INVESTIGATION OF THE FLOW IN AN IDEALISED INLET PORT/POPPET VALVE ASSEMBLY

by

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This thesis describes an experimental and computational investigation of steady air flow in an enlarged-scale version of an axisymmetric intake valve/port assembly for a low-speed model reciprocating engine.

The experimental investigation involved (a) assessment of the overall performance of the assembly in terms of its coefficient of discharge and the flow patterns at its outlet, the latter from flow visualisation studies, and (b) intensive and detailed hot-wire measurements of the mean and fluctuating velocity field within, and at the exit of, the assembly.

The measurements revealed the existence of four distinct flow regimes in the outlet passage occurring within different ranges of lift. These are characterised by the extent of separation provoked at the seat and/or valve corners and strongly influence the discharge coefficient and the velocity profiles at the valve exit. The turbulence at the valve exit was found to be nearly isotropic (excluding regions of separated flow) with intensity levels very low compared to those generated in the shear layer of the intake jet. The measurements also indicated that the pressure drop across the valve has minor effects on its performance, except at very small lifts.

In the computational study, an existing finite-volume method embodying a boundary-fitted curvilinear non-orthogonal grid system was used. The $k$-$\varepsilon$ turbulence model was used to close the system of time-averaged governing equations.

Comparisons between measurements and predictions showed close agreement in the mean velocity and flow direction within the low-to-medium range of lifts. In the high-lift range, the flow separations
observed in the measurements were not reproduced in the predictions, leading to qualitative and quantitative discrepancies in the valve outlet. These are attributed to deficiencies in the turbulence model to take proper account of the strong pressure gradient and streamline curvature prevailing in the flow field.
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NOMENCLATURE

Roman Letters

\(\mathbf{A}\):
\(A_{i,j}, A_{i}, A^{i}\):
\(A(i,j), A(i), A^{(i)}\):
\(A_{i}\):
\(A\):
\(a_{i}, a^{i}\):
\(a(i), a^{(i)}\):
\(C_{p}\):
\(C_{pd}\):
\(C_{ps}\):
\(C_{d}\):
\(C_{i}\):
\(C_{1}, C_{2}, C_{3}, C_{\mu}\):
\(D\):
\(D_{i}\):
\(D_{i}\):
\(E\):
\(e\):
\(e_{i}, e^{i}\):
\(e_{(i)}, e^{(i)}\):

: arbitrary tensor
: contravariant, covariant and mixed tensor components \((i,j = 1,2,3)\)
: contravariant, covariant and mixed physical tensor components \((i,j = 1,2,3)\)
: cell face area \((i = e,w,n,s)\)
: area or constant
: arbitrary vector
: contravariant and covariant vector components \((i = 1,2,3)\)
: contravariant and covariant physical vector components \((i = 1,2,3)\)
: discharge coefficient
: dynamic discharge coefficient
: static discharge coefficient
: drag coefficient
: convective coefficient \((i = e,w,n,s)\)
: \(k-\varepsilon\) model constants
: port diameter
: deformation (rate of strain) tensor
: diffusion coefficient \((i = e,w,n,s)\)
: anemometer output
: fluctuations in anemometer output
: natural and dual natural base vectors \((i = 1,2,3)\)
: normalised natural and its reciprocal base vectors \((i = 1,2,3)\)
\( \delta \): distribution function

\( G \): generation rate of turbulence energy

\( g \): metric tensor determinant

\( g_{ij}, g^{ij}, g_i \): covariant, contravariant and mixed metric tensor components \((i, j = 1, 2, 3)\)

\( g_{(ij)}, g^{(ij)}, g_{(i)} \): covariant, contravariant and mixed physical metric tensor components \((i, j = 1, 2, 3)\)

\( h \): coefficient of normal cooling

\( \mathbb{1} \): unit tensor

\( \mathbf{e}_j, \mathbf{e}^j \): Cartesian base vectors \((j = 1, 2, 3)\)

\( \{ i \}_j, \{ i \}_{jk} \): Christoffel's symbols \((i, j, k = 1, 2, 3)\)

\( \{ i \}_{jk} \): physical Christoffel's symbols \((i, j, k = 1, 2, 3)\)

\( k \): turbulence kinetic energy or coefficient of tangential cooling

\( L \): dimensionless valve lift

\( l \): valve lift

\( \ell_m \): turbulence length scale

\( \dot{m} \): mass flow rate

\( n \): engine speed

\( \Delta P \): dimensionless pressure drop across the valve

\( p \): pressure

\( \Delta p \): pressure drop across the valve

\( Pe \): Peclet number

\( Q \): magnitude of velocity vector

\( q_i \): fluctuations of velocity vector \((i = 1, 2, 3)\)

\( q \): flux vector

\( Re \): Reynolds number

\( Ri \): Richardson number

\( t \): seat width
\( T \): temperature

\( \overline{T} \): stress tensor

\( u \): axial velocity

\( \dot{u} \): dimensionless velocity

\( u_\tau \): shear velocity

\( u' \): fluctuations of axial velocity component

\( u'_r \): relative fluctuating components of velocity vector \((i = 1, 2, 3)\)

\( v \): volume

\( \nu \): velocity vector

\( \nu^{(i)} \): contravariant physical velocity vector components \((i = 1, 2, 3)\)

\( \nu' \): fluctuations of radial velocity component

\( \omega' \): fluctuations of traverse velocity component

\( x^i \): general coordinates \((i = 1, 2, 3)\)

\( y \): distance from the wall

\( y^+ \): dimensionless distance from the wall

\( y^i \): Cartesian coordinates \((i = 1, 2, 3)\)

**Greek Letters**

\( \alpha \): angle between general coordinator directions, seat angle, or mean flow angle from axial direction

\( R_\phi \): exchange coefficient

\( \gamma \): ratio of specific heats

\( \Delta \): difference

\( \nabla^{(j)} \): covariant differentiation operator \((j = 1, 2, 3)\)

\( \Delta/\Delta x^{(j)} \): differential operator \((j = 1, 2, 3)\)

\( \delta_j^i \): Kronecker delta \((i,j = 1, 2, 3)\)
\( \delta \) : angle between probe axis and mean velocity vector or boundary layer thickness
\( \varepsilon \) : dissipation rate of turbulence energy
\( \theta_x, \theta_y \) : angles between general and Cartesian coordinate directions
\( \kappa \) : turbulence model constant
\( \lambda \) : surface roughness
\( \mu \) : dynamic viscosity
\( \nu \) : kinematic viscosity
\( \pi_i \) : dimensionless groups
\( \rho \) : density
\( \rho_{ij} \) : curvature of coordinate lines \((i,j = 1,2,3)\)
\( \sigma \) : Prandtl/Schmidt number
\( \tau \) : shear stress
\( \tau_{(ij)} \) : anisotropic part of stress tensor
\( \tau_w \) : wall shear stress
\( \psi \) : stream function
\( \psi \) : yaw angle
\( \omega \) : under-relaxation factor

**Subscripts**
- act. : actual
- B : boundary
- cal : calculated
- e : effective or exit
- e, w, n, s : cell faces
- i : inlet or inner
- m : measured, molecular or mid-height
- NI : normal
N2 : binormal
ne, nw, se, sw : cell corners
0 : orifice plate or outer
P, E, W, N, S, NE, NW, SE, SW : compass notation for grid points
T : tangential
t : turbulent
th : theoretical
us : upstream
CHAPTER 1
INTRODUCTION

1.1 BACKGROUND

Since the early days of reciprocating internal combustion engines, designers have recognised that the intake system plays an important role in determining engine performance as it can, by careful design, improve the volumetric efficiency and burning rate and, consequently, the specific power output. Although the valve/port assembly forms only part of the intake system, its characteristics are the most influential factors on the induction process and the consequent crucial process of combustion.

It is not difficult to appreciate the severe demands placed on the inlet valve; remaining open for an extremely limited period of time, even in low-speed engines, it has to ensure the highest possible breathing capacity and make the most of the pressure difference between the cylinder and the intake manifold which, normally, does not exceed atmospheric level. To add to the difficulty, there is always a definite upper limit to the size of the valve in an engine of a given cylinder diameter.

The early investigations of the flow through intake valves were hampered by the hostile environment of the combustion chamber and the lack of adequate instrumentation. The majority of these investigations thus aimed at improving the breathing capacity through modified valve/port assembly designs, and provided data on their global performance. These data typically comprise quantities such as the pressure drop, $\Delta p$, across the valve and the coefficient of discharge, $C_D$, and its dependence on various geometrical parameters and operating conditions (see, for example, Tanaka [1929] and Waldron [1939]).
However, the fact that the details of the valve flow, such as the mean air velocity, turbulence level and swirl, are also of paramount importance for combustion and engine performance in general was discovered early in the history of reciprocating engines. Clerk [1912], for example, demonstrated the effect of reducing the intake-generated turbulence on slowing the rate of combustion. Alcock [1934] also showed that the performance of diesel engines could be improved by imparting swirl to the inlet air.

One of the earliest attempts to study the influence of the induction process on the subsequent in-cylinder air motion was that of Lee [1939], who used feathers to visualise the flow in a transparent-cylinder model engine under motoring conditions. He observed the presence of recirculation patterns created by the intake jet, whose strength increased with the inlet velocity, swirl and engine speed. In a subsequent study of combustion in the same engine, Rothrock & Spencer [1939] asserted the significance of Lee’s observation by showing that the flame speed increased with the same parameters due to the resulting augmentation of the air motion.

The rôle of the in-cylinder air motion in a typical engine cycle commences from the very beginning of induction where the incoming flow pattern is shaped by the geometrical features of the valve/port assembly. These are so influential in determining the engine performance that considerable effort has been devoted to their design, as will be shown later.

During the induction stroke, the inlet jet generates large-scale and turbulent motions within the cylinder, as described in more detail in the following sections, which in turn determine the extent of mixing between the fresh charge and residuals. At or shortly before inlet valve closure, the air motion inside the cylinder starts to decay at a
rate which depends on its structure during induction, and which
determines its effectiveness in performing its subsequent functions.

One of these functions is the "charge preparation", loosely
defined as fuel heating, evaporation and the mixing sequence which
takes place in diesel and other stratified charge engines. This
depends for its success on the existence of adequate bulk and turbulent
motions. The former is mainly created by the induction swirl and often
supplemented by geometrical devices designed to redistribute and/or
regenerate in-cylinder air motion, such as piston bowls and prechambers.

A second closely related function is to assist in conditioning the
mixture for ignition. In compression-ignition engines, this is mainly
a matter of achieving the required degree of mixing at the stage when
ignition temperatures are reached.

With the recent increasing demands on engine economy, performance
and emission control, there has been growing awareness of the
limitations of the cut-and-try methods of engine development. This,
in turn, led to extensive research being directed towards providing
fundamental understanding of the various engine flow processes,
including those in the valve/port assembly.

Induction, like all other reciprocating engine processes, is
inherently unsteady and spatially confined by time-varying boundaries
of generally complex geometry which produce three-dimensional spatial
variations. Although experimental investigations of flows in valve/
port assemblies in engine environments are desirable for determining
their behaviour under actual operating conditions, this is a difficult,
arduous and expensive task. This is why in the past, and currently,
valve and port designs have been, and are usually, tested under steady-
state conditions on the assumption that the results are applicable to
engine situations (see, for example, Kastner, et al. [1964]). The
validity of this approach has been investigated recently by, for example, Vafidis & Whitelaw [1984a], Bicen, et al. [1985] and El Tahry, et al. [1986], who measured the flow field at the valve exit in model engines motored at low speeds ranging between 200 and 1500 rpm. The close correspondence observed between steady and unsteady flow results suggested that the actual flow patterns at the valve exit can be predicted with reasonable accuracy from the less expensive, more convenient steady flow tests.

Apart from these and similar investigations which concentrate on the exit of the inlet valve (see, for example, Vafidis [1982] and Bicen [1983]), it seems that very little attention has been directed so far towards measuring the flow field in valve/port assemblies. This is probably due to the complex geometry and the limited dimensions of the intake passage which make the introduction of measuring probes rather difficult if not impossible. This is particularly noticeable in pressure-ignition engines, where the induction passage is often shaped so as to impart swirl to the air as it enters the cylinder (Monaghan & Pettifer [1981]).

In basic research projects, this difficulty may be overcome by either simplifying the geometry of the port duct, scaling it up to reduce the blockage effects caused by the measuring probes, and/or using optical measuring techniques like laser-Doppler anemometry. The former two options are further discussed in Chapter 4.

In addition to the recent substantial increase in the number of experimental investigations of the flow in reciprocating engines, the past decade has also witnessed a parallel increase in the assembly of mathematical models of engine flow processes associated with the development of more versatile and efficient numerical solution methods. These models, although imperfect and still under development, offer a
potentially cost-effective alternative means of gaining a more comprehensive knowledge of the various flow processes and the influence of key variables, thereon allowing less costly experimental programmes to be planned.

The mathematical models applied to engine flow processes generally fall under one of the following two categories:

(i) "Phenomenological" models; and
(ii) "Multi-dimensional" models.

Phenomenological models are basically structured from a formulation of the first law of thermodynamics in which time is the only independent variable and, thus, often referred to as "zero-dimensional" models. They are mainly concerned with predicting the global characteristics of the engine and the influence of different design parameters. This is done by solving, in addition to the energy equation, the conservation equations of mass and momentum and the equation of state in conjunction with separate submodels of the cycle's important processes and phenomena, such as the flow through ports and valves, in-cylinder air motion, heat transfer and combustion (see, for example, Blumberg, et al. [1979]).

The submodels are based on fundamental physical formulations and/or experimental data. Many of the engine phenomena, however, are sufficiently complex to allow for adequately accurate submodels to be formulated, and simplifying assumptions are usually introduced (Tabaczynski [1983]).

In applications to valves and ports, the flow is usually modelled by the equation for subsonic adiabatic homentropic flow through nozzles (Benson, et al. [1975]). The coefficient of discharge, determined experimentally usually from steady-state tests, is then used to yield
the effective area of the particular valve or port considered. In a more recent model of inlet valve flow, Rabbit [1984] attempted to relax some of these over-simplifications by including compressibility and wave-induced effects in the port area as further detailed in subsection 1.5.3.1.

Despite these simplifications, phenomenological models often provide useful information on many of the engine phenomena, such as induction-generated turbulence and its effect on the burnrate (Davis, et al. [1984]), in-cylinder air motion during induction, compression and combustion (Davis & Kent [1979], Borgnakke, et al. [1981], and Davis & Borgnakke [1982]), and combustion and emission control (Blumberg, et al. [1979]). They also offer a cost-effective method for extensive parametric operating and design evaluations. However, the fact that they do not provide detailed geometry-dependent spatial resolution of the state variables, such as velocities, temperatures, etc., restricts their rôle as a versatile design tool.

In the more fundamental multi-dimensional models, the partial differential equations governing the flow are solved numerically on a computational grid fitted to the flow field boundaries and covering the entire enclosed volume. The solution thus provides detailed information on the spatial and temporal distribution and evolution of the various flow parameters (see, for example, El Tahry [1984] and Gosman, et al. [1984a,b]).

Regardless of the rigour with which the governing equations are solved, phenomenological submodels are still required for describing turbulence, heat transfer and chemical kinetics. The level of the accuracy of these submodels represents one of the key factors for the success of multi-dimensional methods.

Insofar as the flow in inlet valve/port assemblies is concerned,
it appears that very few multi-dimensional predictions have been carried out. Nevertheless, in the rapidly increasing number of in-cylinder flow predictions, detailed information on the flow parameters at the outlet of the intake valve are required as boundary conditions. Recent investigations by, for example, Gosman, et al. [1984b] and Ahmadi-Befrui [1985] have clearly indicated the strong influence of these conditions on the overall accuracy of the in-cylinder flow field predictions, particularly during induction and early compression.

1.2 THE PROBLEM AND OBJECTIVES

As noted above, the majority of investigations related to inlet valve/port assemblies in reciprocating engines have been mainly concerned with their global characteristics and their effect on the engine breathing. The details of the valve flow, despite their early identified influence on the subsequent in-cylinder flow processes, have to date received little attention. The recent evidence on the validity of steady-state tests in characterising the actual unsteady flow through the valve were essentially obtained from measurements carried out across its exit plane. It is not difficult, therefore, to recognise the shortage in detailed quantitative information on the flow field in inlet valve/port assemblies under both steady and reciprocating conditions.

The present thesis addresses this problem by investigating, experimentally and computationally, the two-dimensional flow field in an idealised axisymmetric valve/port assembly under steady-state conditions. The configuration of the assembly selected for these investigations has been extensively used in reciprocating engine research at Imperial College (Vafidis [1984a,b] and Bicen, et al. [1985]).
1.3 THE PRESENT CONTRIBUTION

The work presented in the thesis contributes to the objectives mentioned above through:

1. The assessment of the overall performance of the selected valve/port assembly from the measurements of its coefficient of discharge at different valve lifts and pressure drops across the valve.

2. Identification of the various flow regimes which occur in the valve outlet passage within the various ranges of lift by visualising the flow in this region.

3. Detailed hot-wire measurements of the magnitude and direction of the mean velocity vectors and the turbulent normal and shear stresses within and at the exit of the port flow passages in different lift configurations. The problem of probe accessibility in the complex flow passage was partly solved by:

   (i) scaling up the dimensions of the assembly while preserving geometrical and dynamic similarity of the original configuration; and

   (ii) developing a versatile HWA technique for interpreting the signals of X-wire probes which does not require alignment of the probe axis with the mean velocity vector.

4. The application of a two-dimensional finite-volume method, which was devised by Demirdzic [1982] to predict the flow field in the experimental configurations. The method employs a non-orthogonal
curvilinear grid which is fitted to the complex shape of the boundaries of the valve/port assembly.

5. Assessment of the numerical predictions through extensive comparison with the experimental data.

1.4 THESIS OUTLINE

The remainder of this chapter is divided into two sections devoted to a survey of the previous experimental and theoretical studies related to intake valve/port assemblies. In Section 1.5, the measuring techniques used in engine research are first reviewed, together with a comparison of the capabilities and shortcomings between hot-wire anemometry and laser-Doppler anemometry. This is followed by a review of the experimental investigations of the characteristics and performance of valve/port assemblies and their influence on the engine performance. In Section 1.6, a review of the theoretical studies is presented which, in line with the objectives mentioned above, focuses primarily on multi-dimensional methods. In view of the limited previous applications of these methods to valve flows, the review concentrates on the observed effects of the imposed boundary conditions at the exit of the inlet valve on the in-cylinder flow predictions.

Chapter 2 of the thesis is concerned with the presentation of the governing conservation equations in a general coordinate-free form. The chapter begins with a review of the basic tensor operations in terms of physical vector and tensor components. The derivation of the time-averaged forms of these equations applicable to turbulent flows is also presented, together with a review of the different approaches to the 'closure problem', from which a selection is made. The transport
equations of the selected turbulence model are then presented in the chosen framework.

In Chapter 3, the numerical solution procedure is outlined. The grid arrangement is first presented and the discretised equations are obtained by integrating the governing differential equations over the respective control volumes. Some optional differencing schemes are then discussed from the viewpoints of accuracy, stability, and economy and the choice of differencing scheme used in the present study. Also discussed are features of the discretised equations related to the grid structure; and the formation and implementation of the boundary conditions. Finally, the method employed for generating the grid and the evaluation of the various geometrical parameters featuring in the governing equations are presented.

Chapter 4 describes the experimental apparatus and the measuring techniques used. A dimensional analysis is first presented in order to establish the criteria for preserving general similarity between the original port configuration and the scaled-up valve/port assembly. Details of the experimental arrangement and test section are then presented. Special attention is focused on hot-wire anemometry techniques and the method developed for interpreting the signals of X-wire probes is described. Tests of the method over a wide range of mean velocities and turbulence intensity levels are presented and discussed.

Chapter 5 is devoted to the presentation and discussion of the experimental results. The measurements of the coefficient of discharge are first examined and explained with reference to the various flow regimes detected in the flow visualisation studies. The X-wire measurements of the mean velocity and turbulent stress fields are next presented, discussed and compared with other investigations.
In Chapter 6, the results of the computational study are presented and compared with the experimental data. The geometrical properties of the computational grid are first examined with reference to the limitations imposed on them by the shape of the passage boundaries. This is followed by a discussion of the boundary conditions implemented at the inlet and the exit of the solution domain, and the results of the grid refinement tests. The predictions of the mean velocity, turbulence and pressure fields are next presented and discussed with emphasis on the influence of the valve lift on their development. The predictions are then compared with the experimental results and attempts were made to explain the discrepancies observed.

Finally, Chapter 7 provides a summary of the main achievements and conclusions of the thesis, and makes suggestions for possible future work.

The thesis includes two appendices; Appendix A presents a review of general tensor calculus for readers who are not familiar with the subject, and Appendix B contains the estimates of measurement uncertainties, calibration data for the hot-wire probes and the derivation of the response equations of the X-wire probes.

1.5 REVIEW OF PREVIOUS RELATED WORK

1.5.1 Measuring Techniques

The experimental techniques used to investigate steady and unsteady flows in research and production engines may be generally classified as:

(i) probe methods; and
(ii) optical methods.
Probe methods, including pressure probes, spark probes, swirl vane anemometers and hot-wire anemometry, have been extensively used in the past to characterise the inlet flow as well as the velocity field during induction and compression. Recent development in laser technology has given birth to a whole new range of optical methods for measuring velocity as well as other variables.

A detailed and critical review of both categories of measuring techniques is given by Arcoumanis [1984]. The two techniques most widely used in engine measurements are Hot-Wire Anemometry (HWA) and Laser-Doppler Anemometry (LDA). The intention here is to summarise the capabilities and shortcomings of both techniques without going into the details of their principles of operation.

**Hot-wire anemometry:**

HWA is based on the relationship between the convective heat transfer from the surface of a heated sensor and the velocity of the flow past it. In constant temperature anemometry (CTA), the sensor is maintained at a constant temperature by a feedback control system that adjusts the current through the sensor; the anemometer output is the corresponding voltage variation. This output is a non-linear function of the velocity component perpendicular to the sensor (King [1914]), the temperature difference between the sensor and the flow, and the fluid properties, such as thermal conductivity, viscosity and density (Perry [1982]).

Witze [1980], in assessing the validity of HWA measurements in a motored reciprocating engine, concluded that hot-wire probes, besides being intrusive and delicate, possess the following further disadvantages:
(i) Individual probes require calibration over the whole range of velocities and temperatures encountered in the engine environment: this is an impractical task in view of the time consumed and the frequent probe damage.

(ii) Variation of the flow temperature from the calibration conditions usually leads to serious inaccuracies in the measured mean velocities and turbulence quantities.

(iii) The validity of the linearisation approximation used to separate turbulent fluctuations from mean velocity limits the allowable turbulent intensities to about 20%. Above this limit, which is often exceeded in engine flows, excessive errors can be expected (Witze [1977,1980]).

(iv) Directional insensitivity results in limited ability to resolve the velocity components without a prior knowledge of the mean velocity direction.

On the other hand, Perry [1982], in an extensive analysis of HWA, argued that it provides a continuous signal, good frequency response and high signal-to-noise ratio. In the optimum conditions of low turbulence levels and low compression ratios, Witze [1980] showed that HWA can provide accurate measurements during the intake and exhaust strokes. However, during compression and expansion strokes, where temperature and density change dramatically, serious errors are incurred in HWA as compared to LDA measurements.

**Laser-Doppler anemometry:**

The principle of LDA operation is that two coherent laser beams, generated by passing a single laser through a beam splitter, are passed through a focusing lens and made to intersect at the measuring
location in the flow. An interference fringe pattern is formed in the intersection of "measuring volume", and the particles deliberately introduced to the flow scatter light as they cross the fringe. Collection of the scattered light by additional lenses and the measurement of its frequency with the aid of a photomultiplier allow direct determination of the particle velocity normal to the fringe. The flow direction may also be determined by introducing a frequency shift between the two incident laser beams.

In his critical comparison between HWA and LDA techniques in engine applications, Witze [1980] argued that LDA, besides being non-intrusive and applicable to combustion processes, offers the following advantages:

(i) No calibration is needed, regardless of the flow under consideration.
(ii) Directional sensitivity and the capability of resolving the velocity components and the associated turbulent fluctuations are both available.
(iii) High accuracy achievable at all turbulence intensity levels.

The main drawbacks of LDA are:

(i) Low signal intermittency and signal-to-noise ratio.
(ii) Relatively large uncertainties due to velocity gradients within the measuring volume (up to 10% in the mean velocity and 15% in the rms of its fluctuations (Vlachos [1977])).
(iii) Unresolved uncertainties associated with the light-scattering particles which may result in large errors in recirculation regions (Nakayama [1983]).
(iv) The need for optical access to the flow field which restricts the extent to which the technique can be used in complex geometry passages.

In reciprocating engine measurements, the variation of the velocity in the measuring volume during the sampling period represents an additional source of uncertainty. This was investigated theoretically by Morse [1977] and experimentally by Rask [1981] and Arcoumanis, et al. [1982a], who suggested that a "sampling window" of 5° crank angle was a good choice with respect to both the rate of data acquisition and the level of error.

1.5.2 Experimental Investigations

In the following review, the experimental investigations of the induction process and its influence on the engine performance are grouped under two main categories, namely:

1. Investigations of the influence of the geometrical characteristics of the valve/port assembly and operating conditions on the engine volumetric efficiency.

2. Investigations of the effects of the induction system design on the detailed in-cylinder flow field.

Summaries of both categories of investigations and their main findings are given in Table 1.1.

1.5.2.1 Valve parameters and operating conditions

Investigations in this group examined the influence upon the performance of the valve/port assembly, usually characterised
by its coefficient of discharge, $C_D$, of one or more of the following factors:

(i) The pressure drop across the valve.

(ii) The geometry of the valve and port.

(iii) Steady and transient flow conditions.

The pressure drop across the valve: The first study of the performance of a poppet valve/port assembly was probably that carried out by Lucke [1906], in which an attempt was made to establish a relationship between the pressure drop, $\Delta p$, across the assembly and its coefficient of discharge, $C_D$. The study indicated that $C_D$ was independent of the relative pressure drop, $\Delta P^*$, in the range between 0.01 and 0.07. This conclusion was later supported by the findings of Nutting & Lewis [1918] and by the extensive investigation of Tanaka [1929] of the characteristics of poppet valves. Woods, et al. [1942] argued, on the basis of measurements of Reynolds, et al. [1938], that in low-speed engines, i.e. below 3000 rpm, the maximum value of $\Delta p$ should be around 0.3 and the flow through the valve can be treated as incompressible without appreciable loss of accuracy. Following this approach, their measurements indicated that the changes in $C_D$ are negligibly small over a wide range of $\Delta p$ up to 0.4, except for special designs similar to configuration C in Figure 1.1. The performance of this particular design approached that of a perfect venturi as a result of the large pressure recovery in its gradually diverging outlet. They concluded

* $\Delta P$ is defined as the ratio between $\Delta p$ and the pressure upstream of the valve.

** $C_D$ in Figs 1.1, 1.5 and 1.7 is defined differently by different authors.
therefore that, with the exception of such extreme designs which are usually impractical, tests at low $\Delta P$ (typically of order 0.02) should give good indications of the behaviour of $C_D$ over the range of actual operation.

More recently, Vafidis & Whitelaw [1984a] showed that for very small lifts, i.e. $L/D^* < 0.05$, viscous effects become significant within the passage formed by the valve and seat faces, causing $C_D$ to decrease as $\Delta P$, and consequently Reynolds number based on the lift and the exit velocity, decrease. The same conclusion was reached by Vafidis [1982], who showed that in this range of lifts, $C_D$ increases rapidly with Reynolds number in the range between $0.5 \times 10^3$ to $4 \times 10^3$. Above the latter value, viscous effects became less significant and $C_D$ thereafter remained nearly unchanged.

(ii) The geometry of the valve and port: The results of the study carried out by Lucke [1906] indicated that the geometry of the valve and port plays a significant rôle in determining the overall performance of the valve. Although no geometrical details were given, the so-called "conical-seated" valve showed performance superior to that of the "flat-seated" valve. Nutting & Lewis [1918] investigated the merits of using inlet valves in pairs over single valves and applied the concept of geometrical and dynamic similarity to valves of different sizes. They concluded from their measurements of $C_D$ that valve/port assemblies perform similarly under conditions of complete similarity. This same concept was applied in the present study, as outlined in Chapter 4.

The geometrical aspects of valves and their

* $L \equiv$ valve lift; $D \equiv$ port diameter.
influence on the performance of valve/port assemblies were extensively investigated by Tanaka [1929]. The investigation covered the seat angle, α, the seat width, S, the valve-fillet radius, R, and head angle, θ, Figure 1.2. The valve models were tested under steady flow conditions in an axisymmetric passage discharging into a cylinder 2.7 times the diameter of the port.

From the results of those tests, Tanaka [1929] postulated four distinct flow regimes in the valve outlet to explain the discontinuities observed in the variation of the mass flow rate with lift, Figure 1.3a. In régime I, occurring at small lifts up to \( \ell/D \approx 0.13 \), the flow remained attached to both the valve and seat faces, as illustrated schematically in Figure 1.4. Régime II occurred at slightly higher lifts, \( 0.13 < \ell/D < 0.21 \), in which the flow detached from the valve face. At still higher lifts, \( 0.21 < \ell/D < 0.25 \), the flow detached from the seat face as well, giving rise to régime III. In régime IV, the flow became attached again to the valve face and Tanaka argued that the extent of the recirculation zone would increase as the ratio between the diameter of the port and the diameter of the downstream cylinder increased.

The effect of varying the seat angle, α, was examined by the variation it produced in the mass flow rate through the valve at a fixed pressure drop. Extracts from these measurements are shown in Figure 1.3b which indicate that 30° and 45° seat angles gave roughly the same results up to \( \ell/D \approx 0.18 \), above which the 45° design shows slightly improved performance. Both these designs are, however, seen to be significantly superior to that with a 60° seat angle over the entire range of lifts shown.

Similarly, the effects of rounding off the sharp corners of the valve and seat faces, corners a, b, c and d in Figure
were examined and the results shown in Figure 1.3c show an increase in the mass flow rate of about 32%. The major improvement, amounting to about 26%, resulted from rounding off corners a and b which led to suppression of the separation on the valve face and delay in that on the seat face to higher lift. The flow régimes mentioned above were therefore reduced to only two, namely, régimes I and IV.

In the subsequent investigations of Dennison, et al. [1931], Wood, et al. [1942] and Kastner, et al. [1964], attempts were made to reduce the tendency of the flow to separate from the valve and seat faces by (i) rounding off the sharp corners, (ii) shaping the flow passage as a converging-diverging nozzle, and (iii) avoiding abrupt area changes. Although these attempts produced considerable improvements into $C_p$, they also resulted in complex configurations which proved to be impractical for production engines.

The effects of the seat angle, $\alpha$, and the fillet radius, $R$, c.f. Figure 1.1, on $C_p$ were further examined by Annand [1969] and Wallace [1968], whose results supported the conclusions of Tanaka [1929] in that the 60° seat angle is inferior to the 30° and 45° seat angles. The results of Annand [1969], furthermore, supported the use of the 45° seat angle design because of its superior performance at high lifts, despite the better low-lift performance and mechanical properties of the 30° design. This conclusion was based on the observation that, in the range of medium to high lifts, which is normally the range of effective valve operation (Taylor [1977]), the 45° seat angle produced higher values of $C_p$. The results also indicated that a fillet radius, $R$, Figure 1.2, of 0.2 $D$ has no measurable effect on $C_p$, while above this value the divergence of the outlet passage increased and led to a considerable fall in $C_p$.

The effect of the maximum valve lift on $C_p$ was
investigated by Fukutani & Watanabe [1982], who concluded that if the inlet "angle-area", $A_{ia}^*$, was kept constant, the valve with lower maximum lift always showed higher discharge coefficient under both steady and unsteady conditions. This is attributed to the increase in the width of the recirculation zone on the valve face, regime IV in Figure 1.4, at higher lifts and the consequent reduction in $C_p$.

Recent extensive measurements of the in-cylinder flow field under various operating conditions by Bicen [1983], Vafidis & Whitelaw [1984b], Bicen, et al. [1984] and Bicen, et al. [1985] confirmed the existence of the four flow regimes postulated by Tanaka [1929]. Vafidis & Whitelaw also confirmed that the improvement in the performance induced by rounding off the sharp corners of the valve and seat is mainly due to the reduction of these regimes to only two, namely, regimes I and IV.

(iii) Steady and transient flow conditions: As noted above, most of the observations and conclusions on the performance of inlet valve/port assemblies have been drawn from steady-state tests. The values of $C_p$ obtained under these conditions were considered to be applicable to the actual transient conditions prevailing in production engines. This approximation was based mainly on the assumption that the mass of air to be accelerated in the inlet passage is very small and the flow would respond almost instantaneously to changes in the valve opening and the pressure difference between the cylinder and the

$$A_{ia}^* = \int_{\theta_{io}}^{\theta_{ic}} A_{i}(\theta) \, d\theta,$$

where $A_{i}(\theta)$ is the outlet area of the valve, $\theta$ is the crank angle, and $(\theta_{io} - \theta_{ic})$ is the valve opening period.
induction manifold (see, for example, Woods & Khan [1965] and Wallace [1968]).

In the investigations made to examine the validity of this assumption, the average steady-state, or static, discharge coefficient, \( C_{D_s} \), is usually defined as:

\[
C_{D_s} = \frac{1}{\theta_{ic} - \theta_{io}} \int_{\theta_{io}}^{\theta_{ic}} C_D(\theta) \, d\theta
\] (1.1)

where \( \theta_{io} \) and \( \theta_{ic} \) are the crank angles at the opening and closure of the valve, respectively, and \( C_D \) is the steady discharge coefficient measured at fixed lifts and expressed in terms of the crank angle, \( \theta \).

The transient, or dynamic, discharge coefficient, \( C_{D_d} \), on the other hand, is defined as the ratio between the actual and isentropic flow rates under the unsteady conditions.

Waldron [1939] examined the effects of varying the maximum lift and the engine speed on \( C_{D_d} \) as compared to \( C_{D_s} \). In this investigation, the inlet passage was left open to the atmosphere and the cylinder head was mounted directly to a large air-tight low-pressure tank. The valve operating mechanism was fitted with cams of different maximum lifts and driven at constant speeds ranging from 100 to 1200 rpm, corresponding to engine speeds of 200 to 2400 rpm, respectively.

The mass flow rates were determined by monitoring the variation of pressure in the delivery tank during a fixed period of valve operation. The actual mass flow rate was calculated in terms of the initial and final values and the known volume of the tank, while the theoretical mass flow rate was determined from the isentropic flow relation in terms of the registered pressure values during the operating period.
Figure 1.5a shows extracts from the measurements of Waldron [1939], illustrating $C_{D,\Delta}$ and $C_{D,d}$ plotted against Reynolds number based on the valve lift at the average velocity at its exit. From these results, he suggested that $C_{D,\Delta}$ could be applied to transient flow situations with accuracy sufficient for engineering purposes.

Stanitz, et al. [1946] examined the effects of varying the pressure drop, $\Delta P$, across the valve and engine speed on $C_{D,d}$ using an experimental arrangement similar to that described above, with the pressure in the delivery tank being maintained constant. The actual mass flow rate was measured by orifice plate fitted to its exit and the isentropic mass flow rate was calculated in terms of the pressure difference between the inlet, which was left open to the atmosphere, and the tank.

The measurements were carried out at engine speeds ranging between 800 and 3600 rpm at a series of constant $\Delta P$ ranging between 0.03 and 0.35. The results shown in Figure 1.5b indicate that at high $\Delta P$, both $C_{D,\Delta}$ and $C_{D,d}$ remain nearly the same over a wide range of speeds, while at small pressure drops, $C_{D,d}$ becomes progressively less than $C_{D,\Delta}$ as the engine speed increases.

Kastner, et al. [1964], in a similar investigation covering wider ranges of engine speeds and pressure drops (namely, between 800 and 7000 rpm for the former and 0.03 to 0.65 for the latter), observed similar trends. They attributed the fall in $C_{D,d}$ at high speeds and low $\Delta P$ to the overriding effect of the increase in the energy required to accelerate the air column in the inlet passage. They further found that a relation between $C_{D,d}$ and $C_{D,\Delta}$ exists such that:

$$1 - C_{D,d} = \frac{Kn}{\Delta P} \left( \frac{\dot{m}}{A_2} \right)$$

(1.2)
where $C_{d_4} = \frac{C_{p_d}}{C_{p_\delta}}$, $n$ is the engine speed, $\dot{W}$ is the isentropic flow rate, $A_2$ is the minimum or throat area of the valve (c.f. equations (5.5)), and $K$ is a coefficient, typically $= 3 \times 10^{-7}$, constant for all sizes of geometrically similar valves with inlet passages of equal length.

Fukutani & Watanabe [1982] also attempted to correlate $C_{p_d}$ and $C_{p_\delta}$ through the coefficient $\left(\frac{v_a}{\overline{v}_v}\right)$, where $v_a$ is the isentropic mean velocity of the inlet flow, and $\overline{v}_v$ is the average velocity of the valve during its opening period, given respectively by:

$$v_a = \sqrt{2\Delta p/\rho}$$

$$\overline{v}_v = \frac{2\ell_{\text{max}}}{[\theta_{iC} - \theta_{i0}/360] \cdot (60/n)} \quad (1.3)$$

The measurements of Fukutani & Watanabe [1982] were carried out at $\Delta P$ between 0.01 and 0.39 over a range of speeds between 400 and 8000 rpm. The results shown in Figure 1.5c indicated that as $\left(\frac{v_a}{\overline{v}_v}\right)$ increases, $C_{p_d}$ approaches $C_{p_\delta}$ due to the reduction in the relative velocity of the piston and, consequently, the energy required to accelerate the air column in the inlet passage. At $v_a/\overline{v}_v > 100$, both $C_{p_d}$ and $C_{p_\delta}$ became and remained nearly the same. Since this value is usually exceeded in production engines, they concluded that $C_{p_\delta}$ provides a good indication of the valve performance under transient conditions.

Further evidence of the validity of steady-flow results in characterising the performance of valve/port assemblies under reciprocating conditions have been obtained from the recent LDA measurements of the in-cylinder flow field during induction (Bicen & Whitelaw [1983], Vafidis & Whitelaw [1984a] and Bicen, et al. [1985]).
The measurements of Bicen & Whitelaw [1983] were carried out under both steady and reciprocating conditions in a model engine motored at 200 rpm and fitted with a 60° seat angle sharp-edged centrally-located valve/port assembly. The measurements across the valve exit plane covered lifts up to \( L = 0.42 \). At small lifts up to \( L = 0.1 \), the steady-state measurements showed a fully attached flow, régime I in Figure 1.4, with nearly uniform velocity distribution across the valve exit. Between \( L = 0.2 \) and 0.3, the mean velocity profiles indicated the separation at the valve face shown in Figure 1.4 in régime II. At \( L = 0.42 \), separation at the seat face occurred as well, régime III, and in these latter régimes the velocity distribution in the region of the outflow differed markedly from the uniform profiles observed at smaller lifts.

The profiles of the transient ensemble mean velocity were found to be in close agreement with those obtained under steady-state conditions. However, the turbulence intensity levels showed high sensitivity to the flow unsteadiness and were about 100% higher than the steady-state measurements.

These findings were supported by Vafidis & Whitelaw [1984a] and Bicen, et al. [1985], whose measurements were carried out under conditions closely resembling those mentioned above. These latter measurements further confirmed the effect of rounding off the valve and seat corners in reducing the flow régimes in the valve outlet to only régimes I and IV in Figure 1.4.

Arcoumanis, et al. [1986] made use of these findings in an investigation of the effect of engine speed on the ensemble-average velocities at the valve exit in a modified production engine motored between 500 and 3000 rpm. They divided the valve opening period between 14° BTDC and 260° ATDC into opening,
maximum lift and closing periods and carried out single point measurements at the valve exit of the radial mean velocity and turbulent intensity at different speeds. The measuring points were selected so as to avoid the steep velocity gradients in the vicinity of the valve and seat faces and the recirculation zone on the seat face which were detected in the measurements of Bicen, et al. [1985].

During the opening period, between crank angles $\theta = 25^\circ$ to $50^\circ$ and lifts $L = 0.09$ to $0.166$, the mean velocity at the valve exit was found to increase linearly with the average piston speed between 1000 and 3000 rpm. At 500 rpm, the ratio between the mean velocity and the average piston speed was generally higher than that observed at higher engine speeds and approached the values obtained from incompressible flow calculations.

During the maximum lift period, between $\theta = 80^\circ$ to $140^\circ$ and $L = 0.24$ to $0.27$, the mean velocity was also found to increase linearly with the piston speed. The ratio between them, however, fell from about 12 during the opening period to about 7 and remained nearly at this value between 500 and 1500 rpm. In this range of engine speeds, the mean velocities at the valve exit almost coincided with the results obtained from incompressible flow calculations. This was attributed to the limited variation of the valve lift and to the validity of the quasi-steady assumption for the flow in the valve.

The measurements during the closing period, between $\theta = 170^\circ$ to $200^\circ$ and $L = 0.166$ to $0.08$, showed more pronounced effects of the engine speed. Below 1500 rpm, an outflow through the valve occurred shortly after the piston reached Bottom-Dead-Centre; as incompressible flow calculations would suggest. The onset of this outflow was delayed to after $\theta = 200^\circ$ as the engine speed increased to
3000 rpm. This is probably due to the increasing inertia of the flow entering the cylinder which helps to sustain the induction process. As the outflow decreased, the volumetric efficiency improved, reflecting a nearly 50% increase in the peak cylinder pressure between 250 and 2250 rpm.

Turbulence intensity levels at the measuring locations were generally found to decrease with engine speed. This was due to the developing nature of the flow and the need for a finite time for turbulence to be transported from the boundary layers to the core of the inlet jet.

El Tahry, et al. [1987] also reached similar conclusions from the comparison between the HWA measurements in the curtain area of the inlet valve in a model engine motored between 500 and 1500 rpm and the earlier steady-state measurements of Khalighi, et al. [1986]. These may be summarised in the following:

(i) Over a period spanning about 40% of the intake stroke and centred at its middle, the inlet velocity profiles are little affected by the unsteadiness of the flow and have the same general shapes as those determined under steady flow conditions. This period represents the most influential phase of the stroke since most of the cylinder charge is inducted during it.

(ii) In the early and late induction, the velocity profiles are sensitive to the engine speed, and continuously developing and changing their shapes, and exhibit quantitative differences from steady flow profiles, particularly significant at very
small lifts.

(iii) Turbulence intensity levels at the valve exit generally decrease with the increase in engine speed.

1.5.2.2 Design aspects of the induction system

The induction system, consisting of the intake manifold and the valve/port assembly, usually includes means of imparting swirl or otherwise directing the flow as it enters the cylinder. There are several types of swirl-producing inlet ports, all of which conform to two basic categories, depending on whether the swirl is produced upstream or downstream of the inlet valve: pre-valve and post-valve swirl. Pre-valve swirl is generated by helical-type ports (Monaghan & Pettifer [1981]) or by guide vanes located upstream of the valve (Morse, et al. [1980b] and Arcoumanis, et al. [1983]). Post-valve swirl, on the other hand, can be produced by directed ports (Wigley & Hawkins [1978] and Hirotomi, et al. [1981]), or by masked valves (Arnold, et al. [1972] and Wakisaka [1979]). The only configuration that produces a two-dimensional swirl-free flow field in the cylinder is that of a centrally-located axisymmetric valve/port assembly.

An extensive study of axisymmetric engine configurations has been carried out at Imperial College on motored engines operated at low speeds between 200 rpm and 675 rpm (see, for example, Morse, et al. [1980a], Bicen [1983], Arcoumanis [1984], and Bicen, et al. [1985]). The information obtained in this range of speeds is believed to be representative to the real engine performance and can provide a basis for understanding the influence of the inlet
conditions on the structure of the in-cylinder mean flow field.

During early induction, the in-cylinder flow field is dominated by an annular inlet jet impinging on either the cylinder wall or the piston head, depending on the valve seat angle and the piston location, to form two strong recirculation zones near the cylinder head corner and behind the valve, as shown in Figure 1.6. The development of the flow during the induction stroke results in the growth of the main vortex behind the valve as a result of the piston displacement and the entrainment from the inlet mass. In the absence of inlet swirl, the mean flow decays rapidly near the end of induction due to the diminishing velocity of the inlet jet and the effect of turbulent stresses. The vortex structure, however, persists till the end of induction (Bicen [1983], and Arcoumanis [1984]).

The effect of the valve seat angle and maximum lift on the structure of the in-cylinder flow field was investigated by Ekchian & Hoult [1979] and by Arcoumanis, et al. [1982b,c]. The results of the latter indicated that, for a seat angle of 30°, the inlet jet remained attached to both the valve and seat faces, suppressing the vortex formation near the cylinder head corner.

According to Ekchian & Hoult [1979], this only happened at very small lifts, while at higher lifts the inlet jet becomes bistable, i.e. at times it separates from the cylinder head and sometimes it remains attached. The results of Arcoumanis, et al. [1982b,c] and Ekchian & Hoult [1979] suggested that for seat angles of 45° and 60°, the general vortex-pair flow pattern described above

* These predicted velocity fields are in good agreement with the measurements of Ahmadi-Befrui, et al. [1982] and qualitatively similar to the measurements of Bicen [1983].
would prevail throughout the whole range of lifts.

The effect of reducing the maximum valve lift was found by Arcoumanis, et al. [1982c] to increase the velocity of the inlet jet, as would be expected, producing stronger mean flow and, in particular, large vortices.

The influence of the valve seat angle on the in-cylinder flow field subsequent to induction was also investigated by Arcoumanis, et al. [1982c]. They concluded that, although the flow structure during induction differed for different seat angles and, accordingly, inlet jet trajectories, by the time the valve closed the mean flow structure had already collapsed and the mean velocity became and remained similar during compression.

A small displacement in the axis of the inlet valve with respect to the cylinder axis produces asymmetry in the velocity profiles around the valve periphery (Khalighi, et al. [1986]), leading to three-dimensionality in the in-cylinder flow field. The measurements of Morse, et al. [1980b] indicated that a vortex-pair structure was formed in the diametral plane, whose strength and size varied along the cylinder axis. As this and other investigations showed (see, for example, Arcoumanis [1984]), the flow structure in the axial plane was similar to that of the coaxial valve configuration, except for the circumferential differences due to the off-centre positioning of the valve. This was also confirmed in the flow visualisations of Hirotomi, et al. [1981], who showed that for different offset port designs, the vortex-pair structure persisted throughout the induction process. Of the two vortices, the regular one which follows the direction of the main flow from the inlet port increases in size as the valve lift increases.

The structure of the in-cylinder flow field during
and after induction was also found to be strongly influenced by the method of swirl generation (Tindall & Williams [1977], Haghoogie, et al. [1982], and Gosman, et al. [1984a]). Gosman, et al. [1984a], for example, in a study of the history effects of the induction process, concluded that centrally-located shrouded valves generate a complex multi-vortex flow structure, the most important component of which is a strong long-lived tumbling motion. The strength of this tumbling motion was found to be sustained and amplified during compression, leading to an increase in turbulence generation and intensity levels. The latter were appreciably higher than those produced by unshrouded valves.

1.5.3 Theoretical Studies

1.5.3.1 Zero-dimensional methods

In this approach, as mentioned earlier, the spatial variations within the flow field are disregarded and attention is focused on the temporal variations during the engine cycle. In an application to the induction processes, Rabbit [1984] proposed a sub-model to characterise the flow in the inlet valve by treating the incylinder domain as a valve-lift-dependent control volume. From the analysis presented of the losses term in the total energy integral equation as applied to this control volume, he deduced a definition of the total coefficient of discharge, $C_{DT}$, of the valve in the form:

$$C_{DT}^2 = C_{D_d}^2 + C_{D_w}^2 - 1$$ (1.4)

where $C_{D_d}$ is the dynamic coefficient defined earlier at constant mass flow rate, and $C_{D_w}$ is the so-called "wave-induced coefficient" which accounts for transient losses.
Rabbit [1984] further argued that at low-Mach number valve flows, below 0.3, \( C_{dp} \) is indistinguishable from the static coefficient, \( C_{d\delta} \), defined by equation (1.1). As the flow Mach number increases, he showed that the ratio of the two coefficients is proportional to the density ratio \( \rho_e/\rho_p \). With this ratio being difficult to determine directly from measurements, Rabbit suggested isentropic approximation of the flow in the valve such that:

\[
C_{dp} = C_{d\delta} \left( \frac{T_a}{T_\delta} \right)^{1/(\gamma-1)} \sqrt{\frac{A_p}{A_e}}
\]  

(1.5)

where \( T_a \) and \( T_\delta \) are the temperatures at the inlet of the port duct and the valve exit, respectively, \( A_p \) is the piston area, and \( A_e \) is the area of the valve exit.

Determination of \( C_{dp} \), on the other hand, is not as simple and requires extensive experimental measurements to cover the widest possible range of transient conditions. Rabbit [1984] suggested that, alternatively, the wave-induced losses be approximated as being the result of the wave motion inside the cylinder, which could be determined analytically.

1.5.3.2 Multi-dimensional methods

As noted earlier, the majority of flow processes in reciprocating engines are transient and occur in domains with moving boundaries of varying degrees of geometrical complexity: this description certainly applies to the flow through the inlet valve/port assembly. In multi-dimensional methods, a computational grid is

* Subscripts \( \varepsilon \) and \( p \) refer, respectively, to the valve exit and the piston surface.
superimposed on the flow field and the differential equations governing the flow are solved numerically over the whole domain. The solution thus provides spatial and temporal resolution of the various parameters of the flow.

When analysing flows in complex geometries, one of the most important decisions to be made is the shape of the computational grid which strongly influences the accuracy and economy of the solution procedure, as shown below. For complex domains, there are two main options for the grid arrangement (Demirdzic [1982]), namely:

(i) arbitrary grid arrangements; and
(ii) regular grid arrangements.

In arbitrary grid arrangements, the solution domain is divided into arbitrary-shaped control volumes or "cells", usually quadrilateral (Hirt, et al. [1974]) or triangular (Baliga & Patankar [1980]). The finite-difference equations are then formulated from the integral conservation of mass, energy, momentum, etc., as applied directly to the control volumes. In the absence of definite coordinate directions associated with the grid, the governing equations are usually formulated in terms of Cartesian velocity components (Hirt, et al. [1974]).

Although this approach provides relative flexibility in distributing the grid nodes and requires minimum geometrical information in the form of the Cartesian coordinates of the control volume vertices, it needs an elaborate indexing scheme to identify neighbouring cells and can cause, under certain conditions, decoupling between adjacent cells, as discussed later in Chapter 2.

The regular grid arrangements, on the other hand,
are formed by the intersections of families of surfaces which can be characterised by a system of coordinates. The discretised equations are usually obtained from the governing differential equations derived in terms of the particular coordinate system employed. In finite-difference/finite-volume methods, the regular grid approach is often preferred because it leads to difference equations whose banded matrices are amenable to solution by efficient implicit schemes. Moreover, it results in easier definitions of fluxes and stresses, as well as much simpler indexing schemes.

For regular grid arrangements associated with domains of complex geometry, the following main options are available:

(i) Non-conforming orthogonal grids.
(ii) Boundary-fitted orthogonal curvilinear grids.
(iii) Boundary-fitted non-orthogonal curvilinear grids.

(i) **Non-conforming orthogonal grids:** In this approach, the solution domain is covered with a regular orthogonal coordinate system, Cartesian or cylindrical polar, for example. Although the governing equations here assume their simplest forms, and the storage requirements for geometrical quantities are minimal, this approach poses strong constraints on the distribution and spacing of the grid lines (Blottner & Roache [1971]). Moreover, the curved boundaries are usually approximated by a step-wise grid arrangement which makes the treatment of boundary nodes extremely difficult and, sometimes, inaccurate (Koutmos & McGuirk [1983]).
(ii) **Boundary-fitted orthogonal curvilinear grids:**

Here, an orthogonal curvilinear grid which conforms to the flow boundaries is generated, with the coordinate surfaces being orthogonal to each other and to the boundary. The governing equations in terms of the coordinate system are slightly more complex than their counterparts in Cartesian coordinates. Although not general, this approach extended the applicability of numerical solution methods to a wider range of problems. Its main drawback is that it needs a special means for generating the grid, such as conformal mapping or the solution of differential equations expressing the transformation relationship between the grid coordinates in the physical and transformed planes (Pope [1978]). In addition to being costly, the grid-generation procedure usually limits the control on the distribution of the grid lines which may result in poor resolution of the flow where the grid spacing is unavoidably excessive. More details on this approach can be found in the works of Pope [1978] and Antonopuolos [1979].

(iii) **Boundary-fitted non-orthogonal curvilinear grids:**

In this approach, the boundary-fitted curvilinear grid is formed by families of coordinate surfaces intersecting at (nearly) arbitrary angles. The resulting grid does not suffer from the drawbacks mentioned above and quite arbitrary shapes of solution domains can be handled with relative flexibility in controlling the grid distribution as well as its geometrical properties (Demirdzic [1982]). However, the governing equations in terms of the arbitrary coordinate system are considerably more complex and contain extra terms arising from the grid curvature and non-orthogonality. In order to enable more explicit control over the properties of the grid, Demirdzic [1982] suggested for two-dimensional flow problems that one of the coordinate families be
straight lines. Parameswaran [1985] further suggested that these straight lines also be parallel. The main disadvantage of non-orthogonal curvilinear grids lies in the fact that the accuracy and stability of the method of solution may be influenced by the geometrical properties of the grid (Peric [1985]).

Concerning the problem of moving boundaries, Hirt, et al. [1970] developed an approach in which a Lagrangian grid was caused to move with the flow. The method proved capable of handling simple flow cases but in complex flows, particularly those exhibiting recirculation, severe grid distortion prevented a solution from being obtained. For general non-orthogonal curvilinear coordinates, Warsi [1981] and Demirdzic [1982] adopted similar approaches to derive the conservation equations in non-Eulerian coordinate systems. Demirdzic [1982] further showed that the "space conservation law" of Trulio & Trigger [1961] had to be solved simultaneously with the other fluid flow conservation equations.

In an application of multi-dimensional methods to unsteady flow in induction manifolds, Isshiki, et al. [1985] presented a study of the influence of helical intake port configurations on the volumetric efficiency and swirl intensity in a pressure-ignition model engine motored at 1800 rpm. A three-dimensional Cartesian grid was used and the curved walls of the passage were approximated by a step-wise boundary, as shown in Figure 1.7a. The grid was relatively coarse and no refinement was attempted in order to assess the error involved in this approximation. The outlet of the solution domain was taken as the port curtain area, as shown in Figure 1.7b, across which the mean velocity was specified from HWA measurements. This choice of the outlet boundary was not justified, although it excluded from the calculations the outlet
passage of the valve/port assembly, which proved, as noted earlier, to be crucial in determining the characteristics of the flow entering the cylinder.

In order to assess the accuracy of the predictions, Isshiki, et al. [1985] carried out comparison with steady-state measurements of $C_D$ for the port configurations shown in Figure 1.7c. The sample results of this comparison shown in Figure 1.7d indicates significant qualitative and quantitative discrepancies in the low-to-medium range of lifts. While the qualitative differences were attributed to the change of the flow pattern in the valve outlet, which was not included in the calculations, no account for the quantitative differences was given. It is further believed that the agreement observed in the medium-to-high lift range is only coincidental, as may also be concluded from Figure 1.7e.

