Novel Methodology for the Optimisation of Turbocharger Turbine Design for Improved Engine Performance

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This thesis is submitted for the award of Doctor of Philosophy (PhD)

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April 2020
I declare the work in this thesis to be my own, and that relevant citations are included to acknowledge the work of others

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Abstract

Turbocharging has established itself as a key technology in the downsizing of internal combustion engines, with the aim of reducing fuel consumption and tailpipe emissions. For a turbocharger to be effective, both the compressor and the turbine need to be carefully designed to match the engine’s requirements. This thesis presents a methodology for modelling the effect of turbine design on the engine performance, enabling an optimised match to be found. This ultimately leads to a turbine design which is tailored to the engine it is used for, as opposed to being selected from already available components.

The process for selection of a turbocharger turbine has in recent years become enhanced through the use of 1-D engine modelling, a method which uses the 1D Euler equations to predict the instantaneous gas exchange in the intake and exhaust systems of the engine. The turbocharger is modelled using maps of its performance parameters which are usually determined experimentally on a test bench. The methodology presented in this thesis replaces these maps with a meanline model which, using basic design parameters, solves the velocity triangles of the turbine for the mean flow path, and implements loss models to predict turbine performance.

The meanline model developed for this work was validated against CFD results and test data, for turbine designs stemming from a parametric 3D turbine model. Three turbine design parameters were sampled and calculated by CFD, providing a dataset which enabled an assessment of the predictive capability of the meanline model. Subsequently, six turbine designs with variations of the same parameters were manufactured using rapid prototyping techniques, and tested on a cold testing facility. The results showed that the meanline model was able to predict efficiency and mass flow to within 3% across a range of designs and operating points, given data of a baseline design for calibration.

The model was integrated into a 1-D model of a turbocharged gasoline engine and used to predict the impact of the meanline turbine design parameters on three aspects of engine performance: fuel consumption, low end torque and transient response. The insight given into the effect of each design parameter in the study allowed three optimised designs to be selected for the engine, representing three different trade-offs on performance, with fuel savings of up to 0.6%.
Acknowledgements

I would firstly like to thank Ricardo Martinez-Botas for motivating and inspiring me throughout these years. I am very grateful to have been able to complete my PhD under his supervision. I also want to thank Peter Newton for his untiring help, ideas and academic insight. Without his support I would never have progressed. I want to acknowledge Toyota Motor Europe, in particular Martin Halamek, for the interesting collaboration leading into this work and I want to thank Chiong and Srithar from the LoCARtic Centre at UTM for their advice and their huge generosity in providing me with data from their facility.

I want to thank Harminder for his help and patience in getting components manufactured and set up, and for his seemingly magical skills in solving all experimental crises. I am also grateful to all of the members of the Turbo group for helpful discussions, a great environment in the office and some fun trips to conferences. Thanks goes to Pepe and Eva for helping me on the test rig, Miles and Bijie for some insightful conversations, and Karim for the relentless banter.

Particular thanks goes to Piotr Luczynski and Carola Freytag from the RWTH Aachen for a great deal of help and hard work during the final year, and a very fruitful collaboration. It was a pleasure working with you guys!

Of course I must thank also the many people who have supported me in different ways throughout these years, but especially the Ladanyi and Freely families for generously accommodating me in your homes for such long periods. Those were without a doubt the best times!

Finally I want to thank my parents and the rest of my family for their enduring love and support, and of course my fiancée Assunta who has stood by me throughout these years despite the distance and the difficulties. You are wonderful, and I am eternally grateful to you.
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## Nomenclature

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<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tr>
<td>$C_p$</td>
<td>Specific heat capacity at constant pressure</td>
<td>J kg$^{-1}$ K$^{-1}$</td>
</tr>
<tr>
<td>$A$</td>
<td>Area</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$b$</td>
<td>Blade height</td>
<td>m</td>
</tr>
<tr>
<td>$Bl$</td>
<td>Blockage</td>
<td></td>
</tr>
<tr>
<td>$C$</td>
<td>Flow velocity</td>
<td>m s$^{-1}$</td>
</tr>
<tr>
<td>$d$</td>
<td>Diameter</td>
<td>m</td>
</tr>
<tr>
<td>$h$</td>
<td>Specific enthalpy</td>
<td>J kg$^{-1}$</td>
</tr>
<tr>
<td>$K$</td>
<td>Calibration constant</td>
<td></td>
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<tr>
<td>$L$</td>
<td>Enthalpy loss</td>
<td>J kg$^{-1}$</td>
</tr>
<tr>
<td>$l$</td>
<td>Length</td>
<td>m</td>
</tr>
<tr>
<td>$\dot{m}$</td>
<td>Mass flow rate</td>
<td>kg s$^{-1}$</td>
</tr>
<tr>
<td>$Ma$</td>
<td>Mach number</td>
<td></td>
</tr>
<tr>
<td>$MFP$</td>
<td>Mass flow parameter</td>
<td>kg s$^{-1}$ K$^{0.5}$ Pa$^{-1}$</td>
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<tr>
<td>$N$</td>
<td>Rotational speed</td>
<td>RPM</td>
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<tr>
<td>$N_{bl}$</td>
<td>Number of blades</td>
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<tr>
<td>$N_s$</td>
<td>Reduced speed</td>
<td>RPM K$^{-0.5}$</td>
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<tr>
<td>Symbol</td>
<td>Description</td>
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<tr>
<td>$P$</td>
<td>Pressure</td>
<td>Pa</td>
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<tr>
<td>$PR$</td>
<td>Pressure ratio</td>
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</tr>
<tr>
<td>$r$</td>
<td>Radius</td>
<td>m</td>
</tr>
<tr>
<td>$R$</td>
<td>Specific gas constant</td>
<td>J kg$^{-1}$ K$^{-1}$</td>
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<tr>
<td>$s$</td>
<td>Specific entropy</td>
<td>J K$^{-1}$ kg$^{-1}$</td>
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<tr>
<td>$T$</td>
<td>Temperature</td>
<td>K</td>
</tr>
<tr>
<td>$TR$</td>
<td>Temperature ratio</td>
<td>-</td>
</tr>
<tr>
<td>$U$</td>
<td>Blade velocity</td>
<td>m s$^{-1}$</td>
</tr>
<tr>
<td>$W$</td>
<td>Velocity in rotating frame of reference</td>
<td>m s$^{-1}$</td>
</tr>
<tr>
<td>$z$</td>
<td>Axial position</td>
<td>m</td>
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**Abbreviations**

- BMEP: Break mean effective pressure
- BSFC: Break specific fuel consumption

**Greek symbols**

- $\beta$: Flow angle in rotating frame rad
- $\beta_{bl}$: Rotor blade angle rad
- $\gamma$: Ratio of specific heats -
- $\mu$: Dynamic viscosity Pa s
- $\theta$: Rotational angle rad
- $\alpha$: Flow angle in stationary frame rad
- $\eta$: Efficiency -
- $\phi$: Camber angle rad
\( \psi \) Azimuth angle, cone angle \( \text{rad} \)

\( \rho \) Density \( \text{kg m}^{-3} \)

\( \varepsilon \) Clearance \( \text{m} \)

**Indices**

\( ' \) in relative frame of reference

**Subscripts**

0 stagnation state

\( h \) blade hub

\( m \) meridional component

\( r \) radial component

\( s \) isentropic state

\( t \) blade tip

\( t - s \) total to static

\( \theta \) tangential component
Chapter 1

Introduction

1.1 Global Context

The drive towards a reduction of greenhouse gas emissions has become a major factor in the development of internal combustion engines and alternative propulsion technologies in recent years. According to the International Energy Agency, global CO$_2$ emissions in the transport sector amounted to 8.04 billion tons, accounting for nearly 25% of total worldwide emissions (Figure 1.1). The importance of reducing these emissions for the mitigation of human induced climate change is well established, and governments are introducing increasingly stringent legislation to help achieve this goal.

The EU Commission estimates that greenhouse gas emissions from cars and vans account for around 16% of total EU emissions (European Commission 2017). Starting in 2020, new legislation has been implemented aiming for a 23% reduction in greenhouse gas emissions from road transport by the year 2030. This target is to be achieved by enforcing a manufacturer fleet-wide reduction in CO$_2$ emissions of 15% starting from 2025 and 37.5% from 2030 (European Commission 2019).

Experts predict that, despite incentivisation towards powertrain electrification as a means of reducing CO$_2$ emissions, internal combustion engines will continue to be of huge importance in the future, and that research towards their improvement remains a priority (Reitz et al. 2020). In the same publication, the authors list the engine gas exchange process and turbocharging as key elements in the endeavour towards more efficient engines.
One reason turbocharging has become a growing trend in the automotive industry is the fact that it can be used to increase the power density of an engine. This enables a 'downsizing' of engines while delivering the same performance, and because some of the more significant losses such as friction scale with size, this can lead to an increased overall engine efficiency. Some successful downsized engine design concepts using turbocharging have resulted in fuel savings of up to 23% (Turner et al. 2014).

1.2 Forced Induction & Turbocharging

Forced induction refers to the process of compressing air prior to it entering the internal combustion engine, forcing it into the cylinder at higher than ambient pressure (Watson & Janota 1982). This allows the engine to breathe a higher air mass flow rate, and consequently means that more fuel can be burnt to achieve a higher specific power output. Turbocharging is a form of forced induction where the power required to compress the air is derived from the high pressure exhaust gases of the engine through a turbine (as
opposed to the supercharger which draws compressor power from the crankshaft, leading to increased losses).

The energy used to power the turbine is the 'blow-down' energy, resulting from the high pressure expulsion of combustion products during exhaust valve opening. As shown in the P-V diagram in Figure 1.2, for a 4-stroke engine the blow-down energy consists of an ideal, adiabatic expansion to ambient pressure from condition 1-2 (so the area of 1-2-3), plus the work done by the piston in displacing the remaining gas (3-4-5-6 in the P-V diagram).

![P-V diagram of a four stroke engine cycle](image)

Figure 1.2: P-V diagram of a four stroke engine cycle, with hatching in the area which makes up the blow down energy. (Watson & Janota 1982)

The concept of turbocharging was invented by Alfred Büchi and patented in 1905 (Büchi 1905). However, harnessing this blow-down energy is no easy task, and Büchi’s early attempts failed until 1925, when he designed the first turbocharger in the form it is commonly used today, with the supercharging compressor attached directly to the turbine shaft in a mechanically self-contained unit. The typical modern setup for a turbocharger is shown in Figure 1.3. Hot exhaust gases enter the turbine and are expanded to ambient conditions, powering the compressor on the other side of the shaft, which compresses air from ambient to the required boost pressure.

Despite the use of otherwise wasted energy, the presence of a turbocharger itself is not necessarily an indication of higher engine efficiency, and a poorly matched unit can in fact have an adverse effect, even if the primary goal of increasing engine power may be achieved (Watson & Janota 1982). This is linked to numerous factors, including the
3.5 bar is required to boost a 2.0 litre engine to achieve 35 bar BMEP for a 60% level of downsizing. At these conditions, the technical challenges involved are vast. A more aggressive downsizing practice or those carried out on larger engines will inherently require higher boost pressure. This brings about other issues such as charge-air cooling, high exhaust gas temperatures, high cylinder pressure, knock mitigation and control, vehicle drivability, mechanical and thermal stresses.

In order to achieve sufficient boost from the boosting systems, larger turbochargers which operate at extremely high speeds and able to withstand higher temperatures will be required.

Forced Induction Systems and Turbochargers

As mentioned earlier, forced induction systems play a central role in engine downsizing development to provide the necessary boost pressure. Forced induction technologies are nothing new as far as the automotive engines are concerned. Spark ignition (SI) engines have seen the use of turbochargers or superchargers in niche markets and applications such as in performance and racing vehicles as well as in several passenger cars and it has become difficult to find successful NA compression ignition engines in the market today.

Forced induction systems are methods to increase the mass flow rate of air entering the combustion chamber in order to increase the efficiency of the engine. The types of forced induction systems most widely used in automotive engines include turbochargers and superchargers. The present work focuses on the analysis and performance of the former. A

Figure 1.3: Diagram of a typical turbocharger with radial compressor and turbine, connected by a shaft (from Raunekk & Stonecypher (2009))

overall efficiency of the turbocharger, and the mass flow characteristics of both turbine and compressor which will in turn determine boost pressure in the intake manifold $P_2$, and the back pressure in the exhaust manifold $P_3$. The back pressure of the turbine determines the pumping losses of the engine, while $P_2/P_3$ will determine how well the exhaust gases are expelled from the cylinder, both of which impact the overall engine efficiency. The primary source of fuel savings due to turbocharging is however the aforementioned ability to downsize an engine which fulfils the same load requirements as its naturally aspirated equivalent.

Modern turbochargers in the automotive sector predominantly use a radial compressor and turbine, such as those shown in Figure 1.3. The turbine and compressor are connected directly by a shaft which passes through a bearing housing onto which the volutes of both compressor and turbine are mounted. This leads to a compact and robust design.

The setup of a typical turbocharged gasoline engine system is shown in the schematic in Figure 1.4. The exhaust gases leave the cylinder through valves and pass through the exhaust manifold before entering the turbine which uses the energy in the gas to drive the compressor. When boosting occurs, the pressure between the compressor and the throttle is thus higher than ambient. Because gasoline engines operate at near stoichiometric air-to-fuel ratio, the throttle is used to regulate the pressure in the intake manifold so that
An intercooler is situated between the compressor and the throttle. This component extracts heat from the air after compression, to reduce temperature and thus increase the charge density. This allows more mass to enter the cylinder on induction, as well as reducing in-cylinder temperature.

To control the boost at the outlet of the compressor, turbocharged engines often include a turbine bypass valve called a wastegate. Opening of the wastegate relieves the back pressure in the exhaust manifold, and therefore reduces the power output of the turbine. The wastegate usually takes the form of a flap valve which is integrated with the turbine volute, and actuated pneumatically using the compressor outlet pressure. The boost controlling valve shown in the figure uses the boost pressure upstream of the throttle to regulate the pressure acting on a diaphragm inside the wastegate actuator.

### 1.2.1 Turbocharger Matching

For the engine to fulfil its performance requirements, the turbocharger design must be matched correctly. Conventionally, this process means that that the compressor and turbine of the turbocharger are selected on the basis of their performance characteristics. These are matched to the requirements of the engine at full load so that the desired
boost is generated, before considering other aspects of engine performance such as fuel economy and transient response (Gugau & Roclawski 2014). The interplay between the various performance parameters and constraints, makes this a complex task linked with performance trade-offs.

Usually, matching in the automotive sector is limited to the selection of already existing components. This has several advantages, one being that a database of component performance maps can be generated prior to matching, so that their individual performance characteristics are known. For the turbine, the parameter which is matched is the swallowing capacity, and therefore a finite number of designs with increments of mass flow rate are available for selection. Larger increments are determined by the rotor diameter, while smaller increments are achieved by modifying the area of the volute (the $A/r$ ratio). This means that the potential pool of turbine matches available for selection might be limited to one or two rotor designs, with the match being fine-tuned through the selection of a volute.

Although the individual components will have been optimised on their own, with significant resources invested into 3D rotor optimisation for instance, this often focuses on the efficiency and rotor inertia at a constrained swallowing capacity and a limited number of operating points (Mueller et al. 2012, 2019). While optimisation of these parameters will usually manifest itself in engine performance improvements, there is the potential for optimisation of the match itself, considering both swallowing capacity and map efficiency at relevant engine operating conditions. In short, a turbocharger design tailored to the engine. This thesis presents a novel methodology for achieving this kind of tailored turbine design, which is optimised to requirements of engine performance.

### 1.2.2 1-D Engine Modelling

A tool which has become a key component of the turbocharger matching process is the prediction of engine performance through engine modelling. Several methods exist to achieve this with different levels of complexity and cost. In this thesis, a gas dynamic 1-D model is used within the software package GT-Power.

1-D modelling solves the Euler equations for compressible flow in one dimension within
the intake and exhaust systems of the engine by discretising the components along their length and assuming a uniform flow across their cross-section. Under the assumption of quasi-one-dimensional flow, the gas dynamics within the manifolds can be modelled (Winterbone & Pearson 2000). Models considering combustion and heat transfer are used to predict the instantaneous pressure within the cylinder (Watson et al. 1980), setting up the boundary conditions for exhaust and intake manifolds, and predicting the engine torque (Watson 1984). Once calibrated, a 1D engine model correlates well with tested data, which means that it can be used for optimisation of various engine parameters, including the turbocharger performance (Shingne et al. 2010).

1.2.3 Turbine Modelling

The turbocharger in a 1-D engine model is conventionally modelled using performance maps of the key performance parameters. For the turbine, maps consist of isentropic total-to-static efficiency and mass flow parameter plotted against the pressure ratio along lines of constant speed. Several speeds, covering the full operating range of the turbocharger are included. In most applications, turbine maps are generated experimentally for the full stage, with the compressor and turbine already attached, and the compressor acting as a loading device. The SAE standard for experimental determination of turbocharger maps (Society of Automotive Engineers 1995) dictates that these are measured with a turbine inlet temperature of 600°C. The reason for this setup is that under the assumption of a constant turbine inlet temperature, heat transfer effects and bearing friction are already accounted for, and need not be considered separately.

The operating range for which the turbine can be tested is constrained by the surge and choke limits of the compressor, meaning that only a narrow range of turbine pressure ratio can be tested for each speed. Due to the pulsating conditions in the engine exhaust, instantaneous turbine pressure ratios generally exceed this range and therefore extrapolation of turbine maps becomes necessary in gas dynamic engine modelling. Extrapolation is most simply done with a parabolic fit as is common practice in GT-Power, however this is prone to error and sensitive to the available range of data points which can impact the engine performance prediction (Pesiridis et al. 2012).
A method which could be used alternatively is meanline modelling. This is a physics based approach which resolves the velocity triangles of the for the mean flow path and implements loss models to account for the different loss mechanisms within the turbine. These models can be calibrated to available data to give a good overall fit to the aerodynamic performance (Romagnoli & Martinez-Botas 2011)

Although a relatively well established method and widespread in the preliminary design phase of turbines, its implementation in engine modelling has not been widely reported and is therefore an interesting line of investigation. As some of the fundamental turbine design parameters are input into the model, an advantage in its use is the possibility of conducting engine performance optimisations using the turbine design, through integration with an engine model, as is demonstrated in this thesis.

The full width of the map can be covered, and with the use of additional models to predict the rotor inertia, the turbine design can be optimised to multiple objectives, including fuel consumption, transient performance and torque. This multi-objective approach is an entirely novel way of looking at the turbocharger matching process.

The approach rests on the models ability to predict not just off-design performance for a given turbine, but the performance across the range of designs which are modelled. Although some work has been done on the ability of meanline models to predict performance over a range of different turbines, (Baines 1998, Meroni et al. 2018) it remains unclear how well design modifications of a calibrated baseline model are represented.

### 1.3 Thesis Objectives

The overarching question answered in this thesis can be summarised as:

*Is it possible to optimise turbocharged engine performance by using a low order turbine model integrated into a 1-D engine model to predict the effect of turbine design changes on various engine performance parameters?*

To answer this question, the work can be structured into the following objectives:
1. Development of a low order model for the turbocharger turbine based on physical principles: The requirements for this model must be the ability to robustly output turbine performance for a given design and operating condition at low computational cost, implementing a number of different loss models from literature, and including a novel rotor exducer blockage model. The model needs to include a novel method of automated fitting of the calibration parameters to a given set of datapoints. To enable integration with an engine model, a requirement for the meanline code is the ability to output turbine maps in a format readable for GT-Power.

2. Investigation of the low order turbine meanline model: To use the meanline model for turbine design optimisation within an engine modelling environment, its accuracy across designs and operating conditions has to be evaluated using both CFD and experimental data. A part of this objective includes the evaluation of different loss models which were available for selection in the code. The novelty in this objective is a better understanding of how well a meanline model predicts design changes and extrapolates data used for calibration.

3. Methodology supporting the integration of the meanline model into an engine simulation tool: To enable the study of the effect of turbine design on engine performance, a novel approach for integration of the meanline model into the turbine component of a GT-Power engine model is required.

4. Study and evaluation of the engine peformance based on the integrated turbine model: The final objective is to use the integrated meanline model to study the effect of turbine designs on three key engine performance parameters: Fuel economy, transient response and low end torque. Part of this objective is to develop an optimised turbine design for the turbocharger.
1.4 Thesis Outline

1.4.1 Chapter 1: Introduction

This chapter introduces the motivation for the work and the core concepts covered in the thesis. The key objectives and outline of the thesis are presented.

1.4.2 Chapter 2: Literature Review

The scientific literature which is relevant to the thesis is covered, detailing the work already done in the field and leading up to the contributions presented in the thesis. Research into turbocharger matching is summarised, as well as the work done in the field of full engine performance modelling. Finally literature relating more specifically to the modelling of the turbocharger turbine is reviewed.

1.4.3 Chapter 3: Meanline Model Development

This chapter outlines the development of a new radial turbine meanline model. The physics of the model as well as a comprehensive selection of loss models from the literature are presented and summarised. This is followed by the description of the algorithms which ensure the speed and stability of the code and the methods used for calibration of the loss models. The chapter includes the description of a novel exducer blockage model for radial turbine meanline models.

1.4.4 Chapter 4: Parametric Turbine Study

A parametric 3D turbine model is introduced, which is capable of generating turbine geometries for meshing and subsequent CFD analysis. Using design of experiment methods, turbine designs are sampled, and calculated using CFD. The resulting dataset is used to validate the accuracy of the meanline model across the design space, and inform a best practice approach with regards to the selection of loss models. Subsequently the model is used to predict how different sources of loss interact with changing design and operating parameters. This chapter gives a novel insight into the accuracy of radial turbine meanline models, and how to best apply them in a design optimisation setting.
1.4.5 Chapter 5: Testing of Selected Turbine Designs

A testing campaign is reported, detailing the manufacture of six turbine designs using rapid prototyping techniques, and subsequent testing on the existing Imperial College cold flow radial turbine testing facility. The results are used for validation of the meanline model as well as the CFD approach used for the study presented in Chapter 4. Additionally, insight was gained into the sensitivity of the meanline model prediction when calibrated to a sparse dataset in a narrow range of operation.

1.4.6 Chapter 6: Engine Performance Optimisation

This chapter presents the development and calibration of a 1D engine model in GT-Power, using engine data for a Proton CAMPRO CFE turbocharged engine. The meanline model developed in the previous chapters is integrated with the engine model, allowing the effect of turbine design on engine performance to be studied. The engine model controller is adjusted for the prediction of fuel economy, transient performance and low end torque in three separate models, allowing the comparison of different engine performance parameters. This novel approach was used to find three optimum designs which are analysed in more detail.

1.4.7 Chapter 7: Conclusions & Further Work

The final conclusions are summarised, along with the rest of the work, outlining the fulfillment of the objectives presented in Chapter 1. The potential for further work stemming from the presented results is shown.
Chapter 2

Literature Review

A review of the literature relevant to this thesis is structured into four parts:

1. Turbocharger Matching and Optimisation: this includes literature on the basics of turbocharger matching, the methods used, as well as some more recent developments in model based matching

2. Engine Modelling: Some history and context, and the methods used for low order modelling of engine systems, both in steady state and transient operation.

3. Turbine Modelling: This includes a dedicated review on specific turbine modelling methods and integration in the broader context.

4. Meanline Modelling: As this features prominently in the present thesis, a review on the developments in this field is included.

2.1 Turbocharger Matching & Optimisation

As the turbocharger has a significant impact on engine performance, a large body of work is devoted to its improvement. Broadly speaking this improvement can happen on two levels: Optimisation of individual components such as the compressor, turbine and bearings, or the selection of turbocharger components to properly fit to the requirements engine, commonly referred to as turbocharger matching.
A basic conventional matching procedure for a diesel engine is described by Watson & Janota (1982). The first step in finding an appropriate match for the engine is the selection of a compressor. Using the fundamental engine parameters (swept volume \(V_{sw}\), speed \(N\), BMEP, intake manifold density \(\rho_m\), volumetric efficiency \(\eta_{vol}\)), the mass flow and boost pressure of the engine can be determined. Selection of the compressor is then done by analysis of compressor maps provided by turbo manufacturers. These maps can be used to determine whether the operating range of the engine (mass flow and boost) is compatible with the compressor. A turbine match is subsequently selected to provide the required power to the compressor, so that constant speed and load operating lines can be approximated on the compressor map as seen in Figure 2.1. Selection of a different compressor might be desirable to shift engine operation into the highest efficiency region or to increase the margin to the surge region.

![Figure 2.1: Compressor map of pressure ratio against reduced mass flow rate with superimposed engine operating lines for a diesel engine. From Watson & Janota (1982).](image)

Selection of the turbine will have an effect on the engine operating lines, as the turbine efficiency and swallowing capacity will determine the turbine pressure ratio and power at a given engine speed. As a smaller turbine will have a higher specific power, it will
deliver more boost, shifting the engine operating lines as is seen in Figure 2.2. These initial calculations can allow the engine designer to select viable turbocharger designs for a given engine platform, using databases of available performance maps provided by manufacturers. Some of the aspects to be considered when matching the turbocharger, are the maximum cylinder and manifold pressures, maximum turbocharger speed and the compressor surge and choke limits among others.

![Figure 2.2: Compressor map with superimposed engine operating lines for a diesel engine, comparing the effect of two different turbines matches. From Watson & Janota (1982).](image)

Matching the turbocharged gasoline engine differs from the diesel engine, as in the former engine load is primarily controlled by throttling of the intake flow between the compressor and the intake manifold. As explained by Hiereth & Withalm (1979), The compressor pressure ratio at high speeds remains the same for all loads, resulting in the engine operating lines bunching together. At lower speeds, where the compressor no longer delivers maximum boost, do the engine speed lines fan out.

The complex interplay between the turbocharger and engine has generally meant that matching is a highly iterative procedure with some degree of trial and error, meaning that
several different viable solutions are selected and tested until a final match is found. An example of this kind of process was presented by Korakianitis & Sadoi (2005). A simple theoretical matching was performed, leading to the selection of three turbochargers. These were tested on the engine, and the effects on steady state engine performance were studied, finding that the different matches were suited to different engine operating conditions.

The advent of computer models to predict the processes inside the turbocharged internal combustion engine revolutionised this process by allowing initial optimisations to be made without the cost of prototyping and engine testing. The literature on the development of these engine models is outlined in Section 2.2 below, while the present section focuses on the application of these models to turbocharger matching.

Turbocharger design studies using computational engine models where conducted by Benson (1971), Watson et al. (1983), Bozza et al. (1990), Shingne et al. (2010) and Pohorelsky et al. (2012) among others. These studies made use of existing maps to computationally compare different turbocharger designs and find a suitable match for steady state engine performance.

While the matching of the steady state engine load curves with a focus on optimised fuel economy is important, mention should be made of the effect of turbocharger matching on the transient performance of the engine. This generally refers to how quickly the engine
responds to a load change. One of the key issues in turbocharged engine performance is the "turbo-lag", a delay between application of the load step and the point at which the engine delivers the required power. This is caused on the one hand by the time taken for exhaust and intake manifolds to fill (a problem not encountered in naturally aspirated engines which have no boost or back pressure), and on the other hand due to the inertia of the turbocharger rotating assembly, which needs to accelerate during this process.

Using an engine performance model, Watson (1981) showed the effect of turbocharger inertia for a load step, while Schorn et al. (1987) coupled the engine to a vehicle model to show acceleration under a load step, as seen in Figure 2.4. Hong & Watson (1988) in one of the first published examples of a transient model of a turbocharged spark ignition engine, showed the effect of turbine inertia under a gradual torque demand change. The aforementioned work by Bozza et al. (1990) analysed the performance of two potential turbocharger matches using an engine model and included the modelling of transient performance on a load step with imposed engine acceleration.

The publications mentioned above used already existing maps for tested turbocharger designs in their analysis of engine performance. In an attempt to optimise turbine and compressor design for a given engine, Zhuge et al. (2009) and Chen et al. (2009) developed a new methodology, integrating a turbine and compressor meanline model into an engine simulation in order to evaluate the effects of individual turbine design parameters. The model was used to re-match the turbocharger of a 1.8 litre gasoline engine by designing a new compressor and turbine using the meanline method. The resulting designs showed a marked improvement in torque and fuel economy over the full range of speed at full load, with a 4% BSFC reduction at 5500 RPM.

A similar approach was taken by Qiu et al. (2013) who integrated meanline models of the compressor and turbine with a very simplified engine model to perform matching calculations across the engine operating range. The method included wastegate and EGR modelling as well as prediction of variable geometry stators. This allowed for the inverse design of compressor and turbine geometries, which was demonstrated by retrofitting a 2100 kW Diesel engine with a new turbocharger. Fuel flow was reduced by up to 4.5% while torque was slightly improved at lower engine speeds, compared to the original turbocharger match.
In a more simplified approach, Ismail et al. (2015) used a scaling factor to change the swallowing capacity of the turbine in a 1.8 litre gasoline engine, modelled in the engine modelling software GT-Power. Scaling the mass flow rate of the turbine was seen as equivalent to changing the size of the volute, and thus the effective area of the turbine, while assuming that the efficiency of the turbine remains unchanged. The turbine size was increased to shift the speed at which the wastegate opens to the point at which the highest full load engine torque is required, reducing the back pressure and therefore the residual gas fraction in the cylinder. Further increase in turbine size would mean that the required engine torque is not achieved. This rematching, however, resulted in a torque deficit at lower speeds which was overcome by the introduction of a divided manifold turbine entry.

The use of CFD for generating maps to be used for turbine matching was demonstrated by Gugau & Roclawski (2014). Based on theoretical matching using 1D tools, and blade optimisation with 3D CFD, the authors generated maps of four different turbine designs using CFD. An integration procedure was developed to predict average unsteady turbine performance for steady state matching procedures. This included the consideration of turbine inertia and its impact on transient performance.

Integrating a meanline model into an engine, in an approach similar to that of Qiu et al. (2013), Sanaye et al. (2015) used a genetic algorithm to optimise eight compressor and turbine design parameters, for the sum of compressor and turbine losses. The authors however did not report what effect this would have on the overall engine performance.

Kapoor et al. (2018) on the other hand focused on the effect of turbine design on the performance of the engine at full load and part load, using a meanline model of the turbine integrated into GT-Power. A genetic algorithm was used to optimise seven turbine design parameters by simulating the engine performance using the meanline predicted turbine map. The authors found that the turbine exit angle had the largest impact on BSFC, followed by the exducer tip radius. While an improvement of BSFC was seen, no consideration was taken for transient performance.
Figure 2.4: Effect of turbocharger inertia on transient engine performance. This was modelled using an engine model coupled to a vehicle model to simulate a realistic load step. Three values of rotor inertia were modelled, as shown in the topmost graph. From Schorn et al. (1987).
2.2 Engine Modelling

The increasing availability of computing resources in the middle of the last century quickly manifested itself in the development of computational models of internal combustion engines. This would accelerate the ongoing reduction of fuel consumption by allowing the optimisation of engine components at an earlier stage of development, as shown in the previous section.

The majority of these models divide the engine into its individual components (intake manifold, exhaust manifold, cylinder, valves, compressor turbine, etc.) and solve the energy and continuity equations for each. This has led to a very large body of literature with a focus on different aspects of modelling. One class of models uses the "filling and emptying" assumption to model different components of the engine. This means that intake and exhaust manifolds are each considered as one control volume with homogenous gas properties as shown in Figure 2.5, which fills and empties during the engine cycle. An example of such a model for a turbocharged diesel engine was presented by Streit & Borman (1971) with a very thorough review of the existing literature, such as the work by Whitehouse et al. (1962) and McAulay et al. (1965) which set the path for modelling of heat transfer and combustion effects.

Watson & Marzouk (1977) presented another example of a filling and emptying model for the modelling of transient response of a turbocharged diesel engine. This was one of the first examples of such a model for the analysis of transient engine performance. The model was developed further by the same author in Watson (1981). Heat transfer in the cylinder, manifolds and charge air cooler was modelled, as well as the combustion in the cylinder using an empirical correlation from Watson et al. (1980). The turbocharger was modelled using steady state maps of the components, as well as including the mechanical dynamics due to inertia.

The model showed good correlation with experimental results, making it an effective tool for the study of engine design. High importance was given to the effects of the turbocharger match on the engine, and a parameter study was conducted on the impact of sizing the turbine giving insight into the tradeoff between turbine inertia and efficiency/mass flow effects.
A second class of models stem from groundbreaking work by Benson and others in Benson & Woods (1960) and Benson et al. (1964). The authors developed a computational model for capturing the unsteady effects in 1-dimensional pipes using the method of characteristics. The resulting model, termed the "mesh method of characteristics" was able to spacially resolve the instantaneous pressure in intake and exhaust ducts, giving it a distinct advantage over the filling and emptying method, albeit at a higher computational cost. Using this method enabled the modelling of cylinder to cylinder variations in the exhaust manifold, enabling a better estimation of scavening and therefore better exhaust manifold design.

