The Development of Multiple Criteria Decision Making Methods with Applications to the Selection Problem in Mining and Mineral Processing

Fernando

Department of Earth Science and Engineering
Imperial College London

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Doctor of Philosophy

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To my father, mother, mother in law and my wife who always remind me that "the fear of the LORD is the beginning of knowledge, but fools despise wisdom and instruction" (Proverbs 1:7).

O lavdate Dominvm! - o, praise the Lord!
Praedicate Devm - give notice of the God
Amate creatorem, - love the Creator,
Qvi creavit mvndvm - who created the world
O lavdate Dominvm! - o, praise the Lord!

Jesvs Christvs in crvce - Jesus Christ at the cross
De vita decessit dolens - painful he departed this life
Sed de morte resvrrexit, - but from the death,
 lvx mvndi nova - a new light for the world arose

O lavdate Dominvm! - o, praise the Lord!
Filivm Jesvm Christvm - and His Son Jesus Christ
Omnivm redemptorem - the redeemer of everyone
Et Spiritvm Sanctvm! - and the Holly Spirit
O lavdate Dominvm! - o, praise the Lord!

(Michael Weikath, 1998)
The only certainty is uncertainty.
(Gaius Plinius Secundus, AD 23/24–79)
Declaration

I hereby confirm that the work in this thesis is my own and I give appropriate references and citations whenever I refer to, describe or quote from the work of others, whether published or unpublished.

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Fernando
November 2020
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Working on the PhD has been a challenging and overwhelming experience. The completion of my PhD thesis would not have been possible without the help and support of many key individuals.

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Abstract

Over the past decade, decision makers in the mining and mineral processing industry have had to deal with multiple challenges such as varying metal prices, depleting resource quality, and complying with stricter environmental regulations. Besides, the mining and mineral processing industry also needs to consider the use of renewable energy in order to comply with strict environmental regulations. In addition, since the use of renewable energy has costs associated to the additional equipment installation and maintenance, optimising the equipment selection to minimise energy costs is of vital importance. Therefore, decision makers in the sector often face problems that involve multidisciplinary knowledge that take into account technical, social, economic and environmental aspects.

Multiple Criteria Decision Making (MCDM), which is a part of operations research, has become extremely useful to overcome a variety of decision making problems in mining and mineral processing. Although a very large number of MCDM methods have been developed, the effectiveness of these methods still depends on the objective of the decision making process (i.e. selection, sorting, ranking, description) and the nature of the problem.

An exhaustive literature review on the applications and trends of MCDM for the selection problem in mining and mineral processing has been conducted in this work. The literature review indicates that conventional MCDM methods have been frequently criticised on several drawbacks including its inability to quantify the uncertainty in data and information, the occurrence of the rank reversal phenomenon, and its difficulty in aggregating several judgements and preferences from multiple decision makers including related uncertainty on their judgements or preferences, as well as the robustness of MCDM methods in dealing with non-homogenous data (i.e. quantitative and qualitative).

This thesis presents the development of four new MCDM methods and their application to the selection problem (i.e. determining the best option from a set) in mining and mineral processing by taking into account the role of uncertainties in the decision making process. This work mainly demonstrates the value of applying the concept of constrained fuzzy arithmetic in fuzzy extension of conventional MCDM methods when the input data that need to be analysed are difficult to define precisely. In order to showcase the capability of the developed methods, three case studies on the selection problem in the mining industry were conducted. Furthermore, the robustness of the developed methods are shown by conducting sensitivity analyses and comparing their results to those obtained from existing methods.
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Nomenclature

Acronyms / Abbreviations

AHP        Analytic Hierarchy Process
ANP        Analytic Network Process
CFAHP      Constrained Fuzzy Analytic Hierarchy Process
CI         Consistency Index
CR         Consistency Ratio
CSP        Concentrated Solar Power
CTL        Coefficient of Technical Level
DEMATEL    Decision Making Trial and Evaluation Laboratory
ELECTRE    ELimination Et Choix Traduisant la REalité (ELimination and Choice Expressing REality)
FAHP       Fuzzy Analytic Hierarchy Process
FMCMDM     Fuzzy Multiple Criteria Decision Making Method
FPCM       Fuzzy Pairwise Comparison Matrix
FAHP       Fuzzy Shannon Entropy
FAHP       Fuzzy Set Theory
FTOPSIS    Fuzzy Technique for Order of Preference by Similarity to Ideal Solution
GHG        Greenhouse Gas
GIS        Geographical Information System
IC-FSAHP   Integrated Constrained Fuzzy Stochastic Analytic Hierarchy Process
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<td>IC-FSE</td>
<td>Integrated Constrained Fuzzy Shannon Entropy</td>
</tr>
<tr>
<td>LEC</td>
<td>Levelised Energy Cost</td>
</tr>
<tr>
<td>MADM</td>
<td>Multiple Attribute Decision Making Method</td>
</tr>
<tr>
<td>max</td>
<td>maximum</td>
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<tr>
<td>MCDM</td>
<td>Multiple Criteria Decision Making Method</td>
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<td>MCGDM</td>
<td>Multiple Criteria Group Decision Making Method</td>
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<td>ME</td>
<td>Mining Equipment</td>
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<td>min</td>
<td>minimum</td>
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<td>MM</td>
<td>Mining Method</td>
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<td>MODM</td>
<td>Multiple Objective Decision Making Method</td>
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<td>MOORA</td>
<td>Multi-Objective Optimisation Method by Ratio Analysis</td>
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<td>MPE</td>
<td>Mineral Processing Equipment</td>
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<td>MPM</td>
<td>Mineral Processing Method</td>
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<td>MPOP</td>
<td>Mineral Processing Operation Parameter</td>
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<td>Mineral Processing Plant Location</td>
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<td>MSL</td>
<td>Mining Site Location</td>
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<td>MT</td>
<td>Mining Technology</td>
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<td>NV</td>
<td>Normalised Vector</td>
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<td>OW</td>
<td>Onshore Wind</td>
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<td>PCM</td>
<td>Pairwise Comparison Matrix</td>
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<td>PROMETHEE</td>
<td>Preference Ranking Organisation Method for Enrichment Evaluation</td>
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<td>PV</td>
<td>Photovoltaic</td>
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<td>SE</td>
<td>Shannon Entropy</td>
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<td>TFN</td>
<td>Triangular Fuzzy Number</td>
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<td>TODIM</td>
<td>TÔmada de Decisão Interativa e Multicritério (Interactive and Multicriteria Decision Making)</td>
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Nomenclature

TOPSIS  Technique for Order of Preference by Similarity to Ideal Solution
VIKOR  VIsekriterijumska optimizacija i Kompromisno Rešenje
WASPAS  Weighted Aggregated Sum Product Assessment
WSM  Weighted Sum Model
Chapter 1

Introduction

1.1 Research background

The success of a mining and mineral processing operation or project is not dominated by a single parameter (i.e. a technical or economic consideration). Decision makers in mining and mineral processing often face complex problems that involve multidisciplinary knowledge (e.g. technical, environmental, social and economic aspects) and complicated challenges (e.g. varying metal prices, depleting resource quality, stricter customer requirements, complying to stricter environmental regulations). It is thus often difficult to systematically assess and solve a decision making problem.

A holistic analysis that incorporates quantitative and qualitative elements should be used to derive a reliable solution in the decision making process. This holistic analysis must be carried out in a scientific manner rather than depending solely on intuition and experience. Failure to do so may not directly affect the project in the early years; however, the accumulation of this latent impact may harm several aspects in the operation stage of the project, such as resource utilisation, productivity and cost efficiency, and even in the post-production stage, such as environmental responsibility, which is much more difficult to manage. Thus, a holistic analysis is one of the key factors for the sustainability of mining and mineral processing projects.

One way to perform a holistic analysis is to apply multiple criteria decision making (MCDM). MCDM is a part of operations research that helps decision makers to resolve problems when multiple conflicting criteria are involved and need to be evaluated. Since the 1970s, MCDM research has developed immensely, leading to MCDM methods having been widely used in many fields, as evidenced by recent literature reviews on: energy, supply chain management, material, and quality management [1], natural resources management [2], environmental [3], chemical engineering [4], engineering [5], manufacturing [6], civil engineering [7, 8], software development [9], economics [10], e-learning systems [11], mining and mineral processing [12], and renewable energy [13].
MCDM has been designed to overcome four types of problems [14, 15]:

1. The choice problem, in which MCDM is used to select the best option from a set of alternatives.

2. The sorting problem, in which MCDM is used to assign a set of alternatives to categories that have been designed a priori.

3. The ranking problem, in which MCDM is used to order the alternatives partially or completely.

4. The description problem, in which MCDM is used to elaborate the alternatives, build a set of criteria and determine the performance of all or some alternatives taking into account additional information.

More than a hundred MCDM methods have been developed, the number rapidly growing since many of the original MCDM methods have been modified by the original authors of the work or by other researchers [16]. Such modifications often try to overcome different challenges for specific applications in real-life problems that require enhanced methods. Although a large number of methods have been developed, their effectiveness on the decision making process is dependent on the characterisation of the decision making problems, with no single MCDM method able to solve all problems.

An MCDM method usually involves six stages [17, 15, 18], namely: (i) selecting the criteria; (ii) weighting the local importance of each criterion with respect to the goal, which is of critical importance as it affects the subsequent stages; (iii) scoring the local preference of each alternative with respect to criteria; (iv) calculating the global weighted scores of alternatives; (v) ranking of the feasible alternatives based on the global weighted scores; (vi) selecting the most suitable alternative, i.e. the highest global weighted score.

There are two main groups of methods to derive the criteria weights and alternatives’ scores in MCDM, namely subjective and objective methods. The subjective method relies on decision makers’ judgements and can be achieved by means of pairwise comparisons of qualitative or quantitative data. The Analytic Hierarchy Process [18] is the most frequently used subjective weighting method. The objective methods, on the other hand, are derived from the computation of quantitative data without involving any consideration of decision makers’ judgements. The Shannon Entropy (SE) method [19] is one of the most frequently used objective methods for deriving the criteria weights.

Conventional AHP and SE have been criticised on their inability to quantify uncertain data or information [20, 21]. In addition, conventional AHP has been criticised on the following limitations:

- the occurrence of the rank reversal phenomenon, which is caused by the addition or deletion of an alternative [22],
the lack of ranking discrimination, which is responsible for their failure to determine the best alternative when scores of two or more alternatives are too close [23],

- its inability to handle agreement or disagreement among decision makers [24],

- its difficulty in assigning robust criteria weights and alternatives’ scores that integrate subjective preferences and objective information [25].

A vast amount of research has been carried out to overcome the aforementioned limitations, particularly the limitation due to uncertainty. Three types of approaches have been attempted to overcome the uncertainty:

- Appropriate sensitivity analysis has been applied for a crisp deterministic MCDM method [26].

- Linguistic approximation including fuzzy sets and systems [27], grey systems [28], and rough sets [29] have been applied, thus combining the linguistic method and an MCDM method [30].

- A stochastic approach, such as the use of Monte Carlo simulations, has also been employed to reduce decision uncertainties [31].

In the case of rank reversal, several approaches for avoiding this phenomenon have been proposed by Belton and Gear [22] and Leskinen and Kangas [32]. Similarly, several methods for enhancing the discrimination of alternatives have been developed for the cases in which it is difficult to make a decision due to scores of two or more alternatives being too close [23] and a variety of consensus approaches have been used to handle agreement or disagreement between decision makers [33, 34]. Ma et al. [35] and Wang and Lee [36] have proposed methods that combine subjective and objective approaches for weighting criteria, which have been applied in several cases in order to enhance the accuracy of weights.

However, studies to develop one extended MCDM method which might overcome the aforementioned drawbacks for the selection problem are rare. Each solution has only solved a particular limitation. There is therefore a need for MCDM methods that are able to analyse complex problems with multiple conflicting criteria under uncertainty to determine the most suitable solution to all decision makers.

1.2 Research aims and objectives

The aim of this research is to develop robust MCDM methods for the selection problem in mining and mineral processing. To this end, the following research studies and objectives are critically considered:
1. A comprehensive literature review and survey on the application of MCDM for the choice problem in mining and mineral processing.
   The objective is to understand and critically evaluate the current application of MCDM methods and tools for the choice problem in mining and mineral processing. Eight electronic databases were selected to provide a comprehensive application of MCDM for the choice problem in mining and mineral processing, namely Web of Science, Scopus, Elsevier Science Direct, Taylor and Francis Online, Springer Link, Wiley Online Library, The Institute of Materials, Minerals, and Mining (IoM3) and Google Scholar. Only articles published in academic journals by these eight databases have been considered.

2. The development of robust MCDM methods.
   The objective is to develop robust MCDM methods for the selection problem, so a chosen alternative might be applied efficiently considering technical, social, economic and environmental criteria. The modified method must overcome the shortcomings of existing MCDM methods identified in the literature. Thus, the confidence that decision makers place on the decision outcomes would increase.

3. The application of the developed MCDM methods to case studies.
   The aim of this study is to examine the usability of each of the developed MCDM methods to solve a specific case of the selection problem in mining and mineral processing. The capability of applying the developed MCDM methods was explored.

4. The robustness analyses of the developed MCDM methods.
   The objective is to demonstrate the robustness of the developed MCDM methods via sensitivity analysis and results comparison against other existing methods.

1.3 Thesis layout

This PhD thesis comprises six chapters which present the main findings of this doctoral project followed by one chapter that summarises the key conclusions and future work directions. Chapter 1 states the research objectives and associated research issues of this study whilst chapters 2–6 address the research objectives. Chapter 7 summarises the conclusions, discusses the broader implications of this study and recommends possible future work.

Chapter 1 presents an overview of this study. The motivation, research objectives, research questions, and research hypotheses are identified and discussed. In addition, the thesis layout and publication status of this work are outlined in this chapter.

Chapter 2 presents an extensive literature review on the applications and trends of MCDM methods for the choice problem in mining and mineral processing. It focuses on the following key topics: (i) overview of MCDM methods; (ii) growth in published articles based on publication year; (iii) distribution by publication journal; (iv) distribution by application area;
(vi) distribution by MCDM methods; (vii) MCDM software used; (viii) analysis of geographic distribution of MCDM papers; (ix) critical remarks; and (x) future research directions.

Chapter 3 documents the development of a new MCDM method for the choice problem, the integrated constrained fuzzy stochastic analytic hierarchy process (IC-FSAHP). IC-FSAHP combines constrained Fuzzy AHP (FAHP), Fuzzy Set Theory (FST), a modified beta-PERT distribution, and Monte Carlo simulation. IC-FSAHP takes advantage of the characteristics of the aforementioned approaches to overcome the drawbacks and concerns of conventional AHP for the selection problem by multiple decision makers under uncertainty. The methodological development of IC-FSAHP is outlined in this chapter. This chapter also presents an investigation into the performance of IC-FSAHP through a series of computational experiments. First, IC-FSAHP was applied to a case study in the equipment selection problem in mineral processing. Second, the comparative analysis between the results that were obtained from IC-FSAHP and those obtained from other existing methods (i.e. AHP and FSAHP) was carried out. Third, the suitability of IC-FSAHP under different levels of disagreement among decision makers was studied. Fourth, the performance of IC-FSAHP with regard to the number of decision makers and uncertainty as well as rank discrimination was assessed. Fifth, the performance of IC-FSAHP on the occurrence of rank reversal due to the addition and deletion of one alternative as well as the use of other preference scales, including its effects on the selection problem, was studied.

Chapter 4 showcases a new comprehensive sensitivity analysis method that takes into account the main notions of IC-FSAHP. The concept of the developed sensitivity analysis method is provided in this chapter. The developed method was applied to examine the effects of changing the uncertainty levels of the judgements in the Fuzzy Pairwise Comparison Matrices (FPCMs) and disagreement among decision makers under the various uncertainty levels of the judgements. A case study for the selection of mineral processing equipment using IC-FSAHP was conducted, the results of which were used to perform the sensitivity analysis proposed.

Chapter 5 presents the development of a new criteria weighting method: the Integrated Constrained Fuzzy Shannon Entropy (IC-FSE). IC-FSE can be used to determine criteria weights when the input quantitative data that need to be analysed are difficult to define precisely, and thus need to be presented in fuzzy numbers. IC-FSE combines the concepts of constrained fuzzy arithmetic, Triangular Fuzzy Number (TFN), and the Shannon Entropy method. The applicability of IC-FSE in weighting the sustainability criteria of renewable energy technologies is also showcased in this chapter. The reliability, superiority and robustness of IC-FSE is demonstrated through the following computational experiments: first, the reliability of IC-FSE compare to stochastic methods is demonstrated; second, the superiority of the normalisation procedure used in IC-FSE is presented by comparing it to common existing procedures; third, the robustness of IC-FSE is discussed via comparing the results obtained from IC-FSE to those obtained from the only other existing FSE method.
Chapter 6 presents an integrated MCDM method that considers both quantitative and qualitative data under uncertainty in the context of group decision making. The integrated method combines a subjective method (i.e. Integrated Constrained Fuzzy Stochastic AHP (IC-FSAHP)) and objective methods (i.e. Integrated Constrained Fuzzy Shannon Entropy (IC-FSE), Normalised Vector (NV) and Weighted Sum Model (WSM)). The capability of the integrated method developed is showcased and the results obtained from the method are analysed. A methodology for the carrying out of sensitivity analysis by varying the coefficient factors of subjective and objective weights, including the assessment of its results, is presented in this chapter.

Chapter 7 summarises the main findings of this thesis in the context of the project aim and objectives, and provides suggestions for future research directions.

1.4 List of publications

This thesis contains chapters edited from five peer-reviewed journal publications completed during this PhD project. Whilst the previous section describes the links between the chapters, chapters 2–6 are presented as stand-alone journal style manuscripts in this thesis. The author of this thesis is the first author on all five manuscripts, and is responsible for all the observations, interpretations and discussion therein.


5. Chapter 6 - Sitorus, F., Brito-Parada, P.R.,. The selection of renewable energy technologies using subjective and objective multiple criteria decision making methods. J Clean Prod (under review).
Chapter references


Chapter 2

Multi-Criteria Decision Making for the Choice Problem in Mining and Mineral Processing: Applications and Trends

Abstract

Despite the fact that the potential of multi-criteria decision making (MCDM) to overcome a variety of problems in mining and mineral processing has been widely recognised, no literature review in these fields has been conducted. This manuscript addresses this issue by providing a comprehensive overview of the applications and trends of MCDM methods for the choice problem (i.e. determining the best option from a set) in mining and mineral processing. 90 articles published between 1999 and 2017 were selected following a searching methodology and eligibility criteria detailed in this manuscript. In addition, for the purpose of the survey, different types of selection problems were identified. The results show that there are two phases of growth in the application of MCDM techniques to the choice problem in mining and mineral processing. The first phase, from 1999 to 2007, shows a very low number of publications with only a moderate increase by the end, whereas the second phase, from 2007 to 2017, shows a significant growth in the number of published articles. The review also shows that the most addressed problem has been the selection of mining methods, while the Analytical Hierarchy Process (AHP) has been the most used MCDM method. The rise in the application of hybrid MCDM methods is also discussed. This review paper provides insight into the current state of applications of MCDM in mining and mineral processing and discusses pathways for future research directions in the development of MCDM methods that would benefit these fields.
2.1 Introduction

Decision makers in the mining and mineral processing industry often face complex problems; solving these problems frequently involves multidisciplinary knowledge including technical, economic, environmental and social aspects, as well as politics and regulations. The complexity arises because the decision maker must take into account various objectives and criteria from different stakeholders. The decision maker must also consider various risks associated to geological data, mining methods, new technology, land acquisition, resource nationalism, commodity prices, and market conditions, for which information is either difficult to obtain or not always accurate. In addition, the imprecise nature of the information may lead to a decision made under uncertainty.

Many trade-offs should be considered in the decision-making (DM) process in a complex problem under conflicting multiple criteria and uncertainty. The decision maker needs a tool that incorporates both quantitative and qualitative analyses in a scientific manner rather than depending solely on intuition and experience. Failure to do so may not directly affect the project in its early years; however, the accumulation of this latent impact may harm several aspects in the operation stage, such as resource utilisation, productivity of the operation, and cost efficiency, and even in the post-mining stage, such as environmental responsibility, which is much more difficult to manage. Incorporating quantitative and qualitative analyses prudently and appropriately is thus a key factor for the sustainability of mining and mineral processing projects and the decision maker should therefore make use of the best tools available to inform this process.

Multi-Criteria Decision Making (MCDM) is a part of operations research that supports the decision maker to resolve problems when multiple conflicting criteria are involved and need to be evaluated. MCDM is a practical and powerful tool that may be used either under certainty or uncertainty and that facilitates the incorporation of quantitative and qualitative analyses in a scientific manner.

Since the 1970s, MCDM research has developed immensely, leading to MCDM techniques having been widely applied in many fields, as evidenced by recent literature reviews on:

- Energy, supply chain management, material, quality management, etc [1]
- Natural resources management [2]
- Environmental [3]
- Chemical engineering [4]
- Engineering [5]
- Manufacturing [6]
- Civil engineering [7, 8]
2.1 Introduction

- Software development [9]
- Economics [10]

According to Roy [12, 13], MCDM has been designed to tackle four types of problematics:

1. The choice problematic, in which MCDM is used to select the best option from a set of alternatives.

2. The sorting problematic, in which MCDM is used to assign a set of alternatives to the categories that have been designed a priori.

3. The ranking problematic, in which MCDM is used to order the alternatives partially or completely.

4. The description problematic, in which MCDM is used to elaborate the alternatives, build a set of criteria and determine the performance of all or some alternatives taking into account additional information.

It is relevant to note that for this review, the term problem is used instead of problematic. This survey will focus on the choice problem because this problem has a relatively higher impact in terms of cost and benefit in mining and mineral processing. An incorrect decision in selecting the best alternative in this industry may result in considerable losses during operation – a situation that is difficult to fix.

Approximately three hundred papers in the application of MCDM methods in mining and mineral processing conclude that MCDM provides a significant improvement in the DM process to resolve the aforementioned problems. Despite the fact that the application of MCDM methods for the selection of the best alternative in mining and mineral processing has been growing since 1999, no comprehensive survey has previously been conducted to review the specific applications, trends and challenges. The aim of this manuscript is therefore to survey the state of the art of the use of MCDM methods for the choice problem in mining engineering and mineral processing.

For the purpose of this survey, four main applications of MCDM for the choice problem in mining engineering have been identified: mining equipment selection, mining method selection, mining technology selection, and mining site selection; in addition, other selected relevant applications that fall outside the aforementioned ones are also discussed. Similarly, four applications of MCDM for the choice problem in mineral processing are considered, namely the selection of: the location of the processing plant location, the processing equipment, the processing method, and the operating parameters. This review will outline the type of study carried out (i.e. case study, concept or both), discuss the problem addressed, outline the MCDM method and the software employed. It will also provide information on relevant
journals where work in the area has been published as well as statistics on the geographical origin of the work and the number of scientific outputs by year. It is expected that the information provided in this study will be useful for researchers and practitioners, both users and potential users of MCDM in mining and mineral processing.

The remainder of the manuscript is organised as follows: Section 2.2 provides a technical overview of MCDM methods; Section 2.3 describes the methodology used in this survey; Section 2.4 presents the results of the survey; Section 2.5 discusses the results by field and application as well as critical remarks, and Section 2.6 provides conclusions and future research directions.

2.2 Overview of MCDM methods

2.2.1 MCDM classification

MCDM methods can be classified in two categories [14]:

- Multiple attribute decision making (MADM)
  MADM is mostly applied for selection problems and is always associated with a limited number of alternatives and preference ranking (i.e. the decision space is discrete). A finite number of proposed alternatives is evaluated with respect to different weighted attributes and a preference ranking is obtained that describes the performance of each alternative to meet the objective with respect to the attributes.

- Multiple objective decision making (MODM)
  MODM is generally used for design or planning, where the number of alternatives is infinite (i.e. the decision space is continuous). The aim of MODM is to design the optimal alternative by considering the various interactions within the given constraints. By attaining satisfactory levels of several objectives, the best alternative is obtained.

In terms of the type of data and information that the method uses, Kahraman [15] classified MCDM as:

- Crisp MCDM methods
  Used when all relevant information and data are fully available and these are precise, so the decision maker has sufficient knowledge of the decision circumstances.

- Fuzzy MCDM (FMCDM)
  Used when some or all information and data are not clearly defined. This may be due to unquantifiable, incomplete or unobtainable information, or due to lack of knowledge [16].
2.2 Overview of MCDM methods

2.2.2 General stages of MCDM

In general, a decision making process to tackle MCDM problems involves three stages [13, 17–19], namely:

1. Structuring a decision problem
   This stage should plan the type of actions that should be taken for each of the following activities [17]:
   - identifying decision maker(s);
   - defining the goal(s);
   - analysing the feasible alternatives that will be evaluated;
   - determining the criteria for evaluating the consequences of each alternative.

   The aforementioned activities in this stage are similar among all MCDM methods [19] and provide information that is then used as the basis for determining what type of MCDM method can be applied [13, 18] or to develop a new method to solve the problem.

2. Determining and applying an MCDM method
   In order to control the applicability of MCDM methods to the problem, an MCDM method should not be chosen before the decision problem has been structured [13, 18]. This stage can cover the following activities:
   - weighting the importance of each criterion with respect to the goal;
   - scoring the preference of each alternative with respect to criteria;
   - calculating the overall weighted scores of alternatives;
   - ranking of all feasible alternatives based on the overall weighted scores.

   The differences among MCDM methods lie in this second stage [19].

3. Determining the final recommendation
   The higher the overall weighted score is the more preferable the alternative will be [17]. The obtained results should be examined further by performing a sensitivity analysis that can be used to answer "what-if" questions that decision makers might have [19, 17].

2.2.3 MCDM methods

Over the years, numerous MCDM methods have been proposed in the literature; these methods are different in the type of research questions they aim to address, the type of problem, the theoretical background, and type of outcomes obtained. There is no particular MCDM method that can be applied to all types of problems, since methods have been
designed for specific cases, with their associated benefits and limitations. As part of this review, a brief description of the most relevant methods to the choice problem is presented, including some key references that cover in depth discussions of each methodology and their applications.

### 2.2.3.1 Analytic Hierarchy Process (AHP)

The AHP, originally designed by Saaty [20], provides a systematic process to incorporate factors such as logic, experience or knowledge, emotion, and a sense of optimisation into a decision making methodology. This method simplifies a multi-criteria complex problem into a hierarchy structure; according to Saaty and Vargas [21], hierarchy is defined as a representation of a complex problem in a multi-level structure where the first level is the goal/aim/objective, followed by sub-levels, criteria, and sub-criteria, and down to the last level of the alternatives. With this approach, a complex problem can be deconstructed into sections and then arranged into a form of hierarchy so that the problem will appear more structured and systematic.

The AHP method comprises four main stages [22]. First, structuring the model into a hierarchy; second, conducting the comparative judgment of the criteria, sub-criteria, and alternatives with respect to their importance through pairwise comparisons; third, summarising the result of the pairwise comparisons in an evaluation matrix; finally, synthesising the order of preferences of the alternatives that are obtained from the normalised evaluation matrix. A detailed description of the AHP method can be found in Saaty and Vargas [21]. With over thirty-five years of existence, AHP has been applied in a broad range of application areas [23]) and its methodology has been further developed in order to overcome some limitations, particularly in problem modelling, pair-wise comparisons, judgement scales, derivation methods, consistency indices, incomplete matrix, synthesis of the weights, sensitivity analysis and group decisions [24–28].

The Analytic Network Process (ANP) is a generalisation of the AHP that deals with dependencies [29]. Many real-life MCDM problems might involve the interaction and dependency between different criteria, as well as between different sub-criteria in the form of the inner and outer dependencies, or in the form of feedbacks from alternatives to criteria [29]. The ANP method allows modelling all these interactions, dependencies and feedbacks between the aforementioned elements in the network [30, 31].

The ANP method comprises four main stages [31]. First, structuring the model into a network. Second, performing the pairwise comparisons on each cluster with respect to their importance. The comparative judgment among criteria of a cluster must also be examined pairwise. Third, developing a super matrix according to the network that reflects a relationship between two clusters in a system. Finally, synthesising the order of preferences
of the alternatives that are attained from the normalised super matrix. The wide application of the ANP in real life problems was summarised by Saaty and Vargas [32].

2.2.3.2 Elimination Et Choix Traduisant la Réalité (ELECTRE) Family

The ELECTRE was initially created in the 1960s [33, 34] as a response to limitations of existing decision-making methods for resolving the choice problem. ELECTRE overcomes those limitations by introducing the concept of outranking relation [35]. The outranking relation [33] means a complete order of alternatives in ELECTRE are modelled by using pairwise comparisons among alternatives under each one of the criteria separately [36].

Since the introduction of the method, eight further variations have been applied for solving MCDM problems, namely ELECTRE I, IS, Iv, II, III, IV, III-H and Tri. All these methods were developed on the same fundamental concept but differ in their stages. Each of the ELECTRE family methods has a specific function regarding the type of problem. The ELECTRE I [34] was originally developed for solving the choice problem. By introducing an indifference threshold and a veto threshold into the ELECTRE I, the ELECTRE IS [37] and the ELECTRE Iv [36] were developed, respectively. The ELECTRE II [38], which is an updated version of the original, was designed to overcome the ranking problem. The main difference between ELECTRE I and ELECTRE II is the way in which the outranking relation is defined; ELECTRE II defines two outranking relations, namely the strong outranking and the weak outranking, whereas ELECTRE I consider only one type of outranking. The ELECTRE III [39], which can be considered a fuzzy outranking relation, was developed to deal with the ranking problem, while the ELECTRE IV [40], which is a variant of the ELECTRE III, was developed to deal with situations in which no weights of the criteria are introduced. The weights of the criteria are not introduced is in either of these because they are difficult to define or because decision makers do not want to determine them. In terms of developing outranking relations, the ELECTRE III and ELECTRE IV methods both use different areas of preference (i.e. strictly, weakly, hardly, and indifferent). However, a main difference between both methods lies in their distillation procedures. In ELECTRE IV, the number of criteria in different areas of preference is used, whereas in ELECTRE III, a value of a membership function is used. In addition, for the case when decision makers face a problem with hierarchical structure of criteria, ELECTRE-III-H [41, 42] was developed in order to help defining a local preference model at each level of the hierarchy. The ELECTRE Tri [43], ELECTRE Tri-C, [44] and ELECTRE Tri-nC [45], which are the most recent methods of the ELECTRE family, were developed to overcome the sorting problem. A detailed description of the ELECTRE family was described by Roy [46], and their methodologies were presented in detail by Figueira et al. [36, 47]. A recent review of applications of the ELECTRE family methods can be found in Govindan and Jepsen [48].
2.2.3.3 Preference Ranking Organisation Method for Enrichment Evaluation (PROMETHEE) Family

The PROMETHEE method, which was initially proposed by Brans [49], is another outranking method for a finite set of alternatives that is to be ranked and selected. The original method was further extended by Brans and Vincke [50]. A finite set of predetermined alternatives are evaluated under multiple criteria. Each independent criterion is weighted, and an appropriate preference function should be selected. The preference function describes the difference between the evaluations of an alternative to another into a preference degree [51].

Since its introduction, six methods developed within the PROMETHEE family have been applied for solving MCDM problems. Similarly to the ELECTRE family, each of the PROMETHEE methods has a specific role with respect to the type of problem.

- **PROMETHEE I** [49] was initially designed for partial ranking of the alternatives.
- **PROMETHEE II** [52, 50, 51] was developed to provide a complete ranking of the alternatives. An extension of the PROMETHEE II method, namely the PROSA (PROMETHEE for Sustainability Assessment) [53] method was developed in order to obtain a lower degree of criteria compensation.
- **PROMETHEE III** [52] was developed to enhance indifferences in order to rank the alternatives based on the overlapping intervals. The overlapping intervals are obtained from the computation of interval flows.
- **PROMETHEE IV** [52] was designed for complete or partial ranking of the alternatives when the set of viable solutions is continuous.
- **PROMETHEE V** [54] was developed for a continuous problem by means of using constraints to maximise the total outranking flow of the alternatives.
- **PROMETHEE VI** [55] allows the preferences of the decision maker to be included, therefore this method allows variations in the criteria weights.

A detailed description of the methodology of the PROMETHEE family methods can be found in Brans and Mareschal [56]. Behzadian et al. [57] extensively reviewed the literature on the PROMETHEE methodologies and their applications. It is worth noting that like the ELECTRE method, an extension of the PROMETHEE method to a hierarchical form, namely the hierarchical PROMETHEE method [41] was developed in order to assist decision makers when defining the preference relations in a particular level of the hierarchy.
2.2 Overview of MCDM methods

2.2.3.4 Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)

TOPSIS, which was originally developed by Hwang and Yoon [14], is another well-known MCDM method that evaluates the performance of alternatives based on the distance from the ideal solution. The preferred alternative must have the shortest distance from the positive ideal solution and the longest distance from the negative ideal solution.

Hwang and Yoon [14] proposed six main stages for implementing TOPSIS. First, a decision matrix is developed and normalized. Second, a weighted normalized decision matrix is constructed. Third, the positive and negative ideal solutions are determined. Fourth, the separation measures for each alternative are calculated. Fifth, the relative closeness to the ideal solution is calculated. Finally, the alternatives are ranked from best to worst according to the closeness to the ideal solution. Behzadian et al. [58] provide a useful insight into the TOPSIS methodologies and its applications, while an outline of the development of the TOPSIS was presented by Zyoud and Fuchs-Hanusch [23].

2.2.3.5 Visekriterijumska optimizacija i Kompromisno Rešenje (VIKOR)

The VIKOR method, which was proposed firstly by Opricovic [59], was originally designed to deal with the ranking and the choice problems. The idea of the VIKOR method is to determine the best alternative, which is a compromise solution, among feasible alternatives considering conflicting criteria based on the closeness to the ideal solution. The compromise solution is the one with the closest distance to the ideal solution. An in depth description of the procedure used on the VIKOR methods was presented by Opricovic and Tzeng [60].

Since its introduction, the use of VIKOR to real world problems has been substantial, which is reflected in recent literature review manuscripts on the topic. The VIKOR and its wide applications was surveyed by Yazdani and Graeml [61]; a systematic review of methodologies and applications of the VIKOR was provided by Mardani et al. [62]; and a literature review of the VIKOR and its fuzzy extensions on applications can be found in Gul et al. [63]. Other studies have presented a comparative analysis of the VIKOR to other methods. A comparison between VIKOR and TOPSIS was carried out by Opricovic and Tzeng [60], and a comparison of the VIKOR to TOPSIS, ELECTRE, and PROMETHEE was conducted by Opricovic and Tzeng [64].

2.2.3.6 Weighted Aggregated Sum Product Assessment (WASPAS)

The WASPAS method, which was created by Zavadskas et al. [65], is proposed for solving the ranking and the choice problems under conflicting multicriteria. This method was constructed based on the combination of the Weighted Sum Model (WSM) and the Weighted Product Model (WPM) in order to increase the ranking accuracy. The detailed procedure of the original WASPAS and its application were described in detail by Zavadskas et al. [65].
The WASPAS is a recently developed technique in comparison to other conventional MCDM methods such as AHP, ELECTRE and TOPSIS; however, the application of the WASPAS for solving real MCDM problems has rapidly developed since its introduction [66–72]. A study that focuses on the robustness of WASPAS and compares it to other MCDM methods, namely MOORA and MULTIMOORA, for solving real life problems can be found in Zavadskas et al. [73].

2.2.3.7 Hybrid MCDM methods

Employing a single MCDM method is often not enough to overcome real world decision problems. The integration of two or more MCDM methods by utilising the strengths of each one often supports the decision maker to obtain a more reliable result. Belton and Stewart [74] suggest that when integrating MCDM methods one must identify commonalities among the methodologies and distinguish the strengths of each method. Various hybrid MCDM methods have been applied to a wide range of application areas [1, 75, 5]. There are three types of hybrid MCDM methods identified in this survey: an integrated MCDM method with fuzzy sets theory (FST), an integrated MCDM method with other MCDM method(s), and an integrated MCDM with other method(s).

One of the improvement to the techniques is the combination of crisp MCDM methods with fuzzy set theory (FST). In the case when the decision maker faces uncertainty, it is difficult to use crisp MCDM methods to accurately evaluate the relative importance of criteria and sub-criteria, and the performance ratings of alternatives with respect to a criterion. This difficulty arises because the evaluation could only be stated by using a linguistic term instead of crisp or real values. Further development in crisp MCDM methods is required to solve this difficulty. The FST was introduced into the field of MCDM by Bellman and Zadeh [76] and Zimmermann [77] and has been widely applied in some areas [15, 78]. Since the introduction of FST into MCDM (FMCDM), a large number of FMCDM methods have been applied and developed to solve a variety of problems, with promising outcomes. Examples of such FMCDM methods found in the literature include fuzzy AHP (FAHP) [15], fuzzy TOPSIS (FTOPSIS) [15], Yager method [79–81], Fuzzy ELECTRE [82], Fuzzy PROMETHEE [83], an integration of fuzzy DEMATEL, fuzzy ANP and fuzzy VIKOR [84], and an integration of FAHP and fuzzy MOORA [85]. The wide application of FMCDM to real life problems was recently surveyed by Kahraman et al. [78] and Mardani et al. [75].

The integration of an MCDM method with other MCDM method(s), either combined with the FST or not, has been frequently proposed and implemented by researchers. The integration of two or more MCDM methods, which is developed for making more robust method, has been mainly developed for improving weaknesses of one method by adopting the strength of another method to an integrated proposed method [74]. A general feature of
2.3 Survey Methodology

this integration is that one method is employed to weighting the criteria, whereas another method is adopted to rank the alternatives [86, 84, 87].

An integrated MCDM method with other method(s), has been frequently proposed and used for solving MCDM problems in wide application areas. The integration of MCDM method and other method(s) has been mainly developed for improving limitations of one MCDM method that is not solved either by utilising other MCDM methods or combining with the FST. This integration is done through adopting the strength of another method to an integrated proposed method. For example, a stochastic approach has been used to handle the uncertainty and imprecision in data on MCDM [88, 89]; Geographical Information System (GIS) was combined with an MCDM method to select the most feasible location[90]; Coefficient of Technical Level (CTL) was adopted to an MCDM method to select the most appropriate equipment [91].

2.3 Survey Methodology

Figure 2.3.1 shows the framework of searching criteria and the selection of eligible papers that was applied in this survey.

2.3.1 Searching methodology

The first row in Figure 2.3.1 shows the searching methodology of this survey. Eight electronic databases were selected to provide a comprehensive application of MCDM for the choice problem in the mining and mineral processing field. These databases are Web of Science, Scopus, Elsevier Science Direct, Taylor and Francis Online, Springer Link, Wiley Online Library, The Institute of Materials, Minerals, and Mining (IoM3) and Google Scholar. Only articles that were published in English and in peer-reviewed journals identified in the aforementioned eight databases were taken into account.

For the searching process, there were three general themes that were used: 'decision making', 'mining', and 'mineral processing'. The following keywords were used for the 'decision making' theme: decision making, MCDM, MCDA, AHP, ANP, TOPSIS, PROMETHEE, and ELECTRE. Searches for the 'mining' theme consisted of the following keywords: mining industry, mining method, mining technology, mining equipment, and mining site. Finally, keywords for the 'mineral processing' theme were: mineral processing, processing equipment, and processing location. Several examples of the phrases used for the searching process are: “Decision making in mining industry”, “MCDM in mineral processing”, “AHP for the selection of mining equipment”.

Fig. 2.3.1 The framework of searching criteria and the selection of eligible papers for this survey.
2.3 Survey Methodology

2.3.2 Selection of eligible papers

One of the most crucial elements of a literature review is the definition of inclusion and exclusion criteria. The second row to the fourth row in Figure 2.3.1 shows the stages of the selection of eligible papers for this survey. These criteria describe the steps taken to select eligible papers [9]. Three stages of inclusion and exclusion criteria were carried out.

1. The title and keywords of obtained articles, which had been produced from searching stage, were reviewed;
2. The abstract of articles, which had been passed from the first stage, were assessed;
3. The full manuscript of articles, which had been passed from the second stage, were read.

After the three stages for the selection of eligible papers were conducted, the results were deemed appropriate for this survey.

In summary, 295 potentially relevant articles, which were obtained from the first and second stages, were assessed in the third stage. Titles, abstracts and full text of 295 articles were screened, and irrelevant papers were eliminated, therefore a total of 90 eligible articles remained.

2.3.3 Classification scheme

This review classified the selected articles based on:

1. Type of study
   - Concept study: work that has developed and proposed MCDM methods is considered a concept study;
   - Case study: those which have used MCDM for a particular problem are classified as case study;
   - Review: the review corresponds to manuscripts that have reviewed several previous studies from the literature.

2. MCDM method
   - Individual MCDM method such as AHP, TOPSIS, ELECTRE, PROMETHEE, VIKOR, and WASPAS;
   - Hybrid MCDM method, such as FAHP, FTOPSIS, and Yager;
   - Review, it means if the use of MCDM methods for applications that referred from previous literature was discussed, but no actual case study made by the author was presented;
• Other, it means if the use of several MCDM methods for an application and compared.

3. Application area

• Mining; comprising five sub-fields: the selection of mining equipment (ME), mining method (MM), mining technology (MT), mining site location (MSL) and other;

• Mineral processing; comprising four sub-fields: the selection of mineral processing plant location (MPPL), mineral processing equipment (MPE), mineral processing method (MPM), and mineral processing operation parameter (MPOP).

2.4 Results from the survey

2.4.1 Growth in published articles based on publication year

The growth of published papers for the application of MCDM methods for the choice problem in the mining and mineral processing fields is presented in Figure 2.4.1. The number of publications has increased from one paper in early 1999 to a total of 90 papers by the end of 2017, of which 69 correspond to the mining field while 21 papers correspond to the mineral processing field.

The figure shows different trends in the number of published papers. In the years 1999 to 2007, the number of published papers in mining and mineral processing was below 2 papers per year, with the exception of 2004, then rising again in the last decade. In the years 2008 to 2017, the publication continued regularly in both mining and mineral processing. The application of MCDM in mining started in 2000, however, no significant increase in publications is found until 2007. In 2001, 2003, 2005, and 2007 there was no single publication in the application of MCDM in mining. The application of MCDM in mineral processing started in 1999, yet no significant increase in the number of publications is found until 2014. In 2000, 2001, 2002, 2003, 2004, 2006, 2007, 2009 and 2012, there is no single publication in the application of MCDM in mineral processing.

The data reveal a gradual increase in the number of publications, with the number of papers published with applications to mining greater than those with a focus on mineral processing. While the exact reasons for this difference are difficult to ascertain, a ten year gap can be observed if we consider the first occurrence of at least four publications in these fields. With wider availability of MCDA software, further development in the application of MCDM methods in mineral processing field should be expected.
2.4 Results from the survey

Fig. 2.4.1 Number of published articles on the application of MCDM for the choice problem in the mining and mineral processing fields according to the year of publication.

