A Fully-Decentralized Consensus-Based ADMM Approach for Component-Wise ACOPF

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***Abstract*—Decentralized active and reactive power coordination of the emerging smart grid with increasing integration of small-scaled distributed energy resources (DER) is becoming more and more challenging. However, the existing optimization techniques either require a central coordinator in their calculation procedure or are not able to realize component-level problem decomposition. In this work, we propose a novel consensus-based ADMM approach, which characterizes a two-level iterative solution procedure and solves the coordination problem in a fully decentralized way. The optimal network-constrained power coordination is achieved through solely the information exchange between neighboring DER and bus agents and completely avoid the need for a central coordinator. The convergence and effectiveness of the algorithm has been validated in case studies using the IEEE xx-bus test system.**

***Index Terms*—Distributed optimization, consensus-based algorithm, ADMM, optimal power coordination, DCOPF.**

NOMENCLATURE

A. Indices and Sets

$t \in T$	Index and set of time periods.
$k \in K$	Index and set of ADMM iterations.
$r \in R$	Index and set of consensus iterations.
$i, i' \in \mathcal{I}$	Index and set of distributed energy resources (DER).
$b, b' \in \mathcal{B}$	Index and set of buses in the network.
$bb' \in \mathcal{E}$	Index and set of lines in the network.
$\mathcal{B}_b$	Set of buses connected to bus b.
$\mathcal{I}_b$	Set of DER connected to bus b.

B. Parameters

$\mathcal{F}_i$	Feasible operation set of DER i.
$\overline{s}_{bb'}$	Thermal capacity of line bb' (xx).
$\theta_{ref,t}$	Angle of the reference bus at period t.
$\underline{V}_b, \overline{V}_b$	mInimum and maximum allowable voltage magnitude at bus b.
$B_{b,b'}$	$(b,b')^{\text{th}}$ element of the susceptance matrix $\boldsymbol{B}$.
$G_{b,b'}$	$(b,b')^{\text{th}}$ element of the xx matrix $\boldsymbol{G}$.
$W^b_{i,i'}$	$(i,i')^{\text{th}}$ element of the communication matrix $\boldsymbol{W}^b$ on bus b.
ϵ^{λ}	Learning gain constants for the update of consensus variables λ, α and β (£/kW^{2}h).
$\varepsilon^{pri}, \varepsilon^{dual}$	Tolerances for primal and dual residuals.
$\varepsilon^{\lambda}, \varepsilon^{e}$	Tolerances for price and power mismatch (kW).

C. Variables

$\boldsymbol{p}^{der}_i$	Vector of active injections $p^{der}_{i,t}$ of DER i (kW).
$\boldsymbol{q}^{der}_i$	Vector of reactive injections $q^{der}_{i,t}$ of DER i (kW).
$\boldsymbol{\theta}_b$	Vector of voltage angle $\theta_{b,t}$ at bus b ().
$\boldsymbol{\theta}^b_{b'}$	Vector of voltage angle $\theta^b_{b',t}$ at bus b' estimated by bus b ().
$\boldsymbol{V}_b$	Vector of voltage magnitudes $V_{b,t}$ at bus b ().
$\boldsymbol{V}^b_{b'}$	Vector of voltage magnitudes $V^b_{b',t}$ at bus b' estimated by bus b ().
$\boldsymbol{p}_{bb'}$	Vector of active power flow $p_{bb',t}$ on line bb' (kW).
$\boldsymbol{q}_{bb'}$	Vector of reactive power flow $q_{bb',t}$ on line bb' (kW).
$s^{sen}_{bb',t}$	Apparent power flow leaving the reference sending bus on line bb' at time period t (kW).
$s^{rec}_{bb',t}$	Apparent power flow reaching the reference receiving bus on line bb' at time period t (kW).
$\boldsymbol{\lambda}^b_i, \boldsymbol{\lambda}^b_b$	Vector of active prices $\lambda^b_{i,t}$ and $\lambda^b_{b,t}$ estimated by DER i and bus b (£/kWh).
$\boldsymbol{\mu}^b_i, \boldsymbol{\mu}^b_b$	Vector of active prices $\mu^b_{i,t}$ and $\mu^b_{b,t}$ estimated by DER i and bus b (£/kWh).
$\boldsymbol{e}^q_i, \boldsymbol{e}^q_b$	Vector of active power imbalances $e^q_{i,t}$ and $e^q_{b,t}$ estimated by DER i and bus b (kW).
$\boldsymbol{e}^q_i, \boldsymbol{e}^q_b$	Vector of active power imbalances $e^p_{i,t}$ and $e^p_{b,t}$ estimated by DER i and bus b (kW).

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I. INTRODUCTION

A. Background and Motivation

IN smart electric power grids, the emerging communication and control technology together with the proliferation of small-scale distributed energy resources (DER) in distribution networks significantly increase the complexity of effective power coordination and stimulate the need for advanced optimization techniques [1]–[3]. The traditional centralized control paradigm [xx]-[xx] although provides the optimal coordination solution, suffers from significant communication and computational scalability challenges as well as privacy concerns by DER owners who are not generally willing to disclose any private information [xx]-[xx]. Thus, they may no longer be appropriate for controlling and managing such a distributed energy system. Distributed control approaches, on the other hand, enable optimal coordination on the basis on solely bilateral information exchange between neighboring agents, thereby perform distributed calculations and avoid the above mentioned limitations. In this context, significant efforts have been made in the existing literature towards the development of distributed control approaches.

The idea of exploiting consensus among neighboring agents has recently attracts continuous interests for solving optimal power coordination problems, which is essentially an optimization problem with the objective of total operational cost minimization (or social welfare maximization). This type of approach aims at reaching an agreement among agents regarding certain quantities associated with the coordination problem, referred to as consensus variables, through an iterative process [4]. In the vast majority of relevant papers, the consensus variables are electricity prices. By applying consensus-based algorithms, authors in [5]–[7] proposed fully decentralized methods to determine optimal power coordination through information exchange solely between dispatchable generators (DG) and in [8]–[10], the controllable DER also includes price-elastic demands. Authors in [11], [12], solved power coordination also with the consideration of line losses.

However, previous work [5]–[12] employing consensus-based algorithms do not take into account the network constraints (i.e. power flow constraints) in the coordination process since the nodal prices, which encompass both energy generation and congestion-related costs, could be unequal for DER located differently in the network due to the presence of line congestion. As a result, DER owners may not be able to agree on uniformed electricity prices across the network, hindering significantly the applicability of the consensus-based algorithms for network-constrained coordination, namely optimal power flow (OPF) problems [13].

In the view of the above challenges, there is an extensive literature on developing optimization methods for the distributed/decentralized solution of OPF problems. Those algorithms are characterized by different decomposition techniques, which separate the original centralized OPF problem to sub-problems with each sub-problem solved by different agents in a sequential or a parallel manner. Depending on different application of interests, those agents, who are responsible to solve sub-problems and control devices in its physical regions, can be large portions of a power system including several buses connected by transmission lines, individual buses connected by multiple distributed resources or individual DER owners. The OPF problem decomposition level correlated to the above three types of agents are classified by regional-level, bus-level and component-level, respectively.

