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A Mathematical Model for Reduction of Trim Loss in Cutting Reels at a Make-to-Order Paper Mill

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Abstract: One of the main issues in a paper mill is the minimization of trim loss when cutting master reels and stocked reels into reels of smaller required widths. The losses produced in trimming at a paper mill are reprocessed by using different chemicals which contributes to significant discharge of effluent to surface water and causes environmental damage. This paper presents a real-world industrial problem of production planning and cutting optimization of reels at a paper mill and differs from other cutting stock problems by considering production and cutting of master reels of flexible widths and cutting already stocked over-produced and useable leftover reels of smaller widths. The cutting process of reels is performed with a limited number of cutting knives at the winder. The problem is formulated as a linear programming model where the generation of all feasible cutting patterns determines the columns of the constraint matrix. The model is solved optimally using simplex algorithm with the objective of trim loss minimization while satisfying a set of constraints. The solution obtained is rounded in a post-optimization procedure in order to satisfy integer constraints. When tested on data from the paper mill, the results of the proposed model showed a significant reduction in trim loss and outperformed traditional exact approaches. The cutting optimization resulted in minimum losses in paper trimming and a lesser amount of paper is reprocessed to make new reels which reduced the discharge of effluent to the environment.

Keywords: process optimization for waste reduction; combinatorial optimization; integer programming; waste generation

1. Introduction

Cutting stock problem has many applications and is one of the representative combinatorial optimization problems. In a paper mill, master reels of different widths and internal and external diameters are produced while orders are accepted for reels of smaller widths grouped by grade, thickness and type of paper. A pattern is a feasible combination of smaller required widths cut from master reels. A limited number of knives at winder cut master reels into reels of smaller required widths which are either sold as such or sheeted into required dimensions. An optimal solution to the

problem is the number of times a feasible pattern associated with master reels, over-produced reels, and useable leftovers are cut to satisfy ordered quantities while considering minimization of waste. The involvement of various operational and technological constraints in the planning process causes the trim loss minimization problem more interesting and difficult.

As requested by a paper mill at Peshawar, a system has been developed and described in this paper for the support of its production planning which focuses on the production of master reels and cutting master reels, over-produced reels and leftovers to fill an order. Being part of COOL, which is a broader system, this work intends to support the application of an optimizing plan for the production of paper and stock management.

New mathematical models and approaches are developed to solve the constraints that arise in practical situations at a paper mill. Though there are various types of cutting stock problems, a problem which has not been frequently studied and can be included in the broad family of cutting stock problem, consists of considering master reels of flexible widths, over-produced reels of smaller widths and useable leftovers when objects are cut to satisfy demands. In cutting master reels, over-produced reels and leftovers are stocked for future use and are not wasted, therefore, the problem is called cutting stock and useable leftover problem. A one-dimensional approach has been devised and trim loss at the end of the reels is disregarded in the considered problem formulation. The same approach can also be applied in other industries as well, such as steel [1,2] wood, leather, and plastic industry [3].

A limited number of cutting knives at winder cut the reels into smaller reels of required widths. All feasible patterns are generated and the knives are adjusted in such a way that trim loss is reduced. Knife setting can benefit in reducing pollution load by minimizing trim loss and reducing reprocessing of paper trimming. In the reprocessing and recycling of paper, a harsh chemical called bleach is used for the purpose of paper brightness, which is harmful to both environment and health. Chlorine and chlorine compounds are used in the pulp bleaching which react with lignin by different chemical processes and a large number of organochlorine chemicals are produced and discharged to surface water [4]. These chemicals were reported to increase the risk of hormone-related cancers including lung, stomach, prostate, and breast cancer [5]. Several other types of sludge and solid wastes are generated in paper mills due to wastewater treatment and deinking. The recycling of one ton of paper produces about 300 kg of sludge in a paper mill [6].

In this paper, an original solution method is proposed for a particular cutting stock problem with the main objective of trim loss minimization and reduction in discharge of effluent to the environment. The production and cutting of master reels of flexible widths are considered and stocked over-produced and leftover reels are cut to fill an order previously ignored. Various operational and technological constraints are considered in the production and cutting process and all feasible cutting patterns are generated by combining smaller required widths. The developed linear programming model with the objective of waste minimization is solved through simplex algorithm. In order to satisfy the integer constraints, the solution obtained is rounded by a new post-optimization procedure, if the initially obtained solution contains non-integer decision variables. Although the problem still poses challenges in practice, so by solving real-life problems, the proposed model provides a benchmark for paper mills while cutting master reels and stocked reels to meet a demand for reels of smaller required widths. The paper mill has confirmed that by implementing the proposed method, significant improvements in terms of trim loss minimization and reduction in discharge of effluents into the environment has been achieved and also it outperformed other traditional approaches.

The remainder of the paper is outlined as follows. A brief literature review is provided in Section 2, where studies that are closely related to the context of our problem are referred. In Section 3, a methodology developed to solve the problem is described. In order to illustrate the solution procedure, a small example is considered throughout Section 4. The results are presented and discussed in Section 5 while Section 6 presents the conclusion and future research directions.

2. Literature Review

Since there is very rich literature on trim loss minimization problems, therefore, studies that are most relevant to our current work have been reviewed in this section.

In literature, various types of cutting stock problems have been investigated [7–10]. Effective algorithms for solving cutting stock problems are beneficial to reduce production cost and improve material utilization [11]. Correia et al. [12] presented reel and sheet cutting at a paper mill. To satisfy customer orders, the reels were first cut into smaller widths which were then cut into finished reels or sheets. The proposed solution procedure comprises of three stages. At the first stage, all feasible cutting patterns were enumerated. The second stage solved the linear programming model and a rounding procedure was used in the third stage for obtaining integer solution. Chauhan et al. [13] discussed the roll assortment optimization in a paper mill. The production of large reels on cyclical basis was considered and the reels produced were then cut into smaller size. The problem was formulated as a binary nonlinear programming model and two solution methods were presented. The first method was column generation and fast pricing heuristic based on branch-and-price algorithm while the second was marginal cost heuristic.

I. Muter et al. [14] considered a two-stage extension of one-dimensional cutting stock problem and an exact simultaneous column-and-row generation algorithm was presented for its solution. The pricing sub-problem was a knapsack problem and three algorithms were proposed for the problem including branch-and-price. The performances of the three algorithms were compared by conducting extensive computational experiments which revealed that a hybrid algorithm which integrated the other two algorithms was performing best out of all the three algorithms. For a bi-objective one-dimensional trim loss problem, H. K. Alfares et al. [15] presented a new model and an efficient solution algorithm. The two objectives of the model were: the minimization of trim loss and the minimization of the number of partially-cut large objects. A two-stage least-loss algorithm was used to determine the patterns which minimized the trim loss and near-optimum solution was produced. Computational tests confirmed the efficiency and near-optimality of the proposed algorithm. In the context of furniture production, M. Vanzela et al. [16] studied the integrated lot sizing and cutting stock problem. A mathematical model was proposed and tested that captured the production process of the selected factory. A column-generation solution method was proposed to solve the problem and an intensive computational study was conducted using real data. It has been shown that the integrated approach performed well both in terms of reduction in total cost of raw as well as inventory costs of small pieces.