Due to the shortage in detailed and reliable information on flows in valve/port assemblies, the growing number of in-cylinder predictions had to rely, until recently, on assumed boundary conditions at the inlet valve during induction (see, for example, Gosman, et al. [1979], Ramos & Sirignano [1980], Ahmadi-Befrui, et al. [1982], and Wakisaka, et al. [1986]). The errors involved in these assumptions limit the accuracy of multi-dimensional methods in predicting the in-cylinder flow field and processes (Johns [1980], and Gosman, et al. [1984b]).

Attempts have been made recently to eliminate this source of uncertainty by employing realistic boundary conditions at the inlet valve from the growing amount of data obtained from in-cylinder measurements during induction. Ahmadi-Befrui [1985], for example, used the mean velocity profiles at the valve exit measured by Bicen [1983] to estimate the reduction in the effective outflow area resulting
from the flow separation at the valve and seat faces. He calculated
the induction velocity across this area from the mass flow rate
measurements assuming a plug flow distribution. The predictions of
the in-cylinder flow parameters obtained with these boundary conditions
were then compared with those based on a fully attached flow at the
valve exit and the $C_p$ measurements of Kastner, et al. [1964]. The
predicted mean velocities and turbulence intensities during induction
obtained with the former set of boundary conditions were found to be in
closer agreement with the measurements of Bicen [1983].

Further details on the applications of multi-
dimensional methods to in-cylinder flow field predictions are given in
the reviews of Arcoumanis [1984] and Ahmadi-Befrui [1985].

1.6 CLOSURE

In the review presented above, the capabilities and shortcomings
of the measuring techniques most widely used in engine research,
namely, Hot-Wire Anemometry and Laser-Doppler Anemometry, have been
examined. Although LDA has the main advantage of being non-intrusive,
it also suffers from optical accessibility problems in complex-
geometry passages similar to that considered here. For isothermal
incompressible flows, HWA offers a reliable inexpensive and well
developed means of measuring mean and fluctuating velocity fields over
a reasonably wide range of turbulence intensities. It has been
selected for the present investigation with full awareness of its
limitations, as will be discussed later in Chapters 4 and 5.

Experimental studies of the induction process have been extensively
reviewed with respect to:

(i) The global performance of inlet valve/port assemblies,
characterised by the discharge coefficient, as a measure of the engine volumetric efficiency.

(ii) The details of the flow at the exit of the inlet valve and their influence on the structure of the in-cylinder flow field.

Steady-state tests show that the discharge coefficient is strongly dependent on lift and the geometrical details of the valve/port assembly. Of these, the seat angle and the shape of valve and seat corners, sharp or rounded, are most influential in the development of distinct flow patterns in the valve outlet. These patterns differ within different ranges of lift and are characterised by the extent of separation provoked at either or both the valve and seat corners.

Comparison with reciprocating test results at up to 8000 rpm indicate close agreement between steady and unsteady discharge coefficients at relatively high pressure drops across the valve. At small pressure drops and high engine speeds, the unsteady coefficient becomes generally smaller due to the relative increase in the energy required to accelerate the air column in the inlet passage.

Detailed velocity measurements at the valve exit in model engines motored at low speeds, between 200 and 750 rpm, indicate the presence of the flow patterns mentioned above. They also confirm the effects of varying the seat angle and rounding off the sharp corners on the discharge coefficient. Within this range of engine speeds, the agreement observed between steady and unsteady velocity profiles suggests the validity of the quasi-steady assumption for valve flow.

At higher speeds, this agreement becomes restricted to the valve dwell period during which most of the charge is admitted into the cylinder. During early opening and late closure, the flow through
the valve is affected by the proximity of the piston and its high accelerations near both TDC and BDC. However, the outflow through the valve observed during late closure at low speeds reduces with the increase in piston speed, allowing induction to extend beyond 180° ATDC.

The in-cylinder flow field during induction and early compression was found to be strongly influenced by valve lift, velocity profile and trajectory of the intake jet, off-centre positioning of the valve and the design of the valve/port assembly and intake passage. The latter aspect is often exploited in diesel engines to impart swirl to the incoming flow and ensure bulk and turbulent motions adequate for preparing and conditioning the charge for ignition.

Theoretical studies of the flow in inlet valves have also been reviewed with the conclusion that no serious multi-dimensional predictions have so far been published. It has also been shown that the accuracy of in-cylinder multi-dimensional predictions depends to a great extent on the accurate specification of the spatial and temporal variations of the flow parameters at the exit of the inlet valve.
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<td>(a) In small-lift range, $C_D$ increases rapidly with lift and remains almost constant in high-lift range. For fixed pressure drop, $C_D$ decreases with increasing lift. (b) Unsteady $C_D$ is generally smaller than steady-state $C_D$. (c) Conical seated valves are generally superior to flat-seated valves.</td>
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| Wakisaka, et al. [1986] | In-cylinder three-dimensional prediction of the swirl behaviour in a motored model engine with various intake valve and piston cavity configurations. | Similar to Watkins [1977] with rectilinear grid in the piston cavity, eddy viscosity based on the SGS model | (a) Off-centre position of the valve generates complicated flow patterns which are highly sensitive to inlet conditions.  
(b) For directed port, the swirl development during induction is slower than for helical port.  
(c) The swirl at the end of induction is different from solid body and axisymmetric swirls.  
(d) The decay of swirl and squish velocities during compression is little influenced by cavity configuration. |
CHAPTER 2

CONSERVATION EQUATIONS IN

GENERAL CURVILINEAR COORDINATES

2.1 INTRODUCTION

One of the main problems encountered in many practical flow situations is the geometrical complexity of the flow domain. The early attempts to handle complex geometries were by fitting curved boundaries into simple rectilinear grids: this proved to be cumbersome, expensive, and/or unsuccessful. The obvious alternative, therefore, was the use of boundary-fitted coordinates. In this connection, two options exist, namely, curvilinear orthogonal coordinates and general non-orthogonal curvilinear coordinates.

Although the transport equations in orthogonal curvilinear coordinates are a little more complicated than their cartesian counterpart, the coordinate system itself possesses several drawbacks. Firstly, the coordinate mesh has to be generated by some means, conformal mapping for example. This can be restrictive, cumbersome and sometimes costly. Secondly, the grid lines, so generated, often distribute themselves into an inconvenient and inefficient manner in regard to the degree of resolution required for different regions of the flow domain. Thirdly, for regions with non-orthogonally intersecting boundaries, orthogonal coordinates cannot be generated and special treatments have to be developed.

On the other hand, the transport equations in general non-orthogonal coordinates, and their discretised counterpart, are more complicated and usually contain cross-derivatives in pressure and other variables which, without due care, can cause major computational problems. The coordinate mesh, however, is much easier to generate and the mesh distribution can be
controlled and optimised to a greater extent. Furthermore, the possibility exists in aligning the grid lines with the flow stream lines and thus alleviating the effect of "false diffusion" which affects most numerical methods. Therefore, judicious grid arrangement can lead to improved accuracy for a given number of points; a compensation for the additional computations due to the grid non-orthogonality.

This chapter is concerned with the presentation of the conservation equation in a general coordinate-free form. Firstly, for the purpose of the derivation, and subsequent manipulation, of the governing equations, Section 2.2 presents a review of the basic operations of tensor calculus in terms of the physical components of vectors and tensors. For the reader who is not familiar with general tensor analysis, an introduction to the subject is given in Appendix A. Most of the presented material is given without formal proofs. These proofs, however, can be found in references given in the text.

In Section 2.3, the governing differential equations are derived from their respective integral forms. When simple coordinate systems are used, the choice of the vector and tensor components and the form of the conservation equations are obvious and usually not discussed. In the case of general curvilinear coordinates, however, there are many options which may affect the validity, accuracy, and the cost of a numerical solution procedure. In this context, the different forms of the conservation equations are discussed. Following a review of previous work on numerical solution procedures, this section ends with a selection for the forms of the governing equation to be adopted in the present study.

Section 2.4 outlines the derivation of the time-averaged version of the governing equations applicable to turbulent flow. A review of the different approaches to the "closure problem" of the resulting set of equations is also given and a selection is made. The transport equations
incorporated in the selected turbulence model are also presented in the chosen framework. Finally, a general form for the governing equations, suitable for further manipulation, is given.

Section 2.5 summarises the conclusions drawn from the preceding sections.

2.2 TENSOR CALCULUS IN TERMS OF PHYSICAL COMPONENTS

Vectors and tensors that occur in physical problems are usually assigned physical dimensions. The covariant and contravariant components often fail to bear the physical dimensions of the field to which they belong and they are, therefore, sometimes difficult to interpret physically. This is mainly due to the fact that, in Euclidean three-dimensional space, general curvilinear coordinates, unlike Cartesian coordinates, do not always have the dimensions of length. For example, in plane polar coordinates, where*

\[ x^1 = r, \quad x^2 = \theta \]  \hspace{1cm} (2.1)

the first has the dimension of length, while the second has no dimension. Thus, for the velocity field \( v(x^1, x^2) \), the contravariant velocity component

\[ v^1 = \frac{dx^1}{dt} = \frac{dr}{dt} \quad , \quad v^2 = \frac{dx^2}{dt} = \frac{d\theta}{dt} \] \hspace{1cm} (2.2)

would not all have the same physical dimensions of length/time. With the metric tensor given by

---

* Upper and lower indices are generally used throughout the thesis to designate coordinates and components of vectors and tensors. Exponents are identified with parentheses when confusion with superscripts might arise.
\[ g_{11} = 1, \quad g_{22} = (r)^2, \quad g_{i,j} = 0 \quad (i = 1, 2 \text{ and } i \neq j) \quad (2.3) \]

the covariant velocity components are in no better situation since*

\[ v_1 = g_{1j} v^j = v^1, \quad v_2 = g_{2j} v^2 = (r)^2 \frac{d\theta}{dt} \quad (2.4) \]

Thus, both \( v_2 \) and \( v^2 \) do not have the significance of what is usually understood by a "velocity component".

Another unfavourable property of the covariant and contravariant components is that they, in general, change from point to point in relation with the geometrical properties of the coordinate system employed. For example, if a uniform-velocity parallel flow is studied in plane polar coordinates, the radial and circumferential velocity components, according to equations (2.2), become:

\[ v_r = \frac{dr}{dt} = v^1 = \text{const}, \quad v_\theta = r \frac{d\theta}{dt} = r v^2 = \text{const} \quad (2.5) \]

Thus, the contravariant velocity component

\[ v^2 = v^2(r) = \frac{\text{const}}{r} \quad (2.6) \]

approaches infinity as \( r \) approaches zero, a situation which can introduce serious errors in numerical solutions.

Several attempts have been made to overcome the drawbacks of covariant and contravariant components by defining vectors and tensors in

* The Einstein summation convention is used throughout the thesis, i.e. matching upper and lower indices are summed over the range of their values, unless otherwise stated.
terms of their physical components. Ricci & Levi-Civita [1901] introduced the following set of quantities:

\[ v^i \sqrt{g_{ii}}, \quad v^i \sqrt{g^{ii}}, \quad \frac{v_i}{\sqrt{g_{ii}}} \quad \text{and} \quad \frac{v_i}{\sqrt{g^{ii}}} \]  

(2.7)

to represent the components of the vector \( v \). These four sets have the same dimensions as \( v \) itself with different geometrical interpretation, as shown in Figure 2.1. In their treatment of tensors of order higher than one, Ricci & Levi-Civita did not attempt to define quantities of direct physical significance, and in their applications to mathematical physics they were content to derive forms of various classical equations valid in general coordinate systems.

Muranghan [1928], Synge & Schild [1949], Green & Zerna [1950], Ollendorf [1950] and Truesdell [1953] defined the physical components of tensors of the second order. These definitions differ one from another, except in orthogonal coordinate systems where they all reduce to McConell's components (McConell [1931]). None of the definitions, however, was found to be satisfactorily adaptable for general applications in fluid mechanics. This is because these definitions either lack generality (McConell [1931]), increase the degree of complexity in handling the equations (Muranghan [1928] and Truesdell [1953]), or because they do not bear physical significance (Synge & Schild [1949], Green & Zerna [1950] and Ollendorf [1950]).

Ericksen [1960], as a special case of his method of anholonomic components of tensors, introduced a definition for physical vector and tensor components with respect to the natural basis field of the coordinate system. Although the approach is general and flexible, its full potentials have not been recognised. Truesdell [1977] also employed this definition and suggested that equations be derived first in terms of
covariant and contravariant components and then the results be converted
in terms of physical components. This, however, usually leads to
cumbersome, complex and inconvenient expressions.

In the present study, the definition of physical components
introduced by Demirdzic [1982] has been employed. In this, the physical
components are related to normalised natural base vectors. This enables
the derivation of expressions directly in terms of physical components,
while retaining the simplicity of mathematical operations featured by
covariant and contravariant components.

2.2.1 Definition of Physical Components

Consider the normalised base vectors

\[ \xi(i) = \frac{\xi_i}{\sqrt{\gamma_{ii}}} \] (no summation) (2.8)

as the basis in a three-dimensional Euclidean space. These vectors are
non-dimensional unit vectors colinear with \( \xi_i \).

Vectors and tensors are expressed in terms of their contra-
variant components relative to the base vectors as follows:

\[ a = a^{(i)} \xi(i) \] (2.9)

\[ A = A^{(ij)} \xi(i) \xi(j) \]

where, according to the definitions of contravariant and covariant
components, equations (A.9) to (A.12), and equation (2.8)

\[ a^{(i)} = \sqrt{\gamma_{ii}} a^i \] (no summation) (2.10)
\[ A_{ij} = \sqrt{g_{ii}} \sqrt{g_{jj}} A^{ij} \quad \text{(no summation)} \quad (2.11) \]

With \( \varepsilon_{\{i\}} \) being non-dimensional, the components \( a_{\{i\}} \) have the same dimensions as the vector \( a \) and conform with the usual interpretation as directed line segments that add by the parallelogram rule. These components, however, coincide with Ricci & Levi-Civita's components along the coordinate lines as shown in Figure 2.1. Accordingly, \( a_{\{i\}} \) and \( A_{\{ij\}} \) are called contravariant physical vector and tensor components, respectively.

The vectors

\[ \varepsilon_{\{i\}} = \sqrt{g_{ii}} \varepsilon^i \quad (2.12) \]

form the reciprocal basis associated with equation (2.8). It is to be noted that while \( \varepsilon_{\{i\}} \) are non-dimensional and colinear with \( \varepsilon^i \), they are not unit vectors. Vectors and tensors, in terms of their covariant components relative to the reciprocal base vectors, are expressed as

\[ a = a_{\{i\}} \varepsilon_{\{i\}} \quad (2.13) \]

and

\[ A = A_{\{ij\}} \varepsilon_{\{i\}} \cdot \varepsilon_{\{j\}} \quad (2.14) \]

where,

\[ a_{\{i\}} = \frac{a_i}{\sqrt{g_{ii}}} \quad \text{(no summation)} \quad (2.15) \]

\[ A_{\{ij\}} = \frac{A_{ij}}{\sqrt{g_{ii}} \sqrt{g_{jj}}} \quad \text{(no summation)} \quad (2.16) \]

Although the components \( a_{\{i\}} \) still have the same dimensions as \( a \), they are not regarded as vector components in the usual sense because
\( e^{(i)} \) are not unit vectors. These components, however, coincide with Ricci & Levi-Civita's projections on the coordinate lines. The geometrical representation of covariant and contravariant components of vectors is illustrated in Figure 2.1.

2.2.2 Physical Metric Tensor and Christoffel's Symbols

The definition of the metric of a three-dimensional Euclidean space in terms of the physical components of the metric tensor is, analogous to that given by equations (A.14) to (A.16), as follows:

\[
\begin{align*}
g_{(ij)} &= e^{(i)} \cdot e^{(j)} = \frac{g_{ij}}{\sqrt{g_{ii}} \sqrt{g_{jj}}} \quad \text{(no summation)} \\
g^{(ij)} &= e^{(i)} \cdot e^{(j)} = g^{ij} \frac{1}{\sqrt{g_{ii}} \sqrt{g_{jj}}} \quad \text{(no summation)} \\
g^{[i]} &= e^{[i]} \cdot e^{[j]} = \delta^{[i]}_{[j]} \\
\end{align*}
\]

where \( \delta^{[i]}_{[j]} \) is the "Kronecker delta" defined by

\[
\begin{align*}
\delta^{[i]}_{[j]} &= 1 \quad , \quad i = j \\
\delta^{[i]}_{[j]} &= 0 \quad , \quad i \neq j \\
\end{align*}
\]

The arc length differential is also defined as

\[
\begin{align*}
dS^2 &= g_{(ij)} \, dx^{[i]} \, dx^{[j]} \\
\end{align*}
\]

The physical components of the metric tensor are symmetric and can be used for raising and lowering the indices of the physical components of vectors and tensors in the same way as for covariant and
contravariant components. For example,

\[ \varepsilon_i(e) = g_{i(m)} \varepsilon^{(m)} \]

\[ \varepsilon_i(\ell) = g_{(i(m)} \varepsilon_{(m)} \]

\[ \varepsilon_i(\ell) = g_{(i(m]} \varepsilon_{(m)} \]

\[ A_{[i]j} = g_{(i(m)} A_{[j]} = g_{i(m)} g_{(i(n]} A_{(mn)} \]

(2.21)

The derivative of the unit vector \( \varepsilon_i(\ell) \) is given by,

\[ \frac{\partial \varepsilon_i(\ell)}{\partial x^j} = \frac{1}{\sqrt{g_{jj}}} \frac{\partial}{\partial x^j} \left( \frac{\varepsilon_i}{\sqrt{g_{ii}}} \right) \]

(2.22)

With the help of equations (A.36), (A.39), (2.7) and (2.8), the above expression may be expanded into

\[ \frac{\partial \varepsilon_i(\ell)}{\partial x^j} = \frac{1}{\sqrt{g_{ii}}} \left\{ \varepsilon^m_{\ell j} \varepsilon_m - \frac{1}{\sqrt{g_{ii}}} \frac{1}{\sqrt{g_{jj}}} \left\{ \varepsilon^n_{\ell j} g_{in} \varepsilon_i \right\} \right\} \]

(2.23)

where \( \varepsilon^m_{\ell j} \) and \( \varepsilon^n_{\ell j} \) are the Christoffel's symbols given by equation (A.37). Since, similar to equation (2.20), we may write

\[ \varepsilon_i(m) = g_{i(m} \varepsilon_{m} \]

(2.24)

equation (2.23) is re-written in the form,

\[ \frac{\partial \varepsilon_i(\ell)}{\partial x^j} = \sqrt{g_{nn}} \left[ \varepsilon^m_{\ell j} - g_{i(m} g_{in} \varepsilon^n_{\ell j} \right] \varepsilon_{(m)} \]

(2.25)

This expression is further reduced to,
by introducing the definition:

\[
\{^m_j\} = \sqrt{\frac{g_{mn}}{g_{ij}g_{jk}}} \left[ \{^m_i\} - g_{im} \frac{\partial n}{\partial i} \right] (2.27)
\]

The analogy between equations (2.26) and (A.36) suggests that the quantities represented by \(\{^m_j\}\) be regarded as the physical Christoffel's symbols. It is to be noted, however, that the expanded forms of \(\{^m_j\}\) do not contain any of the equal-indices symbols. Another important property of the physical Christoffel's symbols is that they are, unlike \(\{^m_j\}\), not in general symmetric with respect to indices \(i\) and \(j\), thus

\[
\{^m_j\} \neq \{^m_i\} (2.28)
\]

### 2.2.3 Algebraic Operations

Scalar multiplication, summation, dot and tensor products, transposition, and symmetrisation and alteration, when performed with the physical components of vectors and tensors, yield expressions similar to those given by equations (A.23) to (A.35). For example,

\[
S \frac{\partial}{\partial x} = S a^{(i)} \frac{\partial}{\partial x^{(i)}} = p^{(i)} \frac{\partial}{\partial x^{(i)}} (2.29)
\]

\[
S \frac{A_{ij}}{\partial x} = S A^{(ij)} \frac{\partial}{\partial x^{(i)}} \frac{\partial}{\partial x^{(j)}} = p^{(ij)} \frac{\partial}{\partial x^{(i)}} \frac{\partial}{\partial x^{(j)}}
\]
\[ \mathbf{a} \cdot \mathbf{b} = a^{(i)} b^{(j)} \varepsilon_{(i)} \cdot \varepsilon_{(j)} = g_{(i)(j)} a^{(i)} b^{(j)} = a^{(j)} b^{(j)} \]  

\[ \mathbf{a} \cdot \mathbf{\bar{b}} = a^{(i)} b^{(j)} \varepsilon_{(i)} \cdot \varepsilon_{(j)} = a^{(i)} b^{(j)} \varepsilon_{(i)} \cdot \varepsilon_{(j)} \]  

\[ \mathbf{a} \cdot \mathbf{\bar{B}} = g_{(i)(j)} a^{(i)} B^{(jk)} \varepsilon_{(k)} \]  

\[ \mathbf{A} \cdot \mathbf{\bar{B}} = g_{(jk)} A^{(ij)} B^{(k\ell)} \varepsilon_{(\ell)} \cdot \varepsilon_{(k)} = A^{(ij)} B^{(jk)} \varepsilon_{(i)} \cdot \varepsilon_{(k)} \]  

\[ \mathbf{A} \cdot \mathbf{\bar{B}} = g_{(jk)} A^{(ij)} B^{(k\ell)} = A^{(i)} B^{(k)} \]  

\[ (A^T)_{(ij)} = A^{(ij)} \]  

\[ (A^T)_{(ij)} = A^{(ij)} \]  

\[ (A^T)_{(jk)} = A^{(j)} \]  

\[ (A^T)_{(j)} = A^{(j)} \]  

### 2.2.4 Covariant Derivative

The derivative of a vector field, defined as,

\[ \frac{\partial a}{\partial x^{(j)}} = \frac{\partial}{\partial x^{(j)}} (a^{(i)} \varepsilon_{(i)}) = \frac{\partial a^{(i)}}{\partial x^{(j)}} \varepsilon_{(i)} + a^{(i)} \frac{\partial \varepsilon_{(i)}}{\partial x^{(j)}} \]  

may be reduced, using equation (2.26), into

\[ \frac{\partial a}{\partial x^{(j)}} = \nabla^{(j)} a^{(i)} \varepsilon_{(i)} \]  

where

\[ \nabla^{(j)} a^{(i)} = \frac{\partial a^{(i)}}{\partial x^{(j)}} + a^{(m)} \Gamma^{(i)}_{mj} \]
is the covariant derivative of the contravariant physical component of the vector. Expanding the right-hand side of equation (2.35), in terms of equations (2.10) and (2.27), and substituting equation (A.39) into the resulting expression, one obtains

$$\nabla_{(j)} a_{(i)} = \sqrt{g_{jj}} \left[ \frac{\partial a_{(i)}}{\partial x^{(j)}} + a^{m} \left\{ \frac{a_{(m)}}{g_{jj}} \right\} \right] = \sqrt{g_{jj}} \nabla_{j} a_{i} \quad (2.36)$$

In a similar manner, an expression for the covariant derivative of the covariant physical component of a vector may be obtained in the form

$$\nabla_{(j)} a_{(i)} = \frac{\partial a_{(i)}}{\partial x^{(j)}} - a_{(m)} \left\{ \frac{a_{(m)}}{g_{jj}} \right\} \quad (2.37)$$

This can alternatively be written as

$$\nabla_{(j)} a_{(i)} = \frac{1}{\sqrt{g_{ll}} \sqrt{g_{jj}}} \nabla_{j} a_{l} \quad (2.38)$$

The covariant derivatives of the contravariant and covariant physical components of tensors are defined, respectively, by

$$\frac{\partial A}{\partial x^{(k)}} = \nabla_{(k)} A^{(i)j} \xi^{(i)} \xi^{(j)} \quad (2.39)$$

$$\frac{\partial A}{\partial x^{(k)}} = \nabla_{(k)} A^{(i)j} \xi^{(i)} \xi^{(j)} \quad (2.40)$$

where

$$\nabla_{(k)} A^{(i)j} = \frac{\partial A^{(i)j}}{\partial x^{(k)}} + A^{(mj)} \left( \xi^{(i)} \right)_{m}^{(k)} + A^{(im)} \left( \xi^{(j)} \right)_{m}^{(k)} = \sqrt{g_{ll}} \sqrt{g_{jj}} \nabla_{k} A^{(i)j} \quad (2.41)$$

and
From these last two expressions and the definition (2.27), it can be shown that the physical components of the metric tensor retain the property expressed by equation (A.48), namely

$$
\nabla(k) \ g^{ij} = \nabla(k) \ g^{ij} = 0 \quad (2.43)
$$

### 2.2.5 Divergence and Gradient

The divergence of vector and tensor fields, in terms of their physical components, are given, respectively, by

$$
div \mathbf{a} = \varepsilon^{(j)} \frac{\partial \mathbf{g}}{\partial x(j)} = \nabla(j) \ a^{(j)} \quad (2.44)
$$

and

$$
div \ A^{(i)} = \varepsilon^{(k)} \frac{\partial A^{(i)}}{\partial x(k)} = \nabla(j) \ A^{(ij)} \ v^{(l)}(i) \quad (2.45)
$$

Substituting equation (2.10) into equation (A.54), one obtains an alternative expression for the divergence of a vector field in the form

$$
div \ a = \sqrt{g_{jj}} / g \ \frac{3}{\partial x(j)} \ (\sqrt{g} g_{jj} \ a^{(j)}) \quad (2.46)
$$

which may be reduced to

$$
div \ a = \frac{\Delta a^{(j)}}{\Delta x(j)} = \frac{\Delta}{\Delta x(j)} [g^{(jm)} a_{(m)}] \quad (2.47)
$$

The differential operator $\Delta/\Delta x(j)$ is the so-called "strong conservation form of the divergence operator", defined by
\[
\frac{\Delta}{\Delta x(j)} \{ \ldots \} = \sqrt{\frac{\delta_{ij}}{g}} \frac{\partial}{\partial x(j)} \left[ \sqrt{g}g_{ij} \right] \{ \ldots \} \quad (2.48)
\]

The divergence of a tensor field may also be expressed in terms of the differential operator (2.48) to give

\[
div \frac{A}{\partial x} = \frac{\Delta}{\Delta x(j)} \{ A(ij) \} e(ij) = \frac{\Delta}{\Delta x(j)} \{ g^{(jm)} A_{(im)} \} e(ij) \quad (2.49)
\]

The gradients of scalar, vector and tensor fields in terms of the physical components, are given, respectively, by

\[
\begin{align*}
\text{grad} S &= e(j) \frac{\partial S}{\partial x(j)} = g^{(jm)} \frac{\partial S}{\partial x(j)} e(m) \quad (2.50a) \\
\text{grad} \varphi &= e(j) \frac{\partial \varphi}{\partial x(j)} = e(j) \cdot \nabla(j) a(j) e(j) \\
\text{grad} A &= e(km) \frac{\partial A}{\partial x(km)} = g^{(km)} \nabla(km) A_{(ij)} e(m) - e(ij) - e(j) \quad (2.50c)
\end{align*}
\]

2.3 TRANSPORT EQUATIONS

For a fluid element having a control volume \(dV\), the equations of conservation of mass, a scalar \(\phi\), and linear momentum may be written, respectively, in the integral forms (Schlichting [1968])

\[
\begin{align*}
\frac{d}{dt} \int_V \rho \, dV &= \int_V S_m \, dV \\
\frac{d}{dt} \int_V \rho \phi \, dV &= \int_\Gamma \phi \frac{\partial n}{\partial \Gamma} \, dA + \int_V S_\phi \, dV 
\end{align*}
\]
and

\[
\frac{d}{dt} \int_V \rho \psi \, dV = \oint_A \mathbf{T} \cdot \mathbf{n} \, dA + \int_V \mathbf{S} \cdot dV \tag{2.53}
\]

In the above equations, \( S \) represents a source or a sink of the respective dependent variable and \( \mathbf{n} \) represents the unit vector normal to the area element \( dA \) pointing outwards. To close this system of equations, one has to provide constitutive relations defining the stress tensor \( \mathbf{T} \) and the flux vector \( \mathbf{q} \) in terms of the basic dependent variables.

For a Newtonian isotropic fluid, the stress tensor, based on Stokes' hypothesis (Schlichting [1968]), is given by

\[
\mathbf{T} = - (p + \frac{2}{3} \mu \text{div} \mathbf{v}) \mathbf{I} + 2 \mu \mathbf{D} \tag{2.54}
\]

where \( p \) is the thermodynamic pressure, \( \mu \) is the dynamic viscosity, \( \mathbf{I} \) is a unit tensor, and \( \mathbf{D} \) is the rate of strain or deformation tensor.

The diffusion flux vector \( \mathbf{q} \) is usually represented by a Fourier-type expression of the form (Roache [1982])

\[
\mathbf{q} = \Gamma_\phi \text{grad} \phi \tag{2.55}
\]

where \( \Gamma_\phi \) is the diffusion coefficient pertinent to the particular \( \phi \).

The set of equations (2.51) to (2.53) may, however, be reduced to the general coordinate-free form

\[
\frac{d}{dt} \int_V \rho \psi \, dV = \oint_A \Sigma \cdot \mathbf{n} \, dA + \int_V \sigma \psi \, dV \tag{2.56}
\]

where \( \psi \) and \( \sigma \) are tensor fields of the same order zero or one, and \( \Sigma \) is a tensor field of one order higher than \( \psi \). The physical significance of the different terms of equation (2.56) is given in Table 2.1.
The Reynolds transport theory (Aris [1962]), expressed in the form

$$\frac{d}{dt} \int_V \eta \, dV = \int_V \left[ \frac{\partial \eta}{\partial t} + \text{div} \{ \eta \mathbf{v} \} \right] dV$$  \hspace{1cm} (2.57)

when applied to equation (2.56) yields

$$\int_V \left[ \frac{\partial}{\partial t} (\rho \psi) + \text{div} (\rho \psi \mathbf{v}) \right] dV = \oint_A \Sigma \cdot n \, dA + \int_V \sigma_\psi \, dV$$ \hspace{1cm} (2.58)

Furthermore, the divergence theorem, expressed in the form

$$\oint_A \Sigma \cdot n \, dA = \int_V \text{div} \Sigma \, dV$$ \hspace{1cm} (2.59)

when applied to equation (2.58) gives

$$\int_V \left[ \frac{\partial}{\partial t} (\rho \psi) + \text{div} (\rho \psi \mathbf{v} - \Sigma) \right] dV = \int_V \sigma_\psi \, dV$$ \hspace{1cm} (2.60)

Integration of equation (2.60) over the control volume $dV$ yields the general conservation equation in the differential coordinate-free form

$$\frac{\partial}{\partial t} (\rho \psi) + \text{div} (\rho \psi \mathbf{v} - \Sigma) = \sigma_\psi$$ \hspace{1cm} (2.61)
The conservation equations of mass, scalar $\phi$ and linear momentum may thus be rewritten, respectively, as

$$\frac{\partial \rho}{\partial t} + \text{div} (\rho \mathbf{v}) = S_m \quad (2.62)$$

$$\frac{\partial (\rho \phi)}{\partial t} + \text{div} (\rho \phi \mathbf{v} - q) = S_\phi \quad (2.63)$$

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \text{div} (\rho \mathbf{v} \cdot \mathbf{v} - \mathbf{T}) = S_\mathbf{v} \quad (2.64)$$

In terms of their physical contravariant components, the velocity and diffusion flux vectors and the stress tensor are given by

$$\mathbf{v} = v^i \mathbf{e}_i$$

$$\mathbf{q} = q^i \mathbf{e}_i$$

$$\mathbf{T} = \tau^{ij} \mathbf{e}_i \otimes \mathbf{e}_j$$

The definition (2.50a) for the gradient of a scalar, when substituted into equation (2.55), yields the following expression for the diffusion flux

$$q^i = g^{jm} \frac{\partial \phi}{\partial x^m} \mathbf{e}_i$$

Also, substitution of the definition (2.47) into equation (2.54) produces the following form for the stress tensor,

$$\tau^{ij} = \left[ - \left( p + \frac{2}{3} \mu \frac{\partial \mathbf{v}^i}{\partial x^j} \right) g^{ij} + 2 \mu \mathbf{D}^{ij} \right] \mathbf{e}_i \otimes \mathbf{e}_j$$

The deformation tensor is defined as the symmetric part of the
velocity gradient (Schlichting [1968]). From equation (2.50b)

\[
\text{grad } \nu = [g^{(jm)} \nu^{(j)}] \varepsilon^{(j)} - \varepsilon^{(j)}
\]  

(2.68)

the symmetric part of which is given by

\[
\nu^{(ij)} = \frac{1}{2} [g^{(jm)} \nu^{(j)} + g^{(im)} \nu^{(m)}]
\]  

(2.69)

Therefore, the stress tensor may be represented by

\[
\frac{T}{\rho} = \left[ - (p + \frac{2}{3} \mu \frac{\Delta \nu}{\Delta x}) g^{(ij)} + \mu \left( g^{(jm)} \nu^{(j)} + g^{(im)} \nu^{(m)} \right) \right] \varepsilon^{(i)} - \varepsilon^{(j)}
\]  

(2.70)

or, alternatively, as

\[
\frac{T}{\rho} = \{- p \nu^{(ij)} + \tau^{(ij)} \} \varepsilon^{(i)} - \varepsilon^{(j)}
\]  

(2.71)

where

\[
p = p + \frac{2}{3} \mu \frac{\Delta \nu}{\Delta x}
\]  

(2.72)

and

\[
\tau^{(ij)} = \mu \left( g^{(jm)} \nu^{(j)} + g^{(im)} \nu^{(m)} \right)
\]  

(2.73)

With definitions (2.47) and (2.48) for the divergence of vectors and tensors, the conservation equations (2.62) to (2.64) may be obtained in the following forms

\[
\frac{\partial p}{\partial t} + \frac{\Delta}{\Delta x} \{ \rho \nu^{(j)} \} = S_m
\]  

(2.74)

\[
\frac{\partial}{\partial t} \{ \rho \phi \} + \frac{\Delta}{\Delta x} \{ \rho \phi \nu^{(j)} - \phi \left[ g^{(jm)} \frac{\partial \nu}{\partial x} \right] \} = S_{\phi}
\]  

(2.75)
\[ \frac{\partial}{\partial t} \left( \rho v^i \right) + \frac{\Delta}{\Delta x^j} \left[ \left( \rho v^i v^j \right) - \tau^{ij} \right] \phi^i = S_v^i \phi^i \]  

(2.76)

where

\[ \tau^{ij} = - \rho g^{ij} + \tau^{ij} \]  

(2.77)

and the operator \( \Delta/\Delta x^j \) is given by equation (2.48).

Inspection of equations (2.74) and (2.75) shows that the terms under the differential operator \( \Delta/\Delta x^j \) are those under the divergence operator in equation (2.61). This means that the integration with respect to \( x^j \) results in integrals of exact differentials which depend only on the boundary values of the flow parameters. In terms of the discretised equation, this means conservative discretisation (Roache [1982]). Accordingly, equations (2.74) and (2.75) are said to be in a strong conservation-law form (Vinokur [1974]).

On the other hand, the momentum equation is usually resolved into specific directions before it can be solved. Furthermore, the directions of resolution are usually taken to be normal to the velocity components. Therefore, equation (2.76), when resolved in the direction of \( \phi^i \), yields

\[ \frac{\partial}{\partial t} \left( \rho v^i \right) + \frac{\Delta}{\Delta x^j} \left[ \rho v^i v^j - \tau^{ij} \right] + \left( \rho v^i \right) \phi^i = S_v^i \]  

(2.78)

The directions of resolution are not, in general, spatially constant. Moreover, the physically conserved quantity is the momentum vector itself and not its components. Thus, the Coriolis and centrifugal forces \( \left( \rho v^i \right) \phi^i \) and the stress counterpart \( \tau^{ij} \phi^i \), which appear in equation (2.78) as undifferentiated terms, serve to redistribute the conserved momentum vector components between the coordinate directions.
With the presence of undifferentiated terms, there is no obvious method by means of which a conservative discretisation of equation (2.78) could be achieved. Equation (2.76) is, therefore, said to be in a weak conservation-law form (Vinokur [1974]), while the set of equations (2.74), (2.75) and (2.78) is said to be in a semi-strong conservation-law form.

From the above discussion, it is clear that in order to obtain a strong conservation form of the momentum equation, it must be resolved into spatially invariant directions. This may generally be achieved by referring to an arbitrary constant basis to express either the base vectors \( \mathbf{e}_i \) (Andersen, et al. [1968]), or the components of the velocity vector \( \mathbf{v} \) and the stress tensor \( \mathbf{T} \) (MacCormack & Paullay [1972], and Rizzi & Inouye [1973]).

The strong conservation form of the momentum equation obtained by resolving the velocity vector and stress tensor into their Cartesian components is the one most extensively used. The vast majority of the calculation procedures adopting this approach have been devised for high Mach number flow problems, a review of which is presented by Hollanders & Viviand [1980].

For the solution of flow problems in arbitrary domains at all speeds, a finite volume method was first developed at Los Alamos Scientific Laboratory (Hirt, et al. [1974]). The method, named "ICED-ALE", is an extension of the Arbitrary Lagrangian-Eulerian procedure of Hirt [1970] for irregular geometries and/or moving boundaries. It employs the Implicit Continuous-Eulerian (ICE) technique of Harlow & Amsden [1971].

The ICED-ALE computational grid consists of trapezoidal cells with the velocity components located at the cell vertices. The scalar variables, such as pressure, density, etc., are assigned to the cell centre as shown in Figure 2.2. The governing equations do not contain curvature terms and the only geometrical information needed is the...
Cartesian coordinates of the cell vertices. However, with the discretised momentum equation being independent of the position of the vertex of the computational cell, decoupling between velocities at adjacent vertices may occur (Hirt, et al. [1974]). To eliminate this undesirable feature, an artificial restoring body force is introduced on each vertex to keep more in line with neighbouring vertices (Butler [1970]).

Although the method was mostly used for high Mach number flows, some applications to subsonic flow problems (Harlow & Amsden [1975], and Daly [1976]) and engine flow predictions (Boni, et al. [1976], El-Tahry [1982], and El-Tahry [1984]) have been made.

Another procedure which uses Cartesian velocity components, and consequently strong conservation form of the transport equations, is the TURF method of Wachspress [1979]. The method uses an arbitrary curvilinear grid whose geometrical variables are calculated by employing a local coordinate transformation defined relative to the cell node and four neighbouring nodes, Figure 2.3.

The solution procedure is based on the SIMPLE algorithm of Patankar & Spalding [1972]. As with the ICED-ALE, the TURF method suffers from the possibility of decoupling between adjacent nodes. Moreover, the coupling of the finite difference equations between the odd and even columns/rows is relatively weak and arises only from the cross derivatives due to the grid non-orthogonality.

To avoid the problems associated with the Cartesian basis, vector and tensor components may alternatively be expressed in terms of a locally fixed basis. In this case, the directions of the flux vectors depend on the local orientation of the coordinate system. Vinokur [1974] takes the natural base vector at the central point $P$ of each subdomain or cell, Figure 2.4, as the fixed basis. The base vectors at the cell faces are then expressed in terms of the locally fixed basis $\mathbf{e}_m$ such that
\[ e_{\{i\}} = \beta_{\{i\}} \left[ e_{\{m\}} \right]_P \]  

(2.79)

The momentum equation in this case takes the form

\[
\frac{\partial}{\partial t} \left[ \rho \, v_{\{i\}} \right] \beta_{\{i\}} + \frac{\Delta}{\Delta x_{\{j\}}} \left[ \left\{ \rho \, v_{\{i\}} \, v_{\{j\}} - T_{\{ij\}} \right\} \beta_{\{i\}} \right] \left[ e_{\{m\}} \right]_P = S_{\{i\}} \beta_{\{i\}} \left[ e_{\{m\}} \right]_P 
\]

(2.80)

with the coefficients \( \beta_{\{i\}} \) defined, according to equation (2.79), by

\[ \beta_{\{i\}} = e_{\{i\}} \left[ e_{\{m\}} \right]_P \]  

(2.81)

At the cell node, \( \beta_{\{i\}} \) reduces to the Kronecker delta and consequently the time derivative term is reduced to only one velocity component. For a closely spaced grid or when the curvature of the grid lines does not vary steeply from one cell to another, the following approximation may be made at the cell faces:

\[ \beta_{\{i\}} = \delta_{\{i\}} \quad \text{(no summation)} \]  

(2.82)

For a grid arrangement where the velocity components are located at the cell faces, this approximation reduces the number of the convection terms in equation (2.80) and promotes the velocity component \( v_{\{i\}} \) as the dominant variable. It should be kept in mind, however, that at a cell face, common to two cells, the value of \( \beta_{\{i\}} \) determined with respect to one cell node is not necessarily equal to its value calculated with respect to the other node. This leads to discontinuities in the coefficient \( \beta_{\{i\}} \) and consequently to discontinuities in the fluxes crossing the cell.
boundaries. Therefore, an unconditional overall momentum balance can no longer be ensured.

Liu [1976] introduced a method for solving three-dimensional unsteady compressible flow employing the governing equations in their semi-strong conservation form in terms of the contravariant components. After establishing the finite difference equations in terms of the contravariant components, the corresponding equations in terms of the physical contravariant components are obtained and solved using the method of Viecelly [1971]. The method has been applied to a few simple flow configurations with variable success.

Demirdzic, et al. [1980] extended Vinokur's approach by expressing the non-physical contravariant components in terms of the locally fixed basis \( e^i_\xi \). With the governing equations written in their semi-strong form, a solution procedure, based on the SIMPLE algorithm, was presented for the grid arrangement shown in Figure 2.5. The method has been successfully applied to some two-dimensional flow situations. In some cases, however, numerical errors occurred mainly due to the use of non-physical vector and tensor components referred to earlier in Section 2.2.

Demirdzic [1982] introduced a modification of the method described above for solving unsteady three-dimensional subsonic flow problems. In order to avoid the numerical errors mentioned above, the vectors and tensors were expressed in terms of their contravariant physical components which were referred to a locally fixed basis. Parameswaran [1985] adopted the same approach for the derivation of the governing equations and introduced a constraint on the coordinate system by fixing one of the coordinate directions. Consequently, the momentum equation, when resolved in that direction, did not contain any of the curvature term. This constraint, however, calls for special care when handling non-parallel inlet and exit boundaries and restricts the application to
relatively simple geometries.

In the present study, the method of Demirdzic [1982] is employed in its general form. With the momentum equation resolved into the directions normal to the velocity components, the governing equations take the semi-strong form:

\[
\frac{3p}{3x} + \frac{\Delta}{\Delta x} \left[ \rho v^i \right] = S_m
\]

\[
\frac{3\phi}{3x} + \frac{\Delta}{\Delta x} \left[ \rho \phi \right] = \sum_{j} \left[ \frac{\partial}{\partial x_j} \left( \rho \phi \right) \right] = S_\phi
\]

\[
\frac{3\mathcal{I}}{3x} + \frac{\Delta}{\Delta x} \left[ \rho \mathcal{I} \right] = \sum_{j} \left[ \frac{\partial}{\partial x_j} \left( \rho \mathcal{I} \right) \right] = S_v
\]

(2.83)

2.4 TURBULENCE MODELLING

The flow regimes of practical importance are almost always turbulent. In principle, there is no need to adopt a special practice for turbulent flow calculations for the transport equations apply equally to turbulent flows as to laminar ones. Turbulent flows are, however, unsteady, random and three-dimensional and important details of turbulence are small-scale in character. Therefore, in any numerical procedure devised for solving the equations describing turbulent flow, the mesh size and time-step must be such as to resolve the small-scale turbulence processes. In terms of the speed and storage capacity of present-day computers, this requirement cannot be met for all but simple flows.

In most engineering applications, the time-averaged effects of turbulence are of greater importance than instantaneous details, even when the mean flow is unsteady. Therefore, predictions of turbulent flows are
based mainly on the time-averaged transport equations in which direct calculations of small-scale turbulence is bypassed.

In order to rewrite the transport equations in time-averaged forms, the instantaneous dependent variables are expanded into mean and fluctuating quantities such that (Hinze [1975])

$$v(t) = \overline{v}(t) + v'(t)$$

$$\rho = \overline{\rho} + \rho'$$

$$p = \overline{p} + p'$$

(2.84)

$$\phi = \overline{\phi} + \phi'$$

where the overbar denotes the mean component and the prime denotes the fluctuations.

For stationary flows, the mean values are constant and independent of time. For unsteady incompressible flow, the Reynolds time-averaging process, defined by

$$\overline{\psi} = \lim_{t_a \to \infty} \frac{1}{t_a - t_0} \int_{t_0}^{t_a} \psi(t) \, dt$$

may be used to obtain the mean values of equations (2.84) (Donaldson & Sullivan [1972]).

In the case of unsteady compressible flow, where the density changes are to be taken into account, the mass or Favre-averaging, first suggested by Favre [1965], or a combination of the Reynolds and Favre-averaging, proposed by Rubesin & Rose [1973], may be used. In all cases, the time-averaging interval must be sufficiently long compared to the time-scale of
turbulence but small when compared to the time-scale of the changes within the flow field.

If the fluctuations in density and laminar viscosity are assumed to be negligibly small, which is usually justifiable in non-reacting and non-buoyant flows, the substitution of equations (2.84) into the transport equations and rearrangement yields (Demirdzic [1982]):

\[
\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial}{\partial x} \left( \bar{\rho} \bar{v}(i) \right) = \bar{S}_m
\]

\[
\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial}{\partial x} \left[ \left( \bar{\rho} \bar{v}(j) \right) - \left( \bar{\rho} \bar{v} \right) \right] = \bar{S}_\phi
\]

\[
\frac{\partial (\bar{\rho} \bar{v}(i))}{\partial t} + \frac{\partial}{\partial x} \left[ \left( \bar{\rho} \bar{v}(i) \right) \bar{v}(j) \right] = \bar{S}_v(i) \left( m \ \bar{v}(m) \bar{v}(j) \right) - \left( \bar{v}(i) \bar{v}(j) \right)
\]

Equations (2.86) are similar to equations (2.83), except for the appearance of the second order correlations, i.e.

\[
q_{x}^{(j)} = - \bar{\rho} \bar{v}(i) \bar{v}(j)
\]

which represents the turbulent diffusion fluxes (Hinze [1975]) and the turbulent Reynolds stresses,

\[
\tau_{x}^{(ij)} = - \bar{\rho} \bar{v}(i) \bar{v}(j)
\]

The process of time-averaging thus produces statistical correlations involving fluctuating velocities and scalars of unknown magnitudes. Additional information about these correlations is required in order to close the system of equations (2.86). The process of approximating them
in terms of the averaged quantities is known as "turbulence modelling".

Generally, turbulence models can be classified under one of the following:

(i) eddy viscosity models
(ii) Reynolds stress models
(iii) large-eddy simulation

(i) Eddy Viscosity Models

Most turbulence models of practical use are based on the Boussinesq concept of 'turbulent' or 'eddy' viscosity (Boussinesq [1877]). The concept assumes that, in analogy to viscous stresses in laminar flows, the Reynolds stresses are proportional to the mean velocity gradient, as follows (Hinze [1975]):

\[-\rho \overline{v'_i v'_j} = \mu_t \rho \overline{\epsilon_i \epsilon_j} \]  (2.89)

Here, \(\overline{\epsilon_i \epsilon_j}\) is the deformation tensor defined by equation (2.69), and \(\mu_t\) is the eddy viscosity which depends strongly on the state of turbulence.

The first models of this kind, known as 'zero equation' models, of which the Prandtl mixing length hypothesis (Prandtl [1925]) is the most well-known example, relate turbulent transport terms to mean flow quantities. This is done through empirical formulae which imply that these models contain very specific experimental data as an essential ingredient. Accordingly, even for the few flows to which they could be applied, these early models lacked universality.

In order to provide more generality and to reduce the need for extensive experimental work, more elaborate models were developed. These
discard a direct algebraic link between the turbulent transport terms and the mean motion, and employ instead differential transport equations for the turbulence quantities. They are often characterised by the number of such equations which they contain.

Bradshaw, et al. [1967] provided what was perhaps the first widely used one-equation turbulence model for boundary layer prediction. In this model, a transport equation for the turbulence kinetic energy is solved in conjunction with the transport equations of the mean motion.

Nee & Kovasznay [1969] also proposed a one-equation turbulence model which entails the solution of a transport equation for the effective viscosity. Since the length-scale of turbulence which appears in these models must be algebraically prescribed, they have been found to be only marginally superior to zero-equation models.

To allow transport effects on the turbulence length-scale to be accounted for, various two-equation models were introduced (Harlow & Nakayama [1968], Rodi & Spalding [1970], Ng & Spalding [1970], Spalding [1970], and Jones & Launder [1972]). Each of these provides, in addition to an equation for the turbulent kinetic energy \( k \), an equation for a length-scale related variable. Among these variables, the rate of dissipation \( \varepsilon \) of turbulence energy is favoured partly because of the ease with which its exact equation is derived and partly because of the fact that \( \varepsilon \) appears directly as an unknown in the equation of \( k \). The turbulent viscosity is linked to the aforementioned variables via

\[
\nu_t = C_\mu \frac{k^2}{\varepsilon}
\]  

(2.90)

It should be noted that from \( k \) and \( \varepsilon \), there can be deduced a length-scale \( \ell \) (\( \equiv k^{3/2}/\varepsilon \)) proportional to that of the energy-containing motion.
Reviews of the above models can be found in Launder & Spalding [1972], Bradshaw [1978], Rodi [1980], and Bradshaw, et al. [1981].

(ii) Reynolds Stress Models

The models mentioned so far assume that the local state of turbulence can be characterised by one velocity scale, and that the Reynolds stresses can be related to this scale via expressions similar to equation (2.90). These relations often imply that the transport of the individual stresses is not accounted for. Models involving transport equations for the non-vanishing components of the stress tensor have also been developed. In the process of deriving these equations, higher order correlations appear as extra unknowns. To close the system of equations at this level, the unknown correlations are approximated in terms of the other determinable quantities (Rotta [1951], Lumley [1972], Launder, et al. [1975], and Gibson, et al. [1981]). At the stress tensor level of closure, seven turbulence equations (for the six components of stresses and the length-scale) are generated in conjunction with those of the mean flow motion. Some higher level closures, however, introduced additional transport equations for third order correlations (André, et al. [1977]).

The Reynolds stress models provide a more complete description of turbulence than the lower order models mentioned above. However, in all closure approximations and assumptions, there exist uncertainties, even in the case of homogeneous flow, which the model predicts reasonably well. These approximations, which are also restrictive in that they often assume high turbulent Reynolds numbers, remoteness from walls and uniform density, are still subject to extensive investigation and development. In addition to being computationally expensive, they sometimes have not proved to be superior to simpler, less expensive models when applied to complex flows (Thompson [1983], El-Tahry [1984], and
Further details on the Reynolds stress models can be found in Reynolds [1976], Rodi [1980], and Gibson, et al. [1984].

(iii) **Large-Eddy Simulation (LES)**

This is a completely different approach for turbulence modelling which does not aim at simulating turbulent stresses and fluxes in the mean flow equations. It is based on the premise that the small-scale turbulence is universal and much less problem-dependent than the large-scale turbulence. If large-scale variables can be defined (Kwak, et al. [1975]), then the solution of the full three-dimensional time-dependent governing equations may be carried out on a computational grid whose time and spatial dimensions characterise the large-scale eddies (Ferziger & Leslie [1979], and Antonopolous-Domis [1981]). The small-scale eddies that cannot be resolved with the chosen numerical grid are approximated through 'subgrid-scale' modelling (Clark, et al. [1979]).

Although the Large-Eddy Simulation approach is computationally expensive, as would be expected, and still in the development stage, it has already shown promising results (Kaned & Leslie [1983]).

### 2.4.1 Choice for the Present Study

In the present study, the two-equation \( k-\varepsilon \) model of Jones & Launder [1972] is used. This defines the turbulent viscosity via equation (2.90) and the coefficient of scalar diffusion as:

\[
\Gamma_{\phi t} = \frac{\nu_{t}}{\sigma_{\phi t}} \tag{2.91}
\]

where \( \sigma_{\phi t} \), the turbulent Prandtl/Schmit number, is an empirical coefficient.
The formal definitions of turbulence kinetic energy and its rate of dissipation are given, respectively, by (Corrsin [1953]):

\[ k = \frac{1}{2} \sum \nu' \langle \xi \rangle \nu' (\xi) \]  
\[ \varepsilon = \frac{1}{\rho} \tau' : \text{grad} \nu' \]  
(2.92)

The definitions of the scalar product of tensors, equations (2.31), and the gradient of a vector, equations (2.50), together with the assumption of isotropy, allow an alternative expression for \( \varepsilon \) to be written in the form:

\[ \varepsilon = \nu \sum g_{\langle \xi m \rangle} g_{\langle j n \rangle} \nu_{\langle m \rangle} \nu_{\langle j \rangle} \nu_{\langle \xi \rangle} \]  
(2.93)

where the stress tensor \( \tau' \) is represented, in terms of the fluctuating velocity components, by an expression similar to equation (2.73), namely

\[ \tau_{\langle \xi j \rangle} = 2 \nu \sum g_{\langle j m \rangle} \nu_{\langle m \rangle} \nu_{\langle \xi \rangle} + g_{\langle \xi m \rangle} \nu_{\langle m \rangle} \nu_{\langle j \rangle} \]  
(2.94)

The \( k-\varepsilon \) model contains the following differential equations for \( k \) and \( \varepsilon \):

\[ \frac{3}{\Delta x} (\overline{p} \ k) = \frac{\Delta}{\Delta x(\overline{j})} (\overline{p} \ k \overline{v}(j)) - \frac{\nu}{\sigma_k} g_{\langle j m \rangle} \overline{a_k} \frac{\overline{a_k}}{\Delta x(\overline{m})} = G - \overline{p} \ v \]  
(2.95)

\[ \frac{3}{\Delta x} (\overline{p} \ \varepsilon) + \frac{\Delta}{\Delta x(\overline{j})} (\overline{p} \ \varepsilon \overline{v}(j)) - \frac{\nu}{\sigma_\varepsilon} g_{\langle j m \rangle} \overline{a_\varepsilon} \frac{\overline{a_\varepsilon}}{\Delta x(\overline{m})} \]  
\[ = C_1 \ \varepsilon \ G - C_2 \ \overline{p} \ \varepsilon^2 - C_3 \ \overline{p} \ \varepsilon \frac{\Delta v_{\langle m \rangle}}{\Delta x(\overline{m})} \]  
(2.96)

where \( C_1, C_2, C_3, \sigma_k \) and \( \sigma_\varepsilon \) are further empirical coefficients. The
values employed in the present study for the set of empirical constants appearing in equations (2.90), (2.95) and (2.96) are given in Table 2.2 (Launder & Spalding [1974]).

**TABLE 2.2**

Values of the Constants Used in the $k$-$\epsilon$ Model

<table>
<thead>
<tr>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_{\mu}$</th>
<th>$\sigma_k$</th>
<th>$\sigma_{\epsilon}$</th>
<th>$\kappa$</th>
<th>$E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.44</td>
<td>1.92</td>
<td>1.0</td>
<td>0.09</td>
<td>1.0</td>
<td>1.22</td>
<td>0.4187</td>
<td>9.0</td>
</tr>
</tbody>
</table>

The effective viscosity $\mu$ and diffusivity $\Gamma_\phi$ appearing in the above equations are defined as:

$$\mu = \mu_t + \mu_m$$

(2.97)

and

$$\Gamma_\phi = \Gamma_\phi_t + \Gamma_\phi_m$$

where the subscript $m$ refers to molecular values.

