

1 **Magnetic flux circulation in the Saturnian magnetosphere as constrained by**
2 **Cassini observations in the inner magnetosphere**

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11

12 **Abstract**

13 In steady state, magnetic flux conservation must be maintained in Saturn's
14 magnetosphere. The Enceladus plumes add mass to magnetic flux tubes in the inner
15 magnetosphere, and centrifugal force pulls the mass-loaded flux tubes outward. Those
16 flux tubes are carried outward to the magnetotail where they deposit their mass and
17 return to the mass loading region. It may take days for the magnetic flux to be carried
18 outward to the tail, but the return of the nearly empty flux tubes can last only several
19 hours, with speeds of inward motion around 200km/s. Using time sequences of Cassini
20 particle count rate, the difference in curvature drift and gradient drift is accounted for
21 to determine the return speed, age and starting dipole L-shell of return flux tubes.
22 Determination of this flux-return process improves our understanding of the magnetic
23 flux circulation at Saturn, and provides insight into how other giant planets remove the
24 mass added by their moons.

25 **1. Introduction**

26 The plasma and magnetic flux circulations in the Saturnian magnetosphere, a
27 fast-rotating system with an inner plasma source [e.g., Burger et al., 2007; Holmberg
28 et al., 2012], are usually considered to be driven intrinsically by the Vasyliunas cycle
29 [Vasyliunas, 1983]. Recently, evidence for a solar wind-driven Dungey cycle
30 [Dungey, 1963] was also found at Saturn [Badman and Cowley, 2007]. In both cycles,
31 the cold plasma stretches the magnetic flux tubes outward, and eventually is allowed
32 to escape from the magnetosphere by reconnection. Meanwhile, conservation of total
33 magnetic flux requires the emptied flux tubes to return to the mass-loading region by
34 buoyancy.

35 The return magnetic flux is well distinguishable using the Cassini magnetometer
36 and plasma data: Over 700 such events, from 5 to 18 R_s ($R_s = 60268$ km is Saturn's
37 radius), have been identified and catalogued by Lai et al. [2016]. During these events,
38 in magnetic field strength, such return flux tubes feature intermittent short-duration
39 increase/decrease with abrupt onset and recovery, while in plasma data, cold plasma is
40 depleted, replaced by hot and tenuous plasma. In addition to the radially inward
41 relaxation, these flux tubes may also be diverted azimuthally by the rotating
42 ionosphere [Burch et al., 2005].

43 Currently, two different types of flux-return processes are proposed: A simple
44 picture expects the empty flux tubes to return directly to the inner magnetosphere,
45 after they are formed in tail reconnection, which usually takes place beyond 20~30 R_s
46 [Smith et al., 2016]. As one piece of evidence, the statistical study by Lai et al. [2016]
47 found no boundary from 4 to 18 R_s in the distribution of the flux tube properties. In
48 the other picture, Thomsen et al. [2015] postulated a plasma boundary at around 8.6
49 R_s using case studies, and more recent statistical results set this boundary at ~10 R_s
50 [Thomsen & Coates, 2019]. They propose that the return flux tubes would stop at the
51 plasma boundary. To penetrate further, an "interchange" process is proposed in their
52 model to swap magnetic flux within this boundary. Empty return flux tubes outside
53 the plasma boundary would interchange with flux tubes loaded with cold plasma
54 inside the plasmasphere. This process has been employed in some

55 magnetohydrodynamic simulations [e.g., Winglee et al., 2013]. Hence, determination
56 of the starting dipole L shell (L_S) of the return flux tubes is an effective way to
57 distinguish the two different proposed processes.

58 The history of the return process should be recorded in the hot plasma population
59 inside the return flux tubes until they encounter another significant change in their
60 environment conditions, such as entry into the mass loading region near Enceladus
61 [Jia et al., 2016]. After drifting in the curved magnetic field, higher energy ions arrive
62 earlier at a spacecraft and vice versa for electrons, forming a monotonic curve in their
63 energy spectrum, respectively, as sketched in Figure 2 in Hill et al. [2005]. The
64 authors fitted these slopes linearly, related them to the age of return flux tubes, which
65 are identified from plasma data only, and found most of the flux tubes to be less than
66 2 hours old [Hill et al., 2005; Chen and Hill, 2008]. Based on the estimated age,
67 whether the injection locations are organized by the Saturnian longitude or local time
68 is still under debate [Hill et al., 2005; Chen and Hill, 2008; Kennelly et al., 2013;
69 Azari et al. 2019].

70 By assuming conservation of the first adiabatic invariant and introducing a
71 constant radial return speed, Paranicas et al. [2016 and 2020] used a similar method to
72 model the energy dispersion of electrons. However, with an extra unknown variable
73 employed, the radial velocity and L_S have a wide range of solutions. The authors
74 thus compared the measured phase-space density to its background average to obtain
75 L_S and then the radial velocity and age. It is found that a return speed between 0 and
76 50km/s with an average at 22km/s results in an energy-time dispersion of electrons,
77 which matches observations the best. In addition, the authors also found that for
78 “fresh” events, the age of the observed flux tubes is 1~2hr, and L_S is about 1Rs
79 radially outward from the point of observation, supporting the two-step return theory.

