a reduced order observer (Kravaris and Chung, 1986, Kravaris and Chung, 1987, Soroush and Kravaris, 1992b). This method only integrates the differential equations for the unmeasured states. The values for the remaining states are used as inputs and are obtained from process measurements. This method has the advantage of possibly being satisfactory for open loop unstable systems (Sistu, 1992) and reduces computation time associated with the observer. However, as plant measurements are required this method will be susceptible to measurement error and noise.

Alvarez and Alvarez, 1988 suggest an extension to the basic closed loop observer, for state space linearisable systems, which uses derivatives of the measured output to calculate estimates of the states. Few details of this method are given. It is anticipated that estimates must be very sensitive to measurement noise. An equivalent method is presented in Daoutidis and Kravaris, 1992a for input output linearisable systems. This method uses measurements of the states and derivatives (up to $r - 1$) in conjunction with a reduced order open loop observer to obtain the states. A more detailed discussion is given in Section 2.1.3.

For systems which are unstable or have slow dynamics it is necessary to use closed loop observers. That is, the observer incorporates the error between process measurements and the observed values. There is no general theory available for the design of closed loop

observers for nonlinear systems.

Alvarez and Alvarez, 1988 present a method of state observation based on sliding surfaces. This technique integrates the autonomous system (ie. no inputs or disturbances) with linear correction factors for the error between the measurements and estimates of the measurable states. For the specific application considered by Alvarez and Alvarez, 1988, this method appears to work well. However, it is not clear how general or how robust the method is.

Krener and Isidori, 1983 designed a closed loop observer by transforming the open loop model into a linear system, with nonlinear output injection. The observer is used to cancel the nonlinear components resulting in linear error dynamics and therefore exponential convergence. This method is suitable for only a very limited class of systems. The restrictions on the type of system are given by Krener and Isidori, 1983 and Krener and Respondek, 1985. This method is used by Kantor, 1989 and Limqueco and Kantor, 1990 for a continuously stirred tank reactor system, to observe the reactor concentration using measurements of the reactor temperature. The method is limited to a two state description of the process assuming a first order irreversible reaction and no measured or unmeasured changes to the feed concentration.

The observer gains must be chosen as a trade-off between speed of convergence and robustness. Kravaris and Chung, 1987 present the only available tuning method, which involves linearising the model around an operating point and using linear tuning rules. To obtain physically meaningful estimates from the observer, good initial estimates of the states must be used. Limqueco and Kantor, 1990 discuss a method for initialisation based on a qualitative description of the process. The method amounts to choosing values for the states from the normal operating region.

This review is far from a complete review of observers for nonlinear systems because a full review is beyond the aims of this thesis. The focus of the review is on the use of observers in conjunction with geometric nonlinear control techniques. Therefore, other suitable techniques may exist which to date have not been used in conjunction with geometric nonlinear control.

Deadtime Compensation

Most systems in the process industries will contain deadtime due to transportation and/or measurement delays. In state space form a system with delays in the manipulated input may be represented by,

$$\begin{aligned}\dot{x} &= f(x) + g(x)u(t - t_d) \\ y &= h(x)\end{aligned}\tag{2.35}$$

where, x is the vector of state variables, $x \in \mathbb{R}^n$,

u is the manipulated input, $u \in \mathbb{R}$,

y is the output variable, $y \in \mathbb{R}$,

f is a vector field, $f \in \mathbb{R}^n$,

g is a vector fields, $g \in \mathbb{R}^n$,

h is a vector field, $h \in \mathbb{R}$.

The effective relative order for such a system is infinite as the manipulated input, u , at time t does not directly affect any derivative of the output, y , at time t (Henson and Seborg, 1991a).

Kravaris and Wright, 1989 present a method for dead time compensation of single input single output systems with transportation delays in the manipulated input. This method uses an extension of the Smith predictor (Smith, 1957) in an input output linearisation framework to produce a linear $v - y$ system with deadtime, as follows.

$$\sum_{k=0}^r \beta_k \frac{d^k y}{dt^k} = v(t - t_d)\tag{2.36}$$

The deadtime is not actually removed from the closed loop system as a causal state feedback is used (in conjunction with predictive action). An external linear controller is also used and this must be compensated for deadtime as well. The structure of the overall nonlinear controller is shown in Figure 2.7. This method is applicable to systems which are open loop stable and have a stable inverse for the equivalent model without deadtime.

Kravaris and Wright, 1989 also consider the robust stability of the closed loop system to errors in the dead time and unstructured multiplicative uncertainty.

The method of Kravaris and Wright, 1989 is the only technique for deadtime compensation of the general input output linearisation technique. Bartee *et al.*, 1989 present a method for nonlinear input output models based on the reference system control framework. Lee, 1991 presents a method based on the generic model control framework for nonlinear input output models. This method results in a noncausal controller and the future values for the outputs are approximated by integrating forward the system model. The reference system control and generic model control frameworks are only able to exactly linearise systems with a relative order of one. The same method is extended for

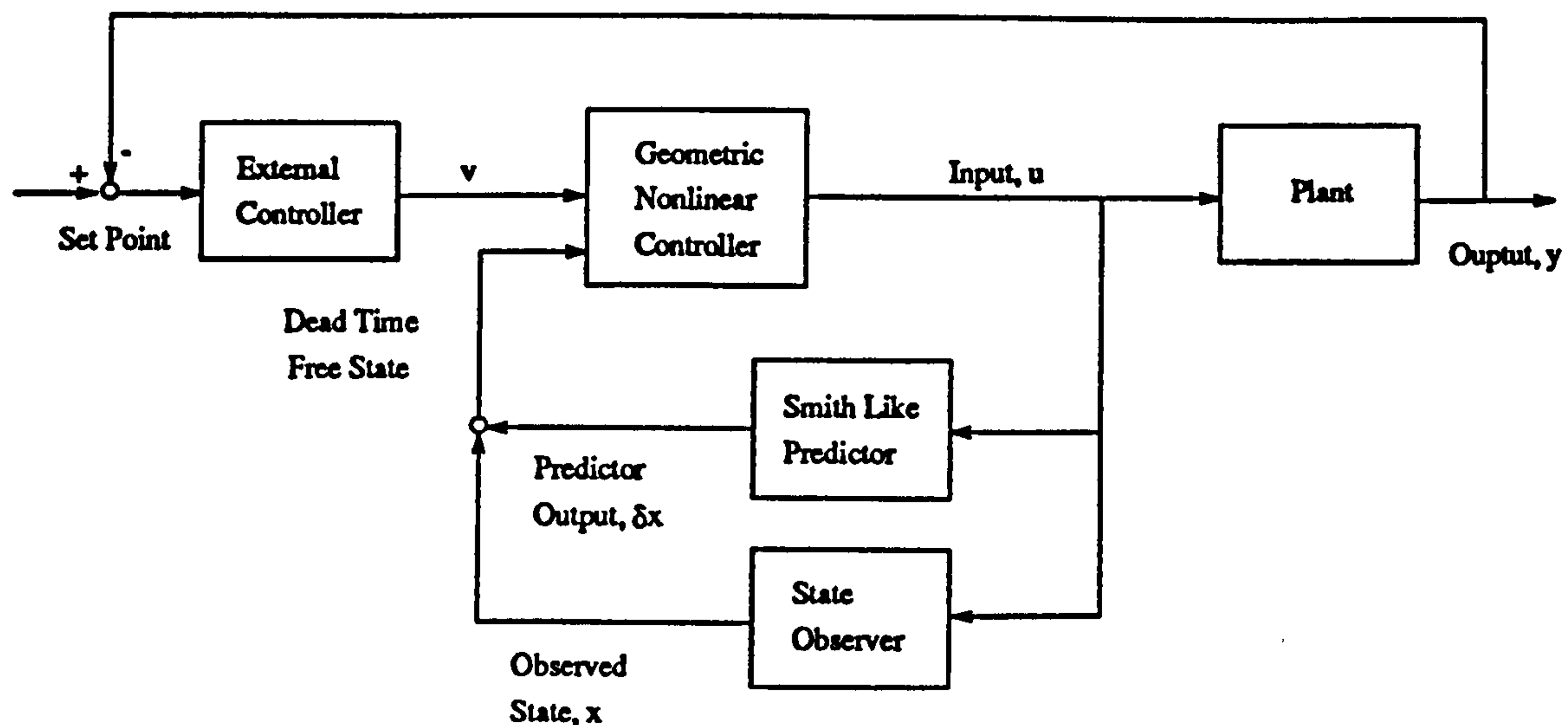


Figure 2.7: Nonlinear controller with deadtime compensator.

reference systems synthesis of systems with the relative order equal to the system order in Bartee *et al.*, 1989.

Henson and Seborg, 1991a state that the method of Kravaris and Wright, 1989 has two flaws: the compensator must be perfectly initialised and the compensator requires future values to be initialised. Even given these restrictions, it appears to be the most suitable method for directly accounting for process deadtime.

Discrete Time Implementation

A controller in a real time control scheme will require a discrete time implementation to coordinate with the data sampling function. This point has been made by a number of sources, concluding that the effect is to destabilise the closed loop system. The simulation implementations in the literature use a single step ahead calculation for the manipulated variable with a very fast sampling period and a sufficiently detuned control scheme to avoid potential stability problems.

Soroush and Kravaris, 1992a set the foundation for analysing the single step ahead approach by specifying the nonlinear control methods based on differential geometry in a discrete setting. This paper generalises the work of a number of sources (Grizzle, 1986, Glad, 1987 and Nijmeijer and van der Shaft, 1990) for a discrete time state space system of the form,

$$\begin{aligned} x(k+1) &= \Phi_{disc}[x(k), u(k)] \\ y(k) &= h[x(k)] \end{aligned} \quad (2.37)$$

Soroush and Kravaris, 1992a then give details of the discrete time analogs for relative order, zero dynamics, minimum phase, input/output linearisation, open loop observers (full and reduced order) and Smith prediction. This work allows discrete time descriptions to be used and provides the foundation for future analyses of systems with piecewise constant control action. A simulation example of a CSTR is given in Soroush and Kravaris, 1992a to demonstrate the application of the results. A continuous model is derived and then approximated as a discrete model using Euler's method (first order approximation: $\frac{dx}{dt} = (x(k+1) - x(k))/\text{SamplePeriod}$). The simulations use a relatively fast sample period and the effect of different sampling periods is not considered.

Soroush and Kravaris, 1992a consider a discrete system of the form given in Equation 2.37. However, the conversion of a continuous system with piecewise constant control action to a discrete system is nontrivial. For example consider a linear system as given in Equation 2.1. Chen, 1984 shows that the discrete form of this system with piecewise constant control action is,

$$\begin{aligned} x(k+1) &= \exp(AT)x(k) + \left(\int_0^T \exp(A\tau) d\tau \right) Bu(k) \\ y(k) &= cx(k) \end{aligned} \quad (2.38)$$

The extension to nonlinear systems is correspondingly more complicated. The real problem for a discrete implementation is determining what sample interval can be chosen without an unacceptable loss in performance.

The foundation for the analysis of sampled data schemes for geometric nonlinear control has been set, though further work must be done in this area to relate closed loop performance to the sample period. The best available method still remains a trial and error detuning of the closed loop system.

Bounded Control

Geometric nonlinear control methods are based on the assumption that unbounded control action is available. In a practical situation this is not a valid assumption and often bounds on the control action may become the limiting factor. In particular, specification of faster closed loop responses requires correspondingly greater control action. So the speed of response will ultimately be limited by the saturation limits on the control action.

The effect of controller saturation on a closed loop system can be realised as,

1. degradation of performance,

2. existence of undesired critical points,
3. oscillatory behaviour,
4. limit cycling.

Degradation in performance may simply correspond to a closed loop response which is slower than specified. In a more severe case the closed loop system may exhibit unexpected oscillatory behaviour due to the controller swapping between saturation limits for a finite amount of time. While undesirable, these cases will ultimately return to the specified operating point. The case of limit cycling represents a constant swapping between the controller saturation limits for an unlimited period, without return to the closed loop operating point.

These situations have been studied in detail in Alvarez *et al.*, 1990a and Alvarez *et al.*, 1991 for the specific situation of single input CSTR system modelled by two state equations. A characterisation is given for the situations in which limit cycling will occur. Of particular note it was found that input output linearisation of a nondistinguished output reduces the oscillatory behaviour and the possibility of limit cycling, due to the presence of the internal dynamics.

In the CSTR system the existence of undesired critical points has been found (Alvarez *et al.*, 1990a) in closed loop systems with controller saturation. For open loop systems exhibiting ignition/extinction behaviour (ie. three steady state operating points - one unstable) Alvarez *et al.*, 1990a Alvarez *et al.*, 1991 characterise the conditions for the existence of undesired critical points.

Lee, 1991 presents a method explicitly handling constraints within the GMC structure by defining slack variables representing the deviation of the output response from the desired linear response. The control problem is then specified as a nonlinear optimisation problem allowing deviations from the desired linear response to avoid manipulated input constraint violations. This approach requires the specification of a large number of weighting functions and the solution of a nonlinear optimisation problem on each iteration.

More recently an anti-windup mechanism has been designed by Aoufoussi *et al.*, 1992 to modify the reset action of the regulator during saturation. Simulation studies on a fluidised bed reactor by Aoufoussi *et al.*, 1992 demonstrated that the new constant was easy to tune and that the anti-windup mechanism improved the response of the closed loop system.

Typically, applications detune the controller to avoid or at least limit the amount of controller saturation (Alvarez *et al.*, 1989). However, the anti-windup mechanism of Aoufoussi *et al.*, 1992 is logical given the experiences of linear systems. It does not provide a general answer to the problem of controller saturation, though is the best available technique.

Summary of the Geometric Nonlinear Controller Structure

The basic structure of all the nonlinear control methods based on geometric methods is given in Figure 2.8. The feedback part will be called the geometric nonlinear control. The plant in this diagram may represent the physical plant or a model of the plant, depending on the method considered.

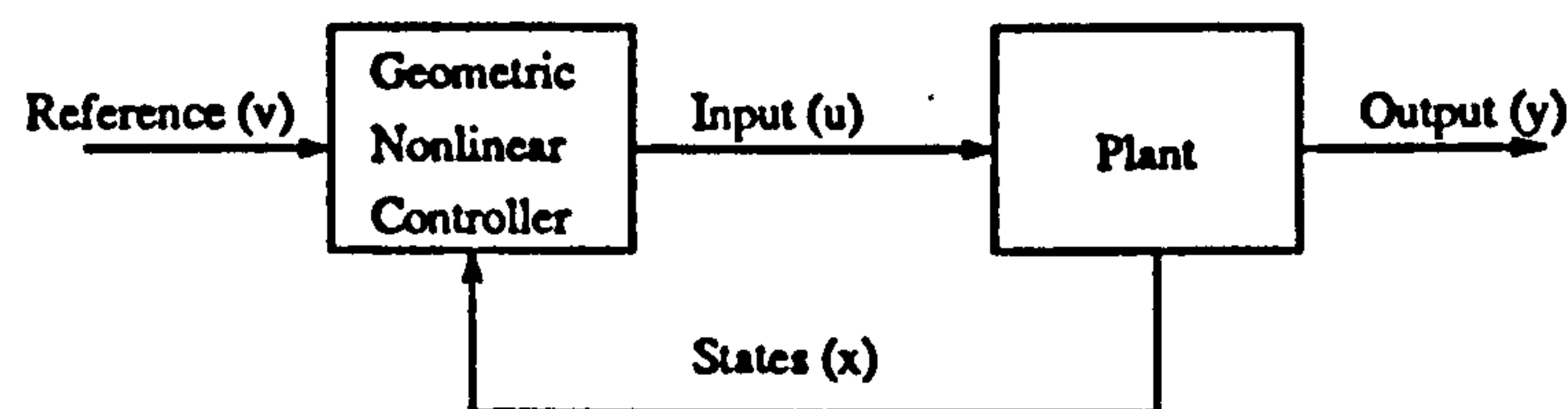


Figure 2.8: Basis to the geometric nonlinear controller structure.

As has been shown, the closed loop system is a realisation of the system inverse and can be transformed to a linear system. The function of the geometric nonlinear controller is in effect to make the system follow,

$$v = \beta_0 y + \beta_1 \frac{dy}{dt} \dots + \beta_r \frac{d^r y}{dt^r} \quad (2.39)$$

There are a number of theoretical limitations for obtaining a geometric nonlinear controller for a system. The first is that all the functions in the model description must be sufficiently differentiable. Secondly the relative order of the system must be well defined and less than infinity. These restrictions are met for most system descriptions. In addition, the system must be minimum phase. If a system is non minimum phase then a number of possibilities exist. Another output could be considered (candidates can be found using the state space method), the RSS method could be used, a Smith like predictor could be used or an auxiliary output could be used.

Possibly the biggest theoretical constraint for applying geometric nonlinear control in the process industries is that a set of ordinary differential equations must be used as the description of the system. No general method for considering differential and algebraic equations has been found in the literature.

Once the basic geometric nonlinear controller has been identified there are a number of practical issues which must be addressed before implementing the controller on an experimental system. In particular, the robustness of the controller to modelling errors, unmeasured disturbances and process dead time must be accounted for. The functions of linearisation and robustness are normally considered independently by including a regulator external to the geometric nonlinear controller. This structure is shown in Figure 2.9.

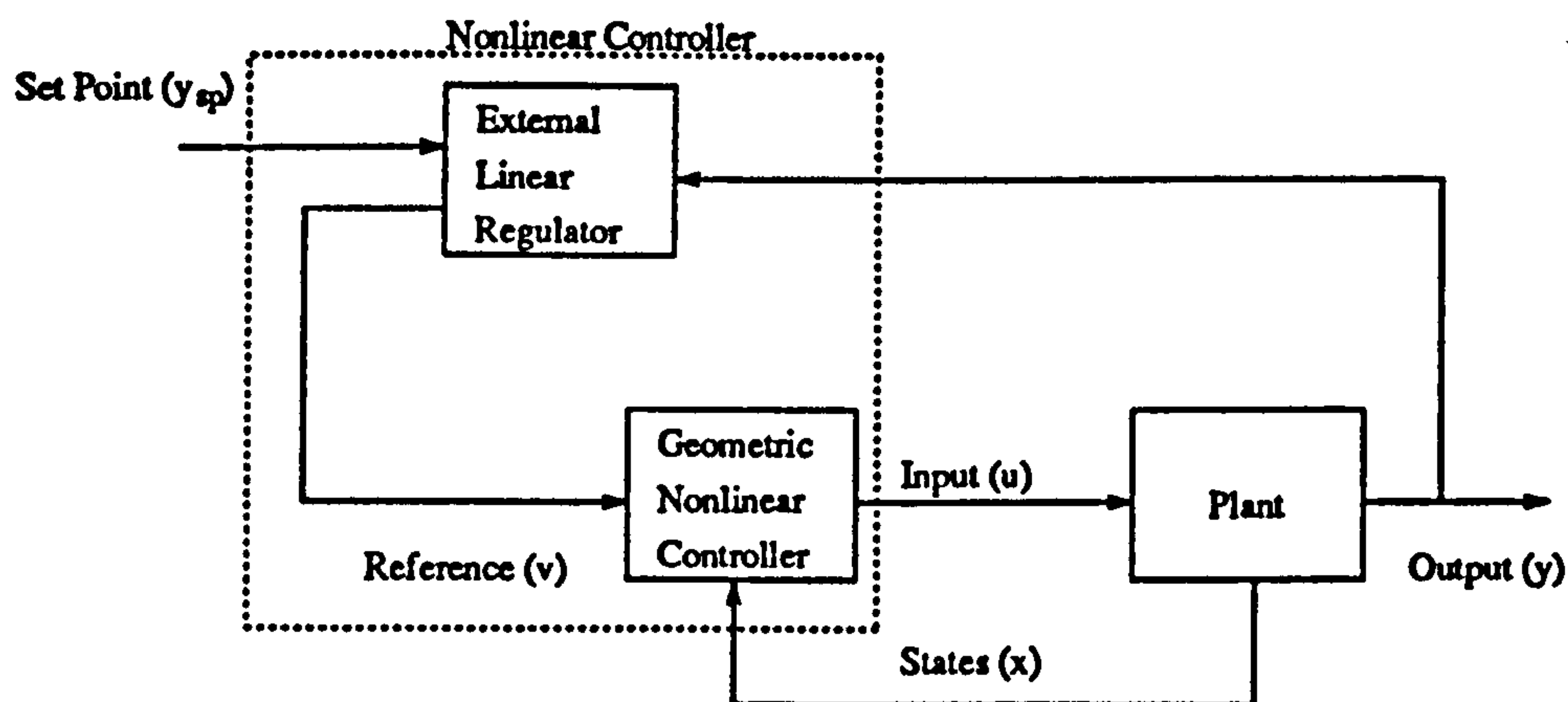


Figure 2.9: Nonlinear controller structure including regulator.

In general this regulator could be of any form, though typically a P or PI regulator is used. The inclusion of a P regulator derives no advantage over the basic geometric nonlinear controller as this regulator simply shifts the poles specified by the geometric nonlinear controller. In other words, the specification of the gain for a P regulator does not introduce an extra degree of freedom. Use of a PI regulator does remove any steady state offset and so is of additional benefit.

As the measurement of all system states is generally not possible for a practical situation, an observer will typically be included in the nonlinear controller structure. This structure is shown in Figure 2.10.

Ideally a closed loop observer will be used (as represented by the dotted arrow). However, this option is rarely available given the present technology. Therefore, the plant model will typically be used as an open loop observer (or a reduced order observer if partial plant measurement is available) for the unmeasured states.

The linearisation techniques are not able to explicitly handle manipulated variable constraints. One method is presented for avoiding constraints by defining a nonlinear optimisation problem to be solved on each iteration. The method used in most of the applications involves detuning the closed loop system to avoid constraints. A simple ex-

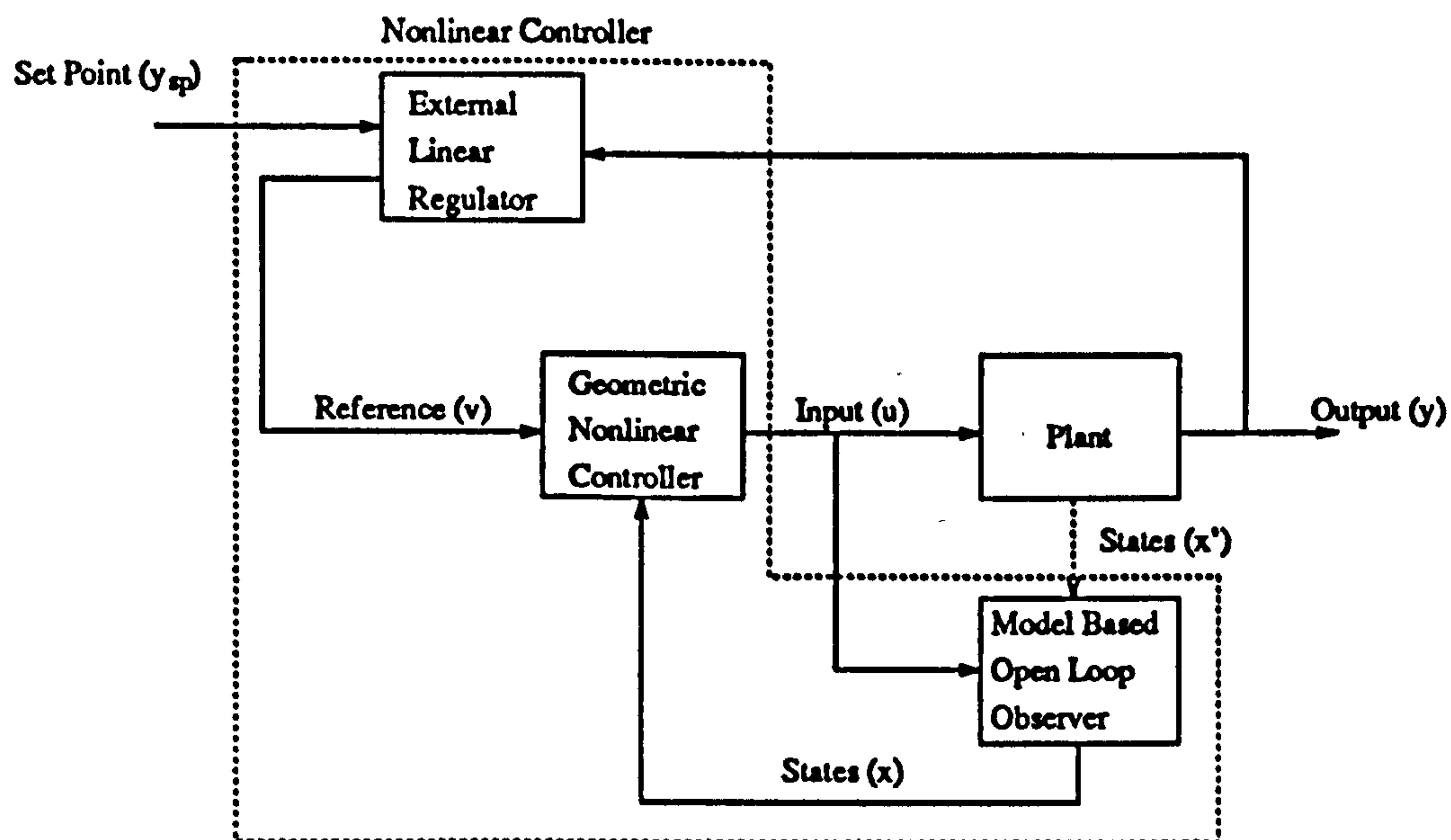


Figure 2.10: Nonlinear controller structure with observation.

tension of the anti-windup mechanism, as used for standard PID controllers, has been presented.

For systems with an ill-defined relative order, a state dependent controller saturation has also been presented.

Most of these elements are summarised in one overall controller structure in Figure 2.11, with the geometric nonlinear controller as the central element.

The problem of choosing tuning constants, both for the basic geometric nonlinear controller and the external regulator (and for the observer if a closed loop observer is used), is not only fundamental to any application but also remains unsolved. Attempts to ensure robustness have resulted in methods which are of very little practical use. What is needed at the moment is a general set of guidelines to initialise and help the designer through a trial and error procedure for determining a set of controller constants.

It can be seen that the theoretical limitations of the geometric nonlinear control techniques still allow consideration of a large class of systems in the process industries. Even though a number of the theoretical and practical issues remain unsolved, there exists sufficient scope for practical applications of the geometric nonlinear control method.

2.2 Review of Simulation Applications

Nonlinear controller designs based on differential geometric techniques have been applied to a number of chemical engineering systems. A review of these applications is discussed in the following sections. This review includes a tabulated summary of the

applications and an explanation of the abbreviations used in the tables is given in Table 2.1.

2.2.1 Continuously Stirred Tank Reactor Applications

A continuously stirred tank reactor (CSTR) system has been the most commonly used example in the literature to demonstrate the applicability of differential geometric techniques.

Of the configurations available, the most commonly used is a single CSTR with a single first order irreversible, exothermic reaction:



The outlet temperature is controlled through manipulation of the temperature of the cooling jacket. The disturbances to the system are the feed composition and temperature. The system is modelled with a mass balance on the component A and an energy balance. In most cases the dynamics of the surrounding jacket are also considered. The rate of reaction is related to the composition of A through an Arrhenius relationship. The choice of parameters can have a large effect on the open loop behaviour of the system. For example Limqueco and Kantor, 1990 present three sets of parameters corresponding to strong parametric sensitivity, ignition/extinction behaviour and nonlinear oscillations (limit cycle).

A summary of the reviewed applications of the geometric nonlinear control techniques is given in Table 2.2.

In the literature, consideration has also been given to a number of other reaction systems occurring in a CSTR. For example, a parallel reaction system,



or two reactions in series,



In all cases the system equations are based on mass and energy balances in the CSTR and an energy balance for the surrounding jacket (if considered). The reaction dynamics are typically represented by an Arrhenius equation. A number of sources in the literature have considered the control of multiple CSTRs in series.

A summary of the reviewed articles concerning the application of geometric nonlinear control to CSTRs with other than a first order irreversible reaction and to multiple CSTRs is summarised in Tables 2.3 and 2.4.

The work on applying linearisation techniques to CSTR systems has been fairly

Abbreviation	Description
Method I/O S/S GMC IMC RSS Config. (mimo) (multiloop)	Input output linearisation State space linearisation Generic model control Internal model control Reference system structure Analysing control configurations Multi input multi output system Multi loop system
Internal Stability Checked Decomp Equiv. Output Adaptive	Stability of internal dynamics checked Decomposition of system into minimum and nonmimum phase Uses a statically equivalent output Adaptive strategy
Disturbance Decoupling F/F Robust Regulation	Feed Forward External regulator tuned for robust regulation
Robustness Track Dist. Model Noise Meas. No. Sim. Phase Plane	Setpoint tracking Disturbance rejection Modelling error Measurement noise Measurement error No simulation results presented Phase plane results
Observed States Open Closed Reduced Open Kalman Derivative	Open loop observer Closed loop observer Reduced order open loop observer Kalman filter Observer based on output derivatives
Adaptive Adaptive	Adaptive strategy used
Discrete Implementation Discrete	Discrete sampling simulated
Dead Time Smith Detuned	Smith like predictor used Controller detuned to account for dead time
Controller Saturation Simulated Analysed Anti wind.	Simulated, but not accounted for by the controller Effect analysed theoretically Anti-windup mechanism used

Table 2.1: Abbreviations for Tables 2.2 to 2.10.

Reference	Method	Internal Stability	Disturb. Decoup.	Robustness	Observed States	Adaptive	Discrete Imp.	Controller Saturation
Riggs and Rhinehart, 1990 & Riggs and Rhinehart, 1988	GMC & IMC			Track Model			Discrete	
Parrish and Brosilow, 1986	I/O		F/F	Track Dist. Model		Adaptive		
Limquenco and Kantor, 1990	I/O			Track Dist.	Closed			
Kantor, 1989	S/S			Track	Closed			Simul.
Hoo and Kantor, 1985	S/S			Phase Plane				
Alvarez <i>et al.</i> , 1989	S/S			Track Dist. Model				Simul.
Sistu, 1992	I/O		F/F	Track Dist. Model	Open		Discrete	Simul.
Daoutidis and Kravaris, 1991	I/O	Checked		No. Sim.				
Kantor, 1986a	S/S			No. Sim.				
Alvarez <i>et al.</i> , 1991 & Alvarez <i>et al.</i> , 1990a	S/S			Phase Plane				Analysed
Bartusiak <i>et al.</i> , 1989	RSS		F/F	Track Dist.				
Calvet and Arkun, 1990 & Calvet and Arkun, 1989b	S/S		Robust Regulation	Dist.				
Calvet and Arkun, 1988	S/S		F/F	Track Dist.				Simul.
Bhat <i>et al.</i> , 1991a	RSS	Checked		Track Dist.				

Table 2.2: Summary of the applications of geometric nonlinear control to CSTRs with a first order, irreversible reaction.

Reference	Method	Internal Stability	Disturb. Decoup.	Robustness	Observed States	Adaptive	Discrete Imp.	Dead Time
Cott and Macchietto, 1989 CSTR : $A + B \rightarrow C$ $2 A + C \rightarrow D$	GMC			Track Model.		Adaptive		
Kantor, 1986a CSTR : $A \rightarrow B \rightarrow C$ $2 A \rightarrow D$	I/O & S/S			No Simul.				
Kravaris and Palanki, 1988b CSTR : $A \rightleftharpoons B \rightarrow C$	I/O	Checked		No Simul.				
Kravaris and Wright, 1989 CSTR : $A \rightarrow B \rightarrow C$	I/O			Track Model	Open			Smith
Daoutidis and Kravaris, 1992b CSTR : $A + B \rightarrow C + D$	Config.			No Simul.				
Daoutidis and Kravaris, 1992a CSTR : $A \rightleftharpoons B$	I/O			Track	Reduced Order			
Kravaris and Daoutidis, 1990 CSTR : $A \rightarrow B \rightarrow C$ $2 A \rightarrow D$	I/O	Decomp.		Track				
Wright and Kravaris, 1992 CSTR : $A \rightarrow B \rightarrow C$ $2 A \rightarrow D$	I/O	Equivalent Output		Track Dist.	Open			

Table 2.3: Summary of the application of geometric nonlinear control to reaction systems with multiple CSTRs and CSTRs with reactions other than first order irreversible reactions (Part 1).

Reference	Method	Internal Stability	Disturb. Decoup.	Robustness	Observed States	Adaptive	Discrete Imp.	Dead Time
Henson and Seborg, 1990a 2 CSTRs : $A \rightarrow B$	I/O			Dist. Model				
Daoutidis and Kravaris, 1992b 2 CSTRs : $A \rightarrow B$	Config			No Simul.				
Daoutidis and Kravaris, 1989 3 CSTRs : $A \rightarrow B$	I/O		F/F	Track Dist Noise				
Economou <i>et al.</i> , 1986 & Economou and Morari, 1985 CSTR : $A \rightleftharpoons B$	IMC			Track Dist.				
Adebekun, 1992 CSTR : $A \rightarrow B \rightarrow C$	I/O	Checked		Phase Plane				
Sorouch and Kravaris, 1992a CSTR : $A \rightarrow B$ $A \rightarrow C$ $A \rightarrow D$	I/O		F/F	Track Dist	Open		Discrete	

Table 2.4: Summary of the application of geometric nonlinear control to reaction systems with multiple CSTRs and CSTRs with reactions other than first order irreversible reactions (Part 2).

extensive. Many of the controllers show impressive results, though the comparisons are not necessarily fair. For example, a number of comparisons are made between a simple PI controller (requiring a single measurement) and a nonlinear controller requiring multiple measurements. The ability of the nonlinear controllers to provide a good servo response, given model errors and noise, is well demonstrated. Their ability to reject disturbances which have a relative order greater than the input relative order is also well demonstrated. However, it has been found when the relative order of the disturbances is smaller than the input relative order, the regulatory response of the controller is less satisfactory.

Normal steady operation of a CSTR does not require a great range of operating conditions, so the complexity of the nonlinear controller is not always required (one notable exception is when the CSTR exhibits ignition/extinction behaviour). The real strength of the geometric nonlinear technique for these applications appears to be for tracking setpoint changes over a wide range of operating conditions.

2.2.2 Batch Reactor Applications

The application of geometric nonlinear control techniques to batch reactor systems has been considered by a number of sources in the literature. The basic system generally is a stirred tank reactor modelled by a mass and energy balance. The aim is to achieve a particular predetermined temperature profile throughout the batch. One application to a semi batch reactor has been reviewed. This system aims to achieve a predetermined product composition profile by controlling the rate of addition of one of the reactants. A summary of the reviewed articles considering these applications is given in Table 2.5.

These examples demonstrate the applicability of the geometric nonlinear control techniques to simulated batch reactor systems. However, for practical implementation it has been recognised that an observer for the composition of reactants (the controlled output, temperature, is generally measurable) or adaptation of the heat generation term are required. Geometric nonlinear control techniques work well for batch and semi batch reactors because the main control problem is setpoint tracking over a wide range of operating conditions. Disturbances are considered, though their effect is fairly small compared to the total range of conditions considered. Also, the methods appear to be stable despite modelling errors.

Reference	Method	Internal Stability	Disturb. Decoup.	Robustness	Observed States	Adaptive	Discrete Imp.	Controller Saturation
Bhat <i>et al.</i> , 1990 Batch : $A \rightarrow B \rightarrow C$	RSS			Track Model Noise				
Kravaris and Chung, 1986 & Kravaris and Chung, 1987 Batch : $A \rightarrow B \rightarrow C$	I/O			Track Dist. Model Noise	Reduced Open			Detuned
Chidambaram and Malleswararao, 1992 Semi Batch : $A + B \rightarrow C$ $A + 2 B \rightarrow D$	RSS			Track Model				Simul.

Table 2.5: Summary of the application of geometric nonlinear control to batch and semi batch reactors.

2.2.3 Biochemical Systems Applications

The application of differential geometric techniques to a number of biochemical systems has been considered in the literature. The reviewed systems are either continuous reactors or fed batch reactors. The basic modelling of such systems is well described in many texts, such as Bailey and Ollis, 1986, and the equations have the following form.

$$\begin{aligned}\dot{X} &= \mu(X, S_{ub})X - DX \\ \dot{S}_{ub} &= -\theta_{sub}(X, S_{ub})S_{ub} - D(S_{ub,f} - S_{ub})\end{aligned}\quad (2.40)$$

where, X is the concentration of cells in the reactor,

S_{ub} is the concentration of substrate in the reactor,

$S_{ub,f}$ is the concentration of substrate in the feed,

D is the dilution rate,

μ is the specific growth rate,

θ_{sub} is the substrate consumption rate.

In some applications this basic situation is complicated by consideration of extra balances corresponding to a combination of: additional cell types, inhibitors and products. Henson and Seborg, 1990c show that such systems are very nonlinear and that there is a gain sign change around the equilibrium operating point. This makes such systems very difficult to control with conventional linear theories and very good candidates for nonlinear control techniques.

For both continuous and fed batch reactors the control objective is to maximise the production of a certain cell type or product by varying the feed flowrate.

From an application point of view, two major problems arise. Firstly, the actual structure of the system is very difficult to determine. In particular, determining a relationship for the specific growth rate is very a difficult problem. Commonly a form of Monod kinetics is assumed, though this is at best an approximation. Secondly, measuring the states of the system is very difficult. Some biological sensors exist, though few of these are available for on-line measurement. Often remote sensors are used in biological systems. However, these sensors introduce a significant delay into the system and do not measure the conditions within the reactor directly.

A summary of the reviewed applications of geometric nonlinear control techniques to biochemical processes is given in Table 2.6.

These applications are generally very idealised. Biochemical systems are difficult to model and the required control objective is often difficult to identify. The applications

Reference	Method	Internal Stability	Disturb. Decoup.	Robustness	Observed States	Adaptive	Discrete Imp.
Dochain and Bastin, 1984 CSTR - Cell/Substrate/Product	I/O		F/F	Track Model Noise		Adaptive	Discrete
Pomerleau <i>et al.</i> , 1989 Fed Batch - Cell/Substrate/Product	I/O			Track Dist. Noise		Adaptive	Discrete
Niranjan and San, 1988 Fed Batch - Cell/Substrate	S/S			Dist. Noise	Kalman		
Hoo and Kantor, 1986a CSTR - Mixed Culture	S/S (mimo)			Track Noise	Kalman		
Hoo and Kantor, 1986b CSTR - Cell/Substrate/Prod.	S/S			Track Model Noise	Kalman		
Henson and Seborg, 1990b & Henson and Seborg, 1991b CSTR - Cell/Substrate/Prod.	I/O & IMC			Track Dist. Model			
Henson and Seborg, 1990c & Henson and Seborg, 1992 CSTR - Cell/Substrate/Prod.	I/O & S/S			Track Dist. Model			
Kantor, 1986a - CSTR Mixed Culture	S/S			No. Sim.			
Kravaris, 1988 - CSTR Cell/Substrate	I/O	Checked		No. Sim.			
Alvarez and Alvarez, 1988 - CSTR and Fed Batch	S/S		F/F	Dist. Model	Deriv.		
Lien and Wang, 1990 - CSTR Cell/Substrate/Prod.	I/O			Track Model			
Bhat <i>et al.</i> , 1991b - Fed Batch - Cell/Substrate/Prod.	RSS			Track Dist. Model			
Dochain, 1992 CSTR - Mixed Culture	I/O	Adaptive		Track Dist.		Adaptive	

Table 2.6: Summary of the application of geometric nonlinear control to biochemical systems.

have shown some very good results. However, the testing of a real controller under real conditions is required to judge if the assumptions and simplifications, used to obtain the state feedback functions, are justifiable.

2.2.4 pH System Application

Simulation applications of geometric nonlinear control to pH systems have been given in two references. Such systems are difficult to control using linear techniques due to the large changes in gain across the operating region and particularly near the usual operating point of pH 7. Therefore, pH systems should be a prime candidate for nonlinear controllers.

The basis of all the systems is a CSTR with a base and acid feed streams. The main control objective is to maintain a pH setpoint by manipulating the rate of base addition. Henson and Seborg, 1991a also control the height of liquid in the reactor by manipulating the acid flowrate (in this case the acid and buffer streams are separate).

The modelling of pH systems, as dealt with in these references, is varied. Shukla *et al.*, 1993 consider a simple, one state model representing a strong base/strong acid system which is not representative of a realistic system. Henson and Seborg, 1991a consider a three state system, representing the height of liquid, a charge balance and a carbonate ion concentration.

A summary of these applications is given in Table 2.7.

The basic geometric nonlinear control techniques are shown to operate well under ideal situations. However, in all cases, bad performance is reported for control with unmeasured disturbances and modelling errors. Henson and Seborg, 1991a use disturbance decoupling to remove the effect of the disturbances and obtain good control. Shukla *et al.*, 1993 adapt the acid feed composition parameter to obtain a robust controller for unmeasured disturbances, modelling errors and large sampling intervals.

It appears that the geometric nonlinear controllers are able to effectively account for the large gain nonlinearity in pH systems. However, a severe reduction in performance is experienced given small errors.

2.2.5 Polymerisation Systems Applications

A number of simulation applications of geometric nonlinear controllers to polymerisation reaction systems have been reviewed. These applications are based on free radical polymerisation or copolymerisation reactions. The polymerisation system has a methyl methacrylate monomer with a benzoyl peroxide or azobisisobutyronitrile initiator

Reference	Method	Internal Stability	Disturb. Decoup.	Robustness	Observed States	Adaptive	Dead Time	Discrete Imp.	Controller Saturation
Henson and Seborg, 1991a	I/O (mimo)		F/F	Track Dist.					
Shukla <i>et al.</i> , 1993	IMC			Dist. Model		Adaptive			Simulated

Table 2.7: Summary of the application of geometric nonlinear control to pH systems.

and benzene or toluene as the solvent. The copolymerisation reaction is of styrene and acrylonitrile with xylene as the solvent. Batch, fed batch and continuous reactors are all considered.

A summary of the applications of geometric nonlinear control to polymerisation systems is given in Table 2.8.

In general good set point tracking and disturbance rejection are obtained with little interaction for the multi input multi output configurations. Comparisons with standard linear controllers show the geometric nonlinear controllers to work more effectively in just about all situations, due to the strong nonlinearities in polymerisation systems.

2.2.6 Miscellaneous Applications

Geometric nonlinear control methods have been applied to a number of other systems from the process industries. The reviewed applications are distillation, evaporators, heat exchangers (and heat exchanger networks), a liquid liquid extraction unit, a fixed bed reactor, dissolved oxygen content and level controllers. A summary of these applications is given in Tables 2.9 and 2.10.

In general, geometric nonlinear control is shown to work well in all of these applications. More details can be found in the references listed in Tables 2.9 and 2.10.

2.2.7 Summary

This section has presented a summary of a wide range of the simulation applications of the various geometric nonlinear control techniques in the literature. The focus of this summary has been applications to simulations of systems from the process industries. Effective control has been demonstrated on many systems which traditionally are difficult to control using standard linear controllers. In some cases excellent control is achieved with relatively simple models of the process. Certainly the results have warranted a call for experimental applications of the geometric nonlinear control technique. A review of the available experimental applications of geometric nonlinear control is given in the next section.

2.3 Review of Experimental Applications

The use of geometric nonlinear control techniques has been considered theoretically and in simulation for a number of systems from the process industries. The results of these studies have highlighted problems and led to advances in the theory and understand-

Reference	Method	Internal Stability	Disturb. Decoup.	Robustness	Observed States	Adaptive	Discrete Imp.	Controller Saturation
Kravaris <i>et al.</i> , 1989 Batch	I/O			Track Dist. Noise	Reduced Order			
Kravaris and Soroush, 1990 Fed Batch	I/O (mimo)			Track Dist. Noise				
Alvarez <i>et al.</i> , 1990b CSTR	I/O (mimo)		F/F	Track Dist. Model				
Daoutidis <i>et al.</i> , 1990 CSTR	I/O (mimo)		F/F	Dist. Model				Detuned
Daoutidis and Kravaris, 1991 CSTR	I/O (mimo)	Checked		No Simul.				
Rao <i>et al.</i> , 1992 Fed Batch	RSS			Track Model				
Adebekun and Schork, 1989a CSTR	I/O (mimo)			Track Dist Model	Kalman			
Adebekun and Schork, 1989b CSTR	I/O (mimo)			Track Noise Model				

Table 2.8: Summary of the application of geometric nonlinear control to polymerisation systems.

Reference	Method	Internal Stability	Disturb. Decoup.	Robustness	Adaptive	Discrete Imp.	Cont. Sat.
Riggs, 1990 - Distillation	GMC & IMC (multiloop)			Track Dist. Model			Simul.
Alsop and Edgar, 1990 - Distillation	S/S			Track Dist. Model		Discrete	
McLellan et al., 1990a - Distillation	I/O		F/F	No Sim.			
Rani and Gangiah, 1991 - Distillation	GMC			Track Dist.	Adaptive	Discrete	
Lee and Zhou, 1990 - Distillation	GMC			Track Model		Discrete	Simul.
Cott et al., 1989 - Distillation	GMC (mimo)		F/F	Track Dist Model	Adaptive	Discrete	
Newell and Lee, 1989 (mimo) - Forced circul. evaporator	GMC		F/F	Track Dist.			
Lee and Zhou, 1990 - Forced circul. evaporator	GMC			Track		Discrete	Simul.
Daoutidis and Kravaris, 1992a - Evaporator	Config.						
Malleswararao and Chidambaram, 1992 - Heat Ex.	RSS			Track Dist. Model		Discrete	

Table 2.9: Summary of the application of geometric nonlinear control to a number of process industries systems (part 1).

Reference	Method	Internal Stability	Disturb. Decoup.	Robustness	Adaptive	Discrete Imp.	Cont. Sat.
Riggs and Rhinehart, 1990 & Riggs and Rhinehart, 1988 - Heat Ex.	GMC & IMC			Track		Discrete	
Daoutidis and Kravaris, 1992a - Heat Ex. Net.	Config.						
Aoufoussi <i>et al.</i> , 1992 - Fixed Bed Reactor	I/O	Checked		Track Dist. Model	Adaptive		Anti Wind.
Perez-Correa <i>et al.</i> , 1991 - Dissolved O ₂	I/O		F/F	Track Dist. Model	Adaptive	Discrete	
Ogunnaike, 1986 - Level Control	I/O			Track			

Table 2.10: Summary of the application of geometric nonlinear control to a number of process industries systems (part 2).

ing of geometric nonlinear control. The state of the art in geometric nonlinear control is reviewed in Section 2.1 and the simulation applications are summarised in Section 2.2.

Some of the simulation examples in the literature consider systems with characteristics similar to a practical situation. For example, measurement noise, time delays, model mismatch and unmeasured disturbances. However, the development of any control method from a purely theoretical concept to an industrially useful technique requires experimental verification of its applicability and identification of its deficiencies. The call for experimental implementation of nonlinear controllers has been made in a number of references, including Bequette, 1991. Furthermore, the request calls for a fair comparison between the different nonlinear control techniques and standard linear control techniques.

The aim of this thesis is based on this need for experimental verification of the geometric nonlinear control technique. In particular, at the onset of this work (September 1990) only one experimental application had been referred to in the literature, Cott *et al.*, 1989. This paper mentions an application of GMC to an industrial depropaniser, splitting C_3 from a mixed hydrocarbon stream. Acceptable regulatory and good servo responses are reported, though few details are given.

In the past three years, three experimental applications of GMC, three experimental applications to multi input multi output systems and one experimental application of the input output linearisation technique to a relative order two process have been reported. These applications are reviewed in this section.

2.3.1 Relative Order One Processes - GMC

One technique which has received a lot of attention in the literature has been GMC. Three applications of the GMC method are considered below.

pH Control

One application of geometric nonlinear control to an experimental size, multi point base injection acid neutralisation system has been reviewed. This system considered by Mahuli *et al.*, 1992 accepts an acidic stream of variable composition and flowrate and neutralises the acid through the addition of base at two points. The pH is measured before each addition point and at the outlet point. The flowrate of base is the manipulated variable and the split of base between the two addition points is fixed. The pH at the outlet is the controlled variable.

The basic model of the process is a combination of two nonlinear steady state models. Each model has three adjustable parameters and represents a separate region

of operation. Dynamics are artificially added to the model through a first order lag to represent the dominant time constant of the experimental system.

Generic model control is used and an implicit, algebraic control law results (because the system has a relative order of one). As with the simulation applications of pH control, an adaptive scheme has been built around the GMC controller. This system performs two functions. Firstly, it judges which of the two models best describes the given conditions. Secondly it refits the three parameters for the corresponding model. As the model is a steady state model this update is only performed when the systems is judged to be at steady state (as determined from the slope of the filtered process measurement).

The experimental implementation is subjected to variations in the inlet acid concentration, acid type and flowrate. In all situations the controller is seen to work very well.

The experiments performed are a good test of the control system as there is a fair amount of measurement noise and model mismatch (between steady state points). However, no justification for the tuning parameters is given and most of the results are based on fairly idealised, well characterised acidic solutions. Also there will probably be a fair amount of drift in the pH meters and it is not clear how this will affect accuracy of the steady state model. In an industrial situation there will probably be many changes to the inlet conditions and these changes may not allow the adaptive routine to update as often the presented results.

Partially Simulated Reactor

Kershenbaum and Kittisupakorn, 1993 considered an experimental application of GMC to a partially simulated exothermic reactor (PARSEX). This system uses a batch tank, charged with water to represent a mixture of liquid reactants. A simulation of a chemical reaction is run in parallel to represent a fictitious reaction occurring in the tank. The simulation is used to control rate of steam sparge to the reactor, representing the heat of reaction. The control problem is to follow a pre-set temperature profile by manipulating the flowrate of cooling water to the jacket.

The simulated reaction scheme is assumed to be two parallel reactions,



The overall model of the batch reactor has six states, representing the mass

balances of the four components and energy balances for the reactor and the jacket.

A GMC controller is used, based on a simplified model of one state, representing the temperature of the reactor (ie. jacket side dynamics are ignored). One of the parameters in this model is the heat released by reaction. An extended Kalman filter is used to estimate this parameter. The model used for the extended Kalman filter contains a simplified description of the chemical kinetics.

The controller and estimator are tested on the experimental system. The closed loop experimental system demonstrates reasonable set point tracking and parameter estimation, despite process measurement noise and parametric error.

Blast Furnace Stove Rig

Labossiere and Lee, 1991 have applied GMC to the control of a laboratory-scale blast furnace stove. In this paper control objectives over two times scales are identified.

The basic operation is a stove containing a chequerwork of bricks which are heated up with combustion gases during the on-gas phase. This stored heat is used to heat up a cold stream of air in the on-blast phase. In this phase the rate of air to the stove and the rate of bypass air can be manipulated to maintain a constant total flowrate and temperature of air. GMC is applied to this on-blast phase control problem.

This is a multi input multi output control problem. However Labossiere and Lee, 1991 repose the problem so that the manipulated variables are the ratio of the two flowrates and the stove air flowrate. A simple steady state model is then derived for the energy balance around the system which is augmented with a simple lag to represent the bulk thermal dynamics of the system. In this way the final temperature is related directly to the ratio of the two flowrates, resulting in a single input single output system. A basic generic model controller is designed for this loop.

A simple mass balance is also performed resulting in a single dynamic relationship between the total flowrate of air, the flowrate to the stove and the ratio of the bypass and stove flowrates. The ratio setpoint from the first controller is fed forward to the second controller resulting in another single input single output system (with the total flowrate as the controlled variable and the flowrate of air to the stove as the manipulated variable) with a single disturbance (the ratio of flowrates). Another generic model controller is designed for this system.

The controller was tested on the experimental system in response to single set point changes and sequences of set point changes in both parameters. In all cases the GMC was able to track the set point changes better than PID loops and with less interaction.

The work of Labossiere and Lee, 1991 represents a fair test between GMC and PID. The GMC controller is able to accommodate the main gain nonlinearity in the system and this is well demonstrated.

The system is fairly simple, with little dead time and little process noise. There is some question of how well this experimental system represents the operation of an industrial blast furnace stove.

2.3.2 Multi Input Multi Output Systems

Three multi input multi output experimental applications have been reviewed. These are a simple temperature, level control system, a pH, level control system and a distillation column system.

Temperature and Level Control

A simple experimental system consisting of a stirred vessel with manipulated feed flowrate and rate of heat addition is considered in Nakamoto and Watanabe, 1991. The objective of the control problem is to meet setpoints for both the temperature and the height of liquid in the vessel. The system is modelled by two states with two inputs and two outputs. Both of the states are directly measured from the experimental apparatus.

The design of the controller is not complicated and no account of how the final tuning parameters were arrived at is given. The experiments demonstrate the effectiveness of the controller for step changes in the setpoint, both with and without an external PI regulator. In both cases a good response is obtained. However, this system has very little noise, disturbance rejection is not considered, there is no apparent dead time and model mismatch is not considered in detail.

pH and Level Control

Nakamoto and Watanabe, 1991 also consider the control of pH and liquid level in a strong acid, strong base system. The manipulated variables are the feed flowrate of strong acid and strong base. The model of the system has two states, two inputs and two outputs. The experimental results consider set point tracking only and demonstrate an acceptable response.

This system is very similar to the temperature and level control system considered in the same paper. The system is quite simple and relatively well described by the model equations (ie. very little model mismatch). The apparatus introduces very little noise and there is no apparent dead time in the system. The rejection of disturbances is not

considered. The tuning of the nonlinear controller is not discussed and it is not clear why different control constants are not used to obtain a better closed loop performance.

Binary Distillation Column

The application of a multi input multi output geometric nonlinear controller to binary distillation columns has been considered by a group of researchers at École des Mines de Paris. Their work is presented by Creff *et al.*, 1991, Rouchon *et al.*, 1991, Rouchon, 1992, with most details appearing in the paper by Levine and Rouchon, 1991.

The theoretical approach of these papers is directed either at binary or pseudo binary distillation columns. Levine and Rouchon, 1991 consider a binary distillation column, starting with a general set of equations based on the mass balance on each tray. It is assumed in the models that the liquid inflow to each stage is equal to the liquid outflow and the vapour inflow is equal to the vapour outflow. These assumptions are very important from the point of view of the structure of the model because the manipulated inputs (boilup rate and reflux flow) affect each of the state equations directly. As a result, the relative order for each state variable is one.

The controller is implicitly based on a realisation of the inverse, similar to the method of RSS. Therefore, each of the outputs must have a relative order of one. The choice of reference system is not discussed, though an example describes a reference which is equivalent to proportional feedback.

Some results from the application of the nonlinear controller on a refinery depropaniser (with 42 trays) are presented in Levine and Rouchon, 1991, Rouchon *et al.*, 1991 and Creff *et al.*, 1991. Very few details about the design, tuning and application of the controller are given. It is not possible to conclude more than the fact that the method has been applied in an industrial setting and appears to work well.

2.3.3 Relative Order Two Processes

Stirred Tank Reactor

The most detailed presentation of an experimental implementation is given in Soroush and Kravaris, 1992b. The experimental system considered in this work is a batch polymerisation reactor. The reactor is a 3 L stirred tank with an external jacket for temperature control. The system is controlled both by the rate of coolant addition to the jacket and by the amount of heat added to the coolant before entering the jacket. The two input problem is reduced to a single input problem by introducing a coordination rule

between the two.

The objective for control is to follow a predetermined temperature profile in the reactor. The controller is used to start up the reactor and provide control for the length of the batch (6.5hrs). Linear control is generally not suitable to the application due to the large range of operating conditions with a correspondingly large range of gains and time constants.

The system is described by four states, representing the concentration of the initiator, concentration of the monomer, temperature of the reactor and the temperature of the jacket. The system has a relative order of two and an input output linearisation is used as the basis of the controller. To this is added an external PI controller and partial observer.

Controller constraints are handled in the implementation by switching off the integral action during startup and when a constraint is hit during normal operation. The measurement of the temperatures is directly fed back to the controller, though the composition measurements are obtained from an observer. The control interval is 5 sec, which is much less than the time constant of the system.

Basic tuning guidelines are presented which set the response of the linearised plant equal to the slowest modes of the open loop system. Then the external regulator is tuned for fast and offsetless tracking.

The controller is tested for setpoint tracking, under unmeasured disturbances, modelling errors and for a simplified model. The performance of the controller is compared to a PID controller tuned by trial and error. The nonlinear controller is shown to be much better than the PID controller.

This application is very complete, though is slightly idealised as there appears to be very little noise, no apparent dead time and a small time constant. The comparison with PID control is not completely fair as the nonlinear controller requires the measurement of two temperatures and only one is used for the PID controller. Also, there is some question about the tuning of the PID controller, particularly as the closed loop response does not saturate the controller (and the nonlinear controller does).

2.3.4 Summary

This section has reviewed a number of experimental applications of geometric nonlinear control techniques. In all cases the nonlinear controller worked well and any comparisons with a standard linear technique demonstrated that the nonlinear control technique is more effective. However, except for the work on the binary distillation column

(for which there is very little available information) the experiments are fairly idealised.

There is still the need for rigorous test on a complicated system approaching conditions in industry. Important characteristics would include,

1. significant measurement noise,
2. significant measurement error,
3. significant process dead time,
4. significant model mismatch,
5. complicated process model,
6. tight manipulated variable constraints.

Such a test should consider general methods for the tuning of the parameters and a generalised approach to controller design.

2.4 Graph Theory

Graph theory is a branch of mathematics used to study the structure of a system and the relationship between objects of the system. It is potentially a valuable tool to be used with geometric nonlinear control techniques as these techniques involve a change of co-ordinates (ie. objects of the system) so that the structure of the system is changed to some prescribed form.

For example, if the feedback transformation is used to input output decouple a multiple input, multiple output system then the structure of the system is changed from,

- all the outputs affected by all the inputs,

to

- each output is affected by only one input.

Basic transformation theory makes no assumptions about the structure of the system and requires the calculation of many partial derivatives to determine how the inputs and outputs are related. Graph theory provides a method for determining the relevant structural information without computing partial derivatives. Once it is understood how the inputs and outputs are related it is then possible to calculate only the relevant partial derivatives required to solve the feedback transformation problem.

The most obvious use for graph theory is for quick calculation of the generic relative order for a system. This technique has been used by Daoutidis and Kravaris,

1992b as a technique for nonlinear controllability analysis and in Daoutidis and Kravaris, 1989 for analysing the requirements for disturbance decoupling of a system.

Basic graph theory and how it may be used for computation of feedback transformations is discussed below.

2.4.1 State Space Systems and Graph Theory

Consider a system represented by the following state and output equations.

$$\begin{aligned}\dot{x} &= f(x) + \sum_{i=1}^m u_i g_i(x) \\ y_i &= h_i(x) \quad i = 1, \dots, p\end{aligned}\tag{2.43}$$

where, x is the vector of state variables, $x \in \mathbb{R}^n$,

u is the vector of manipulated inputs, $u \in \mathbb{R}^m$,

y is the vector of outputs, $y \in \mathbb{R}^p$,

$f(x)$ is a vector field, $f(x) \in \mathbb{R}^n$,

$g(x)$ is a set of vector fields, $g(x) \in \mathbb{R}^{n \times m}$,

$h(x)$ is a vector field, $h(x) \in \mathbb{R}^p$.

The nodes of the system are defined as follows.

1. Input Nodes - u_i $i = 1, \dots, m$.
2. Intermediate Nodes - x_i $i = 1, \dots, n$.
3. Output Nodes - y_i $i = 1, \dots, p$.

Directed arcs exist between various nodes under the following conditions.

1. An arc exists from u_i ($i = 1, \dots, m$) to x_k ($k = 1, \dots, n$) if $g_i(x)|_k \neq 0$.
2. An arc exists from x_k ($k = 1, \dots, n$) to x_j ($j = 1, \dots, n$) if $\frac{\partial}{\partial x_k} f_j(x) \neq 0$.
3. An arc exists from x_k ($k = 1, \dots, n$) to y_i ($i = 1, \dots, p$) if $\frac{\partial}{\partial x_k} h_i(x) \neq 0$.

A path is a subset of the graph made up of a number of arcs joining two nodes. The value $path(u_j, y_i)$ is the minimum number of directed arcs forming a path from u_j to y_i .

Finally, $path_i$ is defined as,

$$path_i = \min_{1 \leq j \leq m} path(u_j, y_i) \quad (2.44)$$

2.4.2 Relative Order

The vector relative order of a multiple input, multiple output system is defined as follows.

Vector relative order = $r = (r_1, \dots, r_p)$,

r_i is a unique integer such that,

$$\exists j \in \{1, \dots, m\} : L_{g_j} L_f^{r_i-1} h_i(x) \neq 0 \quad (2.45)$$

and

$$\forall j \in \{1, \dots, m\}, \forall count \in \{0, \dots, r_i - 2\} : L_{g_j} L_f^{count} h_i = 0 \quad (2.46)$$

Calculation of the vector relative orders would normally involve the following steps.

1. Set $count = 0$.
2. Compute Lie derivative in Equation 2.46, for each input, u_j .
3. If the derivative is zero for all inputs, then increase $count$ and repeat step 2. If the derivative is non zero for one of the inputs then the relative order is $count$.
4. Repeat steps 1, 2 and 3 for each output, y_i .

2.4.3 Graph Theory and Relative Order

The relative order may also be calculated by finding a path between a given output node and any input node on the system graph.

For example, consider the following system, as used by Kasinski and Levine, 1984 and Geromel *et al.*, 1984.

$$\begin{aligned}
\dot{x}_1 &= f_1(x_1, x_2, x_3) + u g_1(x_1, x_2, x_3)|_1 \\
\dot{x}_2 &= f_2(x_1, x_2) \\
\dot{x}_3 &= f_3(x_1, x_2, x_3) \\
y &= h(x_2)
\end{aligned} \tag{2.47}$$

The graph of this system is shown in Figure 2.12. From this it can be seen that $path_1 = 3$. This corresponds to a relative order of two (ie. $r_1 = 2$) as the arc joining the state node to the output node does not represent an integration.

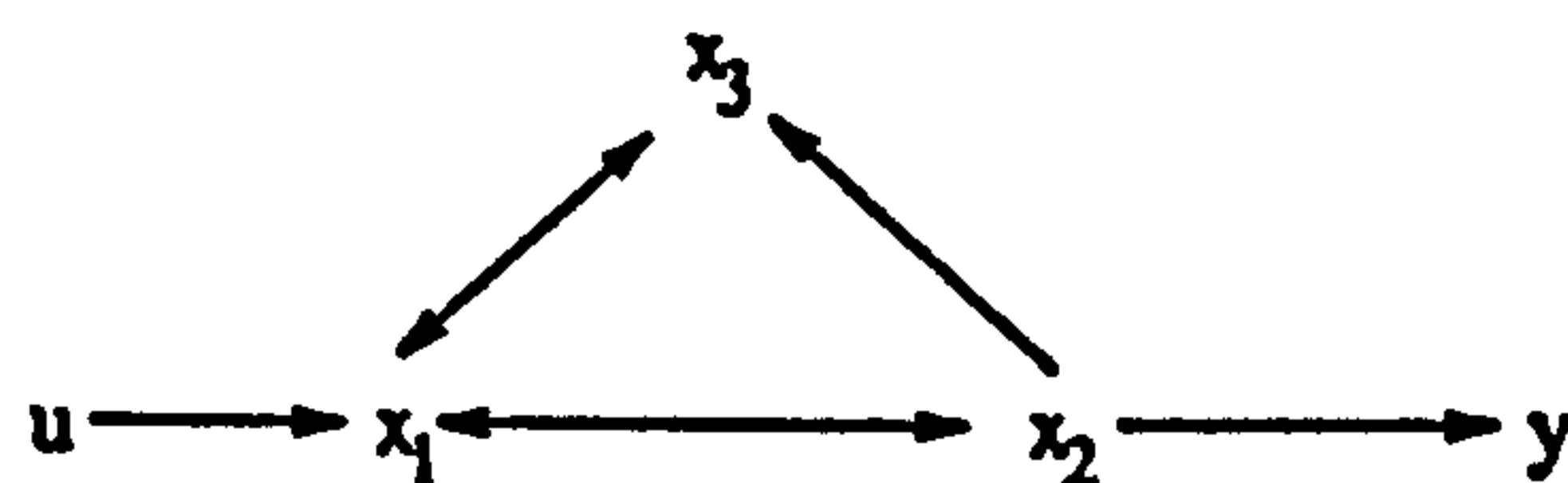


Figure 2.12: System graph for system described in Equation 2.47.

The main result presented by Kasinski and Levine, 1984: an integer ν_i is used to represent a lower bound on the relative order.

ν_i $i = 1, \dots, p$ is the integer satisfying,

$\exists j \in \{1, \dots, m\}, \exists k_{\nu_i}, \dots, k_o \in \{1, \dots, n\}$ such that,

$$g_j(x)|_{k_{\nu_i}} \frac{\partial f_{k_{\nu_i-1}}(x)}{\partial x_{k_{\nu_i}}} \dots \frac{\partial f_{k_o}(x)}{\partial x_{k_1}} \frac{\partial h_i(x)}{\partial x_{k_o}} \neq 0 \tag{2.48}$$

and

$\exists j \in \{1, \dots, m\}, \forall r < \nu_i, \forall k_r, \dots, k_o \in \{1, \dots, n\};$

$$g_j(x)|_{k_r} \frac{\partial f_{k_{r-1}}(x)}{\partial x_{k_r}} \dots \frac{\partial f_{k_o}(x)}{\partial x_{k_1}} \frac{\partial h_i(x)}{\partial x_{k_o}} = 0. \tag{2.49}$$

Then,

$$\nu_i = path_i - 2 \quad i = 1, \dots, p \tag{2.50}$$

$$\nu_i \leq r_i \quad i = 1, \dots, p \tag{2.51}$$

Furthermore, generically the following is true.

$$\nu_i = r_i \quad i = 1, \dots, p \quad (2.52)$$

A generic system is described in Geromel *et al.*, 1984 as a system with functions $f(x), g_i(x), h_i(x)$ depending locally on the same variables as those of the original system. In other words, a nongeneric system relies on an exact cancellation of two mathematical expressions which are only equal with particular combinations of parameter values.

The result of the work by Geromel *et al.*, 1984 has been the development of a relatively easy method for calculating the vector of relative orders for generic systems and obtaining a starting point for calculating the relative orders of nongeneric systems.

The following algorithm for computing the relative order has been taken from Kasinski and Levine, 1984 .

The computation of the feedback law is performed in two parts.

1. Determine minimal paths, $path_i$.
2. Calculate vector relative order, r .

System Graph and Minimal Paths

The first step involves determining $path_i$ and the subset U_i of $u = (u_1, \dots, u_n)$ corresponding to the minimal paths.

1. Start from $i = 1, count = 0$.
2. Build every directed arc with $\frac{\partial h_i(x)}{\partial x_{k_{count}}} \neq 0 \quad k_{count} \in \{1, \dots, n\}$.
3. Test if $(u_j, x_{k_{count}})$ are incident by checking if $g_{jk_{count}}(x) \neq 0$.
4. If $(u_j, x_{k_{count}})$ are not incident (ie. $g_j(x)|_{k_{count}} = 0 \quad \forall j$) then increase $count$ by 1. If $count = n - 1$ then $\nu_i = +\infty$. Otherwise, with the new value for $count$ build every directed arc with $\frac{\partial}{\partial x_{k_{count}}} g_j(x)|_{k_{count-1}} \neq 0 \quad k_{count-1}, k_{count} \in \{1, \dots, n\}$. Go back to step 3.
5. If $(u_j, x_{k_{count}})$ are incident then $path_i = count, \nu_i = count - 2$ and $u_j \in U_j$.
6. Repeat the above steps for all $i = 1, \dots, p$.

Vector Relative Order

The values ν_i give a lower bound on the relative orders and equal the relative order for a generic system. The following algorithm checks for genericity and computes the relative order for non generic systems.

1. Start from $i = 1$.
2. Compute $L_{g_j} L_f^{\nu_i} h_i(x) \quad \forall j$ such that $u_j \in U_i$.
3. If $\exists j \in 1, \dots, p$ such that $L_{g_j} L_f^{\nu_i} h_i(x) \neq 0$ with $u_j \in U_i$ then,

$$r_i = \nu_i$$
4. If $L_{g_j} L_f^{\nu_i} h_i(x) = 0 \quad \forall j$ such that $u_j \in U_i$ then $r_i > \nu_i$.
 Compute $L_{g_j} L_f^{\text{count}} h_i \quad \forall \text{count} > \nu_i, \forall j = 1, \dots, m$ until one of the values is non zero ($r_i = \text{count}$) or $\text{count} = n - 1$ and all the values are zero ($r_i = +\infty$).
5. Repeat the above for $i = 1, \dots, p$.

2.4.4 Summary

This section has introduced some basic concepts from graph theory which are useful for analysing nonlinear systems as part of the geometric nonlinear controller design procedure. Graph theory pictorially represents the states, manipulated inputs, disturbance inputs, outputs (all called nodes) and the connections between them. These connections have a direction and therefore the connections are represented by arrows. This naturally leads to the definition of a path length between various nodes of the graph.

The relative order is shown to be generically equal to the shortest path between an input and any of the outputs, for the multi output case. For single input single output systems the generic relative order is the shortest path between the input and the output. Under specific conditions cancellation may result in a non generic situation and therefore, the shortest path is a lower bound on the relative order for a general system.

2.5 Conclusions

This chapter has introduced geometric nonlinear control and the recent advances in theory. The combined result is an overall controller structure which is able to control many different systems from the process industries. A wide range of applications to simulated examples from the process industries have been reviewed. These applications demonstrate the use of the various approaches to geometric nonlinear control and the recent advances. The real strength of the geometric nonlinear techniques is demonstrated for setpoint tracking. Effective regulatory control often requires feedforward or adaptive schemes to be considered. The conclusion is that geometric nonlinear control techniques are potentially suitable for the process industries.

To further validate the applicability of geometric nonlinear control and to bring the controller one step closer to an industrially acceptable method a number of experimental applications have been performed in the past three years. These applications have also been reviewed, demonstrating that under experimental conditions the geometric nonlinear control methods work well. Comparisons between these techniques and standard linear techniques have demonstrated the potential advantages for using geometric nonlinear control.