The mesh method of characteristics was implemented in a full engine model by the
same authors in Benson (1971) and Benson & Baruah (1973), showing use cases from industry partners where the model enabled initial matching of the turbocharger, as well as a parametric study on the effect of exhaust valve timing on engine performance. The turbocharger compressor and turbine were modelled in a quasi-steady fashion using steady state maps.

The method of characteristics was later applied to an automotive turbocharged spark ignition engine by Watson (1984). In this model, pipes in the exhaust and intake where modelled with a homentropic formulation of the method of characteristics to reduce computing cost, and connected to volumes instead of flow junctions. This essentially made it a filling and emptying model with some consideration of wave action to allow modelling of effects due to manifold piping lengths. The model included a dynamic model of the wastegate poppet valve, to allow later use in transient simulation where valve inertia and damping could have an impact. Transient simulation was subsequently implemented in Hong & Watson (1988) showing good comparison with engine testing data. Turbocharger inertia, valve timing, and the effect of using variable geometry turbocharger designs where studied using the model.

2.3 Turbine Modelling

An important aspect in the low order modelling of turbocharged engines, is the treatment of the turbine. Two major topics exist in this respect:

1. Modelling the unsteady performance of the turbine and the boundary conditions used in filling and emptying/wave action models

2. The modelling of turbine steady state/map performance in the application of the above turbine models

The filling and emptying models (Watson & Marzouk (1977) and Watson (1984) among others) simply applied the turbine steady state maps as boundary conditions to the exhaust manifold volume, making the so called quasi-steady assumption - meaning that the turbine performance is instantaneously the same as it would be in steady state operation with the same boundary conditions at a given time step. As early as 1965
however, work by Wallace & Blair (1965) and Benson & Scrimshaw (1965) showed that
the quasi-steady condition does not accurately predict turbine power, though Wallace &
Blair (1965) over-predicted the power while Benson & Scrimshaw (1965) under-predicted
it.

Based on an analytical steady state radial turbine model presented by Wallace (1958),
Wallace & Adgey (1967) developed a model which used the method of characterics to
model the unsteady behaviour of the different turbine components, namely the volute,
nozzle, interspace and rotor passage, the rotor passage being treated as a rotating duct.
This approach was further developed in Wallace & Miles (1970) who compared the method
to experimental results from a pulsating flow rig, as well as a quasi-steady approach, show-
ing improved prediction. The authors showed how the steady state characteristics of the
turbine were implemented as a direct boundary condition to the method of characteristics
for quasi-steady prediction.

Farrashkhalvat & Baruah (1980) presented a similar approach for the generation of
boundary conditions using steady state turbine performance characteristics, and showed
a method of extrapolating both the mass flow parameter and turbine efficiency.

Chen & Winterbone (1990) where the first authors to use a meanline model (see
Section 2.4 for meanline modelling literature) for the prediction of steady state turbine
performance, coupling this with a 1-D duct to account for the unsteady effects due to the
volute and assuming a quasi-steady performance. This method was extended in Chen
et al. (1996) and Abidat et al. (1998) showing a better prediction of unsteady mass flow
rate and power compared to the fully quasi-steady method.

Taking account of the unsteady effect due to accumulation of mass within the vo-
lute, Baines et al. (1994) modelled the volute as a simple filling and emptying device
of equivalent volume upstream of a turbine boundary condition. The turbine boundary
consisted of the steady state turbine performance predicted by a meanline model. The
model neglected the effect of wave action in the volute, but the authors concluded that
this approach was sufficient for prediction of unsteady effects.

Although research has been done for more complex methods of 1-D modelling of
the volute (Chiong et al. 2014), a method similar to the one developed by Chen &
Winterbone (1990) has become state of the art for use in the GT-Power engine modelling
software (Gamma Technologies 2016) used for the work presented in this thesis. A duct of equivalent length $l_{vol}$ and diameter $d_{vol}$ is placed upstream of the turbine node to account for unsteady effects. The equivalent dimensions are calculated with the following equations:

\[
l_{vol} = l_1 + \frac{r_1 + r_2}{2}\pi
\]

\[
d_{vol} = \sqrt{\frac{4V_{vol}}{\pi l_{vol}}}
\]

where $V_{vol}$ is the total volume of the volute and the other dimensions are as shown in Figure 2.6.

![Figure 2.6: Diagram of a radial turbine volute indicating the dimensions used for the equivalent duct calculations (from Gamma Technologies (2016)).](image)

In light of the common use of steady state maps to predict the quasi-steady component of turbine performance (sometimes referred to as "rotor maps" although strictly speaking this is not correct as the rotor requires the volute to set up the velocity triangles at the leading edge), research has been conducted on methods of map generation, either by extrapolation of existing data, or by predictive modelling.
As the standard for turbocharger steady state testing is to test the full stage, i.e. turbine and compressor already attached, at high turbine inlet temperatures (600°C) (Society of Automotive Engineers 1995), the range of available turbine performance is often very narrow, limited by the surge and choke lines of the attached compressor. As engine pulses cover a wider range of turbine operation, extrapolation of available experimental is necessary, and work has been done in this area to improve extrapolation quality.

Jensen et al. (1991) proposed a polynomial fit for predicting efficiency, where the coefficients are a function of turbine speed, while the mass flow was calculated using an equivalent turbine area for isentropic nozzle equation, dependent on speed and pressure ratio. The authors found that the efficiency fit the data with +/- 1%, while the mass flow parameter fit was +/-2%, with higher speed and pressure ratio fitting less well than low speed and pressure ratio. A similar approach was used by Moraal & Kolmanovsky (1999), including a factor for variable geometry turbines, and using a slightly different quadratic polynomial approach to fit the efficiency, showing good results.

The meanline model used by authors such as Chen et al. (1996) and Abidat et al. (1998) has already been mentioned as well as the model presented by Wallace (1958). However other authors have presented alternative physically based methods for extrapolation or prediction of turbine performance. Serrano et al. (2008) developed such a model, predicting the degree of reaction for different variable geometry settings to predict performance. Similarly, Payri et al. (2012) developed a physically based extrapolation model, using effective areas and velocity triangles, showing good prediction against wide experimental data of both mass flow rate and efficiency. This was extended by Serrano et al. (2016) who demonstrated the ability of prediction of non-measured VGT settings after calibration with data.

De Bellis et al. (2013) and Bozza & De Bellis (2014) developed yet another 1-D model of a radial inflow turbine, and used a genetic algorithm to fit the calibration constants to both mass flow and efficiency prediction. The model was later applied to unsteady flow (De Bellis et al. 2014), showing that improvements in unsteady modelling can be achieved if the whole turbine domain is modelled using 1D ducts, as opposed to what the authors refer to as a 'time-delay' pipe upstream of a mapped turbine component.
Despite the significant work done in the domain of physically based turbine modelling, the engine modelling software GT-Power uses a mathematical fit to extrapolate experimental map data (Gamma Technologies 2016). Although other methods may offer better accuracy/certainty outside of measured data, this offers a very simple approach, requiring only experimental maps and no geometric information, making it particularly suitable for conventional matching procedures where this information would often not be available to the engine designer.

Several authors have investigated different effects related to turbine mapping with regards to engine performance prediction. So for instance, Winkler & Ångström (2007) investigated the effect of implementing a meanline model for extrapolation of turbine maps. Both the conventional GT-Power method and the meanline model were compared to experimental data, showing only small differences between the two approaches, although it should be mentioned that the dataset for fitting both models covered a relatively large portion of the operating range, leading to less reliance on extrapolation accuracy.

Pesiridis et al. (2012) made a similar comparison to investigate the effect of turbine map extrapolation errors on engine performance. This was done by using two different data inputs of the same turbine to inform the GT-Power extrapolation procedure. One dataset was a wide map obtained from testing at the Imperial College turbine dynamometer facility (Szymko 2006), while the second dataset was a reduced version of the first, containing points from the wide map which were deemed to fall into the range which would realistically be available from conventional turbocharger testing. The two maps resulted in a marked difference in engine performance, with the narrow dataset causing a maximum 9.8% underprediction of full load BMEP (at mid range engine speed), and 10.8% overprediction of BSFC.

2.4 Meanline Modelling

The literature on some of the approaches for steady state turbine map generation for use in an engine modelling context has been surveyed in the previous section. The steady state modelling method which features in the present thesis is meanline modelling, so a brief
overview of the relevant literature is included here. Further details on the implementation of such a model are covered in Chapter 3.

In many tasks related to turbomachinery design, the performance of the turbine is represented by maps relating the four parameters described in the following function, which are derived from the non-dimensional parameters as shown by Whitfield & Baines (1990):

$$f \left( PR, \eta, \frac{m \sqrt{T_{01}}}{P_{01}}, \frac{N}{\sqrt{T_{01}}} \right) = 0$$

A detailed derivation of the above equation is given in Section 3.2. These parameters can be derived experimentally, however the advantages of using models to generate maps have led to a body of work from the last decades. Early analytical models for the prediction of off-design radial turbine performance were developed by Wallace (1958) followed by Wallace et al. (1969). Concurrently, NASA started developing a set of oft-cited models (Futral & Wasserbauer 1965) with detailed descriptions of the algorithms used. These were adapted from earlier models of axial turbine stages by Whitney & Stewart (1962). The model by Futral & Wasserbauer (1965) used an iterative procedure to solve the equations resulting from the analysis of velocity triangles for a nozzled radial turbine stage.

The model divided the turbine domain into stations along the flow at the mean blade height and included viscous and incidence losses, showing good results over a range of speed and pressure ratio when compared to experimental data. The viscous losses were assumed to be proportional to the mean kinetic energy of the flow and included a calibration parameter for the nozzle and rotor, while the incidence loss to the rotor was taken to be the circumferential component of the kinetic energy in the relative frame of reference.

This modelling approach was further developed by Wasserbauer & Glassman (1975) who improved the method of incidence loss modelling by taking consideration of the fact that optimum rotor incidence is usually positive and using the correlation established by Stanitz (1952) to calculate optimum incidence angle before evaluating the loss. Glassman (1976) included a tip clearance loss assuming it to be a fractional loss equal to the ratio of radial clearance $\varepsilon_r$ to the exducer passage height:
Meitner & Glassman (1980) applied the above methods to a turbine with a pivoting vane stator, and developed a clearance flow model for the stator vane end-clearance. Meitner & Glassman (1983) developed an interesting approach by dividing the exducer into sectors and assuming radial equilibrium, rather than modelling only at the mean exducer radius. Mass flow and power are calculated for each sector and summed to provide the turbine performance.

A new model, developed by Baines (1998) was based on the NASA modelling approach, but used a novel approach for the modelling of tip clearance and viscous losses. This was to give a better fit to data from different turbine designs without recalibration of the model, as the viscous loss coefficient in the previous models is dependent on the turbine geometry. The model for the rotor viscous losses remained proportional to the mean relative kinetic energy in the rotor, but the coefficient was divided into a frictional loss component and a secondary flow component. Geometric parameters are included to account for curvature in the blade passage leading to entropy generation, and hydraulic diameter and length to better model viscous losses. The tip clearance model explicitly models the flow through the gap, assuming the kinetic energy of this flow to be fully lost. The author found an improved prediction of turbine efficiency when comparing to the design and off-design performance data of 30 different turbine designs available in the open literature, as reproduced in Figure 2.7. The loss models developed (described in more detail in Section 3.3.2), have become state of the art for radial turbine meanline models.

Suhrmann et al. (2010) conducted a survey of different rotor loss models available in the literature and compared these to CFD results of a small turbine stage (31mm inlet tip radius) with variation of several design parameters including the number of blades, tip clearance and exit angle. The authors subsequently developed a correlation for exit angle deviation from the blade and made recommendations for the selection of loss models to be used for turbine performance prediction.

Experimental results of the aerodynamic performance of a nozzled and a nozzleless
mixed flow turbine were used by Romagnoli & Martinez-Botas (2011) to develop and validate a meanline model. Due to the nature of the test facility, a very wide range of velocity ratio was available, allowing a deeper insight into the effectiveness of the model. The authors observed good prediction of efficiency after introduction of a fitting parameter for incidence loss. Subsequently a breakdown of the different losses through the operating range of the turbine was presented.
Figure 2.7: Efficiency prediction vs. efficiency measurement for the NASA loss model system and the novel system, for a range of turbines from open literature from Baines (1998). The numbers labelling the points refer to the different turbine geometries. The figures show an improvement in efficiency prediction by the model presented in the paper.
2.5 Summary

The literature review has covered various aspects related to turbocharging, from the matching of the turbocharger, to low order engine and turbine modelling, with a view of the interplay between them. This has demonstrated an increasing trend of moving from a fundamental matching method as described by Watson & Janota (1982) using already manufactured and tested components, to more model based matching approaches such as those demonstrated by Qiu et al. (2013) and Kapoor et al. (2018). A gap of knowledge which exists here, is how the optimisation of multiple objectives can add further improvements to engine performance, in contrast to optimisation of only the fuel economy without consideration of other important performance aspects such as transient response and low end torque. This gap is addressed in Objectives 3 and 4 of the thesis, as shown in Chapter 6.

This methodology requires a predictive method of turbine performance modelling, such as meanline modelling. Meanline modelling can allow a more refined approach to turbine design and optimisation, beyond the simple scaling of the volute area as demonstrated by Ismail et al. (2015), with multiple design parameters, such as rotor exit angle and rotor exducer diameter, directly affecting engine performance. There is a gap in understanding of this effect on both steady state and transient engine performance, and how the design can be optimised, which is discussed in Chapter 6 and also relates to Objective 4 of the thesis.

Although a large body of literature exists on the topic of meanline modelling, as shown both above and in Chapter 3, little structured research can be found on its efficacy when moving from the 1D domain (that is, a meanline designed turbine) to the 3D domain and finally experimentally tested designs. This would help to evaluate to what extent a calibrated meanline model can extrapolate to different turbine designs when the design parameters are changed. This gap is addressed through Objectives 1 and 2, as discussed in Chapters 4 and 5. In addition to this, there is a lack of proper consideration of the rotor exducer blade geometry in the literature, which is addressed by the development of a new exducer blockage model in Chapter 3.

A further gap of knowledge is how well the meanline model extrapolates the perfor-
mance of a given design to operating conditions beyond available data. As Pesiridis et al. (2012) showed, the width of available data can have a significant impact on extrapolation of turbine maps using other methods. As limited data is often available for calibration, the robustness of the meanline method for extrapolation needs to be determined. This also relates to Objective 2 and is discussed in Section 5.5.1
Chapter 3

Meanline Model Development

3.1 Overview

This chapter describes the development of a reduced order radial turbine model for the prediction of turbine performance at design and off-design conditions, as a function of basic design parameters. Its primary purpose in the context of this work is the generation of turbine maps for integration into a turbocharged engine model.

The chapter is structured as follows:

1. **Dimensional Analysis:** A brief analysis of the non-dimensional groups defining turbine performance is presented. The resulting four quasi-non-dimensional parameters used to describe turbine performance maps throughout this thesis are defined.

2. **Fundamentals of low order turbine performance prediction:** This section describes the meanline modelling approach used in the present work for the prediction of turbine performance. This includes an introduction of the concept followed by consideration of the equations used for modelling the volute and the rotor, and and a detailed description of the loss models used. A novel exducer blockage model is introduced.

3. **Meanline model code structure:** In this section the implementation of the meanline model in a code is shown, with focus on its structure and the algorithms
used for solving the model. A key factor in development was the robustness and speed of the code.

4. **Meanline model calibration**: This sections presents the novel calibration procedure which was developed for automatic fitting to input data.

### 3.2 Dimensional Analysis

The performance of a turbine can be fundamentally described by the functional relationship of parameters in the following equation (Whitfield & Baines 1990):

\[
f(d_2, N, \dot{m}, P_{01}, P_{03}, T_{01}, T_{03}, R, \gamma, \mu) = 0
\]  

(3.1)

where \(d_2\) is a characteristic linear dimension, commonly taken to be the rotor tip diameter for radial turbines and the positions 1 and 3 refer to the turbine inlet and outlet respectively. A dimensional analysis can be performed, reducing these parameters to the following non-dimensional groups:

\[
f\left(\frac{P_{01}}{P_3}, \frac{T_{01}}{T_3}, \frac{\dot{m}\sqrt{RT_{01}}}{P_{01}d_2^2}, \frac{Nd_2}{\sqrt{RT_{01}}}, \frac{\dot{m}}{\mu d_2}, \gamma\right) = 0
\]  

(3.2)

\(P_{01}/P_3\) is the total-to-static pressure ratio, \(PR_{t-s}\). In turbocharging, this definition of pressure ratio is commonly used, as the kinetic energy leaving the turbine stage is not recovered. Similarly, \(T_{01}/T_3\) is the total-to-static temperature ratio \(TR_{t-s}\). An important performance parameter is the efficiency of the turbine. In this work the total-to-static isentropic efficiency \(\eta_{t-s}\) is used:

\[\eta_{t-s} = \frac{T_{01} - T_{03}}{T_{01} - T_{3s}}\]  

(3.3)

were \(T_{3s}\) is the outlet temperature for an isentropic process. As \(\eta_{t-s}\) is a function of \(PR_{t-s}\) and \(TR_{t-s}\), the latter can be replaced by it.

The third non-dimensional group is the mass flow parameter of the turbine. As \(R\) is considered to remain constant, and turbines of the same size are generally compared, it
is common for both $R$ and $d_2^2$ to be removed from the equation to give the following mass flow parameter:

$$MFP = \frac{\dot{m}\sqrt{T_{01}}}{P_{01}}$$  \hfill (3.4)

Similarly, $R$ and $d_2$ are removed from the fourth non-dimensional group, which shows the reduced speed $N_s$:

$$N_s = \frac{N}{\sqrt{T_{01}}}$$  \hfill (3.5)

The last two groups in Equation 3.2 are a Reynolds number and the ratio of specific heats. Both are commonly neglected when using turbocharger maps as their effect is assumed small within the analysis. This leaves four parameters defining the turbine performance:

$$f (PR, \eta_{t-s}, MFP, N_s)$$  \hfill (3.6)

The relationship between the four groups is commonly described by the use of maps, showing lines of $\eta$ vs. $PR$ and $MFP$ vs. $PR$ at constant reduced speed $N_s$, over a range of discreet speeds.

A commonly used non-dimensional parameter in turbine analysis is the isentropic velocity ratio $U_2/C_{is}$, where $U_2$ is the rotor inlet blade tip speed, and $C_{is}$ is the velocity which would be achieved in an isentropic expansion:

$$\frac{C_{is}^2}{2} = \frac{\gamma R T_{01}}{\gamma - 1} \left(1 - \left(\frac{P_3}{P_{01}}\right)^{\frac{\gamma - 1}{\gamma}}\right)$$  \hfill (3.7)

A useful feature of the isentropic velocity ratio is that the $\eta_{t-s}$ vs. $U/C_{is}$ lines of constant speed fall on top of each other and do not vary so strongly with turbine speed $N_s$, as shown in Figure 3.1. For radial inflow turbines, the theoretical peak efficiency can be shown to fall on a value of $U/C_{is} = 0.7$, which is a good approximation in most cases.
3.3 Fundamentals of Low Order Turbine Performance Prediction

The flow inside a turbine is complex and very much 3-dimensional. Prediction of the turbine performance parameters outlined in the section above can be achieved with varying degrees of complexity based on what assumptions can be made, but at the most simple level it is possible to do this based on a fundamental understanding of the physical processes which occur in the flow, using 1D equations and velocity triangles.

The flow through a turbine inherently comes with sources of entropy, as boundary layer friction and other flow features such as secondary flows cause losses. Although these losses are complex in nature, there are some key sources of loss which have been isolated and correlated to the flow characteristics of the turbine at a given operating point. These losses are based on physical understanding of the processes in the turbine and use semi-empirical correlations to predict a given loss. This enables a prediction of the overall entropy generated in the different stages of a turbine, and thus allows the operating conditions of the turbine to be modelled.

To achieve the low-order performance prediction of a turbine stage, modelling is usu-
Figure 3.2: Cross section of a nozzleless radial turbine stage, with the stations used for meanline modelling labelled as 1 to 3

ally done for the mean line of flow through the turbine, hence a common term for this way of modelling is "meanline modelling". The method resolves the thermodynamic conditions and velocity triangles at various stages of the turbine and includes functions describing the aforementioned sources of loss. The velocity triangles for the rotor stage in the present work are shown in Figure 3.3. The performance of the turbine can thus be predicted over a range of operating conditions. Initially developed for axial turbines (Whitney & Stewart 1962) and subsequently adapted for radial machines by NASA (Futral & Wasserbauer 1965), it has been commonplace in the preliminary design of radial turbines for decades.

Figure 3.2 shows the stations which are used for meanline modelling of a nozzleless radial turbine. The model solves the basic equations of turbomachinery by resolving velocity triangles at each station, and predicting losses between these stations. For nozzleless radial turbines such as the ones used in the present work, this typically amounts to three stations: Volute inlet (1), rotor inlet (2) and rotor outlet (3). The detailed description of this method is subdivided into volute modelling and rotor modelling below.

### 3.3.1 Volute Modelling

The primary role of the volute model is to set up the velocity triangle at the inlet to the rotor, as seen in Figure 3.3. The volute scroll is commonly modelled as a free vortex flow, an assumption which comes close to the true physics for a well designed scroll (Japikse & Baines 1997).
Figure 3.3: Velocity Triangles for a turbomachine
For an ideal volute this therefore means:

\[ r C_\theta = K \]  \hspace{1cm} (3.8)

where \( K \) is a constant.

For volute meanline modelling, Station 1 is taken as the plane normal to the circumferential direction, just upstream of the tongue as seen in Figure 3.2. Through continuity:

\[ C_1 \rho_1 A_1 = C_m \rho_2 A_2 = \dot{m} \]  \hspace{1cm} (3.9)

where the volute outlet plane is taken as a cylindrical surface so that:

\[ A_2 = 2 \pi r_2 b_2 \]  \hspace{1cm} (3.10)

where \( b_2 \) is the channel height, determined by the rotor inlet blade height.

Considering purely aerodynamic performance and therefore neglecting heat transfer effects in the volute, it can be said that \( T_{01} = T_{02} \). For an ideal volute (no losses) \( P_{01} = P_{02} \).

The other equations used to model the volute are some of the basic equations of thermodynamics:

\[ \frac{P_i}{P_{0i}} = \left(1 + \frac{\gamma - 1}{2} M_a^2 \right)^{-\frac{\gamma}{\gamma - 1}} \]  \hspace{1cm} (3.11)

\[ \frac{T_i}{T_{0i}} = \left(1 + \frac{\gamma - 1}{2} M_a^2 \right)^{-1} \]  \hspace{1cm} (3.12)

\[ C_i = M a_i \sqrt{\gamma R T_i} \]  \hspace{1cm} (3.13)

\[ P = \rho R T \]  \hspace{1cm} (3.14)

where \( i \) refers to an arbitrary meanline station. From the velocity triangles it can be
deduced that:

\[
\tan (\alpha_2) = \frac{C_{\theta_2}}{C_{m_2}} \tag{3.15}
\]

and

\[
C_{m_2} = C_2 \cos (\alpha_2) \tag{3.16}
\]

The flow at the scroll inlet is assumed to be circumferential, with no radial or axial component. Therefore:

\[
C_{\theta_1} = C_1 \tag{3.17}
\]

With a given total temperature and total pressure at volute inlet, as well as \( Ma_2 \), the above equations can be solved fully, setting up the velocity triangle at the rotor inlet.

Substituting Equations 3.8 and 3.15 into Equation 3.9 and rearranging accordingly gives the following relationship:

\[
\tan \alpha_2 = \left( \frac{\rho_2}{\rho_1} \right) \frac{2\pi b_2}{A_1/r_1} \tag{3.18}
\]

This shows that, assuming little change in the ratio of densities, the volute exit angle \( \alpha_2 \) is largely dependent on the commonly called “\( A/r \) ratio” (referring to the value of \( A_1/r_1 \)), and the blade height \( b_2 \). Therefore the \( A/r \) ratio is conventionally used as a design parameter to determine the swallowing capacity of a radial turbine with a given rotor.

### 3.3.1.1 Volute Losses

In the volute, the literature generally only considers losses due to wall friction. Losses due to the volute tongue are acknowledged but not modelled explicitly and therefore assumed as included in wall friction losses. For small automotive turbocharger turbines, the tongue is relatively pronounced because of mechanical constraints caused by thermal stresses and highly pulsating flow. This has been shown to impact volute loss, as some of the flow recirculates and mixes with the main inlet flow (Suhrmann et al. 2012). Because of their complexity, these losses are difficult to model explicitly in low order models.
and are therefore compensated by the volute pressure loss coefficient. The loss models included in the presented code are outlined here:

**Pressure loss coefficient:** The pressure loss coefficient from Japikse & Baines (1997), shown in the equation below models the total pressure drop due to friction with the volute walls. It typically has a value of \(0.1 - 0.3\)

\[
K_{pl} = \frac{P_{01} - P_{02}}{P_{02} - P_2}
\]  
(3.19)

**Swirl Loss Coefficient:** The coefficient \(K_{sw}\) accounts for the effect of wall friction on the free vortex flow, by modifying the free vortex equation (Japikse & Baines 1997). It typically has a value of 0.85-0.95.

\[
r_1C_{\theta 1} = K_{sw}r_2C_{\theta 2}
\]  
(3.20)

**Blockage Factor:** This parameter accounts for the reduction in effective flow area due to the boundary layers in the flow (Japikse & Baines 1997). It forms part of the continuity equation and has typical values of 0.85-0.95. The blockage factor is also used for the model of the rotor, where it not only represents the area reduction due to boundary layers, but also due to the thickness of the blade. The subscript \(i\) in the following equation represents the station for which the blockage is calculated.

\[
\dot{m}_i = \rho_iA_i(Bl_i)C_i
\]  
(3.21)

### 3.3.2 Rotor Modelling

The rotor component is also solved with basic flow equations, as well as the Euler turbo-machinery equation, which can be rearranged to state that:

\[
h_2 + \frac{w_2^2 - U_2^2}{2} = h_3 + \frac{w_3^2 - U_3^2}{2}
\]  
(3.22)

where \(w\) is the relative flow velocity and \(U\) is blade velocity. The position of station three is taken to be at the RMS radius of the exducer, so \(r_3 = \sqrt{(r_{3h}^2 + r_{3d}^2)/2}\). The
The continuity equation for the rotor exit takes the following form from the velocity triangle:

\[ \dot{m}_3 = w_3 \cos \beta_3 B l_3 \rho_3 \]  

(3.23)

where \( A_3 \) is the area of the blade span:

\[ A_3 = \pi (r_{3t}^2 - r_{3h}^2) \]  

(3.24)

The rotor losses can be applied as either internal or external losses. Internal losses are modelled to cause an entropy rise in the flow, and are therefore visible in the h-s diagram of the flow. This can be seen in Figure 3.4 which shows a turbine process. The internal losses are modelled as enthalpy losses and factored into the following equation:

\[ h_{3s, \text{rotor}} = h_3 - L_{\text{int}} \]  

(3.25)

where \( h_{3s, \text{rotor}} \) refers to the isentropic rotor exit enthalpy when referenced to the rotor inlet at station 2 (shown in Figure 3.4), and stems from the following equation:

\[ \left( \frac{P_3}{P_{02}} \right)^{\frac{\gamma-1}{\gamma}} = \frac{h_{3s, \text{rotor}}}{h_{02}} \]  

(3.26)

The entropy rise due to internal losses, means that there is a corresponding pressure loss in the flow. For a constant mass flow this affects the pressure drop, which in turn means that internal losses impact the mass flow parameter prediction of the model.

Some of the loss functions from literature, such as the disk friction loss outlined in more detail below, are applied as external losses. This means they are subtracted from the final power of the turbine, and are therefore not visible in the h-s diagram, nor do they have an effect on the mass flow rate. As they impact the power of the turbine however, they manifest themselves in the measured efficiency and should therefore be modelled.
Figure 3.4: h-s diagram of a turbine expansion process. The dashed isobar refers to the pressure at the rotor inlet station in the meanline model. Positions 1-3 are as shown in the diagram on the right.

3.3.2.1 Rotor Exducer Blockage Considerations

While the inlet of a radial inflow turbine has a constant radius across the span, the exducer is axial, leading to a significant variation in flow from hub to shroud. Radial turbines used in turbocharging are typically designed using the radial fibred condition. This means that an axial section of a blade will always be straight and radial, as seen in Figure 3.5a, so that no torsion is generated and the centrifugal force runs along the length of the blade. The primary reason for this is to reduce centrifugal stresses. The azimuthal position of a blade fibre is defined by the camberline as seen in Figure 3.5b, so that the full blade can be built.

As seen in Figure 3.5c, the blade exit angle for a radial fibred blade becomes:

\[
\tan \beta = r \frac{d\theta}{dz} \tag{3.27}
\]
Figure 3.5: Diagrams depicting how a radial fibred blade is built and how the exducer blade angle is derived. $z$ refers to the axial coordinate of the rotor.

...where $d\theta/dz$ is the camberline gradient, described in more detail in Section 4.3.2, and $r$ is the radial position. The exit angle of the rotor is therefore variable across the span, while the meanline model requires a single value. The common way of considering the exducer, is to model the mean line at the RMS average radius, ie.:

$$r_3 = \sqrt{r_{3u}^2 + r_{3h}^2}$$

(3.28)

This means that the area on both sides of $r_3$ is equal. The exit angle $\beta_3$ at this radius is calculated using Equation 3.27, and the equation for continuity is formulated for the exducer station:

$$\dot{m}_3 = W_3\rho_3 A_3 Bl_3 \cos \beta_3$$

(3.29)

where $A_3 \cos \beta_3$ could be considered the equivalent exducer throat area, $A_{th}$. This
method neglects the effect of the spanwise variation of $\beta_3$ on the throat area, as well as the blade thickness variation from hub to tip. A novel model has been developed, which considers the effect of spanwise variation through an integral of throat area across the span.

Considering just the effect of exit angle first, at the exducer of a turbine with zero thickness blades an annular differential element of the throat area can be defined:

$$dA_3 = 2\pi r \cos \beta dr$$

(3.30)

where Equation 3.27 can be substituted to give

$$\cos \beta = \cos \left( \tan^{-1} \left( r \frac{d\theta}{dz} \right) \right) = \frac{1}{\sqrt{(r \frac{d\theta}{dz})^2 + 1}}$$

(3.31)

so that the following integral can be formed:

$$A_{th} = 2\pi \int_{r_{th}}^{r} \frac{r}{\sqrt{(r \frac{d\theta}{dz})^2 + 1}} dr$$

(3.32)

This solves to the following equation for the equivalent rotor exducer throat area:

$$A_{th} = 2\pi \left( \sqrt{(r_{3h} \frac{d\theta}{dz})^2 + 1} - \sqrt{(r_{th} \frac{d\theta}{dz})^2 + 1} \right)$$

(3.33)

To include blade thickness in the above consideration in a somewhat universally applicable manner, some assumptions about the design of the blade can be made. If it is assumed that there is a linear variation in blade thickness from hub to tip, an axial blade element can be drawn as can be seen on the left hand side in Figure 3.6, the blade thickness being defined as $r \delta \theta_{el}$. A simplified throat area resulting from the thickness of the blade can be seen in the radial section of the exducer on the right hand side in Figure 3.6. The following equation can thus be deduced for the throat area differential:

$$dA_{th} = \frac{r \left( 2\pi - \delta \theta_{el} N_{bl} \right)}{\sqrt{(r \frac{d\theta}{dz})^2 + 1}} dr$$

(3.34)

where $N_{bl}$ is the number of blades, and $\delta \theta_{el}$ is the blade element thickness as shown in
Figure 3.6 and calculated by the following equation:

$$\delta \theta_{el} = \left( \frac{r_t \delta \theta_{el,t} - r_h \delta \theta_{el,h}}{r_t - r_h} \right) \left( 1 - \frac{r_h}{r} \right) + \frac{r_h \delta \theta_{el,h}}{r}$$  \hspace{1cm} (3.35)

Figure 3.6: Diagram showing three adjacent rotor blades with a linear reduction in $\delta \theta_{el}$ from hub to tip, in the axial projection, and two adjacent blades in the radial projection at the exducer of the rotor.

Equation 3.34 can be integrated analytically, resulting in the following Equation:

$$A_{th} = \left[ \frac{r (BN \delta \theta_{el,h} - 2r_h \delta \theta_{el,h} N_{bl} + 2\pi r)}{2 \sqrt{\left( \frac{d \delta \theta}{dz} \right)^2 + 1}} \right]_{r_3h}^{r_3h}$$  \hspace{1cm} (3.36)

where

$$B = \frac{r_t \delta \theta_{el,t} - r_h \delta \theta_{el,h}}{r_t - r_h}$$  \hspace{1cm} (3.37)

If Equation 3.28 is used to calculate the average radius, and $\beta$ is calculated at the cor-
responding position using Equation 3.27, an additional blockage factor can be introduced to account for the area difference due to thickness and angle variations:

\[ Bl_{th} = \frac{A_{th}}{A_3 \cos \beta_{3,rms}} \]  

(3.38)

The continuity equation for the rotor exit, including blockage factors then becomes:

\[ \dot{m}_3 = W_3 \rho_3 A_3 Bl_3 Bl_{th} \cos \beta_3 \]  

(3.39)

The parameter \( Bl_{th} \) is determined by the rotor geometry and can be considered as accounting for the blockage effects relating to blade thickness and angle, while the parameter \( Bl_3 \) is included as a calibration parameter, to account for boundary layer thickness (which should be small at the exducer) and to correct errors in the underlying assumptions of the above analysis. This parameter should generally be close to unity and be independent of blade design, offering an improvement to the original form of the blockage factor as seen in Equation 3.29, where a change in blade design would require re-calibration of \( Bl_3 \).