80 After repeating their work, we further analyzed the difference between gradient
81 drift and curvature drift that depends on the pitch angle of the particles when they
82 pass the equator. We find that even particles in one energy channel of the detectors

83 should still be drifting at different speeds due to their different pitch angles, as shown
84 in Figure 1, for example. Such a dispersive drift in particles of different pitch angles
85 creates a trapezoid shape in their count rate across the return flux tube, which gives us
86 extra constraint and thus a unique solution.

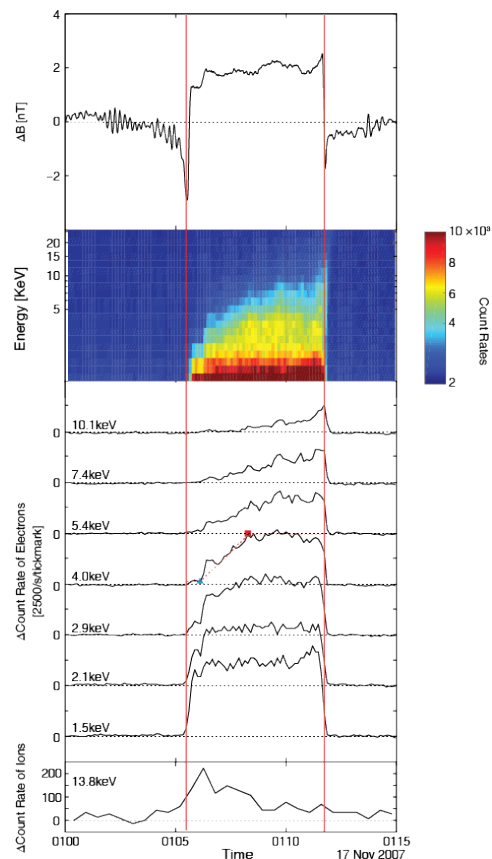
87 In this study, we revisit the detailed structures in count rate distributions of
88 ions/electrons at each energy channel available, and fit these structures with our new
89 method that incorporates more physics to determine the return speed u , the L_S and
90 age of the empty flux tubes on detection. The data we used is described in Section 2;
91 model and results are shown in Section 3; and Section 4 concludes this study with a
92 discussion of the results.

93 **2. Data**

94 Three data sets are used in this study: Magnetic field data of one-second
95 resolution measured by the Cassini magnetometer (MAG) [Dougherty et al., 2013], 4-
96 second resolution electron spectrometer (ELS) data (energy range from 0.58eV to
97 26.04keV) and 16-second resolution ion mass spectrometer (IMS) data (energy range
98 from 1.19eV to 46.34keV) from Cassini plasma spectrometer (CAPS) [Young et al.,
99 2004].

100 Figure 1 shows the magnetic field and plasma count rate during the same event
101 studied by Paranicas et al. [2016] (this paper is referenced as Paranicas16 hereafter).
102 Since the measurement was obtained on 17 November 2007, we name it the Nov07
103 event hereafter. Although we use a lower energy dataset, a “straight-back blade”
104 shape similar to Figure 1 in Paranicas16 is shown in the time-energy spectrum of
105 Figure 1. In the panels below, the effect of differential drifting between particles
106 conducting gradient and curvature drifts shows up in each energy channel of the
107 electron count rate: From bottom to top, the shape of the count rate inside the tube
108 transforms gradually from a rectangle for relatively low energy electrons to a
109 trapezoid, and eventually into a triangle for high energy electrons. The trapezoid-
110 shaped profiles are visible from 2.1 keV to 5.4 keV. As an example, a blue triangle

111 and a red square are added to mark the slanted side along the profile of 4keV
 112 electrons. The two points are defined at the two vertices of the best fitted trapezoid
 113 (red dashed line), whose two parallel sides are parallel to the x-axis. Both vertices are
 114 inside the flux tube, to be consistent with our theory. The final result is found not
 115 sensitive to the location of these two points, so some points can be estimated
 116 manually. As energy increases, drifting enhances, so the slanted side moves to the
 117 right, and the slope decreases, until the 10.1keV energy channel, at which the top spot
 118 (red) touches the boundary of the flux tube, leaving a triangular shape in the flux tube.
 119 The same evolution of these shapes is found in the ion distribution during other events
 120 (not shown). At the boundary of the flux tubes, the sharp changes in magnetic field
 121 only affects the counts of electrons with energy below $\sim 700\text{eV}$ (not shown), and thus
 122 this effect is not discussed further.