A. R. Pitombeira-Neto et al. [17] presented an integer linear programming model for the scheduling and one-dimensional cutting stock problem with heterogeneous orders. A novel metaheuristic algorithm was proposed based on a fix-and-optimize strategy hybridized with a random local search. The performance of the proposed metaheuristic was compared with IBM CPLEX by carrying out extensive computational experiments which showed that the proposed metaheuristic performed better than CPLEX in larger instances while in small instances, CPLEX showed a marginal advantage. In steel industries, D. Tanir et al. [18] studied the one-dimensional cutting stock problem with divisible items. The main objectives were to minimize both the number of welds used to recombine small pieces and the trim loss. A nonlinear integer formulation was first developed for the NP-hard problem and to fulfill the needs of the company, a heuristic based on dynamic programming was also designed. The computational experiments revealed that the algorithm performed well on real life instances.

To the best of our knowledge, the first attempt for the evaluation of usable leftovers in one-dimensional cutting stock problem can be found in work of L. V. Kantorovich [19]. The minimization of scrap method was applied to a problem and presented the solution which gave minimum waste under the given conditions. It was shown that analogous solutions can be obtained for other such problems with several modifications of the conditions. Afterwards, A. R. Brown [20] distinguished two main forms of solution methods for the resource-usage problems and presented that in which form solutions have been approached and implemented in different problem settings. G. Scheithauer [21] investigated the handling of utilizable leftovers for one-dimensional cutting stock problem to solve the

continuous relaxation issue, using the column generation method. Z. Sinuany-Stern and I. Weiner [22] presented an algorithm for an optimal solution of the cutting stock problem but the algorithm was not efficient for large problems as it was developed for a small metal workshop.

In order to solve the problem of minimizing trim loss in one-dimensional roll cutting for different roll lengths, M. Gradisar et al. [23] proposed a sequential heuristic procedure which lead to an optimal solution. A computer program COLA was designed which was based on the proposed algorithm and the proposed procedure was used in other industries for similar problems with slight modifications. M. Gradisar et al. [24] presented an analysis which considered the sequential heuristic procedure to optimize a one-dimensional cutting stock problem for the case when all stock lengths were different. A lexicographic approach was applied to solve the bicriterial multidimensional knapsack problem with side constraints and based on the proposed algorithm, the computer program CUT was developed which was an improved and generalized version of COLA.

To solve the cutting-and-reuse problem aroused in a European gear belt production plant, C. Arbib et al. [25] presented a decision support tool and a mathematical model based on linear programming and column generation was developed for a particular one-dimensional cutting-and-reuse problem. The main objectives of optimization were: quality control, setup minimization, workload equalization and trim loss minimization. Leon Kos et al. [26] presented a hybrid genetic algorithm which was used as heuristics that provided quality packing for cutting and was also prone to minimize the trim loss. A. C. Cherri et al. [27] presented some characteristics of a desirable solution to the cutting stock problem with useable leftover. Modifications on classical heuristic methods were proposed to solve the cutting stock problem to find a solution that satisfies those characteristics.

K. C. Poldi and M. N. Arenales [28] reviewed some already published heuristics and proposed other heuristic methods for obtaining integer solution for one-dimensional cutting stock problem with multiple stock length. The proposed greedy rounding heuristics resulted in significant objective function value gaps. Y. Cui and Y. Yang [29] presented a heuristic algorithm for one dimensional cutting stock problem with residual lengths which was efficient in shorter stock to reduce trim loss and bar cost. The algorithm was easy to code and efficient in increasing the average leftover length. M. E. Berberler et al. [30] proposed a new mathematical model for one-dimensional problem and a new dynamic programming algorithm was applied on the developed model. A. C. Cherri et al. [31] considered the one-dimensional cutting stock problem with useable leftovers. M. N. Arenales et al. [3] presented a new mathematical model for cutting stock problem using leftover which consisted of partially cutting the stocked items and keeping the remnants to be cut in the next periods. All other dimensions were considered straightforward and computational experiments were performed for a one-dimensional case.

In recent times, remarkable works have been presented to solve one-dimensional cutting stock problem with useable leftover. R. Andrade et al. [32] presented two bi-level mathematical programming models to represent non-exact two-stage, two-dimensional residual bin-packing problem. L. Tomat and M. Gradišar [33] proposed a method to control stock and to calculate the near optimal amount of usable leftover. Instead of randomly generated, usable leftovers were used from previous orders in the next order and trim loss in sequential orders was reduced and was explained with the help of computational outcomes. F. Clautiaux et al. [34] proposed diving heuristic based on column generation and used dynamic programming to solve pricing problem. Most recently, S. Sumetthapiwat et al. [35] proposed a mathematical model which utilized column generation method and integer solution finding strategies to solve one-dimensional cutting stock problem with useable leftover.

One of the most important industrial sectors in the world is the paper industry [36]. This industry is placed third after the chemical and metal industries in terms of water consumption [37]. Due to high consumption of water, the paper industry results in high generation of wastewater [36]. Generally, a paper mill produces effluents of volume ranging between 1.5 to 60 m³ per ton of paper produced [37]. Additionally, when 1 m³ of wastewater is discharged from a paper mill, it contains 0.9 m³ of potentially reusable water and generates a reusable sludge of mass 0.7 kg [38]. The treatment

of effluents consequently produces a large amount of sludge ranging between 40 to 50 kg of dry sludge per ton of paper produced, of which 30% is biological sludge and 70% is primary sludge [36,39]. Usually the primary and secondary sludge originates from sedimentation and biological treatment, respectively [40].

Due to recent increasing restrictive environmental legislation, different challenges regarding management of waste have been faced by the paper industry. Without adequate treatment, industrial wastewater has an adverse effect on the aquatic environment. Land applications is one of the most commonly practiced methods for sludge management [41]. However, unplanned landfilling is also commonly practiced within paper mills which leads to negative outcomes regarding air, water, and soil [42].

Nowadays, people are facing a serious problem of environmental pollution and one of the challenges is its protection. The paper industry is one of the important industrial sectors but it causes an adverse impact on the environment as it discharges wastewater containing toxic pollutants [43,44]. Monte et al. [45] presented the data regarding waste generation from few European paper mills and this is shown in Table 1.

Table 1. Generation of waste from few European pulp and paper mills.

	Paper Mill			
	SCA	Norske Skog	Stora Enso	Holmen
Mill production (millions of tonnes)	9.9	4.8	15.1	2.3
Total waste generated (kg/ton product)	163	163	155 (dry)	160
Recovered waste (kg/ton product)	115	138	-	136
Waste sent to landfill (kg/ton product)	47	16	22	23 (wet)
Hazardous waste (kg/ton product)	0.3	1.5	0.3	0.2

In a paper mill, waste paper is produced during the cutting process of reels. The waste paper is reprocessed to make new reels. During reprocessing, chlorine and chlorine dioxide are used for bleaching of pulp and chlorinated organic substances are discharged to the environment [46,47]. Cutting optimization minimizes the production of waste paper and less paper is reprocessed. As a result, minimum effluents are discharged to the environment from the paper mill.

In this paper, we present a new mathematical model for reduction of trim loss in cutting reels at a make-to-order paper mill. The main differences from other models proposed in the literature are the consideration of cutting master reels of flexible widths and stocked over-produced and leftovers reels to fill a customer demand. Master reels are normally produced of standard size, which on cutting into smaller reels to fill an order results in trim losses while the production of master reels of flexible widths mostly results in no trim loss and hence no reprocessing of paper. The cutting of master reels into smaller reels results sometime in over-produced reels and leftover reels, which are stocked to meet future demands. Our mathematical model considers these over-produced reels and leftover reels in stock to meet a demand with minimum or no trim loss. So, trim loss minimization and provision of stocked reels are important aspects in solving cutting stock problems which are considered in this model.