### 3.3.2.2 Rotor Losses

Five sources of loss are considered for the rotor, with several models available for selection for each loss within the code. An outcome of Chapter 4, is the analysis of which of these models leads to the best prediction.

**Passage Loss:** The passage loss models the losses due to wall friction in the rotor (boundary layer losses). Two passage loss models were made available in the code, and can be chosen through the input. The first loss model is from Wasserbauer & Glassman (1975) and assumes the passage loss to be proportional to the mean relative velocity in the blade passage:

\[ L_p = K_p \left( \frac{w_2^2 \cos^2 i + w_3^2}{2} \right) \]  

(3.40)

where \( i \) is the rotor incidence with respect to the optimum incidence angle, as shown in Equation 3.48.
The second available passage loss model was developed by Baines (1998) and includes geometric parameters.

\[ L_p = K_p \left[ \left( \frac{L_H}{D_H} \right) + 0.68 \left( 1 - \left( \frac{r_3}{r_2} \right)^2 \right) \frac{\cos \beta_3}{b_3/l_c} \right] B \]  

(3.41)

where \( L_H \) is the mean hydraulic length defined as:

\[ L_H = \frac{\pi}{4} \left[ \left( l_z - \frac{b_2}{2} \right) + \left( r_2 - r_3 - \frac{b_3}{2} \right) \right] \]  

(3.42)

and \( D_H \) is the mean hydraulic diameter:

\[ D_H = \frac{1}{2} \left[ \left( \frac{4\pi r_2 b_2}{2\pi r_2 + N_b b_2} \right) \left( \frac{2\pi (r_3^2 - r_3^3)}{\pi (r_3 - r_3^3) + N_b b_3} \right) \right] \]  

(3.43)

The hydraulic length and diameter allow the model to better correlate frictional losses to the velocity as a function of geometry, reducing the importance of the calibration parameter \( K_{pl} \) and making the model more universally applicable.

The model accounts for secondary flow losses by considering the blade loading and the turning of the flow in the tangential plane. The parameter \( l_c \) is the rotor blade chord which is approximated by:

\[ l_c = \frac{l_z}{\cos \beta} \]  

(3.44)

where

\[ \tan \beta = 0.5 \left( \tan \beta_{3d} + \tan \beta_{3d} \right) \]  

(3.45)

**Incidence Loss** As the flow incidence to the rotor changes, a change in entropy gain occurs. There is an optimum incidence angle which is often calculated by the following equation from Stanitz (1952):

\[ \beta_{inc} = \tan^{-1} \left[ \tan (\alpha_2) \left( 1 - \frac{1}{\lambda} \right) \right] \]  

(3.46)
where
\[
\lambda = 1 - \frac{1.98}{Z}
\] (3.47)
and \(Z\) is the number of blades.

The loss due to incidence is then calculated as a function of deviation from the optimum angle which is determined by the following relation:

\[
i = \beta_2 - \beta_2\beta - \beta_{inc}
\] (3.48)

Three incidence loss models were made available in the code: The first is from Wasserbauer & Glassman (1975):

\[
L_{inc} = \frac{w_2^2 \sin^2 i_2}{2}
\] (3.49)

The fundamental theory is that the kinetic energy due to the velocity component perpendicular to the optimum incidence is lost.

Mizumachi et al. (1979) found from cascade tests, that for incidence angles larger than 45° the incidence loss increases at a faster rate due to separations on the leading edge. This led to a new formulation for the incidence loss for high incidence angles:

\[
L_{inc} = \begin{cases} 
\frac{1}{2} K_{inc} w_2^2 \sin^2 i_2 & \text{for } i_2 < \frac{\pi}{4} \\
\frac{1}{2} K_{inc} w_2^2 (0.5 + i_2 - \pi/4) & \text{for } i_2 \geq \frac{\pi}{4}
\end{cases}
\] (3.50)

The parameter \(K_{inc}\) was introduced by Romagnoli & Martinez-Botas (2011) to calibrate the model to existing data, as it was found that the loss was underestimated without it.

The third model is from Meitner & Glassman (1980). The authors introduced the exponent shown here as \(K_{inc}\) and found that values of 1.75 for positive incidence and 2.5 for negative incidence gave the best fit with data.

\[
L_{inc} = \frac{1}{2} w_2^2 \left(1 - \cos(i_2)^{K_{inc}}\right)
\] (3.51)

A shortcoming of this model is that moving away from an exponent of 2 reduces the physical basis of the model, as it is no longer models the kinetic energy in the tangential
Tip Clearance Loss  Tip clearance losses occur because of flows crossing through the clearance between blade and shroud. Two mechanisms dictate this flow: The pressure difference between the pressure to the suction side of the blade causes fluid to flow through the gap, and at the same time, considering the relative frame, the counter-rotating shroud wall causes a “scraping” flow in the opposite direction. Tip clearance can have a significant impact on the turbine efficiency and has therefore been studied in great detail. Five tip clearance models were made available in the code. The first three losses relate to the ratio of tip clearance to the total blade span.

The loss models by Rodgers (1968) (Equation 3.52) and Krylov & Spunde (1967) (Equation 3.53) are drawn from empirical results:

\[ \Delta \eta_{t-s} = 0.1 \frac{\varepsilon_r}{r_3t - r_{3h}} \]  

\[ \Delta \eta_{t-s} = 2 \frac{\varepsilon_r}{b_2} \left( \frac{r_3}{r_2} - 0.275 \right) \]  

The loss estimate by Glassman (1976) assumes that the tip clearance loss as a proportion of the total enthalpy drop is equal to the fraction of tip clearance and passage height at the exducer:

\[ L_{cl} = \Delta h_{t-t} \varepsilon_r \]  

A more sophisticated approach is to explicitly model the flow through the clearance gap, and assume that the kinetic energy of the flow is fully lost, leading to a pressure drop. Two such models are integrated into the code. The first, by Spraker (1987) uses the following equation to model the mass flow through the gap:

\[ \dot{m}_{cl} = \frac{1}{2} \rho U_t \varepsilon_r l_t N_{bl} K_{cl} \]  

where \( \dot{m}_{cl} \) is the clearance flow, \( l_t \) is the length of the gap, \( N_{bl} \) is the number of blades, and \( K_{cl} \) is the calibration coefficient, which should be close to 1 according to
Spraker (1991) and was determined to be 1.5 by experiment according to Whitfield & Baines (1990). Assuming a velocity equal to the blade passing velocity, the clearance loss can then be calculated by the following equation:

\[ L_{cl} = \frac{1}{2} \frac{\dot{m}_{cl}}{\dot{m}} U_t^2 \]  

(3.56)

The second model from Baines (1998) considers the radial and axial clearance separately, also including a “cross-coupling” term to account for interaction between radial and axial clearance flows:

\[ L_{cl} = \frac{U_3^3 N_{bl}}{8\pi} \left( K_x \varepsilon_x C_x + K_r \varepsilon_r C_r + K_{xr} \sqrt{\varepsilon_x \varepsilon_r} C_x C_r \right) \]  

(3.57)

where

\[ C_x = \frac{1 - \left( \frac{r_3 t}{r_2} \right)}{C_{m2} b_2} \]  

(3.58)

and

\[ C_r = \left( \frac{r_3 t}{r_2} \right) \frac{l_z - b_2}{C_{m3} r_3 b_3} \]  

(3.59)

\( l_z \) refers to the axial length of the rotor. The author found that coefficient values of \( K_x = 0.4, K_r = 0.75 \) and \( K_{xr} = -0.3 \) gave the most satisfactory results.

**Trailing Edge Loss**  Trailing edge losses are caused by the sudden expansion of the flow going the blade passage past the trailing edges to the unbladed region. Due to the difficulty of implementation for choked flow which was a necessary requirement, the current work includes only one trailing edge loss model from Meroni et al. (2018) summarised in Equation 3.60

\[ L_{te} = \frac{1}{2} w_3^2 \left( \frac{N_{bl} t_{te}}{2\pi r_3 \cos \beta_3} \right)^2 \left( 1 + \frac{\gamma - 1}{2} (M_{\alpha_3}^3)^2 \right)^{-\frac{\gamma}{\gamma - 1}} \]  

(3.60)

Where \( t_{te} \) is the trailing edge blade thickness. This equation derives from Glassman (1995), who modelled a total pressure loss in the relative frame using the following equation:
This relates the total pressure loss to the physical blockage due to blade thickness at the exducer. The derivation of the corresponding enthalpy loss was done according to the approach outlined in Horlock (1960).

The literature on the trailing edge loss for radial turbines is sparse and the phenomenon is generally poorly understood, but it seems that the loss is generally small (Meitner & Glassman 1983). Many meanline models in the literature do not include a trailing edge loss at all. However, for completeness it is included in the present work.

**Disk friction**  Disk friction, or windage loss, is the loss that occurs due to wall friction on the back face of the rotor. This is generally very small and is modelled with the following relation from Japikse & Baines (1997):

\[
L_{df} = 0.25 \rho U_t r_t^2 K_{df} / \dot{m}
\]

where

\[
K_{df} = \begin{cases} 
3.7(\epsilon_b/r_t)^{0.1}/Re_r^{0.5} & \text{for } Re_r < 3 \times 10^5 \\
0.102(\epsilon_b/r_t)^{0.1}/Re_r^{0.2} & \text{for } Re_r > 3 \times 10^5 
\end{cases}
\]

and \(Re_r\) is the rotor Reynolds’s number \(Re_r = U_t r_t / \nu\). The disk friction is modelled as an external loss, as it does not affect the flow going through the rotor directly, but causes a loss at the shaft. This means it has an effect on the efficiency.

### 3.3.3 Choked Flow

For higher pressure ratios, a turbine will experience choke, after which the mass flow parameter will not increase. For a nozzled turbine there are two locations where choke can occur, namely in the nozzle and at the exducer. If choke happens in the nozzle first, a further reduction in exit pressure will eventually lead to choke in the rotor exducer (in the rotating frame of reference). For a nozzleless turbine, choke only occurs in the rotor exducer exit. The choking mass flow is dependent on the rotational speed of the
Figure 3.7: Diagram showing choke and the mode of expansion after the rotor exducer throat. The flow expands by deflecting in the axial direction so that the exit angle $\beta_3$ is lower than the throat blade angle $\beta_{th}$, effectively forming a converging-diverging nozzle.

This is because at higher rotor speeds and for a given mass flow rate and turbine inlet total pressure, the total pressure in the relative frame of reference $P'_0$ is higher. The pressure ratio $P'_0/P_3$ will determine whether the rotor is choked, so that at a higher speed choking will be reached at a lower mass flow parameter.

When the pressure ratio is increased beyond choke, the flow will deflect at the exducer allowing a supersonic expansion to occur, as seen in Figure 3.7, which is assumed to be isentropic (Whitfield & Baines 1990). This is explicitly modelled by fixing the mass flow rate once choke has occurred and reducing $\beta_3$ to further increase the expansion ratio. This approach of modelling the rotor exducer flow beyond the choking point was taken by Meitner & Glassman (1983).

### 3.3.4 Summary of geometric parameters and loss models

The tables below give a summary of the geometric parameters required to resolve a radial turbine stage using the meanline approach presented here, as well as the volute and rotor loss models available for selection in the code.
### Table 3.1: Meanline geometric input parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_2$</td>
<td>inlet tip radius</td>
</tr>
<tr>
<td>$A_1$</td>
<td>Area of station 1 at the scroll inlet</td>
</tr>
<tr>
<td>$r_1$</td>
<td>radial distance of the centroid station 1 to the axis</td>
</tr>
<tr>
<td>$(d\theta/dz)_3$</td>
<td>Camberline gradient at the exducer</td>
</tr>
<tr>
<td>$r_{3t}$</td>
<td>exducer tip radius</td>
</tr>
<tr>
<td>$r_{3h}$</td>
<td>exducer hub radius</td>
</tr>
<tr>
<td>$b_2$</td>
<td>inlet blade height</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>tip clearance</td>
</tr>
<tr>
<td>$N_{bl}$</td>
<td>Number of blades</td>
</tr>
</tbody>
</table>

### Table 3.2: Volute Loss Functions

<table>
<thead>
<tr>
<th>Loss</th>
<th>Equation</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure loss</td>
<td>$K_{pl} = \frac{P_{01} - P_{02}}{P_{02} - P_2}$</td>
<td>Japikse &amp; Baines (1997)</td>
</tr>
<tr>
<td>Swirl Loss</td>
<td>$r_1C_{\theta 1} = K_{sw} r_2C_{\theta 2}$</td>
<td>Japikse &amp; Baines (1997)</td>
</tr>
<tr>
<td>Loss No.</td>
<td>Equation</td>
<td>Ref.</td>
</tr>
<tr>
<td>---------</td>
<td>--------------------------------------------------------------------------</td>
<td>------------------------------------------------</td>
</tr>
<tr>
<td>Passage</td>
<td></td>
<td>Wasserbauer &amp; Glassman (1975)</td>
</tr>
<tr>
<td>1</td>
<td>$L_p = K_p \left( \frac{u_2^2 \cos^2 \gamma + \nu_2^2}{2} \right)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\left( \frac{L_H}{D_H} \right) + 0.68 \left( 1 - \frac{f_3}{f_4} \right) \left( \frac{W_2^2 + W_3^2}{b_3/l} \right)$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$L_p = \frac{1}{2} K_p \left( \frac{L_H}{D_H} \right) + 0.68 \left( 1 - \frac{f_3}{f_4} \right) \left( \frac{W_2^2 + W_3^2}{b_3/l} \right)$</td>
<td>Baines (1998)</td>
</tr>
<tr>
<td>Incidence</td>
<td></td>
<td>Futral &amp; Wasserbauer (1965)</td>
</tr>
<tr>
<td>1</td>
<td>$L_{inc} = \frac{u_2^2}{2} \sin^2 (\gamma)$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$L_{inc} = K_{inc} \frac{u_2^2}{2} \left( 0.5 + i_2 - \pi/4 \right)$ for $i_2 &gt; \pi/4$</td>
<td>Mizumachi et al. (1979)</td>
</tr>
<tr>
<td>3</td>
<td>$L_{inc} = \frac{u_2^2}{2} \left( 1 - \cos (\gamma) \right) \frac{K_{inc}}{2}$</td>
<td>Metner &amp; Glassman (1980)</td>
</tr>
</tbody>
</table>
Table 3.4: Rotor Tip Clearance and Trailing Edge Loss Functions

<table>
<thead>
<tr>
<th>Loss No.</th>
<th>Equation</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$m_{cl} = \frac{1}{2} \rho U_t \varepsilon_b LN_{bl} K_d$, $L_{cl} = \frac{1}{2} \left( \frac{m_{cl}}{m} \right) U_t^2$</td>
<td>Spraker (1987)</td>
</tr>
<tr>
<td>2</td>
<td>$L_{cl} = \frac{U_t^3 N_{bl}}{8\pi} \left( K_x \varepsilon_x C_x + K_r \varepsilon_r C_r + K_{xr} \sqrt{\varepsilon_x \varepsilon_r C_x C_r} \right)$</td>
<td>Baines (1998)</td>
</tr>
<tr>
<td>3</td>
<td>$\Delta \eta_{t-s} = 0.1 \frac{\varepsilon_r}{r_{3t} - r_{3h}}$</td>
<td>Rodgers (1968)</td>
</tr>
<tr>
<td>4</td>
<td>$\Delta \eta_{t-s} = 2 \frac{\varepsilon_r}{b_2} \left( \frac{r_3}{r_2} - 0.275 \right)$</td>
<td>Krylov &amp; Spunde (1967)</td>
</tr>
<tr>
<td>5</td>
<td>$L_{cl} = \frac{\Delta h_{t-s} \varepsilon_r}{r_{3t} - r_{3h}}$</td>
<td>Glassman (1976)</td>
</tr>
</tbody>
</table>

Tip Clearance

$$L_{te} = B \left[ \frac{N_{bl} t}{2\pi r_3 \cos \beta_3} \right]^2 \frac{1}{2} w_4^2$$

where: $B = \left( 1 + \frac{\gamma - 1}{2} (Ma_t')^2 \right)^{\gamma/(\gamma - 1)}$

3.4 Meanline Model Code Structure

For the present work, a meanline model was developed in FORTRAN 90. The reason for this was the possibility of integrating FORTRAN user models in the engine modelling software GT-Power, used for the next stage of the project. An additional advantage of FORTRAN is a very high computing speed. It is also straightforward to integrate with C++ and other languages if necessary. Emphasis was placed on the stability of the code, as this was paramount for use in optimisation.

3.4.1 Meanline Function

The meanline model can be described as solving the turbine efficiency and mass flow parameter for a given turbine design and calibration parameter set, as a function of operating condition (reduced speed and pressure ratio), as shown in the following equation:

\[
(\eta_{t-s}, MFP) = MEANLINE\left(\frac{N}{\sqrt{T_0}}, \frac{P_{01}}{P_3}\right)
\]  

(3.64)

This function cannot be solved directly, and a series of algorithms are included within the code to solve the equations numerically. The simplest way of structuring the code is to model the volute and rotor as a function of Mach number at the volute outlet, \( M_{a2} \), and reduced speed, \( \frac{N}{\sqrt{T_{01}}} \), with pressure ratio, efficiency and mass flow parameter as the output. In the code, the volute and rotor models are bundled into the CORE subroutine:

\[
(\eta_{t-s}, MFP, \frac{P_{01}}{P_3}) = CORE\left(\frac{N}{\sqrt{T_{01}}}, M_{a2}\right)
\]  

(3.65)

The CORE function has the shape shown in the graph in Figure 3.8 for a constant speed. The asymptote at maximum \( M_{a2} \) indicates that the function is approaching a choked state in the rotor exducer. The CORE subroutine will recognise this and return a choked flag when \( M_{a2} \) is higher than this value.

The MEANLINE routine contains an algorithm to solve the function for the required \( P_{01}/P_3 \). This algorithm will begin with the bisection method and once two real values higher than the required pressure ratio have been found, the algorithm will initialize the secant method which is significantly faster, but only sufficiently stable when the above
condition is met. If the required pressure ratio is in the choked region, the bisection will converge on the value of $Ma_2$ at which the rotor is choked, and then initialise the “choked mode”. The reader is referred to Section 3.4.4 for a detailed explanation of how the code handles choking in the rotor.

Figure 3.9 shows a flowchart of the MEANLINE routine.

### 3.4.2 Volute Subroutine

The volute subroutine solves the volute equations shown in Section 3.3, with integration of the loss functions, for a given outlet Mach number $Ma_2$ and inlet total pressure $P_{01}$ which is known from the input pressure ratio. The equations are arranged in such a way that the following function can be solved numerically:

$$\frac{2 (\dot{m}_1 - \dot{m}_2)}{m_1 + \dot{m}_2} = f(Ma_1)$$  \hspace{1cm} (3.66)
The equations are structured as follows:

The equation for the pressure loss coefficient (Equation 3.19) is rearranged to give

\[ P_{02} = \frac{P_{01}}{K_{pl} (1 - P_2/P_{02}) + 1} \]  

(3.67)
where

\[
\frac{P_2}{P_{02}} = \left( 1 + \frac{\gamma - 1}{2} Ma_2^2 \right)^{\frac{\gamma - 1}{\gamma}}
\]  

(3.68)

Meanline modelling does not commonly include heat transfer effects, and if significant, they are included after the calculation of adiabatic performance. The assumption of no heat transfer and no work in the volute, means that

\[T_{01} = T_{02}\]  

(3.69)

\[T_2\] and \(C_2\) can subsequently be derived using Equations 3.12 and 3.13 respectively, and \(P_2\) is solved with Equation 3.68. The ideal gas law (Equation 3.14) is then used to solve \(\rho_2\).

For a given value of \(Ma_1\), Equations 3.12 - 3.14 can be used to solve \(T_1\), \(C_1\) and \(\rho_1\).

The flow at the scroll inlet can be assumed to be completely circumferential, and so \(C_1 = C_{\theta 1}\). Equation 3.20 can then be re-written to give

\[r_1 C_1 = K_{sw} r_2 C_2 \sin \alpha_2\]  

(3.70)

and continuity dictates that:

\[\rho_1 C_1 A_1 Bl_1 = \rho_2 C_2 A_2 Bl_2 \cos \alpha_2\]  

(3.71)

Rearranging and substitution of Equations 3.70 and 3.71, yields:

\[\tan \alpha_2 = \frac{r_1 K_{sw} A_2 \rho_2 Bl_2}{r_2 A_1 \rho_1 Bl_1}\]  

(3.72)

Subsequently, all variables required for calculation of mass flow at both the inlet and the outlet have been calculated:

\[\dot{m}_1 = \rho_1 A_1 Bl_1 C_1\]  

(3.73)

\[\dot{m}_2 = \rho_2 C_2 A_2 Bl_2 \cos \alpha_2\]  

(3.74)
This allows Equation 3.66 to be solved numerically using the bisection and secant methods. When solved, the function outputs all the relevant variables needed to solve the rotor equations. Figure 3.10 shows a flowchart of the VOLUTE subroutine algorithm.

3.4.3 Rotor Subroutine

The rotor subroutine solves the equations between station 2 and 3, using the outputs of the volute subroutine. The function which is solved is:

\[
\frac{\dot{m}_3 - \dot{m}_2}{m_2} = f(Ma'_{03}) \tag{3.75}
\]

From velocity triangles it can be deduced that:
\[ W_{\theta 2} = C_{\theta 2} - U_2 \]  

(3.76)

and

\[ W_2 = \sqrt{W_{\theta 2}^2 + C_{m 2}^2} \]  

(3.77)

where \( U_2 \) is the blade velocity at the inlet to the rotor. From this, total enthalpies in the relative frame of reference at rotor inlet and outlet can be established:

\[ h'_{02} = h_2 + 0.5 W_2^2 \]  

(3.78)

\[ h'_{03} = h'_{02} + 0.5 (U_3^2 - U_2^2) \]  

(3.79)

For a given value of \( Ma_3' \), the following equations hold:

\[ T_3 = T_{03}' \left( 1 + \frac{\gamma - 1}{2} Ma_03^2 \right)^{-1} \]  

(3.80)

\[ W_3 = Ma_03^2 \sqrt{\gamma RT_3} \]  

(3.81)

\[ C_{m3} = W_3 \cos(-\beta_3) \]  

(3.82)

Subsequently the internal rotor losses can be calculated, leading to the equation:

\[ T_{3s} = T_3 - \frac{L_{int}}{C_p} \]  

(3.83)

Equation 3.26 is rearranged to give:

\[ P_3 = P_{02} \left( \frac{T_{3s}}{T_{02}} \right)^{\gamma/\gamma} \]  

(3.84)

Through the ideal gas law, \( \rho_3 \) can be calculated, which makes all variables needed to solve the function in Equation 3.75 available:
\[ \dot{m}_3 = C_{m3} \rho_3 A_3 Bl_3 \quad (3.85) \]

The shape of the function in Equation 3.75 is shown in Figure 3.11, for different operating points. The algorithm used to solve it utilises a combination of secant method and bisection, depending on the requirements of the operating point. When the maximum of the curve lies below 0, the mass flow determined by the volute subroutine is higher than the choking mass flow, therefore there is no solution. In that case the Rotor subroutine returns a “rotor choked” flag to the code.

![Figure 3.11: Shape of the ROTOR function at different values of \( Ma_2 \) with solution marked](image)

With the exception of no solution when the input of \( Ma_2 \) causes a mass flow higher than choke, the function has two solutions. The lower solution is subsonic, when the rotor is not choked. The higher one is supersonic for a choked rotor, when the code is in choke mode. The method used for choke handling is outlined in the next section.
Figure 3.12: Flowchart for ROTOR subroutine
3.4.4 Choking

When the pressure ratio to be solved is in the choked region, the MEANLINE subroutine converges on the choked pressure ratio of the turbine using the bisection method, ie. it determines that there is no un-choked solution and finds the choking point. The subroutine then initialises the “choked mode”: $Ma_2$ is fixed at the choked value and an algorithm attempts to find the exit angle $\beta_3$ for which the pressure ratio is solved. This goes by the assumption of an isentropic expansion from the throat onwards, leading to supersonic flow (see Section 3.3.3). The function solved by the CORE subroutine then becomes:

$$\left( \eta_{t-s}, MFP, \frac{P_{o1}}{P_3} \right) = CORE \left( \frac{N}{\sqrt{T_{o1}}}, \delta \beta_3 \right) \quad (3.86)$$

The algorithm then solves the function for the required pressure ratio. The minimum possible exit angle $\beta_3$ is 0, when the turbine is fully loaded, and power can no longer be increased. Energy from any further increases in pressure ratio is fully lost.

In choked mode the ROTOR subroutine behaves differently, in that it now solves for the higher value of $Ma_3'$, giving the supersonic solution. The shape of the ROTOR function at different values of $\beta_3$ is seen in Figure 3.13. As $\beta_3$ is decreased, the solution for $Ma_3'$ becomes higher, resulting in a lower exit pressure.
Figure 3.13: Shape of the $ROTOR$ function in choked mode, for different values of $\beta_3$. The points where the line crosses the abscissa are shown

### 3.5 Calibration Procedure

The inputs to the meanline code can be categorised into geometric parameters, operating parameters and calibration parameters. The calibration parameters are used in conjunction with the loss models outlined in Section 3.3. These are based on the empirical observation of physical processes occurring within the turbine and must therefore be fit using available data. A common way of doing this is through a manual process of trial and error, using value ranges from literature for each parameter (Romagnoli 2010).

The calibration forms an optimisation problem where the objective function is the error between the model and input data for a given set of calibration coefficients, and needs to be minimised. This means that calibration of the model can be achieved with an algorithm enabling automation of the fitting process. An automated fitting procedure was implemented into the code which utilises a genetic algorithm to minimise a weighted average of the error between both efficiency and mass flow parameter prediction to the input data. This is determined by the following objective function:
\[
\sum_{i=1}^{n_{\text{points}}} \left( \frac{|\eta_i,\text{data} - \eta_i,\text{model}|}{Y \eta_{\text{data}}} + \frac{Y |MFP_{i,\text{model}} - MFP_{i,\text{model}}|}{MFP_{\text{data}}} \right) = f(K_{\text{cal}}) \tag{3.87}
\]

where \(i\) refers to the position in the input data vector and \(n_{\text{points}}\) is the number of input data points being calibrated to. \(Y\) is the weighting factor which can be used to control the importance of efficiency fit compared to mass flow parameter fit. \(K_{\text{cal}}\) is the vector which contains the calibration parameters, the number of which can vary depending on the selection of models.

Calculation of the objective function therefore involves the meanline performance calculation of all input data points with the given calibration parameter vector. The genetic algorithm used was an openly available FORTRAN function, PIKAIA, described in Charbonneau (1995). Genetic algorithms are a form of evolutionary algorithms which mimic the natural selection process present in evolution to maximise an objective function by modification of the input parameters. It was selected in this case because its ease of application and stability. Due to the low computational cost of the function this was feasible despite the existence of more efficient optimisation algorithms.

Though PIKAIA offers several detailed settings for how the genetic algorithm is run, most of these were set at their default value as their sensitivity was small. The settings which were changed on a case by case basis were the number of generations and the population of each generation. These settings represent a compromise between how well the model is fit, to how long the fitting process takes.

The fitting routine for the calibration parameters requires two text-based input files: the datafile containing the performance parameters of the turbine, and the parameter file, which contains geometric and calibration parameters.

The geometric and calibration parameters are input to the model as a single vector \(K_{\text{input}} = \{K_{\text{geom}}, K_{\text{cal}}\}\). The parameter input file of the calibration routine has a row for each position in the vector \(K_{\text{input}}\) and a column for selection which parameters are to be used for calibration, as well as columns for the value if not calibrated and the lower and upper bounds for calibration.

As the calibration and geometric parameters are treated in the same way, the proce-
dure can be used to fit the geometric parameters of the turbine to desired performance characteristics, providing a tool for preliminary turbine design. This is done by giving the calibration parameters default values, and setting lower and upper bounds for the geometric parameters instead. The genetic algorithm then finds the set of geometric parameters $K_{geom}$ which best fit the input datapoints.

Examples of calibrations of the calibration parameters to data are found in the following chapters of this thesis (e.g. Section 5.5.1). The time taken for these calibrations is dependent on the number of calibration data points used, and the number of generations and population setting of the genetic algorithm. For nine calibration points, 100 generations, and a population of 100, the total time taken for calibration of the model was 20s.
Chapter 4

Parametric Turbine Study

4.1 Overview

The purpose of the work shown in this chapter, was to determine the accuracy of the meanline model described in Chapter 3, and thus evaluate its suitability for turbine design optimisation on an engine modelling level. This novel insight was achieved through a parametric study using CFD.

The key objectives described are as follows:

1. Development of 3-D Parametric Model: A 3-D parametric model of a nozzleless radial turbine stage was developed on the basis of an existing turbocharger turbine, allowing the generation of turbine geometries for a given set of geometric input parameters from the meanline model. This was necessary for setting up CFD calculations, and for later manufacture of turbine designs

2. Numerical Experiment: This section describes the numerical experiment which was set up on the basis of the 3D parametric model to evaluate the turbine performance over a range of designs. Two designs of experiment are described, as well as the numerical setup for CFD.

3. Meanline model validation: Data from the numerical experiment was used for calibration and evaluation of the meanline model performance. Different loss models outlined in Chapter 3 were investigated, as well as the extent to which the model can extrapolate to different designs once calibrated.
4.2 Introduction

To allow the engine level optimisation of turbine design using 1-D engine modelling, a performance prediction method with low computational cost is required for the turbine. The meanline model presented in Chapter 3 addresses the need for low order prediction of aerodynamic performance. This would allow optimisation of the geometric input parameters of the meanline model. However, the ability to accurately predict turbine performance across the design space and the full operating range, enabling optimisation of the design, remains largely unvalidated.

Several approaches would be possible for achieving this validation, including the use of existing experimental data for a range of turbines. This has been done in literature (Baines 1998, Meroni et al. 2018) however, the detail which can be achieved with such an approach is limited, and does not provide a strong basis for the proposed use of the model, as a key requirement is for the model to respond to small changes of the meanline geometric parameters. This requires a uniform design approach, and validation over a relevant sample of designs.

A feasible approach for validating the response of a low order model would be the use of computational fluid dynamics (CFD) models in a computational experiment. This was the approach taken in this thesis. While not as accurate as an experimental study, CFD is able to resolve the performance of the turbine within good accuracy, and can predict trends of efficiency and mass flow parameter due to design changes. This means that it provided the ideal tool to enable inexpensive and time efficient sampling of turbine performance in the design space set by design constraints, giving a novel insight into meanline model accuracy when calibrating using the genetic algorithm procedure presented in the previous chapter.

To enable the use of sampling methods for an experiment of numerical calculations, the generation of 3D turbine designs was required. The development of a 3D parametric turbine model proved to be a good approach, as it ensured a unified design process with good comparison between designs, and stable automatic geometry generation. The input parameters to the parametric model were the geometric input parameters of the meanline model.
Automated geometry generation and meshing tools allowed a design of experiment for numerical calculations within the design space and across the whole operating range. Two sampling methods were used, namely a full factorial sampling for three geometric parameters, with a fixed set of turbine operating points, as well as a latin hypercube sample using stratified sampling of three geometric parameters and the two operating parameters (turbine speed and pressure ratio). The results could be compared to the performance prediction of the meanline model and thus provided an insight into the predictive capability of the model.

Detailed data and design information for a Proton CAMPRO Engine were available, and therefore the turbine models developed in this chapter are tailored towards this engine. This would enable integration of the validated models into an engine model for use in optimisation, and the potential for future engine testing of optimised designs. To do this, the turbine of the stock turbocharger was reverse engineered using 3D and CT scanning methods, and the design used as a basis for the parametric turbine geometry, such that the geometry generated with the baseline set of parameters, was near equivalent to that of the stock turbine.

To align the operating range of the turbine with that of the radial turbine dynamometer at Imperial College for experimental validation, scaling up of the turbine was required. The turbine models developed in this chapter are therefore for the scaled up turbine size, with a rotor inlet diameter of 76mm.

4.3 Development of 3-D Parametric Model

The parametric turbine model which was developed consists of two separate components, the volute and the rotor. The purpose of the model was to generate a geometry as a function of the geometric meanline parameters which were outlined in Chapter 3 and are repeated in Table 4.1.