2.4.2 Distribution by publication journal

Table 2.4.1 shows the distribution of articles on the application of MCDM for the choice problem in mining and mineral processing according to the journal of publication. The 90 articles selected for this review are distributed in 45 journals. It can be noted from table 1 that the majority of the papers have been published in journal specialised in mining. There seems to be more interest in the application of MCDM from researchers in mining than from those in mineral processing.

There were 4 journals out of 45 journals in the list that have published more than 5 papers, whereas other 40 journals have published less than 5 papers. In addition, 32 journals have published only one paper. *The Journal of The Southern African Institute of Mining and Metallurgy* contributed 15 papers (16.67 %) to the total number of published articles. It was followed by *Mining Technology* (10%), *International Journal of Mining Science and Technology* (5.56%) and *Journal of Mining Science* (5.56%). It is interesting to note that despite the topic being relevant to more general journals, almost no papers have been published in these, an exception being *Expert Systems with Applications* (2.22%), where two papers were published in the period of interest to this survey.
Table 2.4.1 Distribution of articles on the application of MCDM for the choice problem in mining and mineral processing according to the Journal

<table>
<thead>
<tr>
<th>No</th>
<th>Journal title</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The Journal of The Southern African Institute of Mining and Metallurgy</td>
</tr>
<tr>
<td>2</td>
<td>Mining Technology</td>
</tr>
<tr>
<td>3</td>
<td>International Journal of Mining Science and Technology</td>
</tr>
<tr>
<td>4</td>
<td>Journal of Mining Science</td>
</tr>
<tr>
<td>5</td>
<td>Journal of Mining and Environment</td>
</tr>
<tr>
<td>6</td>
<td>Acta Montanistica Slovaca</td>
</tr>
<tr>
<td>7</td>
<td>Arch. Min. Sci</td>
</tr>
<tr>
<td>8</td>
<td>Gospodarka Surowcami Mineralnymi - Mineral Resources Management</td>
</tr>
<tr>
<td>9</td>
<td>International Journal of Mining, Reclamation and Environment</td>
</tr>
<tr>
<td>10</td>
<td>Tunnelling and Underground Space Technology</td>
</tr>
<tr>
<td>11</td>
<td>Archives of Mining Sciences</td>
</tr>
<tr>
<td>12</td>
<td>Expert Systems with Applications</td>
</tr>
<tr>
<td>13</td>
<td>Hydrometallurgy</td>
</tr>
<tr>
<td>14</td>
<td>Anais da Academia Brasileira de Ciências</td>
</tr>
<tr>
<td>15</td>
<td>Applied Soft Computing</td>
</tr>
<tr>
<td>16</td>
<td>Arabian Journal for Science and Engineering</td>
</tr>
<tr>
<td>17</td>
<td>Asia-Pacific Journal of Operational Research</td>
</tr>
<tr>
<td>18</td>
<td>Bulletin of the Earth Sciences Application and Research Centre of Hacettepe University</td>
</tr>
<tr>
<td>19</td>
<td>Engineering Economics</td>
</tr>
<tr>
<td>20</td>
<td>Environmental Earth Sciences</td>
</tr>
<tr>
<td>21</td>
<td>Informatica</td>
</tr>
<tr>
<td>22</td>
<td>Int. J. Industrial and Systems Engineering</td>
</tr>
<tr>
<td>23</td>
<td>International Journal of Applied Engineering Research</td>
</tr>
<tr>
<td>24</td>
<td>International Journal of Coal Science &amp; Technology</td>
</tr>
<tr>
<td>25</td>
<td>International Journal of Surface Mining, Reclamation and Environment</td>
</tr>
<tr>
<td>26</td>
<td>International Research Journal of Applied and Basic Sciences</td>
</tr>
<tr>
<td>27</td>
<td>IOP Conference Series: Earth and Environmental Science</td>
</tr>
<tr>
<td>28</td>
<td>J. Mater. Environ. Sci</td>
</tr>
<tr>
<td>29</td>
<td>Journal of Applied Science and Agriculture</td>
</tr>
<tr>
<td>30</td>
<td>Journal of Central South University</td>
</tr>
<tr>
<td>31</td>
<td>Journal of Civil Engineering and Management</td>
</tr>
<tr>
<td>32</td>
<td>Journal of Environmental Management</td>
</tr>
<tr>
<td>33</td>
<td>Journal of Intelligent &amp; Fuzzy Systems</td>
</tr>
<tr>
<td>34</td>
<td>Journal of Multidisciplinary Engineering Science and Technology (JMEST)</td>
</tr>
<tr>
<td>35</td>
<td>Kuwait J. Sci</td>
</tr>
<tr>
<td>36</td>
<td>Mathematical Problems in Engineering</td>
</tr>
<tr>
<td>37</td>
<td>Mineral Processing and Extractive Metallurgy Review</td>
</tr>
<tr>
<td>38</td>
<td>Minerals Engineering</td>
</tr>
<tr>
<td>39</td>
<td>Mining Science and Technology</td>
</tr>
<tr>
<td>40</td>
<td>Mining Science and Technology (China)</td>
</tr>
<tr>
<td>41</td>
<td>REM – Revista Escola de Minas</td>
</tr>
<tr>
<td>42</td>
<td>Science Research</td>
</tr>
<tr>
<td>43</td>
<td>The International Journal of TRANSPORT and LOGISTICS</td>
</tr>
<tr>
<td>44</td>
<td>International Journal of Mining and Geo-Engineering</td>
</tr>
<tr>
<td>45</td>
<td>Mining Science</td>
</tr>
<tr>
<td>Total</td>
<td>90  100%</td>
</tr>
</tbody>
</table>

2.4.3 Distribution by application area

Figure 2.4.2 presents a distribution of the published MCDM articles for the choice problem according to the application area. 9 choice problems are categorised into two main application areas. 5 choice problems are included in mining and 4 choice problems are included in mineral processing.
2.4 Results from the survey

Fig. 2.4.2 Distribution of published MCDM papers for the choice problem in mining (a) and mineral processing (b).

In the case of the mining field, MCDM methods have been used most widely for the selection of the mining method: 33 of 69 articles (47.83%). This application was followed by the mining equipment selection problem (24 articles or 47.83%). Two studies were categorised as "other", either because the article could fall into more than one category of application or if the topic was not among the four application areas. For further details on these papers, the reader is referred to Table A.1 in Appendix A.

In the case of mineral processing, 7 out of 21 articles (33.33%) describe the utilisation of MCDM methods for the plant location selection problem. The second most researched topic was mineral processing equipment selection, with 6 papers (28.57%).

Figure 2.4.3 shows the number of published articles on the application of MCDM in mining and mineral processing according to the application area based on year of publication. Different trends can be seen from the figure. In the case of mining, the mining method selection and mining equipment selection problems have been studied every year since 2000 and 2001, respectively. The highest number of publications for the aforementioned selection problems is 5 papers per year. In the case of mineral processing, there is no constant application area problem that has been studied every year. The highest number of publications per year is 3 papers that studied the application of MCDM for the mineral processing method selection problem.
Fig. 2.4.3 Number of published articles on the application of MCDM for the choice problem in the mining (a) and mineral processing (b) fields according to the application area based on year of publication.

### 2.4.4 Distribution by MCDM methods

This section presents the distribution of MCDM methods used for solving the choice problem in mining and mineral processing. Table 2.4.2 shows the number of published articles according to the type of MCDM method employed.

Table 2.4.2 Distribution of the number of published articles on the application of MCDM methods for the choice problem in mining and mineral processing according to the method used and the type of application.

<table>
<thead>
<tr>
<th>Application area</th>
<th>Sub-field</th>
<th>AHP</th>
<th>TOPSIS</th>
<th>ELECTRE</th>
<th>PROMETHEE</th>
<th>VIKOR</th>
<th>Hybrid</th>
<th>Review</th>
<th>Other</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mining</strong></td>
<td>Mining equipment</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>15</td>
<td>0</td>
<td>3</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>Mining method</td>
<td>8</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>14</td>
<td>2</td>
<td>7</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>Mining technology</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Mining site location</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Other</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td><strong>Mineral</strong></td>
<td>Processing plant location</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Processing equipment</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td><strong>Processing</strong></td>
<td>Processing method</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Operation parameter</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td>26</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>41</td>
<td>2</td>
<td>14</td>
<td>90</td>
</tr>
</tbody>
</table>
2.4 Results from the survey

The most frequently used individual method is AHP (26 articles). AHP has been chosen mostly because:

- the process is logic and easy to understand [23];
- the process allows the selection of the best alternatives with respect to each criterion to be described in a simple mathematical form [23];
- the process evaluates quantitative and qualitative criteria as well as alternatives on the same preference scale [23];
- the computation processes are straightforward [23];
- the process facilitates consistency check on the importance of the criteria and the preferences of alternatives and even provides the degree of consistency in the judgements [92];
- AHP is supported by user-friendly software packages such as Expert Choice [93] and Super Decision [94]. In addition, AHP can also be implemented in Microsoft Excel easily.

It can also be observed that a large proportion of the published papers (41 out of 90 articles, or 45.6%) make use of hybrid methods. The use of hybrid methods in the literature surveyed is further analysed in Table 2.4.3, which shows the number of published articles according to the hybrid MCDM methods applied. Publications on AHP based methods (AHP in combination with one or more MCDM methods) dominate with 17 out of 41 articles (41.5%), followed by the FAHP method with 8 articles (19.5%). This fact shows that another advantage of AHP is that the method can be combined with other operation research (OR) techniques or other methods to solve decision making problems that cannot be tackled with a single method due to particular constrains [92, 95];

Table 2.4.3 Distribution of the number of published articles on the application of hybrid MCDM methods for the choice problem in mining and mineral processing according to the method used and the type of application.

<table>
<thead>
<tr>
<th>Application area</th>
<th>Sub-field</th>
<th>MCDM &amp; FST</th>
<th>MCDM &amp; Other MCDM (Based)</th>
<th>MCDM &amp; Other Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>FAHP</td>
<td>FTOPSIS</td>
<td>PROMETHEE</td>
</tr>
<tr>
<td>Mining</td>
<td>Mining equipment</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Mining method</td>
<td>5</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Mining technology</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Mining site location</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Processing plant location</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Mineral Processing</td>
<td>Processing equipment</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Processing method</td>
<td>0</td>
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<td>0</td>
</tr>
<tr>
<td></td>
<td>Operation parameter</td>
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<td>0</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>8</td>
<td>7</td>
<td>4</td>
</tr>
</tbody>
</table>
As can be seen from Table 2.4.2 and Table 2.4.3, the most frequently used MCDM method for the choice problem in mining and mineral processing is AHP; the second ranked and the third ranked methods are the hybrid AHP-based (in combination with other MCDM methods) and the FAHP, respectively. In mining, for the sub-field with the highest number of publications, namely the selection of the mining method, the MCDM technique most frequently used is AHP, with 8 publications, followed by the FAHP and AHP based methods (5 articles and 4 articles, respectively). The survey shows that in mineral processing the use of AHP is strongly preferred over other methods, either used individually or as a hybrid method.

Fig. 2.4.4 Number of published articles on the application of MCDM for the choice problem in mining (a) and mineral processing (b) according to the method used based on year of publication. The data for papers that use AHP, FAHP and AHP-based methods is presented as separate plots (c and d) for clarity.
2.4 Results from the survey

Figure 2.4.4.a) and Figure 2.4.4.b) show the number of published articles on the application of MCDM in mining and mineral processing according to the MCDM methods used based on year of publication. These figures show the trends for the different MCDM methods used. In the case of mining, studies that used AHP can be found every year during the period surveyed but it is only over the last decade that papers using a wide variety of MCDM methods have been published. For mineral processing, on the other hand, there were no publications using AHP from 1999 to 2004. It is only from 2010 that the use of AHP, FAHP or AHP-based methods has been reported, with the exception of 2012, albeit only one publication each year.

Figure 2.4.4.c) and Figure 2.4.4.d) show the number of published articles of the application of AHP and its hybrid used in mining and mineral processing based on year of publication. In the case of mining, AHP was the most MCDM method used from 2000-2008, FAHP and AHP based were used initially in 2009. Since this year until 2016, the variety of AHP was used almost every year except 2012. In the case of mineral processing, AHP was used almost every year since 2010 except 2012. A little variation of AHP hybrid was used in this field.

2.4.5 MCDM software used

Figure 2.4.5 shows the distribution of published articles for the choice problem in mining and mineral processing according to the software used to support the MCDM methods. There are many software packages available [96, 97], including commercial, open source and freeware. Of the 90 publications that are the focus of this review, only 30 mention the software used. Figure 2.4.5 shows the distribution of those 30 articles by the software used. The software Expert Choice dominates at 10 articles, followed by Matlab at 5 articles; Decision Lab and Microsoft Excel, are reported in 3 of the papers. It is noted that all the software reported in the published studies is commercial software. However, the number of articles that do not state the type of software used is high (60 out of 90 articles).

Fig. 2.4.5 Distribution of software used to support MCDM methods for the choice problem in (a) mining and (b) mineral processing.
2.4.6 Analysis of geographic distribution of MCDM papers

The distribution of MCDM articles for the choice problem in mining and mineral processing according to the geographic region is shown in Table 2.4.4. For this analysis, the country of origin is selected on the basis of its first author’s affiliation at the time of publication. The analysis shows that whereas 18 countries have contributed to the literature on MCDM for the choice problem in mining, for mineral processing the number of countries is only 8. Iran has been the most prolific country in this area of research, with 27 articles published on mining and 10 articles on mineral processing. Turkey is ranked second, with 15 articles on mining and 3 on mineral processing.

If grouped by continent, Asia clearly leads with 60 out of 90 published articles. This is similar to what has been found by other recent reviews on the application of MCDM in other fields [58, 63, 98, 1, 75, 62, 23], where resulting distributions by author’s affiliation at the time of publication also showed Asia as the continent with the highest number of published papers.

Table 2.4.4 Distribution of MCDM articles in mining and mineral processing according to country and continent

<table>
<thead>
<tr>
<th>Country</th>
<th>Continent</th>
<th>Mining</th>
<th>Mineral Processing</th>
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</thead>
<tbody>
<tr>
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<td>10</td>
</tr>
<tr>
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<td>Asia</td>
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<tr>
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<td>3</td>
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</tr>
<tr>
<td><strong>Total</strong></td>
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</table>

Figure 2.4.6.a) and Figure 2.4.6.b) show the number of published articles of the application of MCDM in mining and mineral processing according to the country and Figure 2.4.6.c) and Figure 2.4.6.d) show the number of publications according to the continent based on the year of publication.
From Figure 2.4.6.a) and b) it can be seen that studies in mining have been published every year, whereas for mineral processing there are several years with no publications. While Iran is the most prolific country, there is only three publications from this country in the period 1999-2007; it is only over the last decade that their number of publications has increased. From 2008, the number of countries that have contributed to the literature on the application of MCDM for the choice problem in both mining and mineral processing has increased considerably.

Figure 2.4.6.c) shows that previous to 2008, it was only two continents that contributed to the topic of interest in mining, namely Asia and America. From 2008, contributions from Europe, Africa and Australia can be found, although Asia remains as the main contributor by far. Similarly, from Figure 2.4.6.d) it can be seen that it is only from 2011 that other continents other than Asia have published on the topic of interest in mineral processing. It is
interesting to note that the maximum number of publications from Asia in any given year is 8 for mining and 3 for mineral processing.

According to Kubler et al. [98], Asia has been the most productive country in terms of publications on the application of FAHP to MCDM problems. This is in line with the prediction made by [99] that AHP would be the most frequently used method for MCDM in countries such as Iran and Turkey, where the use of AHP was found to be widely spread. This review has found the same to be true for articles published on the application of MCDM in mining and mineral processing.

Figure 2.4.7 shows the world mining production from 1999 to 2016. The data of this figure is adopted from Reichl et al. [100]. This figure shows that Asia has been leading the world mining production since 1999 and the production increment is evident until 2013, after which it has remained more or less constant. This trend in production is similar to that in the number of published articles on the use of MCDM for the choice problem in mining and mineral processing.

![Fig. 2.4.7 World mining production 1999 - 2016 by continent (adapted from Reichl et al. [100].)](image)

### 2.5 Discussion

Conventional crisp MCDM methods have been successfully applied to the choice problem in mining and mineral processing in the cases where all relevant information and data have
been acquired accurately. When that is not the case, and to account for uncertainty in the data, other MCDM methods have been developed, e.g. combining crisp MCDM with FST or with other techniques in Operational Research. This section discusses the published work on the use of MCDM techniques for the choice problem in mining and mineral processing. In both application areas, individual and hybrid MCDM methods have been used in all the sub-fields identified as part of this review.

2.5 Discussion

2.5.1 Mining

2.5.1.1 Mining equipment selection

The equipment selection process is the task of assessing the alternatives being considered and choosing the best alternative under multiple criteria. The DM process of equipment selection in mining includes an integrated evaluation of multi-disciplinary knowledge [101]. Several studies that are presented below have revealed that the equipment selection process is a key factor in mine design and production planning as it affects the sustainability of a mining operation.

AHP has been the most frequently employed individual method for crisp MCDM problems involving mining equipment selection [102, 101, 103, 104]. Other individual methods that have been used are the ELECTRE [105] and the PROMETHEE [106]. To account for uncertainties in the models, such as lack of complete geological data or risks associated to mining, hybrid MCDM methods are usually applied. In terms of hybrid methods, AHP has also been the most frequently used method, in combination with a variety of other methods. Adebimpe et al. [107], Aghajani Bazzazi et al. [108], Bazzazi et al. [109, 110], Komiljenovic and Keceojić [91], Lashgari et al. [111], Ozfirat et al. [112], Wang and Tu [113], Yazdani-Chamzini [114] have all reported the use of a hybrid AHP based method. Three other hybrid MCDM methods that have been used are: FAHP [115, 116], Yager [117, 118], and FTOPSIS [119].

In addition, it is relevant to point out that only three studies have presented comparisons of individual MCDM methods for the selection of mining equipment: a comparison of the Weighted Product Model, ELECTRE and PROMETHEE by de Sousa Junior et al. [120] and a comparison of AHP and Yager by both Başçetin [121] and Yavuz [122].

The task of selecting equipment in mining could involve choosing the type of equipment, the equipment size or the number of units required. However, the authors have found from this survey that the only task for which MCDM has been applied is the selection of equipment. The following equipment selection problems have been found in the literature:

- the selection of an excavating machine, using AHP [102] and using FMCDM [117],
- the selection of a loading-hauling system, using VIKOR Aghajani Bazzazi et al. [108], FAHP [115, 109], AHP [101], FTOPSIS [110], hybrid Yager and AHP [121],
- the selection of a tunnelling machine, using PROMETHEE [106],
• and the selection of a fan in an underground mining site, using AHP [103].

In terms of the software used to support the decision maker in the selection of equipment in mining, four different software packages were identified in this survey:

• Expert Choice [101, 121, 109, 110, 91] for supporting AHP.
• EQS (Equipment Selection) software [118] for supporting Yager.
• EQUIPSELECTOR [107] for supporting FTOPSIS.
• An implementation of Yager in Matlab [121].

2.5.1.2 Mining method selection

The selection of a mining method is the process of choosing a suitable mining technique which is technically feasible for the ore-body geometry and the ground conditions and which presents the least difficulties [123]. The main objective of this process is maximising the company’s profit by improving the recovery of the resources and minimising the operation cost. Several conventional approaches for evaluating appropriate mining methods for an ore deposit have been developed and applied, e.g. the Nicolas technique, the modified Nicolas technique, and the UBC methodology. Nonetheless, neither of these methods take into account criteria weights that impact on the selection of the mining method [124], which is something that MCDM methods do.

For the purpose of this survey, the application of MCDM techniques to the mining method selection problem has been identified based on the type of resources mined:

• For coal resources, AHP [125], FAHP [126] and Yager [127] have been employed for the selection of the underground mining method, while Yavuz [128] compared AHP and Yager in this context.

• In the case of bauxite, the use of AHP [123, 129, 130], FAHP [131], TOPSIS [132], a hybrid FAHP and TOPSIS [133], and a hybrid Monte Carlo and AHP (MAHP) [134] has been reported for the selection of the optimum mining method. All the above correspond to studies for bauxite mines in Iran. It is also noted from the literature that the application of different MCDM methods for the same case studies might yield similar results. For example, three MCDM methods, AHP [123, 129], TOPSIS [132] and MAHP [134] were employed for the selection of a suitable mining method for the Golbini No. 8 deposit in the Jajarm Bauxite Mine, Iran. The evaluation of predetermined alternatives was conducted under the same criteria and alternatives and all the studies resulted in the selection of the same mining method.
For selecting a mining method for iron ore operations, TOPSIS [130], FAHP [135], and FTOPSIS [136] have been used. In addition, two articles that compare several MCDM methods have been published, namely a comparison between the GREY and the TODIM methods [137], and the comparison between the FDominance method and Yager [124].

In the case of copper mines, FAHP [138] and FTOPSIS [136] have been employed for the selection of the proper mining method.

For lead-zinc mines, three studies have focused on the selection of a suitable mining method in Iran using MCDM: Samimi et al. [130] used PROMETHEE, Hayati et al. [139] used VIKOR, while a hybrid FAHP and TOPSIS method was adopted by Shariati et al. [140].

For chromite, two studies have compared MCDM methods for the appropriate selection of mining method in Turkish mines, namely a comparison between AHP and Yager [141] and a comparison of AHP, the Bellman-Zadeh method, and TOPSIS [142].

In the case of platinum ore, three studies were found that used AHP [143–145].

In the case of an Iranian salt mine, FAHP [146] has been used for the selection of the optimum mining method.

This survey revealed that there are eight software packages that have been used in the literature on the application of MCDM for the selection of the mining method:

- Expert Choice [147, 130] for supporting AHP.
- Criterium Decision Plus [148] for supporting AHP.
- Decision Lab [148] for supporting PROMETHEE.
- Fuzzy Decision Making (FDM) [136] for supporting FTOPSIS.
- An implementation of AHP in OPL Studio 3.7 [126].
- UMMS [141] for supporting AHP.
- An implementation of AHP in Microsoft Excel [144, 125, 149].
- An implementation of Yager in MATLAB [124].

The selection of mining methods is the only sub-field considered in this manuscript for which review papers have been published. Mahase et al. [150] performed a survey of 150 case studies on the application of MCDM methods in mine planning, which were obtained from different journal sources and conference proceedings. Kant et al. [151] conducted a critical
review of studies on the application of MCDM methods (i.e. AHP, FAHP, TOPSIS and PROMETHEE) and the Nicholas technique for the selection of an optimum stoping method in hard rock underground mining.

2.5.1.3 Mining technology selection

One of the keys to increase the competitiveness of the mining industry is the use of new technologies. These technologies could consist of new procedures, systems or integration of activities within an enhanced single system.

Two individual MCDM methods have been used to tackle the selection of mining technology namely AHP [152, 153] and PROMETHEE [154]. Dessureault and Scoble [152] demonstrated the use of AHP to appraise the impact of a new blasthole drilling technology. Petit and Fraser [153] also employed AHP but to assess alternative energy delivery systems for drilling in hard rock mines. PROMETHEE, on the other hand, was used by Vujic et al. [154] for analysing, assessing, and selecting an Excavator (bucket chain) – Conveyor – Spreader (ECS) system in surface mining.

Hybrid MCDM methods have also been used to select mining technologies. Stojanovic et al. [155] reported the use of a hybrid AHP based method for the selection of an optimal technology for surface mining, whereas Bouhedja et al. [156] demonstrated the use of a hybrid PROMETHEE based method to choose secondary breakage process technologies for limestone quarries.

There are three software packages that have been employed in the literature on the application of MCDM for the selection of mining technologies:

- Criterium Decision Plus [155] for supporting AHP.
- ELECTRE [155] for supporting ELECTRE.
- Promcalc [154] for supporting PROMETHEE.

2.5.1.4 Mining site selection

Selecting a mine site is a critical decision in the early years of a mining project. The selection of a mining site is the task of evaluating a potential mining area for the mine facilities. The selection is governed by multiple conflicting factors such as resource availability, logistics, costs and socio-economic-environmental aspects.

Two studies reported the use of AHP to select the potential mining site [157, 158]. Dey and Ramcharan [157] developed a systematic framework based on AHP for selecting a site for the expansion of limestone quarry operations. Straka et al. [158] employed AHP for the selection of a mine waste storage location. The only reported study on the use of a hybrid method for this type of selection problem has been for selecting the location of a tailing impoundment site using a hybrid AHP method [159].
Some studies have reported the use of multiple MCDM methods, not as hybrid methods but applying the different techniques individually for the selection problem and then combining the results employing different strategies. An example is the work by Hekmat et al. [160], who employed three individual MCDM methods, namely the simple additive weighting (SAW) method, TOPSIS and AHP for the selection of waste dump sites in open pit mines. The authors classified the aforementioned MCDM methods as subjective (AHP) or objective (SAW and TOPSIS) based on the way that the criteria weights were determined. The final rank was obtained by means of an average-scoring approach of the subjective and objective methods. Similarly, Hudej et al. [161] proposed to employ four individual MCDM methods, i.e. PROMETHEE, ELECTRE, AHP and VIKOR, for the selection of the main mine shaft location. While PROMETHEE and VIKOR proposed location "D", ELECTRE and AHP both proposed location 'B', thus leaving the decision makers without a clear choice. A ranking pondering technique was then employed, but the result was still a tie between the same two locations. A more comprehensive analysis of the criteria for both alternatives was conducted using the SWOT (Strengths, Weaknesses, Opportunities, Threats) method, which then allowed the selection of the best location.

Only two software packages have been reported in the literature on the application of MCDM for the selection of a mining site:

- Expert Choice [157, 160] for supporting AHP.
- e-MDM [158] for supporting AHP.

### 2.5.1.5 Other type of selection problems

For the purpose of this survey, two studies were classified into other type of problems either because they covered different problems or the topic would not fit the classification. Kazakidis et al. [162] reported the application of AHP using Expert Choice for five case studies, each dealing with a different selection problem, namely investment analysis of recent technology, ground support design, tunnelling systems design, shaft location selection, and mine-planning risk assessment. Yavuz et al. [163] also used AHP but for evaluating the support system alternatives in an underground mine.

### 2.5.2 Mineral Processing

#### 2.5.2.1 Processing plant location selection

The selection of the location for a processing plant is the task of evaluating a potential area for the mineral processing facilities. AHP has been the most frequently employed method for the selection of processing plant location, whether it is employed individually [164–166] or as a hybrid FAHP method [167]. However, after comparing the Yager and the AHP methods for the plant location selection problem, Yavuz [168] suggests employing Yager instead of
AHP if many attributes and sub-attributes need to be considered. This is because while AHP requires many pairwise comparisons to be conducted, fewer matrices are required when employing the Yager method.

The use of hybrid methods for the selection of processing plant locations include the work by Safari et al. [169] and Bakhtavar and Lotfian [170]. The former used FTOPSIS for the selection of an iron processing plant location while the latter compared FAHP and gray MCDM methods for the selection of a copper processing plant site.

2.5.2.2 Processing equipment selection

The mineral processing equipment selection is the task of assessing the equipment being considered and choosing the best equipment under multiple criteria. The equipment referred to in this section covers all the machinery used from the ore transportation stage to the concentrate stages. Three examples of processing equipment selection were found in the literature: the selection of ore transportation, the selection of the optimum modification parameter for belt conveyor construction, and the selection of primary crusher equipment.

Basçetin and Kesimal [171] employed Yager to select an optimal mined coal transportation system to a power station under inadequate knowledge and imprecise information. Owusu-Mensah and Musingwini [172] concluded that AHP is a useful method for the selection of ore transportation equipment from an underground mine to the mill. Despodov et al. [173] proposed AHP for the selection of equipment to transport ore to a processing facility. AHP has also been used to select suitable modifications of belt conveyor construction parameters in order to optimise the transportation of raw materials [174].

In the case of the selection of a primary crusher, two articles have reported the use of different MCDM techniques for the exact same mine site under the same alternatives and criteria. Rahimdel and Ataei [175] used AHP for the primary crusher selection problem under crisp conditions, whereas Rahimdel and Karamoozian [176] carried out a similar study but using FTOPSIS to incorporate uncertainty. It is interesting to note that both studies resulted in the same type of primary crusher being selected.

2.5.2.3 Processing method selection

The selection of processing methods that result in low capital and operational costs is a very important task in mineral processing. The DM process of selecting a processing method is complex due to many criteria that may affect the efficiency of the processes.

Individual MCDM and hybrid MCDM have also been proposed and used for the mineral processing method selection problem. In the case of individual MCDM, Kursunoglu et al. [177] employed AHP for the selection of the best leaching method to process low grade nickel ore, Montazeri and Taji [178] used TOPSIS for the selection of a coke making method by ranking and comparing traditional and industrial methods. In the case of hybrid MCDM,
several studies have proposed novel methods, such as the application of an integrated Delphi Analytical Hierarchy Process (DAHP) and FTOPSIS was proposed by Shahab et al. [179] for the selection of an alunite processing method. Other examples include the integration of pairwise comparison, group decision making, interval grey numbers and the Ratio system approach of the Multi-Objective Optimization by Ratio Analysis (MOORA) method, developed by Stanujkic et al. [180] for the selection of a grinding circuit design. Finally, an integrated WASPAS and single-valued neutrosophic set (SVNS) was developed by Zavadskas et al. [70] for the selection of a lead-zinc flotation circuit design.

2.5.2.4 Operational parameter selection

This task involves the selection of optimal operational parameters such as temperature, pressure, retention time, and particle size, among a set of feasible alternatives. This selection problem might occur in the early stage of the project and in the operation stage. The appropriate decision from an early stage of the project might deliver a more efficient operation and guarantee a proper resource utilisation during the production stage. Additionally, the appropriate selection of parameters in the operation stage might improve the performance of the operation at a process or equipment level, and might positively effect the subsequent processes.

Three previous studies have utilised individual MCDM and hybrid MCDM methods for the selection of operational parameters. In the case of the use of an individual MCDM method, Baral et al. [181] used TOPSIS to select the best leaching operation parameter system for the extraction of a rare earth metal; MATLAB R2010a was used to implement TOPSIS to this end. In the case of hybrid MCDM, Kostovic and Gligoric [182] employed FTOPSIS for the selection of the optimum sulphide mineral collector and dose rate in a flotation study. Savic et al. [183], on the other hand, proposed an integration of the subjective AHP, the Objective Entropy Weight (OEW), and the PROMETHEE methods, the latter implemented in Decision Lab, for the selection of the optimal zinc concentrates for blending in zinc production.

2.5.3 Critical remarks

Decisions in mining and mineral processing are often made by a group of multiple decision makers or experts [143], which is referred to as group decision-making (GDM). GDM is the process of determining the most optimal outcome (i.e. judgement, preference, and alternative) that is most acceptable for the group based on the opinions of multiple decision makers or experts [184]. Since it has been indicated from this section that AHP was applied more than any other individual MCDM method, this paper highlighted the application of AHP in GDM. The application of AHP in GDM has attracted a great deal of attention in mining [125, 103, 104, 129, 123, 185] and mineral processing [164, 165, 177, 172, 175, 166]. It is worth
highlighting that disparities among decision-makers, such as competencies, abilities, and compliances \[186\] can lead to a biased opinion as a group. Therefore, applying the appropriate algorithm to aggregate various opinions obtained from decision makers in order to determine the weights of criteria as well as the preferences of alternatives is of vital importance. Simply aggregating decision makers’ opinions either by arithmetic mean approach \[123, 129\] or geometric mean approach \[185\] is not enough because those approaches assume that the decision makers are equally important. A weight on a decision maker’s opinion based on the importance assigned to them should be therefore applied in the calculation (i.e. weighted arithmetic mean and weighted geometric mean approaches) when the disparities among decision makers are taken into account \[187\]. Despite the fact that the disparities among decision makers can affect the final outcomes, several authors either in the mining field \[125, 103, 104\] or mineral processing field \[164, 165, 177, 172, 175, 166\] have failed to mention how to aggregate the various opinions obtained from decision makers. In addition, the weighted mean approaches still cannot capture the various opinions between decision makers.

Another concern that has been noted in the application of AHP for the choice problem in mining and mineral processing is insufficient rank discrimination. Insufficient rank discrimination, which occurs when the overall weighted scores of two or more alternatives are very close, is an undesired phenomenon that is often faced by decision makers when using AHP. There is a scarcity of literature concerning the insufficient rank discrimination in AHP. In fact, only one article in mining by Ataei et al. \[134\] describes a method to enhance the insufficient rank discrimination.

There are two main issues in the application of FAHP for the choice problem in mining and mineral processing, namely the calculation of the inconsistency of a fuzzy pairwise comparison matrix (FPCM) and the occurrence of rank reversal. It has been noted that even though the AHP based hybrid method was the most frequently applied MCDM hybrid method, there have been no studies on this topic in mining and only one in mineral processing \[167\] has discussed the measurement of inconsistency in FPCMs. Bejari et al. \[167\] applied the traditional consistency index (CI) and consistency ratio (CR) formulas to measure inconsistency of FPCMs; CI and CR were evaluated for the most likely value in fuzzy number. Although the extension formulas of CI and CR to measure inconsistency of FPCMs are available \[188, 189\], Bejari et al. \[167\] did not mention the reason for applying the traditional formulas. Furthermore, even though the occurrence of rank reversal in MCDM methods has led to much attention, no study has been conducted to investigate the occurrence of rank reversal in the application of FAHP for the choice problem in mining and mineral processing.

It has been noted as well that the AHP based hybrid method was the most frequently applied MCDM hybrid method. AHP in hybrid methods is mainly used for weighting the relative importance of subjective criteria. However, in the case when it is difficult to acquire reliable subjective criteria, decision makers could consider using objective criteria. Shannon entropy \[190\] is one of the most frequently used objective methods for obtaining the weights
of criteria. The application of entropy for the choice problem have been studied in both mining [108, 110, 191] and mineral processing [183], although these studies did not take into account uncertainty, which in reality is often involved in the decision making process.

Furthermore, MCDM problems often involve the interaction and dependency between objects (i.e. criteria, sub-criteria, and alternatives) that are present in the hierarchical structure. Despite the fact that ANP is capable of addressing the interaction and dependency between objects in the hierarchical structure, there is a paucity of studies investigating the application of ANP in mining and mineral processing. In fact, only one article, Lashgari et al. [111], has described the application of ANP for the selection of equipment in mining.

It was also noted that there is no single MCDM method able to solve all types of selection problems in mining and mineral processing. Many studies in the literature have studied the application of MCDM methods for a particular case but no study has previously been conducted to develop a general framework for the choice problem in mining and mineral processing. Such a framework would allow the decision maker to approach the choice problem in different ways, e.g. single or group DM, under or crisp or uncertain conditions, or deal with quantitative or qualitative data (or both).

Finally, it can be concluded from the aforementioned viewpoints that MCDM is already a powerful tool to overcome the choice problem in mining and mineral processing. However, there is much scope for research in the development of MCDM algorithms and frameworks to improve the applicability of MCDM methods to solve more complex problems.

2.6 Conclusions and future research directions

2.6.1 Literature review conclusions

Since their introduction almost two decades ago, MCDM methods have become useful for a wide range of applications in mining and mineral processing. Of great significance is the role they perform in the DM process for multi-conflicting criteria under crisp and uncertainty.

This survey shows that the application of MCDM methods for the choice problem in mining and mineral processing, both individual and hybrid methods, has increased over the last fifteen years. Several studies have implemented hybrid MCDM methods to increase the performance of the DM process or to overcome limitations of individual methods. It was also noted that several authors have used a number of different MCDM methods for the same selection problem in order to compare the results.

The distribution of articles according to their publication year from 1999 to 2017 showed a general increase in the number of studies published although a cyclic trend was observed. The application area for which the largest number of MCDM studies were published was mining, with 69 out of 90 articles, while the type of problem that was addressed the most was the mining method selection problem, with 33 articles out of the 69 articles on mining.
The fact that 36% of the publications deal only with the mining method selection problem clearly shows that there is a wide range of opportunities to apply MCDM methods in other type of selection problems in mining and mineral processing.

This survey also found that AHP is the most used MCDM method for solving the choice problem in mining and mineral processing, and that Expert Choice, which is based on the AHP method, is the most used MCDM software. It was also shown that the country with the largest contribution to the literature on these topics has been Iran, while Asia has been the continent with the largest number of papers published.

There are two main techniques that can be found in the literature to increase the accuracy of preference ranking for the choice problem, namely unity and uniformity. The former is the integration of an MCDM method with other MCDM method(s) while the latter applies different individual MCDM methods to a same problem with respect to the same alternatives and attributes and then aggregates the results. This literature survey found that for mining and mineral processing selection problems, the unity technique has been the most commonly used. This review article provides an overview of the application of MCDM methods in mining and mineral processing. While the focus of this review was the use of MCDM for the choice problem, there is scope for also considering the ranking, description and sorting problems in mining and mineral processing. The authors also acknowledge that MCDM techniques have been applied in areas not covered by this survey, such as mining geology, post-mining activities, as well as health, safety and the environment, which might be the focus of future studies. However, the methodology presented here could be used as a guideline for further research into the topic by academics and practitioners.

2.6.2 Future research directions

As it has been discussed in Section 2.5, there is scope to improve MCDM methods and its applications in real-world decision problems, which would benefit the mining and mineral processing communities. Certain directions of future research are summarised as follows:

1. In order to derive more acceptable and accurate outcomes when applying AHP in GDM, further research into the application of appropriate algorithms to aggregate various decision makers’ opinions is required. In addition, there is also a need to develop algorithms for capturing the various opinions between decision makers. For this purpose, a combination of AHP with stochastic simulations can be developed. Furthermore, this hybrid method could be extended as well under a fuzzy environment for the case when uncertainty is involved;

2. Like traditional AHP, measuring CI and CR of each FPCM is of vital importance in FAHP. It has been indicated that no study has been conducted to scrutinise the effectiveness of using traditional CI and CR for measuring inconsistency check between
FPCM. It would therefore be useful to re-examine the use of traditional CI and CR into FAHP. In addition, an extension formula of traditional CI and CR for FAHP needs to be developed. The results obtained from the extended formula could then be compared to those obtained using currently available formulas;

3. Even though FAHP was the most frequently applied FMCDM method for the choice problem in mining and mineral processing, no study has been conducted to examine the occurrence of rank reversal in the application of FAHP. It would thus be beneficial to carry out the analyses of rank reversal in FAHP in future studies in order to obtain more robust outcomes.
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Chapter 3

An Integrated Constrained Fuzzy Stochastic Analytic Hierarchy Process Method with Application to the Choice Problem

Abstract

The ability of the Analytical Hierarchy Process (AHP) when applied to the choice problem in the context of group decision making under uncertainty has been often criticised. AHP is not able to fully capture the various opinions and the uncertainty associated with the lack of information. This work develops an integrated constrained fuzzy stochastic analytic hierarchy process (IC-FSAHP) method in order to deal with the aforementioned drawbacks. IC-FSAHP combines two existing fuzzy AHP (FAHP) methods and further extends its applicability by implementing stochastic simulations. A case study has been conducted in order to assess the ability of IC-FSAHP; the results showed that IC-FSAHP is able to capture the uncertainty and multiple DMs’ opinions. This paper also discusses the effect that the number of DMs has in enhancing rank discrimination. Besides, the possibility of the occurrence of rank reversal because of the use of IC-FSAHP has been analysed. The results showed that the ranking of alternatives was preserved throughout the changes in the number of alternatives, however, rank reversal occurred in the case of changes in judgements scales. By comparing the $U$-uncertainty in fuzzy global priorities obtained using IC-FSAHP to that obtained using an existing FSAHP method, we show that our method is capable of minimising the risk of losing important knowledge during the computations. We also discuss how IC-FSAHP can decrease the uncertainty and increase the reliability of the decisions by means of robust computations.
3.1 Introduction

Multiple criteria decision making (MCDM) is an advanced field of operations research that can be used to solve problems when multiple alternatives are evaluated with respect to multiple conflicting criteria. One of the problems that can be tackled by MCDM is the choice problem because MCDM can be used to determine the best alternative from a finite number of predetermined alternatives \( (A_1, A_2, \ldots, A_m) \) under multiple criteria \( (C_1, C_2, \ldots, C_n) \) [1]. Since the best alternative is chosen, the ranking of worse alternatives can be ignored. The MCDM methods have been effectively applied in various fields such as natural resources management [2], environmental science [3], economics [4] and finance [5], civil engineering [6], manufacturing [7], sustainable renewable energy development [8], and mining and mineral processing [9]. A number of literature reviews have showcased the importance of MCDM methods and have reported numerous integrated MCDM methods [10–13]. In addition, a great deal of attention is currently being given to the application of MCDM by multiple decision makers (DMs), which is referred to as multiple criteria group decision making (MCGDM) [14–16]. A larger number of preferences, judgements and insights from multiple DMs can lead to an increase in confidence in the selection process. It is worth noting that when aggregating the preferences, judgements and priorities of criteria and alternatives, biased opinions due to disparities among decision makers, such as competencies, abilities, and compliances [17] should be taken into account. Therefore, selecting the appropriate aggregation method for weighting criteria judgement as well as evaluating the preference of alternatives is of vital importance.

One of the most widely used MCDM methods is the Analytic Hierarchy Process (AHP) that was developed by Saaty [18]. Despite its popularity, the application of AHP has been always criticised when uncertainty is present. Uncertainty is mostly associated to the stages of the decision making process that deal with the assessment and calculation of weighting criteria and scoring of alternatives [19]. The most frequent cause of uncertainty is the lack of information [20] that may lead to a lack of knowledge or imprecise human judgements and preferences. This lack of knowledge often leads to difficulty in determining DMs’ preferences with a crisp value precisely. Therefore, the combination of AHP and fuzzy language expressions, most widely known as the Fuzzy Analytic Hierarchy Process (FAHP), has been developed and applied widely in order to minimise uncertainty. Although a number of FAHP methods with sophisticated mathematical formulas have been developed [21–25], uncertainty can be still present in its mathematical functions. For example, significant differences in results have been noticed when normalised fuzzy weights were computed by using three different formulas, namely the Wang and Chín’s formula [25], the Ishizaka’s formula [23] and the fuzzy eigenvector’s formula [24]. Krejčí [24] showed that the fuzzy eigenvector’s formula produced the least uncertain result. Therefore, there is scope to improve FAHP in order to minimise uncertainty as much as possible.
3.1 Introduction

Other problems with AHP are the occurrence of rank reversal and insufficient rank discrimination. Belton and Gear [26] initially noticed that rank reversal occurs because of the addition or deletion of an alternative. Dede et al. [27] highlighted that the uncertainty level, various weight deriving methods, and alternative preference scales may cause rank reversal. In the case of the uncertainty level, the involvement of a great number of DMs may reduce uncertainty [27] and rank reversal can be avoided. Moreover, the rank reversal caused by a weight deriving method could be avoided by applying the geometric mean aggregation [28, 29]. In contrast to earlier findings, however, Franek and Kresta [30] showed that no evidence of rank reversal was detected because of alternative preference scales. Further research is therefore required to clarify the relationship between alternative preference scales and rank reversal. In addition, unlike AHP, no work has focused on investigating rank reversal in FAHP.

