A COMPARATIVE STUDY ON THE FAILURE CRITERIA FOR PREDICTING THE DAMAGE INITIATION IN FIBRE-REINFORCED COMPOSITES

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Abstract

In this research, a review is performed to explore the advantages and disadvantages of different failure criteria for fibre reinforced composites. Widely-used failure criteria, such as the Maximum stress criterion, Hashin criterion, Puck's criterion, LaRC03 and Northwestern University (NU) criteria are reviewed based on the relevant literature. A comparison is performed of these failure criteria, using the analytical results obtained from a MATLAB programme and numerical results obtained from an Abaqus finite element model. The applicability and reliability of these failure criteria for predicting damage in thermoplastic laminates, i.e. AS4 carbon fibre reinforced Polyether-ether-ketone (PEEK), are evaluated based on the analytical and numerical results. The numerical results reveal that the Maximum Stress criterion provides the most conservative prediction, whilst the Hashin and Northwestern University (NU) criteria give reasonable and sensible results with an acceptable running time. Puck and LaRC03 criteria deliver more accurate predictions, but with longer running times.

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1. Introduction

Past studies have been undertaken on failure criteria for composite laminates including a world-wide failure exercise (WWFE) [1-7]. Notwithstanding, there are still many challenges to overcome, such as the complex failure mechanisms of composite materials, inadequate understanding of the mechanisms and difficulties in developing tractable models of the failure modes. Due to the complex failure behaviours exhibited in composites, even for a simple unidirectional laminate, it is difficult to predict the full range of observed behaviours [8].

For composite design, engineers conduct many experiments to get a comprehensive understanding of the failure mechanisms of composite laminates under different loading conditions and laminate constraints. Conducting uniaxial and pure shear tests to acquire the failure envelope of the laminates can considerably reduce the costs. The function of a lamina failure criterion is to determine the onset and failure modes of composite laminates, which might be in a state of combined stress [9].

In the aviation industry, materials with higher strength or good performance in various situations are required. Compared with the conventional thermoset composites, thermoplastic composites have superior toughness enabling better design of lightweight damage tolerant structural components. This kind of material can be remelted and reshaped by reheating above the processing temperature. The highest working temperature of carbon fibre reinforced Polyether-ether-ketone (CF/PEEK) composite with 30% reinforcements can reach 310 ℃ [10,11].

Most of the past failure criteria research work were based on fibre reinforced thermoset composites. J.D. Schaefer et al. [12] carried out some experiments on IM7/8552 composite specimens over the range of quasi-static to dynamic strain rates and found a set of apparent yield criteria, which could predict the matrix-dominated yielding of composites. Isaac M. Daniel et al. [13] proposed a new yield/failure theory, based on the experimental results obtained from IM7/8552 and AS4/3501-6 carbon/epoxy, to predict lamina yielding and failure under multi-axial states of stress. Xi Li et al. [14] evaluated the applicability of five different failure criteria and damage evolution methods in finite element analysis (FEA) for T700GC/M21 carbon/epoxy laminate under low-velocity impact. Some modelling work have been done for thermoplastic material involving failure criteria by Sun and Chen [15], Tan et al. [16] and Liu et al. [17] Good correlation between the experimental and numerical results was claimed in their researches, but none of these have compared the different applicability and accuracy of various failure criteria for thermoplastic composites.

In this study, the advantages and limitations of different failure criteria are discussed based on the literature review. The comparison of the predictive capability was conducted using the analytical results and numerical results obtained from MATLAB and Abaqus, respectively. Some suggestions were provided for engineering application, according to the analytical and numerical comparison.

2. Brief review of failure criteria

2.1. The category of failure criteria

Numerous failure theories have been proposed and are available for composite structural design [18]. One can categorise these failure criteria from different aspects. In this paper, the failure theories are discussed based on four categories: Macro versus Micro, Stress-based versus Strain-based, Mode-independent versus Mode-dependent and Non-Interactive versus Interactive.

2.1.1. Macro versus micro

Failure criteria can be classified into macroscopic failure criteria and microscopic failure criteria. The results of WWFE indicate that the top five failure criteria ranked [3] are those based on the macroscopic observations, including Zinoviev’s model [19,20], Bogetti’s model [21,22], Puck [23,24], Cuntze [25,26] and Tsai criterion [27]. Therefore, in this study, the main interest is on macroscopic failure criteria.

2.1.2. Stress-based versus strain-based

Failure criteria, employing the stress state of materials to predict the failure, are categorised as stress-based criteria. Likewise, those criteria based on the strain state are classified as strain-based criteria. The failure criteria based on the stress or strain components specify the different failure modes with respect to various failure mechanisms such as fibre breakage, fibre buckling, matrix cracking and shear failure. One typical strain-based criterion is the Maximum strain criterion [27,28]. The examples of stress-based criteria are Tsai-Hill theory [29], Hashin [30], Puck [23,24], and LaRC [31,32], Hoffma [33] etc.

2.1.3. Mode-independent versus mode-dependent [34]

The failure criterion, presented as a mathematical curve/surface in stress/strain space that predicts the occurrence of material damage, but does not directly identify the failure modes or the nature of damage, could be categorised as mode-independent criterion in which the equations are typically polynomials.

A mode-dependent failure criterion generally consists of several different equations or sets of equations each of which defines the event of a particular failure mode. Hashin–Rotem criterion [35]，Hashin criterion [30], Puck criterion [23,24] and the series criteria of LaRC [31,32] are some examples of mode-dependent failure criteria.

2.1.4. Non-interactive versus interactive

By comparing lamina stresses (or strains) with corresponding strengths separately, the non-interactive failure criteria are able to predict failure loads and modes. For example, failure prediction in transverse tension is not influenced by the presence of longitudinal shear. These sometimes could be referred to as "mode-independent criteria" since these criteria do not directly identify the failure modes or the nature of the damage, such as max-stress [28], Tsai-Hill [29] etc. The criteria which predict the failure loads by using a single quadratic or higher order polynomial equation involving all stresses (or strains) components are classified as the interactive failure criteria. The modes of failure are determined directly by the satisfied failure equations. These criteria are also referred to as "mode-dependent failure criterion". Hashin-Rotem [35], Hashin [30], Sun [9], Puck [23,24], and the series criteria of LaRC [31,32] etc., are all classified under this group.

2.2. Review of some widely-used failure criteria

In this study, five representative failure criteria (Max-Stress, Hashin, Puck, LaRC03 and Northwestern University (NU) are reviewed. Characteristics, abilities and evolution of these failure criteria are summarised in Table 1.

For Max-Stress criterion, only two indexes are involved which could distinguish the fibre and matrix failure modes separately. The Max-Stress criterion was found by Jiang et al. [36] to be applicable for corrugated plate and square tube crush models. Gliszczynski and Kubiak [37], in investigating thin-walled C-shaped cross-sections composite beam subjected to pure bending, found that the best agreement of numerical and experimental results was obtained using the Max-Stress criterion reduced to the fibre direction.