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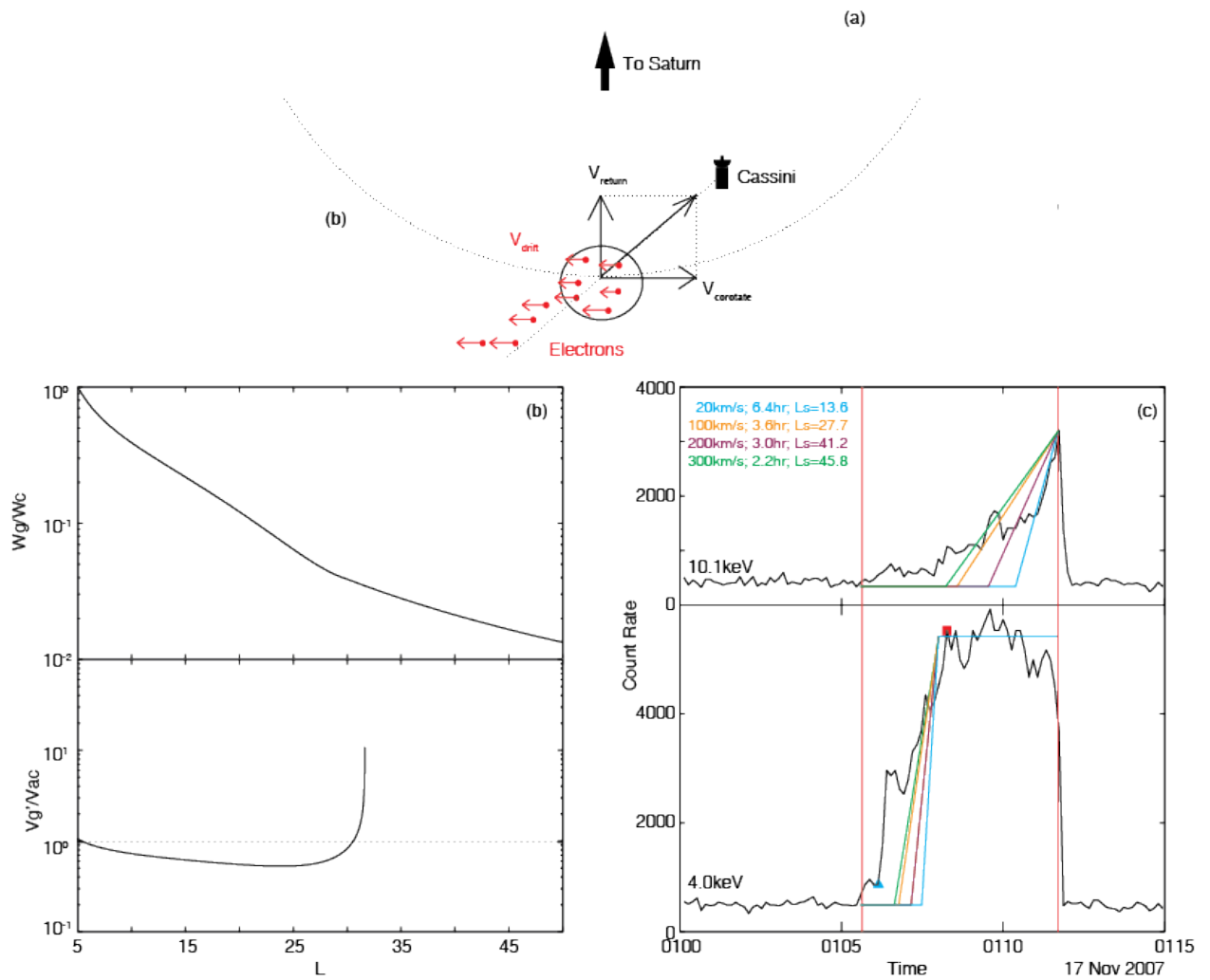
125 Figure 1. Detrended magnetic field and plasma profiles during the event on 17
126 November 2007 [Paranicas et al., 2016]. The two vertical red lines mark the boundary
127 of the flux tube that is empty of water group ions and has “floated” inward from the
128 tail. The sinusoidal oscillations in the top panel are ion cyclotron waves induced by
129 water group ions added by the Enceladus plume. This event was detected at a Saturn-
130 centric distance of 5.9Rs, a latitude of 3.19°, and a local time of 21.03 Hours.

131 3. Model and results

132 In Paranicas16, only gradient drift is considered, assuming that particles with the
133 same energy drift at the same speed. This is inconsistent with the observed trapezoid-
134 shaped electron counts, which requires multiple drift motions to elongate the returning
135 flux rope. In a curved magnetic field, a charged particle naturally drifts at a speed
136 combining two different mechanisms: Gradient drift V_g and curvature drift V_c :

$$137 \quad V_g = W_{\perp} \frac{\mathbf{B} \times \nabla \mathbf{B}}{qB^3}, \quad V_c = 2W_{\parallel} \frac{\mathbf{B} \times \nabla \mathbf{B}}{qB^3}, \quad (1)$$

138 where W_{\perp} is the energy of the particle thermal velocity component perpendicular to
139 the ambient magnetic field, and W_{\parallel} is the parallel energy [Kivelson and Russell,
140 1995].



