# A fuzzy Einstein-based decision support system for public transportation management at times of pandemic

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## ARTICLE INFO

## ABSTRACT

Optimal decision-making has become increasingly more difficult due to their inherent complexity exacerbated by uncertain and rapidly changing environmental conditions in which they are defined. Hence, with the aim of improving the uncertainty management and facilitating the weighting criteria, this paper introduces an improved fuzzy Einstein Combined Compromise Solution (CoCoSo) methodology. Such a CoCoSo model improves previous CoCoSo proposals by using nonlinear fuzzy weighted Einstein functions for defining weighted sequences. In addition, it proposes a novel algorithm for determining the criteria weights based on the fuzzy logarithmic function, therefore it allows decision-makers a better perception of the relationship between the criteria, as it considers the relationships between adjacent criteria; high consistency of expert comparisons; and enables the definition of weighting coefficients of a larger set of criteria, without the need to cluster (group) the criteria. Nonlinear fuzzy Einstein functions implemented in the fuzzy Einstein CoCoSo methodology enable the processing of complex and uncertain information. Such characteristics contribute to the rational definition of compromise strategies and enable objective reasoning when solving real-world decision problems. The efficiency, effectiveness, and robustness of the proposed fuzzy Einstein CoCoSo model are illustrated by a case study to create a conceptual framework to evaluate and rank the prioritization of public transportation management at the time of the COVID-19 pandemic. The results reveal its good performance in determining the transportation management systems strategy.

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## 1. Introduction

We have recently seen how pandemics among other disasters can affect countries all around the world in various negative ways. Transportation is one of the most affected fields because of the stay-at-home restrictions and strict lockdown measures. In the early stages of the COVID-19 pandemic, private vehicle usage and public transportation usage have decreased dramatically [1]. Afterward, the decrease in private vehicle use has picked up from the bottom, but the public transportation demands are still very low compared to the pre-pandemic stages [2]. People are avoiding closed and crowded areas, which is lowering the increase in the demand for public transportation. Therefore, there is a need for new management methods to re-adjust public transportation according to the current pandemic conditions and increase the demands [3,4].

Due to the COVID-19 pandemic, the demand for public transportation has decreased significantly [5]. Consequently, there is a potential for increased traffic congestion in urban areas, as people who stopped using public transportation primarily switch to private vehicles [6]. In addition to private vehicles, studies indicate that micro-mobility services such as bike-sharing have attracted a significant amount of demand since the pandemic, which can be linked to a portion of the decline in public transportation [7]. Therefore, people are abandoning public transportation and adopting new behaviors, such as cycling and driving private vehicles. These passengers may not use public transportation even after the pandemic, putting public transportation operators in a dangerous position. Despite the fact that public transportation systems are services for the public, they incur high operating expenses due to vehicle maintenance, fuel consumption, and taxes.

If the pandemic continues or even if it ends and demand does not return to pre-pandemic levels, the revenues generated from commuters will not be sufficient to cover operating expenses. In light of the ongoing pandemic, new public transportation management strategies are required to ensure commuters' safety while simultaneously reducing operating costs.

In this study, a fuzzy CoCoSo method by applying Einstein norms is proposed to improve two issues: (i) the definition of weighted sequences and compromise alternative strategies by using fuzzy nonlinear Einstein functions, and (ii) the proposal of a new algorithm for determining the weight coefficients of the criteria based on the fuzzy logarithmic function. We can summarize the advantages and novelty of fuzzy Einstein CoCoSo methodology as follows:

- The fuzzy Einstein CoCoSo methodology has an original nonlinear model for defining the significance of the criteria, thus eliminating the need to apply additional auxiliary tools. This algorithm enables objective expert reasoning and the rational definition of the relationship between the criteria and allows decision-makers to better understand the relationships between the criteria because it considers the relationships between adjacent criteria.
- The proposed methodology for determining criteria weights eliminates the problem of defining the relationships between remote criteria, which often leads to inconsistent results in subjective models, such as the Analytic Hierarchy Process (AHP) and Best Worst Method (BWM) [8] in which there are many criteria (more than eight). This limitation is due to the small scale range in AHP and BWM models. A nine-point scale limits the expression of expert preferences to a maximum ratio of 9:1, which further imposes inconsistencies in comparisons. However, with the methodology proposed in this paper, this limitation is eliminated, by comparing adjacent criteria for obtaining weighting coefficients in cases where there are many criteria, since the model is not limited to the use of a predefined scale. This allows decision-makers to objectively express their preferences and the relationships between the criteria.
- Nonlinear fuzzy Einstein functions implemented in the fuzzy Einstein CoCoSo methodology allow the processing of complex and uncertain information.
- A flexible nonlinear function for fusion weighted alternative strategies has been proposed in the fuzzy Einstein CoCoSo methodology. The introduction of nonlinear fuzzy Einstein functions enables nonlinear information processing and improves flexibility in decision making. These characteristics of the proposed multi-criteria framework contribute to the rational definition of compromise strategies and enable objective reasoning when solving real-world decision problems.
- The proposed fuzzy Einstein CoCoSo methodology has an improved methodology for calculating a compromise index of alternatives. The new approach to integrating aggregate strategies has been proposed, as the classical approach proposed by Yazdani et al. [9] has anomalies that can lead to wrong results.
- The introduction of additional stabilization parameters has improved the flexibility of fuzzy Einstein functions. The effectiveness, efficiency, and robustness of the proposed fuzzy Einstein CoCoSo model were confirmed through application in a complex case study for choosing the best transportation management systems strategy involving seven decision-makers.

The rest of this study is constructed as the following. Section 2 gives the literature review investigating previous studies regarding the effects of the pandemic on travel behavior. Section 3 includes the problem definition, the definition of alternatives by which the alternatives and the criteria are defined. Sections 4 and 5 presents the proposed methodology and the case study, respectively. Section 6 is the results and discussion part, and Section 7 provides the conclusion and future directions of the study.

## 2. Preliminaries

In this section, first, a brief overview of different studies on public transport management is introduced, and afterward, a revision of the necessary concepts to better understand our proposal is provided.

### 2.1. Overview of the studies on public transport management

Pandemics affect many aspects of the cities all around the world, such as transportation, socio-economics, and general management activities. COVID-19 is not the first pandemic faced by the world. However, traveling to other countries is easier, and cheaper, and hence the growing interactions between the people of different countries have made the rapidness of the spreading of the virus greater than other pandemics in history [10]. The continuous and rapid spreading of the virus has led authorities to search for new and adapted ways of managing urban matters like public transportation [11].

Mobility is one of the most affected fields by pandemics. The fear of getting infected, the need for social distancing, staying away from crowded and closed areas and the stay-at-home restrictions of authorities are the primary reasons behind the negative effect of any pandemic on mobility. In a study regarding the changes in mobility in the USA, the correlation between stay-at-home restrictions, infection levels, and average distance traveled by the individuals is investigated [12]. According to the comparison between current and pre-pandemic data of average distance traveled by the individuals, it is observed that with an increase of 0.003% in the local infection rate, mobility is reduced by 2.31%. It is also seen that, with the official application of a stay-at-home restriction, mobility is seen to be decreasing by 7.87%. In a different study, the effects of social distancing on the mobility of people in the UK are investigated [13]. The mobility data of Google is used to analyze the changes in mobility after the pandemic outbreak. Mobility in six different areas is investigated, namely supermarkets, grocery shops, workplaces, retail areas, and transit stations. Comparing the current and pre-pandemic data, an overall reduction of 63% is observed in mobility. Mobility in retail and recreational areas has seen the sharpest reduction by 85%. Second, the most reduction is observed at transit stations by 75%. According to the analysis, mobility in transit stations and hence public transportation has seen to be facing a great decline.

There are many studies regarding the effects of pandemics on the use of public transportation. One common result of these studies is that the demand for public transportation decreases a lot and that there is a need for new management and planning methods for the public transportation systems. In a study regarding the transportation mode used in Germany, the transportation mode preferences of the individuals are investigated at the time of the strictest lockdown measures [6]. According to the results of a representative survey, it is seen that the usage of private vehicles has increased considerably, whereas the usage of public transportation has decreased drastically. It is also observed that one-third of the car-free households wanted to have a car rather than using public transportation, which shows that people are leaning toward owning a private vehicle more than ever. In another study about public transport usage in the USA, the decline in demand and the correlative aspects related to this decline are

investigated [5]. According to the analysis of the study, there is a decline in public transportation demand of around 60% and 80% after WHO (World Health Organization) declared COVID-19 as a pandemic.

It is also observed that as the population ratio of essential workers and vulnerable populations, such as people over 45 years, increases, the decline in demand for public transportation increases. Alongside the increase in private vehicle usage, the demand for bike-sharing systems (BSS) is seen to be staying strong, which makes the demand for public transportation decrease. In a different study, the current and pre-pandemic data of Citi Bike and subway usage in NYC are compared [14]. It is seen that initially, both bike and subway usage decreased drastically. However, demand for Citi Bike has reached back to the pre-pandemic levels, but subway usage is still very low compared to pre-pandemic levels, which shows that people care for social distancing while traveling. This is one of the major reasons that there is a great demand decline in public transportation [15]. In another study, which analyzes the link between bike-sharing and subway use in NYC during the COVID-19 pandemic, it is seen that there is evidence of a modal shifting from the subway to bike-sharing systems [7]. Therefore, considering all these studies, it is possible to say that there is still time until the demand levels for public transportation reach back to the pre-pandemic levels, so re-adjustment and re-planning of public transportation systems is seeming to be essential.

Some studies investigate planning methods for public transportation systems to provide an optimized operation. Several of these studies are applied management styles of the authorities, and implemented. A study investigates the different physical, social distancing restrictions such as 1, 1.5, and 2 m in the subway transportation system and analyzes if the social distancing restrictions compensate the passenger demand, allocation of resources like train availability, and passenger waiting times [16]. The model proposed in the system allocates the trains according to the demand distribution of lines. Results show that as the distance increases, the number of trains, the frequency of the trips, and the number of service-denied passengers must also be increased. Therefore, there is a trade-off between safety because of the applied social distancing and economics and public acceptability because of the increased number of trains, frequency, and denied passengers. In another study, demand management of the public transportation systems under social distancing measures is investigated [17]. Five different demand management methods are presented in the study, namely inflow control with queueing, time and space-dependent pricing, capacity reservation with booking, slot auctioning, and tradeable travel permit schemes. Results of the study show that, due to the individual limitations, these management methods should not be applied alone, but a group of them can be applied simultaneously to control the occupancy rates of the public transit vehicles.

In another study, various planning and managing methods for public transportation systems are proposed [18]. One proposal is that authorities should create bus lines since the capacity of the vehicles is to be reduced because of social distancing. Congestion has the potential to be more severe in the post-pandemic stage since people use private vehicles more, so public transit vehicles must work at higher frequencies to compensate for the passenger demand, but congestion slows down the process. Another proposal is to divert the departure times of the companies so that passenger demand becomes more spread, which makes it easy for the public transportation system to compensate for the demand. Alongside the studies about re-planning the public transportation systems, a study states that some authorities started investing in non-car transportation infrastructure such as bike lanes [19]. These acts promote the usage of bikes as a traveling mode, which reduces the demand for public transportation.

### 2.2. Preliminaries on Fuzzy Einstein T-norms and T-conorms

Fuzzy set theory [20] and different types of generalization of fuzzy sets [21–27] are most commonly used to process uncertain information in multi-criteria models. When using traditional fuzzy theory in decision-making models, researchers most widely use triangular fuzzy numbers [28], as they allow for efficient and straightforward processing of uncertain information. Triangular fuzzy numbers are represented by the membership function $\mu_E(\delta): R \rightarrow [0,1]$ as follows [29,30]:

$$\mu_E(\delta) = \begin{cases} \dfrac{\delta - d}{d - g} & d \leq \delta \leq s \\ 1 & \delta = s \\ \dfrac{g - \delta}{g - s} & s \leq \delta \leq g \\ 0 & otherwise \end{cases} \quad (1)$$

where $d$ and $g$ mean the lower and upper bounds of the fuzzy number $E$, and $s$ is the modal value for $E$.

**Definition 1** ([31])**.** Let $\rho_1$ and $\rho_2$ be any two real numbers. Then the Einstein T-norm and T-conorm for $\rho_1$ and $\rho_2$ are defined as follows:

$$t(\rho_1, \rho_2) = \frac{\rho_1 \rho_2}{1 + (1 - \rho_1)(1 - \rho_2)} \quad (2)$$

$$t^c(\rho_1, \rho_2) = \frac{\rho_1 + \rho_2}{1 + \rho_1 \rho_2} \quad (3)$$

where $(\rho_1, \rho_2) \in [0, 1]$.

Following the Einstein T-norm and T-conorm, Einstein operations with fuzzy numbers are defined.

**Definition 2.** Suppose that $\rho_1 = (\rho_1^{(d)}, \rho_1^{(s)}, \rho_1^{(g)})$ and $\rho_2 = (\rho_2^{(d)}, \rho_2^{(s)}, \rho_2^{(g)})$ are two triangular fuzzy numbers (TFNs), and let it be $f(\rho_i) = \left( f\left(\rho_i^{(d)}\right), f\left(\rho_i^{(s)}\right), f\left(\rho_i^{(g)}\right) \right) = \left( \rho_i^{(d)} / \sum_{i=1}^{n} \rho_i^{(d)}, \rho_i^{(s)} / \sum_{i=1}^{n} \rho_i^{(s)}, \rho_i^{(g)} / \sum_{i=1}^{n} \rho_i^{(g)}, \right)$ TFN function, then some operational laws of TFNs based on the Einstein T-norm and T-conorm can be defined as follows

(1) Addition:

$$\rho_1 + \rho_2 = \begin{pmatrix} (\rho_1^{(d)} + \rho_2^{(d)}) \dfrac{f\left(\rho_1^{(d)}\right) + f\left(\rho_2^{(d)}\right)}{1 + f\left(\rho_1^{(d)}\right) f\left(\rho_2^{(d)}\right)}, \\ (\rho_1^{(s)} + \rho_2^{(s)}) \dfrac{f\left(\rho_1^{(s)}\right) + f\left(\rho_2^{(s)}\right)}{1 + f\left(\rho_1^{(s)}\right) f\left(\rho_2^{(s)}\right)}, \\ (\rho_1^{(g)} + \rho_2^{(g)}) \dfrac{f\left(\rho_1^{(g)}\right) + f\left(\rho_2^{(g)}\right)}{1 + f\left(\rho_1^{(g)}\right) f\left(\rho_2^{(g)}\right)} \end{pmatrix} \quad (4)$$

(2) Multiplication "×"

$$\rho_1 \times \rho_2 = \begin{pmatrix} (\rho_1^{(d)} + \rho_2^{(d)}) \dfrac{f\left(\rho_1^{(d)}\right) f\left(\rho_2^{(d)}\right)}{1 + \left(1 - f\left(\rho_1^{(d)}\right)\right)\left(1 - f\left(\rho_1^{(d)}\right)\right)}, \\ (\rho_1^{(s)} + \rho_2^{(s)}) \dfrac{f\left(\rho_1^{(s)}\right) f\left(\rho_2^{(s)}\right)}{1 + \left(1 - f\left(\rho_1^{(s)}\right)\right)\left(1 - f\left(\rho_1^{(s)}\right)\right)}, \\ (\rho_1^{(g)} + \rho_2^{(g)}) \dfrac{f\left(\rho_1^{(g)}\right) f\left(\rho_2^{(g)}\right)}{1 + \left(1 - f\left(\rho_1^{(g)}\right)\right)\left(1 - f\left(\rho_1^{(g)}\right)\right)} \end{pmatrix} \quad (5)$$

