Essays on Financial Intermediation

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Doctor of Philosophy

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Declaration of Originality

I certify that this thesis and the research to which it refers, are the product of my own work, and that any ideas or quotations from the work of other people, published or otherwise, are fully acknowledged in accordance with the standard referencing practices of the discipline.
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ABSTRACT
Zaici Li

In the first two chapters of the dissertation, I address the research question: why does the issuance of private-label mortgage back securities in the US remain stagnant while all other types of securitization recovered with the economic fundamental years ago? The first chapter presents the static model and the second chapter presents the dynamic model. The third chapter addresses the research question: Why do credit and liquidity conditions vary at the same time and how do they interact along with the business cycle?

I provide a dynamic model of shadow banking and securitization in which short-lived shocks can generate slow recovery. In the model, intermediaries’ capital and risk perceptions play key roles in driving securitization dynamics. The model features a self-reinforcing effect between high risk perceptions and low securitization. High risk perceptions reduce intermediaries’ willingness to hold junior tranches, leading to low levels of securitization. Since securitization generates a positive informational externality, low issuance slows down learning and risk perception remains high, resulting in very slow recovery. The model can explain why the private label residential mortgage backed securities (PL RMBS) market remains stagnant. During the financial crisis, the massive write-downs of subprime mortgage related securities not only wiped out intermediaries’ capital, leading to the collapse of issuance in all major securitization markets but also sharply increased risk perception in PM RMBS market. The higher risk perceptions in PL RMBS market have kept issuance low until now while other securitization markets recovered after intermediaries’ recapitalization.

Why do credit and liquidity conditions vary at the same time and how do they interact along with the business cycle? To address these questions, we propose a dynamic equilibrium model where risk-averse banks originate long-term projects and distribute them to risk-neutral investors. The key friction is the information asymmetry between banks and investors. During recession, credit quality deteriorates, making it more costly for banks with good projects to distribute and increasing the severity of information asymmetry.

JEL classification: E44, G01, G23

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I. Chapter 1: The Static Model of Securitization

A. Introduction

The boom and bust of shadow banking plays a central role in the recent credit cycles. In the shadow banking system, financial intermediaries originate and trade risky loans, assemble them into diversified portfolios and tranche the portfolios to create safe assets outside regulated banking system and this process is referred as securitization. Since the financial crisis, securitization experienced a deep downturn with slow recovery. In Europe, the issuance of securitized assets remains low until now (Figure 1). In the US, securitization recovery is faster but has not rebounded to pre-crisis levels. Meanwhile the issuance of residential mortgage backed securities (RMBS) shows little sign of recovery (Figure 2). These stylized facts remain puzzling. Why does securitization remain stagnant after economic fundamentals gain momentum? Why does the recovery occur in some securitization markets but not occur in the other markets? To provide a plausible explanation, I propose a parsimonious dynamic model of shadow banking and securitization to rationalize both slow recovery and heterogeneous recovery patterns as observed since the financial crisis.

The model can also generate the dynamics of risk-taking, total issuance volume and safe debt creation, which are consistent with the stylized facts in securitization markets.

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2 As securitization is believed to fuel the 2007-2008 financial crisis, there are many policy changes, such as reform of mortgage markets and money market funds, capital requirement. These changes definitely have an impact on the recovery of securitization markets. In this paper, I provide a model to show that even in absence of all these reforms and policy changes, the recovery in securitization markets can be very slow.
loans into diversified portfolios and issue safe (senior) tranches to outside investors against the underlying portfolios. They hold the junior tranches (toxic wastes), hence providing insurance against the aggregate risk. Second, intermediaries learn about the likelihood of aggregate risk through securitization and the learning positively depends on issuance. Securitization transforms illiquid loans into securities traded in financial markets where agents can learn from each other. Therefore when issuance is large learning is faster.

![Securitization Market Issuance](image)

**Figure 2.** The graph plots the issuance index of residential mortgage backed securities, commercial mortgage backed securities and collateralized debt obligation and collateralized loan obligation from 2000 to 2018 in the US. The issuance index on Y-axis is the issuance volume each year normalized by that in 2000. Source: The Securities Industry and Financial Markets Association

The combination of the two assumptions generates the following three key insights:

- Securitization depends on both intermediaries’ perception of aggregate risk (risk perception for convenience) and capital, which represents their willingness and ability to provide insurance respectively. Both low risk perception and high capital are necessary to sustain high issuance volume. But either an increase in risk perception or a decrease in intermediaries’ capital can cause the collapse of issuance.

- The economy features a dynamic self-reinforcing effect between high risk perception and low securitization. High risk perception discourages intermediaries’ willingness to hold the junior tranches, leading to low level of securitization. Since learning positively depends on issuance volume, learning slows down and risk perception remains high in the future. Therefore the recovery from high risk perception can very slow due to the self-reinforcing effect.

- Low level of capital constrains intermediaries’ ability to provide insurance, leading to low securitization. In this scenario, recovery needs capital accumulation. If risk perception is low, intermediaries are confident about the economic fundamentals. The speed of capital accumulation is fast despite low level of capital. After several periods of capital accumulation,
learning resumes and issuance recovers.

To sum up, the model provides a belief-driven (risk perception) explanation on the slow recovery of securitization and can rationalize heterogeneous recovery patterns based on capital and risk perception channels. The mechanism offers a plausible explanation on the stagnancy of PL RMBS since the financial crisis. Intermediaries such as AIG and Leman Brothers operated in all major securitization markets. During financial crisis, these institutions were insolvent due to the massive write-downs on subprime mortgage related securities hence they were not able to provide insurance, leading to collapse of issuance in major securitization markets. 

As documented by Benmelech & Dlugosz (2010), the downgrade of subprime mortgage backed securities was much more severe and widespread than other types of securities, implying sharp increase of risk perception in PL RMBS market. The self-reinforcing effect between high risk perception and low securitization keeps PL RMBS low while other markets recovered after intermediaries were recapitalized.

I start with a static equilibrium model of securitization. In the model, an intermediary can invest in low-risk and high-risk loans and can finance these loans with its own capital as well as issuing debt. In line with actual empirical evidence (Bernanke, Bertaut, Demarco & Kamin (2011), Krishnamurthy & Vissing-Jorgensen (2012) and Pozsar (2014)), outside investors are only interested in riskless debt and their demand is large (investors are assumed to be infinitely risk-averse and with large endowment). Both types of loans are subject to institution-specific idiosyncratic risk but have different exposure to the aggregate risk. The low-risk and high-risk loans can be understood as prime and subprime mortgages and the loss-given-default of high-risk loans is higher provided hit by the aggregate shock. To meet the demand for safe debt, intermediaries diversify their portfolios by trading loans with each other to eliminate idiosyncratic risk, similar to Diamond (1984) and issue riskless debt to investors. Essentially, intermediaries are providing insurance against aggregate risk by hold the toxic wastes (junior tranches), whereby they pledge the returns of underlying loans in the worst state as collateral for risk-free debt and earn the upside in better states of the world.

The static model results show that intermediaries’ risk perception and capital play key roles in determining intermediaries’ insurance provision hence the level of securitization. Risk perception can be understood as intermediaries’ willingness to provide insurance while capital can be understood as their ability. My analysis of the static equilibrium yields the following implications:

- If risk perception is low, intermediaries are optimistic about the fundamentals. Provided they are not constrained by capital, intermediaries expand their balance sheet by securitizing high-risk loans (after exhausting limited supply of low-risk loans) and increase the exposure to aggregate risk, creating more riskless debt. The key result is that both low risk perception and high capital are necessary to sustain high level of securitization.

3In Benmelech & Dlugosz (2010), they document that between 2007 and 2008 64% downgrades of structural financial products were related to home equity loans and first mortgages. Most of the downgrades were more than eight notches, meaning from AAA directly to junk bond. In other securitization markets, the downgrades were usually less than four notches. The phenomenon provides a suggestive evidence that the increase of risk perception is larger than other markets.
If risk perception is high, intermediaries are pessimistic about the underlying portfolio’s fundamentals, discouraging their willingness to provide insurance. If intermediaries fall short of capital, insurance provision is constrained. Either scenario can lead to the contraction of intermediaries’ balance sheet, reducing the riskless debt issued to the investors. In other words, either an increase in risk perception or a decrease in capital can result in the collapse of securitization.

When risk perception is above certain threshold, intermediaries are so pessimistic that the expected return of junior tranches is lower than the opportunity cost of their capital. Intermediaries are not willing to provide insurance regardless of the capital position. Therefore securitization does not occur in the equilibrium.

Then I consider a dynamic version of the static equilibrium where intermediaries’ capital and risk perception are endogenized. As shown by Eisfeldt, Lustig & Zhang (2017), securitized assets such as MBS and CDOs are complex assets, whose risk factors are not well understood. As intermediaries are exposed to the aggregate risk (factors) in securitization markets, they progressively learn the risk through signals generated by securitization. The key assumption in the model is that the precision of the signals is increasing in the size of intermediaries’ balance sheet (total volume of loans securitized). This assumption fits particularly well securitization, which converts illiquid loans into securities traded in financial markets. Market participants form expectation and trade, facilitating social learning of aggregate risk factors. When more loans are securitized, intermediaries can get more precise estimation of aggregate risk and more confident to update their risk perception. As a result, learning is fast when issuance is large.

The dynamic model features a dynamic self-reinforcing interaction between high risk perception and low securitization. As illustrated by Figure 3, high risk perception discourages intermediaries’ willingness to provide insurance, leading to low level of securitization. Since learning positively depends on issuance, intermediaries are less confident to adjust their risk perception. Learning slows down and high risk perception remains in the future. The self-reinforcing effect can lead to sluggish recovery of securitization.

I consider the recovery dynamics from different levels of risk perception but the same level of capital. The model illustrates that the recovery under higher risk perception is much slower. The reason is that the self-reinforcing effect between high risk perception and low securitization results in both slow decrease in risk perception (slow learning effect) and slow capital accumulation even though the economic fundamental gains momentum.

The dynamic model also delivers several predictions about the risk-taking (defined as market share of high-risk loans), total issuance volume and safe debt creation in securitization markets.

- Together with strong cumulative performance of underlying loans, risk perception decreases and capital increases hence intermediaries provide more insurance. As a result, there are

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4There are very few papers on risk factors of securitized assets unlike risk factors in equity or bond markets. As far as my knowledge goes, the only papers analyzing pricing of securitized assets (agency MBS) are Diep, Eisfeldt & Richardson (2017) and Chernov, Dunn & Longstaff (2017). There is no paper analyzing the risk factors in non-agency MBS.
endogenous increases in risk-taking, total issuance volume and safe debt creation in securitization markets. These predictions are consistent with stylized facts in securitization markets during expansion, such as increasing market share of subprime mortgage backed MBS, increasing total issuance of securitized assets and increasing AAA bonds creation.

- The economy may feature a permanent "risk perception trap" depending on parameters. When risk perception increases above a certain level, intermediaries are not willing to provide insurance and securitization does not occur. If learning is only conducted through issuance, intermediaries are not able to adjust their risk perception which will perpetuate the pessimism.
- The model also features a transitory "capital trap". When the level of capital falls zero, intermediaries are unable to provide insurance hence securitization stops. However it is important to notice that capital trap is transitory since recapitalizing the intermediaries can revive securitization.

The paper contributes to the understanding of dynamics of securitization and shadow banking system and provides a useful framework from the supply side of securitization. In the cross-sectional dimension, the model provides a rationale behind the observed heterogeneous recovery patterns, namely issuance PL RMBS market remains stagnant since financial crisis while other securitization markets recovered years ago. In the time-series dimension, the model also sheds light on the dynamics of risk-taking, total issuance volume and safe debt creation in securitization markets.

B. Related Literature

My analysis relates to a large theoretical literature on social learning. The form of learning, depending on the level of economic activity, has been studied by Veldkamp (2005), Van Nieuwerburgh & Veldkamp (2006), Ordonez (2013), Fajgelbaum, Schaal & Taschereau-Dumouchel (2017) and Kurlat (2018). These theories focus on agents observing the level of aggregate productivity and make production, investment or trading decisions. This paper focuses instead on agents who learn risk factors in securitization markets and studies the consequence for securitization dynamics.

The paper also relates to a large literature on securitization and shadow banking. I group the literature into two groups, namely asymmetric information and moral hazard associated with
originate-to-distribute banking model and belief-driven credit cycle models. The first strand of papers includes DeMarzo & Duffie (1999), DeMarzo (2004), Parlour & Plantin (2008) and Chemla & Hennessy (2014). These papers analyze asymmetric information between originator (or sponsors) and investors and its implication on market liquidity of securitized assets. On the other hand, the key mechanism in my model is intermediaries’ learning about risk factors in securitized asset markets, which leads to slow recovery.

The second strand of literature is closely related to my paper and focuses on belief-driven credit cycle in securitized asset markets, including Gennaioli, Shleifer & Vishny (2013), Moreira & Savov (2017) and Ordonez (2018). In Moreira & Savov (2017), securitization dynamics is driven by uncertainty dynamics. When uncertainty spikes, liquidity deteriorates, leading to less issuance in the market. The paper leaves one important question unanswered: why did issuance of PL RMBS stay depressed long after measures of uncertainty (like the VIX) had recovered? I offer an explanation based on the asymmetric learning mechanism. Gennaioli et al. (2013) argue that securitization emerges as risk-sharing mechanism. The possibility of crisis arises due to neglected tail risk. However agents in my story are fully rational and securitization expansion is driven by decreasing risk perception and increasing intermediaries’ capital. The shadow banking system can collapse under rational expectation.

The paper also belongs to the macro-finance literature, an important strand of which focus on the scarcity of expert’s net worth, such as Carlstrom & Fuerst (1997), Kiyotaki & Moore (1997), He & Krishnamurthy (2013), Brunnermeier & Sannikov (2014) and Rampini & Viswanathan (2018). In these papers, the common assumption is market segmentation between experts and households. Moreover experts’ net worth is scarce and external financing is constrained by collateral constraint. During financial crisis, expert’s net worth is impaired and recovery requires accumulation of net worth by expert which is slow. The slow recovery has real effects, such as risk premia (He & Krishnamurthy (2013)) and investment (Kiyotaki & Moore (1997) and Brunnermeier & Sannikov (2014)).

In our framework, we maintain the market segmentation assumption but introduce learning (time varying risk perception) which affects capital accumulation. This distinction has important policy implication. In He & Krishnamurthy (2013) and Brunnermeier & Sannikov (2014), recapitalization always helps the economy recover from crisis since increase in expert’s net worth help mitigate capital mis-allocation. On the other hand, in our model, the effectiveness of recapitalization depends on both confidence and capital. Recapitalization helps market recovery when intermediaries fall short of capital while it’s not effective when risk perception is high. One prominent example is when the economy falls into confidence trap, recapitalizing the intermediaries doesn’t help recovery at all.

The rest of the paper is organized as follows. Section I.C describes the model setup. Section I.D characterizes the static equilibrium and presents comparative statics. Section II.A characterises the dynamics of state variables and Section II.B presents the short-run and long-run equilibrium. Section II.D discusses the main assumptions of the model and presents testable predictions from
our theoretical analysis. Section II.E concludes.

C. Model Setup

C.1. Agents and Preferences

There are two types of agents, financial intermediaries and outside investors. Each type lives for two periods and in an overlapping-generation structure. They have access to financial markets in the first period and only consume in the second period.

**Outside Investors**: In each period there is a measure-one continuum of investors. Investors have absolute preference for safe assets as in Gennaioli, Shleifer and Vishny (2013). At time $t$, their utility function is expressed as follows

$$U_{i,t} = c_t + E_t[\min_{\omega \in \Omega} c_{t+1,\omega}]$$

where $c_t$ is the consumption at time $t$ and $c_{t+1,\omega}$ is the consumption in state $\omega$ at time $t+1$. Investors are infinitely risk-averse in the sense that they value future stochastic consumption streams at their worst-case scenario. Investors have large endowments and their expected required return for future consumption is normalized to one. The assumptions of infinite risk aversion and deep pockets reflect the large demand for safe assets (shortage in the supply of safe assets), as documented by Krishnamurthy and Vissing-Jørgensen (2012, 2015) and Greenwood and Vayanos (2014).

**Financial Intermediaries**: In each period there is also a measure-one continuum of risk neutral financial intermediaries. They are endowed with $w_t < 1$ units of consumption good, which captures the idea that financial intermediary’s capital is scare. At time $t$, their utility function is characterised by

$$U_{int,t} = c_t + \frac{1}{1+r} E_t[c_{t+1,\omega}]$$

where $c_t$ and $c_{t+1,\omega}$ are intermediaries’ consumption at $t$ and $t+1$. Financial intermediaries are impatient and prefer early consumption. $r$ captures the opportunity cost of financial intermediary’s capital and reflects empirically documented feature intermediary’s capital is more expensive.

C.2. Investment Technology

There are two types of constant-return-to-scale investment technologies, referring to low-risk and high-risk loans (denoting $L$ and $H$ respectively). Here both types of loans are used as underlying investment for securitization. Both of the technologies are risky and subject to both institutional-specific idiosyncratic risk and aggregate risk. The idiosyncratic risk is the same for both types.

---

5In Section II.A, $w_t$ will be endogenized.

6Here financial intermediary’s capital refers to their net worth and should not be confounded with physical capital (capital goods that firms used for production).

7There is still debate on the social value of financial intermediary’s capital. We take the view that financial intermediary’s capital is more expensive than debt and don’t take stance on the debate.

8I call technology as assets and therefore asset and technology are interchangeably in the paper.
of loans: by investing $i_j$ amount of consumption goods at time $t$ into individual loan, at $t + 1$ intermediary $j$ obtains the amount of $\tilde{Y}_{ij}$, where $\tilde{Y}$ is a random variable with binomial distribution $\tilde{Y} = \begin{cases} Y & \text{with probability } p \\ 0 & \text{with probability } 1 - p \end{cases}$ (1)

For individual loan, it defaults with probability $1 - p$. The return on individual loan is i.i.d across intermediaries. We assume the intermediaries can only diversify idiosyncratic risk by buying loans issued by other intermediaries. Thus the intermediaries can’t diversify their idiosyncratic risk through its own projects and they have to buy those from other intermediaries.

The aggregate risk corresponds to the risk factors that are not diversifiable in securitization markets. For example, the risk factors of private label MBS include interest rate risk, prepayment risk and credit risk. In the following context, aggregate shock and aggregate risk are interchangeable for convenience. The two types of loans are both subject to aggregate risk but they have different exposure to aggregate risk, characterised by the parameter $\rho_L$ and $\rho_H$. After diversification, the return of asset pool is

$$\tilde{R}_i = \begin{cases} (1 - \rho_i)pY & \text{with probability } \theta_t \\ pY & \text{with probability } 1 - \theta_t \end{cases}$$

where $i \in \{L, H\}$

Here realization of aggregate shock is denoted as $\xi_t = 1$ and no realization denoted as $\xi_t = 0$. When the aggregate risk doesn’t realize ($\xi_t = 0$), the returns of low-risk and high-risk asset pools are the same. However, their payoff differs when the aggregate shock hits the market ($\xi_t = 1$). The loss-given-default (measured by $\rho_i$) is higher for high-risk loans. Therefore expected return of low-risk loans is higher than that of high-risk loans. So the financial intermediaries strictly prefer to securitize low-risk loans which however are in limit supply, $\int_{j \in J} i_{L,j} dj \leq 1^9$ There is no other storage technology in the model. To expand investment beyond this limit, the intermediaries must securitize high-risk loans. The high-risk loans are of infinite supply. Figure 4 shows return of investment technology in the economy.

ASSUMPTION 1: $\rho_L < \rho_H$

Assumption 1 means H-asset has higher exposure to aggregate risk.

ASSUMPTION 2: $(1 - \rho_H)pY < (1 - \rho_L)pY < 1$

Assumption 2 means when aggregate shock hits the economy, the intermediaries have to use their own money to pay back the households.

We assume that the aggregate shock realizes with probability $\theta_t$, which we will endogenize with Bayesian learning in Section II A.

---

9Mathematically this assumption is similar at least qualitatively to the situation that there is infinite supply of both high-risk and low-risk loans. But high-risk loans have no search cost while investing in low-risk loans incurs a increasing, convex search or screening cost.
DEFINITION 1: $\theta_t$ is defined as intermediaries’ perception of the aggregate risk.

I refer to $\theta_t$ as risk perception for convenience. When $\theta_t$ is high, intermediaries’ belief of aggregate risk realization is high and vice versa.

C.3. Uncertainty and Learning

Securitized assets created by private sectors such as private label MBS, ABS and CDOs are still in their early stage. Risk factors in these markets are still not well understood. Correspondingly, intermediaries must learn about these risk factors. For example, Diep et al. (2017) and Chernov et al. (2017) empirically show that prepayment risk is priced by specialized MBS investors. In the model, I don’t explicitly model risk factors instead intermediaries are learning about the likelihood of aggregate shock.

Each period, the securitization assets can perform well which is denoted by $\xi_t = 0$. Alternatively, it can be hit by the aggregate shock ($\xi_t = 1$). Initially the likelihood of aggregate shock is uncertain, but intermediaries know that the market can be strong or weak and that strong market are less prone to aggregate shock than are weak ones. More precisely, when the market is strong, the probability of aggregate risk realization is $\overline{\theta}$. When it is weak this probability is $\hat{\theta} > \theta$. Formally we have Assumption 3 as below

ASSUMPTION 3: $\hat{\theta} > \theta$

Assumption 3 implies that weak markets are more exposed to aggregate shocks than strong ones.

Throughout the paper, I assume the occurrence of aggregate shock is not directly observable. Instead intermediaries observe a noisy ex-post signal $m_t \in \{0, 1\}$ with the conditional probabilities

$$
Pr(m_t = 1|\xi_t = 1) = Pr(m_t = 0|\xi_t = 0) = \gamma(s_t)
$$

$$
Pr(m_t = 0|\xi_t = 1) = Pr(m_t = 1|\xi_t = 0) = 1 - \gamma(s_t)
$$

(2)
where \( s_t = i_{L,t} + i_{H,t} \) is the total issuance volume. The following property is imposed on the function \( \gamma(s_t) \):

**ASSUMPTION 4:** \( \gamma(s_t) \) is an increasing function of \( s_t \). For \( s_t \in [0, +\infty) \), we have \( \gamma(s_t) \in [\frac{1}{2}, 1] \).

\( \gamma(s_t) \) measures the precision of the ex-post signal. The larger \( \gamma \) is, the more precise \( m_t \) is. When \( \gamma = \frac{1}{2} \), the ex-post signal \( m_t \) is not informative to learn the realization of aggregate shock. When \( \gamma = 1 \), the realization of aggregate shock can be observed directly. The intuition behind Assumption 4 is that as more loans are securitized, a large cross-sectional sample is generated and intermediaries can get more precise estimation of the aggregate risk.