The most well-recognized distributed/decentralized optimization algorithms can be divided into two main categories. The first category is based on Lagrange decomposition, including dual decomposition (DD), analytical target cascading (ATC), alternating direction of multipliers (ADMM) and auxiliary problem principle (APP). DD applied in [14]–[17] constitutes a two-level iterative process. However, as shown by Fig. 1, the second update process of DD requires a central coordinator to collect global information to account for imbalances in supply and demand and drive local decisions to the optimal solution of OPF. Moreover, the convergence of DD is not guaranteed even for convex problems [17].

Authors in [13], [18] solved OPF by applying ATC, which decomposes the OPF into sub-problems solved by agents in different hierarchies iteratively. At each iteration,

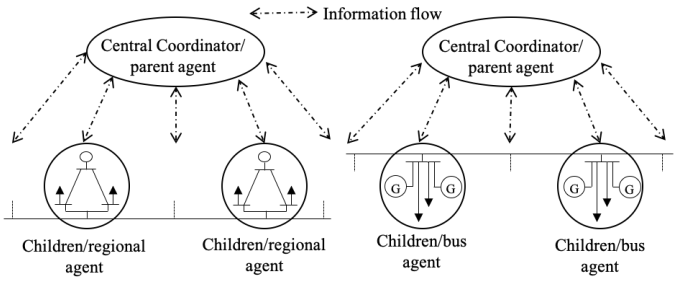

Fig. 1. Type 1 algorithm implementation structure: distributed OPF with the existence of a central coordinator.

the parent-level and children-level sub-problems linked by the shared variables (which in the majority of the literature are voltages and voltage angles associated with nodal power balance equations of two buses connected by tie-lines) are solved sequentially. Then, in [19], two variants of ATC are further developed in order to make the sequential calculation process of ATC into parallel. However, 1) as shown by the implementation structure of ATC in Fig. 1, the parent agent needs to collect copies of shared variables from its children agents to guarantee their equivalence; 2) ATC and its variants may not be able to deal with systems with meshed networks [20].

ADMM, developed by [21], with widespread use constitutes another important methodology framework for solving OPF in a distributed manner. ADMM decomposes the augmented Lagrangian function of the centralized OPF to sub-problems. Then, agents solve their sub-problems independently over sequential iterations with each of them holding copies of the shared variables (which usually are the voltages and voltage angles associated with nodal power balance equations of two buses connected by tie-lines). ADMM characterizes implementation structure shown by Fig. 1, where a central coordinator exists in the second update process to gather the updated shared variables from all agents and coordinate those variables to ensure their identity. ADMM is applied to solve OPF with AC [22], [23] and DC [24], [25] power flow formulation, respectively. Authors in [26] solve DCOPF also taking into the consideration of security constraints. To decompose DC-OPF to the component-level, authors in [27], [28] proposed a variant of ADMM, however, a central coordinator still exists in the computational process.

APP developed by [29] is applied in [30] to solve OPF in a decentralized way by decomposing the centralized problem to regional/bus-level sub-problems. However, as shown by Fig. 2, the regional/bus agents, who collects information from its connected buses/DER components for the calculation of sub-problems, still acts as a central entity.

Rather than based on the augmented Lagrangian decomposition technique as DD, ATC, ADMM and APP, the optimal condition decomposition (OCD) and consensus+innovation (C+I) algorithms are developed to decompose OPF based on the Karush-Kuhn-Tucker (KKT) necessary conditions for local optimality. Both OCD and C+I characterize iterative solution procedure where at each iteration the decomposed

TABLE I
Comparison of different distributed optimization algorithms and relevant references

ref	Algorithm	Network constraints	Implementation Structure	Level of OPF decomposition	Time Periods	Maximum number of agents in case studies
[5]-[12]	consensus-based	✗		component-level	each DER owner	25
[13]	ATC	✓	Type 1 Fig.1	regional-level	single	5
[16]	DD	✓	Type 1 Fig.1	regional-level	single	30
[17]	DD	✓	Type 1 Fig.1	bus-level	single	
[18]	DD	✓	Type 1 Fig.1	regional-level	multiple	
[19]	ATC	✓	Type 1 Fig.1	regional-level	single	

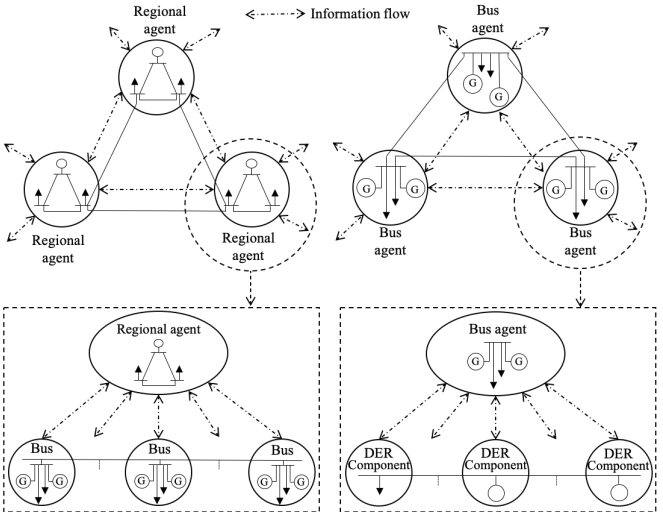

Fig. 2. Type 2 algorithm implementation structure: decentralized OPF with regional-level a) and bus-level b) decomposition.

sub-problems are solved independently. OCT is used in [31] and [13], for decentralized calculation of both AC and DC OPF problems, and in [32] C+I method is applied to solve OPF with DC power flow formulation in a decentralized way. However, 1) the applicability of OCD requires the satisfaction of the convergence condition, which is not always hold and hard to prove with complex and meshed systems; 2) C+I algorithm is originally designed for distributed parameter estimation in sensor networks [33], [34] and is not theoretically guaranteed to converge to the OPF optimum, moreover, it has only been used to solve DCOPF problems; 3) both OCD and C+I decompose OPF only to regional/bus-level, as shown by Fig. 2, the regional and bus agents still act as a central coordinator in their update processes.

Table I summarizes and compares the aforementioned distributed/decentralized optimization algorithms and the relevant literature for solving power coordination problems. It can be observed that:

- A solely consensus-based algorithm [5]–[12] is inherently unable to solve optimal power coordination problems when taking into the consideration of network constraints since a line congestion could result in unequal marginal costs.
- DD [XX]-[XX], ACT [xx]-[xx] and ADMM [xx]-[xx] require a central entity to manage distributed computation in the certain step of the algorithm.
- Although APP, OCD and C+I algorithms completely avoid the need for a central coordinator, their application in [xx]-[xx] only decompose the original centralized OPF problem to regional/bus-level sub-problems. The regional/bus agent still acts as a central entity who collects global information from its connected buses/DER devices for the calculation of sub-problems.
- Moreover, most of the above works are focusing on decomposing the OPF to regional/bus-level sub-problems and none of them accounts for component-level fully decentralized optimization.