(3) Scalar multiplication, where $\varsigma > 0$

$$\varsigma \cdot \rho_1 = \begin{pmatrix} \rho_1^{(d)} \dfrac{\left(1+f\left(\rho_1^{(d)}\right)\right)^{\varsigma} - \left(1-f\left(\rho_1^{(d)}\right)\right)^{\varsigma}}{\left(1+f\left(\rho_1^{(d)}\right)\right)^{\varsigma} + \left(1-f\left(\rho_1^{(d)}\right)\right)^{\varsigma}}, \\[2em] \rho_1^{(s)} \dfrac{\left(1+f\left(\rho_1^{(s)}\right)\right)^{\varsigma} - \left(1-f\left(\rho_1^{(s)}\right)\right)^{\varsigma}}{\left(1+f\left(\rho_1^{(s)}\right)\right)^{\varsigma} + \left(1-f\left(\rho_1^{(s)}\right)\right)^{\varsigma}}, \\[2em] \rho_1^{(g)} \dfrac{\left(1+f\left(\rho_1^{(g)}\right)\right)^{\varsigma} - \left(1-f\left(\rho_1^{(g)}\right)\right)^{\varsigma}}{\left(1+f\left(\rho_1^{(g)}\right)\right)^{\varsigma} + \left(1-f\left(\rho_1^{(g)}\right)\right)^{\varsigma}} \end{pmatrix} \qquad (6)$$

(4) Power, where $\varsigma > 0$

$$\rho_1^{\varsigma} = \begin{pmatrix} \rho_1^{(d)} \dfrac{2f\left(\rho_1^{(d)}\right)^{\varsigma}}{\left(2-f\left(\rho_1^{(d)}\right)\right)^{\varsigma} + f\left(\rho_1^{(d)}\right)^{\varsigma}}, \\[2em] \rho_1^{(s)} \dfrac{2f\left(\rho_1^{(s)}\right)^{\varsigma}}{\left(2-f\left(\rho_1^{(s)}\right)\right)^{\varsigma} + f\left(\rho_1^{(s)}\right)^{\varsigma}}, \\[2em] \rho_1^{(g)} \dfrac{2f\left(\rho_1^{(g)}\right)^{\varsigma}}{\left(2-f\left(\rho_1^{(g)}\right)\right)^{\varsigma} + f\left(\rho_1^{(g)}\right)^{\varsigma}} \end{pmatrix} \qquad (7)$$

**Definition 3.** Let $\rho_j = (\rho_j^{(d)}, \rho_j^{(s)}, \rho_j^{(g)})$; $(j = 1, 2, \ldots, n)$, represents a set of TFNs, and $\varpi_j \in [0, 1]$ represents the weight coefficient of $\rho_j = (\rho_j^{(d)}, \rho_j^{(s)}, \rho_j^{(g)})$, $(j = 1, 2, \ldots, n)$, which fulfills the requirement that it is $\sum_{j=1}^{n} \varpi_j = 1$. Then we can define fuzzy weighted averaging (FWA) operator and fuzzy weighted geometric averaging (FWGA) operator:

$$FWA(\rho_1, \rho_2, \ldots, \rho_n) = \sum_{j=1}^{n} \varpi_j \rho_j = \left( \sum_{j=1}^{n} \varpi_j^{(d)} \rho_j^{(d)}, \right.$$
$$\left. \sum_{j=1}^{n} \varpi_j^{(s)} \rho_j^{(s)}, \sum_{j=1}^{n} \varpi_j^{(g)} \rho_j^{(g)} \right) \qquad (8)$$

$$FWGA(\rho_1, \rho_2, \ldots, \rho_n) = \prod_{j=1}^{n} (\rho_j)^{\varpi_j} = \left( \prod_{j=1}^{n} \left(\rho_j^{(d)}\right)^{\varpi_j^{(d)}}, \right.$$
$$\left. \prod_{j=1}^{n} \left(\rho_j^{(s)}\right)^{\varpi_j^{(s)}}, \prod_{j=1}^{n} \left(\rho_j^{(g)}\right)^{\varpi_j^{(g)}} \right) \qquad (9)$$

## 3. Problem definition

Pandemic periods have various adverse impacts on public transportation since crowds provide viruses to transmit and infect many people. Therefore, people get worried about traveling with a bunch of people who inevitably have the possibility of carrying the virus. Thus, ridership in public transportation decreased. To overcome these negative effects on public transportation, some public transportation management methods should be implemented on public transportation. As an example, in COVID-19 times, the obligation to wear face masks and reduce capacities in public transportation has been implemented by decision-makers. However, there is still a gap in effectively prioritizing these alternative methods according to their advantages. In this study, four different management methods, namely doing nothing, adjusting public transportation schedule based on work hours, capacity reduction in public transportation, and increasing the ridership

in sharing based modes such as bicycle sharing, scooter sharing, car sharing, are present in the order of priority based on economic, health and environmental, social, and transportation aspects by using fuzzy MCDM. These alternatives are deduced with the support of academic specialists in the field by interpreting the responses of policies implemented in pandemic times, and by studying publications on this subject.

*Alternative 1 ($A_1$). Do nothing*

With this alternative, the management of public transportation systems is not adjusted according to the conditions of the COVID-19 pandemic. Management style is kept the same as it was before the pandemic.

*Alternative 2 ($A_2$). Adjusting public transportation schedule based on the work hours*

One of the public transportation management methods is scheduling the time timetables of public transportation lines according to the work hours. Because of the high demands faced at work hours, adjusting public transportation schedules based on these hours has the potential to reduce the waiting time of the workers. However, there is a possibility of increasing the waiting times of commuters, who are traveling outside of work hours.

*Alternative 3 ($A_3$). Capacity reduction in public transportation*

Implementation of capacity reduction to public transportation systems is related to the application of social distancing in public transit vehicles, which decreases the risks of commuters getting infected. The application of this alternative has the potential to increase the number of services denied commuters and therefore the waiting times [16].

*Alternative 4 ($A_4$). Increasing the ridership in sharing based modes such as bicycle sharing, scooter sharing, car sharing*

After the covid-19 outbreak, many people started staying out of closed areas and keeping the social distance from others not to get infected. This has increased the demands for micro-mobility services such as bicycle sharing and scooter sharing [7]. However, since these transportation systems are sharing services, frequent disinfection of the vehicles is a concern for the users. Also, when the travel distances increase, micro-mobility services lose sufficiency.

### Criteria

Since pandemics affect public transportation negatively, some actions should be taken to eliminate these adverse impacts. In this study, solutions to public transportation in pandemics are investigated with the help of experts in public transportation in the academy, by evaluating the responses of policies implemented by decision-makers and municipalities during pandemic periods, and by analyzing the publications on this issue. Thus, with the help of extensive literature reviews, the alternatives and criteria in this study are defined.

(1) **Economic Aspect**

$C_1$. Operation cost (cost): Due to the COVID-19 pandemic, the need for frequently disinfecting the public transit vehicles increases the operating costs. Since public transit vehicle drivers are at very high risk, there is a need for a budget for the compensation. Also, the reduction in capacities of the vehicles increases the financial burden on the operators because the revenues of each trip reduce [18].

$C_2$. Subsidy from the government (benefit)**:** This criterion is related to countries providing subsidies to the public transport operators. Since there is a great decline in public transportation demand, revenues have decreased to a great extent. Therefore, subsidies are of big importance for the operators to keep providing services of good quality. Also, workers, who are at higher risks than others such as public transportation drivers, need extra

payments, which puts an additional financial burden, and this extra payment can be provided the subsidies.

$C_3$. Ridership of public transportation (cost): In alternatives except for the do nothing alternative, ridership of public transportation is highly affected. The reduction in capacities or the frequencies of public transportation lines can increase the use of private vehicles and reduce the share of public transportation, which is the case in some countries [32].

**(2) Health and Environmental Aspect**

$C_4$. The severity of the pandemic (cost): As the mortality of the coronavirus increases by a mutation or another reason, the importance of social distancing also increases, which makes it more important to manage public transportation to keep the commuters healthy and uninfected.

$C_5$. The health of the drivers (benefit): Since public transportation drivers are of great importance in keeping the public transportation systems working, their health is very important. The alternative, which increases the usage of ride-sharing systems and the capacity reduction alternative, reduces the in-vehicle crowd and has a positive effect on drivers' health.

$C_6$. Spread of the COVID-19 virus (cost): This criterion is related to the spreading speed of the coronavirus, regardless of the mortality rate of the virus. As the virus spreads and more cities are affected, total usage of public transportation decreases to a great extent. Also, as the speed of spreading increases, strict lockdown measures are taken by the authorities, which accompanies a decline in public transportation usage [33].

$C_7$. Air pollution (cost): This criterion is defined as the air pollution arising from the vehicles in traffic. Alternatives that are not in favor of the commuters such as the ones that increase waiting times, increase the number of service-denied commuters due to decreased capacities, and concentrate the trips at determined time intervals, reduces the satisfaction level of the commuters and reduces the ridership. This has a high potential for increasing the usage of private vehicles. As the number of private vehicles in traffic increases, air pollution also increases.

**(3) Social Aspect**

$C_8$. Income level of the population (benefit): If the income level of the population is high, public transportation management adjustments do not affect the people at great levels since it is likely that private vehicle ownership is high [34]. However, if the income levels are low and people do not own private vehicles, the adjustments have great importance. For example, when the capacities are reduced, commuters have to wait for more to use the public transportation services since they do not have another option.

$C_9$. Disadvantaged and vulnerable groups (cost): This criterion is related to the groups such as disabled people that had difficulty in using public transportation even before the covid-19 pandemic. These groups are affected by the public transportation planning adjustments even more so than others.

$C_{10}$. Public acceptance (benefit): If the alternative method increases the waiting times and the number of service-denied passengers, the citizens will probably be dissatisfied, so public acceptance is not achieved, and eventually ridership decreases. This also reduces the chances of the municipality being elected again in the following elections.

**(4) Transportation Aspect**

$C_{11}$. Personal mobility (benefit): This criterion is related to the effects of each alternative on the mobility of the commuters. The capacity reduction has the potential to not be able to compensate the demand, which increases waiting times and the number of refused passengers. Adjustments according to work hours have the potential to make passengers not be able to find a public transportation vehicle outside of the work hours. Do-nothing

alternative is costly in means of resource allocation and having a risk to spread the virus, but has the potential to provide better mobility since demand is compensated better. Bicycle sharing and scooter sharing alternatives are very beneficial in terms of micro-mobility, but at long distances to travel, they lose sufficiency.

$C_{12}$. Traffic congestion (cost): If the alternatives are applied properly and demands are compensated effectively, commuters are likely to continue using public transportation. However, if the alternatives are implemented without consideration, there is a potential of not being able to compensate the demand, which makes commuters shift to using private vehicles, which eventually increases traffic congestion [35].

$C_{13}$. Sustainability of the service (benefit): This criterion is related to the applied alternatives and them being environmentally friendly or not. If the applied alternative makes the commuters stop using public transportation and a shift to private vehicle use takes place, the alternative is not a sustainable one. However, if the applied alternative is efficient and commuters keep on using public transportation, the alternative is sustainable.

## 4. Proposed methodology

The Fuzzy Einstein Combined Compromise Solution (CoCoSo) framework (see Fig. 1) represents an improvement on the traditional CoCoSo methodology [9] by applying the Einstein T-norm and T-conorm in a fuzzy environment. The application of Einstein functions in the CoCoSo model improved the performance of the traditional CoCoSo methodology. The traditional CoCoSo method applies linear weighted functions to calculate a weighted sequence of alternatives and define trade-off strategies. Linear weighted functions can, in extreme values at the position of the most influential criteria in the decision matrix, lead to radical changes in the values of compromise significance of alternatives. This feature of the traditional CoCoSo method can lead to a wrong decision in such situations. The application of nonlinear weighted Einstein functions eliminates this anomaly of the CoCoSo model, which contributes to the objectification of decision making.

Besides the application of Einstein norms and fuzzy sets, the fuzzy Einstein CoCoSo methodology has been improved by implementing a novel methodology for determining the weight coefficients of the criteria, that is based on defining the relationship between the criteria using the fuzzy logarithmic additive function, and this is an indispensable element of the fuzzy Einstein CoCoSo framework. Fuzzy Einstein CoCoSo methodology does not need to apply added models for determining the weighting of criteria has been eliminated. The proposed methodology for determining weight coefficients can be either a multi-criteria decision-making (MCDM) tool itself or an adjunct to other MCDM models for rational and objective decision-making. In the following section, we have proposed the mathematical background of the proposed fuzzy Einstein CoCoSo methodology.

### 4.1. Fuzzy Einstein Cocoso method

The Fuzzy Einstein CoCoSo methodology is based on processing information in the initial home matrix. The information in the home matrix is defined based on expert assessments of alternatives under specified criteria. After determining the aggregated home matrix, the expert estimates were normalized to translate the information into an interval [0,1]. The next step defines the weighting coefficients of the evaluation criteria used to calculate the fuzzy Einstein nonlinear functions. Weighted strategies were created based on the defined fuzzy Einstein functions, which in the last step were aggregated into the final criterion function defined for each alternative. Based on the specified criteria

functions, the ranking of alternatives was performed. The fuzzy Einstein CoCoSo method is realized through six steps which are presented in the following part.

Suppose that in a multi-criteria problem, there are $c$ alternatives denoted by $G_i$ ($i = 1, 2, \ldots, c$) and $k$ of the criterion $H_j$ ($j = 1, 2, \ldots, k$). Also, suppose that $e$ experts $T_b$ ($b = 1, 2, \ldots, e$) take part in the research. Then, based on Definitions 1–3 and arithmetic operations with TFN, we can define an algorithm for applying the fuzzy Einstein CoCoSo method.

*Step 1.* Based on the defined fuzzy scale, experts from the set $T_b$ ($b = 1, 2, \ldots, e$) evaluated alternatives $G_i$ ($i = 1, 2, \ldots, c$). An initial decision matrix $\Phi^l = \left[\theta_{ij}^l\right]_{c \times k}$ ($1 \leq l \leq e$) was formed for each expert $T_l$ ($1 \leq l \leq e$). The values $\widetilde{\theta}_{ij}^l = \left(\theta_{ij}^{(d)l}, \theta_{ij}^{(s)l}, \theta_{ij}^{(g)l}\right)$ from the matrix $\Phi^l$ were obtained on the basis of the fuzzy linguistic scale. Using the fuzzy Einstein weighted averaging (FEWAA) operator, expression (13) calculates the aggregated initial decision matrix $\Phi = \left[\widetilde{\theta}_{ij}\right]_{c \times k}$ elements. Values $\widetilde{\theta}_{ij} = \left(\theta_{ij}^{(d)}, \theta_{ij}^{(s)}, \theta_{ij}^{(g)}\right)$ from the matrix obtained using fuzzy Einstein weighted geometric aggregation (FEWGA), expression (13).