Hashin's criterion utilises four different indexes to distinguish the failure modes - fibre tension, fibre compression, matrix tension and matrix compression. Hashin's criterion are already amenable to computational procedure and have been used for several decades. Gu and Chen [38] affirmed the amenability arising from the criterion's ease of use though indicating that it does not always correlate well to experimental results. Chaht [39] using the Hashin criterion to simulate stratified composite material damage using shell elements. Li and Ju [40] found that including the shear stress term in the Hashin fibre tension failure criterion led to an underestimation of failure strength.

The following phenomenological failure criteria are all based on the Hashin's criterion. The NU and Puck criteria are two typical examples. By comparing the normal stress and shear stress, the NU theory considers the transverse failure as two main types, the normal stress dominated, and the shear stress dominated. Schaefer et al. [41] obtained good prediction of first-ply-failure (FPF) of embedded plies using the NU Theory. Reinoso et al. [42] used a 3D-version of the Puck failure criterion in creating an anisotropic damage (based on ply failure mechanisms) model for laminated fiber-reinforced composites. They found good agreement with experimental data in an open hole tension test.

In the Puck criterion, the fracture plane (introduced by Hashin) was incorporated for matrix compression by applying Mohr-Coulombs theory. Puck's criterion can also show the interaction of transverse compression and in-plane shear load.

The LaRC03 criterion, developed by Davila, involved in-situ strength and six non-empirical equations to predict the failure of fibre reinforced laminates. In this criterion, fibre alignment is idealized as a local region of waviness in which fibre kinking angle was involved. This criterion was more recently used by Chang et al. [43] to evaluate the macroscale fracture of unidirectional fiber-reinforced composites.

The mathematical expressions of these failure criteria range from simple equation to polynomial. These are often related to the failure mechanism phenomenon. The review of these failure criteria, are summarised as follows:

(1) Some damage criteria are phenomenon based. The prediction of these phenomenon-based criteria are accurate for describing different failure of composites. In the past, the validation of these criteria was focused on in-plane stress condition. There-axial characterisation results have been recently applied to validate these failure criteria.

(2) For fibre failure under tension loading condition, these criteria are almost fixed on or based on the Max- Stress/Strain criterion. The basic expression for matrix tension failure is also not changed too much from Hashin's criterion till now except the concept of in-situ strength was integrated, like LaRC criteria. Those failure criteria introduced the in-situ strength (not only in-situ longitude and transverse shear strength but also the in-situ transverse tension and compression strength) to replace the strength of composite laminates to obtain more accurate prediction, especially for those with different laminate angle in embedded laminates.

(3) The evolution for failure criteria is concentrated on the fibre and matrix failure under compression loading condition. For fibre failure, when $\sigma_{11} < 0$, the concept of fibre kinking was introduced from Puck's criterion, in which the compressive stress in fibre, instead of in composite laminate, was employed to get the failure condition. Then LaRC03 criterion involve the hypothesis of a kinking band along with the failure criterion for matrix tension and matrix compression to establish the criterion for fibre failure.

(4) For matrix compression, Hashin introduced the concept of fracture plane and pointed out the angle of fracture plane that could be obtained from Mohr failure theory which in his mind was complicated.so he employed the quadratic approximation to obtain the failure model. Then Puck followed with his step to get the fracture plane, and developed the expression for matrix compression failure, which involved the interaction of transverse compression and in-plane shear stresses.

2.3. Summary

Based on the previous discussion, some advantages and limitations of the selected damage criteria are summarised below:

(1) Maximum stress/strain criterion

Advantages: Maximum Stress and Strain criterion employed the simplest expressions to distinguish the fibre and matrix failure modes. If the basic strength value of a ply was examined the failure envelope will be obtained. These criteria are widespread in the engineering field. For predicting fibre failure in laminates in a simulation problem, maximum stress/ strain criterion is useful.

Limitations: The Poisson's effect for Maximum Stress criterion and the fibre, matrix failure mechanisms and other factors which could influence the failure of composites are not involved both in Maximum Stress and Maximum Strain criterion, so the prediction may not be acceptable for FE simulation under some loading conditions.

(2) Hashin criterion

Advantages: Hashin criterion is a phenomenon-based theory which can distinguish between the different failure modes. It is the simplest way to approximate an assumed interaction between different effects once a simple linear interaction is discarded. Hashin criterion was also amenable to computational procedure and widely used in the engineering field because of the effective equation and conservative prediction.

Limitations: This criterion could not clearly indicate the phenomenon that moderate transverse compression could increase the apparent shear strength of a ply. In addition, Hashin fibre compression criterion does not account for the effects of in-plane shear, which significantly reduces the effective compressive strength of a ply.

(3) Puck's criterion

Advantages: Puck's criterion use varying expressions to describe the different failure modes reasonably. The phenomenon that moderates transverse compression could increase the apparent shear strength of a ply was illuminated.

Limitations: There are as many as eleven material parameters， some of which are not physical and may be difficult to quantify without considerable experience in a particular material system. The fracture angle for thermoplastic laminates requires tests.

(4) LaRC03 criterion

Advantages: Same as Puck's criterion, LaRC03 criterion also employ varying expressions (six indexes) to predict different failure modes under in-plane stress states. This criterion can also illuminate the interaction between σ_{22} and τ_{12}. Introducing the conception of in-situ strength could produce a more accurate prediction of the laminate's strength.

Limitations: Too many parameters involved in this criterion make it difficult to use in the engineering field. The calculation of fracture angle is iterative which makes it time consuming. The calculation of in-situ strength for different laminates is inconclusive.

(5) NU theory

Advantages: NU theory based on linear behaviour, is a rather simple and accurate way to obtain the strength of laminates in 3D stress state. New materials can be easily evaluated by simply conducting a few macroscopic tests on the lamina and establishing the strain rate dependence of three basic lamina strengths, i.e., transverse tensile, compressive strengths and in-plane shear strength.

Limitations: The parameters involved in this criterion are based on the moduli of the laminate, which haven't been validated in other type of composite laminates with different moduli. The prediction may exceed the real failure strength of laminates under combined loading condition.

3. Comparison of the predictive capability between different failure criteria

The review has shown that the phenomenon-based failure criteria have better capability compared to others. Thus, this study focused on comparing the capability of the phenomenon-based failure criteria. The comparison was conducted in two ways, one is the analytical comparison, based on the envelops proceed from MATLAB programme, and the other is the numerical comparison, according to the computational results attained from the finite element models. The material employed in this study was the AS4 carbon fibre reinforced PEEK composite. The material properties are presented in Table 2.

3.1. Comparison of analytical results

In order to compare the envelopes of different failure criteria, a MATLAB programme was developed to identify these envelopes based on the discussed damage criteria. The failure envelops of various criteria subjected to different combined loading conditions ($\sigma_{11}-\sigma_{22}, \sigma_{11}-\tau_{12}$,$\sigma_{22}-\tau_{12}$),were plotted using MATLAB and shown in Fig. 1, Fig. 2 and Fig.3, respectively.