B. Scope and Contributions

In order to fill this knowledge gap, this paper proposes a novel fully decentralized consensus-based ADMM algorithm to tackle optimal power coordination problems considering network constraints. The term "fully decentralized" signifies the absence of the need of a central control entity (e.g. the central coordinator in the calculation process of DD, ATC and ADMM, and the regional/bus-level energy management agent who is responsible for sub-problems and collects global information from its connected DER with the implementation structure shown in Fig. 2). The algorithm can be applied to coordination problems with both AC and DC power flow formulation and networks with arbitrary typologies. Although the implementation of APP, OCD and C+I do not need the central coordinator in the regional/bus-level OPF decomposition, the solely efforts of them cannot realize component-level optimization (the reason will be further explained in Section III). Therefore, the proposed algorithm involves a two-level iterative process, where in the upper-level, a variant of ADMM algorithm is applied to decompose the OPF to each bus agent and in the lower-level, a consensus-based algorithm is used and the nodal balance constraint is full-filled based on interactions with neighboring DER agents connected at the same bus. The implementation structure of the algorithm is shown in Fig. 3. In this algorithm, each bus and each DER component of the system is modeled as an agent with communication capability. By using information exchange between neighboring agents, the consensus-based ADMM algorithm converges asymptotically to the optimal solution of power coordination considering network constraints.

To the best of our knowledge, this is the first time to use ADMM to ad- dress the DEDP with nonquadratic cost function (E-exponential function) in distributed manner.

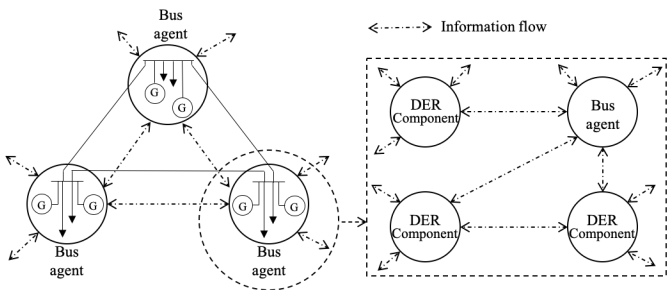

Fig. 3. Implementation structure for the fully decentralized algorithm with component-level decomposition.

II. Formulation of the Network-Constrained Coordination Problem

The optimal active and reactive power scheduling in the smart grid is important to both economic and efficient operation of the system. In this work, the OPF problem is formulated as a social welfare maximization problem (1) in multiple time periods, assuming that C_i constitutes a cost function if DER i is a generator and a utility function if DER i is a load (C_i equals zero if DER i is an inflexible generator or an inflexible load). This problem is subject to the nodal active (3) and reactive (4) power balance constraints (the Lagrangian multipliers of which constitute the locational active and reactive trading prices, respectively), the voltage (6) and thermal (7)-(8) limits of the distribution network, and the individual operation constraints of DER (9).

$$\min_{U} \sum_{i\in\mathcal{I}} C_i(\boldsymbol{p}_i^{der}, \boldsymbol{q}_i^{der}) \tag{1}$$

$$U = \{\boldsymbol{p}_i^{der}, \boldsymbol{q}_i^{der}, \boldsymbol{p}_{bb'}, \boldsymbol{q}_{bb'}, \boldsymbol{s}_{bb'}^{sen}, \boldsymbol{s}_{bb'}^{rec}, \boldsymbol{V}_b, \boldsymbol{\theta}_b\} \tag{2}$$

$$\text{s.t.:} \sum_{i\in\mathcal{I}_b} p_{i,t}^{der} - \sum_{bb'\in\mathcal{E}} p_{bb',t}(\boldsymbol{V}_b, \boldsymbol{\theta}_b) = 0 : \lambda_t, \forall t \tag{3}$$

$$\sum_{i\in\mathcal{I}_b} q_{i,t}^{der} - \sum_{bb'\in\mathcal{E}} q_{bb',t}(\boldsymbol{V}_b, \boldsymbol{\theta}_b) = 0 : \mu_t, \forall t \tag{4}$$

$$\theta_{ref,t} = 0, \forall t \tag{5}$$

$$\underline{V}_b \le V_{b,t} \le \overline{V}_b = 0, \forall b, \forall t, \tag{6}$$

$$s_{bb',t}^{sen}(\boldsymbol{V}_b, \boldsymbol{\theta}_b) \le \overline{s}_{bb'}, \forall bb', \forall t \tag{7}$$

$$s_{bb',t}^{rec}(\boldsymbol{V}_b, \boldsymbol{\theta}_b) \le \overline{s}_{bb'}, \forall bb', \forall t \tag{8}$$

$$(\boldsymbol{p}_i^{der}, \boldsymbol{q}_i^{der}) \in \mathcal{F}_i, \forall i \in \mathcal{I} \tag{9}$$

Following the formulation in [xx], the active and reactive power flow on the branch connecting bus b and b' at each time period t is formulated as:

$$\begin{aligned} &p_{bb',t}(\boldsymbol{V}_b, \boldsymbol{\theta}_b) \\ &= V_{b,t}^2 G_{b,b} + V_{b,t}V_{b',t}(G_{b,b'}\cos(\theta_{b,t}-\theta_{b',t})) + B_{b,b'}\sin(\theta_{b,t}-\theta_{b',t}), \\ &q_{bb',t}(\boldsymbol{V}_b, \boldsymbol{\theta}_b) \\ &= -V_{b,t}^2 B_{b,b} + V_{b,t}V_{b',t}(G_{b,b'}\sin(\theta_{b,t}-\theta_{b',t})) - B_{b,b'}\cos(\theta_{b,t}-\theta_{b',t}). \end{aligned} \tag{10}$$

III. Proposed Consensus-Based ADMM Algorithm

As discussed in Section I, most of the existing work for solving OPF in a distributed/decentralized way decompose the OPF problem to regional/bus-level sub-problems and none of them considering component-level decomposition. The applied algorithms either require a central coordinator in the certain step of the algorithm, or cannot decompose OPF into component-level sub-problems. The reason behind this is that there are two types of coupling variables associated with nodal power balance constraints in OPF problem formulation. The first type constitutes voltages and voltage angles between adjacent buses connected by tie lines and the second type includes active and reactive power injections of DER components connected at the same bus. The applied APP [xx]-[xx] and OCD [xx]-[xx] algorithms first duplicate the first type coupling variables in each region or bus connected by tie-lines. Then, the centralized OPF can be decomposed to sub-problems and solved by regional/bus agents involving only local variables. However, in the computation progress of each sub-problem, the second type of coupling variables have not been decoupled and regional/bus agents still need to gather DER power injections to satisfy the nodal power balance constraint. Therefore, to decompose the OPF to component-level, we propose a consensus-based ADMM algorithm, which solves OPF in a fully decentralized way and involves a two-level iterative process. In the upper-level, a variant of ADMM algorithm is applied to decompose the OPF to each bus agent and in the lower-level, a consensus-based algorithm is used and the nodal balance constraint is full-filled based solely on interactions with neighboring DER agents connected at the same bus. The optimal solution of OPF is achieved through the information exchange between neighboring DER and bus agents and completely avoid the need for a central coordinator.