Intermediaries use past realization of the ex-post signals \( \{m_t\} \) to conduct Bayesian learning about the state of the market. At the beginning of the first period (\( t = 1 \)), they begin with prior probability \( \pi_1 \) that the securitization market is weak. For \( t > 1 \), denoted by \( \pi_t \) the updated probability that the market is weak given the history of ex-post signals and the total issuance volume \( \{m_r, s_r\}_{r=1}^{\tau} \). As shown by Figure 5, there are two layers of learning. Using the ex-post signal \( m_t \), intermediaries can infer the realization of aggregate risk \( \xi_t \), upon which the state of the economy \( \pi_t \) can be learned. The dynamics of the probability \( \pi_t \) that the securitization market is strong is one-to-one mapping with that of the updated probability of the aggregate shock

\[
\theta_t = \pi_t \bar{\theta} + (1 - \pi_t) \theta
\]  

(3)

In the dynamic model, \( \theta_t \) is one of the state variables. When the market is strong for sure (\( \pi_t = 1 \)), we have \( \theta_t = \bar{\theta} \). When the market is weak for sure (\( \pi_t = 0 \)), we have \( \theta_t = \theta \).
C.4. Timeline of the Model

Within each period $t$, the sequence of actions is the following:

- Intermediaries start with belief of aggregate risk with probability $\theta_t$
- Intermediaries give investors take-it-or-leave-it offer. Each intermediary $j$ raises $k_t$ unit of riskless debt.
- Using its own wealth $w_t$ and the resources raised $k_t$, intermediaries decide on consumption policy $c_t$, investment policies $\{i_{L,t}, i_{H,t}\}$ and tranching policies $\{k_t, e_t\}$.
- Uncertainty resolves and consumption takes place at the end of the period.

D. Static Equilibrium

Since all the agents arrive in overlapping-generation structure, it is possible to define an equilibrium of static economy that takes place each period. The equilibrium will depend on the risk perception $\theta_t$ and intermediary’s capital $w_t$ that they have upon entering the period. Investors have absolute preference for safe assets and are not endowed with diversification technology. If they invest directly in the risk loans, the expected return is zero. So the risky asset is worthless for them.

Securitization in our model refers to the process of pooling loans into diversified portfolio and tranching the pool into senior and junior tranches. Senior tranches are held by investors who demand safe asset and junior tranches are held by intermediaries who provide insurance against aggregate risk.

<table>
<thead>
<tr>
<th>Low-risk Loans $i_{L,t}$</th>
<th>Senior Tranche $k_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>High-risk Loans $i_{H,t}$</td>
<td>Junior Tranche $e_t$</td>
</tr>
</tbody>
</table>

**Table I** Financial Intermediary’s Balance Sheet

Financial intermediary’s balance sheet is illustrated as Table I. On the asset side, intermediary can invest in low-risk $i_{L,t}$ and high-risk loans $i_{H,t}$. Since financial intermediaries are risk-neutral and low-risk assets have higher expected return, they will first invest in low-risk assets until there is no supply. After that, they begin to invest in high-risk assets. On the liability side, the diversified pool of loans are divided into senior $k_t$ and junior tranches $e_t$. Trading with each other can diversify idiosyncratic risk but not the aggregate risk while tranching facilitates risk-sharing between intermediary and household. Formally we have Lemma \cite{15}
LEMMA 1: Given any underlying loan investment policies \( \{i_{L,t}, i_{H,t}\} \), senior tranche is always risk-free and held by investors. Intermediaries hold junior tranche and provide insurance against aggregate risk.

The proof of Lemma 1 is simple and heuristic. Households are infinite risk-averse and intermediaries are risk-neutral. Therefore the equilibrium allocation is intermediaries provide full insurance to households. The allocation can be implemented with the following two steps

- Intermediaries trade with each other to diversify institutional-specific idiosyncratic risk.
- Intermediaries divide the pool of loans into senior (safe) tranche held by households and junior (risky) tranche held by intermediaries.

D.1. Equilibrium Characterization

**Intermediaries’ Optimization:** At time \( t \), financial intermediaries maximize their utility by choosing consumption investment and tranching policies \( \{c_t, c_{t+1}, i_{L,t}, i_{H,t}, k_t, e_t\} \). The optimization of intermediary is

\[
\max_{\{c_t, c_{t+1}, i_{L,t}, i_{H,t}, k_t, e_t\}} \quad c_t + \frac{1}{1+r} E_t[c_{t+1}]
\]

s.t. \( c_t + e_t = w_t \)

\[
\begin{align*}
\quad & c_{t+1} = \tilde{R}_L i_{L,t} + \tilde{R}_H i_{H,t} - k_t \\
\quad & i_{L,t} + i_{H,t} = k_t + e_t \\
\quad & k_t \leq i_{L,t}(1-\rho_L)pY + i_{H,t}(1-\rho_H)pY \\
\quad & i_{L,t} \in [0, 1] \quad i_{H,t} \in [0, +\infty) \\
\quad & c_t \geq 0 \quad c_{t+1} \geq 0
\end{align*}
\] (4)

Here I explain the constraints in the optimization

- The first constraint \( c_t + e_t = w_t \) concerns intermediaries’ budget, meaning that intermediaries use their capital either for consumption \( c_t \) or holding the junior tranche \( e_t \).
- The second constraint \( c_{t+1} = \tilde{R}_L i_{L,t} + \tilde{R}_H i_{H,t} - k_t \) is intermediaries’ consumption at \( t+1 \). Idiosyncratic risk doesn’t appear here since it is eliminated through the pooling of risky loans. Consumption at \( t+1 \) equals payoff of low-risk \( \tilde{R}_L i_{L,t} \) and high-risk loans \( \tilde{R}_H i_{H,t} \) minus the repayment to investors \( k_t \). Return on safe debt is one because intermediaries are assumed to have all the bargaining power.
- The third constraint \( i_{L,t} + i_{H,t} = k_t + e_t \) is the balance sheet constraint of financial intermediary\(^{10} \) On the asset side, intermediaries securitize low-risk \( i_{L,t} \) and high-risk \( i_{H,t} \) loans. On the liability side, safe tranches \( k_t \) are issued to outside investors and junior \( e_t \) tranches are held by intermediaries.

\(^{10}\)In reality, the securitization vehicle is special purpose vehicles. They are created to isolate risk and are bankruptcy remote. The main advantage of SPV is to avoid costly bankruptcy.
• The fourth constraint $k_t \leq i_{L,t}(1-\rho_L)pY + i_{H,t}(1-\rho_H)pY$ is the collateral constraint. $k_t$ is the amount that intermediaries need to repay the investors. Intermediaries can pledge to investors their holdings of securitized loans when aggregate shock hits the economy $i_{L,t}(1-\rho_L)pY + i_{H,t}(1-\rho_H)pY$. Intermediaries essentially take a levered position of the underlying portfolio and pledge the proceeds in the worst aggregate state as collateral for riskless debt. When the aggregate shock hits the market, intermediaries must be able to repay the investors. The collateral constraint also imposes a cap on the maximum amount of safe debt intermediaries can create.

• The fifth constraint $i_{L,t} \in [0,1]$ $i_{H,t} \in [0,\infty)$ is the feasibility constraint, meaning that there is unit supply of low-risk loans while there is unlimited supply of high-risk loans.

• The last constraints $c_t \geq 0$ and $c_{t+1} \geq 0$ rule out negative consumption.

The following lemma characterises the property of collateral constraint at the optimum.

**Lemma 2:** The collateral constraint $k_t \leq i_{L,t}(1-\rho_L)pY + i_{H,t}(1-\rho_H)pY$ is always binding at the optimum.

The intuition of binding collateral constraint is that intermediaries never over-insure the outside investors. They never pledge extra collateral more than the proceeds in the worst aggregate state because the intermediaries’ capital is more expensive.

Technically the intermediaries’ optimization is linear (in both objection function and constraint) therefore I get corner solutions. The equilibrium allocation varies depending on both risk perception and capital. To solve the optimization, it is useful to define expected return on intermediary’s capital (denoted as $EROC$).

$$ EROC(\theta_t, w_t) = \frac{\text{expected income from underlying loans}}{\text{intermediaries’ position in junior tranches}} - \frac{\text{repayment}}{k_t} $$

The numerator is the expected profit accrued to intermediaries. With probability $1-\theta_t$ the market performs well and payoff of the underlying loans is $pY(i_{L,t} + i_{H,t})$. With probability $\theta_t$ the aggregate shock hits the market. The payoff of underlying loans is $(1-\rho_L)pYi_{L,t} + (1-\rho_H)pYi_{H,t}$. $k_t$ is the repayment to investors and $e_t$ is intermediaries’ holding of junior tranches.

$EROC$ measures the ex-ante profitability of the junior tranches. Since intermediaries are risk neutral, they are willing to hold the risky junior tranche provided the expected profit is larger or equal than the opportunity cost of their capital. Formally, the following property holds

**Lemma 3:** Intermediaries’ consumption can be characterised by the comparison between intermediaries’ $EROC_t$ and the opportunity cost of their capital $1+r$.

• When $EROC_t < 1+r$, intermediaries strictly prefer to consume at time $t$ than to hold the junior tranches. There is no insurance provision by intermediaries. We have $c_t = w_t$ and $e_t = 0$. 

17
• When $EROC_t = 1 + r$, intermediaries are indifferent between consumption at $t$ and holding junior tranche. We have $e_t \in [0, w_t]$ and $c_t = w_t - e_t$.

• When $EROC_t > 1 + r$, intermediaries strictly prefer to hold junior tranche and they provide full insurance. We have $c_t = 0$ and $e_t = w_t$.

In the model, the infinitely risk-averse investors value risky assets at the lowest possible value and are not endowed with diversification technology, implying that the only way to transfer resource to next period is through securitization of risky loans. Therefore when $EROC_t \leq 1 + r$, no loans are securitized because intermediaries are not willing to hold the junior tranches. In other words, securitization is only possible provided intermediaries are willing to provide insurance by holding junior tranches ($EROC_t \geq 1 + r$). Combining equation 5 with the collateral constraint, we get

$$EROC(\theta_t, w_t) = \frac{(1 - \theta_t)(\rho_L i_{L,t} + \rho_H i_{H,t})pY}{e_t}$$

From equation 6, it is easy to see the following property holds

LEMA 4: Given any investment profile $\{i_{L,t}, i_{H,t}\}$, $EROC$ is decreasing in both risk perception $\theta_t$ and intermediaries’ position in junior tranches $e_t$.

The proof is straight forward.

The next step is to analyze how the investment profile $\{i_{L,t}, i_{H,t}\}$ on the asset side of intermediaries’ balance sheet affects $EROC(\theta_t, w_t)$. As the expected return of low-risk loans is higher than that of high-risk loans, investment in high-risk loans lowers the overall expected return of the underlying pool of loans. To see this, the expected return of underlying pool of loans $E_t(\tilde{R})$ can be written as the weighted average expected return of low-risk and high risk loans.

$$E_t(\tilde{R}) = E_t(\tilde{R}_L) \cdot \frac{i_{L,t}}{i_{L,t} + i_{H,t}} + E_t(\tilde{R}_H) \cdot \frac{i_{H,t}}{i_{L,t} + i_{H,t}}$$

Intermediate always prefer to securitize low-risk loans first until the unit supply is exhausted. If intermediaries only invest in low-risk loans, the expected return of underlying pool of loans is $E_t(\tilde{R}_L)$. As more high-risk loans are securitized, $E_t(\tilde{R})$ decreases and gets closer to $E_t(\tilde{R}_H)$. Essentially intermediaries are risk neutral and hold a lever position of the underlying pool of loans hence high $E_t(\tilde{R})$ increases the profitability of the junior tranches $EROC$. Then based on the risk perception $\theta_t$, we can get three risk perception regimes and they are characterised as follows

PROPOSITION 1: Suppose intermediary’s capital is strictly positive $w_t > 0$, there are two thresholds $\theta_L$ and $\theta_H$ satisfying $\bar{\theta} \leq \theta_L < \theta_H \leq \bar{\theta}$ such that, in the equilibrium, there are three risk perception regimes depending on intermediaries’ risk perception $\theta_t$.

- In the low risk perception regime $\theta_t \in [\bar{\theta}, \theta_L]$, intermediaries are so optimistic that they always prefer to hold the junior tranches than to consume today. Hence intermediaries invest all their
capital \( w_t \) into the junior tranches \( e_t \). Formally we have

\[
EROC(\theta_t, w_t) > 1 + r \quad \text{and} \quad e_t = w_t
\]

- In the medium risk perception regime \( \theta_t \in (\theta_L, \theta_H) \), \( EROC(\theta_t, w_t) \) is weakly larger than its opportunity cost

\[
EROC(\theta_t, w_t) \geq 1 + r
\]

- In the high risk perception regime \( \theta_t \in (\theta_H, \bar{\theta}) \), intermediaries are so pessimistic that they always prefer to consume today than to hold the junior tranches. Intermediaries are not willing to provide insurance, hence securitization does not occur. Formally we have

\[
EROC(\theta_t, w_t) < 1 + r \quad \text{and} \quad e_t = 0
\]

A graphic illustration of the three risk perception regimes is

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( \theta_L )</th>
<th>( \theta_H )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Risk Perception</td>
<td>Medium Risk Perception</td>
<td>High Risk Perception</td>
</tr>
<tr>
<td>( EROC &gt; 1 + r )</td>
<td>( EROC \geq 1 + r )</td>
<td>( EROC &lt; 1 + r )</td>
</tr>
</tbody>
</table>

**Figure 6. Market Confidence Regimes**

The details of proof are described in the Appendix and here we presents the intuitions. Intermediaries always prefer to securitize low-risk loans due to their higher expected return. \( \theta_H \) is defined as the maximum risk perception, at which intermediary is willing to hold the junior tranche when only low-risk loans are securitized. When risk perception is too high \( \theta_t > \theta_H \), securitization can’t be sustained because intermediaries are not willing to provide insurance against correlation risk as \( EROC(\theta_t, w_t) < 1 + r \). Mathematically, \( \theta_H \) is defined by the following system of equations

\[
\begin{align*}
    k_t + e_t &= i_{L,t} \\
    k_t &= i_{L,t}(1 - \rho_L)pY \\
    e_t &= w_t \\
    \frac{[(1-\theta_H)pY + \theta_H(1-\rho_L)pY]i_{L,t} - k_t}{e_t} &= (1 + r)
\end{align*}
\]

The first equation is the balance sheet constraint and the second is collateral constraint. Here intermediaries only securitize in low-risk loans therefore \( i_{H,t} \) doesn’t appear. The third equation means all intermediaries’ capital is used to hold the junior tranche. The fourth equation means \( EROC_t = 1 + r \). The system has four linear equations with four unknowns hence under mild conditions the solution exists and is unique.

Following the same logic, suppose intermediaries only securitize high-risk loans, which have larger exposure to correlation risk. Intermediaries are willing to hold the junior tranche if the risk
perception is low enough. \( \theta_L \) is defined as the maximum risk perception at which intermediaries are willing to hold junior tranche when only high-risk loans are securitized. Mathematically, we have

\[
\begin{align*}
  k_t + e_t &= i_{H,t} \\
  k_t &= i_{H,t}(1 - \rho_H)pY \\
  e_t &= w_t \\
  \frac{(1-\theta_L)pY + \theta_L(1-\rho_H)pY}{e_t}k_t &= (1 + r)
\end{align*}
\]

The system is the same as the previous one except that only high-risk loans are securitized. Intermediaries prefer to securitize low-risk loans first. It is easy to see when \( \theta_t < \theta_L \), \( EROC(\theta_t, w_t) > 1 + r \).

In the medium risk perception regime \( \theta_t \in [\theta_L, \theta_H) \), intermediaries’ capital starts to play a role. When intermediaries are endowed with abundant capital, in the equilibrium they first securitize low-risk loans until the supply is exhausted. Then high-risk loans are securitized until \( EROC(\theta_t, w_t) = 1 + r \). Therefore we can define \( e^*(\theta_t) \) as the minimum capital level that intermediaries are not capital constrained for \( \theta_t \in [\theta_L, \theta_H) \). Mathematically we have

\[
\begin{align*}
  k_t + e^* &= i_{H,t} + i_{L,t} \\
  k_t &= i_{H,t}(1 - \rho_H)pY + i_{L,t}(1 - \rho_L)pY \\
  i_{L,t} &= 1 \\
  EROC(\theta_t, e^*) &= (1 + r)
\end{align*}
\]

the first equation is balance sheet constraint and the second equation is collateral constraint. Here both types of loans are securitized. The third equation \( i_{L,t} = 1 \), meaning unit supply of low-risk loans is exhausted. The fourth equation means intermediaries are indifferent between consumption and holding the junior tranche. Note this is not a linear system of equations because \( \theta_t \) and \( e^* \) are not linear and solving the system we can get a hyperbolic function \( e^*(\theta_t) \). This is formally established in the following proposition

**PROPOSITION 2:** Suppose intermediaries’ capital \( w_t \) is strictly positive. For any level of risk perception in the medium risk perception regime \( (\theta_t \in [\theta_L, \theta_H)) \), there exists a threshold \( e^*(\theta_t) \) such that

- **When capital \( w_t \) is higher than \( e^*(\theta_t) \), intermediaries are not capital-constrained. They provide partial insurance, investing \( e^*(\theta_t) \) in the junior tranches and consuming the rest. Formally we have**
  
  \[
  e_t = e^*(\theta_t) \quad \text{and} \quad c_t = w_t - e^*(\theta_t)
  \]

- **When capital \( w_t \) is higher than \( e^*(\theta_t) \), intermediaries are capital-constrained. They provide full insurance, investing all their capital in the junior tranches and consuming nothing. Formally**

\[\text{Here solving the system of equations, we want to get } e^*(\theta_t) \text{ while in the previous two cases, we want to express } \theta_H \text{ in terms of parameters } \{r, p, Y, \rho_L, \rho_H\}\]
we have
\[ e_t = w_t \quad \text{and} \quad c_t = 0 \]
where \( e^*(\theta_t) \) is a hyperbolic function
\[ e^*(\theta_t) = \frac{\mu}{\theta_t - \alpha} + \beta \]
and \( \mu, \alpha \) and \( \beta \) are defined in the Appendix.

It is straightforward to see when intermediaries are capital constrained \( w_t < e^*(\theta_t) \), the expected return on intermediary’s capital \( EROC(\theta_t, w_t) \) is strictly larger than its opportunity cost.

At last, equilibrium properties of securitization policies \( \{i_{L,t}, i_{H,t}\} \) are analyzed in the capital constrained region. Suppose risk perception \( \theta_t \) is smaller than \( \theta_H \), meaning intermediaries are willing to securitize high-risk loans. If intermediaries are moderately capital-constrained, high-risk loans are securitized in the equilibrium. On the other hand, if intermediaries are severely capital constrained, they are only able to provide insurance to low-risk loans. \( w^* \) is defined as the exact capital level, at which the supply of low-risk loans is exhausted and no high-risk loans are securitized. Formally we have the following proposition

**PROPOSITION 3:** In the low and medium risk perception regime where \( \theta_t \in [\underline{\theta}, \theta_H] \), there exists a threshold \( w^* \) such that

- When intermediaries’ capital \( w_t \) is higher than \( w^* \), the supply of the low-risk loans is exhausted and some high-risk loans are securitized. Formally we have
  \[ i_{L,t} = 1 \quad \text{and} \quad i_{H,t} > 0 \]
- When intermediaries’ capital \( w_t \) is lower or equal than \( w^* \), intermediaries only securitize a part of the low-risk loans and no high-risk loan is securitized. Formally we have
  \[ i_{L,t} \leq 1 \quad \text{and} \quad i_{H,t} = 0 \]

With the four thresholds derived \( \{\theta_L, \theta_H, e^*(\theta_t), w^*\} \), four regions can be obtained depending on risk perception and capital \( \{\theta_t, w_t\} \). This can be clearly illustrated in Figure 7

- In Region 1 where \( \{\theta_t, w_t\} \in (\theta_H, \bar{\theta}] \times (0, +\infty) \), risk perception is too high so intermediaries are not willing to hold junior tranche \( EROC(\theta_t, w_t) < 1 + r \). No insurance is provided hence securitization does not occur in the equilibrium even though intermediaries are endowed with large amount of capital. I name this region as no-securitization region.
- In Region 2 where \( \{\theta_t, w_t\} \in (\theta_L, \theta_H] \times (e^*(\theta_t), +\infty) \), risk perception is in the medium regime while capital is relatively abundant. Intermediaries securitize the whole supply of low-risk loans and some high-risk loans until \( EROC(\theta_t, w_t) = 1 + r \). In this region, intermediaries’ upfront consumption \( c_t \) is positive. I name this region as \( \theta \)-constrained region.
• In Region 3 where \( \{\theta_t, w_t\} \in (\theta, \theta_{H\dagger}) \times (w^*, e^*(\theta_t)) \), risk perception is in the medium or low regime while intermediaries are moderately capital-constrained, meaning that the whole supply of low-risk loans as well as some high-risk loans are securitized. But the difference with \( \theta \)-constrained region is \( EROC(\theta_t, w_t) > 1 + r \). In this region, intermediaries’ upfront consumption is zero. We name this region as moderate \( w \)-constrained region.

• In Region 4 where \( \{\theta_t, w_t\} \in (\theta, \theta_{H\dagger}) \times (0, w^*) \), risk perception is in the medium or low regime while intermediaries are severely capital-constrained. In this region, only part of low-risk loans are securitized and we have \( EROC(\theta_t, w_t) > 1 + r \), implying intermediaries’ upfront consumption is zero. We name this region as severe \( w \)-constrained region.

Figure 7. There are four equilibrium regions of securitization: Region 1 as no securitization region, Region 2 as \( \theta \)-constrained region, Region 3 as Moderate \( w \)-constrained region and Region 4 as severe \( w \)-constrained region.

D.2. Comparative Statics

In this section, we define three measures in the model on risk-taking, size and safe debt issuance as follows

• Define measure of risk-taking \( \sigma_t \) as the share of high-risk loans securitized by intermediaries

\[
\sigma_t = \frac{i_{H,t}}{i_{H,t} + i_{L,t}}
\]

\( \sigma_t \) can be mapped to market share of subprime mortgage backed MBS in reality. The higher
\( \sigma_t \), intermediaries’ capital is more exposed to aggregate risk. It measures the vulnerability of the financial sector.

