

OPTIMAL GENERATION EXPANSION PLANNING
FOR ELECTRIC POWER SYSTEMS

by

Rozali bin Mohamed Ali, MSc, DIC

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Thermal Power Section
Department of Mechanical Engineering
Imperial College of Science and Technology
London

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ABSTRACT

This thesis is concerned with the determination of the optimal strategies of development for an electric power system, from the point of view of the choice and sequencing of additions to and removals from existing generating capacity.

An original methodology is presented for the determination of such strategies in the medium term. The basis of the model is a dynamic program, with traceback in a decision tree, using features of the branch-and-bound and implicit enumeration algorithms of mathematical programming to reduce computational requirements to manageable levels. By employing this approach the discrete and limited nature of acceptable alternatives and other awkward real-life constraints can be fairly readily accommodated.

The model is designed for the planning of small to medium-sized power systems, where such discrete effects can be significant. It is also usable for the inspection and evaluation of projected installation schedules in the medium term.

The thesis also describes an original loading simulation model which utilizes a chronological daily load curve rather than a load duration curve for system load description. The model, which makes use of several dynamic programs in its solution, takes into consideration discontinuous feasible regimes of operation of each individual unit comprising the system and treats explicitly the provision of spinning reserve, and the chronological availability of hydro energy. Although computationally less efficient than models using the load duration curve, this model achieves greater accuracy of simulation for mixed hydro-thermal systems, particularly those containing pumped storage stations and large individual units, and is suitable for medium-term system planning and design.

The two models have been incorporated into a package of computer programs and used to examine strategies of optimal expansion for two hypothetical power systems. The results of these two case studies are presented and discussed.

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INTRODUCTION

The electricity supply industry is a key one in the economy of any nation. Electricity as an energy source is clean, and for practical purposes has no competitor for industrial power drives, space cooling, and lighting. Its use is widespread for space heating, cooking, and a wide variety of domestic applications. Electricity supply is therefore concerned with the provision of a vital input to all sectors of the national economy.

Historically the organization of electricity supply began with small private or municipally-owned utilities supplying local areas or industries. Recognition of possible returns to scale led to the merging or expansion of these utilities into large centrally organized concerns, now either privately owned or largely autonomous public bodies. Concomitant with this development was the adoption of a fundamental approach to electricity supply: that of a grid network supplying numerous diverse loads via an interconnected system of generators. This approach to electricity supply is now almost universally used throughout the world.

The electricity supply industry is a natural monopoly, it being generally held that competition would be wasteful of resources. Its size in relation to the national economy and the all-pervasive nature of the commodity it produces has meant that close regulation of the operation and conduct of the industry is necessary. Electricity supply is very much in the public eye, and society is generally very critical of shortfalls in the quality or standard of service provided.

Owing in no small part to the scale of the capital investments involved, careful forward planning has always been employed, in one form or another, by the industry. Constraints on development are often severe, and sometimes because of these the investment criteria employed may not be obviously coincident with those of commercial concerns.

Chapter One of the thesis discusses the operational strategy of power supply, the nature of the loads served, and the problems entailed by these. There is an important link between problems of system operation and design and planning. Chapter Two in contrast concentrates on economic considerations, financial problems, and other constraints on planning. Whilst both these Chapters may be open to criticism for lack of detail from specialists in either field, the objective has been mainly to provide the essential perspective necessary for an appreciation of the problems of system planning. Chapter Three draws the threads together, discusses planning approaches, and suggests a systematic methodology for planning studies.

The succeeding chapters are devoted to the solution of a particular problem: that of determining an installation framework within which detailed feasibility studies can be approached and installation schedules developed. Chapter Four first outlines three possible methods, involving linear programming and dynamic programming. An original two-phase solution is then outlined, which utilizes continuous function dynamic programming together with traceback in a decision tree.

Chapter Five deals with loading simulation models; this begins with an appraisal of models based on the load duration curve. Their advantages and disadvantages are discussed, and the case for a more accurate model based upon chronological load data is then argued. The theory for such a model is then set out. This model explicitly takes into consideration the provision of spinning reserve and constraints on energy production and reservoir levels for hydro plant.

In Chapter Six a brief discussion of a set of computer programs applicable for generation expansion planning in the medium term, applying the theory of the previous two chapters, is given. Results of two case studies are also presented.

Chapter Seven concludes the thesis, with an appraisal of the method, and a discussion of possible avenues for further development.

The scope of the thesis is therefore limited to solution of the problems of expansion or development of a power system for electricity supply on a national or regional scale through a grid system, in the medium term. As such it is not, except in the wider sense, an energy policy study. The development of large centralized organizations dealing almost exclusively with electricity supply has not been without its critics. There are those who suggest that the fundamental function of such giant establishments is the reinforcement of their own existence by expansion to accommodate an increase in demand and consumption that they themselves stimulate, although this may be an unduly cynical view. It is frequently pointed out that since the overall thermal efficiency of conversion of the energy in primary fuels to electricity seldom exceeds 30% on average, for every kilowatt of electricity generated, two kilowatts or so are rejected as waste heat. Unfortunately the structure of the organization of electricity supply means that the electricity supply industry is seldom in a position to lead the way towards more complete energy utilization. We will not be concerned here with studies of this sort, important as they may be; the viewpoint taken is very much that from within the supply industry, as it were. An effort has been made however to recognize and accommodate any constraints which might possibly be imposed as a result of separate energy policy studies, political considerations, or other factors.

Discussion is frequently directed towards the problems of system planners of the smaller systems in general, and those in developing nations in particular. These have not always received much attention in the literature.

PART I

CHAPTER ONE. TECHNICAL CONSTRAINTS ON SYSTEM PLANNING

An electric power system consists of a number of generators interconnected by a transmission network, which also serves as part of the distribution system, in order to satisfy the demand created by diverse loads. This procedure is preferable to that of individual generators connected to individual loads for important technical and economic reasons, some of which will become more apparent in this and the next chapter.

The demand for electricity varies over a wide range with time. Whilst this in itself is not unusual in the public service industries, problems in the design, operation and planning of electric power systems, fundamental and largely unique to the industry, arise from the fact that at the present day there is no feasible large scale technology available for the direct storage of electricity. Consequently demand has to be met exactly at each instant of time, as and when it occurs.

Inspection of the variation of demand with time for typical systems reveals a distinct daily cyclic variation. Peaks of demand occur around midday and early evening, the evening peak being usually the larger of the two. In between these peaks the valleys correspond to slack periods of demand, the lowest demand level during a 24-hour period occurring usually in the early hours of the morning. The magnitude and timing of the peaks and depressions for any particular power system are remarkably consistent from day to day, and significant departures from the general short term trends occur only usually on weekends and public holidays, for obvious reasons. Special events can cause an abnormal surge in demand. In the United Kingdom, an increase of demand of 1000 MW or more within two minutes or so can be expected in the CEGB system at the end of a popular television programme /11/ *. In the West German RWE grid in 1971 the live telecast of the return of Apollo 14 caused an increase of 1200 MW above normal peak levels, representing some 8% of

* Such numbers refer to references listed in the Bibliography (Appendix C).

total installed system capacity /37/. The magnitude, duration, and timing of the peaks and depressions will vary in detail from system to system; peaks may lengthen or even merge to form plateaus, the ratio of maximum and minimum demand levels to average demand and the load factors will be different, but the general shape remains recognizable. These chronological daily load curves reflect the general lifestyle of the society which the power system seeks to serve. Daily load factors range from about 40% up to 75%.

For geographical regions where there is a well defined seasonal climatic change in temperature, as in non-tropical zones, an annual cyclic variation in demand is discernable. This is caused by the widespread use of electric energy for space heating, resulting in annual peaks of demand in the winter months. The increasing use of electricity for space cooling (air-conditioning) has led in recent years to the existence of annual peaks during summer months in certain areas of the United States. For obvious reasons such clear-cut variation is uncommon in tropical climates. Annual system load factors vary from 35% to 70%.

The lack of direct storage capability of electric energy leads to the necessity for the total system installed capacity to be at the least equal to the maximum expected demand. In practice for various technical reasons to be discussed later it has to be significantly greater than this. Clearly this requirement means that much of the system's installed capacity will lie idle for much of the time. This accounts for many of the problems in planning and operation of power systems and has unfortunate consequences - contributing to the capital intensiveness of the electricity supply industry.

Before further discussion on the operation of power systems, types of generating plant will be briefly described. Apart from their physical sizes and rated capacities, generators in contemporary power systems differ essentially only in their sources of motive power*. Many types may be distinguished by virtue of principle of operation, fuel usage, or function. Four main types will be recognized here for convenience in exposition, and this classification will be adhered to throughout this thesis.

- Firstly, plant specifically designed for, and applicable to, the satisfaction of short term peaks of demand. Peaking plant in common use today are Joule cycle gas turbines (either purpose-built industrial machines or modified derated aviation units) and internal combustion diesel engines. The capital costs of such plant are low, whereas running costs are high because of the refined fuel necessary. Plant of this type is relatively uncomplicated and consist essentially of the prime mover coupled directly or through reduction gearing to the generator, with governor control to ensure constant speed operation. This simplicity reflects to the low capital costs, and peaking plant can be installed rapidly, within one to three years. Lifetimes** are comparatively short, from 10 to 20 years, and unit sizes are relatively small, between 1 MW to about 70 MW.

The function of such plant is suggested by the name: to support the system during short periods of high demand. It is well suited to the task. Start-up times are short - from standstill to synchronization within two minutes - and the plant is compatible with the demands of intermittent operation at short notice.

* Sources which do not employ rotating electromagnetic machinery (eg thermoelectric, thermionic, certain solar devices, or fuel cells) have yet to be proven feasible for electricity generation on a large scale.

**These are the lifetimes generally used for economic comparisons in planning studies. As a general rule accounting lifetimes are somewhat shorter, and actual physical lifetimes are in fact often uncertain, and can sometimes be significantly longer.

In recent years peaking plant has also been installed to perform other useful functions, the rationale probably being that if their installation is necessary and their utilization low, they might well be profitably used for other functions. Auxiliary gas turbines are now installed in many large power stations to supplement or replace station auxiliary supplies. In this capacity the normal peaking function can be discharged when required. In the event of system disturbance, emergency, or other abnormal conditions, the gas turbines can automatically be started up via the operation of frequency-sensitive relays and take over generator auxiliary supplies for station security. These gas turbines can also enable station start-up in the (catastrophic) event of total system failure, independently of the main supply network. Gas turbines have also been used as non-passive power factor correction devices, in which case the gas turbine is coupled to its generator through a clutch drive. This use, as a synchronous reactive power generator, is becoming increasingly important, augmenting shunt reactor installations on systems with very long high voltage transmission lines.

A virtue of the Joule cycle is that such stations do not require a large supply of cooling water. This applies to diesel engines as well. There are therefore few restrictions on site availability for such plant, since they are also quite compact in layout.

- Secondly, steam turbines operating in a Rankine cycle, with the heat input derived from the combustion of coal, oil, peat, natural gas, or even town refuse. Such plant is variously called thermal, fossil (-fuelled), or even just conventional plant. None of these labels is precise, but all are used widely and interchangeably in the literature without confusion. Steam conditions at turbine entry are now almost invariably superheated, and in some cases supercritical. Reheat in one or more stages is widely employed, particularly for the larger unit sizes. Much ancillary plant is associated with the cycle, and a large modern power station of this type, with duplicated standby plant for important functions, monitoring and control equipment, and multiplicity of safety devices and interlocks, is a complex

system indeed. This contributes to the capital costs, which are considerably greater than those of peaking plant. On the other hand combustion is carried out in large watertube boilers and this removes the need for refined ashfree fuels which restrict the operation of gas turbines and diesel engines. Thus running costs are significantly lower than those of peaking plant. For coal-burning stations, arrangements will have to be made at the design and planning stage for the supply and storage of fuel and the disposal of ash. This problem may not be so serious with oil or gas-burning stations.

Unit sizes encompass a wide range, from about 20 MW to over 600 MW. Plant lifetimes average 30 years. In contrast to peaking plant, flexibility of operation is limited. Start-up times are considerably longer, and several hours may be required for a cold start of a large thermal station. A boiler may be banked (filled with cold or heated water), in which case a set may be brought onto bars within an hour or so. Load following capability varies inversely with unit size, and large thermal units cannot generally be operated with confidence at much below about 30% rated capacity. Large unit sizes are therefore best used for continuous steady base-load duty. Intermittent operation is undesirable and often the smaller thermal units in a power system are operated on a two-shift schedule; plant is started up twice in a 24-hour period. Operation in this way can be used to satisfy mid-merit load.

Construction times vary according to size of station and local conditions, between three to six years being fairly typical figures. Thermal stations have been criticised for their contribution to atmospheric pollution, both in respect of particulate matter (soot, ash) and gaseous pollutants (sulphur dioxide, oxides of nitrogen). Costs of such stations have risen owing to the need to install electrostatic precipitators, or the need to burn low-sulphur fuels. The Rankine cycle requires large amounts of cooling water. Availability of direct natural heat sinks is restricted to large rivers, lakes, and the sea. The atmosphere can only be used with cooling towers which increases capital costs and brings into consideration questions of visual

amenity. Hence there are often site restrictions, particularly for large stations.

- Thirdly, nuclear plant, consisting of steam turbines operating in a Rankine cycle as with thermal plant, but with the heat input to the cycle derived from the controlled nuclear fission of uranium-235. Such plant now forms a significant proportion of installed generation capacity in many developed country systems, and are increasing into prominence as acceptable alternatives for power system expansion in developing countries. It is generally only considered economic to install nuclear plant with turbogenerator unit sizes from about 300 MW to over 1000 MW. Thermal efficiencies, especially for water-cooled designs, are low, but the unit cost of energy of nuclear fuel is low compared to that of fossil fuels, particularly in the wake of the general petroleum price rises of 1973. This makes running costs low and to a large extent offsets the very high capital costs of construction of such stations, engendered by the need for stringency in design and construction for secure containment of the nuclear reaction. The low fuel cost also makes running costs relatively insensitive to price fluctuations.

The biological hazards of nuclear radiation mean that a high degree of caution for security in control and operation is entailed, and this requires specialist highly trained personnel for operation of nuclear plant. Siting problems arise since nuclear plant may not be located close to densely populated areas (often also the load centres) and must be sited on seismologically stable ground. Owing to low cycle efficiencies and large station outputs thermal pollution of lakes and rivers can be a constraining factor on station siting. On-line refuelling is possible although the need for this is infrequent. The disposal of radioactive waste products poses problems which are currently the subject of controversy and continuing research.

Plutonium fuelled fast breeder reactors which attain a much more efficient utilization of fuel are under development but are not yet a proven commercially feasible possibility. Various configurations of thermal reactors are available, these may be

cooled by light water (PWR, BWR), heavy water (CANDU-PHWR), or carbon dioxide gas (MAGNOX, AGR). The stable operating range and load following capability are restricted. This is partly due to the large unit sizes used and partly due to the necessity of maintaining stability within the reactor core. Nuclear plant is invariably therefore relegated exclusively to base load duties. An indirect result of the rapid increase in nuclear generating capacity in many systems has been the displacement of large thermal stations originally designed and intended for base load duty to two-shift operation. Construction times for nuclear stations are between four and seven years. Delays in construction have been quite common, especially with prototype designs, and nuclear plant has in many cases been subject to heavy cost inflation during construction in recent years. This has also been due to the increasingly high standards of safety demanded by authorities and the public. Transportation of reactor containment vessels for light water reactors and shafts for the larger turbogenerators may pose unusual problems because of their bulk and weight. Strict quality control is necessary during fabrication, and these components cannot be fabricated on site.

- Fourthly, hydroelectric power plant. In contrast to the previous three types, these stations derive their motive power from the potential and kinetic energy in the waters of lakes and rivers. Hydroelectric schemes involve the enlargement of existing lakes, or the creation of artificial reservoirs, by the construction of dams. This enables controlled exploitation of hydraulic energy by the regulation of flow through a water turbine. Depending on local conditions and head, this may be an impulse (Pelton), a reaction (Francis), or a mixed-flow (Kaplan) turbine. The size of the reservoir associated with a hydroelectric station determines, in the first case, its likely function. A small reservoir (a run-of-river station) is best used to satisfy steady base loads, output being dependent mainly upon the hydraulic inflow rate. With a large reservoir (storage station) however the hydraulic energy may be more profitably employed for meeting peak demands. Despite the

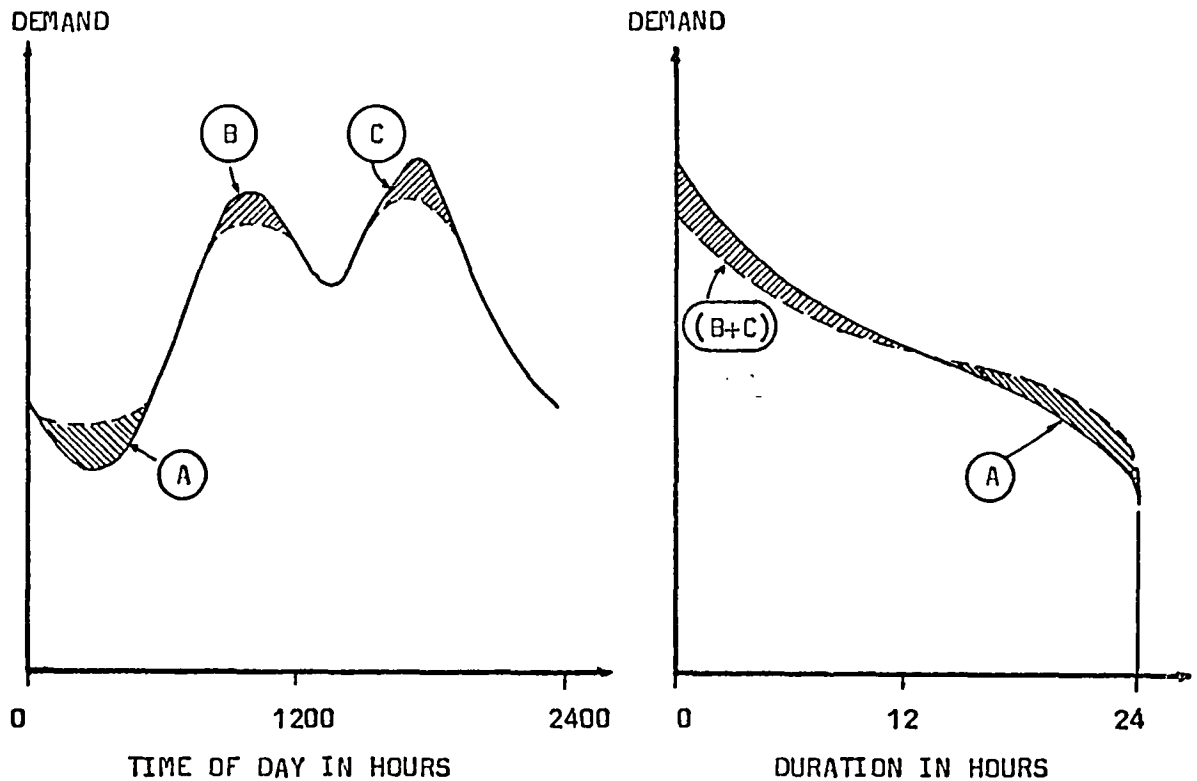
simplicity of the principle of operation and of the machinery, construction costs are extremely high due to the magnitude of the civil engineering works involved. Siting problems exacerbate these high costs since useful sites are rarely found near load centres and long transmission lines will have to be installed. For obvious reasons the number of available sites is strictly limited, depending upon the local topographical conditions and the availability of a predictable and reliable supply of hydraulic energy.

Because of the simplicity of form maintenance costs are relatively low, and running costs are of course virtually zero. In contrast to the other three plant types however hydro plant is limited in energy production by the rate of inflow of water into its catchment area. This can mean that a station may not be able to maintain full output for any appreciable length of time without dangerous depletion of water levels in the reservoir. Such plant is also vulnerable to variations in inflow which may be both significant and regular. This happens where there is a well defined wet/dry seasonal variation or where hydraulic inflow is derived from the annual melting of ice and snow on high mountains. Hydro capacity may therefore be severely curtailed during dry seasons or abnormal dry spells.

Hydroelectric plant demonstrates great flexibility in operation within the limits imposed by energy shortage. Sluice gates may be opened rapidly and at short notice and start-up times as a consequence are very short. Response to load changes is also rapid, and the usable stable range of operation of water turbines can be very wide. For these reasons hydro stations often perform regulatory or corrective functions in mixed hydrothermal systems. Unit sizes vary from under 1 MW to over 100 MW, depending upon local conditions (inflow, head). Construction times vary up to about seven years, depending upon the scale of the project and its accessibility. Useful lifetimes can extend up to fifty years.

In many cases the generation of electricity by hydro power forms part of an integrated multipurpose development scheme. This could include ore extraction, public water supply, irrigation

FIGURE 1-1 EFFECT OF PUMPED STORAGE (DAILY CYCLE)



A = energy stored by pumped storage hydro in pumping mode

B, C = energy produced by pumped storage hydro operated in generating mode.

$$A = \eta (B + C) \quad (\eta = \text{pumped storage cycle efficiency})$$

or flood control.

A special development of hydro plant involves separate pumps and turbines, or reversible pump-turbines, installed between two reservoirs at different levels. Such pumped storage schemes constitute the only present-day case where large scale storage of electrical energy can, albeit indirectly, be achieved. During slack periods of demand when the system demand can be economically met by the running of other plant in the system the pumps are operated, transferring water from the lower to the higher reservoir. The energy required for this operation is drawn from the rest of the system operating at low incremental

cost. During peak periods of demand water flowing from the upper to the lower reservoir generates power through the turbines which effectively displace other plant in the system, which under these conditions is operating at high incremental cost. Used in this way the pumped storage stations can even out the peakiness of the load demand curve perceived by the remainder of the system (Fig 1-1). Despite some losses in the process (typical turnaround efficiencies are between 70% and 80%) this can lead to significant economies and convenience in system operation, particularly when used in conjunction with nuclear or large thermal plant /12/. It should be noted that this is an energy storage device. It does nothing to reduce the total installed capacity required to meet the maximum expected load. Neither does it reduce the total energy to be generated by the system (in fact, because of cycle efficiencies less than unity, more energy has to be generated by the system). Cycles of pumped storage operation are generally biased to satisfy daily cycles, although some harmonic weekly variation is often also evident.

Hydroelectric schemes are often criticised for their installation in areas of great natural beauty. Landscaping is often necessary to forestall objections from the public. Local flora and fauna can also be adversely affected by large scale hydro development.

There are of course significant subdivisions within each of these four basic categories, and also types that transcend the categories. Combined cycle stations which combine the Rankine and Joule cycles for higher thermal efficiencies have in recent years been installed in the United States. Such stations, usually fuelled by natural gas, attain a much higher thermal efficiency than either peaking or thermal plant. Recently more attention has been focussed on methods of power generation using renewable energy resources - the sun, wind, waves, or tides. In January 1977 an experimental solar power station was linked into the French grid. Solar generation of electricity seems destined to have a significant impact on the character of

future power systems either by supplanting existing plant types or by conditioning the demand for electricity. Geothermal stations do not require large heat inputs but usable sites are very limited. Attempts are continuing to achieve large scale storage of electricity, both directly (batteries, fuel cells) and indirectly. These latter methods include the use of superconducting magnets, flywheels, or the interruption in time of the basic Joule or Rankine cycles by storage of compressed air, heat, or boiler feedwater. Such methods of storage appear to be within the bounds of present-day feasible technology, and because of the grid philosophy of electricity supply can as with pumped storage facilitate economic and convenient system operation.

The four basic categories have been listed in increasing order of capital costs, and, as it happens, in decreasing order of running costs. Almost without exception power stations and units benefit from an increase in size. In the first case this is evidenced by returns to scale in respect of capital and construction costs. The 'six-tenths power rule' is common in industrial plant costing calculations. For various reasons the exponent for electric power stations seems to be somewhat greater than this; a figure of 0.85 has been suggested. These reasons include the increasing steam conditions (temperature and pressure at turbine entry) with size, the inclusion of refinements to the cycle (reheat, more stages of feedheating) and recently the use of two-shaft turbogenerators.

These increased steam conditions and cycle refinements mean an improved thermal efficiency. Hence there are returns to scale with respect to running costs as well.

In the early days of the power industry transmission was by direct current lines. The advantages of AC transmission were quickly recognized and up to fairly recently AC transmission was exclusively used. Economies are effected with the use of higher voltages on long lines. In the UK the supergrid employs a voltage of 400 kV, the remainder of the grid system operating at voltages of 132 kV and 275 kV. In the US longer lines and greater power flows mean somewhat higher voltages, up to 765 kV. At such high voltages problems arise due to hysteresis losses, insulation breakdown, and corona discharge. Towers for high voltage transmission lines are huge structures which impair visual amenity.

High voltage DC transmission has begun to be reused. This requires the installation of heavy-duty and reliable rectifiers and inverters. This can be an advantage since it enables the linkage of two systems operating asynchronously without large damaging power flows. Charging current losses are lower at high voltage than AC, and HVDC transmission demonstrates definite advantages where cables rather than lines are being considered, as in areas of great natural beauty, under water, or in congested built-up areas. Recent advances in cryogenics and research into superconductivity will prove to be important in the development of HVDC transmission. As contemporary examples of HVDC transmission: a link of 1500 MW capacity exists in the US, spanning 850 miles at a voltage of 750 kV; the submarine link connecting the systems of the CEEGB and the EDF under the English Channel has a relatively slender capacity of 160 MW and a voltage of 200 kV.

The consequences of failure to meet demand are severe. Operators of power systems have to be aware for some time in advance of the likely levels of demand. An excess of demand over supply is evidenced by a reduction in system frequency. To avoid large damaging power flows and the operation of generators in potentially dangerous operating regions the system frequency has to be stabilized. Initial action is often voltage reduction by means of tap-changers on generator and inter-network

transformers. If this action is insufficient to maintain frequency within the desired limits selective load shedding may be necessary. The time scale within which demand may rise unpredictably can be very short indeed; far shorter than the time required to start-up, synchronise and load even the fastest-starting unit in the system. However an increase in demand can be met almost instantaneously by increasing the output of units already loaded and synchronized. It is therefore usual practice to run several or all units at below their continuous maximum rating (CMR) in order to handle safely sudden increases in demand levels. This reserve capacity on tap, as it were, is termed spinning reserve.

Spinning reserve also fulfils a second important function in addition to the regulation of frequency and voltage. All plant has a finite though small probability of failure. An adequate spinning reserve has to be made available always to cover the possibility of accidental outage of generating units and transmission infeeds. If this is not available a single major fault condition can cause serious system disturbance, with line and generator failures spreading in cascade, as happened with the New York blackouts of 1965.

Sophisticated arrangements are incorporated into contemporary power systems to ensure continuity of supply. Large amounts of spinning reserve are always maintained. On-line monitoring of power flows with analogue or digital equipment provides continual assessment of system stability. Frequency-sensitive relays which operate circuit breakers (sometimes with automatic reclosing) or which start up emergency generating plant are widely used. Research into on-line security assessment and correction using large high-speed digital computers is continuing.

A survey by Cash and Scott of security practices in power supply and planning in European countries showed that spinning reserve carried varied between 0% and 12% of demand. The lower figure is misleading, since many systems relied for reserve in emergencies on inter-system or international interconnections:

" There is general agreement that spinning reserve should cover errors in load estimation and the loss of the largest generator. It appears, however, that the double contingency may not always be covered simultaneously, international interconnection being relied upon in part. " (/9/).

For a large system, 10% of demand is likely to be well in excess of the capacity of the largest generating unit or transmission infeed. For the CEGS:

" In total, the present day national reserve at time of daily system peak in winter amounts to 1500 MW and this rises to 1900 MW during spring and autumn because of the larger possible errors in demand estimation. Off-peak spinning reserves carried on conventional generating plant range from 400 MW to 900 MW to cover the loss of a 500 MW infeed which is at present the largest single risk on the system ... The low figure of 400 MW applies when the pumped storage plant is pumping, as immediate relief of 300 MW can be obtained by tripping this plant. " (/20/)

The installed capacity for the CEGS system when the above paragraph was written (1968) was about 42 GW.

In contrast to this, 10% of demand for a smaller system in a developing country may be well below the CMR of the largest unit. Furthermore such systems are usually not interconnected with neighbouring systems. A single unit may be supplying up to 50% of the total instantaneous demand during low load periods /47/.

It is CEGS practice to provide spinning reserve by operating at 75% CMR the highest cost plant generating at the time, although:

" ... the reserve carried on conventional generating plant is spread throughout the system with larger proportions in those sections of the system receiving power transfers " (/20/)

Ninety percent of this reserve can be attained within a few seconds and full output can be attained within five minutes. For other systems spinning reserve may be distributed over many or all units, determined only by economic factors.

The need to provide spinning reserve encourages the enlargement of systems or the interconnection of neighbouring systems. In the US in 1964 approximately 97% of the electricity supply industry's generating capacity was linked for the purposes of transmission in five large networks. The Federal

Power Commission in its survey of that year commented, however:

" In many interconnected situations ... the depth of coordinated system planning has not progressed to the point of taking full advantage of joint planning and generating capacity. "

Clearly the problem of providing adequate amounts of spinning reserve in small independent systems is aggravated by the presence of large unit sizes. But there are often good reasons for the installation of such units. Firstly, the advantage of economies of scale in capital costs. Secondly, increased thermal efficiencies and reduced production costs. There is now however evidence that in recent years the improvement in efficiencies has been tapering off:

" The 600 MW, 800 MW, and 1300MW units built and to be built between 1966 and 1973 will all have the same heat rates* because of technological factors. In fact heat rates of the larger units may even be increased in an effort to improve reliability. So, economies of scale due to improved heat rates from larger fossil-fuelled thermal plants (or, for that matter, other sources of improved heat rates) seem to be mainly in the past - resulting from changes from much smaller units and more primitive technologies. " (/13/)

Thirdly, the scarcity of suitable sites may encourage the installation of larger and larger units in the limited space available.

Against these powerful reasons is set the demands on system security in operation arising from the installation of large units, mainly the need for large amounts of spinning reserve. Moreover, the availability of the large units has in many respects been disappointing. The US journal 'Electrical World' regularly lists the outage rates of major classes of power stations in that country. In 1971 the average unavailability of the twenty newest stations was 18.6%. In the previous annual survey the outage rate for the same class was 11.3%. Whilst these figures could just as well be interpreted as a reflection of declining standards of design, construction, or operation, there is general agreement that higher outage rates are associated with larger unit sizes. The disruptive effects

* an inverse measure of thermal efficiency.

TABLE 1-1 ANNUAL LOAD FACTORS FOR NUCLEAR PLANT - 1976

(A) BY REACTOR TYPE			(B) BY COUNTRY	
PHWR	77.67	(82.41)	Canada	82.2
Magnox	64.94	(63.40)	FRG	69.7
PWR	61.38	(56.93)	UK	68.1
BWR	59.20	(55.71)	USA	56.9
			Sweden	56.5
			Japan	56.1
			France	55.3

- (1) Values tabulated are the percentage average annual load factors for the 12-month period ending December 1976.
- (2) Figures in parentheses in Table (A) are load factors weighted according to plant output.
- (3) Source: Howles /32/.

of forced outages of large units in small systems are so great that they may well outweigh any advantages in capital or production costs.

The load factors for nuclear plant in operation have also been below early expectations. In the US in 1972 the average nuclear plant load factor for new plant was 61% /12/. In Japan in 1974 nuclear plant availability was a mere 50% owing to shutdowns for plant tests and repairs /27/. Recent figures for average nuclear plant load factors according to reactor type and country of installation are tabulated in Table 1-1.

New developments are of course always subject to teething troubles and this may partly explain the disappointing availability rates. But nuclear plant in economic evaluations would be less competitive with such low load factors, in stark contrast to the 70% - 80% load factors frequently quoted and often used in planning calculations.

An inflexible adherence to a programme of installation of large units can also have undesirable indirect effects on the scheduling of existing plant in the system. Pandit, et al

/47/, discussing the rapid expansion of developing country systems, point out that:

" It is not unusual in such rapidly growing systems to come across a newly installed plant which was base loaded initially to about 80 - 85% annual load factor and which is required to operate on a two-shift basis with an annual operating load factor as low as 40 - 50% in a short period of five years ... Units ... are required to perform peaking duties within 6 to 8 years after initial base load operations. "

It is difficult to find general agreement on the optimum unit size to be installed in small systems in the face of the conflicting requirements of economy and security. In the last analysis the decision depends very much on the value attached by planners of systems to the operating standards that they are hoping to achieve.

Economy in operation is accomplished by merit order loading. All units or stations in the system are ranked according to economy in operation, in increasing order of running cost. The merit order table then provides the basis for economic plant scheduling. Low demands are met by first loading plant with low production costs (high in the merit order). As demand increases plant with progressively higher generating cost are loaded. In this way optimal, in the sense least-cost, allocation of load is ensured.

Often the merit order cannot be strictly adhered to. At any particular time certain units may not be available, or their outputs curtailed because of outages for repair, tests, or routine maintenance. Hydro plant may not be readily available because of water shortage. Furthermore large units in particular cannot be started up from rest easily or rapidly, and stop-start operation of such units has to be avoided even if this distorts merit order loading. Load following capability is often restricted and outputs of units may not be variable over the whole or even a large part of their CMRs. Finally in the interests of system security restrictions on certain power flows and the need to provide spinning reserve raise yet more problems. As mentioned previously it is CEEB practice to carry spinning

reserve on low-merit plant. The Union for the Coordination of the Production and Transport of Electric Power (UCPTE) on the other hand recommend that:

" ... the reserve in principle should be distributed over as many units as possible, including those with the lowest operating costs. " (/9/)

For the above and related reasons out-of-merit operation may often be necessary. The plant duration curve and the load duration curve may no longer be coincident.

We now move on to a discussion of the optimal static plant mix. The following approach is taken from Berrie /5/. Two components are associated with the cost of a plant in a system. Initially (a) capital costs are expended. Over its lifetime (b) running costs are incurred. The ranking of total plant costs therefore depends on both these components and also the degree of plant utilization. Total annual costs can be expressed as:

$$\begin{array}{lcl} \text{annual} & & \\ \text{cost} & = & A + Bx \end{array}$$

where A = annuitized capital charges per kW per year

B = operating cost of plant if fully utilized,
per kW per year

x = plant annual load factor

The relationship, a straight line for each plant type, is shown in Fig 1-2. Figure 1-2A demonstrates that at low load factors peaking plant is best used (line aa'). At load factor x_3 however the use of thermal plant (line bb') is more economical. This in turn is displaced at load factor x_2 by nuclear plant; finally at load factors x_1 and greater the use of run-of-river hydro plant is desirable. The load factors projected onto an annual load duration curve relate the ranking of the plant types to their required capacities (Fig 1-2B). Capacities c_1 , c_2 , c_3 , and c_4 then represent the optimal static plant mix.

This approach provides at least prima facie evidence that

FIGURE 1-2A PLANT COST CHARACTERISTICS

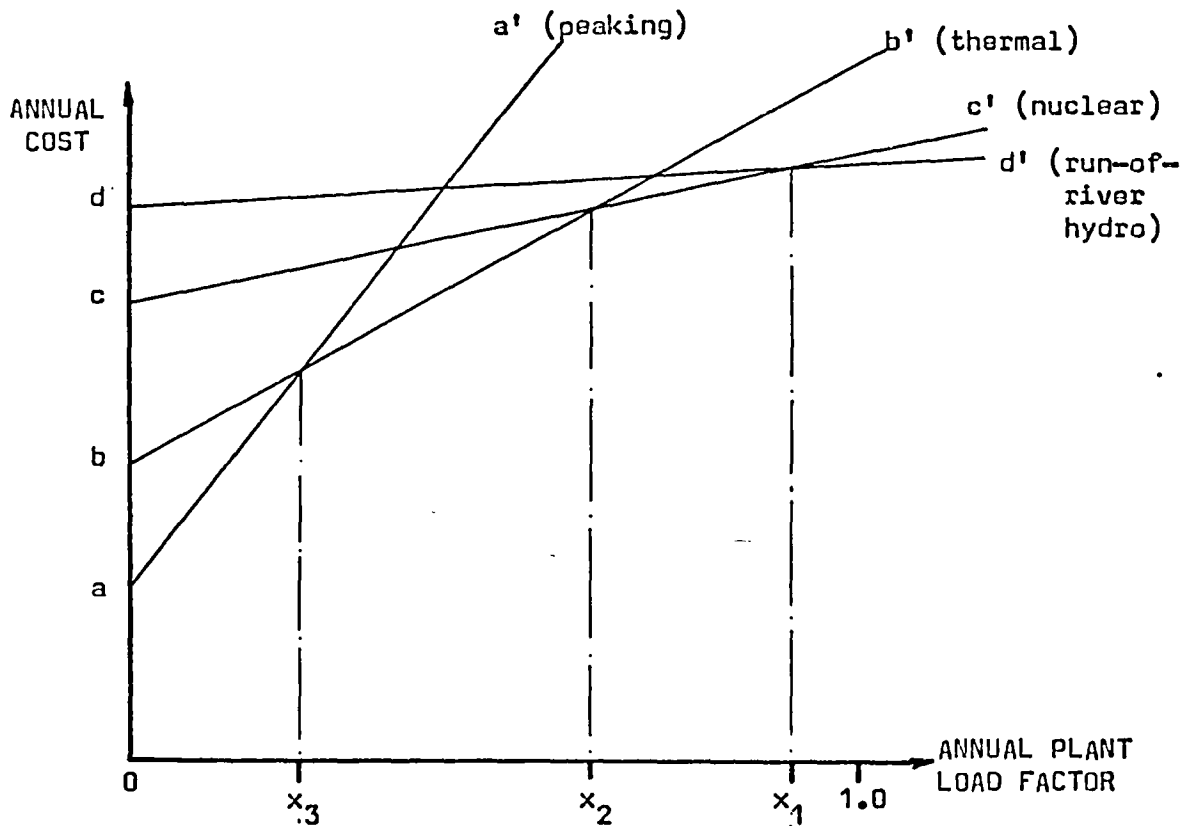
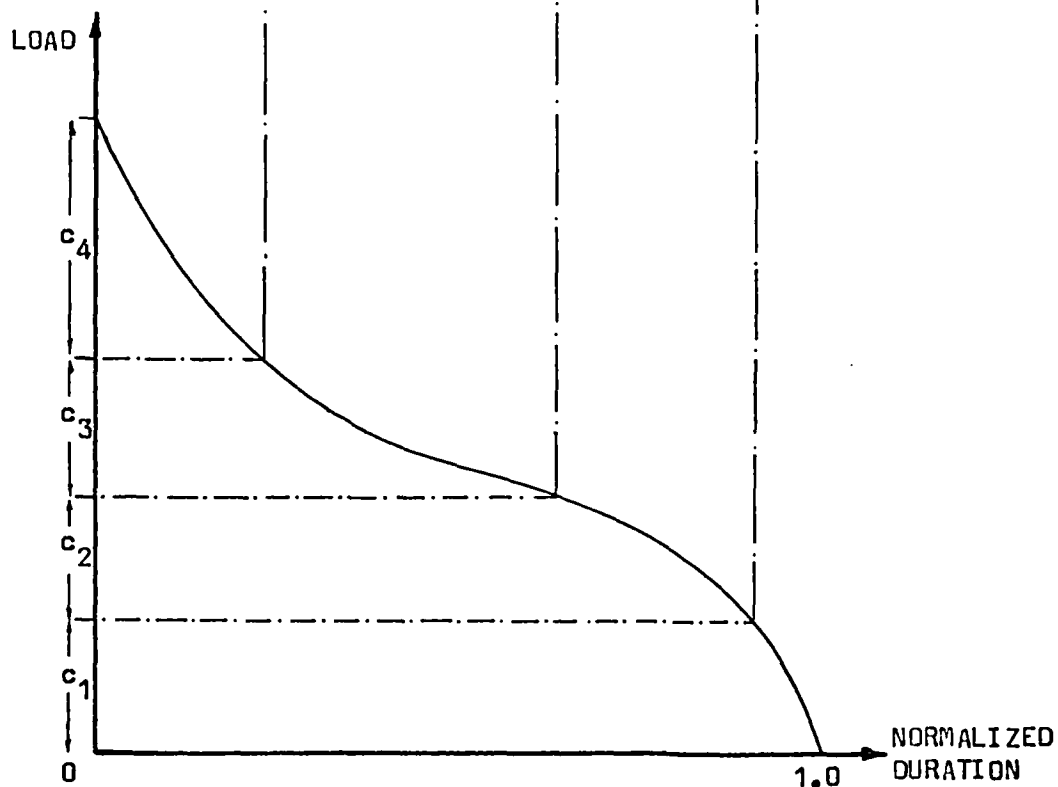


FIGURE 1-2B OPTIMAL STATIC PLANT MIX



in the first place a mix of plant types is desirable, insofar as cost is concerned, and an indication of the relative proportions of the different plant types to be installed for optimality. It is not beyond credibility to imagine a system comprising solely plant of a single type (all gas turbine, or all hydro), and secure and stable operation may well be achieved, albeit with difficulty. But overall system costs could be cheaper and hence system composition is not optimal. Though this fact is familiar to all system planners it is an unusual state of affairs and stems basically from the need to meet demand instantaneously and the inability to store electricity in any appreciable quantity. Note that the model may not be valid with pumped storage plant or with hydro plant with large-capacity reservoirs.

The simple model is of course open to criticism. Not only has no allowance been made for hydro storage, but also for restrictions in hydro energy. Determination of the optimal mix does not tell us what plant sizes to install. For example 2000 MW of thermal capacity may be achieved in a variety of ways when considering the wide range of discrete unit sizes currently available. Transforming the static situation to a dynamic one raises many further problems. An optimal static mix may be determined for each year spanning a period of consecutive years, and if these mixes vary greatly the question of what strategy to adopt in planting and retirements arises. Perhaps a suboptimal mix for one or two years out of five may produce a lower total expenditure over the whole five-year period.

Nevertheless the principles of this model have been further developed and used in a successful solution of the dynamic plant mix planning problem./49/. Despite its simplicity, at least in this explanatory exposition, the discussion of the optimal plant mix problem provides a useful bridge linking the problems of system operation to those of system planning.

Hitherto the problem under discussion has been directed towards system operation. The solution of this kind of problem has received much attention, and plant scheduling and load despatching studies have led to very useful tools in the operation of power systems: the deployment of existing stations or units and the operational strategy adopted in response to variation of demand in daily and annual cycles. The system planner on the other hand has to take into consideration yet another temporal scale - that of demand variation from year to year, perhaps over decades. Because of this he can rarely pay detailed attention to the problems of system operation, at least explicitly.

The blanket assumption is generally made that an adequate level of security in system operation can be achieved in day to day operation if the system is overplanted by a certain margin. For economy this margin should be as small as possible, although to ensure reliability and continuity of supply it should not be too small. The margin, known as the gross planning plant margin, is set over and above the projected demand level, to allow for:

- statistical considerations of possible errors in load estimation, and deviations from the standard weather conditions relative to which demand forecasts are made,
- delays in construction,
- planned outages for maintenance. This can take on greater import where there is little seasonal climatic variation and hence no obviously suitable time for the programmed outage of large-capacity plant in the planned maintenance schedules,
- provision of spinning reserve, bearing in mind statistical estimates of the probability of failure of major individual components of the system,
- water shortages during driest expected annual conditions, of particular importance for predominantly hydroelectric systems.

The gross plant margin varies with local practice. The Cash and Scott survey revealed widely divergent values among the European countries surveyed, ranging from 9% (Hungary, Austria), 17% (France, FRG, UK*), 25% (Denmark-Zealand, Netherlands,

* currently (1977) 20%, with a likelihood of it being increased yet further.

Portugal), and with some Swiss utility operators quoting values up to 50%. Gross plant margins for developing countries are not so comprehensively documented but there is little reason to suppose a wide divergence from the above values.

A formidable problem facing the system planner is the estimation of future demand levels, and perhaps in addition the shape and characteristics of the demand curves in the future. It has been common to use historical data to project future peak or average demand levels related to a standard weather condition (the 'average cold spell' in the UK). For predominantly hydro-electric systems energy consumption or average load levels are more important.

There is general agreement that, remarkably, there is a continuous and steady increase in peak demand and energy consumption with time, although recent evidence suggests that the rate of rise is tapering off, at least in some developed economies. Thus the traditional method of extrapolation of historical data fitted to an exponential curve may not now provide sufficiently accurate estimates. The modelling of the price elasticity of demand requires more sophisticated techniques.

Mount, et al /39/ typify the increasingly popular econometric approach. The demand for electricity, it is argued, is conditioned by five causal factors, these being (a) population, (b) income, (c) the price of electricity, (d) the price of competitive substitute fuels, and (e) the price of complementary products, such as household electrical appliances. The results of such a model, applied to various sectors of the economy in the US, are shown in Table 1-2.

Such models are flexible and can be used to examine a variety of scenarios. Comerford and Michaelson /14/ describe a model in use by the Public Service Electric and Gas Utility of New Jersey, and its use to evaluate demand levels in response to, among other things, income taxes, tax credits, even the impact of a large consumer (steel) strike. Most models of this sort confirm a negative price elasticity of demand, suggesting that usage of electricity may be approaching saturation,

TABLE 1-2 LONG RUN ELECTRICITY DEMAND ELASTICITIES

(Evaluated at 1971, average US factor levels)

	Residential	Commercial	Industrial
Population	0.99	1.03	1.01
Income	0.20	0.86	0.51
Price of Electricity	-1.20	-1.36	-1.82
Price of Gas	0.19	0.06	0.00
Price of appliances	-0.42	-	-

at least in some sectors of the economy in the United States, where most of these models have been applied.

The United States of course has the highest per capita consumption of electricity in the world. The general conclusions of such models are therefore not directly applicable to the conditions prevailing in developing countries. Unfortunately there is a paucity of comprehensive disaggregate data in these economies which makes implementation of such models difficult and their results suspect. There appears however to be a general growing awareness that mere extrapolation of historical data, or simple time-series analysis alone may not be adequate. Other considerations may be important. Development plans can have a significant effect on the growth of demand, both directly and indirectly. Population and GNP growth rates have to be taken into account. Contracts to large individual customers can have a large effect on demand. For large regions geographically disaggregate models will be necessary.

On the whole the study of the growth of demand levels and energy consumption is currently deservedly receiving more detailed treatment. But there have been no published studies to this author's knowledge which examine the change in shape of the load curves under the impact of economic growth, changes in incomes and income distribution, and any other possible causal factors. The

importance of accurate load projections cannot be understated, for it is a peculiar feature of the electricity supply industry that decisions on investment have to be based upon predictions of load several years into the future and beyond. Once committed a programme of investment is very inflexible. No other industry has to place such great reliance upon forecasts so far into the future. The repercussions of erroneous forecasting are severe. Overestimation results in a large amount of capital investment which could otherwise have been more gainfully employed in other sectors of the national economy lying idle. As will be made clear in the next chapter this is a serious matter for many developing nations. On the other hand an underestimation of demand can retard national social and economic development, an equally undesirable state of affairs.

CHAPTER TWO. ECONOMIC, FINANCIAL, AND OTHER CONSIDERATIONS

The electricity supply industry is generally regarded as a natural monopoly. Two principal reasons for this are the increasing returns to scale evident in system development, and the presence of direct physical links between producer and consumer. Duplication of effort as would occur with the presence of competition would be wasteful. It should be noted however that competition between fuel industries, such as for example gas and electricity, can be and often is fierce. The increasing returns to scale with respect to cost and efficiency have led almost invariably to the existence of a few large individual power suppliers.

The development of the fuel industries in general, and the electricity supply industry in particular, has vital effects on a nation's economic well-being. Suppliers of electric power generally exist as large private companies (USA, FRG), nationalized or statutory bodies (UK), or as largely autonomous government departments (many developing countries). Whatever the form of organization such undertakings are always subject to very strict government or public regulation. Such regulation is necessary because of:

" The importance of these industries, as measured not merely by their own sizeable share in total national output, but also their very great influence, as suppliers of essential inputs to other industries, on the size and growth of the entire economy. These industries constitute a large part of the 'infrastructure' uniquely prerequisite to economic development ... On the one hand, they condition the possibilities of growth ... On the other hand, because many of them are characterized by great economies of scale, their own costs and prices depend in turn on the rate at which the economy and its demand for its services grows. As general economic growth proceeds, the contribution of these industries to future expansion is thus enhanced by their own progressive realization of these economies of scale, in a cumulative and self-reinforcing process. " (/35/)

Several tools are available to regulatory bodies or governments for the supervision and control of the operation of such enterprises, of varying degrees of effectiveness. Three of the principle ones will be described in the paragraphs to follow.

- Firstly, pricing policies and levels. It is a fundamental proposition of welfare economics that in order to ensure optimal resource allocation both between industries and within an industry itself, outputs should be priced at marginal cost. In principle this should simulate the so-called "invisible hand of competition" where monopolistic or oligopolistic producers are concerned. Marginal cost pricing of electricity has received considerable attention in the literature /25,46,50,56/. It is usual to distinguish between short run marginal cost (SRMC) where an incremental demand can be met without the need to install additional capacity, and long run marginal cost (LRMC) where capacity additions are necessary. Incremental demands in the short term are met by loading plant in merit order whereas those in the long term are met by the growth and development with time of the system as a whole.

For decreasing-cost industries, of which electricity supply is widely held to be one, adherence to marginal cost pricing can lead to a failure to recoup investment. This is viewed with dismay by utility operators although from the viewpoint of the welfare economist it is not at all inconsistent with optimal allocation of resources throughout the economy. Farrell /22/ however contends that electricity supply is no longer a decreasing cost industry and recent evidence does appear to uphold this view to a certain extent, though only for developed economies. Few will deny that the electricity supply industry in developing countries, still in its infancy and with much to gain from development, expansion and the introduction of an increased level of technological sophistication, can but benefit from an increase in scale. The Hotelling solution to this dilemma is to raise the required subsidy through lump-sum taxes; another more usual approach is to follow marginal cost theory in determining the structure but not the level of electricity tariffs, this level being set so as to yield a sufficient return on investment. The economists' objection is that it leads to distortions in allocation elsewhere in the economy, although it does still prevent non-optimal internal cross-subsidization, provided it is applied accurately.

Insofar as the planning process is concerned the distinction between SRMC and LRMC does not appear to be as critical as was once held. Bates and Fraser:

" As a number of writers have now pointed out, there is not a choice between short-run and long-run marginal cost ... There is nothing paradoxical about the cost of a margin of output being the same whether or not there is an impact of new capital as long as it is realized that it is not within the year of output increment that capital costs are accounted but over a plant's lifetime ... " (/2/)

In other words, in the dynamic context SRMC and LRMC are equivalent.

Difficulties arise in practice when formulating a suitable tariff structure. There are of course an infinite number of marginal costs, depending on total system output at a particular instant. Kahn /35/ also points out that the notion of price is meaningless when unrelated to a particular standard of service. The problem when constructing a tariff is one of identification of causality and responsibility amongst diverse customers with widely differing requirements. Turvey /56/ lists the following considerations with respect to tariff structure:

- a connection charge for each new consumer equal to the capital cost wholly and exclusively incurred in connecting his premises to the system,
- a rate per kW of contribution to peak at each voltage level of the distribution system, equal to the average annuitized cost per kW increment of capacity at that voltage level, grossed up by the capacity margin. These charges might vary with power factor, or might be levied per kVA,
- an annual charge equal to the marginal annual cost per consumer of consumer service, meter-reading, billing, and account collecting,
- a contribution to general overheads and to the annual cost of running, repairing and maintaining the system,
- marginal running cost per kWh, grossed up to allow for incremental transmission and distribution losses, and varying from hour to hour as the total generating load changed,
- incremental capacity costs per kWh of contribution to peak generation.

This is a comprehensive list indeed, including as it does considerations of consumer voltage level, distribution and administrative costs, and power factor. In practice, unsurprisingly, it is extremely difficult to construct a tariff that accurately reflects marginal costs without over-complication. Such a detailed tariff would undoubtedly lead to consumer resistance and administrative problems. It would also fail to perform an important function - to guide the consumer towards regulating his own consumption both for his own and the system's mutual benefit.

The use of multipart tariffs is universal. The CEBG Bulk Supply Tariff currently (1976/77) operative in the UK has the following components:

(a) Service charges

allocated to each Area Board, a sum equal to the ascertained annual charges and expenses of providing the bulk supply point capacity to meet its electricity requirements.

(b) Demand/capacity charges

A basic demand is defined for each Area Board, reflecting its average demand over the year when total system demand is within $\pm 1\%$ of 90% of maximum system demand. (The precise definition of this basic demand is somewhat involved). The following charges are then applied:

- (i) A basic capacity charge for every kW of the Area Board's basic demand,
- (ii) A peak capacity charge (or rebate) for every kW by which the Area Board's peak demand exceeds (or falls short of) its basic demand.

(c) Running charges

These are levied per kWh supplied:

- (i) a peak rate for every kWh supplied during the two half-hours of greatest system demand over the 12-hour period before 1600 hrs; and similarly for the 12-hour period after 1600 hrs.
- (ii) a night rate for every kWh supplied from 0000 hrs to 0800 hrs (unless charged under (i) above) together with a surcharge for every kWh supplied when demand exceeds that during the half-hour of minimum system demand on that day (the 'night valley demand').
- (iii) a standard rate for every kWh supplied not charged under (i) or (ii) above.

A fuel cost adjustment clause is included to reflect changes in the national fuel cost.

Retail tariffs can take several forms. An attempt can be made to duplicate the structure of the Bulk Supply Tariff by imposing a maximum demand tariff (per kW of demand, charged retrospectively over, say, a period of three months) and a running charge per kWh of consumption. This is used by most Area Boards in the UK for large industrial or commercial consumers. For domestic and other consumers tariffs are much simplified. A two-part tariff can be imposed, involving a standing charge related to, for example, the rateable value of the premises, or its installed capacity, together with a running charge per kWh consumed. Also common for domestic consumers is a block tariff, the first component of which is charged at a higher rate to recover the standing charge, and with decreasing block rates for subsequent consumption. Because of their simplicity these retail tariffs lose much of the accuracy of bulk supply tariffs in reflecting marginal costs of supply. It has also been claimed that decreasing block tariffs are promotory and encourage profligate use of electricity, and cases have been presented for the use of inverted block tariffs (increasing block charges) for domestic use, aimed towards energy conservation /10/. Crew /15/ mentions an imaginative tariff used in Norway:

" The consumer pays a standing charge, related to his estimated annual demand. When the load is below the maximum of his subscribed demand he pays at a low rate. When it rises above he pays a surcharge for the excess units. "

Additional metering is required, but the advantage of such a form of tariff is that it assists the individual consumer to regulate his own demand and consumption according to criteria that he can readily understand.

The restructuring of tariffs has been generally used to good effect to modify daily cycles of demand. Promotional off-peak rates and advertising of off-peak storage heating appear to have succeeded in transferring to other times of the day those demands which can be deferred.

According to the price theory of economics, of course, prices should function as conditioners of demand and signals from consumer to producer. As has been mentioned in the previous chapter only recently has there been any evidence of a significant price elasticity of demand, in the United States and other countries with a high per capita consumption. It is generally accepted that no such mechanism is currently operative in developing economies although there is no confirmation of this in the published literature. Suppliers have a responsibility to stabilise prices and price trends. One feature of the supply-demand relationship and price mechanisms in electricity supply is the very long lags associated with response to price changes. Prices should not be changed too frequently since the consumers of this commodity are often making long term decisions. Builders may choose to install electric heating systems and householders may decide to buy electric cookers on the basis of price levels or trends at a particular date but they cannot readily communicate their disapproval if the status quo with respect to competitive fuels changes after a year or so.

The pricing of electricity is often one of the more controversial issues which arise between electricity supply authorities and their regulatory bodies. Overregulation is often the norm rather than the exception. It is easy to see why, since a price rise in such a pervasive commodity can be a prime contributory factor to an inflationary spiral. On the other hand it is counterproductive to impose a restriction on future investment in this way. Investment requirements will somehow have to be raised if development is not to be curtailed, perhaps subsidized by taxation elsewhere in the economy. Subsidy via pricing levels is considered by some to be more equitable since at least this ensures that electricity consumers contribute themselves to future system development according to the aggregate of their individual levels of consumption.

- The second tool for the supervision and control of regulated industries consists of directives concerning investment criteria and independent appraisal of investment programmes. The use of some form of discounted cash flow (DCF) is almost universal. Frequently a test discount rate (TDR) is decreed. The choice of a suitable TDR is not purely academic*, and several possibilities have been discussed very comprehensively by Henderson /30/, including the following:

- a rate reflecting the rate of interest on long term government securities,
- a social time preference rate (STP),
- a rate reflecting the opportunity cost of capital,
- a hybrid cost-benefit procedure, combining an STP rate of discount with social opportunity costs.

The use of a positive TDR has rarely been challenged and is nowadays invariably used, representing as it does a trade-off factor less than unity for future against present resource use.

The use of a high discount rate (10 - 15%) allows for the imposition of some degree of capital rationing. As will be seen this may be critical. The use of opportunity costs however may achieve this same object, and high TDRs can lead to a myopic view of project appraisal. The cynic would remark that the life of power stations often exceeds that of many business concerns and even governments, and the public interest endures longer still. So it is often argued that the use of an STP rate (3 - 5%) is apt. Capital rationing can be imposed by the evaluation of opportunity costs. This approach on the other hand can lead to premature overcommitment of financial resources and inflexibility to change in response to technological progress.

Perhaps fortunately the choice of the TDR is usually not open to the system planner, being set by the regulatory body,

* In the early days of nuclear power the debate on the economic viability of such plant as opposed to conventional methods of electricity generation hinged to a great extent upon the discount rate used for the economic appraisal. Lower TDRs made nuclear plant marginally desirable, and vice versa.

government, or financing authority*. The planner has to be aware however of the wide range of values that the TDR may assume, and the effect that these may have on the shape of alternative investment programmes. In the UK the test discount rate is currently set at 10%, this value is arrived at as a reflection of:

" ... the minimum rate of return to be expected on a marginal low-risk project undertaken for commercial reasons. " (/31/)

It remains to point out two virtues of a positive TDR, the first of which has already been implied above. Progress, even if not leading to major breakthroughs in the technology of power supply, does lead to improvements in existing techniques of design and manufacture, resulting in increased efficiency, reduced cost, or both. Hence there are economies related not directly to scale, but to time. A positive TDR does (in a rather clumsy way) allow for this. Moreover the forecasting of future costs and demand levels is an uncertain process and the use of a positive discount rate does at least provide a weighting factor to allow for risk and uncertainty. This is of course not to be interpreted as an excuse for laxity in the determination of input data.

Investment programmes are usually submitted to some higher authority for inspection and approval. Investment reviews in the UK involve the Electricity Council, the relevant Ministry or Ministeries, and the Treasury (financing authority). Programmes which may be optimal from the viewpoint of the electricity supply industry may not be so in the wider framework of national policy and may have to be altered at this stage to accommodate these wider interests. Energy sector planning, cost-benefit analysis and integration within government macroeconomic planning are considerations which may render alterations necessary. Sometimes the data which was used for the planning studies may be held to

* or, indeed, all three. Fortunately there does not seem to be much evidence of a conflict of interests between these parties although the prospect of such an occurrence cannot, of course, be totally discounted.

be at fault, or may be rendered invalid by further considerations which may not have been available to the system planner for a variety of reasons.

- The final tool to be described here is the regulation of the financial performance of the industry. The need for this is fairly obvious in view of the monopolistic position of electricity supply. Depending on prevailing conditions this can take the form of an exhortation to attain at least a certain financial surplus, or a restriction on permitted rate of return on investment. Posner /50/, and others, have commented that the imposition of financial targets in conjunction with marginal cost pricing and a set TDR in fact leads to an overdetermined problem. The UK White Paper of 1967 /31/ recognizes this point and makes clear that the attainment of a financial target should be the prime consideration:

" The alternative would be an indefensible lack of control over the return achieved on a very substantial public investment. There are, inevitably, simplifications and compromises to be made ... Nevertheless, despite these difficulties the system of financial objectives has proved its value in practice: it cannot provide a perfect formula for the resolution of all pricing and investment decisions, but it does give a framework within which such decisions can be reached. "

From the viewpoint of local management a financial surplus is desirable. It is important both for internal morale and public relations that the undertaking be seen to be doing well.

Kahn /35/ points out that restriction of profits in regulated bodies with monopolistic tendencies can sometimes, paradoxically, lead to a coincidence of the interests of the monopoly producer and its consumers. When profits are restricted to a percentage of the rate-base, it is to the advantage of the producer to extend that base, sometimes beyond the point where it would be strictly necessary in the presence of competition. When this occurs it often takes the form of an increased standard of service with eventual benefit to the consumer.

We now turn to an examination of the capital intensiveness of the electricity supply industry and its implications. The industry is generally agreed to be extremely capital intensive, ranking with water-resources projects in this respect. Garfield and Lovejoy /25/ quote a capital turnover ratio (gross revenue divided by capital investment) of 0.30 for power utilities in the US, compared to an average value of 2.00 for manufacturing industries in that country. In the UK capital investment for the nationalized fuel industries can amount to £1000 million annually, and this accounts for about 10% of the gross capital formation (GCF). Of this well over half is incurred by capital investment in the electricity supply industry. Power expansion in developing countries accounts for 7 - 8% of GCF on average. This scale of finance, as may well be imagined, raises special problems of its own.

Schmitt and Monig /54/ have examined the capital requirements of the electricity supply industry in the Federal Republic of Germany. They identify a shift towards capital intensity due to accelerated installation of large nuclear plant. Total installed capacity is expected to rise from 61.9 GW in 1973 to 128 GW in 1985, and owing to retirements of existing plant during this period some 73.5 GW of new capacity will have to be installed. A conservative estimate of the cost of this expansion, taking into account rises in the costs of labour and capital, is 89×10^9 DM. They comment that:

" ... the electric power industry's capital requirements over the next ten years is so large that there are those in the electricity industry and the banking sector who doubt that it can be met at all. "

Friedmann /23/, arguing the case for urgent mobilization of financial resources for power expansion in developing countries points out that:

" As a result of various factors - continued faster than GNP growth of power demand, less scope for economies of scale, faster than average price increases in equipment and heavy construction and a particularly rapid shift to higher cost plant (nuclear, lignite, hydro) - the proportion of investment taken by power expansion in less developed countries is projected to increase by about half, from about 7 - 8% in the past decade to

about 10 - 12% in the next. In terms of public rather than total investment this means an increase from about 17% to about 25%, indicating that a difficult reallocation of public investment funds will be required. We are not sure that this fact is fully grasped by decision makers in countries and developing institutions. "

An analysis of expenditure for power expansion in developing countries suggests a ratio of 60% generation, 40% transmission and distribution. Of this foreign exchange comprises roughly 60% of generation expenditure and 35% of transmission expenditure. Thus on average half the capital expenditure for power expansion in developing nations requires foreign exchange, often in scarce supply. It has been perceptively pointed out that the 'energy crisis' precipitated by the OPEC oil embargo in late 1973 has manifested itself to oil-importing developing nations as a financial crisis which continues to worsen with global recession, inflation, and depressed prices for primary products. The increased costs of energy systems as imported by these countries means that relative to the rest of the world little progress can be made in real terms. Friedmann quotes part of the text of an address by the President of the World Bank in 1975:

" Even to achieve the wholly inadequate levels of economic activity of 1975, the developing countries have had to borrow large sums from the private capital markets. They will have to continue to do so in the future. But there are limits to credit-worthiness and these make it unlikely that they will be able to borrow significantly more in real terms, over the next five years, than they are doing now ...

" The great increase in external capital flows ... from the levels of 1969 - 73 has been and will remain essential to sustain economic growth in less developed countries. These new levels represent between 2.5 and 3% of GNP (vs past figures of 1.5 to 2%), 12 to 15% of gross capital investment (vs 8 - 10%) and about 30% of exports earnings (vs 20%). "

The increase of capital intensiveness has thus been aggravated by the events of 1973. Price rises in petroleum fuels have encouraged developing countries to accelerate development of hydro resources and the installation of high capital cost nuclear plant, in some cases where such plant is only marginally desirable. This has been blamed for difficulties in mobilization of capital. But perhaps this is a result of the

general trend of technological development in the power industry (perhaps in industry as a whole) which has always been one of increasing capital-intensiveness - larger and larger generating units, ever-increasing transmission voltages and line lengths. The logical results has been the present-day state of affairs: the enormous scale of capital resources required by the power industry.

Furthermore for a developing nation a significant proportion of this is a foreign exchange component and power expansion may have to be curtailed in some cases owing to a lack of funds.

In the Federal Republic of Germany and the United States funds for power development are raised by recourse largely to private capital markets. This source was at least partially employed some years ago in the United Kingdom, with the issue of British Electricity Stock guaranteed by the Treasury. The nationalized fuel industries in the UK are encouraged as far as possible to be self-financing. Manifestly this is not easily achieved. Table 2-1 summarises the capital requirements of the British electricity industry from 1970. Under the 1968 Gas and Electricity Act the electricity industry was permitted to borrow in international capital markets, this appears to have been done at least to some extent. Most loans however are granted by the Treasury at comparatively low interest rates.