In this section, we first introduce the standard ADMM algorithm in Section III-A, followed by the detailed discussion on reformulation of ACOPF problem in a suitable form for the application of ADMM algorithm in Section III-B. Section II-C discussed the application of the standard ADMM algorithm for solving ACOPF with the central coordinator. The proposed consensus-based ADMM algorithm is introduced in Section III-D, which achieves fully decentralized ACOPF with local problems solved by component-level agents.

A. Standard ADMM Algorithm

ADMM [21] is an algorithm that solves optimization problems in the form of (xx)-(xx), where $\boldsymbol{x} \in \mathbf{R}^{q1}$, $\boldsymbol{z} \in \mathbf{R}^{q2}$,$A_1 \in \mathbf{R}^{q3\times q1}$, $A_2 \in \mathbf{R}^{q3\times q2}$, and $\boldsymbol{c} \in \mathbf{R}^{q3}$. $c_1(x)$ and $c_2(z)$ are convex functions of $\boldsymbol{x}$ and $\boldsymbol{z}$, respectively.

$$\min_{\boldsymbol{x},\boldsymbol{z}} c_1(\boldsymbol{x}) + c_2(\boldsymbol{z}) \tag{11}$$

$$s.t.: \quad \boldsymbol{A}_1 \cdot \boldsymbol{x} + \boldsymbol{A}_2 \cdot \boldsymbol{z} = \boldsymbol{c} \tag{12}$$

The augmented Lagrange function of (xx)-(xx) is written as (xx), where $\boldsymbol{u} = (\frac{\rho}{2}) \cdot \boldsymbol{\nu}$ is the *scaled dual variable*, $\boldsymbol{\nu}$ are multipliers of (xx), $\rho > 0$ is a predefined parameter, and $\|\cdot\|$ represents the l_2-norm of a vector [xx]. ADMM is an iterative process consists of steps (xx)-(xx), where k is the index of

ADMM iterations.

$$L_\rho(\boldsymbol{x},\boldsymbol{z},\boldsymbol{u}) := c_1(\boldsymbol{x}) + c_2(\boldsymbol{z}) + (\frac{\rho}{2})\cdot \parallel A_1\cdot\boldsymbol{x} + A_2\cdot\boldsymbol{z} - \boldsymbol{c} + \boldsymbol{u} \parallel_2^2 \quad (13)$$

$$\boldsymbol{x}^{k+1} := \arg\min_{\boldsymbol{x}} L_\rho(\boldsymbol{x},\boldsymbol{z}^k,\boldsymbol{u}^k) \quad (14)$$

$$\boldsymbol{z}^{k+1} := \arg\min_{\boldsymbol{z}} L_\rho(\boldsymbol{x}^{k+1},\boldsymbol{z},\boldsymbol{u}^k) \quad (15)$$

$$\boldsymbol{u}^{k+1} := \boldsymbol{u}^k + A_1\cdot\boldsymbol{x}^{k+1} + A_2\cdot\boldsymbol{z}^{k+1} - \boldsymbol{c} \quad (16)$$

The primal residual and dual residual, that characterize the convergence of the algorithm, at iteration $k+1$ are defined as:

$$\boldsymbol{pri}^{k+1} = A_1\cdot\boldsymbol{x}^{k+1} + A_2\cdot\boldsymbol{z}^{k+1} - \boldsymbol{c} \quad (17)$$

$$\boldsymbol{dual}^{k+1} = \rho A_1^T\cdot A_2(\boldsymbol{z}^{k+1} - \boldsymbol{z}^k) \quad (18)$$

ADMM algorithm converges to the optimal solution $(\boldsymbol{x}^*,\boldsymbol{z}^*)$ and the optimal Lagrange multiplier $\boldsymbol{\nu}^*$ of problem (xx)-(xx), with:

$$\lim_{k\to\infty} \parallel \boldsymbol{pri}^{k+1} \parallel_2^2 = 0 \text{ AND } \lim_{k\to\infty} \parallel \boldsymbol{dual}^{k+1} \parallel_2^2 = 0,$$

under the assumption: 1) The functions $c_1 : \mathbb{R}^{q1} \to \mathbb{R}\cup\{+\infty\}$ and $c_2 : \mathbb{R}^{q2} \to \mathbb{R}\cup\{+\infty\}$ are closed, proper and convex; 2) The unaugmented Lagrangian L_0 has a saddle point; 3) The matrices A_1, A_2 have full column ranks [xx].

B. Reformation of ACOPF for Application of ADMM

In a transmission network, buses connected by the transmission lines are called adjacent buses. The difficulty in decomposing ACOPF to sub-problems solved by different bus agents lies in power flow formulations (xx), which induce couplings in both bus voltage and voltage angle between adjacent buses (as shown by a simple two-bus power system example in Fig.4). To decouple the first type coupling variables in the

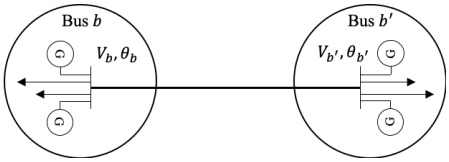

Fig. 4. A two-bus system showing the coupling variables

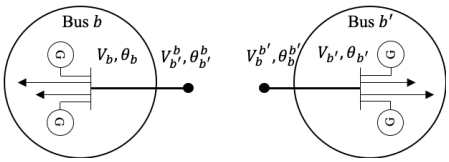

Fig. 5. A two-bus system showing the duplication of the coupling variables

OPF formulation and decompose (xx) to small sub-problems solved by each bus agent individually, the coupling variables (bus voltage and voltage angle) at each bus are duplicated at all its adjacent buses. For example, as illustrated by Fig. 5, variables $\boldsymbol{V}_b$ and $\boldsymbol{\theta}_b$ of bus b are duplicated at its adjacent bus b' as $\boldsymbol{V}_b^{b'}$ and $\boldsymbol{\theta}_b^{b'}$, respectively. Similarly, variables $\boldsymbol{V}_{b'}$ and $\boldsymbol{\theta}_{b'}$ of bus b' are duplicated at its adjacent bus b as $\boldsymbol{V}_{b'}^b$ and $\boldsymbol{\theta}_{b'}^b$. Accordingly, the power flow equation (xx) at time period t becomes:

$$\begin{aligned} & p_{bb',t}(\boldsymbol{V}_b,\boldsymbol{V}_{b'}^b,\boldsymbol{\theta}_b,\boldsymbol{\theta}_{b'}^b) \\ = \;& V_{b,t}V_{b',t}^b(G_{b,b'}\cos(\theta_{b,t}-\theta_{b',t}^b)) + B_{b,b'}\sin(\theta_{b,t}-\theta_{b',t}^b) \\ & q_{bb',t}(\boldsymbol{V}_b,\boldsymbol{V}_{b'}^b,\boldsymbol{\theta}_b,\boldsymbol{\theta}_{b'}^b) \\ = \;& V_{b,t}V_{b',t}^b(G_{b,b'}\sin(\theta_{b,t}-\theta_{b',t}^b)) - B_{b,b'}\cos(\theta_{b,t}-\theta_{b',t}^b). \end{aligned} \quad (19)$$