*Step 2.* Normalization of matrix elements $\Phi = \left[\widetilde{\theta}_{ij}\right]_{c \times k}$ is performed depending on whether the criterion $H_j$ ($j = 1, 2, \ldots, k$) belongs to *Benefit* (B) or *Cost* (C) type, expression (10).

$$\widetilde{\zeta}_{ij} = (\zeta_{ij}^{(d)}, \zeta_{ij}^{(s)}, \zeta_{ij}^{(g)}) = \begin{cases} \zeta_{ij}^{(l)} = \dfrac{\theta_{ij}^{(d)}}{\theta_j^+}; \ \zeta_{ij}^{(m)} = \dfrac{\theta_{ij}^{(s)}}{\theta_j^+}; \\[2mm] \zeta_{ij}^{(u)} = \dfrac{\theta_{ij}^{(g)}}{\theta_j^+} & \text{if} \quad j \in B, \\[4mm] \zeta_{ij}^{(l)} = \dfrac{\theta_j^-}{\theta_{ij}^{(d)}}; \ \zeta_{ij}^{(m)} = \dfrac{\theta_j^-}{\theta_{ij}^{(s)}}; \\[2mm] \zeta_{ij}^{(u)} = \dfrac{\theta_j^-}{\theta_{ij}^{(g)}} & \text{if} \quad j \in C. \end{cases}$$

(10)

where $\widetilde{\theta}_{ij} = \left(\theta_{ij}^{(d)}, \theta_{ij}^{(s)}, \theta_{ij}^{(g)}\right)$ represent the elements of the initial decision matrix $\Phi = \left[\widetilde{\theta}_{ij}\right]_{c \times k}$, $\widetilde{\zeta}_{ij} = (\zeta_{ij}^{(d)}, \zeta_{ij}^{(s)}, \zeta_{ij}^{(g)})$ represent the elements of the normalized matrix $\Psi = \left[\widetilde{\zeta}_{ij}\right]_{c \times k}$.

The elements $\theta_j^+$ and $\theta_j^-$ from expression (10) are obtained by applying the expression (11) and (12):

$$\theta_j^+ = \max_{1 \leq j \leq c}(\theta_{ij}^{(d)}, \theta_{ij}^{(s)}, \theta_{ij}^{(g)})$$

(11)

$$\theta_j^- = \min_{1 \leq j \leq c}(\theta_{ij}^{(d)}, \theta_{ij}^{(s)}, \theta_{ij}^{(g)})$$

(12)

where $\theta_{ij}^{(d)}, \theta_{ij}^{(s)}$ and $\theta_{ij}^{(g)}$ represent the elements of the initial decision matrix $\Phi$.

*Step 3.* Applying Definitions 1–3 define the weighted sequences of alternatives. The fuzzy Einstein CoCoSo model defines two weighted sequences, whose values are used to calculate alternative aggregation strategies. The first weighted sequence is defined using the fuzzy Einstein weighted averaging function ($EQ_i$), while the second weighted sequence is determined using the fuzzy Einstein weighted geometric averaging function ($EP_i$).

**Theorem 1.** *Let* $(\widetilde{\zeta}_1, \widetilde{\zeta}_2, \ldots, \widetilde{\zeta}_k)$ *be a set of normalized elements of the matrix* $\Psi = \left[\widetilde{\zeta}_{ij}\right]_{c \times k}$ *represented by fuzzy numbers* $\widetilde{\zeta}_{ij} = (\zeta_{ij}^{(d)}, \zeta_{ij}^{(s)}, \zeta_{ij}^{(g)})$, *(i = 1, 2, \ldots, c; j = 1, 2, \ldots, k), let* $\varsigma > 0$ *and let* $\varpi_j = (\varpi_1, \varpi_2, \ldots, \varpi_k)^T$ *be a fuzzy vector of weight coefficients of the criterion, then the fuzzy* $EQ_i$ *function can be defined:*
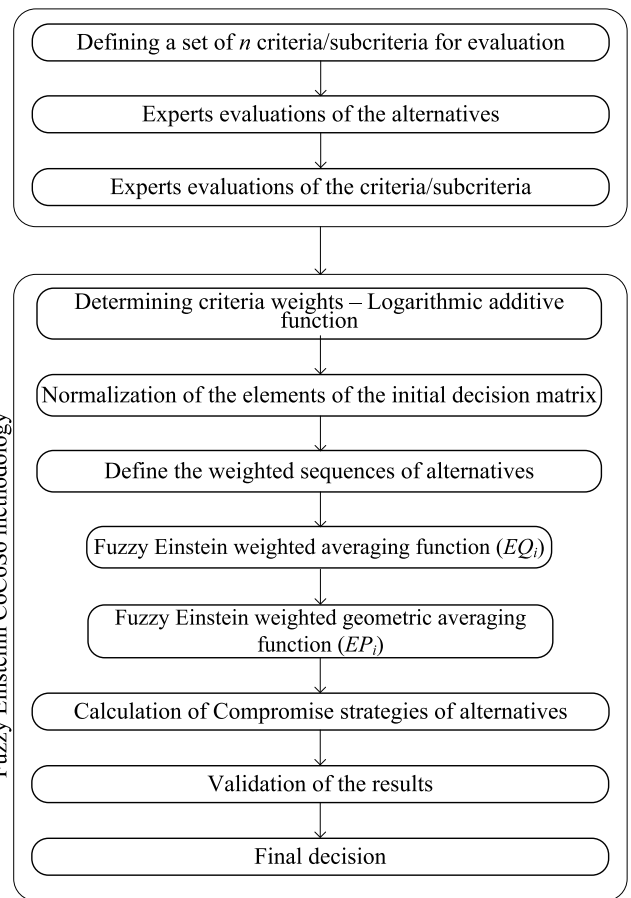
$$EQ_i = \left(EQ_i^{(d)}, EQ_i^{(s)}, EQ_i^{(g)}\right)$$



**Fig. 1.** Fuzzy Einstein CoCoSo framework.

$$= \begin{pmatrix} \sum_{j=1}^{k} \left(\zeta_{ij}^{(d)}\right) \dfrac{\prod_{j=1}^{k}\left(1+f\left(\zeta_{ij}^{(d)}\right)\right)^{\varpi_j^{(d)}} - \prod_{j=1}^{k}\left(1-f\left(\zeta_{ij}^{(d)}\right)\right)^{\varpi_j^{(d)}}}{\prod_{j=1}^{k}\left(1+f\left(\zeta_{ij}^{(d)}\right)\right)^{\varpi_j^{(d)}} + \prod_{j=1}^{k}\left(1-f\left(\zeta_{ij}^{(d)}\right)\right)^{\varpi_j^{(d)}}}, \\[4mm] \sum_{j=1}^{k} \left(\zeta_{ij}^{(s)}\right) \dfrac{\prod_{j=1}^{k}\left(1+f\left(\zeta_{ij}^{(s)}\right)\right)^{\varpi_j^{(s)}} - \prod_{j=1}^{k}\left(1-f\left(\zeta_{ij}^{(s)}\right)\right)^{\varpi_j^{(s)}}}{\prod_{j=1}^{k}\left(1+f\left(\zeta_{ij}^{(s)}\right)\right)^{\varpi_j^{(s)}} + \prod_{j=1}^{k}\left(1-f\left(\zeta_{ij}^{(s)}\right)\right)^{\varpi_j^{(s)}}}, \\[4mm] \sum_{j=1}^{k} \left(\zeta_{ij}^{(g)}\right) \dfrac{\prod_{j=1}^{k}\left(1+f\left(\zeta_{ij}^{(g)}\right)\right)^{\varpi_j^{(g)}} - \prod_{j=1}^{k}\left(1-f\left(\zeta_{ij}^{(g)}\right)\right)^{\varpi_j^{(g)}}}{\prod_{j=1}^{k}\left(1+f\left(\zeta_{ij}^{(g)}\right)\right)^{\varpi_j^{(g)}} + \prod_{j=1}^{k}\left(1-f\left(\zeta_{ij}^{(g)}\right)\right)^{\varpi_j^{(g)}}} \end{pmatrix},$$

(13)

*where* $\varpi_j = (\varpi_1, \varpi_2, \ldots, \varpi_k)^T$ *is the vector of the weight coefficients of the criteria, while* $f\left(\widetilde{\zeta}_j\right) = \left(\zeta_j^{(d)} / \sum_{j=1}^{k} \zeta_j^{(d)}, \zeta_j^{(s)} / \sum_{j=1}^{k} \zeta_j^{(s)}, \zeta_j^{(g)} / \sum_{j=1}^{k} \zeta_j^{(g)}\right)$. *Then* $EQ_i$ *represents the fuzzy Einstein weighted averaging function. The proof for* Theorem 1 *is presented in* Appendix.

**Theorem 2.** *Let* $(\widetilde{\zeta}_1, \widetilde{\zeta}_2, \ldots, \widetilde{\zeta}_k)$ *be a set of normalized elements of the matrix* $\Psi = \left[\widetilde{\zeta}_{ij}\right]_{c \times k}$ *represented by fuzzy numbers* $\widetilde{\zeta}_{ij} = (\zeta_{ij}^{(d)}, \zeta_{ij}^{(s)}, \zeta_{ij}^{(g)})$, *(i = 1, 2, \ldots, c; j = 1, 2, \ldots, k), let* $\varsigma > 0$ *and let* $\varpi_j = (\varpi_1, \varpi_2, \ldots, \varpi_k)^T$ *be a fuzzy vector of weight coefficients of the criterion, then the fuzzy* $EP_i$ *function can be defined as follows:*

$$EP_i = \left(EP_i^{(d)}, EP_i^{(s)}, EP_i^{(g)}\right)$$

$$= \begin{pmatrix} \sum_{j=1}^{k} \left(\zeta_{ij}^{(d)}\right) \dfrac{2\prod_{j=1}^{k}\left(f\left(\zeta_{ij}^{(d)}\right)\right)^{\varpi_j^{(d)}}}{\prod_{j=1}^{k}\left(2-f\left(\zeta_{ij}^{(d)}\right)\right)^{\varpi_j^{(d)}}+\prod_{j=1}^{k}\left(f\left(\zeta_{ij}^{(d)}\right)\right)^{\varpi_j^{(d)}}}, \\[2em] \sum_{j=1}^{k} \left(\zeta_{ij}^{(s)}\right) \dfrac{2\prod_{j=1}^{k}\left(f\left(\zeta_{ij}^{(s)}\right)\right)^{\varpi_j^{(s)}}}{\prod_{j=1}^{k}\left(2-f\left(\zeta_{ij}^{(s)}\right)\right)^{\varpi_j^{(s)}}+\prod_{j=1}^{k}\left(f\left(\zeta_{ij}^{(s)}\right)\right)^{\varpi_j^{(s)}}}, \\[2em] \sum_{j=1}^{k} \left(\zeta_{ij}^{(g)}\right) \dfrac{2\prod_{j=1}^{k}\left(f\left(\zeta_{ij}^{(g)}\right)\right)^{\varpi_j^{(g)}}}{\prod_{j=1}^{k}\left(2-f\left(\zeta_{ij}^{(g)}\right)\right)^{\varpi_j^{(g)}}+\prod_{j=1}^{k}\left(f\left(\zeta_{ij}^{(g)}\right)\right)^{\varpi_j^{(g)}}} \end{pmatrix},$$

$$(14)$$

where $\varpi_j = (\varpi_1, \varpi_2, \ldots, \varpi_k)^T$ is the vector of the weight coefficients of the criteria, while $f\left(\widetilde{\zeta_j}\right) = \left(\zeta_j^{(d)}\Big/\sum_{j=1}^{k}\zeta_j^{(d)}, \zeta_j^{(s)}\Big/\sum_{j=1}^{k}\zeta_j^{(s)}, \zeta_j^{(g)}\Big/\sum_{j=1}^{k}\zeta_j^{(g)}\right)$. Then $EP_i$ represents the fuzzy Einstein weighted geometric averaging function. The proof for Theorem 1 is presented in Appendix.

**Step 4.** Determination of weight coefficients of criteria. Weighting coefficients of criteria were defined by applying logarithmic additive evaluation of expert assessments.

**Step 4.1.** Defining priority vectors. The priority vector $\Omega^l = \left(\psi_{H1}^l, \psi_{H2}^l, \ldots, \psi_{Hk}^l\right)$ is formed based on expert preferences. Expert $T_l$ ($1 \leq l \leq e$) assigns a value from the fuzzy scale to each criterion from the set $H_j$ ($j = 1, 2, \ldots, k$), by assigning the highest value from the fuzzy scale to the criterion that has the greatest significance, while to the criterion that has the lowest significance assigns the lowest value from the scale.

**Step 4.2.** Defining the absolute anti-ideal point ($\tau_{AIP}$). The absolute anti-ideal point is defined by applying the expression (15).

$$\tau_{AIP} < \min\left(\psi_{H1}^l, \psi_{H2}^l, \ldots, \psi_{Hn}^l\right) \tag{15}$$

**Step 4.3.** Defining the ratio vector $\Theta^l$ for the expert $T_l$ ($1 \leq l \leq e$). Using expression (16), the relationship between the elements of the vector $\Omega^l$ and $\tau_{AIP}$ is determined.

$$\xi_{Hj}^l = \frac{\psi_{Hj}^l}{\tau_{AIP}} \tag{16}$$

where $j = 1, 2, \ldots, k$, while $\psi_{Hk}^l$ represents the element of the priority vector $\Omega^l$ for the expert $T_l$ ($1 \leq l \leq e$).

Thus we obtain the vector of the relation $\Theta^l = \left(\xi_{H1}^l, \xi_{H2}^l, \ldots, \xi_{Hk}^l\right)$ for the expert $T_l$ ($1 \leq l \leq e$).

**Step 4.4.** Determination of weights vector $\widetilde{\varpi}_j = (\widetilde{\varpi}_1, \widetilde{\varpi}_2, \ldots, \widetilde{\varpi}_n)^T$. By applying expression (17), we obtain the values of the weighting coefficients of the criteria for the expert $t$ ($1 \leq t \leq k$).

$$\widetilde{\varpi}_j^l = \frac{\ln\left(\xi_{Hj}^l\right)}{\ln\left(\eta^l\right)} \tag{17}$$

where $\xi_{Hj}^l$ represents the element of the relation vector $\Theta^l$ for the expert $T_l$ ($1 \leq l \leq e$), while $\eta^l = \prod_{j=1}^{k}\xi_{Hj}^l$.

By applying the fuzzy Einstein weighted averaging operator (18), we obtain the aggregated fuzzy vector of weight coefficients

$$\widetilde{\varpi}_j = (\widetilde{\varpi}_1, \widetilde{\varpi}_2, \ldots, \widetilde{\varpi}_n)^T.$$

$$\widetilde{\varpi}_j = \begin{pmatrix} \sum_{j=1}^{e}\left(\varpi_{ij}^{(d)l}\right) \dfrac{\prod_{j=1}^{e}\left(1+f\left(\varpi_{ij}^{(d)l}\right)\right)^{1/e}-\prod_{j=1}^{e}\left(1-f\left(\varpi_{ij}^{(d)l}\right)\right)^{1/e}}{\prod_{j=1}^{e}\left(1+f\left(\varpi_{ij}^{(d)l}\right)\right)^{1/e}+\prod_{j=1}^{e}\left(1-f\left(\varpi_{ij}^{(d)l}\right)\right)^{1/e}}, \\[2em] \sum_{j=1}^{e}\left(\varpi_{ij}^{(s)l}\right) \dfrac{\prod_{j=1}^{e}\left(1+f\left(\varpi_{ij}^{(s)l}\right)\right)^{1/e}-\prod_{j=1}^{e}\left(1-f\left(\varpi_{ij}^{(s)l}\right)\right)^{1/e}}{\prod_{j=1}^{e}\left(1+f\left(\varpi_{ij}^{(s)l}\right)\right)^{1/e}+\prod_{j=1}^{e}\left(1-f\left(\varpi_{ij}^{(s)l}\right)\right)^{1/e}}, \\[2em] \sum_{j=1}^{e}\left(\varpi_{ij}^{(d)l}\right) \dfrac{\prod_{j=1}^{e}\left(1+f\left(\varpi_{ij}^{(s)l}\right)\right)^{1/e}-\prod_{j=1}^{e}\left(1-f\left(\varpi_{ij}^{(s)l}\right)\right)^{1/e}}{\prod_{j=1}^{e}\left(1+f\left(\varpi_{ij}^{(s)l}\right)\right)^{1/e}+\prod_{j=1}^{e}\left(1-f\left(\varpi_{ij}^{(s)l}\right)\right)^{1/e}} \end{pmatrix}$$

$$(18)$$

where $1 \leq l \leq e$, while $e$ represents the number of experts

**Step 5.** Using expressions (19)–(21), the aggregation strategies are calculated as follows:

(a) Additive normalized strategy for combining Einstein functions:

$$\widetilde{\chi}_i = \frac{EQ_i + EP_i}{\sum_{i=1}^{c}(EQ_i + EP_i)} \tag{19}$$

(b) Relative relations of the significance of Einstein functions:

$$\widetilde{\alpha}_i = \frac{EQ_i}{\min_{1\leq i\leq c}(EQ_i)} + \frac{EP_i}{\min_{1\leq i\leq c}(EP_i)} \tag{20}$$

(c) Compromising significance of Einstein functions:

$$\widetilde{\beta}_i = \frac{\gamma EQ_i + (1-\gamma)EP_i}{\gamma \max_{1\leq i\leq b}(EQ_i) + (1-\gamma)\max_{1\leq i\leq b}(EP_i)}; 0 \leq \gamma \leq 1. \tag{21}$$

where the coefficient $\gamma$ represents the corrective parameter for the calculation of compromise values of Einstein functions.