TABLE 2-1 CAPITAL REQUIREMENTS OF THE BRITISH*
ELECTRICITY SUPPLY INDUSTRY

Financial year	Gener- ation £10 ⁶	Trans- mission £10 ⁶	Area Boards £10 ⁶	Other £10 ⁶	Total £10 ⁶	Internal Financing Ratio %
1970/71	180.9	45.2	143.6	5.0	374.7	53
1971/72	200.6	36.3	138.9	8.9	383.8	70
1972/73	213.9	32.0	139.8	10.8	396.5	78
1973/74	222.2	33.7	140.9	11.4	408.2	62
1974/75	280.6	40.6	162.8	15.0	499.0	48
1975/76	366.6	55.3	181.9	16.6	620.4	87

Source: Handbook of Electricity Supply Statistics
(Electricity Council, 1976)

* England-Wales and Scotland.

Developing countries of course have limited recourse to domestic private capital. The bulk of their financing derives from government funds where forthcoming, bilateral loans, or multilateral development aid organizations. In particular generous bilateral loans have been made available at favourable rates of interest for the specific financing of nuclear stations. For example, from 1975 20% of the loans for power development made available by the US Eximbank were diverted towards the financing of the construction of nuclear plant /23/. As was pointed out in previous paragraphs it is likely however that within the next decade or so adequate funding from these sources may no longer be available. Friedmann suggests more imaginative approaches to the raising of capital for developing countries, and recommends recourse to the large international (Eurocurrency) capital markets. International currency markets are an important source of funding for many developing nations now. The Morgan Guaranty Trust has estimated that the total external debts of oil-importing developing countries at the end of 1975 amounted to $\$ 150 \times 10^9$, of which some $\$ 60 \times 10^9$ consisted of liabilities to commercial banks*. In early 1977 a loan agreement for the financing of part of the second stage of development of the Ko-Ri power station in South Korea was signed in London. The following extract is from a report in the Guardian, 1 March 1977:

" More and more the problems of arranging the finance of these multinational consortia projects are proving just as great as the purely technical or economic ones. One of the American bankers at yesterday's signing ... commented that he foresees the day when all such projects will be negotiated by a tripartite group of financiers, technologists and economists ...

" The problems of finding finance on this scale are indeed immense. The consortium signing yesterday's contracts covered only $\$ 139$ millions of the $\$ 700$ millions yet it involved nine major international merchant banks plus the ECGD** and the Eximbank ... "

* "World Financial Markets", Morgan Guaranty Trust, New York 1976

** the (UK) Export Credits Guarantee Department.

Much may still have to be done however by many developing nations to establish their credit-worthiness:

" Even in much easier times, the (World) Bank's experience with public utilities in developing nations has shown that their financing problems have been due more to inadequate domestic funding than any lack of foreign loans. One of the major weaknesses has been poor pricing policies, low and badly structured tariffs, and regulatory delays in granting appropriate adjustments for inflation and other courses ... (which) fall entirely under the control of local public utilities management and political authorities of developing nations. " (/23/)

Hopefully therefore the ill wind aroused by the energy crisis may yet encourage efficient organization, regulation, and rational centralised energy policies in these nations.

Finally in this chapter brief consideration will be given to some further constraints on system planning which may not immediately be classifiable as caused by either engineering or financial factors.

For several reasons it is often desirable to limit the relative proportions of fuel usage. In engineering considerations of planning, peaking, thermal and nuclear plant are assumed to have no limits imposed on their utilization by energy or fuel shortage. When regarding primary fuel availability this may not be so. A limitation on plant mix can be imposed as a result of this which may render the mix non-optimal. A chairman of the CEEB is on record as having stated quite bluntly that:

" ... since the early 1950s pressure has been put on us to burn the least economic fuel ... " (/55/)

This seems at first sight to be a peculiar way of optimizing fuel usage but in fact there are often sound reasons for doing so. In the UK the need has periodically arisen to support the local primary fuel industry, coal. The CEEB as a major customer has a far-reaching effect on the growth of both complementary and competitive fuel industries. It also has a profound effect on the level of employment, both directly and indirectly. As a result, often against its will, the electricity industry can be used as a political tool. Moreover in the UK as in most

industrialised nations there is a strong almost symbiotic relationship between the CEEB and major power plant manufacturers. A stable and regular equipment ordering programme is conducive to stability in the power plant manufacturing industry, and this in turn can give rise to considerable export potential and can therefore aid balance of payments problems.

The overdependence of a nation on imported fuel can render it vulnerable to international politically-directed action, as is known only too well since the events of 1973. President Nixon's ill-fated Project Independence was instituted in the United States essentially for reasons of economic and political freedom from imported energy sources. Started in 1974, it aimed for energy self-sufficiency by 1980, but has since to all intents and purposes been abandoned.

Further constraints may constrain plant mix or fuel usage. As mentioned above a mix which is suboptimal with respect to the internal criteria of the power industry may aim for overall national optimality in fuel usage. An indirect subsidy may be granted to local heavy industry, or to encourage the introduction of a new level of technology and to develop local expertise in the case of developing nations. For certain decisions national pride and prestige may be crucial. The question of nuclear proliferation is always the underlying and sometimes the overriding consideration when the pros and cons of the introduction of nuclear technology to a nation are being weighed. A project may have functions and yield social benefits not primarily related to its power production capability. It may stimulate regional development in depressed areas. It may be installed in conjunction with desalinization plant, or it may be mainly intended for irrigation or flood control. Social cost-benefit analysis is required for proper evaluation of such projects and this is normally outside the terms of reference of the system planner. Planting programmes have to be carefully timed to harmonize with national development plans. Bilateral aid terms may dictate the installation of certain stations, or their dates of installation. Some projects can have undesirable often indirect environmental effects and hence impose non-monetary

but nevertheless very real penalties. Finally in recent years there has been increased insistence that a purely economic, in the sense least-cost, evaluation may not be appropriate for certain types of project. Such has been the case in the continuing nuclear power controversy.

This plethora of constraints makes life no easier for the system planner. Unfortunately constraints, particularly those last mentioned, though often important are all too often very vaguely defined. Perhaps it is too much to ask the planner to take into account all possible repercussions of his planning decisions. Nevertheless he is open to criticism if he neglects to do so. The usual approach is to ignore these difficulties when examining broad long-term strategies and to distort the optimal strategies so as to accommodate as far as possible these awkward constraints, when considering the short-term planning programmes.

CHAPTER THREE. APPROACHES TO SYSTEM PLANNING

Forward planning is desirable for any organized activity. In the case of electricity supply it is necessary for many important reasons which have been developed in the previous chapters. The principle ones will be recapitulated below:

- The long construction and lead times associated with large power stations and transmission projects. Typical lead times for large power projects are of the order of five years, and this presumes a well-organized machinery for dealing with the problems of logistics, sequencing and coordination during all stages of planning and construction. The lead time will of course vary according to local conditions, and is frequently somewhat longer in developing countries. Delays unfortunately are not uncommon. Some large power projects have attained notoriety and entered popular mythology for their constructional delays. Consequences of such delays can be serious. System security may be imperilled and low merit plant will have to be used thus increasing production costs. Delays also mean vulnerability to cost inflation during construction. It should be noted that the quoted lead times do not include provision for the training of personnel where necessary. Many developing nations are in no position to install large generating units or nuclear plant because of the lack of operating experience and trained manpower. When such training is included lead times can extend to ten years or more.

- The interactive nature of system operation arising from the interconnection of stations by a grid network. This means that the installation of a unit or group of units cannot be considered in isolation. The operation of the system as a whole with and without the proposed installations has to be examined. The problem of assessment of system performance is more difficult than appears at first sight with the small independent systems existing in developing countries where the effects of the installation of large units (advantageous because of economies of scale) may not be immediately obvious in terms

of system security and reliability of supply, and merit very careful study.

- The long physical lifetimes of stations, transmission lines and power plant equipment in general. Stations last from ten to fifty years or more, although lower values are often used for project evaluation and book-keeping purposes. To gauge the performance of a power station over its useful lifetime requires a planning horizon extending perhaps decades into the future. The actual useful lifetime of a station is by no means certain. Frequently it is itself greatly dependent on the future development of the system as a whole, and must be viewed in this context.
- The capital intensiveness of the electricity supply industry. Large amounts of capital are required for investment in system expansion. Planning should include preparation for the mobilisation of capital, and due attention will need to be paid to the need for foreign exchange. Restrictions may have to be applied because of a lack of funds. All these considerations demand assiduous forward planning.
- Publicly regulated bodies are answerable to their regulatory bodies or financing institutions for their financial performance. Quite apart from considerations of corporate strategy, price levels and tariffs have to be determined in the light of future system development and growth of demand. Frequent price changes are unacceptable both to consumers and regulatory bodies. Tariff levels have to be directed towards a return on investment that is adequate yet not to be interpreted as an abuse of monopoly position. Forward planning and sensitivity analysis can provide useful aids to judgement.
- The position of the electricity supply industry both as an indicator and stimulator of the general national economic condition. Planners have not only to be aware of developments in the technology of power supply but also of likely or planned trends in government macroeconomic policy. An increase in the availability of electrical energy is frequently an essential

prerequisite for accelerated industrial activity and the success of major industrial projects is dependent upon an adequate reliable supply of electric power. The planning of system expansion has to be integrated carefully into any national development plans, often before the Plan itself is made available for publication.

It is of course a fallacy to imagine that the planner has to fix his ideas to start at a point in time five years or so in the future. The conditions upon which estimates were made and previous decisions taken are continually changing; patterns and trends are constantly shifting. Berrie /5/ mentions several means of trimming plant margins to accommodate these shifts; he calls these 'regulators' of the planning planning margin. Firstly, peaking plant can be installed in well under the oft-quoted five-year lead time. As a very last resort a crash programme of installation of gas turbines or diesel plant can be undertaken to maintain system performance, although such ad hoc measures are bound to have costly repercussions enduring well beyond the initial period for which corrective measures were required. Secondly, existing construction schedules may be rephased. Unfortunately it is always a risky business to deliberately slow down construction schedules - it would lay the electricity authority open to criticism from major contractors and could well involve further financial penalties due to cost inflation. It is doubtful also whether construction programmes can be accelerated to any significant extent. On the contrary, it is often the delays in existing construction schedules themselves which precipitate the need for corrective action. Delivery dates of plant on order may be deferred to some extent but there is little leeway in this respect. Finally, the rescheduling of plant retirements. Power stations are rarely used to the point of dilapidation, they are usually retired when their individual load factors are so low that it is no longer economic to maintain them in a suitable state of repair. Short-term shifts in expansion strategies and load demand levels can therefore be accommodated fairly readily by postponing or by putting forwards retirement dates of old stations.

When evaluating development strategies for any power system (the timing and scale of additions to and removals from the system, and the operating strategy adopted), four principal factors are generally considered:

- capital cost of new plant, P
- operating cost of the system, Q
- revenue from sales of electricity and related activities, R
- standard of system security (quality of supply), S

From the contents of the previous chapters it is clear that these requirements are often conflicting: P and Q should be as low as possible, whilst it is desirable that R and S be as high as possible. For example a development strategy which entails the planting of largely nuclear plant leads to high P, low Q, and owing to the generally large unit sizes can lead to a degraded quality of supply in small independent systems. Peaking plant on the other hand gives rise to low P but high Q, while S is not likely to suffer from this kind of strategy. Thus in the most general terms a multi-objective optimization problem exists for system planning. A utility function or objective function can be proposed, this is to be optimized (maximized or minimized). For example:

$$\text{minimize } U(P, Q, R, S)$$

where a typical objective function could be:

$$U(P, Q, R, S) = aP + bQ - cR + X(S)$$

$X(S)$ is a penalty function which assigns a penalty cost to a degraded quality of supply. Constants a, b, and c assign relative weights to the values of P, Q, and R. By using such a form of utility function all four objectives have been transformed onto a single scale of effectiveness. Quite apart from the question of the validity of such an assumption it will also be appreciated that the separate objectives may not be completely independent with respect to each other and hence a simple additive utility function may not be acceptable. As a further comment it is pointed out that the evaluation of P, Q, R, and S can be difficult problems in themselves.

Instead of optimizing with respect to the four objectives it is usual in system planning studies to satisfy with respect to one or more of them. Thus:

minimize $U'(P, R, Q)$

subject to $S \geq A$

The implicit assumption has been made that since P , Q , and R are generally measured on the same scale of effectiveness (monetary units) an additive form of utility function is satisfactory for the three variables. It is sufficient that the quality of supply never degrades to a level less than the index A in the constraint. This formulation treats objective S as a 'hard' constraint - one that can never be violated in any feasible solution. It is equivalent to assuming function $X(S)$ in the previous formulation as a barrier function: $X(S) = \infty$ if $S < A$, and $X(S) < \infty$ otherwise.

Even this form of objective function is not without its disadvantages. Although it may be superficially true that P , Q , and R are measured in equivalent units, yet problems of economics and financing mean that there is a very real distinction between money capital needed for investment in new plant and machinery, and other monetary outlays expended in system operation. In recognition of this a shadow cost of capital is often used to reflect this distinction.

Further constraints may now be added to limit the rate of capital expenditure, or to pay due attention to rationing of foreign exchanges, and to allow for more detailed considerations with respect to system development.

The objective is now the minimization of the discounted present value of the sum of all net monetary outlays over a predefined planning period, which may extend to infinity. The constraints imposed relate to system security, capital limitations and so on. Then, using DCF:

minimize $PW (P + Q - R)$

subject to:

- quality of supply to be attained and/or maintained at all times
- and any other considerations to reflect real-life restrictions.

The problem is still cast in very broad terms, and it is common at this stage to make several simplifying assumptions:

The revenue from sales of electricity, it is argued, is not causally influenced by the planting strategy adopted and can therefore be dropped from the objective function since it can be regarded as a constant. Thus the problem reduces to one of cost-minimization, and the removal of one degree of freedom makes it at once much simpler. Two points arise from this assumption which deserve elucidation. Firstly, a feasible solution may not exist to the problem as formulated. This can arise, for example if there is insufficient capital for system development and a degradation of supply will have to be accepted. The assumption will not hold under such conditions and the whole problem shifts emphasis to one of how best, with limited funds, to satisfy conflicting requirements of demand - a problem of resource allocation. This form of the problem will not be pursued further, suffice to say that such a state of affairs may not be uncommon in developing economies. In the second case, assuming a feasible solution exists, the planting strategy itself is dependent on assumed parameters in the DCF evaluation, notably the TDR. Alternative strategies may be proposed which may be different in detail and almost certainly in the value of the objective function owing to the use of differing TDRs. In the event that one or the other of the alternatives is accepted and executed, it is hardly likely that the revenue will have been the same. The industry can at least attempt, with no little likelihood of success, to justify a tariff level which ensures an adequate return on investment (not necessarily the TDR, because of regulatory considerations). Hence in practice revenue often adjusts to fit the strategy. It has been suggested that in the United Kingdom a low discount rate was implicitly used for evaluation of the nuclear power programme, and the consumers of electricity in that country financed at least part of the national nuclear power development programme.

It is unfortunate that the planning problem is so complex or so large that it is necessary to neglect revenue in the appraisal of investment strategies. This breach has not escaped the notice

of planners. Berrie /5/ notes that a cost-minimization approach constitutes part of a planning loop which is not closed. Turvey /56/ disapproves of the fact that "in too many undertakings the commercial side of the work is divorced from the planning side intellectually as well as organizationally". Nevertheless despite these important considerations a cost-minimization approach will be adhered to in this thesis; it will be assumed that due regard will be paid to all ramifications resulting from this assumption.

Transmission constraints are often ignored when assessing system performance in planning studies. It is assumed that planting at least the required capacity dictated by the gross plant margin will result in feasible and safe operation. Limitations on power flows and the necessity of reactive power generation are often neglected until the very last stages of planning. The model usually dealt with is the so-called 'point' model, or 'infinite busbar' model, which consists of the generating units in the system interconnected by a lossless conductor of infinite capacity, serving a point load of unity power factor. Interest is therefore focussed on the scale and sequence of additions to generating capacity - a generation expansion model. It should be pointed out that this assumption does not necessarily mean that transmission costs are ignored; some costs of transmission development arising from the installation of particular stations at particular sites can be readily accommodated by the inclusion of these costs with station capital costs. The use of point models is widespread and again is caused by the complexity of the problem and the need for simplification. Traditionally too planning has been dealt with by separate departments dealing with the generation and transmission aspects and such demarcations are difficult to erase. Perhaps also the transmission characteristics of different systems vary from one to another so widely that a generalised model, beloved of the mathematical programmer, cannot be easily constructed. However with the advent of the use of large digital computers in system planning there seems to be no strong reason why a simplified load flow model, specific to a particular system, should not be incorporated as one of the

constraints or in production cost models before the final stage of detailed feasibility studies. In this respect a small system is a positive advantage. Load flow studies for large networks can be of such a degree of complexity that the systems must be decomposed into separate subsystems for analysis.

Siting problems have proved awkward to handle and are often neglected or simplified greatly. Most electric utilities do however face severe problems as regards power station siting. Quite obviously the choice of hydro sites is strictly limited, but less obviously so are those of large thermal or nuclear stations. Neglect of this point can lead to peculiar results. A recent planning study /21/ outlined a possible scenario of power development for Singapore, involving the installation of 2600 MW of nuclear capacity by 1989. Presuming these plant are to be sited on land, one wonders how much attention was paid to the problems of siting such plant on a small, densely populated island*. Of course conditions such as these are usually screened out on further investigation, but it would be better if they were not admitted as feasible alternatives in the first place.

It is often not a tenable assumption that station capital costs are proportional to capacity, especially for hydro plant and even for other types as well. One reason is the returns to scale with respect to capital cost. But site shortages and development can give rise to other powerful reasons. Turnkey projects are still uncommon in many parts of the world and sites are seldom planted up to full capacity in a short period. It is in fact normal to develop sites in stages, with a few units installed at each stage. Full development in one stage is neither necessary nor desirable, and often full site capacity is reached many years after the commissioning date of the first sets. Because of this development costs are significantly greater for the first stage, involving as it does initial site preparation, much of the civil works (station buildings, etc)

* 1500 km², population over 2 million.

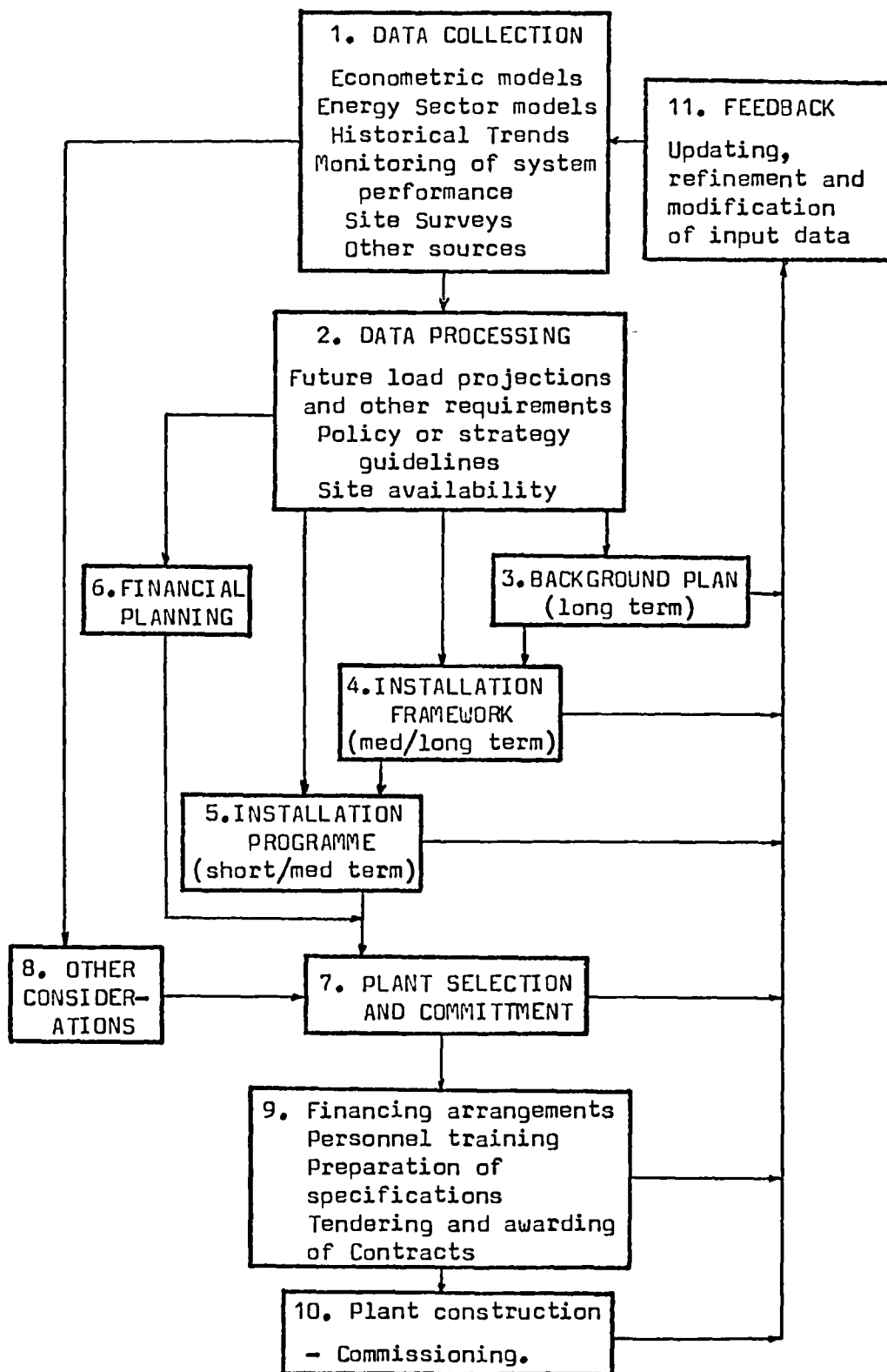
and the bulk of transmission development associated with the station. Future installations will be cheaper per unit capacity. Often future installations themselves will be conditional upon prior installations. The introduction of certain units at a site means that there is a strong incentive for installing units of the same size or even the same manufacturer for future development at that location, in the interests of standardization. Again, a site may be available for the introduction of either nuclear or base load thermal plant, but seldom both. Less attention seems to have been paid to the existence of discrete unit sizes than is justifiable. Selection of the appropriate unit sizes is far from irrelevant especially for the smaller system. Few planning studies have paid attention to such considerations of siting until the final stages. It may well be advantageous to narrow down the range of alternatives at a much earlier stage.

Finally, for the most part perfect foreknowledge of load data and costs is assumed. There have been notable exceptions to this, in both generation planning /7/ and transmission planning /29/.

The remainder of this chapter will be devoted to a discussion of a planning methodology for a systematic and orderly approach to the development and expansion of a power system, directed towards the construction of a rational framework within which the planner can make logical moves, within constraints and considerations which have been outlined in the previous chapters. Such a methodology, perhaps different in detail or emphasis, is used by electricity supply authorities throughout the world. A possible procedure is flowcharted in Fig 3-1.

Planning is a continuous rather than a once-and-for-all process. There is continual feedback of results, refinement of data, and frequent reiteration of critical stages in the procedure before usable results can be deemed to have been obtained. Sensitivity analysis may have to be performed. The recommendations of the planner are then presented for an executive decision on installation of plant to be made, but the planner's interest

FIGURE 3-1 A PLANNING METHODOLOGY



does not necessarily end there. After the decision has been taken the operation of executive machinery for carrying out the installation in turn provides input data for future planning studies.

Box 1 in Fig 3-1 represents the collection of data. The sources of such data are many. Econometric models will provide statistical estimates of demand levels and perhaps shapes in the future. Macroeconomic models may be required to provide and indication of trends in the state of the economy which in turn will condition demand. These models will also provide indices of economic activity which will aid in arriving at a decision for a suitable TDR, identification of inflationary or deflationary trends, and likely changes in the costs of labour and capital. Primary fuel availability will have to be examined. Monitoring of system performance often provides a very useful check on the validity of previous assumptions which may unjustifiably have been taken for granted, and site surveys too have to be undertaken well in advance of any proposed installations. Sites far from population and load centres in particular have to be costed with care. An eye has always to be directed towards imminent or likely developments in the technology of power supply. Political directives may have to be sought and these in some cases may encourage or suppress certain avenues of development. The collation of input data is a lengthy and time-consuming process but its importance cannot be understated. Scanty data hurriedly assembled and casually checked have too often been used for planning studies, the results of which have often been wide off the mark in retrospect. Data collection ideally should involve not only the utility operators and planners but also relevant government departments, major primary fuel suppliers, major industrial customers, development and aid organizations, and other interested parties.

Box 2 represents the checking and processing of input data. Load projections from separate independent sources will have to be checked and correlated. Cost projections will have to be related to indices of economic activity and shadow costs if used will have to be evaluated. This stage not only involves

the marshalling of a mass of numbers but also their compression where necessary and translation into a form suitable for use by mathematical programming models. Again care taken at this stage is time well spent.

Boxes 3 to 6 represent various stages in the formal planning process. Financial planning is accorded a box on its own, highlighting its substantial and increasing importance in the planning of system development. The investigation of trends of development of the system is accorded three separate boxes, representing a steady narrowing down of alternatives with an increase of accuracy and detail.

The need for a model for treatment of long-term trends (Box 3) is widely accepted. Such a model yields what has been called the background plan, which provides the backdrop against which more detailed plant installation strategies can be evaluated. Planning horizons for background plans are far into the future, perhaps 20 to 30 years. A desirable attribute of such a model is flexibility in use, and not necessarily pinpoint accuracy. This is because the primary function of such a model should be to present a general idea of the effects of variation of the major parameters on the shape of the system in the future and trends in costs. Despite their simplicity useful conclusions can be drawn from such models. Jenkin /34/ describes a conceptually simple model which uses elements of functional algebra in its solution and which is amenable to parametric analysis and for this reason very useful. The model can also be used for simple sensitivity analysis. Intelligently directed use of marginal analysis can also aid investigations into the structure of the problem. Many global methods have been described in the literature. The model CHOSE /49/, used by the CEEGB, checks for optimality with Kuhn-Tucker conditions within an iterative framework based upon the load duration curve and merit-order loading. Nicholson /40/ describes a solution to the plant mix problem which uses the projected gradient method of mathematical programming. Seglari and Laughton /3/ typify the linear programming approach which has been used in one form or another since the 1950s. Dynamic programming is also well

suited to the serial nature of system development, Petersen /48/ describes such a model. These last four models are more complex than the first and much more lengthy in terms of computer utilization, not least because they include much more detail. As such they lose much of the flexibility and usefulness of the simpler models and can only be used to good effect where access to large high-speed digital computing facilities is readily available. Doubt has been expressed as to the need for such complexity in models peering so far into the future. Berrie /5/ makes the point that planning models should be "properly organized and be only as complex as the standard of data available and the problem being solved requires".

Box 5 represents the stage of detailed feasibility studies. By this stage the range of possibilities will have been narrowed down to perhaps three options in the near future and horizons are pitched to about five or ten years in the future. Greater care has to be taken at this stage in cost estimation, site availability, discrete unit sizes, load projections, and the accuracy of the input data will be better and more reliable. Explicit allowance can be made for transmission constraints, the provision of spinning reserve, transportation of fuels where this may be a constraining factor, and perhaps even consideration of the scheduling of planned outages. A detailed, possibly hour-by-hour simulation of system operation, both under normal (steady-state) and fault (transient) conditions is required. Thus the determination of the installation programme is usually directed to the consideration of the installation of a single unit or set of units in the near future. This has to be so restricted since when taking into account type, size, and date of installation the total number of feasible avenues of development can be quite great even with such a short planning period.

The case for an intermediate step between Box 3 and Box 5, called here the installation framework, is now presented. The general trend in the evolution of system planning models has been a steady increase in sophistication in models of the background plan, essentially plant mix models. They can now handle a wider range of constraints, but the inclusion of

these facilities can all too often obscure long-term trends in development which can themselves provide very useful guides for the planner. An alternative approach is suggested (Box 4). The model for the background plan should only include as much detail as is necessary to gauge these long-term trends, and to provide indications of recommended patterns of development. A conscious effort should be made to ensure that it is versatile, flexible, and easily used. The guidelines suggested by this model serve as part of the input data for the installation framework model, more restricted in scope, but not as narrow as that for the installation programme. This model will aim for the medium to long-term and will therefore take into account the interaction of proposed installations in system operation over much of their lifetimes instead of restricting the horizon to the immediate future. It will take into account constraints on the plant mix (as dictated by the background plan), discrete unit sizes of generating plant, sites and restrictions on site capacity, variations in plant retirement schedules, and a possibly more detailed loading simulation of system operation under steady-state conditions. Constraints on finance can be included, and perhaps even a simple load flow model used as a constraint on possible alternatives, provided that the system is not too complex in structure. As implied by the name, the model will aim at providing a framework of possible development strategies within which immediate installations can be studied in greater detail.

The results of the four planning models can then be presented for the executive decision to be made for the immediate future (Box 7). Extraneous factors which cannot be handled by mathematical models can be introduced at this stage although hopefully with efficient planning they will not be too great in number. Mathematical models of course can only provide rational bases for decision making, and the executive decision on plant installation will still be very much a matter of judgement tempered by experience.

The planner's job can strictly be held to end there, but he retains an interest in future steps. Arrangements have to

made to carry out the decision. Loans have to be sought, justified, and arranged. Personnel training may be necessary. Contract specifications have to be prepared, tenders evaluated, and contracts awarded. Arising from discussions with contractors some specifications and projected commissioning dates may have to be modified and this will have relevance to future decision-making. Upon satisfactory conclusion of paperwork construction can commence. Even then changes can occur in commissioning dates due to unforeseen delays and the planner will have to evaluate the impact of such delays. The time between decision and implementation takes many years even when a well organized experienced department is involved.

Throughout this whole lengthy process the feedback of information provides part of the data for future planning studies. System planning is therefore very much a fulltime occupation. The actual mechanics of the process vary widely from one organization to another, and range from extremes of the simplistic, ruled by hunches and intuitive judgements, to the overcomplex, ruled by the mammoth computer. Intelligently used however computer-based planning models can ease the burden on the planner, and can be a useful tool in his armoury. At worst, they can at least provide a rationalization for his recommendations.

The remainder of this thesis is concerned with the solution of a particular problem - that of determination of the installation framework. The theory, features, and performance of such a model will be presented.

PART II

CHAPTER FOUR. THE INSTALLATION FRAMEWORK

A brief recapitulation of the requirements and basic features of a model for determination of the installation framework for power system expansion or development is given below:

- It will be a point model, with no explicit treatment of transmission losses in system operation. However some attempt may be made to include costs of transmission development and losses in the construction and operating costs of particular stations at particular sites.
- The objective of the model will be cost minimization within imposed constraints relating to a particular standard of system security. Present worth arithmetic will be used to weight all expenditures according to their times of occurrence.
- The planning horizon will be in the medium-term future - from about 10 to 20 years. For this horizon it will be necessary to gauge the effects of interaction between different plants in system operation.
- Estimates of system operating costs obtained from a loading simulation model will be as accurate as possible commensurate with the improved quality of input data due to the somewhat reduced horizon. Plant availability rates and the costs of provision of adequate levels of spinning reserve will be taken into consideration.
- The existence of discrete unit sizes will be recognized. Only a limited number of sites may be available and there may be restrictions on minimum or maximum planted capacities at each site.
- Guidelines, for example on plant mix, may be imposed as a result of previous investigations into planning development scenarios using a long-term model to determine a background plan.
- Constraints may be imposed on the amount and rate of capital expenditure for system development, which in turn may be differentiated into local and foreign components.

- Both optimal planting and retirement schedules may be sought.
- Escalation factors for capital and fuel costs may be introduced to adjust prices relative to one another during the course of the planning period.
- Predetermined or committed installations, to be introduced into the system during the planning period (as a result of previously taken decisions) must be accommodated.
- It may be necessary to apply strict constraints on planting dates for particular projects. For example the development of a certain site may be constrained to be expedited before a given date, in between two given dates, or only after a given date.
- Constraints may also be imposed on the sequencing of planting at certain sites. Some alternative units may only be available after prior installation of other particular units at a site. In this way it will be possible to allow for the greater costs of initial site development and the reduced costs of follow-on units. Furthermore this type of constraint may be applicable where (a) a site may be developed with either (say) nuclear or base-load thermal units, but not both, or (b) in the interests of standardization, where the introduction of a particular unit type and size dictates future site development with identically sized units of the same type.

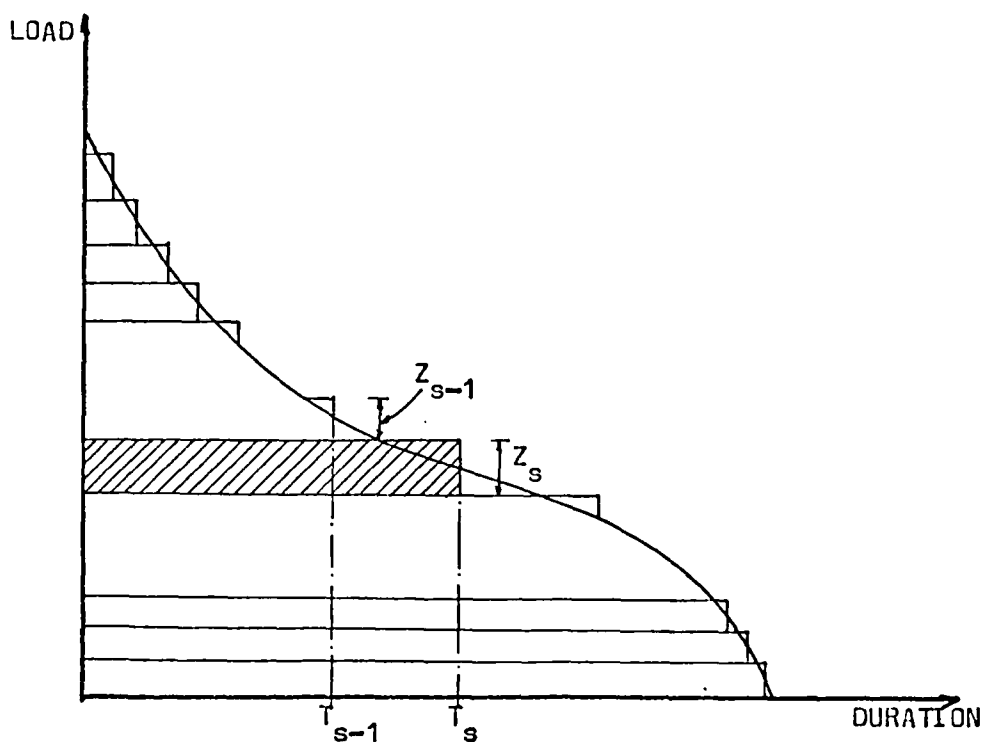
Solution of the installation framework problem is best done with a computer-based model. As will be seen however the size of the problem in such formulations is formidable and frequently solution may be limited by the computational requirements. Initially linear programming, both continuous and integer, will be described and discussed briefly (Sections 4.1 and 4.2). The approach of dynamic programming is then presented in Section 4.3. Finally an original two-phase method is outlined, which combines dynamic programming with a decision tree search (Section 4.4).

4.1 LINEAR PROGRAMMING

A linear programming (LP) approach to the determination of an installation framework will be examined in this section. LP techniques are amongst the most useful of the many mathematical programming tools available today, and were one of the first of the formal operational research techniques to be applied to aid system planning, with much early work being focussed on the solution of the plant mix problem. Early formulations were troubled by an excessive number of constraints which made solution difficult and time-consuming. Recent developments have achieved a significant reduction in the number of constraints to be handled. The Improved Z-Substitutes method to be outlined below is one such formulation.

A plant in what follows is defined as an independent unit, or a group of units of identical performance, size and vintage. The load duration curve for each subperiod (year) during the study is approximated to by a histogram of S horizontal load blocks (Fig 4.1-1).

FIGURE 4.1-1 LDC HISTOGRAM FOR THE IMPROVED Z-SUBSTITUTES METHOD



NOTATION

- A plant availability factor
- C capital costs for newly-introduced plant, per MW capacity.
This includes fixed costs, summed and present-worthed over
estimated plant lifetime.
- D minimum total installed capacity of system
- d_{ns} demand associated with block s , year n , in the load duration
curve histogram, Fig 4.1-1.
- F fixed costs for existing plant, per MW capacity.
- I number of plant types.
- L_i number of sites available for plants of type i .
- M number of plants in the system prior to start of the study.
- N number of subperiods (years) in the planning period.
- O operating costs of plant, per MWh.
- P nominal capacity of plant, MW.
- PWF present worth factor.
- r reserve factor, to allow for the provision of spinning reserve.
- S number of load blocks in the LDC histogram, Fig 4.1-1.
- T_s Duration associated with block s in the LDC histogram, h.
(Fig 4.1-1)
- Z reduction in output of an individual plant from block s
in the LDC histogram to block $(s+1)$.

Plant is divided into two classes: those which are installed prior to the start of the planning study, and those which are introduced during the course of it.

A' , O' , P' , and Z' refer to plant installed prior to the commencement of the study (existing plant).

FORMULATION

Objective function:

$$\begin{aligned} \text{minimize } \sum_{n=1}^N (PWF)_n \left\{ \sum_{m=1}^M F_{mn} P'_{mn} + \sum_{i=1}^I \sum_{l=1}^{L_i} C_{inl} P_{inl} \right. \\ \left. + \sum_{m=1}^M \sum_{s=1}^S O'_{mn} Z'_{mns} T_s + \sum_{i=1}^I \sum_{l=1}^{L_i} \sum_{s=1}^S O_{inl} Z_{inls} T_s \right\} \end{aligned} \quad \dots(4.1-1)$$

subject to constraints on:

total installed capacity:

$$\sum_{m=1}^M P'_{mn} + \sum_{j=1}^n \sum_{i=1}^I \sum_{l=1}^{L_i} P_{ijl} \geq D_n (1+r_n) \quad \text{for } n = 1, 2, \dots, N \quad \dots(4.1-2)$$

instantaneous demand:

$$\sum_{m=1}^M \sum_{k=s}^S Z'_{mnk} + \sum_{j=1}^n \sum_{i=1}^I \sum_{l=1}^{L_i} \sum_{k=s}^S Z_{ijlk} \geq d_{ns} \quad \begin{aligned} &\text{for } n = 1, 2, \dots, N \\ &\text{and } s = 1, 2, \dots, S \end{aligned} \quad \dots(4.1-3)$$

individual plant operating capacities:

(a) existing plant:

$$\sum_{s=1}^S Z'_{mns} \leq \Lambda'_{mn} P'_{mn} \quad \begin{aligned} &\text{for } n = 1, 2, \dots, N \\ &\text{and } m = 1, 2, \dots, M \end{aligned} \quad \dots(4.1-4a)$$

(b) newly installed plant:

$$\sum_{s=1}^S Z_{inls} \leq \sum_{j=1}^n A_{ijl} P_{ijl} \quad \begin{aligned} &\text{for } n = 1, 2, \dots, N \\ &i = 1, 2, \dots, I \\ &\text{and } l = 1, 2, \dots, L_i \end{aligned} \quad \dots(4.1-4b)$$

The inclusion of appropriate slack and surplus variables, together with non-negativity constraints, will produce the standard LP form, amenable to solution by the Simplex method.

NOTES

1. The use of substitute variables Z enables a significant reduction in the number of constraints to be handled by the LP.

If an individual plant's contribution to block s of the LOC is U_s (existing plant) or U'_s (new plant), then by definition:

$$Z'_s = U'_s - U'_{s+1}$$

and $Z_s = U_s - U_{s+1}$

Now if $Z \geq 0$, then constraint (4.1-3) has replaced:

$$\sum_{m=1}^M U'_{mns} + \sum_{j=1}^n \sum_{i=1}^I \sum_{l=1}^{L_i} U_{ijls} \geq d_{ns}$$

for $n = 1, 2, \dots, N$

and $s = 1, 2, \dots, S$

and constraint (4.1-4a) has replaced:

$$U'_{mns} \leq A'_{mn} X'_{mn} \quad \text{for } n = 1, 2, \dots, N$$

$m = 1, 2, \dots, M$

and $s = 1, 2, \dots, S$

and constraint (4.1-4b) has replaced:

$$U_{inls} \leq \sum_{j=1}^n A_{ijl} X_{ijl} \quad \text{for } n = 1, 2, \dots, N$$

$i = 1, 2, \dots, I$

$l = 1, 2, \dots, L_i$

and $s = 1, 2, \dots, S$

The introduction of the Z variables has made the number of constraints for individual plant operating capacity constraints (4.1-4a and 4.1-4b) independent of the number of load blocks S .

The assumption that $Z \geq 0$ is a valid one under most conditions provided that the merit order of loading does not vary from one block to the next. The reader is referred to Beglari and Laughton /3/ for a formal justification and more complete discussion on the Improved Z -Substitutes method.

Compared to simple conventional LP formulations the introduction

of the Z variables can reduce the number of constraints by up to 80%, enabling a reduction of CPU time by factors of between two and five /3/.

2. Escalation factors can be introduced into appropriate terms of the objective function to allow for inflationary or deflationary effects in capital or operating costs. These may be differentiated according to fuel usage or plant mix.

3. The evaluation of system operating costs is integrated into the objective function in this LP formulation. There is therefore no necessity for a separate loading simulation model. This integration has only been achieved by a linearized modelling of system operation. The model does not allow for part load efficiencies of individual plants, nor for the cost of provision of spinning reserve. This can lead to errors in the estimation of production costs (eg for over-planted systems).

Furthermore the basic formulation does not take into account energy limitations for hydro plant. This shortcoming may be remedied by the inclusion of further constraints (these will be linear) on the energy production of such plant.

4. A constraint on permissible plant mix may be imposed:

$$(D_{\min})_{in} \leq \sum_{m \in M_i} p'_{mn} + \sum_{j=1}^n \sum_{l=1}^{L_i} p_{ijl} \leq (D_{\max})_{in}$$

for $n = 1, 2, \dots, N$

and $i = 1, 2, \dots, I$

where $(D_{\min})_{in}$ and $(D_{\max})_{in}$ are the minimum and maximum permissible planted capacities of plant type i in year n , imposed by the background plan. M_i is the set of existing plant of type i .

5. Constraints on maximum site capacity can be accommodated:

$$\sum_{j=1}^n p_{ijl} \leq (p_{\max})_{il} \quad \text{for } n = 1, 2, \dots, N$$

$$i = 1, 2, \dots, I$$

$$\text{and } l = 1, 2, \dots, L_i$$

where $(p_{\max})_{il}$ is the maximum site capacity for site l , plant type i .

6. Optimal retirement programmes may be obtained by imposing the following constraints:

Let X_m^i be the nominal capacity of plant m , which is to be retired at some date prior to year N_m . Then:

$$p_{mn}^i \leq X_m^i \quad \text{for } n = 1, 2, \dots, N_m$$

$$\text{and } p_{mn}^i = 0 \quad \text{for } n = N_m + 1, N_m + 2, \dots, N$$

In a similar fashion predetermined installations effective during the planning period may be introduced at the appropriate dates.

7. An upper limit on the annual capital expenditure, or the cumulative capital expenditure over a number of consecutive years, may be catered for by imposing linear inequality constraints as required.

8. Conditional installations may prove difficult (though not impossible) to handle since LPs become unwieldy with conditional constraints, dichotomies, or multiple-fold alternatives /26/.

9. The assumption that plant costs are proportional to installed capacity may not be acceptable in view of the significant economies of scale available with power plant equipment. The LP formulation described here is a development of a plant mix formulation where the existence of discrete unit sizes is not acknowledged. To include this as a constraint some form of mixed-integer linear program (MILP) will be necessary.

Early uses of LPs for application to power system planning were inhibited mainly by the number of constraints to be handled in practical cases. It appears that the CEEGB developed nonlinear formulations primarily because of this computational burden. In recent years however exceedingly powerful and efficient software packages have been made available for the solution of

large LPs and the CEGB now uses linear programming because of the inherent simplicity and flexibility /34/.

It may be noted that for the plant mix development problem all costs and constraints are linear in nature with the single exception of the estimate of system production costs. Even this may be made linear if a sufficiently large number of load blocks is used to approximate the LDC. The drawback however is still that to include accurate simulation of system operation and to accommodate realistic constraints entails the penalty of an extremely large number of constraints, leading to a prohibitively large LP. As an example, a problem spanning 20 years with five existing plants, 4 possible plant types and three locations for each plant type (a very modest-sized problem) the basic formulation leads to an LP with 560 constraints, with 10 load blocks used in the LDC histogram. If further constraints are now added to handle other considerations the number of constraints will be increased considerably. Moreover if an MILP solution is sought this may well increase the size of the LP and therefore computational requirements to a prohibitive level.

4.2 ZERO-ONE PROGRAMMING

The formulation of Section 4.1, designed from the outset as the solution to the dynamic plant mix problem, rapidly increases in size and complexity if attempts are made to modify it to obtain the installation framework. However the advantages of retaining an LP-type format are considerable, and it might be advisable to examine alternative formulations of the same problem which may be more easily solved whilst remaining within a linear framework.

The ideas within the formulation to be described arise from the observation that in practice the number of choices open to the planner at this intermediate stage in the planning process is fairly limited. At this stage several sites will have been identified for evaluation, certain unit sizes and types will have been earmarked as possible candidates, and the problem is principally one of determination of optimal sequencing and coordination of plant installations*. If the number of alternatives is fairly small, say less than about 50, a zero-one LP format may be attempted.

NOTATION

The notation of Section 4.1 is retained, with the following differences:

- F annual fixed charges for existing plant
- C capital costs, and fixed costs summed and present-worthed over plant lifetime, for newly installed plant.

From these definitions the costs are no longer assumed to be proportional to plant capacity, and the objection of 4.1 (9) is overcome. These costs are now associated with a particular plant or candidate for installation, and the quantities P are now no longer independent variables but constants associated with particular plant.

* Oatman and Hamant /43/ recognize this fact and evaluate a large number of alternative development strategies for solution of the planning problem. Exhaustive enumeration can lead to an extremely large problem, but in this approach the planner's experience and judgement are used to reduce enumeration to manageable levels.

FORMULATION

The objective function, similar to before, is:

$$\begin{aligned} \text{minimize } \sum_{n=1}^N (PWF)_n \left\{ \sum_{j=n}^N \sum_{m=1}^M X'_{mj} F_{mn} + \sum_{i=1}^I \sum_{l=1}^{L_i} X_{inl} C_{inl} \right. \\ \left. + \sum_{m=1}^M \sum_{s=1}^S 0'_{mn} Z'_{mns} T_s + \sum_{i=1}^I \sum_{l=1}^{L_i} \sum_{s=1}^S 0_{inl} Z_{inls} T_s \right\} \end{aligned} \quad \dots(4.2-1)$$

subject to the following constraints on:

total installed capacity:

$$\sum_{j=n}^N X'_{mj} P'_{mn} + \sum_{i=1}^I \sum_{l=1}^{L_i} P_{il} \sum_{j=1}^n X_{ijl} \geq D_n (1+r_n) \quad \text{for } n = 1, 2, \dots, N \quad \dots(4.2-2)$$

instantaneous demand:

$$\sum_{m=1}^M \sum_{k=s}^S Z'_{mnk} + \sum_{j=1}^N \sum_{i=1}^I \sum_{l=1}^{L_i} \sum_{k=s}^S Z_{ijlk} \geq d_{ns} \quad \begin{aligned} &\text{for } n = 1, 2, \dots, N \\ &\text{and } s = 1, 2, \dots, S \end{aligned} \quad \dots(4.2-3)$$

individual plant capacity constraints:

(a) existing plant:

$$\sum_{s=1}^S Z'_{mns} \leq A'_{mn} P'_{mn} \sum_{j=n}^N X'_{mj} \quad \begin{aligned} &\text{for } n = 1, 2, \dots, N \\ &\text{and } m = 1, 2, \dots, M \end{aligned} \quad \dots(4.2-4a)$$

(b) newly installed plant:

$$\sum_{s=1}^S Z_{inls} \leq A_{il} P_{il} \sum_{j=1}^n X_{ijl} \quad \begin{aligned} &\text{for } n = 1, 2, \dots, N \\ &\quad i = 1, 2, \dots, I \\ &\text{and } l = 1, 2, \dots, L_i \end{aligned} \quad \dots(4.2-4b)$$

zero-one variables:

$$X'_{mj} = 0,1 \quad \begin{array}{l} \text{for } m = 1,2,\dots,M \\ \text{and } j = 1,2,\dots,N \end{array} \quad \dots(4.2-5a)$$

$$\text{and } X_{ijl} = 0,1 \quad \begin{array}{l} \text{for } i = 1,2,\dots,I \\ \quad \quad \quad l = 1,2,\dots,L_i \\ \text{and } j = 1,2,\dots,N \end{array} \quad \dots(4.2-5b)$$

The X variables denote plant status:

if $X'_{mj} = 1$ plant m is retired in year j,

if $X_{ijl} = 1$ plant type i, location l, is installed in year j.

To prevent a plant being installed or retired more than once during the planning period:

$$\sum_{n=1}^N X'_{mn} \leq 1 \quad \text{for } m = 1,2,\dots,M \quad \dots(4.2-6a)$$

$$\text{and } \sum_{n=1}^N X_{iln} \leq 1 \quad \begin{array}{l} \text{for } i = 1,2,\dots,I \\ \text{and } l = 1,2,\dots,L_i \end{array} \quad \dots(4.2-6b)$$

NOTES

1. This formulation represents a zero-one program - the independent variables X and X' are constrained to assume the values 0 and 1 only. It is not however a true binary LP because the independent variable Z and Z' are still continuous. Special-purpose methods are available for the solution of linear 0-1 programs conveniently and sometimes more economically than for the corresponding MILPs /26/. Thus solution can be easier though the actual number of constraints can be very much greater.
2. In a sense this formulation can be identified more closely with the problem - the question of assigning values of 0 or 1 to the independent variables X and X' corresponds to the planning decision of (choose)/(not choose), or (retire)/(not retire).
3. Incorporation of factors for inflationary or deflationary effects can be handled as before (Section 4.1 (2)); similarly hydro energy restrictions (4.1 (3)), plant mix constraints (4.1 (4)), and site capacity constraints (4.1 (5)).
4. Optimal retirement dates for existing plant may be determined by the imposition of the following constraints:

If a plant is to be retired at some date in or before year N_m :

$$\sum_{n=1}^{N_m} X'_{mn} = 1$$

Note that this form is somewhat simpler than the equivalent for the MILP.

5. Constraints on dates of installation can be readily handled by similar constraints on the variables X . If for example a plant (type i , location l) must be installed before or in year N_1 :

$$\sum_{n=1}^{N_1} X_{inl} = 1$$

Alternatively if this plant is not to be installed before or in year N_1 then:

$$\sum_{n=1}^{N_1} X_{in1} = 0$$

These general ideas can be readily extended to accommodate a variety of possibilities. The following constraint dictates that the plant must be installed between years N_1 and N_2 inclusive:

$$\sum_{n=N_1}^{N_2} X_{in1} = 1$$

Similarly if the plant is only available for installation between years N_1 and N_2 inclusive, but at no other time:

$$\sum_{n=1}^{N_1-1} X_{in1} = 0 \quad \text{and} \quad \sum_{n=N_2+1}^N X_{in1} = 0$$

In this way a variety of constraints with respect to permissible installation dates may be catered for.

6. For conditional installations a Boolean truth table may be constructed to map permissible combinations of plant installations and dates of installations, and this enables (in principle) complete generality in this respect, although this will add considerably to problem size.

The existence of special software packages makes the method attractive. The formulation is flexible and can be made to reflect actual capital costs more accurately. Constraints on sites and the sequencing of planting programmes can be handled. A complete consideration of all possible combinations is hampered however by the size of the problem, even where a small number of candidates for addition is involved. The approach also suffers for possibly inaccurate evaluation of system production costs due to the linearization of the LDC, neglect of part load plant efficiencies, and costs of provision of spinning reserve.

4.3 DYNAMIC PROGRAMMING

In contrast to the previous two formulations, a general nonlinear approach is now described. This approach may be necessary for more accurate estimates of system operating costs and handling of real-life constraints. The interactive nature of system operation entails discontinuous and nonlinear constraints and leads to a significantly nonlinear function for system production costs.

Dynamic programming is well suited to the idea of planning additions to the system and retirements of existing plant, being as it is a staged serial process. Dynamic programming also has the desirable attribute of being able to handle a wide variety of constraints which need not necessarily be linear or even continuous. A global solution is always achieved. Appendix A outlines a derivation of the dynamic programming algorithm together with a brief discussion on its solution, advantages, and limitations.

Examples of the use of dynamic programming for system planning whilst recognizing the existence of discrete unit sizes are the approaches of Booth /7/, Dale /17/, and the WASP computer package used by the International Atomic Energy Agency /21/. All these formulations employ the forward programming algorithm, the basic approach of which will be described in the sections to follow.

There are two phases to the solution of a dynamic program, firstly a build-up phase in which optimal recursive state functions and policy functions are determined, and secondly a traceback phase where they are used to generate the optimal sequence of decisions and the cost of this optimal policy. For the forward programming algorithm the build-up phase moves forward in time, whilst the traceback phase moves backward in time.

NOMENCLATURE

- C capital cost function,
 $C_n(\underline{d}_n)$ is the cost of implementing decision $\underline{d}$ at year n .
- $\underline{d}_n$ decision vector, operative from the beginning of year n .
- F recursive state function,
 $F_n(\underline{s}_n)$ is defined as the cost of expanding or developing the system in an optimal manner from state $\underline{s}_1$ at year 1 to state $\underline{s}_n$ at year n .
- $\underline{G}$ recursive policy (vector) function,
 $\underline{G}_n(\underline{s}_n)$ is the optimal decision vector to be applied at year n for expansion along an optimal path, if the current state of the system is described by $\underline{s}_n$.
- N number of time intervals (eg years) in the planning period.
 $N =$ horizon year.
- R running cost function,
 $R_n(\underline{s})$ is the cost of system operation in year n for the system described by state vector $\underline{s}$.
- $\underline{s}_n$ state vector, describing the system at the beginning of time interval n .
- Z objective function, representing the discounted present worth of all monetary outlays over the planning period. This is to be minimized.

The functions C and R are so defined as to include the present worth factor and any inflationary or deflationary effects relative to year 1, or any other base year.

The state vector $\underline{s}$ and decision vector $\underline{d}$ are for the time being not defined with precision. For the moment it will be assumed that these vectors provide a sufficiently complete description of the system and the additions and removals at each year during the planning period.

FORMULATION

$$\text{minimize } Z = \sum_{n=1}^N \left\{ C_n(\underline{d}_n) + R_n(\underline{s}_n) \right\} \quad \dots(4.3-1)$$

with the transition function:

$$\underline{s}_{n-1} = \underline{s}_n - \underline{d}_n \quad \text{for } n = 1, 2, \dots, N \quad \dots(4.3-2)$$

$$\text{and constraints: } \underline{s}_n \in S_n \quad \dots(4.3-3a)$$

$$\text{and } \underline{d}_n \in D_n(\underline{s}_n) \quad \dots(4.3-3b)$$

This dynamic program can be solved by the following recursive functional equation (see Appendix A):

$$\begin{aligned} F_n(\underline{s}_n) &= \min_{\underline{d}_n} \left\{ C_n(\underline{d}_n) + R_n(\underline{s}_n) + F_{n-1}(\underline{s}_n - \underline{d}_n) \right\} \\ &= \min_{\underline{d}_n} \left\{ C_n(\underline{d}_n) + F_{n-1}(\underline{s}_n - \underline{d}_n) \right\} + R_n(\underline{s}_n) \end{aligned}$$

for $n = 2, 3, \dots, N \quad \dots(4.3-4)$

$$\begin{aligned} \text{with the initial condition: } F_1(\underline{s}) &= 0 \quad \text{if } \underline{s} = \underline{s}_1 \\ &= \infty \quad \text{otherwise} \end{aligned} \quad \dots(4.3-5)$$

Rewriting Eqn 4.3-4 as:

$$F_n(\underline{s}_n) = C_n(\underline{d}_n^*) + F_{n-1}(\underline{s}_n - \underline{d}_n^*) + R_n(\underline{s}_n) \quad \dots(4.3-6)$$

and letting:

$$G_n(\underline{s}_n) = \underline{d}_n^* \quad \dots(4.3-7)$$

Then, (a) by definition, $F_N(\underline{s}_N)$ provides the solution to the problem:

$$Z = \min \left[F_N(\underline{s}_N) \right] = F_N(\underline{s}_N^*) \quad \dots(4.3-8)$$

and (b) the policy function G_n will provide the optimal strategy of expansion, via recursive application of:

$$\underline{d}_{n-1}^* = G_{n-1}(\underline{s}_n^* - \underline{d}_n^*) \quad \text{for } n = N, N-1, \dots, 2 \quad \dots(4.3-9)$$

from Eqns 4.3-2 and 4.3-7

$\underline{d}_n^*, n = 2, 3, \dots, N$ is the optimal sequence of decisions
- the optimal policy for least-cost expansion.

NOTES

1. In principle the solution to this formulation is straightforward. Initially $n = 2$ (since the system is completely defined at the start of the planning period when $n = 1$) and Eqn (4.3-4) is solved for every feasible state $\underline{s}_n$. These functions are then stored for each year n in quantized form. Index n is then incremented and the process repeated till $n = N$. This is the build-up phase.

The solution to the problem is given by Eqn (4.3-8). The optimal policy will be obtained in the traceback phase by the recursive application of Eqn (4.3-9).

2. In practice, as so often is the case with dynamic programming formulations, problems in implementation of the method of solution can arise due to computer core or CPU time limitations.

The optimal values which define the functions F_n and G_n have to be computed for every feasible state vector at year n , and this can take on formidable proportions as even with a small system the number of possible states can be considerable. Many of these can be rejected as strictly infeasible (Eqn (4.3-3a)) or undesirable (eg excessive overplanting) but a large number may still remain for any practical problem. This can lead to prohibitively large core storage requirements and excessively long computation times.

For this reason careful definition of the state and decision vectors is necessary. Too much detail will increase the number of feasible states and hence the computational burden, whilst insufficient detail will degrade accuracy.

3. The constraints of Eqns (4.3-3) have been presented in generalized terms. This is virtue of the dynamic programming approach - no specific form of constraint is required. A variety of constraints can be accommodated, these need neither to be linear nor continuous. In fact since constraints will tend to reduce the number of feasible states and decisions, the solution of a dynamic program is often easier the greater the

number of constraints, as long as they can be coded appropriately for the computer to handle efficiently.

4. For this dynamic programming formulation the estimate of system operating costs is not integral with the minimization expression at each stage. There is thus a need for a separate loading simulation model* for determination of $R_n(\underline{s}_n)$. A separate nonlinear model can be constructed for this problem to give more accurate estimates of production costs. This is of course an optimization problem in itself. It should be noted however that $R_n(\underline{s}_n)$ will have to be evaluated for every feasible state at each year and hence in the interests of efficient solution it cannot be made too complex or too time-consuming.

5. A backward solution to this problem is admissible since the transition function (Eqn (4.3-2)) is linear and easily inverted with slight redefinition of the decision vector (see next Section). The advantage of a forward programming approach however is that the final state function $F_N(\underline{s})$ gives an indication of the sensitivity of the objective function to different final state vectors. Thus the K-best suboptimal solutions can be obtained for K different final system states. It is not generally possible to obtain the suboptimal sequences of decisions for the same final state without an increase in storage capacity. Arguably this latter information would be more useful.

5. The effect of truncation at the end of the planning period can also be corrected with the use of forward programming. Booth /7/ describes how this may be done by approximating annual costs beyond the horizon year to a geometric progression and summing this series to infinity. The usefulness of this refinement is perhaps doubtful since in any real-life application the focus of attention is weighted to a period well before the planning horizon.

* So called because they simulate the loading of the system in response to projected demand curves in order to arrive at an estimate of system operating costs.

7. Constraints on the dates of installation of particular plants at particular sites are easily handled by appropriate inclusion into the constraints of (4.3-3b). Finite lifetimes can be catered for with the forward programming model. However installations which are dependent upon previous decisions may not be so readily accommodated. The Markovian restriction on the applicability of dynamic programming may prevent this, if the prior decision upon which future decisions are dependent cannot be directly related to the state vectors at every stage n . To do this could require a high-dimensional state vector which in turn imposes a great computational burden.

Dynamic programming is potentially a powerful tool for the determination of the installation framework. The limitations due to computer size or time are very real, however. Details of a program outlined by Dale /17/ show that only a maximum of 150 states a year could be accommodated. Whilst this may appear to be a large number, note that a system which may contain up to 10 different single units can be constructed in $2^{10} = 1024$ possible ways if no constraints are operative. Real-life systems are considerably more complex than this. The WASP program had the number of feasible states curtailed because of core restrictions /21/ and in such cases a true global solution may not be identified. In principle a more accurate simulation of the loading of the system may be used to estimate production costs but in practice this cannot be made too comprehensive without increasing computing time to a prohibitive level.

A further significant drawback is the inability to handle conditional constraints which do not affect consecutive years, owing to the non-Markovian nature of such constraints. For the same reason constraints on the cumulative rate of capital expenditures cannot be handled without an increase in dimensionality of the state vector.

4.4 A TWO-PHASE SOLUTION

The approach of dynamic programming holds promise for the solution of the installation framework problem. Discrete unit sizes can be handled easily and the algorithm mirrors closely the actual planning process of selection of candidates for installation from a finite set of discrete values at various points along the temporal axis up to the planning horizon. In principle it is possible to simulate the loading of a system and hence estimate production costs more accurately than with linearized models.

Limitations in application arise not from the principles involved but from the method used in solution - the standard computational algorithm. This necessitates the evaluation of a great many feasible states and leads to computational restrictions in practice. Owing to these it may not be possible to examine the full feasible range of possibilities even with sophisticated methods of data compaction in state description. Attempts to handle conditional and cumulative constraints are thwarted by the necessary increase in dimensionality in the state vectors.

In this section an original alternative two-phase solution is proposed, which attempts to overcome the shortcomings of the forward dynamic programming approach. This uses simple continuous-function dynamic programming in the first phase of solution. In recognition of the potential size of the problem, the emphasis in this phase is not the attainment of great accuracy but merely the determination of valid estimates of optimal costs of development and optimal policies of expansion over the whole range of feasible states (this guarantees a global optimum). The state and policy functions generated in this phase contain much useful information. They are employed in the second phase of solution where a logically structured search is carried out for an optimal policy, guided by the estimates generated during the build-up phase. This traceback phase treats the problem as a discrete-valued one, and it is

here that close attention is paid to accurate determination of both capital and production costs. The full range of constraints including those which are non-Markovian, can be dealt with during traceback fairly readily. The guidelines of optimal development contained in the state and policy functions of the continuous-function dynamic program also have a further function in this method. They can be used to implicitly evaluate and perhaps reject a large number of feasible development strategies at an early stage of the search through the use of simple rules akin to those of the branch-and-bound and implicit enumeration algorithms of mathematical programming /26/.

NOMENCLATURE

- C capital costs of implementing decisions.
 $C_n(\underline{d}_n)$ or $C_n(U_n)$ = the cost of implementing decisions $\underline{d}_n$ or U_n at year n .
- $\underline{d}_n$ decision vector, to take effect from the beginning of year $(n+1)$. $\underline{d}_n \subseteq U_n$
- F state function for dynamic program, build-up phase.
 $F_n(\underline{s}_n)$ = cost of system expansion in an optimal manner from state $\underline{s}_n$ at year n to the horizon year N .
- $\underline{G}$ policy function for dynamic program, build-up phase.
 $\underline{G}_n(\underline{s}_n)$ = decision vector to be applied at year n if the system is in state $\underline{s}_n$, in order to achieve optimal expansion from year n to horizon year N .
- N number of time intervals (years) in the planning period.
- R function for system operating costs.
 $R_n(\underline{s}_n)$ or $R_n(X_n)$ = cost of operating the system described by $\underline{s}_n$ or X_n , in year n .
- $\underline{s}_n$ state vector for system in year n . $\underline{s}_n \subseteq X_n$
- U_n decisions implemented at year n (plant installations)
- X_n state of system in year n .
- Z_n cumulative cost associated with state X_n . Defined as the total monetary outlay (capital costs of decisions + running costs of system) from year 1, through the path by which state X_n was attained, to year n .

Functions C_n and R_n are present-worthed and adjusted for inflationary or deflationary effects accordingly at year n .

$\underline{s}_n$ and $\underline{d}_n$ are simplified state and decision vectors. In this formulation the elements of these vectors represent the installed capacities of each plant type. Dimensionality is therefore equal to the number of plant types. X_n and U_n on the other hand are the corresponding complete state and decision descriptions.

CONTINUOUS-FUNCTION DYNAMIC PROGRAM

The following formulation for a dynamic program differs from that of Section 4.3 in two important respects:

- (a) It uses the backward programming algorithm. The reason for this will become apparent later.
- (b) Instead of dealing with generalised state and decision vectors, simple low-dimensional vectors are used.

