Advancements in Mode-Locked Fibre Lasers and Fibre Supercontinua

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In memory of Catherine Kelleher (1951 – 2009)  
who died during the writing of this thesis
For Soomi.
Abstract

The temporal characteristics and the spectral content of light can be manipulated and modified by harnessing linear and nonlinear interactions with a dielectric medium. Optical fibres provide an environment in which the tight confinement of light over long distances allows the efficient exploitation of weak nonlinear effects. This has facilitated the rapid development of high-power fibre laser sources across a broad spectrum of wavelengths, with a diverse range of temporal formats, that have established a position of dominance in the global laser market. However, demand for increasingly flexible light sources is driving research towards novel technologies and an improved understanding of the physical mechanisms and limitations of existing approaches.

This thesis reports a series of experiments exploring two topical areas of ongoing research in the field of nonlinear fibre optics: mode-locked fibre lasers and fibre-based supercontinuum light sources.

Firstly, integration of novel nano-materials with existing and emerging fibre-based gain media allows the demonstration of ultrafast mode-locked laser sources across the near-infrared in a conceptually simple, robust, and compact scheme. Extension to important regions of the visible is demonstrated using nonlinear conversion.

Scaling of pulse energies in mode-locked lasers can be achieved by operating with purely positive dispersion for the generation of chirped pulses. It is shown unequivocally, through a direct measurement, that the pulses generated in ultra-long mode-locked lasers can exist as highly-chirped dissipative soliton solutions of the cubic (and cubic-quintic) Ginzburg Landau equation. The development of a numerical model provides a framework for the interpretation of experimental observations and exposes unique evolution dynamics in extreme parameter ranges. However, the practical limitations of the approach are revealed and alternative routes towards achieving higher-energy are proposed.

Finally, an experimental and numerical study of the dependence of continuous-wave pumped supercontinua on the coherence properties of the pump source shows an optimum exists that can be expressed as a function of the modulation instability period. A new and simplified model representing the temporal fluctuations expressed by continuous-wave lasers is proposed for use in simulations of supercontinua evolving from noise.

The implications of the experiments described in this thesis are summarised within the broader context of a continued research effort.
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1 Introduction

Advances in fibre lasers are a direct result of numerous developments in the field of photonics over the last fifty years: in particular, the addition of rare-earth ions into silicate glass fibre hosts [Koe64]; the fabrication of low-loss single transverse mode optical fibres [Kap70]; the suggestion and demonstration of dispersion engineered and double-clad fibres [Coh79, Coh82, Sni88, Kni96]; and the realisation of high-power, highly efficient integrated pump diodes [Gap90] have contributed significantly. In parallel, progress has been made in advancing understanding of light’s interaction with matter, thus facilitating benefits from steps forward in fibre technology. The increasing importance of laser applications in academic, commercial, industrial and medical areas requires continued progress in high-performance photonic devices. This thesis is concerned with attempts to further advance two areas of topical fibre optic research: mode-locked lasers and supercontinuum generation.

A basic scheme for enhancing the peak power of a laser system is to operate in a pulsed rather than continuous-wave (CW) mode. By coupling the phases of the longitudinal cavity modes, a pulse can be formed simply using passive optical elements. Recent interest in materials that exhibit a broadband saturable component of absorption have been investigated as saturable absorbers in mode-locked fibre lasers, thus promoting the development of ultrashort pulse sources at new wavelengths. Obviously, this requires optical media that provides amplification in new wavelength regions. Gain can be achieved using either actively doped fibres or exploiting nonlinear processes in passive fibres.

Due to the tight confinement of the optical mode of a fibre, high optical intensities can be sustained over long interaction lengths (while retaining a compact system footprint), thus weak nonlinear interactions can be observed (and exploited) at relatively modest power levels. The strength and nature of the nonlinearity is mediated by the sign and magnitude of the linear dispersion. By managing both the dispersive and nonlinear properties, a large degree of control can be exercised over the light which propagates within the fibre, providing a route to temporal and spectral control for the development of flexible light sources.
1 Introduction

1.1 Overview

This thesis is organised as follows. In chapter 2 novel nano materials that exhibit a saturable component of absorption are utilised to mode-lock fibre lasers across the near infrared, and the linear and nonlinear response of the materials are characterised. Chapter 3 focuses attention towards novel gain media for ultrafast lasers, as well as exploiting existing technologies in new configurations. The dynamics of mode-locked lasers are studied numerically in chapter 4. In addition, experimental results of solitary wave formation in a gain-guided laser, subject to normal dispersion are presented. Ultra-long mode-locked lasers are studied experimentally and numerically in chapter 5. A new regime of pulse parameters is identified, and the numerical model developed in chapter 4 is used to interpret experimental observations. In chapter 6 attention is turned towards the spectral rather than temporal manipulation of light in optical fibres, with experiments demonstrating how pump source coherence affects the generation of optical solitons from a continuous wave, through modulation instability. Thus, an optimum condition is defined for efficient generation of continuous-wave pumped supercontinua. Finally, chapter 7 concludes the thesis, with a discussion of the broader implications of the research and a summary of how the ideas presented can be developed as part of continued research effort.

The rest of this short introductory chapter is intended to provide a broad overview of background material, in particular regarding the basic theory of light propagation in dielectric fibre waveguides, that will be further developed in the relevant chapters.

1.2 Linear and nonlinear propagation in fibres

Using Snell’s law it is understood how a light ray can be contained in a dielectric medium such as glass. This provides an initial picture of how light energy can be transferred through an optical fibre. A brief review of electromagnetic theory leads to an expression for the electric field, which provides a more complex introduction to fibre modes. This is followed by further discussion of conventional and speciality optical fibres; looking in particular at their light guiding properties. This provides a basis upon which to consider both linear and nonlinear mechanisms that affect the propagation of light in fibres.

1.2.1 Conventional optical fibres

In 1890 Tyndall demonstrated that light could be guided in a water jet through total internal reflection [Hec02]. However, it was not until the 1960s that the idea of a communication system based on the propagation of light within dielectric waveguides was seriously
1.2 Linear and nonlinear propagation in fibres

considered [Wil98]. The optical fibre, essentially an optical waveguide: a structure that can efficiently guide light, has subsequently attracted much attention.

Conventional optical fibres rely on total internal reflection to guide light. In its simplest form a conventional optical fibre consists of a central dielectric core, with refractive index \( n_{\text{core}} \), surrounded by a dielectric cladding layer of a lower refractive index, \( n_{\text{clad}} \) (see Fig. 1.1). This structure is fundamental to the light guiding properties. Without turning to Maxwell’s equations, it is straightforward to convince ourselves using ray theory, Snell’s law and Fresnel’s equations that light reaching the core-cladding interface will be totally internally reflected (and hence energy transferred down the fibre) when the angle between the direction of propagation and the core-cladding interface is smaller than some critical value.

**Total internal reflection**

Snell’s law describes the angles of incidence \( \theta_i \) and reflection \( \theta_r \) of light rays passing through a boundary between two isotropic media [Wil98]:

\[
\begin{align*}
\theta_i &= \theta_r \\
\frac{\sin \theta_i}{\sin \theta_t} &= \frac{n_2}{n_1}
\end{align*}
\]

where \( \theta_i \) is the angle of the transmitted wave and \( n_1 \) and \( n_2 \) are the refractive indices of the two media. In the specific case of fibres we can consider \( n_1 = n_{\text{core}} \) and \( n_2 = n_{\text{clad}} \). Fresnel’s equations describe the magnitudes of the transmitted and reflected electric fields. Increasing \( \theta_i \) increases the amplitude of the reflected component of the field and thus decreases the transmitted component. Moreover, \( \theta_i \) (the angle made with the normal to the surface) increases. As \( \theta_i \) increases the transmitted ray gradually approaches...
tangency with the boundary [Hec02]. When $\theta_i = 90^\circ$, from Equation 1.2

$$\sin \theta_i = \theta_{\text{crit}} = \frac{n_t}{n_i}$$

(1.3)

where $n_i$ and $n_t$ are the refractive indices of the adjoined media. Beyond this critical angle all of the available energy from the incident ray appears in the reflected beam, this phenomenon is called *total internal reflection*.

**Electromagnetic waves**

From Maxwell’s equations it can be shown that the net electric field $E$ is a solution of the wave equation [Agr07]:

$$\nabla^2 E - \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} E = \mu_0 \frac{\partial^2}{\partial t^2} P$$

(1.4)

where $P$ is the induced polarisation, $\mu_0$ is the permeability of free space and $\epsilon_0$ is the permittivity of free space. The relation between $E$ and $P$ is through the electric susceptibility function $\chi_L$; in linear, isotropic media [Buc04]

$$P = \epsilon_0 \chi_L E$$

(1.5)

A more general expression says that the polarisation is a convolution of the electric field in time

$$P(t) = \epsilon_0 \int_{-\infty}^{t} \chi_L(t-\tau)E(\tau) d\tau$$

(1.6)

For a linear system it is more convenient to express Equation 1.6 in the frequency domain, as the convolution integral simply becomes a product

$$P(\omega) = \epsilon_0 \chi_L(\omega)E(\omega)$$

(1.7)

Explicitly in Equation 1.7 we can see that $\chi_L$ is a function of frequency. In linear media $\chi_L(\omega)$ is independent of field strength and can be expressed in terms of a relative permittivity $\epsilon_\tau(\omega) = 1 + \chi_L(\omega)$. The refractive index of a material is given by

$$n(\omega) = \sqrt{\epsilon_\tau(\omega)} = \sqrt{1 + \chi_L(\omega)}$$

(1.8)

In optics the variation of $n$ with frequency gives rise to chromatic dispersion. The case when the electric susceptibility, and thus the refractive index is not independent of field strength, and the interesting effects that arise as a result, will be considered below.
1.2 Linear and nonlinear propagation in fibres

Guidance mechanism

The basic structure of a conventional step index fibre has been considered and it has been shown that the index contrast between the core and cladding leads to total internal reflection at the core-cladding interface. Based on an understanding of electromagnetic waves, considering the propagation vector of the electric field in the fibre provides a fuller appreciation of the dynamics.

Given the axial invariance of optical fibre, the electric field for a given frequency can be written as

\[ E(r, z, t) = E_m(r) \exp\left[ i(\omega t - \beta z) \right] \] (1.9)

where \( E_m \) is the modal field distribution, \( z \) is the axial distance through the fibre, \( \omega \) is the angular frequency, \( r \) is the transverse position vector and \( \beta \) is the axial component of the wavevector, known as the propagation constant. The propagation constant is invariant, regardless of transverse and axial position and the magnitude of the propagation vector is constant in any material at \( k_0 n \), where \( n \) is the local refractive index. Consequently, the magnitude of the transverse propagation constant has been fully constrained such that

\[ k_t = \left( k_0^2 n^2 - \beta^2 \right)^{\frac{1}{2}} \] (1.10)

From Equation 1.10 it is clear to see that if the following condition is satisfied

\[ n_{\text{clad}} k_0 \leq \beta \leq n_{\text{core}} k_0 \] (1.11)

\( k_t \) is real in the core and imaginary in the cladding, resulting in a guided field confined to the core. From equation 1.9 it can be seen that the variation in field along \( z \) is given by the phase factor \( \exp(i \beta z) \). To fully satisfy modal confinement the field at \( z = z_1 \) and \( z = z_2 \), two arbitrary points on axis, must only differ by \( \exp(i \beta (z_2 - z_1)) \).

The value of \( \beta \) together with its corresponding field distribution constitute a mode of the fibre and are eigenvalues and eigenvectors respectively of the fibre propagation equation [Sny83, Yar07]. A finite number of modes satisfy Equation 1.10 and these are classified as guided modes. For a conventional step index fibre, the number of supported modes is expressed by the \( V \)-parameter

\[ V = k_0 a \left( n_{\text{core}}^2 - n_{\text{clad}}^2 \right)^{\frac{1}{2}} \] (1.12)

where \( a \) is the radius of the core. The number of guided modes a fibre can support scales with \( V \). For \( V < 2.405 \) the fibre supports only the fundamental mode. Single mode fibres
have a small core size to wavelength ratio and a small index difference between the core and cladding. However, to obtain strong waveguide dispersion, the refractive index contrast should be high; because of this conflict it is not possible to produce conventional step index fibres that are both anomalously dispersive and single mode at wavelengths shorter than 1.27 \( \mu \text{m} \) - the zero dispersion wavelength of silica glass. Significant effort has been directed towards overcoming this constraint. Perhaps the most influential and recent innovation is the photonic crystal fibre (PCFs), where the structural flexibility allows a large degree of control over the waveguide contribution to the overall dispersion. Although PCFs do not feature explicitly in the following experimental chapters, they are an important technology in the field of nonlinear fibres optics because of their uniquely controllable dispersive and nonlinear properties, and are widely used in supercontinuum generation (discussed in chapter 6) and optical pulse compression (considered in chapter 5), and thus are worthy of at least brief discussion.

1.2.2 Photonic crystal fibre

In photonic crystals light can be totally suppressed at certain wavelengths, regardless of propagation direction and polarisation. This so-called photonic bandgap arises because of the periodicity of the dielectric. This section briefly reviews the unique light guiding properties that make these periodic structures exciting optical devices.

Guidance mechanism

Solid core PCFs guide light due to modified total internal reflection: the effective refractive index of the cladding is lower than that of the core and is determined by the band structure of the surrounding holes [Zol05]. The band gap effect originates from the periodicity of the cladding region, preventing transmission of frequencies which destructively interfere due to multiple reflections from the multiple air-silica interfaces.

In all PCFs the photonic bandgap effect is used to achieve specific values of the propagation constant, \( \beta \) that prevent real transverse components existing in the cladding region. All values of the propagation constant \( \beta \) greater than a specific value \( \beta_{\text{max}} \) cannot propagate in the cladding. This is analogous to conventional step index fibres in which light with \( \beta > n_{\text{clad}} k_0 \) cannot propagate in the cladding, where \( n_{\text{clad}} \) is the refractive index of the cladding and \( k_0 \) is the free-space wavevector.
1.2 Linear and nonlinear propagation in fibres

1.2.3 Pulse propagation in fibres

The standard equation used to describe the propagation of pulses in optical fibres is the nonlinear Schrödinger equation (NLSE) [Mol06]

\[ i \frac{\partial u}{\partial z} + \frac{1}{2} \frac{\partial^2 u}{\partial t^2} \pm |u|^2 u = 0 \] (1.13)

Equation 1.13 is derived from Maxwell’s equations and adapted to field propagation in single mode optical fibre. This environment provides spatial constraint in the transverse dimensions \(x\) and \(y\), this simplifies the description of the field as appropriate averages can be applied over these dimensions. Consequently, the NLSE equation involves only distance along the propagation direction \(z\) in time \(t\). The absolute magnitude of \(u(z, t)\) represents the amplitude envelope of the propagating pulse. With \(u(z, t)\) being a complex quantity, it also contains phase information that is important in determining the way a pulse propagates. The term containing the second derivative with respect to time describes the effects of chromatic dispersion, this issue will be discussed in greater detail in section 1.2.5. It should be noted that this term does not affect the frequency spectrum of the pulse, it merely acts to broaden (or narrow) it in the time domain. The third term on the left is the nonlinear term, effectively the product of the pulse’s intensity envelope with \(u\), that acts to broaden it spectrally. It will be shown in section 1.2.7 that this is due to the intensity dependent refractive index. Equation 1.13 has a special solution [Mol06]

\[ u(z, t) = \text{sech}(t) \exp(i z/2) \] (1.14)

known as the fundamental soliton. Optical solitons will be considered further in section 1.2.7.

1.2.4 Losses in optical fibres

Fused silica glass transmits electromagnetic radiation over a wide range of wavelengths from the ultra-violet, with an absorption edge at about 0.2 \(\mu\)m, to the deep infra-red absorption edge at about 2.5 \(\mu\)m [Gha98]. Within this transmission window optical loss is limited by Rayleigh scattering, water absorption loss related to the vibrational modes of Si-OH bonds at 1.38 \(\mu\)m and waveguide imperfections.

In standard single mode optical fibres this loss has reached a physical limit due to Rayleigh scattering, with very low loss (less than 0.4 dB km\(^{-1}\)) expected over the range 1.2 to 1.7 \(\mu\)m. The telecoms industry is dictated by a minimum loss (less than 0.2 dB km\(^{-1}\)) in such fibres at 1.55 \(\mu\)m.

It is perhaps not surprising that photonic crystal fibres exhibit higher losses than con-
1 Introduction

Conventional silica fibre because of imperfections at the air-silica interfaces and fluctuations in the hole size. The contribution from water absorption loss at 1.38 \( \mu \)m is particularly significant, leading to losses at this wavelength in excess of 100 dB km\(^{-1}\). Confinement loss at long wavelengths, where the field effectively becomes too large for the waveguide, can also contribute to increased optical loss in PCFs.

1.2.5 Dispersion

In optics chromatic dispersion describes the phase velocity dependence on frequency (or wavelength), this is implicit through the relation

\[ v_p = \frac{\omega}{k} = \frac{c}{n(\omega)} \quad (1.15) \]

where \( v_p \) is the phase velocity of the propagating wave and \( c \) is the speed of light in a vacuum. In single mode optical fibres there are two sources of dispersion. Intrinsic material dispersion that occurs because of a frequency dependent refractive index and waveguide dispersion due to the fact that the field of a given mode propagates in both the core and cladding region - seeing an index contrast. These two dispersion contributions will be dealt with independently before considering their cumulative effect on the overall dispersion profile.

Material dispersion

The bound electrons of a dielectric medium oscillate with different amplitudes depending on the frequency of the incident electromagnetic wave. For a collection of \( N \) atoms with a resonant frequency \( \omega_0 \), driven at some frequency \( \omega \) by an electric field of constant amplitude \( E_0 \), the refractive index is given by

\[
n = \left(1 + \chi_L \right)^{\frac{1}{2}} = \left(1 + \frac{Ne^2}{m\epsilon_0(\omega^2 - \omega_0^2 + i\gamma\omega)} \right)^{\frac{1}{2}} \quad (1.16)\]

where \( e \) is the electronic charge, \( m \) is the electron mass and \( \gamma \) is some damping coefficient. This single resonance condition is a simple approximation and the refractive index of a dielectric medium is better described by the Sellmeier equation [Yar07]

\[
n^2(\omega) = 1 + \frac{Ne^2}{m\epsilon_0} \sum_{j} \frac{f_j}{\omega_j^2 - \omega^2 + i\gamma\omega} \quad (1.17)\]

where \( f_j \) are known as oscillator strengths: the strength of oscillations at the \( j_{th} \) resonant frequency. Equation 1.17 describes explicitly the refractive index dependence on
A theoretical treatment of waveguide optics is based on Maxwell’s equations. In section 1.2.1 a plane wave solution to the wave equation describing the field distribution of a monochromatic wave with a definite frequency and wavevector was considered. However, the finite duration of an optical pulse results in a finite spread of frequencies. Conveniently, because of the linear nature of Maxwell’s equations the propagation of an optical pulse in a linear medium can be described in terms of an appropriate superposition of plane waves with different frequencies [Yar07]. In a linear medium the induced polarisation is proportional to the electric field (Equation 1.5). If the material is dispersive, the phase velocity of the propagating plane wave is a function of frequency. Physically this means different frequency components propagate at different speeds. This can result in the distortion of an optical pulse with finite bandwidth.

This situation can be described by considering the summation of many monochromatic plane wave components, polarised in the transverse direction. Each plane wave is a solution to Maxwell’s equations. If \( A(k) \) is the wave amplitude then the electric field of the pulse is given by [Yar07]

\[
E(z, t) = \int_{-\infty}^{\infty} A(k) \exp \left( i \left( \omega(k) t - k z \right) \right) dk
\]

\( \beta \) was previously defined as the \( z \) component of the wavevector, known as the mode propagation constant. \( \beta \) is related to the effective mode index by

\[
\beta = n_{\text{eff}} \frac{\omega}{c} = \frac{2\pi n_{\text{eff}}}{c}
\]

The effects of dispersion on optical pulses (such as 1.18) can be expressed by a Taylor series expansion of the mode propagation constant about a central frequency \( \omega_0 \) [Agr07]

\[
\beta(\omega) = \beta_0 + \beta_1(\omega - \omega_0) + \frac{1}{2} \beta_2(\omega - \omega_0)^2 + \frac{1}{6} \beta_3(\omega - \omega_0)^3 + \ldots
\]

where

\[
\beta_m = \left( \frac{d^m \beta}{d\omega^m} \right)_{\omega = \omega_0} \quad (m = 0, 1, 2, \ldots)
\]

The first order parameter \( \beta_1 \) is related to the group velocity \( v_g \) by

\[
v_g = \frac{1}{\beta_1}
\]

This describes the overall velocity of the pulse envelope. The second order term gives rise to the group velocity dispersion (GVD), responsible for pulse broadening. It is common
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to characterise the GVD using the so-called dispersion parameter $D$, which has units of ps nm$^{-1}$ km$^{-1}$ and is related to $\beta_2$ by [Mol06]

$$D = - \frac{2\pi c}{\lambda^2} \frac{d^2 \beta}{d\omega^2} = - \frac{2\pi c}{\lambda^2} \beta_2 = \frac{\lambda}{c} \frac{d^2}{d\lambda^2} \left( n_{\text{eff}}^{\text{WG}} + n_{\text{eff}}^{\text{Mat}} \right)$$

(1.23)

where $n_{\text{eff}}^{\text{WG}}$ and $n_{\text{eff}}^{\text{Mat}}$ are the waveguide and material contributions to the total refractive index.

**Waveguide dispersion**

Waveguide dispersion is related to the geometry of the optical fibre and occurs because the fundamental mode of a single-mode optical fibre propagates in both the core and cladding regions where it sees an index difference. This results in an effective mode index. The mode field distribution is strongly frequency (wavelength) dependent. And thus the mode index varies as a function of wavelength. The propagation constant of a fibre mode varies from values close to the cladding wavevector at low frequencies (long wavelengths) to values close to the wavevector of the core at high frequencies (short wavelengths). This variation in $\beta$ as a function of frequency leads to non-zero derivatives.

Waveguide dispersion effects can be tailored to counterbalance material dispersion to achieve nearly zero chromatic dispersion anywhere within the 1.3-1.6 $\mu$m low-loss spectral region [Coh85].

**Total dispersion**

The dispersion of a dielectric waveguide mode is a function of the material and of the waveguide properties. If the refractive index difference between the core and cladding is small these effects can be considered in isolation [Glo71]. To a first approximation the total dispersion is the sum of the two contributions:

$$D = D_m + D_w$$

(1.24)

where $D_m$ is the material dispersion and $D_w$ is solely the waveguide dispersion. It has been shown that the assumption of additivity of material and waveguide dispersion is not quite correct [Mar79]. In standard fibre, because of the small contribution from waveguide dispersion to the total dispersion of the $LP_{01}$ mode, even a large percentage error in the waveguide dispersion has little influence. However, what makes PCF so interesting is the potential to exploit the large waveguide contribution, providing control of the dispersion profile.

Higher order dispersion terms can be important particularly around $\lambda_{ZD}$, the zero-
1.2 Linear and nonlinear propagation in fibres

dispersion wavelength, where the GVD is small or negligible ($\beta_2 \approx 0$). In this case of an optical pulse propagating close to the point of zero GVD, the third order dispersion term $\beta_3$ becomes the dominant effect, leading to asymmetric broadening and the development of temporal oscillatory wings.

1.2.6 Birefringence

In an optically anisotropic material, propagating waves of different polarisations will exhibit different phase velocities [Kam81]. This property is known as birefringence. Ideally, optical fibres are non-birefringent: fibres with cylindrical symmetry can support two degenerate modes that are polarised in two orthogonal directions. However, there is inevitably some residual mechanical stress as a result of the drawing process that randomly changes the core shape, breaking the cylindrical symmetry, making the fibre slightly birefringent. This marginal birefringent property of single mode optical fibre leads to a gradual and uncontrollable change in the polarisation of the propagating field and polarisation mode dispersion (PMD) [Shi85]. The magnitude of the modal birefringence is given by [Agr07]

$$B_m = \left| \frac{\beta_x - \beta_y}{k_0} \right| = \left| n_{\text{eff}_x} - n_{\text{eff}_y} \right|$$

(1.25)

where $n_{\text{eff}_x}$ and $n_{\text{eff}_y}$ are the effective modal refractive indices of the respective orthogonal polarisation states. The *beat length* is defined as the period in which the two modes experience complete coupling and is a function of wavelength:

$$L_B = \frac{2\pi}{\left| \frac{\beta_x - \beta_y}{k_0} \right|} = \frac{\lambda}{B_m}$$

(1.26)

The larger of the two mode indices means a smaller group velocity on axis and hence is termed the *slow axis*. For the same reason, the *fast axis* has a smaller effective mode index and larger group velocity.

In continuous wave systems this evolving polarisation is harmless [Agr07], however in pulsed applications it is often desirable for fibres to transmit light without changing the polarisation state. In such *polarisation-maintaining* (PM) fibres a large amount of birefringence is intentionally introduced into the fibre through modifications to the structure so that small fluctuations are inconsequential.

1.2.7 Nonlinear effects in optical fibres

The response of any dielectric to electromagnetic radiation becomes nonlinear for intense fields [Agr07]. Under such circumstances the motion of bound electrons becomes
anharmonic and the total polarisation $\mathbf{P}$ induced by electric dipoles is not linear in the electric field. The relationship between the field and the induced polarisation is given by the more general relation [Yar07]

$$\mathbf{P}_i = \varepsilon_0 \chi_{ij} E_j + 2d_{ijk} E_j E_k + 4\chi_{ijkl} E_j E_k E_l + ...$$ (1.27)

where $\mathbf{P}_i$ is the $i$th component of the instantaneous polarisation and $E_i$ is the $i$th component of the instantaneous electric field of optical beams $(j, k, l)$. $\chi_{ij}$ is the linear susceptibility, while $d_{ijk}$ and $\chi_{ijkl}$ are the second-order and third-order nonlinear susceptibilities, respectively.

The second-order nonlinear susceptibility, $d_{ijk}$ is responsible for second-harmonic generation and sum-frequency mixing. The symmetric nature of the SiO$_2$ molecule means that second-order nonlinear effects are not normally observed in silica glass, however, defects or colour centres inside the fibre core can contribute to second-harmonic generation, in certain cases [Agr07].

The lowest-order nonlinear effects observed in optical fibres are governed by the third-order susceptibility $\chi_{ijkl}$.

Nonlinear refraction

The efficiency of parametric processes such as third-harmonic generation and four-wave mixing are strongly dependent on the phase matching condition. As a result, most nonlinear effects observed in optical fibres originate from nonlinear refraction: the intensity dependence of the refractive index

$$n(\omega, |E|^2) = n(\omega) + n_2 |E|^2$$ (1.28)

where $n(\omega)$ is the linear refractive index given by the Sellmeier function (Equation 1.17), $|E|^2$ is the intensity and $n_2$ is the nonlinear-index coefficient, a function of the third-order susceptibility $\chi_{ijkl}$ [Agr07].

Self-phase and cross-phase modulation

Self-phase modulation (SPM) describes the self-induced phase shift of an optical pulse as it propagates in optical fibre. The phase difference of an optical field is related to the fibre length by

$$\Delta \phi = \left[n(\omega) + n_2 |E|^2\right] k_0 L$$ (1.29)
where $L$ is the fibre length (all other parameters have been previously defined). From Equation 1.29 we can see that because of the time dependent intensity of an optical pulse, there is a corresponding modulation to the local refractive index and this in turn causes a time dependent phase delay to the same pulse (SPM) or a co-propagating pulse (XPM).

Clearly linear and nonlinear processes do not act in isolation, ones dominance over the other depends largely on the width and peak power of the pulse entering the fibre. It is the interplay between dispersion and nonlinearity that leads to interesting optical effects in fibres. There are two definite regimes of operation that have distinctly different behaviour: the normal dispersion regime where $\beta_2 > 0$, and therefore the dispersion parameter $D < 0$; and the anomalous dispersion regime where $\beta_2 < 0$, and $D > 0$. The interaction between SPM and normal dispersion results in the temporal and spectral broadening of a propagating pulse. In the anomalous dispersion regime the resulting dynamics from the cooperation of SPM and dispersion leads to the generation of optical solitons.

**Stimulated Raman scattering**

The nonlinear effects arising from the third-order nonlinear susceptibility $\chi_{ijkl}$ discussed thus far have been elastic in nature: no energy is exchanged between the electromagnetic field and the dielectric medium. Raman scattering is an inelastic scattering process of photons from vibrational and rotational modes of silica. This process can be viewed as the annihilation of a pump photon leading to the creation of a frequency downshifted photon (the Stokes wave) and a phonon at the appropriate energy and momentum to comply with the conservation of energy. The process can be stimulated through the coherent coupling of the pump and Stokes wave, further enhancing molecular excitation. An anti-Stokes signal can be generated but is much weaker than the Stokes wave because the molecules must be in an initial state of excitation. The amorphous nature of fused silica means molecular vibrational frequencies spread out into bands that overlap leading to a continuum. The Raman gain curve is broad (over 40 THz wide) with a peak at 13 THz downshift from the pump.

**Four wave mixing, modulation instability and optical solitons**

Four wave mixing (FWM) and modulation instability (MI) are important parametric processes and principal dynamics in the evolution of supercontinua (depending on the pump conditions). FWM describes any third-order nonlinear process in which the interaction of three fields leads to the generation of a fourth [Buc04]. It can generate new frequency components in spectral regions with no previous spectral overlap with the pump pulse. The process can be understood as a coupling between four waves, through the real part
of $\chi_{ijkl}$. Due to energy conservation the frequencies of the four waves are constrained to the condition

$$\omega_1 + \omega_2 = \omega_3 + \omega_4$$  \hspace{1cm} (1.30)

Similarly, conservation of momentum requires the phase mismatch between the pump waves to be zero:

$$\Delta k = \beta(\omega_1) + \beta(\omega_2) - \beta(\omega_3) - \beta(\omega_4) + 2\gamma P = 0$$  \hspace{1cm} (1.31)

where the last term, $2\gamma P$ ($P$ being the pump power), accounts for the nonlinear phase shift due to the intensity dependent refractive index. It is common to assume $\omega_1 = \omega_2$ (a degenerate pump) and expand $\Delta k$ using a Taylor series expansion around the common pump frequency, so the lowest order dispersion terms can be observed.

MI is a feature of propagation in any nonlinear, anomalously dispersive medium. The NLSE can be considered to describe a localised potential, originating from the Kerr effect term [Mol06]. Solitons form as a result of being trapped in this potential. As the potential becomes stronger, in proportion to the square of the field amplitude, the noise fluctuations on continuous wave radiation can become self-trapping and evolve towards the fundamental soliton condition. Ultimately, a train of ultra-short pulses, with peak powers orders of magnitude greater than the pump power, are generated. In CW supercontinuum generation, Raman self-scattering of such solitons, generated from break-down of the pump field through MI, is a primary mechanism leading to extreme spectral expansion.
2 Ultrafast fibre laser technology
part 1: novel saturable absorber devices

It is unequivocal that the development of ultrafast lasers has had major scientific impact, recognised by the award of two Nobel prizes for ultrafast science: the time-resolved observation of transition states of molecules [Zew96]; and the generation of frequency combs for metrology [Jon00]. The Kerr-lens mode-locked Ti:Sapphire laser, first demonstrated in 1991 [Spe91], has proven to be the work-horse in experiments requiring a source of femtosecond pulses in research labs the world over. However, compact and reliable systems, requiring little to no technical knowledge of optics, delivering short pulses are more appropriate for applications outside the research domain. Optical fibres provide a convenient platform for the monolithic integration and miniaturisation of lasers producing a turn-key source of short, intense pulses of light.

This chapter describes the development of ultrafast fibre lasers using novel saturable absorber devices, in particular emphasis focuses on the use of single-wall Carbon nanotubes, in systems based on established rare-earth amplifier technology: namely double-clad Ytterbium- (Yb) and Erbium-doped (Er) fibres, utilising widely available, high-power integrated multi-mode pump diodes. Section 2.1 provides a succinct history of significant developments in ultrafast fibre laser technology over the last 40 years, with specific attention paid to advances in passively mode-locked schemes. The basic theories of active and passive mode-locking are briefly reviewed for completeness in section 2.2, although passively mode-locked fibre lasers will be the sole experimental focus throughout. In section 2.3 single-wall nanotubes are introduced and their optical properties characterised; experimental results demonstrating their use as a saturable absorber in fibre lasers are presented. Section 2.4, concludes this chapter with a discussion of other novel technologies that can be used to mode-lock fibre lasers, with multi-wall Carbon nanotubes and graphene having received significant attention, perhaps because of the recent award of a Nobel prize in a third related area of physics – for experimental studies regarding graphene [Nov04]. Results of mode-locking ultrafast lasers incorporating these new nano materials are presented.
Results outlined in this chapter have been published in the following journal articles and conference proceedings [Tra09b, Tra10b, Tra11a, Zha11b, Sun11, Sun10b, Has11].

2.1 Introduction

A laser can be described as ultrafast if it emits pulses with a temporal duration less than, or of the order of several tens of picoseconds. This is by no means a strict or rigid definition; from text to text (and within specific communities) the term will be used with a degree of flexibility: pulses maybe as long as a nanosecond. However, here I refer to ultrafast systems as mode-locked oscillators producing pulses with durations less than 100 ps, to distinguish them from chirped pulse oscillators, generating pulses on the nanosecond-scale, and described in detail in chapter 5.

2.1.1 A brief history

The development of ultrafast optics was stimulated by the proposal of laser mode-locking [DiD64, Har64, Yar65]: the simultaneous oscillation of a vast number of highly coherent, phase-locked longitudinal modes of a laser cavity, periodically emitting intense pulses containing the entire energy of the light field, resulting from short-lived, constructive interference between oscillating waves. Over the last 45 years, since the first demonstration of passive mode-locking of a laser (together with a Q-switched output, also known as Q-switched mode-locking) [Moc65], the field of ultrafast science has diverged greatly. The laser development can be loosely differentiated into two broad categories: bulk and fibre-based systems, depending on the nature of the gain medium used to promote laser action. For a thorough review of the evolution of state-of-art ultrafast bulk laser systems I refer to [Bra00, Kel03]. In what proceeds is a brief history of fibre-based mode-locked lasers.

Reference is made to mode-locking of a fibre laser in a passive context in experiments reported as early as 1983 [Dzh83], with the generation of giant pulses from a partially mode-locked Neodymium-doped glass fibre laser. However, demonstration of a fully passively mode-locked fibre laser, utilising a nonlinear amplifying loop mirror (NALM), and generating pulses with picosecond durations, was not realised until 1990 [Dul90, Dul91]. A number of active-schemes based on amplitude and phase modulation (both optical and electrical) pre-dated this first passive report [Alc86, Dul88, Kaf89], but were limited in pulse duration by the switching ability of the electronic modulator or the optically injected pump pulse. The report by Duling [Dul90, Dul91], where the emitted pulses were 2 ps in duration, was 18 years after Ippen and co-workers reported the first continuous-wave (CW) mode-locked operation of a dye-laser [Ipp72], utilising a saturable dye cell,
generating pulses as short as 1.5 ps. What is striking is despite similar performance in terms of pulse duration, furious research into fibre-based systems continued because of marked advantages of fibre technology [Dig01].

Although the technique of using a NALM in figure-eight lasers provided a means of generating short pulses, the pulse trains were typically unstable: not emitting pulses fixed at the fundamental repetition frequency of the cavity [Wu93]; or possessing a low-power continuous background component. Other interferometric techniques based on the fibre nonlinearity (namely the optical Kerr effect – or the intensity dependence of the refractive index) were proposed, yielding similarly quasi-continuous pulse trains ∼50 ms in duration [Wig90].

In addition to figure-eight lasers, utilising both amplifying and non-amplifying loop mirrors (NALM/NOLM), and additive pulse schemes proposed by Blow and Wood [Blo88] and reviewed in Ref. [Hau00], other techniques were developed to provide passive intensity discrimination to promote the phase-locking of longitudinal cavity modes. Nonlinear polarization rotation- (NPR or nonlinear polarization evolution- (NPE)) based ultrafast fibre lasers were simultaneously demonstrated by a number of groups in 1992 [Mat90, Mat92b, Tam92, Nos92a]. NPE had been used extra-cavity to clean the pulses emitted from early mode-locked oscillators. While a regular train of subpicosecond pulses can be reliably generated from NPE lasers, the scheme precludes the sole use of polarization maintaining fibre for a linearly polarized output. In addition, due to thermal and stress induced birefringence, environmental stability for long-term mode-locking in harsh conditions is compromised. Despite this NPE remains a favourable method for the passive mode-locking of fibre lasers, particularly where high-intracavity powers demand robustness (high average-power damage thresholds) from constituent system components [Wis08]. Prior to the availability of high-power (and highly efficient) single-stripe pump diodes, and subsequently high-power multi-mode diodes to pump double-clad fibre geometries [Fer96a], Ti:Sapphire lasers were widely used to provide sufficiently high-power optical excitation of the rare-earth fibres used in the early mode-locked fibre laser examples based on NALM/NOLM-type techniques, among others [Fer94, Fer97].

In 1991 Zirngibl et al. demonstrated transform-limited, picosecond pulse operation of an erbium-doped fibre laser passively mode-locked by an InGaAs/GaAs-on-GaAs superlattice which exhibited a fast saturable absorption [Zir91]. The system was optically pumped with a low-power 1480 nm diode delivering just 28 mW of pump power, sufficient to achieve self-starting mode-locked operation. This first demonstration of a passively mode-locked fibre laser using a semiconductor saturable absorber (SESA) paved the way for compact and efficient, low-power ultrafast fibre lasers. After over 20 years of intensive research, accelerated by momentum provided by the telecommunication
boom in the 1990s, ultrafast fibre technology can be delivered in a small package to a broad and growing applications base [Fer09]. Now a commercial success, innovative advances in ultrafast lasers are still being pursued [Web03].

This short history has tracked the development of, and highlighted the major advances in mode-locking techniques. Little has been said about the nature and effect of the lasing medium or the duration, energy, and evolution dynamics of the pulses generated. These aspects will be discussed in detail in chapters 3 and 4.

### 2.2 The basics of mode-locking

The requirement that an electromagnetic field be unchanged after one round-trip of a resonant cavity imposes the modal condition that supported frequencies correspond to a wavelength that is an integer multiple of the cavity length: \( v_j = j c / 2 n L \), where \( j \) is an integer that indexes the modes, \( c \) is the vacuum speed of light, \( n \) is the average refractive index of the cavity and \( L \) is the resonator length. If multiple longitudinal modes of the cavity oscillate or lase simultaneously, a short-pulse can be established if a fixed phase relationship exists between the resonant modes. This process of phase-locking, leading to the formation of pulse train is illustrated in Fig. 2.1.

While mode-locking refers to a coupling between the longitudinal modes of a resonant cavity, it is important to differentiate between a number of distinct pulsating regimes: namely continuous-wave (CW); and Q-switched mode-locking (QML). CW mode-locking refers to the generation of a periodic pulse train of equal pulse energy (or small fluctuations of the pulse-to-pulse energy arising from noise considerations). This regime is achieved through applying a loss modulation to the cavity using a saturable absorber (SA), or a mechanism which mimics the action of a saturable absorber. The inclusion of this element introduces a Q-switching tendency, driving the laser into the Q-switched mode-locking regime, where the pulse-to-pulse energy undergoes a large modulation at a frequency defined by the lifetime of the active gain element – in the case of rare-earth doped fibre lasers, the active ion lifetime is typically of the order of 1 ms corresponding to a modulation \( \sim 1 \) kHz. The relatively long upper state lifetime allows a large inversion to be achieved, making rare-earth-doped fibres attractive low-noise amplifiers, but the high-energy storage makes them susceptible to Q-switching instabilities. Operating a laser in the Q-switched mode-locked regime provides a means of increasing the energy in bunches of emitted pulses, but is often undesirable when a stable peak-power and energy is required at a high-repetition rate (>10 MHz). Kärtner et al., subsequently augmented by Höninger et al., developed a theory describing the stability limits of passive CW mode-locking in solid-state bulk lasers [Kae95, Hon99], deriving a simple criterion
2.2 The basics of mode-locking

\[ I(t) = |A(t)|^2, \]

where \( A \) is the electric field amplitude, for one, two and three modes. 2.1b Temporal intensity for one thousand modes with a fixed and random phase. A pulse train (or mode-locked state) is established under a fixed phase relationship; a quasi continuous-wave signal is observed for random phasing between modes. It is clear that in the mode-locking regime a significant enhancement to the peak power can be achieved. The marginal intensity noise on the pulse train is a result of aliasing – an effect that can also occur in experimental measurements, with diagnostics of limited resolution.

Figure 2.1: Illustration of the formation of a pulse train due to constructive interference between longitudinal modes. 2.1a the temporal intensities, \( I(t) = |A(t)|^2 \), where \( A \) is the electric field amplitude, for one, two and three modes. 2.1b Temporal intensity for one thousand modes with a fixed and random phase. A pulse train (or mode-locked state) is established under a fixed phase relationship; a quasi continuous-wave signal is observed for random phasing between modes. It is clear that in the mode-locking regime a significant enhancement to the peak power can be achieved. The marginal intensity noise on the pulse train is a result of aliasing – an effect that can also occur in experimental measurements, with diagnostics of limited resolution.
for the threshold level for the transition from QML to CW mode-locking:

\[ E_p^2 > E_{sat,g}E_{sat,a}\Delta R \]  
(2.1)

where \( E_p \) is the pulse energy, \( E_{sat,g} \) is the saturation energy of the gain medium, \( E_{sat,a} \) is the saturation energy of the absorber, and \( \Delta R \) is its modulation depth. While this expression needs to be modified to include soliton effects and gain filtering (which appear to reduce the critical energy for CW mode-locking, increasing the stability against QML), and was not explicitly derived for fibre-based systems – where the parameters differ considerably – it does provide a qualitative understanding of the dynamic in terms of experimentally accessible parameters. Reduction of the QML threshold can be achieved by careful design of the saturable absorber device, where, from Equation 2.1 it is clear the saturation energy\(^1\) (or fluence – the energy per unit area) and modulation depth play a vital role. However, stable CW mode-locked operation is always a careful balance of many coupled effects: for example naively reducing the modulation depth of the saturable absorber will reduce the lasers tendency towards QML, but will also seriously reduce or even prohibit self-starting operation, and even lead to increased pulse durations. In fibre lasers, where the gain efficiency of the active fibre is high and nonlinear and dispersive effects are strong, a large modulation depth (typically >10\%) is required. This immediately constrains the materials available for use as saturable absorbers in fibre systems. While semiconductor structures can be grown with multiple quantum well devices to increase the modulation depth to the necessary level, the added complexity puts a mechanical strain on the structure and can lead to limited lifetime and absorber damage. The development of ultrafast lasers based on alternative materials with saturable absorbing properties is outlined in this chapter. All systems discussed focus on achieving a stable CW mode-locked state.

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\(^1\)Saturation energy is typically defined as the amount of pulse energy required to reduce the gain (for an amplifying gain medium) or loss (for a saturable absorber) by \(1/e\) of its initial value. However, a more intuitive definition says that the saturation energy is that which supports equal population \( N \) of the lower and upper energy levels, such that

\[ \frac{N}{2} F_{sat} (\sigma_{em} + \sigma_{abs}) = \frac{N}{2} h\nu \]

\[ F_{sat} = E_{sat} A = \frac{h\nu}{(\sigma_{em} + \sigma_{abs})} \]

where \( N \) is the fractional level population, \( F \) is the fluence, \( \sigma_{em} \) and \( \sigma_{abs} \) are the emission and absorption cross-section, \( h \) is Planck’s constant, and \( \nu \) is the photon frequency.
2.2 The basics of mode-locking

2.2.1 Active mode-locking

Actively mode-locking lasers will not be discussed in this chapter beyond a short description of the basic mechanism, given here for completeness.