Other contributions of this paper are the consideration of various operational and technological constraints in the production and cutting process. Operational constraints include the combination of reels of identical weight of paper per unit width and combination of reels of identical internal and external diameters. Additionally, to optimize the use of available machinery at the paper mill, the minimum and maximum widths are imposed to the cutting patterns. Operational constraints include edge trim loss at the two ends, maximum diameter of the master reels, and the limitation of cutting blades at winder. The limited cutting blades has been considered in the mathematical model and feasible patterns are generated according to the number of available cutting blades.

The linear programming model with the objective of trim loss minimization is solved through simplex algorithm which normally results in non-integer solution. The constraints of the model are regarding meeting the customer demand, provision of stocked reels, and limitation on cutting blades at winder. A non-integer solution is rounded by a new post-optimization procedure which is relatively easy and takes less computational time.

The minimization of trim loss and provision of stocked reels results in reduced reprocessing of paper. In the reprocessing of paper, harsh chemicals are used for different purposes. These chemicals are discharged in wastewater to the environment which are harmful and can increase the risk of different types of cancers and also affect aquatic life badly. The paper mill has confirmed that by implementing the proposed method, significant improvements in terms of trim loss minimization and reduction in discharge of effluents into the environment has been achieved and it outperformed other traditional approaches.

However, the trim loss minimization problem has been analyzed in various contexts in literature. Our study proposes a new mathematical model for the minimization of trim loss in cutting master reels of flexible widths into smaller reels, provision of stocked over-produced and leftover reels to fill an order, limitation of cutting blades in the feasible pattern generation, consideration of various operational and technological constraints, rounding the non-integer solution by a new post-optimization procedure and the minimization of chlorinated compounds by minimizing the reprocessing of paper trimming to protect environment.

The next section contains the mathematical model for the considered problem.

3. Mathematical Model

The mathematical model for minimizing trim loss while cutting master reels of flexible widths and stocked reels of smaller widths to fill a given order is characterized by the following indices, parameters, and decision variables.

Indices

$a = 1, 2, \dots, l$	master reels of width a
$b = 1, 2, \dots, m$	stocked reels of width b
$c = 1, 2, \dots, n$	feasible combination in master reels of width W_a
$d = 1, 2, \dots, r$	feasible combination in stocked reels of width W_b
$i = 1, 2, \dots, u$	required width i

Parameters

W_a	width of master reels
W_b	width of stocked reels
A_{ai}	number of times a required width i appears in a feasible pattern of W_a
B_{bi}	number of times a required width i appears in a feasible pattern of W_b
T_{ac}	trim loss in cutting feasible pattern c of W_a
T_{bd}	trim loss in cutting feasible pattern d of W_b
N_b	available number stocked reels of W_b
L_i	length of smaller demanded widths i
L_a	length of master reels of width W_a
L_b	length of stocked reels of width W_b
S_i	shortages in an order for required width i
E_i	surpluses in an order for required width i
D_i	demand for reels of smaller width i
w_i	width of demanded reels i
k_i	available number of knives to cut an order i

Decision Variables

- X_{ac} number of master reels of pattern c associated with width W_a
- X_{bd} number of stocked reels of pattern d associated with width W_b

The problem can verbally be summarized as determining the decision variables that can fill the required orders with minimum trim loss area. Mathematically, the objective function for trim loss minimization for first phase can be stated as follows:

$$\text{Minimize : } Z = \sum_{a=1}^l W_a \sum_{c=1}^n X_{ac} + \sum_{b=1}^m W_b \sum_{d=1}^r X_{bd} \tag{1}$$

Subject to the constraints which deal directly with satisfying the demand for reels of smaller widths:

$$\sum_{a=1}^l A_{ai} \sum_{c=1}^n X_{ac} + \sum_{b=1}^m B_{bi} \sum_{d=1}^r X_{bd} \geq D_i \quad i = 1, 2, \dots, u \tag{2}$$

$$\sum_{d=1}^r X_{bd} = N_b \quad \text{if } w_i = W_b \text{ and } D_i \geq N_b \quad i = 1, 2, \dots, u \quad b = 1, 2, \dots, m \tag{3}$$

$$\sum_{d=1}^r X_{bd} = D_i \quad \text{if } w_i = W_b \text{ and } D_i < N_b \quad i = 1, 2, \dots, u \quad b = 1, 2, \dots, m \tag{4}$$

$$\sum_{d=1}^r X_{bd} \leq N_b \quad \text{if } w_i \neq W_b \quad i = 1, 2, \dots, u \quad b = 1, 2, \dots, m \tag{5}$$

$$\sum_{i=1}^u A_{ai} - 1 \leq k \quad a = 1, 2, \dots, l \tag{6}$$

$$\sum_{i=1}^u B_{bi} - 1 \leq k \quad b = 1, 2, \dots, m \tag{7}$$

$$X_{ac}, X_{bd} \geq 0 \quad a = 1, 2, \dots, l \quad b = 1, 2, \dots, m \quad c = 1, 2, \dots, n \quad d = 1, 2, \dots, r \tag{8}$$

The objective function (1) minimizes the trim loss and satisfies an order by cutting master reels of widths W_1, W_2, \dots, W_l and stocked reels of widths W_1, W_2, \dots, W_m into required smaller widths w_1, w_2, \dots, w_u . The constraints (2) ensure that the number of required widths of type i to be cut from the master reels and stocked reels must be greater than or equal to the order requirements while the Equations (3) and (4) are used when a required width in a set of order matches with the width of stocked reels. The inequalities (5) are used in case of non-matching widths in a set of orders and stocked reels, and impose that the quantity of stocked reels of widths W_1, W_2, \dots, W_m chosen for cutting to meet an order must be less than or equal to its maximum number in stock N_1, N_2, \dots, N_m . Constraints (6) and (7) satisfy the condition that the available number of knives k can cut master reels or stocked reels into $k + 1$ pieces simultaneously. Finally, the inequalities (8) are the non-negativity restrictions.

In the second phase, all non-integer decision variables, if obtained in the first phase, are converted into integers. However, if there is single non-integer decision variable, it is just rounded up.

All non-integer decision variables between two successive integers y and $y + 1$ are converted into integers by removing the fractional part $frac(y) = y - \lfloor y \rfloor$ from the entire column vector of decision variables resulted in first phase. Consider coefficients of all decision variables in inequalities (2) as matrix of order $i \times (a + b)$ and the pre-decimal parts of all decision variables as matrix of order $(a + b) \times 1$. The multiplication of these two matrices results in a column vector F of order $i \times 1$, which is used to calculate shortages S_i or surpluses E_i . Figure 1 shows a simplified flow chart of the algorithm.

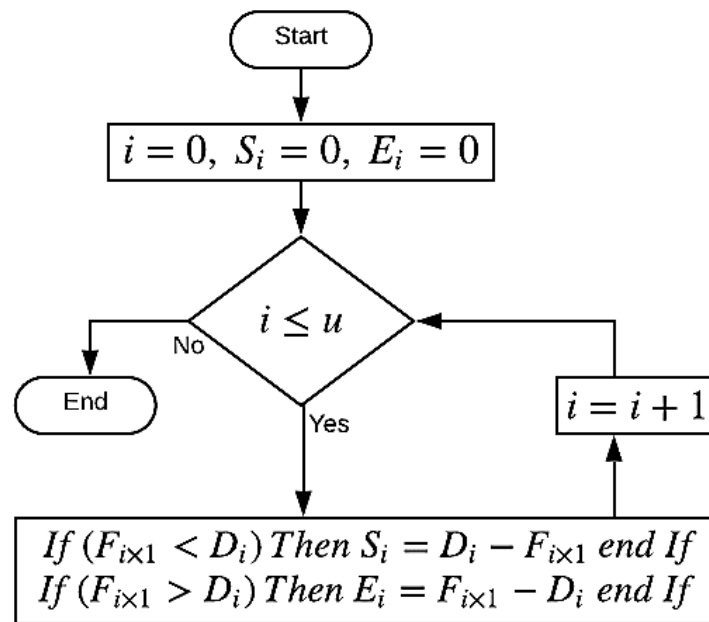


Figure 1. Shortages and surpluses algorithm.