The basis of this approach follows that of Petersen /48/. It is a solution to the plant mix problem:

$$\text{minimize } \sum_{n=1}^N \left\{ C_n(\underline{d}_n) + R_n(\underline{s}_n) \right\} \quad \dots(4.4-1)$$

with the transition function:

$$\underline{s}_{n+1} = \underline{s}_n + \underline{d}_n \quad \text{for } n = 1, 2, \dots, N \quad \dots(4.4-2)$$

and the constraints:

$$\underline{s}_n \in S_n \quad \dots(4.4-3a)$$

$$\text{and } \underline{d}_n \in D_n(\underline{s}_n) \quad \dots(4.4-3b)$$

Solution is by the backwards recursion formula:

$$F_n(\underline{s}_n) = \min_{\underline{d}_n} \left\{ C_n(\underline{d}_n) + F_{n+1}(\underline{s}_n + \underline{d}_n) \right\} + R_n(\underline{s}_n) \quad \text{for } n = N, N-1, \dots, 1 \quad \dots(4.4-4)$$

$$\text{initial condition: } F_{N+1}(\underline{s}_n) = 0, \text{ for all } \underline{s}_n \quad \dots(4.4-5)$$

Rewriting Eqn (4.4-4) as:

$$F_n(\underline{s}_n) = C_n(\underline{d}_n^*) + F_{n+1}(\underline{s}_n + \underline{d}_n^*) + R_n(\underline{s}_n) \quad \text{for } n = N, N-1, \dots, 1 \quad \dots(4.4-6)$$

Then the policy function for stage n is defined as:

$$\underline{G}_n(\underline{s}_n) = \underline{d}_n^* \quad \text{for } n = N, N-1, \dots, 1 \quad \dots(4.4-7)$$

1. The existence of discrete unit sizes and all implications arising from this has been simply ignored in this dynamic program. The functions $R_n(\underline{s}_n)$ and $C_n(\underline{d}_n)$, and hence the state and policy functions, are now assumed to be continuous with respect to states and decisions. Solution is by construction of a grid in the multidimensional state space $\underline{s}_n$ and evaluation of the values of the state and policy functions at each point in this grid by application of Eqns (4.4-4) and (4.4-7). The computational burden for this phase of the problem will be firstly dependent on the dimensionality of the state and decision vectors, and secondly on the number of points in the grid. A coarse grid lessens the computational burden at the expense of accuracy.

2. Despite the simplified state and decision descriptions care will nevertheless have to be taken in view of this computational requirement. If each component of the state vector is quantized to M levels, the number of times Eqn (4.4-4) will have to be solved over a N -year period for a 4-dimensional state vector will be NM^4 (eg if $M = 10$, $N = 20$, $NM^4 = 2 \times 10^5$) - a formidable prospect. The simulation model for evaluation of the production costs $R_n(\underline{s}_n)$ cannot therefore be too elaborate nor too time-consuming.

3. Petersen /48/ outlines methods by which computational requirements can be reduced significantly. The first arises from the fact that the capital cost function C_n is additively separable with respect to the components of the decision vector, hence the 4-dimensional minimization can be reduced to a sequence of four uni-dimensional minimizations. The second will be valid if a particular linear form of the function C_n is acceptable. Using these methods Petersen achieves, with 20 grid divisions for each element of the 4-dimensional state vector, and N (probably) = 20 years, a computation time reduced "from about 50 days to the order of 10 minutes" (UNIVAC 1108 computer). The reader is referred to the original cited paper for details of how these economies are achieved.

4. The use of backward recursion means that many constraints will have to be ignored. Those which can be handled in this phase are:

- minimum total installed capacity at each year,
- upper and lower bounds on installed capacity for each plant type at each year (imposed by the background plan*),
- upper limits on annual capital expenditures or operating costs, or both, but not cumulative constraints,
- minimum or maximum plant capacities that can be installed in any one year,
- escalation factors for capital and production costs, perhaps differentiated according to plant type.

Those constraints which are ignored are by no means insignificant. They include the existence of discrete unit sizes, installation dates, individual site requirements and all constraints which may arise from siting considerations, those selections for installation which are conditional upon prior choices, and cumulative capital expenditure constraints. Moreover treatment of finite individual plant lifetimes, retirements, and correction for truncation effects at the planning horizon are also neglected.

5. This formulation, for a simplified continuous-function dynamic program, can be regarded as a relaxation of the complete installation framework problem. Now for the general case of two related mathematical programs:

Problem A: $\min f(\underline{x})$, subject to $\underline{x} \in S_A$

Problem B: $\min f(\underline{x})$, subject to $\underline{x} \in S_B$

Then if $S_A \subseteq S_B$, problem B is said to be a relaxation of problem A (conversely, problem A is a restriction of problem B), and if $\underline{x}^A$ and $\underline{x}^B$ are the solutions to problems A and B respectively, then:

$$f(\underline{x}^A) \geq f(\underline{x}^B)$$

This important result is of great significance for the solution

* the dynamic program itself may be used to determine the background plan, although it is a singularly inefficient way of doing so.

of many integer or discrete-value mathematical programs /26/.

In the context of the installation framework problem, the following useful conclusion may be drawn:

- That the solutions to the N subproblems imbedded in the simplified continuous-function dynamic program are always at most an underestimate of the true solution of the installation framework problem, since the constraint set for this latter problem is a subset of the constraint set for the former.

The next phase of solution will utilize this conclusion and the property of invariant imbedding to guide a logically-directed serial search during the traceback phase.

THE DECISION TREE

Decisions applied over the planning period can be regarded as branches in a decision tree, connecting the states of the system (nodes in the tree) at various years in the planning period (levels in the tree) - Fig 4.4-1. An expansion strategy or policy can therefore be regarded as a path in the tree from level 1 (initial year) to level N (horizon year). It is clear that even for a modest-sized system with a restricted set of feasible decisions the total number of nodes and branches can be very large indeed.

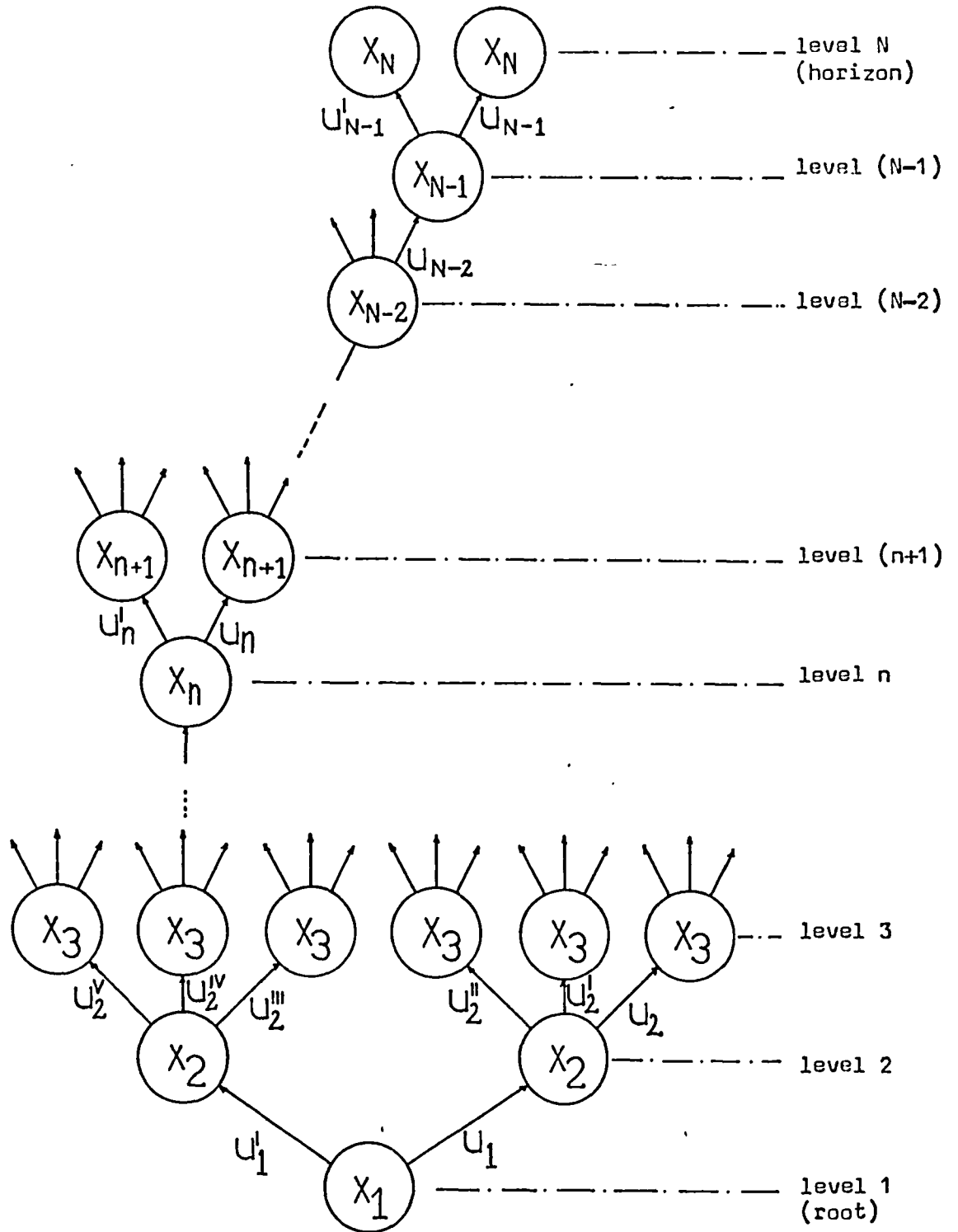
1. The objective will be to conduct a search along the decision tree whilst at the same time terminating the search along unpromising paths as soon as possible. Information from the state and policy functions derived previously will be used to do this.

2. Let the state of the system in any year be denoted by X_n , X'_n , X''_n , etc, and the decisions (by definition active from year $n+1$ onwards) by U_n , U'_n , U''_n , etc, as in Fig 4.4-1. In this special form of directed graph the branches only interconnect nodes between consecutive levels, and there is no interconnection between nodes at the same level. Each node will have in general several successor nodes, but always only one predecessor node. A node is therefore uniquely defined by the path leading to it from the root node (at level 1). Nodes at the same level can represent identical system states but they are considered to be unique since they were arrived at through different paths.

3. Associated with each node is a cost $R_n(X_n)$ - the cost of operating the system X_n in year n . Similarly, associated with each branch is a cost $C_n(U_n)$ - the cost of implementing the decision U_n in year n . The cumulative cost of a node is defined:

$$\begin{aligned} Z_n(X_n) &= \sum_{i=1}^n R_i(X_i) + C_i(U_i) \\ &= Z_{n-1}(X_{n-1}) + R_{n-1}(X_{n-1}) + C_{n-1}(U_{n-1}) \end{aligned}$$

FIGURE 4.4-1 DECISION TREE



$Z_n(X_n)$ is therefore the total cost of arriving at state X_n at year n , via the decision set U_i , $i = 1, 2, \dots, n-1$. At the initial root node therefore:

$$Z_1(X_1) = 0$$

Furthermore, from the definition, $Z_{N+1}(X_{N+1})$ is the objective function, to be minimized.

4. Since decisions involve the introduction of new plant into the system (or at the least no additions at all), each decision can never involve negative monetary outlays. In addition, system operating costs R_n are always positive. Therefore:

$$Z_n(X_n) > Z_{n-1}(X_{n-1})$$

Moving from a lower to a higher level in the tree (away from the root, forwards in time) can never reduce the value of the cumulative cost Z_n along the current path. From this we can put forward two important rules to guide the search:

Suppose that a feasible path to the horizon has been established, with a cumulative cost at the horizon $Z_{N+1}(X_{N+1}) = Z_{\min}$. Call the decision set which defines this path $U_{\min}$.

- (a) If the cumulative cost $Z_n(X_n)$ at any node exceeds $Z_{\min}$, where $n \leq N$, then no further emanations from this node can improve on the current best path $U_{\min}$, since all subsequent branches and nodes cannot involve negative monetary outlays.
- (b) If $\underline{s}_n$ is the state vector representing X_n , then $F_n(\underline{s}_n)$, by definition, is the least cost of expanding the system from this state at this point in time to the planning horizon. There is no cheaper way. Thus if:

$$Z_n(X_n) + F_n(\underline{s}_n) \geq Z_{\min} \quad \text{where } \underline{s}_n \subseteq X_n$$

then any subsequent path through node X_n to the horizon cannot improve on the current best path $U_{\min}$.

A lower bound has therefore been established for node X_n .

If this exceeds $Z_{\min}$, then all emanations from this node have been implicitly evaluated and discarded as unfavourable paths for expansion.

5. The solution to the continuous-function dynamic program can also provide a useful indication for the selection of branches at each node, through use of the policy functions. Consider the case of a node X_n , represented by the state vector $\underline{s}_n \subseteq X_n$. The recommended decision vector to be applied for this year for optimal expansion is given by:

$$\underline{d}_n = \underline{G}_n(\underline{s}_n)$$

This use of the policy function can therefore direct branching to those choices at each year which are most likely to be favourable.

Owing to the neglect of many constraints in the simplified dynamic program it should however be noted that neither F_n nor $\underline{G}_n$ can provide accurate estimates, and they are only to be used as guides for decisions or for the establishing of lower bounds and implicit evaluation of vertices. The evaluation of capital and operating costs and the handling of all constraints will be carried out in chronological sequence during the branching process and search in the decision tree.

6. This second phase of solution as outlined above may best be regarded as a development of the straightforward traceback process of the dynamic programming standard computational algorithm.

THE TRACEBACK ALGORITHM

This algorithm summarises the discussion of the previous subsection.

INITIALIZE: set $n = 1$, $Z_{\min} = \infty$

STEP 1: Determine the state vector $\underline{s}_n$ corresponding to state X_n . Calculate $R_n(X_n)$.

- (a) Invoke state function to determine the optimal cost of expanding from this state at this year to horizon.
(BOUNDING) If $Z_n(X_n) + F_n(\underline{s}_n) \geq Z_{\min}$, go to STEP 3.
- (b) Invoke policy function to determine the recommended decision for implementation this year, $\underline{d}_n = G_n(\underline{s}_n)$.
- (c) Identify all feasible decisions U_n from this node.
Go to STEP 2.

STEP 2: Select an unexplored decision closest to $\underline{d}_n$. If all branches have been explored, go to STEP 3. Designate the selected branch as explored, and calculate $C_n(U_n)$, where U_n is the selected branch.

(BRANCHING) Set $n = n + 1$

Calculate $X_n = X_{n-1} + U_{n-1}$

and $Z_n(X_n) = Z_{n-1}(X_{n-1}) + R_{n-1}(X_{n-1}) + C_{n-1}(U_{n-1})$

(BOUNDING) If $Z_n(X_n) \geq Z_{\min}$, go to STEP 3.

If $n = N+1$, go to STEP 4.

Otherwise, go to STEP 1.

STEP 3: (BACKTRACKING) Set $n = n - 1$

If $n = 0$ then the search is completed and all branches have explicitly or implicitly been explored. TERMINATE.
Otherwise go to STEP 2.

STEP 4: The current path provides a better value of the objective function than $Z_{\min}$.

set $Z_{\min} = Z_{N+1}(X_{N+1})$ and $\underline{U}_{\min} = U_i, i = 1, 2, \dots, N$

Go to STEP 2.

NOTES

1. The search procedure is flowcharted in Fig 4.4-2. It normally follows the chronological sequence of decision-making, but backtracks when the horizon is reached or when it is found that future branching from the current node will be unprofitable.

The non-Markovian restrictions which limit the direct application of dynamic programming in Section 4.3 can be readily accommodated during the traceback phase of this method as outlined. Cumulative constraints on capital expenditures can be checked by reference to previous decisions and states along the current path, and this may render certain decisions at the current year infeasible. Dates of installation can be dealt with when deciding upon the feasible branches at any node. Conditional installations may best be taken into account by selecting possible decisions from a pool of available plant at each level in the decision tree; this pool of candidates will undergo transformations from year to year according to decisions made along the current path.

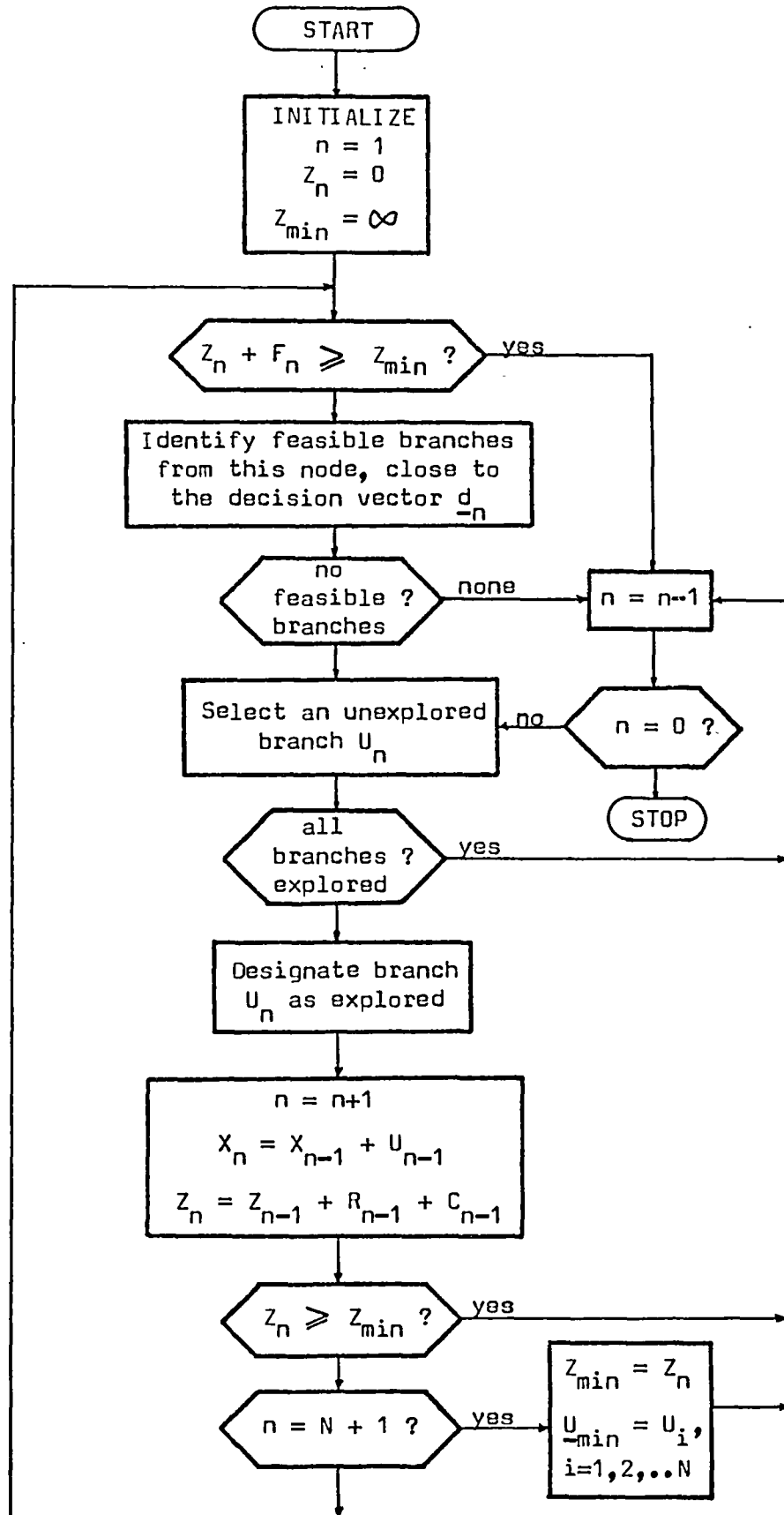
2. Computational requirements are fairly modest. At each stage during the process only a single current path is under consideration and for each level in the tree only the following information needs to be stored:

- state X_n , decisions U_n , and the costs R_n and C_n associated with these,
- cumulative cost of the node, Z_n ,
- pool of candidates for selection at year n ,
- functions F_n and G_n .

In addition to the above, the current best path, $U_{\min}$, and its cost at the horizon $Z_{\min}$ has to be available.

It will also be possible with little increase in storage requirements to detail the K -best optimal paths. For branching and bounding, the worst (K^{th}) of these will be used for comparisons with current values. Note that in contrast to forward dynamic programming these are the K -best paths and not the K -best final states.

FIGURE 4.4-2 FLOWCHART FOR TRACEBACK SEARCH



3. Initially, with $Z_{\min} = \infty$, the search locates a feasible path to the horizon (a feasible expansion strategy). This may not necessarily be optimal, although it is likely to be close to the optimum due to the guidance when branching from the policy functions.

Upon reaching the horizon, the search then backtracks a year at a time, moving again towards the horizon along branches which may improve upon the objective function, and backtracking when branches cease to be promising. With efficient bounding the unfavourable moves can be isolated quite early on in the search. Cumulative errors in the build-up phase are likely to be of lesser magnitude close to the horizon (because this phase moves backwards in time) and this is most useful since it is in this region where there are the greatest number of branches and bounding needs to be efficient.

The search terminates when the root node of the tree has been reached, and when all branches from this node have been implicitly or explicitly explored. The solution is $Z_{\min}$, and the optimal expansion strategy $U_{\min}$.

4. States X_n and decisions U_n are now complete state and decision descriptions, no longer simplified state and decision vectors. In this phase there is no dimensionality restriction as with the dynamic program of Section 4.3.

5. The total number of states for which $R_n(X_n)$ has to be evaluated will be much less during the traceback search than during the build-up phase of the forward dynamic program of Section 4.3. The loading simulation model can therefore include somewhat more detail than before.

6. Computational requirements for the build-up phase are much less extensive than before since it will now be appreciated that accuracy in this phase is not the prime objective. It may be noted that the models for evaluation of $R_n(\underline{s}_n)$ and $R_n(X_n)$ need not be the same, although their results must be consistent for proper bounding. This should be the case since essentially

the two models solve the same problem.

For a 4-dimensional state vector, if the number of quantized levels for each component of the state vector is halved, core storage and CPU time requirements are reduced by a factor of $2^4 = 16$; this indicates the possible savings in computational effort.

7. Moreover the build-up phase need only be carried out once in many real-life planning applications. The state and policy functions so assembled will contain all the necessary information for branching and bounding during traceback for all problems relating to the same system for which all constraints handled during the first phase are unchanged (eg TDR, plant mix and installed capacity constraints, load curves and escalation factors).

With this data traceback searches may be conducted with variations in the additional constraints which were ignored during build-up. In fact these turn out to be those which are often varied for marginal analysis-type runs: data for candidates for selection (number available, type, dates of availability), site considerations (minimum or maximum planting limits, sequencing of installations), retirements of existing plant, and constraints on capital expenditures.

At the extreme, when all additions to the system throughout the planning period are committed with respect to dates of installation and number available, and when all retirement dates of existing plant are fixed, the traceback search degenerates to a simulation model of power system development.

CHAPTER FIVE. LOADING SIMULATION MODELS

This chapter describes solutions to the problem of the simulation of the operation of an electric power system, treated as a point model of an interconnected group of generating units, supplying a point load of unity power factor. Many successful models have been developed to this end, and in contemporary power systems often provide indispensable tools for economic plant scheduling in large complex networks.

The models to be described here have a different purpose. They are designed for incorporation into a computer-based power system planning or development package. In this context a model is required to supply an assessment of the likely minimum cost of operation of a power system within constraints as imposed by the demand and its variation with time, individual plant operating restrictions, availability of hydro energy, and so on. Used as a subroutine within a computer package the model would be accessed a great many times during a single planning study. Hence the prime requirements: accuracy coupled with speed of execution and a modest central memory core requirement. Unlike a model for plant scheduling, details of the actual individual operating conditions for each plant or unit in the system are not normally of interest, it is sufficient to be sure that each plant or unit is always operated within prescribed limits.

Whilst the primary function of such loading simulation models* is an estimate of system operating costs, they are often also used to check for feasibility of operation of a particular configuration of units. Although the total installed capacity of a system may exceed expected maximum demand, operation may in fact be infeasible when the need for provision of spinning reserve and plant outage rates are taken into account. The simulation model should identify such infeasible cases.

The load to be supplied by the system can be presented in the form of a chronological load curve (CLC) which defines the variation with time of system demand levels over a particular

* often simply called simulation models in the literature.

period - the simulation period (usually 24 hours, hence a daily load curve). Alternatively a load duration curve may be used, this is a transformation of the CLC which provides a powerful tool for the analysis and modelling of system operation. Coordinates on the LDC relate the demand level to the total number of hours (the duration) for which this demand level is equalled or exceeded over the simulation period.

To date the most widely employed simulation models for planning applications have been those which use the LDC, and their variants. In their simpler forms these models are straightforward. Their basis derives from the concept of merit-order loading: if plant is loaded onto the LDC in order of decreasing efficiency, yielding a plant duration curve which matches the LDC, the resultant cost of operation is the minimum cost of operation to satisfy the given load conditions. Section 5.1 outlines such a model suitable for cases where the system is described in terms of the plant mix. A more sophisticated version of these LDC-based models is described in Section 5.2, this is a probabilistic model which attempts to include the effects of individual plant outage rates. For several reasons LDC-based models may not provide a sufficiently accurate simulation of system operation; these reasons are discussed in Section 5.3.

Apart from models utilizing the LDC, formal mathematical programming procedures have been applied to the problem of the simulation of system operation following the CLC, these include the Pontryagin Maximum Principle /28/ and Dynamic Programming /24/. Such models do not usually suffer from the main drawbacks of LDC models (which arise from the loss of chronological data) since the CLC is used for load description. However because of computer time or core limitations their use has hitherto been restricted to programs for plant scheduling in real time or short-term horizon operation, rather than for planning purposes.

The remainder of the chapter outlines the theory for a computationally efficient procedure, based exclusively on dynamic programming and taking advantage of properties inherent in Bellman's Principle of Optimality and the standard computational algorithm (Appendix A), the structure and nature of the problem,

together with a few fairly realistic assumptions. The basis of this original method is as follows:

- The system is divided into two subsystems, the hydro and nonhydro (thermal) subsystems. Dynamic programming is used to solve what is essentially an allocation problem for each subsystem. The information acquired from these computations fully describes the performance of the two subsystems with respect to power output, spinning reserve, and operating cost.
- The system performance functions so obtained are used in the solution of a further dynamic program which traces the optimal trajectory of reservoir levels for minimal cost of operation, within constraints imposed by chronological demand and spinning reserve requirements, and safe limits on reservoir levels.

Computational problems associated with dynamic programs arise with multidimensional state and decision vectors. For all the dynamic programs above the dimensionality is unity.

Section 5.4 lists the notation to be used in subsequent sections. The dynamic program for the optimal reservoir trajectory is outlined in Section 5.5. In Section 5.6 the basic dynamic programs for the system performance functions are described. The efficiency of these dynamic programs can be greatly enhanced by utilizing the concepts of operating lines and operating values - Section 5.7, and the improved formulations which result for the hydro and thermal subsystems are presented in Section 5.8 and Section 5.9 respectively. Finally a brief appraisal of the complete simulation model is given in Section 5.10.

5.1 A SIMPLE PLANT MIX SIMULATION MODEL

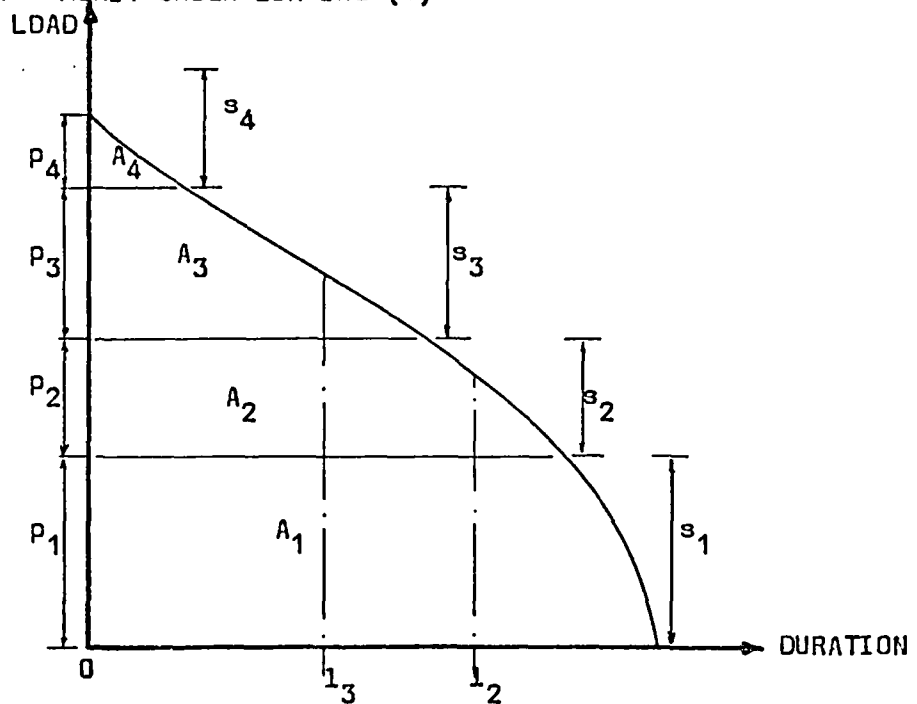
Here the problem of estimation of system operating costs when the plant mix of the system is specified is dealt with. The input to the model will be an annual load duration curve and a state vector $\underline{s}$, dimensionality equal to the number of plant types in the system, each element of which is equal to the installed capacity for that plant type.

Let there be four plant types: s_1 = hydro, s_2 = nuclear, s_3 = thermal, and s_4 = peaking. The types have been ordered in increasing operating cost order. Let c_i be the operating cost per unit energy output of plant type i . It is assumed that this value is the average for all plants of each particular plant type, and that individual variations do not transcend the plant mix categories. Then following merit order loading, plant is loaded into the LDC as shown in Fig 5.5-1, successively up to the limit of installed capacity, until peak demand is met. The minimum cost of operation is then:

$$\sum_{i=1}^4 c_i A_i$$

where A_i is the area enclosed by plant type i in the LDC as in Fig 5.1-1.

FIGURE 5.1-1 MERIT ORDER LOADING (1)



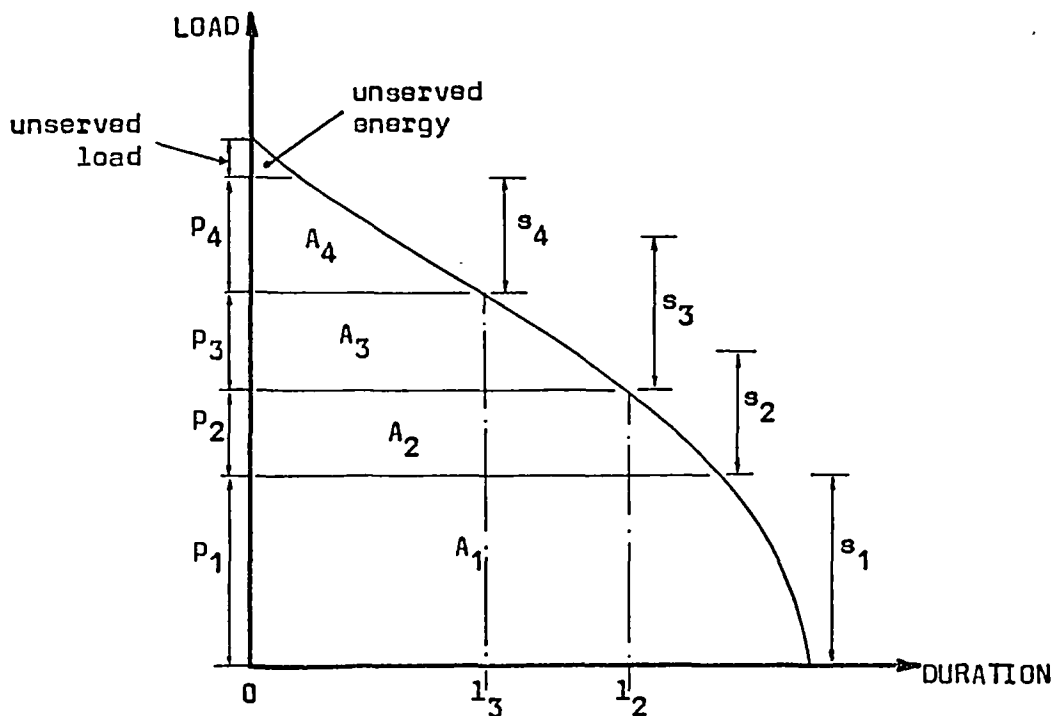
There are several objections to this simple form. Firstly, certain plant types are inherently unsuitable to operation at low load factors and hence a minimum annual load factor may be applied as a constraint. Thus for example minimum load factors l_2 and l_3 applied to nuclear and thermal plant respectively may mean that these plant are not loaded to their capacity limit. Hence in Fig 5.1-2, $p_2 \leq s_2$ and/or $p_3 \leq s_3$, according to whether the load factor constraints are operative. From Fig 5.1-2 it will be seen that for this example system operation now becomes infeasible owing to insufficient peaking capacity to meet peak loads.

Secondly, energy limited hydro plant has not been dealt with realistically; since owing to this restriction it may not be possible to use all the hydro complement to satisfy base loads even though hydro plant has the lowest operating costs. Thus a proportion of hydro plant may be used to displace that of the peaking complement - Fig 5.1-3. The constraints operative will be:

$$A_1 + A_5 \leq E_H \quad \text{and} \quad p_1 + p_5 \leq s_1$$

where E_H is the limited energy available from hydro plant.

FIGURE 5.1-2 MERIT ORDER LOADING (2)



Then for the merit order loading, first peaking hydro plant is loaded until:

$$P_5 > s_1 \quad \text{or} \quad A_5 > E_p$$

then base hydro plant is loaded until:

$$P_1 > s_1 - P_5 \quad \text{or} \quad A_1 > E_B + E_p - A_5$$

The remainder of the LOC is then filled by loading with nuclear, thermal, and finally peaking plant in that order (merit order).

A third possible objection is the neglect of spinning reserve. In practice some of the plant will have to be loaded below rated capacity (CMR). In the absence of further information it will be impossible to decide how spinning reserve is to be distributed amongst the various plant types. An approximation can be made by reducing the CMR of each plant type by a (possibly weighted) factor according to the amount of spinning reserve desired.

The general ideas of this simple model can be extended to the subdivision of plant type categories into individual stations or even units, which are then loaded in merit order with due attention paid to the need to attain minimum annual load factors, hydro energy limitations, and the provision of spinning reserve.

5.2 PROBABILISTIC SIMULATION

Probabilistic simulation models represent an attempt to incorporate some of the uncertainties inherent in power system operation into a formal framework. The use of such models has gained widespread acceptance and they are used in the dynamic programming formulations of Booth, and the WASP package (Section 4.3).

It is recognized that accurate matching of the load duration curve and the plant duration curve does not occur, particularly over fairly long (monthly or annual) simulation periods. The reason for this is the finite outage rate of individual units, which means that usually no single unit can be fully utilized over the simulation period.

It is convenient in probabilistic simulation work to interchange the X and Y axes of the conventional LDC, as in Fig 5.2-1. If the Y-axis (duration) is now normalized, then the LDC, $F_0(x)$, represents the proportion of the simulation period H for which demand equals or exceeds the associated demand level x. It may also therefore be regarded as the probability that this demand level is exceeded or equalled in the simulation period. This concept is basic to the approach of probabilistic simulation.

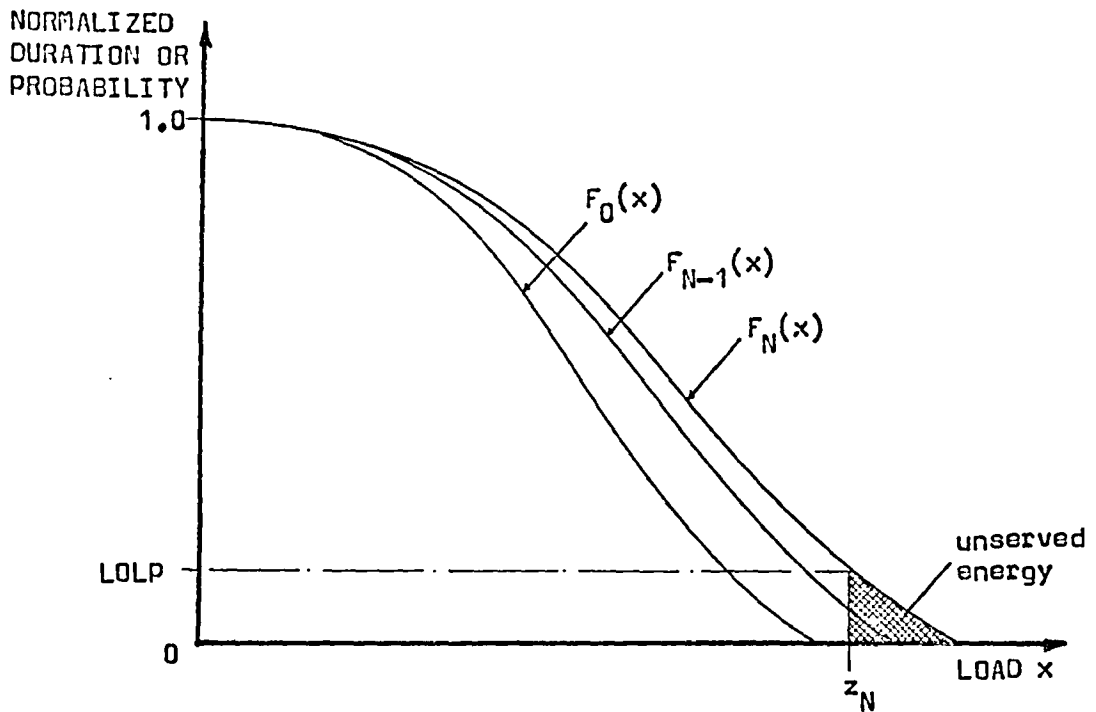
Let the system comprise N units, each of capacity x_n and operating cost per unit energy c_n . Let the cumulative capacity of n plants be z_n , ie $z = \sum_{j=1}^n x_j$. Furthermore, let the availability of plant n be p_n . For the simplest case, where stochastism in demand data is ignored, and where a plant is regarded to be either available or not, then the probability of its being available is p_n , and the probability of its being unavailable is $(1-p_n)$. The basic probability of load distribution will be conditioned by the accumulated outage rates of all units in the system. Suppose that the LDC $F_0(x)$ is defined by a grid in the (X,Y) plane, with grid size in the X-direction dx.

Then successive convolved probability distributions $F_n(x)$ each for a system of n units can be obtained recursively from:

$$F_n(x) = p_n F_{n-1}(x) + (1 - p_n) F_{n-1}(x - dx) \quad \text{for all } n \in N$$

$$\text{where } F(x) = 0 \text{ if } x \leq 0.$$

FIGURE 5.2-1 LOSS-OF-LOAD AND LOSS-OF-ENERGY PROBABILITIES



The final convolved distribution $F_N(x)$ represents the probability distribution of load for the whole system of N units. By definition, the installed capacity of the system is z_N , and the value of $F_N(z_N)$ is the probability that this system installed capacity is equalled or exceeded - the loss-of-load probability (LOLP).

Furthermore the area:

$$H \int_{z_N}^{\infty} F_N(x) dx$$

is the expected value of the energy that the system cannot supply.

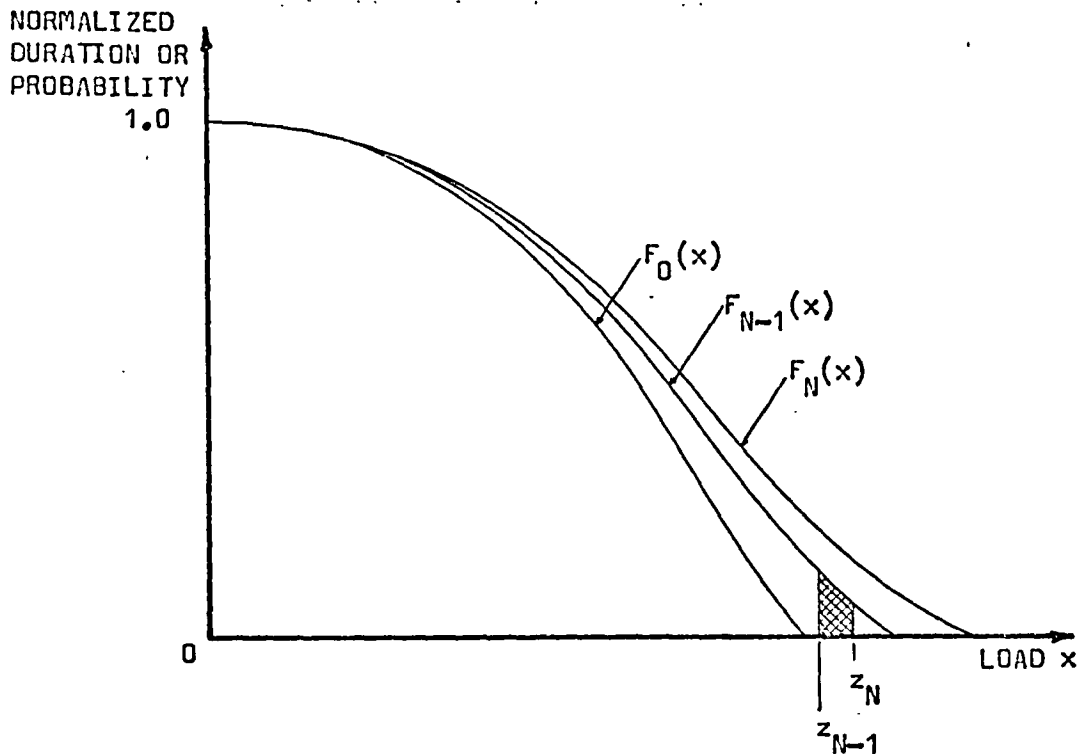
Either of these two quantities provides a useful measure of the inability of the system to meet a required load distribution (or load duration curve), which proves very useful in the context of system planning, where quick evaluation of different plant configurations with respect to system security may be required.

Suppose now that the least efficient plant is removed from the system. The probability distribution for the remaining $(N-1)$ plants is now given by the inverted form of the original recursive formula:

$$p_N F_{N-1}(x) = F_N(x) - (1-p_N) F_{N-1}(x-dx)$$

$$\text{where } F_{N-1}(x) = 0 \quad \text{if } x \leq 0$$

FIGURE 5.2-2 EVALUATION OF SYSTEM OPERATION COST



and the expected cost of generation of energy for this unit is:

$$c_N p_N H \int_{z_{N-1}}^{z_N} F_{N-1}(x) dx$$

In order to estimate system operating costs, therefore, plant are successively removed from the system in reverse merit order, the new probability distributions for the reduced system computed, and the individual operating costs evaluated and summed, one plant at a time. When all plants have been removed, $F_{N-N} = F_0$ is the original load duration curve, and the total operating cost over the simulation period will have been determined (Fig 5.2-2).

Note that the build-up of the convolved distributions is independent of the order in which plant are considered. It would appear a more efficient and convenient procedure to develop the convolved distributions in merit order, and to evaluate operating costs at the same time. However by starting with F_N and working in reverse merit order there are some important advantages:

- By removing from the system any unit, the performance of the system (LOLP, etc) without that unit may be evaluated. Thus the impact of major plant outages (eg for maintenance) may be obtained

for a variety of conditions.

- Treatment of hydro energy limitations may be attained using a peak-shaving method. If storage hydro plant is removed from the system of N units, the distribution F_{N-1} shows the expected energy generation from this hydro plant to be:

$$E_N = p_N H \int_{z_{N-1}}^{z_N} F_{N-1}(x) dx$$

Now if this energy E_N exceeds that available from the hydro unit, due to energy shortage, then the distribution F_{N-1} will have to be further modified for the remaining $N-1$ plants in the system before the process is continued. The reader is referred to Booth /8/ for an exposition of the general method and its extensions to accommodate hydro and pumped storage constraints, together with details of applications of the model.

Spinning reserve requirements can be handled as in the previous section by loading all or some plant to less than CMR. Part-load efficiencies of individual plants can be accommodated by multiblock representation of individual plants. Thus a large unit may be represented by two load blocks - a base-load block and a peak-load block. It may be noted that because of the estimation of operating costs in reverse merit order certain constraints such as those imposed by minimum load factors can prove awkward. These will have to be accommodated by further modification of the probability distributions according to whether or not the constraint is violated, in a similar manner as that used for treatment of energy-limited hydro.

5.3 COMMENTS ON LOC-BASED MODELS

Simulation models utilizing the LOC have many advantages. Such models are fairly easily programmed for computer use, are economic in terms of computational effort, and provide reasonably accurate estimates over a wide range of variation of major parameters (system composition, load curves). They do however suffer from several disadvantages if more careful simulation and more accurate estimates of operating costs are required. These shortcomings arise mainly from the loss of chronological data when the transformation from CLC to LOC is made:

- The LOC, being a cumulative integral along the CLC, loses the time sequence of load variation. This can lead to possible inaccuracies where large storage capacity hydro plant is concerned. With a large reservoir there is, in effect, a stock-control or inventory problem - whether to use the hydro energy as and when it becomes available, or to store it for later use to greater advantage. Pumped storage stations bring in a further consideration - the optimum time sequence of reservoir replenishment and depletion as a result of the pumping/generating strategy adopted. Reservoir level trajectories will have to be determined for most efficient system operation since these in turn will affect the operating strategy for the remainder of the system and hence operating costs. Consider two CLCs, one with well defined midday and evening peaks, and the other with these peaks merging to an afternoon-evening plateau. It is not too difficult to conceive of two cases which yield very similar LOCs, yet with differences in the optimal trajectory of reservoir levels which may be significant. Whilst for the first case there may be time for a degree of reservoir replenishment, in the second case this may not be so. Hence overall system operating costs may differ, and this difference cannot be identified with the use of an LOC. Furthermore the situation may arise whereby because of too rapid reservoir depletion hydro plant may not be available when required, and system peaks may not be met at all (although in practice this condition is probably more apparent than real).

Thus with LOC-based models only the net energy transfers

integrated over the simulation period may be modelled; for systems with large reservoir storage capacity, or with pumped storage plant, the actual utilization of the hydro complement may present quite a different picture depending upon the chronological sequence of demand.

- Secondly there is no satisfactory explicit allowance for the provision of spinning reserve. As was mentioned in Chapter 1, spinning reserve requirements vary not only from one system to another according to local practice, but also with both the magnitude and the rate of change of demand. Thus again because of the loss of chronological data it will be impossible for an LDC-based model to pay adequate attention to the spinning reserve requirement.

The distribution of spinning reserve amongst the units in the system itself constitutes an optimization problem, requiring an iterative solution for the models of Sections 5.1 and 5.2. Obviously to provide spinning reserve by loading all units to a fixed fraction of CMR is not an accurate reflection of the actual strategy of system operation. On the other hand the assumption that the provision of such reserve costs nothing is manifestly untrue. Spinning reserve considerations are probably more critical in smaller systems since there is likely to be a wide disparity in unit size and fewer units on line at any one time.

- Thirdly, there are important constraints, often neglected, on the loading of individual units. Large units in particular, whether nuclear or thermal, cannot be safely operated at low loads, and it is also desirable for such units to maintain a steady continuous output. A satisfactory overall load factor for a unit as determined from its position on the LDC does not always guarantee that the unit is not operated intermittently. Such constraints can be particularly important in the mid-merit zone.

The remainder of this chapter presents the theory for a somewhat more complex model, which simulates the loading of a system according to chronological demand and spinning reserve requirements. By doing so it attempts to overcome most of the disadvantages listed above.

5.4 NOMENCLATURE

c_k	operating cost function for thermal unit k
C, C'	recursive cost functions for thermal subsystem
C_{TH}	performance function for thermal subsystem
d, D	demand level, MW
f	hydro inflow rate, MW equivalent, allowing for losses in conversion
G	subset of units in subsystem that are operated in generating mode. $G \subseteq S$
HY	set of units comprising the hydro subsystem $HY \subseteq S$
N	number of units in system or subsystem
D	subset of units in subsystem that are not operated (off). $D \subseteq S$
p	power output of unit or subsystem, MW
P	subset of units in hydro subsystem that are operated in pumping mode. $P \subseteq HY \subseteq S$
q, Q	reservoir or pseudoreservoir level, MWh equivalent, allowing for losses in conversion.
r, R	spinning reserve level, MW
R_{HY}	performance function for the hydro subsystem
S	set of units comprising the system
T	number of time intervals in CLC over simulation period
ΔT_t	time interval between points t and (t+1) on CLC (hours)
TH	set of units comprising the thermal subsystem. $TH \subseteq S$
z	operating value of unit or subsystem, MW
Z, Z'	recursive operating value functions for hydro or thermal subsystem

A unit is defined as the smallest unit of capacity that can be independently operated.

Subscripts HY and TH refer to the hydro and thermal subsystems respectively.

5.5 THE OPTIMAL RESERVOIR TRAJECTORY

In this section the problem of simulation of system operation is cast into a form amenable to solution by dynamic programming. System operation can clearly be thought of as a control problem, where decisions are applied, in the form of output levels of individual units, in such a way as to minimize total system operation costs over the period spanned by the CLC. Constraints which have at all times to be satisfied include the provision of adequate levels of power and spinning reserve, individual plant operating conditions, and permissible safe reservoir levels. The obvious dynamic programming formulation which would result would entail a state vector of dimensionality equal to the number of reservoirs in the system, and decision vector of dimensionality equal to the number of units or groups of units. Obviously for practical cases this dynamic program would be computationally well beyond the capability of all but the largest computers and totally unsuitable for a planning loading simulation model.

To overcome the problem of dimensionality in the decision vector, the system is now regarded as comprising two subsystems which may be operated independently of each other: the hydro subsystem, consisting of those units whose operation directly affects reservoir levels, and the thermal subsystem, which consists of all remaining plant within the system. The following transition function can now be presented, relating the state vector $\underline{q}$ (reservoir levels) to the decisions p_i (hydro output) at time t , to the state vector at time $(t+1)$:

$$q_j(t+1) = q_j(t) + (r_j - \sum_{i \in J} p_i) \Delta T_t$$

where q_j refers to the level of reservoir j , and J is the set of hydro units associated with this reservoir.

In view of the computational penalties associated with a multidimensional state vector (Appendix A) the following assumption is made:

- That for the purpose of simulation all reservoirs in the system may be combined and treated as a single aggregate pseudoreservoir.

By making this assumption some accuracy has at once been lost, insofar as it cannot now be certain that individual reservoir levels remain at all times within their safe operating bounds.

The transition function for the system with a single aggregate pseudoreservoir is now:

$$q(t+1) = q(t) + (f - p_{HY}(t))\Delta T_t$$

This is now a unidimensional problem with q the state variable and p_{HY} (power output of hydro subsystem) the decision variable.

The above equation presumes that the hydro inflow rate to the reservoirs is constant, and this may not be true since inflow rate may vary with time. More significantly perhaps it may vary in real cases according to prior operation of upstream hydro stations when several hydro stations are connected in cascade on a single river. Unfortunately an attempt to handle this form of variation in $f(t)$ will render the system non-Markovian in nature, and thus cannot be handled by dynamic programming. However if the pretence of a pseudoreservoir is maintained the problem does not arise, and in this formulation it will be ignored.

Secondly, a pumped storage cycle efficiency of unity has been assumed; this is not generally justifiable. Now if the bounds on $p_{HY}(t)$ are given by:

$$p_A \leq p_{HY}(t) \leq p_B \quad \text{where } p_A \leq 0 \leq p_B$$

Then if $p_{HY}(t) = p_A$, losses in the pumped storage cycle will mean that the net decrease in reservoir level (ignoring for the moment hydro inflow) will be $e_{PS}p_{HY}(t)$, where e_{PS} is the weighted average pumped storage cycle efficiency. Since $p_{HY}(t)$ is negative, this means reservoir replenishment.

But if $p_{HY}(t) = p_B$, then no hydro plant can be pumping, and the net decrease in reservoir level will be $p_{HY}(t)$.

In between these extremes the net efficiency of the storage cycle cannot be ascertained without a greater knowledge of the

actual operating strategy adopted for the hydro subsystem, but a linear variation may be suggested. Let the net pumped storage cycle efficiency be $G(p)$:

$$G(p_{HY}(t)) = e_{PS} + (1 - e_{PS}) \left(\frac{p_{HY}(t) - p_A}{p_B - p_A} \right)$$

The transition function now becomes:

$$q(t+1) = q(t) + [f(t) - p_{HY}(t)G(p_{HY}(t))] \Delta T_t$$

Constraints are applicable on the reservoir levels, power demand, and spinning reserve. For the moment, consideration of the constraints on individual operating levels for the units in the system will be deferred to a later section. The constraints for the dynamic program here are:

instantaneous demand:

$$p_{TH}(t) + p_{HY}(t) = D(t)$$

instantaneous spinning reserve:

$$r_{TH}(t) + r_{HY}(t) = R(t)$$

permissible safe reservoir bounds:

$$Q_{min} \leq q(t) \leq Q_{max}$$

The objective function which is to be minimized is the cost of operating the system over the simulation period. Assuming that there is no variable cost of operation associated with the hydro subsystem and that there is no interconnexion with neighbouring systems and hence no import or export of energy, then this operating cost is incurred wholly by the thermal subsystem. It is reasonable to suppose that this cost is a function of the power output and spinning reserve that the subsystem is called upon to provide. The objective function is then:

$$\min \sum_{t=1}^T C_{TH}(p_{TH}(t), r_{TH}(t)) \Delta T_t$$

Now for the hydro subsystem, the function $R_{HY}(p_{HY})$ is defined as the spinning reserve available from this subsystem when delivering power output p_{HY} . A moment's reflection will disclose that there may not in fact be a unique value of reserve

for any value of p_{HY} , although as will be seen later there are not an infinite number of values. For the moment however it will be assumed that the function R_{HY} can be adequately defined.

Several simplifications can now be made. From the instantaneous demand and spinning reserve requirements:

$$p_{TH}(t) = D(t) - p_{HY}(t)$$

$$\begin{aligned} \text{and } r_{TH}(t) &= R(t) - r_{HY}(t) \\ &= R(t) - R_{HY}(p_{HY}(t)) \end{aligned}$$

It is a reasonable assumption that a CLC spanning a period which is an integer multiple of a day is cyclic. Hence reservoir levels, demand and reserve levels are equal at the start and end of the simulation period. The cyclic nature of the constraints and the trajectory means that an initial condition for the dynamic program may be set at any instant, not necessarily coincident with the start of the CLC. For example the initial condition may be $q(t') = q_0$ at the time of maximum $D(t)$, or maximum $D(t) + R(t)$; time instant t' becomes stage 1 in the dynamic program.

The simplified dynamic program is now recapitulated here for convenience. Note that in this case the transition function is not easily inverted, and hence only backward programming as applicable for solution.

$$\text{minimize } \sum_{t=1}^T C_{TH}[D(t) - p_{HY}(t), R(t) - R_{HY}(p_{HY}(t))] \Delta T_t$$

with the transition function:

$$q(t+1) = q(t) + [f(t) - p_{HY}(t)G(p_{HY}(t))] \Delta T_t \quad t = 1, 2, \dots, T$$

and the constraint:

$$q_{\min} \leq q(t) \leq q_{\max} \quad t = 1, 2, \dots, T$$

Other constraints on p_{HY} may implicitly be operative from the definitions of the performance functions C_{TH} and R_{HY} .

The dynamic program is solved by backwards recursion:

$$F(q) = \min_{p_{HY}} \left\{ C_{TH}(D(t) - p_{HY}(t), R(t) - R_{HY}(p_{HY})) \Delta T_t \right. \\ \left. + F_{t+1} q - [(f(t) - p_{HY}(t)G(p_{HY}(t))) \Delta T_t] \right\} \\ \text{for } t = T, T-1, \dots, 1$$

with the initial condition:

$$F(q) = 0 \quad \text{for } q = q_0 \\ = \infty \quad \text{otherwise.}$$

and the solution to the dynamic program is $F_1(q_0)$.

The above formulation has a unidimensional state vector which ensures a rapid and compact solution. Since what is required in this application is the cost of operation (value of the objective function) and not the sequence of decisions which give rise to it (the optimal policy), a single backwards build-up phase will usually suffice for the required solution.

5.6 SYSTEM PERFORMANCE FUNCTIONS

The function $R_{HY}(p_{HY})$ was defined in the previous section as the spinning reserve available from the hydro subsystem when delivering a power output p_{HY} . As was then pointed out, this may not necessarily be a unique function of output. We now make it so by applying the following assumption:

- That the hydro subsystem is operated in such a manner that, for whatever level of output the subsystem produces, the spinning reserve provided is maximal for that level of output.

An optimization problem has now been created for this subsystem.

For the system performance functions strict constraints can be imposed on the allowable output levels that individual units can produce. For the most general case, that of a reversible pump-turbine, the following bounds on the range of feasible operating conditions may be identified:

- The 'maximum' pumping capability, p_1
- The 'minimum' safe pumping capability, p_2
- The minimum safe generating level, p_3
- The nominal rating (CMR) of the unit, p_4
- The overload capacity of the unit, p_5 . This is the output that the unit is capable of supplying under emergency conditions. It is assumed that this output can be maintained indefinitely, although in normal operation p_4 is never to be exceeded.

From these definitions:

$$p_1 \leq p_2 \leq 0 \leq p_3 \leq p_4 \leq p_5$$

These parameters will be used to relate the output of any single unit to the spinning reserve that it provides. A unit $k \in S$ can be operated in one of three modes - pumping (for pumped storage plant only), off, or generating:

$k \in P$, unit pumping, $p_1(k) \leq p(k) \leq p_2(k) < 0$

In this mode the spinning reserve available is $-p(k)$, since the unit may be tripped instantaneously and thus immediately increase net system output by an amount $-p(k)$.

$k \in O$, unit off, $p(k) = 0$

In this mode both power output and spinning reserve are zero. The unit may only be brought onto bars after a finite time interval.

$k \in G$, unit generating, $0 < p_3(k) \leq p(k) \leq p_4(k)$

Under these conditions when producing power output $p(k)$, the spinning reserve available is $p_5(k) - p(k)$, it being assumed that the unit can be brought up to its full overload level within a very short, almost instantaneous time interval.

For a pumping unit alone, $k \notin G$; for a generating unit alone (this includes all thermal plant), $k \notin P$.

A dynamic program for maximizing reserve from the hydro subsystem may now be formulated:

$$\text{find } R_{HY}(P_{HY}) = \max \sum_{k=1}^{N_{HY}} r(k)$$

$$\text{subject to: } \sum_{k=1}^{N_{HY}} p(k) \geq P_{HY}$$

where $p(k)$ is the output of hydro unit k , and the values of $r(k)$ are defined as:

$$\begin{aligned} r(k) &= -p(k) & \text{if } p_1(k) \leq p(k) \leq p_2(k) & \quad (k \in P) \\ r(k) &= 0 & \text{if } p(k) = 0 & \quad (k \in O) \\ \text{and } r(k) &= p_5(k) - p(k) & \text{if } p_3(k) \leq p(k) \leq p_4(k) & \quad (k \in G) \end{aligned}$$

- the above definitions also define the feasible region for $p(k)$.

The problem can be regarded as an allocation problem (Appendix A), soluble by the following forwards recursion formula:

$$F_k(p) = \max_{p(k)} \left\{ r(k) + F_{k-1}(p - p(k)) \right\} \quad \text{for } k = 1, 2, \dots, N_{HY}$$

with the initial condition: $F_0(p) = 0$

and the required solution is: $R_{HY}(p) \equiv F_{N_{HY}}(p)$

This is a unidimensional state, unidimensional decision dynamic program.

The function $C_{TH}(p_{TH}, r_{TH})$ was defined in Section 5.4 as the cost of operating the thermal subsystem in order to provide output p_{TH} with spinning reserve r_{TH} . It is tacitly assumed that this subsystem is operated in an optimal manner, that is:

- The thermal subsystem is operated at minimum cost.

In a similar way as before, the function may be obtained as the results of an allocation problem solved by dynamic programming, as follows:

$$C_{TH}(p_{TH}, r_{TH}) = \min \sum_{k \in G_{\underline{C}_{TH}}} c_k(p(k))$$

subject to the constraints:

$$\sum_{k=1}^{N_{TH}} p(k) \geq p_{TH}$$

and:

$$\sum_{k \in G_{\underline{C}_{TH}}} (p_5(k) - p(k)) \geq r_{TH}$$

For solution, the recursive relation is:

$$F_k(p, r) = \min_{p(k)} \left\{ c_k(p(k)) + F_{k-1}(p-p(k), r-r(k)) \right\}$$

for $k = 1, 2, \dots, N_{TH}$

where $r(k) = 0$ if $p(k) = 0$ ($k \in O$)

and $r(k) = p_5(k) - p(k)$ if $p_3(k) \leq p(k) \leq p_4(k)$ ($k \in G$)

The above definitions for $r(k)$ also define the feasible region for $p(k)$.

The initial condition is: $F_0(p, r) = 0$

and the required solution is: $C_{TH}(p_{TH}, r_{TH}) \equiv F_{N_{TH}}(p_{TH}, r_{TH})$

This is a two-dimensional state, unidimensional decision dynamic program.

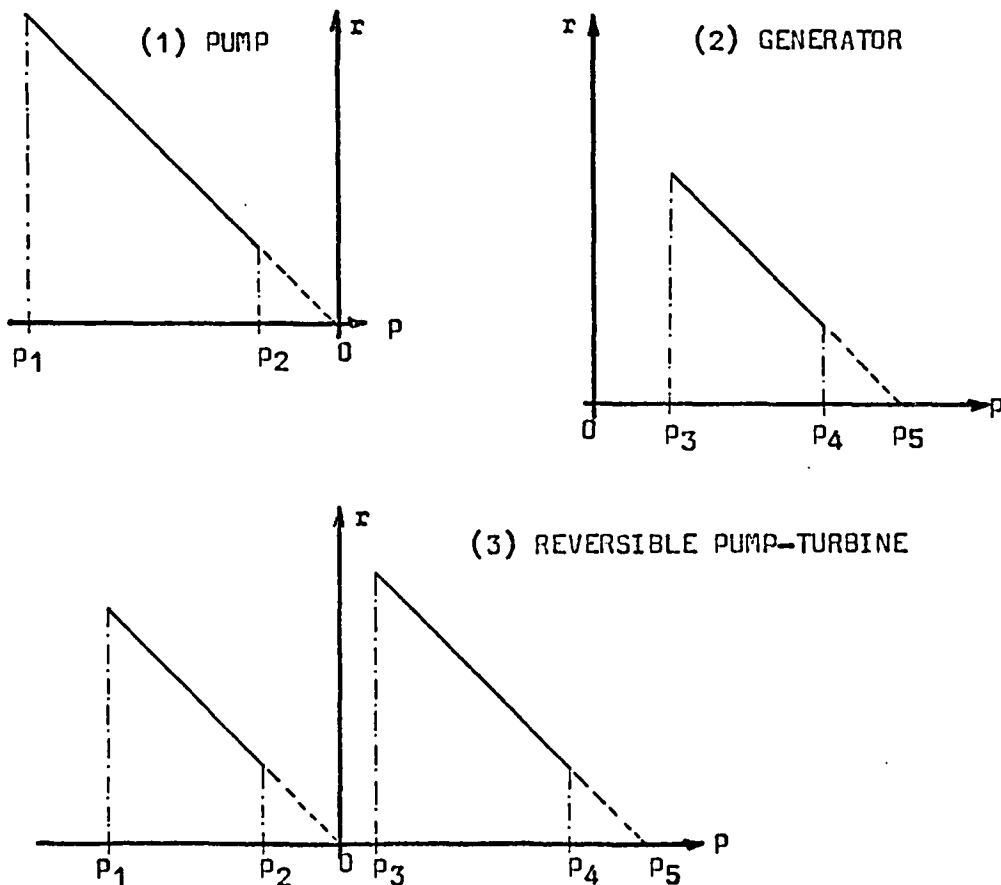
5.7 OPERATING LINES AND OPERATING VALUES

From the definitions of spinning reserve as presented in Section 5.6, according to the mode (pumping, off, or generating) in which a unit is operating, the relation between a unit's output and the spinning reserve it provides may be plotted on the (p,r) plane; hence Fig 5.7-1.

The lines so obtained will be called operating lines for the unit. These lines define the spinning reserve available from the unit as a function of the output it produces. From the definitions the lines are always linear, with a gradient of -1 . Clearly for a single unit the spinning reserve available is a unique function of its output.

Turning now to the case of a system comprising more than one unit, in general the spinning reserve available from the system

FIGURE 5.7-1 OPERATING LINES FOR SINGLE UNITS



will no longer be a unique function of system output. It will in fact depend upon the status of every unit in the system - whether off, pumping, or generating. The status of each unit determines the system operating line which is active for that particular set of conditions, and hence the relationship between output and spinning reserve for the system. The following general rules will apply in regard to unit status and spinning reserve:

- If a unit is off, it makes no contribution to the spinning reserve provided by the system.
- If a unit k is pumping, supplying $-p(k)$ MW to the system, the spinning reserve of the system is increased by $p(k)$ MW.
- If a unit k is generating, supplying $p(k)$ MW to the system, the spinning reserve available from the system is augmented by this unit's contribution, $p_s(k) - p(k)$ MW.

Mathematically the spinning reserve provided by a system S is given by:

$$\sum_{k \in P \subseteq S} p(k) + \sum_{k \in G \subseteq S} (p_s(k) - p(k))$$

Figure 5.7-2 shows the individual operating lines for three generating units, the operating lines for the system which consists of these three units are tabulated in Table 5.7-1, and graphed on the (p,r) plane in Fig 5.7-3.

For the most general case, a system of N units each of which may be operated in one of three possible modes, there are 3^N operating lines (this includes the origin, which corresponds to the case for all units being off). Each operating line is valid between lower and upper bounds which are determined by the status of each unit in the system:

$$\text{lower bound} = \sum_{k \in P} p_1(k) + \sum_{k \in G} p_3(k)$$

$$\text{upper bound} = \sum_{k \in P} p_2(k) + \sum_{k \in G} p_4(k)$$

These parameters are also tabulated in Table 5.7-1.