Phenomenologically, active and passive mode-locking can be described in the same way: an applied periodic loss or phase-change modulation to a resonant cavity exhibiting gain, where the frequency of the modulation $\Omega_m$ is coincident with the frequency separation of axial modes of the resonator, and given by

$$\Omega_m = \frac{4\pi}{T_R} = \frac{2\pi c}{n_{\text{eff}} L}$$

(2.2)

when neglecting high-order transverse modes – a valid assumption in a single-mode fibre waveguide – and assuming a travelling wave ring resonator; where $T_R$ is the cavity round-trip time, $c$ is the vacuum speed of light, $n_{\text{eff}}$ is the effective refractive index and $L$ is the resonator length. Viewed in the frequency domain, the application of a sinusoidal modulation to the central mode $\omega_0$ produces sidebands at $\omega_0 \pm \Omega_m$ that injection lock adjacent modes, which in turn lock their neighbours until the entire gain bandwidth (or all modes above threshold) are phase-locked [Hau00]. The frequency of the modulation and the bandwidth of the gain directly determine the duration of the pulse, and are related by the Kuizenga-Siegman formula [Sie70]

$$\tau^4 = \frac{2g_0}{M\Omega_m^2 \Omega_g^2}$$

(2.3)

where $g_0$ is the saturated single-pass gain, $M$ is the modulation parameter and $\Omega_g$ is the gain bandwidth.

The two main methods of active mode-locking are amplitude and phase modulation. Acousto-optic and electro-optic modulators have been widely used because of their ease of integration with a fully fibreised format. The frequency of modulation has to be synchronous to the round-trip time of the resonant cavity for stable operation, this presents an added degree of complexity, with additional feedback electronics often used to adaptively correct for frequency mismatch. Pulse durations from actively mode-locked systems are naturally going to be limited by the switching ability of the drive electronics, which puts a lower bound on achievable pulse durations. Although subpicosecond pulses have been demonstrated with careful control of soliton pulse shaping effects [Jon96], susceptibility to soliton instability and pulse timing because of Gordon-Haus jitter can be a problem.
2.2.2 Passive mode-locking

The shortest events known to man have been generated using passively mode-locked lasers. Two major categories of passive mode-locking exist: *pseudo* saturable absorber techniques depend on the intrinsic nonlinear index or nonlinear birefringent properties of the fibre; and *real* saturable absorber schemes, where passive modulation of the cavity is applied through the nonlinear absorptive properties of a material medium: typically semiconductors; organic dyes; or more recently Carbon-based nanomaterials. The intrinsic response time of these materials dictates the ultimate speed of its switching potential and the duration of pulses emitted from the mode-locked system, although additional pulse-shaping can result in pulses with durations shorter than the recovery time of the absorber. The saturable absorber can be either *fast* or *slow* depending on the recovery time of the device relative to the duration of the pulse circulating in the cavity. For the generation of short pulses in a passively mode-locked fibre laser a fast saturable absorber is required, but often a slow absorber has superior self-starting performance. Figure 2.2 shows schematically the pulse-shaping action of a fast saturable absorber. The assumption is made that the gain is approximately time independent, and thus constant during the passage of the pulse – a valid assumption for fibre systems exhibiting long gain relaxation times compared to the period of the cavity. A window of net gain exists when the absorber is bleached by the intense pulse peak, consequently amplifying the pulse centre while attenuating its wings. The theory describing the transmission $s(t)$ through a fast saturable absorber was developed by Haus in 1975 [Hau75b]

$$s(t) = \frac{s_0}{1 + \frac{I(t)}{I_{sat}}} \tag{2.4}$$
where $s_0$ is the unsaturated loss, $I(t)$ is the time-dependent intensity and $I_{\text{sat}}$ is the saturation intensity of the absorber.\(^2\) By assuming that the saturation is relatively weak, normalizing the power contained in the mode, such that $P(t) = I(t) A_{\text{eff}} = |a(t)|^2$, where $A_{\text{eff}}$ and $a(t)$ are the mode area and amplitude respectively, and using the solution to the master mode-locking equation developed by Haus in 1991 [Hau91] (and discussed in detail in chapter 4) the expected duration of a pulse from a mode-locked laser based on a fast saturable absorber is given by

$$\tau^2 = \frac{2g_0}{\delta |A_0|^2 \Omega^2_g}$$

(2.5)

here $\delta$ is the self-amplitude modulation (SAM) coefficient. The role of the coefficient $M \Omega^2_m$ in Equation 2.3 is replaced by $\frac{\delta |A_0|^2}{\Omega^2_g}$ in Equation 2.5, and is proportional to the curvature of the loss modulation against time. This suggests that in the case of passive mode-locking the modulation curvature increases, as the pulse duration reduces, further shortening the pulse until the duration is limited by the finite bandwidth of the gain or a spectral limiting filter. For active mode-locking the modulation curvature is constant with pulse duration; consequently shorter pulses can be obtained from passive systems.

The model for a slow saturable absorber has to consider the change in gain during the passage of the pulse, consequently the analytic description of mode-locking with this device is more complex. The window of net gain that promotes pulse formation exists between the saturation of the absorber and the saturation of the gain (if the gain is non-saturating, i.e. approximately constant, then the pulse duration is limited to the recovery time of the saturable absorber). For experimental results presented in this chapter the saturable absorber is fast, i.e. the recovery time is short relative to the duration of the pulse, and so I refer to Ref. [Hau75a] for a fuller discussion of slow saturable absorber mode-locking. Gain saturation effects in mode-locked lasers will be revisited in chapter 4.

### 2.3 Single-wall Carbon nanotubes as a saturable absorber device

Ijima’s seminal paper in 1991, reporting the synthesis of helical microtubules of graphitic Carbon, generated unprecedented interest in Carbon-based nanostructures, and is often cited as the first discovery of Carbon nanotubes (CNTs) [Iji91]. The potential of CNTs for new optical devices was realised in theoretical and experimental reports by Margulis

---
\(^2\) Saturation intensity is the optical intensity required in the steady-state to reduce the absorption to half of its unbleached value.
Figure 2.3: A two-dimensional hexagonal lattice of Carbon atoms (planar graphene sheet) illustrating the chiral vector, the unit cell, and one possible chirality, with indices (5,2), corresponding to a metallic tube. And a schematic illustration of a rolled sheet forming a SWNT with (5,3) chirality.

and Sizikova [Mar98b], and Kataura et al. [Kat99] in the late 1990’s. These reports showed that the first and second lowest transitions in the density of states (DoS) could exhibit semiconducting properties, where the band gap was determined by the diameter of the tube geometry.

CNTs are structures from the fullerene family consisting of a honeycomb sheet of sp$^2$-bonded Carbon atoms rolled seamlessly into itself to form a cylindrical tube-like structure [Mar11b]. A single-wall nanotube (SWNT) consists of a single atomic layer. The unrolled sheet, known as graphene and independently possessing interesting optical properties, is a semimetal. The rolling of this single atomic layer into a tube adds an extra level of confinement to form a quasi-one-dimensional structure: up to centimeters in length, with diameters below 3 nm. The electronic properties of SWNTs are governed by their chiral vector that indicates the orientation of the tube axes relative to the organisation of the honeycomb lattice, such that

$$\vec{c} = n\vec{a}_1 + m\vec{a}_2$$

(2.6)

where $\vec{a}_1$ and $\vec{a}_2$ are two real-space unit vectors and $n$ and $m$ are integers describing the atomic coordinates of the one-dimensional unit cell [Kes04]. Schematics illustrating the unit cell, the chiral vector and chiral angle $\Theta$, and a three-dimensional rendering of a SWNT with (5,3) chirality are shown in Fig. 2.3. Depending on their chirality they can behave like direct bandgap semiconductors or metals. If the congruence relation $n \equiv m \pmod{3}$, the SWNTs are considered metallic, which suggests for a collection of SWNTs with a random distribution of $(n, m)$ indices two thirds of the population will exhibit semiconducting properties. SWNTs that behave like semiconductors have an optical absorption determined by their electronic bandgap. Broadband absorption arises because of the large distribution of diameters formed during the synthesis process [Kat99].
2.3 Single-wall Carbon nanotubes as a saturable absorber device

A zone-folding method can be used to calculate the electronic band structure of SWNTs using the two-dimensional energy dispersion relations for \( \pi \) bands of graphite, subject to periodic boundary conditions [Sai92, Kat99, Sai00]. Kataura designed the Kataura plot to relate the nanotube diameter to its bandgap energy, rationalizing the electronic properties of all nanotubes (defined by their \((n,m)\) indices) in a given diameter range. Figure 2.4 shows this dependence for semiconducting and metallic SWNTs with diameters from \( \sim 0.25 \text{ nm} - 3.0 \text{ nm} \) for a nearest-neighbour overlap integral \( \gamma_0 = 2.9 \text{ eV} \).

In direct bandgap semiconductors a variation of the incident light intensity results in a change in absorption and refractive index, arising from the third-order nonlinear susceptibility. For high optical intensities the upper energy states fill-up, preventing further absorption. This process is known as saturable absorption and has been discussed in the context of mode-locking of ultrafast lasers in section 2.2.2. Semiconducting SWNTs exhibit a very high third-order nonlinearity and a fast recovery time making them an attractive material for new saturable absorber devices.

The saturation intensity, assuming only absorption processes from the ground state, is related to the relaxation time \( \tau \) by

\[
I_{\text{sat}} = \frac{h \nu}{\sigma_{\text{abs}} \tau}
\]

where \( h \) is Planck’s constant, \( \nu \) is the photon frequency, and \( \sigma_{\text{abs}} \) is the absorption cross section (as defined in footnote 1). From Equation 2.7 it is clear to see that as \( \tau \) gets shorter the saturation intensity gets larger. It has already been established that a fast relaxation time is a desirable property of a material used as a fast saturable absorber in a mode-locked laser if short pulses are to be achieved. It has been shown that SWNTs
satisfy this condition, exhibiting a fast characteristic electronic transition relaxation time of \( \sim 800 \text{ fs} \) [Che02]. In isolation the relaxation time of a single semiconducting SWNT is much longer (of the order of tens of picoseconds [Wan04]). The ultrafast response time is based on the coupling of excited electrons to metallic transitions in SWNT bundles that form during the synthesis process and exist even in highly purified samples. However, this combination of slow and fast relaxation components makes SWNT-based devices very effective saturable absorbers.

### 2.3.1 Functionalising SWNTs for use as saturable absorbers in photonic devices

Nanotubes can be synthesised from a graphite precursor in a number of ways: arc discharge; laser ablation; and chemical vapour deposition are the three main methods. It is now possible to disperse SWNTs in different solvents [Has09]. These processed SWNTs can then be embedded into polymer composites for deployment as an integrated photonic device. All SWNTs used in experiments described in this chapter have been functionalised with a host matrix to form a nanotube-polymer composite, typically a film, that can be handled easy and used in development without suffering significant degradation. Optical grade nanotube-based polymeric devices were prepared by two research groups based at the University of Cambridge and the General Physics Institute, Moscow that formed two separate collaborative partnerships with the Femtosecond Optics Group. While both groups used unique synthesis and fabrication processes, the general steps in the development of a SWNT-based polymer composite, for use in ultrafast fibre lasers, are briefly summarised below. For details of each specific preparation process I refer to Refs. [Has09] and [Che07].

The tendency for SWNTs to form bundles can be beneficial when optimizing the range over which the device could potentially exhibit nonlinear absorption: an inhomogenous blend of SWNTs with differing diameters embedded in a single polymer film can promote broadband operation, but can also significantly increase unsaturated losses when the device is selectively excited. Moreover, aggregates with dimensions comparable to the excitation wavelength (\( \sim 1–2 \text{ \mu m} \)) can suffer from enhanced scattering losses [Has09]. Thus, prior to preparation of the composite film, ultrasonication, in the presence of dispersants and in a liquid medium, is used to debundle the largest aggregates. The insoluble materials are removed by vacuum filtration or ultracentrifugation. If the host polymer is not used as the initial dispersant, it is then dissolved in the supernatant. Finally, the solvent is gradually evaporated leaving the SWNTs embedded in the host matrix. A typical recipe for this process is as follows:

**step–i:** 2 mg of CNTs are sonicated in 20 ml deionized water for 2 hr at 10–
2.3 Single-wall Carbon nanotubes as a saturable absorber device

12°C.

**step–ii:** The CNT dispersion is then centrifuged at 20,000 g using a MLA-80 fixed-angle rotor (Beckman) for 2 hr.

**step–iii:** The top 70% is decanted to obtain an aggregation-free CNT dispersion.

**step–iv:** This is then mixed with polyvinyl alcohol (PVA) in deionized water with a homogeniser and drop cast in a Petri dish.

**step–v:** Slow evaporation at room temperature in a desiccator yields a ∼50 μm CNT-PVA composite.

The chiralities, and consequently the tube diameters, can then be determined using Raman spectroscopy and Photoluminescence Excitation spectroscopy [Jor09, Wei03].

### 2.3.2 Wideband mode-locking of fibre lasers using both $E_{11}$ and $E_{22}$ transitions in single-wall Carbon nanotubes

The use of SWNTs as saturable absorbers for initiating and maintaining mode-locking has created wide interest since the first demonstration in 2003 [Set03, Set04c]. Fibre lasers utilizing SWNTs have been demonstrated at a range of wavelengths [Son05, Goh05, Roz06a, Nic07, Tau08, Sol08, Kie08, Wan08, Kiv09b]. This interest arises from the key properties of SWNTs for mode-locking lasers [Set04a, Set04b, Yam04]: sub-picosecond characteristic transition times; a high damage threshold; environmental stability; and all-fibre integration. Combined, these properties make SWNTs competitive with conventional fibre mode-locking techniques such as NPE [Mat92a], or SESAs [Sou93].

The great majority of mode-locked lasers demonstrated use the fundamental ($E_{11}$) transition of semiconducting nanotubes, which corresponds to the single real gap in the electron DoS [Bac02]. In contrast, only a few groups have reported saturable absorption and mode-locking of bulk lasers, using the second transition ($E_{22}$) [Yim08, Sch08a], corresponding to a pseudo-gap [Bac02]. It is of fundamental interest that a second electronic transition can be used for saturable absorption as it significantly increases the absorption energy for a given nanotube diameter [Set04b], and opens the possibility of mode-locking shorter wavelength systems, below 1 μm, towards the visible spectral region, something that is hard to achieve on the $E_{11}$ transition with currently realizable nanotube dimensions (0.3 nm [Zha08]). Also the shorter $E_{22}$ lifetime [Man05], may affect mode-locking behaviour. Figure 2.5 schematically illustrates the DoS of semiconducting SWNTs, characterised by sharp van Hove singularities arising from the quasi-one-dimensional nature of the structure, and the suggested mechanism governing saturable absorption of the $E_{22}$ transition [Ma04]. After excitation, the electron must pass to the
Density of one-electron states

Figure 2.5: Schematic of the levels involved in saturable absorption of \( E_{22} \).
\( E_F \) = Fermi level.

Figure 2.6: Linear transmission spectrum of the Carbon nanotube film used in the saturable absorption and mode-locking experiments. The wavelengths at which the saturable absorption was measured, and mode-locking was achieved are shown by the red circles. The \( E_{11} \) and \( E_{22} \) transition absorptions are also indicated.

lower energy transition, corresponding to the real gap \( E_{11} \). This has been illustrated by photoluminescence (PL) measurements, where strong signals have been observed only for \( E_{11} \) transitions of different nanotubes [Bac02] under excitation at wavelengths corresponding to \( v_n \rightarrow c_n \) transitions: the \( n \)th electronic interband transition.

Linear and nonlinear optical properties

A transparent carboxymethyl cellulose film, embedded with homogeneously dispersed individual SWNTs [Che07], was used as a saturable absorber. The same film has already been used to mode-lock Er- and Tm-fibre lasers reported in Refs. [Tau08, Sol08]. The transmittance spectrum of this film is shown in Fig. 2.6. The clean two peaked spectrum
2.3 Single-wall Carbon nanotubes as a saturable absorber device

Figure 2.7: Schematic of the experimental z-scan configuration. CO - coupler, POW - power meter, SCF - small core fibre, SA - scanning arm, ATS - automated translation stage, R - reference, T - transmitted, L1 and L2 - focusing lenses, L3 - collection lens, NT - nanotube sample.

is an indication of the purity of the nanotubes embedded in the film matrix, as the width of the absorption bands is related to the diameter distribution of the nanotubes. The absorption centered at 1.75 μm is due to the E11 transition, and was sufficiently broad to allow mode-locking at 1.55 μm. The peak around 1.0 μm is due to the E22 transition [Kat99], even when taking into account excitonic effects [Wei03].

The nonlinear absorption of the SWNT film was characterised using a Z-scan method. The setup is shown in Fig. 2.7. The pump light is passed through a fused fibre coupler to split 7% of the power to a reference power meter. The remaining power is then coupled into a small core, high NA fibre (Nufern UHNA 3), the output of which is imaged by a pair of identical aspheric lenses. An automated translation stage moves the sample arm and nanotube film through the focus. A third lens collects the light transmitted through the sample to a second power meter. To pump the E11 transition an amplified mode-locked Erbium fibre laser with a repetition rate of 14 MHz and pulse duration of 3 ps was used. The E22 transition was excited using a mode-locked Ytterbium fibre laser followed by an amplifier and grating compressor system to produce 0.9 ps pulses at a repetition rate of 47.5 MHz. The resulting peak intensities at the Z-scan focus were approximately 125 MW cm⁻² and 2.6 GW cm⁻² for the E11 and E22 transitions respectively. To connect the lateral sample position with a certain peak intensity, the standard model of Gaussian beam propagation was used. Given that the beam waist at the output of the small-core fibre was, in the ideal case, one-to-one imaged after the second lens, the beam radius w scales as

$$w(z) = w_0 \sqrt{1 + \left( \frac{z - z_f}{z_0} \right)^2}$$

where w₀ is the beam waist (half the mode field diameter (MFD) of the fibre), z₀ = πw₀²/λ is the Raleigh length, and z_f is the focal position (determined to be the centre of the transmission maximum). The incident peak power was determined using the duty cycle
Figure 2.8: Saturable absorption measurements by the Z-scan method.

and the output average power for each respective excitation source. The corresponding lateral change in peak intensity for the on axis Gaussian beam approximation is then given by

$$I(z) = \frac{2P_0}{\pi w(z)^2}$$

(2.9)

An optical chopper was used to reduce the thermal loading on the SWNT sample, maintaining an average power of well below ~6 mW comparable to the damage threshold of the composite material. The signature of thermal damage was an asymmetry of the transmitted power as a function of $z$-position. Errors due to peak power fluctuations arising from noise on both excitation sources were minimized by internally integrating the measured signals over a period of 500 ms. In addition, five readings were taken for each $z$-position, and the mean intensity computed. Automation of both the stepping of the sample in $z$ and the reading of the corresponding average power, using computer control, was employed to eliminate human error, maximise data redundancy, and increase the acquisition time.

The results of the Z-scan measurements are shown in Fig. 2.8. It should be noted that there is some uncertainty as to the accuracy of the intensity scale due to uncertainty in the exact pump pulse shape. The red fit curve is based on the instantaneous saturable absorption model (similar to Equation 2.4):

$$\alpha(t) = \frac{\Delta\alpha}{1 + \frac{I(t)}{I_s}} + (1 - \alpha_{ns}).$$

(2.10)

where $\Delta\alpha$ is the modulation depth and $\alpha_{ns}$ accounts for non-saturable losses. Due to
the fact that for both the $E_{11}$ and $E_{22}$ transition the pump pulses are significantly longer than the transition lifetimes, by a factor of $\sim 8$ and $\sim 23$ respectively [Man05], the instantaneous model is appropriate. The normalised modulation depths ($\Delta \alpha$) of 13% and 15% respectively for the $E_{11}$ and $E_{22}$ transitions are very similar, however, the saturation intensities ($I_{\text{sat}}$) of $\sim 10 \text{ MW cm}^{-2}$ and $\sim 220 \text{ MW cm}^{-2}$ are different by an order of magnitude. This difference is expected, and can almost entirely be accounted for, from the respective transition lifetimes of $\sim 400 \text{ fs}$ and $\sim 40 \text{ fs}$ [Man05].

The $E_{22}$ modulation depth ought to be sufficient to mode-lock a fibre laser, and the increased saturation intensity, although larger than the $E_{11}$ transition of nanotubes, and also of that of SESAs, is still significantly smaller than required in NPE-based systems, and should not present a problem for fibre lasers. In the following section results demonstrating the use of the $E_{22}$ transition to mode-lock a fibre laser are presented.

**Experimental setup**

Two Erbium and Ytterbium fibre ring lasers were constructed to test mode-locking on both the $E_{11}$ and $E_{22}$ transitions. The experimental setups for the lasers are shown in Fig. 2.9. Both lasers were constructed from isotropic single-mode fibre. The polarization controllers were included to check the output pulse dependence on polarization state. The Carbon nanotube films were sandwiched between two FC-APC connectors. In the Ytterbium ring laser a fibre integrated circulator and chirped Bragg grating with a dispersion of 35.7 ps/nm was included to compensate for the normal dispersion of the other cavity components; due to this extremely high value the overall cavity dispersion was strongly anomalous. The Erbium system was naturally anomalously dispersive and no compensation was required to operate in the soliton regime. The output coupler was

![Figure 2.9:](image_url)

**Figure 2.9:** Experimental setup of the mode-locked ring lasers. The Erbium based $E_{11}$ transition laser. The Ytterbium based $E_{22}$ transition laser. Acronyms defined as follow: EDFA - Erbium doped fibre amplifier; YDFA - Ytterbium doped fibre amplifier; SWNT-SA - single-walled Carbon nanotube saturable absorber; PC - polarization controller; OC - fused fibre output coupler; ISO - isolator; C - circulator; CFBG - chirped fibre Bragg grating.
50% for the Erbium laser and 15% for the Ytterbium laser. In both setups exactly the same nanotube film was used.

**Results**

Both laser setups were self-starting upon reaching threshold, and mode-locking was maintained through approximately 10% tuning of the pump power. For increasing powers Q-switching was observed. The pulse trains exhibited no transient dynamics and remained stable with a single pulse per cavity round-trip. The polarization control had little effect on the pulse train and output characteristics. Figure 2.10 shows the results obtained with the Erbium ring laser. The FWHM of the second-harmonic intensity autocorrelator signal was 1.1 ps, which corresponds to 0.7 ps if we assume a sech$^2$ pulse shape. No attempt was made to correctly tune the cavity dispersion and nonlinearity for short pulse operation, although that can lead to considerably shorter pulses [Tau08]. The central wavelength of the pulse was 1.565 $\mu$m and the spectral width 3.12 nm. From inspection of Fig. 2.6 it is clear that this laser was mode-locked using the short wavelength edge of the E$_{11}$ transition of the nanotubes. Figure 2.11 shows the results obtained with the Ytterbium ring laser. For this system the FWHM autocorrelation duration was 10.0 ps, which corresponds to 6.5 ps assuming a sech$^2$ pulse shape. Again, no attempt was made to fully dispersion compensate the cavity, rather the chirped Bragg grating ensured operation in the strongly anomalous regime [Fer95]. However, strong third-order dispersion, arising from the finite length of the CBFG [Err00], causes the appearance of resonant sidebands in the optical spectrum [Hau93] (see Fig. 2.11b).
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sidebands represents a coupling of energy from the pulse to the continuum (or background), resulting in a broadening of the pulse in time (see Fig. 2.11a). Full cavity dispersion compensation with bulk gratings or photonic band-gap fibre should lead to shorter pulse operation [Goh05, Nic07]. The central laser wavelength was 1.066 µm and the spectral bandwidth was 0.18 nm. This corresponds to a time-bandwidth product of 0.31 indicating that the output pulses were not strongly chirped. The average output power was 170 µW. The repetition rate of the laser was 15.2 MHz.

While the wavelength of the Erbium laser clearly indicates it is operating on the E\(_{11}\) transition (see Fig. 2.6), the Ytterbium laser wavelength is too short for this transition and must be operating on the second absorption peak of the nanotubes, i.e. the E\(_{22}\) transition. Despite this the laser exhibited similar qualitative behavior as the Erbium system operating on the E\(_{11}\) transition.

The ability to saturate the fast E\(_{22}\) transition raises a number of possibilities. Firstly, the larger transition energy of the E\(_{22}\) level means that mode-locking at shorter wavelengths is viable, potentially into the visible spectral region. Secondly, because the electron falls to the E\(_{11}\) level after E\(_{22}\) excitation, the E\(_{11}\) saturable absorption could be controllable via the E\(_{22}\) transition. Consequently, it should be possible to exploit this effect to passively synchronise a two-colour fibre laser mode-locked using both transitions.

Figure 2.11: Temporal and spectral properties of the Ytterbium ring laser operating on the E\(_{22}\) transition.

![Autocorrelation and Optical spectrum graphs](image-url)
2.3.3 Passive synchronisation of a two-colour mode-locked fibre laser using single-wall Carbon nanotubes

In certain applications, for example pump-probe processes, Raman spectroscopy, and difference-frequency mixing, tunable single wavelength sources are not sufficient for the requirements\cite{Gan06}; thus synchronised two-wavelength mode-locked lasers with a locked repetition rate have emerged and have been demonstrated using different methods: active synchronisation \cite{Sch03, Kim08} that applies electronic feedback to control the cavity length, active-passive hybrid synchronisation \cite{Yos05} and passive synchronisation \cite{Fur96}, where the nonlinear interaction of the two beams promotes coupling. The relative timing jitter, which currently has the most advanced reported results of attoseconds \cite{Kim08, Yos05}, provides an important metric for the stability of the systems synchronisation and is required for optimal performance in the majority of applications. Previous reports have shown that solid-state passively synchronised laser systems, using cross-phase modulation (XPM), result in low timing jitter and large cavity mismatch \cite{Yos06}. The merits of fibre systems have been discussed previously and include, environmental stability, compactness, and high efficiency. Recently, the passive synchronisation of fibre lasers was achieved using a number of techniques: using XPM \cite{Rus04b, Rus04a, Hsi09, Yan09} in a common fibre length or using a shared saturable absorber in a master-slave configuration \cite{Wal11}, where the master injection mode-locks the slave. In addition, saturable absorbers have been used to stabilize mode-locking through direct pump modulation \cite{Gui02}.

A single SWNT-based polymeric saturable absorber film, to mode-lock two disparate wavelengths in the near-IR, was demonstrated in section 2.3.2; and the potential of this device for passive synchronisation of a two-colour fibre laser was suggested because of the coupled relaxation of the $E_{11}$ and $E_{22}$ excited state transitions. In the proceeding sections experimental results are presented using the SWNT device introduced in section 2.3.2 for this application.

Experimental setup

The configuration of the two-color all-fibre laser is shown in Fig. 2.12, with the top and bottom parts presenting the Er-laser and Yb-laser, respectively. In the Er-laser, a fibre amplifier module (consisting of 1.5 m of double-clad Er-doped fibre, counter pumped by a 4 W multi-mode diode at 980 nm) providing a noise seed and amplification in a band around 1.55 $\mu$m was followed by an inline optical isolator. The output signal was delivered through a 50:50 fused-fibre coupler, and a polarization controller was added to adjust the polarization state within the cavity, but was not fundamental to the mode-
locking action. The cavity length of the Er-laser could be changed by a maximum of 9 cm through a fibre-pigtailed optical delay line, with a corresponding optical delay of 300 ps. The Er-laser cavity contained ~15 m of single mode fibre (SMF28), where the approximate group velocity dispersion (GVD) value is $-17 \text{ ps}^2 \text{ km}^{-1}$ [Agr07]. The maximum GVD of Er-doped fibre at a wavelength $\lambda = 1.534 \mu\text{m}$ is 0.009 $\text{ps}^2 \text{ dB}^{-1}$ [Mat91]. Assuming a maximum small-signal gain of ~25 dB for a single-pass of the ring cavity the net GVD ($\beta_2$) of the cavity can be estimated. Based on the length of active and passive fibre, the net GVD is approximately $-0.0094 \text{ ps}^2$. Although the average cavity dispersion is anomalous, the magnitude of $\beta_2$ is low and consequently the soliton period (the length scale over which a soliton will form) is long; in this regime stable soliton-operation is not guaranteed. The Yb-laser, one-half of the two-color laser, was constructed from a fibre amplifier module (consisting of 0.6 m of double-clad Yb-doped fibre, counter pumped by a 4 W multi-mode diode at 980 nm) generating the noise seed and amplification around $1.06 \mu\text{m}$, a 20% output fused-fibre coupler, and a polarisation controller. A polarization independent inline fibre circulator was employed to incorporate a chirped fibre Bragg grating, providing a negative dispersion of $-21.6 \text{ ps}^2$, and ensuring unidirectional propagation. The strong anomalous dispersion of the CFBG dominated all other contributions in the cavity and ensured that the laser operated in the average-soliton regime\(^3\).

Both lasers shared a transparent carboxymethyl-cellulose thin film ($\sim 10 \mu\text{m}$ thick, and synthesized by an arc-discharge technique [Obr99]), with homogeneously embedded in-

\(^3\)In a fibre laser segments of the cavity possess both positive and negative dispersion. In addition, fibre segments are passive or active: the gain fibre provides lumped amplification. On a single round-trip a soliton pulse is perturbed by both the gain and the changing dispersion. The average (or guiding centre) soliton regime refers to a stable condition for soliton propagation, where the period of any perturbation is short compared to the soliton length.
dividual SWNTs [Che07]; this film was characterised and used to mode-lock an Er and Yb fibre laser in section 2.3.2. The shared saturable absorber device was incorporated into the shared section of the double-cavity using two 1060/1550 wavelength division multiplexers (WDM). The linear and nonlinear response of the SWNT saturable absorber was plotted in Figs 2.6 and 2.8, respectively. It has been established that the two prominent absorption features at 1.8 $\mu$m and 1.0 $\mu$m, correspond to the $E_{11}$ and $E_{22}$ electronic transitions in the density of states.

Results

Both lasers operated in the soliton-regime, achieving a synchronised repetition rate of 13.08 MHz. The broadband absorption range of the SWNTs ensures the stable mode-locking behavior at 1 $\mu$m and 1.5 $\mu$m. The nonlinear coupling effects between two energy states of a SWNT result in the stable synchronisation of pulses generated in the two-color laser for several hours and a large tolerable cavity length mismatch of $\sim$1400 $\mu$m.

Measurements of the pulse repetition rate were carried out with a fast photodiode connected to a radio frequency analyser (RF analyser). Stable, self-starting mode-locking was obtained in both lasers independently using the same SWNT device. In the non-synchronised state, the spectral and temporal intensity of the Er-laser pulses were maintained while the cavity length was adjusted through the delay line. The systems behaved like two independent lasers, with their spectral and temporal properties equal to the two decoupled systems presented in section 2.3.2 (see Figs. 2.10 and 2.11).

When the repetition rate of the Er-laser was finely tuned using the delay line to twice
the value of the Yb-laser’s, a passive synchronisation was achieved. As the pump power of the Yb-laser varied, two modes of synchronisation were observed: either the repetition rate was locked at the fundamental frequency or the second cavity harmonic frequency, with the latter observed as the pump power increased. When synchronised at the repetition rate of both their fundamental frequency, the radio frequency spectra were measured by an RF analyser, with the value of 6.54 MHz and 13.08 MHz in Yb-laser and Er-laser, respectively. The corresponding single pulse energy was 1.9 pJ for the Yb-laser and 15 pJ for the Er-laser, assuming all the energy is contained within the body of the pulse.

To obtain the higher harmonic frequencies of the synchronised laser, the output pulses from both lasers were injected into a 1060/1550 WDM, and Fig. 2.14 shows the measured frequency components detected from the output port of the WDM.

Figure 2.15 shows the repetition rates of both lasers as a function of the cavity length tuning of the Er-laser through the variable delay line. In the fundamental mode of the synchronised operation, where there is only one mode-locked pulse inside the cavities, both lasers remained at the repetition frequency imposed by the Yb-laser and obtained for the maximum detuned frequency of 1200 Hz, corresponding to a tolerable cavity length mismatch of ∼1400 μm. It is noted that the transition from the nonsynchronised mode to the synchronised regime described above does not modify the pulse parameters within experimental accuracy; however, a small decrease (∼5%) in the threshold power of both lasers was observed. This could be attributed to the relaxation dynamics of the E_{22} transition of the saturable absorber device, excited by a 1 μm photon: the electron in the CNT matrix nonradiatively decays via the E_{11} level, coupling the saturable losses to the 1.55 μm transition. This model of saturable decay of the E_{22} transition has been illustrated by photoluminescence measurements in Ref. [Bac02] that show only a strong
signal at the corresponding $E_{11}$ transition. Thus, in the synchronised mode, the $E_{11}$ state is occupied and saturated by both lasers and results in a decrease of threshold. Different from [Rus04b, Rus04a, Hsi09, Yan09], XPM is not expected to contribute to the synchronised operation because of low intracavity peak powers and a short shared interaction length; the nonlinear coupling effects between the $E_{11}$ and $E_{22}$ states of the SWNT support the synchronisation.

To further qualify the synchronisation, the timing jitter between the two lasers was measured by cross-correlation technique [Miu02]. The output pulses of the Er- and Yb-laser were amplified to 2.84 mW and 6.03 mW, corresponding to 217.2 pJ and 921.6 pJ, by an Er-amplifier and Yb-amplifier, respectively. By adjusting the variable delay line in one arm of the cross-correlator, the cross-correlation trace was obtained and recorded on an oscilloscope, with the FWHM of 6.5 ps, shown in Fig. 2.16.

In order to quantify the timing jitter between the two synchronised lasers, the half-maximum intensity of the cross-correlation trace was recorded with 8000 points in one second, corresponding to the Nyquist frequency of 4 kHz. The calibrated power spectral density (PSD) and the integrated root mean square (RMS) timing jitter of 600 fs, in a Fourier frequency range from 1–4 kHz, are shown in Fig. 2.17. Beyond 1 kHz, the jitter PSD decays while the integrated RMS timing jitter appears to saturate ($\sim 530$ fs RMS jitter from 1 Hz–1 kHz), indicating that the jitter was mainly contributed by lower frequency noise. Compared to previous reports [Zho09] the jitter is large. Reducing the duration of the pulses will reduce the degree of pulse jitter.

**Figure 2.15:** Repetition rates of the synchronised Yb- (blue circles) and Er-laser (black circles) under different cavity length mismatches. The synchronised repetition rate is 13.08 MHz and twice the repetition rate of the Yb-laser is plotted. The diagonal line (red circles) presents the cavity length-dependent repetition rates of the nonsynchronised Er-laser.
2.3 Single-wall Carbon nanotubes as a saturable absorber device

**Figure 2.16:** Cross-correlation function of the two synchronised, all-fibre mode-locked lasers. The red dashed curve shows a fit to the experimental data.

**Figure 2.17:** Sum-frequency generation intensity-noise power spectral density (PSD) and integrated RMS (I-RMS) timing jitter of the synchronised lasers.
2.4 Other novel polymer composite technologies: double-wall Carbon nanotubes and graphene

The application of SWNTs for ultrafast mode-locking of fibre lasers has been widely studied. In this chapter the basic operation of a single device has been extended by using the $E_{22}$ transition. However, other Carbon-based polymeric composites can be used to diversify the operation of a single saturable absorber device; this is the subject of this final experimental section in this chapter.

2.4.1 Double-wall nanotube-based saturable absorbers

Here, it is proposed that the double-shell structure of DWNTs could be used to achieve saturable absorption using both the distinct resonant absorption features expressed by the inner and outer shells.

Summary of the preparation process of a DWNT polymer film

The CNT-based polymer saturable absorber device used here to demonstrate this was prepared using solution processing [Has09]. The CNTs were grown by catalytic chemical vapor deposition [Fla03] (CCVD). Analysis of the purified samples by transmission electron microscopy (TEM) reveals the presence of $\sim$90% DWNTs, $\sim$8% SWNTs, and $\sim$2% triple-wall nanotubes (TWNTs) [Oss05]. The diameter distribution for DWNTs is $\sim$0.8–1.2 nm for the inner and $\sim$1.6–1.9 nm for the outer diameters, as determined by Raman spectroscopy and TEM. This wide diameter distribution can potentially enable broadband operation. The mean diameters of the inner and outer wall were $\sim$1.1 nm.
2.4 Other novel polymer composite technologies: double-wall Carbon nanotubes and graphene

Figure 2.19: Schematics of two ultrafast lasers based on novel saturable absorber technologies: double-wall Carbon nanotubes and graphene.

and ~1.8 nm, respectively. The purified nanotubes are then dispersed in water with sodium dodecylbenzene sulfonate surfactant, and mixed with an aqueous polyvinyl alcohol (PVA) solution to obtain a homogeneous and stable dispersion, free of aggregates. Slow evaporation of water from this mixture produces a CNT-PVA composite ~50 μm thick. Optical microscopy reveals no CNT aggregation or defect in the composite, thus avoiding scattering losses.

Figure 2.18 shows the linear transmission profile of the DWNT film. The vertical lines denote the wavelengths corresponding to the expected real $E_{11}$ and pseudo $E_{22}$ transitions in the DoS based on Ref. [Wei03] for the mean tube diameters of the inner and outer shell. The outer wall transitions are highlighted with a red line and the inner wall with a blue line. What is noticeable is that the $E_{22}$ of the outer wall is expressing itself more strongly than the $E_{11}$ transition of the inner wall. This suggests that it could be expected that the outer wall plays a more significant role in the saturable absorption process, although the interaction between the inner and outer wall remains unclear.

**Experimental setup**

The performance of the DWNT-based saturable absorber was evaluated for two independent lasers: one operating at 1.55 μm, based on an Er-doped fibre amplifier; and one operating at 1.06 μm, based on a Yb-doped fibre amplifier. The basic construction of the cavities was qualitatively similar to Figs. 2.9a and 2.9b. The explicit components comprising the Yb-laser are shown schematically in Fig. 2.19a. A 2 mm$^2$ piece of the DWNT-polymer composite was integrated into the cavity using the favoured butt-coupled method: the film is sandwiched between two FC/PC connectors to form a robust fibreised package, typically suffering an insertion loss (including the linear transmission of the polymer film) of ~4 dB. Dispersion compensation to operate in the soliton regime was provided by a CFBG, incorporated using a fibre-pigtailed circulator. However, similar to the results discussed in section 2.3.2, no attempt was made to balance the dispersion map of
Results

Both lasers operated stably at their fundamental repetition frequency and produced 4.85 ps (Yb) and 532 fs (Er) near transform-limited pulses. Their temporal performance is summarised in Fig. 2.20a (Yb) and 2.20b (Er). These preliminary experiments show unequivocally that DWNTs can also be used as a mode-locker in ultrafast fibre lasers, and that their properties could be useful for extending the operational bandwidth of a single device. Further experiments are required to characterise fully the nature of the interaction between the two walls, and to establish how the geometry affects the device’s electronic properties.

2.4.2 Graphene: the universal saturable absorber?

The most coveted prize in Physics was recently awarded for experimental studies regarding graphene [Nov04]: a single atomic layer of sp²-hybridised carbon, forming a honeycomb crystal lattice [Bao09]. Due to its unique electronic and optical properties, that can be described in terms of massless Dirac Fermions with a linear dispersion relation near the Fermi energy, it quickly became apparent that this new material could be applied to the next generation of photonic devices [Has09]. Since 2009 a variety of lasers exploiting the saturable properties of graphene have been realised [Sun10a]. It is the linear dispersion of the Dirac electrons in graphene that means that for any excitation an electron-hole pair will always be in resonance with the incident light. Due to the ultrafast carrier
2.4 Other novel polymer composite technologies: double-wall Carbon nanotubes and graphene

**Figure 2.21:** The linear dispersion relation of graphene: a three panel illustration of the saturable absorption process. The solid black arrow indicates an optical interband transition. The photogenerated carriers thermalise and cool on a subpicosecond scale to form a hot Fermi-Dirac distribution; an equilibrium electron-hole distribution can be finally approached through intra-band photon scattering and electron-hole recombination. Under high-intensity optical excitation, the photo-generated carriers cause the states near the conduction and valence band to fill, preventing further absorption through Pauli blocking [Bao09].

Dynamics and large absorption per layer (~2.3%), graphene should exhibit fast saturable properties over a broad wavelength range, without the need for bandgap engineering or chirality/diameter control, as is the case with SESAs and CNTs [Sun10a]. The photoexcited electron kinetics, which allow graphene to uniquely exhibit saturable absorption without expressly possessing a discrete bandgap, are illustrated in Fig. 2.21 and were discussed in detail in Ref. [Sun10a] in the context of mode-locking lasers. Under weak optical excitation, electrons from the valence band excited into the conduction band undergo rapid thermalisation, cooling to form a hot Fermi-Dirac distribution [Bao09]. Phonon scattering further cools the thermalised carriers before electron-hole recombination restores an equilibrium distribution. If the optical excitation intensity is high enough, the optical transition dynamics become nonlinear, with an increasing concentration of photogenerated carriers causing the states near the edge of the conduction and valence bands to fill, blocking further absorption thus imparting transparency to light at photon energies just above the band-edge. The band filling results because the carriers can be described as Dirac Fermions (with half integer spin) that adhere to the Pauli exclusion principle, where no two identical fermions can simultaneously occupy the same
Wavelength tuning in lasers employing graphene-based saturable absorbers, has been achieved by exploiting fibre birefringence [Bao10, Zha10a]. However, fibre birefringence is sensitive to temperature fluctuations and other environmental instabilities [Agr07], making this approach non-ideal for long-term-stable systems; a key requirement for mode-locked lasers used in practical applications. Outlined in this section are results demonstrating an ultrafast tunable fibre laser mode-locked by a graphene-based saturable absorber, with stable mode-locking over 34 nm, insensitive to environmental perturbations. The tuning range is limited only by the tunable filter.

**Summary of the preparation process of a graphene-based saturable absorber**

The saturable absorber was prepared as described in Ref. [Sun10a]. Graphene flakes were exfoliated by mild ultrasonication, with sodium deoxycholate surfactant. The dispersion was then enriched with single (SLG) and few layer graphene (FLG), and mixed with polyvinyl alcohol (PVA; Wako chemicals). The top 70% of the dispersion was decanted for characterisation using absorption and Raman spectroscopy [Fer96b], and composite fabrication. The absorption spectrum (measured using a PerkinElmer spectrometer by collaborators at the University of Cambridge) of the centrifuged dispersion diluted to 10% is shown in Fig. 2.22. Apart from the strong absorption feature in the UV region due to an exciton-shifted van Hove singularity [Kra10, Ebe08], the spectrum is featureless. Figure 2.23 shows a TEM image of a folded single layer graphene (SLG) flake. TEM statis-

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A more rigorous definition of the exclusion principle states that the wavefunction for two identical fermions is anti-symmetric. Equally, particles with integer spin (bosons) have symmetric wavefunctions.
2.4 Other novel polymer composite technologies: double-wall Carbon nanotubes and graphene

Figure 2.23: TEM image of a folded graphene flake (courtesy of the University of Cambridge).