141

142 Figure 2. (a) A cartoon illustrating the combination of drift motion of electrons and
 143 the return motion of the flux tubes. (b) Comparisons of curvature and gradient drift
 144 motions in a stretched magnetosphere: Top panel plots the ratio of the energies related
 145 to different drift motions, while bottom panel plots the calculated ratio between drift
 146 speeds V_g'/V_{ac} , with the dashed line marking unity. (c) Modeled electron counts at
 147 different return speeds, u , of the flux tube (color-coded) compared to the
 148 observations: upper panel shows time series for 10.1 keV electrons, while lower panel
 149 shows result for 4.0 keV electrons.

150 In Saturnian magnetosphere, for the electrons, both drifts are in the direction
 151 against the corotating plasma. The relative motions of the flux tube and electrons are
 152 summarized in Figure 2(a). To determine the return history and match the observed
 153 curve of count rate, it is necessary to figure out the leading drift motion, which

154 depends not only on the corresponding energy, but also on the bouncing process. For
155 a particle with energy $W_{\parallel}=W_{\perp}$, as can be seen from equation (1), its instantaneous
156 curvature drift speed V_c is twice the gradient drift speed V_g at the equator. However,
157 the bounce-averaged “pure” curvature drift speed (V_{ac}) for 0° pitch angle particle is
158 about 0.7 times the “pure” gradient drift for 90° pitch angle in a dipole field [Hamlin
159 et al. 1961].

160 The real magnetosphere field that returning particles drift in is usually
161 significantly from a dipole field [Kivelson and Russell, 1995]. To assess the effect of
162 the stretched field on the drift process, we run a steady-state global
163 magnetohydrodynamic (MHD) simulation of Saturn’s magnetosphere using the
164 BATS-R-US code, with 0° magnetic declination [Connerney et al., 1982] and similar
165 mass loading as presented by Jia and Kivelson [2012], but on a coarse Cartesian grid,
166 to generate a set of stretched magnetic field lines stretched by the fast rotating
167 planet (also see Appendix). From observations, the field strength inside the flux tube
168 is about 4.5% increase over the background [Lai et al., 2016]. Therefore, it is safe to
169 use the background field strength and curvature offered by this model in the following
170 calculation.

171 At L_d , where the event is detected, electrons in the same energy channel are
172 assumed to have the same energy $W(L_d)$. However, when the return process is traced
173 backward, the inferred energy $W(L)$ of those electrons varies depending on their
174 pitch angle $\theta(L)$. In the equatorial plane, the estimated loss cones of particles are less
175 than 10° in this part of the Saturnian magnetosphere, so it is safe to neglect the loss
176 cone effect and investigate a $v = v_{\perp}$, or “perpendicular” (90° pitch angle) case and an
177 approximately $v = v_{\parallel}$, or “perpendicular” (90° pitch angle) case as two extremities.
178 In addition, the calculated flux tube return process where the drifts take place lasts at
179 least several hours, which is much slower than the gyroperiod and bounce periods of
180 electrons of at most several minutes. Therefore, we can assume that the first adiabatic
181 invariant is conserved for “perpendicular” case during its relatively slow gradient

182 drift, and the second adiabatic invariant is conserved for the “parallel” case during its
 183 curvature drift, respectively. With the calculated known field strength and geometry
 184 using our magnetospheric MHD model, the variation of “perpendicular”/“parallel”
 185 (W_g/W_c) energy as a function of the radial distance is shown in top panel of Figure
 186 2(b): in order to end up with the same energy at L_d , a “parallel” particle needs to start
 187 with and always keep at a higher energy: $W_c(L) > W_g(L)$.

188 To estimate V_{ac} , the bounce averaged curvature drift speed, in the stretched
 189 magnetic field, we launch test particles with different pitch angles θ from 0.1° to
 190 85.1° at the equator, with a step size of 5° , at an L shell. For each of such particles i ,
 191 following the procedures in Hamlin et al. [1961], we calculate the coordinates of the
 192 pair of mirror points. Between this pair of mirror points, we integrate the angular
 193 displacement Φ resulting from both gradient and curvature drift in a bounce period
 194 (T_b) to obtain the bounce-averaged angular drift speed $\omega_{ai} = \Phi(\theta_i, L)/T_b(\theta_i, L)$
 195 using equation (1). In addition, to calculate the extreme case of parallel electrons, we
 196 then extrapolate the drift speed ratio $R(\theta, L) = \frac{\omega_a(\theta, L)L}{V'_g(L)}$ to $\theta=0^\circ$ to obtain $R(0^\circ, L) =$
 197 $\frac{V_{ac}(L)}{V'_g(L)}$. Here V'_g is the averaged gradient drift speed. As an example, at $L=5$, where the
 198 magnetic field is almost dipolar, this ratio is $R(0^\circ, 5) = 0.8$, close to the 0.7 value in a
 199 pure dipole magnetic field [Hamlin et al. 1961].