Based on the envelopes plotted, the following information was elicited:

(1) Under unidirectional loading condition, all the failure criteria could meet at four points which are the material strength value ($X_T, X_C, Y_T, Y_C, \pm S_{12}$ respectively).

(2) For Max-Stress criterion, the envelope is the simplest one for which the shape are all square under different combined in-plane stress state, and the boundary of these three envelopes are the values of material strength.

(3) For Hashin's criterion, the failure envelope coincides with the curve of Max-Stress when subjected to the loading condition of $\sigma_{11}-\sigma_{22}$,but under $\sigma_{11}-\tau_{12}$, the situation changed. When $\sigma_{11}<0$, the envelope is same as the Max-Stress criterion, which indicated that it doesn't account for the interaction of in-plane shear and longitude compressive stress, while for $\sigma_{11}>0$, the curve looks like a parabola. Under the load of $\sigma_{22}-\tau_{12}$, the envelope is totally enclosed by the Max-Stress's curve and delivered the most conservative prediction of failure when $\sigma_{22}<0$.

(4) It is more complicated to plot Puck criterion's envelope due to the large number of parameters used. When subjected to $\sigma_{11}-\sigma_{22}$ and $\sigma_{11}-\tau_{12}$ loading conditions, there were two similar curves which were enclosed by the square envelope of Maximum Stress criterion.

(5) Same as Puck criterion, the LaRC03 also involves a large quantity of parameters compared to other criteria discussed above. For the loading condition of $\sigma_{11}-\sigma_{22}$, when $\sigma_{11}\geq 0$, the LaRC03 criterion has the same curve with the Max-Stress's curve. When $\sigma_{11}\leq 0$, the curve changed significantly, especially in the third quadrant. For the loading condition of $\sigma_{11}-\tau_{12}$ with $\sigma_{11}\geq 0$, LaRC03 criterion predict the same results as Max-Stress criterion, but when $\sigma_{11}\leq 0$, it has the most conservative prediction within these six different criteria. For the loading condition of $\sigma_{22}-\tau_{12}$, LaRC03 has similar prediction as Puck's criterion.

(6) For NU criterion, the envelope is same as Max-Stress criterion and Hashin criterion when subjected to the loading condition of $\sigma_{11} - \sigma_{22}$, and same as Hashin criterion when the loading condition is $\sigma_{11} - \tau_{12}$. When bearing the $\sigma_{22} - \tau_{12}$ load, like Puck and LaRC03, NU criterion can illuminate the phenomenon that moderate transverse compression could increase the apparent shear strength of a ply but it has the most conservative prediction when $\sigma_{22} > 0$, within the analysed criteria.

3.2. The effects of fracture plane angle on failure criteria

When measuring the failure of matrix under compressive loading condition, the determination of the fracture angle is key for both the Puck and the LaRC03 criteria. The angle (α), between σ_{22} and σ_n, has to be determined, as the risk of fracture in the stress action plane, related to this angle, reaches its global maximum. Puck determined that, when subjected to transverse compression, most unidirectional graphite-epoxy composites fail by transverse shear along a fracture plane oriented at $\alpha_0 = 53 \pm 2°$. When employing the criterion of Puck and LaRC03 to predict the initiation failure of the laminates, this typical fracture angle was often used. For thermoplastic material, no such experiments have been performed, and no typical fracture angle has been measured. Taking LaRC03 criterion as an example, the exact angle of α_0 have great effects on the failure envelope. For the in-plane stress state, when under the transverse compression loading condition, the failure envelopes of LaRC03 criterion are quite different from each other when $\alpha_0 = 51°$, $\alpha_0 = 53°$ and $\alpha_0 = 55°$, which could be seen in Fig. 4. The reason is that some of the parameters implemented in LaRC03 are based on the fracture angle α_0.

The Puck's criterion is a semi-empirical approach as all the parameters used in this criterion were already recommended [23,24]. All these parameters were obtained based on thermoset composites with the fracture angle of 53°. There will be uncertainty when employing Puck criterion to predict the onset failure for thermoplastic composites, as no experimentally fracture angle is available for thermoplastic composites. In this paper, as a preliminary study, the fracture angle of 53° for thermoset composites was used to first develop the predictive model for thermoplastic composite.

3.3. Comparation of numerical results

To further compare the predictive capability of the above failure criteria, a finite element model, incorporating damage models based on different failure criteria, was developed. In the FE model, virtual three-point bend was performed on a thermoplastic composite specimen, from which the numerical results were extracted for comparison [48]. The reason for employing three-point bend is that lots of stress interaction exist (not only for tensile stress, but also compression and shear stress) in this type of test, which results in the different failure modes more related to engineering applications. Moreover, the model is simple enough to avoid additional geometric complexities that increase simulation time.

3.3.1. Finite element model

The FE model was developed in Abaqus 2018. The dimensions of the three-point bend specimen were 120 mm x 13 mm x 3 mm, and the radius of loading nose and fixed supports were both 5mm, while the support span was 96 mm for three point bending test. During the experiments, a displacement control was applied by setting the loading speed as 1 mm/s. Generally, in three-point bending tests, multiple damage modes could occur, such as fibre tension/compression, matrix tension/compression, and delamination in interlaminate, with strong and complex interactions between various failure modes. Delamination propagation in composite structures is a 3D phenomenon because a delamination frequently propagates in a nonself-similar fashion and may kink into other plies and propagate along another ply interface [14]. The presence of microcracks will often interfere with the direction and shape of delamination growth. Therefore, 3D elements should be used as shown in Fig.5. The element type was C3D8R solid elements with size of 1 mm × 1 mm. The interfacial failure between composite plies was modelled using the Abaqus in-built cohesive surface model and the general contact algorithm was used to govern global contact with the modelling set-up [48].

3.3.2. Brief overview of the damage model

3.3.2.1. The constitutive law

An extended 3-D EP model has been used to capture the EP material response prior to the initiation of matrix cracking, etc, since this enables a more accurate prediction of the impact behaviour of the composite laminate and allows the modelling of any permanent indentation arising from the impact to be simulated. The constitutive relation for the EP model can be obtained by combining the classic elastic model with the extended plastic model and is given by [49]:

$$\begin{Bmatrix} d\varepsilon_{11} \\ d\varepsilon_{22} \\ d\varepsilon_{33} \\ d\varepsilon_{12} \\ d\varepsilon_{13} \\ d\varepsilon_{23} \end{Bmatrix} = \begin{bmatrix} 1/E_{11} & -\nu_{21}/E_{11} & -\nu_{31}/E_{11} & 0 & 0 & 0 \\ -\nu_{12}/E_{22} & 1/E_{22} & -\nu_{32}/E_{22} & 0 & 0 & 0 \\ -\nu_{13}/E_{33} & -\nu_{23}/E_{33} & 1/E_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{23} \end{bmatrix} \begin{Bmatrix} d\sigma_{11} \\ d\sigma_{22} \\ d\sigma_{33} \\ d\sigma_{12} \\ d\sigma_{13} \\ d\sigma_{23} \end{Bmatrix} + \begin{Bmatrix} d\varepsilon_{11}^{p} \\ d\varepsilon_{22}^{p} \\ d\varepsilon_{33}^{p} \\ d\varepsilon_{12}^{p} \\ d\varepsilon_{13}^{p} \\ d\varepsilon_{23}^{p} \end{Bmatrix} \quad (1)$$