Figure 2.24: Raman spectrum of a flake deposited on a Si wafer (courtesy of the University of Cambridge).

Raman spectrometry suggests that the dispersion consists of \(~26\%\) SLG, \(~22\%\) bi-layer graphene (BLG) (\(~40\%\) of which are folded SLG flakes), with the remainder consisting multi-layer flakes. The Raman spectrum of a representative exfoliated graphene flake is shown in Fig. 2.24. For a full descriptive interpretation of the Raman spectrum I refer to Ref. [Sun10b]. However, the essential feature illustrated by the Raman spectrum is that it strongly indicates that the layers in the FLG flakes behave like electronically decoupled SLG, retaining the linear dispersion of Dirac fermions [Sun10b, Fer96b].

Slow water evaporation in a dessicator produced free-standing, 50 \(\mu m\) thick graphene-PVA (GPVA) composites [Has09, Sun10a]. The linear transmission properties of the GPVA film were measured directly using a spectrophotometer, and the spectrum is plotted in Fig. 2.25a. Similarly to the profile of the absorption of the graphene dispersion, the GPVA transmission is featureless from 0.5 \(\mu m\)–2.0 \(\mu m\) (excluding the sharp edge at 0.75 \(\mu m\)).
due to a change of detector, illustrating that there is some mis-calibration in the absolute value of transmission). The power dependent absorption at six excitation wave-

![Figure 2.25](image-url): Linear transmission (2.25a) spectrum of the GPVA film. Power-dependent absorption (2.25b) at six wavelengths. Input repetition rate: 38 MHz; pulse duration: 580 fs. (courtesy of the University of Cambridge)

lengths is shown in Fig. 2.25b; measured using an all-fibre setup described in detail in Ref. [Has09]. When the average power is increased to 5.35 mW (corresponding to an intensity of 266 MW/cm²), the corresponding absorption decreases by ∼4.5% at 1558 nm. Although it is not possible to fully characterise the modulation depth and saturation intensity from this data, due to limitations of the pump system, it is clear to see that the GPVA film does exhibit an intensity dependent absorption component.
2.4 Other novel polymer composite technologies: double-wall Carbon nanotubes and graphene

Figure 2.26: Temporal and spectral characteristics of the GPVA-based ultrafast laser. Fig. 2.26a shows the spectrum (on a linear scale) as a function of the cavity tuning filter, limited only by the finite bandwidth of the Er gain profile. Fig. 2.26b shows a typically autocorrelation trace. The variation in the pulse duration across the spectral tuning range is shown in Fig. 2.27.

Experimental setup

The packaged GPVA absorber was integrated into an Er-based laser, shown schematically in Fig. 2.19b, and consisting of the common components of a ultrafast fibre laser: an optical isolator; a fused fibre output coupler; a polarisation controller; and a 1.2 m length of Er-doped fibre (Fibercore), counter pumped by a 976 nm diode. A broadband (12.8 nm) tunable bandpass filter (TBPF) was included to provide control of the lasing wavelength. The broadband filter was necessary to allow the broadest bandwidth, supporting the shortest duration pulses. The TBPF allowed continuous tunability from 1530 nm–1555 nm.

Results

Single-pulse, fundamental mode-locked operation was observed for a pump power threshold level of ~20 mW. The linear output spectra (recorded using an Anritsu MS9710B optical spectrum analyser), for six wavelengths across the range over which stable CW mode-locking of the laser was achieved, is shown in Fig. 2.26a. Characteristic soliton sidebands can be observed, due to periodic intracavity perturbations, confirming operation in the average-soliton regime. A typical autocorrelation trace is shown in Fig. 2.26b; assuming a sech^2 pulse-shape, the deconvolved duration is ~1 ps. Figure 2.27 plots the durations and corresponding time-bandwidth products as a function of operation wavelength. The output pulse duration remained approximately constant, with near transform-limited
The stability of the mode-locking was evaluated by inspection of the fundamental RF frequency of the cavity, shown in Fig. 2.28. The narrow spike at \( \sim 8 \) MHz suggests that the pulse jitter is comparable to conventional mode-locked fibre lasers, where mode-locking is achieved using SESA or NPE schemes. In addition, the high pedestal suppression of 80 dB indicates low amplitude noise fluctuations. Compared to previously reported graphene-based mode-locked lasers this represents a significant improvement in combined noise performance and near transform-limited short-pulse operation over 35 nm of tunable bandwidth [Bao10, Zha10a].
2.5 Summary

In this chapter I have presented the characterisation of the linear and nonlinear optical properties of SWNTs embedded in polymeric films. Specifically, the saturable absorption properties of the $E_{11}$ and $E_{22}$ transitions of a high-purity film of single-wall Carbon nanotubes have been established. The modulation depths were found to be similar, but the saturation intensity was one order of magnitude smaller for the $E_{11}$ compared to the $E_{22}$ transition. This can be accounted for by the similar disparity in transition lifetimes. The first demonstration of mode-locking of a fibre laser on both transitions was achieved. This raised the potential of exploiting the relaxation dynamics to passively synchronise a two-colour fibre laser.

The passive synchronisation between an Er- and Yb-laser, through use of a common SWNT saturable absorber, was demonstrated for the first time. A cavity mismatch tolerance of $\sim 1400 \mu m$ was achieved and the RMS timing jitter was 600 fs from 1 Hz to 4 kHz.

Manipulation of the absorption spectrum of a SWNT-polymer film, through careful selection of the correct diameter/chirality tubes during the fabrication process, presents the possibility of synchronising lasers of other colours.

It was also shown, for the first time, that double-wall nanotubes embedded in a similar polymeric matrix were effective saturable absorber devices, with similarly wideband operation potential.

Finally, a broadly tunable low-noise ultrafast laser, mode-locked using a graphene-based saturable absorber, was developed. The unique optical properties of graphene represent, for the first time, the possibility of a universal saturable absorber device.
3 Ultrafast fibre laser technology
part 2: novel gain media

The use of rare-earth-doped glass fibres as an optically amplifying medium can be traced back five decades to the inception of the field of laser physics at the beginning of the 1960’s, where trivalent Neodymium (Nd) was used as an active ion in a host matrix of barium crown glass in the form of clad rods three inches long and as small as thirty two micrometres in diameter [Sni61]. However, because of the lack of availability of suitable, compact pump sources glass-based fibre lasers did not enjoy wide-spread application until Poole et al. succeeded in incorporating rare-earth ions into single-mode Silica-based optical fibres [Poo85].

The fibre laser supports improved thermal management, reducing thermal loading, operates in a confined diffraction-limited mode, is compatible with single and multimode fibre integrated pump diodes, and offers a large gain bandwidth critical for the generation of short optical pulses. In addition to these favourable properties, the unique feature of a spatially confined mode over a long interaction length allows efficient exploitation of weak nonlinear interactions with the fibre waveguide. Through management and enhancement of both the linear and nonlinear properties expressed by fibres, a high degree of control can be exercised over the light which propagates within it; this has lead to the realisation of highly engineered sources of ultrashort pulses across a broad spectrum of wavelengths, with a wide range of temporal formats.

This chapter follows on from the previous, continuing the discussion of ultrafast fibre-based technology, with emphasis on novel gain media. The introduction of rare-earth activated fibres, in particular Erbium-doping, revolutionised modern telecommunications, offering a platform upon which a fully integrated transmission line could be realised over thousands of kilometers. Similarly, more recently Ytterbium-doped fibre lasers have displaced traditional methods in areas as diverse as machining and welding, mining, and ground-to-air defense. Despite these two dominant technologies, the need for broader wavelength access demands continued research and development into new areas. In section 3.1 I present some basic but pertinent theory, and provide a brief history and review of fibre-based amplifiers used in ultrafast fibre laser systems. Bismuth activated fibre for the generation and amplification of short pulses is considered in section 3.2.
First results showing the potential of this medium for the direct amplification of picosecond pulses are presented. The development of passively mode-locked lasers, based on Bismuth-active fibre, for master-oscillator power amplifier schemes and frequency conversion is discussed. Finally, the intrinsic Raman gain of silica fibre, doped only with germanium-oxide to enhance the nonlinear coefficient, is explored as a potential amplifier in a passively mode-locked laser. The obvious advantage of this approach is that gain is not restricted to the specific emission band of an active-ion, and combined with a suitable broadband saturable absorber could represent a new paradigm in ultrafast fibre laser technology.

Results presented in this chapter have been published in the following journal articles and conference proceedings [Cha11a, Kel10a, Kel10b, Cha11b, Cas11a, Cas11b].

3.1 Introduction

The principle of optical amplification depends on the ability to create an inverted population of excited electrons in a material, either using an electrical or optical pump (or supply of energy). The subsequent spontaneous decay of an electron, governed by a characteristic relaxation time, and emission of a photon with energy equal to the magnitude of the transition is unavoidable\(^1\). However, effective optical amplifiers have long relaxation times and can consequently store large amounts of useful energy. The passage of a photon through an inverted medium, with a resonant frequency, can result in the emission of a second photon through the stimulated decay of an excited-state electron. The ability to create a population inversion for laser action depends on the level structure of the medium and the frequency of the pump signal. Three common models exist to describe the lasing process: three level systems; quasi-three-level systems; and four level systems.

3.1.1 The basics: three- and four-level systems

In a three level system the lower lasing transition is the ground state; an absorbed pump photon promotes an electron to a higher energy level, which subsequently undergoes rapid non-radiative relaxation to the longer-lived upper laser level. In such a medium, for instance glasses doped with rare-earth active ions, net-gain is achieved when over half the ions are pumped into the upper laser level. Thus three-level systems typically exhibit high pump thresholds. Lower pump thresholds can be achieved using four-level systems, where the lower laser level has an energy higher than the ground state and rapidly de-

\(^1\)This is an over simplification for the purposes of the discussion: in the case of Yb:Er co-doping for example the excited Yb ion does not emit radiation.
populates through multiphonon transitions. Fig. 3.1 illustrates the transition dynamics involved in a four-level laser system. Nd-doped yttrium aluminium garnet (Nd:YAG) is possibly the most successful example of a four-level solid-state gain medium. Many rare-earth activated fibres, such as Yb and Er, are quasi-three level systems: the lower lasing transition lies very close to the ground state, such that in thermal equilibrium a considerable population occurs. This causes stimulated absorption for unpumped media and raises the transparency threshold.

### 3.1.2 Review of advances in fibre gain media

After the successful inclusion of rare-earth active ions in single-mode fibres [Poo85], compact, efficient and robust fibre lasers were demonstrated using single-mode fibre-integrated pump diodes [Mea85]. The development of double-clad fibre architecture allowed the use of low-cost, high-power multi-mode pump diodes, immediately facilitating a step increase in the available power, and marking the thermal superiority of a fibre scheme over bulk counter-parts. Rapid advances in the efficiency of diode laser technology has lead to similarly rapid advance in the power delivered by a fibre laser, with multiple kilowatts continuous-wave at 1 μm, and diffraction limited performance now possible from a Yb-doped system [Gap05].

The inclusion of Ytterbium ions (Yb$^{3+}$) in a silicate glass host was first reported in 1962 by Etzel et al. [Etz62]. However, early fibre laser research focused on Nd (typically the trivalent ion Nd$^{3+}$) and Er (typically Er$^{3+}$) dopants [GN89b, Mea87, Dig01, Des94]. Interest in Er-doping$^2$ of single-mode glass fibres is obvious: because of the need for in-band absorption cross section of Yb, leading to shorter pump absorption lengths and higher gain. The mechanism depends on efficient coupling of the energy from the excited Yb to the Er ion. This is now a widely

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$^2$It was quickly realised that co-doping Er active fibre with Yb ions could be used to exploit the larger absorption cross section of Yb, leading to shorter pump absorption lengths and higher gain. The mechanism depends on efficient coupling of the energy from the excited Yb to the Er ion. This is now a widely
amplification coincident with the loss minimum of telecom fibres [Mea87]. However, the strongest laser transition of Nd$^{3+}$-doped fibre provides gain in a region of the near-infrared (near-IR) overlapping with Yb-doped fibre, around 1.064 $\mu$m, forcing direct competition between the two technologies [Han88]. Nd$^{3+}$ possesses a purely four-level structure offering lower pump thresholds. Prior to the availability of high-power integrated pump diodes, fibre lasers were pumped with dye or Ti:sapphire systems, among others, around 800 nm. Nd$^{3+}$ absorbs strongly at 808 nm or 869 nm, but does not absorb at 980 nm, where Yb$^{3+}$ expresses strong absorption. Thus, Yb-systems can offer significant gain efficiency enhancements, when pumped with now commercially available diodes at 980 nm, due to the lower quantum defect, despite having a quasi three-level structure. In addition, due to the simple nature of the available transition states$^3$, high concentration Yb-doping can be applied with reduced quenching difficulties, leading to shorter amplifier lengths, particularly advantageous for ultrafast lasers where dispersion management is essential for short pulse operation. The high solubility of the Yb ion in glass$^4$ compared to Nd (and Er) also increases the doping homogeneity, reducing clustering that can also lead to quenching of the gain, due to ion-ion energy transfer. A long upper-state lifetime (1-2 ms) and a broad gain bandwidth contribute to the fact that Yb-doped fibre amplifiers have all but supplanted Nd-doped fibre systems [Pas97].

3.2 Bismuth activated fibres for ultrafast lasers and amplifiers

Yb- and Er- (or co-doped Er/Yb-) doped fibre amplifiers are, and will continue to be widely used. However, not all regions of the near-IR spectrum (from $\sim$0.8–2.0 $\mu$m) can be addressed with these media; and for specific applications certain wavelengths are required. The most obvious omission lies in the range 1.1–1.45 $\mu$m, coincident with the second telecoms window (at 1.3 $\mu$m), where loss in silica fibre is low and chromatic dispersion is weak: 1.27 $\mu$m being the material dispersion minimum (i.e. point of zero GVD). This section explores new avenues that present alternatives to current fibre-based tech-

$^3$The level structure of Yb$^{3+}$ involves only one excited state manifold $^2F_{5/2}$ when pumped by a near-IR photon from the ground-state manifold $^2F_{7/2}$. Amplification involves sub-levels of both the ground and excited state, and consequently it is considered a quasi-three-level system. It is worth noting that Yb-doped fibres can suffer from excessive photodarkening, limiting the useful lifetime of active fibres. Photodarkening is strongest at shorter wavelengths, and appears to depend on the doping density and excitation level.

$^4$Typically pure silica is not the sole host because of low solubility of rare-earth ions in the glass matrix. Low solubility leads to: clustering of ion dopants; quenching, i.e. large decrease in the lifetime of electronic levels of active ions; and low gain. Doping with other elements improves solubility of the glass. Alumino-silicate, germano-silicate, phospho-silicate and borosilicate are among the common hosts [Dig01].
nology operating in the 1.1–1.4 μm band, such as praseodymium-(Pr$^{3+}$) doped fluoride fibre amplifiers [Car91, Ohi91], struggling to receive widespread acceptance or commercial validation, due in part to issues with integration of fluoride fibres with silica-fibre componentry, among other unfavourable characteristics.

3.2.1 Review of early results from the literature

Near-infrared luminescence from Bismuth-(Bi) doped silica glass was first observed by Fujimoto et al. in 2001 [Fuj01] (Fig. 3.2b). This initial discovery promoted further research, resulting in the demonstration of broadband infrared luminescence in many other glass hosts, such as germanate, phosphate and borate [Pen04, Men05, Pen05, Suz06, Ren07c, Ren07a, Ara07, Ren07b]. Different groups have tentatively assigned this near-IR luminescence phenomena to the electronic transition of Bi$^{3+}$, Bi$^{+}$, Bi$^{2+}$ or to clusters of Bi atoms [Sun09]. To date experimental evidence remains inconclusive, but studies of Bi-doped crystals, where spectroscopy of active ions is more clearly expressed due to the definite majority of doping sites, suggest that near-IR emission should be from Bi$^{+}$ infrared active centres, instead of others [Sun09]. Recently it has been shown by Sharonov et al. that near-IR fluorescence is not specific to solely Bi-ions [Sha08]. Other 6p and 5p ions, such as Pb, Sn and Sb, exhibit similar behaviour when excited at 514, 680, 810 and 980 nm, indicating similar active centres. In all cases aluminium (Al) was used as a codopant and was found to significantly enhance the fluorescence in the germanate glass host [Sha08]. This study indicates the potential of extending the range of element-doped glasses available for CW and pulsed laser operation in the near-IR region of the electromagnetic spectrum.

Favourable properties of Bi-doped glass fibre include: a broad emission band (up to 400 nm [Ara07]) in the region 1.1–1.4 μm; a broad-band absorption spectrum (∼0.5–1.1 μm), with four dominant absorption peaks (Fig. 3.2a) at 300 nm, 500 nm, 700 nm and 800 nm, coincident with many well establish pump sources; and a long luminescence lifetime (∼1 ms [Dia05]). Such properties make Bi-doped glasses a potentially attractive gain medium, particularly for the generation and amplification of short pulses [Fir11]. Although Pr-doped optical fibre also operates in this wavelength window, with Pr-doped fluoride fibre amplifiers receiving significant attention [San96], fluoride glass is weak both in chemical durability and mechanical strength [Men05] and remains problematic to integrate with silica fibre technology. In contrast, Bi-doping can not only be embedded into a silicate glass host, but can also be drawn into a Bi-doped silica glass fibre [Dvo05]. Dvoyrin et al. were the first to move from bulk glasses and demonstrate the potential of similar luminescence behaviour in silica glass optical fibres [Dvo05], by doping the core with Bismuth using modified chemical vapour deposition (MCVD). They reported broad-
3.2 Bismuth activated fibres for ultrafast lasers and amplifiers

![Figure 3.2: The absorption (3.2a) and emission (3.2b) spectrum of the Bi-doped pure Silica glass made by Fujimoto et al., optically excited at three wavelengths: 500 nm; 700 nm; and 800 nm. Adapted from [Fuj01].](image)

band emission of 200 nm FWHM, with the peak in the region 1.1–1.2 µm. This confirmed that such fibres were good candidates for CW and pulsed laser sources and amplifiers in the spectral range 1.1–1.4 µm.

Dianov et al. experimentally confirmed the use of Bi-doped fibre as a new optical amplifier with the first report of a CW Bi-doped aluminosilicate glass fibre laser in 2005, obtaining slope efficiencies as high as 14.3% [Dia05], with emission at 1.215 µm. The active fibre was pumped using a Nd:YAG laser at 1.064 µm, with the resonator formed by two fibre Bragg gratings (FBGs) written into germanosilicate fibre. However, the broadband absorption spectrum of Bi-doped fibre permits pump sources including Yb-doped fibre lasers, leading to the possibility of significantly more efficient all-fibre Bi-doped lasers. Such systems were realised by Dianov et al., Razdobreev et al. and Rulkov et
in 2007 [Dia07b, Raz07, Rul07]. All employed a linear cavity configuration, with feedback and lasing wavelength selection provided by FBGs, one acting as a high reflector and pump coupler and one providing output coupling (optimum ∼50%). Similar lengths of Bi-doped fibre (∼80 m), fabricated using MCVD, with similar absorption and luminescence profiles, were fusion spliced into the cavity with quoted losses as low as 0.1 dB. Record optical-to-optical efficiencies of 24% for lasers of this type were reported [Raz07], and the potential for frequency doubling into the visible was demonstrated [Dia07b, Rul07].

Despite successes, Bi-activated glass fibre is not yet a mature technology. One limitation that restricts the efficiency is the high unsaturable loss that appears to be temperature dependent [Dia07b]. Cooling the active fibre to 77 K, using liquid nitrogen (LN₂), could provide a means of accessing more gain and result in a marked increase in system performance. Until 2008, all Bi-doped fibres had been pulled from preforms fabricated by a thermodynamically equilibrated MCVD method. Motivated by the production of Er-activated fibre preforms, Bufetov et al. proposed an alternative synthesis method: surface-plasma chemical vapour deposition (SPCVD) [Buf08]. Despite a reduced efficiency, lasing in the reported Bi-fibre laser was achieved with significantly reduced lengths of active fibre than had been previously possible, due to an order of magnitude increase in the number of active Bi centres, without significant clustering. The promise of shorter gain fibres extends the range of possible applications of Bi active fibres beyond use in the CW regime, in particular for the generation and amplification of short pulses.

### 3.2.2 Amplification of picosecond pulses in Bismuth-doped fibre amplifiers

In this section the single-pass amplification of picosecond (ps) pulses in single-mode Bi-doped fibre amplifiers is demonstrated for the first time. In addition, it is confirmed that Bi active fibre can also support the amplification of high-frequency modulated signals, thus proving its potential as a possible candidate for deployment in future telecommunication networks.

#### Experimental setup

Two Bi-doped fibres were examined: both aluminosilicate glass (core composed of 97 mol.% SiO₂ and 3 mol.% Al₂O₃), with a core diameter of 8 μm and a core/cladding refractive index contrast of 5 × 10⁻³, fabricated using a surface-plasma chemical vapour deposition (SPCVD) process [Bu08]. The fibres differ in their core doping concentration of Bi active centres. The first (hereafter referred to as fibre 1) has a core Bi center content of
3.2 Bismuth activated fibres for ultrafast lasers and amplifiers

0.002 mol.%, and the second (hereafter referred to as fibre 2) has a content of 0.004 mol.%. Fibre 2 also has a significantly reduced OH content (as can be seen by the enhanced transmission around 1.45 µm in Fig. 3.3). The attenuation spectra for both fibres is shown in Fig. 3.3; the specifics of the fabrication technology, absorption spectra and near-infrared luminescence can be found in Ref. [Buf08] and [Gol10] respectively.

![Attenuation spectra for the two Bi-doped aluminosilicate fibres.](image)

**Figure 3.3:** Attenuation spectra for the two Bi-doped aluminosilicate fibres.

Measurements of the gain was conducted by counter-pumping the active fibre using a commercial CW Yb fibre laser operating at 1.06 µm with a maximum output power of 6.5 W. The experimental setup is shown in Fig. 3.4; the amplifier was constructed from a 30 m length of active fibre, fusion spliced at either end to wavelength division multiplexers (WDMs) for pump combination and extraction.

![Schematic of the Bi-active fibre amplifier unit (BiDFA), core pumped through a fused fibre WDM with a CW Yb-doped fibre laser.](image)

**Figure 3.4:** Schematic of the Bi-active fibre amplifier unit (BiDFA), core pumped through a fused fibre WDM with a CW Yb-doped fibre laser.

A spectrally filtered ps pulse-pumped supercontinuum (SC) source was used to provide a train of ps pulses selectable within the gain bandwidth of both Bi fibre amplifiers (Fig. 3.5a). The SC source was constructed from a low-power commercial ps mode-locked oscillator (operating at ~1065 nm), the output of which was amplified in an Yb-doped fibre amplifier (YDFA) and directly fusion spliced to a 60 m length of PCF, with a zero dispersion wavelength (ZDW) of 1038 nm, providing low anomalous dispersion at the pump wavelength, and a nonlinear coefficient $\gamma(\lambda = 1060 \text{ nm}) = 11 \text{ W}^{-1} \text{ km}^{-1}$. The
fibre output from the SC source was collimated and spectrally filtered in an air-gap, with a band-pass filter centered at either 1160 or 1180 nm, and recoupled to fibre. The SC source provided signal levels with average spectral powers of -4 and -7 dBm respectively, suitable for two independent small-signal gain saturation measurements within the gain bandwidth of both Bi-doped fibre amplifiers. The output spectrum, and corresponding filtered spectral profiles of the SC source are shown in Fig. 3.6. Autocorrelations of both selected wavelength ranges were taken (shown in Fig. 3.7a and 3.7b) indicating an average full width half maximum (FWHM) pulse duration of 2.5 ps for the 1160 nm band and 1.6 ps for the 1180 nm band.

**Results**

The small-signal gain was measured for both wavelengths in both fibres by recording the spectrum and total output power for each pump level, so that the output power could be integrated within the -3 dB width of the input spectrum. As previously noted in [Dvo06], and subsequently in [Kal09], the performance of Bi-doped fibre amplifiers is heavily influenced by fibre temperature. Subsequently, Gumenyuk et al. reported cryogenic cooling of Bismuth-doped fibre forced four-level laser behaviour [Gum11], and was necessary to obtain sufficient gain in the 1.18 μm band for the development of laser systems using these active fibres. Accordingly, the optical gain was measured with the fibres cryogenically cooled in a liquid N₂ bath (~77 K), and also at room temperature (~300 K) for
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\[ \lambda_c = 1.16 \mu m \]
\[ \lambda_c = 1.18 \mu m \]

\textbf{Figure 3.6:} Pulse-pumped supercontinuum generated in a 60 m length of PCF; and the corresponding filtered spectral profiles used as the signal inputs for the small-signal gain saturation measurements of the two Bi-doped fibre amplifiers.

The small-signal gain saturation curves are plotted in Fig. 3.8. In the case of fibre 1, the gain saturates above \( \sim 2.5 \) W pump power for both signal wavelengths, giving a maximum small-signal gain of 21.2 dB and 15.7 dB for 1160 nm and 1180 nm respectively, when cryogenically cooled. At room temperature, the corresponding maximum gain is 6.3 dB and 5.5 dB. At maximum pump power, gain in fibre 2 under cryogenic cooling was not fully saturated, with peak values of 21.8 dB and 16.1 dB for 1160 nm and 1180 nm respectively. At room temperature, the gain in fibre 2 saturates before the amplifier reaches transparency. This suggests that there is a distinct change in the electronic properties of the active centres when cryogenically cooled.

Autocorrelations of the output pulses, when the amplifier was operating in the saturated regime under cryogenic cooling, are shown, along with the autocorrelations of the input pulses, in Fig. 3.7. All traces have been fitted with a sech\(^2\) function, from which the inferred FWHM pulse duration can be extracted.

In both amplifier fibres, the pulses were subject to normal dispersion, and were broadened to over 10 ps in duration. A significant spike is apparent on the output autocorrelations. This is attributed to the amplification of non-solitonic radiation generated in the supercontinuum, which acquires an anomalous chirp in the PCF, and is subsequently compressed as it is amplified in the normally dispersive Bi-doped amplifier fibre.

The gain of the amplifier for a high-frequency, modulated input signal was characterised using a CW Raman fibre laser operating at 1178 nm (shown in Fig. 3.5b), modulated at 10 GHz using a fibre-pigtailed Mach-Zehnder amplitude modulator. The Raman
Figure 3.7: Autocorrelations of input and output pulse from each amplifier fibre, at 1.16 μm and 1.18 μm, under maximum pump power.
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Figure 3.8: Small-signal gain saturation curves for picosecond pulse input.

Laser was based on an Yb-doped fibre laser pumping a length of HNLF, with normal dispersion at both the pump wavelength and at 1178 nm. Cascaded feedback of multiple Stokes orders results in efficient transfer of energy from the pump to a Stokes shifted frequency. Here, two narrow-band, but highly reflecting FBGs at the first Stokes shift form a high-Q linear cavity around the HNLF. A second pair of FBGs, with unbalanced reflection, at the second Stokes shift forms a partially transmitting resonant cavity sufficient to initiate laser action around the second Stokes wavelength of 1178 nm. A band pass filter was employed to suppress residual pump/Stokes lines in the output of the Raman laser. This CW signal at 1178 nm was then modulated using a Mach-Zehnder amplitude modulator (MZAM) to produce a 10 GHz sinusoidal signal with an average power of -6 dBm. The small-signal gain saturation curves for the two Bi-doped fibres were measured, as described above, and are shown in Fig. 3.9. The gain in fibre 1 was found to saturate above ~2 W at 14.4 dB, whilst the gain was not saturated in the case of the fibre 2, with a peak at
17.7 dB. The temporal profile of the high-frequency pulse train before and after amplification is shown in Fig. 3.10 for the two fibres, respectively. What is clear is the sinusoidal signal can be amplified with no noticeable distortion to the high-frequency waveform.

These measurements confirm that Bi activated aluminosilicate-based fibre is a suitable medium for the amplification of short pulses, down to ~1 ps in duration, corresponding to a bandwidth of ~1.5 nm at 1.2 μm. As such, this technology can be effectively used in the development of ultrafast mode-locked lasers operating at wavelengths unavailable with other fibre-based silicate-glass amplifiers.

It is well known that the fluorescence bandwidth of Bismuth active fibre is extremely broad (up to hundreds of nanometers). If the entire bandwidth could support the amplification of short pulses, it would represent real potential for generating very short pulses.
from Bi-doped fibre-based oscillators. However, little is known about the nature of the active centres involved in the lasing process, and the homogeneity of the medium remains unclarified. Experiments are needed to explore further the nature of the coupling between regions of the gain spectrum. In the following sections preliminary experiments, demonstrating the use of Bi-doped fibres in mode-locked lasers, are presented.

### 3.2.3 Bismuth-doped all-fibre soliton laser

Bismuth doped fibre systems, passively mode-locked using a semiconductor saturable absorber mirror (SESAM), have been reported in Refs. [Dia07a, Kiv09a, Kiv10]. The first CW mode-locked Bi-doped fibre laser, reported by Dianov et al. in 2007 [Dia07a], generated pulses with a duration of \( \sim 50 \) ps, limited by the narrow-band FBG used for pulse stabilisation, and the normal GVD of the cavity. Kivistö et al. used a broader bandwidth chirped FBG (CFBG) to compensate the normal-dispersion, achieving soliton pulses with a duration approaching 1 ps [Kiv09a]. The advantages of using nanomaterials, such as single-wall nanotubes, embedded in polymer-composite films for saturable absorption in passively mode-locked lasers was outlined in the previous chapter. In this section a SWNT-based saturable absorber is used to mode-lock a Bi-doped fibre ring laser, for the first time.

In Bi-doped lasers emitting below 1.3 \( \mu \text{m} \) the cavity GVD at the wavelength of operation is inherently normal. Consequently, dispersion compensation is needed to achieve soliton-like operation, for the generation of near transform-limited pulses. A number of schemes are available to manage the cavity dispersion. Bulk gratings, or prism pairs can be used to provide a variable amount of anomalous dispersion (dependent on the separation of the optical elements), but lose the advantage of a fibre format. Through careful control of the waveguide contribution to the overall dispersion, a PCF can be designed to possess a zero-dispersion wavelength (ZDW) well below 1.27 \( \mu \text{m} \), the natural material ZDW point of silica. While this retains the fibreised approach, coupling in and out of PCF fibres, with small cores (and a large mode-field mismatch) and delicate cladding structures that collapse under excessive heating, can result in high insertion losses. Such loss can only be tolerated in fibre-based systems, where the roundtrip gain is high. However, in addition to dispersion PCFs typically have a strong nonlinearity which can be unhelpful for achieving stable mode-locked operation. PCFs with a hollow-core, that confine light using a photonic band-gap, overcome the problem of high nonlinearity, but again suffer from problems with coupling to and from other fibres, not least because of the index change from a glass to air core. A FBG with an aperiodic index modulation (resulting in a shift of the Bragg wavelength with position) can be fabricated with a specific chromatic dispersion profile. This element can be introduced into a fully fibreised cavity,
without significant insertion loss, to compensate the normal GVD.

The term soliton laser is widely used to describe a class of ultrashort pulse lasers generating near transform-limited pulses, generally requiring some form of dispersion compensation. Soliton lasers have been intensively studied since the first demonstration by Mollenauer and Stolen in 1984 [Mol84], and represent an elegant means of counteracting the nonlinear phase accumulated by an ultrashort pulse, resulting in the emission of the shortest pulses supported by the effective bandwidth of the amplifying cavity. Given that the luminescence spectrum of Bi active fibre is broad compared to other active fibre technologies, it is expected that Bi-doped fibre lasers could support the generation of very short pulses if the dispersion of the cavity is correctly compensated. The proceeding experiment represents preliminary investigations into the suitability of Bi-doped fibre for the development of ultrashort pulse soliton lasers.

**Experimental setup**

The configuration of the Bi-doped fibre mode-locked laser is outlined in Fig. 3.11, containing the main elements common to all ultrafast passive fibre systems: gain, intensity dependent loss, and feedback. The intensity dependent loss is provided by a SWNT-based saturable absorber. It has already been established that for semiconducting nanotubes the size of the band-gap defines the resonant wavelength where saturable absorption exists. The size of the band-gap is controlled in several ways, for example by changing the growth methods and conditions [Has09]. In partnership with our collaborators at the University of Cambridge, the SWNT-SA was optimised for operation at the laser wavelength of 1.178 µm. Although this was not coincident with the peak of the gain of the Bi-doped fibre – the previous section clearly demonstrating that both fibres tested exhibited higher gain at 1.16 µm – FBGs were only available at 1.178 µm. To match the laser wavelength SWNTs with ~0.9 nm diameter are needed [Wei03]. CoMoCAT\(^5\) SWNTs

\(^5\)CoMoCAT is a catalytic process for the synthesis of nanotubes developed specifically to obtain a high-yield of highly selective single-wall structures, with a small distribution of tube diameters, by inhibiting the rapid sintering of Cobalt (Co) that occurs at the high temperatures required for the formation of nan-
3.2 Bismuth activated fibres for ultrafast lasers and amplifiers

with the correct diameter were selected to fabricate a SWNT-polyvinyl alcohol (PVA) composite, as described in Ref. [Has09] and references therein. The linear transmission spectrum of the composite is shown in Fig. 3.12a. A strong absorption peak at ~1.178 μm is evident, corresponding to the first transition (E_{11}) of (7,6) SWNTs [Wei03]. Another band at ~1.028 μm is also present, assigned to the first transition of (6,5) SWNTs [Wei03]. The corresponding E_{22} transitions for both identified species present weak absorption features around 600 nm, in the visible region. The nonlinear optical properties of the device, excited at 1.065 μm, are shown in Fig. 3.12b, measured using the z-scan approach described in the previous chapter. The modulation depth at 1.065 μm is ~16%. Given that the linear absorption is equivalent at 1.18 μm, it is reasonable to assume that the modulation depth will be at least 16% at 1.18 μm. The SWNT-SA film was integrated into the cavity using the standard butt-coupling approach: sandwiching the substrate between two APC fibre connectors.

A 30 m length of Bi-doped active fibre (denoted fibre 1 in the previous section), core-pumped through a custom wavelength division multiplexer (WDM) using a commercial 10 W Yb-doped fibre laser, provided gain around 1.18 μm. Residual pump light was coupled out of the cavity with a second WDM. The alumosilicate-core Bi-doped fibre preform was fabricated using an SPCVD process and drawn into a single-mode fibre compatible with Corning HI-1060 (Flexcore) to facilitate direct fusion splicing to passive cavity components with low loss: typically less than 0.1 dB. Such a long length of active fibre was needed because of low pump absorption and a low gain coefficient [Kiv09a, Seo07]. In addition, it was necessary to cryogenically cool the active fibre to access enough gain to overcome the losses, as the gain is strongly temperature dependent, as illustrated by the measurements in the previous section.

Unidirectionality was imposed by a fibre-pigtailed optical circulator, which also acted as a means of incorporating the CFBG into the cavity for compensation of the normal cavity GVD. A fused-fibre output coupler extracted 5% of the laser light per roundtrip. Typically fibre laser systems can support much higher output coupling ratios, however the high-Q cavity was necessary for the system to lase due to the low roundtrip gain at 1.18 μm (~16 dB).

Results

Self-starting, fundamental CW mode-locking was achieved, with a repetition rate of 5 MHz defined by the round-trip time of the long cavity (~40 m). The mode-locking threshold was reached for a pump power of ~200 mW. The average output power was typically

---

...nanotubes, using a Molybdenum (Mo) catalyst. Large Co clusters, resulting from uncontrolled sintering, results in the production of multi-wall nanotubes and graphite.
Figure 3.12: 3.12a SWNT-SA linear transmission spectrum. The MLL operation wavelength is indicated with a vertical blue dotted line. The vertical red dotted line illustrates the excitation wavelength of 1.065 µm used in the nonlinear saturation measurement of the sample shown in 3.12b.

10–15 µW, corresponding to pulse energies of ~3 pJ. Fig. 3.13a shows the intensity autocorrelation trace of the output pulses, fitted with the autocorrelation shape expected for a sech² pulse. The corresponding spectrum, centered at 1177 nm (defined by the pass-band of the CFBG) expressing prominent solitonic sidebands and a full width half maximum (FWHM) bandwidth of 0.35 nm, is plotted in Fig. 3.13b. The spikes on the long-wavelength edge can be attributed to high-order dispersion due to the CFBG. The deconvolved pulse duration (calculated from the sech² autocorrelation function) was 4.7 ps, giving a time-bandwidth product of 0.36, near transform-limited for a sech². The
3.2 Bismuth activated fibres for ultrafast lasers and amplifiers

**Figure 3.13:** Temporal and spectral properties of the Bi-doped fibre soliton laser.

(a) Autocorrelation. 
(b) Optical spectrum. 
(c) Electrical spectrum.
achieved pulse width is limited by the intracavity dispersion: given the soliton relation

\[ T_0 = \sqrt{\frac{|\beta_2|}{\gamma P_0}} \] (3.1)

where the FWHM duration of the soliton \( \tau = 1.76T_0 \), for large values of \( \beta_2 \) the FWHM duration of the soliton pulse \( \tau \) becomes longer. Sub-picosecond pulses should be obtainable with more balanced dispersion compensation. Although reliable CW mode-locking was achieved for a fixed pump power, because of the low gain of the active fibre and relatively high modulation depth of the SWNT-SA (\( \sim 16\% \) at 1.065 µm) the system would become unstable with elements of Q-switching for increased pump power.

The radio frequency (RF) spectrum of the fundamental mode-locking harmonic is shown in Fig. 3.13c. The peak to pedestal extinction is at least 50 dB at the resolution limit of the device (30 Hz), and limited by the noise floor of the RF analyser. The linewidth is again device limited, indicating low temporal inter-pulse jitter. Measurement of the pulse train on a 400 MHz analogue oscilloscope confirmed stable CW mode-locking in the average-soliton regime, with no signs of transient effects nor Q-switching.

This system represents a necessary development step in the realisation of efficient, short-pulse lasers based on Bismuth fibre technology, and operating in the second telecoms window. However, because of the low average output power, the low peak pulse power limits the use in many practical applications. High-power picosecond and femtosecond fibre systems are very attractive, but such sources are non-trivial to realise in practice. One difficulty arises due to the effect of dispersion on a short pulse propagating in several meters of fibre. Secondly, the relatively large nonlinearity of a fibre amplifier compared to a bulk solid-state competitor can cause the pulse to broaden, distort or even break-up (fragment in time). The combination of self-phase modulation (SPM) based nonlinear spectral broadening and dispersion can result in the rapid increase of the pulse duration even for several picosecond pulses. Another challenge is posed by stimulated Raman scattering (SRS), which results in wavelength shift, energy loss, noise build-up and waveform distortion of the high-power pulses after the SRS threshold is reached.

To control nonlinearity and dispersion in fibre systems and access high peak powers, the setup for high power ultrashort pulse generation is conceptually divided into at least three parts. The first part typically consists of a stable, low-power mode-locked fibre laser. While a mode-locked fibre laser can scale well itself (this issue will be discussed further in chapter 5), using a carefully managed dispersion map or large mode area (LMA) fibres for reduced nonlinearity, due to the relative complexity of the mode-locking dynamics this approach is not often preferred. Thus, for improved laser performance a booster fibre am-

\[ \text{80} \]
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The pulse compressor forms the third part of the system. The mode-locked seed, pulse amplifier and output compressor can be regarded as building blocks of a basic high-power ultrafast laser system. In the proceeding section the development of a Bi-doped MOPFA is considered and preliminary demonstrations are discussed.

3.2.4 Bismuth-doped all-fibre MOPFA

Master oscillator power fibre amplifier (MOPFA) schemes are a proven route to power scale mode-locked fibre lasers, and is a commonly used approach in the Ytterbium and Erbium gain bands, having obvious applications, such as frequency conversion, where a high duty factor is desirable to take advantage of the peak-power dependence inherent to the nonlinear process. The applicability of Bi-doped fibre for MOPFA schemes was demonstrated in the previous two sections. Here the output of a stable, low-power mode-locked oscillator is amplified in a Bi-doped fibre amplifier, and used to pump an unoptimised length of periodically poled Lithium Niobate (PPLN) crystal to demonstrate its potential for frequency doubling.

Experimental setup

An overview of the MOPFA system is shown in Fig. 3.14a. The first stage comprised a SESAM-based mode-locked oscillator, operating at 1177 nm and shown schematically in Fig. 3.14b. A linear cavity was preferred over a ring cavity as it minimised the round-trip losses allowing the use of a shorter length of active fibre (~5 m), and resulted in robust CW mode-locked operation, initiated and maintained by the SESAM. Although SWNTs embedded in polymer composite films have been widely used as the saturable absorber element in ultrafast lasers described in this thesis, for linear cavities a SESAM is more convenient as it can also be used as the highly reflecting end mirror\(^6\). Details of the SESAM are given in Refs. [Dia07a, Kiv08]. The 5 m length of alumosilicate Bismuth-doped fibre (denoted fibre 2 previously) was end pumped at 1.06 \(\mu\)m through a CFBG high-reflector, which also acted to provide strong anomalous dispersion to the otherwise normally dispersive cavity for operation in the average-soliton regime. The pump laser was a com-

\(^6\)A SESAM with a specific resonant absorption and a strongly reflecting dielectric Bragg mirror at the operating wavelength was designed and fabricated by collaborative partners at Tampere University of Technology, Finland.
3 Ultrafast fibre laser technology part 2: novel gain media

Figure 3.14: Configurations of: 3.14a the MOPA system; 3.14b the mode-locked oscillator; 3.14c the power fibre amplifier unit. L1, lens one; PPLN, periodically-poled lithium niobate; BS, beam-splitter; D, detector; all other acronyms previously defined.

A WDM, placed centrally in the cavity, extracted \( \sim 10\% \) of the laser light per pass and provided two output ports, from which the pulses could be simultaneously monitored and coupled directly to the next stage. A two-stage amplifier scheme was employed: comprising a pre-amplifier; and a power amplifier. Each stage consisted of a core-pumped BiDFA, shown in Fig. 3.14c, and similar in essence to the amplifier constructed and tested in section 3.2.2. Both featured a 30 m length of Bi-doped alumsilicate fibre, counter pumped by a 10 W CW Yb-doped fibre laser. Fused fibre WDMs were used for pump combination and extraction. The pre-amplifier was based on fibre 1 from section 3.2.2,

\(^7\)Cryogenic cooling enhanced the operation of the seed oscillator, but was not essential to mode-locked operation in this case due to reduced round-trip losses over the ring cavity described in the previous section.
and was pumped with 2 W for operation in the saturated regime. The second amplifier stage was based on fibre 2 from section 3.2.2, and was pumped with 5 W.

Two optical circulators were used in conjunction with a matched pair of narrow-band (unchirped) FBGs, with 0.2 nm passband and highly reflective (HR) at 1177 nm. The first optical circulator prevented backwards propagating amplified spontaneous emission (ASE) generated in the pre-amplifier perturbing the stable operation of the seed oscillator. In addition, the narrow-band FBG spliced to port two of the circulator (see Fig. 3.14a) reflects only the centre of the pulse spectrum, rejecting the spectral wings that contains dispersive wave components generated due to perturbations of the soliton pulse in the seed resonator and the high-order dispersion introduced by the highly chirped FBG that provides compensation of the cavity dispersion. The action of the spectral filter also results in a broadening of the pulse in time that lowers the pulse peak power and reduces the amount of nonlinear spectral broadening introduced in the pre-amplifier that can degrade the pulse profile.