200 We repeat such θ extrapolation for each of the integer L between 5 and 40, and
 201 the variation of $\frac{V'_g}{V_{ac}}$ ratio as a function of L is shown in the bottom panel of Figure
 202 2(b). Inside $\sim 29R_s$, due to the higher energy of the “parallel” particles, V_{ac} is slightly
 203 larger than V'_g . However, close to the tail reconnection X-line where the magnetic
 204 field lines are highly stretched, the local radius of curvature is huge off the equator,
 205 decreasing V_{ac} to almost zero. Therefore, beyond $29R_s$, only perpendicular particles
 206 undergo significant gradient drift motions. The ratio $\frac{V'_g}{V_{ac}}$ changes from below to
 207 above unity implies that depending on the L_s location, the leading particles in drift

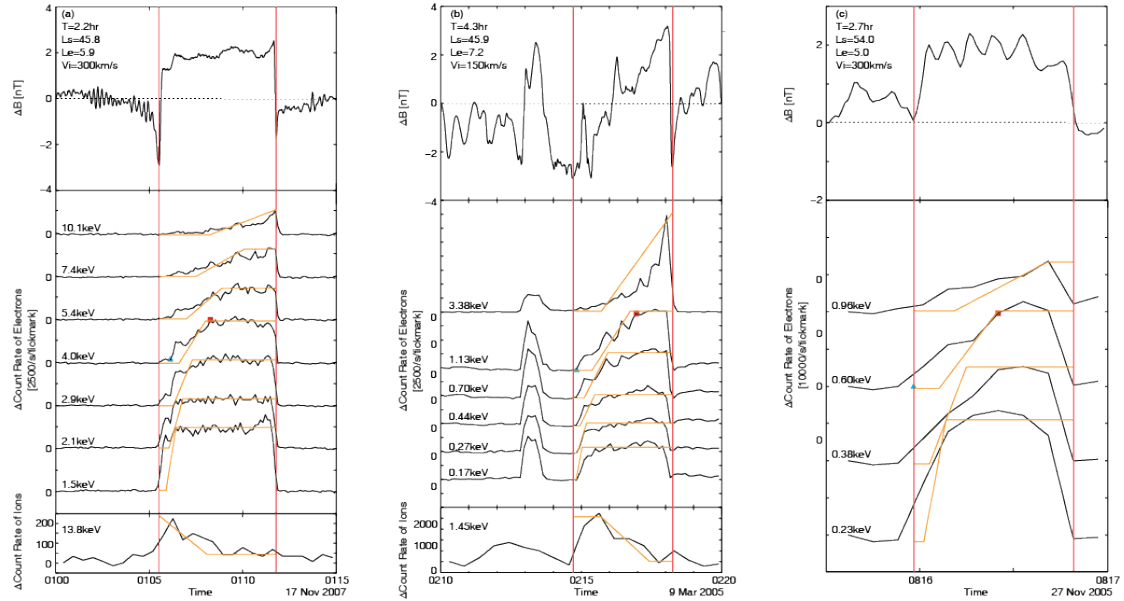
208 motions can be either parallel or perpendicular ones: If $L_s < 29$, “parallel” particles
209 dominated by curvature drift always drift fastest and would be the first to get out of
210 the flux tube while perpendicular particles drift the slowest and would be the ones
211 closest to the “trailing” boundary; while if L_s is well beyond 34, perpendicular
212 particles will lead the drift. Between the two extreme pitch angles, particles with
213 intermediate pitch angles drift at a speed between V_{ac} and V_g' , to form the slope in
214 count rates.

215 Based on the Magnetosphere Imaging Instrument (MIMI) data set, Paranicas16
216 defined a critical energy, with which electrons gradient drift a distance just equals the
217 diameter of the flux tube. In our model, when curvature drift is also considered, in
218 addition to the distance where fastest electrons drift (red square in Figure 1), the
219 distance corresponding to slowest electrons (blue triangle in Figure 1) also sets a
220 constraint. In this situation, the L_s and age of the flux tube can be determined
221 without comparing the PSD profile. The Nov07 event is used to illustrate our
222 procedure and the results are shown in Figure 2(c). The duration of this Nov07 event
223 is 363.6s, and an 80% sub-corotating speed gives a diameter of the flux tube of $d_0 =$
224 1.77×10^7 m. First, we assume several different averaged return speeds u (km/s).
225 We then examine the 10.1 keV electrons whose top of the slope reaches the right-
226 hand side boundary, indicating that the faster particles at that energy have drifted
227 entirely out of the flux tube. Two electrons with energy $W_g = 10.1\text{keV}$ and $W_c =$
228 10.1keV are released at $L_d = 5.9$. A 1-second time step is chosen to trace the
229 electrons backward: at i th second, the two electrons are at $L = L_d + iu/R_s$ with
230 drifting speed $V_g'(L)$ and $V_{ac}(L)$, respectively. This tracing process stops when the
231 faster electron has crossed the distance of d_0 and the corresponding travel time T is
232 then the age of the flux tube and $L_s = L_e + uT$. For each travel time T , the drift
233 distance d of the slower particles can also be integrated. The results are compared in
234 Figure 2(c): with 200km/s and 300km/s injection speeds and L_s beyond 40, the
235 top of the slope corresponds to the gradient drift (perpendicular particles), while the