Another undesired phenomenon that can be faced by DMs is insufficient rank discrimination. Insufficient rank discrimination occurs when the overall score of two or more alternatives are very close and often leads to failure in determining the best alternative [31]. Despite its importance in the selection of the best alternative, little attention has been paid to enhancing rank discrimination.

Even though some work has been carried out to overcome each of the aforementioned drawbacks individually, no attempt has been made to develop an extended AHP that might do so in a single method. There have been only a few attempts to combine FAHP and stochastic simulations (FSAHP) in order to overcome the lack of information and biased opinions in MCGDM for a selection problem. For example, Jing et al. [32] proposed a hybrid FSAHP method by combining an FAHP that was developed by Buckley [21], a fuzzy set theory (FST), a beta-PERT distribution and a Monte Carlo simulation. Another type of FSAHP was developed by Jato-Espino et al. [33] by combining Integrated Value Model for Sustainable Assessments, FAHP, FST, a beta-PERT distribution, and a Monte Carlo simulation. In the aforementioned studies, each object (i.e. criteria and alternatives) in the hierarchical structure was independent from the rest and therefore AHP was applied. When interdependencies between objects in the hierarchy need to be taken into account, Analytic Network Process (ANP) [34, 35], which is a generalisation of the AHP, must be used. Promentilla et al. [36] proposed a combination of ANP, FAHP, FST, a beta-PERT distribution, and a Monte Carlo simulation.

The FSAHP methods discussed above [33, 32, 36] allow the DMs to emphasise the most likely preference. However, when DMs do not want to give too much emphasis on the most likely preference, Jing et al. [37] proposed another type of language expression, namely interval judgement, that was combined with a probabilistic distribution, lexicographic goal programming, and a Monte Carlo simulation. Despite the particular benefits offered by all the aforementioned methods, they do not always guarantee a less uncertain outcome because they do not consider the reciprocity interactions of pairwise comparisons among the elements in fuzzy pairwise comparison.
matrices (FPCMs) during the computations. In response to the limitations described above, this study proposes a new selection MCDM method, Integrated Constrained Fuzzy Stochastic Analytic Hierarchy Process (IC-FSAHP), that combines constrained FAHP, FST, a modified beta-PERT distribution, and a Monte Carlo simulation. IC-FSAHP takes advantage of the best characteristics of these methods to overcome the drawbacks and concerns of conventional AHP for the selection problem by multiple DMs under uncertainty.

This work also investigates the performance of IC-FSAHP, for which we have designed and conducted extensive computational experiments. First, IC-FSAHP was applied to a case study in the equipment selection problem in mineral processing. The case study, adapted from Rahimdel and Ataei [38], selects the best crushing equipment under uncertainty by multiple experts. Second, the comparative analysis between the results that were obtained from IC-FSAHP and those obtained from other existing methods (i.e. AHP and FSAHP) was carried out. Third, the suitability of IC-FSAHP under different levels of disagreement among DMs was studied; four schemes of the level of disagreement among DMs were applied. Fourth, the performance of IC-FSAHP on the relationships between the number of DMs and uncertainty as well as rank discrimination were assessed. Six schemes of DMs were applied, namely 4, 5, 6, 7, 10 and 20 DMs. Fifth, the performance of IC-FSAHP on the occurrence of rank reversal due to the addition and deletion of one alternative as well as the use of other preference scales including its effects on the selection problem was studied.

The remainder of the manuscript is organised as follows: Section 3.2 provides theoretical background of FST, constrained FAHP, Monte Carlo simulation, and modified beta-PERT distribution; Section 3.3 describes the proposed method; Section 3.4 presents the performance of IC-FSAHP for the selected case study. The computational experiments of the proposed IC-FSAHP are presented in Section 3.5 (comparative analyses between IC-FSAHP with other existing methods), Section 3.6 (the effects of disagreement among DMs), Section 3.7 (the effects of the number of DMs on a decision making process) and Section 3.8 (rank reversal analyses). Finally, Section 3.9 provides the conclusions.

3.2 Theoretical background

This section discusses the key theoretical aspects behind the fuzzy set theory (FST), Monte Carlo simulation, Modified Beta-PERT distribution, as well as AHP and modifications to this method. The following sub-sections are provided as background for the development of the method in Section 3.3.

3.2.1 Fuzzy set theory

The fuzzy set theory (FST) [39], initially introduced into decision making by Bellman and Zadeh [40], is used to represent the vagueness of statements in natural language as real
numbers that have membership function over the range 0 and 1. 0 describes absolutely unlikely or false statements and 1 describes absolutely likely or true statements. In the case of the application of FST in AHP, Laarhoven and Pedrycz [41] proposed the first combined FST and AHP method by using a triangular fuzzy number (TFN).

A TFN consists of three real numbers and its membership function is conditioned in the following form:

\[
f(x) = \begin{cases} 
  \frac{x-c_L}{c_M-c_L}, & \text{if } c_L < x < c_M; \\
  1, & \text{if } x = c_M; \\
  \frac{c_U-x}{c_U-c_M}, & \text{if } c_M < x < c_U; \\
  0, & \text{otherwise.}
\end{cases}
\]

(3.1)

Where \(c_L\) and \(c_U\) are defined as the lower and upper boundary values of the TFN \(\tilde{A}(x)\), and \(c_M\) is termed as the middle value of \(\tilde{A}(x)\). Suppose that we have \(c_L, c_M, c_U = 3, 4,\) and \(5,\) respectively. Then the graph of this TFN is shown in Figure 3.2.1.

![Fig. 3.2.1 Membership function of a TFN (3, 4, 5).](image)

Since its introduction, TFN is the most widely used FST number to represent the information when dealing with uncertainty in real-world applications. The popularity of TFN is mostly because it is easy to implement and thus leads to straightforward computation. In this paper, all the positive TFNs are taken into account so that the positivity property of pairwise comparison is maintained. Therefore, the lowest membership number for a given TFN must be higher than zero.

Suppose that TFN \(\tilde{A}\) and TFN \(\tilde{B}\) are defined as, \(\tilde{A} = (c_L^{A}, c_M^{A}, c_U^{A})\), \(\tilde{B} = (c_L^{B}, c_M^{B}, c_U^{B})\), the arithmetic operations between these two TFNs are as follows:

- Addition of \(\tilde{A} + \tilde{B} = (c_L^{A} + c_L^{B}, c_M^{A} + c_M^{B}, c_U^{A} + c_U^{B}).\) (3.2)
- Multiplication of \(\tilde{A}\) and \(\tilde{B} = (c_L^{A}c_L^{B}, c_M^{A}c_M^{B}, c_U^{A}c_U^{B}).\) (3.3)
- Division of \(\tilde{A}\) and \(\tilde{B} = (c_L^{A}/c_L^{B}, c_M^{A}/c_M^{B}, c_U^{A}/c_U^{B}).\) (3.4)
- Reciprocation \(\tilde{A} = (1/c_A^{U}, 1/c_A^{M}, 1/c_A^{L}).\) (3.5)
Furthermore, the mathematical forms of the comparison of two TFNs are denoted in the following form:

- \( \tilde{A} \geq \tilde{B} \) if \( c_L^A \geq c_L^B, c_M^A \geq c_M^B, c_U^A \geq c_U^B \).  
  \[ (3.6) \]

- \( \tilde{A} > \tilde{B} \) if \( c_L^A > c_L^B, c_M^A > c_M^B, c_U^A > c_U^B \).  
  \[ (3.7) \]

In order to obtain a crisp number from a TFN, the centre-of-area (COA) defuzzification technique [42] is applied in this paper. The centre of area COA \( \tilde{A}(x) \) of a TFN \( \tilde{A}(x) = (c_L^A, c_M^A, c_U^A) \) is formulated as follows:

\[
COA \tilde{A}(x) = \frac{(c_U^A - c_L^A) + (c_M^A - c_L^A)}{3} + c_L^A.
\]

### 3.2.2 Stochastic modelling

Depending on the nature of real-world decision problems, such as qualitative or quantitative information and the level of certainty of such information, the DMs would most likely have a different insight on the preference, judgement, and priority scales. This latent inconsistency increases uncertainty due to dispersed and biased opinions in MCGDM. This inconsistency needs to be taken into account in the development of a more robust MCGDM method. As indicated in the literature, this uncertainty could be modelled by stochastic simulation; Eskandari and Rabelo [43] showed that a stochastic approach might capture and handle the uncertainty behaviour in AHP. By using stochastic simulation, the most likely results that match the real problem would be achieved, and in addition, the probability of overlaps amongst alternatives caused by the DMs’ various insights on the priority scale might be measured. A Monte Carlo simulation has been also applied in AHP to enhance rank discrimination of alternatives [44]. Due to the aforementioned advantages, a Monte Carlo simulation will be applied in this study.

Monte Carlo simulations are commonly used to address variable data under uncertainty. This type of simulation relies on the statistical representation of available information and provides a variety of possible results. Such results are attained from the substitution of TFN values that replicate the outcomes (e.g. the local weight of each criterion and the priority of each alternative), a procedure that is repeated multiple times. The TFN values in each repetition, or iteration, are substituted from a set of random numbers that are generated from a probability distribution. The iteration often takes place hundreds or thousands of times and the obtained outcomes from each iteration are recorded. Therefore, the probabilities of different results could be determined.

As previously mentioned, the Monte Carlo simulation involves random sampling that is derived from a probability distribution. Choosing the probability distribution that is used to model the DMs’ opinion is then of vital importance. The type of input that is going to be modelled is a basis for choosing the type of probability distribution. For the
3.2 Theoretical background

purpose of this study, we divided the opinions from DMs on each pairwise comparison into three designated individual groups, namely minimum, maximum and mode values. Vose [45] recommends using either triangular or beta-PERT distributions for processing these types of inputs. Compared with a triangular distribution, a beta-PERT distribution is more likely to produce normal distributions with fewer data required [32].

3.2.2.1 Modified beta-PERT distribution

An extension of the modified beta-PERT distribution [45] is suggested in this work, in order to obtain a mean ($\mu$) that is more representative. In standard beta-PERT, the value of $\gamma$, which controls the weight of the mode value, is 4; the mean can thus be simply determined by

$$mean = \mu = \frac{min + (\gamma \cdot mode) + max}{\gamma + 2} = \frac{min + (4 \cdot mode) + max}{6}.$$ (3.9)

For the purpose of this paper, the denominator of the formula for the mean ($\mu$) is modified to be the number of DMs ($E$) and the equation is simply formulated as:

$$mean = \mu = \frac{min + ((E - 2) \cdot mode) + max}{E}.$$ (3.10)

The shape parameter $\alpha$ and $\beta$ for the modified beta-PERT distribution [45] are formulated as follow:

$$\alpha = \left(\frac{mean - min}{max - min}\right) \left(\frac{(mean - min) - (max - min)}{stdev^2} - 1\right),$$ (3.11)

$$\beta = \alpha \left(\frac{max - mean}{mean - min}\right),$$ (3.12)

where

$$stdev = \frac{max - mean}{E}.$$ (3.13)

3.2.2.2 Random number generation

In order to generate random numbers ($random_a, random_b, random_c$) [32] in a TFN for each pairwise comparison, Equations (3.14)–(3.16) are then used respectively.

$$random_a = min_a + random.beta(\alpha_a, \beta_a)(max_a - min_a),$$ (3.14)

$$random_b = min_b + random.beta(\alpha_b, \beta_b)(max_b - min_b),$$ (3.15)

$$random_c = min_c + random.beta(\alpha_c, \beta_c)(max_c - min_c),$$ (3.16)

where $min$ and $max$ reflect the lowest and highest opinions from DMs on each pairwise comparison; random.beta denotes a standard Python function for generating a random
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number parameterised by $\alpha$ and $\beta$; $random_a, random_b, random_c$ are a generated TFN, consisting of minimum, medium and maximum random numbers, respectively.

3.2.3 Analytical Hierarchy Process (AHP)

Saaty [18] proposed the analytical hierarchy process (AHP) that incorporates several aspects that are involved when solving an MCDM problem, such as logic, experience, knowledge, and emotion. AHP involves a deconstruction process of an MCDM problem into a hierarchy that often comprises three-level structures from the top to the bottom that describe the goal, criteria, and alternatives, respectively. The main notion of AHP is in the pairwise comparison matrix (PCM) that is constructed from the comparison of each two objects (i.e. criteria or alternatives) in the same level of the hierarchy with respect to each object in the higher level of the hierarchy. For example, suppose that the importance of criteria, $C_1, C_2, \ldots, C_n$ with respect to the goal is represented as $w_1, w_2, \ldots, w_n$, then PCM $A = \{a_{ij}\}^{(p)}_{(i,j=1)}$ (p describes objects, either criteria or alternatives: p (n) represents criteria and p (m) represents alternatives) could be described as follows:

$$
A = \left( \frac{w_i}{w_j} \right) = a_{ij} = \begin{bmatrix}
C_1 & C_2 & \cdots & C_n \\
C_1 & (w_1/w_1) = 1 & (w_1/w_2) = a_{12} & \cdots & (w_1/w_n) = a_{1n} \\
C_2 & (w_2/w_1) = 1/a_{12} & (w_2/w_2) = 1 & \cdots & (w_2/w_n) = a_{2n} \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
C_n & (w_n/w_1) = 1/a_{1n} & (w_n/w_2) = 1/a_{2n} & \cdots & (w_n/w_n) = 1
\end{bmatrix}
$$

In Equation (3.17) $a_{ij}$ describes how many times each criterion i is more important than another criterion j with respect to the goal. In the case when an object compares with itself (i=j), then $a_{11} = a_{22} = \ldots = a_{nn} = 1$. Additionally, another important notion in the PCM is reciprocity, when the criterion $C_i$ is $a_{ij}$ times more important than the criterion $C_j$ so the criterion $C_j$ is $a_{ij}$ times less important than the criterion $C_i$.

In order to check the consistency of each PCM, Saaty [18] developed the formula for measuring the consistency ratio (CR) of PCM $A = \{a_{ij}\}^{(p)}_{(i,j=1)}$ in the following form:

$$
CR = \frac{CI}{RI} = \frac{\lambda_{max} - p}{RI(p - 1)}, \quad (3.18)
$$

where CI represents the degree of acceptable inconsistency, RI is the random consistency index, p is the number of compared objects (p = n for criteria and p = m for alternatives), and $\lambda_{max}$ is the maximal eigenvalue of the pairwise comparison matrix. PCM A is acceptably inconsistent if CR < 0.1. When the PCM A is acceptably inconsistent, the weights of criteria and local scores of alternatives are then measured simultaneously; otherwise, if CR $\geq 0.1$, the
DMs should reconsider their judgements on the importance of each criterion and preference of each alternative.

There are two frequently used techniques to compute the weight of an object from a PCM: (i) the Saaty’s eigenvector technique [46] and (ii) the logarithmic least squares technique (LLSM), that is usually termed as a geometric mean technique [47]. In the case of Saaty’s eigenvector technique, the weights of the objects \( w_{p1}, w_{p2}, \ldots, w_{pi} \) are defined as components of the normalised principal eigenvector corresponding to the maximum eigenvalue of a PCM.

When \( a_{ij} = 1/a_{ji}, a_{ik}a_{kj} = a_{ij} \) for all \( k \), the principal eigenvectors \( \tilde{w}_{pi} \) are obtained from this equation:

\[
A\tilde{w}_{pi} = \lambda_{\text{max}}\tilde{w}_{pi}, \quad (3.19)
\]

and to derive the normalised principal eigenvector \( w_{pi} \), the following equation is then applied:

\[
w_{pi} = \frac{\tilde{w}_{pi}}{\sum_{i} \tilde{w}_{pi}}, \quad i = 1, 2, \ldots, p. \quad (3.20)
\]

Unlike Saaty’s eigenvector technique, in the geometric mean technique the solution is given by normalising the geometric means of the elements in each row of the pairwise comparison matrix. For example, when \( a_{ij} = 1/a_{ji}, a_{ik}a_{kj} = a_{ij} \) for all \( k \), the weights of the objects \( w_{pi} \) are obtained from this equation:

\[
w_{pi} = \frac{\sqrt[p]{\prod_{j=1}^{p} a_{ij}}}{\sum_{k=1}^{p} \sqrt[p]{\prod_{j=1}^{p} a_{kj}}}, \quad i = 1, 2, \ldots, p. \quad (3.21)
\]

Since the weights of the objects from both these methods are obtained from the normalisation of all components, the sum of the weights should be one, i.e. \( \sum_{i=1}^{p} w_{pi} = 1 \).

In order to obtain the total scores \( \tilde{S}_{mk} \) of the \( k-th \) alternative, \( k \in \{1, 2, \ldots, m\} \), the local priorities of the \( k-th \) alternative with respect to each criterion are aggregated according to the following formula:

\[
\tilde{S}_{mk} = \sum_{i=1}^{n} w_{C_i} w_{ki}, \quad (3.22)
\]

where \( w_{C_i} \ (i = 1, 2, \ldots, n) \) is the weight of criterion \( i \) and \( w_{ki} \ (k = 1, 2, \ldots, m) \) is the respective local priority of the \( k-th \) alternative on criterion \( i \). Furthermore, the score of the \( k-th \) alternative \( (S_{mk}) \) is obtained by means of the distributive mode normalisation technique. This normalisation technique divides the overall scores of the \( k-th \) alternative by the total overall scores of the alternatives. The score of each alternative is obtained as follows:

\[
S_{mk} = \frac{\tilde{S}_{mk}}{\sum_{i=1}^{m} \tilde{S}_{mk}}. \quad (3.23)
\]
Based on the score of each alternative \((S_{m1}, S_{m2}, \ldots, S_{mk})\), the alternatives are then ranked from the highest score to the lowest. The alternative with the highest score denotes the best alternative.

### 3.2.4 Constrained Fuzzy AHP (CFAHP)

FAHP is used when DMs find it difficult to determine precisely their preferences with a crisp value due to the lack of information and imprecise human judgements and preferences. Buckley [21], Chang [48] and Laarhoven and Pedrycz [41] developed the three most popular FAHP methods that are widely employed and have broad influence on the theories and applications of FAHP. Those aforementioned FAHP methods are mainly distinguished by their procedures for deriving fuzzy weights from a fuzzy pairwise comparison matrix (FPCM).

An FPCM expresses the configurations of fuzzy AHP problems. An FPCM is constructed by means of FST instead of crisp values and is set up based on fuzzy arithmetic. For example, suppose that the importance of criteria, \(C_1, C_2, \ldots, C_n\) with respect to the goal is represented as \(w_1, w_2, \ldots, w_n\) in TFN then FPCM \(A = \{a_{ij}\}_{i,j=1}^{n}\), \(a_{ij} = (a_{ijL}, a_{ijM}, a_{ijU})\) (p describes objects, either criteria or alternatives; p(n) represents criteria and p(m) represents alternatives; the subscripts \(L, M, U\) represent the lowest, middle, and upper numbers in a TFN) could be described as follows:

\[
A = \left(\begin{array}{cccc}
C_1 & C_2 & \cdots & C_n \\
C_1 & (w_1/w_1) = 1,1,1 & (w_1/w_2) = a_{12} & \cdots & (w_1/w_n) = a_{1n} \\
C_2 & (w_2/w_1) = 1/a_{12} & (w_2/w_2) = 1,1,1 & \cdots & (w_2/w_n) = a_{2n} \\
\vdots & \vdots & \ddots & \ddots & \ddots \\
C_n & (w_n/w_1) = 1/a_{1n} & (w_n/w_2) = 1/a_{2n} & \cdots & (w_n/w_n) = 1,1,1 \\
\end{array}\right)
\]  

(3.24)

To calculate each non-diagonal fuzzy element on upper right components of \((1, 1, 1)\), the assessment of the importance of one criterion over another, e.g. \((w_1/w_2)\) is determined by the division of two TFNs; and for the lower triangular components of \((1, 1, 1)\), e.g. \((w_2/w_1)\), the importance of criteria is determined from the reciprocal of \((w_1/w_2)\). For example, for the importance of criteria of \(C_1\) to \(C_2\) \((w_1/w_2)\), if the TFNs for \(w_1\) and \(w_2\) are \((a_1, b_1, c_1)\) and \((a_2, b_2, c_2)\), respectively, then \(w_1/w_2 = (a_1/c_2, b_1/b_2, c_1/a_2)\) and its reciprocal values, \(w_2/w_1 = (a_2/c_1, b_2/b_1, c_2/a_1)\). This procedure is applied as well for constructing FPCMs of each alternative with respect to each criterion.

Like traditional AHP, the FAHP method should also measure the consistency index (CI) and consistency ratio (CR) of each FPCM. Despite its popularity, neither Buckley [21], nor Chang [48], nor Laarhoven and Pedrycz [41] presented the measurement of the maximal eigenvalue, CI, and CR of each FPCM. In this work, a formula to measure CI and CR will be based on the traditional AHP. This approach is frequently used to measure CI and CR in
3.2 Theoretical background

fuzzy environments [49]. Since this approach requires crisp values, these are generated from the defuzzification of TFN from each component in FPCMs by means of COA, as has been formulated in Equation (3.8). Equation (3.11) is then used to calculate CI and CR. As in traditional AHP, in order to determine whether the FPCM is adequately inconsistent or not, the crisp CR value must be compared with the limit constant value of 0.1. The tolerable inconsistent FPCM \( \tilde{A} \) should have the crisp CR value less than 0.1 and the fuzzy weights of the criteria and fuzzy local priorities of the alternatives are then derived.

There are several techniques to derive the fuzzy weights of the criteria and fuzzy local priorities of the alternatives, such as Buckley’s technique [21] and the constrained FAHP technique [22]. The fuzzy weights of the criteria and fuzzy local priorities of the alternatives that are obtained by means of constrained FAHP produce less vague results compared to those obtained using Buckley’s technique due to a mistake in its normalisation procedure [50]. In addition, the constrained FAHP technique takes into account the reciprocity interactions among the elements of the TFNs in the FPCM \( \tilde{A} \) [51]. Therefore, the constrained fuzzy AHP technique will be applied in the method developed in this paper. Furthermore, another advantage of using constrained FAHP for measuring the weights is that this technique is based on the geometric mean. As discussed earlier, the use of the geometric mean has been proven to help in avoiding rank reversal.

The fuzzy weight of each criterion or alternative in a TFN, \( \tilde{w}_p^C = (w_{pL}^C, w_{pM}^C, w_{pU}^C) \), \( p = 1, \ldots, m \) or \( n \) (where the superscript \( C \) represents the constrained fuzzy arithmetic), is formulated as follows:

\[
w_{pL}^C = \min \left\{ \frac{\sqrt[p]{\prod_{j=1}^{p} a_{ij}}}{\sum_{k=1}^{p} \sqrt[p]{\prod_{j=1}^{p} a_{kj}}} : \begin{cases} a_{kj} \in [a_{kjL}, a_{kjU}], & \forall j > k, \\ a_{jk} = \frac{1}{a_{kj}}, & \forall j < k, \\ a_{jj} = 1, & \forall j \end{cases} \right. \tag{3.25}
\]

\[
w_{pM}^C = \left\{ \frac{\sqrt[p]{\prod_{j=1}^{p} a_{ijM}}}{\sum_{k=1}^{p} \sqrt[p]{\prod_{j=1}^{p} a_{kjM}}} \right\}, \tag{3.26}
\]

\[
w_{pU}^C = \max \left\{ \frac{\sqrt[p]{\prod_{j=1}^{p} a_{ij}}}{\sum_{k=1}^{p} \sqrt[p]{\prod_{j=1}^{p} a_{kj}}} : \begin{cases} a_{kj} \in [a_{kjL}, a_{kjU}], & \forall j > k, \\ a_{jk} = \frac{1}{a_{kj}}, & \forall j < k, \\ a_{jj} = 1, & \forall j \end{cases} \right. \tag{3.27}
\]

However, these equations would be computationally demanding, particularly when deriving the lowest and highest values of a fuzzy weight \( (w_{pL}^C, w_{pU}^C) \). In fact, \( \frac{p(p-1)}{2} \) variables have two values and \( 2^{\frac{p(p-1)}{2}} \) FPCM combinations need to be analysed. For instance, when deriving the fuzzy weights of 7 criteria, 21 variables have two values and \( 2^{21} \) FPCMs need to be evaluated. It is worth noting that there is a trade-off between the accuracy of the decision and the complexity of the computation.
An Integrated Constrained Fuzzy Stochastic Analytic Hierarchy Process Method with Application to the Choice Problem

After deriving the fuzzy weights of the criteria and fuzzy local priorities of the alternatives, the global priorities or overall scores of the alternatives are derived. In order to ensure that the sum of the calculated weights of the criteria is 1 and to maintain reciprocity, the equations that have been developed by Krejčí et al. [51] are used in this paper. These equations are constructed from the combination of constrained fuzzy arithmetic and the formula of the weighted average, as in traditional AHP. The fuzzy global priority of the $k$–th alternative, $k \in \{1, 2, \ldots, m\}$ in a TFN $(S^L_{mk}, S^M_{mk}, S^U_{mk})$ can be expressed by the following formulae:

$$S^L_{mk} = \min \left\{ \sum_{i=1}^{n} \sqrt[n]{\prod_{j=1}^{n} a_{ij} w_{ipL}}; \begin{array}{l}
  a_{kj} \in [a_{kjL}, a_{kjU}], \forall j > k, \\
  a_{jk} = \frac{1}{a_{kj}}, \forall j < k, \\
  a_{jj} = 1, \forall j
\end{array} \right\},$$

(3.28)

$$S^M_{mk} = \sum_{i=1}^{n} \sqrt[n]{\prod_{j=1}^{n} a_{ij2} w_{ipM}},$$

(3.29)

$$S^U_{mk} = \max \left\{ \sum_{i=1}^{n} \sqrt[n]{\prod_{j=1}^{n} a_{ij} w_{ipU}}; \begin{array}{l}
  a_{kj} \in [a_{kjL}, a_{kjU}], \forall j > k, \\
  a_{jk} = \frac{1}{a_{kj}}, \forall j < k, \\
  a_{jj} = 1, \forall j
\end{array} \right\},$$

(3.30)

where $S^L_{mk}$, $S^M_{mk}$, $S^U_{mk} = k = 1, 2, \ldots, m$, are the are the lowest, middle and highest values of the fuzzy global priority of the $k$–th alternative with respect to criteria $i = 1, 2, \ldots, n$.

The TFN of the global priority of each alternative is then defuzzified by means of COA in order to obtain its crisp value. Furthermore, the obtained crisp value of each alternative $\tilde{S}_{mk}$ is normalised by means of Equation (3.23) in order to attain the score of the $k$–th alternative $(S_{mk})$. These normalised scores of alternatives are then ranked from the highest to the lowest in order to obtain the best alternative. The alternative with the highest score is chosen as the best alternative.

3.3 The Developed Integrated Constrained Fuzzy Stochastic AHP (IC-FS-AHP) Method

An integrated constrained fuzzy stochastic AHP (IC-FS-AHP) method is proposed here to solve selection problems with multiple DMs. IC-FS-AHP has the ability to capture uncertainty caused by the lack of information and biased insights in various DMs’ judgements. IC-FS-AHP involves five major stages: (1) defining the problem notions, (2) computing the local fuzzy weights of the criteria and local fuzzy priorities of the alternatives, (3) computing the global priorities of the alternatives, (4) synthesising the results, and (5) ranking. A conceptual framework of IC-FS-AHP is shown in Figure 3.3.1.

The detailed steps in every stage of IC-FS-AHP are presented as follows:
Fig. 3.3.1 The framework of the proposed IC-FSAHP method for the choice problem.

Stage 1: The main objective of the first stage is to define the notions of a decision problem. The problem notions such as the goal, criteria, and alternatives, as well as the number of DMs, need to be defined clearly at the beginning. The criteria and alternatives could be determined through discussions, meetings and literature reviews. Then, the defined
problem is structured in a three-level top-down hierarchy containing the goal, criteria, and alternatives, respectively.

The importance of the criteria and the preference of the alternatives over others are rated by means of a scale of seven linguistic variables. For example, in the case of rating the importance of criteria, the following linguistic variables are used: Extremely Unimportant (EU), Very Unimportant (VU), Unimportant (U), Fair (F), Important (I), Very Important (VI) and Extremely Important (EI). In the case of rating the preference of alternatives, a scale of seven linguistic variables are used: Very Low (VL), Low (L), Medium Low (ML), Medium (M), Medium-High (MH), High (H) and Very High (VH). Each scale level is valued by means of FST to anticipate the influence of uncertainty on judgements or preferences. As indicated in the theoretical background, TFN is used in this paper.

![Fig. 3.3.2 Membership functions of the TFN variables for linguistic judgements that are used in judging the importance of the criteria and preference of the alternatives.](image)

By using TFNs, the DMs are expected to feel more prudent in their opinions than using crisp values. Each TFN contains three boundary numbers containing the lowest, middle and highest values. For example, the ordered number of TFN to describe EU or VL is 0.5, 1, 2, VU or L is 2, 3, 4, U or ML is 3, 4, 5 and so forth. Figure 3.3.2 shows the membership functions for TFN scale levels.

**Stage 2:** The three main aims of the second stage are to conduct the pairwise comparison of each object over others, to model these pairwise comparisons, and to generate the fuzzy weights of the criteria and fuzzy local priorities of the alternatives. Pairwise comparison of each object with respect to the objects in the higher hierarchy level are then conducted by DMs. The DMs’ opinions on each pairwise comparison are divided into three designated individual groups, namely minimum, maximum and mode values. The modified beta-PERT distribution is used in order to generate random numbers that model the values of judgements and preferences of DMs. Equations (3.10) – (3.16) are used to generate the random numbers. The generated random numbers are set as inputs in FPCMs.

The inconsistency check on each FPCM is then performed. The TFNs of the generated random numbers are defuzzified by using the COA method that is formulated in Equation
(3.8) in order to obtain the crisp values. The CI and CR of each defuzzified FPCM are then determined by using Equation (3.18). Each defuzzified FPCM must have CR less than 0.1, termed as acceptable inconsistent FPCM, in order to conduct further steps, otherwise, the judgements and preferences step on criteria and alternatives should be repeated. The last step in Stage 2 uses the original FPCMs instead of the defuzzified values.

The fuzzy weights of the criteria and fuzzy local priorities of the alternatives are then computed from the acceptable inconsistent FPCMs by means of constrained FAHP that is developed by Enea and Piazza [22]. The fuzzy weights of the criteria and fuzzy local priorities of the alternatives are computed by using Equations (3.25) – (3.27). The obtained fuzzy weights of the criteria are then defuzzified by means of Equation (3.8) in order to obtain their crisp values. Furthermore, the obtained crisp value of each criterion $\tilde{w}_C^i$ is normalised by means of Equation (3.23) in order to attain the weight of the $i$-th criteria $(w_C^i)$.

**Stage 3**: In the third stage, the global priorities of the alternatives in TFN and the crisp overall score of each alternative are determined. The former is obtained by means of Equations (3.28) – (3.30), whereas to obtain the crisp overall score, the fuzzy global priority of each alternative is defuzzified by means of COA. Furthermore, the obtained crisp value of each alternative $\tilde{S}_m^k$ is normalised by using Equation (3.23) in order to attain the normalised overall score of the $k$-th alternative $(S_m^k)$. These normalised scores of alternatives are then ranked from the highest to the lowest in order to obtain the best alternative.

**Stage 4**: In the fourth stage, a probability distribution of DMs’ judgements and preferences is synthesised by means of Monte Carlo simulations. The simulations result in:

- the random numbers for each component of the FPCMs,
- the fuzzy weights of the criteria,
- the fuzzy local priorities of the alternatives,
- the fuzzy global priorities of the alternatives,
- the defuzzified fuzzy weights of the criteria and defuzzified global priorities of the alternatives,
- the normalised weights of the criteria and normalised global priorities of the alternatives.

The simulation runs for a number of iterations (e.g. 1000) and the results of each iteration are then recorded.

**Stage 5**: In the fifth stage, the weights of the criteria and overall scores of the alternatives are obtained and plotted as probability distributions in histogram and violin plots. Based on the global priorities or overall scores of the alternatives’ histogram, the alternatives are then ranked from the highest to the lowest on the basis of the global priority or overall score. The highest probability on the highest score is the best alternative and selected as the solution of a selection problem.
It is worth noting that IC-FSAHP could be applied to another type of ordered fuzzy numbers (e.g. trapezoidal fuzzy number). This could be done by generalising the IC-FSAHP formulas for calculating the lowest and highest values of a TFN (e.g. $w_{pL}^C$ and $w_{pU}^C$) to calculate the lowest and highest boundary values of other ordered fuzzy numbers.

3.4 IC-FSAHP for equipment selection in mineral processing

In this section, IC-FSAHP is applied to a case study on equipment selection in mineral processing in order to highlight the advantages and performance of the method. Equipment selection is the task of scrutinising various machinery options being considered and choosing the best equipment with respect to multiple conflicting criteria such as technical, economic, environmental and social aspects [9]. The selection of appropriate equipment in mineral processing is always of vital importance. The use of the right equipment has high impact with regards to cost and benefits and thus the proper equipment selection can avoid losses in production as well as excessive costs associated to equipment troubleshooting [9].

3.4.1 Background

A decision problem for primary crusher selection in an iron mine is used here as a case study in order to demonstrate the applicability of IC-FSAHP. The data for this case study was adapted from Rahimdel and Ataei [38]. For the purpose of this study, an implementation of IC-FSAHP in Python 3 was used to make calculations.

DMs in the Golegohar iron mine wanted to determine the most appropriate primary crusher for performing comminution in its production process. After conducting an analysis on this selection problem, six evaluation criteria were identified, namely capacity ($C_1$), feed size ($C_2$), product size ($C_3$), abrasion index ($C_4$), rock compressive strength ($C_5$), and the application of the primary crusher to mobile plants ($C_6$) [38]. For the purpose of this study, five equipment alternatives in Rahimdel and Ataei [38] were considered: Gyratory ($A_1$), Double toggle jaw ($A_2$), Single toggle jaw ($A_3$), High-speed roll ($A_4$), and Low-speed sizer ($A_5$). For conducting the evaluation of these alternatives Rahimdel and Ataei [38] reported the use of a decision maker committee of three DMs, denoted by $D_1$, $D_2$, and $D_3$.

3.4.2 Application of IC-FSAHP

The hierarchy structure of this selection problem is illustrated in Figure 3.4.1.

The scale of seven linguistic variables that is depicted in Figure 3.3.2 was used to evaluate the importance of criteria with respect to the goal and the preferences of alternatives with respect to each criterion. The pairwise comparison assessments that were determined by DMs are presented in Table 3.4.1 and Table 3.4.2, respectively.
3.4 IC-FSAHP for equipment selection in mineral processing

Fig. 3.4.1 Hierarchy structure of the selection of a primary crusher in this case study.

Table 3.4.1 Assessment of the importance of six criteria with respect to the goal by three DMs [38].

<table>
<thead>
<tr>
<th>Experts</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
<th>$C_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_1$</td>
<td>EI</td>
<td>VI</td>
<td>VI</td>
<td>VI</td>
<td>I</td>
<td>U</td>
</tr>
<tr>
<td>$D_2$</td>
<td>EI</td>
<td>VI</td>
<td>EI</td>
<td>VI</td>
<td>VI</td>
<td>I</td>
</tr>
<tr>
<td>$D_3$</td>
<td>EI</td>
<td>EI</td>
<td>VI</td>
<td>I</td>
<td>I</td>
<td>I</td>
</tr>
</tbody>
</table>

Table 3.4.2 Assessment of the preference of five alternatives with respect to each criterion by three DMs (adapted from Rahimdel and Ataei [38]).

<table>
<thead>
<tr>
<th>$A_m$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
<th>$C_6$</th>
</tr>
</thead>
</table>

In this work, the assessments were aggregated by using the modified beta-PERT distribution in order to generate random numbers. For example, the individual preference groups (i.e. minimum, maximum and mode values) of alternative $A_1$ with respect to criterion $C_2$ were H (6, 7, 8), VH (8, 9, 9) and VH (8, 9, 9). By using Equations (3.14)–(3.16), the random numbers of the corresponding TFN obtained are 6.901, 8.983, 8.995. Table 3.4.3 and Table 3.4.4 show the random TFNs that were aggregated from Table 3.4.1 and Table 3.4.2, respectively.

To attain the FPCMs that are described in Equation (3.24), the elements in each FPCM were derived from fuzzy comparisons using Equation (3.4) for upper triangular FPCM and
Equation (3.5) for lower triangular FPCM. For example, the FPCM of alternatives $A_1$, $A_3$, and $A_5$ with respect to $C_1$ is shown in Equation (3.31).  

\[
\begin{bmatrix}
A_1 \\
A_3 \\
A_5
\end{bmatrix}
\begin{bmatrix}
1, 1, 1 \\
(8.000, 9.000, 9.000) \\
(8.000, 9.000, 9.000)
\end{bmatrix}^{-1}
\begin{bmatrix}
(8.000, 9.000, 9.000) \\
(5.166, 6.303, 7.303) \\
(5.166, 6.303, 7.303)
\end{bmatrix}^{-1}
\begin{bmatrix}
A_1 \\
A_3 \\
A_5
\end{bmatrix}
\begin{bmatrix}
(8.000, 9.000, 9.000) \\
(8.000, 9.000, 9.000) \\
(8.000, 9.000, 9.000)
\end{bmatrix}^{-1}
\begin{bmatrix}
1, 1, 1 \\
(8.000, 9.000, 9.000) \\
(8.000, 9.000, 9.000)
\end{bmatrix}^{-1}
\]

(3.31)

Since the consistencies of all FPCMs were less than 0.1, and therefore acceptable, it was possible to then compute the fuzzy weights of the criteria, fuzzy local and global priorities of the alternatives. The number of iterations used for this case study was 1000.

### 3.4.3 Results

Histogram and violin plots were used to visualise the distribution and probability density of the results. Figure 3.4.2.a) presents the probability density of the weight of each criterion with respect to the goal. The histogram bar plot shows that capacity ($C_1$) (0.198–0.217) was the most important criterion that needs to be prioritised. In addition, Figure 3.4.2.b) shows the violin plot of the weight of each criterion with respect to the goal and reveals that feed size ($C_2$) and product size ($C_3$) have little probability to be the most important criteria, which is evidenced by the fact that only very thin sections of their distribution overlap with that of $C_1$.

The Pearson’s correlation coefficients between the weights of $C_1$ and $C_2$, $C_1$ and $C_3$, and $C_2$ and $C_3$ are -0.081, -0.029, and -0.243, respectively. Since the correlation coefficients are...
3.4 IC-FSAHP for equipment selection in mineral processing

Fig. 3.4.2 a) Probability distributions of the weights of the criteria; and b) Violin plots of the weights of the criteria after 1000 iterations.

negative, as one criterion increases the other criterion has a tendency to decrease, with the effect being more significant as the correlation coefficient approaches -1 [52]. It is worth noting that even though $C_2$ and $C_3$ have a weak correlation, they have large overlaps with each other and thus capture the fact that the DMs had different points of view in terms of these criteria.

Figure 3.4.3.a) shows the probability density of the overall score of each alternative with respect to the criteria. It shows that the high-speed roll crusher ($A_4$) (0.216–0.225) was clearly the most attractive crusher without any overlap. In addition, Figure 3.4.3.b), which shows the corresponding violin plot, reveals that the gyratory ($A_1$), single toggle jaw ($A_3$), low-speed sizer ($A_5$) and double toggle jaw ($A_2$) crushers were the least preferable crushers with significant overlaps between each other. The overlaps indicate that DMs as a group did not provide a clear preference. The Pearson’s correlation coefficients between the weights of $A_1$ and $A_3$, $A_1$ and $A_5$, and $A_3$ and $A_5$ were -0.379, -0.397, and -0.139, respectively. It is worth noting that even though $A_1$ and $A_5$ have a moderate negative correlation, they have small overlaps with each other. Table 3.4.5, which provides the summary of final rank for each alternative, shows clearly the overlaps between each other. Although for the purposes of the equipment selection the ranking of the sub-optimal options could be ignored, the aim here was to highlight that IC-FSAHP can capture inconsistencies of DMs as a group, which can provide valuable information that in some cases might be critical.

It is worth noting that the proposed IC-FSAHP method succeeded in solving an MCGDM problem under a fuzzy environment when the DMs’ opinions have unimodal value. However, the DMs’ opinions sometimes have two or more most likely values. Therefore, there is a scope to further extend IC-FSAHP for the case when multimodal values are involved. In addition, in order to improve the robustness of IC-FSAHP, the uncertainty in the algorithm could be
An Integrated Constrained Fuzzy Stochastic Analytic Hierarchy Process Method with Application to the Choice Problem

Fig. 3.4.3 a) Probability distributions of the overall scores of the alternatives; and b) Violin plots of the overall scores of the alternatives after 1000 iterations.

Table 3.4.5 Summary of the final ranking of each alternative after 1000 iterations.

<table>
<thead>
<tr>
<th>$A_m$</th>
<th>Rank 1</th>
<th>Rank 2</th>
<th>Rank 3</th>
<th>Rank 4</th>
<th>Rank 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>0</td>
<td>956</td>
<td>44</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>23</td>
<td>977</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0</td>
<td>44</td>
<td>918</td>
<td>38</td>
<td>0</td>
</tr>
<tr>
<td>$A_4$</td>
<td>1000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$A_5$</td>
<td>0</td>
<td>0</td>
<td>38</td>
<td>939</td>
<td>23</td>
</tr>
<tr>
<td>Total</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
</tbody>
</table>

minimised by developing an appropriate technique to check the consistency of FPCMs. Such a technique should also be able to examine the consistency of the process in attaining the fuzzy weights of the criteria and fuzzy local priorities of the alternatives from the FPCMs. Moreover, it would be valuable to improve the method described in this work so that it can be used to solve a selection problem in group decision making under incomplete fuzzy preference relations. Further studies to develop such an extended method will be the subject of future work.

3.5 Comparative analyses between IC-FSAHP and other existing methods

In order to demonstrate the advantages of IC-FSAHP, we assess in this section the performance of AHP and FSAHP to the case study. Emphasis is put on applying the crisp and fuzzy judgements during the evaluations. For the purpose of this study, the direct
application of crisp judgements in AHP was conducted by means of two approaches, namely traditional AHP and stochastic AHP. On the other hand, an existing FSAHP method was applied to the case study, in which the judgements of DMs used TFN. The comparative analyses of the results obtained from the aforementioned methods are also discussed.

3.5.1 AHP

As previously mentioned in Section 3.1, the AHP method is used when DMs have no doubt in judging their preferences on the criteria and alternatives. For this reason, this assessment assumed that DMs were confident in their judgements. The importance of a specific criterion and the preference of each alternative over others are rated by means of a scale of seven crisp judgements. Table 3.5.1 shows the scale of relative importance that was used in this assessment.

In addition, since multiple DMs are involved in the case study, the aggregation of DMs’ judgements is of vital importance. On account of this reason, we considered applying two aggregation approaches, namely geometric mean and stochastic.