- Define measure of intermediary balance sheet size \( s_t \) as total volume of loans securitized by intermediaries.

\[
s_t = i_{L,t} + i_{H,t}
\]

\( s_t \) can be mapped to total issuance volume of PL RMBS in MBS market. Higher \( s_t \) means larger volume of issuance.

- Define measure of safe debt issuance \( k_t \) as the size of senior tranche. \( k_t \) refers to AAA tranches issued in PL RMBS market.

In this section, I present comparative statics of the three measures defined above in different regions, which are derived in the Appendix. The tables show sign of derivatives of \( \{\sigma_t, s_t, k_t\} \) with respect to \( \{\theta_t, w_t\} \) within each region.

- Table II summarizes the comparative statics in No-securitization Region (Region 1). Risk perception is too high and no loans are securitized in the equilibrium. A marginal change in \( \{\theta_t, w_t\} \) has no effect on \( \{\sigma_t, s_t, k_t\} \).

**Table II  Comparative Statics in No-securitization Region**

<table>
<thead>
<tr>
<th>Derivative</th>
<th>( \partial \sigma_t )</th>
<th>( \partial s_t )</th>
<th>( \partial k_t )</th>
<th>( \partial \sigma_t )</th>
<th>( \partial s_t )</th>
<th>( \partial k_t )</th>
<th>( \partial \sigma_t )</th>
<th>( \partial s_t )</th>
<th>( \partial k_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sign</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

- Table III summarizes the comparative statics in \( \theta \)-constrained Region (Region 2). In this region, all supply of low-risk loans and some of high-risk loans are securitized. Therefore in the equilibrium we have

\[
\sigma_t = \frac{i_{H,t}}{i_{H,t} + 1}
\]

\[
s_t = i_{H,t} + 1
\]

\[
k_t = (1 - \rho_L)pY + (1 - \rho_H)pYi_{H,t}
\]

We can see that all three measures \( \{\sigma_t, s_t, k_t\} \) are strictly increasing in \( i_{H,t} \). In \( \theta \)-constrained region, capital \( w_t \) is relative abundant and risk perception \( \theta_t \) is the binding constraint. Marginal change in \( w_t \) has no effect on \( i_{H,t} \) and marginal decrease in \( \theta_t \) will increase \( i_{H,t} \) in the equilibrium, hence increasing all \( \{\sigma_t, s_t, k_t\} \).

- Table IV summarizes the comparative statics in Moderate \( w \)-constrained Region (Region 3). In this region, all supply of low-risk loans and some of high-risk loans are securitized. The equations for \( \{\sigma_t, s_t, k_t\} \) are exactly the same as previous case and they are all increasing function of \( i_{H,t} \). The difference is that capital \( w_t \) becomes the binding constraint in Moderate
Table III  Comparative Statics in $\theta$-constrained Region

<table>
<thead>
<tr>
<th>Derivative</th>
<th>$\frac{\partial \sigma_t}{\partial \theta_t}$</th>
<th>$\frac{\partial s_t}{\partial \theta_t}$</th>
<th>$\frac{\partial k_t}{\partial \theta_t}$</th>
<th>$\frac{\partial \sigma_t}{\partial \theta_t}$</th>
<th>$\frac{\partial s_t}{\partial w_t}$</th>
<th>$\frac{\partial k_t}{\partial w_t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sign</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

$w$-constrained Region. Marginal change in $\theta_t$ has no effect on $i_{H,t}$ while marginal increase in $w_t$ will increase $i_{H,t}$ in the equilibrium, hence increasing all $\{\sigma_t, s_t, k_t\}$.

Table IV  Comparative Statics in Moderate $w$-constrained Region

<table>
<thead>
<tr>
<th>Derivative</th>
<th>$\frac{\partial \sigma_t}{\partial \theta_t}$</th>
<th>$\frac{\partial s_t}{\partial \theta_t}$</th>
<th>$\frac{\partial k_t}{\partial \theta_t}$</th>
<th>$\frac{\partial \sigma_t}{\partial \theta_t}$</th>
<th>$\frac{\partial s_t}{\partial w_t}$</th>
<th>$\frac{\partial k_t}{\partial w_t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sign</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
</tbody>
</table>

- Table VI summarizes the comparative statics in Severe $w$-constrained Region (Region 4). In this region, intermediaries are severely capital-constrained so only part of low-risk loans can be securitized in the equilibrium. The three measures $\{\sigma_t, s_t, k_t\}$ can be written as

$$\sigma_t = 0$$

$$s_t = i_{L,t}$$

$$k_t = (1 - \rho_L) p Y i_{L,t}$$

We can see $\{s_t, k_t\}$ are increasing in $i_{L,t}$. Marginal change in $\theta_t$ has no effect on $i_{L,t}$ while marginal increase in $w_t$ will increase $i_{L,t}$ in the equilibrium, hence increasing all $\{\sigma_t, s_t, k_t\}$.

Table V  Comparative Statics in Severe $w$-constrained Region

<table>
<thead>
<tr>
<th>Derivative</th>
<th>$\frac{\partial \sigma_t}{\partial \theta_t}$</th>
<th>$\frac{\partial s_t}{\partial \theta_t}$</th>
<th>$\frac{\partial k_t}{\partial \theta_t}$</th>
<th>$\frac{\partial \sigma_t}{\partial \theta_t}$</th>
<th>$\frac{\partial s_t}{\partial w_t}$</th>
<th>$\frac{\partial k_t}{\partial w_t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sign</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
</tbody>
</table>

- Table summarizes the comparative statics of intermediaries’ opportunity cost $r$ on the threshold values $\{\theta_L, \theta_H, e^*(\theta_t)\}$. We can see the threshold values $\{\theta_L, \theta_H, e^*(\theta_t)\}$ are decreasing with the opportunity cost of capital $r$. The intuition is that provided intermediaries’ capital is more expensive (increase in $r$) securitization becomes more costly.
Table VI  Comparative Statics in Severe \( w \)-constrained Region

<table>
<thead>
<tr>
<th>Derivative</th>
<th>( \frac{\partial \theta_L}{\partial r} )</th>
<th>( \frac{\partial \theta_H}{\partial r} )</th>
<th>( \frac{\partial \varepsilon^{\ast}(\theta_e)}{\partial r} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sign</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

D.3. Implication of Static Model

IMPLICATION 1: Insurance provision in securitization market depends on intermediaries’ risk perception and capital. Both low risk perception and high level of capital are needed to sustain high level of securitization. Either an increase in risk perception or a decrease in capital can lead to the drop of issuance volume in securitization markets.

In the model, perception of aggregate risk \( \theta_t \) negatively affects the expected return of junior tranches and measures intermediaries’ willingness to provide insurance. Intermediaries’ capital \( w_t \) measures the ability to provide insurance.

Mapping the model to reality, insurance provider such as AIG, Bear Stearns and Leman Brothers were operating in all major securitization markets. Their capital was wiped out by massive write-downs of subprime mortgage products and hence unable to provide insurance. According to our theory, this is the reason that issuance in securitization markets also fell to nearly zero during financial crisis.

IMPLICATION 2: In securitization markets, risk-taking, total issuance volume and safe debt production are increasing with intermediaries’ capital and decreasing with risk perception.

In the model, insurance provision is high when risk perception \( \theta_t \) is low and capital \( w_t \) is high. Mapping this to reality, before 2007 market participants are very optimistic and there is no insolvency concern about major financial institutions. Therefore we see large market share of subprime mortgage related products in PL RMBS and large issuance volume of both MBS and AAA tranches.

IMPLICATION 3: Stringent capital regulation on financial intermediaries increases the opportunity cost of capital \( (1 + r) \), reducing risk-taking, total issuance volume and safe debt production in securitization markets.

After financial crisis, capital requirement is strengthened and intermediaries’ capital becomes more expensive. This leads to lower level of securitization.
II. Chapter 2: The Dynamic Model of Securitization

A. Dynamics of State Variables

A.1. Intermediary Capital Dynamics

Now I present the dynamic model by introducing dynamics of key state variables \{\theta_t, w_t\}. Both agents arrive in overlapping generation structure. To endogenize capital dynamics, I assume intermediaries arrive at \( t + 1 \) inherit all the investment income from their previous generation. The capital dynamics can be characterised by the following equation

\[
    w_{t+1} = \begin{cases} 
    pY(i_{L,t} + i_{H,t}) - k_t & \text{with } 1 - \theta_{\text{true}} \\
    0 & \text{with } \theta_{\text{true}} 
    \end{cases}
\]

where \( \theta_{\text{true}} \) is the true data generation process of aggregate risk. After realization of aggregate risk, capital \( w_{t+1} \) falls to zero and securitization stops until intermediaries are recapitalized. To generate interesting dynamics, we assume that if \( w_t = 0 \) intermediaries are recapitalized with \( w \) with probability \( \kappa \). In the following proposition, expected speed of capital accumulation is defined and its properties are characterised.

**PROPOSITION 4: Speed of Capital Accumulation**

The expected speed of capital accumulation is defined as

\[
    \rho^w(\theta_t, w_t) = \frac{E_t(w_{t+1})}{w_t} = EROC(\theta_t, w_t) \frac{e_t}{w_t}
\]

The properties of \( \rho^w(\theta_t, w_t) \) are characterised as follows

- \( \rho^w(\theta_t, w_t) \) is decreasing both in \( \theta_t \) and \( w_t \)
- In \( \theta \)-constrained region, \( \rho^w(\theta_t, w_t) < 1 + r \)
- In Moderate and Severe \( w \)-constrained regions, \( \rho^w(\theta_t, w_t) > 1 + r \)

The proof is in the Appendix. The expected speed of capital accumulation can be decomposed into two parts, \( EROC(\theta_t, w_t) \) is the expected return on intermediaries’ capital and \( \frac{e_t}{w_t} \) is the fraction of capital for investment (in junior tranche). In \( \theta \)-constrained Region, intermediaries securitize loans until \( EROC(\theta_t, w_t) = 1 + r \) and consume part of their capital \( \frac{e_t}{w_t} < 1 \), hence \( \rho^w(\theta_t, w_t) < 1 + r \). In Moderate and Severe \( w \)-constrained Regions, capital is the binding constraint and we have \( EROC(\theta_t, w_t) > 1 + r \) and \( \frac{e_t}{w_t} = 1 \), hence \( \rho^w(\theta_t, w_t) > 1 + r \). We can see that expected speed of capital accumulation is higher when intermediaries are relatively constrained by capital.

A.2. Risk Perception Dynamics

As illustrated by Coval, Jurek and Strfford (2009), correlation structure of underlying loans is the key to price securitized assets. Correspondingly, intermediaries learn the likelihood of aggregate risk hitting the market. To endogenize \( \theta_t \) dynamics, standard learning is introduced as a useful
benchmark. Then we construct asymmetric learning model based on standard learning model by adding ex-post signal.

**Standard Learning**

There are two states in the economy, strong \((z_t = s)\) and weak \((z_t = w)\). The securitization market can perform well if there is no correlation shock hitting the market \((\xi_t = 0)\). Alternatively, it can be hit by correlation shock \((\xi_t = 1)\). Initially the likelihood of correlation shock is uncertain, but intermediaries know that the market can be strong or weak and that strong market are less prone to correlation risk than are weak ones. More precisely, when the market is strong, the probability of correlation risk realization is \(\bar{\theta}\). When it is weak this probability is \(\bar{\theta} > \bar{\theta}\). Formally we have Assumption 5 as below

**ASSUMPTION 5:** \(\bar{\theta} > \bar{\theta}\)

In standard learning model, history of aggregate risk realization, denoted by \(\hat{H}_t = \{\xi_0, \xi_1, \cdots, \xi_t\}\), is assumed to be observable[12] Hence intermediaries use \(\hat{H}_t\) to conduct Bayesian learning about the strength of the market. The economy begins with prior probability \(\pi_0 = Pr(z_0 = s)\) that the economy is weak. For \(t \geq 1\), let \(\hat{\pi}_t\) be the updated probability that economy is weak given the history of aggregate risk realization

\[
\hat{\pi}_{t+1} = Pr(z_{t+1} = w|\hat{H}_{t-1})
\]

When aggregate risk doesn’t realize, the probability that the economy is weak \(\hat{\pi}_{t+1}\) is revised downward to

\[
\hat{\pi}_{t+1} = d(\hat{\pi}_t) = \frac{\hat{\pi}_t(1 - \bar{\theta})}{\hat{\pi}_t(1 - \theta) + (1 - \hat{\pi}_t)(1 - \bar{\theta})} < \hat{\pi}_t
\]

where \(d()\) is the Bayesian updating function with no realization of aggregate risk. If aggregate risk realizes, the probability that the economy is weak \(\hat{\pi}_{t+1}\) is revised upward to

\[
\hat{\pi}_{t+1} = u(\hat{\pi}_t) = \frac{\hat{\pi}_t\bar{\theta}}{\hat{\pi}_t\theta + (1 - \hat{\pi}_t)\bar{\theta}} > \hat{\pi}_t
\]

where \(u()\) is the Bayesian updating function with realization of correlation risk. Thus \(\bar{\theta} > \bar{\theta}\) is a key assumption, implying weak economy are more prone to aggregate risk than strong economy. The probability that the economy is strong \(\hat{\pi}_t\) is one-to-one mapping with that of the probability of aggregate shock realization \(\hat{\theta}_t\)

\[
\hat{\theta}_t = \hat{\pi}_t\bar{\theta} + (1 - \hat{\pi}_t)\bar{\theta}
\]

In dynamic model with standard learning, \(\hat{\theta}_t\) is one of the state variables. When the market is weak for sure \((\hat{\pi}_t = 1)\), we have \(\hat{\theta}_t = \bar{\theta}\). When the market is strong for sure \((\hat{\pi}_t = 0)\), we have \(\hat{\theta}_t = \bar{\theta}\). The structure of correlation risk is summarized in Figure 8.

**Asymmetric Learning**

[12]Throughout the paper, belief \(\{\hat{\pi}_t, \hat{\theta}_t\}\) and history \(\hat{H}_t\) in standard learning model is with hat. Belief \(\{\pi_t, \theta_t\}\) and history \(H_t\) are notations for asymmetric learning model.
Figure 8. Aggregate Risk Structure in Standard Learning Model

I present elements of asymmetric learning model. Here I assume that the history of aggregate risk realization $\hat{H}_t$ is not observable. As illustrated by Figure 9, at the end of each period intermediaries observe an ex-post signal $m_t \in \{0, 1\}$ with the conditional probabilities

$$
Pr(m_t = 1|\xi_t = 1) = Pr(m_t = 0|\xi_t = 0) = \gamma(s_t)
$$

$$
Pr(m_t = 0|\xi_t = 1) = Pr(m_t = 1|\xi_t = 0) = 1 - \gamma(s_t)
$$

(10)

where $s_t = i_{L,t} + i_{H,t}$ is the total issuance volume. The following property is imposed on function $\gamma(s_t)$

ASSUMPTION 6: $\gamma(s_t)$ is an increasing function of $s_t$. For $s_t \in [0, +\infty)$, we have $\gamma(s_t) \in [\frac{1}{2}, 1]$

$\gamma(s_t)$ measures the precision of the ex-post signal. The larger $\gamma$ is, the more precise $m_t$ is. When $\gamma = \frac{1}{2}$, the ex-post signal $m_t$ is not informative to learn the realization of the shock. When $\gamma = 1$, the realization of aggregate shock can be observed directly. The intuition of Assumption 6 is when more loans are securitized, intermediaries can get more precise estimation of aggregate risk.

The difference with standard learning is summarized as follows

- As shown by Figure 9 there are two layers of learning. Using the ex-post signal $m_t$, intermediaries can infer the realization of aggregate risk $\xi_t$, upon which the state of the economy $\pi_t$ can be learned.
- Since the history of aggregate risk realization is not observable, intermediaries’ information set is different. In asymmetric learning model, we assume the history of ex-post signal and size, denoted by $H_t = \{m_\tau, s_\tau\}_{\tau=0}^t$ can be observed by the intermediaries.

Note that the history of ex-post signal alone $\{m_\tau\}_{\tau=0}^t$ is not enough to infer the state of economy since $\{s_\tau\}_{\tau=0}^t$ contains the precision of $\{m_\tau\}_{\tau=0}^t$.

LEMMA 5: Denote $\phi_1(s_t) = Pr(\xi_t = 1|m_t = 1, H_{t-1})$ and $\phi_0(s_t) = Pr(\xi_t = 0|m_t = 0, H_{t-1})$, we
have $\phi_1(s_t)$ and $\phi_0(s_t)$ are increasing functions of $s_t$. $\phi_1(s_t)$ and $\phi_0(s_t)$ can be written as

\[
\phi_1(s_t) = \frac{\gamma(s_t)\theta_t}{\gamma(s_t)\theta_t + [1 - \gamma(s_t)](1 - \theta_t)}
\]
\[
\phi_0(s_t) = \frac{\gamma(s_t)(1 - \theta_t)}{\gamma(s_t)(1 - \theta_t) + [1 - \gamma(s_t)]\theta_t}
\]

The proof is straightforward with Bayes’ Rule. We intuitively interpret $\phi_1(s_t)$ and $\phi_0(s_t)$ and their properties. Conditioning on the history of ex-post signal and size until $t \ H_{t-1} = \{m_{\tau}, s_{\tau}\}_{\tau=0}^{t-1}$, $\phi_1(s_t)$ is the probability of aggregate shock realization ($\xi_t = 1$) if the ex-post signal is one $m_t = 1$ and $\phi_0(s_t)$ is the probability of no aggregate risk realization $\xi_t = 0$ if the ex-post signal is zero $m_t = 0$. They measure the probability that ex-post signal is correct. When more loans are securitized ($s_t$ is large), the ex-post signal $m_t$ are more precise ($\gamma(s_t)$ is large) and also more informative to infer correlation risk $\xi_t$. The intermediaries are more confident about their estimation of aggregate risk. With $\phi_0(s_t)$ and $\phi_1(s_t)$, Bayesian updating with asymmetric learning can be characterised by the following proposition.

**PROPOSITION 5: Bayesian Updating with Asymmetric Learning**

When the ex-post signal $m_t = 1$, the probability that the economy is weak is revised upward to

\[
\pi_{t+1}^+ = \phi_1(s_t)u(\pi_t) + [1 - \phi_1(s_t)]d(\pi_t) \geq \pi_t
\]

When the ex-post signal $m_t = 0$, the probability that the economy is weak is revised downward to

\[
\pi_{t+1}^- = \phi_0(s_t)d(\pi_t) + [1 - \phi_0(s_t)]u(\pi_t) \leq \pi_t
\]
where \( u() \) is the Bayesian updating function with realization of aggregate risk \( \xi_t = 1 \) and \( d() \) is the Bayesian updating function without realization of aggregate risk \( \xi_t = 0 \) in the standard learning model.

The proof is done with Bayes Rule and details can be found in the Appendix. As shown by Figure 9, there are two layers of learning. The ex-post signals \( m_t \) is used to learn the realization of aggregate risk \( \xi_t \), upon which the state of the economy \( \pi_t \) is inferred. Specifically, we have

- When intermediaries observe \( m_t = 1 \), with probability \( \phi_1(s_t) \) the aggregate risk realizes \( (\xi_t = 1) \) hence the probability of weak economy is revised upward to \( u(\pi_t) \) while with probability \( 1 - \phi_1(s_t) \) there is no realization of aggregate risk \( \xi_t = 0 \) hence the probability of weak economy is revised downward to \( d(\pi_t) \). To sum up, when \( m_t = 1 \) is observed the probability of weak economy is adjusted upward to \( \pi_t^{+} \).

- When intermediaries observe \( m_t = 0 \), with probability \( \phi_0(s_t) \) there is no realization of aggregate risk \( (\xi_t = 0) \) hence the probability of weak economy is revised downward to \( d(\pi_t) \) while with probability \( 1 - \phi_0(s_t) \) aggregate risk realizes \( (\xi_t = 1) \) hence the probability of weak economy is revised upward to \( u(\pi_t) \). To sum up, when \( m_t = 0 \) is observed the probability of weak economy is adjusted downward to \( \pi_t^{-} \).

**COROLLARY 1:** When \( \gamma(s_t) = \frac{1}{2} \), the equality in equation (11) and (12) holds.

When \( \gamma(s_t) = \frac{1}{2} \), the ex-post signal \( (m_t) \) is not informative about the realization of aggregate risk \( \xi_t \). Intermediaries are not able to infer the state of the economy through the ex-post signal and stop updating \( \pi_t \). Similar to standard learning model, the probability that the economy is strong \( \pi_t \) maps one-to-one to the probability of correlation risk realization \( \theta_t \).

\[
\theta_t = \pi_t \bar{\theta} + (1 - \pi_t) \bar{\theta} \tag{13}
\]

To illustrate the asymmetric learning, we define the speed of learning \( \rho^\theta(s_t) \)

**DEFINITION 2:** The speed of learning \( \rho(s_t) \) is defined as

\[
\rho^\theta(s_t) = \frac{E_t[\pi_t^{-} - \pi_t]}{E_t[\pi_t^{+} - \pi_t]} = \frac{\theta_t[\pi_t - \pi_t^{-}] + (1 - \theta_t)[\pi_t^{+} - \pi_t]}{\theta_t[d(\pi_t) - \pi_t] + (1 - \theta_t)[u(\pi_t) - \pi_t]} \tag{14}
\]

where both conditional expectations in the numerator and denominator are taken conditional on the history of \( H_t = \{m_t, s_t\}_t^{\tau=1} \).

The properties of \( \rho(s_t) \) are characterised by the following proposition.

**PROPOSITION 6:** \textbf{Asymmetric Bayesian Learning}

Suppose \( \gamma(s_t) \) is an increasing function of \( s_t \) and the range of \( \gamma(s_t) \) is between \( \frac{1}{2} \) and \( 1 \), we have \( \rho(s_t) \in [0, 1] \) and is an increasing function of \( s_t \).

The proof is in the Appendix. Proposition 6 illustrates the key mechanism of the paper, asymmetric Bayesian learning. \( \rho^\theta(s_t) \) measures intermediaries’ learning inertia. The numerator
E[π_{t+1} - π_t] is the expected absolute change in intermediaries’ risk perception conditioning on \( H_t \). It is easy to prove that \( E_t[|π_{t+1} - π_t|] \) is increasing in \( s_t \), implying when more loans are securitized, ex-post signal \( m_t \) is more precise. Therefore intermediaries have more precise estimation of aggregate risk and are more confident to update their risk perception in response to ex-post signal. The denominator \( E_t[|π_{t+1} - π_t|] \) is the expected absolute change in intermediaries’ risk perception when the aggregate shock at \( t + 1 \) is observable. It is easy to see \( E_t[|π_{t+1} - π_t|] \) is independent of the total issuance volume \( s_t \) since there is no imprecision concern of ex-post signal. It is intuitive that

\[
E[π_{t+1} - π_t] \leq E_t[|π - π_t|] \Rightarrow \rho^θ(s_t) \in [0, 1]
\]

\( \rho^θ(s_t) \) characterises the asymmetry in learning, meaning that learning is faster when more loans are securitized and vice versa.