The rate of generation of turbulence energy $G$ is given by

$$G = \frac{\tau_{ij}}{\tau} : \text{grad} \, \bar{v}$$

(2.98)

and may alternatively be written as

$$G = g_{(ik)} \tau_{ij} \bar{v}_j \bar{v}_k$$

(2.99)

by employing the definitions (2.31) and (2.50). The stress tensor $\tau_{ij}$
is represented by an expression similar to (2.70). Its components, however, are given by:

\[ T_{ij}^{(t)} = \nu_{ij}^{(t)} [g^{(jm)} v^{(m)} \vec{v}^{(i)} + g^{(im)} v^{(m)} \vec{v}^{(j)}] \]

\[ - \frac{2}{3} \left[ \rho k + \nu_{ij} \frac{\Delta \vec{v}^{(m)}}{\Delta x^{(m)}} \right] g^{(ij)} \quad (2.100) \]

From the above discussion, it follows that the forms of the governing equations (2.83) remain valid in the case of turbulent flow, provided that the dependent variables are taken to be the time-averaged quantities and that \( \nu \) and \( \Gamma_{\phi} \) are the effective exchange coefficients defined by equations (2.97). The set of equations (2.83), together with equations (2.90), (2.95) and (2.96), thus represent a closed system that can be solved numerically.

The entire set of transport equations, just assembled, may be written in the general form:

\[ \frac{\partial}{\partial x} \left[ \rho \psi \right] + \frac{\Delta}{\Delta x^{(j)}} \left[ \rho \psi v^{(j)} - \Gamma_{\psi} g^{(jm)} \frac{\partial \psi}{\partial x^{(m)}} \right] = S_{\psi} \quad (2.101) \]

where \( \psi \) stands for the dependent variable under consideration. The definitions for the exchange coefficient \( \Gamma_{\psi} \) and the source term \( S_{\psi} \) pertinent to particular variables are given in Table 2.3.

2.5 CONCLUDING REMARKS

The contravariant physical vector and tensor components in relation to a normalised natural basis are selected in accordance with the recommendations of Demirdzic [1982] for the representation of vectors and tensors involved in the governing equations.

The semi-strong conservation-law form of the governing equations is
### TABLE 2.3

Definitions for the Exchange Coefficients and Source Terms of the General Transport Equation (2.101)

<table>
<thead>
<tr>
<th>$\psi$</th>
<th>$\Gamma_\psi$</th>
<th>$S_\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$</td>
<td>0</td>
<td>$S_m$</td>
</tr>
</tbody>
</table>
| $\nu(\ell)$ | $\mu$ | \[
\frac{\Delta}{\Delta x(j)} (\tau[ij] - g[jm] \frac{\partial u(\ell)}{\partial x(m)}) - g[ij] \frac{\partial p}{\partial x(j)} + S_u(\ell) \\
- \left[ \nu[m] \nu[j] - \tau[mj] \right]
\]
| $\phi$ | $\Gamma_\phi$ | $S_\phi$ |
| $k$   | $\frac{\nu}{\sigma_k}$ | $G - \rho \varepsilon$ |
| $\varepsilon$ | $\frac{\nu}{\sigma_\varepsilon}$ | $C_1 \frac{\varepsilon}{k} G - C_2 \rho \varepsilon^2 + C_3 \rho \varepsilon \frac{\Delta v(m)}{\Delta x(m)}$ |

\[
\tau[ij] = \nu \left[ g[jm] \nu[m] \nu[\ell] + g[ijm] \nu[m] \nu[j] \right]
\]

\[
P = p + \frac{2}{3} (\rho k + \nu \frac{\Delta v(m)}{\Delta x(m)})
\]

\[
G = g[ijk] \left[ \frac{\tau[ij]}{\lambda} - \frac{2}{3} g[ij] \left( \rho k + \nu \frac{\Delta v(m)}{\Delta x(m)} \right) \nu[j] \nu[k] \right]
\]
selected for the present study. This form is obtained by resolving the momentum equation in the non-orthogonal coordinate directions. The physical components of vectors and tensors are taken to be defined with respect to a locally fixed basis.

For applications to turbulent flows, an evaluation of turbulence modelling shows that higher level models, such as Reynolds stress models and large-eddy simulation, though conceptually universal and potentially superior to two-equation models, are computationally expensive and still in the development stage. The $k-\varepsilon$ model was thus chosen to close the system of time-averaged conservation equations, and the transport equations for $k$ and $\varepsilon$ are presented in their coordinate-free forms.

Finally, a general form of the transport equations is given for subsequent use in the presentation of the procedure of numerical solution in Chapter 3.
CHAPTER 3
NUMERICAL SOLUTION

3.1 INTRODUCTION

In this chapter, a method is described for the numerical solution of the governing equations presented in the preceding chapter.

In Section 3.2, the partial differential governing equations for the case of two-dimensional steady incompressible flow are written in an expanded form suitable for discretisation.

The grid arrangement is introduced in Section 3.3. The discretised equations are then obtained by integrating the governing equations over their respective control volumes.

In Section 3.4, some differencing schemes are discussed from the point of view of numerical stability, accuracy and economy. A choice is accordingly made of the differencing scheme to be used in the present study.

Section 3.5 examines some of the features of the discretised equations with attention being focused on the cross-derivative diffusion terms arising from the grid non-orthogonality. Also discussed are the discretisation of the source terms and the interpolation practices adopted for calculating different variables at different locations.

In Section 3.6, the general form of the discretised equations is presented. The procedure that links momentum and continuity equations through the pressure correction equation is then described, followed by an outline of the iterative method used for solving the discretised equations.

In Section 3.7, the treatment of the boundary conditions is discussed.

Finally, the geometry of the computational grid is dealt with in Section 3.8. A method for the grid generation is first presented,
followed by a discussion of the local coordinate transformation and the computation of the geometrical parameters involved in the governing and discretised equations. The bulk of the material in this chapter, it should be noted, follows closely the work of Demirdzic [1982].

3.2 EXPANDED FORM OF THE GOVERNING EQUATIONS

The governing equations derived in Chapter 2 are given here in their expanded forms for subsequent use in the presentation of the method of solution. The equations are written for the case of steady two-dimensional flow in general Eulerian frame of references. Allowance is made for a flow in the $x^3$ direction but must be either plane fully developed, i.e. $\partial p/\partial x^3$ is independent of $x^1$ and $x^2$, or axially symmetric ($\partial p/\partial x^3 = 0$).

In terms of the definitions given in Table 2.3, equations (2.101) may be written as:

(a) **Continuity Equation**

$$\frac{\Delta}{\Delta x^{[1]}} \left[ \rho \upsilon^{(1)} \right] + \frac{\Delta}{\Delta x^{[2]}} \left[ \rho \upsilon^{(2)} \right] = 0$$  \hfill (3.1)

(b) **Momentum Equation**

(i) $x^1$-direction:

$$\frac{\Delta}{\Delta x^{[1]}} \left[ \rho \upsilon^{(1)} \upsilon^{(1)} \right] + \frac{\Delta}{\Delta x^{[2]}} \left[ \rho \upsilon^{(2)} \upsilon^{(1)} \right] - \frac{\Delta}{\Delta x^{[1]}} \left[ \mu \upsilon \frac{\partial \upsilon^{(1)}}{\partial x^{[1]}} \right]$$

$$- \frac{\Delta}{\Delta x^{[1]}} \left[ \mu \upsilon^{(2)} \frac{\partial \upsilon^{(1)}}{\partial x^{[2]}} \right] - \frac{\Delta}{\Delta x^{[1]}} \left[ \mu \upsilon^{(1)} \frac{\partial \upsilon^{(1)}}{\partial x^{[2]}} \right]$$

$$- \frac{\Delta}{\Delta x^{[2]}} \left[ \mu \upsilon^{(2)} \frac{\partial \upsilon^{(1)}}{\partial x^{[1]}} \right] = - g^{(11)} \frac{\partial p}{\partial x^{[1]}} - g^{(12)} \frac{\partial p}{\partial x^{[2]}}$$  \hfill (contd.)
\[
\frac{\Delta}{\Delta x(1)} \left[ \tau^{(11)} - \mu \left( g^{(11)} \frac{\partial u^{(1)}}{\partial x(1)} + g^{(12)} \frac{\partial u^{(1)}}{\partial x(2)} \right) \right] + \frac{\Delta}{\Delta x(2)} \left[ \tau^{(12)} - \mu \left( g^{(21)} \frac{\partial u^{(1)}}{\partial x(1)} + g^{(22)} \frac{\partial u^{(1)}}{\partial x(2)} \right) \right] \\
- \left\{ \begin{array}{l}
(1 \ 1) \ (\rho \ v^{(1)} \ v^{(1)} - \tau^{(11)}) - (1 \ 2) \ (\rho \ v^{(1)} \ v^{(2)} - \tau^{(12)}) \\
(2 \ 1) \ (\rho \ v^{(2)} \ v^{(1)} - \tau^{(21)}) - (2 \ 2) \ (\rho \ v^{(2)} \ v^{(2)} - \tau^{(22)}) \\
(3 \ 3) \ (\rho \ v^{(3)} \ v^{(3)} - \tau^{(33)})
\end{array} \right. 
\]

(3.2a)

\[(11) \ x^2\text{-direction:}\]

\[
\frac{\Delta}{\Delta x(1)} \left[ \rho \ v^{(1)} \ v^{(2)} \right] + \frac{\Delta}{\Delta x(2)} \left[ \rho \ v^{(2)} \ v^{(2)} \right] - \frac{\Delta}{\Delta x(1)} \left[ \mu \ g^{(11)} \frac{\partial v^{(2)}}{\partial x(1)} \right] \\
- \frac{\Delta}{\Delta x(2)} \left[ \mu \ g^{(22)} \frac{\partial v^{(2)}}{\partial x(2)} \right] - \frac{\Delta}{\Delta x(1)} \left[ \mu \ g^{(12)} \frac{\partial v^{(2)}}{\partial x(2)} \right] \\
- \frac{\Delta}{\Delta x(2)} \left[ \mu \ g^{(21)} \frac{\partial v^{(2)}}{\partial x(1)} \right] = - g^{(22)} \frac{\partial p}{\partial x(2)} - g^{(21)} \frac{\partial p}{\partial x(1)} \\
+ \frac{\Delta}{\Delta x(1)} \left[ \tau^{(21)} - \mu \left( g^{(11)} \frac{\partial v^{(2)}}{\partial x(1)} + g^{(12)} \frac{\partial v^{(2)}}{\partial x(2)} \right) \right] \\
+ \frac{\Delta}{\Delta x(2)} \left[ \tau^{(22)} - \mu \left( g^{(21)} \frac{\partial v^{(2)}}{\partial x(1)} + g^{(22)} \frac{\partial v^{(2)}}{\partial x(2)} \right) \right] \\
- \left\{ \begin{array}{l}
(1 \ 1) \ (\rho \ v^{(1)} \ v^{(1)} - \tau^{(11)}) - (1 \ 2) \ (\rho \ v^{(1)} \ v^{(2)} - \tau^{(12)}) \\
(2 \ 1) \ (\rho \ v^{(2)} \ v^{(1)} - \tau^{(21)}) - (2 \ 2) \ (\rho \ v^{(2)} \ v^{(2)} - \tau^{(22)}) \\
(3 \ 3) \ (\rho \ v^{(3)} \ v^{(3)} - \tau^{(33)})
\end{array} \right. 
\]

(3.2b)
(iii) \( x^3 \)-direction:

\[
\begin{align*}
\frac{\Delta}{\Delta x(1)} (p v(1) v(3)) + \frac{\Delta}{\Delta x(2)} (p v(2) v(3)) - \frac{\Delta}{\Delta x(1)} (\mu g(11) \frac{\partial v(3)}{\partial x(1)}) \\
- \frac{\Delta}{\Delta x(2)} (\mu g(22) \frac{\partial v(3)}{\partial x(2)}) - \frac{\Delta}{\Delta x(1)} (\mu g(12) \frac{\partial v(3)}{\partial x(2)}) \\
- \frac{\Delta}{\Delta x(2)} (\mu g(21) \frac{\partial v(3)}{\partial x(1)}) = \frac{\Delta}{\Delta x(1)} \left[ \tau(31) - \mu \left( g(11) \frac{\partial v(3)}{\partial x(1)} + g(12) \frac{\partial v(3)}{\partial x(2)} \right) \right] \\
+ \frac{\Delta}{\Delta x(2)} \left[ \tau(32) - \mu \left( g(21) \frac{\partial v(3)}{\partial x(1)} + g(22) \frac{\partial v(3)}{\partial x(2)} \right) \right] \\
- \left( \frac{\partial^3}{\partial x(1)^3} \right) (p v(1) v(3) - \tau(13)) - \left( \frac{\partial^3}{\partial x(1)^3} \right) (p v(2) v(3) - \tau(23)) \quad (3.2c)
\end{align*}
\]

(c) **Turbulence Kinetic Energy Equation**

\[
\begin{align*}
\frac{\Delta}{\Delta x(1)} (p v(1) k) + \frac{\Delta}{\Delta x(2)} (p v(2) k) - \frac{\Delta}{\Delta x(1)} \left( \frac{\mu}{\sigma_k} g(11) \frac{\partial k}{\partial x(1)} \right) \\
- \frac{\Delta}{\Delta x(2)} \left( \frac{\mu}{\sigma_k} g(22) \frac{\partial k}{\partial x(2)} \right) - \frac{\Delta}{\Delta x(1)} \left( \frac{\mu}{\sigma_k} g(12) \frac{\partial k}{\partial x(2)} \right) \\
- \frac{\Delta}{\Delta x(2)} \left( \frac{\mu}{\sigma_k} g(21) \frac{\partial k}{\partial x(1)} \right) = G - \rho \varepsilon \quad (3.3)
\end{align*}
\]

(d) **Dissipation Rate of Turbulence Energy Equation**

\[
\begin{align*}
\frac{\Delta}{\Delta x(1)} (p v(1) \varepsilon) + \frac{\Delta}{\Delta x(2)} (p v(2) \varepsilon) - \frac{\Delta}{\Delta x(1)} \left( \frac{\mu}{\sigma_\varepsilon} g(11) \frac{\partial \varepsilon}{\partial x(1)} \right) \\
- \frac{\Delta}{\Delta x(2)} \left( \frac{\mu}{\sigma_\varepsilon} g(22) \frac{\partial \varepsilon}{\partial x(2)} \right) - \frac{\Delta}{\Delta x(1)} \left( \frac{\mu}{\sigma_\varepsilon} g(12) \frac{\partial \varepsilon}{\partial x(2)} \right) \\
- \frac{\Delta}{\Delta x(2)} \left( \frac{\mu}{\sigma_\varepsilon} g(21) \frac{\partial \varepsilon}{\partial x(1)} \right) = C_1 \frac{\varepsilon}{k} G - C_2 \rho \frac{\varepsilon^2}{k} \quad (contd.)
\end{align*}
\]
The components of the isotropic and anisotropic parts of the stress tensor are given by:

\[ P = \rho + \frac{2}{3} \left[ \rho \left( k + \mu \left( \frac{\Delta u^{(1)}}{\Delta x^{(1)}} + \frac{\Delta u^{(2)}}{\Delta x^{(2)}} \right) \right) \right] \]  

(3.5a)

\[ \tau^{(11)} = 2\mu \left( g^{(11)} v^{(1)} v^{(1)} + g^{(12)} v^{(2)} v^{(1)} \right) \]

\[ \tau^{(12)} = \tau^{(21)} = \mu \left[ g^{(11)} v^{(1)} v^{(2)} + g^{(22)} v^{(2)} v^{(1)} + g^{(12)} v^{(2)} v^{(2)} + g^{(21)} v^{(1)} v^{(1)} \right] \]

(3.5b)

\[ \tau^{(13)} = \tau^{(31)} = \mu \left[ g^{(11)} v^{(1)} v^{(3)} + g^{(22)} v^{(2)} v^{(2)} + g^{(33)} v^{(3)} v^{(1)} \right] \]

\[ \tau^{(22)} = 2\mu \left[ g^{(22)} v^{(2)} v^{(2)} + g^{(21)} v^{(1)} v^{(2)} \right] \]

\[ \tau^{(23)} = \tau^{(32)} = \mu \left[ g^{(22)} v^{(2)} v^{(3)} + g^{(21)} v^{(1)} v^{(3)} \right] \]

\[ \tau^{(33)} = 2\mu g^{(33)} v^{(3)} v^{(3)} \]

The rate of generation of turbulent kinetic energy \( G \) is given by:

\[ + C_3 \rho \varepsilon \left( \frac{\Delta u^{(1)}}{\Delta x^{(1)}} + \frac{\Delta u^{(2)}}{\Delta x^{(2)}} \right) \]  

(3.4)
\[ G = \sum_{x} \left[ (v_{(1)}^1 + v_{(2)}^2)^2 + (v_{(1)}^2 + v_{(2)}^1)^2 \right. \]
\[ + \frac{g_{(11)} g_{(22)} g_{(33)}}{g} (v_{(1)}^1 - v_{(2)}^1)^2 \]
\[ + g_{(12)} (v_{(1)}^1 - v_{(2)}^2)^2 + 2 (v_{(3)}^1)^2 \]
\[ + (v_{(1)}^3 + v_{(3)}^1)^2 + g_{(12)} v_{(3)}^1 v_{(2)}^2 \]
\[ + \frac{g_{(11)} g_{(22)} g_{(33)}}{g} (v_{(2)}^1)^2 + \frac{g}{g_{(11)} g_{(22)} g_{(33)}} v_{(3)}^1 v_{(2)}^2 \]
\[ - g_{(12)} v_{(1)}^2 (v_{(3)}^1)^2 \]
\[ - \frac{2}{3} \left[ \rho k + \mu \left( \frac{\Delta u_{(1)}}{\Delta x_{(1)}} + \frac{\Delta u_{(2)}}{\Delta x_{(2)}} \right) \right] \]
\[ \frac{\Delta u_{(1)}}{\Delta x_{(1)}} + \frac{\Delta u_{(2)}}{\Delta x_{(2)}} \] (3.6)

where the covariant derivatives are given by:

\[ v_{(1)}^1 = \frac{\partial v_{(1)}}{\partial x_{(1)}} + v_{(1)}^1, \]
\[ v_{(1)}^2 = \frac{\partial v_{(2)}}{\partial x_{(1)}} + v_{(1)}^2, \]
\[ v_{(1)}^3 = \frac{\partial v_{(3)}}{\partial x_{(1)}} \] (3.7)

\[ v_{(2)}^1 = \frac{\partial v_{(1)}}{\partial x_{(2)}} + v_{(1)}^1, \]
\[ v_{(2)}^2 = \frac{\partial v_{(2)}}{\partial x_{(2)}} + v_{(2)}^2, \]
\[ v_{(2)}^3 = \frac{\partial v_{(3)}}{\partial x_{(2)}} \]

(contd.)
\[ \nabla_{(3)} v^{(1)} = v^{(3)} \begin{pmatrix} l_{3}^{1} \end{pmatrix} \]

\[ \nabla_{(3)} v^{(2)} = v^{(3)} \begin{pmatrix} l_{3}^{2} \end{pmatrix} \]

\[ \nabla_{(3)} v^{(3)} = v^{(1)} \begin{pmatrix} l_{1}^{3} \end{pmatrix} + v^{(2)} \begin{pmatrix} l_{2}^{3} \end{pmatrix} \] (3.7)

### 3.3 DISCRETISATION PROCEDURE

The discretised equations are driven using the finite-volume formulation method. For this purpose, the solution domain is divided into contiguous quadrilateral curvilinear control volumes or cells with the cell faces coinciding with the coordinate surfaces. These are called main or 'scalar' control volumes where at their centres* the scalar variables, such as pressure, density, etc., are located. For the momentum components, the staggered cell arrangement of Figure 3.1 is used. The velocity components \( v^{(1)} \) and \( v^{(2)} \) are displaced in the \( x^1 \) and \( x^2 \) directions, respectively, to lie in the middle of the scalar cell faces. The velocity cells, therefore, have two faces coinciding with scalar cell faces and the other two are formed by the coordinate surfaces passing through the nodes of scalar cells. For a uniformly spaced grid, the velocity components are located at the centres of their respective cells.

The staggered arrangement was first used by Harlow & Welch [1965] to avoid the possibility of decoupling between adjacent velocities and pressures (see Patankar [1980]). A further advantage of this

* The definitions of the various geometrical aspects and parameters of the computational grid are given in Section 3.8.
arrangement lies in the fact that the velocity components are located at the scalar cell faces where the convective fluxes are calculated. Moreover, the pressure, and other scalars, are located at the velocity cell faces where the gradients of pressure and other scalars are needed.

At the boundaries of the solution domain, the cell arrangement is modified in such a way that, Figure 3.1,

(i) a row of scalar nodes is introduced whose locations coincide with those of the velocity components crossing the boundary,
(ii) a row of boundary tangential velocity components is introduced located at the corners of the scalar cells.
(iii) There exists a half cell pertaining to the velocity component crossing the boundary for which a momentum balance is not made (Vasilic-Melling [1977]).

Compass notation is used to indicate the locations of the neighbouring points associated with a typical cell with respect to the central node P, Figure 3.2. For velocity cells, similar labelling is used. The discretised equations will be represented in relation to the 9-point 'computational molecule' of Figure 3.2.

The steady-state form of the transport equations (2.101), when integrated over the volume of the cell containing the variable in question, becomes

\[ \int_V \nabla \cdot (\rho \mathbf{v}^j \psi - \Gamma_\psi g^{jm} \frac{\partial \psi}{\partial x^m}) \, dv = \int_V S_\psi \, dv \quad (3.8) \]

Applying Gauss's Divergence theorem, and for the cell illustrated in Figure 3.2, equation (3.8) may alternatively be written as,
where the summation is over the four faces of the cell and the subscript \( n_i \) denotes the flux component normal to the respective cell face.

The left-hand side of equation (3.9) represents the integral over the cell faces of the convective and diffusive fluxes \( F \). This may be decomposed into

(a) \[ F_1 = \sum_{i} \int_{A_i} \left( \rho \, v_i \psi - \Gamma_\psi \, g^{(ij)} \frac{\partial \psi}{\partial x(m)} \right) \, n_i \, dA_i \] (3.10a)

and,

(b) \[ F_2 = \sum_{i} \int_{A_i} \left( \Gamma_\psi \, g^{(jm)} \frac{\partial \psi}{\partial x(m)} \right) \, n_i \, dA_i \] (3.10b)

The component \( F_1 \) represents the sum of the normal convective and diffusive fluxes, while \( F_2 \) represents the cross-derivative diffusion flux resulting from the grid non-orthogonality. Using the Mean-Value theorem, one may approximate \( F_1 \) by the following expression,

\[
F_1 = \left[ \rho \, v_{11} \psi - g^{(11)} \Gamma_\psi \frac{\psi_{E} - \psi_{W}}{\delta x(1)} \right]_e (A \sin \alpha)_e \\
- \left[ \rho \, v_{11} \psi - g^{(11)} \Gamma_\psi \frac{\psi_{P} - \psi_{W}}{\delta x(1)} \right]_w (A \sin \alpha)_w \\
+ \left[ \rho \, v_{22} \psi - g^{(22)} \Gamma_\psi \frac{\psi_{N} - \psi_{P}}{\delta x(2)} \right]_n (A \sin \alpha)_n \\
- \left[ \rho \, v_{22} \psi - g^{(22)} \Gamma_\psi \frac{\psi_{P} - \psi_{S}}{\delta x(2)} \right]_s (A \sin \alpha)_s 
\] (3.11)

where subscripts \( e, w, n, \) and \( s \) denote average values over the
corresponding cell face. Thus, for example, the value of $\psi_e$ is evaluated as a weighted average of its values at the six surrounding locations, i.e.

$$
\psi_e = \alpha_e \left[ \beta_e \psi_P + (1 - \beta_e) \psi_S \right] \gamma_e 
+ \left[ \beta_e \psi_P + (1 - \beta_e) \psi_N \right] (1 - \gamma_e) 
+ (1 - \alpha_e) \left[ \beta_e \psi_E + (1 - \beta_e) \psi_{NE} \right] \gamma_e 
+ \left[ \beta_e \psi_E + (1 - \beta_e) \psi_{SE} \right] (1 - \gamma_e) 
\tag{3.12}
$$

where $\alpha$, $\beta$ and $\gamma$ are spatial weighting factors which will be discussed later. These factors have to be associated with the cell faces rather than the nodes in order to ensure continuity of fluxes and, consequently, conservation of physically conserved quantities.

The $F_2$ component of equation (3.10) may be approximated by

$$
F_2 = (g^{12}) \Gamma_\psi \frac{A \sin \alpha}{\delta x^{(2)}} \psi_e \left( \psi_{ne} - \psi_{se} \right) 
- (g^{12}) \Gamma_\psi \frac{A \sin \alpha}{\delta x^{(2)}} \omega \left( \psi_{nw} - \psi_{sw} \right) 
+ (g^{21}) \Gamma_\psi \frac{A \sin \alpha}{\delta x^{(1)}} n \left( \psi_{ne} - \psi_{nw} \right) 
- (g^{21}) \Gamma_\psi \frac{A \sin \alpha}{\delta x^{(1)}} \delta \left( \psi_{se} - \psi_{sw} \right) 
\tag{3.13}
$$

where $\psi_{ne}$, $\psi_{nw}$, $\psi_{se}$ and $\psi_{sw}$ are weighted sums of the values of $\psi$ at the four surrounding nodes, e.g.
\[ \psi_{ne} = [\delta_{1P} \psi_{NE} + (1 - \delta_{1P}) \psi_{N}] \delta_{2P} \]

\[ + [\delta_{1P} \psi_{E} + (1 - \delta_{1P}) \psi_{P}] (1 - \delta_{2P}) \]  

(3.14)

The spatial interpolation factors \( \delta_{1P} \) and \( \delta_{2P} \) are defined with respect to the cell node \( P \) in Figure 3.2 as follows,

\[ \delta_{1P} = \frac{(\delta x)^{[1]}_{PE}}{(\delta x)^{[1]}_{PE}} \]  

(3.15)

\[ \delta_{2P} = \frac{(\delta x)^{[2]}_{PN}}{(\delta x)^{[2]}_{PN}} \]

The treatment of the source \( S_{\psi} \) in equation (3.8) depends on the variable in question and the associated source terms. For the present, this will simply be integrated into

\[ \int_{V} S_{\psi} \, dV = \delta_{\psi} \]  

(3.16)

Based on the approximations represented by equations (3.11), (3.12) and (3.16), the discretised form of equation (3.7) may be written in the compact form,

\[ a_{P} \psi_{P} = \sum_{K} a_{K} \psi_{K} + \delta_{\psi} + \delta_{m} \]  

(3.17)

where \( K \) denotes summation over the eight points surrounding \( P \), i.e. \( K \in E, \omega, N, S, NE, NW, SE \) and \( SW \). The corresponding coefficients are given by
\[ a_E = [D_e - (1 - \alpha_e) \{1 - \beta_e\} C_e] \]
\[ + \{ (1 - \alpha_d) \beta_d \gamma_d C_d - \alpha_n \beta_n (1 - \gamma_n) C_n \} \]
\[ + \{ \delta_1p \{1 - \delta_2p\} (E_e + E_n) - \delta_1p \delta_2s \{E_e + E_d\}\} \]

\[ a_w = [D_w + \alpha_w (1 - \beta_w) C_w] + \{ (1 - \alpha_d) \beta_d (1 - \gamma_d) C_d - \alpha_n \beta_n \gamma_n C_n \} \]
\[ + \{ (1 - \delta_1w) \delta_2s \{E_w + E_d\} - (1 - \delta_1w) \{1 - \delta_2p\} \{E_w + E_n\}\} \]

\[ a_N = [D_n - (1 - \alpha_n) \{1 - \beta_n\} C_n] \]
\[ + \{ (1 - \alpha_w) \beta_w \gamma_w C_w - \alpha_e \beta_e (1 - \gamma_e) C_e \} \]
\[ + \{ \delta_2p \{1 - \delta_1p\} \{E_n + E_e\} - \delta_2p \delta_1w \{E_n + E_w\}\} \]

\[ a_S = [D_d + \alpha_d (1 - \beta_d) C_d] + \{ (1 - \alpha_w) \beta_w (1 - \gamma_w) C_w - \alpha_e \beta_e \gamma_e C_e \} \]
\[ + \{ (1 - \delta_2s) \delta_1w \{E_d + E_w\} - (1 - \delta_2s) \{1 - \delta_1p\} \{E_d + E_e\}\} \]

\[ a_{NE} = [- \{1 - \alpha_n\} \beta_n \gamma_n C_n - (1 - \alpha_e) \beta_e \gamma_e C_e]\]
\[ + \{ \delta_1p \delta_2p \{E_n + E_e\}\} \]

\[ a_{NW} = [- \{1 - \alpha_n\} \beta_n (1 - \gamma_n) C_n + \alpha_w \beta_w (1 - \gamma_w) C_w] \]
\[ - \{ (1 - \delta_1w) \delta_2p \{E_n + E_w\}\} \]

\[ a_{SE} = [\alpha_d \beta_d (1 - \gamma_d) C_d - (1 - \alpha_e) \beta_e (1 - \gamma_e) C_e] \]

(contd.)
Here, the coefficients of convection $C_\ell$ and diffusion $D_\ell$ and $E_\ell$ are defined, respectively, by

\[ C_\ell = \{p v \{f\} A \sin a\} \ell \]

\[ D_\ell = \{g \{f\} r \frac{A \sin a}{\delta x \{f\}} \ell \] \hspace{1cm} \text{(3.19)}

\[ E_\ell = \{g \{m\} r \frac{A \sin a}{\delta x \{m\}} \ell \]

where $\ell$ stands for the respective cell faces $e$, $w$, $n$ and $\delta$.

The choice of the spatial weighting factors $a_\ell$, $\beta_\ell$, and $\gamma_\ell$ is important for the accuracy and stability of the solution procedure. It is useful, therefore, to discuss the structure of the coefficients of equations (3.18) from this point of view.

The principal coefficients $a_E$, $a_W$, $a_N$ and $a_S$ may be decomposed into three distinct parts as in the following example.

\[ a_E^I = \{D_e - (1 - a_e) \{1 - \beta_e\} C_e\} \] \hspace{1cm} \text{(3.20a)}

\[ a_E^{II} = \{(1 - a_\delta) \beta_\delta \gamma_\delta C_\delta - a_n \beta_n (1 - \gamma_n) C_n\} \] \hspace{1cm} \text{(3.20b)}
\[ a^E_{III} = [\delta_1 p (1 - \delta_2 p) (E_e + E_n) - \delta_1 p \delta_2 s (E_e + E_s)] \]  \hspace{1cm} (3.20c)

I - this is the main part comprising convective and diffusive flux approximations across the corresponding cell face only (i.e. the east for \( a^E \) coefficient);

II - this part consists of the contributions from the convective fluxes across the intersecting faces (i.e. the north and south faces for the example above). For \( \beta_4 = 0 \), this part vanishes;

III - this part arises from the approximation of the cross-derivative diffusion and consequently vanishes for orthogonal grids.

The corner coefficients \( a_{NE}, a_{SE}, a_{NW}, a_{SW} \), which are usually less important than the principal coefficients, contain only the latter two parts. In circumstances, however, involving a highly non-orthogonal grid or large non-alignment of grid lines with flow streamlines, they can play an important rôle.

3.4 CHOICE OF THE DIFFERENCING SCHEME

In the following, a discussion of the more popular differencing schemes is presented. It is to be noted, however, that all these schemes have been defined and employed mainly in conjunction with orthogonal coordinate systems.

Until recently, differencing schemes referring to five-point
computational molecules* have been mainly used. In these schemes, ψ is evaluated at the cell faces as weighted average of the immediately nearest nodes only, e.g.

\[ \psi_c = \alpha_c \psi_P + (1 - \alpha_c) \psi_E \] (3.21)

The assumption of a piecewise-linear profile of \( \psi \) leads to the 'Central Differencing' Scheme. If the cell faces are assumed to lie mid-way between \( P \) and the respective principal nodes, then

\[ \alpha_i = \frac{1}{4} \quad (i = e, w, n, s) \] (3.22)

This formulation gives the highest formal accuracy as measured by Taylor Series truncation error (second order) for the case of two-point schemes. Nevertheless, it leads to unrealistic results when the local Peclet number, defined by

\[ Pe_i = \frac{C_i}{D_i} \quad (i = e, w, n, s) \] (3.23)

exceeds two in magnitude (Patankar [1980]). This can be explained by the fact that for \( |Pe_i| > 2 \), even the main part of the coefficient (3.20) may become negative which implies that the Scarborough interim (Scarborough [1958]), defined as

\[ \frac{\sum |a_M|}{\sum |a_P|} \begin{cases} \leq 1 & \text{at all nodes} \\ < 1 & \text{at one node at least} \end{cases} \] (3.24)

* This implies that \( \beta_i = 1 \) and, hence, the definition of \( \gamma_i \) becomes irrelevant as follows from equation (3.12).
is not satisfied and that the iterative process may fail to produce a solution. This is the main reason for the central differencing scheme to be limited to low Reynolds number problems.

A scheme which does not suffer from this problem is 'Upwind Differencing', first proposed by Courant, et al. [1952]. The scheme ensures that the convection contributions cannot produce negative coefficients by defining $a_i$ such that

$$a_i = \begin{cases} 1 & Pe_i > 0 \\ 0 & Pe_i < 0 \end{cases} \quad (3.25a)$$

For small Peclet numbers, however, a one-dimensional analysis suggests that the upwind differencing scheme is less accurate than the central differencing one.

In an attempt to combine the advantages of both schemes, Spalding [1972] proposed the 'Hybrid Differencing' scheme. The coefficients $a_i$ are defined here by

$$a_i = \begin{cases} 1 & |Pe_i| < 2 \\ 1 & Pe_i > 2 \\ 0 & Pe_i < -2 \end{cases} \quad (3.25b)$$

The comparison between the levels of accuracy associated with the above schemes on basis of the Taylor series expansion is valid only for an extremely fine grid, or, in other words, for a very small value of the Peclet number. Under this condition, and on the grounds of the Taylor series analysis, it may be shown that the central differencing scheme is of second order accuracy, while the upwind and hybrid schemes are only first order accurate (Patankar [1980]). In most practical problems, large values of the Peclet number occur. Consequently,
Taylor series analysis becomes a misleading argument since higher order Taylor series approximations do not necessarily mean higher levels of accuracy (Raithby [1976a] and Stubley, et al. [1980]). In such a case, the upwind differencing scheme is more accurate than the central differencing one.

The above argument suggests the use of the upwind differencing scheme in one-dimensional flow problems. In multi-dimensional problems, however, smearing of the profiles is observed which is attributed to the so-called 'false' or 'numerical' diffusion originating from the fact that interpolation is performed along the grid lines rather than the streamlines. The question of false diffusion attains importance only in the case of large Peclet numbers since, at small Peclet numbers, the real diffusion is relatively large.

A scheme that suffers less from numerical diffusion, called 'Skew Upwind Differencing' (SUD), has been proposed by Raithby [1976b]. This employs a nine-point computational molecule and retains all the advantages of the upwind differencing scheme, except that of unconditionally positive coefficients. The false diffusion is alleviated by taking into account the direction of the velocity vector while evaluating \( \psi \) at the cell faces. For example, the spatial weighing factors of equation (3.12) are given by, see Figure 3.3(a),

\[
\alpha_e = \begin{cases} 
1 & \left( \frac{v_e}{v_1} \right) > 0 \\
1 + \frac{|v_e^{(1)}|}{v_1} & \left( \frac{v_e}{v_1} \right) < 0
\end{cases} \\
\beta_e = 1 - \min \left[ 1, \frac{v_2}{v_1} \frac{\delta x_1}{\delta x_2} \right] P_e, \text{ for } v_1, v_2 > 0 \\
\gamma_e = \begin{cases} 
1 & \left( \frac{v_e}{v_2} \right) > 0 \\
1 + \frac{|v_e^{(2)}|}{v_2} & \left( \frac{v_e}{v_2} \right) < 0
\end{cases}
\]

(3.26)
Some other nine-point schemes that suffer less from false diffusion have been proposed (Leonard [1979], Stubley, et al. [1980], and Gosman & Lai [1982]).

All nine-point schemes, except that of Gosman & Lai [1982], suffer from the possibility of producing negative coefficients, even when applied to orthogonal grids. This, in turn, may lead to unbounded solutions, a case which makes these schemes less stable than five-point molecules (Han, et al. [1981]). Moreover, a nine-point molecule scheme is significantly more complicated, especially in the case of non-orthogonal grids, for which they have not been fully tested.

For the above reasons, and since the flexibility of non-orthogonal grids usually allows for fairly good alignment of the grid lines with the main flow direction, the upwind differencing scheme is employed in the present study.

3.5 THE DISCRETISED EQUATION

The choice made in the previous section implies that the only remaining source of negative coefficients in the discretised equation is the cross-derivative diffusion terms. The treatment of these terms is discussed in the following. The discretisation of the source terms and the interpolation practices are also discussed. The evaluation of the geometrical quantities appearing in these terms is given later in Section 3.8.

3.5.1 Cross-Derivative Diffusion Terms

The presence of the contributions due to $E_\perp$ in part III of equation (3.20), in addition to making the coefficient matrix nine-diagonal, allows for the possibility of negative coefficients. In an attempt to reduce this possibility, Demirdzic [1982] introduced an
interpolation formula for evaluating $\psi$ which produced unconditionally positive corner coefficients. The principal coefficients, on the other hand, are positive if a criterion of the form

$$|\cos \alpha| \max \left[ \frac{\delta x^{(1)}}{\delta x^{(2)}}, \frac{\delta x^{(2)}}{\delta x^{(1)}} \right] < 1$$

is satisfied, where $\alpha$ is the angle between the grid lines, and $\delta x^{(1)}$ and $\delta x^{(2)}$ are the cell dimensions.

Equations (3.13) to (3.15) show that the cross-derivative diffusion terms are proportional to the product of spatial interpolation factors $\delta$ and the diffusion coefficient $E$. These coefficients are usually relatively small and vanish for orthogonal grids. For example, equation (3.19) (through equation (3.97)) shows that

$$\frac{E}{\delta} = - \left[ \cos \alpha \left( \frac{\delta x^{(1)}}{\delta x^{(2)}} \right) \right]$$

Therefore, the cross-derivative diffusion terms are taken into account 'implicitly' by absorbing them into the source term $S$. This approach reduces the coefficient matrix to five-diagonal with unconditionally positive coefficients.

The discretised equation may thus be written in the general form

$$a_p \psi_p = \sum_M a_M \psi_M + b$$

where the summation on $M$ is confined to the four principal nodes $E$, $W$, $N$ and $S$. The finite difference coefficients are given by

$$a_E = D_e - (1 - a_e) C_e$$

(3.30) (contd.)
where the $\alpha_i$ are evaluated from equation (3.24) and the summation on $K$ is over the four points surrounding $P$.

3.5.2 Differencing of Source Terms

From equation (3.17), it may be concluded that the source terms $S_\psi$ comprise all the terms appearing on the right hand side of equations (3.1) to (3.4). The discretised equations (3.30) represent a set of non-linear algebraic equations which, as will be shown later, are solved using methods for linear equations. Therefore the dependence of $S_\psi$ on the respective dependent variable is linearised in the following manner

$$S_\psi = S_1 + S_2 \psi_P$$

The differencing of the source terms of equations (3.2) to (3.4) is given below in terms of equation (3.31). Many of these terms are similar in form, in which case only a representative example is given.
(a) The momentum equation

Since the source terms of the momentum equations have a similar structure, only the $v^{(1)}$ momentum equation is considered. The treatment of the pressure gradient term is dealt with separately in Section 3.6.

The integral divergence of the stress tensor is approximated as, e.g.

$$\int \frac{\Delta}{\Delta x^{(1)}} \tau^{(11)} \, dV = \tau^{(11)}_e A_e - \tau^{(11)}_w A_w \quad (3.32)$$

where

$$\tau^{(11)}_e = 2\nu_e \left[ g^{(1)} \left( \frac{\partial v^{(1)}}{\partial x^{(1)}} \right) + (1 \ 1) v^{(1)} + (2 \ 1) v^{(2)} \right]$$

$$+ g^{(12)} \left( \frac{\partial v^{(1)}}{\partial x^{(2)}} + (1 \ 2) v^{(1)} + (2 \ 2) v^{(2)} \right)$$

and

$$\left[ \frac{\partial v^{(1)}}{\partial x^{(1)}} \right]_e = \frac{v^{(1)}_e - v^{(1)}_p}{\delta x^{(1)}}$$

$$\left[ \frac{\partial v^{(1)}}{\partial x^{(2)}} \right]_e = \frac{v^{(1)}_e - v^{(1)}_p}{\delta x^{(2)}}$$

The stress tensor components are approximated as, e.g.

$$\int (1 \ 1) \tau^{(11)} \, dV = (1 \ 1)_p \tau^{(11)}_p v_p \quad (3.34)$$

where

$$\tau^{(11)}_p = 2\nu_p \left[ g^{(1)} \left( \frac{\partial v^{(1)}}{\partial x^{(1)}} \right) + (1 \ 1) v^{(1)} + (2 \ 1) v^{(2)} \right]$$

$$+ g^{(12)} \left( \frac{\partial v^{(1)}}{\partial x^{(2)}} + (1 \ 2) v^{(1)} + (2 \ 2) v^{(2)} \right)$$

(contd.)
and

\[
\frac{\partial \mathbf{u}^{(1)}}{\partial x} P = \frac{\mathbf{v}^{(1)}_e - \mathbf{v}^{(1)}_w}{\delta x P} \quad \quad (3.35)
\]

\[
\frac{\partial \mathbf{u}^{(1)}}{\partial x} P = \frac{\mathbf{v}^{(1)}_n - \mathbf{v}^{(1)}_\delta}{\delta x P}
\]

All the above terms are incorporated into the \( S_1 \) part of equation (3.31). The contributions to the \( S_2 \) part are obtained from the approximation of the centrifugal and Coriolis inertia terms, e.g.

\[
\int \left[ (G - \rho \varepsilon) \mathbf{v}^{(1)} \right] dV = \left[ (G - \rho \varepsilon) \mathbf{v}^{(1)} \right]_P \mathbf{v}^{(1)}_P \quad (3.36)
\]

The term between the square brackets is taken into \( S_2 \) if negative, otherwise the whole right-hand side is added to \( S_1 \).

(b) Turbulence model equations

The following practices, suggested by Demirdzic [1982], are used for the linearisation and discretisation of the source terms in equations (3.3) and (3.4).

1. \( k \)-equation:

\[
S_k = \int \left[ (G - \rho \varepsilon) \right] dV
\]

\[
= 2 \left[ (G - \rho \varepsilon) C_{\mu} \frac{\partial k}{\mu \varepsilon} \right]_P k_p
\]

\[
= S_{1k} + S_{2k} - k_p
\]

where use has been made of equation (2.90).
(ii) \( \epsilon \)-equation:

\[
S_\epsilon = \int_V \left[ C_1 \frac{\epsilon}{\kappa} - C_2 \rho \frac{\epsilon^2}{\kappa} + C_3 \rho \epsilon \frac{\Delta u(m)}{\Delta x(m)} \right] dV
\]

\[
= \left[ C_1 \frac{\epsilon}{\kappa} V \right]_p - \left[ C_2 \rho \frac{\epsilon^2}{\kappa} \right]_p \epsilon_p + \left[ C_3 \rho \frac{\Delta u(m)}{\Delta x(m)} V \right]_p \epsilon_p
\]

\[
= S_{1\epsilon} + S_{2\epsilon} \epsilon_p
\]

The term within the square brackets in equation (3.38), which vanishes for constant density flow, is incorporated into \( S_{2\epsilon} \) if negative and into \( S_{1\epsilon} \) if positive.

3.5.3 Interpolation Practices

Where reference is made to \( \psi \) values at locations other than the nodal points, linear interpolation is employed. For this purpose, the spatial interpolation factors defined by equation (3.15) are used.

For the staggered grid arrangement of Figure 3.3, the interpolation practices for scalar and momentum cells are considered separately. Since most of the variables are treated in the same way, only representative examples are given.

(a) Scalar cells

At the faces of the scalar cells, the values of scalars other than \( \psi \) (e.g. density, pressure, etc.) are linearly interpolated as follows, e.g.

\[
\rho_e = (1 - \delta_1 p) \rho_p + \delta_1 p \rho_E
\]
The velocity components at the scalar node and cell face, Figure 3.3(a), are likewise interpolated as follows, e.g.

\begin{align}
\nu_p^{[1]} &= \frac{1}{2} (\nu_e^{[1]} + \nu_w^{[1]}) \\
\nu_n^{[1]} &= (1 - \delta_{2p}) \nu_p^{[1]} + \delta_{2p} \nu_N^{[1]}
\end{align}

\text{(3.40)}

(b) \text{Velocity cells}

The values of the velocity components at locations associated with the \( v^{[1]} \) cell, Figure 3.3(b), are approximated by, e.g.

\begin{align}
\nu_e^{[1]} &= \frac{1}{2} (\nu_p^{[1]} + \nu_E^{[1]}) \\
\nu_n^{[1]} &= (1 - \delta_{2p}) \nu_p^{[1]} + \delta_{2p} \nu_N^{[1]}
\end{align}

\text{(3.41)}

\begin{align}
\nu_e^{[2]} &= \frac{1}{2} (\nu_{ne}^{[2]} + \nu_{se}^{[2]}) \\
\nu_n^{[2]} &= \frac{1}{2} (\nu_{ne}^{[2]} + \nu_{nw}^{[2]}) \\
\nu_p^{[2]} &= \frac{1}{2} (\nu_{ne}^{[2]} + \nu_{nw}^{[2]} + \nu_{se}^{[2]} + \nu_{sw}^{[2]})
\end{align}

while scalar variables other than \( \psi \) are approximated by, e.g.

\begin{align}
\rho_p &= \frac{1}{2} (\rho_e + \rho_w) \\
\rho_n &= (1 - \delta_{2p}) \rho_p + \delta_{2p} \rho_N
\end{align}

\text{(3.42)}
3.6 SOLUTION PROCEDURE

Based on the discussion given in the previous section, the discretised equations, equations (3.29), may now be assembled as

\[ a_p \psi_p = \sum_{M} a_M \psi_M + b \]  \hspace{1cm} (3.43)

where the summation on \( M \) is over the four neighbouring points \( E, W, N \) and \( S \). The coefficients \( a_M \) are given by equation (3.30), and

\[ a_p = \sum_{M} a_M - (S_m + S_{2\psi}) \] \hspace{1cm} (3.44)

\[ b = S_{1\psi} + \sum_{K} a_K^{III} (\psi_K - \psi_p) \] \hspace{1cm} (3.45)

Before a solution of equation (3.43) is sought, the treatment of the pressure terms is first discussed.

With the pressure gradients forming part of the source terms in the momentum equation, it is not possible to obtain the velocity field unless the pressure field is known. In the case of high Mach number flows, the density can be regarded as the dominant dependent variable in the continuity equation. The pressure may thus be evaluated from an equation of state. Therefore, the continuity, momentum and turbulence model equation, together with the equation of state, represent a closed set of equations for the unknowns \( \rho, v^{(1)}, v^{(2)}, v^{(3)}, k \) and \( \varepsilon \).

This approach is not applicable for constant density flow problems, since the density can no longer be regarded as a dependent variable in the continuity equation. This is also true in variable density low subsonic flows where local variations of pressure do not affect the density considerably. Therefore, some other means for obtaining the
The pressure field has to be established (see, for example, Harlow & Amsden [1971], Patankar & Spalding [1972], Patankar [1980] and Issa [1982]).

3.6.1 The Pressure Correction Equation

The discretised momentum equations for each of the velocity components normal to the faces of a scalar cell are written in the form of the following example, Figure 3.3,

\[ a_v v_v^{[1]} = \sum_M a_M v_M^{[1]} + b - (g^{[11]} \sin a) e \{p_E - p_p\} \]

\[- (g^{[12]} \sin a) e A_{PE} \{p_ne - p_extra\} \]  
(3.46)

where the pressure gradient terms are extracted from the source term.

If the above equations are solved for a guessed pressure field \(p^*\), they yield a provisional velocity fields \(v^*[1]\) and \(v^*[2]\) which, by definition, satisfy the momentum equations. The correct pressure and velocity fields which satisfy momentum and continuity equations simultaneously may be expressed in terms of the provisional and correction values \((\tilde{v}^{[1]}, \tilde{v}^{[2]}, \tilde{p})\) such that

\[ p = p^* + \tilde{p} \]

\[ v^{[1]} = v^*[1] + \tilde{v}^{[1]} \]  
(3.47)

\[ v^{[2]} = v^*[2] + \tilde{v}^{[2]} \]

The substitution of equation (3.47) into equation (3.46) yields expressions similar to equation (3.46) in terms of the primed
quantities, for example,

\[ a_e \hat{v}_e^{(1)} = \sum_M a_M \hat{v}_M^{(1)} - (g^{11}) A \sin \alpha \cdot e \ (\bar{p}_E - \bar{p}_p) \]

\[ - (g^{12}) \sin \alpha \cdot e \cdot A_{PE} \ (\bar{p}_{ne} - \bar{p}_{se}) \]  

(3.48)

In the SIMPLE procedure (Semi Implicit Method for Pressure Linked Equations) of Patankar & Spalding [1972], the first term on the right hand side is omitted (the justification for this is discussed later). In the extension to the procedure developed by Demirdzic [1982] to cover non-orthogonal coordinate systems, the resulting expression is combined with equation (3.47) to give

\[ d_e = \left( g^{11} \ A \sin \alpha \right)_e \]  

(3.49)

\[ e_e = \frac{1}{a_e} \]  

(3.50)

An equation for \( \bar{p} \) is obtained from the continuity equation by substituting equation (3.49) and the similar expressions for \( v_w^{(1)} \), \( v_n^{(2)} \) and \( v_\Delta^{(2)} \) into equation (3.17), after setting \( \psi = 1 \) and \( S_\psi = \Gamma_\psi = 0 \), to give

\[ a_p \bar{p}_p = \sum_K a_K \bar{p}_K + b \]  

(3.51)

where the summation on \( K \) is over the eight nodes surrounding \( P \), and
\[ a_E = (p^* A \sin \alpha)_e \left[ d_e + \delta_1 p \left( 1 - \delta_2 p - \delta_2 s \right) e_e \right] \]

\[ + (p^* A \sin \alpha)_e \delta_1 p \left( 1 - \delta_2 p \right) e_n - (p^* A \sin \alpha)_\delta \delta_1 p \delta_2 e_\delta \]

\[ a_W = (p^* A \sin \alpha)_w \left[ d_w - \left( 1 - \delta_1 w \right) \left( 1 - \delta_2 p - \delta_2 s \right) e_w \right] \]

\[ - (p^* A \sin \alpha)_n \left( 1 - \delta_1 w \right) \left( 1 - \delta_2 p \right) e_n \]

\[ + (p^* A \sin \alpha)_\delta \left( 1 - \delta_1 w \right) \delta_2 e_\delta \]

\[ a_N = (p^* A \sin \alpha)_n \left[ d_n + \delta_2 p \left( 1 - \delta_1 p - \delta_1 w \right) e_n \right] \]

\[ + (p^* A \sin \alpha)_e \delta_2 p \left( 1 - \delta_1 p \right) e_e - (p^* A \sin \alpha)_w \delta_2 p \delta_1 w e_w \]

\[ a_S = (p^* A \sin \alpha)_\delta \left[ d_\delta - \left( 1 - \delta_2 s \right) \left( 1 - \delta_1 p - \delta_1 w \right) e_\delta \right] \]

\[ - (p^* A \sin \alpha)_e \left( 1 - \delta_2 s \right) \left( 1 - \delta_1 p \right) e_e \]

\[ + (p^* A \sin \alpha)_w \left( 1 - \delta_2 s \right) \delta_1 w e_w \]

\[ a_{NE} = \left[ (p^* A \sin \alpha)_n e_n + (p^* A \sin \alpha)_e e_e \right] \delta_1 p \delta_2 p \]

\[ a_{NW} = - \left[ (p^* A \sin \alpha)_n e_n + (p^* A \sin \alpha)_w e_w \right] \left( 1 - \delta_1 w \right) \delta_2 p \]

\[ a_{SE} = - \left[ (p^* A \sin \alpha)_\delta e_\delta + (p^* A \sin \alpha)_e e_e \right] \delta_1 p \left( 1 - \delta_2 s \right) \]

\[ a_{SW} = \left[ (p^* A \sin \alpha)_\delta e_\delta + (p^* A \sin \alpha)_w e_w \right] \left( 1 - \delta_1 w \right) \left( 1 - \delta_2 s \right) \]

(contd.)
The term \( b \) in the above equation is the local mass source evaluated from the starred velocities. If \( b \) is everywhere zero, this means that the starred velocities satisfy the continuity equation and no pressure correction is needed.

The primed values, therefore, serve in a temporary way to procure a satisfactory linkage between velocity and pressure fields so that they may ultimately satisfy both momentum and continuity equations simultaneously. Accordingly, any approximations made in the process of the derivation of this equation is acceptable as long as a converged solution is reached.

One such approximation is the omission of the terms \( \sum a_M \tilde{u}_M^{(1)} \) in equation (3.48). If such terms were retained, they would have introduced an implicit influence of the pressure correction of more distant grid nodes on local velocity corrections, leading to more complicated discretised equations. In practice, this approximation usually does not prevent the convergence of the solution procedure but under-relaxation is often needed (Wachspress [1979]).

The presence of the coefficients \( e_L \), arising from the cross pressure-gradient terms, prevents the finite difference coefficients (3.52) from being unconditionally positive. To avoid the possibility, and the consequences, of having negative coefficients, a further approximation, suggested by Demirdzic [1982], is to neglect the contribution of \( e_L \) to the coefficients (3.52). This leads to a pressure correction equation of a form similar to equation (3.43),

\[
\begin{align*}
ap &= \sum a_K' \\
b &= - (C_e^* - C_w^* + C_n^* - C_z^*)
\end{align*}
\]
where the summation is over the four principal nodes E, W, N and S, and

\[ a_p \bar{p}_p = \sum_{M} a_M \bar{p}_M + b \]  

(3.53)

where \( a_E \) = \( (p^* A \sin \alpha)_E \)

\( a_W \) = \( (p^* A \sin \alpha)_W \)

\( a_N \) = \( (p^* A \sin \alpha)_N \)

\( a_S \) = \( (p^* A \sin \alpha)_S \)

(3.54)

\[ b = -[C^e_c - C^w_w + C^e_n - C^s_s] \]

The velocity corrections accordingly reduce to, for example,

\[ \bar{v}_e^{(1)} = d_e \left( \bar{p}_p - \bar{p}_E \right) \]  

(3.55)

The overall solution procedure can now be summarised in the following steps:

1. Guess a pressure field, \( p^* \).
2. Solve the momentum equations (3.43) to obtain \( v^*[1] \), \( v^*[2] \) and \( v^*[3] \) and the quantities \( d_i \) and \( b \) used to calculate the coefficients of the pressure correction equations.
3. Solve the pressure correction equation (3.53) to obtain \( \bar{p} \).
and hence \( p \) from equation (3.47).

