Supplementary material

A dynamic flotation model for predictive control incorporating froth physics. Part II: Model calibration and validation

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Tables

Note: In both tables, i refers to the number of mineralogical classes, while K refers to the number of bubble size classes in the pulp phase. A detailed model development is presented in Part I of this paper.

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Table 1: Variables of the model, nomenclature, units, and total number of variables of the dynamic model.

No.	Variable	Symbol	Units	Number of variables
1	Air recovery	α	$-$	1
2	Axial dispersion	D_{axial}	$-$	1
3	Bubble size in the froth (overflowing)	$d_{b,froth}$	m	1
4	Bubble surface area flux	S_b	$1/s$	1
5	Bursting rate	v_b	m/s	1
6	Concentrate flowrate	Q_{conc}	m^3/s	1
7	Control valve output for the tailings flowrate	u_v	$-$	1
8	Entrainment factor	ENT_i	$-$	i
9	Feed flowrate	Q_{feed}	m^3/s	1
10	Flotation rate constant	k_i	$1/s$	i
11	Froth depth	h_f	m	1
12	Froth recovery	$R_{f,i}$	$-$	i
13	Froth velocity over the lip	v_f	m/s	1
14	Froth-phase air recovery	α^*	$-$	1
15	Gas flowrate into the cell (total)	$Q_{air,in}$	m^3/s	1
16	Gas flowrate for each bubble size class	$Q_{air,in}^k$	m^3/s	K
17	Gas free pulp height	h_0	m	1
18	Gas hold-up for each bubble size class	ε_0^k	$-$	K
19	Gas velocity out of the pulp for each bubble size class	$v_{g,\text{out pulp}}^k$	m/s	K
20	Gas volume of each bubble size class in the pulp phase	V_{gas}^k	m^3	K
21	Head grade (mineral concentration)	$C_{i,f}$	kg/m^3	i
22	Interfacial gas rate	v_g^*	m/s	1
23	Length of Plateau borders per volume of foam	λ_{out}	$1/m^2$	1
24	Mean bubble size in the interface	$d_{b,int}$	m	1
25	Particle size per mineralogical class	$d_{p,i}$	m	i
26	Physical parameter	k_1	m/s	1
27	Proportion of bubble size classes in the pulp	$\Psi_{d_{b,pulp}}^k$	$-$	K
28	Pulp flowrate out of the cell	$Q_{pulp,out}$	m^3/s	1
29	Pulp height	h_p	m	1
30	Pulp volume in the cell	V_{pulp}	m^3	1
31	Residence time in the froth	τ_f	s	1
32	Settling velocity	v_{set}	m/s	i
33	Slurry content	ϵ	$-$	1
34	Slurry density	ρ_{pulp}	kg/m^3	1
35	Slurry viscosity	μ_{pulp}	kg/ms	1
36	Solid concentration in the tailings	$C_{i,tailings}$	kg/s	i
37	Solid flowrate in the concentrate due to entrainment	$m_{i,ENT}$	kg/s	i
38	Solid flowrate in the concentrate due to true flotation	$m_{i,TF}$	kg/s	i

Table 1 continued from previous page

39	Solid mass flowrate in the feed	$m_{i,feed}$	kg/s	i
40	Solid mass flowrate in the tailings	$m_{i,tailings}$	kg/s	i
41	Solid mass in the pulp phase	M_i	kg	i
42	Superficial gas velocity	j_g	m/s	1
43	Tailings flowrate	$Q_{tailings}$	m^3/s	1
44	Total gas hold-up	$\varepsilon_{0,total}$	$-$	1
45	Total gas velocity out of the pulp	$v_{g,\text{out pulp}}^{total}$	m/s	1
46	Total gas volume in the pulp phase	V_{gas}	m^3	1

Total number of variables $29 + 12\,i + 5\,K$

Table 2: Model equations for each variable, including their classification. The last column refers to the total number of equations for each variable.