The second circulator introduces isolation between the pre-amplifier and the power-amplifier, again to suppress unwanted backward propagating ASE generated in the power-amplifier stage. In this configuration the amplifier is double-passed to maximise the effective gain length, and amplification of the pulse over noise components, by using a second matched FBG to reject ASE accumulated in the first pass of the power-amplifier. Finally, a polarisation controller on the output allows control of the polarisation state.

Results

The background-free intensity autocorrelation of the pulses generated in the seed oscillator is shown in Fig. 3.15a. A sech\(^2\) fit implies a deconvolved FWHM pulse duration of 3.9 ps. Figure 3.15b shows the corresponding output spectrum (both pre and post filtering), where discrete spikes on the long-wavelength edge can be attributed to third-order dispersion (TOD) introduced by the CFBG [Hau93]. Also evident are sidebands characteristic of deviation from operation in the average-soliton regime [Kel91, Kel92]. The spectrum post filtering (i.e after passage through the first circulator) is shown in red in Fig. 3.15b; it is clear the non-solitonic components have been heavily suppressed. Without this strong filtering, the gain provided by the pre- and power amplifier is heavily depleted by the dispersive components, and the pulse becomes effectively swamped. In addition, the filtering acts to both suppress ASE and temporally broaden the pulse in time, thus increasing the threshold for the onset of strong SPM induced nonlinear spectral broadening and Raman Stokes scattering that occurs in the long amplifier fibres. The radio frequency (RF) power spectrum of the fundamental harmonic of the seed cavity is plotted in Fig. 3.15c. A high peak-to-pedestal suppression ratio of 60 dB indicates good
Figure 3.15: Temporal and spectral properties of the low-power Bi-doped seed oscillator.
stability of the pulse train and the narrow width of the beat note suggests low pulse-to-pulse temporal jitter. Higher harmonics are shown in Fig. 3.16.

**Figure 3.16:** RF spectrum of the seed oscillator showing the harmonic frequencies of the cavity round trip time.

Under 5 W of pumping the final stage amplifier delivered an average output power of 150 mW. The temporal and spectral profiles of the output pulses are shown in Fig. 3.17. The extra-cavity spectral filtering resulted in an increase in the duration of the pulses to 28 ps, corresponding to a peak power of 580 W. Due to spatio-temporal structure (i.e. a time-dependent spectrum) in the pulses emitted from the oscillator (caused by perturbations to the soliton pulse), the second-harmonic (SHG) intensity autocorrelation also exhibits structure after spectral filtering. This structure is initially masked, but selecting a narrow-band of frequencies results in selecting only a portion of the pulse in time, resulting in the appearance of wings on the output autocorrelation trace. This highlights a limitation of SHG autocorrelation for the characterisation of complex temporal waveforms, other techniques such as frequency resolved optical gating overcome some of these short-falls [Tre93], but are more complex measurements to conduct.

The spectral profile shows clear nonlinear spectral broadening (attributed to SPM) due to the relatively high-peak powers in the long lengths of amplifier fibre (total length ~100 m due to the double-pass configuration of the second stage). The inset to Fig. 3.17b shows the output spectrum on a broader span, with the low level contribution from the ASE and the undepleted back-scattered pump around 1.06 μm.

**Frequency doubling in PPLN**

Light sources operating at the wavelength of 1178 nm have been developed previously because the doubled frequency of 589 nm coincides with a narrow atomic absorption line of sodium that exists in the upper atmosphere. As such, lasers operating at this
wavelength have been used by astronomers to excite artificial stars in the sky to use as a light source for the correction of ground-based adaptive optic systems – this technique is known as laser guide star. Typically dye-laser systems have been deployed for this purpose. Recently, alternatives have been suggested [Geo05], the most successful of which is based on a narrow-linewidth, CW diode laser seed amplified in multiple fibre-based Raman amplifiers [Fen09]. However, Bi-doped fibre laser have also been proposed [Rul07], but doubling efficiencies have been limited by low-peak power and relatively broad spectral linewidths. The enhanced peak power obtained using the MOPFA architecture means doubling to the orange part of the visible spectrum should be possible with improved efficiencies, despite the typically broader linewidth due to pulsed operation.

To demonstrate the potential of the Bi-doped fibre MOPFA for the realisation of visible sources an unoptimised, 20 mm long PPLN crystal was used to frequency double the output, which was collimated using a 3.3 mm focal length lens. The collimated output was focused into the crystal with an 88 mm focal length lens, giving a focal beam waist of 44 µm and a confocal length of 10 mm, shorter than the physical length of the crystal. Both lenses and the input face of the crystal were anti-reflection coated at the fundamental wavelength of 1177 nm. The output face of the crystal was uncoated. Phase matching was achieved through temperature control of the crystal (polling period 9.27 µm, phase-matched temperature 106.6°C), and a fibre strainer polarisation controller was used to ensure that the arbitrary polarisation state of the MOPFA output was optimally polarised along the z-axis of the crystal.

The frequency doubled power at ∼589 nm is shown as a function of fundamental signal power in Fig. 3.18. The relation is seen to have an initially quadratic dependence on
3.2 Bismuth activated fibres for ultrafast lasers and amplifiers

![Figure 3.18: Frequency doubling power curve.](image)

**Figure 3.18:** Frequency doubling power curve.

![Figure 3.19: Spectral and spatial properties of the frequency doubled output.](image)

**(a)** Frequency doubled spectrum.  
**(b)** Far-field image.

**Figure 3.19:** Spectral and spatial properties of the frequency doubled output.

Fundamental power, but above ~80 mW the doubled power scales linearly with the fundamental. This is due to the increased spectral broadening of the MOPFA output through SPM with increasing output power, which reduces the proportion of the fundamental spectrum that is within the phase-matched bandwidth of the crystal. The corresponding spectrum of the second harmonic radiation, recorded using an optical spectrum analyser, is shown in Fig. 3.19a. The peak of the spectrum is located at 588.75 nm, which agrees with the location of the peak of the MOPFA output spectrum at 1177.5 nm. The FWHM of the doubled spectrum is 0.16 nm, implying a phase-matched bandwidth of 0.32 nm at the fundamental frequency. No direct $M^2$ measurement of the beam profile was conducted, but a far-field image of the beam pattern, shown in Fig. 3.19b, suggests the power is concentrated within a largely circular mode.
The maximum doubled power was 13.7 mW, corresponding to a conversion efficiency of \( \sim 9\% \), with respect to the total fundamental power. This figure is competitive with previously reported results [Rul07]. Improved efficiency could be achieved using a shorter crystal with a broader phase-matching bandwidth. A further increase in frequency conversion efficiency can be realised by implementing the Bi-doped MOPFA in a polarisation maintaining configuration. In addition, the application of large-mode area (LMA) fibre technologies, already established and widely used with rare-earth doped fibre amplifiers, are readily applicable to Bi-doped silica fibre amplifiers. The use of such schemes would enhance the performance of Bi-doped MOPFA systems and move them closer to becoming a viable technology.

Bi-doped fibre expresses gain in the second telecoms window because of an active dopant, that absorbs and re-emits pump light. Under intense optical excitation un-doped glass fibres respond nonlinearly, with both an instantaneous and delayed (absorptive) contribution. Raman scattering is an absorptive nonlinearity resulting in the downshifting of pump photons to a frequency determined by the transition to a higher vibrational energy state of electrons in a molecule, mediated by the generation of an optical phonon. This so-called Stokes radiation, as a result of Raman scattering, can be employed to provide gain at unconventional wavelengths, including the second telecoms window. This technique has been successfully used to augment telecom networks. The use of Raman gain in ultrafast lasers is explored in the following section, offering the potential for fully wavelength independent short pulse sources.

### 3.3 Exploiting intrinsic fibre nonlinearity: an ultrafast laser based on Raman gain

Nonlinear effects in fibre are well understood. They are controlled and enhanced to manipulate the content of the light which propagates inside the waveguide, such that a high degree of temporal and spectral modification can be exercised. In the closing chapter of this thesis supercontinuum generation is discussed: the generation of a broad spectrum (or continuum) of frequencies, while preserving spatial (and at times temporal) coherence [Dud06, Dud10]. Perhaps the most notable result in the field of nonlinear fibre optics was the observation of temporal optical solitons in fibre, reported in the seminal contribution by Mollenauer et al. in 1980. Subsequently, there has been many advances in the field of nonlinear fibre optics and the development of short pulse sources [Agr01, Tay92, Agr07].

In this chapter the ultrafast fibre laser, from the perspective of the optically amplifying medium, has been discussed, and a novel technology has been considered. In the final
3.3 Exploiting intrinsic fibre nonlinearity: an ultrafast laser based on Raman gain

Section of this chapter the use of Raman gain for the development of ultrashort pulse passively mode-locked fibre lasers is proposed.

3.3.1 The basics: Raman amplification in silica fibre

The effect of spontaneous Raman scattering in silica glass fibres was introduced in chapter 1. The process of Raman amplification is based on stimulated Raman scattering (SRS): if two beams of different wavelength (but equal polarisation) propagate through a Raman active medium, the longer wavelength signal can experience amplification at the expense of the shorter wavelength signal; this process is mediated by the excitation of optical phonons equal in energy to the difference between the two signals. If the intensity of the generated Stokes wave is sufficiently high, it can act as a pump for a further Raman Stokes component downshifted in frequency; leading to cascaded Stokes orders, where the majority of the energy from the pump signal is effectively transferred to the last excited Stokes order. The waveguiding property of fibre supports high power den-

![Raman gain spectrum](image)

**Figure 3.20:** Raman gain spectrum for fused silica at a pump wavelength of $\lambda_p = 1 \mu$m.

sities over long lengths, making the process of SRS very efficient and the observation of multiple Stokes orders possible even for relatively modest powers. Given the amorphous nature of fused silica there are many available vibrational states that can participate in the Raman scattering process, leading to a very broad spectrum over which Raman gain exists: approximately 40 THz for a 1 $\mu$m pump beam, with a maximum at 13.2 THz (see Fig. 3.20, after Ref. [Sto89]); a number of models describing the Raman response function of fused silica have been proposed [Sto89, Hol02], and will be discussed further in chapter 4. A molecule in a high vibration state can interact with a lower energy photon to upshift the photon frequency, resulting in anti-Stokes Raman scattering. However, the requirement for a pre-existing phonon makes this process extremely inefficient unless
a very high population of phonons (with the correct energy) are generated by a strong Stokes component. Consequently, anti-Stokes waves are weak in silica fibres.

The growth of the Stokes wave can be written explicitly:

\[
\frac{dI_s}{dz} = g_R I_p I_s
\]

where \( I_s \) is the Stokes intensity, \( I_p \) is the pump intensity, \( z \) is the length and \( g_R \) is the Raman gain coefficient. The Raman gain coefficient originates from the imaginary part of the third-order nonlinear susceptibility, and is related to the spontaneous Raman scattering cross section. Equation 3.2 holds for CW and quasi-CW signals, where the characteristic time-scale is \( > 1 \) ns. The threshold power for the onset of Raman scattering can also be simply expressed as:

\[
P_{\text{threshold}}^0 = \frac{16 A_{\text{eff}}}{g_R L_{\text{eff}}}
\]

where \( A_{\text{eff}} \) is the effective area of the guided mode and \( L_{\text{eff}} \) is the effective length. Using Equation 3.3 it can be calculated that the Raman threshold in a one hundred metre long standard single-mode silica fibre, with an effective core size of 10 \( \mu m^2 \) pumped at 1 \( \mu m \), is \( \sim 16 \) W. This threshold level can be reduced by doping the core of the optical fibre with elements that possess a larger Raman scattering cross section.

In the context of ultrashort pulse sources Raman gain possesses some interesting properties: firstly, Raman gain exists in every fibre; secondly, it is nonresonant, such that gain is available across the entire transparency window of the fibre (\( \sim 0.3-2.0 \) \( \mu m \) for silica glass); thirdly, the gain spectrum is broad and can be modified by use of multiple pump lines (also improving gain flatness). However, long fibre lengths, a fast response time (and no associated gain lifetime, preventing saturation for CW pump schemes) leads to unavoidable sources of noise, and a relatively high pump threshold present challenges when using Raman gain in a resonant cavity for the generation of short pulses. For the development of mode-locked lasers noise considerations are particularly pertinent. Due to the long fibre lengths in Raman amplifiers, one major source of noise arises from double Rayleigh scattering (DRS): two scattering events (one backward and one forward) occur due to microscopic nonuniformity of the glass fibre; backward propagating ASE generated in the distributed amplifier can be reflected by DRS, the reflected signal can then

---

8The effective length \( L_{\text{eff}} \) is a characteristic length scale over which nonlinear effects cannot be neglected. For short lengths \( L_{\text{eff}} \) is usually equal to the physical length of the fibre. However, if the fibre is long, attenuation limits the effective length, given by [Der97]

\[
L_{\text{eff}} = \frac{1}{\alpha} \left[ 1 - \exp(-aL) \right]
\]

where \( \alpha \) is the fibre attenuation coefficient (with units of \( m^{-1} \)) and \( L \) is the physical fibre length.
be amplified by stimulated Raman scattering. Such noise sources can reduce signal-to-noise ratios (SNRs), or even prohibit mode-locking due to phase disturbance. This effect is reduced in rare-earth-based systems, where gain fibres can be one or several orders of magnitude shorter. A second major source of noise arises because of the short upper state lifetime (∼3–6 fs [Isl02]), resulting in effectively instantaneous gain. Thus, coupling of pump fluctuations to the signal is strong, hence low noise (typically narrow linewidth) laser sources make suitable pump systems. Pump noise contributions can be reduced using counter-propagating geometries to introduce an effective upper-state lifetime equal to the transit time through the fibre.

### 3.3.2 Review of short pulse Raman lasers

In 1962 Woodbury and Ng were the first to observe stimulated Raman scattering\(^9\), but miscalculated the radiation as a new emission line of Ruby [Woo62]. Later that year, Eckardt et al. described the effect more fully: under intense optical excitation a nonlinear effect exists that results in the efficient transfer of energy from the pump to the Stokes wave [Eck62]. Stolen et al. observed Raman scattering in optical fibres for the first time in 1972. It was realised that despite the relatively small Raman scattering cross section, the Raman process in silica fibres could be successfully exploits to provide optical gain, and Raman amplifiers were deployed across new long-haul fibre-optic transmission systems [Isl02]. After Mollenauer’s early soliton experiments in fibre [Mol80], Kafka and Baer showed that sub-picosecond pulses could be generated in a Raman fibre laser, by synchronously pumping a resonant cavity in the normal dispersion regime, but close enough to the ZDW such that the generated Stokes wave appeared in the anomalous region of the fibre and experienced soliton pulse-shaping effects [Kaf87]. This idea of soliton Raman lasers was developed to cover other regions of the near-IR [GN89a]. The major drive was to reduce the pulse durations and increase the wavelength coverage, little attention was paid to the quality of the generated pulses: i.e. amplitude and timing stability; and reduction of large pedestal components. However, these parameters are extremely important in the application of lasers to physical experiments [Kel89]. Passively mode-locked lasers have been demonstrated to deliver high-quality, low-noise trains of regular picosecond and sub picosecond pulses [Lef02, Pas04, Pas10]. In addition, pulse-shaping is not dependent on a synchronous pumping scheme, requiring accurate timing. To date mode-locked fibre lasers are typically based on rare-earth activated fibres, with little effort directed toward passively mode-locking Raman lasers.

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\(^9\)Spontaneous Raman scattering was discovered by Raman in 1928 [?], and is typically a very weak effect, emitting nearly isotropically only 1 part in 10\(^6\) of the incident radiation. In contrast, stimulated Raman scattering, excited by a concentrated beam of laser light, can result in a strong scattering effect in the forward and backward directions.
3.3.3 Passively mode-locked Raman laser using a nanotube-based saturable absorber

In telecommunications, Raman based amplification allows operation beyond the spectral limits of rare-earth devices [Che98]. Consequently similar techniques can be applied to ultrafast fibre lasers. The most attractive feature of Raman based amplification in silica fibre is that gain is available at any wavelength across the transparency window of the medium (300–2300 nm), given a suitable pump source [Che98]. With advances in high power fibre laser pump technology and in cascaded Raman fibre lasers, efficient pump systems are now available throughout this entire band. There have been a number of reports utilizing Raman gain in ultrafast mode-locked sources [Sch06, Che05, Agu10, Cha10b, Cha10a]. However, to date none of these systems have reached a level of performance comparable with state-of-the-art rare-earth based lasers. In Ref. [Sch06] dissipative four-wave mixing was used for mode-locking, generating a pulsed laser with a very high-repetition rate. While this is useful for some applications, the high repetition rate limits the delivered peak power. Nonlinear loop mirrors [Che05, Agu10] and nonlinear polarization evolution (NPE) [Cha10a] have also been used to provide saturable absorption, but such systems suffer from instabilities due to fluctuations in ambient temperature, and often exhibit poor self-starting performance. Recently, a Raman mode-locked laser using a semiconductor saturable absorber mirror (SESAM) was reported [Cha10b]. While the use of a SESAM improves self-starting and robustness against environmental perturbations, there is limited spectral operation from a single device. In addition, the fabrication cost of SESAMs at non-standard wavelengths is high. Availability of a broadband saturable absorber (SA) to achieve mode-locking at any desired wavelength across the transmission window of silica is an essential pre-requisite to fully exploit the flexibility of Raman amplification in ultrafast sources across the visible and near infrared.

It was established in chapter 2 that the introduction of saturable absorbers based on nano materials has moved the field a step closer to a fully universal device. Combining both Raman gain and a CNT- (or even graphene-) based SA presents a versatile approach to the development of wavelength flexible ultrafast lasers.

Experimental setup

The all-fibre geometry is shown in Fig. 3.21. The cavity (Fig. 3.21a) consists of a 100 m length of single-mode highly nonlinear fibre (OFS Raman Fiber), with an enhanced germanium oxide (GeO$_2$) concentration for an increased Raman gain coefficient (2.5 W$^{-1}$ km$^{-1}$), core pumped through a WDM by a CW 15 W Er-doped fibre ASE source at 1555 nm. No synchronous pumping was necessary, resulting in a significantly less complex system.
3.3 Exploiting intrinsic fibre nonlinearity: an ultrafast laser based on Raman gain

The CNT-polymer SA device was prepared by solution processing \[Has09\]. The Carbon nanotubes were grown by catalytic chemical vapor deposition \[Fla03\]. After purification by air oxidation at 450\(^\circ\)C, followed by HCl washing, the remaining Carbon-encapsulated catalytic nanoparticles were removed \[Oss05\]. Analysis of the purified samples by transmission electron microscopy (TEM) revealed the presence of 90\% double wall Carbon nanotubes (DWNTs), \sim 8\% single wall Carbon nanotubes (SWNTs) and \sim 2\% triple wall Carbon nanotubes (TWNTs) \[Oss05\]. The diameter distribution for DWNTs was \sim 0.8–1.2 nm for the inner and \sim 1.6–1.9 nm for the outer shells, as determined by Raman spectroscopy and TEM. This wide diameter distribution can potentially enable broadband operation, essential for the large wavelength coverage offered by Raman amplification. The purified nanotubes were dispersed using a tip sonicator (Branson 450 A, 20 kHz) in water with sodium dodecylbenzene sulfonate surfactant, and mixed with aqueous polyvinyl alcohol (PVA) solution to obtain a homogeneous and stable dispersion, free of aggregates. Slow evaporation of water from this mixture produces a CNT-PVA composite \sim 50 \mu m thick. Optical microscopy reveals no CNT aggregation or defect in the composite, thus avoiding scattering losses. This same device was used in chapter 2. Self-starting mode-locked operation was initiated and maintained by integrating the CNT-based SA into the cavity between a pair of fibre connectors. A polarisation insensitive inline optical isolator and fibre-based polarisation controller were employed to stabilise mode-locking. Light was extracted from the unidirectional cavity through a 5\% fused fibre coupler. To pre-
vent high-levels of undepleted pump power damaging the passive cavity components, a second WDM was used to couple out residual pump light.

**Results**

Stable quasi-continuous wave mode-locking was supported over a range of pump powers above the lasing threshold at 9.5 W. Increasing pump power resulted in spectral broadening and break-up of single-pulse operation. The temporal pulse intensity profile (on a logarithmic scale and fitted with a sech^2), the optical spectrum (on a linear and logarithmic scale), and the electrical (RF) spectrum of the fundamental harmonic of the cavity are plotted in Fig. 3.22. The FWHM pulse duration is \(\sim 308\) ps (Fig. 3.22a), the strong asymmetry of the pulse profile is largely attributed to the fact that the average output power for stable mode-locking, with a single pulse per round trip, was only \(\sim 0.08\) mW, and the corresponding induced photo-current was close to the noise-floor of the diagnostics. The laser operates at the first Stokes order of \(\sim 1668\) nm from a pump at 1555 nm (Fig. 3.22b). The spectrum has a square shape, with a -3 dB bandwidth of 1.38 nm (-10 dB bandwidth of 1.92 nm). The square shaped spectrum is a recognizable feature of lasers operating in the dissipative soliton regime [Wis08, Kel09c, Kel09a]. Such systems generate pulses carrying a large and predominantly linear chirp [Kel09a], and are suitable for compression. The RF trace (Fig. 3.22c) shows a significant pedestal, containing \(\sim 0.57\)% of the total pulse energy, indicating that the cavity is prone to long-term temporal instabilities and fluctuations of the pulse to pulse energy. This relatively high noise contribution, compared to state-of-the-art rare-earth based systems, is expected due to the high level of pumping power. However, the narrow line-width of the peak at 1.72 MHz, corresponding to the round-trip time of the cavity, indicates low pulse timing jitter.

Significant interest in normal dispersion mode-locked lasers is stimulated by the possibility of overcoming the pulse energy limits imposed by soliton propagation, with a linearly chirped pulse structure commonly known as a dissipative soliton, now routinely generated in all-fibre geometries [Kel09a, Wis08, Ren08b]. In particular, giant-chirp oscillators (GCOs) have been proposed as a means of pre-chirping the pulse frequencies directly in the oscillator to simplify the chirped-pulse amplification (CPA) design [Ren08b, Fer04, Cho06, Kob08, Ren08a] (such systems will be the subject of chapter 5). The pulses emitted from the oscillator are 45 times transform-limited, implying a significant chirp that should be compressible, provided the pulse is coherent. To test the degree of compressibility of the pulses generated in this Raman-based ultrafast laser, a 10 km length of Ge-doped fibre, with a ZDW of 1320 nm, was used to provide anomalous dispersion sufficient to de-chirp the pulses (see Fig. 3.21b). In addition, counter-pumping the compressor fibre with the undepleted pump power from the seed oscillator, provides simul-
3.3 Exploiting intrinsic fibre nonlinearity: an ultrafast laser based on Raman gain

Figure 3.22: Temporal and spectral properties of the Raman-based ultrafast laser pre-compression. Inset to 3.22b shows the optical spectrum on a logarithmic scale.
taneous amplification through Raman gain, thus forming a compact GCO-type master-oscillator power fibre amplifier (MOPFA) solution that could potentially be replicated throughout the transmission window of silica fibre, provided a suitable pump source was available.

The autocorrelation trace of the compressed and amplified pulse, its spectrum and the RF trace of the fundamental cavity harmonic after compression are shown in Fig. 3.23. The pulses are successfully compressed ~150 times from 308 ps to 2 ps, and no significant pedestal is observed on the autocorrelation traces. The average output power after amplification and compression is 5 mW, corresponding to the 18 dB gain provided by the amplifier, which requires 6 W supplied by the undepleted power from the oscillator stage. Importantly, the spectral shape is preserved (see Fig. 3.23b) indicating linear compression, although noticeable degradation of the optical signal-to-noise ratio due to ASE in the amplifier fibre is apparent. The electrical spectrum (Fig. 3.23c) of the amplified and compressed signal is naturally centered at 1.72 MHz and shows a 6% increase in the noise pedestal compared to the input signal. The corresponding pulse peak power after compression is ~1.4 kW. Scalability of the peak power to higher levels should be possible by decoupling the amplifier from the compressor to prevent the onset of nonlinear spectral broadening as the peak powers increase.

### 3.4 Summary

In this chapter, ultrafast fibre lasers have been discussed from the context of the optical amplifying medium, and two technologies have been considered. The amplification of picosecond pulses at 1160 nm and 1180 nm in a Bi-doped alumosilicate fibre amplifier, using an Yb pump laser in a core-pump configuration, was demonstrated for the first time. The gain in a 30 m length of fibre, subject to cryogenic cooling, was over 20 dB for input pulses centred spectrally at 1160 nm. A mode-locked soliton laser based on this technology was demonstrated at 1178 nm, using a SWNT-based SA, also for the first time. For accessing high-peak powers in all-fibre systems, MOPFA configurations are widely used. The first Bi-doped fibre MOPFA, producing picosecond pulses, with 580 W peak power was developed and used to demonstrate frequency doubling to 589 nm, with ~9% optical-to-optical efficiency.

While Bi-doped fibre offers a number of attractive properties, it currently remains a laboratory curiosity and has not attracted commercial interest. It is hard to imagine that it will supplant Raman amplifiers in the second telecom band unless significant improvements to the technology are realised: particularly overcoming the need for cryogenic cooling to access sufficient gain for practical purposes; and the ability to fabricate
3.4 Summary

Figure 3.23: Temporal and spectral properties of the Raman-based ultrafast laser post compression. Inset to 3.23b shows the optical spectrum on a logarithmic scale.
shorter length (potentially double-clad) fibres possessing higher levels of pump absorption and lower non-saturable losses. Understanding the active centres involved in the lasing process will be key to progress in this area.

Raman gain has been successfully employed in telecom networks for many years, but has been less widely used as an amplifier in mode-locked laser systems. A fibre laser based on Raman gain and passively mode-locked with a DWNT-based saturable absorber was proposed for the first time. Chirped pulses were amplified and compressed to 2 ps with 5 mW average power, corresponding to a peak power in excess of 1 kW. This system represents the possibility of realising ultrafast all-fibre lasers at all wavelengths across the near-infrared. The success of this approach will be determined by the steps taken in the coming years to improve the performance properties, of the mode-locked systems, to ensure equivalence with state-of-the-art rare-earth based lasers. Reducing the intensity noise and improving the long-term-stable operation will be of particular importance. The ability to accurately model such systems and to fully understand the nature of the mode-locking dynamic will be necessary, and is the subject of ongoing work.

Given the current status of both technologies, it is perhaps foreseeable that combining Bi-active and Raman-active fibres could be the most practical means of maximising their potential: low-power Bi-doped fibre oscillators have demonstrated good long-term-stable performance; and Raman amplification provides high gain at room temperature, albeit in relatively long lengths of fibre.
4 Dynamics, modelling and simulation of mode-locked fibre lasers

The performance of any mode-locked laser – a fibre laser in particular – is largely determined by the overall dispersion and nonlinearity of the cavity. In addition, a finite gain bandwidth and gain saturation contribute to the laser dynamics. Together with the action of the saturable absorber, an ultrashort pulse can be initiated, stabilised and maintained. Depending on the sign of the dispersion and the magnitude of the nonlinearity, the saturable absorber has a limited effect on the steady-state pulse duration; while other mechanisms dominate pulse shaping. The aim of this chapter is to provide an overview of these effects.

In section 4.1 the qualitative description of mode-locking in four distinct regimes (differentiated by their dispersion map) is introduced, and some key results from the literature are reviewed. The theory of mode-locking is established in section 4.2: two models, both of which permit analytic solutions (and consider the average cavity dynamics) are briefly described, before a full numerical model based on experimentally accessible parameters is presented that aims to capture the dynamics of pulse evolution within a mode-locked cavity, consisting of a number of essential components. This model is then applied to a real system in section 4.3, where a low-noise, polarisation-maintaining, gain-guided dissipative soliton Ytterbium fibre laser is developed.

Results presented in this chapter have been published in the following journal articles [Kelona, Peded, Pedon].

4.1 Introduction

The spectral filtering of a finite bandwidth gain combined with the fact that a saturable absorber device preferentially transmits higher intensity spikes, promotes the formation of a pulse from noise in a mode-locked laser. As the duration of the pulse narrows and approaches the picosecond scale, dispersion and nonlinearity become dominant effects on the steady-state properties of the pulse. A great body of work has been devoted to
understanding the dynamics of mode-locked lasers in order to scale, improve and maximise performance. Four broad regimes of operation exist (see Fig. 4.1), depending on the cavity dispersion map and the magnitude of the nonlinearity.

**Soliton lasers**

The soliton laser, where the cumulative round-trip nonlinear phase shift is compensated by anomalous dispersion induced linear chirp, is perhaps the most widely studied ultra-short pulse system. The soliton pulse is a solitary-wave solution of the NLSE (see Equation 1.13). The NLSE belongs to a special class of nonlinear equations that can be solved using the inverse scattering (transformation) method\(^1\) – similar in essence to the Fourier transform method – where the switching between domains is utilised. An initial value pulse-shape (or potential) is transformed to obtain a description of the pulse in terms of eigenvalues and eigenfunctions of a scattering problem [Tay92, Agr07]. The evolution of the eigenvalues is described by a linear system of equations; the final pulse-shape can be reconstructed from the inverse scattering equations. The discrete eigenvalues of the direct scattering transform correspond to the soliton content. In 1972, Zakharov and Shabat showed that the imaginary part of the bound-state eigenvalues defines the soliton energy, while the real part corresponds to the velocity [Zak72]. The number of discrete eigenvalues indicates the number of solitons; as such, any pulse can be thought of as a superposition of many solitons. A subset, whose soliton order \(N\) is an integer, where \(N\)

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\(^1\)I refer to the monographs by Taylor and by Agrawal for a full discussion and mathematical treatment of the inverse scattering problem [Tay92, Agr07]
is given by

\[ N = \left( \frac{\gamma P_0 T_0^2}{|\beta_2|} \right)^{\frac{1}{2}} \] (4.1)

correspond to a high-order soliton, and forms a bound-state of the NLSE that undergoes a periodic temporal evolution.\(^2\) The total soliton energy has to be equal to or less than the total pulse energy. Energy not in the soliton will disperse on propagation. The simplest soliton solution is the \textit{fundamental} or first-order soliton (so-called because it does not change shape on propagation) when \( N = 1 \), where the pulse envelope corresponds to a hyperbolic-secant function that, in its canonical form, is written as [Agr07]:

\[ u(T, z) = E_0 \text{sech}(T) \exp \left( \frac{iz}{2} \right) \] (4.2)

where \( T = (t - z/\nu_g) \), \( \nu_g \) is the group velocity, \( t \) is the physical time, \( z \) is the propagation distance, and \( E_0 \) is the peak soliton amplitude, importantly this parameter also determines the soliton width. The width of the soliton scales as \( T_0/E_0 \), where \( T_0 \) is the characteristic width, i.e. inversely with the pulse amplitude. The width of the soliton \( T_0 \) is related to the FWHM of the pulse by \( T_{\text{FWHM}} = 1.76 T_0 \). The accumulated phase shift on propagation in an anomalously dispersive fibre is constant over time (or frequency); the soliton remains unchirped and the pulse-shape is preserved. However, perturbations to the amplitude of the solitary-wave introduced by gain or loss results in a change in the soliton width, such that the relation \( E_0 T_0 = \sqrt{|\beta_2|/\gamma} \) is preserved. This suggests that the peak power required to maintain a fundamental soliton is [Tay92, Agr07]

\[ P_0 = \frac{|\beta_2|}{\gamma T_0^2}. \] (4.3)

Soliton-like pulses can be formed in a laser cavity with net anomalous dispersion, consisting of segments of positive and negative dispersion; the soliton experiences perturbations through interaction with the differing regions of dispersion. In addition to perturbations introduced by periodic amplification through a gain medium, a stable soliton pulse can be formed if the length scale of the generated soliton is long compared to the length scale of the disturbance – such operation is known as guiding-centre or average-soliton operation and was theoretically developed in Refs. [Has90, Kel91]. The length

\(^2\)Subject to perturbations described by GNLS-type equations high-order solitons can no-longer propagate as a bound state and break-up (or fragment), this process is called soliton fission and is commonly observed in the evolution of supercontinua in certain pump regimes.
scale of a soliton is characterised by [Agr07]

\[ z_0 = \frac{\pi}{2} L_D = \frac{\pi}{2} \frac{T_0^2}{|\beta_2|} \]  

(4.4)

where \( L_D \) is the dispersion length, given by \( L_D = \frac{T_0^2}{|\beta_2|} \). For a bound-state soliton of the NLSE, with soliton order \( N = 3 \), the pulse contracts, splits into two distinct pulses at \( z_0/2 \), and recovers over a period equal to \( z_0 \).

In practical cases, in a soliton laser, under perturbation the soliton adjusts to maintain its shape, shedding radiation. The soliton field interacts with the non-solitonic (dispersive-wave) radiation, at certain frequencies the two waves become phase matched giving rise to characteristic sidebands observed in the optical spectrum [Kel91, Kel92]. This regime of operation was discussed previously in chapters 2 and 3. The sidebands are useful for determining the net cavity GVD [Den94], but are otherwise undesirable as they signify a loss of energy from the soliton.

The maximum pulse energy of a soliton laser is limited to tens of picojoules [Tam94]. An increase in the pumping power, and consequently the pulse energy, results in wave-breaking [And92], which leads to multi-pulse operation. Despite limited pulse energies, soliton-lasers are desirable because of the high pulse quality and near bandwidth-limited operation.

**Stretched-pulse (or dispersion-managed solitons)**

A soliton becomes highly unstable when the period of perturbation approaches approximately \( 8z_0 \) [Mol86, Nos92b, Kel92, Tam93]. Given that the soliton is perturbed by the gain in a soliton laser at least once per round-trip of the cavity (of length \( L \)), this applies a constraint such that the shortest stably supported soliton must satisfy \( 8z_0 > L \) [Mol86]; typically short pulse operation is observed for \( L \approx z_0 \) [Tam93]. For pulses with durations \( T_0 << 100 \text{ fs}, z_0 << 1 \text{ m} \) in standard fibre, presenting a potential practical difficulty. Decreasing the cavity GVD increases \( z_0 \), but lowers the soliton energy, because \( E_{\text{soliton}} \propto \beta_2 \).

The stretched pulse fibre laser was proposed to circumvent this compromise [Tam93], and comprises sections of positive and negative dispersion fibre, such that in one round-trip the pulse chirps from positive to negative and back. A short pulse (~100 fs) broadens significantly (up to an order or magnitude) in the positive dispersion segment and recompresses in the anomalous segment, thus lowering the average peak power compared to a transform-limited pulse of equal bandwidth, reducing the effect of the nonlinearity. With careful design of the cavity dispersion the pulse can be extracted at the point of minimum duration after anomalous compression, while experiencing maximum input of energy from the gain element when the duration is at a maximum. This behaviour of periodic
broadening and compression over a single round-trip is referred to as *breathing* and the solutions are known as dispersion-managed solitons, with a temporal shape often closer represented by a Gaussian rather than hyperbolic-secant pulse shape.

Stretched-pulse fibre lasers can tolerate significantly higher (by up to an order of magnitude) nonlinear phase shifts than the soliton laser (see Fig. 4.1), and can be operated with low net anomalous or normal GVD. Typically, stable operation is observed for low anomalous dispersion, with higher-pulse energies achievable in the low-normal dispersion regime (with operation often restricted to Q-switch mode-locking [Lim03]). Pulses exceeding the nanojoule level have been demonstrated with this approach, where durations can be as short as 52 fs [Lim03]. This represents an order of magnitude improvement over the soliton laser.

**Self-similar pulse evolution**

Solitary-wave propagation of short optical pulses in amplifying, dispersive media was first proposed by Bélanger *et al.* in 1989 [Bel89]. Subsequently, Anderson *et al.* showed that short pulses in normal GVD fibres, subject to strong gain, converge asymptotically towards a parabolic temporal profile, possessing a strong, but linear chirp [And93]. Since, this type of pulse has been generated in mode-locked lasers in order to again scale the energy of the emitted pulses [Dud07] (and references therein). In such systems, where pulse-shaping is dominated by net normal dispersion and high gain, solutions are referred to as self-similar pulses or *similaritons*, and have proven to be robust even at high pulse powers [Dud07].

Similaritons are asymptotic solutions of the NLSE in the high-intensity limit [Dud07], where the pulse chirp increases monotonically in the fibre, thus causing an exponential increase in both the temporal and spectral widths. Such a solution cannot be stable in a system with feedback (where periodic boundary conditions exist [Wis08]) without bound. In a fibre laser the restricting mechanism is the finite bandwidth of the gain. In contrast to both static solitons in a soliton laser and dynamic solitons in stretched-pulse systems, self-similar oscillators support pulses that are positively chirped throughout the cavity. The linear chirp means that the pulses can be dechirped externally to near transform-limited duration. Similariton pulses can tolerate much higher nonlinear phase shifts than dispersion-managed solitons (see Fig. 4.1), and consequently larger pulse energies can be extracted (> 10 nJ) [Buc05].

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3Breathing is used in reference to the break-up and recombination of a bound, high-order soliton into its constituent fundamental solitons. The *breathing ratio* is also often the preferred term used to described the degree of temporal (or spectral) variation exhibited by a pulse on a single pass through a resonant cavity. Given the dual meaning, the term is exercised with caution.
In similariton lasers a dispersive element is required to partially compensate the round-
trip chirp induced by the long length of positive GVD fibre (where self-similar evolution
occurs).\textsuperscript{4} Linear dispersive delay elements typically comprise bulk gratings, loosing the
inherent advantages of a fibreised format. While PCF-type fibres can be used, preserving the
fully fibre scheme, typically integration with standard fibre can compromise the
system performance. Consequently, ultrafast lasers without any (anomalous) dispersion
compensation represent a potential performance advantage.

**All-normal dispersion lasers**

Operating without (anomalous) dispersion compensation requires pulse-shaping to be
non-solitonic [Wis08]. It is well known that bulk lasers operating without dispersion com-
ensation generate longer duration and highly chirped pulses [Goo89, Spe91]; a char-
acterisation of Kerr-lens mode-locked Ti:Sapphire lasers operating without dispersion
compensation was conducted by Proctor \textit{et al}. [Pro93]. Stable pulses in positive GVD
cavities do not depend on soliton pulse-shaping. Gain dispersion acts as a pulse short-
ening process for chirped pulses by clipping the leading and trailing edge. This action is
balanced by the temporal broadening due to the normal GVD. In the frequency domain,
the narrowing of the spectrum due to gain dispersion is compensated by the generation
of spectral bandwidth through SPM.

Early examples of ultrafast fibre lasers operating with all-normal dispersion, exploiting
the shaping of chirped pulses by the action of a spectral limiting element, include [dM04a].
Significant theoretical interest in this class of pulse solutions, now widely recognised as
dissipative solitons, is evidenced by the recent monograph by Akhmediev and Ankiewicz
dedicated to the subject [Akh05], as well as numerous review articles [Kal05, Kal06].

Recently, all-normal dispersion (AND\textsubscript{i}) ultrafast fibre lasers have received much atten-
tion because Yb-doped fibre has proved to be the dominant fibre technology, and Yb
systems operate below the natural region of anomalous dispersion in silica glass fibres.
Thus, simple compact and alignment free fibre based systems, without anomalous ele-
ments, are desirable. In addition, it has been demonstrated that dissipative solitons are
robust against large nonlinear phase shifts per cavity pass, and as such, can support the
generation of high energy, linearly chirped pulses that can be dechirped extra-cavity, re-
sulting in high-energy femtosecond pulses [Cho06, Lef10, Lec10].

The dissipative soliton is an analytic solution of the cubic (or cubic-quintic) Ginzburg
Landau equation (CGLE or CQGLE) and will be considered in the proceeding discussion.

\textsuperscript{4}In this case the amplifier is crucial to preserving the pulse, but is often not used in similariton lasers to
chirp the pulse as in the case of self-similar amplifiers, due to the requirement for periodic boundary con-
ditions imposed by the gain. Thus, self-similar evolution is decoupled from the amplifier that provides
the necessary gain filtering for stabilisation.
4.2 Theory of pulse propagation in a mode-locked fibre laser

Furthermore, normal dispersion ultrafast lasers will be revisited later in this chapter and will be the major point of discussion in chapter 5.

4.2 Theory of pulse propagation in a mode-locked fibre laser

The (1+1) dimensional nonlinear Schrödinger equation (Equation 1.13), in its canonical form, describes the salient features of pulse propagation in an optical fibre, subject to chromatic dispersion and self-phase modulation. For a rigorous derivation of the basic (i.e. NLSE) and more complete or generalised NLSE (GNLSE), using both time [Blo89] and frequency domain [Fra91] formulations, I refer to Refs. [Blo89, Mam90, Fra91, Agr07, Lae07]; a complementary commentary on the formulation of propagation equations for the evolution of optical pulses in nonlinear dispersive media is provided by Refs. [Agr07, Tra10f].

In addition to the effects of SPM and GVD included in the NLSE, in a mode-locked fibre laser the light also experiences periodic (bandwidth-limited) amplification and intensity dependent loss (due to saturable absorption). In the proceeding section I review extensions to the NLS equation to include a dynamic description of mode-locking behaviour and outline a basic numerical scheme for modelling such systems.

4.2.1 The Haus master mode-locking model

Haus developed an extended NLS equation, also known as the master mode-locking model (or complex Ginzburg-Landau equation (CGLE)), to describe the average pulse evolution dynamics within a mode-locked laser cavity [Hau91, Hau00]. The resulting complex Ginzburg-Landau-type equation is one of the most studied in engineering mathematics/physics and can be applied to many nonlinear systems involving a description of the amplitude evolution of unstable modes [Bal08].

In a laser system, amplification is provided by stimulated emission in a length of active fibre. The effect of the gain is considered in the frequency domain, and can be approximated by a parabolic frequency dependence near its peak of the form

$$\Delta g(\omega) = \frac{g(z)}{1 + \left(\frac{\omega}{\omega_g}\right)^2} \approx g(z) \left[1 - \left(\frac{\omega}{\omega_g}\right)^2\right]$$  \hspace{1cm} (4.5)

5One dimension of time and one dimension of space
The perfectly homogeneously saturating gain dynamics can be described by:

\[ g(z) = \frac{2g_0}{1 + \left| \frac{u}{e_0} \right|^2} \] (4.6)

where \( g_0 \) is the unsaturated (or small signal) gain, \( e_0 \) is the gain saturation parameter and \( \|u\| = \int_{-\infty}^{\infty} |u|^2 \, dt \) is the total field energy. The change in the time-domain field due to the gain can be determined by:

\[ u(t) = \mathcal{F}^{-1} \{ \tilde{u}(\omega) g(\omega) \} \] (4.7)

where \( \mathcal{F} \) represents the Fourier transform and \( \tilde{u}(\omega) = \mathcal{F} \{ u(t) \} \) is the spectrum of the temporal field.

The intensity discrimination, necessary for promoting mode-locking, acts as a small perturbation to the NLS equation and is incorporated in a phenomenological way:

\[ s(t) = s_0 + I(t) - I_{sat} \] (4.8)

where \( s_0 \) is the unsaturated loss, \( I(t) \) is the dependent intensity, and \( I_{sat} \) is the saturation intensity of the device. This function was used to fit the data measured experimentally (using a z-scan technique) in chapter 2. If the saturation is relatively weak, the expression can be approximated by:

\[ s(t) = \delta - \beta |u|^2 \] (4.9)

Here, \( \delta \) is the linear loss coefficient and \( \beta \) characterises the strength of the nonlinear (cubic) loss/gain.