4. Correct the velocities using equation (3.55) and the similar expressions for \( v^{(2)} \) and \( v^{(3)} \).

5. Solve the discretised equations (3.43) for other \( \psi \)'s, such as the turbulence quantities.

6. Treat the corrected pressure \( p \) as a new guessed pressure \( p^* \) and return to step 2, repeating the whole procedure until a converged solution is reached.

3.6.2 Solution of the Discretised Equations

The discretised equations (3.43) and the pressure correction equation (3.54) consist of a set of non-linear coupled equations comprising subsets for each of the unknowns. These equations are solved using the following iterative technique.

First, the equations are linearised and the subsets are decoupled by assuming that all quantities entering the coefficients are temporarily known. Thus, subsets of linear algebraic equations are obtained for each unknown with five-diagonal coefficient matrices and unconditionally positive coefficients.

Each of these subsets is then solved by an alternating direction line iteration procedure that employs the tridiagonal-matrix algorithm (TDMA) (Ames [1977]). This is called "inner iteration".

After the subsets for all the unknowns are solved in this manner (i.e. steps 2 to 6 of the previous section are completed), it is said that one "outer iteration" is completed. The finite difference coefficients are then updated using the new values of the dependent variables and the whole procedure is repeated until convergence criteria are satisfied.
A converged solution is said to be reached if the sum over the whole solution domain of the normalised absolute residuals, defined with respect to the general discretised equation (3.43) as

\[ R_\psi = \left( \sum_M a_M \psi_M + b - a_p \psi_p \right)/N_\psi \]  

(3.56)

has fallen below a specified level, i.e.

\[ \sum_V |R_\psi| < \lambda_\psi \]  

(3.57)

where \( \lambda_\psi \) is usually of the order of \( 10^{-3} \).

The normalisation factors \( N_\psi \) in equation (3.56) are given in Table 3.1 as the total influx of the variable in question across the boundaries of the solution domain. Other choices, however, may be more appropriate depending on the nature of the problem under consideration.

It is important to note that full convergence of the inner iterations is not necessary because of the need to update the non-linear coefficients and usually only one iteration is performed. The exception is the pressure correction equation whose convergence is of special importance for the success of the solution procedure, and for which a sufficient number \( I \) of inner iterations is performed to satisfy the criterion (Van Doormal & Raithby [1984]),

\[ \frac{[R_p]^I}{[R_p]^0} \leq \lambda_p \]  

(3.58)

Here, \([R_p]^0\) and \([R_p]^I\) are the absolute sum of mass sources \( b \) in the pressure correction equation (defined in equations (3.54)) before and after \( I \) iterations, respectively, and \( \lambda \) is typically 0.3.
TABLE 3.1

Normalisation Factors of the Residual Sums

<table>
<thead>
<tr>
<th>Equation</th>
<th>Normalising factors N₁ψ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure correction</td>
<td>Total mass influx Σ (C₁)n *</td>
</tr>
<tr>
<td>Momentum</td>
<td>Total momentum inflow Σ (C₁ u(j))n</td>
</tr>
<tr>
<td>Scalar ψ</td>
<td>Total inflow Σ (C₁ ψ)n</td>
</tr>
<tr>
<td>Turbulence kinetic energy k</td>
<td>Total inflow of turbulence kinetic energy Σ (C₁ k)n</td>
</tr>
<tr>
<td>Turbulence dissipation</td>
<td>Total inflow Σ (C₁ c)n</td>
</tr>
</tbody>
</table>

* The definition of C₁ is given by equation (5.19), and the suffix n refers to the component normal to the boundary.

In the inner iterations or in the overall iterative scheme, the non-linearities and the coupling of the equations, as well as the grid non-orthogonality, may impair or even prevent convergence. This is usually avoided by slowing down the changes in the values of the dependent variables between successive iterations. Equation (3.43) can be written as,

$$\psi_p = \psi_p^* + \left[ \sum_M a_M \psi_M + b \right] / a_p - \psi_p^*$$  \hspace{1cm} (3.59)

where the term within the square brackets represents the change in \( \psi_p \). This is modified by introducing an under-relaxation factor \( \omega_\psi \) (0 < \( \omega_\psi \) < 1) such that
\[
\psi_p = \psi_p^* + \omega_\psi \left[ \left( \sum_M a_M \psi_M + b \right)/a_P - \psi_p^* \right] 
\] (3.60)

which may alternatively be written as,

\[
\left( \frac{a_p}{a_\psi} \right) \psi_p = \sum_M a_M \psi_M + b + \left( 1 - \omega_\psi \right) \frac{a_P}{a_\psi} \psi_p^* 
\] (3.61)

In an investigation of the optimum value of the under-relaxation factor for the pressure correction, Peric [1985] established the relationship

\[
\omega_p = 1 - \omega_v \{I\} 
\] (3.62)

where \(\omega_v \{I\}\) is the velocity under-relaxation factor. This expression is valid for both orthogonal and non-orthogonal grid arrangements.

The rate of convergence was found to increase as \(\omega_v \{I\}\) became closer to unity; a trend which held up to a certain problem-dependent limiting value. For a substantially non-orthogonal grid, Peric [1985] suggested that \(\omega_p\) should be reduced below the value obtained from equation (3.61) in order to obtain the maximum rate of convergence. Table 3.2 shows the values of under-relaxation factors employed for the various dependent variables.

**TABLE 3.2**

**Under-Relaxation Factors**

<table>
<thead>
<tr>
<th>(\psi)</th>
<th>(\nu {1})</th>
<th>(\nu {2})</th>
<th>(\bar{p})</th>
<th>(k)</th>
<th>(\varepsilon)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\omega_\psi)</td>
<td>0.8</td>
<td>0.5</td>
<td>0.15</td>
<td>0.7</td>
<td>0.7</td>
</tr>
</tbody>
</table>
3.7 THE BOUNDARY CONDITIONS

The flow field resulting from the solution of the governing partial differential equations (e.g. equation (2.101)) are known to be characterised mainly by the boundary and initial conditions as well as by the flow parameters such as Reynolds number. The boundary conditions are often given as either specified values of the dependent variables or as specified normal gradients at the boundary (Roache [1982]). This section provides the discretised forms of the conditions imposed at the various kinds of boundaries encountered in the present study. These are:

(i) Wall boundary
(ii) Inflow and outflow boundaries
(iii) Free-stream boundary

Wall Boundary

At the solid boundary, the no-slip condition applies and the velocities at the solid wall are set equal to zero, Figure 3.4.

The near-wall regions are distinguished from other parts of the flow by two important features, first, very steep gradients prevail normal to the wall for most flow parameters, and second, viscous effects can be dominant so that the effects of molecular viscosity can influence the turbulence field. It is possible to account for these effects by using a closely spaced grid and adopting a "low Reynolds number" version of the $k-\varepsilon$ turbulence model (see, for example, Jones & Launder [1972]). Such an approach is computationally expensive and is normally adapted only when the flow details in the near-wall zone are of interest.

An alternative approach which uses "wall function", described by
Launder & Spalding [1974], is often employed to link the solution in the interior domain with the flow next to the wall.

The main assumption of the wall function concept is that uniform shear stress prevails in the near-wall region which, in turn, implies that generation and dissipation of turbulence energy are in balance. This region, however, is conventionally conceived to be made up to three layers (Hinze [1975]) defined according to the value of the dimensionless distance \( Y^+ \) given by,

\[
y^+ = \frac{\rho U_T Y}{\mu}
\]  

(3.63)

where \( Y \) is the distance normal to the wall and \( U_T \) is the friction velocity defined as,

\[
U_T = \sqrt{\tau_w \rho}
\]  

(3.64)

with \( \tau_w \) being the wall shear stress.

The buffer zone between the viscous sublayer \((Y^+ < 5)\) and the inertial region \((Y^+ > 30)\) may, however, be dispensed with by extending the above sublayers to a common location at \( Y^+ \) of 11.63. This treatment, however, is adequate for most engineering applications.

For points lying within the viscous sublayer, the relation

\[
U_B^+ = y_B^+
\]  

(3.65)

is used (Gosman & Idrisah [1976]), while the "logarithmic law of the wall":

\[
U_B^+ = \frac{1}{\kappa} \ln \left( E y_B^+ \right)
\]  

(3.66)

is used for points within the inertial sublayer. The dimensionless
velocity $u^+$ is defined by:

$$u^+_B = \frac{u_B}{u_\tau} \tag{3.67}$$

where $B$ refers to the boundary node under consideration, $u_B$ is the velocity component parallel to the wall, and $\kappa$ and $E$ are empirical coefficients of the wall function whose values are given in Table 2.2.

The wall shear stress given by

$$\tau_w = \mu \frac{\partial u}{\partial y} \tag{3.68a}$$

may alternatively be written as

$$\tau_w = \mu \left[ e_1^2 \cdot (\nu \cdot e_1) \right] \tag{3.68b}$$

which, upon simplification, yields

$$\tau_w = \mu \left[ \frac{\partial}{\partial y} (\nu_1 + \nu_2 \cos \alpha) \right] \tag{3.68c}$$

Assuming further that $\nu_2 \cos \alpha$ is small compared to $\nu_1$, the following expressions may be obtained for the wall shear stress (Demirdzic [1982]),

$$\tau_w = \frac{\rho_B u_B C_\mu^{1/4} k_B^{1/2} \kappa}{\ln (E v_B^+)} \tag{3.69}$$

* $e_1^{[i]}$ are the normalised dual base vectors defined by $e_1^{[i]} = e_1^{i \sqrt{g_{ij} g_{kl}}}$, and their reciprocal bases are given by $e_1^{[i]} = e_{1i}^{\sqrt{g_{ij} g_{kl}}}$, see Figure 3.9.
and the local dissipation and generation of turbulence energy,

\[ \varepsilon_B = C_{11} \frac{3^3/4}{u} \frac{k_3^{3/2}}{\kappa y_B} \]  \hfill (3.70)

\[ C_B = \tau_w \left( \frac{3u}{\partial y} \right) \]  \hfill (3.71)

with the dimensionless distance from the wall \( y^+ \) given by

\[ y^+ = \rho_B \frac{C_{14}^{1/4} k_3^{1/2}}{y_B} \]  \hfill (3.72)

Assuming that the first interior \( \chi^1 \) grid line to be fairly parallel to the wall, then

\[ y_B = \delta x_B^{[2]} \sin \alpha \]  \hfill (3.73)

where \( \delta x_B^{[2]} \) is the distance from the wall to point \( B \) along the \( x^2 \) coordinate line.

The derivative of the velocity vector in the direction normal to the wall is given by

\[ \frac{\partial u}{\partial y} = \nabla \cdot v \cdot e^{[2]} = \frac{1}{\sin \alpha} \left( \nabla^{[2]} v^{[1]} - \cos \alpha \nabla^{[1]} v^{[1]} \right) \]  \hfill (3.74)

In the near-wall region, the following assumption is made:

\[ v^{[2]} \ll v^{[1]} \]  \hfill (3.75)

Assuming further that the thickness of this region is small compared to the radius of curvature of the wall, then

\[ u_B = v^{[1]} \]  \hfill (3.76)
\[
\frac{\partial u}{\partial y} = \frac{1}{\sin \alpha} \frac{\partial v}{\partial x}
\]  
(3.77)

(i) Momentum equation

In the case of laminar flow, i.e. \( V^+ < 11.63 \), a linear profile is assumed to apply to the tangential velocity and the wall shear stress is given by (Gosman & Idriah [1976]),

\[
\tau_w = \mu_m \frac{u_B}{V^+}
\]  
(3.78)

This is included in the linearised source term such that

\[
S = \text{existing terms} + S_{2B} u_B
\]  
(3.79)

where,

\[
S_{2B} u_B = - \int_{A_w} \tau_w \, dA = - \left( \frac{\mu_m}{V_t^+} A_w \right) u_B
\]  
(3.80)

and \( A_w \) is the area of the wall subject to the shear stress.

For \( V_t^+ > 11.63 \), \( \tau_w \) is calculated from equation (3.68) and linearised as in equation (3.62) where,

\[
S_{2B} u_B = - \int_{A_w} \tau_w \, dA = - \left( \frac{\rho_B C_U 1/4 k_B 1/2 \kappa}{\ln (E Y_t^+)} A_w \right) u_B
\]  
(3.81)

(ii) Turbulence model equations

For the calculation of \( S_k \) in equation (3.37), the following expressions are used for the generation \( G \) and the dissipation \( \varepsilon \) of turbulence energy in the near-wall region,
\[ G_B = \int |\tau_w| \left( \frac{\partial U}{\partial y} \right) dV = |\tau_w| \left( \frac{\partial U}{\partial y} \right)_B v_B \quad (3.82) \]

and

\[ \tau_B = \frac{1}{v_B} \int \epsilon_B dV = \left\{ \begin{array}{ll}
C^{3/4} k_B^{3/2} \frac{v_B^+}{v_B^*} & , \quad v_B^* < 11.63 \\
C^{3/4} k_B^{3/2} \frac{\ln (E v_B^*)}{v_B^*} & , \quad v_B^* > 11.63
\end{array} \right. \quad (3.83) \]

where \( \left( \frac{\partial U}{\partial y} \right)_B \) in equation (3.82) is determined from equation (3.77).

For the \( \epsilon \)-equation, the value of \( \epsilon \) at the boundary node \( B \) is fixed at that given by equation (3.69), irrespective of the value of \( v_B^+ \). This is done through the source term \( S_\epsilon \) in the following manner,

\[ S_{\epsilon B} = L - L \epsilon_B \quad (3.84) \]

where \( L \) is a large number of the order \( 10^{30} \).

(iii) Pressure correction equation

Since the velocity component intersecting the wall is prescribed (zero for impermeable walls), it follows that the gradient \( \partial \rho / \partial x^j = 0 \). This is implemented by setting the appropriate coefficient \( a_M \) in the pressure correction equation to zero.

Inflow and Outflow Boundaries

The values of all variables (except pressure) at the inflow boundary are usually explicitly specified. This is often done in terms of experimental data which are not necessarily related to the coordinate system and/or the grid arrangement employed for the
calculations. Hence, care must be taken in applying such experimental
data to the computational grid.

The conditions at the outflow boundaries, on the other hand, are
either explicitly specified or, more often, unknown. In the case of
unknown outlet boundary conditions, the outflow boundary should be
placed in regions where the flow is orientated outwards and the Reynolds
number is sufficiently large so that the conditions at the outlet do not
influence the solution. In such cases, only a pressure boundary
condition is required and this can be indirectly imposed by specifying
the velocity component normal to the outflow boundary. This component
can be obtained by some form of extrapolation from the interior
velocities in such a way as to ensure an overall mass balance. For
example,

\[ v_B^{[i]} = v_{B-1}^{[i]} + \delta v^{[i]} \]  

(3.85)

where subscripts \( B \) and \( B-1 \) denote boundary and next to the boundary
velocity nodes, respectively. The velocity increment \( \delta v^{[i]} \) normal to
the boundary is calculated from an overall mass balance, thus,

\[ \delta v^{[i]} = \left( \dot{m}_{in} - \int_{A_B} \rho_B v_{B-1}^{[i]} \sin \alpha_B dA_B \right) / \int_{A_B} \rho_B \sin \alpha_B dA_B \]  

(3.86)

where \( \dot{m}_{in} \) is the total mass inflow.

**Free-Stream Boundary**

Free-stream boundaries, unlike wall boundaries for example, are not
completely defined by the geometry of the flow domain. The choice of
their position and the conditions along them are entirely problem-
dependent. An example of the treatment of free-stream boundaries is
given later in Chapter 6.
3.8 COMPUTATIONAL GRID AND RELATED GEOMETRICAL PARAMETERS

It is evident from the analysis presented in Chapter 2 that the geometrical parameters of the computational grid form an essential ingredient in the discretised governing equations. Therefore, geometrical information on the grid, such as inter-node spacing, coordinate line curvature, etc., must be provided before a solution of the discretised equations is sought.

However, for the computational grid to assist in obtaining accurate numerical solutions, it must exhibit certain properties governed mainly by the flow parameters, such as flow direction, streamline curvature, steepness of gradients, etc. Ideally, the grid should satisfy the following requirements:

1. Alignment of the grid lines with the streamlines in order to minimise numerical diffusion.
2. High concentration of grid lines in regions of steep gradients where high resolution is required.
3. Smoothness of grid lines to avoid discontinuities in curvature terms (c.f. equations (3.91)).
4. Optimum values of the cell dimensions, Figure 3.5, which maintain (i) the aspect ratio \( \delta x_p^{[1]} / \delta x_p^{[2]} \) as close to unity as possible, (ii) the expansion ratios \( (\delta x_E^{[1]} / \delta x_p^{[1]}) \) and \( (\delta x_N^{[2]} / \delta x_p^{[2]}) \) not larger than around 10.0 so as to enhance both the stability and accuracy of the solution procedure (Roache [1982]).
5. Minimum departure from non-orthogonality to reduce the influence of the cross-derivative diffusion terms.

It is often difficult to satisfy the above requirements simultaneously and a compromise is, therefore, usually made.
3.8.1 Grid Generation

The term 'grid generation' here refers to the process of determining the physical coordinates of the main control volumes from which all grid-geometry information is deduced. In principle, this can be achieved by any convenient way, including drawing the grid lines by hand. However, for the accurate representation of the various geometrical quantities, a numerical procedure has to be devised for the generation of the grid.

In the present study, the method developed by Demirdzic [1982] is adopted. The method consists of two steps:

(i) specification of the location of pairs of points on two opposite boundaries (e.g. points $B_1$ and $B_2$), Figure 3.6;
(ii) specification of a function $f$ that distributes the grid points along the straight lines joining the above pairs.

The coordinates of the main control volume vertices are thus obtained from

$$y^i = y_{B_1}^i + f (y_{B_2}^i - y_{B_1}^i) \quad (i = 1, 2) \quad (3.87)$$

Each of the above steps provides a strong measure of control over the spacing of grid lines in each of the coordinate directions. Moreover, specifying different distribution functions for different grid lines helps to obtain smooth grids, even for boundaries with discontinuities. A disadvantage of this method is that it restricts one family of grid lines to be straight. This can sometimes cause unfavourable properties in some regions of the flow domain, as discussed later in Chapter 6.
Following Demirdzic [1982], the coordinates of point $M$ lying in the middle of the scalar cell face, Figure 3.7, are obtained by passing an isoparametric parabola through three consecutive vertices on a coordinate line. Two of these points, $V_1$ and $V_2$, are the vertices of the scalar cell concerned, while the third point, $V_3$, is the vertex of the adjacent left or right cell, $V_{3L}$ or $V_{3R}$, respectively. This, in turn, enables the handling of boundary cells where either $V_{3R}$ or $V_{3L}$ do not exist.

The coordinates of point $M$ are thus obtained from,

$$y_M^1 = \frac{B}{A} (y_{V_2}^1 - y_{V_1}^1) + \frac{C}{A} (y_{V_2}^2 - y_{V_1}^2)$$

$$y_M^2 = \frac{B}{A} (y_{V_2}^2 - y_{V_1}^2) - \frac{C}{A} (y_{V_2}^1 - y_{V_1}^1)$$

where

$$A = (y_{V_2}^1 - y_{V_1}^1)^2 + (y_{V_2}^2 - y_{V_1}^2)^2$$

$$B = \frac{1}{2} [(y_{V_2}^1)^2 - (y_{V_1}^1)^2 + (y_{V_2}^2)^2 - (y_{V_1}^2)^2]$$

The coefficient $C$ is defined in accordance with the location of the third vertex, i.e. for $V_3 = V_{3R}$,

$$C = \frac{1}{8} \left[ (y_{V_2}^2 - y_{V_1}^2) (3y_{V_1}^1 + 6y_{V_2}^1 - y_{V_{3R}}^1) - (y_{V_2}^1 - y_{V_1}^1) (3y_{V_1}^2 + 6y_{V_2}^2 - y_{V_{3R}}^2) \right]$$

and for $V_3 = V_{3L}$,

$$C = \frac{1}{8} \left[ (y_{V_2}^2 - y_{V_1}^2) (3y_{V_1}^1 + 6y_{V_2}^1 - y_{V_{3L}}^1) - (y_{V_2}^1 - y_{V_1}^1) (3y_{V_1}^2 + 6y_{V_2}^2 - y_{V_{3L}}^2) \right]$$
The coordinates of the central point \( P \) are defined as the intersection of the straight lines joining the mid-points of each pair of opposing faces, taking into account the curvature of the coordinate lines. Thus,

\[
C = \frac{1}{8} \left[ (v_{12}^2 - v_{12}^1) (3v_{11}^1 + 6v_{11}^0 - v_{11}^3) \\
- (v_{12}^1 - v_{12}^0) (3v_{11}^2 + 6v_{11}^0 - v_{11}^3) \right] (3.91)
\]

Each scalar control volume is thus defined in terms of the nine points of Figure 3.2.

3.8.2 Geometrical Properties of the Coordinates System

In order to determine the geometrical parameters appearing in the finite-difference equations, a coordinate transformation relationship of the form (A.1) has to be established. In the present case, however, only discrete information about the coordinate system \( x^i \) is available in the form of the Cartesian coordinates \( y^i \) of the different points defining the computational grid. Therefore, the coordinate transformation is carried out on a local basis in terms of the nine points of Figure 3.8 as recommended by Demirdzic [1982].

For each scalar cell, an isoparametric quadratic interpolation function of the form,
\[
y^i = y^i(x^1, x^2) = c_1^i x^1 + c_2^i x^2 + c_3^i (x^1)^2 + c_4^i (x^2)^2 + c_6^i x^1 x^2 + c_7^i (x^1)^2 x^2 + c_8^i x^1 (x^2)^2 + c_9^i (x^1)^2 (x^2)^2
\]

(3.93)

is used. The values \(x^i = \pm 1\) are assigned to the sides of the scalar cells and the values \(x^i = 0\) for the sides of the velocity cells, Figure 3.8. The constants \(C_j^i\) can thus be determined and the coordinate transformation formula (3.93) becomes,

\[
y^i = y_p^i + \frac{y^i - y_p}{2} x^1 + \frac{y^i - y_p}{2} x^2
\]

\[
+ \frac{y^i - y_p}{2} x^1 (x^1)^2 + \frac{y^i - y_p}{2} (x^2)^2
\]

\[
+ \frac{\Delta y^i - \Delta y_p}{4} x^1 x^2 + \frac{\Delta y^i - \Delta y_p}{2} [x^1]^2 [x^2]^2
\]

\[
+ \frac{\Delta y^i - \Delta y_p}{2} x^1 x^2
\]

(3.94)

where

\[
\nu^i_n = \frac{y^i_{ne} + y^i_{nw}}{2}
\]

\[
\nu^i_s = \frac{y^i_{se} + y^i_{sw}}{2}
\]

\[
\nu^i_e = \frac{y^i_{ne} + y^i_{se}}{2}
\]

\[
\nu^i_w = \frac{y^i_{nw} + y^i_{sw}}{2}
\]

(3.95)

\[
\Delta y^i_e = y^i_{ne} - y^i_{se}
\]

\[
\Delta y^i_w = y^i_{nw} - y^i_{sw}
\]
The physical Christoffel's symbols are related, through equation (2.26), to the rate of change of the base vectors along the coordinate lines. The derivative of the unit vector $\mathbf{e}(I)$ along the coordinate line $x_1$, Figure 3.9, may be expressed as,

$$\frac{\partial \mathbf{e}(I)}{\partial x(I)} = \frac{\partial \mathbf{e}(I)}{\partial \theta_x} \cdot \frac{\partial \theta_x}{\partial x(I)} = \rho_{11} \mathbf{e}(2)$$

(3.96)

where

$$\rho_{11} = \frac{\partial \theta_x}{\partial x(I)}$$

(3.97)

is the curvature of the coordinate line $x_1$.

Substituting equation (2.26) into equation (3.96) and taking the dot product with $\mathbf{e}(2)$ on both sides, then

$$\rho_{11} \mathbf{e}(2) \cdot \mathbf{e}(2) = \{_{m}^{1} \}_{m} \mathbf{e}(m) \cdot \mathbf{e}(2)$$

(3.98)

However, for the case of planar geometry, equations (2.17) show that,

$$g(11) = g(22) = 1$$

$$g(12) = \cos \alpha$$

(3.99)

$$g(11) = g(22) = \frac{1}{\sin^2 \alpha}$$

$$g(12) = -\frac{\cos \alpha}{\sin^2 \alpha}$$

Equation (3.98) may thus be reduced to

$$\rho_{11} = \{_{1}^{2} \} \sin \alpha$$

(3.100)
Similarly, the following relationships between the physical Christoffel's symbols and the geometrical parameters of the coordinate system are obtained,

\[
\begin{align*}
\{1_{12}\} &= \frac{\rho_{22}}{\sin \alpha} = \frac{1}{\sin \alpha} \frac{\partial^2 y}{\partial x^2} \\
\{2_{11}\} &= \frac{\rho_{11}}{\sin \alpha} = \frac{1}{\sin \alpha} \frac{\partial^2 x}{\partial y^2} \\
\{1_{21}\} &= \frac{\rho_{21}}{\sin \alpha} = \frac{1}{\sin \alpha} \frac{\partial^2 y}{\partial x^2} \\
\{2_{12}\} &= \frac{\rho_{12}}{\sin \alpha} = \frac{1}{\sin \alpha} \frac{\partial^2 x}{\partial y^2} \\
\{1_{11}\} &= -\rho_{11} \frac{\cos \alpha}{\sin \alpha} = -\frac{\cos \alpha}{\sin \alpha} \frac{\partial^2 x}{\partial y^2} \\
\{1_{22}\} &= -\rho_{22} \frac{\cos \alpha}{\sin \alpha} = -\frac{\cos \alpha}{\sin \alpha} \frac{\partial^2 y}{\partial x^2} \\
\{2_{12}\} &= -\rho_{12} \frac{\cos \alpha}{\sin \alpha} = -\frac{\cos \alpha}{\sin \alpha} \frac{\partial^2 x}{\partial y^2} \\
\{2_{11}\} &= -\rho_{21} \frac{\cos \alpha}{\sin \alpha} = -\frac{\cos \alpha}{\sin \alpha} \frac{\partial^2 y}{\partial x^2} \\
\end{align*}
\]

(3.101)

The first two expressions obviously vanish when the corresponding coordinate lines are straight, while the second two expressions vanish if the corresponding coordinate lines are parallel. The last four Christoffel's symbols vanish for orthogonal coordinate systems.

3.8.3 Basic and Cell-Related Quantities

From the definition of the metric tensor components, equation (A.14), it follows that for planar geometry
\[ g_{11} = \left( \frac{\partial y_1}{\partial x_1} \right)^2 + \left( \frac{\partial y_2}{\partial x_1} \right)^2 \]
\[ g_{22} = \left( \frac{\partial y_1}{\partial x_2} \right)^2 + \left( \frac{\partial y_2}{\partial x_2} \right)^2 \]  
(3.102)

\[ \cos \alpha = \frac{g_{12}}{\sqrt{g_{11}} \sqrt{g_{22}}} = \frac{\frac{\partial y_1}{\partial x_1} \frac{\partial y_2}{\partial x_1} + \frac{\partial y_1}{\partial x_2} \frac{\partial y_2}{\partial x_2}}{g_{11}^{-1/2} g_{22}^{-1/2}} \]

The rate of change of \( \theta_x \) and \( \theta_y \) with respect to \( x_1 \) and \( x_2 \) may be obtained as follows,

\[ \tan \theta_x = \frac{\partial y_2}{\partial x_1} \frac{\partial y_1}{\partial x_1} \]  
(3.103)

Differentiating with respect to \( x_1 \) and using the relation

\[ \cos \theta_x = \frac{\partial y_1}{\partial x_1} \]  
(3.104)

one obtains

\[ \frac{\partial \theta_x}{\partial x_1} = \frac{\partial^2 y_2}{\partial x_1^2} \cdot \frac{\partial y_1}{\partial x_1} - \frac{\partial^2 y_1}{\partial x_1^2} \cdot \frac{\partial y_2}{\partial x_1} \]  
(3.105)

Similarly,

\[ \frac{\partial \theta_x}{\partial x_2} = \frac{\partial^2 y_2}{\partial x_2^2} \cdot \frac{\partial y_1}{\partial x_2} - \frac{\partial^2 y_1}{\partial x_2^2} \cdot \frac{\partial y_2}{\partial x_2} \]  
(3.106)

\[ \frac{\partial \theta_y}{\partial x_1} = \frac{\partial^2 y_1}{\partial x_1^2} \cdot \frac{\partial y_2}{\partial x_1} - \frac{\partial^2 y_2}{\partial x_1^2} \cdot \frac{\partial y_1}{\partial x_1} \]  
(3.107)

\[ \frac{\partial \theta_y}{\partial x_2} = \frac{\partial^2 y_1}{\partial x_2^2} \cdot \frac{\partial y_2}{\partial x_2} - \frac{\partial^2 y_2}{\partial x_2^2} \cdot \frac{\partial y_1}{\partial x_2} \]  
(3.108)

The derivatives on the right hand side can easily be
obtained from the coordinate transformation formula (3.94).

For planar geometry, the elementary volume is given by

\[ dV = \sqrt{g} \, dx^1 \, dx^2 = dx^{[1]} \, dx^{[2]} \, \sin \alpha \]  

(3.109)

where the metric \( g \) is defined by,

\[ g = \det |g_{ij}| = \begin{vmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{vmatrix} = g_{11} \, g_{22} \, \sin^2 \alpha \]  

(3.110)

and the differential arc length by,

\[ dx^{[i]} = \sqrt{g_{ii}} \, \delta x^i \]  

(3.111)

The volume of the scalar cell thus becomes

\[ V_p = \frac{1}{2} \int_{-1}^{1} \int_{-1}^{1} \sin \alpha \, dx^{[1]} \, dx^{[2]} \]  

(3.112)

which is difficult to calculate analytically and may be approximated, as proposed by Demirdzic [1982], by the area of the trapezoidal cell, Figure 3.5,

\[ V_p = (\delta x^{[1]} \, \delta x^{[2]} \, \sin \alpha)_p \]  

(3.113)

The cell face areas may also be approximated by,

\[ A_e = \frac{1}{2} \int_{-1}^{1} \left( \sqrt{g_{22}} \right)_{x^1=1} \, dx^2 = (\delta x^{[2]})_e \]  

(3.114)

\[ A_n = \frac{1}{2} \int_{-1}^{1} \left( \sqrt{g_{11}} \right)_{x^2=1} \, dx^1 = (\delta x^{[1]})_n \]
The accuracy of these approximations is consistent with the overall accuracy of the solution procedure (Demirdzic [1982]).

The geometrical quantities at locations where they are not defined are interpolated linearly using the interpolation factors (3.15). For example,

\[ A_e = (\delta x^{(2)})_e = (1 - \delta_{IP}) \delta x_P^{(2)} + \delta_{IP} \delta x_E^{(2)} \quad (3.15) \]

On the other hand, the volume of the momentum cells and the cell face areas \( \delta x_e^{(1)} \), \( \delta x_n^{(1)} \), \( \delta x_w^{(1)} \), \( \delta x_s^{(1)} \) are obtained exactly from, for example,

\[ V_e = \frac{1}{2} (V_p + V_E) \quad (3.16) \]

\[ \delta x_e^{(1)} = A_{PE} = \frac{1}{2} (\delta x_P^{(1)} + \delta x_E^{(1)}) \]

In the case of an axisymmetric geometry (for which \( g_{13} = g_{31} = 0 \) and \( g_{33} = r^2 \)), the following additional non-zero components of the physical metric tensor require to be calculated,

\[ g_{(33)} = g^{[33]} = 1 \quad (3.17) \]

as well as the following Christoffel's symbols

\[ \left\{_{1}^{3_{3}} \right\} = -\frac{1}{r \sin^2 \alpha} \left( \frac{\partial r}{\partial x^{(1)}} \right) - \cos \alpha \left( \frac{\partial r}{\partial x^{(2)}} \right) \]

\[ \left\{_{2}^{3_{3}} \right\} = -\frac{1}{r \sin^2 \alpha} \left( \frac{\partial r}{\partial x^{(2)}} \right) - \cos \alpha \left( \frac{\partial r}{\partial x^{(1)}} \right) \quad (3.18) \]

\[ \left\{_{3}^{3_{1}} \right\} = \frac{1}{r} \frac{\partial r}{\partial x^{(1)}} \quad , \quad \left\{_{3}^{3_{2}} \right\} = \frac{1}{r} \frac{\partial r}{\partial x^{(2)}} \]
The metric in this case becomes,

\[
g = \begin{bmatrix}
g_{11} & g_{12} & 0 \\
g_{21} & g_{22} & 0 \\
0 & 0 & \kappa^2
\end{bmatrix} = g_{11} g_{22} \kappa^2 \sin^2 \alpha \tag{3.119}
\]

and the elementary volume,

\[
dV = \sqrt{g} \, dx^1 \, dx^2 \, dx^3 = dx^{(1)} \, dx^{(2)} \, \kappa \sin \alpha \tag{3.120}
\]

where

\[
dx^3 = d\theta = 1 \tag{3.121}
\]

and \( \theta \) is the polar angle of the cylindrical polar coordinate system.

The cell volume is approximated by

\[
V_p = \int_{-1}^{1} \int_{-1}^{1} \sqrt{g_{11} g_{22}} \, \kappa \sin \alpha \, dx^1 \, dx^2 = (\delta x^{(1)} \delta x^{(2)} \kappa \sin \alpha)_p \tag{3.122}
\]

and the cell face areas by

\[
A_e = (\delta x^{(2)} \kappa)_e \tag{3.123}
\]

\[
A_n = (\delta x^{(1)} \kappa)_n
\]

3.9 CLOSURE

In this chapter, the discretising procedure of the governing equations, derived in Chapter 2 for the case of two-dimensional planar or axisymmetrical flow, has been presented. Special attention has been focused on the treatment of the cross-derivative diffusion and
pressure terms arising from grid non-orthogonality. Following the proposals of Demirdzic [1982], a partial solution to the problem of negative coefficients due to the cross-derivative diffusion terms has been described. It has also been shown that a stable solution is obtained by suppressing the possibly negative contributions of the cross-derivative terms in the pressure correction equation. Based on the same proposals, a discussion is presented of the linearisation and discretisation of the source terms of the various dependent variables.

The solution procedure of the discretised equations has been described which, for reasons of economy and stability, incorporates the upwind differencing schemes.

The analysis and implementation of boundary conditions have been outlined for the various types of boundaries encountered in the present study.

In covering the geometrical aspects of the solution procedure, the grid generation procedure has been outlined. Although the procedure offers precise control over the smoothness and spacing of the grid lines, departure from orthogonality and/or alignment of grid lines with the flow streamlines are sometimes difficult to control. For the calculation of the geometrical quantities, a local coordinate transformation in terms of a nine-point computational molecule has been described, together with the various approximations involved.
4.1 INTRODUCTION

Due to the unsteady features of the induction process in reciprocating engines and the associated interaction of the various parameters of the engine, experimental investigation of the flow in the intake valve/port assembly becomes an expensive and arduous task. However, in the discussion presented in Chapter 1, it has been shown that probing the flow field under quasi-steady conditions provides an insight into the performance of the valve/port assembly and into the development of flow field in the valve exit with a reasonable degree of accuracy and with considerable economy and convenience. On the other hand, the dimensions and the complex geometry of the intake port in real engines do not allow easy access of the measuring probe into the flow field. This difficulty may be overcome by scaling up the test section and assuming the port duct to be straight. This chapter describes the experimental arrangements and measuring techniques employed for investigating the flow in the selected valve/port assembly under such conditions.

A preliminary discussion of the variables involved in characterising the flow is given in Section 2.4. Dimensional analysis is then performed to assemble these variables into dimensionless groups in order to establish the criteria for preserving dynamic and geometrical similarities between the scaled-up test section and the real valve/port assembly.

Details of the experimental arrangement and test section are given in Section 4.3. The techniques for measuring pressure, temperature and the mass rate of flow are also described.
Section 4.4 is concerned with the hot-wire anemometry used for determining the mean velocity field and the turbulent stresses. In order to assess the performance of the analogue system used for processing the anemometer signals, measurements of the mean velocity and the root-mean square of its fluctuations were carried out in the shear layer of a free jet and are also described in this section.

A new method for interpreting the signals of X-wire probes is also presented which eliminates the requirement of aligning the probe with the mean flow direction imposed by most existing methods. The method has been tested for sensitivity and numerical stability over a wide range of flow angles, mean velocities and turbulence intensities. This section ends with some practical considerations and experimental precautions adopted throughout the measurements.

Section 4.5 summarises the main findings and conclusions of the chapter.

4.2 SIMILARITY CONSIDERATIONS AND DIMENSIONAL ANALYSIS

As was outlined in Chapter 1, hot-wire anemometry has been selected for the measurement of mean velocity and turbulence quantities. In order to reduce the influence of flow blockage due to the finite sizes of the probe, probe holder and support on the flow parameters, the dimensions of the selected valve/port assembly had to be scaled up. The limits on the scaling were mainly dictated by the mass flow rate and the pressure that the available air supply could provide. A compromise, however, had to be made and a scaling factor of four was selected.

With the geometry of the experimental model being thus scaled,
dynamic, kinematic and geometric similarity must be preserved in order for the experimental results to be representative of the actual flow conditions. This can be achieved by adopting dimensional analysis to obtain the dimensionless groups characterising the flow.

In the present investigation, the coefficient of discharge, $C_D$, defined as

$$C_D = \frac{\dot{m}_{act}}{\dot{m}_{th}}$$  \hspace{1cm} (4.1)

is considered as the dependent variable, with $\dot{m}_{act}$ and $\dot{m}_{th}$ being the actual and theoretical mass flow rates, respectively. These are generally expressed as,

$$\dot{m} = \rho \left( Q, \frac{A}{Y} \right)$$  \hspace{1cm} (4.2)

The theoretical mass flow rate is defined as that occurring in an isentropic flow. Under this condition, the first law of thermodynamics leads to (Shapiro [1958]),

$$Q_2^2 = Q_1^2 + \frac{2\gamma}{\gamma - 1} \frac{p_1}{\rho_1} \left[ 1 - \frac{p_2}{p_1} \right]^{\gamma - 1/\gamma}$$  \hspace{1cm} (4.3)

where subscripts 1 and 2 refer to the inlet and outlet planes of the test section, and $\gamma$ is the ratio of specific heats.

From the isentropic flow relation

$$p/p_1^\gamma = \text{const.}$$  \hspace{1cm} (4.4)

the density at plane (2) may be obtained from

$$\rho_2 = \rho_1 \left( \frac{p_2}{p_1} \right)^{1/\gamma}$$  \hspace{1cm} (4.5)
The area $A_2$ at the valve exit is known to be a function of the valve geometry $G_v$, and lift $\ell$ and may therefore be represented by the functional relationship,

$$A_2 = \delta_1(\ell, G_v) \quad (4.6)$$

Substitution of equations (4.3), (4.5) and (4.6) into equation (4.2) yields the following expression for the theoretical mass rate of flow

$$\dot{m}_{th} = p_1 \left( \frac{p_2}{p_1} \right)^{1/\gamma} \left[ Q_1^2 + \frac{2\gamma}{\gamma - 1} \frac{p_1}{\rho_1} \left[ 1 - \left( \frac{p_2}{p_1} \right)^{\gamma - 1/\gamma} \right] \delta(\ell, G_v) \right] \quad (4.7)$$

However, with the pressure drop across the valve $\Delta p$, being defined as,

$$\Delta p = p_1 - p_2 \quad (4.8)$$

equation (4.7) may be replaced by the following functional relationship,

$$\dot{m}_{th} = \phi(Q, p, \Delta p, \rho, \ell, \gamma, G_v) \quad (4.9)$$

where the flow parameters $Q$, $p$ and $\rho$ are those at the inlet section.

On the other hand, for a real flow situation, the first law of thermodynamics takes the form,

$$Q_2^2 = Q_1^2 + \frac{1}{(\gamma - 1)} \left( \frac{p_1}{\rho_1} - \frac{p_2}{\rho_2} \right) + \sum \text{losses} \quad (4.10)$$

* $G_v$ is a dimensionless function relating the different geometrical parameters of the flow configuration. The details of this function are, however, immaterial to the present analysis.
Under the present test conditions, the flow may be assumed adiabatic and, hence, the losses may be attributed mainly to boundary layer friction. The latter is known to be a function of a representative mean velocity, $Q$, a characteristic length, $D$, the molecular viscosity, $\nu$, the surface roughness, $\lambda$, the density, $\rho$, and the passage length, $x$ (Nikuradse [1933]). Experimental investigations have also shown that the flow pattern in the valve seat area, characterised by the pressure drop $\Delta p$ and the valve geometry $G_V$ and lift $\ell$, contributes to these losses (see, for example, Kastner, et al. [1964]). Therefore, the losses term in equation (4.10) may be expressed as,

$$\sum \text{losses} = \delta_2(\rho, Q, D, x, \nu, \lambda, \Delta p, \ell, G_V)$$

(4.11)

Substitution of this expression into equation (4.10) yields,

$$Q_2^2 = Q_1^2 + \frac{1}{[\gamma - 1]} \left[ \frac{p_1}{\rho_1} - \frac{p_2}{\rho_2} \right] + \delta_2(\rho, Q, D, x, \nu, \lambda, \Delta p, \ell, G_V)$$

(4.12)

The present investigation is concerned with low-speed engines, in which the flow is subsonic everywhere in the induction system (Taylor [1977]). For a flow Mach number of less than 0.3, the compressibility effects may be neglected (Fukutani & Watanabe [1979], and Rabbit [1984]) and equation (4.12) may thus become

$$Q_2^2 = Q_1^2 + \frac{\Delta p}{\rho_2 [\gamma - 1]} + \delta_2(\rho, Q, D, x, \nu, \lambda, \Delta p, \ell, G_V)$$

(4.13)

Substitution of this equation, together with equation (4.6), into equation (4.2) gives the following expression for the actual mass flow rate

$$\dot{m}_{\text{act}} = \psi(\rho, Q, D, x, \nu, \lambda, \Delta p, \ell, G_V, \gamma)$$

(4.14)
where \( \rho, Q \) and \( P \) are the respective quantities at the inlet section.

From equations (4.1), (4.9) and (4.14) the variables involved in the present investigation may be assembled, together with their dimensions into the list shown in Table 4.1, with \( L, M \) and \( T \) being the fundamental dimensions of length, mass and time, respectively.

**TABLE 4.1**
The List of Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent</strong></td>
<td>( C_D )</td>
</tr>
<tr>
<td>( Q )</td>
<td>( LT^{-1} )</td>
</tr>
<tr>
<td>( P )</td>
<td>( ML^{-1}T^{-2} )</td>
</tr>
<tr>
<td>( \Delta P )</td>
<td>( ML^{-1}T^{-2} )</td>
</tr>
<tr>
<td>( D )</td>
<td>( L )</td>
</tr>
<tr>
<td>( t )</td>
<td>( L )</td>
</tr>
<tr>
<td><strong>Independent</strong></td>
<td></td>
</tr>
<tr>
<td>( x )</td>
<td>( L )</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>( L )</td>
</tr>
<tr>
<td>( \rho )</td>
<td>( ML^{-3} )</td>
</tr>
<tr>
<td>( u )</td>
<td>( ML^{-1}T^{-1} )</td>
</tr>
<tr>
<td>( G_v )</td>
<td>( - )</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>( - )</td>
</tr>
</tbody>
</table>

According to Buckingham's Pi theory (Langhaar [1951]), the above set of variables should form nine separate dimensionless groups. These are,

\[
\pi_1 = C_D \tag{4.15a}
\]
as the dependent group, and

\[ \pi_2 = \frac{p}{\rho} \rho^2 \]

\[ \pi_3 = \frac{\Delta p}{\rho} \rho \Delta \rho \]

\[ \pi_4 = \frac{\Delta p}{p} \]

\[ \pi_5 = \frac{\xi}{D} \]

\[ \pi_6 = \frac{x}{D} \]

\[ \pi_7 = \frac{\lambda}{D} \]

\[ \pi_8 = G_v \]

\[ \pi_9 = \gamma \]

as the independent groups, and are related through the following relationship,

\[ \pi_1 = \delta(\pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8, \pi_9) \] \hspace{1cm} (4.16)

An alternative and more significant form of \(\pi_2\) may be obtained by substituting the equation of state,

\[ p = \rho R T \] \hspace{1cm} (4.17)

and multiplying the resulting expression by \(\pi_9\) to yield,
which represents the flow Mach number based on the inlet conditions (Shapiro [1958]), accounting for the compressibility effects. As mentioned above, this could be neglected in low-speed engine applications.

The incompressible flow assumption further implies a relationship between the Reynolds number, \( \pi_3 \), and the relative static pressure drop, \( \pi_4 \) (Shames [1982]); only one of them may thus be kept in the list of independent groups. Since \( \Delta P \) is easier to determine from direct measurements, it has been kept to serve as the criterion for preserving dynamic similarity.

The groups \( \pi_5, \pi_6, \pi_7 \) and \( \pi_8 \) are the dimensionless lift, \( L \), the passage length, \( X \), the relative roughness, \( A \), and the geometry of the valve/port assembly. These groups represent different geometrical aspects of the test section and may thus be employed as criteria for geometrical similarity. However, for the same fluid flowing in a geometrically similar passage, they are no longer variables and, together with \( \pi_9 \), may be dropped out of equation (4.16). The coefficient of discharge, therefore, reduces to,

\[
C_D = \delta(L, \Delta P) \tag{4.19}
\]

This expression is further referred to later in the experimental investigation outlined in Chapter 5.

As outlined in the discussion presented in Chapter 1, the quasi-steady induction process takes place at fixed flow rate and valve opening. Under such conditions, and employing the incompressible flow
analysis presented above, the pressure drop across the valve may be estimated for various lifts and simulated engine conditions. In Appendix B, details of the calculation procedure are given for the motored model engine of Vafidis & Whitelaw [1984a] which was operated at a speed of 300 rpm. The estimated dimensionless pressure drop \( \Delta P \) is seen from Figure B.1 to vary between \( 5.6 \times 10^{-2} \) and \( 2.54 \times 10^{-4} \) as the dimensionless lift \( L \) increases from 0.02 to 0.3. According to the similarity criteria given by \( \pi_2, \pi_3 \) and \( \pi_4 \) of equation (4.15), and with a scaling-up factor of 4.0, this corresponds to a range of pressure drops between \( 3.5 \times 10^{-3} \) and \( 1.56 \times 10^{-5} \) on the present valve/port assembly. The upper limit considered in the present investigation was increased to \( 8 \times 10^{-3} \) to cover simulated rotational speeds of up to about 500 rpm. Due to the difficulty of maintaining a reasonable degree of accuracy in measurements performed at the lower limit, this was increased to \( 3 \times 10^{-3} \). The results of the measurements carried out within this range may, however, be extrapolated to cover a wider range as is discussed later in Chapter 5.

4.3 EXPERIMENTAL ARRANGEMENT AND MEASURING TECHNIQUES

Figures 4.1 and 4.2 show photographs of the flow passage and instrumentation, and the test section, while a schematic diagram of the test rig is shown in Figure 4.3. Air was supplied by an 11 kW centrifugal fan fitted on its suction side with an electrostatic precipitator (Honeywell electrostatic air cleaner Y503B). At a mass flow rate of about 3 kg/s, the efficiency of the air cleaner was about 95\% and it removed particles and hydrocarbons in the air down to 0.04 \( \mu \text{m} \) in diameter.

The fan discharged into a \( 1.2 \times 0.60 \times 1.2 \) m settling tank which served to alleviate high frequency fluctuations in the flow. Low
frequency, or long term variations, were compensated for by adjusting a
gate valve fitted at the fan outlet.

The settling tank delivered into a 0.1 m diameter pipe which led to
the 0.285 m diameter test section inlet through a diffuser, followed by
a 2 m long entrance length, Figure 4.3. In order to reduce the flow
losses in the diffuser, its cone angle was made to be 15°. For this
angle and with exit-to-inlet area ratio of about 8.5, a diffuser
efficiency of about 82% could be achieved (Pankhurst & Holder [1968]).
Space limitations, however, prevented increasing the diffuser length to
reach the maximum efficiency of about 85% associated with the given
area ratio and a cone angle of 10°.

**Inlet Conditions to Test Section**

Special attention was focused on ensuring uniformity of the flow
at the inlet of the test section. For this purpose, velocity profiles
were measured at different locations along the flow passage in both
vertical and horizontal diameters. Figure 4.4 shows the velocity
profiles at the diffuser inlet which was about 20 pipe diameters
downstream of the 90° elbow. The non-uniformities observed are
typical of the curvature induced secondary flows arising from elbows
and bends (Enayet [1983]). These were removed by placing a mesh screen
across the pipe at about 6 diameters downstream of the elbow. The mesh
had an area ratio of 52% and a wire diameter to spacing ratio of 0.33.

The effect of the mesh on the velocity profiles at the diffuser
inlet is shown in Figure 4.5. In order to remove any non-uniformities
created by the diffuser itself, a similar mesh was placed at its exit,
together with a honeycomb at the middle of the entrance length. The
velocity profiles thus obtained at the inlet to the test section were
fairly uniform, as may be concluded from Figure 4.6.
Test Section

The valve/port assembly considered in the present investigation is geometrically similar to that fitted to the model reciprocating engine studied by Vafidis & Whitelaw [1984a] scaled up, as mentioned previously, by a factor of 4.0. Details of the test section are shown in Figure 4.7, while the geometry and dimensions of the valve and seat are given in Figure 4.8. The flow was led into the straight port duct through a converging passage whose outlet was carefully rounded to eliminate the possibility of flow separation at the port duct inlet. The relatively long port duct was meant to facilitate detecting the development of the flow as it approaches the valve exit region. The test section shown in Figure 4.7 was made of perspex, except for the converging passage, to allow visual access to the flow passage and help preventing accidental damage of the hot-wire probes. The valve, made of aluminium, was mounted on a metal rod supported at both ends by spider arrangements. The flow discharged into a coaxial cylinder 0.285 m in diameter and one diameter long. The valve was held at a fixed opening by locking its shaft in the hub of the downstream spider support.

Flow Rate Determination

For the measurement of the mass flow rate, a $D \times D/2$ square edged orifice plate of area ratio 0.644 was used. It was manufactured and installed, as shown in Figure 4.3, in accordance with BS1042. When metering a compressible fluid such as air, an upper limit exists above which compressibility effects influence the measurements and must be taken into account. In the present case, unambiguous metering was limited by a mass flow rate of about 0.6 kg/s (see BS1042). The pressure drop across the orifice plate $\Delta p_0$ and the absolute pressure upstream of the plate, $p_{u6}$, see Figure 4.3, were both measured using a
water differential micro-manometer with 0.1 mm maximum resolution. The air temperature upstream of the plate was measured by a Cu-CuNi thermocouple connected to a high-resolution digital voltmeter (Solatron LM1420). Figure 4.9 shows the calibration curve for the thermocouple.

The pressure drop across the valve, \( \Delta p \), was taken to be the difference in static pressure between points (a) and (b) in Figure 4.3 and was determined using a digital micro-manometer (Furness Control MDC FC002). At point (a), the velocity head was small compared to that at the valve exit and could accordingly be neglected, as will further be shown in Chapter 5. The pressure at point (b) was assumed to represent the prevailing pressure inside the downstream cylinder.

The mass flow rate was determined according to the procedure outlined in BS1042. The uncertainty in the measured mass flow rate, based on the analysis given in BS1042 and outlined in Appendix B, was found to be about 2.5%. This produced an error in the coefficient of discharge, \( C_d \), of about 3.2% due to the uncertainties in the measured quantities used for calculating the theoretical mass flow rate, namely, the pressure drop, \( \Delta p \), and the valve lift, \( L \).

4.4 HOT-WIRE ANEMOMETRY

This section is concerned with the description of the constant-temperature hot-wire anemometry techniques used for determining the velocity field and the turbulence characteristics of the flow in the present study. Details of the principles and practices of hot-wire anemometry are outside the scope of this section, and for that the reader is referred to the reviews of, for example, Perry [1982], Compte-Bellot [1977], Bradshaw [1971], and the bibliography of Freymuth [1978].

Single normal and X-wire probes DISA 55P11, 55P14, 55P61 and 55P63 were employed in the present study. Tungsten platinum wires of 5 \( \mu m \)
diameter and length to diameter ratio, \( \ell/d \), of 250 were used for the heated sensors. The anemometer units DISA 55D01 were operated in the constant temperature mode with an overheated ratio of 1.8 and were adjusted to obtain a cut-off frequency of 120 kHz at a flow speed of about 10 m/s, as determined by the square-wave test recommended by Freymuth [1978]. Blocks of the analogue systems used for processing the signals of both the single and X-wire probes are shown in Figures 4.10.

Calibration of the probes was carried out in the potential core of a free jet using a Pitot-static probe over a range of flow speeds covering the actual velocities to be measured. A typical calibration curve for a single wire probe in the range from about 10.0 to 20.0 m/s is shown in Figure 4.11.

The relationship between the anemometer voltage and the magnitude of the effective cooling velocity vector, \( \dot{Q}_c \), was expressed by King's law of cooling (King [1914]),

\[
E^2 = E_0^2 + \dot{E} \dot{Q}_c^n
\]  

It is generally recognised that this relationship is not universal (Perry [1982]), although it may be applied with a good degree of accuracy over a limited range of velocities (Hinze [1975]).

The sensitivity of measurements of the velocity fluctuations, \( q \), is defined as the slope of the calibration curve, i.e.

\[
e/q = \frac{\partial E}{\partial \dot{Q}_c}
\]  

where \( e \) is the fluctuations of the anemometer voltage.

Differentiation of equation (4.20) and substitution into equation
(4.21) yields

\[
\frac{e}{q} = \frac{n}{2} \left[ 1 - \frac{E_0^2}{E^2} \right] \frac{E}{Q_e} \tag{4.22}
\]

At turbulence intensities higher than about 10%, the non-linearity of the calibration curve produces considerable distortion in the local derivative \( \partial E/\partial Q_e \) (Bradshaw [1971]). To compensate for this distortion, the anemometer signal was linearised using an analogue lineariser (DISA 55D10).

The exponent \( n \) in equation (4.20) was varied between 0.4 and 0.55. The values of \( E_0 \) and \( B \) were determined by fitting \( E^2 \) and \( Q_e^n \) to a straight line using the least square method (Draper & Smith [1966]). Figure 4.12, together with Table B.1, show the linearised calibration curve of Figure 4.11 and the maximum uncertainties in both \( E_0 \) and \( B \). This procedure was carried out for each set of calibration points to determine the optimum value of the exponent \( n \) which produced the straight line of best fit.

The linearised signal of the anemometer was fed into a root mean square meter (DISA 55D35) to determine \( \sqrt{q^2} \) from the AC component of the signal. Simultaneously, the linearised signal was integrated using a true integrator (DISA 52B30) to determine the mean velocity \( Q_e \). The integration time depended mainly on the local level of turbulence and ranged between 20 seconds and a maximum of 100 seconds.

In order to assess the measuring techniques and the performance of the instruments, measurements of longitudinal mean velocity and its fluctuations were performed in the self-similar region of a round jet using a normal single wire probe and the system of Figure 4.10a. This particular flow regime was chosen mainly because it exhibits universal characteristics independent of the initial conditions at the nozzle (Abramovich [1963]). Furthermore, the high turbulence intensities and
steep gradients prevailing in shear flows provide suitable test conditions for the required assessment.