Variable	Equation	Type of equation	Number of equation(s) i
Conservation of mass	$\frac{dM_i}{dt} = m_{i,feed} - m_{i,tailings} - m_{i,TF} - m_{i,ENT}$	Phenomenological	i
	Pulp phase		
Air flowrate of each bubble size class	$Q_{air,in}^k = Q_{air,in}\,\Psi_{d_{b,pulp}}^k$	Semi-empirical	K
Flowrate out of the flotation tank	$Q_{\text{pulp,out}} = Q_{\text{tailings}} + Q_{\text{conc}}$	Phenomenological	1
Gas holdup	$\dfrac{d\left(\frac{\varepsilon_0^k}{1-\varepsilon_{0,\text{total}}}\right)}{dt} = \dfrac{1}{h_0}\left(\dfrac{Q_{air,in}^k}{A} - v_{g,\text{out pulp}}^k \varepsilon_0^k\right)\left(\dfrac{\varepsilon_0^k}{1-\varepsilon_{0,\text{total}}}\right)$	Phenomenological	K
Gas volume of each bubble size class	$V_{gas}^k = \dfrac{\varepsilon_0^k}{1-\varepsilon_0^k}h_0 A_{cell}$	Phenomenological	K
Gas-free pulp height	$\dfrac{dh_0}{dt} = \dfrac{Q_{feed}}{A_{cell}} - \dfrac{Q_{pulp,out}}{A_{cell}}$	Phenomenological	1
Proportion of each bubble size class	$\Psi_{d_{b,pulp}}^k = \dfrac{(\text{frequency bubble size class})_k}{\sum_{k=1}^K (\text{frequency bubble size class})_k}$	Empirical	K
Pulp density	$\rho_{\text{pulp}} = \phi\rho_{\text{sol}} + (1-\phi)\rho_{\text{water}}$	Phenomenological	1
Pulp height	$h_p = \dfrac{h_0}{1-\varepsilon_{0,\ \text{total}}}$ ODE form (combining Eqs. for h_0 and gas holdup): $\dfrac{dh_p}{dt} = \dfrac{1}{A_{cell}}\left(Q_{feed} - Q_{pulp,out}\right)\left(\dfrac{1}{1-\sum_{k=1}^K \varepsilon_0^k}\right) + \dfrac{h_0}{\left(1-\sum_{k=1}^K \varepsilon_0^k\right)}\sum_{k=1}^K \dfrac{d\varepsilon_0^k}{dt}$	Phenomenological	1
Pulp viscosity	$\mu_{\text{pulp}} = \mu_{\text{water}}\exp\left(\dfrac{2.5\phi}{1-0.609\phi}\right)$	Empirical	1
Pulp volume (liquid + gas)	$V_{pulp} = h_0 A_{cell} + V_{gas}$	Phenomenological	1
Solid mass flowrate in the feed	$m_{i,feed} = C_{i,f}Q_{feed}$	Phenomenological	i
Solid mass flowrate in the tailings	$m_{i,tailings} = C_{i,tailings}Q_{tailings}$	Phenomenological	i
Superficial gas velocity	$j_g = Q_{air}A_{cell}$	Phenomenological	1
Tailings particle concentration	$C_{i,tailings} = \dfrac{M_i}{V_{pulp}}$	Phenomenological	i
Total gas holdup	$\varepsilon_0^{total} = \sum_{k=1}^K$	Phenomenological	1
Total gas volume in the pulp phase	$V_{gas} = \sum_{k=1}^K V_{gas}^k$	Phenomenological	1
Total upward gas velocity	$v_{g,\text{out pulp}}^{\text{total}} = \sum_{k=1}^K v_{g,\text{out pulp}}^k \varepsilon_0^k$	Semi-empirical	1
Upward gas velocity	$v_{g,\text{out pulp}}^k = \dfrac{g\rho_{\text{pulp}}(d_{b,pulp}^k)^2}{18\mu_{\text{pulp}}(1-\varepsilon_0^k)^{1.39}}$	Semi-empirical	K
Volumetric fraction of solids	$\phi = \dfrac{\rho_{\text{solids}}}{\rho_{\text{water}}-\rho_{\text{solids}}+\frac{\rho_{\text{solids}}}{\Phi_{\text{solids}}}}$	Phenomenological	1