Since the value of the same variables duplicated in different buses must be identical, a global variable $\boldsymbol{z} \equiv [\boldsymbol{zV}, \boldsymbol{z\theta}]$ is introduced, voltage and voltage angle consistency constraints are added to guarantee the equivalence of the problem before and after the introduction of duplicated variables. For instance, for the two-bus system in Fig. 5, the global variable for bus b and b' includes $\boldsymbol{z}_b = \boldsymbol{z}_{b'} = [\boldsymbol{zV}_b, \boldsymbol{zV}_{b'}, \boldsymbol{z\theta}_b, \boldsymbol{z\theta}_{b'}]$, where $\boldsymbol{zV}_b$ and $\boldsymbol{z\theta}_b$ bundles $V_b, V_b^{b'}$ and $\theta_b, \theta_b^{b'}$, respectively; $\boldsymbol{zV}_{b'}$ and $\boldsymbol{z\theta}_{b'}$ bundles $V_{b'}, V_{b'}^b$ and $\theta_{b'}, \theta_{b'}^b$, respectively. Consistency constraints (xx) and (xx) are included in the constraints of ACOPF problem for bus b and b' respectively.

$$\begin{aligned} V_b - zV_b = 0, \quad \theta_b - z\theta_b = 0 \\ V_{b'}^b - zV_{b'} = 0, \; \theta_{b'}^b - z\theta_{b'} = 0 \end{aligned} \quad (20)$$

$$\begin{aligned} V_{b'} - zV_{b'} = 0, \; \theta_{b'} - z\theta_{b'} = 0 \\ V_b^{b'} - zV_b = 0, \; \theta_n^{b'} - z\theta_b = 0 \end{aligned} \quad (21)$$

For the sake of discussion, we define $\boldsymbol{x}_b \equiv [\boldsymbol{x}_b^{\boldsymbol{L}}, \boldsymbol{x}_b^{\boldsymbol{F}}, \boldsymbol{x}_b^{\boldsymbol{C}}], \forall b$ as decision variables of bus b, which is divided into three categories: local variables $\boldsymbol{x}_b^{\boldsymbol{L}} \equiv [\boldsymbol{p}_i^{der}, \boldsymbol{q}_i^{der}, \forall i \in \mathcal{I}_b, \forall b]$, flow variables $\boldsymbol{x}_b^{\boldsymbol{F}} \equiv [\boldsymbol{p}_{bb'}, \boldsymbol{q}_{bb'}, \boldsymbol{s}_{bb'}^{sen}, \boldsymbol{s}_{bb'}^{rec}, \forall bb']$ and coupling variables $\boldsymbol{x}_b^{\boldsymbol{C}} \equiv [\boldsymbol{V}_b, \boldsymbol{V}_{b'}^b, \boldsymbol{\theta}_b, \boldsymbol{\theta}_{b'}^b, \forall b' \in \mathcal{B}_b]$. Accordingly, the above OPF problem can be formulated as the summation of objectives for the individual buses (xx), where $f_b(\boldsymbol{x}_b) = \sum_{i\in\mathcal{I}_b} C_i(\boldsymbol{p}_i^{der}, \boldsymbol{q}_i^{der}).\boldsymbol{x}_b$ satisfying (xx)-(xx) at bus b is represented by (xx). Constraint (xx) guarantees that the same variable duplicated in different buses are identical to each other.

$$\min_{\boldsymbol{x}_b} \sum_{b\in\mathcal{B}} f_b(\boldsymbol{x}_b) \quad (22)$$

$$\text{s.t.:} \quad \boldsymbol{x}_b \in \mathcal{X}_b, \forall b \quad , \quad (23)$$

$$\boldsymbol{x}_b^{\boldsymbol{C}} - \boldsymbol{z}_b = 0, \forall b. \quad (24)$$

To reformulate ACOPF in the similar form as (xx), indicator functions g_b for sets $\mathcal{X}_b, \forall b$ are defined as:

$$g_b(\boldsymbol{x}_b) = \begin{cases} 0 & \text{if } \boldsymbol{x}_b \in \mathcal{X}_b \\ +\infty & \text{otherwise,} \end{cases}, \forall b\,. \quad (25)$$

Then, problem (xx)-(xx) can be written in the following equivalent form:

$$\min_{\boldsymbol{x}_b} \sum_{b\in\mathcal{B}} f_n(\boldsymbol{x}_b) + g_b(\boldsymbol{x}_b) \quad (26)$$

$$\text{s.t.:} \quad \boldsymbol{x}_b^C - \boldsymbol{z}_b = 0, \forall b \quad (27)$$

C. ADMM-based Distributed ACOPF

With the above illustration, the original centralized OPF problem (xx)-(xx) can be reformulated in a separable form

(xx) with objective function and constraints related to each independent bus. We formulate the augmented Lagrangian function for (xx) as (xx), by applying ADMM introduced in Section III-A (xx)-(xx), problem (xx)-(xx) can be solved by individual bus agents in parallel through a three-step iterative process.

$$L_\rho = \sum_{b\in\mathcal{B}} f_b(\boldsymbol{x}_b) + g_b(\boldsymbol{x}_b) + (\frac{\rho}{2})\cdot \parallel \boldsymbol{x}_b^C - \boldsymbol{z}_b + \boldsymbol{u}_b \parallel_2^2 . \quad (28)$$

***Step 0**: (Initialization)* The decision variables, global variables and the scaled dual variables at each bus b are initialized as:

$$\boldsymbol{x}_b^0, \boldsymbol{z}_b^0, \boldsymbol{u}_b^0, \forall b.$$

***Step 1**: (Update $\boldsymbol{x}_b^{k+1}$)* At iteration k, the update rule of $\boldsymbol{x}_b^{k+1}$ with fixed $\boldsymbol{z}_b^k$ and $\boldsymbol{u}_b^k$ can be operated by individual bus agents in parallel as written by (xx).

