Shear enhancement in RC cantilevers with multiple point loads

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Abstract

The shear resistance of reinforced concrete beams is enhanced by arching action when loads are applied to their top face within around twice the beam effective depth (d) of supports. Previous experimental investigations into shear enhancement have almost exclusively considered simply supported beams with single-point loads applied within 2d of supports. Such academic tests are unrepresentative of practice where loading and support conditions are usually more complex. For example, balanced cantilever cross-head girders of bridges and continuous beams can have multiple point loads applied to the flexural tension face within 2d of supports. Such cases have not previously been investigated experimentally. The paper describes an experimental program carried out to investigate shear enhancement in balanced cantilever beams subjected to pairs of concentrated loads within the failing shear span. The shear resistance of the cantilever beams was found to be slightly less than matching simply supported beams, with the difference greatest for beams without shear reinforcement. A strut and tie model is developed for cantilever beams with pairs of concentrated loads applied to the tension face within 2d of the supports. Measured beam strengths are also compared with the predictions of BS 8110, EC2, fib Model Code 2010 and nonlinear finite element analysis.

Keywords: Shear enhancement, multiple point loads, strut and tie, design standards.

List of notation

- $a_v$: Clear shear span
- $d$: Beam effective depth
- $A_{sw}$: Cross-sectional area of each set of stirrups
- $n$: Number of stirrups within the central ¾ of the clear shear span
- $f_{yw,d}$: Yield strength of stirrups
- $V_{Rd,c}$: Design shear resistance attributed to concrete
- $V_{Rd,s}$: Design shear resistance attributed to shear reinforcement
- $V_{Rd}$: Design shear resistance
- $V_{Ed}$: Design shear force
- $k_p$: Reduction factor for shear resistance attributed to concrete
- $\varepsilon_x$: Longitudinal strain at mid-height of effective shear depth
Effective shear depth
Partial factor for concrete
Characteristic compressive strength of concrete
Aggregate size factor
Reduction factor accounting for state of strain in beam web
Strength reduction factor
Maximum aggregate size
Width of beam for calculation of shear resistance
Design bending moment
Maximum design bending moment
Inclination of compressive stress field
Minimum inclination of the compressive stress field
Principal tensile strain
Design shear resistance provided by shear reinforcement
Strength reduction factor for concrete
Horizontal component of force in strut \( i \)
Maximum possible shear strength
Number of stirrups in shear span \( i \)
Force in stirrup set \( i \)
Vertical projection strut \( i \) between nodes
Horizontal projection strut \( i \) between nodes
Width of top-loading plate
Minimum distance from beam surface to centroid of flexural reinforcement
Ratio of horizontal component of force in strut \( i \) to the total flexural compressive force
Depth to neutral axis from extreme compression fibre
Strut width, normal to its centreline, at strut-node interface
Horizontal projection of strut-node interface
Vertical projection of strut-node interface
Mean uniaxial concrete compressive strength
Concrete elastic modulus
Tensile fracture energy
Introduction

The shear strength of reinforced concrete beams is significantly enhanced by arching action when loads are applied to the top face of the beam within around twice the beam effective depth (d) of supports. Many researchers have investigated this phenomenon for simply supported beams with single-point loads applied within 2d of supports as outlined by Reineck and Todisco (2014) and Todisco et al. (2016). However, apart from the tests of Brown and Bayrak (2007), Vollum and Fang (2014, 2015) and Pastore and Vollum (2019a, b), which considered beams loaded on their flexural compression face, little attention has been given to the behaviour of beams with multiple point loads applied either entirely or partly within 2d of supports. This research considers shear enhancement in cantilever beams with pairs of point loads applied to the tension face (top) both entirely and partly within 2d of supports. This type of loading can arise in structures like balanced cantilever cross-head girders of bridges as well as adjacent to the internal supports of continuous beams.

Shear enhancement in beams loaded within 2d of supports is most simply modelled using simplified semi-empirical methods like that of EC2 (BSI 2004). Alternatively, and more rationally, shear enhancement can be modelled with strut and tie (STM) models (Sagaseta and Vollum 2010, Vollum and Fang 2014) or kinematic models like the two-parameter kinematic theory of Mihaylov et al. (2013) and the five spring model (Mihaylov, 2015). This research was motivated by differences in the principal compressive stress trajectories obtained with nonlinear finite element analysis (NLFEA) for simply supported and cantilever beams loaded with pairs of point loads on their top face. These differences are illustrated in Figure 1 for two notionally identical beams, without shear reinforcement, tested in this study. Both beams were symmetrically loaded about their centreline. The shear spans and widths of loading plates were identical in each test but beam AT0 (0.5/0.5) was tested as a balanced cantilever while beam AC0 (0.5/0.5) was simply supported. Consequently, beam AT0 (0.5/0.5) was loaded on its flexural tension face and beam AC0 (0.5/0.5) on its flexural compression face. The paper investigates, seemingly for the first time, the influence on shear resistance, and STM geometry, of the difference in stress fields evident in Figure 1. Details of the NLFEA procedure used to determine the stress fields in Figure 1 can be found later in the paper under the heading “Comparison between NLFEA, STM and observed behaviour”.
Figure 1: Stress fields and critical shear cracks at failure for beams AC0(0.5/0.5) and AT0(0.5/0.5) loaded respectively on the a) compression face and b) tension face.

**Research Significance**

Shear enhancement is investigated experimentally for the first time in balanced cantilever beams loaded with two-point loads in the failing shear span. Comparisons are also made with the shear strength of comparable simply supported beams for the loading arrangement indicated in Figure 1. Based on NLFEA studies, a STM is also developed for the tested cantilever beams which reduces to that of Sagaseta and Vollum (2010) for beams with single-point loads. The geometry of this STM differs from that of Vollum and Fang (2014) due to the difference in stress field obtained with NLFEA for simply supported and cantilever beams. The strengths of the tested beams are compared with the predictions of NLFEA and the shear enhancement provisions given in EC2 (BSI 2004), BS 8110 (BSI 1997) and fib Model Code 2010 (fib 2010). The influence of loading arrangement on shear enhancement is also studied parametrically using NLFEA and STM.

**Experimental program**

Twelve beams were tested to investigate the influence of loading arrangement on shear enhancement. All the beams measured 2800 mm long by 250mm wide by 500mm deep. The beams were tested in three series of four, depicted A to C, using the loading arrangements shown in Figure 2. Details of the beam geometry and reinforcement are provided in Figure 3 and Figure 4, which should be read in conjunction with Table 1. The flexural tension reinforcement, in all the beams, consisted of two layers of three bars of 25 mm diameter ($\rho = 2.36\%$). The effective depth measured to the centroid of the tension reinforcement was 429.5 mm. Two 16 mm diameter bars were provided in the flexural compression zone of all beams ($\rho' = 0.322\%$). In all the tests, additional shear reinforcement was provided in the left-hand shear span, as depicted in Figure 4, to ensure that failure occurred in the right-hand shear span. Where provided, shear reinforcement in the critical shear span consisted of 8 mm diameter links at 200 mm centres. The test ID describes the test series (A, B and C), loading face (T or
C), spacing of the shear reinforcement in mm (0 or 200) and loading ratio. For example, BT200 (0.3/0.7):

“B” – test Series
“T” – loaded on the tension face
“200” spacing of 8 mm diameter stirrups in critical shear span (0 depicts no shear reinforcement)
“0.3/0.7” ratio of inner (closest to support) and outer loads.