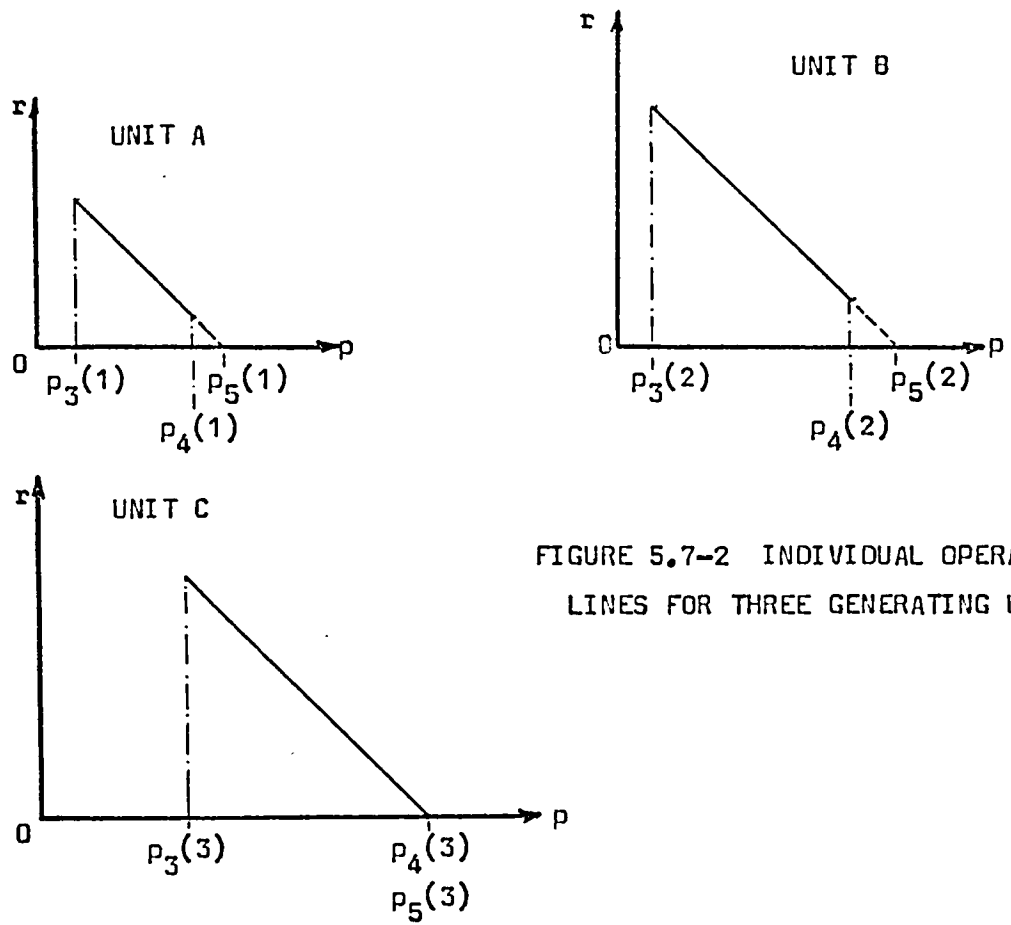


FIGURE 5.7-2 INDIVIDUAL OPERATING LINES FOR THREE GENERATING UNITS

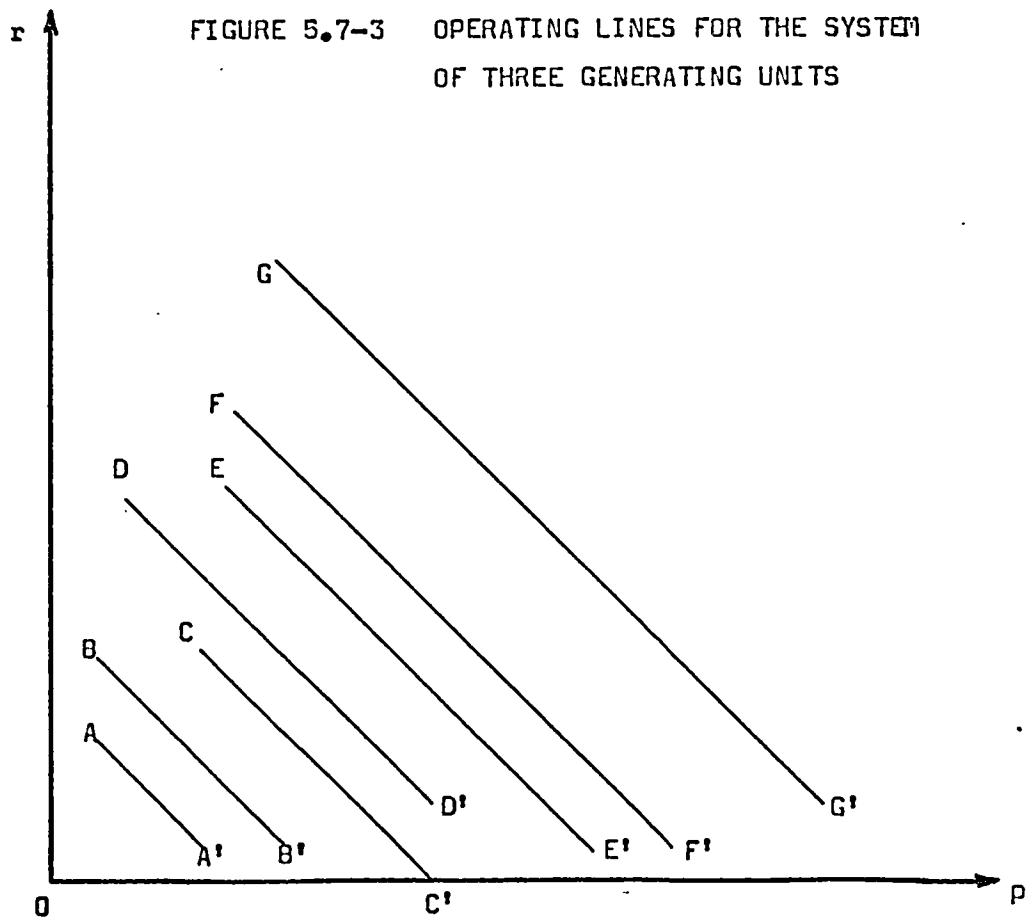


FIGURE 5.7-3 OPERATING LINES FOR THE SYSTEM OF THREE GENERATING UNITS

It can be more convenient to replace spinning reserve r with a new variable, called the operating value z of a unit or a system, defined simply as:

$$z = p + r$$

The operating lines for the system of three units of Fig 5.7-2, plotted this time on the (p, z) plane, are shown in Fig 5.7-4. Operating lines are now horizontal straight lines, along any one operating line the operating value z is constant. A change in the operating status of one or more of the units in the system will cause another operating line to become active. There are a finite number of operating lines for any finite system.

FIGURE 5.7-4 OPERATING LINES FOR THE SYSTEM OF FIG 5.7-2, ON THE (p, z) PLANE

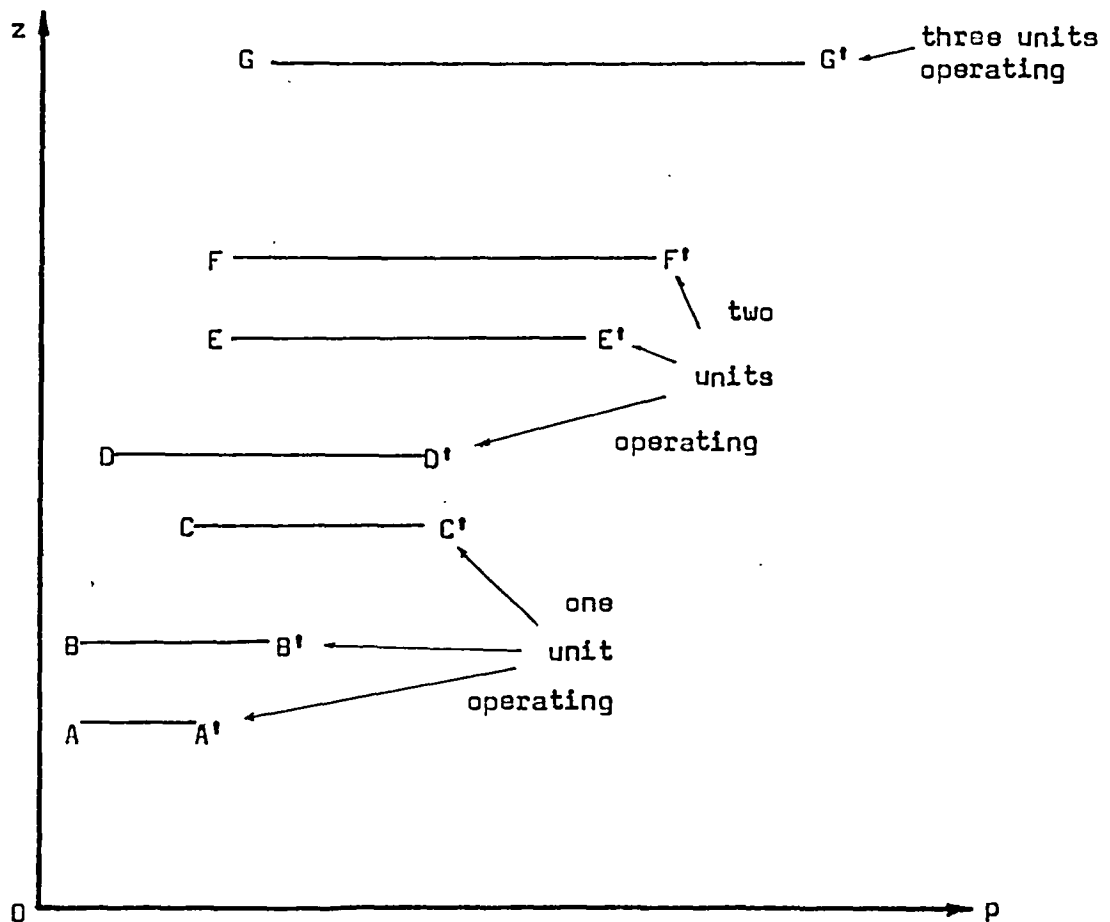


TABLE 5.7-1 OPERATING LINES FOR THE SYSTEM OF FIG 5.7-2

LINE	STATUS OF UNIT			OPERATING VALUE	LOWER BOUND	UPPER BOUND
	A	B	C			
O	O	O	O	0	0	0
AA'	G	O	O	$d_5(1)$	$d_3(1)$	$d_4(1)$
BB'	O	G	O	$d_5(2)$	$d_3(2)$	$d_4(2)$
CC'	O	O	G	$d_5(3)$	$d_3(3)$	$d_4(3)$
DD'	G	G	O	$d_5(1) + d_5(2)$	$d_3(1) + d_3(2)$	$d_4(1) + d_4(2)$
EE'	G	O	G	$d_5(1) + d_5(3)$	$d_3(1) + d_3(3)$	$d_4(1) + d_4(3)$
FF'	O	G	G	$d_5(2) + d_5(3)$	$d_3(2) + d_3(3)$	$d_4(2) + d_4(3)$
GG'	G	G	G	$d_5(1) + d_5(2)$ $+ d_5(3)$	$d_3(1) + d_3(2)$ $+ d_3(3)$	$d_4(1) + d_4(2)$ $+ d_4(3)$

In terms of the operating value, the following rules apply according to the status of each unit in the system:

- If a unit is off, or pumping, it makes no contribution to the operating value of the system.
- If a unit k is generating, regardless of its output, the operating value of the system is increased by $p_5(k)$.

The operating value of a system S is therefore given by:

$$z = \sum_{k \in G \subseteq S} p_5(k)$$

and the spinning reserve of the system, when aggregate system output is p , is:

$$r = z - p$$

where:

$$\sum_{k \in P} p_1(k) + \sum_{k \in G} p_3(k) \leq p \leq \sum_{k \in P} p_2(k) + \sum_{k \in G} p_4(k)$$

Finally, the operating lines for the system of Figure 5.7-1 are shown in Figs 5.7-5 and 5.7-6. Although many of the lines are coincident they are drawn separate for clarity.

FIGURE 5.7-5 OPERATING LINES FOR THE SYSTEM OF FIG 5.7-1
ON THE (p,r) PLANE

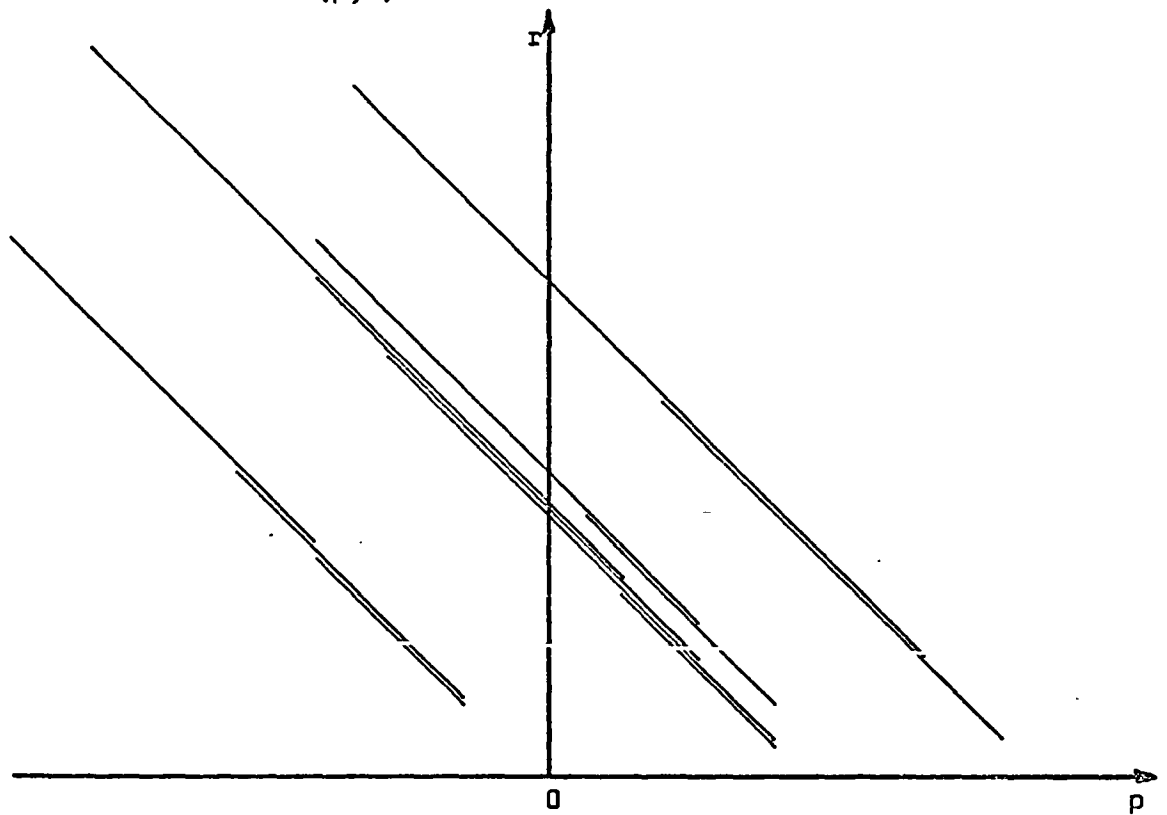
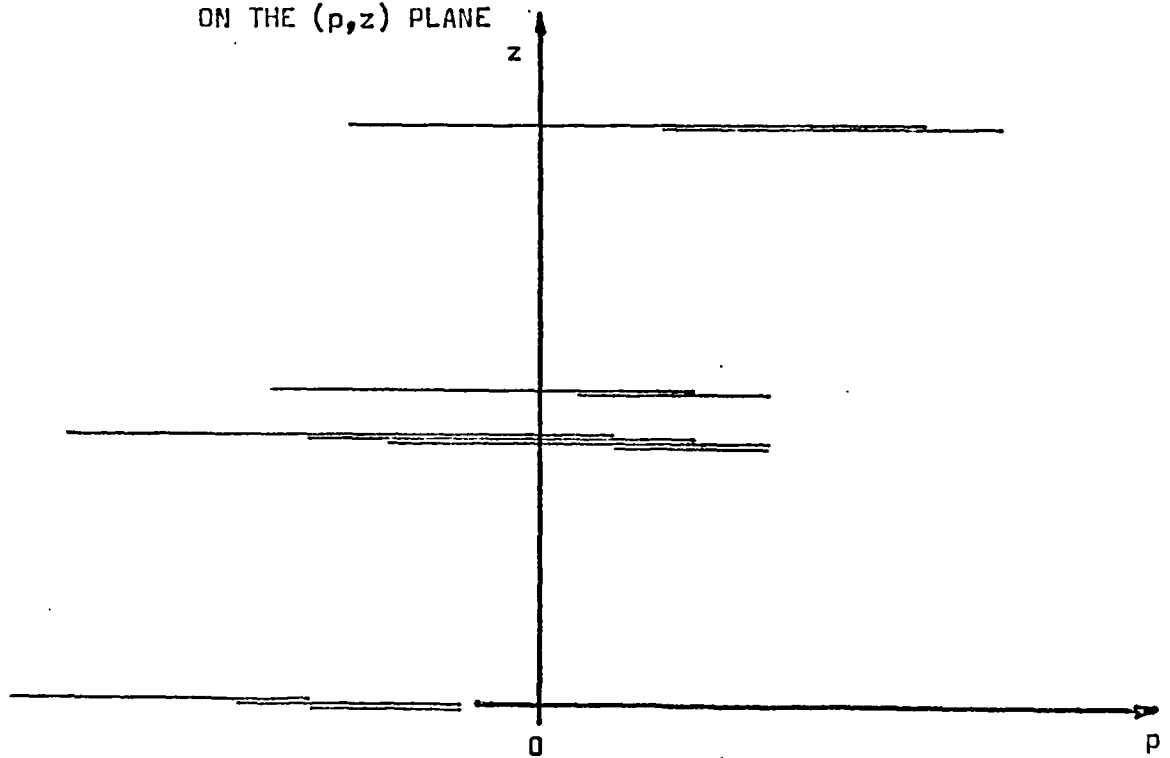


FIGURE 5.7-6 OPERATING LINES FOR THE SYSTEM OF FIG 5.7-1
ON THE (p,z) PLANE



5.8 IMPROVED FORMULATION FOR THE HYDRO SUBSYSTEM

Armed with the concept of operating lines and operating values, the dynamic program of Section 5.6 for the hydro subsystem may now be recast into a more efficient form.

The discussion is first directed towards an example of a hypothetical hydro/pumped storage system consisting of one pumping unit, one reversible pump-turbine, and two generating units (details and parameters in Table 5.8-1). Unit A can be operated in one of two modes (pumping, off), unit B in one of three (pumping, off, generating), and units C and D each in one of two modes (off, generating). There are thus $2 \times 3 \times 2 \times 2 = 24$ possible operating lines for this subsystem. These are listed in Table 5.8-2 and graphed on the (p, z) plane in Fig 5.8-1, from which it can be seen that many of the lines are coincident, or merge, so that for this system there are effectively just eight operating lines.

Now if the system be operated at maximum operating value, the trajectory sought is the line AA"BB"DD"FF"GG"HH" in Fig 5.8-1. Note that lines CC' and EE' do not form part of the maximum operating value trajectory.

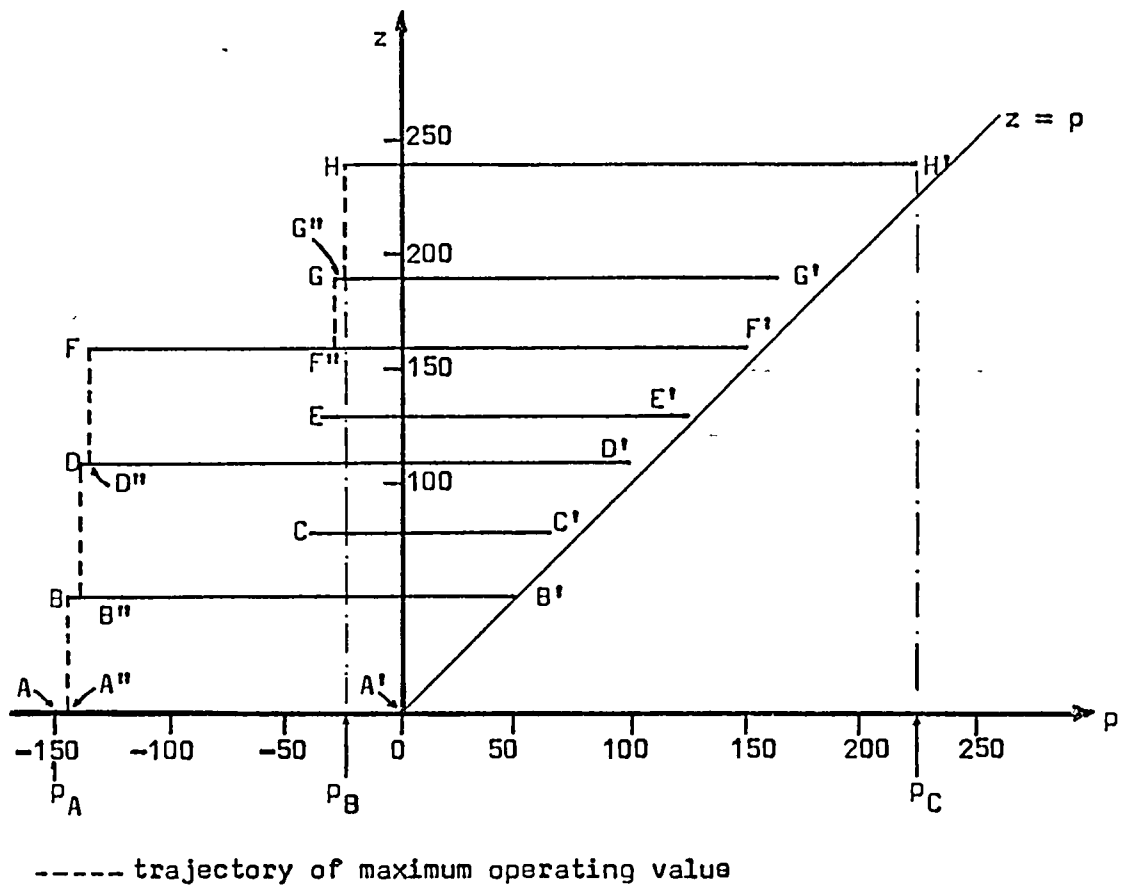
TABLE 5.8-1 DATA FOR HYPOTHETICAL HYDRO/PUMPED STORAGE SYSTEM OF FOUR UNITS

UNIT	TYPE	OPERATING LIMITS (MW)				
		P ₁	P ₂	P ₃	P ₄	P ₅
A	pump	-50	-10	-	-	-
B	reversible pump-turbine	-100	-10	10	75	80
C	generator	-	-	5	50	50
D	generator	-	-	10	100	110

TABLE 5.8-2 OPERATING LINES FOR THE SYSTEM OF TABLE 5.8-1

LINE NO	STATUS OF UNIT				LOWER BOUND (MW)	UPPER BOUND (MW)	OPERATING VALUE (MW)
	A	B	C	D			
1	O	O	O	O	0	0	0
2	O	O	O	G	10	100	110
3	O	O	G	O	5	50	50
4	O	O	G	G	15	150	160
5	O	P	O	O	-100	-10	0
6	O	P	O	G	-90	90	110
7	O	P	G	O	-95	40	50
8	O	P	G	G	-85	140	160
9	O	G	O	O	10	75	80
10	O	G	O	G	20	175	190
11	O	G	O	O	15	125	130
12	O	G	G	G	25	225	240
13	P	O	O	O	-50	-10	0
14	P	O	O	G	-40	90	110
15	P	O	G	O	-45	40	50
16	P	O	G	G	-35	140	160
17	P	P	O	O	-150	-20	0
18	P	P	O	G	-140	80	110
19	P	P	G	O	-145	30	50
20	P	P	G	G	-135	130	160
21	P	G	O	O	-40	65	80
22	P	G	O	G	-30	165	190
23	P	G	G	O	-35	115	130
24	P	G	G	G	-25	215	240

FIGURE 5.8-1 OPERATING LINES FOR THE SYSTEM OF TABLE 5.8-1



The determination of the trajectory of maximum operating value can be formulated as a dynamic program. Now from the definition of operating value, the trajectories of maximal spinning reserve and maximal operating value as a function of subsystem output are identical. Furthermore it will only be necessary to restrict attention to a particular range of subsystem output. This comes about as follows:

Firstly define the following points on the p-axis for the hydro subsystem:

$$P_A = \sum_{k \in HY} P_1(k)$$

$$P_B = \sum_{k \in HY} P_3(k)$$

$$P_C = \sum_{k \in HY} P_4(k)$$

p_C is the maximum output that the subsystem can ever attain under normal operating conditions. From p_B to p_C for all systems, the maximum operating value remains constant, equal to $\sum_{k \in HY} p_5(k)$. This is the operating value of the system when all units (that are capable of being so operated) are operated in generating mode. The system can never attain a higher operating value, and for any system output within this range can never provide a greater spinning reserve. Attention can therefore be solely directed to the range $p_A \leq p_{HY} < p_B$. In this region, for almost all practical cases, the upper bound constraints on the operating lines are never active, the next higher operating line coming into effect before an upper bound is reached (see Fig 5.8-1). Hence the following simplified optimization problem:

$$Z_{HY}(p_{HY}) = \max \sum_{k \in G_{CHY}} p_5(k)$$

subject to the single constraint (lower bound only):

$$\sum_{k \in P_{CHY}} p_1(k) + \sum_{k \in G_{CHY}} p_3(k) \leq p_{HY}$$

This problem may be solved using dynamic programming, which will at the end of the build-up phase yield the solution as a function in feasible state (p) space.

The recursive relation is:

$$F_k(p) = \max \left\{ \begin{array}{l} F_{k-1}(p) \\ F_{k-1}(p - p_1(k)) \\ F_{k-1}(p - p_3(k)) + p_5(k) \end{array} \right\} \quad \begin{array}{l} \dots k \in O \text{ (off)} \\ \dots k \in P \text{ (pumping)} \\ \dots k \in G \text{ (generating)} \end{array}$$

for $k = 1, 2, \dots, N_{HY}$

With the initial condition:

$$F_0(p) = 0$$

and the solution is:

$$Z_{HY}(p_{HY}) \equiv F_{N_{HY}}(p_{HY})$$

This is a greatly more efficient form than that presented for the hydro subsystem in Section 5.6. Instead of having to conduct a search over the feasible operating range for each unit (decision space) for each quantized value of p , it will now only be necessary to make a scalar comparison of at most three values. The formulation in fact is that of a ternary dynamic program.

The required function $R_{HY}(p_{HY})$ can now be obtained from the following rules:

- (a) if $p_A \leq p_{HY} < p_B$, $R_{HY}(p_{HY}) = Z_{HY}(p_{HY}) - p_{HY}$
- (b) if $p_B \leq p_{HY} \leq p_C$, $R_{HY}(p_{HY}) = \sum_{k \in HY} p_5(k) - p_{HY}$
- (c) if $p_{HY} < p_A$ or $p_{HY} > p_C$ $R_{HY}(p_{HY}) = \infty$
(infeasible operation)

During the course of solution of this dynamic program the constraints on individual operating ranges for each unit in the subsystem have been satisfied. The function $R_{HY}(p_{HY})$ is therefore only defined for feasible operating strategies for this subsystem.

5.9 IMPROVED FORMULATION FOR THE THERMAL SUBSYSTEM

The two-dimensional dynamic program for the thermal subsystem as outlined in Section 5.6 is here made more efficient by breaking down the problem into two separate unidimensional dynamic programs.

PROBLEM A. Consider first the following problem:

$$\text{Find } C(p_{TH}) = \min_{k \in G_{CTH}} \sum c_k(p(k))$$

subject to the
single constraint:

$$\sum_{k \in G_{CTH}} p(k) \geq p_{TH}$$

The solution to this problem yields the minimum cost trajectory for operation of the thermal subsystem when the constraint imposed by the need to provide spinning reserve is simply ignored. It may be obtained using the following recursive formula:

$$F_k(p) = \min_{p(k)} [c_k(p(k)) + F_{k-1}(p - p(k))] \quad \text{for } k = 1, 2, \dots, N_{TH}$$

where $p(k) = 0$ or $p_3(k) \leq p(k) \leq p_4(k)$

with the initial condition: $F_0(p) = 0$

The solution being: $C(p_{TH}) \equiv F_{N_{TH}}(p_{TH})$

During the build-up phase, let:

$$F_k(p) = [c_k(p^*(k)) + F_{k-1}(p - p^*(k))] \quad \text{for } k = 1, 2, \dots, N_{TH}$$

and define a second recursive state function:

$$\begin{aligned} F'_k(p) &= F'_{k-1}(p) && \text{if } p^*(k) = 0 \\ \text{or } F'_k(p) &= [F'_{k-1}(p - p^*(k)) + p_5(k)] && \text{if } p_3(k) \leq p^*(k) \leq p_4(k) \end{aligned}$$

For these auxiliary state functions, the initial condition is:

$$F'_k(p) = 0$$

At the N_{TH}^{th} stage, let:

$$Z'(p_{TH}) \equiv F'_{N_{TH}}(p_{TH})$$

The function $Z'(p_{TH})$ provides a measure of the operating value (and hence spinning reserve) available from the thermal subsystem when operated along the minimum cost trajectory and producing output p_{TH} .

PROBLEM B. In a manner analogous to that of Section 5.8, the maximum reserve trajectory for the thermal subsystem can be obtained as:

$$Z(p_{TH}) \equiv F_{N_{TH}}(p_{TH})$$

$$\text{where: } F_k(p) = \max \left\{ \begin{array}{ll} F_{k-1}(p) & \dots k \in O \\ F_{k-1}(p - p_3(k)) + p_5(k) & \dots k \in G \end{array} \right\}$$

with the initial condition: $F_0(p) = 0$

This is a binary dynamic program since for this subsystem no units can be operated in pumping mode. Again as in Problem A, an auxiliary state function for each stage k is defined, giving an estimate of the operating cost along the maximum reserve trajectory:

$$C'(p_{TH}) \equiv F'_{N_{TH}}(p_{TH})$$

$$\text{where: } F'_k(p) = F'_{k-1}(p) \quad \text{if } k \in O$$

$$\text{or } F'_k(p) = [F'_{k-1}(p - p_3(k)) + c_k(p_3(k))] \quad \text{if } k \in G$$

with the initial condition: $F'_0(p) = 0$

Let:

$$p_A = \sum_{k \in TH} p_3(k)$$

$$\text{and } p_B = \sum_{k \in TH} p_4(k)$$

It should be noted that functions $Z(p)$ and $C'(p)$ are only defined for $p \leq p_A$. Furthermore, the maximum reserve and minimum cost trajectories meet at $p = p_B$.

Therefore, if $p_A < p \leq p_B$:

$$(a) \quad Z(p) = Z(p_A)$$

(b) $C'(p)$ can be estimated by linear interpolation:

$$C'(p) = C'(p_A) + \left[\frac{p - p_A}{p_B - p_A} \right] (C(p_B) - C'(p_A))$$

The general shape of the functions C , C' , Z , and Z' are shown in Fig 5.9-1. These can provide sufficient information to estimate the operating cost of the thermal subsystem, $C_{TH}(p_{TH}, r_{TH})$, as follows:

The arguments of the function correspond to the coordinates $(p_{TH}, p_{TH} + r_{TH}) = (p_{TH}, z_{TH})$ on the (p, z) plane of Fig 5.9-1. With reference to this figure, the point will lie in one of four regions:

$$\text{REGION A:} \quad p_{TH} \leq z_{TH} \leq Z'(p_{TH})$$

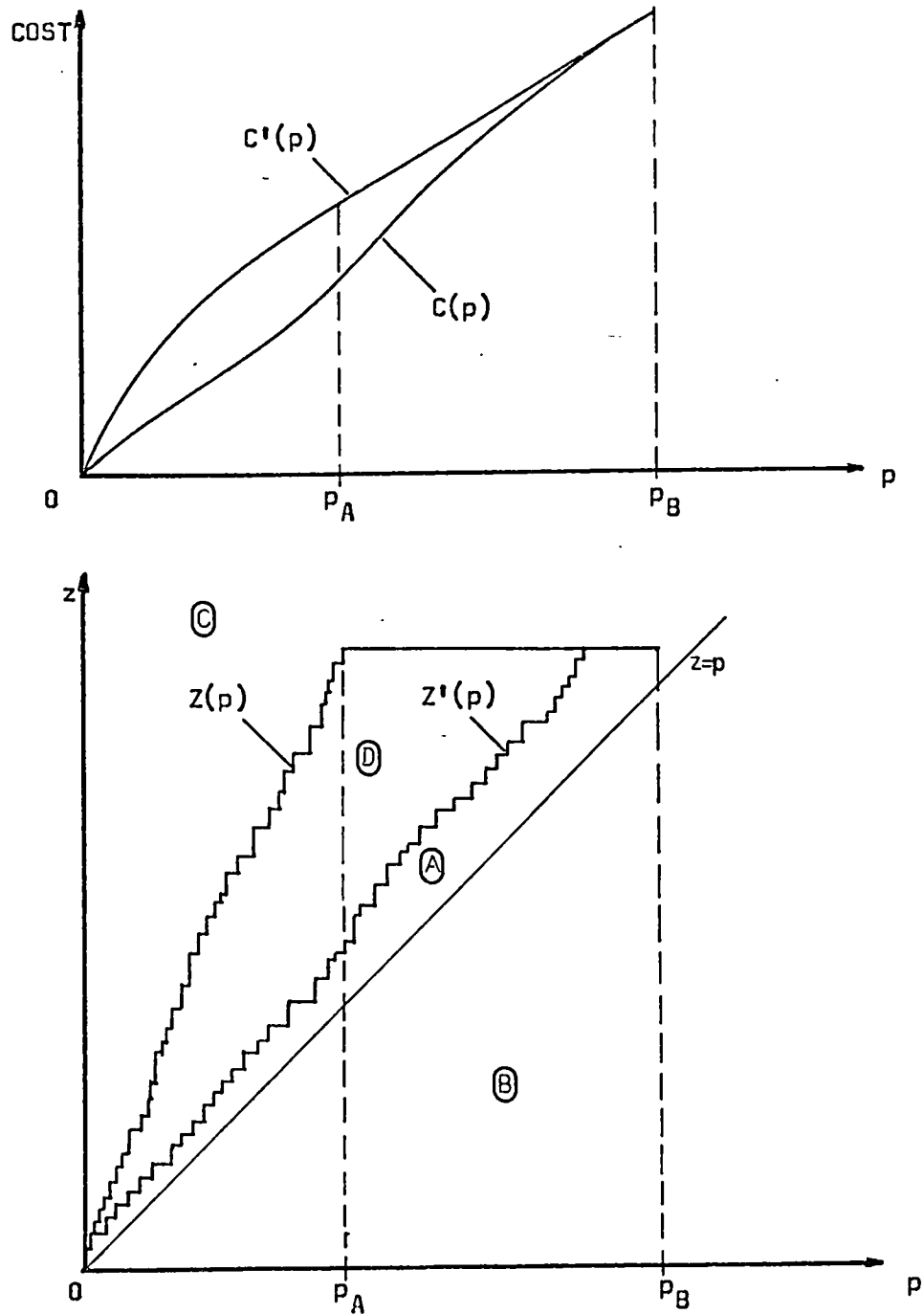
It follows that the spinning reserve required is equalled or exceeded by operating the subsystem along the minimum cost trajectory. It cannot, by definition, be operated more cheaply. Hence for this region:

$$C_{TH}(p_{TH}, r_{TH}) = C(p_{TH})$$

$$\text{REGION B:} \quad z_{TH} < p_{TH}, \text{ which implies that } r_{TH} < 0.$$

This is an infeasible region, and $C_{TH}(p_{TH}, r_{TH}) = \infty$

FIGURE 5.9-1 FUNCTIONS C , C' , Z , AND Z'
FOR THE THERMAL SUBSYSTEM



$C(p)$ = cost along minimum cost trajectory
 $C'(p)$ = cost along maximum reserve trajectory
 $Z(p)$ = operating value along maximum reserve trajectory
 $Z'(p)$ = operating value along minimum cost trajectory

REGION C: $z_{TH} > Z(p_{TH})$.

Operation of the thermal subsystem along the trajectory of maximum reserve will still not provide the required spinning reserve. This too is therefore an infeasible region, and

$$C_{TH}(p_{TH}, r_{TH}) = \infty$$

REGION D: $Z'(p_{TH}) < z_{TH} \leq Z(p_{TH})$

Operation is feasible, but the operating cost is not explicitly defined by either of the functions $C(p_{TH})$ or $C'(p_{TH})$. An estimate can be arrived at by linear interpolation in the z - direction:

$$C_{TH}(p_{TH}, r_{TH}) = C(p_{TH}) + \left[\frac{z_{TH} - Z'(p_{TH})}{Z(p_{TH}) - Z'(p_{TH})} \right] (C'(p_{TH}) - C(p_{TH}))$$

Although an approximation, this relation was found to give reasonably close estimates to the required operating cost of the thermal subsystem. Alternative methods of arriving at estimates of $C_{TH}(p_{TH}, r_{TH})$ are outlined in Appendix B.

5.10 APPRAISAL

In Sections 5.4 to 5.9 the theory for a simulation model based upon the CLC was developed and presented. In comparison to LDC-based models such as those of Sections 5.1 and 5.2 the CLC-based model is much more complex. However since (albeit in a very simplified manner) it attempts to mirror actual system operation it is likely to provide more accurate estimates of production costs. By doing so it also overcomes most of the shortcomings of LDC-based models which derive from the loss of chronological data; of those discussed in Section 5.3, only that of intermittent operation of individual units cannot be directly accommodated.

All this has only been achieved at the penalty of increased computational effort, and the CLC-based model is likely to require a computing time an order of magnitude or so longer than a fairly comprehensive LDC-based model. Hence such a model will have to be used sparingly - it would be unsuitable, for example, for the dynamic programming formulation outlined in Section 4.3 of Chapter 4.

Several simplifying assumptions were made in the course of development of the model which require further discussion:

- The assumption that system operation can be adequately modelled by consideration of a single aggregate pseudoreservoir. The validity of this assumption will depend on the structure of individual systems but can generally be held to be acceptable, particularly if it is borne in mind that the objective is a rapid estimate of operating costs and not accurate simulation of detailed operating strategies. As has been mentioned the assumption does not guarantee that the levels of each individual reservoir are within their safe operating limits. Moreover with such an assumption accurate treatment of hydro units in cascade along a single river system is impossible. Errors may also arise because of inaccuracies in estimation of true pumped storage cycle efficiencies, and also the implicit assumption of a pumped storage cycle over the simulation period. But it will now be realized that such an assumption is necessary for

computational economy - an n-reservoir problem not only entails an n-dimensional state vector in the dynamic program for optimal reservoir trajectories, but also requires separate system performance functions for n independent hydro subsystems.

- A further assumption which requires clarification lies in the definition of the system performance functions for the hydro subsystem: that the subsystem is operated along the trajectory of maximum reserve. This is difficult to justify formally; it is merely pointed out here that in practice for many mixed hydro-thermal systems the hydro complement is often largely used for regulatory functions*. If the premise of hydro subsystem operation along the trajectory of maximum reserve is accepted, the model will seek the most economic reservoir trajectory within this constraint. It should be noted that the operating line along which a system operates does not affect the rate at which reservoirs are replenished or depleted, this is solely dependant upon subsystem output. If it does turn out to be more economic to supply the required spinning reserve partly or wholly from the thermal subsystem, such a condition will be duly identified and costed accordingly.

- The third assumption to be discussed here is the definition of spinning reserve available from a unit. It was assumed that a unit could be brought up to its overload level within a very short, almost instantaneous time interval, irrespective of its prior output capacity. This is manifestly untrue since generally a few minutes will be required for a unit's overload capacity to be attained. The quantity r used in previous sections may more precisely be denoted online rather than spinning reserve. In the construction of the model however the objective has been to make explicit allowance for the provision of spinning reserve and to develop a rational and consistent method for estimating the cost of providing it. Input data may be adjusted upward in respect of the spinning reserve requirement if it is considered necessary to neutralize errors arising from this assumption.

* not specifically for reasons of economic system operation but because of the favourable physical characteristics of water turbines for regulation.

The overall structure of the model will now be clear. The main problem - of tracing the optimal trajectory of reservoir levels for minimal operating cost - is a time serial optimization problem with two separate optimization problems incorporated into its constraints. These subproblems can be viewed as allocation problems, for which in general dynamic programming may not be the best or most efficient means of solution. The advantages however are significant:

- Constraints on feasible operating regions for individual plants turn out to be discontinuous (although linear) and thus can easily be accommodated through the use of a numerical method of solution, as is the case with the standard computational algorithm. There is no particular special form required for the individual cost functions for thermal plant (c_k) and although these can be expected to be linear, quadratic or even more awkward forms may be used to allow for lower part-load efficiencies.
- From the use of the standard computational algorithm, results are obtained approximated in state space. Thus it turns out that the functions $Z_{HY}(p)$ for the hydro subsystem, and $C(p)$, $Z(p)$, $C'(p)$ and $Z'(p)$ for the thermal subsystem are conveniently very close to the form in which they are required for application to the optimal trajectory of reservoir levels problem. To use, for example, linear programming for the solution of the subsystem problems each time a value for $R_{HY}(p_{HY})$ or $C_{TH}(p_{TH}, r_{TH})$ is required would impose an enormous computational burden since dynamic programming is used for the solution of the reservoir problem.
- Since in planning applications the individual operating levels for each plant in the system are not normally of interest only a single build-up phase is required for the subsystem performance functions. Offline storage of the policy functions will not normally be necessary.
- Furthermore since for any particular subsystems the recursive functions will remain unchanged unless plant is removed or installed, the results of a single build-up phase can be used for the solution of a variety of reservoir trajectory problems, without the performance functions having to be recalculated.

In this respect the problem-solving approach differs from that of an LDC-based model, where each simulation is principally dependent upon the number of units in the system. In this multiple dynamic programming approach the evaluation of system performance functions will be proportional to the number of units in each subsystem, but as long as the system remains unchanged the solution of all reservoir trajectory problems for that system but with different chronological load curves will be independent of system size.

- When used in the planning environment, in the context of system development or expansion, where the system is altered in the course of time by the addition or removal of small numbers of units, the property of invariant imbedding gives rise to significant economies in computation which partly offset the apparent CP time penalty of using a complex model.

Consider as an example the function $Z_{HY}(p)$, arrived at from the state function $F_{N_{HY}}(p)$. During the course of derivation of this function, the functions $F_k(p)$, $k = 1, 2, \dots, N_{HY}$ represented the performance functions for the N_{HY} subsystems each comprising the first k units in the computational sequence.

Therefore if it is required to add M units to a system of N units it will only be necessary to compute state functions for a further M stages; the initial condition for this computation at stage $(N + 1)$ is simply $F_N(p)$ - which has already been evaluated.

Similarly if it is required to retire unit j in the system of N units it will only be necessary to recompute the state functions from stage j to stage $(N - 1)$ (the number of units in the new system). The initial condition for this sequence of recursive functional relations is $F_{j-1}(p)$, since the state functions for all imbedded subsystems preceding unit j in the computational sequence are unaffected by its removal.

In this way significant savings in computation time can be achieved when the model is used in the planning environment.

In Section 5.7, the ideas of operating lines and operating values were introduced, and these proved useful for reducing the computational requirement for the system performance functions. An approximate idea of the economies effected by the use of the improved formulations of Sections 5.8 and 5.9 can be obtained from Table 5.10-1. The improved formulation for the hydro subsystem does not reduce core requirements but speeds up computation particularly where there are few reversible pump-turbines in the subsystem. The improved formulation for the thermal subsystem results in very significant savings both in core and CP requirements.

Successive grid reduction can be used for the solution on the reservoir trajectory problem. This method has the advantage of more rapid convergence and faster solution. Another useful advantage is early identification of infeasible cases. The model can therefore be used not only for its primary function of cost

TABLE 5.10-1 COMPUTATIONAL ECONOMIES FROM THE USE OF THE IMPROVED FORMULATIONS

PROBLEM	CENTRAL MEMORY CORE PER STAGE	CP TIME PER STAGE
Hydro subsystem:		
Basic	100	1 000 Δt
Improved	100	200 Δt to 300 Δt (see note 4)
Thermal subsystem:		
Basic	10 000	100 000 Δt
Improved	400	1 200 Δt

Notes:

- (1) All state variables quantized to 100 levels.
- (2) Ten iterations assumed for unidimensional decision searches. For Fibonacci or Golden Section searches this will reduce the interval of uncertainty to about 1%.
- (3) Δt = time required for a single scalar comparison.
- (4) Depending upon the number of reversible pump-turbines in the hydro subsystem.

estimation but also to check for feasibility of operation. For example, system simulation may be carried out for the same system but with four different CLCs corresponding to:

- most severe demand (eg winter)
- most severe (demand + reserve), (eg winter, as above, or for worst-case planned or forced outages)
- reduced reservoir inflows, and perhaps a reduced permissible range of reservoir levels, to simulate system loading during dry-season conditions.

For the above cases it will be sufficient to check that the system is capable of operation under such chronological constraints and an accurate solution is not required, hence a coarse grid can be used. If the system passes these tests satisfactorily, a final simulation to a greater degree of accuracy can be conducted, corresponding to:

- a curve which is adjudged to be representative of an annual 'average' CLC, for cost estimation purposes.

The result of this last loading simulation yields the required estimated production cost.

A disadvantage of the model is its inability to remove a single unit, other than the last in the computational sequence, from the system without extensive recalculation of the recursive state functions. Therefore it is not conveniently possible to evaluate system performance accurately under conditions of planned outage of particular units.

The model is completely deterministic and no provision has been made for stochastic effects with regard to plant performance, load curves, hydrological inflows, and so on. It is possible to accommodate these to a degree. Dealing first with the effect of plant forced outage rates, it is common to reduce CMRs and thus consider the nominal rating as a guaranteed output level. More rigorously, however, noting the stage-by-stage nature of the construction of the recursive state functions, the Markov chain-like structure can be exploited to evaluate the cumulative effects of plant forced outage rates, in a similar manner to that used in Section 5.2 (probabilistic simulation). This will

not impose a great computational penalty.

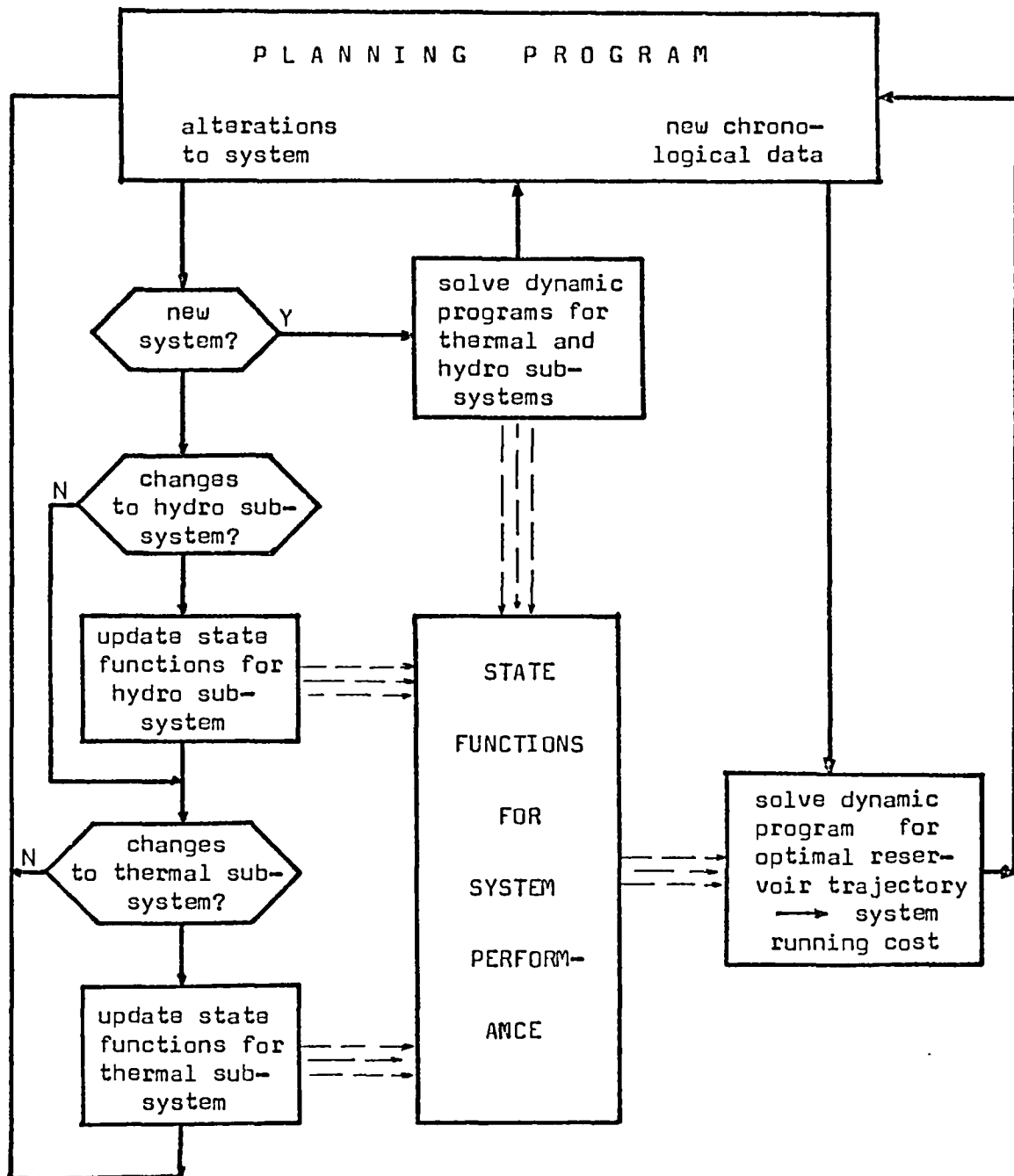
Turning next to the effect of uncertainties in load variation and hydrological inflows (stochastism in chronological constraints) a methodology exists for the treatment of such dynamic programs. Unfortunately stochastic dynamic programming /27,36/ imposes great computational penalties, although alternative simplifications as suggested by Henault, et al /29/ may reduce the computational burden.

Whether or not stochastic effects are formally included into the model the nature of the computational process remains unchanged, and the model retains all its advantages. To date it has not been considered necessary by the author to incorporate such a level of sophistication into the model; in the planning context it is felt unwarranted in view of the quality of the input data.

Owing to the complexity of the model it is generally unsuitable for application to the loading simulation of large networks with many units. In such cases LDC-based models should provide acceptable results since the system can be regarded as more 'homogeneous' - less sensitive to the effects of individual plant loadings. The model is however eminently suitable when applied to small or medium-sized systems where such effects can be important and erroneous results may be obtained from the application of LDC-based models. In such systems the effects of individual plant loadings can be significant, particularly where the rating of a unit is a significant fraction of total installed capacity.

A flowchart showing the suggested layout for the use of the CLC-based model in the planning environment is shown in Fig 5.10-1.

FIGURE 5.10-1 USE OF THE SIMULATION MODEL IN THE PLANNING ENVIRONMENT



CHAPTER SIX. CASE STUDIES

A set of computer programs has been written incorporating the theory outlined in Chapter Four (Section 4.4) and Chapter Five (Sections 5.1 and 5.4 to 5.9) for the determination of the installation framework for power system generating planning. In this chapter two case studies will be presented, and their results analysed and discussed. To facilitate inspection of the results a brief description of the main features of the package is given below. More complete details on the use of the programs are to be found in /53/.

1. The package consists of four main independent programs, to be executed in sequence. This format has been adopted to encourage data checking and inspection of results at intermediate stages, to prevent possible wastage of computing resources. The principal functions of the four programs are:

- Input data processing and simple data checking;
- Assembly of the recursive state and policy functions for the continuous function dynamic program (build-up phase);
- Traceback phase with branch-and-bound search for the discrete function dynamic program;
- Detailed output listings of optimal expansion strategies.

2. The subperiod for planning studies is a year. A single consistent monetary unit is used throughout; all capital costs and operating cost coefficients are to be expressed in terms of this unit. Constraints applied on capital expenditure are differentiated according to local and foreign components and are cumulative over any given number of consecutive years.

3. A plant is defined as a group of one or more units of identical size, performance, and vintage. Each plant is uniquely defined by a name, the number of units it contains, and its date of installation.

4. Parameters supplied as items of input data for each plant refer to a single unit comprising the plant. Capacity parameters (in MW) follow the definitions given in Section 5.6, three being required for non-hydro plants and five for hydro plants. Reservoir capacities and hydrological inflows for the latter again refer to a single unit of the plant, and are measured in MWh or MW equivalent respectively. Cost coefficients for non-hydro plant define the operating cost for each unit according to the quadratic relation:

$$\begin{array}{lcl} \text{operating} & & \\ \text{cost} & = & c_1 + c_2 p + c_3 p^2 \end{array} \quad \text{monetary units per hour}$$

p being the instantaneous output of the unit in MW, and

c_1 , c_2 , and c_3 are the constant cost coefficients.

These definitions apply both to plant in the initial system (the system at the start of the planning period) and the plant in the pool (candidates for addition during the planning period).

5. For plant in the pool of candidates for selection the following parameters are additionally specified:

- M The minimum number of units to be installed over the planning period;
- N The maximum number of units that may be selected over the planning period;
- J The first year in which this plant becomes available for selection;
- K The last year of selection, by which some of the units of the plant must have been installed.

This is to be interpreted as follows:

- Between years J and K inclusive, at least M and not more than N units of the plant must be installed. Not less than M units may be installed in the first year that units of the plant are selected.

These four parameters may be used to describe a variety of common conditional-type constraints which apply in real life

sequencing of site development:

- (i) If the optimum number of units to be installed is to be determined in the study, then $J = K =$ programmed year of installation, and $M < N$.
- (ii) If on the other hand the optimum date of implementation of a project is required, then $M = N =$ number of units to be installed, and $J < K$.
- (iii) If $M = N$ and $J = K$ then this represents a committed addition to the system of M units of the plant in year J . Thus plant currently on order or under construction and due to be commissioned during the planning period, can be introduced to the system at the appropriate dates.
- (iv) If $K >$ horizon year, then the installation of the plant is optional, if $K \leq$ horizon year, its installation is mandatory, before or in year K .
- (v) If $K < J$, the plant will never become available for selection at any date during the planning period (unless it is released to the pool by another selection - see following paragraphs).

6. In addition to the above, any plant in the pool can release up to two other plants to the pool, and remove up to two other plants from the pool:

- (i) If plant A releases plant B, then the latter becomes available for selection the year after the date of installation of plant A. This feature is included to accommodate site cost considerations and sequencing of site development - the greater costs of initial site development and planting, and the usually cheaper costs of follow-on units.
- (ii) If plant A removes plant B from the pool, then the selection of plant A disqualifies plant B as a candidate for future selection. This feature may be used to accommodate up to three mutually exclusive choices available for planting at a single site (eg different

plant types, or different unit sizes).

- (iii) As a special case a plant may renew itself to the pool. In such cases parameter N defines the upper limit to the number of units that may be installed at any one year and not over the whole planning period.
- (iv) A plant may remove itself from the pool.

7. The first program in the package inspects all capital cost data, and a least-squares fit is performed for each plant type, to a function of the form:

$$\begin{array}{lcl} \text{capital} & & \\ \text{cost} & = & a_1 + a_2 p^{a_3} \end{array} \quad \begin{array}{l} p \text{ being the nominal capacity} \\ \text{of the unit in megawatts.} \end{array}$$

This fit is obtained using only first-order information by means of a numerical search procedure (a modification of the Nelder-Mead flexible polyhedron search). In this way upper and lower bounds on each of the coefficients a_1 , a_2 , and a_3 can be imposed. These cost coefficients are used for capital cost estimation (function $C_n(d_n)$ in Section 4.4) during assembly of the recursive functions. Dummy plants (which never become available for selection at any time during the whole of the planning period) may be supplied to the pool to direct the numerical least-squares fit search to an acceptable set of coefficients.

8. Three daily load curves are used for loading simulation of the system at each year during the planning period. The first of these is a curve representing conditions of most severe demand or (demand + reserve) expected during this subperiod; the second is a typical load curve encountered during dry-season operation. A flow factor is to be supplied for each of these curves, this is a factor by which aggregate reservoir inflows are to be multiplied before loading simulation.

These first two curves serve only for checking of feasibility of system operation under such "difficult" conditions. The third daily load curve, representing an average for the year, is used for evaluation of system production costs in that year. For this

curve the flow factor is assumed to be unity.

An annual load duration curve is generated from the third ("average") daily load curve, and then each of the curves is reduced to a skeleton curve for efficiency in chronological loading simulation.

9. During the assembly of the recursive state and policy functions an LDC-based loading simulation model is used. In this (the second) program a penalty cost is applied for every MW by which demand cannot be satisfied, and for every MWh by which energy cannot be supplied. Penalty costs are assumed to be five times average cost of peaking capacity and five times average production costs of peaking plant. Escalation factors for penalty costs per MW are assumed equal to those for peaking plant, escalation factors for penalty costs per MWh are required as input data.

During traceback (the third program in the package) the user can select simulation of system operation using either an LDC-based model (Section 5.1) or a CLC-based model (Sections 5.4 to 5.9), the former being somewhat less accurate though significantly faster in execution than the latter. The use of an LDC-based loading simulation model during traceback is intended for preliminary exploratory runs.

When CLC-simulation is employed, system performance functions are updated every five years during the traceback search. This is done to correct the running cost functions for inflationary or deflationary effects in respect of fuel costs.

10. For computational economy installations are limited to a single plant, any number of units, each year. This assumption is acceptable for a small or medium-sized system, and also enables some simplifications to be made during the build-up phase. All selections are deemed to take effect in the year following chosen date of plant installation, whilst capital costs associated with these additions apply in the year of selection. No salvage value is associated with plant retirements.

11. All programs are compiled by the Imperial College FORTRAN Compiler, and run on the Imperial College Computer System under KRONOS 2.1.0-06 operating system. In the discussion of the case studies which follow, all references to computer CP time are in equivalent CP seconds on a CDC 6600. None of the programs requires a field length greater than 60 000₈ (25 000 decimal) for compilation, loading, or execution.

12. All costs mentioned in the discussion or listed in computer output are discounted and present-valued to year one of the planning study.

6.1 CASE STUDY ONE.

This case study investigates the installation framework for a comparatively small system, with a total installed capacity at the start of the planning period of 946.5 MW - approximately 40% hydro, 54% conventional fossil-fuelled thermal plant, 6% peaking plant, and no installed nuclear capacity. Over the planning period of twenty years, programmed removals of existing plant amount to 120 MW thermal (years 5 and 8) and 60 MW peaking (year 10). These and the plant mix constraints dictate the installation over the next twenty years of at least 220 MW hydro 540 MW thermal, and 200 MW peaking capacity. A test discount rate of 10% is stipulated, and the fifteen best (least-cost) strategies of expansion are required. The monetary unit used throughout corresponds to US\$ 10^3 .

Details of the initial system are given in Table 6.1-1, and of the pool of candidates in Tables 6.1-2. For convenience the following code has been adopted in naming the plants:

1st character	Site identifier (A to J inclusive) for plants in the pool. X denotes existing plant, Z denotes plants which are renewable.
2nd and 3rd	Plant type (HY, NU, TH, or PK)
4th	Alternatives at a single site
5th	Whether a <u>C</u> ommitted installation, a <u>F</u> ree choice, or <u>R</u> elaxed to the pool by a prior selection.

"Sites" Z are peaking selections, either gas turbines (ZPK1 & ZPK2) or diesels (ZPK3 and ZPK4). Four 30 MW units of either type are to be installed in years 5 or 6 (ZPK1C and ZPK3C) and subsequent planting of peaking units is to be with the same type as initially selected - gas turbines or diesels. Units sizes for future installations are 30 MW and 60 MW. Apart from the committed installations in year 5 or 6, all other peaking options renew themselves to the pool.

A summary of the remaining site details and constraints is given in Table 6.1-3. Two of the sites can be planted with nuclear capacity, although this is optional and thermal units

may be selected instead. It is one of the objectives of this study to determine whether to introduce nuclear units within the next twenty years. These units, it has been decided, can be installed from year 10 onwards (at site E), provided a decision is made now.

Other items of input data are graphed in Figures 6.1-1 to 6.1-4, which are generally self-explanatory. The system was in all years required to supply a spinning reserve amounting to 20% (maximum demand), 10% (dry season), and 10% (average) of system demand. The flow factor for the maximum demand curve was unity, and for the dry season curve 0.7. Table 6.1-4 shows the applied capital expenditure constraints. It may be noted that in view of plant costs in the pool these constraints are fairly strict.

The performance of the least-squares fit procedure is graphed in Fig 6.1-5. This shows that owing to different site development costs and the cheaper costs of follow-on units there is considerable disparity in the cost per MW of many units of the same type. The least-squares fit algorithm did not converge for peaking units, but as can be seen from Fig 6.1-5 the capital cost coefficients are satisfactory. The cost coefficients were:

$$\begin{aligned} \text{hydro: } & 1.237 \times 10^4 + 1.166 \times 10^3 (p)^{1.000} \\ \text{nuclear: } & 8.271 \times 10^3 + 1.031 \times 10^3 (p)^{0.999} \\ \text{thermal: } & 2.812 \times 10^3 + 6.698 \times 10^2 (p)^{0.969} \\ \text{peaking: } & 4.424 \times 10^3 + 2.666 \times 10^2 (p)^{1.000} \end{aligned}$$

Data processing and checking required a CP time of approximately 4 seconds.