The common concerns for geometric nonlinear control arise for the following reasons.

1. For most applications it is very difficult to obtain a proof of robust input output stability.
2. Theoretical proofs of internal stability are difficult to obtain for anything but low order systems.
3. Most of the mathematics associated with geometric nonlinear control is fairly involved and complicated.

With regard to the first point, most of the applications consider the issues of robustness in very little detail. Use is made of external regulators to improve robustness, though generally detuned responses are used to avoid instabilities and manipulated variable constraints. However, it is interesting to note that even though stability and robustness are important issues from a theoretical point of view, very few of the applications reported stability problems (degradation of performance has been reported, but this is a separate issue).

Much work in the literature has addressed the second point: internal stability. Most of the applications in the literature do not attempt to prove internal stability and apparently by chance just about all of the examples are internally stable. There appears to be some characteristic of process systems which causes them to be either internally stable or internally unstable for most of the operating region. This certainly cannot be taken as a rule, only an observation.

The final concern is still relevant for the design of geometric nonlinear controllers. One exception is GMC, which despite its limitations has been a very popular technique in the literature. The reasons for this seem to be due to presentation. GMC avoids most of the mathematical theory on which it is implicitly based and presents an almost complete and easy to understand package. The same is required for the general approach

to geometric nonlinear control. The use of graph theory has potential for being a major part of this.

In conclusion, there is a need for further research in many areas associated with the geometric nonlinear control technique. This review has highlighted two needs. Firstly, there is the need for detailed applications on more complex systems, representative of an industrial application. In this case complexity refers to both the physical system and mathematical model. This application should include a fair comparison with a standard linear control technique. Secondly, there is a need for a unified approach to controller design using the geometric nonlinear control. This approach should ultimately be able to handle most given systems from the process industries and must be presented in an understandable fashion.

Chapter 3

Case Study

This chapter describes the modelling and operation of the CO₂ absorption /desorption pilot plant in the Department of Chemical Engineering and Chemical Technology, Imperial College, University of London. This system is representative of systems in the process industries. The characteristics making it representative of the process industries include,

1. The system is a combination of smaller, well defined unit operations.
2. The unit operations, distillation columns and heat exchangers, are standard to and commonly used in the process industries.
3. The system is described by a combination of mass and energy balances.
4. The model of the system considers vapour-liquid equilibrium data, chemical reaction and mass transfer effects.
5. The main control problem is to meet a certain product requirement (residual CO₂ composition in product stream) while minimising the use of the manipulated variable (steam flow rate), subject to manipulated variable constraints, measured disturbances (eg. feed CO₂ composition and recirculation flow rate) and unmeasured disturbances (eg. steam quality and feed temperature).

This chapter begins with a mathematical description of the pilot plant. The description starts with a summary of the chemistry involved in a CO₂ and monoethanolamine (MEA) solution. This is followed by a description of the pilot plant in three sections: the absorption section, the heat exchanger section and the desorption section. A simplified schematic, showing the arrangement of these sections, is given in Figure 3.1.

Details of the experimental system follow the mathematical description. Including a description of the supervisory controls and details of the measurements are given.

Finally, the specification of the model is completed by assigning values to all of the model parameters. Three of the parameters are fitted using steady state experimental results.

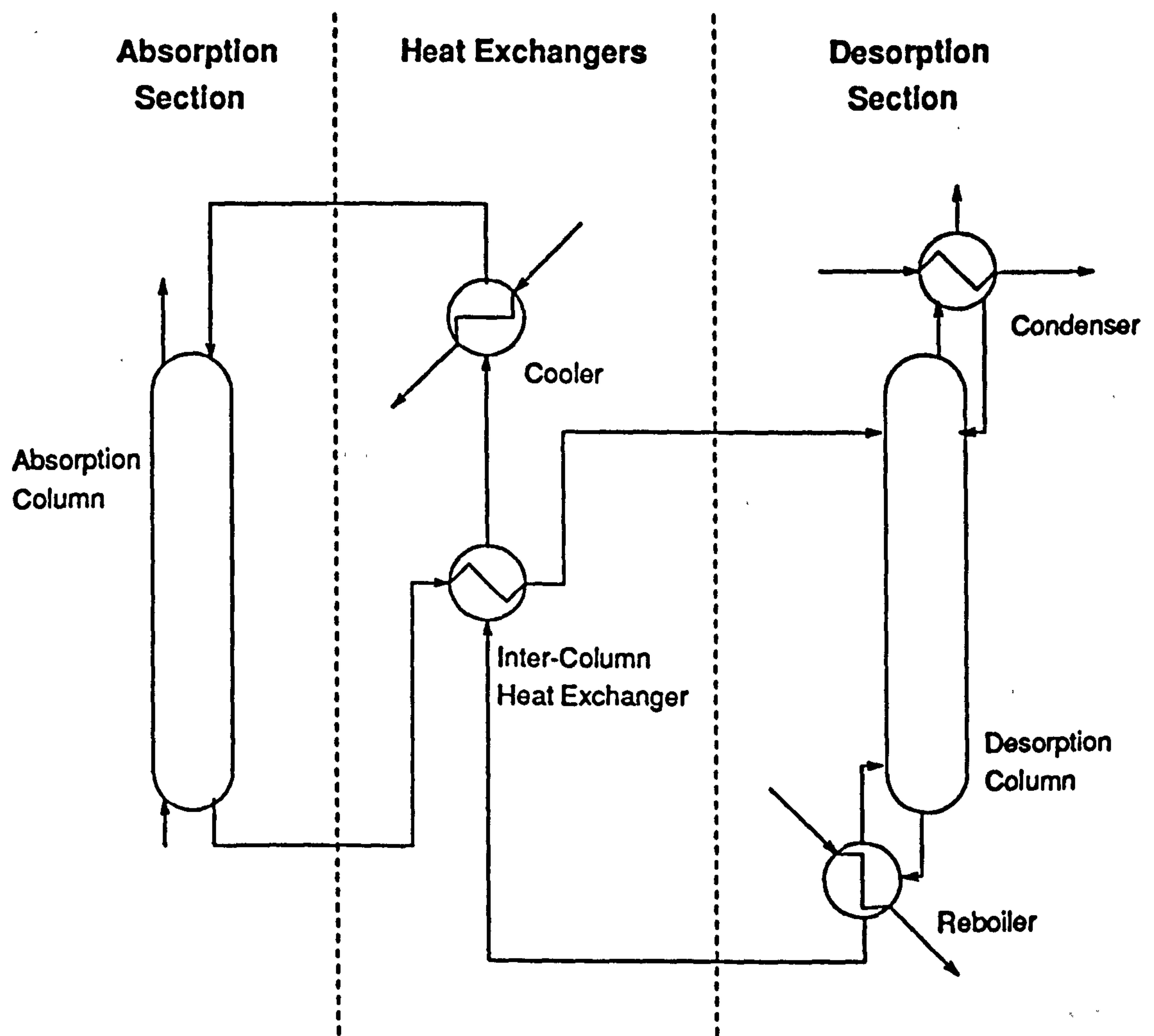


Figure 3.1: Schematic of pilot plant sections.

3.1 Chemistry

The absorption of CO₂ into an aqueous MEA solution takes advantage of the liquid phase reaction between CO₂ and MEA. The reaction mechanism has been investigated by a number of authors and it has been concluded that the most significant reaction involves the formation of a carbamate salt.



This reaction follows second order reaction kinetics with a reaction rate defined by,

$$r_{\text{rxn}} = k_r[\text{CO}_2][\text{HO}(\text{CH}_2)_2\text{NH}_2] \quad (3.2)$$

At absorption conditions this reaction is considered irreversible. However, the desorption process relies on the reverse reaction.

The quantity of CO₂ in solution is expressed in terms of the loading of the MEA in solution, through the carbonation ratio, α .

$$\alpha = \frac{[\text{CO}_2]}{[\text{HO}(\text{CH}_2)_2\text{NH}_2]} \quad (3.3)$$

At higher MEA loadings (α approaching 0.5) there is a secondary reaction of the carbamate with water to form bicarbonate.



For carbonation ratios less than 0.47 this reaction can be ignored.

3.2 Absorption Section

This section describes the operation and modelling of the absorption column of the pilot plant. The absorption column takes a gas feed stream which is a mixture of N₂ and CO₂ (3–5% CO₂). This is passed counter currently with an aqueous monoethanolamine (MEA) solution in a packed column. The CO₂ is absorbed by the solution with the amount of absorption enhanced by the chemical reaction between the CO₂ and MEA.

From the top of the column, the nearly pure gas stream (100 to 1000 ppm CO₂) is passed through a cyclone to knock out entrained liquid. This stream is passed back to the column where it is mixed with fresh CO₂ and becomes the feed stream for the column. For

modelling purposes, this recycle is ignored and the feed stream is considered independent of the outlet conditions.

The absorption column is a packed column containing five packed sections, with intermediate liquid distributors. Each section is 1.4m high, packed with 1 inch ceramic pall-rings, making a total height of 9.1m, including the unpacked sections at the top and bottom of the column.

The aqueous MEA solution comes from the desorption section of the pilot plant and is partially loaded with CO_2 ($\alpha = 0.17$ to 0.27). The spent solution ($\alpha = 0.22$ to 0.32) is sent to the desorption section for regeneration.

A summary of the nomenclature used for the absorption section model is given in Figure 3.2.

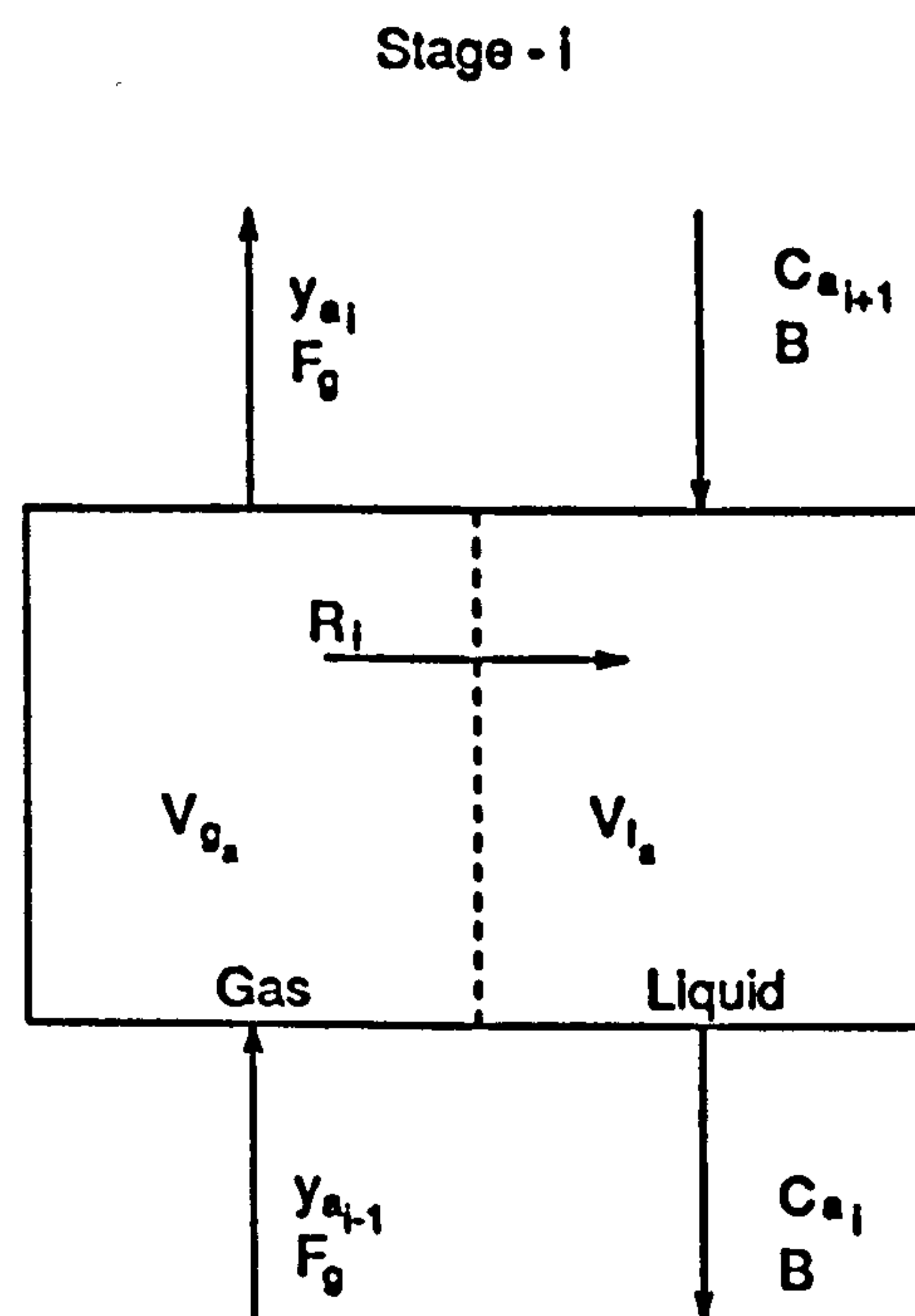


Figure 3.2: Absorber nomenclature.

3.2.1 Absorption Column

Modelling of the CO_2 and MEA absorption systems has been considered in a number of sources in the literature. Vanichseni, 1977 considered a plug flow reactor model for steady state parameter estimation. Burkett, 1977 compared a plug flow model, a dispersion model and a mixing cell model as descriptions for the absorber. There is very little liquid phase dispersion and Burkett, 1977 concluded that a plug flow representation is suitable for normal conditions. A liquid phase Peclet number of about 2 is quoted for

the conditions in Burkett, 1977 suggesting 200 cells would be required in the mixing cell model to match the dispersion characteristics. The results of Burkett, 1977 show that runs with 100 and 50 cells have steady state solutions very close to the plug flow solution.

A lumped parameter model (ie. mixing cell model) of the absorption column will be used for this work as a differential algebraic description is required by the geometric nonlinear control technique. The large number of mixing cells suggested by Burkett, 1977 is unsuitable for the application of the technique. An approximation to the dynamics of the absorber has been used by Disli, 1993 where a limited number of cells are used to represent the absorption column. This description of the absorption system will be used for this work.

Each cell of the model is of fixed volume and is made up of two phases: a liquid and gas phase. It is assumed that the two phases are at the same temperature and that the pressure is constant throughout the column. None of the N_2 is absorbed into the liquid stream and the MEA is considered involatile. Given these conditions it is only necessary to use two states to describe each cell of the absorber, corresponding to the liquid and gas phase CO_2 compositions.

For this model the following assumptions are made (Disli, 1993).

1. The absorption column is adiabatic.
2. There is no pressure drop along the absorption column.
3. The liquid phase is perfectly mixed and the gas phase is in plug flow.
4. The gas phase is at pseudo steady state.
5. The contribution of gas hold up to energy balance is negligible.
6. There are no radial temperature or composition gradients.
7. The temperature of the liquid and gas phases are equal.
8. The physical properties are constant throughout the column.

With these assumptions the gas and liquid phase mass balances are as follows.

Liquid Phase

$$(V_{l,i}) \frac{dC_{a,i}}{dt} = \left(\frac{mwB}{\rho_l} \right) (C_{a,i+1} - C_{a,i}) + R_i \quad (3.5)$$

Gas Phase

$$(V_{g.}) \frac{dy_{a_i}}{dt} = F_g(y_{a_{i-1}} - y_{a_i}) - R_i \quad (3.6)$$

where, R_i is the amount of CO_2 absorbed in each cell.

3.2.2 Absorption Rate

The rate of absorption of CO_2 in MEA has been considered by a number of sources in literature. The model for absorption used in this work is based on the work of Burkett, 1977, where the local absorption rate is modelled as a second order reaction using the Higbie (ie. penetration) theory. The fundamental description of this model was solved analytically by Brian *et al.*, 1961 to provide an equation for the local absorption rate based on an enhancement factor.

During the discussion of the absorption rate a distinction will be made between the bulk and interfacial properties. The corresponding nomenclature is summarised in Figure 3.3.

The enhancement factor, ψ , is defined as the ratio of the rate of absorption with chemical reaction to the rate of physical absorption. The solution by Brian *et al.*, 1961 involves two dimensionless groups: ψ_i defined as the limiting value of the enhancement factor for instantaneous reaction and $\sqrt{M}$ defined as the ratio of the reaction rate to the diffusion rate.

$$\psi = \frac{\bar{R}}{k_L C_{\text{interface}}} \quad (3.7)$$

$$\psi_i = 1 + \frac{C_{\text{bulk}}}{2C_{\text{interface}}} \sqrt{\frac{D_M}{D_c}} \quad (3.8)$$

$$\sqrt{M} = \frac{\sqrt{D_c k_r C_{\text{bulk}}}}{k_L} \quad (3.9)$$

The solution by Brian *et al.*, 1961 is,

$$\psi = \frac{\sqrt{M \left(\frac{\psi_i - \psi}{\psi_i - 1} \right)}}{\tanh \sqrt{M \left(\frac{\psi_i - \psi}{\psi_i - 1} \right)}} \quad (3.10)$$

Using the following expression to represent gas side diffusion,

$$\bar{R} = k_g(p_{\text{bulk}} - p_{\text{interface}}) = \psi k_L C_{\text{interface}} \quad (3.11)$$

and a solubility coefficient, H , for the equilibrium of interfacial compositions,

$$p_{interface} = H C_{interface} \quad (3.12)$$

As reported by Disli, 1993, this relationship is incorrectly reported in Burkett, 1977. The local absorption rate, $\bar{R}$, can be written as,

$$\bar{R} = \frac{\psi k_L p_{bulk}}{H + \frac{\psi k_L}{k_g}} \quad (3.13)$$

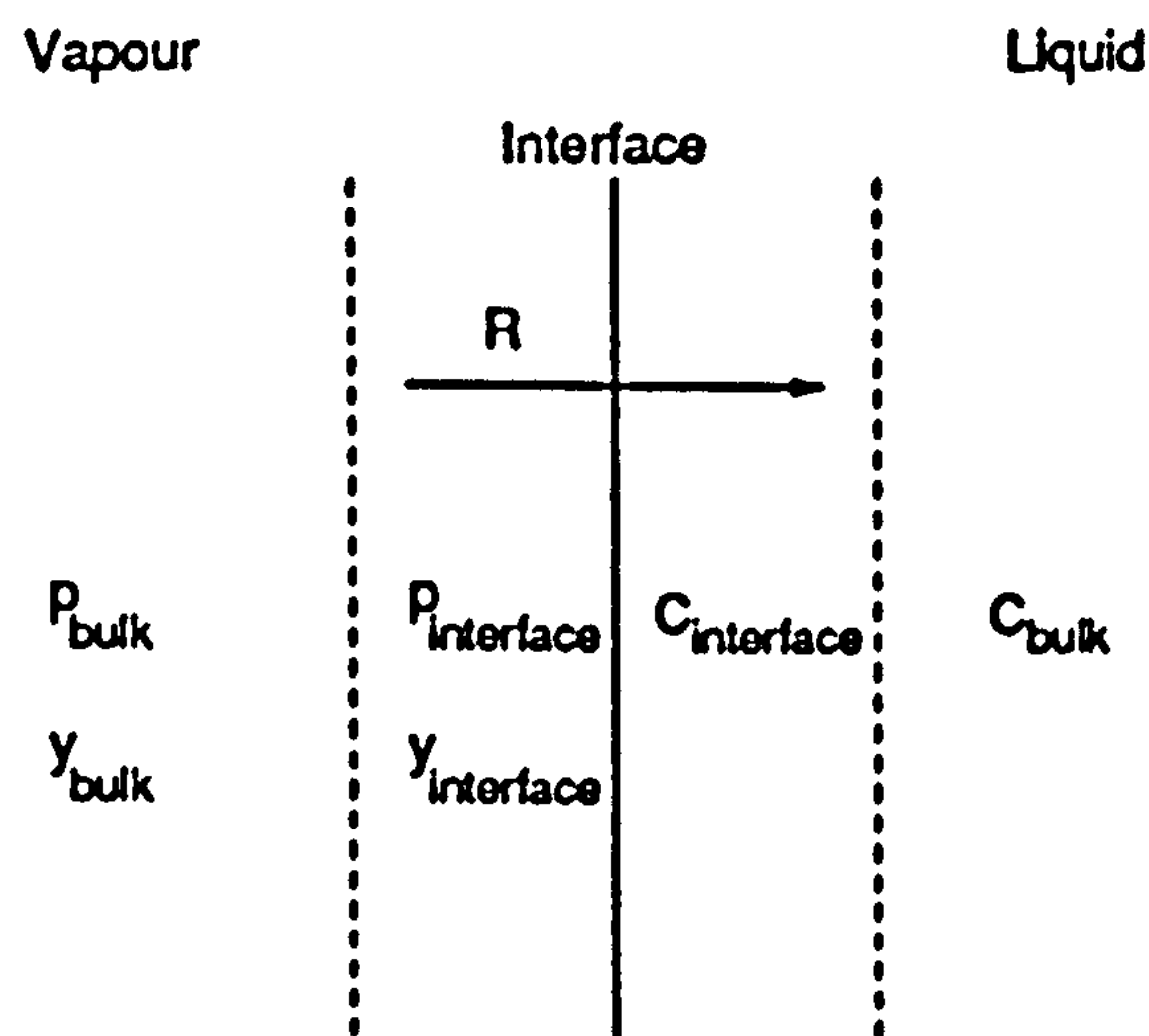


Figure 3.3: Interface nomenclature.

Burkett, 1977 simplifies the correlation of Brian *et al.*, 1961 for conditions where the reaction rate is sufficiently greater than the diffusion rate ($\sqrt{M} \geq 5$), by assuming that the reaction is pseudo first order. Under these conditions the enhancement factor is approximated by the ratio of the reaction rate to the diffusion rate.

$$\psi = \sqrt{M} \quad (3.14)$$

The transfer flux is then approximated by the following equation.

$$R = C_{interface} \sqrt{D_c k_r C_{meas, ...}} \quad (3.15)$$

This is equivalent to assuming that the absorption rate is independent of the liquid phase mass transfer coefficient.

Burkett, 1977 suggests an extension of this simplification for all conditions, by approximating the enhancement factor with the following equation.

$$\psi = k''\sqrt{M} \quad (3.16)$$

This approximation results in the following relationship for the transfer flux in terms of bulk properties.

$$\bar{R} = \frac{k''\sqrt{D_c k_r C_{meafree}}}{H + \frac{k''\sqrt{D_c k_r C_{bulk}}}{k_g}} p_{bulk} \quad (3.17)$$

Finally, Burkett, 1977 demonstrates that assuming low gas resistance, the following approximation equation 3.17 is an adequate representation for calculating exit concentrations, even away from pseudo steady state conditions.

$$\bar{R} = \frac{k'' p_{bulk}}{H} \sqrt{D_c k_r C_{meafree}} \quad (3.18)$$

This equation is suitable for a plug flow description of the absorber. To convert this expression to a term representing the total amount of absorption in a cell it is necessary to integrate a differential mass balance over the length of a cell. Disli, 1993 considers the following mass balance over a differential height in the gas phase, dz (See Figure 3.4).

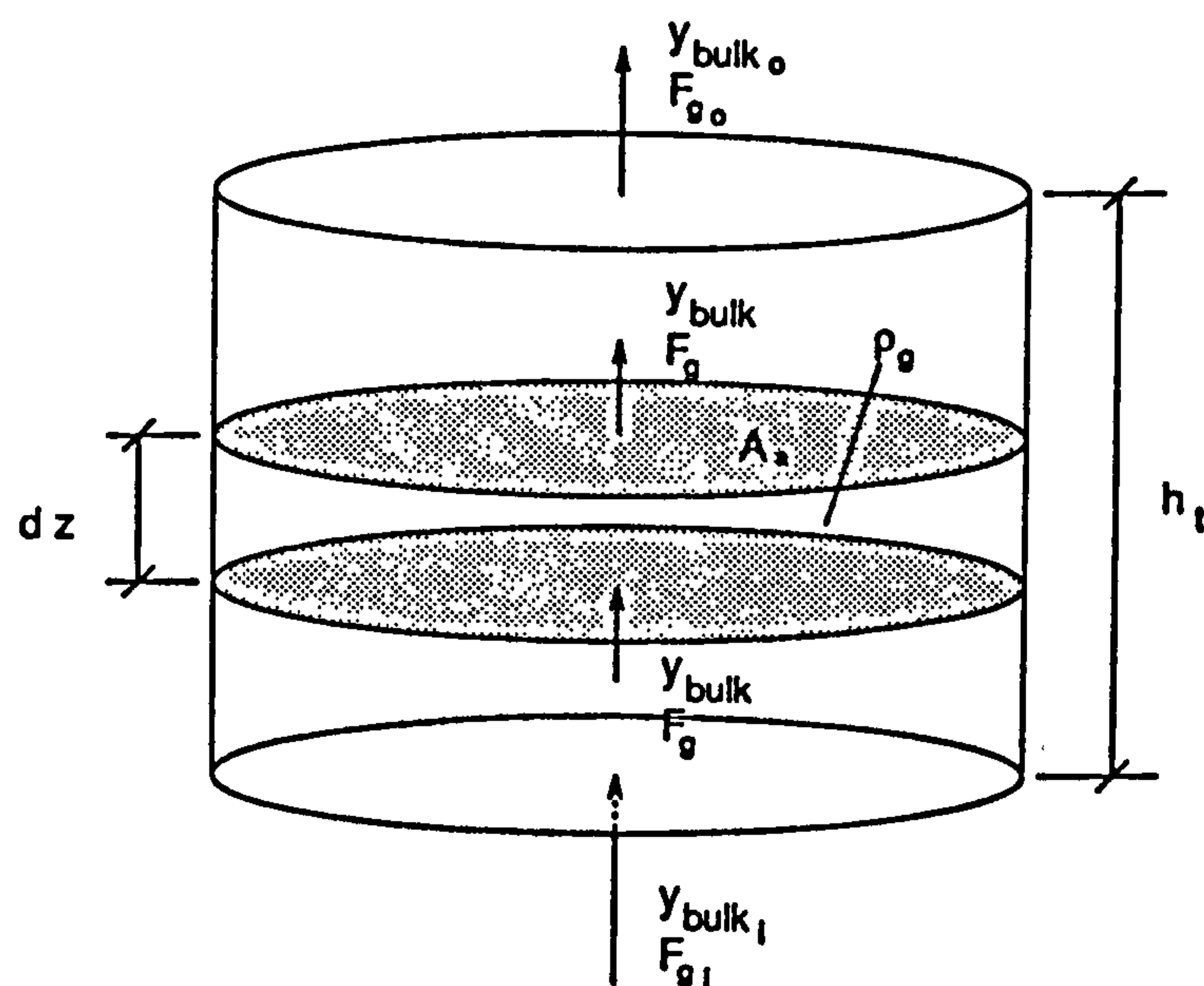


Figure 3.4: Schematic of gas phase in an absorber.

$$A_a dz \left(\frac{d\rho_g y_{bulk}}{dt} \right) = -d(F_g y_{bulk}) - \beta y_{bulk} dz \quad (3.19)$$

where β is defined as $\beta = \frac{a_{int} A_a k'' P}{H} \sqrt{D_c k_r C_{meafree}}$.

Assuming that the terms $\frac{\rho}{F_g}$ and $\frac{d}{dt}(\rho_g y_{bulk})$ are both independent of height, then the total amount of absorption in a cell of length h_i is,

$$R_{stage} = F_{g,i} y_{bulk,i} M' + \left(1 - \frac{M'}{e^{M'} - 1}\right) (F_{g,i} y_{bulk,i} - F_{g,i} y_{bulk,o}) \quad (3.20)$$

where, $M' = \beta \frac{h_i}{F_{g,i}}$.

For small M' , corresponding to a small amount of absorption over the length of the cell, Equation 3.20 reduces to,

$$R_{stage} = PK h_i \frac{(F_{g,i} y_{bulk,i} - F_{g,i} y_{bulk,o})}{2F_{g,i}} \sqrt{C_{mea,rea}} \quad (3.21)$$

where $K = \frac{e_{ia} A_a k''}{H} \sqrt{D_c k_r}$.

The exit bulk gas composition is equal to the composition on a particular stage (ie. $y_{bulk,o} = y_{a,i}$). Also, the amount of free (ie. unloaded) MEA on the stage is found from a molar balance based on Equation 3.1.

$$C_{mea,rea} = C_{mea} - 2C_{a,i} \quad (3.22)$$

where, C_{mea} is the total concentration of MEA in solution.

Finally, it is assumed that there is no accumulation of mass on a stage,

$$F_{g,i} = F_{g,o} = F_g \quad (3.23)$$

Therefore, the total absorption in a stage i is,

$$R_i = P \frac{(y_{a,i} + y_{a,i-1})}{2} h_i K \sqrt{C_{mea} - 2C_{a,i}} \quad (3.24)$$

3.3 Desorption Section

This section describes the operation and modelling of the desorption section of the pilot plant. The unit operations in this section are the desorption column, reboiler and condenser.

The desorption section accepts a loaded, aqueous MEA solution from the absorption column, representing the spent MEA solution. The energy for separation of the CO_2 from the solution is supplied by condensing saturated water vapour passing up the desorption column. The water vapour is produced in the reboiler, by superheated steam at approximately 140 kN/m^2 . The MEA solution in the reboiler circulates by free convection.

The condenser condenses the water vapour coming from the top tray of the desorber to avoid the loss of water and therefore changes in the concentration of the MEA solution. The condensate is returned to the desorber at the same point as the feed stream.

The dynamic modelling of desorption processes has not been considered by many sources in the literature. Fundamental dynamic models have been described by Burkett, 1977 and Disli, 1993. These models have been based on experimental data from the same pilot plant used in this work. A steady state model of the same system has been considered by Albrecht, 1977 and Escobillana *et al.*, 1991. The modelling of the desorption column, reboiler and condenser will be considered in the following discussion, including relevant details from these works. A summary of the nomenclature used for the model is given in Figure 3.5.

3.3.1 Desorption Column

The desorption column will be treated as a general trayed column, with the numbering of the trays starting at the bottom. Each tray of the column is modelled by two states, representing the liquid phase composition and the tray temperature. The vapour phase holdup is assumed insignificant compared to that of the liquid phase.

For this model, the following assumptions are also made.

1. The liquid phase is well mixed.
2. The vapour phase is uniformly mixed on a tray.
3. The MEA is non-volatile.
4. The temperatures of the liquid and vapour phases are equal.
5. There is no heat loss from the desorption column.
6. There is no accumulation of mass on each tray.

Given these conditions, the i^{th} tray of the desorption column may be represented by the following equations.

Tray i

Overall mass balance:

$$L_{i+1} + f_{i-1} = L_i + f_i \quad (3.25)$$

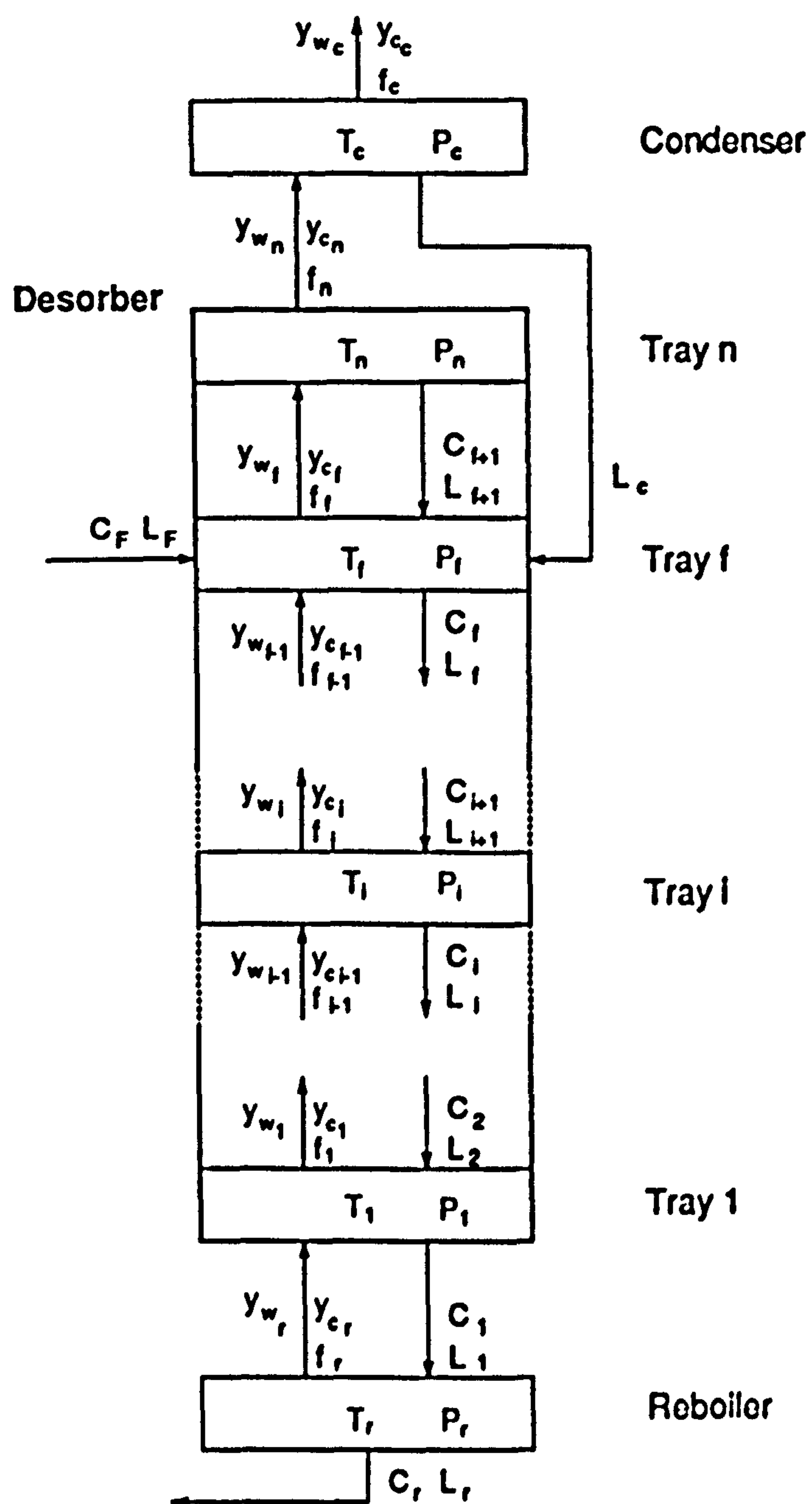


Figure 3.5: Desorber nomenclature.

CO₂ mass balance:

$$(V_i) \frac{dC_i}{dt} = \frac{mw}{\rho_l} (L_{i+1}C_{i+1} - L_iC_i) - (f_i y_{c_i} - f_{i-1} y_{c_{i-1}}) \quad (3.26)$$

Energy balance:

$$\begin{aligned} \left(\frac{V_i C_{pl} \rho_l}{mw} \right) \frac{dT_i}{dt} = & f_{i-1} y_{w_{i-1}} (H_{vap} + C_{pv} T_{i-1}) - f_i y_{w_i} (H_{vap} + C_{pv} T_i) \\ & + f_{i-1} y_{c_{i-1}} (H_{rxn} + C_{pc} T_{i-1}) - f_i y_{c_i} (H_{rxn} + C_{pc} T_i) \\ & + L_{i+1} C_{pl} T_{i+1} - L_i C_{pl} T_i \end{aligned} \quad (3.27)$$

This is true for all trays in the desorption column except for the bottom, top and feed trays. For the bottom tray the subscripts must be changed as follows.

Bottom Tray

Overall mass balance:

$$L_2 + f_r = L_1 + f_1 \quad (3.28)$$

CO₂ mass balance:

$$(V_1) \frac{dC_1}{dt} = \frac{mw}{\rho_l} (L_2 C_2 - L_1 C_1) - (f_1 y_{c_1} - f_r y_{c_r}) \quad (3.29)$$

Energy balance:

$$\begin{aligned} \left(\frac{V_1 C_{pl} \rho_l}{mw} \right) \frac{dT_1}{dt} = & f_r y_{w_r} (H_{vap} + C_{pv} T_r) - f_1 y_{w_1} (H_{vap} + C_{pv} T_1) \\ & + f_r y_{c_r} (H_{rxn} + C_{pc} T_r) - f_1 y_{c_1} (H_{rxn} + C_{pc} T_1) \\ & + L_2 C_{pl} T_2 - L_1 C_{pl} T_1 \end{aligned} \quad (3.30)$$

As there is no liquid flow to the top tray, the description for the top tray is simplified as follows.

Top Tray (*n*)

Overall mass balance:

$$f_{n-1} = L_n + f_n \quad (3.31)$$

CO₂ mass balance:

$$(V_l) \frac{dC_n}{dt} = -\frac{mw}{\rho_l} (L_n C_n) - (f_n y_{c_n} - f_{n-1} y_{c_{n-1}}) \quad (3.32)$$

Energy balance:

$$\begin{aligned} \left(\frac{V_l C_{pl} \rho_l}{mw} \right) \frac{dT_n}{dt} = & f_{n-1} y_{w_{n-1}} (H_{vap} + C_{pv} T_{n-1}) - f_n y_{w_n} (H_{vap} + C_{pv} T_n) \\ & + f_{n-1} y_{c_{n-1}} (H_{resn} + C_{pc} T_{n-1}) - f_n y_{c_n} (H_{resn} + C_{pc} T_n) \\ & - L_n C_{pl} T_n \end{aligned} \quad (3.33)$$

Finally, the description for the feed tray must be augmented to include the feed and reflux streams as follows.

Feed Tray (*f*)

Overall mass balance:

$$L_{f+1} + L_c + L_F + f_{f-1} = L_f + f_f \quad (3.34)$$

CO₂ mass balance:

$$(V_l) \frac{dC_f}{dt} = \frac{mw}{\rho_l} (L_{f+1} C_{f+1} + L_F C_F - L_f C_f) - (f_f y_{c_f} - f_{f-1} y_{c_{f-1}}) \quad (3.35)$$

Energy Balance:

$$\begin{aligned} \left(\frac{V_l C_{pl} \rho_l}{mw} \right) \frac{dT_f}{dt} = & f_{f-1} y_{w_{f-1}} (H_{vap} + C_{pv} T_{f-1}) - f_f y_{w_f} (H_{vap} + C_{pv} T_f) \\ & + f_{f-1} y_{c_{f-1}} (H_{resn} + C_{pc} T_{f-1}) - f_f y_{c_f} (H_{resn} + C_{pc} T_f) \\ & + L_{f+1} C_{pl} T_{f+1} + L_F C_{pl} T_F + L_c C_{pl} T_c - L_f C_{pl} T_f \end{aligned} \quad (3.36)$$

3.3.2 Reboiler

The reboiler is also described by two states, representing the reboiler temperature and liquid phase CO₂ composition. The assumptions for the reboiler model are the same as those listed for the desorber model.

A detailed description of the accumulation of liquid in the reboiler would require a combination of mass, energy balances including pressures. To date this has not been done for the pilot plant. Disli, 1993 introduces an artificial level controller to balance the liquid level in the reboiler. However, for this work, the change in the liquid accumulation in the

reboiler will not be considered. The dynamics of the liquid accumulation are relatively quick and the contribution of changes to the liquid height are difficult to predict from experimental data. Therefore, the flow into the reboiler will equal the flow from the reboiler and the height must be specified separately.

The model equations are as follows.

Overall mass balance:

$$L_1 = L_r + f_r \quad (3.37)$$

CO₂ mass balance:

$$(A_d h_l) \frac{dC_r}{dt} = \frac{mw}{\rho_l} (L_1 C_1 - L_r C_r) - (f_r y_{c,r}) \quad (3.38)$$

Energy balance:

$$\left(\frac{A_d h_l C_{pl} \rho_l}{mw} \right) \frac{dT_r}{dt} = - f_r y_{w,r} (H_{vap} + C_{pv} T_r) - f_r y_{c,r} (H_{rxn} + C_{pc} T_r) + L_1 C_{pl} T_1 - L_r C_{pl} T_r + H_{st} S \quad (3.39)$$

3.3.3 Condenser

The function of the condenser is to lower the partial pressure of the water to limit the loss of MEA from the desorption column. The condensate is collected in the bottom of the condenser and returned via level control to the desorption column. The residence time of the condenser is of the order of a few minutes. Burkett, 1977 considers three different dynamic models for the condenser, all of which ignore the hold up of the gas phase. However, the effect of the condensation return on the system is minor and as a result Disli, 1993 ignores the hold up of both the liquid and vapour phases. This approach will be taken in this work, resulting in a purely steady state description for the condenser.

In addition the following assumptions are made in the condenser model.

1. The liquid and vapour phases are at the same temperature.
2. The condensate contains no CO₂.
3. The liquid and vapour streams contain no MEA.

Based on these assumptions, the following overall and CO₂ mass balances describe the condenser.

$$L_c = f_n - f_c \quad (3.40)$$

$$f_c y_{c,c} = f_n y_{c,d} \quad (3.41)$$

3.3.4 Vapour Liquid Equilibrium

The vapour in the desorption section is assumed to be made up of CO_2 and H_2O . By definition, the sum of the CO_2 and H_2O vapour molar fractions on each stage of the desorption column, in the reboiler and in the condenser must be 1.

$$1.0 = y_{c,c} + y_{w,c} \quad (3.42)$$

$$1.0 = y_{c,i} + y_{w,i} \quad \forall i = 1, \dots, n_d \quad (3.43)$$

$$1.0 = y_{c,r} + y_{w,r} \quad (3.44)$$

Water

The water vapour is assumed to be in equilibrium with the liquid phase. The Antoine equation is used to determine the water vapour pressure at a given temperature. This is combined with Raoult's law to determine the partial pressure of the water vapour at the conditions given in the desorption section.

Raoult's law:

$$y_w P = x_w P^{sat} \quad (3.45)$$

where x_w is the mole fraction of water in the liquid phase.

This is assumed to be a correct representation for all of the unit operations in the desorption section.

$$y_{w,c} P_c = x_{w,c} \exp \left(A_{ant} - \frac{B_{ant}}{C_{ant} + T_c} \right) \quad (3.46)$$

$$y_{w,i} P_i = x_{w,i} \exp \left(A_{ant} - \frac{B_{ant}}{C_{ant} + T_i} \right) \quad (3.47)$$

$$y_{w,r} P_r = x_{w,r} \exp \left(A_{ant} - \frac{B_{ant}}{C_{ant} + T_r} \right) \quad (3.48)$$

Carbon Dioxide

The partial pressure of CO₂ in the vapour is dependent on physical equilibrium, chemical reaction equilibrium and mass transfer resistance.

There are no data available in the literature on the equilibrium between CO₂, carbamate, bicarbonate and MEA under desorption conditions. Equilibrium partial pressure data for CO₂ over an aqueous MEA solution have been found in three sources: Albrecht, 1977, Weiland *et al.*, 1982 and Burkett, 1977.

Burkett, 1977 fits the experimental data from two sources in the literature with the following equation.

$$p_c^{eq} 760.0 = \exp [(-47.91 + 112.8\alpha) + (0.7615 - 1.668\alpha)(T) + (-0.0029 + 0.0070\alpha)(T)^2] \quad (3.49)$$

The experimental data are taken over a temperature range of 80 °C to 120 °C and carbonation ratios up to 0.5.

Albrecht, 1977 fits the experimental data from what appears to be one relevant source in the literature with the following equation.

$$\ln p_c^{eq} = \frac{-13030.42146}{T + 273.0} + 2 \ln \left(\frac{\alpha}{1 - 2\alpha} \right) + 32.97816 \quad (3.50)$$

Albrecht, 1977 states that this equation is valid for a range of carbonation ratios from 0.13 to 0.35. No constraints are given on the temperatures, though the only relevant data supplied is at 95 °C.

Weiland *et al.*, 1982 fit the experimental data of eight sources with the following equation.

$$\ln p_c^{eq} = -3.70 - 4.83\alpha + 2 \ln \left(\frac{\alpha}{1 - 2\alpha} \right) + (-9.71 - 8.18\alpha + 25.2\alpha^2) \left(\frac{1000.0}{T + 273.0} - 3.0 \right) \quad (3.51)$$

A comparison of these curves, coupled with the water equilibrium data, at a pressure of 1.2 atm, is given in Figure 3.6.

From this figure a significant difference between the data of Albrecht, 1977 and the data of both Weiland *et al.*, 1982 and Burkett, 1977 can be seen. The data of Albrecht, 1977 is based on a very limited number of experimental results. Therefore, these data are rejected and the problem becomes one of choosing between Weiland *et al.*, 1982 and

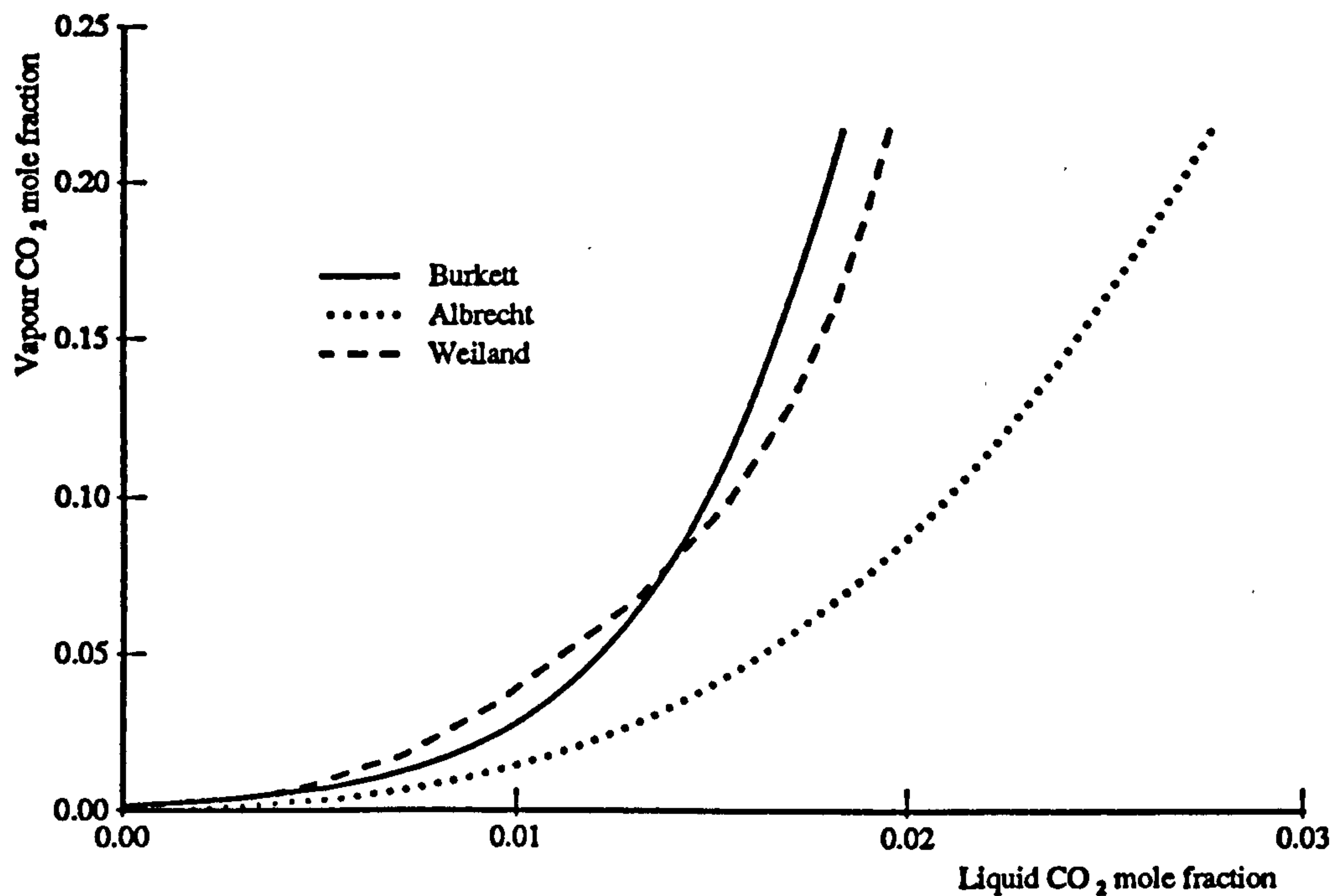


Figure 3.6: Vapour liquid equilibrium data at 1.2 atm of Burkett, 1977, Albrecht, 1977 and Weiland *et al.*, 1982 (Data of Burkett, 1977 used for modelling).

Burkett, 1977. To remain consistent with Disli, 1993 and because the work of Burkett, 1977 was done on the same apparatus as the work for this thesis, the equilibrium relationship of Burkett, 1977 is used.

The equilibrium data represents the conditions at the interface of the vapour and liquid phases. To represent the bulk properties in the desorption section the effects of mass transfer resistance and chemical reaction must also be accounted for. The lack of kinetic data makes it difficult to determine which model should be used for mass transfer and whether or not the desorption reaction is in the fast or instantaneous reaction regimes. In fact, Weiland *et al.*, 1982 state that the reaction regime changes over the length of the column.

Weiland *et al.*, 1982 consider a film model enhanced by a reversible, instantaneous chemical reaction. The resulting model is related to physical parameters and is intended for use as an *a priori* prediction for column design.

Burkett, 1977 considers a surface renewal model enhanced by a reversible, instantaneous chemical reaction. The resulting model represents the effects of mass transfer and chemical reaction in a Murphree efficiency term for the equilibrium tray model. The stage efficiency may be related to physical parameters and operating conditions. However,

Burkett, 1977 suggests a simplified approach based on assuming a single stage efficiency for the whole column.

The specification of the desorption section model is completed by the following equations relating the bulk vapour CO₂ compositions to the equilibrium compositions.

$$\begin{aligned} y_{c,i} P_i &= E_{s,i} y_{c,i}^* P_i + (1 - E_{s,i}) y_{c,i-1} P_{i-1} \quad \forall i = 2, \dots, n \\ y_{c,1} P_1 &= E_{s,1} y_{c,1}^* P_1 + (1 - E_{s,1}) y_{c,r} P_r \end{aligned} \quad (3.52)$$

The reboiler is assumed to be an equilibrium stage and it follows that,

$$y_{c,r} = y_{c,r}^* \quad (3.53)$$

3.3.5 Vapour Phase Tray Hydraulics

To complete the specification of the desorption column and reboiler, a representation of the vapour phase hydraulics is given. Due to a fairly significant variation in the vapour flowrate throughout the column, the assumption of constant vapour flowrate is not considered valid. The representation of the vapour phase hydraulics has been taken from Sinnott, 1983. In this representation, the pressure drop across the tray is made up of three factors: the pressure drop of vapour through the dry orifices, the pressure drop due to the head of water and a residual pressure loss due to the formation of bubbles and the reduction in density of liquid due to the formation of froth.

The residual pressure drop is relatively small and will not be considered further. The resulting equation for the pressure drop across the tray is,

$$\Delta P = \rho_l g_{cont} (h_w + h_{ow}) + K_{DH} \rho_v v_n^2 \quad (3.54)$$

From design data the weir height, h_w , and liquid height above the weir, h_{ow} , were found to be 38.4 mm and 12.5 mm. Based on the equation for flow through an orifice Disli, 1993 uses a dry head constant of 7.116×10^{-6} . Based on these values Equation 3.54 becomes,

$$\Delta P = 4.929 \times 10^{-3} + 7.116 \times 10^{-6} \rho_v v_n^2 \quad (3.55)$$

Assuming the vapour to be an ideal gas, Disli, 1993 computes a vapour density of 0.77 kg/m^3 . Further, from design data the vapour flow area per plate was found to be

0.00203 m^2 . Based on all of these assumptions, the pressure drop can be directly related to the vapour flowrate by,

$$\Delta P = 4.929 \cdot 10^{-3} + 7.116 \cdot 10^{-6} \cdot 0.77 (1.1515 \cdot 10^4 f_v)^2 \quad (3.56)$$

3.4 Inter Column Heat Exchangers

To complete the structure of the model of the full experimental system requires descriptions for the inter column heat exchanger and the cooler. The operations of these two units are assumed to be steady state. The outlet compositions and flowrates are set equal to the inlet conditions.

$$B = L_r \quad (3.57)$$

$$C_{a_1} = C_r \quad (3.58)$$

$$L_f = B \quad (3.59)$$

$$C_f = C_{a_1} \quad (3.60)$$

The desorber side outlet temperature of the inter column heat exchanger is set constant as part of the problem specification. As energy balances are not considered in the absorber model, temperatures around the cooler are not modelled.

3.5 Simulation Technique

The complete model is set up within the Speedup environment. This allows solutions for steady state and dynamic problems. The most recent version of Speedup (Ver. 5.4) provides parameter estimation routines for both steady state and dynamic parameter estimation. In addition there is an external data interface (EDI) allowing the connection between Speedup and the pilot plant control system.

A complete listing of the Speedup model is given in Appendix B. A more detailed description of Speedup may be found in Prosys Technology, 1989.

3.6 Experimental Details

All experiments were performed on the CO_2 absorption/desorption pilot plant, located in the Department for Chemical Engineering and Chemical Technology at Imperial College. The system is supervised by a personal computer based control system, Paragon,

running on NAGA 486 (33 MHz) personal computers. This system provides the data collection facilities, supervisory control routines and operator interfaces. More details regarding Paragon are found in Intec, 1991.

The beginning of each experiment involved starting up the pilot plant and waiting for steady state operation. At the completion of each run the plant was shut down to a safe state. Details regarding the start up and shut down procedures are given in Appendix G.

A detailed schematic of the pilot plant, including the position of measurement devices and automatic control valves are given in Figure 3.7. The following sections discuss the basic control loops used to regulate the pilot plant along with a description of the measurement devices and accuracy.

3.6.1 Basic Control of Pilot Plant

The pilot plant has two major operating configurations: open loop and closed loop. In open loop operation the CO_2 product stream from the top of the condenser is vented and all of the feed CO_2 is made up from bottled CO_2 . In closed loop operation all of the feed CO_2 is made up from the recycle of CO_2 product from the top of the desorber column. During all experimental runs the plant is run open loop, though during start up and shut down the plant is run closed loop to conserve CO_2 . Attempts to perform experiments with a partial recycle and partial CO_2 make up were abandoned due to large variations in the CO_2 product composition and the significant transport delay between the CO_2 bottle and the composition meter ADC1.

The functions of the basic regulator control loops will be discussed in three sections: specification of inlet conditions, maintaining the mass balance and maintaining the heat balance.

Inlet Conditions

The two most important inlet conditions are the steam flowrate and the feed gas CO_2 composition. The steam flowrate is the manipulated variable for the control structure and the feed gas composition is the "adjustable" disturbance. The other inlet conditions are assumed constant during normal operation.

The steam flowrate is controlled with a PI loop from ADF3 to ADV5. In general this measurement can be maintained within 0.2 g/s on either side of the set point. Sharp changes in the steam supply conditions have been noticed leading to an unacceptable

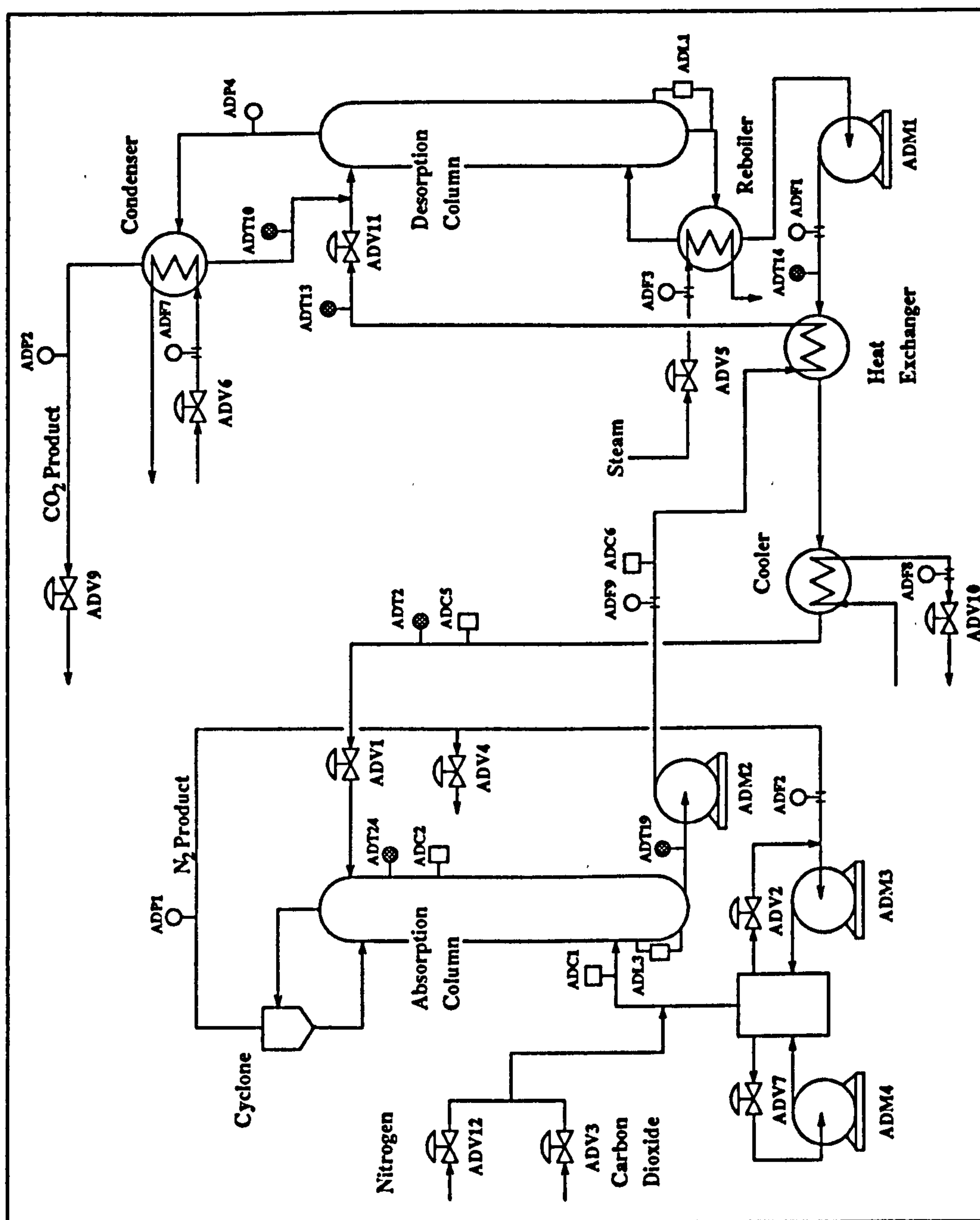


Figure 3.7: Pilot plant schematic.

disturbance in the steam flowrate. The control loop settles to a steady state within 3–4 minutes of a set point change.

The feed CO_2 composition is controlled with a PI loop from ADC1 to ADV3. The composition is susceptible to random fluctuations in the feed gas flowrate (ADF2) and the supply conditions of the CO_2 . The transport delay between the CO_2 bottle and the composition meter, ADC1, is of the order of 1 minute. The control loop is able to maintain the composition within 0.2% CO_2 of the setpoint composition. Maintaining a sample flowrate to ADC1 of approximately 1 L/min is important as large drops in this value can increase the measurement delay and destabilise the control loop.

The gas feed flowrate is controlled with a PI loop from ADF2 to ADV2. The flow measurement from ADF2 is very susceptible to the build up of condensation in the sample lines. As a result, the control loop is closed for only short periods after checking the condition of the flow measurement. At other times the valve position is maintained at the last closed loop position. This measurement was found to fluctuate by up to 1 g/s from the setpoint.

The circulation flowrate is controlled with a PI loop from ADF1 to ADV1. Under normal conditions very little fluctuation in this measurement is seen.

The remaining input to the system is a makeup of N_2 . The position of ADV12 is set constant to maintain a minimal amount of makeup.

Mass Balance

To complete the mass balance for the system requires the specification of an exit flowrate for the CO_2 and N_2 streams.

The removal of CO_2 from the system is achieved by maintaining a particular pressure in the desorption section. This is controlled with a PI loop from ADP2 to ADV9. Few problems are encountered with this control loop during normal operating conditions.

The removal of N_2 from the system is only required to balance the N_2 added as the makeup. This removal is controlled by maintaining a particular pressure in the absorber, with a PI loop from ADP1 to ADV4. Few problems are encountered with this control loop, during normal operating conditions.

The amount of MEA solution in the absorption and desorption sections of the pilot plant is controlled via the level of MEA solution in the bottom of the absorber. This is achieved with a PI loop from ADL3 to ADV11.

Energy Balance

All the energy to the system is supplied through the steam to the reboiler. The heat is removed through the inter column cooler and the condenser at the top of the desorber.

The cooler's exit temperature is controlled with a cascade from ADT2 to a PI loop from ADF8 to ADV10. A number of problems have been experienced with maintaining a constant temperature. However, all of the results presented in this work have the temperature maintained within 0.7 °C of the setpoint.

The exit temperature of the condenser is controlled with a cascade from ADT10 to a PI loop from ADF7 to ADV6. Maintaining a given temperature in the condenser is very difficult due to the design of the condenser. For reasonably low temperatures, the effect of changes in temperature is minor. Therefore, variations in the temperature from 36 °C to as high as 60 °C, with most values around 54 °C, are considered acceptable.

3.6.2 Measurements and Error Analysis

At each steady state point 17 plant measurements are recorded, representing the specified inputs, system states and other operating conditions. These measurements will be discussed in groups representing the quantities measured.

Flow Measurement

The three flowrates recorded at each steady point correspond to the steam flowrate (ADF3), the recirculation flowrate (ADF9) and the gas feed flowrate (ADF2). The steam flowrate is also recorded for each of the dynamic runs. Each of these flows is measured with an orifice plate and a differential pressure transmitter. The measurement error for each of these values is associated with calibration errors. A summary of these measurements and their errors is given in Table 3.1

Device	Range	Units	Absolute Error ($\pm$)	Notes
ADF2	0 - 63.6	<i>g/s</i>	1	Calibration by Disli, 1993
ADF3	0 - 40.0	<i>g/s</i>	1.5	Calibrated by timed collection
ADF9	0 - 0.50	<i>g/s</i>	0.02	Checked against ADF1

Table 3.1: Flow measurements and errors.

Temperature Measurement

The six temperatures recorded at each steady state correspond to the temperature of the MEA solution entering the absorber (ADT2), the temperature of the condensate from the condenser (ADT10), the temperature of the MEA solution entering the desorber (ADT13), the temperature of the reboiler (ADT14), the temperature at the bottom of the absorber (ADT19) and the temperature at the top of the absorber (ADT24). Each of these measurements is made with electrical resistance thermocouples. These devices measure the temperature very precisely. However, each device was only calibrated to read to within an error of 0.5 °C . A summary of these measurements is given in Table 3.2.

Device	Operating Values	Units	Absolute Error ($\pm$)
ADT2	44.5 - 45.5	°C	0.4
ADT10	35.5 - 62.0	°C	0.4
ADT13	92.0 - 95.0	°C	0.4
ADT14	104 - 108	°C	0.4
ADT19	46.0 - 48.0	°C	0.4
ADT24	43.0 - 45.0	°C	0.4

Table 3.2: Temperature measurements and errors.

Pressure Measurement

The pressure measurements recorded at each steady state correspond to the absorber pressure (ADP1) and the desorber pressure (ADP4). Both of these are measured with differential pressure transmitters. These measurements are required only as a rough check of operating conditions and any measurement error is considered minor.

Level Measurement

The level measurements recorded at each steady state correspond to the liquid level in the bottom of the absorber (ADL3) and the level in the reboiler (ADL1). The levels are measured with a differential pressure transmitters. They were calibrated at the beginning of all the experiments to within 0.03 m using sight glass measurements.

Composition Measurement

Four compositions were measured at each steady state and during each dynamic run. These corresponded to the inlet gas composition (ADC1), outlet gas composition (ADC2), clean MEA liquid pH (ADC5) and the loaded MEA liquid pH (ADC6).

The inlet gas composition is measured with an infra-red gas analyser. No indication of the accuracy of the instrument is given, and the calibration error is less than 1 % of full scale deflection. As the random variation in the measurement is about 3 % of full scale deflection, instrument and calibration errors will be considered insignificant and the measurement error will be taken to be 3 % of full scale deflection (fsd. = 10.0 % CO₂).

The exit gas composition is measured in a separate infra-red gas analyser. Despite calibration being accurate to within about 1 % of full scale deflection, calibration curves supplied with the instrument suggest a maximum error of 8 % of full scale deflection on the range used for these experiments. Therefore the measurement error will be taken to be 8 % of full scale deflection (fsd. = 1.0 % CO₂).

Both of the liquid compositions are inferred from measurements of the pH. With careful calibration of these instruments, an accuracy of about 0.05 pH may be achievable. However, these instruments are susceptible to measurement drift and the effect of changes in sample temperature (even with temperature compensation). As the pH meters are not recalibrated daily, a more realistic measurement accuracy for the pH meters would be 0.15 pH (cf. McMillan, 1991). The pH measurements are converted to carbonation ratios through a correlation from Disli, 1993. The relationship, valid for carbonation ratios in the range 0.1 to 0.4, is,

$$\alpha = -0.28315\text{pH} + 3.0926 \quad (3.61)$$

This equation fits the data of Disli, 1993 and Vanichseni, 1977 to an accuracy of 0.01 (carbonation ratio). Therefore, the accuracy of carbonation ratios determined from pH measurements will be 0.04.

These measurements are summarised in Table 3.3.

Device	Operating Values	Units	Absolute Error ($\pm$)
ADC1	0.0 - 10.0	% CO ₂	0.3
ADC2	0.0 - 1000.0	ppm	8
ADC5	0.0 - 14.0	pH	0.15
ADC6	0.0 - 14.0	pH	0.15
α 5	0.1 - 0.4	-	0.04
α 6	0.1 - 0.4	-	0.04

Table 3.3: Composition measurements and errors.

3.7 Model Parameters

Sections 3.1 to 3.4 set out a mathematical description of the complete pilot plant. For simulation, a number of physical constants and plant parameters must be set. Most of these are set with a reasonable degree of confidence from data available in the literature and from physical reasoning based on the dimensions of the plant. Three parameters are set through steady state parameter estimation of the experimental data: the absorption coefficient in the absorption column, a coefficient for the pressure drop across each stage in the desorber and the stage efficiency in the desorption column.

The source of all the plant parameters and the estimation procedure are discussed in the following sections, beginning with a discussion on the choice of the number of trays for the absorber and desorber model and the choice of vapour, liquid equilibrium data for the desorber.

3.7.1 Number of Trays

To define the structure of the absorption column and desorption column models requires the specification of the number of trays used to represent each.

As the absorber is a packed column, modelled without equilibrium stages, the number of stages is chosen to provide a good fit of the experimental data. The experience of Disli, 1993 with operation of the same plant suggested that a minimum of 3 equilibrium stages could be used for the absorption column. To minimise the number of differential equations 3 equilibrium stages are used for this work, with deviations between experimental results and the model accounted for in the absorption parameter. Experience with the experimental data from the desorber showed that a minimum of three stages could be used.

3.7.2 Fixed Parameters

Water Vapour - Antoine Equation and Raoult's Law

The Antoine equation parameters for the water vapour in the desorption column, reboiler and condenser, were obtained from DOW, 1962. The water mole fraction in the liquid phase, required by Raoult's law, is calculated on a CO₂ free basis. Therefore, the mole fraction throughout the desorption column and reboiler is calculated to be 0.9513 and the mole fraction in the condenser is 1.0.

These values are summarised in Table 3.4

Parameter	Value	Notes
A_{ant}	18.379	DOW, 1962
B_{ant}	3880.1	DOW, 1962
C_{ant}	230.0	DOW, 1962
x_{w_e}	1.0	No MEA in condenser
x_{w_i}	0.9513	$\forall i = 1, \dots, n_{desorber}$
x_{w_r}	0.9513	CO ₂ free basis

Table 3.4: Antoine equation parameters.

Physical Properties

The values for the physical properties of H₂O, CO₂ and MEA are summarised in Table 3.5.

Parameter	Value	Units	Description	Source
mw	20.13	$\frac{kg}{kmol}$	Mol. Wgt. of MEA	Periodic Table
ρ_l	1000.0	$\frac{kg}{m^3}$	Density of H ₂ O liquid	-
ρ_g	0.0752	$\frac{kg}{m^3}$	Density of gas in absorber	Ideal gas law
H_{st}	45400.0	$\frac{kJ}{kmol}$	Heat Vap. of H ₂ O	Perry and Green, 1984
H_{vap}	45400.0	$\frac{kJ}{kmol}$	Heat Vap. of H ₂ O	Perry and Green, 1984
H_{rxn}	73400.0	$\frac{kJ}{kmol}$	Heat of Reaction	Kohl, 1985
C_{p_e}	47.09	$\frac{kJ}{kmolK}$	Heat Capacity of CO ₂	Perry and Green, 1984
C_{p_v}	35.4	$\frac{kJ}{kmolK}$	Heat Capacity of H ₂ O vapour	Perry and Green, 1984
C_{p_l}	79.2	$\frac{kJ}{kmolK}$	Heat Capacity of H ₂ O liquid	Perry and Green, 1984

Table 3.5: Physical property data.

Design Data

A number of parameters are obtained from the physical dimensions of the system. These are summarised in Table 3.6.

Parameter	Value	Units	Description
V_l	0.0334	m^3	Cell liquid volume in desorber
V_{l_e}	0.01884	m^3	Cell liquid volume in absorber
V_{g_e}	0.01413	m^3	Cell gas volume in absorber
A_d	0.1995	m^2	Cross sectional area of absorber
h_t	2.5	m	Cell height in absorber

Table 3.6: Desorber physical parameters.

Operating Conditions

A number of parameters used in the model are taken from conditions used in the experimental system. These are summarised in Table 3.7.