where $d\varepsilon_{ij}$ $(i,j = 1,2,3)$ is the incremental total strain tensor and $d\sigma_{ij}$ $(i,j = 1,2,3)$ is the incremental stress tensor. The parameters ν_{ij} $(i,j = 1,2,3, i \neq j)$ are the Poisson's ratios, E_{ii} $(i,j = 1,2,3, i = j)$ are the Young's moduli either for tension or compression loading, which are generally considered to be similar for composite laminates [50], and G_{ij} $(i,j = 1,2,3,\ i \neq j)$ are the shear moduli. The parameter $d\varepsilon_{ij}^{p}$ $(i,j = 1,2,3)$ represents the incremental plastic strain tensor and is given by:

$$\begin{Bmatrix} d\varepsilon_{11}^{p} \\ d\varepsilon_{22}^{p} \\ d\varepsilon_{33}^{p} \\ d\varepsilon_{12}^{p} \\ d\varepsilon_{13}^{p} \\ d\varepsilon_{23}^{p} \end{Bmatrix} = \frac{An}{{\sigma_{eff}}^{1-n}} \begin{Bmatrix} 0 \\ 3(\sigma_{22} - \sigma_{33})/2\sigma_{equ} \\ 3(\sigma_{33} - \sigma_{22})/2\sigma_{equ} \\ 3a_{66}\sigma_{12}/2\sigma_{equ} \\ 3a_{66}\sigma_{13}/2\sigma_{equ} \\ 3a_{44}\sigma_{23}/2\sigma_{equ} \end{Bmatrix} d\sigma_{equ} \tag{2}$$

where the equivalent stress, σ_{equ}, is given by:

$$\sigma_{equ} = \sqrt{\frac{3}{2}(\sigma_{22}^2 + \sigma_{33}^2) - 3\sigma_{22}\sigma_{33} + 3a_{44}\sigma_{23}^2 + 3a_{55}\sigma_{13}^2 + 3a_{66}\sigma_{12}^2} \tag{3}$$

and the relationship between the equivalent stress, σ_{equ}, and the equivalent plastic strain, ε_{equ}^{p}, can be expressed as a power-law function, given by [51]:

$$\varepsilon_{equ}^{p} = A{\sigma_{equ}}^{n} \tag{4}$$

In these equations a_{44}, a_{55} and a_{66} are coefficients indicating the extent of anisotropy in the plastic behaviour. For transversely isotropic solids which are linearly elastic in the fibre direction, i.e. a unidirectional fibre-reinforced composite, then a_{44}=2 and a_{55}= a_{66}. Now, the coefficient a_{66} can be readily determined from off-axis tension and compression stress versus strain experiments conducted at different values of the off-axis angle using a unidirectional composite. From such off-axis experiments, when different off-axis angles are employed using a unidirectional composite, then the non-linear coefficients A and n may also be determined by fitting to the measured plots of σ_{equ} versus ε_{equ}^{p}.

3.3.2.2. Damage initiation model

As mentioned above, for fibre damage, most of the failure criteria are based on Maximum stress criterion. As a result, the Maximum stress criterion was employed in the developed damage model to predict fibre failure, which are shown in Eq. (5) and (6).

Fibre Tension ($\sigma_{11} \geq 0$): $$F_{1t} = \frac{\sigma_{11}}{X_T} \tag{5}$$

Fibre Compression ($\sigma_{11} < 0$): $$F_{1c} = \frac{\sigma_{11}}{X_C} \tag{6}$$

As the variance between different failure criteria is part of matrix damage prediction, five different failure criteria were employed to capture the damage initiation in matrix. Refer to Eqs. (7) to (22).

(1) Maximum stress criterion

Transverse failure ($|\sigma_{22}| \geq |\sigma_{33}|$):

Tension-dominated: $$F_{2t} = \frac{\sigma_{22}}{Y_T} \tag{7}$$

Compression-dominated: $$F_{2c} = \frac{\sigma_{22}}{Y_C} \tag{8}$$

Through-thickness failure ($|\sigma_{33}| \geq |\sigma_{22}|$):

Tension-dominated: $$F_{3t} = \frac{\sigma_{33}}{Z_T} \tag{9}$$

Compression-dominated: $$F_{3c} = \frac{\sigma_{33}}{Z_C} \tag{10}$$

(2) Hashin criterion

Matrix Tension ($\sigma_{22} + \sigma_{33} \geq 0$):

$$F_{2t} = \frac{1}{Y_T^2}(\sigma_{22} + \sigma_{33})^2 + \frac{1}{S_{23}^2}(\tau_{23}^2 - \sigma_{22}\sigma_{33}) + \frac{1}{S_{12}^2}(\tau_{12}^2 + \tau_{13}^2) \tag{11}$$

Matrix Compression ($\sigma_{22} + \sigma_{33} < 0$):

$$F_{2c} = \frac{1}{Y_C}\left[\left(\frac{Y_C}{2S_{23}}\right)^2 - 1\right](\sigma_{22} + \sigma_{33}) + \frac{1}{4S_{23}^2}(\sigma_{22} + \sigma_{33})^2 + \frac{1}{S_{23}^2}(\tau_{23}^2 - \sigma_{22}\sigma_{33}) + \frac{1}{S_{12}^2}(\tau_{12}^2 + \tau_{13}^2) \tag{12}$$

(3) Puck criterion

Matrix Tension $\sigma_n \geq 0$: $$F_{2t} = \left(\frac{\sigma_n}{R_{\perp}^{(+)A}}\right)^2 + \left(\frac{\tau_{nt}}{R_{\perp\perp}^A}\right)^2 + \left(\frac{\tau_{nl}}{R_{\perp\parallel}^A}\right)^2 \tag{13}$$

Matrix Compression $\sigma_n < 0$: $$F_{2c} = \left(\frac{\tau_{nt}}{R_{\perp\perp}^A - P_{\perp\perp}^{(-)}\sigma_n}\right)^2 + \left(\frac{\tau_{nl}}{R_{\perp\parallel}^A - P_{\perp\parallel}^{(-)}\sigma_n}\right)^2 \tag{14}$$

With

$$\sigma_n = \sigma_{22}\cos^2\theta + \sigma_{33}\sin^2\theta + 2\tau_{23}\sin\theta\cos\theta$$

$$\tau_{nt} = (\sigma_{33} - \sigma_{22})\sin\theta\cos\theta + \tau_{23}(\cos^2\theta - \sin^2\theta)$$

$$\tau_{nl} = \tau_{31}\sin\theta + \tau_{21}\cos\theta$$

For carbon fibre: $$P_{\perp\parallel}^{(-)} = 0.3, P_{\perp\perp}^{(-)} = 0.2, R_{\perp}^{(+)A} = Y_T, R_{\perp\parallel}^A = S_{21}, R_{\perp\perp}^A = \frac{Y_C}{2(1 + P_{\perp\perp}^{(-)})}$$