<table>
<thead>
<tr>
<th>The intensity of importance or preference</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>EU/VL</td>
</tr>
<tr>
<td>3</td>
<td>VU/L</td>
</tr>
<tr>
<td>4</td>
<td>U/ML</td>
</tr>
<tr>
<td>5</td>
<td>F/M</td>
</tr>
<tr>
<td>6</td>
<td>I/MH</td>
</tr>
<tr>
<td>7</td>
<td>VH/H</td>
</tr>
<tr>
<td>9</td>
<td>EI/VH</td>
</tr>
</tbody>
</table>

3.5.1.1 AHP based on an unweighted geometric mean approach for aggregating DMs’ opinions

The use of AHP in which the aggregation of DMs’ opinions is based on an unweighted geometric mean approach involves four major stages: (1) defining the problem notions, (2) computing the local weights of the criteria and local priorities of the alternatives, (3) computing the global priorities of the alternatives, and (4) ranking the alternatives. A conceptual framework of the considered AHP method is shown in Figure 3.5.1.

Since most of the stages of AHP in Figure 3.5.1 are explained in Section 3.2.3, this subsection only highlights a technique that is used in aggregating the DMs’ judgements. We assumed that the DMs are equally important and we therefore simply aggregate their opinions \( w_{Aggregated} \) by using the unweighted geometric mean approach that is formulated
Fig. 3.5.1 The stages of AHP in which the aggregation of DMs' opinions is based on an unweighted geometric mean approach.

in Equation (3.32).

\[ w_{Aggregated_p} = \frac{n_v}{\prod_{l=1}^{nu} w_{DM_l}} , \quad l = 1, 2, \ldots, nu, \]  

(3.32)
where $w_{Aggregated_p}$ represents the aggregated opinions from DMs that describe the importance or preference of the $p$–th object (i.e. criteria or alternatives); $w_{DM_l}$ is a judgement that is obtained from the $l$–th DM and $nu$ is the number of DMs.

After the computation of all stages has been completed, the weights of the criteria and global priorities of the alternatives are obtained and shown in Table 3.5.2 and Table 3.5.3, respectively. It is worth noting that the rankings of criteria and alternatives that were obtained from AHP were comparatively similar to those obtained from IC-FSAHP. More detailed findings of the results are discussed below.

As can be seen from Table 3.5.2, the ranking of criteria is determined as $C_1 \succ C_2 = C_3 \succ C_4 \succ C_5 \succ C_6$. An interesting finding of the results is that the second best criteria were $C_2$ and $C_3$, which had the same weight. The fact that both $C_2$ and $C_3$ had the same weight was also shown from the results that were obtained from IC-FSAHP in Figure 3.4.2. However, Figure 3.4.2 shows this in more detail, since the number of overlaps can be observed. In addition, the overlaps among all criteria are clearly shown in Figure 3.4.2. The overlaps show that IC-FSAHP is able to capture the variation of DMs’ opinions. On the other hand, AHP cannot capture such variation; this is because simply aggregating the DMs’ opinions into one crisp value using the unweighted geometric mean approach can increase the risk of losing valuable information about a problem during the computations. Furthermore, the use of the unweighted geometric mean formula often assumes that the DMs are equally important but in real-life applications this is not necessarily the case.

Table 3.5.2 Weights of the criteria that were obtained from AHP.

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
<th>$C_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>0.212</td>
<td>0.179</td>
<td>0.179</td>
<td>0.157</td>
<td>0.149</td>
<td>0.124</td>
</tr>
</tbody>
</table>

Table 3.5.3 Global priorities of the alternatives that were obtained from AHP.

<table>
<thead>
<tr>
<th></th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
<th>$A_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Score</td>
<td>0.201</td>
<td>0.183</td>
<td>0.2</td>
<td>0.222</td>
<td>0.194</td>
</tr>
</tbody>
</table>

As can be observed from Table 3.5.3, the ranking of alternatives is determined as $A_4 \succ A_1 \succ A_3 \succ A_5 \succ A_2$. An interesting finding of the results is the score between $A_1$ and $A_3$ was very close. In the case when the best score and the second best score alternatives are very close, DMs often face difficulties in determining the best alternative because there is no other supporting information. On the other hand, more detailed information (i.e. the number of overlaps) can be seen from the results that were obtained from IC-FSAHP in Figure 3.4.3. Such additional information may provide further insight for the DMs to determine the final recommendation.
3.5.1.2 AHP based on stochastic simulations for aggregating DMs’ opinions

The use of AHP in which the aggregation of DMs’ opinions is based on stochastic simulations involves five major stages: (1) defining the problem notions, (2) computing the local weights of the criteria and local priorities of the alternatives, (3) computing the global priorities of the alternatives, (4) synthesising the results by means of a Monte Carlo simulation, and (5) ranking the alternatives. A conceptual framework of the considered stochastic AHP method is shown in Figure 3.5.2.

In general, most of the stages of AHP in Figure 3.5.2 are similar to those in IC-FSAHP, previously explained in Section 3.3. The main difference is that in the generation of random
numbers for the stages of AHP in Figure 3.5.2, crisp judgements are applied. The minimum (a), mode (b) and maximum (c) values of the crisp judgements that are obtained from 3 DMs are aggregated into three individual groups. Equations (3.9)–(3.13) as well as Equation (3.33) are used to generate crisp random numbers.

\[
\text{random}_{\text{crispa}} = \text{min}_{\text{crispa}} + \text{random.beta}(\alpha_{\text{crispa}}, \beta_{\text{crispa}})(\text{max}_{\text{crispa}} - \text{min}_{\text{crispa}}),
\]

where \(\text{min}_{\text{crispa}}\) and \(\text{max}_{\text{crispa}}\) reflect the lowest and highest opinions from DMs on each pairwise comparison; \(\text{random.beta}\) denotes a standard Python function for generating a random number parameterised by \(\alpha_{\text{crispa}}\) and \(\beta_{\text{crispa}}\); \(\text{random}_{\text{crispa}}\) is a generated crisp random number.

After the computation and simulations of all stages have been completed, the weights of the criteria and global priorities of the alternatives are obtained and shown in Figure 3.5.3 and Figure 3.5.4, respectively. Histogram and violin plots were used to visualise the distribution and probability density of the results obtained from stochastic AHP. It is worth highlighting that the ranking of criteria and alternatives that were obtained from stochastic AHP was similar to those obtained from IC-FSAHP.

Figure 3.5.3.a) and Figure 3.5.3.b) present the probability density and the violin plot of the weight of each criterion with respect to the goal. As can be observed from Figure 3.5.3.a) and Figure 3.5.3.b), for most of the points, the ranking of criteria can be determined as \(C_1 > C_3 > C_2 > C_4 > C_5 > C_6\). The ranking of criteria was similar to that observed from Figure 3.4.2.a) and Figure 3.4.2.b).

Figure 3.5.4.a) and Figure 3.5.4.b) present the probability density and the violin plot of the overall score of each alternative with respect to the criteria. The ranking of the preference...
of alternatives was similar to that observed from Figure 3.4.3.a) and Figure 3.4.3.b). As can be observed from Figure 3.5.4.a) and Figure 3.5.4.b), for most points, the ranking of alternatives can be determined as $A_4 \succ A_1 \succ A_3 \succ A_5 \succ A_2$. However, in terms of the occurrence of overlaps, the results obtained from stochastic AHP were much higher than those obtained from IC-FSAHP. We can observe from Figure 3.5.4.a) and Figure 3.5.4.b) that, for some points, $A_4$ could be the second best alternative. On the other hand, $A_4$ is found to be the best alternative without any overlap in Figure 3.4.3.a) and Figure 3.4.3.b).

As previously mentioned in Section 3.4.3, the overlaps indicate the doubt of the DMs, as a group, when providing their preference. An interesting finding that is thus worth noting is that even though each DM has clear preferences due to the use crisp judgements, the results as a group were relatively vague compared to those obtained from IC-FSAHP. This is probably due to the fact that IC-FSAHP can eliminate solutions that are not feasible and therefore minimise the uncertainty using a larger amount of information or data.

### 3.5.2 FSAHP based on Buckley’s technique

In this work we compared the results that were obtained from IC-FSAHP and those obtained from an FSAHP method based on Buckley’s technique. The main differences between the proposed IC-FSAHP and FSAHP are in the computation of the fuzzy weight of each criterion or alternative in a TFN $(w^C_{iL}, w^C_{iM}, w^C_{iU})$ that is formulated in Equations (3.25) – (3.27) and the computation of the fuzzy global priority of each alternative in a TFN $(S^L_m, S^M_m, S^U_m)$ that is formulated in Equations (3.28) – (3.30). In the case of the FSAHP method, the fuzzy weight of each criterion or alternative in a TFN, $\tilde{w}_i^B = (w^B_{iL}, w^B_{iM}, w^B_{iU})$, i
=1,...,p (the superscript $B$ refers to the Buckley’s technique), is computed from an FPCM $A$, as follows:

$$w_{pL}^B = \frac{\sqrt[\alpha]{\prod_{j=1}^{p} a_{ijL}}}{\sum_{k=1}^{p} \sqrt[\alpha]{\prod_{j=1}^{p} a_{kjU}}},$$

(3.34)

$$w_{pM}^B = \frac{\sqrt[\alpha]{\prod_{j=1}^{p} a_{ijM}}}{\sum_{k=1}^{p} \sqrt[\alpha]{\prod_{j=1}^{p} a_{kjM}}},$$

(3.35)

$$w_{pU}^B = \frac{\sqrt[\alpha]{\prod_{j=1}^{p} a_{ijU}}}{\sum_{k=1}^{p} \sqrt[\alpha]{\prod_{j=1}^{p} a_{kjL}}},$$

(3.36)

where $w_{pL}^B$, $w_{pM}^B$ and $w_{pU}^B$ are the lowest, middle and highest values of the fuzzy weight of the $i$–th object (criteria or alternatives) with respect to a higher level of a hierarchy.

In addition, the fuzzy global priority of the $k$–th alternative in a TFN $(S_{m_k}^L, S_{m_k}^M, S_{m_k}^U)$, $k \in 1, 2, \ldots, m$ is formulated in the following equations:

$$S_{m_k}^L = \sum_{i=1}^{n} w_{kL}^B \times w_{iL}^B,$$

(3.37)

$$S_{m_k}^M = \sum_{i=1}^{n} w_{kM}^B \times w_{iM}^B,$$

(3.38)

$$S_{m_k}^U = \sum_{i=1}^{n} w_{kU}^B \times w_{iU}^B,$$

(3.39)

where $S_{m_k}^L$, $S_{m_k}^M$ and $S_{m_k}^U$ are the lowest, middle and highest values of the fuzzy global priority of the $k$–th alternative with respect to criteria $i = 1, 2, \ldots, n$.

Figure 3.5.5.a) and Figure 3.5.5.b) present the probability density and the violin plot of the weight of each criterion with respect to the goal obtained from FSAHP. As can be observed from Figure 3.5.5.a) and Figure 3.5.5.b), for most of the points, the ranking of criteria can be determined as $C_1 > C_2 > C_3 > C_4 > C_5 > C_6$. The ranking of the importance of criteria in Figure 3.5.5.a) and Figure 3.5.5.b) was slightly different from the ranking observed from Figure 3.4.2.a) and Figure 3.4.2.b), which is $C_1 > C_3 > C_2 > C_4 > C_5 > C_6$.

Figure 3.5.6.a) and Figure 3.5.6.b) display the probability density of the weight of the overall score of each alternative with respect to each criterion obtained from FSAHP. The order of the preference of alternatives was slightly different from the order that was observed from Figure 3.4.3.a) and Figure 3.4.3.b). As can be observed from Figure 3.5.6.a) and Figure 3.5.6.b), for most of the points, the ranking of alternatives can be determined as $A_4 > A_3 > A_1 > A_5 > A_2$ whereas in Figure 3.4.3.a) and Figure 3.4.3.b) the ranking is $A_4 > A_1 > A_3 > A_5 > A_2$. 
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Fig. 3.5.5 a) Probability distributions of the weights of the criteria; and b) Violin plots of the weights of the criteria that were obtained from FSAHP after 1000 iterations.

Fig. 3.5.6 a) Probability distributions of the overall scores of the alternatives; and b) Violin plots of the overall scores of the alternatives that were obtained from FSAHP after 1000 iterations.

Based on the aforementioned findings, further analysis needs to be carried out to determine which method results in more accurate solutions. One of the methods that can be used to assess the difference of uncertainty of the fuzzy results obtained from FSAHP and IC-FSAHP is to determine the $U$-uncertainty. $U$-uncertainty is a natural generalisation of the Hartley function from classical set theory to FST that was proposed by Higashi and Klir [53]. For the case of TFN, $U$-uncertainty can be calculated by using the formula developed by Enea
3.6 The effects of disagreement among DMs

and Piazza [22].

\[
U(A) = -1 + \frac{1 + S_{m_k}^U - S_{m_k}^L}{(S_{m_k}^U - S_{m_k}^L)} \ln \left(1 + S_{m_k}^U - S_{m_k}^L\right),
\]

(3.40)

where \(S_{m_k}^L\) and \(S_{m_k}^U\) = \( k = 1, 2, \ldots, m, \) are the lowest and highest values in a TFN of the fuzzy global priority of the \( k \)th alternative with respect to criteria \( i = 1, 2, \ldots, n. \)

\textit{U-uncertainty} of the IC-FSAHP and FSAHP methods were calculated by using Equation (3.40). Table 3.5.4 shows \textit{U-uncertainty} of IC-FSAHP and FSAHP on the first iteration. It clearly shows that \textit{U-uncertainty} of IC-FSAHP was lower than \textit{U-uncertainty} of FSAHP and hence than the uncertainty is also lower for IC-FSAHP.

Table 3.5.4 \textit{U-uncertainty} values of IC-FSAHP and FSAHP on the first iteration.

<table>
<thead>
<tr>
<th>(A_m)</th>
<th>IC-FSAHP</th>
<th>FSAHP</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A_1)</td>
<td>0.021</td>
<td>0.212</td>
</tr>
<tr>
<td>(A_2)</td>
<td>0.017</td>
<td>0.199</td>
</tr>
<tr>
<td>(A_3)</td>
<td>0.016</td>
<td>0.230</td>
</tr>
<tr>
<td>(A_4)</td>
<td>0.016</td>
<td>0.249</td>
</tr>
<tr>
<td>(A_5)</td>
<td>0.024</td>
<td>0.209</td>
</tr>
</tbody>
</table>

Figure 3.5.7 shows the violin plot of \textit{U-uncertainty} of IC-FSAHP and FSAHP after 1000 iterations. It is clear that the distribution of the values of \textit{U-uncertainty} for the alternatives were scattered when FSAHP was applied whereas they were concentrated when IC-FSAHP was used. For example, the value of \textit{U-uncertainty} for \(A_1\) obtained from FSAHP ranges from 0.146 to 0.251 whereas that obtained from IC-FSAHP has extremely narrow range values, from 0.017 to 0.022. These results demonstrate that IC-FSAHP can produce more precise results than FSAHP, probably due to the fact that the former takes into account the constraints. In this way, we eliminate solutions that are not feasible and minimise the uncertainty using a larger amount of information or data.

3.6 The effects of disagreement among DMs

In this section, we display the applicability of IC-FSAHP in dealing with the variation in the level of disagreement among DMs (\(D_{DMS}\)) when evaluating the importance of six criteria with respect to the goal. We applied four levels of disagreement that represent the gap between the lowest and highest values among DMs’ opinions, namely \(L_1, L_2, L_3, L_4.\) Table 3.6.1 shows the variation of the importance of the criteria with respect to the goal, when three DMs are involved.

The histogram and violin plots of the results show that the distribution and probability density of the weight of criteria and alternatives’ scores were affected by \(D_{DMS}.\) Figure 3.6.1
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Fig. 3.5.7 Violin plots of the U-uncertainty values of IC-FSAHP and FSAHP after 1000 iterations.

Table 3.6.1 Variation of the importance of six criteria with respect to the goal, when three DMs are involved.

<table>
<thead>
<tr>
<th>$D_{DMs}$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
<th>$C_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1$</td>
<td>(I, EI, EI)</td>
<td>(U, U, I)</td>
<td>(U, I, U)</td>
<td>(VI, VI, F)</td>
<td>(VU, F, VU)</td>
<td>(I, EI, EI)</td>
</tr>
<tr>
<td>$L_2$</td>
<td>(F, EI, EI)</td>
<td>(U, U, VI)</td>
<td>(U, VI, U)</td>
<td>(VI, VI, U)</td>
<td>(VU, I, VU)</td>
<td>(F, EI, EI)</td>
</tr>
<tr>
<td>$L_3$</td>
<td>(U, EI, EI)</td>
<td>(U, U, EI)</td>
<td>(U, EI, U)</td>
<td>(VI, VI, VU)</td>
<td>(VU, VI, VU)</td>
<td>(U, EI, EI)</td>
</tr>
<tr>
<td>$L_4$</td>
<td>(VU, EI, EI)</td>
<td>(VU, VU, EI)</td>
<td>(VU, EI, VU)</td>
<td>(VI, VI, EU)</td>
<td>(EU, VI, EU)</td>
<td>(VU, EI, EI)</td>
</tr>
</tbody>
</table>

presents the probability density of the weight of each criterion with respect to the goal for all cases. The histogram bar plots in Figure 3.6.1 show that when $D_{DMs}$ was much higher, the distribution of the criteria’s weight becomes wider. The wide distribution means the uncertainty level of the results is high. Figure 3.6.2 depicts the violin plots of the weight of each criterion with respect to the goal for all cases, which reveal that as $D_{DMs}$ increases, the density of higher probability decreases and the weight of criteria becomes more scattered.

Figure 3.6.3 presents the probability density of the total scores of each alternative with respect to the criteria for all cases. Like in Figure 3.6.1, the histogram bar plots in Figure 3.6.3 show that when $D_{DMs}$ was much higher, a distribution of the overall scores of the alternatives becomes wider and thus the uncertainty level of the results increases. Figure 3.6.4 displays the violin plots of the total scores of each alternative with respect to the criteria for all cases. Like in Figure 3.6.2, the violin plots in Figure 3.6.4 reveal that the density of higher probability decreases and the overall scores of alternatives become more scattered as $D_{DMs}$ increases.

Based on the aforementioned explanations, it can be concluded that the distribution becomes wider and more scattered, and thus uncertainty increases as $D_{DMs}$ increases.
3.6 The effects of disagreement among DMs

Fig. 3.6.1 Probability distributions of the weights of the criteria with respect to the goal after 1000 iterations for a) $L_1$, b) $L_2$, c) $L_3$, d) $L_4$.

Fig. 3.6.2 Violin plots of the weights of the criteria with respect to the goal after 1000 iterations for a) $L_1$, b) $L_2$, c) $L_3$, d) $L_4$. 
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Fig. 3.6.3 Probability distributions of the overall scores of the alternatives with respect to the criteria after 1000 iterations for a) $L_1$, b) $L_2$, c) $L_3$, d) $L_4$.

Fig. 3.6.4 Violin plots of the overall scores of the alternatives with respect to the criteria after 1000 iterations for a) $L_1$, b) $L_2$, c) $L_3$, d) $L_4$. 
3.7 The effects of the number of DMs

Furthermore, it is worth mentioning that for the case when $D_{DM}$ is extremely high (i.e. there is no most likely opinion), IC-FSAHP would not be a suitable method since it requires a most likely opinion to perform the computations. In such circumstances, further discussions among DMs would be required to obtain a most likely opinion so that IC-FSAHP can be used.

3.7 The effects of the number of DMs

The relationship between the number of DMs and uncertainty as well as rank discrimination are analysed in this section. To this end, six cases were considered, namely 4, 5, 6, 7, 10 and 20 DMs. For the purpose of this study, it has been assumed that the minimum, maximum and mode values in the assessments of the objects by three DMs is similar for the cases when 4, 5, 6, 7, 10 and 20 DMs are involved.

The histogram and violin plots of the results show that the distribution and probability density of the alternatives’ scores were affected by the number of DMs. Figure 3.7.1 presents the probability density of the weight of each criterion with respect to the goal for all cases. The histogram bar plots in Figure 3.7.1 show that the increase in the number of DMs makes distributions of criteria weights sharper and more concentrated around their mode values. Figure 3.7.2 depicts the violin plots of the weight of each criterion with respect to the goal for all cases. These violin plots reveal that feed size ($C_2$) and product size ($C_3$) have no probability to be the most important criteria, contrary to what was observed for the case study in Section 3.4 where 3 DMs were considered.

Figure 3.7.3 shows the probability density of the total scores of each alternative with respect to the criteria for all cases. As in Figure 3.7.1, the plots in Figure 3.7.3 clearly demonstrate that the distribution of the total scores of each alternative becomes more concentrated as the number of DMs increases. In addition, Figure 3.7.4, which shows the violin plots of the total score of each alternative with respect to the criteria for all cases, clearly reveal that the gyratory ($A_1$), single toggle jaw ($A_3$), low-speed sizer ($A_5$) and double toggle jaw ($A_2$) crushers were not the most preferable alternatives. It is also observed that contrary to the case with 3 DMs, in the case when 20 DMs are involved there is no longer an overlap between the aforementioned alternatives. The absence of overlap may indicate that the DMs as a group were confident about favouring one alternative over the other alternatives.

Figure 3.7.5 provides the number of overlaps on the final ranking of $A_1$, $A_3$, and $A_5$ based on the number of DMs. This figure shows that the absence of overlaps between these alternatives started when 7 DMs are involved. However, the rank discrimination is still insufficient because the range of the overall scores of $A_1$, $A_3$, and $A_5$ is still very close (as observed in Figure 3.7.3.d).

These results showed that by increasing the number of DMs in a group, when the minimum, maximum and mode values of the assessments do not vary, the distribution becomes sharper.
and concentrated around the mode, and thus less uncertain. This condition can be achieved by means of the participation of a larger number of DMs. The involvement of a higher number of experts and DMs will reflect their preferences more strongly, not only reducing uncertainty but also enhancing rank discrimination between the alternatives. This is probably due to the fact that when the minimum, maximum and most likely values of the opinions do not vary, a larger number of DMs would place more emphasis on the most likely values.
3.7 The effects of the number of DMs

Fig. 3.7.1 Probability distributions of the weights of the criteria with respect to the goal after 1000 iterations for a) 4, b) 5, c) 6, d) 7, e) 10, f) 20 DMs.

Fig. 3.7.2 Violin plots of the weights of the criteria with respect to the goal after 1000 iterations for a) 4, b) 5, c) 6, d) 7, e) 10, f) 20 DMs.
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Fig. 3.7.3 Probability distributions of the overall scores of the alternatives with respect to the criteria after 1000 iterations for a) 4, b) 5, c) 6, d) 7, e) 10, f) 20 DMs.

Fig. 3.7.4 Violin plots of the overall scores of the alternatives with respect to the criteria after 1000 iterations for a) 4, b) 5, c) 6, d) 7, e) 10, f) 20 DMs.
3.8 The rank reversal analyses

Even though the occurrence of rank reversal in AHP has led to much criticism, there is a paucity of studies investigating its occurrence in the application of AHP under fuzzy environment [9, 54]. This section assesses whether the rank reversal phenomenon occurs when using IC-FSAHP for cases when one alternative is added, one alternative is deleted and also when the preference scale is changed, considering a square root and a power preference scale. The effects of the rank reversal phenomenon on the selection problem are also discussed. Furthermore, for the sake of clarity in the results in terms of the absence of overlap, and to take advantage on rank discrimination, the case of 10 DMs was considered in this experiment.

In an attempt to check rank reversal due to the addition of one alternative, another type of crusher analysed by Rahimdel and Ataei [38] is used in this Section. The new alternative, termed as A6, was better than all previous alternatives in terms of preferences. The three individual preference groups of $A_6$, namely minimum, maximum and mode values with respect to each criterion are presented in Table 3.8.1. In addition, another scenario to check rank reversal due to the addition of an alternative, which is the addition of the same alternative, was conducted. To this end, alternative $A_4$ was duplicated and referred to as $A_4^*$. 

Table 3.8.1 Three individual preferences groups of $A_6$, namely minimum, maximum and mode values with respect to each criterion.

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
<th>$C_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>H</td>
<td>MH</td>
<td>MH</td>
<td>ML</td>
<td>MH</td>
<td>MH</td>
</tr>
<tr>
<td>Max</td>
<td>VH</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>Mode</td>
<td>VH</td>
<td>MH</td>
<td>MH</td>
<td>M</td>
<td>H</td>
<td>H</td>
</tr>
</tbody>
</table>
The addition of alternative $A_6$ had no effect on the weights of the criteria, which are the same as those in Figure 3.7.1.e) and Figure 3.7.2.e). This is to be expected, since the addition of an alternative should not have any bearing on the criteria. The best alternative in the original case (see: Figure 3.7.3.e) and Figure 3.7.4.e)), however, was expected to become the second best when $A_6$ was introduced. As can be seen from Table 3.8.2, the alternative $A_6$ was better than $A_4$ for 984 of the overlaps and $A_4$ was better than $A_6$ for 16 of the overlaps. A general overlap that was presented as well in Figure 3.8.1.a) and Figure 3.8.1.b) captures biased opinions between DMs since the difference of preference between alternatives $A_4$ and $A_6$ was small. It is worth highlighting that the rank order of alternatives $A_4$ and $A_1$ is $A_4 \succ A_1$ was retained after $A_6$ had been introduced. For this particular case, the ranking in the case of addition of an alternative was therefore not reversed.

Furthermore, similar to the addition of $A_6$, the addition of alternative $A_4^*$ had no effect on the weights of criteria, which are the same as those in Figure 3.7.1.e) and Figure 3.7.2.e). The best alternative shown in Figure 3.7.3.e) and Figure 3.7.4.e) before the addition of $A_4^*$ was expected to be similar to that after its addition. As can be seen in Figure 3.8.2.a) and Figure 3.8.2.b), the results for $A_4$ and $A_4^*$ were almost the same, with their probability distributions almost completely overlapping. This overlap captures the fact that the new alternative $A_4^*$ was in fact the same as $A_4$. Moreover, the ranking of other alternatives remained the same despite the introduction of $A_4^*$. We may conclude, therefore, that there was no rank reversal for the case of addition of the same alternative.

Table 3.8.2 Summary of the final ranking of each alternative after the addition of one alternative ($A_6$) after 1000 iterations.

<table>
<thead>
<tr>
<th>$A_m$</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>$A_1$</td>
<td>0</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0</td>
</tr>
<tr>
<td>$A_4$</td>
<td>16</td>
</tr>
<tr>
<td>$A_5$</td>
<td>0</td>
</tr>
<tr>
<td>$A_6$</td>
<td>984</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>1000</td>
</tr>
</tbody>
</table>

In an attempt to check rank reversal due to the deletion of one alternative, the third-ranked alternative that was shown in Figure 3.7.3.e) and Figure 3.7.4.e) ($A_3$) was deleted. For the case in which alternative $A_3$ was deleted, it can be observed from Figure 3.8.3.a), Figure 3.8.3.b) and Table 3.8.3 that $A_4$ remained the best alternative. The deletion of $A_3$ also resulted in $A_5$ being the third ranked option without any overlap. The ranking of alternatives $A_1$, $A_2$, $A_4$ and $A_5$ was thus preserved.

It is worth noting that this work is the first attempt in the literature to check the occurrence of rank reversal in FAHP or FSAHP methods. We have shown that, for the case
3.8 The rank reversal analyses

Fig. 3.8.1 a) Probability distributions of the overall scores of the alternatives calculated after the addition of one alternative ($A_6$); and b) Violin plots of the overall scores of the alternatives calculated after the addition of one alternative ($A_6$) after 1000 iterations.

Another attempt to check rank reversal is to change the preference scale [55] or the linguistic term that is converted into a TFN. In this work, root square and power scales were used, the values of which are shown in Table 3.8.4.

Figure 3.8.2 a) Probability distributions of the overall scores of the alternatives calculated after the addition of one alternative ($A_4^*$); and b) Violin plots of the overall scores of the alternatives calculated after the addition of one alternative ($A_4^*$) after 1000 iterations.

being considered, no rank reversal phenomenon occurred with the addition and deletion of an alternative.

Another attempt to check rank reversal is to change the preference scale [55] or the linguistic term that is converted into a TFN. In this work, root square and power scales were used, the values of which are shown in Table 3.8.4.

Figure 3.8.4.a) and Figure 3.8.4.b) show the weights of the criteria obtained after applying the root square scale while Figure 3.8.6.a) and Figure 3.8.6.b) show the weights of criteria obtained after applying the power scale. Even though the weights of the criteria that were
Table 3.8.3 Summary of the final ranking of each alternative after the deletion of one alternative ($A_3$) after 1000 iterations.

<table>
<thead>
<tr>
<th>$A_m$</th>
<th>Rank 1</th>
<th>Rank 2</th>
<th>Rank 3</th>
<th>Rank 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>0</td>
<td>1000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1000</td>
</tr>
<tr>
<td>$A_4$</td>
<td>1000</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$A_5$</td>
<td>0</td>
<td>0</td>
<td>1000</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
</tbody>
</table>

Fig. 3.8.3 a) Probability distributions of the overall scores of the alternatives calculated after the deletion of one alternative ($A_3$) ; and b) Violin plots of the overall scores of the alternatives calculated after the deletion of one alternative ($A_3$) after 1000 iterations.

Table 3.8.4 The linguistic preference scales used in this work and their TFNs based on root and power scales.

<table>
<thead>
<tr>
<th>Linguistic preference scale</th>
<th>TFN (root square)</th>
<th>TFN (power)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EU or VL</td>
<td>$\sqrt{1/2}, 1, \sqrt{2}$</td>
<td>$1/4, 1, 4$</td>
</tr>
<tr>
<td>VU or L</td>
<td>$\sqrt{2}, \sqrt{3}, 2$</td>
<td>4, 9, 16</td>
</tr>
<tr>
<td>U or ML</td>
<td>$\sqrt{3}, 2, \sqrt{5}$</td>
<td>9, 16, 25</td>
</tr>
<tr>
<td>F or M</td>
<td>$2, \sqrt{5}, \sqrt{6}$</td>
<td>16, 25, 36</td>
</tr>
<tr>
<td>I or MH</td>
<td>$\sqrt{5}, \sqrt{6}, \sqrt{7}$</td>
<td>25, 36, 49</td>
</tr>
<tr>
<td>VI or H</td>
<td>$\sqrt{6}, \sqrt{7}, \sqrt{8}$</td>
<td>36, 49, 64</td>
</tr>
<tr>
<td>EI or VH</td>
<td>$\sqrt{8}, 3, 3$</td>
<td>64, 81, 81</td>
</tr>
</tbody>
</table>

obtained from both these scales were different from the weights that were obtained from the initial scale, which are shown in Figure 3.7.1.e) and Figure 3.7.2.e), the order of importance of criteria was similar. This similarity shows that the order of importance of criteria was not
affected by the change of linguistic judgement scales. However, the change of linguistic scales has affected the ranking of alternatives.

Figure 3.8.5.a) and Figure 3.8.5.b) show the probability distributions of the overall scores of the alternatives calculated after applying the root square judgement scale in histogram and violin plots, respectively. As can be seen from Figure 3.8.5.a) and Figure 3.8.5.b), \( A_4 \) was the best alternative, this result fits with the results that are shown in Figure 3.7.3.e) and Figure 3.7.4.e). However, the rank of other alternatives was partially different. For example, the alternative \( A_1 \) was better than alternative \( A_3 \) in Figure 3.7.3.e) and Figure 3.7.4.e) on the other hand in Figure 3.8.5.a) and Figure 3.8.5.b) alternative \( A_3 \) was better than \( A_1 \). In the case of alternatives \( A_5 \) and \( A_2 \), \( A_5 \succ A_2 \) without overlaps in Figure 3.7.3.e) and Figure 3.7.4.e) but becomes \( A_5 \succ A_2 \) hereafter with a large number of overlaps while applying the root square judgement scale. Table 3.8.5 shows the amount of these overlaps. So, even though the best alternative was preserved, the order of other alternatives was reversed. This phenomenon is referred to as partial rank reversal. In this case, since the main aim of the choice problem is to select the best alternative, DMs can select confidently \( A_4 \) as the solution.

Figure 3.8.7.a) and Figure 3.8.7.b) show the probability distributions of the overall scores of the alternatives calculated after applying the power judgement scale in histogram and violin plots, respectively. As can be observed in Figure 3.8.7.a) and Figure 3.8.7.b), \( A_1 \) was the best alternative with overlaps with \( A_4 \). Table 3.8.6 shows the amount of these overlaps. The ranking of the five alternatives after applying the power scale was totally different. For example, in Figure 3.7.3.e) and Figure 3.7.4.e) the ranking was \( A_4 \succ A_1 \succ A_3 \succ A_5 \succ A_2 \) without overlaps on the other hand in Figure 3.8.7.a) and Figure 3.8.7.b) the ranking was...
An Integrated Constrained Fuzzy Stochastic Analytic Hierarchy Process Method with Application to the Choice Problem

Fig. 3.8.5 a) Probability distributions of the overall scores of the alternatives calculated after applying the root square judgement scale; and b) Violin plots of the overall scores of the alternatives calculated after applying the root square judgement scale after 1000 iterations.

Table 3.8.5 Summary of the final ranking of each alternative after applying the root square judgement scale after 1000 iterations.

<table>
<thead>
<tr>
<th>$A_m$</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>$A_1$</td>
<td>0</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0</td>
</tr>
<tr>
<td>$A_4$</td>
<td>1000</td>
</tr>
<tr>
<td>$A_5$</td>
<td>0</td>
</tr>
</tbody>
</table>

$A_1 \succ A_4 \succ A_5 \succ A_3 \succ A_2$ with a large number of overlaps between in $A_1$ and $A_4$. This phenomenon is referred to as total rank reversal.

Table 3.8.6 Summary of the final ranking of each alternative after applying the power judgement scale after 1000 iterations.

<table>
<thead>
<tr>
<th>$A_m$</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>$A_1$</td>
<td>834</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0</td>
</tr>
<tr>
<td>$A_4$</td>
<td>165</td>
</tr>
<tr>
<td>$A_5$</td>
<td>1</td>
</tr>
</tbody>
</table>

Total 1000 1000 1000 1000 1000
3.8 The rank reversal analyses

Fig. 3.8.6 a) Probability distributions of the weights of the criteria calculated after applying the power judgement scale; and b) Violin plots of the weights of the criteria calculated after applying the power judgement scale after 1000 iterations.

Fig. 3.8.7 a) Probability distributions of the overall scores of the alternatives calculated after applying the power judgement scale; and b) Violin plots of the overall scores of the alternatives calculated after applying the power judgement scale after 1000 iterations.

It can be concluded from these experiments that rank reversal occurred partially after applying the root square scale while total rank reversal occurred after applying the power scale. Therefore, determining the TFNs of judgement scales is of paramount importance. The step of the decision making process that deals with the choice of an appropriate judgement scale should be discussed clearly and carefully at the beginning of a decision making process.
3.9 Conclusions

This paper has proposed an integrated constrained fuzzy stochastic AHP (IC-FSAHP) method for selecting the best alternative under multiple conflicting criteria and uncertainty when multiple DMs are involved. The IC-FSAHP method involves five main stages. The aim of the first stage is to define the decision problem notions as clearly as possible. The goal of the second stage is to measure the weight of criteria and local priorities of alternatives. In this stage, the uncertainty of the DMs’ judgement, due to the lack of information in comparing pairwisely each object with others, is quantified by using linguistic judgements that are converted into TFNs. In addition, the TFNs obtained by multiple DMs have been regarded as input values into the modified beta-PERT distribution in order to generate random TFNs that represent the aggregation of DMs’ opinions. The objective of the third stage is to compute the global priorities of all alternatives. Furthermore, in the fourth stage, Monte Carlo simulations for the FPCMs are carried out; for these simulations, the TFNs obtained from the modified beta-PERT distribution have been regarded as input values. The simulations have been shown to capture the uncertainty of evaluations due to various judgements and biased insights from multiple DMs. Finally, the overall scores of alternatives are ranked on the basis of performance rating distributions on the scores. The alternative with the highest probability on the highest score is then chosen as the best alternative.

The results obtained from a case study showed IC-FSAHP’s ability to reduce uncertainty caused by lack of knowledge. In addition, IC-FSAHP was able to capture scattered opinions from multiple DMs caused by various judgements and biased insights. IC-FSAHP produced more accurate and precise results and it was shown to be more suitable for real-life group decision making problems under uncertainty than AHP and FSAHP methods. In the case of rank reversal analyses, the results showed that the ranking of alternatives was preserved throughout the changes in the number of alternatives. Since the rank order was stable and robust, no undesired rank reversal occurred due to the addition or deletion of an alternative. Moreover, it was shown that a higher confidence in the results can be obtained by increasing the number of DMs due to the fact that when the minimum, maximum and mode values of the opinions do not change, a higher number of DMs would place more emphasis on the mode values. This work reveals that IC-FSAHP could increase the certainty and reliability of the decisions in the group decision making process by means of robust computations that take into account the reciprocity of pairwise comparisons in a fuzzy environment and the interactions among the elements.

It is worth noting that in the case of changes in judgements scales, rank reversal occurred either using the root square or power scales. This shows that choosing the right preference scale is of vital importance and needs to be determined carefully during the first stage of the IC-FSAHP method. Furthermore, it is important to bear in mind that when the level of
disagreement among DMs in judging the preferences reaches no most likely opinion, further discussions among DMs are required in order to agree the most likely judgement.

Finally, as it has been discussed in Section 3.4, there is scope to improve the robustness of IC-FSAHP and extend the applicability of the method to solve more complex problems in terms of uncertainty. Some possibilities for future research are summarised as follows:

1. The development of an extended IC-FSAHP method able to deal with multimodal values in DMs’ opinions. An investigation into the efficiency of such an extended method will also be relevant.

2. The development of appropriate algorithms for checking the consistency of FPCMs and for examining the consistency of the process in attaining the fuzzy weights of the objects from the FPCMs. The results that will be obtained from the proposed algorithms could then be compared to those obtained in this paper.

3. The development of an extended IC-FSAHP method for dealing with incomplete fuzzy preference relations in assessing an object over others. The analyses carried out in this paper could also be followed to examine the robustness of such an extended method.
Chapter references


Chapter 4

Equipment selection in mineral processing - a sensitivity analysis approach for a fuzzy multiple criteria decision making model

Abstract

Selecting the most suitable mineral processing equipment among feasible alternatives with respect to multiple conflicting criteria is considered a Multiple Criteria Decision Making (MCDM) problem. For example, a type of crusher that might allow a very high throughput is less likely to be used in a mobile plant, so trade-offs between these type of criteria need to be clearly defined in the decision making process. One of the most frequently used MCDM methods is the Analytical Hierarchy Process (AHP) method, which relies on judgements from decision makers that allow for comparisons to be made between alternatives (e.g. the type of equipment) or criteria (e.g. the characteristics of the equipment that are of interest). However, AHP is not able to capture the uncertainty associated with the various decision makers’ judgements and the lack of precise information. An integrated constrained fuzzy stochastic analytic hierarchy process (IC-FSAHP) is a new hybrid MCDM method that can be used to overcome the aforementioned limitations of AHP. In order to understand the robustness of AHP based methods, a sensitivity analysis of the decision making results is required. However, sensitivity analyses are not often carried out for fuzzy AHP methods, arguably because of the complexity of some of the procedures involved, the computation time required and the limited resources available to do so. The main objective of this paper is therefore to propose a new sensitivity analysis approach by applying an additional fuzzification factor and disagreement level of decision makers in order to model uncertainty. For this purpose, a case study for the selection of primary crushers was considered. Five types of primary crushers
were evaluated with respect to six criteria to showcase the applicability of the proposed approach to assess IC-FSAHP. The results obtained showcase that the proposed sensitivity analysis approach is capable of providing extensive and useful “what-if” information on the decision making results.

4.1 Introduction

Mineral processing equipment selection is the task of examining various machinery alternatives being considered and choosing the most suitable equipment that involves multiple criteria such as technical, environmental, and socio-economic aspects [1]. The selection of suitable equipment in mineral processing is always of paramount importance. Using appropriate equipment has a significant impact in terms of cost and benefits and therefore a proper equipment selection can avoid unnecessary costs associated to equipment troubleshooting as well as losses in production [1].

However, it is often challenging to determine the most suitable equipment to use. Decision makers need to evaluate multiple alternatives under many criteria that are often in conflict with each other. For example, a type of crusher that might achieve very large throughput is less likely to be part of mobile plant, so trade-offs need to be clearly defined during the evaluation stage in the decision making process.

Selecting the best equipment among the alternatives with respect to multiple conflicting criteria is considered a Multiple Criteria Decision Making (MCDM) problem. MCDM is an advanced subject of operations research that can be used to solve problems (e.g. a selection problem) when the feasible alternatives ($A_1, A_2, \ldots, A_m$) are evaluated under many criteria ($C_1, C_2, \ldots, C_n$) in a scientific transparent manner [2]. MCDM methods have been effectively applied in various fields such as mining and mineral processing [1], manufacturing [3], environmental science [4], and natural resources management [5].

In recent years a number of MCDM methods have been effectively used for selecting the best mineral processing equipment, covering machinery used from the ore transportation stage to the concentration stage, such as the selection of ore transportation system, the selection of flotation machines, and the selection of primary crusher equipment. Despodov et al. [6] applied the Analytic Hierarchy Process (AHP), which was developed by Saaty [7], to determine ore transportation equipment for a processing facility. Owusu-Mensah and Musingwini [8] used AHP to select the best equipment to transport ore from an underground mine to the mill. Başçetin and Kesimal [9] used the Yager method to determine an optimal mined coal transportation system to a power station under lack of knowledge and insufficient information. Štirbanović et al. [10] assessed applications of the TOPSIS and VIKOR methods in choosing a rougher flotation machine for the processing of copper sulphide ore. Furthermore, two articles have presented the use of different MCDM methods for the same mine site under the same alternatives and criteria but different uncertainty conditions: Rahimdel and Ataei
applied AHP to determine the best primary crusher, whereas Rahimdel and Karamoozian used Fuzzy TOPSIS to take into account uncertainty. It is worth noting that both studies recommended the same type of primary crusher being chosen.

In addition, a great deal of attention has recently been given to applications of MCDM in mineral processing in which multiple decision makers are involved; this is referred to as multiple criteria group decision making (MCGDM). The most optimal outcome (i.e. judgement, preference, and the rank of alternatives) in MCGDM is the most acceptable for the group and is based on the opinions of a number of decision makers or experts. It is worth highlighting that when aggregating the judgements of criteria and preferences of alternatives, biased opinions caused by disparities among decision makers (i.e. abilities, competencies, and compliances) must be taken into account. Therefore, evaluating the importance of criteria and the preference of alternatives are of vital importance in the decision making process.