The baseline turbocharger for the Proton CAMPRO engine was used as the basis for the parametric design, such that for the baseline set of geometric input parameters, a geometry very similar to that of the baseline turbine is generated. The parametric model was developed in the software package CAESES which is well suited for parametric design,
Table 4.1: Meanline geometric input parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_2$</td>
<td>inlet tip radius</td>
</tr>
<tr>
<td>$A_1/r_1$</td>
<td>A over r ratio at the volute inlet section</td>
</tr>
<tr>
<td>$\left(\frac{d\theta}{dz}\right)_3$</td>
<td>Camberline gradient at the exducer</td>
</tr>
<tr>
<td>$r_3t$</td>
<td>exducer tip radius</td>
</tr>
<tr>
<td>$r_3h$</td>
<td>exducer hub radius</td>
</tr>
<tr>
<td>$b_2$</td>
<td>inlet blade height</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>tip clearance</td>
</tr>
<tr>
<td>$N_{bd}$</td>
<td>Number of blades</td>
</tr>
</tbody>
</table>

and can be integrated with the relevant CFD packages, as well as being able to output geometries in the most commonly used CAD formats.

A full model consists of a large number of parameters, some of which are explained in detail in the following sections. To ensure the model responds reasonably to the meanline geometric input parameters, all of the other parameters must either be made subordinate to these through functions, or kept constant.

It is generally convenient to make all of the parameters relative to the inlet tip radius $r_2$ so that the whole turbine model scales. The reason for this, is that turbine size can vary considerably depending on the application, but the relative proportions of the design parameters tend to be comparable. Scaling consequently makes a more universally applicable model.

To enable good integration and comparison with the baseline engine, detailed knowledge of both the volute and the rotor geometries of the baseline turbine was required. The baseline turbocharger, shown in Figure 4.2 comes from a common supplier, chosen by the manufacturers of the engine. No CAD information was available for the volute, while a CAD model of the rotor was made available. Additionally, standard hot map data were available, showing the performance of the turbine as can be seen in Figure 4.1.

This provided a challenge as the efficiency for such a hot map is strongly affected by heat transfer and bearing friction, making a comparison with aerodynamic performance
difficult. However, the mass flow parameter, is independent of heat transfer and can therefore be used to validate the generated turbine design.

To generate a good 3D baseline model, the turbine had to be partially reverse engineered. This process and the analysis of both the rotor and the volute design are described in detail in the sections below.

Figure 4.1: Hot map of the baseline turbocharger turbine, showing mass flow parameter and efficiency over a range of speed lines
4.3.1 Volute Design

4.3.1.1 Volute design fundamentals

The volute, or scroll, is the only inlet stator component of a nozzleless radial turbine. It is designed to generate a flow field around the inlet of the rotor, at the intended velocity triangles. To achieve this, the flow enters the volute in the tangential direction. A radial pressure gradient turns the flow towards the rotor while the swirl component distributes it around the periphery.

As described in Chapter 3, the flow in an ideal volute should form a free vortex:

\[ r C_\theta = K \]  

(4.1)

Continuity dictates that the mass flow through a cross section of the scroll at azimuth angle \( \psi \) is:

\[ \dot{m}_\psi = \rho_\psi A_\psi C_{\theta\psi} \]  

(4.2)

A uniform distribution of mass flow around the volute would mean that when \( \dot{m} \) is
the total mass flow rate,

$$\dot{m}_\psi = \dot{m} \left(1 - \frac{\psi}{2\pi}\right).$$  \hspace{1cm} (4.3)$$

Combining these equations lead to the assertion that

$$\frac{A_\psi}{r_\psi} = \frac{\dot{m}}{\rho_\psi K} \left(1 - \frac{\psi}{2\pi}\right)$$  \hspace{1cm} (4.4)$$

which means that for an incompressible flow with small variations in density, the ratio $A_\psi/r_\psi$ must be a linear function of azimuth angle (Whitfield & Baines 1990). This is a very commonly used guideline in the design of volutes.

The inlet $A_1/r_1$ ratio is determined by the overall design requirements for the turbine, and is one of the preliminary parameters in controlling the swallowing capacity of the turbine. A linear reduction with azimuth angle then determines the $A_\psi/r_\psi$ ratio around entire azimuth.

In terms of surface modelling, it would then make sense to model the volute of a radial turbine by its circumferential cross sections, as shown in Figure 4.3, with each section being determined by the required $A_\psi/r_\psi$. In general terms, the equation for the parametric scroll surface can then be written as:

$$x, y, z = f(\psi, u)$$  \hspace{1cm} (4.5)$$

where $u$ is the parametric position along the curve describing the section at the azimuth angle $\psi$. In essence this means that a volute will generally be modelled as a series of cross sections, swept around the azimuth. A simple parametric volute design will fully define the section as a function of one or more parameters, with $A_\psi/r_\psi$ being the driving parameter of a given cross section at $\psi$. This means the other parameters are set to values which fulfil the required $A_\psi/r_\psi$. 


4.3.1.2 Baseline Turbine Volute Analysis

As detailed geometric information for the turbine volute was required to create the parametric model, and also extract the basic geometric parameters for the meanline analysis, a Computed Tomography Scan (CT-Scan) was performed at the Imaging and Analysis Facility in the Natural History Museum in London. CT-scanning can be used to create a 3D mesh model of a specimen, through the layering of many X-ray projections. This is challenging for high density materials such as the cast iron of the baseline turbine volute, due to a phenomenon called beam hardening which creates artefacts in the image. This meant that a 3D faceted model of the surfaces could not be built from the CT-Scan data, and so another process was required to utilise the information from the scan.

The scan data was used to produce 100 circumferential slices of the volute, around the azimuth. Figure 4.4 shows 3 sections at different azimuth angles. The effect of beam hardening can clearly be seen affecting the quality of the images, in particular for the section at $\theta = 302.4^\circ$.

Additionally to the circumferential sections, 1170 images of axial sections were extracted from the scan data. The mid-span section is shown in Figure 4.5. This helped
(a) Positions of cross sections shown in Subfigures below

(b) Section at $\theta = 50.4^\circ$  
(c) Section at $\theta = 201.6^\circ$  
(d) Section at $\theta = 302.4^\circ$

Figure 4.4: Example circumferential CT-scan sections at 3 positions around the azimuth establish some design factors which were more difficult to extract from the circumferential sections. This included in particular, the tongue design (tongue radius and distance from rotor leading edge) and the inlet design from inlet flange to tongue. To best fit the baseline turbine performance, it was decided to replicate the inlet design. This was also to ease the potential future manufacture of a turbocharger prototype for engine testing as the inlet position would be in place.

Although the quality of the CT-scan data made it impossible to automatically create a 3D model of the volute directly, and the use of edge detection algorithms proved insufficient for creating cross sections of the volute for further analysis, the human eye can very clearly detect the edges, and so cross sections at 8 azimuthal positions were traced manually using Bezier curves. The resulting traces were filled with plain red colour
as shown in Figure 4.6, which subsequently allowed for calculation of the area and the centroid position using computational image processing in MATLAB.

The motivation for this treatment of the volute sections was twofold: Firstly it allowed for the calculation of $\frac{A_\psi}{r_\psi}$ ratio for each section, and subsequently establish the relationship of $\frac{A_\psi}{r_\psi}$ to azimuth angle $\psi$. Secondly, this approach facilitated the analysis of clean sections to help find a parametric logic in the cross-section design.

Figure 4.7 shows the aforementioned relationship of $\frac{A_\psi}{r_\psi}$ to azimuth angle $\psi$. This has been established as being a key relationship for the volute design, and a driving parameter for the recreation of the volute design. At the inlet, $A_1/r_1$ is one of the meanline geometric parameters, and therefore important to establish. Figure 4.7 clearly shows the line intercepting $\psi = 0$ at $A_\psi/r_\psi = 12$ mm.

As suggested in Section 4.3.1.1, the function of $A_\psi/r_\psi$ to azimuth angle $\psi$ is linear. It is to be expected that $A_\psi/r_\psi$ does not converge to zero as is evident. The reason for this are design constraints. As the volute is subjected to high temperatures, and because the tongue region is thinner than the surrounding area, thermal stresses can occur. This, coupled with high pressure pulses, can cause cracks in the tongue. To avoid this, the tongue radius, and the distance of the tongue from the rotor have a minimum value. The higher the distance from the wheel, the higher the value of $A_\psi/r_\psi$ before the tip of the tongue.
Figure 4.6: Manually traced circumferential sections at 8 positions around the azimuth
Figure 4.7: Relationship between $A_\psi/r_\psi$ and $\psi$ for the baseline volute, from the CT-Scan
4.3.1.3 Parametrisation of Volute Scroll Geometry

The objective of the parametric volute design was the generation of a volute geometry as a function of the meanline geometric input parameters. Only two of these parameters affect the volute, namely the $A_1/r_1$ ratio and the blade height $b_3$. For validation of the meanline model, and for basic turbine design, all other volute design parameters are subordinate, either as a function of $A_1/r_1$ and $b_3$ or predetermined and constant.

As shown in Section 4.3.1.1, the first step in generating a parametric volute design is the parametrisation of the cross section. No obvious parametrisation could be deduced from the sections of the baseline volute which suggests that the design may not have been parametrised at all, or underwent more complex optimisation. An approximation was however possible, and the final parametrised section for the volute scroll is described in Figure 4.8.

![Diagram showing the parametric design of the azimuthal volute cross-section. All parameters except for $r_b$ remain constant for all designs once defined.](image)

In total, the section can be fully defined with the 7 parameters $x_a$, $y_a$, $r_a$, $\beta_a$, $h_b$, $r_b$, and $r_c$. For a given volute design, the majority of these were dormant and fixed at the values relative to the baseline $r_2 = 19.25\text{mm}$ These values were found to be consistent.
with the baseline supplier volute. \( y_a \) is a function of blade height \( b_3 \), and \( r_b \) is varied around the azimuth to generate the scroll shape according to the given \( A_\psi/r_\psi \) trend. This trend is linear, going from the given \( A_1/r_1 \) of the volute at the inlet, to a fixed \( A_{\text{end}}/r_{\text{end}} \) at \( \psi = 360^\circ \). This allows the basis of volute scroll to be built using a swept surface through cross sections around the azimuth.

A complicated but critical procedure in volute design is the insertion of the volute tongue. In this design, the volute tongue was given a fixed radius and a distance from the inlet tip radius \( r_2 \). These two parameters were taken from the baseline volute, and measured \( r_{\text{tongue}} = 1 \) mm with the tongue being positioned 5 mm from the rotor tip radius. The shape of the tongue is cut out of the parametrised scroll, and subsequently filled with a tangent surface, as shown in Figure 4.9. The final scroll geometry is then fully defined as is seen in Figure 4.3.

![Graphic to show the trimming of the initial parameterised scroll surface, and subsequent insertion of the tongue](image)

(a) Parametrised scroll prior to tongue insertion  (b) Trimming of original scroll geometry  (c) Insertion of tongue geometry

The inlet was designed to fit with the flange of the baseline turbine, and connect tangent to the scroll surface. The final volute is shown in Figure 4.10.

4.3.2 Rotor Design
4.3.2.1 Rotor Design Background

The rotor of a turbocharger turbine is typically subjected to extreme conditions, running at very high speeds and high temperatures. This means that the structural integrity is paramount and a key consideration during design. One common method of minimising mechanical stress and the risk of catastrophic failure is the use of a radial fibre design. This condition means that an axial section of a blade will always be straight and radial, so that no torsion is generated and the centrifugal force runs along the length of the blade.

This simplification means that only the curves of hub and shroud, projected onto the meridional \((r - z)\) plane, and the camberline, both shown in Figure 4.11, are needed to fully define a zero-thickness blade.

For a differential blade element at point 1 with the coordinates \((z_1, r_1)\), the camber angle \(\phi\) is the angle of the blade element in its radial, so \((r_1\theta - z)\) projection. The equation for the camber angle is:

$$\tan (\phi_1) = r_1 \left( \frac{d\theta}{dz} \right)_1$$

(4.6)
The cone angle \( \psi_1 \) is the angle of the projection of the streamline going through point 1, on the meridional plane. For the aforementioned blade element, this can be given as:

\[
\cos \psi_1 = \frac{dz}{\sqrt{dr^2 + dz^2}}
\]  

(4.7)

The blade angle \( \beta_1 \) for the blade element can be described as the angle between the streamline and the streamline’s meridional projection which results in

\[
\tan \beta_1 = \frac{r_1 d\theta}{\sqrt{dr^2 + dz^2}}
\]  

(4.8)

Equations 4.6 and 4.7 can be substituted into Equation 4.8 yielding

\[
\tan \beta_1 = \tan \phi_1 \cos \psi_1
\]  

(4.9)

Blade angle is particularly relevant at inlet and outlet, because at these locations it
is often a defined parameter in the preliminary blade design. In the present work the blade angle at the exducer is one of the meanline input parameters being considered in optimisation, and therefore forms an input to the parametric turbine design. For a radial turbine design, the inlet blade angle will always be zero. Where the cone angle $\psi$ is zero, such as at the axial rotor exit, Equation 4.9 reduces to

$$\tan \beta_1 = \tan \phi_1 = \frac{r_1 d\theta}{dz}$$

(4.10)

The geometric representation of a real blade needs consideration of the blade thickness which is neglected in the above analysis. This can be achieved by applying a blade thickness $r \delta \theta$ as a function of $z$ and $r$, as shown for an axial blade section in Figure 4.12. The only remaining aspect to consider is the treatment of the trailing edge, which is explained in greater depth in Section 4.3.2.2.

![Figure 4.12: Axial section of a radial fibred blade, showing the blade thickness $r_1 \delta \theta_1$ at radius $r_1$](image)

A blade surface can consequently be described as a sweep of streamline blade sections as shown in Figure 4.13.
4.3.2.2 Parametrisation of Rotor Design

To design a parametric radial fibre rotor for CFD analysis and manufacture, the hub and shroud curves, the camberline, the axial blade section and the trailing edge needed to be parametrised. As for the volute, the objective was to generate a model which can ultimately be a function of the meanline geometric parameters. Again, this was done to give a good representation of the baseline rotor, shown in Figure 4.14, which required some analysis of the geometry. First it was established that the baseline rotor does indeed have a radial fibre blade design, as can be seen from 3 axial sections of the rotor in Figure 4.15

It was also found that the function for blade thickness was constant through the axis, making thickness only a function of radius $r$. Therefore the thickness of the blade is modelled using a linear reduction of the arc length between pressure and suction side of the blade at a given radius as shown in Figure 4.12 so that $\delta \theta$ at a given radius can be evaluated with the equation

$$\delta \theta = K_1 + \frac{K_2}{r}$$

(4.11)

where $K_1$ and $K_2$ are input parameters to give the desired thickness profile.

The camberline was extracted from the CAD model at the tip of the blade, and
parametrised as shown in Figure 4.16. It consists of 3 segments. The first, shown in orange, is a horizontal line with no change in $\theta$, so that the leading edge is vertical. The length of this segment is controlled by the parameter $z_{p1}$. The last section, shown in blue, is a straight line for the exducer angle. The gradient of this line forms an input to the camberline, and thus determines the exit angle of the rotor. The length of this segment is controlled by the parameter $z_{p3}$. The first and last segments are connected by a B-spline curve, shown in green, which has the three control points shown in red and is therefore tangent to both segments. To be tangent to the horizontal first segment, the middle control point is on the $z$-axis. Its position is controlled by the parameter $z_{p2}$.

The parametrisation of the hub and shroud lines is shown in Figure 4.17. Both the hub and the shroud line consist of a b-spline starting at the leading edge, and a straight vertical line from the b-spline to the trailing edge. Four control points are used for the b-splines, and one additional point to control the training edge lines. In total there are 10 points, which would mean 20 parameters to account for the whole geometry. However, as can be seen, several of the points are constrained either in the $r$ direction or the $z$ direction, which means that 13 parameters are needed to fully describe the hub and shroud lines.

The trailing edge of the rotor requires separate consideration as the pressure and suction side of the blade would otherwise connect with a sharp surface. There are different methods of achieving this, but generally it makes sense to draw the trailing edge on its
Figure 4.15: Axial sections of baseline rotor blades at 3 values of $z' = z/z_{\text{max}}$, shown superimposed on a single rotor blade in the lower image.

cylindrical projection $(r\theta - z)$ as shown in Figure 4.18. It was determined that the supplier baseline rotor uses a constant trailing edge radius of 0.2mm. It consists of an arc with said radius, which draws from the pressure side camberline of the blade to its tangent in the $r\theta$ direction. From there a b-spline is drawn to connect to the suction side camberline, which is cut back by the distance $d_{zt}$.

An important factor in the parametric turbine design is the response of the geometry to the meanline geometric parameters, which form the primary input to the turbine model. In this case the relevant parameters would be the value of $d\theta/dz$ at the exducer (driving the exit angle), $b_3$, $r_{3h}$ and $r_{3t}$.

The four parameters of the camberline needed to be reduced to being a function of just $d\theta/dz$. This was achieved by treating $z_{p1}$, $z_{p2}$ and $z_{p3}$ as fixed parameters. The input of $d\theta/dz$ then impacts the camberline accordingly, leading to a different $\theta$ at the exducer, commonly referred to as the wrap angle. Figure 4.19 shows the camberline for three different values of $d\theta/dz$, resulting in three values of $\beta_3$ at the average exducer radius $r_3$.

Similarly, the 13 free parameters of the hub and shroud lines were reduced to being
functions of $b_2$, $r_{3h}$ and $r_{3t}$ only. This was done by setting up proportional relationships between the b-spline control points. The parametric definition of the hub/shroud curves, camberline, axial section and trailing edge allowed the generation of a fully defined rotor blade. Using the CAD software, the blade was connected with the hub surface and a fillet drawn at the connection, a necessary feature for avoiding high stresses.
Figure 4.17: Parametrisation of the hub and shroud lines. Straight lines are black while b-splines are indicated by an orange line. The red points are the control points for the curves.

Figure 4.18: Diagram of the trailing edge in the \( r\theta - z \) projection

Figure 4.19: Camberlines for three values of \( \beta_3 \) at \( r_{3av} \)
4.4 Numerical Experiment

To help evaluate the predictive capability of the meanline model, and explore the effect of the geometric parameters on turbine performance, two numerical experiments were executed using two different sampling methods: Latin hypercube sampling with 450 samples and a full factorial containing 300 samples. For each sample, the geometry was generated using the parametric design approach described above, and meshing tools were used to generate a CFD setup to be run in ANSYS CFX. To make this generation of CFD setups feasible, an automated approach was developed for meshing, setting up, running and evaluating the CFD results.

4.4.1 Design of experiments

Before setting up a design of experiments, the geometric parameters to be used in the investigation needed to be selected. As the number of samples increases drastically with each additional parameter, it was decided to select three parameters which have a strong impact on turbine performance: $A/r$ ratio of the volute, the rotor exducer tip radius $r_{3t}$ and the exducer camberline gradient $d\theta/dz$ (determining the exit angle). The rotor tip radius $r_2$, which scales the full turbine stage, was found to scale the performance parameters with good accuracy within the range which would be set for an optimisation. This would make it a largely redundant parameter in the design of experiment.

The operating parameters were the reduced speed $N/\sqrt{T_{01}}$ and pressure ratio $PR$ of the turbine, as would be used in a conventional turbine map. Thus the designs of experiments where used to create a sample of the following function:

$$f \left( A/r, r_{3t}, d\theta/dz, N/\sqrt{T_{01}}, P_{01}/P_3 \right) \quad (4.12)$$

To enable later comparison with turbine testing on the turbine test facility at Imperial College, the turbine was scaled up by a factor of 1.974 for this investigation, giving a inlet tip diameter of 76mm. Nevertheless, the geometric parameters are given relative to the baseline size of 38.5mm.
4.4.1.1 Latin Hypercube Sample

The first design of experiment made use of the latin hypercube sampling method (Ye 1998) to generate a sample of 450 points with the three design and two operating parameters. Latin hypercube sampling is a statistical method for which the range of each parameter is split into as many equal discrete sectors as there are points in the sample, and one point is randomly placed in each sector, generating a vector of points. Subsequently, the order of each parameter vector is randomised giving an evenly distributed sample. This procedure guarantees that the full range of each parameter is evenly covered, making it more reliable than a simple random sample and more efficient than a full factorial design of experiments. The design parameter ranges for the sample are shown in Table 4.2.

Table 4.2: Geometric parameter bounds for the Latin Hypercube design of experiment

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{A_1}{r_1} )</td>
<td>mm</td>
<td>9</td>
<td>13</td>
</tr>
<tr>
<td>( \frac{d\theta}{dz} )</td>
<td>deg mm(^{-1})</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>( r_{3t} )</td>
<td>mm</td>
<td>15.15</td>
<td>18.15</td>
</tr>
</tbody>
</table>

As the peak efficiency point with respect to pressure ratio is strongly linked to the rotational speed, using an equal range of pressure ratio for the sample selection would not have yielded a desirable result with very low pressure ratios at high speed (relative to the design point) and very high pressure ratios at low speed. The parameter \( U/C_{ts} \) provides some help as the speed lines become very similar for different speeds. However, as seen in Figure 4.20, a linearly placed selection along \( U/C_{ts} \) would result in a disproportionately high number of points in the low power region of the map.

Additionally, a constant lower bound value of \( U/C_{ts} \) would cause either very high maximum pressure ratios at high speeds, or low maximum pressure ratios at low speeds, both resulting in a skewed sample.

To resolve the first issue, a parabolic function was used to skew the sample of \( U/C_{ts} \). Figure 4.21 shows a sample of 50 points from the speed line shown in Figure 4.20, both with a linear spacing of \( U/C_{ts} \) and with parabolic spacing. Although the parabolic \( U/C_{ts} \)
Figure 4.20: Turbine Power vs. $U/C_{is}$ for an example speed line. It is evident that an even spacing of $U/C_{is}$ samples would result in a disproportionately high number of low power points.

sample does not lead to a fully linear power sample, the power is more evenly distributed, and the high power region is well represented, therefore resulting in a suitable sample.

To ensure a reasonable pressure ratio range, the lower bound of $U/C_{is}$ was set to its value for a pressure ratio of 3.5 at a given speed, and thus made dependent on the speed of the sample point. The upper bound of $U/C_{is}$ was set to 0.8, just higher than the peak efficiency point at 0.7. This was done as the stability of CFD convergence decreased drastically for $U/C_{is}$ values higher than 0.8 as the gradient of both efficiency and mass flow rate against the pressure ratio becomes steep in this region and more complex flow structures and separations begin to dominate. The reduced rotational speed ranged from 1784 to 3318 RPM $K^{-0.5}$. The resulting operating conditions can be seen in Figure 4.22 which shows the sample on axes of Pressure ratio and reduced speed. The upper bound of $U/C_{is}=0.8$ can clearly shows the corresponding pressure ratio (effectively the pressure ratio lower bound) being dependent on reduced speed.

The intention of this design of experiments was to generate a dataset which gives an overview of the parametrised turbine performance, within the set geometric bounds. The dataset would make it possible to analyse the performance of the developed meanline model beyond the baseline design, as well as opening the possibility of training a neural network to represent the turbine performance.
Figure 4.21: Power for a sample of 50 points from a speed line, using a linear selection of $U/C_{ts}$ and a sample corrected by a parabolic function.

Figure 4.22: LHS sample of 450 points showing pressure ratio vs. reduced speed for each sample point.
4.4.1.2 Full Factorial Sample

To better separate the effect of design and operating parameters, a second design of experiment was set up using a full factorial approach. In terms of its usefulness, this type of dataset has higher likelihood of redundant information than the latin hypercube design, making it less efficient. As the same discrete values for each parameter are used throughout, gaps can appear making it less suitable for regression fitting or neural network training. However, it also provides a deeper insight into the interaction between variables and for the present study it provided the additional advantage of enabling consideration of each operating condition individually, allowing more conclusions to be made about the geometric extrapolation capability of the meanline model. Three operating points were selected on the same speed line (low power, peak efficiency, and high PR), as shown in Table 4.3.

Table 4.3: Operating conditions used for the full factorial design of experiment

<table>
<thead>
<tr>
<th>OP number</th>
<th>Reduced speed RPM K^{−0.5}</th>
<th>Pressure ratio</th>
<th>U/C_{is}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2773.5</td>
<td>1.4</td>
<td>0.81</td>
</tr>
<tr>
<td>2</td>
<td>2773.5</td>
<td>1.6</td>
<td>0.69</td>
</tr>
<tr>
<td>3</td>
<td>2773.5</td>
<td>2.7</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Each operating point was run with a given full factorial set of designs for which the design parameter ranges and number of discrete values are shown in Table 4.4. The total number of designs was consequently 100, making a dataset consisting of 300 points.

4.4.2 Numerical setup

All CFD calculations in this investigation were conducted in Ansys CFX (version 19.1), due to the large amount of experience held within the research group for this code, and because of its ease of use for turbomachinery applications. The ANSYS suite contains all of the tools required for meshing of the components, calculation on a high performance computing system and subsequent analysis. The turbine consisted of two primary com-
Table 4.4: Design parameter bounds for the full factorial design of experiment, showing the number of discreet values used for each parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Lower bound</th>
<th>Upper bound</th>
<th>No. Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{A_1}{r_1}$</td>
<td>mm</td>
<td>9</td>
<td>13</td>
<td>5</td>
</tr>
<tr>
<td>$\left(\frac{d\theta}{dz}\right)_3$</td>
<td>deg mm$^{-1}$</td>
<td>5</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>$r_{3at}$</td>
<td>mm</td>
<td>15.15</td>
<td>18.15</td>
<td>4</td>
</tr>
</tbody>
</table>

ponents, the volute and the rotor, which were treated as separate domains and meshed individually.

4.4.2.1 Volute Domain

The volute was meshed using ICEM CFD. An unstructured tetrahedral mesh was used, with prism layers at the walls to give control over the first element offset and thus enable a sufficient boundary layer mesh resolution. The mesh generation was automated using the scripting language native to ICEM. The process was integrated through CAESES, the software used for parametric model generation. For a given set of parameters, CAESES generated a geometry and exported it in the ICEM native geometry format (tetin), and subsequently executed the ICEM macro script. The meshing process involved the reading of the geometry file, generation a tetrahedral mesh, subsequent meshing of the prism layers at the walls, followed by automatic quality correction procedures in ICEM which ensured that the limit for minimum angle (14°) in the mesh was adhered to. The settings for first element offset and the growth law for the prism layers were input through the interface in CAESES. The final mesh was output in the CFX native mesh format, and stored. Figures 4.23 and 4.24 show the volute mesh and an axial section of it.
Figure 4.23: Volute mesh
Figure 4.24: Axial section of volute mesh, around mid-height of the rotor inlet.
4.4.2.2 Rotor Domain

The rotor was meshed using ANSYS TurboGrid, a meshing tool developed for turbomachinery. Within Turbogrid, the blade geometry is modelled using lines of the blade profile at several sections of the span from hub to shroud, as well as the hub and shroud lines. All lines are input through text files which are read by Turbogrid. As for the volute meshing, this process was integrated through CAESES. A macro in CAESES was used to extract the relevant hub, shroud and blade profiles, which were subsequently written to files. Also generated by CAESES was a macro script for Turbogrid with integration of the relevant parameters, including mesh refinement etc. Turbogrid generates a structured hexahedral mesh of the rotor domain.

The hub and shroud lines were extended to include the rotor exit domain. Separation of the rotor and rotor exit domain was done within Turbogrid at a fixed axial position, and a separate exit domain generated.

Figure 4.25: Rotor Mesh
4.4.2.3 Setup & Boundary Conditions

A full rotor model was used, as recommended for turbomachines with a volute component. This is because errors can occur in the calculation due to circumferential variation in the flow when the mixing plane boundary condition is used (necessary for single passage models). To avoid the computational cost of running a transient simulation, the frozen rotor interface was used between the volute and the rotor. This provides good accuracy and is preferred to the mixing plane interface which has been shown to behave poorly for devices with volutes (ANSYS Inc. 2018).

For the inlet of the volute, a total pressure and temperature boundary condition was used, while a static pressure boundary condition was applied to the rotor outlet. This meant that of the turbine performance parameters, pressure ratio and reduced speed was input to the CFD model, and the outlet pressure set to atmospheric (or the measured pressure when comparing to experimental data).

4.4.2.4 Turbulence & Boundary Layer Modelling

Turbulence modelling is one of the key assumptions in CFD. Due to the very small time and length scales present in turbulent flow, turbulence structures cannot be resolved directly within reasonable computational cost, and simplifying models are therefore used to account for turbulent effects. Several such models are available for selection in CFX. The model used in the present study was the SST turbulence model first proposed by Menter (1994). This is a widely used and robust two-equation eddy-viscosity turbulence model, and is commonly regarded as being well suited to highly separated flows (ANSYS Inc. 2018).

Modelling of the boundary layer was done using the scalable wall function available in CFX, as defined by Grotjans & Menter (1998). This is similar to the standard wall functions originally proposed by Launder & Spalding (1974) without the same limitations on near wall mesh spacing, so that the mesh can be arbitrarily refined for increasing accuracy (ANSYS Inc. 2018).
4.4.2.5 Mesh Sensitivity

To ensure reasonable accuracy of the CFD calculations, a mesh sensitivity study was conducted. The purpose of the study was to establish the sensitivity of the model to the number of elements in the mesh. This was controlled primarily by the first element offset, a higher resolution at the wall leading to an increase in the number of prism layers and therefore the total number of elements. In addition, the seed size of the initial tetrahedral mesh, as well as the element size at the surface boundaries of the geometry had an impact on the number of elements.

Similarly, in the rotor mesh, the number of elements was also controlled by the first element offset from the wall, set within Turbogrid. The total number of elements then results from the growth factor for each subsequent layer, which can be set to target the refinement of the mesh.

The mesh study was conducted by comparing the torque and mass flow of three different meshes at two operating conditions, one at peak efficiency and the other at high pressure ratio. The meshes were refined according to target values of $y^+$ with the meshes exhibiting values less than 30, 5 and 1 in order of increasing number of elements. The resulting mass flow and torque of the models is tabularised in Table 4.5.

<table>
<thead>
<tr>
<th>N. Elements</th>
<th>Mass flow rate (kgs$^{-1}$)</th>
<th>Torque (Nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OP1</td>
<td>OP2</td>
</tr>
<tr>
<td>3791106</td>
<td>0.2150</td>
<td>0.4921</td>
</tr>
<tr>
<td>5595586</td>
<td>0.2138</td>
<td>0.4927</td>
</tr>
<tr>
<td>12589492</td>
<td>0.2136</td>
<td>0.4919</td>
</tr>
</tbody>
</table>

The highest percentage difference with respect to the finest mesh was thus 0.7% in the torque (for the second mesh at high pressure ratio), and 0.6% in the mass flow rate (for the coarsest mesh at the peak efficiency point). The time taken up to convergence for the three meshes was 2, 5 and 13 hours respectively. For the intended application, the mesh sensitivity study showed that the coarsest mesh would give sufficiently accurate results, with the lowest computational cost. The meshing parameters used to generate
this setup were therefore applied for all geometries investigated in the present study.

4.4.2.6 Convergence

Two conditions were used to evaluate convergence of the models. The first was the RMS value of the momentum residuals which are output by CFX, which needed to be below $10^{-4}$ to be considered sufficient for prediction of global parameters. The residual plot of a typical run in CFX is shown in Figure 4.26.

The second criterion was the fluctuation of total-to-static efficiency, which was output as a monitor point. A maximum fluctuation of +/-0.2% was decided as the threshold for being considered sufficiently converged for the purpose of the investigation. Efficiency development for a typical run is shown in Figure 4.27. As can be seen, in this case the fluctuation is less than +/-0.05%.
Figure 4.26: Screenshot of the monitor of RMS residuals of mass, as well as momentum in the U, V and W direction, vs. the simulation timestep.

Figure 4.27: Screenshot of the monitor of total-to-static efficiency convergence, vs. the simulation timestep.
4.4.3 Results of latin hypercube sample

Figures 4.28 and 4.29 show the results of $\eta_{ts}$ vs $U/C_{is}$ and $MFP$ vs. $PR$, respectively. It can be seen that the parameter $U/C_{is}$ accounts for a very large part of the variation in efficiency. While the overall range in efficiency for the dataset is 0.45 to 0.81, the range at $U/C_{is} = 0.7$, the commonly used value for the design point of the turbine, is only around 0.73 to 0.81, showing that the operating condition has a much higher effect on the efficiency than the design. The exception to this are the low efficiency points indicated in the figures.

![Figure 4.28: Scatter plot of LHS results, showing $U/C_{is}$ vs. $\eta_{ts}$ for each sample point. Low efficiency points at high $U/C_{is}$ are indicated by red circles.](image)

As can be seen in Figure 4.28, there was a cluster of points (as indicated) in the LHS which showed an unusually low efficiency. A more detailed analysis of the CFD results for these points, showed that a large degree of secondary flows was present as seen in Figure 4.30. In the figure, Case 1 the $A/r$ ratio was 9.33mm and $r_{3t}$ was 17.21mm, while Case 2 had an $A/r$ ratio of 10.19mm and $r_{3t}$ of 15.52mm. Case 1 exhibits a clear passage vortex which causes a large amount of loss.