In the static model part, I have shown that securitization does not occur in the equilibrium due to two reasons, sufficient high risk perception or insufficient capital. To examine the dynamic effects, we have the following two definitions. Depending on the parameters, the model may have the features when risk perception is sufficiently high \( \theta_t > \theta_H \), intermediaries find it too costly to hold junior tranches. No insurance against aggregate risk is provided hence securitization does not occur in the equilibrium. If the only source of information intermediaries learn aggregate risk is securitization \( \gamma(0) = \frac{1}{2} \), future learning stops and the high risk perception perpetuates. I refer this situation as risk perception trap and formally we have the following definition.

DEFINITION 3: **Risk Perception Trap**
Securitization is trapped if the intermediaries’ risk perception is high \( (\theta_t \geq \theta_H) \) and learning stops \( \phi(0) = \frac{1}{2} \).

When intermediaries have zero capital \( w_t \), they are not able to provide insurance to households. In this scenario, securitization is trapped due to lack of capital until intermediaries are recapitalized. Formally we refer this feature as capital trap

DEFINITION 4: **Capital Trap**
Securitization is trapped due to lack of intermediary’s capital if intermediary’s capital \( w_t \) is zero.

B. **Dynamics Equilibrium**

B.1. **Short-run Dynamics**

Now we turn to the short-run dynamics of the model. Given the rare disaster nature of aggregate risk, we assume there is no aggregate shock realizing in the short run and focus on the dynamic feedback effect between risk perception and capital accumulation.

PROPOSITION 7: **Dynamic Effect between Risk Perception and Capital Accumulation**
Suppose there is no correlation shock realizing in the short run, risk perception \( \theta_t \) is below \( \theta_H \) and intermediary capital \( w_t \) is above zero. We have
1. If risk perception and intermediaries’ capital \( \{ \theta_t, w_t \} \) is in \( \theta \)-constrained region, intermediaries’ capital next period \( w_{t+1} \) is strictly decreasing in current risk perception \( \theta_t \).

2. If risk perception and intermediaries’ capital \( \{ \theta_t, w_t \} \) is in Moderate or Severe \( w \)-constrained region, risk perception next period \( \theta_{t+1} \) is strictly decreasing in current capital \( w_t \).

Proposition 7 shows the dynamic feedback effect between decreasing risk perception and intermediary capital accumulation, implying lower risk perception in current period \( \theta_t \) facilitates capital accumulation (higher capital next period \( w_{t+1} \)) and higher capital today \( w_t \) also helps reduce risk perception next period \( \theta_{t+1} \).

For 1 in Proposition 7, Suppose there is no aggregation risk realization and the evolution of intermediary capital is

\[
w_{t+1} = pY(i_{L,t} + i_{H,t}) - k_t
\]

Combining with collateral constraint \( k_t = (1 - \rho_L)pYi_{L,t} + (1 - \rho_H)pYi_{H,t} \), we can get the representation of capital accumulation

\[
w_{t+1} = (\rho_Li_{L,t} + \rho_Hi_{H,t})pY
\]

It is straightforward to see that capital accumulation is increasing in the volume of loans securitized. In \( \theta \)-constrained Region, a marginal decrease in risk perception positively affects the volume of loans securitized therefore increase capital accumulation. To sum up, when risk perception is low intermediaries are willing to provide more insurance hence more loans are securitized. Intermediaries effectively have a levered position on the underlying loans hence more capital is accumulated next period.

\[
\theta_t \uparrow \implies s_t \uparrow \implies w_{t+1} \uparrow
\]

For 2 in Proposition 7, suppose there is no realization of aggregate shock and with probability \( \gamma(s_t) \) intermediaries observe ex-post signals \( m_t = 0 \). The probability of weak economy \( \pi_{t+1} \) is adjusted downward to

\[
\pi_{t+1}^{-} = \phi_0(s_t)d(\pi_t) + [1 - \phi_0(s_t)]u(\pi_t) \leq \pi_t
\]

\( \phi_0(s_t) \) is increasing in \( s_t \), implying risk perception decreases more when more loans are securitized. In Moderate and Severe \( w \)-constrained Regions, intermediaries are constrained by capital. A marginal increase in capital positively affects the volume of loans securitized therefore decrease risk perception next period. To sum up, when capital is high intermediaries are able to provide more insurance hence more loans are securitized. Intermediaries get more precise estimation of correlation risk and are more confident to downward revise risk perception.

\[
w_t \uparrow \implies c_t \uparrow \implies \theta_{t+1} \uparrow
\]

PROPOSITION 8: When \( \{ \theta_t, w_t \} \in [\theta_L, \theta_H] \times (0, +\infty) \), risk-taking \( (\sigma_t) \), size of securitized loans \( (s_t) \) and safe debt issuance \( (k_t) \) in shadow banking grow as long as the economy is not hit by aggregate
Proposition 8 illustrates the dynamics of risk-taking $\sigma_t$, total issuance volume $s_t$ and safe debt creation $k_t$. As no shock realized (here we also assume the ex-post signal $m_t = 1$), intermediaries’ risk perception decreases and capital increases hence they expand their balance sheet by securitizing more high-risk loans, increasing the exposure to aggregate risk. As more insurance is provided, more safe debt is created.

**Effects of Risk Perception on Recovery Dynamics**

In this subsection, we illustrate the effect of different levels of risk perception on recovery dynamics. Suppose there are two markets, referred as blue and black markets respectively, that are identical in all aspects at the beginning of $t = 1$. The aggregate shock hits both markets simultaneously with different levels of intensity at the end of $t = 1$, which is modelled as different levels of increase in risk perception but the same level of capital decrease. Figure 10 illustrates the recovery dynamics of risk perception, capital, speed of learning, speed of capital accumulation, upfront consumption, risk-taking, total issuance volume and safe debt production. In terms of Proposition 5, the recovery evolves in several phases, as risk perception and capital transit through various parts of first $w$-constrained Region and then $\theta$-constrained Region.

On impact the increase in risk perception and decrease in capital results in a drop of insurance provided by intermediaries, leading to a drop in the market share of high-risk loans (risk-taking $\sigma_t$), total issuance volumes and safe debt creation in both markets, as shown in Panel B2, B3 and B4. It is worthwhile to mention that equilibrium allocations in both markets are the same despite different levels of risk perception. The intuition is that the joint allocation of risk perception and capital evolves into $w$-constrained Region and capital level is the same for both market. As shown in Panel A3, the speed of learning also falls as fewer loans are securitized and ex-post signals become very noisy.

After $t = 3$, the equilibrium allocations in the two markets begin to diverge. As illustrated by Figure 10, the recovery in black market is much faster than the blue market. The underlying mechanism driving the divergence is analyzed theoretically in Proposition 7, namely the dynamics feedback effect between low risk perception and capital accumulation. At $t = 4$, the risk perception and capital in blue market evolves into $\theta$-constrained Region while those in black market remains in $w$-constrained Region. Capital accumulation is slower as intermediaries in blue markets are less willing to hold junior tranches. As a result, the recovery in blue market becomes slower than the black market, including risk-taking, total issuance and safe debt creation.

The divergence of recovery is dynamically amplified by the endogenous divergence in speed of learning. When less loans are securitized, ex-post signals are more precise and intermediaries are more confident to downward revise their risk perception. As shown in Panel A3 and B3, speed of learning increases together with total issuance in blue market. Compare with the black market, the recovery of blue market is much slower. With higher level and slow decrease of risk perception,
intermediaries in blue market are less willing to provide insurance. Instead they increase their consumption, as shown in Panel B1, leading to less capital accumulation shown in Panel A2.

In the numerical example, the blue market correspond to PL RMBS market while black market correspond to other securitization markets. Financial intermediaries, like AIG and Leman Brothers, were operating in all major securitization markets in all major securitization markets. During financial crisis, massive write-downs of subprime mortgage related securities wiped out intermediaries’ capital. As a result, issuance in all major private securitization markets collapsed to nearly zero. The difference in recovery patterns is also clearly illustrated in the Figure 10. In PL RMBS market, risk perception increased sharply as the blue market while the increase of risk perception in other markets are relatively mild. I believe the mechanism proposed offers a reasonable explanation for the stagnancy of issuance in PL RMBS market.

**Effects of Capital on Recovery Dynamics**

In this subsection, we illustrate the effect of different levels of capital on recovery dynamics. Suppose there are two markets, referred as blue and black markets respectively, that are identical in all aspects at the beginning of $t = 1$. The aggregate shock hits both markets simultaneously with different levels of intensity at the end of $t = 1$, which is modelled as the same level of increase in risk perception and different levels of capital decrease. Figure 11 illustrates the recovery dynamics of risk perception, capital, speed of learning, speed of capital accumulation, upfront consumption, risk-taking, total issuance volume and safe debt creation. In terms of Proposition 5, the recovery evolves in several phases, as risk perception and capital transit through various parts of first $w$-constrained Region and then $\theta$-constrained Region.

On impact the increase in risk perception and decrease in capital results in a drop of insurance provided by intermediaries, leading to a drop in the market share of high-risk loans (risk-taking $\sigma_t$), total issuance volumes and safe debt creation in both markets, as shown in Panel B2, B3 and B4. After the shock at $t = 2$, the equilibrium allocations in both markets become different immediately. The reason is that after the correlate shock intermediaries’ capital and risk perception evolves into severe $w$-constrained Region, meaning capital is the binding constraint. Different levels of capital imply different levels of risk-taking, total issuance volume and safe debt creation. Correspondingly, collapse in the blue market is deeper than black market.

As less loans are securitized in the blue market, the ex-post signals are less precise and the corresponding speed of learning decreases more as shown by Panel A3. The risk perception in the blue and black markets begin to diverge but magnitude is very small, as shown by Panel A1. Comparing the two markets, we can see that the recovery of blue market quickly catches up with the black market, as shown in Panel B1, B2 and B3. The reason is that between $t = 3$ and $t = 7$, risk perception and capital of black market evolves into $\theta$-constrained Region while those of blue market remains in $w$-constrained Region. As a result, capital accumulation becomes slower in black market, as illustrated by Panel A4.
Figure 10. The figure compares recovery dynamics of shadow banking sector from different confidence levels after the realization of correlation shock at $t = 0$. At $t = 1$, the blue line starts with $\{w_1 = 0.06, \theta_1 = 0.5574\}$ and the black line starts with $\{w_1 = 0.16, \theta_1 = 0.5574\}$. Panel A traces out the two paths of confidence and capital in space $w_t$ vs $\theta_t$. Panel B1 shows the evolution of intermediary consumption $c_t$ over time. Panel B2 shows the evolution of risk-taking $\sigma_t$ over time. Panel B3 shows the evolution of shadow banking size $s_t$ over time. Panel B4 shows the evolution of safe debt issuance $k_t$ over time. The parameter values are: $\{p_Y = 1.1, \rho_H = 0.7, \rho_L = 0.2, r = 0.02\}$. $s_t$ is parameterized as $s_t = 1 - \exp(-5s_t)$. 
Figure 11. The figure compares recovery dynamics of shadow banking sector from different confidence levels after the realization of correlation shock at $t = 0$. At $t = 1$, the blue line starts with $\{w_1 = 0.06, \theta_1 = 0.5574\}$ and the black line starts with $\{w_1 = 0.16, \theta_1 = 0.5574\}$. Panel A traces out the two paths of confidence and capital in space $w_t$ vs $\theta_t$. Panel B1 shows the evolution of intermediary consumption $c_t$ over time. Panel B2 shows the evolution of risk-taking $\sigma_t$ over time. Panel B3 shows the evolution of shadow banking size $s_t$ over time. Panel B4 shows the evolution of safe debt issuance $k_t$ over time. The parameter values are: $\{pY = 1.1, \rho_H = 0.7, \rho_L = 0.2, r = 0.02\}$. 
The two numerical examples shed light on the different impact of risk perception and capital on recovery dynamics. Under reasonable parameter values, recovery from high risk perception takes much longer time than that due to capital shortage. The analysis also yields implication on the effectiveness of policies that help recover the securitization markets. When risk perception is high, intermediaries are not willing to provide insurance. Therefore recapitalizing the intermediaries are not effective to recover the securitization markets.

B.2. Long-run Dynamics with Aggregate Risk

The long-run properties of the model depend on whether there is risk perception trap and if so, on initial conditions and the size of the trap.

**PROPOSITION 9: Properties of Long-run Equilibrium**

Suppose realization of correlation shock is i.i.d and characterised by $\theta_{\text{true}}$ where $\theta_{\text{true}} = \bar{\theta}$. The long-run equilibrium of the economy can be characterised

1. Suppose there is capital trap but no risk perception trap. Given the initial risk perception $\theta_0$ is smaller or equal than $\theta_H$, the asymptotic properties are characterised as follows
   
   - The asymptotic risk perception converges to the true distribution, that is $\lim_{t \to \infty} \theta_t = \theta_{\text{true}}$.
   - When $t \to \infty$, risk-taking $\sigma_t$, total issuance volume $s_t$ and safe debt creation $k_t$ are only determined by capital $w_t$.
   - Capital trap alone is transitory and disappears after recapitalization.

2. Suppose risk perception trap exists. The initial risk perception $\theta_0$ is larger than $\theta_H$, then the economy will remains in the trap, that is $\theta_t = \theta_0$ and $s_t = 0$ for all $t$. This means risk perception trap is permanent.

3. Suppose there is no risk perception trap and the initial risk perception $\theta_0$ is smaller or equal than $\theta_H$, long-run equilibrium can be characterised by two possibilities
   
   - Risk perception will converge to the true distribution $\lim_{t \to \infty} \theta_t = \theta_{\text{true}}$. The asymptotic measures of risk-taking $\sigma_t$, total issuance volume $s_t$ and safe debt creation $k_t$ are only determined by intermediaries’ capital $w_t$.
   - The economy falls into risk perception trap before reaching infinity and asymptotic size of securitization market is zero.

Proposition 3 establishes the condition under which the economy falls into risk perception traps or intermediaries eventually learn the true distribution of correlation risk $\theta_{\text{true}}$ in the long run. Part 1 says that if parameters are such that there is only capital trap but no risk perception trap, learning always takes place. Eventually, large sample will accumulate and intermediaries will learn the true distribution $\theta_{\text{true}}$ perfectly. Under this scenario, the asymptotic properties of securitization markets, including risk-taking, total issuance volume and safe debt creation are only determined...
by intermediaries’ capital. When $t \to \infty$, securitization markets still exhibit boom-and-bust cycle, which is driven by capital.

We take the residential MBS market as an example. Even though issuance in the market fell to zero, intermediaries can still get some information through labor and housing market. But the links between the two markets and MBS market are not well understood. As a result, intermediaries are not confident enough to adjust their model of risk perception when observing the signals and learning becomes very slow.

Part 2 says that if there is risk perception trap and the economy starts in the state ($\theta_t > \theta_H$) where risk perception is too high to sustain securitization, then learning never takes place and confidence remains low forever. This implies risk perception trap is permanent and mathematically $\theta_t = \theta_H$ is an absorbing state. The size of shadow banking is zero and will remain zero forever.

If there is risk perception trap and the economy begins outside it $\theta_t \in [\theta_L, \theta_H]$, some learning will take place at first. Since the sequence of ex-post signal realization that intermediaries learn from is random, there is a positive probability that eventually intermediaries learn the true distribution and also that the economy falls into confidence trap, depending on the realization of ex-post signal sample path. Part 3 illustrates the two possible long-run equilibrium allocations.

- Part 3(a) says that if the economy doesn’t fall into confidence trap in the long run, intermediaries eventually fully learn the true distribution of correlation shock,

  There exists a finite $\tau$, for each $t \geq \tau$ we have $\theta_t = \theta_{true}$

implying that after intermediaries perfectly learn the true distribution, arrival of more ex-post signals will not change their risk perception. After risk perception stabilizes, shadow banking sector still shows the typical boom-and-bust pattern. However here it is intermediary capital alone contributing to the boom-and-bust pattern. Even though securitization may halt after realization of correlation shock, this is temporary due to its capital trap nature.

- Part 3(b) says that in the long run, the economy falls into risk perception trap before intermediaries learn the true distribution of correlation shock. Mathematically we have

  There exists a finite $t$ such that $\theta_t > \theta_H$

When enough negative ex-post signals ($m_t = 1$) are incorporated into intermediaries’ belief, risk perception $\theta_t$ rises strictly $\theta_H$ hence intermediaries are not willing to hold junior tranche. As no loans are securitized, learning stops and pessimism perpetuates.

The reason for the two possible long-run outcomes is that the model features a form of dynamic feedback effect between high risk perception and low securitization. High risk perception at time $t$ deters securitization, which results in slower learning and risk perception at time $t + 1$ remains high. The persistence in high risk perception contributes to slow recovery in securitization markets. The model also features path dependence: realization of ex-post signals in the early stages of the learning process may have long-term consequences for how securitization markets develop.
Figure provides an example where the economy may or may not fall into confidence trap depending on the sample path of realization of correlation shock.

Whenever risk perception \( \theta_t \) rises above \( \theta_H \), intermediaries are not willing to hold junior tranches and securitization stops, hence learning stops as well. The initial confidence \( \theta_0 < \theta_H \), so the economy begins outside confidence trap. And the initial capital is \( w_0 \in (0, w^*) \), \( w_0 > 0 \) meaning the economy begins outside capital trap and we also assume \( w_0 \) is small. Therefore when the economy begins, securitization can be sustained but the size of shadow banking sector is small. This allows us to show the whole credit cycle in the shadow banking sector. The figure shows two possible sample paths of correlation shock in the long run.

The red lines represent the realization of aggregate shock where the economy does not fall into confidence trap. The long-run equilibrium has the following features.

- **Risk Perception Dynamics**: The risk perception dynamics are quite different between early and late stage. At early stage of shadow banking development, belief is very sensitive to the arrival of ex-post signals. As the economy running for a long time, belief converges to the true distribution, meaning the intermediary’s model is very close to the true data-generating process. Belief is stable and not sensitive to the arrival of ex-post signals.

- **Shadow Banking Recovery**: After aggregate shock hitting the economy, the recovery patterns in the early and late stage are quite different, namely much slower recovery in the early stage. The intuition is that the dynamic feedback effect between high risk perception and low securitization is more pronounced in the early stage. Suppose in the early stage after realization of correlation shock, rise perception increases to very high level and securitization falls, generating noisy ex-post signals for future learning. Learning slows down and risk perception remains high for a long time. On the other hand, in the late stage, risk perception is close to the true value and remains very stable even after correlation shock realization. The dynamic feedback effect disappears in the long run.

- **Capital Dynamics**: In the long run after risk perception stabilizes, securitization market is still volatile due to the effect of capital dynamics. Here the capital accumulation effect is similar to Brunnermeier and Sannikov (2014) where agents’ belief is fixed and economy recovery involves capital accumulation.

The blue lines represent the realization of aggregate shock where the economy falls into risk perception trap. This is more likely in the early stage than late stage when risk perception is more sensitive to ex-post signal arrival. We can see that when aggregate shock realizes in the early stage and risk perception \( \theta_t \) rises above \( \theta_H \), intermediaries are not willing to hold junior tranches and securitization stops, leading to the stop of learning and perpetuation of pessimism. Note that when risk perception \( \theta_t \) is below but close to \( \theta_H \), the ex-post signals becoming very noisy, so the learning process slows down before stopping entirely.
C. Implication of Dynamic Model

IMPLICATION 4: Risk perception is decreasing with underlying loans’ performance and intermediary’s capital is increasing with the performance. Therefore risk-taking, total issuance volume and safe debt creation in securitization markets are increasing in the cumulative performance of underlying loans.

From 2000 to 2007, all securitization markets enjoyed a rapid expansion with low underlying default rate during the period. As in the model, risk perception is decreasing with the time with good performance and intermediaries’ capital is increasing with cumulative performance. As intermediaries provide more insurance, securitization expands. In securitization markets, market share of securities backed by risky loans is increasing and total issuance volume and AAA rated bond issuance are also increasing.

IMPLICATION 5: Recovery from high risk perception can be very slow due to the self-reinforcing effect between high risk perception and low securitization. On the other hand, recovery from low level of capital is relatively faster.

A reasonable measure of risk perception in securitization markets are downgrades of securities backed by certain underlying loans. As documented Benmelech and Dlugosz (2009), 64% of all downgrades in 2007 and 2008 were tied to securities that had home equity loans (HELS) or first mortgages as collateral and the downgrades were more than eight notches. For securities backed by other types of collateral, the downgrades were mild (less than four notches). The evidence indicates that there was larger increase of risk perception in PL RMBS market than other markets. Consistent with the theory, recovery of PL RMBS market is much slower than other markets. The issuance in PL RMBS market remains stagnant while issuance in other markets begin to recover since 2010.

IMPLICATION 6: The effectiveness of recapitalization policy depends on both risk perception and capital. When risk perception is relatively high, recapitalizing intermediaries does not help recover issuance in securitization markets.

My theory also speaks to the effectiveness of recapitalizing intermediaries on securitization market recovery. In the model, when risk perception is high recapitalization is not effective. The implication is also consistent with the fact that recapitalizing intermediaries is not able to help issuance recovery in PL RMBS market.

D. Discussion

D.1. Mapping Agents in the Model to Reality

There are two groups of agents in the model, intermediaries and households. Households in the model are assumed to be infinitely risk-averse and with deep pockets. This assumption represents the large demand of private-produced safe assets. As illustrated by Gennaioli et al. (2013) and
the supply of public-produced safe asset is not enough to meet the social demand of safe assets. One prominent role of shadow banking sector is to supply private-produced safe assets. Financial intermediaries take two roles in the model

- The financial intermediary in the model takes the junior tranche in the securitization vehicle and are providing insurance against correlation risk. Effectively this is the risk-sharing role between households and intermediaries.
- The financial intermediary in the model is also uniquely endowed with the diversification technology, similar to Diamond (1984). Since the households are infinitely risk-averse, without intermediary they don’t value individual asset.

In reality, the first role maps to agents who are holding junior tranches, providing tail risk insurance to AAA tranche holders. These agents are usually securitization sponsors (bank holding companies) and hedge funds. The rationales for securitization sponsors holding junior tranches are signalling or regulatory arbitrage views. The former argument is in the context of adverse selection and formalized by Gorton & Pennacchi (1990), DeMarzo & Duffie (1999) and DeMarzo (2004). According to latter argument, banks pursue securitization through off-balance-sheet vehicles to circumvent capital requirement for risky investment and court on government bail-out when things go badly. This view is formalized by Acharya, Schnabl & Suarez (2013) and Plantin (2014). Hedge funds operate CDO and take the equity tranche to make profits, referred as CDO arbitrageurs.