One of the most widely used MCDM methods is AHP. AHP involves a construction process of an MCDM problem into a hierarchy that frequently comprises three-level structures from the top to the bottom that define the goal, criteria, and alternatives, respectively. The main notion of AHP is in the pairwise comparison matrix (PCM) that is constructed from the pairwise comparison of each criterion to another criterion with respect to the goal and the comparison of each alternative to another alternative. The elements of the PCM have their reciprocity values, e.g. when the criterion \( C_i \) is \( n \) times more important than the criterion \( C_j \), so the criterion \( C_j \) is \( \frac{1}{n} \) times less important than the criterion \( C_i \). Another notion of AHP is the inconsistency ratio of each PCM; a PCM is acceptably inconsistent if the ratio value is less than 0.1. A detailed description of AHP can be found in Saaty and Vargas.

Despite its popularity, the application of AHP has been often criticised when uncertainty caused by the lack of information and biased insights in various decision makers’ judgements is present. The extension of AHP by combining it with the fuzzy set theory is one of the most popular techniques to overcome the uncertainty problem caused by the lack of information.

An integrated constrained fuzzy stochastic analytic hierarchy process (IC-FSAHP), which has been developed by Sitorus et al., takes advantage of the best characteristics of two existing fuzzy AHP (FAHP) methods and further extends their applicability by means of stochastic simulations. The main notions of IC-FSAHP are preserving the reciprocity interactions among the elements in the fuzzy pairwise comparison matrices (FPCMs) for each hierarchy level under the constraints and complying with tolerable inconsistency ratios of the FPCMs during the computations. Sitorus et al. showed that IC-FSAHP is able to
reduce uncertainty and not only resulted in more accurate and precise results than AHP and its variants, but also increased the reliability of decisions taken under uncertainty by means of MCGDM.

In order to understand the robustness of IC-FSAHP, the results that are obtained from IC-FSAHP need to be scrutinised further by performing a sensitivity analysis. The results that are obtained from the sensitivity analysis could be used to answer “what-if” questions that decision makers might have. In addition, a sensitivity analysis should be carried out because the importance of criteria and the preference of alternatives that are obtained from subjective insights of decision makers could affect the results. Their subjective insights and potential disparities may result in uncertainty. Furthermore, the results obtained from IC-FSAHP may not offer sufficient information and data to decision makers, and thus it might be difficult to make a final decision. A sensitivity analysis can therefore deliver more insights to decision makers to finalise the decision based on more comprehensive information.

Various approaches have been developed for performing sensitivity analysis on AHP and its variants. Leonelli [22] classified the approaches into three main categories: numerical incremental analysis [23–26], mathematical models [27–29], and probabilistic simulations [30, 31]. The numerical incremental analysis approach is the most frequently used approach in the literature [32] and in associated software tools, such as Super Decisions, ExpertChoice, MakeItRational, TransparentChoice, Total Decision, Intelligent Decision Systems, HIPRE 3+, and SIMUL8. However, the aforementioned approaches do not take into account the main notions of IC-FSAHP. In addition, it can be argued that sensitivity analyses are not often carried out for fuzzy AHP methods due to the complexity of some of the methodologies involved, the time required for the computations and the limited resources available to do so (i.e. the lack of software available to perform the analyses). While there is no need for sensitivity analyses when the difference between the first and second ranked alternatives is large, a suitable method is still required when the best alternative is not clear. Moreover, it is also noted that there is no study in the literature that performs a sensitivity analysis for the selection of mineral processing equipment when using MCDM methods.

In response to the aforementioned limitations, this study proposes a new comprehensive sensitivity analysis approach that takes into account the main notions of IC-FSAHP. By using the proposed approach, it was possible to assess the effects of changing the uncertainty levels of the judgements in the FPCMs. In addition, it was possible to examine the effects of disagreement among decision makers under the various uncertainty levels of the judgements. For the purpose of this work, a case study for the selection of mineral processing equipment using IC-FSAHP was considered, the results of which were used to perform the sensitivity analysis proposed.

The contributions of this paper are: (1) a fallacy concept of existing sensitivity analysis methods is addressed and a new approach is proposed; (2) for the first time, a sensitivity analysis for the selection of mineral processing equipment by multiple decision makers
under uncertainty is performed. It is worth mentioning that none of the aforementioned contributions have been presented in the literature.

The remainder of the manuscript is organised as follows: Section 4.2 provides the methodology of the proposed sensitivity analysis procedure employed in this study; Section 4.3 presents the application of the proposed approach to a case study; the detailed results and findings as well as discussion on the case study are provided in this section; Finally, Section 4.4 provides some final remarks and conclusions on the usability of the suggested sensitivity analysis approach and its use for assessing what if scenarios.

4.2 Methodology

Prior to performing the sensitivity analysis, the initial results obtained by from IC-FSAHP are required. There are five major steps in applying the IC-FSAHP method:

1. defining the notions of a decision problem,
2. determining the local fuzzy weights of the criteria and the local fuzzy priorities of the alternatives,
3. determining the global priorities of the alternatives,
4. synthesising the outcomes,
5. ranking the order of the alternatives.

A detailed description of these steps can be found in Sitorus et al. [21].

The judgement and preference of the criteria and alternatives over others are evaluated by using a scale of seven linguistic variables. In the case of evaluating the importance of the criteria, the following linguistic variables are applied: Extremely Unimportant (EU), Very Unimportant (VU), Unimportant (U), Fair (F), Important (I), Very Important (VI) and Extremely Important (EI). Furthermore, when evaluating the preference of the alternatives, the following linguistic variables are applied: Very Low (VL), Low (L), Medium Low (ML), Medium (M), Medium-High (MH), High (H) and Very High (VH).

For the purpose of this study, each element in the scale is valued by means of triangular fuzzy numbers (TFNs). Each TFN contains three ordered numbers (i.e. the lower, middle and upper values). For example, the ordered number of a TFN in describing EU or VL is 1, 2, 3; VU or L is 2, 3, 4; U or ML is 3, 4, 5; and so forth. Figure 4.2.1 shows the membership functions for the TFN scale levels.

4.2.1 Sensitivity Analysis

The main aim of a sensitivity analysis is to understand when the input data (i.e. preference, judgements, degrees of fuzziness and disagreement between decision makers) are changed into
Equipment selection in mineral processing - a sensitivity analysis approach for a fuzzy multiple criteria decision making model

Fig. 4.2.1 Membership functions of the TFN variables that are used in evaluating the criteria and alternatives.

new values, how the outcomes obtained from IC-FSAHP change or to ensure the consistency of the final results. The main notions of the sensitivity analysis approach proposed in this work are the variation in the uncertainty levels of the judgements and the change of disagreement level among decision makers under the various uncertainty levels of the judgements. Two main changes are therefore required to perform the sensitivity analysis: changes on fuzzification factor ($\theta$) and changes on disagreement level among decision makers. The proposed approach may thus convey additional insights to the decision makers during the decision making process. The priority of each alternative is determined based on the global weights under different conditions.

Based on the aforementioned concepts, it is very important to analyse the changes in the fuzzification factor ($\theta$) in the construction of a TFN. A TFN has three numbers (i.e. $c_{LA}^L$, $c_{LA}^M$, and $c_{LA}^U$) that are ordered from the lowest to highest values. As it can be seen from Figure 3.2.1, the difference between $c_{LA}^L$ and $c_{LA}^M$, $c_{LA}^M$ and $c_{LA}^U$ is 1. This difference is referred to as the fuzzification factor ($\theta$) of a TFN. Thus, the initial rating of decision makers’ judgements have $\theta = 1$. The fuzzification factor ($\theta$) represents uncertainty; the value of $\theta$ being directly proportional to the level of uncertainty. The difference among 3 ordered numbers can be expressed by the following equations:

$$c_{LA}^L = c_{LA}^M - \theta,$$

(4.1)

$$c_{LA}^U = c_{LA}^M + \theta.$$  

(4.2)

This study proposes to extend the fuzzification factor in order to increase uncertainty. Equations (4.1) and (4.2) can thus be extended to:

$$c_{LA}^L = c_{LA}^M - (\theta + \theta'),$$

(4.3)

$$c_{LA}^U = c_{LA}^M + (\theta + \theta'),$$  

(4.4)
where $\theta'$ is the additional fuzzification factor. In this paper, two different $\theta'$ values (i.e. 0.4 and 0.8) were applied in order to analyse the final decision making results that take into account an acceptable inconsistency ratio. Even though other $\theta'$ values (i.e. 0.1, 0.2, 0.3, 0.5, 0.6 and 0.7) were considered and analysed in this work, the results did not vary significantly and, for clarity, only the results obtained for $\theta'$ values of 0.4 and 0.8 are presented. It is important to highlight, however, that from a practical point of view the approach developed in this work can consider any values of $\theta'$ from 0 to $\theta$, and it is suggested that a range of values is tested when applying this method.

In the second change required for the sensitivity analysis is the variation in the level of disagreement among decision makers when examining the pairwise comparisons. The level of disagreement is determined from the difference between the most unlikely and most likely judgements that are obtained from decision makers on each pairwise comparison. The variations were conducted by considering the judgement of the criteria and alternatives simultaneously in order to see how the extreme changes may affect the initial solutions that are obtained from IC-FSAHP. The initial judgements in the pairwise comparisons will be used as a basis of disagreement level among decision makers. There are two disagreement levels used in this study: 2 and 4. Each of the disagreement levels is carried out with the variations of the fuzzification factors. After the first change and the second change are conducted, random numbers are generated based on a probability distribution for each pairwise comparison matrix.

It is worth highlighting that the proposed sensitivity analysis approach in this work could be applied as well to other types of hybrid fuzzy AHP methods. This could be done by generalising the algorithms of additional fuzzification factor in the lower and higher values of a TFN.

4.3 Case study

4.3.1 Background

For the purpose of this study, a decision problem for primary crusher selection in an iron mine is considered as a case study in order to show the initial solutions that are obtained from IC-FSAHP. This case study used data adapted from Rahimdel and Ataei [11]. For the purpose of this study, an implementation of IC-FSAHP in Python 3 was used.

Suppose that decision makers in an iron mine site wanted to select the most suitable primary crusher for carrying out comminution in its production process. For this purpose, six criteria were evaluated for the goal: capacity ($C_1$), feed size ($C_2$), product size ($C_3$), abrasion index ($C_4$), rock compressive strength ($C_5$), and the application of the primary crusher to mobile plants ($C_6$) [11]. Furthermore, five feasible crusher alternatives in Rahimdel and Ataei [11] were assessed with respect to the criteria, namely Gyratory ($A_1$), Double toggle
jaw ($A_2$), Single toggle jaw ($A_3$), High-speed roll ($A_4$), and Low-speed sizer ($A_5$) crushers. Three decision makers, denoted by $D_1$, $D_2$, and $D_3$, were involved in assessing the criteria and alternatives.

After the goal, criteria and alternatives are defined, the pairwise comparisons of the objects (i.e. criteria or alternatives) can be conducted. For the purpose of the implementation of IC-FSAHP, the scale of seven linguistic variables that is shown in Figure 4.2.1 was used to compare pairwisely the importance or preference of an object over others. The assessments of the pairwise comparison of the objects are presented in Tables 4.3.1 and 4.3.2.

Table 4.3.1 The importance of the criteria with respect to the goal by three decision makers [11]

<table>
<thead>
<tr>
<th>Decision makers</th>
<th>Capacity ($C_1$)</th>
<th>Feed size ($C_2$)</th>
<th>Product size ($C_3$)</th>
<th>Abrasion index ($C_4$)</th>
<th>Rock compressive strength ($C_5$)</th>
<th>The application of the primary crusher to mobile plants ($C_6$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_1$</td>
<td>EI</td>
<td>VI</td>
<td>VI</td>
<td>VI</td>
<td>I</td>
<td>U</td>
</tr>
<tr>
<td>$D_2$</td>
<td>EI</td>
<td>VI</td>
<td>EI</td>
<td>VI</td>
<td>VI</td>
<td>I</td>
</tr>
<tr>
<td>$D_3$</td>
<td>EI</td>
<td>EI</td>
<td>VI</td>
<td>I</td>
<td>I</td>
<td>I</td>
</tr>
</tbody>
</table>

Table 4.3.2 The preference of the alternatives with respect to each criterion by three decision makers [11]

<table>
<thead>
<tr>
<th>Crusher ($A_m$)</th>
<th>Capacity ($C_1$)</th>
<th>Feed size ($C_2$)</th>
<th>Product size ($C_3$)</th>
<th>Abrasion index ($C_4$)</th>
<th>Rock compressive strength ($C_5$)</th>
<th>The application of the primary crusher to mobile plants ($C_6$)</th>
</tr>
</thead>
</table>

In this study, violin plots were used to showcase the probability density of the fuzzy local weights of the criteria and the fuzzy global scores of the alternatives after 1000 iterations. Figure 4.3.1.a) shows the violin plot of the weight of each criterion with respect to the goal that were obtained by using IC-FSAHP, which indicates that capacity ($C_1$) was the most significant criterion that needs to be highlighted. In addition, feed size ($C_2$) and product size ($C_3$) have very little likelihood to be prioritised. Figure 4.3.1.b) displays the probability density of the overall scores of alternatives with respect to the criteria that were obtained by using IC-FSAHP. The high-speed roll crusher ($A_4$) was clearly the most suitable crusher without any overlap. In addition, gyratory ($A_1$) and single toggle jaw ($A_3$) crushers were next in line as preferred crushers with a high number of overlaps between each other, whereas the low-speed sizer ($A_5$) and double toggle jaw crusher ($A_2$) were the least suitable alternatives with significant overlaps between each other.
4.3 Case study

Fig. 4.3.1 Violin plots of a) the weights of the criteria, and b) the overall scores of the alternatives obtained from IC-FSAHP after 1000 iterations.

4.3.2 Sensitivity analysis

As previously mentioned in Section 4.2.1, the sensitivity analysis was carried out by varying the additional fuzzification factor \( \theta' \), formulated in Equations (4.3) and (4.4), and changing the disagreement level of decision makers when evaluating the objects. In this work, two \( \theta' \) values, namely 0.4 and 0.8, and two levels of disagreement that refer to the difference between the lowest and highest values among decision makers' opinions, namely \( DL_2 \) and \( DL_4 \), were applied.

Table 4.3.3 shows the variation of the importance of the criteria with respect to the goal when three DMs are involved. Tables 4.3.4 and 4.3.5 show the variation of the preference of the five alternatives with respect to the criteria, when the disagreement level of three DMs is 2 and 4, respectively.

Table 4.3.3 The variation of the importance of six criteria with respect to the goal, when two disagreement levels of three DMs are involved.

<table>
<thead>
<tr>
<th>( DL_x )</th>
<th>Capacity ( (C_1) )</th>
<th>Feed size ( (C_2) )</th>
<th>Product size ( (C_3) )</th>
<th>Abrasion index ( (C_4) )</th>
<th>Rock compressive strength ( (C_5) )</th>
<th>The application of the primary crusher to mobile plants ( (C_6) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_2 )</td>
<td>(I, EI, EI)</td>
<td>(I, I, EI)</td>
<td>(I, I, EI)</td>
<td>(F, VI, VI)</td>
<td>(F, F, VI)</td>
<td>(U, I, I)</td>
</tr>
<tr>
<td>( L_4 )</td>
<td>(U, EI, EI)</td>
<td>(U, U, EI)</td>
<td>(U, U, EI)</td>
<td>(VU, VI, VI)</td>
<td>(VU, VU, VI)</td>
<td>(EU, I, I)</td>
</tr>
</tbody>
</table>

4.3.3 Results

As previously mentioned in Section 4.2.1, the additional fuzzification factor represents the increase in uncertainty on the judgement values obtained from DMs; considering an additional fuzzification factor is therefore always desirable. The value of the additional fuzzification
Table 4.3.4 The variation of the preference of five alternatives with respect to the criteria, when the disagreement level of three DMs is 2 ($DL_2$).

<table>
<thead>
<tr>
<th>Crusher ($A_m$)</th>
<th>Capacity ($C_1$)</th>
<th>Feed size ($C_2$)</th>
<th>Product size ($C_3$)</th>
<th>Abrasion index ($C_4$)</th>
<th>Rock compressive strength ($C_5$)</th>
<th>The application of the primary crusher to mobile plants ($C_6$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-speed sizer ($A_5$)</td>
<td>(MH, MI, VH)</td>
<td>(MH, VH, VH)</td>
<td>(ML, ML, VH)</td>
<td>(VL, MI, ML)</td>
<td>(L, M, L)</td>
<td>(VL, MI, VH)</td>
</tr>
</tbody>
</table>

Table 4.3.5 The variation of the preference of five alternatives with respect to the criteria, when the disagreement level of three DMs is 4 ($DL_4$).

<table>
<thead>
<tr>
<th>Crusher ($A_m$)</th>
<th>Capacity ($C_1$)</th>
<th>Feed size ($C_2$)</th>
<th>Product size ($C_3$)</th>
<th>Abrasion index ($C_4$)</th>
<th>Rock compressive strength ($C_5$)</th>
<th>The application of the primary crusher to mobile plants ($C_6$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gyratory ($A_1$)</td>
<td>(ML, VH, VH)</td>
<td>(ML, VH, VH)</td>
<td>(L, L, H)</td>
<td>(VL, MI, ML)</td>
<td>(L, H, H)</td>
<td>(ML, VL, VL)</td>
</tr>
<tr>
<td>High-speed roll ($A_4$)</td>
<td>(ML, ML, VH)</td>
<td>(L, L, H)</td>
<td>(VL, MI, ML)</td>
<td>(VL, MI, VH)</td>
<td>(ML, ML, VH)</td>
<td>(ML, VL, ML)</td>
</tr>
<tr>
<td>Low-speed sizer ($A_5$)</td>
<td>(ML, VH, VH)</td>
<td>(ML, VH, VH)</td>
<td>(ML, VH, VH)</td>
<td>(VL, MI, ML)</td>
<td>(L, H, L)</td>
<td>(H, L, L)</td>
</tr>
</tbody>
</table>

factor is mainly affected by the lack of information or knowledge and thus more information should be attained in order to minimise it. In this work, the results obtained when the lack of information and knowledge increases are examined.

In addition, the gap between the lowest and highest judgement scales among DMs’ opinions in judging the preferences is represented by the level of disagreement among DMs. The disagreement levels indicate the uncertainty of the DMs’ judgement due to disparities among DMs, such as competencies and abilities. In this work, the results obtained when the level of disagreement among DMs increases are analysed. It is important to point out that, in this work, the level of disagreement among DMs in judging the preferences still reaches a most likely opinion. For a case in which no most likely opinion were reached, further discussions among DMs would be required in order to reach an agreement.

The violin plots of the results show that the probability density of the weights of the criteria and alternatives’ scores were not affected by the variation of the additional fuzzification factors ($\theta'$) and the levels of disagreement among decision makers in terms of the first ranked alternative. The detailed findings of the analysis are presented and discussed below.

Figures 4.3.2.a) and b) present the probability densities of the weight of each criterion with respect to the goal that were obtained from IC-FSAHP at the additional fuzzification factors ($\theta'$) of 0.4 and 0.8 for $DL_2$, respectively. The violin plot shown in Figure 4.3.2.a) shows
that capacity ($C_1$) was the most prioritised criterion in all cases. This outcome is similar
to the outcome shown in Figure 4.3.1.a). The overlap of the thickest section of feed size
($C_2$), product size ($C_3$) and abrasion index ($C_4$) overlap with capacity ($C_1$) reveals that feed
size ($C_2$), product size ($C_3$) and abrasion index ($C_4$) have a high probability to be the most
important criterion. Unlike in Figure 4.3.1.a), in Figure 4.3.2.a), not only feed size ($C_2$) and
product size ($C_3$) have the possibility to be the most prioritised criterion, but also abrasion
index ($C_4$), rock compressive strength ($C_5$) and the application of the primary crusher to
mobile plants ($C_6$) have a very small probability to be the most important criterion. This
difference happened because the disagreement level of decision makers was higher than those
shown in Figure 4.3.1.a). The higher disagreement level resulted in increased uncertainty
and a corresponding increase in the number of overlaps between criteria. In addition, the
plot shown in Figure 4.3.2.b) is very similar to that in Figure 4.3.2.a), indicating that the
variation of the additional fuzzification factor has little effect on the weights of the criteria.

Fig. 4.3.2 Violin plots of the weights of the criteria obtained from IC-FSAHP at the additional
fuzzification factor ($\theta'$) of a) 0.4, and b) 0.8 for $DL_2$ after 1000 iterations.

Figures 4.3.3.a) and b) showcase the probability density of the total scores of the alter-
natives with respect to the criteria that were obtained from IC-FSAHP at the additional
fuzzification factor ($\theta'$) of 0.4 and 0.8 for $DL_2$, respectively. The violin plot presented in
Figure 4.3.3.a) shows that the high-speed roll crusher ($A_4$) was the most preferred crusher
with a very high number of overlaps with the single toggle jaw crusher ($A_3$) and a high
number of overlaps with the low-speed sizer ($A_5$). This finding is different from the results
that are shown in Figure 4.3.1.b) because the uncertainty is higher and the weights of the
criteria are different. In the case of $DL_2$, there are four criteria (i.e. $C_1$, $C_2$, $C_3$ and $C_4$) that
share almost similar weights in the highest prioritised criterion, whereas the results in Figure
4.3.1.b) are mainly affected from capacity ($C_1$) and feed size ($C_2$).
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Fig. 4.3.3 Violin plots of the overall scores of the alternatives obtained from IC-FSAHP at the additional fuzzification factor ($\theta'$) of a) 0.4, and b) 0.8 for $DL_2$ after 1000 iterations.

Furthermore, the violin plot displayed in Figure 4.3.3.b) show that the number of overlaps between high-speed roll ($A_4$) and single toggle jaw ($A_3$) crushers is a little bit higher than those shown in Figures 4.3.3.a). Like in Figure 4.3.3.a), it can be seen in Figure 4.3.3.b) that the gyratory ($A_1$), low-speed sizer ($A_5$) and double toggle jaw ($A_2$) crushers were the least preferable alternatives with significant overlaps between each other. This finding shows that the variation of the additional fuzzification factor has an insignificant effect on the overall scores of the alternatives.

Figures 4.3.4.a) and b) present the probability densities of the weight of each criterion with respect to the goal that were obtained from IC-FSAHP at the additional fuzzification factor ($\theta'$) of 0.4 and 0.8 for $DL_4$, respectively. In terms of median points in violin plots, which are shown in Figures 4.3.4.a), capacity ($C_1$) was still the most important criterion. However, in terms of overlaps, there is a high number of overlaps among the criteria. This result is totally different from the results that are shown in Figures 4.3.1.a) and 3). This provides further evidence that the higher disagreement level makes the uncertainty higher and thus the number of overlaps between criteria increases. Moreover, the plots shown in Figure 4.3.4.b) are similar to those in Figure 4.3.4.a), which confirms that the variation of the additional fuzzification factor has a little impact on the weights of the criteria.

Figures 4.3.5.a) and b) present the violin plots of the overall scores of the alternatives with respect to the criteria that were obtained from IC-FSAHP at the additional fuzzification factor ($\theta'$) of 0.4 and 0.8 for $DL_4$, respectively. The violin plot in Figures 4.3.5.a) show that the high-speed roll crusher ($A_4$) was the best crusher with a very high number of overlaps with gyratory crusher ($A_1$) and a high number of overlaps with alternatives low-speed sizer ($A_5$) and single toggle jaw crusher ($A_3$). This result is different from the outcomes shown in Figures 4.3.1.b) and 4.3.3.a) because the uncertainty is extremely high, and the weights of
Fig. 4.3.4 Violin plots of the weights of the criteria obtained from IC-FSAHP at the additional fuzzification factor ($\theta'$) of a) 0.4, and b) 0.8 for $DL_4$ after 1000 iterations.

The criteria are different. In addition, the violin plot shown in Figure 4.3.5.b) is quite similar to that in Figure 4.3.5.a). As in previous instances, this outcome further strengthens the conclusion that the alteration of the additional fuzzification factor results in little impact on the final recommendations.

Fig. 4.3.5 Violin plots of the overall scores of the alternatives obtained from IC-FSAHP at the additional fuzzification factor ($\theta'$) of a) 0.4, and b) 0.8 for $DL_4$ after 1000 iterations.

Based on the aforementioned explanations, the most important criterion and the most suitable crusher were not altered by varying the additional fuzzification factor not by increasing the disagreement level among decision makers. However, as the level of disagreement among decision makers increases, the density of higher probability decreases and thus the weight of each criterion becomes more scattered. Therefore, even though the most prioritised criterion
and the most suitable crusher were never altered, the ranks of other particular objects (criteria or alternatives) were not the same.

4.4 Conclusions

A novel sensitivity analysis approach has been proposed to understand the robustness of an integrated constrained fuzzy stochastic analytic hierarchy process (IC-FSAHP) method that can be applied to the selection of the most suitable equipment in mineral processing, considering a primary crusher as a case study. The proposed approach allows decision makers to identify the effect of input data changes on the aggregated results for equipment ranking and takes into account the main notions of IC-FSAHP, i.e. preserving the reciprocity interactions among the elements in the fuzzy pairwise comparison matrices and checking the inconsistency ratios of the fuzzy pairwise comparison matrices. The proposed sensitivity analysis approach applies an additional fuzzification factor ($\theta'$) and allows to consider different levels of disagreement among decision makers to model uncertainty.

Since the level of uncertainty of the variables does not affect the final recommendation, the results obtained from the sensitivity analysis indicate that IC-FSAHP could be confidently used for the selection of mineral processing equipment. It was noted that the rank of the most suitable crusher, which was the high-speed roll crusher, remains the same irrespective of changes in the additional fuzzification factor ($\theta'$) or disagreement level of decision makers. IC-FSAHP can also be used in other fields to address the selection problem when dealing with many decision makers under uncertainty.

Furthermore, the results have shown the usability of the proposed sensitivity analysis approach in a mineral processing equipment selection problem under uncertainty with multiple decision makers. This approach allows decision makers to define and calculate the uncertainty involved in a multiple criteria decision making problem with more flexibility and reliability. Moreover, it can be concluded from the results that the proposed sensitivity analysis approach is capable of providing extensive and useful “what-if” information on the decision making results.
Chapter references


Chapter 5

A multiple criteria decision making method to weight the sustainability criteria of renewable energy technologies under uncertainty

Abstract

Selecting the most suitable renewable energy technology among feasible alternatives considering conflicting criteria is a Multiple Criteria Decision Making (MCDM) problem. One of the essential stages in the methods used to solve such problems is determining the appropriate weight of each criterion to be considered. The Shannon Entropy method is a frequently used MCDM method to calculate the criteria weights, however it is not suitable to solve problems for which uncertainty in the input data exists. This paper presents a new extended Shannon Entropy method: the Integrated Constrained Fuzzy Shannon Entropy (IC-FSE) method, by which criteria weights are obtained from uncertain input data. To show the applicability of IC-FSE, an illustrative example for the selection of a renewable energy technology in the mining industry is presented, in which three alternative renewable energy technologies, onshore wind, solar photovoltaic and concentrated solar power, were evaluated with respect to technical, social, economic and environmental categories. The results show that IC-FSE can effectively provide appropriate fuzzy solutions for weighting the sustainability criteria for renewable energy technologies. The superiority of this method is showcased by demonstrating that IC-FSE results are more robust than those obtained using other existing methods. The methodology presented can be applied broadly in the renewable energy sector to ensure better informed decision making processes.
5.1 Introduction

Current energy supply is dominated by non-renewable sources, i.e. fossil fuels, most of which are relatively inexpensive to extract [1], but the impact on sustainable development of the extensive use of fossil fuels has raised concerns about the security, reliability, affordability, and environmental impact of energy supplies [2]. One way to respond to these challenges is to significantly increase the use of renewable energy sources and technologies (e.g. wind and solar) that are clean and sustainable [3].

However, it is often challenging to determine the most suitable renewable energy technology to be utilised. Decision makers need to evaluate multiple renewable energy source alternatives with respect to multiple criteria, which often conflict with each other. For example, a renewable energy technology that might help achieve strict environmental regulations (e.g. low GHG emissions, decreasing the reliance on fossil fuel sources) will have costs associated to the additional equipment installation and maintenance, so trade-offs need to be clearly identified during the decision making process.

The selection of the most suitable renewable energy source among a set of feasible alternatives considering multiple conflicting criteria is a Multiple Criteria Decision Making (MCDM) problem. MCDM is a powerful method to find the best alternative when multiple conflicting criteria are involved and need to be evaluated in a scientific manner [4]. MCDM methods have been successfully applied in many fields such as natural resources management [5], environmental science [6], and mining and mineral processing [7], as well as in the area of renewable energy technologies over the past decade.

Researchers’ interest in applying MCDM methods in renewable energy area continues to grow, as evidenced by recent literature reviews [8–13]. MCDM methods have been used to: investigate and select the most appropriate site location for renewable energy projects [14, 15], evaluate and select the most suitable photovoltaic technology under uncertainties [16], assess, compare and rank the sustainability of various renewable energy technologies for national-scale assessments under uncertainties [17, 2], analyse and rank the sustainability of different energy storage technologies [18, 19], and evaluate the sustainability of concentrated solar power projects [20].

In general, an MCDM method involves five stages [21, 4, 22, 23], namely: (i) weighting the local importance of each criterion with respect to the goal, which is of critical importance as it affects the subsequent stages; (ii) scoring the local preference of each alternative with respect to criteria; (iii) calculating the global weighted scores of alternatives; (iv) ranking of the feasible alternatives based on the global weighted scores; (v) selecting the most suitable alternative, i.e., the highest global weighted score. The final recommendation obtained from any MCDM method usually depends on the criteria against which a set of feasible alternatives are evaluated, the weights (i.e. importance) of the criteria, the local preference scores of
A multiple criteria decision making method to weight the sustainability criteria of renewable energy technologies under uncertainty

There are two main methods to determine the criteria weights in MCDM, namely subjective and objective weighting methods. The subjective weighting method relies on decision makers’ judgements and can be achieved by means of pairwise comparisons of qualitative or quantitative data. The Analytic Hierarchy Process [22] is the most frequently used subjective weighting method. Although the criteria weights are often obtained using only subjective weighting methods, it is sometimes difficult to obtain reliable judgements from the decision makers, in which case objective weighting methods should be considered.

The objective criteria weights are obtained from the computation of quantitative data, using an algorithm to derive the weights without involving any consideration of decision makers’ judgements. The Shannon Entropy method [24] is one of the most frequently used objective methods for deriving the criteria weights. However, in the area of renewable energy, there is a paucity of studies investigating the application of the Shannon Entropy method for objective criteria weighting, an exception being the evaluation of the sustainability of concentrated solar power technologies conducted by Simsek et al. [20].

In cases where the precise values of the relevant input data are available, the evaluation and computation stages in the Shannon Entropy method are applied to express all criteria data and their corresponding weights as crisp values. However, it is often the case that input data are associated with significant uncertainties. For example, Troldborg et al. [2] showed that the data for total power generation, GHG emissions, area requirements and the levelised energy cost, varied widely for each of the eleven renewable energy technologies evaluated. Such variation often results in imprecise input data, which significantly affects the final results and leads to an inaccurate final recommendation [25]. In order to resolve this issue, the imprecise input data can be stated using fuzzy set theory (FST) [26].

A fuzzy set is a group of elements that have degrees of membership over the range 0 and 1, where 0 describes absolutely unlikely or false statements and 1 describes absolutely likely or true statements. The extension of the Shannon Entropy method by means of FST, it is called the fuzzy Shannon Entropy (FSE) method. In the case when the input data have minimum, most likely and maximum values, a type of ordered fuzzy set, a triangular fuzzy number (TFN), could be used to represent such data.

TFN is one of the most cited fuzzy set types applied to MCDM methods in the literature [27], however, only one fuzzification of the Shannon Entropy method by means of TFN has been proposed [28]. Given that imprecise data is the most common cause of uncertainties in real-world sustainability assessments of renewable energy technologies [12], a more detailed investigation into the reliability and the robustness of the existing extension of the Shannon Entropy method with TFN proposed by Kacprzak [28] is necessary.

Kacprzak [28] developed an FSE method that makes use of standard fuzzy arithmetic. The use of standard fuzzy arithmetic in MCDM methods, however, can lead to inapplicable
results for many real-world engineering MCDM problems [29–33]. These inapplicable results arise because a combined method does not take into account the additional information available in real-life situations and ignores known constraints, thus can increase the risk of losing important information about a problem during the computations and result in overestimation of the fuzzy solutions [34, 30, 25]. This is an important issue, since most real-life applications are not constraint-free. Constrained fuzzy arithmetic, on the other hand, can be considered more powerful than standard fuzzy arithmetic [30–32, 35]. There is therefore scope to improve the fuzzification of the Shannon Entropy method using TFN in order to minimise uncertainty as much as possible by means of constrained fuzzy arithmetic.

In response to the limitations described above, this study proposes a new criteria weighting method: the Integrated Constrained Fuzzy Shannon Entropy (IC-FSE). IC-FSE combines the concepts of constrained fuzzy arithmetic, TFN, and the Shannon Entropy method by taking advantage of their most useful characteristics that can be used to robustly determine the criteria weights for the selection of renewable energy technologies.

The contribution of this study is sixfold: (1) the gap in the Shannon Entropy method literature for problems involving uncertainty due to imprecise input data is addressed; (2) a new MCDM method, IC-FSE, is developed to account for imprecise input data and represent the data by using TFN; (3) the applicability of IC-FSE in weighting the sustainability criteria of renewable energy technologies is showcased; (4) the reliability of IC-FSE compare to stochastic methods is demonstrated; (5) the superiority of the normalisation procedure used in IC-FSE is presented by comparing it to common existing procedures; (6) it is demonstrated that the results obtained from IC-FSE are more robust than those obtained from the only other FSE method reported in the literature. IC-FSE is thus a robust, reliable and superior MCDM method that can be applied broadly in the renewable energy sector to support the decision making process when there is uncertainty in the data.

The remainder of the manuscript is organised as follows: Section 5.2 provides renewable energy alternatives and criteria in mining and mineral processing; Section 5.3 provides the theoretical background of TFN, standard fuzzy arithmetic, and constrained fuzzy arithmetic; Section 5.4 describes the proposed IC-FSE method; Section 5.5 presents an example of the applicability of the IC-FSE method; Finally, Section 5.6 provides the final conclusions. Appendix C presents an analysis of normalisation procedures, while in Appendix D provides the comparison of the results obtained from IC-FSE and the only other existing method.

5.2 Renewable energy technologies and criteria in mining and mineral processing

Mining operations are often located in remote areas where the deposits of mineral resources (coal, metals, industrial minerals, etc.) are found. The remoteness of mine sites frequently
results in limited accessibility to energy sources. Because of such circumstances, fossil fuel (e.g. fuel oil, diesel oil, etc.) is in many cases the only feasible choice to power mobile equipment’s internal combustion engines and electric power generators [36]. The mining industry still relies heavily on non-renewable energy sources [37]. It is worth noting that mining operations are very energy intensive; energy costs account for 30-50% of all operating costs [38, 37].

As other industrial operations, mining operations are responsible for producing GHG emissions not only from the use of fossil energy sources for electricity generation but also from operating equipment. As the global demand for metals and minerals steadily increases and the process routes to extract them require larger amounts of energy (due to the need of processing lower grade and finely disseminated ores), higher emissions are produced [39, 40]. In addition, there is a definite correlation between emissions from the mining industry and health risks (e.g. cardiovascular and respiratory diseases) on surrounding communities [41].

In order to address the aforementioned challenges, a number of mining companies worldwide have started to pay more attention to the use of renewable energy technologies in their operations to adhere to the principles of sustainable development [42, 43]. Since energy requirements in mining operations are relatively constant while most renewable energy sources are intermittent, a hybrid scheme that combines renewable sources and diesel generators or electricity from a grid can also be considered for implementation, since energy storage facilities are still relatively expensive [44].

The capability of IC-FSE will be showcased by applying it to weighting the sustainability criteria of renewable energy technologies for mining operations. Section 5.2.1 and Section 5.2.2 describe the alternatives and criteria used in this work.

5.2.1 Identification of feasible renewable energy sources

Three renewable energy technologies that have been successfully implemented in the mining industry [45, 37] were considered to be compared. The feasible renewable energy technologies that were considered are summarised as follows:

**Onshore wind (OW) - A1:** wind energy is harvested from the movement of air masses to drive wind turbines that provide mechanical power, which is converted to electricity [46]. A number of mining companies have implemented wind power systems at operating mines in Argentina, Canada, and Chile. This has also been done at abandoned mines in the USA to provide electricity to households near the site. The generated power varies from 2 MW to 115 MW in the operating mines and from 29 MW to 237 MW in the abandoned mines [45].

**Concentrated solar power (CSP) - A2:** CSP uses reflective surfaces to focus sunlight into a beam to heat a working fluid in a receiver; the steam produced from the heat is utilised to drive a turbine that provides mechanical power, which is converted to electricity [47]. The total installed capacity of CSP in the mining industry in 2016 was 39 MW [37].
Solar photovoltaic (PV) - $A_3$: solar photovoltaic energy is harvested from the thermal radiation emitted by sunlight by means of photovoltaic cells, which is converted into electric current [48]. A number of mining companies have applied PV technology either at operating mines in the USA, Chile, Australia, South Africa, and Suriname or at abandoned mines in the USA, Germany, Canada, and Korea, where it has been used to provide power to nearby households and for acid mine drainage treatment. The generated power varies from 1 MW to 10.6 MW in the operating mines and from 1 MW to 166 MW in the abandoned mines [45].

5.2.2 Evaluation criteria

There are several studies that list a number of sustainability criteria for assessing renewable energy technologies by means of MCDM methods (e.g. [49, 10, 50, 20, 51, 11, 52, 13]). Wang et al. [13] and Simsek et al. [20] present comprehensive lists of frequently used sustainability criteria in MCDM for renewable energy technologies. Those comprehensive lists were used as a basis for determining the criteria considered in the current work. For the purpose of the study, only quantitative criteria were considered. Six evaluation criteria were selected and are summarised in Table 5.2.1 and further described below.

Table 5.2.1 The evaluation criteria and sources of data used for the current work.

<table>
<thead>
<tr>
<th>Main Categories</th>
<th>Criteria</th>
<th>Units</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Technical</td>
<td>$C_1$: Capacity factor</td>
<td>%</td>
<td>[53, 54]</td>
</tr>
<tr>
<td></td>
<td>$C_2$: Water consumption</td>
<td>l/MWh</td>
<td>[55, 56]</td>
</tr>
<tr>
<td>Environmental</td>
<td>$C_3$: GHG emissions</td>
<td>gCO2eq/kWh</td>
<td>[2]</td>
</tr>
<tr>
<td></td>
<td>$C_4$: Area requirement</td>
<td>m²/kW</td>
<td>[2]</td>
</tr>
<tr>
<td>Economic</td>
<td>$C_5$: Levelised Energy Cost</td>
<td>£/MWh</td>
<td>[2]</td>
</tr>
<tr>
<td>Social</td>
<td>$C_6$: Prospective jobs creation</td>
<td>Jobs/annual GWh</td>
<td>[57, 58]</td>
</tr>
</tbody>
</table>

It is important to mention that quantitative data for the criteria selected were attained from the literature and for consistency, correspond to the same geographical region, i.e. the UK.

1. Technical:

Capacity factor ($C_1$) was considered as an important technical criterion. Capacity factor, which is measured as a percentage, defines the ratio between the actual electrical energy production generated by a power plant and the maximum electrical energy output that can be generated over a period of time [51]. A large capacity factor is always desirable; it is important to consider that capacity factors of different power plants vary extensively. The capacity factors of onshore wind, CSP and PV are mainly affected by the weather. For example, when the wind speed is high, the average power generation capacity of wind power plants is high, leading to a greater capacity factor. In addition, since CSP and PV are affected by sunlight, so in summer when daylight
time is longer than in winter, the average power generation capacity of CSP and PV power plants is high, and thus leading to a greater capacity factor.

Based on the literature [53, 54], typical capacity factors for onshore wind, CSP and PV in the UK are about 24–34 %, 17–25 %, and 5-12%, respectively. It is worth mentioning that the capacity factor of PV is the lowest when compared to all other types of power generation.

2. Environmental:

Three environmental criteria are used to reflect the effect of renewable energy technologies on environmental sustainability in the mining industry. Three environmental criteria were considered in this work, namely water consumption, GHG emissions and area requirement.

2.1. Water consumption ($C_2$).

Water consumption, which is measured in l/MWh, is the amount of withdrawn water obtained from the water reservoir, such as surface water or groundwater, that is not returned to its source during the life cycle of electricity generation [55]. A small amount of water consumption is always desirable. Typical water consumption for onshore wind, CSP and PV in the UK are about 0.4–34 l/MWh, 303–644 l/MWh, and 38-795 l/MWh, respectively [55]. It is worth noting that the maximum estimation of water consumption of PV is higher than that of CSP. The reason for this high use is that when processing silicon into PV equipment, many stages that use much more water than the rest of the manufacturing processes, such as the production of steel components [55], are involved.

2.2. GHG emissions ($C_3$).

The GHG emissions criterion is one of the most frequently used criteria when evaluating the sustainability of renewable energy technologies [13]. GHG emissions, which are reported as (g CO$_2$eq/kWh), were estimated on the basis of CO$_2$ and CH$_4$ emissions of each renewable energy technology, from the commissioning of a plant to the full operation of the technology and the dismantling of the system [59]. A small amount of GHG emissions is always favoured. Typical GHG emissions for onshore wind, CSP and PV in the UK are about 5–70 gCO$_2$eq/kWh, 15–150 gCO$_2$eq/kWh, and 20–200 gCO$_2$eq/kWh, respectively [2].

2.3. Area requirement ($C_4$).

Another environmental criterion used in this study is the land area required to implement each of the renewable energy technologies. The land area required by each renewable energy technology, which is expressed as $m^2$/kW, is of great concern for their evaluation in the mining industry. This criterion is important because of concerns that the implementation of renewable energy technologies can
often be competing with agriculturally arable land [60] and thus destabilise the flora, the fauna and the ecosystem [61]. Therefore, the smallest area required is always preferred. Based on the literature [2], typical area requirement values for onshore wind, CSP and PV in the UK are about 10–1200 m²/kW, 10–100 m²/kW, and 10–500 m²/kW, respectively.

3. Economic:
The economic criterion is of paramount importance for assessing the sustainability of renewable energy technologies in numerous MCDM studies. The economic criterion considered in the literature often include the following: capital expenditure (CAPEX), operation and maintenance (O&M) expenditure (OPEX), and fuel costs and levelised energy cost (LEC) [20, 13]. In this work, LEC (\(C_5\)), which is measured in £/MWh, was considered as an economic criterion because all the costs over an assumed project’s financial life and duty cycle (i.e. CAPEX, OPEX, fuel costs, financing costs, as well as an assumed capacity factor for each plant type) are included in the LEC calculation [47]. Not only that, but LEC is also influenced by the characteristics of the technology, such as efficiency, annual energy production, duration, and energy source [2]. A small amount of LEC is always desirable. Based on the literature [2], typical area requirement values for onshore wind, CSP and PV in the UK are about 25–125 £/MWh, 50–450 £/MWh, and 50–600 £/MWh, respectively. It is clearly noticed that onshore wind has the lowest LEC values, while the solar-based technologies have high LEC values. Since performance variations due to the maturity of technology affects the LEC values [2], LEC values of solar based technologies will tend to decrease in the future if the technologies become more mature.