The coefficient $\gamma$ defines the intensity of the influence of $EQ_i$ and $EP_i$ functions on the final decision. The values of $0 \leq \gamma \leq 0.5$ increase the intensity of the influence of the $EP_i$ function in the aggregation strategy, while the values of $0.5 \leq \gamma \leq 1$ increase the intensity of the influence of the $EQ_i$ function. From this analysis, it is obvious that the value of the parameter $\gamma$ affects the stability of the proposed solution, so it is recommended to adopt the value of $\gamma = 0.5$ for the calculation of the initial solution in expression (21). This ensures equal intensity of influence of both Einstein functions on the final decision. To eliminate the influence of subjectivity on the final decision, it is necessary to analyze the influence of the variation of the parameter $\gamma$ in the interval $0 \leq \gamma \leq 1$ on the stability of the initial solution.

**Step 6:** The rank of alternatives is defined based on a compromise index of alternatives ($\Upsilon_i$). In the following section, a new approach to integrating aggregate strategies is proposed, as the classical approach proposed by Yazdani et al. [9] has anomalies that can lead to wrong results. Based on expressions (19)–(21) it is clear that $\widetilde{\chi}_i < 1$, $\widetilde{\alpha}_i \geq 2$, and $\widetilde{\beta}_i < 1$. If we were to use the classical approach to strategy integration [9], the value of $\widetilde{\alpha}_i$ would have a greater impact on the final result than $\widetilde{\chi}_i$ and $\widetilde{\beta}_i$. However, in practice, $\widetilde{\alpha}_i$ may be the least important of the three trade-off strategies considered. By applying expression (22), the mentioned inconsistencies are eliminated, and the compromise index of alternatives is defined objectively and consistently:

$$\Upsilon_i = \frac{\widetilde{\chi}_i + \widetilde{\alpha}_i + \widetilde{\beta}_i}{1 + \left\{\partial_1\left(\frac{1-f(\widetilde{\chi}_i)}{f(\widetilde{\chi}_i)}\right)^\delta + \partial_2\left(\frac{1-f(\widetilde{\alpha}_i)}{f(\widetilde{\alpha}_i)}\right)^\delta + \partial_3\left(\frac{1-f(\widetilde{\beta}_i)}{f(\widetilde{\beta}_i)}\right)^\delta\right\}^{1/\delta}};$$

$$\delta \geq 0 \tag{22}$$

where $\partial_1, \partial_2, \partial_3 \in [0, 1]$ and $\partial_1 + \partial_2 + \partial_3 = 1$. The optimal alternative has the highest value of $\Upsilon_i$.

**Table 1**
The hierarchy of the evaluation criteria.

| Main-criteria | | Sub-criteria | Type |
|---|---|---|---|
| $C_1$:Economic aspect | | | |
| | $C_{11}$ | Operation cost | Cost |
| | $C_{12}$ | Subsidy from the government | Benefit |
| | $C_{13}$ | Ridership of public transportation | Cost |
| $C_2$:Health and Environmental aspect | | | |
| | $C_{21}$ | Severity of the pandemic | Cost |
| | $C_{22}$ | Health of the drivers | Benefit |
| | $C_{23}$ | Spread of the COVID-19 virus | Cost |
| | $C_{24}$ | Air pollution | Cost |
| $C_3$:Social aspect | | | |
| | $C_{31}$ | Income level of the population | Benefit |
| | $C_{32}$ | Disadvantaged and vulnerable groups | Cost |
| | $C_{33}$ | Public acceptance | Benefit |
| $C_4$:Transportation aspect | | | |
| | $C_{41}$ | Personal mobility | Benefit |
| | $C_{42}$ | Traffic congestion | Cost |
| | $C_{43}$ | Sustainability of the service | Benefit |

**Table 2**
Fuzzy scale for evaluating alternatives [36].

| Linguistic terms | Membership function |
|---|---|
| Absolutely low (AL) | (1, 1, 1) |
| Very low (VL) | (1, 2, 3) |
| Low (L) | (2, 3, 4) |
| Medium low (ML) | (3, 4, 5) |
| Equal (E) | (4, 5, 6) |
| Medium high (MH) | (5, 6, 7) |
| High (H) | (6, 7, 8) |
| Very high (VH) | (7, 8, 9) |
| Absolutely high (AH) | (8, 9, 9) |

# 5. Case study

Public transportation management and planning need to be reconsidered by the municipalities because of the COVID-19 pandemic regulations. New management methods are required to adapt old management systems during pandemic times. Since commuter psychology and behavior have changed a lot from the beginning of the pandemic because of safety aspects such as avoiding crowded and closed areas, public transportation ridership is decreasing sharply. This requires the need for public transportation systems to be re-planned.

As a case study, we consider the public transportation management efforts of a municipality of a metropolitan city with a high population is selected. In the selected city, the government has taken strict measures such as stay-at-home restrictions to slow down the spreading of the coronavirus. However, both the strict regulations and the travel behaviors of the citizens are affecting public transportation ridership negatively. Therefore, the governing authority in the city is ready to take action, yet it is required to prioritize the remedies. By conducting a thorough literature survey, operating costs, demand management, capacity management, and health of the commuters and the drivers are seen to be the most important aspects, which created a base for the creation of the alternatives. Do nothing alternative refers to keeping the old management method. Adjusting public transportation schedule based on the work hours is the second alternative, which proposes to concentrate the trips according to the work hours. Capacity reduction in public transportation is the third alternative, which aims to make commuters travel while keeping the social distance. Increasing the ridership in sharing-based modes such as bicycle sharing, scooter sharing, car sharing is the last alternative that aims to increase the isolation of people while traveling. After the alternatives are set, criteria, which are selected as the most important aspects for the management of public transportation in pandemic times, are determined by conducting a thorough literature survey. Finally, experts are consulted to gather their opinions and assessments of the alternatives based on the criteria in order to determine the relative importance of each criterion and the advantage prioritization of the alternatives through in-person interviews.

## 5.1. Proposed model results

In the case study discussed in this paper, four alternatives were evaluated using the fuzzy Einstein CoCoSo model: Alternative 1 - Do nothing; Alternative 2 - Adjusting public transportation schedule based on the work hours; Alternative 3 - Capacity

reduction in public transportation and Alternative 4 - Increasing the ridership in sharing based modes such as bicycle sharing, scooter sharing, car sharing. For evaluating alternatives in the multi-criteria model, 13 criteria are identified and grouped within four clusters as given in Table 1.

The evaluation of alternatives using the fuzzy Einstein CoCoSo model is presented through the following steps:

*Step 1*: Seven experts $T_b$ $(b = 1, 2, \ldots, 7)$ evaluated the alternatives using the fuzzy scale given in Table 2.

Based on expert assessments, seven expert initial matrices $\Phi^l = \left[\widetilde{\theta}_{ij}^l\right]_{4\times 13}$ $(1 \leq l \leq 7)$ were formed in which the evaluation of alternatives concerning 13 criteria was presented. Expert estimates of alternatives are given in Table 3.

Using the fuzzy Einstein weighting function, the expert preferences were averaged, and an aggregated decision matrix was formed in Table 4. Since seven experts participated in the research, the value $\varpi_j = 1/7$ was adopted for the weighting coefficients in the fuzzy Einstein weighting function.

The following section presents the fuzzy Einstein weighting function for the fusion of expert preferences at the $G_2 - C_{11}$ position. Based on the fuzzy linguistic values from Tables 3 and 2, expert preferences were defined according to the following: $\widetilde{\theta}_{21}^1 = (2, 3, 4)$, $\widetilde{\theta}_{21}^2 = (2, 3, 4)$, $\widetilde{\theta}_{21}^3 = (1, 2, 3)$, $\widetilde{\theta}_{21}^4 = (1, 1, 1)$, $\widetilde{\theta}_{21}^5 = (2, 3, 4)$, $\widetilde{\theta}_{21}^6 = (2, 3, 4)$ and $\widetilde{\theta}_{21}^7 = (4, 5, 6)$. As previously emphasized, since seven experts participated in the research, a vector of expert weight coefficients was adopted according to the following $\varpi_b = 1/7$ $(b = 1, 2, \ldots, 7)$. Then, by applying the fuzzy Einstein weighting function, we get the value at position $G_2 - C_{11}$ according to the following:

$$EWA\left(\theta_{21}\right) =$$

$$\begin{cases} \widetilde{\theta}_{12}^{(d)} = \begin{pmatrix} 2+2+1+ \\ 1+2+2+4 \end{pmatrix} \\ \times \dfrac{\begin{pmatrix} (1+0.13)^{0.14} \cdot (1+0.23)^{0.14} \cdot \\ (1+0.19)^{0.14} \cdot \ldots \cdot (1+0.03)^{0.14} \end{pmatrix} - \begin{pmatrix} (1-0.13)^{0.14} \cdot (1-0.23)^{0.14} \cdot \\ (1-0.19)^{0.14} \cdot \ldots \cdot (1-0.03)^{0.14} \end{pmatrix}}{\begin{pmatrix} (1+0.13)^{0.14} \cdot (1+0.23)^{0.14} \cdot \\ (1+0.19)^{0.14} \cdot \ldots \cdot (1+0.03)^{0.14} \end{pmatrix} + \begin{pmatrix} (1-0.13)^{0.14} \cdot (1-0.23)^{0.14} \cdot \\ (1-0.19)^{0.14} \cdot \ldots \cdot (1-0.03)^{0.14} \end{pmatrix}} \\ \widetilde{\theta}_{12}^{(s)} = \begin{pmatrix} 3+3+2+ \\ 1+3+3+5 \end{pmatrix} \\ \times \dfrac{\begin{pmatrix} (1+0.13)^{0.14} \cdot (1+0.18)^{0.14} \cdot \\ (1+0.21)^{0.14} \cdot \ldots \cdot (1+0.05)^{0.14} \end{pmatrix} - \begin{pmatrix} (1-0.13)^{0.14} \cdot (1-0.18)^{0.14} \cdot \\ (1-0.21)^{0.14} \cdot \ldots \cdot (1-0.05)^{0.14} \end{pmatrix}}{\begin{pmatrix} (1+0.13)^{0.14} \cdot (1+0.18)^{0.14} \cdot \\ (1+0.21)^{0.14} \cdot \ldots \cdot (1+0.05)^{0.14} \end{pmatrix} + \begin{pmatrix} (1-0.13)^{0.14} \cdot (1-0.18)^{0.14} \cdot \\ (1-0.21)^{0.14} \cdot \ldots \cdot (1-0.05)^{0.14} \end{pmatrix}} \\ \widetilde{\theta}_{12}^{(g)} = \begin{pmatrix} 2+2+1+ \\ 1+2+2+4 \end{pmatrix} \\ \times \dfrac{\begin{pmatrix} (1+0.13)^{0.14} \cdot (1+0.18)^{0.14} \cdot \\ (1+0.20)^{0.14} \cdot \ldots \cdot (1+0.07)^{0.14} \end{pmatrix} - \begin{pmatrix} (1-0.13)^{0.14} \cdot (1-0.18)^{0.14} \cdot \\ (1-0.20)^{0.14} \cdot \ldots \cdot (1-0.07)^{0.14} \end{pmatrix}}{\begin{pmatrix} (1+0.13)^{0.14} \cdot (1+0.18)^{0.14} \cdot \\ (1+0.20)^{0.14} \cdot \ldots \cdot (1+0.07)^{0.14} \end{pmatrix} + \begin{pmatrix} (1-0.13)^{0.14} \cdot (1-0.18)^{0.14} \cdot \\ (1-0.20)^{0.14} \cdot \ldots \cdot (1-0.07)^{0.14} \end{pmatrix}} \end{cases}$$

$$= (2.01, 2.87, 3.72)$$

The aggregation of the remaining values from Table 4 was performed in a similar manner.

**Table 3**
Expert evaluation of alternatives.

| Crit. | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $C_{11}$ | E; H; VH; H; E; ML; VL | L; L; VL; AL; L; L; E | VL; L; ML; MH; VL; VL; L | MH; E; H; AH; ML; L; H |
| $C_{12}$ | AL; AL; AL; AL; E; ML; E | H; AH; EH; H; H; MH; H | H; H; VH; AH; VH; VH; VH | E; H; MH; L; VH; AH; ML |
| $C_{13}$ | AL; ML; L; E; MH; H; H | AL; VL; VL; L; VL; E; ML | VL; VL; AL; ML; MH; AH; VL | MH; L; ML; H; VH; VH; VH |
| $C_{21}$ | AL; L; AL; AL; VL; AL; VL | L; AL; L; L; E; ML; L | AL; AL; AL; AL; H; H; VL | AL; VL; L; H; VH; AH; AL |
| $C_{22}$ | L; VL; AL; AL; VL; AL; AL | AL; ML; L; L; MH; MH; H | H; H; VH; AH; H; VH; VH | H; VH; MH; MH; H; H; VL |
| $C_{23}$ | AL; AL; AL; AL; VL; AL; AL | VL; L; E; AL; MH; H; VL | VL; VL; AL; AL; H; AH; L | L; L; MH; VH; H; MH; AL |
| $C_{24}$ | E; MH; VH; AH; E; E; AL | MH; MH; VH; MH; L; ML; MH | E; E; VH; L; MH; VL; MH | ML; L; ML; MH; VH; VH; AL |
| $C_{31}$ | AL; AL; AL; AL; E; ML; E | E; L; VL; L; L; VL; H | MH; ML; VL; L; L; VL; VH | VH; VH; MH; MH; H; H; VH |
| $C_{32}$ | E; H; AH; AH; H; MH; ML | H; VH; L; H; L; E; L | H; VH; H; H; VL; ML; AL | H; AH; VH; H; VL; L; AH |
| $C_{33}$ | AL; AL; AL; AL; L; VL; H | AH; H; AH; AH; VH; MH; MH | H; H; MH; AH; VL; MH; E | H; VH; L; VL; VH; H; VH |
| $C_{41}$ | VL; L; AL; AL; L; L; VH | H; AH; AH; H; H; MH; E | L; H; VH; H; L; VL; ML | MH; H; L; E; VH; AH; AH |
| $C_{42}$ | VH; AH; H; MH; VL; AL; ML | AL; AL; AL; AL; H; H; L | L; VL; L; AL; AL; AL; H | E; E; MH; MH; VH; AH; AL |
| $C_{43}$ | E; AL; AL; AL; L; VL; VL | L; H; MH; AH; MH; MH; MH | L; H; L; L; VL; VL; H | H; VH; H; L; ML; L; AH |