As can be seen in Figure 4.31 which shows the LHS points on axes of $A/r$ ratio and $r_{3t}$, this was related to a low value of $A/r$ ratio and a high value of $r_{3t}$. This meant that a corner of the cube of the variable $A/r$ ratio, $r_{3t}$ and $U/C_{is}$ was affected. Considering
Figure 4.29: Scatter plot of LHS results, showing $U/C_{is}$ vs. $eta_{ts}$ for each sample point. Low efficiency points at high $U/C_{is}$ are indicated by red circles.

Figure 4.30: Streamlines for two turbine designs at high $U/C_{is}$ operating conditions, showing a passage vortex in Case 1.

the same points in Figure 4.29 of the mass flow parameter, shows that these points have a low mass flow parameter.

As these effects were not included in the meanline code presented in Chapter 3, the points indicated in Figure 4.28 were excluded from the initial analysis of the meanline model in the sections below. The error that would occur due to these points is revisited in Section 4.5.2.1.
Figure 4.31: Scatter plot of LHS, showing $A/r$ vs. $r_{3t}$ for each sample point. The low efficiency points identified from Figure 4.28 at high $U/C_{ts}$ are indicated by red circles.

### 4.5 Meanline model validation

The three main outcomes of the analysis using both numerical datasets with regards to the meanline model can be summarised as follows:

1. Comparison of the loss models outlined in Chapter 3, showing which models give the lowest error when calibrated. The models which best fit the dataset were selected. The LHS dataset was used.

2. Analysis of the model prediction with regards to different input parameters using the LHS dataset.

3. Analysis of the models effectiveness in predicting the effect of turbine design changes, assessing its suitability as a low order optimisation tool, using the full factorial dataset.

#### 4.5.1 Loss model comparison

For five of the rotor loss categories described in Section 3.3, several models from literature are included in the code for selection. This primarily includes the passage loss (2 models), incidence loss (3 models), and tip clearance loss (5 models). As not all of the meanline
models in literature include a trailing edge loss (Suhrmann et al. 2010), this is considered as having two options, namely on and off.

A “brute-force” approach was used for the evaluation of loss models, meaning that the automatic calibration procedure was used to calibrate to the same dataset using each possible combination of loss models. This resulted in 48 different calibrated loss model combinations, as shown in the matrix in Table 4.6.
Table 4.6: Matrix of loss models used, numbered according to Tables 4.7 & 4.8

<table>
<thead>
<tr>
<th>Combination</th>
<th>Passage</th>
<th>Incidence</th>
<th>Tip clearance</th>
<th>Trailing edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>OFF</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>ON</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>OFF</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>ON</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>OFF</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>ON</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>OFF</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>ON</td>
</tr>
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<td>9</td>
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<tr>
<td>10</td>
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<tr>
<td>11</td>
<td>1</td>
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<tr>
<td>12</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>ON</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>2</td>
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<td>Loss No.</td>
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<td>Ref.</td>
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<tr>
<td>---------</td>
<td>----------</td>
<td>-----</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Passage</td>
<td>$L_p = K_p \left( \frac{w_2^2 \cos^2 i + w_3^2}{2} \right)$</td>
<td>Wasserbauer &amp; Glassman (1975)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Passage</td>
<td>$L_p = \frac{1}{2} K_p \left[ \left( \frac{L_H}{D_H} \right) + 0.68 \left( 1 - \left( \frac{r_3}{r_4} \right)^2 \right) \frac{\cos \beta_b}{b_3/l_c} \right] (W_2^2 + W_3^2)$</td>
<td>Baines (1998)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Incidence</td>
<td>$L_{inc} = \frac{w_2^2 \sin(i_2)^2}{2}$</td>
<td>Futral &amp; Wasserbauer (1965)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Incidence</td>
<td>$L_{inc} = \frac{K_{inc}}{2} w_2^2 (0.5 + i_2 - \pi/4)$ for $i_2 &gt; \pi/4$</td>
<td>Mizumachi et al. (1979)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Incidence</td>
<td>$L_{inc} = \frac{w_2^2 (1 - \cos(i_2) K_{inc})}{2}$</td>
<td>Meitner &amp; Glassman (1980)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4.8: Rotor Tip Clearance and Trailing Edge Loss Functions

<table>
<thead>
<tr>
<th>Loss No.</th>
<th>Equation</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\dot{m}<em>{cl} = \frac{1}{2} \rho U_t \varepsilon_b L N_k K_d$, $L</em>{cl} = \frac{1}{2} \left( \frac{\dot{m}_{cl}}{m} \right) U_t^2$</td>
<td>Spraker (1987)</td>
</tr>
<tr>
<td>2</td>
<td>$L_{cl} = \frac{U_t^3 N_k}{8 \pi} \left( K_x \varepsilon_x C_x + K_r \varepsilon_r C_r + K_{xr} \sqrt{\varepsilon_x \varepsilon_r} C_x C_r \right)$</td>
<td>Baines (1998)</td>
</tr>
<tr>
<td>3</td>
<td>$\Delta \eta_{t-s} = 0.1 \frac{\varepsilon_r}{r_3 t - r_3 b}$</td>
<td>Rodgers (1968)</td>
</tr>
<tr>
<td>4</td>
<td>$\Delta \eta_{t-s} = 2 \frac{\varepsilon_r}{b_2} \left( \frac{r_3}{r_2} - 0.275 \right)$</td>
<td>Krylov &amp; Spunde (1967)</td>
</tr>
<tr>
<td>5</td>
<td>$L_{cl} = \frac{\Delta h_{t-t} \varepsilon_r}{r_3 t - r_3 b}$</td>
<td>Glassman (1976)</td>
</tr>
</tbody>
</table>

Trailing Edge

$L_{te} = B \left( \frac{N_k t}{2 \pi r_3 \cos \beta_3} \right)^2 \frac{1}{2} w_4^2$

where: $B = \left( 1 + \frac{\gamma - 1}{2} (Ma')^2 \right)^{\gamma/(\gamma-1)}$ | Meroni et al. (2018), Glassman (1995) |
4.5.1.1 Calibration using baseline design data

Calibration was initially conducted using a dataset of 8 points from the baseline design. These were calculated using the same CFD procedure, at the operating conditions shown in Table 4.9. For visualisation, Figure 4.35 further below shows the calibration operating conditions on speed lines calculated using the meanline model.

Table 4.9: Operating conditions used for calibration of the meanline model

<table>
<thead>
<tr>
<th>OP number</th>
<th>Reduced speed</th>
<th>Pressure ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RPM K^{-0.5}</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2773</td>
<td>1.60</td>
</tr>
<tr>
<td>2</td>
<td>2773</td>
<td>2.70</td>
</tr>
<tr>
<td>3</td>
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<td>1.18</td>
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<td>1664</td>
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<tr>
<td>8</td>
<td>3328</td>
<td>3.00</td>
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</table>

The LHS dataset was used for comparing the meanline model to CFD, using the root mean square value of the error (RMSE) both of efficiency (in percentage points) and of the mass flow parameter, normalised (nRMSE) by the mean mass flow parameter of the LHS dataset.

Figure 4.32 shows the RMSE of efficiency $\eta_{ts}$ and nRMSE for each calibrated combination of loss models as numbered in Table 4.6.

The figure gives an indication of the overall influence of loss model selection, with the nRMSE of MFP varying from 0.89 to 1.58% while the RMSE of efficiency varied from 1.26 to 2.01 percentage points. There is therefore a clear motivation for careful model selection to achieve the highest possible accuracy.

To give a deeper insight into the effect of individual models, the chart in Figure 4.32 can be rearranged to show each model for a given loss next to each other, for all possible
combinations of the other loss models. This analysis is shown in Figures 4.33 and 4.34 for passage loss and tip clearance models respectively, showing a strong dependence on the selection of these two models.

Considering passage loss first (Figure 4.33), it can clearly be seen that in most cases the model by Wasserbauer & Glassman (1975) performs better in the prediction of efficiency than the model by Baines (1998), whereas in the prediction of $MFP$, the performance is more dependent on the selection of the other models. In particular, positions 1 & 2 in Figure 4.33 referring to model combinations 1-2 & 25-26 in Figure 4.33 show a better $MFP$ prediction with the passage loss model by Baines (1998), with model combination 25 showing the lowest overall $MFP$ error.

Similar analysis can be made for the tip clearance models (Figure 4.34), where it is
clearly visible that the models by Krylov & Spunde (1967) and Glassman (1976) perform worse in all cases for the prediction of $\eta$. Out of the other two models, the model by Spraker (1987) clearly performs best. This is also the case for the prediction of $MFP$, although two model combinations show the tip leakage model by Baines (1998) performing better (Both using the incidence loss model by Mizumachi et al. (1979), but different passage loss models.)

The two best performing model combinations for $MFP$ prediction were 25 & 26. These both used the passage loss model by Baines (1998), the tip clearance model by Spraker (1987), and the incidence loss model by Futral & Wasserbauer (1965), while 25 is without and 26 with the implementation of trailing edge loss. All both combinations
Figure 4.34: RMS error of $\eta_{ts}$ and $MFP$ for different model combinations showing the tip clearance loss models next to each other

perform well in terms of efficiency prediction. It was found however, that these combinations calibrated to a volute pressure loss coefficient of $K_{pl} = 0.19$ and 0.20 respectively, resulting in the volute loss being of the same order of magnitude as the rotor loss, which is contrary to observed results. The loss model combination with the best compromise in this respect was combination 2, with the same models as 26 except for the passage loss model which was by Wasserbauer & Glassman (1975) and a volute pressure loss of $K_{pl} = 0.16$ giving a more realistic division of losses. The loss models used are outlined in Table 4.10 along with the values used for the calibration parameters. The root mean square prediction error for this model selection was 1.00% for the mass flow parameter
Table 4.10: Summary of the models used for the most accurate combination of loss models, including values of the corresponding calibration coefficients.

<table>
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<tr>
<th>Component</th>
<th>Loss</th>
<th>Source</th>
<th>Equation</th>
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</thead>
<tbody>
<tr>
<td>Volute</td>
<td>Pressure</td>
<td>Japikse &amp; Baines (1997)</td>
<td>$K_{pl} = 0.16$</td>
</tr>
<tr>
<td></td>
<td>Swirl</td>
<td></td>
<td>$K_{sw} = 0.92$</td>
</tr>
<tr>
<td></td>
<td>Blockage</td>
<td></td>
<td>$Bl_1 = 0.95$</td>
</tr>
<tr>
<td>Rotor</td>
<td>Passage</td>
<td>Wasserbauer &amp; Glassman (1975)</td>
<td>$K_p = 0.2$</td>
</tr>
<tr>
<td></td>
<td>Incidence</td>
<td>Wasserbauer &amp; Glassman (1975)</td>
<td>$K_{inc} = 1.20$</td>
</tr>
<tr>
<td></td>
<td>Tip clearance</td>
<td>Spraker (1987)</td>
<td>$K_{cl} = 1.65$</td>
</tr>
<tr>
<td></td>
<td>Trailing edge</td>
<td>Glassman (1995)</td>
<td>n/a</td>
</tr>
<tr>
<td></td>
<td>Blockage</td>
<td>Japikse &amp; Baines (1997)</td>
<td>$Bl_2 = 0.91$</td>
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<tr>
<td></td>
<td></td>
<td>Hohenberg (Chapter 3)</td>
<td>$Bl_3 = 1.25$</td>
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and 1.33 percentage points for efficiency.

Figure 4.35 shows the prediction of the baseline design at the speedlines used for calibration. As can be seen, the fit of the meanline model to the calibration points is very close. The maximum efficiency deviation is 2.6 percentage points for the low pressure ratio point at the lowest speed. However, the gradient of efficiency to pressure ratio is very steep at this point, and the proximity of the point to the line is close. The peak efficiency point of the highest speed has an error of 0.78 percentage points.
4.5.1.2 Calibration with geometry variations

Further analysis was done to assess whether the models calibrate to a higher accuracy when data across multiple geometries in the design are used in the calibration process. As a large number of points was available from the LHS dataset, a random subset of 100 points was used for calibration. The dataset was used to calibrate the same model combinations shown in Table 4.6, and compared to the baseline calibrations, as shown in Figure 4.36. Which dataset is used for calibration has a striking effect on the quality of fit for mass flow parameter across most of the loss model combinations, while the efficiency is less strongly affected.

The difference can be attributed to the fact that a model which is calibrated to the baseline dataset can be overfit with the blockage factors, which can influence the mass flow parameter independently of efficiency. This reduces the physical correctness of the model, so that extrapolation to other designs leads to greater errors. Calibration to a dataset spanning the design space will thus lead to a worse fit for the baseline design, but an overall better fit to the parametrised turbine design concept.
Figure 4.36: RMS error of $\eta_{ts}$ and $MFP$ for different model combinations numbered according to Table 4.6, showing results for both the baseline and the LHS calibration dataset. $MFP$ error is normalised by the mean value of the CFD calculations from the LHS.

Interestingly, the model selected as giving the best fit on the basis of calibration using the baseline design (Combination 2), gives the best $MFP$ fit among the combinations using the tip clearance model by Spraker (1987) (therefore also giving a good fit for efficiency). The $MFP$ and efficiency RMS errors were 0.85% and 1.23 percentage points respectively (compared to 1.0% and 1.33 percentage points using the baseline calibration dataset). Table 4.11 shows the comparison of loss and blockage calibration parameters for the two calibration datasets. The only larger differences can be seen in the Swirl parameter $K_{sw}$ and the incidence loss parameter $K_{inc}$.

The difference in fit is relatively small and the conclusion of this analysis is that model combination 2, calibrated using the baseline design only as shown in Table 4.10 provides a good fit to the data, even when compared to calibration sets using a dataset which
spans the design space. This points towards a good physical representation of the models which were used, and comes at a much lower cost than when multiple designs need to be used for calibration. Therefore this calibration set was used for the remaining analysis in this section.

### 4.5.2 Parameter analysis

#### 4.5.2.1 Low Efficiency Points

Analysis of the LHS results showed that there was a drop in efficiency for points with low $A/r$ ratio, high $r_{3t}$ and high $U/C_{is}$ (Figures 4.28 and 4.31). This drop in efficiency is caused by an increase in secondary flow losses at these conditions. The right hand subfigure in Figure 4.37 shows the error in efficiency prediction by the meanline model, indicating the points which fall under the low efficiency criterion, also indicated in the left hand figure. As can be seen, the drop in efficiency is not predicted by the meanline model as it does not include the secondary flow losses which occur for certain designs at high $U/C_{is}$.

Baines (1998) suggests increasing the passage loss by a factor of 2.0 if

$$\frac{r_2 - r_{3t}}{r_{3t} - r_{3h}} < 0.2$$

(4.13)
Figure 4.37: Graphs showing the efficiency of all points in the LHS dataset on the left and the corresponding error in efficiency prediction on the right both vs. $U/C_{is}$. The points with high error are indicated by red circles.

referring to a secondary loss due to separations caused by small radii of curvature at the the shroud line. However, as seen in Figure 4.38 showing the LHS points with $U/C_{is} > 0.7$, the position of the affected points is not only defined by high $r_{3t}$ bus also by low $A/r$ ratio, forming a triangular region.

The region must be defined in terms of the input parameters, to either enable the meanline model to model them explicitly, or to define a boundary outside of which the model is to be considered less accurate. As can be seen in Figure 4.38, these points can be captured by defining the region as being bound by the following set of inequalities:

$$
\begin{align*}
\frac{A/r - (A/r)_{min}}{11.5 - (A/r)_{min}} + \frac{r_{3t} - r_{3t,\text{max}}}{15.5 - r_{3t,\text{max}}} & \leq 1 \\
U/C_{is} & \geq 0.7
\end{align*}
$$

(4.14)

Six points from the LHS dataset which do not exhibit the secondary flow losses are included in this region. This is illustrated in Figure 4.39 which shows the selected points, clearly showing that six points are selected which have an error of less than 3%.

Increasing the passage loss by a factor of 2.0 as suggested by Baines (1998) only has a small effect on the efficiency, lowering the error by less than 2 percentage points. However,
Figure 4.38: Scatter plot of LHS, showing $A/r$ vs. $r_{3t}$ for each sample point with $U/C_{ls} > 0.7$. The low efficiency points identified from Figure 4.37 are indicated by red circles, as well as the region bound by the inequalities shown in Equation 4.14.

much larger multipliers result in a potentially unphysical result, in addition to adding losses to designs which don’t exhibit the secondary flow behaviour. It was found that a value of 2.5 for the passage loss multiplier in the region defined by the set of inequalities in Equation 4.14 gave a good overall result.

The resulting error is reduced from a maximum of 11% to 7.3%, as can be seen in Figure 4.40, However, the points which are erroneously captured by the region have a negative error of up to -5.6% as a result. Although this approach was helpful in reducing the large error seen in some of the points, the region in question remains difficult to model, as the effects are beyond the capacity of the meanline model. Therefore care needs to be taken when working with designs which have high $r_{3t}$ and low $A/r$ ratio.
Figure 4.39: Scatter plot of LHS, showing $\Delta \eta_{t-s}$ vs. $U/C_{is}$ for each sample point. The points excluded by the set of inequalities in Equation 4.14 are indicated by red circles.

Figure 4.40: Scatter plot of LHS results, showing the meanline model efficiency error $\Delta \eta_{ts}$ vs. $U/C_{is}$ for each sample point. The model including the correction factor to account for secondary flow losses is shown.
4.5.2.2 Regression Analysis of Error

One of the questions to be answered was how well the meanline model responds to the different input parameters which were included in the designs of experiments. These can be represented by the function reiterated here:

\[ f \left( \frac{A}{r}, r_{3t}, d\theta/dz, \frac{N}{\sqrt{T_{01}}}, \frac{P_{01}}{P_3} \right) \]  

(4.15)

The LHS dataset provided an effective means of doing this, as it allowed a multiple linear regression analysis of the error to be performed. The following function was fit to the efficiency error using the least squares method:

\[ \Delta \eta_{ts} = A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4 + A_5x_5 + A_6 \]  

(4.16)

where \( x_1 - x_5 \) are \( A/r \) ratio, \( r_{3t} \) and \( d\theta/dz \), \( N_s \) and \( U/C_{is} \) respectively, while \( \Delta \eta_{ts} \) is the error of the meanline prediction of \( \eta_{ts} \) with respect to the CFD results from the LHS (in percentage points). The coefficients \( A_1 - A_6 \) are fit using an algorithm and can be compared to assess the importance of each parameter. It should be noted that the low efficiency points analysed in the previous section were excluded in the regression analysis.

This basic analysis of the contribution of each geometric parameter to the error in the model can be summarised with Figure 4.41, which shows scatter plots of the meanline prediction error compared to the CFD results from the LHS dataset of efficiency \( \eta_{ts} \) vs. each geometric parameter. The result of the linear regression is shown by a red line, which represents the variation due to the given parameter while the other parameters are fixed at their mean value.

As can be seen in the scatter plots, all of the parameters present a trend which is captured by the multiple linear regression fit, however the variable which can account for the most error in the model is \( d\theta/dz \). Table 4.12 shows the coefficients \( A_i \) of each variable in the multiple linear regression fit, along with the normalised coefficients \( A_i(x_{i,max} - x_{i,min}) \) which can be used to compare the impact of each parameter to the error in the dataset.

This confirms that \( d\theta/dz \) has the highest impact, accounting for a difference of 2.56
percentage points error between the upper and lower bound values. By comparison, the
other two geometric parameters and the operating parameters show a lower significance.
The trend with respect to $U/C_{is}$ is primarily due to an over-prediction of efficiency by the
meanline model at high $U/C_{is}$. Interestingly, the points exhibiting errors higher than 2%
nearly all have an $A/r$ ratio of less than 10.5, showing that a combination of low power
and low $A/r$ is modelled less accurately.

Table 4.12: Multiple regression analysis coefficients for $\Delta \eta_{ts}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Coefficient A</th>
<th>Normalised coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A/r$</td>
<td>mm</td>
<td>-0.340</td>
<td>-1.36</td>
</tr>
<tr>
<td>$r_{3\mu}$</td>
<td>mm</td>
<td>-0.283</td>
<td>-0.85</td>
</tr>
<tr>
<td>$d\theta/dz$</td>
<td>° mm$^{-1}$</td>
<td>0.642</td>
<td>2.57</td>
</tr>
<tr>
<td>$N_{red}$</td>
<td>RPM K$^{-0.5}$</td>
<td>-6.76E-04</td>
<td>-1.04</td>
</tr>
<tr>
<td>$U/C_{is}$</td>
<td>-</td>
<td>3.23</td>
<td>1.62</td>
</tr>
</tbody>
</table>

The trend correlating with changes in $d\theta/dz$ (i.e. the exit flow angle) was observable
to a similar degree for all loss model combinations outlined in Section 4.5.1. This suggests
that some physical effects in the flow, which influence the efficiency are not being
considered. The trends show that the meanline model over-predicts the efficiency at high
$d\theta/dz$ (which is equivalent to a high exit angle at a given span radius). For the blade
passage, this means a higher curvature and higher blade loading. This would have a
twofold effect: Higher passage losses as the flow is turned more significantly, and higher
losses stemming from the tip clearance due to the higher blade loading, although both are
difficult to quantify well. A correction of this would require more detailed description
of the phenomena.

Correcting the error for the $d\theta/dz$ trend and plotting the resulting points vs. $U/C_{is}$ as
seen in Figure 4.42, shows that at $U/C_{is} < 0.6$, the error is reduced, while at $U/C_{is} > 0.6$
the correction causes an increase in error. This shows that the trend with respect to
$d\theta/dz$ is dependent on operating points, and does not affect the low power region of the
map.
Figure 4.41: Scatter plots of efficiency prediction error $\Delta \eta_{ts}$ with respect to each of the varied input parameters of the dataset.
Figure 4.42: Scatter plot of LHS results, showing the meanline model efficiency error $\Delta \eta_{ls}$ vs. $U/C_{is}$ for each sample point. The black points are corrected for the error caused by $d\theta/dz$. 
Conducting the same analysis with the mass flow prediction also yields trends although these are less significant as seen in Table 4.13 and Figure 4.43. The $MFP$ prediction error $\varepsilon_{MFP}$ is determined from the following equation:

$$\varepsilon_{MFP} = \frac{MFP_{\text{meanline}} - MFP_{\text{CFD}}}{MFP_{\text{CFD}}}$$  \hspace{1cm} (4.17)

Table 4.13: Multiple regression analysis coefficients for $\varepsilon_{MFP}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Coefficient A</th>
<th>Normalised coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A/r$</td>
<td>mm</td>
<td>-0.37</td>
<td>-1.51</td>
</tr>
<tr>
<td>$r_{3t}$</td>
<td>mm</td>
<td>-0.28</td>
<td>-0.85</td>
</tr>
<tr>
<td>$d\theta/dz$</td>
<td>° mm$^{-1}$</td>
<td>0.36</td>
<td>1.43</td>
</tr>
<tr>
<td>$N_{\text{red}}$</td>
<td>RPM K$^{-0.5}$</td>
<td>1.04E-04</td>
<td>0.16</td>
</tr>
<tr>
<td>$PR$</td>
<td>-</td>
<td>-0.51</td>
<td>-1.21</td>
</tr>
</tbody>
</table>

With a total of 1.51% across the range being attributable to the $A/r$ ratio, it is the parameter which correlates with the most error. Additionally, a trend with $d\theta/dz$ is visible. $PR$ has a negative trend in error with increasing value, however closer examination of the scatter plot shows that the cause of this is mostly a negative error at pressure ratios higher than 2.6 which points towards an under-prediction of choking mass flow.

Overall, the analysis shows that most of the error can not be attributed to the any of the parameters investigated, and is more likely to have a more complex source. It is however not very high in the first place, so that further investigation would not yield much improvement.
Figure 4.43: Scatter plots of MFP prediction error $\varepsilon_{MFP}$ with respect to each of the varied input parameters of the dataset.
4.5.3 Extrapolation of design

To further analyse the model’s ability to extrapolate to different designs, the full factorial dataset was used as it enabled the comparison of different designs at the same operating conditions. This meant that the meanline prediction error could be shown in the context of the overall change in performance due to the design parameter changes.

Figure 4.44 shows the mass flow parameter normalised by the meanline baseline value for each pressure ratio, as predicted by CFD on the abscissa and by the meanline model on the ordinate.

The operating points are compared on the 50% speed line (2773.5 RPM K−0.5) at pressure ratios of 1.4, 1.6 and 2.7.

![Figure 4.44: Points from the full factorial DOE, showing the mass flow parameter predicted by the meanline model vs. CFD, both normalised by the meanline baseline prediction.](image)

It can be seen that the mass flow parameter is well predicted, with all except 3 points at $PR = 1.6$ (lowest mass flow points) having an error of less than +/- 3%. The mass flow parameter meanline prediction error has a trend, with an over-prediction at lower mass flow designs and an under-prediction at higher mass flow designs. This is in line with
the scatter plots analysed in Figure 4.43, which show that the trends in mass flow error with all three geometry parameters, have a negative gradient with increasing swallowing capacity. Although the error is small with respect to the individual parameters, it is compounded towards the extremes of swallowing capacity. Nevertheless, the overall error in meanline prediction with respect to the CFD model is small.

The same analysis was done for the efficiency as shown in Figure 4.45 of the comparison for the two points higher pressure ratios ($PR = 1.6 \text{ & } 2.7$) only. Although the error in prediction is of a similar order to the mass flow parameter, the change in efficiency is small between operating points, making the correlation less strong. The error of the high $PR$ point is less than +/-3% for all but two points. These are both under-predicted and situated at the extremes of the sampling space, with $r_{3t} = 18.15$ and $d\theta/dz = 5$. Similarly, five points fall have an error higher than 3% at $PR = 1.6$. These all have $r_{3t} = 18.15$ and $A/r = 9$, therefore at the highest exducer tip radius and lowest $A/r$. Although the secondary flow correction has been set to correct the losses of points with a $U/C_{is} > 0.7$, the point with $PR = 1.6$ has a value of $U/C_{is} = 0.69$ and is therefore already affected by this increase in loss, hence an over-prediction by the meanline model.

![Figure 4.45: Meanline $\eta_{t-s}$ vs. CFD $\eta_{t-s}$ for each point at two operating conditions.](image)

The low pressure ratio point ($PR = 1.4$) showed much larger errors, as seen in Figure
4.46. Here the secondary flows discussed in Section 4.5.2.1 become important, as $U/C_{is} = 0.848$, well above the 0.7 limit set for the secondary flow region.

Figure 4.46: Meanline $\Delta \eta_{t-s}$ vs. CFD $\Delta \eta_{t-s}$ for each point at low pressure ratio (PR=1.4). $\Delta \eta_{t-s}$ refers to the difference from the baseline design.

The figure shows the meanline correction with and without correction for secondary flows. As can be seen, the CFD results show a significant reduction in efficiency for some points, which the model is only partially able to capture. The correction however does improve the prediction, bringing a large portion of the points within the +/- 5% error band. For the most extreme cases however, even the corrected model has an error of up to 11%, so caution is advised if this part of the map is of high importance, which will often not be the case as the majority of energy conversion occurs at higher pressure ratios during an engine pulse.

To put the two parameters $MFP$ and $\eta_{t-s}$ into perspective, it was helpful to perform the same analysis for turbine power $\dot{W}$. This parameter is key in modelling the turbine within an engine, as it determines the the power delivered to the compressor and thus the boost of the engine. The importance of the efficiency on its own is limited to the prediction of the exhaust temperature, which can be important for the simulation of exhaust emissions, but is not the main focus of matching calculations.
Figure 4.47 shows the prediction of power change with respect to the baseline, as calculated by the meanline model vs. CFD. As can be seen, the trend is strong, and the meanline model is able to well represent changes in geometry. With the exception of a cluster of points with $PR = 1.6$ and low power, the maximum prediction error for the two higher $PR$ points is 5.5%. The points with a higher error than this had an $A/r$ ratio of 9, and $r_{3t}$ above 16, where under-prediction of secondary flow losses can occur. Several points at $PR = 1.4$ have a higher error, due to the efficiency prediction errors outlined above. As already mentioned, analysis of the instantaneous turbine operating conditions during an engine cycle will determine whether this is important to consider or otherwise.

![Figure 4.47: Normalised power predicted by the meanline model vs. normalised power predicted by CFD, for all designs points in the full factorial DOE at three pressure ratios.](image)

4.5.4 Predicted distribution of losses in turbine

An interesting application of meanline modelling is the ability to visualise the impact of different losses with respect to various parameters. The most common way of showing this was first presented by Rohlik (1968) who plotted the “distribution of available energy based on total-to-static pressure ratio”, i.e. the distribution of losses above the total to static efficiency at peak efficiency points against the specific speed.
The total to static efficiency is repeated here as part of the explanation of the methodology used for generating the plots:

\[ \eta_{ts} = \frac{T_{01} - T_{03}}{T_{01} - T_{3s}} \]  

(4.18)

The losses can be divided into the kinetic energy loss and the sum of enthalpy losses as follows:

\[ T_{03} = T_{3s} - \frac{1}{C_p} \sum L - \frac{C_3^2}{2 * C_p} \]  

(4.19)

Where \( L \) refers to the enthalpy loss due to each of the losses calculated in the meanline model. The losses in the rotor are given as enthalpy losses, but to be able to relate the volute pressure loss to the above equation, the increase in entropy across the volute and the increase in entropy due to each of the rotor losses is calculated. Assuming a linear isobar in the T-s diagram, equivalent enthalpy losses can be calculated:

\[ L = \frac{T_3 - T_{3s}}{s_3 - s_1} \Delta s_L \]  

(4.20)

where \( \Delta s_L \) refers to the entropy increase calculated from a given loss function. Subsequently, all of the losses and the kinetic energy leading to the total-to-static efficiency \( \eta_{ts} \) can be divided and plotted as shown in Figures 4.48 to 4.51.

As can be seen in Figure 4.48 efficiency decreases with increasing \( A/r \) ratio, primarily due to an increase in passage loss and exit kinetic energy. Both can be attributed to the higher velocities as a consequence of higher mass flow rate. The incidence loss counteracts this trend somewhat, as \( A/r \) has a major impact on volute exit angle \( \alpha_2 \), impacting the incidence to the rotor.

Increasing the exducer blade tip radius \( r_{3t} \) (Figure 4.49) leads to an increase in efficiency, which is primarily thanks to a decrease in exit kinetic energy. This is due to the increased average exducer blade velocity leading to a higher (more axial) absolute exit flow angle \( \alpha_3 \) and therefore decreased exit velocity in the absolute frame of reference. This trend would be reversed at low power (i.e. \( U/C_{is} > 0.7 \)) where \( \alpha_3 \) is positive, so that an increase in its value leads to an increase in exit velocity. The increase in mass
flow rate due to a larger exducer area at higher $r_{3t}$ leads to a higher mass flow rate at the same pressure ratio. Analysis of the rotor inlet velocity triangle shows that the resulting increase in $C_2$ will lead to a higher incidence, hence there is an increase in incidence loss.

Finally, Figure 4.50 shows that increasing $d\theta/dz$ leads to a lower incidence loss due to the reduction in mass flow rate. The increased degree of reaction leads to a higher exit velocity, which manifests itself in slightly higher passage and kinetic energy losses. Overall, the efficiency increases although this flattens for higher values of $d\theta/dz$. The above error analysis showed a trend in efficiency prediction, the cause of which would change the trends shown in Figure 4.50 due to sources of loss which are not captured by the meanline model.

Figure 4.51 shows the division of losses with respect to operating condition for the baseline design. As expected, the largest loss is the passage loss, except at lower power where the clearance and incidence losses gain in importance. The incidence loss is lowest at $U/C_{ts} = 0.77$, the point where the incidence angle is optimal, and increases to both sides of this point. The modelled clearance leakage mass flow rate is mostly a function of blade tip speed, meaning that for a constant speed, the specific enthalpy loss will increase as the power decreases. The kinetic energy loss is very dependent on the exit angle in the absolute frame of reference, and therefore peaks just above $U/C_{ts} = 0.7$ where the circumferential component of exit velocity, i.e. the exit swirl, is zero.
Figure 4.48: Division of meanline model losses with respect to $A/r$ ratio

Figure 4.49: Division of meanline model losses with respect to $r_{3t}$
Figure 4.50: Division of meanline model losses with respect to $d\theta/dz$

Figure 4.51: Division of meanline model losses with respect to $U/C_{ls}$
4.6 Summary & Conclusions

In this chapter the evaluation of a meanline model by comparison with CFD model is presented. The primary purpose was to establish the feasibility of using such a model as a means of low order optimisation of turbine design for turbocharged engine performance. This was achieved through the use of a 3D parametric turbine model which was based on the manufacturer’s turbocharger of the engine used in this work. The parametric model used the meanline model geometric input parameters to generate geometries, and thus it was possible to conduct a parametric study of the turbine design space. By means of designs of experiments, three key turbine design parameters ($A/r$ ratio, exducer tip radius $r_{3t}$ and exducer camberline gradient $d\theta/dz$) and the two operating parameters (reduced speed and pressure ratio) were sampled and numerically investigated using CFD.