4. Social:
The social criterion has been of vital importance for people’s acceptance of the application of renewable energy technologies, and has been considered in more detail over the past few decades [20, 13]. Prospective jobs creation, \((C_6)\), which is expressed as jobs/annual GWh, is the most frequently used social criterion in the literature [20, 13]; it allows decision makers to consider socioeconomic aspects when deciding which technology can improve the living standards of the surrounding community [61]. This criterion takes into account the potential jobs created during the life cycle of renewable energy technology, from construction and operation until decommissioning. Therefore, the greatest number of job created is favoured. Based on the literature [2], typical prospective job creation values for onshore wind, CSP and PV in the UK are about 0.1–0.6 jobs/annual GWh, 0.2–0.7 jobs/annual GWh, and 0.2–1.3 jobs/annual GWh, respectively.
5.3 Theory

This section discusses the key theoretical aspects behind the TFN, standard fuzzy arithmetic, and constrained fuzzy arithmetic. The following sub-sections are provided as background for the development of the IC-FSE method in Section 5.4.

5.3.1 Triangular Fuzzy Number (TFN)

FST [26] is used to represent the vagueness of statements in natural language into real numbers that have membership function over the range 0 and 1. Bellman and Zadeh [62] first introduced FST into decision making as an approach that can effectively solve problems in a fuzzy environment. This has been followed by numerous approaches in which FST has been applied to existing MCDM methods to solve a variety of problems, with promising outcomes. The following are examples of such approaches found in the literature: fuzzy AHP (FAHP) [63, 64], fuzzy TOPSIS [65, 66], and Fuzzy ELECTRE [67]. The wide application of hybrid FST and MCDM methods to real world problems was surveyed by Kahraman et al. [27] and Mardani et al. [68].

TFN is the most widely used FST for presenting the imprecise input data in real-life MCDM applications because it is easy to apply and thus leads to a straightforward calculation. Kacprzak [28] proposed the first combination of TFN with the Shannon Entropy method. The membership function of TFN $\tilde{A}(x)$ is arranged in the following form:

$$f(x) = \begin{cases} 
\frac{x-c_{LA}}{c_{UA}-c_{LA}}, & \text{if } c_{LA} < x < c_{MA}; \\
1, & \text{if } x = c_{MA}; \\
\frac{c_{MA}-x}{c_{UA}-c_{MA}}, & \text{if } c_{MA} < x < c_{UA}; \\
0, & \text{otherwise},
\end{cases}$$

(5.1)

where $c_{LA}$ and $c_{UA}$ are termed as the left and right membership function of TFN $\tilde{A}(x)$ or the lowest and highest boundary values of TFN $\tilde{A}(x)$, while $c_{MA}$ is defined as the middle value of TFN $\tilde{A}(x)$. Suppose that we have $c_{LA}, c_{UA}, c_{MA} = 2, 3, \text{and } 4$, respectively, then this TFN can be presented graphically as shown in Figure 5.3.1.

In order to obtain a crisp number result of the TFN, the centre-of-area (COA) defuzzification technique, presented by Tzeng and Huang [69], is applied in this paper. The centre of area COA $\tilde{A}(x)$ of a triangular fuzzy number $\tilde{A}(x) = (c_{LA}, c_{MA}, c_{UA})$ is formulated in the following form:

$$COA \tilde{A}(x) = \frac{(c_{UA} - c_{LA}) + (c_{MA} - c_{LA})}{3} + c_{LA}.$$  

(5.2)
5.3 Theory

Fig. 5.3.1 The membership functions of TFN (2, 3, 4).

5.3.2 Standard fuzzy arithmetic

In standard fuzzy arithmetic, basic arithmetic operations on real or crisp numbers are extended to operations on TFNs. In this paper, all the positive TFNs are taken into account since all input data are positive numbers. Therefore, the lowest membership number for a TFN is higher than zero.

Suppose that two positive TFNs, namely TFN \( \tilde{A} \) and TFN \( \tilde{B} \) are defined as, \( \tilde{A} = (c^L_A, c^M_A, c^U_A) \), \( \tilde{B} = (c^L_B, c^M_B, c^U_B) \), the standard arithmetic operations between these two TFNs are as follows:

- **Addition (+):** \( \tilde{A} + \tilde{B} = (c^L_A + c^L_B, c^M_A + c^M_B, c^U_A + c^U_B) \).
  \( (5.3) \)

- **Multiplication (x):** \( \tilde{A} \) and \( \tilde{B} = (c^L_A c^L_B, c^M_A c^M_B, c^U_A c^U_B) \).
  \( (5.4) \)

- **Division (/):** \( \tilde{A} \) and \( \tilde{B} = (c^L_A / c^L_B, c^M_A / c^M_B, c^U_A / c^U_B) \).
  \( (5.5) \)

Moreover, the comparison of two TFNs are represented in the following form:

- \( \tilde{A} \geq \tilde{B} \) if \( c^L_A \geq c^L_B, c^M_A \geq c^M_B, c^U_A \geq c^U_B \).
  \( (5.6) \)

- \( \tilde{A} \leq \tilde{B} \) if \( c^L_A \leq c^L_B, c^M_A \leq c^M_B, c^U_A \leq c^U_B \).
  \( (5.7) \)

5.3.3 Constrained fuzzy arithmetic

The aforementioned notions of standard fuzzy arithmetic operations can be implemented only in the case when there is no interaction between the fuzzy numbers [35]. For the case when the interaction between fuzzy numbers is involved, the notions of constrained fuzzy arithmetic that are presented in the next paragraphs should be applied.

Suppose that \( f \) is a continuous function and \( \mathbb{R} \) is the set of real numbers, \( f: \mathbb{R}^n \to \mathbb{R} \), and suppose that \( \tilde{A}_i = (A^L_i, A^M_i, A^U_i), i = 1, 2, \ldots, n \), where \( n \) is the amount of positive TFNs. Then, \( \tilde{A} = f_F (\tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \ldots, \tilde{A}_n) \) is a TFN \( \tilde{A} = (A^L, A^M, A^U) \) whose significant values are derived from the following form:

\[
A^L = \min \{ f (A_1, A_2, A_3, \ldots, A_n); A_i \in [A^L_i, A^U_i], i = 1, 2, \ldots, n \},
\]

(5.8)
A multiple criteria decision making method to weight the sustainability criteria of renewable energy technologies under uncertainty

\[ A^M = f (A^M_1, A^M_2, A^M_3, \ldots, A^M_n), \quad (5.9) \]

\[ A^L = \max \{ f (A_1, A_2, A_3, \ldots, A_n); A_i \in [A^L_i, A^U_i], i = 1, 2, \ldots, n \}. \quad (5.10) \]

These basic notions of constrained fuzzy arithmetic will be applied to the TFN arithmetic operations throughout this work.

### 5.4 Integrated Constrained Fuzzy Shannon Entropy (IC-FSE)

The Shannon Entropy is a widely used MCDM method to obtain the criteria weights by means of objective weight methods. It was initially introduced by Shannon [24] and various modifications have been developed, particularly in normalising a decision matrix [70–72]. This work applies the normalisation procedure that was developed by Nijkamp and Delft [70] mainly because the outcomes obtained by applying this procedure have the lowest degree of uncertainty in comparison to the procedures that were developed by Weitendorf [72] and Voogd [71] (see: Appendix C).

This section describes the application of constrained fuzzy arithmetic to extend the Shannon Entropy method. This is possible when it is difficult to acquire reliable subjective weights, the data that need to be analysed are difficult to be defined precisely, and the input data is presented as ordered fuzzy numbers, such as triangular fuzzy numbers (TFNs).

IC-FSE involves six major steps: (1) defining the problem notions (e.g. determining alternatives and criteria) and developing a fuzzy decision matrix, (2) normalising the fuzzy decision matrix, (3) determining the fuzzy entropy values, (4) computing the local fuzzy criteria weights, (5) defuzzifying the results obtained in step 4, and (6) normalising the crisp values acquired in step 5 in order to obtain the final criteria weights. The mathematical equations for each step of the IC-FSE method are presented below:

1. Developing a fuzzy decision matrix that presents the ratings of different alternatives, \( \widetilde{A}_j \) \( (\widetilde{A}_1, \widetilde{A}_2, \ldots, \widetilde{A}_m) \) with respect to predetermined criteria, \( \widetilde{C}_i \) \( (\widetilde{C}_1, \widetilde{C}_2, \ldots, \widetilde{C}_n) \); the rating of each alternative is expressed in TFN.

\[ \widetilde{X} = [\widetilde{x}_{ji}]_{m \times n} = \begin{bmatrix} C_1 & C_2 & \cdots & C_n \\ A_1 & \begin{bmatrix} \widetilde{x}_{11} & \widetilde{x}_{12} & \cdots & \widetilde{x}_{1n} \\ \widetilde{x}_{21} & \widetilde{x}_{22} & \cdots & \widetilde{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{x}_{m1} & \widetilde{x}_{m2} & \cdots & \widetilde{x}_{mn} \end{bmatrix} \end{bmatrix} \text{.} \quad (5.11) \]

Note: \( \widetilde{x}_{ji} \) is the rating of the alternative \( j \) with respect to criterion \( i \). For example: \( \widetilde{x}_{11} \) is the rating of the alternative 1 with respect to criterion 1 and the elements of \( \widetilde{x}_{11} \) are \( x_{11L}, x_{11M}, x_{11U} \), where the superscript \( L, M, \) and \( U \) refer to the lowest, middle and highest boundary values of TFN \( \widetilde{x}_{11} \), respectively.
2. Normalising the fuzzy decision matrix. The normalised elements of TFN $\tilde{r}_{ji} = (r_{jiL}^C, r_{jiM}^C, r_{jiU}^C)$, $i = 1, \ldots, n$ and $j = 1, \ldots, m$ (where the superscript $C$ represents the constrained fuzzy arithmetic and the sub-subscript $L$, $M$, and $U$ refer to the lowest, middle and highest boundary values) are obtained by using the following equations:

$$r_{jiL}^C = \min \left\{ \frac{x_{ji}}{\sqrt{\sum_{j=1}^{m} x_{ji}^2}}; \ x_{ji} \epsilon [x_{jiL}, x_{jiU}] \right\},$$  \hspace{1cm} (5.12)\n
$$r_{jiM}^C = \left\{ \frac{x_{ji}^M}{\sqrt{\sum_{j=1}^{m} x_{ji}^2}} \right\},$$  \hspace{1cm} (5.13)\n
$$r_{jiU}^C = \max \left\{ \frac{x_{ji}}{\sqrt{\sum_{j=1}^{m} x_{ji}^2}}; \ x_{ji} \epsilon [x_{jiL}, x_{jiU}] \right\}. $$ \hspace{1cm} (5.14)

3. Determining the fuzzy entropy value $(\tilde{e}_i) = (e_{iL}^C, e_{iM}^C, e_{iU}^C)$ of each criterion through the following formulae:

$$\tilde{e}_i = - \frac{\sum_{j=1}^{m} \tilde{f}_{ji} \times \ln \tilde{f}_{ji}}{\ln m},$$  \hspace{1cm} (5.15)\n
$$\tilde{f}_{ji} = \frac{r_{ji}}{\sum_{j=1}^{m} r_{ji}},$$  \hspace{1cm} (5.16)\n
$$e_{iL}^C = \min \left\{ - \frac{\sum_{j=1}^{m} \left( \frac{r_{ji}^L \times \ln \frac{r_{ji}^L}{\sum_{j=1}^{m} r_{ji}^L}}{\ln m} \right); \ r_{ji} \epsilon [r_{jiL}, r_{jiU}] \right\},$$ \hspace{1cm} (5.17)\n
$$e_{iM}^C = - \frac{\sum_{j=1}^{m} \left( \frac{r_{ji}^M \times \ln \frac{r_{ji}^M}{\sum_{j=1}^{m} r_{ji}^M}}{\ln m} \right)}{\ln m},$$  \hspace{1cm} (5.18)\n
$$e_{iU}^C = \max \left\{ - \frac{\sum_{j=1}^{m} \left( \frac{r_{ji}^U \times \ln \frac{r_{ji}^U}{\sum_{j=1}^{m} r_{ji}^U}}{\ln m} \right); \ r_{ji} \epsilon [r_{jiL}, r_{jiU}] \right\}. $$ \hspace{1cm} (5.19)

If $\tilde{f}_{ji}$ are all same, then the fuzzy entropy value of each criterion is the maximum $(\tilde{e}_i)$. If $\tilde{f}_{ji}$ is 0, then $\tilde{f}_{ji} \times \ln \tilde{f}_{ji}$ is 0.

4. Computing the fuzzy entropy weight $(\tilde{w}_i) = (w_{iL}^C, w_{iM}^C, w_{iU}^C)$ of each criterion using the following equations:

$$\tilde{w}_i = 1 - \frac{\tilde{e}_i}{\sum_{i=1}^{n} \tilde{e}_i},$$ \hspace{1cm} (5.20)\n
$$w_{iL}^C = \min \left\{ \left( \frac{1 - e_i}{\sum_{i=1}^{n} e_i} \right); \ e_i \epsilon [e_{iL}^C, e_{iU}^C] \right\},$$ \hspace{1cm} (5.21)
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\[ w_{C_iM} = \frac{1 - e_i^M}{\sum_{i=1}^{n} e_i^M} \tag{5.22} \]

\[ w_{Ci} = \max \left\{ \left( \frac{1 - e_i}{\sum_{i=1}^{n} e_i} \right); e_i \epsilon [\epsilon_{C_iU}, \epsilon_{C_iL}] \right\} \tag{5.23} \]

5. Defuzzifying the obtained fuzzy entropy weights by using Equation (5.2) in order to obtain the crisp values \((\tilde{C}W_i)\).

6. Normalising the obtained crisp value of each criterion by means of the distributive mode normalisation technique that is shown in Equation (5.24) in order to attain the weight of the \(i-th\) criterion \((ObW_i)\).

\[ ObW_i = \frac{\tilde{C}W_i}{\sum_{i=1}^{n} \tilde{C}W_i} \tag{5.24} \]

### 5.5 IC-FSE for weighting the sustainability criteria of renewable energy technologies in the mining industry

A numerical example of the use of IC-FSE is presented in this section. IC-FSE is used to determine the weights of sustainable criteria in the selection of renewable energy technologies in the mining industry. Three feasible renewable energy technologies were evaluated with respect to six criteria, comprising one technical, three environmental and two socio-economic criteria. For the purpose of this study, an implementation of IC-FSE in Python 3 was used.

The sources of quantitative data for the selected criteria are presented in Table 5.2.1 and the data are summarised in Table 5.5.1. Columns and rows in Table 5.5.1 result in a fuzzy decision matrix that is expressed in TFN.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Capacity factor (%)</th>
<th>Water consumption (l/MWh)</th>
<th>GHG emissions (g CO₂eq/kWh)</th>
<th>Area requirement ((m^2)/kW)</th>
<th>Levelised energy cost (£/MWh)</th>
<th>Prospective jobs (Jobs/annual GWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OW</td>
<td>(24, 28, 34)</td>
<td>(0.4, 4.4, 34)</td>
<td>(5, 15, 70)</td>
<td>(10, 200, 1200)</td>
<td>(25, 70, 125)</td>
<td>(0.1, 0.2, 0.6)</td>
</tr>
<tr>
<td>CSP</td>
<td>(17, 24, 25)</td>
<td>(303, 606, 644)</td>
<td>(15, 40, 150)</td>
<td>(10, 40, 100)</td>
<td>(50, 200, 450)</td>
<td>(0.2, 0.4, 0.7)</td>
</tr>
<tr>
<td>PV</td>
<td>(5, 11, 12)</td>
<td>(38, 307, 795)</td>
<td>(20, 60, 200)</td>
<td>(10, 150, 500)</td>
<td>(50, 340, 600)</td>
<td>(0.2, 0.6, 1.3)</td>
</tr>
</tbody>
</table>

#### 5.5.1 Fuzzy entropy weights

By using Equations (5.12) – (5.14), the normalised decision matrix in TFN was obtained, the results of which are shown in Table 5.5.2. The fuzzy entropy values \(\tilde{\epsilon}_i\) were obtained by applying Equations (5.15) – (5.19), while Equations (5.20) – (5.23) were used to derive the fuzzy entropy weights. The fuzzy entropy weights were then defuzzified by using Equation
(5.2). In order to obtain the crisp criteria weights, Equation (5.24) was then used to normalise the defuzzified entropy weights. The results of the fuzzy entropy values, the fuzzy entropy weights, and the crisp entropy weights are presented in Table 5.5.3. Based on the normalised crisp entropy weights ($ObW_i$), the rank of each criterion is water consumption $>$ area requirement $>$ GHG emissions $>$ levelised energy cost $>$ prospective jobs creation $>$ capacity factor.

Table 5.5.2 The normalised fuzzy decision matrix.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Capacity factor ($C_1$)</th>
<th>Water consumption ($C_2$)</th>
<th>GHG emissions ($C_3$)</th>
<th>Area requirement ($C_4$)</th>
<th>Levelised energy cost ($C_5$)</th>
<th>Prospective jobs ($C_6$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OW</td>
<td>(0.65, 0.73, 0.89)</td>
<td>(0.0004, 0.01, 0.1)</td>
<td>(0.02, 0.2, 0.9)</td>
<td>(0.02, 0.8, 1.0)</td>
<td>(0.03, 0.2, 0.9)</td>
<td>(0.1, 0.3, 0.9)</td>
</tr>
<tr>
<td>CSP</td>
<td>(0.4, 0.6, 0.7)</td>
<td>(0.4, 0.9, 1.0)</td>
<td>(0.1, 0.5, 1.0)</td>
<td>(0.01, 0.2, 1.0)</td>
<td>(0.1, 0.5, 1.0)</td>
<td>(0.1, 0.5, 1.0)</td>
</tr>
<tr>
<td>PV</td>
<td>(0.1, 0.3, 0.4)</td>
<td>(0.1, 0.5, 0.9)</td>
<td>(0.1, 0.8, 1.0)</td>
<td>(0.01, 0.6, 1.0)</td>
<td>(0.1, 0.8, 1.0)</td>
<td>(0.2, 0.8, 1.0)</td>
</tr>
</tbody>
</table>

Table 5.5.3 The fuzzy entropy values ($\tilde{e}_i$), fuzzy entropy weights ($\tilde{w}_i$), defuzzified entropy weights ($ObW_i$) and the ranking of criteria obtained from the IC-FSE.

<table>
<thead>
<tr>
<th>Capacity factor ($C_1$)</th>
<th>Water consumption ($C_2$)</th>
<th>GHG emissions ($C_3$)</th>
<th>Area requirement ($C_4$)</th>
<th>Levelised energy cost ($C_5$)</th>
<th>Prospective jobs ($C_6$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>($\tilde{e}_i$)</td>
<td>(0.67, 0.8, 0.83)</td>
<td>(0.16, 0.45, 0.55)</td>
<td>(0.26, 0.75, 0.79)</td>
<td>(0.07, 0.72, 0.87)</td>
<td>(0.31, 0.72, 0.83)</td>
</tr>
<tr>
<td>($\tilde{w}_i$)</td>
<td>(0.04, 0.11, 0.23)</td>
<td>(0.12, 0.31, 0.5)</td>
<td>(0.06, 0.14, 0.4)</td>
<td>(0.04, 0.16, 0.44)</td>
<td>(0.05, 0.16, 0.38)</td>
</tr>
<tr>
<td>($ObW_i$)</td>
<td>(0.1)</td>
<td>(0.26)</td>
<td>(0.17)</td>
<td>(0.18)</td>
<td>(0.16)</td>
</tr>
<tr>
<td>Rank</td>
<td>6</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

5.5.2 Stochastic analysis

In order to capture the uncertainty due to the imprecise input data, the Shannon Entropy method was applied by means of Monte Carlo simulations. Each of the criteria values in Table 5.5.1 was randomly sampled for a large number of iterations (i.e. 15,000) and the generated random number was used as input for the Shannon Entropy method. A probabilistic ranking of the weight of each criterion could, therefore, be acquired.

In order to generate a random number ($random_a$) in a TFN for all of the criteria values, Equation (5.25) was used.

$$\text{random}_a = \text{np.random.triangular} \left(\text{min}_a, \text{mode}_a, \text{max}_a\right),$$  (5.25)

where np.random.triangular denotes a standard function that generates a random number by triangular probability density functions with a mode$_a$ equal to the most likely value and min$_a$ and max$_a$ equal to the lower and upper bound values that are presented in Table 5.5.1.

A violin plot was used to visualise the probability density of the simulation results. Figure 5.5.1 shows the violin plot of the weight of each criterion and reveals that the water
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Consumption criterion \((C_2)\) has the highest high probability to be the most important criterion with some overlaps with other five criteria.

![Fig. 5.5.1 Violin plot of the weight of each criterion after 15,000 iterations.](image)

### 5.5.3 Discussion

Table 5.5.3 shows the results obtained by applying IC-FSE using the minimum, most likely and maximum values from Table 5.5.1. Based on the normalised crisp entropy weights, which are presented in Table 5.5.3, the weight of each criterion is is water consumption \(\succ\) area requirement \(\succ\) GHG emissions \(\succ\) levelised energy cost \(\succ\) prospective jobs creation \(\succ\) capacity factor.

It can be concluded from the results that the three most important criteria are related to environmental aspects of renewable energy technologies. The total overall weights of these criteria are 0.61. This finding indicates that when choosing renewable energy technologies, water consumption, area requirement, and GHG emissions must be carefully considered, taking into account the specific conditions of the region.

A very interesting finding in the results is that the capacity factor criterion has the lowest weight among all criteria, whereas generally in the literature, this criterion is frequently identified as the most important one. This is probably because the capacity factor’s values among the three alternatives are very similar, while the environmental aspects’ values vary widely. In addition, since the capacity factor affects levelised energy cost values, so the weight of the levelised energy cost criterion becomes low as well.

Figure 5.5.1 shows the violin plot of the weight of each criterion as calculated from 15,000 Monte Carlo simulations. The results of Monte Carlo simulations applied here explicitly present the uncertainty in the weight of criteria through the overlaps between each criterion. The overlaps indicate that over the range of possible values, any ranking of the five criteria is possible. In fact, all of the criteria have been found to be the most important criterion and the least sustainable criterion in a number of simulations. It is therefore worth noting that
the results can indicate the potential variability of the criteria weights in terms of the various data inputs and highlight the effect that the data inputs can have on the results. In addition, based on the results of Monte Carlo simulations, there is a relatively high tendency for water consumption ($C_2$) to be the most important criterion, whereas the least sustainable criterion is the capacity factor ($C_1$). Such a tendency is relatively similar to the results obtained from IC-FSE.

It is evident from the aforementioned outcomes that our proposed method, IC-FSE, can be used to assess, weight, compare and rank different criteria in a scientific transparent manner by means of objective methods. Applying IC-FSE can, therefore, be useful to inform the weight of each criterion in order to guide sustainable renewable energy strategies and their implementation. Nevertheless, the results from this work show a limitation in the use of IC-FSE for assessing and weighting the sustainability criteria in evaluating different renewable energy technologies due to the level of uncertainty involved in terms of input data. The input data used in this work were originally collected at a national scale and are therefore relatively generic. From a practical point of view, if a specific renewable energy project in the mining industry were to be considered, the degree of uncertainty in terms of input data is very likely to be smaller than that in the present example. IC-FSE can still be used, by substituting all values in Table 5.5.1 as required, and one would expect the set of feasible renewable energy technologies would most probably change. For example, if the mining company is located near a river and there is a high potential to build a hydroelectric power plant (HEPP), then a HEPP might be added into a set of feasible alternatives.

It should be noted that there will always be uncertainties involved in assessing the weights of sustainability criteria in the selection of renewable energy technologies. Uncertainties are mostly associated with the stages of the decision making process [73] and even in its mathematical functions [25]. Therefore, care should be taken when considering the outcomes of MCDM methods.

The uncertainty that is associated with the input data or information can be handled by means of FST, as done in this study, and minimised through prudent assessments of the available data sources in order to ensure that the inputs are representative and reliable. In addition, in order to obtain further information, Monte Carlo simulations can be used to address the associated uncertainties, as done in this work. Nonetheless, IC-FSE has been shown to provide less vague results. In addition, uncertainty often exists in mathematical functions of MCDM methods and thus overestimation of fuzzy results could occur. A method should be therefore capable of minimising the undesired overestimation of results. Undesired results can be avoided by reducing the risk of losing valuable information, which can be achieved by taking into account the interactions among the elements in the calculations. This work showcases the ability of IC-FSE in reducing uncertainty due to mathematical functions by comparing the results obtained from IC-FSE to those obtained from other existing methods (see: Appendix D).
Based on the aforementioned explanation, it can be concluded that the proposed IC-FSE method succeeded in weighting the sustainability criteria in a fuzzy environment based on quantitative data only. However, real-life MCDM problems often also involve qualitative data. It should also be pointed out that this study does not take into account the interaction and dependency between objects (i.e. criteria, sub-criteria, and alternatives). Such dependencies can be accounted for using another MCDM method, such as analytic network process. There is therefore scope to further extend IC-FSE for the case when qualitative and quantitative data, as well as the dependency between objects, are involved.

5.6 Conclusions

This paper demonstrates the value of applying the concept of constrained fuzzy arithmetic in a fuzzy extension of Shannon Entropy. A hybrid multiple criteria decision making method is developed, namely the Integrated Constrained Fuzzy Shannon Entropy method (IC-FSE). The developed method can be used to determine criteria weights when credible subjective weights are difficult to acquire and the input data that need to be analysed are difficult to define precisely, and thus need to be presented in fuzzy numbers.

In this study, IC-FSE was applied to weight the sustainability criteria of renewable energy technologies in the mining industry. Three feasible renewable energy technologies, namely onshore wind, concentrated solar power, and solar photovoltaic, were examined with respect to six sustainability criteria. The selected criteria were capacity factor, water consumption, GHG emissions, area requirement, levelised energy cost, and prospective jobs creation. The criteria weighting was assessed using data collected from the literature and applied to an illustrative example for the mining industry in the UK. To deal with the uncertainty in the input data, triangular fuzzy numbers were applied to define each of the criteria values. IC-FSE was then used to compute the criteria weights.

The results demonstrate that the environmental criteria associated to renewable energy technologies was the most important aspect to consider. In particular, water consumption was the highest ranked criterion, followed by area requirement and GHG emissions. The latter will continue to be a particularly relevant criterion for the mining industry due to the expected increase in energy demand, combined with the large scale of mining operations. In addition, the technical criterion, i.e. the capacity factor, had the lowest weight among all criteria and as a consequence, the levelised energy cost criterion was also low.

The uncertainty in the input data was also assessed using Monte Carlo simulations to determine probability distributions of criteria weights. The results from the Monte Carlo simulations were then compared to those from IC-FSE in order to obtain insight into the applicability of the latter in overcoming the uncertainty. The results in Sections 5.5.1 and 5.5.2 showed that the first and second rank as well as the last rank of criteria were similar. However, it is worth noting that although the uncertainty that is associated with the input
data or information can be addressed by means of IC-FSE or Monte Carlo simulations, IC-FSE yields more precise results.

Moreover, when there is uncertainty associated to the method, which can be described mathematically, the method should be capable of minimising the risk of losing important data during the computations and minimising the overestimation of the results. The superiority of IC-FSE over other existing normalisation methods to minimise uncertainty was showcased, demonstrating that IC-FSE’s normalisation procedure results in more precise fuzzy weights. In addition, the results acquired from IC-FSE were compared to those obtained from the only other existing method in the literature for criteria weighting that also uses triangular fuzzy numbers, showing that IC-FSE maintains the order of TFN properly. The results derived from IC-FSE show that an appropriate order of TFN in the fuzzy weights of objective criteria with less uncertainty is achieved by taking into account all the information about the uncertainty in the computation process.

In this study, the proposed IC-FSE method succeeded in weighting the sustainability criteria in a fuzzy environment based on quantitative data alone. There is scope to further extend IC-FSE for the case when both qualitative and quantitative data are available, as well as for when there exists a dependency between criteria and alternatives. Further studies to develop such an extended method will be the subject of future work.

This study demonstrates that IC-FSE is a robust method to weight criteria using quantitative and uncertain data. IC-FSE has a broad application potential in other fields to support decision makers in the selection problem when dealing with objective weights under uncertainty.
Chapter references


Chapter references


Chapter 6

The selection of renewable energy technologies using subjective and objective multiple criteria decision making methods

Abstract

The use of renewable energy technologies is a key factor for sustainable development but their selection from several alternatives is a difficult task that relies on the careful assessment of relevant criteria. While Multiple Criteria Decision Making (MCDM) methods have been used successfully in various renewable energy technology selection problems, the decision process becomes more challenging when preferential judgements are made on the basis of non-homogenous and imprecise input data, and when there is uncertainty due to disparities among decision makers. This paper presents a hybrid MCDM method capable of overcoming these problems by taking into account quantitative and qualitative data under a probabilistic environment in the context of group decision making. In this method, qualitative data is fuzzified and used along with quantitative data to develop a hybrid model. A coefficient factor allows decision makers to vary the weight of each quantitative model so that the resultant criteria weights and overall alternatives’ scores consider both subjective considerations and objective information. An example is presented to showcase the usability of the method developed for ranking and evaluating renewable energy technologies in the mining industry. In addition, the impact of different coefficient factors on the final results was assessed by means of sensitivity analysis. The results indicate that the method developed is able to minimise the loss of valuable objective information, caused by the subjective bias of qualitative weights during the evaluations, by adjusting the coefficient factors of the hybrid model during the calculations.
6.1 Introduction

Energy-generating technologies that depend on non-renewable fossil fuels result in significant environmental challenges, such as increasing greenhouse gas (GHG) emissions, which lead to climate change [1]. In response to these challenges, it is important to better exploit renewable energy technologies (e.g. wind and solar), which are low-cost, clean and sustainable [2].

The selection of renewable energy technologies is a complex and multidisciplinary problem that mainly refers to the performance of the technologies concerning multiple criteria such as environmental, social, technical and economic. In order to evaluate holistically and select the technologies that have a higher performance appropriately, decision makers need to have methodological tools that incorporate both quantitative and qualitative analyses of the multiple criteria. Decision makers should, therefore, make use of the best tools available to evaluate the performance criteria of renewable energy technologies. Choosing the best renewable energy technology to use among various alternatives considering conflicting criteria is thus considered a Multiple Criteria Decision Making (MCDM) problem.

Since the 1970s, a variety of MCDM methods have been developed and extensively applied in many fields and for a wide range of case studies [3]. An MCDM selection problem is often arranged as a decision matrix in which alternatives are evaluated with respect to conflicting criteria. Most of MCDM methods have algorithms for determining the criteria weights that represent the relative importance or significance of each criterion to others. Algorithms are also applied to determine the weights of alternatives, usually referred to as alternatives’ scores, which represent the relative preference of each alternative to others. The weighting methods in MCDM can be classified into two types: subjective methods, which are obtained from decision makers’ opinions (i.e. qualitative), and objective methods, which are acquired purely from calculations (i.e. quantitative).

Many studies have demonstrated the successful application of MCDM methods for the selection of renewable energy technologies, as evidenced by extensive literature reviews provided by Wang et al. [4] and Kumar et al. [5]. There is, however, a paucity of studies addressing the development and application of MCDM methods for problems in which preferential judgements are made based on non-homogeneous data (i.e. quantitative and qualitative) [6–8], uncertain input data (i.e. probabilistic) [9–11], and uncertainty caused by different decision makers’ opinions [12, 13].

Even though there is a need for adequate mathematical tools to support the decision making process under the aforementioned circumstances, there has been no study on the development of decision tools that can be used to overcome complex selection problems, such as the selection of renewable energy technology. Consequently, in view of this lack of existing methods, the main research questions that need to be addressed are as follows:
1. What are the most suitable subjective and objective methods that can be used for the selection of renewable energy technologies?

2. What are the main shortcomings of the most suitable subjective and objective methods identified?

3. What are the notions and the applicability of the combined subjective and objective weights into a single framework?

In line with the aforementioned research questions, the objective of this work is to propose a systematic MCDM method for evaluating renewable energy technologies from a sustainability perspective and identifying the most appropriate alternative considering non-homogenous data (i.e. quantitative and qualitative), uncertainties due to imprecise input data and disparities among decision makers. An integrated MCDM method is presented that takes into account quantitative and qualitative data under uncertainty in the context of group decision making. To this end, the proposed method combines a subjective method (i.e. Integrated Constrained Fuzzy Stochastic Analytic Hierarchy Process (IC-FSAHP)) [14, 13] and objective methods (i.e. Integrated Constrained Fuzzy Shannon Entropy (IC-FSE) [15], Normalised Vector (NV) [16] and Weighted Sum Model (WSM)) [17]. In order to showcase its capabilities, the method developed is applied to an example for the ranking and evaluation of renewable energy technologies in the mining industry.

The contribution of this paper is fourfold: (1) the gap in MCDM methods for problems involving quantitative and qualitative data under uncertainties due to imprecise data and different opinions among decision makers is addressed; (2) a new method that combines IC-FSAHP, IC-FSE, NV, and WSM is presented; (3) the usability of the method developed is showcased and the outcomes obtained from the method are analysed; (4) a methodology for the carrying out of sensitivity analysis by varying the coefficient factors of subjective and objective weights, including the assessment of its results, is presented. It is demonstrated that the proposed method is a robust MCDM method that can be applied broadly in the renewable energy sector to support the process of decision making when there is uncertainty in the non-homogenous input data.

6.2 Literature review

6.2.1 MCDM methods

An MCDM method involves four important steps [18], namely: (i) determining the local criteria weight; (ii) calculating the local alternatives’ score; (iii) measuring the overall weighted alternatives’ scores; (iv) selecting the best alternative which has the greatest overall weighted score. The final results obtained from any MCDM method mainly depend on
the criteria, the criteria weights, the local and overall alternatives’ scores, and the specific algorithm applied for calculating the criteria weights and the alternatives’ scores [3].

There are two main groups of MCDM methods for deriving the criteria weights and the alternatives’ scores, namely subjective and objective methods. In the subjective methods, the criteria weights of and the alternatives’ scores are acquired from decision makers’ judgements and preferences via pairwise comparisons. One of the most widely applied subjective methods is the Analytic Hierarchy Process (AHP), developed by Saaty [19]. Despite its popularity, the application of AHP has been frequently criticised when uncertainty caused by the lack of information and uncertainty caused by various decision makers’ opinions are present.

The extension of AHP by coupling it with the fuzzy set theory is one of the most popular techniques to overcome the uncertainty problem caused by the lack of information. Moreover, stochastic simulation can be coupled with fuzzy AHP methods in order to capture uncertainty caused by opinions from multiple decision makers. Sitorus et al. [13] showed that IC-FSAHP is capable of minimising uncertainty and not only yielded more precise results than AHP and its variants, but also enhanced the reliability of decisions taken under uncertainty by means of multiple criteria group decision making.

Unlike the subjective methods, the criteria weights and the alternatives’ scores in objective methods are obtained from the computation of quantitative data, using mathematical algorithms or models to derive the weights and scores without considering the decision makers’ judgements and preferences. One of the most widely used objective methods is the Shannon Entropy method (SE) [20]. Regardless of its popularity, the application of SE has been often criticised when uncertainty caused by the imprecise input data is present. The extension of SE by combining it with the fuzzy set theory is one of the most popular techniques to overcome the uncertainty problem. The IC-FSE method, which has been developed by Sitorus and Brito-Parada [15], is able to reduce uncertainty and resulted in more accurate and precise results than existing methods.

It is worth noting that both weighting methods (i.e. for subjective and objective weights) have limitations. In order to comprehensively take into account the decision makers’ opinions and reduce subjectivity, the opinions of decision makers and the objective information should be comprehensively considered to determine the criteria weights and alternatives’ scores by means of combining both subjective and objective weights into a single framework. The notions and the applicability of the combined subjective and objective weights into a single framework are interesting and important aspects to study. One of the aims of this work is to present the notions and the applicability of such a combined method.

6.2.2 Renewable energy technology selection

The use of renewable energy technologies (e.g. wind power and solar power) has gained enormous interest due to an increasing environmental awareness and the need to avoid the
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negative impacts of non-renewable fossil fuels on the environment (i.e. producing GHG emissions which lead to global warming and climate change) [21]. Expanding the application of renewable energy technologies in many sectors, particularly in large energy consuming industries, is of vital importance. Renewable energy technologies are environmentally friendly and able to compete with fossil fuels in a wide variety of applications at reasonable prices [22, 23].

The use of renewable energy sources is a key factor for sustainable development [24]. However, there are several aspects linked to the implementation of renewable energy technologies that need to be considered. Among those factors are their high initial cost [25], potential capacity limitations (e.g. inconsistent energy source input) [15], infrastructure management (e.g. land area required) [26], and social impacts management (e.g. the acceptance and understanding by the public of some renewable energy technologies) [26].

Moreover, it is vital to understand and assess the trade-offs between the aforementioned aspects associated with different renewable energy technologies. The selection of a renewable energy technology for a given application often requires a careful management of conflicting technical, environmental, and socio-economic criteria [15]. For example, the use of renewable energy technologies reduces GHG emissions but can be costly and may have impacts on land use or habitats. There is thus a need for adequate tools that can deal with these conflicts and trade-offs when evaluating and selecting the most suitable renewable energy technologies at a given location.

In many cases, the selection of the most suitable renewable energy technology involves several challenges, such as non-homogenous input data and uncertainties due to either imprecise input data or to divergent opinions from decision makers. These challenges can make the selection process significantly more difficult. The benefit of MCDM, compared to single criterion decision analysis, is that the methods take into account multiple conflicting criteria to attain an integrated decision result [5].

MCDM methods have been successfully applied in a number of different aspects in renewable energy, such as selection of renewable energy sources employing Fuzzy TOPSIS [22], renewable energy technologies evaluation using Fuzzy VIKOR [23], optimal sites selection for photovoltaic solar farms using two different MCDM methods, namely TOPSIS and ELECTRE TRI [24], sustainable energy planning strategies evaluation by means of an integrated AHP and Fuzzy TOPSIS method [25], and the evaluation of a renewable energy project performance using an extended TODIM [27]. In all the aforementioned cases, MCDM supported decision makers in determining the importance of criteria and the preference of alternatives, and in making a proper selection based on the rank order of the alternatives.

As previously mentioned in Section 6.2.1, MCDM methods can be classified into subjective and objective methods, depending on the type of weighting considered. Table 6.2.1 shows successful examples of the most frequently used subjective (i.e. AHP based) and objective (i.e. SE based) methods for the selection of renewable energy technology.
Table 6.2.1 Examples of the successful application of subjective and objective MCDM methods to various renewable energy technology selection problems.

<table>
<thead>
<tr>
<th>Subjective methods</th>
<th>Objective methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Evaluating and selecting the most suitable photovoltaic technology by applying AHP [28].</td>
<td>Evaluating the sustainability of concentrated solar power technologies using SE [29].</td>
</tr>
<tr>
<td>2. Selecting various renewable energy technologies using Fuzzy AHP (FAHP) [30].</td>
<td>Weighting the sustainability criteria of wind and solar power technologies using Fuzzy SE [15].</td>
</tr>
<tr>
<td>3. Selecting the best site location for an integrated solar–wind power plant by applying AHP [31].</td>
<td></td>
</tr>
<tr>
<td>4. Analysing the sustainability of various type of energy storage by applying FAHP [32].</td>
<td></td>
</tr>
</tbody>
</table>

Van de Kaa et al. [28] and Yunna and Geng [31] successfully applied AHP, the most frequently used subjective method, to determine the optimal photovoltaic system among five alternatives and the most suitable location for a hybrid solar-wind power station, respectively. It should be noted that Van de Kaa et al. [28] and Yunna and Geng [31] assumed that the input data, all criteria weights and alternatives’ scores were expressed as crisp values. However, it is often the case that selection problems are associated with uncertainties due to imprecise input data, and thus all criteria weights and alternatives’ scores are expressed in fuzzy numbers. In this regard, Tasri and Susilawati [30] and Ren [32] showed the successful development and application of combined Fuzzy AHP methods to select the most suitable renewable energy technology to be applied in Indonesia and to select the optimal type of energy storage to be used, respectively. It should be noted that while Tasri and Susilawati [30] and Ren [32] made use of experts’ opinions to determine the importance of criteria and preference of alternatives, they did not take into account uncertainties associated to the various opinions from the experts. Sitorus et al. [13] developed a method that makes use of stochastic simulations to capture the various opinions from the experts in assessing selection problems that are associated with the imprecise input data.

Simsek et al. [29] successfully implemented SE, the most frequently used objective method, to evaluate the sustainability of concentrated solar power technologies. It is worth mentioning that Simsek et al. [29] assumed that all input data were expressed as crisp values. As previously mentioned, the input data to be analysed are often imprecise and thus the use of crisp SE is not sufficient. Sitorus and Brito-Parada [15] developed a method that combines ordered fuzzy numbers and the SE method (i.e. IC-Fuzzy SE) to weight the sustainability criteria of wind and solar power technologies.

It is worth highlighting that the successful examples discussed above did not consider the case when non-homogeneous data, uncertainties due to imprecise input data and disparities among decision makers are involved. There is scope to develop a method capable of dealing with such cases.
6.2.3 Renewable energy in the mining sector

Mining operations are very energy intensive, with energy costs typically accounting for 30–50% of all operating costs [33]. Mining operations are often located in remote areas where the mineral deposits are discovered. Due to the remoteness of mine sites, accessibility to energy sources is usually limited, which results in fossil fuels being the only readily available option to power equipment [34]; in fact, this contributes to the mining industry heavy dependence on non-renewable energy sources [33].

Mining operations are noticeably responsible for producing greenhouse gas (GHG) emissions not only from the use of fossil fuels for operating equipment but also for power generation. Moreover, as the global demand for minerals continues to increase and the process to extract and separate them require greater amounts of energy (due to the need of mining lower grade and finely disseminated ores), greater emissions are produced [35]. In order to address the aforementioned concerns, many mining companies have started to give greater consideration to the use of renewable energy technologies in their operations [36–38].

MCDM methods have been successfully used in a number of different aspects in the mining industry, including the assessment of mine closure risk using an integrated AHP, PROMETHEE, and TOPSIS method [39], sustainable water management in a mining complex by means of AHP [40], corporate social responsibility strategies evaluation in the mining industry employing fuzzy DEMATEL [41]. While previous studies in the literature have emphasised the importance of applying MCDM methods in evaluating and selecting renewable energy technologies, no study has yet done so for the mining industry.