$$\begin{aligned}\boldsymbol{x}_b^{k+1} &= \arg\min_{\boldsymbol{x}_b} L_\rho(\boldsymbol{x}_b, \boldsymbol{z}_b^k, \boldsymbol{u}_b^k) \\ &= \arg\min_{\boldsymbol{x}_b} f_b(\boldsymbol{x}_b^L) \\ &+ g_b(\boldsymbol{x}_b) + (\frac{\rho}{2})\cdot \parallel \boldsymbol{x}_b^C - \boldsymbol{z}_b^k + \boldsymbol{u}_b^k \parallel_2^2\end{aligned} \quad (29)$$

***Step 2**: (Update $\boldsymbol{z}_b$)* In this step, the updated coupling variables of individual bus agents from the first step are sent to a central operator as shown in Fig. 2 . The central operator updates global variables $\boldsymbol{z}_b, \forall b$ via:

$$\begin{aligned}\boldsymbol{z}_b^{k+1} &= \arg\min_{\boldsymbol{z}_b}(\boldsymbol{x}_b^{k+1}, \boldsymbol{z}_b, \boldsymbol{u}_b^k) \\ &= \frac{\boldsymbol{x}_b^{\boldsymbol{C},k+1} + \sum_{b'\in\mathcal{B}} \boldsymbol{x}_{b'}^{\boldsymbol{C},k+1}}{1 + \sum_{b'\in\mathcal{B}} 1}\end{aligned} \quad (30)$$

For example for the two-bus system in Fig. 6, the central coordinator collects voltage $V_b^k, V_{b'}^k$, voltage angle $\theta_b^k, \theta_{b'}^k$ variables and their duplicated variables $V_b^{b',k}, V_{b'}^{b,k}, \theta_b^{b',k}, \theta_{b'}^{b,k}$ from bus b and bus b'. Then, it updates the global variables based on (xx), which are the average of all coupling variable bundled by $\boldsymbol{z}_b, \boldsymbol{z}_{b'}$, and sends the updated $\boldsymbol{z}_b^{k+1}, \boldsymbol{z}_{b'}^{k+1}$ back to bus b and b', respectively.

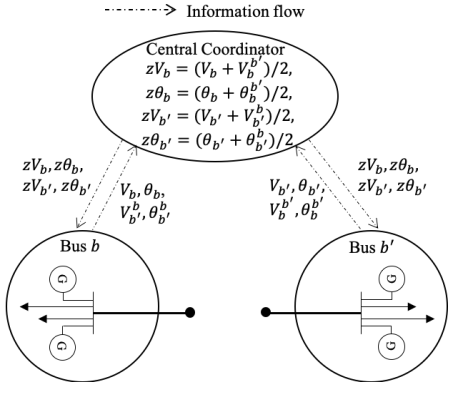

Fig. 6. A two-bus system showing the duplication of the coupling variables

***Step 3**: (Update $\boldsymbol{u}_b$)* After receiving $\boldsymbol{z}_b^{k+1}, \forall b$, each bus agent calculates $\boldsymbol{u}_b^{k+1}, \forall b$ by using the following update rule:

$$\boldsymbol{u}_b^{k+1} = \boldsymbol{u}_b^k + \boldsymbol{x}_b^{C,k+1} - \boldsymbol{z}_b^{k+1}. \quad (31)$$

Then, we compute the primal residual and dual residual using:

$$\boldsymbol{pri}_b^{k+1} = \boldsymbol{x}_b^{C,k+1} - \boldsymbol{z}_b^{k+1}, \forall b \quad (32)$$

$$\boldsymbol{dual}_b^{k+1} = \rho(\boldsymbol{z}_b^{k+1} - \boldsymbol{z}_b^k), \forall b, \quad (33)$$

the algorithm (xx)-(xx) terminates when:

$$\parallel \boldsymbol{pri}_b^{k+1} \parallel_2^2 \leq \varepsilon^{pri} \text{ AND } \parallel \boldsymbol{dual}_b^{k+1} \parallel_2^2 \leq \varepsilon^{dual}, \forall b,$$

where ε^{pri} and ε^{dual} are feasible tolerances. Detailed process of ADMM based distributed ACOPF with the existence of the central operator is described in Algorithm 1.

Algorithm 1 ADMM Algorithm

1: **Initialization:** Set $k = 0$ and initialize: $\boldsymbol{x}_b^0, \boldsymbol{z}_b^0, \boldsymbol{u}_b^0, \forall b$.
2: **while** $\parallel \boldsymbol{pri}_b^{k+1} \parallel_2^2 > \varepsilon^{pri}$ or $\parallel \boldsymbol{dual}_b^{k+1} \parallel_2^2 > \varepsilon^{dual}, \forall b$ **do**
3: **Step 1**: Given $\boldsymbol{z}_b^k$ and $\boldsymbol{u}_b^k$, each bus agent b update $\boldsymbol{x}_b^{k+1}$ using (xx).
4: **Step 2**: Each bus agent send the updated $\boldsymbol{x}_b^{k+1}$ to the central coordinator. The central operator update the global variables $\boldsymbol{z}_b^{k+1}, \forall b$ using (xx) and send the updated $\boldsymbol{z}_b^{k+1}, \forall b$ to each bus agent.
5: **Step 3**: Given $\boldsymbol{x}_b^{k+1}$ and $\boldsymbol{z}_b^{k+1}$, $\boldsymbol{u}_b^{k+1}$ is updated using (xx).
6: Compute the primal residual and dual residual using (xx)-(xx). $k = k + 1$
7: **end while**

However, as discussed in Section I-B, Section III-C and shown by Fig. 2, it is obvious that a central coordinator exists in the second step of ADMM. Moreover, the bus agent still requires global information from its connected DER agents in the first update process of ADMM algorithm. Specifically, the update of $\boldsymbol{x}_b$ in (xx) can be written in its equivalently form as (xx)-(xx) with function $h_b(\boldsymbol{x}_b^C)$ defined by (xx). The solution of (xx)-(xx) require each bus agent to collect active and reactive power injections from all its connected DER devices for the satisfaction of demand-supply balance constraints (xx) and (xx).

$$\min_{\boldsymbol{x}_b} f_b(\boldsymbol{x}_b^L) + h_b(\boldsymbol{x}_b^C) \quad (34)$$

$$\text{s.t.:} \quad (5) - (9) \quad (35)$$

$$\sum_{i\in\mathcal{I}_b} p_{i,t}^{der} - \sum_{bb'\in\mathcal{E}} p_{bb',t}(\boldsymbol{x}_b^C) = 0 : \lambda_t, \forall t \quad (36)$$

$$\sum_{i\in\mathcal{I}_b} q_{i,t}^{der} - \sum_{bb'\in\mathcal{E}} q_{bb',t}(\boldsymbol{x}_b^C) = 0 : \mu_t, \forall t \quad (37)$$

$$h_b(\boldsymbol{x}_b^C) = (\frac{\rho}{2})\cdot \parallel \boldsymbol{x}_b^C - \boldsymbol{z}_b^k + \boldsymbol{u}_b^k \parallel_2^2 \quad (38)$$

D. Proposed Consensus-based ADMM for Fully Decentralized ACOPF

To eliminate the central coordinator in the second update process of ADMM and decompose the OPF to component-level, we propose a consensus-based ADMM algorithm for the fully decentralized solution of OPF, which involves a two-level iterative process.