The jet emerged from a nozzle 12.5 mm in diameter at a Reynolds number of about $4 \times 10^4$. The issuing velocity of the jet was monitored by the pressure drop across the nozzle which was maintained constant to within 2%. Figure 4.13 shows the velocity profile across the exit plane of the nozzle where the turbulence intensity was found to be constant at about 0.2%.

The measurements were taken at 55, 66, 77, 88 and 96 nozzle diameters downstream of the jet exit. After checking the axial-symmetry of the jet, Figure 4.14, the traverses were made along one radius only. Figure 4.15 shows the mean velocity, $U$, normalised by the centreline velocities $U_m$ at the respective locations, plotted against the radius, $r$, normalised by the distance measured from the nozzle. The figure also shows the curve representing the average of the data points obtained by Wignanski & Fiedler [1969], referred to henceforth as WF. The agreement between the two sets of measurements appears to be good, except near the outer edge of the shear layer. There, the intermittency factor drops steeply and the integration time must increase in order to obtain a representative time average of the signal. As mentioned earlier, the maximum integration time available with the DISA 52B30 integrator is limited to 100 seconds which, evidently, was not sufficient to alleviate the scatter in the data points. This scatter is believed to cause the rate of spread of the jet, represented by the slope of the straight line of Figure 4.16, to increase from 0.09, as determined by WF, to about 0.092.

The decay of the mean velocity along the centreline of the jet is shown in Figure 4.17. The measurements follow the linear distribution required by the theory for self-similar flows (Abramovich [1963]), and
agree well with the measurements of Rodi [1975]. Except for a small shift in the location of the virtual origin, the results of WF obey the same decay law down to $x/D = 57$ where the slope of their line changes. This is difficult to understand.

Perry [1982] demonstrated that normal single wire probes are sensitive only to the longitudinal mean velocity, $\bar{U}$, and its fluctuation, $u'$. Figure 4.18 shows the profiles of the rms values of these fluctuations normalised by the centreline velocities at the respective measuring locations. The observed scatter is again believed to be due to the short integration time associated with the rms voltmeter which also produced the shift from the solid curve representing the measurements of WF. The agreement, however, is good with the differences ranging between 1% and 2.5%.

It may be concluded from the above analysis that the hot-wire system is performing reasonably well, except for the distortion in the signal resulting from the limitations on the integration times. This is particularly noticeable when a high level of fluctuations with low intermittency factors occur. Such situations, however, are not expected to prevail in the actual flow of concern of the present study.

4.4.1 Directional Response of X-Wire Probes

As was outlined above, normal single wire probes are capable only of determining the magnitude and fluctuations of the cooling velocity vector normal to the sensor and aligned with the probe axis. Small misalignment may generally be allowed but must be confined to the limits where the aerodynamic disturbances induced by the prongs do not influence the heat transfer from the sensor (Champagne, et al. [1967], Perry [1982], and Merati & Adrian [1984]).

For directional measurements and determination of the
Reynolds stresses, slant and/or X-wire probes must be used. Here, the heated sensors are initially held inclined to the direction of the mean flow at an angle more or less known. Different methods have been developed for the processing and interpretation of the signals of such probes (Champagne & Sleicher [1967], Rodi [1975], Ribeiro [1976], Acrivilelis [1977], and Thompson [1983]). Although these methods cover a wide range of turbulence intensity levels, their applicability is limited to situations where the probe axis is aligned with the direction of the mean flow. Such a restriction, however, represents an obstacle to hot-wire measurements in flow domains with complex geometries similar to that considered here.

In the following, a new method for interpreting the signals of X-wire probes is outlined. This method, which is an extension to Ribeiro's [1976] approach, was developed to enable the measurement of the mean velocity, flow direction and Reynolds stress without the restriction of the alignment of the probe axis and the flow direction.

Consider the hot-wire shown in Figure 4.19 in a general velocity field and subject to the velocity vector $\mathbf{Q}$. Heat transfer and temperature measurements made with inclined wires indicated that they respond to all three components of the velocity vector (Champagne, et al. [1967]). The effective cooling velocity vector, $Q_c$, may therefore be expressed as the sum of three cooling velocity components such that

$$Q_c = \left( u_{N1}^2 + h^2 u_{N2}^2 + k^2 u_T^2 \right)^{1/2} \quad (4.23)$$

where $h$ and $k$ are the coefficients of normal and tangential cooling, respectively. Rodi [1975] suggested the second term on the right hand side to account for the influence of aerodynamic disturbance due to the prongs and stem of the probe. Müller [1982] showed that $h$ is constant
for a yaw angle, $\psi$, between $\pm 10^\circ$ and is determined from (Jørgensen [1971]),

$$k^2 = \frac{\bar{Q}_c(\psi=10^\circ) - \bar{Q}_c(\psi=0^\circ)}{\bar{Q}^2 \sin^2 (10^\circ) \cos^2 (10^\circ)}$$  \hspace{1cm} (4.24)

which gave a typical value of 1.10.

The coefficient of tangential cooling, $k$, is defined as (Müller [1982]),

$$k = \left[ \left( \frac{\bar{Q}_c}{\bar{Q}^2} - \frac{\cos^2 \delta}{\sin^2 \delta} \right) \right]^\frac{1}{2}$$  \hspace{1cm} (4.25)

Most researchers working with inclined hot-wire probes employ single-valued $k$ obtained from the data of Champagne, et al. [1967] as a function of the wire's length to diameter ratio, $\ell/d$. Müller [1982] further showed that $k$ depends also on the magnitude and direction of the mean velocity vector $\bar{Q}$. In the following analysis, the approach of variable $k$ is adopted based on the sensitivity calibrations carried out by Müller [1982].

For the two-dimensional flow situation shown in Figure 4.20, the velocity vector $\bar{Q}$ lies in the plane containing both wires (1) and (2) (separated in the figure for clarity). The response equations in terms of the time-averaged velocity components are written for the two wires, respectively, as

$$\bar{Q}_{c1} = \bar{Q} \cos (\alpha_1 - \delta) \left[ 1 + k_1^2 \tan^2 (\alpha_1 - \delta) \right]^\frac{1}{2}$$  \hspace{1cm} (4.26a)

$$\bar{Q}_{c2} = \bar{Q} \cos (\alpha_2 + \delta) \left[ 1 + k_2^2 \tan^2 (\alpha_2 + \delta) \right]^\frac{1}{2}$$  \hspace{1cm} (4.26b)

Calibration of the probes was performed in the potential core of a free jet with the velocity vector $\bar{Q}$ aligned with the probe
axis. Equations (4.26) became, for these circumstances,

\[
\bar{Q}_{e1} = \bar{Q} \cos \alpha_1 \left[1 + k_1^2 \tan^2 \alpha_1\right]^{1/2}
\]

(4.27a)

\[
\bar{Q}_{e2} = \bar{Q} \cos \alpha_2 \left[1 + k_2^2 \tan^2 \alpha_2\right]^{1/2}
\]

(4.27b)

To account for the effects of prong interference and the difficulty of accurately measuring the angles \(\alpha_1\) and \(\alpha_2\), the calibration method suggested by Bradshaw [1971] for an apparent effective angle, \(\alpha_e\), was carried out. This method is based on the assumption that, with \(k^2\) typically of the order of 0.02, the term \(k^2 \tan^2 \alpha\) in equations (4.27) is small and can be neglected, allowing the equations to reduce to the general form,

\[
\bar{Q}_{ei} = \bar{Q} \cos \alpha_{ei} \quad , \quad (i = 1, 2)
\]

(4.28)

where \(i\) refers to the wire being considered. Substitution of this into equation (4.20) yields,

\[
\bar{Q} \cos \alpha_{ei} = \left[\left\{\left[E^2 - E_0^2\right]/B\right\}_{\delta=0}\right]^{1/n} \quad , \quad (i = 1, 2)
\]

(4.29)

If the angle \(\delta\) between the velocity vector \(\bar{Q}\) and the probe axis is increased by \(\Delta \delta\), then

\[
\bar{Q} \cos \left(\alpha_e + \Delta \delta\right)\hat{i} = \left[\left(E^2 - E_0^2\right)/B\right]^{1/n} \hat{i} \quad , \quad (i = 1, 2)
\]

(4.30)

Dividing equation (4.30) by equation (4.29) and rearranging, one may obtain the following expression for \(\alpha_{ei}\).
Plotting the left hand side of this expression against 

- \sin \Delta \delta \_i \ for \ both \ wires \ and \ for \ various \ values \ of \ \Delta \delta \_i \ yields \ two 
straight lines whose slopes determine \( \alpha_{\text{el}} \). Typical calibration of a 
DISA 55P61 X-wire probe for the effective wire angles is shown in 
Figure 4.21, where equal increments \( \Delta \delta \_i \) of \( 2^\circ \) were taken in the range 
\( \pm 13^\circ \) using the probe support mechanism illustrated in Figure 4.22. 
The effective angles, \( \alpha_{\text{el}} \) and \( \alpha_{\text{e2}} \), are typically \( 49^\circ \) and \( 48^\circ \), 
respectively.

The effective wires' angles, \( \alpha_{\text{el}} \), thus obtained were found 
to have an effect on the probe calibration, shown in Figures 4.23 and 
4.24 and Table B.2, similar to that produced by assuming nominal angles, 
\( \alpha \_i \), of \( \pm 45^\circ \) and taking \( k \_i \_2 \tan^2 \alpha \_i \), neglected previously in equations 
(4.27), into account. In such a case, equations (4.26), which are used 
to determine the magnitude and direction of the mean velocity, become

\[
\overline{q}_{\text{e1}} = \frac{1}{\sqrt{2}} \overline{q} \left[ \cos \delta + \sin \delta \right] \left[ 1 + k^2 \left( \frac{1}{1 + \tan \delta} \right)^2 \right]^{\frac{1}{2}} \tag{4.32a}
\]

\[
\overline{q}_{\text{e2}} = \frac{1}{\sqrt{2}} \overline{q} \left[ \cos \delta - \sin \delta \right] \left[ 1 + k^2 \left( \frac{1}{1 - \tan \delta} \right)^2 \right]^{\frac{1}{2}} \tag{4.32b}
\]

Replacing the terms between the square brackets by

\[
A_1 = \left[ 1 + k^2 \left( \frac{1}{1 + \tan \delta} \right)^2 \right]^{\frac{1}{2}} \tag{4.33a}
\]

and

\[
A_2 = \left[ 1 + k^2 \left( \frac{1}{1 - \tan \delta} \right)^2 \right]^{\frac{1}{2}} \tag{4.33b}
\]

and solving for \( \delta \), then
The flow angle is determined by an iterative method. Starting with \( \delta = 0 \), the values of \( Q_e^1 \), \( Q_e^2 \), \( A_1 \) and \( A_2 \) are calculated from equations (4.32) and (4.33), respectively, and then substituted into equation (4.34) to yield a modified \( \delta \). With this new value of \( \delta \), the procedure is repeated until it converges to within 1%. The mean velocity, \( \bar{Q} \), is determined with reference to the wire subject to the smallest tangential velocity using one of equations (4.32). This reduces the influence of the uncertainties in the coefficient of tangential cooling (Thompson [1983]).

In terms of the instantaneous velocity components, the response equation, equations (4.23), are rewritten in the form,

\[
Q_{ei} = \bar{Q} \cos \gamma_i \left( \frac{U_{N1}}{\bar{Q} \cos \gamma_i} \right) \left[ 1 + \left( \frac{h^2 U_{N2}^2 + h_i^2 U_i^2}{\bar{Q}^2 \cos^2 \gamma_i} \right) \left( \frac{\bar{Q}^2 \cos^2 \gamma_i}{U_{N1}^2} \right) \right]^{\frac{1}{2}} \quad \text{\((i = 1, 2)\)}
\]

(4.35)

where, see Figure 4.20,

\[
\gamma_i = \alpha_i \pm \delta \quad \text{\((4.36)\)}
\]

For convenience, equation (4.35) is expressed in the more general form,

\[
Q_e = \bar{Q} \cos \gamma_i \left( 1 + F_{1i} \right) \left[ 1 + F_{2i} \left( 1 + F_{1i} \right)^{-2} \right]^{\frac{1}{2}} \quad \text{\((i = 1, 2)\)}
\]

(4.37)

where
\[ 1 + F_{i\ell} = \frac{U_{N1}}{\overline{Q}} \cos \gamma_{i\ell} \quad (4.37) \]

\[ F_{2\ell} = \frac{(h^2 U_{N2}^2 + k_{i\ell}^2 U_T^2)}{\overline{Q} \cos^2 \gamma_{i\ell}} \]

With reference to Figure 4.25, the velocity components \( U_{N1}, U_{N2} \) and \( U_T \) are

\[ U_{N1} = (\overline{Q} + q_1) \cos \gamma_{i\ell} - q_2 \sin \gamma_{i\ell} \]

\[ U_{N2} = q_3 \quad , \quad (i = 1, 2) \quad (4.38) \]

\[ U_T = (\overline{Q} + q_1) \sin \gamma_{i\ell} + q_2 \cos \gamma_{i\ell} \]

where \( q_1, q_2 \) and \( q_3 \) are the fluctuation components of the velocity vector in the three principal directions relative to \( \overline{Q} \).

Substitution of equations (4.38) into equations (4.37) yields

\[ F_{i\ell} = \frac{q_1}{\overline{Q}} - \frac{q_2}{\overline{Q}} \tan \gamma_{i\ell} \quad , \quad (i = 1, 2) \quad (4.39) \]

\[ F_{2\ell} = \frac{h^2}{\cos^2 \gamma_{i\ell}} + \frac{k_{i\ell}^2}{\cos^2 \gamma_{i\ell}} \left[ \frac{(\overline{Q} + q_1)^2 \sin^2 \gamma_{i\ell}}{\overline{Q}^2} \right] \]

\[ + \frac{2q_2}{\overline{Q}^2} \left[ (\overline{Q} + q_1) \cos \gamma_{i\ell} \cos \gamma_{i\ell} + q_2^2 \cos^2 \gamma_{i\ell} \right] \quad , \quad (i = 1, 2) \quad (4.40) \]

Equations (4.39) and (4.40) may alternatively be written in the general form

\[ F_{i\ell} = b_{i\ell} u_i^1 + \delta_{i\ell} u_i^2 + \delta_{3i\ell} u_i^3 \quad (4.41) \]
where
\[ u_1^i = \frac{q_1}{Q} \]
\[ u_2^i = \frac{q_2}{Q} \]
\[ u_3^i = \frac{q_3}{Q} \]

and
\[ \delta_{1i} = 1 \]
\[ \delta_{2i} = -\tan \gamma_i, \quad (i = 1, 2) \]
\[ \delta_{3i} = 0 \]

Equation (4.40) may also be written as

\[ F_2 = g_{0i} + g_{1i} u_1^i + g_{2i} u_2^i + g_{3i} u_3^i + g_{4i} u_1^{i2} + g_{5i} u_2^{i2} \]
\[ + g_{6i} u_3^{i2} + g_{7i} u_1^i u_2^i + g_{8i} u_2^i u_3^i + g_{9i} u_3^i u_1^i, \quad (i = 1, 2) \]

(4.44)

where
\[ g_{0i} = k_i^2 \tan^2 \gamma_i \]
\[ g_{1i} = g_{2i} = 2g_{0i}, \quad (i = 1, 2) \]
\[ g_{3i} = 0 \]
\[ g_{4i} = k_i^2 \tan^2 \gamma_i \]

(contd.)
\[ g_{5i} = k_i^2 \]
\[ g_{6i} = h^2 \sec^2 \gamma_i \] \quad (i = 1, 2) \hspace{1cm} (4.45)\
\[ g_{7i} = 2k_i^2 \tan^2 \gamma_i \]
\[ g_{8i} = g_{9i} = 0 \]

The non-linear term in equation (4.37), expressed by

\[ T_i = [1 + F_{2i} (1 + F_{1i})^{-2}]^{\frac{1}{2}} \] \hspace{1cm} (4.46)

may be approximated by a binomial expansion for low turbulence intensities. Ribeiro [1976] suggests that it be fitted to curves which represent the response over limited ranges of turbulence intensities and flow angles. This approach has been adopted here and the non-linear term, approximated in two stages, yields

\[ H_i = \frac{1}{1 + F_{1i}} = C_{H0i} + C_{H1i} F_{1i} + C_{H2i} F_{1i}^2 \] \hspace{1cm} (4.47)

neglecting third and higher order terms, and

\[ T_i = (1 + F_{2i} H_i^2)^{\frac{1}{2}} = C_{G1i} + C_{G2i} F_{2i} H_i^2 \] \hspace{1cm} (4.48)

Substitution of equations (4.47) and (4.48) into equations (4.37) yields the following form of the response equations.
\[ Q_{\ell i} = \overline{Q} \cos \gamma_i c_{G1i} (1 + F_1 + C_{1i} F_2 + C_{1,i} F_1^2 F_2^{2i} + C_{2,i} F_1^{2i} F_2^{2i}) \]

\[ \text{where the coefficients } C_{ij} \text{ are given by} \]

\[ C_{ji} = \left[ \frac{C_{G2}}{C_{G1}} \right] C_{Hji} \quad (j = 0, 1, 2; \ i = 1, 2) \]

These coefficients are determined as follows.

(i) An estimate is first made for \( u'_1, u'_2 \) and \( u'_3 \). With the magnitude and direction of the mean velocity vector, \( \overline{Q} \) and \( \delta \), being known, \( F_1 \) and \( F_2 \) are calculated from equations (4.41) and (4.44) and then substituted into equation (4.37) to obtain \( Q_{\ell i}/\overline{Q} \cos \gamma_i \).

(ii) A range of turbulence intensities \( u'_1, u'_2 \) and \( u'_3 \) around the above estimate is taken (at least 3 values for each component). The value of \( F_{1i} \) obtained by substituting the above turbulence intensities into equation (4.41) are then curve fitted to obtain \( C_{Hji} \) of equation (4.47).

(iii) The above turbulence intensities are also substituted into equation (4.44) and the values of \( F_2 \) thus obtained, together with the coefficients determined in the previous step are curve fitted to determine the coefficients \( C_{G1} \) and \( C_{G2} \) of equation (4.48).

(iv) The coefficients \( C_{ji} \) are then obtained by substituting \( C_{Hji} \) and \( C_{G1} \) and \( C_{G2} \) from the above steps into equation (4.50).
Ribeiro [1976] has demonstrated that curves could be fitted to equations (4.47) and (4.48) over a wide range of turbulence intensities, provided that rectification effects due to flow reversal are avoided. In the present analysis, however, the coefficients $C_{Hf1}$ and $C_{G1}$ and $C_{G2}$ were obtained by linear regression over the required range of turbulence intensities. The maximum discrepancy between the true and approximated response was found to be less than 0.7% for turbulence intensities around 30% (Thompson [1983]).

The mean squares of the fluctuations of the effective cooling velocities are defined as

$$
\overline{q_1^2} = \overline{\sigma_{e1}^2} - \overline{\sigma_{e1}^2}
$$

(4.51)

and

$$
\overline{q_2^2} = \overline{\sigma_{e2}^2} - \overline{\sigma_{e2}^2}
$$

(4.52)

These may be obtained from equation (4.49) in the forms

$$
\frac{\overline{q_1^2}}{\overline{q^2}} = \cos^2 \gamma_1 \left[ A_1 \overline{u_1^2} + B_1 \overline{u_2^2} + C_1 \overline{u_1 u_2} + D_1 \overline{u_3 u_2} + R_1^* \right]
$$

(4.53)

$$
\frac{\overline{q_2^2}}{\overline{q^2}} = \cos^2 \gamma_2 \left[ A_2 \overline{u_1^2} + B_2 \overline{u_2^2} + C_2 \overline{u_1 u_2} + D_2 \overline{u_3 u_2} + R_2^* \right]
$$

(4.54)

In the present analysis, a third equation for the cross-correlation, defined as

$$
\overline{q_1 q_2} = \overline{\sigma_{e1} \sigma_{e2}} - \overline{\sigma_{e1}} \overline{\sigma_{e2}}
$$

(4.55)

has been introduced. This was also obtained from equation (4.49) in the form
\[
\frac{q_1 q_2}{Q^2} = \cos \gamma_1 \cos \gamma_2 \left( A_3 \overline{u_1'^2} + B_3 \overline{u_2'^2} + C_3 \overline{u_1' u_2'} + D_3 \overline{u_3'^2} + R^*_3 \right) \quad (4.56)
\]

The coefficients \(A_3\), \(B_3\), and \(C_3\), and the terms \(R^*_3\) are functions of the two sets of coefficients of equation (4.50) and are given in Appendix B. \(R^*_3\) contains third and higher order correlations which can only be obtained by solving further equations containing still higher order correlations. An approximation is, therefore, required to close these equations and that adopted here is that the probability density function of the turbulence fluctuations has a Gaussian distribution such that odd-order correlations vanish. This was done without appreciable loss of accuracy since the maximum error in the values of \(\overline{u_1' u_1'}\) is less than 0.7% as calculated by Thompson [1983] using third and higher order moments measured by Simpson, et al. [1981]. Even-order moments are, furthermore, assumed to be functions of second order moments only (Ribeiro [1976]). As this approximation is applied to fourth and higher order correlations, it is not expected to produce appreciable error, as will be shown later.

The stresses \(\overline{u_1'^2}, \overline{u_2'^2}\) and \(\overline{u_1' u_2'}\) are obtained by solving equations (4.53), (4.54) and (4.56) simultaneously with \(q_1^2, q_2^2\) and \(q_1 q_2\) being the values measured by the rms voltmeters and the cross-correlator in Figure 4.10, respectively. Initially, \(R^*_3\) are set equal to zero and the stresses are obtained using Gauss-Jordan elimination. \(R^*_3\) are then updated and equations (4.53), (4.54) and (4.56) are solved

* A similar equation for \(q_3^2/Q^2\) has to be included if \(u_3'^2\) is to be determined. Here, \(u_3'^2\) was assumed to be of the same order of magnitude as \(u_1'^2\) and \(u_2'^2\) and was taken as their mean. This assumption is further discussed later.
again. The procedure is repeated until the stresses converge to within 1% which usually required 4 to 7 iterations.

The calculated stresses are then resolved into the duct coordinates of Figure 4.25 using the relations,

\[
\frac{u_{12}^2}{Q^2} = u_{12}^2 \cos^2 \delta + u_{22}^2 \sin^2 \delta - 2 u_{12} u_{22} \sin \delta \cos \delta \quad (4.57)
\]

\[
\frac{v_{12}^2}{Q^2} = u_{12}^2 \sin^2 \delta + u_{22}^2 \cos^2 \delta + 2 u_{12} u_{22} \sin \delta \cos \delta \quad (4.58)
\]

\[
\frac{u_{1}v_{1}}{Q^2} = (u_{12}^2 - u_{22}^2) \sin \delta \cos \delta + (\cos^2 \delta - \sin^2 \delta) u_{1} u_{2} \quad (4.59)
\]

Measurements using the experimental arrangement of Figure 4.10b were carried out in the shear layer of a free jet within the developing zone. The probe was held inclined to the flow direction using the probe support mechanism of Figure 4.22. The inclination was increased in steps of 5° to cover the range from 0° to 35°. In each position, the mean velocity \(\overline{\mathbf{u}}\), the flow direction \(\delta\) and the Reynolds stresses \(\overline{u_{12}^2}, \overline{v_{12}^2}\) and \(\overline{u_{1}v_{1}}\) were calculated using the procedure outlined above. The aim was to check the accuracy and the sensitivity of the developed method to changes in the flow direction. The investigation also covered the effects of the magnitude of mean velocity and the level of turbulence intensity on the calculated stresses.

The probe was initially aligned with the flow direction at the location where the required mean velocity and turbulence intensity occur. The latter were roughly estimated from (Champagne & Sleicher [1967]),

\[
\frac{u_{12}^2}{Q^2} = \frac{1}{4} \left[ \frac{q_{1}^2}{Q_{c1}^2} + \frac{2 q_{1} q_{2}}{Q_{c1} Q_{c2}} + \frac{q_{2}^2}{Q_{c2}^2} \right] \quad (4.60)
\]
The probe was then held at the same spatial position but inclined to the mean flow direction and the measurements were repeated to cover the range of inclinations mentioned above. To ensure that each set of measurements was being taken at the same point in the flow field, two perpendicular microscopes were used to fix the initial location of the X-wires of the probe.

Figures 4.26 and 4.27 show the mean velocity $\bar{\Omega}$ and the calculated flow angle $\delta_{\text{cal}}$ relative to the axial direction plotted against the measured angle $\delta_{\text{m}}$ for different turbulence intensities. The latter was varied by moving the probe to different locations within the flow field.

The calculated and measured angles remain almost equal up to values of about 20°, where the calculated flow angle becomes progressively underestimated, particularly as both the mean velocity and turbulence intensity increase. With reference to equation (4.34), this is believed to be due to the increased uncertainty in the value of $k_z^2$ as the angle $\alpha + \delta$ in Figure 4.20 increases (Muller [1982]). This does not seem to influence the mean velocity appreciably, Figure 4.26, since the response equation of wire (1), equation (4.26), which is used for calculating $\bar{\Omega}$, refers only implicitly to $k_z^2$ through the calculated flow angle.

The Reynolds stresses $\bar{u}^2$, $\bar{v}^2$, and $\bar{u}'\bar{v}'$ are plotted in Figures 4.28 to 4.30, respectively, against the measured flow angle. For the ranges of mean velocities and turbulence intensities shown, the

\[
\frac{v'^2}{\bar{Q}} = \frac{1}{4} \left( \frac{1 + k^2}{1 - 3k^2 + 4k^4} \right) \left[ \frac{q_{1}^{2}}{Q_{c1}} - \frac{2 q_{1} q_{2}}{Q_{c1} Q_{c2}} + \frac{q_{2}^{2}}{Q_{c2}} \right] \tag{4.61}
\]

\[
\frac{u'^2}{\bar{Q}} = \frac{1}{4} \left( \frac{1 + k^2}{1 - k^2} \right) \left[ \frac{q_{1}^{2}}{Q_{c1}} - \frac{q_{2}^{2}}{Q_{c2}} \right] \tag{4.62}
\]
calculated stresses remain almost constant up to about 25°, where they start decreasing slightly. This is believed to be due to the combined uncertainties in the tangential cooling coefficient and the flow angle mentioned above. The shear stresses $u'^2 v'^2$, however, show higher relative sensitivity to those uncertainties than the normal stresses $u'^2$ and $v'^2$.

Further comparisons with data obtained using the formulae derived by Champagne, et al. [1967] for X-wire probes, equations (4.60) to (4.62), are shown in Figures 4.31a to 4.31c. At flow angles up to 20°, close agreement is observed in the values of the normal stresses $u'^2$ and $v'^2$, Figures 4.31a and 4.31b. The slightly higher values obtained by the present method are believed to arise from the inclusion of third and fourth order correlations, neglected in equations (4.60) to (4.62), as well as the terms containing $u'^3$ in the calculations.

Comparison of shear stress values, on the other hand, shows that equation (4.62) is incapable of accurately representing $u'^2 v'^2$ in cases where the slightest misalignment exists between the probe axis and the flow direction.

Figures 4.32 show the effects of neglecting higher order moments and the different assumptions for $u'^3$ on the normal and shear stresses $u'^2$, $v'^2$ and $u'^2 v'^2$, respectively. With $R^5$ being set equal to zero, the absolute differences in the calculated stresses increase steadily with the flow angle, Figure 4.32a. On the other hand, varying $u'^3$ between ±10% of the mean of $u'^2$ and $u'^2$ produces the differences shown in Figure 4.32b. In both cases, however, the shear stress shows higher sensitivity to changes of the flow angle.

An error of 2.5% was introduced into the input signals to examine the characteristics of error propagation in the method of
calculation. Table 4.2 gives the maximum uncertainties in the different calculated values which show boundness in the error propagation behaviour, despite the higher level of uncertainty associated with the shear stress. The values in Table 4.2 were, furthermore, found to be independent of the turbulence level and the magnitude of the mean velocity.

**TABLE 4.2**

Maximum Uncertainties in the Calculated Quantities for a 2.5% Error in Input

<table>
<thead>
<tr>
<th>$\Delta \delta_m^\circ$</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{Q}$ %</td>
<td>1.85</td>
<td>1.9</td>
<td>1.96</td>
<td>1.98</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td>$\delta_{cal}^\circ$</td>
<td>0.43</td>
<td>0.51</td>
<td>0.70</td>
<td>0.68</td>
<td>0.68</td>
<td>0.68</td>
</tr>
<tr>
<td>$\frac{u'^2}{\bar{U}}$ %</td>
<td>2.00</td>
<td>2.70</td>
<td>2.70</td>
<td>2.40</td>
<td>2.60</td>
<td>2.70</td>
</tr>
<tr>
<td>$\frac{v'^2}{\bar{V}}$ %</td>
<td>2.40</td>
<td>2.70</td>
<td>2.60</td>
<td>2.30</td>
<td>2.30</td>
<td>2.10</td>
</tr>
<tr>
<td>$\bar{u}' \bar{v}'$ %</td>
<td>7.40</td>
<td>8.00</td>
<td>8.60</td>
<td>8.60</td>
<td>6.60</td>
<td>6.60</td>
</tr>
</tbody>
</table>

**4.4.2 Practical Considerations**

Velocity gradients along the wire axis can have a significant effect on the indicated velocity because of the averaging of heat transfer (Gessner & Moller [1971], Ronald [1976], and Perry [1982]). In order to reduce the effects of gradient broadening, the magnitude of the mean velocity was determined from a single normal wire perpendicular to the velocity gradient and probes of length to diameter ratio of 250 were used.
In measurements carried out in the vicinity of conductive walls, corrections for additional heat loss from the wire become important at values of $Y^*$ (see equation (3.72)) less than 6 (Bhatia, et al. [1982]), but no corrections were required here.

The effect of anemometer drift caused by changes in flow temperature was compensated for by repeating the calibration if the air temperature changed by more than 1°C. Furthermore, the accuracy of calibration was checked periodically during a set of measurements against the velocity obtained by a Pitot-static probe. Calibration was repeated if a difference of more than 2% was noticed.

Contamination of the wire surface by oxidation and/or particle deposition may cause drift in the anemometer voltage (Bradshaw [1971]). To prevent this, the air supply was filtered, as mentioned earlier. Moreover, the probes were cleaned regularly. This was done by soaking the probe in acetone every 8 hours of operation, unless difficulties in calibration indicated the need for more frequent cleaning. To avoid changes in the oxide layer on the heated surface, probes were burnt-in prior to measuring by operating them in still air for at least 2 hours when new or after cleaning.

4.5 CLOSURE

Dimensional analysis of the variables characterising valve flow yielded nine dimensionless groups which served as similarity criteria. For geometrically similar passages, these were reduced to only two groups, namely, the relative pressure drop and valve lift. The dependence of the coefficient of discharge on both groups is investigated in the next chapter.

The experimental apparatus and the measuring technique were presented, together with an analysis of the uncertainties encountered. 
in the measurement of the mass rate of flow.

The hot-wire anemometry techniques used for measuring mean velocities and turbulence quantities were also outlined. A new method has been described for interpreting the signals of X-wire probes which takes into account the variation of the directional sensitivity of the heated sensors. This enabled the measurement of the mean velocity, flow angle and the turbulent stresses, $\overline{u'^2}$, $\overline{v'^2}$ and $\overline{u' \cdot v'}$, without the restriction of aligning the probe with the mean flow direction. Test measurements were carried out and the new method showed good numerical stability in the calculated mean velocity and turbulent stresses over a wide range of flow angles, turbulence intensities and mean velocities. It also proved to be superior to similar methods which employ X-wire probes for the determination of the stresses $\overline{u'^2}$, $\overline{v'^2}$ and $\overline{u' \cdot v'}$. The effect of the estimated value for the transverse normal stress component was also investigated and a 20% difference in $\overline{w'^2}$ was shown to produce differences in $\overline{u'^2}$ and $\overline{v'^2}$ of about 5% and 3%, respectively. The shear stress $\overline{u' \cdot v'}$, on the other hand, proved to be more sensitive to $\overline{w'^2}$, where the difference started at about 4% and increased rapidly with the flow angle to reach about 25% at an angle of 25°.

Investigation of the error propagation behaviour was carried out by introducing an error of 2.5% into the input signals. The calculation procedure showed boundness in error propagation and the resulting maximum errors in $\overline{u'^2}$ and $\overline{v'^2}$ were found to be 2.7%, while that in $\overline{u' \cdot v'}$ mounted to 8%.
CHAPTER 5
EXPERIMENTAL RESULTS AND DISCUSSION

5.1 INTRODUCTION

In this chapter, the results of the experimental investigation of the flow in the selected valve/port assembly are presented and discussed. The details of the techniques employed for measuring the various parameters reported here, as well as the description of the test rig, were given in the previous chapter.

In Section 5.2, the global performance of the valve/port assembly is presented in the form of measurements of the coefficient of discharge, $C_D$. These were conducted on the basis of the results of the dimensional analysis outlined in Chapter 4. To provide a better understanding of the behaviour of $C_D$ and its connection with the flow pattern in the valve outlet, visualisation of the flow was performed using a surface paint technique and is also reported in this section.

The detailed hot-wire measurements of the mean velocity and turbulence characteristics are presented and discussed in Section 5.3. The features of the mean velocity field are described in Subsection 5.3.1 in the light of the single and X-wire measurements and the information provided by the flow visualisation results of the preceding section.

In Subsection 5.3.2, the X-wire measurements of the turbulent normal and shear stresses are presented. These were determined by the method for processing the probe signals described in Subsection 4.4.1.

Finally, Section 5.4 summarises the main findings.
5.2 THE COEFFICIENT OF DISCHARGE

The coefficient of discharge, $C_D$, was defined by equation (4.1) as the ratio between the actual and theoretical mass flow rates at a given overall pressure drop, $\Delta P$. The dimensional analysis of Section 4.2 for low-speed engine valve flow under quasi-steady conditions showed that the coefficient of discharge for a particular valve/port assembly may be expressed as,

$$C_D = \delta(\Delta P, L)$$  \hspace{1cm} (5.1)

Here, $\Delta P$ and $L$ are, respectively, the dimensionless static pressure drop across the valve based on the exit conditions, and the dimensionless valve lift referred to the port diameter, given by equation (4.15).

In order to determine the functional relationship (5.1) for the present valve/port system, the actual and theoretical mass flow rates were determined for different lifts and static pressure drops. In these experiments, the dimensionless lift, $L$, ranged from 0.02 to 0.35, covering the range of operations for most practical engines (Taylor [1977]). The dimensionless static pressure drop across the valve was varied in the range from $3 \times 10^{-3}$ to $8 \times 10^{-3}$ in steps of $1 \times 10^{-3}$. As outlined in Section 4.2, according to the similarity criteria represented by equation (4.15) and the geometric scale-up factor of four, this range corresponds to the range 0.48 to 1.28 m water measured on the motored engine model investigated by Vafidis & Whitelaw [1984a] which was fitted with a geometrically similar valve/port assembly.

The measured mass flow rate, $\dot{m}$, is plotted in Figures 5.1 as a function of the dimensionless lift, $L$, and pressure drop, $\Delta P$: $\dot{m}$ was
The theoretical mass flow rate is defined as (Shames [1982]),

\[ \dot{m}_{th} = \rho_e Q_e A_e \]  

(5.2)

where \( A_e \) and \( Q_e \) are, respectively, the area of and the mean velocity normal to the exit plane \( E-E \) in Figure 5.2.

The mean velocity, \( Q_e \), in the above expression was determined by assuming isentropic flow between the inlet and outlet planes, \( I-I \) and \( E-E \), respectively, in Figure 5.2, such that (Shapiro [1958]),

\[ Q_e = \frac{1}{\sqrt{1 - \frac{A_e}{A_i}^2}} \sqrt{\frac{2\gamma T_i}{T_e}} \cdot \frac{p_i}{p_e} \left( 1 - \frac{p_e}{p_i} \right)^{\gamma - 1/\gamma} \]  

(5.3)

where the subscripts \( i \) and \( e \) refer to the inlet and exit planes, respectively, across which the static pressure drop, \( \Delta p \), is being measured, and \( \gamma \) is the ratio of specific heats. In order to relate the coefficient of discharge to the bulk in-cylinder conditions, \( p_e \) was taken as the prevailing pressure inside the cylinder measured at point (b) in Figure 4.3.

However, for the low-Mach number conditions of the present measurements, \( Q_e \) may alternatively be determined to an error of less than 0.5% from the incompressible flow relation

\[ Q_e = \frac{1}{\sqrt{1 - \frac{A_e}{A_i}^2}} \sqrt{\frac{2\Delta p}{\rho}} \]  

(5.4)

where \( \Delta p \) is the static pressure drop between the inlet and exit planes.

The exit area, \( A_e \), in equation (5.2) has been variously defined by previous researchers. For example, Kastner, et al. [1964] considered...
the outlet passage formed by the valve and seat faces (cf. Figure 5.2) as a converging diverging nozzle and defined $A_e$ as its minimum, or throat, area. Since the geometry of this nozzle is a function of the valve lift, the following three configurations were identified by the above authors according to the minimum flow area associated with each. In the notation of Figure 5.3a, these configurations are:

1) $2t/D > l/D > 0$

In this range of lifts, the minimum area corresponds to a frustrum of a cone of height $l/2$ perpendicular to the valve face. This is calculated as

$$A_e = \frac{\pi l}{\sqrt{2}} [D + l/2]$$  \hspace{1cm} (5.5a)

ii) $[t + \sqrt{\left(\frac{D^2 - d^2}{4(D + t)}\right)^2 - t^2}] / D > l/D > 2t/D$

Here, the cone of minimum area is no longer perpendicular to the valve face and $A_e$ is given by

$$A_e = \pi (D + t) \sqrt{(l - t)^2 + t^2}$$  \hspace{1cm} (5.5b)

iii) $l/D > [t + \sqrt{\left(\frac{D^2 - d^2}{4(D + t)}\right)^2 - t^2}] / D$

The minimum area now becomes constant and equal to the port duct area minus the cross-sectional area of the valve stem, i.e.

$$A_e = \pi/4 (D^2 - d^2)$$  \hspace{1cm} (5.5c)
The variation of $A_e$ with valve lift based on the above definitions is illustrated in Figure 5.3b. The application of these definitions to the measurements of Kastner, et al. [1964], Vafidis & Whitelaw [1984a,b,c], and Bicen, et al. [1985] produced values for the coefficient of discharge greater than unity which are unrealistic. This was due to the fact that the pressure, $p_e$, at the minimum area was taken to be that at the exit plane, plane E-E in Figure 5.2, which was in turn equated to the pressure inside the cylinder.

For isentropic flow, however, the correct pressure at the nozzle throat, $p_e'$, may be obtained by first determining the critical throat area, $A^*$, from the following expression (Oswatitch [1965]),

$$A^* = A_e \sqrt{\left( \frac{2}{\gamma-1} \right) \left( \frac{\gamma+1}{2} \right)^{\gamma+1/\gamma-1} \left( \frac{p_e'}{p_0} \right)^{2/\gamma} \left[ 1 - \left( \frac{p_e'}{p_0} \right)^{\gamma-1/\gamma} \right]} \quad (5.6)$$

where $p_0$ is the stagnation pressure at the nozzle exit ($p_e + \Delta p$), and $A_e$ and $p_e$ are the area and static pressure at the nozzle exit plane, respectively. With the value of $A^*$ being determined, $A_e$ in equation (5.6) is replaced by the minimum flow area, $A_e$, calculated from either of equations (5.5), and the corrected pressure at the nozzle throat, $p_e'$, may thus be determined by solving the equation

$$A^* = A_e \sqrt{\left( \frac{2}{\gamma-1} \right) \left( \frac{\gamma+1}{2} \right)^{\gamma+1/\gamma-1} \left( \frac{p_e'}{p_0} \right)^{2/\gamma} \left[ 1 - \left( \frac{p_e'}{p_0} \right)^{\gamma-1/\gamma} \right]} \quad (5.7)$$

This analysis is valid, as mentioned above, for compressible flow situations. Under the low-Mach number conditions considered here, the flow may be treated as incompressible and the application of the equations of conservation of mass and energy between the exit and minimum area planes yields the following expression for $p_e'$ (Shames [1982]),
where \( Q_E \) is determined from either equation (5.3) or equation (5.4).

Tanaka [1929] and Annand [1969], on the other hand, defined \( A_c \) in equation (5.2) as the peripheral valve lift, or curtain, area given by (cf. Figure 5.3),

\[
p_e = p_E - \frac{p}{2} \left[ \frac{Q_E^2}{A_c^2} - 1 \right]
\]

and took \( p_e \) to be the pressure prevailing inside the cylinder. Since the streamlines in the outlet passage should, ideally, remain attached and parallel to the flow boundaries, i.e. the valve and seat faces, it is suggested that definition (5.9) be modified into

\[
A_2 = \pi (D + 2t) L \sqrt{2}
\]

where the seat angle is 45° in the present configuration.

The variation with valve lift of the exit area \( A_c \) as determined from equations (5.9) and (5.10) are also shown in Figure 5.3b.

Figure 5.4 shows an example of the values of the coefficient of discharge obtained from the present measurements based on the various definitions given above for \( A_c \) and \( p_e \). Also shown are extracts from the measurements of Vafidis & Whitelaw [1984a] carried out on a geometrically similar valve/port assembly. The values of \( C_D \) based on the minimum flow area given by equations (5.5) and the corrected pressure obtained from either equation (5.7) or equation (5.8) were found to be identical to those obtained by employing the normal curtain area given by equation (5.10) and the prevailing in-cylinder pressure. These are plotted on a single curve in Figure 5.4. The calculations
based on the minimum flow area and the in-cylinder pressure are seen to produce unrealistically high \( C_D \) values for both the present measurements and those of Vafidis & Whitelaw [1984a] which are further seen to be in close agreement. The relatively lower values obtained with equation (5.9) are due to an overestimated exit area used for calculating the theoretical mass flow rate from equation (5.2).

Since the calculation of \( A_e \) from equation (5.10) is simple and straightforward, this definition was adopted here and \( p_e \) was taken as the prevailing pressure inside the cylinder.

In Figure 5.5, the coefficient of discharge, \( C_D \), as defined above, is plotted against the dimensionless lift \( L \) for various values of the dimensionless pressure drop, \( \Delta P \), across the valve. At small lifts, between \( L = 0.02 \) and \( 0.07 \), \( C_D \) is seen to increase steadily with an increase in both \( \Delta P \) and \( L \). At the lowest lift, ie \( L = 0.02 \), \( C_D \) increases rapidly from about 0.71 to 0.86 as \( \Delta P \) increases from \( 3 \times 10^{-3} \) to \( 8 \times 10^{-3} \). At this lift Reynolds number, based on the exit conditions and defined as

\[
Re = \frac{Q_e L}{\nu}
\]  

is seen in Figure 5.6 to increase from \( 2 \times 10^3 \) to \( 4.5 \times 10^3 \) between the lower and upper values of \( \Delta P \). This increase in \( Re \) is apparently associated with significant reduction in the thickness of the boundary layers on the valve and seat faces and, consequently, in the viscous effects in the outlet passage [Batchelor (1983)]. This in turn leads to the improvement in \( C_D \) noted above. At higher lifts Figure 5.6 further shows that both the initial value and the rate of change of \( Re \) with \( \Delta P \) increase due to the increase in \( \dot{m} \) with \( L \) (c.f. Figure 5.1). However the variation of the coefficient of discharge with Reynolds number, illustrated in Figure 5.7 shows that at \( Re = 1 \times 10^4 \), the rate of
increase of $C_D$ with $Re$ becomes and remains very slow, suggesting that the flow in the outlet passage has become fully turbulent.

However, Figure 5.5 shows that at $L = 0.07$, $C_D$ reaches a maximum of about 0.925 for $\Delta P = 3 \times 10^{-3}$ which increases to 0.945 for $\Delta P = 8 \times 10^{-3}$ as a result of the higher $Re$ associated with higher pressure drops, as shown in Figure 5.6. Figure 5.5 further shows that an increase in the lift to $L = 0.09$ is associated with a slight drop in the coefficient of discharge which will be explained later. At still higher $L$, $C_D$ recovers relatively slowly to a slightly higher maximum of about 0.95 at $L = 0.17$. This may be explained on the basis that the nearly linear variation of Reynolds number with lift shown in Figure 5.6 between $L = 0.02$ and 0.175 indicates that the average velocity at the valve exit, $Q_e$ (cf. equation (5.11)), remains nearly unchanged within this range of lifts. Therefore, the drop in the friction factor associated with the increase of $Re$ (Shames [1982]), together with the increase in lift, lead to the reduction of the friction losses in the outlet passage.

At $L = 0.18$, $C_D$ starts to fall rapidly and continues to drop for still higher lifts. This may be explained by a change of the flow pattern in the outlet passage, as will be further discussed below.

The results presented in Figures 5.5 to 5.7 show that, while $C_D$ exhibits definite features at various fixed lifts, its values are almost independent of the pressure drop and the associated variations in Reynolds number, except, as mentioned earlier, at very low lifts. With reference to the analysis outlined in Section 4.2, on the basis of which the relationship (5.1) was established, it may be concluded that, for a valve/port assembly of given geometry, the lift represents the most influential parameter characterising its overall behaviour. Use was made of this observation in the velocity field measurements
discussed in the following section.

The behaviour of $C_p$ discussed above suggests that the range of lifts considered may be divided into the four regions shown in Figure 5.8. Within each of these, a distinct flow pattern prevails in the outlet passage, as was first noted by Tanaka [1929]. In order to determine these patterns, the flow was visualised by observing the motion of a thin layer of white-pigmented oil mixture over the valve and seat faces, as well as over the surface of a 1.5 mm thick aluminium plate inserted radially in the valve crown. The plate was shaped to follow the port surface and extended into the outlet passage. It was covered with black plastic to enhance the contrast between the surface and the oil mixture, as shown in Figures 5.10 to 5.12. The mixture consisted of titanium dioxide powder emulsified in silicon oil using oleic acid in the proportions 3, 8 and 1 by volume, respectively. Medical grade paraffin (kerosen) was added to control the thickness of the mixture.

The flow visualisation photographs shown in Figures 5.9 to 5.12 were obtained under a static pressure drop, $\Delta P$, of $6 \times 10^{-3}$. Within the various regions defined in Figure 5.8, these plates show different flow patterns, as will now be described.

(a) $0.07 > L > 0$

For small lifts, the boundaries of the outlet passage are so close that the flow is forced to remain attached to both the valve and seat faces, as seen in Figures 5.9. The coefficient of discharge in this range increases with lift, as noted earlier, as a result of the increase in Reynolds number and the consequent reduction in the viscous effects.
The photographs for this region, Figures 5.10b and 5.10c, show a vena contracta in the outlet passage resulting from a recirculation bubble attached to the valve face, which is well defined in Figure 5.10a by the thick layer of paint on this face. The size and location of this recirculation bubble are further seen to vary with lift in Figures 5.10c and 5.10d. The separation of the flow occurs at the valve entry corner due to the sharp curvature there.

The flow reattaches to the valve face at a location that is seen from Figures 5.10c and 5.10d to depend on the lift. The restriction of the flow passage thus created causes $C_D$ to vary in this lift range, as shown in Figure 5.8, which suggests that the maximum restriction to the flow is reached at $l = 0.09$ where $C_D$ drops to a minimum of about 0.91.

The measurements of Annand [1969] and Vafidis and Whitelaw [1984a], and Bicen, et al. [1985] for geometrically similar, but sharp-edged, valves suggested that the separated flow in their cases did not reattach on the valve face and thus extended downstream to the exit plane. Consequently, a considerable drop in the value of the coefficient of discharge was observed over the remaining range of lifts studied.

Figures 5.10 also show that the flow after the vena contracta expands and fills the outlet passage, and thereby produces partial recovery of the pressure.

It should be mentioned, however, that in the measurements of Vafidis & Whitelaw [1984a] and Bicen, et al. [1985] in the original round-edged valve/port assembly on which the present model is based, and those of Kastner, et al. [1964] in a geometrically similar valve, seemed not to have produced this particular flow pattern, since in their case $C_D$ continued to increase, joining regions
A and C in Figure 5.8: this can be seen in the measurements of Vafidis & Whitelaw [1984a] illustrated in Figure 5.4. These differences are believed to be due to slight dissimilarities in the rounding of the valve corner to which the flow is believed to be strongly sensitive.

(c) $0.175 > L > 0.12$

The flow pattern in this lift range, shown in Figures 5.11, is almost identical to that of the previous region, except for the presence of a thin separation zone near the rounded seat corner, indicating the tendency of the flow to separate at the seat corner. This, however, does not seem to have an appreciable effect on $C_p$ which is seen from Figure 5.8 to remain nearly constant at about 0.95.

(d) $L > 0.175$

The effectiveness of the valve in forcing the flow to remain attached to the seat has now diminished. Accordingly, the separation at the seat corner, observed in the previous regime, now extends to the exit plane, as shown in Figures 5.12. The resulting reduction in the effective flow area observed in Figure 5.12c leads to the observed drop in the coefficient of discharge seen in Figure 5.4 and the subsequent plots. As the lift increases, the size of this recirculation zone also increases, causing a further drop in $C_p$.

Even after the lift exceeds the limiting value given in equation (5.5c), beyond which the exit area becomes constant and equal to the port area (cf. Figure 5.3), the continuing fall in $C_p$ indicates the continuous increase with lift of the size of the separation zone in the outlet passage.

The various flow patterns presented above are schematically
illustrated in Figures 5.13. The development of these patterns will be further discussed when the detailed measurements of the velocity field are presented.

5.3 VELOCITY FIELD MEASUREMENTS

The foregoing discussion of the $C_P$ measurements suggests that the flow pattern in the outlet passage and its strong dependence on $L$ determines the overall behaviour of this valve/port assembly. However, for lifts higher than 0.2, the flow pattern shown in Figure 5.13d remains unchanged, as may be seen from Figures 5.12, despite the changes in the size of the separation zone. Therefore, the detailed measurements of the flow field were performed for different dimensionless lifts ranging between 0.05 and 0.25 ($\ell = 5 \text{ mm to } 25 \text{ mm}$). The influence of the pressure drop across the valve, and the associated changes in Reynolds number, were further seen to be minor over most of the lift range examined in the previous section. Accordingly, a single value for the pressure drop of $5 \times 10^{-4}$ was chosen which corresponded to Reynolds numbers ranging between $2.5 \times 10^{4}$ and $3.6 \times 10^{4}$ at the lowest and highest lifts, respectively. An additional set of measurements was performed for a pressure drop of $3 \times 10^{-3}$ at selected lifts and locations along the flow passage to investigate the relatively large effect of pressure drop at very low lifts observed earlier.

For the measurements of the mean velocity and turbulence quantities, the hot-wire analogue systems described in Section 4.4 and shown in Figures 4.10 were used. The single-wire measurements were performed to provide more accurate information on the magnitude of the mean velocity vector since they suffered less from the broadening effects mentioned in Section 4.4.2 than the X-wire system. The latter measurements, on the other hand, provided the two components of the
velocity vector, as well as the turbulent normal and shear stresses. For this purpose, the method developed in Section 4.4.1 was adopted for processing the probe signals.

It is generally recognised that hot-wire measurements fail to provide reliable information on the flow parameters if rectification of the signal occurs due to reversal of the direction of the flow at the measuring point. Therefore, the flow visualisation photographs shown in Figures 5.9 to 5.12 were used for approximately determining the regions of validity of the hot-wire measurements.

The measuring locations along the flow passage are shown in Figure 5.14. The measurements along the port duct were intended to provide information about the flow as it approaches the valve exit region. Due to space limitations, the L-shaped probes DISA 55 P14 and 55 P63 were used within this part of the test section with the probe support being inserted radially in the flow field through 6 mm diameter holes in the duct wall.

Because of the complex geometry of the boundaries of the port exit, certain regions of the flow domain were inaccessible to the probes and no measurements could be performed there. Moreover, traverse 11 in Figure 5.14 was accessible only at $L = 0.25$; and at $L = 0.05$, the large dimensions of the probe relative to the outlet passage made measurements possible only across the valve exit plane.

At the measuring locations near and in the port exit region, Figure 5.2, the yaw angle, $\psi$, of the flow relative to the single-wire sensor shown in Figure 4.19 was expected to be considerable. It is difficult to account for the effect of this yaw angle on the probe response without \textit{a priori} knowledge of its value (Merati & Adrian [1984]). Therefore, the single-wire probe was calibrated for different yaw angles, Appendix B, and within a range of velocities
between 10 m/s and 20 m/s, a yaw angle of 45° produced a maximum calibration error in the mean velocity of about 3%.

In the outlet passage, the measuring points were more closely spaced in order to detect the rapid changes the flow undergoes in this region. Here, the single and X-wire straight probes DISA 55 P11 and 55 P61, respectively, were placed within the outlet passage with their axes parallel to the valve and seat faces.

5.3.1 The Mean Velocities

The measured (with the X-wire probe) profiles of the dimensionless mean velocity, \( Q/U \), at the inlet section, station 1 in Figure 5.14, are shown in Figure 5.15 for the various lifts and \( \Delta P = 5 \times 10^{-3} \). The normalising velocity, \( U \), is taken as the radially averaged mean velocity determined from,

\[
U = \frac{\sum_{i=1}^{n} (r_i \cdot U_i) \cdot \Delta r}{(R_o^2 - R_i^2)}
\]  

(5.12)

where \( U_i \) is the axial mean velocity component at radius \( r_i \), \( \Delta r \) is the radial distance between two successive points, and \( n \) is the number of measuring points across the inlet section whose inner and outer radii are \( R_i \) and \( R_o \), respectively. The mean flow angle measured from the axial direction was negligibly small, typically ±0.4°, and \( U_i \) in the above expression could thus be replaced by the respective magnitude, \( Q_i \), of the mean velocity vector without appreciable loss of accuracy.

Figure 5.16 shows comparison between \( U \) values for different lifts as determined from the hot-wire measurements according to equation (5.12) and from flow rate measurements. The observed differences are attributed to the difficulty of achieving highly accurate calibration of the hot-wire system at such low speeds (Seifert & Graichen [1982])
which may further explain the increasing difference at smaller lifts.

The mean velocity profiles in Figure 5.15 indicate the presence of a potential core-like region in the central part of the inlet section and show considerable difference in the thickness of the boundary layer on the upper and lower walls. This is due to the relatively short distance between the leading edge of the lower surface (cf. Figure 4.7) and the measuring location which does not allow the boundary layer to become as developed at the latter. At lower lifts, the thickness of the boundary layer on the lower wall is seen to increase slightly as a result of the associated decrease in Reynolds number (Schlichting [1968]).

The mean velocity distributions across the upper and lower boundary layers are illustrated in Figure 5.17 in the form of Clauser's chart (Clauser [1954]). They indicate that across both boundary layers, the velocity distributions obey the universal logarithmic law (Hinze [1975]),

$$\frac{U_m}{U_\tau} = 2.44 \ln \frac{U_\tau Y}{\nu} + 4.9 \quad (5.13)$$

where $U_m$ is the free stream velocity, taken here as the mid-height mean velocity, $Y$ is the distance from the wall, and $U_\tau$ is the shear velocity obtained from (Clauser [1954]),

$$U_\tau = \frac{U_m}{\sqrt{2/C_\delta}} \quad (5.14)$$

The values of $C_\delta$ shown in Figures 5.17 agree, on the upper wall, with smooth pipe data (Nikuradse [1933]), while on the lower wall they indicate an increase in the relative roughness to about 0.002.