236 bottom of the slope corresponds to the curvature drift (parallel particles). In
237 comparison, with 20km/s and 100km/s injection speeds and L_s inside 29,
238 parallel particles with curvature drift move faster. The best fit is defined as the
239 shortest distance to both the fastest moving point (red square in Figure 1) and the
240 slowest one (blue triangle in Figure 1). In the Nov07 case, a 300km/s injection
241 speed with a $L_s = 45.8$ and travel time $T = 2.2\text{hr}$ returns the best fit. It should be
242 noted that for simplification, the slope as a straight line is a special case of the count
243 curve, which relies on the initial pitch angle distribution of the particles.

244 Two more events detected near equator with well-defined slopes are studied, as
245 shown in Figure 3. The best fit is obtained with an injection speed of 150km/s and
246 300km/s, respectively. In both cases, the perpendicular particles under gradient drift
247 move faster than the parallel ones. Figure 3(a) is, again, the Nov07 event, with all
248 energy channels modeled. As shown, all profiles can be fitted well with a constant
249 velocity.

250 During the returning process of these flux tubes, the evolution of their plasma
251 contents should disturb the magnetic field. Hence, Lai et al. [2016] have identified the
252 flux tubes with “well defined” magnetic field signature as younger events. Compared
253 with the remaining 700 such events, the magnetic field profile during this Nov07
254 event is among the most well-defined [Paranicas et al., 2016], indicating a younger
255 flux tube by the above standard. In this study, our calculated travel time, T , shows that
256 it is the youngest among the three events shown in Figure 3, consistent with the age
257 predictions by Lai et al. [2016]. On the other hand, the case in Figure 3(b) is judged to
258 be “old” from the magnetic field data [Lai et al., 2016], and its modeled travel time is
259 also the longest in this study. Such consistency obtained with two independent
260 methods studying two different data sets strongly supports our picture of particle drift
261 during the flux return processes. We note that the case in Figure 3(c) lasts only 1
262 minute, and the ion data are not shown due to their time resolution of 32 seconds
263 being too low to judge the degree of fitting with our model.



264

265 Figure 3. Comparisons of modeled distributions of electrons and ions against
 266 Cassini measurements during three flux tube events. All orange lines are modeled by
 267 assuming a constant return speed varies from 150km/s to 300km/s.

268 4. Discussion and conclusions

269 The Nov07 event was previously studied by Paranicas16, considering an upper
 270 cutoff energy (defined as the energy with which electrons can drift out of the flux tube
 271 completely) of 60keV and a return speed of $u = 18\text{km/s}$. With only the gradient
 272 drift considered, the starting L-shell and travel time are found to be $L_s = 7.2$ and
 273 $T = 1.2\text{hr}$, respectively. In addition, the 9 March 2005 event shown in Figure 3(b)
 274 has also been studied in Paranicas16, to arrive at an upper cutoff energy at 27keV
 275 and return speed of $u = 18\text{km/s}$; a starting L shell of $L_s = 8.55$; and a travel time
 276 of $T = 1.3\text{hr}$.

277 We note that the method adopted by Paranicas16 to fit the whole energy
 278 spectrum may bring in a large uncertainty above or below their line fit. Depending on
 279 the chosen dataset and the range of color code, the determined upper cutoff energy
 280 can be six times smaller (second panel in Figure 1). This artificial bias can be
 281 excluded if the count curves are employed as proceeded in our model. Here, with both
 282 drifts considered, fits the count curve of relatively lower energy particles ($<\sim 10\text{keV}$)

283 and returns a set of much larger L_s and longer travel times. Another major difference
284 is that Paranicas16 method only calculated the gradient drift but use that drift distance
285 to fit the counterpart of the blue triangle in Figure 1, which is turned out to be
286 completed by curvature drift, as determined with our method. In addition, the data that
287 we use are at the lower end of the energy spectrum and consist of higher counts,
288 associated with lower statistical errors.

289 We acknowledge our significant difference in both method and result with
290 Paranicas16. Their L_s is determined by matching the phase space density (PSD) of
291 the particles inside the injection with the radial profile of the nominal phase space
292 density. We note that there are two over-simplifications in Paranicas16's assumption:
293 First, in their Figure 4, the PSD line is assumed to grow monotonically with L .
294 However, this is inconsistent with the figure. In the same figure, the slope of the
295 averaged PSD line decreases toward 0 as L increases. Hence, we conclude that with
296 Paranicas16's analysis, greater L origin of the flux tubes are inherently excluded with
297 this method. Second, at the location of detection in the inner magnetosphere,
298 including the range of L shells plotted in their Figure 4, the background plasma
299 surrounding the flux tubes is magnetospheric plasma constituting the plasma torus. In
300 contrast, as stated by Hill [2016], the plasma inside the returning flux tube is probably
301 the planetward exhaust from tail reconnection, which should be the entrance of
302 external plasma [Vasyliunas 1983]. There is no reason for the PSD of the torus
303 plasma to match the PSD of solar wind plasma: Searching for a matched PSD of a
304 stream of particles from an external source in the averaged PSD of Saturn's torus
305 particles may not return where the external particles are launched. Therefore, we
306 conclude that what Paranicas16 have found is not necessarily the L_s of flux tube
307 origin. Instead, our method does not have these problems.