The second role maps to sponsor of securitization, who pools asset together to diversify idiosyncratic risk. As illustrated by Pozsar, Adrian, Ashcraft & Boesky (2010), shadow credit intermediation process includes loan warehousing and ABS warehousing, corresponding to the diversification role of financial intermediary in the model.

D.2. Asymmetric Learning

As illustrated by Coval, Jurek & Stafford (2009a) and Coval, Jurek & Stafford (2009b), the key to price structure financial product is default correlation parameter. Unlike the previous literature, such as Veldkamp (2005), Van Nieuwerburgh & Veldkamp (2006) and Ordonez (2013) where agents are learning the first moment, instead in our model intermediaries are learning about the change of default correlation risk (second moment), which in reality proves much more challenging. The default correlation corresponds to correlation shock in the model, which intermediaries need to learn from the ex-post signals.

This assumption is the key driver of the model’s dynamics. It says there is a form of positive informational externality that depends on the total volume of loans securitized in the shadow banking sector. The larger the size of shadow banking sector, learning is faster, namely intermediaries are more confident to update their risk perception. The intuition behind the assumption is that one prominent function of securitization in shadow banking sector is creating financial markets for previously non-tradable assets and generate observations for future learning. We model this feature in a reduced-formed way, namely the precision of ex-post signals depends on the size of shadow
banking sector, measured by total volume of loans securitized in the equilibrium.

E. Conclusion

I develop a dynamic theory of securitization and shadow banking, where financial intermediaries assemble loans into diversified portfolios and issue safe debt to outside investors. They hold the toxic wastes (junior tranches) and provide insurance against aggregate shock. In the model, I show that intermediaries’ risk perception and capital play key roles in determining insurance provision, hence securitization. The model shows that both low risk perception and high capital are necessary to sustain high level of securitization. Either an increase in risk perception or a decrease in capital can lead to the collapse of securitization, whereas the recovery patterns are very different.

The key assumption of the model is learning positively depends on issuance volume. The rationale behind the assumption is that given the complexity of securitized assets, intermediaries need to learn about aggregate risk factors through observation generated by securitization. If the sample is small, estimation is not precise and learning slows down. Under the assumption, the model features a self-reinforcing effect between high risk perception and low level of securitization. High risk perception decrease intermediaries’ willingness to provide insurance, leading to low level of securitization. Since learning depends on issuance, low securitization results in slow learning and high risk perception persists.

Central to our aggregate implication is the fact that my model features two state variables, intermediaries’ risk perception and capital. Their joint dynamics result in compelling dynamics. The model shows that the recovery from downturns associated with high risk perception can be much slower than the recovery from that associated with insufficient intermediaries’ capital due to the self-reinforcing effect between high risk perception and low securitization. This offers a plausible explanation for the stagnancy of private label RMBS. Before the financial crisis, intermediaries such AIG and Leman Brothers were operating in all major securitization markets. The massive write-downs of subprime mortgage related securities resulted in the insolvency of theses intermediaries, leading to the collapse of issuance in all major securitization markets. The sharp rise of risk perception in PL RMBS market keeps issuance low until now while other securitization markets recover after intermediaries are recapitalized.

The theory is also consistent with the key stylized facts in securitization market during expansion. I show that together with good cumulative performance of underlying portfolios, there are endogenous increases of market share of securities backed by risky loans, total issuance volume and safe debt creation. My model provides a useful framework for the analysis of securitization dynamics from financial intermediaries' perspective. It would be interesting for future research to incorporate the model into a fully fledged DSGE model to examine the general equilibrium effect.
REFERENCES


24(1), 161–208.

journal of economics 105(1), 87–114.

Bernanke, B. S., Bertaut, C. C., Demarco, L. & Kamin, S. B. (2011), ‘International capital flows and
the return to safe assets in the united states, 2003-2007’, FRB International Finance Discussion
Paper (1014).

105(6), 1883–1927.


The Review of Financial Studies .


Chernov, M., Dunn, B. R. & Longstaff, F. A. (2017), ‘Macroeconomic-driven prepayment risk and

Coval, J. D., Jurek, J. W. & Stafford, E. (2009a), ‘Economic catastrophe bonds’, American Eco-


of Finance 71(2), 809–870.


III. Chapter 3: Originate-to-distribute Banking, Endogenous Market Liquidity and Credit Cycle

A. Introduction

Discussion on financial stability effect of leverage lending markets, which is the non-investment grade segment of syndicated loan market, has been on the top of regulators’ agenda. In May 2019, the Federal Reserve had issued a fresh warning about high levels of lending to highly indebted US companies, saying it could exacerbate a downturn were one to happen. Before that, several financial regulators around the world, including International Monetary Fund, Bank of International Settlement and Bank of England had sounded alarm on the rapid growth of the market. According to the estimate from Bank of England, there is more than 2.2 trillion dollars in leverage loan outstanding globally with 1.3 trillion dollar in the US. Also the share of cov-lite loans in the market increase dramatically from less than 30% in 2007 to more than 80% in 2018.

The debate whether rapid expansion of leverage loan market would undermine financial stability is still ongoing. On the one hand, former Federal Reserve chair Janet Yallen argued that “huge deteriorate” in underwriting standards may generate systemic risk, undermining stability of whole financial system and triggering financial crisis as 2008. On the other hand, many practitioners argue that the market remains resilient despite rapid growth. Rating of collateralised loan obligations are more conservative when compared to CDO before financial crisis in 2008 and ultimately most of the risky loans are held by institutional investors with stable liability structure.

In order to shed light on the debate, the paper addresses the following research questions: What are the potential amplification mechanism (systemic risk) in the market? What happens in the market if there is an economic downturn?

We built a dynamic general equilibrium model of syndicate lending market and calibrate to US leverage loan market data from 2001 to 2018 to study these questions. We obtains three main results

* We identify syndication arranger’s pipeline risk as the main potential amplification mechanism. Pipeline risk is defined as the risk that arrangers can’t place the loans to potential investors and have to hold them on their own balance sheet.
* We find that arrangers with large retention will reduce new origination since arrangers have limited risk-bearing capacity, consistent with empirical finds by Bruche, Malherbe and Meisenzahl (2018). In our calibrated results show that 1% increase in arrangers’ retention leads to 3% decrease of credit supply.
* We also find that arrangers’ retention share is significantly counter-cyclical. In recessions, arranger retains a much larger share of risky loan compared in expansions. On the other hand, credit supply is significantly pro-cyclical. This is consistent with the empirical finds with Ivashina and Scharfstein (2010).

We believe our findings are useful to inform policy debate on financial stability concerns on
leverage loan market, especially on the design of marco-prudential policy. In leverage loan market, investors who ultimately holding the loans are institutional investors (such as mutual funds, insurance companies and pension funds) however arrangers are usually regulated banks or broker-dealers with limited risk capacity and liability structure vulnerable to runs. If arrangers’ syndication pipelines are blocked simultaneously in financial recession, there will be large negative spillover effects in both leverage loan markets and other markets which the arrangers operates.

Our model is constructed to highlight the primary role of arrangers in the syndicate lending markets as well as their potential procyclical effect on credit supply over the business cycle. The building blocks of the model is

- Entrepreneurs live for two periods and are overlapping-generation agents who operate Cobb-Douglas production technology. They need to borrow capital to finance production. To capture the idea loans are usually long-term and risky, we assume the production technology pays off only after two periods and is subject to failure risk.
- Arrangers are infinite-lived agents with log utility function. The assumption reflects arrangers (in reality are regulated banks and broker-dealers) have limited risk-bearing capacity. They originate loans to entrepreneurs with their own wealth and then sell the loans to investors. The combination of long-lived arrangers with overlapping-generation loan structure can generate endogenous loan retention and sales decision. This assumption captures the implication arrangers could increase capital turnover and supply more credit to the economy.
- Investors are risk neutral and have deep pockets. Arrangers can share risk and increase their asset turnover by selling loans to investors. To keep the model tractable, we abstract from fluctuations in investors’ demand, which would exacerbate our results.
- The key assumption in the model is information asymmetry between arrangers and investors. Before selling loans to investors, the arrangers observe a private signal about loan’s quality. Here we also capture heterogeneous loan quality (referring to high-type and low-type in the model). We also assume the market is anonymous and focus only on pooling equilibrium.
- For simplicity, business cycle is modeled as a Markov process with two states (expansion and recession). In recession, there are more loans default than in expansion.

The key mechanisms at work are presented here. First we describe the economics problem arrangers need to solve at individual level.

- When types of arrangers are observable, there are two markets for both types of arrangers. Full risk-sharing is achieved and both types of arrangers sell all the loans to investors. Credit supply is at first-best level.
- When types of arrangers are not observable, different types of arrangers have to sell loans at the same price. At individual arranger level, the combination of overlapping-generation structure long-term asset and information symmetry between arrangers and investors leads to endogenous decision of retention. The arrangers trades off the benefit of risk-sharing and obtaining new capital with the cost of selling loans to investors.
Low-type arrangers face no trade-offs. They benefit from the pooling equilibrium price since they are selling overvalued assets. They will sell all the loans.

High-type arrangers face the trade-off between risk-sharing and obtaining new funds with the cost of selling undervalued securities. They will choose to retain a fraction of risky loan on their own balance sheet.

- At aggregate level, the proportion of high-type banks determines the market liquidity.
  - Strategic complementary among high-type’s retention decision
  - Dynamic feedback loop between market liquidity and credit supply
    * Financial amplification mechanism: business cycle ⇒ credit cycle
  - Possibility of multiple equilibrium: self-fulfilling financial crisis
    * Financial fragility: extreme amplification
    * Coordination failure
  - Calibration to leverage loan market

B. Literature Review

B.1. Theoretical Literature

This is the closest literature to our paper focusing on the real effect of adverse selection in financial market. [Eisfeldt (2004)](Eisfeldt2004) analyses an economy where agents hold different vintages of overlapping generation long-term risky assets. Asset sales facilitates diversification but are plagued with adverse selection of quality. In her model, the degree of adverse selection is defined as the
illiquidity of assets. Her model focuses only on the deterministic steady state and cycles since there is no aggregate shock. The main contribution of the paper is to add business cycle risk and provide quantitative calibration to leverage loan market.

Kurlat (2013) analyzes an economy where entrepreneurs have heterogeneous investment opportunities and need to sell existing capital to fund investment projects. The quality of capital is only known by the entrepreneurs and negative aggregate shock leads to worsening of adverse selection. Based on Kurlat (2013), Bigio (2015) introduces limited commitment friction in labor market. They find that increasing dispersion in asset quality can generate large economic fluctuations. Similar to our paper, they focus on the amplification of aggregate shock through adverse selection in the market. The key difference is that we focus on different markets. In their models, the information asymmetry is between entrepreneurs and the incentive to sell assets is to fund new investment. Technically, there is no interaction between different vintages. In our paper, we analyze originate-to-distribute banking model and the information asymmetry is between lead arranger and investors. The overlapping generation structure allows us to model the interaction between distributing ongoing loans and originating new loans, interpreted as pipeline risk in leverage syndication market.

Daley & Green (2016) studies a dynamic model with stochastic arrival of information and strategic delay of trading serves as a costly signaling device. Traders rationally expect future market liquidity, leading to a feedback effect between prices and market liquidity. Even though our model shares the feedback effect, in their model anticipation of future illiquidity leads to costly delay of trading and in our model anticipation of future illiquidity leads to more retention and less new origination.

Another strand of literature studies the feedback effect between long-term investment decision and liquidity anticipation, including Heider, Hoerova & Holthausen (2010), Bolton, Santos & Scheinkman (2011) and Malherbe (2014). They share the idea that anticipation of future illiquidity in long-term asset may trigger ex-ante cash hoarding behaviour. In Plantin (2009), anticipation of future illiquidity, investors choose to retain high-type assets on their balance sheet instead of distributing to investors. However, this is a bang-bang solution as arrangers distribute all the asset or nothing. We do not model cash hoarding by banks. In our model, we focus on credit fluctuation over the business cycle, which is missing from these models.

Balance sheet effect have been a central object in the research of macroeconomics with financial friction. Papers on balance sheet effect without aggregate shock includes Bernanke & Gertler (1990), Kiyotaki & Moore (1997) and Carlstrom & Fuerst (1997). Papers on balance sheet effect with aggregate shock: Lorenzoni (2008), Brunnermeier & Sannikov (2014) and Dávila & Korinek (2018). Technically, the underpinning frictions in these models is price-dependent collateral constraint of productive agents, which generates capital fire-sales during crisis. There are two differences. First in their models, the key assumption to generate capital fire-sale is heterogeneity in productivity while in our model there is no productivity heterogeneity. Second even though pecuniary externality exits in both our and their models, the underlying microeconomics foundation is
completely different. Their models rely on limited commitment of productive agent while ours rely on asymmetric information.

The idea that market liquidity affects incentives to screen asset quality is explored by Parlour & Plantin (2008), Chemla & Hennessy (2014) and Vanasco (2017). In their models, origination of high quality assets require costly effort. At the same time, the originators get private information about the assets which negatively affects market liquidity when distributing to investors. These papers highlight the trade-off between incentives to originate high quality loans and market liquidity. Comparing with these papers, loan quality is exogenous. The main contribution of our paper is the analysis of credit cycle and the quantitative exercise. Chari, Shourideh & Zetlin-Jones (2014) analyses the volume of new issuance in an originate-to-distribute model with adverse selection and reputation concern. With reputation effect, low-type originators have stronger incentive to mimic high-types’ decision, leading to pooling equilibrium and persisting adverse selection. Instead of reputation effect, our paper focuses on the feedback effect between adverse selection when arrangers distribute existing loans to investors and their incentive to originate new loans. Even though we assume the distribution market is anonymous, it generates qualitative similar effect when reputation effects are taken into account.

B.2. Empirical Literature

Ivashina (2009), in a study of syndicated loan market, finds that when the share of the loan retained by the lead arrange rises, other syndication participants accept a lower unit return on the loan, suggesting that adverse selection exists between lead arranger and syndication investors. In MBS market, Downing, Jaffe & Wallace (2009) finds that loans that banks held on their balance sheet yielded higher returns on average than did similar loans that they securitize and sold. Drucker & Mayer (2008) argue that underwriters of prime mortgage-back securities are better informed than buyers. Mian & Sufi (2009) present evidence that securitized loans were more likely to default than non-securitized loans.

Ivashina & Scharfstein (2010) examines cyclically of credit supply in originate-to-distribute banking model. They found a strong positive relationship between lead arranger retention share and credit tightening, which is consistent with the prediction of our model. Bruche, Malherbe & Meisenzahl (2017), in a study of leverage loan market, finds that arrangers of a syndicated loan face the risk of retaining loan which can’t be placed to investors, which they refer to pipeline risk. They also find that this type of retention is associated with subsequent reduction in arranging activities.

C. Model

We propose a dynamic equilibrium model of syndicate loan market, in which arrangers intermediate between entrepreneurs and investors. Arrangers originate loans to entrepreneurs with their own wealth and sell them to investors. The key assumption is information asymmetry between arrangers and investors on the quality of loans. A central feature of the model is that asymmetric information endogenously amplifies credit supply over the business cycle.
C.1. Agents

There are 4 groups of agents in the model: entrepreneurs, originators, investors and workers.

- There is a continuum of entrepreneurs. Each of them runs a competitive firm, which produces consumption good combining capital and labor.
- Originators (referring to O-agents) are the only agents endowed with the ability to originator loans to firms. Loans can be consider as capital input for the firms. After origination, they can choose whether to sell to investors or keeping them on the balance sheet.
- Investors (referring to I-agent) are assumed to be with deep pockets and their only investment opportunity is to buy assets from originators.
- Workers supply unit labor to the firms and have no access to financial market.

Arrangers intermediate between entrepreneurs and investors. To capture the market institution that arrangers usually provide some guarantee to borrowers, we assume arrangers originate loans with their own wealth and then sell them to investors. Arrangers are the most important player in the model and we devoted a separate sector to introduce them.

Investors are risk-neutral agents with deep pockets. Their required expected return is \( \beta \) per period, which can be interpreted as exogenous return on their alternative investment technology. They provide a perfectly elastic supply of fund to the arrangers. Due to unmodeled reason, they can’t directly lend to the entrepreneurs and their only investment opportunity in the model is to buy loan from arrangers. In leverage loan market, most of investors are institutional investors such as mutual fund, pension fund and insurance company. The assumptions here capture that institutional investors don’t have the expertise in screening and monitoring entrepreneurs but they have enough risk capacity for risky loan exposure.

Workers play very limited role in the model. They are risk-neutral agents and supply their unit labor in perfect inelastic fashion. To keep the model tractable, we also assume workers have no access to investment technology and they are constrained to consume their wage in hand-to-month fashion.

C.2. Firms

The entrepreneurs are active for two period and each of them run a competitive firm index by \( i \in [0, 1] \)[13]. Each firms operates an constant-return-to-scale technology that transform physical capital and labor into consumption goods. To capture the idea loans are usually long-term, we assume the technology needs input at \( t \) and produce output at \( t + 2 \). The technology is described as follows

\[
y_{i,t+2} = (1 - z_{i,t+2})[AF(k_{it}, n_{it}) + (1 - \delta)k_{it}] + z_{i,t+2}(1 - \lambda)k_{it}
\]

[13] Since entrepreneurs and firms are matched one-to-one, the term entrepreneur and firm means the same thing and is interchangeable in the paper.
where the binary $z_{i,t+2} \in \{0,1\}$, realized at $t+2$, indicates whether firms’ production succeeds ($z_{i,t+2} = 0$) or fails ($z_{i,t+2} = 1$). The parameter $\delta$ and $\lambda$, where $\delta < \lambda$, are depreciation rate of physical capital when firms production succeeds or fails. The higher depreciation rate when production fails allows us to capture loss-given-default rate as empirical observed in leverage lending market. The net output in case of successful production is the production of total productivity factor $A$ and Cobb-Douglas production function

$$F(k_{it}) = k_{it}^\alpha n_{it}^{1-\alpha}$$

where $\alpha \in (0,1)$ is the elasticity of physical capital. In case of production failure, there is no net output and the physical capital used for production depreciates at rate $\lambda$.

In order to model loan market, we assume that entrepreneurs are born with no initial endowment and physical capital needs to be purchased in advance. The contract between arrangers and entrepreneurs is standard debt contract. Under the assumption, entrepreneurs need to borrow from arrangers to finance the purchaseament of physical capital and the size of loan is $k_{it}$. The entrepreneurs are also protected by limited liability and to keep the model tractable, we assume that workers’ wage is paid after the realization of output. In case of successful production after two periods, entrepreneurs pay back $R k_{it}$ to arrangers and investors and pay workers for their wage $W$. In case of failed production, the arrangers, on behalf of investors will seize all the remaining capital $(1-\lambda)k_{it}$ and the workers get zero wage. To sum up, the contract between arrangers and entrepreneurs is $\min\{R k_{it}, (1-\lambda)k_{it}\}$, where $R$ will be endogenously determined in the equilibrium.

### C.3. Arrangers

**Preferences** There is a unit-mass continuum of heterogeneous arrangers indexed by $j$. Arranger’s preferences over a stochastic stream of consumption $\{c_{j,t}\}$ are given by

$$E[\sum_{t=0}^{\infty} \beta^t (1 - \epsilon)^t u(c_{j,t})]$$

where $\beta < 1$ is the time discount factor and $\epsilon$ is the probability that an arranger dies. $u(c) = \log c$ is the Bernoulli utility function. Choice of logarithm utility is to keep the model simple and captures the idea arrangers usually have limited risk capacity. This is consistent with the reality that arrangers in leverage loan market are regulated banks, subject to all kinds of prudential regulations.

**Loan Origination Market and Business Cycle** The loan market is perfectly competitive and arrangers intermediate between entrepreneurs and investors. Arrangers are randomly matched with entrepreneurs. At each period, arrangers decide the amount of loan originated to entrepreneurs. Loan are subject to credit risk because firms’ production is risky and they pay off only after two periods. In the period following origination, arrangers receive a private signal which indicates the conditional probability $q \in \{q_h, q_l\}$ where $q_h > q_l$ that the entrepreneur’s’ production
Lending unit consumption good at $t$

High-type

$q_h$ → Repayment $R$

$1 - q_h$ → Repayment $1 - \lambda$

Low-type

$q_l$ → Repayment $R$

$1 - q_l$ → Repayment $1 - \lambda$

Figure 12. Timeline of Risky Loan

is successful. We refer to the arrangers with high success conditional probability $q_h$ as high-type arrangers and the arrangers with low success conditional probability $q_l$ as low-type arrangers. The signal is private to arrangers and not observed by investors. The information asymmetry between arrangers and investors are consistent with the empirical finds by Ivashina (2009) in syndicate loan market.

We model business cycle as a Markov process with two states, expansion and recession which we denotes $s \in \{e, r\}$. The transitional probabilities of the Markov Chain are given as

$$\pi_{ss'} = Pr(s_{t+1} = s'|s_t = s), \text{ for } s, s' = \{h, l\}$$

We also assume that the business cycle variable $s$ is observable for all the agents in the model and is the only aggregate shock in the model. The proportion of arrangers receiving the high signal $q_h$ depends on the state of the economy. In expansion, $q_e$ proportion of the arrangers are high-type and $(1 - q_e)$ proportion becomes low-type. The unconditional probability of successful production (non-default loan) in expansion is $q_e q_h + (1 - q_e)q_l$. In recessions, $q_r$ proportion of arrangers are high-type and $(1 - q_r)$ proportion becomes low-type. The unconditional probability of successful production in recession is $q_r q_h + (1 - q_r)q_l$. To capture the empirical facts that default probability is higher in recession than expansion, we assume there are more high-type arrangers in recession than expansion ($q_e > q_r$).

The arrangers are long-lived and they hold overlapping generation loans. Arrangers enter period $t$ with both ongoing loans (originated at $t - 1$) and matured loans (originated at $t - 2$). These two variables are endogenous state variables in the model. Hence we refer arrangers’ income from matured loan as cash-in-hand ($m$) and arrangers’ position in on-going loans as on-going $y_o$. Arrangers face the same investment opportunities but their cash-in-hand, position of ongoing loans and quality of their ongoing loans are heterogeneous.

**Loan Selling Market** In addition to access to loan origination market, the arrangers also have
access to an anonymous competitive market in which they can sell the shares of ongoing loans to investors. Selling risky loans to investors is efficient in two ways.