Each series of four tests had separate objectives as described below. The clear shear spans in Series A and B are the same as in Vollum and Fang’s (2014) beams that had the same depth but were either 160 mm or 165 mm wide. Vollum and Fang’s beams with stirrups also had two layers of 25 mm diameter bars. Series A was designed to investigate the influence of loading face on the shear resistance of otherwise identical beams without and with shear reinforcement. Beams AC0 (0.5/0.5) and AC200 (0.5/0.5) were simply supported and loaded on their flexural compression face while beams AT0 (0.5/0.5) and AT200 (0.5/0.5) were balanced cantilevers loaded on their tension face. Each shear span was loaded with two nominally equal point loads positioned within 2d of the adjacent support as indicated in Figure 2 and Table 1.
Figure 2 Test setup and loading arrangements a) beams loaded in the compression face, b) beams loaded in the tension face within 2d and c) beams loaded in the tension face partly outside 2d
Figure 3 Cross-section for beams loaded in the a) compression and b) tension faces

Table 1: Description of beams and test configurations

<table>
<thead>
<tr>
<th>Beam</th>
<th>$f_c$ (MPa)</th>
<th>$f_{ct}$ (MPa)</th>
<th>Shear span (mm)</th>
<th>Load Ratio</th>
<th>Plates dimensions (mm)</th>
<th>Rig dimension (mm)</th>
<th>Span to depth ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$a_{v1}$</td>
<td>$a_{v2}$</td>
<td>$P_1/P$</td>
<td>$P_2/P$</td>
<td>$L_1$</td>
</tr>
<tr>
<td>AC0 (0.5/0.5)</td>
<td>29.8</td>
<td>2.5</td>
<td>325</td>
<td>700</td>
<td>0.5</td>
<td>0.5</td>
<td>100</td>
</tr>
<tr>
<td>AC200 (0.5/0.5)</td>
<td>30.3</td>
<td>2.6</td>
<td>325</td>
<td>700</td>
<td>0.5</td>
<td>0.5</td>
<td>100</td>
</tr>
<tr>
<td>AT0 (0.5/0.5)</td>
<td>30.6</td>
<td>2.7</td>
<td>325</td>
<td>700</td>
<td>0.6</td>
<td>0.4</td>
<td>100</td>
</tr>
<tr>
<td>AT200 (0.5/0.5)</td>
<td>31.2</td>
<td>2.8</td>
<td>325</td>
<td>700</td>
<td>0.6</td>
<td>0.4</td>
<td>100</td>
</tr>
<tr>
<td>BT200 (0.5/0.5)</td>
<td>28.4</td>
<td>2.6</td>
<td>325</td>
<td>700</td>
<td>0.5</td>
<td>0.5</td>
<td>100</td>
</tr>
<tr>
<td>BT200 (1.0/0)</td>
<td>28.9</td>
<td>2.7</td>
<td>325</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>BT200 (0/1.0)</td>
<td>29.2</td>
<td>2.8</td>
<td>700</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>BT200 (0.3/0.7)</td>
<td>28.8</td>
<td>2.7</td>
<td>325</td>
<td>700</td>
<td>0.3</td>
<td>0.7</td>
<td>100</td>
</tr>
<tr>
<td>CT0 (1.0/0)</td>
<td>26.9</td>
<td>2.6</td>
<td>512.5</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>CT200 (1.0/0)</td>
<td>28.0</td>
<td>2.6</td>
<td>512.5</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>CT0 (0.6/0.4)</td>
<td>28.9</td>
<td>2.6</td>
<td>512.5</td>
<td>1100</td>
<td>0.6</td>
<td>0.4</td>
<td>500</td>
</tr>
<tr>
<td>CT200 (0.6/0.4)</td>
<td>28.3</td>
<td>2.6</td>
<td>512.5</td>
<td>1100</td>
<td>0.6</td>
<td>0.4</td>
<td>500</td>
</tr>
</tbody>
</table>
Figure 4  Reinforcement details for beams: a) AC0 (0.5/0.5/), b) AC200 (0.5/0.5), c) AT0 (0.5/0.5), d) AT200 (0.5/0.5) and beams in Series B, e) CT0 (1.0/0), f) CT200 (1.0/0), g) CT0 (0.6/0.4) and h) CT200 (0.6/0.4)

Series B considered the influence of varying the ratio between the inner and outer loads on the shear resistance of balanced cantilever beams with stirrups positioned at 200 mm centres. The loads were placed at the same positions as in Series A but in two of the tests, the critical shear span was loaded with only one point load to give load ratios of 0/1.0 and 1.0/0. Beam BT200 (0.5/0.5) was notionally a duplicate of beam AT200 (0.5/0.5) but the applied loading ratios differed as shown in Table 1 for reasons explained subsequently.

Series C compared the shear resistance of beams loaded either entirely within 2d of the support or partly within 2d of the support. These tests were motivated by the very different strength predictions given for this loading case by the shear enhancement methods of the superseded UK code BS8110 and EC2. Series C included beams without and with shear reinforcement. The critical shear span of beams CT0 (1.0/0) and CT200 (1.0/0) was loaded within 2d of the support by a single point load positioned at a = 712.5 mm (a/d = 1.66) where “a” is the shear span measured between the centreline of the support and load. The dimension a = 712.5 mm was selected to be midway between the pair of point loads in Series A and B. Hence, the loading arrangement in tests CT (1.0/0) is statically equivalent to that in tests AT (0.5/0.5) and BT (0.5/0.5). The critical shear span of beams CT0 (0.6/0.4) and CT200 (0.6/0.4) was also loaded with a concentrated load positioned at a = 1300 mm (a/d=3.0) where shear enhancement is minimal.

Material properties

The beams were cast in three groups of four with each group cast from a single batch of ready-mixed concrete specified to have strength class C25/30, consistency class S3 and limestone aggregate with a maximum size of 20 mm. A total of 18 (36 for Series A) 100 mm cubes, 12 cylinders (100 mm diameter × 200 mm long) for compressive strength, and 12 cylinders (150 mm diameter × 300 mm long) for split
cylinder tensile strength were cast with each series of beams. Half of the specimens were cured in water at 20°C with the remainder cured in air alongside the beams. Cubes were tested at regular intervals to establish the development of compressive strength with time as well as the cube strength at the time of testing each beam. The cylinders were tested on the same day as the first and last beam test of each series. Compressive cylinder strengths were estimated for intermediate beam tests by interpolation making use of the strength development curve obtained from the cube tests. The resulting compressive and tensile strengths are given in Table 1.