(4) LaRC03 criterion

Matrix Tension ($\sigma_{22} \geq 0$):

$$F_{2t} = (1-g)\frac{\sigma_{22}}{Y_{is}^T} + g\left(\frac{\sigma_{22}}{Y_{is}^T}\right)^2 + \left(\frac{\tau_{12}}{S_{is}^L}\right)^2 \quad (15)$$

Matrix Compression ($\sigma_{22} < 0$):

$$\begin{cases} F_{2c} = \left(\frac{\tau_{eff}^T}{S_{23}}\right)^2 + \left(\frac{\tau_{eff}^L}{S_{is}^L}\right)^2, \sigma_{11} \geq Y_C \\ F_{2c} = \left(\frac{\tau_{eff}^{mT}}{S_{23}}\right)^2 + \left(\frac{\tau_{eff}^{mL}}{S_{is}^L}\right)^2, \sigma_{11} < Y_C \end{cases} \quad (16)$$

With

$$g = \frac{G_{IC}}{G_{IIC}} = \frac{\Lambda_{22}^0}{\Lambda_{44}^0}\left(\frac{Y_{is}^T}{S_{is}^L}\right)^2, Y_{is}^T = \sqrt{\frac{8G_{IC}}{\pi t \Lambda_{22}^0}}, S_{is}^L = \sqrt{\frac{8G_{IIC}}{\pi t \Lambda_{44}^0}}, \Lambda_{22}^0 = 2\left(\frac{1}{E_2} - \frac{\upsilon_{21}^2}{E_1}\right), \Lambda_{44}^0 = \frac{1}{G_{12}}$$

$$\tau_{eff}^T = \langle -\sigma_{22}\cos\theta\,(\sin\theta - \eta^T\cos\theta)\rangle, \tau_{eff}^L = \langle \cos\theta\,(|\tau_{12}| + \eta^L\sigma_{22}\cos\theta)\rangle,$$

$$\tau_{eff}^{mT} = \langle -\sigma_{22}^m\cos\theta\,(\sin\theta - \eta^T\cos\theta)\rangle, \tau_{eff}^{mL} = \langle \cos\theta\,(|\tau_{12}^m| + \eta^L\sigma_{22}^m\cos\theta)\rangle,$$

$$\sigma_{11}^m = \sigma_{11}\cos^2\varphi + \sigma_{22}\sin^2\varphi + 2|\tau_{12}|\sin\varphi\cos\varphi$$

$$\sigma_{22}^m = \sigma_{11}\sin^2\varphi + \sigma_{22}\cos^2\varphi + 2|\tau_{12}|\sin\varphi\cos\varphi$$

$$\tau_{12}^m = (\sigma_{22} - \sigma_{11})\sin\varphi\cos\varphi + |\tau_{12}|(\cos^2\varphi - \sin^2\varphi)$$

$$\varphi = \frac{|\tau_{12}| + (G_{12} - X_C)\varphi^C}{G_{12} + \sigma_{11} - \sigma_{22}}, \quad \varphi^C = \tan^{-1}\left(\frac{1 - \sqrt{1 - 4\left(\frac{S_{is}^L}{X_C} + \eta^L\right)\left(\frac{S_{is}^L}{X_C}\right)}}{2\left(\frac{S_{is}^L}{X_C} + \eta^L\right)}\right), \quad \alpha_0 = 53^\circ$$

$$\eta^T = \frac{-1}{\tan 2\alpha_0}, \quad \eta^L = \frac{S^L\cos 2\alpha_0}{Y_C\cos^2\alpha_0}, \quad S^T = Y_C\cos\alpha_0\left(\sin\alpha_0 + \frac{\cos\alpha_0}{\tan 2\alpha_0}\right)$$

(5) NU criterion

Transverse failure ($|\sigma_{22}| \geq |\sigma_{33}|$):

Tension-dominated ($|\sigma_{22}| \geq |\tau_{12}(\tau_{23})|$ and $\sigma_{22} \geq 0$):

$$F_{2t} = \frac{\sigma_{22}}{Y_T} + \left(\frac{E_{22}}{2G_{12}}\right)^2\left(\frac{\tau_{12}}{Y_T}\right)^2 + \left(\frac{E_{22}}{2G_{23}}\right)^2\left(\frac{\tau_{23}}{Y_T}\right)^2 \leq 1 \quad (17)$$

Compression-dominated ($|\sigma_{22}| \geq |\tau_{12}(\tau_{23})|$and $\sigma_{22} < 0$):

$$F_{2c} = \left(\frac{\sigma_{22}}{Y_C}\right)^2 + \left(\frac{E_{22}}{G_{12}}\right)^2\left(\frac{\tau_{12}}{Y_C}\right)^2 + \left(\frac{E_{22}}{G_{23}}\right)^2\left(\frac{\tau_{23}}{Y_C}\right)^2 \leq 1 \quad (18)$$

Shear-dominated ($|\sigma_{22}| \leq |\tau_{12}(\tau_{23})|$):

$$F_{2s} = \left(\frac{\tau_{12}}{S_{12}}\right)^2 + \left(\frac{\tau_{23}}{S_{23}}\right)^2 + \frac{2G_{12}}{E_{22}}\frac{\sigma_{22}}{Y_C} \leq 1 \quad (19)$$

Through-thickness failure ($|\sigma_{33}| \geq |\sigma_{22}|$):

Tension-dominated

$$F_{3t} = \frac{\sigma_{33}}{Z_T} + \left(\frac{E_{33}}{2G_{13}}\right)^2\left(\frac{\tau_{13}}{Z_T}\right)^2 + \left(\frac{E_{33}}{2G_{23}}\right)^2\left(\frac{\tau_{23}}{Z_T}\right)^2 \leq 1 \quad (20)$$

$(|\sigma_{33}| \geq |\tau_{13}(\tau_{23})|$ *and* $\sigma_{33} \geq 0)$:

Compression-dominated $(|\sigma_{33}| \geq |\tau_{13}(\tau_{23})|$ *and* $\sigma_{33} < 0)$:

$$F_{3c} = \left(\frac{\sigma_{33}}{Z_C}\right)^2 + \left(\frac{E_{33}}{G_{13}}\right)^2 \left(\frac{\tau_{13}}{Z_C}\right)^2 + \left(\frac{E_{33}}{G_{23}}\right)^2 \left(\frac{\tau_{23}}{Y_C}\right)^2 \leq 1 \quad (21)$$

Shear-dominated $(|\sigma_{33}| \leq |\tau_{13}(\tau_{23})|)$:

$$F_{3s} = \left(\frac{\tau_{13}}{S_{13}}\right)^2 + \left(\frac{\tau_{23}}{S_{23}}\right)^2 + \frac{2G_{13}}{E_{33}}\frac{\sigma_{33}}{Z_C} \leq 1 \quad (22)$$