- Since arrangers are risk averse and investors are risk neutral, selling risky loans to investors can improve risk sharing.
- Arrangers have limited wealth (compare with deep-pocket investors) and need to originate loans with their own wealth, selling shares of ongoing loan improve efficiency of their limited wealth. The arrangers can recycle their capital one period before loans pay off and provide more credit to the entrepreneurs.

We abstract from security design problem and assume that the contract between arrangers and investors is equity contract. This is consistent with institutions in syndicate loan market. In terms of Figure [12] arrangers can sell shares of ongoing loans (denoting $y_s \in \{0, y_o\}$) which pay off at period $t + 2$ at period $t + 1$ for price $P$. The selling price reflects the average loan quality sold in the market. Since the quality of ongoing loans is private information to arrangers, the selling market is illiquid due to adverse selection. The severity of adverse selection is endogenously determined in the equilibrium.

The information structure is as follows: Arrangers have private information about the quality of ongoing loans (they know their own type) but the investors can’t observe the quality. When arrangers originate new loans, they have no private information about their quality. The timeline of loan is described in Figure [12]. Different from canonical real business cycle model in which production lasts for one period, in our model loans lasts for two periods and are affected by two subsequent aggregate shock before paying off as illustrated by Figure [13]. Conditioning on state of the economy, loan qualities are independent across arrangers and over time. Moreover and importantly we assume the selling market is anonymous, meaning signalling by retention is not possible. Therefore we focus on the pooling equilibrium price.

C.4. Timeline

Each period $t$, the sequence of actions is described as follows

- Aggregate state of the economy $s_t$ is realized and observed by all the agents
- Quality of ongoing loans is privately observed by arrangers
- Arrangers decide how much share of ongoing loans selling to investors

![Figure 13. Overlapping Generation Structure of Loans](image-url)
Arrangers decide on the amount of new lending to the firms. Firms produce by combining capital and labor to produce consumption.

D. First-Best Benchmark: Observable Types

Before characterizing equilibrium under asymmetric information, it is useful to analyze outcomes if arranger’s types are observable. Here we describe arranger’s optimization in recursive form. Individual state variable \( \{m, y_o\} \) and aggregate state variable \( \{s\} \). Denote high-type and low type’s value functions as \( V_h \) and \( V_l \).

The expectation is on both \( \bar{R}_h \) and \( s' \).

For high-type,

\[
V_h(m, y_o, s) = \max_{\{c_h, y'_{o,h}, y_{s,h}, m'_h\}} \{u(c_h) + \beta E[V_h(m'_h, y'_{o,h}, s'), s]|s\} \\
\text{s.t.} \quad c_h + y'_{o,h} = m + y_{s,h}P_h \\
\quad m'_h = (y_o - y_{s,h})\bar{R}_h \\
\quad y_{s,h} \in [0, y_o]
\]

High-type policy functions

\[
c_h = (1 - \beta)(m + y_oP_h) \\
y'_{o,h} = \beta(m + y_oP_h) \\
y_{s,h} = y_o
\]

For low-type,

\[
V_l(m, y_o, s) = \max_{\{c_l, y'_{o,l}, y_{s,l}, m'_l\}} \{u(c_l) + \beta E[V_l(m'_l, y'_{o,l}, s'), s]|s\} \\
\text{s.t.} \quad c_l + y'_{o,l} = m + y_{s,l}P_l \\
\quad m'_l = (y_o - y_{s,l})\bar{R}_l \\
\quad y_{s,l} \in [0, y_o]
\]

Low-type policy functions

\[
c_l = (1 - \beta)(m + y_oP_l) \\
y'_{o,l} = \beta(m + y_oP_l) \\
y_{s,l} = y_o
\]

The total credit supply in state \( s \) is

\[
y' = q^*y'_{o,h} + (1 - q^*)y'_{o,l} \\
y' = \beta m + \beta y_o[q^*P_h + (1 - q^*)P_l] \\
y' = \beta m + \beta y_o[q^*q_h + (1 - q^*)q_l]R + [q^*(1 - q_h) + (1 - q^*)(1 - q_l)](1 - \lambda)
\]
The total credit demand in state $s$ is

$$R = A \alpha k^{\alpha - 1} + (1 - \delta) \Rightarrow k = \left( \frac{R - 1 + \delta}{A \alpha} \right)^{\frac{1}{\alpha - 1}}$$

Market clearing condition is $y'_o = k$, where $y'_o(R)$ is linear increasing function in $R$ while $k(R)$ in decreasing function. So there is a unique equilibrium when types are observable.

Here we present the comparative statics of parameters on equilibrium credit supply

<table>
<thead>
<tr>
<th>$\frac{dk^{FB}}{dy}$</th>
<th>$A$</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$\lambda$</th>
<th>$q_h$</th>
<th>$q_l$</th>
<th>$q^c$</th>
<th>$q^r$</th>
<th>$\pi_{ee}$</th>
<th>$\pi_{rr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**E. Equilibrium with Asymmetric Information**

**E.1. Firm’s Problem**

Entrepreneurs running the firms maximize their profits. They borrow to form capital and hire workers for production. They are only active for two periods so their optimization is static as follows

$$\max_{\{k_{it}, n_{it}\}} AF(k_{it}, n_{it}) + (1 - \delta)k_{it} - Rk_{it} - Wn_{it}$$

In a competitive equilibrium, marginal productivity of physical capital equals interest rate and marginal productivity of labor equals wage. Since the entrepreneurs are protected by limited liability, when the production fails, $(1 - \lambda)k_{it}$ is seized by arrangers. As a result, in an optimal contract, entrepreneurs’ repayment is

$$\tilde{R} = \begin{cases} 
A \alpha k_{it}^{\alpha - 1} + (1 - \delta) & \text{if } z_{it+2} = 0 \\
1 - \lambda & \text{if } z_{it+2} = 1
\end{cases}$$

The labor market is competitive and workers are hired at the beginning of the period. Workers supply unit labor in perfectly elastic fashion. Therefore workers’ wage is

$$\tilde{W} = \begin{cases} 
A(1 - \alpha)k_{it}^{\alpha} & \text{if } z_{it+2} = 0 \\
0 & \text{if } z_{it+2} = 1
\end{cases}$$

**E.2. Arrangers’ Problem**

At the beginning of each period, arrangers’ asset include income from matured loans (referring to cash-in-hand $m$) and positions in ongoing loans $y_o$. The quality of ongoing loans is heterogeneous (referring to high-type and low-type). They allocate fund to consumption and new lending to
entrepreneurs. They also decide on share of ongoing loans to sell. The optimization problem for individual arranger is

$$\begin{align*}
\max_{\{c_t, y_{o,t}, y_{s,t}\}} & \quad \mathbb{E}_t \left[ \sum_{t=1}^{\infty} \beta^t (1-\epsilon)^t \log c_t \right] \\
\text{s.t.} & \quad c_t + y_{o,t+1} \leq (y_{o,t-1} - y_{s,t-1}) \hat{R}_{t-1} + y_{s,t} P \\
& \quad y_{s,t} \in [0, y_{o,t}]
\end{align*}$$

(17)

Consumption in period $t$ is denoted as $c_t$, $y_{o,t+1}$ is the amount of new loans originated to entrepreneurs. $(y_{o,t-1} - y_{s,t-1})$ is the amount of loan retained by arrangers that are originated two periods ago. $y_{s,t}$ is the amount of ongoing loans selling to the investors. $P$ is the loan selling price and $\hat{R}_{t-1}$ is the return of retained loan two periods ago. The last constraint $y_{s,t} \in [0, y_{o,t}]$ means short-selling of loans are not allowed. The Expectation is taken on $\{\hat{R}, q, s\}$, where $q$ is types of arrangers and $s$ is the aggregate shock representing the business cycle. $\{s_t\}$ follows a Markov chain. The stochastic process $\{\hat{R}_t, q_t\}$ is independently distributed across arrangers and across time. $\{\hat{R}_t\}$, $\{q_t\}$ and $\{s_t\}$ are also mutually independent.

It is convenient to write the arranger’s optimization in recursive form. The individual state variables are $\{m, y_{o}, q\}$, where cash-in-hand $m = (y_{o} - y_{s}) \hat{R}$ coming from matured loans, $y_{o}$ is the position of ongoing loans in the arrangers’ portfolio and $q$ is the signal about the quality of those projects. Aggregate state variable is $X_t = \{M_t, Y_o, s_t\}$. The Bellman equation for an individual arranger is

$$\begin{align*}
v(m, y_{o}, q, X) & = \max_{\{c, y_{o}', m', y_{s}\}} \log c + \beta(1-\epsilon)\mathbb{E}[v(m', y_{o}', q', X')|X] \\
\text{s.t.} & \quad c + y_{o}' = m + y_{s} P \\
& \quad m' = (y_{o} - y_{s}) \hat{R} \\
& \quad y_{s} \in [0, y_{o}]
\end{align*}$$

(18)

PROPOSITION 10: The arranger’s Bellman equation (18) can be characterised as follows

1. There exists a unique value function $v(m, y_{o}, q, X)$ as the solution to optimization (18). The associated policy functions $c(m, y_{o}, q, X)$, $y_{o}'(m, y_{o}, q, X)$ and $y_{s}(m, y_{o}, q, X)$ are also unique
2. When the last constraint $y_{s} \in [0, y_{o}]$ is always slack or binding, the policy functions $c(m, y_{o}, q, X)$, $y_{o}'(m, y_{o}, q, X)$ and $y_{s}(m, y_{o}, q, X)$ are linear in $\{m, y_{o}\}$.

We first put the constraint $y_{s} \in [0, y_{o}]$ aside. Key to this proposition is budget constraints are linear and objective function is homothetic. Alvarez and Stokey (1998) show that the standard properties of dynamic programming on bounded spaces apply to homogeneous dynamic programming problems such as the ones here. Therefore the solution is unique and policy functions are linear. This implies the following result

COROLLARY 2: Price and Aggregate Quantities do not depend on distribute of endogenous state variables $\{m, y_{o}\}$ only on the aggregate amount $\{M, Y_o\}$.
Now we consider the constraint $y_s \in \{0, y_o\}$. In the next session, we show that under symmetric information constraint for both types is always binding and under asymmetric information the constraint for low-type is always binding and for high-type is slack in the equilibrium.

Taking first-order and envelop conditions and combining them, we can get the Euler equations for new lending and selling. The Euler equations are

$$u'(c) = \beta^2 (1-\epsilon)^2 \mathbb{E}[u'(c')\tilde{R} + \nu^0 |s]$$

$$u'(c)\rho = \beta(1-\epsilon)\mathbb{E}[u'(c')|s] + \nu^0 - \nu^0$$

where $\nu^0$, $\nu^0$ and $\nu^0$ are the Lagrangian multiplier associated with the constraint $y_s \in \{0, y_o\}$. In words, equation 19 states that marginal utility of consumption today equals marginal utility of originating new loan to entrepreneurs and keep them on their own balance sheet after two periods when the loans mature (undistributed loans). Taking the constraint into account, there may be corner solution. Equation 20 means that marginal utility from selling additional unit of ongoing loan equals the marginal utility of keeping them on balance sheet. If the return is less than the cost then $y_s = 0$ and if the return is greater than the cost then $y_s = y_o$.

### E.3. Competitive Equilibrium Under Symmetric Information

Before characterizing equilibrium under asymmetric information, it is useful to consider the equilibrium allocation if arrangers’ types are observable. This benchmark setting is particularly useful when we analyse welfare loss associated with asymmetric information. Here we define the competitive equilibrium under observable types.

DEFINITION 5: A competitive equilibrium with observable types consists of prices $\{\tilde{R}(X), \tilde{W}(X), P_H, P_L\}$; A law of motion $\Gamma(X)$ and associated transition density $\Pi(X'|X)$; Two value functions $V(m, k_o, q_i, X)$ for $i = H, L$; Policy functions $\{c^w(X), c(m, k_o, q_i, X), k_s(m, k_o, q_i, X), k'_o(m, k_o, q_i, X), m'(m, k_o, q_i, X)\}$ for $i = H, L$ such that

1. Firms maximize profit
2. Workers consume their wage $c^w(X) = \tilde{W}(X)$
3. Policy functions $\{c(m, k_o, q_i, X), k_s(m, k_o, q_i, X), k'_o(m, k_o, q_i, X), m'(m, k_o, q_i, X)\}$ and value functions $V(m, k_o, q_i, X)$ for $i = H, L$ solves Programming 23 taken prices as given
4. Investors make zero expected profit under selling price $\{P_H, P_L\}$
5. Law of motion $\Gamma$ is consistent with aggregation of individual decision
6. All the markets clear

Since arrangers’ types are observable, there are two markets selling high-type and low-type loans respectively. The arrangers sell all the loans to investors and efficient risk-sharing is achieved.

### E.4. Agents

There are 4 groups of agents in the model: entrepreneurs, originators, investors and workers.
There is a continuum of entrepreneurs. Each of them runs a competitive firm, which produces consumption good combining capital and labor.

Originators (referring to O-agents) are the only agents endowed with the ability to originate loans to firms. Loans can be consider as capital input for the firms. After origination, they can choose whether to sell to investors or keeping them on the balance sheet.

Investors (referring to I-agent) are assumed to be with deep pockets and their only investment opportunity is to buy assets from originators.

Workers supply unit labor to the firms and have no access to financial market.

E.5. Flow of Funds and Trading Environment

Firms’ overlapping generation structure and adverse selection are the two basic ingredients of my model. We consider an economy, where firms lives for 2 period and have an overlapping generation structure. Investment in capital and labor at time \( t \) generates output at \( t + 2 \) and this reflects real investment are usually long-term. We also assume investment opportunities for O-agents are not contractable so they need to use their own capital for new lending. They can’t borrow directly from outside investors. In reality, even though banks use leverage to originate new loans, they need to finance part of the lending with scarce bank capital due to regulatory requirements. This is a parsimonious way to model the limited scale of lending. The quality of loans is heterogeneous. They are either high or low quality (I refer the O-agent with high quality loan as high-type O and those with low quality low as low-type O), where the payoff of high-type first-order stochastic dominates the low-type. In order to increase asset turnover and improve risk sharing, O-agents can sell loans to outside investors. However, O-agents are privately informed about quality before selling to investors and there is adverse selection between the O-agents and investors.

Here we can define three important concepts.

- By credit, we refer to the amount of new lending offered to firms as input for production, measuring the O-agents’ willingness to lend.
- By liquidity, we means the liquidity in the selling market, defined as the proportion of trading not due to private information as Eisfeldt (2004). Effectively market liquidity measures the severity of adverse selection in the loan selling market.
- Macroeconomics fundamental is defined as proportion of low quality loans in the economy and therefore more defaults in the economy. In expansion, there are less low quality loans than recessions. Meaning the adverse selection is less severe in expansion.

As standard model of adverse selection, the equilibrium selling price not only clears the market but also reflects the average quality of loans trading in the marketing. There is cross subsidy from the high-types to the low-types. Therefore the high-types prefer to partially retain loans on their own balance sheet and the amount of retention increases with the severity of adverse selection. Furthermore, the increasing of retention today deteriorates the market liquidity tomorrow. The intuition is more retention of high quality loans today generates more cash-in-hand tomorrow and
the high-types tomorrow don’t participate in the selling market, worsening the adverse selection in the market and driving down the price tomorrow.

F. Solving the Model

We are going to solve the Euler equations sequentially thanks to the assumption of logarithm utility. The Euler equations are

\[ u'(c) P = E[\beta u'(c') \tilde{R}'] \]  
\[ u'(c) = E[\beta^2 u'(c'') \tilde{R}'' ] \]

Equation 21 means marginal benefit from selling the asset equalling marginal cost of keeping the asset on the balance sheet tomorrow (the trade-off between selling and keeping). Equation 22 means marginal benefit of consuming today equalling the marginal cost of investing (the trade-off between consumption and investment). Since the originators have logarithm utility, the consumption and investment policy functions implied by equation 22 are

\[ c = (1 - \beta)(m + k_s^P) \]
\[ k_o = \beta(m + k_s^P) \]

Specifically we have for high-type Os

\[ c_H = (1 - \beta)(m + k_{H,s}^P) \]
\[ k_{H,o} = \beta(m + k_{H,s}P) \]

For low-type, we have

\[ c_H = (1 - \beta)(m + k_o^P) \]
\[ k_{H,o} = \beta(m + k_o P) \]

F.1. Solving the Model Numerically

The model is solved using Euler equation iteration

1. Define consumption and lending functions for low-type arrangers

\[ c_L = (1 - \beta)(m + y_o P) \]
\[ y'_{o,L} = \beta(m + y_o P) \]

2. Guess consumption function for high-type \( c_H(m, y_o) \). The consumption function is linear in \( m \) and \( y_o \)
3. Here we define 2 functions (origination function and distribution function)

\[ y_{o,H}' = \frac{\beta}{1 - \beta} c_H \]
\[ y_{s,H} = \frac{1}{P}(\frac{c_H}{1 - \beta} - m) \]

4. Compute credit supply
   - High-type credit supply
     \[ y_{o,H}' = \frac{\beta}{1 - \beta} c_H \]
   - Low-type credit supply
     \[ y_{o,L}' = \beta(m + y_o P) \]
   - Total credit supply
     \[ y_o' = qy_{o,H}' + (1 - q)y_{o,L}' \]

5. Compute the interest rate and wage

\[ R = A \alpha (y_o')^{(\alpha - 1)} + (1 - \delta) \]
\[ W = A(1 - \alpha)(y_o')^{\alpha} \]

6. Update high-type’s consumption function through Euler equation iteration

\[ u'(c_H)P = \mathbb{E}[\beta u'(\tilde{c}')\tilde{R}_H] \]

where the expectation is taken on both return and type tomorrow. In the case with aggregate shock, the expectation is also taken on aggregate state.
Figure 14. The figure compares recovery dynamics of shadow banking sector from different confidence levels after the realization of correlation shock at $t = 0$. At $t = 1$, the blue line starts with $\{w_1 = 0.06, \theta_1 = 0.5574\}$ and the black line starts with $\{w_1 = 0.16, \theta_1 = 0.5574\}$. Panel A traces out the two paths of confidence and capital in space $w_t$ vs $\theta_t$. Panel B1 shows the evolution of intermediary consumption $c_t$ over time. Panel B2 shows the evolution of risk-taking $\sigma_t$ over time. Panel B3 shows the evolution of shadow banking size $s_t$ over time. Panel B4 shows the evolution of safe debt issuance $k_t$ over time. The parameter values are: $\{pY = 1.1, \rho_H = 0.7, \rho_L = 0.2, r = 0.02\}$. $\phi(s_t)$ is parameterized as $\phi(s_t) = 1 - \exp(-5.5s_t)$.
Figure 15. The figure compares recovery dynamics of shadow banking sector from different confidence levels after the realization of correlation shock at $t = 0$. At $t = 1$, the blue line starts with $\{w_1 = 0.06, \theta_1 = 0.5574\}$ and the black line starts with $\{w_1 = 0.16, \theta_1 = 0.5574\}$. Panel A traces out the two paths of confidence and capital in space $w_t$ vs $\theta_t$. Panel B1 shows the evolution of intermediary consumption $c_t$ over time. Panel B2 shows the evolution of risk-taking $\sigma_t$ over time. Panel B3 shows the evolution of shadow banking size $s_t$ over time. Panel B4 shows the evolution of safe debt issuance $k_t$ over time. The parameter values are: $\{pY = 1.1, \rho_H = 0.7, \rho_L = 0.2, r = 0.02\}$. $\phi(s_t)$ is parameterized as $\phi(s_t) = 1 - \exp(-5.5s_t)$. 
Figure 16. The figure compares recovery dynamics of shadow banking sector from different confidence levels after the realization of correlation shock at $t = 0$. At $t = 1$, the blue line starts with $\{w_1 = 0.06, \theta_1 = 0.5574\}$ and the black line starts with $\{w_1 = 0.16, \theta_1 = 0.5574\}$. Panel A traces out the two paths of confidence and capital in space $w_t$ vs $\theta_t$. Panel B1 shows the evolution of intermediary consumption $c_t$ over time. Panel B2 shows the evolution of risk-taking $\sigma_t$ over time. Panel B3 shows the evolution of shadow banking size $s_t$ over time. Panel B4 shows the evolution of safe debt issuance $k_t$ over time. The parameter values are: $\{pY = 1.1, \rho_H = 0.7, \rho_L = 0.2, r = 0.02\}$. $\phi(s_t)$ is parameterized as $\phi(s_t) = 1 - \exp(-5.5s_t)$. 

![Retention Ratio Diagram](image)
G. Discuss

G.1. Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total productivity factor</td>
<td>$A$</td>
<td>2</td>
</tr>
<tr>
<td>Physical capital elasticity</td>
<td>$\alpha$</td>
<td>0.5</td>
</tr>
<tr>
<td>Depreciation rate for non-default projects</td>
<td>$\delta$</td>
<td>0.1</td>
</tr>
<tr>
<td>Depreciation rate for default projects</td>
<td>$\lambda$</td>
<td>0.35</td>
</tr>
<tr>
<td>Default probability of high-type projects</td>
<td>$q_h$</td>
<td>0.95</td>
</tr>
<tr>
<td>Default probability of low-type projects</td>
<td>$q_l$</td>
<td>0.5</td>
</tr>
<tr>
<td>Proportion of high-type projects in expansion</td>
<td>$q^e$</td>
<td>0.975</td>
</tr>
<tr>
<td>Proportion of high-type projects in recession</td>
<td>$q^r$</td>
<td>0.9</td>
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<tr>
<td>Transitional probability from expansion to expansion</td>
<td>$\pi_{ee}$</td>
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<tr>
<td>Transitional probability from recession to recession</td>
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<tr>
<td>Arranger’s discount factor</td>
<td>$\beta$</td>
<td>0.95</td>
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Table IX: Endogenous Variables in the Model

<table>
<thead>
<tr>
<th>Description</th>
<th>Variable Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Arranger</strong></td>
<td></td>
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<tr>
<td>Income from Matured Loan (Cash-in-hand)</td>
<td>$m$</td>
</tr>
<tr>
<td>Position in Ongoing Loan (Ongoing Loan)</td>
<td>$y_{o}$</td>
</tr>
<tr>
<td>High-type consumption</td>
<td>$c_{h}$</td>
</tr>
<tr>
<td>Low-type consumption</td>
<td>$c_{l}$</td>
</tr>
<tr>
<td>High-type selling of ongoing loans</td>
<td>$y_{s,h}$</td>
</tr>
<tr>
<td>Low-type selling of ongoing loans</td>
<td>$y_{s,l}$</td>
</tr>
<tr>
<td>High-type origination of new loan</td>
<td>$y_{o,h}'$</td>
</tr>
<tr>
<td>Low-type origination of new loan</td>
<td>$y_{o,l}'$</td>
</tr>
<tr>
<td>Total origination of new loan</td>
<td>$y_{o}'$</td>
</tr>
<tr>
<td>High-type cash-in-hand tomorrow</td>
<td>$m_{h}'$</td>
</tr>
<tr>
<td>Low-type cash-in-hand tomorrow</td>
<td>$m_{l}'$</td>
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<tr>
<td><strong>Production sector</strong></td>
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<tr>
<td>Physical capital for production</td>
<td>$k$</td>
</tr>
<tr>
<td>Labor for production</td>
<td>$n$</td>
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<tr>
<td>Interest rate of non-default project</td>
<td>$R$</td>
</tr>
<tr>
<td>Wage for worker</td>
<td>$W$</td>
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<tr>
<td><strong>Loan selling market</strong></td>
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<tr>
<td>Loan selling price (pooling)</td>
<td>$P$</td>
</tr>
<tr>
<td>High-type price (separating)</td>
<td>$P_{h}$</td>
</tr>
<tr>
<td>Low-type price (separating)</td>
<td>$P_{l}$</td>
</tr>
<tr>
<td>Proportion of high-type in the market</td>
<td>$\theta$</td>
</tr>
</tbody>
</table>

The model is calibrated to leverage loan market therefore is characterised by the specific institutions in the market.