The reinforcement properties were obtained from tensile tests in which the axial strain was measured using a digital video extensometer. The resulting mechanical properties are listed in Table 2 in which \( f_{yk,0.2} \) depicts the 0.2% offset yield strain, \( f_{tk} \) the ultimate tensile strength and \( \varepsilon_1 \) is the strain at maximum force. The reinforcement elastic modulus is assumed to be 200 kN/mm² in this paper.

Table 2: Reinforcement properties for the experimental program

<table>
<thead>
<tr>
<th>Bar size</th>
<th>( f_{yk,0.2} ) (MPa)</th>
<th>( f_{tk} ) (MPa)</th>
<th>( \varepsilon_1 ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 mm</td>
<td>570</td>
<td>670</td>
<td>8.89</td>
</tr>
<tr>
<td>16 mm</td>
<td>550</td>
<td>670</td>
<td>10.81</td>
</tr>
<tr>
<td>10 mm</td>
<td>520</td>
<td>650</td>
<td>9.15</td>
</tr>
<tr>
<td>8 mm</td>
<td>560</td>
<td>689</td>
<td>5.36</td>
</tr>
</tbody>
</table>

**Instrumentation**

The applied loads and reactions were measured with load cells positioned as shown in Figure 2. Detailed measurements of displacements and crack kinematics, which are not reported here, were obtained by means of Digital Image Correlation (DIC). Transducers were also used as a cross-check on displacements calculated with DIC. Up to 24 strain gauges were fixed to the reinforcement at locations selected based on nonlinear finite element analysis.

**Test results and discussion**

The beams were loaded in displacement control to capture the post-failure response. For beams loaded on the tension face, the displacement of the left-hand actuator was fixed at zero while loading the right-hand actuator in displacement control. All the tested beams failed in the right-hand shear span as intended. In all beams, diagonal shear cracks formed prior to failure. With exception of CT0(0.6/0.4), failure was characterised by penetration of the critical shear crack through the flexural compression zone. The width of diagonal cracks was relatively uniform until near the crack ends where the width was zero prior to the crack end reaching the beam edge. The beam failure loads are listed in Table 3, which shows the critical failure plane highlighted in bold, with further details given below.

**Series A:** Series A considered the influence of loading face for matching pairs of beams without and with shear reinforcement. Due to rotational friction in the loading arrangement, which was eliminated...
in subsequent tests, the loading ratios in tests AT0 (0.5/0.5) and AT200 (0.5/0.5) were 0.58/0.42 rather than 0.5/0.5 as intended. In subsequent tests, the loading ratios were as intended including beam BT 200 (0.5/0.5) which was identical to beam AT200 (0.5/0.5) apart from a small difference in concrete strength. Table 3 shows that despite the difference in loading ratio, the outermost failure loads were fairly similar in beams AT200 (0.5/0.5) and BT200 (0.5/0.5) at 308 kN and 324 kN respectively. Table 3 also shows that the shear force at failure of the critical outer shear span (P₂) of comparable beams in Series A was noticeably less for tension than compression face loading. Possible reasons for this are discussed subsequently.

Table 3: Experimental failure load and estimations of Pcalc/Ptest using design codes and STMs

<table>
<thead>
<tr>
<th>Beam ID</th>
<th>STM</th>
<th>RHS Failure loads* (kN)</th>
<th>STM Pcalc/Ptest (total)</th>
<th>Pcalc/Ptest (total)</th>
<th>FE Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>P Inner</td>
<td>P Outer</td>
<td>P Total</td>
<td>P Flexure</td>
</tr>
<tr>
<td>AC0 (0.5/0.5)</td>
<td>2</td>
<td>269</td>
<td>269</td>
<td>539</td>
<td>834</td>
</tr>
<tr>
<td>AC200 (0.5/0.5)</td>
<td>2</td>
<td>371</td>
<td>371</td>
<td>742</td>
<td>839</td>
</tr>
<tr>
<td>AT0 (0.5/0.5)</td>
<td>3</td>
<td>260</td>
<td>189</td>
<td>449</td>
<td>883</td>
</tr>
<tr>
<td>AT200 (0.5/0.5)</td>
<td>3</td>
<td>425</td>
<td>308</td>
<td>733</td>
<td>890</td>
</tr>
<tr>
<td>BT200 (0.5/0.5)</td>
<td>3</td>
<td>324</td>
<td>324</td>
<td>648</td>
<td>820</td>
</tr>
<tr>
<td>BT200 (1.0/0)</td>
<td>1/3</td>
<td>610</td>
<td>610</td>
<td>1169</td>
<td>780</td>
</tr>
<tr>
<td>BT200 (0/1.0)</td>
<td>1/3</td>
<td>0</td>
<td>423</td>
<td>423</td>
<td>640</td>
</tr>
<tr>
<td>BT200 (0.3/0.7)</td>
<td>3</td>
<td>158</td>
<td>370</td>
<td>528</td>
<td>737</td>
</tr>
<tr>
<td>CT0 (1.0/0)</td>
<td>1/3</td>
<td>441</td>
<td>441</td>
<td>802</td>
<td>802</td>
</tr>
<tr>
<td>CT200 (1.0/0)</td>
<td>1/3</td>
<td>543</td>
<td>543</td>
<td>815</td>
<td>815</td>
</tr>
<tr>
<td>CT0 (0.6/0.4)</td>
<td>3</td>
<td>227</td>
<td>151</td>
<td>378</td>
<td>603</td>
</tr>
<tr>
<td>CT200 (0.6/0.4)</td>
<td>3</td>
<td>301</td>
<td>201</td>
<td>502</td>
<td>598</td>
</tr>
</tbody>
</table>

Average: 0.91 0.87 0.78 0.49 0.67 0.54 0.75 1.00
St Dev: 0.20 0.14 0.16 0.06 0.10 0.11 0.13 0.09
COV %: 0.22 0.17 0.21 0.11 14 20 18 9.19

Notes: a) critical failure plane indicated in bold
Figure 5 shows the crack patterns at failure for beams in Series A. Different line types are used to distinguish the first, second, third and fourth cracks to form. Cracks which developed at or post-failure are also indicated. In all cases, the critical shear crack ran between the inside edges of the support and outer loading plate. Several diagonal cracks formed in the beams with shear reinforcement, unlike the beams without shear reinforcement, where one dominant shear crack formed. Diagonal cracking initiated near the supports of the simply supported beams and extended towards the outer loading plate to form the critical shear crack. Conversely, in the cantilever beams, diagonal cracking initiated in the inner shear span near beam mid-height and from there extended outwards towards the support and inner loading plate. These cracks did not form part of the critical shear crack which developed in the outer shear span subsequently. Figure 6 uses superposition to compare the crack patterns of comparable beams loaded on the tension and compression face. The crack patterns are inverted for the two beams loaded on the tension face. Figure 6 shows that the crack patterns were very similar in comparable beams loaded on the tension and compression faces despite the apparent reduced strength of beams loaded on the tension face, which is discussed subsequently.
**Series B:** Figure 7 shows the development of cracking in Series B. In all these tests, diagonal cracking initiated in the inner shear span. The critical diagonal crack formed subsequently and extended from the outermost loading plate to the support. As in series A, failure arose as a result of the critical shear crack penetrating the flexural compression zone.