The master mode-locking equation includes the effects of the amplifier and the saturable absorber into the NLS equation, and is written as follows [Hau00, Bal08]

\[ i \frac{\partial u}{\partial z} + \frac{D \partial^2 u}{\partial T^2} + \left( \gamma - i \beta \right) |u|^2 u + i \delta u - i g(z) \left( 1 + \tau \frac{\partial^2}{\partial T^2} \right) u = 0 \] (4.10)

where \( u(z, T) \) is the slowly varying electric field envelope, \( z \) is the propagation distance, \( T = t - z/v_g \) is the retarded time, where \( t \) is the physical time and \( v_g \) is the group velocity, \( D \) is the dispersion parameter (\( D > 0 \) for anomalous GVD), \( \beta \) is the cubic saturation parameter; \( \delta \) is the linear attenuation; \( g \) is the bandwidth-limited gain, and \( \tau \) is related to the filter bandwidth.

---

6A constant gain can be applied using simply \( g(z) = g_0 \).

7The Fourier transform is defined as \( \mathcal{F} \{ u(z, t) \} = \int_{-\infty}^{\infty} u(z, t) \exp \left\{ i(\omega - \omega_0) t \right\} dt \). Given that \( \omega \) is angular frequency (\( \omega = 2\pi v \)), the transform pair is non-symmetric, and the inverse Fourier transform is given by \( \mathcal{F}^{-1} \{ \tilde{u}(z, \omega) \} = u(z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{u}(z, \omega) \exp \left\{ -i(\omega - \omega_0) t \right\} d\omega \).

8Expanding the denominator using a Taylor series, and retaining the first two terms [Abl09].
4.2 Theory of pulse propagation in a mode-locked fibre laser

Overview of a basic numerical solution

While Equation 4.10 permits analytic solutions for $\pm D$ (in a limited parameter space), typically nonlinear partial differential equations require numerical integration [Agr07]. Equation 4.10 can be solved numerically using a pseudospectral scheme, known as the (reduced) split-step Fourier method (SSFM) as follows [Wei86, Agr07].

$$\frac{\partial u}{\partial z} = (\hat{D} + \hat{N}) u \quad (4.11)$$

where the operators $\hat{D}$ and $\hat{N}$ are given by

$$\hat{D} = i \frac{D}{2} \frac{\partial^2 u}{\partial T^2} - \delta u + g(z) \left( 1 + \tau \frac{\partial^2}{\partial T^2} \right) u \quad (4.12)$$

$$\hat{N} = (i \gamma + \beta) |u|^2 u \quad (4.13)$$

The solution to Equation 4.11 over a small step $h$ in $z$ is approximated by [Hul07]

$$u(z+h, T) \approx \exp \left( \frac{h}{2} \hat{D} \right) \exp \left( \int_z^{z+h} \hat{N}(z') \, dz' \right) \exp \left( \frac{h}{2} \hat{D} \right) u(z, T) \quad (4.14)$$

This split step scheme is known as the symmetric split step method (SSSFM) and evaluates the full nonlinear step in the middle of two half dispersive steps. Essentially the smaller the step size the smaller the global error of the solution. However, the computation cost of a small step size is expensive when large parameter grids (in time and frequency) are involved. There are many approaches used to approximate the nonlinear term, described by the integral in the middle exponential of Equation 4.14; the simplest (and least accurate) approximates it with $\exp(h \hat{N})$. With the SSSFM it has been shown that the error is second-order in the step size $O(h^2)$ when $\hat{D}$ and $\hat{N}$ do not commute [Hul07]. In this thesis, a fourth-order Runge-Kutta (RK4) method is used to integrate the nonlinear step, with fourth-order global accuracy $O(h^4)$ [Hul07].

Computational codes implementing pseudospectral methods depend on the discrete Fourier transform (DFT), or more specifically a fast Fourier transform (FFT): an efficient algorithm to compute the DFT quickly, rendering the same result as evaluating the DFT directly; and for which there are many highly-optimised numerical codes available embedded as built-in functions in open-source and proprietry scientific computing languages, such as Matlab and Scipy (or scientific Python). The FFT is necessary because the dispersive operator, containing second-order differentials in $T$, has an analytic solution in the frequency domain using the rule [Arf05, Agr07]

$$\mathcal{F} \left\{ \frac{\partial^n}{\partial T^n} u(T) \right\} = (-i \omega)^n \tilde{u}(\omega) \quad (4.15)$$
Equation 4.15 imposes implicit boundary conditions on the function $u(T)$, such that $u(T) \to 0$ as $T \to \pm\infty$. This is reasonable for soliton solutions where the function and its derivatives tend to zero as $T = \pm\infty$. However, in most cases periodic boundary conditions (with periodicity $\mathcal{T}$) are appropriate [Wei86]

$$u(z, T + \mathcal{T}) = u(z, T), \quad -\infty < T < \infty, \quad z > 0$$  \hspace{1cm} (4.16)

In fact, due to the nature of split-step algorithms, periodic boundary conditions are implicit. However, this does not pose a problem as long as the condition that the system size is greater than the phenomena under study is satisfied [Rob97]. Reducing the number of FFTs needed per step, reduces computational cost and increases speed of the overall numerical algorithm. A modified RK4 method for solving the general NLSE (or GNLSE) was proposed by Hult [Hul07]. In this thesis this method was adopted for large scale (i.e. $>2^{14}$ points per grid) mode-locking simulations discussed in detail in chapter 5.

Another approach to retain numerical accuracy while improving computation speed is to use an adaptive, rather than a fixed, step size $h$ [Sin03]. An adaptive step algorithm adjusts the step size to be the largest possible, while retaining a minimum relative local error (RLE) level. The RLE is estimated using the principle of conservation of energy: calculating the percentage change in the energy between one coarse step of $2h$ and two fine steps of $h$. The step size $h$ is reduced until the RLE value is below a threshold level. It is not straightforward to implement adaptive step strategies when employing the reduced SSFM or SSSFM, due to difficulties evaluating an estimate of the RLE; higher-order integration schemes are more easily compatible with adaptive step algorithms [Fra91]. The approach outlined by Sinkin et al. was adopted in all numerical schemes [Sin03]. A broader discussion of numerical approaches to solving GNLSE-type equations is given in the recent monographs [Agr07, Tra10f].

### Dynamics of the CGLE

Equation 4.10 is an attractive model because it permits exact solutions in both the negative (soliton-like) and positive (dissipative soliton-like) dispersive regime, and thus has been extensively analytically studied [Hau91, SC96, SC97, Hau00, Ska06, Leb06, Bal08, Ren08b, Zav09, Din11]. To illustrate the behaviour of the master mode-locking model, Equation 4.10 was numerically integrated, with gain saturation given by Equation 4.6.

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9The modelling of supercontinuum generation (discussed in chapter 6), in particular continuous-wave pumped supercontinua, is extremely computationally demanding. A separate high-speed C code, using optimised FFT libraries (Fastest Fourier Transform in the West (FFTW)) was available courtesy of Dr J. C. Travers.
and numerical parameters $D = 2; \gamma = 4; \delta = 0.1; \tau = 1.5; \text{ and } e_0 = 1.0$. The initial condition for all simulations was a broad, low-amplitude sech-shape pulse.\textsuperscript{10} In all cases, the third axis in the evolution plots has been normalised to one and is not shown for clarity.\textsuperscript{11} Three distinct regimes are summarised in Fig. 4.2. Figure 4.2a shows the evolution for $\beta = 0.01$ and $g_0 = 1.5$; the amplitude continues to fluctuate and a steady-state is never achieved. The pulse parameters (Fig. 4.2b), recording the FWHM temporal duration (red circles) and the L2-Norm (black circles) after each roundtrip, show a chaotic evolution. In this case, the saturable absorption is insufficient to overcome radiation mode instabilities [Bal08]. Figure 4.2c evolves to a steady-state pulse solution, given an arbitrary initial condition, $\beta = 0.25$ and $g_0 = 0.88$. This is the case of the bright soliton, where the pulse readjusts its amplitude, width and energy to reach a stable equilibrium. In Fig. 4.2e, where $\beta = 0.35$ and $g_0 = 1.1$, the energy (L2-Norm) increases without bound and the pulse undergoes self-similar collapse. In this regime the nonlinear gain is too high to be compensated by the linear attenuation and the pulse rapidly grows, resulting in blow-up of the solution.

While the master mode-locking model encapsulates the essential features of mode-locked operation, the region over which stable pulse solutions exist is limited and does not extend to the full set of parameters where stable operation is observed in physical systems. It was proposed by Moores that augmenting the equation to include a quintic (or fifth-order) saturation term extended the region of dynamic stability [Moo93], this model is known as the cubic-quintic Ginzburg-Landau equation (CQGLE) and is equally widely studied (see Refs. [SC96, Kap02] and references therein) in contexts not confined to nonlinear wave optics.

### 4.2.2 The cubic-quintic Ginzburg-Landau equation

The cubic-quintic Ginzburg-Landau equation can be written as follows

$$i \frac{\partial u}{\partial z} + \frac{D}{2} \frac{\partial^2 u}{\partial T^2} + \left( \gamma - i \beta \right) |u|^2 u + i \mu |u|^4 u + i \delta u - g(z) \left( 1 + \tau \frac{\partial^2}{\partial T^2} \right) u = 0, \quad (4.17)$$

which is Equation 4.10, with a nonlinear quintic saturation term $\mu$. The quintic saturation parameter stabilises the nonlinear growth preventing blow-up and extending the region of stable parameter space. While the CQGLE remains an average model, it represents a broader set of the dynamics observed in physically realisable systems.

Figure 4.4 shows some typical dynamics of the CQGLE, with a saturating gain model

\textsuperscript{10}The dynamic is weakly dependent on initial condition: all simulations can be seeded from white-noise and follow the same qualitative evolution.

\textsuperscript{11}It is also worth noting that the evolution is not always viewed from the same perspective.
Figure 4.2: Mode-locking dynamics of the master mode-locking equation, with gain saturation. The common parameter values for simulation were: $D = 2$; $\gamma = 4; \delta = 0.1; \tau = 1.5$; and $e_0 = 1.0$. 4.2a Evolution dynamics with $\beta = 0.01$; and $g_0 = 1.5$. 4.2c Evolution dynamics with $\beta = 0.25$; and $g_0 = 0.88$. 4.2e Evolution dynamics with $\beta = 0.35$; and $g_0 = 1.1$. 

(a) Temporal evolution. 
(b) Pulse parameters. 
(c) Temporal evolution. 
(d) Pulse parameters. 
(e) Temporal evolution. 
(f) Pulse parameters.
4.2 Theory of pulse propagation in a mode-locked fibre laser

Figure 4.3: Phase-space plots showing the attractor dynamics of the CGLE system for parameters given in Fig. 4.2, respectively: 4.3a corresponds to parameters of Fig. 4.2a; 4.3b corresponds to parameters of Fig. 4.2c; and 4.3c corresponds to parameters of Fig. 4.2e. The red circle denotes the leading point in phase-space and the black circle the trailing point.
and numerical parameters \( D = 2; \gamma = 4; \delta = 0.1; \tau = 1.5; \) and \( e_0 = 1.0 \). In Fig. 4.4a, where \( \beta = 0.1; \mu = 0.02; \) and \( g_0 = 0.9 \), a quasi-periodic oscillation prevents localised pulse formation. A stable pulse is formed for \( \beta = 0.1; \mu = 0.1; \) and \( g_0 = 1.5 \) (Fig. 4.4c). For large values of \( \beta \), and with sufficient energy, Equation 4.10 was susceptible to blow-up. However, for \( \beta = 0.35; \mu = 0.2; \) and \( g_0 = 1.5 \), the quintic saturation stabilises the nonlinear gain, resulting in a multi-pulse solution (Fig. 4.4e).

Thus far, the discussion has been restricted to regimes in the anomalous dispersion region, where transform-limited soliton-like pulses tend to form. However, the CQGLE (and the CGLE, although in a limited range of parameters) supports stable pulse solutions in the normal dispersion regime \((D < 0)\). Figure 4.6 shows the temporal and spectral evolution of a stable solution of the CGQLE, with numerical parameters \( D = -1; \gamma = 1; \delta = 0.2; \tau = 1.5; e_0 = 1.0; \beta = 0.5; \mu = -0.1; \) and \( g_0 = 3.0 \). This is the case of the so-called dissipative soliton, characterised by a broad sech-shaped temporal envelope, a square-shape spectral intensity profile, and often possessing a large (linear) chirp. Such pulses can be many times-transform limited, and although have a typically lower amplitude (and lower peak power) than the characteristic soliton, prove to be more resistant and robust to perturbations and break-down as their energy is scaled. Consequently, mode-locked lasers operating in the normal dispersion regime have been the subject of intense study.

### 4.2.3 A piece-wise numerical model: an empirical implementation

The master mode-locking model (Equation 4.10) and the CQGLE (Equation 4.17) capture the essential qualitative features and operating regimes of mode-locked lasers with various dispersion maps, using a normalised coordinate system. However, it is sometimes useful and instructive to be able to represent the parameters and design of experimental setups, and explore the dynamic properties of individual components in a laser system. In addition, when higher-order nonlinear terms are relevant\(^{12}\) the extended NLSE no-longer permits analytic solutions (even in limited parameter ranges), necessitating the use of numerical integration schemes (as described previously), negating the need to make some necessary approximations. To this end, an empirical model – based on a modified NLS equation – was developed using a heuristic approach, where a complex field was propagated through each of the components in the laser cavity using a modular-based design. A representative flow of cavity components is illustrated

\(^{12}\)For the purposes of this thesis higher-order nonlinear terms, describing physical effects such as Raman scattering and optical shock formation, are neglected due to the limited spectral bandwidth of the pulses. An exception is made in chapter 6 where I consider supercontinuum generation in optical fibres, and the inclusion of such terms is necessary to capture all of the contributing effects.
4.2 Theory of pulse propagation in a mode-locked fibre laser

Figure 4.4: Mode-locking dynamics of the cubic-quintic Ginzburg-Landau equation, with gain saturation. The common parameter values: $D = 2; \gamma = 4; \delta = 0.1; \tau = 1.5; \text{ and } e_0 = 1.0$. 4.4a Evolution dynamics with $\beta = 0.1; \mu = 0.02; \text{ and } g_0 = 0.9$. 4.4c Evolution dynamics with $\beta = 0.1; \mu = 0.1; \text{ and } g_0 = 1.5$. 4.4e Evolution dynamics with $\beta = 0.35; \mu = 0.2; \text{ and } g_0 = 1.5$. 

(a) Temporal evolution.  
(b) Pulse parameters.  
(c) Temporal evolution.  
(d) Pulse parameters.  
(e) Temporal evolution.  
(f) Pulse parameters.
Figure 4.5: Phase-space plots showing the attractor dynamics of the CQGLE system for parameters given in Fig. 4.4, respectively: 4.5a corresponds to parameters of Fig. 4.4a; 4.5b corresponds to parameters of Fig. 4.4c; and 4.5c corresponds to parameters of Fig. 4.4e. The red circle denotes the leading point in phase-space and the black circle the trailing point.
4.2 Theory of pulse propagation in a mode-locked fibre laser

Figure 4.6: Normal dispersion mode-locking dynamics of the cubic-quintic Ginzburg-Landau equation, with gain saturation. The parameter values for the numerical simulation were: \( D = -1; \gamma = 1; \delta = 0.2; \tau = 1.5; e_0 = 1.0; \beta = 0.5; \mu = -0.1; \) and \( g_0 = 3.0. \)

in Fig. 4.8. Here, I call this model the piece-wise numerical scheme, and it is easily extendible to include effects such as inhomogeneously broadened gain media [Kom06, Yan07], long term temporal instabilities due to Q-switching [Pas04, Men07], and slow saturable absorption [Hau75a].

Propagation in passive and active fibre

The complex field spectral envelope, \( \tilde{A}(z, \Omega), \)

\[ \partial_z \tilde{A}(z, \Omega) = i \frac{\beta_2}{2} \Omega^2 \tilde{A}(z, \Omega) - \frac{a(\Omega)}{2} \tilde{A}(z, \Omega) + i \gamma \mathcal{F} \{ |A(z, \tau)|^2 A(z, \tau) \} \]  

\( \text{GVD} \quad \text{loss/gain} \quad \text{SPM} \)

It has already been stated that higher-order nonlinearities, namely shock formation and Raman terms can be neglected from any NLSE formulation in this case due to the nar-

\[ u(z, T) = \frac{A(z, T)}{\sqrt{P_0}} \]  

\[ \text{where } P_0 \text{ is the pulse peak power. Here, } \tilde{A}(z, \Omega) \text{ is simply } \mathcal{F} [A(z, T)] \]

\[ \text{Note a change of notation for brevity, such that } \partial_z \tilde{A}(z, \Omega) = \frac{\partial}{\partial z} \tilde{A}(z, \Omega). \]
Figure 4.7: Pulse parameters and phase-space plot showing the attractor dynamics of the CQGLE system for parameters given in Fig. 4.6. In Fig. 4.7b the red circle denotes the leading point in phase-space and the black circle the trailing point.
4.2 Theory of pulse propagation in a mode-locked fibre laser

Figure 4.8: Flow of components in a typical piece-wise numerical model.

![Flow of components in a typical piece-wise numerical model](image)

Figure 4.9: Measured fluorescence spectra from a typical Yb-doped fibre pumped at 980 nm. The data is fitted with both a parabolic and Lorentzian function.

...row bandwidth (and consequently longer duration) of the pulses; in addition, although higher-order dispersion was accounted for, it was rarely included in simulations of basic mode-locked systems, where $\beta_2 \gg \beta_3$. For passive fibre (assuming negligible losses over short lengths) $\alpha = 0$ and for active fibre $\alpha \neq 0$. In addition, the gain (or loss) of the active fibre segment can have an arbitrary spectral profile. $A(z, T) = \mathcal{F}^{-1} \{ \tilde{A}(z, \Omega) \}$ is the Fourier transform of the spectral envelope. As in the previous section, $\Omega = \omega - \omega_0$ is the frequency with respect to the central pulse frequency $\omega_0$, and $T = t - \beta_1 z$ is the time frame moving with the group velocity of the pulse.

Gain profile, saturation and dispersion

The amplifier fibre can be modelled to include a parabolic gain profile, as in section 4.2.1 (see Equation 4.5), that shows strong agreement with the experimentally measured fluorescence spectrum for a typical Yb-doped fibre amplifier (see Fig. 4.9). The spectral shape of the gain is particularly important for short pulses (≤1 ps), as their spectrum is...
wide enough such that all spectral components cannot be amplified by the same amount because of gain roll-off – this phenomenon is usually referred to as gain dispersion [Agr91]. The relationship between the gain and the refractive index is mediated through the Kerr term (or intensity dependent refractive index), that is responsible for SPM (see Equation 4.19). Consequently, gain dispersion is accompanied by gain-induced GVD, and can lead to pulse compression in the time-domain (depending on the nature of the input pulse), and gain-induced SPM, leading to spectral broadening when the amplifier operates in the saturated regime [Agr91].

The peak gain can be modelled with 

\[ g = \frac{g_0}{1 + E/E_{\text{sat}}} \]

where \( g_0 \) is the small signal gain, and \( E \) and \( E_{\text{sat}} \) are the input and input saturation energies of the amplifier respectively. This expression for the gain dynamics is equivalent to Equation 4.6. This model of the gain assumes that the amplifier fully recovers before the next pulse arrives, as such the saturation effect is based on the energy of a single pulse. In fibre-amplifiers, where the upper-state lifetimes are long (typically > 1 ms), a superior model includes gain storage, such that [Pas04]

\[
\tau g \frac{\partial}{\partial T} g = -\frac{g - g_0}{\tau g} - \frac{g P}{E_{\text{sat}}} 
\]

where \( \tau g \) is the gain recovery time (spontaneous lifetime of the upper laser level) and \( P \) is the average intracavity power (over one round-trip). Using this model, relaxation oscillations and timing noise arising from intensity fluctuations can be investigated. However, the improved validity is computationally expensive and in most cases the basic gain saturation model was used.

While many of the fibre gain media discussed in this thesis are appropriately modelled with a perfectly homogeneously saturating gain, it is sometimes necessary to include inhomogeneous saturation (in particular in Bi-doped fibres where the active centres involved in the lasing process are less well understood, and inhomogeneous broadening may contribute). A naive model of inhomogeneously broadened gain media can be developed using a summation over an arbitrary, but odd number of Lorentzian modes (of arbitrary spacing), each of which represents an independent active centre involved in the lasing process, with an associated bandwidth and weighted saturation energy [Kom06, Yan07].
4.2 Theory of pulse propagation in a mode-locked fibre laser

Figure 4.10: Functional form of Equation 4.21 for parameters: $\alpha_{\text{unsat}} = 0.5$; $\alpha_{\text{sat}} = 0.20$; and $P_{\text{sat}} = 1000$.

**Saturable absorber**

Previously it was necessary to make the approximation of a weak saturation effect in order to permit analytic solutions to Equations 4.10 and 4.17. Here, I use the saturable transmission operator $\hat{T}$ directly on the temporal field

$$\hat{T} = (1 - \alpha_{\text{unsat}}) \left( 1 - \left( \frac{\alpha_{\text{sat}}}{\rho + P_{\text{sat}}} \right) \right)$$

(4.21)

where $\alpha_{\text{unsat}}$ is the unsaturable loss (i.e. linear attenuation), $\alpha_{\text{sat}}$ is the saturable loss (equal to the modulation depth) and $P_{\text{sat}}$ is the saturation power of the absorber device. Equation 4.21 allows empirically estimated parameters to be used directly in the numerical model, and has a form that can be compared with experimental data (see Fig. 4.10). Figure 4.10 shows the transmission profile for a typical saturable absorber device with a modulation depth of 20%, a saturation power of 1 kW and a linear attenuation of 3 dB.

**Filters**

Although the amplifier has a spectral limiting effect due to the parabolic shape of the gain, it is sometimes necessary to include a discrete bandpass filter into a laser cavity to stabilise mode-locked operation. This has proven to be particularly pertinent when the cavity has a normal dispersion map [Bal08, Kel09c, Kel09a]. A simple Gaussian filter

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15 The saturated and unsaturated regime correspond to whether the pulse energy in the amplifier is comparable to or much less than its saturation energy.

16 An odd number of modes ensures that gain is available at the centre frequency – this model is still under development.
function was applied to the spectral field, with the following form

$$f_{\text{Gauss}}(\Omega) = \Gamma_{\text{trans}} \left[ \frac{1}{\sqrt{2\pi}\sigma} \exp \left( -\frac{1}{2} \left( \frac{\Omega}{\sigma} \right)^2 \right) \right]$$

(4.22)

where $\sigma$ is the filter bandwidth and $\Gamma_{\text{trans}}$ defines the maximum transmission of the filter, where $0 \leq \Gamma_{\text{trans}} \leq 1$.

In addition to a discrete bandpass filter element, it was found that a transmission based saturable absorber device of finite thickness, when integrated into a laser cavity between two FC/APC connectors introduced a weak Fabry-Perot etalon. Although this effect did not noticeably influence the pulse dynamics, for good quantitative agreement between simulation and experiment it was necessary to include a description of this effect in the model. The Fabry-Perot element modified the complex spectral field with the following function (see Fig. 4.11 for the graphical form of Equation 4.23):

$$f_{\text{FP}}(\Omega) = \frac{1}{1 + F \sin \left( \frac{(\delta-\phi)}{2} \right)^2}$$

(4.23)

where $F$ is the finesse, given by

$$F = \frac{4R}{(1-R)^2}$$

(4.24)

where $R$ is the reflectivity coefficient; $\delta$ and $\phi$ represent the phase difference between...
4.3 Gain-guided soliton propagation in normally dispersive mode-locked lasers

succeeding reflections, and are given by:

$$\delta = 2\pi \left( \frac{\lambda_c^2}{\lambda} \right) \left( \frac{1}{\Delta \lambda} - \frac{1}{\lambda_c} \right)$$ (4.25)

$$\phi = \frac{2\pi \lambda_c^2}{\Delta \lambda - 1}$$ (4.26)

where \( \lambda_c \) is the centre wavelength and \( \Delta \lambda \) is the fringe spacing.

The effect of this element on the steady-state pulse formation will be considered in more detail in section 4.3.

4.3 Gain-guided soliton propagation in normally dispersive mode-locked lasers

Stable, polarised fibre lasers with compact and simple design are in great demand for a variety of applications, such as spectroscopy, wavelength conversion, and optical communications [Fer02, Agr01]. Yb-doped fibres, possessing a broad gain bandwidth, are an attractive medium for ultrafast pulse generation. Previously, such lasers required complex dispersion-compensation setups. It is now routine to operate Yb fibre lasers without dispersion compensating elements, in the normally dispersive regime, to overcome the limits imposed by conservative soliton propagation [Wis08]. A large body of work, both theoretical and experimental, has been devoted to understanding the evolution of pulse structures in such dissipative systems [Wis08, Cho08b, Cho06, Ort10]. Instead of the usual dynamic of a balance between anomalous group velocity dispersion (GVD) and electronic Kerr nonlinearity, leading to the stable formation of a solitary wave, as in a soliton laser, ANDi systems support temporal dissipative solitons, characterised by an internal energy flow that underlies the balance of amplitude and phase modulation needed to form a soliton-pulse solution [Akh05]. Hence, the pulse shaping mechanism in ANDi lasers is strongly dependent on dissipative processes, such as linear gain (and loss) and nonlinear saturable absorption, resulting in self-amplitude modulation [Wis08, Cho08b].

Currently, ultrafast ANDi lasers generating chirped pulses typically need intracavity filters to mimic the action of a saturable absorber (SA), maintaining pulse formation in the steady-state [Wis08]. Such components (e.g. free-space [Wis08] or fibre [Kie08] based) can give rise to extra instabilities [Agr01] and increase the system complexity [Agr01]. However, previous work by Zhao and co-workers [Zha07b] has shown that the need for a discrete filtering element can be relaxed when the gain bandwidth is sufficiently narrow, leading to the formation of the so-called “gain-guided” soliton [Zha07b], typically found in Er-doped fibre lasers, where the peak of the gain spectrum, around 1530 nm, is
Dynamics, modelling and simulation of mode-locked fibre lasers

Figure 4.12: Overview of the ring cavity. All acronyms have been previously defined. It is worth noting that the passive fibre is polarisation-maintaining (PM), rather than non-PM (isotropic) as in previous cases.

relatively narrow. Consequently, additional filter components are not needed.

Nonlinear polarization evolution (NPE) has been the widely employed mechanism to mode-lock ADi lasers, allowing scalability of the pulse energy, without damage to passive components [Lef10]. However, such systems can suffer from instabilities due to environmental fluctuations [Wis08, Zha07b, Cho08b, Ren08a]. In addition, polarisation-maintaining fibres, which can offer increased stability [Liu10] and a polarised output, are not employable in NPE lasers [Agr01]. CNTs have been widely employed in this thesis to mode-locked ultrafast fibre lasers, and their unique properties have been widely discussed (in particular in chapter 2).

In this section I discuss the development of a polarised, low-noise gain-guided Yb-doped fibre laser, mode-locked by CNTs. In particular, attention is directed towards the role of spectral filtering on the stabilisation of valid pulse solutions, and I show that gain-guided dissipative soliton pulses can be supported in such lasers, where the single-pass gain bandwidth is up to ~ 55 nm. Extensive numerical simulations, based on the model described in the previous section, are used to clarify the solitary-wave evolution dynamics, and define regions where stable pulses can be guided by the broad bandwidth gain medium.

4.3.1 Experiment

Setup

An overview of the laser setup is shown in Fig. 4.12. The cavity was constructed from polarisation maintaining fibre for a stable polarised output and to be consistent with the linear one-dimensional nature of the numerical model. The cavity consisted of 0.9 m of double-clad Yb-doped fibre pumped with a 4 W multi-mode diode laser at 980 nm; a broadband output coupler, with a 3 dB transmission bandwidth greater than 150 nm, which coupled 30% of the light out; and a CNT-based saturable absorber [Has09, Kel09c], adhered to the facet of a FC-APC (fibre connector with angled physical contact) with in-
4.3 Gain-guided soliton propagation in normally dispersive mode-locked lasers

![Image of optical spectrum and autocorrelation](image)

**Figure 4.13:** Spectral and temporal properties of the PM dissipative soliton Yb-doped fibre laser.

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dex matching gel, and butt coupled to a second FC-APC forming a transmission-style device. The length of passive fibre in the cavity was 5.35 m including the contribution from the coupler in the ring. The operation wavelength of CNT-based saturable absorbers (CNT-SA) depends on the CNT band-gap energy [Has09]. To match the operation wavelength of Yb-doped fibre lasers (∼1060 nm), CNTs of ∼0.8 nm diameter are required [Wei03]. Fabrication of the SA device was undertaken by collaborative partners at the University of Cambridge, where controlled production of SWNTs was achieved using catalytic decomposition of CO on bimetallic Co-Mo catalysts (CoMoCaT) [Kit00]. The sample predominantly consisted of (6, 5), (7, 5), (7, 6) tubes, as determined by Raman spectroscopy, transmission electron microscopy (TEM) and photoluminescence [Kit00, Bac02]. The average tube diameter was ∼0.75 nm to 0.9 nm, corresponding to a gap of ∼1.0 eV to 1.3 eV [Wei03]. Details of the fabrication process can be found in [Has09]. The linear and nonlinear absorption spectrum of the CNT film were plotted in Fig.3.12.

The linear absorption spectrum shows a peak ∼1028 nm, close to the desired operation wavelength, in correspondence to the first transition of (6, 5) and (7, 5) CNTs, with 0.757 nm and 0.829 nm diameters [Wei03]. Another band at ∼1170 nm can be seen, due to (7, 6) CNTs, of 0.895 nm diameter [Wei03].

**Results**

When the laser was mode-locked and in a stable single-pulse mode of operation the measured data was recorded. It was possible to get the laser to stay in the same stable state for several hours without readjustment of the cavity pump power. The output power out of
the ring cavity was −1.4 dBm. Fig. 4.13a shows the output spectrum of the laser. Without the inclusion of a discrete spectral filter and within a specific power range above threshold the laser could operate in a mode with more than one line oscillating. However, in single-pulse operation the laser operated at a wavelength of 1060.0 nm, with a 3 dB bandwidth of 0.15 nm. The low level oscillations present in the spectrum arise due to a Fabry-Perot (FP) effect at the nanotube interface. However, because of the low finesse (< 4% at each interface, attributed to Fresnel reflections) it has a very weak filtering effect. The power transmission of the CNT-SA device was measured using a low-power broadband ASE source over the spectral range 1000 nm to 1140 nm, and the response curve is shown in Fig. 4.14. Small (∼2%) periodic modulations due to the resonant cavity formed in the CNT-SA device are clear. This filtering effect is included in the numerical modelling described below, and shall show that this effect is not critical to the dynamic of the cavity. However, it is important to note that, although weak, this FP effect will be present in all mode-locked oscillators adopting transmission style SA devices of finite thickness. As such it is important to augment existing laser models to include a description of this effect for better understanding of experimental observations, and good quantitative agreement between performance parameters.

\[ \Phi_{\text{NL}} \]

\[ \text{Power transmission ratio} \]

\[ \text{Wavelength (µm)} \]

**Figure 4.14:** Power transmission through the saturable absorber device, consisting of the polymeric nanotube-doped film embedded between two FC-APC connectors.

The degree of accumulated nonlinear phase shift (\( \Phi_{\text{NL}} \)) per cavity pass, which is proportional to the peak power of the field inside the laser cavity, is important to the mode of laser operation, and the resulting pulse spectral and temporal profiles. Previous work by Chong et al. [Cho08b], and others [Cho06, Ren08a, Bal08] has classified the properties of the pulse solutions in such lasers for a range of \( \Phi_{\text{NL}} \) values. This laser operates in a regime exhibiting low intra-cavity peak power: although self-phase modulation broad-
4.3 Gain-guided soliton propagation in normally dispersive mode-locked lasers

ens the spectrum to balance spectral narrowing due to gain dispersion, the accumulated nonlinear phase shift is small ($\Phi_{NL} \ll \pi$). This value is low for ANDi lasers, which can tolerate much larger values ($\Phi_{NL} \gg \pi$) in the presence of strong spectral filtering. In a gain-guided system rapid spectral broadening cannot be controlled by the gain dispersion alone: single-pulse operation collapses, leading to multi-pulsing. However, the spectral shape observed is consistent with the classifications outlined in Ref. [Cho08b] for lasers of this type, with normal dispersion maps.

The measured autocorrelation of the output pulse is shown in Fig. 4.13b. The autocorrelation matches that of a sech$^2$ pulse, implying a FWHM temporal duration of 13.9 ps. The time-bandwidth product is 1.11, which suggests the pulses are moderately chirped, expected for mode-locked fibre lasers with normal dispersion maps [dM04a, Wis08].

![Relative intensity noise spectra](image)

**Figure 4.15:** Relative intensity noise spectra: fundamental harmonic of the lasing cavity (4.15a); and higher harmonic frequencies (4.15b). Note that the RIN level is device limited around the higher-harmonics.
Figure 4.15 shows the relative intensity noise (RIN) performance of the laser. The RIN was measured using the approach outlined in Ref. [Der97]:

\[
\text{RIN} = \frac{(\Delta P)^2}{(P_{\text{avg}})^2} \tag{4.27}
\]

where the electrical spectrum corresponds to an amplified equivalent of \((\Delta P)^2\) and the dc photocurrent, that can be simultaneously monitored, corresponds to an electrical equivalent of \((P_{\text{avg}})^2\), when squared and multiplied by the impedance (assuming unity gain and a 50 \(\Omega\) input impedance). Fig. 4.15a shows the RIN around the fundamental harmonic of the cavity; and Fig. 4.15b shows the higher beat-frequency harmonics in the electrical spectrum.\(^{17}\) The repetition rate of the cavity was 33 MHz, corresponding to the cavity round-trip time. The low RIN level (\(-177 \text{ dBc Hz}^{-1}\)) suggests good pulse-to-pulse quality, this was confirmed using an analogue oscilloscope to view the pulse train stability; slow modulations of the pulse train amplitude were not observed. From

\[
\Delta E = \sqrt{\frac{\Delta P \Delta f}{\Delta f_{\text{Res}}}} \tag{4.28}
\]

given in Ref. [vdL86], where \(\Delta E\) is the relative pulse-to-pulse energy fluctuation, \(\Delta P\) is the relative peak spectral intensity, \(\Delta f\) and \(\Delta f_{\text{Res}}\) are the spectral width and resolution, the calculated \(\Delta E\) from the fundamental harmonic is \(3.6 \times 10^{-3}\), highlighting the stability of the generated pulses.

**4.3.2 Numerical model**

To investigate the pulse formation dynamics in this ultrafast laser, in the presence of all-normal cavity GVD and gain dispersion, and without a discrete filter, numerical simulations of the system were performed using the model developed in section 4.2.3. Two sets of numerical simulations were conducted to determine the influence of the FP effect from the nanotube interface on the overall pulse dynamic: the first set excluded the FP element; the second set included the FP element. In Fig. 4.8 an overview of the numerical model is shown, illustrating the flow of cavity components represented by the equations presented previously. The simulation model consisted of solving the nonlinear Schrödinger equation for the different cavity components and using the output from one component as the input to the other. The simulation was started from noise and after some thousands of iterations a stable pulse was obtained. The Raman effect was disregarded due to the small bandwidth of the pulse. The simulation frame was centred

\(^{17}\)Note that due to the large frequency coverage the resolution of the electrical spectrum analyser is reduced.

The low level RIN noise of the laser is limited by the electrical noise floor of the diagnostic.
4.3 Gain-guided soliton propagation in normally dispersive mode-locked lasers

at 1060 nm, with a time window of 200 ps divided into $2^{12}$ points. For the doped fibre the following parameters were used: length 0.9 m; dispersion $\beta_2 = 0.018$ ps$^2$m$^{-1}$; gain bandwidth of 56.8 nm; small signal gain of 20 dB; a saturation energy of 90 pJ for the case with the FP, and 60 pJ for the case without the FP; and a nonlinear coefficient of 0.003 W$^{-1}$m$^{-1}$. The loss element represents contributions from the output coupler and the additional losses in the cavity. The value of the total loss was 7 dB. The two passive fibre components were identical with a length of 2.675 m, a dispersion of $\beta_2 = 0.018$ ps$^2$m$^{-1}$, and a nonlinear coefficient of 0.003 W$^{-1}$m$^{-1}$. The saturable absorber had a modulation depth of 10%, a saturation power of 4.2 W for the case with the FP element and 2.3 W for the case without the FP element, and a linear loss of 3 dB, based on values obtained from experimental measurements [Tra11a]. When included the FP element had a reflectivity of 4% and a fringe spacing of 1.2 nm.

It was possible to numerically obtain mode-locking in a single pulse regime without the use of a bandpass filter in the cavity, both with and without the effect of the FP from the nanotube interface. The average power in the output arm of the coupler in the simulated cavity was $-2.90$ dBm for the case with the FP and $-2.94$ dBm for the case without. The temporal and spectral properties of the two sets of simulations are summarised in Fig. 4.16. In Fig. 4.16a the calculated autocorrelation of the pulse extracted at the coupler position is shown (FP not included in the simulation). The temporal duration and shape closely matches that of the physical system, illustrating that dissipative soliton pulses can be supported by a cavity where a broad gain bandwidth (such as Yb) is the dominant spectral profile. In Fig. 4.16b the pulse extracted at the coupler position is shown in the frequency domain (FP not included).

Figure 4.16c and 4.16d show the corresponding spectral and temporal profiles (or calculated autocorrelation function of the field), with the inclusion of the FP element. In this case, the spectral bandwidth was 0.19 nm and the background spectral oscillations observed experimentally (Fig. 4.13a) are reproduced. Without the FP element, the bandwidth increases to 0.60 nm and the oscillations disappear. It is clear that the FP effect, introduced unintentionally by the chosen integration scheme for the transmission style SA device, results in a narrowing of the laser spectrum and the presence of low level spectral modulations. Quantitatively better agreement between simulation and experiment, with the inclusion of this effect in the model, confirms this hypothesis. What is also clear is that the FP element is not fundamental to the formation dynamics of a steady-state pulse, in this case.

To confirm that the output pulses are true dissipative solitons,\textsuperscript{18} possessing a linear low-coherence, noise-burst-type pulses that are incompressible are also often observed in the positive dispersion regime; such pulses will be discussed further in chapter 5.

\textsuperscript{18}
Figure 4.16: Calculated autocorrelation function and corresponding optical spectrum of the simulated time-domain output pulse. In 4.16c and 4.16d the FP element was included.
4.3 Gain-guided soliton propagation in normally dispersive mode-locked lasers

![Graph showing temporal intensity and phase](image)

**Figure 4.17**: Temporal intensity (blue curve) and temporal phase (red curve) of the simulated output pulse, with the FP element included in the model. The black dashed curve is a weighted fit to the temporal phase.

Chirp, plotted in Fig. 4.17 is the temporal field intensity and corresponding temporal phase. The phase profile is parabolic (with high-order terms). A quadratic variation in the phase across the pulse represents a linear ramp in frequency as a function of time – the pulses are linearly chirped [Tre00]. In this case the chirp is negative, and can be simply compensated extra-cavity to obtain a near transform-limited pulse. It should be noted that the phase is meaningless when the pulse intensity is zero. Therefore, a weighted fit to the temporal phase \( \phi(t) \), based on the pulse intensity, was applied, with the following form

\[
\phi(t) = \phi_0 + \frac{\phi_1 t^2}{2!} + \frac{\phi_3 t^3}{3!} + \frac{\phi_4 t^4}{4!} + \frac{\phi_5 t^5}{5!} \tag{4.29}
\]

The functional form of the temporal phase (see Fig. 4.17) has negligible cubic phase, but residual quartic phase; such high-order phase distortions arise due to nonlinear processes, such as SPM (in this case).

In Fig. 4.18 the FWHM of the field intensities in both the temporal and spectral domains, for a pulse over one cavity evolution in the steady-state mode-locking regime, are plotted for the case with and without the FP element. The FP effect is found to have only a small affect on the overall temporal dynamics of the laser: lowering the average steady-state duration by \( \sim 1.8\% \); this is not unexpected given that the pulse is positively chirped and the element also acts to limit the laser bandwidth. Although the simulations with the FP effect are more accurate, and better represent the parameters of the physical system, the comparison here indicates that the effect simply narrows the spectrum, introduces background spectral oscillations and marginally reduces the pulse duration; it does not
Figure 4.18: Temporal (4.18a) and spectral (4.18b) evolution of the steady-state pulse over one round-trip of the laser ring cavity, through each of the cavity elements. The red curve corresponds to the case without the FP element and the blue the case with the FP element included in the numerical model.
4.3 Gain-guided soliton propagation in normally dispersive mode-locked lasers

significantly influence the dynamics of mode-locked operation in the regime of normal dispersion: the pulse shaping remains largely dominated by gain dispersion. It should be noted that the affect of the FP etalon could be reduced with the use of angle polished fibre connectors and index matching gel, at the expense of a possible increase in the net insertion loss of the device.

A second observation from Fig. 4.18 is that the pulse does not significantly change shape on propagation around the cavity, in the steady-state: the breathing ratio is low. This is perhaps less expected given that pulse shaping is non-solitonic, and often in such systems there is significant spectral (and temporal) variation over a single cavity pass. This can be explained by a low value of $\Phi_{NL}$, and demonstrates that solitary wave solutions exist in GNLS-type equations, in the regime of positive GVD. In the following section I use numerical simulations to determine bounds on stable operation of mode-locked lasers of this type for varying filter bandwidth. In addition, I compare performance between soliton and dissipative soliton-type systems.

**Bounds on stable operation**

For low peak power (low $\Phi_{NL}$) ANDi mode-locked fibre lasers I have shown that the gain bandwidth (even for broad media, such as Yb) is sufficient to provide a filtering effect strong enough to stabilise pulse formation; this is the case of the so-called gain-guided dissipative soliton. However, if a high peak power exists inside the cavity, it will contribute to broadening of the spectrum by SPM resulting in a higher nonlinear phase shift. In this case, a discrete filter is required for stable operation [Cho06, Cho08b, Ren08a, Bal08]. Here, I consider further the role spectral filtering plays in the dynamics of ANDi lasers.

All simulations were based on the physical system shown schematically in Fig. 4.12, with the exception of the addition of a discrete Gaussian filter of arbitrary bandwidth.

The details and numerical parameters are provided above. Regions of stable operation were identified for a range of Gaussian filter bandwidths (0.1 nm–10 nm), for a corresponding range of amplifier saturation energies from 6 pJ–300 pJ. The simulations ran for up to 8000 iterations, or until a steady-state had converged. Algorithms were developed to identify and classify the output pulse-shape according to the following set of sequential tests.