The first design of experiments used the latin hypercube sampling method for both the geometric and operating parameters, generating a dataset of 450 points. This allowed the overall accuracy of the model (both in terms of efficiency and mass flow parameter) to be analysed, including the possible choices of loss models from literature, and the best set of models was suggested. In addition, it gave insight into secondary flows occurring at low turbine power, leading to significantly increased losses for designs with a high exducer tip radius $r_{3t}$ and a low $A/r$ ratio. These secondary flows are not modelled by the meanline model, and analysis of the numerical results enabled on the one hand the definition of boundaries within which the meanline model performs well, and on the other, the inclusion of additional losses outside of these boundaries to reduce the error.

While comparison to the CFD results within these boundaries showed a maximum efficiency error of around +/- 3 percentage points, analysis of the geometric parameters revealed that a large part of this error (+/- 1.6 percentage points) could be attributed to the parameter defining the exit angle, $d\theta/dz$. It was also found that in the low power region, for $U/C_{1s}$ values higher than 0.7, the performance was more difficult to predict.

A second design of experiments was conducted using a full factorial approach so that the same operating conditions were investigated for each design. This meant that ability of the model to predict design changes could be analysed, showing that at two pressure ratios higher than the design point, changes in mass flow parameter predicted by the
meanline model were very well correlated to those predicted by CFD, while the efficiency correlation was less strong, although the errors were of the same order of magnitude. This was primarily because the efficiency changes between two designs were lower at around +/- 7% than mass flow changes (+/-30%), making them more difficult to predict well. Translating the efficiency and mass flow parameter to turbine power, showed that the comparison between geometries is well represented by the meanline model, giving an error of less than 5.5% at pressure ratios higher than the peak efficiency point.

A low pressure ratio point was analysed, showing that due to higher complexities in the flow physics at this operating point, the prediction of the meanline model was less good. This needs to be accounted for when the model is used as an optimisation tool, as its feasibility will depend on the operating points covered. During an engine pulse, a large range of power can be covered, and low power errors will have a lower relative importance than high power errors.
Chapter 5

Testing of Selected Turbine Designs

5.1 Overview

In this chapter a testing campaign of turbine designs is reported. The chapter is structured as follows:

1. **Description of the Testing Facility:** The radial turbine testing facility at Imperial College is described in detail, showing the setup, instrumentation and calculations required for generating maps of the turbine performance parameters. Analysis of the uncertainty in the measurements is included.

2. **Testing of the turbine designs:** The designs of the tested turbines are described, along with the methods used for manufacture. Additionally, the operating conditions tested are shown and the results presented.

3. **Analysis of results:** The results of the testing are analysed with respect to the meanline model and CFD. Novel insights into the accuracy of the meanline model given various datasets for calibration are presented, as well as an overall experimental validation of the model. Comparison to CFD showed the requirement of including wall roughness for accurate representation of the tested designs, and this is described in detail.
5.2 Introduction

In this chapter an experimental campaign of different turbine designs using the turbine performance facility at Imperial College is presented. Although of interest on their own, the results of the campaign were used to further validate the meanline modelling approach presented in Chapter 3. Furthermore, the CFD methods used in Chapter 4 could be validated, ensuring that trends due to design changes were captured correctly and therefore a proper assessment of the meanline model could be made.

A testing matrix of six turbine designs was set up, varying the parameters $A/r$, $d\theta/dz$ and $r_3t$. The 3D parametric CAD model introduced in Chapter 4 was implemented to generate manufacturable designs of both the volute and the rotor of the turbine, so that these could interface easily with the existing facility. Four volutes were rapid prototyped using SLS, while the three rotors were manufactured out of aluminium using a 5-axis CNC machine. Thanks to interchangeability of components, this resulted in six turbine designs which were tested, so that maps covering a wide range of operation were generated.

The results were subsequently used to analyse the meanline model prediction. The first part of this analysis investigated the sensitivity of the self-calibration procedure in the code to the operating conditions and number of input calibration points. Comparison to experimental results for the baseline design allowed an evaluation of the meanline model extrapolation quality to be made, providing a novel and useful insight into the impact of input data range on the model calibration. The second part of the analysis applied the baseline calibrated meanline model to the other designs which were tested, to determine the ability of the model to extrapolate to different designs.

Finally a comparison was made with the CFD setup used in Chapter 4. CFD was used to predict the performance of the tested turbine designs at selected operating conditions of the experimental data. This includes a discussion on the need to include surface roughness in the CFD calculations to match the experimental data.
5.3 Testing Facility

The testing facility used for the acquisition of data was the fully equipped turbine dynamometer ("turbo-rig") at Imperial College London. The origins of the facility date back to the work by Dale & Watson (1986) but significant changes were made by Szymko (2006) who fitted the rig with an eddy current dynamometer, removing the operating restrictions dictated by the more conventional method of loading with a compressor and therefore enabling the measurement of torque and speed for a wider range of operation. A unique feature of the test rig, is that it is a cold flow facility, so that adiabatic conditions can be assumed, and true aerodynamic efficiency measured.

As already described in greater detail in Section 3.2, the performance of a turbocharger radial turbine is commonly reduced to four parameters:

$$\eta_{ts}, \frac{m\sqrt{T_{01}}}{P_{01}d_1} = f \left( \frac{Nd_1}{\sqrt{T_{01}}}, \frac{P_{01}}{P_4} \right)$$

(5.1)

which are generally formatted as maps of constant speed lines, defining efficiency and mass flow parameter as a function of pressure ratio. A commonly used dimensionless parameter for the analysis of turbocharger turbine performance is the turbine blade speed ratio:

$$\frac{U}{C_{is}} = f \left( \frac{Nd_1}{\sqrt{T_{01}}}, \frac{P_{01}}{P_4} \right)$$

(5.2)

Efficiency is often plotted against the blade speed ratio as the speed lines fall on top of each other, the peak efficiency of most radial turbines falling at a blade speed ratio $U/C_{is} = 0.7$. A disadvantage of this representation is the disproportionate visual importance given to the low power region of the map, which is inflated due to the functional relationship between blade speed ratio and pressure ratio.

The use of non-dimensional parameters to describe turbine performance, allows cold flow testing to be conducted with the principal of similitude applied to model the corresponding hot conditions. This reduces the complexity of the testing procedure as the high temperatures under engine conditions are not encountered. For the present work, it also allowed the use of 3D printed polymer components.
Shown below, is the setup of the turbine testing facility used to experimentally establish maps of the turbine performance parameters.

### 5.3.1 Test rig layout

The layout of the test rig is shown in Figure 5.1. Air is supplied by 3 centrally housed screw compressors at a pressure of 4 bar, and a maximum mass flow rate of 1.2 kg/s. Though the test rig is a cold flow facility, after entering the lab the air passes through a 72kW electric heater so that the temperature of the air can controlled to enter the turbine at 330K-350K. This is to avoid condensation at the rotor outlet, which would occur at lower temperatures.

Following the heating stack, the air flows through the main control valves, a 4 inch
electrically actuated ball valve in parallel with a 1 inch bypass valve, also electrically actuated which can be used for finer control control of the flow. To allow testing of twin and double entry turbines, the flow is split into two separate limbs. Each has an additional control valve to allow partial admission testing, though these were kept fully open at all times for the work presented here. For the measurement of mass flow rate, two V-cone differential pressure type flow meters are placed into the flow, one for each limb. These are used due to their low sensitivity to upstream flow disturbances when compared to orifice plate systems.

After measurement of the mass flow, the air passes through the measuring plane, where instrumentation for pressure and temperature measurements is positioned. As for the mass flow rate, these measurements are taken in each limb, before the flow is passed through a conversion duct which unites the two flows when a single entry setup is used. The turbine inlet flange attaches to this conversion duct, and the flow is led through the turbine. An exit duct is placed at the outlet of the turbine to enable measurement of the exit static pressure, after which the flow exits to atmosphere.

The turbine rotor shaft is connected to the high speed eddy dynamometer system. Attached to the shaft is a magnetic rotor, which loads the turbine through the magnetic forces caused by induction of eddy currents in two adjacent metallic stator plates, as shown in Figure 5.2.

![Figure 5.2: Turbine rotor attached to shaft, showing the magnetic rotor and adjacent stator plates (from Szymko (2006))](image)
The load on the turbine is controlled by the distance of stator plates from the magnetic rotor, which is set by two high precision stepper motors. The volute is mounted onto the dynamometer. The entire system is gimballed, allowing the measurement of torque directly behind the turbine wheel through the reaction force on a load cell, as seen in Figure 5.3.

![Figure 5.3: Schematic of the dynamometer gimbal system, showing the load cell connection (from Szymko (2006))](image)

As the bearings are positioned within the gimballed system, the dynamometer can measure the true aerodynamic performance of the turbine, accounting for bearing friction. The speed is measured by a 20-toothed encoder attached to the turbine shaft which is read by an optical sensor. Through the measurement of speed and torque, the turbine power can be evaluated. The dynamometer allows turbine speeds of up to 60 000 RPM and can dissipate a maximum power of 60kW. A velomitor is mounted to the side of the dynamometer to monitor vibrations. A fast fourier transform plot is available to the operator of the dynamometer, showing whether vibrations are within an acceptable limit.
5.3.2 Instrumentation

5.3.2.1 Pressure

The pressure measurements at the inlet and outlet of the turbine were taken using wall tappings connected by a pressure line to a 24 channel Scanivalve system. Two high accuracy strain gauge pressure sensors are used, a Druck PDCR 22 for low pressure measurements (<350 mbar) and a Druck PDCR 23D for higher pressures (<3.5 bar). A valve system exposes the pressure sensors to each channel of the system sequentially allowing 24 different pressures to be measured. Each channel has two ports so that differential pressure can be measured, or one port left open to atmosphere for gauge pressure measurement. To derive absolute pressure, the atmospheric pressure of the test cell was measured using a dial barometer. A LabView interface program was used with Transmission Control Protocol connection to log pressure data.

The pressure transducers in the Scanivalve were calibrated using a Druck DPI 610 portable pressure calibrator, with pressure correlating linearly to the transducer voltage reading as shown in Figure 5.4 which shows a typical calibration curve.

![Calibration curve for the high pressure scanivalve transducer.](image)

5.3.2.2 Temperature

Temperature is measured at two points along the flow path, one for calculation of the mass flow rate (see below), and the other for measurement of the temperature in the
measuring plane just upstream of the turbine flange.

This measurement was taken using a T-type thermocouple positioned at the centre of each the measuring plane. Temperature measurements of this kind require the use of a recovery factor, as the measurement falls somewhere between the total and static temperature. The recovery factor takes the form shown in the following equation:

$$r = \frac{T_a - T_m}{T_{0m} - T_m}$$  \hspace{1cm} (5.3)$$

where $T_a$, $T_m$ and $T_{0m}$ are the acquired, static and total temperatures, respectively. The static temperature can thus be determined by the following relation:

$$T_m = \frac{T_a}{1 + r \left( \frac{\gamma - 1}{2} \right) M a^2}$$  \hspace{1cm} (5.4)$$

where $Ma$ is the mach number of the flow which results from the mass flow rate:

$$Ma_m = \frac{\dot{m}}{A_m P_m} \sqrt{\frac{R T_m}{\gamma_m}}$$  \hspace{1cm} (5.5)$$

Since the mach number and $\gamma$ are themselves a function of temperature, an iterative procedure was implemented to solve for both.

5.3.2.3 Mass flow rate

The mass flow rate passing through the turbine was measured using two McCrometer v-cone differential pressure flow meters (McCrometer 2017), one in each limb leading up to the turbine. As shown in Figure 5.5, this device works by placing a a conical obstruction into the flow with a port in its wake. The differential pressure between this low pressure port, labelled $L$ in the figure, and a port in the free flow, labelled $H$ is measured using a Siemens Sitrans P DSIII differential pressure transmitter, while the absolute pressure at the high pressure port is also required and measured by the Scanivalve. Free flow temperature is measured using an E-type thermocouple. The mass flow rate going through the flow meter can the be determined using the following equation:

$$\dot{m} = F_a C_d Y k_1 \sqrt{\rho \Delta P}$$  \hspace{1cm} (5.6)$$
where $F_a$ is the material thermal expansion factor, $C_d$ is the discharge coefficient, $Y$ is the gas expansion factor and $k_1$ is a flow coefficient. $\Delta P$ is measured using the differential pressure sensor, while $\rho$ can be determined from the measurements of free flow pressure and temperature.

The material thermal expansion factor is dependent on the temperature and the thermal expansion coefficient of the v-cone material but generally deviates less than 0.1% from 1.0.

The discharge coefficient is a calibrated parameter, given in tabulated form as a function of Reynolds number. For the full range of turbine mass flow to be measured, the discharge coefficient was constant to the 4 significant figures given by the manufacturer, and therefore a constant value of discharge coefficient was used. The gas expansion factor is calculated by the following equation:

$$Y = 1 - (0.649 - 0.696\beta^4)\frac{U_1\Delta P}{\gamma P}$$

where $U_1$ is a unit conversion constant given by the manufacturer and $\beta$, referred to as the beta-ratio, is a function of the v-cone geometry.

The flow coefficient $k_1$ from Equation 5.6 is calculated as a function of pipe diameter and $\beta$, with a value of $k_1 = 0.0139$ given for both v-cones used.

5.3.2.4 Torque

As already explained above, the torque determined using a gimballed system to measure the reaction of the dynamometer using an Interface Miniature Beam Load Cell transducer
(Interface 2018). The signal of the load cell is delivered to a bridge module on the National Instruments Compact RIO system, which in turn interfaces with LabView to log the data. The load cell measurement is calibrated at the beginning of each test set by applying a known reaction force to the gimballed system. This is done by hanging four known masses, accurately measured using a precision scale with a resolution of 0.01g, at 250 mm from the dynamometer axis and recording the corresponding voltage. This is done adding and removing the weights in succession, resulting in two linear calibration lines of applied torque to load cell signal voltage, as shown in Figure 5.6 for an example calibration. The gradient of these lines is averaged and used for calculating torque from a given voltage reading, while the offset voltage at zero torque is measured at the end of each point when the turbine reaches standstill.

![Sample calibration curve of the torque measuring load cell](image)

**Figure 5.6: Sample calibration curve of the torque measuring load cell**

**5.3.2.5 Speed**

Turbine speed is measured by the 20-toothed encoder attached to the back of the shaft as shown in Figure 5.7. The encoder rotates through a Omron EE-SX1103 photomicrosensor,
a light gate which is triggered by the passing of an encoder tooth. By measurement of the tooth passing frequency, the speed of the rotor can be determined.

![Figure 5.7: Speed encoder used for measurement of turbine speed](image)

5.3.2.6 Derived Variables

The parameters shown in Equation 5.1 are derived from the measured variables outlined above. They require the total pressure and temperature at the inlet $P_{01}$ and $T_{01}$, derive from the measured parameters. Additionally, to determine efficiency, the turbine power $\dot{W}$ is needed.

Total temperature and pressure are derived from the measured static temperature and pressure using the isentropic flow equations, with the Mach number $Ma$ coming from Equation 5.5, the measuring plane being taken as the inlet of the turbine:

$$T_{01} = T_1 \left( 1 + \frac{\gamma - 1}{2} Ma_1^2 \right)$$  \hspace{1cm} (5.8)

$$P_{01} = P_1 \left( 1 + \frac{\gamma - 1}{2} Ma_1^2 \right)^{-\frac{\gamma}{\gamma-1}}$$  \hspace{1cm} (5.9)

This allows the calculation of mass flow parameter, reduced speed and pressure ratio. The total to static isentropic efficiency $\eta_{t-s}$ is calculated from the measured power of the turbine:

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\[
\eta_{ts} = \frac{\dot{W}}{mC_p T_{01} \left( 1 - \left( \frac{P_{01}}{P_{b1}} \right)^{\frac{T_{b1}}{T_{01}}} \right)}
\]  

(5.10)

where the denominator is equivalent to the total to static isentropic power. The measured power is derived from the measured torque and speed of the rotor.

### 5.3.3 Uncertainty

During analysis of the experimental parameters it is important to consider the measurement uncertainty, and the impact it has on the results. The uncertainty in the measurements of the turbine performance parameters was calculated on the principle of the propagation of error in the measured quantities. These measured quantities were the primary variables acquired by the instrumentation, such as static pressure and temperature.

#### 5.3.3.1 Uncertainty as standard deviation

The uncertainties in this work are defined as the error band bounding the 95.4% confidence interval, such that the true value \( y_{true} \) will lie within \( \pm \Delta y_i \). Assuming a normal distribution in the precision of measurements, this equal to two standard deviations of the measured value \( y_i \). Therefore:

\[
y_{true} = y_i \pm \Delta y_i
\]  

(5.11)

For most of the variables which were measured, a linear calibration curve was determined, relating the measured physical property \( y \) to the directly measured signal \( x_i \) (usually a voltage or current):

\[
y_i = ax_i + b
\]  

(5.12)

The measured points will not fall exactly on this line due to various sources of error such as electrical interference, thermal disturbances etc. The deviation of the points from the fit line can be used to determine the uncertainty of the measurement. The standard
deviation of points from a linear function as shown by Kirkup (1994) is:

$$\sigma = \left(\frac{1}{n-2} \sum_{i=1}^{n} (y_i - ax_i - b)\right)$$ \hspace{1cm} (5.13)

Thus, if the deviation of values from the trend line is assumed to follow a normal distribution, the 95.4 % confidence interval of $\Delta y_i = \pm 2\sigma$ can be established.

### 5.3.3.2 Propagation of uncertainty to derived parameters

Most of the experimental parameters presented in this work are a function of several individual measured variables, taking the following form:

$$Y = f(x_1, x_2, ..., x_n)$$ \hspace{1cm} (5.14)

where the variables $x_1$ to $x_n$ are measured results. The uncertainty in the parameter $Y$ must therefore be dependent on the uncertainty in its constituent variables ($\Delta x_i$). The propagation of error through the function is determined by the Root Sum Square (RSS) uncertainty as initially described by Kline & McClintock (1953) and further developed by Moffat (1988), with the following equation:

$$\Delta Y_{RSS} = \left[ \sum_{i=1}^{n} \left( \Delta x_i \frac{\partial Y}{\partial x_i} \right)^2 \right]^{1/2}$$ \hspace{1cm} (5.15)

where $n$ is the number of input variables $x_i$, and $Y$ is the derived parameter. $\Delta x_i$ is the error in each input variable.

### 5.3.3.3 Uncertainty in Measurements

**Pressure:** The standard deviation of the high pressure Scanivalve transducer from the linear calibration line was 112Pa, resulting in an uncertainty of $\pm 225$ Pa. Similarly, the low pressure transducer had an uncertainty of $\pm 86$ Pa.

Further sources of uncertainty stem from the shape of the pressure tappings as shown by Szymko (2006), using correlations shown in Benedict (1984). The error due to the pressure tapping interaction with the flow was determined to be less than $\pm 100$ Pa for
the most extreme point for the high pressure transducer, and negligible for measurements using the low pressure transducer.

Furthermore, an uncertainty from the pressure calibration unit was shown to be $\pm 113 \, Pa$, while the absolute pressure measurement is calculated from the measured gauge pressure and measurement of the atmospheric pressure which has an uncertainty of $\pm 50$.

The overall uncertainty in the pressure measurement can therefore be determined:

$$\Delta P_{abs} = \sqrt{\left( \Delta P_{scani} \frac{\partial P_{abs}}{\partial P_{scani}} \right)^2 + \left( \Delta P_{tap} \frac{\partial P_{abs}}{\partial P_{tap}} \right)^2 + \left( \Delta P_{calib} \frac{\partial P_{abs}}{\partial P_{calib}} \right)^2 + \left( \Delta P_{atm} \frac{\partial P_{abs}}{\partial P_{atm}} \right)^2} \quad (5.16)$$

This results in an overall uncertainty of $\pm 257 \, Pa$ for the high pressure transducer and $\pm 150 \, Pa$ for the low pressure transducer.

**Mass flow rate:** The mass flow rate was measured by the V-cone meters in both limbs. It was calculated using Equation 5.6, so that the uncertainty becomes

$$\Delta \dot{m} = \sqrt{\left( \Delta Y \frac{\partial \dot{m}}{\partial Y} \right)^2 + \left( \Delta \rho \frac{\partial \dot{m}}{\partial \rho} \right)^2 + \left( \Delta (\Delta P) \frac{\partial \dot{m}}{\partial \Delta P} \right)^2} \quad (5.17)$$

The errors in the calibration parameters of the supplier ($F_a, C_d, k_1$) were considered negligible.

**Temperature** The thermocouples used for measuring temperature were calibrated using the method outlined by Szymko (2006) and Newton (2013). The calibration procedure showed an uncertainty of less than $\pm 0.5 \, K$.

**Torque** The torque was measured using a load cell which read the reaction force on the gimbaled system. This was calibrated by loading the gimbal with weights to generate a known torque. Calibration was done prior to testing, with system was warmed up to the operating temperature and all auxiliary systems running (cooling water, bearing oil, air for pressurisation of the bearing system) to avoid any systematic errors which could occur.
as a result of switching these on. By calibration and through turbine runs the overall uncertainty in the torque measurement was assessed to have a value of +/- 0.062 Nm.

5.3.3.4 Uncertainty in Performance Parameters

The equations for calculating the performance parameters can then be used to establish their uncertainties:

**Pressure ratio**

\[
\Delta PR = \sqrt{\left( \Delta P_{01} \frac{\partial PR}{\partial P_{01}} \right)^2 + \left( \Delta P_{4} \frac{\partial PR}{\partial P_{4}} \right)^2}
\]  

(5.18)

The uncertainty in measured pressure ratio is low, ranging from 0.08% - 0.33%.

**Efficiency**

\[
\Delta \eta_{t-s} = \sqrt{\left( \Delta \dot{m} \frac{\partial \eta_{t-s}}{\partial \dot{m}} \right)^2 + \left( \Delta T_{01} \frac{\partial \eta_{t-s}}{\partial T_{01}} \right)^2 + \left( \Delta PR \frac{\partial \eta_{t-s}}{\partial PR} \right)^2 + \left( \Delta \tau \frac{\partial \eta_{t-s}}{\partial \tau} \right)^2 + \left( \Delta N \frac{\partial \eta_{t-s}}{\partial N} \right)^2}
\]  

(5.19)

The efficiency uncertainty is highly dependent on the measured torque, as torque has an absolute measurement uncertainty of +/- 0.062 Nm. This means that at low torque, the efficiency uncertainty can reach +/-7% while at high torque it will be +/-0.9%.

**Mass flow parameter**

\[
\Delta MFP = \sqrt{\left( \Delta \dot{m} \frac{\partial MFP}{\partial \dot{m}} \right)^2 + \left( \Delta T_{01} \frac{\partial MFP}{\partial T_{01}} \right)^2 + \left( \Delta PR \frac{\partial MFP}{\partial PR} \right)^2}
\]  

(5.20)

The overall uncertainty in mass flow parameter \((MFP)\) ranges from 0.87% to 1.9%.

5.4 Testing of Turbine Geometries

To investigate the design space established in the previous chapters experimentally, six turbine design were selected, varying the parameters investigated using CFD in Chapter
4 ($A/r$ ratio, $r_{3t}$ and $d\theta/dz$). These designs were manufactured using rapid prototyping techniques, and tested for the full operating range as outlined in the section above. This was done to give insight into the turbine performance with modification of meanline parameters, and to validate the meanline model and CFD results experimentally.

5.4.1 Selection of Turbine Designs

Each geometric parameter was varied, in the direction of increasing efficiency compared to the baseline design, according to the meanline model analysis shown in Section 4.5.4. This meant that $A/r$ ratio was decreased, and $r_{3t}$ and $d\theta/dz$ were increased, as can be seen in the test design matrix in Table 5.1.

The changes were maximised to within a reasonable value, which resulted in Design 2 (decreased $A/r$), Design 3 (increased $d\theta/dz$) and Design 5 (increased $r_{3t}$). Design 3 resulted in a different rotor, whereas Design 2 resulted in a different volute. These were designed to be interchangeable, so that Design 4 with decreased $A/r$ and increased $d\theta/dz$ could be tested with the parts already designed. An additional volute was designed to fit the rotor of Design 5 with a decreased value of $A/r$ to investigate the efficiency reduction at low power due to the secondary flows observed in CFD (Section 4.4.3).

Table 5.1: Test Geometry Matrix

<table>
<thead>
<tr>
<th>Des. No.</th>
<th>$A_1/r_1$</th>
<th>$d\theta/dz$</th>
<th>$r_{3t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[mm]</td>
<td>[$^\circ$ mm$^{-1}$]</td>
<td>[mm]</td>
</tr>
<tr>
<td>1*</td>
<td>12</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td></td>
<td>16.15</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>7</td>
<td>18.15</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Baseline
5.4.2 Design & Manufacture

Two processes were used for the manufacture of the selected designs. The four volutes were 3D printed using SLS with glass filled nylon material (PA12). This material fulfils the tensile stress requirements under pressure (increased by the addition of glass spheres) while being available at a low cost. Internal flow geometries were generated with the parametric CAD model developed in Chapter 4 and a part designed by the thickening of surfaces to create a solid component which could be mounted on the rig. According to the manufacturer, the manufacturing tolerance using this material was ±0.2mm, deemed sufficient to not have a large impact on performance.

The three rotors were manufactured on a 5-axis CNC mill, and consisted of Al7075-T6 aluminium alloy with a tensile yield strength of 503 MPa, to ensure mechanical integrity at high speeds. The rotors were anodised, increasing the hardness of the surface and therefore reducing the susceptibility to erosion. The three manufactured rotors are shown in Figure 5.8.

![Figure 5.8: Manufactured turbine rotors.](image-url)
5.4.3 Testing Procedure

For testing, the manufactured components were mounted onto the rig, all fasteners tightened and the instrumentation connected. A photo of the rig setup can be seen in Figure 5.9.

The two parameters which control the operating point of the test rig are the intake valve opening which controls the mass flow rate, and the clearance of the dynamometer stator plates with the magnetic rotor, which controls the loading torque at a given speed. Testing was conducted in speed lines. For each speed, points were taken for different plate clearances to generate a map of points at different torques. For each speed line between 9-10 points were taken, from the lowest torque, limited by the bearing losses of an unloaded dynamometer, to the highest torque which is limited by either the minimum stator plate clearance or the maximum allowable pressure of the rig such that designs with a lower mass flow parameter were not able to reach the highest torque which the dynamometer can load. The speeds tested were 20 000, 30 000, 40 000, 50 000 and 60 000 RPM, at an inlet temperature of 325K. 60 000 RPM corresponds to a speed of 98 000 RPM at 873.15K, the standard temperature for measuring hot turbine maps.

5.4.4 Results

Figures 5.10a to 5.11b show all of the experimental results, with each figure showing all tested designs at one speed. As expected, the effect of turbine design was higher on the mass flow than on efficiency. Several insightful details can be observed from the mass flow parameter results.

While Design 3 (high \( \frac{d\theta}{dz} \), baseline \( A/r \) & \( r_3 \)) has a higher mass flow rate than Design 6 at 20 000 RPM, this trend shifts so that at 40 000 RPM the mass flow parameter of the two designs is almost identical and at speeds higher than 40 000 RPM, Design 6 has a higher mass flow rate. Similarly, the difference in mass flow parameter between Design 1 and Design 5 grows as the speed increases. Another way of viewing this is to consider all speedlines for a design on individual axes, as shown in Figure 5.12. It can clearly be seen that the speed lines bunch closer together at high pressure ratio for Design 6. This would be typical for a nozzled turbine design, where choke occurs within
the nozzle throats and not the rotor exducer so that the choking mass flow parameter is therefore unaffected by turbine speed. A plausible explanation for this behaviour in the test cases would be the larger effective exducer area of Design 6 due to the higher tip radius. Designs 3 & 4 have the lowest effective exducer area due to their higher blade exit angle, and therefore comparison between Design 3 & 6 shows a particularly large difference.

The efficiency changes less significantly than the mass flow parameter, though a few observations can be made. The most obvious is the low efficiency of Designs 5 & 6 (high exducer tip radius $r_{3t}$) at low power. This effect was previously observed in the CFD results in Chapter 4 and was found to be due to a significant increase in secondary flows at high exducer tip radius and lower $A/r$ ratio. Design 6 with a lower $A/r$ ratio is therefore affected more significantly, while at high speed the effect disappears for Design 5. While at low power ($U/C_{is} > 0.7$), Designs 5 & 6 perform very poorly in comparison to the other designs, at high power their efficiency is higher. The peak efficiency point is shifted to a lower $U/C_{is}$ which can be desirable in some applications.
Figure 5.10: Experimental speed lines for turbine designs 1-6, as shown in Table 5.1, for speedlines from 20 000 - 40 000 RPM
Figure 5.11: Experimental speed lines for turbine designs 1-6, as shown in Table 5.1, for speedlines of 50 000 and 60 000 RPM
Figure 5.12: Experimental speed lines for turbine designs 3 and 6, as shown in Table 5.1, showing all speed lines
5.5 Analysis

5.5.1 Meanline Model Calibration

To evaluate the predictive capability of the meanline model using the experimental results of the baseline design, the model needed to be calibrated using a selection of data points and subsequently compared to the remaining points. This gave rise to the question of the model's sensitivity to the selection of data when using the self-calibration procedure presented in Chapter 3. When the turbine is loaded with a compressor as is typically the case when generating turbocharger maps, the range of operation is limited to a narrow band of $U/C_{is}$ around the design point, and therefore extrapolation to operating regions outside of the available data becomes important. The much wider operating range available in the experimental data from the testing facility allowed an investigation into how well the meanline model is able to perform this extrapolation, providing a valuable insight into its feasibility in an industrial setting.

To evaluate this, the meanline model was calibrated using three different sets of points from the experimental data:

1. The peak efficiency point on all speed lines.

2. The peak efficiency points and the two adjacent operating points on each speed line.

3. A selection of points covering the full range of speed and load tested. These points are shown in Figure 5.13

It should be noted that lowest measured speed (20 000 RPM) was excluded from the calibration sets as the meanline model had difficulty fitting to the design point efficiency at this speed, meaning that the inclusion of points would have led to a reduction in overall calibration quality. This can be justified because the speed is very low, and falls well outside the range of a typical turbocharger map.

Set 1 represents a situation where very few data points are available for a given turbine, such as when CFD is used during initial design studies. Set 2 is a more common situation in turbocharging, where testing of maps on a test bench with the compressor
Figure 5.13: Experimentally determined speedlines of the baseline design (Design 1) with points used for calibration of the meanline model indicated by bold circles attached, results in the narrow range of available turbine data around the design point. The points used for Set 3 cover the full range of operation which was tested. This is an unlikely scenario in most applications as testing for such a wide range requires a specialised facility. Therefore calibration to this set represents a measure of how accurate the meanline model can be given a wide set of operating conditions.

The calibration parameter values for each of the sets are shown in Table 5.2. As can be seen, Set 2 and Set 3 have relatively similar values for most parameters, while Set 1 varies significantly for $K_{pl}$, $K_{inc}$ and $K_{cl}$. The resulting difference in prediction can be seen in Figures 5.14 and 5.15 for mass flow parameter and efficiency at two speeds.

The first thing to note is that the fit at 50 000 RPM is in fact quite good for both mass flow parameter and efficiency with all calibration sets. With the exception of the two lowest pressure ratio points, the error in mass flow with Set 3 is less than 1%, and less than 2% with Sets 1 and 2. Similarly for the efficiency, the error is less than 1% for Set 3 and less than 0.3% for Set 2. The error with Set 1 increases at higher power, with a value of 1.6% at the lowest $U/C_{is}$.

The lower speed is less well predicted by the meanline model with all calibration
Table 5.2: Values of calibrations coefficients of meanline model, calibrated with three different sets of calibration points.

<table>
<thead>
<tr>
<th>Component</th>
<th>Loss</th>
<th>Source</th>
<th>Set 1</th>
<th>Set 2</th>
<th>Set 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volute</td>
<td>Pressure</td>
<td>Japikse &amp; Baines (1997)</td>
<td>0.25</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>Swirl</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Blockage</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rotor</td>
<td>Passage</td>
<td>Wasserbauer &amp; Glassman (1975)</td>
<td>0.22</td>
<td>0.24</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>Incidence</td>
<td>Wasserbauer &amp; Glassman (1975)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tip clearance</td>
<td>Spraker (1987)</td>
<td>2.00</td>
<td>1.00</td>
<td>1.24</td>
</tr>
<tr>
<td></td>
<td>Blockage</td>
<td>Japikse &amp; Baines (1997)</td>
<td>0.92</td>
<td>0.92</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>Blockage</td>
<td>Hohenberg (Chapter 3)</td>
<td>0.97</td>
<td>0.97</td>
<td>0.99</td>
</tr>
</tbody>
</table>
Figure 5.14: Experimentally acquired speed line of mass flow parameter for the baseline design, compared to the meanline model prediction after calibration with three different sets of data points.

Figure 5.15: Experimentally acquired speed line of efficiency for the baseline design, compared to the meanline model prediction after calibration with three different sets of data points.

sets. It is evident that there is a small shift in the position of peak efficiency such that
the points between \( U/C_{ts} = 0.4 \) and 0.7 are over-predicted by as much as 2.2% with calibration Sets 2 and 3. This shift is more significant with Set 1 (up to 4%), which can in part be attributed to the higher value of the clearance loss calibration parameter \( K_{cl} \). As the predicted clearance loss decreases with lower \( U/C_{ts} \) (see analysis in Figure 4.51 in Chapter 4), a higher clearance loss parameter will have a proportionately higher impact at higher \( U/C_{ts} \), shifting the peak efficiency to a lower value.