Considering all decision variables non-integer in the first phase, which is a rare case, the new objective function and constraints for the second phase are given as:

$$\text{Minimize : } Z = \sum_{a=1}^l \sum_{c=1}^n X_{ac} + \sum_{b=1}^m \sum_{d=1}^r X_{bd} \tag{9}$$

Subject to:

$$\sum_{a=1}^l A_{ai} \sum_{c=1}^n X_{ac} + \sum_{b=1}^m B_{bi} \sum_{d=1}^r X_{bd} \geq S_i \quad i = 1, 2, \dots, u \tag{10}$$

$$X_{ac}, X_{bd} \in \{0, 1\} \quad a = 1, 2, \dots, l \quad b = 1, 2, \dots, m \quad c = 1, 2, \dots, n \quad d = 1, 2, \dots, r \tag{11}$$

The objective function (9) minimizes the summation of all non-integer decision variables resulted in the first phase of the solution. The constraints (10) limit the shortages whereas constraints (11) are binary restrictions on the decision variables. Finally, the decision variables obtained in the second phase are added with pre-decimal parts of the corresponding decision variables in the first phase.

After the optimal integer solution is obtained, other measures are calculated by the equations given below. The total area of an order is calculated using Equation (12) while total master reels and stocked reels used are enumerated utilizing Equations (13) and (14), respectively. The gross area of the selected master and stocked reels can be estimated using Equations (15) and (16), respectively. Finally, Equations (17) and (18) evaluate the net area of selected master reels and stocked reels, respectively.

The problem falls in the domain of linear programming because the objective as well as all the constraints are linear in the decision variables. The non-linear models can either be converted to linear models using approximation techniques or they can be solved using non-exact/meta-heuristic approaches which requires higher computational requirements. Furthermore, our developed algorithm solves the linear problem in adequate amount of time.

$$\text{Total area of an order} = \sum_{i=1}^u L_i w_i D_i \tag{12}$$

$$\text{Total master reels used} = \sum_{a=1}^l \sum_{c=1}^n X_{ac} \tag{13}$$

$$\text{Total stocked reels used} = \sum_{b=1}^m \sum_{d=1}^r X_{bd} \tag{14}$$

$$\text{Gross area of master reels} = \sum_{a=1}^l L_a W_a \sum_{c=1}^n X_{ac} \tag{15}$$

$$\text{Gross area of stocked reels} = \sum_{b=1}^m L_b W_b \sum_{d=1}^r X_{bd} \tag{16}$$

$$\text{Net area master reels} = \sum_{a=1}^l L_a (W_a - T_a) \sum_{c=1}^n X_{ac} \tag{17}$$

$$\text{Net area of stocked reels} = \sum_{b=1}^m L_b (W_b - T_b) \sum_{d=1}^r X_{bd} \tag{18}$$

The trim loss comprises of both reusable and unusable reels. All those reels that cannot be sold are transferred for recycling while those greater than a certain width are considered as reusable reels for the next orders.

The benefits of the proposed model for paper mill industry are: it takes less computational time for its solution due to linearity, it is used to solve the trim loss minimization problem for flexible widths of master reels, it is used to consider the cutting of stocked over-produced and leftover reels to fill an order, it considers the limitation of cutting blades at winder and it is used to minimize the total cost in terms of master reels and stocked reels. The constraints of the model are used to meet the demand and provide limited stocked reels to customers. As the model minimizes trim loss, thus the reprocessing of paper trimming is minimized and as a result, the discharge of chlorinated compounds and other toxic organics to the environment are minimized. Additionally, the model finds a better solution and proclaims considerable economical and operational benefits to the industry.

4. Solution Procedure

The solution procedure adopted is divided into several stages, as presented in Figure 2. First, to fill an order by setting knives to the desired widths, all feasible cutting patterns for all widths of master reels and stocked reels are generated. The cutting patterns must also be feasible in terms of operational and technological constraints. All evaluated cutting patterns are then used as columns in a linear programming model and the problem is solved using the simplex algorithm. Finally, the solution obtained is made integer if the solution algorithm results in one or more than one non-integer decision variables.

As regards costs, it is always of interest to know how much savings one made due to the optimization effort as such inclusion of such information would have added value to the current study. However, once organizations or industries are approached for possible areas of improvement, while they readily share their problems, they are reluctant to share the finance related information. That is why the cost/savings information in monetary terms could not be included. However, the same information is included in terms of reels of paper saved through application of this study.

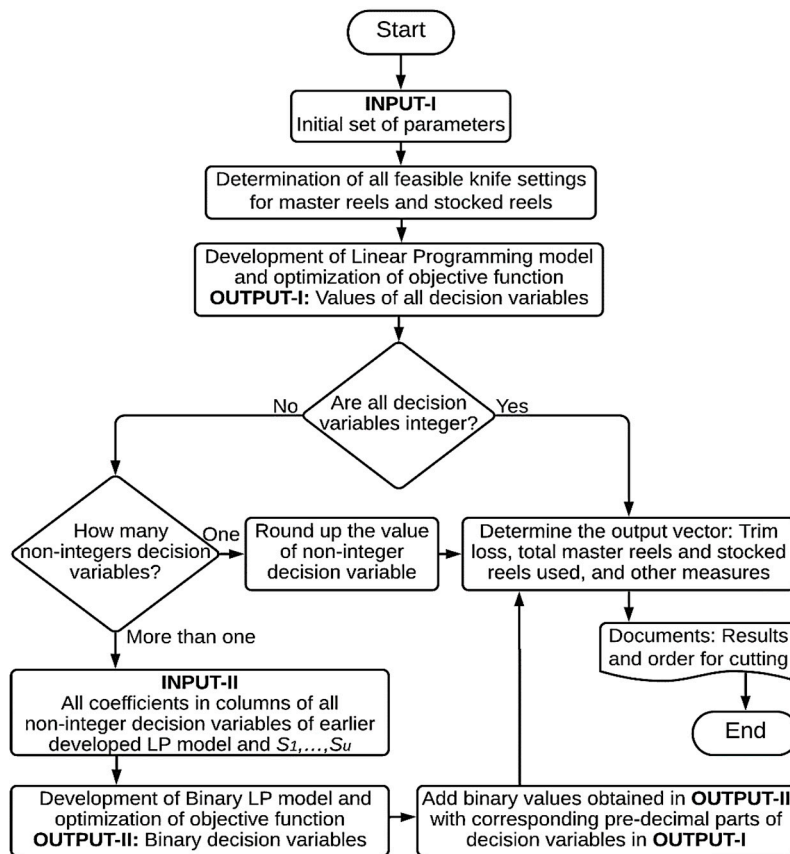


Figure 2. Optimal cutting algorithm.