TABLE 6.1-1 CASE STUDY ONE
SYSTEM AT YEAR 1

SYSTEM CONTAINS 6 THERMAL PLANTS

PLANT NAME	PLANT TYPE	NO.OF UNITS	YEAR IN	YEAR OUT	CAPACITY (MW)			COST COEFFICIENTS		
					MINIMUM	NOMINAL	OVERLOAD			
XTH1	THERML	3	0	15	5.000	26.000	30.000	5.000E-03	2.530E-02	0
XTH2	THERML	3	0	20	5.000	27.500	30.000	5.000E-03	2.640E-02	0
XTH3	THERML	4	0	8	.001	10.000	10.000	1.000E-03	2.870E-02	0
XTH4	THERML	4	0	26	20.000	55.000	60.000	1.000E-02	2.330E-02	0
XTH5	THERML	4	0	5	5.000	20.000	20.000	3.000E-03	2.530E-02	0
XPK1	PEAKING	3	0	10	.001	20.000	20.000	0	7.110E-02	0

SYSTEM CONTAINS 3 HYDRO PLANTS

					CAPACITY (MW)			OVERLOAD	RESERVOIR	
					PUMPING NOMINAL	MINIMUM	GENERATING NOMINAL		CAPACITY M4	INFLOW M4
XHY1	HYDRO	2	0	27	0	0	.001	50.000	55.000	250.000
XHY2	HYDRO	4	0	30	0	0	.001	50.000	50.000	0
XHY3	HYDRO	4	0	21	0	0	.001	20.000	20.000	0

TABLE B.1-2A CASE STUDY ONE
CANDIDATES FOR ADDITION, YEAR 1

PLANT NAME	PLANT TYPE	TOT. NO AVAILBL MIN MAX	YEARS AVAILABLE FIRST LAST	LIFE YRS	CAPITAL FOREIGN	COST LOCAL	CAPACITY (MW) MIN NOM BULD	COST COEFFICIENTS	
ATH1C	THERM	2 2	1 1	25	4.77E+04	2.40E+04	50.0 115.0 120.0	2.500E-02 2.310E-02	0
ATH2C	THERM	1 1	2 2	25	4.87E+04	2.33E+04	50.0 115.0 120.0	2.500E-02 2.310E-02	0
CTH1F	THERM	5 6	4 7	25	2.57E+04	7.80E+03	20.0 55.0 60.0	1.200E-02 2.311E-02	0
CTH1R	THERM	2 6	21 15	25	2.46E+04	8.60E+03	20.0 55.0 60.0	1.200E-02 2.311E-02	0
CTH2F	THERM	2 2	4 7	25	5.22E+04	1.57E+04	50.0 115.0 120.0	2.700E-02 2.250E-02	0
CTH2R	THERM	2 3	21 15	25	4.56E+04	2.15E+04	50.0 115.0 120.0	2.700E-02 2.250E-02	0
CTH3F	THERM	2 2	4 7	25	7.53E+04	2.89E+04	100.0 180.0 200.0	4.300E-02 2.210E-02	0
CTH3R	THERM	1 3	21 15	25	7.29E+04	3.02E+04	100.0 180.0 200.0	4.300E-02 2.210E-02	0
OTH1F	THERM	4 4	6 13	25	3.04E+04	5.01E+03	20.0 55.0 60.0	1.000E-02 2.310E-02	0
OTH1R	THERM	2 6	21 17	25	2.81E+04	6.86E+03	20.0 55.0 60.0	1.000E-02 2.310E-02	0
OTH2F	THERM	2 2	6 13	25	5.23E+04	1.76E+04	50.0 115.0 120.0	2.450E-02 2.250E-02	0
OTH2R	THERM	2 3	21 17	25	4.60E+04	2.36E+04	50.0 115.0 120.0	2.450E-02 2.250E-02	0
OTH3F	THERM	2 2	6 13	25	7.53E+04	3.37E+04	100.0 180.0 200.0	4.200E-02 2.205E-02	0
OTH3R	THERM	1 3	21 17	25	7.02E+04	3.84E+04	100.0 180.0 200.0	4.200E-02 2.205E-02	0
ETH1F	THERM	2 2	10 21	25	2.10E+05	4.14E+04	200.0 400.0 500.0	1.250E-01 2.032E-02	0
ETH1R	THERM	1 4	21 21	25	2.20E+05	3.56E+04	200.0 400.0 500.0	1.250E-01 2.032E-02	0
ENU1F	NUCLR	2 2	10 21	25	4.27E+05	1.45E+05	400.0 550.0 600.0	1.300E-01 6.140E-03	0
ENU1R	NUCLR	1 4	21 21	25	4.35E+05	8.12E+04	400.0 550.0 600.0	1.300E-01 6.140E-03	0
ENU2F	NUCLR	2 2	10 21	25	5.47E+05	2.49E+05	500.0 750.0 800.0	1.500E-01 6.000E-03	0
ENU2R	NUCLR	1 3	21 21	25	5.47E+05	1.60E+05	500.0 750.0 800.0	1.500E-01 6.000E-03	0
ETH1F	THERM	2 2	15 21	25	2.18E+05	6.04E+04	200.0 400.0 500.0	1.250E-01 2.032E-02	0
ETH1R	THERM	1 4	21 21	25	2.20E+05	5.60E+04	200.0 400.0 500.0	1.250E-01 2.032E-02	0
ENU2F	NUCLR	2 2	15 21	25	4.17E+05	2.05E+05	400.0 550.0 600.0	1.300E-01 6.140E-03	0
ENU2R	NUCLR	1 4	21 21	25	4.37E+05	1.39E+05	400.0 550.0 600.0	1.300E-01 6.140E-03	0
ENU3F	NUCLR	2 2	15 21	25	5.34E+05	2.90E+05	500.0 750.0 800.0	1.500E-01 6.000E-03	0
ENU3R	NUCLR	1 3	21 21	25	5.37E+05	2.30E+05	500.0 750.0 800.0	1.500E-01 6.000E-03	0
ZPA1F	PEAKNG	2 6	21 21	15	4.69E+03	4.89E+03	.0 30.0 30.0	0 7.000E-02	0
ZPA2F	PEAKNG	2 6	21 21	15	8.18E+03	1.55E+03	.0 30.0 30.0	0 7.350E-02	0
ZPA2R	PEAKNG	1 3	21 21	15	9.43E+03	9.44E+03	.0 60.0 60.0	0 6.700E-02	0
ZPA3F	PEAKNG	1 3	21 21	15	1.55E+04	3.10E+03	.0 60.0 60.0	0 7.000E-02	0
ZPA3C	PEAKNG	4 4	5 6	15	4.60E+03	4.80E+03	.0 30.0 30.0	0 7.000E-02	0
ZPA3C	PEAKNG	4 4	5 6	15	8.17E+03	1.55E+03	.0 30.0 30.0	0 7.350E-02	0
DUM1F	NUCLR	4 4	21 1	0	1.10E+05	0	.0 100.0 0	0 0	0
DUM1F	NUCLR	4 4	21 1	0	3.10E+05	0	.0 300.0 0	0 0	0
DUM1F	THERM	4 4	21 1	0	4.15E+05	0	.0 750.0 0	0 0	0
DUM1F	PEAKNG	4 4	21 1	0	7.46E+04	0	.0 250.0 0	0 0	0
DUM1F	PEAKNG	4 4	21 1	0	1.40E+05	0	.0 500.0 0	0 0	0

TABLE 6.1-2A (CONTD)

PLANT NAME	PLANT TYPE	TOT. NO AVAILBL		YEARS AVAILABLE		LIFE YRS	CAPITAL FOREIGN	COST LOCAL	CAPACITY (MW)				BULD	RESERVOIR	
		MIN	MAX	FIRST	LAST				PUMPING NO*	MIN	MIN	GENERATING NO*		CAPCTY M4*	INFLOW M4
BHYIC	HYDR0	1	1	3	3	50	1.05E+05	5.57E+04	0	0	.0	100.0	100.0	300.0	80.0
BHYIC	HYDR0	2	2	4	6	50	4.45E+04	2.42E+04	0	0	.0	100.0	100.0	0	80.0
GHYIF	HYDR0	3	3	7	9	50	5.13E+04	3.05E+04	0	0	.0	30.0	30.0	100.0	32.5
GHYIR	HYDR0	2	4	21	11	50	2.13E+04	1.20E+04	0	0	.0	30.0	30.0	0	0
HNYIF	HYDR0	4	4	8	21	50	6.05E+04	2.13E+04	0	0	.0	40.0	50.0	25.0	40.0
HNYIR	HYDR0	1	4	21	21	50	2.43E+04	1.20E+04	0	0	.0	40.0	50.0	0	0
HNYIF	HYDR0	2	2	8	21	50	1.05E+05	4.50E+04	0	0	.0	80.0	100.0	50.0	60.0
HNYIR	HYDR0	1	2	21	21	50	4.55E+04	2.63E+04	0	0	.0	80.0	100.0	0	0
JNYIF	HYDR0	4	4	10	21	50	6.33E+04	3.27E+04	0	0	.0	45.0	50.0	250.0	75.0
JNYIR	HYDR0	1	4	21	21	50	3.03E+04	1.03E+04	0	0	.0	45.0	50.0	0	0
JNYIF	HYDR0	2	2	10	21	50	1.21E+05	6.06E+03	0	0	.0	60.0	100.0	500.0	150.0
JNYIR	HYDR0	1	2	21	21	50	5.64E+04	3.06E+04	0	0	.0	90.0	100.0	0	0
JNYIF	HYDR0	2	2	10	21	50	1.67E+05	8.16E+04	0	0	.0	145.0	150.0	500.0	150.0
JNYIR	HYDR0	1	1	21	21	50	8.76E+04	5.43E+04	0	0	.0	145.0	150.0	0	0
JNYIF	HYDR0	4	4	15	21	50	5.25E+04	2.70E+04	0	0	.0	45.0	50.0	60.0	25.0
JNYIR	HYDR0	1	4	21	21	50	2.01E+04	1.21E+04	0	0	.0	45.0	50.0	0	0
JNYIF	HYDR0	2	2	15	21	50	9.90E+04	5.55E+04	0	0	.0	90.0	100.0	100.0	50.0
JNYIR	HYDR0	1	2	21	21	50	4.21E+04	2.37E+04	0	0	.0	90.0	100.0	0	0

TABLE 6.1-28 CASE STUDY ONE
POOL MODIFICATIONS, YEAR 1

SELECTION OF CTH1F	RELEASES CTH1R	TO POOL, AND REMOVES CTH2F	CTH3F	FROM POOL
SELECTION OF CTH1R	RELEASES CTH1F	TO POOL, AND REMOVES NONE		FROM POOL
SELECTION OF CTH2F	RELEASES CTH2R	TO POOL, AND REMOVES CTH1F	CTH3F	FROM POOL
SELECTION OF CTH3F	RELEASES CTH3R	TO POOL, AND REMOVES CTH1F	CTH2F	FROM POOL
SELECTION OF DTH1F	RELEASES DTH1R	TO POOL, AND REMOVES DTH2F	DTH3F	FROM POOL
SELECTION OF DTH2F	RELEASES DTH2R	TO POOL, AND REMOVES DTH3F	DTH1F	FROM POOL
SELECTION OF DTH3F	RELEASES DTH3R	TO POOL, AND REMOVES DTH1F	DTH2F	FROM POOL
SELECTION OF ETH1F	RELEASES ETH1R	TO POOL, AND REMOVES ENU2F	ENU3F	FROM POOL
SELECTION OF ENU2F	RELEASES ENU2R	TO POOL, AND REMOVES ENU3F	ETH1F	FROM POOL
SELECTION OF ENU3F	RELEASES ENU3R	TO POOL, AND REMOVES ETH1F	ENU2F	FROM POOL
SELECTION OF FTH1F	RELEASES FTH1R	TO POOL, AND REMOVES FNU2F	FNU3F	FROM POOL
SELECTION OF FNU2F	RELEASES FNU2R	TO POOL, AND REMOVES FNU3F	FTH1F	FROM POOL
SELECTION OF FNU3F	RELEASES FNU3R	TO POOL, AND REMOVES FTH1F	FNU2F	FROM POOL
SELECTION OF GHY1F	RELEASES GHY1R	TO POOL, AND REMOVES NONE		FROM POOL
SELECTION OF HHY1F	RELEASES HHY1R	TO POOL, AND REMOVES HHY2F		FROM POOL
SELECTION OF HHY2F	RELEASES HHY2R	TO POOL, AND REMOVES HHY1F		FROM POOL
SELECTION OF IHY1F	RELEASES IHY1R	TO POOL, AND REMOVES IHY2F	IHY3F	FROM POOL
SELECTION OF IHY2F	RELEASES IHY2R	TO POOL, AND REMOVES IHY3F	IHY1F	FROM POOL
SELECTION OF IHY3F	RELEASES IHY3R	TO POOL, AND REMOVES IHY1F	IHY2F	FROM POOL
SELECTION OF JHY1F	RELEASES JHY1R	TO POOL, AND REMOVES JHY2F		FROM POOL
SELECTION OF JHY2F	RELEASES JHY2R	TO POOL, AND REMOVES JHY1F		FROM POOL
SELECTION OF ZPK1F	RELEASES ZPK1R	TO POOL, AND REMOVES ZPK3F	ZPK4F	FROM POOL
SELECTION OF ZPK3F	RELEASES ZPK3R	TO POOL, AND REMOVES ZPK1F	ZPK2F	FROM POOL
SELECTION OF ZPK2F	RELEASES ZPK2R	TO POOL, AND REMOVES ZPK3F	ZPK4F	FROM POOL
SELECTION OF ZPK4F	RELEASES ZPK4R	TO POOL, AND REMOVES ZPK1F	ZPK2F	FROM POOL
SELECTION OF ZPK1C	RELEASES ZPK1F	ZPK2F		FROM POOL
SELECTION OF ZPK3C	RELEASES ZPK3F	ZPK4F		FROM POOL

TABLE 6.1-3 SITE DETAILS FOR CASE STUDY ONE

SITE	STAGE	DATES	COMMENTS
A	1	year 1	Thermal plant currently under construction, due to be introduced into system during the planning period: 2 x 115 MW in year 1, and 1 x 115 MW in year 2.
	2	year 2	
B	1	year 3	Hydro plant currently under construction, 1 x 100 MW to be commissioned in year 3, to be followed by a further two identical units in years 4 - 6; the optimum date is to be established
	2	years 4 to 6	
C	1	years 4 to 7	An available thermal site. Initial planting with (a) 5 or 6 x 55 MW, or (b) 2 or 3 x 115 MW, or (c) 2 x 180 MW. Subsequent planting to be with units of the same rating. The site is to be fully developed by year 15.
	2	by year 15	
D	1	years 6 to 13	Similar to site C. Again optimum unit sizes and planting dates are sought. Full development of this site is compulsory by year 17.
	2	by year 17	
E		from year 10	A large site earmarked for thermal or nuclear development. Initially a choice of (a) 2 x 480 MW thermal or (b) 2 x 550 MW nuclear, or (c) 2 x 750 MW nuclear sets. Follow-on units are to be of the same type and size; up to 4 x 480 MW thermal, 4 x 550 MW nuclear, or 3 x 750 MW nuclear. Development of this site is optional.

TABLE 6.1-3 (CONTD)

SITE	STAGE	DATES	COMMENTS
F		from year 15	A further large site available for development with large thermal or nuclear units. Except for dates of availability, the site is similar to site E.
G	1 2	year 7 to 9 by year 11	A hydro site which must be planted with 30 MW units. Optimal dates of installation and the number to be installed at the second stage are to be determined.
H		from year 8	An optional hydro site with 40 MW or 80 MW choice of unit size, these being mutually exclusive alternatives.
I		from year 10	Another optional hydro site, with the choice of 45 MW, 90 MW, or 145 MW unit sizes. As with all other sites, subsequent site development will be with the same unit sizes as initial planting.
J		from year 15	Similar to sites H and I, with the choice of 45 MW or 90 MW units.

TABLE 6.1-4 CAPITAL EXPENDITURE CONSTRAINTS
FOR CASE STUDY ONE

i	F_i US\$ 10^6	L_i US\$ 10^6	K_i
1 to 4	250	200	3
5 to 9	400	300	3
10 to 14	500	400	3
15 to 20	500	400	3

To be interpreted as follows:

- At year i , the maximum capital expenditure over the past K_i years, including year i , is not to exceed F_i (foreign component) or L_i (local component).

FIGURE 6.1-1A CASE STUDY ONE
CHRONOLOGICAL LOAD CURVES OVER PLANNING PERIOD

1. MAXIMUM DEMAND

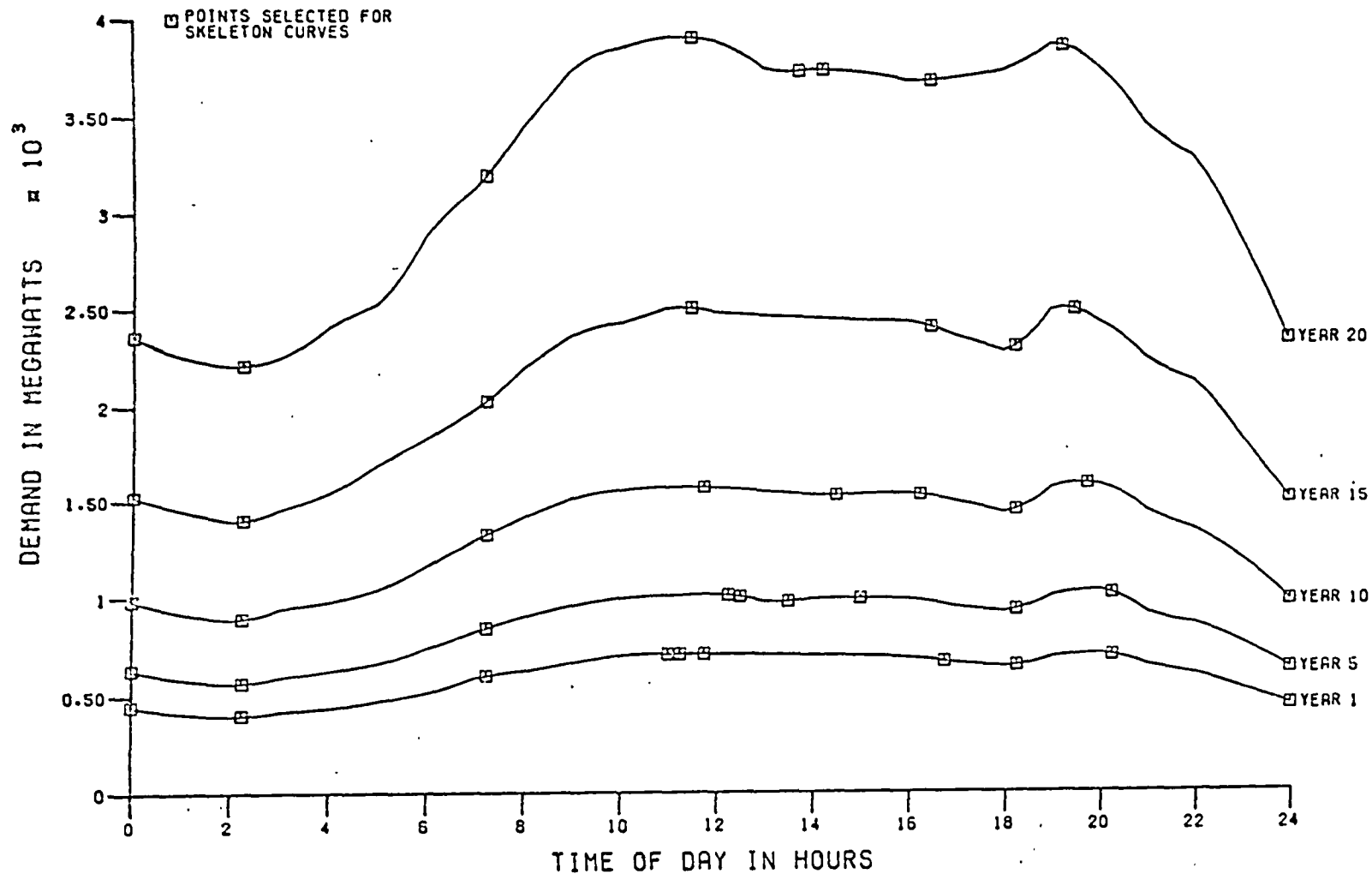


FIGURE 6.1-1B CASE STUDY ONE
CHRONOLOGICAL LOAD CURVES OVER PLANNING PERIOD

2. DRY SEASON

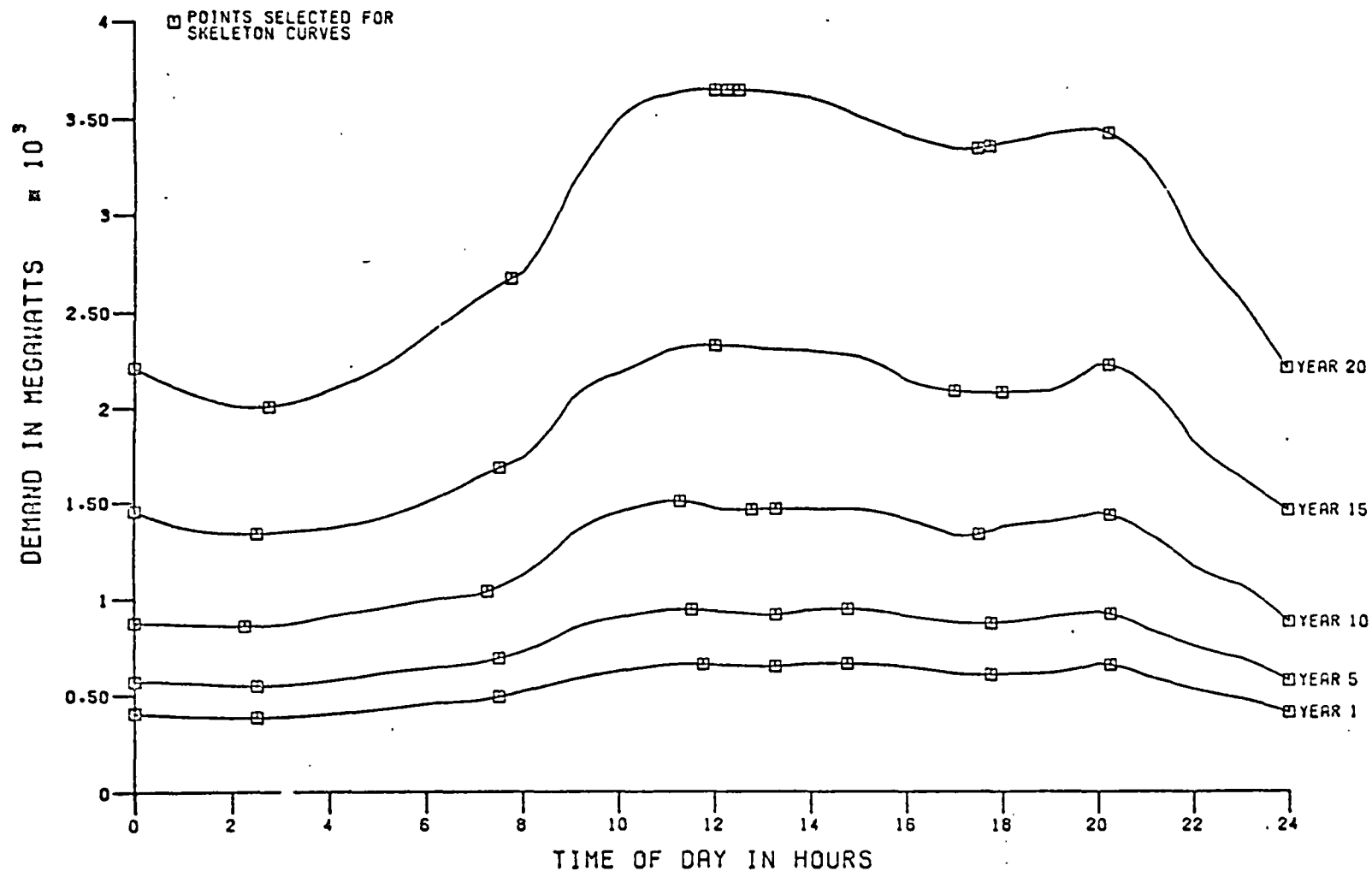


FIGURE 6.1-1C CASE STUDY ONE
CHRONOLOGICAL LOAD CURVES OVER PLANNING PERIOD

3. AVERAGE

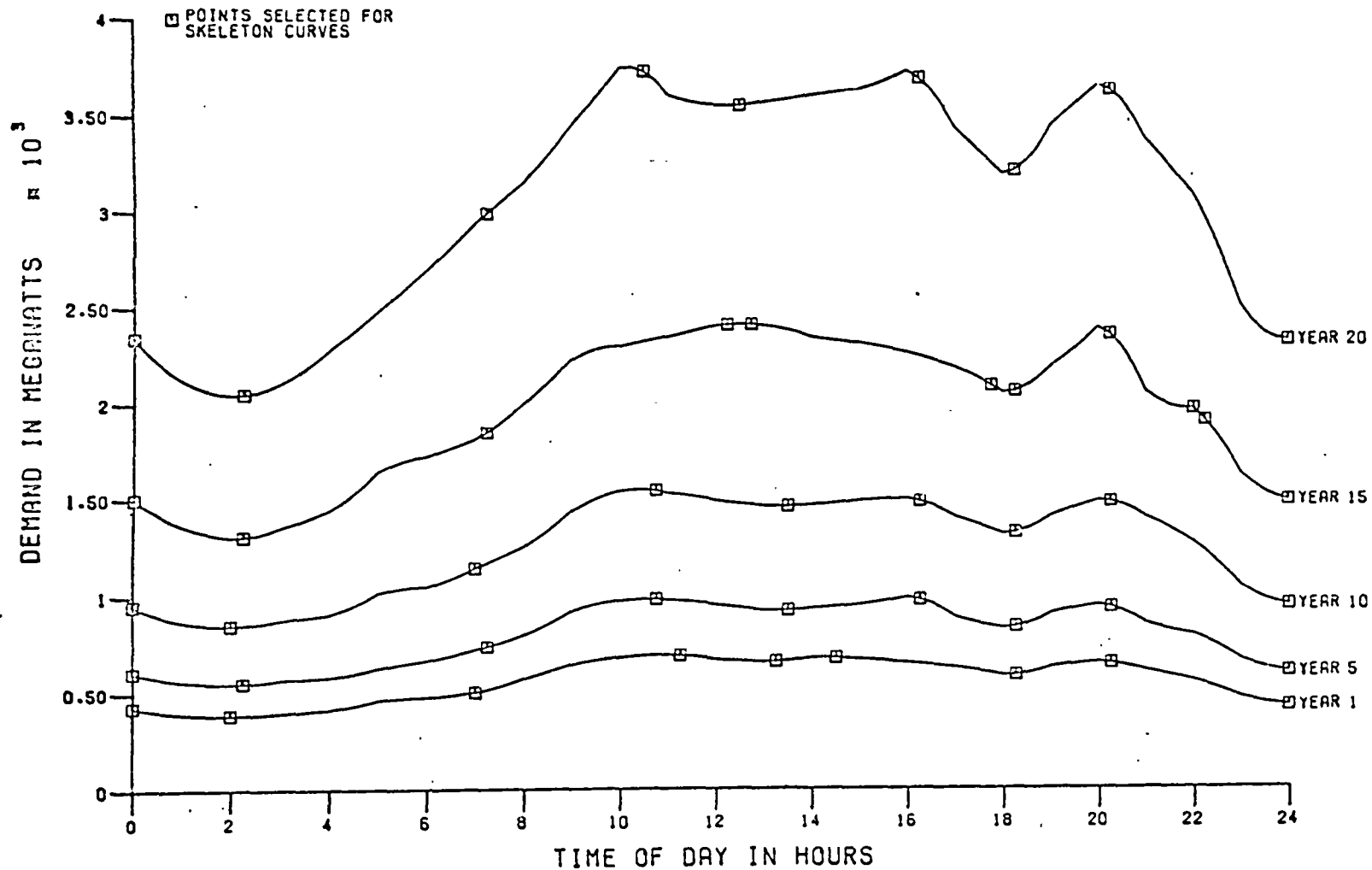


FIGURE 6.1-2 CASE STUDY ONE

ANNUAL LOAD DURATION CURVES OVER PLANNING PERIOD

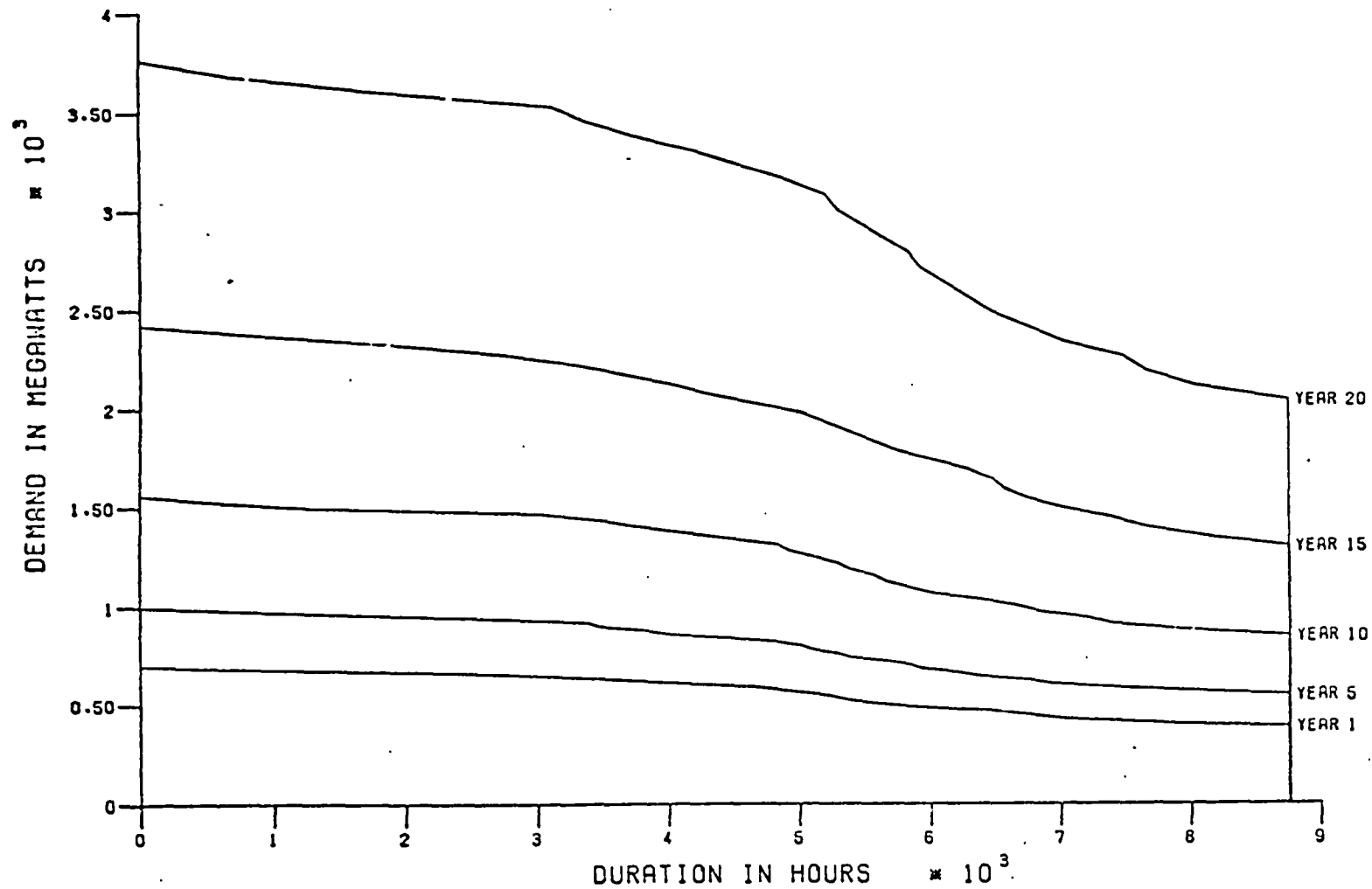


FIGURE 6.1-3A CASE STUDY ONE

PATH CONSTRAINTS OVER PLANNING PERIOD

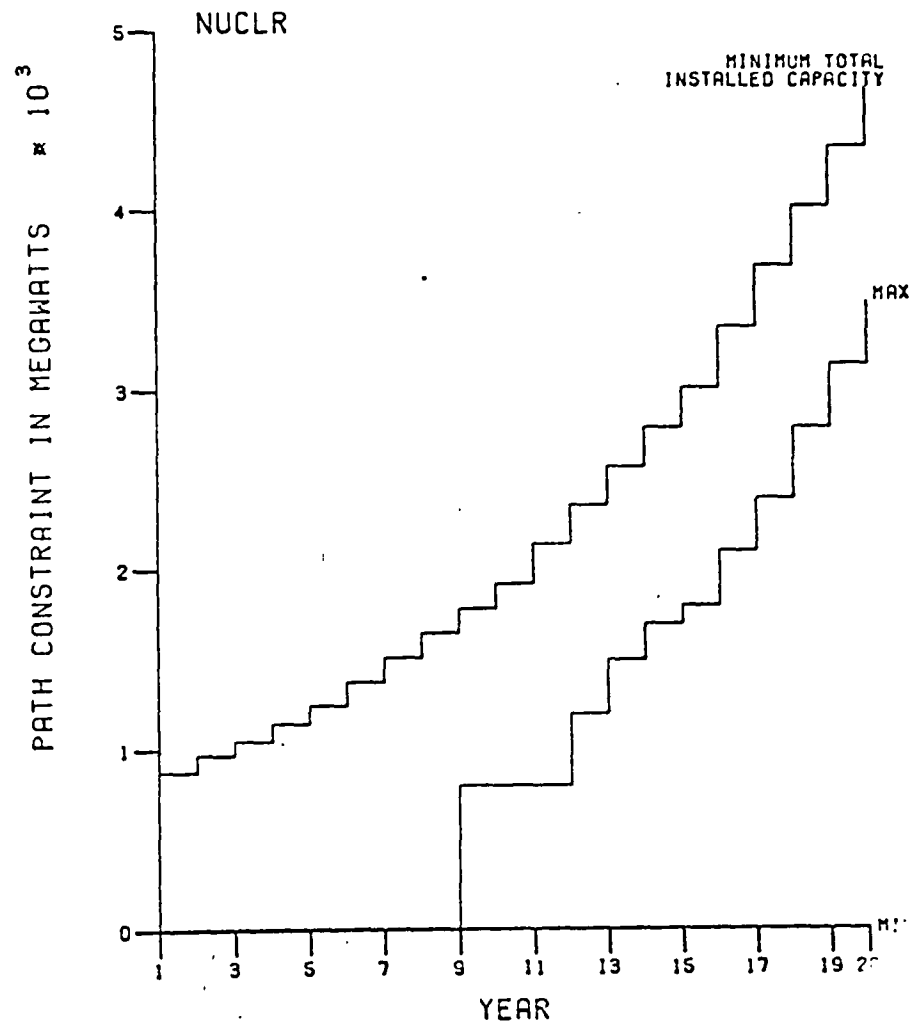
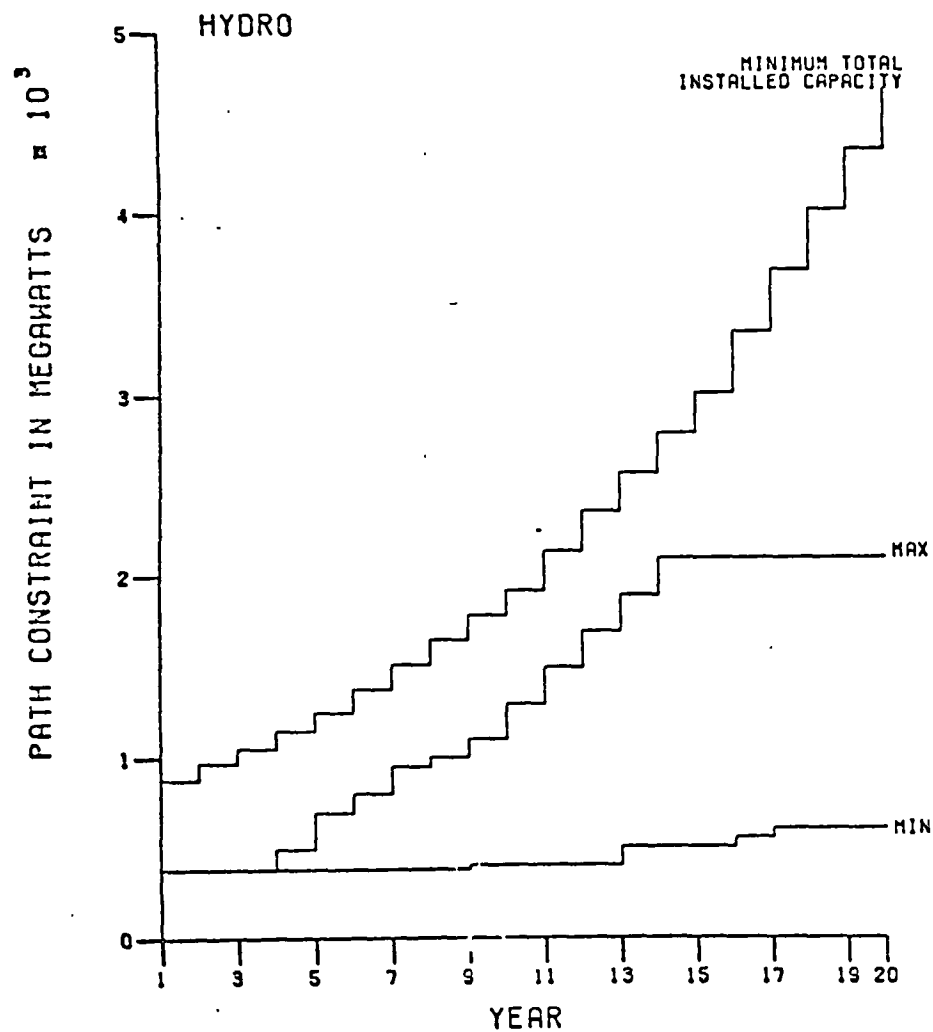


FIGURE 6.1-3B CASE STUDY ONE

PATH CONSTRAINTS OVER PLANNING PERIOD

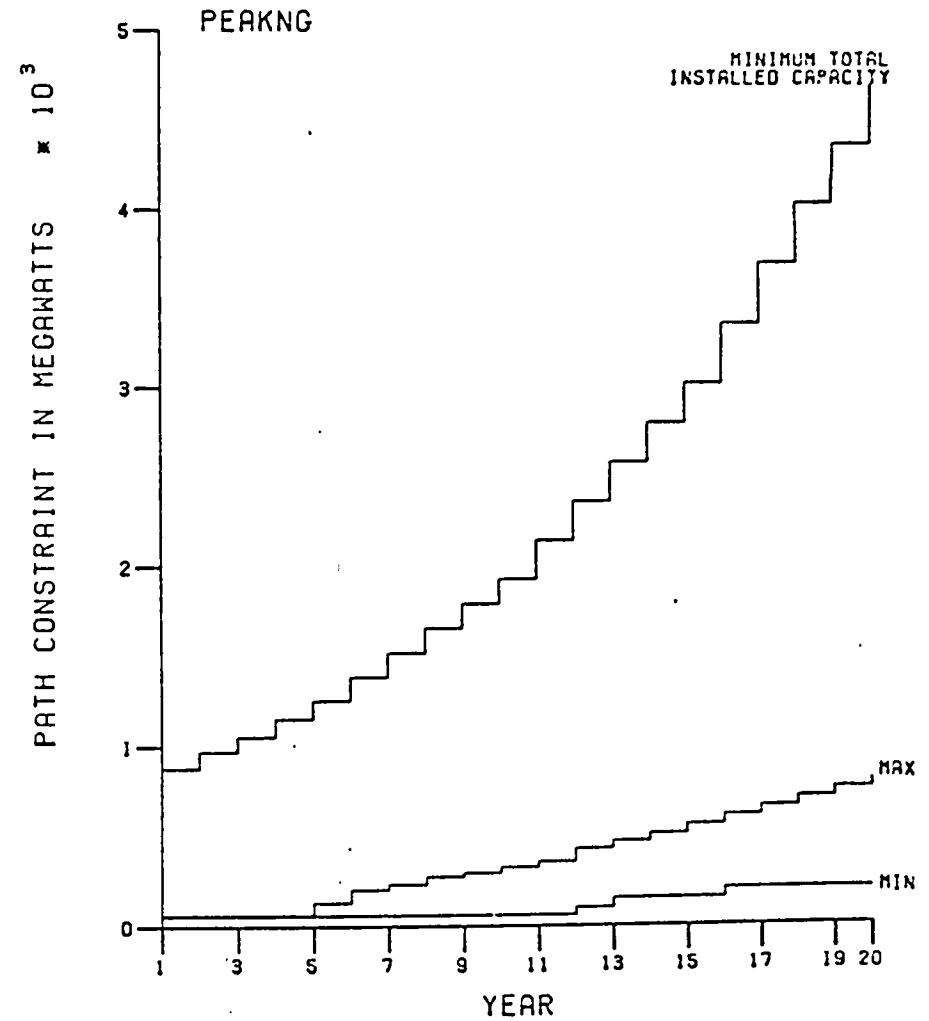
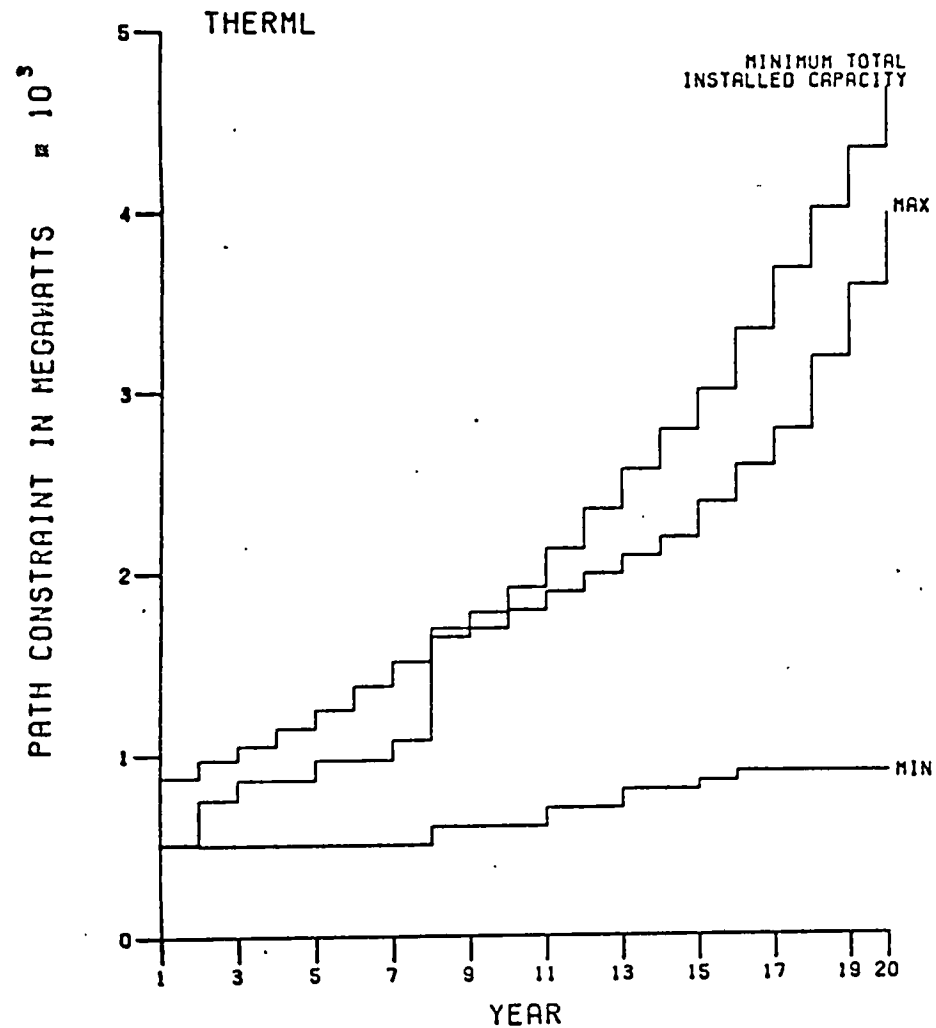


FIGURE 6.1-4 CASE STUDY ONE
DISCOUNT AND ESCALATION FACTORS

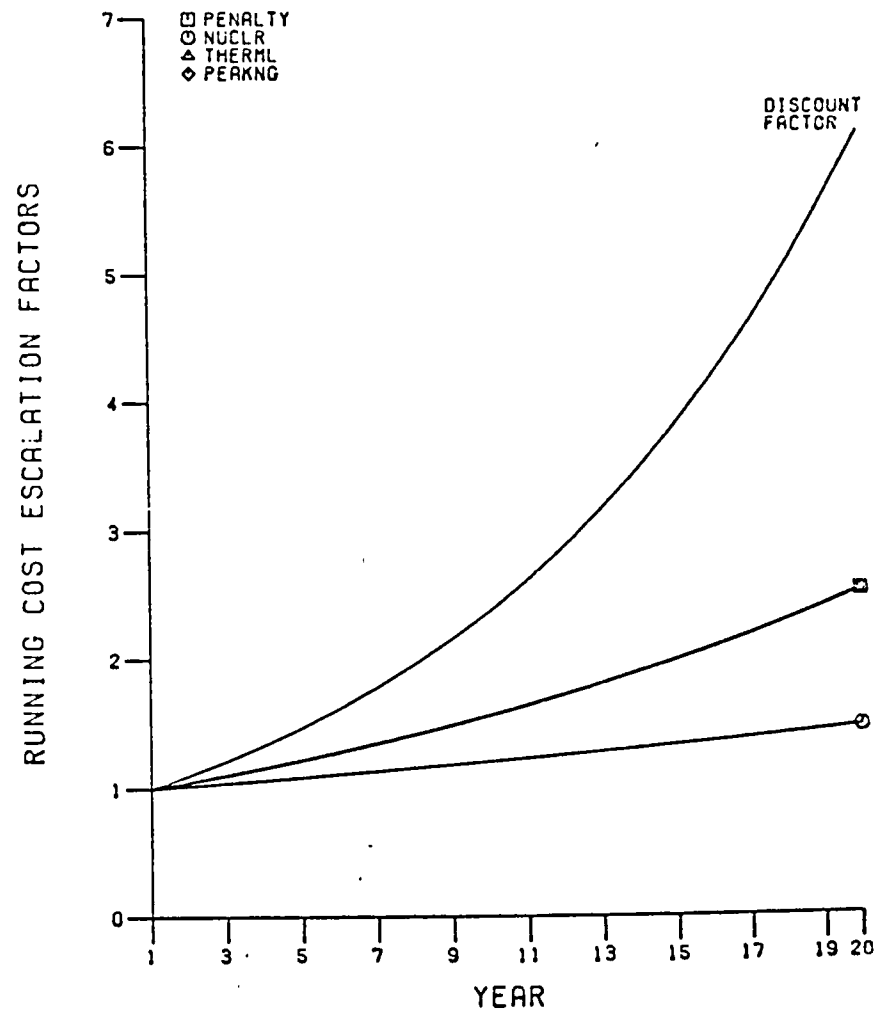
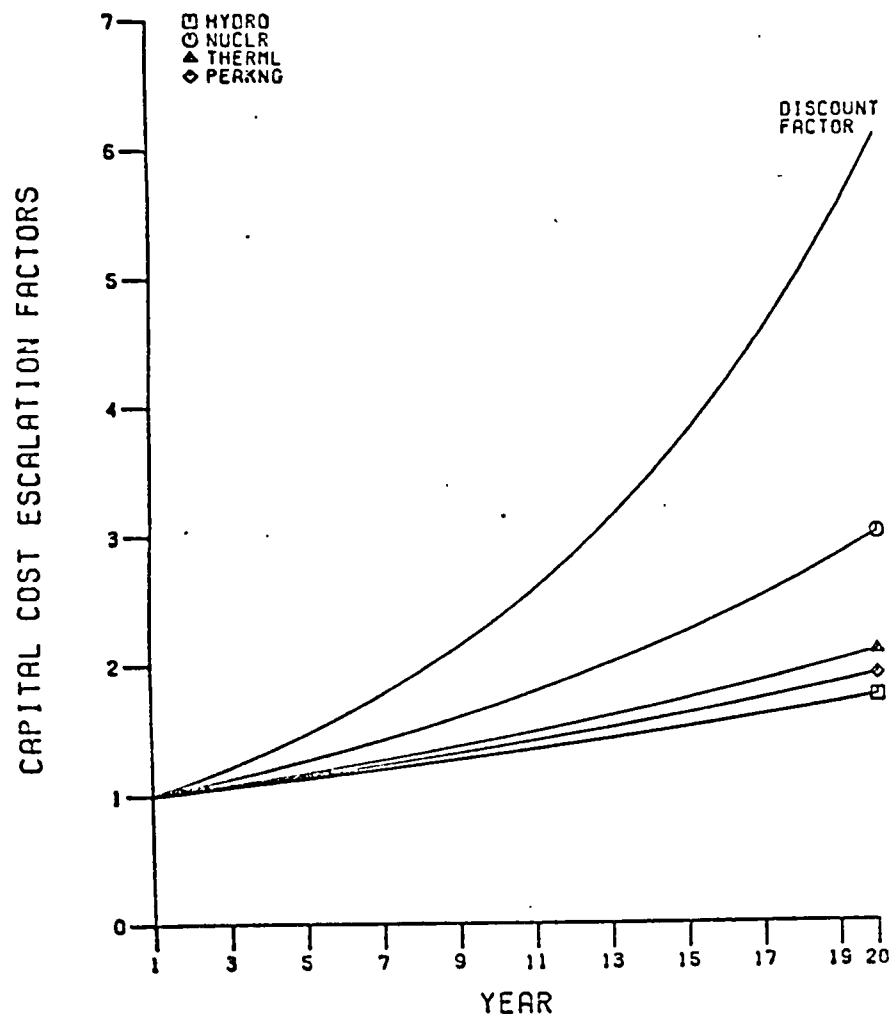
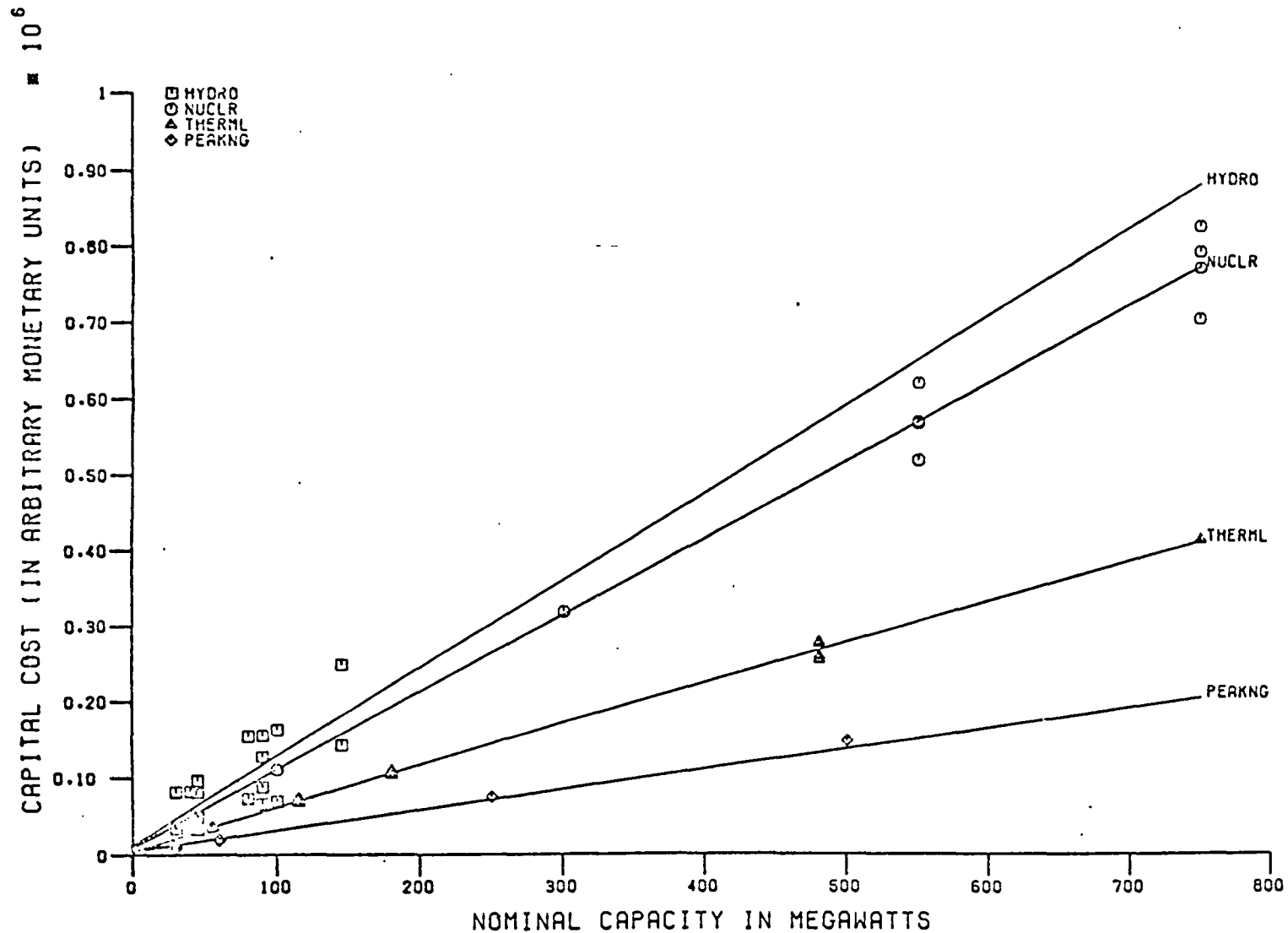


FIGURE 6.1-5 CASE STUDY ONE
LEAST-SQUARES FIT FOR CAPITAL COSTS



The assembly of all state and policy functions required a CP time of 175 seconds. The optimal expansion policy (from initial state at year 1) and expansion costs to horizon as predicted by these recursive functions are shown in Table 6.1-5. Total cost of optimal system development - costs of installation and costs of system operation - is predicted to be not greater than US\$ 1.66×10^9 over the twenty-year planning period, involving net additions to the system over this period of 1627 MW hydro, 44 MW nuclear, 1073 MW thermal, and 740 MW peaking capacity. This produces a state vector at the horizon:

$$\underline{s}_{20} = \begin{pmatrix} 2007 \text{ MW hydro} \\ 44 \text{ MW nuclear} \\ 1579 \text{ MW thermal} \\ 800 \text{ MW peaking} \end{pmatrix}$$

Tables 6.1-6 show similar optimal expansion strategies, not from the initial state at year one, but from randomly generated initial feasible states at randomly generated start years within the planning period, to the horizon. Inspection of Tables 6.1-5 and 6.1-6 reveals the following general trends:

- Largely hydro installation is recommended in the first half of the planning period, whilst the second half features the planting of mainly thermal and peaking capacity. This is a reflection of truncation effects at the planning horizon, a shortcoming which can only be satisfactorily overcome with a steady-state or infinite-horizon model. Hydro (and also nuclear) plants are associated with high capital costs coupled with benefits which are only fully realized over the whole life of the plant. In many cases plant lifetimes will extend to well beyond the planning horizon. Owing to truncation at the end of the planning period the full benefits of the installation of such plant are not recognized close to the horizon.
- Truncation at the horizon is also responsible for the fact that installation of substantial nuclear capacity is not recommended. Plant mix constraints (Fig 6.1-3A) preclude

the installation of nuclear plant until year 10, at which date the myopic view taken by this model causes it to select thermal or peaking capacity in preference to nuclear. If however the planting strategy is forced to deviate from that recommended in Table 6.1-5 in such a way that some nuclear capacity has been installed after year 10, under certain conditions the continuance of a nuclear planting programme can be advocated. This will happen if there is a shortfall in hydro capacity and excessive thermal and peaking capacity; in such cases nuclear planting is preferred to hydro. The policies of Tables 6.1-6 have been selected to illustrate this.

- Annual planting programmes are generally quite small and seldom conform to actual options for addition as defined in the pool of Table 6.1-2. It will never be possible, for example, to install 44 MW of nuclear capacity in year 17. Moreover the sequencing of installations will in general not satisfy the constraints contained in the pool data. Actual planting strategies will therefore be distortions of the optimal policy of Table 6.1-5, and may in the event differ significantly from the continuous-function idealized case. The traceback searches to be described in the following paragraphs will demonstrate to what extent this occurs.

TABLE 6.1-5 CASE STUDY ONE: OPTIMAL EXPANSION STRATEGY FROM STATE FUNCTIONS
 =====

YEAR	EXPANSION COST TO HORIZON	HYDRO	STATE (M\$) NUCLR	STATE (M\$) THERML	PEAKNG	HYDRO	DECISION (M\$) NUCLR	DECISION (M\$) THERML	PEAKNG
1	1.653907E+06	300.000	.000	506.501	60.000	0	0	0	0
2	1.615095E+06	300.000	.000	506.501	60.000	0	0	0	.001
3	1.569275E+06	300.000	.000	506.501	60.001	106.027	0	0	0
4	1.396974E+06	465.027	.000	506.501	60.001	101.521	0	0	0
5	1.224710E+06	587.549	.000	506.501	60.001	172.000	0	0	112.65
6	1.038047E+06	759.549	.000	506.501	172.632	190.451	0	0	J
7	8.78942E+05	950.000	.000	506.501	172.632	0	0	100.000	0
8	8.250147E+05	950.000	.000	606.501	172.632	105.000	0	0	0
9	7.624375E+05	1055.000	.000	606.501	172.632	156.328	0	0	76.000
10	7.191735E+05	1211.328	.000	606.501	243.632	179.000	0	100.000	0
11	5.472547E+05	1390.328	.000	706.501	243.632	0	0	32.246	0
12	6.890404E+05	1390.328	.000	706.750	243.632	445.000	0	65.095	42.000
13	4.117378E+05	1835.328	.000	823.846	293.632	0	0	0	45.952
14	5.178144E+05	1835.328	.000	823.846	336.584	0	0	50.000	4.680
15	4.747046E+05	1835.328	.000	873.646	341.271	0	0	46.154	111.000
16	4.197532E+05	1835.328	.000	920.000	452.271	171.743	0	0	0
17	3.592871E+05	2007.071	.000	920.000	452.271	0	43.000	232.037	198.039
18	2.951056E+05	2007.071	43.000	1152.837	650.311	0	0	242.454	99.689
19	2.202711E+05	2007.071	43.000	1395.291	750.000	0	.501	184.171	50.000
20	1.185032E+05	2007.071	43.501	1578.462	800.000	0	0	0	50.000

TABLE 6.1-6A CASE STUDY ONE, OPTIMAL EXPANSION STRATEGIES FROM STATE FUNCTIONS
 =====

YEAR	EXPANSION COST TO HORIZON	HYDR0	STATE NUCLR	(M0 THERML	PEAKNG	HYDR0	DECISION (M0 NUCLR	THERML	PEAKNG
19	1.217431E+05	1591.824	1001.273	3187.558	222.460	0	0	0	0
20	7.164413E+04	1591.824	1001.273	3187.558	222.460	0	0	0	0
YEAR	EXPANSION COST TO HORIZON	HYDR0	STATE NUCLR	(M0 THERML	PEAKNG	HYDR0	DECISION (M0 NUCLR	THERML	PEAKNG
20	6.310278E+04	1313.336	1359.017	2516.199	404.507	0	0	0	46.019
YEAR	EXPANSION COST TO HORIZON	HYDR0	STATE NUCLR	(M0 THERML	PEAKNG	HYDR0	DECISION (M0 NUCLR	THERML	PEAKNG
14	2.578750E+05	1380.812	1470.588	1096.684	376.767	0	0	0	0
15	2.503793E+05	1380.812	1470.588	1096.684	376.767	0	0	50.000	0
16	2.141404E+05	1380.812	1470.588	1146.684	376.767	0	0	0	0
17	1.736906E+05	1380.812	1470.588	1146.684	376.767	0	0	0	33.315
18	1.593003E+05	1380.812	1470.588	1146.684	407.082	0	109.000	0	39.966
19	1.283970E+05	1380.812	1579.588	1146.684	447.078	0	46.000	0	101.191
20	8.142772E+04	1380.812	1625.588	1146.684	548.269	0	0	0	208.563
YEAR	EXPANSION COST TO HORIZON	HYDR0	STATE NUCLR	(M0 THERML	PEAKNG	HYDR0	DECISION (M0 NUCLR	THERML	PEAKNG
18	3.378948E+05	1059.954	25.151	1392.756	372.960	0	0	230.761	256.301
19	2.110931E+05	1059.954	25.151	1623.517	629.260	0	0	73.827	170.740
20	1.404997E+05	1059.954	25.151	1697.344	800.000	0	0	0	50.000
YEAR	EXPANSION COST TO HORIZON	HYDR0	STATE NUCLR	(M0 THERML	PEAKNG	HYDR0	DECISION (M0 NUCLR	THERML	PEAKNG
16	2.897113E+05	808.463	1628.004	1613.587	233.717	8.312	0	0	0
17	2.509300E+05	816.776	1628.004	1613.587	233.717	292.926	0	0	41.918
18	1.690155E+05	1109.702	1628.004	1613.587	275.635	0	149.242	1072.000	0
19	9.972200E+04	1109.702	1777.247	2605.587	275.635	0	0	0	0
20	5.817252E+04	1109.702	1777.247	2685.587	275.635	0	0	0	63.000
YEAR	EXPANSION COST TO HORIZON	HYDR0	STATE NUCLR	(M0 THERML	PEAKNG	HYDR0	DECISION (M0 NUCLR	THERML	PEAKNG
15	1.682611E+05	1798.624	1631.107	1934.525	494.345	0	0	0	0
16	1.467592E+05	1798.624	1631.107	1934.525	494.345	0	0	0	0
17	1.227720E+05	1798.624	1631.107	1934.525	494.345	0	0	0	0
18	9.729273E+04	1798.624	1631.107	1934.525	494.345	0	0	0	0
19	7.056971E+04	1798.624	1631.107	1934.525	494.345	0	0	0	0
20	3.904081E+04	1798.624	1631.107	1934.525	494.345	0	0	0	0

TABLE 6.1-68 CASE STUDY ONE: OPTIMAL EXPANSION STRATEGIES FROM STATE FUNCTIONS

YEAR	EXPANSION COST TO HORIZON	HYDRO	STATE NUCLR	(M\$) THERML	PEAKING	HYDRO	DECISION NUCLR	(M\$) THERML	PEAKING
3	1.501350E+06	280.000	.000	725.832	60.001	0	0	0	0
4	1.445731E+06	290.000	.000	725.832	60.001	185.000	0	0	0
5	1.197014E+06	565.000	.000	725.832	60.001	0	0	0	0
6	1.209305E+06	565.000	.000	725.832	60.001	385.000	0	0	43.650
7	8.224377E+05	650.000	.000	725.832	103.658	0	0	0	0
8	8.294514E+05	650.000	.000	725.832	103.658	74.211	0	0	100.518
9	7.743360E+05	1024.211	.000	725.832	204.176	79.580	0	0	0
10	7.810585E+05	1103.701	.000	725.832	204.176	186.885	0	100.000	0
11	5.715557E+05	1290.675	.000	825.832	204.176	0	0	.000	0
12	7.604954E+05	1290.675	.000	825.832	204.176	447.173	0	51.603	5.432
13	4.346190E+05	1737.848	.000	877.435	209.608	0	0	11.038	95.808
14	5.760591E+05	1737.848	.000	888.473	305.415	0	0	50.000	12.340
15	5.510220E+05	1737.848	.000	938.473	317.755	0	0	50.000	166.070
16	4.460146E+05	1737.848	.000	938.473	483.755	184.665	0	54.000	7
17	3.813322E+05	1922.513	.000	1042.473	483.755	0	.000	217.158	77.887
18	3.427406E+05	1922.513	.000	1259.631	561.640	177.487	0	178.537	188.360
19	1.784637E+05	2100.000	.000	1438.168	750.000	0	0	77.632	50.000
20	1.046360E+05	2100.000	.000	1516.000	800.000	0	0	0	50.000
YEAR	EXPANSION COST TO HORIZON	HYDRO	STATE NUCLR	(M\$) THERML	PEAKING	HYDRO	DECISION NUCLR	(M\$) THERML	PEAKING
12	8.475152E+05	666.107	505.870	1522.665	333.007	0	425.000	0	3.686
13	5.692161E+05	666.107	930.870	1522.665	336.703	29.206	0	0	27.254
14	6.060493E+05	666.107	930.870	1522.665	364.057	42.333	23.926	433.000	0
15	5.227112E+05	737.646	954.796	1955.665	364.057	52.427	0	0	0
16	4.397627E+05	790.073	954.796	1955.665	364.057	11.279	0	0	0
17	3.811505E+05	801.351	954.796	1955.665	364.057	0	83.228	760.662	42.203
18	2.692004E+05	801.351	1043.024	2716.327	406.260	0	0	0	.714
19	2.120554E+05	801.351	1043.024	2716.327	406.674	0	0	802.000	0
20	1.051911E+05	801.351	1043.024	3518.327	406.974	0	0	0	0
YEAR	EXPANSION COST TO HORIZON	HYDRO	STATE NUCLR	(M\$) THERML	PEAKING	HYDRO	DECISION NUCLR	(M\$) THERML	PEAKING
17	2.057807E+05	1549.064	470.060	2637.357	367.222	0	0	0	0
18	2.509572E+05	1549.064	470.060	2637.357	367.222	0	0	0	0
19	1.806316E+05	1549.064	470.060	2637.357	367.222	0	0	0	2.871
20	1.065214E+05	1549.064	470.060	2637.357	370.093	0	0	0	62.638

TRACEBACK SEARCH NO 1

This was an exploratory search with LDC-simulation, and required a CP time of 27 seconds. A skeleton listing of the fifteen optimal planting schedules produced by the program is given in Tables 6.1-7. From these it can be seen that apart from the fixed installations at site A and the first stage of development of site B, the fifteen schedules show the following features:

- Site B, second stage planting to be expedited at the earliest possible date, year 4.
- For site C, 115 MW units are preferred. For initial site development, two units are to be installed in year 7 (the latest permissible date), with two follow-on units in year 14, unless site D is planted with 180 MW units, in which case three follow-on units are to be installed in the second stage of development of site C, in year 14.

Optimal strategies Nos 9, 12, 13, and 14 also feature a further single follow-on unit at site C in year 19.

- In contrast, there is no particular preference shown for any unit size at site D, all three options being featured. First stage development of this site is to be at the latest permissible date (year 13), with 4 x 55 MW, 2 x 115 MW, or 2 x 180 MW. For the second stage development, if 55 MW units are chosen, two follow-on units are to be installed in year 17, followed by another four units in year 19; if 115 MW unit sizes are selected then two follow-on units at year 17 (schedule No 15 also featuring a further follow-on unit in year 19); if 180 MW units, then two such units in year 16 constitute the second stage site development.
- For site E, 480 MW thermal units are preferred to the nuclear options. Initial planting is in year 15 (two units) with a single follow-on unit in year 18; unless site D is planted with 180 MW units, in which case initial development of site E will be in year 17, with no subsequent site development before year 20.

- Initial development of site G will be as late as possible (year 9). The second stage of development involves the planting of a further four units, also at latest permissible date, year 11.
- For site H, 80 MW unit sizes are preferred, two units to be installed in year 8. For strategies Nos 1 to 4 inclusive a single follow-on unit is to be installed in year 19.
- Site I, 145 MW units, two to be installed in year 10.
- Sites F and J are not to be developed in the next twenty years.
- No significant preference is shown for diesel or gas turbines for the peaking complement, neither for the 30 MW or 60 MW available unit sizes. Gas turbines however appear to be marginally more economical.
- No installations are to be made in year 5 and year 16 or 18.

The total costs of system development range from US\$ 3.535×10^9 to US\$ 3.545×10^9 . These costs and the installed capacities at the end of the planning period are summarized for convenience and comparisons in Table 6.1-11. Details of the best, 8th best, and 15th best development schedules are reproduced in Tables 6.1-8.