The mass flow parameter can be seen to be well predicted at low speed when calibrated with Sets 2 and 3 with the error being less than 1.5%. The mass flow prediction when calibrating with Set 1 has a higher error, which increases with pressure ratio, resulting in errors of up to 4.3%.

Overall, it can be said that Set 2 and Set 3 lead to very similar calibrations, with neither providing an evidently better fit than the other. This leads to the conclusion that a narrow data range would in fact be sufficient for a good calibration in this case, and that the meanline model is capable of extrapolating to the extremes of performance, particularly at high power.

It is also evident that calibration with just peak efficiency points, as was done with calibration Set 1, does lead to increased errors. This is because the model is unable to establish the loss trends properly such that errors in the calibration parameters occur.

### 5.5.2 Extrapolating to different designs

The above analysis showed the accuracy of the meanline model for operating point extrapolations using a single turbine design. A further outcome of this work was to evaluate the capability of the meanline model to predict the performance of modified turbine designs once calibrated. This was done by applying the model calibrated using Set 3 (data spanning the full width of operation of the baseline design), to the other five designs which were tested to generate speed lines. The subsequent speed lines were compared to the experimental points.
5.5.2.1 Mass Flow Parameter Prediction

Comparison of the prediction of mass flow through the meanline model is shown in Figure 5.16. At low speeds (20 000 and 30 000 RPM) the meanline model can be seen to under-predict the mass flow parameter as the pressure ratio increases, for all cases except Design 6 where the mass flow parameter is generally slightly over-predicted. Even for Design 6 however, the over-prediction decreases as pressure ratio increases. This is likely to be due to the difficulty in modelling incidence losses correctly at such low $U/C_{is}$ (at 20 000RPM and $PR = 2.0$, $U/C_{is}$ has a value of 0.23). The error in mass flow parameter prediction for Design 4 at 30 000 RPM and PR=2.97 is -2.9%

At higher speeds, the mass flow is well predicted, in particular for the baseline design which the model was calibrated to. The only exception is the prediction of Designs 3 and 4 (low $A/r$), where mass flow is over-predicted in the mid-range of pressure ratio from 1.5 to 2.5. This results in a maximum error of around 2% with respect to the measured mass flow parameter.

To account for losses due to secondary flows in some regions of the design space at low power, the passage flow loss has a multiplier of 2.5, as explained in Section 4.5.2.1. Though the desired effect of this is to model the efficiency correctly, it also has an impact on the mass flow parameter, as can be seen in the speed lines of Design 6 which falls into the region in question. As the correction multiplier comes into force at values of $U/C_{is} > 0.7$, a step in mass flow parameter appears in the meanline prediction. This causes an under-prediction of mass flow parameter at low power which becomes more significant at higher speeds, causing an error of nearly -7% at 60 000 RPM. This suggests that the passage flow multiplier does not fully capture the physics of the secondary flow losses. Care must therefore be taken if such designs are to be considered, especially if the region of operation above $U/C_{is} = 0.7$ is of interest. In many turbocharging applications, the overall impact of this will not be significant.
Figure 5.16: Measured Mass flow parameter of turbine designs 1-6 (shown with markers and solid lines) compared to meanline prediction (dashed lines) at all measured speeds.
5.5.2.2 Efficiency Prediction

Figure 5.17 shows the meanline prediction of efficiency compared to experimental results for each design at a speed of 50,000 RPM. Extrapolation of efficiency to the other designs gave satisfactory results, and consideration has to be taken of the fact that, despite relatively large design changes being made, the efficiency changes were quite low, venturing into the region of experimental uncertainty at low torques.

Going from Design 1 to Design 2 ($A/r$ from 12.0 to 10.0) a slight efficiency increase was predicted. At low $U/C_{is}$ this is reflected in the measurement, though to a lesser extent than predicted by the meanline, while at values of $U/C_{is}$ of 0.7 and higher, the efficiency actually decreased, albeit by around 1 percentage points, the error at peak efficiency being 2.3% percentage points.

A similar result is seen for the prediction of Design 3 ($d\theta/dz$ from 7 to 9), although here the error remains at low $U/C_{is}$. This reflects the error with respect to $d\theta/dz$ established using the CFD numerical experiment, where an over-prediction of efficiency determined at lower $U/C_{is}$ for increasing $d\theta/dz$. The most significant error is seen for Design 4, where the meanline model predicts a peak efficiency increase of near 2.8%, while the tested performance is almost identical to that of the baseline design. The errors in the prediction of Designs 3 and 4 are in line with the investigation of CFD, which showed the meanline prediction error trends with respect to $d\theta/dz$ and $A/r$ ratio.

Interestingly Design 5 is very well predicted by the meanline model which captures the trend of higher efficiency at low $U/C_{is}$ and lower efficiency at high $U/C_{is}$ almost perfectly. Similarly, the meanline predicts the higher efficiency region of Design 6. It seems however, that the lower value of $A/r$, also seen in Design 2, causes losses which are not captured by the model. This could be due to higher velocities at the smaller volute inlet area, especially passing the tongue which is relatively large due to the design coming from a small turbocharger turbine.

As already discussed above, the efficiency of Design 6 deteriorates above $U/C_{is}$ values of 0.6, due to large vortices within the passage flow. The meanline model accounts for this by multiplying the passage loss by factor of 2.5, which leads to a very good efficiency fit at the speed shown. The remaining speed lines are shown in Appendix A.
Figure 5.17: Measured efficiency of turbine designs 1-6 compared to meanline prediction (dashed lines) and meanline prediction of the baseline design at a turbine rotational speed of 50 000 RPM. Design 1 measurements are shown in all plots for reference.
5.5.3 Comparison to CFD Setup

To analyse the validity of the CFD setup presented in Chapter 4, the model was setup to calculate the performance of all tested turbine designs at selected operating points. This comparison can be seen in Figures 5.18 and 5.19 for efficiency and mass flow parameter respectively. The original CFD setup showed significant differences in efficiency with the data. The efficiency error varied from 1.9-5.5%, while the mass flow parameter varied only from -0.4-1.2%. For most designs, with exception of the baseline, a decreasing trend in efficiency error with decreasing $U/C_{ls}$ can be observed so that the error at design point is slightly higher than at high power. Taking Design 2 as an example, it can be observed that the efficiency error at $U/C_{ls} = 0.7$ is 5.5% while it is 3.6% at $U/C_{ls} = 0.4$.

![Graphs showing efficiency comparison](image)

These differences to CFD for results from the testing facility are not entirely unusual (Padzillah 2014), and are often attributed to effects such as the windage losses which
are not considered in the CFD setup due to the larger computational cost associated with modelling the back face cavity of the turbine rotor. However a key difference to the usual setup within the experiment, was the use of SLS 3D printing for manufacture of the volutes, which to the touch were found to have a rougher surface than the cast iron volutes commonly used. To investigate possible discrepancies due to the roughness of the material, a fragment of the inner surface of one of the volutes volute was investigated under a laser scanning microscope to determine its roughness and hence estimate the impact this would have on the results.

Figure 5.20 shows the roughness profile along one plane, determined by the microscope. The value of $R_{rms}$ (root mean squared roughness) was found to be $17.6\mu m$. It has been shown that a roughness profile such as the one seen in the figure, can be represented by a sand grain roughness, causing the effect equivalent to a layer of closely packed spheres of a given diameter $h_s$ at the wall (Schlichting 1979). In CFX, the sand
grain roughness is modelled by shifting the logarithmic velocity profile downwards by a function of the roughness height. The reader is referred to the CFX-Solver Theory Guide (ANSYS Inc. 2018) for a detailed description of this process. The challenge which arises is the selection of equivalent sand grain roughness based on the measured roughness profile. Several authors have shown that the equivalent sand grain roughness of a surface can be approximately determined by 

\[ h_s \approx 3R_{rms} \]  

(Zagarola & Smits 1998, Shockling et al. 2006). For the investigated material, this came to a value of approximately 55\(\mu\)m. This sand grain roughness was applied to the models, with the setup otherwise remaining exactly the same, and the models were re-run, with the results also shown in Figures 5.18 and 5.19.

![Roughness profile](image)

Figure 5.20: Roughness profile of SLS glass filled PA12 material surface measured by a laser scanning microscope.

The efficiency of the resulting CFD calculations were found to be closer to the data after implementation of roughness in CFD. The efficiency error varied from 0.4-3.0% with less variation in error for different operating conditions. This means that roughness had a slightly higher impact on the calculations at higher \(U/C_{is}\), explaining why the calculation error was higher at these points with the standard CFD setup. The predicted mass flow parameter changed, though by a small amount (<1% in all cases) resulting in an error of -0.8-1.4%.

Although the resulting efficiencies can be considered within an acceptable range, it seems the sand grain roughness was under-estimated by the approach as the efficiency was still higher than the experimental result. Though this can result from other deficiencies in the model such as the mesh, the relationship between sand grain roughness and a measured surface roughness profile is complex and not only dependent on the root
mean square roughness (the shape of the peaks plays a role too) so an under-estimation is plausible. The most feasible, rigorous method of experimentally determining the sand grain roughness would be the manufacture and instrumentation of a pipe of the same material, and measurement of the pressure drop across it for a range of mass flow rates. Comparison with CFD of the same pipe would allow the calibration of sand grain roughness and subsequent application to the turbine CFD. This was however beyond the scope of the present work.

The range of error in both mass flow and efficiency with implementation of sand grain roughness give confidence in the mesh, volute rotor-interface and other aspects of the CFD setup. Although there remains a small error, the trends were well captured and the errors close to experimental uncertainty. The work presented in Chapter 4 was carried out without the inclusion of roughness calculations. This can be justified because the intended application of the meanline model is for use with standard turbocharger manufacturing processes which have surfaces which are commonly assumed to be smooth in CFD. Therefore it can be assumed that the model is able to represent the performance characteristics within the accuracy required for comparison with the meanline model.

5.6 Summary & Conclusion

In this chapter, the testing and subsequent analysis of six prototyped turbine designs (one baseline and five variations) is presented. The designs where manufactured using SLS 3D printing for the volutes and a 5-axis CNC mill for the rotors. The designs were tested in the cold flow testing facility at Imperial College London, for a wide range of operating conditions.

The results were used primarily for validation of the meanline model. The results from the baseline design where used to calibrate the turbine model. Three different approaches were used to select calibration points for the automatic calibration procedure. It was found that while inputing the performance of only the peak efficiency point for each speed line led to errors in the prediction, calibration points from a narrow range around the peak efficiency point did not have a significantly higher error than if the full width covered by the data was used. This leads to the conclusion that the narrow range
often available from turbocharger testing would be sufficient to calibrate the model, and that the extrapolation would be accurate.

The calibrated model was subsequently applied to the five other designs. Comparison with the experimental data showed that the mass flow for different designs was very well predicted by the meanline model across the operating range. The fit with the efficiency showed a slightly higher error, especially when multiple design parameters were modified. Nevertheless, the efficiency error across the operating range was less than 3%. This was deemed sufficient for an accurate representation of design parameter impact on engine performance.

The CFD approach used in the previous Chapter was validated by applying it to the designs which were tested. An efficiency error of up to 5.5% was determined, which was attributed to the neglect of surface roughness in the calculations. The surface roughness of the 3D printed volutes was measured and an equivalent sand roughness calculated, bringing the error to less than 3%.
Chapter 6

Engine Performance Optimisation

6.1 Overview

This chapter describes a turbine design optimisation procedure for several objectives relating to engine performance. To achieve this, the developed meanline model was integrated into the 1-D model of an existing engine, and the modification of design parameters explored. The chapter is structured as follows:

1. Development and Calibration of Engine Model: Engine test data for the existing baseline engine was used for calibration of the GT-Power engine model. Subsequently control strategies were developed for the model for the evaluation of key engine performance objectives.

2. Integration of Meanline model: The meanline model developed in the chapters above was integrated into the engine model to allow evaluation of turbine design effects on engine performance.

3. Analysis of Engine Performance: The integrated meanline methodology was used for generating a range of maps for different designs, allowing firstly the analysis of individual parameters from a new perspective, and subsequently the optimisation of parameters for different performance outcomes.
6.2 Introduction

There are several aspects of engine performance which are of importance to engine design, and oftentimes trade-offs are necessary to arrive at a result which fulfils all requirements. In recent years, reduction of emissions has become a key factor and engine designs have become increasingly tailored towards the goal of reducing fuel consumption and tailpipe emissions. Within the automotive sector, the reduction of emissions is somewhat limited by the requirements of other engine performance aspects, most notably the torque as well as transient engine performance. These parameters are key to designing an engine which ensures drivability and therefore must be taken into consideration.

This becomes an important factor in the turbocharger matching process, including the selection of a turbine: If the selected turbine is too large, the torque at low engine RPM will be lowered and the rotor inertia will cause reduction in transient performance. If the turbine is too small, the effect can be a reduction in fuel economy due to higher pumping losses. The conventional matching process involves the selection of a correct size turbine from a catalogue of options, for which tested maps are available, giving the swallowing capacity and efficiency of the turbine over a range of operating conditions. These available turbines have a limited range of rotors with different diameters often referred to as 'frame sizes', and for each rotor, the $A/r$ ratio of the volute is varied to achieve a range of swallowing capacities.

The rotors are optimised for standalone efficiency, without direct consideration of engine performance. However, a highly efficient rotor with a volute to match the mass flow requirements of the engine design, may not actually be an optimum match giving the best engine performance. There is therefore scope for tailoring a turbocharger design to the engine by consideration of the effect of turbine design on the whole system performance.

This kind of optimisation can be achieved with the use of 1-D modelling tools, which predict the system performance of the engine by simulating the air flow and gas dynamics in the components of the engine. A common commercial software for this kind of modelling is GT-Power, which provides a suite of tools for the simulation of engine performance. Modelling of the turbocharger is achieved by the use of maps. These typically come from a hot gas stand which requires extrapolation of the turbine performance by
To evaluate the effect of turbine design on engine performance by modelling, a means of generating the turbine maps for different designs is necessary. One way of doing so is the use of the meanline model developed in the previous chapters of this thesis, enabling evaluation of the impact of the meanline design parameters on engine performance.

This analysis was conducted on the basis of an existing engine, the Proton CAMPRO CFE which is a turbocharged gasoline engine. An engine model was set up and calibrated using engine test bench data with the stock turbocharger. Subsequently a methodology was developed for integrating the meanline predicted turbine performance maps into the turbine component of the engine model. This included the modelling of rotor inertia and the use of a low complexity bearing and heat transfer model.

Three different control models were developed for the engine, to evaluate different aspects of engine performance:

1. **Fuel Consumption:** The engine model was set up to target a pre-set torque at a fixed engine speed using a throttle controller, while a wastegate controller was used to limit the compressor outlet pressure. This enabled evaluation of engine BSFC at a given operating condition.

2. **Low end torque:** The engine model was set up to run with a wide open throttle, using a wastegate controller to limit the maximum compressor outlet pressure. This model was run at 1500RPM to evaluate the low end torque of the engine for a given turbine design. At this engine speed, the boost limit was not reached so that the wastegate remained closed.

3. **Transient response:** A constant speed transient model was set up with a load step from low torque to wide open throttle. Transient response was measured as the time taken to reach a target torque from the beginning of throttle opening.

Integrating meanline generated turbine maps into each of the above models allowed an evaluation of the effect of turbine design on different key engine performance parameters in a novel manner. Maps were generated for a sample of 500 different designs. The three parameters investigated in the previous chapters were sampled (volute $A/r$ ratio, turbine
exducer tip radius $r_{3t}$ and exducer camberline gradient $d\theta/dz$), with the addition of the turbine wheel radius $r_2$. This parameter scales the full turbine stage, and has a large impact on engine performance due to its effect on swallowing capacity, rotor speed and rotor inertia. To conclude the Chapter, three optimised designs are selected from the sample and analysed.

### 6.3 Development of Proton CAMPRO engine model

#### 6.3.1 Description of Engine Model

The engine used for this study was the Proton CAMPRO CFE engine, which is a 4-cylinder, 1.56L turbocharged gasoline engine with port injection (pictured in Figure 6.1.) The engine is wastegated and the throttle situated after the intercooler, just upstream of the intake manifold.

![Figure 6.1: Picture of the Proton CAMPRO engine](image)

A 1-D GT-Power model of this engine was available from previous work at the authors
institution (Ismail 2017) and was used as a starting point of this engine model. The component map of the final engine model is shown in Figure 6.2.

![Figure 6.2: GT-Power engine component map with relevant components highlighted.](image)

Steady state and transient engine performance data including in-cylinder pressure measurements were available from engine dynamometer testing at the LoCARtic Centre at the University of Technology Malaysia. To accommodate the engine on the test rig with corresponding instrumentation, and to improve performance at the high temperatures within the test cell, some of the components of the engine were modified from the stock setup. These included the piping between the compressor and the throttle as well as the intercooler, which was enlarged to reduce knocking. In light of these changes, the engine ECU map was calibrated for all engine operating conditions. This included calibration
of the intake valve timing, lambda, ignition timing, and the wastegate opening. All changes (both piping and the ECU map) were implemented in the GT-Power model before calibration, as was the manufacturer hot map for the turbocharger which was used in the testing.

6.3.2 Available Data

Both steady state data covering the full operating range (full load and part load) as well as transient data were available. Steady state data was available for a range of speeds and throttle settings, as is best summarised by Figure 6.3. Measurements of pressure and temperature were available at the following positions in the engine:

1. Compressor inlet
2. Compressor outlet
3. Intercooler inlet
4. Intercooler outlet
5. Intake manifold
6. Turbine inlet
7. Turbine outlet

Additionally, measurements of air mass flow rate, fuel mass flow rate as well as engine torque and speed were available. It should be noted that turbocharger rotational speed was not included in the data.

In addition to the steady state data with time averaged values, in-cylinder pressure measurements were available for the steady state operating points.

Constant speed transient runs were carried out at different speeds with various load steps. The data used in the present work was limited to one transient run, from a throttle position of 7.5% to wide open throttle at 1500 RPM. For this run, the same measurements from the steady state points were available with a temporal resolution of 0.2 seconds.

Finally, data for a WLTC driving cycle was available with the same temporal resolution and measurements as the transient load steps. The speed and torque profiles of this driving cycle are shown in Figure 6.4.
6.3.3 Engine Model Calibration

The model was calibrated using the available data. The GT-Power manual gives detailed information for calibration of an engine model which involves the fitting of pressure loss coefficients and heat transfer effects, so that the engine is well represented. Typical calibration parameters include wall temperatures in the pipes, as well as pressure loss coefficients of the pipes and discharge coefficients of orifices between pipes. These are matched to produce the measured values at various points in the engine, typically for a set of steady state engine operating points at full load and part load conditions.

The pressure loss and discharge coefficients remain the same at all operating conditions. Once calibrated, the model can predict performance outside of the calibrated points and therefore be used to investigate engine design parameters.
Figure 6.4: Instantaneous engine speed and torque measurements for the WLTC driving cycle carried out on the engine dynamometer.

6.3.3.1 Steady State Engine Model Calibration

The baseline engine model which was used for both full load and part load calibration was set up to run at a constant speed, with a wastegate controller to target the required torque at each operating point. 3-dimensional maps were used for lambda, ignition timing and intake valve timing, as a function of engine speed and throttle position, according to the ECU settings from the tested engine. Combustion was modelled using the commonly used SI-Wiebe model to determine the burn rate of the fuel. The input parameters for the Wiebe function were calibrated for each steady state operating point using in-cylinder
pressure measurements, and subsequently a map generated as a function of engine speed and load.

Figures 6.5, 6.6, 6.7 and 6.8 show the result of the calibrated model at full load for BSFC, air mass flow rate, compressor outlet pressure and turbine inlet pressure, respectively. As the engine torque was targeted by the wastegate controller, it fit to within 0.5% of the engine data for all points and is therefore not shown.

It can be seen that the BSFC is over-predicted at low engine speed, and under-predicted at high speed, similarly to the mass flow rate. This is explained by a similar trend in the turbine inlet pressure. This is too low at higher engine speeds, which leads to a lower back pressure and therefore reduced mass flow rate required for the same engine torque. It is evident that there was a source of error, either in the model or in the data preventing a fit within 5%. However, in the absence of turbocharger speed measurements, further investigation proved difficult, and the calibration was accepted as being sufficiently good for the proposed analysis.

Figure 6.5: BSFC of the calibrated engine model for full load engine operation across all engine speeds, compared to the available engine data
Figure 6.6: Air mass flow rate of the calibrated engine model for full load engine operation across all engine speeds, compared to the available engine data.

Figure 6.7: Compressor outlet pressure of the calibrated engine model for full load engine operation across all engine speeds, compared to the available engine data.
Figure 6.8: Turbine inlet pressure of the calibrated engine model for full load engine operation across all engine speeds, compared to the available engine data.
6.3.3.2 Transient Engine Model Calibration

The available data recorded a load step from 7.5% throttle position to wide open throttle at 1500 RPM, therefore a constant speed transient. The instantaneous torque of the engine fluctuates due to the cylinder ignition events. Capturing this requires a high time resolution, which was not available in the data. As a result the data resolution caused some noise in the torque profile as can be seen in Figure 6.9.

An equivalent transient model was set up in GT-Power. The engine was run for several cycles to stabilise at the lower BMEP from the data using the throttle controller. Subsequently a load demand was simulated by fully opening the throttle at the same rate determined from the data. The result can be seen in Figure 6.9, showing a well predicted torque profile by the engine model. The fluctuations in the modelled torque prior to $t = 0s$ show the throttle controller targeting the lower torque of the engine. This is followed by a rapid increase in torque, as the high pressure air stored in between the compressor and the throttle empties into the intake manifold. After around 0.3 seconds the torque response slows down as the inertia of the turbocharger resists acceleration, and therefore boosting is delayed. This is what is commonly understood as 'turbo lag'. To enable correct modelling of this part of the transient, the inertia of the turbocharger rotating components was calculated from a CAD model, with explicit modelling of the density of different components, and implemented in GT-Power.
Figure 6.9: Transient load step at 1500 RPM, comparing the recorded engine torque with that predicted by the model. The model output from GT-Power gives the cycle averaged performance. A line indicating 95% of the peak torque is shown.

6.4 Modelling the Objective Parameters

As described previously, three primary parameters were used as criteria to evaluate engine performance: Fuel economy (BSFC), transient response and low end torque. To achieve comparability of results for different turbine designs, different engine model control strategies were used as outlined below. These were based on the calibrated engine model, changing only the engine control mechanisms.

6.4.1 Fuel Consumption

To evaluate fuel consumption for each turbine design in the design of experiment, the engine model was run at 2 part load points. These were selected on the basis of the WLTC cycle shown in Figure 6.4 above, which was run on the engine dynamometer. The data-points of the WLTC, recorded at a resolution of 0.2s are shown on axes of torque vs. engine speed in Figure 6.10.

These data-points allow numerical integration of the fuel flow using the midpoint rule,
to give the total fuel consumed within the drive cycle:

\[ m_f = \sum_{i=1}^{N} (\dot{m}_{fi} \Delta t) \]  

(6.1)

The product of fuel mass flow rate \( \dot{m}_f \) and the length of the time step \( \Delta t \) thus gives the fuel consumed at each time step, i.e. each data point. If the Torque vs. Speed map of the engine is discretised into a grid, the fuel consumed during each point within each region of the grid can be summed to give the total amount of fuel consumed within each region during the drive cycle. The regions can then be shown as in Figure 6.11, with the colour of each square being equivalent to the fuel consumed within that region as a percentage of the total fuel consumed.

Plotting in this way, revealed areas of the torque vs. speed map where a larger amount of fuel was burnt during the drive cycle. This led to the selection of two representative operating points for the driving cycle, indicated in the figure, to limit the computational cost of modelling a full drive cycle for the evaluation of fuel economy. These are shown again in Table 6.1. Average fuel consumption was then evaluated as the arithmetic average of the two modelled points.

As the engine performance requirements remain constant, the torque and speeds cov-
Figure 6.11: Engine Torque vs. Speed map, divided into rectangular regions which are coloured according to the percentage of fuel burned within that region during the WLTC drive cycle. The steady state engine torque lines of constant throttle opening shown in Figure 6.3 are superimposed on the image for reference.

Table 6.1: Two operating conditions used to represent the fuel economy of the engine.

<table>
<thead>
<tr>
<th>Point</th>
<th>Engine Speed (RPM)</th>
<th>Engine Torque (N-m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2500</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td>3750</td>
<td>100</td>
</tr>
</tbody>
</table>

Valve and ignition timings were kept the same as the baseline calibration. While it is acknowledged that a new turbine match would ultimately lead to recalibration of the
engine ECU settings, this effect was neglected in the analysis and assumed to be small.

6.4.2 Low End Torque

Low end torque refers to the engine torque at low engine speed and wide open throttle (full load). In the present thesis this was taken to be at 1500 RPM. To compare low end torque of the engine using different turbine designs, a simple full load (wide open throttle) model was run at 1500 RPM. To ensure good comparability between turbine matches, and the possibility of covering full load at other speeds, a wastegate controller was implemented to limit the compressor outlet pressure to 170 kPa. However, this functionality did not feature in the actual optimisation, where the compressor outlet always remained below the threshold and the wastegate was therefore kept closed.

This use of the wastegate was not in line with the original engine data, as it can be seen that the full load line (throttle fully open) was calibrated to give a flat torque curve (controlled by the wastegate) starting at 2000RPM. The compressor outlet pressure at full load (seen in Figure 6.7) thus continues increasing from 170 kPa at 2000 RPM to a maximum of 180 kPa at 3500 RPM, and therefore limiting it to 170 kPa leads to a torque decrease with increasing engine speed above 2000 RPM. The result of this is shown in Figure 6.12. Up to 2000 RPM, the torque is close to that of the data, as the wastegate is either shut, or limiting the compressor outlet pressure to the same value as in the data. However, at higher speeds, the pressure continues to be limited to 170 kPa, where the data has an increasing pressure. This effect has no impact on the parameter of the objective function, which is the full load torque at 1500 RPM, and therefore only becomes important once higher speeds are compared.

6.4.3 Transient Response

To predict transient response, the same model shown in Section 6.3.3.2 was used, with a throttle controller to target a torque of 40Nm at 1500RPM and a subsequent throttle opening to simulate a load step. The parameter used to evaluate the transient response of a given turbine match was the time taken reach a torque of 135.85 Nm, equivalent to 95% of the torque reached in the tested transient, starting from the beginning of the
Figure 6.12: Full load torque of the engine data, compared to that of the low end torque control strategy.

throttle opening event.
6.5 Integrated Turbine Modelling

The objective of the work was to determine the impact of turbine design changes on engine performance, enabling an optimisation of the turbine match. This was done through integration of the meanline model presented in prior chapters for the prediction of turbine performance. The meanline model was calibrated using CFD results for the baseline design. The code was then set up to output the turbine performance parameters in a format that was readable for GT-Power, namely through the "TurbineMapGrid" template. The data input through this template is read from a text file and subsequently interpolated by GT-Power, therefore the full operating range of the turbine needs to be covered, as can be done with the meanline model.

To ensure the turbine performance was equivalent to that measured by a hot gas stand, bearing friction and heat transfer effects were accounted for in a separate model. Nevertheless, perfect modelling of the full stage including all effects is very difficult, and the engine performance through meanline model integration differed enough to make comparison between the hot map data of the baseline, and an optimised design with a meanline model unrealistic.

All comparisons made relative to the engine performance using the meanline prediction of the baseline turbine design. The underlying assumption in this approach was that, although the modelled absolute performance of a given turbine design would differ from the equivalent performance with a hot map, the differences seen between turbine designs would be reflected in the model. This rests on the assumption that heat transfer and bearing losses would not change significantly between turbine designs, as the modelled operating conditions are the same.

6.5.1 Integration of meanline model for baseline design

Integration of the meanline model into GT-Power required calibration of the meanline calibration parameters. This was done using CFD results from the baseline design, generated by the parametric model presented in Chapter 4, with the same CFD setup, at 9 different operating conditions.

The resulting calibration can be seen in Figure 6.13. The fit was good, with a max-
imum error of 1.1% in efficiency and 1.7% in mass flow parameter, both for the high pressure ratio point at the lowest speed.

A hot gas stand map of the baseline turbocharger was available and the meanline model was run at the same operating points for comparison, as is shown in Figure 6.14. The mass flow parameter prediction through the meanline model was very close to that measured on the gas stand for all speeds, adding confidence to the parametric CAD model. The aerodynamic efficiency predicted by the meanline model was more than 5% higher than that of the hot map. The reason for this is that hot map turbine efficiency is determined by the total enthalpy rise in the compressor with the following equation:

\[ \eta_{t,\text{hot}} = \frac{\dot{m}_c (h_{02} - h_{01})}{\dot{m}_t (h_{03} - h_{4s})} \]  

where \( \dot{m}_c \) and \( \dot{m}_t \) are the compressor and turbine mass flow rate respectively, and \( h_{02} - h_{01} \) is the total specific enthalpy rise through the compressor while \( h_{03} - h_{4s} \) is the total-to-static isentropic specific enthalpy drop through the turbine. This way of
calculating the efficiency when testing at engine conditions is effective, as the power delivered to the compressor at similar turbine inlet conditions will already include bearing losses and heat transfer effects, such that these do not need to be explicitly modelled. This rests on the underlying assumption that bearing friction and heat transfer effects do not change significantly at different engine operating conditions, and therefore this way of representing turbocharger performance is sufficient.

The meanline maps are however adiabatic and do not include bearing losses, and therefore consideration needs to be made of these effects to avoid large discrepancies to the hot map calibrated engine model after meanline model integration.
Figure 6.14: Mass flow parameter and total to static efficiency maps of the baseline design. The hot map data is shown, and compared to the meanline prediction.
6.5.1.1 Bearing loss modelling

To account for bearing losses in the turbocharger, a simple bearing model was implemented using Petroff’s equation for journal bearing torque:

\[ \tau = \frac{\pi \mu \omega LD^3}{4 \varepsilon_r} \]  

(6.3)

where \( \mu \) is the oil viscosity, \( \omega \) is the rotational speed, \( \varepsilon_r \) is the fluid film thickness, and \( L \) and \( D \) are the bearing length and diameter. For a floating ring journal bearing such as the one used, the equation must be solved for the inner and outer surfaces of the floating ring, which was assumed to rotate at a constant speed, giving an equilibrium of inner and outer surface torque:

\[ (\omega_{\text{shaft}} - \omega_{\text{ring}})K_{\text{inner}} = \omega_{\text{ring}}K_{\text{outer}} \]  

(6.4)

where

\[ K = \left( \frac{LD^3}{\varepsilon_r} \right) \]  

(6.5)

and \textit{inner} and \textit{outer} refer to the inner and outer surfaces of the floating ring. The overall friction torque acting on the shaft within the journal bearing subsequently resolves to the following equation:

\[ \tau = \omega_{\text{shaft}} \frac{\pi \mu K_{\text{inner}}K_{\text{outer}}}{4 K_{\text{inner}} + K_{\text{outer}}} \]  

(6.6)

To model the thrust bearing friction, the Petroff equation can be modified to give the friction torque for a plain double-sided thrust bearing:

\[ \tau = \frac{\pi \mu \omega (r_2^4 - r_1^4)}{\varepsilon_x} \]  

(6.7)

where \( r_1 \) and \( r_2 \) are the inner and outer radii of the bearing, and \( \varepsilon_x \) is the axial clearance. For the bearing system used in the mapped turbocharger, the total bearing friction torque could thus be modelled with the following equation:
\[ \tau = \omega_{\text{shaft}} \pi \mu \left( \frac{1}{4} \frac{K_{\text{inner}} K_{\text{outer}}}{K_{\text{inner}} + K_{\text{outer}}} + K_{\text{thrust}} \right) \]  \hspace{1cm} (6.8)

where \( K_{\text{thrust}} = \frac{(r_2^4 - r_1^4)}{\varepsilon_x} \). The relevant variables for the constants could be obtained from a CAD model of the bearing housing, shown in Figure 6.15. Because the oil film thickness of the thrust bearing is a complex function related to the speed and axial thrust, this was assumed to have a constant value of \( \varepsilon_x = 0.04\text{mm} \). The value of \( \mu \) was taken from the SAE 0W-30 standard of engine oil, at a temperature of 96°C, giving \( \mu = 9.9 \text{ mPa-s} \).

Assuming an outlet static pressure of 101kPa and an inlet total temperature of 600°C (the common conditions when generating hot maps), the meanline predicted efficiency could be corrected for bearing losses, resulting in the efficiencies shown in Figure 6.16. The corrected efficiency is much closer to that measured on the gas stand, except at the lowest speed of 2350 RPM \( K^{0.5} \). At higher pressure ratios for each speed line, the hot map

Figure 6.15: Section view of the bearing housing used in the baseline turbocharger
efficiency drops, where the meanline efficiency with bearing friction is still increasing. A possible cause of this is the assumption that the bearing friction is only a function of speed, while the pressure ratio is likely to increase the axial load on the thrust bearing, causing an increase in bearing losses at higher pressure ratios. To model this would require a level of complexity which was impossible to achieve with the available information, therefore the effect was neglected.