To illustrate the solution procedure, a small real industrial example is solved which is concerned with production planning of grey back paper in master reels of widths 60 to 70 inches and cutting of limited stocked reels of widths 35, 30, 25, 22, and 18 inches, available in stock. The thickness of paper and grade are 1.0 mm and 350 g/m², respectively. The production requisition involved and stocked reels are described in Tables 2 and 3, respectively.

Table 2. Data of a small industrial example.

Production Requisition	Widths (in)	Paper Type	Number of Reels	External Diameter (in)	Internal Diameter (in)
PR 881	17	GB *	30	60	6
PR 882	21	GB	25	60	6
PR 901	25	GB	40	60	6
PR 909	27	GB	35	60	6
PR 915	30	GB	35	60	6
PR 916	34	GB	30	60	6

* Grey Back.

Table 3. Stocked over-produced and leftover reels.

Production Requisition	Widths (in)	Paper Type	Number of Reels	External Diameter (in)	Internal Diameter (in)
UL 014	35	GB *	05	60	6
UL 084	30	GB	10	60	6
UL 101	25	GB	08	60	6
UL 106	22	GB	01	60	6
UL 107	18	GB	03	60	6

* Grey Back.

The operational constraints of identical paper type, internal and external diameters of reels have to be considered in making new master reels and in the selection of stocked reels to fill a given order.

4.1. Generation of Feasible Cutting Patterns

Feasible cutting patterns are generated by combining the required widths 17, 21, 25, 27, 30, and 34 inches. The inequalities given below are used to enumerate all feasible cutting patterns. Constraints (19) are used to ensure that a cutting pattern is feasible if the summation of combination of required widths in a pattern is less than or equal to the width of master reel and also greater than the difference of width of master reel and minimum width in a set of order. Constraints (20) are similar in nature to constraints (19) but are used for over-produced and useable leftovers in stock. Equations (21) and (22) are simply written for equating trim loss in cutting master and stocked reels, respectively.

$$W_a \geq \sum_{i=1}^u A_{ai} > W_a - \min w_i \quad a = 1, 2, \dots, l \tag{19}$$

$$W_b \geq \sum_{i=1}^u B_{bi} > W_b - \min w_i \quad b = 1, 2, \dots, m \tag{20}$$

$$Trim Loss = W_a - \sum_{i=1}^u A_{ai} \quad a = 1, 2, \dots, l \tag{21}$$

$$Trim Loss = W_b - \sum_{i=1}^u B_{bi} \quad b = 1, 2, \dots, m \tag{22}$$

All feasible patterns must satisfy technological constraints imposed on the cutting of master reels, over-produced reels and useable leftovers, into reels of smaller required widths. One of the technological constraints is the number of cutting knives at winder and those patterns which do not satisfy this constraint are not considered as feasible cutting patterns.

4.2. Linear Programming Model

At this stage of the solution procedure, a linear programming model is developed and implemented. The model includes constraints related to the satisfaction of ordered quantities. The over-produced reels are stocked for later usage or negotiated with client and those produced of width less than the minimum acceptable limit are either recycled or utilized in the form of less valuable products than the ordered items. To fill the order, the objective function of the model minimizes the trim loss area in cutting master reels of widths 70, 69, 68, 67, 66, 65, 64, 63, 62, 61, and 60 inches, and stocked reels of widths 35, 30, 25, 22, and 18 inches.

$$\text{Minimize : } Z = 70 \sum_{c=1}^{24} X_{70,c} + \dots + 60 \sum_{c=215}^{232} X_{66,c} + 35 \sum_{c=233}^{238} X_{35,c} + 30 \sum_{c=239}^{243} X_{30,c} + 25 \sum_{c=244}^{246} X_{25,c} + 22 \sum_{c=247}^{249} X_{22,c} + 18 \sum_{c=250}^{250} X_{18,c} \tag{23}$$

The obtained optimal solution comprises of four non-integer decision variables as presented in Table 4. The optimal objective function value is 5050.

Table 4. Non-integer optimal solution.

Pattern Number	Width (in)	Cutting Pattern	Number of Reels
035	69	$0 \times 17 + 2 \times 21 + 0 \times 25 + 1 \times 27 + 0 \times 30 + 0 \times 34$	12.50
071	68	$0 \times 17 + 0 \times 21 + 0 \times 25 + 0 \times 27 + 0 \times 30 + 2 \times 34$	07.25
079	67	$1 \times 17 + 0 \times 21 + 2 \times 25 + 0 \times 27 + 0 \times 30 + 0 \times 34$	16.00
202	61	$2 \times 17 + 0 \times 21 + 0 \times 25 + 1 \times 27 + 0 \times 30 + 0 \times 34$	07.00
214	71	$0 \times 17 + 0 \times 21 + 0 \times 25 + 1 \times 27 + 0 \times 30 + 1 \times 34$	15.50
229	60	$0 \times 17 + 0 \times 21 + 0 \times 25 + 0 \times 27 + 2 \times 30 + 0 \times 34$	12.50
243	30	$0 \times 17 + 0 \times 21 + 0 \times 25 + 0 \times 27 + 1 \times 30 + 0 \times 34$	10.00
246	25	$0 \times 17 + 0 \times 21 + 1 \times 25 + 0 \times 27 + 0 \times 30 + 0 \times 34$	08.00

The solution is not implementable due to non-integer decision variables. An integer algorithm is used to solve the problem and the post-optimization procedure is described in the next subsection.

4.3. Integer Solution

The optimal solution of the linear programming model contains four non-integer decision variables. By considering its pre-decimal parts (12, 07, 15, and 12), the shortages in each order are calculated and are given in Table 5. The objective function is simply minimization of the summation of non-integer decision variables. To make the decision variables integer, a separate algorithm has also been explained in Figure 1 of Section 3.

Table 5. Non-integer decision variables.

$X_{69,035}$	$X_{68,071}$	$X_{61,214}$	$X_{60,229}$	Shortages	Excesses
0	0	0	0	0	0
2	0	0	0	1	0
0	0	0	0	0	0
1	0	1	0	1	0
0	0	0	2	1	0
0	2	1	0	1	0

The model is solved for binary values of the decision variables. The resulting values are then added with the pre-decimal parts of the corresponding decision variables to get the final integer solution, as can be seen in Table 6.

Table 6. Final optimal solution.

Decision Variable	Non-Integer	Pre-Decimal	Binary	Integer
$X_{69,035}$	12.50	12	1	13
$X_{68,071}$	07.25	07	0	07
$X_{67,079}$	16.00 *	—	—	16
$X_{61,202}$	07.00 *	—	—	07
$X_{61,214}$	15.50	15	1	16
$X_{60,229}$	12.50	12	1	13
$X_{30,243}$	10.00 *	—	—	10
$X_{25,246}$	08.00 *	—	—	08

* already integer.

The final solution leads to over-production of one reel of each width 21, 27, and 30 inches, which may be stocked for later usage or negotiated with the client. The optimal objective function value is 5128.

While cutting the pattern 229 thirteen times, 26 new reels of width 30 inches are produced. Thus, only 9 stocked reels of width 30 inches are picked to fill the order. By doing this, the objective function value reduces to 5098.

5. Results and Discussion

The marketing department of the company provided the data used in the computational results and it corresponds to real problems solved at the paper mill as discussed earlier. The algorithm was implemented using both FORTRAN and MATLAB programming languages on a computer and the results were obtained with a Core i7 and 1.99 GHz in reasonable computational time.