First the output (temporal) field intensity was tested for a CW mode by checking if the minimum power value at any point in the time domain window was above 10% of the peak power of the field intensity. If this was true the mode-locked solution was classified as (quasi) CW. However, if this test failed the second test was performed to check if

\[19\] Note the FP effect arising from the transmission-style SA device was neglected in all cases.
Figure 4.19: Identified pulse-types. All on an equal scale. Inset to 4.19a shows a magnified y-axis.
4.3 Gain-guided soliton propagation in normally dispersive mode-locked lasers

![Image](image.png)

**Figure 4.20:** Parameter maps. 4.20a Regions of stability for energy saturation as a function of filter bandwidth, defined according to the pulse types categorised in Fig. 4.19: where the value -1 is assigned (quasi) CW (Fig. 4.19a); 1 is assigned to a single pulse (Fig. 4.19b); 2 is assigned multi-pulse (Fig. 4.19c); and 0 is assigned to an unclassified pulse-shape. 4.20b Regions of stability for energy saturation as a function of passive fibre GVD (note a 0.9 m length of positive GVD amplifier fibre means net cavity zero GVD requires $\sim -0.0162 \text{ ps}^2 \text{ km}^{-1}$ of passive fibre). 4.20c Pulse temporal FWHM duration as function of GVD.
the output was unclassified (or did not possess a uniform symmetric temporal profile). This was achieved by checking how many times the temporal trace crossed a threshold value, set to 5% of the peak power of the pulse. If the returned value was larger than 2, the pulse was grouped as unclassified in the first pass. However, this classification could be updated depending on the outcome of the third test, which checked for a multi-pulse output. The multi-pulse test was performed by evaluating the number of times the temporal field intensity crossed a threshold value, set to 95% of the peak power of the pulse. If this value was integer of 2 and corresponded to the value of the previous test, then the classification was updated to a multi-pulse output. All the pulses that failed the above tests were classified as single-pulse solutions. It is believed that this is a robust way to identify truly single-pulse solutions. Figure 4.19 illustrates the three main pulse-type classifications: CW; single-pulse; and multi-pulse (in this case double-pulse).

Large scale ensemble simulations for the grid of filter bandwidths and amplifier saturation energies were performed. The results were past to the classification software for grouping; and the corresponding stability map is plotted in Fig. 4.20a. Each point on the map consists of an average over ten simulations. The CW mode is assigned as the colour black, the unclassified pulse shape is assigned the colour dark grey, the multi-pulse output is assigned as the colour white, and the single pulse output is assigned as the colour light grey. It is worth noting that if only one of the ensemble of ten simulations passed a low-order test (i.e. the CW test is the lowest order and the single-pulse is the highest order), all were assigned the lowest order grouping, resulting in the most stringent conditions for single-pulse classification.

Three clear regions are evident in Fig. 4.20a: a region below the mode-locking threshold where CW operations exists, for very narrow filter bandwidths this threshold level is never overcome; above the mode-locking threshold single-pulse operation is achieved within a limited range of saturation energies that broadens with a broader bandwidth filter; as the energy is increased within the cavity the multi-pulse threshold is reached. It is worth noting that at the boundaries between these distinct regions the classification of the mode of operation is problematic and an unclassified pulse-shape is often assigned. This could in fact be physical, given that instabilities often occur in the transition between modes of operation, i.e. single-pulsing and multi-pulsing.

In addition to exploring stable operation regimes within a space of discrete filter bandwidths, the filter bandwidth was fixed (to 10 nm) and the dispersion of the passive fibre within the cavity was varied. For a large negative GVD value the passive fibre could compensate the normal GVD of the short length amplifier fibre for operation in the average-soliton regime. The same ensemble set was computed; the resulting map of pulse characterisations for the space of passive fibre GVD (in the range \(-0.04 \text{ ps}^2 \text{ m}^{-1} \rightarrow +0.04 \text{ ps}^2 \text{ m}^{-1}\)
is shown in Fig. 4.20b. The boundary (just below the zero GVD value) where net anomalous dispersion resulted in soliton operation is clear. Two qualitative observations can be made: firstly, a soliton laser has a lower mode-locking threshold compared to lasers operating in the ANDi regime, and this threshold value (in both cases) increases with increasing dispersion; secondly, a soliton laser is more susceptible to wave-breaking or break up of the single-pulse solution, leading to multi-pulsing.

Figure 4.20c shows the same ensemble map, but displaying the calculated FWHM of the field intensity, rather than the output pulse groupings. It is clear that in the regions where the laser operates in either the CW mode or multi-pulses the algorithm used to determine the FWHM of the field intensity fails. However, in the regions where a single-pulse solution exists, in both the positive and negative dispersion regime, three characteristics are clear. Firstly, the duration of a soliton pulse is much shorter than that of a dissipative soliton pulse in the normal dispersion region. This is expected given that the soliton pulses are close to transform-limited, and the dissipative solitons are expected to be chirped. Secondly, although the duration of the soliton increases with increasing (negative) dispersion, it is unclear on this scale. More noticable is the fact that the duration of the dissipative soliton pulse increases with increasing (positive) GVD. This is because the pulse is becoming increasingly chirped in the cavity. This characteristic of dissipative soliton pulses in ANDi lasers will be discussed further in chapter 5.

4.4 Summary

In this chapter I have discussed in detail the role of dispersion and nonlinearity on the evolution of pulses in a mode-locked fibre laser. A number of nonlinear partial differential equations (all variants of the NLSE), describing mode-locking in fibre lasers, have been reviewed; and a numerical model has been developed. The numerical model describes the complex interaction between the dispersion, nonlinearity, gain and loss of a medium acting on an electric field. This model was used to explore the dynamics of an experimental system developed to demonstrate gain-guided dissipative soliton operation in a Yb-doped fibre laser for the first time. The role of spectral filtering on the mode-locking dynamics was explored. It was also found that when using a transmission-style saturable absorber device, the well-known mode-locking equations needed to be augmented to achieve good qualitative agreement between simulation and experiment. This is widely applicable to all mode-locked fibre lasers utilising such a SA device.

Computational modelling is a powerful tool to complement experimental investigation, and it will be widely used in the proceeding chapters to gain an insight and understanding of the dynamics involved.
5 Chirped pulse fibre laser sources

An optical pulse is chirped if the wavelength of the carrier changes continuously throughout the pulse, in particular if the change is monotonic [Tre69a]. In this chapter, I consider the development of all-normal dispersion mode-locked fibre lasers, producing chirped pulses. The pulses can be many-times transform limited, carrying a significant chirp; such mode-locked systems have become known as giant-chirp oscillators (GCOs). In section 5.1, I introduce this concept further and discuss techniques used to visualise these temporal structures. Section 5.2 considers the generation of nanosecond pulses, using both lumped and distributed positive dispersion (stretcher) elements. It is important however, to make the distinction between GCOs that produce temporally coherent, chirped pulses and mode-locked lasers operating in the noise-burst emission regime, where the duration of the pulse envelope can be of the order of several nanoseconds. The coherence properties of the nanosecond pulses are fully characterised in section 5.3, where the pulse spectrogram is measured directly. Aspects regarding compression of the pulses generated in section 5.2 and characterised in 5.3 are briefly discussed in section 5.4, and a practical scheme for accessing higher-energy, while retaining high-quality pulses, is proposed. The dynamics of chirped pulse formation, subject to strong positive dispersion, in GCO-type systems in considered in section 5.5.

Results presented in this chapter have been published in the following journal articles and conference proceedings [Kel09c, Kel09a, Kel09b, Kel10c, Zha11a, Kelona].

5.1 Introduction

5.1.1 A brief history

Throughout the first two decades of technological advancement in short-pulse laser development, from the first demonstrations in the mid 1960s that focussed primarily on solid-state systems, little attention was directed towards correcting for accumulated frequency chirp induced by the self-phase modulation of high intensity intra-cavity pulses. Subsequently, it was shown that picosecond scale optical pulses emitted from solid-state Nd:glass lasers possessed frequency swept carriers and could therefore be compressed extra-cavity, using a diffraction grating pair, to a duration closer to the reciprocal of their
bandwidth [Tre68]. In addition, the diffraction grating pulse compressor was used in conjunction with second harmonic intensity autocorrelation to demonstrate substructure and asymmetry of the chirped pulses generated in early Nd:glass lasers [Tre69b] – these measurements represented early attempts to resolve both the amplitude and phase information of ultrashort pulses, experiments which today have lead to the widely accepted techniques of FROG (and XFROG) for the full characterisation of femtosecond pulses [Tre93, Tre97, Tre00].

In 1973 Hasegawa and Tappert proposed theoretically the generation of optical solitons in single-mode optical fibres1 [Has73a]: temporally localised packets of light that form through a subtle balance between the mutual interaction of anomalous dispersion and self-phase modulation, resulting in a transform-limited pulse. The experimental observation of optical solitons was reported in 1980 [Mol80], revolutionising modern telecommunications and directing the focus of short pulse laser research towards resonating solitons in a cavity, subject to control of both dispersion and nonlinearity, to form a soliton laser [Mol84]. The term soliton-laser is now widely applied to generic dispersion-compensated ultrashort pulse fibre lasers, although the generated pulses are generally not solitons according to the strict mathematical definition [Tay03].

With the objective of power-scaling fibre-based systems to pulse parameters competitive with solid-state counterparts, it has become clear that the single-soliton pulse is not an impervious solution to increasing amounts of accumulated nonlinear phase shift, with scaling pulse energies. A fundamental threshold exists where the single-pulse condition collapses and the temporal waveform breaks-down into sub-pulses at aharmonic cavity frequencies [And93]. This threshold has been found to occur for single-pulse energies above \( \sim 100 \) pJ in standard fibre. In the previous chapter, a number of schemes were reviewed that aim to overcome the limit imposed by operation in the soliton-regime. In such cases, the pulse is encouraged to stretch and re-compress over a single pass of the cavity through careful control of segments of positive and negative dispersion, in order to lower the average peak power and increase the effective nonlinear threshold, where wave-breaking may occur. In the natural limit, the pulse remains chirped throughout the cavity; such operation can be achieved with an all-normal dispersion map and has proven a suitable route to obtain the highest energy pulses from mode-locked fibre lasers [Wis08, Lef10, Ort10, Bau10].

It is natural to extend the magnitude of positive cavity dispersion in order to stretch the pulse in time, such that the pulse can accept the maximum input of energy without distortion. There have been a number of experimental demonstrations of ANDi mode-

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1This seminal contribution was published as a two letter pair considering stationary nonlinear optical pulses, subject to both negative [Has73a] and positive [Has73b] dispersion, where bright and dark soliton pulses exist, respectively.
Figure 5.1: Schematic representation of the algorithm applied to compute the spectrogram: a specific gate function samples a portion of the temporal waveform (in this case a chirped Gaussian pulse); the spectrum of the gated waveform is then computed (or measured); the gate function is swept sequentially through the pulse waveform, recording the spectrum at each delay step.

locked lasers, with net cavity dispersion $\beta_2 \leq 1 \text{ ps}^2$ [dM04a, Cho06, Wis08]. Such systems, have produced uncompressed output pulses with durations up to $\sim 150 \text{ ps}$ [Ren08b, Wis08]. It has been confirmed analytically that these correspond to linearly chirped, dissipative soliton solutions of the cubic-quintic Ginzburg Landau equation [Ren08a, Ren08b]. It is necessary to clarify nomenclature at this stage: ANDi refers to lasers with a completely positive dispersion map that can generate chirped dissipative soliton solutions; GCOs (or giant-chirp oscillators) is the colloquial term for a sub-class of (usually ANDi) lasers where the cavity dispersion $\beta_2 \geq 1 \text{ ps}^2$, and the generated pulses possess a significant chirp (and have a large time-bandwidth product). This definition is for the purposes of discussion in this thesis; in fact the term is often used with a degree of flexibility in the literature.

5.1.2 Representation of optical pulses in time-frequency space – the pulse spectrogram

The time-dependent spectrum – or spectrogram – provides an intuitive picture of co-localised spectral and temporal components of a light field, and is widely used in optics research: to view ultrashort pulses (through FROG and XFROG measurements) [Tre00]; explore the complex dynamics involved in supercontinua [Tra10c]; and establish the coherence properties of chirped pulses [Kel09a]. In addition, the sonogram – analogous to the optical spectrogram – is widely used in acoustics to analyse phonetic waveforms. Figure 5.1 illustrates the process of generating the optical spectrogram of a complex pulse waveform: a gate function spectrally samples a portion of the temporal waveform; the gate function is swept through the waveform, recording the corresponding spectrum for all values of gate position (or delay), to recover the full pulse spectrogram [Tre00].
mathematical description of the optical spectrogram \( S(z, T, \omega) \) is given by [Tre00, Tra10c]

\[
S(z, T, \omega) = \left| \int_{-\infty}^{\infty} A(z, \tau) \mathcal{G}(\tau - \tau') \exp(-i\Omega \tau) d\tau \right|^2
\]  

(5.1)

corresponding to a windowed Fourier transform, where \( \mathcal{G}(\tau - \tau') \) is the variable-delay gate function.\(^2\) The spectrogram is widely used in this chapter (and throughout the thesis) in particular to view, and gain insight from, numerically simulated results, where perfect diagnostics can be applied. In such cases, a Gaussian window is used as the gate function, with a characteristic duration \( \tau_{\text{gate}} \) much less than the time-scale of interest. For small values of \( \tau_{\text{gate}} \), the spectrogram can be used to determine the spectral content of \( A(z, T) \) at time \( \tau' \), over the duration of \( \tau_{\text{gate}} \). However, due to the reciprocal relationship between time and frequency the spectrum is not exact, as the resolution is also inversely related in each domain. The duration of the gating function is chosen appropriately depending on whether the dynamics evolve faster in time or frequency.\(^3\)

### 5.2 Nanosecond pulse generation

Passively mode-locked fibre lasers with all-normal dispersion cavities have been widely investigated as a means of obtaining high pulse energies [Ren08b], generating linearly chirped, dissipative soliton pulses that can be dechirped close to the inverse of their bandwidth [Wis08]. Initiation of a regular pulse train from noise and pulse stabilization in ANDi mode-locked fibre lasers relies on the intensity dependent loss provided by the nonlinear action of a saturable absorber in combination with a bandwidth limited gain preventing pulse spreading. The single-pulse energy can be increased by simply increasing the cavity length (hence lowering the resulting fundamental repetition frequency) for a constant average intra-cavity power [Ren08b]. Consequently, pulses with unchirped durations of \( \sim 150 \) ps (dechirped to 670 fs) and energies of 1 J, after amplification, illustrate the usefulness of this approach. With few exceptions (Refs. [Kel09c, Kel09a, Tia09b, Tia09a] in particular), NPE has been the preferred mechanism for initiating and maintaining mode-locking in lasers with large positive dispersion maps, where the intention is to generate high-energy pulses, because of the intrinsic robustness of the optical elements involved. However, NPE is not compatible with extended (normally dispersive) cavity lengths, if dissipative soliton emission is the desired mode of operation: typically, due to excessive polarisation mode-dispersion (PDM) (induced by accumulated birefringence in long (even isotropic) fibre lengths) a single short-pulse splits into sub-pulses;

\(^2\)It should be noted that in XFROG/FROG type-measurements the pulse is gated by a delayed copy of itself, and in its simplest form is the spectrally resolved autocorrelation [Tre00].

\(^3\)This becomes particularly pertinent in chapter 6, where I discuss supercontinua.
5 Chirped pulse fibre laser sources

Figure 5.2: Two all-fibre schemes for generating chirped, dissipative soliton pulses, in all-normal dispersion fibre lasers – giant chirp oscillators (GCOs). Dispersive fibre, DF; all other acronyms previously defined.

similarly a long (narrow-band) pulse, where the duration exceeds the PMD, cannot be supported [Hor97b]. The only stable mode of operation is the formation of noise-like pulses [Mat92b, Hor97a, Hor97b, Hor98, Kan98, Kob08, Kob09, Kob10]. Such pulses, are characterised by a broad, rounded spectrum, a broad temporal envelope (typically several nanoseconds in duration) and low-coherence; and have been useful for applications such as optical coherence tomography (OCT). However, noise-like pulses can only be partially compressed extra-cavity, due to incoherence between coherent modal clusters [Kob09], and cannot be considered dissipative solitons of the cubic (and cubic-quintic) Ginzburg Landau equation [Akh05].

The saturation properties of a SWNT-SA are independent of cavity length, and consequently provides a robust device for initiating stable dissipative soliton pulses in an elongated normally dispersive mode-locked cavity [Kel09c, Kel09a]. Absence of a spectral filtering element results in extremely chirped pulses, up to 750 times transform limited duration. These are suitable for compression, provided the duration is chosen to match external compressor schemes. The experimental generation of such pulses, in all-fibre configurations using both distributed and lumped dispersive elements, is the subject of the following sections.

5.2.1 Using a fibre-based dispersive element

Experimental setup

The experimental setup of the all-fibre system used to demonstrate the stable generation of nanosecond dissipative soliton pulses, using a SWNT-SA to initiate mode-locking and a fibre-based dispersive element to stretch the pulses, is illustrated in Fig. 5.2a. The laser was based on a 2 m Yb-doped fibre amplifier unit operating around 1.06 \( \mu \text{m} \), be-
low the point of zero material group velocity dispersion in silica fibre for a naturally all-normally dispersive cavity. The output of the amplifier was directly fusion spliced to a SWNT-PolyVinyl Alcohol (PVA) saturable absorber device that comprised a PVA film embedded with CoMoCat SWNTs [Res02] and integrated by clamping it between two FC-APC fibre connectors [Roz06b, Sca07]. The pure PVA film had high transparency between 400 nm and >1300 nm, and did not contribute to the saturable absorption properties of the SWNT-PVA composite. Typical recovery times are sub-picosecond [Gam08], significantly shorter than the generated mode-locked pulses, even for short cavities where the magnitude of the positive dispersion is low and the chirp induced stretching is minimum. The absorber had a damage threshold of $\sim 0.16 \text{ mW} \mu \text{m}^{-2}$ at 1.06 $\mu \text{m}$. Full details of such SWNT-SA devices, including their linear and nonlinear optical properties, used to mode-lock fibre lasers across the near-infrared, were provided in chapter 2.

Although the cavity consisted of isotropic fibre, a fibre-strainer polarization controller was used to provide a degree of control over the state of polarisation in the cavity. It was found that, although the laser mode-locked for all settings of the polarisation controller, tuning achieved a state where the intensity noise was significantly reduced. It is believed that this is due to strain induced loss for certain polarisation settings. Resolving the two orthogonal components of polarisation extra-cavity showed an approximately equal distribution of power along both $x$ and $y$ axes, suggesting no dominant linear polarisation. It is important to emphasise that NPE did not contribute to pulse-shaping in this laser. A fused fibre coupler provided a 15% output for diagnostic analysis of both the temporal and spectral domain. A fibreised polarisation insensitive, inline optical isolator was used to impose unidirectional propagation. Finally, a single-mode optical fibre, with a length (varied using cut-back and fusion splices) between approximately 1 m and 1200 m, provided a means of controlling the magnitude of positive cavity dispersion (intrinsically coupled to the fundamental repetition frequency). It is assumed that the cavity fibre (including the gain fibre in the amplifier unit) had a dispersion coefficient of $\sim 30 \text{ ps nm}^{-1} \text{ km}^{-1}$ and a nonlinearity of $\sim 3 \text{ W}^{-1} \text{ km}^{-1}$, at 1.06 $\mu \text{m}$. In contrast to Ref. [Ren08b], neither inline spectral filters nor polarisation selective components were used. Thus, the lasing wavelength was defined by the overlap of the gain and spectral loss profiles of the laser components, and the dynamic filtering effect of the saturable absorber. The fully single-mode fibre format of the cavity prevented higher-order modal effects contributing to the temporal broadening mechanism, which could not be negated in Ref. [Kob08]. The total round-trip loss of the cavity was approximately 11 dB, not including losses due to the addition of the dispersive fibre. The major contribution was attributed to loss across the SWNT-SA device (3 dB of which is the linear attenuation of the composite film). Subsequently, with the use of correct index matching gels and FC-
PC connectors rather than FC-APC connectors, insertion losses were reduced.

**Experimental results**

Figures 5.3a and 5.3b show the autocorrelation and corresponding spectrum of the output pulses generated in the shortest tested cavity, with length $L \approx 9.5 \text{ m}$. The autocorrelation FWHM is $\sim 30 \text{ ps}$ corresponding to a deconvolved pulse duration of $\sim 20 \text{ ps}$, assuming a sech$^2$ profile (a hyperbolic secant fit is shown in red in Fig. 5.3a). The spectral FWHM of 0.47 nm was centred at 1.062 $\mu$m. The output pulses were expected to be chirped, as the transform limited duration for the corresponding spectral bandwidth, at the centre frequency of the pulse, is $\sim 2.5 \text{ ps}$, assuming a sech$^2$ profile. Mode-locking was self starting, with a limited dependence on the polarisation controller for stable operation. However, the output pulse shape did not depend on polarisation state. Mode-locking could not be initiated without the inclusion of the SWNT-SA. The repetition rate of 21 MHz corresponded to the round-trip time of the cavity. With the approximation that all the energy is contained within the temporal bit-slot occupied by the pulse, the single-pulse energy was $\sim 1.75 \text{ pJ}$.

Figures 5.3c and 5.3d plot comparative temporal and spectral properties of the dissipative soliton pulses generated in the longest tested cavity, with length $L \approx 1130 \text{ m}$. The measured temporal intensity profile and sech$^2$ pulse-shape fit are in good agreement with analytic mode-locking theory, subject to positive group velocity dispersion [Hau91, Akh05, Ren08a]. In the regime of stable operation, the FWHM pulse duration was $\sim 1.7 \text{ ns}$, with a repetition frequency of 177 kHz (corresponding to the cavity round-trip time). Assuming again that the energy of the pulse is much larger than the energy of the continuum (or background), the single-pulse energy was $\sim 0.18 \text{ nJ}$, corresponding to the approximate threshold level where single-pulse operation collapses in a soliton laser. The spectral FWHM of 0.52 nm, centred at 1.059 $\mu$m, indicates strong chirp, with a time-bandwidth product of 236. This is $\sim 750$ times the transform limit. A linear pulse chirp, with a magnitude of 0.14 nm ns$^{-1}$, was experimentally measured using a 1 m monochromator and streak camera (this measurement will be discussed in detail in a proceeding section of this chapter), indicating compressibility down to near bandwidth limited duration (2–3 ps). Such compression would represent a dramatic enhancement to the duty cycle and the corresponding peak power, even for moderate average power levels. A chirp naturally arises in this mode-locking regime due to the balance of nonlinearity, dispersion, and saturable absorption in the presence of gain saturation and gain dispersion [Hau91, Ren08b]. However, the magnitude of chirp carried by the generated pulses

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4 In fact this is a poor approximation given the round-trip time of the cavity. However, this puts an upper bound on the expected single-pulse energy.
5.2 Nanosecond pulse generation

Figure 5.3: Temporal and spectral pulse properties of two chirped pulse lasers with cavity length $L \approx 9.5$ m (5.3a and 5.3b) and $L \approx 1130$ m (5.3c and 5.3d). Two orders of magnitude increase in the cavity length (21 MHz – 177 kHz) has resulted in an increase in the pulse duration by approximately two orders of magnitude (20 ps – 1.7 ns); the spectral bandwidth is approximately constant (0.5 nm 3dB width), implying a largely linear (dispersive) broadening process.
was previously not recognised to correspond to a stable mode-locking regime.

Figures 5.4a (experimental) and 5.4b (numerical) show how the steady-state output pulse duration evolves as a function of the total cavity length: $L_{\text{total}} = L_{\text{min}} + L_{\text{dispersive}}$, where $L_{\text{min}}$ is the minimum length of fibre including the necessary active and passive components, and $L_{\text{dispersive}}$ is the additional length of passive fibre added to temporally stretch the pulse. There is strong correspondence between measured and simulated data, illustrating validity of the numerical model (discussed in greater detail in section 5.5).

Secondly, a positive trend between increasing cavity length and increasing pulse duration is clear in both the experimental and numerical data. The numerical data presented in Fig. 5.4b suggests a strong linear relationship between these two quantities. The deviation from a perfect linear scaling, particularly for longer cavity lengths, observed in Fig. 5.4a is physical and can be explained. The considerable variation in the duration of pulses with cavity lengths longer than $\sim 750$ m, corresponds to either a marginal increase or decrease of the pump power level or a change in the settings of the polarisation controller, which also changes the energy of the circulating pulses by modifying the loss profile of the cavity.

It is important to emphasise that the mode-locking threshold increases with increasing cavity length. In the simulation, a stable single-pulse is maintained with a logarithmic ramp in the amplifier saturation energy (to remain at an operating point at, or above the mode-locking threshold) for a logarithmic increase in the cavity length. This is equivalent to increasing the amplifier pump power in the experiment, which is also necessary to preserve single-pulse mode-locking for increasing cavity length. It is clear that the energy affects the corresponding pulse duration: an increase in the pump energy usually leads to a decrease in duration. Figure 5.5 shows the output pulse duration and energy plotted on a normalised length scale $N L$ that removes the dependence on increasing pump threshold level, and attempts to isolate the impact of the increasing cavity length (and consequently, increasing dispersion) on the output pulse duration, using the following normalisation

$$N L = E \left( \frac{L}{E_{\text{sat}}} \right)$$

where $E$ is the pulse energy and $E_{\text{sat}}$ is the saturation energy of the amplifier. The strong linear dependence of the duration on the cavity length remains clear (see Fig. 5.5). In addition, the energy also scales approximately proportionately with increasing cavity length (consequently decreasing repetition frequency).

These results demonstrate the capability to access a broad range of pulse durations ($\sim 20$ ps to 2 ns) by simply cavity length tuning. The output peak power was $\sim 70$ mW and remained approximately constant with increasing length, corresponding to an intra-cavity peak power of $\sim 0.5$ W. With increasingly long fibre lengths, long duration pulses,
5.2 Nanosecond pulse generation

**Figure 5.4:** The evolution of the steady-state pulse duration as a function of cavity length: 5.4a experimental data; 5.4b data from numerical simulation. In Fig. 5.4b the black circles correspond to the duration of the output pulse envelope. The red circles correspond to the minimum duration at half maximum – and indicate the degree of temporal coherence: if this time-scale is equal to the duration of the envelope the pulse is coherent, consisting of a single structure within the time-window of the simulation.
and increasing pulse energies loss of radiation from the main pulse spectrum to a Raman shifted Stokes component could degrade pulse quality. Although the systems developed here operated below the stimulated Raman threshold, Raman scattering cannot be overlooked in other ultra-long cavity, chirped-pulse, high-energy systems, such as Refs. [Kob08, Kob09, Kob10], where peak powers exceeded 1 kW.

Figure 5.6 shows the fundamental radio frequency spectrum and the pulse-train emitted from the longest tested cavity (where $L = 1130$ m). A peak to pedestal extinction of $\sim 50$ dB (measured at a resolution of 30 Hz), suggests low intensity noise. This was confirmed by observing the pulse-train on a 400 MHz analogue oscilloscope, with no transient effects nor signs of Q-switched behaviour (see Fig. 5.6b). In addition, the narrow-band spectrum in the RF trace implies low pulse to pulse timing error (or pulse jitter) – this is perhaps surprising given the kilometer long cavity, with only a single pulse circulating at the fundamental round-trip time.

This experiment illustrates that stable mode-locking can be achieved, in the presence of strong normal dispersion, without NPE contributing to pulse-shaping, and without a discrete filtering element. A positive relationship between the cavity length and the pulse duration is established. Increasing the cavity length is synonymous with increasing the magnitude of positive cavity dispersion. Increasing the dispersion by addition of normally dispersive passive fibre is a simple scheme. However, the change in the cavity length $\Delta L$, and the consequent increase in the duration of the emitted pulse $\Delta \tau$, is directly coupled to the repetition frequency of the laser $\nu_{\text{rep rate}} = c / L$. This is not always desirable when higher repetition rates of long duration pulses are necessary. In addition, the extra fibre length introduces nonlinearity. The nonlinearity could be necessary for stabilisation of a giant-pulse, in this case. In the proceeding section, I establish whether
5.2 Nanosecond pulse generation

![Fundamental electrical spectrum](image1)

(a) Fundamental electrical spectrum.

![Pulse train](image2)

(b) Pulse train.

**Figure 5.6:** Electrical spectrum of the fundamental cavity harmonic and the pulse train emitted from the chirped pulse laser, with cavity length $L = 1130$ m.
the giant-pulse formation dynamics in chirped pulse lasers are dominated by dispersion or nonlinearity, using a lumped rather than distributed dispersion (with an associated nonlinearity).

### 5.2.2 Using a strongly chirped FBG as the dispersive element

The generation of chirped nanosecond-scale pulses directly in a normally dispersive oscillator was demonstrated in the previous section [Kel09c]. A long length (>1 km) of passive, dispersive, single-mode fibre provided a distributed dispersion that chirped the pulses on successive round-trips of the cavity, leading to the steady-state formation of a regular train of highly-linearly chirped dissipative solitons. It was also shown that the duration of the emitted pulses is proportional to the length of dispersive fibre, with durations from tens of picoseconds to nanoseconds achievable for lengths from 20 m to >1 km. The addition of dispersive fibre to temporally stretch the pulses in time intrinsically couples the increase in duration to a reduction in the fundamental repetition frequency of the laser: limiting nanosecond operation to the hundreds of kilohertz regime.

In the proceeding experimental discussion the dispersive fibre element was replaced by a chirped FBG, incorporated into the cavity using a three-port fibreised optical circulator. The optical circulator also functioned as the unidirectional element, replacing the need for a discrete inline optical isolator (see Fig. 5.2b). The grating provided a large lumped dispersion, with negligible increase to the overall nonlinearity of the cavity, when illuminated with a low peak power pulse.

This approach permits the generation of long, highly-chirped pulses at higher (megahertz) repetition rates. In addition, it suggests that the dynamics of long pulse formation in highly normally dispersive mode-locked lasers is dominated by dispersive, rather than nonlinear effects, in this case.

**Experimental setup**

An overview of the experimental configuration is shown in Fig. 5.2b. The laser setup is the same as Fig. 5.2a, with the exception that the dispersive fibre is replaced by a chirped FBG (providing strong positive dispersion), and the optical isolator is rendered redundant because of the inclusion of an optical circulator, used to incorporate the CFBG. The grating provided a lumped dispersion of 35.7 ps nm\(^{-1}\) (or -35.7 ps nm\(^{-1}\) depending on orientation), with negligible increase in the overall nonlinearity of the cavity, and had a spectral bandwidth of 3.36 nm and transmission of -27.7 dB (highly reflecting). It is worth highlighting the difference between uniform and nonuniform fibre Bragg gratings: unchirped or chirped gratings.
5.2 Nanosecond pulse generation

**Fibre Bragg Gratings**

Ultra-violet (UV) illumination of a photosensitive fibre core (typically hydrogen loaded germanosilicate glass) results in permanent structural changes, causing a modification to the refractive index of the medium. By placing a phase-mask between the light source and the sample, or using interference between two UV sources, a specific defect pattern can be written in the core of the fibre. A uniform grating, with periodic modulation of the core refractive index reflects light of a wavelength that coincides with the stop band of the device, centred at the Bragg wavelength $\lambda_B$ [Agr01]. Close to the band edge, strong absorption is related to a large dispersion through the Kramers-Kronig relations.\(^5\) However, large GVD is associated with high TOD, causing distortion of optical pulses. By changing the period of refractive index modulation along the length of a FBG, the corresponding Bragg wavelength shifts with distance through the grating, resulting in a delay between reflected spectral components. Such aperiodic structures written into fibre are known as chirped fibre Bragg gratings, and can exhibit a controllably large amount of GVD, and have been widely used in optical pulse compression and for the compensation of dispersion-induced broadening over long-haul fibre-optic transmission systems (see Ref. [Agr01] and references therein). For a chirped grating with a stop-band bandwidth of $\Delta\lambda_{\text{stop}}$ and an effective length $L_g$, the magnitude of dispersion $D_g$ (with units of $\text{ps nm}^{-1}$) can be estimated by considering the delay introduced by the total shift in the Bragg wavelength $\Delta\lambda_B$, and is given by

$$D_g = \left[ \frac{\bar{n} \lambda^2}{2c \Delta \lambda_B^2} \right]$$ (5.3)

where $\bar{n}$ is the average refractive index, $c$ is the vacuum speed of light and $\lambda$ is the centre wavelength of the stop-band; the factor of two accounts for the reflection. CFBGs have been widely used throughout this thesis for intra-cavity dispersion compensation. CFBGs will be briefly revisited later in this chapter, when I discuss potential schemes for dechirping (or compressing) optical pulses with large time-bandwidth products.

**Experimental results**

Stable single-pulse mode-locking was achieved at the fundamental repetition frequency of the cavity (6.6 MHz), with a corresponding single-pulse energy of 15 pJ. Figure 5.7 shows a summary of the spectral and temporal performance of this laser system. The spectrum (see Fig. 5.7a), centered at 1068.2 nm, has a FWHM of 0.06 nm corresponding to a transform limited pulse duration of 20 ps, resulting in a time-bandwidth product of 5.

\(^5\)I refer to Ref. [Boy03] for a discussion and mathematical description of the Kramers-Kronig relations.
Figure 5.7: Spectral and temporal pulse properties of the nanosecond chirped pulse oscillator, employing a lumped dispersive element. 5.7a and 5.7b correspond to experimental measurements. 5.7d and 5.7c correspond to numerical simulation of the physical system.
5.2 Nanosecond pulse generation

18, indicating that the generated pulses are strongly chirped. The temporal intensity profile was measured using a fast photodiode (with a 15 ps rise-time) and a 50 GHz sampling oscilloscope. Figure 5.7b shows that the FWHM pulse width is 1.15 ns, and possesses a hyperbolic secant functional form. This is the expected pulse shape of a dissipative soliton of the cubic (and cubic-quintic) Ginzburg Landau equation, predicted by analytic theory of mode-locking initially developed by Haus et al. and later augmented by Renninger et al. for application specific to the normal dispersion regime [Hau91, Ren08a].

In Fig. 5.7a the contrast ratio between the lasing peak and the ASE background is \( \sim 30 \) dB, limited by ASE generated because the laser operation wavelength, defined by the pass-band of the CFBG, did not overlap with the peak gain of the amplifier. The electrical spectra of both the fundamental and higher cavity harmonics are plotted in Fig. 5.8. The narrow-spectrum and relatively large extinction ratio \( \sim 45 \) dB at the fundamental cavity harmonic (see Fig. 5.8a) suggest low temporal jitter, and low intensity noise. In Fig. 5.8b no noticeable beat frequency shift, which would normally indicate longer-term temporal instabilities, can be observed.

The mode-locking performance was dependent on the action of the SWNT-SA and the filtering effect of the CFBG, which also contributes to a very narrow-band spectrum. The pulse duration was defined by the large lumped normal dispersion of the CFBG. The large time-bandwidth product means compression is impractical with standard schemes, such as bulk gratings. As an example for an input pulse with a spectrum equal to our measured spectrum (0.06 nm, centred at 1068.0 nm), but a temporal duration two orders of magnitude shorter (11.5 ps) the required grating separation for a pair of bulk compression gratings, with 1200 lines mm\(^{-1}\) and an incident angle of 45 degrees, would be approximately 10 m. As the separation scales linearly with the duration (and assuming infinitely large gratings), given the small bandwidth, we would need a separation greater than 1 km to compensate a nanosecond pulse, assuming a linear chirp!

**Numerical results**

Numerical simulations of the laser system solved a modified nonlinear Schrödinger equation, excluding higher order dispersion, shock formation and Raman terms (the details of the model were outlined in section 4.2.3 of chapter 4). The CFBG was modelled by an equivalent short-length passive fibre with a GVD parameter and length equal to the magnitude of the dispersion \( D_g \) and the physical length \( L_g \) of the chirped grating used in the experiment. The nonlinear coefficient of the CFBG was assumed to be equal to

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\( ^6 \) The calculation of this example was based on theory of pulse compression in diffraction gratings developed by Treacy in Ref. [Tr69a] and discussed in detail in Ref. [Agr01]. A basic code was available courtesy of Dr J. C. Travers that solved the simple analytic expressions outlined in Refs. [Tr69a] and [Agr01] for chosen input parameters defining the properties of the grating pair and the incident light pulse.
Figure 5.8: Electrical spectrum of the fundamental cavity harmonic and higher beat-frequency harmonics.

Figure 5.9: Calculated spectrogram of the chirped output pulse from the simulation of the physical system. Colour scale: -30 dB - 0 dB.
that of standard fibre of an equivalent core-size ($\gamma = 0.003 \, \text{W}^{-1} \, \text{km}^{-1}$), high-order contributions to the dispersion were neglected, given the dominance of the $\beta_2$ term, and a Gaussian filter, with a bandwidth and centre wavelength equal to the pass-band of the grating, represented the frequency response of the passive optical device.

The simulated spectral and temporal profiles (see Figs. 5.7c and 5.7d) are in good agreement with experimental results for similar intra-cavity energies: 65 pJ and 60 pJ (taken straight after the OC) for simulation and experiment respectively. It should be noted that the experimental spectrum was recorded on a broader wavelength span to show the contribution from the ASE, and thus because of the limited resolution of the diagnostics the square shape to the spectral profile is not evident. The calculated spectrogram of the simulated output pulse is shown in Fig. 5.9 and confirms that linearly chirped dissipative solitons, with a duration and spectral width equivalent to experimental observations, are a solution of this system of equations in this extreme parameter space, characterised by a large lumped dispersion.

Control of the intra-cavity pulse evolution in ANDi lasers can be reduced to three key parameters [Cho08b]: the nonlinear phase shift $\Phi_{NL}$, the effective spectral filtering bandwidth and the GVD. In this cavity, the dominant contribution to the spectral filtering and the GVD is attributed to the CFBG; for observed peak powers during stable mode-locking the round-trip $\Phi_{NL} \ll \pi$, explaining the narrow bandwidth spectrum. A low value value of $\Phi_{NL}$ is expected, given the low intra-cavity average power at which the mode-locking threshold is reached. This threshold is largely defined by the saturation power ($\sim 6 \, \text{W}$) and subsequent damage power ($\sim 5 \, \text{mW}$ in single-mode fibre) of the saturable absorber device [Tra11a].

Stable operation of the experimental system and convergence of simulations to well matched pulse parameters confirm that nanosecond pulses can be generated in a normally dispersive mode-locked laser, using a large lumped dispersion, provided by a CFBG element. In this case the duration is decoupled from the fundamental repetition frequency; as such, the generation of long pulses is not limited to the sub-MHz regime. In addition, this experiment suggests that giant-pulse dynamics are dominated by the magnitude of the dispersion rather than the nonlinearity, and evolve linearly in this case.

### 5.3 Measurement of pulse chirp

Although numerical simulations support dissipative soliton solutions in the extreme parameter ranges equivalent to the physical systems described above, experimental measurements, so far, have not conclusively eliminated the possibility that the nanosecond pulses exist as incoherent bursts of noise, given the resolution limitations of temporal
diagnostics. Consequently, an important question remains: are the experimentally generated pulses true dissipative solitons carrying a linear chirp or an incoherent bunch of sub-pulses with a nanosecond envelope centred at the fundamental frequency of the cavity? The answer has important implications for the significance and potential application of large dispersion GCO laser, for example as the front end of a simplified all-fibre, passive chirped pulse amplification scheme [Ren08b]. Measurement of the optical pulse in the time-frequency domain can be used to obtain the pulse spectrogram, from which the chirp, phase, and degree of temporal coherence can be retrieved [Tre00].

**Experimental setup**

An overview of the approach taken to measure the pulse spectrogram is shown in Fig. 5.10. The pulses generated by the longest cavity laser tested in section 5.2.1, with a duration of $\sim1.7$ ns, were characterised in the time-frequency domain using the following scheme. The output of the GCO was bulk-coupled into a meter-long monochromator, incorporating a 1200 line mm$^{-1}$ grating, with a resolution limit of $\sim0.05$ nm at 1 $\mu$m. The spectrally selected output of the monochromator was then focused into a synchronously scanning picosecond streak camera. The pulse intensity as a function of delay was measured using the streak camera, as a function of wavelength selected by sequential tuning of the grating angle and correspondingly the transmitted wavelength of the monochromator.
5.3 Measurement of pulse chirp

Figure 5.11: Spectrograms of a coherent nanosecond pulse produced in a chirped pulse fibre laser. 5.11a Experimentally measured; 5.11b calculated using the Haus master equation; 5.11c calculated from the result of a full numerical simulation of the physical system.
Figure 5.12: Experimentally measured pulse shape (logarithmic scale) including sech^2, sech and Gaussian fits.

Results

The measured spectrogram is shown in Fig. 5.11a. The spectrogram confirms a distinct chirp across the pulse. The full laser spectrum was also resolved temporally using the synchronously scanning streak camera, to characterise the temporal intensity shape (with a resolution of \( \sim 15 \) ps). The full temporal pulse shape, fitted with a Gaussian, sech and sech^2 function, is plotted on a logarithmic scale in Fig. 5.12. The squared hyperbolic secant pulse shape is in excellent agreement with the measured data (even 30 dB from the peak level), and with analytical mode-locking theory [Hau91, Ren08b].

The temporal phase was fitted using the measured pulse shape (see Fig. 5.12), adding a random initial phase and employing an iterative phase-retrieval algorithm, similar to that used in FROG [Tre97], to reproduce the measured spectrogram. The algorithm converged to the same parameters from a range of random initial phase conditions. On expanding the retrieved phase a second-order temporal phase value of \( \phi_2 = 1.2 \times 10^{-4} \) ps^{-2}, a negligible third order phase value, and a quartic phase value of \( \phi_4 = -4.4 \times 10^{-12} \) ps^{-4} was obtained. These values confirm the strong linear chirp of the pulse, with a residual quartic phase component, suggesting marginal nonlinear distortion (due to SPM) as seen in [Ren08a]. From the retrieved phase the expected spectral broadening of 0.40 nm was determined, in good agreement with the experimentally measured value of 0.49 nm. Considering only second-order phase, the expected broadening would be 0.57 nm and thus the quartic component acts to reduce the chirp.

The spectrogram was stable over the measurement period of several hours. The temporal stability was confirmed using single-shot measurements on a 50 GHz sampling oscilloscope. In addition, the measured autocorrelation, over a limited time window of
Measurement of pulse chirp

100 ps centred on the pulse, did not possess a coherence spike, indicating no noise substructure on timescales down to ∼50 fs. This confirmed the temporal and spectral coherence of the pulse structure and that the nanosecond pulse did not constitute a noise burst, typical of many long-pulse NPE-based systems [Hor97a]. The long-term stability of the pulse-train was previously characterised, through the electrical spectrum (see Fig. 5.6a).

The large linear chirp naturally arises in this mode-locking regime due to the balance between nonlinearity, dispersion and saturable absorption [Hau91, Ren08a, Ren08b], in the presence of gain saturation and gain dispersion. These effects were treated as perturbations to the NLSE in chapter 4, where simplified equations describing the evolution of optical pulses in mode-locked lasers were introduced. The analytical theory, first developed in Ref. [Hau91] and augmented in Ref. [Ren08b], was used to compute the expected spectrogram for parameters closely matching the experimental conditions. Although it is well known that the inclusion of a quintic loss term damps the growth of the nonlinear gain, and has improved the robustness and stability of the master mode-locking model [Bal08], strong quantitative agreement, using the simple cubic complex Ginzburg-Landau (or Haus master) equation was found, even in the nanosecond regime. The following best-estimate values were used in the analytic model outlined in Ref. [Hau91]: the second derivative of the propagation constant was 0.018 ps$^2$ m$^{-1}$; the nonlinear parameter was 0.0035 W$^{-1}$ m$^{-1}$; the resonator length was 1200 m; the central wavelength was 1057.64 nm; the gain bandwidth was 20 nm; the net system gain (or loss) was 11 dB; the minimum intra-cavity pulse energy was 500 pJ; and the saturable absorption coefficient $\beta = 0.035$ was tuned to give the best quantitative agreement with the measured chirp value. The resulting analytic spectrogram is shown in Fig. 5.11b.