Where $F_{it}(i=1,2,3)$, $F_{ic}(i=1,2,3)$ and $F_{is}(i=1,2,3)$ are the failure indexes for tensile-dominated, compression-dominated and shear-dominated failure in three material directions, respectively. X_T, Y_T, Z_T and X_C, Y_C, Z_C are the allowable tensile and compression strength along three material directions respectively. S_{12},S_{13}andS_{23}represent allowable shear strengths in the corresponding principal material directions. θ is the angle of fracture plane. Further, σ_n, τ_{nt}, τ_{nl} are the normal and shear stress acting on the fracture plane. *p* stands for slope parameters of the fracture curves. *R* stands for fracture resistance. τ_{eff}^{T}, τ_{eff}^{L} are the effective shear stress along transverse and longitude direction of the fracture plane while τ_{eff}^{mT}, τ_{eff}^{mL} are the effective shear stress in the misalignment frame. Y_{is}^{T}, S_{is}^{L} are the transversal in-situ compressive strength and longitudinal in-situ shear strength with subscript to indicate in-situ. t is the thickness of the composite plate.

3.3.2.3. Damage propagation model

Corresponding to the damage initiation model, damage variables for the tensile, compressive and shear failure of the fibre and matrix are defined to indicate the growth of the intralaminar damage in a composite ply. The damage propagation model is developed based on the energy dissipated during the damage process and the linear material softening. A general form of the damage variable, d, for a particular type of damage initiation, is given by [25]:

$$d = \frac{(\varepsilon^f - \varepsilon_p)(\varepsilon - \varepsilon^0)}{(\varepsilon - \varepsilon_p)(\varepsilon^f - \varepsilon^0)} \quad (23)$$

and here the strain, ε, is the combined strain in the composite ply. The strain values, ε^0 and ε^f, are the combined strains corresponding to the initiation of damage and final failure, respectively. The term ε_p is the combined plastic strain. The strains at failure may be determined from the respective values of the tensile, $G_{Ic}|_{ft}$, and compressive, $G_{Ic}|_{fc}$, intralaminar ply fracture energies in the longitudinal fibre-direction; and the tensile, $G_{Ic}|_{mt}$, compressive, $G_{Ic}|_{mc}$, and shear, $G_{IIc}|_{ms}$, intralaminar ply fracture energies in the transverse directions. Next, there damage variable were used to degrade the elasticity matrix to form the damaged elasticity matrix, C_d, which will then be employed to calculate the degraded stress in the damaged materials. The details of this intralaminar damage propagation model have been previously presented in [14,23,31,32,49,52].

3.3.2.4. Model implementation

The flow chart of the developed FE model is schematically shown in Fig.6, where the sub-flowchart of the composite damage model is also highlighted. The flowchart shows one computation step for a single element. The computation process was performed for every appropriate single element for mechanical response and progressive failure of thermoplastic composites.

3.3.2.5. Simulation results

The loading response obtained from the experiment and the numerical simulations is shown in Fig.7. At the initial stage, all the predictions started with the linear elastic response up to damage initiation followed by the maximum load and a significant load drop. These six loading responses yielded reasonable agreement with the experimental results. This indicated that all the studied failure criteria have acceptable capability to predict the global mechanical response for thermoplastic laminates under three-point bend.

Table 3 shows the comparison of maximum loads obtained from the experiment and simulations along with approximate run time. The Puck and LaRC03 criteria have very similar load predictions, 1.06 and 1.05 kN respectively, which correlated with experimental results. Due to the iterative calculation for fracture angle in both Puck and LaRC03 criteria, the simulation time of both criteria were 26 and 28 hours respectively, longer than the other three criteria. Maximum stress criterion predicted the most conservative results, with a load prediction of 0.98 kN and running time of 16 hours. Hashin and NU criteria could get reasonable results, 1.03 and 1.04 kN respectively, for thermoplastic laminate under three-point bend, with acceptable running time of 17 and 16 hours, respectively.

4. Conclusions

In this paper, five representative macroscopic failure criteria were reviewed. A comparative study was performed by analysing the theoretical failure envelopes plotted using a MATLAB programme and numerical results were extracted using the finite element (FE) model.

Based on the analytical and numerical results, the following conclusions were drawn:

- Maximum Stress/Strain, Hashin and Northwestern University (NU) criteria are often employed as these required less parameters, which made implementation easier as a user subroutine. The Hashin criterion does not clearly indicate the interaction between σ_{11} and σ_{22}, and there are still some discussions on the parameter selection, for example, transverse shear strength. Northwestern University (NU) criterion can illuminate the interaction between transverse load σ_{22} and the in-plane shear and τ_{12}, whilst it has a conservative prediction for fibre and matrix tensile failure.
- Puck and LaRC03 failure criteria can reveal more failure mechanisms in a reasonable way, which not only indicated the interaction between σ_{11} and τ_{12}, σ_{22} and τ_{12}, but also involved the effects of fracture orientation, fibre kinking and in-situ strength. The limitation is the parameters used in these two criteria require considerable experience for application.
- The numerical results showed that the Maximum Stress criterion provides the most conservative prediction, while the Hashin and NU criteria give reasonable results which are reasonable and sensible, with an acceptable running time. Puck and LaRC03 criteria delivered more accurate predictions but with longer running times.

It is considered this review of different failure criteria applied to fibre composite materials is useful in identifying the most appropriate damage criteria to be employed for modelling failure in composites.

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Figures

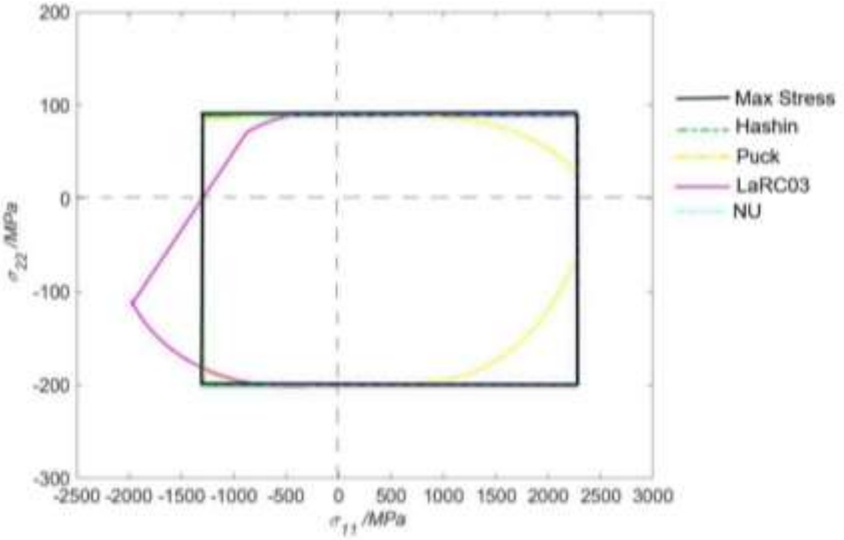


Fig. 1. The theoretical failure envelopes of AS4/PEEK under $\sigma_{11} - \sigma_{22}$.