The paper mill managers contacted our institute to improve their system. The issues which the mill managers were facing were: spending more time on calculating the feasible pattern generation, finding a width of master reels that can result in minimum trim loss, decision about provision of stocked reels to fill an order, etc. It was a very interesting study, and also it was a convenient sample as the industry was close to our institute and it was easy to collect data and share findings. It was also easy for us to trace the results of implementation.

For the purpose of comparative analysis and to validate the solution procedure adopted, two cases were considered. Case 1 was master reels with adjustable widths and Case 2 was master reels with fixed widths.

The number of patterns of master reels decreased when more stocked reels were selected to meet demand and vice versa. Therefore, for longer widths of master reels in Case 2, the number of patterns of master reels remained equal to that of Case 1, but for shorter widths, the number decreased gradually as represented in Figure 3a. The same was anticipated for stocked reels in both cases, but the number increased gently for shorter widths of master reels in Case 2, as can be observed in Figure 3b.

With greater width of the master reel, a lesser number of master reels were used to fill an order and vice versa. Hence, in Case 1, reels of different widths of master reels were selected for cutting, which resulted in slightly higher number of master reels used than only few master reels of fixed width, while in Case 2, the number of master reels selected for cutting increased with decrease in the width of master reels as shown in Figure 3c. In cutting stocked reels to meet demand, initially the number of reels selected for cutting remained same in both cases, but then increased with decrease in the widths of master reels in Case 2, as can be seen in Figure 3d.

Increasing the area of master reels for cutting resulted in a decrease in the area of stocked reels and conversely decreasing the area of master reels resulted in an increase in the area of stocked reels. Thus, there is an inverse relationship between master reels and stocked reels in Case 2 and the total width of master reels selected for cutting in Case 1 was higher than some results of Case 2 as one can see in Figure 3e, while the total width of stocked reels remained same in both cases but then diverged for smaller widths of Case 2 as compared in Figure 3f.

As already known, a solution is considered best when both master reels and stocked reels are cut to meet demand with no or minimum trim loss. Here, the proposed model resulted in no trim loss in cutting master reels and stocked reels. As depicted in Figure 3g, in Case 1, the trim loss in cutting master reels and stocked reels was zero while in Case 2, the loss was much higher than Case 1 as seen in Figure 3h.

The optimal solution in Case 1 produced few over-produced reels and no trim loss. Therefore, the value of the objective function in Case 1 was less than all other values of objective function in Case 2 as presented in Figure 3i.

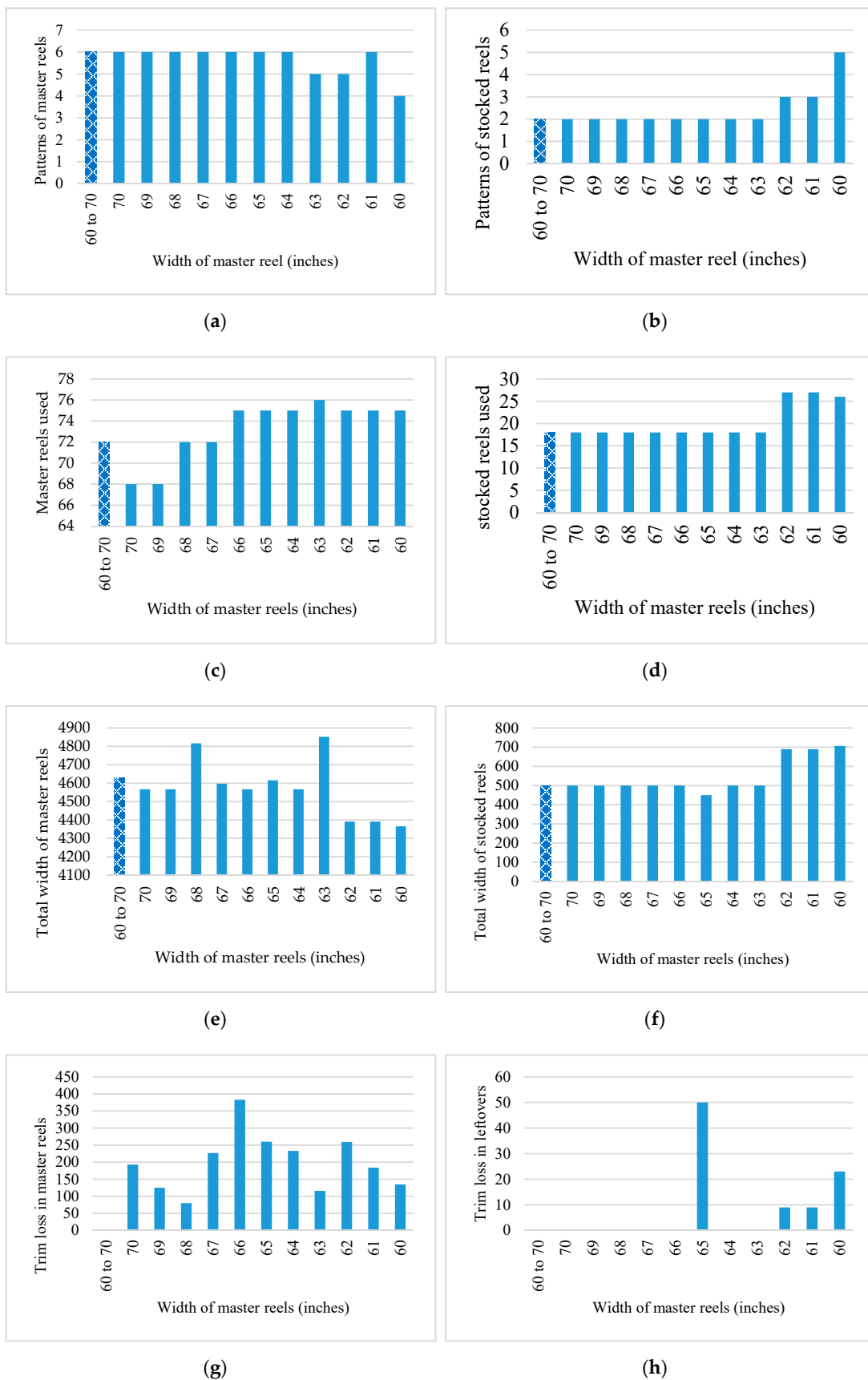


Figure 3. Cont.

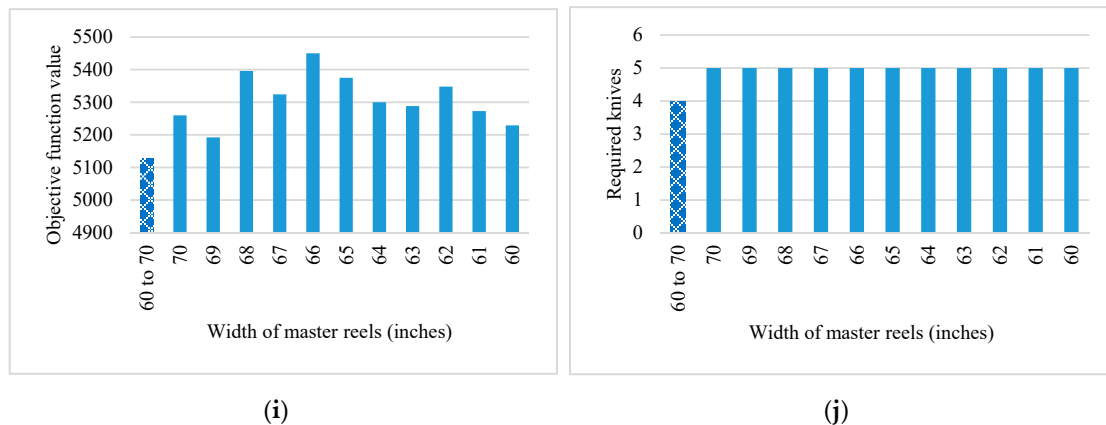


Figure 3. Comparison of the outcomes of adjustable and fixed widths of master reels.