There are, however, other tools that have been used in the literature to evaluate the performance of renewable energy technologies and select the best alternative in the mining industry. Mostert [42] adopted the triple bottom line (i.e. financial, social, and environmental) accounting method to evaluate the sustainability of a project in order to select the best renewable energy technology in the mining industry. The financial, social, and environmental values were engineered in order to determine a monetary value for a renewable energy project. However, Mostert [42] recommends that a monetary value alone is not sufficient to base a decision on, and a combination of qualitative measures to be used in conjunction with the triple bottom line are advocated. A different decision-making approach to implement renewable energy technologies in the mining industry, namely the use of cost analysis and SWOT analysis, was applied by Zharan and Bongaerts [43]. Both Mostert [42] and Zharan and Bongaerts [43] considered mainly the financial value on a decision. It is worth noting, however, that there were no multiple conflicting criteria involved in their evaluation. Because of the complexity of decision analysis, primarily in terms of problem analysis and structuring, the aforementioned tools (i.e. those in Mostert [42] and Zharan and Bongaerts [43]), tend not to be sufficient to support decision makers in the evaluation of more complex selection problems. An appropriate tool, such as MCDM methods, would therefore be required to
better support decision makers in selecting renewable energy technologies in the mining industry.

6.3 Research framework

In line with the challenges discussed in Sections 6.1 and 6.2, the research framework for this study, as shown in Figure 6.3.1, is as follows: first, the development of novel hybrid MCDM method including the workflows and equations of the novel method is presented; second, the applicability of the novel hybrid method in an illustrative example is showcased; finally, the conclusions of the current work are provided.
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Fig. 6.3.1 Flowchart of the research in this work.
6.4 Methodology

The following sub-sections discuss the key theoretical aspects behind Triangular Fuzzy Number (TFN), Integrated Constrained Fuzzy Stochastic AHP (IC-FSAHP), Integrated Constrained Fuzzy Shannon Entropy (IC-FSE), Normalised Vector (NV), and Weighted Sum Model (WSM)) and the proposed hybrid method. TFN is used to represent the uncertain data as ordered real numbers, consisting of minimum, medium and maximum numbers, IC-FSAHP is used to calculate the criteria weights and the alternatives’ scores in a subjective manner, while an objective procedure makes use of IC-FSE to determine the criteria weights, with the NV and WSM methods used to calculate the local and overall alternatives’ scores, respectively.

6.4.1 TFN

The order of membership function of TFN \( \tilde{A}(x) \) is expressed in the following form:

\[
f(x) = \begin{cases} 
\frac{x-c_L^A}{c_M^A-c_L^A}, & \text{if } c_L^A < x < c_M^A; \\
1, & \text{if } x = c_M^A; \\
\frac{c_U^A - x}{c_U^A - c_M^A}, & \text{if } c_M^A < x < c_U^A; \\
0, & \text{otherwise},
\end{cases}
\]  

(6.1)

where \( c_L^A \) and \( c_U^A \) are the lowest and highest values of TFN \( \tilde{A}(x) \), while \( c_M^A \) is the middle value of TFN \( \tilde{A}(x) \).

Figure 6.4.1 shows an example of a TFN which has a membership function of 2, 3, and 4.

![Fig. 6.4.1 Membership function of a TFN (2, 3, 4).](image)

In this work, a crisp number value of the TFN is obtained using the centre-of-area (COA) defuzzification approach, proposed by Tzeng and Huang [44], and is expressed in the following form:

\[
COA \ \tilde{A}(x) = \frac{(c_U^A - c_L^A) + (c_M^A - c_L^A)}{3} + c_M^A.
\]  

(6.2)
6.4.2 Subjective weighting method

In the current work, IC-FSAHP was used as a subjective weighting method for obtaining the criteria weights and the overall alternatives’ scores. Sitorus et al. [13] showed that IC-FSAHP was able to reduce uncertainties caused by imprecise input data and various judgements among decision makers.

The following steps to apply the IC-FSAHP method are: (i) the notions of a decision problem are defined; (ii) the local fuzzy criteria weights and the local fuzzy alternatives’ scores are calculated; (iii) the overall alternatives’ scores are calculated; (iv) the results are synthesised; (v) the alternatives are ranked. Figure 6.4.2 shows the workflow of the IC-FSAHP method. In the reader is referred to Sitorus et al. [13] for a full description of the steps.
Fig. 6.4.2 The workflow of the IC-FSAHP method [13].
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For the simplicity of data collection and analysis, a scale of seven linguistic variables used in Sitorus and Brito-Parada [14] was applied in this work instead of did a survey at the beginning. Figure 6.4.3 presents the membership functions for the TFN scale levels.

![Membership functions of the TFN scale levels](image)

Fig. 6.4.3 Membership functions of the TFN scale levels used in evaluating the criteria and alternatives (note: EU: Extremely Unimportant, VU: Very Unimportant, U: Unimportant, F: Fair, I: Important, VI: Very Important, EI: Extremely Important, VL: Very Low, L: Low, ML: Medium Low, M: Medium, MH: Medium High, H: High, and VH: Very High).

It is worth noting that the weights of criteria and the overall scores of alternatives obtained from IC-FSAHP are expressed as $W_i^S$ and $OSc_k^S$, respectively. $W_i^S$ is the defuzzified value of $W_i^{S,L}, W_i^{S,M}$ and $W_i^{S,U}$, which are obtained from the following equations:

$$W_i^{S,L} = \min \left\{ \frac{\sqrt[n]{\prod_{j=1}^{n} a_{ij}^L}}{\sum_{i=1}^{n} \sqrt[n]{\prod_{j=1}^{n} a_{ij}^L}} ; \quad a_{ij} \in [a_{ijL}, a_{ijU}], \quad \forall j > i, \quad a_{ji} = \frac{1}{a_{ij}}, \quad \forall j < i, \quad a_{jj} = 1, \quad \forall j \right\} , \quad (6.3)$$

$$W_i^{S,M} = \frac{\sqrt[n]{\prod_{j=1}^{n} a_{ij}^M}}{\sum_{i=1}^{n} \sqrt[n]{\prod_{j=1}^{n} a_{ij}^M}} , \quad (6.4)$$

$$W_i^{S,U} = \max \left\{ \frac{\sqrt[n]{\prod_{j=1}^{n} a_{ij}^U}}{\sum_{i=1}^{n} \sqrt[n]{\prod_{j=1}^{n} a_{ij}^U}} ; \quad a_{ij} \in [a_{ijL}, a_{ijU}], \quad \forall j > i, \quad a_{ji} = \frac{1}{a_{ij}}, \quad \forall j < i, \quad a_{jj} = 1, \quad \forall j \right\} , \quad (6.5)$$

where the superscript $S$ represents the subjective weighting method, the subscripts $L$, $M$ and $U$ describe the lowest, middle, and highest numbers in TFN and $a_{ij}$ represents the extent to which a criterion $i$ is more important than another criterion $j$ ($i = j = 1, 2, \ldots, n$) with respect to the goal.

Furthermore, $OSc_k^S$ is the defuzzified value of $OSc_{k,L}^S, OSc_{k,M}^S$ and $OSc_{k,U}^S$, which are determined from the following formulas:

$$OSc_{k,L}^S = \min \left\{ \frac{\sum_{i=1}^{n} \sqrt[n]{\prod_{j=1}^{n} a_{ij}^L} \cdot LSc_{k,L}}{\sum_{i=1}^{n} \sqrt[n]{\prod_{j=1}^{n} a_{ij}}} ; \quad a_{ij} \in [a_{ijL}, a_{ijU}], \quad \forall j > i, \quad a_{ji} = \frac{1}{a_{ij}}, \quad \forall j < i, \quad a_{jj} = 1, \quad \forall j \right\} , \quad (6.6)$$
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\[\text{OSc}_k^S = \frac{\sum_{i=1}^{n} \sqrt[n]{\prod_{j=1}^{n} a_{ij}^M} \text{LSc}_k^S}{\sum_{i=1}^{n} \sqrt[n]{\prod_{j=1}^{n} a_{ij}^M}}, \quad (6.7)\]

\[\text{OSc}_{k,U}^S = \max \left\{ \frac{\sum_{i=1}^{n} \sqrt[n]{\prod_{j=1}^{n} a_{ij}^L} \text{LSc}_{k,U}^S}{\sum_{i=1}^{n} \sqrt[n]{\prod_{j=1}^{n} a_{ij}^L}} ; \quad a_{ij} \in [a_{ijL}, a_{ijU}] , \forall j > i, \quad a_{ji} = \frac{1}{a_{ij}} , \forall j < i, \quad a_{jj} = 1, \forall j \right\}, \quad (6.8)\]

where \(\text{LSc}\) describes the local scores of alternatives which are acquired by using equations (6.3)–(6.5) (i.e. \(a_{ij}\) represents the extent to which each alternative \(i\) is more important than an alternative \(j\) \((i = j = 1, 2, \ldots, m)\) with respect to each criterion \((1, 2, \ldots, n)\), and \(k\) represents the \(k\)-th alternative \((k = 1, 2, \ldots, m)\).

### 6.4.3 Objective weighting method

In the objective weighting method, three approaches were applied for determining the criteria weights, the local and overall alternatives’ scores. IC-FSE was used for calculating the criteria weights while NV and WSM were used for determining the local and overall alternatives’ scores, respectively.

#### 6.4.3.1 Objective criteria weighting method

IC-FSE was used to determine the criteria weights when subjective weights are difficult to be acquired and the input data that need to be evaluated are difficult to be defined precisely (thus need to be presented in TFN). Sitorus and Brito-Parada [15] showcased that IC-FSE was able to produce precise fuzzy weights with less uncertainty and could maintain the order of TFN properly.

IC-FSE involves six major steps: (1) defining the problem notions (e.g. determining alternatives and criteria) and developing a fuzzy decision matrix, (2) normalising the fuzzy decision matrix, (3) determining the fuzzy entropy values, (4) computing the local fuzzy criteria weights, (5) defuzzifying the results obtained in step 4, and (6) normalising the crisp values acquired in step 5 in order to obtain the final weights of criteria. Figure 6.4.4 presents the framework of the IC-FSE method, a detailed explanation of which can be found in Sitorus and Brito-Parada [15].
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Fig. 6.4.4 The framework of the IC-FSE method [15].

It is worth mentioning that the weights of criteria obtained from IC-FSE are expressed as $W_{ei}^O$. $W_{ei}^O$ is the defuzzified value of $W_{emiL}^O$, $W_{emiM}^O$ and $W_{emiU}^O$, which are obtained from the following equations:

$$W_{emiL}^O = \min \left\{ \left( \frac{1 - e_i}{\sum_{i=1}^n e_i} \right); \ e_i e \left[ e_{iL}, e_{iU} \right] \right\}, \quad (6.9)$$

$$W_{emiM}^O = \frac{1 - e_i^M}{\sum_{i=1}^n e_i^M}, \quad (6.10)$$

$$W_{emiU}^O = \max \left\{ \left( \frac{1 - e_i}{\sum_{i=1}^n e_i} \right); \ e_i e \left[ e_{iL}, e_{iU} \right] \right\}, \quad (6.11)$$

where the superscript $O$ represents the objective weighting method and $e_i$ is the fuzzy entropy value of the $i-th$ criterion ($i = 1, 2, \ldots, n$).

6.4.3.2 Objective local priorities of alternatives scoring method

Each alternative is evaluated with regard to its data corresponding to every criterion. The local scores of alternatives ($LSc_{k}^O$) are obtained from the NV method.

a. For the beneficial criteria that should be maximised, such as potential total power generation ($C_1$) and prospective jobs creation ($C_5$), the direct NV method formulated in the following equation is applied:
6.4 Methodology

\[ LSc^O_k = \left( \frac{x_{ki}}{\sum_{k=1}^{m} (x_{ki})} \right). \]  
\[ \text{(6.12)} \]

b. For the non-beneficial criteria that should be minimised, such as GHG emissions \((C_2)\), area requirement \((C_3)\) and LEC \((C_4)\), the reciprocal NV method formulated in the equation below is applied:

\[ LSc^O_k = \left( \frac{1/x_{ki}}{\sum_{k=1}^{m} (1/x_{ki})} \right). \]  
\[ \text{(6.13)} \]

The superscript \(O\) in \(LSc^O_k\) represents the objective weighting method and \(x_{ki}\) is the defuzzified rating of the \(k-th\) alternative with respect to the \(i-th\) criterion.

### 6.4.3.3 Objective overall priorities of alternatives scoring method

Based on the aforementioned description, each criterion has an objective weight obtained from IC-FSE \((We^O_i)\) and each alternative has a local score obtained from NV \((LSc^O_k)\). The overall alternatives’ scores \((OSc^O_k)\) are obtained by aggregating the local alternatives’ score with the criteria weights by means of the WSM formulated in the following form:

\[ \overset{\text{O}}{\text{OSc}}_k = \sum_{i=1}^{n} We^O_i LSc^O_k. \]  
\[ \text{(6.14)} \]

Furthermore, the normalised overall score of the \(k-th\) alternative \((OSc^O_k)\) is obtained using the distributive mode approach expressed in the equation below.

\[ OSc^O_k = \left( \frac{\overset{\text{O}}{\text{OSc}}_k}{\sum_{k=1}^{m} \overset{\text{O}}{\text{OSc}}_k} \right). \]  
\[ \text{(6.15)} \]

### 6.4.4 Proposed combined method

In line with the descriptions in Sections 6.4.1, 6.4.2 and 6.4.3, in the case when decision makers need to use both objective and subjective weighting methods, the following combined methodology is proposed.

a. For the combined weights of criteria \((We^C_i)\), the following equation is applied:

\[ We^C_i = \left( \alpha We^S_i \right) + \left( \beta We^O_i \right); \quad \alpha + \beta = 1. \]  
\[ \text{(6.16)} \]

b. For the combined overall scores of alternatives \((OSc^C_k)\), the equation below is applied:

\[ OSc^C_k = \left( \alpha OSc^S_k \right) + \left( \beta OSc^O_k \right); \quad \alpha + \beta = 1. \]  
\[ \text{(6.17)} \]
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The superscript $C$ in $We_i^C$ and $OSc_k^C$ represents the combined subjective and objective weights while $\alpha$ and $\beta$ are the coefficient factors given to the subjective and objective weights, respectively. The coefficient factors $\alpha$ and $\beta$ thus enable decision makers to determine how much importance they intend to assign to the subjective and objective weights. In this paper, $\alpha = \beta = 0.5$ was used for the base case calculations. In order to show the impact of the changes of coefficient factor $\alpha$ on the final results, six values of $\alpha$ were considered for a sensitivity analysis, namely 0, 0.2, 0.4, 0.6, 0.8, 1.

The detailed flowcharts of the proposed novel hybrid MCDM method for weighting the criteria and scoring the alternatives are shown in Figures 6.4.5 and 6.4.6, respectively.

Fig. 6.4.5 Flowchart of proposed novel hybrid MCDM method for weighting the criteria.
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There is a need for adequate MCDM methods to support decision makers in selecting renewable energy technologies in the mining industry when preferential judgements are made.
based on non-homogenous data (i.e. quantitative and qualitative), uncertain input data (i.e. probabilistic), and uncertainty because of different decision makers’ opinions. In this section, the applicability of the novel MCDM method is showcased.

### 6.5.1 The notions of the selection problem

Suppose that a mining company would like to select the most suitable renewable energy technology for one of its operations. For this purpose, five criteria were considered: potential total power generation \( C_1 \), GHG emissions \( C_2 \), area requirement \( C_3 \), levelised energy cost (LEC) \( C_4 \), and prospective jobs creation \( C_5 \). Furthermore, three feasible alternatives were examined, namely Onshore wind (OW) — \( A_1 \), Concentrated solar power (CSP) — \( A_2 \), Solar photovoltaic (PV) — \( A_3 \). Sections 6.5.1.1 and 6.5.1.2, provide the detailed description of different criteria and alternatives considered in this work.

In this work, the decision making process was conducted through an objective assessment first, followed by subjective judgements. For the purpose of this work, an implementation of the method in Python 3 was used.

#### 6.5.1.1 Sustainability criteria

Five sustainability criteria \( C_i \) were selected and are summarised in Table 6.5.1 and further described below. It is worth to emphasise that quantitative data for the criteria selected were obtained from the literature and, for consistency, correspond to the same geographical region, i.e. the UK.

Table 6.5.1 The evaluation criteria and sources of quantitative data used for the current work.

<table>
<thead>
<tr>
<th>Main Categories</th>
<th>Criteria</th>
<th>Units</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Technical</td>
<td>( C_1 ): Potential total power generation</td>
<td>TWh/yr</td>
<td>[10]</td>
</tr>
<tr>
<td>Environmental</td>
<td>( C_2 ): GHG emissions</td>
<td>gCO2eq/kWh</td>
<td>[10]</td>
</tr>
<tr>
<td></td>
<td>( C_3 ): Area requirement</td>
<td>( m^2/kW )</td>
<td></td>
</tr>
<tr>
<td>Economic</td>
<td>( C_4 ): Levelised Energy Cost</td>
<td>$/MWh</td>
<td>[10]</td>
</tr>
<tr>
<td>Social</td>
<td>( C_5 ): Prospective jobs creation</td>
<td>Jobs/annual GWh</td>
<td>[10]</td>
</tr>
</tbody>
</table>

1. Technical:

Potential total power generation \( (C_1) \) was considered as an important technical criterion. Potential total power generation \( (\text{TWh/yr}) \) \( (C_1) \) is the quantity of energy that can be delivered by each of the renewable energy technologies per year [10]. The great value of the potential total power generation is always preferred.

2. Environmental:
Two environmental criteria are used to account for the effect of renewable energy technologies on environmental sustainability in the mining industry. Two environmental criteria were considered in this work, namely GHG emissions and area requirement.

2.1. GHG emissions ($C_2$):

The GHG emissions criterion is one of the most frequently used criteria when assessing renewable energy technologies [4]. GHG emissions, which are measured in gCO$_2$eq/kWh, are estimated by CO$_2$ and CH$_4$ emissions of each renewable energy technology, from the commissioning of a power plant to its full operation and the dismantling stage of the power plant [45]. The target should be eliminating GHG emissions or reducing them as much as possible.

2.2. Area requirement ($C_3$):

The extension of land required by each renewable energy technology, which is reported as $m^2$/kW, is of vital importance for their evaluation in the mining industry because of concerns that the implementation of renewable energy technologies can frequently be competing with agriculturally arable land [46] and thus destabilise the ecosystem [47]. The decision making process would therefore always favour alternatives that require the smallest area.

3. Economic:

The economic considerations are of utmost importance for evaluating the sustainability of renewable energy technologies in various MCDM studies. In this work, levelised energy cost (LEC) ($C_4$), which is expressed as $$/MWh, was considered as an economic criterion because all the costs over an assumed project’s financial life and duty cycle are included in the LEC calculation [48]. The aforementioned costs include capital expenditure (CAPEX), operation and maintenance (O&M) expenditure (OPEX), fuel costs, financing costs, as well as an assumed capacity factor for each plant type. In addition, LEC takes into account the attributes of the technology, such as energy source, annual energy production, efficiency, and duration [10]. Reducing LEC is always advantageous.

4. Social:

A range of social aspects have been of enormous significance for people’s acceptance of the implementation of renewable energy technologies. Prospective jobs creation ($C_5$), which is reported as jobs/annual GWh, is the most commonly used social criterion in the literature [4]; it allows decision makers to consider socioeconomic aspects when determining which technology can enhance the living standards of the surrounding population [47]. This criterion considers the prospective jobs generated during the life cycle.
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of a renewable energy technology, from construction and operation to decommissioning.
A large number of jobs created is, of course, desirable.

6.5.1.2 Feasible renewable energy technologies

Three renewable energy technologies that have been successfully applied in the mining
industry [49, 33] were considered as alternatives \( A_i \) for the assessment in this work and are
summarised as follows:

1. Onshore wind (OW) — \( A_1 \)

Wind energy is harvested from the movement of air masses to drive wind turbines
that provide mechanical power, which is converted into electricity [50]. Several mining
companies have applied OW power systems at operating mines in Argentina, Canada,
and Chile. This has also been implemented at abandoned mines in the USA to provide
electricity to households surrounding the site. The generated power in the operating
mines varies from 2 MW to 115 MW and in the abandoned mines from 29 MW to 237
MW [49].

2. Concentrated solar power (CSP) — \( A_2 \)

Concentrated solar power utilises reflective surfaces to concentrate sunlight into a beam
to heat a working fluid in a receiver and produce the steam that is employed to drive a
turbine that provides mechanical power, which is then converted to electricity [48]. The
installed capacity of concentrated solar power in the mining industry in 2016 was 39
MW [33]. Even though the existing installed capacity is relatively low, several mining
companies in Chile have investigated a future potential concentrated solar power plant
installation with high capacity (up to 50 MW) to support their operations [51].

3. Solar photovoltaic (PV) — \( A_3 \)

Solar photovoltaic energy is another renewable source of electricity generation harvested
from the thermal radiation produced by sunlight through photovoltaic cells, which is
converted into electric current [52]. Several mining companies have implemented solar
photovoltaic power systems at operating mines in the USA, Chile, Australia, South
Africa, and Suriname. Solar photovoltaic technology has also been implemented at
abandoned mines in the USA, Germany, Canada, and Korea, where it has been used
for acid mine drainage treatment and to provide power to households near the site.
The power generated in the operating mines varies from 1MW to 10.6 MW and in the
abandoned mines from 1 MW to 166 MW [49].
6.5 Application of the method developed to the selection of renewable energy technologies in the mining industry

6.5.2 Input data

The sources of quantitative data for the five criteria are presented in Table 6.5.1 and the data are summarised in Table 6.5.2. These data were used as a basis for obtaining the criteria weights and the alternatives’ scores using the objective weight. Columns and rows in Table 6.5.2 result in a fuzzy decision matrix that is expressed in TFN.

Table 6.5.2 The minimum, most likely and maximum values for each of the considered renewable energy technologies with respect to each criterion.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Total power generation (TWh/yr)</th>
<th>GHG emissions (g CO\textsubscript{2}eq/kWh)</th>
<th>Area requirement (m\textsuperscript{2}/kW)</th>
<th>Levelised energy cost ($/MWh)</th>
<th>Prospective jobs (Jobs/annual GWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OW</td>
<td>(25, 45, 125)</td>
<td>(5, 15, 70)</td>
<td>(10, 200, 1200)</td>
<td>(32, 90, 160)</td>
<td>(0.1, 0.2, 0.6)</td>
</tr>
<tr>
<td>CSP</td>
<td>(2.5, 5, 20)</td>
<td>(15, 40, 150)</td>
<td>(10, 40, 100)</td>
<td>(64, 256, 576)</td>
<td>(0.2, 0.4, 0.7)</td>
</tr>
<tr>
<td>PV</td>
<td>(2.5, 20, 70)</td>
<td>(20, 60, 200)</td>
<td>(10, 150, 500)</td>
<td>(64, 435, 768)</td>
<td>(0.2, 0.6, 1.3)</td>
</tr>
</tbody>
</table>

In order to obtain the criteria weights and the alternatives’ scores using the subjective weight, the hierarchy of this renewable energy technologies selection problem was constructed and is presented in Figure 6.4.4. In addition, four experts (two from academia and two from the mining industry), denoted by $E_1$, $E_2$, $E_3$ and $E_4$, were asked for their judgements and preferences through a survey conducted via online questionnaires in November 2019. The pairwise comparison of the criteria and the alternatives that were examined by the four experts are shown in Table 6.5.3 and Table 6.5.4, respectively. Furthermore, the scale of linguistic variables, shown in Figure 6.4.2, was applied to compare pairwisely the significance of the criteria and preference of the alternatives.

![Fig. 6.5.1 Hierarchy structure for choosing the most suitable renewable energy technology in the mining industry.](image-url)
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Table 6.5.3 The experts’ assessment of the significance of the criteria with respect to the goal.

<table>
<thead>
<tr>
<th>Experts</th>
<th>Total power generation (TWh/yr)</th>
<th>GHG emissions (g CO₂eq/kWh)</th>
<th>Area requirement (m²/kW)</th>
<th>Levelised energy cost ($/MWh)</th>
<th>Prospective jobs (Jobs/annual GWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E₁</td>
<td>VI</td>
<td>El</td>
<td>VI</td>
<td>VI</td>
<td>I</td>
</tr>
<tr>
<td>E₂</td>
<td>El</td>
<td>El</td>
<td>El</td>
<td>El</td>
<td>El</td>
</tr>
<tr>
<td>E₃</td>
<td>I</td>
<td>I</td>
<td>U</td>
<td>El</td>
<td>I</td>
</tr>
<tr>
<td>E₄</td>
<td>VI</td>
<td>VI</td>
<td>U</td>
<td>El</td>
<td>VI</td>
</tr>
</tbody>
</table>

Table 6.5.4 Preference assessment of the alternatives with respect to each criterion by four experts.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Total power generation (TWh/yr)</th>
<th>GHG emissions (g CO₂eq/kWh)</th>
<th>Area requirement (m²/kW)</th>
<th>Levelised energy cost ($/MWh)</th>
<th>Prospective jobs (Jobs/annual GWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV</td>
<td>(VH, VH, M, H)</td>
<td>(VH, VH, H, MH)</td>
<td>(VH, VH, H, M)</td>
<td>(VH, VH, H, MH)</td>
<td>(VH, VH, M, H)</td>
</tr>
</tbody>
</table>

6.5.3 Results

Based on the workflow of the IC-FSAHP method [13], shown in Figure 6.4.2, the assessments shown in Table 6.5.3 and Table 6.5.4 were aggregated by using the modified beta-PERT distribution [13] in order to generate random numbers. The number of iterations used for this work was 1000. Table 6.5.5 and Table 6.5.6 show the first iterations of the random TFNs that were aggregated from Table 6.5.3 and Table 6.5.4, respectively.

Table 6.5.5 Results from the first iteration of random TFNs aggregated from the assessment of the importance of the criteria with respect to the goal, using the modified beta-PERT distribution.

<table>
<thead>
<tr>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>C₄</th>
<th>C₅</th>
</tr>
</thead>
</table>

Table 6.5.6 Results from the first iteration of random TFNs aggregated from the assessment of the preference of alternatives with respect to each criterion, using the modified beta-PERT distribution.

<table>
<thead>
<tr>
<th>A₁</th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>C₄</th>
<th>C₅</th>
</tr>
</thead>
</table>

To obtain the Fuzzy Pairwise Comparison Matrices (FPCMs) that are described in Figure 6.4.2, the elements in each FPCM, \(a_{ij} = (a_{ijL}, a_{ijM}, a_{ijU})\), were derived from the division formula of two TFNs for upper triangular FPCM [13] and the reciprocation formula of a TFN for lower triangular FPCM [13]. For example, the FPCM of alternatives \(A₁, A₂,\) and \(A₃\) with respect to \(C₁\) is shown in Equation 6.18.
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Since the consistency ratios of all FPCMs were less than 0.1, and thus acceptable, it was possible to then calculate the fuzzy criteria weights \( \left( W_{e_i}^S \right) \) fuzzy alternatives local priorities \( \left( L_{Sc}^S \right) \) and overall scores \( \left( O_{Sc}^S \right) \).

Based on the workflow of the IC-FSE method \([15]\), presented in Figure 6.4.4, the normalised decision matrix in TFN, the fuzzy entropy values \( \left( \tilde{e}_i \right) \), fuzzy entropy weights \( \left( \tilde{w}_i \right) \), normalised crisp entropy weights \( \left( W_{e_i}^O \right) \), and the ranking of criteria obtained from the IC-FSE were presented in Table 6.5.7 and Table 6.5.8.

### Table 6.5.7 The normalised fuzzy decision matrix.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Total power generation ( (C_1) )</th>
<th>GHG emissions ( (C_2) )</th>
<th>Area requirement ( (C_3) )</th>
<th>Levelised energy cost ( (C_4) )</th>
<th>Prospective jobs ( (C_5) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Onshore wind</td>
<td>( (0.32, 0.89, 1.0) )</td>
<td>( (0.02, 0.2, 0.9) )</td>
<td>( (0.02, 0.79, 1.0) )</td>
<td>( (0.03, 0.18, 0.87) )</td>
<td>( (0.07, 0.25, 0.89) )</td>
</tr>
<tr>
<td>CSP</td>
<td>( (0.02, 0.22, 0.62) )</td>
<td>( (0.07, 0.54, 0.99) )</td>
<td>( (0.01, 0.16, 0.99) )</td>
<td>( (0.08, 0.50, 0.99) )</td>
<td>( (0.12, 0.57, 0.94) )</td>
</tr>
<tr>
<td>PV</td>
<td>( (0.02, 0.40, 0.94) )</td>
<td>( (0.12, 0.82, 0.10) )</td>
<td>( (0.01, 0.59, 1.0) )</td>
<td>( (0.11, 0.85, 0.10) )</td>
<td>( (0.25, 0.79, 0.99) )</td>
</tr>
</tbody>
</table>

### Table 6.5.8 The fuzzy entropy values \( \left( \tilde{e}_i \right) \), fuzzy entropy weights \( \left( \tilde{w}_i \right) \), normalised crisp entropy weights \( \left( W_{e_i}^O \right) \) and the ranking of criteria obtained from the IC-FSE.

<table>
<thead>
<tr>
<th>Total power generation ( (C_1) )</th>
<th>GHG emissions ( (C_2) )</th>
<th>Area requirement ( (C_3) )</th>
<th>Levelised energy cost ( (C_4) )</th>
<th>Prospective jobs ( (C_5) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (\tilde{e}_i) ) ( (0.14, 0.73, 0.73) )</td>
<td>( (0.26, 0.75, 0.79) )</td>
<td>( (0.07, 0.72, 0.87) )</td>
<td>( (0.31, 0.72, 0.82) )</td>
<td>( (0.43, 0.78, 0.82) )</td>
</tr>
<tr>
<td>( (\tilde{w}_i) ) ( (0.08, 0.21, 0.55) )</td>
<td>( (0.06, 0.19, 0.49) )</td>
<td>( (0.04, 0.22, 0.53) )</td>
<td>( (0.05, 0.21, 0.46) )</td>
<td>( (0.06, 0.17, 0.42) )</td>
</tr>
<tr>
<td>( (W_{e_i}^O) ) ( (0.22) )</td>
<td>( (0.20) )</td>
<td>( (0.21) )</td>
<td>( (0.19) )</td>
<td>( (0.17) )</td>
</tr>
</tbody>
</table>

By using Equation 6.12 and Equation 6.13, the objective local priorities of alternatives \( \left( L_{Sc}^O \right) \) with respect to criteria were obtained by means of NV, the results of which are shown in Table 6.5.9. Moreover, Table 6.5.10 shows the overall scores of alternatives \( \left( O_{Sc}^O \right) \) obtained from WSM, calculated by using Equation 6.14 and Equation 6.15.
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Table 6.5.9 Objective local priorities ($LSc_k^O$) of alternatives with respect to criteria obtained NV.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Total power generation ($C_1$)</th>
<th>GHG emissions ($C_2$)</th>
<th>Area requirement ($C_3$)</th>
<th>Levelised energy cost ($C_4$)</th>
<th>Prospective jobs ($C_5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Onshore wind</td>
<td>(0.61)</td>
<td>(0.58)</td>
<td>(0.08)</td>
<td>(0.65)</td>
<td>(0.19)</td>
</tr>
<tr>
<td>CSP</td>
<td>(0.10)</td>
<td>(0.24)</td>
<td>(0.75)</td>
<td>(0.21)</td>
<td>(0.34)</td>
</tr>
<tr>
<td>PV</td>
<td>(0.29)</td>
<td>(0.19)</td>
<td>(0.17)</td>
<td>(0.14)</td>
<td>(0.47)</td>
</tr>
</tbody>
</table>

Table 6.5.10 The overall scores of alternatives ($OSc_k^O$) obtained from WSM.

<table>
<thead>
<tr>
<th>Total power generation ($C_1$)</th>
<th>GHG emissions ($C_2$)</th>
<th>Area requirement ($C_3$)</th>
<th>Levelised energy cost ($C_4$)</th>
<th>Prospective jobs ($C_5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>($\hat{e}_i$)</td>
<td>(0.14, 0.73, 0.73)</td>
<td>(0.07, 0.72, 0.87)</td>
<td>(0.31, 0.72, 0.82)</td>
<td>(0.43, 0.78, 0.82)</td>
</tr>
<tr>
<td>($\hat{w}_i$)</td>
<td>(0.08, 0.21, 0.55)</td>
<td>(0.04, 0.22, 0.53)</td>
<td>(0.05, 0.21, 0.46)</td>
<td>(0.06, 0.17, 0.42)</td>
</tr>
<tr>
<td>($\hat{W}_{Sc}^O$)</td>
<td>(0.22)</td>
<td>(0.20)</td>
<td>(0.19)</td>
<td>(0.17)</td>
</tr>
<tr>
<td>Rank</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

Furthermore, by applying Equation 6.16 and Equation 6.17, the results of overall criteria weights and overall alternatives’ scores from the first iteration for $\alpha=0.5$ were obtained, the results of which are shown in Table 6.5.11 and Table 6.5.12, respectively.

Table 6.5.11 Results of overall criteria weights obtained from the first iteration for $\alpha=0.5$.

<table>
<thead>
<tr>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.200)</td>
<td>(0.214)</td>
<td>(0.177)</td>
<td>(0.228)</td>
<td>(0.181)</td>
</tr>
</tbody>
</table>

Table 6.5.12 Results of overall alternatives’ scores obtained from the first iteration for $\alpha=0.5$.

<table>
<thead>
<tr>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.318)</td>
<td>(0.325)</td>
<td>(0.357)</td>
</tr>
</tbody>
</table>

Violin plots were used to show the probability density of the local criteria weights and the overall alternatives’ scores for different $\alpha$ after 1000 iterations. Figures 6.5.2.a) and 6.5.2.b) show the violin plots of the criteria weights and the overall alternatives’ scores obtained from the proposed combined method for $\alpha=0.5$ after 1000 iterations. The results indicate that total power generation ($C_1$) was the highest prioritised criterion and the onshore wind technology ($A_1$) was the most suitable alternative. Moreover, Figure 6.5.2.b) showcases that the ranking of alternatives can be determined as onshore wind ($A_1$) $\succ$ concentrated solar power ($A_2$) $\succ$ solar photovoltaic ($A_3$).
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6.5.4 Sensitivity analysis

A sensitivity analysis of the decision making results was conducted by applying different values of $\alpha$ in the interval $[0, 1]$ to the proposed combined method. In this study, six values of coefficient $\alpha$ were used, namely 0, 0.2, 0.4, 0.6, 0.8, and 1.

In the case when decision makers intend to obtain results from only applying the objective weighting method, which means that the final recommendations do not take into account the subjective judgements or preferences obtained from decision makers, $\alpha=0$ is then applied. Tables 6.5.8 and 6.5.10 show the criteria weights ($We_i^O$) and the overall alternatives’ scores ($OSc_k^O$) obtained from the objective weighting method, respectively. It can be seen from Tables 6.5.8 and 6.5.10 that the most prioritised criterion is total power generation ($C_1$) and the rank of each alternative is in the following order: onshore wind ($A_1$) $\succ$ concentrated solar power ($A_2$) $\succ$ solar photovoltaic ($A_3$). In addition, it is worth highlighting that for this case study when $\alpha=0$ the most important criterion and the rank of each alternative are similar to those of when $\alpha=\beta=0.5$.

On the other hand, when the subjective weighting method needs to be used, $\alpha=1$ is applied. By applying $\alpha=1$ or fully subjective weighting method, the final outcomes consider only the subjective judgements or preferences obtained from decision makers. Figures 6.5.3.a) and 6.5.3.b) show the violin plots of the criteria weights and the overall alternatives’ scores obtained from the proposed combined method for $\alpha=1$ after 1000 iterations. LEC ($C_4$) was the most prioritised criterion and solar photovoltaic ($A_3$) is the most suitable renewable energy technology. Moreover, Figure 6.5.3.b) shows that the ranking of alternatives can be determined as solar photovoltaic ($A_3$) $\succ$ concentrated solar power ($A_2$) $\succ$ onshore wind ($A_1$).
It can be concluded that by applying the fully subjective weight, the final recommendations are different from those obtained when applying the fully objective weight.

![Fig. 6.5.3 Violin plots of a) the criteria weights, and b) the overall alternatives scores obtained from the proposed combined method for α=1 after 1000 iterations.](image)

After obtaining the results from the fully objective weighting method (α=0) or the fully subjective weighting method (α=1), four other values of coefficient α were used for further analysis, namely 0.2, 0.4, 0.6, 0.8. Figures 6.5.4 and 6.5.5 provide violin plots of the criteria weights and the overall alternatives’ scores based on the various values of α, respectively. Figure 6.5.4 showcases that when the coefficient α increases, the importance of the criteria is slightly changed (i.e. the most prioritised criterion was changed from total power generation (C_1) to LEC (C_4)). This means that the influence of the objective weights on the importance of the criteria increases when α is increased. In addition, Figure 6.5.5 shows that the increase of the coefficient α does not change the most suitable alternative (i.e. the onshore wind technology (A_1)), thus indicating that the objective weights have a powerful influence on the overall alternatives’ scores.
6.5 Application of the method developed to the selection of renewable energy technologies in the mining industry

Fig. 6.5.4 Violin plots of the criteria weights obtained from the proposed combined method for a) $\alpha=0.2$; b) $\alpha=0.4$; c) $\alpha=0.6$; d) $\alpha=0.8$ after 1000 iterations.

Fig. 6.5.5 Violin plots of the alternatives’ scores obtained from the proposed combined method for a) $\alpha=0.2$; b) $\alpha=0.4$; c) $\alpha=0.6$; d) $\alpha=0.8$ after 1000 iterations.
Based on the example presented above, it is worth concerning that the uncertainty is not only associated with the imprecise input data, which can be minimised by means of TFN, and associated with the different decision makers’ opinions, which can be captured by Monte Carlo simulations, as done in this study. The uncertainty can be also associated with ill-judged assessments when decision makers do not take into account the available data sources and mostly use their subjective opinion in decision making analysis.

For example, Figures 6.5.2–6.5.5 as well as Tables 6.5.8 and 6.5.10 show that the criteria weights and the overall alternatives’ scores obtained by different coefficient factors ($\alpha$) result in different outcomes that reflect both subjective preferences and the objective weight. The results from Figures 6.5.2, 6.5.4 and 6.5.5 show that the objective weight dominates the final criteria weights and overall alternatives’ scores. The best alternative shown in these figures is the onshore wind ($A_1$) technology. This result is completely different from that shown in Figure 6.5.3, when the fully subjective weighting method was applied, resulting in the solar photovoltaic ($A_3$) technology being the best alternative.

The difference in the results obtained can arguably be linked to the fact that the experts did not consider the objective information. Their judgements and preferences were made on the basis of their knowledge and experience. This circumstance can lead to potential bias during the evaluation and affect the final results. For example, a very interesting finding can be observed in the evaluation of the renewable energy technologies considered with respect to the LEC criterion. Despite the quantitative data presented in Table 6.5.2, showing that solar based technologies (solar photovoltaic ($A_3$) and concentrated solar power ($A_2$)) have higher LEC than the onshore wind ($A_1$) technology, Table 6.5.4 indicates that the experts regard solar based technologies as not being dissimilar to onshore wind with regards to LEC. Therefore, choosing the right coefficient factor for the objective weight is critical to avoid subjective bias during the evaluations.

This work also showcases the applicability of the proposed hybrid approach in capturing uncertainty due to ill-judged assessments. The proposed hybrid method determines criteria weights and alternatives’ scores by solving a comprehensive mathematical programming model which considers both subjective and objective factors. It overcomes the shortcomings which possible arise in either a subjective weighting approach or an objective weighting approach.

6.5.5 Discussion

It is evident from the aforementioned outcomes that the proposed method can be used to evaluate different criteria and alternatives under uncertainties in the context of group decision making in a scientific transparent manner by means of subjective weights (i.e. Integrated Constrained Fuzzy Stochastic Analytic Hierarchy Process (IC-FSAHP)) and objective weights (i.e. Integrated Constrained Fuzzy Shannon Entropy (IC-FSE), Normalised Vector (NV), and Weighted Sum Model (WSM)). It is worth noting that when evaluating such complex
selection problems, one criterion (i.e. cost or monetary value) is not sufficient to base a decision on. In fact, multiple criteria that are often conflicting are involved. Therefore, the triple bottom line approach or the cost analysis proposed by Mostert [42], Zharan and Bongaerts [43], respectively, which are based on a monetary value, are unable to assess the complex selection problem comprehensively. A combination of MCDM methods with the triple bottom line or with cost analysis is thus suggested in order to analyse the problem holistically.

In MCDM problems, uncertainty due to imprecise input data are often present and quantifying such input data is challenging. In the current study, the proposed combined method is capable of quantifying these types of data by means of triangular fuzzy numbers. This evidenced that the proposed hybrid method is superior to those developed by Ma et al. [6] and Rao and Patel [7], which did not take into account the risk of imprecise input data.

Furthermore, the proposed method can capture inconsistencies of decision makers as a group, which are caused by decision makers having different points of view in judging their preference, by means stochastic methods in IC-FSAHP. This feature was missing in the methods proposed by Ma et al. [6], Rao and Patel [7] and Rao et al. [8]. Since the proposed hybrid method in this work does not have the aforementioned shortcomings, it is deemed superior to other MCDM methods in its capability of dealing with uncertainties.

Further, combining subjective and objective weighting methodologies enhances the capability of the proposed method in terms of determining the criteria weights and alternatives’ scores through the use of coefficient factors. The coefficient factors are able to be adjusted for balancing the decision makers’ opinions and the objective information or quantitative data involved, and thus reduce the subjectivity of decision makers in assessing the selection problem.

Regarding the coefficient factors, decision makers or experts should discuss and adjust the coefficients that will be used. They can freely choose adjusting coefficients according to the particular characteristics of the decision makers or experts and input data. Selecting the adjusting coefficients depends on the background, expertise and experience of decision makers or experts and the availability of quantitative data. For example: if the decision makers or experts have a lot of experience with the high success rate on the selection of renewable energy technologies in the same type of mineral being processed and in the same country, it is possible to use a very low coefficient on objective weights (<0.5) and the very high coefficient on subjective weights (>0.5).

It should also be indicated that this work does not consider the interaction and dependency between criteria, sub-criteria, and alternatives. Such dependencies can be handled by using another MCDM method, such as Analytic Network Process (ANP). There is therefore scope to further extend the proposed method for the case when non-homogeneous data and uncertainties due to imprecise input data and various decision makers’ opinions, as well as the dependency between criteria and alternatives, are involved.
6.6 Conclusions

This paper demonstrates the value of combining the concept of subjective and objective weighting methodologies when decisions are made based on non-homogenous data (i.e. quantitative and qualitative), uncertain input data (i.e. probabilistic), and uncertainty because of different decision makers’ opinions. An integrated multiple criteria decision making (MCDM) method is presented in this paper; the method is used to deal with the process of decision making for the selection of a renewable energy technology. The method considers subjective judgements and preferences as well as objective data under uncertainties in the context of group decision making. The method supports decision makers in weighting the criteria and scoring the alternatives considering both subjective and objective weights. The method applies a two-quantitative model which comprises subjective weighting method (i.e. Integrated Constrained Fuzzy Stochastic AHP (IC-FSAHP)) and objective weighting methods (i.e. Integrated Constrained Fuzzy Shannon Entropy (IC-FSE), Normalised Vector (NV) and Weighted Sum Model (WSM)). Each quantitative model has a coefficient factor that allows decision makers to adjust the weight of each model.