Table 2 continued from previous page

Variables	Equations	Classification	Number of equation(s)
	Pulp-froth interface		
Interfacial bubble size	$d_{b,int} = \dfrac{\sum_{k=1}^{K} v_{gas,out\,pulp}^k \varepsilon_0^k}{\sum_{k=1}^{K} \frac{v_{gas,out\,pulp}^k \varepsilon_0^k}{d_{b,pulp}^k}}$	Phenomenological	1
Interfacial gas velocity	$v_g^* = \dfrac{dh_p}{dt} + v_{g,\,out\,pulp}^{total}$	Phenomenological	1
	Froth phase		
Air recovery	$\alpha = \dfrac{v_f t_{lip} h_{over}}{Q_{air,in}}$	Phenomenological	1
Air recovery as a function of busting rate	$\alpha^* = \dfrac{v_g^* - v_b}{v_g^*}$	Phenomenological	1
Axial dispersion	$D_{axial} = \dfrac{j_g^{1.5}}{\sqrt{k_1 (\sqrt{3}-\frac{\pi}{2})Pe}}$	Phenomenological	1
Bubble surface area flux	$S_b = \dfrac{6 v_g^*}{d_{b,int}}$	Phenomenological	1
Bursting rate	$v_b = a + bj_g + cj_g^2$	Semi-empirical	1
Concentrate flowrate	$Q_{conc} = \left\{ \begin{array}{l} \frac{A_{cell} v_g^{*2} \lambda_{out}(1-\alpha^*)\alpha^*}{4k_1} \text{ if } \alpha \geq 0.5 \\ \frac{A_{cell} v_g^{*2} \lambda_{out}}{A_{cell} v_g^{*2} \lambda_{out}} \frac{k_1}{} \text{ if } \alpha < 0.5 \end{array} \right.$	Phenomenological	1
Entrainment factor	$ENT_i \approx \left\{ \begin{array}{l} \exp\left(\frac{-v_{set,i}^{1.5} h_f}{D_{axial}\sqrt{v_g^*(1-\alpha^*)}}\right) \text{ if } \alpha \geq 0.5 \\ \exp\left(\frac{-2 v_{set,i}^{1.5} h_f}{D_{axial}\sqrt{v_g^*}}\right) \text{ if } \alpha < 0.5 \end{array} \right.$	Phenomenological	i
Froth height	$h_f = h_T - h_p$	Phenomenological	1
Froth recovery	$R_{F,i} = \left\{ \begin{array}{l} \left(\frac{\alpha^*(1-\alpha^*)v_g^*}{v_{set,i}}\right)\frac{f}{2}\left(\frac{d_{b,\,int}}{d_{bfroth}}\right)^f \text{ if } \alpha \geq 0.5 \\ \left(\frac{v_g^*}{v_{set,i}}\right)\frac{f}{2}\left(\frac{d_{b,\,int}}{d_{bfroth}}\right)^f \text{ if } \alpha < 0.5 \end{array} \right.$	Phenomenological	i
Froth residence time	$\tau_f = \dfrac{h_f}{v_g^*}$	Phenomenological	1
Length Plateau border out of the froth per volume	$\lambda_{out} = \dfrac{k_\lambda}{d_{bfroth\,out}^2}$	Phenomenological	1
Overflowing froth bubble size	$ODE: \dfrac{d}{dt} d_{b,froth} = Cd_{b,froth}^{1-n}$ $Analytical\,solution: d_{bfroth\,out} = \left(nC\tau_f + d_{b,int}^n\right)^{1/n}$	Phenomenological	1
Overflowing froth velocity	$v_f = \dfrac{Q_{concentrate}}{\epsilon l_{lip} h_{over}}$	Phenomenological	1
Physical constant	$k_1 = \dfrac{\rho_{pulp} g}{3C_{PB}\mu_{pulp}}$	Phenomenological	1
Settling velocity	$v_{set,i} = \dfrac{g(\rho_{solid,i} - \rho_{water})d_{p,i}^2}{18\mu_{pulp}} \dfrac{(1-\phi)^{4.65}}{3}$	Phenomenological	i

Table 2 continued from previous page

Variables	Equations	Classification	Number of equation(s)
Slurry content	$\epsilon \approx \{ \begin{array}{l} \frac{v_g^*}{k_1} \left(1 - \alpha^*\right) \lambda_{out} \text{ if } \alpha < 0.5 \\ \frac{v_g^*}{2k_1} \lambda_{out} \text{ if } \alpha \geq 0.5 \end{array}$	Phenomenological	1
Solid mass flowrate in the concentrate due to entrainment	$m_{i,ENT} = Q_{conc} ENT_i C_{tailings,i}$	Phenomenological	i
Solid mass flowrate in the concentrate due to true flotation	$m_{i,\text{true flotation}} = V_{cell} k_i R_{f,i} C_{tailings,i}$	Phenomenological	i
Specific rate constant	$k_i = P_i S_b$	Phenomenological	i
Total number of equations:			$26 + 10i + 5K$