**Table 4**
Aggregated decision matrix.

| Crit. | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $C_{11}$ | (4.44,5.44,6.44) | (2.01,2.87,3.72) | (2.17,3.16,4.15) | (4.87,5.87,6.72) |
| $C_{12}$ | (2.16,2.60,3.04) | (6.29,7.29,8.14) | (6.86,7.86,8.71) | (5.02,6.01,6.87) |
| $C_{13}$ | (3.87,4.73,5.59) | (1.87,2.73,3.58) | (2.92,3.77,4.46) | (5.30,6.30,7.30) |
| $C_{21}$ | (1.15,1.58,2.02) | (2.29,3.15,4.01) | (2.49,2.93,3.36) | (3.77,4.48,5.05) |
| $C_{22}$ | (1.15,1.58,2.02) | (3.45,4.30,5.16) | (6.72,7.72,8.57) | (5.15,6.15,7.15) |
| $C_{23}$ | (1.00,1.15,1.30) | (2.89,3.74,4.60) | (2.93,3.64,4.19) | (4.02,4.88,5.74) |
| $C_{24}$ | (4.73,5.59,6.30) | (4.58,5.58,6.58) | (4.02,5.01,6.01) | (4.03,4.88,5.74) |
| $C_{31}$ | (2.16,2.60,3.04) | (3.73,4.72,5.72) | (3.04,4.03,5.02) | (6.15,7.14,8.14) |
| $C_{32}$ | (5.73,6.72,7.43) | (4.16,5.16,6.16) | (4.31,5.17,6.02) | (5.45,6.45,7.16) |
| $C_{33}$ | (1.92,2.35,2.78) | (6.72,7.72,8.29) | (5.02,6.01,6.87) | (5.16,6.16,7.16) |
| $C_{41}$ | (2.35,3.05,3.76) | (6.15,7.15,7.86) | (3.89,4.88,5.88) | (5.73,6.73,7.44) |
| $C_{42}$ | (4.46,5.32,6.03) | (2.62,3.06,3.50) | (2.05,2.62,3.19) | (4.88,5.73,6.45) |
| $C_{43}$ | (1.59,2.17,2.74) | (5.15,6.15,7.01) | (2.89,3.88,4.88) | (4.88,5.88,6.73) |

*Step 2:* Normalization of the decision matrix elements $\Phi = \left[\tilde{\theta}_{ij}\right]_{4\times 13}$ was performed using expression (10). The normalized matrix $\Psi = \left[\tilde{\zeta}_{ij}\right]_{4\times 13}$ is presented in Table 5. Normalization of the element at position $G_2 - C_{11}$ in the matrix $\Psi = \left[\tilde{\zeta}_{ij}\right]_{4\times 13}$ was performed using expression (10) as follows:

$$\tilde{\zeta}_{2,11} = (\zeta_{2,11}^{(d)}, \zeta_{2,11}^{(s)}, \zeta_{2,11}^{(g)}) = \begin{cases} \zeta_{2,11}^{(l)} = \frac{\theta_{C11}^-}{\theta_{2,11}^{(d)}} = \frac{2.01}{6.44} = 0.31; \\ \zeta_{2,11}^{(m)} = \frac{\theta_{C11}^-}{\theta_{2,11}^{(s)}} = \frac{2.01}{5.44} = 0.37; \\ \zeta_{2,11}^{(u)} = \frac{\theta_{C11}^-}{\theta_{2,11}^{(g)}} = \frac{2.01}{4.44} = 0.45. \end{cases}$$

$$= (0.31, 0.37, 0.45)$$

The normalization of the remaining elements of the decision matrix was performed similarly. The normalized matrix $\Psi = \left[\tilde{\zeta}_{ij}\right]_{4\times 13}$ is presented in Table 5.

*Step 3:* The calculation of the weight coefficients of the criteria was performed using the fuzzy logarithmic additive methodology as follows.

*Step 3.1:* Based on expert assessments, a priority vector $\Omega^l = \left(\psi_{H1}^l, \psi_{H2}^l, \ldots, \psi_{Hk}^l\right)$, $1 \leq l \leq 7$, was defined for each expert. The five-point fuzzy scale presented in Table 6 was used to show expert estimates.

One priority criterion vector is defined for each expert as given in Table 7.

*Step 3.2:* Absolute anti-ideal point $\gamma_{AIP} = (0.4, 0.5, 0.6)$ is defined using expression (15).

*Step 3.3:* Using expression (14), the vectors of the ratio $\Theta^l = \left(\xi_{H1}^l, \xi_{H2}^l, \ldots, \xi_{Hk}^l\right)$, $(1 \leq l \leq 7)$ were defined for experts, Table 8.

*Step 3.4:* Using expressions (17) and (18), we obtain fuzzy vectors of weight coefficients of the criteria as given in Table 9.

Local vectors of cluster/criterion weight coefficients represent the significance of clusters/criteria within the considered group.

Global weight values were obtained by multiplying the local weight coefficients of the cluster by the local coefficients of the criteria. Global weighting factors represent the significance of the total criteria within the global group of 13 criteria.

*Step 4:* The fuzzy Einstein CoCoSo model defines two weighted sequences obtained by applying the following fuzzy functions: *(1)* fuzzy Einstein weighted averaging function ($EQ_i$) and *(2)* fuzzy Einstein weighted geometric averaging function ($EP_i$). $EQ_i$ and $EP_i$ functions were calculated using expressions (13) and (14):

$$EQ_i = \begin{matrix} G_1 \\ G_2 \\ G_3 \\ G_4 \end{matrix} \begin{bmatrix} (0.380, 0.435, 0.661) \\ (0.566, 0.714, 0.778) \\ (0.534, 0.661, 0.738) \\ (0.503, 0.629, 0.627) \end{bmatrix};$$

$$EP_i = \begin{matrix} G_1 \\ G_2 \\ G_3 \\ G_4 \end{matrix} \begin{bmatrix} (0.340, 0.399, 0.606) \\ (0.532, 0.680, 0.731) \\ (0.506, 0.630, 0.691) \\ (0.453, 0.568, 0.550) \end{bmatrix}$$

Global fuzzy vectors of criterion weight coefficients were used to calculate $EQ_i$ and $EP_i$ functions. The calculation of the $EQ_i$ function, expression (13), for alternative $G_3$ is shown in Box I. The value of the $EP_i$ function for alternative $G_3$, expression (14), is obtained as follows

$$EP_3 = \left(EP_3^{(d)}, EP_3^{(s)}, EP_3^{(g)}\right) =$$
$$\begin{cases} EP_3^{(d)} = \begin{pmatrix} 2.17 + 6.86 + 2.92 \\ + \cdots + 2.05 + 2.89 \end{pmatrix} \\ \quad \times \frac{2\left((0.076)^{0.018} \cdot (0.108)^{0.0166} \cdot (0.059)^{0.0199} \cdot \ldots \cdot (0.066)^{0.0194}\right)}{\begin{pmatrix} (2-0.07)^{0.018} \cdot (2-0.113)^{0.0166} \cdot \\ (2-0.061)^{0.0199} \cdot \ldots \cdot (2-0.06)^{0.0194} \end{pmatrix} + \begin{pmatrix} (0.07)^{0.018} \cdot (0.113)^{0.0166} \cdot \\ (0.061)^{0.0199} \cdot \ldots \cdot (0.06)^{0.0194} \end{pmatrix}} \\ EP_3^{(s)} = \begin{pmatrix} 3.16 + 7.86 + 3.77 \\ + \cdots + 2.62 + 3.88 \end{pmatrix} \\ \quad \times \frac{2\left((0.13)^{0.074} \cdot (0.18)^{0.077} \cdot (0.21)^{0.077} \cdot \ldots \cdot (0.05)^{0.068}\right)}{\begin{pmatrix} (2-0.13)^{0.074} \cdot (2-0.18)^{0.077} \cdot \\ (2-0.21)^{0.077} \cdot \ldots \cdot (2-0.05)^{0.068} \end{pmatrix} + \begin{pmatrix} (0.13)^{0.074} \cdot (0.18)^{0.077} \cdot \\ (0.21)^{0.077} \cdot \ldots \cdot (0.05)^{0.068} \end{pmatrix}} \\ EP_3^{(g)} = \begin{pmatrix} 4.15 + 8.71 + 4.46 \\ + \cdots + 3.19 + 4.88 \end{pmatrix} \\ \quad \times \frac{2\left((0.091)^{0.332} \cdot (0.098)^{0.339} \cdot (0.063)^{0.341} \cdot \ldots \cdot (0.068)^{0.234}\right)}{\begin{pmatrix} (2-0.091)^{0.332} \cdot (2-0.098)^{0.339} \cdot \\ (2-0.063)^{0.341} \cdot \ldots \cdot (2-0.068)^{0.234} \end{pmatrix} + \begin{pmatrix} (0.091)^{0.332} \cdot (0.098)^{0.339} \cdot \\ (0.063)^{0.341} \cdot \ldots \cdot (0.068)^{0.234} \end{pmatrix}} \end{cases}$$

$$= (0.506, 0.63, 0.691)$$

*Step 5:* Using expressions (19)–(21), fuzzy aggregation strategies were calculated as follows:

$$\tilde{\chi}_i = \begin{matrix} G_1 \\ G_2 \\ G_3 \\ G_4 \end{matrix} \begin{bmatrix} (0.134, 0.177, 0.332) \\ (0.204, 0.296, 0.396) \\ (0.193, 0.274, 0.375) \\ (0.178, 0.254, 0.309) \end{bmatrix};$$

**Table 5**
Normalized matrix.

| Crit. | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $C_{11}$ | (0.312,0.369,0.452) | (0.54,0.701, 1.000) | (0.484,0.637,0.928) | (0.299,0.343,0.413) |
| $C_{12}$ | (0.248,0.299,0.349) | (0.722,0.836,0.934) | (0.787,0.902, 1.000) | (0.576,0.690,0.788) |
| $C_{13}$ | (0.335,0.396,0.484) | (0.523,0.687, 1.000) | (0.420,0.498,0.641) | (0.257,0.298,0.354) |
| $C_{21}$ | (0.568,0.725,1.000) | (0.286,0.364,0.500) | (0.341,0.392,0.460) | (0.227,0.256,0.304) |
| $C_{22}$ | (0.134,0.184,0.235) | (0.402,0.502,0.602) | (0.783,0.900,1.000) | (0.601,0.718,0.834) |
| $C_{23}$ | (0.771,0.873,1.000) | (0.218,0.267,0.346) | (0.238,0.275,0.341) | (0.174,0.205,0.249) |
| $C_{24}$ | (0.637,0.719,0.849) | (0.611,0.720,0.877) | (0.668,0.801,1.000) | (0.700,0.823,0.998) |
| $C_{31}$ | (0.266,0.319,0.373) | (0.458,0.580,0.703) | (0.373,0.494,0.616) | (0.755,0.877,1.000) |
| $C_{32}$ | (0.560,0.619,0.727) | (0.676,0.807,1.000) | (0.691,0.806,0.966) | (0.582,0.646,0.764) |
| $C_{33}$ | (0.231,0.283,0.336) | (0.811,0.931,1.000) | (0.605,0.726,0.828) | (0.623,0.743,0.864) |
| $C_{41}$ | (0.298,0.388,0.478) | (0.782,0.909,1. 000) | (0.494,0.621,0.748) | (0.729,0.856,0.946) |
| $C_{42}$ | (0.340,0.386,0.459) | (0.586,0.669,0.781) | (0.642,0.783,1.000) | (0.318,0.357,0.420) |
| $C_{43}$ | (0.227,0.309,0.391) | (0.736,0.878, 1.000) | (0.413,0.554,0.696) | (0.697,0.839,0.960) |

$$EQ_3 = \left(EQ_3{}^{(d)}, EQ_3{}^{(s)}, EQ_3{}^{(g)}\right) =$$

$$\begin{cases} EQ_3{}^{(d)} = \begin{pmatrix} 2.17 + 6.86 + 2.92 \\ + \cdots + 2.05 + 2.89 \end{pmatrix} \dfrac{\begin{pmatrix} (1+0.076)^{0.018} \cdot (1+0.108)^{0.0166} \cdot \\ (1+0.059)^{0.0199} \cdot \ldots \cdot (1+0.066)^{0.0194} \end{pmatrix} - \begin{pmatrix} (1-0.07)^{0.018} \cdot (1-0.113)^{0.0166} \cdot \\ (1-0.061)^{0.0199} \cdot \ldots \cdot (1-0.06)^{0.0194} \end{pmatrix}}{\begin{pmatrix} (1+0.07)^{0.018} \cdot (1+0.113)^{0.0166} \cdot \\ (1+0.061)^{0.0199} \cdot \ldots \cdot (1+0.06)^{0.0194} \end{pmatrix} + \begin{pmatrix} (1-0.07)^{0.018} \cdot (1-0.113)^{0.0166} \cdot \\ (1-0.061)^{0.0199} \cdot \ldots \cdot (1-0.06)^{0.0194} \end{pmatrix}} \\[3em] EQ_3{}^{(s)} = \begin{pmatrix} 3.16 + 7.86 + 3.77 \\ + \cdots + 2.62 + 3.88 \end{pmatrix} \dfrac{\begin{pmatrix} (1+0.13)^{0.074} \cdot (1+0.18)^{0.077} \cdot \\ (1+0.21)^{0.077} \cdot \ldots \cdot (1+0.05)^{0.068} \end{pmatrix} - \begin{pmatrix} (1-0.13)^{0.074} \cdot (1-0.18)^{0.077} \cdot \\ (1-0.21)^{0.077} \cdot \ldots \cdot (1-0.05)^{0.068} \end{pmatrix}}{\begin{pmatrix} (1+0.13)^{0.074} \cdot (1+0.18)^{0.077} \cdot \\ (1+0.21)^{0.077} \cdot \ldots \cdot (1+0.05)^{0.068} \end{pmatrix} + \begin{pmatrix} (1-0.13)^{0.074} \cdot (1-0.18)^{0.077} \cdot \\ (1-0.21)^{0.077} \cdot \ldots \cdot (1-0.05)^{0.068} \end{pmatrix}} \\[3em] EQ_3{}^{(g)} = \begin{pmatrix} 4.15 + 8.71 + 4.46 \\ + \cdots + 3.19 + 4.88 \end{pmatrix} \dfrac{\begin{pmatrix} (1+0.091)^{0.332} \cdot (1+0.098)^{0.339} \cdot \\ (1+0.063)^{0.341} \cdot \ldots \cdot (1+0.068)^{0.234} \end{pmatrix} - \begin{pmatrix} (1-0.091)^{0.332} \cdot (1-0.098)^{0.339} \cdot \\ (1-0.063)^{0.341} \cdot \ldots \cdot (1-0.068)^{0.234} \end{pmatrix}}{\begin{pmatrix} (1+0.091)^{0.332} \cdot (1+0.098)^{0.339} \cdot \\ (1+0.063)^{0.341} \cdot \ldots \cdot (1+0.068)^{0.234} \end{pmatrix} + \begin{pmatrix} (1-0.091)^{0.332} \cdot (1-0.098)^{0.339} \cdot \\ (1-0.063)^{0.341} \cdot \ldots \cdot (1-0.068)^{0.234} \end{pmatrix}} \end{cases}$$

$$= (0.534, 0.661, 0.738)$$

**Box I.**

**Table 6**
Fuzzy scale for evaluating criteria.

| Linguistic terms | Membership function |
|---|---|
| Very low (VL) | (1, 1, 2) |
| Low (L) | (1, 2, 3) |
| Medium (M) | (2, 3, 4) |
| High (H) | (3, 4, 5) |
| Very high (VH) | (4, 5, 5) |

$$\widetilde{\alpha}_i = \begin{matrix} G_1 \\ G_2 \\ G_3 \\ G_4 \end{matrix} \begin{bmatrix} (1.224, 2.000, 3.524) \\ (1.869, 3.346, 4.198) \\ (1.772, 3.100, 3.978) \\ (1.627, 2.870, 3.268) \end{bmatrix};$$

$$\widetilde{\beta}_i = \begin{matrix} G_1 \\ G_2 \\ G_3 \\ G_4 \end{matrix} \begin{bmatrix} (0.467, 0.589, 1.143) \\ (0.728, 1.000, 1.374) \\ (0.692, 0.927, 1.301) \\ (0.623, 0.839, 1.041) \end{bmatrix}$$