In the upper-level, a variant of ADMM algorithm is proposed to remove the central coordinator, where in the second update process of standard ADMM, we let the bus agent with the original voltage and voltage angle variables collect the duplicated variables from its adjacent buses. Then, each bus agent updates the global variable correlates to the original variable using (xx) and sends the updated global variable back to its adjacent buses. In this step, a communication network consistent with the transmission network exists for the information exchange between buses. For the example two-bus system shown in Fig. 7, bus b with original voltage V_b and voltage angle θ_b collects the duplicated variables $V_b^{b'}$ and $\theta_b^{b'}$ from its adjacent agent b'. It updates zV_b and $z\theta_b$ using (xx) and sends the updated global variables to b'. Bus b' has the similar calculation and communication process as b as shown by Fig. 7.

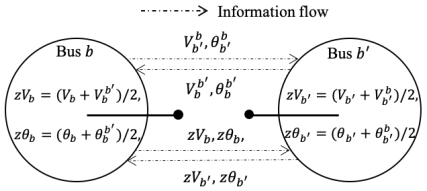

Fig. 7. A two-bus system showing the communication of adjacent buses with coupling variables

As discussed in Section I-A, consensus-based algorithms solve optimization problems through the iterative, bilateral exchange of information between the neighboring agents until they reach an agreement regarding certain consensus variables. Therefore, in the lower level, driven by the approach adopted by the vast majority of relevant papers, by selecting local active and reactive prices at bus b as consensus variables, the first update step of the ADMM algorithm (xx)-(xx) can be decomposed to component-level sub-problems and solved by each DER/bus agent using the consensus-based algorithm in a fully decentralized way. Define the set $\mathcal{H}_b := \{\mathcal{I}_b, b\}$ at bus $b \in \mathcal{B}$ with $|\mathcal{H}_b|$ represents the size of $\mathcal{H}_b$. Each agent $h \in \mathcal{H}_b$ estimates the values of these prices and determines its optimal response accordingly. Furthermore, they update these estimates by exchanging information with their neighboring agents, i.e. the DER/bus with which they are connected through the communication network. The network at each bus can be represented by a communication matrix $\boldsymbol{W}^b = [W^b_{h,h'}]_{|\mathcal{H}_b|\times|\mathcal{H}_b|}$, each element $W^b_{h,h'}$ of which represents the relative weight of the estimates transmitted by the neighboring agent $h' \in \mathcal{H}_b$ in the estimates' update of agent $h \in \mathcal{H}_b$. For space limitation reasons, the derivation of $\boldsymbol{W}^b, \forall b$ is provided in Appendix A.

Specifically, the consensus-based algorithm [5]-[12] involves a four-step iterative process, which is outlined below (considering that the first step update of ADMM addressed is the one expressed by (xx)-(xx)).

Step 0*: (Initialization)* The active and reactive prices, local variables, coupling variables and power mismatches estimated by each agent $h \in \mathcal{H}_b$ at bus b are initialized as:

$$\boldsymbol{\lambda}_h^0, \boldsymbol{\mu}_{\boldsymbol{h}}^0, \boldsymbol{p}_h^{der,0}, \boldsymbol{q}_h^{der,0}, \boldsymbol{x}_b^{C,0}, \boldsymbol{e}_h^{p,0}, \boldsymbol{e}_h^{q,0}, \forall h \in \mathcal{H}_b, \forall b.$$

Step 1*: (Price update):* At each iteration r, each DER/bus agent $h \in \mathcal{H}_b, \forall b$ updates its active and reactive price estimates based on its neighboring DER/bus estimates and a correction term which is proportional to its power mismatch estimates, weighted by learning gain constants ϵ^λ and ϵ^μ. The rationale behind this correction term lies in micro-economic principles: when the overall demand is higher than the overall supply, i.e. the power mismatch is positive, the price should be increased, and vice versa.

$$\lambda_{h,t}^{r+1} = \sum_{h' \in I} W^b_{h,h'} \lambda^r_{h',t} + \epsilon^\lambda e^r_{h,t}, \forall h \in \mathcal{H}_b, \forall b, \tag{39}$$

$$\mu_{h,t}^{r+1} = \sum_{h' \in I} W^b_{h,h'} \mu^r_{h',t} + \epsilon^\mu e^r_{h,t}, \forall h \in \mathcal{H}_b, \forall b. \tag{40}$$

Step 2*: (Response Optimization):* In this step, each dispatchable DER i determines its optimal responses to its estimated electricity prices by independently solving its economic surplus maximization problem subject to its operation constraint and the bus agent b solves a minimization subproblem subject to network constraints (xx)-(xx). (Note, non-dispatchable DER cannot change their outputs and demands according to their estimated prices, or equivalently, their responses are equal to their fixed outputs and demands, respectively.)

$$\begin{aligned}&(\boldsymbol{p}_i^{der,r+1}, \boldsymbol{q}_i^{der,r+1})\\ &= \underset{(\boldsymbol{p}_i^{der}, \boldsymbol{q}_i^{der})}{\arg\min}\ \lambda_{i,t}^{r+1} p_{i,t}^{der} + \mu_{i,t}^{r+1} q_{i,t}^{der} + C_h(\boldsymbol{p}_i^{der}, \boldsymbol{q}_i^{der}),\\ &\text{s.t.:(9)}\end{aligned} \tag{41}$$

$$\begin{aligned}&\boldsymbol{x}_b^{C,r+1}\\ &= \arg\max_{\boldsymbol{x}_b^C} \lambda_{h,t}^{r+1} \sum_{bb' \in \mathcal{E}} p_{bb',t}(\boldsymbol{x}_b^C) + \mu_{h,t}^{r+1} \sum_{bb' \in \mathcal{E}} q_{bb',t}(\boldsymbol{x}_b^C)\\ &\quad - h_n(\boldsymbol{x}_b^C),\ \text{s.t.:(xx)-(xx)}\end{aligned} \tag{42}$$

Step 3*:* (Power mismatch update) Each agent $h \in \mathcal{H}_b, \forall b$ updates its power mismatch estimates based on its neighboring agent estimates and a correction term which is given by the difference between its optimal responses at the two most recent iterations (for non-dispatchable DER, this difference is zero by definition and therefore is not included in this step).