Parameter	Value	Units	Description
C_{mea}	2.5	-	MEA molar concentration
h_l	0.31	m	Height of liquid in reboiler
T_c	54.3	°C	Condenser temperature
$T_{l,d}$	94.0	°C	Temp. liquid to desorber
P_c	1.06	bar	Pressure of condenser
P_a	1.89	bar	Pressure of absorber
$F_{g,a}$	0.0111	$\frac{kmol}{s}$	Gas flow to absorber
B	0.0148	$\frac{kmol}{s}$	Recirculation flowrate

Table 3.7: Nominal experimental operating conditions.

The remaining two operating parameters are the gas feed CO_2 composition ($y_{c,a}$) and the steam mass flowrate (S_{mass}). These parameters are varied on each simulation run.

3.7.3 Steady State Parameter Estimation

Three parameters are set using the nonlinear, least squares, steady state parameter estimation routines of Speedup. The parameters set are the absorption coefficient in the absorption column (K), a coefficient for the pressure drop across each stage in the desorber (plt) and the stage efficiency in the desorption column (E_{st}). Experimental data corresponding to the CO_2 gas composition leaving the absorber ($y_{a,s}$), the reboiler CO_2 liquid composition (C_r) and the reboiler temperature (T_r) for steady state points at seven different steam flowrates, are used to fit these parameters. The values at each steam flowrate correspond to an average of all results recorded at that particular steam flowrate. A complete listing of the experimental values is given in Appendix C. The weightings used for the different measurements reflect the importance of the measurement and its relative accuracy.

The results of the parameter estimation are given in Table 3.8.

The parameter standard deviations are computed by Speedup from the scatter in the experimental data as compared to the model and are independent of the *a priori* estimates of the measurement standard deviations.

The sum of squared weighted errors is 0.001164. Given that only seven measurement sets are used for the fit and that three different parameters are fitted from seven

Parameter	Estimated Value	Standard Deviation
E_{st}	0.776	0.163
K	0.291×10^{-3}	0.0012×10^{-3}
plt	14.32	1.62

Table 3.8: Parameter estimation results.

sets of three measurements, it is difficult to find a meaningful value for χ^2 for comparison. However, this value is very small suggesting that a significant fit has been obtained.

Plots of all the experimental points and the simulation results given the estimated parameters are given in Figures 3.8 to 3.10. The most important measurement, absorber CO_2 gas composition, is well fitted with the estimated parameters, with the simulated values falling within the scatter of the experimental results. The reboiler liquid composition is also well fitted, again with the simulation results falling within the scatter of the experimental results. The simulated reboiler temperatures do not fit the experimental data well at higher steam flowrates. This is partly due to the low weighting placed on the reboiler temperature measurements and partly due to the large scatter in the experimental results.

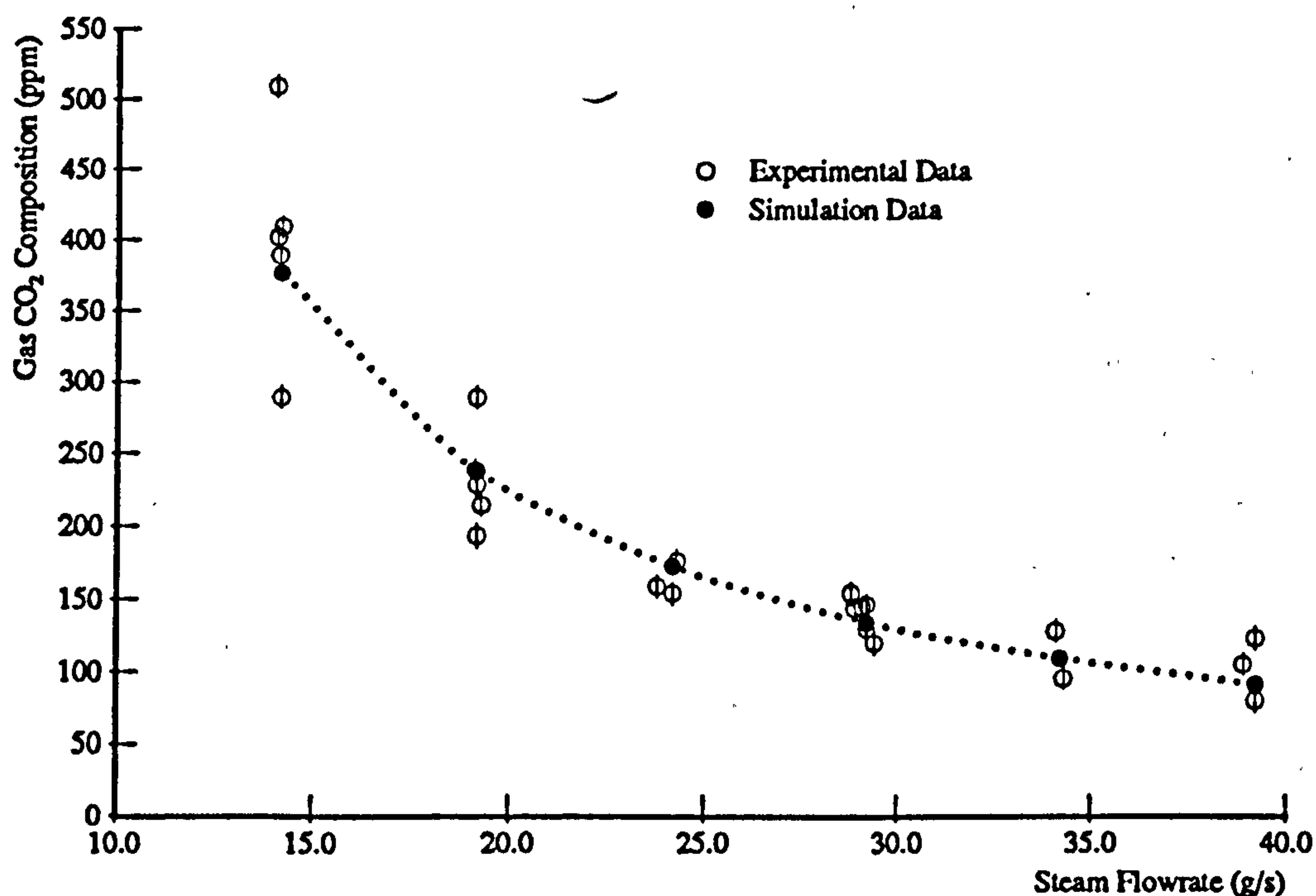


Figure 3.8: Comparison of experimental and simulation steady state results - gas composition (ppm CO_2).

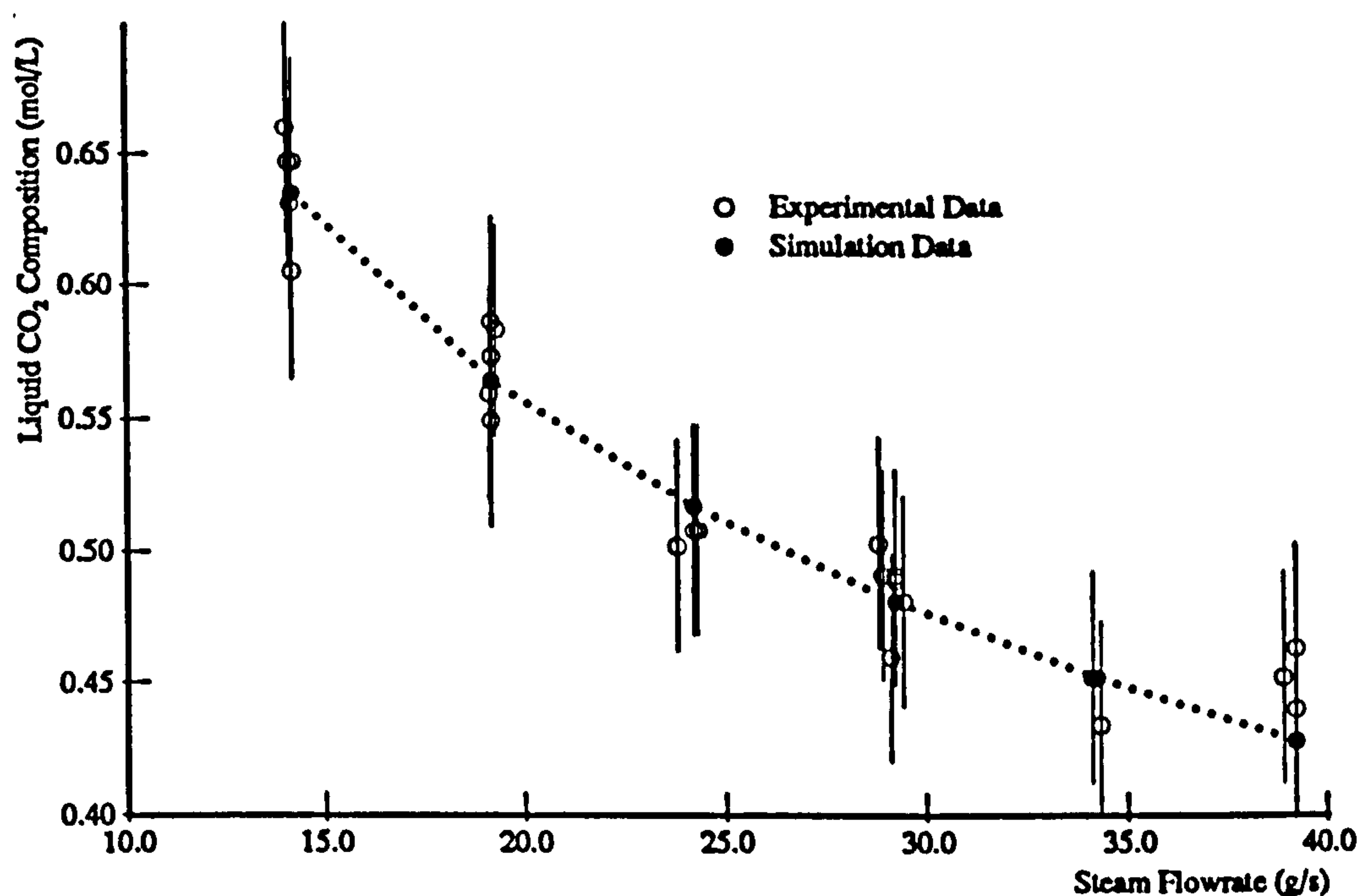


Figure 3.9: Comparison of experimental and simulation steady state results - reboiler CO₂ composition.

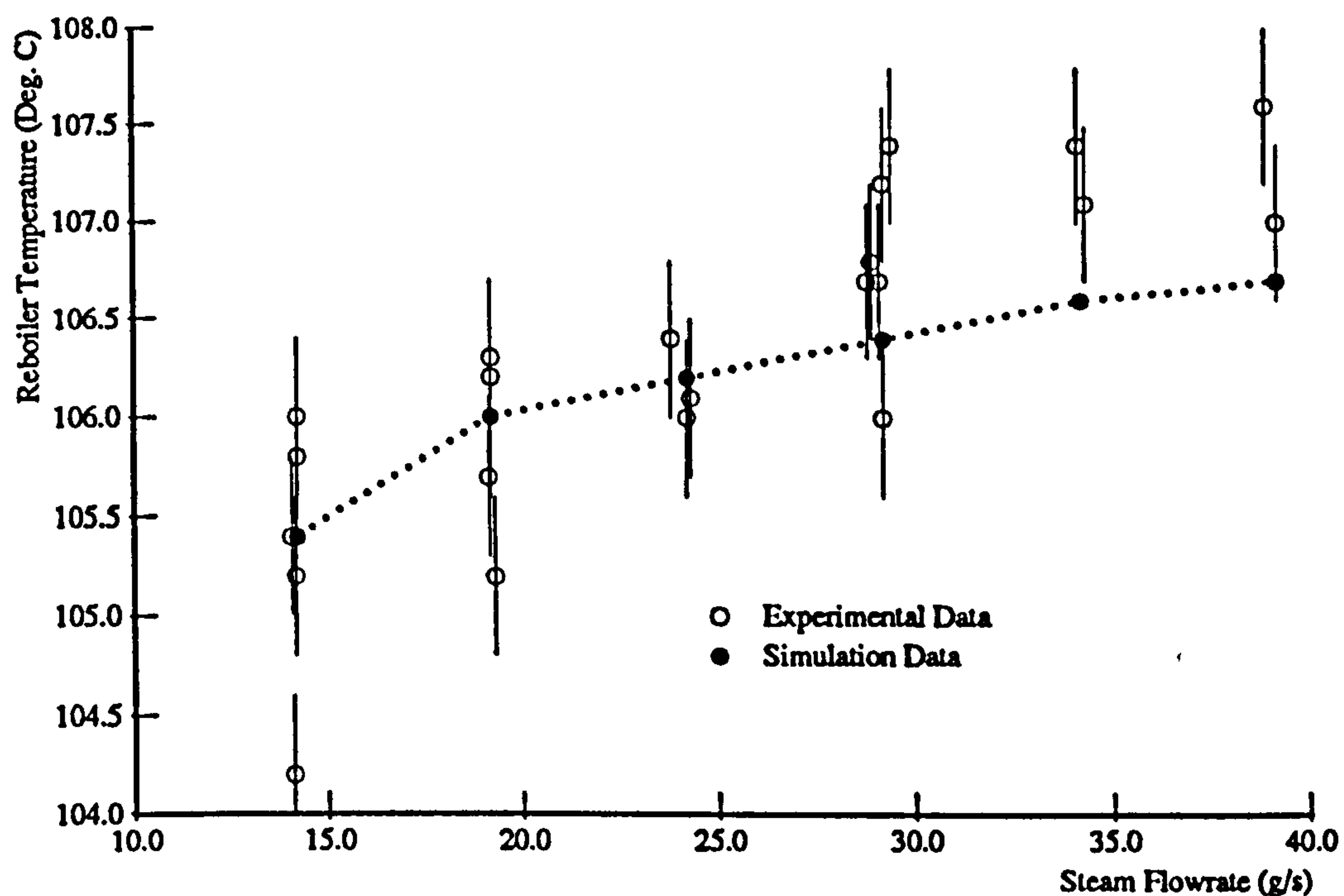


Figure 3.10: Comparison of experimental and simulation steady state results - reboiler temperature.

In terms of the model based controller, the gas compositions are the most important measurement to be fitted. So the parameters listed in Table 3.8 are taken as an adequate fit of the steady state experimental data.

3.7.4 Open Loop Dynamics

It is also necessary to check the dynamic response of the model against the results of open loop dynamic experiments.

The responses of the gas composition to a number of different step changes in the steam flowrate have been recorded. As a check of the dynamic response of the model these results are compared to simulation results, using the parameters discussed in the previous sections. Minor differences in the dynamic response may be accounted for by errors in the design parameters (Table 3.6).

Both the simulation and experimental responses are given in Figures 3.11, 3.12 and 3.13. From these responses it can be seen that the main dynamic characteristics of the system are captured in the model. Some error in the predicted steady state positions is expected due to experimental scatter (reflected in the experimental steady state results). The experimental results have a significant dead time which is not accounted for in the model. This dead time is due to a number of factors: lag in obtaining a new steam flowrate, transport delay in the circulation system and measurement delay at ADC2. Given that a purely differential algebraic system must be used (due to the limitations of the geometric nonlinear control technique), and that a small order model is required transport delays will not be modelled.

3.8 Summary

The chapter has described a mathematical model of the CO₂ absorption/ desorption system. This system has been chosen as the case study for this work as representative of the type of systems found in the process industries.

A summary of the experimental system is given, including details of the supervisory control structure and plant measurements. The system model is completed through the specification of the model parameters. Three of these parameters were fitted using steady state experimental data.

The final result is a full mathematical description of the system, which is used to design a geometric nonlinear controller in the next chapter.

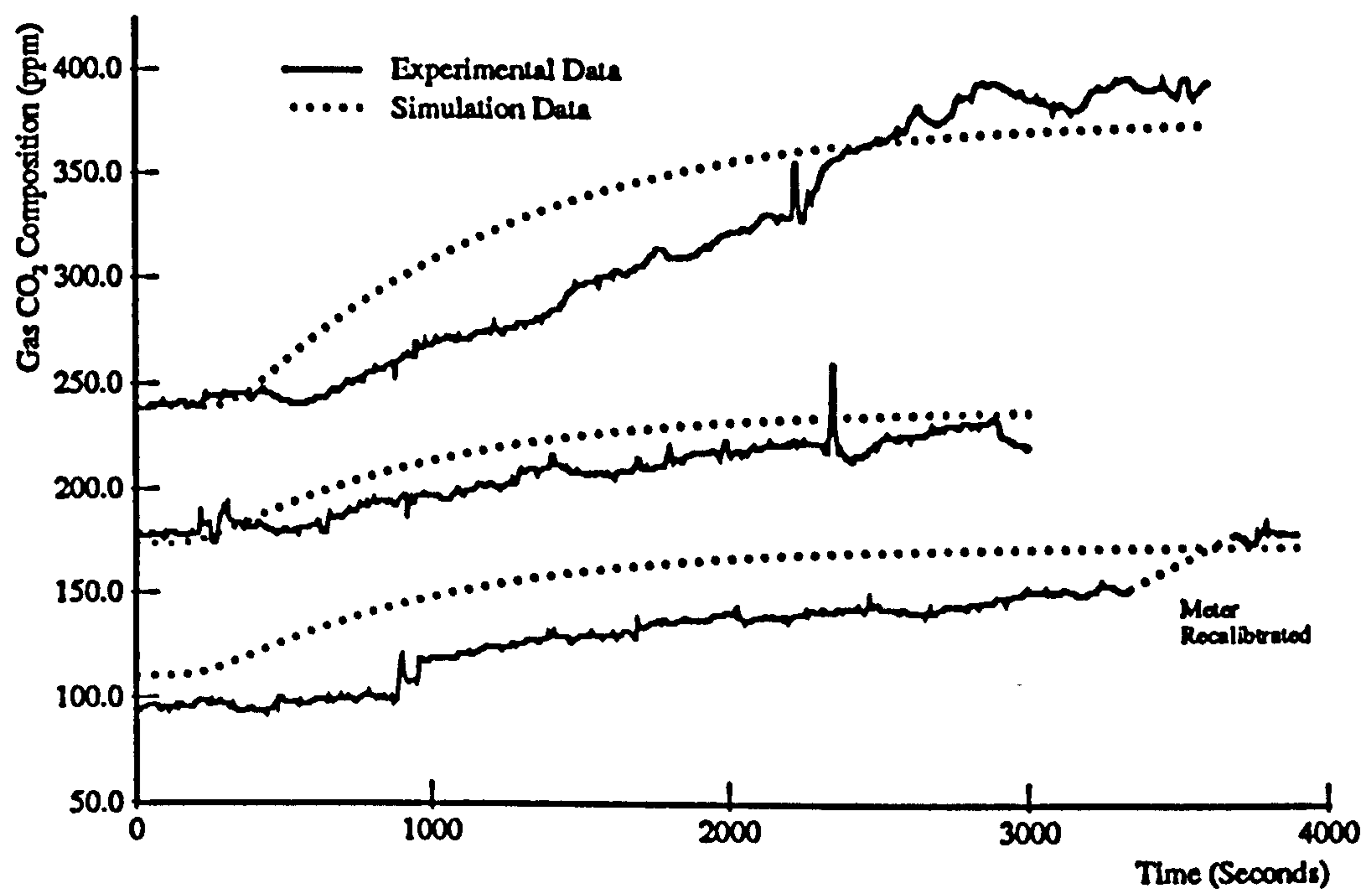


Figure 3.11: Comparison of experimental and simulated dynamic responses of the gas CO₂ composition, to step changes in the steam flowrate (part 1).

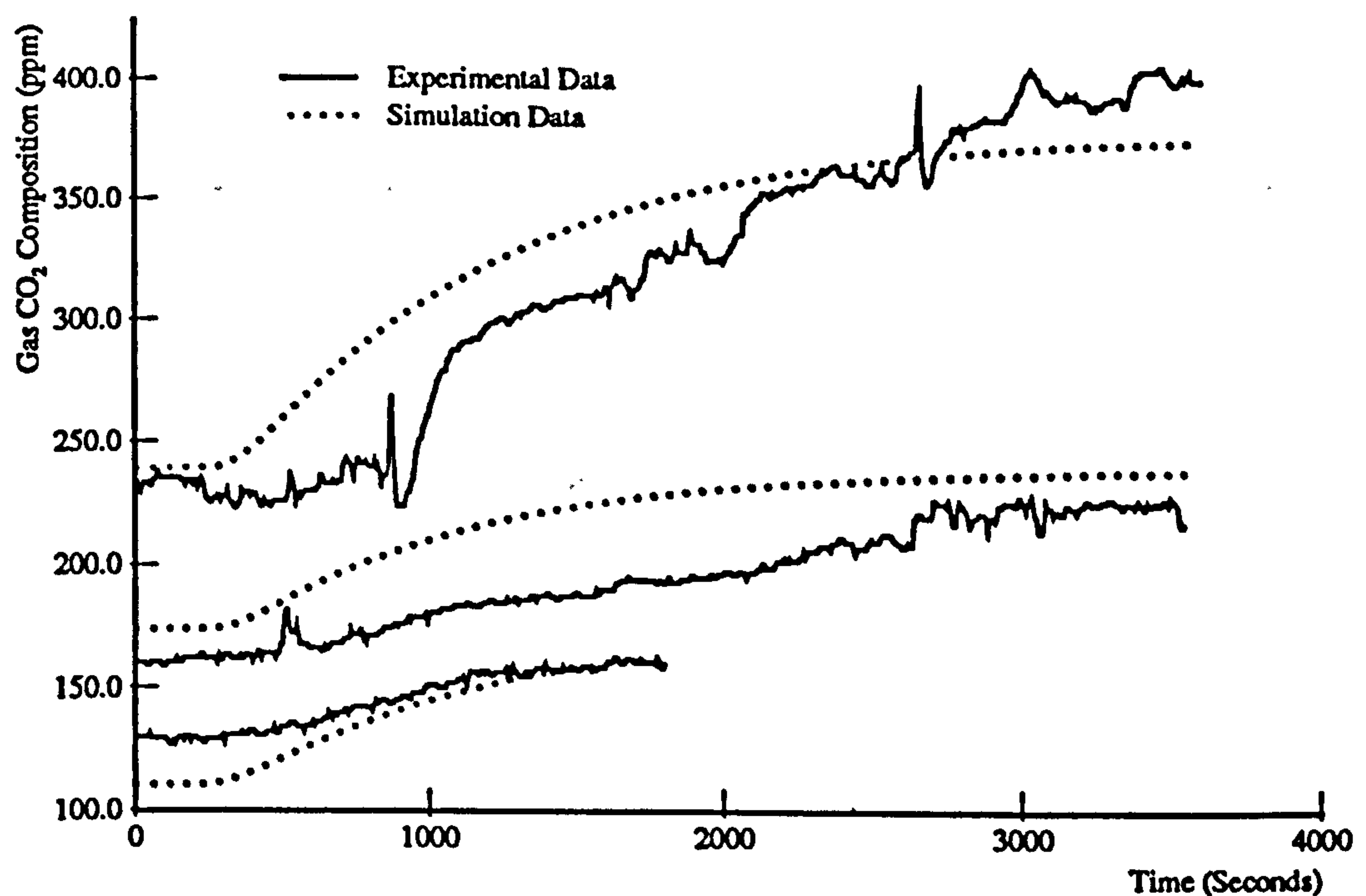


Figure 3.12: Comparison of experimental and simulated dynamic responses of the gas CO₂ composition, to step changes in the steam flowrate (part 2).

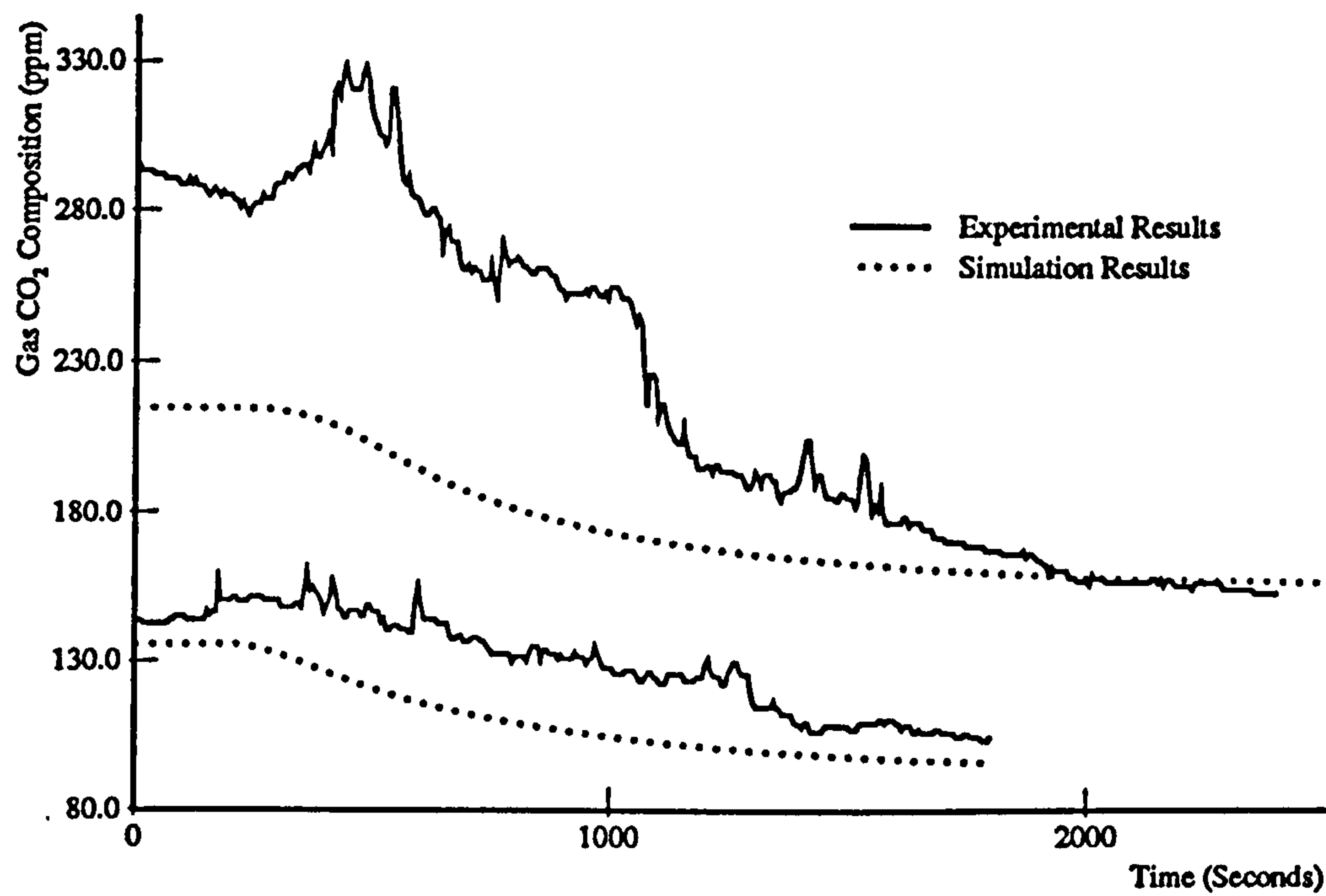


Figure 3.13: Comparison of experimental and simulated dynamic responses of the gas CO₂ composition, to step changes in the steam flowrate (part 3).

Chapter 4

Controller Design

This chapter considers the design of a geometric nonlinear controller for a chemical engineering system. The design begins with a state space description of the system. This description is used to obtain the feedback law required for a realisation of the inverse. This feedback law is very complex which raises two questions. Is such a complicated answer required and can the size of the controller be reduced?

An explanation for the size of the controller is given in Section 4.3. This is followed by a logical approach to reducing the size of the controller through model reduction. The approach uses graph theory to identify key states which have the greatest contribution to the complexity of the controller. The approximation of the dynamics of less significant states to pseudo steady state is justified on the basis of a singular perturbation argument. This results in differential equations being replaced by algebraic equations and associated algebraic variables. In general it will not be possible to substitute these new algebraic equations into the remaining differential equations, resulting in a differential and algebraic (DAE) description of the system. Previous work on geometric nonlinear control has been limited to ordinary differential descriptions of the system. However, Section 4.4 presents an extension to the basic geometric nonlinear control technique which allows the use of differential and algebraic equations in the system description. This extension allows geometric nonlinear control to be applied to a much broader class of mathematical descriptions.

Finally, the reduced model of the case study is used to produce a greatly reduced controller.

4.1 System Description

The basic geometric nonlinear control design requires a state space description of the process of the following form,

$$\begin{aligned}\dot{x} &= f(x) + g(x)u \\ y &= h(x)\end{aligned}\tag{4.1}$$

In other words, a set of differential equations in control affine form is required. This basic case has been extended to include systems which are not control affine by Henson and Seborg, 1990a, allowing systems of the following form,

$$\begin{aligned}\dot{x} &= f(x, u) \\ y &= h(x)\end{aligned}\tag{4.2}$$

The work of Daoutidis and Kravaris, 1989 has allowed the decoupling of disturbances when the system description includes measured disturbances. For this situation a system description of the following form must be available,

$$\begin{aligned}\dot{x} &= f(x, u) + w(x)d \\ y &= h(x)\end{aligned}\tag{4.3}$$

Work presented in Section 4.4.1 of this chapter demonstrates that a differential algebraic description which is not control affine may be used.

$$\begin{aligned}\dot{x} &= f(x, a, u) \\ 0 &= f(x, a, u) \\ y &= h(x, a)\end{aligned}\tag{4.4}$$

With this extension to the theory the designer may now use a generic description which covers a large class of systems.

Geometric nonlinear control directly uses the system model in the design of the controller. This naturally leads to the problem of deciding how complex the description of the system should be. There is a significant argument for using a very simple model to,

1. minimise the effort of producing and maintaining a complicated, nonlinear model,
2. minimise the number of states which must be accurately represented in the model and accurately measured on-line,
3. minimise the complexity of the controller.

On the other hand, a more complicated model would provide more information about the plant. Therefore, the problem becomes one of minimising the complexity, while

providing sufficient information for the controller. To decide how much information is sufficient it is necessary to look at what a geometric nonlinear controller does.

Under the ideal conditions of no controller saturation, no model mismatch and no mathematical irregularities, geometric nonlinear control will force a nonlinear plant to behave linearly with prescribed characteristics. If the gain between the manipulated input and the system output is nonlinear, a geometric nonlinear controller will adjust the amount of control action to ensure that the gain between a new reference input and the output is always constant. If the speed of response of the system is dependent on the operating conditions, the controller will use more or less control action to ensure that the speed of response is constant throughout the operating region. If the plant demonstrates multiplicity the controller will ensure operation is maintained at a given point, independent of the existence of other steady state points. With disturbance decoupling, the effect of measured disturbances may also be removed.

The complexity of the model should reflect the purpose of the final controller. For example, if the main function of the controller is to take account of a large gain nonlinearity, then the model should represent this gain nonlinearity adequately. In this case, consideration could be given to using a steady state, nonlinear model with a number of artificial lags to represent the major dynamics of the response.

For the purpose of this work, a detailed, nonlinear model of the process is available. Therefore, the basic complexity of the model is predetermined. However, as will be shown in the next section, the resulting controller is very large. Because of this, consideration is given to reducing the model complexity with the aim of reducing controller complexity.

4.2 Controller for Full Model

The full model of the case study is detailed in Chapter 3. A Speedup input file of this model is given in Appendix B and a graph of the system is given in Figure 4.1.

From the graph it can be seen that the system is generically of relative order four. As a result, algebraic expressions for the first four derivatives of the output are required. These have been found using Mathematica routines. A summary of the Mathematica routines, input file and usage are given in Appendix D. The results from Mathematica are algebraic expressions for the derivatives in Fortran 77 form. These expressions are too large to warrant inclusion in this thesis, though their sizes, both in terms of the number of lines of Fortran code and the number of ASCII characters, are summarised in Table 4.1.

Expressions of this size can not be reasonably considered for control purposes.

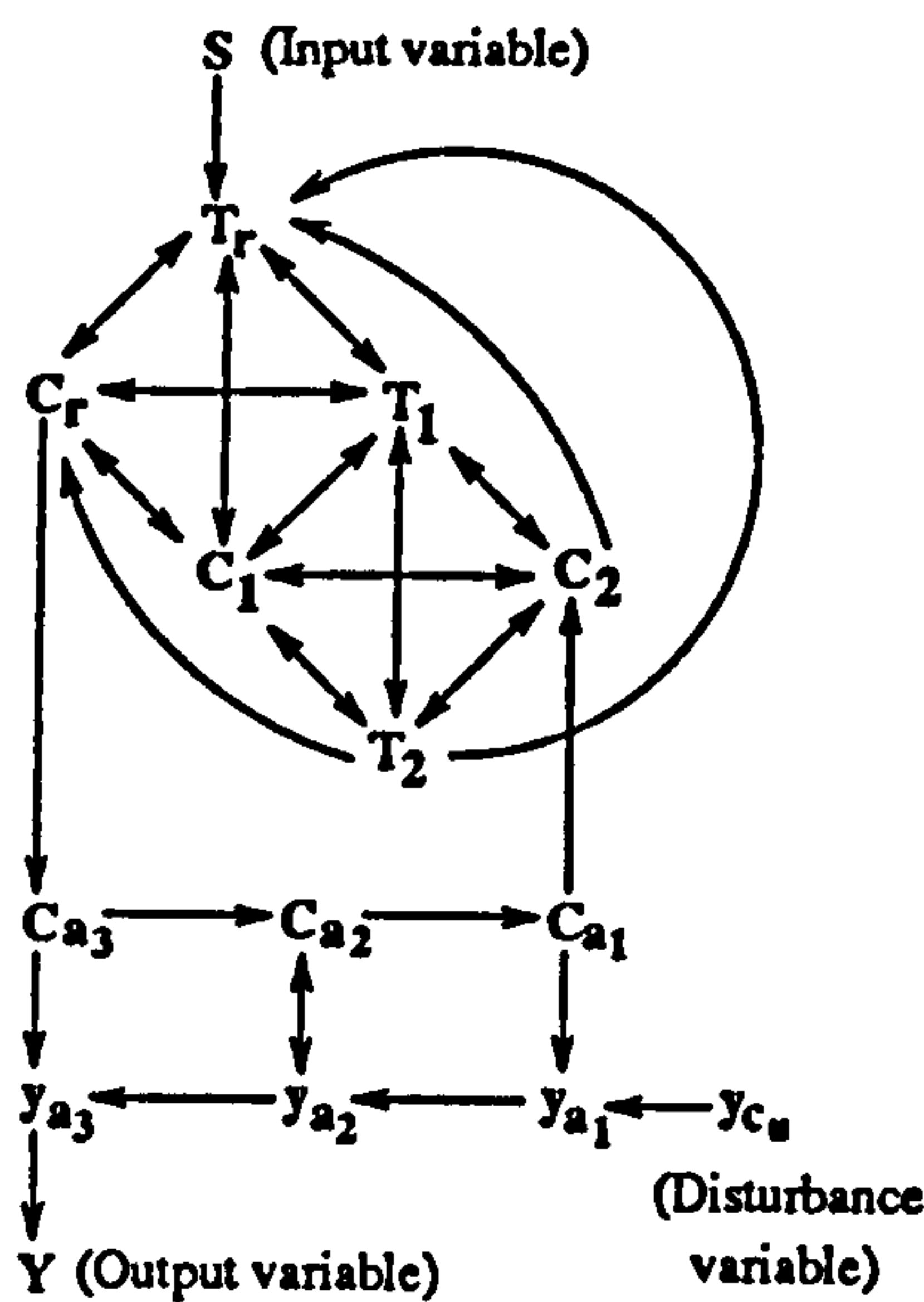


Figure 4.1: System graph for full model.

Derivative	Lie Algebra Notation	Size (Lines)	Size (Characters)
0	h	1	3
1	$L_j h$	2	94
2	$L_j^2 h$	7	435
3	$L_j^3 h$	112	5763
4	$L_j^4 h$	2374	122892

Table 4.1: Controller size for full model.

As previously mentioned, this raises two questions. Why is such a complicated answer required and can the size of the controller be reduced?

4.3 Model Reduction for Relative Order

The geometric nonlinear controller design is based on an algebraic inverse of the dynamic description of the plant. The realisation requires the definition of a state feedback (see Section 2.1.2) based on analytical expressions for all the derivatives of the output from the zeroth order to a maximum order determined by the structure of the model. The complexity of these expressions is dependent on two factors.

1. The complexity of the expressions for each of the required derivatives of the output.
2. The maximum order of derivative required for the model inverse.

Computation of a particular order of derivative of the output involves taking the partial derivative with respect to each of the states, multiplying each of the terms by an expression and then summing all of the terms together (See the definition of the Lie derivative in Appendix A). As a result, higher orders of derivative of the output have progressively larger and more complex analytical expressions. Therefore, the maximum benefit, in terms of reducing the controller complexity, will be obtained by reducing the maximum order of derivative required by the geometric nonlinear controller.

Model reduction without reducing the relative order may derive some benefit through reducing the complexity of the expressions for the required derivatives.

Section 2.4 demonstrates that graph theory can be used to provide a clear representation of the structure of a model. It is shown that the relative order of a system is related to the shortest path between the input and the output nodes of the system graph. Reducing the length of the shortest path will result in a reduction of the relative order of the model. The length of a path in the system graph is characterised by the number of state nodes it passes through. Therefore, reducing the length of the shortest path between the input node and output node is achievable by removing a state node from the shortest path. The following points about the shortest path of the system graph need to be made.

1. Reducing the shortest path on the system graph is not the only method for reducing the relative order. Relying on non generic cancellation of parallel contributions may also reduce the relative order. However, model changes required for such a result are rarely justifiable.

2. Paradoxically, the shortest path can be reduced by increasing the complexity of the model. If new interactions between the states are introduced the relative order may also be reduced.
3. If there are multiple shortest paths through the system graph, the length of each must be reduced to achieve a reduction in the relative order.

The removal of a state node from the system graph is equivalent to removing the dynamics of the corresponding state (ie. assuming that the state is at pseudo steady state).

In summary, the system graph can be used to identify key states, through which the shortest path(s) between the input node and the output node passes. Removal of these nodes will result in a reduction of the relative order of the model and will correspondingly lead to a reduction in the size of the analytical inverse of the system. The justification for removal of a node will be considered in the next section.

4.3.1 Techniques for Model Reduction

In this section four methods for model reduction will be considered. In order of presentation they are: modal approximation, quasi steady state approximation, fast mode perturbations and slow mode perturbations.

Modal Approximation

Modal approximation is a method of model reduction aimed at removing the parts of the model which reach steady state quickly, relative to the rest of the model. Friedly, 1972 presents a method for modal approximation of linear systems. This method considers a linear system of the form,

$$\frac{dx}{dt} = Ax + Bu \quad (4.5)$$

This can be transformed into a canonical system of the form,

$$\frac{d\xi'}{dt} = \Lambda\xi' + D^T Bu \quad (4.6)$$

where, x is the vector of untransformed states,

ξ' is the vector of transformed states,

u is the vector of inputs,

A, B are matrices,

Λ is a matrix with the eigenvalues of A down the diagonal,
 E is the matrix of the eigenvectors of A ,
 $D^T = E^{-1}$.

This transformation extracts each of the modes of the original system and associates each mode with an element of the transformed vector. Removal of a particular mode of the system corresponds to the removal of the differential equations associated with one of the transformed states, ξ' , and replacing it with an algebraic equation.

This method is limited to linear systems because modal decomposition only applies to linear systems.

Quasi Steady State for Linear Systems

A similar method, also presented in Friedly, 1972, considers linear systems and removes one of the states of the original, untransformed system. This method is called pseudo (or quasi) steady state approximation and replaces a differential equation of the original system with an algebraic equation.

Fast Mode Singular Perturbations

Singular perturbation techniques decompose a system on the basis of time scales. These techniques are used almost intuitively whenever models are developed. However, an explicit statement of the method helps with the analysis of a complex system. The main aim of the singular perturbation technique is to decompose a nonlinear model into two sets of differential equations represented by,

$$\begin{aligned}\dot{x}' &= f'(x', x'', u, \varepsilon, t) \\ \varepsilon \dot{x}'' &= f''(x', x'', u, \varepsilon, t)\end{aligned}\tag{4.7}$$

where, x' and x'' are decompositions of the original state vector, $x' \in \mathbb{R}^{n'}$, $x'' \in \mathbb{R}^{n''}$,
 f' and f'' are decompositions of the original differential equations, $f' \in \mathbb{R}^{n'}$, $f'' \in \mathbb{R}^{n''}$,
 u is the control vector, $u \in \mathbb{R}^p$,
 ε represents a set of small parameters,
 $n' + n'' = n$,
 t is time.

Model reduction based on fast mode perturbations approximates ε to 0 giving the reduced model,

$$\begin{aligned}\dot{x}' &= f'(x', x'', u, 0, t) \\ 0 &= f''(x', x'', u, 0, t)\end{aligned}\tag{4.8}$$

This is a set of differential and algebraic equations. The initial response of the reduced system will not match the response of the full model precisely. However, the approximation improves over time, as the system approaches steady state.

The quasi steady state approximation method for linear systems, discussed above, is the subset of the fast mode singular perturbation method for linear systems. Therefore, the size of the eigenvalues of the linear approximation of a nonlinear model will give an indication of the existence of fast and slow modes in the nonlinear system. The approximation $\varepsilon = 0$ is equivalent to approximating the states x'' to pseudo steady state and corresponds to the removal of the fast eigenvalues from the linear approximation of the nonlinear system.

The identification of suitable small parameters is not always obvious and physical intuition is usually used. The use of singular perturbation arguments is explicitly described in many articles, and implicitly used in just about all approaches to modelling of systems in the process industries. A review of a large number of applications of singular perturbation techniques to control problems is given in Kokotovic, 1984. In the literature on geometric nonlinear control, singular perturbation techniques are used to reduce the size of models in the papers by Rouchon *et al.*, 1988 and Levine and Rouchon, 1991. Rouchon *et al.*, 1988 consider a counter-current liquid/liquid extractor, modelled as a number of stages. These stages are aggregated to form three artificial states. The dynamics of the original states are ignored on the basis that they are fast compared to the dynamics of the aggregated states. Levine and Rouchon, 1991 consider the modelling of a binary distillation column as a number of stages. Again aggregated states are formed and the dynamics of the original states are ignored. Even though tray hydraulics were not considered as part of the original model, Levine and Rouchon, 1991 state that the hydraulic dynamics are much faster than the composition dynamics (eg. Depropaniser column has a time constant for hydraulics of the order 5 *min* and 30 *min* for the composition dynamics). Therefore, tray hydraulics could also be removed based on singular perturbation arguments.

Slow Mode Singular Perturbations

The technique of fast mode singular perturbations introduces the idea that some parts of the model are fast over the time scale of interest. Slow mode singular perturbations

techniques remove those parts of the model which do not vary much over the time scale of interest. In this case, the approximate model initially matches the full model and then moves away from the full model over time, ultimately reaching a different steady state.

This technique has been used by Niranjana and San, 1988 on a model of a bioreactor. A simplified version of the model used by Niranjana and San, 1988 is,

$$\begin{aligned}\dot{x}_1 &= f_1(x_1, x_4, u) \\ \dot{x}_2 &= f_2(x_1, x_2) \\ \dot{x}_3 &= f_3(x_2, x_3) \\ \dot{x}_4 &= f_4(x_1, x_4)\end{aligned}\tag{4.9}$$

The dynamics associated with x_4 (the biomass concentration) are removed on the basis that the biomass dynamics are much slower than the substrate dynamics (x_1, x_2 and x_3 vary on the time scale of the substrate dynamics). The reduced system is involutive and a state space linearisation controller is designed. To account for the slow mode error a Kalman filter estimating the biomass concentration, x_4 , is used. The graph of this system is given in Figure 4.2.

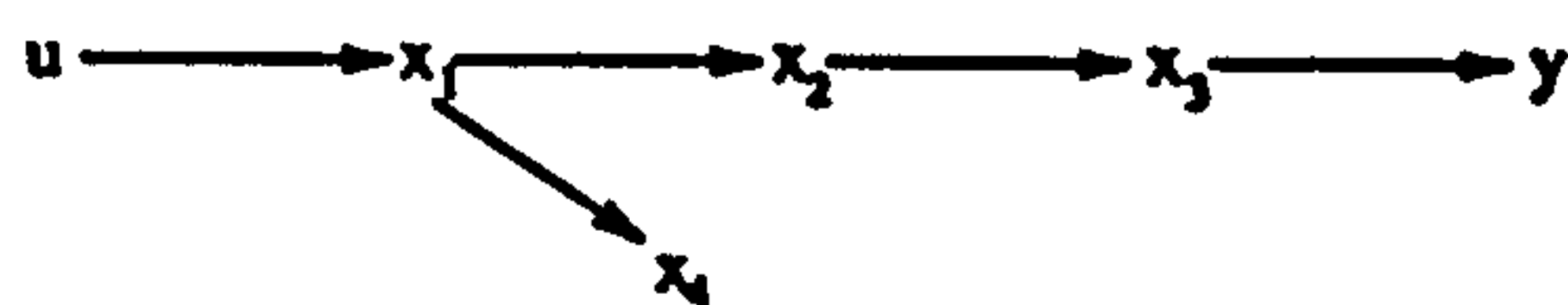


Figure 4.2: System graph for system in Niranjana and San, 1988.

It is clear from the system graph that the removal of the node x_4 results in a three state system with x_3 as a distinguished output.

Key State Identification

The slow mode and fast mode singular perturbation arguments allow the replacement of differential equations with algebraic equations. However, identifying small parameters or associating states with fast (or slow) modes is not always intuitive or straightforward.

The discussion on graph theory and relative order characterises key states for consideration as pseudo steady states. A simple and scale independent method for justifying the removal of these states involves comparing the eigenvalues of the full model with the eigenvalues of the model with each of the key states removed. If the slower modes of a reduced system are largely unaffected, then the corresponding state can be removed.

When considering a nonlinear model the effect of the removal of a state variable on the system modes over the whole operating region should be considered. However, for many nonlinear systems the effect of a state on the modes is qualitatively the same throughout the operating region. Further justification for removal of the state can be obtained by identifying small parameters as discussed in the fast mode singular perturbations technique.

Similarly, the key states can be considered for removal through a slow mode perturbation argument.

As has already been stated, the removal of a state node introduces a new algebraic equation. As the differential equations will in general be nonlinear in the state variables, this procedure will result in a set of differential and algebraic equations to be used for the controller design.

4.3.2 Case Study Relative Order Reduction

A method of reducing the complexity of the analytical inverse of a model and hence of the resulting geometric nonlinear control law has been presented in the previous sections. In order to demonstrate the power of this method, it will be applied to the case study.

The first step involves identifying the key states using the system graph. The graph of the full model of the case study is shown in Figure 4.1. The shortest path between the input node (S) and the output node (Y) follows the route: $S \rightarrow T_r \rightarrow C_r \rightarrow C_a \rightarrow y_a \rightarrow Y$ and therefore the system is generically of relative order four. To reduce the relative order of the system, nodes associated with T_r , C_r , C_a , or y_a , must be removed.

The contribution of each of these states to the dynamics of the overall system is found by examining the poles of the linear approximation of the nonlinear system at a nominal operating point. This nominal operating point corresponds to a steam flowrate of 24.2 g/s and a feed gas composition of 4.0 % CO₂. The poles of the full model and four reduced models, corresponding to the removal of C_r , C_a , y_a , and both y_a , and T_r , are listed in Tables 4.2 and 4.3.

From these results it can be seen that the states y_a , and T_r make a negligible contribution to the slow poles of the linear approximation of the system. This suggests that assuming y_a , and T_r are at pseudo steady state will have a negligible effect on the dynamic response of the output variable. A simulation of the nonlinear system with the y_a , and T_r dynamics removed is shown in Figure 4.3. This response corresponds to a step change in the steam flowrate from 24.2 g/s to 35.0 g/s initially, followed by a step to 15.0

Full Model	C_r Removed	C_{a_s} Removed
-26.1637	-23.7354	-26.1637
-10.1324	-6.1490	-10.1324
-1.1491	-0.5782	-1.1491
-0.1832	-0.1832	-0.1826+0.0030i
-0.1810	-0.1810	-0.1826-0.0030i
-0.1749	-0.1749	-0.1737
-0.0276		
-0.0211+0.0081i	-0.0259+0.0062i	-0.0247+0.0046i
-0.0211-0.0081i	-0.0259-0.0062i	-0.0247-0.0046i
-0.0086+0.0077i	-0.0116+0.0096i	-0.0107+0.0075i
-0.0086-0.0077i	-0.0116-0.0096i	-0.0107-0.0075i
-0.0015	-0.0021	-0.0016

Table 4.2: Poles (s^{-1}) of linear approximation of full and reduced models (part 1).

Full Model	y_{a_s} Removed	y_{a_s} and T_r Removed
-26.1637	-26.1637	-22.2676
-10.1324	-10.1324	-3.7401
-1.1491	-1.1491	
-0.1832		
-0.1810	-0.1816	-0.1816
-0.1749	-0.1748	-0.1748
-0.0276	-0.0276	-0.0276
-0.0211+0.0081i	-0.0211+0.0081i	-0.0211+0.0081i
-0.0211-0.0081i	-0.0211-0.0081i	-0.0211-0.0081i
-0.0086+0.0077i	-0.0086+0.0078i	-0.0086+0.0078i
-0.0086-0.0077i	-0.0086-0.0078i	-0.0086-0.0078i
-0.0015	-0.0015	-0.0015

Table 4.3: Poles (s^{-1}) of linear approximation of full and reduced models (part 2).

State	Parameters	State/Parameters
y_{a_s}	$\rho_g V_{g_s}$	0.0164
C_{a_s}	V_{l_s}	27.6
T_r	$\frac{C_{p1}\rho_l}{A_d h_l m w}$	0.00168
C_r	$A_d h_l$	8.26

Table 4.4: Parameter and state values used to justify singular perturbation arguments for reduction of full model.

g/s at 5000 sec. From these results, the conclusion that the dynamics associated with T_r and y_{a_s} are essentially instantaneous appears to be valid.

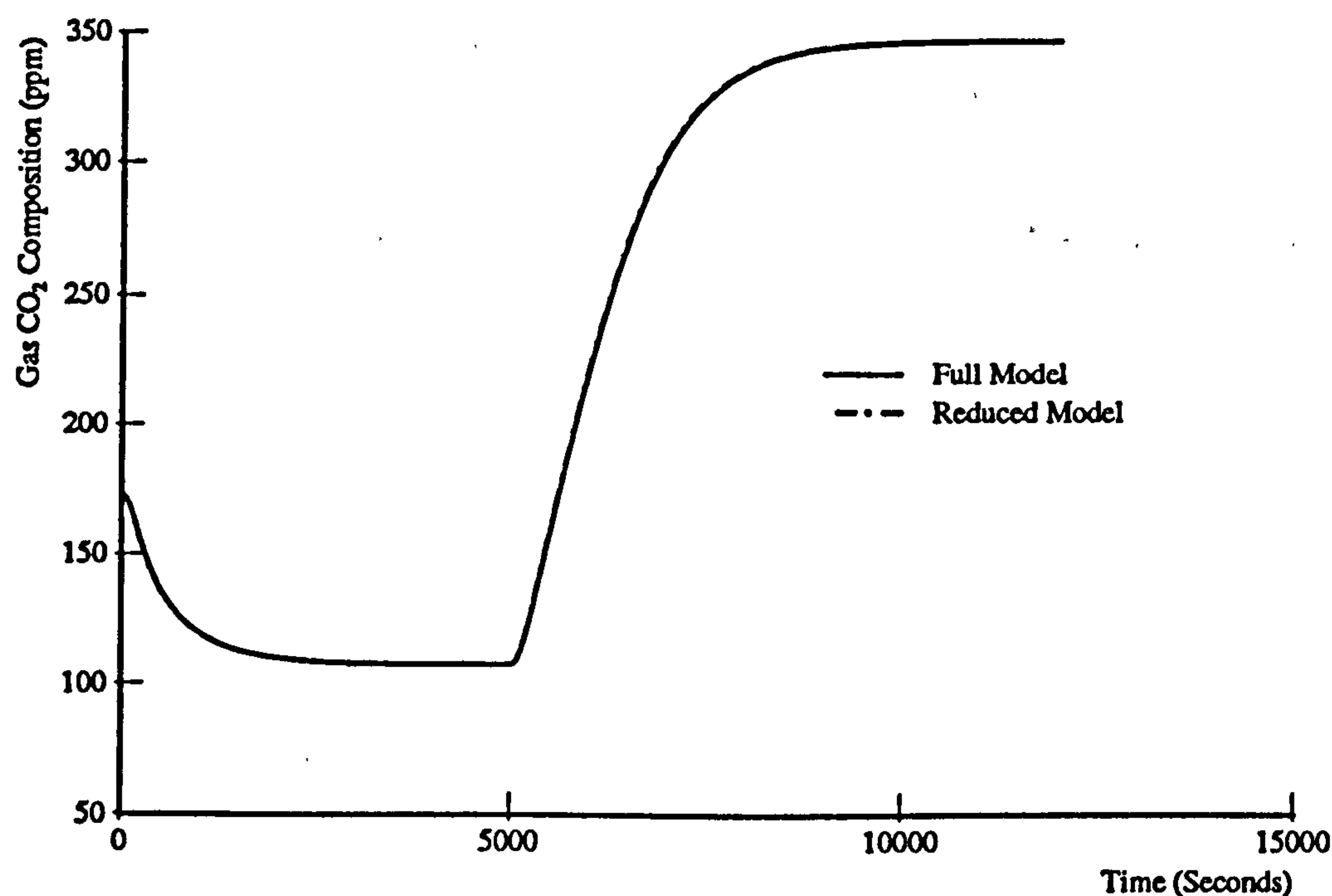


Figure 4.3: Simulation comparison of full and reduced models.

Examining the system equations associated with these four states also allows the identification of two small parameter sets, consistent with the singular perturbation approach. The time constants associated with each of the key states are given in Table 4.4. In addition, the table includes the values of the state variables at the nominal operating point divided by the identified time constant. The nominal values of state variables are included to scale the results. These results demonstrate that the time constants associated with the states T_r and y_{a_s} are very small and by singular perturbation arguments, the corresponding differential equations can be replaced by algebraic equations.

The physical argument for the removal of the $y_{a,}$ dynamics is that the gas phase mass holdup at the top of the absorber is small. The physical argument for the removal of the T_r dynamics is that the liquid phase thermal capacity in the reboiler is small relative to the heat flows.

Removing the dynamics associated with the states $y_{a,}$ and T_r effectively turns these two state variables into a further two algebraic variables. The corresponding algebraic equations are,

$$0 = -f_r y_{w,r} (H_{vap} + C_{p,r} T_r) - f_r y_{c,r} (H_{ren} + C_{p,r} T_r) + L_1 C_{p,l} T_1 - L_r C_{p,l} T_r + H_{s,l} S \quad (4.10)$$

$$0 = F_{g,s} (y_{a,2} - y_{a,3}) - P_s \frac{y_{a,2} + y_{a,3}}{2} h_i K \sqrt{C_{m,s} - 2C_{a,s}} \quad (4.11)$$

Equation 4.11 is linear in the new algebraic variable $y_{a,}$ and may therefore be substituted into the existing differential equations. However, equation 4.10 contains linear, bilinear and nonlinear (through the algebraic variables $y_{w,}$ and $y_{c,}$) functions of T_r . Therefore, it is convenient to describe the reduced model with ten state variables and one algebraic variable, T_r . The system graph of the reduced model is given in Figure 4.4. The shortest path between the input node and the output node follows the route: $S \rightarrow C_r \rightarrow C_{a,3} \rightarrow Y$. Therefore, the reduced model has a relative order of two.

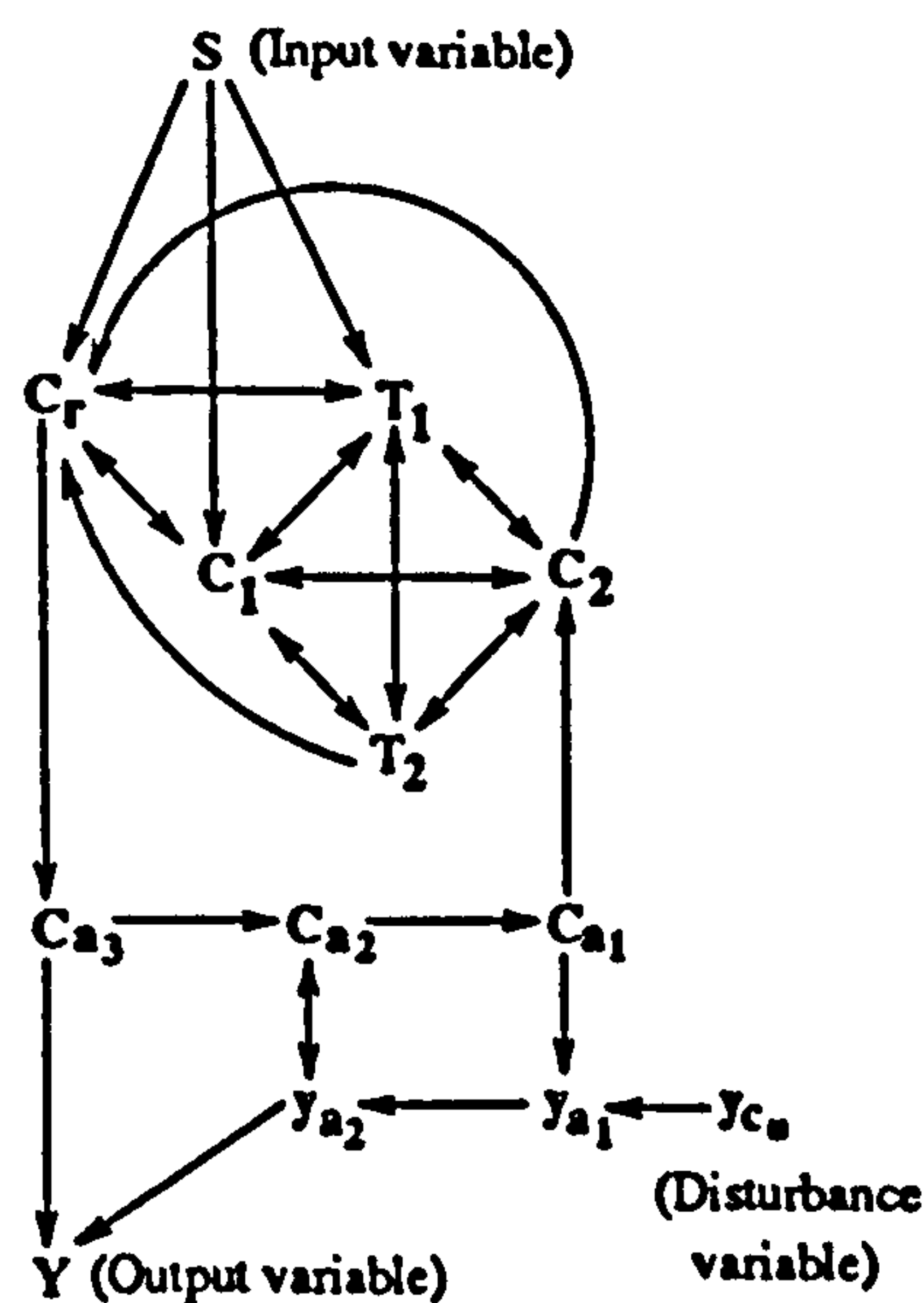


Figure 4.4: Reduced model system graph.

4.4 Geometric Nonlinear Control and DAE Systems

Section 2.1.4 describes the geometric nonlinear control technique. In summary this technique takes a nonlinear, differential equation description of a system and in effect solves the following problem.

Solve for u ,

$$\beta_r \frac{d^r y}{dt^r} + \dots + \beta_1 \frac{dy}{dt} + \beta_0 y = v \quad (4.12)$$

This is done by deriving relationships for each of the derivatives of the output variable, in terms of the state and input variables. For the case of a control affine system it is possible to solve this equation algebraically, to produce an explicit relationship for the manipulated input variable, u , in terms of the states and new reference input, v .

The basic geometric nonlinear control technique is limited to the use of a set of ordinary differential equations for a mathematical description of the system. This form of model is rather restrictive for modelling in the process industries, where a combination of differential and algebraic equations is often more convenient for describing the system. Also, the model reduction method discussed in Section 4.3 will often lead to new algebraic equations which can not be substituted into the differential equations. Therefore, there is a need to extend the basic geometric nonlinear control technique to include algebraic equations.

4.4.1 Differential Algebraic Systems

This section describes the extension of the basic geometric nonlinear control technique to include algebraic equations in the mathematical model.

Consider a system described by the following set of equations.

$$\dot{x} = f(x, a, u) \quad (4.13)$$

$$y = h(x, a) \quad (4.14)$$

$$0 = f(x, a, u) \quad (4.15)$$

where, x is the vector of state variables, $x \in \mathbb{R}^n$,

a is the vector of algebraic variables, $a \in \mathbb{R}^{n'}$,

u is the input variable, $u \in \mathbb{R}$,

y is the output variable, $y \in \mathbb{R}$,

$f(x, a, u)$ is a vector field, $f(x) \in \mathbb{R}^n$,

$f(x, a, u)$ is a set of algebraic functions, $f(x, a, u) \in \mathbb{R}^{n'}$,
 $h(x, a)$ is a vector field, $h(x) \in \mathbb{R}$.

The main objective of geometric nonlinear control is to express time derivatives of the output variable, in terms of the state variables. These are then combined to produce an equation of the following form,

$$\beta_r \frac{d^r y}{dt^r} + \dots + \beta_1 \frac{dy}{dt} + \beta_0 y = v \quad (4.16)$$

The complication for differential algebraic equations arises when defining the time derivatives of the output variable. The aim is to define the time derivatives in terms of the state (or differential) and algebraic variables. The first derivative of the output variable can be defined by the following equation,

$$\frac{dy}{dt} = \frac{\partial h(x, a)}{\partial x} f(x, a, u) + \frac{\partial h(x, a)}{\partial a} \frac{da}{dt} \quad (4.17)$$

An expression for the derivative of the algebraic variables, $\frac{da}{dt}$ as a function of the states, x , the algebraic variables, a , and the input, u (the notation, $\frac{da}{dt}(x, a, u)$ will be used to represent this fact), is required.

The first derivative of the algebraic variable is found as follows,

$$\frac{da}{dt} = - \left(\frac{\partial f(x, a, u)}{\partial a} \right)^{-1} \left[\left(\frac{\partial f(x, a, u)}{\partial x} \right) \frac{dx}{dt} + \left(\frac{\partial f(x, a, u)}{\partial u} \right) \frac{du}{dt} \right] \quad (4.18)$$

A solution is always found to this equation, providing that $\frac{\partial}{\partial a} f(x, a, u)$ is non-singular. This is equivalent to the condition that the algebraic equations are independent.

The relative order represents the number of times the output variable must be differentiated to depend explicitly on the manipulated input. For systems involving algebraic equations the relative order must be extended to include implicit relationships between the manipulated input and a particular derivative. Appendix E derives conditions for which the system,

$$0 = f(x, a, u)(x, a, u) \quad (4.19)$$

$$v = g(x, a, u) \quad (4.20)$$

has a unique implicit function, Ξ , such that,

$$u = \Xi(x, v) \quad (4.21)$$

Based on these conditions, the extension of the definition of relative order for differential algebraic systems (not necessarily control affine), using the notation introduced above, is,

r s.t.

$$\begin{aligned} & \left. \begin{aligned} & \frac{\partial}{\partial u} \left(\frac{d^r y}{dt^r}(x, a, u) \right) \neq 0 \\ \text{and/or } & \frac{\partial}{\partial a} \left(\frac{d^r y}{dt^r}(x, a, u) \right) \left(\frac{\partial}{\partial a} f(x, a, u) \right)^{-1} \left(\frac{\partial}{\partial u} f(x, a, u) \right) \neq 0 \end{aligned} \right\} \quad (4.22) \\ & \left. \begin{aligned} & \frac{\partial}{\partial u} \left(\frac{d^i y}{dt^i}(x, a, u) \right) = 0 \\ \text{and } & \frac{\partial}{\partial a} \left(\frac{d^i y}{dt^i}(x, a, u) \right) \left(\frac{\partial}{\partial a} f(x, a, u) \right)^{-1} \left(\frac{\partial}{\partial u} f(x, a, u) \right) = 0 \end{aligned} \right\} \quad \forall i = 1, \dots, r-1 \end{aligned}$$

The notation $\frac{d^i y}{dt^i}(x, a, u)$ represents an algebraic expression for the i^{th} time derivative of the output variable as a function of the state variables, x , algebraic variables, a , and the manipulated input u . The derivation of these algebraic expressions will be discussed next.

Consider the zeroth derivative of the output variable. The relative order of the system will be greater than 0 if,

$$\left. \begin{aligned} & \frac{\partial}{\partial u} h(x, a, u) = 0 \\ \text{and } & \frac{\partial}{\partial a} h(x, a, u) \left(\frac{\partial}{\partial a} f(x, a, u) \right)^{-1} \left(\frac{\partial}{\partial u} f(x, a, u) \right) = 0 \end{aligned} \right\} \quad (4.23)$$

The first time derivative of the output variable can be defined as,

$$\frac{dy}{dt} = \frac{\partial h(x, a)}{\partial x} \frac{dx}{dt} + \frac{\partial h(x, a)}{\partial a} \frac{da}{dt} + \frac{\partial h(x, a)}{\partial u} \frac{du}{dt} \quad (4.24)$$

Substituting Equation 4.18 gives,

$$\begin{aligned} \frac{dy}{dt} &= \frac{\partial h(x, a)}{\partial x} \frac{dx}{dt} - \frac{\partial h(x, a)}{\partial a} \left(\frac{\partial}{\partial a} f(x, a, u) \right)^{-1} \left[\frac{\partial}{\partial x} f(x, a, u) \frac{dx}{dt} + \frac{\partial}{\partial u} f(x, a, u) \frac{du}{dt} \right] \\ &+ \frac{\partial h(x, a)}{\partial u} \frac{du}{dt} \end{aligned} \quad (4.25)$$

If the relative order of the system is greater than 0, from Equation 4.23 it follows that $\frac{\partial h(x, a)}{\partial a} \left(\frac{\partial}{\partial a} f(x, a, u) \right)^{-1} \left(\frac{\partial}{\partial u} f(x, a, u) \right)$ and $\frac{\partial h(x, a)}{\partial u}$ are both zero. Therefore, the above expression simplifies to,

$$\frac{dy}{dt} = \frac{\partial h(x, a)}{\partial x} \frac{dx}{dt} - \frac{\partial h(x, a)}{\partial a} \left(\frac{\partial}{\partial a} f(x, a, u) \right)^{-1} \left(\frac{\partial}{\partial x} f(x, a, u) \right) \frac{dx}{dt} \quad (4.26)$$

Then substitute Equations 4.13 to obtain,

$$\frac{dy}{dt} = \left[\frac{\partial h(x, a)}{\partial x} - \frac{\partial h(x, a)}{\partial a} \left(\frac{\partial f(x, a, u)}{\partial a} \right)^{-1} \frac{\partial f(x, a, u)}{\partial x} \right] f(x, a, u) \quad (4.27)$$

To simplify the notation an augmented version of the Lie derivative will be defined.

$$L'_{f^r} h(x, a) = \left[\frac{\partial h}{\partial x}(x, a) - \left(\frac{\partial h(x, a)}{\partial a} \left(\frac{\partial f(x, a, u)}{\partial a} \right)^{-1} \frac{\partial f(x, a, u)}{\partial x} \right) \right] f(x, a) \quad (4.28)$$

Higher orders for this augmented Lie derivative (up to $i = r$) can be defined as follows,

$$L''_{f^r} h(x, a) = \left[\frac{\partial L'_{f^r} h}{\partial x}(x, a) - \frac{\partial L'_{f^r} h(x, a)}{\partial a_1} \left(\frac{\partial f(x, a, u)}{\partial a} \right)^{-1} \frac{\partial f(x, a, u)}{\partial x} \right] f(x, a) \quad (4.29)$$

Using this notation, the first time derivative of the output becomes,

$$\frac{dy}{dt} = L'_{f^r} h(x, a, u) \quad (4.30)$$

and the relative order of the system will be greater than 1 if the following is true.

$$\text{and } \left. \begin{aligned} \frac{\partial}{\partial u} L'_{f^r} h(x, a, u) &= 0 \\ \frac{\partial}{\partial a} L'_{f^r} h(x, a, u) \left(\frac{\partial f(x, a, u)}{\partial a} \right)^{-1} \left(\frac{\partial f(x, a, u)}{\partial u} \right) &= 0 \end{aligned} \right\} \quad (4.31)$$

If the relative order is greater than 1, then it follows that the second time derivative of the output is,

$$\frac{d^2 y}{dt^2} = L''_{f^r} h(x, a, u) \quad (4.32)$$

The extension to determining expressions for higher derivatives of the output variable is obvious.

$$\frac{d^i y}{dt^i} = L''_{f^r} h(x, a, u) \quad \forall i \leq r \quad (4.33)$$

where, r is the relative order of the system defined as follows,

$$\begin{aligned} \text{or } & \left. \begin{aligned} \frac{\partial}{\partial u} (L''_{f^r} h(x, a, u)) &\neq 0 \\ \frac{\partial}{\partial a} (L''_{f^r} h(x, a, u) \left(\frac{\partial f(x, a, u)}{\partial a} \right)^{-1} \left(\frac{\partial f(x, a, u)}{\partial u} \right)) &\neq 0 \end{aligned} \right\} \\ \text{and } & \left. \begin{aligned} \frac{\partial}{\partial u} (L''_{f^r} h(x, a, u)) &= 0 \\ \frac{\partial}{\partial a} (L''_{f^r} h(x, a, u) \left(\frac{\partial f(x, a, u)}{\partial a} \right)^{-1} \left(\frac{\partial f(x, a, u)}{\partial u} \right)) &= 0 \end{aligned} \right\} \quad \forall i = 1, \dots, r-1 \end{aligned} \quad (4.34)$$

Using the above method for calculating expressions for the time derivatives of the outputs and the definition of the relative order, the geometric controller becomes,

u s.t.

$$\begin{aligned} \beta_r \frac{d^r y}{dt^r}(x, a, u) + \beta_{r-1} \frac{d^{r-1} y}{dt^{r-1}}(x, a) + \dots + \beta_1 \frac{dy}{dt}(x, a) + \beta_0 h(x, a) &= v \\ 0 &= g(x, a, u) \end{aligned} \quad (4.35)$$

Using the augmented Lie derivative notation, this can be written as,

u s.t.

$$\begin{aligned} \beta_r L_{f_f}^r h(x, a, u) + \beta_{r-1} L_{f_f}^{r-1} h(x, a) + \dots + \beta_1 L_{f_f}' h(x, a) + \beta_0 h(x, a) &= v \\ 0 &= g(x, a, u) \end{aligned} \quad (4.36)$$

4.5 Reduced Controller

The reduced model is of relative order two. As a result, analytical expressions for only the first and second time derivatives of the output are required. The derivation of these expressions is discussed in Appendix D. The final controller is listed in Appendix F and the sizes of the individual terms in the controller, both in terms of the number of lines and the number of ASCII characters in the Fortran code, are summarised in Table 4.5.