**Table 7**
Priority vectors.

| Clusters | $T_1$ | $T_2$ | $T_3$ | $T_4$ | $T_5$ | $T_6$ | $T_7$ |
|---|---|---|---|---|---|---|---|
| $C_1$: Economic aspect | VH | M | VL | H | L | L | VL |
| $C_2$: Health and Environmental aspect | M | L | VH | VL | M | VL | L |
| $C_3$: Social aspect | H | H | M | M | H | H | M |
| $C_4$: Transportation aspect | L | VL | H | L | H | M | VH |
| **$C_1$: Economic aspect** | | | | | | | |
| $C_{11}$ | L | VH | M | VL | L | H | H |
| $C_{12}$ | M | M | L | L | VH | M | L |
| $C_{13}$ | VL | L | H | M | H | VH | M |
| **$C_2$: Health and Environmental aspect** | | | | | | | |
| $C_{21}$ | H | M | L | M | VL | M | M |
| $C_{22}$ | M | H | M | H | M | H | L |
| $C_{23}$ | L | VL | L | L | L | VL | H |
| $C_{24}$ | VH | VH | H | VH | H | VH | VH |
| **$C_3$: Social aspect** | | | | | | | |
| $C_{31}$ | H | VH | M | VH | H | VH | M |
| $C_{32}$ | M | H | VH | M | M | H | VH |
| $C_{33}$ | L | L | VL | VL | L | L | L |
| **$C_4$: Transportation aspect** | | | | | | | |
| $C_{41}$ | VH | VH | VH | VH | VH | H | L |
| $C_{42}$ | L | L | H | L | M | L | M |
| $C_{43}$ | M | VL | VL | M | VL | M | VH |

**Table 8**
The vectors of the ratio.

| Clusters | $T_1$ | $T_2$ | $T_3$ | $T_4$ | ... | $T_7$ |
|---|---|---|---|---|---|---|
| $C_1$ | (6.67,10,12.5) | (3.33,6,10) | (1.67,2.0,5.0) | (5.0,8.0,12.5) | ... | (1.67,2.0,5.0) |
| $C_2$ | (3.33,6.0,10) | (1.67,4,7.5) | (6.67,10,12.5) | (1.67,2.0,5.0) | ... | (1.67,4.0,7.5) |
| $C_3$ | (5.0,8.0,12.5) | (5,8,12.5) | (3.33,6.0,10.0) | (3.33,6.0,10) | ... | (3.33,6.0,10) |
| $C_4$ | (1.67,4.0,7.5) | (1.67,2,5) | (5.0,8.0,12.5) | (1.67,4,7.5) | ... | (6.67,10,12.5) |
| $C_1$:Economic aspect | | | | | | |
| $C_{11}$ | (1.67,4.0,7.5) | (6.67,10,12.5) | (3.33,6,10) | (1.67,2.0,5.0) | ... | (5.0,8.0,12.5) |
| $C_{12}$ | (3.33,6.0,10) | (3.33,6.0,10) | (1.67,4.0,7.5) | (1.67,4.0,7.5) | ... | (1.67,4.0,7.5) |
| $C_{13}$ | (1.67,2.0,5.0) | (1.67,4.0,7.5) | (5.0,8.0,12.5.0) | (3.33,6.0,10) | ... | (3.33,6.0,10) |
| $C_2$:Health and Environmental aspect | | | | | | |
| $C_{21}$ | (5.0,8.0,12.5) | (3.33,6.0,10) | (1.67,4.0,7.5) | (3.33,6.0,10) | ... | (3.33,6.0,10) |
| $C_{22}$ | (3.33,6.0,10.0) | (5.0,8.0,12.5) | (3.33,6.0,10) | (5.0,8.0,12.5) | ... | (1.67,4.0,7.5) |
| $C_{23}$ | (1.67,4.0,7.5) | (1.67,2.0,5.0) | (1.67,4.0,7.5) | (1.67,4.0,7.5) | ... | (5.0,8.0,12.5) |
| $C_{24}$ | (6.67,10,12.5) | (6.67,10,12.5) | (5.0,8.0,12.5) | (6.67,10,12.5) | ... | (6.67,10,12.5) |
| $C_3$:Social aspect | | | | | | |
| $C_{31}$ | (5.0,8.0,12.5) | (6.67,10,12.5) | (3.33,6.0,10) | (6.67,10,12.5) | ... | (3.33,6.0,10) |
| $C_{32}$ | (3.33,6.0,10.0) | (5.0,8.0,12.5) | (6.67,10,12.5) | (3.33,6.0,10) | ... | (6.67,10,12.5) |
| $C_{33}$ | (1.67,4.0,7.5) | (1.67,4.0,7.5) | (1.67,2.0,5.0) | (1.67,2.0,5.0) | ... | (1.67,4.0,7.5) |
| $C_4$:Transportation aspect | | | | | | |
| $C_{41}$ | (6.67,10,12.5) | (6.67,10,12.5) | (6.67,10,12.5) | (6.67,10,12.5) | ... | (1.67,4.0,7.5) |
| $C_{42}$ | (1.67,4.0,7.5) | (1.67,4.0,7.5) | (5.0,8.0,12.5) | (1.67,4.0,7.5) | ... | (3.33,6.0,10) |
| $C_{43}$ | (3.33,6.0,10) | (1.67,2.0,5.0) | (1.67,2.0,5.0) | (3.33,6.0,10) | ... | (6.67,10,12.5) |

**Table 9**
Aggregated fuzzy vectors of weight coefficients of the criteria.

| Criteria | Local | Global |
|---|---|---|
| $C_1$:Economic aspect | (0.11, 0.228, 0.483) | – |
| $C_{11}$ | (0.165, 0.325, 0.689) | (0.018, 0.074, 0.332) |
| $C_{12}$ | (0.150, 0.338, 0.702) | (0.017, 0.077, 0.339) |
| $C_{13}$ | (0.181, 0.338, 0.705) | (0.020, 0.077, 0.341) |
| $C_2$:Health and Environmental aspect | (0.102, 0.216, 0.464) | – |
| $C_{21}$ | (0.117, 0.228, 0.456) | (0.012, 0.049, 0.211) |
| $C_{22}$ | (0.142, 0.268, 0.497) | (0.015, 0.058, 0.231) |
| $C_{23}$ | (0.074, 0.184, 0.418) | (0.008, 0.04, 0.194) |
| $C_{24}$ | (0.200, 0.321, 0.534) | (0.020, 0.069, 0.248) |
| $C_3$:Social aspect | (0.163, 0.301, 0.559) | – |
| $C_{31}$ | (0.238, 0.396, 0.679) | (0.039, 0.119, 0.380) |
| $C_{32}$ | (0.224, 0.383, 0.668) | (0.037, 0.115, 0.374) |
| $C_{33}$ | (0.075, 0.222, 0.524) | (0.012, 0.067, 0.293) |
| $C_4$:Transportation aspect | (0.128, 0.257, 0.506) | – |
| $C_{41}$ | (0.251, 0.422, 0.702) | (0.032, 0.108, 0.355) |
| $C_{42}$ | (0.131, 0.314, 0.617) | (0.017, 0.081, 0.312) |
| $C_{43}$ | (0.150, 0.266, 0.581) | (0.019, 0.068, 0.294) |

When calculating the compromise values of the Einstein function, expression (21), the value $\gamma = 0.5$ was adopted. This ensured equal intensity of the influence of both Einstein functions on the final decision.

*Step 6:* The ranking of alternatives was performed based on the value of the compromise index of alternatives ($\Upsilon_i$). The compromise index of alternatives is calculated based on the expression (22), where the value $\delta = 1$ and $\partial_1 = \partial_2 = \partial_3 = 0.33$.

$$
\begin{array}{cccc}
 & \widetilde{\Upsilon_i} & \Upsilon_i & Rank \\
G_1 & (1.039, 1.521, 2.776) & 0.815 & 4 \\
G_2 & (1.585, 2.544, 3.306) & 1.768 & 1 \\
G_3 & (1.503, 2.357, 3.133) & 1.543 & 2 \\
G_4 & (1.381, 2.183, 2.576) & 1.241 & 3
\end{array}
$$

Based on the results, we can define the initial ranking of alternatives according to the following $G_2 > G_3 > G_4 > G_1$.

### 5.2. Checking the stability of the results

The stability of the obtained results was performed through three phases. In the first phase, the analysis of the influence of the parameter $\gamma$ on the final ranking results was performed. In the second phase, the influence of changing the weights of the criterion weights on the ranking results was analyzed, while in the third phase, the results of the fuzzy Einstein CoCoSo model were compared with the results of the fuzzy CoCoSo model and the crips CoCoSo model.

*(a) The impact of parameter $\gamma$ on the ranking results*
The parameter $\gamma$ is used in expression (21) to determine the intensity of the influence of $EQ_i$ and $EP_i$ functions on the final decision. As previously emphasized, adopting values from the interval $0 \leq \gamma \leq 0.5$ increases the intensity of the influence of the $EP_i$ function in the aggregation strategy, while adopting values from the interval $0.5 \leq \gamma \leq 1$ increases the intensity of the influence of the $EQ_i$ function. When calculating the initial rank ($G_2 > G_3 > G_4 > G_1$), the experts decided that in the final
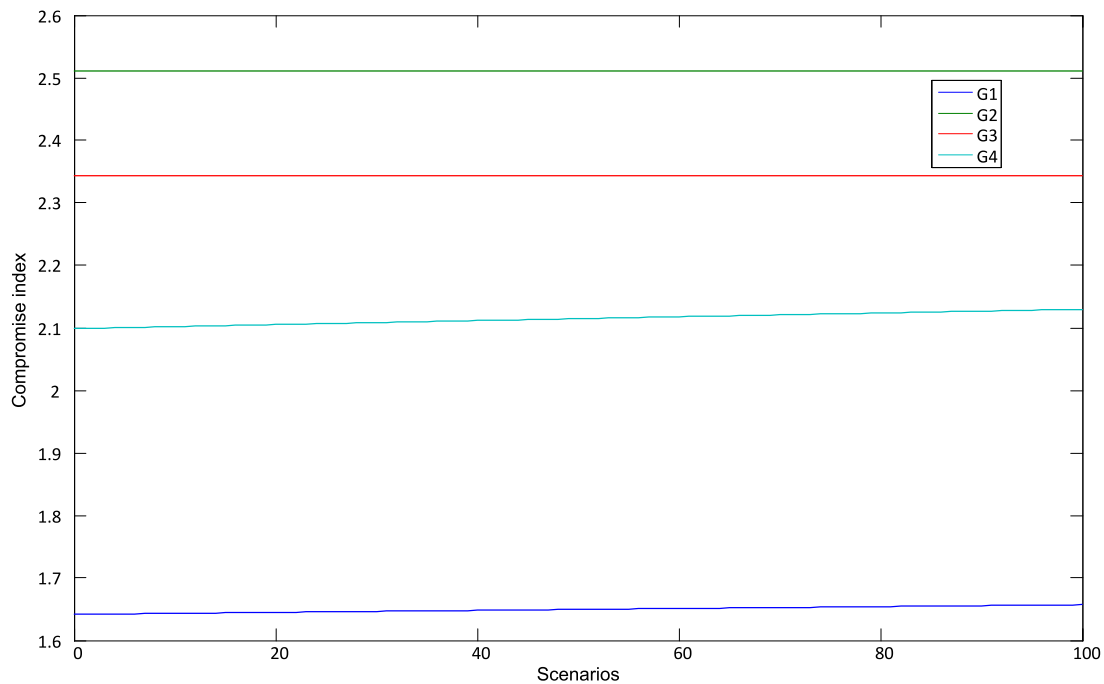
**Fig. 2.** The influence of the parameter $\gamma$ on the results.

decision, both functions have equal influence, so the value $\gamma = 0.5$ was adopted.

In the next part, 100 scenarios were formed in which the change of the parameter $\gamma$ was simulated. In the first scenario, the value of the parameter $\gamma = 0$ was adopted, which completely eliminated the influence of $EQ_i$; that is, the ranking was performed based on the $EP_i$ function. In each subsequent scenario, the value of the parameter $\gamma$ was increased by 0.01. Fig. 2 shows the influence of the parameter $\gamma$ on the change in the value of the compromise index of alternatives.

The simulation of the change of the parameter $\gamma$ in the interval $0 \leq \gamma \leq 1$ showed that the increase in the value of the parameter $\gamma$ affects the growth of Einstein functions ($EQ_i$ and $EP_i$). The extreme application of the influence of Einstein functions on the ranking results was simulated through the presented scenarios. Thus, in the first scenario ($\gamma = 0$), the impact of $EQ_i$ was eliminated, i.e. the $EP_i$ function was favored. On the other hand, in the last scenario ($\gamma = 1$), the impact of $EP_i$ was eliminated, i.e., the $EQ_i$ function was favored. In the remaining scenarios (S2–S99), the influence of $EQ_i$ was gradually increased while at the same time, the influence of $EP_i$ was proportionally reduced. Through this experiment, the influence of both functions on the ranking results was observed. From Fig. 2, we notice that the compromise values of the functions increase evenly with the change of the parameter $\gamma$. There is no extreme growth in trade-offs of alternatives that could cause changes in rankings. This leads us to conclude that there is a sufficient advantage between the compromise values of the alternatives, i.e., that the initial rank is credible regardless of the adopted value of the parameter $\gamma$.

*(b) Change of a vector of weight coefficients of criteria*

In multi-criteria models, decisions are often made in dynamic conditions during which there are significant changes in the model's input parameters. To simulate the dynamic environment, the change of the value of the fuzzy vector of the weight coefficients of the criteria is analyzed in the following part. Based on the recommendations of Kahraman [37], new weight vectors were generated, and their influence on changes in the rankings of alternatives was analyzed. New vectors of weight coefficients

were obtained on the basis of variation of the value of the most influential criterion ($C_{31}$).

The amount of change in the generated vector of weight coefficients was defined based on the methodology of Kahraman [37]. Since TFNs for uncertainty processing were used in this study, for all three segments of the triangular fuzzy number, the limits of change of the weight coefficient of the most influential criterion $C_{31}$ were defined, according to the following: *(1)* lower bound $-0.039 \leq \Delta w_s \leq 0.226$; *(2)* modal value $-0.119 \leq \Delta w_s \leq 0.883$ and (3) upper bound $-0.379 \leq \Delta w_s \leq 1.00$. Each of the defined intervals is divided into 27 sequences that represent scenarios. In each scenario, based on the obtained limit values and proportions for defining the relationship between the weights of the criteria [37], new vectors of weight coefficients were generated in Fig. 3.

After calculating the new vectors of weight coefficients, the influence of each of the generated vectors on the change of the compromise values of the alternatives was analyzed. The influence of new vectors of weight coefficients on the change of compromise values of alternatives is shown in Fig. 4.

New vectors of criterion weight coefficients lead to a change in the trade-offs of the alternatives. Such changes show a sufficient sensitivity of the fuzzy Einstein CoCoSo model to changes in the values of the weighting coefficients of the criteria, which is one of the significant characteristics of the MCDM model. The biggest changes occur in the third-ranked alternative ($G_4$), which in scenario S27 significantly approached the second-ranked alternative ($G_3$). In S27, the compromise significance of alternatives $G_4$ and $G_3$ is $\Upsilon_3 = 2.33$ and $\Upsilon_4 = 2.30$. Ranks of alternatives through scenarios are shown in Fig. 5.