In the converging passage, the flow accelerates and, as a result of the passage asymmetry, the profiles of the magnitude of the
mean velocity vector, $Q$, assume the shapes shown in Figure 5.18a at station 2. The mean flow angle, $\alpha$, measured from the axial direction in an anti-clockwise direction, illustrated in Figure 5.18b, varies almost linearly in the radial direction, except near the upper and lower boundaries. The deviation from the expected variation near the upper wall, represented by the dashed line, is believed to emerge from the disturbances in the flow caused by the probe stem and around the hole through which the probe was inserted into the passage. This deviation, increasing and extending in the radial direction with smaller lifts, further indicates an increasing uncertainty in the measurements in this region.

In the following discussion, the measurements of the mean velocity in the port duct and the outlet passage are presented and discussed for the $L = 0.25$ configuration first and then the other lifts are considered.

For incompressible flow, the stream function, $\psi$, is defined by (Batchelor [1983]),

$$\psi = \int U r \, dr$$  \hspace{1cm} (5.15)

where $U$ is the axial velocity component at a distance $r$ from the axis of symmetry. In the port duct, this expression was approximated by (see Figure 5.19a),

$$\psi = \sum_{i=1}^{n} (U \Delta r) \Delta r \Delta \alpha$$  \hspace{1cm} (5.16)

and in the outlet passage, Figure 5.19b, as
\[ \Psi = \sum_{l=1}^{n} (U \Delta x)_l \cdot r_l \]  \hspace{1cm} (5.17)

where \( n \) is the number of measuring points across traverse \( I \).

When calculating \( \Psi \) from either equations (5.16) or (5.17), next to a wall boundary a parabolic velocity distribution was assumed for simplicity. In the port duct, the maximum error produced by this approximation in the value of the total discharge \( \dot{V} \) determined from

\[ \dot{V} = 2\pi \Psi \]  \hspace{1cm} (5.18)

ranged between 0.5% and 6.5%, as compared with the orifice plate measurements. The largest differences occurred at stations crossing recirculation zones in the port exit, as further explained below. In the outlet passage, however, assessment of the error produced by approximation (5.17) was not possible due to the presence of the recirculation zones on either or both the valve and seat faces shown in Figures 5.9 to 5.13. As was mentioned earlier, hot-wire measurements cannot provide reliable information in recirculating flows due to signal rectification effects. It is possible, however, to determine the approximate locations of the streamline bounding the recirculation zone adjacent to one surface, provided that the flow remains attached to the other form which the integration (5.15) commences. From the discussion presented in Section 5.2, it was found that the separation at the seat corner is the dominant feature of the flow pattern in the outlet passage for lifts higher than 0.17. The recirculation bubble attached to the valve face was further found to be relatively small compared to the dimensions of the outlet passage and was accordingly neglected, and the summation (5.17) thus proceeded from the valve to
the seat face.

Based on the mean velocity measurements and the calculation procedure outlined above, the mean velocity vectors and the streamlines for \( L = 0.25 \) are illustrated, respectively, in Figures 5.20 and 5.21. In addition to the separation from the seat corner, Figure 5.21 shows a recirculation zone attached to the root of the valve crown, whose presence was also detected from the analysis of the turbulent stresses measurements presented in the next subsection. It should be remembered that the sizes of both recirculation zones are only approximate due to the high and unquantifiable levels of uncertainty associated with the steep velocity gradients and the reversal of the flow at and across their boundaries. They are used, however, in the following discussion to roughly identify the regions of invalid measurement. Also in Figure 5.21, the estimated streamlines, in the regions of the flow where no measurements were performed, are plotted as dashed lines.

Figures 5.22 illustrate the profiles of the mean flow angle, \( \alpha \), and the dimensionless magnitude of the mean velocity vector, \( Q/\bar{U}_3 \), along the port duct and the outlet passage. The normalising velocity, \( \bar{U}_3 \), is the average mean velocity at the port inlet, station 3, determined according to equation (5.12) and given in Figure 5.23 for the various lifts. Figure 5.22a shows a nearly uniform velocity distribution across most of the port inlet, except near the upper wall where a slight increase in the velocity occurs as a result of the local acceleration along the upstream convex curvature (Smits, et al. [1979]). In the constant cross-sectional area zone between stations 5 and 9, the velocity profiles exhibit a gradual deformation in the form of deceleration near the valve stem and acceleration near the port wall. This is attributed mainly to the adverse pressure gradient associated with the streamline curvature induced by the crown surface. The
influence of this pressure gradient apparently extends upstream of the port exit region, between stations 9 and 12, where the streamlines suffer appreciable curvature, causing the steady and gradual deceleration near the valve stem. With the cross-sectional area being constant, this deceleration is compensated for by an acceleration, and accordingly a favourable pressure gradient, near the port wall.

The port duct between stations 5 and 9 may therefore be considered as divided by the mid-height streamline, along which the mean velocity remains nearly unchanged, into a lower region dominated by a positive streamwise pressure gradient and an upper region of negative pressure gradient. This observation was supported later on by the pressure field results obtained from the numerical predictions presented in Chapter 6. At this point, however, no effects of the adverse pressure gradient, other than the deceleration mentioned above, could be detected from the mean velocity profiles; it will be seen later that it leads to separation from the valve stem and the formation of the recirculation zone shown in Figure 5.21 on the crown root.

In the port exit region, the flow accelerates due to the continuous reduction in the cross-sectional area. The flow near the port wall approaches the entrance of the outlet passage, station 12, in an almost axial direction, Figure 5.20, while fluid next to the crown surface is expected to leave it nearly radially. The flow near the port wall, in diverting towards the outlet passage, transfers part of its axial momentum to the fluid next to the valve side, causing the diversion of the velocity vectors there from the expected radial direction as shown from the flow angle distribution at station 12 in Figure 5.22b.

The development of the profiles of the magnitude and direction of the mean velocity vectors in the outlet passage indicates
that the deflection of the flow towards the valve face continues to station 15, which is half way to the exit plane. Accordingly, the flow next to the valve face is forced to remain parallel to it and if it separates at the entrance corner, it soon reattaches, as shown earlier in Figure 5.13. On the other hand, the flow next to the seat face at this lift is encouraged to separate at the seat corner and remains detached, as also shown in Figure 5.13 and in Figure 5.21.

The approximate locations of the streamline bounding the recirculation zone next to the seat face are identified at the different stations in Figure 5.22b by the vertical dashed lines as determined from Figure 5.21. Excluding this region of recirculation, Figure 5.22b shows that towards the exit of the outlet passage, the high cross-stream velocity gradients gradually smooth out. At the exit plane itself, the velocity distribution becomes nearly uniform and unidirectional, with the vectors parallel to the valve and seat faces.

The velocity vectors and the streamlines for the other configurations investigated, namely $L = 0.2, 0.15, 0.10$ and $0.05$, are illustrated in Figures 5.24 and 5.25, respectively. In the constant cross-sectional area length of the port duct, they generally do not indicate appreciable differences from those shown in Figures 5.20 and 5.21 for $L = 0.25$. However, for the smaller lifts, Figures 5.25b, 5.25c and 5.25d, show that the point of separation on the valve stem shifts upstream to lie between stations 6 and 7. Evidence supporting this observation is discussed in the following subsection.

In the port exit region at the lower lifts, the valve crown causes an increasingly earlier outwards diversion of the axial flow approaching the outlet passage, as shown from the velocity vectors in Figures 5.24 at its entrance. At $L = 0.2$, this results in the reduction in the size of the recirculation zone near the seat face.
shown in Figure 5.25a. Accordingly, the effective outlet area at the valve exit increases, leading to the improvement in the coefficient of discharge shown in Figure 5.5. For $L = 0.15$, the flow which separates at the seat corner is now forced to reattach to the seat face forming the thin recirculation bubble shown in Figure 5.25b and leading to the further increase observed in $C_D$. At the smaller lift of 0.1, the velocity vectors next to the seat face are seen in Figure 5.24c to be nearly parallel to it at the entrance of the outlet passage and separation round the seat corner is thereafter suppressed. However, the recirculation bubble on the valve face at these lifts, shown in Figures 5.10 to 5.12, was apparently too thin to be detected in the present measurements.

As explained earlier, for the $L = 0.05$ configuration, no measurements could be carried out in the outlet passage, except at its exit. In order to obtain measurements as close to the walls as possible, these were performed across a traverse 1 mm above plane E-E in Figure 5.2. The velocity vectors across this traverse, shown in Figure 5.25d, indicate a nearly uniform unidirectional flow, except at the point closest to the cylinder head. This is believed to be an influence of the large recirculation zone extending over the cylinder head and part of its wall which was detected in previous measurements and predictions (see, for example, Bicen [1983], Bicen, et al. [1985], and Ahmadi-Befrui [1985], as well as in the present predictions discussed in the next chapter).

The profiles of the magnitude and direction of the mean velocity vectors for these configurations are shown in Figures 5.26 to 5.29 with the normalising velocity being the average mean velocity at the port inlet shown in Figure 5.23. As the lift decreases, the profiles at corresponding stations in the port duct, Figures 5.26a to
5.29a exhibit two main features:

(i) a steady increase in the level and rate of change of the flow direction, particularly within the lower part of the passage; and

(ii) earlier deformation in the velocity profiles associated with an increase in the acceleration and deceleration in the upper and lower parts of the passage, respectively.

The changes in the mean velocity profiles with lift are more clearly evident in Figure 5.30, where the profiles are assembled at selected stations along the port duct and normalised by the local mid-height mean velocities.

The curvature of the streamlines induced by the crown surfaces, starting earlier with lower lifts, causes the positive pressure gradient near the valve surface to extend upstream in the port duct. This, in turn, leads to the observed earlier deceleration within the lower part of the passage and, consequently, the acceleration near the upper wall. Also, under the extending influence of this adverse pressure gradient, the point of separation on the valve stem recedes upstream, producing, it is believed, an increase in the width of the recirculation zone attached to the crown root. The presence of this recirculation may be detected at small lifts from the irregular velocity distributions very close to the valve, e.g. at stations 6 and 7 for \( L = 0.1 \) and 0.05, Figures 5.28a and 5.29a, respectively. In the port exit region, and as a result of the reduction in the cross-sectional area with smaller lifts, the flow at the corresponding stations accelerates more rapidly while suffering an increasingly sharper deflection towards the outlet passage.
The velocity profiles in the outlet passage for various lifts, illustrated in Figures 5.26b to 5.29b, show trends similar to those observed in Figure 5.22b for \( L = 0.25 \). These include the relatively higher accelerations round the valve and seat corners and the continuing deflection of the flow towards the valve face down to about station 15. The mean flow angle profiles at station 12 further show the flow near the seat face becoming more aligned with it at lower lifts and thus reducing the possibility of separation at the seat corner. Beyond station 15, the steep cross-stream velocity gradients gradually diminish, and at the exit plane the profiles invariably indicate an almost unidirectional uniform velocity parallel to the valve and seat faces. As a result of the reduction in the cross-sectional area associated with smaller lifts, the acceleration of the flow into the outlet passage increases. Interestingly, the reduction in the effective outlet area at high lifts, namely \( L = 0.25 \) and 0.2, due to the presence of recirculation regions, cause the maximum mean velocity at the valve exit to vary relatively little over the whole range of lifts, typically between 30.9 m/s for \( L = 0.25 \) and 31.7 m/s for \( L = 0.05 \).

As was mentioned in the beginning of this section, single-wire measurements were carried out for the purpose of providing an independent check on the X-wire measurements of the mean velocity. Comparisons between the two measurements are shown in Figure 5.31a, which illustrates the profiles of the dimensionless magnitude of the mean velocity vectors for \( L = 0.1 \). This particular lift was chosen mainly because separation did not occur in the outlet passage and any differences there should be attributed to effects other than the uncertainties associated with reversal of the flow.

The profiles shown in Figure 5.31a show a fairly good
agreement between the measurements of the two probes in the port duct where the flow is nearly axial, i.e. down to station 7. The difference seen at station 7 at the first point near the lower wall is probably due to the separation of the flow in this region. Between stations 8 and 10, the yaw angle between the mean flow and the single wire sensor increases steadily, as shown in Figure 5.28a. The calibration errors in the single-wire measurements (cf. Figure B.2) are believed to be responsible for the differences seen in Figure 5.31a at these stations. This is supported by the evidence that these differences are significant only within the lower part of the passage where the flow angle assumes its highest values. A maximum difference of 4.5% occurs at station 8 where the flow angle is 8°: this increases to about 10% for a flow angle of 30° at station 10. These figures are considerably higher than those reported earlier, and shown in Figure B.2, due to the signal linearisation effects included in the present measurements.

These calibration errors are further believed to produce the small differences (<4.0%) in the velocity profiles across the entrance of the outlet passage. Further along this passage, the flow becomes more aligned with its boundaries, and hence the probe axis, and these differences diminish. Across the exit plane, where the flow is nearly unidirectional and parallel to the valve face, the agreement between both measurements improves markedly.

Figures 5.31b also illustrate profiles of the mean velocity and the flow angle in the vicinity of the valve exit extracted from the LDA in-cylinder measurements of Vafidis & Whitelaw [1984a]. These were obtained across a traverse 0.037D above the exit plane and at lifts different from those considered in the present investigation, namely, 0.157, 0.225 and 0.3. Therefore, close comparison between these and the present measurements was not attempted and they are
referred to here to roughly validate the trends and flow patterns at
the exit plane detected in the latter.

Examination of the profiles shown in Figure 5.31b for
$L = 0.157$ and those obtained from the present measurements at $L = 0.15$
indicates fully attached flow in both cases. It also indicates close
agreement in the magnitudes and directions of the mean velocity vectors.
At $L = 0.225$, the LDA mean velocity measurements show the zone of
reversed flow near the cylinder head which was detected previously at
$L > 0.2$ (cf. Figures 5.21 and 5.25). The width of this zone is seen
to lie somewhere between those at $L = 0.20$ and $L = 0.25$, roughly
identified by the vertical dashed lines at both lifts. In the region
of outflow, the profiles at $L = 0.20$, 0.225 and 0.25 are nearly similar
and show a fairly uniform unidirectional flow at the valve exit. The
same trends are further seen to prevail at $L = 0.3$, except for the
relatively slight increase in the width of the recirculation zone over
that observed at $L = 0.25$. The slower rate of decrease in $C_D$ between
$L = 0.25$ and 0.30 shown in Figure 5.4 may therefore be attributed to
the slower rate of increase in the relative width of the recirculation
zone as compared to that observed between $L = 0.2$ and 0.25.

5.3.2 The Turbulent Stresses

The turbulent normal and shear stresses, $\overline{u'^2}$, $\overline{v'^2}$ and $\overline{u'v'}$,
were determined using the method described in Chapter 4 for processing
the X-wire signal. The principal axes for resolving the calculated
stresses were taken in the axial and radial directions, except in the
outlet passage where they were taken in the directions parallel and
normal to the valve and seat faces.

In the calculation procedure for signal processing outlined
in Section 4.4.1, the lateral stress component $\overline{w'^2}$ had to be known from
measurements or given an assumed value. Measuring $\overline{w'^2}$ using an X-wire array requires precise manipulation of the probe orientation within the flow field (Thompson [1983]). This proved to be difficult in the present investigation due to the complex geometry of the flow passage and the configuration of the probes used. Accordingly, $\overline{w'^2}$ was assumed here to be the mean of $u'^2$ and $v'^2$, broadly in accordance with the boundary layer measurements of Klebanoff [1954]. The uncertainties in the calculated stresses resulting from different assumptions for $\overline{w'^2}$ were investigated earlier in Section 4.4, where a ±10% difference in $\overline{w'^2}$ from the above value produced a difference of ±2.5% in $u'^2$ and $v'^2$ and a difference of ±5% in $u'^v'$. 

The profiles of the normalised normal and shear stresses at the inlet section, station 1 in Figure 5.14, are shown in Figure 5.32 for the various lifts with the normalising velocity, $Q_m$, being the respective mid-height mean velocity. In the potential core region, turbulence is nearly isotropic with $u'^2$ and $v'^2$ falling to about 0.15%. These residual normal stresses are due to the upstream mesh screens (cf. Figure 4.3). The stress distributions near the upper and lower walls shown in Figures 5.32a and 5.32b are typical of those in boundary layers along smooth walls under zero pressure gradient (Klebanoff [1954]). The asymmetry evident in the thickness of the boundary layers and the maximum values of $v'^2$ and $u'^v'$ is a result of the boundary layer on the lower surface being less developed as argued in the previous subsection. Figures 5.32a and 5.32b further show almost identical profiles due to the fact that at these lifts, the Reynolds number is nearly the same in both cases (cf. Figure 5.15). At lower lifts, the Reynolds number decreases and the distributions of $u'^2$, $v'^2$ and $u'^v'$ near the lower wall shown in Figures 5.32c, 5.32d and 5.32e indicate an extension in the buffer region to cover the
measuring points near the wall (Hinze [1975]). In this region, \( y^* \), defined by equation (3.63), falls below 30, as may also be concluded from Figure 5.17, associated with a reduction in the three stress components. Across the boundary layer on the upper wall, the stress distributions do not seem to undergo significant changes with the decrease in Reynolds number within the range shown in Figures 5.15.

In the converging passage downstream of the inlet section, the turbulence structure of the flow comes under the influence of:

(i) the streamwise acceleration due to area changes and the consequent decay of turbulence featured in the reduction of the relative normal and shear stresses (Jones [1973] and Bradshaw [1973]); and

(ii) the cross-stream velocity gradients arising from the passage asymmetry (cf. Figure 5.18a) and the associated rise in the turbulent energy which is transferred from the mean motion through shear stresses (Bradshaw [1973]).

At station 2, these counteracting effects are seen from Figure 5.33 to produce appreciable reductions in the thickness of the boundary layer on the lower wall, indicated by the nearly equal values of \( u'^2 \) and \( v'^2 \) at the first measuring points: a situation which normally occurs on flat surfaces at \( \gamma/\delta = 0.8^* \) (Hinze [1975]). Figures 5.33 also show that in the central part of the passage, where cross-stream velocity gradients have been generated, all three stress components increase with \( v'^2 \) being generally 50% higher than \( u'^2 \).

Furthermore, as the streamwise acceleration decreases at lower lifts,

* \( \delta \equiv \) displacement thickness of the boundary layer.
the effect of the additional mean strain, $\partial u/\partial y$, becomes more apparent in the form of the steady increase in the turbulent stresses shown in Figures 5.33b, 5.33c and 5.33d. On the upper wall, the boundary layer was either disturbed by the probe stem (cf. Figure 5.18b) or was too thin to be detected in these measurements. In any case, no information on the stress distribution within it could be obtained.

In the following discussion, attention is focused first on the development of the turbulent stresses in the port duct and the outlet passage for the $L = 0.25$ configuration. The discussion will then be extended to the other lifts.

Figures 5.34 illustrate the distribution of the relative normal and shear stresses across stations 3 to 11 in the port duct. The normalising velocity is taken as the local mid-height, or free stream, mean velocity, $U_n$, at the respective stations. Across station 3, isotropy in the central part of the port is seen to be restored and is maintained downstream to station 8 with nearly constant relative stresses of about 0.02%. The departure from isotropy and the changes observed in $u^{12}$, $v^{12}$ and $u^r v^r$ between stations 8 and 11 are discussed further below. The development of the boundary layer on the valve stem under the increasing influence of the adverse pressure gradient discussed in the previous subsection, is featured in the steady, gradual increase in all three stress components between stations 3 and 7. The nearly equal values of $u^{12}$ and $v^{12}$ at stations 3 and 4 indicate that the measuring points closest to the valve stem lie within the outer region of the boundary layer where turbulence becomes isotropic (Hinze [1975]). As the thickness of the boundary layer increases in the downstream direction, $u^{12}$ becomes progressively higher than $v^{12}$, as shown at stations 5, 6 and 7.

Downstream of station 7, the adverse pressure gradient next
to the valve boundary becomes more influential as argued earlier. It was expected to maintain the normal stresses nearly unchanged and slightly increase the shear stress near the wall (Müller [1982a]): however, anomalous steep increases in $u'^2$, $v'^2$ and $u'v'$ occur between stations 7 and 9 and are believed to result from an excessively high distortion in the probe signal in this region. This could be explained only as a result of the separation of the flow from the valve stem somewhere between stations 7 and 8 and the formation of a recirculation zone extending into station 9 and probably farther downstream along the crown surface. The exact extent of this recirculation zone could not be determined from the available data due to signal rectification effects discussed in Chapter 4. Figure 5.21, however, illustrates a crude estimate of this recirculation in the form of the closed dashed contour attached to the crown root: this, it should be stressed, does not have any quantitative significance and was only drawn to roughly identify a region of high measurement uncertainties. This region of uncertainty, and the fact that no close enough measurements could be performed near the valve crown at stations 10 and 11, have together prevented the determination of the details of stress development along the valve.

The turbulent stresses measurements along the upper wall, shown in Figures 5.34, are also believed to be subject to the unavoidable disturbances caused by the probe stem around the holes through which it was inserted into the passage. This view is supported by the comparison with single-wire measurements in this region, discussed later.

In the port exit region between traverses 8 and 12, the turbulence structure becomes subject to the interaction between:
(i) the decay of relative turbulence intensities due to the streamwise acceleration (Jones [1973]) caused by the reduction in cross-sectional area;

(ii) the increase in turbulence energy transferred from the mean motion by the additional mean strains, \( \partial u/\partial y \), developed under the influence of the asymmetric pressure field (Bradshaw [1976], and Townsend [1980]);

(iii) the destabilising effects of the concave streamline curvature induced by the crown surface and the associated increase in normal and shear stresses (Smits, et al. [1979], and Hoffman, et al. [1985]); and

(iv) the collapse of turbulence due to the sharp convex curvature of the streamlines round the valve and seat corners (Smits, et al. [1979], and Muck, et al. [1985]).

It has been found difficult to establish from the available data an account for the magnitudes of the individual effects on the turbulent stresses. However, their combined effects are seen from Figures 5.34 to produce a departure from isotropy first observed at station 9 and continued, at an increasing rate, at stations 10 and 11, and asymmetric increases in \( \overline{u'^2} \), \( \overline{v'^2} \) and \( \overline{u'^2v'^2} \) seen between stations 10 and 11.

At the entrance of the outlet passage, station 12, the relative stress profiles, illustrated in Figure 5.35, indicate an appreciable collapse in all three stress components, except near the seat corner where the velocity gradient across the separation zone is necessarily large. The evolution of these profiles through the outlet passage indicates a steady increase in \( \overline{u'^2} \), \( \overline{v'^2} \) and \( \overline{u'^2v'^2} \) along the valve face as the boundary layer there develops towards the exit plane.
Similar increases are also observed near the boundary of the recirculation zone, identified by the vertical dashed lines at the various stations, extending gradually across the outlet passage under the influence of the large cross-stream velocity gradients prevailing there. Excluding this region, the turbulence across the rest of the outlet passage is seen to remain nearly isotropic downstream to the exit plane.

The profiles along the port duct of the individual stress components for the configurations $L = 0.2, 0.15, 0.1$ and 0.05 are assembled in Figures 5.36 to 5.38, normalised by the local mid-height mean velocity, $Q_m$. Comparison with the respective profiles at stations 3 to 6 in Figure 5.34 shows that for lower lifts, and, consequently, smaller Reynolds numbers, the relative stresses generally increase in the port duct, particularly near the lower wall. This is believed to be due to:

(i) the smaller rates of decay in the relative turbulence intensities in the converging passage at lower mean velocities; and

(ii) the increase in the thickness of the boundary layer at smaller Reynolds number.

For $L = 0.2$, where the average mean velocity in the port duct, and consequently Reynolds number, are nearly the same as those for $L = 0.25$, the relative stresses profiles are almost identical downstream to station 7. The steep increase in $\overline{u'v'}$, $\overline{v'^2}$ and $\overline{u'^2}$ between stations 7 and 8 indicates again the separation of the flow from the valve surface somewhere between these two stations.

For lifts smaller than 0.2, this sudden apparent rise in
stresses is seen to take place between stations 6 and 7, suggesting that the point of separation recedes upstream with decreasing lift. This is believed to be due to the extended influence of the adverse pressure gradient featured in the earlier and steeper deformation in the mean velocity profiles at smaller lifts (cf. Figure 5.30). As noted earlier, this region of recirculation is roughly identified for the various configurations by the dashed contour lines attached to the valve stem and crown root in Figures 5.25.

As the effects of streamline curvature and area changes mentioned above occur earlier in the port duct with lower lifts, the stress profiles accordingly exhibit changes in the stress distributions across the port exit as early as station 7. These are featured in the departure from isotropy in the central part of the passage, and the steady increase in $\overline{u'^2}$, $\overline{v'^2}$ and $\overline{u'v'}$ is particularly significant near the crown surface between stations 7 and 10. At the entrance of the outlet passage, the profiles shown at station 12 in Figures 5.39 to 5.41 invariably indicate collapse of relative turbulent stresses, except near the seat corner for $L = 0.2$ and 0.15, where separation occurs. Within the outlet passage, Figures 5.39 to 5.41 generally show a steady increase in all relative stresses along the valve face higher than that observed earlier for $L = 0.25$. This increase, exhibiting inconsistent trends, is believed to be due to the change in the size and location of the recirculation bubble attached to the valve face which was seen from Figures 5.9 to 5.12 to persist for all lifts higher than 0.07.

For $L = 0.2$, the stress profiles are closely similar to those shown in Figure 5.35 for $L = 0.25$, despite the considerable difference between the two configurations in the size of the separated zone on the seat face. This may again be explained as a result of the
high measurement uncertainties in this region. At $L = 0.15$, the recirculating zone round the seat corner was too thin to be probed, yet the effect of the large velocity gradient in its vicinity is manifested in the profiles by the relatively high stresses near the seat face, which are seen to prevail downstream to the exit. For $L = 0.1$, where no separation at the seat corner occurs, the relative normal and shear stresses across most of the passage remain very small and of order $0.015\%$. At this lift, no measurements could be carried out close to the boundaries of the outlet passage, and for the smallest lift of 0.05 only the exit plane could be probed, across which the relative stresses are seen from Figures 5.39 to 5.41 to be similar to those for $L = 0.10$.

Comparisons between the single and X-wire measurements of the relative turbulence intensity, $\sqrt{\bar{u}^2/\bar{u}_m^2}$, are shown in Figures 5.42 at selected stations along the port duct for the $L = 0.25$ configuration. Both measurements are seen to agree fairly well in the central part downstream to station 10 with a maximum difference in $\sqrt{\bar{u}^2/\bar{u}_m^2}$ of about $0.35\%$. Along the valve stem, the agreement is also seen to be good down to station 7, beyond which the separation discussed earlier renders both measurements as unreliable. This may explain the large differences shown at stations 8, 9 and 10. Furthermore, at stations 10 and 11, the single-wire probe becomes progressively misaligned with the mean velocity vector, particularly near the crown surface. Accordingly, its sensor responds to both the axial and radial fluctuation components, $u'$ and $v'$, respectively, causing the increase shown in the difference between single and X-wire measurements.

Along the upper wall, the single-wire measurements are seen to be generally higher than the respective X-wire measurements. With the flow being axial in this region and parallel to the port wall, these differences can only be attributed to the disturbances in the
flow caused by the probe's stems around the holes in the port wall, through which they were inserted. Therefore, both measurements in this region are considered as involving an unquantified uncertainty and are treated as such in the further discussion presented in the next chapter.

Comparison between both measurements in the outlet passage, shown in Figure 5.43, also indicate good agreement across most of the traverses shown. Close to the boundary of the recirculation zone next to the seat face, the agreement deteriorates as a result of the high velocity gradients and measurement uncertainties prevailing there.

5.4 CONCLUSIONS

An assessment of the overall and detailed characteristics of the selected valve/port assembly under quasi-steady state conditions has been carried out through measurements of the discharge coefficient and velocity and turbulent stress components.

Various definitions of the coefficient of discharge, \( C_D \), were discussed and a modified definition was introduced on the basis of its physical significance and used for the presentation of the results. The measurements of \( C_D \) indicated that the performance of the valve/port assembly is influenced mainly by the valve lift, \( L \). The variation of \( C_D \) with lift suggested the existence of four distinct flow regimes in the outlet passage:

1) at very low lifts, \( L < 0.07 \), the flow remains attached to the valve and seat faces;
2) for \( L > 0.07 \), the flow separates downstream of the valve corner, forming a recirculation bubble on the valve face which thereafter persists with varying size and location;
3) at \( 0.12 < L < 0.17 \), another separation occurs at the seat corner
which reattaches to the seat face, forming a thin recirculation zone round the seat corner; and

iv) for \( L > 0.17 \), the second separation zone extends to the exit plane and dominates the flow in the outlet passage as its width increases with the increase in lift.

These régimes were verified by visualisation of the flow in the outlet passage. The measurements of \( C_D \) further showed that the performance of the valve/port assembly is independent of the pressure drop across the valve (for \( \Delta P \) in the range between \( 3 \times 10^{-3} \) and \( 8 \times 10^{-3} \)), except at very low lifts \( (L < 0.05) \), where viscous effects in the outlet passage become significant.

The mean velocity measurements indicated that the flow develops in the port duct under the influence of an asymmetric pressure field across the flow passage. Near the valve, an adverse streamwise pressure gradient prevails induced by the curvature of the streamlines along the crown surface. This causes the flow to decelerate, and eventually separate from the valve stem forming a recirculation bubble which extends along the crown surface. Near the port wall, a favourable streamwise pressure gradient is created as the flow accelerates there to compensate for the deceleration along the valve surface. The influence of this pressure field was found to extend upstream in the port duct beyond the region where appreciable streamline curvature occurs. At lower lifts, this led to an upstream shift in the point of separation on the valve stem.

The separation at the seat corner which dominated the performance of the valve/port assembly at high lifts was found to be due to the diminishing influence of the crown on diverting the approaching axial flow into the outlet passage. The measurements of the mean flow angle
at the entrance of the outlet passage showed that the velocity vectors next to the seat face become more aligned with it at lower lifts and the possibility of separation at the seat corner is thus reduced.

The mean velocity distribution across the exit plane was found to be nearly uniform for lifts below 0.2, where no recirculation existed at the valve exit, with the velocity vectors being parallel to the valve and seat faces. This observation is important as it provides reliable inlet boundary conditions for in-cylinder numerical predictions in engines fitted with similar valve/port assemblies. The knowledge of the flow direction during the induction stroke becomes particularly important if the piston comprises a combustion bowl.

At \( L = 0.2 \) and 0.25, similar trends in the mean velocity distribution were also observed at the exit plane, excluding the regions of recirculation. Although the sizes of these regions were not small relative to the lifts at which they occurred, the amount of recirculating mass within them would be expected to be very small compared to the total outflow. Thus, with \( C_p \) being known, the inlet boundary conditions at the valve exit at high lifts could be approximated by a plug-type velocity distribution over a distance corresponding to the effective outflow area.

The comparison between the single and X-wire measurements of the magnitude of the mean velocity vectors showed good agreement, except in regions of recirculation where the measurement uncertainties are very high. Also, where misalignment between the axis of the single-wire probe and the mean flow direction prevailed, calibration errors incorporated in the single-wire measurements gave rise to a difference of about 10% between both measurements at a yaw angle of 32°.

Examination of the development of the turbulent normal and shear stresses along the valve stem showed an anomaly in the measurements
featured in a sudden and steep increase in all three stress components between two successive measuring stations. This was attributed to the separation of the flow from the valve stem: however, no precise information could be obtained regarding its exact location and size from the available data.

Across most of the port duct, turbulence was nearly isotropic in the constant cross-sectional area portion and where the streamline curvature and the cross-stream velocity gradients were small. In the port exit region, the turbulence structure becomes subject to the simultaneous influences of the streamwise acceleration due to the reduction in the cross-sectional area, the destabilising effects of streamline curvature, and the additional mean strain. The interaction between these effects was featured in a steady increase in the normal and shear stresses as well as in a gradual departure from isotropy within the port exit.

At the entrance of the outlet passage, appreciable decay in the three relative stress components was observed, resulting from the steep streamwise acceleration there. The development of the turbulent stresses within the outlet passage was further found to be influenced by the growth of the boundary layer on the valve face and the persisting steep cross-stream velocity gradients across the recirculation zone next to the seat face. At low lifts, where no separation occurred, turbulence remained isotropic across most of the outlet passage with low turbulence intensity levels, typically about 0.15%.

Comparison between single and X-wire measurements of the rms of the streamwise velocity fluctuations, $\sqrt{u'^2}$, showed significant differences along the port wall. This is believed to be due to disturbances around the measuring holes caused by the stems of the probes. The measurements in this region are therefore considered as unreliable.
CHAPTER 6

COMPUTATIONAL INVESTIGATION AND

COMPARISON WITH EXPERIMENTS

6.1 INTRODUCTION

The numerical method employed for solving the transport governing equations allows for a variety of computational grid arrangements to be employed for a given flow domain. The selection of a particular grid arrangement is, however, limited by the geometrical properties of its control volumes which may affect the accuracy and stability of the solution, as outlined earlier in Chapter 3. In Section 6.2, the geometrical properties of the computational grid employed for the present study are discussed. The discussion is aimed mainly at emphasising the restrictions imposed by the shape of the domain boundaries on the simultaneous achievement of the favourable properties outlined in Section 3.8.

In Section 6.3, the boundary conditions prescribed at the inlet and the outlet of the solution domain are discussed. The treatment of other boundaries is as presented in Section 3.7.

In Section 6.4, the results of the grid-independence tests are presented and the choice is made of the final grid arrangement employed.

Section 6.5 is devoted to the presentation and discussion of the numerical predictions and comparison with the experimental results. The presentation is arranged such that the results of the highest lift configuration are first discussed and comparisons are then drawn for the other configurations considered. This strategy allows the effects of the valve lift on the development of the flow parameters in the various regions of the valve/port assembly to be clearly identified.

In subsection 6.5.1, the overall features of the predicted flow
fields are presented and the similarities and differences with experimental results are broadly outlined. The predicted mean velocity and turbulence fields are then examined in detail in the comparisons with experimental data presented in subsection 6.5.2. Attempts to explain the discrepancies found between them are made in subsection 6.5.3.

Section 6.6 summarises the main findings and conclusions of this chapter.

6.2 THE DOMAIN OF SOLUTION AND COMPUTATIONAL GRID

The solution domain is shown in Figure 6.1 bounded by the port duct inlet I-I and the planes E-F and F-F inside the cylinder. With attention being focused on the flow field between the port duct inlet and the valve exit plane E-E, the in-cylinder region E-E-F-F was included in the solution domain to avoid the uncertainty in the boundary conditions at the valve exit as further explained in subsection 6.3.2. For reasons of economy, the grid in this region was taken to be relatively coarse, except in the vicinity of the valve exit where attempts were made to avoid sharp discontinuities in the grid properties.

When generating the grid, the solution domain was first divided into the zones numbered 1-5 in Figure 6.2 which are bounded by the discontinuities in the upper and/or lower boundaries. The spacing and number of grid nodes were then specified in a preliminary manner within each zone in the streamwise direction according to the criteria outlined in Section 6.4. In the cross-stream direction, the nodes were distributed such that the grid spacing contracted towards the walls to ensure higher resolution, where large cross-stream gradients were expected to prevail. The same distribution function was employed
for all cross-stream grid lines, except across those defining the sharp corners, $S_1$ and $S_2$ in Figure 6.2, where different functions were used to improve the smoothness of the grid.

As was outlined in Section 3.8, the accuracy, stability and rate of convergence of the numerical algorithm are influenced by the following properties of the grid:

(i) alignment with the streamlines;
(ii) smoothness of the grid lines;
(iii) grid non-orthogonality; and
(iv) grid spacing and relative cell dimensions.

In the following, a discussion of these properties is presented with reference to the grid arrangement shown in Figure 6.2. This discussion is intended to point out regions where some of the favourable properties had to be sacrificed.

(i) **Alignment With the Streamlines**

Skewness of the grid and streamlines was identified earlier in Chapter 3 to give rise to numerical diffusion, which affects the accuracy of the solution, especially with lower-order differencing schemes. Although curvilinear grids offer a great advantage in that close alignment of grid lines and streamlines can often be procured, *a priori* knowledge of the streamlines pattern is required at the time of generating the grid. This not being the case here, optimisation of the grid in this regard had to be an adaptive process carried out in the course of the flow calculations (Dwyer [1984], and Eiseman [1985]). Peric [1985] suggested performing the adaptation manually by obtaining a solution on an initial grid and then using it to improve alignment.
for a further calculation. This is only possible if the grid generation procedure employed offers this facility which is not the case with the present method outlined earlier in Section 3.8.

In the course of the grid refinement process, it was possible to introduce minor adjustment to the grid in this regard. However, in order not to sacrifice other favourable properties, such as smoothness of the grid and limited degree of non-orthogonality, it was not possible to avoid misalignment everywhere in the solution domain, as will be seen later in Section 6.5.

(ii) **Smoothness of the Grid Lines**

The term 'smoothness' refers to the spatial rates of change in the directions of the grid lines from one cell to another. These are represented by (c.f. Figure 3.9) $\theta_x/\Delta x^{(1)}$, $\theta_y/\Delta x^{(1)}$, $\theta_x/\Delta x^{(2)}$ and $\theta_y/\Delta x^{(2)}$ given by equations (3.105) to (3.108) and referred to henceforth as $\theta_{xx}$, $\theta_{yx}$, $\theta_{xy}$ and $\theta_{yy}$, respectively. As was shown in Section 3.8, $\theta_{xx}$ and $\theta_{yy}$ are the curvatures of the grid line families $x^1$ and $x^2$, respectively, while $\theta_{xy}$ and $\theta_{yx}$ measure the parallelism between two successive grid lines of the same family $x^1$ or $x^2$, respectively. With the $x^2$ grid lines being straight, $\theta_{yy}$ consequently vanishes.

Discontinuities in the grid smoothness affect the accuracy of the linear interpolation formulae used to obtain the geometrical parameters and the dependent variables at locations other than the scalar nodes (c.f. subsections 3.5.3 and 3.8.3). Higher-order interpolation practices may help to reduce such influence but these are computationally expensive and can produce negative coefficients in the discretised equations (Demirdzic [1982], and Peric [1985]). An alternative to using higher-order interpolations is to ensure a
sufficiently fine grid where the smoothness is expected to suffer discontinuities, as further discussed in Section 6.4.

Abrupt changes in $\theta_{xx}$ were found to occur across the grid lines defining the various zones of Figure 6.2 as a result of the discontinuities in the boundaries, i.e. the sudden curvature of the valve crown and the seat corner and the sudden change in curvature at the valve corner. It is evident that these cannot be avoided; however, in order to suppress further oscillations in $\theta_{xx}$ across the boundaries of the different zones, these were treated as boundary grid lines in the manner described in Section 3.8.

Similar discontinuities also occurred in $\theta_{xy}$ and $\theta_{yx}$ at the same locations as those in $\theta_{xx}$. Attempts made to eliminate them were not successful and only produced different grid arrangements with similar features at different locations. Increasing the grid intensity in both coordinate directions helped to reduce these abrupt changes but this was constrained mainly by economy considerations.

(iii) **Non-Orthogonality**

As outlined in Section 3.6, the grid non-orthogonality gives rise to cross-derivative pressure gradients and diffusion terms, which in turn may adversely affect accuracy and delay convergence.

In zone 1 between $I = 1$ and 19, c.f. Figure 6.2, an orthogonal grid could be established. In zones 2 and 3, between $I = 20$ and 44, departure from orthogonality could be maintained within reasonable limits, i.e. ±10°, across most of the passage. However, between $I = 45$ and 50, sudden and rapid increase in the angle between the coordinate lines occurred as a result of the sudden sharp curvature of the grid lines in this region.

In the outlet passage, between $I = 50$ and 65, the angle
between the coordinate lines increased in the streamwise direction and reached a maximum of 135° at the valve exit.

(iv) Grid Spacing

On the basis of the Taylor series truncation analysis, the upwind differencing scheme employed in the present calculations loses one degree of accuracy if the grid spacing is non-uniform. Maintaining uniform grid spacing in complex geometry domains is difficult to achieve in addition to being computationally expensive. This is mainly due to the fact that in regions of the flow where steep gradients occur, the grid has to be fine enough to enable fine resolution of the field parameters, whereas in regions of small gradients a coarser grid becomes more adequate. Therefore, the constraint on grid uniformity had to be overlooked.

It is recommended, however, that the grid expansion ratios in both coordinate directions, defined earlier in Section 3.8, be as close to unity as possible (Roache [1982]). The streamwise expansion ratio was generally kept between 0.85 and 1.2, except across the boundaries of the different zones of Figure 6.2. In the cross-stream direction, the expansion ratio varied between 0.87 and 1.15.

Furthermore, Peric [1985] recommended that the cell aspect ratio, also defined in Section 3.8, remains as close to unity as possible and does not exceed 10 in order to maintain a reasonably fast rate of convergence. This was generally kept below 5 over most of the solution domain. In the few cells near the inlet and exit boundaries, this constraint on the aspect ratio was overlooked in favour of the economy associated with fewer cell nodes. This, however, led to weaker coupling between the pressure correction equations in the streamwise direction and slow rate of convergence. This was remedied
by increasing the number of inner iterations on the pressure correction equation, as suggested by Doormal & Raithby [1984] and independently by Peric [1985], and by heavily under-relaxing the pressure correction, as outlined in Section 3.6.

6.3 THE BOUNDARY CONDITIONS

In the following, the boundary conditions across the inlet I-I and the free stream boundary E-F are discussed separately. The treatment of the exit F-F and wall boundaries were presented earlier in Chapter 3.

6.3.1 Inlet Boundary Conditions

The inlet boundary conditions specification consists of the distributions of the magnitude \( Q \) and direction \( \alpha \) of the mean velocity vector across the inlet section as well as the turbulent energy \( k \) and its rate of dissipation \( \varepsilon \). The distributions of \( Q \) and \( \alpha \) were obtained from the measured profiles at station 3 (c.f. Figures 5.22 and 5.26 to 5.29). This was done by interpolating between the measuring points to determine the respective values at the nodes of the computational grid. Similarly, \( k \) was determined from the measured normal turbulent stresses \( \overline{u'^2} \) and \( \overline{v'^2} \), illustrated in Figures 5.34 and 5.36 to 5.38, via the following expression:

\[
k = \frac{1}{2} \left[ \overline{u'^2} + \overline{v'^2} + \frac{1}{2} (\overline{u'^2} + \overline{v'^2}) \right]
\]  

(6.1)

where \( \overline{w'^2} \) has been approximated as \( (\overline{u'^2} + \overline{v'^2})/2 \) according to the assumption and uncertainty analysis discussed in subsection 4.4.1.

The rate of energy dissipation, \( \varepsilon \), may also be obtained from the measurements by approximating equation (2.89) by:
and substituting the eddy viscosity, \( \nu \), from equation (2.90) to yield (Thompson [1983]):

\[
\rho \left( \overline{u' v'} \right) = - \nu \frac{\Delta U}{\Delta Y} \tag{6.2}
\]

where \( U \) and \( V \) are, respectively, the axial component of the mean velocity vector and the radial distance measured from the surface of the valve stem.

The accuracy of the mean velocity and shear stress measurements used to obtain \( \epsilon \) across the inlet section is evidently important, particularly in the boundary layers where large gradients prevail. In this connection, the comparisons between X-wire and the more accurate single-wire measurements of the mean velocity illustrated in Figures 5.31a showed a difference of less than 2% across the port duct inlet. However, other measurement uncertainties rendered the distribution of \( \epsilon \) obtained from equation (6.3) across the boundary layers at station 3 as subject to large uncertainties as summarised below:

1. The boundary layer on the valve stem was too thin to allow for intensive measurements to be carried out close to the surface leading to high levels of uncertainty in the calculation of the velocity gradient there.

2. The distributions of \( U \) and \( \overline{u' v'} \) in the vicinity of the valve surfaces were such that equation (6.3) would produce negative, and even infinite, \( \nu \) at some points; a situation which is physically unrealistic and frequently
arises in wall jets on straight and curved surfaces (see, for example, Gibson, et al. [1981], Gibson & Younis [1982], and Dakos, et al. [1984]).

3. The unquantified measurement uncertainties near the port duct wall due to the probe interference effects are as discussed in the previous chapter.

Alternatively, Escudier's formulae (Escudier [1966]) for the distribution of the mixing length, $\ell_m$, in boundary layers were employed for the determination of $\varepsilon$. These formulae are:

\[
\frac{\ell_m}{\delta} = \eta \frac{y}{\delta} \quad , \quad \frac{y}{\delta} < \lambda / \eta \\
\frac{\ell_m}{\delta} = \lambda \quad , \quad \frac{y}{\delta} > \lambda / \eta 
\]  

(6.4)

where $\delta$ is the thickness of the boundary layer, defined as the distance from the wall to the point where the mean velocity is 99% of the free stream value; and $\eta$ and $\lambda$ are constants whose values are 0.41 and 0.09, respectively. Launder & Spalding [1972] argued that the above formulae are valid for velocity profiles with maxima similar to those featured in wall jets, provided that $\eta$ is modified to 0.435.

On the valve stem, the thickness of the boundary layer, as defined above, was determined from the mean velocity profiles, c.f. Figures 5.22 and 5.26 to 5.29. Due to the large measurement uncertainties near the upper wall, the boundary layer thickness there was taken equal to that on the valve stem. The effects resulting from this approximation will be discussed later in Section 6.5.

Outside the boundary layers, the measured turbulent stresses illustrated in Figures 5.34 and 5.36 to 5.38 indicate a rapid drop in the relative turbulence intensity levels to about 1.5%. This implied
that the turbulence, and hence the shape of the length scale profile across this part of the inlet section, were unlikely to appreciably influence the development of the flow field in the downstream direction. Accordingly, and for simplicity, \( \ell_m \) in this potential core-like region was taken to be uniform.

The distribution of \( \varepsilon \) across the inlet boundary may now be determined by substituting \( \ell_m \) into the relationship (Launder & Spalding [1974]):

\[
\varepsilon = C_\mu \frac{k^{3/2}}{\ell_m}
\]

where \( C_\mu \) is an empirical coefficient given in Table 2.2.

6.3.2 Exit Boundary Conditions

Due to the lack of detailed information on the in-cylinder flow field, it was not possible to prescribe exact boundary conditions at the outlet of the solution domain. The approach adopted here was to determine the boundary values along E-F, Figure 6.1, iteratively by extrapolating them from the interior of the solution domain as follows:

1. The mean velocities, turbulence energy and length scale were initially approximated by their respective values prevailing along the boundary of the shear layer of a self-preserving plane free jet (Townsend [1980]).

2. After a converged solution was reached with this initial approximation, the field values were then extrapolated linearly to obtain a new set of boundary values along E-F.

3. Another solution was then sought with the solution obtained in the previous step being used as the initial guess of the flow field (c.f. Section 3.6).
4. Steps 2 and 3 above were repeated until an overall convergence was achieved, established by monitoring the changes in the profiles of the various variables across the valve exit plane. This usually took between 4 and 6 boundary adjustments (step 2 above), as shown in Figures 6.3.

In order to investigate the uniqueness of this procedure, another set of initial boundary values along E-F was employed based on the predictions of Ahmadi-Befrui [1985] of the in-cylinder flow field in an engine equipped with a similar valve/port assembly. In these predictions, a uniform mean velocity distribution across the valve exit plane was assumed which resulted in streamwise velocities along E-F higher than those obtained from the free jet assumption associated with lower turbulence energy and rate of dissipation. The final profiles across the valve exit plane obtained after 6 boundary adjustments were nevertheless found to be almost identical to those obtained with the free jet initial approximation, as shown in Figures 6.4. At the first two nodes next to the valve face, the initial approximation of Ahmadi-Befrui [1985] is seen, however, to produce higher streamwise velocity and turbulence energy amounting to about 8.5% and 22% in the former and latter, respectively.

The boundary F-F in Figure 6.1 was treated as an outflow boundary in the manner described in Section 3.7 with the entrainment mass across E-F being included in the overall mass balance represented by equation (3.86).
6.4 GRID REFINEMENT AND GRID DEPENDENCE TESTS

Tests were carried out to examine the sensitivity of the solution to the number and spacing of the grid nodes in both the streamwise and cross-stream coordinate directions. These tests were performed in two steps by first altering the grid density in the streamwise direction to obtain a grid-independent solution and then refining in the cross-stream direction. The conclusions presented below are for the $L = 0.25$ configuration, but they are expected to be equally valid for smaller lifts due to the increase in grid node intensity and the reduced complexity of the flow.

A preliminary grid arrangement was specified in both coordinate directions in accordance with the results of the experimental investigation. In the streamwise direction, close spacing was used in regions where rapid changes in the flow field occur, notably in the port exit and the outlet passage regions (c.f. Figure 5.14). In order to exercise local control over the grid, the spacing and the number of grid nodes within each of the zones defined in Figure 6.5 were altered separately starting from the inlet zone to obtain a grid-independent solution at their respective exit planes. This was done for zones 1, 2 and 3. For zone 4, it was felt that strict grid independence was not necessary because of its limited influence on the flow upstream of the valve exit plane. However, in order to avoid sharp discontinuities in the grid properties between zones 3 and 4, the grid in the latter was made to smoothly expand in the streamwise direction.

Figures 6.6 illustrate the results of the grid refinement tests in the streamwise direction, represented by the zone exit-plane profiles of the mean velocity components and turbulence energy normalised by the mid-height mean velocity vector, $U_m$, at the valve exit, and the relative turbulent viscosity. The latter was chosen for comparison
instead of the length scale as it appears explicitly in the governing equations. The profiles in Figure 6.6 are plotted across the outlets \( Z_1, Z_2 \) and \( Z_3 \) shown in Figure 6.5 with \( N_{11}, N_{12} \) and \( N_{13} \) being the number of grid nodes in the streamwise direction in each zone, respectively.

It is evident from these profiles that the grid arrangement for which \( N_{11} = 19, N_{12} = 25 \) and \( N_{13} = 20 \) ensures a solution very nearly independent of the grid distribution in the streamwise direction.

The results of the grid refinement attempts in the cross-stream direction are represented in Figures 6.7 across \( Z_2 \) and \( Z_3 \) for total numbers of cross-stream nodes \( N_J = 22, 26, 30 \) and \( 32 \). Because of limitations on computer time and memory, the 30 node arrangement was selected, especially since the 32 nodes produced differences of only 2\% in \( k \) and \( \mu_t \) and even less differences in the mean velocities.

6.5 FLOW FIELD PREDICTIONS

The analysis presented in Chapter 5 of the experimental measurements indicated that the flow field was most significantly influenced by the lift-dependent geometry of the passage. Varying the pressure drop across the valve was found to have only minor quantitative effects on the detailed flow field and the overall performance of the valve/port assembly. The predictions were therefore performed for the five lift configurations \( L = 0.05, 0.10, 0.15, 0.20 \) and 0.25 at a single value of the pressure drop, \( \Delta P \), of \( 5 \times 10^{-3} \) for which detailed measurements were reported in the previous chapter.

The global characteristics of the predicted flow fields are presented in the following subsection, with emphasis on the qualitative similarities and differences with the expected features detected in the measurements. In subsection 6.5.2, detailed comparisons with the experimental data are carried out and attempts to explain the
discrepancies observed follows in subsection 6.5.3.

6.5.1 Presentation of Results

Figures 6.8 and 6.9 illustrate, respectively, the mean velocity vectors and streamlines for \( L = 0.25 \). Both figures show that the recirculation bubbles on the valve crown and face and the extended separation at the seat corner, which were detected in the measurement at this lift (c.f. Figures 5.12 and 5.21), were not reproduced in the predictions. There is predicted a large vortex near the cylinder head corner, but it does not extend into the outlet passage. Reasons for these discrepancies and their consequent effects on the predictions will be discussed in detail in the following subsections.

For the same configuration, Figure 6.10 illustrates the contours of selected isobars normalised by \( \rho \overline{U}_3^2 \). The cross-stream profiles of the dominant streamwise velocity component are plotted in Figure 6.11 at selected locations along the flow passage, including the boundaries of the various zones of Figure 6.2. In the straight part of the port duct, the streamwise pressure gradient is negligibly small and the velocity profiles 1, 2, 3 and 4 in Figure 6.11 are thus nearly identical and show an almost uniform distribution. The spacing between, and the values associated with, the isobars upstream of zone 2 indicate an adverse streamwise pressure gradient prevailing near the valve stem and across most of the lower part of the port duct. This is created by the curvature of the streamlines induced by the crown surface and the fact that it extends into the straight part of the port duct suggests that the streamline curvature in zone 2 actually starts

\* \( \overline{U}_3 \) is the average axial mean velocity at the inlet section given by equation (5.16) and plotted in Figure 5.23.
earlier in zone 1. This adverse pressure gradient causes the flow in the lower region to decelerate, as shown in profiles 4, 5 and 6 in Figure 6.11. With the cross-sectional area being constant between profiles 4 and 5, this deceleration is compensated for by the acceleration and consequent favourable streamwise pressure gradient near the port wall.

Similar trends, it is recalled, were observed in the measured velocity profiles in this part of the passage. However, in the measurements, the adverse pressure gradient near the valve stem was found to lead to separation and the formation of the recirculation bubble shown in Figure 5.21 extending along the crown surface. In the calculations, there is a tendency towards separation which can be seen in the predicted shear stress, \( \tau_w \), along the valve surface, which is normalised by \( \rho U_3^2 \) and plotted in Figure 6.12 against the cross-stream grid number, \( I \) (c.f. Figure 6.2). This is seen to fall steadily in this region, but the rate of reduction is too small for separation to take place before the flow accelerates again due to the now favourable pressure gradient. Along the port wall in the same region, i.e. between \( I = 5 \) and 30, \( \tau_w \) is seen from Figure 6.12 to increase steadily as a result of the acceleration there.

The flow acceleration due to the reduction in the cross-sectional area as it enters the port exit region leads to the simultaneous increase in \( \tau_w \) on both the valve and port surfaces shown in Figure 6.12 downstream of \( I = 30 \). The increasing deformation in the velocity profiles 6 and 7 further indicates higher acceleration near the port wall.

At the seat and valve corners, Figure 6.10 indicates steepening pressure gradients of both signs created by the relatively sharp streamline curvature at the former and the rapid change in
curvature around the latter, as well as by the continuing reduction in the cross-sectional area. These pressure gradients are manifested in the changes in profiles 6, 7 and 8 in Figure 6.11 which show rapid acceleration around both corners followed by almost similar deceleration downstream of the seat corner. The transfer of streamwise momentum towards the valve face seen from profiles 7 and 8 does not allow similar rapid deceleration downstream of the valve corner to take place. In Figure 6.12, the wall shear stress is seen to increase steeply towards and around both corners and then fall equally rapidly downstream of them, although again not sufficiently fast to produce the separation detected in the measurements.

Along the remainder of the valve and seat faces, $\tau_w$ continues to fall at relatively slower rates due to the deceleration resulting from the gradual increase in the cross-sectional area. In this part of the outlet passage, the predicted streamlines become straight and almost parallel to the valve and seat faces, as seen from Figure 6.9. Thus, the rotational accelerations created by their curvature vanish and the cross-stream pressure gradients gradually relax, causing the velocity profiles to become progressively more uniform towards the exit plane, as seen from profiles 8, 9 and 10.