The proposed method was applied to the selection of renewable energy technologies in the mining industry, which faces an increase in energy demand as high grade ores are depleted and the demand for metals and minerals, including those required for renewable energy technologies, increases. The large scale of mining operations makes it very important to consider renewable energy options in order to contribute to the sustainability of the operations.

Three renewable technology alternatives, namely onshore wind, concentrated solar power, and solar photovoltaic, were assessed taking into account both subjective considerations and objective information with respect to five sustainability criteria. The selected criteria were potential total power generation, GHG emissions, area requirement, levelised energy cost, and prospective jobs creation. An objective weight was obtained using data compiled from the literature, whereas a subjective weight was obtained from the judgements and preferences of four experts. The proposed method was then employed to compute the criteria weights and the alternatives’ scores.

The results when the same coefficient factors of subjective and objective weights were applied show that total power generation \( (C_1) \) was the most important criterion and the onshore wind technology \( (A_1) \) was the most suitable alternative. Furthermore, from the sensitivity analysis results, it can be summarised that the criteria weights and the overall alternatives’ scores obtained by various coefficient factors yield different results. Despite the different results, the ranking of alternatives obtained with the proposed method reflects both subjective preferences and the objective weight. Moreover, the proposed method is also able to minimise the loss of valuable objective information, which is caused by the subjective bias
of qualitative weights during the evaluations, by adjusting the coefficient factors of both quantitative and qualitative data in the two-quantitative model during the calculations.

The outcomes have shown the usability of the proposed method in selecting renewable energy technologies in the mining industry in a fuzzy environment based on quantitative and qualitative data in the context of group decision making. In addition, the method can be used in other areas to support decision makers in the selection problem under the aforementioned circumstances.

Nevertheless, the outcomes from this work show that there is some uncertainty in the quantitative input data used. The input data used in this study were originally compiled at a national scale and are therefore relatively generic. In terms of the application of objective weighting methods, if a specific renewable energy project in the mining industry were to be considered, the degree of uncertainty in terms of input data is very likely to be lower than that in the present example. The proposed method can be applied by substituting all values in Table 6.5.2 as required, and amending the set of feasible renewable energy technologies to be considered. For example, if a mining company is located nearby geothermal energy resources and there is a high potential to build a geothermal power plant (GPP), then a GPP might be added into a set of feasible alternatives.

The use of coefficient factors can be extended to other MCDM methods that combine objective and subjective weighting. It is worth highlighting that in such cases, the outcome for the preferred renewable energy system might differ to that obtained in the present study, unless there is a high level of consistency in the process of decision making.

This study has shown that the proposed hybrid method is a robust method to identify and screen the criteria, weight the criteria and rank the alternatives when decision makers face a complex problem that requires to consider non-homogeneous input data and uncertainties due to imprecise input data and different decision makers’ opinions. The proposed method has a broad application potential in other sectors to support decision makers in dealing with a selection problem with the aforementioned characteristics. In addition, there is scope to further extend the proposed hybrid method for the case when there exists a dependency between criteria and alternatives. Further studies to develop such an extended method will be the subject of future work.
Chapter references


Chapter 7

Conclusions, research contributions and future research directions

The main goal of this PhD work was to develop robust Multiple Criteria Decision Making (MCDM) methods that are suitable for decision problems with multiple conflicting criteria under uncertainty (e.g. non-homogenous data, imprecise input data, and when there is uncertainty due to disparities among decision makers). The developed methods should be able to support decision makers in determining the most suitable solution for the selection problem in mining and mineral processing.

This chapter presents the conclusions from the research carried out and states the contributions of this PhD project. In addition, research directions for further development of the presented work and possible extensions for future applications are outlined.

7.1 Conclusions

Multiple Criteria Decision Making (MCDM) methods have become useful for a wide range of applications in mining and mineral processing, with a number of MCDM methods proposed for decision making in these areas. A comprehensive literature review showed that the conventional Analytic Hierarchy Process (AHP) and the Shannon Entropy method (SE), which are the two frequently applied MCDM methods, have a limitation on their inability to quantify the uncertainty of data or information. Furthermore, conventional AHP has several limitations: (i) the occurrence of the rank reversal phenomenon; (ii) the lack of ranking discriminations; (iii) their difficulty to capture the uncertainty of various judgements among decision makers; (iv) their inability in determining robust criteria weights and alternatives’ scores that combine subjective preferences and objective information.

In order to overcome the aforementioned limitations, novel MCDM methods have been developed as part of the work in this PhD project: the Integrated Constrained Fuzzy Stochastic AHP (IC-FSAHP), the Integrated Constrained Fuzzy Shannon Entropy (IC-FSE),
Conclusions, research contributions and future research directions

and an integrated subjective and objective MCDM method. These methods were shown to outperform the conventional AHP and SE in terms of handling and visualising uncertainty.

A decision problem for primary crusher selection in an iron mine (i.e. five types of primary crushers evaluated with respect to six criteria) was used as a case study in order to demonstrate the applicability of IC-FSAHP. The results obtained from the case study showed IC-FSAHP’s ability to reduce uncertainty caused by lack of knowledge. In addition, IC-FSAHP was able to capture scattered opinions from multiple decision makers caused by various judgements and insights. IC-FSAHP produced more precise results and it was shown to be more suitable for real-world group decision making problems under uncertainty than the AHP and FSAHP methods. In the case of rank reversal analyses, the results showed that the ranking of alternatives was preserved throughout the changes in the number of alternatives. Since the rank order was stable and robust, no undesired rank reversal occurred due to the addition or deletion of an alternative. Moreover, it was shown that a higher confidence in the results can be obtained by increasing the number of decision makers due to the fact that when the minimum, maximum and mode values of the opinions do not change, a higher number of decision makers would place more emphasis on the mode values.

In order to understand the robustness of IC-FSAHP, a new sensitivity analysis approach was developed. This approach allows decision makers to identify the effect of input data changes on the aggregated results for equipment ranking and takes into account the main notions of IC-FSAHP, i.e. preserving the reciprocity interactions among the elements in the fuzzy pairwise comparison matrices and checking the inconsistency ratios of the fuzzy pairwise comparison matrices. The proposed sensitivity analysis approach applies an additional fuzzification factor and allows to consider different levels of disagreement among decision makers to model uncertainty. In addition, the approach can be applied to other fuzzy AHP based methods. The case study for the selection of primary crushers was considered in order to show the applicability of the sensitivity analysis approach. The sensitivity analysis results obtained showed that the level of uncertainty of the variables does not affect the final recommendation, the results obtained from the sensitivity analysis indicate that IC-FSAHP could be confidently used for the selection of mineral processing equipment.

Regarding the developed objective method, IC-FSE, an illustrative example for the selection of a renewable energy technology in the mining industry was presented in order to show its applicability. Three alternative renewable energy technologies, onshore wind, solar photovoltaic and concentrated solar power, were evaluated with respect to technical, social, economic and environmental criteria. The results show that IC-FSE can effectively provide appropriate fuzzy solutions for weighting the sustainability criteria for renewable energy technologies. The superiority of this method was showcased by demonstrating that IC-FSE results are more robust than those obtained using other existing methods.

The integrated subjective and objective MCDM method developed considers subjective judgements and preferences as well as objective data under uncertainties in the context
of group decision making. The method applies a two-quantitative model which comprises subjective method (i.e. Integrated Constrained Fuzzy Stochastic AHP (IC-FSAHP)) and objective methods (i.e. Integrated Constrained Fuzzy Shannon Entropy (IC-FSE), Normalised Vector (NV) and Weighted Sum Model (WSM)). Each quantitative model has a coefficient factor that allows decision makers to adjust the weight of each model.

The developed integrated method was applied to the selection of renewable energy strategies in the mining industry. Three renewable energy alternatives, namely onshore wind, concentrated solar power, and solar photovoltaic, were assessed taking into account both subjective considerations and objective information with respect to five sustainability criteria. It is worth mentioning that for the simplicity of data collection and analysis, the number of criteria considered in Chapter 6 was different from those considered previously in Chapter 5. An objective weight was obtained using data compiled from the literature, whereas a subjective weight was obtained from the judgements and preferences of four experts. The proposed method was then employed to compute the criteria weights and the alternatives’ scores.

The outcomes have shown the usability of the developed integrated method in selecting renewable energy technologies in the mining industry. The method was showcased in a fuzzy environment based on quantitative and qualitative data and in the context of group decision making. Furthermore, the proposed method is capable of minimising the loss of valuable objective information, which is caused by the subjective bias of qualitative weights during the evaluations, by adjusting the coefficient factors of both quantitative and qualitative data in the two-quantitative model during the calculations. In addition, this robust MCDM method can be applied broadly in other selection problems to support the process of decision making when there is uncertainty in the non-homogenous input data in the context of group decision making.

7.2 Research contributions

The five main scientific contributions of this research are summarised as follows.

1. An exhaustive literature review identified and described the application and trends of the most commonly applied MCDM methods for the choice problem in mining and mineral processing.

2. A novel subjective MCDM method, IC-FSAHP, was developed as a solution for decision making for the choice problem under uncertainty in the context of group decision making that can avoid rank reversal, enhance rank discriminations and provide precise results.
3. A novel sensitivity analysis for IC-FSAHP, which can be also used for other fuzzy AHP based methods, was proposed. The approach takes into account the main notions of AHP, i.e. preserving the reciprocity interactions among the elements in the fuzzy pairwise comparison matrices and checking the inconsistency ratios of the fuzzy pairwise comparison matrices.

4. A novel weighting criteria based on the objective method for decision making, IC-FSE, was developed to support the decision making process when there is uncertainty in the input data.

5. A novel integrated MCDM method that considers subjective judgements and preferences as well as objective data under uncertainties in the context of group decision making was developed.

7.3 Future research directions

The fuzzy MCDM methods (i.e. IC-FSAHP, IC-FSE and the integrated subjective and objective method) and sensitivity analysis developed in this study have addressed a number of relevant issues in this research area. The corresponding ranking results were validated for selected case studies. In order to solve more complex problems in terms of uncertainty, there is scope to improve the robustness of IC-FSAHP and IC-FSE and thus extend the applicability of the methods. Some possibilities for future research are summarised as follows:

1. Since IC-FSAHP succeeded in solving an MCDM problem under uncertainty when the decision makers’ opinions have unimodal value, there is a scope to further extend IC-FSAHP for the case when multimodal values are involved.

2. The development of appropriate algorithms for checking the consistency of fuzzy pairwise comparison matrices and for examining the consistency of the process in attaining the fuzzy weights of the objects from the fuzzy pairwise comparison matrices. The results obtained from the proposed algorithms could then be compared to those obtained in this work.

3. The development of an extended IC-FSAHP method for dealing with incomplete fuzzy preference relations in assessing an object (i.e. criterion and alternative) over others. The analyses carried out in this work could also be followed to examine the robustness of such an extended method.

4. There is scope to further extend IC-FSE for the case when both qualitative and quantitative data are available, as well as for when there exists a dependency between criteria and alternatives.
Appendix A

Scientific journal articles on the application of MCDM methods for the choice problem in mining
Scientific journal articles on the application of MCDM methods for the choice problem in mining

Table A.1 Scientific journal articles on the application of AHP for the choice problem in mining.

<table>
<thead>
<tr>
<th>No</th>
<th>Authors</th>
<th>Type of study</th>
<th>Type of selection problem</th>
<th>Problem addressed</th>
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</thead>
<tbody>
<tr>
<td>1.</td>
<td>Kazakidis et al. [1]</td>
<td>Case study</td>
<td>Other</td>
<td>A series of case studies in different mining scenarios to demonstrate the application of AHP.</td>
</tr>
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<td>2.</td>
<td>Straka et al. [2]</td>
<td>Case study</td>
<td>Mining Site selection</td>
<td>The application of AHP to select an appropriate storage location for mines waste.</td>
</tr>
<tr>
<td>4.</td>
<td>Acaroglu et al. [4]</td>
<td>Case study</td>
<td>Mining equipment selection</td>
<td>The application of AHP to select the most suitable roadheaders.</td>
</tr>
<tr>
<td>5.</td>
<td>Samanta et al. [5]</td>
<td>Case study</td>
<td>Mining equipment selection</td>
<td>The application of AHP to select a mobile surface mining machine for excavating, transporting, or loading coal.</td>
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<tr>
<td>8.</td>
<td>Ataei et al. [8]</td>
<td>Case study</td>
<td>Mining method selection</td>
<td>The application of AHP to select a suitable mining method.</td>
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<td>9.</td>
<td>Mohsen et al. [9]</td>
<td>Case study</td>
<td>Mining method selection</td>
<td>The application of AHP to select an appropriate mining method.</td>
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<td>10.</td>
<td>Kluge and Malan [10]</td>
<td>Case study</td>
<td>Mining method selection</td>
<td>The application of AHP to select a backfill support system in a new platinum project.</td>
</tr>
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<td>12.</td>
<td>Petit [12]</td>
<td>Case study</td>
<td>Mining method selection</td>
<td>The application of AHP to select the most suitable mining technology to support the business optimisation for platinum mining projects and operations.</td>
</tr>
<tr>
<td>15.</td>
<td>Yavuz et al. [15]</td>
<td>Case study</td>
<td>Other</td>
<td>The application of AHP to select the optimum support design for the main transport road for deep coal seam panels.</td>
</tr>
<tr>
<td>17.</td>
<td>Dey and Ramcharan [17]</td>
<td>Case study</td>
<td>Mining site selection</td>
<td>The application of AHP to select site location for the expansion of limestone quarry operations.</td>
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<td>18.</td>
<td>Petit and Fraser [18]</td>
<td>Case study</td>
<td>Mining technology selection</td>
<td>The application of AHP to select an alternative energy-delivery system for stoping in narrow-reef hard rock mines.</td>
</tr>
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Table A.2 Scientific journal articles on the application of TOPSIS for the choice problem in mining.

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<tr>
<td>1</td>
<td>Ataei et al. [19]</td>
<td>Case study</td>
<td>Mining method selection</td>
<td>The application of TOPSIS to select a suitable mining method.</td>
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</table>

Table A.3 Scientific journal articles on the application of ELECTRE for the choice problem in mining.

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<td>1</td>
<td>Bodziony et al. [20]</td>
<td>Case study</td>
<td>Mining equipment selection</td>
<td>The application of ELECTRE III to select the off-highway dump truck in an open pit mining.</td>
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</table>

Table A.4 Scientific journal articles on the application of PROMETHEE for the choice problem in mining.

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<th>Type of study</th>
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<td>1</td>
<td>Vujic et al. [21]</td>
<td>Case study</td>
<td>Mining technology selection</td>
<td>The application of PROMETHEE II to select the technological system in an open pit mine.</td>
</tr>
<tr>
<td>2</td>
<td>Elevli and Demirci [22]</td>
<td>Case study</td>
<td>Mining equipment selection</td>
<td>The application of the PROMETHEE to select the most suitable underground ore transport system.</td>
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Table A.5 Scientific journal articles on the application of VIKOR for the choice problem in mining.

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<td>1</td>
<td>Hayati et al. [23]</td>
<td>Case study</td>
<td>Mining method selection</td>
<td>The application of VIKOR to select the optimal mining block size.</td>
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Table A.6 Scientific journal articles on the review of MCDM methods for the choice problem in mining.

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<th>Type of selection problem</th>
<th>Problem addressed</th>
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<td>1</td>
<td>Mahase et al. [24]</td>
<td>Case study</td>
<td>Mining method selection</td>
<td>Mine planning and related case studies are identified and categorized according to the MCDM methods.</td>
</tr>
<tr>
<td>2</td>
<td>Kant et al. [25]</td>
<td>Case study</td>
<td>Mining method selection</td>
<td>Several available approaches to identify a suitable stoping method in hard rock underground mine are reviewed critically.</td>
</tr>
</tbody>
</table>
Table A.7 Scientific journal articles on the application of hybrid MCDM methods for the choice problem in mining.

<table>
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<th>No</th>
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<th>Type of study</th>
<th>Type of selection problem</th>
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<tbody>
<tr>
<td>1.</td>
<td>Ebrahimabadi [26]</td>
<td>Case study</td>
<td>Mining equipment selection</td>
<td>The application of FAHP to select the proper roadheader.</td>
</tr>
<tr>
<td>2.</td>
<td>Aghajani Bazzazi et al. [27]</td>
<td>Case study</td>
<td>Mining equipment selection</td>
<td>The application of AHP to select the suitable loading-haulage equipment.</td>
</tr>
<tr>
<td>3.</td>
<td>Naghadehi et al. [28]</td>
<td>Case study</td>
<td>Mining method selection</td>
<td>The application of FAHP to select the optimum mining method.</td>
</tr>
<tr>
<td>4.</td>
<td>Ghazikalayeh et al. [29]</td>
<td>Case study</td>
<td>Mining method selection</td>
<td>The application of FAHP to select the proper mining method.</td>
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<tr>
<td>5.</td>
<td>Özfirat [30]</td>
<td>Case study</td>
<td>Mining method selection</td>
<td>The application of FAHP to select an underground mining method.</td>
</tr>
<tr>
<td>6.</td>
<td>Karimnia and Bagloo [31]</td>
<td>Case study</td>
<td>Mining method selection</td>
<td>The application of FAHP to select an optimum mining method.</td>
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<tr>
<td>7.</td>
<td>Azadeh et al. [32]</td>
<td>Concept and case study</td>
<td>Mining method selection</td>
<td>The application of FAHP for the selection mining method based on modifying Nicholas technique.</td>
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<tr>
<td>8.</td>
<td>Gligoric and Gligoric [33]</td>
<td>Concept</td>
<td>Mining method selection</td>
<td>The development of an integrated dynamic model based on FTOPSIS in the process of strategic decision-making and mine design.</td>
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<tr>
<td>9.</td>
<td>Yavuz [34]</td>
<td>Case study</td>
<td>Mining equipment selection</td>
<td>The selection of a truck for mining operations in an open pit coal mine by means of FTOPSIS.</td>
</tr>
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<td>11.</td>
<td>Başçetin et al. [36]</td>
<td>Concept and case study</td>
<td>Mining equipment selection</td>
<td>The development of a computer programme software called EQS (EQuipment Selection) based on Yager method. EQS is applied to the selection of the underground mining method.</td>
</tr>
<tr>
<td>12.</td>
<td>Acaroglu et al. [37]</td>
<td>Case study</td>
<td>Mining equipment selection</td>
<td>The selection of roadheaders for coal basin by using Yager.</td>
</tr>
<tr>
<td>No</td>
<td>Authors</td>
<td>Type of study</td>
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<tr>
<td>13.</td>
<td>Karadogan et al. [38]</td>
<td>Case study</td>
<td>Mining method selection</td>
<td>The selection of underground mining method by using Yager.</td>
</tr>
<tr>
<td>14.</td>
<td>Bazzazi et al. [39]</td>
<td>Case study</td>
<td>Mining equipment selection</td>
<td>The selection of loading-haulage equipment in open pit by using a combined AHP and FTOPSIS.</td>
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<tr>
<td>15.</td>
<td>Aghajani Bazzazi et al. [40]</td>
<td>Concept and case study</td>
<td>Mining equipment selection</td>
<td>The selection of the suitable loading-haulage equipment in open pit mines using an integrated AHP, entropy method, and VIKOR.</td>
</tr>
<tr>
<td>16.</td>
<td>Adebimpe et al. [41]</td>
<td>Case study</td>
<td>Mining equipment selection</td>
<td>The selection of mine equipment using a combined AHP and FTOPSIS.</td>
</tr>
<tr>
<td>17.</td>
<td>Komljenovic and Kecojevic [42]</td>
<td>Concept and case study</td>
<td>Mining equipment selection</td>
<td>The development of the selection methodology for bulk material handling systems using the combination of Coefficient of Technical Level (CTL) [43] and AHP.</td>
</tr>
<tr>
<td>18.</td>
<td>Lashgari et al. [44]</td>
<td>Concept and case study</td>
<td>Mining equipment selection</td>
<td>The selection of the equipment for loading and hauling system by using the combination of FAHP, ANP, FTOPSIS.</td>
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<td>19.</td>
<td>Yazdani-Chamzini [45]</td>
<td>Concept and case study</td>
<td>Mining equipment selection</td>
<td>The selection of the optimum handling system by using the combination of FAHP and FTOPSIS.</td>
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<td>20.</td>
<td>Ozfiras et al. [46]</td>
<td>Concept and case study</td>
<td>Mining equipment selection</td>
<td>The development of a general methodology employing FAHP and FGP for a roadheader selection in the mining industry.</td>
</tr>
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<td>21.</td>
<td>Bazzazi et al. [47]</td>
<td>Concept and case study</td>
<td>Mining equipment selection</td>
<td>The development of a combined AHP, entropy, and FTOPSIS for the selection of the most suitable ore transportation system,</td>
</tr>
<tr>
<td>22.</td>
<td>Bogdanovic et al. [48]</td>
<td>Concept and case study</td>
<td>Mining method selection</td>
<td>The selection of mining method by using an integrated AHP and PROMETHEE.</td>
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### Table A.7 – continued from previous page

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<tr>
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<th>Problem addressed</th>
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<tr>
<td>23</td>
<td>Wang and Tu [49]</td>
<td>Concept and case study</td>
<td>Mining equipment selection</td>
<td>The selection of an appropriate mechanized mining technical process (MMTP) for thin coal seam mining in by using an integrated AHP and FPROMETHEE.</td>
</tr>
<tr>
<td>24</td>
<td>Shariati et al. [50]</td>
<td>Concept and case study</td>
<td>Mining method selection</td>
<td>The selection of the best mining method by using an integrated FAHP and TOPSIS.</td>
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<td>25</td>
<td>Mikael et al. [51]</td>
<td>Concept and case study</td>
<td>Mining method selection</td>
<td>The selection of the optimum mining method by using an integrated FAHP and TOPSIS.</td>
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<td>26</td>
<td>Stojanovic et al. [52]</td>
<td>Case study</td>
<td>Mining technology selection</td>
<td>The selection of an optimal technology for surface mining using an integrated AHP and ELECTRE in the open pit coal mine.</td>
</tr>
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<td>27</td>
<td>Yari et al. [53]</td>
<td>Case study</td>
<td>Mining method selection</td>
<td>The selection of the most suitable blasting pattern using an integrated AHP and TOPSIS.</td>
</tr>
<tr>
<td>28</td>
<td>Bouhedja et al. [54]</td>
<td>Case study</td>
<td>Mining technology selection</td>
<td>The selection of a secondary breakage process of the oversized blocks using the integrated centroid weight method (CWM) and PROMETHEE.</td>
</tr>
<tr>
<td>29</td>
<td>Wang et al. [55]</td>
<td>Case study</td>
<td>Mining equipment selection</td>
<td>The selection of an auxiliary transportation mode in a fully-mechanized face in a nearly horizontal thin coal seam by using an integrated entropy and FPROMETHEE</td>
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<td>30</td>
<td>Golestanifar and Aghajani Bazzazi [56]</td>
<td>Concept and case study</td>
<td>Mining site selection</td>
<td>The establishment of a framework for the selection of tailing impoundment site based on combined FAHP and FTOPSIS.</td>
</tr>
<tr>
<td>31</td>
<td>Ataei et al. [57]</td>
<td>Concept and case study</td>
<td>Mining method selection</td>
<td>The selection of the optimum mining method by using a combined Monte Carlo simulation and AHP (MAHP).</td>
</tr>
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<th>Type of selection problem</th>
<th>Problem addressed</th>
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<tr>
<td>32.</td>
<td>Javanshirgiv and Safari [58]</td>
<td>Case study</td>
<td>Mining method selection</td>
<td>The selection of an underground mining method using FTOPSIS.</td>
</tr>
</tbody>
</table>

Table A.8 Scientific journal articles on the comparisons of MCDM methods for the choice problem in mining.

<table>
<thead>
<tr>
<th>No</th>
<th>Authors</th>
<th>Type of study</th>
<th>Type of selection problem</th>
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<tbody>
<tr>
<td>1.</td>
<td>de Sousa Junior et al.</td>
<td>Case study</td>
<td>Mining equipment selection</td>
<td>The comparison of WPM, ELECTRE I and PROMETHEE II for the selection of a hauling truck to transport run of mine (ROM) ore.</td>
</tr>
<tr>
<td>2.</td>
<td>Yavuz [60]</td>
<td>Concept and case study</td>
<td>Mining equipment selection</td>
<td>The comparison between AHP and Yager method for the selection of the wheel loader.</td>
</tr>
<tr>
<td>3.</td>
<td>Basçetin [61]</td>
<td>Concept and case study</td>
<td>Mining equipment selection</td>
<td>The comparison between Yager and AHP for the selection of an optimal loading hauling system for coal production.</td>
</tr>
<tr>
<td>4.</td>
<td>Samimi et al. [62]</td>
<td>Case study</td>
<td>Mining method selection</td>
<td>The selection of an ore extraction system by using AHP, TOPSIS, and PROMETHEE.</td>
</tr>
<tr>
<td>5.</td>
<td>Yavuz and Alpay [63]</td>
<td>Case study</td>
<td>Mining method selection</td>
<td>The comparison of AHP, the Bellman-Zadeh Method, and TOPSIS for the selection of an optimal underground mining method.</td>
</tr>
<tr>
<td>6.</td>
<td>Dehghani et al. [64]</td>
<td>Concept and case study</td>
<td>Mining method selection</td>
<td>The comparison between grey and TODIM for the selection of the mining method.</td>
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<tr>
<td>7.</td>
<td>Hekmat et al. [65]</td>
<td>Concept and case study</td>
<td>Mining site selection</td>
<td>The comparison of SAW, TOPSIS and AHP for the selection of a waste dump site location.</td>
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<td>8.</td>
<td>Hudej et al. [66]</td>
<td>Concept and case study</td>
<td>Mining site selection</td>
<td>The comparison of PROMETHEE, ELECTRE, AHP and VIKOR for the selection of the main mine shaft location.</td>
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Scientific journal articles on the application of MCDM methods for the choice problem in mining

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<tr>
<td>10.</td>
<td>Yavuz [68]</td>
<td>Concept and case study</td>
<td>Mining method selection</td>
<td>The comparison between AHP and Yager for the selection of the mining method.</td>
</tr>
<tr>
<td>11.</td>
<td>Alpay and Yavuz [69]</td>
<td>Concept and case study</td>
<td>Mining method selection</td>
<td>The comparison between AHP and Yager for the selection of an underground mining method.</td>
</tr>
<tr>
<td>12.</td>
<td>Kabwe [70]</td>
<td>Case study</td>
<td>Mining method selection</td>
<td>The comparison between AHP and Yager to select the best underground mining method.</td>
</tr>
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</table>
Appendix B

Scientific journal articles on the application of MCDM methods for the choice problem in mineral processing
Table B.1 Scientific journal articles on the application of AHP for the choice problem in mineral processing.

<table>
<thead>
<tr>
<th>No.</th>
<th>Authors</th>
<th>Type of study</th>
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<tbody>
<tr>
<td>1.</td>
<td>Safari et al. [71]</td>
<td>Case study</td>
<td>Mineral processing plant location selection</td>
<td>The selection of mineral processing plant location by using AHP.</td>
</tr>
<tr>
<td>2.</td>
<td>Ataei [72]</td>
<td>Case study</td>
<td>Mineral processing plant location selection</td>
<td>The selection of alumina cement plant location by using AHP.</td>
</tr>
<tr>
<td>3.</td>
<td>Ataei [73]</td>
<td>Case study</td>
<td>Mineral processing plant location selection</td>
<td>The selection of alumina cement plant location by using AHP.</td>
</tr>
<tr>
<td>4.</td>
<td>Rahimdel and Ataei [74]</td>
<td>Case study</td>
<td>Mineral processing equipment selection</td>
<td>The selection of the best primary crusher by using AHP.</td>
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<tr>
<td>5.</td>
<td>Owusu-Mensah and Musingwini [75]</td>
<td>Case study</td>
<td>Mineral processing equipment selection</td>
<td>The application of AHP to evaluate mined ore transport options.</td>
</tr>
<tr>
<td>6.</td>
<td>Despodov et al. [76]</td>
<td>Case study</td>
<td>Mineral processing equipment selection</td>
<td>The selection of an optimal system for ore transportation from an open-pit mine to a processing facility by using AHP.</td>
</tr>
<tr>
<td>7.</td>
<td>Andrejiová et al. [77]</td>
<td>Case study</td>
<td>Mineral processing equipment selection</td>
<td>The selection of suitable and feasible modifications of belt conveyor construction parameters by using AHP.</td>
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<tr>
<td>8.</td>
<td>Kursunoglu et al. [78]</td>
<td>Case study</td>
<td>Mineral processing method selection</td>
<td>The selection of the best leaching method for processing lateritic nickel ore by using AHP.</td>
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Table B.2 Scientific journal articles on the application of TOPSIS for the choice problem in mineral processing.

<table>
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<th>Type of study</th>
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<tr>
<td>1.</td>
<td>Baral et al. [79]</td>
<td>Concept and case study</td>
<td>Mineral processing operation parameter selection</td>
<td>Optimization of leaching parameters for the extraction of rare earth metal using TOPSIS.</td>
</tr>
<tr>
<td>2.</td>
<td>Montazeri and Taji [80]</td>
<td>Case study</td>
<td>Mineral processing method selection</td>
<td>Ranking and comparing of traditional and industrial coke making by using TOPSIS.</td>
</tr>
</tbody>
</table>

Table B.3 Scientific journal articles on the comparison of MCDM methods for the choice problem in mineral processing.

<table>
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<tbody>
<tr>
<td>1.</td>
<td>Yavuz [81]</td>
<td>Concept and case study</td>
<td>Mineral processing plant location selection</td>
<td>The selection of a plant location in the natural stone industry by comparing Yager with AHP.</td>
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<td>2.</td>
<td>Bakhtavvar and Lotfian [82]</td>
<td>Case study</td>
<td>Mineral processing plant location selection</td>
<td>The selection of a mineral processing plant site location in a copper mine industry by comparing FAHP with gray MCDM.</td>
</tr>
</tbody>
</table>
Table B.4 Scientific journal articles on the application of hybrid MCDM methods for the choice problem in mineral processing

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<th>Type of selection problem</th>
<th>Problem addressed</th>
</tr>
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<tbody>
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<td>1</td>
<td>Bejari et al. [83]</td>
<td>Case study</td>
<td>Mineral processing plant location selection</td>
<td>The selection of chromite processing plant location using FAHP.</td>
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<tr>
<td>2</td>
<td>Rahimdel and Karamoozian [84]</td>
<td>Case study</td>
<td>Mineral processing equipment selection</td>
<td>The selection of the best primary crusher by using FTOPSIS.</td>
</tr>
<tr>
<td>3</td>
<td>Kostovic and Gligoric [85]</td>
<td>Concept and case study</td>
<td>Mineral processing operation parameter selection</td>
<td>The selection of the optimum sulfide mineral collector and dose rate in a flotation study using FTOPSIS.</td>
</tr>
<tr>
<td>4</td>
<td>Basçetin and Kesimal [86]</td>
<td>Case study</td>
<td>Mineral processing equipment selection</td>
<td>The selection of an optimal transportation system to a power station by using Yager.</td>
</tr>
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<td>5</td>
<td>Safari et al. [87]</td>
<td>Concept and case study</td>
<td>Mineral processing plant location selection</td>
<td>The selection of plant location by using FTOPSIS.</td>
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<td>6</td>
<td>Shahab et al. [88]</td>
<td>Concept and case study</td>
<td>Mineral processing method selection</td>
<td>The selection of Alunite processing method by using an integrated Delphi, AHP, and FTOPSIS.</td>
</tr>
<tr>
<td>7</td>
<td>Savic et al. [89]</td>
<td>Concept and case study</td>
<td>Mineral processing operation parameter selection</td>
<td>The determination of the optimal mixture of zinc concentrates for zinc factory by using an integrated AHP, OEW, and PROMETHEE/GAIA.</td>
</tr>
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<td>8</td>
<td>Zavadskas et al. [90]</td>
<td>Concept and case study</td>
<td>Mineral processing method selection</td>
<td>The selection of lead-zinc flotation circuit design by using a combined WASPAS and SVN.</td>
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<tr>
<td>9</td>
<td>Stanujkic et al. [91]</td>
<td>Concept and case study</td>
<td>Mineral processing method selection</td>
<td>The selection of a grinding circuit (GC) design based on ratio system part of MOORA.</td>
</tr>
</tbody>
</table>
Appendix C

A comparison of a normalisation procedure used in IC-FSE with another existing method based on the Shannon Entropy method
For deriving fuzzy weights of objective criteria from a decision matrix by means of the Shannon Entropy method, a number of normalisation procedures have been reported in the literature. This section assessed the two most frequently used procedures for normalising a fuzzy decision matrix based on the Shannon Entropy method, namely the Voogd [92] and Weitendorf [93] ratios [94].

The Voogd ratios are formulated in the equation below:

$$\tilde{r}_{ji} = \left( \frac{x_{ji}}{\sum_{j=1}^{n} x_{ji}} \right).$$  \hspace{1cm} (C.1)

The Equations (5.12)–(5.14) can then substituted by Equation (C.1) in order to obtain fuzzy criteria weights based on the Voogd normalisation procedure.

The Weitendorf ratios are expressed in Equations (C.2) and (C.3) below.

(a) For the beneficial criteria that should be maximised, such as capacity factor ($C_1$) and prospective jobs creation ($C_6$), the following equation is applied:

$$\tilde{r}_{ji} = \frac{x_{ji} - \min(x_{ji})}{\max(x_{ji}) - \min(x_{ji})}. \hspace{1cm} (C.2)$$

(b) For the non-beneficial criteria that should be minimised, such as water consumption ($C_2$), GHG emissions ($C_3$), area requirement ($C_4$) and LEC ($C_5$), the equation below is then applied:

$$\tilde{r}_{ji} = \frac{\max(x_{ji}) - x_{ji}}{\max(x_{ji}) - \min(x_{ji})}. \hspace{1cm} (C.3)$$

Equations (C.2) and (C.3) were used to substitute Equations (5.12)–(5.14) in order to obtain fuzzy criteria weights based on the Weitendorf normalisation procedure. A fundamental problem arises when the lowest and/or most likely and/or highest numbers in TFN are the same. The denominator in such a scenario will be zero and the fraction is therefore undefined. In this situation, it is impossible to compute further calculations. For example, in the study presented here, the lowest number in TFN for onshore wind, CSP and PV with respect to area requirement is 10 $m^2$/kW. For this reason, this work only compared the results obtained from the Voogd normalisation procedure.

Table C.1 shows the results for fuzzy entropy values, fuzzy entropy weights, and crisp entropy weights that are obtained from IC-FSE by using the Voogd normalisation procedure. Based on the crisp entropy weights, the rank of each criterion is water consumption $\succ$ area requirement $\succ$ GHG emissions $\succ$ levelised energy cost $\succ$ prospective jobs creation $\succ$ capacity factor, which is similar to the rank that was obtained from the Nijkamp and Delft’s normalisation procedure shown in Table 5.5.2. However, it is worth noting that by employing the Nijkamp and Delft’s normalisation procedure in Equations (5.12)–(5.14), the distance between the middle values and lower and upper values of the fuzzy entropy values and the fuzzy entropy weights are smaller than those obtained by applying the Voogd’s normalisation procedure. Figure C.1 shows the comparison of fuzzy entropy weights ($\tilde{w}_i$) of water consumption and area requirement that are obtained from the Nijkamp and Delft’s normalisation procedure and the Voogd normalisation procedure. As can be seen from Figure C.1, the smaller area of TFN was produced from the Nijkamp and Delft’s normalisation. Thus, the Nijkamp and Delft’s normalisation results in less vague fuzzy entropy weights than those acquired by the Voogd’s normalisation.
A comparison of a normalisation procedure used in IC-FSE with another existing method based on the Shannon Entropy method

Table C.1 The fuzzy entropy values ($\tilde{e}_i$), fuzzy entropy weights ($\tilde{w}_i$), defuzzified entropy weights ($ObW_i$) and the ranking of criteria obtained from the Voogd normalisation procedure.

<table>
<thead>
<tr>
<th></th>
<th>Capacity factor ($C_1$)</th>
<th>Water consumption ($C_2$)</th>
<th>GHG emissions ($C_3$)</th>
<th>Area requirement ($C_4$)</th>
<th>Levelised energy cost ($C_5$)</th>
<th>Prospective jobs ($C_6$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{e}_i$</td>
<td>(0.8, 0.94, 0.96)</td>
<td>(0.2, 0.6, 0.71)</td>
<td>(0.32, 0.89, 0.93)</td>
<td>(0.09, 0.86, 1.0)</td>
<td>(0.38, 0.86, 0.96)</td>
<td>(0.55, 0.92, 0.96)</td>
</tr>
<tr>
<td>$\tilde{w}_i$</td>
<td>(0.01, 0.06, 0.31)</td>
<td>(0.09, 0.42, 0.81)</td>
<td>(0.02, 0.12, 0.63)</td>
<td>(0.0, 0.15, 0.66)</td>
<td>(0.01, 0.15, 0.59)</td>
<td>(0.01, 0.08, 0.51)</td>
</tr>
<tr>
<td>$ObW_i$</td>
<td>(0.083)</td>
<td>(0.285)</td>
<td>(0.166)</td>
<td>(0.174)</td>
<td>(0.162)</td>
<td>(0.13)</td>
</tr>
<tr>
<td>Rank</td>
<td>6</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Fig. C.1 The comparison of fuzzy entropy weights ($\tilde{w}_i$) of of water consumption and area requirement criteria that are obtained from the Nijkamp and Delft’s normalisation procedure and the Voogd normalisation procedure.
Appendix D

A comparison of IC-FSE with another existing method based on ordered triangular fuzzy numbers
The main differences between the proposed IC-FSE and the existing method based on ordered fuzzy numbers [95] are in the normalisation procedure that is formulated in Equations (5.12)–(5.14), the computation of the fuzzy entropy values \( \tilde{e}_i \) that is formulated in Equations (5.15)–(5.19), and the computation of the fuzzy entropy weights \( \tilde{w}_i \) that is formulated in Equations (5.20)–(5.23). In the Kacprzak’s method, the Voogd’s normalisation procedure is applied to normalise elements of TFN in a fuzzy decision matrix. Equations (D.1)–(D.3) show the algorithms for finding \( r_{ji}^K = \left( r_{ji}^L, r_{ji}^M, r_{ji}^U \right) \), \( i = 1, \ldots, n \) and \( j = 1, \ldots, m \) (where the superscript \( K \) represents the fuzzy arithmetic based on the Kacprzak method and the sub-subscript \( L, M, \) and \( U \) refer to the lowest, middle and highest boundary values).

\[
\begin{align*}
    r_{ji}^K &= \left\{ \frac{x_{ji}}{\sum_{j=1}^{m} (x_{ji})} \right\}, \quad (D.1) \\
    r_{ji}^K &= \left\{ \frac{x_{ji}}{\sum_{j=1}^{m} (x_{ji})} \right\}, \quad (D.2) \\
    r_{ji}^K &= \left\{ \frac{x_{ji}}{\sum_{j=1}^{m} (x_{ji})} \right\} \quad (D.3)
\end{align*}
\]

In addition, the algorithms for obtaining fuzzy entropy values, \( \tilde{e}_i = \left( e_{iL}^K, e_{iM}^K, e_{iU}^K \right) \), based on the Kacprzak’s method are formulated in the following equations:

\[
\begin{align*}
    e_{iL}^K &= \left\{ \frac{\sum_{j=1}^{m} \left( r_{ji}^L \times \ln \sum_{j=1}^{m} r_{ji}^L \right)}{\ln m} \right\}, \quad (D.4) \\
    e_{iM}^K &= \left\{ \frac{\sum_{j=1}^{m} \left( r_{ji}^M \times \ln \sum_{j=1}^{m} r_{ji}^M \right)}{\ln m} \right\}, \quad (D.5) \\
    e_{iU}^K &= \left\{ \frac{\sum_{j=1}^{m} \left( r_{ji}^U \times \ln \sum_{j=1}^{m} r_{ji}^U \right)}{\ln m} \right\} \quad (D.6)
\end{align*}
\]

while the algorithms for computing the fuzzy entropy weight \( \tilde{w}_i = \left( w_{iL}^K, w_{iM}^K, w_{iU}^K \right) \) of each criterion are expressed in the following equations:

\[
\begin{align*}
    w_{iL}^K &= \left\{ \frac{1 - e_{iL}^K}{\sum_{i=1}^{n} e_{iL}^K} \right\}, \quad (D.7) \\
    w_{iM}^K &= \left\{ \frac{1 - e_{iM}^K}{\sum_{i=1}^{n} e_{iM}^K} \right\}, \quad (D.8) \\
    w_{iU}^K &= \left\{ \frac{1 - e_{iU}^K}{\sum_{i=1}^{n} e_{iU}^K} \right\} \quad (D.9)
\end{align*}
\]

Table D.1 shows the fuzzy entropy values \( \tilde{e}_i \) and fuzzy entropy weights \( \tilde{w}_i \) obtained from the Kacprzak’s method. As can be observed from Table D.1, the lower, middle and upper values of \( \tilde{e}_i \) and \( \tilde{w}_i \) do not follow the notion of TFN that is presented in Equation (5.1). Thus, the results do not represent a fuzzy number; in fact, they are just a triplet of real numbers. In addition, since such triplet of real numbers does not represent a TFN, the fuzzy entropy weights \( \tilde{w}_i \) cannot be represented graphically and cannot be compared to those obtained from IC-FSE. It can be concluded that by taking into account the interactions among the elements in the calculations, the application of constrained fuzzy arithmetic in IC-FSE results in a correct estimation of the fuzzy entropy weights \( \tilde{w}_i \).
Table D.1 The fuzzy entropy values ($\tilde{e}_i$) and fuzzy entropy weights ($\tilde{w}_i$) obtained from the Kacprzak’s method.

<table>
<thead>
<tr>
<th></th>
<th>Capacity factor $(C_1)$</th>
<th>Water consumption $(C_2)$</th>
<th>GHG emissions $(C_3)$</th>
<th>Area requirement $(C_4)$</th>
<th>Levelised energy cost $(C_5)$</th>
<th>Prospective jobs $(C_6)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\tilde{e}_i)$</td>
<td>(0.86, 0.94, 0.93)</td>
<td>(0.33, 0.6, 0.71)</td>
<td>(0.89, 0.89, 0.93)</td>
<td>(1.0, 0.86, 0.72)</td>
<td>(0.96, 0.86, 0.86)</td>
<td>(0.96, 0.92, 0.95)</td>
</tr>
<tr>
<td>$(\tilde{w}_i)$</td>
<td>(0.14, 0.06, 0.08)</td>
<td>(0.67, 0.42, 0.32)</td>
<td>(0.11, 0.12, 0.08)</td>
<td>(0.0, 0.15, 0.31)</td>
<td>(0.04, 0.15, 0.15)</td>
<td>(0.04, 0.08, 0.06)</td>
</tr>
</tbody>
</table>
Appendices references


Appendices references


Appendices references