Derivative	Lie Algebra Notation	Size (Lines)	Size (Characters)
0	h	1	3
1	$L_f h$	13	727
2	$L_f^2 h$	156	8223

Table 4.5: Controller size for reduced model.

A comparison with Table 4.1 shows that there is a vast reduction in the size of the controller, compared with that based on the full model. This is an effective demonstration of the strength of the model reduction technique discussed in Section 4.3 for reducing the size of a geometric nonlinear controller. The reduction in size is at the expense of,

1. information concerning the dynamics of y_a , and T_r (which are both relatively fast),

2. information about the third and fourth time derivatives of the output variable (which have a relatively minor effect on the time scale of the open loop response of the plant and would be difficult to estimate effectively from plant data),
3. an additional algebraic equation for T_r . Therefore, the control law requires the solution of two algebraic equations.

4.6 Summary

This chapter has considered the structure and complexity of mathematical descriptions that can be used for the design of a geometric nonlinear controller.

A logical approach to controller size reduction through model reduction with the aim of reducing the model relative order is presented. The approach identifies key states using the system graph. A singular perturbation argument is then used to justify the removal of as many of the key states as possible.

An extension of the geometric nonlinear control technique to include differential and algebraic equations has also been presented. The advantage of this is that a much broader class of models may now be used.

The final result from this chapter is a greatly reduced controller which will be used in the following chapters for experimental and simulated implementation.

Chapter 5

Geometric Nonlinear Controller Tuning

This chapter addresses the problem of setting values for the tuning parameters for the geometric nonlinear controller, derived in the previous chapter. Sections 5.2 to 5.4 consider the tuning of the geometric nonlinear controller parameters for a generic system. This leads to the development of a basic set of tuning guidelines, in Section 5.5, which are based on a trade off between a desired output response and reasonable control action.

The tuning procedure is then applied to the case study to obtain an acceptable servo response in Section 5.6. However, it is shown that the resulting parameters produce a closed loop system which is unable to reject unmeasured changes in the feed CO₂ composition. Furthermore, it is shown that no values of the tuning parameters will give an acceptable regulatory response for unmeasured changes in the feed CO₂ composition. It is concluded that this is related to the rates of response of the system to changes in the feed CO₂ composition and the steam flowrate.

By introducing an open loop observer to estimate the states required by the geometric nonlinear controller, it is possible to obtain an acceptable servo and regulatory response. This is demonstrated by considering a generic system.

The final result is a geometric nonlinear control structure augmented with an open loop observer, which has satisfactory servo and regulatory behaviour.

5.1 Generic Systems

This section describes a number of generic systems to be used in this chapter for analysis of the tuning parameters of a geometric nonlinear controller. The most general system to be considered will be the single input, single output, nonlinear system,

$$\begin{aligned}\dot{x} &= f(x) + g(x)u + w(x)d \\ y &= h(x)\end{aligned}\tag{5.1}$$

where, x is the vector of states, $x \in \mathbb{R}^n$,

u is the manipulated input, $u \in \mathbb{R}$,

d is the disturbance input, $d \in \mathbb{R}$,

y is the output variable, $y \in \mathbb{R}$,

$f(x), g(x)$ and $w(x)$ are vector fields, $f(x), g(x), w(x) \in \mathbb{R}^n$,

$h(x)$ is a scalar function, $h(x) \in \mathbb{R}$.

For this the system, the required geometric nonlinear control law is,

$$u = \frac{v - \beta_0 h(x) - \dots - \beta_r L_f^r h(x)}{L_f L_f^{r-1} h(x)} \quad (5.2)$$

The servo response of the closed loop system is set by defining the $r + 1$ tuning parameters, $\beta_0, \dots, \beta_r$. r degrees of freedom corresponding to the r poles of the closed loop system. The remaining parameter determines the gain between the controller input, v , and the output, y . The effect of these parameters on the response of a system with no model mismatch, no controller saturation and no disturbances is well understood. However, it has not been clearly stated how these parameters can affect the closed loop response outside these ideal conditions.

To highlight the basic issues for parameter tuning of geometric nonlinear controllers, some of the analyses in the following sections will use a linear approximation of the system in Equation 5.1.

$$\begin{aligned} \dot{x} &= Ax + bu + wd \\ y &= cx \end{aligned} \quad (5.3)$$

where, x is the vector of states, $x \in \mathbb{R}^n$,

u is the manipulated input, $u \in \mathbb{R}$,

d is the disturbance input, $d \in \mathbb{R}$,

y is the output variable, $y \in \mathbb{R}$,

A is a matrix, $A \in \mathbb{R}^{n \times n}$,

b and w are column vectors, $b, w \in \mathbb{R}^n$,

c is a row vector, $c \in \mathbb{R}^{1 \times n}$.

Results based on this simple system will not generalise to all nonlinear systems. However, they will provide a starting point and guidelines for the tuning of a full nonlinear system.

Many of the expressions derived in the following sections are too complicated, even for the specific case of a linear system, to clearly demonstrate the important points. Therefore, a simple, canonical system has been chosen to simplify the final results and to produce some sample trajectories. The system is,

$$\begin{aligned}
 \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} &= \begin{pmatrix} -a_1 & 1 & 0 \\ 0 & -a_2 & 1 \\ 0 & 0 & -a_3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ b \end{pmatrix} u + \begin{pmatrix} 0 \\ 0 \\ w \end{pmatrix} d \\
 y &= (c \ 0 \ 0) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}
 \end{aligned} \tag{5.4}$$

This system has been chosen due to its simplicity and transparency. It has three poles with values determined by the diagonal elements of the A matrix. The relative order of the manipulated input, u , and the disturbance input, d , are both three and there are no zeros associated with either input. These characteristics are clearly seen in the transfer function representation of the system,

$$y(s) = \frac{cb}{(a_1 + s)(a_2 + s)(a_3 + s)} u(s) + \frac{cw}{(a_1 + s)(a_2 + s)(a_3 + s)} d(s) \tag{5.5}$$

The sample trajectories given in this chapter are based on a open loop system with the following parameter values,

$$\begin{aligned}
 a_1 &= a_2 = a_3 = 1 \\
 w &= b = 1 \\
 c &= 1
 \end{aligned} \tag{5.6}$$

5.2 Servo Response

This section considers the response of the closed loop system to a setpoint change for the ideal situation of no mismatch between the plant and the model. Two configurations of the nonlinear controller will be considered: one with the basic geometric nonlinear controller and no external regulator and the other with an external PI regulator (See Figure 5.1).

These configurations will be considered in two steps. The first step looks at a linear approximation of the closed loop system to derive trajectories of the output, y , response to changes in the setpoint. The second step again looks at a linear approximation of the closed loop system to derive expressions for the amount of control action required.

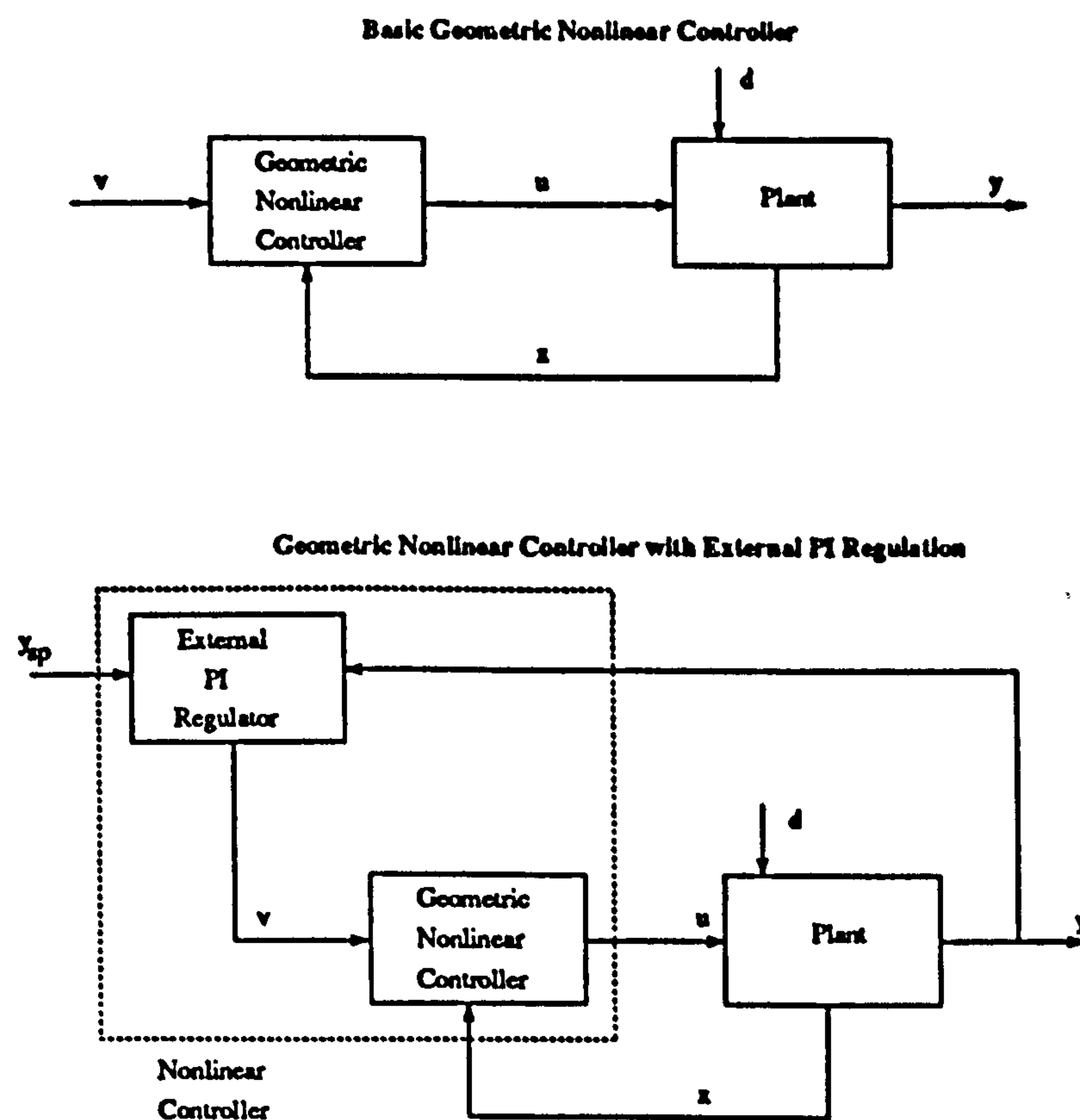


Figure 5.1: Nonlinear controller configurations with and without PI regulation.

5.2.1 Output Trajectories

For the case of the plant and controller models both equalling the nonlinear system in Equation 5.1, the closed loop responses of the output for the two configurations are,

No regulation

$$\frac{y(s)}{v(s)} = \frac{1}{\beta_0 + \dots + \beta_r s^r} \quad (5.7)$$

PI regulation

$$\frac{y(s)}{y_{sp}(s)} = \frac{ks + \frac{k}{\tau}}{\frac{k}{\tau} + (\beta_0 + k)s + \dots + \beta_r s^{r+1}} \quad (5.8)$$

It can be seen that the servo output responses are simple linear systems with r or $r + 1$ poles. The tuning parameters for the geometric nonlinear controller alone are the parameters of the characteristic equation of the requested output. For the configuration

with a PI regulator the combination of the tuning parameters of the geometric nonlinear controller and PI regulator determine the parameters of the characteristic equation.

To obtain a faster servo response for the closed loop system, faster closed loop poles must be specified.

5.2.2 Control Action

The previous conclusion was that as fast closed loop poles as possible should be specified. However, consideration must also be given to the amount of control action required to effect these responses. Manipulated variable trajectories for the linear system in Equation 5.3, and for the two configurations may be determined from,

No regulation

$$\frac{u(s)}{v(s)} = \frac{1}{cA^{r-1}b} (1 - \xi_u(s)) \frac{s^r}{\beta_0 + \dots \beta_r s^r} \quad (5.9)$$

PI regulation

$$\frac{u(s)}{y_{sp}(s)} = \frac{1}{cA^{r-1}b} (1 - \xi_u(s)) \frac{(ks + \frac{k}{\tau})s^r}{\frac{k}{\tau} + (\beta_0 + k)s + \dots + \beta_r s^r} \quad (5.10)$$

where,

$$\xi_u(s) = \frac{cA^r \left(sI - A + \frac{bcA^r}{cA^{r-1}b} \right)^{-1} b}{cA^{r-1}b} \quad (5.11)$$

For the specific case of the simple system given in Equation 5.4, these equations simplify to,

No regulation

$$\frac{u(s)}{v(s)} = \frac{1}{bc} \frac{(a_1 + s)(a_2 + s)(a_3 + s)}{(\beta_0 + \beta_1 s + \beta_2 s^2 + \beta_3 s^3)} \quad (5.12)$$

PI regulation

$$\frac{u(s)}{y_{sp}(s)} = \frac{1}{bc} \frac{(ks + \frac{k}{\tau})(a_1 + s)(a_2 + s)(a_3 + s)}{(\frac{k}{\tau} + (\beta_0 + k)s + \beta_1 s^2 + \beta_2 s^3 + \beta_3 s^4)} \quad (5.13)$$

To demonstrate the characteristics of these responses, sample trajectories are given in Figure 5.2 for the structure with no external regulator and Figure 5.3 for the structure with an external PI regulator. The open loop system has the characteristics given in Equation 5.6. The control parameters are chosen so that all the closed loop poles are equal.

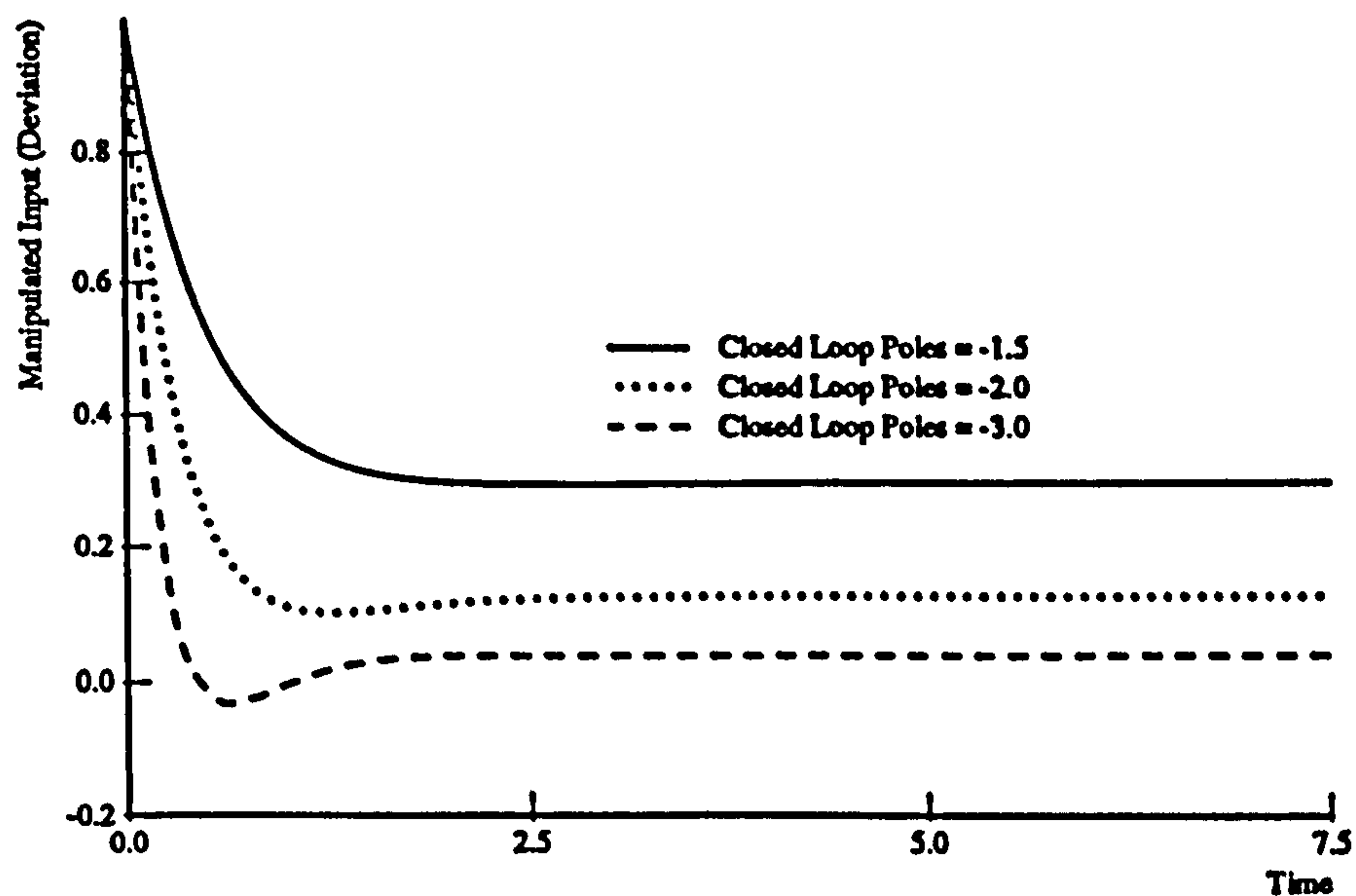


Figure 5.2: Manipulated input deviation trajectory of a simple closed loop system (open loop system given in Equation 5.4) with no external regulator to a step change in the set point, given $a_1 = a_2 = a_3 = w = b = c = 1$. The closed loop poles are all the same.

From these responses it can be seen that to minimise the amount of control action closed loop poles near the open loop poles should be specified. In the limit that the specified closed loop poles equal the open loop poles, the control action will be directly proportional to the setpoint change. Ultimately the poles will be a trade off between the speed of response required from the closed loop system and the limits on the control action.

5.2.3 Summary

In summary, this section shows that when a faster closed loop response is specified greater control action is required for the closed loop servo response. Theoretically, as long as stable closed loop poles are specified a stable closed loop response will be obtained. However, the saturation limits on the control action limit the achievable performance and may lead to limit cycling.

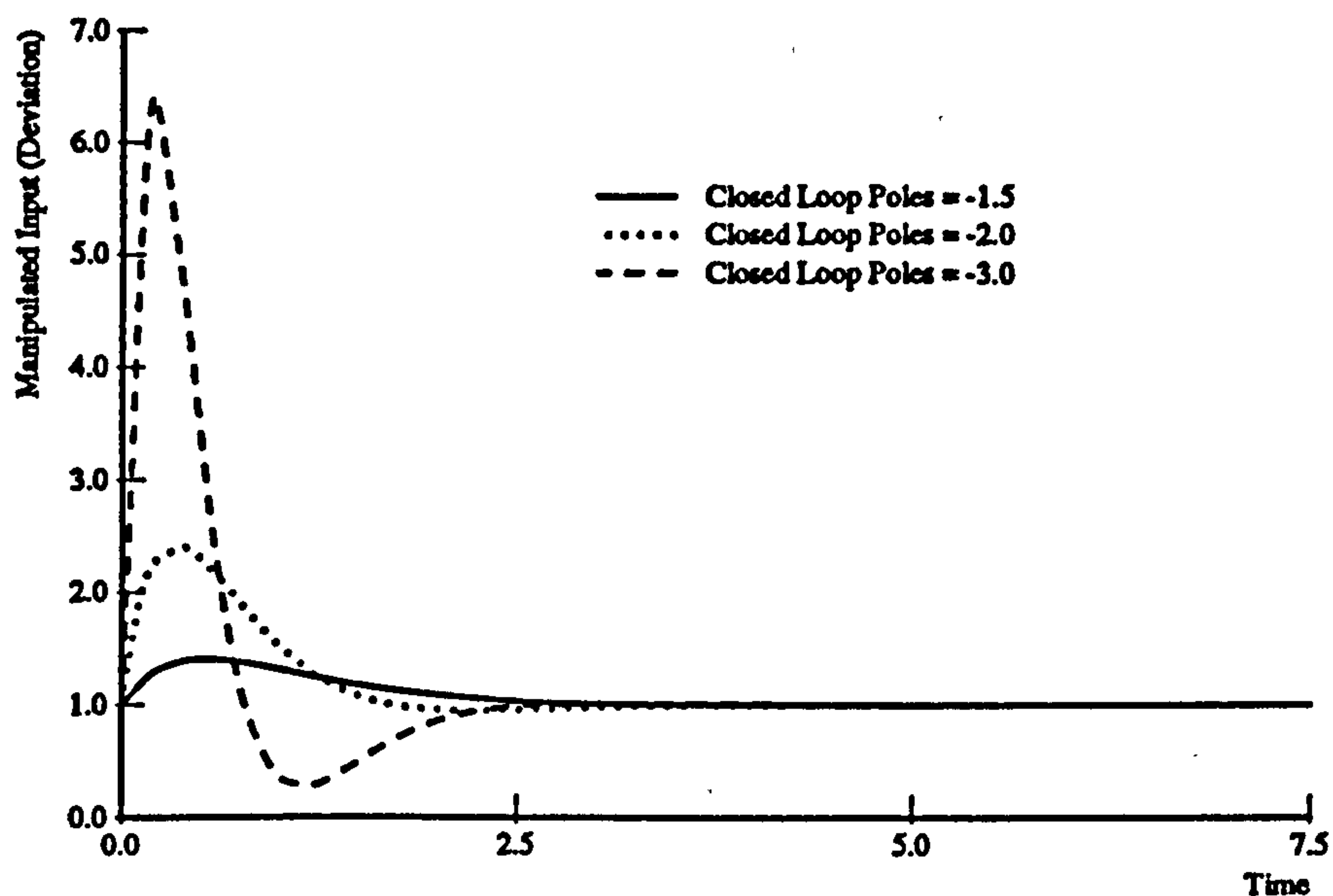


Figure 5.3: Manipulated input deviation trajectory of a simple closed loop system (open loop system given in Equation 5.4) with an external PI regulator to a step change in the set point, given $a_1 = a_2 = a_3 = w = b = c = 1$. The closed loop poles are all the same.

Therefore, the basic tuning objective based on the servo response is to maximise the speed of response of the closed loop system given saturation limits on the manipulated variable.

5.3 Regulatory Response

In a practical application of a controller there will be disturbances affecting the plant. Disturbance decoupling provides a method for accounting for measured and modelled disturbances. In reality, disturbance measurements are noisy and contain some degree of measurement error and delay. In addition, there will be unmeasured and unmodelled disturbances affecting the plant.

The issue of disturbance rejection with geometric nonlinear control is not well understood. The method is aimed at providing a linear response between a reference input, v , and the plant output, y . The response of the system to an unmeasured disturbance in general is nonlinear, with the actual response determined by the structure of the system and how the disturbance affects the open loop system.

This section looks at how the tuning parameters for two configurations of the nonlinear controller, with and without external PI regulation (See Figure 5.1) affect the output and manipulated input trajectories. Producing trajectories for a general nonlinear

system will result in expressions too complicated to be meaningful, so the linear model in Equation 5.3 is used to capture the basic dynamic characteristics. Furthermore, it is assumed that the input and disturbance relative orders are equal.

These results are generalised by looking at the steady state positions of the nonlinear system for differing disturbance relative orders, to a permanent change in an unmeasured disturbance.

5.3.1 Output Trajectories

Given that the plant and controller models are both equal to the linear approximation in Equation 5.3, with input and disturbance relative orders equal, the closed loop responses of the output to a change in an unmeasured disturbance for the two control configurations are,

No regulation

$$\frac{y(s)}{d(s)} = \frac{\beta_r c A^{r-1} w}{\beta_0 + \dots + \beta_r s^r} \quad (5.14)$$

PI regulation

$$\frac{y(s)}{d(s)} = \frac{\beta_r c A^{r-1} w s}{\frac{k}{\tau} + (\beta_0 + k)s + \dots + \beta_r s^{r+1}} \quad (5.15)$$

To demonstrate the characteristics of these responses, sample trajectories are given in Figure 5.4 for the structure with no external regulation and Figure 5.5 for the structure with an external PI regulator. The open loop system has the characteristics given in Equation 5.6. The controller parameters are chosen so that the characteristic equations have equal poles.

The results for these systems demonstrate that for the case of no external regulation a smaller final offset is obtained with faster closed loop poles. For the case of PI regulation, a smaller overshoot and a quicker settling time are obtained with faster closed loop poles. As with the output servo response of the nonlinear controller, it is desirable to have as fast closed loop poles as possible to reject unmeasured disturbances.

5.3.2 Control Action

It is also necessary to look at the manipulated input trajectories for changes in an unmeasured disturbance, for the closed loop system. Given that the plant and controller

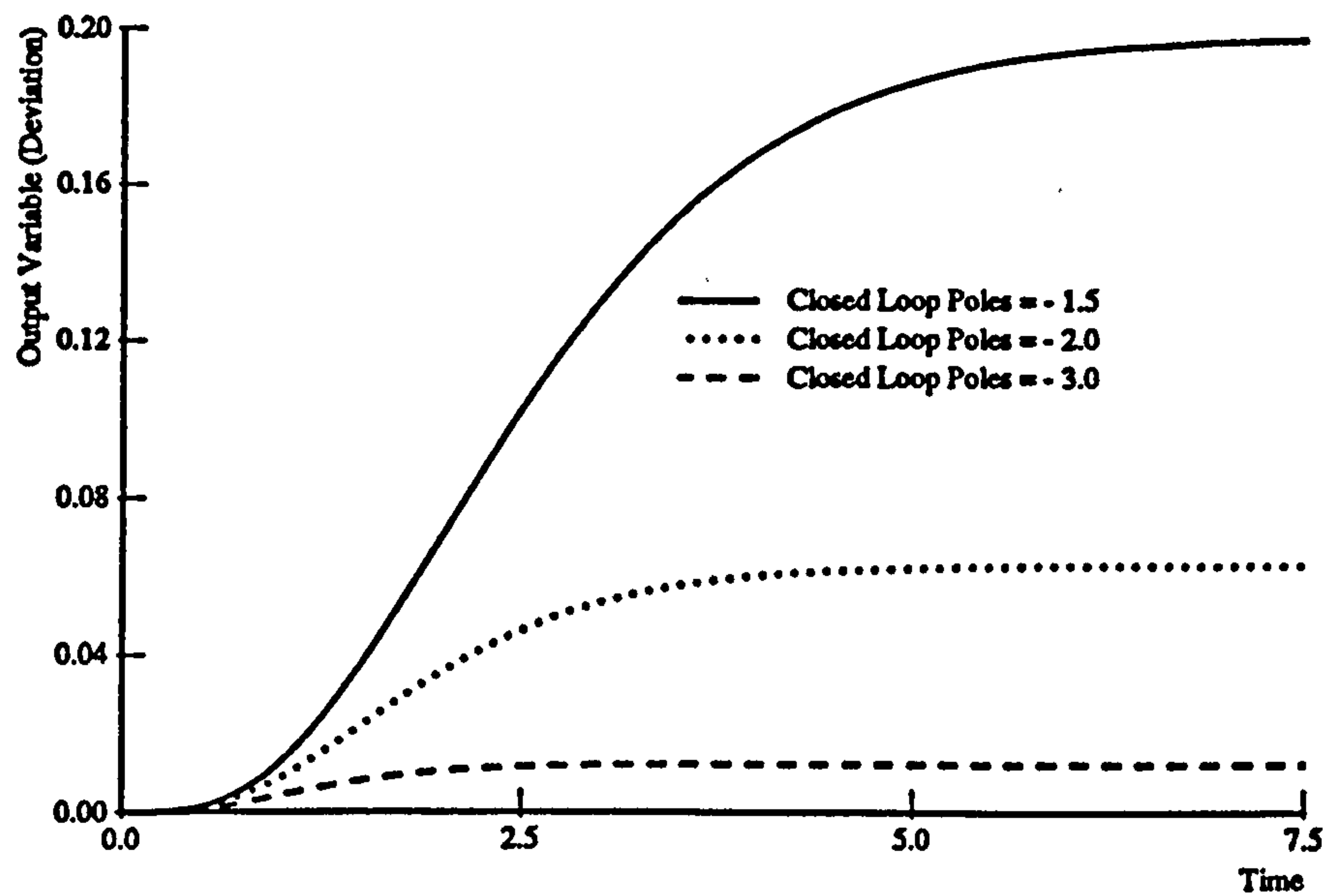


Figure 5.4: Output deviation response of a simple closed loop system (open loop system given in Equation 5.4) with no external regulator to a step changes in the unmeasured disturbance, given $a_1 = a_2 = a_3 = w = b = c = 1$. The closed loop poles are all the same.

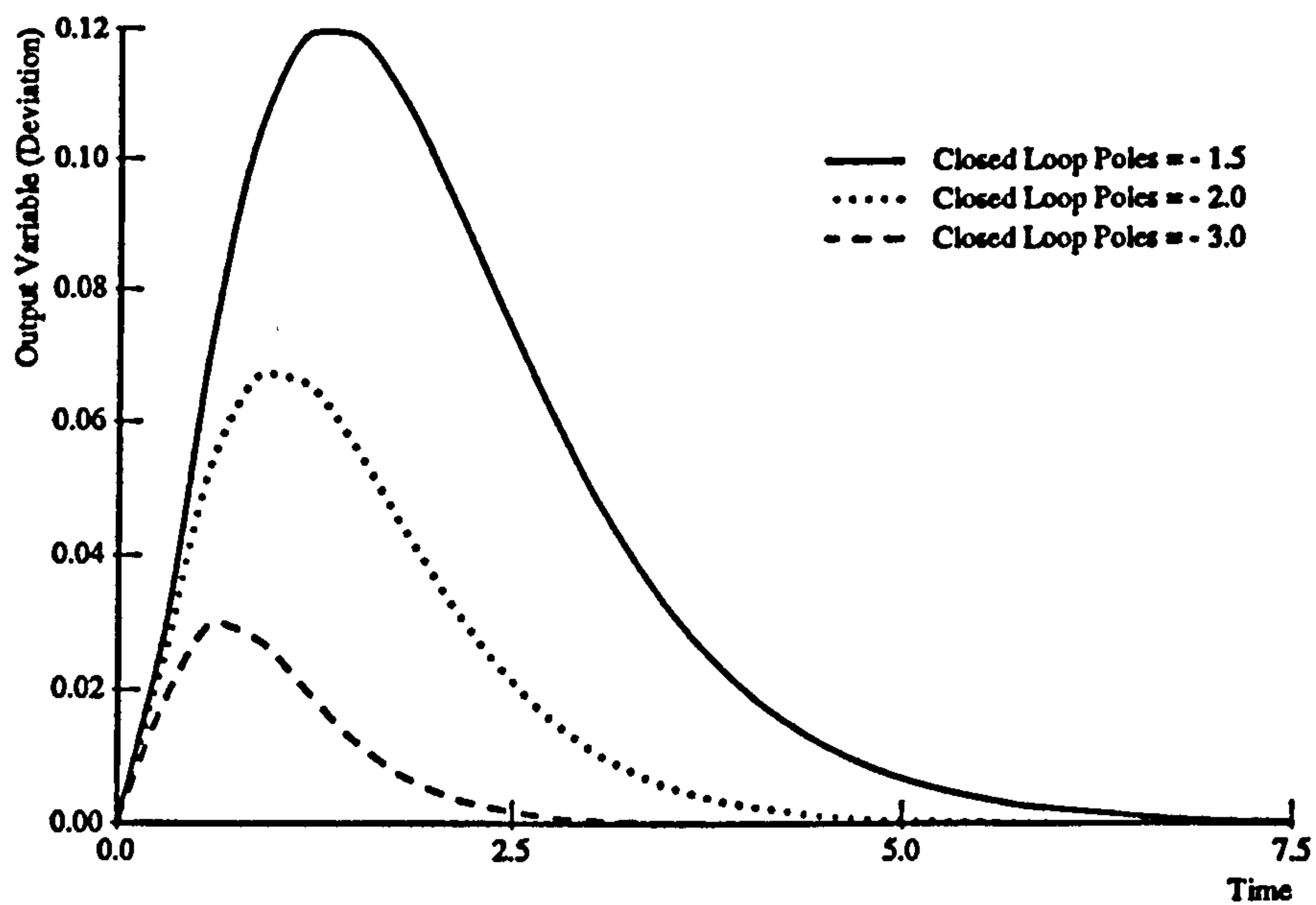


Figure 5.5: Output deviation response of a simple closed loop system (open loop system given in Equation 5.4) with an external PI regulator to a step changes in the unmeasured disturbance, given $a_1 = a_2 = a_3 = w = b = c = 1$. The closed loop poles are all the same.

models are both equal to the linear approximation in Equations 5.3, with disturbance and input relative orders equal, these responses to the two nonlinear control configurations in Figure 5.1 are,

No regulation

$$\frac{u(s)}{d(s)} = \frac{cA^{r-1}w}{cA^{r-1}b} (\xi_u(s) - 1) \frac{\beta_0 + \dots + \beta_{r-1}s^{r-1}}{\beta_0 + \dots + \beta_r s^r} - \xi_d(s) \quad (5.16)$$

PI regulation

$$\frac{u(s)}{d(s)} = \frac{cA^{r-1}w}{cA^{r-1}b} (\xi_u(s) - 1) \frac{\frac{k}{r} + (\beta_0 + k)s + \dots + \beta_{r-1}s^r}{\frac{k}{r} + (\beta_0 + k)s + \dots + \beta_r s^{r+1}} - \xi_d(s) \quad (5.17)$$

where,

$$\xi_u(s) = \frac{cA^r \left(sI - A + \frac{bcA^r}{cA^{r-1}b} \right)^{-1} b}{cA^{r-1}b} \quad (5.18)$$

$$\xi_d(s) = \frac{cA^r \left(sI - A + \frac{bcA^r}{cA^{r-1}b} \right)^{-1} w}{cA^{r-1}b} \quad (5.19)$$

For the specific case of the simple system given in Equation 5.4, these equations simplify to,

No regulation

$$\frac{u(s)}{d(s)} = \frac{w}{b} \left(\frac{\beta_3(s+a_1)(s+a_2)(s+a_3)}{\beta_0 + \beta_1 s + \beta_2 s^2 + \beta_3 s^3} - 1 \right) \quad (5.20)$$

PI regulation

$$\frac{u(s)}{d(s)} = \frac{w}{b} \left(\frac{\beta_3(s+a_1)(s+a_2)(s+a_3)s}{\frac{k}{r} + (\beta_0 + k)s + \beta_1 s^2 + \beta_2 s^3 + \beta_3 s^4} - 1 \right) \quad (5.21)$$

To demonstrate the characteristics of these responses, sample trajectories are given in Figure 5.4 for the structure with no external regulation and Figure 5.5 for the structure with an external PI regulator. The open loop system has the characteristics given in Equation 5.6. The controller parameters are chosen so that the characteristic equations have equal poles.

These results demonstrate that a greater range of the manipulated variable is required when faster closed loop poles are specified.

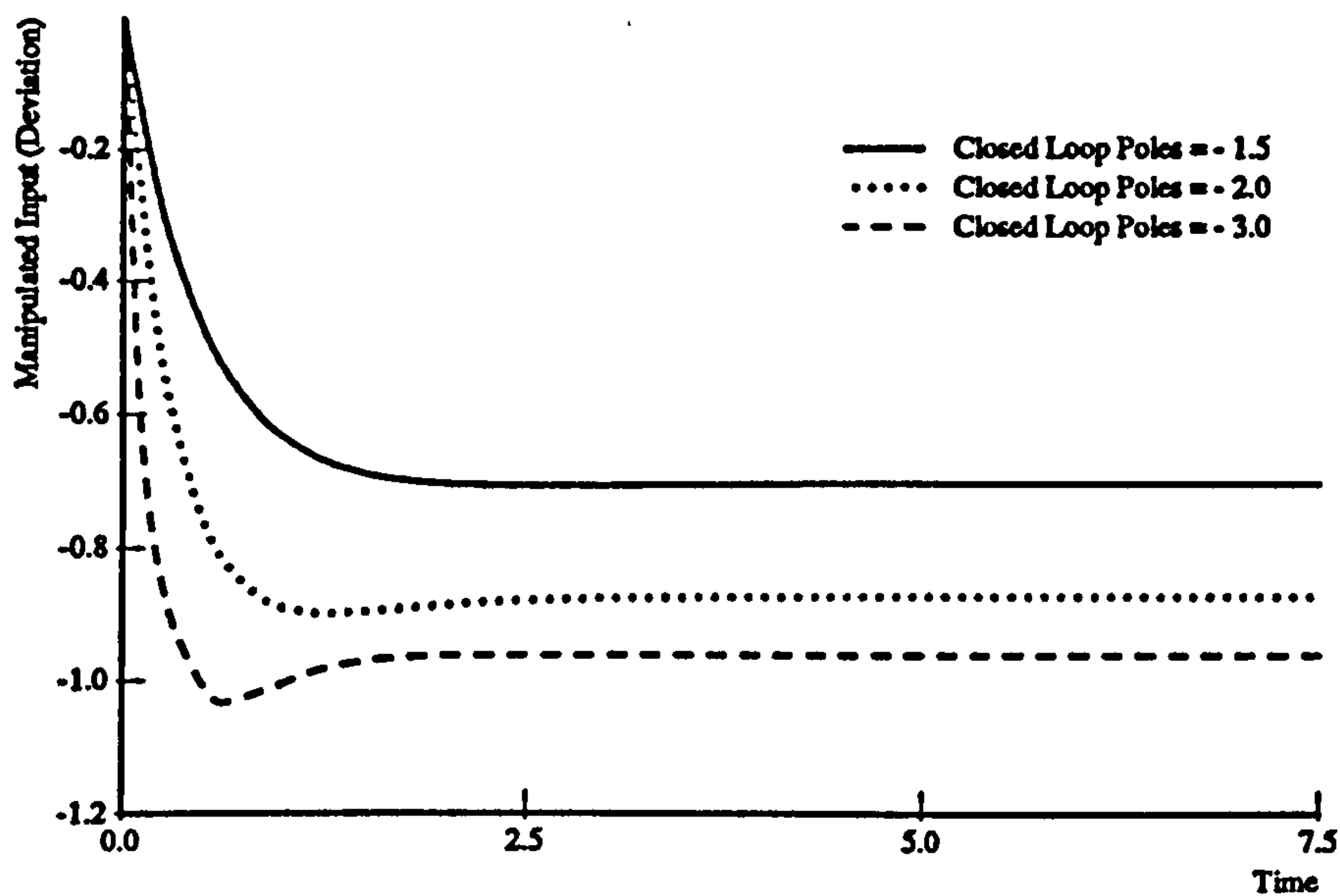


Figure 5.6: Manipulated input deviation trajectory of a simple closed loop system (open loop system given in Equation 5.4) with no external regulator to a step change in the unmeasured disturbance, given $a_1 = a_2 = a_3 = w = b = c = 1$. The closed loop poles are all the same.

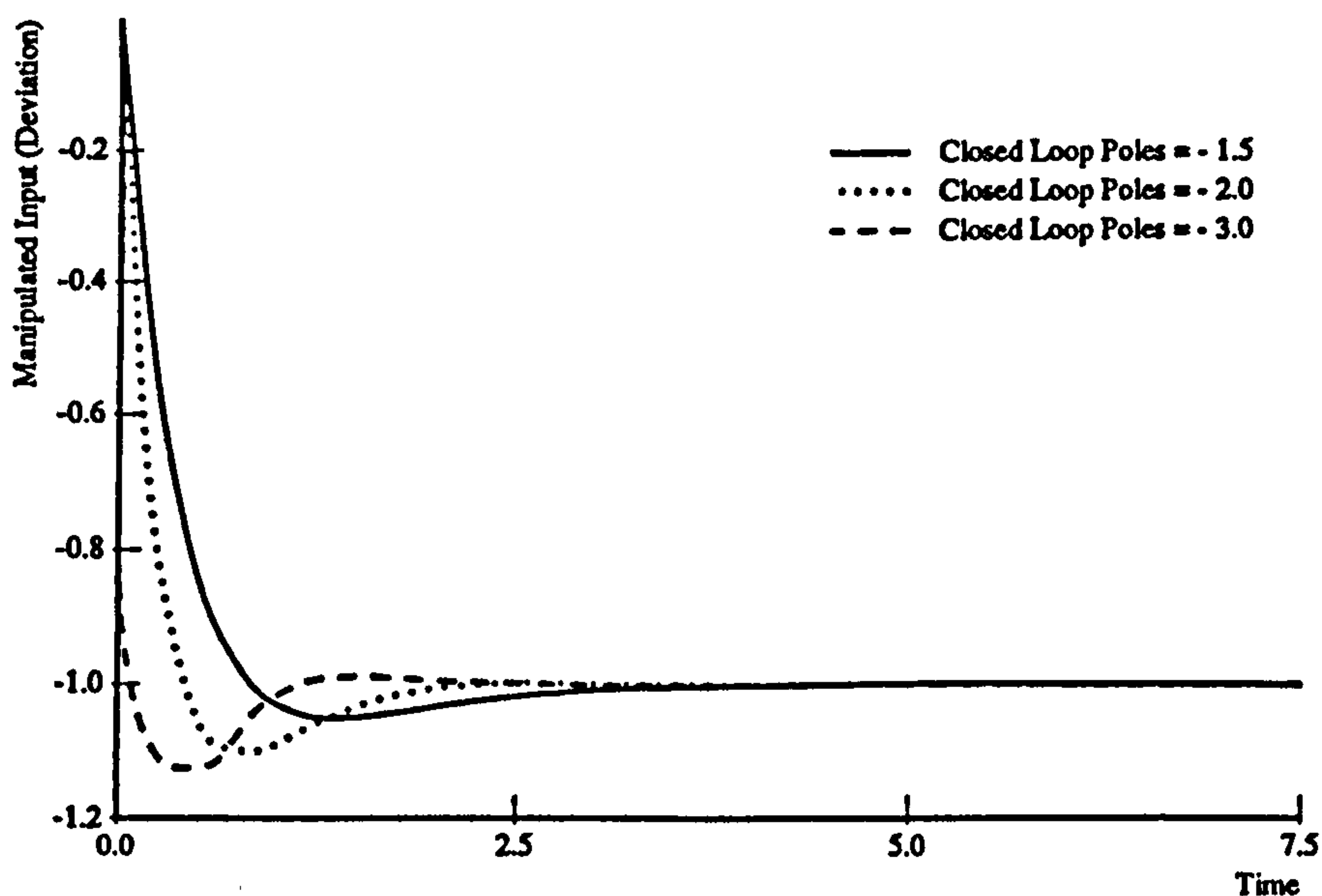


Figure 5.7: Manipulated input deviation trajectory of a simple closed loop system (open loop system given in Equation 5.4) with an external PI regulator to step a change in the unmeasured disturbance, given $a_1 = a_2 = a_3 = w = b = c = 1$. The closed loop poles are all the same.

5.3.3 Nonlinear Steady State Position

Sections 5.3.1 and 5.3.2 look at the manipulated input and output trajectories for linear, closed loop systems. From these responses some generalisations are made concerning the effects of the tuning parameters of the nonlinear controller for the two configurations. Producing equivalent trajectories for a general nonlinear system will result in much more complicated equations than presented in these sections. The basic dynamic characteristics of a nonlinear system will be captured in the linear approximation of the system, so the results of the previous section should in general extend to the nonlinear case. To further support these results the steady state position of the full nonlinear, closed loop system is considered in this section.

The steady state position of the nonlinear, closed loop system has been calculated for the case of no external regulation.

$$h(x|_{ss}) = \frac{v|_{ss}}{\beta_0} + \frac{\beta_r}{\beta_0} L_w L_f^{r-1} w^*(x, u, d)|_{ss} - \sum_{i=1}^{r-1} \frac{\beta_i}{\beta_0} L_f^i h(x|_{ss}) \quad (5.22)$$

The steady state position of the system under PI regulation is,

$$h(x|_{ss}) = \frac{k}{\beta_0 \tau} \int_0^{t|_{ss}} (y_{sp} - y(t)) dt + \frac{\beta_r}{\beta_0} w^*(x, u, d)|_{ss} - \sum_{i=1}^{r-1} \frac{\beta_i}{\beta_0} L_f^i h(x|_{ss}) \quad (5.23)$$

The function $w^*(x, u, d)$ is a function of the states, manipulated input and disturbance input. This function simplifies under certain assumptions regarding the relative order of the manipulated input and disturbance input. Two particular cases will be considered in the following discussion:

1. Disturbance relative order is greater than the input relative order,
2. Disturbance relative order is equal to the input relative order.

Similar results to those presented in the following discussion can be derived for other situations.

Disturbance relative order is greater than the input relative order

Input output linearisation of a system with disturbance relative order greater than the manipulated input relative order achieves perfect disturbance decoupling.

$$\text{ie.} \quad y = y_{sp} \implies y|_{ss} = y_{sp} \quad (5.24)$$

The steady state position for the case of no external regulation is given by,

$$h(x|_{ss}) = \frac{v|_{ss}}{\beta_0} \quad (5.25)$$

For the case of PI regulation, Equation 5.23 simplifies to,

$$h(x|_{ss}) = y_p \quad (5.26)$$

The lack of a disturbance term in these equations reflects the fact that the effect of the disturbance will be exactly accounted for by the controller.

Disturbance relative order is equal to the input relative order

Under the assumption that both the input relative order and the disturbance relative order equal r , the terms $L_f^i h(x|_{ss}) \quad \forall i = 1, \dots, r-1$ are all 0. Therefore, with no external regulation the steady state position is given by,

$$h(x|_{ss}) = \frac{v|_{ss}}{\beta_0} + \frac{\beta_r}{\beta_0} L_w L_f^{r-1} h(x|_{ss}) d|_{ss} \quad (5.27)$$

The final offset in the output, due to the unmeasured variable is,

$$\frac{\beta_r}{\beta_0} L_w L_f^{r-1} h(x|_{ss}) d|_{ss} \quad (5.28)$$

This factor represents the effect of calculating the r^{th} derivative incorrectly. This can be demonstrated by looking at the expression for the r^{th} derivative of the output of an open loop system of relative order r and with a disturbance of relative order r .

$$\frac{d^r y}{dt^r} = L_f^r h(x) + L_g L_f^{r-1} h(x) u + L_w L_f^{r-1} h(x) d \quad (5.29)$$

As the disturbance is assumed to be unmeasured, the last term, $L_w L_f^{r-1} h(x) d$ is not accounted for by the controller. The controller is trying to achieve,

$$L_f^r h(x|_{ss}) + L_g L_f^{r-1} h(x|_{ss}) = 0 \quad (5.30)$$

This represents the model information about the r^{th} derivative (ie. without the disturbance). However, at steady state it is forced to,

$$L_f^r h(x|_{ss}) + L_g L_f^{r-1} h(x|_{ss}) = -L_w L_f^{r-1} h(x|_{ss}) d|_{ss} \quad (5.31)$$

This equation is also important because it demonstrates that the error due to the unmeasured disturbance is propagated through the prediction of the r^{th} derivative. Ideally, at steady state this factor should be zero. However, with a disturbance it is non zero.

The expression in Equation 5.28 is proportional to the ratio β_r/β_0 which is equal to the product of the inverse of the specified closed loop poles.

$$\frac{\beta_r}{\beta_0} = \prod_{i=1}^r \frac{1}{p_i} \quad (5.32)$$

Therefore, to minimise the offset of the output, the poles should be made as large (in magnitude) as possible. This supports the finding of the previous section, that it is desirable to make the closed loop system as fast as possible to minimise the effect of the disturbance.

The expression for the steady state position of the system under PI regulation is,

$$h(x|_{ss}) = \frac{k}{\beta_0 \tau} \int_0^{t_{ss}} (y_{sp} - y(t)) dt + \frac{\beta_r}{\beta_0} L_w L_f^{-1} h(x|_{ss}) d|_{ss} \quad (5.33)$$

By rearrangement the following expression for the integral term results.

$$\int_0^{t_{ss}} (y_{sp} - y(t)) dt = \frac{\beta_0 \tau}{k} h(x|_{ss}) - \frac{\beta_r \tau}{k} L_w L_f^{-1} h(x|_{ss}) d|_{ss} \quad (5.34)$$

The theoretical final output of the closed loop system with an external PI controller is the setpoint, y_{sp} . The trajectory of the output to the setpoint will be affected by the characteristics of the open loop system and the tuning parameters chosen for the nonlinear controller. A general expression for the trajectory of the output of the full nonlinear system has not been derived. However, the magnitude of the offset of the output from the setpoint over time (dynamic offset) is reflected in the above integral term.

The second half of this term, $L_w L_f^{-1} h(x|_{ss}) d|_{ss}$, is the same as for the case of no external regulation. The other part of the expression, $\frac{\beta_r \tau}{k}$, is dependent on the tuning parameters of the geometric nonlinear controller and external PI regulator. It is also possible to relate this expression to the specified poles of the closed loop system as follows,

$$\frac{\beta_r \tau}{k} = \prod_{i=1}^{r+1} \frac{1}{p_i} \quad (5.35)$$

5.3.4 Summary

This section has looked at the linear approximation of the output and manipulated variable responses of the closed loop around a nominal operating point, given changes in the disturbance variable. The characteristics of the system at this point will approximate the characteristics of the closed loop system over a range of operating conditions assuming,

1. that the value of the gain does not change too much and does not change sign,
2. that the dynamics of the system do not change too much and the system does not become unstable,
3. that the parameters do not change to introduce multiplicities.

For many systems these assumptions will be true over a large part of the required operating region (and possibly the whole region). Under these conditions a number of conclusions may be made.

1. A smaller range for the output is obtained if faster closed loop poles are specified.
2. A larger range of the manipulated variable is required if faster closed loop poles are specified.

These results are then compared to the steady state point of the full nonlinear system resulting in the following conclusions.

1. For the case of no external regulation, the steady state offset resulting from an unmeasured disturbance is minimised by specifying faster closed loop poles.
2. If an external PI controller is used the dynamic offset resulting from an unmeasured disturbance is minimised by specifying faster closed loop poles.
3. The effect of unmeasured disturbances with a relative order greater than the input relative order is propagated through the prediction of the higher order derivatives.

These results are qualitative and will not be applicable to all situations. However, these conclusions will lead to some basic guidelines for tuning of the nonlinear controller.

5.4 Model Mismatch

The previous two sections have examined how geometric nonlinear control reacts in servo and regulatory modes. As the method is based on a model of the plant, it is also of interest to consider how differences between the model used for the design of the controller and the actual plant affect the geometric nonlinear controller.

Obtaining a suitable error description is a major problem when considering the effect of model mismatch. A suitable error description should,

1. be compatible with the system description,
2. be computable,
3. lead to some conclusion about its effect.

The main aim of this section is to investigate how the effect of model mismatch is related to the parameters of the controller. The model given in Equation 5.1 is taken as the model representation of the plant. This description will differ from the exact representation of the plant. The plant is described by,

$$\begin{aligned}\dot{x} &= (\hat{f}(x) + \delta f(x)) + (\hat{g}(x) + \delta g(x))u \\ y &= (\hat{h}(x) + \delta h(x))\end{aligned}\tag{5.36}$$

where $\hat{\cdot}$ represents the model (nominal) value.

This representation is used in Calvet and Arkun, 1989b and is introduced in Section 2.1.4.

It is desired to demonstrate how the tuning parameters contribute to the effect of model mismatch on the steady state position of the closed loop nonlinear system. However, using the error description in Equation 5.36 results in very complex equations which make the tuning issues difficult to see. A more convenient error description is as follows,

$$\begin{aligned}\hat{h}(x) &= h(x) - e_0(x) \\ &\vdots \\ L_f L_f^{r-1} \hat{h}(x) &= L_f^r h(x) - e_r(x) \\ L_g L_f^{r-1} \hat{h}(x) &= L_g L_f^{r-1} h(x) - e(x)\end{aligned}\tag{5.37}$$

and,

$$e_f(x) = \frac{L_g L_f^{r-1} h(x) - e(x)}{L_g L_f^{r-1} h(x)} \quad (5.38)$$

This error description can be related to the description in Equation 5.36 through,

$$\begin{aligned} e_0 &= \delta h \\ e_1(x) &= L_f \delta h(x) + L_{\delta f} \hat{h}(x) + L_{\delta f} \delta h(x) \\ e_2(x) &= L_{\delta f} L_f \hat{h}(x) + L_f e_1(x) + L_{\delta f} e_1(x) \\ &\vdots \\ e_r(x) &= L_{\delta f} L_f^{r-1} \hat{h}(x) + L_f e_{r-1}(x) + L_{\delta f} e_{r-1}(x) \\ e(x) &= L_{\delta g} L_f^{r-1} \hat{h}(x) + L_{\delta g} e_{r-1}(x) + L_{\delta g} e_{r-1}(x) \end{aligned} \quad (5.39)$$

Given this error description the steady state position of the closed loop system, without an external regulator, is,

$$y|_{ss} = \frac{v|_{ss}}{\beta_0} + \sum_{i=0}^r \frac{\beta_i}{\beta_0} e_i(x|_{ss}) + \frac{\beta_r}{\beta_0} (e_f(x|_{ss}) - 1) L_f^r h(x|_{ss}) \quad (5.40)$$

The first point of interest about this equation is the similarity with Equation 5.22. This new model mismatch characterisation highlights the fact that, for nonlinear systems, disturbances have the same effect as model mismatch. In linear systems the superposition principle allows the disturbances to be added directly to the output. However, with nonlinear systems the effect of the disturbances propagates through the entire system. A consequence of this is that the error terms $e_1(x), \dots, e_r(x)$ can be seen to have an associated relative order. That is, the relative order of $e_1(x)$ is one and so forth. The effect of each of these terms is equivalent to the effect of a disturbance with a corresponding relative order.

The parameters of the geometric nonlinear controller affect the steady state offset through the ratios, $\frac{\beta_r}{\beta_0}, \dots, \frac{\beta_1}{\beta_0}$. These ratios are the parameters of the specified, closed loop characteristic equation. The parameters of the characteristic equation are related to the poles $(p_1, \dots, p_r)$ through,

$$\begin{aligned} \frac{\beta_0}{\beta_r} &= \prod_{i=1}^r p_i \\ \frac{\beta_1}{\beta_r} &= \sum_{i=1}^r \frac{\beta_0}{\beta_r} \frac{1}{p_i} \end{aligned}$$

$$\begin{aligned}
\frac{\beta_2}{\beta_r} &= \frac{1}{2} \sum_{i=1}^r \sum_{j=1, j \neq i}^r \frac{\beta_0}{\beta_r} \frac{1}{p_i p_j} \\
\frac{\beta_3}{\beta_r} &= \frac{1}{6} \sum_{i=1}^r \sum_{j=1, j \neq i}^r \sum_{k=1, k \notin \{i, j\}}^r \frac{\beta_0}{\beta_r} \frac{1}{p_i p_j p_k} \\
&\vdots \\
\frac{\beta_{r-1}}{\beta_r} &= \sum_{i=1}^r p_i
\end{aligned} \tag{5.41}$$

The ratios of the parameters as they appear in Equation 5.40 are,

$$\begin{aligned}
\frac{\beta_r}{\beta_0} &= \prod_{i=1}^r \frac{1}{p_i} \\
\frac{\beta_1}{\beta_0} &= \sum_{i=1}^r \frac{1}{p_i} \\
\frac{\beta_2}{\beta_0} &= \frac{1}{2} \sum_{i=1}^r \sum_{j=1, j \neq i}^r \frac{1}{p_i p_j} \\
\frac{\beta_3}{\beta_0} &= \frac{1}{6} \sum_{i=1}^r \sum_{j=1, j \neq i}^r \sum_{k=1, k \notin \{i, j\}}^r \frac{1}{p_i p_j p_k} \\
&\vdots \\
\frac{\beta_{r-1}}{\beta_0} &= \prod_{i=1}^r \frac{1}{p_i} \sum_{i=1}^r p_i
\end{aligned} \tag{5.42}$$

Each of these terms converge to zero as the poles, p_i , get large. Therefore, the effect of modelling error on the steady state offset, independent of the associated relative order, is minimised by specifying faster closed loop poles.

As with the effect of unmeasured disturbances, it can be seen that the different modelling error terms propagate through the predictions of the derivatives of the output.

5.4.1 Summary

This section has considered the effect of model mismatch on the steady state position of the closed loop system, given no external regulation.

These results have shown the similarity between the effect of disturbances and model mismatch on the closed loop system. In particular, the error description introduced in this chapter identifies different error terms as having an associated relative order. The effects of these terms propagate through the predictions of the derivatives of the output.

These results do not take into account instabilities due to model mismatch and can not be taken as general. It is intuitive that there must be a trade off between speed of response and robustness. However, these results show that ironically as faster closed

loop poles are specified, the effect of the model mismatch on the closed loop steady state offset is reduced.

5.5 Tuning Procedure Based on Servo Response

The work in Sections 5.2 to 5.4 has considered the responses to changes in the setpoint and unmeasured disturbances of a single input, single output, closed loop system. These results have demonstrated that,

1. a better servo response,
2. a better regulatory response,
3. a reduced steady state offset due to an unmeasured disturbance,
4. a reduced steady state offset due to model mismatch,

are all expected if faster closed loop poles are specified. The constraints on specifying fast closed loop poles are the saturation limits on the control action and the stability limits. The stability limits have been ignored in this analysis because the main problem for the case study is the saturation limits on the control action.

The tuning of geometric nonlinear controllers in the literature has largely been aimed at obtaining particular stability guarantees. The only solution for tuning, given saturation limits, is to detune the controller sufficiently so that the limits are avoided (See Section 2.1.4).

This suggests that a tuning procedure for a geometric nonlinear controller would follow these guidelines.

1. Find the most difficult situation for the controller, where difficult corresponds to a requirement of a large range of control actions.
2. Tune the controller (with saturation) to give a stable response with as fast closed loop poles as possible.

The difficulty with this procedure is in identifying the most difficult situation for the controller. The following tuning procedure is suggested as reasonable for tuning a basic system. It is based on the assumption that the most difficult situation for the controller will be represented by a set point requiring a steady state value for the manipulated variable which is the closest approach to one of the saturation limits. This is not intended to be a general tuning procedure, only a first step.

1. Specify a nominal operating point.

2. Specify saturation limits for the manipulated variable.
3. Based on saturation limits of manipulated variable, compute the feasible operating region.
4. Calculate the slowest steady state poles of the nonlinear system, over the feasible operating region.
5. Specify potential operating points which require manipulated input values closest to the controller saturation limits.
6. Initially tune the parameters so that all the poles of the closed loop system will be the same.
7. Specify a value for the poles of the closed loop system, which is slightly faster than the slowest steady state pole found in step 4.
8. Compute the trajectory of the manipulated input for set point changes to the operating points specified in Step 5, through simulation.
9. If trajectory is too conservative then increase speed of closed loop pole and repeat from step 8.
10. If trajectory is unacceptable then use slower poles and repeat from step 8.
11. At this stage a relatively fast closed loop system should have been obtained, which has a reasonable manipulated input profile even for the hardest servo problem. It is necessary to check the regulatory response of the closed loop system. If an unacceptable regulatory response is obtained, then consideration will have to be given to speeding up the closed loop system, at the expense of increased control action.
12. Finally, the closed loop response can be fine tuned by altering the specification for all but the slowest closed loop.

These guidelines are not applicable to a general system, only to systems where the main instabilities will be due to saturation limits on the control action.

5.6 Controller Tuning for CO₂ Absorption/Desorption System

This section applies the tuning ideas presented in the previous section to the case study. With the basic geometric controller structure (ie. no external regulation) parameters are obtained to give a good servo response. However, a suitable regulatory response is not obtainable.

5.6.1 Servo Response Tuning and Regulatory Response

To begin the tuning of the geometric nonlinear controller, the basic structure (ie. no external regulation) is considered alone. A nominal operating point corresponding to

a steam flowrate of 24.2 g/s was arbitrarily chosen. A minimum operating steam flowrate of 14.0 g/s was chosen due to the limitations of the model and a maximum operating flowrate of 40.0 g/s was chosen to correspond to the physical limitation of the pilot plant. These values along with the corresponding output values are summarised in Table 5.1.

Condition	Steam Flowrate (g/s)	Output (ppm)
Nominal	24.2	174
Maximum	40.0	90
Minimum	14.0	389

Table 5.1: Range of operating conditions.

As the output is monotonically related to the manipulated input throughout the specified range, the values for the output variable listed in Table 5.1 represent the extent of the feasible steady state operating region. The limits on the steam flowrates to be used as steady state operating conditions will be taken to be 32 g/s and 17 g/s. The output values at these conditions correspond to the closest approach of the steady state operating point to the saturation limits of the manipulated input.

The characteristics of the approximation of the system at the nominal operating point are given in Table 5.2.

The slowest pole of the open loop system, at the nominal operating condition, was found to be -0.0015 s^{-1} . After following the trial and error procedure, discussed in the previous section, it was found possible to increase the speed of response of the closed loop system to correspond to a system with both poles at -0.004 s^{-1} . The servo response of the open loop system and the closed loop systems are shown in Figure 5.8. For the worst case step changes in the set point, the manipulated variable is seen to saturate, though an acceptable trajectory is still obtained. Saturation corresponds to a slowing down of the closed loop system and the introduction of nonlinear characteristics to the output response. Therefore, for servo mode the closed loop system is driven to be as fast as can be reasonably obtained.

The next step in the design procedure is to test the regulatory mode of the controller. The first test is to introduce a step change in the disturbance variable y_c , which is the feed stream CO_2 concentration. The response of the system to a variation in this concentration from 4.0 % to 4.01 % is given in Figure 5.9, both for the open loop and closed loop systems.

To show that it is relatively simple to reject this disturbance in the inlet concen-

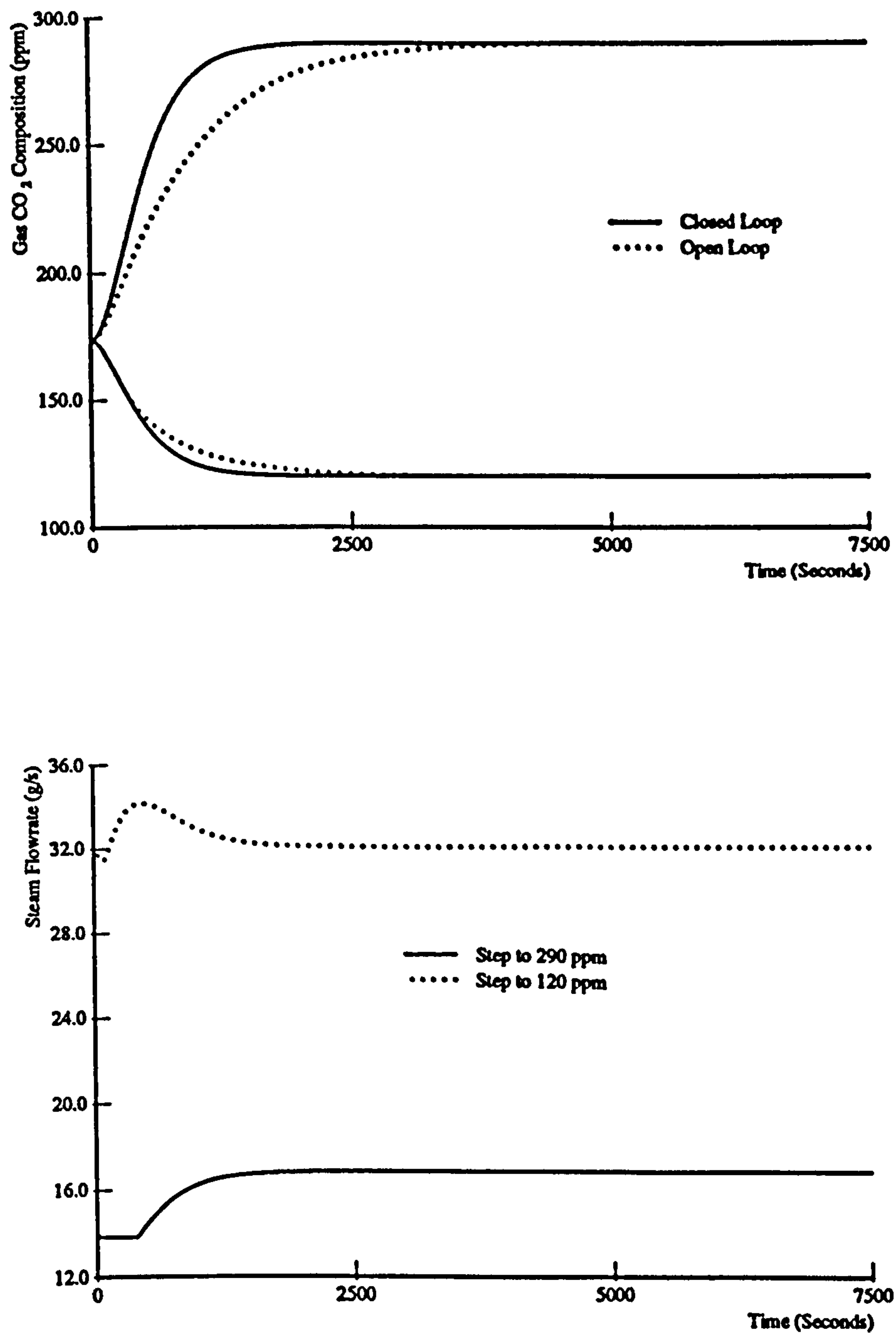


Figure 5.8: Simulated responses of the pilot plant to two step changes in the set point. The responses correspond to a step change in the steam flowrate for the open loop system and a closed loop system with a geometric nonlinear controller with no external controller (closed loop poles at -0.004 s^{-1}).

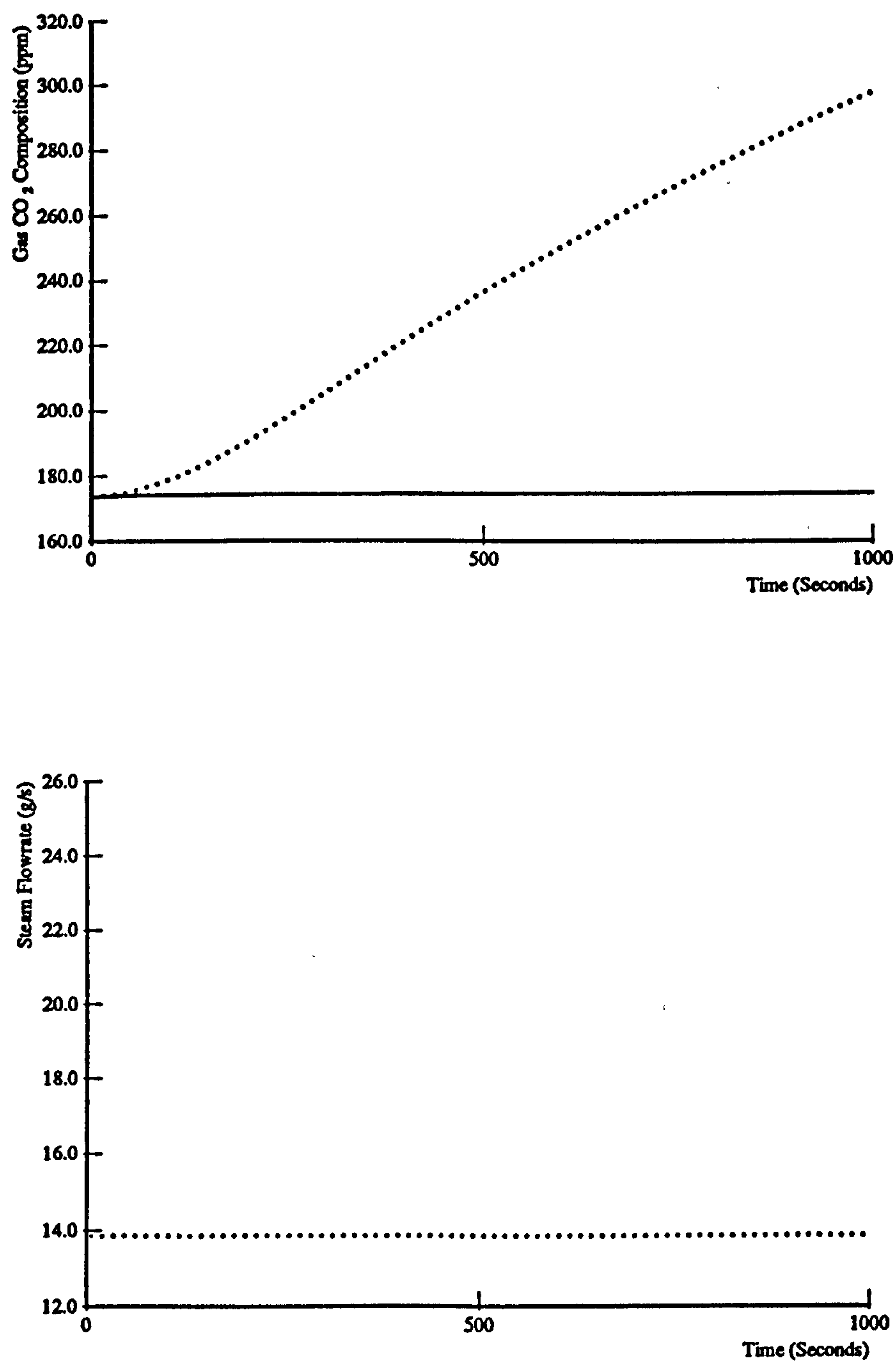


Figure 5.9: Simulated response of the pilot plant to a step change in the unmeasured disturbance, y_{c_u} by 0.01 % CO₂. The response corresponds to the open loop system and a closed loop system with a geometric nonlinear controller with no external controller (closed loop poles at -0.004 s^{-1}) and a controller saturation limit set at 14.0 g/s.