G.2. Skin in the game

Skin in the game plays a very important role in securitization. Deteriorating underwriting standards, induced by low risk retention, has been recognized as contributing to Great Financial Crisis. However, we argue in leverage loan market, skin in the game does not play an important role because the arranger can’t commit to keep a fraction of loan on their balance sheet.

- Leverage loan guidance doesn’t impose mandatory risk retention by arrangers
- Usually at the origination stage, arranger banks on average retain less than 5% of total loan. And they can further re-distribute through the opaque secondary OTC market.
The leverage loan guidance issued by regulators stress the risk management during the transaction, which gives arrangers more incentive to sell them. Also this increases the confidence of loan buyer. Therefore retaining enough skin-in-game is not credible commitment in leverage loan market.

G.3. Different Credit Facilities in Leverage Loan Market

In leverage loan market, there are 2 kinds of credit facilities, term loan and credit line. The syndication arrangement is very different. For term loan, the arrangers usually syndicate to institutional investors while credit lines are distributed to other banks. The usual argument is that institutional investors do not have the expertise in liquidity management.

G.4. Pooling Equilibrium

In the model, we focus only on pooling equilibrium instead of separating equilibrium in loan distribution market. In the previous literature, originator’s retention of junior tranche is a signal of better asset quality. As we discuss above, retention is a creditable commitment therefore focusing only on pooling equilibrium is an legitimate assumption.

G.5. Distribution Market Structure

In leverage loan market, the distribution can happen at both origination stage (primary market) and after the origination stage (secondary market). The primary market includes a book running stage, when the arranger markets the loan to potential investors. This is more similar to a sequential dis-centralized market. The secondary market is an opaque OTC market.

Even though we model the distribution market as a centralized market, change in market structure will not change the results if we assume search cost is zero and arranger has all the bargaining power. The important assumption here is the rational expectation.

G.6. Primary Leverage Loan Market

The main reference here is Bruche, Malherbe and Meisenzahl(2018). However the link between cyclical liquidity in financial markets and credit condition for the real section has not been well understood. I present a unified theory that is the 1st to explain these facts simultaneously.

G.7. Information Asymmetry between Arranger and Syndicate Members

The main reference here is Ivashina(2009). She estimates the cost arising from information asymmetry between the lead bank and members of the lending syndicate. In a lending syndicate, the lead bank retains only a fraction of the loan but acts as the intermediary between the borrower and the syndicate participants. Theory predicts that asymmetric information will cause participants to demand a higher interest rate and that a large loan ownership by the lead bank should reduce this
effect. In equilibrium, however, the asymmetric information premium demanded by participants is offset by the diversification premium demanded by the lead. Using shifts in the idiosyncratic credit risk of the lead bank’s loan portfolio as an instrument, I measure the asymmetric information effect of the lead’s share on the loan spread and find that it accounts for approximately 4% of the total cost of credit.

H. Conclusion

I propose an dynamic equilibrium model where risk-averse banks originate long-term projects and distribute them to risk-neutral investors. The key friction is the information asymmetry between banks and investors. During recession, credit quality deteriorates, making it more costly for banks with good projects to distribute and increasing the severity of information asymmetry. This creates a feedback loop between retention and market liquidity.
REFERENCES


IV. Appendix

A. Appendix for Chapter 1

A.1. Proof of Lemma 1

We prove Lemma 1 by contradiction. Households’ required expected return of senior tranche $R_s \geq 1$. The expected return on intermediaries’ capital is

$$
EROC = \frac{(1 - \theta_t)(i_{L,t} + i_{H,t})pY + \theta_t(1 - \rho_L)i_{L,t}pY + \theta_t(1 - \rho_L)i_{L,t}pY - R_s k_t}{e_t}
$$

Intermediaries are endowed with all the bargaining power. In order to maximize $EROC$, intermediaries will offer households with unit expected return $R_s = 1$.

Suppose senior tranche held by households is risky, meaning the realized return under the worst-scenario case is strictly below one and the expected return for households is

$$
R_s > 1
$$

Therefore we get an contradiction. In the equilibrium, given the loan investment portfolio $\{i_{L,t}, i_{H,t}\}$, senior tranche held by households is risk-free and offer an expected return of one.

A.2. Proof of Lemma 2

Lemma 2 says that intermediaries provide the minimum amount of collateral to keep senior tranche risk-free. First we introduce a few notations. Given the loan investment portfolio $\{i_{L,t}, i_{H,t}\}$, denote

$$
k_t^* = (1 - \rho_L)pY i_{L,t} + (1 - \rho_H)pY i_{H,t}
$$

and the corresponding $e_t^*$ as

$$
i_{L,t} + i_{H,t} = k_t^* + e_t^*
$$

Now suppose the collateral constraint is slack

$$
k_t < (1 - \rho_L)pY i_{L,t} + (1 - \rho_H)pY i_{H,t}
$$

and the corresponding $e_t$ as

$$
i_{L,t} + i_{H,t} = k_t + e_t
$$

It is straightforward to see that given the loan investment portfolio $\{i_{L,t}, i_{H,t}\}$

$$
e_t > e_t^* \quad k_t < k_t^*
$$

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Therefore we have the corresponding $EROC$ and $EROC^*$

$$
EROC^* = \frac{(1 - \theta_t)(i_{L,t} + i_{H,t})pY + \theta_t(1 - \rho_L)i_{L,t}pY + \theta_t(1 - \rho_L)i_{H,t}pY - k_t^*}{e_t} 
$$

$$
EROC = \frac{(1 - \theta_t)(i_{L,t} + i_{H,t})pY + \theta_t(1 - \rho_L)i_{L,t}pY + \theta_t(1 - \rho_L)i_{H,t}pY - k_t}{e_t}
$$

We have $EROC^* > EROC$. Therefore intermediaries never provide extra collateral and the collateral constraint is always binding in the equilibrium.

### A.3. Proof of Lemma 3

The only way for intertemporal consumption transfer is by holding junior tranches for intermediaries. Therefore intermediaries’ consumption at $t + 1$ can be written as the return on intermediaries’ capital times the amount of consumption good invested in junior tranches. Intermediaries’ optimization can be written as

$$
\max_{\{c_t, e_t\}} c_t + \frac{1}{1 + r} E_t[c_{t+1}] 
$$

s.t. $c_t + e_t = w_t$

$$
c_{t+1} = e_t ROC_{t+1}
$$

Rewriting the optimization, we can get

$$
\max_{\{c_t\}} c_t + \frac{1}{1 + r} EROC_t (w_t - c_t) 
$$

$$
\max_{\{c_t\}} \frac{1}{1 + r} EROC_t w_t + \frac{(1 + r) - EROC_t}{1 + r} c_t
$$

We can get

- When $EROC_t < 1 + r$, intermediaries strictly prefer to consume at $t$. We have $c_t = w_t$ and $e_t = 0$.
- When $EROC_t = 1 + r$, intermediaries are indifferent between consumption at $t$ and holding junior tranche. We have $e_t \in [0, w_t]$ and $c_t = w_t - e_t$.
- When $EROC_t > 1 + r$, intermediaries strictly prefer holding junior tranche. We have $c_t = 0$ and $e_t = w_t$.

### A.4. Proof of Proposition 1

Since the expected return of low-risk loans is higher than the expected return of high-risk loans, intermediaries strictly prefer to securitize low-risk loans until the supply is exhausted. $\theta_L$ is the defined the maximum risk perception, at which intermediaries are willing to securitize low-risk
loans. Mathematically, we have

\[
\begin{align*}
&\left\{\begin{array}{l}
k_t + e_t = i_{L,t} \\
k_t = i_{L,t}(1 - \rho_L)pY \\
e_t = w_t \\
\frac{[(1 - \theta_H)pY + \theta_H(1 - \rho_L)pY]i_{L,t} - k_t}{e_t} = (1 + r)
\end{array}\right.
\]

Solving the system of equations, we can get

\[
\begin{align*}
&\left\{\begin{array}{l}
k_t = \frac{(1 - \rho_L)pY}{1 - (1 - \rho_L)pY}w_t \\
i_{L,t} = \frac{1}{1 - (1 - \rho_L)pY}w_t \\
\theta_H = 1 - (1 + r)\left[\frac{1 - \rho_L}{\rho_LpY}\right] = \frac{(1 + r)(pY - 1)}{\rho_LpY} - r
\end{array}\right.
\]

Then we can define $\theta_L$ as the maximum risk perception, at which intermediaries are willing to securitize only high-risk loans

\[
\begin{align*}
&\left\{\begin{array}{l}
k_t + e_t = i_{H,t} \\
k_t = i_{H,t}(1 - \rho_H)pY \\
e_t = w_t \\
\frac{[(1 - \theta_L)pY + \theta_H(1 - \rho_L)pY]i_{H,t} - k_t}{e_t} = (1 + r)
\end{array}\right.
\]

Solving the system we have

\[
\begin{align*}
&\left\{\begin{array}{l}
k_t = \frac{(1 - \rho_H)pY}{1 - (1 - \rho_H)pY}w_t \\
i_{H,t} = \frac{1}{1 - (1 - \rho_H)pY}w_t \\
\theta_L = 1 - (1 + r)\left[\frac{1 - \rho_H}{\rho_HpY}\right] = \frac{(1 + r)(pY - 1)}{\rho_HpY} - r
\end{array}\right.
\]

We can prove that $\theta_H > \theta_L$. Consider the following function

\[
\theta(\rho) = \frac{(1 + r)(pY - 1)}{\rho pY} - r
\]

It is easy to prove that the function $\theta(\rho)$ is a decreasing function of $\rho$. Since I have assumed $\rho_L < \rho_H$, we can get

\[
\theta_L = \theta(\rho_H) < \theta(\rho_L) = \theta_H
\]
The expected return of intermediaries’ capital is defined as follows

\[ EROC(\theta_t, w_t) = \frac{(1 - \theta_t)(i_{L,t} + i_{H,t})pY + \theta_t(1 - \rho_L)i_{L,t}pY + \theta_t(1 - \rho_L)i_{L,t}pY - R_s k_t}{e_t} \]

The function \( EROC(\theta_t, w_t) \) is a decreasing function of \( \theta_t \).

- In low risk perception regime \( \theta_t \in [\theta, \theta_L] \), \( EROC(\theta_t, w_t) \) is always strictly larger than its opportunity cost
  \[ EROC(\theta_t, w_t) > 1 + r \]

- In medium risk perception regime \( \theta_t \in [\theta_L, \theta_H] \), \( EROC(\theta_t, w_t) \) is weakly larger than its opportunity cost
  \[ EROC(\theta_t, w_t) \geq 1 + r \]

- In high risk perception regime \( \theta_t \in [\theta_H, \bar{\theta}] \), \( EROC(\theta_t, w_t) \) is always strictly smaller than its opportunity cost
  \[ EROC(\theta_t, w_t) < 1 + r \]

A.5. Proof of Proposition 2

When \( \{\theta_t, w_t\} \in [\theta, \theta_L] \times (0, +\infty) \), we have

\[ EROC(\theta_t, w_t) > 1 + r \]

When \( \{\theta_t, w_t\} \in (\theta_H, \bar{\theta}) \times (0, +\infty) \), we have

\[ EROC(\theta_t, w_t) < 1 + r \]

When \( \theta_t \in (\theta_L, \theta_H] \), the effect of capital \( w_t \) begins to affect \( EROC(\theta_t, w_t) \). For each \( \theta_t \in (\theta_L, \theta_H] \), we can define \( e^* \) as the threshold that intermediaries are indifferent between consumption and investment in junior tranche. Mathematically we have

\[
\begin{align*}
    k_t + e^* &= i_{L,t} + i_{H,t} \\
    k_t &= (1 - \rho_L)pYi_{L,t} + (1 - \rho_H)pYi_{H,t} \\
    i_{L,t} &= 1 \\
    (1 - \theta_t)pY(i_{L,t} + i_{H,t}) + \theta_t(1 - \rho_L)pYi_{L,t} + \theta_t(1 - \rho_H)pYi_{H,t} - k_t &= 1 + r \\
    e^* &= 1 + r
\end{align*}
\]
Solving the system of equations, we have

\[
\begin{align*}
\dot{i}_{H,t} &= \frac{e^* - [1 - (1 - \rho_L)pY]}{1 - (1 - \rho_H)pY} \\
\dot{k}_t &= \frac{(\rho_H - \rho_L)pY + (1 - \rho_H)pYe^*}{1 - (1 - \rho_H)pY} \\
(1 - \theta_t)pY(\rho_L + \rho_H\dot{i}_{H,t}) &= (1 + r)e^*
\end{align*}
\]

\[
\begin{align*}
\rho_LpY + \frac{\rho_HpY}{1 - (1 - \rho_H)pY}e^* - \frac{[1 - (1 - \rho_L)pY]\rho_HpY}{1 - (1 - \rho_H)pY} - \rho_LpY\theta_t - \frac{\rho_HpY}{1 - (1 - \rho_H)pY}e^*\dot{\theta}_t + \\
+ \frac{[1 - (1 - \rho_L)pY]\rho_HpY}{1 - (1 - \rho_H)pY}\theta_t &= (1 + r)e^*
\end{align*}
\]

\[
\begin{align*}
\frac{(\rho_H - \rho_L)(pY - 1)pY}{1 - (1 - \rho_H)pY} + \frac{(1 + r)(pY - 1) - r\rho_HpY}{1 - (1 - \rho_H)pY}e^* - \frac{\rho_HpY}{1 - (1 - \rho_H)pY}e^*\dot{\theta}_t - \frac{(\rho_H - \rho_L)(pY - 1)pY}{1 - (1 - \rho_H)pY}\theta_t &= 0
\end{align*}
\]

We use method of undetermined coefficient

\[
\begin{align*}
(e^* - \alpha)(\theta_t - \theta_L) &= \beta \\
\dot{\theta}_t - \theta_L e^* - \alpha \dot{\theta}_t + \alpha \theta_L - \beta &= 0
\end{align*}
\]

\[
\begin{align*}
\dot{\theta}_t - \frac{(1 + r)(pY - 1) - r\rho_HpY}{\rho_HpY}\theta_t - \frac{(\rho_H - \rho_L)(pY - 1)pY}{\rho_HpY}\theta_t - \frac{(\rho_H - \rho_L)(pY - 1)pY}{\alpha \theta_L - \beta} &= 0
\end{align*}
\]

We can get

\[
\begin{align*}
\alpha &= \frac{(\rho_H - \rho_L)(pY - 1)}{\rho_H} < 0 \\
\beta &= \alpha(\theta_L - 1) > 0
\end{align*}
\]

The expression for \( e^*(\theta_t) \) is

\[
e^*(\theta_t) = \frac{\beta}{\theta_t - \theta_L} + \alpha
\]
A.6. Proof of Proposition 3

We have used the following system of equations to define $\theta_H$

$$\begin{align*}
  k_t + e_t &= i_{L,t} \\
  k_t &= i_{L,t}(1 - \rho_L)p_Y \\
  e_t &= w_t \\
  \frac{[(1 - \theta_H)p_Y + \theta_H(1 - \rho_L)p_Y]i_L - k_t}{e_t} &= (1 + r)
\end{align*}$$

Solving the system of equations, we can get

$$\begin{align*}
  k_t &= \frac{(1-\rho_L)p_Y}{1-(1-\rho_L)p_Y}w_t \\
  i_{L,t} &= \frac{1}{1-(1-\rho_L)p_Y}w_t \\
  \theta_H &= 1 - (1 + r)\left(\frac{1-(1-\rho_L)p_Y}{\rho_Lp_Y}\right) = \frac{(1+r)(\rho_Y-1)}{\rho_Lp_Y} - r
\end{align*}$$

Here $w^*$ is defined as

$$\begin{align*}
  i_{L,t} &= \frac{1}{1-(1-\rho_L)p_Y}w^* = 1 \\
  w^* &= 1 - (1 - \rho_L)p_Y
\end{align*}$$

We have the following

- When $w_t > w^*$, we have $i_{H,t} > 0$
- When $w_t \leq w^*$, we have $i_{H,t} = 0$

A.7. Static Equilibrium Allocation

In this section, we solve equilibrium allocation in the static model in the four different regions.

- In Region 1 (No securitization) where $\{\theta_t, w_t\} \in (\theta_H, \bar{\theta}] \times (0, +\infty)$, we have $EROC(\theta_t, w_t) < 1 + r$ and equilibrium allocation is

$$\begin{align*}
  i_{L,t} &= 0 \\
  i_{H,t} &= 0 \\
  e_t &= 0 \\
  k_t &= w_t \\
  c_t &= w_t
\end{align*}$$

- In Region 2 ($\theta$-constrained Region) where $\{\theta_t, w_t\} \in (\theta_L, \theta_H] \times [e^*(\theta_t), +\infty)$, in the equilibrium we have $EROC(\theta_t, w_t) = 1 + r$ and equilibrium allocation is

$$\begin{align*}
  i_{L,t} &= 1 \\
  i_{H,t} &= \frac{e^*(\theta_t) - [1 - (1 - \rho_L)p_Y]}{1 - (1 - \rho_H)p_Y} \\
  e_t &= e^*(\theta_t) \\
  k_t &= \frac{(\rho_H - \rho_L)p_Y + (1 - \rho_H)p_Ye^*(\theta_t)}{1 - (1 - \rho_H)p_Y}
\end{align*}$$

- In Region 3 (Moderate $w$-constrained Region) where $\{\theta_t, w_t\} \in [\underline{\theta}, \theta_H] \times [w^*, e^*(\theta_t))$, in the equilibrium we have $EROC(\theta_t, w_t) > 1 + r$, the equilibrium allocation is defined by the
following system of equations

\[
\begin{align*}
k_t + e_t &= i_{L,t} + i_{H,t} \\
k_t &= (1 - \rho_L)pY + (1 - \rho_H)pY_i_{H,t} \\
e_t &= w_t \\
i_{L,t} &= 1
\end{align*}
\]

Solving the system we can the equilibrium allocation

\[
i_{L,t} = 1 \quad i_{H,t} = \frac{w_t - [1 - (1 - \rho_L)pY]}{1 - (1 - \rho_H)pY} \quad c_t = 0 \\
e_t = w_t \quad k_t = \frac{(\rho_H - \rho_L)pY + (1 - \rho_H)pY_w_t}{1 - (1 - \rho_H)pY}
\]

- In Region 4 (Severe \(w\)-constrained Region) where \(\{\theta_t, w_t\} \in [\theta_L, \theta_H] \times [0, w^*]\) in the equilibrium we have \(EROC(\theta_t, w_t) > 1 + r\), the equilibrium allocation is defined by the following system of equations

\[
\begin{align*}
k_t + e_t &= i_{L,t} \\
k_t &= (1 - \rho_L)pY_i_{H,t} \\
e_t &= w_t
\end{align*}
\]

Solving the system we can the equilibrium allocation

\[
i_{L,t} = \frac{w_t}{1 - (1 - \rho_L)pY} \quad i_{H,t} = 0 \quad c_t = 0 \\
e_t = w_t \quad k_t = \frac{(1 - \rho_L)pY_w_t}{1 - (1 - \rho_L)pY}
\]

A.8. Comparative Statics

In this section, we compute the measures of risk-taking \(\sigma_t\), size \(s_t\) and safe debt issuance \(k_t\) in each region. Then we present the comparative statics inside the region.