308 Our high return velocity from 150km/s to 300km/s is reasonable, as Lai et al.
309 [2016] find that the return speed of flux tubes should be about 200km/s at $L=10$, and
310 can be up to 80km/s at $L=5$ by balancing the reconnection rate with the return
311 magnetic flux.

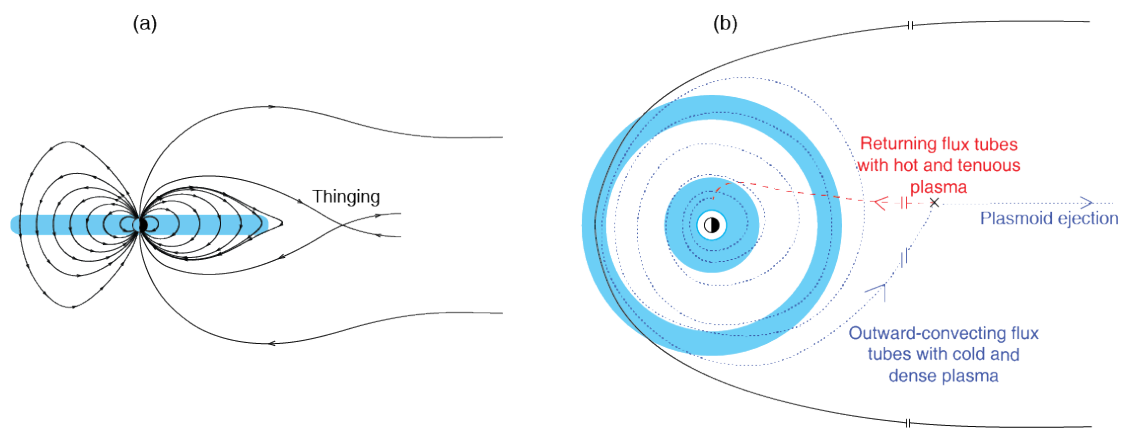
312 As Figure 2(b) shows, the ratio of perpendicular to parallel energy at the equator
313 varies with L. Therefore, the pitch angle distributions at L_d does not necessary
314 resemble their original pitch angle and energy at L_s , which are not available in any
315 observations. We did not follow a full pitch angle range of particles from L_s to L_d
316 in a particular flux tube. Instead we only use the two extreme points of the curve to
317 determine L_s , as particles of close- to- 0° pitch angle and 90° pitch angle at the
318 equator will retain the same drift angle at the same latitude, unless disturbed in a non-
319 adiabatic process. On the other hand, the count of these two extreme types of particles
320 defines the two ends of the line fit, and thus can be compared with the modeled results
321 directly using our method, without requiring the shape of the curve in between, or the
322 unconstrained pitch angle distribution at L_s .

323 Due to the fluctuations and the time-resolution of the data, the two points on the
324 curve of count rate cannot be determined unambiguously. However, as we can see in
325 Figure 2(c), even a slow return speed gives a L_s beyond 10, which is the key
326 conclusion of this study. This is also true in the other two cases. Therefore, our
327 method is not sensitive to the exact location of the two extreme points.

328 All starting L shells L_s that we found are greater than 45, when we use the
329 stretched dipole field. This L_s location is way outside the plasmopause. Rather, all our
330 L_s values are consistent with a direct release from the tail reconnection region.
331 Although the magnetospheres of the gas giants are largely internally driven, the solar
332 wind plays a major role in shaping the magnetic field in distant regions away from the
333 planets [e.g., Arridge et al., 2008]. However, because age $T = R_s \int \frac{dL}{u}$, the relatively
334 slower travel speed in the inner magnetosphere where the field is less influenced by
335 the solar wind contributes more to the total age, so even if there is a warped dynamic
336 magnetotail different from out stretched dipole, we will still arrive at L_s is not in the
337 inner magnetosphere, but close to the tail reconnection site. Similarly, in the Jovian
338 magnetosphere, the non-axisymmetry of the internal field relative to the rotation axis
339 [Russell and Dougherty, 2010 and references therein] is expected to make the

340 geometry more complicated, but this key conclusion of direct return will not change
 341 qualitatively. On the other hand, since the return process consists of a segment in the
 342 mid tail, estimating the local time of release at L_S still faces uncertainties and will
 343 not be discussed here.