$$e_{h,t}^{p,r+1} = \sum_{h \in I} W_{h,h'} e_{h',t}^{p,r} + p_{h,t}^{der,r+1} - p_{h,t}^{der,r}, \forall h \in \mathcal{H}_b, \forall b, \forall t \tag{43}$$

$$e_{h,t}^{q,r+1} = \sum_{h \in I} W_{h,h'} e_{h',t}^{q,r} + q_{h,t}^{der,r+1} - q_{h,t}^{der,r}, \forall h \in \mathcal{H}_b, \forall b, \forall t \tag{44}$$

$$e_{b,t}^{p,r+1} = \sum_{h \in I} W_{h,h'} e_{h,t}^{p,r} - \sum_{bb' \in \mathcal{E}} p_{bb',t}(\boldsymbol{x}_b^{C,r+1}) + \sum_{bb' \in \mathcal{E}} p_{bb',t}(\boldsymbol{x}_b^{C,r}), \forall b, \forall t \tag{45}$$

$$e_{b,t}^{q,r+1} = \sum_{h \in I} W_{h,h'} e_{h,t}^{q,r} - \sum_{bb' \in \mathcal{E}} q_{bb',t}(\boldsymbol{x}_b^{C,r+1}) + \sum_{bb' \in \mathcal{E}} q_{bb',t}(\boldsymbol{x}_b^{C,r}), \forall b, \forall t. \tag{46}$$

Step 4 *(Termination check)* The l_2-norm of the differences in the price estimates between the two most recent iterations and the l_2-norm of the power mismatch estimates are calculated.

If both of these are lower than their pre-specified tolerances $\varepsilon^{\lambda}, \varepsilon^{\mu}$ and $\varepsilon^{ep}, \varepsilon^{eq}$ for all agents $h \in \mathcal{H}_b, \forall b$ (xx), the algorithm terminates. Otherwise, the iteration counter increases by 1 and the algorithm goes back to Step 1.

$$\begin{aligned} &\|\boldsymbol{\lambda_h^r} - \boldsymbol{\lambda_h^{r-1}}\|_2 \leq \varepsilon^{\lambda}, \|\boldsymbol{\mu_h^r} - \boldsymbol{\mu_h^{r-1}}\|_2 \leq \varepsilon^{\mu} \\ &\text{AND } \|\boldsymbol{e_h^{p,r}}\|_2 \leq \varepsilon^{ep}, \|\boldsymbol{e_h^{q,r}}\|_2 \leq \varepsilon^{eq} \, \forall h. \end{aligned} \tag{47}$$

To this end, we have introduced the fully decentralized ADMM-based algorithm, which decomposes OPF to component-level sub-problems without the need for a central coordinator. The detailed process of the fully decentralized ADMM is described in Algorithm 2.

Algorithm 2 Consensus-Based ADMM Algorithm

1: **Initialization:** set $k = 0, r = 0$ and initialize: $\boldsymbol{x}_b^0, \boldsymbol{z}_b^0, \boldsymbol{u}_b^0$, $\lambda_h^0, , \mu_h^0, \boldsymbol{e}_h^{p,0}, \boldsymbol{e}_h^{q,0}, \forall h \in \mathcal{H}_b, \forall b$.
2: **while** $\| \boldsymbol{pri}_b^{k+1} \|_2^2 > \varepsilon^{pri}$ or $\| \boldsymbol{dual}_b^{k+1} \|_2^2 > \varepsilon^{dual}, \forall b$ **do**
3: **Step 1:**
4: **while** $\|\boldsymbol{\lambda_h^r} - \boldsymbol{\lambda_h^{r-1}}\|_2 \leq \varepsilon^{\lambda}, \|\boldsymbol{\mu_h^r} - \boldsymbol{\mu_h^{r-1}}\|_2 \leq \varepsilon^{\mu}$ AND $\|\boldsymbol{e_h^{p,r}}\|_2 \leq \varepsilon^{ep}, \|\boldsymbol{e_h^{q,r}}\|_2 \leq \varepsilon^{eq} \, \forall h$. **do**
5: **Price estimates**: Each DER/bus agent update price estimates using (xx).
6: **Optimal response**: Each DER/bus agent update price estimates using (xx).
7: **Mismatch estimates**: Each DER/bus agent calculate optimal responses to the estimated price using (xx).
8: Compute the l_2-norm of differences in the price estimates and power mismatch estimates between the most recent two iterations. $r = r + 1$
9: **end while**
10: **Step 2:** Each bus collects the duplicated variables from its adjacent buses. The global variables $\boldsymbol{z}_b^{k+1}, \forall b$ are updated using (xx) and then sent to the adjacent buses.
11: **Step 3:** Given $\boldsymbol{x}_b^{k+1}$ and $\boldsymbol{z}_b^{k+1}$, $\boldsymbol{u}_b^{k+1}$ is updated using (xx).
12: Compute the primal residual and dual residual using (xx)-(xx). $k = k + 1$
13: **end while**

IV. Case Studies

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APPENDIX A
GRAPH THEORY NOTATIONS

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be an undirected graph of order I modeling the communication network among the DER, where $\mathcal{V} = \{1, 2, \ldots, I\}$ and $\mathcal{E} = \{(i,h) \mid i,h \in \mathcal{V}\}$ are the non-empty vertex and finite edge set, respectively [35]. The edge (i,h) implies that DER i and h can communicate with each other. The neighbor set of vertex i is denoted by $\mathcal{I}_i \triangleq \{h \mid (i,h) \in \mathcal{E}\}$. A strongly connected undirected communication network $\mathcal{G}$, containing a minimum spanning tree (a path from any DER i to any other DER h) [36], no self-circle and no multiple edges, is assumed in this work.

The *Adjacency Matrix* $\boldsymbol{A} = [A_{i,h}]_{I\times I}$, associated with $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is defined by (A1). The *Laplacian Matrix* $\boldsymbol{L} = [L_{i,h}]_{I\times I}$ and communication weight matrix $\boldsymbol{W} = [W_{i,h}]_{I\times I}$ associated with $\boldsymbol{A}$ are defined by (A2) and (A3), respectively.

$$A_{i,h} = \begin{cases} 1 & \text{if } h \in \mathcal{I}_i \\ 0 & \text{otherwise.} \end{cases}, \tag{A1}$$

$$L_{i,h} = \begin{cases} \sum_{h=1}^{I} A_{i,h}, & \text{if } i = h \\ -A_{i,h}, & \text{otherwise} \end{cases}, \tag{A2}$$

$$W_{i,h} = \frac{\mid L_{i,h} \mid}{\sum_{h=1}^{I} \mid L_{i,h} \mid}. \tag{A3}$$

It is obvious that $\boldsymbol{W}$ is a positive symmetric matrix with sums of entries of each row and each column equal to ones. Therefore, $\boldsymbol{W}$ is a doubly stochastic matrix, satisfying $\boldsymbol{W}\mathbf{1} = \mathbf{1}$ and $\mathbf{1}^T\boldsymbol{W} = \mathbf{1}^T$, where $\mathbf{1}$ is a column vector of ones.