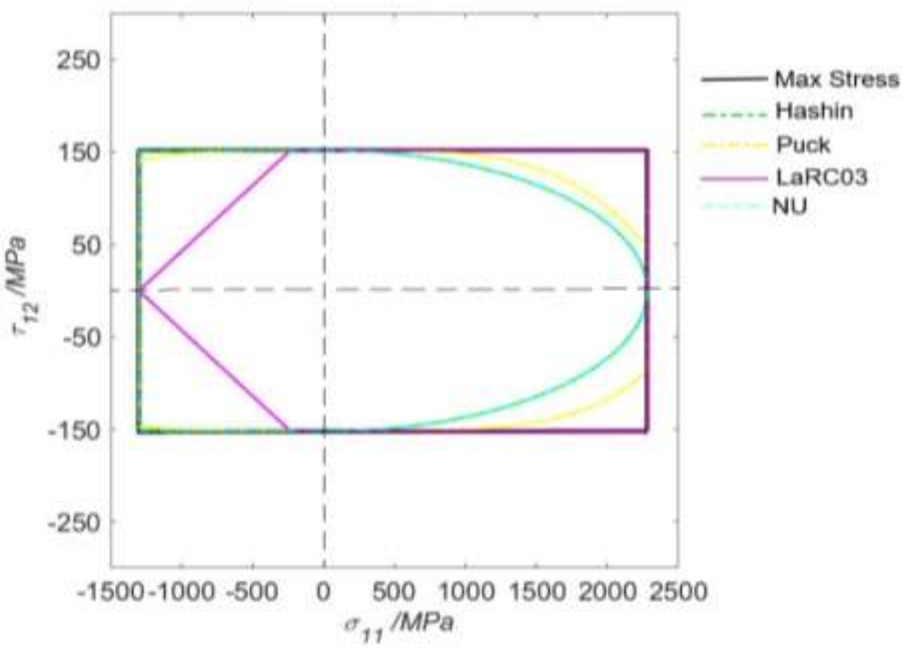


Fig. 2. The theoretical failure envelopes of AS4/PEEK under $\sigma_{11} - \tau_{12}$.

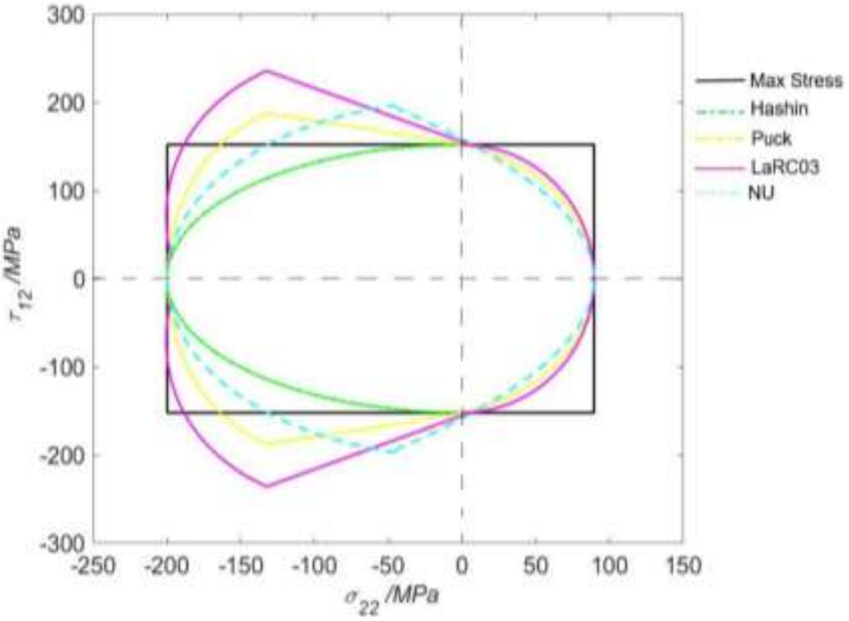
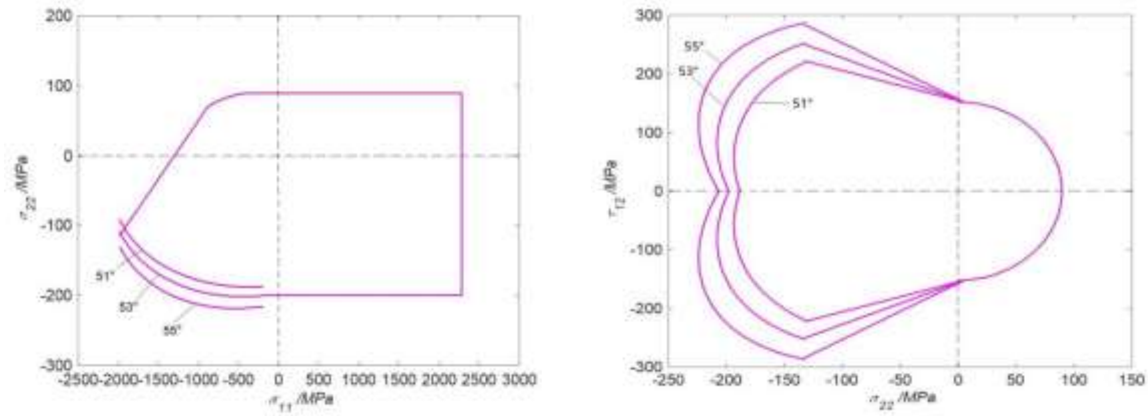


Fig. 3. The theoretical failure envelopes of AS4/PEEK under $\sigma_{22} - \tau_{12}$.



(a) (b)

Fig. 4. Failure envelop for LaRC03 with different α_0: (a) under $\sigma_{11} - \sigma_{22}$ (b) under $\sigma_{22} - \tau_{12}$ loading condition.

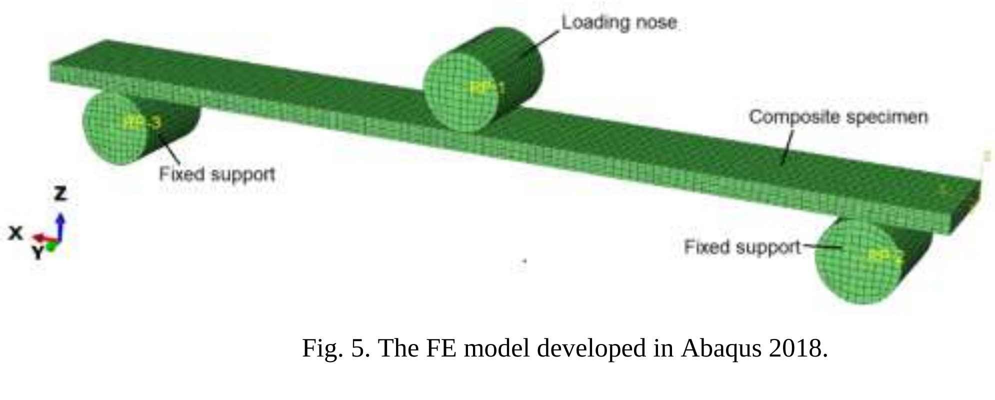


Fig. 5. The FE model developed in Abaqus 2018.