The results from Figs. 4 and 5 confirm that the initial solution is credible since the new vectors of the weight coefficients of the criteria do not lead to changes in the ranks of the alternatives.

*5.3. Comparison of results with Fuzzy and crisp CoCoSo method*

As this is a new approach in the literature that analyzes the possibilities of applying Einstein norms and Einstein functions to improve the performance of the traditional CoCoSo method,
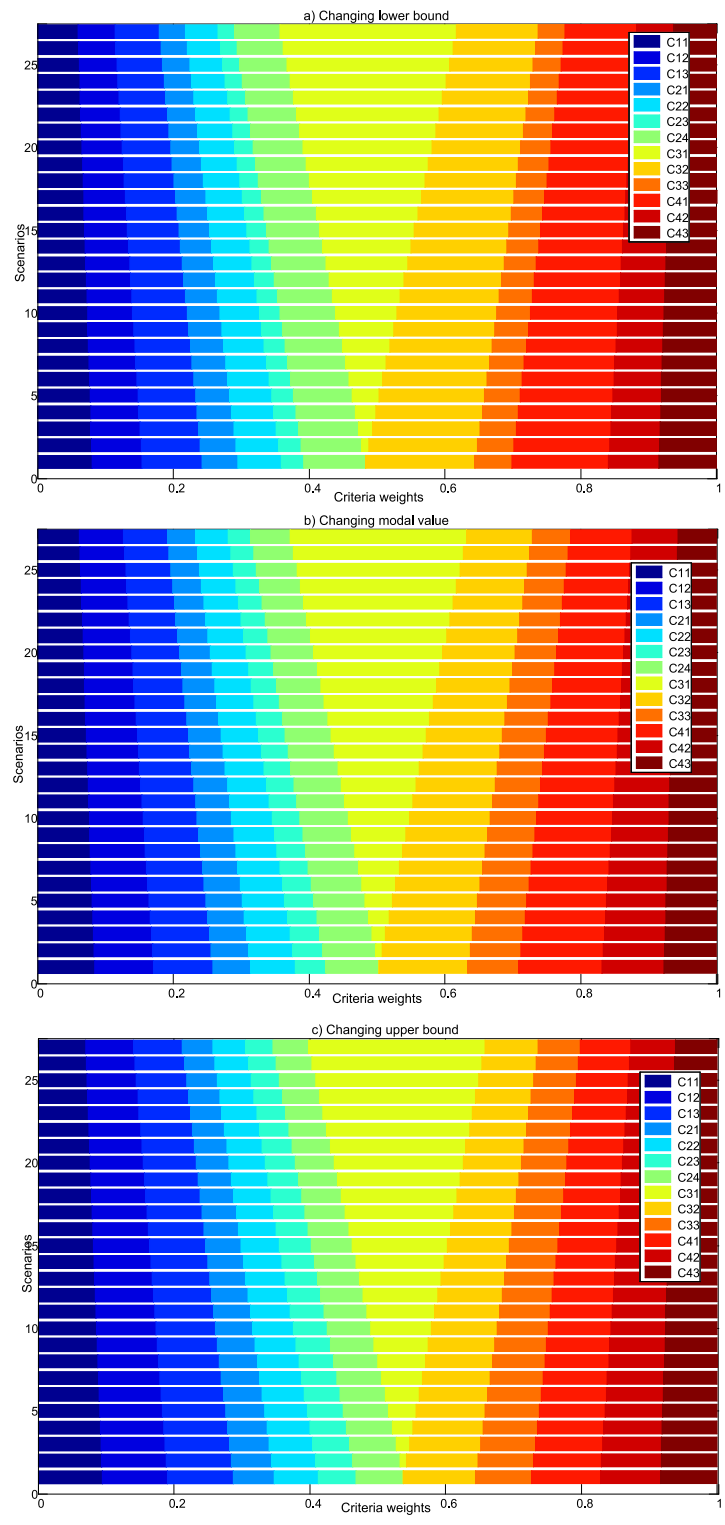
**Fig. 3.** New fuzzy vectors of criteria weights.

the following compares the results with fuzzy and crip Co-CoSo models. When comparing the results of the fuzzy Einstein CoCoSo method with crisp and fuzzy CoCoSo methods, the data from Table 4 were used. Since the crisp CoCoSo method [9] was used for comparison, the fuzzy values from Table 4 were transformed into crisp values using the expression $\theta_{ij}^{crisp} = \left( \theta_{ij}^{(d)} + 2\theta_{ij}^{(s)} + \theta_{ij}^{(g)} \right) \Big/ 4$, where $\widetilde{\theta}_{ij} = \left( \theta_{ij}^{(d)}, \theta_{ij}^{(s)}, \theta_{ij}^{(g)} \right)$. Using

fuzzy Einstein, fuzzy and crisp CoCoSo methods, the criterion functions were obtained, which are given in Table 10.

A graphical representation of the fuzzy Einstein CoCoSo model [38], fuzzy CoCoSo model, and crisp CoCoSo model alternatives is shown in Fig. 6.

The comparative presentation shows that all three applied methodologies suggest the same order of alternatives, i.e., $G_2 > G_3 > G_4 > G_1$. The results obtained using the fuzzy Einstein
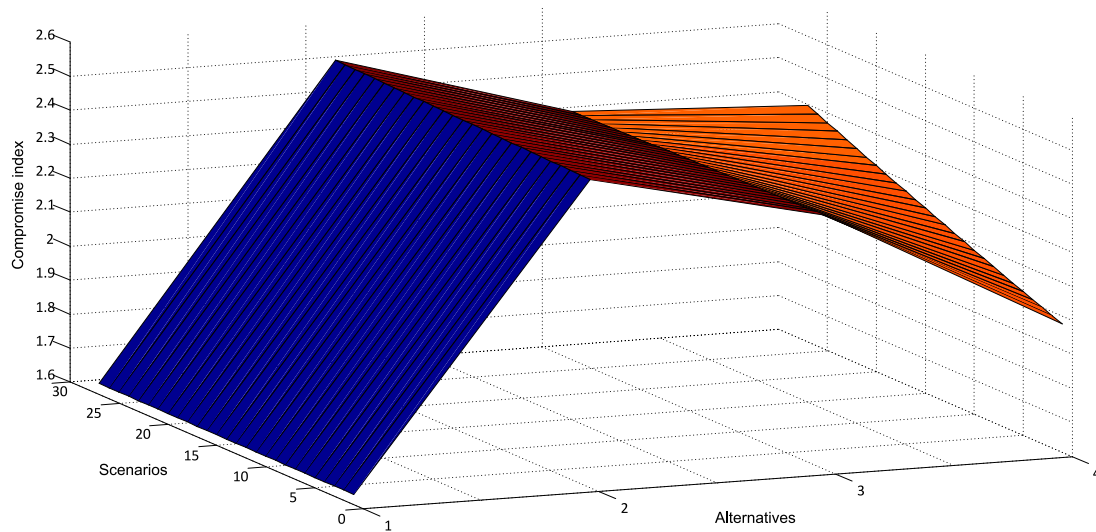
**Fig. 4.** Influence of new fuzzy vectors of criteria weights on the change of compromise significance of alternatives.
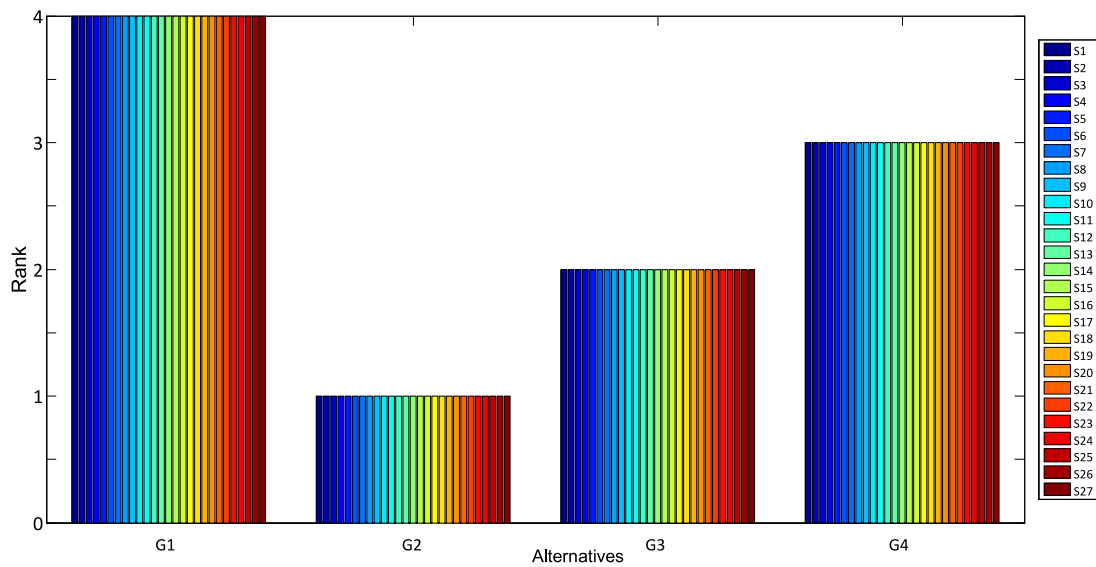


**Fig. 5.** Rank alternatives through 27 scenarios.

**Table 10**
Results of application of different CoCoSo techniques.

| Alt. | Fuzzy Einstein CoCoSo | | Fuzzy CoCoSo | | Crisp CoCoSo | |
|------|------------------------|------|---------------|------|---------------|------|
| | $\Upsilon(G_i)$ | Rank | $\Upsilon(G_i)$ | Rank | $\Upsilon(G_i)$ | Rank |
| G1 | 0.815145 | 4 | 0.731351 | 4 | 1.494571 | 4 |
| G2 | 1.767897 | 1 | 1.267929 | 1 | 2.687359 | 1 |
| G3 | 1.542645 | 2 | 1.166991 | 2 | 2.463667 | 2 |
| G4 | 1.241148 | 3 | 1.037057 | 3 | 2.181222 | 3 |

CoCoSo model and the fuzzy CoCoSo model have lower values of the compromise significance of the alternatives compared to the crisp CoCoSo model. Such changes are a consequence of fuzzy numbers in fuzzy Einstein CoCoSo and fuzzy CoCoSo models.

In addition to the Einstein functions in the fuzzy Einstein CoCoSo model, the original fuzzy methodology for determining the weight coefficients of the criteria was implemented. The proposed methodology belongs to the group of subjective models for determining the weight coefficients of the criteria and is based on defining the relationship between the criteria using a fuzzy logarithmic additive function. In this way, the performance of the CoCoSo methodology has been significantly improved, and the need to apply added subjective or objective models to determine the weight coefficients of the criteria has been eliminated. The characteristics of the fuzzy Einstein CoCoSo methodology compared to traditional fuzzy and crisp CoCoSo models are presented in Table 11.

Based on the presented comparisons between the proposed method and the two existing methods, the following can be pointed out:

(1) Based on the comparison with fuzzy and crip CoCoSo models, it can be concluded that: (*i*) the best alternative according to all methodologies is the same, i.e., alternative $G_2$ is the best alternative; (*ii*) The aggregation functions in the fuzzy and crip CoCoSo models are of a simple, linear character, while in the fuzzy Einstein CoCoSo model
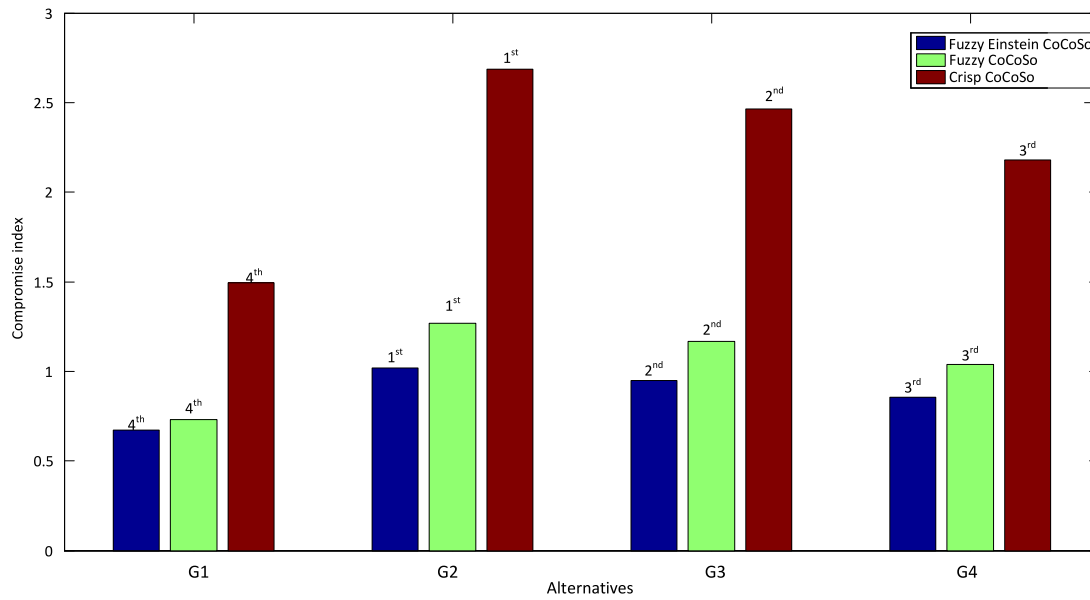
**Fig. 6.** Comparative presentation of the results of compared approaches.

**Table 11**
Characteristics of fuzzy Einstein CoCoSo methodology and traditional CoCoSo models.

| Characteristics | CoCoSo method | Fuzzy CoCoSo method | Fuzzy Einstein CoCoSo method |
|---|---|---|---|
| Flexibility in real applications | No | No | Yes |
| Determining weight coefficients of criteria | No | No | Yes |
| Characteristics of functions for defining weighted sequences | Linear form | Linear form | Nonlinear form |
| Processing of complex information | Partially | Partially | Yes |
| Defined rank alternative | Yes | Yes | Yes |
| Algorithm complexity | No | No | Partially |

nonlinear Einstein functions are used. That is why the fuzzy Einstein CoCoSo method presented in this paper has greater generality and flexibility. Because of this feature, the presented fuzzy Einstein CoCoSo method can be applied more widely.

(2) The ranking results are the same for all three methodologies used to solve the same illustrative example. However, when extreme values appear in the initial decision matrix at the most influential criteria, there are extreme changes in the values of linear functions in the fuzzy and crisp CoCoSo models. This can cause a disproportionate increase in the value of the trade-offs of the alternatives. This is a consequence of the linear character of the weighted sum and weighted product functions used in the traditional CoCoSo method. The application of Einstein functions eliminates this characteristic, which objectifies the decision-making process.

(3) The Fuzzy Einstein CoCoSo model enables decision-makers to determine weighting coefficients using a rational and straightforward methodology. Since there is no need to apply additional MCDM methods to determine the weights of the criteria, the fuzzy Einstein CoCoSo model is a powerful tool suitable for solving real-world decision-making problems.

## 6. Results and discussion

During a pandemic, public transportation demands go through a sharp decrease when the public transportation time schedules are kept the same. Hence, a good deal of public transportation trips has the potential to be conducted for very few people, which

decreases revenues, and the financial burden of operating the public transit vehicles increases to a great extent. As stated in a study, public transportation demands decreased notably after the COVID-19 outbreak, but demands are still concentrated at morning and evening peaks [5], which causes crowded public transit vehicles. However, crowded and closed areas carry a large risk of further increasing the spreading speed of a virus. Therefore, for all these reasons taken into consideration, the do-nothing alternative is chosen to be the least advantageous one.