In beam BT200 (0.5/0.5), the first diagonal crack formed near mid-height of the beam and extended towards the support and inner loading plate. Subsequently, the critical shear crack initiated near the inner edge of the outer loading plate and extended towards the inside edge of the bottom support plate. In beam BT200 (1.0/0), diagonal cracking also initiated in the inner shear span, near the beam mid-height. The critical shear crack developed subsequently at mid-height of the beam and extended parallel to the initial diagonal crack towards the support to the loading plate. In beam BT200 (0/1.0), the critical shear crack developed from a flexural crack. This crack initially extended towards the support plate prior to developing a secondary branch which extended towards the loading plate to form the critical diagonal crack. In beam BT200 (0.3/0.7), with greater load at the outer support, diagonal cracking initiated adjacent to the inner loading plate. The critical shear crack formed subsequently and extended from the outer loading plate towards the inner corner of the support plate.

Figure 8 superimposes the critical shear crack in all the beams of Series B as well as beam AT200 (0.5/0.5) from Series A, which was notionally identical to beam BT200 (0.5/0.5). As previously mentioned, the load ratio in beam AT200 (0.5/0.5) was actually 0.58/0.42 rather than 0.5/0.5 as intended. Two types of crack path, dependent on the loading arrangement, are evident in Figure 8. The crack path is convex in beams BT200 (0.5/0.5) and AT200 (0.58/0.42) but almost straight in beams BT200 (0/1.0) and (0.3/0.7). This suggests that the crack path is diverted by the inner load once it exceeds around 40% of the total failure load but further data are required to confirm this.
Figure 7: Crack patterns for beams in Series B

Figure 8: Critical shear cracks for the second series beams at 95% of Vmax
**Series C:** Figure 9 shows the crack patterns at failure for test Series C. Beams CT0 (1.0/0) and CT200 (1.0/0) were loaded with a single point load at \( a/d = 1.65 \) while beams CT0 (0.6/0.4) and CT200 (0.6/0.4) were loaded with two-point loads positioned at \( a/d = 1.65 \) and \( a/d = 3.0d \). In beams CT0 (1.0/0) and CT200 (1.0/0), the critical shear crack extended from the inner edge of the loading plate to the inner edge of the support. The crack patterns in beams CT0 (0.6/0.4) and CT200 (0.6/0.4) were similar up to around 95% of the failure load but the eventual failure planes differed. The initial crack in beam CT0 (0.6/0.4) extended from the support to the inner loading plate. However, failure was initiated by the sudden development of a crack between the outer loading plate and the support. Subsequently, cracking also propagated along the tension reinforcement at the outer loading plate and around the bend of the flexural reinforcement. In beam CT200 (0.6/0.4), diagonal cracking initiated at mid-height of the beam and extended in two separate lengths along the diagonal linking the inner loading plate and support. As the load approached failure, multiple cracks developed in the inner shear span. These cracks eventually coalesced to form the critical shear crack.

![Crack Patterns](image)

Figure 9: Crack pattern for beams in Series (C).

**Assessment of beam strength using codified sectional methods**

**Sectional approach for shear enhancement**

This section considers the shear enhancement design methods given in the superseded UK code BS8110 (BSI, 1997), EC2 (BSI, 2004) and fib Model Code 2010 (MC2010) (fib, 2013). These methods apply to beams loaded on the top face within \( \alpha_p \leq 2d \). BS 8110 enhances the shear resistance provided
by concrete $V_{Rd,c}$ for beams loaded within $a_v \leq 2d$ by the multiple $\frac{2d}{a_v}$. Conversely, EC2 and MC2010 reduce the contribution to the design shear force of loads applied within $a_v \leq 2d$ by the multiple $\beta = \frac{a_v}{2d}$ where $\beta$ is limited to a minimum of 0.25 in EC2 and 0.5 in MC2010.

For beams with shear reinforcement, BS8110 gives the enhanced shear resistance as:

$$V_{Rd} = \frac{2d}{a_v} n A_{sw} f_y w d$$

Where $n$ is the number of stirrups within the central $\frac{3}{4}$ of the clear shear span, $A_{sw}$ is the cross-sectional area of the vertical legs of each set of stirrups and $V_{Rd,c}$ is the shear resistance of the beam without shear reinforcement.

EC2 requires the reduced design shear force to be less than or equal to the greater of the resistances provided by the concrete alone and the stirrups within the central $\frac{3}{4}$ of the clear shear span as follows:

$$V_{Rd} = \max (V_{Rd,c}, n A_{sw} f_y w d)$$

$$V_{Rd,c} = \frac{0.18}{\gamma_c} k (100 \rho f_{ck})^{1/3} b_w d$$

In which $k = 1 + \frac{200}{d} \leq 2$ and $\rho = \frac{A_s}{b_w d} \leq 0.02$.

There can be significant differences between the enhanced shear resistances given by EC2 and BS8110 as shown in Table 3 for the tested beams. Moreover, application of the EC2 shear enhancement design provisions are unclear for beams with shear reinforcement and multiple point loads applied within $a_v \leq 2d$ as discussed by Vollum and Fang (2015).

MC2010 compares the reduced design shear force with a design shear resistance calculated in the same way as for beams without shear enhancement. MC2010 gives four levels of design approximation for shear of which Levels II and III are intended for the detailed design of existing structures.

MC2010 Level II:

$$V_{Rd} = \max (V_{Rd,c}, V_{Rd,s})$$

$$V_{Rd,c} = k_v \sqrt{f_{ck}} z b_w$$

$$k_v = \frac{0.4}{1 + 1500 \varepsilon_x^2} \frac{1300}{1000 + k_{dg} z}$$

$$k_{dg} = \frac{32}{16 + d_g} \geq 0.75$$

$$\varepsilon_x = \frac{1}{2E_s A_s} \left[ \frac{M_{Ed}}{z} + V_{Ed} \right] \leq \frac{M_{max}}{2E_s A_s z}$$

Where $V_{Rd,s}$ is the shear resistance provided by shear reinforcement, $d_g$ is the maximum aggregate size, $z = 0.9d$, $M_{Ed}$ is the design moment at the critical section for shear and $M_{max}$ is the maximum
beam moment. The upper limit on \( \varepsilon_c \) of \( \varepsilon_x \leq \frac{M_{\text{max}}}{2E_sA_s} \) is not given in MC2010 but is implicit in its design model for shear. According to MC2010, the critical section for shear is located at a distance \( d \) from either the face of the support or the face of the load with the most critical governing. If \( a_v < d \), MC2010 suggests that the shear resistance should be calculated as if the load were at \( d \) from the face of the support.

\[
V_{Rd,s} = \frac{A_{sw}}{s}zf_{yd}\cot\theta \leq V_{Rd,max}
\]

\[
V_{Rd,max} = k_c\frac{f_{ck}}{Y_c}b_wz\sin\theta\cos\theta
\]

\[
\theta_{\text{min}} = 20^\circ + 10,000\varepsilon_x
\]

\[
k_c = k_\varepsilon f_{ec} = \frac{1}{1.2+55\varepsilon_1}\left(\frac{30}{f_{ck}}\right)^{1/3} \text{ where } \frac{1}{1.2+55\varepsilon_1} \leq 0.65 \text{ and } \left(\frac{30}{f_{ck}}\right)^{1/3} \leq 1
\]

\[
\varepsilon_1 = \varepsilon_x + (\varepsilon_x + 0.002)\cot^2\theta
\]