In Case 2, there are trim losses in cutting master reels and stocked reels and as trimming requires an additional knife, thus the number of required knives at winder in Case 1 was less than Case 2 as given in Figure 3j.

The paper mill has validated the proposed model and is in use currently. The method finds a better solution and considerable economical, operational, and environmental benefits are proclaimed. The model significantly reduces trim loss, thereby improves productivity. The model selects minimum possible patterns of master reels for cutting and also chooses stocked reels to fill an order. Additionally, considerable savings have been achieved in terms of energy as well.

While direct cost figure are not known, if we compare our results with the past practices (shared by the concerned industry), this study helped in using minimum number of master reels and selected stocked reels to fill an order. The model results in minimum trim loss and saves paper. By adopting the proposed method, the waste paper produced in the mill was decreased by at least 1.25% which corresponds to several hundred tons of paper a year. Hence, minimizing the loss in paper trimming, the reprocessing of waste paper is decreased and the discharge of wastewater containing toxic pollutants to the environment is reduced.

Although there are a number of studies on trim loss analysis in steel [1,2] wood, leather, and plastic industries [3], this is a first attempt towards the analysis of trim loss in paper mill in this region. The findings will help practitioners in the selection on appropriate parameters of width and length of master reels, number of blades at winder etc. to minimize the trim loss in production and cutting of reels. The results have been shared with the relevant department in the paper mill and the implementation of our findings resulted in reduction of total cost, trim losses, reprocessing of paper, and discharge of effluents to the environment. The proposed algorithm is generic in nature and it can be applied to multiple similar contexts. Hence, this study contributes theoretically as well as practically towards the analysis of trim loss at a paper mill.

The proposed mathematical model results in minimum or no paper trim loss in the paper mill and hence the reprocessing of paper trimming is reduced. Bleaching in the reprocessing of paper trimming affects the environment badly as it discharges chlorine-based toxic organic compounds to the environment. The discharge of chlorinated compounds due to bleaching is one of the important environmental issues. The paper mill highly focuses on the reduction of wastewater discharge containing chlorine-based compounds and other toxic organics. Additionally, because of industrial development, total chlorine-free bleaching is feasible for paper products but cannot produce certain paper grades. However, it is encouraged to adopt these modern process developments in the paper mill wherever feasible. To protect the environment, the paper mill gives stress on the minimization of the generation of effluents, reduction of bleaching, minimization of unplanned wastewater discharges, and on the minimization of sulphur emission to the atmosphere.

6. Conclusions

This article describes a system developed for a particular cutting stock problem occurring in a paper mill at Peshawar. The main goal of the solution procedure developed is to minimize the trim loss while producing and cutting master reels of flexible widths and cutting stocked over-produced reels and usable leftovers while satisfying a number of operational and technological constraints.

Due to combinatorial nature, a solution method based on generation of cutting patterns of master reels, over-produced reels and usable leftovers has been formulated. The selected patterns satisfy the imposed operational and technological constraints and are used as columns of the linear programming model of the problem, which is solved by simplex algorithm. A new post-optimization procedure is solved to make the solution integer if the initially obtained solution contains non-integer decision variables.

The developed linear programming-based model is tested, and the obtained results are quite satisfactory as it reduces several hundred tons of paper waste a year. The method has been successfully employed on a number of problems and optimal solutions have been obtained. The system is now in use at the paper mill and considerable operational, economic, and environmental benefits have been achieved.

The cutting optimization considerably minimized the loss in paper trimming and marginal reduction of pollution load has been achieved. The discharge of effluent from the mill to the environment is reduced by reprocessing a smaller amount of waste of paper to make new reels. In this way, this study contributed to maintain the ecological balance in the environment.

Other approaches to deal with uncertainty in the problem data can be considered as future research directions.

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References

1. De Carvalho, J.V.; Rodrigues, A.G. An LP-based approach to a two-stage cutting stock problem. *Eur. J. Oper. Res.* **1995**, *84*, 580–589. [[CrossRef](#)]
2. Ferreira, J.S.; Neves, M.A.; Castro, P.F. A two-phase roll cutting problem. *Eur. J. Oper. Res.* **1990**, *44*, 185–196. [[CrossRef](#)]
3. Arenales, M.N.; Cherri, A.C.; Nascimento, D.N.d.; Vianna, A. A new mathematical model for the cutting stock/leftover problem. *Pesqui. Oper.* **2015**, *35*, 509–522. [[CrossRef](#)]
4. Solomon, K.R. Chlorine in the bleaching of pulp and paper. *Pure Appl. Chem.* **1996**, *68*, 1721–1730. [[CrossRef](#)]
5. Wolff, M.S.; Toniolo, P.G.; Lee, E.W.; Rivera, M.; Dubin, N. Blood levels of organochlorine residues and risk of breast cancer. *J. Natl. Cancer Inst.* **1993**, *85*, 648–652. [[CrossRef](#)]
6. Balwaik, S.A.; Raut, S. Utilization of waste paper pulp by partial replacement of cement in concrete. *Int. J. Eng. Res. Appl.* **2011**, *1*, 300–309.
7. Becker, K.H.; Appa, G. A heuristic for the Minimum Score Separation Problem, a combinatorial problem associated with the cutting stock problem. *J. Oper. Res. Soc.* **2015**, *66*, 1297–1311. [[CrossRef](#)]
8. Arbib, C.; Marinelli, F.; Ventura, P. One-dimensional cutting stock with a limited number of open stacks: Bounds and solutions from a new integer linear programming model. *Int. Trans. Oper. Res.* **2016**, *23*, 47–63. [[CrossRef](#)]