TABLE 6.1-7A CASE STUDY ONE, TRACEBACK SEARCH NO 1
SKELETON LISTING OF OPTIMAL STRATEGIES 1 TO 10, LDC-SIMULATION
=====

YEAR	NUMBER 1	NUMBER 2	NUMBER 3	NUMBER 4	NUMBER 5	NUMBER 6	NUMBER 7	NUMBER 8	NUMBER 9	NUMBER 10
1	2 X ATH1C	2 X ATH1C	2 X ATH1C	2 X ATH1C	2 X ATH1C	2 X ATH1C	2 X ATH1C	2 X ATH1C	2 X ATH1C	2 X ATH1C
2	1 X ATH2C	1 X ATH2C	1 X ATH2C	1 X ATH2C	1 X ATH2C	1 X ATH2C	1 X ATH2C	1 X ATH2C	1 X ATH2C	1 X ATH2C
3	1 X BHY1C	1 X BHY1C	1 X BHY1C	1 X BHY1C	1 X BHY1C	1 X BHY1C	1 X BHY1C	1 X BHY1C	1 X BHY1C	1 X BHY1C
4	2 X BHY2C	2 X BHY2C	2 X BHY2C	2 X BHY2C	2 X BHY2C	2 X BHY2C	2 X BHY2C	2 X BHY2C	2 X BHY2C	2 X BHY2C
5	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL
6	4 X ZPK3C	4 X ZPK3C	4 X ZPK3C	4 X ZPK3C	4 X ZPK3C	4 X ZPK3C	4 X ZPK3C	4 X ZPK3C	4 X ZPK3C	4 X ZPK3C
7	2 X CTH2F	2 X CTH2F	2 X CTH2F	2 X CTH2F	2 X CTH2F	2 X CTH2F	2 X CTH2F	2 X CTH2F	2 X CTH2F	2 X CTH2F
8	2 X HHY2F	2 X HHY2F	2 X HHY2F	2 X HHY2F	2 X HHY2F	2 X HHY2F	2 X HHY2F	2 X HHY2F	2 X HHY2F	2 X HHY2F
9	3 X GHY1F	3 X GHY1F	3 X GHY1F	3 X GHY1F	3 X GHY1F	3 X GHY1F	3 X GHY1F	3 X GHY1F	3 X GHY1F	3 X GHY1F
10	2 X IHY3F	2 X IHY3F	2 X IHY3F	2 X IHY3F	2 X IHY3F	2 X IHY3F	2 X IHY3F	2 X IHY3F	2 X IHY3F	2 X IHY3F
11	4 X GHY1R	4 X GHY1R	4 X GHY1R	4 X GHY1R	4 X GHY1R	4 X GHY1R	4 X GHY1R	4 X GHY1R	4 X GHY1R	4 X GHY1R
12	6 X ZPK3F	6 X ZPK3F	6 X ZPK3F	6 X ZPK3F	6 X ZPK3F	6 X ZPK3F	6 X ZPK3F	6 X ZPK3F	6 X ZPK3F	6 X ZPK3F
13	2 X DTH2F	2 X DTH2F	2 X DTH2F	2 X DTH2F	2 X DTH2F	4 X DTH1F	4 X DTH1F	2 X DTH2F	2 X DTH2F	2 X DTH2F
14	2 X CTH2R	2 X CTH2R	2 X CTH2R	2 X CTH2R	2 X CTH2R	2 X CTH2R	2 X CTH2R	5 X ZPK3F	2 X CTH2R	5 X ZPK3F
15	2 X ETH1F	2 X ETH1F	2 X ETH1F	2 X ETH1F	2 X ETH1F	2 X ETH1F	2 X ETH1F	3 X CTH2R	2 X ETH1F	3 X CTH2R
16	NULL	NULL	NULL	NULL	NULL	NULL	NULL	2 X DTH2R	NULL	2 X DTH2R
17	2 X DTH2R	2 X DTH2R	2 X DTH2R	2 X DTH2R	2 X DTH2R	2 X DTH2R	2 X DTH2R	2 X ETH1F	2 X DTH2R	2 X ETH1F
18	1 X ETH1R	1 X ETH1R	1 X ETH1R	1 X ETH1R	1 X ETH1R	1 X ETH1R	1 X ETH1R	1 X ETH1R	1 X ETH1R	1 X ETH1R
19	1 X HHY2R	1 X HHY2R	1 X HHY2R	1 X HHY2R	4 X DTH1R	4 X DTH1R	4 X DTH1R	2 X ZPK3F	1 X CTH2R	2 X ZPK3F
20	3 X ZPK4F	3 X ZPK2F	6 X ZPK3F	6 X ZPK1F	3 X ZPK4F	3 X ZPK2F	6 X ZPK3F	6 X ZPK3F	3 X ZPK4F	6 X ZPK1F
TOTAL										
COST	3.5354E+06	3.5360E+06	3.5362E+06	3.5366E+06	3.5427E+06	3.5434E+06	3.5435E+06	3.5438E+06	3.5439E+06	3.5439E+06
	=====	=====	=====	=====	=====	=====	=====	=====	=====	=====

TABLE 6.1-7B CASE STUDY ONE, TRACEBACK SEARCH NO 1
SKELETON LISTING OF OPTIMAL STRATEGIES 11 TO 15, LDC-SIMULATION

YEAR	NUMBER 11	NUMBER 12	NUMBER 13	NUMBER 14	NUMBER 15
1	2 X ATH1C	2 X ATH1C	2 X ATH1C	2 X ATH1C	2 X ATH1C
2	1 X ATH2C	1 X ATH2C	1 X ATH2C	1 X ATH2C	1 X ATH2C
3	1 X BHY1C	1 X BHY1C	1 X BHY1C	1 X BHY1C	1 X BHY1C
4	2 X BHY2C	2 X BHY2C	2 X BHY2C	2 X BHY2C	2 X BHY2C
5	NULL	NULL	NULL	NULL	NULL
6	4 X ZPK1C	4 X ZPK1C	4 X ZPK3C	4 X ZPK1C	4 X ZPK3C
7	2 X CTH2F	2 X CTH2F	2 X CTH2F	2 X CTH2F	2 X CTH2F
8	2 X HHY2F	2 X HHY2F	2 X HHY2F	2 X HHY2F	2 X HHY2F
9	3 X GHY1F	3 X GHY1F	3 X GHY1F	3 X GHY1F	3 X GHY1F
10	2 X JHY3F	2 X JHY3F	2 X JHY3F	2 X JHY3F	2 X JHY3F
11	4 X GHY1R	4 X GHY1R	4 X GHY1R	4 X GHY1R	4 X GHY1R
12	6 X ZPK1F	6 X ZPK1F	6 X ZPK3F	6 X ZPK1F	6 X ZPK3F
13	4 X DTH1F	2 X DTH2F	2 X DTH2F	2 X DTH2F	2 X DTH2F
14	2 X CTH2R	2 X CTH2R	2 X CTH2R	2 X CTH2R	2 X CTH2R
15	2 X ETH1F	2 X ETH1F	2 X ETH1F	2 X ETH1F	2 X ETH1F
16	NULL	NULL	NULL	NULL	NULL
17	2 X DTH1R	2 X DTH2R	2 X DTH2R	2 X DTH2R	2 X DTH2R
18	1 X ETH1R	1 X ETH1R	1 X ETH1R	1 X ETH1R	1 X ETH1R
19	4 X DTH1R	1 X CTH2R	1 X CTH2R	1 X CTH2R	1 X DTH2R
20	6 X ZPK1F	3 X ZPK2F	6 X ZPK3F	6 X ZPK1F	3 X ZPK4F
TOTAL	3.5439E+06	3.5446E+06	3.5447E+06	3.5451E+06	3.5451E+06
COST	*****	*****	*****	*****	*****

TABLE 6.1-0A CASE STUDY ONE, TRACEBACK SEARCH NO 1
OPTIMAL EXPANSION STRATEGY, NUMBER 1

YR	SYSTEM CHANGES FOR THIS YEAR	INSTALLED CAPACITY AT START OF YEAR (M E G A W A T T S)					COST OF FOREIGN	ADDITIONS LOCAL	SYSTEM RUNNING COST	CUMULATIVE COST AT END OF YEAR	YR
		HYDRO	NUCLR	THERML	PEAKING	TOTAL	(ALL COSTS PRESENT-WORTHED TO YEAR ONE)				
1	INSTALL ATH1C 2 X 115 MW THERML	380	0	507	60	947	9.54000E+04	4.96000E+04	5.56456E+04	2.00646E+05	1
2	INSTALL ATH2C 1 X 115 MW THERML	380	0	737	60	1177	4.60436E+04	2.19913E+04	6.44376E+04	3.33118E+05	2
3	INSTALL BHY1C 1 X 100 MW HYDRO	380	0	852	60	1292	9.23246E+04	4.97132E+04	7.29303E+04	5.48006E+05	3
4	INSTALL BHY2C 2 X 100 MW HYDRO	400	0	852	60	1362	7.30510E+04	3.97355E+04	6.63629E+04	7.27236E+05	4
5	NO ADDITIONS THIS YEAR RETIRE XTH5 4 X 20 MW THERML	680	0	852	60	1592			4.13939E+04	7.68620E+05	5
6	INSTALL ZPK3C 4 X 30 MW PEAKING	680	0	772	60	1512	2.41002E+04	4.57225E+03	5.46234E+04	8.52215E+05	6
7	INSTALL CTH2F 2 X 115 MW THERML	680	0	772	180	1632	7.45666E+04	2.24271E+04	6.78117E+04	1.01E02E+06	7
8	INSTALL WHY2F 2 X 80 MW HYDRO RETIRE XTH3 4 X 10 MW THERML	680	0	1002	180	1862	1.37584E+05	5.68008E+04	7.82681E+04	1.28947E+06	8
9	INSTALL GHY1F 3 X 30 MW HYDRO	840	0	962	180	1982	9.09484E+04	5.31862E+04	6.97302E+04	1.50334E+06	9
10	INSTALL IHY3F 2 X 145 MW HYDRO RETIRE ZPK1 3 X 20 MW PEAKING	930	0	962	180	2072	1.84819E+05	9.03069E+04	6.73042E+04	1.94586E+06	10
11	INSTALL GHY1R 4 X 30 MW HYDRO	1220	0	962	120	2302	4.41453E+04	2.48706E+04	5.21815E+04	1.96706E+06	11
12	INSTALL ZPK3F 6 X 30 MW PEAKING	1340	0	962	120	2422	2.51148E+04	4.75891E+03	5.36813E+04	2.05061E+06	12
13	INSTALL DTH2F 2 X 115 MW THERML	1340	0	962	300	2602	5.48902E+04	1.79563E+04	6.06483E+04	2.19211E+06	13
14	INSTALL CTH2R 2 X 115 MW THERML	1340	0	1192	300	2832	4.36863E+04	2.07394E+04	8.21357E+04	2.33097E+06	14
15	INSTALL ETH1F 2 X 480 MW THERML RETIRE XTH1 3 X 20 MW THERML	1340	0	1422	300	3062	1.09540E+05	3.77572E+04	9.46153E+04	2.67079E+06	15
16	NO ADDITIONS THIS YEAR	1340	0	2298	300	3938			1.06081E+05	2.77687E+06	16
17	INSTALL DTH2R 2 X 115 MW THERML	1340	0	2298	300	3938	3.81528E+04	1.94840E+04	1.21600E+05	2.95620E+06	17
18	INSTALL ETH1R 1 X 480 MW THERML	1340	0	2528	300	4168	8.45813E+04	1.37106E+04	1.37076E+05	3.19158E+06	18
19	INSTALL WHY2R 1 X 80 MW HYDRO	1340	0	3008	300	4643	1.40545E+04	8.23671E+03	1.46015E+05	3.36282E+06	19
20	INSTALL ZPK4F 3 X 60 MW PEAKING RETIRE XTH2 3 X 28 MW THERML	1420	0	3008	300	4728	1.46170E+04	2.92340E+03	1.54946E+05	3.53538E+06	20
	QDC-SIMULATION RUN					TOTALS	1.33794E+06	5.33780E+05	1.65866E+06		

TABLE 6.1-23 CASE STUDY ONE, TRACEBACK SEARCH NO 1
OPTIMAL EXPANSION STRATEGY, NUMBER 8

YR	SYSTEM CHANGES FOR THIS YEAR	INSTALLED CAPACITY AT START OF YEAR (M E G A W A T T S)					COST OF FOREIGN	ADDITIONS LOCAL	SYSTEM RUNNING COST	CUMULATIVE COST AT END OF YEAR (ONE)	YR
		HYDRO	NUCLR	THERML	PEAKING	TOTAL					
1	INSTALL ATH1C 2 X 115 MW THERML	380	0	507	60	947	9.54000E+04	4.66000E+04	5.56456E+04	2.00646E+05	1
2	INSTALL ATH2C 1 X 115 MW THERML	380	0	737	60	1177	4.60435E+04	2.19013E+04	6.44376E+04	3.33118E+05	2
3	INSTALL BHY1C 1 X 100 MW HYDRO	380	0	852	60	1292	9.23246E+04	4.97132E+04	7.29333E+04	5.48066E+05	3
4	INSTALL BHY2C 2 X 100 MW HYDRO	480	0	852	60	1392	7.30510E+04	3.97355E+04	6.63629E+04	7.27246E+05	4
5	NO ADDITIONS THIS YEAR RETIRE XTH5 4 X 20 MW THERML	680	0	852	60	1592			4.13039E+04	7.66624E+05	5
6	INSTALL ZPK3C 4 X 30 MW PEAKING	680	0	772	60	1512	2.41002E+04	4.57225E+03	5.49234E+04	8.52215E+05	6
7	INSTALL CTH2F 2 X 115 MW THERML	680	0	772	180	1632	7.45566E+04	2.24271E+04	6.76117E+04	1.01602E+06	7
8	INSTALL MHY2F 2 X 80 MW HYDRO RETIRE XTH3 4 X 10 MW THERML	680	0	1002	180	1862	1.37584E+05	5.68008E+04	7.82681E+04	1.28947E+06	8
9	INSTALL GHY1F 3 X 30 MW HYDRO	840	0	862	180	1982	9.09404E+04	5.31662E+04	6.97302E+04	1.50334E+06	9
10	INSTALL JHY3F 2 X 145 MW HYDRO RETIRE XPK1 3 X 20 MW PEAKING	930	0	862	180	2072	1.84016E+05	9.03069E+04	6.73942E+04	1.84566E+06	10
11	INSTALL GHY1R 4 X 30 MW HYDRO	1220	0	862	120	2302	4.41453E+04	2.48706E+04	5.21815E+04	1.95706E+06	11
12	INSTALL ZPK3F 6 X 30 MW PEAKING	1340	0	862	120	2422	2.51146E+04	4.75891E+03	5.36813E+04	2.05061E+06	12
13	INSTALL OTH3F 2 X 180 MW THERML	1340	0	862	300	2602	7.68260E+04	3.43833E+04	6.66483E+04	2.23047E+06	13
14	INSTALL ZPK3F 5 X 30 MW PEAKING	1340	0	1322	300	2662	1.85206E+04	3.51092E+03	8.13862E+04	2.33390E+06	14
15	INSTALL CTH3R 3 X 115 MW THERML RETIRE XTH1 3 X 20 MW THERML	1340	0	1322	450	3112	6.23814E+04	2.84123E+04	9.83650E+04	2.52405E+06	15
16	INSTALL OTH3R 2 X 180 MW THERML	1340	0	1583	450	3373	6.05309E+04	3.31100E+04	1.13258E+05	2.73095E+06	16
17	INSTALL ETH1F 2 X 400 MW THERML	1340	0	1843	450	3733	1.78372E+05	3.37505E+04	1.26210E+05	3.08929E+06	17
18	NO ADDITIONS THIS YEAR	1340	0	2603	450	4693			1.38792E+05	3.20600E+06	18
19	INSTALL ZPK3F 2 X 30 MW PEAKING	1340	0	2603	450	4693	5.46564E+03	1.03537E+03	1.50403E+05	3.36228E+06	19
20	INSTALL ZPK3F 6 X 30 MW PEAKING RETIRE XTH2 3 X 20 MW THERML	1340	0	2803	510	4753	1.54260E+04	2.82340E+03	1.62436E+05	3.54377E+06	20
	QDC-SIMULATION RUN					TOTALS	1.30563E+05	5.56093E+05	1.60205E+06	*****	

TABLE 6.1-UC CASE STUDY ONE, TRACEBACK SEARCH NO 1
OPTIMAL EXPANSION STRATEGY, NUMBER 15

YR	SYSTEM CHANGES FOR THIS YEAR	INSTALLED CAPACITY AT START OF YEAR (MEGAWATTS)					COST OF FOREIGN	ADDITIONAL LOCAL	SYSTEM RUNNING COST	CUMULATIVE COST AT END OF YEAR	YR
		HYDRO	NUCL	THERM	PEAKING	TOTAL	(ALL COSTS PRESENT-WORTHED TO YEAR ONE)				
1	INSTALL ATH1C 2 X 115 MW THERM	380	0	507	60	947	9.54000E+04	4.06000E+04	5.56456E+04	2.00646E+05	1
2	INSTALL ATH2C 1 X 115 MW THERM	380	0	737	60	1177	4.60435E+04	2.19913E+04	6.44376E+04	3.33118E+05	2
3	INSTALL BHY1C 1 X 100 MW HYDRO	380	0	852	60	1292	9.23246E+04	4.97132E+04	7.29303E+04	5.48086E+05	3
4	INSTALL BHY2C 2 X 100 MW HYDRO	480	0	852	60	1392	7.30510E+04	3.97355E+04	6.63620E+04	7.27236E+05	4
5	NO ADDITIONS THIS YEAR RETIRE XTH5 4 X 20 MW THERM	680	0	852	60	1592			4.13836E+04	7.60620E+05	5
6	INSTALL ZPK2C 4 X 30 MW PEAKING	680	0	772	60	1512	2.41002E+04	4.57225E+03	5.49234E+04	8.52215E+05	6
7	INSTALL DTH2F 2 X 115 MW THERM	680	0	772	180	1632	7.45666E+04	2.24271E+04	6.76117E+04	1.01682E+06	7
8	INSTALL MHY2F 2 X 80 MW HYDRO RETIRE XTH3 4 X 10 MW THERM	680	0	1002	180	1862	1.37504E+05	5.68008E+04	7.82681E+04	1.28947E+06	8
9	INSTALL GHY1F 3 X 30 MW HYDRO	840	0	962	180	1982	9.00484E+04	5.31662E+04	6.97302E+04	1.50334E+06	9
10	INSTALL JHY3F 2 X 145 MW HYDRO RETIRE XPK1 3 X 20 MW PEAKING	930	0	962	180	2072	1.84816E+05	9.03069E+04	6.73942E+04	1.84580E+06	10
11	INSTALL GHY1R 4 X 30 MW HYDRO	1220	0	962	120	2302	4.41453E+04	2.48705E+04	5.21815E+04	1.96706E+06	11
12	INSTALL ZPK3F 6 X 30 MW PEAKING	1340	0	962	120	2422	2.51148E+04	4.75891E+03	5.36913E+04	2.05061E+06	12
13	INSTALL DTH2F 2 X 115 MW THERM	1340	0	962	300	2602	5.49909E+04	1.79560E+04	6.06403E+04	2.18211E+06	13
14	INSTALL DTH2R 2 X 115 MW THERM	1340	0	1192	300	2832	4.36069E+04	2.07394E+04	8.21357E+04	2.33897E+06	14
15	INSTALL ETH1F 2 X 400 MW THERM RETIRE XTH1 3 X 28 MW THERM	1340	0	1422	300	3062	1.69548E+05	3.77572E+04	9.45153E+04	2.67074E+06	15
16	NO ADDITIONS THIS YEAR	1340	0	2298	300	3938			1.06081E+05	2.77687E+06	16
17	INSTALL DTH2R 2 X 115 MW THERM	1340	0	2298	300	3938	3.81528E+04	1.94340E+04	1.21688E+05	2.05620E+06	17
18	INSTALL ETH1R 1 X 400 MW THERM	1340	0	2528	300	4168	8.45913E+04	1.37196E+04	1.37070E+05	3.19158E+06	18
19	INSTALL DTH2R 1 X 115 MW THERM	1340	0	3008	300	4548	1.70521E+04	8.70323E+03	1.40015E+05	3.36636E+06	19
20	INSTALL ZPK4F 3 X 60 MW PEAKING RETIRE XTH2 3 X 28 MW THERM	1340	0	3123	300	4763	1.46170E+04	2.62340E+03	1.61170E+05	3.54507E+06	20
CLDC-SIMULATION RUN)						TOTALS	1.34084E+06	5.30292E+05	1.66488E+06		

TRACEBACK SEARCH NO 2

After several similar exploratory runs, with marginal changes applied to the pool, Traceback Search No 2 was conducted, the pool of Table 6.1-2 being modified by:

- Compulsory development of the second stage at site B in year 14.
- Removal of 55 MW and 180 MW options at both sites C and D, thus standardizing for future system development on the use of 115 MW thermal units at these sites. Site C is to be initially developed in year 7.
- Removal of 750 MW nuclear options at site E; the initial development of this site (with 480 MW thermal or 550 MW nuclear units) constrained to year 15 onwards, although development of the site remains optional.
- Initial development of site G compulsory in year 9.
- Removal of 40 MW options at site H, 45 MW and 90 MW options at site I, and all options at sites F and J.
- Removal of all peaking options except ZPK3C (30 MW gas turbines in years 5 or 6) and ZPK4F (60 MW gas turbines at "renewable" sites), thus standardizing on gas turbines for the future peaking complement.

No changes were made to the dates of availability of the remaining candidates in the pool, nor to the programmed retirement dates for existing plant.

With these modifications a traceback search with CLC-simulation was conducted, this requiring a CP time of 108 seconds. Inspection of the computer list-out of the path taken during the search reveals that the eventual optimal schedules were in fact identified within about 70 seconds, the remainder of the time being required for examination and eventual rejection of other feasible paths. A skeleton listing of the fifteen least-cost development strategies produced by this run is shown in Tables 6.1-9.

Fundamentally the same trends as in Traceback Search No 1 (Tables 6.1-7) are evident, with one significant difference. Second-stage development of site C is brought forward to year 8 (one year after initial planting), displacing the development of site H, which is now no longer featured in any of the optimal development schedules. Any shortfall in total installed capacity as a result of this is counterbalanced by the installation of more peaking capacity, or one more follow-on unit (115 MW) at sites C or D, or a single follow-on unit (145 MW) at site I.

The results of this traceback search evaluate the costs of the development schedules to range from US\$ 3.609×10^9 to US\$ 3.655×10^9 . Detailed analyses of the best, 5th best, and 15th best development schedules identified during this run are given in Tables 6.1-10.

TABLE 6.1-9A CASE STUDY ONE, TRACEBACK SEARCH NO 2
SKELETON LISTING OF OPTIMAL STRATEGIES 1 TO 10, CLC-SIMULATION

YEAR	NUMBER 1	NUMBER 2	NUMBER 3	NUMBER 4	NUMBER 5	NUMBER 6	NUMBER 7	NUMBER 8	NUMBER 9	NUMBER 10
1	2 X ATH1C	2 X ATH1C	2 X ATH1C	2 X ATH1C	2 X ATH1C	2 X ATH1C	2 X ATH1C	2 X ATH1C	2 X ATH1C	2 X ATH1C
2	1 X ATH2C	1 X ATH2C	1 X ATH2C	1 X ATH2C	1 X ATH2C	1 X ATH2C	1 X ATH2C	1 X ATH2C	1 X ATH2C	1 X ATH2C
3	1 X BHY1C	1 X BHY1C	1 X BHY1C	1 X BHY1C	1 X BHY1C	1 X BHY1C	1 X BHY1C	1 X BHY1C	1 X BHY1C	1 X BHY1C
4	2 X BHY2C	2 X BHY2C	2 X BHY2C	2 X BHY2C	2 X BHY2C	2 X BHY2C	2 X BHY2C	2 X BHY2C	2 X BHY2C	2 X BHY2C
5	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL
6	4 X ZPK3C	4 X ZPK3C	4 X ZPK3C	4 X ZPK3C	4 X ZPK3C	4 X ZPK3C	4 X ZPK3C	4 X ZPK3C	4 X ZPK3C	4 X ZPK3C
7	2 X CTH2F	2 X CTH2F	2 X CTH2F	2 X CTH2F	2 X CTH2F	2 X CTH2F	2 X CTH2F	2 X CTH2F	2 X CTH2F	2 X CTH2F
8	2 X CTH2R	2 X CTH2R	2 X CTH2R	2 X CTH2R	2 X CTH2R	2 X CTH2R	2 X CTH2R	2 X CTH2R	2 X CTH2R	2 X CTH2R
9	3 X GHY1F	3 X GHY1F	3 X GHY1F	3 X GHY1F	3 X GHY1F	3 X GHY1F	3 X GHY1F	3 X GHY1F	3 X GHY1F	3 X GHY1F
10	2 X JHY3F	2 X JHY3F	2 X JHY3F	2 X JHY3F	2 X JHY3F	2 X JHY3F	2 X JHY3F	2 X JHY3F	2 X JHY3F	2 X JHY3F
11	4 X GHY1R	4 X GHY1R	4 X GHY1R	4 X GHY1R	4 X GHY1R	4 X GHY1R	4 X GHY1R	4 X GHY1R	4 X GHY1R	4 X GHY1R
12	2 X ZPK4F	2 X ZPK4F	2 X ZPK4F	2 X ZPK4F	2 X ZPK4F	2 X ZPK4F	2 X ZPK4F	2 X ZPK4F	2 X ZPK4F	2 X ZPK4F
13	2 X DTH2F	2 X DTH2F	2 X DTH2F	2 X DTH2F	2 X DTH2F	2 X DTH2F	2 X DTH2F	2 X DTH2F	2 X DTH2F	2 X DTH2F
14	3 X ZPK4F	3 X ZPK4F	3 X ZPK4F	3 X ZPK4F	3 X ZPK4F	3 X ZPK4F	3 X ZPK4F	3 X ZPK4F	3 X ZPK4F	3 X ZPK4F
15	2 X ETH1F	2 X ETH1F	2 X ETH1F	2 X ETH1F	2 X ETH1F	2 X ETH1F	2 X ETH1F	2 X ETH1F	2 X ETH1F	2 X ETH1F
16	1 X ZPK4F	NULL	1 X ZPK4F	1 X ZPK4F	NULL	1 X ZPK4F	1 X ZPK4F	1 X CTH2R	1 X CTH2R	NULL
17	2 X DTH2R	2 X DTH2R	2 X DTH2R	2 X DTH2R	2 X ZPK4F	2 X DTH2R	2 X DTH2R	2 X DTH2R	2 X DTH2R	2 X DTH2R
18	3 X ZPK4F	1 X ETH1R	1 X ETH1R	3 X ZPK4F	1 X ETH1R	3 X ZPK4F	1 X ETH1R	3 X ZPK4F	3 X ZPK4F	1 X ETH1R
19	1 X ETH1R	2 X ZPK4F	1 X ZPK4F	1 X ETH1R	3 X ZPK4F	1 X ETH1R	1 X CTH2R	1 X ETH1R	1 X ETH1R	1 X JHY3R
20	1 X ZPK4F	3 X ZPK4F	3 X ZPK4F	2 X ZPK4F	3 X ZPK4F	1 X CTH2R	3 X ZPK4F	1 X ZPK4F	2 X ZPK4F	3 X ZPK4F
TOTAL COST	3.6083E+06	3.6101E+06	3.6134E+06	3.6147E+06	3.6177E+06	3.6262E+06	3.6322E+06	3.6332E+06	3.6391E+06	3.6422E+06
	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****

TABLE 6.1-9B CASE STUDY ONE, TRACEBACK SEARCH NO 2
SKELETON LISTING OF OPTIMAL STRATEGIES 11 TO 15, CLC-SIMULATION

YEAR	NUMBER 11	NUMBER 12	NUMBER 13	NUMBER 14	NUMBER 15
1	2 X ATH1C	2 X ATH1C	2 X ATH1C	2 X ATH1C	2 X ATH1C
2	1 X ATH2C	1 X ATH2C	1 X ATH2C	1 X ATH2C	1 X ATH2C
3	1 X BHY1C	1 X BHY1C	1 X BHY1C	1 X BHY1C	1 X BHY1C
4	2 X BHY2C	2 X BHY2C	2 X BHY2C	2 X BHY2C	2 X BHY2C
5	NULL	NULL	NULL	NULL	NULL
6	4 X ZPK3C	4 X ZPK3C	4 X ZPK3C	4 X ZPK3C	4 X ZPK3C
7	2 X CTH2F	2 X CTH2F	2 X CTH2F	2 X CTH2F	2 X CTH2F
8	2 X CTH2R	2 X CTH2R	2 X CTH2R	2 X CTH2R	2 X CTH2R
9	3 X GHY1F	3 X GHY1F	3 X GHY1F	3 X GHY1F	3 X GHY1F
10	2 X JHY3F	2 X JHY3F	2 X JHY3F	2 X JHY3F	2 X JHY3F
11	4 X GHY1R	4 X GHY1R	4 X GHY1R	4 X GHY1R	4 X GHY1R
12	2 X ZPK4F	2 X ZPK4F	2 X ZPK4F	2 X ZPK4F	2 X ZPK4F
13	2 X DTH2F	2 X DTH2F	2 X DTH2F	2 X DTH2F	2 X DTH2F
14	3 X ZPK4F	3 X ZPK4F	3 X ZPK4F	3 X ZPK4F	3 X ZPK4F
15	2 X ETH1F	2 X ETH1F	2 X ETH1F	2 X ETH1F	2 X ETH1F
16	1 X ZPK4F	1 X ZPK4F	1 X JHY3R	1 X CTH2R	1 X ZPK4F
17	2 X DTH2R	2 X DTH2R	2 X DTH2R	2 X DTH2R	2 X DTH2R
18	3 X ZPK4F	1 X ETH1R	2 X ZPK4F	3 X ZPK4F	1 X ETH1R
19	1 X ETH1R	1 X JHY3R	1 X ETH1R	1 X ETH1R	1 X CTH2R
20	1 X JHY3R	2 X ZPK4F	1 X ZPK4F	1 X DTH2R	1 X JHY3R
TOTAL					
COST	3.6437E+06	3.6449E+06	3.6511E+06	3.6510E+06	3.6554E+06
	*****	*****	*****	*****	*****

TABLE 6.1-10A CASE STUDY ONE, TRACEBACK SEARCH NO 2
OPTIMAL EXPANSION STRATEGY, NUMBER 1

YR	SYSTEM CHANGES FOR THIS YEAR		INSTALLED CAPACITY AT START OF YEAR (M E G A W A T T S)					COST OF FOREIGN	ADDITIONS LOCAL	SYSTEM RUNNING COST	CUMULATIVE COST AT END OF YEAR ONE	YR
			HYDRO	NUCLR	THERML	PEAKING	TOTAL					
(ALL COSTS PRESENT-40YR-THED TO YEAR ONE)												
1	INSTALL	ATH1C 2 X 115 MW THERML	380	0	507	60	547	9.54000E+04	4.96000E+04	6.94460E+04	2.14447E+05	1
2	INSTALL	ATH2C 1 X 115 MW THERML	380	0	737	60	1177	4.60436E+04	2.19913E+04	7.36564E+04	3.56130E+05	2
3	INSTALL	BHY1C 1 X 100 MW HYDRO	380	0	852	60	1292	9.23246E+04	4.97132E+04	7.89661E+04	5.77142E+05	3
4	INSTALL	BHY2C 2 X 100 MW HYDRO	480	0	852	60	1392	7.30510E+04	3.97355E+04	6.80800E+04	7.59009E+05	4
5	NO ADDITIONS THIS YEAR RETIRE XTH5	4 X 20 MW THERML	660	0	852	60	1592			5.21339E+04	8.11143E+05	5
6	INSTALL	ZPK3C 4 X 30 MW PEAKING	680	0	772	60	1512	2.41002E+04	4.57225E+03	6.85332E+04	9.08349E+05	6
7	INSTALL	OTH2F 2 X 115 MW THERML	680	0	772	180	1632	7.45666E+04	2.24271E+04	7.80625E+04	1.08340E+06	7
8	INSTALL	OTH2R 2 X 115 MW THERML RETIRE XTH3	680	0	1002	180	1862	6.15856E+04	2.90371E+04	8.18025E+04	1.25583E+06	8
9	INSTALL	GHY1F 3 X 30 MW HYDRO	680	0	1192	180	2052	9.09484E+04	5.31862E+04	8.67835E+04	1.48675E+06	9
10	INSTALL	IHY3F 2 X 145 MW HYDRO RETIRE XPK1	770	0	1192	180	2142	1.84819E+05	9.03060E+04	8.08426E+04	1.84272E+06	10
11	INSTALL	GHY1R 4 X 30 MW HYDRO	1060	0	1192	120	2372	4.41453E+04	2.40700E+04	8.10920E+04	1.99282E+06	11
12	INSTALL	ZPK4F 2 X 60 MW PEAKING	1180	0	1192	120	2462	1.58630E+04	3.17261E+03	8.60228E+04	2.09060E+06	12
13	INSTALL	OTH2F 2 X 115 MW THERML	1180	0	1192	240	2612	5.48908E+04	1.79569E+04	1.01714E+05	2.27324E+06	13
14	INSTALL	ZPK4F 3 X 60 MW PEAKING	1180	0	1422	240	2842	2.10655E+04	4.21311E+03	1.03707E+05	2.40223E+06	14
15	INSTALL	ETH1F 2 X 430 MW THERML RETIRE XTH1	1180	0	1422	420	3022	1.89540E+05	3.77572E+04	1.01806E+05	2.74140E+06	15
16	INSTALL	ZPK4F 1 X 60 MW PEAKING	1180	0	2238	420	3890	6.21651E+03	1.24330E+03	1.22967E+05	2.87183E+06	16
17	INSTALL	OTH2R 2 X 115 MW THERML	1180	0	2238	480	3958	3.81520E+04	1.94340E+04	1.31922E+05	3.06139E+06	17
18	INSTALL	ZPK4F 3 X 60 MW PEAKING	1180	0	2528	480	4188	1.65106E+04	3.30212E+03	1.38905E+05	3.22000E+06	18
19	INSTALL	ETH1R 1 X 480 MW THERML	1180	0	2528	660	4368	7.89772E+04	1.29713E+04	1.43405E+05	3.45644E+06	19
20	INSTALL	ZPK4F 1 X 60 MW PEAKING RETIRE XTH2	1180	0	3008	660	4848	4.87234E+03	9.74467E+02	1.46605E+05	3.60889E+06	20
CLC-SIMULATION RUN								TOTALS	1.22406E+06	4.80515E+05	1.85826E+06	

TABLE 6.1-10B CASE STUDY ONE, TRACEBACK SEARCH NO 2
OPTIMAL EXPANSION STRATEGY, NUMBER 5

YR	SYSTEM CHANGES FOR THIS YEAR	INSTALLED CAPACITY AT START OF YEAR (M E G A W A T T S)					COST OF FOREIGN	ADDITIONS LOCAL	SYSTEM RUNNING COST	CUMULATIVE COST AT END OF YEAR	YR
		HYDRO	NUCLR	THERML	PEAKING	TOTAL	(ALL COSTS PRESENT-WORTHED TO YEAR ONE)				
1	INSTALL ATH1C 2 X 115 MW THERML	380	0	507	60	947	9.54000E+04	4.96000E+04	6.94439E+04	2.14447E+05	1
2	INSTALL ATH2C 1 X 115 MW THERML	380	0	737	60	1177	4.60436E+04	2.10913E+04	7.36504E+04	3.56138E+05	2
3	INSTALL BHY1C 1 X 100 MW HYDRO	380	0	852	60	1292	9.23246E+04	4.97132E+04	7.89661E+04	5.77142E+05	3
4	INSTALL BHY2C 2 X 100 MW HYDRO	480	0	852	60	1392	7.30510E+04	3.97355E+04	6.60090E+04	7.59009E+05	4
5	NO ADDITIONS THIS YEAR RETIRE XTH5 4 X 20 MW THERML	680	0	852	60	1592			5.21339E+04	8.11143E+05	5
6	INSTALL ZPK3C 4 X 30 MW PEAKING	680	0	772	60	1512	2.41002E+04	4.57225E+03	6.85332E+04	9.08348E+05	6
7	INSTALL CTH2F 2 X 115 MW THERML	680	0	772	180	1632	7.45666E+04	2.24271E+04	7.80625E+04	1.08340E+06	7
8	INSTALL CTH2R 2 X 115 MW THERML RETIRE XTH3 4 X 10 MW THERML	680	0	1002	180	1862	6.15856E+04	2.90371E+04	8.18025E+04	1.25583E+06	8
9	INSTALL GHY1F 3 X 30 MW HYDRO	680	0	1192	180	2052	9.09484E+04	5.31862E+04	8.67835E+04	1.40675E+06	9
10	INSTALL INY3F 2 X 145 MW HYDRO RETIRE XPK1 3 X 20 MW PEAKING	770	0	1192	180	2142	1.84819E+05	9.03069E+04	8.08426E+04	1.84272E+06	10
11	INSTALL GHY1R 4 X 30 MW HYDRO	1060	0	1192	120	2372	4.41453E+04	2.48706E+04	8.10920E+04	1.99282E+06	11
12	INSTALL ZPK4F 2 X 60 MW PEAKING	1180	0	1192	120	2492	1.58630E+04	3.17261E+03	8.68220E+04	2.09868E+06	12
13	INSTALL DTH2F 2 X 115 MW THERML	1180	0	1192	240	2612	5.48939E+04	1.79569E+04	1.01714E+05	2.27324E+06	13
14	INSTALL DTH2R 2 X 115 MW THERML	1180	0	1422	240	2842	4.51445E+04	2.30545E+04	1.03707E+05	2.44515E+06	14
15	INSTALL ETH1F 2 X 400 MW THERML RETIRE XTH1 3 X 28 MW THERML	1180	0	1652	240	3072	1.99548E+05	3.77572E+04	9.55552E+04	2.77801E+06	15
16	NO ADDITIONS THIS YEAR	1180	0	2528	240	3948			1.33963E+05	2.91197E+06	16
17	INSTALL ZPK4F 2 X 60 MW PEAKING	1180	0	2528	240	3948	1.16983E+04	2.33947E+03	1.31523E+05	3.05753E+06	17
18	INSTALL ETH1R 1 X 400 MW THERML	1190	0	2528	360	4068	8.45913E+04	1.37196E+04	1.38108E+05	3.29395E+06	18
19	INSTALL ZPK4F 3 X 60 MW PEAKING	1130	0	3008	360	4548	1.55350E+04	3.10700E+03	1.42560E+05	3.45516E+06	19
20	INSTALL ZPK4F 3 X 60 MW PEAKING RETIRE XTH2 3 X 28 MW THERML	1180	0	3008	540	4728	1.46170E+04	2.92340E+03	1.45031E+05	3.61773E+06	20
	CLC-SIMULATION RUN					TOTALS	1.22087E+06	4.89471E+05	1.89936E+06		

TABLE 6.1-10C CASE STUDY ONE, TRACEBACK SEARCH NO 2
OPTIMAL EXPANSION STRATEGY, NUMBER 15

YR	SYSTEM CHANGES FOR THIS YEAR	INSTALLED CAPACITY AT START OF YEAR (M E G A W A T T S)					COST OF FOREIGN	ADDITIONS LOCAL	SYSTEM RUNNING COST	CUMULATIVE COST AT END OF YEAR	YR
		HYDRO	NUCLR	THERML	PEAKNG	TOTAL					
1	INSTALL ATH1C 2 X 115 MW THERML	380	0	507	60	947	8.54000E+04	4.86000E+04	6.21450E+04	2.14447E+05	1
2	INSTALL ATH2C 1 X 115 MW THERML	360	0	737	60	1177	4.60436E+04	2.19913E+04	7.36364E+04	3.56138E+05	2
3	INSTALL BMY1C 1 X 100 MW HYDRO	300	0	852	60	1292	9.23246E+04	4.97132E+04	7.83661E+04	5.77142E+05	3
4	INSTALL BMY2C 2 X 100 MW HYDRO	480	0	852	60	1392	7.30510E+04	3.87355E+04	6.80800E+04	7.59009E+05	4
5	NO ADDITIONS THIS YEAR RETIRE XTH5 4 X 20 MW THERML	680	0	852	60	1592			5.21335E+04	8.11143E+05	5
6	INSTALL ZPK3C 4 X 30 MW PEAKNG	680	0	772	60	1512	2.41002E+04	4.57225E+03	6.85332E+04	9.08348E+05	6
7	INSTALL CTH2F 2 X 115 MW THERML	680	0	772	180	1632	7.45666E+04	2.24271E+04	7.80625E+04	1.00340E+06	7
8	INSTALL CTH2R 2 X 115 MW THERML RETIRE XTH3 4 X 10 MW THERML	680	0	1002	180	1862	6.15856E+04	2.80371E+04	8.18025E+04	1.25583E+06	8
9	INSTALL GHY1F 3 X 30 MW HYDRO	690	0	1192	180	2052	9.09484E+04	5.31862E+04	8.67835E+04	1.46575E+06	9
10	INSTALL JHY2F 2 X 145 MW HYDRO RETIRE XPK1 3 X 20 MW PEAKNG	770	0	1192	180	2142	1.84319E+05	9.03066E+04	8.08426E+04	1.84272E+06	10
11	INSTALL GHY1R 4 X 30 MW HYDRO	1060	0	1192	120	2372	4.41453E+04	2.48706E+04	8.10920E+04	1.99282E+06	11
12	INSTALL ZPK4F 2 X 60 MW PEAKNG	1180	0	1192	120	2492	1.58530E+04	3.17261E+03	8.68228E+04	2.05960E+06	12
13	INSTALL DTH2F 2 X 115 MW THERML	1180	0	1192	240	2612	5.48900E+04	1.75569E+04	1.01714E+05	2.27324E+06	13
14	INSTALL ZPK4F 3 X 60 MW PEAKNG	1180	0	1422	240	2842	2.10655E+04	4.21311E+03	1.03707E+05	2.40223E+06	14
15	INSTALL ETH1F 2 X 480 MW THERML RETIRE XTH1 3 X 28 MW THERML	1180	0	1422	420	3022	1.99548E+05	3.77572E+04	1.01866E+05	2.74140E+06	15
16	INSTALL ZPK4F 1 X 60 MW PEAKNG	1180	0	2298	420	3698	6.21651E+03	1.24330E+03	1.22957E+05	2.87183E+06	16
17	INSTALL DTH2R 2 X 115 MW THERML	1180	0	2298	480	3958	3.81528E+04	1.84840E+04	1.31922E+05	3.06139E+06	17
18	INSTALL ETH1R 1 X 480 MW THERML	1180	0	2528	460	4188	8.45913E+04	1.37166E+04	1.38805E+05	3.28800E+06	18
19	INSTALL CTH2R 1 X 115 MW THERML	1180	0	3038	480	4698	1.66149E+04	7.83375E+03	1.45258E+05	3.46821E+06	19
20	INSTALL JHY2R 1 X 145 MW HYDRO RETIRE XTH2 3 X 28 MW THERML	1180	0	3123	480	4783	2.51160E+04	1.55685E+04	1.46463E+05	3.65536E+06	20
(CLC-SIMULATION RUN)						TOTALS	1.24904E+05	5.06389E+05	1.89892E+05		

DISCUSSION

From the preceding results it will be seen that, for this case study, the use of an LDC-based loading simulation model yields estimates of system operating costs some 15% lower than those given by CLC-loading simulation. This is the principal reason for the difference in costs of optimal expansion strategies of Traceback Searches Nos 1 and 2.

Of the two, the CLC-based model provides a more accurate treatment of hydro plant, taking into account the chronological sequence of reservoir depletion and replenishment, and also evaluating costs of provision of spinning reserve. The second traceback run rejects the installation of hydro units at site H on the basis of this more stringent evaluation procedure.

Table 6.1-11 shows that the estimates of total expansion costs from the state function are gross underestimates of the actual expansion costs obtained from the traceback searches. The discrepancies are due to constraints inherent in the pool data, which mean that the required capacity of hydro and peaking plant cannot be installed, and the resulting shortfall in installed capacity has to be made good by the installation of thermal stations.

Nuclear stations do not feature in any of the optimal development schedules. As discussed previously, substantial nuclear capacity is not recommended by the state and policy functions. There is however a second reason for the non-selection of nuclear options. The steady exponential growth in demand means that new system capacity is continually required, as evidenced by the very busy planting schedules of Tables 6.1-7 and 6.1-9. This together with the stringency of the capital expenditure constraints of Table 6.1-4 discourages initial overplanting at any site, especially of large units. In fact the lack of sufficient capital often prevents even the consideration of expensive nuclear units at sites E and F as feasible options.

TABLE 6.1-11 COMPARISON OF RESULTS FOR CASE STUDY ONE

	state and policy functions	traceback search	
		No 1 (LDC)	No 2 (CLC)
expansion cost, US\$	1.66×10^9	3.535×10^9 to 3.545×10^9	3.609×10^9 to 3.655×10^9
installed capacities at end of year 20 (MW):			
hydro	2007	1340	1180 - 1325
nuclear	44	0	0
thermal	1579	2903 - 3123	3008 - 3238
peaking	800	480 - 690	600 - 780
TOTALS	4430	4908 - 4943	4908 - 5023

Subsequent work on this case study would therefore include greater attention paid to cost estimates of those sites selected during these preliminary runs, modifications to the plant mix constraints (Figs 6.1-3) to give greater agreement with the implied constraints in the pool and hence closer estimates of expansion costs, followed by reassembly of the recursive functions before further traceback search runs.

At this stage however some broad conclusions may be drawn:

- Attention must be directed towards early completion of the second stage of site development at site 8.
- The development of site G, which is always deferred to the latest possible date, may not be an economic proposition for the system*. Site I on the other hand appears to occupy an important position in all development strategies.
- The installation of nuclear plant within the next twenty years is not urgent, provided that the development of sites C, D, and E with thermal units remains feasible and economic, and if constraints on capital expenditure remain unchanged.

* This option was included as an example of a multipurpose water resources project; it is more expensive per MW than all other hydro options (see Table 6.1-2)

6.2 CASE STUDY TWO

This second case study concerns a somewhat larger system, with a total installed capacity at the start of the planning period of 7756.8 MW (23% hydro, 10% nuclear, 65% thermal, and 3% peaking). The planning period spans fifteen years, and the five best (least-cost) development strategies are required from the traceback searches. The unit used for all costs and expenditures corresponds to £ 10³.

The system is comfortably overplanted at the beginning of the planning study, and the path constraint (minimum total installed capacity, Figs 6.2-3) ensures that a gross plant margin of at least 25% is maintained. Retirements during the planning period total 90 MW hydro, 448 MW thermal, and 55 MW peaking capacity, and the plant mix constraints enforce the addition during the fifteen-year planning period of at least 65 MW hydro and 135 MW peaking capacity.

The initial system and details of the pool of candidates for selection are in Tables 6.2-1 and 6.2-2. Of the sites to be developed, site A (2 x 600 MW nuclear) and site 8 (4 x 25 MW peaking) represent projects currently in progress, due for completion in years 4 and 2 respectively.

Site C, a hydro site with a maximum installed capacity of 4 x 40 MW, is to be selected between years 5 and 10 inclusive. For this site between two and four units may be chosen, with no subsequent site development.

Sites D and E are large sites earmarked for thermal or nuclear development, to be planted with 600 MW units. First stage of planting is to be with two units, and follow-on units can total up to three. Removal and release pool options for these plants constrain the development of at least one of the sites with thermal capacity (to take advantage of indigenous fossil fuel resources) although the planting of both sites is optional. Site D is available from year 5 onwards, site E from year 8 onwards.

Site F is an optional thermal site to be developed in two stages to an eventual maximum site capacity of 4 x 300 MW. This site is available from year 7 onwards, and first stage development is to be with two 300 MW units.

Sites G (maximum 5 x 20 MW or 4 x 30 MW, inflow 80 MW equivalent) and H (maximum 4 x 50 MW or 3 x 100 MW, inflow 240 MW equivalent) are separate optional hydro sites, available from years 7 and 9 respectively.

Site I is a possible pumped storage site, available from year 6. The implementation of this project is also optional.

Capital expenditure constraints for this case study are given in Table 6.2-3. Other data are in Figures 6.2-1 to 6.2-5. The system was required at all years to supply spinning reserve amounting to 25% (maximum demand) and 15% (dry season and average) of demand. The flow factor for the maximum demand curve 1.00, for the dry season curve 0.85. The least squares fit algorithm yielded the following expressions for the capital cost functions:

$$\begin{array}{ll} \text{hydro:} & 1.442 \times 10^{-2} + 1.029 \times 10^3 (p)^{0.946} \\ \text{nuclear:} & 7.090 \times 10^4 + 7.863 \times 10^2 (p)^{0.924} \\ \text{thermal:} & 4.403 \times 10^4 + 5.955 \times 10^2 (p)^{0.865} \\ \text{peaking:} & 1.338 + 2.660 \times 10^2 (p)^{0.911} \end{array}$$

Data processing and checking required a CP time of approximately 2.5 seconds.

TABLE 5.2-1 CASE STUDY TWO
SYSTEM AT YEAR 1

SYSTEM CONTAINS 14 THERMAL PLANTS

PLANT NAME	PLANT TYPE	NO.OF UNITS	YEAR IN	YEAR OUT	CAPACITY (MW)			COST COEFFICIENTS		
					MINIMUM	NOMINAL	OVERLOAD			
XNU1	NUCLR	6	0	20	10.000	48.300	48.300	3.350E-02	1.410E-03	0
XNU2	NUCLR	1	0	25	200.000	500.000	500.000	3.820E-01	1.600E-03	0
XTH1	THERML	4	0	25	300.000	576.000	576.000	6.110E-01	6.470E-03	0
XTH2	THERML	4	0	20	100.000	208.000	208.000	3.700E-01	8.270E-03	0
XTH3	THERML	2	0	17	150.000	300.000	314.000	5.100E-01	1.040E-02	0
XTH4	THERML	4	0	19	20.000	49.000	51.000	1.100E-01	1.314E-03	0
XTH5	THERML	1	0	19	29.000	57.000	59.000	1.200E-01	1.266E-03	0
XTH6	THERML	2	0	18	48.000	105.000	117.500	2.300E-01	1.363E-03	0
XTH7	THERML	3	0	9	28.000	57.000	60.000	1.700E-01	1.823E-03	0
XTH8	THERML	4	0	4	33.000	55.000	59.000	1.300E-01	1.431E-03	0
XTH9	THERML	2	0	3	15.000	28.500	29.000	1.600E-02	3.830E-03	0
XTH10	THERML	2	0	4	20.000	27.500	27.500	1.700E-02	3.660E-03	0
XPK1	PEAKNG	2	0	16	.001	70.000	70.000	0	2.136E-03	0
XPK2	PEAKNG	1	0	12	.001	55.000	55.000	0	2.035E-03	0

SYSTEM CONTAINS 7 HYDRO PLANTS

					CAPACITY (MW)			OVERLOAD	RESERVOIR	
					PUMPING NOMINAL	MINIMUM	MINIMUM		CAPACITY M3H	INFLOW M3
XHY1	HYDRO	6	0	25	0	0	.001	40.000	0	45.000
XHY2	HYDRO	3	0	8	0	0	.001	30.000	40.000	35.000
XHY3	HYDRO	4	0	16	0	0	.001	60.000	0	60.000
XHY4	HYDRO	4	0	20	0	0	.001	60.000	0	35.000
XHY5	HYDRO	2	0	19	0	0	.001	60.000	0	30.000
XPS1	HYDRO	4	0	30	-110.000	0	.001	100.000	160.000	0
XPS2	HYDRO	2	0	35	-160.000	0	.001	150.000	150.000	2800.000

TABLE 6.2-2A CASE STUDY TWO
CANDIDATES FOR ADDITION, YEAR 1
=====

PLANT NAME	PLANT TYPE	TOT. NO AVAILBL		YEARS AVAILABLE		LIFE YRS	CAPITAL FOREIGN	COST LOCAL	CAPACITY (MW)			COST COEFFICIENTS		
		MIN	MAX	FIRST	LAST				MIN	NOM	OVLD			
ANUIC	NUCLR	2	2	4	4	25	3.11E+05	3.45E+04	400.0	600.0	600.0	4.000E-01	1.650E-03	0
BPKIC	PEAKNG	4	4	2	2	15	2.30E+03	2.70E+03	.0	25.0	25.0	0	2.200E-02	0
OTHIF	THERML	2	2	5	16	25	1.04E+05	1.03E+05	300.0	600.0	600.0	6.900E-01	7.000E-03	0
OTHIR	THERML	1	3	16	16	25	8.64E+04	8.64E+04	300.0	600.0	600.0	6.900E-01	7.000E-03	0
DNUIF	NUCLR	2	2	5	16	25	3.63E+05	4.04E+04	400.0	600.0	600.0	4.110E-01	1.660E-03	0
DNUIR	NUCLR	1	3	16	16	25	2.78E+05	3.11E+04	400.0	600.0	600.0	4.110E-01	1.660E-03	0
ETHIF	THERML	2	2	8	16	25	1.10E+05	1.10E+05	300.0	600.0	600.0	6.870E-01	7.100E-03	0
ETHIR	THERML	1	3	16	16	25	9.17E+04	8.95E+04	300.0	600.0	600.0	6.870E-01	7.100E-03	0
ENUIF	NUCLR	2	2	8	16	25	3.70E+05	4.22E+04	400.0	600.0	600.0	4.110E-01	1.660E-03	0
ENUIR	NUCLR	1	3	16	16	25	2.92E+05	3.25E+04	400.0	600.0	600.0	4.110E-01	1.660E-03	0
FTHIF	THERML	2	2	7	16	25	6.37E+04	6.00E+04	100.0	300.0	300.0	5.650E-01	1.100E-02	0
FTHIR	THERML	1	2	16	16	25	5.81E+04	5.70E+04	100.0	300.0	300.0	5.650E-01	1.100E-02	0
ZPKIR	PEAKNG	2	6	3	16	15	4.50E+03	4.81E+03	.0	50.0	50.0	0	2.200E-02	0

PLANT NAME	PLANT TYPE	TOT. NO AVAILBL		YEARS AVAILABLE		LIFE YRS	CAPITAL FOREIGN	COST LOCAL	CAPACITY (MW)			GENERATING OVLD	RESERVOIR CAPCTY		INFLOW MW
		MIN	MAX	FIRST	LAST				PUMPING NOM	MIN	MIN		NOM	MW	
GHYIF	HYDR0	2	4	5	10	45	3.24E+04	2.16E+04	0	0	.0	40.0	50.0	0	35.0
GHYIF	HYDR0	2	2	7	16	45	9.98E+03	6.40E+03	0	0	.0	20.0	20.0	50.0	40.0
GHYIR	HYDR0	1	3	16	16	45	5.54E+03	3.70E+03	0	0	.0	20.0	20.0	0	0
GHY2F	HYDR0	2	2	7	16	45	1.43E+04	9.20E+03	0	0	.0	30.0	30.0	50.0	40.0
GHY2R	HYDR0	1	2	16	16	45	7.94E+03	5.20E+03	0	0	.0	30.0	30.0	0	0
HHYIF	HYDR0	2	2	9	16	45	3.22E+04	2.16E+04	0	0	.0	50.0	55.0	30.0	120.0
HHYIR	HYDR0	1	2	16	16	45	1.00E+04	1.20E+04	0	0	.0	50.0	55.0	0	0
HHY2F	HYDR0	2	2	9	16	45	5.16E+04	4.10E+04	0	0	.0	100.0	100.0	30.0	120.0
HHY2R	HYDR0	1	1	16	16	45	3.52E+04	2.26E+04	0	0	.0	100.0	100.0	0	0
IPSIF	HYDR0	2	2	6	16	45	7.45E+04	6.00E+04	-75.0	-10.0	.0	100.0	100.0	1000.0	0

TABLE B.2-2B CASE STUDY TWO
POOL MODIFICATIONS, YEAR 1

SELECTION OF CHY1F	RELEASES NONE		TO POOL, AND REMOVES CHY1F		FROM POOL
SELECTION OF DTH1F	RELEASES DTH1R		TO POOL, AND REMOVES ETH1F	DNJ1F	FROM POOL
SELECTION OF DNJ1F	RELEASES DNJ1R	DTH1F	TO POOL, AND REMOVES NONE		FROM POOL
SELECTION OF ETH1F	RELEASES ETH1R		TO POOL, AND REMOVES DTH1F	ENU1F	FROM POOL
SELECTION OF ENU1F	RELEASES ENU1R		TO POOL, AND REMOVES NONE		FROM POOL
SELECTION OF FTH1F	RELEASES FTH1R		TO POOL, AND REMOVES NONE		FROM POOL
SELECTION OF FTH1R	RELEASES NONE		TO POOL, AND REMOVES FTH1R		FROM POOL
SELECTION OF GHY1F	RELEASES GHY1R	GHY2R	TO POOL, AND REMOVES GHY2F		FROM POOL
SELECTION OF GHY1R	RELEASES NONE		TO POOL, AND REMOVES GHY2R		FROM POOL
SELECTION OF GHY2F	RELEASES GHY1R	GHY2R	TO POOL, AND REMOVES GHY1F		FROM POOL
SELECTION OF GHY2R	RELEASES NONE		TO POOL, AND REMOVES GHY1R		FROM POOL
SELECTION OF HHY1F	RELEASES HHY1R	HHY2R	TO POOL, AND REMOVES HHY2F		FROM POOL
SELECTION OF HHY1R	RELEASES NONE		TO POOL, AND REMOVES HHY2R		FROM POOL
SELECTION OF HHY2F	RELEASES HHY1R	HHY2R	TO POOL, AND REMOVES HHY1F		FROM POOL
SELECTION OF HHY2R	RELEASES NONE		TO POOL, AND REMOVES HHY1R		FROM POOL
SELECTION OF ZPK1R	RELEASES ZPK1R		TO POOL, AND REMOVES NONE		FROM POOL

TABLE 6.2-3 CAPITAL EXPENDITURE CONSTRAINTS
FOR CASE STUDY TWO

i	F_i $\pounds 10^9$	L_i $\pounds 10^9$	K_i
1 to 4	1.5	2.0	5
5 to 9	1.5	1.75	5
10 to 15	1.5	1.75	5

To be interpreted as follows:

- At year i , the maximum capital expenditure over the past K_i years, including year i , is not to exceed F_i (foreign component) or L_i (local component).