In MC2010 Level III, \( V_{Rd} = V_{Rd,c} + V_{Rd,s} \) where all the terms are as defined as above except:

\[
k_v = \frac{0.4}{1+15000\varepsilon_x}\left(1 - \frac{V_{Rd}}{V_{Rd,max}^{\theta_{\text{min}}}}\right) \geq 0
\]

**Enhanced shear strength of beams with multiple point loads**

When two point loads are applied within \( a_v \leq 2d \), BS8110 requires failure to be checked for each of the two failure planes shown in Figure 10 as follows:

- Inner Load
- Outer Load
- Inner failure plane
- Outer failure plane

**Figure 10: Possible failure planes for beams loaded with two points within 2d**
For beams with multiple point loads applied on the upper side within 2\(d\) of supports, EC2 appears to require the strength to be calculated as the least of:

**Inner plane:**
\[ V_{Ed} \leq V_{Rd,c} \frac{2d}{a_{v1}} + n_1 A_{sw} f_{yd} \frac{d}{a_{v1}} \]  
\[ \beta_1 P_1 + \beta_2 P_2 \leq V_{Rd1} = \max(V_{Rd,c}, n_1 A_{sw} f_{yd}) \]  
\[ \beta_1 P_1 \leq V_{Rd1} = \max(V_{Rd,c}, n_1 A_{sw} f_{yd}) \]  
\[ \beta_2 P_2 \leq V_{Rd2} = \max(V_{Rd,c}, n_2 A_{sw} f_{yd}) \]  
\[ \frac{\beta_1 P_1}{\max(V_{Rd,c}, n_1 A_{sw} f_{yd})} + \frac{\beta_2 P_2}{\max(V_{Rd,c}, n_2 A_{sw} f_{yd})} \leq 1.0 \]

Vollum and Fang (2015) showed that equation 16 can give nonsensical results for beams with shear reinforcement since the application of an infinitesimally small load at \(a_1\) can falsely cause the inner shear plane to become critical. To circumvent this, Vollum and Fang (2015) suggested calculating the design shear resistance for failure along the inner shear plane as:

\[ \frac{\beta_1 P_1}{\max(V_{Rd,c}, n_1 A_{sw} f_{yd})} + \frac{\beta_2 P_2}{\max(V_{Rd,c}, n_2 A_{sw} f_{yd})} \leq 1.0 \]

Ratios of calculated to predicted failure loads (\(P_{calc}/P_{test}\)) are given in Table 3 for EC2 (calculated with equations 17 and 18 for beams with multiple point loads), BS8110 and MC2010. For purposes of comparison, \(V_{Rdc}\) was calculated with EC2 when applying the BS 8110 shear enhancement method. All the predictions are safe. Overall, MC2010 LoA III and BS8110 give the best estimates of \(P_{calc}/P_{test}\) and EC2 the worst. None of the considered design methods explicitly account for the influence of loading face (i.e. compression or tension) or support condition (i.e. simply supported or continuous). However, the MC2010 predictions depend on the flexural reinforcement strain, which depends on the loading arrangement.

**Evaluation of the shear strength using STM and NLFEA**

Sagaseta and Vollum (2010) developed the STM shown in Figure 11 (depicted STM1 in this paper) for beams loaded on their compression face with concentrated loads within 2\(d\) of supports. Shear strength is typically governed by simultaneous yielding of shear reinforcement and crushing of strut I at the CCT node. STM1 is applicable to all the tested beams with single-point loads applied within \(a_1/d = 2\) of the supports. In the case of cantilever beams, the geometry of STM1 is inverted, in which case the central support corresponds to the top-loading plate in Figure 3. Vollum and Fang (2013) used a similar approach to develop an STM (depicted STM2 in this paper) for beams loaded on their flexural compression face with four-point loads (see Figure 12). The geometry of STM1 and STM2 was developed making use of stress fields derived using NLFEA. In both cases, the load carried by the direct
strut is calculated from vertical equilibrium assuming that stirrups yield at failure. The shear resistance is limited by the strength of the direct strut. Both STM1 and STM2 give reasonable predictions of measured short shear span beam strengths as shown elsewhere (Sagaseta and Vollum, 2010 and Vollum and Fang, 2015).

Figure 11: Sagaseta and Vollum (2010) STM for deep beams with vertical shear reinforcement
STM2 is inapplicable to beams loaded on their tension face since in this case the location of the nodes at the upper end of the direct struts is constrained by the location of the flexural reinforcement. This is illustrated in Figure 13, which shows a possible STM for beams loaded with two loads on the tension face (depicted STM3). For beams without shear reinforcement, STM3 reduces to that shown in Figure 1b.

In STM3, the total load (P) is divided between the inner (P₁) and outer (P₂) loading plates in the proportions \( P₁ = R₁P \) and \( P₂ = R₂P \). If the inner load \( P₁ \) is removed, STM3 reduces to STM1. It should also be noted that STM3 can be readily modified for compression face loading in which case the top nodes become CCC nodes as in Figure 11. The two discrete vertical ties shown in Figure 13 represent the centroid of the stirrups placed within the inner and outer shear spans. The STM equations below are derived for a uniform shear reinforcement ratio within the clear shear span \( a_{v2} \) as adopted in the tests. The geometry of the STM is defined by its node geometry and the horizontal (X) and vertical (Y) projections of each strut. This allows the horizontal components of force \( H_l \) in each strut to be
calculated in terms of their horizontal and vertical projections \((X, Y)\), the applied loads and stirrup forces \((T_{s1}, T_{s2})\) as follows:

\[
\begin{align*}
X_1 &= 0.5a_v + l_{t,2}(1 - \frac{0.5(T_{s1} + T_{s2})}{P_2}) \\
Y_1 &= h - 2d' + \gamma d' - 0.5\beta_1x \\
H_1 &= \frac{X_1}{Y_1}(T_{s1} + T_{s2})
\end{align*}
\]
\[ \beta_1 = \frac{H_1}{C} \]  
\[ \gamma = \frac{H_1}{H_1 + H_2} \]

**Strut (2)**
\[ X_2 = a_{v2} + 0.5 \frac{P_2 - (T_{s1} + T_{s2})}{P_1 + P_2} l_b + 0.5(1 - \frac{T_{s1} + T_{s2}}{P_2}) l_{t,2} \]
\[ Y_2 = h - (\beta_1 + 0.5\beta_2)x - d'(1 - \gamma) \]
\[ H_2 = \frac{X_2}{Y_2} (P_2 - T_{s1} - T_{s2}) \]
\[ \beta_2 = \frac{H_2}{C} \]