The theoretical spectrogram (Fig 5.11b) exhibits very similar structure to the measured spectrogram (Fig. 5.11a). The calculated phase expansion indicates a second-order temporal phase value of $\phi_2 = 9.5 \times 10^{-5}$ ps$^{-2}$, a negligible third order phase value and a quartic phase value of $\phi_4 = -2.2 \times 10^{-12}$ ps$^{-4}$, all of which are in very close agreement with the values extracted from the experimental spectrogram, confirming that the model incorporates all the essential features to describe this nanosecond-pulse GCO. Given that this analytical model does not include inter-pulse jitter or noise contributions (this also explains the larger dynamic range), the agreement with experiment indicates that the measured pulses are not broadened by such processes, as confirmed by the oscilloscope, RF spectrum and autocorrelation measurements discussed above.

In the final panel of Fig. 5.11 the calculated spectrogram of the output pulses generated from full numerical simulations of the physical system, using the model developed in section 4.2.3 of chapter 4, is shown. The numerical results are also in close agreement
5 Chirped pulse fibre laser sources

Figure 5.13: Spectrogram representation of a numerically compressed nanosecond pulse. A significant peak power and duty-factor can be achieved, but the amount of dispersive delay to obtain near transform-limited compression is impractical using standard compression schemes such as bulk grating-based compressors.

with theory and experiment. The numerical model will be used later in this chapter to probe the dynamics of giant-pulse evolution in long-cavity lasers, subject to strong positive dispersion.

5.4 Prospects for amplification and compression

The transform-limited duration of the nanosecond pulses generated in the system outlined in section 5.2.1 is \( \sim 2-3 \) ps. Given the linear nature of the pulse chirp – now confirmed by a direct measurement [Kel09a] – full compression would result in a dramatic enhancement of the duty cycle. This is exemplified numerically, where the correct second-order phase can be applied exactly [Tre69a]; the results of compressed (and uncompressed) pulses are shown in Fig. 5.13 in the time-frequency domain. However, it is not possible to compensate the chirp produced by the second-order phase, over several nanoseconds, with any practical compression scheme – I have already outlined the limitations of using bulk compression gratings.

Two potential compression formats were proposed during the time-frame over which the work in this thesis was conducted, but the experiments are currently ongoing. A brief outline of the concepts is provided here. Firstly, physically long (up to 75 mm) chirped fibre Bragg gratings were suggested for external pulse compression, after a single or multiple stages of amplification. Difficulties encountered with this approach included, amplifying the low-repetition rate pulses without significant degradation to the signal to noise ratio due to ASE generated in the gain stages. Secondly, due to the relatively narrow linewidth and fixed emission wavelength of the laser source, and constraints imposed
upon the fabrication of the correct CFBG to compensate the chirp, it was non-trivial to
operate the laser in a stable regime coincident with the stopband of the grating. Neither
of these issues present a fundamental problem and can be overcome with careful consid-
eration.

The second scheme applies long-period gratings (LPG) as mode-converters to effi-
ciently transfer the energy from the fundamental Gaussian mode to some high-order
(HO) mode [Ram02], which propagates over many hundreds of meters in a HO-mode
fibre subject to significant anomalous dispersion for pulse compression. Subsequent
reconstruction of the fundamental mode can be achieved using a second LPG mode-
converter, with high efficiency [Ram02]. The advantage of this approach is that ano-
malous dispersion is available in the HO-mode fibre at 1 μm. In addition, due to the larger
mode-field diameter of the HO mode the effective nonlinearity is significantly reduced.
However, control over the HO-mode excited when performing fusion welds between the
mode-coverters and compression fibre is limited, and can result in significant loss of sig-
nal power.

Both schemes are worthy of further consideration. However, for practical purposes
and for achieving dramatic increases in the power performances of chirped pulse fibre
laser systems alternative approaches offer significant advantages.

5.4.1 LMA fibres: a practical route to higher-energy

It is clear that given the extreme chirp carried by the nanosecond pulses generated in sec-
tion 5.2 and characterised in section 5.3, compression (and compression without intro-
ducing significant distortion to the pulse through nonlinearity) is complex with current
technology. However, the generation of chirped pulses in all-normal dispersion oscil-
lators (followed by external compression) has been shown to be the preffered route to
access powerful ultrashort pulses in fibre systems [Bau11]. An alternative scheme for
lowering the nonlinear threshold in a mode-locked oscillator, rather than temporally
stretching the pulses, is to use fibres with a larger core size. However, standard (step
index) fibres where \( V > 2.405 \) support multiple transverse modes that can destabilise
mode-locking and are usually less preferred for practical applications. The emergence of
photonic crystal fibres (incorporating rare-earth dopants [Sch08b]) has allowed the scal-
ibility of the core size, while maintaining fundamental single-mode propagation [Kni98,
Mor03]. By utilising a hybrid-fibre design, based on double-clad Yb-doped LMA PCF,

\[ V = \sqrt{\frac{k_0 a}{n_1^2 - n_c^2}} \]  

where \( k_0 = \frac{2\pi}{\lambda} \), \( a \) is the core radius, \( n_1 \) and \( n_c \) are the refractive index of the core and cladding, respectively.

\[ V = \sqrt{\frac{k_0 a}{n_1^2 - n_c^2}} \]  

\[ (5.4) \]
pulse parameters from mode-locked fibre lasers are becoming comparable to state-of-the-art solid state systems [Lec07, Bau10, Lef10, Bau11], with peak powers in excess of 3 MW (and output beam quality of $M^2 = 1.2$ at 50 MHz repetition frequency). Figure 5.14 illustrates the design of such a system.

It is well known that ionically-doped glass filters (such as Schott RG1000) possess a saturable component of absorption that provides a sufficient modulation to mode-locked bulk lasers [Bre64, Win73, Rua95]. Recently, it has been demonstrated that an RG1000 filter can be used to mode-lock a fibre laser, in both the positive and negative dispersion regime [Zha12]. In the positive regime the pulse duration is limited by the switching speed of the saturable absorber, limiting the duration of the pulses to a minimum of tens of picoseconds [Zha12]. For self-starting, short pulse, high-energy operation NPE can be used in conjunction with an IDG-SA to shorten the duration of the pulses in the cavity (see Fig. 5.14). Combining this approach with integrated, high-power fibre pigtailed multi-mode pump diodes (with average output powers up to 60 W), high energy, high peak power, short pulse operation is anticipated in a robust, compact package. This is currently the direction and focus of ongoing research.
5.5 Dissipative soliton formation and dark pulse dynamics in highly-chirped pulse oscillators

Overview

As well as being of practical relevance, GCO-type laser systems, where localised coherent states have been shown to be attracting solutions, represent an interesting nonlinear system, with analogues in other pattern-forming complexes [Rob97, Bab08, Tur09b]. As such, they warrant continued attention purely for fundamental interest.

The integration of active and passive fibre based components exhibiting anomalous dispersion has allowed the extensive study of optical soliton generation, amplification, interaction and transmission in compact fibre laser configurations; providing a convenient experimental environment in which to verify theory. The stability of soliton operation has been demonstrated despite changes to the system parameters such as gain, loss, dispersion and even the inclusion of normally dispersive fibre by operating in a “guiding centre” or average soliton regime: the length scale of the perturbation is less than the characteristic length of the average soliton, such that the soliton is effectively unable to react to the perturbation. In systems where the overall dispersion is normal, dark optical soliton generation is possible [Has73b], although experimentally these have proven to be more difficult to generate and tend to be more unstable than their bright counterparts in the anomalously dispersive regime [Emp87, Wei88, Tay92, All94, Ike97].

Over the past few years, there has been considerable experimental interest directed towards a class of mode-locked laser that exhibit relatively large net normal dispersion – such systems have been the major focus of this chapter. It has been analytically demonstrated that these systems exhibit dissipative soliton solutions, and as a result of the strong normal dispersion the pulses possess a large normal chirp, leading to the description of “giant-chirp oscillators” for this family of devices.

However, controversy existed within the community regarding the quality of the generated pulses in GCO-type lasers. In the previous sections I have shown unequivocally that SWNT-based GCOs can generate true dissipative soliton pulses on the nanosecond-scale, exhibiting predominantly quadratic and quartic chirp, with vanishing tertiary chirp – predicted by theoretical solutions in this regime.

In this section I clarify the dynamics of pulse formation in highly-chirped mode-locked lasers.
lasers (based on the physical system outlined in Fig. 5.2a), using extensive numerical sim-
ulation, demonstrating the evolution from noise structure to a stable, linearly chirped
bright soliton-like solution [Kelona].

Interestingly, the generation and evolution of self-trapped, coherent dark optical soli-
tons in the early stages of the evolution of the bright pulse envelope can be identified.
Similar dark soliton behaviour has been observed in Bose-Einstein condensates [Dum98,
Bur99, Bec08], demonstrating a conceptual link between solitons in these disparate ar-
eas. The dark feature is tracked throughout mode-locking towards a steady-state, and is
shown to decay during the asymptotic evolution of the stable bright pulse [Kelona].

**Brief details and parameters of the model**

I used the numerical scheme developed in section 4.2.3 of chapter 4 to construct a sim-
ulation model of the physical system outlined above and in Refs. [Kel09c, Kel09a]. To
briefly recapitulate, the resonant cavity consisted of: a short-length rare earth doped fi-
bre amplifier (typically ∼2 m); a fast saturable absorber; a linear loss representing the
output coupler and transmission losses through other passive components such as an
isolator; a bandpass filter; and a variable length of single-mode fibre. For direct com-
parison to Refs. [Kel09c, Kel09a], I assumed parameters equivalent to a Yb-doped gain
fibre, operating at a wavelength of 1.06 μm, a SWNT-SA and 1.2 km of single-mode fibre.
Both the length of passive fibre and the amplifier were assumed to have a dispersion of
β_2 = 0.018 ps² m⁻¹ and a nonlinear coefficient of γ = 0.003 W⁻¹ m⁻¹. The amplifier fibre
was modelled (see Equations 4.5 and 4.6) to include a parabolic gain profile, with a band-
width of 40 nm, and had a small signal gain coefficient of g_0 = 30 dB. The parameters
of the saturable absorber were (see Equation 4.21): α_S = 0.05; α_NS = 0.45; and P_sat = 6 W
(consistent with experimental measurements [Tra11a]). Although a discrete spectral filter
was not included in the physical system described above (and in Refs. [Kel09c, Kel09a]), a
10 nm Gaussian filter was used to increase the rate of convergence of the computational
model. A fuller discussion of how the spectral filter effects the dynamics in such lasers
was discussed in chapter 4 and is provided in Refs. [Bal08, Peded, Pedon]. The cavity loss
was 3 dB, and I assumed the entire system was linearly polarised. To accurately repre-
sent the pulse solutions a numerical grid with 2¹⁶ points over a temporal span of 8 ns
was used. The simulations were initiated from a field of white noise equivalent to one
photon per mode [Dud06]. The input field was iterated around the cavity elements until
the parameters of the system stably converged.
5.5 Dissipative soliton formation and dark pulse dynamics in highly-chirped pulse oscillators

Figure 5.15: Temporal and spectral intensities for three quantitatively different stages within the evolution of a stable bright dissipative soliton pulse. Also illustrated is the temporal minimum (red arrow) and maximum (green arrow) width at half maximum, within the time-window of the simulation. When the minimum width is equal to the maximum width the temporal waveform has converged to a coherent single-pulse solution.
5 Chirped pulse fibre laser sources

Results

The model stably converged to a pulse with parameters of the physical system for an intra-cavity peak power of \( \sim 6 \text{ W} \), corresponding to a cavity round trip nonlinear phase shift \( \Phi_{NL} \approx 7\pi \); a moderate value encountered in many conventional normal-dispersion fibre lasers [Cho08b]. However, the total cavity dispersion of \( \sim 21.6 \text{ ps}^2 \), is approximately two orders of magnitude larger than conventional normal-dispersion chirped-pulse fibre lasers [Cho08b]. This large normal dispersion, and in the absence of discrete spectral filtering, will dominate modest nonlinear phase-shift, placing the laser system in the regime of low spectral variation (or breathing) throughout the cavity [Cho08b].

Figure 5.11c shows the calculated spectrogram, confirming that a single coherent pulse is a solution of the system of constituent equations even for extreme positive values of \( \beta_2 \). In addition, it clearly confirms that such pulses are highly-chirped structures. The analytical and experimental spectrograms are plotted in Figs. 5.11b and 5.11a, respectively. The numerical spectrogram has strong quantitative agreement with the experimental measurement; it is perhaps not unexpected that the analytical spectrogram, being a simplified description of the system, only shares the essential qualitative features.

The numerical model can be used to probe the pulse formation dynamics from an initial noise field to a coherent single pulse. Fig. 5.15 shows the characteristic qualitative evolution of the temporal and spectral intensity of the field, with increasing numbers of round trips of the resonant cavity: the left panel shows the temporal profiles; the right, the corresponding spectral profiles. Three distinct regimes exist: in the early stages of pulse formation the degree of pulse coherence is low, with a high density of sub-pulses within a broader pulse envelope. The corresponding spectrum is broad with a rounded top. This noisy-pulse behaviour has been observed previously in the steady-state regime of partially mode-locked (normally dispersive) lasers, where clusters of modes form locally coherent sub-pulses [Dzh83, Zha06, Zha07a, Pot11]. In the steady-state a single pulse exists (see Fig. 5.15e), characterised by a sech\(^2\)-shaped temporal intensity profile and a steep-edged spectrum; such a pulse structure is predicted by analytic theory [Hau91, Ren08a, Kel09a], and has been observed in laboratory experiments [Kel09c]. In the transition between a fully converged, coherent single-pulse and a partially mode-locked noise-burst, a meta-stable phase exists that can provide the conditions for the seeding and generation of self-trapped, dark-solitonic structures (see Figs. 5.15c and 5.15d). Like their bright counterparts, dark solitons can form through a balance of self-phase modulation (SPM) and dispersion, but in the normal rather than anomalous regime. Their characteristic duration is determined by the power of the background from which
they evolve, such that

$$\tau_0 = \sqrt{\frac{\beta_2}{\gamma P_0}}$$

(5.5)

where $\tau_0$ is the characteristic duration of a fundamental soliton, $P_0$ is the power (of the background for dark solitons), and $\beta_2$ and $\gamma$ have their usual meaning. Similarly, high-order dark solitons exist, where their order $N = \sqrt{\gamma P_0 \tau_0^2/|\beta_2|}$. Unlike bright-solitons, high-order dark solitons do not undergo a period evolution due to a repulsive force that prevents the formation of a bound state (even under the influence of no other perturbative effects), resulting in the shedding of energy (or negative energy as it is a dark pulse) on propagation: for $N > 1$ a fundamental dark soliton is the asymptotic solution.

Figures 5.15a, 5.15c and 5.15e also illustrate two important time-scales that are used as a metric to determine convergence of a single-pulse: a minimum time-scale and a maximum time-scale. The maximum time-scale is simply the envelope full-width at half maximum, while the minimum time-scale is the shortest duration at half of the peak value, and gives a measure of the degree of separation of the noise-components. It is clear from Fig. 5.15e that when the minimum time-scale is equal to the maximum time-scale, only one pulse exists within the time-window of the simulation and convergence is complete.\textsuperscript{11}

A typical convergence profile, tracking the evolution of the two time-scale metrics over 4000 round-trips of the cavity is shown in Fig. 5.16. It is clear to see that the pulse envelope converges faster than the substructure. Such an evolution, showing snap-shots in time-frequency space, is plotted in Fig. 5.17, for the parameters given above. The existence of a steady-state solution can exist that does not converge: the temporal properties of the pulse evolve for a stabilised value (or typically a limited range of values) of pulse energy.

\textsuperscript{11}
tence of persistent holes in the temporal intensity profile after 900 round-trips is clear (Fig. 5.17e).

An alternative metric to quantify convergence is the pulse energy and peak power. The equivalent convergence profile of the energy and power is plotted in Fig 5.18a (from the same simulation data set). It is noticeable that the average power rapidly clamps (due to the time-scale of the simple gain model), while the peak power and pulse energy continue to evolve dynamically before stabilising after approximately 2500 iterations of the cavity. Comparing against Fig. 5.16, it is clear that the pulse energy and peak power stabilises before the pulse has fully converged in the time-domain, although small-scale fluctuations around a constant value persist. This suggests that any dark component constitutes a negligibly small loss of energy from the main pulse. For the purposes of convergence tests, implemented to hault long-term simulations after a steady-state is reached, I use the temporal convergence data – the tests require a fixed and equal value of the minimum and maximum time-scale metrics (accurate to within 1%) over a historical period of 150 cavity iterations. Such an algorithm is important for minimising the computational demands of large-scale ensemble simulation sets.

The intermediate phase between a noise-burst and a fully converged pulse, where dark structures can be spontaneously seeded from noise, has not previously been identified in the context of the evolution of a bright soliton-like pulse in a normally dispersive (fibre) laser. Fig. 5.19 shows the temporal intensity profiles of a quasi-stable bright-pulse, supporting a meta-stable dark component. Fig. 5.19b plots the temporal intensity around a dark component on a greatly reduced time-scale. Black crosses represent numerical data extracted from simulation, where the intensity dips to zero within the envelope of the longer, bright pulse. The intensity profile has been fitted with the functional form of the fundamental dark soliton pulse given by [Agr07]

\[
A(z, \tau) = \eta \left[ B \tanh (\tau') - i \sqrt{1 - B^2} \right] \exp \left( i \eta^2 z \right)
\]

where \( \tau' = \eta B \left( \tau - \eta B \sqrt{1 - B^2} \right) \), \( \eta \) is the power of the background and \( B \) determines the depth of the intensity dip: \(|B| \leq 1\), and the intensity drops to zero for \(|B| = 1\) – known as a black soliton to differentiate it from grey solitons. The fitted pulse shape has a characteristic duration \( \tau_0 = 3.14 \text{ ps} \), using Equation 5.5 the power corresponding to a fundamental soliton with this duration is \( \sim 0.6 \text{ W} \), which matches well the background power we observe for the dark structure in Fig 5.19b.

The corresponding temporal phase across the intensity dip (shown with a red dotted curve in Fig 5.19b) shows an abrupt phase step, approximately equal to \( \pi \), expected for a dark soliton, where \(|B| = 1\). This phase change becomes smaller and more gradual
5.5 Dissipative soliton formation and dark pulse dynamics in highly-chirped pulse oscillators

Figure 5.17: Evolution of a bright soliton-like pulse from noise.
5 Chirped pulse fibre laser sources

Figure 5.18: Pulse energy-power evolution dynamics. Fig. 5.18a tracks the convergence of the pulse energy and power in physical space. The phase-space attractor diagram (Fig. 5.18b) depicts a focussing stability: the red and black circle illustrate the leading and trailing edge, respectively. The first 100 points have been plotted in black to highlight the direction of the evolution.
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Figure 5.19: Temporal intensity profiles of a quasi-stable bright-pulse, where meta-stable dark components persist.

Figure 5.20: Temporal intensity evolution of a stable nanosecond-scale single-pulse from noise. A dark component, seeded from the early noisy-pulse phase, happens to propagate at the group-velocity of the bright pulse (on an equal trajectory) for several hundreds of round-trips, corresponding to a physical time on the micro second-scale. Colour scale: 0 - 1
for smaller values of $|B|$. The internal phase also determines the rate at which the dark structure shifts, relative to the group velocity at the centre frequency of the background pulse: a black soliton propagates at the same velocity as the bright pulse that supports it [Ike97, Wei92].

Figure 5.20 shows the full temporal intensity evolution of a bright-pulse developing from noise as a function of iterations of the resonant cavity. The three distinct pulsation regimes that form the evolution can be clearly identified:

- **stage–i:** The noise-burst phase where partial mode-locking of modal clusters results in the development of a low coherence pulse with a broad envelope.
- **stage–ii:** That provides the conditions for spontaneous, self-trapped, meta-stable dark solitons to exist.
- **stage–iii:** The final asymptotic evolution towards a coherent bright pulse with a duration on the nanosecond scale.

In the early stages of the evolution distinctive dark trails, characteristic of dark sub-pulses, can be seen to bleed-off from the main body of the bright pulse. However, one dark pulse, that is seeded from noise sufficiently close to the peak of the background pulse, is seen to survive this noisy-phase, where multiple collisions occur between dark structures. This dark solitonic structure propagates stably for approximately one thousand iterations of the resonant cavity. Due to the long cavity length ($\sim 1200$ m), where the fundamental repetition frequency is $\sim 166$ kHz, this corresponds to a dark soliton lifetime $\tau_{\text{lifetime}} \sim 20.35$ ms (assuming the full decay of the dark pulse up to 3500 iterations of the resonant cavity).

In order to evaluate the likelihood of such a long-lived dark structure I performed an ensemble set of simulations, with equal parameters but initiated from a different random initial white-noise field. The lifetime of the longest lived dark structure for each simulation was simply taken to be proportional to the rate of convergence of the single-pulse solution: when the minimum time-scale within the simulation window is equal to the maximum width of the pulse envelope; under such conditions a dark structure cannot be present. The histogram for an ensemble size of one thousand is plotted in Fig. 5.21. The data was fitted with a skew-normal function that confirms the slight positive distribution, with a kurtosis of 0.15 and a mean lifetime of 1249.24 round trips. The mean lifetime can be expressed as a time-scale of $\sim 7.5$ ms, given a fundamental repetition frequency of 166 kHz. This suggests that the event observed and plotted in Fig. 5.20 is indeed rather rare; perhaps this is not surprising given that the process is noise seeded and, due to the long cavity length, over $10^{12}$ modes have the opportunity to interact,
5.5 Dissipative soliton formation and dark pulse dynamics in highly-chirped pulse oscillators

![Figure 5.21](image)

**Figure 5.21:** Statical distribution of dark-soliton lifetimes. A positive skew indicates that longer lived dark components are less likely to occur. A skew-normal fit to the data is shown with red crosses.

despite the relatively narrow spectral content (∼0.5 nm). Such modal interactions have been widely studied in the context of ultra-long, continuous-wave (CW) lasers, where conceptual links to wave-turbulence have been established [Tur09b].

**Seeding and stability of dark solitons in chirped pulse lasers**

I have shown that under the right conditions spontaneously seeded, meta-stable dark soliton propagation can be supported by a long-duration, chirped pulse, subject to strong positive dispersion, in an ultra-long (km-scale) cavity. Consequently, it is expected that artificial (or stimulated) seeding, with a perfect \( N = 1 \) fundamental dark soliton pulse, could result in stable propagation of a dark soliton. Such a realisation of stable dark soliton formation and propagation in a chirped pulse fibre laser could have important practical implications: as the generation of dark-soliton pulses remains a challenge experimentally.

Figure 5.22 shows the the convolved temporal intensity profile of an \( N = 1 \) dark soliton convolved with a bright-pulse. This waveform was used as the initial state in the simulation of stimulated dark soliton propagation in the laser model (with parameters of the physical system described above). The bright-pulse background was constructed by fitting (a hyperbolic secant function) to a fully converged single-pulse shape. Using the exact parameters of the model that evolved to the fitted pulse, the dark soliton forms a small perturbation to a known stable bright-pulse solution of the system.

![12Although I have shown that similar nanosecond-scale pulses can be generated in shorter cavities, with similar overall dispersion (at MHz repetition frequencies), it is believed the long-cavity contributes to stabilisation of the dark soliton from noise. This suggestion is in need of further investigation.](image)
5 Chirped pulse fibre laser sources

(a) Full temporal intensity profile.    (b) Fundamental dark soliton intensity and phase.

Figure 5.22: A perfect $N=1$ fundamental dark soliton, with a $\pi$ phase step, convolved with a nanosecond-scale bright-pulse.

Figure 5.23: Evolution of a stimulated dark soliton, forming a small perturbation to a known bright-pulse solution of the system. Colour scale: 0 1
The evolution of the dark soliton is shown in Fig. 5.23. It is clear that, in this case, the dark soliton is unstable, decaying after approximately 650 iterations. Secondly, although an $N = 1$ dark soliton was launched (based on the initial peak power of the background) due to the loss of energy from the bright-pulse to the dark-soliton a small drop in peak power causes a (small) perturbation that could contribute to the destabilisation of the dark component. Figure 5.23 represents the evolution for a single set of parameters. Further investigation is required to establish stability limits of dark soliton propagation, supported by the chirped pulses of a mode-locked laser, subject to strong, distributed positive dispersion. Such understanding could potentially direct experiment towards observation in a laboratory system.

Interpretation of the results discussed above could share similarities with another set of solutions observed in mode-locked lasers, with either dispersion-managed or positive dispersion maps: the anti-symmetric high-order soliton [Par99]. A high-order anti-symmetric soliton (in particular Fig. 5.22 could be viewed as a bi anti-symmetric soliton), is described by a Hermite Gaussian function of the form [Par99, Cho08a, Abl09]

$$A(0, T) = A_0 T \exp \left( -\frac{T^2}{T_0^2} \right)$$

(5.7)

The intensity profile exhibits two temporal (and spectral) peaks with $\pi$ phase difference ($\Delta \phi$) between them; as such the pulse can be interpreted as a pair of bound solitons with $\Delta \phi = \pi$ and the required separation [Par99], and can be described within the framework of the CQGLE [Abl09]. Bound solitons with $\Delta \phi = \pi$ have been demonstrated experimentally in a dispersion-managed mode-locked laser [Cho08a].

How robust the analogy between the anti-symmetric soliton and the dark solitons observed in the formation dynamics of chirped pulses in ultra-long lasers is currently subject to debate.

### 5.6 Summary

Chirped pulse fibre lasers present a practical route to accessing high-energy, high-quality pulses and form an interesting system for the study of nonlinear wave dynamics in a non-conservative regime. In this chapter, I have demonstrated that nanosecond-scale pulses can be stably generated in an ultra-long positive dispersion cavity. In addition, nanosecond pulses are not restricted to sub-MHz repetition frequencies, with the use of large lumped dispersion. The chirp, phase and degree of coherence of the nanosecond pulses, generated in an ultra-long GCO, were experimentally characterised through direct measurement of the spectrogram – i.e the time-dependent spectrum. The experi-
mental measurement was in strong agreement with both analytic theory and numerical simulation, confirming that the generated pulses exist as dissipative soliton solutions of the system. Although the pulses have been shown to possess a linear chirp, compression with standard schemes is impractical. Two approaches were discussed that could be employed to dechirp such pulses, with large time-bandwidth products. In addition, an alternative scheme for accessing high-energy was reviewed. Both are the subject of ongoing research.

Finally, the dynamics of long-pulse formation was studied extensively using a numerical model. It was found that a noisy-phase exists in the evolution of the bright-pulse that provides the conditions appropriate for the spontaneous seeding of dark temporal solitons. Some of these dark soliton components were found to be highly persistent, within the framework of the numerical system. Analogies were also made between the dark solitons on a bright-background and high-order, anti-symmetric bound soliton-states, previously observed in mode-locked lasers, subject to a dispersion-managed or positive dispersion map. Experimental observation of the evolution dynamics of nanosecond-scale chirped pulses in ultra-long mode-locked lasers would be a significant achievement, and maybe the best opportunity to confirm the existence of the characteristics suggested by the numerics.
6 Optimising continuous wave supercontinuum generation

An electromagnetic wave that propagates in an optical fibre, where a high optical intensity can be maintained over a long interaction distance, is altered by effects governed by the linear and nonlinear response of the material. The magnitude and nature of the nonlinearity is mediated by the sign of the linear dispersion. Preserving the quality of the transmitted wave becomes a challenge over long distances, or for increasing intensities above a threshold value where the nonlinear response of the medium cannot be neglected. In most situations reducing the onset of strong nonlinearity is necessary, where spectral purity is required; however, in certain cases the converse is true: enhancing the nonlinearity is deliberate in order to exploit the effect of rapid spectral expansion. A source emitting a continuum of frequencies, similar to an incandescent light bulb, but with the spatial (and in some cases temporal) coherence, directionality and brightness of a laser has opened lines of scientific enquiry previously closed using conventional methods of illumination.

The availability of high-power fibre lasers, in conjunction with specifically engineered fibre wave-guides, has allowed the rapid development of monolithic all-fibre supercontinuum light sources exhibiting extremely broad spectral widths. In this chapter I consider the optimisation of supercontinuum in the continuous-wave pumped regime. It may seem somewhat of a departure from the theme of the previous four chapters, which focused on ultrashort and chirped pulse generation and dynamics in mode-locked fibre lasers, but in fact the underlying mathematics is the same: a perturbed NLS equation is central to the discussion.

The structure of the chapter is as follows. Section 6.1 briefly introduces the concept of fibre-based supercontinua and provides a review of recent advances in the field. The theory of CW-pumped supercontinua is presented in section 6.2. An experiment to characterise the effect of pump coherence on the efficiency of solitons generated from modulation instability is outlined in section 6.3. The construction of a suitable numerical model to analyse the experimental configuration is developed in section 6.4, consisting of two components: a model to describe the CW laser; and a model to propagate the laser field in the HNLF. The numerical and experimental results are presented and discussed.
6 Optimising continuous wave supercontinuum generation

in section 6.5 and section 6.6; and finally conclusions drawn in section 6.7.

Results presented in this chapter have been published in the following journal articles and conference proceedings [Kelomb, Kel11].

6.1 Introduction

6.1.1 Overview of fibre-based supercontinuum

Three broad classes of supercontinua exist depending on the pump conditions and the fibre parameters [Tra10a], each with distinct evolution dynamics and output properties:

Low-order soliton pumping in the anomalous dispersion regime

Pumping with pulses that correspond to low-order solitons (approximately \( N < 15 \) [Gen07, Tra10c]) results in dynamics dominated by soliton fission. Practically, this regime is accessed using ultra-short pulses (typically \( \leq 100 \) fs), with several kilo Watts peak power. In the fission regime rapid temporal compression (or spectral expansion) in the early stages of the evolution leads to a high degree of temporal coherence; subsequently, perturbations, such as the soliton-self frequency shift, continually degrade the degree of coherence for increasing propagation length [Dud02]. In this regime the spectral power is often limited, unless a very high repetition rate pump source is used.

Long-pulse, quasi-CW and CW pumping in the anomalous dispersion regime

For continuous-wave or long pulse pumping (i.e. \( > 1 \) ps, or where the effective soliton order \( N > 15 \) [Gen07, Tra10c]) modulation instability dominates the early stages of the continuum evolution. This usually leads to broad supercontinua, with very high average spectral power and spectral flatness, but very low temporal coherence.

Pumping in the positive dispersion regime

In this case, soliton effects are restricted. For long-pulse (i.e. \( > 100 \) ps) or CW pumping Raman scattering is the dominant effect, leading to a discrete cascade of Stokes lines rather than a continuum. Shorter pump pulses, with sufficient power, can generate a coherent supercontinuum through self-phase modulation. An SPM-based continuum is useful for metrological applications and nonlinear pulse compression.\(^1\)

\(^1\)A number of nonlinear pulse compression techniques exist. SPM can be used to linearly chirp a short pulse, generating additional spectral bandwidth. The chirped pulse can be then dispersive compressed to a duration approximately the inverse of the generated bandwidth. Other schemes include high-order soliton and adiabatic soliton compression.
The above is obviously an oversimplification, and in fact the transitions between the regimes are continuous. However, this does provide a qualititative framework useful for grouping the complex interactions that characterise supercontinuum generation in fibre.

For the interested reader the recent monograph on fibre supercontinuum by Taylor and Dudley is an excellent exposé of the subject [Dud10]. In particular the first chapter, Ref. [Tay10], provides an authoritative and accurate account of historical developments in supercontinuum generation. In addition, I refer to the review articles by Genty et al. [Gen07], Dudley et al., and Travers [Tra10c, Tra10a] for a thorough discussion of: historical developments up to the state-of-the-art sources; experimental guidelines; and numerical aspects regarding a detailed treatment of the evolution dynamics in various regimes. In what follows is a brief review of advances in the pursuit of high-average power continuum light sources.

6.1.2 High-average power supercontinuum sources

Supercontinuum generation in an optical fibre, pumped with a relatively low power (typically tens of Watts) continuous wave (CW) source, has proven to produce the flattest spectra and highest spectral powers of all pump configurations [Avd03, Nic03, GH03, dM04b, Tra05, Rul08, Cum08, Tra08b, Kud08, Kud09a, Kud09b, Cha10d, Tra10d].

Persephonis et al reported in Ref. [Per96] the first significant spectral broadening in a 2.3 km length of Ge-doped fibre, generated by a low-power (Watt level) CW pump source, to which the term supercontinuum could be reasonably applied. However, the rapidly developing interest in pulse pumped supercontinua, exhibiting in some cases visible spectral content, meant that it was another seven years before the first CW supercontinuum generated in a PCF was reported [Avd03]. Following the work by Avdokhin et al., many new results were reported, using both conventional highly nonlinear fibres (HNLFs) and more exotic PCFs, with tailored dispersion and enhanced nonlinearity [Nic03, GH03, dM04b, Tra05, Rul08, Cum08, Tra08b, Kud08, Kud09a, Kud09b]. This lead to increasing spectral powers and broader bandwidths, including the demonstration of visible supercontinua pumped by a CW laser [Tra08b, Kud08, Kud09b]. The universal availability of efficient, high-power CW fibre lasers and speciality optical fibres, with highly engineered dispersion profiles, has facilitated all-fibre integrated supercontinuum light sources with spectral powers approaching 100 mW nm$^{-1}$, filling the transmission window of silica [Tra08b, Kud09b].

Modulation instability (MI) is the key mechanism that initiates the formation of high peak power temporal optical solitons from a low power CW field, a process inherent to any anomalously dispersive nonlinear medium [Has80]. It is the subsequent red-shifting of the solitons generated from MI, through self-Raman interaction, influenced by their
local dispersion and collision events between temporally and spectrally coincident solitons, that gives rise to the broad spanning spectra of long-pulse and CW pumped super-continua [Isl89, GN88].

Without special attention paid to the modal content, it is typical for fibre lasers to have thousands of oscillating longitudinal modes [Bab07, Tur09a, Tur11]. Without the inclusion of a mode-locking element, providing a fixed phase relationship between the longitudinal modes, strong stochastic intensity fluctuations will be present in the laser’s output. It has been shown previously that pump noise arising from such intensity fluctuations in a CW laser can strongly influence the shape of the resulting supercontinuum [ML06]. The average duration of these fluctuations, or the coherence time \( \tau_c \), is inversely proportional to the pump bandwidth: the faster the decorrelation of the wave (i.e. smaller \( \tau_c \)), the larger the range of frequencies the wave contains, and the broader the spectral linewidth [Goo85]. In this chapter, I study the effect of the pump coherence on the MI gain and show how this influences the resulting spectral expansion in the evolution of a supercontinuum generated in a length of highly nonlinear fibre.

6.2 Theory

6.2.1 Basic theory of CW continuum generation

The general mechanism and detailed dynamics of the CW continuum formation process have been studied in great detail [Tra10d]. Here, I review the essential points useful for the rest of this chapter.

The evolution dynamics of CW supercontinuum formation can be reduced to three clear stages [Tra09a]:

1. The break up of the input pump source into solitons due to MI, by pumping in the anomalous dispersion region of a nonlinear fibre.

2. The red-shifting of the solitons though Raman self frequency shift and Raman mediated soliton collisions.

3. If MI induced solitons form sufficiently close to the zero dispersion wavelength, they can excite dispersive waves in the normal dispersion region. These can then be further blue-shifted due to the trapping of dispersive waves by red-shifting solitons.

Stage 1 is characterised by the MI period \( T_{\text{MI}} \), given by [Agr07]:

\[
T_{\text{MI}} = \frac{2\pi}{\Delta \omega_{\text{MI}}} = \sqrt{\frac{2\pi^2 |\beta_2|}{P_{\text{cw}} \gamma}}
\]  

(6.1)
6.2 Theory

i.e. $T_{\text{MI}}$ is defined by the fibre parameters: $\beta_2$ the group velocity dispersion and $\gamma$, the nonlinear coefficient; and scales inversely with the pump power.

The MI period is an important parameter for a number of reasons: it defines the number of solitons that are generated per unit of time; with one soliton generated per period, under conditions of energy conservation, $T_{\text{MI}}$ can be used to calculate the energy of each soliton emitted from MI [Tra10d]

$$E_{\text{sol}} = 2P_0\tau_0 = P_{\text{cw}}T_{\text{MI}}$$  (6.2)

From Equation 6.2 it is clear that as the MI period reduces, or the MI bandwidth increases, the soliton duration reduces and the corresponding soliton peak power, $P_0$ increases. The duration of solitons emitted from MI can be estimated by considering the relationship between $\tau_0$ and $P_0$ given by the soliton equation [Agr07]

$$P_0 = \frac{|\beta_2|}{\gamma\tau_0^2}$$  (6.3)

such that

$$P_{\text{cw}}T_{\text{MI}} = \frac{2|\beta_2|}{\gamma\tau_0}$$  (6.4)

Using Equation 6.1 in Equation 6.4 and rearranging for $\tau_0$ gives

$$\tau_0 = \frac{1}{\pi^2}T_{\text{MI}} \approx 0.1T_{\text{MI}}$$  (6.5)

Thus the estimated full width half maximum duration (FWHM) of a soliton emitted from MI is approximately $T_{\text{MI}}/5$ [Tra10d, Dia92].

The extent to which stage 2 occurs depends on the duration of the solitons emitted from the MI process. Shorter solitons have broader bandwidths, which must be sufficiently broad for a significant fraction to overlap with the Raman gain spectrum, to undergo self-scattering (otherwise known as the soliton self-frequency shift). Although MI initiates the spectral broadening in CW supercontinuum generation (where the peak powers are typically $< 100$ W), it is subsequent soliton collisions, and primarily the Raman scattering of solitons formed from MI that provides the majority of the spectral expansion [Isl89, Fro06]. The rate at which the solitons Raman shift is proportional to the inverse of the fourth-order of their duration, such that

$$\frac{\partial \omega}{\partial z} \propto \frac{|\beta_2|}{\tau_0^4} \propto \frac{|\beta_2|}{T_{\text{MI}}^4} \propto \frac{\gamma^2 P_{\text{cw}}^2}{|\beta_2|}$$  (6.6)

The quartic dependence means shorter duration solitons shift significantly further. Note
also that the rate of shift becomes inversely proportional to the dispersion when the MI formation process is accounted for.

In addition, because $T_{\text{MI}}$ is not a fixed quantity when evolving from noise – this is clearly seen in frequency space with the formation of symmetric sidebands either side of the pump frequency, with maxima at $\omega_0 \pm \Delta \omega_{\text{max}}$, but with a linewidth indicating that the MI modulation period is not precisely fixed – solitons are emitted with a distribution of durations, consequently Raman shifting to different positions in frequency space [Van05, Kut05]. It is this variation in the soliton duration that causes the smooth average spectra expected of CW supercontinua. Despite having a high degree of spectral flatness, this also results in low temporal coherence.

### 6.2.2 Designing continuous wave supercontinuum systems

The above analysis can assist in the design of experimental systems for CW supercontinuum. There are three competing demands to be met [Tra10d]:

1. The ratio $|\beta_2|/\gamma$ should be reduced as much as possible, as this both increases the efficiency of the MI process, and also leads to the generation of the shortest possible solitons, leading to maximum Raman self-interaction and maximum red-shift.

2. However, the dispersion slope (and to a lesser extent, the reduction of $\gamma$) should be minimal or negative with increasing frequency, so that solitons undergoing Raman self-frequency shift obtain the maximum red-shift before they adiabatically broaden to a point which prevents further red-shift.

3. For blue-shifted CW supercontinua, the MI induced solitons should propagate close to a zero dispersion point, to generate dispersive waves.

Note that point 1 is often in conflict with point 2; i.e. reducing $|\beta_2|$ at the pump wavelength often suggests one should move close to the first zero dispersion wavelength, as does point 3. However, this can prevent any significant red-shifted continuum from forming due to the consequent large positive dispersion slope. This is unavoidable if one wishes to obtain blue-shifted or visible CW supercontinua; but it has been found that a higher absolute value of $|\beta_2|$ in a fibre with a very flat dispersion profile is optimal for large red-shifted CW continuum formation [Cum08, Kud09a, Kud09b, Tra10d].

### 6.2.3 Role of pump coherence in MI

It is well known that for both instantaneous and noninstantaneous nonlinear media wave incoherence (or partial coherence) affects the efficiency of modulation instability
6.2 Theory

gain [Sol00, Sau05, Tra10e]. One model for optical fibre suggests the following modification to the purely CW MI gain term in the presence of incoherence:

\[
g(\omega) = \left| \beta_{2\omega} \right| \left( \frac{8\gamma P_{cw}}{\beta_{2} - \omega^2} - 2 \left| \beta_{2\omega} \right| \sigma \right)^{1/2} \tag{6.7}
\]

where \( P_{cw} \) is the average pump power and \( \sigma \) is the spectral bandwidth.

Two conclusions can be drawn directly from Equation 6.7: firstly, that the MI gain strictly decreases with increasing pump bandwidth (or increasing incoherence of the pump wave); in fact, MI gain is eliminated completely when the pump bandwidth is equivalent to the MI bandwidth. Secondly, that the maximum MI gain occurs for the most coherent pump source. Oddly, in the limit of infinitesimal pump bandwidth, Equation 6.7 suggests that the MI gain is still higher than that derived for the coherent MI case.

In contrast, it has been experimentally observed that broader pump bandwidths, at least initially, produce more efficient supercontinua [ML06], and can be expected to enhance the MI gain [Tra10d, Tra10e], through the enhancement of instantaneous peak powers in the partially coherent pump source. I suggest that this discrepancy with Equation 6.7 is due to the fact that the increased peak power fluctuations are not fully accounted for in this model of incoherent MI.

The temporal coherence (\( \tau_c \)) of a light source, with a complex electric field \( E(t) \) can be determined by considering the field autocorrelation (AC) given by

\[
\Gamma^{(2)}(\tau) = \int_{-\infty}^{\infty} E(t) E^*(t-\tau) \, dt = \mathcal{F}^{-1} [S(\omega)]
\]

\[
\tau_c \approx \left| \Gamma^{(2)}(\tau) \right|_{\text{FWHM}}^2 \tag{6.9}
\]

where \( \Gamma^{(2)}(\tau) \) is the second-order coherence function [Tre00], which is the Fourier transform of the spectral power \( S(\omega) \) [Goo85]. This relation clearly links the pump spectral bandwidth inversely with the coherence time. However, it says nothing about the temporal intensity fluctuations: all of the spectral width could be (momentarily) due to phase fluctuations.

The intensity AC of a complex, or highly structured waveform, such as a continuous-wave laser, given by [Tre00]

\[
A^{(2)}(\tau) \approx \left| \Gamma^2(\tau) \right|^2 + \int_{-\infty}^{\infty} I_{\text{env}}(t) I_{\text{env}}(t-\tau) \, dt \tag{6.10}
\]

where \( I_{\text{env}}(t) \) is the intensity of the waveform envelope, contains information about the intensity fluctuations. For a purely CW signal, the intensity AC will be a flat line, but for a
more general signal, the AC function possesses a flat pedestal half the height of the peak, and a spike, symmetric about the zero delay ($\tau = 0$) characterising the average-duration of the finest structure of the signal noise.

I therefore conclude that the field autocorrelation is redundant if the spectrum is measured directly, which provides identical information, whereas the intensity autocorrelation characterises the intensity modulations present on the background CW signal. In order to quantify both the coherence time and intensity fluctuations of the experimental system as a function of bandwidth, the spectrum and background-free intensity AC were recorded.