6.5.1.2 Heat transfer modelling

The other effect which can be observed is the increase in hot map efficiency for the lowest speedline, 2350 RPM $K^{0.5}$, due to heat transfer effects. This is contrary to the meanline efficiency with bearing loss correction which drops significantly at low speeds. The map efficiency at this speed is important for the transient model, as the reduced speed can be as low as 1000 RPM $K^{0.5}$ at the starting point of the transient load step. The 10% difference seen between the bearing friction corrected meanline model and the hot map at low speed would cause larger discrepancies in the transient model with the integrated meanline model, and therefore heat transfer should be accounted for.

As shown in Equation 6.2, the turbine efficiency is calculated using the compressor total enthalpy rise $\dot{m}_c(h_{02} - h_{01})$. For a turbocharger tested at hot turbine inlet temperatures, the performance can no longer be assumed to be adiabatic, and therefore the total enthalpy rise in the compressor will include heat transfer to the flow:

$$\dot{m}_c(h_{02} - h_{01}) = \dot{W}_{shaft} + \dot{Q}$$  \hspace{1cm} (6.9)

where $\dot{W}_{shaft}$ is the shaft power and $\dot{Q}$ is the heat transfer into the compressor, primarily from the turbine. Changes in $\dot{Q}$ have been shown by Casey & Fesich (2009) and Sirakov & Casey (2012) to be relatively small across the turbocharger operating range. This means that the effect on efficiency becomes higher at low powers, leading to the observed increase in efficiency on the lowest speed line in the map. This observation was confirmed by Lüddecke et al. (2012) who developed a simplified model for map correction using a constant heat flux into the compressor.

This approach was implemented in the current work, and it was found that a heat
flux of 90W gave satisfactory results as shown in Figure 6.16. At high turbo speeds, the heat flux represented less than 1% of the aerodynamic power and could therefore be considered almost negligible, while at the lowest speed, it had a value of up to 27%, bringing the corrected meanline efficiency within 5% of the hot map efficiency.

![Figure 6.16: Total to static turbine efficiency of the hot map, with the aerodynamic, friction corrected and final (friction and heat transfer corrected) meanline generated map.](image)

6.5.1.3 Inertia modelling

The final consideration of importance for integration of new turbine designs using the meanline model, was the inclusion of rotor inertia, which is of critical importance to the transient response. This was modelled using the rotating components of the turbocharger, as seen in Figure 6.15 of the bearing housing. For the analysis, the baseline turbine rotor was replaced with the one generated by the parametric CAD model, and the inertia of the whole system could be calculated for a given turbine design.
6.5.1.4 Comparison to GT-Power Extrapolation

To model the turbine in the pulsating conditions of the engine, GT-Power has its own extrapolation procedure for narrow hot maps which is described in the flow theory manual (Gamma Technologies 2016). The test bench data is input, and a grid of turbine characteristics is generated during pre-processing for interpolation during the running of the model. The method normalises the points for both mass flow parameter and efficiency, and fits a single function to both.

Implementing the meanline model into GT-Power not only changes the fit to the available data, but also the extrapolation to pressure ratios beyond this range, as the meanline model includes implicit physical modelling. To fully understand the differences between the meanline integration and the standard hot map extrapolation, the full range of pressure ratios need to be considered.

Figure 6.17 shows the comparison between the GT-Power extrapolation of the hot map data and the integrated meanline map at two reduced speeds. It is evident that discrepancies exist, and these are likely to cause differences in engine performance.

The mass flow parameter at low speeds is relatively close, although the meanline model predicts a slightly lower choking mass flow. At higher speed, it can be seen that the prediction of low pressure ratio mass flow is very different. While the meanline model predicts the effect of a pressure ratio of around 1.28 at zero mass flow, which is close to that seen in the testing of the upscaled turbine in Chapter 5, the GT-Power extrapolation has shows a very different trend, going to a pressure ratio of unity at zero mass flow.

The efficiency is also different, though at low pressure ratios the prediction is relatively close. When extrapolating to higher pressure ratios however, the efficiency of the two methods diverges at both low and high speed leading to differences of more than 5% at \( PR \) values higher than 1.5 at low speed.
Figure 6.17: Mass flow parameter and efficiency maps, comparing the performance prediction of the GT-Power extrapolation and the meanline model over the full pressure ratio range.
6.5.2 Engine Performance Comparison

To evaluate the impact of map differences seen between the meanline integrated model and the hot map extrapolation by GT-Power, both maps were implemented in the models of the objective parameters, and the engine performance compared.

The difference in BSFC for both operating points introduced in Table 6.1 is shown in Table 6.2. The fuel consumption is higher at both operating points, with the average BSFC being 0.12% higher.

Table 6.2: Modelled BSFC at both operating points, using the hot map and the meanline model for modelling of the turbine

<table>
<thead>
<tr>
<th>BSFC (g/kWh)</th>
<th>Point 1</th>
<th>Point 2</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hot Map</td>
<td>289.7</td>
<td>270.6</td>
<td>280.2</td>
</tr>
<tr>
<td>Meanline</td>
<td>290.0</td>
<td>271.0</td>
<td>280.5</td>
</tr>
<tr>
<td>∆ (%)</td>
<td>0.10</td>
<td>0.15</td>
<td>0.12</td>
</tr>
</tbody>
</table>

The low end torque model was run across engine speeds, and as can be seen in Figure 6.18. At speeds of 2000 and above, the torque is nearly the same, as the wastegate is open, limiting the compressor outlet pressure. The torque comparison at these conditions is primarily dictated by the back pressure caused by the turbine, which is comparable for both maps. At lower speeds, the model using the integrated meanline map exhibits a higher torque, particularly at 1750RPM where the predicted BMEP is 1.7 Bar higher using the meanline integrated map.

The transient response was also different, as can be seen in Figure 6.19. The response with the meanline integration was faster, primarily due to a higher rise in torque immediately after throttle opening. The reason for this was the higher efficiency of the meanline map at the low speeds at the beginning of the transient. This led to a reduction in time to torque from 2.08 s to 1.72 s, 0.35 s faster.

These differences between the models using the baseline turbocharger hot map and those using the integrated meanline maps to model the turbine mean that the significance of any result is limited to changes with respect to the meanline integrated baseline model.
Figure 6.18: Full load torque curve of the low end torque objective parameter model using a hot map to model the turbine compared to the meanline integration.

Changes in heat transfer effects and bearing losses can be assumed to negligible with the small changes made to the turbine design, such that the differences seen when comparing turbines of different aerodynamic performance using the meanline integrated model are representative of actual engine performance.
Figure 6.19: Transient response of the modelled engine using a hot map to model the turbine compared to the meanline integration.

6.6 Engine Performance Analysis and Optimisation

Integration of the meanline model into the GT-Power engine models for the three objectives, BSFC, transient response time and low end torque, allowed an investigation into the effects of turbine design parameters on the engine performance. The four design parameters which were investigated were turbine wheel size $r_2$, $A/r$ ratio, exducer tip radius $r_{3t}$ and the exit angle, represented by the parameter $d\theta/dz$.

Of these, the wheel size $r_2$ and $A/r$ ratio are the two parameters which are used in traditional matching, because these are varied in the turbochargers provided by manufacturers along with maps. A selection of 'frame sizes' are available, referring to the size of the turbine wheel, and for each frame size, the $A/r$ ratio can varied to change the swallowing capacity according to the requirements determined by the matching calculations for the engine. The other two parameters, $r_{3t}$ and $d\theta/dz$, also have an impact on turbine performance and can therefore be optimised on an engine performance level.

In the presented setup, $r_2$ acts as a scaling parameter, such that the entire geometry scales relative to it. This means that the other parameter values shown in this chapter are
relative to the baseline value of \( r_2 = 19.25 \text{mm} \). The accuracy of \( r_2 \) scaling has not been explicitly investigated in this work, unlike the other parameters. However, it is known that turbine performance scales, provided that the Reynolds number is not significantly changed Whitfield & Baines (1990) as would be the case within the range applied.

6.6.1 Designs of Experiments

The selected parameters were investigated and optimised using a DOE, available for selection in GT-Power. A latin hypercube with 500 designs was generated and meanline maps output for each, within the upper and lower bounds shown in Table 6.3. These maps, along with the inertia determined for each design, were integrated into the engine models with the procedure outlined above. As three objective parameters exist, the optimisation problem was essentially three-dimensional. Analysis was however easier considering two parameters simultaneously. The scatter plots of the DOE results in the analysis sections below therefore plot one parameter against another and are divided into two colours, black for an improvement of the third parameter and grey for deterioration.

Table 6.3: Geometric parameter bounds for the Latin Hypercube design of experiment

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_2 )</td>
<td>mm</td>
<td>18</td>
<td>21</td>
</tr>
<tr>
<td>( \frac{A_1}{r_1} )</td>
<td>mm</td>
<td>9</td>
<td>13</td>
</tr>
<tr>
<td>( \frac{d\theta}{dz}_3 )</td>
<td>deg mm(^{-1})</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>( r_{3t} )</td>
<td>mm</td>
<td>15.15</td>
<td>18.15</td>
</tr>
</tbody>
</table>

In addition to the DOE, sweeps of 20 points of each parameter increasing linearly from the lower to upper bound where conducted, keeping the other parameters constant. This allowed additional insight into the effect of each parameter.
6.6.2 Effect of \( r_2 \) and \( A/r \) ratio

As described, the two design parameters \( r_2 \) and \( A/r \) ratio are most commonly used to match a turbine to the engine. Analysis was made of these two parameters by sweeping each parameter at three different values of the other and running the engine models for the three objective parameters with maps of each design. Figures 6.20 and 6.21 show the transient time to torque (\( \Delta t_{\text{torque}} \)) and the low end torque (\( \Delta BMEP_{1500} \)) vs. BSFC for the resulting sweeps respectively, superimposed on a scatter plot of the full LHS set of designs.

![Figure 6.20: Sweeps of \( r_2 \) and \( A/r \) and their effect on time to torque vs. BSFC. The lines are superimposed onto the results of the latin hypercube sample of designs](image)

Figure 6.20: Sweeps of \( r_2 \) and \( A/r \) and their effect on time to torque vs. BSFC. The lines are superimposed onto the results of the latin hypercube sample of designs.

In both Figures the different effect of \( r_2 \) and \( A/r \) on engine performance can be seen very clearly. In terms BSFC, there is an increase with the reduction of either parameter, which is primarily due to the decrease in swallowing capacity. A smaller turbine requires a higher pressure ratio to deliver the same power to the compressor, therefore causing a
higher back pressure, and an increase in pumping losses. Consequently, a higher air and fuel mass flow rate is required to deliver the targeted torque.

In terms of both transient response time and low end torque, the effect of both parameters differs more significantly. An increase in $A/r$ leads to an increase in transient time to torque, and a reduction in low end torque. Analysis of the turbine performance, seen Figure 6.22 at a reduced speed of 3000 RPM $K^{0.5}$ shows that this is due to an increase in swallowing capacity and a decreasing efficiency. A higher swallowing capacity means that lower power is delivered to the compressor at the same mass flow rate, slowing the transient response and decreasing the maximum torque.

The effect of $r_2$ is markedly different to that of $A/r$ ratio. While the transient response time increases with $r_2$ starting from the baseline size, it does so with decreasing $r_2$ as well, the baseline value coinciding with the optimum transient response for a given $A/r$ ratio. Again, the performance maps shown in Figure 6.23 indicate the reason for this:
a decrease in $r_2$ leads to a decrease in both swallowing capacity and efficiency (while for $A/r$ ratio, the trends are opposed).

The primary mechanism here is scaling, as the truly non-dimensional maps at different turbine radii would be very similar within the range investigated. However, as the rotational speed is dictated by the compressor, it makes sense to compare turbine performance at the same absolute speed. Thus, the larger turbine runs at a higher non-dimensional speed, when at the same compressor speed, with the resulting efficiency differences seen in Figure 6.23. The higher $r_2$ then, the higher the efficiency of the turbine at the pressure ratios covered in the transient run.

This difference in the efficiency map is even more visible in its effect on low end torque, as a decrease in $r_2$ consistently results in a decrease in low end torque, making lines of sweeping $r_2$ and $A/r$ ratio almost perpendicular to each other. The low end torque is seen to be more affected by the impact of efficiency, while the BSFC is mostly affected by the resulting changes in mass flow parameter.

Analysis of Figures 6.20 and 6.21 shows that a Pareto front of optimum designs can be formed using the results of the latin hypercube DOE (analysed in more detail in Section 6.6.4 below). This Pareto front in both plots runs almost parallel to the $A/r$ ratio sweep at the upper bound value of $r_2$. This indicates a clear potential for performance improvement through an increase in $r_2$ with respect to the baseline, while
an change in $A/r$ ratio is more likely to lead to a compromise between the three objective parameters. From the results, it is unclear what the optimum value of $r_2$ is, and further analysis would be required to determine this. However, both figures show the gradient of transient response vs. BSFC and low end torque vs. BSFC changing, suggesting that it will eventually reach a point where further increase in the upper bound value of $r_2$ would no longer lead to a shift in the Pareto front.

6.6.3 Effect of $r_{3t}$ and $d\theta/dz$

Similar sweeps were conducted for the parameters $r_{3t}$ and $d\theta/dz$, keeping all other parameters at baseline value. The resulting effect on engine performance can be seen in Figures 6.24 and 6.25.

To some extent, the effect of $r_{3t}$ is similar to that of $r_2$, with an optimum value for both transient response time and low end torque. Inspection of the turbine performance in Figure 6.26 shows an increase in mass flow with $r_{3t}$ and a decrease in peak efficiency, but an increase in efficiency at high pressure ratios. Increasing $r_{3t}$ thus impacts the transient by increasing the swallowing capacity which in addition to decreasing the rate of pressure build up, lowers the average pressure ratio during a given engine cycle. This leads to a worse performance, while higher mass flow and higher efficiency at higher pressure ratio range lead to improved BSFC. Overall however, the difference caused by $r_{3t}$ is small, with
changes in transient performance of less than 0.2s overall, and only a minor influence on BSFC.

\( d\theta/dz \) on the other hand, has a profound effect on transient performance, and to some extent the low end torque, but a lesser effect on BSFC. Increasing \( d\theta/dz \) starting from the lower bound value, leads to a lower swallowing capacity and a higher efficiency, as seen in Figure 6.27. Both effects will improve transient performance and low end torque, which explains the trends for these parameters. The lower mass flow parameter at high \( d\theta/dz \) leads to an increase in BSFC, despite the higher turbine efficiency. This highlights the importance of swallowing capacity in determining fuel economy.

For the parameter sweeps analysed using \( r_{3t} \) and \( d\theta/dz \), the potential for optimisation is limited, with the baseline values proving to be well positioned. Changes in neither of the two parameters would bring the performance significantly nearer to the Pareto front. However, it is evident that incorrect selection of \( d\theta/dz \) can have a detrimental effect on transient performance primarily due to the lower efficiency of the resulting turbine.
Figure 6.25: Sweeps of $r_{3t}$ and $d\theta/dz$ and their effect on low end torque at 1500RPM vs. BSFC, referenced to the baseline design. The lines are superimposed onto the results of the latin hypercube sample of designs.

Figure 6.26: Maps of MFP and $\eta_{ts}$ at 3000 RPM $K^{-0.5}$ for three different values of $r_{3t}$
Figure 6.27: Maps of MFP and $\eta_{ts}$ at 3000 RPM $K^{-0.5}$ for three different values of $d\theta/dz$. 
6.6.4 Selection of Three Designs

To conclude the analysis, three optimised designs were selected from the LHS sample, as shown in Figures 6.28 and 6.29. Both figures exhibit a Pareto front of optimum designs when considering only two objectives. The three designs were selected from the Pareto fronts of either Figure on the following basis:

**Design 1**: The point with the lowest BSFC, without making a compromise on low end torque with respect to the baseline.

**Design 2**: The point with the lowest BSFC, without a compromise on transient performance with respect to the baseline.

**Design 3**: A point with very little compromise in BSFC, while showing an improvement in low end torque.

![Diagram of LHS points on axes of transient response change vs. BSFC change with respect to the baseline design. Three selected optimised designs are highlighted.](image-url)

Figure 6.28: LHS points on axes of transient response change vs. BSFC change with respect to the baseline design. Three selected optimised designs are highlighted
The parameters for each design are shown in Table 6.4. It is evident that the parameter $r_2$ was maximised for all designs (the upper bound being 21mm), while the other parameters change to target the desired position on the Pareto fronts. While the most important parameter to these positions is the $A/r$ ratio (the $A/r$ sweep at maximum $r_2$ is close to the Pareto front in Figures 6.20 and 6.21), $r_3$ and $d\theta/dz$ also change to fulfil the three criteria determined for the designs.

The impact of the design changes on turbine performance can be seen in Figure 6.30. The optimisation of the meanline parameters led to designs with a higher efficiency, similar for all designs, while the mass flow parameter was different according to the position on the Pareto front. The improvement in efficiency was mostly due to the increased turbine size at the given compressor speed, while the other parameters adjusted to determine the position on the Pareto front. Design 3 had a swallowing capacity similar to that of the baseline, resulting in only a small change in BSFC, while the change in
efficiency led to an improvement of both transient response and low end torque. Designs 2 and 1 have successively higher swallowing capacities, leading to improved BSFC as well as a reduction in low end torque and an increase in transient response time.

Figure 6.30: MFP and $\eta_{t-s}$ at 3000 RPM $K^{-0.5}$ for the baseline design and the three optimised designs.

### 6.6.4.2 Engine Performance

The change in average part load fuel consumption of the optimised designs can be seen in Figure 6.28 above. It is however of additional interest how this divides between the two operating conditions modelled. This division is shown in Table 6.5. It can be seen that the percentage improvement in BSFC is higher for operating point 2 (3750RPM, 100Nm), by around 0.2% for all Designs. This includes Design 3, where there is an increase in
Table 6.5: Change in BSFC relative to the baseline for each optimised design, at both modelled operating conditions

<table>
<thead>
<tr>
<th></th>
<th>OP 1</th>
<th>OP 2</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design 1</td>
<td>-0.51</td>
<td>-0.71</td>
<td>-0.60</td>
</tr>
<tr>
<td>Design 2</td>
<td>-0.27</td>
<td>-0.50</td>
<td>-0.38</td>
</tr>
<tr>
<td>Design 3</td>
<td>0.11</td>
<td>-0.07</td>
<td>0.03</td>
</tr>
</tbody>
</table>

BSFC for OP 1 (1500RPM, 60N-m) and a very small decrease for OP 2. The difference in average BSFC seems to be mostly driven by the swallowing capacity of the turbine, although OP 2 has a slightly lower BSFC with Design 3, despite this design having a slightly lower swallowing capacity than the baseline, suggesting that the OP 2 has more sensitivity to the efficiency change.

The impact on full load performance was examined in more detail by running the selected designs at all speeds with wide open throttle, with the resulting full load BMEP lines shown in Figure 6.31. At 1500 RPM, the difference in torque between the three designs is primarily dependent on the swallowing capacity, while the baseline has a lower torque due to the lower efficiency of the turbine. While Design 3 has the highest torque at 1500RPM, Design 2 does so at 1750RPM. At this operating point, both Designs cause boosting in excess of the wastegate controller allowance, so that the wastegate is opened. With an open wastegate, the compressor outlet pressure is limited to 170 kPa, so that the design with lower swallowing capacity, causing a higher back pressure for the same boost, induces a lower engine torque. This trend is seen for all optimised designs at 2000 RPM and higher. The trend from 1500 RPM is reversed so that the largest turbine has the highest torque. The baseline design, with lower turbine efficiency, had a lower torque than all optimised designs, at all operating points. This highlights the importance of efficiency for the low end torque of an engine.

The transient engine torque for each optimised design is shown in Figure 6.32. The difference between the transient response of the three designs and the baseline is relatively small, at only 0.31s in time to torque between Design 1 and Design 3. From 0.4 to 1s into the transient, the engine with the baseline design has a higher torque than the other
Figure 6.31: Full load BMEP with the baseline turbine and the selected turbine designs. designs due to the lower inertia of the smaller rotor. Due to the difference in maximum torque at 1500 RPM the lines eventually cross, first Design 3 at 1s and then Design 2 at 1.62s. Eventually the torque of Design 1 would overtake as well. Overall, for the changes made to the turbine design, the transient response is less dependent on rotor inertia than on the efficiency and swallowing capacity of the turbine.
6.7 Summary & Conclusions

To investigate the possibility of further optimisation of turbine design for engine performance, a meanline model was integrated into a calibrated GT-Power engine model to predict turbine performance as a function of key design parameters. The model included the effect of bearing friction and heat transfer, as well as rotor inertia. The performance parameters which were investigated were fuel consumption (BSFC), low end torque at 1500RPM, and the transient response of a constant speed load step.

Four turbine design parameters were chosen and sweeps of their value modelled, giving interesting insights into their impact on engine performance. Additionally a latin hypercube sample of 500 designs was generated and modelled, allowing optimisation of the turbine design for the three objective parameters. It was found that an increase in turbine radius would result in an improved engine performance through improvement of efficiency in relevant portions of the turbine map, while the other design parameters adjusted to target different objectives with regards to the engine performance. A 0.6% improvement in fuel economy was possible without compromise of the other performance parameters, while the low end BMEP at 1500 RPM could be raised by 1.2 Bar from 12.6
to 13.8 Bar.

These improvements show that further optimisation of matching is possible if several parameters are taken into account, as opposed to the conventional method of matching using just the volute $A/r$ for scaling of the mass flow after selection of a discrete turbine diameter.
Chapter 7

Conclusions & Future Work

The overarching purpose of the work presented in this thesis was the development of a methodology for optimising turbine design for improved turbocharged engine performance. The approach taken to achieve this was the integration of a radial turbine meanline model into the engine modelling software GT-Power, to allow turbine performance effects within the engine to be modelled as a consequence of turbine design.

To assess the viability of this approach, the developed meanline model was compared to CFD and experimental data. This was achieved using a 3D parametric model of the turbine stage, allowing the generation of meshes for CFD, as well as manufacturable geometries for testing. Using the resulting data, the loss models included in the meanline model were evaluated, and the best combination of models for prediction of the full design space was selected.

The meanline model was integrated into GT-Power, and simple models included to account for bearing friction and heat transfer effects. A engine model for the Proton CAMPRO CFE engine was developed and calibrated using engine dynamometer test data. The meanline model was calibrated to CFD results for the geometry of the baseline turbocharger turbine, and subsequently used within the engine model to assess the effect of key geometry parameters on the fuel consumption, transient response and low end torque. Using a design of experiment approach, three turbine geometries were found as optimised designs, each representing a different compromise on engine performance while showing improvement over the baseline design.
7.1 Conclusions

The introduction outlined the key objectives which were geared towards achieving the goal of optimising the engine performance. The following presents the conclusions which could be made regarding each of the objectives.

1. Development of a low order model for the turbocharger turbine based on physical principles:

This objective involved the development of a meanline model code, able to predict radial turbine performance robustly and at low computational cost. The code was written in the FORTRAN language and was based on the NASA meanline method, although the algorithms used for solving for turbine performance were modified significantly to increase speed and reliability across the full operating range and design space.

The meanline model solves the velocity triangles of the turbine, using the free vortex assumption for the volute. Several sources of loss in radial turbines have been identified, and the model accounts for these using loss models from the literature and were applicable, several different models for a given loss were made available for selection in the code. Additionally, a new blockage model for the rotor exducer was developed.

The main inputs to the code included the meanline geometric parameters required for solving the equations as well as calibration parameters for the models used. A novel feature in the presented code was the inclusion of a self-calibration scheme to fit the model to available data. This was done using a genetic algorithm, which given a set of turbine performance data and bounds for each calibration parameter, finds the optimum set of calibration parameter values to minimise the average error of both mass flow and efficiency prediction.

2. Investigation of the low order turbine meanline model:

To evaluate the accuracy of the meanline model across design changes and for a wide operating range, CFD and turbine testing was conducted. A parametric 3D model
of the turbine was developed, based on the design of a manufacturer turbocharger used for the engine platform which was subsequently analysed. The purpose of the parametric model was to generate a turbine geometry given a set of turbine design parameters, namely those used as inputs to the meanline model.

A large dataset, sampled from three design parameters ($A/r$ ratio, exducer tip radius $r_{3t}$ and exducer camberline gradient $d\theta/dz$) and two operating parameters (reduced speed and pressure ratio) was generated by CFD in an automated process, using the parametric 3D model. The meanline model was calibrated to eight operating points of the baseline design calculated by the same CFD process, and subsequently applied to the geometries and operating conditions sampled in the dataset.

The loss models included in the code were compared and a combination of models which performed best across the dataset was selected, with an RMS error of 1.00% for mass flow parameter prediction and 1.33 percentage points for efficiency.

Additionally, insights were given into error resulting from the different parameters which were sampled, showing that efficiency prediction error generally increased at $U/C_{is}$ values of greater than 0.7, and that the exducer camberline gradient $d\theta/dz$ accounted for a large proportion of the error in the dataset.

The meanline model accuracy was further investigated by testing of six rapid prototyped turbine designs at the Imperial College radial turbine testing facility. The resulting data showed that the meanline model, when calibrated to selected data-points from the baseline design, was able to predict mass flow parameter for the other tested designs within 3% at all operating conditions, while the efficiency was predicted to within 2.8%. The overall conclusion of the investigation was that the accuracy of the meanline model is sufficient for the results of an integration into an engine model to be of significance.

3. Methodology supporting the integration of the meanline model into an engine simulation tool:

To conduct a turbine design investigation and optimisation, an engine model was
developed using engine performance data and turbocharger hot maps from a Proton CAMPRO CFE engine. The control logic of the calibrated model was modified to generate models calculating the three different engine performance parameters which were outlined in the introduction: Fuel consumption, transient response and low end torque. For fuel consumption, two operating conditions were selected, based on where the most fuel is consumed during a WLTC driving cycle for the engine.

The meanline model was then integrated into the engine model, replacing the hot turbine map used for the engine model calibration. Rotor inertia was modelled using the 3D parametric CAD model to allow correct prediction of transient response. A simple friction model was included, as well as a heat flux to account for heat transfer effects. As the complexity of friction and heat transfer phenomena in the hot map are difficult to model correctly given the available measurements, the integration of the meanline model led to discrepancies in the performance parameters when compared to the calibrated baseline model. This caused a difference in BSFC of 0.12%, while the transient time to torque decreased by 0.35 s and the full load BMEP at 1500 RPM increased by 0.6 Bar. These effects can however be assumed to remain constant for the small intended changes of turbine design, and so subsequent changes in engine performance due to turbine design modifications could be compared.

4. Study and evaluation of the engine performance based on the integrated turbine model:

Using the engine model and integrated meanline model, a study was conducted on the effect of different turbine design parameters on engine performance. Four geometric parameters were investigated, showing that while exducer tip radius and exit angle had an impact on engine performance, it was the turbine size (scaling of the full design), and the $A/r$ ratio which showed the most potential for optimisation.

An increase in turbine size by 0.91% (turbine rotor inlet diameter from 38.5mm to 42mm) was found to reduce fuel consumption by 0.5%, while increasing BMEP at 1500 RPM by 0.2 bar but also causing a small increase in transient response
time. This was driven by the larger swallowing capacity and a shift in peak efficiency pressure ratio to more engine relevant values at given rotational speeds. Analysis of the trends showed that further increasing the turbine size would lead to additional improvements in fuel economy, however at a higher cost to the transient response. Changing only the $A/r$ ratio was found to lead to more compromise, with fuel economy improvements only being possible at the expense of the other two parameters.

An optimisation was performed by running the models with a latin hypercube sample of 500 different turbine designs. Plotting low end torque and transient response against BSFC, allowed two Pareto frontiers of optimised designs to be determined. It was found that a 0.6% reduction in fuel consumption was possible, without any reduction of low end torque and only a 0.12s increase in transient time to torque. The fuel saving was limited to 0.4% without any compromise on transient response. As it is likely however that the turbine was originally matched with some margin on the maximum transient response time, it is likely that small increases would still provide an acceptable match.

### 7.2 Future Work

#### 7.2.1 Implementation with nozzled and variable geometry volutes

The present investigation focused on the optimization of a nozzleless turbine. However, nozzled turbine volutes are also commonly used in automotive engines, as well as variable geometry designs, which can change the nozzle area according to the requirements of the engine operating point. In future work, nozzles and variable nozzles can be included in the meanline model, and optimisation of additional parameters conducted. The use of variable geometry turbines adds another layer of engine control which can also be optimised.
7.2.2 Integration of a compressor performance model

Although the present work focused on the effect of turbine matching, the compressor plays a hugely important role on the engine performance. Improvements could be realised by inclusion of a compressor meanline model so that compressor and turbine design parameters could be modelled simultaneously. An improved compressor match would in turn impact the optimum turbine design.

As the mechanical constraints on the compressor wheel do not require radial fibred blading, the outlet blade angle could be used as an additional design parameter. Care would have to be taken in prediction of the compressor operational limits, ie the surge and choke lines, such that optimised matches do not operate outside of these limits, potentially causing damage to the compressor.

7.2.3 Manufacture and engine testing of turbocharger prototypes

Engine testing of one or more turbine designs optimised by the methodology presented in this thesis would complement the work and give confidence in the results. Additional value would be added by including optimised compressor designs, ie. re-matching the full turbocharger stage. Both the baseline model and the optimised designs would have to be manufactured and tested to ensure comparability of the results.

Design, manufacture and testing of new geometries would bring additional challenges as they would have to withstand the high mechanical stresses encountered during engine operation. This work would therefore require the additional modelling of stresses to reduce the risk of catastrophic failure.

7.2.4 Heat transfer modelling

The present work uses a simplified approach to predict the effect of heat transfer on the turbine performance, as a more complex model would have required additional data. Testing of heat transfer effects could provide the data required to generate a more rigorous heat transfer model, enhancing the accuracy of the integrated turbine prediction. This experimental work would require the setup of a new hot gas stand with relevant
instrumentation, and would be further enhanced by taking on-engine heat transfer measurements.

Although it is more common to superimpose heat transfer effects onto adiabatic turbine performance maps (Romagnoli 2010), an alternative method could be the introduction of heat transfer effects into the meanline model itself. This would allow the physical parameters calculated by the meanline model to be used to inform physics based heat transfer correlations and lead to the prediction of diabatic turbine performance.

Integration of a compressor model, and models for the heat transfer through other components in the turbocharger (such as the bearing housing) would give a more realistic representation of overall turbocharger performance than is commonly achieved. As more parameters are modelled, a requirement would be to move away from a map based integration of the meanline model, and introduce a fully integrated model which is solved for every timestep.

7.2.5 Application of machine learning to available datasets

A large amount of turbine performance data was generated as a result of this work, in particular through the CFD study presented in Chapter 4. This data should be used for the application of machine learning tools for predicting turbine performance.

This could for instance be achieved through the implementation of artificial neural networks which are trained using sample points from the CFD dataset, and validated using the remaining points. Given enough training data, an artificial neural network could potentially give a better prediction of performance than the meanline modelling approach, with the limitation that it is likely perform poorly when extrapolating beyond the limits of the data. The use of different subsets of data could give insight into the minimum requirements for training an accurate model, an important factor for determining the computational cost of such a methodology.

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URL: https://doi.org/10.4271/800029

URL: https://doi.org/10.1243/PIME_PROC_1962_176_022_02


Appendix A

Experimental Results

For completeness, the results not shown in Chapter 5 are included here. Figures A.1 and A.2 show the remaining speedlines resulting from the calibration analysis in Section 5.5.1.

Figures A.3 to A.7 show comparison between the meanline prediction and experimental efficiency for all designs and speedlines, analysed in Section 5.5.2.
Figure A.1: Experimentally acquired speed lines of mass flow parameter for the baseline design, compared to the meanline model prediction after calibration with three different sets of data points.
Figure A.2: Experimentally acquired speed lines of efficiency for the baseline design, compared to the meanline model prediction after calibration with three different sets of data points.
Figure A.3: Measured efficiency of turbine designs 1-6 compared to meanline prediction (dashed lines) and meanline prediction of the baseline design at a turbine rotational speed of 20 000 RPM
Figure A.4: Measured efficiency of turbine designs 1-6 compared to meanline prediction (dashed lines) and meanline prediction of the baseline design at a turbine rotational speed of 30 000 RPM
Figure A.5: Measured efficiency of turbine designs 1-6 compared to meanline prediction (dashed lines) and meanline prediction of the baseline design at a turbine rotational speed of 40 000 RPM
Figure A.6: Measured efficiency of turbine designs 1-6 compared to meanline prediction (dashed lines) and meanline prediction of the baseline design at a turbine rotational speed of 50 000 RPM
Figure A.7: Measured efficiency of turbine designs 1-6 compared to meanline prediction (dashed lines) and meanline prediction of the baseline design at a turbine rotational speed of 60 000 RPM