Characteristic	Value
Poles (s^{-1})	-22.3
	-3.74
	-0.182
	-0.175
	-0.0276
	-0.0211 $\pm 0.00806 i$
	-0.00856 $\pm 0.00778 i$
	-0.00146
Zeros (s^{-1})	-22.1
	-3.88
	-0.171 $\pm 0.0168 i$
	-0.0237 $\pm 0.0151 i$
	-0.0179 $\pm 0.00329 i$
Gain (ppm/(g/s))	-9.64

Table 5.2: Linear approximation of reduced model at the nominal operating point.

tration, a simulation run has been done where the value of the inlet concentration is fed forward to the controller using the disturbance decoupling method as discussed in Section 2.1.4. The response of the system with manipulated variable saturation is shown in Figure 5.10, along with the open loop response.

These results demonstrate that the geometric nonlinear controller without feed forward action is unable to reject even a small change in the unmeasured disturbance. As the system is tuned to be as fast as possible, given the servo response restrictions, it must be concluded that the basic geometric nonlinear control method is not suitable for both set point changes and disturbance rejection with the CO₂ absorption desorption process.

5.6.2 Regulatory Response Tuning

The first step in deciding what factors are causing the difficulties for regulatory control is to remove the complication of nonlinearities. This is done by linearising the full system around a nominal operating point corresponding to a steam flowrate of 24.2 g/s. Then the geometric controller is computed using the linear system matrices. Simulations are performed on the closed loop with both poles again placed at $-0.004 s^{-1}$. The response of the system to a step change in the feed stream CO₂ concentration of 0.01 % CO₂ is given in Figure 5.11.

From these results it can be seen that the same characteristics are obtained with a linear system as found previously in the full nonlinear case. Therefore it can be concluded that the problem is associated with the structure of the implementation of the geometric nonlinear controller and not with the nonlinear characteristics of the original system. The

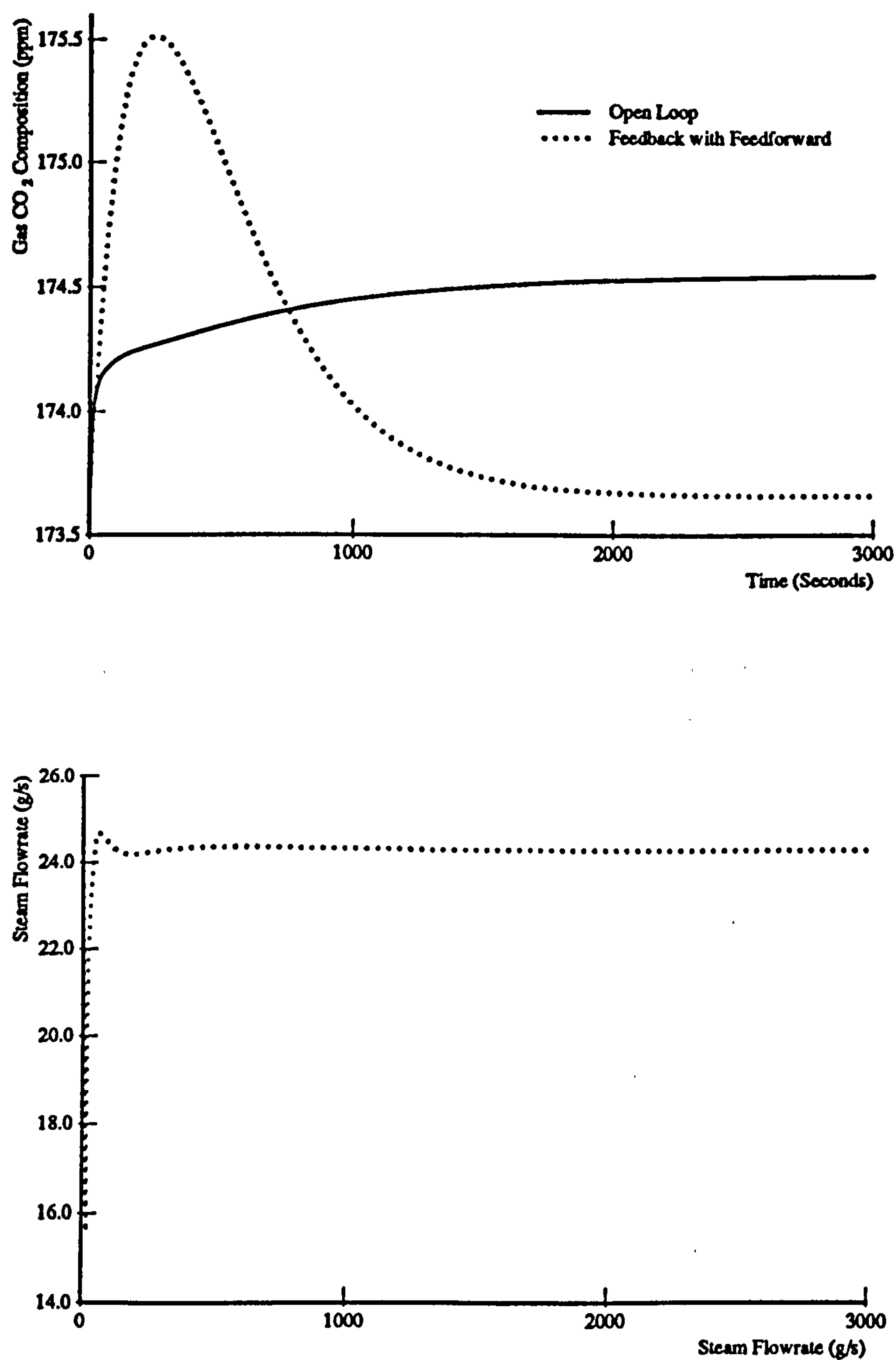


Figure 5.10: Simulated responses of the pilot plant to a step change in the unmeasured disturbance, y_{c_1} by 0.01 % CO₂. The responses correspond to the open loop system and a closed loop system with a geometric nonlinear controller with the disturbance measurement fed forward, but with no external controller (closed loop poles at -0.004 s^{-1}).

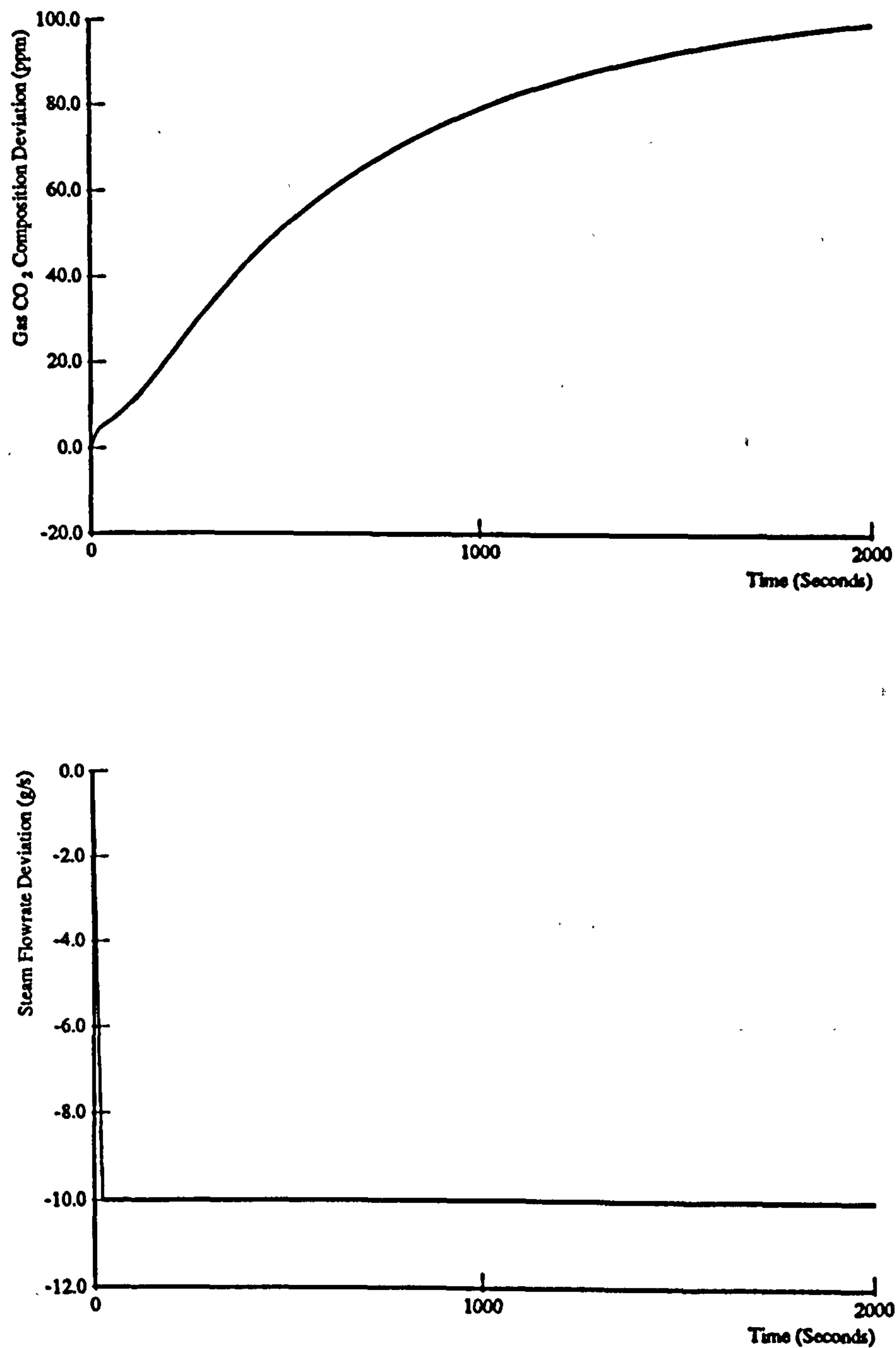


Figure 5.11: Simulated response of the linear approximation of the pilot plant to a step change in the unmeasured disturbance, y_{c_1} by 0.01 % CO₂. The responses corresponds to the closed loop system with a geometric nonlinear controller with no external controller (closed loop poles at -0.004 s^{-1}) and a controller saturation limit set at -10 g/s (Deviation).

manipulated input trajectories obtained in the previous simulation suggest that the ability of the controller is limited by the saturation limits on the steam flowrate. Ultimately this can not be true, as acceptable disturbance rejection is achieved when using a feed forward strategy.

As demonstrated in Figure 4.4 the relative orders with respect to the steam flowrate and feed CO_2 composition are both two. For this situation Equation 5.27 shows that the final offset of the controller is proportional to $\frac{\beta_2}{\beta_0}$. It can be shown that the tuning constants chosen for the basic geometric nonlinear controller configuration result in a very large offset with a steady state steam flowrate beyond the saturation limits set by the problem. To demonstrate, the maximum and minimum saturation limits on the steam flowrate are extended. Due to mathematical restrictions in the nonlinear model, it is not possible to extend the limits for the nonlinear simulation to any significant extent. Therefore the linear approximation of the model and geometric nonlinear controller were used with extended saturation limits.

Using a disturbance deviation of 0.01 % CO_2 and with both the closed loop poles set at -0.004 s^{-1} the steady state deviation in the output was found to be 8 569 ppm. The corresponding deviation in the steam flowrate was -888 g/s. For all four poles at -0.004 s^{-1} the ratio of β_2 to β_0 is 62 500 sec^2 . This explains the large offset in the process output.

Fast Closed Loop Poles

Equation 5.27 predicts that if these ratios are reduced then a smaller offset will be obtained. Both of the ratios are inversely related to the product of the closed loop poles. Therefore it is possible to reduce their values by specifying faster closed loop poles. A number of steady state simulations were done on the linear system with varying closed loop poles. In all cases a disturbance deviation of 0.01 % CO_2 was used and all the closed loop poles were specified to be equal. The results of these simulations are summarised in Table 5.3.

To produce a system with a reasonable steady state offset the ratio, $\frac{\beta_2}{\beta_0}$, should be of the order of magnitude of 1. For example, if both closed loop poles are set to 1 s^{-1} then $\frac{\beta_2}{\beta_0}$ will be 1 sec^2 . The dynamic response of the system with all closed loop poles at -1.0 s^{-1} is given in Figure 5.12.

The initial response to the disturbance is to reduce the steam flowrate well into the unrealistic region (Note: the nominal steam flowrate is 24.2 g/s). This is followed by an increase in the steam flowrate to unrealistically high values. Finally the steam flowrate is returned to the feasible region and ultimately to an output deviation of 0.137 ppm and a steam flowrate deviation of 0.908 g/s.

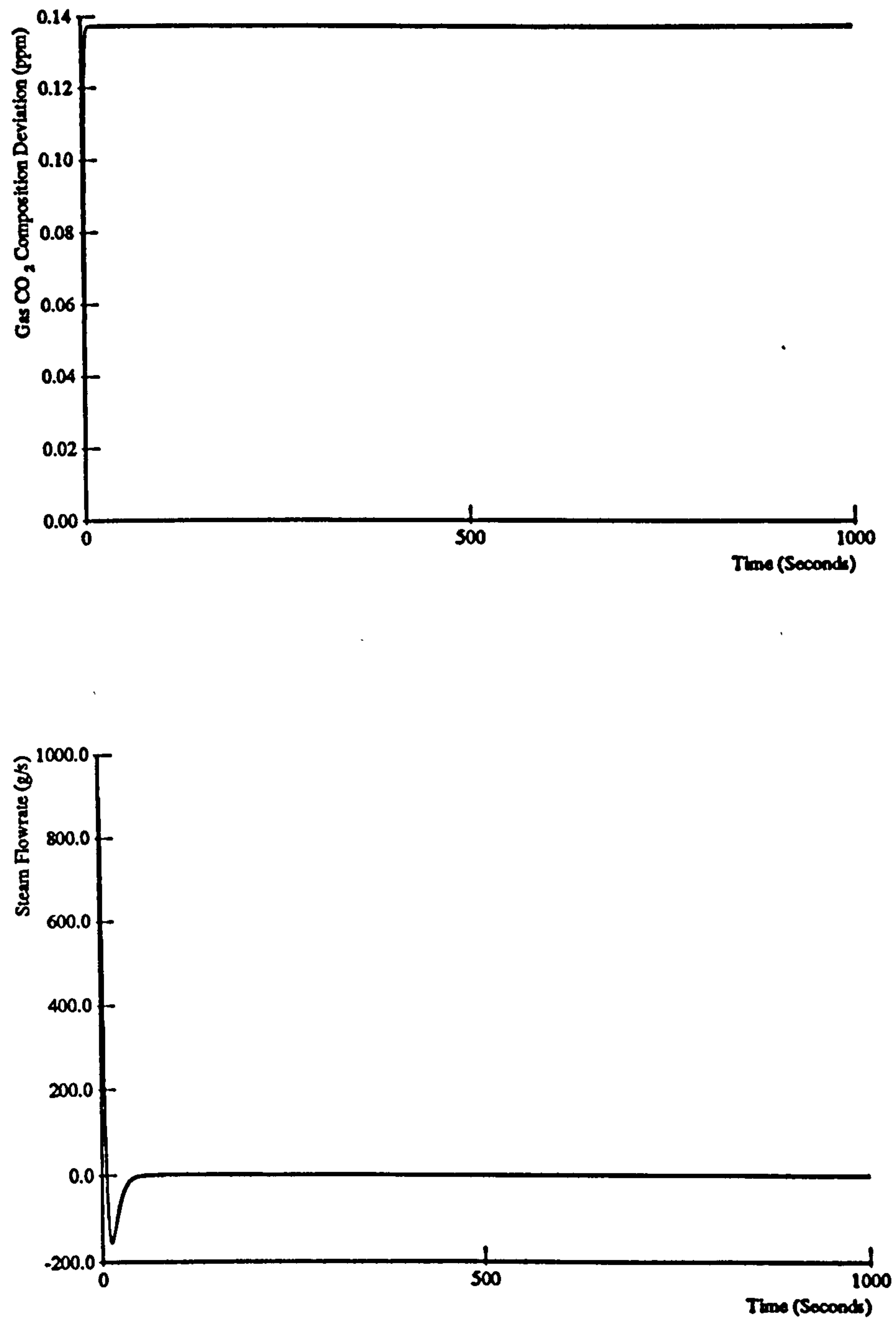


Figure 5.12: Simulated response of the linear approximation of the pilot plant to a step change in the unmeasured disturbance, y_{c_u} by 0.01 % CO₂. The response corresponds to the closed loop system with a geometric nonlinear controller with no external controller (closed loop poles at -0.004 s^{-1}) and no controller saturation limits.

$\frac{\beta_2}{\beta_0}$	$\frac{\beta_1}{\beta_0}$	Output (ppm) (Deviation)	Steam Flowrate (g/s) (Deviation)
62 500	500	8 569	-888
10	500	1.373	0.780
1	500	0.137	0.908
0.1	500	0.014	0.921
62 500	1	8 569	-888
1	1	0.137	0.908

Table 5.3: Steady state offsets of the linear approximation of the closed loop system for different values of the tuning parameters.

Shown in Figure 5.13 is the response of the system with the same tuning constants and the original saturation limits on the steam flowrate.

It can be seen from this figure that the limitations on the steam flowrate lead initially to constant switching between the two saturation limits. This leads to a cyclic response in the output variable, which is ultimately stable. Even though the closed loop response is stable, this is not an acceptable response, especially considering the minute disturbance being rejected.

So even if relatively fast poles are specified for the closed loop it is not possible to obtain a reasonable regulatory response given the saturation limits on the steam flowrate.

Combined Fast and Slow Poles

In summary, the steady state position of the closed loop system with a geometric nonlinear controller and direct state feedback to an unmeasured change in the disturbance is given by Equation 5.22. This implies that the offset of the closed loop system to a constant disturbance for the case of the equal disturbance and manipulated input relative orders, is related to the ratio $\frac{\beta_r}{\beta_0}$. Furthermore, this ratio is proportional to the product of the inverse of the specified closed loop poles. Therefore, to obtain a reasonable regulatory response we want the closed loop system to be as fast as possible. It was found in the previous section, that to obtain a reasonable output response the product of the closed loop poles should be of the order of magnitude of 1.

For the CO₂ absorption desorption system we are limited by quite tight saturation limits on the steam flowrate. A reasonable final steady state offset is obtained, at the expense of unrealistic deviations in the steam flowrate to achieve this speed of response. It is necessary to consider how the controller with the feed CO₂ composition fed forward achieves reasonable disturbance rejection. The answer is apparent from looking at the

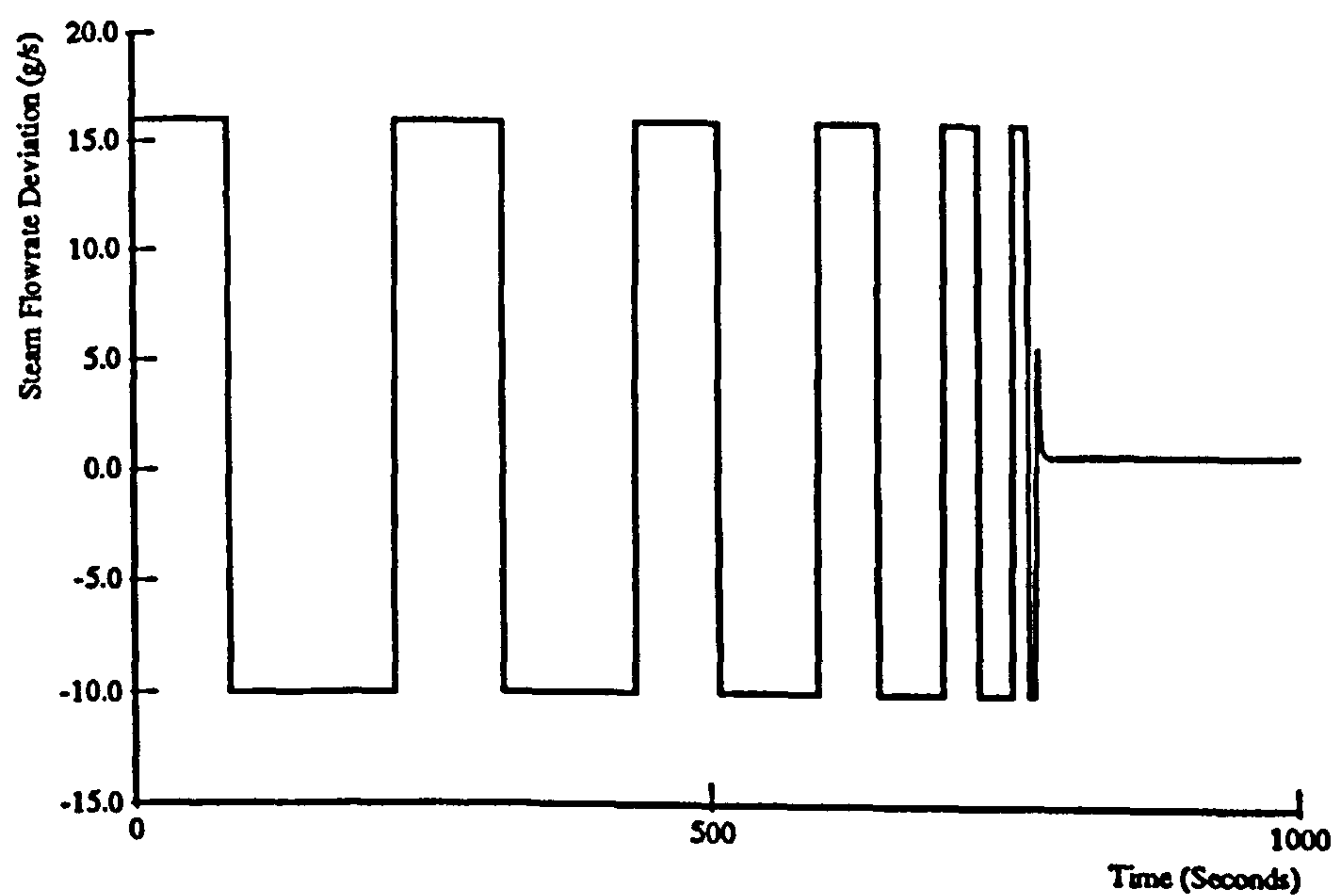
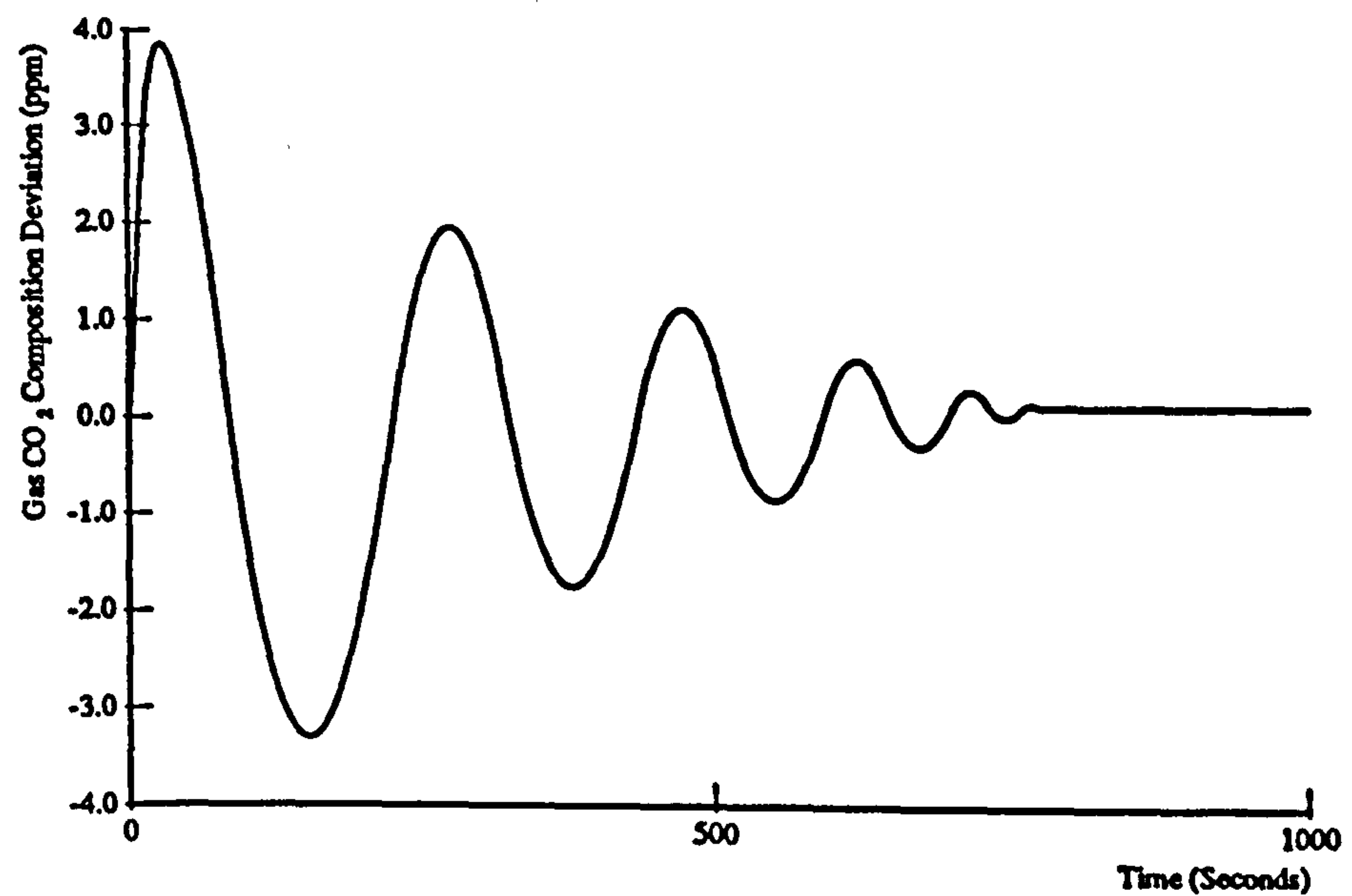


Figure 5.13: Simulated response of the linear approximation of the pilot plant to a step change in the unmeasured disturbance, $y_{c,u}$ by 0.01 % CO₂. The response corresponds to the closed loop system with a geometric nonlinear controller with no external controller (closed loop poles at -0.004 s^{-1}) and controller saturation limits set at -10.0 and 15.8 g/s (Deviation).

steam flowrate response given in Figure 5.10. When compared to the steam flowrate response in Figure 5.9 it can be seen that the response of the controller with feed forward is much slower. Initially it is dominated by the response of the system to the disturbance then the final steady state is eventually reached at a speed of response similar to the open loop dynamics.

This suggests that a reasonable response may be obtained by specifying a combination of a slow and a fast pole. Therefore, the constraints of the tuning problem are,

1. the product of p_1 and p_2 should be 1,
2. the closed loop system should have a response of similar to the open loop system.

This is approximately achieved with poles at -0.00089 and -1.0. The response of the closed loop linear approximation is given in Figure 5.14.

A more reasonable controller response is achieved with these closed loop poles. However, the fast pole still causes an initial saturation of the controller. This response is not acceptable and Section 5.7 investigates what characteristics of the system are causing the problem for the controller.

5.6.3 External Regulation

The discussion in this section has concentrated on the basic geometric nonlinear controller, without an external regulator. Theoretically, the PI regulator should improve the robustness of the basic controller and help to bring the system back to the setpoint. However, the basic geometric nonlinear controller has such a large effect on the response of the system that it dominates and will not allow the benefit from the PI regulator. Experience has shown that a detuned PI regulator has a very slow effect and therefore allows the response of the geometric nonlinear controller to dominate for a long period of time. A PI regulator with a very small integral time can overcome the response of the basic geometric nonlinear controller, though will also require large control action and results in an oscillating system.

5.6.4 Summary

This section has demonstrated that the geometric nonlinear controller with the states directly fed back from the plant is unable to reject unmeasured disturbances. The effect is due to the nonlinear controller and is too large to be effectively removed by an external P or PI regulator. Feedforward control may result in a suitable closed loop

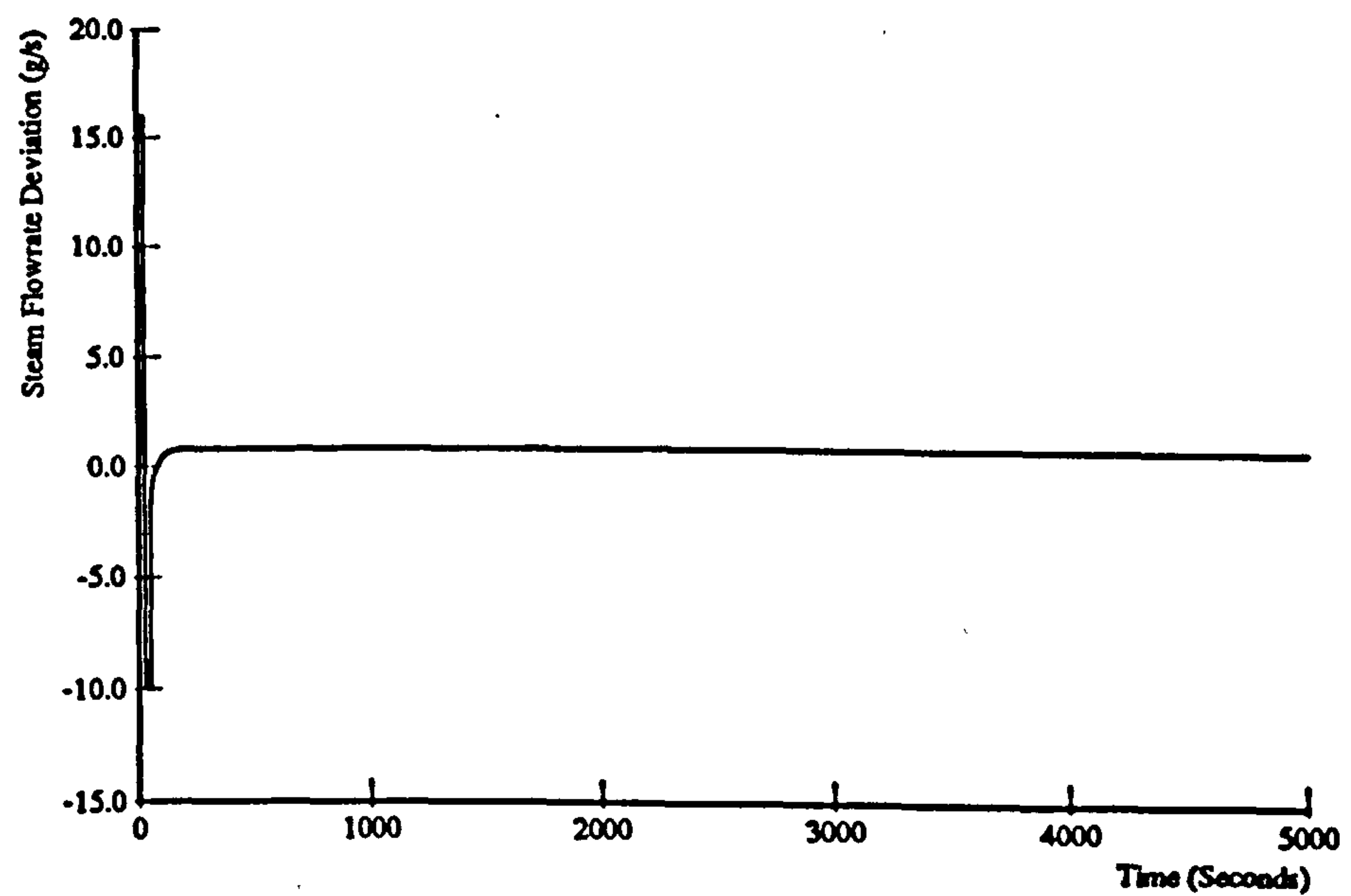
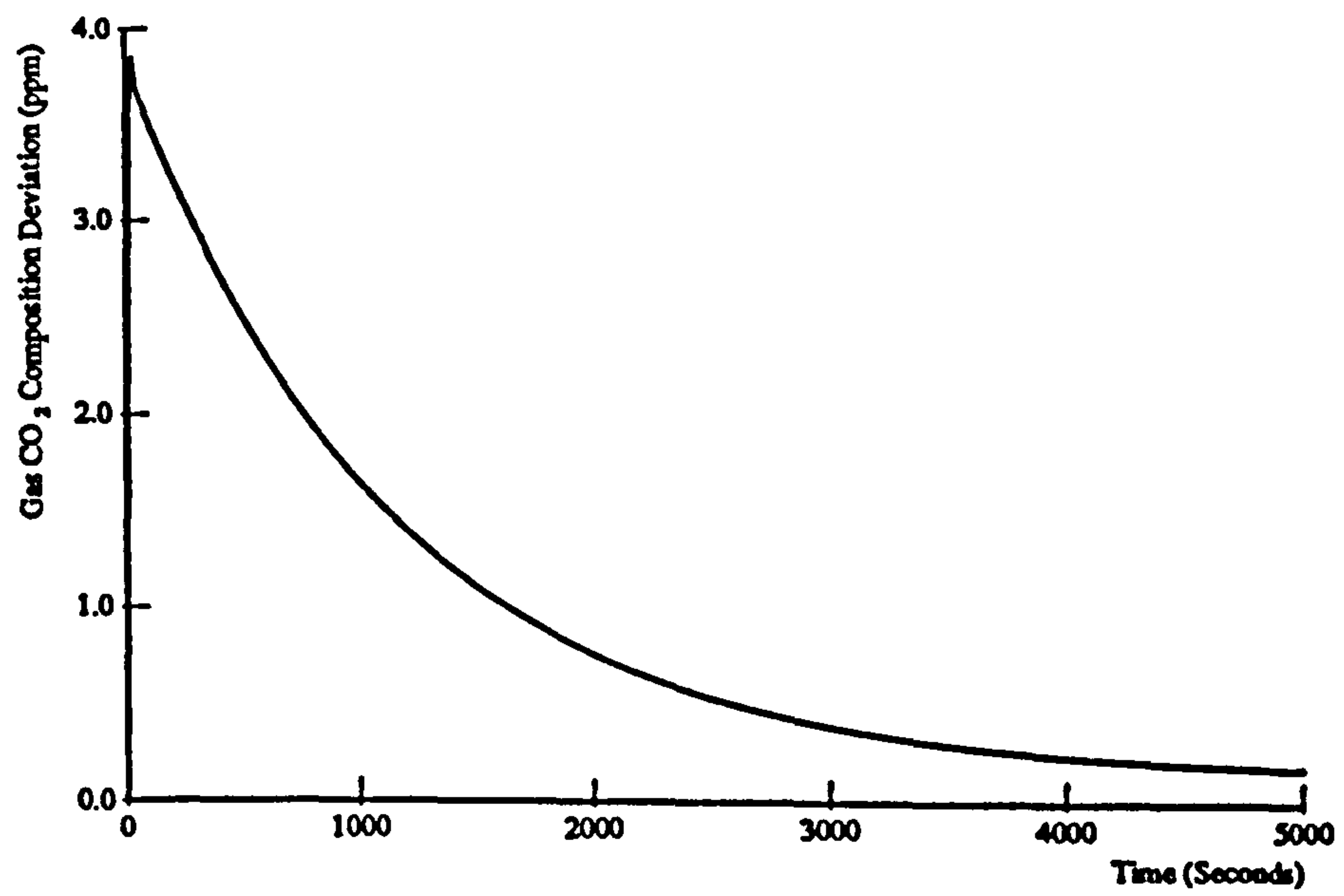


Figure 5.14: Simulated responses of the linear approximation of the pilot plant to a step change in the unmeasured disturbance, y_c , by 0.01 % CO₂. The response corresponds to the closed loop system with a geometric nonlinear controller with no external controller (closed loop poles at -1.0 s^{-1} and -0.00089 s^{-1}) and controller saturation limits set at -10.0 and 15.8 g/s (Deviation).

response, though not all disturbances will be measured in practice. Therefore, another solution to the problem must be found.

5.7 Open Loop Characteristics of CO₂ Absorption Desorption System

The original problem encountered in Section 5.6 with the tuning of the geometric controller for the CO₂ absorption desorption system was due to the specification of slow closed loop poles. However, it was shown in the Section 5.6.2 that even the specification of very fast closed loop poles did not allow a reasonable regulatory response due to the saturation limits on the steam flowrate. In Section 5.6.1 it was shown that rejection of the disturbance is relatively simple and has been achieved using extensions to the basic geometric nonlinear control structure. Furthermore, rejection of disturbances using geometric nonlinear control has been achieved quite effectively in some case studies presented in literature.

The characteristics of the CO₂ absorption desorption system causing poor regulatory control with a geometric nonlinear controller are discussed below.

The first hint of the problem can be seen in Figure 5.12. The geometric nonlinear controller immediately forces the system in a particular direction by dropping the steam flowrate to unrealistically low values. The effect on the output variable is seen as a very dramatic drop. The original drop is countered by a large increase in the steam flowrate to infeasibly large values. This is required to slow down and ultimately stop the momentum of the system created by the first drop in steam flowrate. This is seen in the output as a sudden cessation in the decrease at a value at which it remains for the remainder of the simulation. The steam flowrate continues to change in response to the momentum of the system ensuring that the output is held constant, ultimately obtaining a final steady state value.

These characteristics suggest that the effect of the disturbance on the system is much faster than the manipulated input. To reject the effect of the disturbance and provide the specified speed of response, the system must initially be forced by a large change in the manipulated input. A slower acting disturbance would not require such dramatic control action to be taken. To demonstrate this characteristic consider the following simple linear system.

$$\dot{x} = \begin{pmatrix} -a_1 & 0 \\ a_2 & -a_2 \end{pmatrix} x + \begin{pmatrix} \frac{a_1}{2} \\ 0 \end{pmatrix} u + \begin{pmatrix} 0 \\ a_2 \end{pmatrix} d \quad (5.43)$$

$$y = (1 \ 1)x \quad (5.44)$$

The open loop response of this system in the Laplace domain is,

$$y(s) = \frac{a_2(a_1 + s)}{2(a_1 + s)(a_2 + s)}u(s) + \frac{a_2}{a_2 + s}d(s) \quad (5.45)$$

The reasons for choosing this system are: the gains between the two inputs and the output are both 1, the response of the system to a step change in the manipulated input will be dominated by the slower of the two poles a_1 and a_2 , the response of the system to a step change in the disturbance input is affected only by the pole a_2 . Effectively this isolates the characteristics associated with the two poles of the system. The open loop responses of this system to step changes in the manipulated input and disturbance variables, with $a_1 = 1$ and various values of a_2 , are given in Figure 5.15.

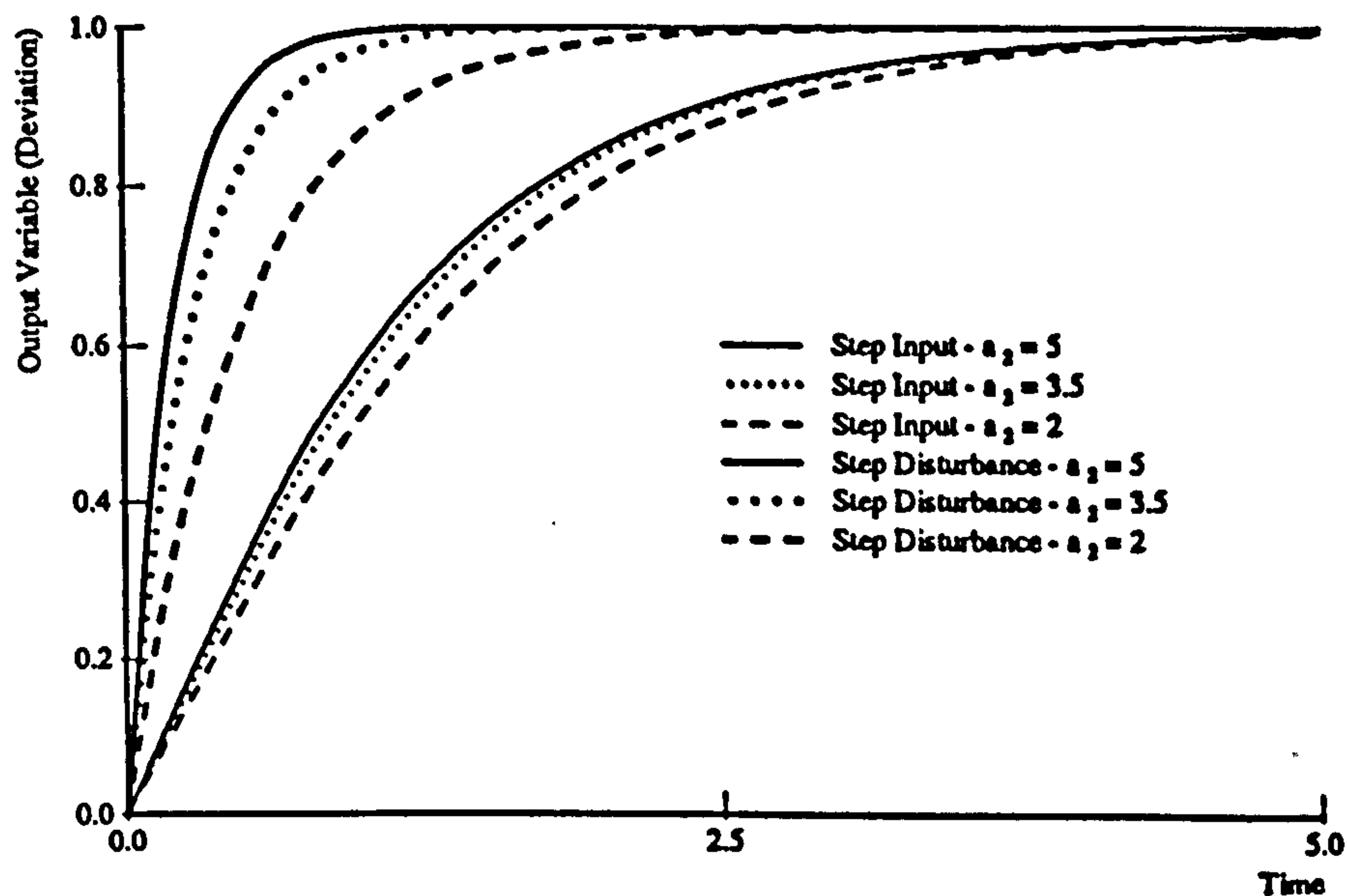


Figure 5.15: Open loop response of a simple system (given in Equation 5.44) for step changes in the manipulated input and the disturbance. The constant $a_1 = 1$ and a_2 is given the values 5, 3.5 and 2.

What is of interest is the regulatory response of this system with a full state feedback modal controller (linear equivalent of a geometric nonlinear controller). For this case an external PI regulator will also be used. The required controller is,

$$\begin{aligned} u &= \frac{v - \beta_0 c x - \beta_1 c ((a_2 - a_1)x_1 - a_2 x_2)}{\beta_1(a_1 a_2 - a_1^2)} \\ v &= -k(1 + \frac{1}{\tau s}) \end{aligned} \quad (5.46)$$

The closed loop response of this system is found easily from Equations 5.8 and 5.14,

$$y(s) = \frac{ks + \frac{k}{\tau}}{\frac{k}{\tau} + (\beta_0 + k)s + \beta_1 s^2} y_p(s) + \frac{\beta_1 a_2 s}{\frac{k}{\tau} + (\beta_0 + k)s + \beta_1 s^2} d(s) \quad (5.47)$$

The control action required to reject disturbances is obtained from Equation 5.17.

$$\frac{u(s)}{d(s)} = \frac{2a_2(a_1 + s)(-\frac{k}{\tau} - \beta_0 s + \beta_1 a_2 s - ks)}{a_1(2a_2 + s)(\frac{k}{\tau} + (\beta_0 + k)s + \beta_1 s^2)} \quad (5.48)$$

To obtain some sample responses the following open and closed loop parameters were used,

$$\begin{aligned} a_1 &= 1 \\ \beta_0 &= \beta_1 = 1 \\ k &= \tau = 1 \end{aligned} \quad (5.49)$$

The trajectories of the output and manipulated input for this system, with various values for a_2 , are given in Figure 5.16.

From these results it can be seen that as the speed of response of the disturbance in the open loop system is increased (relative to the speed of response of the manipulated input), a greater amount of control action is required by the closed loop system.

These characteristics are seen for the case study. Figure 5.17 shows the open loop response of the full nonlinear model to a step change in steam flowrate from 24.2 g/s to 20.3 g/s and a step change in the disturbance input, y_{c_2} , from 4.0 % CO₂ to 4.5 % CO₂.

As can be seen, the speed of response to the disturbance input is much quicker. As a side point it is interesting to ask what characteristics of a system enable the disturbance to have a faster effect than the manipulated input. The simplest way is to look at the graph of the system. For example, the graph of the simple system presented in Equations 5.43 and 5.44 is given in Figure 5.18. The important point is that there exists at least one path between the disturbance input and the output where the speed of response between each node is relatively faster than the slowest speed of response existing between any two nodes in the fastest path between the manipulated input and the output.

This characteristic is seen in the graph of the CO₂ absorption desorption system (Figure 4.4). It can be shown that the speed of response between each node of the following path, $y_{c_2} \rightarrow y_{a_1} \rightarrow y_{a_2} \rightarrow Y$ is relatively fast. However, all paths between the steam flowrate S and the output Y must pass along the directed path $C_r \rightarrow C_a$, and the dynamics associated with this link are significant.

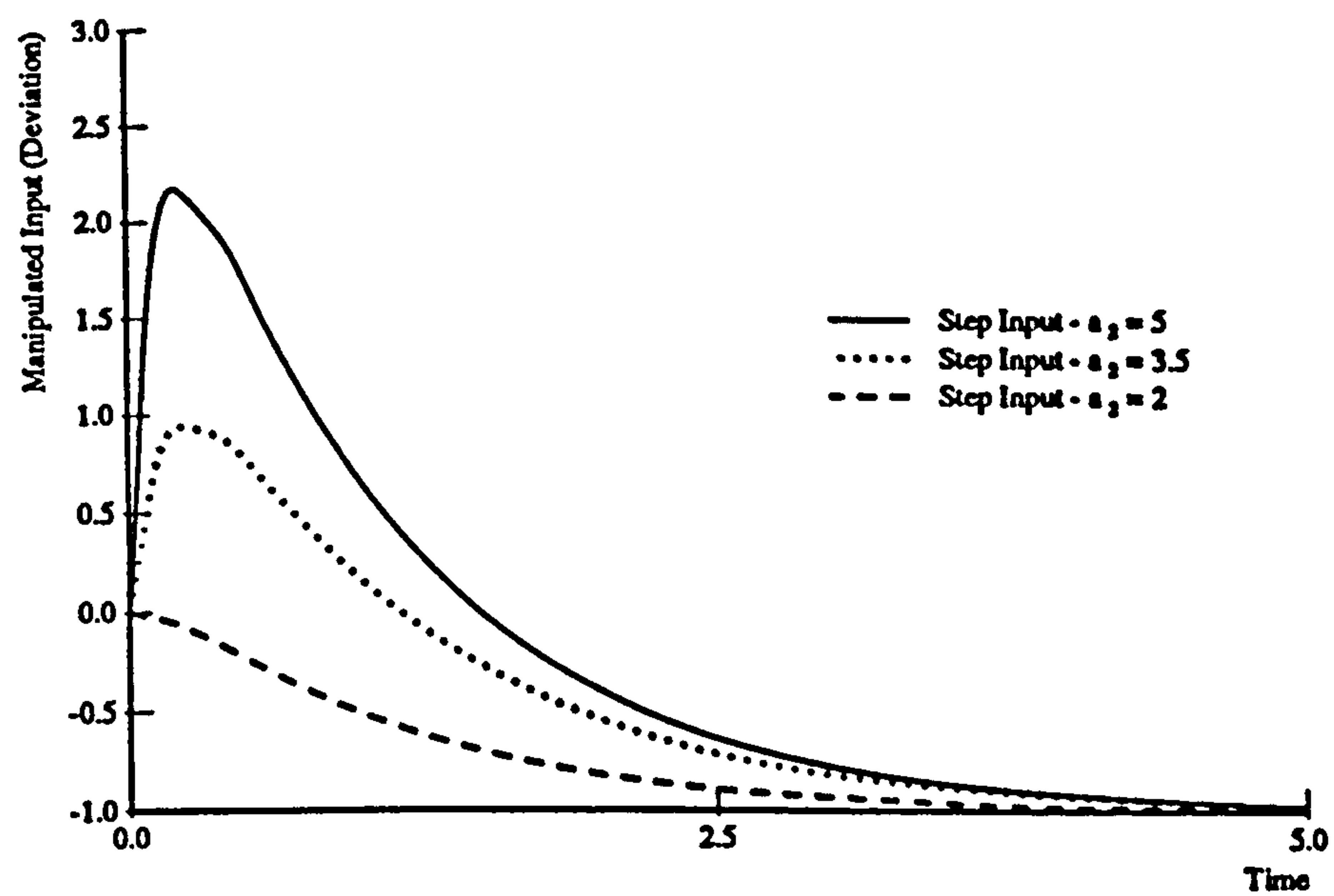
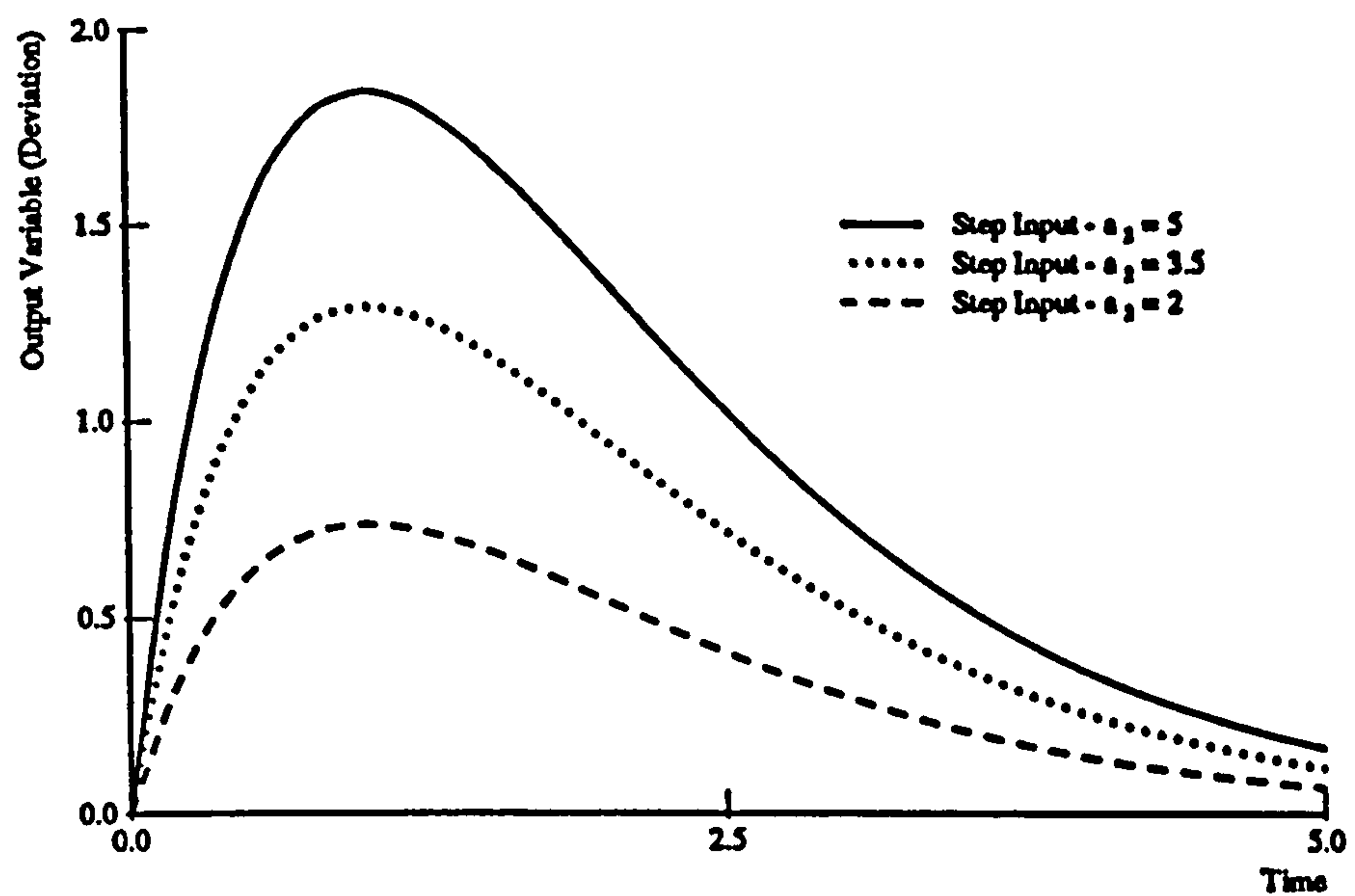


Figure 5.16: Response of a simple closed loop system (open loop system given in Equation 5.44) with an external PI regulator, for step changes in the disturbance. The constant $a_1 = 1$ and a_2 is given the values 5, 3.5 and 2. The controller constants are: $\beta_0 = \beta_1 = k = \tau = 1$.

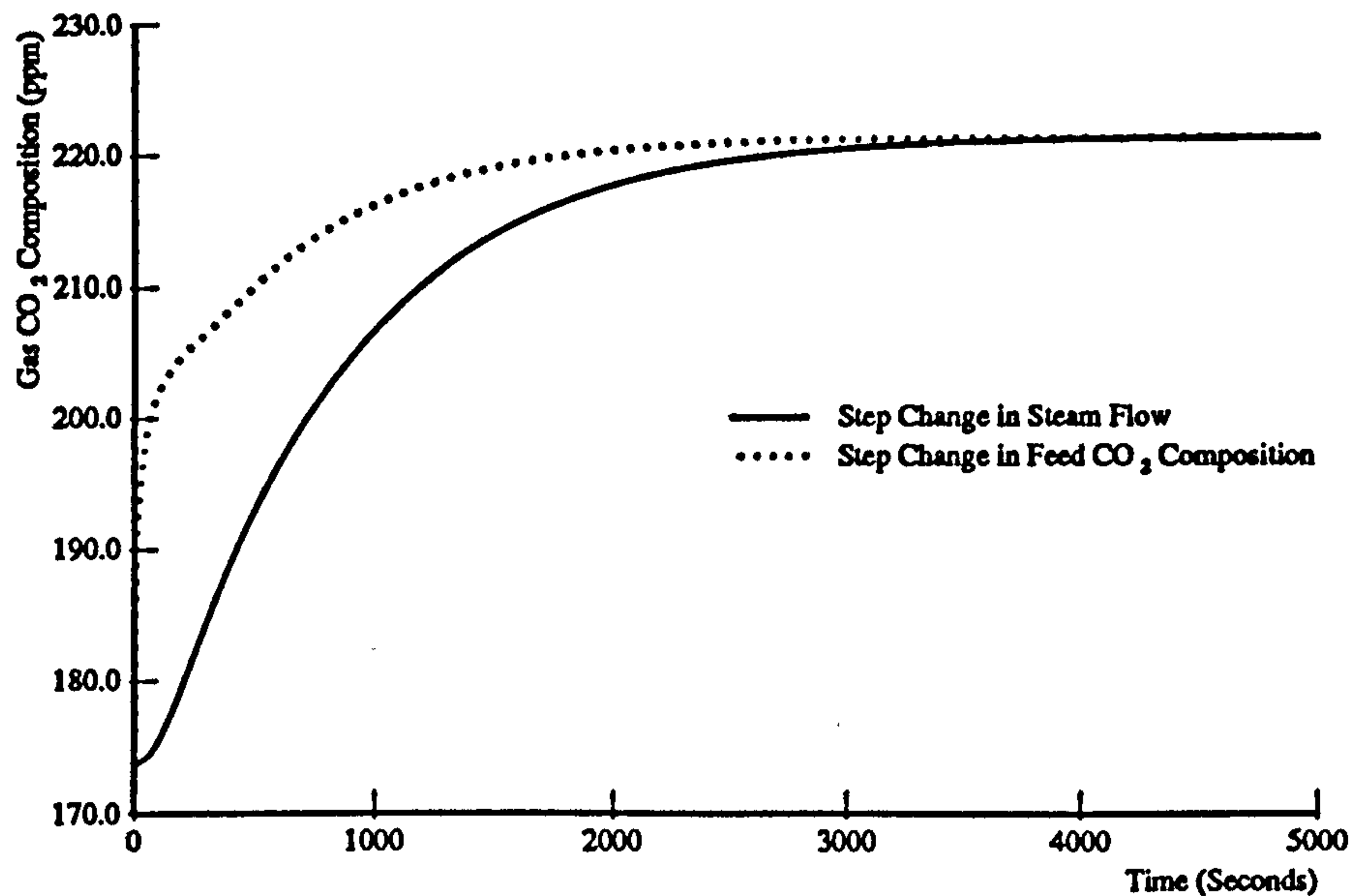


Figure 5.17: Simulated open loop response of the for step a change in the steam flowrate from 24.2 g/s to 20.3 g/s and for a step change in the feed CO₂ composition from 4.0 % CO₂ to 4.5 % CO₂.

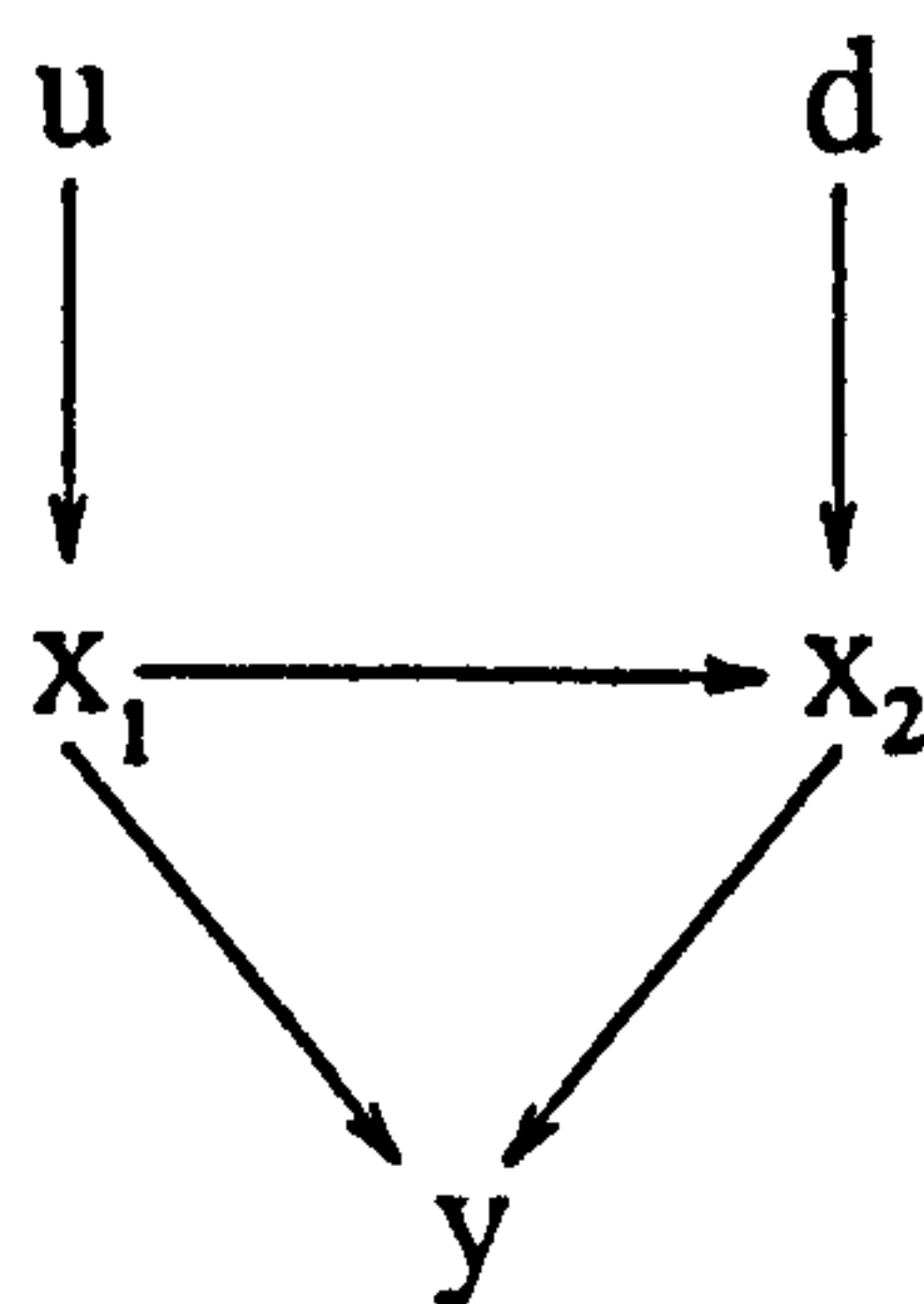


Figure 5.18: System graph for the simple system given in Equation 5.44.

5.8 Geometric Nonlinear Controller with Open Loop Observation

A problem for the geometric nonlinear control technique is identified in the previous section. The problem arises because the range of the output and the range of the manipulated input are dependent on the specified closed loop poles of the nonlinear controller. As faster poles are specified a smaller range of the output is covered and a greater range of the manipulated input is required. The reverse is true if slower poles are specified. This provides a convenient method for tuning the controller for servo mode and these guidelines are discussed in Section 5.5.

However, altering the gain of the nonlinear controller does not have the desired effect on the regulatory mode of the closed loop system. The steady state offset for the nonlinear controller structure with no external PI regulation, given a permanent change in the disturbance or permanent error in the model, is inversely related to the product of the closed loop poles and is independent of the controller gain.

Due to the structure of the case study, it is shown that an acceptable steady state position for the closed loop system can only be obtained through the specification of very fast closed loop poles. Correspondingly, an unacceptably large range of the manipulated variable is required.

A controller with two characteristics is required.

1. The steady state offset of the controller to a permanent change in an unmeasured disturbance is related to the gain of the controller and is independent of the specified closed loop poles.
2. The response of the system to an unmeasured disturbance is similar to the servo response of the closed loop system.

Both of these objectives are achievable by augmenting the basic geometric nonlinear controller structure to include an open loop observer. Such a structure is given in Figure 2.10, for the cases of an external P and PI regulator. The following discussion will demonstrate the characteristics of the structure with an open loop observer by looking at the output and manipulated input trajectories for the servo and regulatory mode of the linear approximation of the closed loop system. This will be followed by a discussion of the steady state position of the full nonlinear system.

5.8.1 Output Response

This section considers the trajectory of the output response of the nonlinear controller structure which includes an open loop observer. For the case of the plant and controller models both equalling the linear approximation in Equation 5.3, the closed loop response of the output for the two configurations in Figure 5.1 are,

P Regulation

$$y(s) = \frac{\xi_o k}{(\beta_0 + \xi_o k) + \beta_1 s + \dots + \beta_r s^r} y_{sp}(s) + \frac{c(sI - A)^{-1} w(\beta_0 + \dots + \beta_r s^r)}{(\beta_0 + \xi_o k) + \beta_1 s + \dots + \beta_r s^r} d(s) \quad (5.50)$$

PI Regulation

$$y(s) = \frac{\xi_o(k s + \frac{k}{\tau})}{\xi_o \frac{k}{\tau} + (\beta_0 + \xi_o k)s + \beta_1 s^2 + \dots + \beta_r s^{r+1}} y_{sp}(s) + \frac{c(sI - A)^{-1} w(\beta_0 + \dots + \beta_r s^r)}{\xi_o \frac{k}{\tau} + (\beta_0 + \xi_o k)s + \beta_1 s^2 + \dots + \beta_r s^{r+1}} d(s) \quad (5.51)$$

where,

$$\xi_o = cA^{r-1} \left(I - A(sI - A + \frac{bcA^r}{cA^{r-1}b})^{-1} \right) b \quad (5.52)$$

For the specific case of Equation 5.4 these equations simplify to,

P Regulation

$$y(s) = \frac{k}{k + \beta_0 + \beta_1 s + \beta_2 s^2 + \beta_3 s^3} y_{sp}(s) + \frac{wc(\beta_0 + \beta_1 s + \beta_2 s^2 + \beta_3 s^3)}{(k + \beta_0 + \beta_1 s + \beta_2 s^2 + \beta_3 s^3)(s + a_1)(s + a_2)(s + a_3)} d(s) \quad (5.53)$$

PI Regulation

$$y(s) = \frac{k s + \frac{k}{\tau}}{\frac{k}{\tau} + (k + \beta_0)s + \beta_1 s^2 + \beta_2 s^3 + \beta_3 s^4} y_{sp}(s) + \frac{wc(\beta_0 + \beta_1 s + \beta_2 s^2 + \beta_3 s^3)}{(\frac{k}{\tau} + (k + \beta_0)s + \beta_1 s^2 + \beta_2 s^3 + \beta_3 s^4)(s + a_1)(s + a_2)(s + a_3)} d(s) \quad (5.54)$$

As can be seen from these trajectories, the closed loop response of the system to an unmeasured disturbance is dependent on the poles of the open loop system. This limits the response of the system to the open loop response for changes in the manipulated input, for both servo and regulatory modes of the nonlinear controller. This is a limitation in terms of the achievable performance. However it means that the overall speed of the regulatory response is independent of the specified closed loop poles and the speed of response to changes in the unmeasured disturbance. Therefore, it is expected that an acceptable manipulated input trajectory will also be obtained.

5.8.2 Nonlinear Steady State Position

The nonlinear steady state position of the generic nonlinear system given in Equation 5.1 controlled by a nonlinear controller and open loop observer based on the following model (ie. both model mismatch and unmeasured disturbances exist),

$$\begin{aligned}\dot{\hat{x}} &= \hat{f}(\hat{x}) + \hat{g}(\hat{x})u \\ \hat{y} &= \hat{h}(\hat{x})\end{aligned}\tag{5.55}$$

with an open loop observer and P regulator the steady state position of the output of the plant is given by,

$$y = y_{sp} - \frac{\beta_0}{k} \hat{h}(\hat{x}|_{ss})\tag{5.56}$$

This equation holds for servo and regulatory modes of the closed loop system. Therefore, the steady state offset is controlled by the gain of the nonlinear controller and can be set independently of the poles of the closed loop system. At steady state, the predictions of the derivatives of the observer output will be zero because the inverse built into the geometric nonlinear controller is always exact for the observer model. The effects of model mismatch and unmeasured disturbances are accounted for by the error between the output of the observer and the required setpoint.

5.8.3 Summary

A geometric nonlinear controller with the states obtained from an open loop observer allows the steady state offset of the closed loop system resulting from unmeasured disturbances and model mismatch to be related to gain of the controller. Most importantly, this specification is independent of the closed loop poles of the system. The effects of model

mismatch and unmeasured disturbances are accounted for by the error between the output of the observer and the required setpoint.

5.9 Conclusions

This chapter considers the tuning of geometric nonlinear controllers for servo and regulatory responses. These results demonstrate qualitatively that the most desirable output response for servo and regulatory actions is obtained if the specified closed loop poles are set to be fast. However, the speed of response is limited by saturation limits on the control action. (Note: Saturation limits are the main issue for the case study, though in other cases the speed of response limit may be stability).

It is demonstrated that the effect of model mismatch is similar to the effect of unmeasured disturbances and equivalent relative orders can be defined for various model error terms. When direct state feedback is used, these errors are propagated through the derivative expressions in the control law. As a result the offset of the closed loop system (steady state offset when no integral action is used and dynamic offset when an external PI regulator is used) is inversely proportional to the specified closed loop poles.

Based on these results a general set of tuning guidelines are designed, aimed at achieving maximum speed of response given saturation limits on the control action. These guidelines are then tested on the simulation of the case study, resulting in a good servo response. However, a suitable regulatory response is not obtainable. It is demonstrated that this is due to two factors. Firstly, the disturbance response on the system is much faster than the manipulated input response. A discussion shows the structural characteristics of such a system. Secondly, as already concluded, the effect of the unmeasured disturbance is related only to the specified closed loop poles. A suitable compromise between speed of response and the control action is not possible.

This problem is overcome by using an open loop observer to obtain the states. In this configuration the offset of the closed loop system is specified by the gain of the controller and is independent of the closed loop poles. This is because the effects of model mismatch and unmeasured disturbances are accounted through the difference between the output of the observer and the required setpoint.

The effectiveness of this configuration in simulation and in practice will be demonstrated in the next chapter.

Chapter 6

Experimental and Simulation Results

Chapter 3 describes the development of a mathematical description of the CO₂ absorption desorption system. This is followed in Chapters 4 by a discussion of how the geometric nonlinear control technique uses the model of the process to design a controller to work on a simulation of the process. Chapter 5 considers the tuning of the nonlinear controller, resulting in an augmented structure with an open loop observer supplying estimates of all the required states. At the completion of this work a controller is obtained which in principle is able to move the system between different setpoints, reject disturbances and remain stable under uncertainty.

Strict mathematical limits on the performance of the controller have not been obtained, due to limitations in stability theory for geometric nonlinear control and the difficulties in adequately representing and quantifying model mismatch. Therefore, the nonlinear controller is tuned using the simulation of the pilot plant and updated using experimental tests. The final control constants are used to test the controller over a wide range of operating conditions, on the pilot plant. Details regarding the experimental implementation of the nonlinear controller are given in Section 6.1. The experimental results are discussed in Section 6.2.