**Strut (3)**
\[ X_3 = z_2 + \frac{(P_2 - T_{s1} - 0.5T_{s2})}{P_1 + P_2} l_b \]
\[ Y_3 = h - (\beta_1 + \beta_2 + 0.5\beta_3)x \]
\[ H_3 = \frac{X_3}{Y_3} T_{s2} \]
\[ \beta_3 = \frac{H_3}{C} \]

**Strut (4)**
\[ X_4 = a_{v1} + 0.5l_{t,1} + \frac{0.5P_1 + P_2 - T_{s1}}{P_1 + P_2} l_b \]
\[ Y_4 = d - (\beta_1 + \beta_2 + \beta_3 + 0.5\beta_4)x \]
\[ H_4 = \frac{X_4}{Y_4} P_1 \]
\[ \beta_4 = \frac{H_4}{C} \]

**Strut (5)**
\[ X_5 = z_1 + l_b(1 - \frac{0.5T_{s1}}{P_1 + P_2}) \]
\[ Y_5 = d - (1 - 0.5\beta_5)x \]
In the above equations, \( \beta_i \) is the ratio of the horizontal component of the force in strut (i) to the total compression force in the flexural compression zone, \( T_{si} \) is the tensile force carried by stirrup set (i), \( z_i \) is the horizontal distance to the centroid of \( T_{si} \) measured from the inside edge of the bottom support plate and \( C = bx f_{flex} \) is the flexural compressive force where \( f_{flex} \) is the flexural compressive strength. The dimensions \( z_1 \) and \( z_2 \) were taken as 0.25\( a_2 \) and 0.75\( a_2 \) respectively in the analysis of the tested specimens. The flexural compression zone depth \( x \) can be calculated from moment equilibrium as follows:

\[
M = P_2 a_2 + P_1 a_1 - 0.5(P_1 + P_2)l_b
\]

\[
\frac{x}{d} = 1 - \sqrt{1 - 2k}
\]

\[
k = \frac{M}{bd^2f_{flex}}
\]

Shear failure in STM3 is assumed to result from yielding of the shear reinforcement and crushing of either strut (2) or strut (4). The strut strength is assumed to be the least of the strengths calculated at each end as the product of the design concrete strength and the strut cross-sectional area normal to its centreline at the strut-node interface. The strut width, normal to its centreline, at the strut-node interface is given by:

\[
w = l\sin\theta + u\cos\theta
\]

In which \( l \) and \( u \) are the horizontal and vertical projections of the strut-node interface (see Table 4).

The axial resistance of each direct strut was calculated in accordance with the recommendations of EC2 and the Modified Compression Field Theory (Vecchio and Collins 1986). Following the example of Sagaseta and Vollum (2010) and Collins and Mitchell (1991), different concrete strengths were adopted at the intersection of struts with CCC and CCT nodes as depicted in Table 4 in which the principal tensile strain \( \varepsilon_1 \) is given by:

\[
\varepsilon_1 = \varepsilon_L + (\varepsilon_L + 0.002)\cot^2\theta
\]

where \( \varepsilon_L \) is the strain in the flexural reinforcement at its intersection with the strut centreline at the CCT node.

The flexural compressive strength \( f_{flex} \) was taken as \( f_{flex} = (1 - \frac{f_c}{250})f_c \) in calculations with EC2 and \( f_{flex} = 0.85f_c \) in calculations with the MCFT.
Table 4: Nodal dimensions (\(u\) and \(l\)) and design concrete strengths for struts 2 and 4

<table>
<thead>
<tr>
<th></th>
<th>Strut 2</th>
<th>Strut 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Top (CCT)</td>
<td>Bottom (CCC)</td>
</tr>
<tr>
<td>Width</td>
<td>(\frac{P_2 - T_s1 - T_s2}{P_2}l_t)</td>
<td>(\frac{P_2 - T_s1 - T_s2}{P_1 + P_2}l_b)</td>
</tr>
<tr>
<td>Height</td>
<td>(2d'(1 - \gamma))</td>
<td>(\beta_2x)</td>
</tr>
<tr>
<td>EC2 strut strength</td>
<td>(0.6 \left(1 - \frac{f_c}{250}\right)f_c)</td>
<td>(\alpha \left(1 - \frac{f_c}{250}\right)f_c)</td>
</tr>
<tr>
<td>MCFT strut strength</td>
<td>(\frac{f_c}{(0.8 + 170\varepsilon_1)})</td>
<td>(0.85f_c)</td>
</tr>
</tbody>
</table>

**Solution procedure for STM3**

The shear resistance in STM3 can be readily obtained using the following procedure in a nonlinear equation solver like the Generalized Reduced Gradient (GRG) solver in Microsoft Excel:

1. Estimate an initial value of \(P\) and calculate \(P_1\) and \(P_2\) (\(P_1 = R_1, P_2 = R_2\) where \(R_1\) and \(R_2\) are known).
2. Calculate the flexural compression depth (\(x\)) with equation 39 and hence the flexural compressive force (\(C\)).
3. Assume stirrups yield and calculate \(X_1, X_2, X_3\) and \(X_4\) using equations 19, 24, 28 and 32.
4. Estimate unknown forces \(H_1, H_2, H_3, H_4\)
5. Calculate \(\beta_1, \gamma, \beta_2, \beta_3, \beta_4\) using equations 22, 23, 27, 31 and 35.
6. Vary \(H_1, H_2, H_3, H_4\) in solver until the estimated and calculated values are equal (calculate \(H_1\) to \(H_4\) using equations 21, 26, 30 and 34).
7. Vary \(P\) until strut (2) or strut (4) fails.

The tested beams were analysed with the appropriate STM as indicated in Table 3, which presents failure loads for strut strengths calculated with both EC2 and the MCFT. All the stirrups in the clear shear span were assumed to yield unless the resulting direct strut force was negative, in which case the stirrup force was calculated from equilibrium assuming that the direct strut force was zero. Results are presented in Table 3 for STM 3 EC2 with \(\alpha = 1.0\) (as adopted by Sagaseta and Vollum (2010) and Vollum and Fang (2015) for beams loaded on their compression face) and \(\alpha = 0.85\) which gives improved predictions for STM3 where failure at the CCC node governs (see Table 4 for definition of \(\alpha\)). As shown in Table 3, the overall accuracy of the STM predictions is good particularly if strut
strengths are calculated using EC2 with \( \alpha = 0.85 \) (mean/covariance of \( P_{\text{calc}}/P_{\text{test}} = 0.87/17\% \)). Where applicable, STM3 EC2 gives good estimates of failure load for the eight applicable beams, although the strength of beam AT0 (0.5/0.5) is overestimated (mean/covariance of \( P_{\text{calc}}/P_{\text{test}} = 0.85/13\% \) for EC2 (\( \alpha = 0.85 \)) and 0.84/23% for MCFT). STM3 EC2 with \( \alpha = 0.85 \) correctly predicts shear failure to occur in the outer shear span due to crushing of strut 2 for all relevant beams.