9. Garraffa, M.; Salassa, F.; Vancroonenburg, W.; Vanden Berghe, G.; Wauters, T. The one-dimensional cutting stock problem with sequence-dependent cut losses. *Int. Trans. Oper. Res.* **2016**, *23*, 5–24. [[CrossRef](#)]
10. Wei, L.; Qin, H.; Cheang, B.; Xu, X. An efficient intelligent search algorithm for the two-dimensional rectangular strip packing problem. *Int. Trans. Oper. Res.* **2016**, *23*, 65–92. [[CrossRef](#)]
11. Chen, Y.; Song, X.; Ouelhadj, D.; Cui, Y. A heuristic for the skiving and cutting stock problem in paper and plastic film industries. *Int. Trans. Oper. Res.* **2019**, *26*, 157–179. [[CrossRef](#)]
12. Correia, M.H.; Oliveira, J.F.; Ferreira, J.S. Reel and sheet cutting at a paper mill. *Comput. Oper. Res.* **2004**, *31*, 1223–1243. [[CrossRef](#)]
13. Chauhan, S.S.; Martel, A.; D'Amour, S. Roll assortment optimization in a paper mill: An integer programming approach. *Comput. Oper. Res.* **2008**, *35*, 614–627. [[CrossRef](#)]
14. Muter, İ.; Sezer, Z. Algorithms for the one-dimensional two-stage cutting stock problem. *Eur. J. Oper. Res.* **2018**, *271*, 20–32. [[CrossRef](#)]
15. Alfares, H.K.; Alsawafy, O.G. A Least-Loss Algorithm for a Bi-Objective One-Dimensional Cutting-Stock Problem. *Int. J. Appl. Ind. Eng.* **2019**, *6*, 1–19.
16. Vanzela, M.; Melega, G.M.; Rangel, S.; de Araujo, S.A. The integrated lot sizing and cutting stock problem with saw cycle constraints applied to furniture production. *Comput. Oper. Res.* **2017**, *79*, 148–160. [[CrossRef](#)]
17. Pitombeira-Neto, A.R.; de Athayde Prata, B. A matheuristic algorithm for the one-dimensional cutting stock and scheduling problem with heterogeneous orders. *TOP* **2019**, *28*, 178–192. [[CrossRef](#)]
18. Tanir, D.; Ugurlu, O.; Guler, A.; Nuriyev, U. One-dimensional cutting stock problem with divisible items: A case study in steel industry. *TWMS J. Appl. Eng. Math.* **2019**, *9*, 473–484.
19. Kantorovich, L.V. Mathematical methods of organizing and planning production. *Manag. Sci.* **1960**, *6*, 366–422. [[CrossRef](#)]
20. Brown, A.R. *Optimum Packing and Depletion*; American Elsevier: New York, NY, USA, 1971.
21. Scheithauer, G. A note on handling residual lengths. *Optimization* **1991**, *22*, 461–466. [[CrossRef](#)]
22. Sinuany-Stern, Z.; Weiner, I. The one dimensional cutting stock problem using two objectives. *J. Oper. Res. Soc.* **1994**, *45*, 231–236. [[CrossRef](#)]
23. Gradišar, M.; Jesenko, J.; Resinovič, G. Optimization of roll cutting in clothing industry. *Comput. Oper. Res.* **1997**, *24*, 945–953. [[CrossRef](#)]
24. Gradišar, M.; Kljajić, M.; Resinovič, G.; Jesenko, J. A sequential heuristic procedure for one-dimensional cutting. *Eur. J. Oper. Res.* **1999**, *114*, 557–568. [[CrossRef](#)]
25. Arbib, C.; Marinelli, F.; Rossi, F.; Di Iorio, F. Cutting and reuse: An application from automobile component manufacturing. *Oper. Res.* **2002**, *50*, 923–934. [[CrossRef](#)]
26. Kos, L.; Duhovnik, J. Cutting optimization with variable-sized stock and inventory status data. *Int. J. Prod. Res.* **2002**, *40*, 2289–2301. [[CrossRef](#)]
27. Cherri, A.C.; Arenales, M.N.; Yanasse, H.H. The one-dimensional cutting stock problem with usable leftover—A heuristic approach. *Eur. J. Oper. Res.* **2009**, *196*, 897–908. [[CrossRef](#)]
28. Poldi, K.C.; Arenales, M.N. Heuristics for the one-dimensional cutting stock problem with limited multiple stock lengths. *Comput. Oper. Res.* **2009**, *36*, 2074–2081. [[CrossRef](#)]
29. Cui, Y.; Yang, Y. A heuristic for the one-dimensional cutting stock problem with usable leftover. *Eur. J. Oper. Res.* **2010**, *204*, 245–250. [[CrossRef](#)]
30. Berberler, M.E.; Nuriyev, U.; Yıldırım, A. A software for the one-dimensional cutting stock problem. *J. King Saud Univ Sci.* **2011**, *23*, 69–76. [[CrossRef](#)]
31. Cherri, A.C.; Arenales, M.N.; Yanasse, H.H.; Poldi, K.C.; Vianna, A.C.G. The one-dimensional cutting stock problem with usable leftovers—A survey. *Eur. J. Oper. Res.* **2014**, *236*, 395–402. [[CrossRef](#)]
32. Andrade, R.; Birgin, E.G.; Morabito, R. Two-stage two-dimensional guillotine cutting stock problems with usable leftover. *Int. Trans. Oper. Res.* **2016**, *23*, 121–145. [[CrossRef](#)]
33. Tomat, L.; Gradišar, M. One-dimensional stock cutting: Optimization of usable leftovers in consecutive orders. *Central Eur. J. Oper. Res.* **2017**, *25*, 473–489. [[CrossRef](#)]
34. Clautiaux, F.; Sadykov, R.; Vanderbeck, F.; Viaud, Q. Pattern-based diving heuristics for a two-dimensional guillotine cutting-stock problem with leftovers. *EURO J. Comput. Optim.* **2019**, *7*, 265–297. [[CrossRef](#)]
35. Sumetthapiwat, S.; Intiyot, B.; Jeenanunta, C. A column generation on two-dimensional cutting stock problem with fixed-size usable leftover and multiple stock sizes. *Int. J. Logist. Syst. Manag.* **2020**, *35*, 273–288. [[CrossRef](#)]

36. Jaria, G.; Silva, C.P.; Ferreira, C.I.; Otero, M.; Calisto, V. Sludge from paper mill effluent treatment as raw material to produce carbon adsorbents: An alternative waste management strategy. *J. Environ. Manag.* **2017**, *188*, 203–211. [[CrossRef](#)]
37. Gopi, A.; Bhattacharyya, K.G. Impact of pulp and paper mill effluents and solid wastes on soil mineralogical and physicochemical properties. *Environ. Monit. Assess.* **2015**, *187*, 98.
38. Molina-Sánchez, E.; Leyva-Díaz, J.C.; Cortés-García, F.J.; Molina-Moreno, V. Proposal of sustainability indicators for the waste management from the paper industry within the circular economy model. *Water* **2018**, *10*, 1014. [[CrossRef](#)]
39. Pratima, B. *Management of Pulp and Paper Mill Waste*; Springer International Publishing: Cham, Switzerland, 2015.
40. Buruberri, L.H.; Seabra, M.P.; Labrincha, J.A. Preparation of clinker from paper pulp industry wastes. *J. Hazard. Mater.* **2015**, *286*, 252–260. [[CrossRef](#)]
41. Pervaiz, M.; Sain, M. Recycling of paper mill biosolids: A review on current practices and emerging biorefinery initiatives. *CLEAN Soil Air Water* **2015**, *43*, 919–926. [[CrossRef](#)]
42. Reckamp, J.M.; Garrido, R.A.; Satrio, J.A. Selective pyrolysis of paper mill sludge by using pretreatment processes to enhance the quality of bio-oil and biochar products. *Biomass Bioenergy* **2014**, *7*, 235–244. [[CrossRef](#)]
43. Gavrilesco, D. Solid Waste Generation In Kraft Pulp Mills. *Environ. Eng. Manag. J.* **2004**, *3*, 399–404. [[CrossRef](#)]
44. Scott, G.M.; Smith, A. Sludge characteristics and disposal alternatives for the pulp and paper industry. In Proceedings of the Tappi International Environmental Conference, Atlanta, GA, USA, 7–10 May 1995; Tappi Press: Atlanta, GA, USA, 1995.
45. Monte, M.C.; Fuente, E.; Blanco, A.; Negro, C. Waste management from pulp and paper production in the European Union. *Waste Manag.* **2009**, *29*, 293–308. [[CrossRef](#)] [[PubMed](#)]
46. Bajpai, P.; Kondo, R. *Biotechnology for Environmental Protection in the Pulp and Paper Industry*; Springer Science & Business Media: Berlin, Germany, 2012.
47. Gullichsen, J. Process internal measures to reduce pulp mill pollution load. *Water Sci. Technol.* **1991**, *24*, 45–53. [[CrossRef](#)]



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