The results of the nonlinear controller are compared with results for a PID controller in Section 6.3.

6.1 Controller Implementation

This section details the implementation of the nonlinear controller and open loop observer for experimentation. A schematic of the implementation is given in Figure 6.1.

The pilot plant includes all the measurement devices, valves and transducers. The locations of the important equipment, including tag names, are given in Figure 3.7. The signals to and from the plant are analog signals in the range 4-20 mA.

These signals are controlled by a personal computer based control system called Paragon. This package manages the basic regulatory control loops, provides an operator interface and performs the basic data logging. The data within Paragon are handled as digital signals. The relevant measurements and regulatory control loops are detailed in

Control System

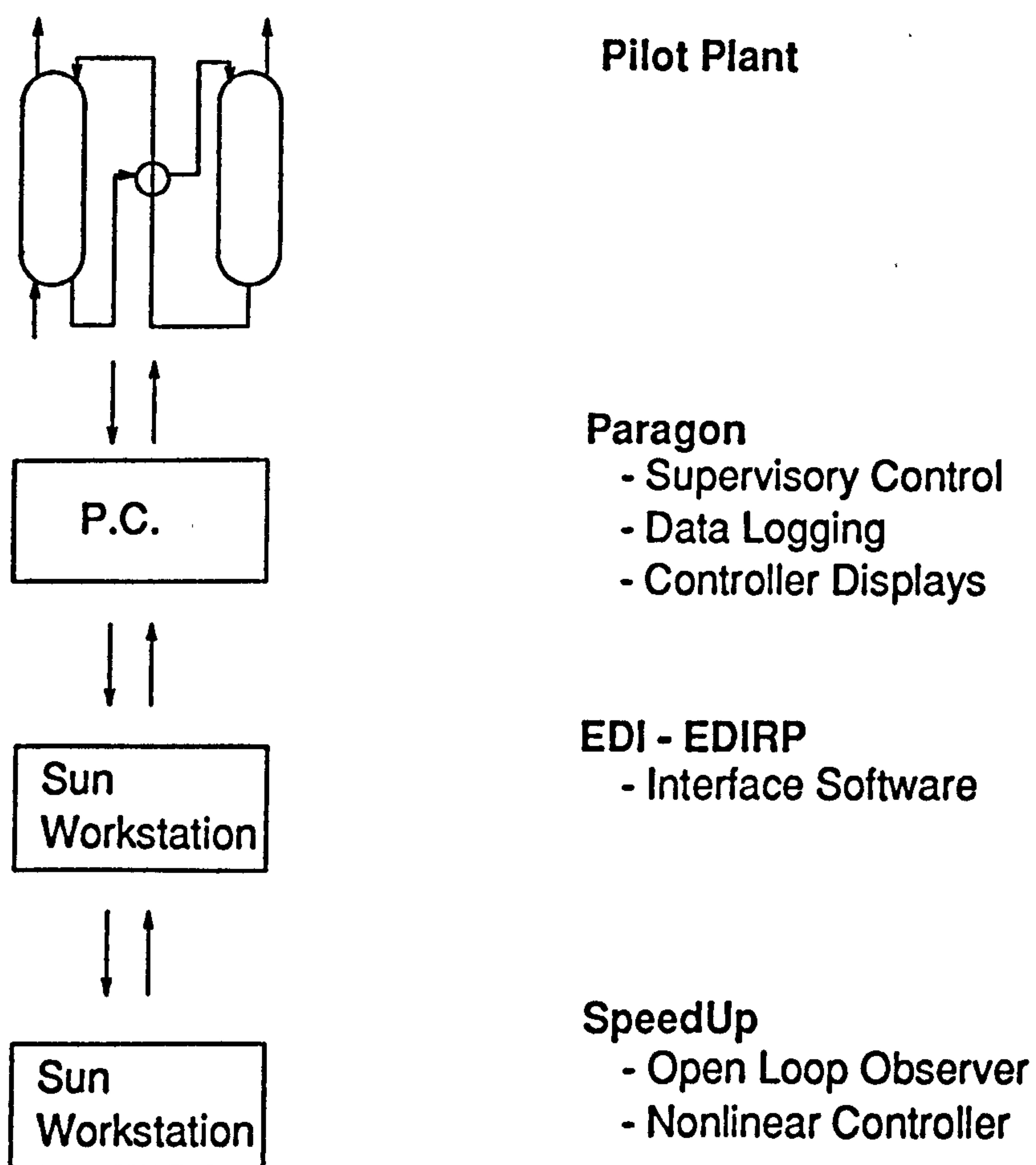


Figure 6.1: Experimental implementation of nonlinear controller.

Section 3.6.1. These functions are updated once every second.

The Paragon software allows great flexibility for designing basic supervisory controllers and operator displays. An existing Paragon environment performing the necessary supervisory control and providing basic operator interfaces was manipulated and added to as necessary to perform the experimental work in this thesis.

The open loop observer and nonlinear controller are implemented within Speedup, running on a Sun Sparc 1 computer. The nonlinear controller includes both the geometric controller and external PI regulator. This requires the setpoint and measurement of the product gas CO_2 composition, ADC 2. Speedup cascades the setpoint for the steam flowrate, ADF 3, to Paragon.

Data are transferred to and from Speedup through the external data interface routines (EDI). These data are transferred between Paragon and EDI, through routines specifically written for this application called EDIRP (external data interface, remote paragon). The data are transferred every eight seconds and the control actions are computed a single step ahead. This function is summarised in Figure 6.2.

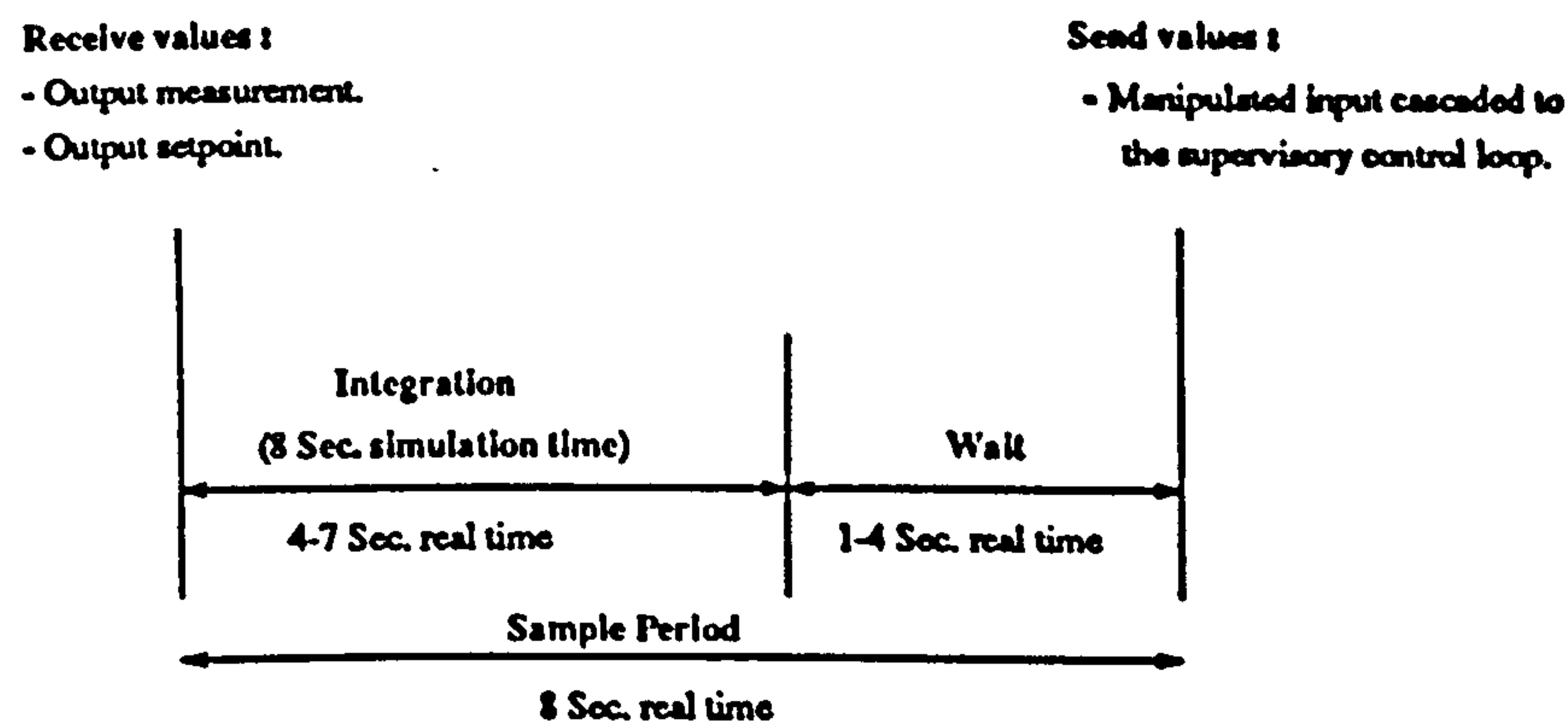


Figure 6.2: Real time data transfer in Speedup.

During the sample period the gas CO_2 composition measurement and setpoint are held constant and the model is integrated forward eight seconds in simulation time. The geometric nonlinear controller is included as part of the model within Speedup. Therefore, during this period the model of the plant (open loop observer) experiences a constantly varying steam flowrate, even though the inputs to the PI regulator are piecewise constant. The control action sent to the plant at the end of the sample period is based on the state measurements predicted for the end of the sample period. This procedure takes approximately four to seven seconds in real time, allowing a one to four second buffer during which Speedup waits before integrating further. This fast sample rate is chosen to avoid problems with a discrete implementation of the controller.

A summary of the simulation and plant values and the values transferred between the simulation and the plant are summarised in Figure 6.3.

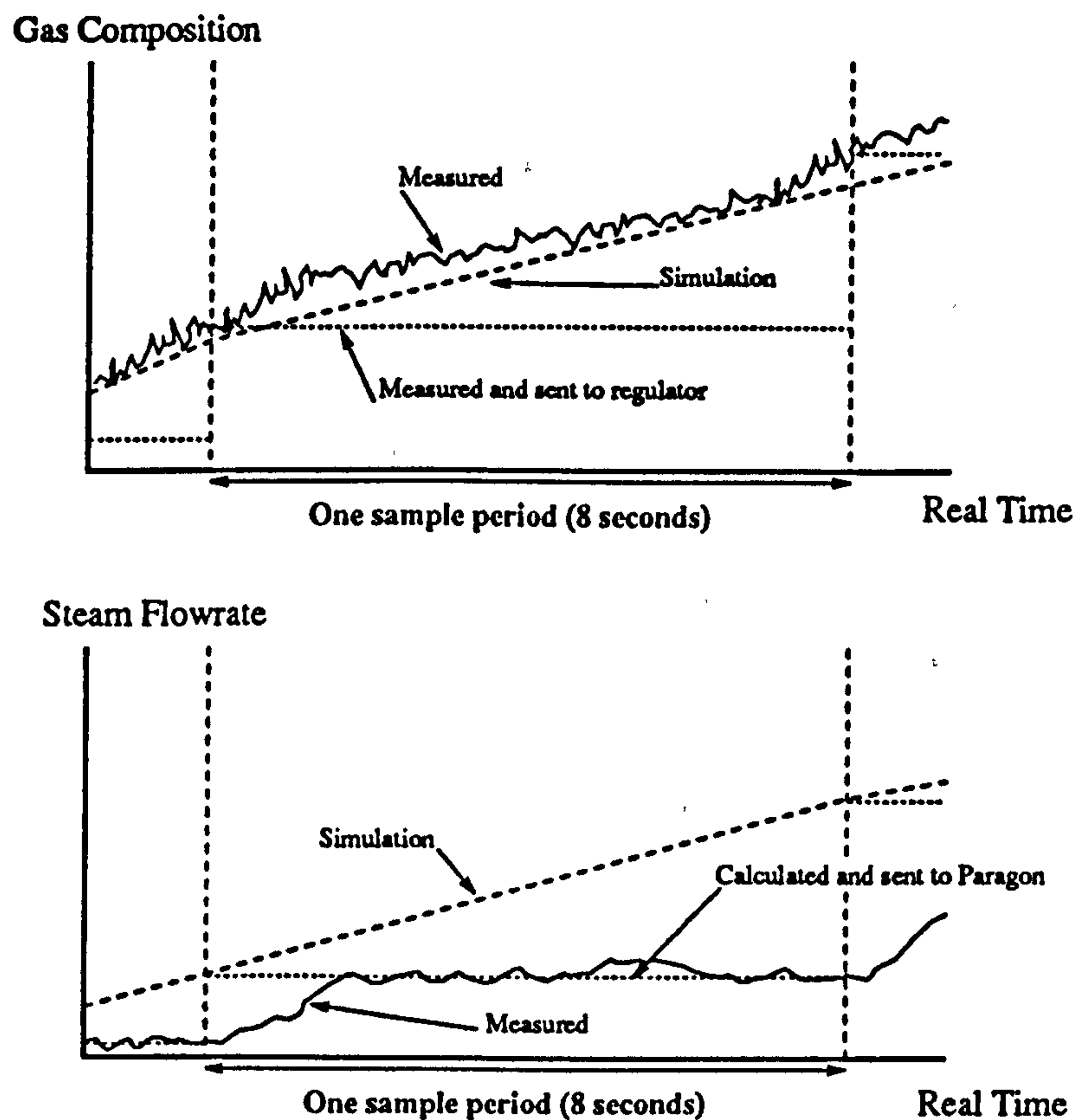


Figure 6.3: Plant and simulation values over time.

6.2 Nonlinear Controller

6.2.1 Simulation Results - Tuning

The nonlinear controller has been implemented with a simulation of the pilot plant within Speedup. The closed loop simulations have been used to tune the controller and test its operation for two setpoint changes and two unmeasured disturbances. All the tests start at the same nominal operating condition, corresponding to a steam flowrate of 24.2 g/s. At this condition, the simulation predicts a gas CO₂ composition of 174 ppm.

The final control arrangement allows the specification of four degrees of freedom: two poles for the transformed (linearised) system; T_{reset} , the reset time for the external PI regulator, and the gain for the external PI regulator.

To obtain initial values for the tuning constants, the characteristics of the linear approximation of the open loop model at the nominal operating point (corresponding to a steam flowrate of 24.2 g/s) were found. These characteristics are given in Table 5.2.

The initial tuning of the nonlinear controller is based on the guidelines given in Section 5.5. The first step in the tuning procedure is to set the two poles of the transformed system approximately equal to the two slowest poles of the linear approximation at the nominal operating point. In this case they were set to $1/125 \text{ s}^{-1}$ and $1/1000 \text{ s}^{-1}$. To obtain a response similar to the nominal open loop system the external regulator was given a gain of 1.0 and a fairly slow reset time of 1000 sec. The responses of the system, with these tuning constants, to four experiments are displayed in Figures 6.4 to 6.7 (called closed loop 1). The experiments are respectively, a setpoint change from 174 ppm to 290 ppm, a setpoint change from 174 ppm to 120 ppm, a step change in the feed composition from 4.0 % to 3.0 % and a step change in the feed composition from 4.0 % to 4.5 %. For the setpoint changes, open loop results are also displayed, corresponding to step changes in the steam flowrate to values required to give steady state gas compositions of 290 ppm and 120 ppm.

As expected, the first setpoint change has a response very similar to the open loop system. The open loop system is faster for the set point change to 120 ppm as the open loop system responds faster to increased steam flowrates. Also, the response of the closed loop system to unmeasured disturbances is seen to be acceptable.

These results demonstrate fairly conservative closed loop characteristics. A faster response can be obtained at the expense of greater control action and some overshoot. Rather than adjusting the poles of the transformed system, the reset time of the external regulator was reduced to 750 sec. The responses of the closed loop system with the amended set of constants is also given in Figures 6.4 to 6.7 (Called closed loop 2).

In all cases the response of the system is faster, with a small amount of overshoot. The response of the closed loop system to a set point change to 290 ppm is significantly faster than the open loop response. The response to a setpoint change to 120 ppm is only marginally slower than open loop. The initial response to unmeasured disturbances is faster than the first set of controller constants, with a return to the setpoint occurring almost 2000 sec earlier. However, the ultimate return to steady state is still the same, as the error dynamics of the closed loop system are limited by the dynamics of the open loop observer. The range of steam flowrates required in each of the experiments is increased, with a short period of time in which the controller is saturated, for the setpoint change to 290 ppm.

Given the guidelines in Section 5.5 the second set of control values gives an acceptable response to the chosen setpoint changes and unmeasured disturbances.

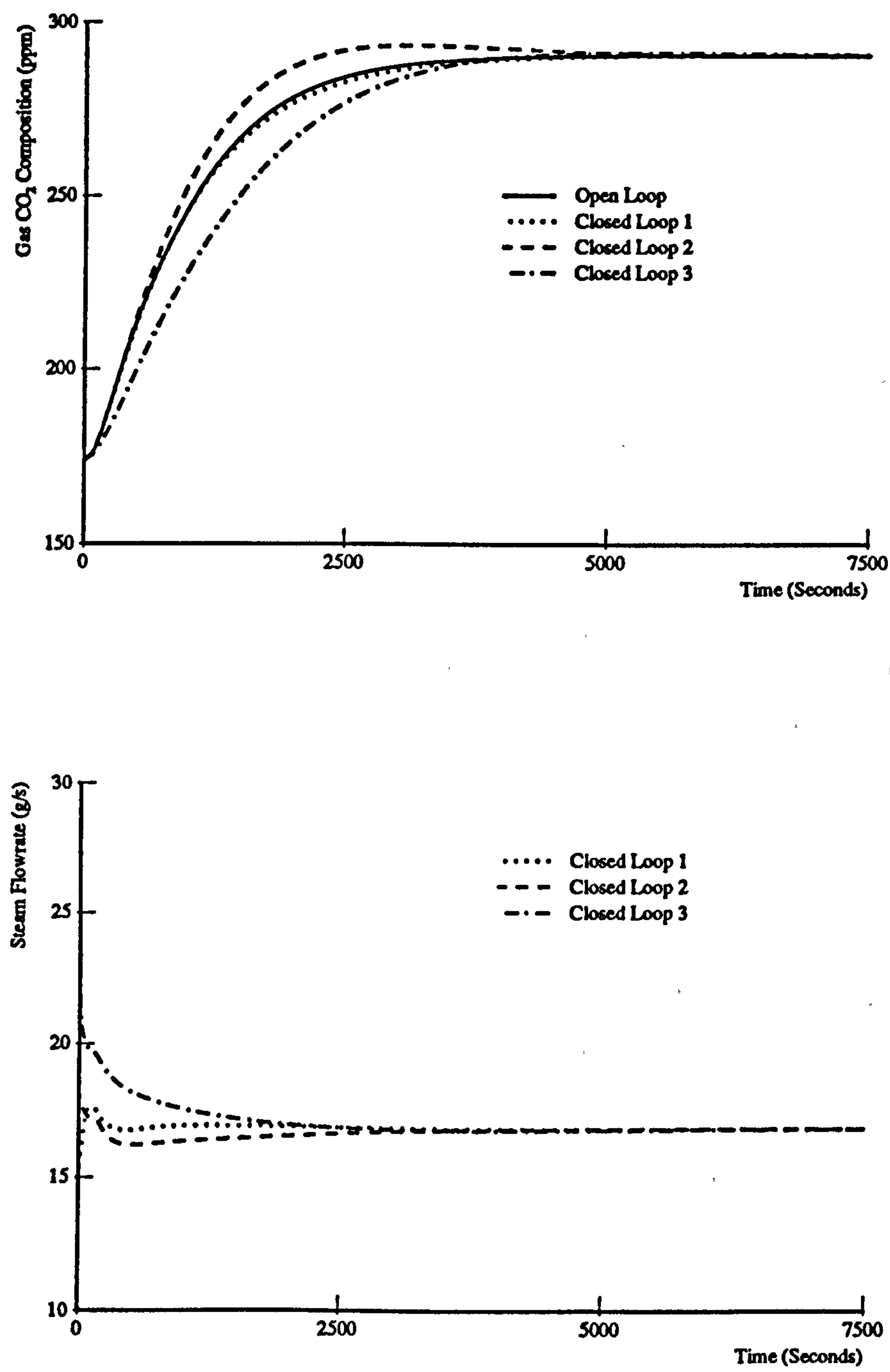


Figure 6.4: Tuning of nonlinear controller in simulation - Setpoint change from 174 ppm to 290 ppm CO₂.

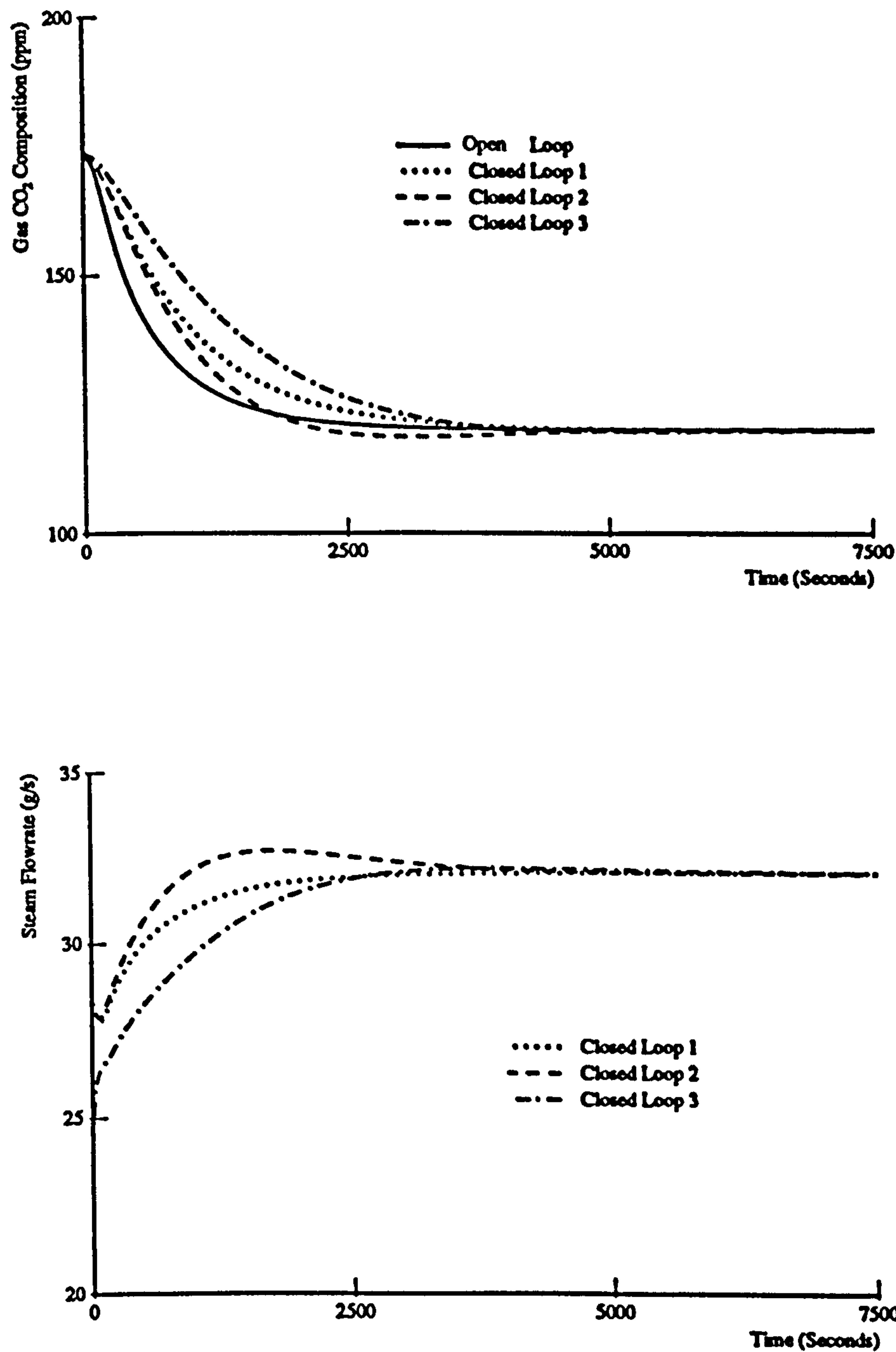


Figure 6.5: Tuning of nonlinear controller in simulation - Setpoint change from 174 ppm to 120 ppm CO₂.

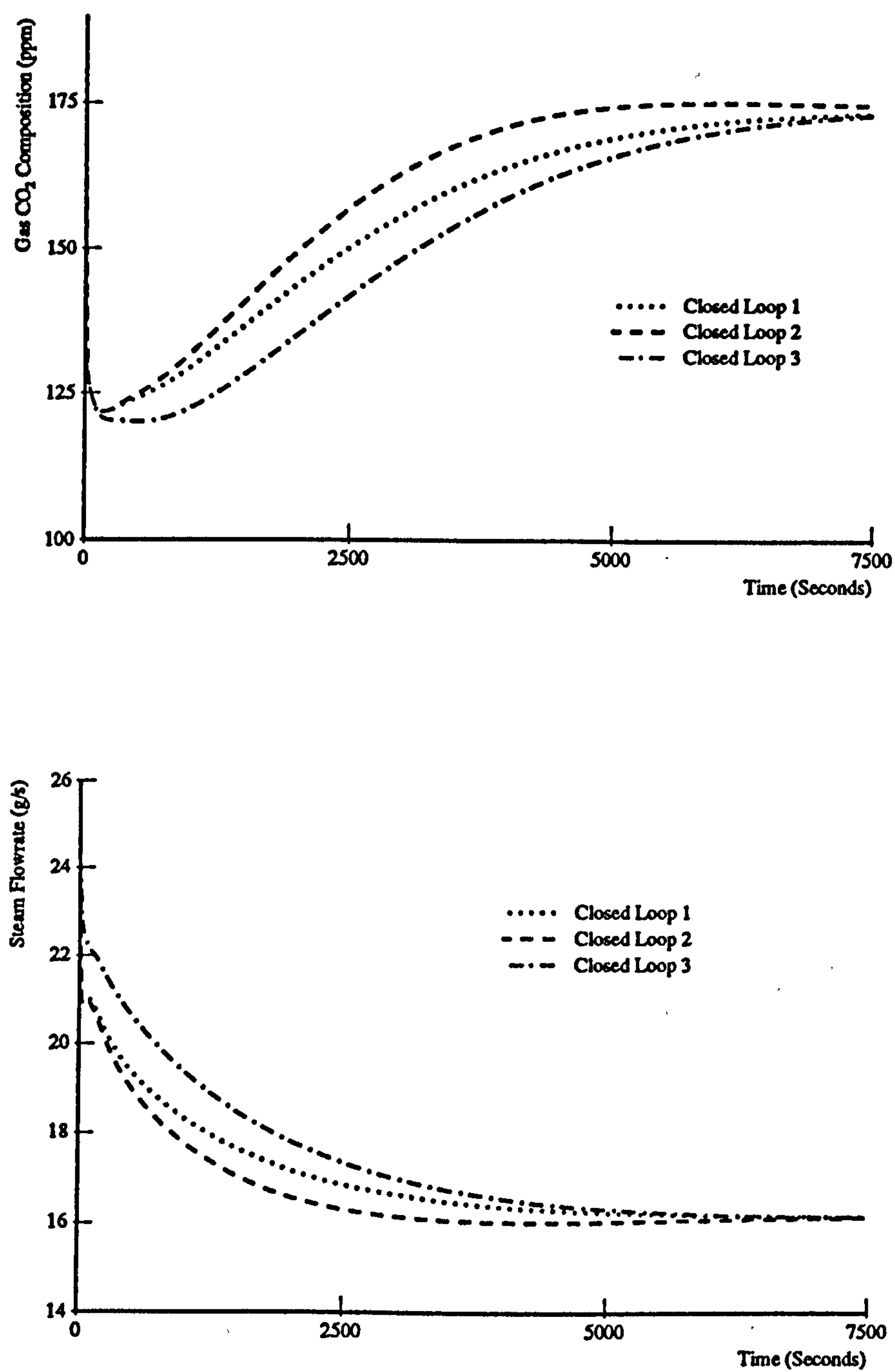


Figure 6.6: Tuning of nonlinear controller in simulation - Rejection of a step change in the unmeasured disturbance y_c from 4.0 % to 3.0 % CO₂.

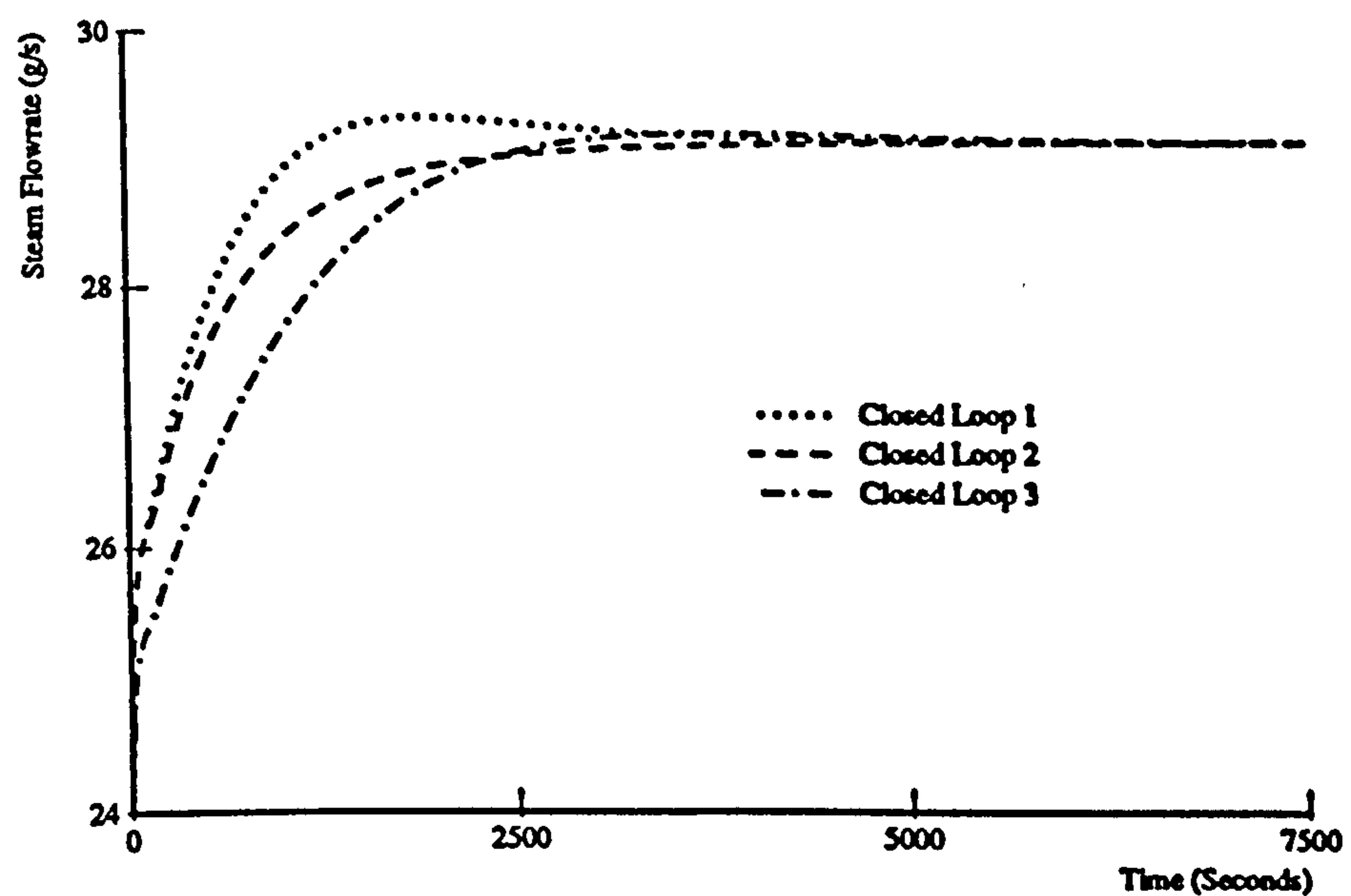
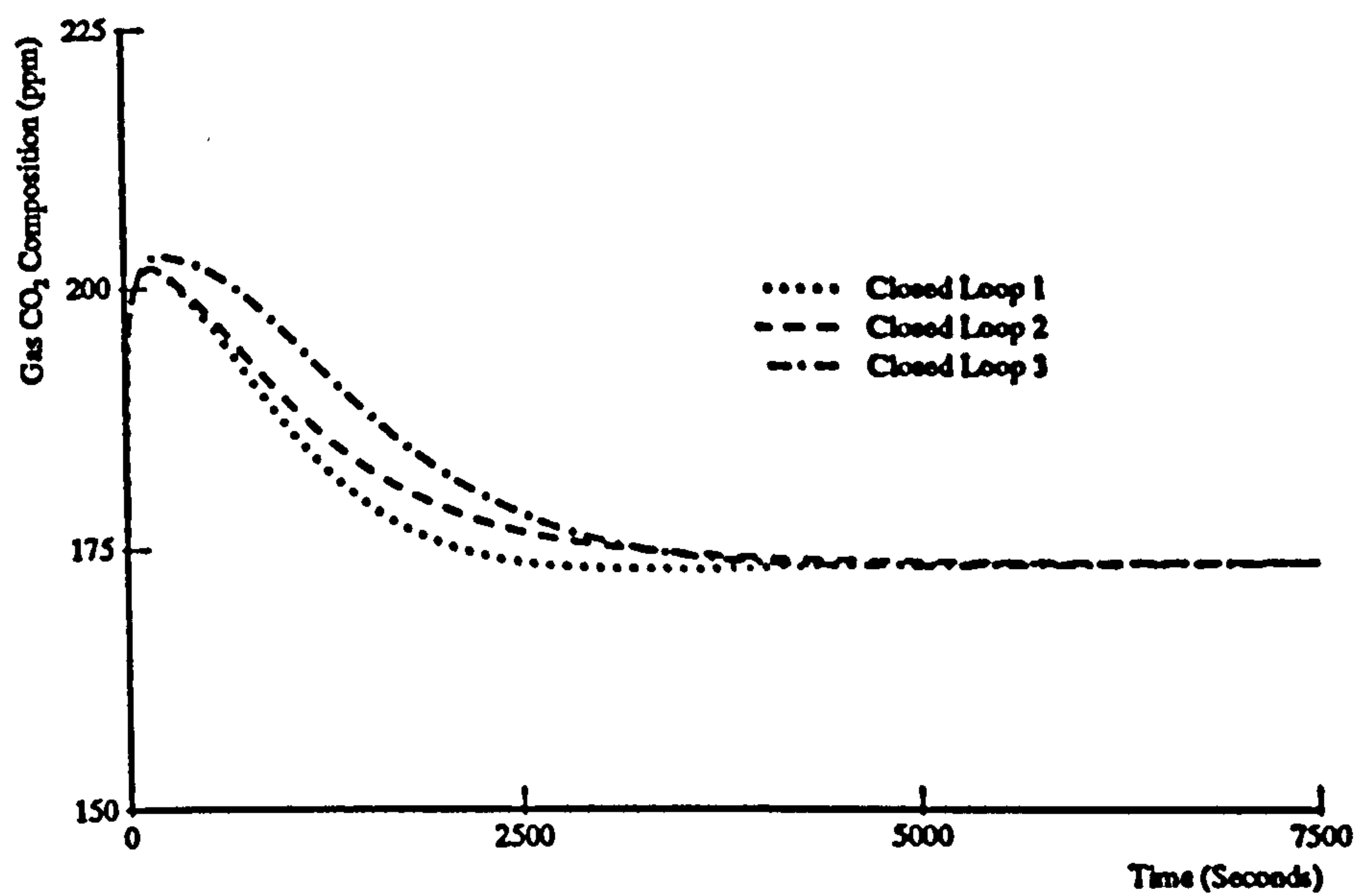


Figure 6.7: Tuning of nonlinear controller in simulation - Rejection of a step change in the unmeasured disturbance y_{c_1} from 4.0 % to 4.5 % CO₂.

6.2.2 Experimental Results - Tuning

The previous section resulted in the following values for the tuning constants for the nonlinear transformation and external regulator: poles at $1/125 \text{ s}^{-1}$ and $1/1000 \text{ s}^{-1}$, external regulator gain = 1.0 and reset time = 750.0 sec. These values were tested experimentally on the pilot plant by introducing a setpoint change to 250 ppm. The plant was started up with a steam flowrate of 24.2 g/s and a corresponding gas composition of approximately 175 ppm. The closed loop response of the system is given in Figure 6.8. The response appears to be marginally stable, though very oscillatory. The decrease in performance for the experimental implementation is mainly due to the unmodelled dead time in the pilot plant. This is noticeable in the difference between the response of the plant and the open loop observer (See Figure 6.8).

To improve the response of the closed loop system the controller was detuned by decreasing the gain of the external regulator and leaving the poles of the transformed system constant. The maximum overshoots of the experimental results were compared to the maximum overshoots of the simulation results to give an indication of the reduction in performance resulting from the experimental implementation. Using this as a rough guideline it was decided to limit the overshoot of the closed loop in simulation to 2 ppm for a setpoint change to 250 ppm, aiming to achieve a maximum overshoot of about 10 ppm (approximately 4 % of the final value) in the experimental system.

The gain of the external PI regulator was adjusted to 0.6, to achieve a maximum overshoot of 2 ppm for a setpoint change to 250 ppm in simulation. The simulated response of the closed loop system with the new gain is shown in Figures 6.4 to 6.7 (Called closed loop 3). Note that a reduction in the overshoot of the signal is accompanied by a reduction in the speed of response.

The experimental results with the transformed poles at $1/125 \text{ s}^{-1}$ and $1/1000 \text{ s}^{-1}$, external regulator gain = 0.6 and reset time = 750 sec, are shown in Figure 6.9. Now the closed loop system is seen to be stable with a maximum overshoot of about 15 ppm (approximately 6 % of the final value, which is only slightly higher than predicted). This represents a good response to a setpoint change and these controller constants are used for the remainder of the experimental results.

6.2.3 Experimental Results - Servo Response

To test the effectiveness and stability of the controller two experiments involving setpoint changes were performed. In each case the plant was started with a steam flowrate of 24.2 g/s and a corresponding gas CO₂ composition of about 175 ppm. Then the set

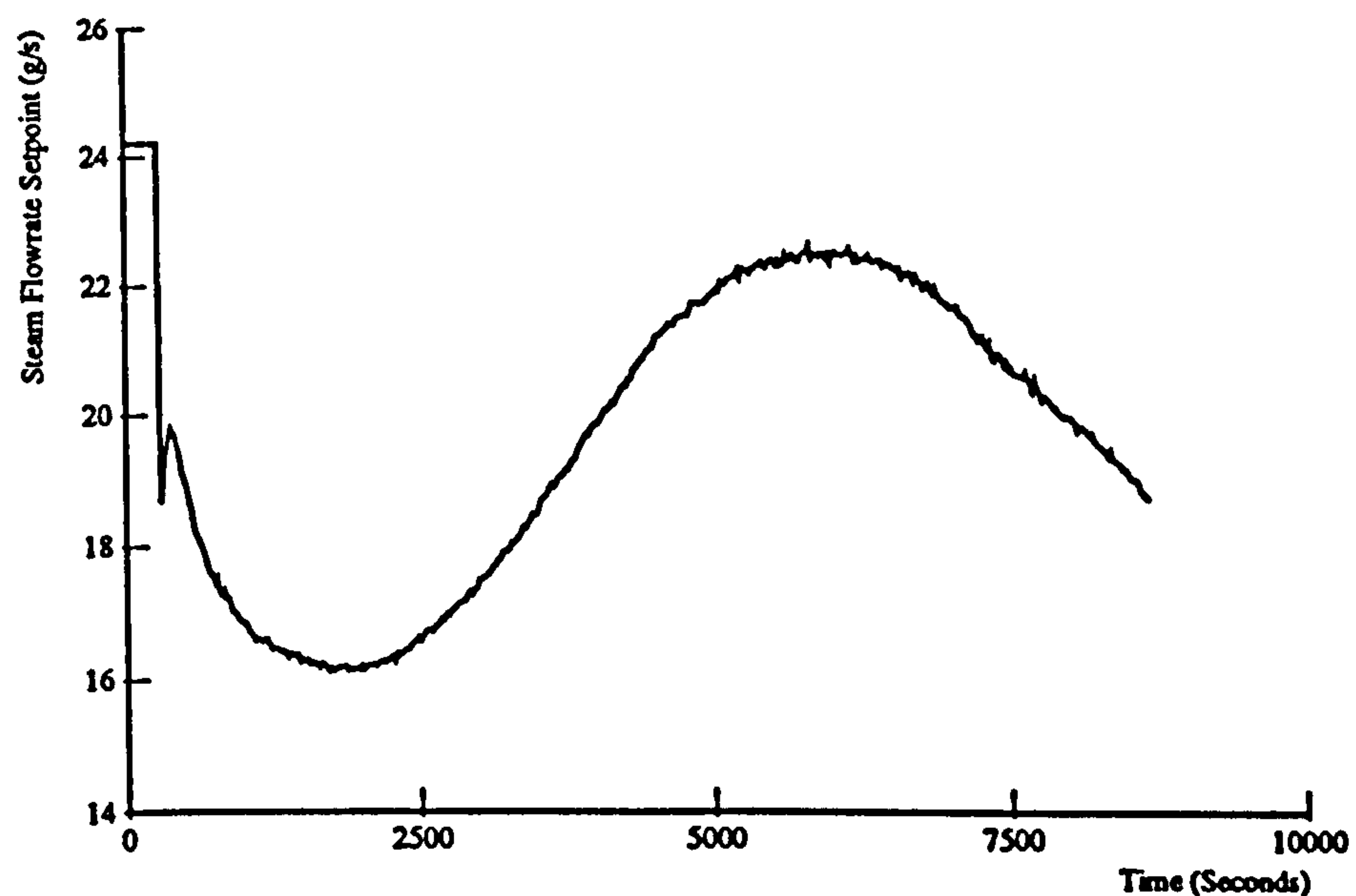
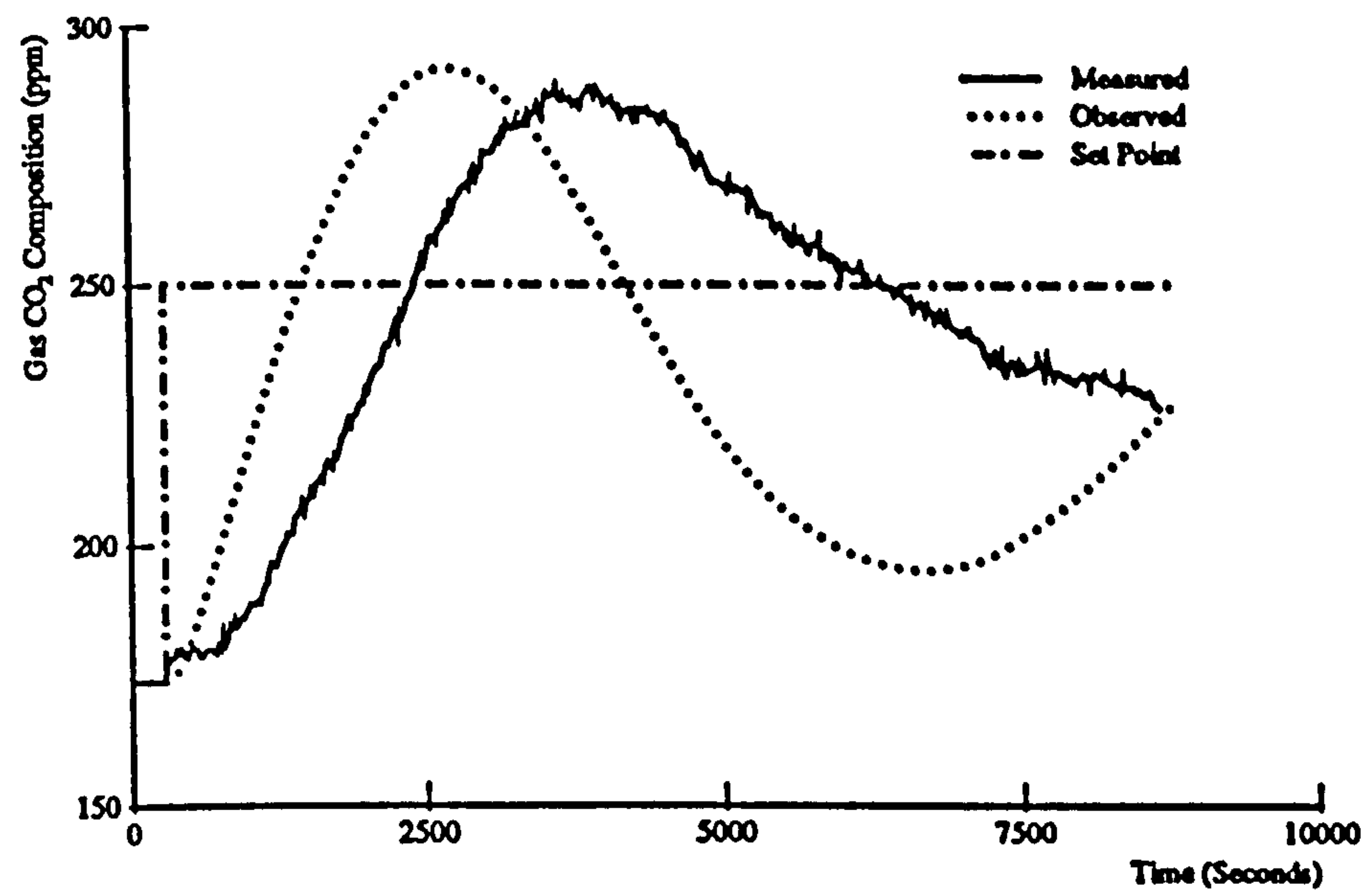


Figure 6.8: Tuning of nonlinear controller in experimentation - Setpoint change from 174 ppm to 250 ppm with gain = 1, $T_{reset} = 750$, poles = $1/125, 1/1000 \text{ s}^{-1}$.

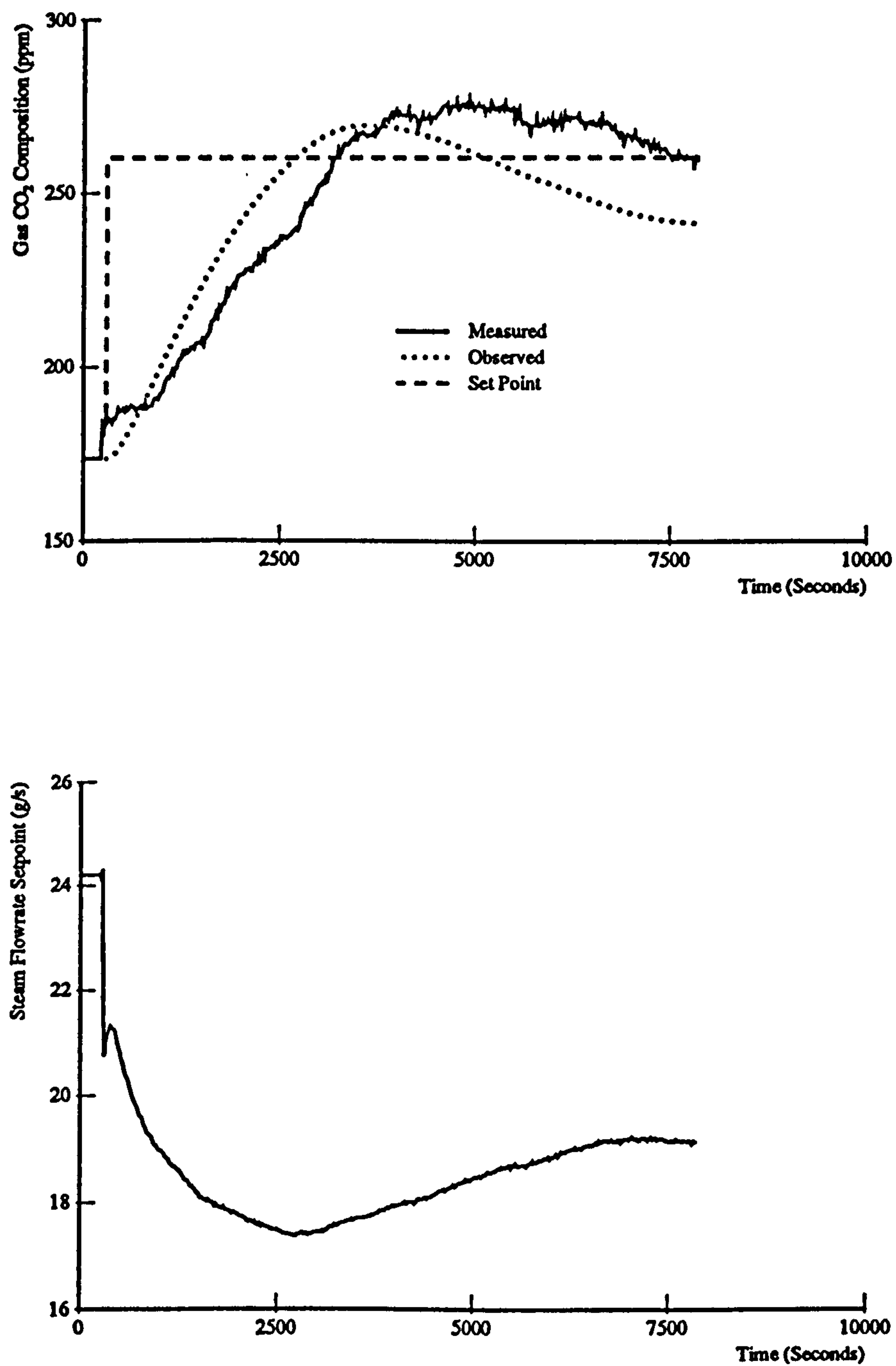


Figure 6.9: Tuning of nonlinear controller in experimentation - Setpoint change from 174 ppm to 260 ppm with gain = 0.6, $T_{reset} = 750$, poles = $1/125 \text{ s}^{-1}$, $1/1000 \text{ s}^{-1}$.

point was stepped to 260 ppm and 130 ppm. The response of the gas CO₂ composition and the output from the nonlinear controller (ie. steam flowrate setpoint) were recorded for both experiments. The results are given in Figure 6.10. The tuning parameters are discussed in the previous section and the results for the setpoint change to 260 ppm are the same as in Figure 6.9.

For the change in setpoint to 260 ppm, the gas composition is moved to 260 ppm in 3500 sec and overshoots by approximately 14 ppm (6 % of final setpoint), with the maximum overshoot occurring at 5000 sec.

For the change in setpoint to 130 ppm, the gas composition is moved to 130 ppm in 4000 sec and overshoots by approximately 9 ppm (7 % of final setpoint), with the maximum overshoot occurring at 6500 sec.

Despite a large change in gain over the range of steam flowrates used, the closed loop system demonstrates a relatively consistent servo response. That is, the new setpoint is first achieved approximately 4000 sec after the setpoint change, with a maximum overshoot of about 7 % occurring approximately 2000 sec later. This consistency is an effective demonstration that the major gain nonlinearity is removed by the geometric nonlinear controller.

To fully test the usefulness of the controller, the regulatory response must also be considered. This is done in the next section.

6.2.4 Experimental Results - Regulatory Response

To experimentally test the regulatory response of the nonlinear controller, two unmeasured changes in the feed CO₂ composition were introduced: from 4.0 % to 4.5 % and 3.0 %. The same controller configuration and tuning constants as used in the previous section have been used for the disturbance rejection responses.

For both experiments the plant was started up with a steam flowrate of 24.2 g/s and feed CO₂ composition of 4.0 % and allowed to come to steady state. The resulting outlet gas composition was used as the setpoint of the controller throughout the experiment. This method was used due to the difficulties in getting exactly the same steady state outlet CO₂ composition (to within 1 ppm) for two different runs, even with the same operating conditions. As a result the different experiments have slightly different setpoints.

The changes in the feed composition were made by step changing the set point of the controller between the inlet CO₂ composition (ADC1) and the rate of CO₂ addition (ADV3). The resulting change in the feed CO₂ composition is not a step and it takes approximately 3-4 minutes for the new composition to be finally obtained.

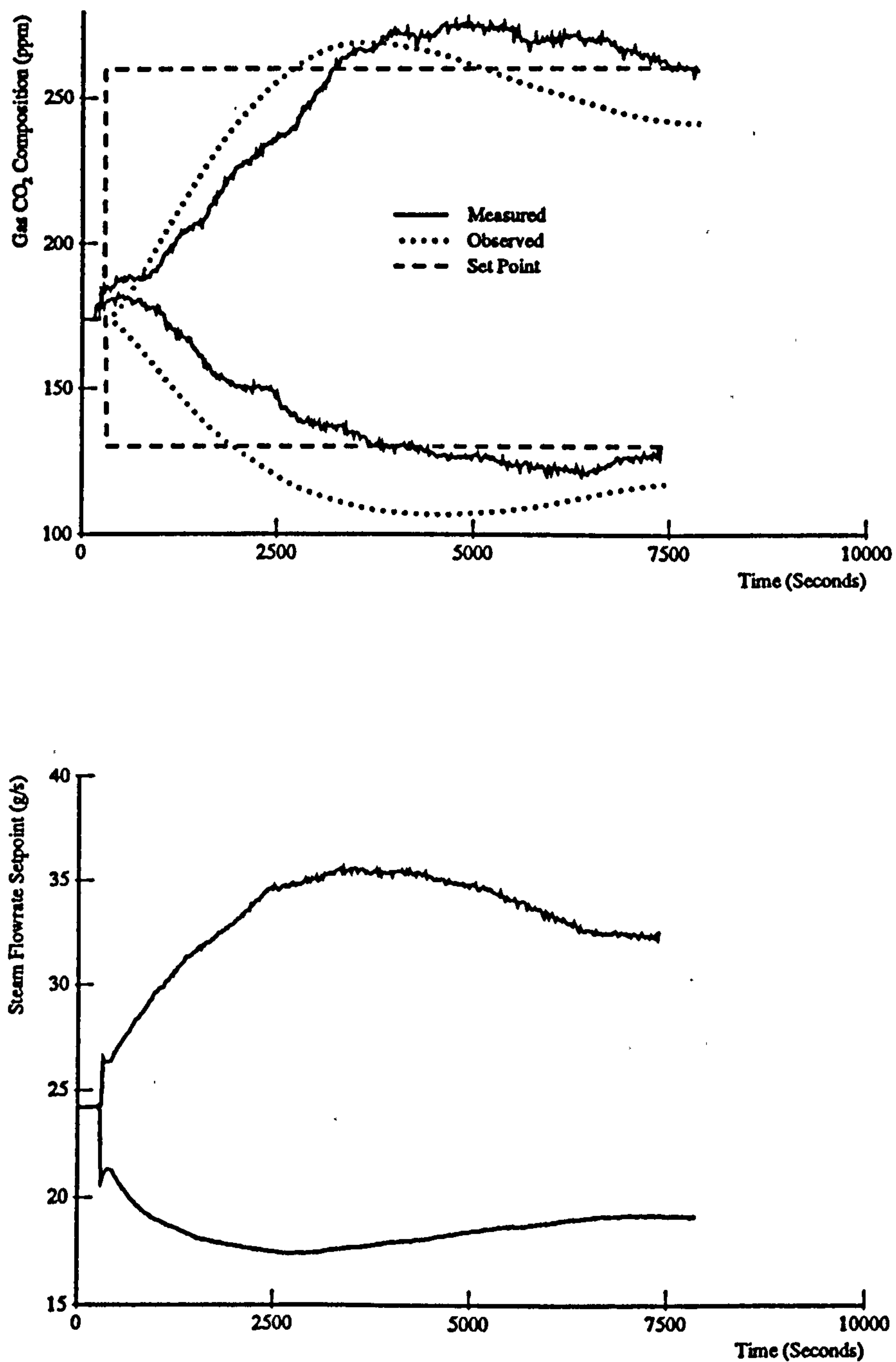


Figure 6.10: Nonlinear controller in experimentation - servo response : - Setpoint change from 174 ppm to 260 ppm and - Setpoint change from 178 ppm to 130 ppm with gain = 0.6, $T_{reset} = 750$, poles = $1/125 \text{ s}^{-1}$, $1/1000 \text{ s}^{-1}$.

The responses of the nonlinear controller to an unmeasured disturbance in the feed gas CO₂ composition from 4.0 % to 3.0 % CO₂ is given in Figure 6.11.

This response has a maximum deviation in the outlet CO₂ composition of -42 ppm occurring at 3000 sec, with the setpoint not achieved after 6000 sec, at which point the experiment was terminated. The observed gas composition increases to 295 ppm after 6000 sec. In this region the gain between the steam flowrate and outlet gas composition is much greater and the controller uses less control action. The plant is still operating in a region with an outlet composition of 130 ppm to 170 ppm, where the gain is not significantly changed. This mismatch in gain between the plant and prediction results in a slower response than might be expected.

The responses of the nonlinear controller to an unmeasured disturbance in the feed gas CO₂ composition from 4.0 % to 4.5 % is given in Figure 6.12.

This response has a maximum deviation in the outlet CO₂ composition of 25 ppm occurring at 2000 sec, with the setpoint achieved again at 4400 sec. The observed gas composition drops to 130 ppm, where the gain between the steam flowrate and outlet gas composition is much lower. The plant is still operating with an outlet gas CO₂ composition in the range 170 ppm to 200 ppm, where the gain is not significantly changed. The mismatch in gains between the plant and prediction results in greater control action and an artificially faster response.

The first point to be made from these results is that the geometric nonlinear controller is able to effectively reject the unmeasured disturbances, without excessive control action. Secondly, it can be seen that there is a significant difference in the speed of response of the closed loop system for the two regions of operation. As discussed, this is caused by the error between the plant and the observer over time. This phenomenon is also seen in the simulation results.

Despite these relatively large errors between the model and plant after unmeasured disturbances, the nonlinear controller is able to effectively reject disturbances. Complete disturbance rejection is potentially available by feeding a measurement of the disturbance both to the observer model and the geometric nonlinear controller. However, an experimental implementation of the nonlinear controller with disturbance decoupling is not considered in this thesis.

6.2.5 Conclusions

The robustness of the controller is demonstrated in these results because an effective regulatory response and consistent servo response is obtained despite,

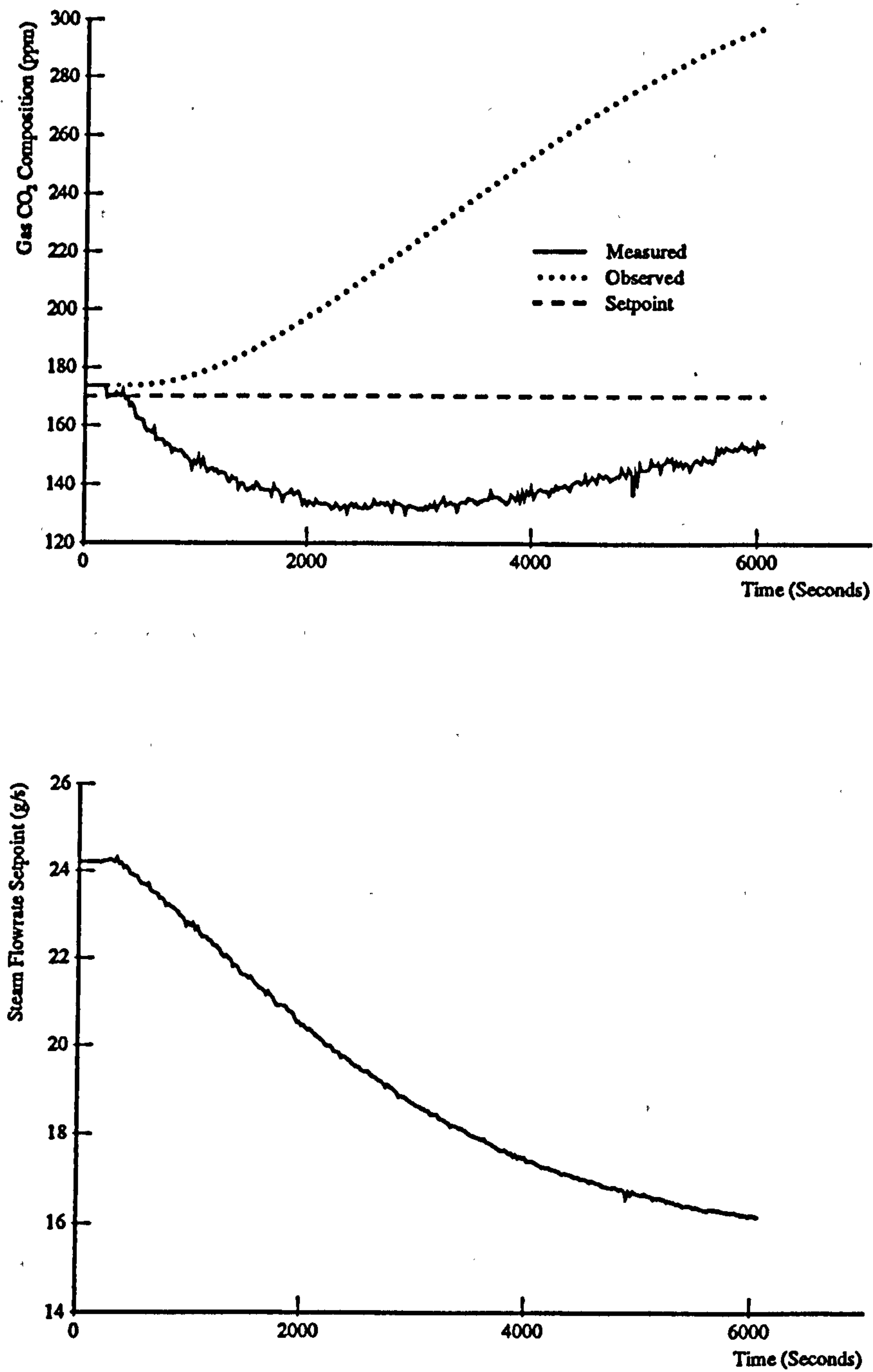


Figure 6.11: Nonlinear controller in experimentation - regulatory response : - Step change in feed composition from 4.0 % to 3.0 % CO₂ and - Setpoint at 176 ppm with gain = 0.6, $T_{reset} = 750$, poles = $1/125 \text{ s}^{-1}$, $1/1000 \text{ s}^{-1}$.

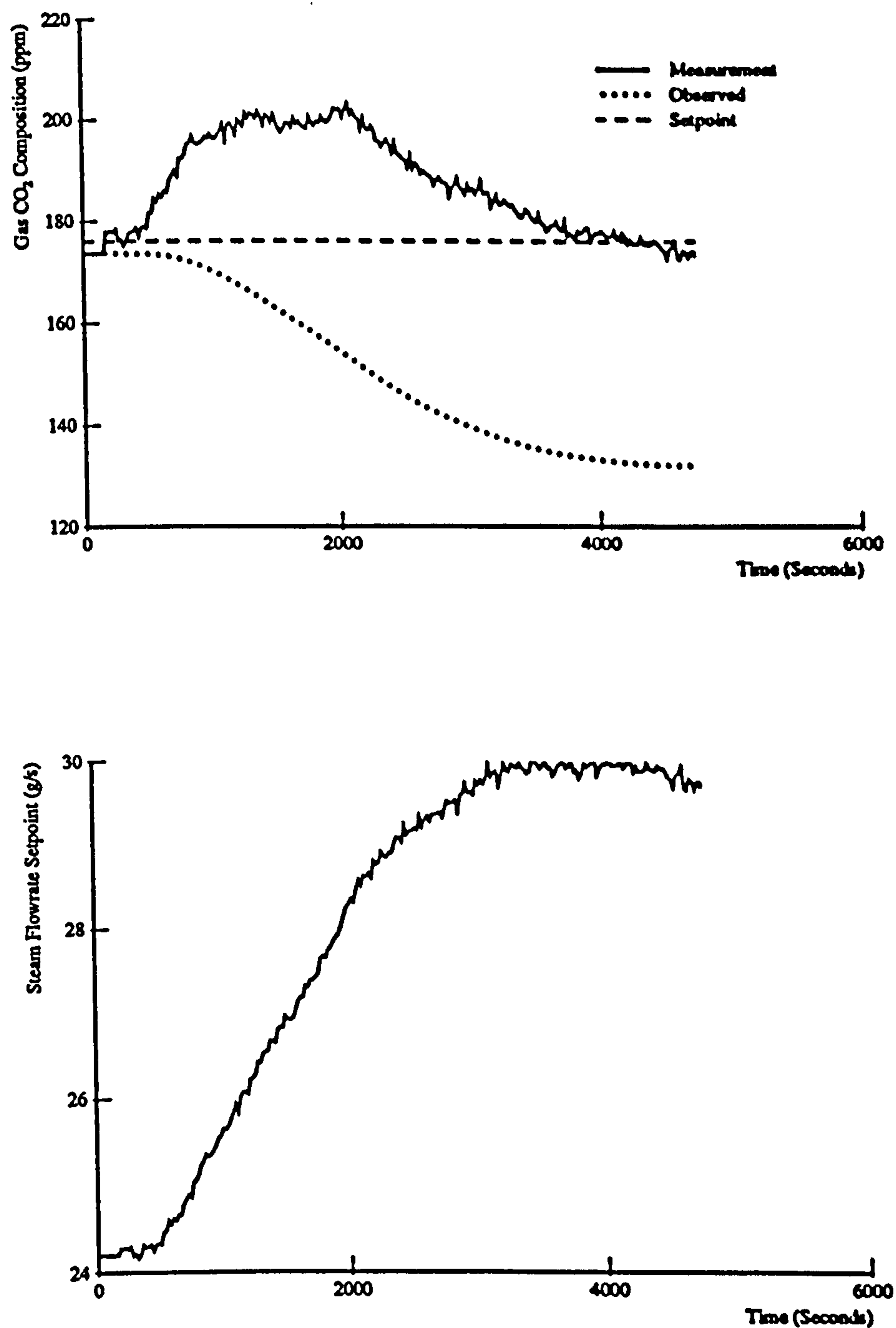


Figure 6.12: Nonlinear controller in experimentation - regulatory response : - Step change in feed composition from 4.0 % to 4.5 % CO₂ and - Setpoint at 170 ppm with gain = 0.6, $T_{reset} = 750$, poles = $1/125 \text{ s}^{-1}$, $1/1000 \text{ s}^{-1}$.

1. noisy measurement signal,
2. model mismatch between observer and the plant,
3. model mismatch between the model used for controller and the plant,
4. significant dead time in plant,
5. unmeasured disturbances.

6.3 PID Controller

6.3.1 Simulation Results - Tuning

As with the nonlinear controller, a PID controller was initially implemented within Speedup for tuning. The tuning was done in two steps. In the first step the IMC tuning rules were used to obtain an initial guess for the PID constants. In the second step these values were adjusted to obtain a closed loop response similar to the nonlinear controller for a setpoint change from 174 ppm to 290 ppm.

For the IMC tuning rules, the system characteristics of the linear approximation of the open loop model at a nominal operating point corresponding to a steam flowrate of 24.2 g/s were used. These characteristics are given in Table 5.2. The IMC tuning rules for a PID controller have been taken from Seborg *et al.*, 1989, which states that for a system with a transfer function,

$$G(s) = \frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)} \quad (6.1)$$

the controller constants are given as follows,

$$\begin{aligned} K_c &= \frac{\tau_1 + \tau_2}{\tau_c} \\ \tau_I &= \tau_1 + \tau_2 \\ \tau_D &= \frac{\tau_1 \tau_2}{\tau_1 + \tau_2} \end{aligned} \quad (6.2)$$

The two poles were taken to correspond to the real parts of the two slowest poles of the linear approximation at the nominal operating point. These values are -0.00146 s^{-1} and -0.00856 s^{-1} . The closed loop time constant is set to 1125 sec, to be approximately consistent with the nonlinear controller setting. This results in a gain of $-73.714 \text{ ppm/(g/s)}$, an integral time constant of 799.4 sec and a derivative time constant of 99.7 sec.

The response of the closed loop simulation to a setpoint change from 174 ppm to 290 ppm is given in Figure 6.13. Also shown in this figure is the response of the nonlinear controller with the tuning parameters that were finally used for experimentation. The IMC parameters produce a response similar to the response obtained by the nonlinear controller with the pre-experimental tuning parameters. However, after experimentation it was found that this response, when coupled with measurement noise and process dead time, produced a response which is only marginally stable and very oscillatory.

Rather than trying the unaltered IMC parameters, the parameters were adjusted to obtain a match between the final nonlinear controller response and the PID response. This was done since the experience of the nonlinear controller suggested that an unsatisfactory response would be obtained with the basic IMC tuning parameters. More importantly, it is desirable to have a consistent response for both controllers in one region, to provide a basis for further comparison. The matching of the responses was done in two steps. The first step involved multiplying the gain by 0.6, to be consistent with the step taken with the nonlinear controller. Then the integral and derivative times were altered through trial and error to match the responses. The final PID controller has a gain of $-44.228 \text{ ppm}/(g/s)$, an integral time of 900.0 sec and a derivative time of 224.7 sec.

The response of the PID controller with the final set of constants to a setpoint change from 174 ppm to 290 ppm is also given in Figure 6.13. From this, the consistency between the nonlinear and PID controller responses can be seen. Figures 6.14, 6.15 and 6.16 show the simulated response to a setpoint change from 174 ppm to 120 ppm, an unmeasured disturbance in the feed CO₂ composition from 4.0 % to 3.0 % and from 4.0 % to 4.5 %, respectively, for both the nonlinear controller and PID controllers.

For the setpoint change to 120 ppm, the PID controller operates significantly slower than the nonlinear controller. This is a result of the smaller gain between the steam flowrate and the gas CO₂ composition for the higher steam flowrates. The nonlinear controller is able to account for this and produces a response with dynamic characteristics very similar to the setpoint change to 290 ppm. Over 10000 sec the PID controller is not able to reach the set point. This is a clear demonstration of the nonlinear controller's ability to remove the nonlinearity from the system.