344 In our simplified model, to figure out the leading drift motion, we use
 345 extrapolated results of “pure” parallel particles. The existence of a finite loss cone
 346 limits the real curvature drift to be slower than that of the “pure” parallel extremity.
 347 However, in the region where L_S locates, the faster particles are under gradient drift
 348 motion as shown in Figure 2(c). Therefore, our conclusion of direct return still holds.



349

350 Figure 4. Illustration of the time-integrated circulation of magnetic flux in the
 351 Saturnian magnetosphere, indicated by our result. The blue ring is the neutral torus
 352 from Enceladus. (a) Black lines are magnetic field lines drawn until around the tail
 353 reconnection site in side view. (b) The black line marks the magnetopause. The blue
 354 dotted curve sketches the radially slow convection of plasma-loaded magnetic flux
 355 tubes spinning outward into the tail. The red dashed line indicates the return of
 356 plasma-depleted magnetic field tubes directly back to the inner magnetosphere at a
 357 higher radial speed. This red line may not end at the dawn side, but is not expected to
 358 wind multiple circles as indicated by the age estimation.

359 We have applied our model to another 10 cases with both magnetic field and
 360 plasma data available. Their starting locations are all found to be $L_S \gg 10$. This result

361 supports the hypothesis that empty flux tubes generated by reconnection in the far-tail
362 region return directly into the inner magnetosphere. On the other hand, we suspect
363 that even if there is a plasma boundary at around $10R_S$, the effective viscosity may not
364 be sufficient to perturb the return motion significantly. Slowing-down of the returning
365 flux tubes is expected only when mass-loading processes become significant, i.e.,
366 inside $6R_S$. A cartoon summarizing our interpreted circulation of magnetic flux in the
367 Saturnian magnetosphere is shown in Figure 4. On the other hand, however, a lot of
368 events studied by Thomsen et al. [2015] and Thomsen and Coates [2019] are not
369 selected based on their magnetic signatures like we did, but on their plasma signatures
370 (second panel in Figure 1), and, therefore, are not included in our study. This different
371 data pool may be a reason for our different conclusions. We leave such distinction to
372 future studies with a new criterion that may accommodate all available data.

373 In summary, we present case studies to model the fine structures in the particle
374 profiles observed by Cassini during its flux tube encounters. Our new method not only
375 incorporates more physics, but also yields results that are consistent with both the age
376 judged by magnetic measurements, and the velocity estimated by outflow speed.
377 More sophisticated models of magnetic field and plasma background, including
378 numerical simulations, should be applied to improve the accuracy of such studies.
379 Last, the method and understandings we applied to Saturn's magnetosphere should
380 also be applicable to observations of the Jovian magnetosphere obtained by Galileo
381 [Kivelson et al., 1997], Juno [Haggerty et al., 2019], and future missions to the Jupiter
382 system, to determine the circulation of plasma and magnetic flux. For the ice giant
383 planets and exoplanets with limited or no in situ observations available, we believe
384 that such a process may be the best paradigm for global circulation estimates.

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392 The return flux tube events are chosen from the list in Lai et al. [2016].

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490 **Appendix: The Saturn model that generates the stretched field lines**

491 The following MHD equations are used in this model:

$$492 \quad \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = \rho_s \quad (\text{A1})$$

$$493 \quad \rho \frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla P - en(\mathbf{u} \times \mathbf{B}) + \mathbf{M}_s \quad (\text{A2})$$

$$494 \quad \frac{\partial P}{\partial t} + (\mathbf{u} \cdot \nabla) P + \gamma P(\nabla \cdot \mathbf{u}) = E_s \quad (\text{A3})$$

$$495 \quad \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) \quad (\text{A4})$$

496 where e is the electric charge on an electron; ρ , \mathbf{u} , and P is the mass density,
497 bulk velocity vector and thermal pressure of the plasma, respectively. Variable $n =$
498 ρ/m is the number density of protons, m is proton mass, and we neglect the electron
499 momentum. \mathbf{B} is the magnetic field vector, and the mass, momentum, and energy
500 source terms (ρ_s , \mathbf{M}_s , E_s) follow the same used by Jia et al. [2012].

501 Equations (A1-A4) are solved numerically using the Michigan BATS-R-US code
502 [Tóth et al., 2012], on a $204.8 \times 102.4 \times 102.4 R_s$ Cartesian grid, with finest resolution
503 $0.1 R_s$. A steady state solution is reached, with the following solar wind conditions
504 launched at the $x = 51.2 R_s$ upstream boundary: $n = 0.0081/\text{cc}$, $T = 10^4 \text{K}$, $U =$
505 $U_x = 615 \text{km/s}$, $\mathbf{B} = (0.00594, 0.00173, 0.00063) \text{nT}$. Saturn's dipole tilt is
506 assumed to be 0. Ionospheric inner boundary is assumed at $r = 3.5 R_s$, where thermal
507 pressure and magnetic field has 0 gradient (float), plasma velocity is 0 at the inner
508 boundary interface, and density is set to constant $n_{\text{in}} = 70/\text{cc}$.