Increasing the ridership in sharing-based modes such as bicycle sharing, scooter sharing, and car-sharing is the second least advantageous method. Micro-mobility systems, such as scooters and bicycle sharing, are good traveling modes for short distances, but very insufficient modes for long distances. Even though sharing-based modes are isolated traveling modes, there is still a need for frequent disinfection of the vehicles, since there is no way of knowing if the previous user was infected or not. Also, increasing the ridership of sharing-based modes decreases the demands for public transportation, which is an undesired aspect since the revenues of public transportation are very low. Hence, decreasing the demand further by promoting micro-mobility may damage the public transportation system seriously. Usage of micro-mobility traveling modes can stay as a habit for the passengers and public transportation demands may not reach back to the levels present before the pandemic. Operation of public transportation is carried out as a public service, but such decreases in demand may make the operational costs unmanageable and the public transportation system may come to a point of collapsing. This alternative is more advantageous than the do-nothing alternative because the health of the passengers is managed better.

The primary reason behind the capacity reduction is related to the application of social distancing in public transit vehicles. Sustaining the social distance in the vehicles is very beneficial to mitigate the speed of spreading of a virus and to provide a safer trip for the commuters. However, as the capacities are reduced, people might need to wait for long durations until they can get on a public transit vehicle, which increases the waiting times to a great extent. The increase in waiting times decreases the passenger satisfaction levels and hence the public acceptance of the management method. For example, at morning and evening peak hours, the frequencies of the trips must be increased to a great level to compensate for the high demand levels. Even if the trips are at the original levels considering the reduced capacities, revenues are decreased proportionally to the social distancing restriction, which increases the financial burden on the operators. Capacity reduction in public transportation alternatives provides safer transportation for passengers. It has the potential to mitigate the spreading speed of a virus and also promotes the use of public transit for people who are avoiding crowded areas. These have made this alternative the second most advantageous one.

The public transportation demands are still larger during morning and evening hours, which correspond to work hours as well, even during pandemic conditions. This makes adjusting the public transportation schedule based on the work hours alternative very beneficial to meet the high demands faced at these peaks. People who are traveling to work and people who are leaving their work can find a public transit vehicle to use easily. Hence, this is an aspect that increases commuters' satisfaction levels and public acceptance. Increasing the accessibility of public transportation services decreases the use of private vehicles, which eventually decreases traffic congestion and air pollution. Taking all these aspects into consideration, experts selected this alternative to be the most advantageous one.

## 7. Conclusion and future directions

This paper fills a gap concerning the selection of a public transportation management method in times of a pandemic. Considering the presented case study, adjusting public transportation schedules based on work hours is seen to be an effective public transportation management method. For different case scenarios, the results may vary, but the application of the methodology of this paper creates a consensus for municipalities to select the most optimum public transportation management method among various methods. One other contribution of this paper is that in the future, the world will most probably face new epidemics, and having such a multi-criteria decision-making algorithm to choose the best public transportation management method helps the municipalities in reacting to the situation in a faster and more ideal way.

The case study discussed in this paper has shown that the fuzzy Einstein CoCoSo framework is a powerful tool for rational and objective decision-making. Besides the obvious advantages of the presented methodology, there are certain limitations. One limitation of the fuzzy Einstein CoCoSo methodology is the inability to represent the mutual relations between the criteria. Therefore, it is necessary to direct future research toward implementing Einstein norms and Bonferroni functions in the CoCoSo methodology. The application of Einstein–Bonferroni functions would enable the presentation of interrelationships between the criteria and further improve the CoCoSo model's flexibility. Also, the evaluation of alternatives was performed, considering the opinions of experts. Therefore, the findings based on one case study cannot be generalized, but it is necessary to direct future research to provide quantitative information. In addition, further research should focus on improving the adaptability of the fuzzy Einstein CoCoSo method by implementing Heronian, Dombi, and Bonferroni functions. Also, an interesting direction for further research is the implementation of neutrosophic, rough, and gray sets in the Einstein CoCoSo methodology.

There are also limitations to the study. One limitation is that if a pandemic continues for a long duration, alternatives might change according to future conditions. Also, different cities have different cultures and habits, which may make other alternatives stand stronger against others. For example, if the population of a city is more suitable for a sharing system, the alternative of increasing the ridership in sharing-based modes such as bicycle sharing, scooter sharing, and car-sharing may move forward in the advantage prioritization. Therefore, assessments must be done according to the individual dynamics of the cities.

The Fuzzy Einstein CoCoSo methodology is an effective, rational, and robust decision-making tool. However, in addition to the obvious advantages, it is necessary to point out the potential limitations of the improved CoCoSo model, which include (i) A complex mathematical apparatus for calculating the relational relationships between criteria, (ii) the complexity of Einstein weighted nonlinear functions for budgeting maintenance strategies, and (iii) computational load and acquisition of the information in a fuzzy form. As this is a model that has an evident potential for rational processing of complex, uncertain, and group information, it is necessary to implement the fuzzy Einstein CoCoSo model in the decision support system. This eliminated the presented limitations, which would make the presented model acceptable for use by a larger number of users.

The fuzzy Einstein CoCoSo methodology has two limitations: (1) The first limitation is the impossibility of representing the mutual relations between the elements of the home matrix. Therefore, it is necessary to direct further research toward implementing Bonferroni and Heronian functions in the Einstein CoCoSo methodology. Applying Bonferroni and Heronian functions with Einstein norms would enable the presentation of interrelationships between criteria and would further enhance the flexibility of the Einstein CoCoSo model; (2) The second limitation of this study is the lack of quantitative attributes in the home matrix. Therefore, the findings based on one case study cannot be generalized, but it is necessary to develop future research to provide quantitative information. In the future, new modifications can be made to the management methods and criteria based on the prevailing conditions. With new virus threats and COVID-19 persisting for an extended period, a demand analysis for emerging alternative modes of transportation (such as e-scooters) can also be conducted. According to the management of alternative transportation methods, then, new alternatives may be developed. Then, new alternatives can be formed according to the management of the alternative transportation methods. Fuzzy sets such as Hesitant Fuzzy Linguistic Term Sets (HFLTS) can be used to handle uncertainties and hesitations in human linguistic assessments

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Appendix

**Proof for Theorem 1.**

Expression (8) is decomposed into segments in order to gradually derive expression (13).

From expressions (6) and (8) we get that:

$$\widetilde{\varpi}_j \cdot \widetilde{\zeta}_j = \left( \varpi_j^{(d)} \cdot \zeta_j^{(d)}, \varpi_j^{(s)} \cdot \zeta_j^{(s)}, \varpi_j^{(g)} \cdot \zeta_j^{(g)} \right)$$

$$\left( \begin{array}{c} \zeta_1^{(d)} \dfrac{\left(1 + f\left(\zeta_{ij}^{(d)}\right)\right)^{\varpi_j^{(d)}} - \left(1 - f\left(\zeta_{ij}^{(d)}\right)\right)^{\varpi_j^{(d)}}}{\left(1 + f\left(\zeta_{ij}^{(d)}\right)\right)^{\varpi_j^{(d)}} + \left(1 - f\left(\zeta_{ij}^{(d)}\right)\right)^{\varpi_j^{(d)}}}, \\[2em] \zeta_1^{(s)} \dfrac{\left(1 + f\left(\zeta_{ij}^{(s)}\right)\right)^{\varpi_j^{(s)}} - \left(1 - f\left(\zeta_{ij}^{(s)}\right)\right)^{\varpi_j^{(s)}}}{\left(1 + f\left(\zeta_{ij}^{(s)}\right)\right)^{\varpi_j^{(s)}} + \left(1 - f\left(\zeta_{ij}^{(s)}\right)\right)^{\varpi_j^{(s)}}}, \\[2em] \zeta_1^{(g)} \dfrac{\left(1 + f\left(\zeta_{ij}^{(g)}\right)\right)^{\varpi_j^{(g)}} - \left(1 - f\left(\zeta_{ij}^{(g)}\right)\right)^{\varpi_j^{(g)}}}{\left(1 + f\left(\zeta_{ij}^{(g)}\right)\right)^{\varpi_j^{(g)}} + \left(1 - f\left(\zeta_{ij}^{(g)}\right)\right)^{\varpi_j^{(g)}}} \end{array} \right) \qquad \text{(A.1)}$$

Then, by applying expression (8) we obtain the fuzzy Einstein weighted averaging function (13):

$$EQ_i = \left(EQ_i^{(d)}, EQ_i^{(s)}, EQ_i^{(g)}\right) == \sum_{j=1}^{k} \widetilde{\varpi}_j \cdot \widetilde{\zeta}_j$$

$$= \left( \begin{array}{c} \sum_{j=1}^{k} \left(\zeta_{ij}^{(d)}\right) \dfrac{\prod_{j=1}^{k}\left(1+f\left(\zeta_{ij}^{(d)}\right)\right)^{\varpi_j^{(d)}} - \prod_{j=1}^{k}\left(1-f\left(\zeta_{ij}^{(d)}\right)\right)^{\varpi_j^{(d)}}}{\prod_{j=1}^{k}\left(1+f\left(\zeta_{ij}^{(d)}\right)\right)^{\varpi_j^{(d)}} + \prod_{j=1}^{k}\left(1-f\left(\zeta_{ij}^{(d)}\right)\right)^{\varpi_j^{(d)}}}, \\[2em] \sum_{j=1}^{k} \left(\zeta_{ij}^{(s)}\right) \dfrac{\prod_{j=1}^{k}\left(1+f\left(\zeta_{ij}^{(s)}\right)\right)^{\varpi_j^{(s)}} - \prod_{j=1}^{k}\left(1-f\left(\zeta_{ij}^{(s)}\right)\right)^{\varpi_j^{(s)}}}{\prod_{j=1}^{k}\left(1+f\left(\zeta_{ij}^{(s)}\right)\right)^{\varpi_j^{(s)}} + \prod_{j=1}^{k}\left(1-f\left(\zeta_{ij}^{(s)}\right)\right)^{\varpi_j^{(s)}}}, \\[2em] \sum_{j=1}^{k} \left(\zeta_{ij}^{(g)}\right) \dfrac{\prod_{j=1}^{k}\left(1+f\left(\zeta_{ij}^{(g)}\right)\right)^{\varpi_j^{(g)}} - \prod_{j=1}^{k}\left(1-f\left(\zeta_{ij}^{(g)}\right)\right)^{\varpi_j^{(g)}}}{\prod_{j=1}^{k}\left(1+f\left(\zeta_{ij}^{(g)}\right)\right)^{\varpi_j^{(g)}} + \prod_{j=1}^{k}\left(1-f\left(\zeta_{ij}^{(g)}\right)\right)^{\varpi_j^{(g)}}} \end{array} \right) \qquad \text{(A.2)}$$

where $\widetilde{\varpi}_j = (\varpi_j^{(d)}, \varpi_j^{(s)}, \varpi_j^{(g)})$ fuzzy is the vector of the weight coefficients of the criterion, while $f\left(\widetilde{\zeta}_j\right) = \left(\zeta_j^{(d)} \middle/ \sum_{j=1}^{k} \zeta_j^{(d)}, \zeta_j^{(s)} \middle/ \sum_{j=1}^{k} \zeta_j^{(s)}, \zeta_j^{(g)} \middle/ \sum_{j=1}^{k} \zeta_j^{(g)}\right)$.

**Proof for Theorem 2.** Expression (9) is decomposed into segments in order to gradually derive expression (14).

From expressions (7) and (9) we get that:

$$\left(\widetilde{\zeta}_j\right)^{\widetilde{\varpi}_j} = \left( \left(\zeta_j^{(d)}\right)^{\varpi_j^{(d)}}, \left(\zeta_j^{(s)}\right)^{\varpi_j^{(s)}}, \left(\zeta_j^{(s)}\right)^{\varpi_j^{(g)}} \right)$$

$$= \left( \begin{array}{c} \zeta_1^{(d)} \dfrac{2f\left(\zeta_1^{(d)}\right)^{\varpi_j^{(d)}}}{\left(2 - f\left(\zeta_1^{(d)}\right)\right)^{\varpi_j^{(d)}} + f\left(\zeta_1^{(d)}\right)^{\varpi_j^{(d)}}}, \\[2em] \zeta_1^{(s)} \dfrac{2f\left(\zeta_1^{(s)}\right)^{\varpi_j^{(s)}}}{\left(2 - f\left(\zeta_1^{(s)}\right)\right)^{\varpi_j^{(s)}} + f\left(\zeta_1^{(s)}\right)^{\varpi_j^{(s)}}}, \\[2em] \zeta_1^{(g)} \dfrac{2f\left(\zeta_1^{(g)}\right)^{\varpi_j^{(g)}}}{\left(2 - f\left(\zeta_1^{(g)}\right)\right)^{\varpi_j^{(g)}} + f\left(\zeta_1^{(g)}\right)^{\varpi_j^{(g)}}} \end{array} \right) \qquad \text{(A.3)}$$

Then, by applying expression (9) we obtain the fuzzy Einstein weighted geometric averaging function (14):

$$EP_i = \left(EP_i^{(d)}, EP_i^{(s)}, EP_i^{(g)}\right) = \prod_{j=1}^{n} \left(\widetilde{\zeta}_j\right)^{\widetilde{\varpi}_j}$$

$$= \left( \begin{array}{c} \sum_{j=1}^{k} \left(\zeta_{ij}^{(d)}\right) \dfrac{2\prod_{j=1}^{k}\left(f\left(\zeta_{ij}^{(d)}\right)\right)^{\varpi_j^{(d)}}}{\prod_{j=1}^{k}\left(2 - f\left(\zeta_{ij}^{(d)}\right)\right)^{\varpi_j^{(d)}} + \prod_{j=1}^{k}\left(f\left(\zeta_{ij}^{(d)}\right)\right)^{\varpi_j^{(d)}}}, \\[2em] \sum_{j=1}^{k} \left(\zeta_{ij}^{(s)}\right) \dfrac{2\prod_{j=1}^{k}\left(f\left(\zeta_{ij}^{(s)}\right)\right)^{\varpi_j^{(s)}}}{\prod_{j=1}^{k}\left(2 - f\left(\zeta_{ij}^{(s)}\right)\right)^{\varpi_j^{(s)}} + \prod_{j=1}^{k}\left(f\left(\zeta_{ij}^{(s)}\right)\right)^{\varpi_j^{(s)}}}, \\[2em] \sum_{j=1}^{k} \left(\zeta_{ij}^{(g)}\right) \dfrac{2\prod_{j=1}^{k}\left(f\left(\zeta_{ij}^{(g)}\right)\right)^{\varpi_j^{(g)}}}{\prod_{j=1}^{k}\left(2 - f\left(\zeta_{ij}^{(g)}\right)\right)^{\varpi_j^{(g)}} + \prod_{j=1}^{k}\left(f\left(\zeta_{ij}^{(g)}\right)\right)^{\varpi_j^{(g)}}} \end{array} \right)$$
$$\text{(A.4)}$$

where $\widetilde{\varpi}_j = (\varpi_j^{(d)}, \varpi_j^{(s)}, \varpi_j^{(g)})$ fuzzy is the vector of the weight coefficients of the criterion, while $f\left(\widetilde{\zeta}_j\right) = \left(\zeta_j^{(d)} \middle/ \sum_{j=1}^{k} \zeta_j^{(d)}, \zeta_j^{(s)} \middle/ \sum_{j=1}^{k} \zeta_j^{(s)}, \zeta_j^{(g)} \middle/ \sum_{j=1}^{k} \zeta_j^{(g)}\right)$.

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