The responses of the closed loop system to an unmeasured feed composition disturbance from 4.0 % to 3.0 % are very similar. This is due to the fact that the PID controller was tuned to obtain a similar response to the nonlinear controller for a setpoint change from 174 ppm to 290 ppm. This setpoint change corresponds to a change in steam flowrate from 24.2 g/s to 16.7 g/s. The rejection of the unmeasured disturbance in the feed

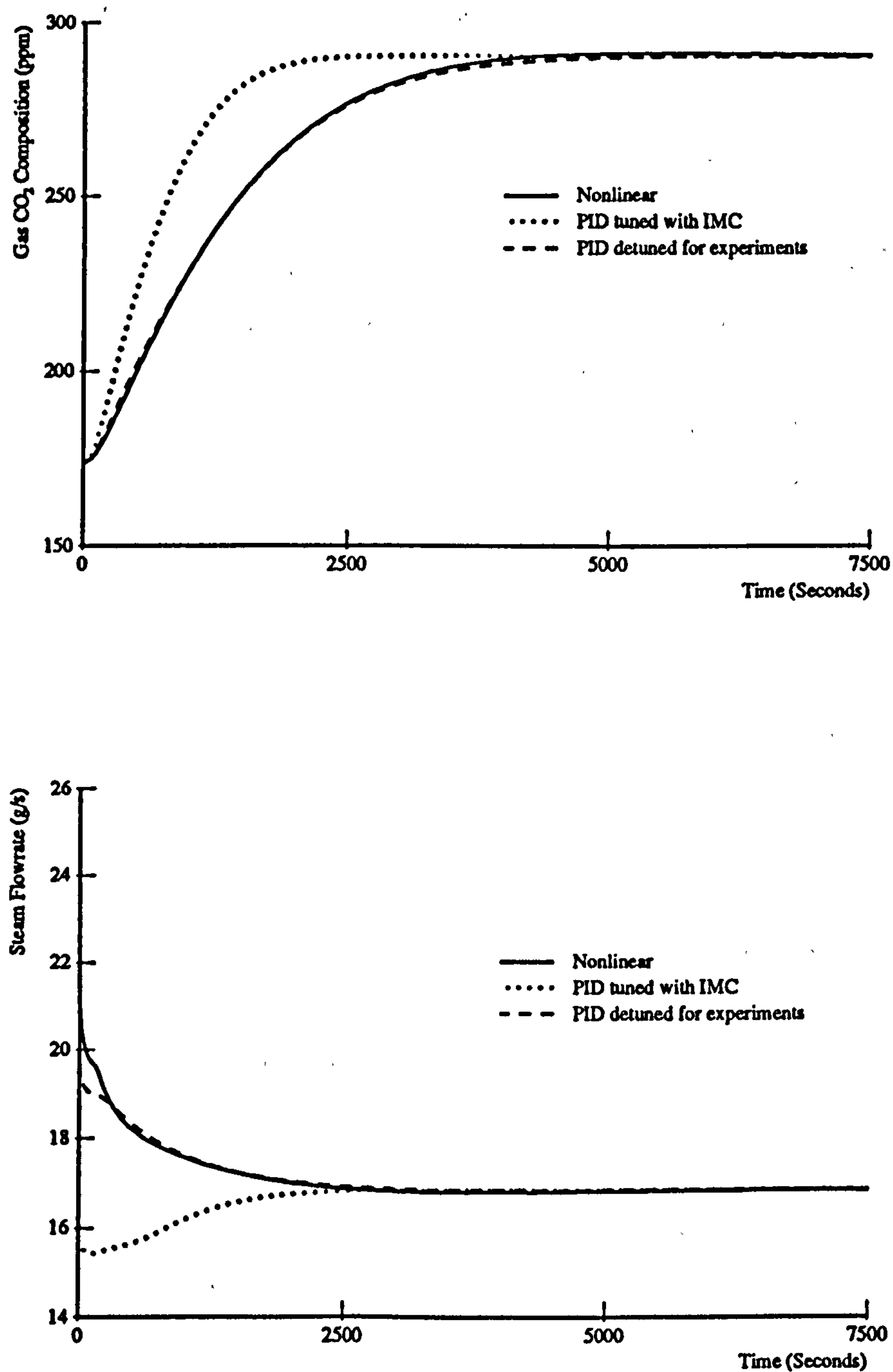


Figure 6.13: Tuning of PID controller in simulation - Setpoint change from 174 ppm to 290 ppm CO₂ with gain = $-73.714 \text{ ppm/(g/s)}$, integral time = 799.4 sec, derivative time = 99.7 sec and gain = $-44.228 \text{ ppm/(g/s)}$, integral time = 900.0 sec, derivative time = 224.7 sec.

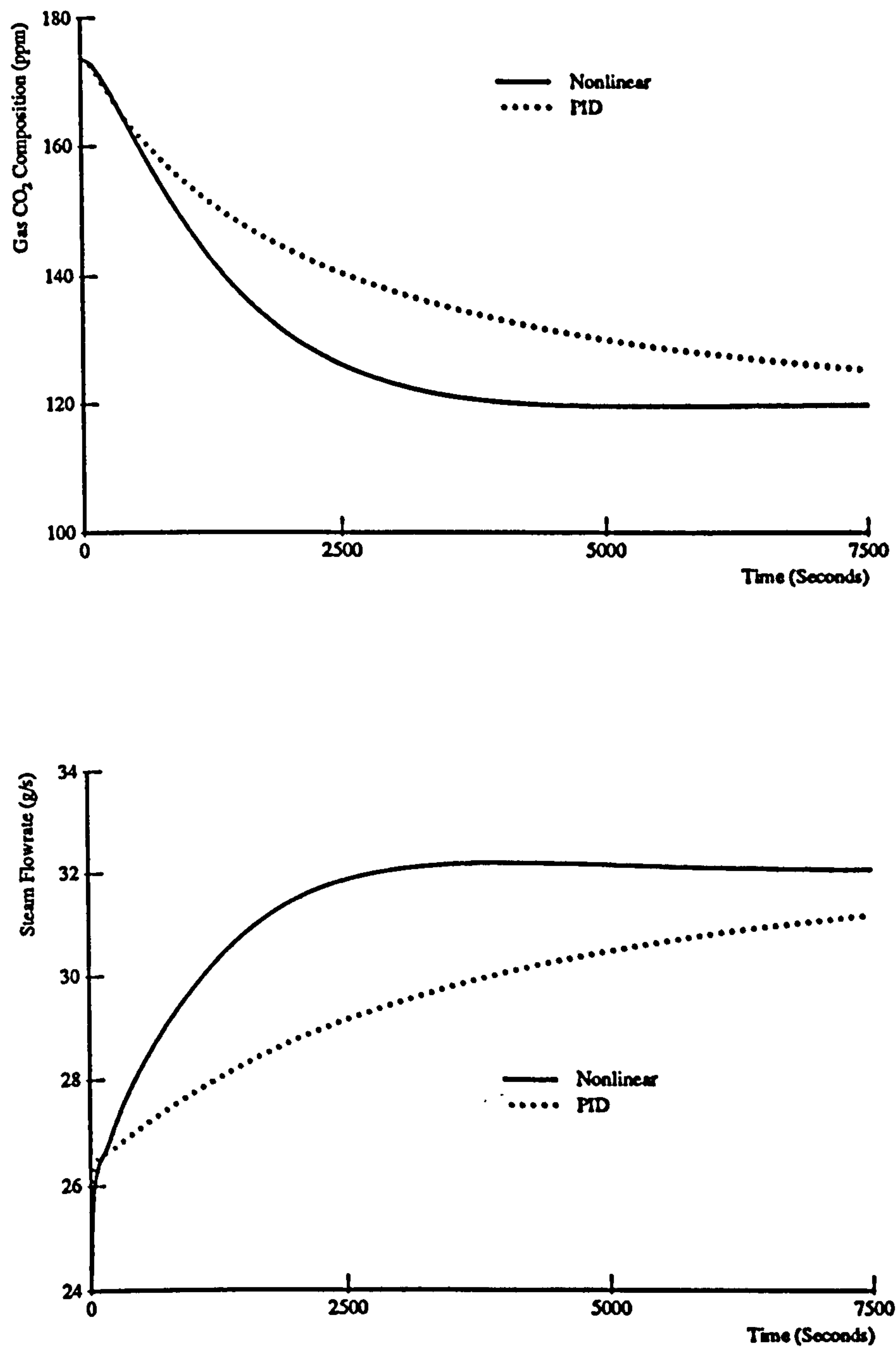


Figure 6.14: Tuning of PID controller in simulation - Setpoint change from 174 ppm to 120 ppm CO₂ with gain = $-44.228 \text{ ppm}/(\text{g/s})$, integral time = 900.0 sec, derivative time = 224.7 sec.

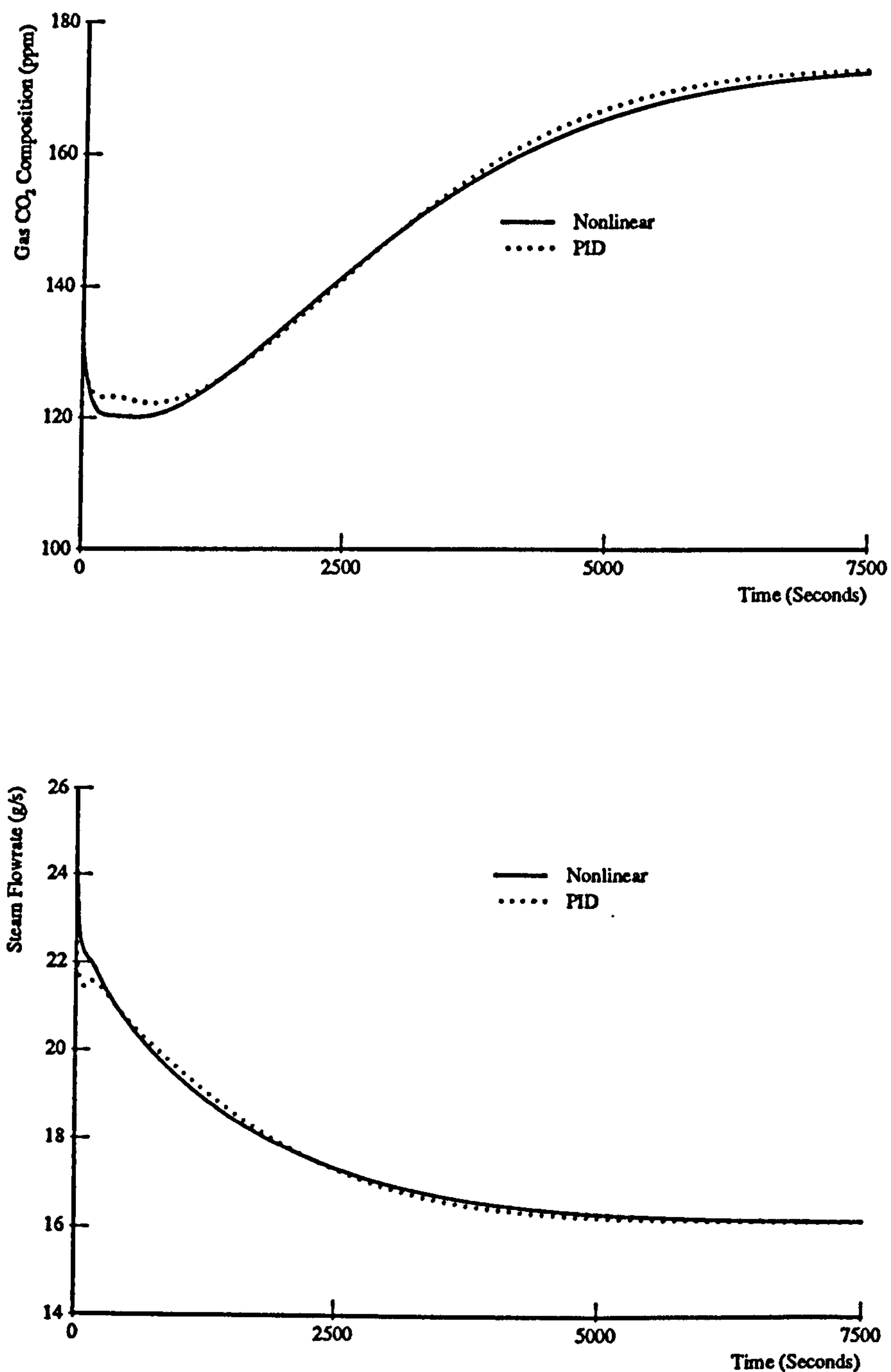


Figure 6.15: Tuning of PID controller in simulation - Rejection of a step change in the unmeasured disturbance y_c , from 4.0 % to 3.0 % CO₂ with gain = $-44.228 \text{ ppm}/(\text{g/s})$, integral time = 900.0 sec, derivative time = 224.7 sec.

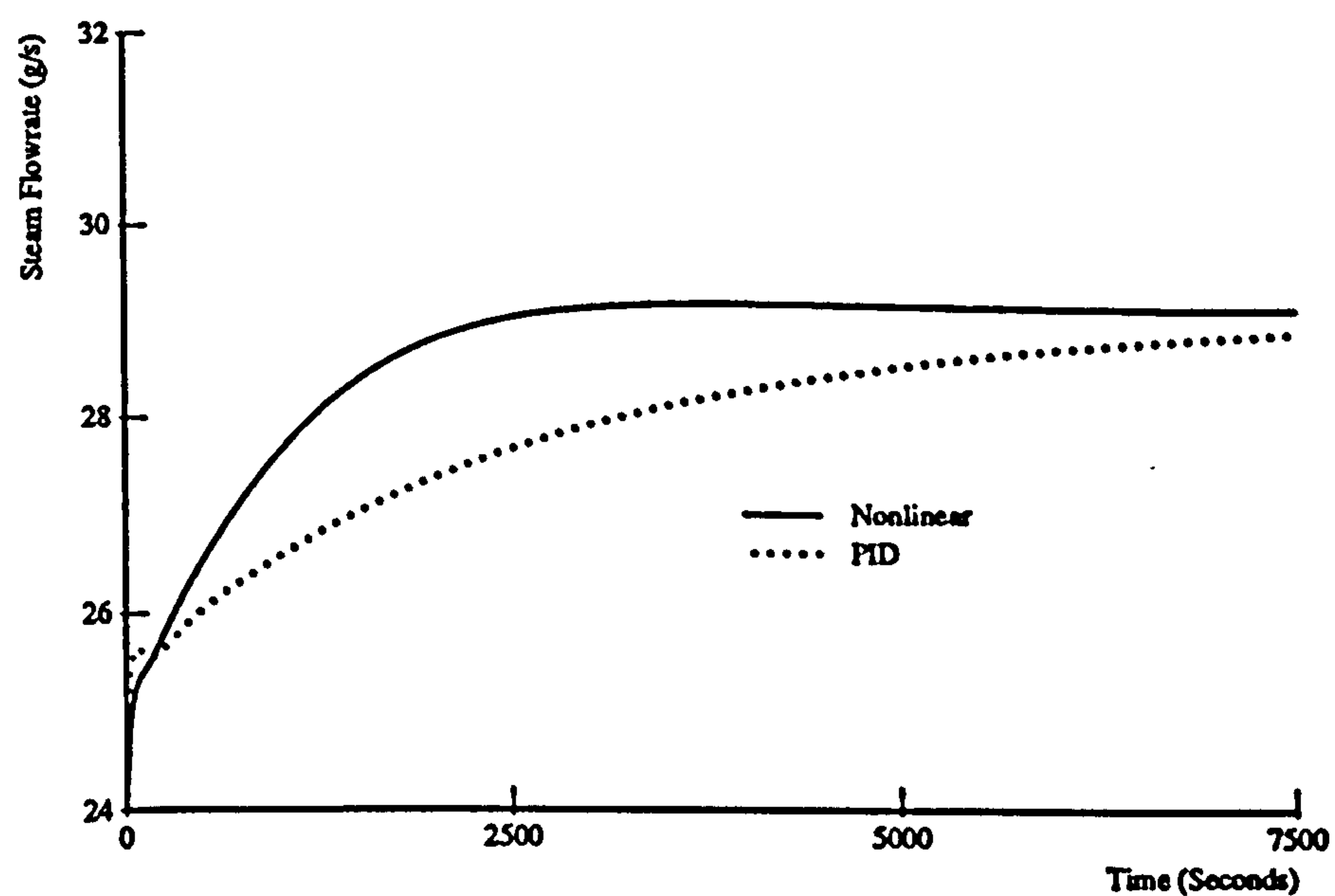
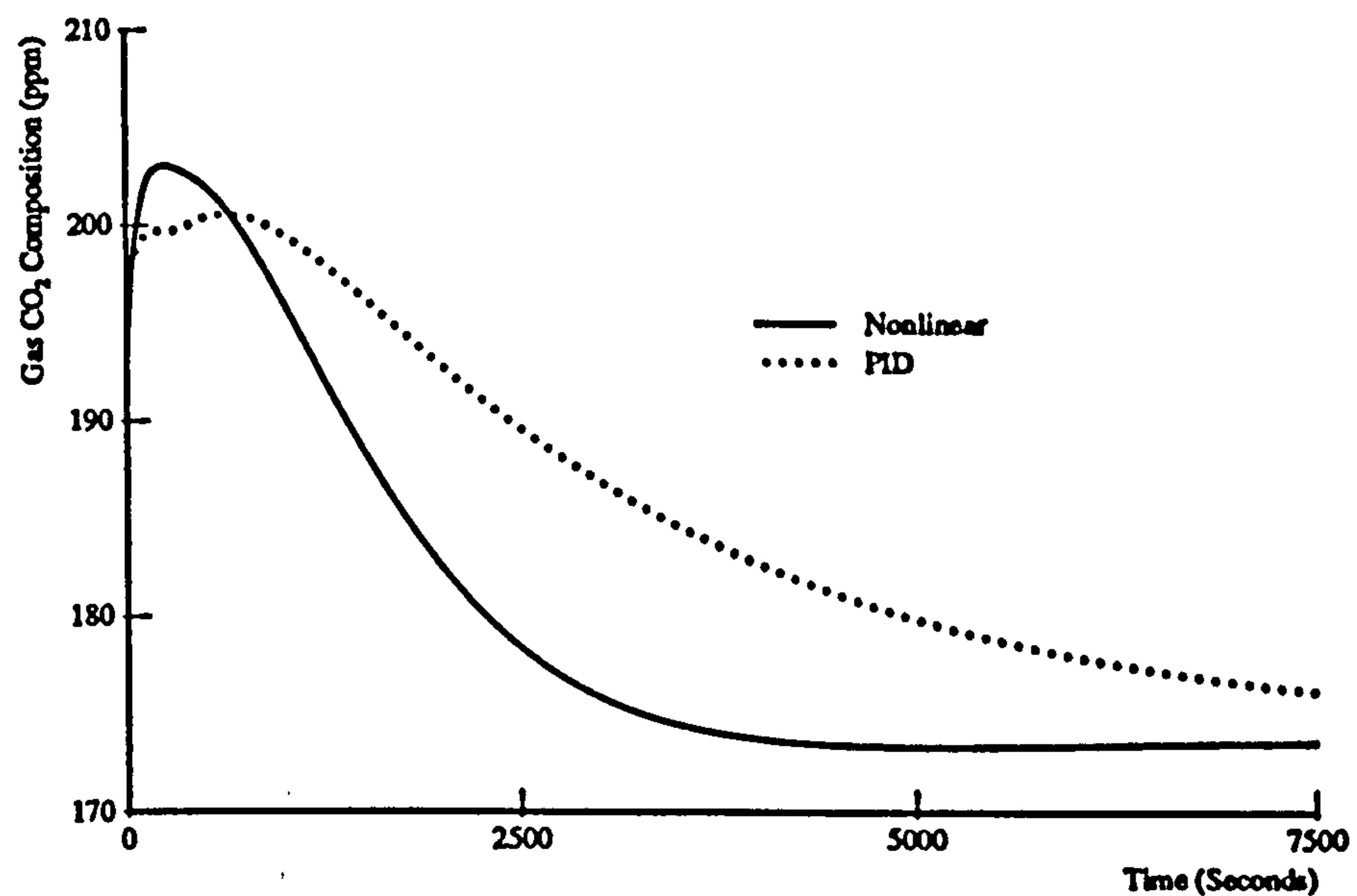


Figure 6.16: Tuning of PID controller in simulation - Rejection of a step change in the unmeasured disturbance y_c , from 4.0 % to 4.5 % CO₂ with gain = $-44.228 \text{ ppm/(g/s)}$, integral time = 900.0 sec, derivative time = 224.7 sec.

composition to 3.0 % requires a change in steam flowrate from 24.2 g/s to approximately 16.0 g/s. The consistency in responses for the rejection of the unmeasured disturbance is expected because the PID controller was tuned to produce a response consistent with the nonlinear controller for this range of steam flowrates.

The response of the PID controller to an unmeasured feed composition disturbance from 4.0 % to 4.5 % is significantly slower than the corresponding nonlinear controller. Again this is expected, because the rejection of this disturbance requires an increase in the steam flowrate to approximately 29.0 g/s. The reduction in the gain between the steam flowrate and gas CO₂ composition in this region is accounted for by the nonlinear controller, but not by the PID controller.

The conclusion from these results is that an effective set of PID controller constants can be obtained to control the gas CO₂ composition using the steam flowrate. The tuned response is set to obtain a consistent servo and regulatory performance for steam flowrates in the region 24.2 g/s to 17.0 g/s. However, a significant reduction in both servo and regulatory performance is obtained for steam flowrates in the region 24.2 g/s to 33.0 g/s. This is due to a reduction in the gain between the steam flowrate and gas CO₂ composition in this region.

6.3.2 Experimental Results - Tuning

Starting with IMC tuning rules and using the experience of the experimental results with the nonlinear controller, the following constants for the PID controller were obtained in the previous section.

$$\text{Gain} = -44.228 \text{ ppm/(g/s)}$$

$$T_I = 900.0 \text{ sec}$$

$$T_D = 224.7 \text{ sec}$$

To test robustness and performance of the PID controller on the experimental system, a test was performed starting with a steam flowrate of 24.2 g/s (gas CO₂ composition of approximately 180 ppm). The setpoint was then changed to 260 ppm and the response of the system is given in Figure 6.17.

The response is similar to that predicted from the simulation results and is very similar to that obtained with the nonlinear controller. The new setpoint is first reached after about 3500 sec and continues to a maximum overshoot of about 15 ppm (approximately 6 % of the setpoint) after 5500 sec. As expected, the nonlinear controller and PID

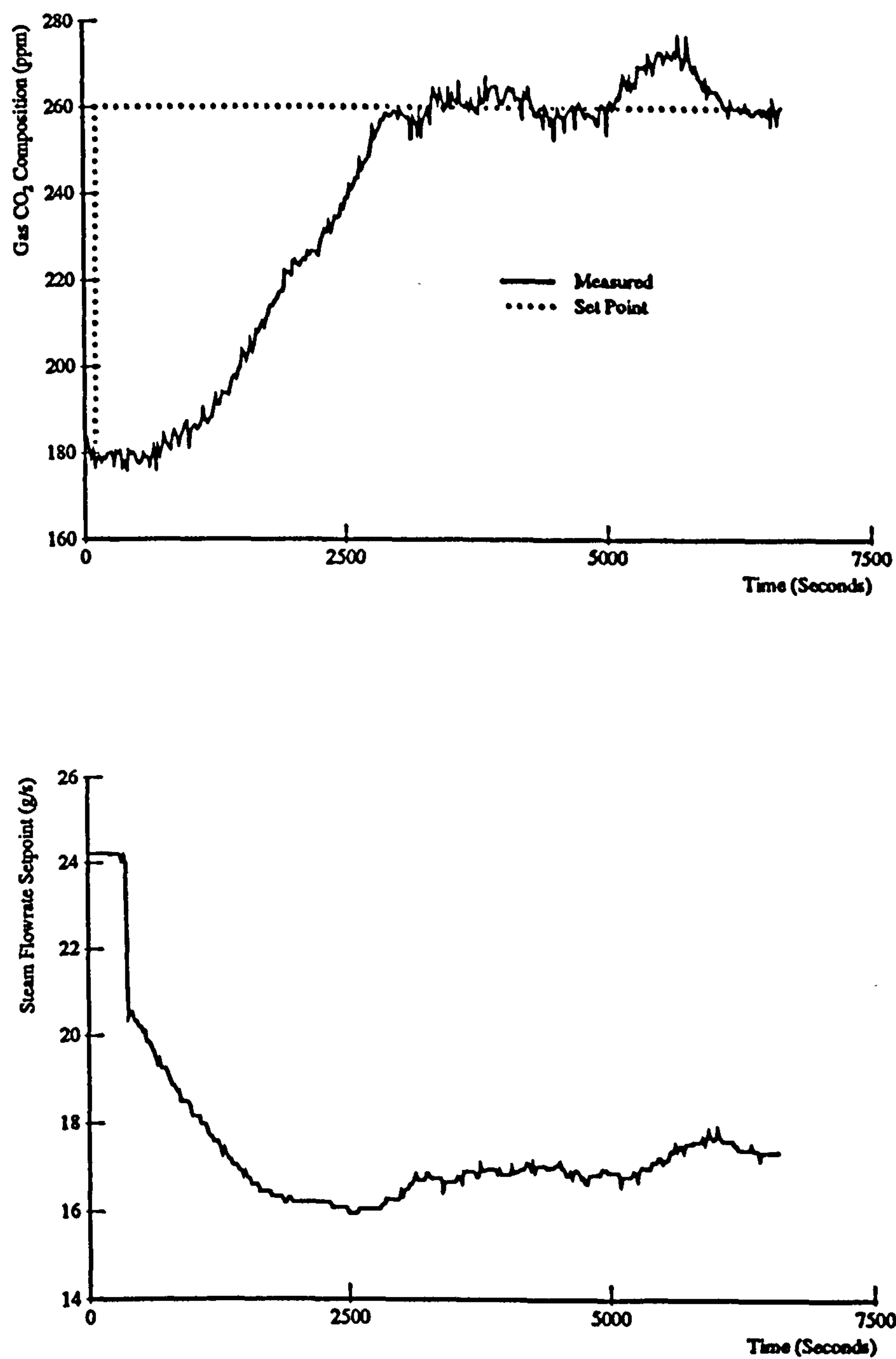


Figure 6.17: Tuning of PID controller in experimentation - Setpoint change from 180 ppm to 260 ppm with gain = $-44.228 \text{ ppm}/(\text{g/s})$, integral time = 900.0 sec, derivative time = 224.7 sec.

controller responses for this setpoint change are consistent. Therefore, these PID controller constants provide a consistent basis for comparison and are used for all the closed loop runs.

6.3.3 Experimental Results - Servo Response

To provide a test between the servo performance of the PID and nonlinear controllers, the same tests were applied to the PID controller as applied to the nonlinear controller: a setpoint change to 260 *ppm* and a setpoint change to 130 *ppm*. The responses of both the PID and nonlinear controllers are given in Figure 6.18.

A summary of these characteristics is given in Table 6.1.

Controller	Set Point (ppm)	Time to setpoint (sec)	Overshoot (% of setpoint)	Time of Max. overshoot (sec)
Nonlinear	260	3500	6	5000
	130	4000	7	6500+
PID	260	3500	6	5500+
	130	9000+	-	-

Table 6.1: Characteristics of servo responses from PID and nonlinear controllers.

These results demonstrate the major characteristics predicted from the comparison between the PID and nonlinear controllers carried out in the simulations. For the setpoint change to 260 *ppm* a similar response for both the nonlinear and PID controllers is obtained. As discussed in Section 6.3.1 the PID controller is tuned to give a response similar to the nonlinear controller for steam flowrates less than 24.2 *g/s*. However, for the setpoint change to 130 *ppm*, the PID controller has a much slower response, with the set point only just reached 9000 *secs* after the change in set point. These results clearly demonstrate that the nonlinear control is able to obtain a far more consistent servo response to setpoint changes by taking into account the change in gain between the steam flowrate and gas CO₂ composition.

6.3.4 Experimental Results - Regulatory Response

To provide a comparison between the regulatory performance of the PID and nonlinear controller, the same changes in the feed CO₂ composition were introduced to the PID controller. For these experiments the plant was started at a steam flowrate of 24.2 *g/s* and a feed CO₂ composition of 4.0 %. The resulting steady state outlet gas

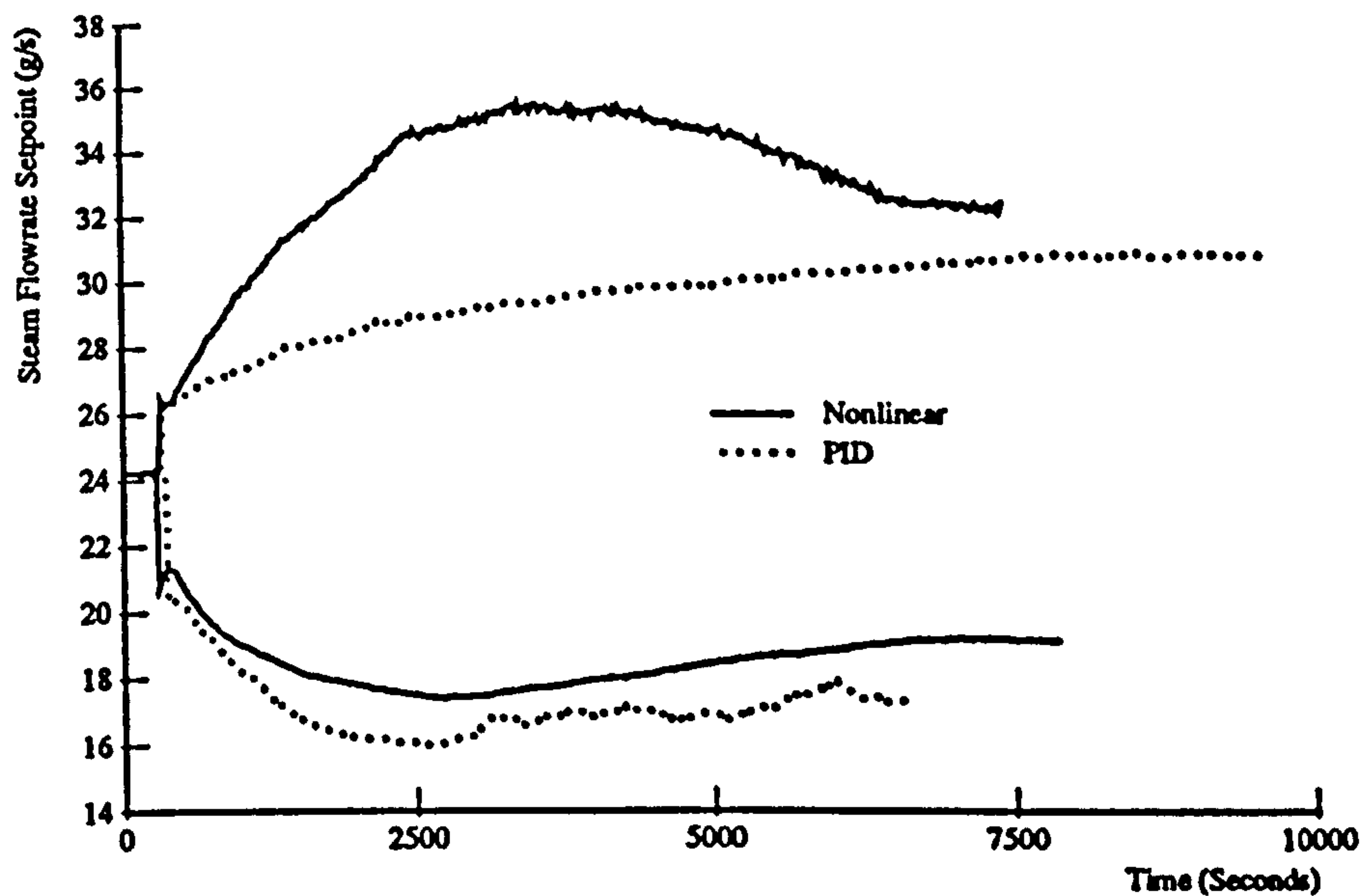
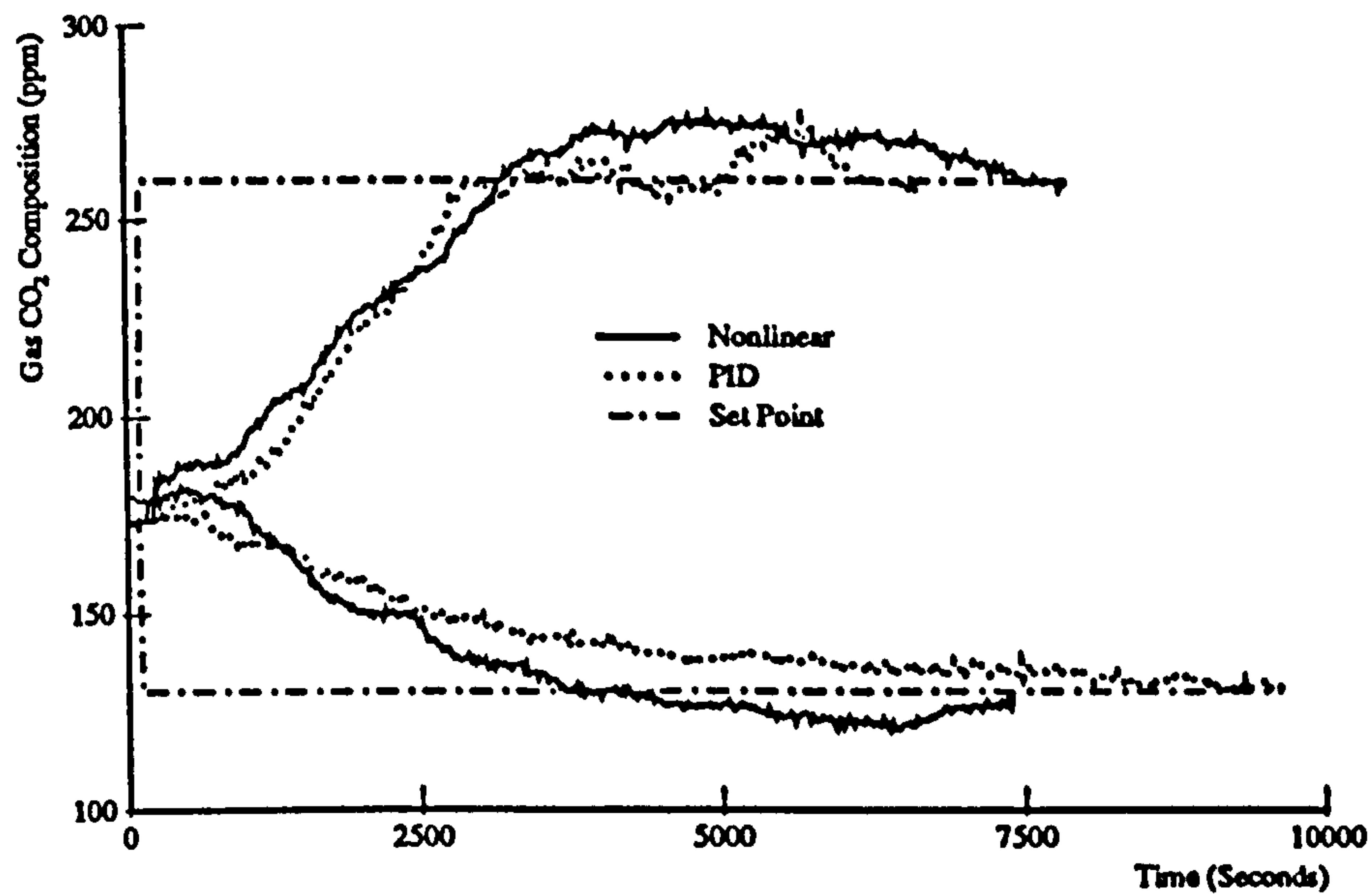


Figure 6.18: Nonlinear and PID controller in experimentation - Servo Response : PID - Setpoint change from 180 ppm to 260 ppm and PID - Setpoint change from 174 ppm to 130 ppm with gain = $-44.228 \text{ ppm}/(\text{g/s})$, integral time = 900.0 sec, derivative time = 224.7 sec and Nonlinear - Setpoint change from 174 ppm to 260 ppm and Nonlinear - Setpoint change from 178 ppm to 130 ppm with gain = 0.6, $T_{reset} = 750$, poles = $1/125 \text{ s}^{-1}$, $1/1000 \text{ s}^{-1}$.

CO₂ composition was used as the setpoint for the experiment.

The responses of the PID and nonlinear controllers to the two disturbances are given in Figures 6.20 and 6.19.

A summary of these characteristics is given in Table 6.2.

Controller	Disturbance (% CO ₂)	Max. Offset (ppm)	Time of Offset (sec)	Time to Setpoint (sec)
Nonlinear	4.0 to 4.5	25	2000	4400
	4.0 to 3.0	42	3000	6000+
PID	4.0 to 4.5	21	1900	6000+
	4.0 to 3.0	43	2700	6800

Table 6.2: Characteristics of regulatory responses from PID and nonlinear controllers.

The major characteristics of these responses are as predicted from the simulation results. The responses of the geometric nonlinear controller to an unmeasured change in feed CO₂ composition from 4.0 % to 4.5 % is much quicker than the PID controller, with a slightly larger maximum offset. This is partly because the nonlinear controller takes account of the nonlinearity and partly due to the error between the observed and measured conditions for the nonlinear controller.

The response of both controllers to the unmeasured change in feed CO₂ composition from 4.0 % to 3.0 % are approximately of the same speed. From the experimental results it appears that the PID controller will actually perform slightly better. This is explained again by the error between the measured and observed conditions for the nonlinear controller, which in this case artificially slows down the closed loop system.

6.4 Summary

This chapter has described the implementation of the nonlinear controller on a CO₂ absorption/desorption pilot plant. From the simulation and experimental results the following conclusions are made.

1. The nonlinear controller provides a stable closed loop response despite,
 - noisy measurement signal,
 - model mismatch between observer and the plant,
 - model mismatch between the model used for controller and the plant,
 - significant dead time in plant,

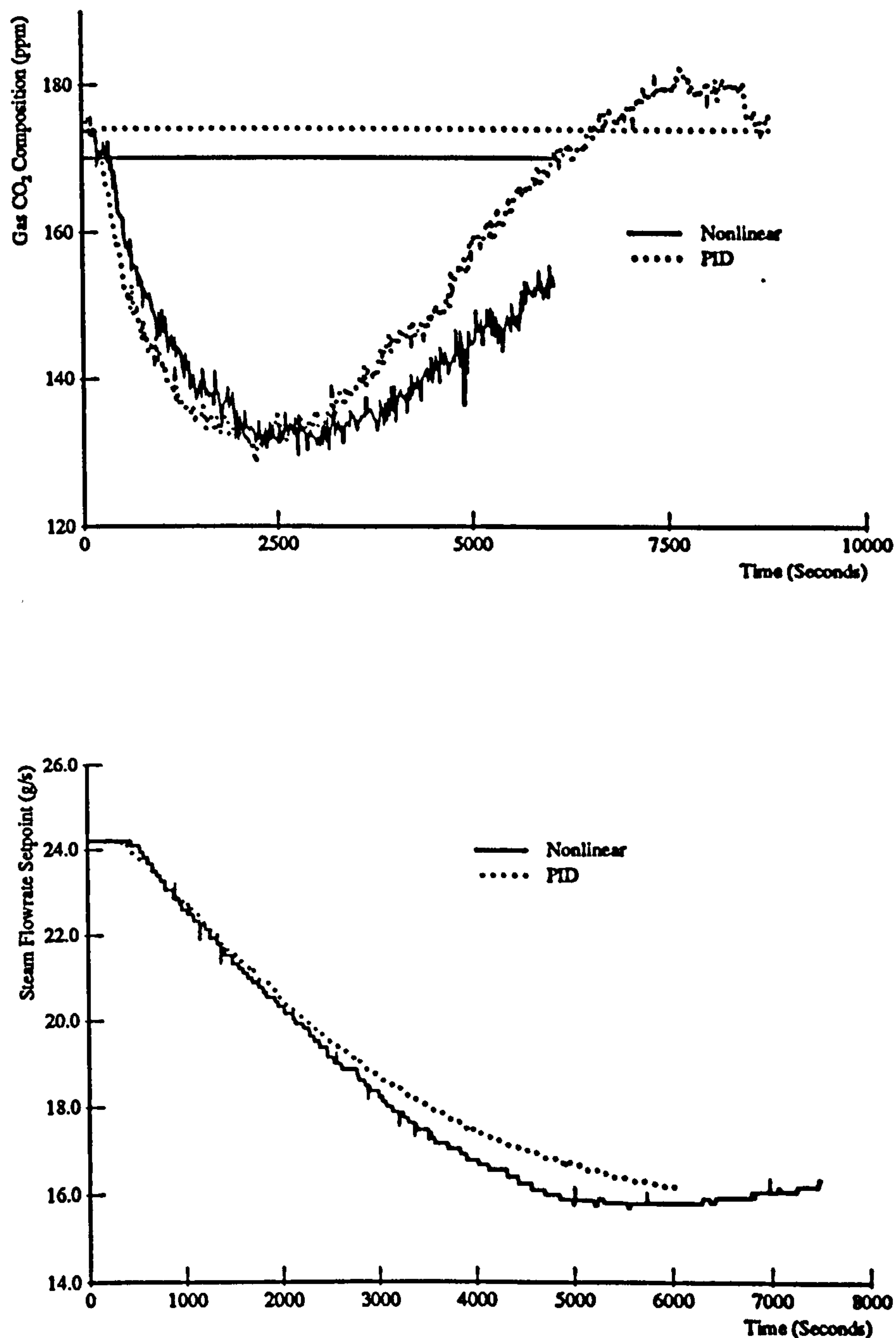


Figure 6.19: Nonlinear and PID Controller in experimentation - regulatory response : - Step change in feed composition from 4.0 % to 3.0 % CO₂, PID - Setpoint at 174 ppm with gain = $-44.228 \text{ ppm}/(\text{g/s})$, integral time = 900.0 sec, derivative time = 224.7 sec and Nonlinear - Setpoint at 170 ppm with gain = 0.6, $T_{reset} = 750$, poles = $1/125 \text{ s}^{-1}$, $1/1000 \text{ s}^{-1}$.

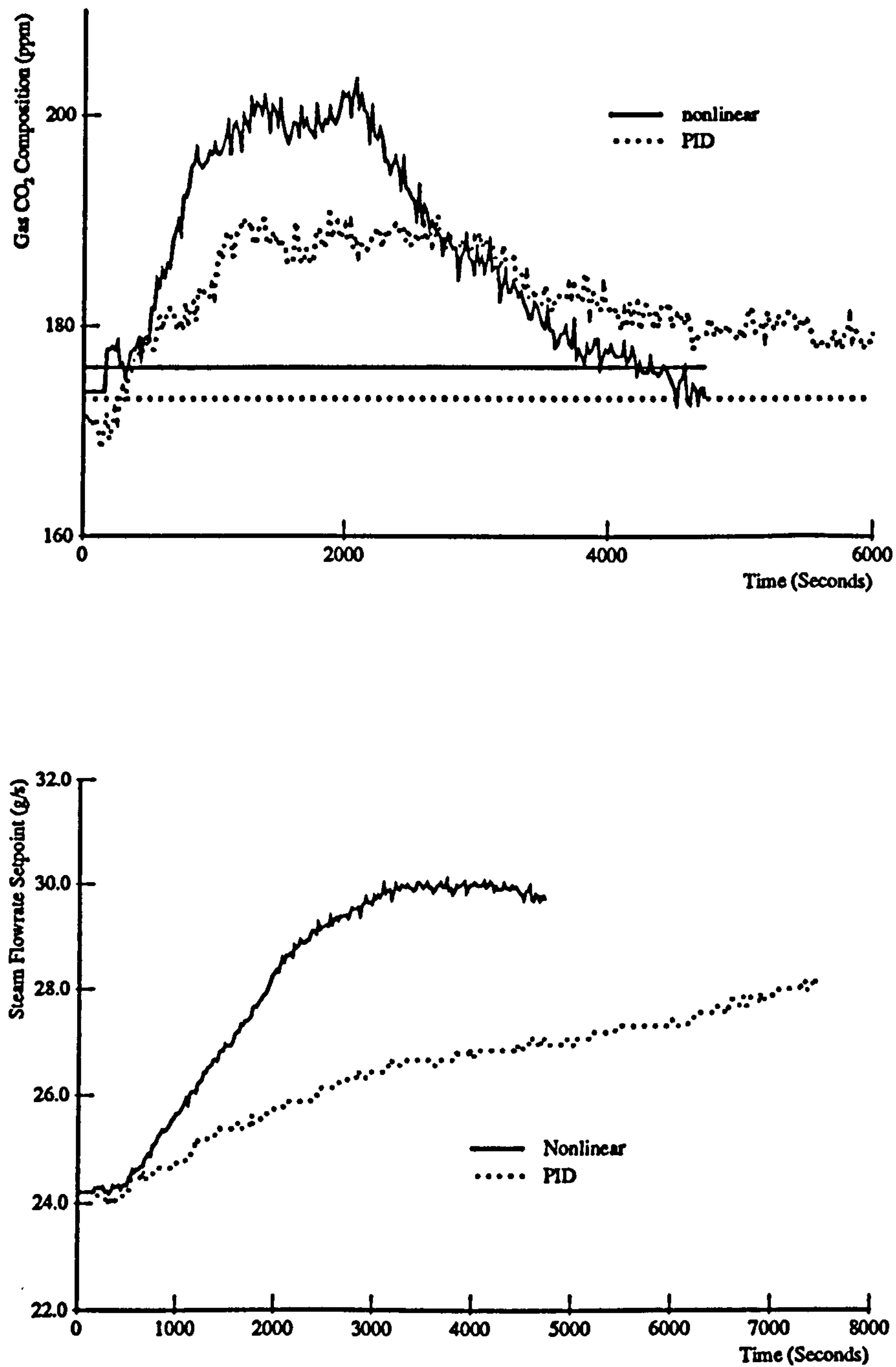


Figure 6.20: Nonlinear and PID controller in experimentation - regulatory response : - Step change in feed composition from 4.0 % to 4.5 % CO₂, PID - Setpoint at 173 ppm with gain = $-44.228 \text{ ppm}/(\text{g/s})$, integral time = 900.0 sec, derivative time = 224.7 sec and Nonlinear - Setpoint at 176 ppm with gain = 0.6, $T_{reset} = 750$, poles = $1/125 \text{ s}^{-1}$, $1/1000 \text{ s}^{-1}$.

- unmeasured disturbances.
2. The nonlinear controller provides a consistent closed loop response to step changes in the setpoint over a wide range of operating conditions. This is an effective demonstration that the nonlinear controller is able to account for the nonlinear gain in the system.
 3. The nonlinear controller is able to reject unmeasured disturbances. However, as the disturbances are unmeasured, a large difference between the observed and measured values for the outlet gas CO₂ composition results. This causes an increase or a decrease in the speed of the closed loop regulatory response, depending on the direction of the error. This phenomenon is demonstrated in the simulation results and experimentally verified in the pilot plant study.

These results are an effective demonstration that the nonlinear controller takes account of the gain nonlinearity in the system. The simulations further suggest that there is a dynamic nonlinearity in the system. Experimentally this would be expected, particularly due to the holdup of the reboiler. However, attention has not been given to this part of the modelling and the ability of the nonlinear controller to account for dynamic nonlinearities has not been used to advantage. Further advantage may be obtained from considering the dynamics of the system in more detail. However, the main control problem for this system appears to be due to the gain nonlinearity.

In addition to implementing the nonlinear controller on the pilot plant, a PID loop was implemented for the same configuration. A number of steps were taken to ensure that a fair comparison was made. Firstly, the nonlinear controller and PID controller use the same amount of information (the gas CO₂ measurement). As an open loop observer is used for the nonlinear controller, measurement of all the states is not required. Secondly, the implementation of both controllers is with a sample rate of 8 sec. Finally there is the question of tuning. In an attempt to obtain a fair comparison the PID controller has been tuned to provide the same closed loop servo performance as the nonlinear controller for steam flowrates in the region 24.2 g/s to 16.7 g/s. This provides a base case for comparison of the PID and nonlinear controllers. This comparison allows the following conclusions to be drawn.

1. The PID and nonlinear controllers can be tuned to give an equivalent closed loop servo response for a given region of operation on the pilot plant.

2. The nonlinear controller gives a better servo response in the operational region for which the PID controller was not tuned.
3. The nonlinear controller is able reject disturbances in the inlet gas CO₂ composition from 4.0 % to 3.0 % better than the PID controller. The apparent advantage of the nonlinear controller is largely due to the model mismatch between the observer and the pilot plant.
4. The PID is marginally better at rejecting disturbances in the inlet gas CO₂ composition from 4.0 % to 4.5 % than the nonlinear controller. The decrease in performance of the nonlinear controller is largely due to the model mismatch between the observer and the pilot plant.

The PID controller could possibly be tuned to obtain a better response for the region with small input output gain. However, a faster closed loop response with the nonlinear controller response could also be obtained in this region, correspondingly with an improvement over the entire region of operation. A detuned response for both controllers is used to avoid manipulated variable constraints. Given the operating conditions used for the experiments and the basic closed loop response used for comparison, the nonlinear controller gives a marginal improvement in the closed loop response. However, a fair comparison of the two methods would require the definition of a specific objective function. Furthermore, the improvement of the nonlinear controller must be significant to justify the expense of designing and maintaining a full dynamic model of the plant.

Chapter 7

Summary and Future Work

This chapter gives a summary of the review, modelling, theoretical, simulation and experimental work in this thesis. This is followed by some general conclusions and suggestions for further areas of work.

7.1 Summary

Recent interest in the use of nonlinear controllers of process systems has led to the development of a class of control techniques called geometric nonlinear control. These techniques are based either explicitly or implicitly on the mathematics of differential geometry. The aim of this thesis has been to assess the applicability of these techniques to the process industries, through the detailed analysis of a case study.

Chapter 2 provides an extensive literature review of the geometric nonlinear control techniques and the recent advances in theory. The result is a combined structure centred on a realisation of the inverse of the dynamic nonlinear system (called the geometric nonlinear controller). The basic controller creates a closed loop system which can be transformed to a linear system. However, there are a number of theoretical and practical limitations to the basic techniques. The additional parts of the combined structure have developed to provide a technique which is applicable to a large number of practical systems.

A review of simulated and experimental applications of geometric nonlinear control techniques to process systems demonstrates that the methods are applicable to a large number of systems. The issues of robust closed loop stability and internal stability are often ignored in the implementations but effective control is still obtained. Comparisons between geometric nonlinear control and standard linear control techniques demonstrate the effectiveness of the nonlinear controllers. The real strength appears to be with set-point tracking, with very good servo responses obtained in most applications. Effective regulatory control often requires feedforward or adaptive schemes.

The literature review highlights two needs: Firstly there is a need for detailed applications on more complex systems, representative of an industrial application, including comparisons with standard techniques. Secondly there is a need for a unified approach

to controller design using geometric nonlinear control. This technique should ultimately be able to handle most given systems from the process industries and must be presented in an understandable fashion.

Chapter 3 describes the CO₂ absorption/desorption system used as the case study for the thesis. This system is seen to be representative of the process industries because of the following.

1. The system is a combination of smaller, well defined unit operations.
2. The unit operations are standard to and commonly used in the process industries.
3. The system is described by a combination of mass, energy balances.
4. The model of the system considers vapour, liquid equilibrium data, chemical reaction and mass transfer effects.
5. The main control problem is to meet a certain product requirement while minimising the use of the manipulated variable, subject to manipulated variable constraints, measured and unmeasured disturbances.

A mathematical model of the system is developed. A detailed description of the pilot plant system is also given. Experimental data are used to estimate three model parameters to complete the specification. The dynamic model was then compared to open loop responses to step changes in the manipulated variable, demonstrating a reasonable match. There is some modelling error, partly due to a significant dead time in the system, not accounted for in the model. In addition, the results demonstrate a large amount of process noise for the gas CO₂ composition measurement.

Chapter 4 examines the complexity of the geometric nonlinear controller and how it is related to the structure of the original model. The complexity of the controller is largely related to the relative order of the system. Therefore, a logical approach to controller size reduction through model reduction should be aimed at reducing the relative order. Graph theory is used to identify key states, the removal of which will reduce the relative order of the system. Singular perturbation arguments are used to justify the removal of as many of the key states as possible.

In general, the removal of a state corresponds to the addition of an algebraic equation. Therefore, an extension to the basic geometric nonlinear control technique is presented, which allows the use differential and algebraic equations as part of the process model. The advantage extends to the process industries in general, which often require

both differential and algebraic equations as part of the mathematical description. The final result of this chapter is a greatly reduced controller.

Chapter 5 considers the tuning of geometric nonlinear controllers for servo and regulatory responses. These results demonstrate qualitatively that the most desirable output response for servo and regulatory actions is obtained if the specified closed loop poles are set to be fast. However, the speed of response is limited by saturation limits on the control action. (Note: Saturation limits are the main issue for the case study, though in other cases the speed of response limit might be stability).

It is demonstrated that the effect of model mismatch is similar to the effect of unmeasured disturbances and equivalent relative orders can be defined for various model error terms. When direct state feedback is used, these errors are propagated through the derivative expressions in the control law. As a results, the offset of the closed loop system (steady state offset when no integral action is used and dynamic offset when an external PI regulator is used) is inversely proportional to the specified closed loop poles.

Based on these results a general set of tuning guidelines are developed, aimed at achieving maximum speed of response given saturation limits on the control action. These guidelines are tested on the simulation of the case study, resulting in a good servo response. However, a suitable regulatory response is not obtainable. It is demonstrated that this is due to two factors. Firstly, the disturbance response on the system is much faster than the manipulated input response. A discussion shows the structural characteristics of such a system. Secondly, as already concluded, the effect of the unmeasured disturbance is related only to the specified closed loop poles. A suitable compromise between the speed of response and the control action is not possible.

This problem is overcome by using an open loop observer to obtain the state information. In this configuration the offset of the closed loop system is specified by the gain of the controller and is independent of the closed loop poles. This is because the effects of model mismatch and unmeasured disturbances are accounted for by the difference between the output of the observer and the required setpoint.

Chapter 6 describes the implementation of the nonlinear controller on a CO₂ absorption/desorption pilot plant. From the simulation and experimental results the following conclusions are made.

1. The nonlinear controller provides a stable closed loop response despite,
 - noisy measurement signal,
 - model mismatch between observer and the plant,

- model mismatch between the model used for the controller and the plant,
 - significant dead time in the plant,
 - unmeasured disturbances.
2. The nonlinear controller provides a consistent closed loop response to step changes in the setpoint over a wide range of operating conditions. This is an effective demonstration that the nonlinear controller is able to account for the nonlinear gain in the system.
 3. The nonlinear controller is able to reject unmeasured disturbances. However, as the disturbances are unmeasured, a large difference between the observed and measured values for the outlet gas CO_2 composition results. This causes an increase or a decrease in the speed of the closed loop regulatory response, depending on the direction of the error. This phenomenon is demonstrated in the simulation results and experimentally verified in the pilot plant study.

These results are an effective demonstration that the nonlinear controller takes account of the gain nonlinearity in the system. The simulations further suggest that there is a dynamic nonlinearity in the system. However, attention has not been given to this part of the modelling and the ability of the nonlinear controller to account for dynamic nonlinearities has not been used to advantage. Further advantage may be obtained from considering the dynamics of the system in more detail. However, the main control problem for this system appears to be due to the gain nonlinearity.

In addition to implementing the nonlinear controller on the pilot plant, a PID loop was implemented for the same configuration. This comparison allows the following conclusions to be drawn.

1. The PID and nonlinear controllers can be tuned to give an equivalent closed loop servo response for a given region of operation on the pilot plant.
2. The nonlinear controller gives a better servo response in the operational region for which the PID controller was not tuned.
3. The nonlinear controller is able reject disturbances in the inlet gas CO_2 composition from 4.0 % to 3.0 % better than the PID controller. The apparent advantage of the nonlinear controller is largely due to the model mismatch between the observer and the pilot plant.

4. The PID is marginally better at rejecting disturbances in the inlet gas CO₂ composition from 4.0 % to 4.5 % than the nonlinear controller. The decrease in performance of the nonlinear controller is largely due to the model mismatch between the observer and the pilot plant.

The PID controller could possibly be tuned to obtain a better response for the region with small input output gain. However, a faster closed loop response with the nonlinear controller could also be obtained in this region, correspondingly with an improvement over the entire region of operation. A detuned response for both controllers is used to avoid manipulated variable constraints. Given the operating conditions used for the experiments and the basic closed loop response used for comparison, the nonlinear controller gives a marginal improvement in the closed loop response. However, a fair comparison of the two methods would require the definition of a specific objective function. Furthermore, the improvement in performance of the nonlinear controller must be significant to justify the expense of designing and maintaining a full dynamic model of the plant.

7.2 Conclusions

A comparison of the results in this thesis with the objectives for control research listed in Section 1.1 lead to the following conclusions.

1. Geometric nonlinear control provides a method for directly using the process model as part of the controller. These models are generally small and with a small relative order to avoid very large controller equations. Hence the model should only represent the important part of the dynamics. However, there is no explicit handling of discontinuities (manipulated variable constraints, state constraints).
2. An extensive discussion of the advantages and limitations of geometric nonlinear control is given in Chapter 2. A general conclusion is that geometric nonlinear control is able to work on systems representative of industrial applications.
3. It is not clear that geometric nonlinear control can be used to directly address the requirements of control. The objective of geometric nonlinear control is to achieve a consistent gain and time constant over the entire region of operation. This makes the design of an external regulator more logical, though this is not necessarily desirable. Ultimately the tuning of the controller is limited by the most sensitive operating re-

gion. Realisation of the requirements for control may actually come from a nonlinear closed loop response.

7.3 Future Research

This work has suggested the following areas for future research.

1. The implementation of the geometric nonlinear controller on the pilot plant could be enhanced through a number of changes. Feeding forward the measurement of the feed composition would provide a more consistent regulatory response over the full range of operating conditions. The specified closed loop poles could be moved to obtain a faster response. In this case some method for dealing with controller saturation should also be used. Partial state feedback could be used, in conjunction with a full or reduced order open loop observer.
2. It has been demonstrated that the rate of response of the open loop system to changes in the manipulated and disturbance inputs affects the regulatory response of the closed loop system with full state feedback. A further characterisation of the problem is required, possibly resulting in an alternative solution to using an open loop observer.
3. The effect of the sampling interval on the performance of the controller should be considered in more detail. This should be done both theoretically and experimentally.
4. The relative order has already been used as an indicator for controllability analysis. However, the effect of relative order and internal dynamics on the closed loop response requires further research. This is particularly important when developing models to be used as part of the controller design.

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Appendix A

Mathematical Preliminaries

Lie Derivative

Consider the smooth vector field $f(x)$ defined on an open set U of $\mathbb{R}^n$, represented by,

$$f(x) = \begin{bmatrix} f_1(x_1, x_2, \dots, x_n) \\ f_2(x_1, x_2, \dots, x_n) \\ \vdots \\ f_n(x_1, x_2, \dots, x_n) \end{bmatrix} \quad (\text{A.1})$$

The smoothness criteria ensure that all the entries $f_1, \dots, f_n$ are real valued functions of $x_1, \dots, x_n$ with continuous partial derivatives of any order. Consider also a real valued function λ defined on U . The differential (or gradient), $d\lambda$, is defined as,

$$d\lambda(x) = \frac{\partial \lambda}{\partial x} \quad (\text{A.2})$$

The Lie derivative is the derivative of a real valued function, λ , along a vector field, $f(x)$, and is written as,

$$\langle d\lambda(x), f(x) \rangle = \frac{\partial \lambda}{\partial x} f(x) = \sum_{i=1}^n \frac{\partial \lambda}{\partial x_i} f_i(x) \quad (\text{A.3})$$

The following notation is used to represent the same value,

$$L_f \lambda = \langle d\lambda(x), f(x) \rangle \quad (\text{A.4})$$

The Lie derivative is also a smooth real valued function. If a second vector field, $g(x)$, is also considered, then a new function may be defined.

$$L_g L_f \lambda(x) = \frac{\partial (L_f \lambda)}{\partial x} g(x) \quad (\text{A.5})$$

Higher order Lie derivatives follow logically and are denoted $L_f^k \lambda(x)$.

Lie Bracket

Consider two smooth vector fields, $f(x)$ and $g(x)$, both defined on U . The Lie bracket (or product) is defined as,

$$[f, g](x) = \frac{\partial g}{\partial x} f(x) - \frac{\partial f}{\partial x} g(x) \quad (\text{A.6})$$

The result is a new, smooth vector field defined on U . The functions $\frac{\partial f}{\partial x}$ and $\frac{\partial g}{\partial x}$ represent the Jacobian matrices of $f(x)$ and $g(x)$ respectively. Repeated Lie brackets of one vector field, $g(x)$, follow easily from the basic definition. The following notation is commonly used to represent repeated Lie brackets;

$$ad_f^k g(x) = [f, ad_f^{k-1} g](x) = [f, [f, \dots [f, g] \dots]] \quad (\text{A.7})$$

Complete Integrability

Consider d smooth, n dimensional vector fields, $f_1, f_2, \dots, f_d$ in U (an open subset of $\mathbb{R}^n$) and a distribution Δ defined as,

$$\Delta(x) = \text{span}\{f_1(x), \dots, f_d(x)\} \quad (\text{A.8})$$

Isidori, 1989 describes a distribution as a subspace spanned by the values, at x , of n smooth vector fields defined on U .

Consider also $n - d$ smooth, real valued functions $\lambda_1, \dots, \lambda_{n-d}$. According to Isidori, 1989 complete integrability of $\lambda_1, \dots, \lambda_{n-d}$ will ensure $n - d$ independent solutions to the following equation,

$$\frac{\partial \lambda_j}{\partial x} \Delta(x) = 0 \quad (\text{A.9})$$

Involutivity

Consider a distribution as defined in equation A.8. The distribution, Δ , is involutive if any pair of vector fields, $f_i(x)$ and $f_j(x)$ ($i \neq j$), have a Lie bracket vector field which also belongs to Δ .

$$(f_i \in \Delta) \text{ and } (f_j \in \Delta) \Rightarrow [f_i, f_j] \in \Delta \quad (\text{A.10})$$

This is equivalent to the definition by Su, 1982.

The set of vector fields, $\{f_1, \dots, f_n\}$ is involutive if,

$$[f_i, f_j] = \sum_{k=1}^m \alpha_{ijk} f_k \quad \text{with} \quad [i \neq j] \quad (\text{A.11})$$

Where, α_{ijk} is a real valued function of x .

The concept of involutivity is important as it is related to the complete integrability of a set of vector fields (ie. a distribution). They are connected through the Frobenius theorem. This theorem is stated as follows (Boothby, 1975);

“A set of linearly independent vector fields is completely integrable if and only if it is involutive.”

Relative Order

A discussion of relative order for single input single output systems is given in Section 2.1.2.

For multiple input, multiple output systems the analogous definition results in a vector of relative orders, $r = (r_1, \dots, r_m)$.

A multi input, multi output system with the form given in equation 2.6 has a vector relative order, $r (r \in \mathbb{R}^m)$, if,

1.

$$\begin{aligned} L_{g_j} L_f^k h_i(x) &= 0 \\ \forall 1 \leq j \leq m \quad \forall 1 \leq i \leq m \quad \forall k < r_i - 1 \quad \forall x \text{ around } x^0. \end{aligned} \quad (\text{A.12})$$

2. The decoupling matrix Δ , as defined below, is not singular at x^0 .

$$\Delta(x) = \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1(x) & \dots & \dots & L_{g_m} L_f^{r_1-1} h_1(x) \\ \vdots & & & \vdots \\ L_{g_1} L_f^{r_m-1} h_m(x) & \dots & \dots & L_{g_m} L_f^{r_m-1} h_m(x) \end{bmatrix} \quad (\text{A.13})$$

Each element of r represents the number of times the output y_i must be differentiated to have at least one of the inputs u appear.

Appendix B

Speedup Model for Case Study

This appendix lists the Speedup model of the case study. This description corresponds to the reduced model as the differential equations associated with T_r and y_a , have been replaced by algebraic equations. The differential equations have been left in as comments.

The SET conditions correspond to the nominal operating point used in the thesis, with a steam flowrate of 24.2 g/s. The PRESET conditions are used to obtain a converged solution, both for steady state and dynamic runs.

TITLE CO2 PILOT PLANT SIMULATION

OPTIONS

EXECUTION

PRINTLEVEL = 2
 TARGET = TERMINAL
 EXTERNAL = OFF
 TIME_STEP = 10
 INTERVALS = 1000
 ABS_TOL = 1.e-6
 REL_TOL = 1.e-6
 SS_TOL = 1.e-8
 OPT_TOL = 1.e-2
 EVENT_TOL = 1.e-6

DECLARE

TYPE

temp	=	50.0	:	10.0	:	300.	unit = "C"
molflow	=	3.5E-03	:	-110	:	110.0	unit = "kmol/s"
molflow1	=	3.5E-03	:	-110	:	110.0	unit = "kmol/s"
molconc	=	0.8	:	0.0	:	1.24999	unit = "kmol/m3"
efficiency	=	0.01	:	0.0	:	1.0	
enthalpy	=	400.	:	-1.E+05	:	1.E+07	unit = "kJ /kmol"
enth	=	20.	:	-1.E+05	:	1.E+07	unit = "kJ/kmol"
hcapacity	=	25.71	:	1.E-07	:	1.E+03	unit = "kJ/kmol.C"
volume	=	7.0E-02	:	1.E-06	:	1.E+02	unit = "m3"
density	=	1.09E-03	:	1.E-06	:	1.E+04	unit = "kg/m3"
positive	=	9.0	:	0.0	:	1.E+08	
absorption	=	-1.27E-04	:	-1.E+04	:	1.E+04	unit = "kmol/s"

```

parameter = 750.e-3 : -10.0 : 10.0
parameter1 = 0.6 : 0.0 : 1.0
pressure = 1.0 : 1.E-06 : 10. unit = "atm"
press_CO2 = 1.E-03 : 1.E-06 : 1.0 unit = "atm"
notype = 2.0 : -1.E+12 : 1.E+12
notype1 = 2.0 : -1.E+12 : 1.E+12
height = 3.0 : 0.0 : 10.0 unit = "m"
ratio = 0.2 : 0.01 : 0.75 unit = "mol/mol"
calc_con = 5.0 : 0.1 : 10.
mol_tran = 3.5E-03 : -2.0 : 10.0 unit = "kmol/s"
velocity = 1.0 : 0.0 : 100.0 unit = "m/s"
ilknur = 3. : 1.e-6 : 100

```

Note - The maximum height may have to be increased if a small
number of units and a tight product concentration are specified.

```

STREAM liquidstream
TYPE
  MOLFLOW,TEMP,MOLCONC

```

```

STREAM gasstream
TYPE
  MOLFLOW,TEMP,EFFICIENCY,EFFICIENCY

```

```

STREAM gasstream1
TYPE
  MOLFLOW,TEMP,EFFICIENCY,EFFICIENCY,EFFICIENCY

```

```

MODEL      DUMB
TYPE
  false      as notype
  false1     as notype
STREAM
  connection 1 false
EQUATION
  false = false1;

```

```

MODEL      FULLCL

```

```

TYPE

```

```

flu,B,Vb,fgc,S,Fgu,S_m      as molflow
fg1,fg2                      as molflow

```

L1,L2,Lc	as molflow
Tc,Tlu	as temp
Tr,Tl1_m,Tl2_m,Tr_m	as temp
Tl1,Tl2	as temp
Cl1,Cl2	as molconc
Cr,Cr_m,Cl1_m,Cl2_m	as molconc
Ca1,Ca2,Ca3	as molconc
Ca1_m,Ca2_m,Ca3_m	as molconc
Hvap,Hrxn,Hst	as enthalpy
Cpl,Cpc,Cpv	as hcapacity
Vl,Vla,Vga	as volume
dl,dg	as density
Cmea,Ad,hl,htra	as positive
Pr,Pc,Pa	as pressure
Pd1,Pd2	as pressure
k_fit,y_fit	as parameter
k,mw,fn1r,fn1d1,fn1d2	as notype
A1,B1,C1,hd1,kdh,plt1,plt2,plt3	as notype
ya1,ya2,ya3,Ycu	as efficiency
ya1_m,ya2_m,ya3_m	as efficiency
ycd1,ycd2	as efficiency
ycd1eq,ycd2eq	as efficiency
ywd1,ywd2	as efficiency
ycr,ycreq,ycc,ywr,y,ywc	as efficiency
Smass,y_reading	as notype
drivef1,drivef2,drivef3	as positive
Ed	as parameter1
Qc,Qr,Qrxn,Qliq,Qvap,Qerror	as notype

STREAM

```

CONNECTION 1 S
CONNECTION 2 Ca1_m,ya1_m
CONNECTION 3 Ca2_m,ya2_m
CONNECTION 4 Ca3_m,ya3_m
CONNECTION 5 Cr_m,Tr_m
CONNECTION 6 Cl1_m,Tl1_m
CONNECTION 7 Cl2_m,Tl2_m
CONNECTION 8 S_m
CONNECTION 9 y
CONNECTION 10 k

```

EQUATION

```
#####
```

```
# Pressure Drop and Vapour Flowrates in Desorber and Reboiler.
```

```

Vb*Vb = ((Pr-Pd1-hd1*plt1)/(plt1*kdh*0.77*(1.1515e4)'2));
fg1*fg1 = ((Pd1-Pd2-hd1*plt2)/(plt2*kdh*0.77*(1.1515e4)'2));
fg2*fg2 = ((Pd2-Pc-hd1*plt3)/(plt3*kdh*0.77*(1.1515e4)'2));
fgc*ycc = fg2*ydc2;
0.5*plt1=plt2=plt3;

```

Liquid Flowrates.

```

L1 = Vb - fgc + flu;
L2 = Vb - fgc + flu;
Lc = fg2 - fgc;
B = flu;

```

Vapour CO2 Composition in Desorber and Reboiler.

```

fn1r = (-47.91+112.8*Cr/Cmea+(0.7615-1.668*Cr/Cmea)*Tr
        + (-0.0029+0.007*Cr/Cmea)*Tr*Tr);
fn1d1 = (-47.91+112.8*Cl1/Cmea+(0.7615-1.668*Cl1/Cmea)*Tl1
        + (-0.0029+0.007*Cl1/Cmea)*Tl1*Tl1);
fn1d2 = (-47.91+112.8*Cl2/Cmea+(0.7615-1.668*Cl2/Cmea)*Tl2
        + (-0.0029+0.007*Cl2/Cmea)*Tl2*Tl2);

ydc1eq = Exp(fn1d1)/(Pd1*760.0);
ydc2eq = Exp(fn1d2)/(Pd2*760.0);
ycreq = Exp(fn1r)/(Pr*760.0);

```

Actual CO2 Compositions based on Tray Efficiency.

```

ydc1*Pd1 = Ed*ydc1eq*Pd1 + (1-Ed)*ycr *Pr ;
ydc2*Pd2 = Ed*ydc2eq*Pd2 + (1-Ed)*ydc1 *Pd1 ;
ycr = ycreq;

```

Vapour H2O Composition.

```

ywd1 = 0.9513*Exp(A1-B1/(C1+Tl1))/(Pd1*760.);
ywd2 = 0.9513*Exp(A1-B1/(C1+Tl2))/(Pd2*760.);
ywr = 0.9513*Exp(A1-B1/(C1+Tr))/(Pr*760.);
ywc = Exp(A1-B1/(C1+Tc))/(Pc*760.);

```

Gas molar fraction definition.

```

1.0 = ydc1 + ywd1;
1.0 = ydc2 + ywd2;
1.0 = ycr + ywr;
1.0 = ycc + ywc;

```

Driving Force across Interface in Absorber.

```
drivef1*drivef1 = Cmea-2.*Ca1;
drivef2*drivef2 = Cmea-2.*Ca2;
drivef3*drivef3 = Cmea-2.*Ca3;
```

Duties.