**Comparison between NLFEA, STM and observed behaviour**

Comparisons were made between the behaviours predicted by STM and NLFEA carried out with DIANA (DIANA 2017). Concrete was modelled with using an orthogonal grid of 25 mm square eight-node quadrilateral isoparametric plane-stress elements (CQ16M). Interface elements were introduced between the loading plates and the concrete to avoid stress concentrations developing at the corners of loading plates (Vollum and Fang, 2015). The interface elements were given normal and shear stiffness of \( 1 \times 10^7 \text{N/mm}^3 \) and \( 1 \times 10^5 \text{N/mm}^3 \) respectively as recommended for similar cases (DIANA 2017). Reinforcement was modelled with discrete embedded elements using the measured stress-strain curve with isotropic strain hardening. A fixed crack total strain model was used in conjunction with the Feenstra parabolic compression curve (DIANA 2017). The compressive fracture energy was taken as \( G_c = 100G_f \) where \( G_f \) is the tensile fracture energy. The reduction in concrete strength due to lateral cracking was modelled using the method of Vecchio and Collins (1993). The Hordijk tensile softening model (DIANA 2017) was used in conjunction with the measured split cylinder tensile strength. Based on calibration studies, a variable shear retention model (Maekawa, Okamura et al. 2003) was used for beams without shear reinforcement. In beams with shear reinforcement, better predictions of shear resistance were obtained with a constant shear retention factor of 0.25 as used by Vollum and Fang (2015) for similar cases. The concrete elastic modulus and tensile fracture energy were calculated using fib Model Code 1990 (fib, 1993) as follows:

\[
E_c = 22 \left( \frac{f_{ck}}{10} \right)^{0.3}  
\]

\[
G_f = G_{f0} \left( \frac{f_{cm}}{f_{cm0}} \right)^{0.7}  
\]

\[
G_{f0} = 0.0204 + 6.625 \times 10^{-4} D_{\text{max}}^{0.95}  
\]

Where \( D_{\text{max}} \) is the maximum aggregate size, \( f_{cm} \) is the mean concrete compressive strength and \( f_{cm0} = 10 \text{MPa} \).

The NLFEA predictions of \( P_{\text{calc}}/P_{\text{test}} \) (where \( P \) is the total load) are presented in Table 3, which shows that the NLFEA predictions are on average more accurate than both the STM and code predictions. It is interesting to compare the measured and predicted strengths of beams CT0 (1.0/0) and CT200 (1.0/0) with their statically equivalent loaded pairs in series A and B. Table 3 shows that the measured strengths of beams CT0 (1.0/0) and AT0 (0.5/0.5) are similar but the strength of CT200 (1.0/0) is less than that of beams AT200 (0.5/0.5) and BT200 (0.5/0.5). This suggests that it is conservative to represent a pair
of point loads positioned within $2d$ of the support by a statically equivalent single point load. This assumption also leads to safe strength predictions of strength for all the considered design methods.

Figure 14 shows the principal stress vectors obtained from the FEA overlaid on the STMs and observed crack pattern for selected beams. Results for beams AC0 (0.5/0.5) and AT0 (0.5/0.5) are shown in Figure 1. The NLFEA stress fields are broadly consistent with the assumed STM geometry particularly for beams without shear reinforcement where direct strutting action is most dominant. There are also clear differences in the NLFEA stress fields for beams loaded on the tension and compression faces which are consistent with the differing STM geometries assumed in STM2 and STM3 (e.g. see beams AC0 (0.5/0.5) and AT0 (0.5/0.5)).

![Figure 14: FEA principal stress, STMs and experimental cracks for the test beams: a) BT200 (0.5/0.5), b) AT200 (0.5/0.5), c) BT200 (1.0/0.0) and d) BT200 (0.3/0.7)](image)

**Shear reinforcement strains**

In the STM, the direct strut contribution is calculated assuming that all stirrups yield. The validity of this assumption was investigated by examining the strains measured in selected stirrups of each beam as well as the results of the NLFEA simulations. Strain gauges were positioned on all the stirrups between the support and outer load at locations where NLFEA suggested strains would be greatest.
Additionally, stirrup strains were estimated from measured crack widths using the method proposed by Sigrist (Sigrist 1995). In practice, the strain gauge positions did not always coincide with the critical shear cracks in the tested beams due to differences between the observed and estimated crack patterns. Therefore, the peak strains would usually have been greater than measured. The difference between observed and NLFEA crack patterns is to be expected since the geometry of the critical shear crack varies randomly between notionally identical beams. Despite this, the measured and calculated strains agree reasonably well as shown in Figure 15 for selected beams.

Table 5 summarises the number and position of stirrups that yielded in each test according to the measured strains, Sigrist and NLFEA. Based on the greatest of the measured and NLFEA strains, at least one stirrup yielded in all the beams. Despite overestimating the extent of stirrup yield, the STM gives reasonable estimates of beam strength as shown in Table 3.

![Figure 15: Comparison of measured and predicted stirrup strains in beams a) BT200(1.0/0) and b) BT200(0.3/0.7)](image-url)
Table 5 Location and number of stirrups yielded in the test and NLFEA

<table>
<thead>
<tr>
<th>Beam ID</th>
<th>Total No. of stirrups in shear span</th>
<th>Location of stirrups that yielded (STR 1 near to the support)</th>
<th>Strain Gauges</th>
<th>Sigrist (1995)</th>
<th>NLFEA</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC200 (0.5/0.5)</td>
<td>4</td>
<td>STR 2, 3</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>AT200 (0.5/0.5)</td>
<td>4</td>
<td>STR 3</td>
<td>STR 3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>BT200 (0.5/0.5)</td>
<td>4</td>
<td>STR 2, 3, 4</td>
<td>STR 2, 3</td>
<td>STR 1</td>
<td>-</td>
</tr>
<tr>
<td>BT200 (1.0/0)</td>
<td>2</td>
<td>STR 2</td>
<td>STR 1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>BT200 (0/1.0)</td>
<td>4</td>
<td>STR 2, 3</td>
<td>STR 2, 3</td>
<td>STR 1, 2, 3</td>
<td>-</td>
</tr>
<tr>
<td>BT200 (0.3/0.7)</td>
<td>4</td>
<td>STR 2, 4</td>
<td>STR 2</td>
<td>STR 2, 4</td>
<td>-</td>
</tr>
<tr>
<td>CT200 (1.0/0)</td>
<td>3</td>
<td>-</td>
<td>-</td>
<td>STR 1, 2</td>
<td>-</td>
</tr>
<tr>
<td>CT200 (0.6/0.4)</td>
<td>6</td>
<td>STR 2</td>
<td>STR 2</td>
<td>STR 2</td>
<td>-</td>
</tr>
</tbody>
</table>

**Prediction of critical failure plane for tested beams**

Dependent on the ratio between the inner ($P_1$) and outer ($P_2$) loads, beams loaded with two-point loads can fail in shear along either the inner or outer shear planes depicted in Figure 10. Both EC2 and BS8110 indicate the location of the critical shear plane as does STM3 where the outer shear span is critical when failure is governed by strut 2. The maximum shear force ($P = P_1+P_2$) is always critical with MC2010. The situation is less clear with the NLFEA, which does not give any direct indication of the critical failure plane. In the tests, failure occurred in the outer shear span of all specimens except CT200 (0.6/0.4). Conversely, BS8110, with the exceptions of AC0 (0.5/0.5) and BT200 (0.3/0.7), and EC2 falsely predict shear failure to be critical in the inner shear span for all specimens with two-point loads apart from CT200 (0.6/0.4) where the failure plane was correctly predicted.