Figures 6.1 and 6.2 show the simulated temporal and spectral field intensities for three pump bandwidths using the model described in more detail in Section 6.4. It is clear that as the spectral bandwidth of the CW pump source increases, the rate of fluctuations in the temporal domain increases: the decorrelation time of the wave increases or the temporal coherence of the field, $\tau_c$ decreases. It is also evident that as $\tau_c$ decreases the associated peak power increases. However, the effect of increased instantaneous power saturates at a certain pump bandwidth. This is confirmed by the clear logarithmic dependence of the peak power enhancement factor, $\Psi_{\text{enhancement}}$ (defined as the ratio of the peak and average power), on the pump linewidth shown in Fig. 6.3. Due to the stochastic nature of the initial noise conditions, as with all simulations performed for this chapter, the peak power enhancement factor was averaged over an ensemble of simulations, each seeded from a different random initial noise condition, for each pump bandwidth. The five-point averaged data is shown in Fig. 6.3 with blue dots, and the logarithmic fit to this averaged data with a solid blue curve. The corresponding coherence time, $\tau_{\text{coherence}}$ is also plotted, showing an inverse log-log dependence on the pump bandwidth: as the linewidth of the pump laser increases from 0.1 nm to 10 nm the coherence time decreases by two orders of magnitude from 100 ps to 1 ps.

I suggest that the optimal pump bandwidth for efficient production of short solitons through MI, and hence efficient CW supercontinuum generation, is not the nearly coherent pump suggested by Equation 6.7, but much broader. For very narrow pump bandwidths the coherence time of the pump source is much longer that the MI period, therefore the peak MI gain is determined by the peak power fluctuations of the pump source. These peak power fluctuations increase as the bandwidth is moderately increased and the coherence time decreased, hence the MI and continuum efficiency should also increase. When the bandwidth increases so much that the pump coherence time is reduced below the MI period, i.e. the pump bandwidth becomes comparable to the MI bandwidth, the gain is reduced by a corresponding amount, and eventually is completely inhibited.
Figure 6.1: Numerically modelled temporal intensity profiles of the input pump source for three pump bandwidths as indicated. The time averaged power is shown with a dotted blue line, and is constant with increasing pump bandwidth to within 1% of a target value of 6.3 W.
Figure 6.2: Numerically modelled spectral intensity profiles of the input pump source for three pump bandwidths as indicated. Note a change of x-axis scale in Fig. 6.2c.
6.3 Experimental Setup

6.3.1 A broadly tunable ASE source

An overview of the laser pump system used in the experiments is shown in Fig. 6.4a. It comprises a chain of two low-power Er-doped fibre amplifiers generating amplified spontaneous emission (ASE) within their gain bandwidth (1545 nm–1575 nm), intersected by a broad, but fixed bandwidth (12 nm FWHM passband), bandpass filter, centered at 1565 nm. The filter acts to increase the signal to noise ratio, by suppressing ASE outside the desired bandwidth. This assembly forms the seed for a 10 W Er-doped power amplifier. A tunable bandwidth filter is employed to control the linewidth of the seed source passing into the power amplifier, allowing continuous control of the pump source bandwidth from ∼0.1 nm–7.0 nm, corresponding to a coherence time range of ∼20 ps ≥ τc ≥ 50 fs. The lower limit on τc is restricted by the imprint of the finite, parabolic gain shaping of successive stages of amplification in the Er-doped fibre amplifiers and not by the maximum allowable bandwidth of the tunable filter.

The spectral and temporal performance of the pump system is shown in Fig. 6.5, where the FWHM spectral bandwidths are as indicated. Greater than 30 dB suppression between the signal and pedestal was achieved. From Fig. 6.5e it is clear to see the effect of multiple stages of amplification and the shape of the gain profile of the amplifiers imposed on the output spectrum – ultimately limiting the minimum coherence time available from this

![Figure 6.3](image-url)

**Figure 6.3:** The relative peak power enhancement factor (where \( \Psi_{\text{enhancement}} = \frac{P_{\text{peak}}}{P_{\text{average}}} \)) as a function of the FWHM pump source bandwidth.

In the following sections I provide experimental and numerical support for this hypothesis.
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![Diagram](attachment:image.png)

(a) Pump system.

(b) Experimental system.

**Figure 6.4:** 6.4a The components of the tunable ASE source. ASE seed: DF, Er-doped fibre amplifier; BPF, bandpass filter ($\Delta \lambda = 12$ nm); Er-doped fibre pre-amplifier. TBPF, tunable bandpass filter ($0.1 < \Delta \lambda \leq 15$ nm); power amplifier: 10 W Er-doped fibre amplifier. 6.4b Experimental system overview. ISO, high-power fibreised optical isolator; HNLF, highly-nonlinear fibre. The cross denotes a permanent welded fusion splice between the output of the isolator and the HNLF for a fully fibre integrated format.

The corresponding AC function for the pump bandwidth shown in Figs. 6.5a, 6.5c and 6.5e is given in Figs. 6.5b, 6.5d and 6.5f, respectively. A two-to-one contrast between the peak and the pedestal of the function shows that the field consists of 100% modulations, where the average duration of the modulation is given by the width of coherence spike. As such, it is clear to see that for the narrowest pump bandwidth, there are fluctuations on the time-scale of tens of picoseconds, reducing to tens of femtoseconds as the bandwidth increases.

### 6.3.2 HNLF details

The output of the tunable ASE source was directly fusion spliced to a high-power, inline, fibre pigtailed optical isolator to prevent spurious back reflections damaging upstream components, due to the high-gain of the final stage amplifier. The output of the isolator was fusion spliced directly to a 50 m length of HNLF (see Fig. 6.4b). The splice was optimised on an arc-discharge fusion splicer using a mode-matching algorithm, with repeatable splice losses of $\sim 0.5$ dB. The details of the HNLF are summarised in Fig. 6.6a:
Figure 6.5: Optical spectra and corresponding measured background-free (non-colinear) intensity autocorrelation trace for three increasing pump source bandwidths: 0.36 nm (6.5a, 6.5b); 1.77 nm (6.5c, 6.5d), 5.28 nm (6.5e, 6.5f).
the calculated zero dispersion wavelength (ZDW) is \(\sim 1.47 \, \mu m\), a second ZDW exists at 2.14 \(\mu m\); the calculated dispersion at the pump wavelength is 2.1 \(\text{ps nm}^{-1} \, \text{km}^{-1}\); and the calculated nonlinear coefficient at the pump wavelength is 9.2 \(W^{-1} \, \text{km}^{-1}\). The MI period for the fibre used in the experiments is plotted in Fig. 6.6b, as a function of the average pump power at a fixed pump wavelength of 1.565 \(\mu m\). At the average power of 6.3 \(W\) (typical for all results presented in this chapter), \(T_{\text{MI}} \approx 1 \, \text{ps}\). Figure 6.7 shows the estimated duration of the solitons emitted from MI, based on Equation 6.5. For the fibre parameters, pump wavelength and power matching the experimental conditions the estimated characteristic duration of MI generated soliton is \(\sim 100 \, \text{fs}\).

### 6.4 Constructing a numerical model

#### 6.4.1 Modelling a CW pump source

In order to investigate the role of pump source coherence in CW-pumped supercontinuum generation (SCG) an appropriate model of the pump system is a prerequisite for use as the initial condition. A suitable numerical model of a CW fibre-based source containing empirically valid fluctuations in the amplitude and phase of the field is a complex problem, and a number of models have been proposed \[V_{\text{an05}, \text{Ko}b05, \text{T}ra08a, \text{T}ur09a, \text{Fro10, Tur11}}\]. In addition to difficulties regarding physical initial conditions, CW supercontinuum simulations are necessarily going to be an approximation due to the finite periodic boundary conditions imposed to make the problem tractable.

Considerable simplification is possible when using a correctly isolated cascaded ASE source, as the entire experimental system is strictly forward propagating, eliminating the need to model cavity mode effects, and allowing the use standard forward propagating generalised nonlinear Schrödinger equation (GNLSE) models to simulate the nonlinear evolution through the amplifier systems. It should be noted that given that ASE and CW laser based supercontinua are essentially identical if the pump bandwidths are matched \[dM04a\], this model should be transferable to laser pump cases.

The experimental pump system (which comprised a chain of Erbium (Er) amplifiers and tunable filters; see Fig. 6.4a) was modelled by iterating an initial white noise field, equivalent to one-photon-per-mode \[Dud06\], through an amplifier stage, followed by a spectral filtering element, until the average power and spectral width match the pump conditions. Examples of the temporal and spectral intensity profiles output from this model are shown in Fig. 6.2.

In order to maintain a constant average power it is necessary to adjust the pump power as the filter bandwidth is increased, as filtering out less power results in a lower insertion loss. In the lab a constant launched pump power was maintained by simply adjusting
Figure 6.6: 6.6a Calculated dispersion curve and corresponding nonlinearity curve. The vertical dotted line corresponds to the experimental pump wavelength of 1.565 µm. 6.6b MI period, and estimated energy of solitons emitted from MI as a function of pump power for the given fibre parameters at the pump wavelength of 1.565 µm. The vertical dotted line denotes the average power of the CW laser source used in the experiments.
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Figure 6.7: The estimated duration of the solitons emitted from MI, for a given pump power in the HNLF.

The pump power of the final amplifier stage as the filter bandwidth was varied, to obtain the required average output power. In the numerical model a similar approach was adopted, using a simple proportional, integral, differential (PID) feedback control loop (implemented in software) to iteratively find the correct value of amplifier saturation energy (analogous to the pump power control in the lab) for a given pump bandwidth, to within 1% of a target time-averaged power.

Figure 6.8: The −3 dB (or FWHM) pump source bandwidth as a function of the tunable bandpass filter bandwidth.

Using an ASE-based source means that the temporal coherence of the pump system can be broadly tuned, within the gain bandwidth of the amplifiers used, with a high degree of control by employing a tunable bandpass filter (TBPF) after a final pre-amplifier stage and before a power amplifier, while ensuring all other parameters of the pump system remain unchanged (i.e. average launched power, central wavelength etc.).
important for isolating parameters that affect the evolution, and ultimately the optimisation of the SCG. A TPBF, with a pass-band tunable from 0.1 nm–15 nm and centred around 1565 nm, compatible with high-power fibre-based Er amplifiers was used. However, given the finite gain bandwidth of Er-doped fibre amplifiers (approximately 30 nm) it was only possible to utilise, in the ideal case, a FWHM pump bandwidth of \(\sim 10\) nm because of the cascaded single-pass gain shaping effect of the amplifier chain. This is illustrated in Fig. 6.8 where the calculated FWHM pump bandwidth is plotted as a function of the TBPF bandwidth. The data was computed using the numerical model of the experimental pump system, with a gain bandwidth of 30 nm specified for all amplifiers in the ASE chain and a perfect parabolic gain profile centred around 1565 nm. The broadest obtainable pump bandwidth is shown with a dashed blue line. However, because the gain profiles of the amplifiers used in the experiment deviate somewhat from being purely parabolic and have slightly shifted gain peaks due to variations in doping concentration, fibre length etc., the broadest bandwidth that could be extracted from the ASE-based pump system was \(\sim 7\) nm.

Although it is not possible to measure the full field of a CW source (as it is for say fs-scale pulses using frequency resolved gating (FROG) techniques, among others [Tre93, Wal96]) it is possible to obtain the average duration of the intensity fluctuations through autocorrelation of the field intensity, given by Equation 6.10. Fig. 6.9 shows the autocorrelation functions – both numerical and experimental – for three pump bandwidths: 0.36 nm (6.9a and 6.9b); 1.77 nm (6.9c and 6.9d), 5.28 nm (6.9e and 6.9f). Comparison of the measured intensity AC with the calculated AC function of the simulated field provides a reliable means of evaluating the accuracy of the numerical model of the experimental pump system. Excellent agreement confirms that the model predicts empirically valid fluctuations in the pump field and can be used to generate the initial noise conditions for the simulation of SCG in a HNLF.

### 6.4.2 Modelling supercontinuum evolution in fibre

Modelling of supercontinuum evolution in optical fibre waveguides can be well described by the one-dimensional generalised nonlinear Schrödinger equation that includes the relevant linear and nonlinear contributions that modify the initial field depending on the parameters of the fibre, such as GVD and nonlinearity [Agr07]. A frequency domain
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Figure 6.9: Second harmonic intensity autocorrelation traces (measured from experiment and calculated from simulation) of the CW input pump source for three pump bandwidths: 0.36 nm (6.9a, 6.9b); 1.77 nm (6.9c, 6.9d); 5.28 nm (6.9e, 6.9f).
6.4 Constructing a numerical model

formulation of the equation can be written as [Lae07, Tra10f]²

\[
\frac{\partial \tilde{A}(z, \omega)}{\partial z} + \left[ \frac{\alpha(\omega)}{2} - i \sum_{k \geq 2} \frac{\beta_k}{k!} (\omega - \omega_0)^k \right] \tilde{A}(z, \omega) = i \gamma \frac{\omega}{\omega_0} \mathcal{F} \left[ A(z, T) \int_{-\infty}^{\infty} R(T') |A(z, T - T')|^2 dT' \right]
\]

(6.11)

where \( \tilde{A}(z, \omega) \) is the (linearly polarized) complex spectral amplitude of the pulse envelope³ which is a function of the propagation distance \( z \), within a retarded time frame \( T = t - z/\nu_g \), moving at the group velocity \( \nu_g \) of the pulse. The nonlinear coefficient is defined in the usual way: \( \gamma = \omega_0 n_2 / (c A_{\text{eff}}) \), with units of \( W^{-1} km^{-1} \), where \( n_2 \) is the nonlinear refractive index, \( c \) the speed of light and \( A_{\text{eff}} \) the effective mode area.

In Equation 6.11, the left hand side terms model linear effects: the power attenuation \( \alpha \); and the dispersive coefficients \( \beta \), to arbitrary order – however, here I consider only GVD (\( \beta_2 \)), third- and fourth-order (TOD/FOD) dispersion (\( \beta_{(3,4)} \)) terms of the Taylor series expansion. The right hand side describes nonlinear effects: the instantaneous contributions to the nonlinearity caused by the electronic Kerr effect, self-steepening and optical shock formation; and the delayed contribution from non-instantaneous effects, namely inelastic Raman scattering. The convolution integral contains an instantaneous electronic and a delayed Raman contribution, given by

\[
R(t) = (1 - f_R) \delta(t) + f_R h_R(t)
\]

(6.13)

where \( \delta(t) \) is the Dirac delta function, \( f_R = 0.18 \) is the fractional contribution of the delayed Raman response, and the response function \( h_R(t) \) can take a number of forms, depending on the complexity (and accuracy) of the desired function [Blo89, Agr07, Hol02]; a multi-vibrational-mode model, developed by Hollenbeck and Cantrell [Hol02], was used in all numerical experiments described in this chapter.

When the duration of the temporal input field is of the order of hundreds of femtoseconds, the time-scale of the simulation is typically performed over a reference frame several picoseconds long. This means that for the case of a CW initial input, only a snap-shot of the field in time is simulated, with a typical temporal grid width of the order of several hundred picoseconds – limited by the requirement to satisfy the Nyquist sampling the-

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²It is often preferred to derive a time-domain GNLS equation, because of analytic similarity to the NLS equation – about which there is vast literature [Sul99]; but in fact a frequency domain formulation has many advantages: allowing more direct treatment of frequency-dependent effects such as dispersion, dispersion of the nonlinearity, loss and the fibre effective mode area [Lae07, Tra10f].

³In this case the approximation from Ref. [Lae07], where

\[
\tilde{A}(z, T) = \mathcal{F}^{-1} \left\{ \tilde{A}(\omega, T) \right\}
\]

(6.12)

was used to account for the frequency dependence of the effective area.
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**Figure 6.10**: The dependence of generated continuum width (10 dB) on the CW pump bandwidth (3 dB) for propagation of 6.4 W average power through 8 m of HNLF. 6.10a Experimental results; 6.10b numerical comparison.

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The spectral output of the HNLF, under a fixed pump power of 6.3 W, for pump bandwidths in the range 0.3 nm–7 nm was recorded using an optical spectrum analyser. The experimentally measured continuum width (10 dB) as a function of the pump source bandwidth (3 dB) is plotted in Fig. 6.10a.

It is evident that initially the continuum width increases as the pump bandwidth increases. Beyond a pump bandwidth of ~3 nm the rate of increase in the corresponding continuum width slows and the spectral expansion begins to saturate. Indeed beyond ~5 nm the spectral expansion appears to start to contract. Due to limitations in the pump
system that have been discussed above, it was not possible to extend further the degree of pump source incoherence and recover the full contraction in the continuum evolution. However, with perfectly coincident gain profiles, a numerical model of the pump system showed that it was possible to obtain a maximum pump bandwidth $\sim 11$ nm. The dependence of the continuum width on the pump source bandwidth calculated using the numerical model is shown in Fig. 6.10b. The additional degree of pump source incoherence (or pump bandwidth, up to $\sim 11$ nm) shows a full saturation of the spectral expansion for pump bandwidths beyond 4 nm and a contraction above 6 nm. Each point on the curve in Fig. 6.10b represents an average over an ensemble of five simulations, each with a unique initial noise field. The peak of the curve is less well defined than demonstrated in the experimental measurement, this can be predominately attributed to the fact that the ensemble size is too small – typically CW SCG simulations will be averaged over $\sim 50$ single-shot simulations. However, the additional accuracy of simulating the initial noise field comes at increased computational cost.

Figure 6.11 shows the computed temporal field intensities for three pump bandwidths: 0.33 nm (6.11a); 2.58 nm (6.11b); 6.24 nm (6.11c). The calculated MI period for the HNLF fibre parameters is shown with the temporal field for comparison. The corresponding (single-shot) output spectra generated after 50 m of propagation in the HNLF are shown in Fig. 6.12. The input pump spectrum is shown with a dashed line for reference (see Fig. 6.12). When the pump source exhibits a long coherence time, the peak power enhancement is low and the corresponding output spectrum shows no continuum evolution after the full propagation length. As the coherence time of the initial field decreases and the peak power is enhanced, a broad continuum is formed after the full propagation length. As long as the pump coherence time is longer than the MI period the intensity noise fluctuations enhance the soliton energies formed through MI and hence the resulting continuum evolution. Additionally, this spectrum shows the largest energy transfer to dispersive waves around 1300 nm [Wai87, Akh95], showing that the most intense and short duration solitons are formed in this case. Figure 6.11c shows that when the intensity fluctuations in the initial time-domain field are much shorter than the MI period the MI efficiency and the continuum expansion is reduced (see the corresponding output spectrum in Fig. 6.12c).
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Figure 6.11: Temporal input field intensities for three pump bandwidths, computed using the CW laser model: 0.33 nm (6.11a); 2.58 nm (6.11b); 6.24 nm (6.11c). The horizontal bars show the modulation instability period, $T_{MI}$, for the HNLF fibre parameters, pump wavelength (1.55 µm) and power (6.3 W) corresponding to the experimental conditions.
Figure 6.12: The corresponding computed single-shot spectra after propagation in the 50 m length of HNLF: pump bandwidths 0.33 nm (6.12a); 2.58 nm (6.12b); 6.24 nm (6.12c). The spectral input pump lines are shown with a dotted curve.
Figure 6.13: Single-shot spectral evolutions as a function of propagation length in the HNLF for three input pump bandwidths: 0.56 nm (6.13a); 4.25 nm (6.13b); 38.66 nm (6.13c). Colour scale: −90 dB to 0 dB.
This dependence is further exemplified in Fig. 6.13, where the spectral intensity evolution is plotted against distance of propagation for three input pump bandwidths: 0.56 nm (6.13a); 4.25 nm (6.13b); 38.66 nm (6.13c). Although the same ASE model was used to generate the initial pump fields for the SCG simulations, here I have relaxed the requirement on the components in the model to fully represent values of experimental parameters to allow the generation of an input field with a FWHM spectral bandwidth of 38.66 nm, and to show the full contraction of the continuum dynamics.

The continuum dynamics are often better observed simultaneously in time and frequency space through the field spectrogram, where the temporal location of spectral components leads to an intuitive view of the nonlinear interaction within the fibre and the evolution of soliton and dispersive processes. The field spectrograms (for the corresponding input pump bandwidths from Fig. 6.13) after 50 m of propagation of the initial field in the HNLF fibre are shown in Fig. 6.14. Solitonic structures can be identified as temporally and spectrally localized hot-spots, while dispersive waves exhibit lower intensity and a temporal chirp. Fig. 6.14b, with a pump bandwidth of 4.25 nm, shows the only significant continuum formation, with the generation and red-shifting of solitons to the long-wavelength edge of the pump wavelength and dispersive wave radiation to the short edge (albeit > 45 dB down from the peak of the pump) indicating the formation of broadband (i.e. ultrashort) solitons. As the red part of the spectrum extends, this dispersive radiation blue-shifts indicating the presence of soliton trapped dispersive waves [Skr05, Tra09a]. In contrast, no solitons are formed in Fig. 6.14a, where the input pump bandwidth is 0.56 nm, although the pump has experienced a small degree of SPM over the full propagation length, seen in the chirping of the temporal intensity peaks. As the pump bandwidth becomes too broad and the average duration of temporal fluctuations in the time-domain field become consistently shorter than the MI period, MI is effectively quenched and the pump wave does not efficiently break-down into a train of fundamental solitons (Fig. 6.14c).
Figure 6.14: Single-shot spectrograms after the full 50 m propagation length of the HNLF for three input pump bandwidths: 0.56 nm (6.14a); 4.25 nm (6.14a); 38.66 nm (6.14a). Colour scale: $-90 \text{ dB}$ to $0 \text{ dB}$. 

(a) Pump bandwidth: 0.56 nm. 
(b) Pump bandwidth: 4.25 nm. 
(c) Pump bandwidth: 38.66 nm.
6.6 Discussion

The modelling of the pump source in the previous sections, which provided the initial conditions for the supercontinuum simulations, was related as closely as possible to the experimental system, to allow direct comparison between numerical and experimental results. In this section I consider a simplified model of the pump system to generate the initial pump conditions, but use the same propagation equations to simulate the evolution of the field in the HNLF. The simplified model allows a broader space of parameters to be investigated.

![Figure 6.15:](image)

**Figure 6.15:** The output spectrum (averaged over 40 shots) obtained by pumping 20 m of fibre ($\gamma = 44 \text{ W}^{-1}\text{km}^{-1}$, $\beta_2 = -0.012 \text{ ps}^2\text{km}^{-1}$) with a 10 W, 1065 nm CW source with the given pump bandwidth. Colour scale: $-30 \text{ dB}$ to $0 \text{ dB}$.

It has been shown that a CW-fibre laser can be well modelled by a sech-shaped spectral intensity profile with an associated random spectral phase to account for intensity fluctuations in the time-domain field [Tra10d], in a manner similar to that described in [Van05]. This simple model allows arbitrarily broad bandwidth pump sources to be modelled by modification to the width of the sech-shaped spectrum. Here, I use this model to provide the initial conditions for simulations performed with only first order dispersion ($\beta_2$)
and no high-order dispersion effects. Although this model exhibits an unphysical dependence on the numerical grid size and ignores any phase relationship between laser modes (accounted for in the previous description of the experimental pump system), it is self-consistent for simulations conducted over constant grids and does not require extensive numerical evaluation.

Figure 6.16: The output supercontinuum width (20 dB level, averaged over 40 shots) obtained by pumping 20 m of fibre \((\gamma = 44 \text{ W}^{-1}\text{km}^{-1})\) with a 10 W, 1065 nm CW source, as a function of pump bandwidth. Each curve has \(\beta_2\) scaled to achieve the \(\sqrt{\gamma/|\beta_2|}\) values shown.

To probe the dependence of the MI/continuum efficiency on pump bandwidth I study a range of fibres designed to have a range of MI periods. I proceed by varying \(\beta_2\) while holding \(\gamma = 44 \text{ W}^{-1}\text{km}^{-1}\) constant. This means that the MI gain and the nonlinear length of the fibre is fixed. In the following an average pump power of 10 W and a fibre length of 20 m was chosen. The results are illustrated on a wavelength scale centred at 1065 nm, similar to Yb fibre lasers (and the fibre parameters are similar to those used in common CW continuum experiments at 1065 nm in photonic crystal fibres), but the results are generally applicable.

Fig. 6.15 shows the output spectra for pumping a fibre with \(\beta_2 = -0.012 \text{ ps}^2\text{km}^{-1}\) with
a range of pump bandwidths. It can be seen that there is a clear optimum pump bandwidth of \( \sim 2 \) nm to obtain the broadest supercontinuum. This corresponds to a coherence time of \( \sim 1.7 \) ps, compared to an MI period of 0.74 ps. Fig. 6.16 shows obtained supercontinuum width (20 dB level) as a function of 3 dB pump bandwidth for a number of fibres with differing values of \( \beta_2 \) designed to show a linear progression in the quantity \( \sqrt{\gamma/|\beta_2|} \) which, for fixed pump power, determines the MI bandwidth and hence MI period. Each curve consists of 20 pump bandwidths logarithmically distributed between 0.03 nm–30 nm, where each point is averaged over 40 single-shot simulations with different random initial noise conditions. It is clear that for each curve there is a variation in obtained supercontinuum width as a function of pump bandwidth, with a gentle peak value.

As described in Section 6.2, in the absence of higher-order dispersion, optimal continuum formation is achieved in a fibre with a high nonlinearity and low value of GVD, consequently a large value of \( \sqrt{\gamma/|\beta_2|} \) is desirable. This is evident in Fig. 6.16 as the continuum bandwidth clearly scales with this quantity. The reason for this scaling is that the MI period, and hence the induced soliton duration, is reduced. As the MI period is reduced the tolerated pump incoherence is higher and hence the range of pump bandwidths over which a continuum will form should increase with increasing \( \sqrt{\gamma/|\beta_2|} \); this is also clear from Fig. 6.16: the curves are wider for increasing \( \sqrt{\gamma/|\beta_2|} \). The peak value in these curves, i.e. the pump bandwidth leading to the broadest continuum, also depends on \( \sqrt{\gamma/|\beta_2|} \), as expected from the preceding analysis. Consequently, it is possible to express an optimum pump bandwidth as a function of the MI bandwidth; the relationship, plotted in Fig. 6.17, is found to be linear where the optimum pump bandwidth is approximately one third of the MI bandwidth: \( \Delta \omega_{pump} \approx 0.3 \Delta \omega_{MI} \).

**Figure 6.17**: The optimum pump bandwidth extracted from (a) compared to the corresponding MI bandwidth, with corresponding linear fit.
Taking the coherence time of the sech² spectrum as $5.53/\Delta\omega_{\text{pump}}$ [Goo85], this implies that the optimum pump coherence time $\tau_c \approx 3T_{\text{MI}}$. This agrees with the original hypothesis: the optimum degree of coherence of the pump source is the minimum (enhancing peak power fluctuations) that remains sufficiently coherent for MI to occur.

### 6.7 Summary

In this chapter, the relationship between the degree of pump coherence and the generation and evolution of a CW supercontinuum from MI has been studied in detail. A model to accurately reproduce the temporal noise field of a CW laser was developed; direct measurements of the intensity fluctuations through the intensity autocorrelation confirmed the validity of the numerical approach. This CW model was used to create the initial conditions for the simulation of CW-pumped supercontinuum in HNLFs. While it is well known that wave incoherence destroys MI, it was shown empirically that partial wave coherence of the pump laser can enhance the instantaneous peak power leading to the generation of shorter and more intense solitons through MI. This results in the formation of a broad spanning supercontinuum that cannot be generated with a coherent input, matching all other parameters. Furthermore, it was shown that an optimal degree of pump coherence exists that results in the greatest amount of spectral broadening of the initial pump spectrum. The optimum pump bandwidth is that which supports the largest possible intensity fluctuations, while remaining sufficiently coherent for MI to occur. It was found that the optimum pump bandwidth shifts linearly with the MI frequency, such that an optimum bandwidth can be defined as approximately one third of the MI frequency. Current theory of incoherent MI does not consider the enhancement of the peak power due to partial coherence of the input wave, and requires augmentation to account for the empirical observations presented in this chapter.
7 Conclusion and Outlook

This thesis has described experiments towards advancing mode-locked fibre lasers and fibre-based supercontinua. A complete summary, highlighting significant results, was provided at the conclusion of each of the preceding experimental chapters. Thus, it is the intention here to provide a review and discussion of advancements made, state the implications within a broader context, and propose the directions of ongoing research. A number of novel nano-materials were investigated to establish their optical properties and their potential as saturable absorbers in mode-locked fibre lasers. In addition, new wavelength regions for ultrashort pulse sources were accessed using emergent actively-doped fibre technology and nonlinear processes to provide gain. The dynamics of mode-locked operation and pulse formation, within the framework of CQGLE-type equations, were understood. Numerical modelling was used to interpret experimental observations and identify a class of coherent solutions in the highly-chirped pulse regime. Extension of the numerical models, to include the complex nonlinear interactions that lead to the evolution of supercontinuum in optical fibres, provided insight into the role of temporal coherence in continuum evolution, subject to a continuous-wave input. Experiments coupled with numerical investigation expressed an optimum pump condition as a function of the modulation instability period.

Single-wall nanotube-based saturable absorber devices were extensively used throughout the thesis for mode-locking fibre lasers. While this technology has received much attention within the academic community since it was first proposed in 2003 [Set03, Set04a, Set04b], it has only in a limited number of cases transferred to commercial systems. The current technology that is predominantly used for passive mode-locking of low-power lasers is the semiconductor saturable absorber mirror (it should also be noted that transmission devices based on quantum wells, but without a Bragg mirror, are also available). However, SESAMs are not so deeply entrenched that the nanotube could not surplant it: they both offer similar performance properties in terms of device lifetime and damage threshold, and it is often widely reported that nanotube fabrication is significantly less complex and more cost effective [Has09]. In addition, nanotubes offer a number of other favourable properties, some of which were clarified in this thesis, namely the potential to operate over a broader spectral band, albeit with higher nonsaturable losses, which could degrade the mode-locking performance.
The application of nanotubes, even in the limited context of optics, is far from monospecific: the unique optical properties, some of which were revealed in experimental work described in this thesis, could have implications in other areas of ultrafast science. For instance, the relaxation dynamics of purified nanotubes, subject to a strong driving field at a specific resonance, could potentially be exploited for electromagnetically induced transparency [Har90, Bol91, Har97, Mar98a], with potential application in areas of quantum optics.

More promising perhaps, for wide-spread adoption in the context of passive mode-locking fibre systems, is the demonstration of few-layer graphene-based polymer composites [Mar11a, Xu11, Cun11]. Although the self-starting performance and reliability of such systems would require improvement to become a competitive technology, the genuine advantages of a universal absorber that can be applied across the visible and near-infrared (potentially extending to longer wavelengths) mean improvements in device performance are worth pursuing.

The combination of Raman gain and a universal saturable absorber to construct an ultrafast fibre laser, with the potential to operate across the transparency window of silica, is a conceptually elegant approach. The validity of such a system was demonstrated at the first Stokes shift from a pump wavelength at $\sim1.55\,\mu\text{m}$. However, significant improvements in the demonstrated noise performance and system efficiency are required before this scheme could be reliably used for its intended application. To this end, it will be crucial to understand fully the dynamic that leads to the formation of a pulse. Initially, it is straightforward to augment existing equations, outlined in this thesis to describe mode-locking behaviour in lasers based on rare-earth dopants, to include an empirical description of a Raman-based ultrafast laser. The second telecoms window (around 1.3 $\mu$m) is perhaps better accessed using low-power picosecond oscillators based on Bismuth-doped silica fibre, with subsequent stages of Raman amplification to scale the power.

Active doping of glass fibres with Bismuth ions has been a consistently emerging technology that has yet to reach maturity. Low pump absorption and three level behaviour prohibit operation at room temperature, unless high pump densities can be achieved. Attempts to increase the doping concentration have resulted in rapid quenching through ion-ion energy transfer and lower gain. Such properties mean that reduced interest in this medium is likely in the forthcoming years, unless significant steps are made towards optimising its operation as a laser material. The realisation of a double-clad fibre geometry, compatible with multi-mode pump diodes, could stimulate renewed attention.

A central theme of this thesis was a numerical code that solved a set of propagation equations suitable to describe, in one dimension, linear and nonlinear effects observed in fibre systems on time-scales from a continuous-wave to the femtosecond regime, in-
cluding broadband processes such as supercontinua. Extension of the model to include an orthogonal axis, with cross coupled terms, would immediately broaden the applicability to include dynamics observed in mode-locked lasers based on NPE. This would provide a useful tool for the experimentalist moving towards the development of high-power ultrafast systems, where saturable absorbers, such as SWNT-based devices, effective in low-power systems become redundant.

The model was sufficient to identify classes of pulse solutions observed in the physical systems described in this thesis, including highly-chirped structures previously not recognised as stable, coherent waveforms. However, the practical limitations of such highly-chirped pulses were also discussed, and an alternative approach in the pursuit of high-energy ultrashort pulses, based on a hybrid-fibre design that has already received attention within the community, was reviewed [Lec07, Lef10]. Interest in large-mode area fibres (where high-order modes are efficiently suppressed using unique fibre designs [Liu07a, Liu07b, Gal07, Gal08]), in order to limit the effect of large nonlinear phase-shifts in high-energy mode-locked systems, will undoubtedly receive increasing attention, as the performance parameters of fibre systems are brought inline with bulk lasers, but with a considerably reduced footprint and cost.

The final experimental chapter of this thesis is intended to provide a guideline for the generation of efficient, high-power supercontinua from continuous-wave pump sources. Supercontinuum in conventional HNLFs and solid-core, index-guiding PCFs has been extensively studied over the last decade. A white-light supercontinuum nearly filling the silica transmission window, pumped by a relatively low-power CW source at \( \sim 1 \mu \text{m} \), has been demonstrated, with high-average spectral power [Kud09b]. New work will no doubt make use of PCF designs, using materials other than silica for enhanced transmission in particular to longer wavelengths for accessing the mid-IR. Significant contributions have already been made in this direction [Xia06, Xia07, Dom08]. At the opposite end of the spectrum, nonlinear optics in gases, where control of phase-matching processes can be used to access the UV using high-power femtosecond lasers, facilitated by advances in hollow-core PCFs, has recently received attention [Hol11, Tra11b]. Advances in high-power fibre-based ultrashort lasers will significantly simplify the pump schemes for such experiments, presenting the possibility of a compact UV source.

High-spectral power density supercontinua is accessible using CW lasers (typically Yb-doped fibre systems) at the expense of the temporal coherence properties, when evolving from modulation instability. However, significant interest, acknowledged by the award of the Nobel Prize in 2005, in the generation of frequency combs, in particular for high-resolution, ultra-sensitive, and rapid acquisition spectroscopy, means that highly coherent broadband sources are desirable [Jon00, Ude02]. In addition, frequency combs for
the calibration of astronomical spectrographs (known as astro-combs) have also recently received attention [Cha10c, Sta11]. The conventional scheme for comb generation uses a phase stabilised ultrafast laser (typically Ti:Sapphire) with a pulse duration of $\sim 10$ fs, giving a spectral bandwidth of about 70 nm at 830 nm [Jon00]. However, where the end goal is strong-field processes, such as high-harmonic generation (HHG) for application in extreme UV (XUV) precision spectroscopy, high peak intensities ($> 10^{14}$ W cm$^{-2}$) are required to reach the ionisation threshold. Such peak intensities necessitates the use of large-scale solid-state amplifiers, limiting the pulse repetition rate to $< 1$ MHz [Bac01, Zha10b]. A low repetition rate comb, reduces the acquisition speed and prevents direct access to the frequency comb structure of the femtosecond oscillator. Although resonant enhancement cavities have been successfully employed to increase the available repetition frequency [Tho08], a compact and reliable source of high energy ($\geq 1 \mu$J), high repetition rate ($\geq 50$ MHz), ultrashort pulses ($\leq 50$ fs) would impact significantly on, and contribute greatly to advances in this field. A fibre-based system could satisfy some [Har07], or all of these criteria; this will be the subject of future work.

Increasing the repetition rate of frequency combs is particularly pertinent in the context of astro-combs, where current high resolution astrophysical spectrographs cannot resolve the comb lines with a mode spacing $< 15$ GHz [Cha10c]. Integrating the source comb with a stabilised Fabry-Perot cavity, with a free-spectral range (FSR) equal to $M$ times the source combs repetition frequency (where $M$ is an integer), filters all but the $M$th mode. The filtered comb can then be used for calibration, but problems with side-mode suppression reduce performance. In addition, an ideal astro-comb would be able to reference the entire bandwidth of the spectrograph ($\sim 300$ nm across the visible spectrum, from $\sim 380$–$690$ nm in the case of the HARPS spectrograph [Sta11]), however, the dispersion over this wavelength range is not constant within the FP cavity (modifying the path-length and the effective FSR). Thus, the useful bandwidth is limited to $\sim 100$ nm. High-repetition rate, coherent supercontinua were recently suggested and demonstrated as a solution to this problem [Sta11]. Further enhancements to the repetition rate, repetition rate flexibility, bandwidth coverage and spectral flatness would represent significant progress in this area.

It would not be unreasonable to proffer the notion that the invention of optical fibre (and the wealth of research subsequently towards optimising lights interaction with this medium) has had an unprecedented impact on the global community, and was perhaps one of the single most important innovations of the twentieth century. Recognition of which, at least in part, resulted in the award of the Nobel Prize in 2009 for [Kao66]

“...groundbreaking achievements concerning the transmission of light in fibers for optical communication.”
Advancements in fibre technology and our understanding of its interaction with light, in increasingly complex geometries, will continue throughout the twenty first century. It is hoped that the experiments and analysis described in this thesis, provide a small step in the right direction.
References


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List of publications

The following publications form the basis of the present thesis:

Journal papers


**Conference contributions**


Additional publications

In addition, the following work was published during the period October 6th 2008 – October 6th 2011, but is outside the scope of this thesis:

**Journal papers**


Conference contributions


Acronyms

An attempt was made to minimise the number of acronyms throughout, to make the thesis accessible to a broad audience. Inevitably, every field has a number of widely accepted acronyms. Although each acronym is defined the first time it appears in the thesis, a complete list of all acronyms is provided here as a reference tool for the reader.

AC  Autocorrelation
ANDi  All-normal dispersion
APM  Additive pulse mode-locking
ASE  Amplified spontaneous emission
BiDFA  Bismuth-doped fibre amplifier
BLG  Bi-layer graphene
BPF  Band-pass filter
CCVD  Catalytic chemical vapour deposition
CFBG  Chirped fibre Bragg grating
CNT  Carbon nanotube
CW  Continuous-wave
CGLE  Cubic Ginzburg Landau equation
CPA  Chirped pulse amplification
CQGLE  Cubic-quintic Ginzburg Landau equation
DF  Doped/dispersive fibre
DL  Delay line
DoS  Density of states
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Full Form</th>
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<tbody>
<tr>
<td><strong>DRS</strong></td>
<td>Double Rayleigh scattering</td>
</tr>
<tr>
<td><strong>DWNT</strong></td>
<td>Double wall nanotube</td>
</tr>
<tr>
<td><strong>EDFA</strong></td>
<td>Erbium-doped fibre amplifier</td>
</tr>
<tr>
<td><strong>FBG</strong></td>
<td>Fibre Bragg grating</td>
</tr>
<tr>
<td><strong>FC-APC</strong></td>
<td>Fibre connector, angle polished connector</td>
</tr>
<tr>
<td><strong>FLG</strong></td>
<td>Few layer graphene</td>
</tr>
<tr>
<td><strong>FROG</strong></td>
<td>Frequency resolved optical gating</td>
</tr>
<tr>
<td><strong>FWHM</strong></td>
<td>Full width at half maximum</td>
</tr>
<tr>
<td><strong>FWM</strong></td>
<td>Four wave mixing</td>
</tr>
<tr>
<td><strong>GCO</strong></td>
<td>Giant chirp oscillator</td>
</tr>
<tr>
<td><strong>GPVA</strong></td>
<td>Graphene polyvinyl alcohol</td>
</tr>
<tr>
<td><strong>GVD</strong></td>
<td>Group velocity dispersion</td>
</tr>
<tr>
<td><strong>GNLSE</strong></td>
<td>Generalised nonlinear Schrödinger equation</td>
</tr>
<tr>
<td><strong>HNLF</strong></td>
<td>Highly nonlinear fibre</td>
</tr>
<tr>
<td><strong>HO</strong></td>
<td>High-order</td>
</tr>
<tr>
<td><strong>HWP</strong></td>
<td>Half-wave plate</td>
</tr>
<tr>
<td><strong>IDG-SA</strong></td>
<td>Ionically-doped glass saturable absorber</td>
</tr>
<tr>
<td><strong>IR</strong></td>
<td>Infrared</td>
</tr>
<tr>
<td><strong>ISO</strong></td>
<td>Optical isolator</td>
</tr>
<tr>
<td><strong>LMA</strong></td>
<td>Large mode area</td>
</tr>
<tr>
<td><strong>MCVD</strong></td>
<td>Modified chemical vapour deposition</td>
</tr>
<tr>
<td><strong>MFD</strong></td>
<td>Mode field diameter</td>
</tr>
<tr>
<td><strong>MI</strong></td>
<td>Modulation instability</td>
</tr>
<tr>
<td><strong>MLL</strong></td>
<td>Mode-locked laser</td>
</tr>
<tr>
<td><strong>MOPFA</strong></td>
<td>Master oscillator power fibre amplifier</td>
</tr>
</tbody>
</table>
**References**

**MOPA** Master oscillator power amplifier

**MWNT** Multi wall nanotube

**MZAM** Mach Zehnder amplitude modulator

**NA** Numerical aperture

**NALM** Nonlinear amplifying loop mirror

**NLSE** Nonlinear Schrödinger equation

**NOLM** Nonlinear optical loop mirror

**NPE** Nonlinear polarisation evolution

**NPR** Nonlinear polarisation rotation

**OC** Output coupler

**OCT** Optical coherence tomography

**PBS** Polarising beam splitter

**PC** Polarisation controller

**PCF** Photonic crystal fibre

**PD** Pump diode

**PL** Photoluminescence

**PMD** Polarisation mode dispersion

**PM** Polarisation maintaining

**PPLN** Periodically poled Lithium Niobate

**PSD** Power spectral density

**PVA** Polyvinyl alcohol

**QML** Q-switched mode-locking

**QWP** Quarter-wave plate

**RF** Radio frequency

**RFL** Raman fibre laser
References

**RMS**  Root mean squared

**SA**  Saturable absorber

**SBS**  Stimulated/spontaneous Brillouin scattering

**SC**  Supercontinuum

**SESA**  Semiconductor saturable absorber

**SESAM**  Semiconductor saturable absorber mirror

**SLG**  Single layer graphene

**SNR**  Signal to noise ratio

**SPCVD**  Surface plasma chemical vapour deposition

**SPM**  Self phase modulation

**SRS**  Stimulated/spontaneous Raman scattering

**SWNT**  Single-wall carbon nanotube

**SWN-SA**  Single-wall carbon nanotube saturable absorber

**SHG**  Second harmonic generation

**TBPF**  Tunable bandpass filter

**TEM**  Transmission electron microscopy

**TOD**  Third order dispersion

**TWNT**  Triple wall nanotube

**UHNA**  Ultra-high numerical aperture

**(X)UV**  (Extreme) Ultra violet

**WDM**  Wavelength division multiplexer

**XFROG**  Cross frequency resolved optical gating

**XPM**  Cross phase modulation

**YDFA**  Ytterbium-doped fibre amplifier

**ZDW**  Zero dispersion wavelength