Contours of the normalised turbulence intensity, $\sqrt{k}/U_3$, and the dimensionless length scale, $L_m/D$, are illustrated, respectively, in Figures 6.13 and 6.14. They generally show a potential core-like region in the central part of the flow with turbulence intensities below 2% and 3.5% in the port duct and the outlet passage, respectively. They also indicate a steady increase in the thickness of the boundary layer along the valve produced by the deceleration mentioned earlier in zone 1 and the inlet of zone 2 (c.f. Figure 6.2).

Along the port wall, $k$ is seen to increase upstream of the
seat corner due to the acceleration, and the associated increase in the cross-stream velocity gradient, there. A similar increase in $k$ is seen to occur near the valve crown along with a steep cross-stream variation in $\ell_m$, which apparently results from the streamwise acceleration along the crown surface.

Downstream of the valve and seat corners, Figures 6.13 and 6.14 indicate a steady increase in the thickness of the boundary layer on the seat face created by the relatively large streamwise deceleration next to it. On the valve face, the boundary layer is seen to remain unchanged in thickness down to the exit plane.

The predictions for $L = 0.20, 0.15, 0.10$ and $0.05$ are shown in Figures 6.15 to 6.18, respectively. The mean velocity vectors and streamlines for the various configurations again show the absence of all separation and recirculation zones detected in the measurements which, in the light of the similar features at $L = 0.25$, is scarcely surprising.

The patterns of the isobars and the streamwise variations in the velocity profiles for the various configurations closely follow the general trends outlined above for $L = 0.25$. However, as the lift decreases, the streamline curvature induced by the crown surface starts earlier in the port duct, causing the influence of the pressure field to extend progressively upstream, as shown in Figures 6.15c to 6.18c. This is also identified in the earlier deformation in the mean velocity profiles at smaller lifts seen from Figures 6.15d to 6.18d. Similar trends were also observed in the measurements, c.f. Figures 5.26a to 5.29a, which further indicated that, under the extending influence of the adverse pressure gradient in the lower part of the port duct, the point of separation on the valve stem moves upstream as the lift decreases. A related trend can be seen in the predicted wall shear
stress variation along the valve surface (Figure 6.19a), for various lifts, in which the point of minimum $\tau_w$ recedes upstream at smaller lifts. On the port duct, Figure 6.19b shows steady increase in $\tau_w$ as a result of the acceleration there.

In the port exit region, the streamwise acceleration due to the reduction in the cross-sectional area increases with the decreasing lift, leading to steeper pressure gradients (Figures 6.15c to 6.18c) and the higher wall shear stresses shown in Figures 6.19.

At the entrance, and downstream, of the outlet passage, the corresponding velocity profiles at different lifts (Figures 6.15d to 6.19d) assume nearly similar shapes as those discussed above for the $L = 0.25$ configuration. At the valve exit plane, the velocity distributions are, more or less, uniform as the large cross-stream gradients gradually smoothed out. The wall shear stresses are also seen from Figures 6.19 to have similar distributions for the different lifts, except for the relatively slower rate of reduction downstream of the seat corner at smaller lifts.

Comparison between the turbulence intensity and length scale contours for the different lifts, shown in Figures 6.15 to 6.18, reveals the following common features:

(i) A potential core-like region occupies most of the central part of the port duct where the turbulence intensity is 2% or less.

(ii) Gradual streamwise increase in the thickness of the layer of relatively high turbulence intensities on the valve stem at a nearly constant rate.

(iii) Nearly constant thickness of this layer over most of the port wall.
(iv) Higher turbulence intensity and cross-stream variations in $\ell_m$ on the crown surface and upstream of the seat corner as the flow accelerates due to the reduction in the cross-sectional area.

(v) Steady streamwise increase in $k$ and $\ell_m$ across the port exit region, particularly near the valve corner.

The most noticeable effects on the turbulence field resulting from reducing the lift are the faster rates of increase of the turbulence intensity and length scale within the port exit region and near the valve corner. This is due to the steeper velocity gradients induced at smaller lifts by the increase in streamline curvature and the rate of area reduction. The values of $k$ and $\ell_m$ thus generated at the entrance of the outlet passage are seen to prevail downstream to the valve exit plane.

6.5.2 Comparison With Experiments

6.5.2.1 Flow field parameters

In order to compare between the predictions and measurements, the former were obtained at the measuring stations shown in Figure 5.14 by bilinear interpolation between the values at the four nodal points shown in Figure 5.20 surrounding the location of interest, $P$. The accuracy of the interpolations is consistent with the overall accuracy of the solution procedure.

As was outlined in Section 6.3, the inlet boundary conditions were taken as the measured profiles of the mean velocity and normal stresses at the inlet of the port duct (i.e. station 3 in Figure 5.14) and from formulae (6.4) for the turbulent length scale. The profiles of the assumed and measured shear stresses for $L = 0.25$,
normalised by \( \rho U_3^2 \), are illustrated in Figure 6.21. The former was calculated from the expression for \( \tau_{ij} \) given in Table 2.3 with \( i \) and \( j \) being substituted for by 1 and 2, respectively*.

The assumed shear stress is seen in Figure 6.21 to differ from that measured near the upper and lower walls. This is due to uncertainties in the length scale assumed according to formulae (6.4), which relies on the proper determination of the boundary layer thickness. This was difficult to define accurately from the measurements. However, these differences were found to diminish rapidly downstream, as will be seen later.

For the \( L = 0.25 \) configuration, Figures 6.22 illustrate the predicted and measured profiles of the normalised magnitude \( Q/U_3 \) and direction \( \alpha \) of the mean velocity vector at selected measuring stations (c.f. Figure 5.14). The profiles of the relative turbulent energy, \( \kappa/U_3^2 \), and shear stress, \( \tau/\rho U_3^2 \), at the same locations are illustrated in Figures 6.23, with the former being determined from the measurements according to equation (6.1).

At station 4, Figures 6.22 show differences between the predicted and measured mean velocities near the upper and lower walls of about 2.5% and 6%, respectively. These differences, which remain nearly unchanged downstream to station 6, are believed to be mainly due to the uncertainties discussed above in the assumed shear stress distribution at the inlet section.

In this part of the port duct, i.e. between the inlet and station 6, the agreements between the measurements and

---

* In the port duct and port exit region, \( \tau \) is resolved in the axial and radial directions, and, in the outlet passage, in the directions parallel and normal to the valve and seat faces.
predictions of $k$ and $\tau$ are seen from Figures 6.23 to be reasonably good, except near the port wall where the disturbances mentioned above influenced the measurements. Within the central part of the passage, differences in $k/\bar{U}_3^2$ are further seen to be negligibly small.

Downstream of station 6, the flow becomes increasingly influenced by the pressure field, as discussed in the previous section: starting at station 7, both the measured and predicted velocity profiles show progressively increasing skewness in the downstream direction. As was noted in subsection 5.3.2, the measured flow apparently separates from the valve stem between stations 7 and 8 forming the recirculation zone illustrated schematically in Figure 5.21. This separation did not develop in the predictions, as may be seen from the present plots and the streamline pattern illustrated in Figure 6.9. However, the influence of the adverse streamwise pressure gradient near the valve boundary is nonetheless evident at stations 7, 8 and 9, where it produces steady reduction in the predicted mean velocity and shear stresses, and an associated increase in the boundary layer thickness. The predicted shear stress at the wall, $\tau_w$, is also seen to decrease steadily in the streamwise direction and Figure 6.12 suggests that separation in this region would have occurred had the rate of reduction in $\tau_w$ been steeper. The reason why this did not occur will be discussed later.

The presence of the aforementioned recirculation zone caused the measured mean velocities to become generally higher than the predicted ones across most of the passage at stations 8, 9 and 10. Moreover, with the predicted flow direction near the valve surface being aligned with it, as seen from the velocity vectors in Figure 6.8, the extent of the recirculation bubble at these stations may be roughly determined as the region across which the measured and predicted flow
directions differ. This is identified in Figures 6.22 and 6.23 by the horizontal dashed lines at stations 8, 9, 10 and 11.

At station 11, the measurements show considerably higher mean velocities and smaller flow angles in the upper part of the port exit region near the entrance of the outlet passage (c.f. Figure 5.14). This is due to the reduction in the effective flow area at the entrance of the outlet passage caused by the separation at the seat corner and the associated effect on the flow direction. This separation was, as already noted, absent in the predictions.

The failure to predict separation at the seat corner and the recirculation bubble on the valve face are the principal cause for the discrepancies between the predictions and the measurements in the outlet passage, shown in Figures 6.22 and 6.23. In the region of valid measurements, the mean velocities are seen to be between 30% to 45% higher than the predictions, due to the reduction in the effective cross-sectional area. Despite these quantitative differences, the computed and measured mean velocity profiles have nearly similar shapes, except in the vicinity of the valve face downstream to station 14. The higher measurements in this region indicate the transfer of axial momentum of the incoming flow towards the valve face continues in the outlet passage longer than predicted. The agreement between the measured and predicted mean flow angle is also seen to improve steadily towards the exit plane, except near the recirculation zones, as should be expected.

The profiles of the predicted and measured $k$ and $\tau$ across the entrance of the outlet passage, station 12 in Figures 6.23, are seen to be in good agreement, except near the separation at the seat corner. Downstream of station 12, however, Figures 6.23 show increasingly higher measured $k$ and $\tau$ than predicted within the region
of valid measurements. This is attributed to the necessarily steeper cross-stream velocity gradients (due to the higher velocities produced by the blockage effect of the separation zone) as compared to the mild gradients predicted in these regions. The differences are also seen to extend gradually across the outlet passage and occupy most of it at the valve exit plane.

Comparisons for $L = 0.20$, not shown here, revealed features and trends similar to those mentioned above, in particular the discrepancies due to the lack of prediction of separation.

Attention will now be focused on the lower lifts investigated, namely, $L = 0.15, 0.10$ and $0.05$, for which separation at the seat corner was either less influential, i.e. for $L = 0.15$, or absent, as for $L = 0.10$ and $0.05$.

The distribution of the assumed and measured shear stresses across the inlet boundary for these lifts is illustrated in Figures 6.24. They show improved agreement near the valve surface over that observed earlier for $L = 0.25$ in Figure 6.21. This is due to a more accurate determination of the boundary layer thickness as this increased with the reduction in Reynolds number at smaller lifts. Thus, differences between the predicted and measured mean velocities near the valve surface at stations 4 and 6, attributed earlier to the uncertainty in the length scale inlet boundary condition, are seen in Figures 6.25a to 6.27a to diminish with the reduction in lift. Across the remainder of the port duct, the differences observed at these two stations, particularly near the port wall, are consistent with those discussed above for $L = 0.25$.

The departure of the predicted profiles of the mean flow from the measurements, induced by the separation from the valve surface, is also seen from Figures 5.25a to 5.27a to start emerging
earlier at station 6. This supports the observation made in Section 5.3 that the point of separation shifts upstream to lie between stations 6 and 7 for \( L = 0.15 \) and 0.10 and probably even earlier for \( L = 0.05 \).

The extent of the recirculation zone across the port duct is roughly indicated by the point at which the measured and predicted flow angles start showing significant differences. This is identified by the horizontal dashed lines near the lower wall at stations 6, 7 and 8. It is to be stressed again that the extent of the recirculation zone thus determined is highly approximate and that the data points shown within it do not bear any significance due to the high level of measurement uncertainties. However, the predicted shear stresses along the valve surface shown in Figures 6.19 indicate that the point of minimum shear stress recedes upstream as the lift decreases.

Between stations 7 and 10, the differences between the predictions and the measurements shown in Figures 6.25a to 6.27a are seen to be consistent with those discussed earlier for \( L = 0.25 \), except for the large differences in the flow angle at station 10 for \( L = 0.05 \), Figure 6.27a. These are attributed to measurement inaccuracies caused by the large flow angles in the lower part which exceed the range of reliable measurements, namely, \( \pm 25^\circ \), as discussed earlier in Section 4.4. They may also be due to uncertainties in the location of the measuring probe, particularly with the steep streamwise gradients prevailing in the port exit region at this lift.

In the outlet passage, Figures 6.25a to 6.27a generally show closer agreement in the mean velocity profiles than that observed for \( L = 0.25 \). This is due to the considerable reduction in the size of the measured recirculation zone next to the seat face at
$L = 0.15$ and the full attachment of the flow to the seat face at $L = 0.10$. At and downstream of station 16, the agreement between the mean velocity and flow angle profiles improves markedly and, across the valve exit plane, they become nearly identical. The differences visible at station 18 for the different lifts, which amount to a maximum of about 3% in the mean velocity and about 2° in the mean flow angle, lie well within the limits of estimated experimental uncertainties.

The comparison between the predicted and measured profiles of the relative turbulent energy and shear stress shown in Figures 6.25b to 6.27b again indicates good agreement in the central part of the port duct downstream to station 8. The profiles also show the departure of predictions from measurements near the valve stem starting at station 6 for the reasons noted above with reference to the mean velocity profiles. For $L = 0.15$ and 0.10, the differences in both $k$ and $\tau$ profiles between stations 7 and 10 are consistent with those discussed for the $L = 0.25$ configuration and shown in Figure 6.23.

For $L = 0.05$, however, substantial discrepancies are seen at station 10 which are difficult to explain and cannot be attributed to the uncertainties mentioned above alone. They are believed to arise from the inability in the turbulence model to take proper account of the steep acceleration and the associated decay in both $k$ and $\tau$ featured in the measurements. The significantly higher predictions apparently indicate the dominance of the increase in the rate of transfer of turbulent energy from the mean motion under the influence of the steepening cross-stream velocity gradients. This evidently continues to the entrance of the outlet passage and the values attained are thereafter maintained downstream to the valve exit plane, as may be seen at station 18 in Figure 6.27b.
For $L = 0.15$ and $0.10$, similar trends are seen to prevail at the entrance of, and downstream of, the outlet passage. The excess in the predicted values of $k$ and $\tau$ over the measurements is, however, seen to increase rapidly as the lift decreases.

### 6.5.2.2 Coefficient of discharge

Comparison between the measured and the predicted coefficients of discharge is illustrated in Figure 6.28. At $L = 0.05$, the predicted $C_D$ is about 9% lower than the measured $C_D$ due to the excessively higher rate of transfer of turbulent energy from the mean motion featured in the predictions. In the mid-lift range, between $L = 0.10$ and $0.15$, the agreement between the measured and predicted improves as a result of the improved agreement in the flow parameters in the outlet passage. In the high-lift range, $L > 0.2$, and due to the failure to reproduce the extended separation on the seat face in the predictions, the predicted $C_D$ assumes a nearly constant value between $L = 0.20$ and $0.25$. At $L = 0.20$, the over-prediction is seen to be about 6.5% and increases to about 25% at $L = 0.25$ as the relative size of the measured recirculation zone in the outlet passage increases, as was noted earlier in the previous chapter.

### 6.5.3 Discussion of Results

The results presented in the preceding subsections revealed qualitative and quantitative discrepancies with the measurements which were particularly significant in the port exit and outlet passage regions. As already noted, these discrepancies arose mainly from suppression of the flow separation at the crown root and that at the seat corner in the medium-to-high lift range.

The inability to reproduce flow separation in the predictions
is believed to be due to deficiencies in the turbulence model to account for the effects of adverse pressure gradient and streamline curvature. These matters will now be examined in detail.

(i) **Adverse pressure gradient**

One of the conclusions of the 1980-81 AFOSR-HTTM Stanford Conference on Complex Turbulent Flows (Rodi, et al. [1982]) was that the effects of adverse pressure gradients on shear layers were not predicted very well by most turbulence models and especially by two-equation and Reynolds stress models. The poor performance of the $k-\varepsilon$ model under the adverse pressure gradient prevailing along the valve surface may be observed from the streamwise variation of the predicted shear stress in Figures 6.23 and 6.25b to 6.27b. This is manifested in the slow response to the flow deceleration and the persistingly high shear stresses at, and in the vicinity of, the valve surface, which force the flow to remain attached where the experiments have indicated separation.

This under-predicted response to the adverse pressure gradient is attributed, as further discussed below, to the steeper rise in the length scale determined by the $\varepsilon$-equation as compared to that under zero pressure gradients. Experimental data (Bradshaw [1969]) indicate that the length scale gradient is nearly independent of the pressure gradient over a wide range.

As outlined in Chapter 3, the wall boundary layer (or, more specifically, that part of the layer where the velocity profile is logarithmic) is bridged in the $k-\varepsilon$ model treatment by wall functions (see Launder & Spalding [1974]), where the velocity at the first grid node is linked to the wall shear stress by the logarithmic law of the wall. Under the Couette flow conditions, where only the simple shear
\( \frac{\partial U}{\partial \gamma} \) exists, the log law reads:

\[
\frac{U}{U_\tau} = \frac{1}{\kappa} \ln \gamma^* + A
\]  

(6.6)

where \( A \) is a function of the wall roughness and varies between 4.9 to 7 for smooth walls (Hinze [1977]).

Implementation of equation (6.6) in the \( \varepsilon \)-equation, equation (2.96), yields the following relationship between the coefficients \( C_1 \) and \( C_2 \) (Launder & Spalding [1974]):

\[
C_1 = C_2 - \frac{\kappa^2}{\sigma_\varepsilon \sqrt{C_\mu}}
\]  

(6.7)

In the derivation of this relationship, it has been assumed that (i) the convection of \( \varepsilon \) is negligible, (ii) the shear stress is constant and the length scale increases linearly with the distance from the wall such that \( \ell_m = \kappa \gamma \), and (iii) the production, \( G \), of turbulence energy balances its rate of dissipation, \( \varepsilon \).

Under adverse pressure gradient conditions, some of the above assumptions become invalid; firstly, the log law does not hold in the form given by equation (6.6) and has to be modified. Mellor [1966] suggested the following form:

\[
\frac{U}{U_\tau} = \frac{1}{\kappa} \ln \gamma^* + B(\xi)
\]  

(6.8)

where \( \xi \) is a pressure parameter defined as \( \nu (\partial p/\partial x)/\rho U_\tau^3 \) and, as it approaches zero, \( B \) approaches \( A \) in equation (6.6). Mellor [1966] showed that \( B \) increases with \( \xi \), producing smaller values of the wall shear stress than that obtained from equation (6.6). Secondly, the shear stress varies across, and in the vicinity of, the boundary layer,
as shown near the valve surface in Figure 6.23, due to the additional strain rate, \( \partial u / \partial x \), and \( G \) and \( \varepsilon \) are thus expected to be no longer in balance.

Rodi & Scheuerer [1986] attributed the poor performance of the \( k-\varepsilon \) model to the fact that the rate of reduction of \( G \) is nearly the same as that of \( \varepsilon \). Consequently, relatively high \( k \) levels persist near the wall, leading to a steep increase in the length scale, \( \ell_m \) (\( \ell_m \equiv k^{3/2}/\varepsilon \)), as shown in Figures 6.14 and 6.15d to 6.18d. This also leads to an over-predicted turbulent viscosity, \( \nu_t \) (\( \nu_t = C_\mu k^2/\varepsilon \)), and shear stresses, c.f. equation (2.94). This argument is supported in the present predictions by the fact that the ratio \( G/\varepsilon \) was found to remain nearly constant at the first grid node between stations 4 and 11. Furthermore, Figures 6.23 and 6.25b to 6.27b indicate that \( k \) in the immediate vicinity of the valve surface undergoes very small changes between these stations.

Some modifications of the \( \varepsilon \)-equation aimed at increasing the values in the boundary layer to suppress the steep increase in \( \ell_m \) there have been published. Hanjalic & Launder [1980], for example, suggested multiplying the irrotational strain rate part in the generation term in the \( \varepsilon \)-equation by a larger empirical coefficient than the rotational strain part. This modification gives rise to larger \( \varepsilon \) values and, consequently, smaller shear stresses near the wall. Although the model predictions show improved response to adverse pressure gradient, they still exhibit over-predicted skin friction and shear stresses in the boundary layer.

Another modification proposed by Hanjalic & Launder (see Launder [1984]) is to replace \( \sigma_\varepsilon \) in the diffusion term in the \( \varepsilon \)-equation by a function of the ratio \( G/\varepsilon \). This brought minor improvements to the model predictions. Launder [1984] further suggested a limitation
on the growth of \( \ell_m \) by simply increasing the computed values of \( \varepsilon \) to yield the experimentally observed values \( \ell_m = 2.5 \gamma \) whenever exceeded. This suggestion is evidently inconsistent with the concept of a two-equation model.

Rodi & Scheuerer [1986] also proposed a modification to the \( \varepsilon \)-equation by assuming a variable shear stress in the vicinity of the wall, such that:

\[
\frac{\tau}{u_e^2} = \frac{C_k}{2} - K y^+ - \frac{1}{\kappa} y^+ \sqrt{C_f/2}
\]  

(6.9)

where \( K \) is an acceleration parameter defined as \( \nu (\partial u_e/\partial X)/u_e^2 \), and \( u_e \) is the free stream velocity. They further assumed that near the wall, \( \ell_m \) is independent of the pressure gradient and may be described, as in the one-equation model of Norris & Reynolds [1975], by:

\[
\ell_m = \frac{\kappa}{C_\mu^3/4} \gamma
\]  

(6.10)

When equations (6.9) and (6.10) are inserted in the \( \varepsilon \)-equation, the relationship (6.7) becomes:

\[
C_1 = C_2 \sqrt{\tau/(\rho u_e^2)} - \frac{\kappa^2}{\nu C_\mu} \left[ 1 - \frac{3}{2} \frac{dp/dX}{\tau/\nu} + \frac{3}{4} \left( \frac{dp/dX}{\tau/\nu} \right)^2 \right]
\]  

(6.11)

Rodi & Scheuerer used for \( C_1 \) the value determined for equation (6.7) in order to calculate \( C_2 \) and, therefore, it was not surprising that the improvement in the predictions were minor.

At the valve and seat corner, the situation becomes much more complicated. In addition to the relatively steep pressure gradients around both, the sharp streamline curvature gives rise to augmentation of all components of the strain rate tensor \( \mathcal{D}_{ij} \) given by
equation (2.69). The effects of streamline curvature on turbulence structure in shear layers and the $k-\varepsilon$ model behaviour in predicting them are discussed below.

(ii) **Streamline curvature**

On the basis of the Prandtl mixing length theory, a simple analysis of the dynamics of a displaced element of fluid subject to centrifugal force indicates that turbulence is suppressed along convex curvatures when the angular momentum of the mean flow increases away from the centre of curvature: along concave curvatures, on the other hand, where the angular momentum increases towards the centre of curvature, turbulence intensifies. These effects give rise to the terms "stabilising" and "destabilising" in relation to the effects of streamline curvature referred to earlier in Section 5.3 (Bradshaw [1973]).

The available turbulence models invariably seem to fail, in one way or another, to reproduce the full effects of curvature on turbulence structure (Bolour-Froushan [1986]). The consistently under-predicted effects of curvature produced by the standard $k-\varepsilon$ model as compared to those obtained by the full Reynolds stress transport closure in earlier studies (see, for example, Irwin & Smith [1975], Gibson [1979], and Gibson & Rodi [1981]) indicated that the curvature-induced extra strain rates produce changes in the higher-order parameters of turbulence structure larger than expected (Bradshaw [1973]). The curvature effects were found to be most significant on the distribution of the length scale across the boundary layer. The measurements of Gillis & Johnston [1983] showed that close to the wall the length scale remains linearly proportional to the distance from the boundary, while away from it, it becomes nearly constant at a distance
shorter than that observed in flat-plate boundary layers. This behaviour suggests that the small scale eddies near the wall respond to the wall as flat and only the large scale eddies away from it are affected by its curvature.

Several attempts have been made to modify the $k-\varepsilon$ model to take special account of curvature effects on the length scale distribution in boundary layers. These generally adopted modified versions of the mixing length model of Bradshaw [1971a] based on the analogy between the effects of streamline curvature and buoyancy on turbulence structure. In most cases, the coefficients $C_1$ and $C_2$ in the $\varepsilon$-equation, equation (2.96), were made functions of suitably defined Richardson number* in an ad hoc fashion (see, for example, Sharma [1975], Launder, et al. [1977], and Rodi [1979]). The validity of these modifications appears, however, to be restricted to the specific

* (i) Gradient Richardson number, $Ri_g$, defined as:

$$Ri_g = \frac{\text{typical body force}}{\text{typical inertia force}}$$

where, for curved flows, the body force is taken as the centrifugal force. Thus, in the 'streamline' $s-n$ coordinates in Figure 6.29, this expression yields:

$$Ri_g = \frac{2 \left( \frac{u_s}{r^2} \right) \frac{\partial u_s}{\partial r}}{\left( \frac{\partial u_s}{\partial r} \right)^2}$$

where $r = R + n$, and $R$ is the local radius of streamline curvature.

(ii) Flux Richardson number, $Ri_f$, defined as:

$$Ri_f = \frac{2 \frac{u_s}{r}}{\left( \frac{\partial u_s}{\partial r} \right) + \left( \frac{u_s}{r} \right)}$$
situations for which they were developed, none of which shared the full complexity of the flow subject to the present investigation, i.e. the combined effects of sharp streamline curvature and steep gradients in both the streamwise and cross-stream directions. Therefore, they were not expected to produce substantial improvement in the predictions and no attempt was made to adopt any of them.

More detailed reviews of the effects of curvature on turbulence and its modelling may be found in Bradshaw [1973] and Bolour-Froushan [1986].

6.6 CONCLUSIONS

The examination of the various geometrical parameters of the computational grid indicated that these were strongly influenced by the shape of the solution domain boundaries, as might be expected. Departure from the desired favourable geometrical properties was therefore inevitable, particularly in the port exit region and the outlet passage, where discontinuities in the boundaries took place in the form of sudden changes in curvature. Skewness between the grid lines and the streamlines was also found to occur in the port exit region and the entrance of the outlet passage, which could only be eliminated through an adaptive grid generation technique. This, in addition to being computationally expensive and may result in deterioration in other properties, was difficult to adopt in conjunction with the grid generation procedure employed in the present investigation.

However, the influence of the geometrical properties of the grid on the overall accuracy of the solution was minimised, together with other possible sources of numerical errors, by refining the grid in both coordinate directions sufficiently to ensure a grid-independent
solution.

In order to resolve the ambiguity in specifying the proper conditions at the valve exit plane, the solution domain was extended inside the cylinder and the flow parameters along the free-stream boundary were extrapolated from inside the field. This was done iteratively from an initial set of boundary values; and by monitoring the changes in the profiles of the various flow parameters at the valve exit plane, the method proved to be convergent and stable. The sensitivity of the final solution to the initial values assumed along the free-stream boundary was also examined and found to be negligibly low.

The predictions of the flow field showed rapid and steep variations in the pressure and mean velocities at the entrance of the outlet passage, particularly round the valve and seat corners. This supports the observation made in Chapter 1 regarding the strong influence of the shape of these corners on the flow pattern developing in the outlet passage and the consequent exit flow pattern and discharge coefficient.

Reducing the lift was found to produce increasingly steeper pressure and mean velocity gradients near the outlet passage. This, however, did result in significant qualitative changes in the outlet flow patterns due to the inadequacies in the turbulent model summarised below. These steep gradients gradually relaxed within the outlet passage, resulting in a more uniform velocity distribution across the valve exit plane.

Comparison with the measured mean velocities and turbulent normal and shear stresses showed close agreement in the early part of the port duct where the flow developed under negligibly small pressure and velocity gradients. In the port exit region, and in the outlet passage, substantial discrepancies emerged due mainly to the suppression
of the separation and recirculation zones, which were detected in the measurements. This is believed to be due to deficiencies in the $k$-$\varepsilon$ turbulent model to take proper account of the effects of:

(i) streamwise mean velocity and pressure gradients; and

(ii) streamline curvatures.

The former resulted in suppression of the recirculation zone attached to the crown root as well as increasingly over-predicted turbulent energy and shear stresses across the entrance of the outlet passage at small lifts.

The interaction between both effects at the valve and seat corners is believed to be responsible for the suppression of the separation at both corners, and a fully attached flow was thus obtained in the outlet passage at all lifts.

Accordingly, the predicted mean velocities in the outlet passage were smaller than those measured at lifts where extended separation took place along the seat face, namely, at $l = 0.25$ and $0.20$. For smaller lifts and as the relative size of the separation zones decreased, the agreement in the profiles of the mean velocity and flow angle progressively improved.

The deficiencies in the turbulence model were traced back to the $\varepsilon$-equation and to the fact that the predicted $\varepsilon$ values are too small relative to the production rate of $k$. This, in turn, leads to a steep increase in the length scale in regions of streamwise pressure gradient and/or sharp streamline curvature producing, via its direct contribution to the turbulent viscosity, excessively high shear stresses and skin friction.

The comparison between the predicted and measured discharge
coefficient, $C_D$, showed discrepancies consistent with those featured in the flow field predictions. At the lowest lift, $L = 0.05$, the predicted $C_D$ was about 9% smaller than the measured one due to the higher rate of transfer of energy from the mean motion to the turbulent eddies featured in the excessively over-predicted turbulent energy and shear stresses in the outlet passage. At $L = 0.10$ and 0.15, the closer agreement between the measured and predicted turbulent parameters in the outlet passage resulted in close agreement in $C_D$ with differences ranging between ±2%. At $L > 0.20$, the measured $C_D$ steadily fell below the predicted $C_D$ as the relative size of the separation on the seat face, which was not reproduced in the predictions, progressively increased with lift. A difference of 6.5% at $L = 0.20$ thus increased to about 25% at $L = 0.25$.

Finally, in the light of the results presented in this chapter, it is evident that multi-dimensional predictions of the more complex flow regimes occurring in practical valve/port assembly configurations will be of limited accuracy until better turbulence models become available.
CHAPTER 7

CONCLUSIONS

7.1 MAIN FINDINGS AND ACHIEVEMENTS

This chapter outlines the main findings and achievements with respect to the objectives of the present study set in Chapter 1 and makes suggestions for future research.

The main contributions of the thesis relate to the detailed flow structure in the assembly and its effect on the discharge coefficient and exit flow characteristics, as determined experimentally; and the assessment of a multi-dimensional prediction method.

The analysis of the coefficient of discharge measurements indicated that:

(i) $C_D$ is nearly independent of the pressure drop across the valve, except at very small lifts, where boundary layer effects in the valve outlet become significant.

(ii) The variation of $C_D$ with lift suggested that, contrary to the results of previous investigations on a geometrically similar assembly, four distinct flow regimes exist in the valve outlet passage:
   (a) Fully attached flow to both the valve and seat faces at very small lifts;
   (b) At slightly higher lifts, flow separation at the valve corner and reattachment again to its face to form a recirculation bubble which persists thereafter;
   (c) In the medium lift range, flow separation at the seat corner, as well as forming a similar recirculation
bubble on the seat face; and

(d) In the high lift range, separation at the seat corner extending downstream to the valve exit and resulting in reduction in the effective outflow area and, consequently, the coefficient of discharge. The development of these régimes was further confirmed by flow visualisation.

From the detailed measurements of the mean velocities, and turbulent normal and shear stresses, it has been found that:

(iii) A recirculation zone attached to the root of the valve crown persists for all lift configurations.

(iv) The large skewness in the mean velocity profiles at the entrance of the valve outlet, induced mainly by the curvature of the streamlines, gradually relaxes towards the valve exit. Excluding the zone of separated flow at high lifts, the mean velocity distribution across the valve exit is nearly uniform at all lifts with the velocity vectors being nearly parallel to the valve and seat faces.

(v) Due to the limitations on the HWA measurements in regions of reversed flow, no detailed information could be obtained in these regions. However, it was possible to determine approximately the extent of the main recirculation zone in the valve outlet by integration of the velocity profiles.

(vi) The reduced tendency of the flow to separate from the seat
corner at small lifts is due to the increasing influence of the valve crown via the pressure field which it generates on diverting the flow into the outlet passage.

(vii) At the entrance of the outlet passage, appreciable decay in turbulence occurs due to the overriding effect of the streamwise acceleration induced by the reduction in the cross-sectional area.

(viii) The development of turbulence within the outlet passage is dominated by the relatively large cross-stream mean velocity gradients near the valve and seat boundaries and in the vicinity of the recirculation zones.

(ix) At the valve exit, the turbulence is nearly isotropic (excluding the regions of recirculating flow).

Concerning the measuring technique employed, a method was developed for interpreting the signals of X-wire probes using the concept of directional sensitivity of the sensors. It proved to be accurate and sensitive over a wide range of flow angles (up to 25°), mean velocities (10 to 30 m/s) and turbulence intensity levels (up to about 20%).

With respect to the computational objective of the thesis, the comparisons presented between the predicted and measured parameters of the flow field indicated deficiencies in the version of the $k$-$\varepsilon$ model of Jones & Launder [1973] in the present application. These were due to the effects of (a) streamline curvature, (b) appreciable normal strain rates, and (c) turbulence decay process associated with rapid accelerations. These effects were found to be of various degrees of
importance in the different regions of the flow passage.

The most noticeable adverse effect of the turbulence model was the suppression of separation and recirculation zones detected in the measurements. Accordingly, the various measured flow regimes in the outlet passage were not reproduced in the predictions. This was reflected in the predicted coefficient of discharge in the form of a nearly constant value over most of the lift range investigated. In the medium-lift range, where separation does not significantly influence the performance of the assembly, close agreement was obtained. At very low lifts, however, the predicted coefficient of discharge was considerably less than the measured one due to the excessively over-predicted turbulence.

7.2 SUGGESTIONS FOR FUTURE WORK

(1) With respect to multi-dimensional computational methods, there is a pressing need for a turbulence model capable of accounting for the separate and interacting effects of pressure gradients and streamline curvatures. Until such a model becomes available, numerical predictions of many of the complex flow regimes in reciprocating engines will remain, to say the least, unsatisfactory.

(2) Although the present study provides useful information on the flow in valve/port assemblies, the results in many respects are restricted to the idealised configuration considered. Further investigations have therefore to be directed towards more practical configurations employed in production engines. In view of the limitations on computational methods, mentioned above, these investigations will have to rely for the time being on experimental measurements.
(3) There is a need for more specific investigations of the effects of varying the geometrical details of the assembly, to which the flow patterns in the valve outlet proved to be sensitive. The information presented in this thesis provides some guidelines for the more important modifications which might be made, e.g. the shape of the crown surface, the rounding off of the corners and the width of the valve and seat faces. Such modifications should be aimed at reducing the tendency of the flow to separate at the seat corner, which is responsible for the deterioration in performance of inlet valves in the range of effective operation, namely, the medium-to-high lift range.
Fig. 1.1 Variation of $C_D$ with $\Delta P$ (Woods et al (1942)).

Fig. 1.2 Geometrical aspects of the valve/port assembly.
Fig. 1.3 Mass flow rate measurements of Tanaka (1929).

(a) Sharp-edged valve and seat.

(b) Effect of varying the seat angle.

(c) Effect of rounding off the valve and seat corners
Fig. 1.4 Flow regimes postulated by Tanaka (1929).
Fig. 1.6 In-cylinder mean velocity fields during induction, compression and early expansion (Predictions of Ahmadi-Befrui (1985)).
Fig. 1.5 Comparisons between steady and transient coefficients of discharge.
Fig. 1.7 Predictions of Isshiki et al (1985).
\[ PA = a^{(1)} = a^1 \sqrt{g_{11}} \]
\[ PB = a^{(2)} = a^2 \sqrt{g_{22}} \]
\[ PC = a^{(1)}_1 = a_1 \sqrt{g_{11}} \]
\[ PD = a^{(2)}_2 = a_2 \sqrt{g_{22}} \]

\{ Ricci and Levi-Civita components \}
\{ Ricci and Levi-Civita projections \}

Fig. 2.1 Physical covariant and contravariant components of a two-dimensional vector.
Fig. 2.2 Two-dimensional ICED-ALE grid.

Fig. 2.3 The TURF grid.
Fig. 2.4 Definition of base vectors in terms of locally fixed basis.

\[(e_{(1)})_w = \beta_{(1)}^{(1)} (e_{(1)})_p + \beta_{(1)}^{(2)} (e_{(2)})_p\]

\[(e_{(2)})_w = \beta_{(2)}^{(1)} (e_{(1)})_p + \beta_{(2)}^{(2)} (e_{(2)})_p\]
Fig. 2.5 Grid and variables arrangements employed by Demirdzic (1980).
Fig. 3.1 Staggered grid arrangement.
Fig. 3.2 Computational module and notation.
Fig. 3.3 Scalar and $v^{(1)}$ computational cells.
Fig. 3.4 Wall boundary.

Fig. 3.5 Dimensions of a scalar cell.
Fig. 3.6 Boundary points and notation for Eqn. 3.87.

Fig. 3.7 Notation for Eqns. 3.88 to 3.92.
Fig. 3.8 Local coordinate transformation.

Fig. 3.9 Local geometrical parameters.
Fig. 4.1 Test rig layout.

Fig. 4.2 Test Section.
Fig. 4.3 Schematic layout of the test rig.
Fig. 4.4 Velocity profiles at the diffuser inlet.

Fig. 4.5 Modified velocity profiles at the diffuser inlet.
Fig. 4.6 Velocity profiles at the inlet of the test section.
Fig. 4.7 Details of the test section.

(Dims. in mm.)
Fig. 4.8 Details of the valve/port assembly.
(Dims. in mm.)

Fig. 4.9 Thermocouple calibration.
(cold junction at 0°C.)
Fig. 4.10 Block diagrams for single and X-wire probes.
Fig. 4.11 Typical calibration curve for a single normal wire probe.

Fig. 4.12 Linearized output of Fig. 4.11.
Fig. 4.13 Velocity profile at the jet nozzle exit.

Fig. 4.14 Velocity profiles at x/d=55.
Fig. 4.15 Normalized velocity profiles along the jet axis.

Fig. 4.16 Jet rate of spread.
Fig. 4.17 Decay of the mean velocity along the jet axis.

Fig. 4.18 Turbulence intensity profiles.
Fig. 4.19 Hot-wire in a general flow field.

Fig. 4.20 Notation for X-wire probes.
Fig. 4.21 Calibration for effective wires' angles.
Fig. 4.22: Probe support for directional calibration and measurements. (Dims. in mm.)
Fig. 4.23 Typical calibration curves for an X-wire probe.
(Probe axis aligned with the mean velocity vector)

Fig. 4.24 Linearized output of Fig. 4.23.
Fig. 4.25 Two-dimensional velocity field.

(a) Instantaneous velocity components

(b) Cooling velocity components

Fig. 4.25 Two-dimensional velocity field.
Fig. 4.26 Mean velocity measurements at different turbulence intensity levels.

Fig. 4.27 Mean flow angle.
Fig. 4.28 Relative normal stress $u^2/\bar{\sigma}^2$

Fig. 4.29 Relative normal stress $v^2/\bar{\sigma}^2$
Fig. 4.30 Relative shear stress $\overline{u v} / \overline{Q^2}$

Fig. 4.31 Comparison with the formulae of Champagne & Sleicher (1967). (cont.)
Fig. 4.31 (concluded)
Fig. 4.32a Effect of neglecting higher order Reynolds stresses.
Fig. 4.32b Effect of different assumptions for the lateral normal stress $\bar{u}_3^2$.

\[ \bar{u}_3^2 = 0.55(\bar{u}_1^2 + \bar{u}_2^2) \]

\[ \bar{u}_3^2 = 0.45(\bar{u}_1^2 + \bar{u}_2^2) \]

Turbulence intensity ~ 10%
$\overline{Q}$ ~ 20.0 m/s
Fig. 5.1 Mass Flow Rate Measurements.
Fig. 5.2 Notation for various regions of the flow passage.
Fig. 5.3a Throat area at different lift configurations
(Eqns. 5.5).

Fig. 5.3b Variation with lift of the throat area based on different definitions.
Fig. 5.4 $C_D$ based on various definitions of $A_2$ and $P_2$

- Throat area (Eqn. 5.5) & in-cylinder pressure
- Normal curtain area (Eqn. 5.10) & in-cylinder pressure
- Throat area (Eqn. 5.5) & corrected pressure
- Curtain area (Eqn. 5.9) & in-cylinder pressure
Fig. 5.5 Variation of $C_D$ with lift
Fig. 5.6 Variation with lift of Reynolds number based on outlet conditions.
Fig. 5.7 Variation of $C_D$ with Reynolds number.
Fig. 5.8 Definition of the regions of various flow regimes in the outlet passage.
Fig. 5.9 Attached Flow ($L \leq 0.07$)

(a) $L = 0.05$
Fig. 5.10 Separation on the valve face ($0.12 \geq L > 0.07$) (cont.)
Fig. 5.11 Separation on both the valve and seat faces
(0.17 ≥ L > 0.12) (cont.)

(a) L = 0.15
Fig. 5.11 (concluded)
Fig. 5.12 Extended separation on the seat face
\((L > 0.17)\) (cont.)

(a) \(L = 0.25\)
(d) $L = 0.30$

Fig. 5.12 (concluded)
(a) Fully attached flow (\( L \leq 0.07 \))

(b) Separation on the valve face (\( 0.07 \leq L < 0.12 \))

(c) Separation on valve and seat faces (\( 0.12 \leq L < 0.17 \))

(d) Extended separation on the seat face (\( L \geq 0.17 \))

**Fig. 5.13** Sketch of the different flow regimes in the outlet passage.
Fig. 5.15 Mean velocity profiles at the inlet section.
Fig. 5.16 Average mean velocity at the inlet section.
Fig. 5.17 Mean velocity distribution in the boundary layers at the inlet section (Eqn. 5.13)
Fig. 5.18 Mean velocity profiles at Station (2).
Fig. 5.19 Definitions for Eqns. 5.16 & 5.17
Fig. 5.20 Mean velocity vectors for L=0.25.
Fig. 5.21 Streamlines for L=0.25.
Fig. 5.22 Mean velocity profiles for $L=0.25$. (cont.)
Fig. 5.22 (concluded).
Fig. 5.23 Average mean velocity at Station (3).
Fig. 5.24 Mean velocity vectors for various configurations. (cont.)
Fig. 5.24 (cont.)

(a) $L = 0.15$

(b) $L = 0.15$

-$\rightarrow$ 30.0 MT/S.
Fig. 5.24 (cont.)

(c) $L = 0.10$

$30.0 \text{ MTS}$
I mill
I Mill
U;
0
C;
V
C;
ei
0)
U-

30.0 MT/S.
(d) L = 0.05

Fig. 5.24 (concluded).
Fig. 5.25 Streamlines for various configurations. (cont.)
Fig. 5.25 (concluded).
Fig. 5.26 Mean velocity profiles for L=0.20. (cont.)
Fig. 5.26 (concluded).

(b) Outlet passage
Fig. 5.27 Mean velocity profiles for L=0.15. (cont.)
Fig. 5.27 (concluded).
Fig. 5.28 Mean velocity profiles for L=0.10. (cont.)
(b) Outlet passage

Fig. 5.28 (concluded).
Fig. 5.29 Mean velocity profiles for L=0.05. (cont.)
(b) Outlet passage

Fig. 5.29 (concluded).
Fig. 5.30 Mean velocity profiles in the port duct. (cont.)
Fig. 5.30 (concluded).
Fig. 5.31b Comparison with LDA measurements of the mean velocity at the valve exit.
Fig. 5.32 Turbulent Stresses profiles at the inlet section. (cont.)
Fig. 5.32 (cont.)
Fig. 5.32 (concluded).
Fig. 5.34 Turbulent stresses profiles along the port duct for L=0.25. (cont.)
Fig. 5.35 Turbulent stresses profiles in the outlet passage for $L=0.25$ (cont.)
Fig. 5.35 (concluded)
Fig. 5.36 Normal stress component $\overline{\sigma_2}$ (cont.)

Station 1

$\overline{\sigma_2} = \frac{(\overline{Q}_n^2)}{10^3}$

- $0.20$
- $0.15$
- $0.10$
- $0.05$

$uR_0$

$0.2$
$0.4$
$0.6$
$0.8$

$1.0$
$2.0$
$3.0$
$4.0$
$5.0$
$6.0$
$7.0$
Fig. 5.37 (concluded).
Fig. 5.38 Shear stress $\frac{{u\bar V}}{{Q^2_m}}$ (cont.)
Fig. 5.39 Normal stress component $\frac{u^2}{Q_m^2}$ in the outlet passage.
Fig. 5.40 Normal stress component $\frac{\nabla^2 \Omega^2_m}{Q_m^2}$ in the outlet passage.
Fig. 5.14. Shear stress $\overline{u'v'^2}$ in the outlet passage.
Fig. 5.42 Comparison between single and X-wire measurements of $\sqrt{u'^2}/Q_m$ in the port duct for $L=0.25$. 
Fig. 5.43 Comparison between single and X-wire measurements of $\sqrt{\bar{u}^2 / Q_m}$ in the outlet passage, for $L=0.25$
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Fig. 6.6 Results of the streamwise grid refinement. (cont.)
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(b) Exit of zone 2

(iii) Relative turbulence energy

(iv) Relative turbulent viscosity

(v) Streamwise velocity component

(vi) Cross-stream velocity component
Fig. 6.6 (concluded).
Fig. 6.7 Results of the cross-stream grid refinement.

(e) Relative turbulence energy
(f) Relative turbulent viscosity
(g) Streamwise velocity component
(h) Cross-stream velocity component

(cont.)
(a) Exit of zone 2

Fig. 6.7 (concluded).
Fig. 6.8 Mean velocity vectors for L=0.25.

Fig. 6.9 Streamlines for L=0.25.
Fig. 6.10 Isobars $\frac{\rho}{\rho} u_3^2$ for $L=0.25$.

Fig. 6.11 Streamwise mean velocity profiles for $L=0.25$.  

= 28.3 m/s
Fig. 6.12 Streamwise distribution of the predicted wall shear stresses for L=0.25.
Fig. 6.13 Turbulence intensity $\sqrt{k/U_3}$ for $L=0.25$.

Fig. 6.14 Dimensionless length scale $l_m/D$ for $L=0.25$. 
Fig. 6.15 Predicted flow field for L=0.20. (cont.)
Fig. 6.15 (cont.)

- Streamwise mean velocity profiles

(c) Isobars \( \frac{p}{\rho U_g^2} \)

(d) Streamwise mean velocity profiles

\[
\begin{align*}
A &= 0.230 \\
B &= 0.180 \\
C &= 0.110 \\
D &= 0.070 \\
E &= 0.023 \\
F &= 0.023 \\
G &= 0.092 \\
H &= 0.110 \\
I &= 0.200 \\
J &= 0.300 \\
K &= 0.400 \\
L &= 0.500 \\
M &= 0.650 \\
N &= 0.850 \\
O &= 1.150
\end{align*}
\]
Fig. 6.15 (concluded).
Fig. 6.16 Predicted flow field for L=0.15. (cont.)
Fig. 6.16 (cont.)

(c) Isobars \( \left( \frac{p}{p_0^2} \right) \)

(d) Streamwise mean velocity profiles

\[ \begin{align*}
A &= 0.180 \\
B &= 0.110 \\
C &= 0.070 \\
D &= 0.023 \\
E &= 0.023 \\
F &= 0.002 \\
G &= 0.200 \\
H &= 0.400 \\
I &= 0.650 \\
J &= 0.850 \\
K &= 1.150 \\
L &= 1.350 \\
M &= 1.500 \\
N &= 1.650
\end{align*} \]
(e) Turbulence intensity $\sqrt{\kappa \overline{u}_3}$

(f) Length scale $l_m/D$

Fig. 6.16 (concluded).
Fig. 6.17 Predicted flow field for $L=0.10$. (cont.)

(a) Mean velocity vectors

(b) Streamlines

- $A = 0.022$
- $B = 0.019$
- $C = 0.015$
- $D = 0.012$
- $E = 0.010$
- $F = 0.008$
- $G = 0.005$
- $H = 0.0035$
- $I = 0.0020$
- $J = 0.0010$
- $K = 0.0001$
- $L = 0.0005$

$= 36.2 \text{ m/s}$
(c) Isobars \( \left( \frac{p}{p_0 U_3^2} \right) \)

(d) Streamwise mean velocity profiles

Fig. 6.17 (cont.)
(e) Turbulence intensity $\sqrt{\kappa / \bar{U}_3}$

(f) Length scale $l_m/D$

Fig. 6.17 (concluded).
Fig. 6.18 Predicted flow field for L=0.05. (cont.)
(c) Isobars \( \frac{p}{p_U^2} \)

(d) Streamwise mean velocity profiles

Fig. 6.18 (cont.)
(e) Turbulence intensity $\sqrt{k}/u_3$

(f) Length scale $l_m/D$

Fig. 6.18 (concluded).
Fig. 6.19 Streamwise distribution of the predicted wall shear stresses for various lifts.
Fig. 6.20 Notation for interpolation between cell nodes.

Fig. 6.21 Shear stress distribution across the inlet section (L=0.25).
Fig. 6.22 Comparison between predicted and measured mean velocities for L=0.25. (cont.)
Fig. 6.22 (concluded).
Fig. 6.23 Comparison between predicted and measured relative
turbulence energies and shear stresses for L=0.25. (cont.)
Fig. 6.24 Shear stress distribution across the inlet section for various lifts.
Fig. 6.25 Comparison between predictions and measurements for $L=0.15$. (cont.)
Fig. 6.25 (cont)
(b) Relative turbulence energy and shear stress

Fig. 6.25 (cont.)
Fig. 6.26 Comparison between predictions and measurements for \( L=0.10 \). (cont.)
Fig. 6.26 (cont.)
Fig. 6.27 Comparison between predictions and measurements for L=0.05. (cont.)
(b) Relative turbulence energy and shear stress

Fig. 6.27 (cont.)
Fig. 6.28 Comparison between predicted and measured $C_D$.

Fig. 6.29 Two-dimensional $s$-$n$ coordinate system (Bradshaw(1973)).

$(s, n)$ coordinates with radius of curvature $R$ a function of $s$, positive for convex $s$-axis.
APPENDIX A

REVIEW OF GENERAL TENSOR CALCULUS

A.1 COORDINATE TRANSFORMATION

In an Euclidean, three-dimensional space, let \( x^i \) and \( \xi^i \) denote two different frames of reference whose coordinates are related by,

\[
\xi^i = \xi^i(x^j) = \xi^i(x^1, x^2, x^3), \quad (i = 1, 2, 3) \tag{A.1}
\]

If the functions involved are single valued, continuous and have continuous derivatives, the Jacobian (Spiegel [1974]),

\[
J = \det \begin{vmatrix}
\frac{\partial \xi^1}{\partial x^1} & \frac{\partial \xi^2}{\partial x^1} & \frac{\partial \xi^3}{\partial x^1} \\
\frac{\partial \xi^1}{\partial x^2} & \frac{\partial \xi^2}{\partial x^2} & \frac{\partial \xi^3}{\partial x^2} \\
\frac{\partial \xi^1}{\partial x^3} & \frac{\partial \xi^2}{\partial x^3} & \frac{\partial \xi^3}{\partial x^3}
\end{vmatrix} \tag{A.2}
\]

is non-zero, and corresponding to each set of coordinates \((\xi^1, \xi^2, \xi^3)\) there exists a unique set \((x^1, x^2, x^3)\) such that,

\[
x^i = x^i(\xi^j) = x^i(\xi^1, \xi^2, \xi^3), \quad (i = 1, 2, 3) \tag{A.3}
\]

The relations (A.1) and (A.3), together with the condition (A.2), define the transformation of coordinates from one frame of reference to another.

The natural base vectors \( e_i \) and \( \xi_i \), which are tangents to the coordinate curves \( x^i \) and \( \xi^i \), respectively, transform according to,
The dual base vectors $\xi^i \xi^j$ and $\xi^i \xi^j$, which are orthogonal to the coordinate surfaces $X^i = \text{const.}$ and $\xi^i = \text{const.}$, transform according to,

$$\xi^i \xi^j = \frac{\partial x^i}{\partial \xi^j} \xi^j$$

and

$$\xi^i \xi^j = \frac{\partial \xi^i}{\partial x^j} \xi^j$$ (A.5)

If $\gamma^i$ is taken to represent the Cartesian frame of reference whose bases $\gamma^m$ and $\gamma^m$ are orthogonal by definition, it can be shown that the natural and dual bases of any frame of reference are orthogonal and reciprocal to each other. This follows directly from (A.4) and (A.5), since (Aris [1962])

$$\xi^i \xi^j \cdot \xi^i \xi^j = \frac{\partial x^i}{\partial y^j} \xi^j \cdot \xi^i \xi^j = \frac{\partial x^i}{\partial y^j} \xi^j \xi^j = \frac{\partial x^i}{\partial x^j} \delta^j = \delta^j$$ (A.6)

where $\delta^j$ is the Kronecker delta defined by

$$\delta^j = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$$ (A.7)

+ Matching upper and lower indices are summed over the range of their values.

* Two base vectors $b^i$ and $b^j$ are said to be reciprocal to each other if $b^i \cdot b^j = \delta^j$.
In the $x^i$ coordinate system, a vector $a$ can be represented in terms of either basis, i.e.

$$ a = a^i e_i $$

or

$$ a = a_i e^i $$  \hspace{1cm} (A.8) $$

To represent this vector in the $\xi^i$ coordinate system, the transformations (A.4) and (A.5) are substituted into (A.8) to give

$$ a = a^i e_i = \frac{\partial x^i}{\partial \xi^j} e_j = a^i e_j $$

or

$$ a = a_i e^i = \frac{\partial x^i}{\partial \xi^j} e^j = a_j e^j $$  \hspace{1cm} (A.9) $$

where

$$ a^i = a_i \frac{\partial x^i}{\partial \xi^j} $$

and

$$ a_j = a_i \frac{\partial x^i}{\partial \xi^j} $$  \hspace{1cm} (A.10) $$

The scalars $a_i$ and $a_j$ are called the covariants of the vector $a$, while $a^i$ and $a^j$ are called the contravariant components.

Similarly, a tensor $A$ can be represented as

$$ A = A^i_j e_i \otimes e_j $$

$$ A = A_{ij} e^i \otimes e^j $$  \hspace{1cm} (A.11) $$

$$ A = A^j_i e^i \otimes e^j $$

where $e_i \otimes e_j$, $e^i \otimes e^j$, $e^i \otimes e_j$ and $e_i \otimes e^j$ are the tensor products of the base vectors or the "dyadics". Transformation formulae for the dyadics
are readily obtained from equations (A.4) and (A.5). The representation of tensor $\mathbf{A}$ in the $\xi$ coordinate system may thus be written as

$$\mathbf{A}_\xi = \alpha_{ij} \xi^i \xi^j$$

(A.12)

$$\mathbf{A}_\xi = \alpha_{ij} \xi^i \xi^j$$

where

$$\alpha_{ij} = \Lambda_{ij} \frac{\partial \xi^j}{\partial x^i}$$

(A.13)

The quantities $(\alpha_{ij}^i)$ and $(\alpha_{ij}^j)$, $(\Lambda_{ij}^i)$ and $(\Lambda_{ij}^j)$ and $(\alpha_{ij})$ are called contravariant, covariant and mixed components of tensor $\mathbf{A}$, respectively.