FIGURE 6.2-1A CASE STUDY TWO
CHRONOLOGICAL LOAD CURVES OVER PLANNING PERIOD

1. MAXIMUM DEMAND

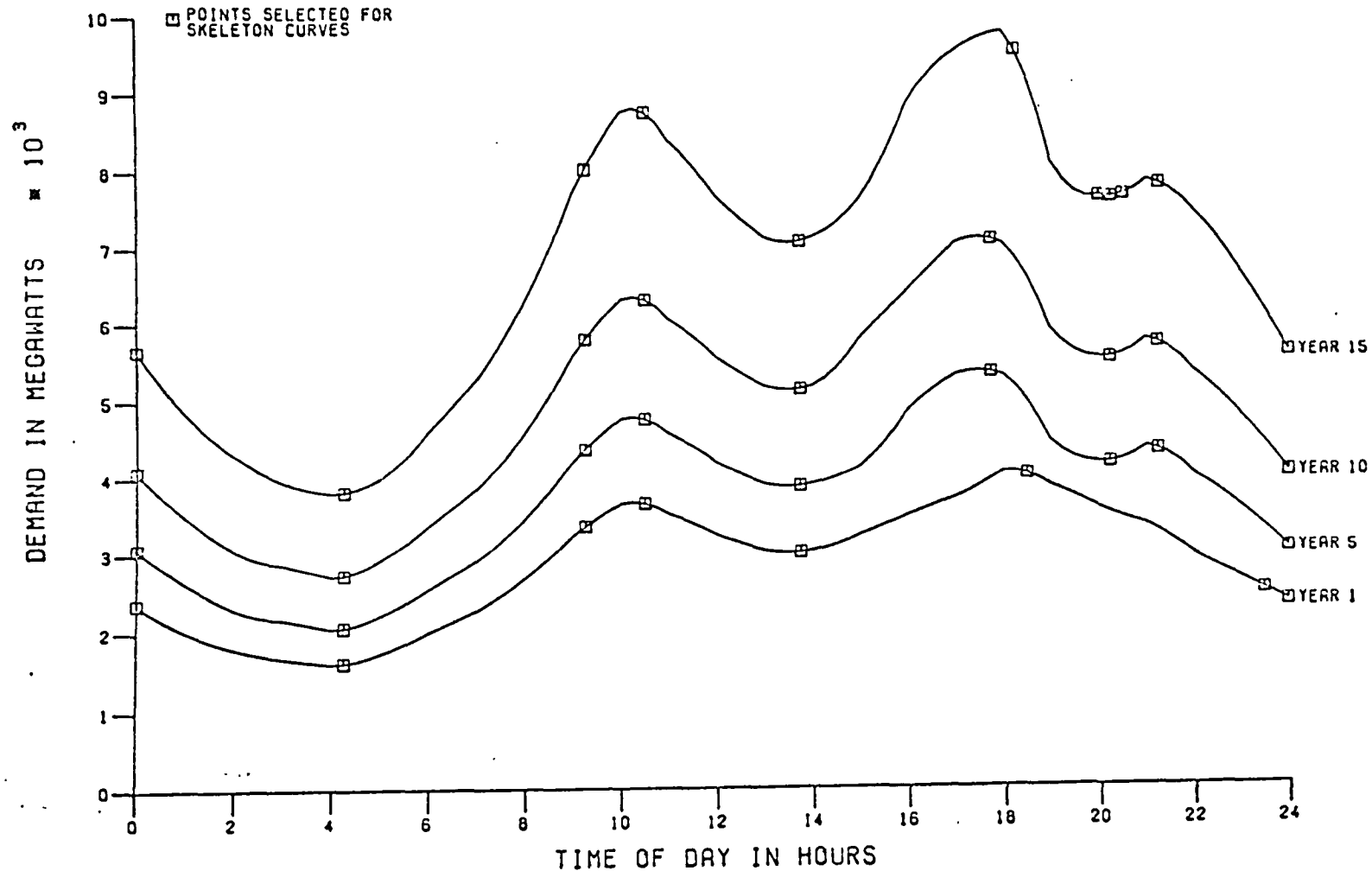


FIGURE 6.2-1B CASE STUDY TWO
CHRONOLOGICAL LOAD CURVES OVER PLANNING PERIOD

2. DRY SEASON

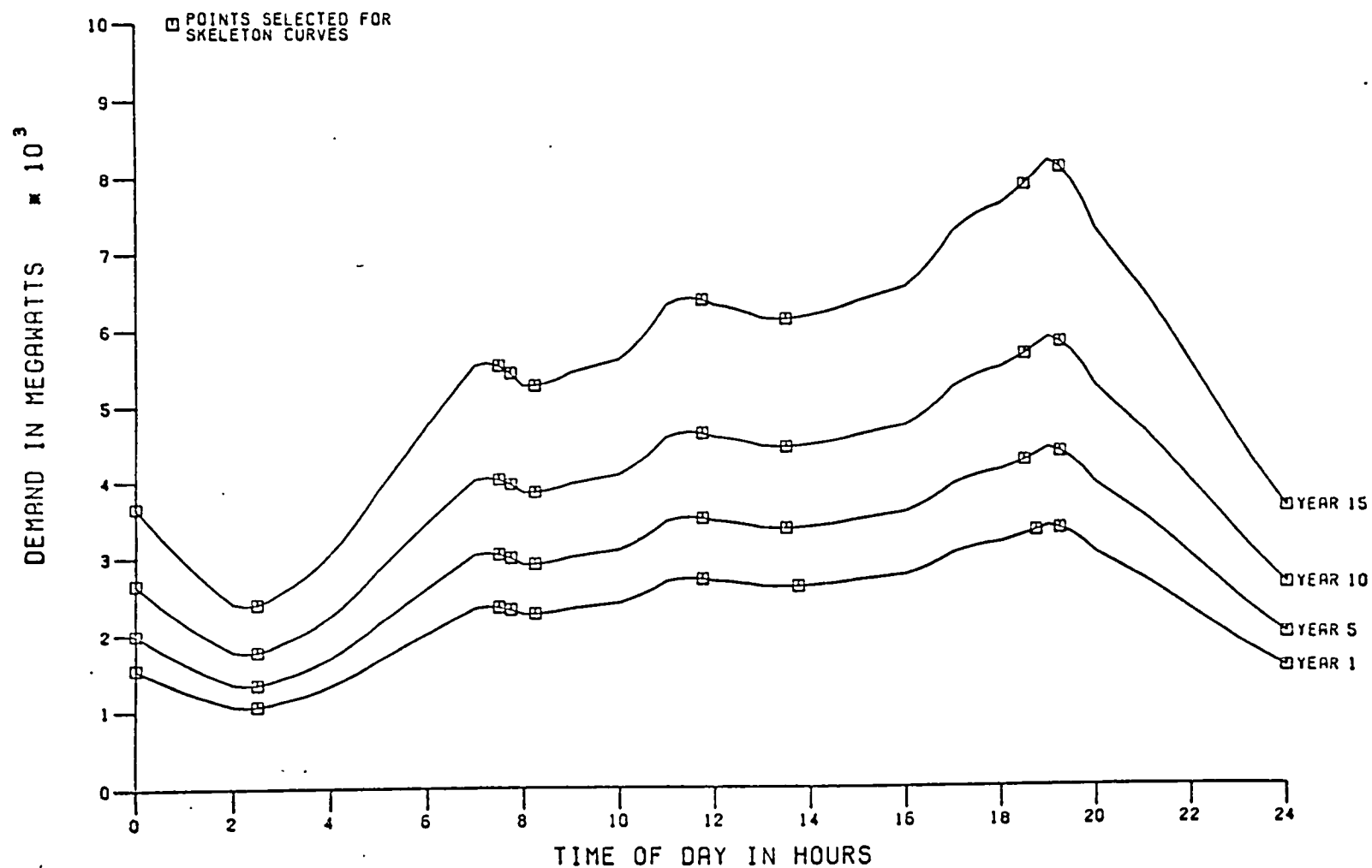


FIGURE 6.2-1C CASE STUDY TWO

CHRONOLOGICAL LOAD CURVES OVER PLANNING PERIOD

3. AVERAGE

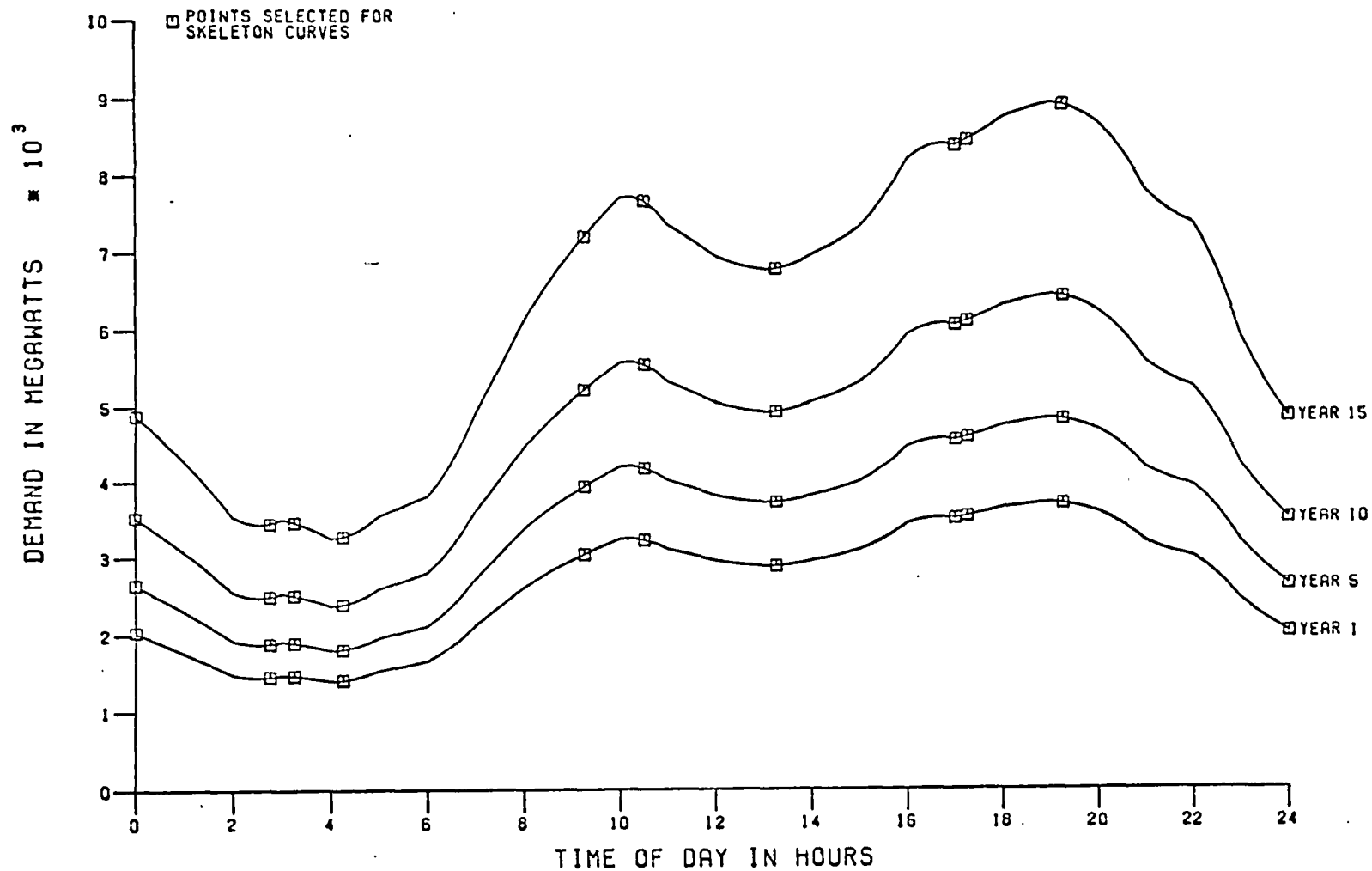


FIGURE 6.2-2 CASE STUDY TWO

ANNUAL LOAD DURATION CURVES OVER PLANNING PERIOD

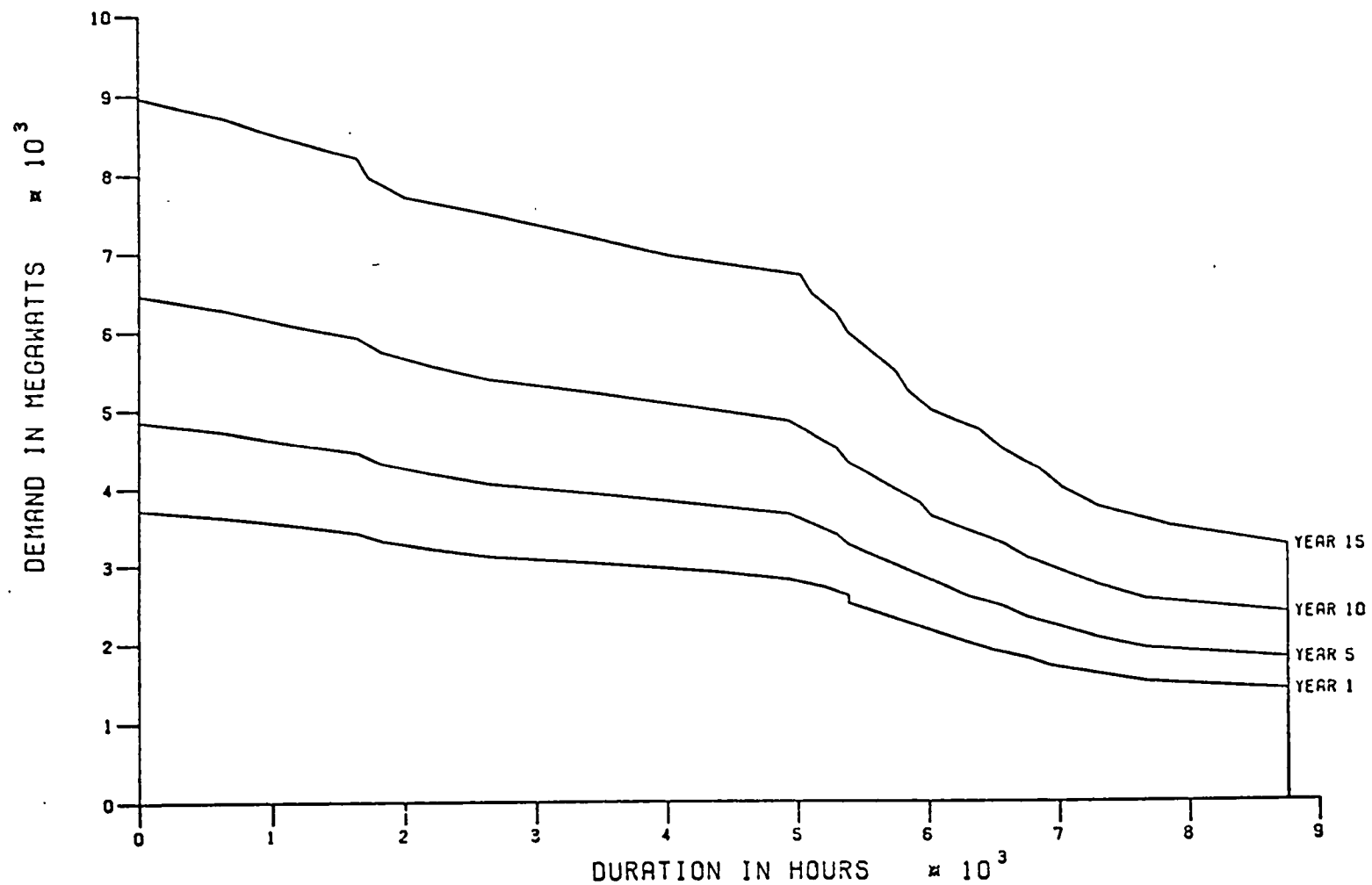


FIGURE 6.2-3A CASE STUDY TWO
PATH CONSTRAINTS OVER PLANNING PERIOD

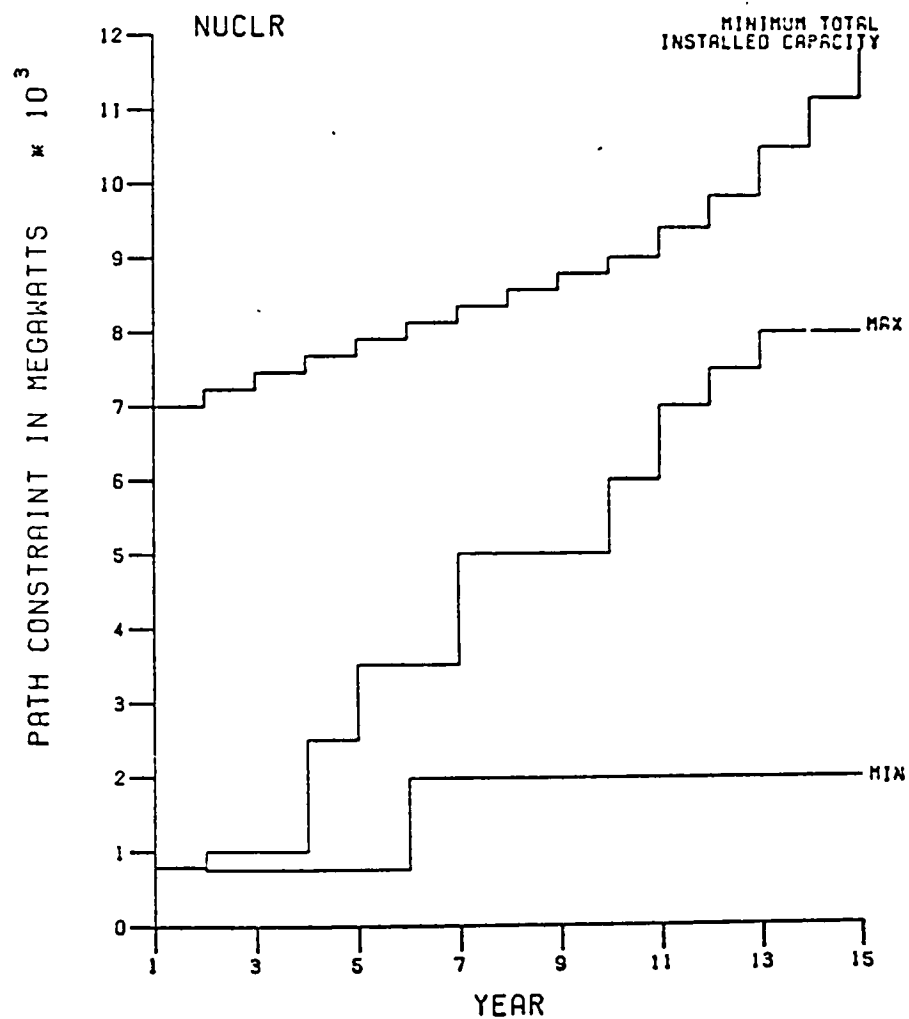
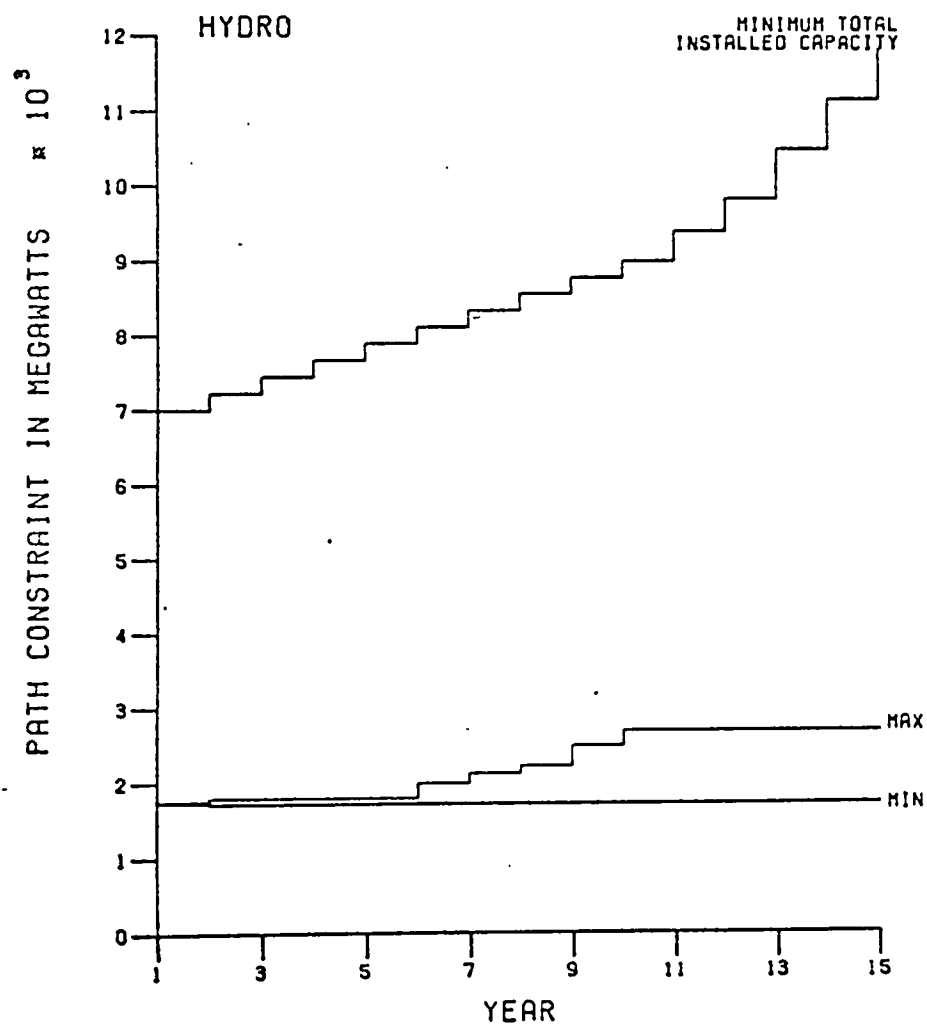


FIGURE 6.2-3B CASE STUDY TWO
PATH CONSTRAINTS OVER PLANNING PERIOD

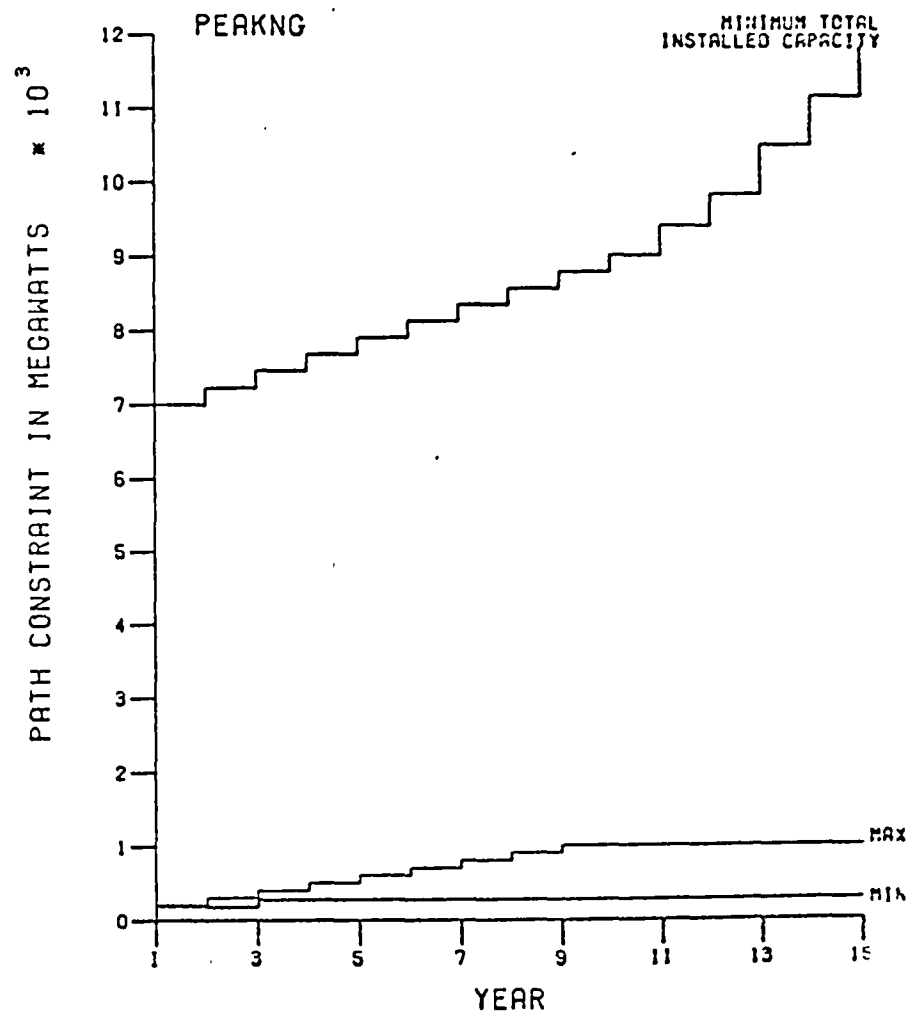
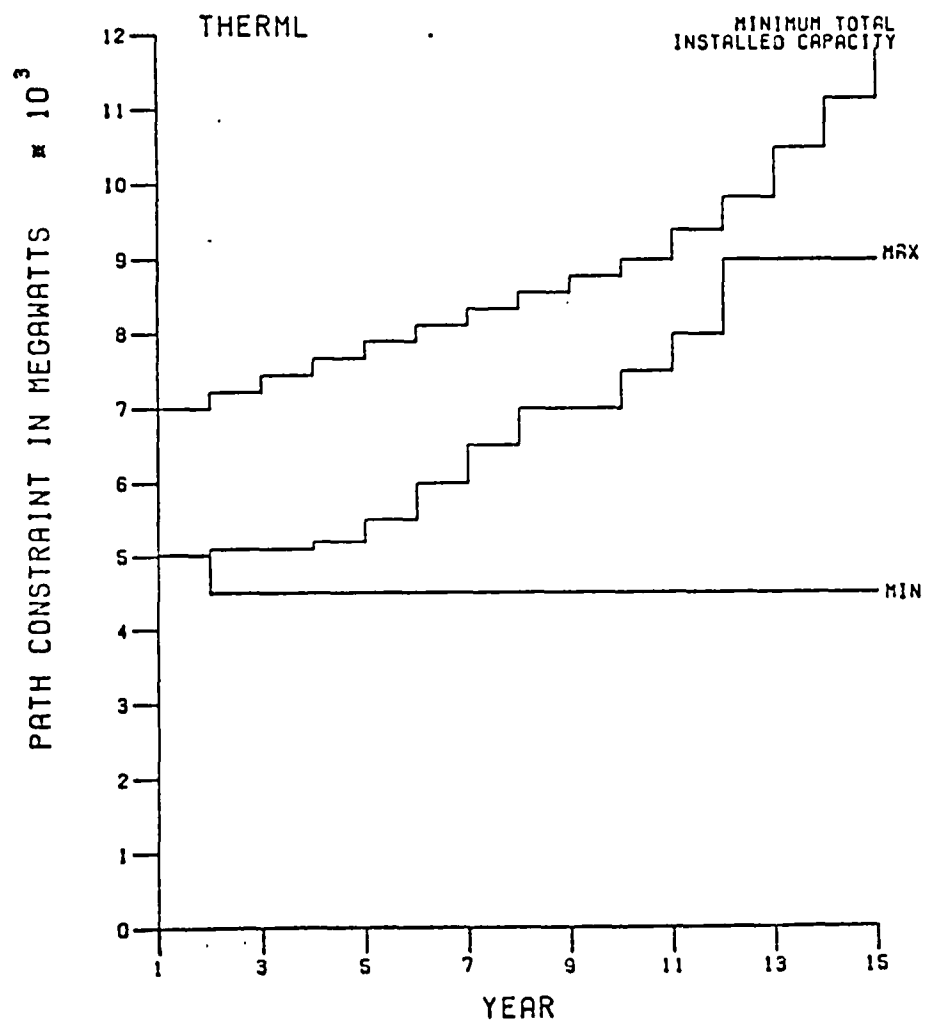


FIGURE 6.2-4 CASE STUDY TWO
DISCOUNT AND ESCALATION FACTORS

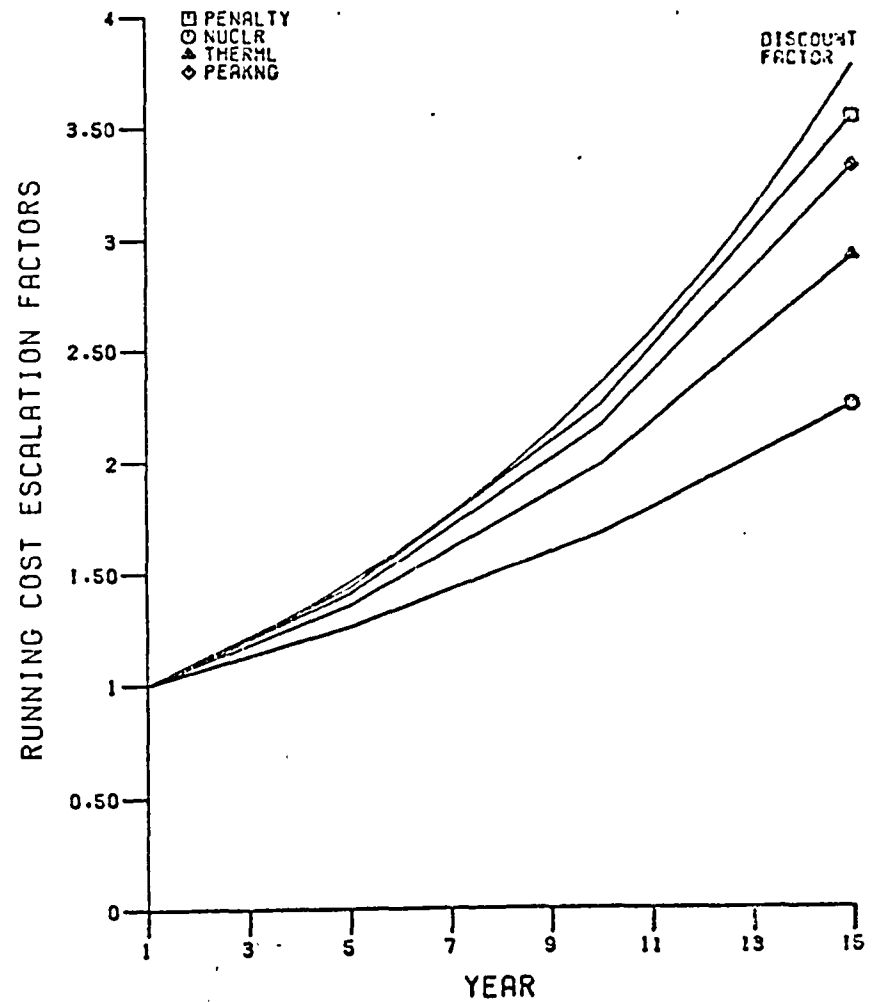
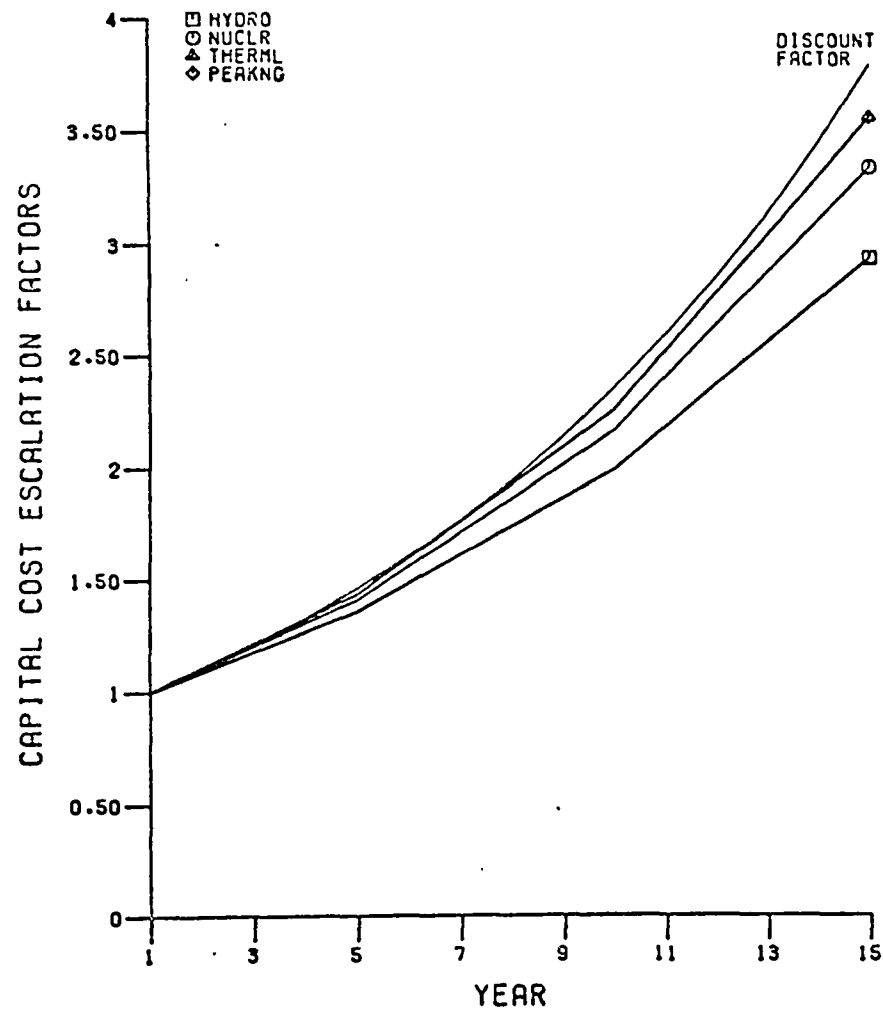
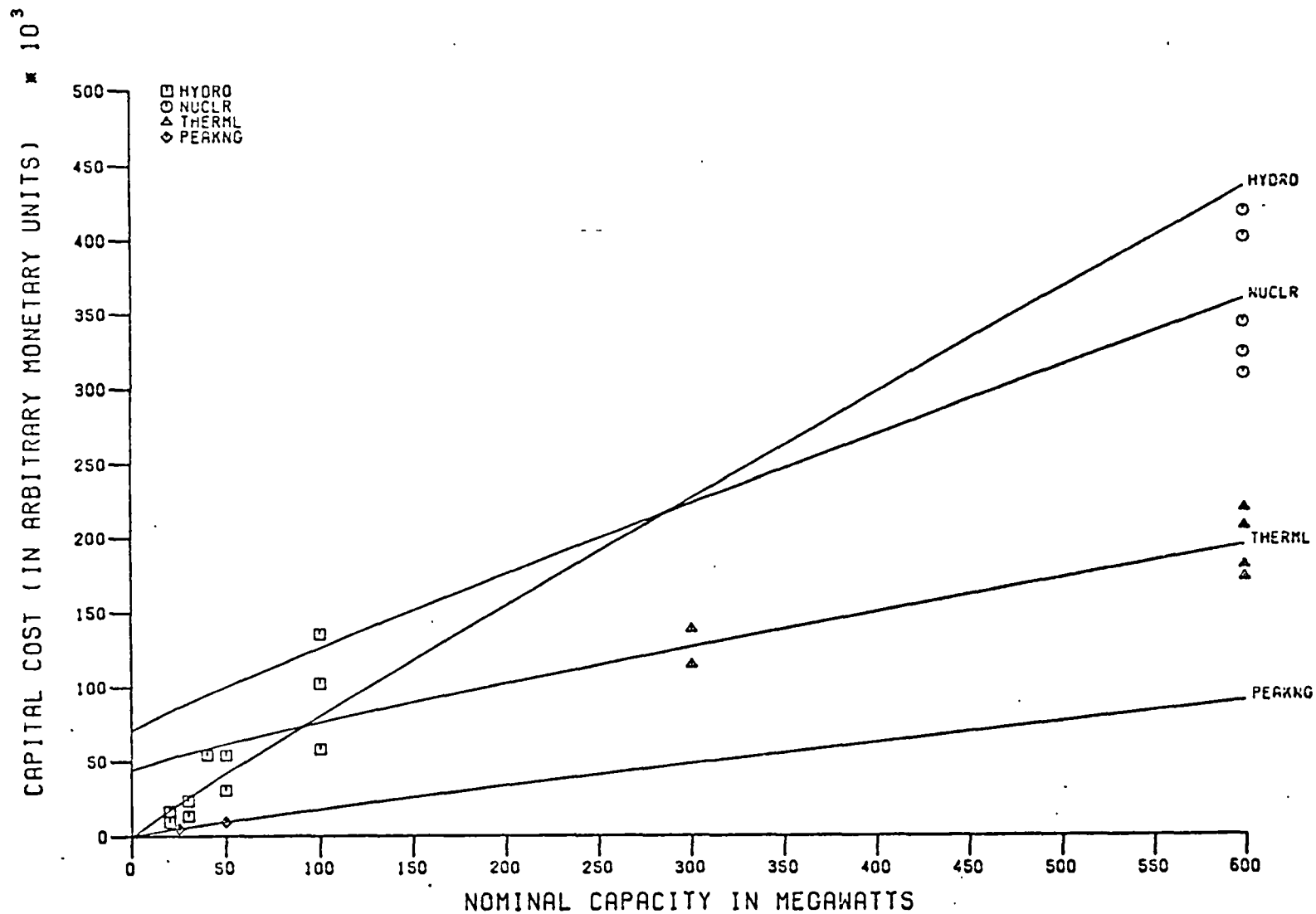


FIGURE 6.2-5 CASE STUDY TWO

LEAST-SQUARES FIT FOR CAPITAL COSTS



The information necessary to define all state and policy functions for all subsequent tracoback searches was assembled in approximately 170 seconds.

The optimal expansion policy from initial state at year 1 is shown in Table 6.2-4, indicating the net addition over the fifteen-year planning period of no hydro, 1627 MW nuclear, 1219 MW thermal, and 802 MW peaking capacity, yielding a final state vector at year 15:

$$\underline{s}_{15} = \begin{pmatrix} 1750 \text{ MW hydro} \\ 2417 \text{ MW nuclear} \\ 6241 \text{ MW thermal} \\ 997 \text{ MW peaking} \end{pmatrix}$$

Truncation effects as discussed under Case Study One in Section 6.1 are discernable in Table 6.2-4, and also in Table 6.2-5, which lists some optimal expansion strategies and costs from randomly generated initial state vectors at randomly generated start years to the horizon.

The total cost of system development from the initial state at year one to the horizon is predicted by the state functions to be not greater than £ 2.7×10^9 .

TABLE 6.2-4 CASE STUDY TWO: OPTIMAL EXPANSION STRATEGY FROM STATE FUNCTIONS

YEAR	EXPANSION COST TO HORIZON	HYDRO	STATE (M\$) NUCLR THERML	PEAKNG	HYDRO	DECISION (M\$) NUCLR THERML	PEAKNG
1	2.660359E+06	1750.000	789.800	5022.001	195.000	0	0
2	2.572493E+06	1750.000	789.800	5022.001	195.000	0	80.000
3	2.446105E+06	1750.000	789.800	5022.001	275.000	0	0
4	2.495731E+06	1750.000	789.800	5022.001	275.000	0	0
5	1.532048E+06	1750.000	2417.207	5022.001	275.000	0	0
6	1.404406E+06	1750.000	2417.207	5022.001	275.000	0	0
7	1.408406E+06	1750.000	2417.207	5022.001	275.000	0	0
8	1.339711E+06	1750.000	2417.207	5022.001	275.000	0	0
9	1.254203E+06	1750.000	2417.207	5022.001	275.000	0	85.564
10	1.140108E+06	1750.000	2417.207	5022.001	360.564	0	166.953
11	1.002004E+06	1750.000	2417.207	5022.001	527.517	0	92.200
12	9.097309E+05	1750.000	2417.207	5102.001	619.717	0	108.939
13	8.675495E+05	1750.000	2417.207	6043.908	728.656	0	113.266
14	5.560355E+05	1750.000	2417.207	6048.908	841.922	0	154.635
15	3.371266E+05	1750.000	2417.207	6241.194	996.558	0	3.442

TABLE 6.2-5 CASE STUDY TWO, OPTIMAL EXPANSION STRATEGIES FROM STATE FUNCTIONS

YEAR	EXPANSION COST TO HORIZON	HYDRO	STATE NUCLR	CHD THERML	PEAKING	HYDRO	DECISION NUCLR	CHD THERML	PEAKING
2	2.564355E+06	1786.710	774.052	4606.799	185.704	0	0	0	80.206
3	2.437220E+06	1786.710	774.052	4606.799	275.000	0	0	0	31.713
4	2.337644E+06	1786.710	774.052	4606.799	306.713	0	1425.305	0	212.708
5	1.633322E+06	1786.710	2324.436	4606.799	519.501	0	125.000	0	0
6	1.439116E+06	1786.710	2324.436	4606.799	519.501	0	0	0	0
7	1.411639E+06	1786.710	2324.436	4606.799	519.501	0	0	0	0
8	1.333460E+06	1786.710	2324.436	4606.799	519.501	0	0	0	0
9	1.233335E+06	1786.710	2324.436	4606.799	519.501	0	0	0	12.373
10	1.131622E+06	1786.710	2324.436	4606.799	531.874	0	0	0	69.514
11	1.013187E+06	1786.710	2324.436	4606.799	631.388	0	0	0	230.206
12	9.716320E+05	1786.710	2324.436	4606.799	911.533	0	0	980.000	88.407
13	7.204003E+05	1786.710	2324.436	5505.799	1000.000	0	0	0	0
14	6.477051E+05	1786.710	2324.436	5568.799	1000.000	0	0	408.435	0
15	4.660656E+05	1786.710	2324.436	5925.234	1000.000	0	0	0	0

YEAR	EXPANSION COST TO HORIZON	HYDRO	STATE NUCLR	CHD THERML	PEAKING	HYDRO	DECISION NUCLR	CHD THERML	PEAKING
3	2.435710E+06	1751.798	841.573	4726.090	381.386	0	0	0	0
4	2.412347E+06	1751.798	841.573	4726.090	381.388	48.212	1536.014	0	0
5	1.522460E+06	1800.000	2379.637	4735.090	381.386	0	125.000	0	0
6	1.427672E+06	1800.000	2504.687	4735.090	381.386	0	0	0	0
7	1.353527E+06	1800.000	2504.687	4735.090	381.386	0	0	0	0
8	1.287770E+06	1800.000	2504.687	4735.090	381.386	0	0	0	200
9	1.207804E+06	1800.000	2504.687	4735.090	381.594	0	0	0	37.168
10	1.101341E+06	1800.000	2504.687	4735.090	418.762	0	0	0	83.391
11	9.870578E+05	1800.000	2504.687	4735.090	502.153	0	0	80.000	165.223
12	8.979370E+05	1800.000	2504.687	4916.090	697.376	0	0	935.335	185.415
13	6.612422E+05	1800.000	2504.687	5751.424	892.791	0	0	0	117.209
14	5.646600E+05	1800.000	2504.687	5751.424	1000.000	0	0	223.246	0
15	4.206377E+05	1800.000	2504.687	5974.670	1000.000	0	0	0	0

YEAR	EXPANSION COST TO HORIZON	HYDRO	STATE NUCLR	CHD THERML	PEAKING	HYDRO	DECISION NUCLR	CHD THERML	PEAKING
4	2.029807E+06	1795.897	1651.828	4662.897	394.671	0	068.000	0	0
5	1.433856E+06	1795.897	2610.828	4662.897	394.671	0	0	0	0
6	1.300210E+06	1795.897	2610.828	4662.897	394.671	0	0	0	0
7	1.310051E+06	1795.897	2610.828	4662.897	394.671	0	0	0	0
8	1.297702E+06	1795.897	2610.828	4662.897	394.671	0	0	0	0
9	1.182652E+06	1795.897	2610.828	4662.897	394.671	0	0	0	21.703
10	1.081036E+06	1795.897	2610.828	4662.897	416.375	0	0	0	58.301
11	9.722543E+05	1795.897	2610.828	4662.897	474.675	0	0	80.000	176.658
12	8.681679E+05	1795.897	2610.828	4742.097	654.533	0	0	942.532	150.715
13	6.427064E+05	1795.897	2610.828	5665.429	835.340	0	0	0	154.677
14	5.569456E+05	1795.897	2610.828	5825.429	970.026	0	0	244.769	29.974
15	4.048003E+05	1795.897	2610.828	5930.197	1000.000	0	0	0	0

TRACEBACK SEARCH NO 1

A total of five traceback runs were conducted, the first four being exploratory searches to examine the effect of marginal changes in the initial system and pool data.

With the pool and initial system of Tables 6.2-1 and 6.2-2, Traceback Search No 1 was conducted; a CP time of 25 seconds was required to identify the optimal planting schedules of Table 6.2-6 as the five lowest-cost expansion strategies. Total costs of system expansion are between £ 4.083 x 10⁹ and £ 4.092 x 10⁹, the final state vector at the horizon being:

$$s_{15} = \begin{pmatrix} 1740 \text{ MW hydro} \\ 1990 \text{ MW nuclear} \\ 7519 \text{ MW thermal} \\ 590 \text{ MW peaking} \end{pmatrix}$$

It will at once be noted from an inspection of Table 6.2-6 that in contrast to the previous case study, a busy planting schedule is not required. Apart from the compulsory introductions to the system in years 2 and 4 (sites B and A respectively), the optimal strategies require the following planting sequences:

- Site C to be developed with 2 x 40 MW in year 8.
- Initial planting of site D with two thermal units in years 5 or 6, two follow-on units in years 12 or 13, and a further follow-on unit in years 11 or 13. This site is therefore to be developed to its full capacity within the next fifteen years.
- A total of seven peaking units are to be installed, between years 11 and 14 inclusive.
- In the remaining years no installations are to be made. Sites E, F, G, H, and I are not selected at all.

A detailed analysis of the best of the five recommended planting schedules is reproduced in Table 6.2-7.

TABLE 6.2-6 CASE STUDY TWO, TRACEBACK SEARCH NO 1
SKELETON LISTING OF OPTIMAL STRATEGIES 1 TO 5, LDC-SIMULATION

YEAR	NUMBER 1	NUMBER 2	NUMBER 3	NUMBER 4	NUMBER 5
1	NULL	NULL	NULL	NULL	NULL
2	4 X BPKIC	4 X BPKIC	4 X BPKIC	4 X BPKIC	4 X BPKIC
3	NULL	NULL	NULL	NULL	NULL
4	2 X ANUIC	2 X ANUIC	2 X ANUIC	2 X ANUIC	2 X ANUIC
5	2 X DTHIF	2 X DTHIF	NULL	NULL	NULL
6	NULL	NULL	2 X DTHIF	2 X DTHIF	2 X DTHIF
7	NULL	NULL	NULL	NULL	NULL
8	2 X CHYIF	2 X CHYIF	2 X CHYIF	2 X CHYIF	2 X CHYIF
9	NULL	NULL	NULL	NULL	NULL
10	2 X ZPKIR	NULL	2 X ZPKIR	NULL	NULL
11	NULL	2 X ZPKIR	NULL	2 X ZPKIR	1 X DTHIR
12	2 X DTHIR	2 X DTHIR	2 X DTHIR	2 X DTHIR	5 X ZPKIR
13	1 X DTHIR	1 X DTHIR	1 X DTHIR	1 X DTHIR	2 X DTHIR
14	5 X ZPKIR	5 X ZPKIR	5 X ZPKIR	5 X ZPKIR	2 X ZPKIR
15	NULL	NULL	NULL	NULL	NULL
TOTAL COST	4.0831E+06 *****	4.0834E+06 *****	4.0877E+06 *****	4.0876E+06 *****	4.0920E+06 *****

TABLE 6.2-7 CASE STUDY TWO, TRACEBACK SEARCH NO 1
OPTIMAL EXPANSION STRATEGY, NUMBER 1

YR	SYSTEM CHANGES FOR THIS YEAR		INSTALLED CAPACITY AT START OF YEAR (M E G A W A T T S)					COST OF FOREIGN	ADDITIONS LOCAL	SYSTEM RUNNING COST	CUMULATIVE COST AT END OF YEAR (ONE)	YR
			HYDRO	NUCL	THERML	PEAKING	TOTAL					
(ALL COSTS PRESENT-WORTHED TO YEAR ONE)												
1	NO ADDITIONS THIS YEAR		1750	790	5022	195	7757			9.46250E+04	9.40250E+04	1
2	INSTALL BPK1C	4 X 25 MW PEAKING	1750	790	5022	195	7757	9.27883E+03	1.08225E+04	1.07620E+05	2.22618E+05	2
3	NO ADDITIONS THIS YEAR RETIRE XTH9 2 X 29 MW THERML		1750	790	5022	295	7857			1.18298E+05	3.41915E+05	3
4	INSTALL ANU1C RETIRE XTH8 RETIRE XTH10	2 X 600 MW NUCL 4 X 55 MW THERML 2 X 28 MW THERML	1750	790	4865	295	7800	8.10586E+05	6.78440E+04	1.30443E+05	1.15080E+06	4
5	INSTALL DTH1F	2 X 600 MW THERML	1750	1990	4690	295	8725	2.04240E+05	2.02285E+05	9.99751E+04	1.65731E+06	5
6	NO ADDITIONS THIS YEAR		1750	1990	5020	295	8925			1.14495E+05	1.77180E+06	6
7	NO ADDITIONS THIS YEAR		1750	1990	5890	295	8925			1.27040E+05	1.89804E+06	7
8	INSTALL CHY1F RETIRE XHY2	2 X 40 MW HYDRO 3 X 30 MW HYDRO	1750	1990	5690	295	8925	5.79793E+04	3.86520E+04	1.38470E+05	2.13395E+06	8
9	NO ADDITIONS THIS YEAR RETIRE XTH7 3 X 57 MW THERML		1740	1990	5890	295	8915			1.48873E+05	2.28282E+06	9
10	INSTALL ZPK1R	2 X 50 MW PEAKING	1740	1990	5719	295	8744	8.81113E+03	9.23345E+03	1.60084E+05	2.46095E+06	10
11	NO ADDITIONS THIS YEAR		1740	1990	5719	395	8844			1.76460E+05	2.64041E+06	11
12	INSTALL DTH1R RETIRE ZPK2	2 X 600 MW THERML 1 X 55 MW PEAKING	1740	1990	5719	395	8844	1.60550E+05	1.68558E+05	1.97010E+05	3.17457E+06	12
13	INSTALL DTH1F	1 X 600 MW THERML	1740	1990	6919	340	10989	8.37734E+04	8.37734E+04	2.15150E+05	3.55727E+06	13
14	INSTALL ZPK1R	5 X 50 MW PEAKING	1740	1990	7519	340	11589	2.19574E+04	2.30098E+04	2.32187E+05	3.83442E+06	14
15	NO ADDITIONS THIS YEAR		1740	1990	7519	590	11839			2.40720E+05	4.08314E+06	15
QDC-SIMULATION RUN							TOTALS	1.16520E+06	6.04250E+05	2.31366E+03		

TRACEBACK SEARCH NO 2

The second LOC-simulation traceback run examines the effect of the postponement of the retirement dates of thermal plants XTH8 and XTH10 of the initial system (Table 6.2-1) by a single year. These stations are now to be withdrawn from service in year 5.

No other changes were applied to the input data, either to the initial system or to the pool of Table 6.2-2.

The CP time required for this run was 25 seconds, and the planning strategies recommended are listed in Table 6.2-8. These schedules are unchanged from those of Traceback Search No 1, but total costs of expansion now range from £ 4.080×10^9 to £ 4.089×10^9 over the fifteen year period - slightly lower than those of the previous run. This is clearly due to the slightly lower costs of system operation in year 5 owing to the retention in service of the two thermal stations XTH8 and XTH10.

Table 6.2-9 lists details of Optimal Strategy No 1 for this traceback search, and a comparison of this with Table 6.2-7 confirms the savings in system operating costs in year 5.

TABLE 6.2-8 CASE STUDY TWO, TRACEBACK SEARCH NO 2
SKELETON LISTING OF OPTIMAL STRATEGIES 1 TO 5, LDC-SIMULATION
=====

YEAR	NUMBER 1	NUMBER 2	NUMBER 3	NUMBER 4	NUMBER 5
1	NULL	NULL	NULL	NULL	NULL
2	4 X BPKIC	4 X BPKIC	4 X BPKIC	4 X BPKIC	4 X BPKIC
3	NULL	NULL	NULL	NULL	NULL
4	2 X ANUIC	2 X ANUIC	2 X ANUIC	2 X ANUIC	2 X ANUIC
5	2 X DTHIF	2 X DTHIF	NULL	NULL	NULL
6	NULL	NULL	2 X DTHIF	2 X DTHIF	2 X DTHIF
7	NULL	NULL	NULL	NULL	NULL
8	2 X CHYIF	2 X CHYIF	2 X CHYIF	2 X CHYIF	2 X CHYIF
9	NULL	NULL	NULL	NULL	NULL
10	2 X ZPKIR	NULL	2 X ZPKIR	NULL	NULL
11	NULL	2 X ZPKIR	NULL	2 X ZPKIR	1 X DTHIR
12	2 X DTHIR	2 X DTHIR	2 X DTHIR	2 X DTHIR	5 X ZPKIR
13	1 X DTHIR	1 X DTHIR	1 X DTHIR	1 X DTHIR	2 X DTHIR
14	5 X ZPKIR	5 X ZPKIR	5 X ZPKIR	5 X ZPKIR	2 X ZPKIR
15	NULL	NULL	NULL	NULL	NULL
TOTAL					
COST	4.0804E+06	4.0807E+06	4.0840E+06	4.0852E+06	4.0893E+06
	=====	=====	=====	=====	=====

TABLE 6.2-9 CASE STUDY TWO, TRACEBACK SEARCH NO 2
OPTIMAL EXPANSION STRATEGY, NUMBER 1

YR	SYSTEM CHANGES FOR THIS YEAR		INSTALLED CAPACITY AT START OF YEAR (M E G A W A T T S)					COST OF FOREIGN	ADDITIONS LOCAL	SYSTEM RUNNING COST	CUMULATIVE COST AT END OF YEAR	YR
			HYDRO	NUCLR	THERML	PEAKNG	TOTAL					
(ALL COSTS PRESENT-WORTHED TO YEAR ONE)												
1	NO ADDITIONS THIS YEAR		1750	790	5022	195	7757			9.48259E+04	9.48259E+04	1
2	INSTALL BPK1C	4 X 25 MW PEAKNG	1750	790	5022	195	7757	9.27883E+03	1.08925E+04	1.07620E+05	2.22618E+05	2
3	NO ADDITIONS THIS YEAR RETIRE XTH9 2 X 29 MW THERML		1750	790	5022	295	7857			1.19298E+05	3.41915E+05	3
4	INSTALL ANU1C	2 X 600 MW NUCLR	1750	790	4965	295	7800	5.10596E+05	5.78440E+04	1.30443E+05	1.15080E+06	4
5	INSTALL DTH1F RETIRE XTH8 RETIPE XTH10	2 X 600 MW THERML 4 X 55 MW THERML 2 X 28 MW THERML	1750	1690	4965	295	9000	2.04246E+05	2.02285E+05	9.72435E+04	1.65456E+06	5
6	NO ADDITIONS THIS YEAR		1750	1690	5890	295	9925			1.14495E+05	1.76907E+06	6
7	NO ADDITIONS THIS YEAR		1750	1690	5890	295	9925			1.27040E+05	1.89611E+06	7
8	INSTALL CHY1F RETIRE XHY2	2 X 40 MW HYDRO 3 X 30 MW HYDRO	1750	1690	5890	295	9925	5.79793E+04	3.86529E+04	1.38470E+05	2.13121E+06	8
9	NO ADDITIONS THIS YEAR RETIRE XTH7 3 X 57 MW THERML		1740	1690	5890	295	9915			1.46873E+05	2.28006E+06	9
10	INSTALL ZPK1R	2 X 50 MW PEAKNG	1740	1690	5719	295	9744	8.81113E+03	9.23345E+03	1.60084E+05	2.45822E+06	10
11	NO ADDITIONS THIS YEAR		1740	1690	5719	395	9844			1.79460E+05	2.63760E+06	11
12	INSTALL DTH1R RETIRE XPK2	2 X 600 MW THERML 1 X 55 MW PEAKNG	1740	1690	5719	395	9844	1.68558E+05	1.68558E+05	1.97049E+05	3.17184E+06	12
13	INSTALL DTH1R	1 X 600 MW THERML	1740	1690	6919	340	10989	8.37734E+04	8.37734E+04	2.15150E+05	3.55454E+06	13
14	INSTALL ZPK1R	5 X 50 MW PEAKNG	1740	1690	7519	340	11589	2.19574E+04	2.30098E+04	2.32187E+05	3.83160E+06	14
15	NO ADDITIONS THIS YEAR		1740	1690	7519	590	11839			2.48720E+05	4.00341E+06	15
CLC-SIMULATION RUN							TOTALS	1.16520E+06	6.04250E+05	2.31006E+06	*****	

TRACEBACK SEARCH NO 3

The third exploratory run investigates the effect of a delay in the programmed commissioning date for one of the 600 MW units at site A. This unit (named ANU2C) is delayed by one year, and is now due to be introduced into the system in year 5.

No other changes were applied to the initial system or the pool of Tables 6.2-1 and 6.2-2.

The CP time required for this run was about 16 seconds, yielding the results shown in Table 6.2-10. comparison of these results with those of Traceback Search No 1 (Table 6.2-6) shows that the three best strategies are similar, except that the installation of the second nuclear unit at site A in year 5 postpones by a year the initial planting of site D, where applicable. Furthermore the sequencing of the planting of follow-on units at this site is altered slightly, although eventually the total capacity installed is the same.

For the 4th and 5th best strategies there are similar rearrangements in the planting schedules, with the added difference of the introduction of an extra 4 x 50 MW peaking capacity in year 15.

Total costs of system expansion for the planning period range from £ 4.126×10^9 to £ 4.161×10^9 , thus in comparison to the strategies of Table 6.2-6 the delay in the commissioning of the single unit at site A imposes cost penalties (present-valued to base year one) amounting to between £ 42.7×10^9 and £ 69.3×10^9 . These penalties are principally due to the greater costs of system operation.

A detailed analysis of the best and 5th best optimal expansion schedules is given in Tables 6.2-11.

TABLE 6.2-10 CASE STUDY TWO, TRACEBACK SEARCH NO 3
SKELETON LISTING OF OPTIMAL STRATEGIES 1 TO 5, LDC-SIMULATION

YEAR	NUMBER 1	NUMBER 2	NUMBER 3	NUMBER 4	NUMBER 5
1	NULL	NULL	NULL	NULL	NULL
2	4 X BPK1C	4 X BPK1C	4 X BPK1C	4 X BPK1C	4 X BPK1C
3	NULL	NULL	NULL	NULL	NULL
4	1 X AMU1C	1 X AMU1C	1 X AMU1C	1 X AMU1C	1 X AMU1C
5	1 X AMU2C	1 X AMU2C	1 X AMU2C	1 X AMU2C	1 X AMU2C
6	2 X DTH1F	2 X DTH1F	2 X DTH1F	2 X DTH1F	2 X DTH1F
7	NULL	NULL	NULL	NULL	NULL
8	2 X CHY1F	2 X CHY1F	2 X CHY1F	2 X CHY1F	2 X CHY1F
9	NULL	NULL	NULL	NULL	NULL
10	2 X ZPK1R	NULL	NULL	2 X ZPK1R	NULL
11	NULL	2 X ZPK1R	1 X DTH1R	NULL	2 X ZPK1R
12	2 X DTH1R	2 X DTH1R	5 X ZPK1R	2 X DTH1R	2 X DTH1R
13	1 X DTH1R	1 X DTH1R	2 X DTH1R	1 X DTH1R	1 X DTH1R
14	5 X ZPK1R	5 X ZPK1R	2 X ZPK1R	5 X ZPK1R	5 X ZPK1R
15	NULL	NULL	NULL	4 X ZPK1R	4 X ZPK1R
TOTAL					
COST	4.1258E+06	4.1260E+06	4.1301E+06	4.1611E+06	4.1613E+06
	*****	*****	*****	*****	*****

TABLE 6.2-11A CASE STUDY TWO, TRACEBACK SEARCH NO 3
OPTIMAL EXPANSION STRATEGY, NUMBER 1

YR	SYSTEM CHANGES FOR THIS YEAR	INSTALLED CAPACITY AT START OF YEAR (M E G A W A T T S)					COST OF FOREIGN	ADDITIONS LOCAL	SYSTEM RUNNING COST	CUMULATIVE COST AT END OF YEAR (ONE)	YR
		HYDRO	NUCLR	THERML	PEAKING	TOTAL					
1	NO ADDITIONS THIS YEAR	1750	790	5022	195	7757			9.48259E+04	9.48259E+04	1
2	INSTALL BPK1C 4 X 25 MW PEAKING	1750	790	5022	195	7757	9.27883E+03	1.08925E+04	1.07620E+05	2.22618E+05	2
3	NO ADDITIONS THIS YEAR RETIRE XTH9 2 X 29 MW THERML	1750	790	5022	295	7857			1.19253E+05	3.41815E+05	3
4	INSTALL ANU1C 1 X 600 MW NUCLR RETIRE XTH9 4 X 55 MW THERML RETIRE XTH10 2 X 28 MW THERML	1750	790	4965	295	7800	3.05298E+05	3.36220E+04	1.30443E+05	8.11578E+05	4
5	INSTALL ANU2C 1 X 600 MW NUCLR	1750	1390	4690	295	8129	2.89366E+05	3.32620E+04	1.23469E+05	1.26760E+06	5
6	INSTALL DTH1F 2 X 600 MW THERML	1750	1090	4690	295	8725	2.07004E+05	2.05014E+05	1.16023E+05	1.70572E+06	6
7	NO ADDITIONS THIS YEAR	1750	1090	5890	295	9925			1.25450E+05	1.92517E+06	7
8	INSTALL CHY1F 2 X 40 MW HYDRO RETIRE XHY2 3 X 30 MW HYDRO	1750	1090	5890	295	9925	5.79793E+04	3.86529E+04	1.40791E+05	2.16250E+06	8
9	NO ADDITIONS THIS YEAR RETIRE XTH7 3 X 57 MW THERML	1740	1090	5890	295	9915			1.51102E+05	2.31369E+06	9
10	INSTALL ZPK1R 2 X 50 MW PEAKING	1740	1090	5719	295	9744	8.81113E+03	9.23345E+03	1.62218E+05	2.43395E+06	10
11	NO ADDITIONS THIS YEAR	1740	1090	5719	395	9844			1.81531E+05	2.67549E+06	11
12	INSTALL DTH1R 2 X 600 MW THERML RETIRE XPK2 1 X 55 MW PEAKING	1740	1090	5719	395	9844	1.68558E+05	1.68558E+05	1.99051E+05	3.21165E+06	12
13	INSTALL DTH1R 1 X 600 MW THERML	1740	1090	6919	340	10989	8.37734E+04	8.37734E+04	2.17076E+05	3.59620E+06	13
14	INSTALL ZPK1R 5 X 50 MW PEAKING	1740	1090	7519	340	11589	2.19574E+04	2.30098E+04	2.34035E+05	3.87529E+06	14
15	NO ADDITIONS THIS YEAR	1740	1090	7519	590	11839			2.50463E+05	4.12578E+06	15
	QDC-SIMULATION RUN					TOTALS	1.16203E+06	6.06319E+05	2.35743E+06		

TABLE 6.2-11B CASE STUDY TWO, TRACEBACK SEARCH NO 3
OPTIMAL EXPANSION STRATEGY, NUMBER 5

YR	SYSTEM CHANGES FOR THIS YEAR	INSTALLED CAPACITY AT START OF YEAR (MEGAWATTS)					COST OF FOREIGN	ADDITIONS LOCAL	SYSTEM RUNNING COST	CUMULATIVE COST AT END OF YEAR	YR
		HYDRO	NUCLR	THERML	PEAKING	TOTAL	(ALL COSTS PRESENT-WORTHED TO YEAR ONE)				
1	NO ADDITIONS THIS YEAR	1750	700	5022	195	7757			9.48256E+04	9.48256E+04	1
2	INSTALL BPK1C 4 X 25 MW PEAKING	1750	700	5022	195	7757	9.27083E+03	1.08925E+04	1.07620E+05	2.22618E+05	2
3	NO ADDITIONS THIS YEAR RETIRE XTH9 2 X 28 MW THERML	1750	700	5022	295	7857			1.18298E+05	3.41915E+05	3
4	INSTALL AMU1C 1 X 600 MW NUCLR RETIRE XTH8 4 X 55 MW THERML RETIRE XTH10 2 X 28 MW THERML	1750	700	4965	295	7800	3.05298E+05	3.39220E+04	1.30443E+05	8.11578E+05	4
5	INSTALL AMU2C 1 X 600 MW NUCLR	1750	1300	4690	295	8125	2.99366E+05	3.32629E+04	1.23469E+05	1.26768E+06	5
6	INSTALL DTH1F 2 X 600 MW THERML	1750	1990	4690	295	8725	2.07004E+05	2.05014E+05	1.16023E+05	1.76572E+06	6
7	NO ADDITIONS THIS YEAR	1750	1990	5890	295	9925			1.29450E+05	1.02517E+06	7
8	INSTALL CHY1F 2 X 40 MW HYDRO RETIRE XHY2 3 X 30 MW HYDRO	1750	1990	5890	295	9925	5.79793E+04	3.86529E+04	1.40791E+05	2.16256E+06	8
9	NO ADDITIONS THIS YEAR RETIRE XTH7 3 X 57 MW THERML	1740	1990	5890	295	9915			1.51102E+05	2.31369E+06	9
10	NO ADDITIONS THIS YEAR	1740	1990	5719	295	9744			1.62218E+05	2.47591E+06	10
11	INSTALL ZPK1R 2 X 50 MW PEAKING	1740	1990	5719	295	9744	8.93012E+03	9.35814E+03	1.81531E+05	2.67573E+06	11
12	INSTALL DTH1R 2 X 600 MW THERML RETIRE XPK2 1 X 55 MW PEAKING	1740	1990	5719	305	9844	1.68558E+05	1.68558E+05	1.98051E+05	3.21160E+06	12
13	INSTALL DTH1R 1 X 600 MW THERML	1740	1990	6919	340	10909	8.37734E+04	8.37734E+04	2.17079E+05	3.59652E+06	13
14	INSTALL ZPK1R 5 X 50 MW PEAKING	1740	1990	7519	340	11589	2.19574E+04	2.30088E+04	2.34039E+05	3.87553E+06	14
15	INSTALL ZPK1R 4 X 50 MW PEAKING	1740	1990	7519	590	11839	1.72258E+04	1.80514E+04	2.50493E+05	4.16130E+06	15
	QLOC-SIMULATION RUN					TOTALS	1.17937E+06	6.24495E+05	2.35745E+06		

TRACEBACK SEARCH NO 4

The last exploratory run with LDC-simulation was then carried out, this time with the installation of the pumped storage candidate made compulsory between years 6 and 10 inclusive. Apart from this one difference, the initial system and pool were the same as in Tables 6.2-1 and 6.2-2.

The results of this run, which required a CP time of 20 seconds, are reproduced in Table 6.2-12. In the five optimal development schedules it is found that the pumped storage project is best introduced into the system in years 6, 7, or 8. This has the effect of causing some significant changes in the basic schedules of Traceback Search No 1 as listed in Table 6.2-6. Comparison of this table with Table 6.2-12 shows that:

- The planting of site C, with two 40 MW units, is brought forward to years 5 or 6.
- Initial development of site D is deferred to year 12, at which date two units are installed. There is now only one follow-on unit needed, to be installed in year 13.
- Site F is now selected, with initial units to be installed in year 10 (2 x 300 MW), followed by a further single 300 MW unit in year 13.
- A total of ten peaking units are required, these are to be installed in years 7, 9, and 11.

The total costs of expansion under these modified constraints now range from £ 4.417×10^9 to £ 4.422×10^9 ; thus a penalty of up to £ 334×10^6 (present-valued over the fifteen-year planning period) is associated with the compulsory installation of the pumped storage units.

These results are not obvious, at least to this author. Contrary to what might be expected, the compulsory development of site I does not postpone the development at site C, rather the reverse. Full development of site D is now no longer the most economic course of action, owing to the generating capacity

now available from site I, the shortfall in total system capacity being counterbalanced by the installation of a total of three 300 MW units at site F in two stages, and the installation of an additional 3 x 50 MW peaking units.

The earlier installations at site C and the introduction of the pumped storage units yield reductions in system operating costs over the middle period when these projects are brought into operation, but the later installations of thermal units at site F and the extra peaking capacity cause somewhat greater system production costs towards the end of the planning period (cf Table 6.2-7 and 6.2-13A), hence total present-valued operating costs over the fifteen-year planning period are more or less the same. The cost penalties due to the enforced installation of the pumped storage station are therefore mainly due to the higher capital costs of development.

TABLE 6.2-12 CASE STUDY TWO, TRACEBACK SEARCH NO 4
SKELETON LISTING OF OPTIMAL STRATEGIES 1 TO 5, LDC-SIMULATION
=====

YEAR	NUMBER 1	NUMBER 2	NUMBER 3	NUMBER 4	NUMBER 5
1	NULL	NULL	NULL	NULL	NULL
2	4 X BPK1C	4 X BPK1C	4 X BPK1C	4 X BPK1C	4 X BPK1C
3	NULL	NULL	NULL	NULL	NULL
4	2 X AMU1C	2 X AMU1C	2 X AMU1C	2 X AMU1C	2 X AMU1C
5	2 X CHY1F	2 X CHY1F	2 X CHY1F	2 X CHY1F	NULL
6	2 X IPS1F	2 X IPS1F	NULL	NULL	2 X CHY1F
7	NULL	2 X ZPK1R	2 X IPS1F	NULL	2 X IPS1F
8	NULL	NULL	NULL	2 X IPS1F	NULL
9	6 X ZPK1R	4 X ZPK1R	6 X ZPK1R	6 X ZPK1R	6 X ZPK1R
10	2 X FTH1F	2 X FTH1F	2 X FTH1F	2 X FTH1F	2 X FTH1F
11	4 X ZPK1R	4 X ZPK1R	4 X ZPK1R	4 X ZPK1R	4 X ZPK1R
12	2 X DTH1F	2 X DTH1F	2 X DTH1F	2 X DTH1F	2 X DTH1F
13	1 X FTH1R	1 X FTH1R	1 X FTH1R	1 X FTH1R	1 X FTH1R
14	1 X DTH1R	1 X DTH1R	1 X DTH1R	1 X DTH1R	1 X DTH1R
15	NULL	NULL	NULL	NULL	NULL
TOTAL					
COST	4.4171E+06	4.4175E+06	4.4197E+06	4.4207E+06	4.4215E+06
	=====	=====	=====	=====	=====

TABLE 6.2-13 CASE STUDY TWO: TRACEBACK SEARCH NO 4
OPTIMAL EXPANSION STRATEGY, NUMBER 1

YR	SYSTEM CHANGES FOR THIS YEAR	INSTALLED CAPACITY AT START OF YEAR (M E G A W A T T S)					COST OF FOREIGN	ADDITIONS LOCAL	SYSTEM RUNNING COST	CUMULATIVE COST AT END OF YEAR	YR
		HYDRO	NUCLR	THERML	PEAKING	TOTAL					
1	NO ADDITIONS THIS YEAR	1750	790	5022	195	7757			9.48259E+04	9.48259E+04	1
2	INSTALL BPKIC 4 X 25 MW PEAKING	1750	790	5022	195	7757	9.27883E+03	1.08925E+04	1.07620E+05	2.22618E+05	2
3	NO ADDITIONS THIS YEAR RETIRE XTH9 2 X 29 MW THERML	1750	790	5022	295	7857			1.19298E+05	3.41915E+05	3
4	INSTALL ANUIC 2 X 600 MW NUCLR RETIRE XTH9 4 X 55 MW THERML RETIRE XTH10 2 X 28 MW THERML	1750	790	4965	295	7800	8.10566E+05	6.78440E+04	1.30443E+05	1.15080E+06	4
5	INSTALL CHY1F 2 X 40 MW HYDRO	1750	1890	4690	295	8725	6.02147E+04	4.01432E+04	9.69751E+04	1.35113E+06	5
6	INSTALL IPS1F 2 X 100 MW HYDRO	1030	1890	4690	295	8805	1.37684E+05	1.12550E+05	1.11163E+05	1.71263E+06	6
7	NO ADDITIONS THIS YEAR	2030	1890	4690	295	9005			1.17778E+05	1.83031E+06	7
8	NO ADDITIONS THIS YEAR RETIRE XHY2 3 X 30 MW HYDRO	2030	1890	4690	295	9005			1.29387E+05	1.95968E+06	8
9	INSTALL ZPK1R 6 X 50 MW PEAKING RETIRE XTH7 3 X 57 MW THERML	1940	1890	4690	295	8915	2.69956E+04	2.82476E+04	1.42211E+05	2.15711E+06	9
10	INSTALL FTH1F 2 X 300 MW THERML	1940	1890	4519	595	9044	1.33799E+05	1.32455E+05	1.54166E+05	2.57753E+06	10
11	INSTALL ZPK1R 4 X 50 MW PEAKING	1940	1890	5119	595	9644	1.78602E+04	1.87163E+04	1.65242E+05	2.79935E+06	11
12	INSTALL OTH1F 2 X 600 MW THERML RETIRE XPK2 1 X 55 MW PEAKING	1940	1890	5119	795	9844	2.02894E+05	2.00943E+05	2.04619E+05	3.40760E+06	12
13	INSTALL FTH1R 1 X 300 MW THERML	1940	1890	6319	740	10689	5.63338E+04	5.52672E+04	2.19772E+05	3.73917E+06	13
14	INSTALL OTH1R 1 X 600 MW THERML	1940	1890	6619	740	11289	8.26632E+04	8.26632E+04	2.54046E+05	4.15855E+06	14
15	NO ADDITIONS THIS YEAR	1940	1890	7219	740	11899			2.58544E+05	4.41700E+06	15
	GLDC-SIMULATION RUN)					TOTALS	1.33820E+06	7.49722E+05	2.32909E+06		

TRACEBACK SEARCH NO 5

This was a traceback run with CLC-simulation. For this run no changes were made to the initial system of Table 6.2-1, but the pool of Table 6.2-2 was modified in such a way as to constrain system development to be identical with that in Table 6.2-6 up to and including year 9.