Figures 16 – a and b show the measured and predicted influences of $P_1/P$ on $P_2$ and $P$ respectively for series B and beam AT200 (0.5/0.5). The failure mode is depicted in Figure 16 by open symbols for failure in the inner shear span and solid symbols for failure in the outer shear span. The STM3 EC2 results are plotted for $\alpha = 0.85$ (see Table 3) which gives the correct failure modes. Figure 16a shows that the shear resistance of the critical outer shear span reduced as $P_1$ was increased while Figure 16b shows the maximum shear force $P$ increased with increasing $P_1/P$ despite the reduction in shear resistance $P_2$ of the outer shear span. This interaction between $P_1$ and $P_2$ is most realistically captured by STM3 EC2 ($\alpha = 0.85$) and NLFEA but all the strength predictions are reasonable. Unsurprisingly, the NLFEA is most accurate since it was calibrated to give a best fit to the current test results. The accuracy of the code methods is less good but this is to be expected since shear enhancement is modelled.
in a simplistic manner. The STM predictions are considered good since the STM were developed from first principles and code recommended strengths were used for the nodes and struts.

Figure 16: Influence of $P_1/P$ on normalised failure loads a) $P_2$ and b) $P$ for Series B & AT200(0.5/0.5)
Figure 17 – a and b show the influence of $P_2/P$ on the measured and predicted failure loads according to BS8110 and EC2 respectively. Strengths are shown in each figure for beams without and with shear reinforcement. Failure is predicted to occur at the least of the loads corresponding to failure along the inner and outer shear planes. Figure 17 also shows the measured strengths of beams in Series A and B in which the outer shear plane was always critical in the tested beams. Beams loaded on the compression face (i.e. beams AC0 (0.5/0.5) and AC200 (0.5/0.5) are identified in the graphs. All other beams were loaded on the tension face. For both BS8110 and EC2, the experimental failure load corresponds most closely to the predicted failure load for failure along the outer shear span. The overall failure load is underestimated because both codes underestimate the shear resistance for failure along the inner shear plane.
Influence of tension face loading on shear resistance

The predicted influence of compression/tension face loading was investigated by comparing the resistance of matching pairs of beams, with four-point loads, identical to those tested in Series A but with all specimens having the same concrete strength of $f_c = 30$ MPa. Partial factors were taken as 1.0 for steel and concrete. Beam strengths were calculated for both tension and compression face loading using NLFEA and STM. The geometrical arrangements for the tension and compression face loading were the same as adopted in test series A. Consequently, the width of the loading plates was 100 mm irrespective of whether the load was applied to the flexural tension or compression face. Similarly, the width of the support plate was 300 mm for cantilever beams and 150 mm for simply supported beams. The consequence of this is that the node geometries of beams with loading ratios (1.0/0) and (0/1.0) are different for tension and compression face loading. This is reflected in the STM results as shown in Figure 18 in which C denotes compression face loading and T tension face loading. The strength of tension face loaded beams was calculated with STM3 EC2 (with $\alpha = 0.85$ for beams without shear reinforcement (i.e. strut strength at CCC node equals $0.85(1-f_d/250)f_{ck}$) and $\alpha = 1$ for beams with shear reinforcement). STM3 is equivalent to STM1 for loading ratios of (1.0/0) and (0/1.0) so long as the inner load is assumed zero. Compression face loaded strengths were calculated with STM1 or STM2 as appropriate. The measured and predicted failure loads $P$ are plotted against $P_2/P$ in Figure 18a and b for
beams without and with shear reinforcement respectively. The experimental and NLFEA results in Figure 18 suggest that tension face loaded beams (T) have lower shear resistance than comparable compression face (C) loaded beams. However, the STM incorrectly predict the shear resistance of the tension face loaded beams to be greater than that of comparable compression face loaded beams. It appears that the adopted STM are overly sensitive to variations in node dimensions as found by Vollum and Fang (2015).
Of previous tests with compression face loading, only those of Vollum and Fang (2015) are comparable with the current campaign. Vollum and Fang (2015) tested five simply supported beams with pairs of concentrated loads applied within 2d of supports. The beam geometry and loading arrangement of these beams were almost identical to beams AC0 (0.5/0.5) and AC200 (0.5/0.5) apart from the beam width being 165 mm instead of 250 mm. A straightforward method of assessing the influence of loading face on shear enhancement is to compare $P_{\text{test}}/P_{\text{calc}}$ for the tests of Vollum and Fang as well as this campaign using BS8110 which of the code methods gives the most consistent predictions of strength. In total, there are seven beams with pairs of equal loads on the compression face and three beams with pairs of equal loads on the tension face. The mean value of $P_{\text{test}}/P_{\text{calc}}$ for BS8110 is 1.50 for beams loaded on the compression face and 1.39 for beams loaded on the tension face. Whilst inconclusive, this suggests that the influence of loading face is small and although present can be neglected in practical design.
Conclusions

This paper investigates the influence of loading arrangement on shear enhancement in top-loaded reinforced concrete beams. A total of 12 beams were tested of which 10 were tested as balanced cantilevers (tension face loading) and two as simply supported beams with four equal point loads (compression face loading). The cantilever beams were loaded with either one- or two-point loads positioned in the critical shear span. The strengths of the tested beams were compared with the predictions of BS8110, EC2, MC2010, NLFEA and STM. Unlike NLFEA and STM, the code design methods do not differentiate between shear enhancement in simply supported beams and cantilevers. The following conclusions are drawn from this study:

− The shear resistance of the tested cantilever beams with two-point loads was slightly less than that of comparable simply supported beams. The difference in strength was greatest for beams without shear reinforcement but appears sufficiently small to be neglected in practical design.

− The ratio $P_1/P_2$ of the inner ($P_1$) to outer ($P_2$) point loads was varied in tests of balanced cantilever beams. Failure occurred in the outer shear span of all the tested beams except CT200 (0.6/0.4). The shear resistance of the outer shear span reduced as $P_1/P_2$ increased but the total failure load ($P = P_1 + P_2$) increased. The influence of $P_1$ on the shear resistance of the outer shear span is captured by STM3 and NLFEA but not by BS8110, EC2 or MC2010.

− BS8110 and EC2 underestimate the shear resistance of the inner shear span. Consequently, EC2 falsely predicts the failure plane of all seven of the tested beams with two-point loads that failed along the outer shear plane. BS8110 performed slightly better and correctly predicts the outer shear plane to be critical for beams AC0 (0.5/0.5) and BT200 (0.3/0.7).

− Of the code methods, MC2010 and BS8110 give significantly better estimates of shear strength than EC2, which is overly conservative for beams with shear reinforcement. The best predictions of shear resistance were obtained with STM and NLFEA. However, STM was found to overestimate the influence of node dimensions on shear resistance. Consequently, STM3 falsely gives greater shear resistances for beams loaded on the flexural tension than compression face.

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References