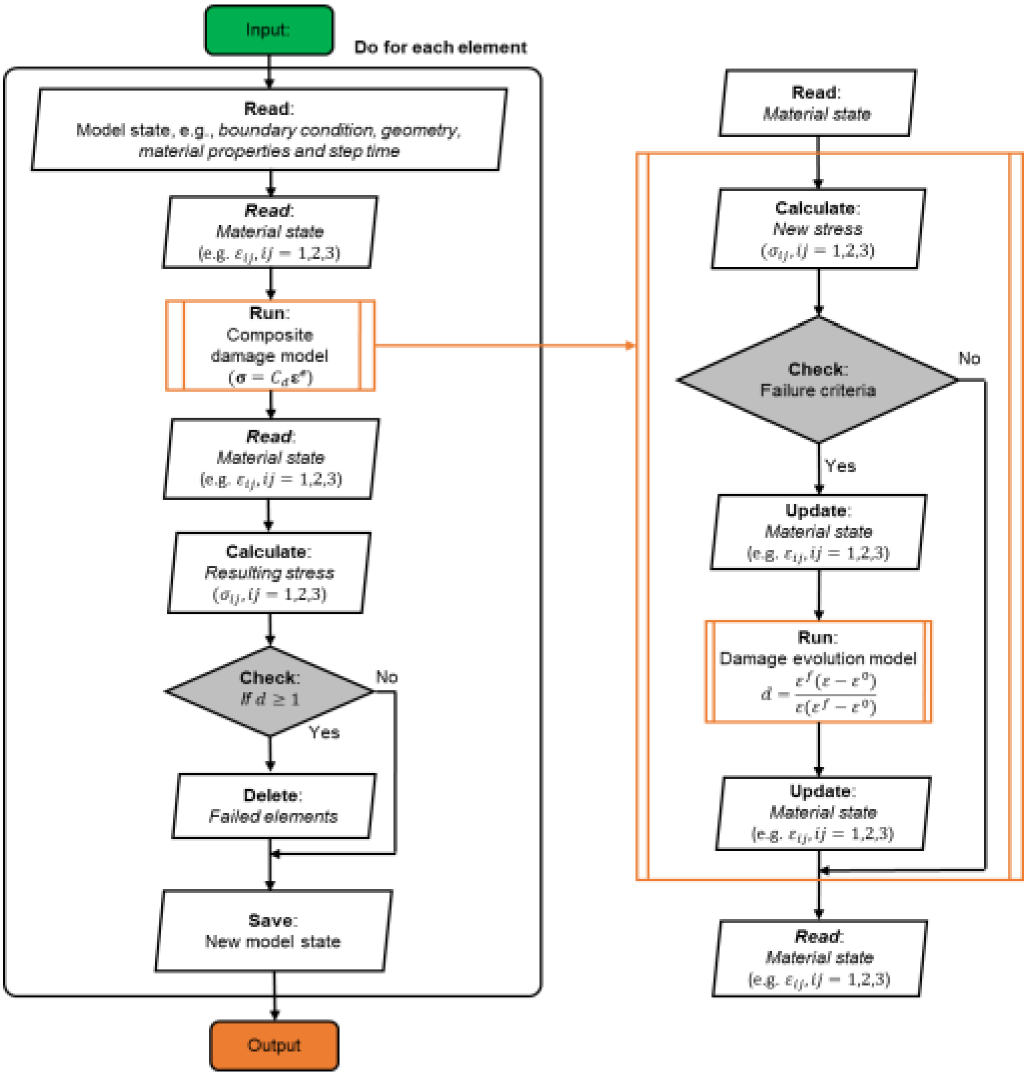


Fig. 6. The flowchart of the main model (left) and the highlighted flowchart of incorporating the failure criteria (right).

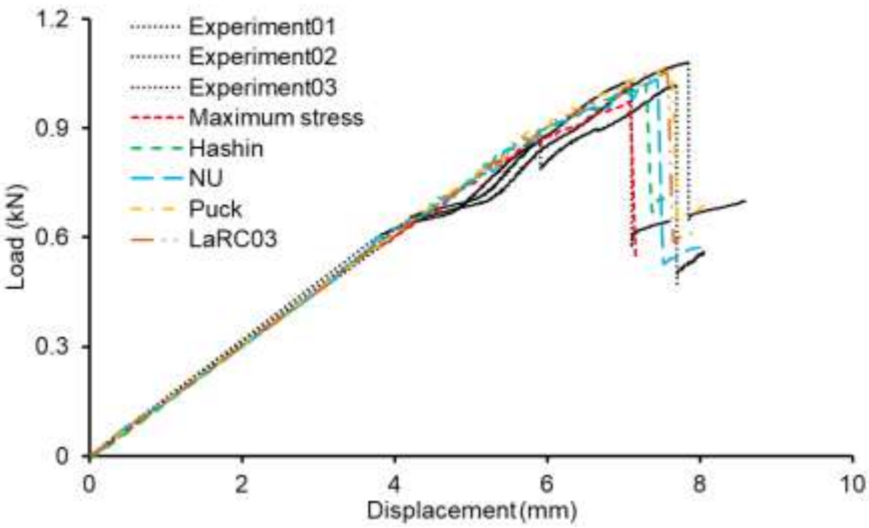


Fig. 7. Comparison of the experimental and predicted loading response for three-point bend tests.

Tables

Table 1. The ability of different failure criterion.

Criteria	Max-Stress (Strain)	Hashin	Puck	LaRc03	NU
Year developed	1957	1973	1998	2003	2008
Fibre tension	√	√	√	√	√
Fibre compression	√	√	√	√	√
Matrix tension	√	√	√	√	√
Matrix compression	√	√	√	√	√
σ_{22} - τ_{12} interaction			√	√	√
Fracture plane			√	√	
Fibre kinking				√	
In-situ strength				√	
Strain rate					√

Table 2. Engineering elastic constants and mechanical properties of AS4/PEEK [44-47].

Properties	Values
Moduli (GPa)	$E_{11} = 127.6;\ E_{22} = 10.3;\ G_{12} = 10.3$
Poisson's ratios	$\nu_{12} = 0.32$
Strengths (MPa)	$X_T = 2280;\ X_C = 1300;\ Y_T = 86;\ Y_C = 200;\ S_{12} = 152$

Table 3. Comparison of maximum load predicted with different failure criteria.

Failure criterion	Maximum stress	Hashin	NU	Puck	LaRC03	Test result
Maximum load (kN)	0.98	1.02	1.03	1.06	1.05	1.01 - 1.07
Running time (hour)	16	17	16	26	28	/