Correction Note to Pathwise Large Deviations for the Rough Bergomi Model

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Abstract

This note corrects an error in the definition of the rate function in [Jacquier et al., Pathwise large deviations for the rough Bergomi model, J. Appl. Prob. 2018] and slightly simplifies some proofs.

1 Corrected rate function

Note that the correct rate function also appears in the PhD thesis [3] (see Proposition 1.4.18), but with a different proof. We first give a slightly simplified proof of Theorem 3.1 in [1]. Any unexplained notation is as in [1]. Let $Y := \int_0^{\cdot} \varphi(u,\cdot)dW_u$ be the Gaussian process from that theorem, and $K_Y : \mathcal{C}^* \to \mathcal{C}$ its covariance operator (definition on p. 5 of [2]). As noted in [1], $\mathcal{I}^\varphi$ is injective by Titchmarsh's convolution theorem. By the factorization theorem (Theorem 4.1 in [2]) and the discussion on pp. 32–33 of [2], it suffices to verify the factorization identity $\mathcal{I}^\varphi(\mathcal{I}^\varphi)^* = K_Y$ to conclude that the RKHS is the image $\mathcal{I}^\varphi\big(L^2([0,1])\big)$. By Fubini's theorem, we have $(\mathcal{I}^\varphi)^*\mu = \int_\cdot^1 \varphi(\cdot,t)\mu(dt)$ for any measure $\mu \in \mathcal{C}^*$. We then compute, for

$\mu, \nu \in \mathcal{C}^*$,

$$\begin{aligned}\mu\big(\mathcal{I}^\varphi(\mathcal{I}^\varphi)^*\nu\big) &= \int_0^1 \int_0^t \varphi(u,t) \int_u^1 \varphi(u,s)\,\nu(ds)\,du\,\mu(dt)\\ &= \int_0^1 \int_0^1 \int_0^{s\wedge t} \varphi(u,t)\varphi(u,s)\,du\,\nu(ds)\,\mu(dt)\\ &= \int_0^1 \int_0^1 \mathbb{E}[Y_t Y_s]\,\nu(ds)\,\mu(dt) = \mathbb{E}[\mu(Y)\nu(Y)],\end{aligned}$$

which proves the theorem.

The second definition in (2.3) of [1] should be replaced by the following one.

Definition 1.1. *For $\Phi : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^{2\times 2}$, define $\mathcal{I}^\Phi : L^2([0,1],\mathbb{R}^2) \to L^2([0,1],\mathbb{R}^2)$ by*

$$\mathcal{I}^\Phi f := \int_0^\cdot \Phi(u,\cdot) f(u) du.$$

The following theorem replaces Theorem 3.2 of [1].

Theorem 1.2. *Let φ_1, φ_2 satisfy Assumption 3.1 of [1], and define $Y_i := \int_0^\cdot \varphi_i(u,\cdot) dW_u^i$, $i = 1, 2$, where W^1 and W^2 are standard Brownian motions with correlation $\rho \in (-1,1)$. Then the RKHS of (Y_1, Y_2) is*

$$\mathcal{H}^\Phi := \{\mathcal{I}^\Phi f : f \in L^2([0,1],\mathbb{R}^2)\},$$

with inner product $\langle \mathcal{I}^\Phi f, \mathcal{I}^\Phi g\rangle = \langle f, g\rangle$, where

$$\Phi = \begin{pmatrix} \varphi_1 & 0 \\ \rho\varphi_2 & \sqrt{1-\rho^2}\varphi_2 \end{pmatrix}.$$

Proof. Analogous to the proof above. Injectiveness of $\mathcal{I}^\Phi$ follows from the Titchmarsh convolution theorem. For a measure $\mu \in (\mathcal{C}^2)^*$, we have $(\mathcal{I}^\Phi)^*\mu = \int_\cdot^1 \Phi^\top(\cdot, t)\mu(dt)$. The factorization identity $\mathcal{I}^\Phi(\mathcal{I}^\Phi)^* = K_{Y_1,Y_2}$ is verified as above. ∎

Theorem 1.2 implies the following corollary, which replaces Corollary 3.2 of [1].

Corollary 1.3. *The RKHS of the measure induced on $\mathcal{C}^2$ by the process (Z, B) is $\mathcal{H}^\Psi$, where*

$$\Psi = \begin{pmatrix} K_\alpha & 0 \\ \rho & \sqrt{1-\rho^2} \end{pmatrix}.$$

Consequently, $\|\cdot\|_{\mathcal{H}^\Psi}$ should replace $\|\cdot\|_{\mathcal{H}_\rho^{K_\alpha}}$ in line 4 of p. 1083 and in the proof of Theorem 2.1 of [1] on p. 1088. The special case $\rho = 0$ requires no separate treatment, and the result agrees with Section 5 of [1].

2 Minor corrections

1. p. 1079, last line of the introduction: replace $\int_0^1$ by $\int_0^\cdot$.
2. p. 1084, definition of topological dual: add "continuous" before "linear functionals".
3. p. 1085, second displayed formula: After the second =, replace f by $\Gamma(f^*)$.
4. In the statement of Theorem 3.4, $\varepsilon\mu$ should be replaced by $\mu(\varepsilon^{-1/2}\,\cdot)$. The speed $\varepsilon^{-\beta}$ resulting from the application of the theorem on p. 1088 is correct, though.
5. First line of p. 1089: Replace $v_0^{1+\beta}$ by $v_0\varepsilon^{1+\beta}$. To make the estimate work for $t = 0$, confine ε to the finite interval $[0, 1]$ instead of $\mathbb{R}^+$ in line -4 of p. 1088.

References

[1] A. Jacquier, M. S. Pakkanen, and H. Stone, *Pathwise large deviations for the rough Bergomi model*, J. Appl. Probab., 55 (2018), pp. 1078–1092.

[2] M. Lifshits, *Lectures on Gaussian processes*, SpringerBriefs in Mathematics, Springer, Heidelberg, 2012.

[3] H. Stone, *Rough volatility models: small-time asymptotics and calibration*, PhD thesis, Imperial College, 2019.