- In Region 1, there is no securitization

- In Region 2, the comparative statics for risk-taking measure \(\sigma_t\)

\[
\sigma_t = \frac{i_{H,t}}{i_{H,t} + i_{L,t}} = \frac{i_{H,t}(\theta_t)}{i_{H,t}(\theta_t) + 1} \\
\frac{\partial \sigma_t}{\partial \theta_t} = \frac{\partial i_{H,t}}{\partial i_{H,t}} \frac{\partial i_{H,t}}{\partial \theta_t} > 0 \\
\frac{\partial \sigma_t}{\partial w_t} = \frac{\partial i_{H,t}}{\partial i_{H,t}} \frac{\partial w_t}{\partial i_{H,t}} = 0
\]
The comparative statics for size measure $s_t$

$$s_t = 1 + i_{H,t}(\theta_t)$$

$$\frac{\partial s_t}{\partial \theta_t} = \frac{\partial i_{H,t}}{\partial \theta_t} > 0$$

$$\frac{\partial s_t}{\partial w_t} = \frac{\partial i_{H,t}}{\partial w_t} = 0$$

The comparative statics for safe debt issuance measure $k_t$

$$k_t = \frac{(\rho_H - \rho_L)pY + (1 - \rho_H)pYe^*(\theta_t)}{1 - (1 - \rho_H)pY}$$

$$\frac{\partial k_t}{\partial \theta_t} > 0$$

$$\frac{\partial k_t}{\partial w_t} = 0$$

- In Region 3, the comparative statics for risk-taking measure $\sigma_t$

$$\sigma_t = \frac{i_{H,t}(w_t)}{i_{H,t}(w_t) + 1}$$

$$\frac{\partial \sigma_t}{\partial \theta_t} = 0$$

$$\frac{\partial \sigma_t}{\partial w_t} > 0$$

The comparative statics for size measure $s_t$

$$s_t = 1 + i_{H,t}(w_t)$$

$$\frac{\partial s_t}{\partial w_t} = \frac{\partial i_{H,t}}{\partial w_t} > 0$$

$$\frac{\partial s_t}{\partial \theta_t} = \frac{\partial i_{H,t}}{\partial \theta_t} = 0$$

The comparative statics for safe debt issuance measure $k_t$

$$k_t = \frac{(\rho_H - \rho_L)pY + (1 - \rho_H)pYw_t}{1 - (1 - \rho_H)pY}$$

$$\frac{\partial k_t}{\partial \theta_t} = 0$$

$$\frac{\partial k_t}{\partial w_t} > 0$$

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• In Region 4, risk-taking measure $\sigma_t$

$$\sigma_t = 0$$

$$\frac{\partial \sigma_t}{\partial \theta_t} = 0$$

$$\frac{\partial \sigma_t}{\partial w_t} = 0$$

The comparative statics for size measure $s_t$

$$s_t = 1 + i_{H,t}(w_t)$$

$$\frac{\partial s_t}{\partial w_t} = \frac{\partial i_{H,t}}{\partial w_t} > 0$$

$$\frac{\partial s_t}{\partial \theta_t} = \frac{\partial i_{H,t}}{\partial \theta_t} = 0$$

The comparative statics for safe debt issuance measure $k_t$

$$k_t = \frac{(\rho_H - \rho_L)pY + (1 - \rho_H)pY w_t}{1 - (1 - \rho_H)pY}$$

$$\frac{\partial k_t}{\partial \theta_t} = 0$$

$$\frac{\partial k_t}{\partial w_t} > 0$$

B. Appendix for Chapter 2

B.1. Proof of Proposition 4

We can write intermediaries’ expected wealth tomorrow as

$$E_t(w_{t+1}) = E_t[ROC_{t+1}e_t]$$

The expected speed of capital accumulation can be written as

$$\rho^w(\theta_t, w_t) = EROC(\theta_t, w_t) \frac{e_t}{w_t}$$

We know that in $\theta$-constrained Region, intermediaries consume a fraction of their capital therefore $\frac{\sigma_t}{w_t} < 1$. In this region, intermediaries will securitize loans until the $EROC(\theta_t, w_t) = 1 + r$. As a result, in $\theta$-constrained Region

$$\rho^w(\theta_t, w_t) < 1 + r$$

In Moderate and Severe $w$-constrained Regions, intermediaries don’t consume at time $t$ and the expected return on intermediaries’ capital is strictly larger than $1 + r$. We have $\frac{\sigma_t}{w_t} = 1$

$$\rho^w(\theta_t, w_t) > 1 + r$$
B.2. Proof of Lemma 4

Denote $\phi_1(s_t) = Pr(\xi_t = 1|m_t = 1, H_{t-1})$ and $\phi_0(s_t) = Pr(\xi_t = 0|m_t = 0, H_{t-1})$. Using Bayes theorem, we have

$$\phi_1(s_t) = Pr(\xi_t = 1|m_t = 1, H_{t-1})$$
$$= \frac{Pr(m_t = 1|\xi_t = 1, H_{t-1})Pr(\xi_t = 1|H_{t-1})}{Pr(m_t = 1|\xi_t = 1, H_{t-1})Pr(\xi_t = 1|H_{t-1}) + Pr(m_t = 1|\xi_t = 0, H_{t-1})Pr(\xi_t = 0|H_{t-1})}$$
$$= \frac{\gamma(s_t)\theta_t}{\gamma(s_t)(1 - \theta_t) + [1 - \gamma(s_t)](1 - \theta_t)}$$

$$\phi_0(s_t) = Pr(\xi_t = 0|m_t = 0, H_{t-1})$$
$$= \frac{Pr(m_t = 0|\xi_t = 0, H_{t-1})Pr(\xi_t = 0|H_{t-1})}{Pr(m_t = 0|\xi_t = 0, H_{t-1})Pr(\xi_t = 0|H_{t-1}) + Pr(m_t = 0|\xi_t = 1, H_{t-1})Pr(\xi_t = 1|H_{t-1})}$$
$$= \frac{\gamma(s_t)(1 - \theta_t)}{\gamma(s_t)(1 - \theta_t) + [1 - \gamma(s_t)](1 - \theta_t)}$$

Denote $\phi_1(s_t) = Pr(\xi_t = 1|m_t = 1, H_{t-1})$ and $\phi_0(s_t) = Pr(\xi_t = 0|m_t = 0, H_{t-1})$. Using Bayes theorem, we have

$$\phi_1(s_t) = Pr(\xi_t = 1|m_t = 1, H_{t-1})$$
$$= \frac{Pr(m_t = 1|\xi_t = 1, H_{t-1})Pr(\xi_t = 1|H_{t-1})}{Pr(m_t = 1|\xi_t = 1, H_{t-1})Pr(\xi_t = 1|H_{t-1}) + Pr(m_t = 1|\xi_t = 0, H_{t-1})Pr(\xi_t = 0|H_{t-1})}$$
$$= \frac{\gamma(s_t)\theta_t}{\gamma(s_t)(1 - \theta_t) + [1 - \gamma(s_t)](1 - \theta_t)}$$

$$\phi_0(s_t) = Pr(\xi_t = 0|m_t = 0, H_{t-1})$$
$$= \frac{Pr(m_t = 0|\xi_t = 0, H_{t-1})Pr(\xi_t = 0|H_{t-1})}{Pr(m_t = 0|\xi_t = 0, H_{t-1})Pr(\xi_t = 0|H_{t-1}) + Pr(m_t = 0|\xi_t = 1, H_{t-1})Pr(\xi_t = 1|H_{t-1})}$$
$$= \frac{\gamma(s_t)(1 - \theta_t)}{\gamma(s_t)(1 - \theta_t) + [1 - \gamma(s_t)](1 - \theta_t)}$$

B.3. Proof of Proposition 5

Under standard learning assumption, we define the following updating function: if there is no realization of correlation shock, risk perception is revised upward

$$\hat{\pi}^+(\pi_t) = \frac{\pi_t(1 - \theta)}{\pi_t(1 - \theta) + (1 - \pi_t)(1 - \theta)} > \pi_t$$

If there is no realization of correlation shock, risk perception is revised downward

$$\hat{\pi}^-(\pi_t) = \frac{\pi_t\theta}{\pi_t\theta + (1 - \pi_t)\theta} < \pi_t$$
Under asymmetric learning model, information set has changed and the history is defined as
\[ H_t = \{ \} \] We can write intermediaries’ expected wealth tomorrow as
\[ E_t(w_{t+1}) = E_t[ROC_{t+1}e_t] \]

The expected speed of capital accumulation can be written as
\[ \rho^w(\theta_t, w_t) = EROC(\theta_t, w_t) \frac{e_t}{w_t} \]

We know that in \( \theta \)-constrained Region, intermediaries consume a fraction of their capital therefore \( \frac{e_t}{w_t} < 1 \). In this region, intermediaries will securitize loans until the \( EROC(\theta_t, w_t) = 1 + r \). As a result, in \( \theta \)-constrained Region
\[ \rho^w(\theta_t, w_t) < 1 + r \]

In Moderate and Severe \( w \)-constrained Regions, intermediaries don’t consume at time \( t \) and the expected return on intermediaries’ capital is strictly larger than 1 + \( r \). We have \( \frac{e_t}{w_t} = 1 \)
\[ \rho^w(\theta_t, w_t) > 1 + r \]
B.4. Recovery Dynamics under Standard and Asymmetric Learning

Asymmetric learning is the key assumption of the model. In the subsection, we compare the recovery dynamics under standard learning and asymmetric learning. In model with standard learning, the ex-post signals are perfectly correlated with realization of correlation shock regardless of total volume. We can see that
Figure 17. The figure compares recovery dynamics of shadow banking sector from different confidence levels after the realization of correlation shock at $t = 0$. At $t = 1$, the blue line starts with $\{w_1 = 0.06, \theta_1 = 0.5574\}$ and the black line starts with $\{w_1 = 0.16, \theta_1 = 0.5574\}$. Panel A traces out the two paths of confidence and capital in space $w_t$ vs $\theta_t$. Panel B1 shows the evolution of intermediary consumption $c_t$ over time. Panel B2 shows the evolution of risk-taking $\sigma_t$ over time. Panel B3 shows the evolution of shadow banking size $s_t$ over time. Panel B4 shows the evolution of safe debt issuance $k_t$ over time. The parameter values are: $\{pY = 1.1, \rho_M = 0.7, \rho_L = 0.2, r = 0.02\}$.
For $\theta_t$ in medium regime

- $w_t \geq e^*(\theta_t) \Rightarrow$ risk perception binding constraint
- $w_t < e^*(\theta_t) \Rightarrow$ capital binding constraint
1. Nosecuritization

Low securitization ($t_k, s_t \uparrow$)

High securitization ($t_k \downarrow, s_t \downarrow$)

Low securitization ($t_k \downarrow, s_t \downarrow$)
1. No securitization

low securitization

low $\rho^\theta$

low securitization

high $\rho^w$
1. No securitization

- High $\rho^\theta$
- High securitization
- Low $\rho^\theta$
- Low securitization

$e^*(\theta_t)$

$\rho^\theta$ increasing

$w^*$

$\theta_L$

$\theta_H$

$\theta$
C. Appendix for Chapter 3

There are 3 groups of agents, originators (denote by O), investors (denote by I) and workers.

1. There is unit-measure continuum of long-lived Os with preference over consumption flows represented by the following utility function

\[ U^O = E\left[ \sum_{t=0}^{\infty} \beta^t \log c_t \right] \]

where \( \beta \) is physical discount factor. Os are the only agents endowed with the ability to lend directly to firms.

2. There is also measure-one continuum of risk-neutral investors (Is) with deep pockets and they share the same physical discount factor with Os. They can’t directly lend to firms however they can buy financial claims from Os in the secondary loan market.

3. There is also measure-one risk-neutral workers in the economy, each of whom supplies one unit of labor inelastically. For simplicity, we assume they have no access to financial markets. Therefore they just consume their wage in hand-to-month fashion.

**Firms, Technology and Information**

Each period, there is a continuum of competitive and ex-ante identical firms, indexed by \( i \in [0, 1] \). To capture the idea real investments are long-term and illiquid, we assume projects undertaken by active firms only payoff after 2 periods and they have an overlapping generation structure. Each firm operates a constant-return-to-scale technology that transform capital \( k_{it} \) and labor \( n_{it} \) into consumption good as follows

\[ y_{it+2} = (1 - z_{it+2})[F(k_{it}, n_{it}) + (1 - \delta)k_{it}] + z_{it+2}(1 - \lambda)k_{it} \]

The binary random variable \( z_{it+2} \in \{0,1\} \), realized at \( t+2 \), indicates whether the production undertaken by firm succeeds \( (z_{it+2} = 0) \) or fails \( (z_{it+2} = 1) \). The parameter \( \delta \) and \( \lambda > \delta \) are the rate at which physical capital depreciates when the firms’ production succeeds and fails, respectively. Here \( 1 - \lambda \) can be interpreted as loss-given-default in the banking literature. The net output function \( F(k_{it}, n_{it}) \) in case of success is characterized by Cobb-Douglas production function.

\[ F(k_{it}, n_{it}) = Ak_{it}^\alpha n_{it}^{1-\alpha} \]

where \( A \) is the productivity level and \( \alpha \in (0,1) \). In case of production failure, there is no output on top of depreciated capital.

For projects starting at \( t \), Os can observe a private signal \( q \in \{q_H, q_L\} \) \( (q_H > q_L) \) of the quality of their loan in the next period at \( t+1 \). \( q \) can be interpreted as conditional probability of success. Henceforth we refer Os with \( q_H \) as high-type and Os with \( q_L \) as low-type. The proportion of high-type denoting as \( q^H \), depend on state of the economy \( s \). The information set of ongoing projects for Os at time \( t \) is shown in Figure 18. For Is, they can’t observe the private signal \( q \in \{q_H, q_L\} \).
They only observe the unconditional probability of success \( q^I_s = q^I_0 q_H + (1 - q^I_0) q_L \). Is’ information set is shown in figure 19.

Since Os are long-lived, they hold overlapping projects. They begin each period with both projects that are ongoing and completed. Os are ex-ante identical, but they differ in terms of current income from completed projects and quality of ongoing projects due to idiosyncratic quality shocks. The only aggregate shock in the model is the state of economy \( s \), where \( s \in \{E, R\} \) is binary random variable (E indicates expansion and R indicates recession).

**Loan Markets**

There are 2 loan markets in the economy, primary and secondary loan markets.

1. **Primary Loan Market**: Each period, firms and Os are randomly matched in a competitive market where firms borrow capital from Os. We also assume contract between firms and Os is standard debt contract and the repayment structure is

\[
\tilde{R} = \begin{cases} 
R & \text{in case of success} \\
1 - \lambda & \text{in case of failure}
\end{cases}
\]
2. **Secondary Loan Market**: Each period, Os also have access to an anonymous competitive market in which they can sell claims of ongoing loans to Is. Since Os have private information about the quality of ongoing loans, the secondary market is illiquid due to adverse selection. The difference of information set is described in figures 18 and 19. Apart from private information, Os have 2 other incentives for selling: risk-sharing and funding recycling. Os are risk-averse and Is are risk-neutral, therefore selling can facilitate better risk-sharing. Meanwhile Os’ investment opportunities come in overlapping structure, selling increase liquidity of Os’ balance sheet, meaning Os can recycle fund from ongoing long-term project and expand their lending capacity. The degree of secondary market liquidity (adverse selection) is jointly determined by all the 3 factors together. If more sellings are private information based, the market is more illiquid. As a result high-type Os prefer to keep loans on their balance sheet instead of selling to Is, leading to less new loans originated.

**Time-line**

1. Os enter date $t$ with cash $x$ and ongoing project $l$
2. State of economy $s$ is revealed as public information
3. Os receive private signal $q$ about the quality of their loan
4. Secondary loan market opens and Os make selling decision to Is
5. Primary loan market opens and firm borrow capital from Os

C.1. **First-Best Benchmark**

Before characterizing equilibrium under asymmetric information, it is useful to analyze the equilibrium if O’s type is observable.

Firm’s optimization is static and they only maximize their profit under the case of successful production

$$\max_{\{k_t, n_t\}} AF(k_t, n_t) - Rk_t - Wn_t$$

Since the labor supply is inelastic, we have

$$R = Ao k_t^{\alpha - 1}$$
$$W = A(1 - \alpha) k_t^\alpha$$

Taken production failure into account, we have the function for wage and interest rate

$$\tilde{R}(X) = \begin{cases} 
R(X) + (1 - \delta) & \text{Success} \\
1 - \lambda & \text{Failure}
\end{cases}$$

$$\tilde{W}(X) = \begin{cases} 
W(X) & \text{Success} \\
0 & \text{Failure}
\end{cases}$$
If \( q \in \{q_H, q_L\} \) were observable for the Is, there will be 2 markets and selling price must be such that I’s expected profit equals zero. For high type O, the selling price is

\[
P_H = \beta[q_H R + (1 - q_H)(1 - \lambda)]
\]

For low-type O, the selling price is

\[
P_L = \beta[q_L R + (1 - q_L)(1 - \lambda)]
\]

Let \( X = \{s, M, K_o\} \) be the aggregate state variable, where \( s \) is state of the economy, \( M \) is aggregate cash and \( K_o \) is the aggregate ongoing loans. Originator solves the Bellman equation

\[
V(m, k_o, q_i, X) = \max_{\{c, k_s, k_o', m'\}} \{u(c) + \beta E[V(m', k_o', q_i', X')|X]\}
\]

s.t.

\[
c + k_o' = m + k_s P_i
\]

\[
m' = (k_o - k_s) \hat{R}
\]

\[
k_s \in [0, k_o]
\]

\[
i \in \{H, L\}
\]

Constraint 23b is O’s budget constraint. The RHS is the available fund, including \( m \) the income from completed loan and \( k_s P_i \) the proceeds coming from loan sales. Constraint 23c keeps track of O’s holding of loans and constraint 23d is the no short-sale constraint.

**Equilibrium Conditions**

Here define competitive equilibrium with observable types.

**DEFINITION 6**: A competitive equilibrium with observable types consists of prices \( \{\hat{R}(X), \hat{W}(X), P_H, P_L\} \); a law of motion \( \Gamma(X) \) and associated transition density \( \Pi(X'|X) \); 2 value functions \( V(m, k_o, q_i, X) \) for \( i = H, L \); policy functions \( \{c^w(X), c(m, k_o, q_i, X), k_s(m, k_o, q_i, X), k_o'(m, k_o, q_i, X), m'(m, k_o, q_i, X)\} \) for \( i = H, L \) such that

1. Firms maximize profit
2. Workers consume their wage \( c^w(X) = \hat{W}(X) \)
3. Policy functions \( \{c(m, k_o, q_i, X), k_s(m, k_o, q_i, X), k_o'(m, k_o, q_i, X), m'(m, k_o, q_i, X)\} \) and value functions \( V(m, k_o, q_i, X) \) for \( i = H, L \) solves Programming 23 taken prices as given
4. Investors make zero expected profit under selling price \( \{P_H, P_L\} \)
5. Law of motion \( \Gamma \) is consistent with aggregation of individual decision
6. All the markets clear

If investors can observe \( q \), both high- and low-type O’s will sell all the ongoing loans to Is to fully diversify the idiosyncratic shocks. Hence \( k_s = k_o \) and \( m = 0 \). Therefore we can reduce the number
of state variables: the aggregate state variable $X = \{s, K_o\}$. We can rewrite O’s optimization as

$$ V(k_o, q_i, X) = \max_{\{c, k_o'\}} \{u(c) + \beta E[V(k_o', q_i', X')|X]\} \quad (24a) $$

s.t. 

$$ c + k_o' = k_o P_i \quad (24b) $$

$$ i \in \{H, L\} \quad (24c) $$

**LEMMA 6: (Properties of Programming 24)**

1. Policy functions to Programming 24, $c(k_o, q_i, X)$ and $k_o'(k_o, q_i, X)$, are both linear in $k_o$. Therefore prices and aggregate quantities do not depend on the distribution of O’s loan holding, only depends on the aggregate loan holding $K_o$.

2. With logarithm utility function, the policy functions are

$$ c(K_o, q_i) = (1 - \beta)K_o P_i $$

$$ k_o'(K_o, q_i) = \beta K_o P_i $$

Linearity of policy functions directly follows Alvarez and Stokey (1998). Constraint in Programming 24 is linear in $k_o$ and the utility function is homothetic. Therefore history of idiosyncratic shocks can be fully summarized in current period aggregate state variables $X$. The aggregate credit supply by O is

$$ K(X) = (1 - \beta)K_o \left[ q_o^s P_H + (1 - q_o^s) P_L \right] $$

Average selling with full participation

Denoting $\bar{P}^{FB} = [q_o^s P_H + (1 - q_o^s) P_L]$.

**C.2. Competitive Equilibrium Under Asymmetric Information**

Here we only consider pooling equilibrium. For simplicity, we assume the secondary market is anonymous therefore signalling by retention is not possible. The firm’s optimization remains the same.

The difference with FB case is that both high- and low-type Os are selling at the same price $P$ in the same market in Programming 25. Under asymmetric information, the selling price is lower than the average selling price with full participation $P < \bar{P}^{FB}$, meaning the high-type is selling undervalued assets while low-type is selling overvalued assets. We will show that in the equilibrium, high-type Os always prefer to retain part of the ongoing loans on their balance sheet.
However low-type Os prefer to sell all of them. The optimization can be characterized.

\[
V(m, k_o, q_i, X) = \max_{\{c, k, k_o, m, f\}} \{u(c) + \beta E[V(m', k'_o, q'_i, X')|X]\}
\]

\[
s.t. \quad c + k'_o = m + k_s P
\]
\[
m' = (k_o - k_s)\tilde{R}_i
\]
\[
k_s \in [0, k_o]
\]
\[
i \in \{H, L\}
\]

(25)

The properties of programming \ref{25} is the same except for high-type Os.

**Lemma 7:** Policy functions to Programming \ref{25} \(c(k_o, q_i, X)\) and \(k'_o(k_o, q_i, X)\), are both linear in \(k_o\). Therefore prices and aggregate quantities do not depend on the distribution of O’s loan holding, only depends on the aggregate loan holding \(K_o\).

The policy function for low type Os will be the same except the selling price being \(P\) instead of \(P_L\).

\[
c_L = (1 - \beta)(M + K_o P)
\]
\[
k'_L = \beta(M + K_o P)
\]
\[
m'_L = 0
\]

For high-type Os, the optimization is more involved

\[
V_H(m, k_o, X) = \max_{\{c, k, k'_o, m'\}} \{u(c) + \beta E[V(m', k'_o, q'_i, X')|X]\}
\]

\[
s.t. \quad c + k'_o = m + k_s P
\]
\[
m' = (k_o - k_s)\tilde{R}_i
\]
\[
k_s \in [0, k_o]
\]
\[
i \in \{H, L\}
\]

(26)

The Euler equations are

\[
\begin{align*}
\frac{u'(C)}{P} &= E[\beta u'(c')\tilde{R}_i] \\
\frac{u'(C)}{C} &= E[\beta^2 u''(c'')\tilde{R}_i]
\end{align*}
\]

(27)

Questions

- Linearity of policy functions, whether the proof is correct
- Intuition: Separation between consumption, investment decision and selling decision. But do not know how to prove. How to guess the functional form of value and policy functions.

**C.3. Competitive Equilibrium Under Asymmetric Information**

Here we present competitive equilibrium when types are private information. The assumption here is that market where arrangers sell loan to investors is anonymous.
For low-type,

$$V_l(m, y_o, s) = \max_{\{c_l, y'_{o,l}, y_{s,l}, m'_l\}} u(c_l) + \beta E[V_l(m'_l, y'_{o,l}, s')|s]$$

s.t.  
\[
\begin{align*}
&c_l + y'_{o,l} = m + y_{s,l}P \\
&m'_l = (y_o - y_{s,l})\bar{R}_l \\
&y_{s,l} \in [0, y_o]
\end{align*}
\]

(28)

The key difference here is here the high-type and low-type are selling at the same price. For low-type the policy functions are the same

\[
\begin{align*}
&c_l = (1 - \beta)(m + y_oP) \\
&y'_{o,l} = \beta(m + y_oP) \\
&y_{s,l} = y_o
\end{align*}
\]

For high-type, the problem is very different now. High-type arranger faces a dynamic optimization because

$$V_h(m, y_o, s) = \max_{\{c_h, y'_{o,h}, y_{s,h}, m'_h\}} u(c_h) + \beta E[V_h(m'_h, y'_{o,h}, s')|s]$$

s.t.  
\[
\begin{align*}
&c_h + y'_{o,h} = m + y_{s,h}P \\
&m'_h = (y_o - y_{s,h})\bar{R}_h \\
&y_{s,h} \in [0, y_o]
\end{align*}
\]

(29)