The CP time required was 113 seconds, and the results are shown in Table 6.2-14. These differ in detail from the strategies advocated by the LDC-simulation run, but follow the same basic trends, involving, after year 9:

- Installation of one follow-on unit at site D in years 11 or 13, and two follow-on units in years 12 or 13.
- A total of seven 50 MW peaking units between years 10 and 14.

The most remarkable difference, however, is in the expansion costs, which range from £ 3.179×10^9 to £ 3.182×10^9 . Examination of the details of the best development strategy (Table 6.2-15A) shows this to be due to discrepancies in the evaluation of production costs between the LDC- and CLC-loading simulation models; estimates from the latter being some 40% lower. Two main reasons can be forwarded for this:

- The system is significantly overplanted during most of the planning period. This means that from the operational point of view there is considerable flexibility in the scheduling of plant with respect to the chronological sequence of load. The CLC-model recognizes and takes advantage of this fact, whereas the LDC-model, which deals with aggregate demand levels, cannot.
- The system contains two large pumped-storage stations and a large hydro storage capability. This too adds to operational versatility which is exploited by the CLC-model. As described in Chapter Five this model determines the trajectory of reservoir levels which yields lowest system operating cost. No proper account is taken of the pumping/generating cycle in the LDC-model.

TABLE 6.2-14 CASE STUDY TWO, TRACEBACK SEARCH NO 5
 SKELETON LISTING OF OPTIMAL STRATEGIES 1 TO 5, CLC-SIMULATION
 =====

YEAR	NUMBER 1	NUMBER 2	NUMBER 3	NUMBER 4	NUMBER 5
1	NULL	NULL	NULL	NULL	NULL
2	4 X BPKIC	4 X BPKIC	4 X BPKIC	4 X BPKIC	4 X BPKIC
3	NULL	NULL	NULL	NULL	NULL
4	2 X ANUIC	2 X ANUIC	2 X ANUIC	2 X ANUIC	2 X ANUIC
5	2 X DTHIF	2 X DTHIF	2 X DTHIF	2 X DTHIF	2 X DTHIF
6	NULL	NULL	NULL	NULL	NULL
7	NULL	NULL	NULL	NULL	NULL
8	2 X CHYIF	2 X CHYIF	2 X CHYIF	2 X CHYIF	2 X CHYIF
9	NULL	NULL	NULL	NULL	NULL
10	NULL	NULL	2 X ZPK1R	2 X ZPK1R	2 X ZPK1R
11	1 X DTH1R	2 X ZPK1R	NULL	2 X ZPK1R	1 X DTH1R
12	5 X ZPK1R	2 X DTH1R	2 X DTH1R	2 X DTH1R	3 X ZPK1R
13	2 X DTH1R	1 X DTH1R	1 X DTH1R	1 X DTH1R	2 X DTH1R
14	2 X ZPK1R	5 X ZPK1R	5 X ZPK1R	3 X ZPK1R	2 X ZPK1R
15	NULL	NULL	NULL	NULL	NULL
TOTAL					
COST	3.1766E+06	3.1791E+06	3.1806E+06	3.1811E+06	3.1816E+06
	=====	=====	=====	=====	=====

TABLE B.2-15A CASE STUDY TWO, TRACEBACK SEARCH NO 5
OPTIMAL EXPANSION STRATEGY, NUMBER 1

YR	SYSTEM CHANGES FOR THIS YEAR	INSTALLED CAPACITY AT START OF YEAR (M E G A W A T T S)					COST OF FOREIGN	ADDITIONS LOCAL	SYSTEM RUNNING COST	CUMULATIVE COST AT END OF YEAR	YR
		HYDRO	NUCLR	THERML	PEAKING	TOTAL	(ALL COSTS PRESENT-WORTHED TO YEAR ONE)				
1	NO ADDITIONS THIS YEAR	1750	790	5022	195	7757			5.50037E+04	5.50037E+04	1
2	INSTALL BPK1C 4 X 25 MW PEAKING	1750	790	5022	195	7757	9.27683E+03	1.08925E+04	5.97265E+04	1.34902E+05	2
3	NO ADDITIONS THIS YEAR RETIRE XTH9 2 X 29 MW THERML	1750	790	5022	295	7857			6.34976E+04	1.98396E+05	3
4	INSTALL ANU1C 2 X 600 MW NUCLR RETIRE XTH8 4 X 55 MW THERML RETIRE XTH10 2 X 28 MW THERML	1750	790	4965	295	7800	6.10596E+05	6.78440E+04	6.70731E+04	9.43912E+05	4
5	INSTALL DTH1F 2 X 600 MW THERML	1750	1990	4690	295	8725	2.04249E+05	2.07285E+05	5.23382E+04	1.40278E+06	5
6	NO ADDITIONS THIS YEAR	1750	1990	5890	295	9925			7.39937E+04	1.47678E+06	6
7	NO ADDITIONS THIS YEAR	1750	1990	5890	295	9925			7.73721E+04	1.55415E+06	7
8	INSTALL CHY1F 2 X 40 MW HYDRO RETIRE XHY2 3 X 30 MW HYDRO	1750	1990	5890	295	9925	5.79793E+04	3.86529E+04	8.00729E+04	1.73006E+06	8
9	NO ADDITIONS THIS YEAR RETIRE XTH7 3 X 57 MW THERML	1740	1990	5890	295	9915			8.30585E+04	1.81395E+06	9
10	NO ADDITIONS THIS YEAR	1740	1990	5719	295	9744			8.60708E+04	1.90003E+06	10
11	INSTALL DTH1R 1 X 600 MW THERML	1740	1990	5719	295	9744	8.40482E+04	8.40482E+04	1.30591E+05	2.19871E+06	11
12	INSTALL ZPK1R 5 X 50 MW PEAKING RETIRE XPK2 1 X 55 MW PEAKING	1740	1990	6319	295	10344	2.23866E+04	2.34506E+04	1.37594E+05	2.36215E+06	12
13	INSTALL DTH1R 2 X 600 MW THERML	1740	1990	6319	490	10539	1.67547E+05	1.67547E+05	1.44666E+05	2.86191E+06	13
14	INSTALL ZPK1R 2 X 50 MW PEAKING	1740	1990	7519	490	11739	8.78266E+03	9.20393E+03	1.47786E+05	3.02769E+06	14
15	NO ADDITIONS THIS YEAR	1740	1990	7519	590	11839			1.50924E+05	3.17862E+06	15
	(CLC-SIMULATION RUN)					TOTALS	1.16487E+06	6.03933E+05	1.40582E+06		

TABLE 6.2-158 CASE STUDY TWO, TRACEBACK SEARCH NO 5
OPTIMAL EXPANSION STRATEGY, NUMBER 5

YR	SYSTEM CHANGES FOR THIS YEAR	INSTALLED CAPACITY AT START OF YEAR (M E G A W A T T S)					COST OF FOREIGN	ADDITIONS LOCAL	SYSTEM RUNNING COST	CUMULATIVE COST AT END OF YEAR ONE)	YR
		HYDR	NUCL	THERM	PEAKING	TOTAL					
1	NO ADDITIONS THIS YEAR	1750	790	5022	195	7757			5.50037E+04	5.50037E+04	1
2	INSTALL BPK1C 4 X 25 MW PEAKING	1750	790	5022	195	7757	9.27883E+03	1.08925E+04	5.97265E+04	1.34902E+05	2
3	NO ADDITIONS THIS YEAR RETIRE XTH9 2 X 29 MW THERM	1750	790	5022	295	7857			6.34975E+04	1.98399E+05	3
4	INSTALL ANU1C 2 X 600 MW NUCL RETIRE XTH8 4 X 55 MW THERM RETIRE XTH10 2 X 28 MW THERM	1750	790	4965	295	7800	6.10595E+05	6.78440E+04	6.70731E+04	9.43912E+05	4
5	INSTALL DTH1F 2 X 600 MW THERM	1750	1990	4690	295	8725	2.04249E+05	2.02285E+05	5.23382E+04	1.40278E+06	5
6	NO ADDITIONS THIS YEAR	1750	1990	5890	295	9925			7.39937E+04	1.47678E+06	6
7	NO ADDITIONS THIS YEAR	1750	1990	5890	295	9925			7.73721E+04	1.55415E+06	7
8	INSTALL CHY1F 2 X 40 MW HYDR RETIRE XHY2 3 X 30 MW HYDR	1750	1090	5890	295	9925	5.79793E+04	3.86529E+04	8.00729E+04	1.73085E+06	8
9	NO ADDITIONS THIS YEAR RETIRE XTH7 3 X 57 MW THERM	1740	1990	5090	295	9915			8.30985E+04	1.81395E+06	9
10	INSTALL ZPK1R 2 X 50 MW PEAKING	1740	1990	5719	295	9744	8.81113E+03	9.23345E+03	8.60708E+04	1.91807E+06	10
11	INSTALL DTH1R 1 X 600 MW THERM	1740	1990	5719	395	9844	8.40482E+04	8.40482E+04	1.31350E+05	2.21752E+06	11
12	INSTALL ZPK1R 3 X 50 MW PEAKING RETIRE XPK2 1 X 55 MW PEAKING	1740	1990	6319	395	10444	1.34320E+04	1.40758E+04	1.36317E+05	2.38434E+06	12
13	INSTALL DTH1R 2 X 600 MW THERM	1740	1990	6319	490	10539	1.67547E+05	1.67547E+05	1.44921E+05	2.86436E+06	13
14	INSTALL ZPK1R 2 X 50 MW PEAKING	1740	1990	7519	490	11739	8.78295E+03	9.20393E+03	1.48003E+05	3.03044E+06	14
15	NO ADDITIONS THIS YEAR	1740	1990	7519	590	11839			1.51192E+05	3.18163E+06	15
	QCLC-SIMULATION RUN)					TOTALS	1.16472E+06	6.03783E+05	1.41313E+06		

6.3 COMMENTS

The results of the two preceding case studies are not intended as propositions of general rules for power system development. Individual systems and their input data vary considerably from one case to another, and also from year to year according to conditions not always foreseeable.

It is of course for this reason that planning studies are continually under review, taking advantage of one of the few luxuries available to the planner: the time he has to examine alternatives and to evaluate them in as much detail as he thinks fit. The exploratory traceback searches of Case Study Two demonstrate the usefulness of the method for the appraisal of the effect of particular constraints. Changes in the retirement dates for existing plant, or compulsory installation of certain choices can have an indirect effect on the overall planning strategy which may not at first sight be obvious. The impact of particular constraints and the cost penalties or gains engendered by them can be evaluated fairly readily. This kind of information can be very useful and is often required for management decision-making.

The package is amenable to further development to improve the efficiency of computation or to suit particular needs. For example the incorporation of a simple load flow model can be readily accomplished to screen the feasibility or otherwise of various alternatives during the branching process. Other improvements currently under active consideration are:

- More sophisticated input data processing to check and perhaps modify input path constraints in accordance with implicit constraints contained in the pool data. This will give closer agreement between predictions of expansion costs by the state functions and the eventual expansion costs. Some care should be taken however to avoid degrading the flexibility of the model for investigations into alternative traceback scenarios.

- At least part of the discrepancy between predicted and eventual expansion costs to the horizon is due to the failure to take into account the costs of replacement of existing plant which is due to be retired during the planning period. This can be taken into consideration and corrections applied either during assembly of the state functions or during traceback, the latter being preferable.
- The incorporation of a probabilistic loading simulation model (Section 5.2) during assembly of the recursive functions and exploratory traceback searches, instead of the crude simplified model of Section 5.1. Whatever form of LDC-model is used, closer attention is to be paid to reducing the discrepancies between predictions of system operating costs between it and the CLC-loading simulation model over the whole range of likely system compositions.
- A separate routine may prove useful for verification of the existence of a path to the horizon through a proposed node. The traceback search is often short-sighted in making selections which do not lead to feasible paths to the horizon, and this is particularly wasteful when the CLC-loading simulation model is used, since much effort can be expended in assembling the system performance functions. Provided some constraints are ignored, this routine will involve finding a basic feasible solution to a linear program, a procedure which is fundamental to the use of the simplex algorithm of linear programming, and for which standard techniques are available and well documented.

At their present state of development, however, the programs suffice a workable tools for use in medium-term system planning, and to demonstrate the efficacy of the two-phase method in overcoming the not inconsiderable computational problems associated with dynamic programming.

CHAPTER SEVEN. CONCLUSIONS

In Part I, the discursive portion of the thesis, an attempt has been made to emphasize some of the principal engineering, economic, and financial considerations pertaining to the planning of bulk electricity supply. The nature of the service provided coupled with the considerable potential economies available from an increase in the scale of operations are crucial factors which encourage the adoption of a grid supply system and the installation of large generating unit sizes, with interconnection of neighbouring supply networks for mutual security and benefit. This is not however without its attendant disadvantages, some of which were outlined in Chapter One. The problems of plant scheduling in response to variation of load demand on an hourly timescale are complex, and these operational considerations carry over to the planning and design of power systems. For small independent systems in particular the simulation of system operation leads to non-linearities and discrete functions owing to the interaction between different plants or plant types and actual physical modes of operation of units. In some respects accurate simulation of system operation for the smaller network can be more difficult than for the large network, for whilst the interaction still leads to a non-linear problem, the effects of the discrete functions are not as significant.

In Chapter Two a discussion of the economic and financial factors to be considered in planning studies showed also how the increase in scale imposes penalties which, in conjunction with the shift towards capital intensiveness apparent in power system equipment price trends, aggravates already serious problems in the financing of power development. For developing countries with limited resources of capital and foreign exchange such considerations render diligent forward planning necessary.

Apart from these, political and other factors will at all times have to be taken into account, for the planner cannot work in a vacuum. The electricity supply industry maintains an

importance position in relation to the national economy and often has to accept its fair share of the responsibility for the continuance of stability in the industries of the suppliers of its capital equipment and primary fuel, and its consumers.

It is therefore not surprising in view of the foregoing that the planning of system expansion and development is an integral part of the management of bulk electricity supply organizations. Clearly with a positive growth rate in demand it is imperative that future expansion of the system be evaluated with care, but even with depressed or even zero growth rates (implying system development rather than expansion) the problem is no less critical. Changes in relative capital and production costs, improvements in the technology of power supply, and the retirements of existing plant mean that new capacity is always required and future system development plans have to be considered carefully for maximum efficiency in resource use. System planning is therefore very much a continuous process and it is by no means unusual for even modest sized electricity supply organizations to maintain full-time planning and construction departments or divisions.

The next few paragraphs (Section 7.1) comprise a general discussion on planning models, from the point of view of their scope and the degree of constraint satisfaction. Section 7.2 discusses the nature and structure of an original two-phase mathematical planning model for the determination of the optimal installation framework in the medium term. Section 7.3 is devoted to a similar appraisal of a new chronological loading simulation model, described in Chapter Five, applicable to the evaluation of mixed hydro-thermal systems in planning studies. An integrated planning approach incorporating these two models and used for the case studies of Chapter Six is discussed in Section 7.4, and the chapter and the thesis concludes with a few final remarks in Section 7.5.

7.1 PLANNING MODELS

The importance of planning models means as a matter of course that great attention has been paid to their construction and solution by system planners. Models for the determination of optimal system development are widespread, and vary considerably from one to another in scope, approach, and usage. Despite extensive work however the problem remains obdurate and a satisfactory comprehensive solution has yet to be obtained from a single model. For several reasons however it is generally agreed that such a model would not justify the effort required for its construction and solution. It must always be borne in mind that input data and constraints will vary with time and will have to be continually updated. Moreover by the end of this century it is certain that there will be important changes in primary fuel availability which will have a profound effect on future optimal composition and perhaps even the functions of electric power systems. Patterns of energy demand and consumption are also likely to undergo significant changes in the foreseeable future. Intensive research is under way on the urgent question of "energy futures" but these questions still remain to a large extent imponderables, or at least extremely difficult to quantify with any degree of confidence.

Under such conditions a staged approach to planning is the only rational approach, for often the best-laid plans can be rendered invalid due to unforeseen and sometimes unpredictable changes in the premises upon which the plans were originally based. Hence a step-by-step approach, representing a steady narrowing down in scope, with a shortening of the planning horizon accompanied by an increase in rigour of constraint satisfaction, is generally adopted in planning studies.

Apart from the size or complexity of the overall planning problem a step-by-step approach is also necessitated by the nature and quality of the input data. Projections of load growth and cost estimates even in the medium-term future are notoriously difficult, and successful development of more

sophisticated planning models as a whole must await the development of more rigorous formal methodologies for the determination of input data. The econometric models of growth of electricity consumption briefly discussed in Chapter One are a useful step in this direction, although they have yet to gain unqualified acceptance from system planners. It is notable that in International Atomic Energy Agency studies of electric power system expansion close attention was directed to the evaluation of projected capital costs, and a set of complex computer programs (ORCOST) was utilized to assemble satisfactory capital cost data /21/. In general it cannot be too heavily stressed that collection and verification of input data demands at least as much time and effort from the planner as any other single part of the planning process.

Any mathematical model is a distortion of reality, and the integrity and usefulness of such a model is to be judged by the accuracy with which it reflects actual events. Inevitably, simplifications have to be made and a close understanding of the nature and structure of the problem is essential for the construction of a practical planning model. One of the main stumbling blocks encountered in the development of such models for planning is the difficulty of dealing satisfactorily with real-life constraints which apply in the planning context, some of which may be critical. For convenience in the discussion of the next few paragraphs, these constraints have been classified into three broad categories:

- (a) Those constraints related to well-defined trends along the temporal axis (eg years during the planning period), examples being the growth of demand (peak, average, or both), energy, cost escalation factors, and the test discount rate selected.
- (b) Those constraints related primarily to siting or planting considerations, such the limited availability of discrete unit sizes, dates of availability of particular options, site capacity limits, and the sequencing of planting or site development.

- (c) Constraints related to the daily or hourly operation of the system in response to given load curves (cycles of demand).

Whilst constraints (a) are characterized by smooth continuous functions, both constraints (b) and (c) feature nonlinear, often discontinuous, and sometimes conditional functions. It is these latter two types which are principally responsible for difficulties in the construction and solution of mathematical programs for optimal power system development.

In Chapter Three engineering-economic models for system development were divided into three classes according to the emplacement of their planning horizon: long-term models (this includes infinite-horizon and steady-state models), medium-term models, and short-term models.

Models of the first class occur most frequently in the published literature. As a general rule these are global optimization models which cater for constraints of type (a), whilst constraints of type (b) are usually ignored, and those of type (c) often greatly simplified for computational ease or economy. The focus of attention is the determination of the optimal plant mix. Such models have been refined to a fairly high level of sophistication, although excessive refinement has in recent years sometimes been brought into question:

" The uncertainties in all these items (input data), and the constraints imposed by them, mean that planning strategy will be determined by judgement at a senior level, based on economic appraisals and sensitivity studies of a fairly simple nature. Complexity and sophistication of mathematical optimization is quite irrelevant.

" All this having been said, it would nonetheless be inappropriate to have no mathematical understanding at all of the economic assessment problems underlying day-to-day planning work ... Having regard to the irrelevance of over-complicated and precise models ... by making sensible simplifying assumptions, a very simple mathematical model can be set up which is quite adequate for indicating in outline the expected long-term development of generating plant in an electricity supply system for given forecasts of parameter values." (/34/)

Whilst the broad generalization contained at the end of the first of the quoted paragraphs is perhaps to be faintly

deplored, the point is well made and timely, for often the quality of the input data to models peering so far into the future does not justify great accuracy. Future development of distant-horizon planning models might well be more profitably directed towards more comprehensive rather than more accurate parametric analysis, or those variants which do not have simple cost-minimization as their objective. Maximization of a social welfare function (/18/), or multiple-objective formulations might be considered more appropriate. Alternatively in recognition of the inherent stochastic nature of most planning data, models which deal specifically with the statistical distribution of major parameters may prove useful. By the use of minimax (eg minimize the maximum expected overall system development costs, within system security constraints) or maximin (eg maximize the minimum expected system security, within development cost constraints) and similar formulations the planner can be guided to a "safe" or low-risk strategy in the face of an uncertain future.

In contrast to these long-term models, short-term models deal with the immediate future, in system planning terms. These models are often more concerned with feasibility rather than optimality, and frequently simplicity and elegance of formulation are luxuries the planner can ill afford. Such models are far less well documented in the published literature, probably because of the wide variation in individual system structure, characteristics, and load. In practice it is in fact often arguable whether an optimization problem, in the generally accepted sense, exists for the near-horizon problem. Often the number of feasible alternatives is very small and the usual approach is to evaluate in detail each of these alternatives, and cost and rank them accordingly. Such models are therefore simulation* models, since the development of the system is being simulated. Short term models deal in detail with constraints of types (b) and (c).

* The term is used in the precise sense, although it is common in system planning work to use the label to identify loading simulation models (Chapter Five). In these paragraphs a qualifier will always be included to avoid confusion.

Part II of the thesis addresses itself squarely to the solution of system development or expansion in the medium term. A model is outlined in Chapter Four which takes into consideration both constraints of types (a) and (b). This model occupies an intermediate position between distant- and near-horizon planning models. Appropriately, it is a two-phase model with global optimization in the first and simulation of system development in the second phase. An appraisal of this original two-phase method follows in the next Section.

Also in Part II, the theory for a loading simulation model which aims at chronological simulation of system operation is presented. This model attains more rigorous satisfaction of constraints of type (c) than comparable models, and is discussed in Section 7.3.

7.2 THE TWO-PHASE PLANNING MODEL

The theory for the two-phase model was set out in Chapter Four. In the first phase of solution a simple continuous function dynamic program is set up and solved. This is a global optimization model and is in its own right a model for optimal power system development, detailing the optimum plant mix to be followed for least-cost system development. The use of dynamic programming for solution imposes a computational burden, but yields benefits which are thoroughly exploited during the second phase of solution, which is a traceback search in a decision tree in which the discrete and non-Markovian nature of certain functions and constraints are accommodated during simulation of system development.

A useful analogy may be drawn between the approach to system planning of this model and the preparations of a hiker for progress across rough country. Having made careful plans for his journey by assembling the relevant state and policy functions, the hiker will at the beginning of his trip have a very good idea of his likely optimal (least-effort) path towards his intended destination, by reference to the state and policy functions. There is of course a limit to the amount of detail that he can or may wish to take into account in his preparations, and during his trek he may encounter obstacles (cliffs, water-courses, forests) not previously catered for, which impede his progress and necessitate detours. Thus during traceback, if forced to deviate from his intended strictly optimal path, reference to the state functions will inform him what resources will be needed to move from his present position to his intended destination, and the policy functions will guide his next move from this position to an optimal path. Note that if his detour has been substantial, the new optimal path may be quite different from the original optimal path.

This is not all, for the state and policy functions can be defined for the whole feasible area which the hiker can conceivably cover. Thus if (for example) he is dropped by helicopter anywhere in this area, reference to the recursive functions immediately

provides an evaluation of his present position and a guide to his next move.

Herein lies the clue to the power of the dynamic programming approach, and also the reason for its computational extravagance. Although greatly more efficient than exhaustive enumeration, nevertheless a very large number of evaluations still have to be made for assembly of the recursive functions. It is for this reason that the state and policy functions contain a mass of information, and this fact is exploited by the traceback search.

It would be unwise to pursue this analogy much further, since the system planning problem as treated in this thesis involves a four-dimensional state vector as opposed to the hiker's position as indicated by his two-dimensional map references; also in the system planning context some planning constraints - corresponding to the hiker's obstacles - may even change according to events or decisions taken along the current path (indicating a non-Markovian process). Furthermore in system planning the temporal scale of years during the planning period is an important feature of the problem.

The traceback search is conducted in a decision tree, which potentially can be computationally demanding, bearing in mind the near-exponential nature of the tree. However the number of branches near the root of the tree (in the near future) is in real cases strictly limited, when typical lead-times for power projects are taken into consideration. Furthermore the information embodied in the recursive functions is used to lessen the computational burden by the use of simple rules akin to those of the branch-and-bound algorithm of mathematical programming. The state functions are used to predict least-cost expansion from any given state at any given year to the horizon; the policy functions are used to recommend near-optimal moves at each year.

The division of the problem into two separate phases of solution each dealing principally with a different level or class of constraint has advantages which prove useful when examining alternative scenarios of development for a single

specific system, which would be the case in practice. This was discussed in Chapter Four and demonstrated in Chapter Six.

The second phase of solution is ideally suited to marginal-type analysis. In Case Study Two of Chapter Six it was shown how the impact of variations in retirement dates for existing plant, delays in commissioning dates for current projects, and compulsory implementation of certain projects, can be evaluated. The advantage of an optimization search during traceback lies in the fact that when marginal changes are applied to parts of a projected expansion schedule the remainder of the schedule need not necessarily remain unaltered but may readjust to give a new lowest-cost strategy under these new conditions. This property neatly resolves a common dilemma facing the system planner: small changes to a proposed programme of development may have indirect effects on future system development, causing shifts in other areas of the planting strategy, but often he cannot be certain as to their nature or scope without a wealth of experience or accurate intuition backed by a great many trial runs. The use of a branch-and-bound search during traceback will identify such effects automatically.

Furthermore since the search is a chronologically structured numerical procedure all nonlinear, discontinuous, or conditional constraints can be accommodated with little difficulty.

The structure of the two-phase model may therefore be summed up as follows:

- The state and policy functions generated during the first (build-up) phase of solution embody the "master plans" for optimal power system expansion, not only from the initial state at the initial year, but also from all feasible states at all years, to the horizon. Thus in this way the recursive functions define optimal policies of development over the planning period. The second (traceback) phase on the other hand conducts a decision tree search to determine the tactics by which the recommended policies are to be realized.

7.3 THE CHRONOLOGICAL LOADING SIMULATION MODEL

If the two-phase planning model discussed in the previous section attempts to bridge the gap between short- and long-term planning models, the loading simulation model described in the concluding section of Chapter Five occupies a similar intermediate position.

Models which accept the load duration curve as a description of the load a system is required to serve are in common use in system planning studies because of their computational economy. Under certain conditions, particularly in the short to medium term, this form of load description may not be entirely satisfactory. As discussed in Section 5.3 the shortcomings of LDC-based loading simulation models are primarily due to the loss of chronological data when the transformation from CLC to LDC is made. This may not constitute a serious disadvantage when the proportion of installed hydro capacity is small, but for mixed hydro-thermal systems such a model can produce erroneous estimates of system operating costs.

Models which simulate the operation of systems in response to chronological load data are widely used for short-term feasibility studies or for plant scheduling in real time but are often much too unwieldy or too time-consuming for planning purposes. They do not however suffer from any of the disadvantages of LDC-models which arise from the loss of chronological data.

Sections 5.4 to 5.9 in Chapter Five describe the theory for an original loading simulation model which accepts as the load description a chronological (daily) load curve. This CLC-based model aims at the computational economy of the LDC-based models combined with the accuracy of CLC-models in reflecting actual strategies of system operation. In the event it falls short of both these aims but nevertheless forms a satisfactory compromise between the two. Although requiring a CP time perhaps an order of magnitude or so more than LDC-models, it is applicable for planning studies in the medium term. It

represents an improvement over existing loading simulation models used for planning in the following respects:

- Systems containing pumped storage stations, or even conventional hydro stations with appreciable reservoir capacity possess an indirect storage capability - by the operation of pumps to store electric energy as hydraulic potential energy, or in the latter case by withholding the expenditure of hydro energy to a later time. For accurate examination of the operational strategy to be adopted for such a system proper treatment of the chronological sequence of demand is essential.
- Limited feasible regimes of operation for each individual unit in the system and part-load efficiencies are taken into account.
- Explicit attention is given to the evaluation of the costs of provision of spinning reserve; this is of particular relevance for small independent systems and those which contain a wide disparity of unit sizes. The chronological sequence of spinning reserve required is also taken into consideration.

Such factors represent facets of system operation which are non-trivial and often neglected or given superficial consideration in long term planning studies but which should be considered in the short or medium term.

Apart from its use in the planning context the model will also be acceptable for the evaluation of other methods of direct or indirect storage of electrical energy on a large scale. As was mentioned in Chapter One greater attention is currently being paid to such methods which can effect significant economies in system operation as well as enhancing system security and quality of supply. Loading simulation following a chronological load curve is again essential for proper evaluation of such accumulator devices.

As discussed in Chapter Two the use of marginal pricing in determining the structure of supply tariffs is fundamentally directed towards the modulation of daily and annual cycles of

demand for the mutual benefit of both producer and consumer. The system performance functions of Sections 5.8 and 5.9 can be used to determine the incremental costs of system operation for the system comprising the two (hydro and non-hydro) subsystems in terms of output, spinning reserve, and instantaneous available hydro energy.

As with the two-phase planning model dynamic programming has been used to full advantage. System performance functions need not be reevaluated if a single particular system is under study, for again they contain "master plans" for system performance evaluation under all feasible conditions. The functions do not have to be completely updated when minor additions or removals are applied to the system - the case in practice - because of the property of invariant imbedding of dynamic programming.

The model as outlined in Chapter Five is in a very basic form, simplified and aimed at maximum computational economy for use in planning studies. The general principles of the model however render it quite flexible and it is amenable to further development or detail improvements for other purposes.

7.4 DETERMINATION OF THE INSTALLATION FRAMEWORK

Chapter Six demonstrated the use of a set of computer programs written to test and verify the two-phase planning model of Chapter Four, with loading simulation by means of the model of Chapter Five. In the following paragraphs some general comments on this integrated model will be offered.

Perhaps the basic problem facing the planner in the medium term is one of selection and timing for future installations. General trends as advocated by long term planning models are often fairly self-evident, but the transition from "policy" to "tactics" is not always so clear. Stated simply, the deferment of a particular installation means that in real terms there is every likelihood of its being cheaper when eventually installed, but this also means that any benefits in terms of reduced production costs or easier system operation are also postponed. The planner's decision has of course to be made not only in the light of system composition and demand, but also any other constraints that may apply. Particular cases can sometimes be easily evaluated but when dealing with a planning strategy over a period of a decade or more, with perhaps interaction between selections themselves the problem becomes extremely difficult to manage.

It is therefore reassuring to have a tool for formal and comprehensive evaluation of all candidates or combinations of candidates which can be ranked accordingly and arranged to form the lowest-cost overall installation framework.

In the computer package a conscious effort has been made to attain more rigorous constraint satisfaction than comparable methodologies. Many awkward constraints are handled, some of which can form overriding considerations in medium-term power system development:

- The existence of discrete unit sizes, economies of scale for larger units, penalties in system operation from the use of too large a unit size in relation to system installed capacity.
- Site considerations: the recognition of a limited number

of available sites, limits to individual site capacities, mutually exclusive options at a single site or group of sites, and the sequencing of site development.

- Constraints on capital expenditures. As Case Study One shows, such constraints can have the effect of discouraging premature overplanting of the system, with the result that system capacity is continually being incremented by small amounts, and difficulty may be experienced in maintaining adequate system capacity to meet load growth owing to the lack of capital resources. This confirms trends often apparent in developing countries.

- The results of separate cost-benefit analyses, which may entail compulsory development of certain sites. Where selections are made mandatory the planner's problem is the determination of suitable dates for project implementation and the creation of a suitable framework of planting within which these compulsory installations may be made.

- Political or other constraints which can have a similar effect on selections or dates of availability of plant. Such constraints can also arise from the results of more general energy policy studies.

All such constraints are bound to distort overall optimal policies of development, and any installation framework is never likely to be optimal at any point in time during the planning period. The best that can be done in the face of such difficulties is to aim for overall optimality over the planning period taken as a whole. The planning model determines such strategies taking into account all the constraints mentioned above.

A disadvantage of the model, discussed in Chapter Six, is the effect of truncation at the end of the planning period. The longevity and high initial costs of hydro and nuclear plant means that they can be discriminated against in favour of lower capital cost plant close to the planning horizon. Such errors can be minimized by imposing strict constraints on plant mix.

Some proposed improvements to the model are listed in Section 6.3, whilst detail improvements to the program structure and code are discussed in /53/.

If the traceback phase of the planning model is regarded primarily as a simulation model of system development, some further extensions to its applicability can be suggested. In the case studies presented, capital expenditure constraints, differentiated according to foreign and local components and applied cumulatively over a set period of consecutive years, have been used. This is in fact one of the simpler forms that can be accommodated, and it is of course a gross simplification of actual constraints that prevail. Since however the traceback search is a numerical procedure, computer programs are readily recoded to suit any specific needs. Further development along these lines could eventually lead to the incorporation of a financial model running in parallel with traceback, simulating annual cash flows and capital reserves over the period of study. This would take into account operating costs, overheads, tariffs and revenue, and the servicing of existing loans for each year of study, thus paying closer attention to actual financial status, which in turn would impose constraints on feasible development options. Such a model would of course tend to be large and time-consuming, especially if used as an optimization procedure, and would really only be warranted with greatly improved quality of input data. But it would be eminently suitable for the simulation of system development in the medium term.

In a similar manner the traceback search can be integrated into a larger model for comprehensive energy policy studies. Again the same strictures apply, and it is doubtful whether in these applications such detailed simulation would be required.

7.5 CONCLUDING REMARKS

Mathematical programming models used in conjunction with digital computers are generally accepted these days as powerful tools which can aid the planner in a logical and rational approach to system evaluation and design. There is a multitude of models available to this end, each with its own peculiar features, advantages or otherwise, and adherents. It is however generally unrewarding to enter into debate on the relative merits and demerits of each particular technique. As Berrie and Anderson point out:

" The actual choice of technique ... may properly rest more on the predilections of the individual, the computing facilities at his disposal, the data, and the size of the system he is analyzing, than on the intrinsic superiority of any one method over another. " (/6/)

Rather therefore than undertake to duplicate the distinguished efforts of other workers in the field of power system planning, an attempt has instead been made to identify specific intermediate problem areas not hitherto the subject of intensive research, and to develop suitable models for power system planning within these terms of reference.

This author believes that a reduced-horizon problem is not a trivial one, and the results of this quest, presented in this thesis, are proffered here as practical complementary additions to the existing body of knowledge on the subject.

APPENDICES

APPENDIX A. DYNAMIC PROGRAMMING

Dynamic programming is a term variously applied to both a methodology of mathematical programming and a computational technique, conceptually simple but with profound and wide-ranging applications to the solution of optimization problems in general. As the name implies, it is applicable to any problem that exhibits a multistage structure. An advantage of the standard computational procedure is that during the course of solution constraints that are not easily handled by gradient-based methods (integer restrictions, discontinuities, or severe nonlinearities) can be readily accommodated. There are disadvantages. Except for the most simple cases the solution requires the use of large high-speed computing machinery, and even with these powerful numerical tools many problems cannot be solved at all within a reasonable time.

A.1 NOTATION

Figure A-1 represents a multistage process. It is visualized as encompassing K stages. An n -dimensional state vector $\underline{s}_k$ represents the state of the system at stage k . An m -dimensional decision vector $\underline{d}_k$ is applied to the system at stage k . The transition function T_k relates the input and output states at stage k with this decision vector, this is an n -dimensional vector function. Associated with each stage is a scalar return function R_k , this is a function of $\underline{s}_k$ and $\underline{d}_k$.

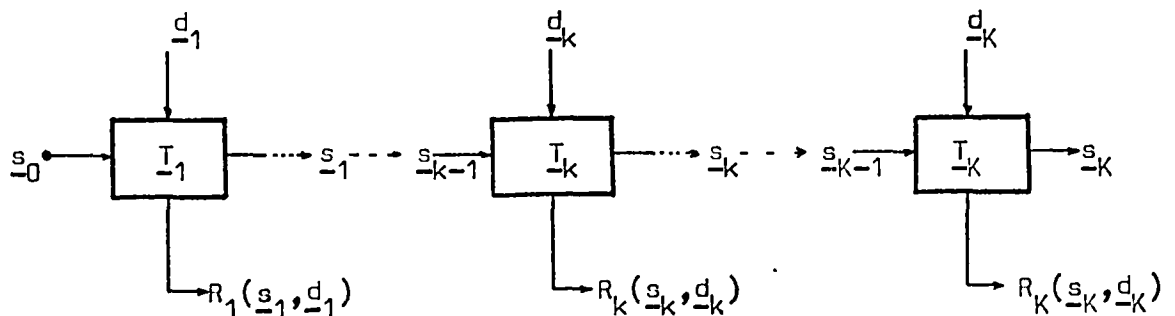


FIGURE A-1. MULTISTAGE PROCESS

A.2 DERIVATION OF THE ALGORITHM

The derivation given below is based upon the approach of Larson /36/. The forward programming algorithm is derived.

$$\text{optimize}^* \sum_{k=1}^K R(\underline{s}_k, \underline{d}_k) \quad \dots(A-1)$$

where states $\underline{s}_k$ and $\underline{s}_{k-1}$, and decision $\underline{d}_k$, are related by a transition function of the form:

$$\underline{s}_{k-1} = \underline{I}_k(\underline{s}_k, \underline{d}_k) \quad \dots(A-2)$$

subject to the following constraints:

$$\underline{s}_k \in S_k \quad \dots(A-3a)$$

$$\text{and } \underline{d}_k \in D_k(\underline{s}_k) \quad \dots(A-3b)$$

Initial and final states $\underline{s}_0$ and $\underline{s}_K$ may or may not be specified.

For solution, an optimal recursive state function F_k is defined:

$$F_k(\underline{s}_k) = \underline{d}_j, j = 1, 2, \dots, k \left(\sum_{j=1}^k R_j(\underline{s}_j, \underline{d}_j) \right) \quad \dots(A-4)$$

By definition, F_k provides the optimal solution (Eqn A-1).

Splitting the summation of Eqn A-4 into two parts:

$$\begin{aligned} F_k(\underline{s}_k) &= \underset{\underline{d}_k}{\text{opt}} \underset{\underline{d}_j, j = 1, 2, \dots, k-1}{\text{opt}} \left\{ R_k(\underline{s}_k, \underline{d}_k) + \sum_{j=1}^{k-1} R_j(\underline{s}_j, \underline{d}_j) \right\} \\ &= \underset{\underline{d}_k}{\text{opt}} \left\{ R_k(\underline{s}_k, \underline{d}_k) + \underset{\underline{d}_j, j = 1, 2, \dots, k-1}{\text{opt}} \sum_{j=1}^{k-1} R_j(\underline{s}_j, \underline{d}_j) \right\} \\ &= \underset{\underline{d}_k}{\text{opt}} \left\{ R_k(\underline{s}_k, \underline{d}_k) + F_{k-1}[\underline{I}_k(\underline{s}_k, \underline{d}_k)] \right\} \quad \dots(A-5) \end{aligned}$$

Eqn A-5 represents the core of the dynamic programming approach, and is a mathematical statement of Bellman's Principle of Optimality:

"An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision". (/4/)

* optimize = maximize or minimize.

A.3 NOTES

1. Figure A-1 can readily be imagined to represent the planning process for power system development, with the separate stages corresponding to points in time (years) up to the horizon year (stage K). State vectors will represent the system at each year, and decision vectors will represent the introductions to or retirements from the system. The costs of installations and system operation will be represented by the return functions R_k .

In a similar way Fig A-1 can also represent a control problem: the determination of what controls to apply ($\underline{d}_k$, $k = 1, 2, \dots, K$) in order to attain a particular final state $\underline{s}_K$ with (say) minimum effort or cost ($\sum_{k=1}^K R_k(\underline{s}_k, \underline{d}_k)$).

Most multistaged processes may be solved with judicious definition of state and decision (or control) variables, even fairly complex networks containing loops and recirculations, using techniques of system decomposition. It is not always immediately obvious however how the algorithm may be applied to the solution of allocation problems. A treatment of these is presented in Section A.5.

2. From the derivation above it will be seen that this form of solution is only applicable for objective functions which are additively separable with respect to the stages k . If this condition is violated the derivation of Eqn A-5 will be invalid.

3. The algorithm can only be employed if the state of the system at each stage k can be completely described by the state vector $\underline{s}_k$. The past history of the process (how the state arose) should be irrelevant as far as future behaviour of the system is concerned. In other words, the system must be Markovian. Most natural systems are inherently Markovian, or can be regarded to be so with sufficient detail in state description. Unfortunately an increase in state dimensionality can often introduce severe computational penalties in solution (see Section A.4).

4. A feature of the algorithm is that minimization is carried out a stage at a time. This stagewise optimization is not however carried out for each stage in isolation, but includes a contribution from the previous stage in the form of the state function F_{k-1} . At the k^{th} stage the problem that is in fact being solved is that for a system consisting of stages 1 to k in the computational sequence. This is the property of invariant imbedding: in a single solution to the dynamic program, solutions for a whole subset of K identically structured problems with different numbers of stages is produced. Often the property of invariant imbedding is of little practical use, although it is thoroughly exploited in the dynamic programs in this thesis.

5. The forward programming algorithm involves the recursive application of Eqn A-5 for $k = 1, 2, \dots, K$. Larson /36/, by redefining the transition function of Eqn A-2:

$$\underline{s}_{k+1} = T_k(\underline{s}_k, \underline{d}_k) \quad \dots(A-6)$$

and the state function of Eqn A-4:

$$F_k(\underline{s}_k) = \min_{\underline{d}_j, j = k, k+1, \dots, K} \left\{ \sum_{j=k}^K R_j(\underline{s}_j, \underline{d}_j) \right\} \quad \dots(A-7)$$

follows an analogous procedure to derive a recursive relationship for the backward algorithm:

$$F_k(\underline{s}_k) = \min_{\underline{d}_k} \left\{ R_k(\underline{s}_k, \underline{d}_k) + F_{k+1} [T_k(\underline{s}_k, \underline{d}_k)] \right\} \quad \dots(A-8)$$

- this formula being applied backwards for $k = K, K-1, \dots, 1$.

The choice of which of the two algorithms, forward or backward, depends largely upon the problem being solved, and the initial conditions given. If the transition function can be readily inverted to give either the form of Eqn A-2 or Eqn A-6 then either forward or backward programming is applicable. Then if the initial condition $\underline{s}_0$ is given forward programming is preferable, conversely if the final condition $\underline{s}_K$ is fixed then a backward solution would be preferable.

6. Whether backward or forward recursion is employed the full solution will have to be carried out in two phases. In the first, build-up, phase, the state functions F_k are assembled and stored

a stage at a time. The end results of this phase, $F_K(\underline{s}_K)$ will provide the required solution of Eqn A-1. If the sequence of decisions that give rise to this optimal solution (the optimal policy $\underline{d}_k$, $k = 1, 2, \dots, K$) it is also necessary during the build-up phase to assemble the policy functions as well as the state functions:

Let Eqn A-4 be written as:

$$F_k(\underline{s}_k) = R_k(\underline{s}_k, \underline{d}_k^*) + F_{k-1} [T_k(\underline{s}_k, \underline{d}_k^*)] \quad \dots(A-9)$$

and define: $\underline{G}_k(\underline{s}_k) = \underline{d}_k^* \quad \dots(A-10)$

The m-dimensional vector functions $\underline{G}_k$ are the policy functions; $\underline{G}_k(\underline{s}_k)$ is the decision to be implemented at stage k, with the system at state $\underline{s}_k$, in order to attain optimality at final stage K.

The second phase of solution, the traceback, uses the policy functions together with the transition functions in order to generate the optimal policy. Let the final condition be $\underline{s}_K^*$ (if this is not specified, then the optimal final state is $\text{opt} [F_K(\underline{s}_K)] = F_K(\underline{s}_K^*)$).

By definition, the optimal decision for this year is:

$$\underline{d}_K^* = \underline{G}_K(\underline{s}_K^*) \quad \dots(A-11)$$

and from the transition function (Eqn A-2):

$$\underline{s}_{K-1} = T_K(\underline{s}_K^*, \underline{d}_K^*) \quad \dots(A-12)$$

Equations A-11 and A-12 are applied recursively for $k = K, K-1, \dots, 1$, thus generating the optimal decision sequence $\underline{d}_k^*$, $k = 1, 2, \dots, K$. This is the optimal policy.

For a forward dynamic program, the build-up phase moves forward (increasing k) whilst the traceback phase moves backward (decreasing k). For a backward dynamic program this order is reversed.

A.4 COMPUTATIONAL ASPECTS

Equation A-5 (or A-8) is a functional equation, in the sense that F_k is an unknown function rather than an unknown quantity. As a general rule such an equation can rarely be solved analytically, except for some special forms and for very simple unconstrained almost trivial cases. The so-called standard computational algorithm, to be described below, is frequently used for the solution of the recursive equations; although perhaps inelegant it has yet to be supplanted by more efficient general methods.

Suppose that the state and decision vectors are discrete, and that each state variable (each component of the state vector) can assume N_k discrete values at stage k . If then none of the constraints of Eqn A-3a is active then the number of feasible states at this stage is N_k^n . Equation A-5 is now solved for each of these feasible state vectors, yielding the value of the state function F_k and policy function G_k for each feasible state. Effectively Eqn A-5 will have to be solved N_k^n times at each stage k .

If each component of the decision vector can assume M_k discrete values, the optimization of Eqn A-5 may be carried out by direct comparison. M_k^m comparisons have to be carried out for each value of the state vector - a total of $M_k^m N_k^n$ (scalar) comparisons at each stage k .

Now if the state and decision variables are continuous they are approximated to discrete values by the construction of an n -dimensional grid for the state vector and an m -dimensional grid for the decision vector, and a similar process to that described above carried out.

The values of F_k thus obtained will be used for the next stage in the solution (the recursive equation for stage $k+1$). This requires storage in high-speed central memory amounting to N_k^n locations. Values of G_k for each quantization of the state vector need not be accessed during the build-up phase and may be stored off-line on peripheral devices (disk or tape) together with the values of F_k for stages previously computed and

TABLE A-1 CORE REQUIREMENTS FOR THE STANDARD COMPUTATIONAL ALGORITHM

Values tabulated are of N_k^n - the number of locations in central memory required for storage of the state functions F_k .

N_k = number of quantized levels for each component of the state vector.

n = dimensionality of the state vector.

	$N_k = 5$	$N_k = 10$	$N_k = 50$	$N_k = 100$
$n = 1$	5	10	50	100
$n = 2$	25	10^2	2500	10^4
$n = 5$	3125	10^5	3.1×10^8	10^{10}
$n = 10$	9.8×10^6	10^{10}	9.8×10^{16}	10^{20}
$n = 50$	8.9×10^{34}	10^{50}	8.9×10^{84}	10^{100}

no longer required in central memory. Thus the off-line storage requirement for a K-stage problem will be $KN_k^n(1 + m)$.

In continuous-variable problems, N_k and M_k should be as large as possible for maximum accuracy. However as these quantities are increased computer core and CPU time requirements rapidly become excessive. Table A-1 does no more than highlight the core requirements; it will be seen that even with a 2-dimensional state vector core requirements can be prohibitively large with a fine grid.

CPU time is proportional to N_k^n ; if quantization of the decision variables and direct scalar comparison is used, it is proportional to $M_k^m N_k^n$. Computing time constraints can also be serious, even to the extent that the solution to a problem can take longer than the real-life situation it seeks to simulate (/41/).

As general rule even with present-day large high-speed digital computing equipment problems with state vectors of dimensionality greater than about four cannot satisfactorily

be resolved with straightforward application of the standard computational algorithm. It should be noted that the bulk of the computational effort is expended during the build-up phase. The reason for this is that all feasible states at all stages will have to be evaluated and inspected, whether or not they are eventually used for generation of the optimal policy during traceback.

Clearly constraints which limit the number of feasible states aid the computational process since fewer states need to be explored. This fact is in direct contrast to most other methods of optimization where the existence of a greater number of constraints hinders solution. Furthermore the use of a direct numerical method of solution allows the handling of many types of constraints - linear, nonlinear, discontinuous, equalities or inequalities, and so on, with equal ease.

Ingenious methods have been proposed and used to reduce the computational burden. Core requirements may be reduced significantly if the state function can be approximated to by a low order polynomial. Optimization procedures more efficient than quantization and search in decision space may be used. Such techniques include Fibonacci or Golden Section search, multivariate search algorithms (eg Powell, Fletcher-Reeves), and even the Simplex method of linear programming (provided that the state function and return function are in suitable form) For special forms of transition function and objective function certain special techniques (eg state increment dynamic programming) can produce solutions efficiently. Other methods such as that of successive approximation, approximation in function space, or policy space, can also reduce the computational burden and have been widely used for multidimensional problems where a steady-state solution is required. Finally, certain heuristic methods have been successful for very large problems which cannot be solved with the standard computational algorithm (eg /18/,/41/).

A.5 ALLOCATION PROBLEMS

Consider the following form of the allocation problem:

$$\text{optimize } \sum_{k=1}^K f_k(x_k) \quad \dots(A-13)$$

$$\text{subject to: } \sum_{k=1}^K g_k(x_k) \leq A \quad \dots(A-14)$$

$$\text{and: } x_k \in X_k \quad \dots(A-15)$$

This problem can be considered to be a mathematical statement of:

"Allocate in an optimal manner (Eqn A-13) a limited quantity of a resource amongst K activities, each of which provides a return (or penalty) $f_k(x_k)$, with constraints on overall (Eqn A-14) and individual (Eqn A-15) permissible allocations".

This problem may also be regarded as a staged process with appropriate definitions of state and decision variables.

Let the stages correspond to the K activities. Let d_k represent the allocated quantity of the resource to activity k ($d_k = x_k$). Since the constraint of Eqn A-14 is additively separable with respect to stages k, at each stage k let the state variable s_k denote the value of this constraint up to and including stage k:

$$s_k = \sum_{j=1}^k g_j(d_j) \quad \dots(A-16)$$

The problem has now been recast into the form:

$$\text{optimize } \sum_{k=1}^K f_k(d_k) \quad \dots(A-17)$$

with the transition function:

$$s_{k-1} = s_k - g_k(d_k) \quad \dots(A-18)$$

and constraints:

$$s_k \leq A \quad \dots(A-19a)$$

$$\text{and } d_k \in X_k \quad \dots(A-19b)$$

The problem is now in a form identifiable as soluble by dynamic programming. Both state and decision vectors are unidimensional, and the required recursive equation for solution is:

$$F_k(s_k) = \underset{d_k \in X_k}{\text{opt}} \left\{ f_k(d_k) + F_{k-1}[s_k - g_k(d_k)] \right\} \quad \dots(\Lambda-20)$$

and the solution to problem Eqns $\Lambda-16$ to $\Lambda-19$ is:

$$\text{opt}[F_K(s_K)]$$

1. The arguments presented above can readily be extended to include multidimensional cases. Thus if there are n commodities to be allocated over K activities ($x_{1k}, x_{2k}, \dots, x_{nk}$ at each activity k), and m constraints, of the form:

$$\sum_{k=1}^K g_{ik}(x_k) \leq A_i \quad \text{for } i = 1, 2, \dots, m \quad \dots(\Lambda-21)$$

then this is a dynamic program with K stages, an n -dimensional state vector, and an m -dimensional decision vector.

2. The unidimensional case can be solved readily with the standard computational algorithm. Multidimensional cases are of course computationally much more demanding.

3. Owing to the nature of the standard computational algorithm, the solution has been obtained for all feasible final states (all $s_K \leq A$). Therefore:

$F_K(A')$ is the solution to the problem if the constraint:

$$\sum_{k=1}^K g_k(x_k) \leq A' \quad \text{replaces the constraint of Eqn } \Lambda-14$$

$$(A' \leq A)$$

4. The property of invariant imbedding means that k subproblems have been solved. Thus:

$F_j(A')$ is the solution to the following problem:

"Allocate in an optimal manner the single resource over j activities such that:

$$(a) \sum_{i=1}^j g_i(x_i) \leq A'; \text{ and } (b) x_i \in X_i, \text{ for } i = 1, 2, \dots, j"$$

5. Allocation problems as a class may be solved in many ways. For the problem of Eqns A-13 to A-15, if the functions f_k and g_k are linear, and if the constraints of Eqn A-15 represent linear equalities or inequalities, then the problem may be solved by linear programming, which computationally may well be more efficient. Even nonlinear forms may be accommodated by the use of multivariate search algorithms. In many cases the use of dynamic programming cannot be recommended since it may not be more efficient or more accurate.

6. It would be hazardous to present general rules without a closer knowledge of the nature and structure of the problem, but dynamic programming may be preferable over alternative formulations in the following cases:

- when the solution to the subset of identically structured problems is also required (exploiting the property of invariant imbedding),
- when a parametric solution is required; viz. when the solution is desired in the form of a function rather than a specific quantity,
- where performance functions are highly nonlinear, discontinuous, or not available in analytic form (eg as the output from a subroutine),
- where constraints are numerous, highly nonlinear, discontinuous or not available in analytic form, or where states and decisions are constrained to discrete or integer values.

APPENDIX B. EXTENSIONS TO THE LOADING SIMULATION MODEL

The exposition of Sections 5.4 to 5.9 has been directed towards the development of a model for evaluation of operating feasibility and operating costs to be used in planning studies. In such a case one of the prime considerations is the minimization of computational effort, in respect of computer CP time and central memory core requirement. In the two sections to follow some extensions to the model will be presented which, albeit at the expense of an increased computational burden may make the model more suitable in other contexts, eg for plant scheduling studies.

Except where stated, notation follows that of Section 5.4.

B.1 RESERVOIR TRAJECTORY

(a) If greater accuracy is required the assumption of a single pseudoreservoir may be relaxed, and consideration be given to two or even more reservoirs. It must however be borne in mind that the computational burden increases very rapidly with dimensionality in the dynamic program.

(b) Where the system is not independent, and export and import of energy is allowed between it and a neighbouring system, a third dummy subsystem may be defined with a cost function $C_X(p_X, r_X)$, which defines the cost of electricity supplied by the neighbouring system. Note that this function may also be defined for negative values of p_X and r_X , in which case the value of C_X itself may be negative - this corresponds to export of energy from the system.

The overall objective still being cost minimization, the objective function, to be minimized, is now:

$$\sum_{t=1}^T [C_{TH}(p_{TH}, r_{TH}) + C_X(p_X, r_X)] \Delta T_t$$

Noting however that the operation of the hydro subsystem is independent of that of both the thermal and dummy subsystems, an equivalent system performance function for these latter two

subsystems:

$$C_{EQ}(p_{EQ}, r_{EQ}) = \min \{ C_{TH}(p_{TH}, r_{TH}) + C_X(p_X, r_X) \}$$

subject to: $p_{EQ} = p_{TH} + p_X$

and: $r_{EQ} = r_{TH} + r_X$

The solution of this relatively simple optimization problem can be through any nonlinear algorithm. A two-dimensional two-stage dynamic program may be used.

The complete reservoir trajectory will now be:

$$\text{minimize } \sum_{t=1}^T C_{EQ} [D(t) - p_{HY}(t), R(t) - R_{HY}(p_{HY}(t))] \Delta T_t$$

with the transition function:

$$q(t+1) = q(t) + [f(t) - p_{HY}(t)G(p_{HY}(t))] \Delta T_t$$

and the constraints:

$$Q_{\min} \leq q(t) \leq Q_{\max} \quad \text{for all } t.$$

The required recursive relationship for solution will be similar to that given in Section 5.5.

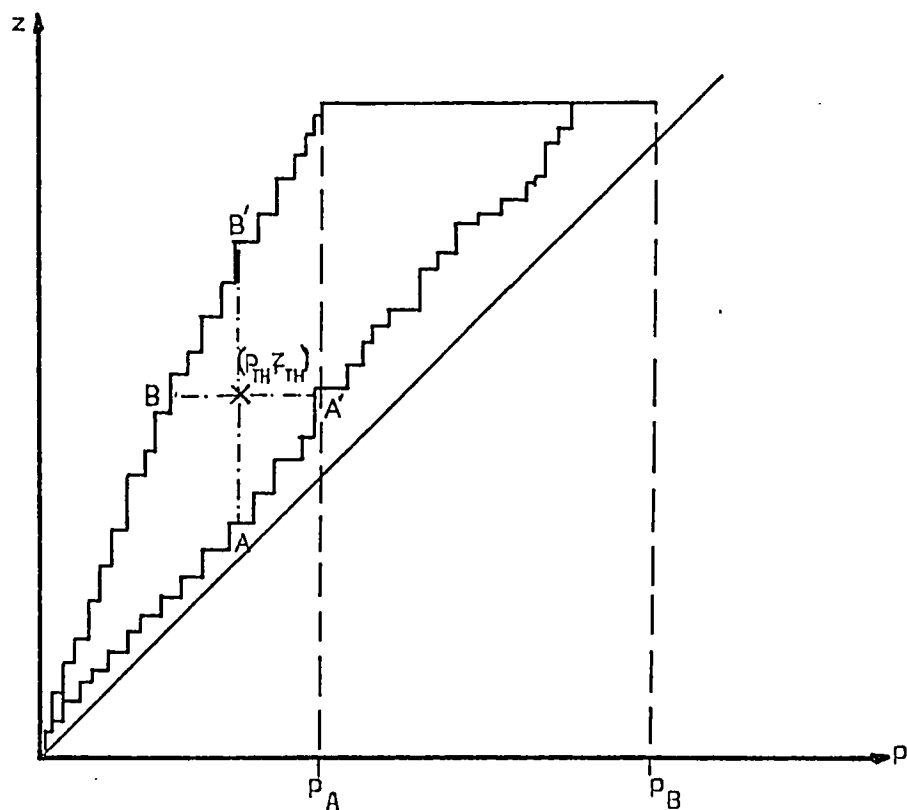
B.2 PERFORMANCE FUNCTION FOR THE THERMAL SUBSYSTEM

Two alternative methods for derivation of the performance function C_{TH} for the thermal subsystem are outlined below:

(a) Consider the functions $Z(p)$ and $Z'(p)$ in Fig 8.1. The cost of operation of the subsystem at point (p_{TH}, z_{TH}) is required.

As a starting point, one of the points A, A', B, B' may be selected. According to the trajectory on which this point lies (maximum reserve or minimum cost), a single traceback pass is conducted to determine the optimal policy for that point. From this policy, the set G of points that are operating is obtained. If it is now assumed that this same set is also operating at the point (p_{TH}, z_{TH}) , then the following problem is to be solved:

FIGURE B.1 ALTERNATIVE FORMULATION FOR THERMAL SUBSYSTEM - 1.



$$\text{minimize } \sum_{k \in G} c_k(p(k))$$

subject to:

$$\sum_{k \in G} p(k) \geq p_{TH}$$

$$\sum_{k \in G} p_5(k) - p(k) \geq z_{TH} - p_{TH}$$

$$\text{and: } p_3(k) \leq p(k) \leq p_4(k)$$

The first two constraints however are equivalent to the single constraint:

$$\sum_{k \in G} p(k) = \max \left\{ p_{TH}, z_{TH} - p_{TH} - \sum_{k \in G} p_5(k) \right\}$$

Note that there are now no discontinuities in the feasible region for the vector $\underline{p}$ (since all $p(k) \neq 0$), and the problem has a single linear inequality constraint with lower and upper bounds on the independent variables. For cases where the functions c_k are linear or quadratic, the problem is a straightforward linear or quadratic program, and may be solved by standard algorithms fairly readily.

If it is found that the set G defines an infeasible case at the point (p_{TH}, z_{TH}) then one of the remaining three alternative points may be selected, and the process repeated.

With this method it will be necessary to store the policy functions as well as the state functions during the build-up phase of the dynamic program, thus there are penalties both in respect of CP time and core requirements. However a simplified form of policy function may be used since all that is needed, even for the minimum cost trajectory, is the status of each unit (off/generating) in order to determine the set G .

(b) Consider the following problem:

$$\text{minimize } \sum_{k \in G \cap \underline{C}_{TH}} c_k(p(k))$$

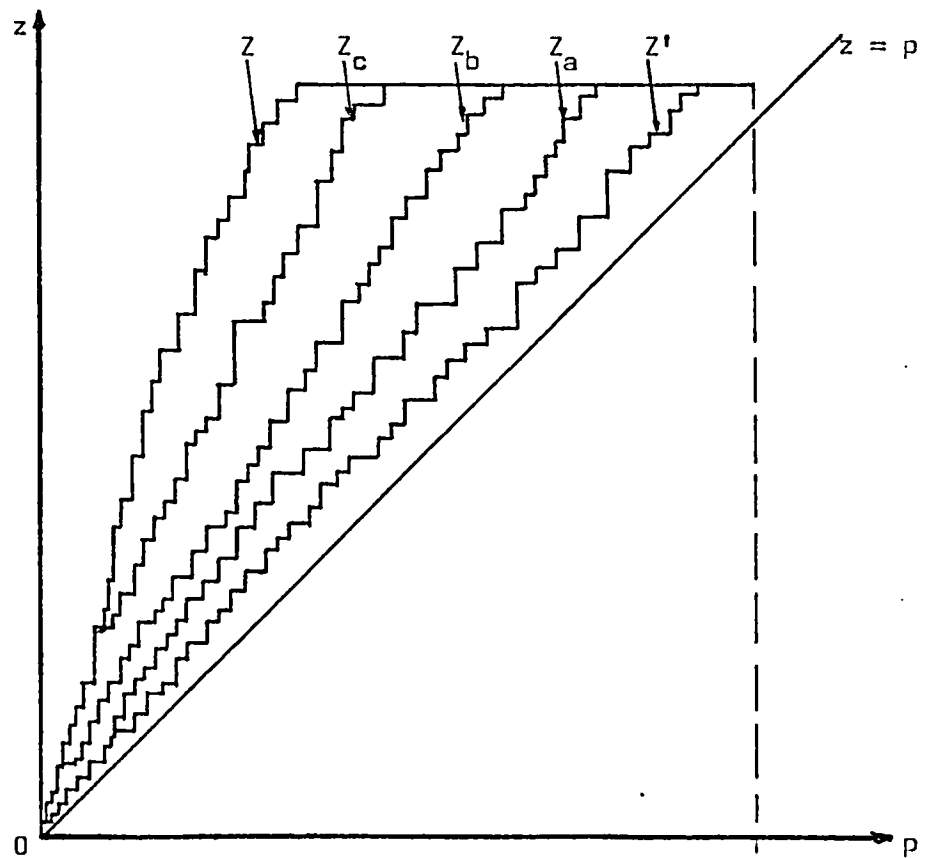
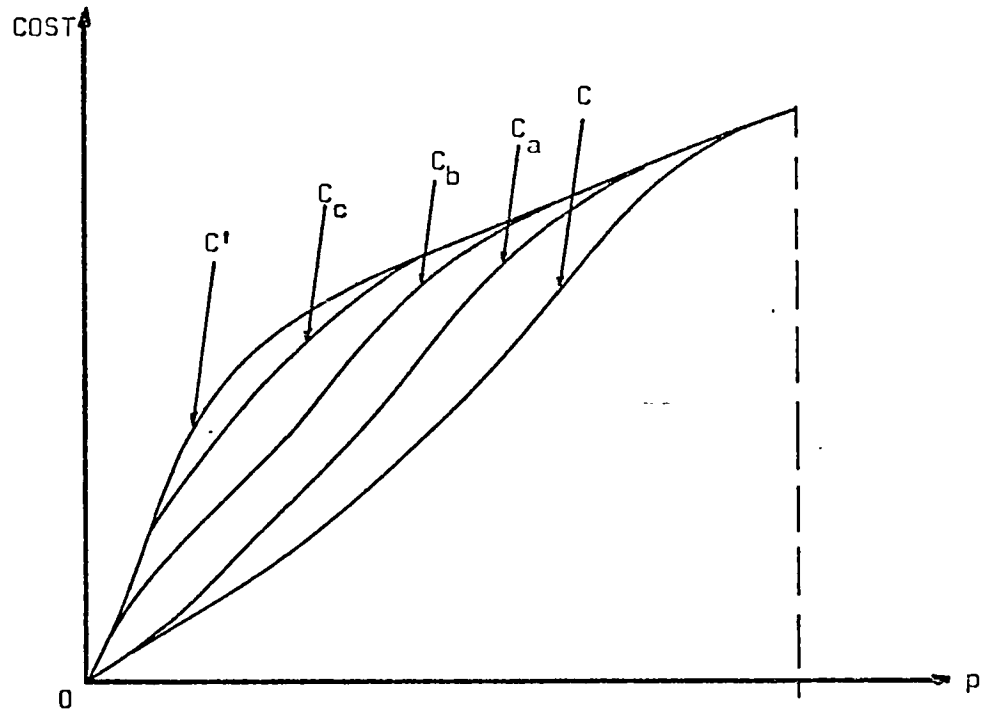
subject to:

$$\sum_{k \in G \cap \underline{C}_{TH}} p(k) \geq x \cdot p_{TH} \quad \text{where } x > 1$$

This problem may be solved by dynamic programming as before, it may be interpreted as the solution of the minimum cost trajectory for the case where the system output is constrained to exceed required system demand by a fixed fraction $(x - 1)$. The case $x = 1$ corresponds to the minimum cost trajectory problem of Section 5.9.

Assume that the above problem is solved for several different values of x , set by the user. A family of minimum cost trajectories and their associated reserve trajectories is obtained. Figure 8.2 shows an example with three functions, the other two curves are, as before, $C(p)$ and $Z'(p)$ for the minimum cost trajectory ($x = 1$) and $C'(p)$ and $Z(p)$ for the maximum reserve trajectory.

FIGURE B.2 ALTERNATIVE FORMULATION FOR THERMAL SUBSYSTEM - 2.



It will now be a straightforward matter to apply linear (or even higher order) interpolation in the z -direction to provide the required estimate of operating cost for any feasible point in the (p,z) plane.

The increased computational burden is evident. Although policy functions need not be stored during build-up, a total of five unidimensional dynamic programs are to be solved for the subsystem.

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