A.2 THE METRIC

The arc length differential in an Euclidean, three-dimensional space is given by (Sokolnikoff [1951])

$$dS^2 = g_{ij} \, dx^i \, dx^j$$

(A.14)

where $g_{ij}$ are the covariant components of the metric tensor and are defined by

$$g_{ij} = \varepsilon_{\xi^i} \cdot \varepsilon_{\xi^j} = \sum_{m=1}^{3} \frac{\partial \xi^m}{\partial x^i} \cdot \frac{\partial \xi^m}{\partial x^j}$$

(A.15)

The contravariant components of the metric tensor are similarly
defined as the dot product of the dual base vectors,

\[ g^{ij} = \xi^i \cdot \xi^j = \sum_{m=1}^{3} \frac{\partial x^i}{\partial \xi^m} \frac{\partial x^j}{\partial \xi^m} \quad (A.16) \]

The mixed components of the metric tensor are similarly defined but, according to equation (A.6), they reduce to the Kronecker delta as follows,

\[ g^{ij} = \xi^i \cdot \xi^j = \delta^i_j \quad (A.17) \]

Moreover, the components of the metric tensor are used for determining the magnitudes of the base vectors as well as the angles between them. For example,

\[ |\xi^i| = \sqrt{g_{ii}} \quad (A.18) \]
\[ |\xi^i| = \sqrt{g^{ii}} \quad (A.19) \]
\[ \cos \theta^{ij} = \frac{g_{ij}}{\sqrt{g_{ii} g_{jj}}} \quad (A.20) \]

where \( \theta^{ij} \) is the angle between the base vectors \( \xi^i \) and \( \xi^j \).

It can also be shown that the metric tensor relates natural and dual bases, as well as contravariant, covariant and mixed components of vectors and tensors. For example,

\[ \xi^i = g^{im} \xi^m \quad (A.21) \]
\[ \xi^i = g^{-im} \xi^m \]
A.3 ALGEBRAIC OPERATIONS

Similar to the familiar definition of scalar multiplication and addition of vectors, the same operations for tensors are defined as (Spiegel [1974])

\[ S \frac{A}{\tau} = (S \lambda^{ij}) \frac{e_{i}}{\tau} \cdot \frac{e_{j}}{\tau} \quad \text{(A.24)} \]

\[ \frac{A}{\tau} + \frac{B}{\tau} = (\lambda^{ij} + \beta^{ij}) \frac{e_{i}}{\tau} \cdot \frac{e_{j}}{\tau} \quad \text{(A.25)} \]

where \( S \) is a scalar. The same rules apply if the tensor is expressed in terms of covariant or mixed components. It is obvious, however, that one can sum only tensors of the same type.

The dot, or inner, product of two vectors \( a \) and \( b \) is a scalar defined by

\[ a \cdot b = g^{ij} a^{i} b^{j} = a^{i} b_{i} = a_{j} b^{j} \quad \text{(A.26)} \]

The dot product of a vector \( a \) and a tensor \( B \) is a vector defined by

\[ a \cdot B = a^{i} b^{jk} g_{ij} \frac{e_{k}}{\tau} \quad \text{(A.27)} \]
The dot product of two tensors $A$ and $B$ is a tensor given by

$$
\frac{A \cdot B}{\nabla} = g_{jk} A^{ij} B^{k\ell} \varepsilon_{i\ell} \varepsilon_{j\ell} = A^{ij} B_{jk} \varepsilon_{i\ell} \varepsilon_{j\ell}
$$

(A.28)

The double dot, or inner, product of two tensors is a scalar defined by

$$
\frac{A:B}{\nabla} = g_{jk} g_{i\ell} A^{ij} B^{k\ell} = A^{i\ell} B_{i\ell}
$$

(A.29)

The tensor, or outer, product of two vectors $a$ and $b$ is a tensor given by

$$
a \times b = a^i b^j \varepsilon_{i\ell} \varepsilon_{j\ell} = a^\ell b^\ell \varepsilon_i \varepsilon_j
$$

(A.30)

The tensor product of the base vectors

$$
I = \varepsilon_{i\ell} \varepsilon_{j\ell} = \varepsilon_i \varepsilon_j
$$

(A.31)

is called the identity or unit tensor. The unit tensor has the property of not changing other vectors or tensors when the dot product on either side is taken.

The tensor product of a vector and a tensor and two tensors are third and fourth order tensors, respectively, whose definition is similar to (A.30).

The transpose of a tensor $A$ is a tensor $A^T$ which is formed by interchanging the indices of $A$ such that,

$$
(A^{ij})^T = A^{ji} , \quad (A_{ij})^T = A_{ji} , \quad (A^{ij})^T = A^{ij}
$$

(A.32)

Tensors $B$ and $C$, defined by
are called symmetric and skew symmetric, respectively. Any tensor $A$ can be expressed as the sum of a symmetric and a skew symmetric tensors such that
\[
A = \frac{1}{2} (A + A^T) + \frac{1}{2} (A - A^T) = B + C
\] (A.34)
where
\[
B = \frac{1}{2} (A + A^T)
\] (A.35)
and
\[
C = \frac{1}{2} (A - A^T)
\] (A.36)

The operations of generating tensors $B$ and $C$ are called symmetrisation and alternation, respectively.

### A.4 CHRISTOFFEL'S SYMBOLS AND COVARIANT DERIVATIVE

Since the base vectors are not generally constants (except in Cartesian coordinate systems), their derivatives form vectors. These derivatives are compactly written in the forms (Sokolnikoff [1951])

\[
\frac{\partial e_i}{\partial x^j} = \{^m_i j\} e^m
\] (A.37)
\[
\frac{\partial e^i}{\partial x^j} = -\{^i_j m\} e^m
\]

The quantities $\{^m_i j\}$ are called Christoffel's symbols of the second kind, simply referred to here as Christoffel's symbols, and are functions of the metric tensor components, e.g.

\[
\{^i_{j k}\} = \frac{1}{2} g^{im} \left( \frac{\partial g_{im}}{\partial x^k} + \frac{\partial g_{km}}{\partial x^j} - \frac{\partial g_{ik}}{\partial x^m} \right)
\] (A.38)
In an Euclidean, three-dimensional space, Christoffel's symbols are symmetric with respect to the indices \( j \) and \( k \). However, for \( \dot{i} = j \), equation (A.38) reduces to

\[
\{ \dot{i} \}_{\dot{k} \dot{k}} = \frac{1}{\sqrt{g}} \frac{\partial \sqrt{g}}{\partial x^k} \tag{A.39}
\]

where \( g \) is the determinant of the metric tensor.

In terms of Christoffel's symbols, the derivatives of the metric tensor components are given by

\[
\frac{\partial g_{\dot{i} \dot{j}}}{\partial x^j} = \frac{1}{\sqrt{g}} \{ \dot{i} \}_{\dot{j}} g_{\dot{k} \dot{m}} \tag{A.40}
\]

The derivative of a continuous vector \( \vec{a} \) is given by

\[
\frac{\partial \vec{a}}{\partial x^j} = \frac{\partial}{\partial x^j} (a^i e_{\dot{i}}) = \frac{\partial a^i}{\partial x^j} e_{\dot{i}} + a^i \frac{\partial e_{\dot{i}}}{\partial x^j} \tag{A.41}
\]

Substituting (A.37) into this expression, one gets

\[
\frac{\partial \vec{a}}{\partial x^j} = \nabla_j a^i e_{\dot{i}} \tag{A.42}
\]

where

\[
\nabla_j a^i = \frac{\partial a^i}{\partial x^j} + a^m \{ m \}_{\dot{i} \dot{j}} \tag{A.43}
\]

Equation (A.43) defines the covariant derivative of the contravariant components of vector \( \vec{a} \). Similarly,

\[
\nabla_j a_{\dot{i}} = \frac{\partial a_{\dot{i}}}{\partial x^j} - a_m \{ m \}_{\dot{i} \dot{j}} \tag{A.44}
\]

is the covariant derivative of the covariant components of \( \vec{a} \).

On the other hand, the derivative of a tensor \( \vec{A} \) is given by
\[ \frac{\partial A}{\partial x^k} = \frac{\partial}{\partial x^k} \left( A^{ij} \frac{e_i}{e_j} \right) \]

\[ = \frac{\partial A^{ij}}{\partial x^k} \frac{e_i}{e_j} + A^{ij} \frac{\partial e_i}{\partial x^k} \frac{e_j}{e_j} + A^{ij} \frac{e_i}{e_i} \frac{\partial e_j}{\partial x^k} \]

\[ = \nabla_k A^{ij} \frac{e_i}{e_j} \frac{e_j}{e_j} \]

(A.45)

where

\[ \nabla_k A^{ij} = \frac{\partial A^{ij}}{\partial x^k} + A^{mj} \{ m \}_{i k} + A^{im} \{ j \}_{m k} \]

(A.46)

is the covariant derivative of the contravariant components of tensor \( A \).

In terms of the covariant and mixed components, the covariant derivatives of tensor \( A \) are defined, respectively, by

\[ \nabla_k A_{\cdot j} = \frac{\partial A_{\cdot j}}{\partial x^k} - A_{mj} \{ m \}_{i k} - A_{im} \{ j \}_{m k} \]

(A.47)

\[ \nabla_k A_{\cdot i} = \frac{\partial A_{\cdot i}}{\partial x^k} + A_{mj} \{ m \}_{k j} - A_{m i} \{ j \}_{m k} \]

(A.48)

Equations (A.46) and (A.47), together with the definition (A.38), yield an important property of the metric tensor, namely,

\[ \nabla_k g^{ij} = \nabla_k g_{ij} = 0 \]

(A.49)

A.5 GRADIENT AND DIVERGENCE

The gradients of scalar, vector and tensor fields are defined, respectively, by (Eisenhart [1940])

\[ \text{grad } S = e^j \frac{\partial S}{\partial x^j} = g^{jm} \frac{\partial S}{\partial x^j} e_m \]

(A.50)
\[
\text{grad} \; \mathbf{a} = e^j \sum_{i} \frac{\partial a_i}{\partial x^j} = \nabla_j a_i \mathbf{e}^i - e^i = g^{im} \nabla_j a^i \mathbf{e}_m - \mathbf{e}_i^i \quad (A.51)
\]

and,
\[
\text{grad} \; A = e^k \sum_{i} \frac{\partial A_{ij}}{\partial x^k} = g^{km} \nabla_k A_{ij} \mathbf{e}_m - \mathbf{e}_i^j \quad (A.52)
\]

Replacing the tensor product by the dot product in (A.51) and (A.52), one obtains the following expressions for the divergence of vector and tensor fields, respectively,
\[
\text{div} \; \mathbf{a} = e^j \sum_{i} \frac{\partial a_i}{\partial x^j} = g^{ij} \nabla_j a_i = \nabla_j a^j \quad (A.53)
\]

and
\[
\text{div} \; A = e^k \sum_{i} \frac{\partial A_{ij}}{\partial x^k} = \nabla_j A_{ij} \mathbf{e}_k - \mathbf{e}_i^k \quad (A.54)
\]

However, with the help of (A.43) and (A.39), equation (A.53) may be rewritten as
\[
\text{div} \; \mathbf{a} = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^j} [\sqrt{g} \; a^j] \quad (A.55)
\]

which is called the strong conservation form of the divergence of the vector field \( \mathbf{a} \). The introduction of the differential operator
\[
\frac{\Delta}{\Delta x^j} (\ldots) = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^j} [\sqrt{g} (\ldots)] \quad (A.56)
\]

reduces equation (A.55) into
\[
\text{div} \; \mathbf{a} = \frac{\Delta a^j}{\Delta x^j} \quad (A.57)
\]

The divergence of a tensor field, equation (A.54), is similarly written in the form
$$\text{div } \mathbf{A} = \frac{\Delta}{\Delta x^j} \left( A^{ij} \frac{e_{\xi}}{e_{\xi}} \right)$$ 
(A.58)
APPENDIX B

B.1 ESTIMATE OF THE PRESSURE DROP ACROSS THE VALVE

Engine and valve/port assembly specifications:

Stroke, \( S = 0.094 \) m
Bore diameter, \( d_b = 0.074 \) m
Speed, \( n = 300 \) rpm
Valve head diameter, \( D_v = 0.033 \) m
Port diameter, \( D = 0.027 \) m
Valve seat angle, \( \alpha = 45^\circ \)

Under quasi-steady conditions, the rate of flow through the valve is set to be the equivalent to the total volume admitted into the cylinder per unit time. For a motored engine at a rotational speed of 300 rpm, 150 swept volume are charged into the cylinder per minute, thus

\[
\dot{V} = \frac{\pi}{4} (0.074)^2 \times 0.094 \times \frac{150}{60} = 1.01 \times 10^{-3} \text{ m}^3/\text{s}. \quad (B.1)
\]

With reference to the incompressible flow analysis presented in Section 4.2, application of the energy and continuity equations, neglecting the upstream velocity head, yields

\[
\dot{V} = C_D A \sqrt{2\Delta p/\rho}. \quad (B.2)
\]

where \( C_D \) is the coefficient of discharge, \( \Delta p \) is the pressure drop across the valve, and \( A \) is the valve exit area given by
\[ A = \pi D \ell \sin \alpha \]  \hspace{1cm} (B.3)

where \( \ell \) is the valve lift.

Assuming the coefficient of discharge \( C_D \) to have an average value of 0.8, equations (B.1), (B.2) and (B.3), in terms of the dimensionless pressure drop \( \Delta P \) and lift \( L \), yield

\[ \Delta P = 2.26 \times 10^{-5}/L^2 \]  \hspace{1cm} (B.4)

This relationship is plotted in Figure B.1 for lifts ranging between 0.02 and 0.3.

**B.2 UNCERTAINTY IN MASS FLOW RATE MEASUREMENTS**

Based on the analysis detailed in BS1042, the uncertainty in the measured mass flow rate \( X_m \) may be represented by

\[
X_m = \left[ X_C^2 + X_R^2 + X_D^2 + X_e^2 + \left( \frac{2}{1 - m^2} \right)^2 X_d^2 + \left( \frac{2m^2}{1 - m^2} \right)^2 X_p^2 \right]^{1/4} + \frac{1}{4} \left( X_p^2 + X_h^2 \right)^{1/2} \]  \hspace{1cm} (B.5)

where

- \( X_C = \pm 2.3\% \) uncertainty in the coefficient of discharge of the orifice
- \( X_R = \pm 0.65\% \) uncertainty in the Reynolds number correction factor
- \( X_D = 0.0 \) (for plastic pipe) uncertainty in pipe size correction factor
- \( X_e = 0.1\% \) uncertainty in compressibility factor
- \( m = 0.644 \) orifice-to-pipe area ratio
- \( X_d = \pm 0.025\% \) uncertainty in orifice diameter
$\chi_D = \pm 0.05\%$ uncertainty in pipe diameter

$\chi_h = \pm 1.0\%$ uncertainty in measurement of pressure drop across the orifice

$\chi_\rho = \pm 0.5\%$ uncertainty in air density calculations

The above figures yield an estimate for the maximum uncertainty in mass flow measurements of about 2.5%.
B.3 CALIBRATION DATA FOR HOT-WIRE PROBES

(1) Single Wire Probe

TABLE B.1
Calibration Data for a Single-Wire Probe

<table>
<thead>
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<th>Q (m/s)</th>
<th>E (volt)</th>
</tr>
</thead>
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</tr>
<tr>
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\[ R = \text{regression coefficient} \]

\[ e_{E_0} = \text{maximum error in } E_0 \ (%\) \]

\[ e_B = \text{maximum error in } B \ (%\) \]
TABLE B.2

Calibration Data for a X-Wire Probe

<table>
<thead>
<tr>
<th>$\overline{Q}$ (m/s)</th>
<th>$\overline{Q}_1$ (m/s)</th>
<th>$\overline{Q}_2$ (m/s)</th>
<th>$\overline{E}_1$ (volt)</th>
<th>$\overline{E}_2$ (volt)</th>
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<td>4.845</td>
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<tr>
<th>$n$</th>
<th>$E_{01}$</th>
<th>$B_1$</th>
<th>$E_{02}$</th>
<th>$B_2$</th>
<th>$R_1$</th>
<th>$R_2$</th>
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<td>1.85</td>
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Fig. B.1 Estimated pressure drop across the valve under quasi-steady conditions.

Fig. B.2 Typical yaw angle calibration for a single-wire probe.
B.4 TERMS IN THE HOT-WIRE RESPONSE EQUATIONS

The approximate equation (4.49) representing the hot-wire response results in the following expression for the instantaneous cooling velocity,

\[
Q_c = CGI \bar{\Omega} \cos \gamma \left[ a_0 + a_1 u_1' + a_2 u_2' + a_3 u_3' + a_4 u_1'^2 + a_5 u_2'^2 + a_6 u_3'^2 + a_7 u_1' u_2' + a_8 u_1' u_3' + a_9 u_2' u_3' + a_{10} u_1'^3 + a_{11} u_1'^2 u_2' + a_{12} u_1'^2 u_3' + a_{13} u_2'^3 + a_{14} u_1'^2 u_2' + a_{15} u_1'^2 u_3' + a_{16} u_3'^3 + a_{17} u_1' u_3'^2 + a_{18} u_2'^2 u_3' + a_{19} u_1' u_2' u_3' + a_{20} u_1'^4 + a_{21} u_1'^3 u_2' + a_{22} u_1'^3 u_3' + a_{23} u_1'^2 u_2'^2 + a_{24} u_1'^2 u_3'^2 + a_{25} u_1'^2 u_2' u_3' + a_{26} u_2'^4 + a_{27} u_1'^3 u_2'^2 + a_{28} u_2'^3 u_3' + a_{29} u_2'^2 u_3'^2 + a_{30} u_1' u_2'^2 u_3' + a_{31} u_3'^4 + a_{32} u_1' u_3'^3 + a_{33} u_2'^3 u_3'^3 + a_{34} u_1' u_2' u_3'^2 \right] \tag{B.6}
\]
The coefficients $a_j$ are given by

\begin{align*}
a_0 &= 1 + c_0 g_0 \\
a_1 &= b_1 + c_0 g_1 + c_1 b_1 g_0 \\
a_2 &= b_2 + c_0 g_2 + c_1 b_2 g_0 \\
a_3 &= b_3 + c_0 g_3 + c_1 b_3 g_0 \\
a_4 &= c_0 g_4 + c_1 b_1 g_1 + c_2 b_1^2 g_0 \\
a_5 &= c_0 g_5 + c_1 b_1 g_2 + c_2 b_2 g_0 \\
a_6 &= c_0 g_6 + c_1 b_3 g_3 + c_2 b_3^2 g_0 \\
a_7 &= c_0 g_7 + c_1 (b_1 g_2 + b_2 g_1) + 2 c_2 b_1 b_2 g_0 \\
a_8 &= c_0 g_8 + c_1 (b_1 g_3 + b_3 g_1) + 2 c_2 b_1 b_3 g_0 \\
a_9 &= c_0 g_9 + c_1 (b_2 g_3 + b_3 g_2) + 2 c_2 b_2 b_3 g_0 \\
a_{10} &= c_1 b_1 g_4 + c_2 b_1^2 g_1 \\
a_{11} &= c_1 (b_1 g_7 + b_2 g_4) + c_2 b_1 (2 b_2 g_1 + b_1 g_2) \\
a_{12} &= c_1 (b_1 g_8 + b_3 g_4) + c_2 b_1 (2 b_3 g_1 + b_1 g_3) \\
a_{13} &= c_1 b_2 g_5 + c_2 b_2^2 g_2
\end{align*}

(B.7)
\[ a_{14} = c_1 (b_1 g_5 + b_2 g_7) + c_2 b_2 (b_2 g_1 + b_2 g_2) \]

\[ a_{15} = c_1 (b_2 g_9 + b_3 g_5) + c_2 b_2 (b_2 g_3 + b_2 g_2) \]

\[ a_{16} = c_1 b_3 g_6 + c_2 b_3^2 g_3 \]

\[ a_{17} = c_1 (b_1 g_6 + b_3 g_8) + c_2 b_3 (b_3 g_1 + b_3 g_3) \]

\[ a_{18} = c_1 (b_2 g_6 + b_3 g_9) + c_2 b_3 (b_3 g_2 + b_3 g_3) \]

\[ a_{19} = c_1 (b_1 g_3 + b_2 g_8 + b_3 g_7) + 2 c_2 (b_2 b_3 g_1 + b_1 b_3 g_2 + b_1 b_2 g_3) \]

\[ a_{20} = c_2 b_1^2 g_4 \]

\[ a_{21} = c_2 b_1 (2 b_2 g_4 + b_1 g_7) \]

\[ a_{22} = c_2 b_1 (2 b_3 g_4 + b_1 g_8) \]

\[ a_{23} = c_2 (b_2^2 g_4 + b_1^2 g_5 + b_1 b_2 g_7) \]

\[ a_{24} = c_2 (b_3^2 g_4 + b_1^2 g_6 + b_1 b_3 g_8) \]

\[ a_{25} = c_2 (2 b_2 b_3 g_4 + 2 b_1 b_3 g_7 + 2 b_1 b_2 g_8 + b_1^2 g_9) \]

\[ a_{26} = c_2 b_2^2 g_5 \]

\[ a_{27} = c_2 b_2 (2 b_1 g_5 + b_2 g_7) \]
\[ a_{28} = c_2 \delta_2 (2 \delta_3 g_5 + \delta_2 g_9) \]
\[ a_{29} = c_2 (\delta_3^2 g_5 + \delta_2^2 g_6 + 2 \delta_2 \delta_3 g_9) \]
\[ a_{30} = c_2 (2 \delta_1 \delta_3 g_5 + 2 \delta_2 \delta_3 g_7 + \delta_2^2 g_8 + 2 \delta_1 \delta_2 g_9) \]
\[ a_{31} = c_2 \delta_3^2 g_6 \quad \text{(B.7)} \]
\[ a_{32} = c_2 \delta_3 (2 \delta_1 g_6 + \delta_3 g_7) \]
\[ a_{33} = c_2 \delta_3 (2 \delta_2 g_6 + \delta_3 g_7) \]
\[ a_{34} = c_2 (2 \delta_1 \delta_2 g_6 + \delta_3^2 g_7 + 2 \delta_2 \delta_3 g_8 + 2 \delta_1 \delta_3 g_9) \]

The constants \( c_0, c_1 \) and \( c_2 \) are given in equation (4.50), while expressions for \( \delta_i \) and \( g_i \) are given in equations (4.43) and (4.45).

The coefficients \( \lambda_i, \beta_i, \gamma_i \) and \( \delta_i \) appearing in the general expression for second order moments, namely, equations (4.53), (4.54) and (4.56), are given by,

\[ A_1 = \{a_i^2\}_1 \]
\[ B_1 = \{a_2^2\}_1 \quad \text{(B.8)} \]
\[ C_1 = \{a_3^2\}_1 \]
\[ D_1 = \{2 a_1 a_2\}_1 \]

(contd.)
\[ A_2 = (a_1^2)_2 \]
\[ B_2 = (a_2^2)_2 \]
\[ C_2 = (a_3^2)_2 \]
\[ D_2 = (a_1 a_2)_2 \]
\[ A_3 = (a_1^1 . a_1^2) \]
\[ B_3 = (a_2^1 . a_2^2) \]
\[ C_3 = (a_3^1 . a_3^2) \]
\[ D_3 = (a_1^1 . a_2^2) + (a_2^1 . a_1^2) \]

where the outer subscript refers to the wire number.

The production terms \( R^* \) are given by,

\[ R^* = p_1 \, u_1^{12} \, u_1^{12} + p_2 \, u_1^{12} \, u_2^{12} + p_3 \, u_1^{12} \, u_3^{12} + p_4 \, u_1^{12} \, u_1^1 \, u_2^1 \]
\[ + p_5 \, u_2^{12} \, u_2^{12} + p_6 \, u_2^{12} \, u_3^{12} + p_7 \, u_2^{12} \, u_1^1 \, u_2^1 \]
\[ + p_8 \, u_3^{12} \, u_3^{12} + p_9 \, u_3^{12} \, u_1^1 \, u_2^1 + p_{10} \, u_1^1 \, u_2^1 \, u_1^1 \, u_2^1 \]
\[ + p_{11} \, u_1^{12} \, u_1^{12} \, u_1^{12} + p_{12} \, u_1^{12} \, u_1^{12} \, u_2^{12} + p_{13} \, u_1^{12} \, u_1^{12} \, u_3^{12} \]
\[ + p_{14} \, u_1^{12} \, u_2^{12} \, u_1^1 \, u_2^1 + p_{15} \, u_1^{12} \, u_2^{12} \, u_2^1 \, u_3^{12} + p_{16} \, u_1^{12} \, u_2^{12} \, u_3^{12} \]

(B.9 contd.)
\[ + p_{17} \frac{u_{1}^{12}}{u_1} \frac{u_{2}^{12}}{u_2} \frac{u_{3}^{12}}{u_3} + p_{18} \frac{u_{1}^{12}}{u_1} \frac{u_{2}^{12}}{u_2} \frac{u_{3}^{12}}{u_3} + p_{19} \frac{u_{1}^{12}}{u_1} \frac{u_{3}^{12}}{u_3} \frac{u_{1}^{1}}{u_1} \frac{u_{2}^{1}}{u_2} \\
+ p_{20} \frac{u_{1}^{12}}{u_1} \frac{u_{2}^{12}}{u_2} \frac{u_{1}^{1}}{u_1} \frac{u_{2}^{1}}{u_2} + p_{21} \frac{u_{2}^{12}}{u_2} \frac{u_{1}^{12}}{u_1} \frac{u_{2}^{12}}{u_2} + p_{22} \frac{u_{2}^{12}}{u_2} \frac{u_{1}^{12}}{u_1} \frac{u_{3}^{12}}{u_3} \\
+ p_{23} \frac{u_{2}^{12}}{u_2} \frac{u_{1}^{12}}{u_1} \frac{u_{2}^{12}}{u_2} + p_{24} \frac{u_{1}^{12}}{u_1} \frac{u_{3}^{12}}{u_3} \frac{u_{3}^{12}}{u_3} + p_{25} \frac{u_{1}^{12}}{u_1} \frac{u_{3}^{12}}{u_3} \frac{u_{1}^{1}}{u_1} \frac{u_{1}^{1}}{u_1} \\
+ p_{26} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{2}^{12}}{u_2} + p_{27} \frac{u_{1}^{12}}{u_1} \frac{u_{3}^{12}}{u_3} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} + p_{28} \frac{u_{3}^{12}}{u_3} \frac{u_{3}^{12}}{u_3} \frac{u_{1}^{12}}{u_1} \frac{u_{2}^{12}}{u_2} \\
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+ p_{32} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} + p_{33} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} + p_{34} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \\
+ p_{35} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} + p_{36} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} + p_{37} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \\
+ p_{38} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} + p_{39} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} + p_{40} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \\
+ p_{41} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} + p_{42} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} + p_{43} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \\
+ p_{44} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} + p_{45} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} + p_{46} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \\
+ p_{47} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} + p_{48} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} + p_{49} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \frac{u_{1}^{12}}{u_1} \\
(B.9 contd.)
The coefficients $P_{ij}$ ($j = 1, 2$) are

\[ P_1 = 2a_4^2 + 6a_1a_{10} \]

\[ P_2 = a_7^2 + 2a_1a_{14} + 2a_2a_{11} \]

\[ P_3 = a_8^2 + 2a_3a_{12} + 2a_1a_{17} \]

\[ P_4 = 6a_1a_{11} + 6a_2a_{10} + 4a_4a_7 \]

(contd.)
\[ p_5 = 2a_5^2 + 6a_2a_{13} \]

\[ p_6 = a_9^2 + 2a_2a_{18} + 2a_3a_{15} \]

\[ p_7 = 6a_1a_{13} + 6a_2a_{14} + 4a_5a_7 \]

\[ p_8 = 2a_6^2 + 6a_3a_{16} \]

\[ p_9 = 2(a_1a_{18} + a_2a_{17} + a_3a_{19} + a_8a_9) \]

\[ p_{10} = a_7^2 + 4a_1a_{14} + 4a_2a_{11} + 2a_4a_5 \]

\[ p_{11} = 15a_{10}^2 + 24a_4a_{20} \]

\[ p_{12} = 3a_1a_{12}^2 + 6a_7a_{21} + 6a_{10}a_{14} + 4a_4a_{23} \]

\[ p_{13} = 3a_1a_{12}^2 + 6a_8a_{22} + 6a_{10}a_{17} + 4a_4a_{24} \]

\[ p_{14} = 24a_4a_{21} + 24a_7a_{20} + 30a_{10}a_{11} \]

\[ p_{15} = 3a_1a_{14}^2 + 6a_7a_{27} + 4a_5a_{23} + 6a_{11}a_{23} \]

\[ p_{16} = a_9^2 + 2a_1a_{15} + 2a_7a_{34} + 2a_8a_{30} + 2a_9a_{25} \]

\[ + 2a_1a_{18} + 2a_4a_{17} \]

\[ p_{17} = 12a_4a_{27} + 16a_7a_{23} + 18a_{10}a_{13} + 18a_{11}a_{14} + 6a_5a_{21} \]

(contd.)
\[ p_{18} = 3a_{17}^2 + 4a_6a_{24} + 6a_8a_{32} + 6a_{12}a_{16} \]

\[ p_{19} = 4a_4a_{34} + 4a_7a_{24} + 6a_8a_{25} + 6a_9a_{22} + 6a_{10}a_{18} \]

\[ + 6a_{11}a_{17} + 6a_{12}a_{19} \]

\[ p_{20} = 12a_{11}^2 + 18a_7a_{21} + 24a_{10}a_{14} + 20a_4a_{23} + 24a_5a_{20} \]

\[ p_{21} = 15a_{13}^2 + 24a_5a_{26} \]

\[ p_{22} = 3a_{15}^2 + 6a_9a_{28} + 4a_5a_{29} + 6a_{13}a_{18} \]

\[ p_{23} = 24a_5a_{27} + 24a_7a_{26} + 30a_{13}a_{14} \] (B.10)

\[ p_{24} = 3a_{18}^2 + 4a_6a_{29} + 6a_{15}a_{16} + 6a_9a_{33} \]

\[ p_{25} = 4a_5a_{34} + 4a_7a_{29} + 6a_8a_{28} + 6a_9a_{30} + 6a_{13}a_{17} \]

\[ + 6a_{14}a_{18} + 6a_{15}a_{19} \]

\[ p_{26} = 12a_{14}^2 + 18a_7a_{27} + 24a_4a_{26} + 20a_5a_{23} + 24a_{11}a_{13} \]

\[ p_{27} = 15a_{16}^2 + 24a_6a_{31} \]

\[ p_{28} = 4a_6a_{34} + 6a_8a_{33} + 6a_9a_{32} + 6a_{16}a_{19} + 6a_{17}a_{18} \]

(contd.)
$p_{29} = 2 \ a_{19}^2 + 4 \ a_4 \ a_{29} + 4 \ a_{12} \ a_{15} + 4 \ a_5 \ a_{24} + 2 \ a_7 \ a_{34} + 4 \ a_8 \ a_{30}$

$+ 4 \ a_9 \ a_{25} + 4 \ a_{11} \ a_{18} + 4 \ a_{14} \ a_{17}$

$p_{30} = 12 \ a_4 \ a_{27} + 8 \ a_7 \ a_{23} + 12 \ a_{10} \ a_{13} + 12 \ a_{11} \ a_{14} + 12 \ a_5 \ a_{21}$

$p_{31} = 96 \ a_{20}^2$

$p_{32} = 15 \ a_{21}^2 + 24 \ a_0 \ a_{23}$

$p_{33} = 15 \ a_{22}^2 + 24 \ a_0 \ a_{24}$

$p_{34} = 19 \ a_{20} \ a_{21}$

$p_{35} = 8 \ a_{23}^2 + 18 \ a_7 \ a_{27}$

$p_{36} = 3 \ a_{25}^2 + 6 \ a_{21} \ a_{34} + 6 \ a_{22} \ a_{30} + 4 \ a_{23} \ a_{24}$

$p_{37} = 72 \ a_{20} \ a_{27} + 84 \ a_{21} \ a_{23}$

$p_{38} = 8 \ a_{24}^2 + 18 \ a_{22} \ a_{32}$

$p_{39} = 24 \ a_{20} \ a_{34} + 24 \ a_{21} \ a_{24} + 30 \ a_{22} \ a_{25}$

$p_{40} = 81 \ a_{21}^2 + 168 \ a_{20} \ a_{23}$

$p_{41} = 15 \ a_{27}^2 + 24 \ a_{23} \ a_{26}$

(contd.)
\[ p_{42} = 3 a_{30}^2 + 4 a_{23} a_{29} + 6 a_{25} a_{28} + 6 a_{27} a_{34} \]

\[ p_{43} = 72 a_{21} a_{26} + 84 a_{23} a_{27} \]

\[ p_{44} = 3 a_{34}^2 + 2 a_{24} a_{29} + 6 a_{25} a_{33} + 6 a_{30} a_{32} \]

\[ p_{45} = 12 a_{21} a_{29} + 18 a_{22} a_{28} + 16 a_{23} a_{34} + 18 a_{25} a_{30} + 12 a_{24} a_{27} \]

\[ p_{46} = 68 a_{23}^2 + 144 a_{20} a_{26} + 126 a_{21} a_{27} \]

\[ p_{47} = 15 a_{32}^2 + 24 a_{24} a_{31} \]

\[ p_{48} = 18 a_{22} a_{33} + 16 a_{24} a_{34} + 18 a_{25} a_{32} \]

\[ p_{49} = 12 a_{25}^2 + 24 a_{20} a_{29} + 18 a_{21} a_{34} + 24 a_{22} a_{30} + 20 a_{23} a_{24} \]

\[ p_{50} = 120 a_{20} a_{27} + 108 a_{21} a_{23} \]

\[ p_{51} = 96 a_{26}^2 \]

\[ p_{52} = 15 a_{28}^2 + 24 a_{26} a_{29} \]

\[ p_{53} = 192 a_{26} a_{27} \]

\[ p_{54} = 8 a_{29}^2 + 18 a_{28} a_{33} \]

\[ p_{55} = 24 a_{26} a_{34} + 24 a_{27} a_{29} + 30 a_{28} a_{30} \]

(contd.)
\[ p_{56} = 81 a_{27}^2 + 168 a_{23} a_{26} \]
\[ p_{57} = 15 a_{33}^2 + 24 a_{29} a_{31} \]
\[ p_{58} = 18 a_{28} a_{32} + 16 a_{29} a_{34} + 18 a_{30} a_{33} \]
\[ p_{59} = 12 a_{30}^2 + 20 a_{23} a_{29} + 24 a_{24} a_{26} + 24 a_{25} a_{28} + 18 a_{27} a_{34} \]
\[ p_{60} = 120 a_{21} a_{26} + 108 a_{23} a_{27} \quad \text{(B.10)} \]
\[ p_{61} = 96 a_{31}^2 \]
\[ p_{62} = 24 a_{31} a_{34} + 30 a_{32} a_{33} \]
\[ p_{63} = 5 a_{34}^2 + 12 a_{24} a_{29} + 12 a_{25} a_{33} + 12 a_{30} a_{32} \]
\[ p_{64} = 12 a_{21} a_{29} + 12 a_{22} a_{28} + 8 a_{23} a_{34} + 12 a_{24} a_{27} + 12 a_{25} a_{30} \]
\[ p_{65} = 20 a_{23}^2 + 48 a_{20} a_{31} + 48 a_{22} a_{32} \]

For \( j = 3 \), the coefficients \( P_{\lambda} \) become,
\[ p_1 = 2(a_4)_1 (a_4)_2 + 3(a_7)_1 (a_{10})_2 + 3(a_{10})_1 (a_7)_2 \]
\[ p_2 = (a_7)_1 (a_{14})_2 + (a_{11})_1 (a_7)_2 + (a_{11})_1 (a_7)_2 + (a_{11})_1 (a_2)_2 \quad \text{(B.11)} \]
\[ + (a_{14})_1 (a_7)_2 \]
\[ \text{(contd.)} \]
$$p_3 = (a_1)_1 (a_{17})_2 + (a_3)_1 (a_{12})_2 + (a_5)_1 (a_8)_2 + (a_{12})_1 (a_3)_2$$

$$+ (a_{17})_1 (a_7)_2$$

$$p_4 = 3(a_1)_1 (a_{17})_2 + 3(a_2)_1 (a_{10})_2 + 2(a_4)_1 (a_7)_2 + 2(a_7)_1 (a_4)_2$$

$$+ 3(a_{10})_1 (a_2)_2 + 3(a_{11})_1 (a_1)_2$$

$$p_5 = 3(a_2)_1 (a_{13})_2 + 2(a_5)_1 (a_5)_2 + 3(a_{13})_1 (a_2)_2$$

$$p_6 = (a_2)_1 (a_{18})_2 + (a_3)_1 (a_{15})_2 + (a_9)_1 (a_9)_2 + (a_{15})_1 (a_3)_2$$

$$+ (a_{18})_1 (a_2)_2$$

$$p_7 = 3(a_1)_1 (a_{13})_2 + 3(a_2)_1 (a_{14})_2 + 2(a_5)_1 (a_7)_2 + 2(a_7)_1 (a_5)_2$$

$$+ 3(a_{14})_1 (a_2)_2 + 3(a_{13})_1 (a_1)_2$$

$$p_8 = 3(a_3)_1 (a_{13})_2 + 2(a_6)_1 (a_6)_2 + 3(a_{16})_1 (a_3)_2$$

$$p_9 = (a_1)_1 (a_{18})_2 + (a_2)_1 (a_{17})_2 + (a_3)_1 (a_{19})_2 + (a_8)_1 (a_9)_2$$

$$+ (a_9)_1 (a_8)_2 + (a_{17})_1 (a_1)_2 + (a_{18})_1 (a_1)_2 + (a_{19})_1 (a_3)_2$$

$$p_{10} = 2(a_1)_1 (a_{14})_2 + 2(a_2)_1 (a_{11})_2 + 2(a_4)_1 (a_5)_2 + 2(a_5)_1 (a_4)_2$$

$$+ (a_7)_1 (a_7)_2 + 2(a_{11})_1 (a_2)_2 + 2(a_{14})_1 (a_1)_2$$

(contd.)
\[ p_{11} = 12(a_4)^1 (a_{20})^2 + 15(a_{10})^1 (a_{10})^2 + 15(a_{20})^1 (a_4)^2 \]

\[ p_{12} = 2(a_4)^1 (a_{20})^2 + 3(a_7)^1 (a_{21})^2 + 3(a_{10})^1 (a_{14})^2 + 3(a_{11})^1 (a_{11})^2 \]
\[ + 3(a_{14})^1 (a_{10})^2 + 3(a_{21})^1 (a_7)^2 + 2(a_{23})^1 (a_4)^2 \]

\[ p_{13} = 2(a_4)^1 (a_{24})^2 + 3(a_8)^1 (a_{22})^2 + 3(a_{10})^1 (a_{17})^2 + 3(a_{12})^1 (a_{12})^2 \]
\[ + 3(a_{17})^1 (a_{10})^2 + 3(a_{22})^1 (a_8)^2 + 2(a_{24})^1 (a_4)^2 \]

\[ p_{14} = 12(a_4)^1 (a_{21})^2 + 12(a_7)^1 (a_{20})^2 + 15(a_{10})^1 (a_{11})^2 \]
\[ + 15(a_{11})^1 (a_{10})^2 + 12(a_{20})^1 (a_7)^2 + 12(a_{21})^1 (a_4)^2 \]

\[ p_{15} = 2(a_5)^1 (a_{23})^2 + 3(a_7)^1 (a_{27})^2 + 3(a_{11})^1 (a_{13})^2 + 3(a_{13})^1 (a_{11})^2 \]
\[ + 3(a_{14})^1 (a_{14})^2 + 2(a_{23})^1 (a_5)^2 + 3(a_{27})^1 (a_7)^2 \]

\[ p_{16} = (a_7)^1 (a_{34})^2 + (a_8)^1 (a_{30})^2 + (a_9)^1 (a_{25})^2 + (a_{11})^1 (a_{18})^2 \]
\[ + (a_{12})^1 (a_{15})^2 + (a_{14})^1 (a_{17})^2 + (a_{15})^1 (a_{12})^2 + (a_{17})^1 (a_{14})^2 \]
\[ + (a_{18})^1 (a_{11})^2 + (a_{19})^1 (a_{19})^2 + (a_{30})^1 (a_8)^2 + (a_{34})^1 (a_7)^2 \]

\[ p_{17} = 6(a_4)^1 (a_{27})^2 + 6(a_5)^1 (a_{21})^2 + 8(a_7)^1 (a_{23})^2 + 9(a_{10})^1 (a_{13})^2 \]
\[ + 9(a_{11})^1 (a_{14})^2 + 9(a_{13})^1 (a_{10})^2 + 9(a_{14})^1 (a_{11})^2 \]

(contd.)
\[ p_{18} = 2(a_6)_1 \{a_{24}\}_2 + 3(a_8)_1 \{a_{32}\}_2 + 3(a_{12})_1 \{a_{16}\}_2 + 3(a_{16})_1 \{a_{12}\}_2 \]
\[ + 3(a_{17})_1 \{a_{17}\}_2 + 2(a_{24})_1 \{a_6\}_2 + 3(a_{32})_1 \{a_8\}_2 \]
\[ p_{19} = 2(a_4)_1 \{a_{34}\}_2 + 2(a_7)_1 \{a_{24}\}_2 + 3(a_8)_1 \{a_{25}\}_2 + 3(a_9)_1 \{a_{22}\}_2 \]
\[ + 3(a_{10})_1 \{a_{18}\}_2 + 3(a_{11})_1 \{a_{17}\}_2 + 3(a_{12})_1 \{a_{19}\}_2 \]
\[ + 3(a_{17})_1 \{a_{11}\}_2 + 3(a_{18})_1 \{a_{10}\}_2 + 3(a_{19})_1 \{a_{12}\}_2 \]
\[ + 3(a_{22})_1 \{a_9\}_2 + 2(a_{24})_1 \{a_7\}_2 + 3(a_{25})_1 \{a_8\}_2 + 2(a_{34})_1 \{a_4\}_2 \]

(B.11)

\[ p_{20} = 10(a_4)_1 \{a_{23}\}_2 + 12(a_5)_1 \{a_{20}\}_2 + 9(a_7)_1 \{a_{21}\}_2 \]
\[ + 12(a_{10})_1 \{a_{14}\}_2 + 12(a_{11})_1 \{a_{11}\}_2 + 12(a_{14})_1 \{a_{10}\}_2 \]
\[ + 12(a_{20})_1 \{a_{5}\}_2 + 9(a_{21})_1 \{a_7\}_2 + 10(a_{23})_1 \{a_4\}_2 \]
\[ p_{21} = 12(a_5)_1 \{a_{26}\}_2 + 15(a_3)_1 \{a_{13}\}_2 + 12(a_{26})_1 \{a_5\}_2 \]
\[ p_{22} = 2(a_5)_1 \{a_{29}\}_2 + 3(a_9)_1 \{a_{28}\}_2 + 3(a_{13})_1 \{a_{18}\}_2 + 3(a_{15})_1 \{a_{15}\}_2 \]
\[ + 3(a_{18})_1 \{a_{13}\}_2 + 3(a_{28})_1 \{a_9\}_2 + 2(a_{29})_1 \{a_5\}_2 \]
\[ p_{23} = 12(a_5)_1 \{a_{27}\}_2 + 12(a_7)_1 \{a_{26}\}_2 + 15(a_{13})_1 \{a_{14}\}_2 \]

(contd.)
\[ p_{24} = 2(a_6)_1 (a_9)_2 + 3(a_9)_1 (a_{33})_2 + 3(a_{15})_1 (a_{16})_2 + 3(a_{16})_1 (a_{15})_2 + 3(a_{16})_1 (a_{18})_2 + 2(a_{29})_1 (a_6)_2 + 3(a_{33})_1 (a_9)_2 \]

\[ p_{25} = 2(a_5)_1 (a_{34})_2 + 2(a_7)_1 (a_{29})_2 + 3(a_8)_1 (a_{28})_2 + 3(a_9)_1 (a_{30})_2 + 3(a_{13})_1 (a_{17})_2 + 3(a_{14})_1 (a_{18})_2 + 3(a_{15})_1 (a_{19})_2 \]

\[ p_{26} = 12(a_4)_1 (a_{26})_2 + 10(a_5)_1 (a_{23})_2 + 9(a_7)_1 (a_{27})_2 + 12(a_{11})_1 (a_{13})_2 + 12(a_{13})_1 (a_{11})_2 + 12(a_{14})_1 (a_{14})_2 + 10(a_{23})_1 (a_5)_2 + 12(a_{26})_1 (a_4)_2 + 9(a_{27})_1 (a_7)_2 \]

\[ p_{27} = 12(a_4)_1 (a_{31})_2 + 15(a_{16})_1 (a_{16})_2 + 12(a_{31})_1 (a_6)_2 \]

\[ p_{28} = 2(a_6)_1 (a_{34})_2 + 3(a_8)_1 (a_{33})_2 + 3(a_9)_1 (a_{32})_2 + 3(a_{16})_1 (a_{19})_2 + 3(a_{17})_1 (a_{18})_2 + 3(a_{18})_1 (a_{17})_2 + 3(a_{19})_1 (a_{16})_2 + 3(a_{32})_1 (a_9)_2 + 3(a_{33})_1 (a_8)_2 + 2(a_{34})_1 (a_6)_2 \]

\[(B.11)\]
\[ p_{29} = 2(a_4)_1 (a_{29})_2 + 2(a_7)_1 (a_{24})_2 + (a_7)_1 (a_{34})_2 + 2(a_8)_1 (a_{30})_2 \]
\[ + 2(a_9)_1 (a_{25})_2 + 2(a_{11})_1 (a_{18})_2 + 2(a_{12})_1 (a_{15})_2 \]
\[ + 2(a_{14})_1 (a_{17})_2 + 2(a_{15})_1 (a_{12})_2 + 2(a_{17})_1 (a_{14})_2 \]
\[ + 2(a_{18})_1 (a_{11})_2 + 2(a_{19})_1 (a_{19})_2 + 2(a_{24})_1 (a_{5})_2 \]
\[ + 2(a_{25})_1 (a_{9})_2 + 2(a_{29})_1 (a_{4})_2 + 2(a_{30})_1 (a_{8})_2 + (a_{34})_1 (a_{7})_2 \]
\[ p_{30} = 6(a_4)_1 (a_{27})_2 + 6(a_5)_1 (a_{21})_2 + 4(a_7)_1 (a_{23})_2 + 6(a_{10})_1 (a_{13})_2 \]
\[ + 6(a_{11})_1 (a_{14})_2 + 6(a_{13})_1 (a_{10})_2 + 6(a_{14})_1 (a_{11})_2 \]
\[ + 6(a_{21})_1 (a_{5})_2 + 6(a_{27})_1 (a_{7})_2 \]
\[ p_{31} = 96(a_{20})_1 (a_{20})_2 \]
\[ p_{32} = 12(a_{20})_1 (a_{23})_2 + 15(a_{21})_1 (a_{21})_2 + 12(a_{23})_1 (a_{20})_2 \]
\[ p_{33} = 12(a_{20})_1 (a_{24})_2 + 15(a_{22})_1 (a_{22})_2 + 12(a_{24})_1 (a_{20})_2 \]
\[ p_{34} = 96(a_{20})_1 (a_{21})_2 + 96(a_{21})_1 (a_{20})_2 \]
\[ p_{35} = 9(a_{21})_1 (a_{27})_2 + 8(a_{23})_1 (a_{23})_2 + 9(a_{27})_1 (a_{21})_2 \]
\[ p_{36} = 3(a_{21})_1 (a_{34})_2 + 3(a_{22})_1 (a_{30})_2 + 2(a_{23})_1 (a_{24})_2 \]

\(\text{(contd.)}\)
\[ \begin{align*}
+ 2(a_{24})_1 (a_{23})_2 + 3(a_{25})_1 (a_{25})_2 + 3(a_{30})_1 (a_{22})_2 \\
+ 3(a_{34})_1 (a_{21})_2 \\
^2 \]

\[ p_{37} = 36(a_{20})_1 (a_{27})_2 + 42(a_{21})_1 (a_{23})_2 + 42(a_{23})_1 (a_{21})_2 \\
+ 36(a_{27})_1 (a_{20})_2 \\
\]

\[ p_{38} = 9(a_{22})_1 (a_{32})_2 + 8(a_{24})_1 (a_{24})_2 + 9(a_{32})_1 (a_{22})_2 \\
\]

\[ p_{39} = 12(a_{20})_1 (a_{34})_2 + 12(a_{21})_1 (a_{24})_2 + 15(a_{22})_1 (a_{25})_2 \\
+ 12(a_{24})_1 (a_{21})_2 + 15(a_{25})_1 (a_{22})_2 + 12(a_{34})_1 (a_{20})_2 \\
\]

\[ (B.11) \]

\[ p_{40} = 84(a_{20})_1 (a_{23})_2 + 81(a_{21})_1 (a_{21})_2 + 84(a_{23})_1 (a_{20})_2 \\
\]

\[ p_{41} = 12(a_{23})_1 (a_{26})_2 + 12(a_{26})_1 (a_{23})_2 + 15(a_{27})_1 (a_{27})_2 \\
\]

\[ p_{42} = (a_{23})_1 (a_{29})_2 + 3(a_{25})_1 (a_{28})_2 + 3(a_{27})_1 (a_{34})_2 \\
+ 3(a_{28})_1 (a_{25})_2 + 2(a_{29})_1 (a_{23})_2 + 3(a_{30})_1 (a_{30})_2 \\
+ 3(a_{34})_1 (a_{27})_2 \\
\]

\[ p_{43} = 36(a_{21})_1 (a_{26})_2 + 42(a_{23})_1 (a_{27})_2 + 36(a_{26})_1 (a_{21})_2 \\
+ 42(a_{27})_1 (a_{23})_2 \\
\]

(contd.)
\[ p_{44} = 2(a_{24})_1(a_{29})_2 + 3(a_{25})_1(a_{33})_2 + 2(a_{29})_1(a_{24})_2 + 3(a_{30})_1(a_{32})_2 + 3(a_{32})_1(a_{30})_2 + 3(a_{33})_1(a_{25})_2 + 3(a_{34})_1(a_{34})_2 \]

\[ p_{45} = 6(a_{21})_1(a_{29})_2 + 9(a_{22})_1(a_{28})_2 + 8(a_{23})_1(a_{34})_2 + 6(a_{24})_1(a_{27})_2 + 9(a_{25})_1(a_{30})_2 + 6(a_{27})_1(a_{24})_2 + 9(a_{28})_1(a_{22})_2 + 6(a_{29})_1(a_{21})_1 + 9(a_{30})_1(a_{25})_2 + 8(a_{34})_1(a_{23})_2 \]

\[ p_{46} = 72(a_{20})_1(a_{26})_2 + 63(a_{21})_1(a_{27})_2 + 68(a_{23})_1(a_{23})_2 + 72(a_{26})_1(a_{20})_2 + 63(a_{27})_1(a_{21})_2 \]

\[ p_{47} = 12(a_{24})_1(a_{31})_2 + 12(a_{31})_1(a_{24})_2 + 15(a_{32})_1(a_{32})_2 \]

\[ p_{48} = 9(a_{22})_1(a_{33})_2 + 8(a_{24})_1(a_{34})_2 + 9(a_{25})_1(a_{32})_2 + 8(a_{34})_1(a_{24})_2 \]

\[ p_{49} = 12(a_{20})_1(a_{29})_2 + 9(a_{21})_1(a_{34})_2 + 12(a_{22})_1(a_{30})_2 + 10(a_{23})_1(a_{24})_2 + 10(a_{24})_1(a_{23})_2 + 12(a_{25})_1(a_{25})_2 \]

(contd.)
\[ p_{50} = 60(a_{20})_1 (a_{27})_2 + 54(a_{21})_1 (a_{23})_2 + 54(a_{23})_1 (a_{21})_2 + 60(a_{27})_1 (a_{20})_2 \]

\[ p_{51} = 96(a_{26})_1 (a_{26})_2 \]

\[ p_{52} = 12(a_{26})_1 (a_{29})_2 + 15(a_{28})_1 (a_{28})_2 + 12(a_{29})_1 (a_{26})_2 \]

\[ p_{53} = 96(a_{26})_1 (a_{27})_2 + 96(a_{27})_1 (a_{26})_2 \]

\[ p_{54} = 9(a_{28})_1 (a_{33})_2 + 8(a_{29})_1 (a_{29})_2 + 9(a_{33})_1 (a_{28})_2 \]

\[ p_{55} = 12(a_{26})_1 (a_{34})_2 + 12(a_{27})_1 (a_{29})_2 + 15(a_{28})_1 (a_{30})_2 + 12(a_{29})_1 (a_{27})_2 + 15(a_{30})_1 (a_{28})_2 + 12(a_{34})_1 (a_{26})_2 \]

\[ p_{56} = 84(a_{23})_1 (a_{26})_2 + 84(a_{26})_1 (a_{23})_2 + 81(a_{27})_1 (a_{27})_2 \]

\[ p_{57} = 12(a_{29})_1 (a_{31})_2 + 12(a_{31})_1 (a_{29})_2 + 15(a_{33})_1 (a_{33})_2 \]

\[ p_{58} = 9(a_{28})_1 (a_{32})_2 + 8(a_{29})_1 (a_{34})_2 + 9(a_{30})_1 (a_{33})_2 + 9(a_{32})_1 (a_{28})_2 + 9(a_{33})_1 (a_{30})_2 + 8(a_{34})_1 (a_{29})_2 \]

\[ p_{59} = 10(a_{23})_1 (a_{29})_2 + 12(a_{24})_1 (a_{26})_2 + 12(a_{25})_1 (a_{28})_2 \]

(contd.)
\[ + 12(a_{26})_1 (a_{24})_2 + 9(a_{27})_1 (a_{34})_2 + 12(a_{28})_1 (a_{25})_2 \]
\[ + 10(a_{29})_1 (a_{23})_2 + 12(a_{30})_1 (a_{30})_2 + 9(a_{34})_1 (a_{27})_2 \]

\[ p_{60} = 60(a_{21})_1 (a_{26})_2 + 54(a_{23})_1 (a_{27})_2 + 60(a_{26})_1 (a_{27})_2 \]
\[ + 54(a_{27})_1 (a_{23})_2 \]

\[ p_{61} = 96(a_{31})_1 (a_{31})_2 \]

\[ p_{62} = 12(a_{31})_1 (a_{34})_2 + 15(a_{32})_1 (a_{32})_2 + 15(a_{33})_1 (a_{32})_2 \]
\[ + 12(a_{34})_1 (a_{31})_2 \]

\[ (B.11) \]

\[ p_{63} = 6(a_{24})_1 (a_{29})_2 + 6(a_{25})_1 (a_{33})_2 + 6(a_{30})_1 (a_{32})_2 \]
\[ + 6(a_{32})_1 (a_{20})_2 + 6(a_{33})_1 (a_{25})_2 + 5(a_{34})_1 (a_{34})_2 \]

\[ p_{64} = 6(a_{21})_1 (a_{29})_2 + 6(a_{22})_1 (a_{28})_2 + 4(a_{23})_1 (a_{34})_2 \]
\[ + 6(a_{24})_1 (a_{27})_2 + 6(a_{25})_1 (a_{30})_2 + 6(a_{27})_1 (a_{24})_2 \]
\[ + 6(a_{28})_1 (a_{22})_2 + 6(a_{29})_1 (a_{21})_2 + 6(a_{30})_1 (a_{25})_2 \]
\[ + 4(a_{34})_1 (a_{23})_2 \]

\[ p_{65} = 24(a_{20})_1 (a_{26})_2 + 24(a_{21})_1 (a_{27})_2 + 20(a_{23})_1 (a_{23})_2 \]

(contd.)
+ 24(a_{26})_1 (a_{20})_2 + 24(a_{27})_1 (a_{21})_2 \quad (B.11)

where the outer subscript refers to the wire number.
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