Condenser

```
Qc = - fg2*ywd2*(Hvap+Cpv*Tl2) - fg2*yd2*(Hrxn+Cpc*Tl2)
      + fgc*ywc *(Hvap+Cpv*Tc ) + fgc*ycc *(Hrxn+Cpc*Tc )
      + Cpl*Lc*Tc;
```

Steam

```
Qr = Hst*S;
```

Reaction

```
Qrxn = -fgc*ycc*Hrxn;
```

Difference in Enthalpy of Liquid Streams

```
Qliq = Cpl*(flu*Tlu-B*Tr);
```

Enthalpy of Vapour Leaving Condenser

```
Qvap = - fgc*ywc *(Hvap+Cpv*Tc ) - fgc*ycc *(Hrxn+Cpc*Tc );
```

Error in Energy Balance

```
Qerror = Qc + Qr + Qliq + Qvap;
```

#####

#####

#

Absorber Equations

#

#####

Liquid Compositions.

```
Vla*$Ca1 = (mw*B)/(dl)*(Ca2-Ca1) + Pa*(Ycu+ya1)*0.5*htra*k*drivef1;
Vla*$Ca2 = (mw*B)/(dl)*(Ca3-Ca2) + Pa*(ya1+ya2)*0.5*htra*k*drivef2;
Vla*$Ca3 = (mw*B)/(dl)*(Cr -Ca3) + Pa*(ya2+ya3)*0.5*htra*k*drivef3;
```

Gas Compositions.

```
(dg*Vga)*$ya1 = Fgu*(Ycu-ya1) - Pa*(Ycu+ya1)*0.5*htra*k*drivef1;
(dg*Vga)*$ya2 = Fgu*(ya1-ya2) - Pa*(ya1+ya2)*0.5*htra*k*drivef2;
```

#

```
# (dg*Vga)*$ya3 = Fgu*(ya2-ya3) - Pa*ya3*htra*k*drivef3;
```

```
#
# Note - This differential equation replace by the following
#         algebraic equation for reduced model.
#
```

$$0 = F_{gu} \cdot (y_{a2} - y_{a3}) - P_a \cdot (y_{a2} + y_{a3}) \cdot 0.5 \cdot h_{tra} \cdot k \cdot drivef3;$$

```
#####
#                               #
#   Reboiler Equations        #
#                               #
#####
```

```
# Liquid Composition.
```

$$(Ad \cdot h_l) \cdot \$Cr = (L1 \cdot Cl1 - B \cdot Cr) \cdot (mw/dl) - Vb \cdot ycr;$$

```
# Temperature Dynamics.
```

```
#
# ((Cpl*dl/mw)*(Ad*hl))*$Tr = Cpl*(L1*Tl1-B*Tr)
#
# Note - This differential equation replace by the following
#         algebraic equation for reduced model.
#
```

$$\begin{aligned} 0 = & Cpl \cdot (L1 \cdot Tl1 - B \cdot Tr) \\ & - Vb \cdot ywr \cdot (Hvap + Cpv \cdot Tr) \\ & - Vb \cdot ycr \cdot (Hrxn + Cpc \cdot Tr) \\ & + Hst \cdot S; \end{aligned}$$

```
#####
#                               #
#   Desorber Equations        #
#                               #
#####
```

```
# Liquid Compositions.
```

$$\begin{aligned} V1 \cdot \$Cl1 &= (mw/dl) \cdot (L2 \cdot Cl2 - L1 \cdot Cl1) - (fg1 \cdot ycd1 - Vb \cdot ycr); \\ V1 \cdot \$Cl2 &= (mw/dl) \cdot (flu \cdot Ca1 - L2 \cdot Cl2) - (fg2 \cdot ycd2 - fg1 \cdot ycd1); \end{aligned}$$

```
# Temperature Dynamics.
```

$$\begin{aligned} (V1 \cdot Cpl \cdot dl/mw) \cdot \$Tl1 &= \\ & Vb \cdot ywr \cdot (Hvap + Cpv \cdot Tr) - fg1 \cdot ywd1 \cdot (Hvap + Cpv \cdot Tl1) \end{aligned}$$

```

+ Vb*ycr*(Hrxn+Cpc*Tr) - fg1*ycd1*(Hrxn+Cpc*Tl1)
+ L2*Cpl*Tl2 - L1*Cpl*Tl1;

```

```

(Vl*Cpl*dl/mw)*$Tl2 =
fg1*ywd1*(Hvap+Cpv*Tl1) - fg2*ywd2*(Hvap+Cpv*Tl2)
+ fg1*ycd1*(Hrxn+Cpc*Tl1) - fg2*ycd2*(Hrxn+Cpc*Tl2)
+ Lc*Cpl*Tc - L2*Cpl*Tl2
+ flu*Cpl*Tlu;

```

```

#####

```

```

# Measurements.

```

```

Ca1_m = Ca1;
ya1_m = ya1;
Ca2_m = Ca2;
ya2_m = ya2;
Ca3_m = Ca3;
ya3_m = ya3;
Cr_m = Cr;
Tr_m = Tr;
Cl1_m = Cl1;
Tl1_m = Tl1;
Cl2_m = Cl2;
Tl2_m = Tl2;
S_m = S;
y = ya3_m;

```

```

# Calculation of Steam Flowrate in g/s.

```

```

Smass = S*1000.0*18.0;

```

```

# Convert CO2 composition to % for Paragon.

```

```

y_reading = y*1000.0;

```

```

k_fit = k*1000.0;
y_fit = y*1000.0;

```

```

****

```

```

FLOWSHEET

```

```

# Dummy connection.

```

```

CONNECTION 1 OF plant

```

```

IS CONNECTION 1 OF dummy

```

```

****

```

UNIT DUMMY IS DUMB

UNIT PLANT IS A FULLCL

OPERATION

SET

PLANT

WITHIN plant

A1 = 18.379

B1 = 3880.1

C1 = 230.0

hdl = 4.9289e-3

kdh = 7.118e-6

Tc = 54.3

Pc = 1.06

mw = 20.13

dl = 1000.0

Hvap = 4.54e4

Hrxn = 7.34e4

Cpc = 47.09

Cpv = 35.4

Cpl = 79.2

plt1 = 14.320

Vl = 3.34e-2

Ad = 0.1995

Vla = 0.01884

Vga = 0.1413

Pa = 1.89

dg = 75.2e-3

htra = 2.5

Cmea = 2.5

k_fit = 0.291

Ed = 0.776

hl = 0.312

flu = 14.8e-3

Fgu = 1.11e-3

Ycu = 0.04

Tlu = 94.0

Hst = 45400.0

Smass = 24.0

INITIAL

WITHIN plant

\$Ca1 = 0.0
\$Ca2 = 0.0
\$Ca3 = 0.0
\$ya1 = 0.0
\$ya2 = 0.0
\$Cr = 0.0
\$Cl1 = 0.0
\$Tl1 = 0.0
\$Cl2 = 0.0
\$Tl2 = 0.0

#-----
P R E S E T from RESULTS
#-----

PRESET

WITHIN DUMMY

FALSE1 = 1.344E-3

WITHIN PLANT

B = 14.800E-3
VB = 1.394E-3
FGC = 51.451E-6
S = 1.344E-3
S_M = 1.344E-3
FG1 = 1.360E-3
FG2 = 1.009E-3
L1 = 16.143E-3
L2 = 16.143E-3
LC = 957.602E-6
TR = 106.261
TL1_M = 104.208
TL2_M = 103.088
TR_M = 106.261
TL1 = 104.208
TL2 = 103.088
CL1 = 0.598
CL2 = 0.632
CR = 0.516
CR_M = 0.516
CL1_M = 0.598
CL2_M = 0.632
CA1 = 0.665
CA2 = 0.545
CA3 = 0.520
CA1_M = 0.665

CA2_M = 0.545
CA3_M = 0.520
PR = 1.205
PD1 = 1.133
PD2 = 1.096
Y_FIT = 0.174
K = 291.000E-6
FN1R = 3.278
FN1D1 = 3.538
FN1D2 = 3.627
PLT2 = 7.160
PLT3 = 7.160
YA1 = 7.904E-3
YA2 = 1.206E-3
YA3 = 173.654E-6
YA1_M = 7.904E-3
YA2_M = 1.206E-3
YA3_M = 173.654E-6
YCD1 = 37.926E-3
YCD2 = 43.811E-3
YCD1EQ = 39.978E-3
YCD2EQ = 45.143E-3
YWD1 = 0.962
YWD2 = 0.956
YCR = 28.976E-3
YCREQ = 28.976E-3
YCC = 0.859
YWR = 0.971
Y = 173.654E-6
YWC = 0.141
Y_READING = 0.174
DRIVEF1 = 1.082
DRIVEF2 = 1.187
DRIVEF3 = 1.208
QC = -42.966
QR = 61.038
QRXN = -3.245
QLIQ = -14.372
QVAP = -3.701
QERROR = 0.0

Appendix C

Experimental results

This appendix presents the steady state experimental measurements used for identification of the plant. A summary of the measurements is given in Table C.1 and the results are given in Table C.2.

Symbol	Measurement	Units
F_3	Steam Flowrate	<i>g/s</i>
C_1	Feed CO ₂ Composition	% CO ₂
C_2	Absorber Product CO ₂ Composition	<i>ppm</i> CO ₂
C_5	pH in MEA Stream to Absorber	pH
C_6	pH in MEA Stream to Desorber	pH
P_4	Pressure in Desorber	<i>bar</i>
P_1	Pressure in Absorber	<i>bar</i>
L_1	Level in the Bottom of Desorber	<i>m</i>
L_3	Level in the Bottom of Absorber	<i>m</i>
F_9	Flowrate of MEA to Desorber	<i>kg/s</i>
F_2	Feed Gas Flowrate to Absorber	<i>g/s</i>
T_2	Temperature of MEA after Cooler	°C
T_{10}	Temperature of Liquid in Condenser	°C
T_{13}	Temperature of MEA entering Desorber	°C
T_{14}	Temperature of MEA between Desorber and Heat Exchanger	°C
T_{19}	Temperature of MEA at the bottom of Absorber	°C
T_{24}	Temperature at the top of Absorber	°C

Table C.1: Steady state experimental measurements

F ₃	C ₁	C ₂	C ₅	C ₆	P ₄	P ₁	L ₁	L ₃	F ₉	F ₂	T ₂	T ₁₀	T ₁₃	T ₁₄	T ₁₉	T ₂₄
14.1	4.0	510	9.990	9.798	107	190	0.53	0.44	0.31	24.6	45.1	35.9	92.7	105.4	46.4	43.6
14.2	4.0	410	10.001	9.834	106	191	0.42	0.44	0.31	25.0	45.9	42.5	93.1	106.0	46.8	44.6
14.2	4.0	390	10.031	9.827	106	189	0.44	0.46	0.31	25.0	44.9	41.0	92.4	105.2	46.2	43.6
14.2	4.0	290	10.068	9.879	106	190	0.55	0.45	0.31	24.5	45.1	49.2	92.4	105.8	46.5	43.8
14.1	4.0	403	10.008	9.84	106	190	0.56	0.45	0.31	31.0	45.1	43.9	92.1	104.2	47.3	43.6
19.2	4.0	290	10.094	9.901	106.5	190	0.45	0.45	0.31	24.8	44.9	50.0	92.4	106.0	46.3	43.6
19.2	3.8	195	10.146	10.022	106.5	190	0.39	0.46	0.30	24.7	45.7	45.3	93.4	106.2	46.2	44.1
19.2	4.0	230	10.112	9.969	106	190	0.39	0.46	0.31	25.0	46.1	48.7	93.3	106.3	47.0	44.9
19.1	4.0	240	10.133	9.966	105	190	0.40	0.44	0.30	25.0	45.1	47.6	92.4	105.7	46.1	43.6
19.3	3.9	216	10.098	9.967	105.5	190	0.47	0.45	0.31	34.0	45.1	52.9	92.7	105.2	46.9	43.6
23.8	3.8	160	10.213	10.075	106	190	0.38	0.45	0.32	25.1	44.9	49.4	93.1	106.4	46.1	43.6
24.2	4.0	155	10.205	10.053	105	190	0.40	0.44	0.31	25.0	45.1	51.1	93.1	106.0	46.1	43.8
24.3	4.0	178	10.205	10.036	105	190	0.39	0.45	0.31	25.0	45.0	51.1	92.9	106.1	46.1	43.6
28.8	4.0	155	10.211	10.005	107	190	0.36	0.45	0.31	24.8	45.1	51.1	93.7	106.7	46.3	43.6
28.9	4.0	145	10.228	10.022	107	190	0.35	0.45	0.31	25.0	45.1	51.8	93.8	106.8	46.3	43.8
29.1	4.0	146	10.272	10.088	107	189	0.48	0.45	0.31	25.0	44.9	51.9	93.4	106.7	46.1	43.6
29.2	4.0	147	10.231	10.071	106	190	0.49	0.45	0.31	24.5	45.1	58.0	93.9	107.2	46.4	43.6
29.4	4.0	121	10.242	10.089	106	190	0.43	0.44	0.31	25.0	45.1	61.2	94.0	107.4	46.4	43.8
29.2	4.0	131	10.228	10.086	106	190	0.48	0.45	0.31	31.5	44.9	54.5	93.3	106.0	47.3	43.6
34.1	3.8	129	10.284	10.126	106	189	0.35	0.45	0.31	25.1	44.9	54.3	94.0	107.4	46.1	43.6
34.3	4.0	96	10.309	10.121	106	190	0.34	0.45	0.31	25.8	44.9	55.9	93.8	107.1	46.1	43.8
38.9	4.0	106	10.283	10.091	107	190	0.30	0.44	0.31	24.7	45.1	56.7	94.0	107.6	46.3	43.6
39.2	4.0	081	10.268	10.141	105.5	190	0.36	0.45	0.31	25.3	45.1	58.6	94.3	107.0	46.4	43.6
39.2	4.0	124	10.300	10.103	106	190	0.38	0.45	0.31	30.0	45.1	61.6	94.5	107.0	47.3	43.6

Table C.2: Steady state experimental results.

Appendix D

Controller Design in Mathematica

This appendix gives details on the method used for obtaining the analytic inverses of both the full and reduced models.

Two routines were written for Mathematica 1.2 (Wolfram, 1988) to perform the algebraic manipulations required to compute the Lie derivatives. The first function, *L*, computes the Lie derivative given the vector field, *f*, and the scalar equation, *h*. The third argument to this function, *n*, is equal to the number of state equations. The answer is given in terms of the vector *x*[*i*] (*i* = 1,...,*n*). If different variables are required then this can be obtained by setting each element of the vector to the required variable. For example, *x*[1] = *C*, will produce a answer with the variable *C* rather than *x*[1].

The second function, *Lr*, computes the Lie derivative given the vector field, *f*, the answer from another Lie derivative operation (*L* or *Lr*) as *h*. The third variable is the number of states. For example the following computes the third Lie derivative of the vector field, *f*, and the scalar function, *h*, with 10 states.

```
Lr[f,Lr[f,L[f,h,10],10],10]
```

The functions are defined by,

```
L/: L[f_, h_, n_] :=
  Apply[Plus, Table[D[Apply[h, Table[x[i], {i, n}]], x[j]], {j, n}]*
    Apply[f, Table[x[i], {i, n}]]]

Lr/: Lr[f_, h_, n_] :=
  Block[{i,j},
    Apply[Plus, Table[D[h, x[j]], {j, n}]*Apply[f, Table[x[i], {i, n}]]]
  ]
```

The input files defining the vector field of derivative equations, *f*(*x*), the output function, *h*(*x*), and the vector of states for the full and reduced models are given below. The exact commands used to compute the Lie derivatives and put them into a file, ('check') are,

Full Model

```
Put[FortranForm[L[f,h,12]], "check"]
Put[FortranForm[Lr[f,L[f,h,12],12]], "check"]
Put[FortranForm[Lr[f,Lr[f,L[f,h,12],12],12]], "check"]
Put[FortranForm[Lr[f,Lr[f,Lr[f,L[f,h,12],12],12],12]], "check"]
```

Reduced Model

```
Put[FortranForm[L[f,h,10]], "check"]
Put[FortranForm[Lr[f,L[f,h,10],10]], "check"]
```

As each derivative is computed the number of characters can be found using an editor (in this case Emacs (Stallman, 1993)).

The input files are as follows.

Full Model

(* Complete Model - Mathematica Format *)

```
x[1] = Ca1
x[2] = Ca2
x[3] = Ca3
x[4] = ya1
x[5] = ya2
x[6] = ya3
x[7] = Cr
x[8] = Tr
x[9] = C11
x[10] = C12
x[11] = T11
x[12] = T12
```

(* Algebraic Equations *)

```
fn1r = ((-47.91+112.8*Cr/Cmea+(0.7615-1.668*Cr/Cmea)*Tr
        + (-0.0029+0.007*Cr/Cmea)*Tr*Tr));
fn1d1 = ((-47.91+112.8*C11/Cmea+(0.7615-1.668*C11/Cmea)*T11
        + (-0.0029+0.007*C11/Cmea)*T11*T11));
fn1d2 = ((-47.91+112.8*C12/Cmea+(0.7615-1.668*C12/Cmea)*T12
        + (-0.0029+0.007*C12/Cmea)*T12*T12));

pcd1eq = Exp[fn1d1]/760.0
pcd2eq = Exp[fn1d2]/760.0
pcreq = Exp[fn1r]/760.0
```

```

pcr = pcreq
pcd1 = Ed*pcd1eq + (1-Ed)*pcr
pcd2 = Ed*pcd2eq + (1-Ed)*pcd1

```

```

pwd1 = 0.9513*Exp[A1-B1/(C1+Tl1)]/(760.0)
pwd2 = 0.9513*Exp[A1-B1/(C1+Tl2)]/(760.0)
pwr = 0.9513*Exp[A1-B1/(C1+Tr)]/(760.0)
pwc = Exp[A1-B1/(C1+Tc)]/(760.0)

```

```

Pd1 = pcd1 + pwd1
Pd2 = pcd2 + pwd2
Pr = pcr + pwr
pcc = Pc - pwc

```

```

ycd1 = pcd1/Pd1
ycd2 = pcd2/Pd2
ycr = pcr /Pr
ycc = pcc /Pc

```

```

ywd1 = pwd1/Pd1
ywd2 = pwd2/Pd2
ywr = pwr /Pr
ywc = pwc /Pc

```

```

plt2 = plt1*0.5
plt3 = plt2

```

```

Vb = ((Pr-Pd1-hd1*plt1)/(kdh*0.77*(1.1515 10^-4)^2))^0.5
fg1 = ((Pd1-Pd2-hd1*plt2)/(kdh*0.77*(1.1515 10^-4)^2))^0.5
fg2 = ((Pd2-Pc-hd1*plt3)/(kdh*0.77*(1.1515 10^-4)^2))^0.5
fgc = (fg2*ycd2)/ycc

```

```

L1 = Vb - fgc + flu
L2 = fg1 - fgc + flu
Lc = fg2 - fgc
B = flu

```

```

drivef1 = (Cmea-2.0*Ca1)^0.5
drivef2 = (Cmea-2.0*Ca2)^0.5
drivef3 = (Cmea-2.0*Ca3)^0.5

```

(* Differential Equations *)

```

f[Ca1_,Ca2_,Ca3_,ya1_,ya2_,ya3_,Cr_,Tr_,Cl1_,Cl2_,Tl1_,Tl2_] = {
((mw*B)/(dl)*(Ca2-Ca1) + Pa*(Ycu+ya1)*0.5*htra*k*drivef1)/Vla,
((mw*B)/(dl)*(Ca3-Ca2) + Pa*(ya1+ya2)*0.5*htra*k*drivef2)/Vla,
((mw*B)/(dl)*(Cr -Ca3) + Pa*(ya2+ya3)*0.5*htra*k*drivef3)/Vla,

```

```

(Fgu*(Ycu-ya1) - Pa*(Ycu+ya1)*0.5*htra*k*drivef1)/(dg*Vga),
(Fgu*(ya1-ya2) - Pa*(ya1+ya2)*0.5*htra*k*drivef2)/(dg*Vga),
(Fgu*(ya2-ya3) - Pa*(ya2+ya3)*0.5*htra*k*drivef3)/(dg*Vga),
((L1*C11-B*Cr)*(mw/dl)-Vb*ycr)/(Ad*hl),
((Cp1*(L1*T11-B*Tr)
 - Vb*ywr*(Hvap+Cpv*Tr)
 - Vb*ycr*(Hrxn+Cpc*Tr)
 + Hst*S)/((Cp1*dl/mw)*(Ad*hl))),
((mw/dl)*(L2 *C12-L1*C11) - (fg1*ycd1-Vb *ycr ))/V1,
((mw/dl)*(flu*Ca1-L2*C12) - (fg2*ycd2-fg1*ycd1))/V1,
((Vb*ywr*(Hvap+Cpv*Tr) - fg1*ywd1*(Hvap+Cpv*T11)
 + Vb*ycr*(Hrxn+Cpc*Tr) - fg1*ycd1*(Hrxn+Cpc*T11)
 + L2*Cp1*T12 - L1*Cp1*T11)/(V1*Cp1*dl/mw)),
((fg1*ywd1*(Hvap+Cpv*T11) - fg2*ywd2*(Hvap+Cpv*T12)
 + fg1*ycd1*(Hrxn+Cpc*T11) - fg2*ycd2*(Hrxn+Cpc*T12)
 + Lc*Cp1*Tc - L2*Cp1*T12
 + flu*Cp1*Tlu)/(V1*Cp1*dl/mw))
}

```

(* Output Equation *)

```
h[Ca1_,Ca2_,Ca3_,ya1_,ya2_,ya3_,Cr_,Tr_,C11_,C12_,T11_,T12_] := {ya3}
```

Reduced Model

(* Complete Model - Mathematica Format *)

```

x[1]  = Ca1
x[2]  = Ca2
x[3]  = Ca3
x[4]  = ya1
x[5]  = ya2
x[6]  = Cr
x[7]  = C11
x[8]  = C12
x[9]  = T11
x[10] = T12

```

(* Algebraic Equations *)

```

fn1r = ((-47.91+112.8*Cr/Cmea+(0.7615-1.668*Cr/Cmea)*Tr
 + (-0.0029+0.007*Cr/Cmea)*Tr*Tr));
fn1d1 = ((-47.91+112.8*C11/Cmea+(0.7615-1.668*C11/Cmea)*T11
 + (-0.0029+0.007*C11/Cmea)*T11*T11));
fn1d2 = ((-47.91+112.8*C12/Cmea+(0.7615-1.668*C12/Cmea)*T12

```

```

      + (-0.0029+0.007*C12/Cmea)*T12*T12));

pcd1eq = Exp[fn1d1]/760.0
pcd2eq = Exp[fn1d2]/760.0
pcreq  = Exp[fn1r]/760.0

pcr = pcreq
pcd1 = Ed*pcd1eq + (1-Ed)*pcr
pcd2 = Ed*pcd2eq + (1-Ed)*pcd1

pwd1 = 0.9513*Exp[A1-B1/(C1+T11)]/(760.0)
pwd2 = 0.9513*Exp[A1-B1/(C1+T12)]/(760.0)
pwr  = 0.9513*Exp[A1-B1/(C1+Tr)]/(760.0)
pwc  = Exp[A1-B1/(C1+Tc)]/(760.0)

Pd1 = pcd1 + pwd1
Pd2 = pcd2 + pwd2
Pr  = pcr + pwr
pcc = Pc - pwc

ycd1 = pcd1/Pd1
ycd2 = pcd2/Pd2
ycr  = pcr /Pr
ycc  = pcc /Pc

ywd1 = pwd1/Pd1
ywd2 = pwd2/Pd2
ywr  = pwr /Pr
ywc  = pwc /Pc

plt2 = plt1*0.5
plt3 = plt2

Vb = ((Pr-Pd1-hd1*plt1)/(kdh*0.77*(1.1515 10^-4)^2))^0.5
fg1 = ((Pd1-Pd2-hd1*plt2)/(kdh*0.77*(1.1515 10^-4)^2))^0.5
fg2 = ((Pd2-Pc-hd1*plt3)/(kdh*0.77*(1.1515 10^-4)^2))^0.5
fgc = (fg2*ycd2)/ycc

L1 = Vb - fgc + flu
L2 = fg1 - fgc + flu
Lc = fg2 - fgc
B = flu

drivef1 = (Cmea-2.0*Ca1)^0.5
drivef2 = (Cmea-2.0*Ca2)^0.5
drivef3 = (Cmea-2.0*Ca3)^0.5

```

(* Two States with dynamics removed *)

$$ya3 = (Fgu*ya2 - Pa*htra*k*0.5*drivef3*ya2)/(Fgu + Pa*htra*k*0.5*drivef3)$$

$$Tr = ((Cpl*L1*Tl1 - Vb*ywr*Hvap - Vb*ycr*Hrxn + Hst*S) / (Cpl*B + Cpv*ywr*Vb + Cpc*ycr*Vb)) *$$

(* Note - Manipulated input (S) appears in above algebraic relationships. So Calculating Lie derivates amounts to finding a relationship with Tr *)

(* Differential Equations *)

```
f[Ca1_,Ca2_,Ca3_,ya1_,ya2_,Cr_,Cl1_,Cl2_,Tl1_,Tl2_] = {
  ((mw*B)/(d1)*(Ca2-Ca1) + Pa*(Ycu+ya1)*0.5*htra*k*drivef1)/Vla,
  ((mw*B)/(d1)*(Ca3-Ca2) + Pa*(ya1+ya2)*0.5*htra*k*drivef2)/Vla,
  ((mw*B)/(d1)*(Cr -Ca3) + Pa*(ya2+ya3)*0.5*htra*k*drivef3)/Vla,
  (Fgu*(Ycu-ya1) - Pa*(Ycu+ya1)*0.5*htra*k*drivef1)/(dg*Vga),
  (Fgu*(ya1-ya2) - Pa*(ya1+ya2)*0.5*htra*k*drivef2)/(dg*Vga),
  ((L1*Cl1-B*Cr)*(mw/d1)-Vb*ycr)/(Ad*hl),
  ((mw/d1)*(L2 *Cl2-L1*Cl1) - (fg1*ycd1-Vb *ycr ))/Vl,
  ((mw/d1)*(flu*Ca1-L2*Cl2) - (fg2*ycd2-fg1*ycd1))/Vl,
  ((Vb*ywr*(Hvap+Cpv*Tr) - fg1*ywd1*(Hvap+Cpv*Tl1)
    + Vb*ycr*(Hrxn+Cpc*Tr) - fg1*ycd1*(Hrxn+Cpc*Tl1)
    + L2*Cpl*Tl2 - L1*Cpl*Tl1)/(Vl*Cpl*d1/mw)),
  ((fg1*ywd1*(Hvap+Cpv*Tl1) - fg2*ywd2*(Hvap+Cpv*Tl2)
    + fg1*ycd1*(Hrxn+Cpc*Tl1) - fg2*ycd2*(Hrxn+Cpc*Tl2)
    + Lc*Cpl*Tc - L2*Cpl*Tl2
    + flu*Cpl*Tlu)/(Vl*Cpl*d1/mw))
}
```

(* Output Equation *)

```
h[Ca1_,Ca2_,Ca3_,ya1_,ya2_,Cr_,Cl1_,Cl2_,Tl1_,Tl2_] := {ya3}
```

Appendix E

Inversion of Differential Algebraic Systems

Aim

It is the aim of this appendix to derive conditions under which the following statement is true.

Given,

$$0 = f(x, a, u) \tag{E.1}$$

$$v = g(x, a, u) \tag{E.2}$$

where, x is the vector of states, $x \in \mathbb{R}^n$,

a is the vector of algebraic variables, $a \in \mathbb{R}^{n'}$,

u is the input, $u \in \mathbb{R}$,

v is the new reference input, $v \in \mathbb{R}$.

The vector field f is a smooth mapping $f : W \rightarrow \mathbb{R}^{n'}$ (where W is open and $W \subset \mathbb{R}^{n+1} \times \mathbb{R}^{n'}$). The function g is the smooth mapping $g : X \rightarrow \mathbb{R}$ (where X is open and $X \subset \mathbb{R}^{n+n'+1}$).

There exists a unique smooth mapping Ξ such that $\Xi : Z \rightarrow \mathbb{R}$ (where Z is open and $Z \subset \mathbb{R}^{n+1}$) is the implicit function,

$$u = \Xi(x, v) \tag{E.3}$$

Derivation

If,

$$\left| \frac{\partial f}{\partial a} \right| \neq 0 \tag{E.4}$$

then by the implicit function theorem (Nijmeijer and van der Shaft, 1990) there exists a unique smooth mapping $f' : V \rightarrow \mathbb{R}^{n'}$ (where V is open and $V \subset \mathbb{R}^{n+1}$) such that,

$$\mathbf{f}(x, \mathbf{f}'(x, u), u) = 0 \iff a = \mathbf{f}'(x, u) \quad (\text{E.5})$$

Equation E.2 can alternatively be written as,

$$0 = \mathbf{g}'(x, a, u, v) \quad (\text{E.6})$$

The function $\mathbf{g}'$ is the smooth mapping $\mathbf{g}' : X' \rightarrow \mathbb{R}$ (where X' is open and $X' \subset \mathbb{R}^{n+n'+1} \times \mathbb{R}$). The algebraic variables are implicitly related to the states, x , and the input, u , through the Equation E.5. This can be substituted into Equation E.6 to give,

$$0 = \mathbf{g}'(x, \mathbf{f}'(x, u), u, v) \quad (\text{E.7})$$

which is equivalent to the implicit function,

$$0 = \mathbf{g}''(x, u, v) \quad (\text{E.8})$$

The function $\mathbf{g}''$ is the smooth mapping $\mathbf{g}'' : X'' \rightarrow \mathbb{R}$ (where X'' is open and $X'' \subset \mathbb{R}^{n+1} \times \mathbb{R}$).

If,

$$\frac{\partial \mathbf{g}''}{\partial u} \neq 0 \quad (\text{E.9})$$

then by the implicit function theorem (Nijmeijer and van der Shaft, 1990) there exists a unique smooth mapping $\Xi : Z \rightarrow \mathbb{R}$ (where Z is open and $Z \subset \mathbb{R}^{n+1}$) such that,

$$\mathbf{g}''(x, \Xi(x, v), v) = 0 \iff u = \Xi(x, v) \quad (\text{E.10})$$

The condition in Equation E.9 is the condition to be satisfied to make the statement in the aim of this appendix correct. This condition must now be translated in terms of $\mathbf{f}$ and $\mathbf{g}$.

The input, u , will be implicitly related to the reference input, v , and states, x , if $\frac{\partial \mathbf{f}''}{\partial u} \neq 0$. This expression is found by,

$$\frac{\partial \mathbf{f}''}{\partial u} = \frac{\partial \mathbf{f}'}{\partial u} + \frac{\partial \mathbf{f}'}{\partial \mathbf{f}'} \frac{\partial \mathbf{f}'}{\partial u} \quad (\text{E.11})$$

The term $\frac{\partial \mathbf{f}'}{\partial u}$ can be computed by taking the full derivative of $\mathbf{f}$ as it appears in Equation E.5 with respect to u .

$$\frac{df}{du} = \frac{\partial f}{\partial u} + \frac{\partial f}{\partial f'} \frac{\partial f'}{\partial u} \quad (\text{E.12})$$

Note that $\frac{df}{du} = 0$ because $f(x, a, u) = 0$.

Therefore,

$$\frac{\partial f'}{\partial u} = - \left(\frac{\partial f}{\partial f'} \right)^{-1} \frac{\partial f}{\partial u} \quad (\text{E.13})$$

and

$$\frac{\partial g''}{\partial u} = \frac{\partial g'}{\partial u} - \frac{\partial g'}{\partial f'} \left(\frac{\partial f}{\partial f'} \right)^{-1} \frac{\partial f}{\partial u} \quad (\text{E.14})$$

Furthermore,

$$\frac{\partial g'}{\partial u} = \frac{\partial g}{\partial u} \quad (\text{E.15})$$

$$\frac{\partial g'}{\partial f'} = \frac{\partial g}{\partial f'} \quad (\text{E.16})$$

By Equation E.5,

$$f' = a \quad (\text{E.17})$$

Therefore, the statement list in the aim of this appendix is correct if,

$$\left| \frac{\partial f}{\partial a} \right| \neq 0 \quad (\text{E.18})$$

$$\text{and } \left\{ \begin{array}{l} \frac{\partial g}{\partial u} \neq 0 \\ \text{and/or } \frac{\partial g}{\partial a} \left(\frac{\partial f}{\partial a} \right)^{-1} \frac{\partial f}{\partial u} \neq 0 \end{array} \right. \quad (\text{E.19})$$

Appendix F

Speedup Model of the Nonlinear Controller

This appendix gives a listing of the geometric nonlinear controller model and PI model used in Speedup. Included is an example OPERATION, UNIT and FLOWSHEET section corresponding to an implementation with open loop observer and an external PI regulator. The SET conditions correspond to a steady state at the nominal operating point and the PRESET conditions are required to obtain a converged solution.

MODEL FULLNONL

TYPE

Ca1,Ca2,Ca3,Cr,C11,C12	as molconc
ya1,ya2,ya3,Ycu	as efficiency
Tr,Tr_plant,T11,T12	as temp
beta0,beta1,beta2	as notype1
u_out_1,u_out_2,u_min,u_max	as notype
u_out	as notype
S	as molflow
u,v	as notype
Lh,Lfh,Lffh	as notype
Smass	as notype
Cpl,Cpc,Cpv,Hvap,Hst,Hrxn	as notype
dl,Ad,mw,h1,hdl	as notype
kdh,A1,B1,C1,Cmea	as notype
Vla,Vga,dg,htra,k,Ed	as notype
Pd1,Pd2,Pc,Pr,Pa	as pressure
pcd1eq,pcd2eq,pcreq,pcd1,pcd2	as pressure
pcr,pcc,pwd1,pwd2,pwr,pwc	as pressure
ycd2,ycr,ycc,ywr	as efficiency
plt1,plt2,plt3	as notype
fn1r,fn1d1,fn1d2	as notype
Fgu,fgc,L1,B,flu	as molflow
Tc	as temp
prec1,prec2,prec3,prec4,prec5	as positive
Vb,fg2	as positive

STREAM

CONNECTION 1 Ca1,ya1
 CONNECTION 2 Ca2,ya2
 CONNECTION 3 Ca3,ya3
 CONNECTION 4 Cr,Tr_plant
 CONNECTION 5 C11,T11

```

CONNECTION 6 C12,T12
CONNECTION 7 v
CONNECTION 8 u_out_1
CONNECTION 9 u_out_2
EQUATION

```

```

# Geometric Controller Equation

```

```

v = beta0*Lh + beta1*Lfh + beta2*Lffh;

```

```

# Relationship between Tr and u. This is required as the
# controller equation in effect computes a particular
# value for Tr through the above geometric controller equation
# and the procedure defined below. This is not to be
# confused with the temperature Tr_plant, which is coming
# from the plant, even though they should solve to be the
# same value. In a real implementation they will differ.

```

```

# Algebraic relationships

```

```

fn1r = ((-47.91+112.8*Cr/Cmea+(0.7615-1.668*Cr/Cmea)*Tr
        + (-0.0029+0.007*Cr/Cmea)*Tr*Tr));
fn1d1 = ((-47.91+112.8*C11/Cmea+(0.7615-1.668*C11/Cmea)*T11
        + (-0.0029+0.007*C11/Cmea)*T11*T11));
fn1d2 = ((-47.91+112.8*C12/Cmea+(0.7615-1.668*C12/Cmea)*T12
        + (-0.0029+0.007*C12/Cmea)*T12*T12));

```

```

pcd1eq = Exp(fn1d1)/760.0;
pcd2eq = Exp(fn1d2)/760.0;
pcreq  = Exp(fn1r)/760.0;

```

```

pcr = pcreq;
pcd1 = Ed*pcd1eq + (1-Ed)*pcr;
pcd2 = Ed*pcd2eq + (1-Ed)*pcd1;

```

```

pwd1 = 0.9513*Exp(A1-B1/(C1+T11))/(760.0);
pwd2 = 0.9513*Exp(A1-B1/(C1+T12))/(760.0);
pwr  = 0.9513*Exp(A1-B1/(C1+Tr))/(760.0);
pwc  = Exp(A1-B1/(C1+Tc))/(760.0);

```

```

Pd1 = pcd1 + pwd1;
Pd2 = pcd2 + pwd2;
Pr  = pcr  + pwr;
pcc = Pc  - pwc;

```

```

ycd2 = pcd2/Pd2;
ycr  = pcr /Pr;

```

```
ycc = pcc / Pc;
```

```
ywr = pwr / Pr;
```

```
plt2 = plt1*0.5;
```

```
plt3 = plt2;
```

```
Vb*Vb = ((Pr-Pd1-hdl*plt1)/(plt1*kdh*0.77*(1.1515e4)'2));
```

```
fg2*fg2 = ((Pd2-Pc-hdl*plt3)/(plt3*kdh*0.77*(1.1515e4)'2));
```

```
fgc = (fg2*ydc2)/ycc;
```

```
L1 = Vb - fgc + flu;
```

```
B = flu;
```

```
Tr = ((Cpl*L1*Tl1-Vb*ywr*Hvap-Vb*ycr*Hrxn+Hst*S)/  
      (Cpl*B+Cpv*ywr*Vb+Cpc*ycr*Vb));
```

```
# Controller Saturation
```

```
u = S;
```

```
IF u < u_min THEN
```

```
    u_out = u_min
```

```
ELSE
```

```
IF u > u_max THEN
```

```
    u_out = u_max
```

```
ELSE
```

```
    u_out = u
```

```
ENDIF
```

```
ENDIF;
```

```
u_out_1 = u_out;
```

```
u_out_2 = u_out;
```

```
# Compute steam flowrate as g/s
```

```
Smass = u_out*1000.0*18.0;
```

```
# Precaculations - included to improve numerical stability
```

```
prec1*prec1 = (-2.*Ca1 + Cmea);
```

```
prec2*prec2 = (-2.*Ca2 + Cmea);
```

```
prec3*prec3 = (-2.*Ca3 + Cmea);
```

```
prec4*prec4 = (-0.00125171*exp(A1 - B1/(C1 + Tl1)) +
```

```
    0.00131579*exp
```

```
    (-47.91 + 112.8*Cr/Cmea + (0.7615 - 1.668*Cr/Cmea)*Tr +
```

```
    (-0.0029 + 0.007*Cr/Cmea)*Tr'2) +
```

```
    0.00125171*exp(A1 - B1/(C1 + Tr)) -
```

```

0.00131579*exp
(-47.91 + 112.8*Cr/Cmea + (0.7615 - 1.668*Cr/Cmea)*Tr +
(-0.0029 + 0.007*Cr/Cmea)*Tr'2)*(1 - Ed) -
0.00131579*exp
(-47.91 + 112.8*Cl1/Cmea +
(0.7615 - 1.668*Cl1/Cmea)*Tl1 +
(-0.0029 + 0.007*Cl1/Cmea)*Tl1'2)*Ed - hdl*plt1);
prec5*prec5 = (0.00125171*exp(A1 - B1/(C1 + T12)) +
0.00131579*
exp
(-47.91 + 112.8*Cl2/Cmea +
(0.7615 - 1.668*Cl2/Cmea)*Tl2 +
(-0.0029 + 0.007*Cl2/Cmea)*Tl2'2)*Ed +
(1 - Ed)*
(0.00131579*
exp
(-47.91 + 112.8*Cr/Cmea +
(0.7615 - 1.668*Cr/Cmea)*Tr +
(-0.0029 + 0.007*Cr/Cmea)*Tr'2)*(1 - Ed) +
0.00131579*
exp
(-47.91 + 112.8*Cl1/Cmea +
(0.7615 - 1.668*Cl1/Cmea)*Tl1 +
(-0.0029 + 0.007*Cl1/Cmea)*Tl1'2)*Ed) -
Pc - 0.5*hdl*plt1);

```

Algebraic expressions for derivatives.

```

Lh = (Fgu*ya2 - 0.5*prec3*Pa*htra*k*ya2)/
(Fgu + 0.5*prec3*Pa*htra*k);
Lfh = ((Fgu - 0.5*prec3*Pa*htra*k)*
(Fgu*(ya1 - ya2) -
0.5*prec2*Pa*htra*k*(ya1 + ya2)))/
(Vga*dg*(Fgu + 0.5*prec3*Pa*htra*k)) +
(0.5*Pa*htra*k*ya2/
(prec3*
(Fgu + 0.5*prec3*Pa*htra*k)) +
0.5*Pa*htra*k*(Fgu*ya2 -
0.5*prec3*Pa*htra*k*ya2)/
(prec3*
(Fgu + 0.5*prec3*Pa*htra*k)'2))*
((-Ca3 + Cr)*flu*mw/dl +
0.5*prec3*Pa*htra*k*
(ya2 + (Fgu*ya2 - 0.5*prec3*Pa*htra*k*ya2)/
(Fgu + 0.5*prec3*Pa*htra*k)))/Vla);
Lffh = ((Fgu - 0.5*prec2*Pa*htra*k)*
(Fgu - 0.5*prec3*Pa*htra*k)*

```

$$\begin{aligned}
& (F_{gu}*(Y_{cu} - y_{a1}) - \\
& \quad 0.5*prec1*Pa*htra*k*(Y_{cu} + y_{a1}))/ \\
& (Vga'^2*dg'^2*(F_{gu} + 0.5*prec3*Pa*htra*k)) + \\
& 0.5*Pa*htra*k*(F_{gu} - 0.5*prec3*Pa*htra*k)* \\
& \quad (y_{a1} + y_{a2})*((-Ca2 + Ca3)*flu*mw/dl + \\
& \quad 0.5*prec2*Pa*htra*k*(y_{a1} + y_{a2}))/ \\
& (prec2*Vga*Vla*dg* \\
& \quad (F_{gu} + 0.5*prec3*Pa*htra*k)) + \\
& flu*mw*(-1.3022e-7*exp \\
& \quad (-47.91 + 112.8*Cr/Cmea + (0.7615 - 1.668*Cr/Cmea)*Tr + \\
& \quad (-0.0029 + 0.007*Cr/Cmea)*Tr'^2)* \\
& \quad prec4/ \\
& \quad ((0.00131579*exp \\
& \quad (-47.91 + 112.8*Cr/Cmea + (0.7615 - 1.668*Cr/Cmea)*Tr + \\
& \quad (-0.0029 + 0.007*Cr/Cmea)*Tr'^2) + \\
& \quad 0.00125171*exp(A1 - B1/(C1 + Tr)))*kdh'0.5) + \\
& mw*(-(Cr*flu) + C11* \\
& \quad (flu + 0.0000989671* \\
& \quad \quad prec4/kdh'0.5 - \\
& \quad \quad 0.0000989671* \\
& \quad \quad (0.00131579* \\
& \quad \quad \quad exp \\
& \quad \quad \quad (-47.91 + 112.8*C12/Cmea + \\
& \quad \quad \quad (0.7615 - 1.668*C12/Cmea)*T12 + \\
& \quad \quad \quad (-0.0029 + 0.007*C12/Cmea)*T12'^2)*Ed + \\
& \quad (1 - Ed)* \\
& \quad (0.00131579* \\
& \quad \quad exp \\
& \quad \quad (-47.91 + 112.8*Cr/Cmea + \\
& \quad \quad (0.7615 - 1.668*Cr/Cmea)*Tr + \\
& \quad \quad (-0.0029 + 0.007*Cr/Cmea)*Tr'^2)*(1 - Ed) + \\
& \quad 0.00131579* \\
& \quad \quad exp \\
& \quad \quad (-47.91 + 112.8*C11/Cmea + \\
& \quad \quad (0.7615 - 1.668*C11/Cmea)*T11 + \\
& \quad \quad (-0.0029 + 0.007*C11/Cmea)*T11'^2)*Ed))*Pc* \\
& \quad prec5/ \\
& ((0.00125171*exp(A1 - B1/(C1 + T12)) + \\
& \quad 0.00131579* \\
& \quad \quad exp \\
& \quad \quad (-47.91 + 112.8*C12/Cmea + \\
& \quad \quad (0.7615 - 1.668*C12/Cmea)*T12 + \\
& \quad \quad (-0.0029 + 0.007*C12/Cmea)*T12'^2)*Ed + \\
& \quad (1 - Ed)* \\
& \quad (0.00131579* \\
& \quad \quad exp \\
& \quad \quad (-47.91 + 112.8*Cr/Cmea +
\end{aligned}$$

$$\begin{aligned}
& (0.7615 - 1.668 \cdot Cr/Cmea) \cdot Tr + \\
& (-0.0029 + 0.007 \cdot Cr/Cmea) \cdot Tr'^2) \cdot (1 - Ed) + \\
& 0.00131579 \cdot \\
& \exp \\
& (-47.91 + 112.8 \cdot Cl1/Cmea + \\
& (0.7615 - 1.668 \cdot Cl1/Cmea) \cdot Tl1 + \\
& (-0.0029 + 0.007 \cdot Cl1/Cmea) \cdot Tl1'^2) \cdot Ed)) \cdot \\
& (-0.00131579 \cdot \exp(A1 - B1/(C1 + Tc)) + Pc) \cdot kdh^{0.5})) / \\
& dl) \cdot (0.5 \cdot Pa \cdot htra \cdot k \cdot ya2 / \\
& (prec3 \cdot \\
& (Fgu + 0.5 \cdot prec3 \cdot Pa \cdot htra \cdot k)) + \\
& 0.5 \cdot Pa \cdot htra \cdot k \cdot (Fgu \cdot ya2 - \\
& 0.5 \cdot prec3 \cdot Pa \cdot htra \cdot k \cdot ya2) / \\
& (prec3 \cdot \\
& (Fgu + 0.5 \cdot prec3 \cdot Pa \cdot htra \cdot k)^2)) / \\
& (Ad \cdot Vla \cdot dl \cdot hl) + (Fgu \cdot (ya1 - ya2) - \\
& 0.5 \cdot prec2 \cdot Pa \cdot htra \cdot k \cdot (ya1 + ya2)) \cdot \\
& ((-Fgu - 0.5 \cdot prec2 \cdot Pa \cdot htra \cdot k) \cdot \\
& (Fgu - 0.5 \cdot prec3 \cdot Pa \cdot htra \cdot k) / \\
& (Vga \cdot dg \cdot (Fgu + 0.5 \cdot prec3 \cdot Pa \cdot htra \cdot k)) + \\
& 0.5 \cdot prec3 \cdot Pa \cdot htra \cdot k \cdot \\
& (1 + (Fgu - 0.5 \cdot prec3 \cdot Pa \cdot htra \cdot k) / \\
& (Fgu + 0.5 \cdot prec3 \cdot Pa \cdot htra \cdot k))) \cdot \\
& (0.5 \cdot Pa \cdot htra \cdot k \cdot ya2 / \\
& (prec3 \cdot \\
& (Fgu + 0.5 \cdot prec3 \cdot Pa \cdot htra \cdot k)) + \\
& 0.5 \cdot Pa \cdot htra \cdot k \cdot \\
& (Fgu \cdot ya2 - 0.5 \cdot prec3 \cdot Pa \cdot htra \cdot k \cdot ya2) / \\
& (prec3 \cdot \\
& (Fgu + 0.5 \cdot prec3 \cdot Pa \cdot htra \cdot k)^2)) / Vla + \\
& (0.5 \cdot Pa \cdot htra \cdot k \cdot (Fgu - 0.5 \cdot prec3 \cdot Pa \cdot htra \cdot k) / \\
& (prec3 \cdot \\
& (Fgu + 0.5 \cdot prec3 \cdot Pa \cdot htra \cdot k)^2) + \\
& 0.5 \cdot Pa \cdot htra \cdot k / \\
& (prec3 \cdot \\
& (Fgu + 0.5 \cdot prec3 \cdot Pa \cdot htra \cdot k))) \cdot \\
& ((-Ca3 + Cr) \cdot flu \cdot mw / dl + \\
& 0.5 \cdot prec3 \cdot Pa \cdot htra \cdot k \cdot \\
& (ya2 + (Fgu \cdot ya2 - 0.5 \cdot prec3 \cdot Pa \cdot htra \cdot k \cdot ya2) / \\
& (Fgu + 0.5 \cdot prec3 \cdot Pa \cdot htra \cdot k))) / Vla) / \\
& (Vga \cdot dg) + ((-Ca3 + Cr) \cdot flu \cdot mw / dl + \\
& 0.5 \cdot prec3 \cdot Pa \cdot htra \cdot k \cdot \\
& (ya2 + (Fgu \cdot ya2 - 0.5 \cdot prec3 \cdot Pa \cdot htra \cdot k \cdot ya2) / \\
& (Fgu + 0.5 \cdot prec3 \cdot Pa \cdot htra \cdot k))) \cdot \\
& (0.5 \cdot Pa \cdot htra \cdot k \cdot (Fgu - 0.5 \cdot prec3 \cdot Pa \cdot htra \cdot k) \cdot \\
& (Fgu \cdot (ya1 - ya2) - \\
& 0.5 \cdot prec2 \cdot Pa \cdot htra \cdot k \cdot (ya1 + ya2))) /
\end{aligned}$$

```

(prec3*Vga*dg*
  (Fgu + 0.5*prec3*Pa*htra*k)'2) +
0.5*Pa*htra*k*(Fgu*(ya1 - ya2) -
  0.5*prec2*Pa*htra*k*(ya1 + ya2))/
(prec3*Vga*dg*
  (Fgu + 0.5*prec3*Pa*htra*k)) +
(0.5*Pa*htra*k*ya2/
  (prec3*
    (Fgu + 0.5*prec3*Pa*htra*k)) +
    0.5*Pa*htra*k*
    (Fgu*ya2 - 0.5*prec3*Pa*htra*k*ya2)/
    (prec3*
      (Fgu + 0.5*prec3*Pa*htra*k)'2))*
(-(flu*mw/dl) +
  0.5*prec3*Pa*htra*k*
  (0.5*Pa*htra*k*ya2/
    (prec3*
      (Fgu + 0.5*prec3*Pa*htra*k)) +
      0.5*Pa*htra*k*
      (Fgu*ya2 - 0.5*prec3*Pa*htra*k*ya2)/
      (prec3*
        (Fgu + 0.5*prec3*Pa*htra*k)'2)) -
    0.5*Pa*htra*k*
    (ya2 + (Fgu*ya2 -
      0.5*prec3*Pa*htra*k*ya2)/
      (Fgu + 0.5*prec3*Pa*htra*k))/
    prec3)/Vla +
(0.5*Pa'2*htra'2*k'2*ya2/
  ((-2.*Ca3 + Cmea)*
    (Fgu + 0.5*prec3*Pa*htra*k)'2) +
    0.5*Pa*htra*k*ya2/
    ((-2.*Ca3 + Cmea)*prec3*
      (Fgu + 0.5*prec3*Pa*htra*k)) +
    0.5*Pa'2*htra'2*k'2*
    (Fgu*ya2 - 0.5*prec3*Pa*htra*k*ya2)/
    ((-2.*Ca3 + Cmea)*
      (Fgu + 0.5*prec3*Pa*htra*k)'3) +
    0.5*Pa*htra*k*
    (Fgu*ya2 - 0.5*prec3*Pa*htra*k*ya2)/
    ((-2.*Ca3 + Cmea)*prec3*
      (Fgu + 0.5*prec3*Pa*htra*k)'2))*
((-Ca3 + Cr)*flu*mw/dl +
  0.5*prec3*Pa*htra*k*
  (ya2 + (Fgu*ya2 - 0.5*prec3*Pa*htra*k*ya2)/
    (Fgu + 0.5*prec3*Pa*htra*k)))/Vla)/Vla);

```

MODEL

PI

TYPE

CO2_set_reading as notype
 CO2_reading as notype
 y,y_set as notype
 error,interror as notype
 v,v_bias as notype
 gain,treset as notype

STREAM

CONNECTION 1 y
 CONNECTION 2 y_set
 CONNECTION 3 v

EQUATION

y = CO2_reading/1000.0;
 y_set = CO2_set_reading/1000.0;
 error = y_set - y;
 \$interror = error;
 v = v_bias + gain*error + gain/treset*interror;

FLOWSHEET

Observer connection to controller

CONNECTION 1 OF controller	IS CONNECTION 2 OF observer
CONNECTION 2 OF controller	IS CONNECTION 3 OF observer
CONNECTION 3 OF controller	IS CONNECTION 4 OF observer
CONNECTION 4 OF controller	IS CONNECTION 5 OF observer
CONNECTION 5 OF controller	IS CONNECTION 6 OF observer
CONNECTION 6 OF controller	IS CONNECTION 7 OF observer

Controller signal to process and observer

CONNECTION 8 OF controller	IS CONNECTION 1 OF plant
CONNECTION 9 OF controller	IS CONNECTION 1 OF observer

Regulator connection to controller

CONNECTION 3 OF regulator	IS CONNECTION 7 OF controller
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Plant connection to regulator

CONNECTION 9 OF plant	IS CONNECTION 1 OF regulator
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OPERATION

SET

PLANT

WITHIN controller

beta0 = 1.0
beta1 = 1125.0
beta2 = 125000.0
u_max = 2.22e-3
u_min = 0.77e-3

A1 = 18.379
B1 = 3880.1
C1 = 230.0
hdl = 4.9289e-3
kdh = 7.118e-6
Tc = 54.3
Pc = 1.06
mw = 20.13
dl = 1000.0
Hvap = 4.54e4
Hrxn = 7.34e4
Cpc = 47.09
Cpv = 35.4
Cpl = 79.2
plt1 = 14.320
Ad = 0.1995
Vla = 0.01884
Vga = 0.1413
Pa = 1.89
dg = 75.2e-3
htra = 2.5
Cmea = 2.5
k = 0.000291
Ed = 0.776
hl = 0.312
flu = 14.8e-3
Fgu = 1.11e-3
Ycu = 0.04
Hst = 45400.0

WITHIN regulator

gain = 0.6
treset = 750.0
v_bias = 0.000173659
CO2_set_reading = 0.173659

WITHIN observer

A1 = 18.379
B1 = 3880.1
C1 = 230.0
hdl = 4.9289e-3
kdh = 7.118e-6
Tc = 54.3
Pc = 1.06
mw = 20.13
dl = 1000.0
Hvap = 4.54e4
Hrxn = 7.34e4
Cpc = 47.09
Cpv = 35.4
Cpl = 79.2
plt1 = 14.320
Vl = 3.34e-2
Ad = 0.1995
Vla = 0.01884
Vga = 0.1413
Pa = 1.89
dg = 75.2e-3
htra = 2.5
Cmea = 2.5
k_fit = 0.291
Ed = 0.776
hl = 0.312
flu = 14.8e-3
Fgu = 1.11e-3
Ycu = 0.04
Tlu = 94.0
Hst = 45400.0

INITIAL

WITHIN observer

\$Ca1 = 0.0
\$Ca2 = 0.0
\$Ca3 = 0.0
\$ya1 = 0.0
\$ya2 = 0.0
\$Cr = 0.0
\$Cl1 = 0.0
\$Tl1 = 0.0
\$Cl2 = 0.0

\$T12 = 0.0

WITHIN regulator
interror = 0.0

PRESET

WITHIN CONTROLLER

TR = 106.261

U_OUT = 1.344E-3

S = 1.344E-3

U = 1.344E-3

LH = 173.659E-6

LFH = -0.0

LFFH = 0.0

SMASS = 24.199

PD1 = 1.133

PD2 = 1.096

PR = 1.205

PCD1EQ = 45.283E-3

PCD2EQ = 49.480E-3

PCREQ = 34.907E-3

PCD1 = 42.959E-3

PCD2 = 48.019E-3

PCR = 34.907E-3

PCC = 0.911

PWD1 = 1.090

PWD2 = 1.048

PWR = 1.170

PWC = 0.149

YCD2 = 43.812E-3

YCR = 28.976E-3

YCC = 0.859

YWR = 0.971

PLT2 = 7.160

PLT3 = 7.160

FN1R = 3.278

FN1D1 = 3.538

FN1D2 = 3.627

FGC = 51.451E-6

L1 = 16.142E-3

B = 14.800E-3

PREC1 = 1.082

PREC2 = 1.187

PREC3 = 1.208

PREC4 = 37.578E-3

PREC5 = 27.193E-3

VB = 1.394E-3

FG2 = 1.009E-3
WITHIN OBSERVER
B = 14.800E-3
VB = 1.394E-3
FGC = 51.451E-6
S = 1.344E-3
S_M = 1.344E-3
FG1 = 1.360E-3
FG2 = 1.009E-3
L1 = 16.142E-3
L2 = 16.142E-3
LC = 957.570E-6
TR = 106.261
TL1_M = 104.208
TL2_M = 103.088
TR_M = 106.261
TL1 = 104.208
TL2 = 103.088
CL1 = 0.598
CL2 = 0.632
CR = 0.516
CR_M = 0.516
CL1_M = 0.598
CL2_M = 0.632
CA1 = 0.665
CA2 = 0.545
CA3 = 0.520
CA1_M = 0.665
CA2_M = 0.545
CA3_M = 0.520
PR = 1.205
PD1 = 1.133
PD2 = 1.096
Y_FIT = 0.174
K = 291.000E-6
FN1R = 3.278
FN1D1 = 3.538
FN1D2 = 3.627
PLT2 = 7.160
PLT3 = 7.160
YA1 = 7.904E-3
YA2 = 1.206E-3
YA3 = 173.659E-6
YA1_M = 7.904E-3
YA2_M = 1.206E-3
YA3_M = 173.659E-6
YCD1 = 37.927E-3

YCD2 = 43.812E-3
YCD1EQ = 39.979E-3
YCD2EQ = 45.145E-3
YWD1 = 0.962
YWD2 = 0.956
YCR = 28.976E-3
YCREQ = 28.976E-3
YCC = 0.859
YWR = 0.971
Y = 173.659E-6
YWC = 0.141
SMASS = 24.199
Y_READING = 17.366E-3
DRIVEF1 = 1.082
DRIVEF2 = 1.187
DRIVEF3 = 1.208
QC = -42.964
QR = 61.036
QRXN = -3.245
QLIQ = -14.372
QVAP = -3.701
QERROR = 0.0

WITHIN REGULATOR

CO2_READING = 17.366E-3
Y = 173.659E-6
Y_SET = 173.659E-6
ERROR = 0
V = 173.664E-6

Appendix G

Plant Operation

This appendix gives the startup and shut down procedures used for the pilot plant. These basic procedures were followed for all experimental runs. The locations of all the automatic and manual valves, and all the instruments are given in Figure G.1.

Startup Procedure

Start Up Procedure

1. Utilities

- Turn on composition meters.
- Turn on instrument air.
- Turn on Nitrogen.
- Check level of MEA in columns (Total > 1.6m).

2. Check Manual Valves

- Open ADMV 101, ADMV 103
- Close ADMV 102
- Open ADMV 1, ADMV 3
- Open ADMV 2
- Open ADMV 8, MV after ADV 9
- Open ADMV 4a, ADMV 4b, ADMV 5
- Open ADMV 6a, ADMV 7 (Quarter turn to maintain level)
- Open ADMV 20
- Open ADMV 304 (After ADF 1)

3. Liquid Circulation

- Loop ADF 1 → ADV 1
- Give ADF 1 set point = 0.3 kg/s
- Loop ADL 3 → ADV 11

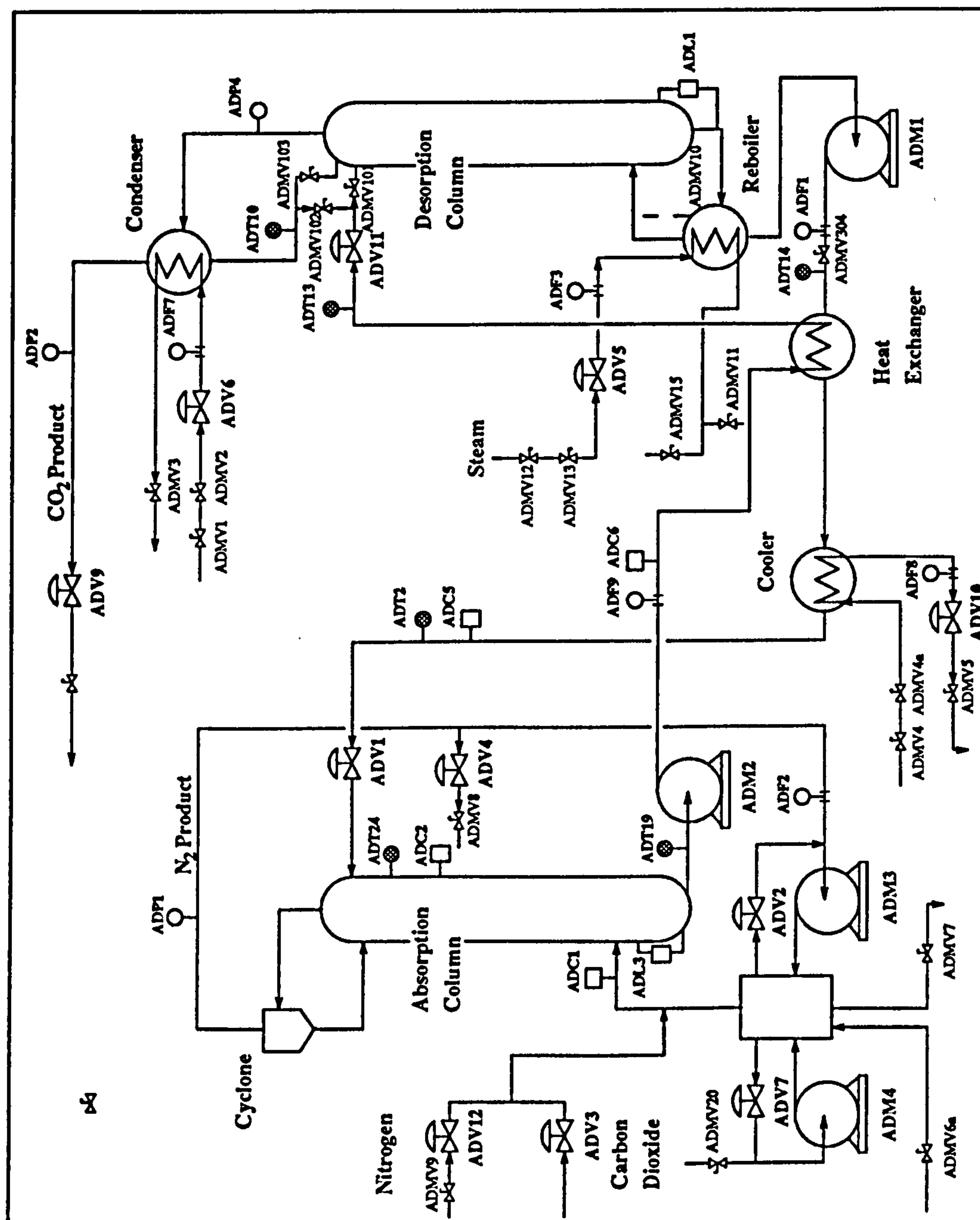


Figure G.1: Schematic of pilot plant sections showing full detail.

- Give ADL 3 set point = 0.55 m
- Start ADM 1
- Check ADL 3 > 0.5 m
- Start ADM 2
- Check ADF 9 and ADF 1 $\approx$ 0.3 kg/s
- Fill ADL 1 to 0.75 m

4. Pressurise Column

- Give ADV 12 set point = 0%
- Give ADV 3 set point = 0%
- Give ADV 9 set point = 0%
- Loop ADP 1 $\rightarrow$ ADV 4
- Give ADP 1 set point = 190kPa
- Open ADMV 9
- Check level in Separation tank.
- Give ADV 2 set point = 100%
- Give ADV 7 set point = 100%
- Start ADM 3
- Start ADM 4
- Loop ADP 2 $\rightarrow$ ADV 7
- Give ADP 2 set point = 110 kPa
- Loop ADF 2 $\rightarrow$ ADV 2
- Give ADF 2 set point = 31 g/s
- Close ADMV 7
- Give ADV 12 set point = 30%
- Check ADP 1 = 185 kPa
- Give ADV 12 set point = 0% (Leak flow is sufficient)

- Close ADMV 9 lightly (Leak flow is sufficient)

5. Start Cooler B4

- Loop ADF 8 → ADV 10
- Check ADF 8 > 0
- Cascade ADT 2 → ADF 8
- Give ADT 2 set point = 45 ° C

6. Start Condenser

- Loop ADF 7 → ADV 6
- Check ADF 7 > 0
- Cascade ADT 10 → ADF 7
- Give ADT 2 set point = 54 ° C

7. Start Reboiler

- Check level in Desorber. If < 0.55 m then
 readjust ADL 3 set point.
- Give ADV 5 set point = 0%
- Open ADMV 10
- Close ADMV 15
- Open ADMV 11
- Open ADMV 12, ADMV 13
- Loop ADF 3 → ADV 5
- Give ADF 3 set point = 30 g/s
- Check for steam at ADMV 10, then Almost fully
 Close ADMV 10
- Check for steam to drain, then Close ADMV 11 and
- Open ADMV 15
- Check ADT 3 = 90 ° C

- Give ADF 3 set point = 24 g/s
- Monitor ADL 1, when boiling occurs it will drop. If it drops too far then Give ADF 3 set point = 0 kg/s, until the level stops dropping.
- Give ADL 3 setpoint = 0.45 m

8. Swap to Open Loop

- Turn on CO₂ bottle.
- Loop ADP 2 → ADV 3
- Give ADP2 setpoint = 108 kPa
- Break ADP 2 → ADV 7
- Ramp ADV 7 to 100%
- Check ADV 7 = 100%
- Close ADMV 20
- Loop ADC 1 → ADV 3
- Give ADC 1 setpoint = 4.0%

Plant Start Up Complete

Shut Down Procedure

Shut Down Procedure

1. Swap to Closed Loop

- Give ADV 7 setpoint = 100%
- Loop ADP 2 → ADV 7
- Give ADP 2 setpoint = 110 kPa
- Open ADMV 20

- Break ADP 2 → ADV 9
- Ramp ADV 9 to 0%
- Check ADV 9 = 0%
- Break ADC 1 → ADV 3

- Turn off CO₂ bottle

2. Shutdown Reboiler

- Break ADF 3 → ADV 5
- Give ADV 5 set point = 0%
- Close ADMV 12, ADMV 13

3. Cool Down Plant

- Break ADT 2 → ADF 8
- Break ADF 8 → ADV 10
- Give ADV 10 set point = 100%
- Break ADF 7 → ADV 6
- Give ADV 6 set point = 100%
- Give ADV1 set point = 0.3 kg/s
- Open ADMV 10
- Check ADT 13 < 70 °C

4. Depressurise Column (Absorber)

- Close ADMV 9
- Break ADP 6 → ADV 12
- Give ADV 12 set point = 0%
- Give ADP 1 set point = 106 kPa
- Check ADP 1 = 106 kPa

- Stop ADM 3
- Stop ADM 4

- Break ADP 2 → ADV 7
- Give ADV 7 set point = 100%
- Break ADF 2 → ADV 2
- Give ADV 2 set point = 0%
- Break ADP 1 → ADV 4
- Give ADV 4 set point = 0%

5. Stop Liquid Circulation

- Stop ADM 1
- Check ADL 3 starts dropping
- Stop ADM 2

- Break ADF 1 → ADV 1
- Give ADV 1 set point = 0%
- Break ADF 9 → ADV 11
- Give ADV 1 set point = 0%

6. Shutdown Condenser

- Give ADV 6 set point = 0%

7. Shutdown Cooler

- Give ADV 10 set point = 0%

8. Close Manual Valves

- Close ADMV 2
- Close ADMV 1, ADMV3
- Close ADMV 8, MV after ADV 9
- Close ADMV 4, ADMV 4a, ADMV 5
- Close ADMV 6a, ADMV 6b
- Close ADMV 9
- Close ADMV 15
- Open ADMV 11

9. Utilities

- Turn off composition meters.
- Turn off and bleed Nitrogen.
- Check level of MEA in columns.

Plant Shut Down Complete

